Flavor Violation in Warped Extra Dimensions
and CP Asymmetries in $B$ Decays

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Abstract

We show that CP asymmetries in $b \to s$ hadronic decays are potentially affected by the presence of massive color-octet particles strongly coupled to the third generation quarks. Theories with warped extra dimensions provide natural candidates in the Kaluza-Klein excitations of gluons in scenarios where flavor-breaking by bulk fermion masses results in the localization of fermion wave-functions. Topcolor models, in which a new gauge interaction leads to top-condensation and a large top mass, also result in the presence of these color-octet states with TeV masses. We find that large effects are possible in modes such as $B \to \phi K_s$, $B \to \eta' K_s$ and $B \to \pi^0 K_s$ among others.
1 Introduction

It is generally believed that extensions of the standard model must address the stability of the weak scale with new physics at energies not far above the TeV scale. The origin of fermion masses, on the other hand, could in principle reside at much higher scales. Nonetheless, it is tempting to consider the possibility that at least part of the fermion masses is dynamically generated at similar scales. This is particularly true when considering the origin of the top quark mass, which is of the order of the weak scale. The correspondingly large Yukawa coupling suggests the presence of a strongly coupled sector associated with the third generation quarks.

Although originally proposed as a solution to the hierarchy problem [1], the existence of warped extra dimensions provides a class of scenarios that may also address fermion masses as dynamical in origin. The Anti de-Sitter metric in a compact extra dimension is responsible for generating a low energy scale (the weak scale) on one of the fixed points, even when there is only one fundamental scale ($\sim M_P$). Although in the earlier versions of this scenario matter and gauge fields were localized on the infrared (TeV) brane, it was soon realized the potential for model building of introducing gauge and matter fields in the bulk [2,3,4]. In particular, it was recognized in Refs. [5,6] that in non-supersymmetric scenarios, the localization of fermion zero modes, which is controlled by order one parameters in the bulk theory, could be used to explain the fermion mass hierarchy. Zero-mode fermions localized toward the TeV brane would have large overlaps with the TeV-localized Higgs field, and therefore an order one Yukawa coupling. On the other hand, the Yukawa couplings of fermions localized on the Planck brane would be exponentially suppressed. Order one flavor breaking in the bulk fermion masses results in an exponentially enhanced hierarchy in the zero-mode Yukawa couplings. On the other hand, and as it is pointed out in Ref. [5], the couplings of the zero-mode fermions to the lightest Kaluza-Klein (KK) excitations of gauge bosons become large for TeV brane localized fermions. Although this might result in tight constraints from electroweak precision measurements [7,8], the actual bounds are quite model dependent, with the most important factors being the choice of “weak” gauge group in the bulk and the localization of the light fermions. However, even if the latter are considered on the Planck brane in order to avoid electroweak bounds, at least part of the third generation should be TeV localized. This constitutes a potentially large source for flavor violation, particularly with third generation quarks.

Such dynamics is also present in purely four dimensional theories. For instance, this is the case in Topcolor theories [9], where a gauge interaction strongly coupled to the third generation quarks is spontaneously broken at the TeV scale. Topcolor theories have to either be supplemented by some additional dynamics (as in Ref. [10]) or by additional matter as in the top see-saw models [11]. However, a rather model independent statement that can be made about these theories, is that they will result in non-universal interactions
Table 1: Most recent data on CP asymmetries in penguin dominated modes, as presented in Ref. [12]. In the SM, we expect $S_f \approx \sin(2\beta)_{\psi K_s} = 0.731 \pm 0.056$, and $C_f \approx 0$.

with a color-octet massive gauge boson, which in turn leads to the presence of flavor changing neutral currents (FCNCs).

