Resonant States of Wave Propagation in Disordered System of Bosonic Particles

A. Kwang-Hua Chu [*]

P.O. Box 30-15, (Xu-Jia-Hui Post Office), Shanghai 200030, PR China

Abstract

We demonstrate the effects of an induced disorder (or a free-orientation \( \theta \) which is related to the relative direction of scattering of particles w.r.t. to the normal of the propagating plane-wave front) upon the possible resonance of the plane (sound) wave propagating in Bose gases by using the quantum kinetic equations. We firstly present the diverse dispersion relations obtained by the relevant Pauli-blocking parameter \( B \) (which describes the Bose particles when \( B \) is positive) and the free-orientation \( \theta \) and then, based on the acoustic analog, address the possible resonant states.

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1 Introduction

Bose-Einstein condensation, as is known, occurs when a macroscopic number of bosons piles down to the same lowest single-particle state. In the presence of a trapping potential, there arises a whole spectrum of energy levels \( (E_i) \) associated with the stationary solutions to the Gross-Pitaevskii equation. Each of these stationary solutions is, by definition, a topological coherent mode [1-2]. The lowest energy level corresponds to the standard Bose-Einstein condensate, while the higher states describe various nonground-state condensates. The latter can be generated by alternating fields, whose frequencies should be in resonance with the related transition frequencies [2-3]. It is feasible to realize an oscillatory modulation of either the trapping potential or of the atomic scattering length. The trapping potential can be either single-well or multi-well. Meanwhile, the theoretically predicted low-energy collective oscillations of the Bose condensate have been experimentally confirmed by laser imaging techniques [1]. Moreover, the dynamics of the collective oscillations of the condensate has been theoretically studied beyond the linear regime, showing strong enhancement of the amplitude dependence of frequencies in presence of resonances (cf. Pitaevskii and Stringari in [1]).

Very recently, optical lattices and atomic transport therein have attracted new attention with the achievement of Bose-Einstein condensation by purely optical means and the observation of a superfluid to Mott insulator transition in a gas of ultracold atoms [4]. However, no detailed study of the dependence of spatial diffusion on the different directions of an anisotropic lattice have been performed so far. In fact, cold atomic samples also constitute an ideal system for the study of complex nonlinear phenomena and allows for the direct observation of wave packet dynamics in real space on a macroscopic scale [1-4]. Above mentioned progresses are closely
relevant to the quantum-mechanic many-body phenomena subjected to extremely confinements. One special interest is related to the dynamic resonances in cold Bose gases [2-3].

On the other hand, emerging interests in the wave propagation in the random, disordered, and granular media under the influence of spatial confinement as well as studies of collision phenomena in rather cold gases, e.g., Bose gases have stimulated intensive researches recently [5-8]. For instance, Varshni investigated the spectra of helium (bosonic particles) at high pressures [5]. The properties of helium atoms confined to move in restricted geometry have been of considerable interest during the last several decades [9-10]. The confined helium atom, however, needs much more complicated calculations than the confined hydrogen atom. Varshni considered the effect of confinement on some of the lower lying excited states and the resulting optical spectrum from transitions between these states. His results could be applicable to (a) bubbles of helium implanted in a variety of materials (say, metals [11]), the physical state of helium in these bubbles had to be inferred by indirect means; (b) high-pressure helium plasmas [12] (to provide a quantitative diagnostic for plasma density); (c) astrophysics [5].

We noticed that acoustical analogs [13-14] considering continuum-mechanic and quantum-mechanic approaches are currently in rapid progress for both theories and measurements. The energy ($E$) adopted in quantum-mechanic formulation directly links to the acoustical frequency ($\omega$) considered in classic (continuum-mechanic) or semi-classic (kinetic) formulation due to the existence of an acoustical analogy [14]. Classical systems could be used to study time-dependent potential fields and nonlinear effects, which are very difficult and time-consuming to treat numerically or analytically in quantum-mechanic ways. Motivated by the need to understand the wave dynamics of cold Bose gases [15-16] at the finite temperature ($T > T_c$) under strong confinement (which is beyond the hard-core approximation we adopted here although our approach can provide the theoretical understanding about how the collision frequency or the rarefaction parameter can tune the wave dispersion), an investigation for observing the microscopic localization (which will induce resonances in confined microdomain) [17-20] using the quantum kinetic model was performed and will be presented here by taking into account the acoustic analog [14].

