Optical measurement of electron spins in quantum dots: Quantum Zeno effects

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We describe the effects of the quantum back action under continuous optical measurement of electron spins in quantum dots. We consider the system excitation by elliptically polarized light close to the trion resonance, which allows for the simultaneous spin orientation and measurement. We microscopically demonstrate that the nuclei-induced spin relaxation can be both suppressed and accelerated by the continuous spin measurement due to the quantum Zeno and anti-Zeno effects, respectively. Our theoretical predictions can be directly compared with the future experimental results and straightforwardly generalized for the pump-probe experiments.

I. INTRODUCTION

The quantum Zeno effect is the well known phenomenon of freezing of the quantum dynamics of the systems under frequent measurements. This resembles the arrow paradox formulated by Zeno of Elea 25 centuries ago [1]. The first prediction of the increase of the decay time of a quasi-stationary state under frequent perturbations was made by Khalfin in the late 1950s [2, 3], and the most popular experimental demonstration of this effect was made for a transition between two $^9$Be$^+$ ground-state hyperfine levels [4]. It is somewhat less known that the system measurements can also accelerate its relaxation, which is called an anti-Zeno effect [5–7].

Nowadays, both quantum Zeno and anti-Zeno effects are considered as an important tool for the quantum computing [8, 9]. On the one hand, it has to be taken into account during the qubit measurement, and on the other hand, it allows one to increase the relaxation time of the qubits. This leads to the growing interest in the investigation of the quantum Zeno effects for many systems promising for the quantum computations: cold atoms [10, 11], trapped ions [12], quantum cavities and waveguides [13–17], NV centers in diamond [18–20], and nuclear spins [21–24].

The paper is organized as follows: In the next section we formulate the model of the system under study. In Sec. III we derive the kinetic equation for the electron spin dynamics in the quantum dots. Then in Sec. IV we present its steady state solution and discuss the quantum Zeno and anti-Zeno regimes in detail. In particular, we find an analytical solution of the kinetic equation in these regimes. In Sec. V we discuss the generalization of our results to the pulsed spin measurements. The results of the work are summarized in Sec. VI.

II. MODEL

We consider an ensemble of identical singly charged quantum dots under continuous illumination by elliptically polarized light, as shown in Fig. 1. The circular component of the light orients electron spins in the quantum dots, while the linear component allows for the measurement of the spin induced Faraday rotation angle $\theta_F$.

Figure 1. Sketch of the ensemble of quantum dots (gray triangles) with the localized electron spins (red arrows) interacted with randomly oriented nuclear spins (black arrows) under illumination by CW elliptically polarized light and the measurement of the spin induced Faraday rotation angle $\theta_F$. The hyperfine interaction with the slowly varying nuclear spin environment [28, 29]. We find that the resonant optical system excitation can lead to both suppression and acceleration of the nuclei-induced spin relaxation due to the quantum Zeno and anti-Zeno effects.

In the same time, the electron spins in quantum dots are believed to be one of the most prominent candidates for scalable quantum computations [25–27]. However, the quantum measurement back action is essentially unexplored experimentally and even theoretically for quantum dots. In this paper we fill this gap and develop a microscopic theory of the quantum Zeno and anti-Zeno effects for electron spins in the quantum dots resonantly measured by the laser light. We suggest the measurement of the steady state spin polarization under continuous excitation of the dots by elliptically polarized light as the simplest experiment that allows one to demonstrate the quantum back action.

The quantum Zeno effect requires slow and non-Markovian spin relaxation. In quantum dots this kind of relaxation is typical in small magnetic fields because of the hyperfine interaction with the slowly varying nuclear spin environment [28, 29]. We find that the resonant optical system excitation can lead to both suppression and acceleration of the nuclei-induced spin relaxation due to the quantum Zeno and anti-Zeno effects.

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singlet trion resonance frequency $\omega_0$. The singlet trion consists of two electrons with antiparallel spins and a heavy hole with the spin projection $J_z = \pm 3/2$ on the structure growth axis $z$.