In this work, we consider the flavor violation induced in these models. In particular, we study the FCNC effects leading to $b \rightarrow s$ transitions induced by massive color-octet gauge bosons that are present in all the specific realizations. In the case of warped extra dimensions, the KK excitations of the gluon field give rise to these interactions provided that the zero-mode fermions of third generation quarks are localized toward the TeV brane, as it is required in order to obtain a large top Yukawa coupling. In Topcolor theories, the dynamics responsible for top-condensation is essentially flavor violating. In both cases, there might be additional flavor violation through color-singlet states. For instance, in some Topcolor models there is a complicated flavor-violating scalar sector, as well as color-singlet gauge bosons. In the case of warped extra dimensions, there is the possibility of flavor violating KK excitations of the $Z$, plus those of other additional color-singlet gauge bosons propagating in the bulk, depending on what the “weak” gauge symmetry is in the bulk. The presence of these extra contributions is model-dependent and therefore we choose to focus on the color-octet gauge boson interactions, which in any event tend to give the largest effects due to the presence of $\alpha_s$. We show that flavor-violating color-octet interactions strongly coupled to the third generation could naturally lead to large effects in $b \rightarrow s\bar{s}s$ and $b \rightarrow s\bar{d}d$ CP asymmetries, such as in $B_d \rightarrow \phi K_s$, $B_d \rightarrow \eta' K_s$, $B_d \rightarrow \pi^0 K_s$ among others. The current data from BABAR and BELLE, shown in Table 1, signal the possibility of a deviation of the value of $\sin(2\beta)$ as extracted in these modes from the one obtained in the $b \rightarrow c\bar{c}s$ golden mode $B_d \rightarrow J/\psi K_s$. Although errors are still large, they are statistics dominated and are expected to be considerably reduced by the time the experiments accumulate 500 fb$^{-1}$.
Several authors have considered various possible new physics sources for deviations in the CP asymmetries of penguin dominated decays \[13\] as the ones in Table 1. These are coming from anomalous $Z$ couplings, a strongly coupled $Z'$ or loop effects involving, for instance, superpartners in some supersymmetric scenario. The distinct and theoretically motivated possibility that the effects are induced by new color-octet currents has not been considered in the literature.

In the next section we derive the flavor-violating interactions in models of warped extra dimensions. In Section 3 we do the same in generic Topcolor models. In Section 4 we predict the CP asymmetry to be observed in these scenarios in various $B$ decay modes as a function of the model parameters. We finally conclude in Section 5.

## 2 Warped Extra Dimensions and Flavor Violation

Recently Randall and Sundrum proposed the use of a non-factorizable geometry in five dimensions \[1\] as a solution of the hierarchy problem. The metric depends on the five dimensional coordinate $y$ and is given by
\[
ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2,
\]
where $x^\mu$ are the four dimensional coordinates, $\sigma(y) = k|y|$, with $k \sim M_P$ characterizing the curvature scale. The extra dimension is compactified on an orbifold $S_1/Z_2$ of radius $r$ so that the bulk is a slice of AdS$_5$ space between two four-dimensional boundaries. The metric on these boundaries generates two effective scales: $M_P$ and $M_P e^{-k\pi r}$. In this way, values of $r$ not much larger than the Planck length ($kr \simeq (11 - 12)$) can be used in order to generate a scale $\Lambda_r \simeq M_P e^{-k\pi r} \simeq O(\text{TeV})$ on one of the boundaries.

In the original RS scenario, only gravity was allowed to propagate in the bulk, with the Standard Model (SM) fields confined to one of the boundaries. The inclusion of matter and gauge fields in the bulk has been extensively treated in the literature \[2,3,4,5,6,7,8\]. Here we are interested in examining the situation when the SM fields are allowed to propagate in the bulk. The exception is the Higgs field which must be localized on the TeV boundary in order for the $W$ and the $Z$ gauge bosons to get their observed masses \[3\]. The gauge content in the bulk may be that of the SM, or it might be extended to address a variety of model building and phenomenological issues. For instance, the bulk gauge symmetries may correspond to Grand Unification scenarios, or they may be extensions of the SM formulated to restore enough custodial symmetry and bring electroweak contributions in line with constraints. In addition, and as it was recognized in Ref.\[5\], it is possible to generate the fermion mass hierarchy from $O(1)$ flavor breaking in the bulk masses of fermions. Since bulk fermion masses result in the localization of fermion zero-modes, lighter fermions should be localized toward the Planck brane, where their wave-function has exponentially suppressed overlap with the TeV-localized Higgs, whereas fermions with order one Yukawa couplings should be localized toward the TeV brane.
This creates an almost inevitable tension: since the lightest KK excitations of gauge bosons are localized toward the TeV brane, they tend to be strongly coupled to zero-mode fermions localized there. Thus, the flavor-breaking fermion localization leads to flavor violating interactions of the KK gauge bosons. In particular, this is the case when one tries to obtain the correct top Yukawa coupling: the KK excitations of the various gauge bosons propagating in the bulk will have FCNC interactions with the third generation quarks, as we will see below in detail.

