Effect of unbalanced magnetic pull and hydraulic seal force on the vibration of large rotor-bearing systems

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Abstract. The influence of unbalanced magnetic pull (UMP) and hydraulic seal force on the vibration of large rotor-bearing systems is studied. The UMP caused by rotor eccentricity imposes important effects on rotating machinery, especially for large generators such as water turbine generator sets, because these machines operate above their first critical speed in some instances and are supported by oil film bearings. A magnetic stiffness matrix for studying the effects of the UMP is proposed. The magnetic stiffness matrix can be generated by decomposing the expression of air gap magnetic field energy. Two vibration models are constructed using the Lagrange equation. The difference between the two models lies in the boundary support condition: one has rigid support and the other has elastic bearing support. The influence of the magnetic stiffness and elastic support on the critical speed of the rotor is studied using Lyapunov nonlinear vibration stability theory. The vibration amplitude of the rotor is calculated, taking the magnetic stiffness and horizontal centrifugal force into account. The unbalanced hydraulic seal force is produced because of the asymmetry of seal clearance. This imbalance is one of the factors that causes self-excited vibration in rotating machinery, and is as important as the UMP for large water turbine generator sets. The rotor-bearing system is supported by an oil film journal bearing, whose characteristic also impose considerable influence on vibration. On the basis of the above-mentioned conditions, a three-dimensional finite element model of the rotating system that includes the oil film journal bearing is constructed. The effect of the UMP and unbalanced hydraulic seal force is considered in the construction, and studied in relation to the magnetic parameters, seal parameters, journal bearing stiffness, and outer diameter of the rotating machine critical speed. Conclusions may benefit the dynamic design and optimized operation of large rotating machinery.

1. Introduction

During the normal operation of a rotor-bearing system, the magnetic field around the rotor is uniform. That is, the air gap is equally distributed around the circumference between the rotor and stator, thereby generating balanced magnetic pull in the rotor. This balance is advantageous to rotor stability. However, some factors such as uneven air gaps, mass unbalance of the rotor, initial displacement of the shaft, and hydraulic unbalance force, may cause unbalanced magnetic pull (UMP). UMP was previously simulated using a linear spring, and its stiffness was approximately evaluated on the basis of experience. Because UMP points to the direction of rotor eccentricity, the stiffness value of the spring is negative. Substantial research has been done on UMP \cite{1-4}. Wan analyzed magnetic field changes, provided the expressions of gap magnetic conductivity and magnetic flux density, and
investigated rotor vibration caused by UMP\cite{5}. Guo et al.\cite{6} obtained UMP through the Maxwell stress integral over the exterior of a rotor or stator, and studied rotor vibration considering pull and mass unbalance force. Chen\cite{7} obtained the UMP by deducing the partial derivative of gap magnetic field energy, and defined the coefficient of the degree one term in the expression as the magnetic stiffness. The author also found that the effect of magnetic stiffness is similar to that of a journal bearing. Ma\cite{8} revealed the influence of bearing stiffness and UMP on the self-vibration of rotor-bearing systems. The rise in excitation current decreased natural frequency. Under a large UMP, the rotor moved to the stator and the thickness of the oil film in the journal bearing decreased, thereby correspondingly changing the dynamic characteristics of the bearing. Therefore, the dynamic characteristics of the journal bearing and rotor eccentricity should be investigated simultaneously. In constructing an eccentricity rotor vibration model, the effect of the elastic support of the journal bearing should be considered.

Seal structure is one of the factors that causes self-excited vibration in rotating machinery. In the current research, seal structure is simulated using an eight-parameter model and the Muszynska model, whereas most studies pay more attention to the Jeffcott rotor model. We analyze the influence of seal parameters on the dynamic characteristics of a system using stability theory and the numerical simulation method\cite{9-11}. The turbine of the system is supported by a water bearing on top, but no support is placed at the bottom. The support condition of the turbine differs from that of the Jeffcott rotor. The shaft system is also influenced by the UMP and elastic support of the oil film journal bearing. The structure and force features are highly complex, so that the current Jeffcott rotor model, which has rigid support, is insufficient.

