Brane-world stars and (microscopic) black holes

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Abstract

We study stars in the brane-world by employing the principle of minimal geometric deformation and find that brane-world black hole metrics with a tidal charge can be consistently recovered in a suitable limit. This procedure allows us to determine the tidal charge as a function of the ADM mass of the black hole (and brane tension). A minimum mass for semiclassical microscopic black holes can then be derived, with a relevant impact for the description of black hole events at the LHC.

1 Introduction

In the Randall-Sundrum (RS) brane-world (BW) models\textsuperscript{1}, the fundamental scale of gravity $M_G$ can be as low as the electroweak scale, which would allow for the production of TeV-scale black holes (BHs) at the Large Hadron Collider (LHC)\textsuperscript{2}. One should however not forget that fully analytical metrics describing BHs in the complete five-dimensional space-time of the RS model\textsuperscript{3} are yet to be found (albeit, the existence of large static BHs was recently established in Ref.\textsuperscript{4}).

Solving the full five-dimensional Einstein field equations with appropriate sources (BH plus brane) is indeed a formidable task (see\textsuperscript{1,4} and References therein). The approach we will follow here is therefore to start from a sensible description of BW stars and employ a limit for

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(roughly speaking) vanishing star radius which (at least formally) allows one to recover the BH geometry found in Ref. [6]. We should warn the reader this method is not free of hindrances. For example, it was shown in Refs. [7] that the collapse of a homogeneous star leads to a non-static exterior, contrary to what happens in four-dimensional General Relativity (GR), and a possible exterior was later found which is radiative [8] (at the classical level, whereas four-dimensional BHs emit Hawking radiation [9], a quantum field theory effect). If one regards BHs as the natural end-state of the gravitational collapse of compact objects, one may conclude that classical BHs in the BW should suffer of the same problem as semiclassical BHs in GR: no static configuration for their exterior might be allowed [10, 11]. Nonetheless, a static configuration was recently found in Ref. [4] (see also Refs. [12, 13] for more explicit examples of static metrics), thus keeping the debate open as to what we should consider a realistic physical description of BHs in models with extra spatial dimensions.

In any case, one may view a static metric (both in GR and in the BW) as an approximate description of BHs which holds for sufficiently short times. This said, although there is no guarantee that our mathematical limit reproduces the physics of gravitational collapse, we shall find interesting consequences which may help to better understand BH physics in the BW. In particular, we shall show that the outer tidal charge \( q \) and ADM mass \( M \) can be both uniquely determined by the BH proper mass \( M \) and brane density \( \lambda = M_P \sigma / \ell_P \). This, in turn, will allow us to estimate the minimum mass of tidally charged BHs, thus filling a crucial gap in the arguments of Ref. [15].

We shall explicitly display the Newton constant \( G_N = \ell_P / M_P \) and denote by \( \ell_G \gg \ell_P \) and \( M_G \ll M_P \) the fundamental five-dimensional length and mass (\( \hbar = M_P \ell_P = M_G \ell_G \)). The “brane density” parameter \( \sigma \) thus has dimensions of an inverse squared length, namely \( \sigma \simeq \ell_G^{-2} \).

2 The brane-world

In the RS model, our Universe is a co-dimension one, four-dimensional hypersurface of vacuum energy density \( \lambda \) [1]. In Gaussian normal coordinates \( x^A = (x^\mu, y) \), where \( y \) is the extra-dimensional coordinate with the brane located at \( y = 0 \) (capitol letters run from 0 to 4 and Greek letters from 0 to 3), the five-dimensional metric can be expanded near the brane as [16]

\[
g^{(5)}_{AB} \simeq g^{(5)}_{AB} \bigg|_{y=0} + 2 K_{AB} \bigg|_{y=0} y + \mathcal{L}_{\hat{n}} K_{AB} \bigg|_{y=0} y^2 ,
\]

where \( K_{AB} \) is the extrinsic curvature of the brane, and \( \mathcal{L}_{\hat{n}} \) the Lie derivative along the unitary four-vector \( \hat{n} \) orthogonal to the brane. Junction conditions at the brane lead to [17]