The action for fermion fields in the bulk is given by

\[ S_f = \int d^4x \, dy \, \sqrt{-g} \left\{ \frac{i}{2} \bar{\Psi} \hat{\gamma}^M \left[ D_M - \hat{D}_M \right] \Psi - \text{sgn}(y) M_f \bar{\Psi} \Psi \right\}, \tag{2} \]

where the covariant derivative in curved space is

\[ D_M \equiv \partial_M + \frac{1}{8} [\gamma^\alpha, \gamma^\beta] V^N_{\alpha} V^M_{\beta}; \tag{3} \]

and \( \hat{\gamma}^M \equiv V^M_\alpha \gamma^\alpha, \) with \( V^M_\alpha = \text{diag}(e^\sigma, e^\sigma, e^\sigma, e^\sigma, 1) \) the inverse vierbein. The bulk mass term \( M_f \) in eqn. (2) is expected to be of order \( k \sim M_P. \) Although the fermion field \( \Psi \) is non-chiral, we can still define \( \Psi_{L,R} \equiv \frac{1}{2} (1 \mp \gamma_5) \Psi. \) The KK decomposition can be written as

\[ \Psi_{L,R}(x,y) = \frac{1}{\sqrt{2\pi r}} \sum_{n=0}^{\infty} \psi_n^{L,R}(x)e^{2\sigma f_n^{L,R}}(y), \tag{4} \]

where \( \psi_n^{L,R}(x) \) corresponds to the \( n \)th KK fermion excitation and is a chiral four-dimensional field. The zero mode wave functions are

\[ f_0^{R,L}(y) = \sqrt{\frac{2k \pi r (1 \pm 2c_{R,L})}{e^{k \pi r (1 \pm 2c_{R,L})} - 1}} e^{\pm c_{R,L} k y}, \tag{5} \]

with \( c_{R,L} \equiv M_f/k \) parametrizing the bulk fermion mass in units of the inverse AdS radius \( k. \) The \( Z_2 \) orbifold projection is used so that only one of these is actually allowed, either a left-handed or a right-handed zero mode. The Yukawa couplings of bulk fermions to the TeV brane Higgs can be written as

\[ S_Y = \int d^4x \, dy \, \sqrt{-g} \frac{\lambda_{ij}^{5D}}{2 M_5} \bar{\Psi}_i(x,y) \delta(y - \pi r) H(x) \Psi_j(x,y), \tag{6} \]

where \( \lambda_{ij}^{5D} \) is a dimensionless parameter and \( M_5 \) is the fundamental scale or cutoff of the theory. Naive dimensional analysis tells us that we should expect \( \lambda_{ij}^{5D} \lesssim 4\pi. \) Thus the 4D Yukawa couplings as a function of the bulk mass parameters are

\[ Y_{ij} = \left( \frac{\lambda_{ij}^{5D} k}{M_5} \right) \sqrt{\frac{1/2 - c_L}{e^{k \pi r (1 - 2c_L)} - 1}} \sqrt{\frac{1/2 - c_R}{e^{k \pi r (1 - 2c_R)} - 1}} e^{k \pi r (1 - c_L - c_R)}. \tag{7} \]
Given that we expect $k \lesssim M_5$ then the factor $\lambda^{5D}_{ij} k / M_5 \simeq O(1)$. In Figure 1 the Yukawa coupling of a left-handed zero-mode fermion is plotted as a function of $c_L$ for a choice of $c_R = -0.4$, i.e. a TeV-brane localized right-handed zero mode. We see that in order to obtain an $O(1)$ Yukawa coupling, the bulk mass parameter $c_L$ should naturally be in the range $[-0.7, 0]$. In other words, the left-handed zero-mode should also be localized toward the TeV brane. This however, posses a problem since it means that the left-handed doublet $q_L$, and therefore $b_L$ should have a rather strong coupling to the first KK excitations of gauge bosons. In Figure 2 we plot the coupling of the first KK excitation of a gauge boson to a zero-mode fermion vs. the fermion’s bulk mass parameter $c$. Thus, the localization of the third generation quark doublet $q_L$ in the range $c_L = [-0.7, 0]$ leads to potentially large flavor violations, not only with the top quark, but also with $b_L$.

