Direct photon production from viscous QGP

A. K. Chaudhuri† and Bikash Sinha‡
Variable Energy Cyclotron Centre,
1/AF, Bidhan Nagar, Kolkata 700 064, India

We simulate direct photon production in evolution of viscous QGP medium. Photons from Compton and annihilation processes are considered. Viscous effect on photon production is very strong and reliable simulation is possible only in a limited $p_T$ range. For minimally viscous fluid $\eta/s=0.08$, direct photons can be reliably computed only up to $p_T \leq 1.3$ GeV. With reduced viscosity ($\eta/s=0.04$), the limit increases to $p_T \leq 2$ GeV.

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I. INTRODUCTION

Recent experiments at Relativistic Heavy Ion Collider (RHIC) have produced convincing evidences that in $\sqrt{s_{NN}}=200$ GeV Au+Au collisions, a collective medium is produced \[1–4\]. Strong elliptic flow in non-central collisions is key evidence to this understanding. Hydrodynamical analysis of experimental charged particles data, also suggests that in Au+Au collisions, a collective medium, with viscosity to entropy ratio close to the AdS/CFT lower bound of viscosity, $\eta/s \geq 1/4\pi$ is produced \[5,6\]. Whether the collective medium is a deconfined Quark-Gluon Plasma (QGP) or not, is still a question of debate. Hopefully, the issue will be settled in Pb+Pb collisions at the Large Hadron Collider (LHC).

Direct photons probe the early medium produced in a collision better than the charged hadrons. Hadrons, being strongly interacting, are emitted from the surface of the thermalised matter and carry information about the freeze-out surface only. They are unaware of the condition of the interior of the matter and can provide information about the deep interior only in an indirect way. In a hydrodynamic model, one fixes the initial conditions of the fluid such that the "experimental" freeze-out surface is correctly reproduced. In contrast to hadrons, photons, being weakly interacting, are emitted from whole volume of the matter. Throughout the evolution of the matter, photons are emitted. Conditions of the produced matter, at its deep interior, are better probed by the photons. Depending on the transverse momentum, direct photons can probe different aspects of heavy ion collisions. A thermalised medium of quarks and gluons, or of hadrons, can produce significant number of thermal photons. They are low $p_T$ photons ($p_T \leq 3$ GeV/c). Low $p_T$ photons can test whether or not, QGP is produced in Au+Au collisions. Hard photons ($p_T > 6$ GeV/c) are of pQCD origin and test the pQCD models. Fast partons from ‘jet’ can interact with thermal partons of QGP and produce photons. At intermediate $p_T$ range, ($3 \leq p_T \leq 6$ GeV/c), interaction jets with QGP could be an important source of direct photons \[8,9\].

In ideal hydrodynamic models, photon production in Au+Au collisions at RHIC energy has been studied extensively \[10,11\]. However, it is now realized that the strongly interacting medium, produced in Au+Au collisions, must be treated as a viscous medium. Gravity dual theories suggest that specific viscosity, i.e. viscosity to entropy ratio of any matter has a lower bound, the so called KSS bound $\eta/s = 1/4\pi$ \[12,13\]. Even though, photons are important probe of QGP matter, viscous effects on photon production are not studied much. Only recently, Dusling \[14\] studied viscous effects on photon production from a QGP medium. However, the model appears to have some inconsistency. In viscous evolution, photon production is affected due to (i) modified fluid evolution and (ii) non-equilibrium correction to equilibrium distribution function. In \[14\], while the non-equilibrium correction to the distribution function was included, modification of the fluid evolution, due to viscosity was neglected. In the present paper, the inconsistency is removed.

II. HYDRODYNICAL EQUATIONS, EQUATION OF STATE AND INITIAL CONDITIONS

In a hydrodynamical model, the invariant distribution of direct photons is obtained by convoluting the photon production rate with space-time evolution of the fluid. We assume that in $\sqrt{s_{NN}}=200$ GeV, Au+Au collisions at RHIC, a baryon free QGP fluid is formed. Space-time evolution of the fluid is obtained by solving 2nd order Israel-Stewart’s theory,

$$ \partial_{\tau} T^{\mu \nu} = 0, $$

$$ D \pi^{\mu \nu} = - \frac{1}{\tau_{\pi}} [\pi^{\mu \nu} - 2\eta \nabla <\pi_{\mu} \pi_{\nu}> - (u^\nu u^\mu + u^\mu u^\nu) D u_\lambda]. $$