\[
K_{\mu\nu} \sim T_{\mu\nu} - \frac{1}{3} (T - \lambda) g_{\mu\nu} ,
\]

where \( T_{\mu\nu} \) is the stress tensor of the matter localized on the brane, and [16]

\[
\mathcal{L}_{\hat{n}} K_{\mu\nu} \sim \mathcal{E}_{\mu\nu} + F_{\mu\nu} ,
\]
where \( \mathcal{E}_{\mu\nu} \) is the (traceless) projection of the Weyl tensor on the brane and \( F_{\mu\nu} = F_{\mu\nu}(\lambda, T) \) a tensor which depends on \( T_{\mu\nu} \) and \( \lambda \).

We recall that junction conditions in GR allow for surfaces with either step-like discontinuities or (infinitely thin) Dirac \( \delta \)-like discontinuities in the stress tensor \([18]\). Since a brane in RS is a thin surface, it generates an orthogonal discontinuity of the extrinsic curvature in five dimensions. Any discontinuities in the BW matter stress tensor would induce further discontinuities in the extrinsic curvature \((2.2)\) parallel to the brane, which should appear in the five-dimensional metric \((2.1)\), and are not allowed by the regularity of five-dimensional geodesics. Moreover, because of the term of order \( y^2 \) in Eq. \((2.1)\), and Eq. \((2.3)\), the projected Weyl tensor must also be continuous on the brane. This regularity requirements can be understood by considering that, in a microscopic description of the BW, matter should be smooth along the fifth dimension, yet localized on the brane (say, within a width of order \( \sigma^{-1/2} \approx \ell_G \), see Ref. [19]). In order to build a physically acceptable model of BW stars, continuity of five-dimensional geodesics should then be preserved by smoothing the matter stress tensor and the projected Weyl tensor across the star surface [11]. Since this renders the problem nearly intractable, we shall apply a simplifying idea which has already proved very effective.

We shall start from the effective four-dimensional equations of Ref. [17]. In general, these equations form an open system on the brane, because of the contribution \( \mathcal{E}_{\mu\nu} \) from the bulk Weyl tensor in Eq. \((2.3)\), and identifying specific solutions requires more information on the bulk geometry and a better understanding of how our four-dimensional space-time is embedded in the bulk. Nonetheless, it is possible to generate the BW version of every spherically symmetric GR solution through the minimal geometric deformation approach \([20]\). This method is based on requiring that BW solutions reduce to four-dimensional GR solutions at low energies (i.e., energy densities much smaller than \( \sigma \)). From the point of view of a brane observer, the five-dimensional gravity therefore induces a geometric deformation in the radial metric component, which is the source of anisotropy on the brane. When a solution of the four-dimensional Einstein equations is considered as a possible solution in the BW, the geometric deformation produced by extra-dimensional effects is minimal, and the open system of effective BW equations is automatically satisfied. This approach was successfully used to generate physically acceptable interior solutions for stellar systems \([21]\), and even exact solutions were found in Ref. \([22]\).

### 2.1 Brane-world star exterior

The metric outside a spherically symmetric BW source should solve the effective four-dimensional vacuum Einstein equations \([17]\),

\[
R_{\mu\nu} - \frac{1}{2} R^\alpha_{\alpha\mu\nu} g_{\mu\nu} = \mathcal{E}_{\mu\nu} \quad \Rightarrow \quad R^\alpha_{\alpha\mu\nu} = 0 \ ,
\]

where we recall that tidal effects are represented by \( \mathcal{E}_{\mu\nu} \). Only a few analytical solutions are known \([4, 12, 13, 14]\), one being the tidally charged metric \([6]\)

\[
ds^2 = e^\nu dt^2 - e^\lambda dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \ ,
\]