The Hamiltonian of the system accounting for the hyperfine interaction with the host lattice nuclei has the form [31]:

$$\mathcal{H} = \frac{\hbar \Omega N}{2} \sum_{i,j=\pm1/2} \sigma^j a_i^\dagger a_j + \sum_{i=\pm3/2} \hbar \omega_0 a_i^\dagger a_i + \hbar \left[ \mathcal{E}_+ e^{-i\omega t} a_{+3/2}^\dagger a_{+1/2} + \mathcal{E}_- e^{-i\omega t} a_{-3/2}^\dagger a_{-1/2} + \text{H.c.} \right] \tag{1}$$

Here $a_i$ ($a_i^\dagger$) are the annihilation (creation) operators of the electron ground states with the spins $S_z = \pm 1/2$ for $i = \pm 1/2$, respectively, and of the trion states with the spins $J_z = \pm 3/2$ for $i = \pm 3/2$, respectively. Further, $\Omega_N$ is the electron spin precession frequency related to the random Overhauser field of the nuclear spin fluctuations [28, 32], and $\sigma$ is the vector composed of the Pauli matrices. We neglect the much weaker hyperfine interaction for holes in trions. Finally, the parameters $\mathcal{E}_\pm$ are proportional to the amplitudes of the $\sigma^\pm$ polarized components of the incident light [33]. According to the optical selection rules, $\sigma^\pm$ polarized light couples the states $S_z = \pm 1/2$ with the states $J_z = \pm 3/2$, respectively, only, see Fig. 2.

The nuclear spin dynamics is much slower than the electron one, which makes the electron spin dynamics non-Markovian, which is necessary for the quantum Zeno and anti-Zeno effects. Theoretically, one can consider the nuclear spins to be frozen and normally distributed [28, 34]. The hyperfine interaction for electrons is isotropic, the distribution function of the random nuclear field has a simple Gaussian form:

$$F(\Omega_N) = \frac{1}{(\sqrt{\pi} \delta)^3} \exp \left( - \frac{\Omega_N^2}{\delta \sigma} \right), \tag{2}$$

where parameter $\delta$ determines the dispersion. Despite the continuous spin orientation, we neglect the dynamic nuclear spin polarization.

Additionally, we take into account the incoherent processes: trion recombination with the rate $\gamma$, electron and trion spin relaxations with the times $\tau_s$ and $\tau_s^T$, respectively, which are unrelated with the hyperfine interaction. These processes are shown in Fig. 2 and can be described theoretically in the density matrix formalism using the Lindblad superoperator [33, 35, 36]

$$\mathcal{L}\{\rho\} = \sum_i \gamma_i \left( O_i^\dagger O_i \rho + \rho O_i^\dagger O_i - 2 O_i \rho O_i^\dagger \right), \tag{3}$$

where $\rho$ is the density matrix of the four level system and the eight operators $O_i$ are taken from the set

$$\{ a_{\pm1/2}^\dagger a_{\mp1/2}, a_{\pm1/2}^\dagger a_{\mp1/2}^\dagger, (a_{+1/2}^\dagger a_{-1/2} + a_{-1/2}^\dagger a_{+1/2})/2, a_{\pm3/2}^\dagger a_{\mp3/2}, (a_{+3/2}^\dagger a_{-3/2} + a_{-3/2}^\dagger a_{+3/2})/2 \}$$

with the corresponding decay rates $\gamma_i$ from the set $\{ \gamma, 1/\tau_s, 1/(2\tau_s), 1/\tau_s, 1/(\tau_s^T), 1/(2\tau_s^T) \}$. The total spin dynamics is described by the quantum master equation

$$\dot{\rho}(t) = \frac{i}{\hbar} [\rho(t), \mathcal{H}] - \mathcal{L}\{\rho(t)\}, \tag{4}$$

where the dot denotes the time derivative.

The main focus of our paper is on the quantum dots, however the same model describes many other systems with the localized electrons: shallow donors in bulk semiconductors [28], electrons localized at the imperfections of quantum wells [37], and electrons in moiré potential of twisted transition metal dichalcogenides bilayers [38–41].

In the next section, we obtain the kinetic equation for the localized electron spin dynamics under weak measurements.

