Semileptonic Decay of $B$-Meson into $D^{**}$ and the Bjorken Sum Rule

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Abstract

We study the semileptonic branching fraction of $B$-meson into higher resonance of charmed meson $D^{**}$ by using the Bjorken sum rule and the heavy quark effective theory (HQET). This sum rule and the current experiment of $B$-meson semileptonic decay into $D$ and $D^*$ predict that the branching ratio into $D^{**}\ell\nu_\ell$ is about 1.7%. This predicted value is larger than the value obtained by various models.
It is well known that the heavy quark effective theory (HQET) is a very useful method to study physics of hadrons containing a heavy quark\[1\]. For example, the HQET is used to determine Kobayashi-Maskawa matrix element $|V_{cb}|$, from experiment of semileptonic decay $B \rightarrow D^* l \nu$\[2\]. The attractive features of the HQET are generally verified in phenomena where both of initial and final states include ground states of heavy hadron. However, the phenomena between a ground state and an excited one, for example branching ratio of $B$ meson semileptonic decay into the excited charmed meson state $B \rightarrow D^{**} l \nu$, where $D^{**}$ means a higher resonance state of charmed meson, are not interpreted in various models of hadrons based on the HQET\[3, 4, 5, 6, 7, 8, 9\]. This discrepancy could be reduced to the models of hadrons as a composite system. It is expected that the models which interpret the $B$-meson branching fractions of the semileptonic decays into $D$, $D^*$ and $D^{**}$s. At present we have no such a model. So it is meaningful to study these processes by a model-independent approach. The purpose of this short note is a test of the HQET applicability to an excited state by dealing with charmed hadron excited states $D^{**}$ without model dependence.

In the heavy quark limit $m_Q \rightarrow \infty$, heavy quark spin and velocity are free from low energy QCD\[1\]. So hadron state is factorized into heavy quark $|Q\rangle$ and light degrees of freedom ($l.d.f.$) $|l f\rangle$ as

$$|\text{hadron}\rangle = |Q\rangle \otimes |l f\rangle. \tag{1}$$

These hadrons are classified by $l.d.f.$ as following

$$
(0^-, 1^-) = |Q(\frac{1^-}{2})\rangle \otimes |l f(\frac{1^-}{2})\rangle,
$$

$$
(0^+, 1^+) = |Q(\frac{1^+}{2})\rangle \otimes |l f(\frac{1^+}{2})\rangle, \tag{2}
$$

$$
(1^+, 2^+) = |Q(\frac{1^+}{2})\rangle \otimes |l f(\frac{3^+}{2})\rangle,
$$

$$
(1^-, 2^-) = |Q(\frac{1^-}{2})\rangle \otimes |l f(\frac{3^-}{2})\rangle,
$$

1
where we use the notation as

\[(J_{-}^{P}, J_{+}^{P}) = |Q(\frac{1}{2}^{+})\rangle \otimes |ldf(j^{P})\rangle,\]

with

\[J_{\pm} = j \pm \frac{1}{2}\]  \hspace{1cm} (3)

and \(J_{\pm}^{P}\) of left hand side denotes spin-parity of heavy hadrons and \(j^{P}\) of right hand side is spin-parity of \(l.d.f.\).

In semileptonic decay conserving light degrees of freedom like \(j^{P} = \frac{1}{2}^{-} \rightarrow \frac{1}{2}^{-}\) such as \(B(0^{-})\) goes to \(D(0^{-})\) or \(D^{*}(1^{-})\), there are 6 independent form factors defined as

\[\langle D(v')|J_{\mu}|B(v)\rangle = f_{+}(y)(v + v')_{\mu} + f_{-}(y)(v - v')_{\mu},\]

\[\langle D^{*}(v')|J_{\mu}|B(v)\rangle = ig(y)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^{\alpha}v^{\beta} - (y + 1)f(y)\varepsilon_{\mu}^{\nu} + (\varepsilon^{*} \cdot v)\{a_{+}(y)(v + v')_{\mu} + a_{-}(y)(v - v')_{\mu}\},\]  \hspace{1cm} (4)