Eq\(1\) is the conservation equation for the energy-momentum tensor, $T^{\mu \nu} = (\varepsilon + p)u^\mu u^\nu - p g^{\mu \nu} + \pi^{\mu \nu}$, $\varepsilon$, $p$ and $u$ being the energy density, pressure and fluid

\[\varepsilon, p\] and \[\pi\] being the energy density, pressure and fluid...
velocity respectively. $\pi^\mu\nu$ is the shear stress tensor (we are neglecting bulk viscosity). Eq 2 is the relaxation equation for the shear stress tensor $\pi^\mu\nu$. In Eq 2, $D = u^\mu \partial_\mu$ is the convective time derivative, $\nabla^\mu \nabla_\nu = \frac{1}{3}(\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{1}{3}(\partial_\mu u^\nu - u^\mu u^\nu)$ is a symmetric traceless tensor. $\eta$ is the shear viscosity and $\tau_\pi$ is the relaxation time. It may be noticed that in a conformally symmetric fluid relaxation equation can contain additional terms [13]. Assuming boost-invariance, Eqs 1 and 2 are solved in $(x, y, \tau)$, coordinates, with the code "AZHYDRO-KOLKATA" [15], developed at the Cyclotron Centre, Kolkata. Details of the code can be found in [6, 7, 16].

Eqs 12 are closed with an equation of state $p(\varepsilon)$. Lattice simulations [17–20] indicate that the confinement-deconfinement transition is a cross over, rather than a 1st or 2nd order phase transition. There is also need to specify the viscosity ($\eta$) and the relaxation time ($\tau_\pi$). A freeze-out temperature is also needed. In the following, we will consider viscous effects on photons from the QGP phase only. The hydrodynamical equations are then solved till the freeze-out temperature $T_F = T_c = 170$ MeV. At the initial time $\tau_i$, initial energy density is assumed to be distributed as 

$$\varepsilon(b, x, y) = \varepsilon_i[(1 - f_{\text{hard}}N_{\text{part}}(b, x, y) + f_{\text{hard}}N_{\text{coll}}(b, x, y)],$$

where $b$ is the impact parameter of the collision. $N_{\text{part}}$ and $N_{\text{coll}}$ are the transverse profile of the average participant and collision number respectively, calculated in a Glauber model. $f_{\text{hard}}=0.13$ is the hard scattering fraction [21]. $\varepsilon_i$ is the central energy density of the fluid in impact parameter $b = 0$ collision. As it will be discussed later, we have simulated Au+Au collisions for a range of initial (central) energy density and initial time. We also assume that initial fluid velocity is zero, $v_x(x, y) = v_y(x, y) = 0$. The shear stress tensor was initialized with boost-invariant value, $\pi_{xx} = \pi_{yy} = 2\eta/3\tau_\pi$, $\pi^{xy}=0$. For the relaxation time, we used the Boltzmann estimate $\tau_\pi = 3\eta/4p$. We also assume that shear viscosity to entropy ratio is a constant throughout the evolution. In the following, we simulate Au+Au collisions for $\eta/s=0.12$.  

### III. PHOTON RATES

As mentioned earlier, in viscous evolution photon rates are modified. The photon rate equations involve distribution functions of quarks and gluons. For example, in $1 + 2 \to 3 + 4$ processes (e.g. Compton and annihilation processes), the general form of photon production rate is,

$$E \frac{dR}{dp} \approx \frac{2}{(2\pi)^3} f_1(E) \int d^3p_2 f_2(E_2) |1 \pm f_3(E_3)|^2 \times \frac{d^3p_3}{(2\pi)^3}$$  

where $M$ is the matrix element for the reaction. $f$'s in Eq.4 are distribution function of quarks and gluons. Unlike an ideal fluid, in viscous evolution, each distribution functions $f$ is modified due to non-equilibrium correction,

$$f_{\text{neq}} \to f_{\text{eq}}(1 + \delta f_{\text{neq}}), \delta f_{\text{neq}} < < 1. \quad (5)$$