### III. DERIVATION OF KINETIC EQUATION

#### A. Absence of excitation

It is useful for the following to consider the spin dynamics in the absence of the incident light, $\mathcal{E}_\pm = 0$, when the trions are not excited. The electron spin is given by the trace of Pauli matrices with the density matrix:

$$S(t) \equiv \text{Tr}[\rho(t)\sigma]/2. \tag{5}$$

![Figure 2: Energy levels of the quantum dot and transitions between them. The two ground states are characterized by the electron spin $S_z = \pm 1/2$, and the two singlet trion states by the hole spin $J_z = \pm 3/2$. Absorption of $\sigma^+$ and $\sigma^-$ photons is shown by the blue and green wavy vertical arrows. The magenta arrows show the trion recombination. The blue and red double arrows show the electron and trion spin relaxations, respectively. Finally, the black double arrow shows the electron spin precession in the random nuclear field.](image)
averaged over the nuclear field distribution function \(2\) 

The equation \([28, 42]\) 

From Eq. (4) in this limit we obtain the standard Bloch equation \([28, 42]\) 

Thus, the electron spin dynamics in the absence of the excitation represents precession in the random nuclear field \(\Omega_N\) and relaxation with the time \(\tau_s\). In what follows we consider the experimentally relevant limit of \(\tau_s \gg 1/\delta\).

For the given initial electron spin orientation along the \(z\) axis, \(S(0) = S_0 e_z\), the solution of the Bloch equation averaged over the nuclear field distribution function \(2\) gives \([28]\)

This dependence is shown by the black curve in Fig. 3. The electron spin precession in the random nuclear field leads to the loss of the two thirds of the spin polarization during the time \(t \sim 1/\delta\). As a result, for \(1/\delta \ll t \ll \tau_s\) the electron spin polarization equals to

This result will be used below as a reference.

**B. Weak excitation**

Now let us describe the effect of the weak system excitation by the continuous light on the spin dynamics. For its description we use the perturbation theory in small parameters \(\mathcal{E}_\pm/\gamma\), which we assume to be real. We consider also the realistic limit of

\[
\gamma \gg 1/\tau_s^T \gg \delta \gg 1/\tau_s,
\]

when the trion recombination is the fastest process in the system (typically, \(\gamma \sim 1 \text{ ns}^{-1}\)) and the spin relaxation in the excited trion state is faster than in that in the ground state.

In zeroth order of the perturbation theory, the four density matrix elements \(\rho_{\uparrow/\downarrow,\uparrow/\downarrow}\) between the two ground states are nonzero. In the absence of excitation the equations for them yield Eq. (6).

In the first order of the perturbation theory, there are 8 equations for the density matrix elements between the two ground and the two excited quantum dots states. From (4) we obtain

\[
\dot{\rho}_{\uparrow,\uparrow} = -\left(\frac{3}{8\tau_s^T} + \gamma - i\omega_0\right)\rho_{\uparrow,\uparrow} + i\mathcal{E}_+\rho_{\uparrow,\uparrow}, \quad (10a)
\]

\[
\dot{\rho}_{\uparrow,\downarrow} = -\left(\frac{3}{8\tau_s^T} + \gamma - i\omega_0\right)\rho_{\uparrow,\downarrow} + i\mathcal{E}_-\rho_{\uparrow,\downarrow}. \quad (10b)
\]

The equations for \(\rho_{\downarrow,\uparrow}\) and \(\rho_{\downarrow,\downarrow}\) can be obtained from (10a) and (10b), respectively, by flipping the spins and exchanging \(\mathcal{E}_+\) and \(\mathcal{E}_-\). Four more equations for the off-diagonal density matrix elements can be obtained from the hermiticity of the density matrix. These equations describe the trion excitation, recombination and spin relaxation. In the adiabatic approximation valid when the trion recombination time \(1/\gamma\) is much shorter than the time scale of the spin dynamics in the ground state these density matrix elements oscillate as \(e^{\pm i\omega_0}\).

This allows us to find these 8 density matrix elements as functions of the ground state matrix elements \(\rho_{\uparrow/\downarrow,\uparrow/\downarrow}\).