where \(v\) and \(v'\) are the velocity of \(B\) and \(D\) or \(D^{*}\), respectively. As the result of heavy quark symmetry, relations among these form factors are obtained and finally there exists only one independent form factor in these processes. This form factor written by \(\xi(y)\) is called as Isgur-Wise function (IW function). Following the HQET we obtain only one independent form factor in each semileptonic process such as \(j^{P} = \frac{1}{2}^{-} \rightarrow \frac{1}{2}^{-}, \frac{3}{2}^{+}, \frac{3}{2}^{-}, \ldots\). In this letter we denote these IW functions as \(\xi_E, \xi_F\) and \(\xi_G\) for \(j^{P} = \frac{1}{2}^{+}, \frac{3}{2}^{+}, \frac{3}{2}^{-}\), respectively. The relations between form factors and IW function are given in Ref.\[9\].

We can derive the Bjorken sum rule on IW functions. In the HQET hadron state is factorized into heavy and light degrees of freedom like in Eq.(1) and the sum of transition rate between heavy hadrons \(H_{Q}\) and \(H'_{Q'}\) is equal to transition rate of heavy quark \(Q\) into \(Q'\) as

\[\sum_{H'_{Q'}} g^{\mu\nu}\langle H'_{Q'}|J_{\mu}|H_{Q}\rangle\langle H_{Q}|J'_{\nu}|H'_{Q}\rangle = g^{\mu\nu}\langle Q'|J_{\mu}(Q)\rangle\langle Q|J'_{\nu}(Q)\rangle \sum_{ldf'}|\langle ldf'|ldf\rangle|^{2}\]
\[ = \langle Q'|J^\mu|Q\rangle\langle Q|J^\mu_\mu|Q'\rangle \]
\[ = -8y, \quad (5) \]

where \( y \equiv v \cdot v' \) and here we use the unitarity relation
\[
\sum_{ldf}|\langle ldf'|ldf\rangle|^2 = 1. \quad (6)
\]

On the other hand in the case of heavy meson transition \( B \rightarrow D_X \), we obtain
\[
\sum_{H' = D_1, D_2} g^{\mu\nu} \langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\nu_\mu|H(v')\rangle = -4y(y+1)|\xi(y)|^2, \\
\sum_{H' = D_1, D_2} g^{\mu\nu} \langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\nu_\mu|H(v')\rangle = -4y(y-1)|\xi_E(y)|^2, \\
\sum_{H' = D_1, D_2} g^{\mu\nu} \langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\nu_\mu|H(v')\rangle = -\frac{8}{3}y^2(y^2-1)(y+1)|\xi_F(y)|^2, \\
\sum_{H' = D_1, D_2} g^{\mu\nu} \langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\nu_\mu|H(v')\rangle = -\frac{8}{3}y^2(y^2-1)(y-1)|\xi_G(y)|^2, \\
\sum_{H' = D_1, D_2} g^{\mu\nu} \langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\nu_\mu|H(v')\rangle = -4y(y+1)|\xi_C(y)|^2, \quad (7)
\]

by the straight calculation\(^{12}\), where index \( C_2 \) denotes radial excitation of \( j^F = \frac{1}{2}^- \). Here, we introduce the hypothesis of resonance saturation which means that it is possible to neglect the contribution from the continuum spectra to semileptonic decay. Under this assumption the relation
\[
1 = \frac{y+1}{2} \left( |\xi(y)|^2 + |\xi_{C_2}(y)|^2 \right) + \frac{y-1}{2} |\xi_E(y)|^2 + \frac{1}{3}y(y^2-1)(y+1)|\xi_F(y)|^2 \\
+ \frac{1}{3}y(y^2-1)(y-1)|\xi_G(y)|^2 + \cdots \quad (8)
\]
is derived by the comparison of Eqs.\(^3\) and \(^4\), where dots means contributions from other higher resonances. This is the simplified Bjorken sum rule\(^{10, 11}\).