3
which has been extensively studied in Refs. [15, 23]. The tidal charge \( q \) and ADM mass \( \mathcal{M} \) are usually treated as independent quantities. However, by studying the interior BW solution of compact stars, it will be possible to obtain a relationship between \( q \) and \( \mathcal{M} \) from the usual junction conditions [18]. For instance, the simplest case of a static, spherically symmetric and uniform distribution with \( \mathcal{E}_{\mu\nu} = 0 \) has already been considered in Ref. [24], where the tidal charge \( q \) and ADM mass \( \mathcal{M} \) were then shown to be related, that is \( q = q(M, R, \rho, \sigma) \) and \( \mathcal{M} = \mathcal{M}(M, \rho, \sigma) \), with \( M \) the total GR mass of a star of radius \( R \) and density \( \rho \). There are however an additional free constant parameter \( \alpha \) and a constraint \( f(M, R, \rho, \sigma, \alpha) = 0 \) relating these functions at the star surface in a non trivial way, thus making it impossible to display a simple relationship between the tidal charge \( q \) and ADM mass \( \mathcal{M} \). In our case, a more realistic BW distribution, with \( \mathcal{E}_{\mu\nu} \neq 0 \), is considered in such a way that these difficulties are overcome, and simple expressions for the tidal charge and ADM mass will be obtained (see Eqs. (2.30) and (2.31) below).

First of all, if no source is present and \( \mathcal{M} = 0 \), or in the GR limit \( \sigma^{-1} \to 0 \), the tidal charge \( q \) should vanish. Let us therefore assume \( q = q(M, \sigma) \). We then consider the junction conditions at the star surface \( r = R \) between the metric given in Eq. (2.5) and a general interior solution of the form (2.5) with \( \nu = \nu_-(r) \) and \( \lambda = \lambda_-(r) \) to be found through the minimal geometric deformation approach. In this approach, from the point of view of a brane observer, the five-dimensional gravity produces a geometric deformation in the radial metric component of a general spherically symmetric metric (2.5) given by

\[
e^{-\lambda} = 1 - \frac{2 \ell_p M(r)}{M_p r} + \text{Geometric Deformation} = f(\nu, \rho, p),
\]

where \( m(r) \) is the standard GR mass function,

\[
m(r) = 4\pi \int_0^r \rho x^2 dx ,
\]

with \( \rho \) the matter energy density. When a solution of the four-dimensional Einstein field equations with density \( \rho \) and pressure \( p \) is considered as a possible BW solution, the geometric deformation in Eq. (2.7) is minimal, and given by

\[
f^*(r) = \frac{8\pi}{\sigma} e^{-I} \int_0^r \frac{2 x e^f}{x \nu' + 4} (\rho^2 + 3 \rho p) dx ,
\]

with

\[
I \equiv \int \frac{2 r \nu'' + r \nu'^2 + 4 \nu' + 4}{r \nu' + 4} dr .
\]
Hence, any perfect fluid solution is altered by the geometric deformation $f^*(r)$ produced by five-dimensional effects at high energies. This geometric deformation produces imperfect fluid effects through the BW solution for the geometric function $\lambda(r)$, which can be written as

$$e^{-\lambda} = 1 - \frac{2 \tilde{m}(r)}{r},$$  \hspace{1cm} (2.11)

where the interior mass function $\tilde{m}$ is given by

$$\tilde{m}(r) = m(r) - \frac{r}{2} f^*(r).$$  \hspace{1cm} (2.12)

Using a generic interior solution $\nu_- = \nu(r)$ and the $\lambda_- = \lambda(r)$ shown in Eq. (2.11), along with the tidally charged metric (2.6), in the matching conditions at the stellar surface $r = R$, we find

$$e^{\nu_R} = 1 - \frac{2 \ell_P M}{M_P R} - \frac{q}{R^2},$$  \hspace{1cm} (2.13)

$$\frac{2 M}{R} = \frac{2 M}{R} \left( 1 - \frac{M}{R^2} \right) \left( \frac{f^* + \frac{q}{R^2}}{f^*} \right),$$  \hspace{1cm} (2.14)

