Adhering 0-branes to 6-branes and 8-branes

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Abstract

A Yang-Mills solution is constructed on $T^6$ which corresponds to a brane configuration composed purely of 0-branes and 6-branes. This configuration breaks all supersymmetries and has an energy greater than the sum of the energies of its components; nonetheless, the configuration is stable classically, at least to quadratic order. An analogous construction is also given for a system of 0-branes and 8-branes on $T^8$. These constructions may prove to be useful for describing 6-branes and 8-branes in M(atrix) theory.
1 Introduction

In the last several years, the discovery that p-brane solitons in string theory can be simply described in terms of Dirichlet membranes [1] has led to a series of remarkable advances in our understanding of string theory. Among other things, D-branes have been used to construct configurations analogous to black holes [2, 3], providing a framework in which one can begin to use string theory to address unresolved questions about information and entropy in black holes.

One of the most important features of D-branes which has been exploited in the recent developments is the tendency of D-branes of various types to bind together into supersymmetric BPS saturated states. Such states are generally either “truly” bound, in which case the energy of the bound state is less than the sum of the energies of the constituent D-branes, or “marginally bound”, in which case the energy of the bound state is equal to the sum of the energies of the constituents. A variety of systems of intersecting D-branes and D-branes at angles have been considered, and their bound states studied. (For a review of some of these developments, see [4].)

In this paper we consider a configuration composed purely of 0-branes and 6-branes. In general, a pointlike 0-brane placed on or near a 6-brane gives rise to a configuration with no supersymmetry. The 0-brane and 6-brane tend to repel one another. It might be expected, therefore, that one could not find a stable configuration comprised only of 0-branes and 6-branes. Nonetheless, we find that by describing a set of four 0-branes which have been smeared out over four 6-branes wrapped on $T^6$ in terms of a gauge field with constant curvature, it is possible to construct a configuration with only 0-branes and 6-branes which satisfies the classical equations of motion and which is classically stable to quadratic order. This construction is related through T-duality to a system of four 3-branes wrapped diagonally around a torus in such a way that each pair of 3-branes is coincident along a single direction. Such a configuration is similar to a system of 3-branes used in [5] to study black holes in four dimensions. It was recently shown that a black hole supergravity solution exists with only 0-brane and 6-brane charges [6, 7]. Presumably at large distances
the brane configuration described here would look precisely like such a black hole.

In Section 2, we review constructions of 0-branes bound to 2-branes and 4-branes. In Section 3 we describe the construction of the system of 0-branes and 6-branes. This construction is carried out using methods completely analogous to those used in the lower dimensional examples in Section 2. In Section 4 we give a brief description of an analogous construction for a system of 0-branes and 8-branes.

A note on conventions: throughout this paper we normalize energies so that in any given configuration the brane of largest dimension has energy equal to its volume. Thus, for example, after compactification on a $T^2$ with unit volume the energy of a 2-brane is 1 and the energy of a 0-brane is $4\pi^2\alpha'$.

2 \hspace{1em} 0-branes on 2-branes and 4-branes

In this section we briefly review the gauge theory descriptions of bound states of 0-branes with 2-branes and 4-branes on tori. For more details see, for example, [4, 8, 9]. (Note that the binding of 0-branes with 0-branes is significantly more subtle [10].) A system of $N$ coincident Dirichlet $(2p)$-branes is described in the low-energy regime by a $U(N)$ super Yang-Mills theory in $2p+1$ dimensions, which can be described as the dimensional reduction of $N = 1$ 10D SYM [11]. In the Yang-Mills theory, a 0-brane attached to the $(2p)$-branes is described by a configuration of the gauge fields with a unit of topological charge proportional to the integral of $F^{\wedge p} = F \wedge \cdots \wedge F$ [12, 13].