In \( B \) meson semileptonic decay, differential decay rate is given as,
\[
\frac{d\Gamma_X}{dy} \equiv \frac{d\Gamma}{dy} (B \rightarrow D_X l\nu) = \frac{G_F^2|V_{tb}|^2}{48\pi^3}m_B^3\sqrt{y^2-1}m_{D_X}^3 W_X(y, r_X)|\xi_X(y)|^2, \quad (9)
\]
where lepton mass is neglected and $W_X(y, r_X)$ is a calculable functions of $y$ and $r_X = \frac{m_{DX}}{m_B}$. In the following analysis, We assume that the excited states, which contribute to the simplified Bjorken sum rule, saturate the $B$-meson semileptonic decay rate (resonance saturation hypothesis) and that these exited states occur in a mass range being small compared with $m_c$. The latter assumption leads us to replace $m_{DX}(X = C_2, E, F, G, \ldots)$ by a common mass $m_{D^{**}}$. Then we can sum up these decay rates for all $D^{**}$ and we get the equality

$$\frac{d\Gamma^{**}}{dy} = \frac{d\Gamma}{dy}(B \to D^{**} l\nu)$$

$$= \sum_{X=C_2,E,F,G,\ldots} \frac{d\Gamma_X}{dy}$$

$$= \frac{G_F^2 |V_{cb}|^2}{48\pi^3} m_B^2 \sqrt{y^2 - 1} \sum_{X=C_2,E,F,G,\ldots} m_{DX}^3 W_X(y, r_X)|\xi_X(y)|^2$$

$$= \frac{G_F^2 |V_{cb}|^2}{24\pi^3} m_B^2 \sqrt{y^2 - 1} m_{D^{**}}^3$$

$$\times \left[(y - 1)(1 + r)^2 + (y + 1)(1 - r)^2 + 4y(1 + r^2 - 2ry)\right]$$

$$\times \left[\frac{y + 1}{2} |\xi_{C_2}(y)|^2 + \frac{y - 1}{2} |\xi_{E}(y)|^2 + \frac{1}{3} y(y^2 - 1)(y + 1)|\xi_{F}(y)|^2$$

$$+ \frac{1}{3} y(y^2 - 1)(y - 1)|\xi_{C_2}(y)|^2 + \cdots\right]. \quad (10)$$

By using Eq.(8), we obtain the $D^{**}$ contribution as

$$\frac{d\Gamma^{**}}{dy} = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} m_B^2 \sqrt{y^2 - 1} m_{D^{**}}^3$$

$$\times \left[(y - 1)(1 + r)^2 + (y + 1)(1 - r)^2 + 4y(1 + r^2 - 2ry)\right]$$

$$\times \left[1 - \frac{y + 1}{2} |\xi_{E}(y)|^2\right]. \quad (11)$$

From this result we can estimate the $D^{**}$ contribution by using the following parameters[13]

$$V_{cb} = 0.040, \quad m_B = 5.279 \text{ GeV}, \quad \tau_B = 1.537 \text{ ps} \quad (12)$$

and for the mass of $D^{**}$ we use the following weighted average mass

$$m_{D^{**}} = \frac{3m_{D_1} + 5m_{D_2}}{8} = 2.444 \text{ GeV} \quad (13)$$
with $m_{D_1} = 2.420 \text{GeV}$ and $m_{D_2} = 2.460 \text{GeV}$. We use the IW function $\xi(y)$ with following three trial functions