This induced flavor violation of KK gauge bosons with $b_L$ (we assume $b_R$ localized on the Planck brane) is, in principle, constrained by the precise measurement of the $Z \rightarrow b\bar{b}$ interactions at the $Z$-pole. For instance, Ref. [14] considers a $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory in the bulk. After electroweak symmetry breaking the $Z$ mixes with its KK excitations, as well as with the KK modes of a $Z'$. This generates $\delta g^b_L \lesssim O(1\%) g^b_L$, compatible with current bounds as long as $c_L \gtrsim 0.3$. This still leaves a large flavor violating coupling of the first KK excitations to the $b_L$, as we can see from Figure 2. Since in general the weak sector of the bulk gauge theory varies from model to model, we will focus on the flavor violating effects of the KK gluons.

In most of these models, only the third generation left-handed quark doublet and $t_R$ are required to be localized toward the TeV brane, whereas $b_R$ could in principle be toward the Planck brane together with the light fermions. In this case, the KK gluons...

Figure 1: Yukawa coupling of a left-handed zero mode fermion for a TeV-brane localized right-handed fermion.
and other KK gauge excitations would not have strong flavor violating couplings with $b_R$. On the other hand, if their bulk mass parameter was similar to the one of $t_R$ or $(t\ b)_L^T$, it would also contribute to flavor violating vertices. This, for instance, would be the case if instead of having $SU(2)_L \times SU(2)_R$ in the bulk, one has $SO(4)$ as the “weak” gauge symmetry [15]. In this case, $(t\ b)_L^T$ and $(t\ b)_R^T$ should have the same bulk mass. Although the case with an exact $SO(4)$ symmetry may not be phenomenologically viable due to constraints from electroweak precision observables [16], an approximate $SO(4)$ could be accommodated. This would still lead to a $b_R$ zero-mode with a TeV-localized wave function, and to interesting phenomenology, both in $B$ physics as well as in electroweak observables.

3 Topcolor Models and Flavor Violation

In Topcolor theories, the flavor violating interactions are intended to become supercritical leading to $\langle \bar{t}_L t_R \rangle \neq 0$, therefore providing a backdrop for top-condensation in a broken gauge theory. The minimal module of a Topcolor model can be viewed as the breaking of $SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c$ at a $O(\text{TeV})$ scale. This leaves not only the usual massless gluons but also a color-octet massive gluon with a typical $O(\text{TeV})$ mass. Third generation left handed quark doublet $(t\ b)_L^T$ and $t_R$ must transform under the stronger $SU(3)_1$, whereas the lighter generations transform under the weaker $SU(2)_2$. The $b_R$ may transform under the stronger group or not depending on the choice of “weak” gauge group (Notice the similarity with the situation in the previous section). Fermions that transform
under $SU(3)_1$ will couple to the heavy gluons with an enhanced coupling given by $\simeq (g_1/g_2)g_c$, with $g_c$ the standard color coupling. The lighter fermions, on the other hand, couple like $(g_2/g_1)g_c$. These non-universal couplings lead to tree-level FCNCs which are particularly important in observables involving the third generation \[17\]. In particular, the exchange of the heavy gluon would lead to four-fermion operators mediating transitions such as $b \rightarrow s\bar{q}q$ with strength $\alpha_s$. Thus, they will be comparable to gluonic penguins resulting in potentially important effects in the CP asymmetries of penguin dominated modes, as we see in the next section.

### 4 Signals in $b \rightarrow s$ Hadronic Decays

As mentioned above, the flavor-changing exchange of KK gluons leads to four-fermion interactions contributing to the quark level processes $b \rightarrow d\bar{q}q$ and $b \rightarrow s\bar{q}q$, with $q = u, d, s$. We are interested in contributions that are typically of $\simeq \alpha_s$ strength due to the fact that in the product of a third generation current times a lighter quark current the enhancement in the former is (at least partially) canceled by the suppression of the latter. At low energies, the $b \rightarrow d_i\bar{q}q$ processes are described by the effective Hamiltonian \[18\]

$$\mathcal{H}_{\text{eff.}} = \frac{4G_F}{\sqrt{2}} V_{ub} V_{ut}^* \left[ C_1(\mu) O_1 + C_2(\mu) O_2 \right] - \frac{4G_F}{\sqrt{2}} V_{tb} V_{tt}^* \sum_{j=3}^{10} C_i(\mu) O_i + \text{h.c.}, \quad (8)$$

where $i = d, s$ and the operator basis can be found in Ref. \[18\].