In this paper, considering the quantum (discrete) kinetic model and the Uehling-Uhlenbeck collision term which could describe the collision of a gas of hard-sphere Bose-particles by tuning a Pauli-blocking parameter $\gamma$ [21] (via a Pauli blocking factor of the form $1 + \gamma N_0$ with $N_0$ being a normalized number density giving the number of particles per cell in phase space), we will investigate the possible resonant states when plane waves propagates in (hard-sphere) Bose gases by introducing a disorder (say, induced by a high pressure or external field) or free-orientation ($\theta$ which is related to the relative direction of scattering of particles with respect to the normal of the propagating plane-wave front) and then obtaining the diverse dispersion relations which can thus be applied to the acoustical analogs [13-14]). This presentation will provide more clues to the studies of the quantum wave dynamics in Bose gases under suitable confined conditions and the possible appearance of the resonant states which are linked to the particles (number) density, induced disorder or relative free-orientation ($\theta$) and their energy states (the analogy between $E$ and $\omega$) [2-3].

The necessary verification of our approach with the previous available approaches [22-27] will be checked firstly. Our preliminary results show that, for the dispersion part (ratio of the
(phase) wave speed to that of hydrodynamical limit), the qualitative agreement (for hydrodynamic regime) with Cowell et al.'s result [26] or Andrews et al.'s result [22] is quite good. As for the damping or attenuation part of ours, the qualitative agreement with Jackson and Zaremba's (m=0 mode) result [28] looks also rather good. Our results show that, as illustrated below, the localized or resonant states will strongly depend on the energy (E or \(\omega\)), the effective scattering cross-section, the number density once the disorder and Pauli-blocking parameters are selected. We also found that, as \(\theta = \pi/4\), \(\theta\) being a disorder parameter, there exist possible resonant states which are similar to those reported in [13-14,17-19,29].

2 Formulations

We firstly make the following assumptions before we introduce the general equations of our model [14,25]:

(1) Consider a gas of identical particles of unit mass and a shape of a disk of diameter \(d\), then each particle \(i, i = 1, \ldots, N\), is characterized by the position of its center \(q_i\) and its velocity \(v_i\).

We also have the geometric limitations: \(|q_i - q_j| \geq d, i \neq j\). This is illustrated schematically in Fig. 1.

(2) Each particle moves in the plane with a relative velocity (in a centre-of-mass coordinate system) belonging to a discrete set \(\mathcal{V}\) of 4 velocities with only one speed (due to conservation of momentum and energy) in the plane (4 possible different directions) during a binary encounter. The velocity modulus \(c\) is a reference speed depending on the reference frame and specific distribution of particles.

(3) The collisional mechanism is that of rigid spheres, that is, the particles scatter elastically and they change their phase states instantaneously, preserving momentum. Only binary collisions are considered, since multiple collisions here are negligible.

The collisions between two particles (say \(i\) and \(j\)) take place when they are located at \(q_i\) and \(q_j = q_i - \text{d}n\), where \(n\) is the unit vector joining their centers. After collisions the particles scatter, preserving momentum, in the directions allowed by the discrete set \(\mathcal{V}\). In other words, particles change according to \((q_i, v_i) \rightarrow (q_i, v_i^*), (q_j, v_j) \rightarrow (q_j, v_j^*)\). The collision is uniquely determined if the incoming velocity and the impact angle \(\psi, \psi \in [-\pi/2, \pi/2]\), are known, which is defined as the angle between \(v_i\) and \(n\) or \(n(\psi) = (\cos (\psi + (k-1)\pi/2), \sin (\psi + (k-1)\pi/2)), k = 1, \ldots, 4\). From the selected velocities we have two classes of encounters, i.e. \(\langle v_i, v_j \rangle = 0\) and \(\langle v_i, v_j \rangle = -c^2\), respectively.

(a) In the first class momentum conservation implies only: encounters at \(\pi/2\) with exchange of velocities \(v_i = v^k \rightarrow v_i^* = v^{k+1}\), \(v_j = v^{k+1} \rightarrow v_j^* = v^k\), \(k = 1, \ldots, 4\), in the case \(\psi \in [-\pi/2, 0]\), and \(v_i = v^k \rightarrow v_i^* = v^{k+3}\), \(v_j = v^{k+3} \rightarrow v_j^* = v^k\), in the case \(\psi \in [0, \pi/2]\).