(I) $1 - \rho^2 (y - 1)$ \hspace{1cm} $\rho = 0.91^{+0.19}_{-0.21}$,
(II) $\exp[-\rho^2 (y - 1)]$ \hspace{1cm} $\rho = 0.99^{+0.27}_{-0.25}$,
(III) $\left(\frac{2}{y+1}\right)2\rho^2$ \hspace{1cm} $\rho = 1.03^{+0.28}_{-0.26}$,

where $\rho$ is a free parameter to reproduce the experimental branching ratio

$$\text{Br}(B \rightarrow D^{(*)} l\nu) = \text{Br}(B \rightarrow D l\nu) + \text{Br}(B \rightarrow D^* l\nu)$$
$$= (1.6 \pm 0.7\%) + (6.6 \pm 2.2\%)$$
$$= (8.2 \pm 2.3\%),$$

and we use $m_{D^{(*)}} = \frac{m_D + 3m_{D^*}}{4} = 1.975 \text{GeV}$ to determine $\rho$.

|     | $D$   | $D^*$ | $D^{**}$ | $\Sigma_X D_X$ |
|-----|-------|-------|----------|----------------|
| (I) | 1.91  | -0.68 | 6.24     | 1.58          |
|     | +0.76 | +1.53 | +1.63    | +0.89         |
| (II)| 1.95  | -0.72 | 6.22     | 1.71          |
|     | +0.72 | -1.61 | +1.61    | +1.05         |
| (III)| 1.96  | -0.67 | 6.21     | 1.77          |
|     | +0.72 | -1.62 | +1.62    | +1.06         |
| Average | 1.94  | -0.68 | 6.22     | 1.69          |
|     | +0.73 | -1.59 | +1.62    | +1.00         |

The $D^{**}$ contribution and $D_X$ total contribution ($D + D^* + D^{**}$) are given in Table 1. Here we use Eq.(11) to obtain $D^{**}$ contribution which gives about 1.7% branching fraction. The direct measurement of $D^{**}$ contribution is $2.7 \pm 0.7\%[13]$. This is a factor 1.6 larger at central value than the theoretical estimation of $D^{**}$ obtained by the simplified Bjorken sum rule Eq.(8). However, the experimental error is still large and the estimated magnitude of $D^{**}$ contribution seems to be consistent with the present experiment. On the other hand, inclusive semileptonic decay branching ratio is experimentally $10.43 \pm 0.24\%$ and the unidentified semileptonic
Table 2: Comparison of $D^{**}$ contribution $(\frac{|V_{cb}|^2 \tau_B}{1.357 \, \text{ps}} \%)$

|                  | ISGW2[7] | SISM[4] | CNP[4] | VO[8] | ours** | Exp.[13] |
|------------------|----------|---------|--------|-------|--------|---------|
| Br($D + D^*$)    | 9.03     | 7.23    | 7.24   | 9.24  | 8.16†  | 8.2± 2.3 |
| Br($D^{**}$)     | 0.96     | 0.33    | 0.53   | 0.96  | 1.69   | 2.7± 0.7 |

*In this table ours data are average in Table 1.
†This is an input value.

branching fraction [13] is

$$\text{Br}(B \rightarrow \text{unknown}) = \text{Br}(\text{inclusive}) - \text{Br}(B \rightarrow D\text{and}D^*\nu) = 2.2 \pm 2.3\%.$$  \hspace{1cm} (14)

This also shows that the resonance saturation hypothesis and the approximate mass degeneracy among the excited charmed mesons($D_{0}^{*}, D_{1}^{*}, D_{2}, D_{c2}, D_{c2}^{*}, \cdots$) might hold in $B$ meson semileptonic decay.

In order to estimate the contribution from each resonance, it is necessary to calculate exclusive decay processes by using hadronic models as given in Ref.[3, 4, 5, 6, 7, 8, 9]. Through these studies there is a tendency that branching fractions into $D_{2}^{*}$ and $D_{1}$ are rather larger compared to the other excited states. The maximum fraction among $D^{**}$s is $D_{2}^{*}$ in Ref.[4, 8, 9] with the magnitude $0.1\% \sim 0.4\%$ and is $D_{1}$ in Ref.[7] with $0.4\%$. To make clear which model is better, further exclusive experiments are needed. One more feature in common with model-dependent analyses given in Table 2 is that relatively small $D^{**}$ fraction is predicted and is inconsistent with experimental value.