The Wilson coefficients $\{C_j(\mu)\}$ contain the short-distance information which, in the SM, arises from integrating out heavy particles such as the $W$ and $Z$ gauge bosons and the top quark. In the SM, the operators $\{O_3 - O_6\}$ are generated from one-loop gluonic penguin diagrams, whereas operators $\{O_7 - O_{10}\}$ arise from one-loop electroweak penguin diagrams. The Hamiltonian describing the $b \rightarrow s\bar{q}q$ decays is obtained by replacing $V_{ts}^*$ for $V_{ts}^*$ in eq. \[8\]. Contributions from physics beyond the SM affect the Wilson coefficients at some high energy scale. Additionally, new physics could generate low energy interactions with the “wrong chirality” with respect to the SM. This would expand the operator basis to include operators of the form $(\bar{s}_R \Gamma b_R)(\bar{q}_\lambda \Gamma q_\lambda)$, where $\Gamma$ reflects the Dirac and color structure and $\lambda = L, R$.

The exchange of color-octet gauge bosons such as KK gluons of the Randall-Sundrum scenario described in Section \[2\] or the top-gluons of Section \[3\] generate flavor-violating currents with the third generation quarks. Upon diagonalization of the Yukawa matrix, this results in FCNCs at tree level due to the absence of a complete GIM cancellation. The off-diagonal elements of the left and right, up and down quark rotation matrices $U_{L,R}$ and $D_{L,R}$ determine the strength of the flavor violation. In the standard model only the left-handed rotations are observable through $V_{\text{CKM}} = U^\dagger_L D_L$. Here, $D_{L,R}^{bs}$, $D_{L,R}^{bd}$, $U_{L,R}^{tc}$, etc., become actual observables.
The tree-level flavor changing interactions induced by the color-octet exchange are described by a new addition to the effective Hamiltonian that in general can be written, for $b \to s$ transitions, as

$$\delta H_{\text{eff}} = \frac{4\pi\alpha_s}{M_G^2} D_{L}^{bs} D_{L}^{bs} |D_{L}^{bs}|^2 e^{-i\omega} \chi (\bar{s}_L \gamma_\mu T^a b_L) (\bar{q}_L \gamma^\mu T^a q_L) + \text{h.c.} \ .$$

(9)

where $\omega$ is the phase relative to the SM contribution; and $\chi \simeq O(1)$ is a model-dependent parameter. For instance in the Randall-Sundrum case, $\chi = 1$ corresponds to the choice of $c_b \simeq 0$ that gives a coupling of the KK gauge boson about five times larger than the corresponding standard model value for that gauge coupling. On the other hand, for Topcolor models $\chi = 1$, as noted in Section 3. Eq. (9) shows the case of left-handed flavor violation interactions. However, in general right-handed flavor violating vertices, leading to “wrong-chirality” operators, can also be present. This is the case, for instance, in the Randall-Sundrum scenario where the right-handed b quark is also localized toward the TeV brane. It is also the case in most Topcolor models. An expression analogous to (9) is obtained by replacing $d$ for $s$ in it. This would induce effects in $b \to d$ processes.

From eq. (9) we can see that the color-octet exchange generates contributions to all gluonic penguin operators. Assuming that the diagonal factors obey $|D_{L}^{bs}| \simeq 1$, these will have the form

$$\delta C_i = -\pi\alpha_s(M_G) \left( \frac{v}{M_G} \right)^2 \left| D_{L}^{bs} \right| e^{-i\omega} f_i \chi \ ,$$

(10)

where $f_3 = f_5 = -1/3$ and $f_4 = f_6 = 1$, and $v = 246$ GeV. This represents a shift in the Wilson coefficients at the high scale. We then must evolve the new coefficients down to $\mu = m_b$ by making use of the renormalization group evolution [18]. The effects described by eq (10) are somewhat diluted in the final answer due to a large contribution from the mixing with $O_2$. Still, potentially large effects remain.