(b) Similarly, \(\langle v_i, v_j \rangle = -c^2\):

(i) Head-on encounters with impact angle \(\psi = 0\) such that \(v_i = v^k \rightarrow v_i^* = v^{k+2}, v_j = v^{k+2} \rightarrow v_j^* = v^k, k = 1, \ldots, 4\).

(ii) Head-on encounters with impact angle \(\psi \neq 0\) such that \(v_i = v^k \rightarrow v_i^* = v^{k+1}, v_j = v^{k+2} \rightarrow v_j^* = v^{k+3}, v_i = v^{k+2} \rightarrow v_i^* = v^{k+1}, v_j = v^{k+1}\), if \(\psi \in [0, \pi/2]\). The schematic presentation is shown in Fig 2. For grazing collisions, that is \(\langle n, v_i \rangle = \langle n, v_j \rangle = \ldots\)
We then assume that the gas (i.e., only a two-body encounter is possible) is composed of identical hard-sphere particles of the same mass [14,25]. The possible velocities of these \(N\) particles are restricted to, e.g., \(\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_p\), \(p\) is a finite positive integer \((p \neq N)\). That is to say, only the velocity space is discretized, the space and time variables are still continuous. The discrete number density of particles are denoted by \(N_i(\mathbf{x}, t)\) associated with the velocity \(\mathbf{u}_i\) at point \(\mathbf{x}\) and time \(t\). If only nonlinear binary collisions and the evolution of \(N_i\) are considered, we have

\[
\frac{\partial N_i}{\partial t} + \mathbf{u}_i \cdot \nabla N_i = F_i = \sum_{j=1}^{n} \sum_{k,l} (A_{kl}^{ij} N_k N_l - A_{kl}^{ij} N_i N_j), \quad i = 1, \cdots, p, \tag{1}
\]

where \((i, j)\) and \((k, l)\) are admissible sets of collisions [14,21,25], \(i, j, k, l \in \Lambda = \{1, \cdots, p\}\), and the summation is taken over all \(j, k, l\), where \(A_{kl}^{ij}\) are nonnegative constants (related to transitional rates) satisfying \(A_{kl}^{ij} = A_{kl}^{ji} = A_{lk}^{ij} = A_{lk}^{ji}, A_{kl}^{ij} (\mathbf{u}_i + \mathbf{u}_j - \mathbf{u}_k - \mathbf{u}_l) = 0, A_{kl}^{ij} = A_{kl}^{ji} [14,25]\). \(F_i\) is the (discrete) approximation of the collisional integral in the conventional continuous kinetic theory.

The conditions defined for the discrete velocity above requires that elastic, binary collisions, such that momentum and energy are preserved.

The collision operator is now simply obtained by joining \(A_{kl}^{kl}\) to the corresponding transition probability densities \(a_{ij}^{kl}\) through \(A_{kl}^{kl} = S|\mathbf{u}_i - \mathbf{u}_j| a_{ij}^{kl}\), where, \(a_{ij}^{kl} \geq 0, \sum_{k=1}^{p} a_{ij}^{kl} = 1, \sum_{l=1}^{p} a_{ij}^{kl} = 1, \forall i, j = 1, \cdots, p\); with \(S\) being the effective scattering or collisional cross-section [14,21,25]. If all \(n\) \((p = 2n)\) outputs are assumed to be equally probable, then \(a_{ij}^{kl} = 1/n\) for all \(k\) and \(l\), otherwise \(a_{ij}^{kl} = 0\). The term \(S|\mathbf{u}_i - \mathbf{u}_j|dt\) is the volume spanned by the particle with \(\mathbf{u}_i\) in the relative motion w.r.t. the molecule with \(\mathbf{u}_j\) in the time interval \(dt\). Therefore, \(S|\mathbf{u}_i - \mathbf{u}_j|N_i\) is the number of \(j\)-particles involved by the collision in unit time. Collisions which satisfy the conservation and reversibility conditions which have been stated above are defined as admissible collisions [14,25].