Further in order to check our model-independent approach we apply this method to $B_s$ semileptonic decay processes. The IW function of $B_s \rightarrow D_{s}^{(*)}\nu$ is the same one of $B \rightarrow D^{(*)}\nu$, because $u$, $d$ and $s$ quarks are treated as l.d.f. in the HQET. In the numerical estimation the parameters are[13]

$$m_{B_s} = 5.375 \, \text{GeV}, \quad \tau_{B_s} = 1.34 \, \text{ps},$$

$$m_{D_s} = \frac{m_{D_s} + 3m_{D^*_s}}{4} = 2.075 \, \text{GeV},$$

$$m_{D^{(*)}_s} = 2.155 \, \text{GeV},$$

$$\tau_{D_{s}^{(*)}} = 1.34 \, \text{ps}.$$
\[ m_{D_s^{(*)}} = \frac{3m_{D_{s1}} + 5m_{D_{s2}^*}}{8} = 2.556 \text{ GeV}, \]

where \( m_{D_{s2}^*} \approx 2568 \text{ MeV} \) is a presumption from other charmed meson masses by using the relation \( m_{D_{s2}^*} - m_{D_{s1}}(2535) = m_{D_{s1}^*}(2456) - m_{D_{s1}}(2423) \). Following the same argument of \( B \to D_X l\nu \) we can give the branching ratios of \( B_s \) to \( D_s, D_s^* \) and \( D_{s^*} \) in Table 3. The predicted branching ratio is similar to ones of \( B \to D_X l\nu \) as shown in Table 1. The largest branching ratio is reduced to a fraction to \( D_s^* \) and a contribution from excited states \( D_{s^*} \) is less than 16% of inclusive ratio. If this is confirmed by experiments it will verify the validity of the analysis using the Bjorken sum rule.

|      | \( D_s \) | \( D_s^* \) | \( D_{s^*} \) | \( \sum X D_{sX} \) |
|------|-----------|------------|--------------|-----------------|
| (I)  | 1.79      | 5.78       | 1.28         | 8.84            |
|      | +0.64     | +1.37      | +0.73        | -1.17           |
| (II) | 1.80      | 5.72       | 1.40         | 8.92            |
|      | -0.59     | -1.41      | +0.88        | -1.12           |
|      | +0.61     | +1.38      | -0.76        | +1.23           |
| (III)| 1.81      | 5.70       | 1.45         | 8.96            |
|      | -0.59     | -1.42      | +0.89        | -1.12           |
|      | +0.61     | +1.39      | -0.78        | +1.22           |
| Average | 1.80     | 5.73       | 1.38         | 8.91            |
|      | +0.62     | +1.38      | +0.83        | -1.14           |
|      |           |            | -0.75        | +1.25           |

In this letter we estimate the semileptonic branching ratios of \( B \) to excited charmed mesons by using a method independent on specific hadron models. The Bjorken sum rule will be checked by measuring the contribution from higher resonance of charmed meson in semileptonic decay of \( B_{u,d} \) and \( B_s \) meson. The prediction is given under the assumption that the semileptonic decay is saturated by the three body decays \( B \to D_X l\nu \) and the continuum contribution is negligible and that the excited states have approximately equal masses. We get \( \text{Br}(B \to D_{s^*} l\nu) = 1.7 \pm 1.0\% \) which seems to be consistent with the experimental value \( 2.7 \pm 0.7\% \). We also estimate \( \text{Br}(B_s \to D_s, D_s^*, D_{s^*} l\nu) \) to be 1.8%, 5.7% and 1.4%, respectively. The predicted branching ratios are similar to \( \text{Br}(B \to D, D^*, D_{s^*} l\nu) \). These evaluations are to be checked by experiments in near future and we expect that the
model-independent approach will be confirmed experimentally.

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