The phase $\omega$ in eq. (9) is in principle a free parameter in most models and it could be large. This is even true in the left-handed sector, since in general $V_{\text{CKM}}$ comes from both the up and down quark rotation. Only if we were to argue that all of the CKM matrix comes from the down sector we could guarantee that $\omega = 0$. Furthermore, there is no such constraint in the right-handed quark sector.

We now examine what kind of effects these flavor violating terms could produce. Their typical strength is given by $\alpha_s$ times a CKM-like factor coming from the $D_{L,R}$ off-diagonal elements connecting to $b$. The fact that the coupling is tree-level is somewhat compensated by the suppression factor $(v/M_G)^2$, for $M_G \simeq O(1)$ TeV. Still, the contributions of eq. (9) are typically larger than the SM Wilson coefficients at the scale $M_W$, and in fact are comparable to the Wilson coefficients at the scale $m_b$. Therefore, they could significantly affect both the rates and the CP asymmetries. In what follows we will make use of $\delta C_i(m_b) \simeq \delta C_i(M_W)$, neglecting the renormalization group effects in these four-quark operators. These typically will result in logarithmic enhancements in terms of $\ln(M_G^2/\mu^2)$,
with \( \mu \) some low energy scale. These effects should be resummed by the renormalization group. We ignore them here in order to make an estimate of the deviations caused by (10).

We are interested in examining the effects of eq. (9) in \( b \to s\bar{s}s \) and \( b \to s\bar{d}d \) pure penguin processes, such as \( B_d \to \phi K_s \), \( B_d \to \eta' K_s \) and \( B_d \to \pi^0 K_s \) in light of the fact that these modes contain only small tree-level contamination of standard model amplitudes. All these decays, constitute a potentially clean test of the standard model since their CP asymmetries are predicted to be a measurement of \( \sin(2\beta)_{\psi K_s} \), the same angle of the unitarity triangle as in the \( b \to c\bar{c}s \) tree-level processes such as \( B_d \to \psi K_s \), up to small corrections [19].

In order to estimate these effects and compare them to the current experimental information on these decay modes we will compute the matrix elements of \( H_{\text{eff}} \) in the factorization approximation [20] as described in Reference [21]. Although the predictions for the branching ratios suffer from significant uncertainties, we expect that these largely cancel when considering the effects in the CP asymmetries. (We examine this expectation below in detail). Thus the CP asymmetries in non-leptonic \( b \to s \) penguin dominated processes constitute a suitable set of observables to test the effects of these color-octet states. As we will see, these observables are the natural place for a first observation of this physics.

In Fig. 3 we plot \( \sin(2\beta)_{\psi K_s} \) vs. the KK gluon mass and for various values of the phase \( \omega \). Here, for concreteness, we have taken \( |D^s_{Lb}| = |V^*_{tb}V_{ts}| \), assumed \( b_R \) is localized on the Planck brane, and \( \chi = 1 \) in order to illustrate the size of the effect. The horizontal band corresponds to the \( B_d \to \psi K_s \) measurement, \( \sin(2\beta)_{\psi K_s} = 0.731 \pm 0.056 \) [12]. Shown are only positive values of \( \omega \), as negative ones increase the value of \( \sin(2\beta) \) contrary to the trend in the data. We see that sizeable deviations from the SM expectation are present for values in the region of interest \( M_G \gtrsim 1 \) TeV. This will be the case as long as \( |D^s_{Lb}| \sim |V_{ts}| \), and \( \chi \simeq O(1) \), both natural assumptions. For \( D^s_{Rb} \), this is valid as long as a significant fraction of the corresponding CKM elements comes from the down-quark rotation. On the other hand, \( \chi \simeq O(1) \) in all the models considered here. In addition, we have not considered the effects of \( D^s_{Rb} \), which could make the effects even larger.