So the objective of the paper is to propose a methodology to study effect of UMP and hydraulic seal force on the vibration of large rotor-bearing systems. We begin the investigation with a discussion of gap magnetic field energy, provide the matrix expression of magnetic stiffness, and establish dynamic differential equations under rigid and elastic support conditions using the Lagrange equation. The influence of the magnetic stiffness and elastic support on the critical speed of the rotor is studied using Lyapunov nonlinear stability theory. The vibration amplitude and track of the rotor are calculated by taking the magnetic stiffness and horizontal centrifugal force into account. On the basis of the foregoing calculations, a 3D finite element model (FEM) of the rotor-bearing system is constructed, in which the magnetic, mechanical, and hydraulic seal vibration sources are considered. The model is supported by three journal bearings. The parameter sensitivity of the magnetic, seal, and journal bearing stiffness, as well as that of the shaft outer diameter, to the critical speed is studied. Conclusions may benefit the dynamic design and optimized operation of large rotating machinery.

2. Stability analyses of nonlinear vibration

2.1. Dynamic differential equations.

Figure 1 shows that $O$ is the inner circle center of the stator, $O_r$ denotes the axis of the shaft, and $O$ and $S$ are the superpositions when the shaft is stationary. $e_0 = SG$ is the mass eccentricity of the rotor, $G$ is the center of gravity, $m_r$ represents the mass, and $e = OS$ denotes the circumgyrate eccentricity. Hence, the kinetic energy and elastic potential energy of the rotor can be expressed as

$$T = \frac{1}{2} m_r (x_G^2 + y_G^2) \quad (1)$$

$$V_e = \frac{1}{2} K_e (x^2 + y^2) = \frac{1}{2} K_e e^2 \quad (2)$$

where $x_G = x + e_0 \cos \omega t$, $y_G = y + e_0 \sin \omega t$.

The gap eccentricity of the generator rotor is shown in figure 2: $O_1$ is the center of the outer circle, $\delta$ denotes the gap, and $e = \sqrt{x^2 + y^2}$ represents the gap eccentricity, where $x, y$ are the eccentricities in coordinate axes $O_x$ and $O_y$, respectively.
Thus, the gap magnetic field energy of the rotor is \[ V_2 = \frac{R L'}{2} \int_0^{2\pi} \left( \sum_{n=1}^{\infty} A_n e^{i\alpha} \cos(\alpha - \beta) \left[ F_{sm} \cos(\omega t - p\alpha) + F_{jm} \cos(\omega t - p\alpha + \theta + \phi + \frac{\pi}{2}) \right] \right) d\alpha \] (3)

The phases of the magnetic potential of the stator and rotor are different. The rotor magnetic potential and gap magnetic derivative rotate synchronously because of rotor rotation. Thus, the function, such as the static state, cannot be calculated. As indicated by the values of electric potential vectors, the phase of the rotor magnetic potential yields an angle of \( \theta + \phi + \pi/2 \) compared with that of the stator. The physical meanings of the other parameters are presented in the nomenclature at the end of the paper.

Taking the first three terms of equation (3) yields

\[
V_2 = \frac{R L' A_0}{2} \int_0^{2\pi} \left(1 + \frac{x^2 + y^2}{\sigma^2} + \frac{x}{\sigma} \right) \cos \alpha + \frac{y}{\sigma} \sin \alpha + \frac{x^2 - y^2}{2\sigma^2} \cos 2\alpha + \frac{xy}{\sigma} \sin 2\alpha \right) \] (4)

where \( \sigma = k' \sigma_0 \), and equation (4) can be simplified as follows:

\[
V_2 = \frac{1}{2} \mathbf{u}' \mathbf{Ku} + \mathbf{u}' \mathbf{F} \] (5)

where \( \mathbf{u} = \{x, y\}' \), and \( \mathbf{K} \) is the magnetic stiffness matrix of the gap magnetic field:

\[
K_{11} = \frac{R L' A_0}{2\sigma^2} \int_0^{2\pi} (1 + \cos(2\alpha)) \left[ F_{sm} \cos(\omega t - p\alpha) + F_{jm} \cos(\omega t - p\alpha + \theta + \phi + \frac{\pi}{2}) \right]^2 d\alpha
\]

\[
K_{12} = K_{21} = \frac{R L' A_0}{2\sigma^2} \int_0^{2\pi} \sin(2\alpha) \left[ F_{sm} \cos(\omega t - p\alpha) + F_{jm} \cos(\omega t - p\alpha + \theta + \phi + \frac{\pi}{2}) \right] \] (6)

\[
K_{22} = \frac{R L' A_0}{2\sigma^2} \int_0^{2\pi} (1 - \cos(2\alpha)) \left[ F_{sm} \cos(\omega t - p\alpha) + F_{jm} \cos(\omega t - p\alpha + \theta + \phi + \frac{\pi}{2}) \right]^2 d\alpha
\]

\( \mathbf{K} \) is the vector of the gap magnetic field energy:

\[
\mathbf{K}_1 = \frac{R L' A_0}{2\sigma} \int_0^{2\pi} \cos \alpha \left[ F_{sm} \cos(\omega t - p\alpha) + F_{jm} \cos(\omega t - p\alpha + \theta + \phi + \frac{\pi}{2}) \right]^2 d\alpha
\]

\[
\mathbf{K}_2 = \frac{R L' A_0}{2\sigma} \int_0^{2\pi} \sin \alpha \left[ F_{sm} \cos(\omega t - p\alpha) + F_{jm} \cos(\omega t - p\alpha + \theta + \phi + \frac{\pi}{2}) \right]^2 d\alpha
\]

The potential energy is composed of elastic potential \( V_1 \) and gap magnetic field energy \( V_2 \):

\[
V = V_1 + V_2
\] (6)

The following equation can be deduced from the Lagrange equation:
is the slenderness ratio, denotes the displacement of the journal, is the polar angle, and are the reaction forces of the oil film in the radial and tangential directions, respectively:

\[
\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s} - \frac{\partial T}{\partial \dot{q}_s} - \frac{\partial V}{\partial \dot{q}_s}\right) = F
\]

(7)

Thus, the equation of rotor motion under rigid support is

\[
\begin{align*}
    m_r\ddot{x} + D_r\dot{x} + (K_x + K_{11})x + K_{12}y &= F_x - \bar{K}_1 \\
    m_r\ddot{y} + D_r\dot{y} + (K_y + K_{22})x + K_{12}y &= F_y - \bar{K}_2
\end{align*}
\]

(8)

in which \( F_x \) and \( F_y \) are the centrifugal forces in the horizontal \( x \) and \( y \) directions, respectively. Hence, the equation of rotor motion under elastic support is

\[
\begin{align*}
    m_r\ddot{x} + D_r\dot{x} + K_{11}x + K_{12}y + K_2(x_r - x_b) &= F_x - \bar{K}_1 \\
    m_r\ddot{y} + D_r\dot{y} + K_{22}y + K_2(y_r - y_b) &= F_y - \bar{K}_2 \\
    m_b\ddot{x}_b + \frac{K_r}{2}(x_b - x) &= -f_x \\
    m_b\ddot{y}_b + \frac{K_r}{2}(y_b - y) &= f_y
\end{align*}
\]

(9)

where \( f_x \) and \( f_y \) denote the oil film forces in the \( x \) and \( y \) directions, respectively:

\[
\begin{bmatrix}
    f_x \\
    f_y
\end{bmatrix} = \begin{bmatrix}
    -P_x \sin \psi + P_y \cos \psi \\
    P_x \cos \psi + P_y \sin \psi
\end{bmatrix} = \frac{1}{\sqrt{X_b^2 + Y_b^2}} \begin{bmatrix}
    -P_x - P_y \\
    P_y - P_x
\end{bmatrix} \begin{bmatrix}
    X_b \\
    Y_b
\end{bmatrix}
\]