2.1 \hspace{1em} 0-branes on 2-branes

A $U(N)$ gauge field in $2 + 1$ dimensions with a total flux of $\text{Tr} \int F = 2\pi k$ corresponds to a system of $k$ 0-branes bound to $N$ 2-branes. Generally, the energy of such a configuration is minimized when the flux is distributed uniformly over the surface of the 2-branes. This is intuitively clear from the relative scaling of the Yang-Mills energy $\int F^2$ and the 0-brane charge $F$. For example, if the size of the 0-brane were reduced by a factor of 2, this would
correspond to a doubling of the flux, and also to a doubling of the integrated Yang-Mills energy.

A particularly simple example of a bound 0-brane and 2-brane is a configuration on a torus $T^2$ with sides of unit length and flux $F = 2\pi$. This can be described explicitly in terms of a $U(1)$ bundle over the torus with first Chern class $C_1 = 1$ and connection

$$A_2(x_1, x_2) = 2\pi x_1.$$ 

This choice of connection can be associated with overlap functions describing the bundle which are trivial in the $x_2$ direction and given by $\Omega_1(x_2) = \exp(2\pi ix_2)$ in the $x_1$ direction. With this choice of connection, the bound state configuration of a 0-brane and a 2-brane can be related through T-duality in the $x_2$ direction to a 1-brane which is diagonally wound on the dual torus. Because the boundary condition in the $x_2$ direction is trivial, we can use an explicit description of T-duality through the relation $X_2 = i\partial_2 + 2\pi \alpha' A_2$ \[14, 15\]. The dual torus has dimensions $(1, 4\pi^2 \alpha')$, and the configuration of the diagonal 1-brane is described by the equation

$$X_2 = 2\pi \alpha' A_2 = 4\pi^2 \alpha' x_1.$$ 

In an analogous fashion, a bound state of $k$ 0-branes and $N$ 2-branes can be described as a configuration in $U(N)$ gauge theory on the torus with constant flux $2\pi k I/N$.

Although the bound 0+2 state can be described in Yang-Mills theory, for a correct calculation of the energy of such a configuration it is necessary to use the Born-Infeld action, which is described for an abelian theory by (up to an overall constant)

$$S = \sqrt{-\det(\eta_{\mu\nu} + 2\pi \alpha' F_{\mu\nu})}.$$ 

A complete definition of the nonabelian Born-Infeld action has not yet been given. For configurations in which all components of the field strength commute, however, it is sufficient to take the above action and trace over the gauge group. (for recent discussions of the problems when the fields do not commute, see \[16, 9\]). The resulting energy when the fields commute is given by

$$E = \text{Tr} \sqrt{1 + 4\pi^2 \alpha'^2 F^2}.$$
The term of order $\alpha'^2$ in an expansion of this expression gives the Yang-Mills contribution to the energy. For bound states of $k$ 0-branes and $N$ 2-branes, the Born-Infeld energy is

$$E = \sqrt{N^2 + (4\pi^2\alpha'k)^2}$$

which is precisely the BPS bound on the energy of such a bound state. This energy is less than the sum of the separate 0-brane and 2-brane energies so that the state is truly bound. This energy is of course also equal to the length of the dual diagonally wrapped 1-brane.

### 2.2 0-branes on 4-branes

Now let us consider a configuration of $k$ 0-branes bound to $N$ 4-branes. As discussed in \cite{13, 17} such a configuration corresponds to a $U(N)$ instanton with instanton number

$$k = \frac{1}{8\pi^2} \int \text{Tr} (F \wedge F) = C_2 - \frac{1}{2} C_1^2.$$

Unlike the previous situation, a 0-brane on a 4-brane can be contracted to a point without increasing or decreasing its energy. This follows from the fact that the Yang-Mills energy $F^2$ and the instanton number $F \wedge F$ both scale quadratically in $F$. In fact, the instanton can be shrunk to a point and then the 0-brane can be removed from the 4-brane without changing the energy of the system. This corresponds to the fact that the 0-brane and 4-brane are marginally bound.