Similarly, the decay \( B_d \to \eta' K_s \) is also dominated by the \( b \to s\bar{s}s \) penguin contribution. In Fig. 4 we plot \( \sin(2\beta)_{\eta' K_s} \) vs. the KK gluon mass and for various values of the phase \( \omega \), for the same choice of parameters as for the previous case. Although the effect in this mode appears even bigger, there are several additional sources of uncertainties. First, within the factorization approximation the effect in \( \sin(2\beta)_{\eta' K_s} \) depends on the quantity

\[
r \equiv \left( \frac{f^{u}_{\eta'}}{f_{K}} \right) \left( \frac{f^{B \to K}(m^{2}_{\eta'})}{f^{B \to \eta'}(m^{2}_{K})} \right),
\]

where \( i f^{u}_{\eta'} p_{\mu} = \langle 0|\bar{q}\gamma_{\mu}\gamma_{5}q|\eta' \rangle \) is the \( \eta' \) decay constant through the \( q = u, d, s \) axial-vector current, and \( f^{B \to P}(q^{2}) \) are the semileptonic form-factors for the \( B \to P \) decays. In Fig. 4
Figure 3: The quantity to be extracted from the CP violation asymmetry in $B^0_d \rightarrow \phi K_s$ vs. the heavy gluon mass and for various values of the decay amplitude phase $\omega$. The curves correspond to $\pi/3$ (solid), $\pi/4$ (dashed) and $\pi/6$ (dot-dash), and $\pi/10$ (dotted). The horizontal band corresponds to the world average value $^{12}$ as extracted from $B_d \rightarrow J/\psi K_s$, $\sin(2\beta)_{J/\psi K_s} = 0.731 \pm 0.056$.

we take the values for the corresponding parameters from Ref. $^{21}$, which results in $r \approx 1$. However, the answer is very sensitive to small variations in $r$, which, for instance, could be provided by our limited understanding of the $B \rightarrow \eta'$ form-factor in eq. $^{11}$. In addition to this source of uncertainty, it is generally expected that this decay mode would receive an enhancement from the large coupling of the QCD anomaly $^{22}$ to the singlet piece in $\eta'$. Several attempts have been made to estimate the size of this effect, suggested first by the large branching fractions with $\eta'$ in the final states. The anomaly contribution not only could introduce an uncertainty of order one in the calculation of the effects in the CP asymmetry, but it might also dilute new physics contributions as long as these do not arise from a modification of the $b \rightarrow sg$ vertex.

We finally consider the effects of the KK gluons in the $b \rightarrow s\bar{d}d$ mode $B_d \rightarrow \pi^0 K_s$. Here, just as for $B_d \rightarrow \phi K_s$, the CP asymmetry can be rather cleanly predicted in the factorization approximation and uncertainties are considerably smaller. In Fig. $^{5}$ we plot $\sin(2\beta)_{\pi^0 K_s}$ vs. he KK gluon mass for various choices of $\omega$. The effect in general appears to be somewhat smaller than the ones in $B_d \rightarrow \phi K_s$ and $B_d \rightarrow \eta' K_s$ for the same choice of $M_G$ and $\omega$.

The flavor violating exchange of the KK gluon also induces an extremely large contri-
Figure 4: The quantity to be extracted from the CP violation asymmetry in $B_d^0 \to \eta' K_s$ vs. the heavy gluon mass and for various values of the decay amplitude phase $\omega$. The curves correspond to $\pi/3$ (solid), $\pi/4$ (dashed) and $\pi/6$ (dot-dash), and $\pi/10$ (dotted). The horizontal band corresponds to the world average value \[12\] as extracted from $B_d \to J/\psi K_s$, $\sin(2\beta)_{\psi K_s} = 0.731 \pm 0.056$.

bution to $B_s - \bar{B}_s$ mixing. This is roughly given by

$$\Delta m_{B_s} \simeq 200 \text{ps}^{-1} \left(\frac{|D_{bs}^L|}{\lambda^2}\right)^2 \left(\frac{2 \text{ TeV}}{M_G}\right)^2 \left(\frac{g_{10}}{5}\right)^2,$$

(12)

where $\lambda \simeq 0.22$ is the Cabibbo angle, and $g_{10} \equiv g_1/g$ represents the enhancement of the zero-mode fermion coupling to the first KK gluon with respect to the four-dimensional gauge coupling, as plotted in Fig. 2. The contribution of eq. (12) by itself is about 10 times larger than the SM one for this natural choice of parameters, and would deem $B_s$ oscillations too rapid for observation at the Tevatron and other similar experiments.

There are also similar contributions to $\Delta m_{B_d}$, when $D_{bs}^L$ is replaced by $D_{bd}^L$. These were examined in Ref. \[23\] in the context of Topcolor assisted technicolor, a much more constrained brand of Topcolor than the one we consider here. The bounds found in Ref. \[23\] can be accommodated as long as $|D_{bd}^L| \lesssim |V_{td}|$ which is not a very strong constraint.