$$\frac{q}{R^4} = \left( \frac{\nu_R'}{R} + \frac{1}{R^2} \right) f^* + 8 \pi \ell_P p_R,$$  \hspace{1cm} (2.15)

where $\nu_R \equiv \nu_- (R), \nu_R' \equiv \partial_r \nu_-|_{r=R}, M = m(R)$ is the total GR mass of the star, $p_R$ the pressure at the surface and $f^* = f^*(R; M, \sigma)$ encodes the minimal geometric deformation undergone by the radial metric component due to bulk effects [20] at $r = R$. In particular, the tidal charge $q$ can be obtained from the second fundamental form of the surface $r = R$, which in the BW is given by

$$p_R + \frac{1}{\sigma} \left( \frac{\rho_R^2}{2} + \rho_R p_R + \frac{2}{k^4} \mathcal{U}_R^- \right) + \frac{4 P_R^-}{k^4 \sigma} = \frac{2 \mathcal{U}_R^+}{k^4 \sigma} + \frac{4 P_R^+}{k^4 \sigma},$$  \hspace{1cm} (2.16)

where $\pm$ denote the limits $r \to R^\pm$ respectively. In our approach, the above reduces to

$$\frac{\ell_P}{M_P} p_R + \left( \frac{\nu_R'}{R} + \frac{1}{R^2} \right) f^* = \frac{2 \mathcal{U}_R^+}{k^4 \sigma} + \frac{4 P_R^+}{k^4 \sigma}.$$  \hspace{1cm} (2.17)

On using the tidally charged metric (2.6) and

$$\mathcal{U}_R^+ = -\frac{P_R^+}{2} = \frac{4 \pi q \sigma}{3 R^4},$$  \hspace{1cm} (2.18)

in Eq. (2.17), we find the expression shown in Eq. (2.15). From Eqs. (2.13) and (2.15), we then obtain the tidal charge as

$$\frac{M_P}{\ell_P} q = \left( \frac{R \nu_R'}{R \nu_R' + 2} \right) \left( 1 - \frac{2 M}{R} \right) \left( \frac{2 M}{R} - \frac{2 M}{R} \right) R^2 + 8 \pi p_R R^4.$$  \hspace{1cm} (2.19)
This charge \( q \) only depends on the interior structure through \( \nu'_R \). We may therefore choose a suitable \( \nu'_R \) in order to obtain \( q = q(M, \sigma) \).

For instance, by taking \( p_R = 0 \) and imposing the boundary constraint (at \( r = R \))

\[
R \nu'_R = -\frac{(M - M)}{(M - M)} - \frac{2MKp}{\sigma R^2 \ell_P},
\]

where \( K \) is a (dimensionful) constant we can fix later, we obtain a simple relation between \( q \) and \( M \) given by

\[
q = \frac{2K}{\sigma R},
\]

(2.21)

where we must have

\[
\frac{MKp}{\sigma R^2 \ell_P} < (M - M) < \frac{2MKp}{\sigma R^2 \ell_P}
\]

(2.22)

to ensure an acceptable physical behaviour in the interior [see Eq. (2.20)]. Remarkably, this expression for \( q \) satisfies all of our requirements, namely:

a) it vanishes for \( M \to 0 \) and for \( \sigma^{-1} \to 0 \), and

b) it vanishes for very small star density, that is for \( R \to \infty \) at fixed \( M \) and \( \sigma \).

Let us further notice that \( q \) diverges for \( R \to 0 \), at fixed \( M \) and \( \sigma \), which one may expect from previous considerations about point-like singularities in the BW. The condition (2.20) then leads to the simple exterior solution

\[
e^\nu = 1 - \frac{2\ell_P M}{MPr} \left( 1 + \frac{MPK}{\ell_P \sigma R r} \right).
\]

(2.23)

It is interesting to note that the solution (2.23) can also be obtained without imposing any ad hoc boundary constraints of the form in Eq. (2.20). We can in fact employ the result that the pressure does not need to vanish at the surface in the BW. From Eq. (2.19), we can then find a value for \( p_R \) leading to Eq. (2.23), namely

\[
4\pi R^3 p_R = \frac{MPMK}{\ell_P \sigma R^2} (2 + R \nu'_R) - (M - M)(1 + R \nu'_R).
\]