For a fixed instanton number $k$ on a compact manifold, the space of gauge field configurations which minimize the Yang-Mills energy can be associated with the moduli space of self-dual (or anti-self-dual, depending upon the sign of $k$) fields $F = \pm \ast F$. The usual way of seeing this relation is to note that $F^2 = F_+^2 + F_-^2$ where $F_\pm$ are the self-dual and anti-self-dual parts of $F$, while $F \wedge F = F_+^2 - F_-^2$. There is another argument for this conclusion, however, which generalizes more naturally to higher dimensions, and which we now describe briefly.

If we are considering configurations on a compact manifold such as $T^4$, subject to the constraint that $\int F = 0$ on all 2-cycles, while the instanton number is fixed to be $k$, then
any gauge field configuration which satisfies

\[ \frac{\delta}{\delta F_{\mu\nu}(x)} \left( \int d^4x \, F^2_{\mu\nu} - \lambda \left( \frac{1}{8\pi^2} \int d^4x \, F^\wedge F - k \right) \right), \]

where \( \lambda \) is a constant Lagrange multiplier, must satisfy the Yang-Mills equations. Thus, any solution of \( F = \lambda * F \) with \( \lambda \) a constant must be a solution of the Yang-Mills equations. Since \( * * F = F \), this relation can only hold for \( \lambda = \pm 1 \), giving the self-dual and anti-self-dual conditions respectively.

An explicit example of a \( 0 + 4 \) configuration with \( N = k = 2 \) on \( T^4 \) is given by a \( U(2) \) bundle with \( C_2 = 2 \) with connection

\[
\begin{align*}
A_1 &= 0 \\
A_2 &= 2\pi x_1 \tau_3 \\
A_3 &= 0 \\
A_4 &= 2\pi x_3 \tau_3
\end{align*}
\]

where

\[
\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

is the usual Pauli matrix. The field strength is given by

\[ F_{12} = F_{34} = 2\pi \tau_3 \]

and is self-dual.

Describing \( A \) as a connection on a bundle with trivial boundary conditions in directions 2 and 4, we can T-dualize in those directions, and we arrive at a system comprising a pair of 2-branes wrapped on \( T^4 \). The embeddings of these 2-branes are given by

\[
\begin{align*}
X_2(x_1, x_3) &= \pm 4\pi^2 \alpha' x_1 \\
X_4(x_1, x_3) &= \pm 4\pi^2 \alpha' x_3
\end{align*}
\]

As discussed in \[18\], this is a configuration of 2-branes intersecting at angles which preserves \( 1/4 \) of the total supersymmetry in the system. One easy way to see this is that if we scale
the original dimensions $L_2, L_4$ of the torus so that the dual torus has all dimensions of equal length then the coefficients $4\pi^2\alpha'$ in the above embedding go to 1. In this case, the 2-branes are perpendicular and are in a 1/4 supersymmetric configuration.

The preservation of supersymmetry in a system of 2-branes intersecting at angles is equivalent to the (anti)-self-duality condition of the dual gauge fields \[18\]. When this condition is satisfied, as in the system just described, there are no tachyonic instabilities in the string theory in the background of the intersecting 2-branes. This is equivalent to the fact that the corresponding gauge theory background is stable and has no negative eigenvalues in the spectrum of fluctuations.

To determine the energy of this system we can again use the Born-Infeld action since the curvatures commute. The energy of the system is given by

$$E = \text{Tr} \sqrt{\det(\delta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} = 2\sqrt{(1 + 4\pi^2\alpha'^2 F_{12}^2)(1 + 4\pi^2\alpha'^2 F_{34}^2)} = 2(1 + 16\pi^4\alpha'^2)$$

which is precisely the energy of two 4-branes and two 0-branes when all the branes are separated to large distances. Thus, this indeed corresponds to a marginally bound state. Note that the energy is also equal to the sum of the areas of the two T-dual 2-branes.