(10)

\( P_x \) and \( P_y \) are the reaction forces of the oil film in the radial and tangential directions, respectively:

\[
\begin{align*}
    P_x &= \frac{\eta LR_s^2}{2c_b} \left(\frac{L}{R_b}\right)^2 (\omega - 2\dot{\psi})G_1(\varepsilon) + 2\varepsilon G_2(\varepsilon) \\
    P_y &= \frac{\eta LR_s^2}{2c_b} \left(\frac{L}{R_b}\right)^2 (\omega - 2\dot{\psi})G_3(\varepsilon) + 2\varepsilon G_4(\varepsilon)
\end{align*}
\]

(11)

where \( \frac{L}{R_b} \) is the slenderness ratio, \( \varepsilon = \sqrt{X_b^2 + Y_b^2}/c_b \) denotes the displacement of the journal, \( \varepsilon = \sqrt{X_b^2 + Y_b^2} \) represents the eccentricity ratio, \( \psi = \arctan \left(\frac{X_b}{Y_b}\right) \) is the polar angle, and

\[
\begin{align*}
    \sin \psi &= \frac{X_b}{\sqrt{X_b^2 + Y_b^2}}, \quad \cos \psi = \frac{Y_b}{\sqrt{X_b^2 + Y_b^2}}, \quad \dot{\psi} = \frac{X_b \dot{Y}_b + Y_b \dot{X}_b}{e^2}, \\
    \ddot{\psi} = \frac{X_b \ddot{Y}_b + Y_b \ddot{X}_b - X_b \dot{Y}_b - Y_b \dot{X}_b}{e^2}, \quad 2\varepsilon = \left(1 - \varepsilon^2\right)^{3/2} \\
    G_1(\varepsilon) &= \frac{2\varepsilon^2}{\left(1 - \varepsilon^2\right)^{3/2}}, \quad G_2(\varepsilon) = \frac{\pi (1 + 2\varepsilon^2)}{2(1 - \varepsilon^2)^{3/2}}, \quad G_3(\varepsilon) = \frac{\pi \varepsilon}{2(1 - \varepsilon^2)^{1/2}}, \quad G_4(\varepsilon) = \frac{2\varepsilon^2}{\left(1 - \varepsilon^2\right)^{3/2}}.
\end{align*}
\]

Coupling terms exist in the stiffness matrix because of the magnetic effect in a rigid support equation such as equation (8). Otherwise, the centrifugal forces in horizontal \( F_x \) and \( F_y \) are the nonlinear functions of vibration amplitude \( x \) or \( y \). Coupling terms are also found in an elastic support motion equation such as equation (9). Furthermore, oil film forces \( f_x \) and \( f_y \) are the nonlinear functions of the journal displacement. Hence, either equation (8) or (9) has strong nonlinear characteristics. Nonlinear vibration stability theory must be used to solve the journal displacement.

2.2. Dimensionless equations

The differential equations of dimensionless motion can be derived under rigid and elastic support:
\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x'' \\
y''
\end{bmatrix}
+ \begin{bmatrix}
\frac{D_x}{m_1\omega} & 0 \\
0 & \frac{D_x}{m_1\omega}
\end{bmatrix}
\begin{bmatrix}
x' \\
y'
\end{bmatrix}
+ \begin{bmatrix}
\frac{K_{11} + K_e}{m_1\omega^2} & \frac{K_{12}}{m_1\omega^2} \\
\frac{K_{21}}{m_2\omega^2} & \frac{K_{22} + K_e}{m_2\omega^2}
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
\frac{F_x}{m_1\delta_{1,\omega^2}} \\
\frac{F_y}{m_2\delta_{2,\omega^2}}
\end{bmatrix}
- \begin{bmatrix}
\frac{1}{m_1\delta_{1,\omega^2}} \\
\frac{1}{m_2\delta_{2,\omega^2}}
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\] (12)