Moreover, all the velocity directions after collisions are assumed to be equally probable. We note that, the summation of \(N_i \left(\sum_i N_i\right)\) : the total discrete number density here is related to the macroscopic density : \(\rho (= m_p \sum_i N_i)\), where \(m_p\) is the mass of the particle [21].

With the introducing of the Uehling-Uhlenbeck collision term [21] in equation (1) \((F_i\) being
\[ F_i = \sum_{j,k,l} A_{kl}^{ij} \left[ N_k N_l (1 + \gamma N_i)(1 + \gamma N_j) - N_i N_j (1 + \gamma N_k)(1 + \gamma N_l) \right], \]

for \( \gamma < 0 \) (normally, \( \gamma = -1 \)) we obtain a gas of Fermi-particles; for \( \gamma > 0 \) (normally, \( \gamma = 1 \)) we obtain a gas of Bose-particles, and for \( \gamma = 0 \) we obtain equation (1) which is for a gas of Boltzmann-particles [14,24-25].

Considering binary collision only, from equation (2), the model of quantum discrete kinetic equation for Bose gases is then a system of \( 2n (= p) \) semilinear partial differential equations of the hyperbolic type (in two dimensional form):

\[
\frac{\partial}{\partial t} N_i + \mathbf{U}_i \cdot \nabla N_i = \frac{c S}{n} \sum_{j=1}^{2n} N_j N_{j+n}(1 + \gamma N_{j+n})(1 + \gamma N_{j+n+1}) - 2c S N_i N_{i+n}(1 + \gamma N_{i+1})(1 + \gamma N_{i+n+1}), \quad i = 1, \cdots, 2n, \tag{3}
\]

where \( N_i = N_{i+2n} \) are unknown functions, and \( \mathbf{U}_i = \mathbf{c}(\cos[\theta + (i - 1)\pi/n], \sin[\theta + (i - 1)\pi/n]) \) are the particles velocities in the centre-of-mass coordinate system; \( \mathbf{c} \) is a reference velocity modulus and the same order of magnitude as the sound speed in the absence of scatters in [13] or [19], \( \theta \) is the orientation starting from the positive \( x \)-axis to the \( u_i \) direction and could be thought of as a disorder induced by high pressure or external field (schematically shown in Fig. 3). For example, there are admissible collisions \((\mathbf{U}_1, \mathbf{U}_3) \leftrightarrow (\mathbf{U}_2, \mathbf{U}_4)\) as \( n = 2 \) [25].

Since passage of the sound wave causes a small departure from an equilibrium state resulting in energy loss owing to internal friction and heat conduction, we linearize above equations around a uniform equilibrium state (particles’ number density: \( N_0 \)) by setting \( N_i(t, x) = N_0 (1 + P_i(t, x)) \), where \( P_i \) is a small perturbation. The equilibrium state here is presumed to be the same as in Refs. [21-22]. After some similar manipulations as mentioned in Refs. [30], with \( B = \gamma N_0 > 0 \) (cf. Chu in [21], \( B \) defines the proportional contribution from the Bose gases; if \( \gamma > 0 \), e.g., \( \gamma = 1 \)), we then have

\[
\left[ \frac{\partial^2}{\partial t^2} + c^2 \cos^2[\theta + (m - 1)\pi/n] \right] \frac{\partial^2}{\partial x^2} + 4c S N_0(1 + B) \frac{\partial}{\partial t} \right] D_m = \frac{4c S N_0(1 + B)}{n} \sum_{k=1}^{n} \frac{\partial}{\partial t} D_k, \tag{4}
\]

where \( D_m = (P_m + P_{m+n})/2 \), \( m = 1, \cdots, n \), since \( D_1 = D_m \) for \( 1 = m \) (mod \( 2n \)).

We start to look for the solutions in the form of plane wave \( D_m = a_m \exp i(kx - \omega t), (m = 1, \cdots, n) \), with \( \omega = \omega(k) \) because it is related to the dispersion relations of quasi one-dimensional plane waves propagating in (monatomic) hard-sphere Bose gases. So we have

\[
(1 + i h(1 + B) - 2\lambda^2 \cos^2[\theta + (m - 1)\pi/n])a_m - \frac{i h(1 + B)}{n} \sum_{k=1}^{n} a_k = 0, \quad m = 1, \cdots, n, \tag{5}
\]

where

\[
\lambda = k c / (\sqrt{2} \omega), \quad h(1 + B) = \frac{4c S N_0(1 + B)}{\omega} \propto 1/K_n,
\]

\( h \) is the rarefaction parameter of the gas; \( K_n \) is the Knudsen number which is defined as the ratio of the mean free path of Bose gases to the wave length of the plane (sound) wave.