3 0-branes on 6-branes

Now that we have reviewed the binding of 0-branes to 2-branes and 4-branes, let us consider the case of 0-branes and 6-branes. As pointed out in \[4\], it is not possible in general to bind a 0-brane to a 6-brane without putting extra energy into the system. Both the short-range and long-range forces between 0-branes and 6-branes are repulsive. (For discussions of 0-brane/6-brane scattering, see \[19, 20\]). In fact, imagine that we could attach a 0-brane to \(N\) infinite 6-branes by producing a gauge configuration with

$$\frac{1}{48\pi^3} \int \text{Tr} (F \wedge F \wedge F) = 1$$
with vanishing $\int \text{Tr} (F \wedge F)$ and $\int \text{Tr} F$, corresponding to an absence of 2-branes and 4-branes. By scaling

$$\tilde{F}(x) = \rho^2 F(\rho x)$$

we would get a new configuration with the same 0-brane charge but with Yang-Mills energy $\tilde{E} = E/\rho^2$. Thus, by taking $\rho \to \infty$, the 0-brane can be shrunk to a point while decreasing the total energy of the system. This corresponds to the fact that generically a 0-brane on a 6-brane will shrink to a point and then will be pushed off the 6-brane.

In spite of these considerations, however, we now proceed to construct an explicit gauge field configuration on $T^6$ which describes a system of four 0-branes and four 6-branes. This configuration satisfies the classical equations of motion and is classically stable, at least to quadratic order. We begin by generalizing the argument given in section 2.2 for solutions of the Yang-Mills equations with fixed topology. Imagine that we have a gauge field on a bundle over $T^6$ with vanishing first and second Chern classes, but with 0-brane charge $k$. Any gauge field configuration which satisfies for some fixed value of $\lambda$ the equation

$$\frac{\delta}{\delta F_{\mu\nu}(x)} \left( \int d^6 x \ F_{\mu\nu}^2 - \lambda \left( \frac{1}{48\pi^3} \int d^6 x \ F \wedge F \wedge F - k \right) \right)$$

must satisfy the Yang-Mills equations. Thus, we can construct moduli spaces of solutions to the Yang-Mills equations by finding solutions to the equation

$$F = \lambda \ast (F \wedge F). \quad (1)$$

We will now proceed to explicitly construct such a solution.

We choose the following connection on $T^6$

$$A_1 = 0 \quad A_2 = 2\pi x_1 \mu_1$$
$$A_3 = 0 \quad A_4 = 2\pi x_3 \mu_2$$
$$A_5 = 0 \quad A_6 = 2\pi x_5 \mu_3$$
where \( \mu_i \) are the following generators of the \( U(4) \) algebra

\[
\begin{align*}
\mu_1 &= \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix} \\
\mu_2 &= \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \\
\mu_3 &= \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\end{align*}
\]

These generators have the properties that

\[
\begin{align*}
\text{Tr} \, \mu_i &= 0 \\
\text{Tr} \, \mu_i \mu_j &= 0, \text{ for } i \neq j \\
\mu_i \mu_j &= |\varepsilon_{ijk}| \mu_k.
\end{align*}
\] (2)

The field strength associated with this connection is

\[
F_{12} = 2 \pi \mu_1 \quad F_{34} = 2 \pi \mu_2 \quad F_{56} = 2 \pi \mu_3
\] (3)