The non-equilibrium correction $\delta f_{\text{neq}}$ depend on dissipative forces as well as on particle momenta. For shear viscosity, it can be obtained as,

$$\delta f_{\text{neq}} = C_{\mu \nu} p_\mu p_\nu \pi^{\mu \nu} = \frac{1}{2(\varepsilon + p)T^2} p_\mu p_\nu \pi^{\mu \nu} \quad (6)$$

It is obvious that non-equilibrium correction to photon rates is non-trivial. For Compton and annihilation processes, in the leading log approximation, the rate equation can be simplified [14],

$$E \frac{dR}{dp} \approx \frac{5}{9\pi} f_{\text{neq}}(E) T^2 \ln \left[ \frac{3.7388E}{g^*T} \right] \quad (7)$$

Non-equilibrium correction to distribution functions inside the phase space integral leads to corrections in the logarithmic term and can be neglected. For equilibrium distribution function, as demonstrated by Kapusta and Lichard [22], the phase integration can be explicitly evaluated and one obtain for both Compton and annihilation processes [14],

$$E \frac{dR}{dp} \approx \frac{5}{9\pi} f_{\text{neq}}(E) T^2 \ln \left[ \frac{3.7388E}{g^*T} \right] \quad (8)$$

The invariant photon distribution then has two parts, equilibrium part ($E \frac{dN_{\text{eq}}}{dp}$) and a non-equilibrium part
The function is assumed to be small, it is essential that, without the non-equilibrium correction to distribution function, are shown separately.

\[ (E \frac{dN_{\text{neq}}}{d^3p}) < (E \frac{dN_{\text{eq}}}{d^3p}) \]  
\[ \text{for non-equilibrium correction} \]

(9)

\[ \frac{dN_{\text{neq}}}{d^3p} \ll \frac{dN_{\text{eq}}}{d^3p} \]

IV. EFFECT OF VISCOSITY ON PHOTON SPECTRA AND ELLIPTIC FLOW

Gravity dual theories indicate that viscosity to entropy ratio of strongly interacting matter is bounded from the lower side, \( \eta/s \geq 1/4\pi \). We first consider effect of minimal viscosity on photon production. We assume that minimally viscous (\( \eta/s = 0.08 \)) QGP fluid is thermalised in the time scale \( \tau_i = 0.6 \text{ fm} \) to initial central temperature \( T_0 = 350 \text{ MeV} \). A large number of charged particle’s data e.g. identified particles spectra, elliptic flow etc. data are explained in hydrodynamical model with similar initial time and temperature scale. In Fig.1, simulated photon spectra in 20-40\% Au+Au collisions, from evolution of ideal and minimally viscous QGP fluid, are shown. The spectra with or without the non-equilibrium correction are shown separately. If non-equilibrium correction to distribution function is neglected, \( p_T \) spectra of photons is increased by a factor of \( \approx 1.2-1.5 \). The increase is largely \( p_T \) independent. In contrast, when non-equilibrium correction is included, photon production is increased more at large \( p_T \) than at low \( p_T \). It is also expected, non-equilibrium correction increases with \( p_T \) (see Eq.6). The arrow in Fig.1 indicate the approximate \( p_T \) when non-equilibrium contribution to photon spectra equals the equilibrium contribution \( N_{\text{eq}} \). For minimally viscous fluid, the equality occur at \( p_T \approx 1.7 \text{ GeV} \). As noted earlier, viscous hydrodynamics is applicable only when \( \delta N_{\text{neq}}/N_{\text{eq}} < 1 \). Evidently, for \( \eta/s = 0.08 \), in a hydrodynamic model, photon production from Compton and annihilation processes can not be reliably computed beyond \( p_T = 1.7 \text{ GeV} \). Indeed, if we assume that viscous hydrodynamics remain reliable only until \( \frac{\delta N_{\text{neq}}}{N_{\text{eq}}} = 0.5 \), the photons from Compton and annihilation processes can be computed only up to \( p_T \approx 1.3 \text{ GeV} \).