Further, in the second order, there are 4 density matrix elements between the two excited trion states. Two of them read

\[
\dot{\rho}_{\uparrow,\uparrow} = -2\gamma\rho_{\uparrow,\uparrow} - \frac{\rho_{\uparrow,\uparrow} - \rho_{\downarrow,\downarrow}}{2\tau_s^T} + i\mathcal{E}_+(\rho_{\uparrow,\uparrow} - \rho_{\downarrow,\downarrow}), \quad (11a)
\]

\[
\dot{\rho}_{\downarrow,\downarrow} = -2\gamma\rho_{\downarrow,\downarrow} + i(\mathcal{E}_-\rho_{\downarrow,\downarrow} - \mathcal{E}_+\rho_{\uparrow,\downarrow}), \quad (11b)
\]

and the other two equations can be obtained by the spin flips and exchange of \(\mathcal{E}_+\) and \(\mathcal{E}_-\). Here the time derivatives should be set to zero, and using Eqs. (10) we find these matrix elements as functions of the density matrix of the ground state.

Finally, the equations of motion for the density matrix elements between the ground states together in zeroth and second orders read
and equation for \( \rho_{\downarrow,\downarrow} \) can be obtained from Eq. (12c) by the complex conjugation.

After substitution of the solution of Eqs. (10) and (11) in Eqs. (12) we obtain the equation for the spin dynamics in the second order in \( \tilde{\omega} \). It can be compactly written as

\[
\dot{S}(t) = \Omega_N \times S(t) - \frac{S(t)}{\tau_s} + \left( g - \frac{S_z(t)}{\tilde{\tau}} \right) e_z - 2\lambda (S_x(t) e_x + S_y(t) e_y) + \tilde{\Omega} e_z \times S(t),
\]

where the parameters \( g, \tilde{\tau}, \lambda, \) and \( \tilde{\Omega} \) represent, respectively, the spin generation rate, additional spin relaxation time, measurement strength, and additional spin precession frequency. They are all proportional to the second power of the incident light amplitude, \( \mathcal{E}_\pm \).

The spin generation rate is given by

\[
g = \frac{\mathcal{E}_+^2 - \mathcal{E}_-^2}{4\tau_s^2 [(\omega - \omega_0)^2 + \gamma^2]}.
\]

Naturally, it is proportional to the circular polarization of the incident light and is the largest at the trion resonance frequency, \( \omega = \omega_0 \). It is also proportional to the trion spin relaxation rate \( 1/\tau_s^2 \), because in the absence of the spin relaxation, the trion excitation and recombination does not produce the spin polarization [43, 44].

Also, excitation by circularly polarized light produces effective magnetic field along the optical axis

\[
\tilde{\Omega} = \frac{(\mathcal{E}_+^2 - \mathcal{E}_-^2)(\omega - \omega_0)}{\gamma^2 + (\omega - \omega_0)^2},
\]

due to the dynamic Stark effect [45, 46]. This field is an odd function of the laser detuning from the trion resonance, \( \omega - \omega_0 \).

Further, the trion excitation accelerates the longitudinal spin relaxation, which is described by the rate

\[
\frac{1}{\tilde{\tau}} = \frac{\mathcal{E}_+^2 + \mathcal{E}_-^2}{2\tau_s^2 [(\omega - \omega_0)^2 + \gamma^2]}.
\]

Clearly, it is proportional to the trion population and the trion spin relaxation rate, \( 1/\tau_s^2 \).

Finally and most importantly, the electron spin measurement by light leads to the quantum back action, which suppresses the coherence between the eigenstates of the observable \( \sigma_z \) [47–49]. This effect is often described phenomenologically [50], while in our model we obtain the microscopic expression for the measurement strength

\[
\lambda = \frac{(\mathcal{E}_+^2 + \mathcal{E}_-^2)\gamma}{2[\omega - \omega_0)^2 + \gamma^2]}.
\]

From Eq. (13) one can see that the corresponding term indeed suppresses the transverse spin components \( S_x \) and \( S_y \) in agreement with the general principles of the quantum mechanics. Microscopically, it is caused by the trion excitation and recombination, which mainly conserves the longitudinal spin component, but not the transverse ones [51].