From the above properties, we can easily see that

\[
\int d^4x \text{ Tr} \, (F \wedge F) = 0
\]

around any 4-cycle on \( T^6 \) and that

\[
\int d^2x \text{ Tr} \, F = 0
\]

around any 2-cycle, while

\[
\int d^6x \text{ Tr} \, (F \wedge F \wedge F) = 4 \cdot 48 \pi^3
\]

corresponding to a charge of four 0-branes. Furthermore, as a result of the last relation in (4), the field strength satisfies (3) and therefore satisfies the Yang-Mills equations.
By choosing boundary overlap functions defining the bundle which are trivial in directions 2, 4, and 6, we can apply T-duality in those three directions, and we get a resulting configuration of four 3-branes wrapped on the dual $T^6$. If we choose the length of dimensions 2, 4 and 6 in the original torus to be $L = 4\pi^2\alpha'$ then the dual torus has unit length in all directions. The four 3-branes wrapped on this torus are described by the equations

$$X_2^{(1)}(x_1, x_3, x_5) = x_1 \quad X_4^{(1)}(x_1, x_3, x_5) = -x_3 \quad X_6^{(1)}(x_1, x_3, x_5) = x_3$$

$$X_2^{(2)}(x_1, x_3, x_5) = x_1 \quad X_4^{(2)}(x_1, x_3, x_5) = -x_3 \quad X_6^{(2)}(x_1, x_3, x_5) = -x_5$$

$$X_2^{(3)}(x_1, x_3, x_5) = -x_1 \quad X_4^{(3)}(x_1, x_3, x_5) = -x_3 \quad X_6^{(3)}(x_1, x_3, x_5) = x_5$$

$$X_2^{(4)}(x_1, x_3, x_5) = -x_1 \quad X_4^{(4)}(x_1, x_3, x_5) = x_3 \quad X_6^{(4)}(x_1, x_3, x_5) = -x_5$$

These 3-branes are all wrapped diagonally and each has volume $2^{3/2}$. The 3-branes are in a configuration which breaks all supersymmetries. This can be verified directly; however, a simple way to see this is to observe that under a coordinate transformation

$$\tilde{x}_1 = x_1 + x_2 \quad \tilde{x}_3 = x_3 + x_4 \quad \tilde{x}_5 = x_5 + x_6$$

$$\tilde{x}_2 = x_1 - x_2 \quad \tilde{x}_4 = x_3 - x_4 \quad \tilde{x}_6 = x_5 - x_6$$

the 3-branes are wrapped parallel to the sets of coordinates

$$(\tilde{1}\tilde{3}\tilde{5}), (\tilde{1}\tilde{4}\tilde{6}), (\tilde{2}\tilde{4}\tilde{5}), (\tilde{2}\tilde{3}\tilde{6})$$

This type of configuration of intersecting 3-branes was previously considered in where a similar configuration was used to study black holes in 4D. As discussed in that paper, for a system of intersecting 3-branes of this type, for half of the 16 choices of orientations for the 3-branes the configuration is 1/8 supersymmetric and for the other half all supersymmetries are broken. The configuration we have constructed here corresponds to a choice of orientations which breaks all supersymmetries. Let us briefly review the argument for the breaking of supersymmetry in this situation. A priori, each brane should break 1/2 of the supersymmetry. However, the conditions on an unbroken supersymmetry given by the four branes are

$$\tilde{\epsilon} = M_1 \epsilon = \tilde{\Gamma}_0 \tilde{\Gamma}_1 \tilde{\Gamma}_3 \tilde{\Gamma}_5 \epsilon = M_2 \epsilon = \tilde{\Gamma}_0 \tilde{\Gamma}_1 \tilde{\Gamma}_4 \tilde{\Gamma}_6 \epsilon$$
\[ M_3 \epsilon = \hat{\Gamma}_0 \hat{\Gamma}_2 \hat{\Gamma}_4 \hat{\Gamma}_5 \epsilon = \quad M_4 \epsilon = \hat{\Gamma}_0 \hat{\Gamma}_2 \hat{\Gamma}_3 \hat{\Gamma}_6 \epsilon \]

The matrices \( M_i \) satisfy the relation

\[(M_1)^{-1} M_2 = -(M_3)^{-1} M_4,\]

which guarantees that all supersymmetries must be broken. One might hope that by simply changing the orientation of one of the branes one could reach a supersymmetric configuration; however, in addition to introducing 2-brane and 4-branes charges into the system, this would invert the orientation of one of the 6-branes in the original configuration, making it impossible to describe this system in terms of a gauge theory on the 6-brane world-volume.