Thus, we see that the flavor-violation effects of the first KK gluon excitation in Randal-Sundrum scenarios where the $SU(3)_c$ fields propagate in the bulk can be significant in non-leptonic $B$ decays and specifically in their CP asymmetries. The dominance of these effects over those induced by “weak” KK excitations, such as KK $Z$ and $Z'$'s, due to the larger coupling would explain the absence of any effects in $b \to s\ell^+\ell^-$ processes, where up to now the data is consistent with SM expectations \[24\]. Deviations in the CP
Figure 5: The quantity to be extracted from the CP violation asymmetry in $B_d^0 \rightarrow \pi^0 K_s$ vs. the heavy gluon mass and for various values of the decay amplitude phase $\omega$. The curves correspond to $\pi/3$ (solid), $\pi/4$ (dashed) and $\pi/6$ (dot-dash), and $\pi/10$ (dotted). The horizontal band corresponds to the world average value [12] as extracted from $B_d \rightarrow J/\psi K_s$, $\sin(2\beta)_{J/\psi K_s} = 0.731 \pm 0.056$.

asymmetries of $b \rightarrow s$ non-leptonic processes would naturally be the first signal of new physics in these scenarios. These very same effects can be obtained by the exchange of the heavy gluons present in generic Topcolor models.

5 Conclusions

We have shown that in Randall-Sundrum scenarios with gauge and matter fields in the bulk, naturally occurring flavor violation is likely to be first observed in non-leptonic $B$ decays. Specifically, in models where obtaining a suitable top Yukawa coupling requires the localization of the third generation left handed doublet and right handed top quark toward the TeV brane, the non-universal interactions of the KK gluons result in tree-level FCNCs inducing $b \rightarrow sq\bar{q}$ decays with ($q = d, s$). In addition, the localization of $b_R$ toward the TeV brane results in extra contributions from flavor changing right handed currents.

We have studied the effects in the CP asymmetries in $B_d \rightarrow \phi K_s$, $B_d \rightarrow \eta' K_s$ and $B_d \rightarrow \pi^0 K_S$, where data is already available. In these three modes it is expected that the CP asymmetry measurements yield the same value of $\sin 2\beta$ as in the vastly studied tree-level decay $B_d \rightarrow J/\psi K_s$. We have seen that for naturally occurring order one relative phases, and for KK gluon masses in the few TeV range, large deviations in the CP asymmetries are expected, as it can be seen in Figs. 3, 4 and 5. Similar deviations should
also be expected in $B_d \to K^+K^-K_s$, as well as in a host of $B_s$ decay modes, where the asymmetry is expected to be negligibly small in the SM. New data to be available in the next few years from BaBar and BELLE might strengthen the case for new physics in these modes beyond the data in Table 1. This, coupled with increasingly precise observations in $b \to s\ell^+\ell^-$ and $b \to s\gamma$ in agreement with SM predictions, would point in the direction of a massive color-octet state strongly coupled to the third generation quarks as the source of the deviation.

We have seen that these effects can also be obtained in generic Topcolor models. It is not possible to distinguish these two sources from $B$ physics alone. This is true of any color-octet flavor-violating gauge interaction that couples strongly to the third generation. Here we focused on these two interesting cases, KK gluons in Randall-Sundrum scenarios and Topcolor. Other model building avenues addressing fermion masses might result in similar effects. In addition, the large contributions to $B_s$ mixing, perhaps rendering $\Delta m_{B_s}$ too large to be observed, is an inescapable prediction in this scenario, as it can be seen in eq (12).¹

Finally, there will also be contributions from the heavy gluons to other non-leptonic $B$ decays, such as $B \to \pi\pi$, etc. These modes have less clean SM predictions. However, if the deviations hinted in Table 1 are confirmed by data samples of $500 \text{fb}^{-1}$, to be accumulated in the next few years, it might prove of great importance to confirm the existence of these effects in less clean modes, perhaps requiring even larger data samples. Even if the heavy gluons are directly observed at the LHC, their flavor-violating interactions will be less obvious there than from large enough $B$ physics samples. Thus, if flavor violating interactions are observed at the LHC, high precision $B$ physics experiments could prove crucial to elucidate what is their role in fermion mass generation.

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