\[
\begin{bmatrix}
x'' \\
y''
\end{bmatrix}
+ \begin{bmatrix}
\frac{D_x}{m_1\omega} & 0 & 0 & 0 \\
0 & \frac{D_x}{m_1\omega} & 0 & 0 \\
0 & 0 & \frac{D_x}{m_2\omega} & 0 \\
0 & 0 & 0 & \frac{D_x}{m_2\omega}
\end{bmatrix}
\begin{bmatrix}
x' \\
y' \\
x'' \\
y''
\end{bmatrix}
+ \begin{bmatrix}
\frac{K_{11} + K_e}{m_1\omega^2} & \frac{K_{12}}{m_1\omega^2} & -\frac{K_1\beta}{m_1\omega^2} & 0 \\
\frac{K_{21}}{m_2\omega^2} & \frac{K_{22} + K_e}{m_2\omega^2} & 0 & -\frac{K_1\beta}{m_2\omega^2} \\
-\frac{K_1\beta}{2m_1\omega^2} & 0 & \frac{K_1\beta}{2m_1\omega^2} \mp \xi F_x & \frac{K_1\beta}{2m_1\omega^2} \mp \xi F_x \\
0 & -\frac{K_1\beta}{2m_2\omega^2} & \frac{K_1\beta}{2m_2\omega^2} \mp \xi F_y & \frac{K_1\beta}{2m_2\omega^2} \mp \xi F_y
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
x' \\
y'
\end{bmatrix}
= \begin{bmatrix}
\frac{F_x}{m_1\delta_{1,\omega^2}} \\
\frac{F_y}{m_2\delta_{2,\omega^2}}
\end{bmatrix}
- \begin{bmatrix}
\frac{1}{m_1\delta_{1,\omega^2}} \\
\frac{1}{m_2\delta_{2,\omega^2}}
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\] (13)

2.3. Nonlinear stability theory

The acceleration and velocity are zero when the system is balanced. Substituting the values of acceleration and velocity into equation (12) and (13) yields the equilibrium point equations. Thus, the equilibrium point can be solved through iteration. According to Lyapunov nonlinear stability theory, the motion stability of a nonlinear system is determined approximately to the latent root of a degree one system. Expanding the status equations at the equilibrium point approximately yields the Jacobian matrixes.

\[
J = D f \left| \begin{bmatrix}
\frac{\partial f_1}{\partial x'} & \frac{\partial f_1}{\partial y'} \\
\frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\
\frac{\partial f_2}{\partial x'} & \frac{\partial f_2}{\partial y'} \\
\frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y}
\end{bmatrix}
\right|
\]

When all the eigenvalues of the Jacobian matrixes have negative real parts, the system is stable. When an eigenvalue of the Jacobian matrix has zero real parts, the system is critical. System balance is therefore disturbed.

2.4. Numerical simulation and analysis

The system parameters are presented in Table 1. The numerical calculation results are shown in Table 2. The table shows that both the magnetic and elastic support stiffness reduce the critical speed and stability of the system. The sizes of the four critical speeds follow the order \( \omega_{eq} > \omega_{eq}' > \omega_{es} > \omega_{es}' \). Therefore, the influence of the elastic support stiffness is greater than that of the magnetic stiffness. The former reduces the critical speed by 26% with the magnetic stiffness, whereas the latter decreases the critical speed by 31.6%. Moreover, the magnetic stiffness has less effect on the critical speed under elastic support.
The tracks of the journal center at the critical speed are shown in figure 3 and figure 4, which illustrates that the magnetic stiffness increases the journal center tracks at either support condition. The magnetic stiffness reduces the system stability, a result that agrees with the conclusion drawn for the calculation of rotation speed. The elastic support stiffness has a greater effect on the increase in rotor amplitude than does magnetic stiffness.