Since all supersymmetries are broken in this configuration we expect that the energy should exceed the minimal BPS energy for a 0 + 6 system. Indeed, to determine the energy of this configuration we can again use the Born-Infeld formula appropriate for diagonal field strengths. The total energy of the configuration is

\[ E = (4 \pi^2 \alpha')^3 4 \sqrt{1 + 1} = 8 \sqrt{2} (4 \pi^2 \alpha')^3 \]

since the fluxes are all equal to \((2 \pi \alpha')^{-1}\) while the volume of \(T^6\) is \((4 \pi^2 \alpha')^3\) with the sides of the torus as above. On the other hand, the energy of four 0-branes and four 6-branes if all are separate is given by

\[ E_{\text{sep}} = 8 (4 \pi^2 \alpha')^3 \]

(where, again, we have normalized 6-branes to have an energy equal to their volume). Since the separated energy is lower than the energy in the combined configuration, this state is not a bound state in the standard terminology. However, the 0-branes have nonetheless adhered to the 6-branes in a relatively stable fashion.

We can also consider the energy in the dual picture. In this case the total volume contained in the diagonally wrapped branes is \( E = 8 \sqrt{2} \) since there are four branes each with volume \(2 \sqrt{2}\). The brane configuration can be characterized by the 3-brane charges along all homology 3-cycles. The charges in all directions other than the 135 and 246 cycles cancel, so that the configuration has the same charges as a system of four branes wrapped
around the 135 cycle and four branes wrapped around the 246 cycle. Such branes would have unit volume and so the total volume of such a perpendicular configuration would be 8, less than the volume of the diagonally wrapped branes.

The configuration we have constructed here is stable, at least to quadratic order, with respect to fluctuations in the gauge field. From the string point of view in the dual 3-brane picture this follows from the fact that the strings stretching between each pair of branes are like strings stretching between two perpendicular 2-branes, since each pair of 3-branes has a single direction in which they are coincident. Thus, there are no tachyonic instabilities in the string theory spectrum around this background. Just as in the 4D case, such an instability would correspond to an unstable direction in the original gauge theory description. Although there are no unstable directions at quadratic order, there are flat directions corresponding to parameters in the moduli space of solutions to $F = \lambda \ast (F \wedge F)$. In the dual 3-brane picture these flat directions correspond to the massless fields in the string spectrum arising from strings stretching between pairs of 3-branes.

4 0-branes on 8-branes

We now briefly describe an analogous construction for a system of eight 0-branes and eight 8-branes on $T^8$. Just as for the 6-brane, a 0-brane attached to an 8-brane will generically contract to a point. This follows from the relative scaling of $F^2$ and $F \wedge F \wedge F \wedge F$. We would expect a solution of the classical Yang-Mills equations corresponding to an adhered state to satisfy the relation $F = \lambda \ast (F \wedge F \wedge F)$. Indeed, considering a system of eight 0-branes and eight 8-branes, such a configuration can be constructed in terms of a constant curvature connection. The four independent components of $F$ can be taken to be proportional to the matrices

$$F_{12} = \frac{1}{2\pi\alpha'} \text{Diag}(1, 1, 1, 1, -1, -1, -1, -1)$$
$$F_{34} = \frac{1}{2\pi\alpha'} \text{Diag}(1, 1, -1, -1, 1, 1, -1, -1)$$
\[
F_{56} = \frac{1}{2\pi\alpha'} \text{Diag}(1, -1, 1, -1, 1, 1, -1)
\]
\[
F_{78} = \frac{1}{2\pi\alpha'} \text{Diag}(1, -1, 1, -1, 1, 1, -1)
\]

Scaling the dimensions of directions 2, 4, 6 and 8 so that after T-duality in those directions we get a $T^8$ with all sides of unit length, we see that the dual configuration to this field strength corresponds to a system of eight 4-branes wrapped diagonally on $T^8$. These 4-branes are arranged so that every pair is perpendicular in either four or eight dimensions. This configuration again breaks all the supersymmetries of the theory.