(2.24)

Note that \( p_R \to 0 \) for \( R \to \infty \) (at fixed \( M, M \) and \( \sigma \)), as well as for \( \sigma^{-1} \to 0 \) (at fixed \( R \)), if \( M \to M \) in the same limit [in fact, see Eq. (2.30) below].
2.2 Brane-world star interior

The star radius \( R \) is still a free parameter. However, it can be fixed for any specific star interiors. For instance, let us consider the exact interior BW solution found in Ref. [22],

\[ e^\nu = A \left(1 + C r^2\right)^4 , \]  

where \( A \) and \( C \) are constants. Using this metric element, with BW matter density

\[ \rho = C_p \left(\frac{M_p}{\ell_p}\right) \frac{C \left(9 + 2 C r^2 + C^2 r^4\right)}{7 \pi \left(1 + C r^2\right)^3} , \]  

where \( C_p = C_p(K) \) is also a constant to be determined for consistency, and vanishing surface pressure

\[ p_R = \left(\frac{M_p}{\ell_p}\right) \frac{2 C \left(2 - 7 C R^2 - C^2 R^4\right)}{7 \pi \left(1 + C R^2\right)^3} = 0 , \]

leads to \( C = \frac{\sqrt{7 - 2}}{2 R^2} \), and, from Eq. (2.8), we find

\[ R = 2 n \left(\frac{\ell_p}{M_p}\right) \frac{M}{C_p} , \]  

with \( n \equiv \frac{56}{43 - \sqrt{37}} \simeq 1.6 \). This result, with \( \frac{1}{2} \)

\[ K = \left(\frac{M_p}{M_G}\right)^2 \frac{\ell_G}{M_G} , \]  

can be used in the constraint (2.20) to obtain \( C_p = (M_G/M_p)^4 \) and the ADM mass

\[ \mathcal{M} = \frac{M^3}{M^2 + n_1 M_G^2} . \]  

Moreover, the expression for the tidal charge (2.21) yields

\[ q = \frac{\ell_G^2 M^2}{n \left(M^2 + n_1 M_G^2\right)} , \]  

where we used \( \sigma \simeq \ell_G^{-2} \) and \( n_1 = \frac{1}{2 \pi} \left(\frac{5 C R^2 + 4}{9 C R^2 + 4}\right) = \frac{31653 - 1007 \sqrt{57}}{175616} \simeq 0.14 \). Upon inverting the relation (2.30) between \( \mathcal{M} \) and \( M \), one could finally express the tidal charge \( q = q(\mathcal{M}, \sigma) \). The explicit expression of \( M = M(\mathcal{M}) \) is however rather cumbersome and we shall not display it here. Moreover, the expressions (2.28) and (2.31) satisfy the bounds in Eq. (2.22), and are therefore associated with a consistent boundary condition (2.20).

\footnote{Different choices of \( K \) (and corresponding \( C_p \)) are also being considered and will be reported elsewhere.}
3 Black hole limit and minimum mass

The ADM mass (2.30) and tidal charge (2.31) do not explicitly depend on the star radius \( R \), and we can therefore assume they are valid in the limit \( R \to 0 \) (or, more cautiously, \( R \ll \ell_G \)). Correspondingly, the exterior metric given by Eq. (2.23) becomes

\[
e^\nu = 1 - \frac{2 \ell_P M^3}{M_P (M^2 + n_1 M_G^2)} r \left(1 + \frac{\ell_G^2 M_P}{2 n \ell_P M r}\right),
\]

which can be used to describe a BH of “bare” (or proper) mass \( M \). Fig. 1 shows the dimensionless ADM mass \( \bar{M} \) from Eq. (3.2) (solid line) and charge \( \bar{q} \) from Eq. (3.3) (dashed line) vs the “bare” mass \( M \).