### Table 1. System parameters.

| Magnetic                        | Saturation | Inner power angle | Stator winding magnetic potential |
|--------------------------------|------------|-------------------|----------------------------------|
| Magnetic permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m | $k_n = 1.102$ | $\theta = 3.06^\circ$ | $F_{im} = 19210$ At |
| Stator inner radius $R_s = 6.24$ m | Rotor effective length $L' = 2.1$ m | Average gap $\delta_0 = 18$ mm | Rotor winding magnetic potential $F_{im} = 24214$ At |

| Seal                          | Axial flow rate | Seal clearance | Loss coefficient | Viscosity coefficient |
|-------------------------------|-----------------|----------------|-----------------|-----------------------|
| Seal length $l = 0.43$ m      | $v = 3.537$ m/s | $C = 2.5$ mm   | $\xi = 1.5$     | $\gamma = 1.3 \times 10^{-3}$ Pa·s |
| Seal radius $R = 2.925$ m     | Pressure drop $\Delta P = 0.5 \times 10^6$ Pa | Exponential coefficient $\lambda = 0.1$ | $n = 2.5$ | $b = 0.2$ | $\tau_0 = 0.5$ |

| Structure | Rotor mass $m_r = 600$ t | Turbine mass $m_t = 300$ t | Shaft radius $R = 0.65$ m | Shaft length $L = 13.6$ m | Rotate speed $\omega = 13.1$ rad/s |
| Journal bearing | Journal mass $m_b = 1000$ kg | Slenderness ratio $L/R_b = 1$ | Journal clearance $c_b = 0.25$ mm | Sommerfeld coefficient $S_0 = 0.001$ |

| Table 2. Numerical calculation results. |
|-----------------------------------------|
| Rigid support                          | Elastic support                     |
| With magnetic stiffness                | Without magnetic stiffness           | With magnetic stiffness | Without magnetic stiffness |
| \(\omega_{cg} = 71.17 \text{ rad/s}\) | \(\omega_{cg}' = 94.2 \text{ rad/s}\) | \(\omega_{ct} = 52.64 \text{ rad/s}\) | \(\omega_{ct}' = 64.37 \text{ rad/s}\) |

Figure 3. Tracks of the journal center at rigid support conditions

Figure 4. Tracks of the journal center at elastic support conditions

### 3. Sensitivity analyses of parameters

The magnetic, seal, and structure parameters of an actual hydro-electric unit are shown in Table 1. The critical speed of the shaft system depends not only on the shaft radius and bearing stiffness, but also on...
the magnetic and hydraulic parameters when the sources of magnetic, hydraulic, and machinery vibration are considered.

### 3.1 Magnetic parameters

Figure 5 and figure 6 show the critical speed curves with the magnetic parameters without seal, linear seal, and nonlinear seal conditions. The magnetic parameter is $R^2 L \Lambda_n / 2 \sigma^2$. The first critical speed reflects the modal shape of the turbine vibration, and the second speed reflects the modal shape of the rotor vibration. As shown in figure 5, the first critical speed decreases with increasing magnetic parameters under all three conditions. Either the linear or nonlinear seal decreases the first speed. Hence, the magnetic stiffness influences the vibration of the turbine even though it exists only at the rotor position. Figure 6 depicts the second speed curves with the magnetic stiffness. The second critical speed rapidly decreases with increasing magnetic parameters. The linear or nonlinear seal has no influence on the vibration of the rotor.

![Figure 5](image1.png)  
Figure 5. Influence of the electromagnetic parameter to first order.

![Figure 6](image2.png)  
Figure 6. Influence of the electromagnetic parameter to second order.

### 3.2 Seal parameters

The linear eight-parameter mode and the nonlinear Muszynska model were considered in this study. The seal parameters studied in this paper are axial flow rate, seal length, seal radius, and seal clearance. Figure 7–14 depict the first and second critical speed curves with the seal parameters.

![Figure 7](image3.png)  
Figure 7. Influence of the axial flow velocity to first order.