Let \( a_m = C / (1 + i h_b - 2\lambda^2 \cos^2[\theta + (m - 1)\pi/n]) \), where \( C \) is an arbitrary, unknown constant,
since we here only have interest in the eigenvalues of above relation. The eigenvalue problems for different $2 \times n$-velocity model reduces to
\begin{equation}
1 - \frac{i h b}{n} \sum_{m=1}^{n} \frac{1}{1 + i h b - 2 \lambda^2 \cos^2 \left[ \theta + \frac{(m-1)\pi}{n} \right]} = 0.
\end{equation}

3 Results and Discussions

We can resolve the complex roots ($\lambda = \lambda_r + i \lambda_i$) from the polynomial equation above and use the numerical way for direct plots. The roots are the values for the nondimensionalized dispersion (positive real part; a ratio of the sound or phase speed with respect to its continuum or hydrodynamical limit) and the attenuation or absorption (positive imaginary part), respectively. $B$ could be related to the occupation number. We plot the main results into figures 4, 5, 6, and 7, respectively. We firstly review the general characteristic dispersion relations for Bose gases before we interpret our present results.

Curves in figures 4 and 5 ($\theta \neq \pi/4$) resemble the conventional dispersion relations of ultrasound propagating in hard-sphere Boltzmann-statistic gases [30]. Present results show that as $B$ or $\theta$ increases, the dispersion ($\lambda_r = k_r c / (\sqrt{2} \omega)$) will reach the hydrodynamical limit (as $h \gg 1$ the wave speed is independent on the $S$ or the s-wave scattering length, this result agrees qualitatively with Cowell et al.’s results [26] (for $B > 0$) or Bruun and Burnett’s [28] (for $B < 0$)) earlier. That is to say, the phase speed of the plane wave in Bose gases (even for small but fixed $h$) increases w.r.t. to the continuum conditions ($h \to \infty$) as the relevant parameter $B$ increases or $\theta$ increases (up to $\pi/4$).

Meanwhile, as illustrated in Fig. 5, there always exist peaks or maxima in our attenuation results (related to the damping of the propagating wave). This agrees qualitatively with that of Jackson and Zaremba’s result (for $m=0$ mode) [28]. Here, considering the Pauli-blocking effect, the maximum or peak absorption (or attenuation $\lambda_i = k_i c / (\sqrt{2} \omega)$) for all the rarefaction parameters $h$ keeps the same for all $B$ (say, $B = 0.3$ and $B = 0.7$). There are only shifts of the maximum absorption state (defined as $h_{\text{max}}$) when $B$ increases. It seems for the same mean free path ($h \propto$ the inverse of $K_n$) or mean collision frequency of the hard-sphere gases (i.e. the same $h$ but $h < h_{\text{max}}$) there will be more absorption in larger $\theta$ cases than those of $\theta = 0$ states when the plane wave propagates.

To apply the acoustic analog, we should now introduce the brief idea [14]. In fact, studies of classical wave mechanical systems have some important advantages over quantum mechanical wave systems even there are similarities in-between [13]. In a mesoscopic system, where the sample size is smaller than the mean free path for elastic scattering, it is satisfactory for a one-electron model to solve the time-independent Schrödinger equation :
\begin{equation}
\frac{-\hbar^2}{2m} \nabla^2 \psi + V'(\vec{r}) \psi = E \psi
\end{equation}
or (after dividing by $-\hbar^2/2m$)
\begin{equation}
\nabla^2 \psi + \left[ a^2 - V(\vec{r}) \right] \psi = 0,
\end{equation}
where \( q \) is an (energy) eigenvalue parameter, which for the quantum-mechanic system is \( \sqrt{2mE/\hbar^2} \). Meanwhile, the equation for classical (scalar) waves is

\[
\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0
\]