![Figure 1: Dimensionless ADM mass \( \bar{M} \) from Eq. (3.2) (solid line) and charge \( \bar{q} \) from Eq. (3.3) (dashed line) vs the “bare” mass \( M \).](image1)

![Figure 2: Dimensionless charge \( \bar{q} \) vs ADM mass \( \bar{M} \).](image2)

The dimensionless ADM mass

\[
\bar{M} = \frac{M}{M_G} = \frac{\bar{M}^3}{M^2 + n_1} \approx \frac{\bar{M}^3}{0.1 + M^2},
\]
and tidal charge
\[ \tilde{q} = \frac{q}{\ell_G^2} = \frac{\bar{M}^2}{n (n_1 + \bar{M}^2)} \approx \frac{\bar{M}^2}{0.2 + 1.6 \bar{M}^2}, \]  
(3.3)
as functions of the dimensionless proper mass \( \bar{M} = M/M_G \). Fig. 2 shows the dimensionless charge \( \tilde{q} \) as a function of the ADM mass \( \mathcal{M} \). Note that, for \( \bar{M} \gtrsim M_G \), the ADM mass \( \mathcal{M} \approx \bar{M} \), whereas the tidal charge saturates to a maximum
\[ q_{\text{max}} \approx 0.6 \ell_G^2, \]  
(3.4)and is practically negligible for macroscopic BHs.

From Eq. (2.31), we can now infer an important result for microscopic BHs. The condition that is usually taken to define a semiclassical BH is its horizon radius \( R_H \) be larger than the Compton wavelength \( \lambda_M \approx \ell_P M_P/M \) (see [15] and References therein). From Eq. (2.6), we obtain the horizon radius
\[ R_H = \frac{\ell_P}{M_P} \left( \mathcal{M} + \sqrt{\mathcal{M}^2 + q \frac{M_P^2}{\ell_P^2}} \right), \]  
(3.5)and the classicality condition \( R_H \gtrsim \lambda_M \) reads
\[ \frac{M}{M_P} \left( \mathcal{M} + \sqrt{\mathcal{M}^2 + q \frac{M_P^2}{\ell_P^2}} \right) \gtrsim 1. \]  
(3.6)
We define the critical mass \( M_c \) as the value which saturates the above bound. In order to proceed, we shall expand for \( M \sim \mathcal{M} \approx M_G \ll M_P \), thus obtaining
\[ \frac{R_H^2}{\lambda_M^2} \approx \frac{M^2}{M_P^2} \frac{q}{\ell_P^2} \approx \frac{\bar{M}^2}{M_P^2} \bar{M}^2 \tilde{q} \ell_P^2 \approx \bar{M}^2 \tilde{q} \approx 1, \]  
(3.7)or \( \bar{M}^4 \approx n (n_1 + \bar{M}^2) \), which yields
\[ M_c \approx 1.3 M_G, \]  
(3.8)or \( \mathcal{M}_c \approx 1.2 M_G \), from Eq. (3.2). This can be viewed as the minimum allowed mass for a semiclassical BH in the BW.

4 Conclusions and outlook

We have analyzed analytical descriptions of stars in the BW to recover the BH metric (2.6) of Ref. [6], with the tidal charge as an explicit function of the ADM mass and brane tension, which was still an open problem. Using the most general junction conditions between a generic interior solution and the tidally charged metric (2.6) in the minimal geometric deformation approach [20],
a simple relationship among the tidal charge, the ADM mass and the brane tension satisfying all physical requirements was found. The simple solution \((2.31)\) for the tidal charge \(q = q(M, \sigma)\) then allowed us to determine the minimum mass of a semiclassical microscopic BH, namely \(M_c \simeq 1.2 M_G\).

Our results could be relevant for the description of BH events at the LHC. For example, the fact that \(M_c \gtrsim M_G\) means that producing BHs, even within TeV-scale gravity, might be beyond the LHC capability (for some data analysis of BH events at the LHC, see Refs. \[25\]).

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