The energy of this configuration is precisely twice the energy that the 0-branes and 8-branes would have if they were completely separated. This can be seen easily in the dual picture, where we have eight 4-branes with volume $4 = (\sqrt{2})^4$. These 4-branes are equivalent in homology to a system of eight 4-branes wrapped in dimensions 1357 and eight 4-branes wrapped in dimensions 2468, which would each have a unit volume. However, as for the 0 + 6 system, this configuration is stable classically, at least to quadratic order.

A system of separated 0-branes and 8-branes can preserve some supersymmetry [4], unlike the situation with 0-branes and 6-branes. In fact, it is believed that 0-branes and 8-branes can form marginally bound states. Nonetheless, in view of the fact that 0-branes tend to become pointlike when embedded in infinite 8-branes, at least at the level of Yang-Mills or Born-Infeld theory, it would be interesting to study configurations of these objects further. Recently eight dimensional Yang-Mills theory has been discussed in a related context [21]. A discussion of 0-branes on 8-branes was also given in [22], and bound states of 0-branes and 8-branes in type I’ theory were described in [23]. In order to achieve a full understanding of bound states of 0-branes and 8-branes in type II string theory it will probably also be necessary to incorporate some aspect of the phenomenon in which a 0-brane passing through an 8-brane produces an extra string [24, 25, 26].
5 Conclusions

We have constructed a configuration of four 0-branes and four 6-branes which have adhered together to form a state with no classical instability at quadratic order. This configuration is dual to a system of four 3-branes wrapped diagonally on $T^6$. The state breaks all supersymmetry. We also constructed an analogous configuration of eight 0-branes and eight 8-branes on $T^8$ which is dual to a system of eight intersecting 4-branes.

One application of the construction described here is to dualize these configurations to get constructions of the 6-brane and 8-brane in M(atrix) theory along the lines of known constructions of 2-branes and 4-branes [27, 13, 28]. Suggestions related to the construction of 6-branes or 8-branes in M(atrix) theory were also discussed recently in [28, 24, 22]. There are a number of tricky issues related to the interpretation of these objects; nonetheless, having an explicit construction of such a configuration might help to understand some of the puzzling aspects of these systems.

Since the configurations which have been described here break supersymmetry and have extra binding energy beyond the energy of their constituents, they presumably do not correspond to truly stable brane configurations. However, because they are stable classically at quadratic order, they may correspond to some kind of long-lived resonances composed of 0-branes and 6-branes or 8-branes. Such metastable states are expected from the supergravity point of view in the $0 + 6$ case when the charges are large [7], but the existence of the configurations constructed here indicates that perhaps a long-lived configuration of 0-branes and 6-branes can be found even for small numbers of branes. It would be interesting to study further the effects of quantum corrections on these configurations. It would also be interesting to understand better the classical moduli spaces of Yang-Mills solutions in which these configurations lie. Presumably these correspond to six-dimensional and eight-dimensional analogues of the well-studied moduli space of self-dual connections on four-manifolds. Because the relevant equations in these cases are nonlinear, this is probably a more difficult mathematical problem. However, at least it is straightforward to compute the dimensions of these moduli spaces; given a point in the moduli space corresponding to a constant curvature
connection, the dimension of the moduli space can be computed by finding the number of physical zero-modes around the given gauge theory background. Generalizing to a space of solutions with arbitrary numbers of 0-branes and 6-branes, for example, the appropriate moduli space of Yang-Mills solutions should correspond to a space whose dimensionality is related to the entropy of the type of $0 + 6$ black hole configuration considered in \cite{6, 7}.

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