In Fig.2 we have shown the simulation results for elliptic flow for photons. At very low \( p_T \) elliptic flow is negative. At low \( p_T \leq 0.5 \text{ GeV} \) present model is not reliable. The logarithmic factor in the photon rate equation is negative when \( E < 0.27 \eta q^2 T \). For fluid (central) temperature \( T_0 = 350 \text{ MeV} \), reliable computation is possible only beyond \( p_T = 0.5 \text{ GeV} \). In ideal fluid evolution, at \( p_T > 0.5 \text{ GeV} \), elliptic flow increases with \( p_T \), till a maxima is reached at \( p_T \approx 2 \text{ GeV} \). At even higher \( p_T \), it decreases again. Note that even the highest \( v_2(p_T) \) is not large, less that \( \sim 2\% \). Compare this value to \( v_2(p_T) \sim 20\% \) for charged particles. In viscous evolution, elliptic flow is reduced further. Incidentally, very small photon elliptic flow is consistent with experiments. In experiments also, photons do not flow [27, 28]. In Fig.2 one observes that if non-equilibrium correction is neglected, elliptic flow in viscous evolution is more than flow in ideal fluid evolution. However, with non-equilibrium correction included, flow is reduced in viscous evolution. It shows that it is important to have a consistent model, otherwise, one can conclude wrongly. The arrow in Fig.2 indicate that approximate \( p_T \) when \( \delta N_{\text{neq}} \approx N_{\text{eq}} \).

Photon spectra from Compton and annihilation processes, for four values of viscosity to entropy ratio, \( \eta/s = 0 \) (ideal fluid), 0.04, 0.08 (AdS/CFT lower bound) and 0.12 are shown in Fig.3. Initial time and temperatures are
\( \tau_i = 0.6 \text{ fm} \) and \( T_i = 350 \text{ MeV} \). As expected, high \( p_T \) yield is increased in more viscous fluid evolution. The arrows in Fig.3 indicate the approximate \( p_T \) when \( \delta N_{\text{eq}} = N_{\text{eq}} \). Photon production can not be computed reliably beyond \( p_T = 1.3, 1.7 \) and \( 2.4 \text{ GeV} \), for QGP viscosity, \( \eta/s = 0.12, 0.08 \) and \( 0.04 \) respectively. Very limited \( p_T \) range over which viscous hydrodynamics remain applicable for photon production raises certain issues, which will be discussed later. We just mention that viscous effect on photon production is much stronger than in charged particles. Indeed, for charged particles, viscous hydrodynamics remain applicable over a much wider \( p_T \) range \([6, 7]\). The reason is understood. Photons are emitted through out the evolution. At early time, shear stress tensors have finite values and non-equilibrium correction to photon production is large. In contrast, charged particles are emitted from the freeze-out surface. Shear stress tensors evolves very fast. Compared to early times, stress tensors at freeze-out are much smaller in magnitude. Accordingly, non-equilibrium correction is small. We have not shown simulation results for elliptic flow as a function of viscosity. As shown earlier, \( v_2(p_T) \) for photons is very small in ideal fluid evolution. It further reduces with \( \eta/s \).

Since photons are emitted throughout the evolution, including the earliest phase of QGP, one hope to extract QGP formation time from photon measurements. Indeed, in \([23, 26]\) it was suggested that photon elliptic flow can be used to constrain QGP formation time. However, from the experiments \([27, 28]\), it appears that photons do not seem to experience flow. One can not possibly extract QGP formation time from elliptic flow measurements. In \([14]\), Dusling suggested that inverse slope of the photon spectra, if measured accurately, can be used to put stringent bound on QGP viscosity and initial (formation) time.

In Fig.4 simulated photon spectra from evolution of minimally viscous (\( \eta/s = 0.08 \)) QGP fluid, for four values of initial time \( \tau_i = 0.2, 0.6, 1.0 \) and \( 1.4 \text{ fm} \) are shown. The initial temperature is obtained from the condition that initial entropy density times the initial time is a constant, \( s_i \tau_i = 60 \text{ fm}^{-3} \). As the initial time is reduced, photon spectra are hardened. Photon yield is also increased. It is easily understood, for fixed \( s_i \tau_i \), initial time and temperature are inversely related. As the initial time is reduced, high \( p_T \) photon production is increased due to increased fluid temperature. The arrows in Fig.4 indicate the \( p_T \) when non-equilibrium correction to spectra is equal to the equilibrium contribution.