The occupancies of the trion states read

\[
\rho_{\hat{\uparrow},\uparrow \uparrow} = \frac{\mathcal{E}_+^2}{2(\omega - \omega_0)^2 + \gamma^2},
\]

So the measurement strength equals to the total trion recombination rate [37, 52]

\[
\lambda = \gamma (\rho_{\hat{\uparrow},\uparrow \uparrow} + \rho_{\downarrow,\downarrow \downarrow}).
\]

For comparison, in the quantum dot-micropillar system in the strong coupling regime, the measurement strength equals to the sum of the trion decay rate and the photon escape rate from the microcavity [16]. This happens because in the steady state the total recombination rate of the excited states equals to the generation rate from the ground state, which unavoidably leads to the loss of the transverse spin components.

We note that the kinetic equation (13) allows one to describe both the steady state and the electron spin dynamics.

**IV. RESULTS**

To describe the quantum Zeno and anti-Zeno effects we consider the steady state of the system.

For simplicity we consider the resonant excitation, \( \omega = \omega_0 \), when the optical field \( \tilde{\Omega} \) vanishes. Also we note that the additional spin relaxation rate \( 1/\tilde{\tau} \) can be neglected in comparison with \( 1/\tau_s \). In this case the kinetic
equation (13) simplifies to
\[
\dot{S}(t) = \boldsymbol{\Omega}_N \times S(t) - \frac{S(t)}{\tau_s} + g e_z - 2\lambda(S_z(t)e_x + S_y(t)e_y).
\] (20)

If the hyperfine interaction were absent, \(\Omega_N = 0\), the steady state spin polarization would be \(S_0 = g\tau_s\) along the \(z\) axis. The ratio of steady state spin polarization \(\langle S_z \rangle\) and \(S_0\) reveals the role of the hyperfine interaction in the electron spin relaxation.

Generally, the steady state solution of Eq. (13) reads
\[
S_z = \frac{S_0[1 + 4\lambda\tau_s + (4\lambda^2 + \Omega_N^2)\cos^2(\theta)\tau_s^2]}{1 + 4\lambda\tau_s + (4\lambda^2 + \Omega_N^2)\tau_s^2 + 2\Omega_N^2\sin^2(\theta)\tau_s^2},
\] (21)

where \(\theta\) is the angle between \(\boldsymbol{\Omega}_N\) and the \(z\) axis, see the inset in Fig. 3. This expression should be averaged over the distribution function (2) to obtain the average spin polarization \(\langle S_z \rangle\).

Figure 4 shows the spin polarization \(\langle S_z \rangle\) as a function of the measurement strength \(\lambda\) (see Appendix A for the details on numerical averaging). One can see that this dependence is nonmonotonic: the spin polarization first decreases and then increases with increase of the measurement strength.

Qualitatively, this behaviour can be understood from Fig. 3. First of all, in the absence of the spin measurement, \(\lambda = 0\), the spin polarization equals to \(\langle S_z \rangle = S_0/3\), which corresponds to the long time limit in Fig. 3, see Eq. (8). The measurement of \(S_z\) projects the spin to the \(z\) axis during the time \(\sim 1/\lambda\). If this time is short (\(\lambda\) is large), this stabilizes the spin because of the initial quadratic decay in Fig. 3. As a result, the nuclear induced spin relaxation becomes inefficient and the steady state spin polarization approaches \(S_0\). This is the quantum Zeno effect.

Moreover, if the measurement time is of the order of \(1/\delta\), the measurements project the spin to the \(z\) axis at the minimum of the curve in Fig. 3, which cancels the spin revival at longer times to \(S_0/3\). As a result, the steady state spin polarization is much smaller than \(S_0\). The acceleration of the nuclear induced spin relaxation is the manifestation of the quantum anti-Zeno effect.

Below we describe the two regimes in more detail and obtain the corresponding analytic expressions for the spin polarization.

### A. Quantum Zeno effect

In the limit of \(\lambda \gg \delta\) the measurement strongly suppresses the transverse spin components, as can be seen from Eq. (20). As a result, the electron spin is almost parallel to the \(z\) axis, as illustrated in Fig. 4(c).