![Figure 8](image4.png)  
Figure 8. Influence of the axial flow velocity to second order.
As shown in the first speed values, the first critical speed decreases with the increase in axial flow rate, seal length, and seal radius regardless of whether the magnetic effect is considered. However, the seal clearance imposes the reverse effect. The increase in seal clearance is advantageous to turbine stability. However, a large clearance may cause flow leakage and decrease unit efficiency. Compared with the
linear seal, the nonlinear type causes the first critical speed to decrease more considerably, and the turbine is much easier to disturb.

As shown in the second speed values, all the seal parameters have no influence on the vibration of the rotor. This result agrees with the conclusion drawn from figure 6. However, the magnetic stiffness has an important influence on rotor vibration, thereby decreasing the second critical speed by 40%.

3.3. Journal bearing stiffness

For the journal bearing of the hydro-electric unit, the journal bearing stiffness changes to a large extent because of the change in the load direction and position of the journal center. Three journal bearings are incorporated into the model. In the sensitivity analysis, the change in bearing stiffness indicates that only one bearing stiffness changes, whereas that of the other bearings are kept constant. The first and second critical speed curves with the bearing stiffness are shown in Figure 15–20.

The value of the first-order critical speed shows that the water bearing significantly influences the first critical speed, whereas the upper and lower bearings have little influence without the magnetic or seal effect. The first critical speed decreases when the upper and lower bearings have a small stiffness under the magnetic condition. The values of the second critical speed illustrate that the water bearing stiffness does not influence the critical second speed regardless of whether the magnetic effect is considered. Furthermore, the increase in the journal bearing stiffness cannot effectively prevent the decrease in the second critical speed because of the magnetic effect.
3.4. Shaft outer radius

The change in the outer radius of the shaft indicates that the outer and inner radii change synchronously, and the thickness is kept constant. Figure 21-22 show the first and second critical speed curves with the shaft outer radius. Both critical speeds increase with rising shaft outer radius. In addition, the increase in the shaft outer radius effectively prevents the decrease in the second critical speed because of the magnetic effect. The second critical speed decreases by 42% because of the magnetic stiffness, at a shaft outer radius of 0.65 m. At a radius of up to 0.75 m, the second critical speed decreases by only 22% (compared with the value obtained at a radius of 0.65 m without the magnetic and seal effects).

4. Conclusion

The magnetic stiffness matrix is introduced to express the energy of the air gap magnetic field. Two vibration models are constructed using the Lagrange equation. The difference of the two models lies in the boundary support condition: one has rigid support and the other has elastic bearing support. The influence of magnetic stiffness and elastic support on rotor critical speed is studied using Lyapunov nonlinear vibration stability theory. The vibration amplitude of the rotor is calculated, in which the magnetic stiffness and horizontal eccentricity force are considered. On the basis of the aforementioned conditions, we construct a 3D FEM model of the shaft system, in which the sources of magnetic, mechanical, and hydraulic seal vibration are considered. The model is supported by three journal
bearings. The parameter sensitivity of the magnetic, seal, and journal stiffness, as well as that of the outer diameter, to the critical speed is studied. The core research conclusions drawn are summarized as follows:

(1) Both magnetic stiffness and elastic support reduce critical speed and system stability, with elastic support exerting a more important influence. Both these factors increase rotor vibration amplitude. (2) The increase in shaft stiffness and journal bearing slenderness ratio is advantageous to system stability, but the increase in magnetic parameters and journal clearance decreases critical speed. The system is also disturbed. (3) Even when magnetic stiffness is applied at the rotor position, it influences turbine vibration. However, the seal imposes no influence on rotor vibration. The increase in axial flow rate, seal length, and seal radius presents disadvantages to turbine stability. Nevertheless, the increase in seal gap is beneficial for turbine vibration. (4) First-order critical speed is determined by water bearing stiffness. The increase in journal guide bearing stiffness cannot prevent the decrease in second-order critical speed, thereby causing magnetic stiffness. (5) The increase in the outer radius of the shaft benefits the system, and especially prevents the decrease in second-order critical speed.

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