or after applying a Fourier transform in time and contriving a system where \( c \) (the wave speed) varies with position \( \vec{r} \)

\[
\nabla^2 \psi + [q^2 - V(\vec{r})] \psi = 0, \tag{7}
\]

here, the eigenvalue parameter \( q \) is \( \omega/c_0 \), where \( \omega \) is a natural (or an eigen-) frequency and \( c_0 \) is a reference wave speed. Comparing the time dependencies one sees the quantum and classical relation \( E = \hbar \omega \) [14]. Thus, the energy \( E \) considered, e.g., in Refs. [19-20] for the resonant localization in quantum system corresponds to the frequency \( \omega \) considered here especially for the resonant states [14] by using the acoustical analogy.

Back to our present figures w.r.t. \( h \), as now the parameter of disorder or free-orientation \( \theta \) dominates. We can observe that, there is a continuous trend as \( \theta \) increases toward \( \pi/4 \approx 0.7854 \). The dispersion (\( \lambda_r \)); a relative measure of the sound or phase speed) keeps increasing while the attenuation or absorption (\( \lambda_i \)) keeps decreasing as \( \theta \) increases from 0. At \( \theta = \pi/4 \), there is no attenuation and dispersion, i.e., \( \lambda_r = 1.0 \) and \( \lambda_i = 0.0 \) [30]. This result also provides a good verification for the experimental side mentioned in [16-18] as there is no loss for this particular case (\( \theta \) being a disorder parameter but fixed as \( \pi/4 \)). We note that around the peak \( \lambda_i \) state \((h_{\text{min}})\) as shown in Fig. 5, there exists a trend for the absence of diffusion (\( \lambda_i \) starts decreasing rapidly) [18-20].

Based on the acoustic analog [14], from the definition of \( h \) or \( Kn \), \( h = f_{\text{collision}}/f_{\text{sound}} \), where \( f_{\text{sound}} \) is related to the classical frequency \( \omega \) as mentioned above (cf. equation (7)) so that it is relevant to the energy \( E \) as defined for the localization, thus we can estimate the localization length from those figures which vary with \( h \); \( f_{\text{collision}} \) is the mean collision frequency of the particles. The localization length defined in [19] is proportional to the (hydrodynamic) mean free path \( l \) (\( l \) also depends on the internal frequency \( E \) as shown in [19]) and, comparing the definition of \( h \) here, is thus related to the inverse of \( h \) (or, say, the frequency) we used. We remind the readers that the temperature \( (T) \) could be linked to the mean free path or mean collision frequency under prescribed conditions [24] and the energy (thus \( \omega \)) could be related to the temperature \( (T) \) with the introduction of the Boltzmann constant \( k_B \). Based on these considerations, the relation for the localization length versus the frequency extracted from our results (especially in Fig. 5; the attenuation or absorption defined here is related to the inverse measure of, say, one wave length; the maximum absorption then corresponds to the minimum localization length in Fig. 5(a) of [19]) is qualitatively similar to that reported in [19]. This observation is now illustrated in figure 6 where we schematically define the localization length as the inverse of the wave absorption : \( 1/\lambda_i \) (per unit wave length). There also shows the exponential decay of the the localization length w.r.t. \( h \) or the inverse of the frequency (a corresponding measure to energy \( E \) in quantum-mechanic sense as already explained before). Thus we can also obtain similar results which resemble that reported in Fig. 5 (a) in [19].

We note that, as the rarefaction parameter \((h = 4cSN_0/\omega)\) is fixed (cf. figure 6 for the schematic
localization length), from its definition and the correspondence between the energy \((E)\) and the frequency \((\omega)\), the product of an effective scattering cross-section \((S)\) (or the s-wave scattering length) and the number density of particles \((N_0)\) must be proportional to \(E\) or \(\omega\) once \(c\) and the localized state are made specific (the Pauli-blocking and disorder effects being excluded). Meanwhile, once \(E\) or \(\omega\) is fixed for the same situation mentioned above, the localized state will strongly depend on the \(S\) and \(N_0\) as well as \(B\) and \(\theta\).