The transverse spin components appear in the first order in the random nuclear field and are given by
\[
S_x = \frac{\Omega_{N,y}S_z}{2\lambda}, \quad S_y = -\frac{\Omega_{N,z}S_z}{2\lambda}.
\] (22)

Then in the second order we obtain from Eq. (20)
\[
\dot{S}_z(t) = g - \frac{S_z(t)}{\tau_s^{\text{eff}}},
\] (23)

where the effective spin relaxation rate is [37]
\[
\frac{1}{\tau_s^{\text{eff}}} = \frac{1}{\tau_s} + \frac{\Omega_N^2 \sin^2(\theta)}{2\lambda}.
\] (24)

In the steady state the electron spin polarization equals to \(g\tau_s^{\text{eff}}\) in agreement with Eq. (21). Averaging it over the distribution function (2) we obtain the average spin polarization in the form
\[
\frac{\langle S_z \rangle}{S_0} = -\nu \text{Ei}(-\nu) \exp(\nu),
\] (25)

where \(\nu = 2\lambda/(\tau_s\delta^2)\) and \(\text{Ei}(x) = -\int_{-\infty}^{\infty} e^{-t}/t \, dt\) is the exponential integral function. This expression is shown by the blue dashed curve in Fig. 4(a). One can see, that it agrees with the numerical calculations very well at \(\lambda/\delta > 1\).

From Eq. (24) one can see that the measurement suppresses the nuclei induced spin relaxation due to the quantum Zeno effect. In particular, in the limit of \(\lambda \to \infty\) one has \(\tau_s^{\text{eff}} = \tau_s\), so the role of the hyperfine interaction is completely suppressed and the spin polarization \(\langle S_z \rangle\) equals to \(S_0\).
Thus the quantum Zeno effect can be used in realistic experimental conditions to increase the spin relaxation time. In the next subsection we describe the opposite quantum anti-Zeno effect, which allows one to accelerate the spin relaxation.

B. Quantum anti-Zeno effect

In the limit of $\lambda \ll \delta$ we obtain from Eq. (21)

$$S_z = \frac{S_0 \cos^2(\theta)}{1 + 2\tau_s \sin^2(\theta)}. \tag{26}$$

This expression can be averaged over the random nuclear fields analytically with the result

$$\langle S_z \rangle = \frac{1}{2\tau_s} \left[ 1 + \frac{2\lambda\tau_s}{\lambda\tau_s - \arctanh \left( \sqrt{\frac{2\lambda\tau_s}{1 + 2\lambda\tau_s}} \right) } - 1 \right]. \tag{27}$$

This dependence of the spin polarization on the measurement strength is shown in Fig. 4(a) by the red dotted curve.

To qualitatively understand this regime, we note that the electron spin dynamics is dominated by the spin precession in the random nuclear field in this case. Therefore, the spin is almost parallel to the direction of $\Omega_N$, which we denote as the $z'$ axis, see Fig. 4(b). From Eq. (20) we find that the dynamics of this component is described by

$$\dot{S}_{z'} = \frac{S_{z'}}{\tau_s} - 2\lambda S_{z'} \sin^2(\theta) + g \cos(\theta). \tag{28}$$

One can see that the spin generation along this axis is suppressed by the factor $\cos(\theta)$, while the measurement-induced relaxation of $S_x$ and $S_y$ contributes to the relaxation of $S_{z'}$ with the factor $\sin^2(\theta)$. This is because the measurement suppresses the transverse spin components, which leads to the small deviation of the average electron spin from the axis $z'$. Then the spin precession in the nuclear field leads to the additional spin relaxation. Acceleration of the spin relaxation, can be described by the effective spin relaxation rate

$$\frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_s} + 2\lambda \sin^2(\theta), \tag{29}$$

which clearly reveals the quantum anti-Zeno effect.

Eqs. (24) and (29) show that the spin relaxation time decreases with decrease of $\lambda$ at $\lambda \gg \delta$ and decreases with increase of $\lambda$ at $\lambda \ll \delta$. So the minimum in the spin relaxation time and the steady state spin polarization unavoidably takes place at the crossover from the quantum Zeno to anti-Zeno regime, see Fig. 4(a). Moreover, one can see that the analytical limiting Eqs. (25) and (27) together describe the spin polarization very well in the whole range of $\lambda$. The crossover between the two curves takes at $\lambda \sim \delta/2$, when the quantum anti-Zeno effect is the strongest.