To obtain the inverse slope parameter \( T_{\text{eff}} \), we have fitted the spectra in the \( p_T \) range \( 1.5 \leq p_T \leq 2.5 \text{ GeV} \) with an exponential \( (dN/d^2p_T \propto e^{-p_T/T_{\text{eff}}}) \). Ideal hydrodynamic simulations for photon spectra suggest that in this \( p_T \) range, QGP photons dominate the spectra \([10, 11]\). In Fig.4 for fluid viscosity, \( \eta/s = 0.04 \) and \( 0.08 \), inverse slope parameter is shown as a function of initial time. Inverse slope parameter \( T_{\text{eff}} \) decreases with increasing initial time, as well as with decreasing viscosity. In Fig.4 the shaded region indicates the experimental slope measured in the PHENIX experiment. For ideal fluid, simulated \( T_{\text{eff}} \) agree with experiment for \( \tau_i \geq 1 \text{ fm} \). For viscous fluid, in the time scale, \( 0.2 - 1.0 \text{ fm} \), inverse slope of the simulated spectra are higher than that observed in experiment. However, we have neglected photons from the hadronic phase. Hadronic photons will dominantly contribute at low \( p_T \), reducing the inverse slope parameter. In other word, in more realistic simulation, \( T_{\text{eff}} \) could be smaller than obtained presently and even for

\[ \begin{align*}
\tau_i &= 0.6 \text{ fm} \quad \eta/s = 0, 0.04, 0.08 \quad \text{and} \quad 0.12 \quad \text{are shown. The initial time is} \quad \tau_i = 0.6 \text{ fm} \quad \text{and initial central temperature is} \quad T_i = 350 \text{ MeV.}
\end{align*} \]
viscous fluid could be in agreement with experiment.

To constrain initial time and viscosity from inverse slope of photon spectra will not be an easy task. As seen in Fig. 5, inverse slope parameter from evolution of low viscosity fluid initialized at small \( \tau_s \) could be confused with \( T_{\text{eff}} \) from high viscosity fluid initialized at large \( \tau_s \). As shown in Fig. 5 for \( \eta/s = 0.04 \pm 0.04 \), depending on the initial time, change in inverse slope parameters is \( \sim 20-80 \) MeV. Depending on the realistic situation, measurement of inverse slope parameter with accuracy \( \delta T_{\text{eff}} \approx 20-80 \) MeV could possibly give a range of values for initial time and viscosity.

Before we summarise, few comments are in order. We have shown that effect of viscosity is quite strong on photon production. Indeed, viscous hydrodynamics in applicable only in a limited \( p_T \) range. For minimally viscous fluid, depending on the initial time, applicability range is less than \( p_T = 1.5 \) GeV. Applicability range is even less in more viscous fluid. Present simulations neglect Bremsstrahlung photons in the QGP phase. However, their inclusion will not improve the situation. One wonders about the photons in the \( p_T \) range 1.5 \( \leq p_T \leq 3 \) GeV. As noted earlier, ideal hydrodynamics simulations indicate that in this \( p_T \) range, thermal photons dominate. However, present simulations indicate that if QGP fluid is minimally viscous, the photons in the \( p_T \) range 1.5 \( \leq p_T \leq 3 \) GeV can not be considered thermal and can not be described hydrodynamically. Do we understand them as pQCD photons? Or are they from a fluid with viscosity much less than the AdS/CFT lower bound? Indeed, in certain gravity dual theories, KSS bound can be violated by \( \sim 36\% \). Limited \( p_T \) range over which photons can be reliably computed in viscous dynamics also indicate that within a same model, one possibly can not explain, charged particle production and photon production.

V. SUMMARY

To summarize, we have studied effect of shear viscosity on Compton and annihilation photons. In viscous dynamics, photon production is modified due to (i) changed space-time evolution of the fluid and (ii) non-equilibrium correction to the equilibrium distribution function. The non-equilibrium correction grows with viscosity as well with transverse momentum. Viscous effects on photon production are strong. Even for AdS/CFT lower bound of viscosity \( \eta/s = 0.08 \), strong viscous correction render the hydrodynamics inapplicable beyond \( p_T \approx 1.5 \) GeV. If QGP viscosity is larger than the AdS/CFT limit, then limitation will be even more. Photon production as a function of initial time, also suggest that if the inverse slope parameter of the photon spectra, is measured within an accuracy of \( \pm 20 \) MeV, one can possibly limit the initial time and viscosity.

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