People might argue that a nonzero \(\theta\) would only make the system anisotropic, but not disordered. We should remind them that the derivation of present quantum kinetic approach was based on the binary encounter of a system of Bose particles. Once the mean free path and the centre of mass coordinate system were introduced (especially when the effective, admissible collision and the microreversibility which neglects the history and the correlations when particles traverse in phase space \([14,21]\) were presumed) the randomness and disorder will occur although they are illustrated implicitly. The results presented here, in fact, also qualitatively resemble that reported in \([29]\) where it was shown that when a periodic medium with a gap (in resulting spectra) is (slightly) randomized (like our disordered case : \(\theta \neq 0\)), possible localization occurs in a vicinity of the edges of the gap (like that of \(\theta = \pi/4\) here; \(\theta = 0\) is implicit) \([29]\). As we only consider plane waves propagating in a hard-sphere gas, which is a kind of hard (Neumann) scatters \([29]\), then it is interesting that our results for the dispersion relation resemble those of Neumann cases (especially Fig. 9) in \([29]\).

To demonstrate the possible resonant states, we summarize our results in figure 7. Our results show that as \(\theta\) increases, the maximum absorption \((\lambda_i)\) will decreases continuously except at \(\theta = \pi/4\) where there is a sudden jump (maximum)! This unusual absorption (or attenuation) peak is similar to those observations found in the \(T_\lambda\) transition (temperature) for liquid helium (Bose liquids) (cf. Figs. 20, 21 in \([31]\)). The interesting result is that, the absorption value obtained for \(B = 0.5\) (Bose gases) is almost twice of that value for \(B = -0.5\) (Fermi gases) at \(\theta = \pi/4\). Note that, once there are Cooper pairings in Fermi gases, we can treat them (atomic pairs) as bosonic particles although the number density of them might be one half of the original. Thus, this resonant state might be relevant to the superfluid phase transition or Cooper pair formation tuned by the disorder. Meanwhile, once we consider the disorder is induced by the magnetic field, this sudden jump in Fig. 7 resembles that of resonance reported in \([32]\).

As we know, when a gas containing many identical particles is confined and cooled, the average momentum can be lowered so far that the typical de Broglie wavelength is larger than the average separation between the particles. In this case, the gas is said to be 'degenerate', meaning that the wave functions of neighboring particles overlap. Degenerate gases exhibit two dramatically different types of behavior, depending on whether the identical particles are bosons (such as photons) or fermions (such as electrons).

To conclude in brief, our calculations here are useful to the understanding of waves propagating in microscopically random, disordered or granular media under strong confinement \([2-3,5-7,9-12]\) and the dispersion management for atomic matter waves \([16]\). The direct relation of our results to the conventional one of static localization is not straightforward but could be understood qualitatively after the application of the acoustical-analog (i.e. the necessary transform from our complex \(\lambda\) or real \(\hbar\) to the conventional \(\omega, E, V\) (potential barrier), and the characteristic
lengths (cf. [13-14,19]) : mean free path, wave length, etc.). Our results show that, as illustrated in figure 6, the resonant states strongly depend on the energy (\(E\) or \(\omega\)), the effective scattering cross-section (\(S\)), the number density (\(N_0\)) once the disorder and Pauli-blocking parameters are selected. We shall study other complicated problems in the future [2-3,33-35].

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[*] The author’s permanent address : P.O. Box 39, Tou-Di-Ban, Xihong Road, Urumqi 830000, PR China.

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4 Appendix : Derivation of Eqn. (4)

From Eqn. (3), after the linearization, we then have, (say, $i=m$)

$$\frac{\partial}{\partial t} P_m + U_m \cdot \frac{\partial}{\partial x} P_m + 2cSN_0[(P_m + P_{m+n}) + \gamma N_0(P_m + P_{m+n} + P_{\Sigma}) + \cdots] =$$

$$\frac{cSN_0}{n} \sum_{k=1}^{2n} (P_k + P_{k+n} + \gamma N_0(P_k + P_{k+n} + P_{\Sigma}) + \cdots),$$

(8)

here, $m = 1, \cdots, 2n$, $P_{\Sigma} = 0$ for $n=2$ because of the restriction for the total perturbations in an equilibrium state and the remaining terms in both sides are higher order terms related to $(\gamma N_0)^2$. The linearized version of above equation (for $n=2$) is

$$\frac{\partial}{\partial t} P_m + U_m \cdot \frac{\partial}{\partial x} P_m + 2cSN_0(P_m + P_{m+n})(1 + \gamma N_0) = \frac{2cSN_0}{n} \sum_{k=1}^{2n} P_k(1 + \gamma N_0).$$