V. DISCUSSION

In this work we considered the continuous spin measurement. It can be always presented as a sequence of weak pulsed measurements with the short repetition period $T_R \ll 1/\lambda$. Generally, the quantum back action of each pulse can be described using the Krauss operator [16, 49, 53]

$$K(s) = \left( \frac{2\eta}{\pi} \right)^{1/4} e^{-\eta(s-\pi/2)^2}, \tag{30}$$

where $\eta = 4\lambda T_R$ and $s$ is a possible outcome of the $S_z$ measurement. The measurement without postselection modifies the density matrix as [49]

$$\rho^{(\text{after})} = \int ds \, K(s) \rho^{(\text{before})} K(s). \tag{31}$$

This is equivalent to

$$S_z^{(\text{after})} = S_z^{(\text{before})}, \quad S_{x,y}^{(\text{after})} = (1 - P) S_{x,y}^{(\text{before})}, \tag{32}$$

In the same time, the spin measurement by resonant linearly polarized optical pulse modifies the spin as [32, 45]

$$S_z^{(\text{after})} = S_z^{(\text{before})}, \quad S_{x,y}^{(\text{after})} = (1 - P) S_{x,y}^{(\text{before})}, \tag{33}$$

where $P$ is the probability of the trion excitation. From the comparison with Eq. (32) one can see that $1 - P = e^{-\eta/2}$. Under the above assumption we obtain

$$\lambda = \frac{P}{2T_R}, \tag{34}$$

which establishes the relation between the power of the pulses, $P$, the repetition period, $T_R$, and the measurement strength $\lambda$. Thus our theory describes also the quantum Zeno and anti-Zeno effects under weak pulsed spin measurements.

Moreover, for the strong probe pulses, $P \lesssim 1$, our theory also qualitatively describes the Zeno effects. In particular, one can consider the $\pi$ pulses, which are described by $P = 1$. In this case, if the repetition period is short, $T_R \ll 1/\delta$, then $\lambda \gg \delta$, and the quantum Zeno effect takes place. This is in agreement with the initial quadratic decay of $\langle S_z \rangle$, shown in Fig. 3. By contrast, at $T_R \sim 1/\delta$ one has $\lambda \sim \delta$, so the quantum anti-Zeno effect takes place. Indeed, in this case the probe pulses project the spin to the $z$ axis in the minimum of the curve in Fig. 3, which accelerates the spin relaxation.

VI. CONCLUSION

To conclude, we have described the effect of the quantum measurement back action under continuous measurement and orientation of electron spins in quantum dots by elliptically polarized light. We have demonstrated that the non-Markovian electron spin dynamics
driven by the hyperfine interaction with the host lattice nuclear spins allows for both quantum Zeno and anti-Zeno effects. For the large light power the nuclear induced spin relaxation is suppressed, which leads to the increase of the steady state spin polarization due to the quantum Zeno effect. For moderate power of light the nuclear induced spin relaxation is accelerated, which results in the suppression of the spin polarization due to the quantum anti-Zeno effect. The theoretical predictions can be directly compared to the future experimental results for ensembles of self-assembled charged quantum dots.

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Appendix A: Numerical details

To average the dimensionless quantity $\langle S(\Omega_N \tau_s, \theta) \rangle = S_z/S_0$ over the distribution function (2) we use the Gauss-Laguerre quadrature to speed up the calculations. Then the average can be calculated as

$$\langle S \rangle = \frac{1}{\sqrt{\pi}} \int dy e^{-y} \sqrt{y} \int d\theta \sin(\theta) S(\sqrt{y} \tau_s \delta, \theta) \approx \frac{2}{\sqrt{\pi}} \sum_{i=1}^{N} w_i \mathbf{F}(y_i \tau_s \delta) \sqrt{y_i}, \quad (A1)$$

where $y = \Omega_N^2/\delta^2$.

$w_i$ and $y_i$ are the weights and the roots according to the Gauss-Laguerre quadrature scheme [54]. The averaging in Eq. (A2) is performed using the Simpson’s rule. In the calculation we use $N = 15$ and check that larger $N$ yields the same results.

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