(9)

In these equations after replacing the index $m$ with $m+n$ and using the identities $P_{m+2n} = P_m$, then we have

$$\frac{\partial}{\partial t} P_{m+n} - U_m \cdot \frac{\partial}{\partial x} P_{m+n} + 2cSN_0(P_m + P_{m+n})(1 + \gamma N_0) = \frac{2cSN_0}{n} \sum_{k=1}^{2n} P_k(1 + \gamma N_0).$$

(10)

Combining above two equations, firstly adding then subtracting, with $A_m = (P_m + P_{m+n})/2$ and $B_m = (P_m - P_{m+n})/2$, we can have

$$\frac{\partial^2}{\partial t^2} A_m - c^2 \cos[\theta + \frac{(m-1)\pi}{n}] \frac{\partial^2}{\partial x} A_m + 4cSN_0A_m(1 + \gamma N_0) = \frac{4cSN_0}{n} \sum_{k=1}^{2n} A_k(1 + \gamma N_0),$$

(11)

$$\frac{\partial^2}{\partial t^2} B_m + c^2 \cos[\theta + \frac{(m-1)\pi}{n}] \frac{\partial^2}{\partial x} A_m = 0, \quad m = 1, \cdots, 2n.$$  

(12)

From $P_{m+2n} = P_m$, and with $A_m = (P_m + P_{m+n})/2$ and $B_m = (P_m - P_{m+n})/2$, we can have $A_{m+n} = A_m$, $B_{m+n} = -B_m$.

After eliminating $B_m$, with $B = \gamma N_0$, we then have

$$\{\frac{\partial^2}{\partial t^2} + c^2 \cos^2[\theta + \frac{(m-1)\pi}{n}] \frac{\partial^2}{\partial x^2} + 4cSN_0(1 + B) \frac{\partial}{\partial t}\} D_m = \frac{4cSN_0(1 + B)}{q} \sum_{k=1}^{n} \frac{\partial}{\partial t} D_k,$$

where $D_m = (P_m + P_{m+n})/2$, $m = 1, \cdots, n$, since $D_1 = D_m$ for $1 = m \pmod{2n}$. 
Fig. 3  Schematic plot for the regular scattering and the disorder-influenced scattering. Plane waves propagate along the $X$-direction. Binary encounters of $U_1$ and $U_3$ and their departures after head-on collisions ($U_2$ and $U_4$). Number densities $N_i$ are associated to $U_i$.

Fig. 4  Disorder or orientational ($\theta$) effects on the dispersion ($\lambda_r$). $h = 4cSN_0/\omega$, $S$ is the effective scattering cross section, $N_0$ is the number density, $B = \gamma N_0$ is the Pauli-blocking parameter. This result agrees with Cowell et al.’s result [26] for the hydrodynamic regime ($h \sim O(10)$ here). Wave speed is independent on $S$ for larger $h$. 

Fig. 5 Disorder or orientational ($\theta$) effects on the attenuation ($\lambda_i$). This result agrees with Jackson and Zaremba’s result ($m=0$ mode) [28]. There always exists a peak or maximum $\lambda_i$.

Fig. 6 Disorder ($\theta$) effects on the localization length ($1/\lambda_i$). $h = 4cSN_0/\omega$. Note that the energy $E$ corresponds to $\hbar\omega$ [13-14,19]. This figure is a schematic type. (cf. the presentation: Fig. 5 (a) for that used in [19].) All units are dimensionless. As $B$ increases, the minimum (for the localization length) state $h_{min}$ decreases, i.e., the temperature becomes much more lower (as the mean free path is rather large).
Fig. 7 Variations of (max.) $\lambda_i$ w.r.t. the disorder or free orientation $\theta$ for $B = \pm 0.5$. The sudden jump at $\theta = \pi/4$ implies the resonant transition in Bose fluids may also occur for Fermi fluids (in bound pairs). $\lambda_i$ for $B = 0.5$ (Bose gases) is almost twice of that value for $B = -0.5$ (Fermi gases) at $\theta = \pi/4$. 