Two-Qubit Gate Operation on Selected Nearest-Neighbor Neutral Atom Qubits

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We have previously discussed the design of a neutral atom quantum computer with an on-demand interaction [E. Hosseini Lapasar, et al., J. Phys. Soc. Jpn. 80, 114003 (2011)]. In this contribution, we propose an experimental method to demonstrate a selective two-qubit gate operation that is less demanding than our original proposal, although the gate operation is limited to act between two neighboring atoms. We evaluate numerically the process of a two-qubit gate operation that is applied to a selected pair of nearest-neighbor, trapped atoms and we estimate the upper bound of the gate operation time and corresponding gate fidelity. The proposed scheme is scalable and, though challenging, is feasible with current experimental capabilities.

I. INTRODUCTION

Quantum information science has rapidly grown with the promise of realizing a quantum computer that will be capable of solving many classically intractable problems by making use of properties of quantum systems, such as superposition and entanglement [1–3]. Similar to classical computers, a quantum computer consists of a memory and a processor, with a set of gates to store and process the information. A number of physical systems have been proposed for practical implementations of quantum computing, such as nuclear magnetic resonance, linear optics, trapped atoms or ions, and superconducting circuits [2, 3]. These physical systems have merits and demerits and, to date, no single system satisfies all the DiVincenzo criteria - the necessary conditions for a physical system to be a realistic candidate for a working quantum computer [4]. A major obstacle in the feasibility of quantum computing is decoherence, which arises from the interaction of the system with its environment.

A system of trapped, neutral atoms is one of the most promising candidates for implementing a scalable quantum computer, since neutral atoms have the advantage of an intrinsically weak interaction with the environment [5, 6]. Coherence times in this system are a few seconds [7], i.e., many orders of magnitude longer than typical operation times. A one-qubit gate operation has already been demonstrated using microwave radiation [8] or a two-photon Raman transition [9, 10], which is a well-established technique by today. However, a two-qubit gate operation - and the relevant high-fidelity control with scalable architectures - is still a serious challenge in a neutral atom-based system. Two-qubit gate operations have been proposed using either short-range collisions [11–13] or long-range dipole-dipole interactions [14, 15]; the latter scheme has been realized with individual atoms in separate dipole traps [16, 17]. Mandel et al. have demonstrated a two-qubit gate operation with neutral atoms using hyperfine state dependent optical lattice potentials [18, 19]. A drawback of this implementation is that the gate acts on all the nearest neighbor pairs in the optical lattice simultaneously and cannot be used for circuit model quantum computation.

We recently proposed a method to apply a two-qubit gate operation on an arbitrarily selected pair of atoms by modifying the method demonstrated by Mandel et al. [20], where the operation is implemented by optical potentials generated by near-field Fresnel diffraction (NFFD) of light at a thin aperture [21] and a state-dependent optical lattice [18]. In order to verify the feasibility of this scheme, the compatibility of the proposed two-qubit operation [20] with the NFFD trap would need to be verified. The proposed experiment is not overly demanding, though may not be suitable for future quantum computer realizations. A similar, but alternative, system has also been proposed, based on trapping of atoms above an array of microtraps [22].

Here, we present a proposal for an experiment towards two-qubit gate operation using neutral atoms. Although the proposed system is less general than our original proposal [20], it should be easier to implement. We first briefly review the design of the neutral atom quantum computer proposed in Ref. [20], and highlight the difficulties associated with the system. Next, we propose a demonstration of a two-qubit operation under simplified situations obtained by

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II. BRIEF REVIEW OF OUR RECENT PROPOSAL

Figure 1 shows the scheme of our recent proposal [20]. Neutral atoms with two hyperfine states, $|0\rangle$ and $|1\rangle$, are trapped in an array of optical potentials generated by NFFD of light at thin apertures, each with a radius comparable to the wavelength of the light [21]. The apertures are made using microfabrication techniques on a silicon substrate. Two-colored laser light is incident on the apertures through a tapered optical fiber attached to each aperture. One laser color is used for trapping the atom and the other is used to control the hyperfine qubit states of the atom, i.e., it provides a one-qubit gate operation to prepare a superposition state $(|0\rangle + |1\rangle)/\sqrt{2}$, by using the two-photon Raman transition [9, 10]. Since each site is equipped with its own gate control laser beam, individual, single-qubit gate operations can be applied on many qubits simultaneously. We assume that the NFFD potential is adjusted so that only one atom is contained within each trap.

In order to implement a two-qubit gate operation between an arbitrary pair of atoms, a one-dimensional optical lattice is superimposed near the substrate by applying a pair of tightly-focused, counterpropagating laser beams along the atom trapping sites, so that the minima of the optical lattice are situated at the space coordinates for the atoms trapped in NFFD potentials. By manipulating the aperture size, one can control the position of an individual atom in a direction perpendicular to the aperture plane to transfer atoms into a one-dimensional optical lattice [20, 23, 24], as shown in Fig. 1(c). A collision in the subspace $|0\rangle|1\rangle$ can be caused by controlling the polarization of
the counterpropagating beams of the optical lattice [Fig. 1(d)], so that this particular component acquires an extra dynamical phase \[11, 12\]. Subsequently, the polarizations are reversed and the atoms are returned to their initial positions in the optical lattice. Finally, the atoms are transferred from the optical lattice to the initial NFFD traps. As an example, we considered two hyperfine states \[|0\rangle = |F = 1, m_F = 1\rangle\) and \[|1\rangle = |F = 2, m_F = 1\rangle\) of \(^{87}\text{Rb}\) atoms to be compatible with this scheme and estimated that the time required for the entangling gate operation is on the order of 10 ms with a corresponding fidelity of 0.886 \[20\].

### III. TWO-QUBIT GATE IMPLEMENTATION

The purpose of this work is to present a quantum computer scheme that is less demanding than our previous proposal \[20\]. There are primarily three difficulties in our previous proposal:

1. Making apertures of variable size,
2. Aligning optical fibers on individual apertures with a size of a few micrometers,
3. Making an optical lattice very close to the surface of the array.

The most challenging part is the aperture size control within microseconds. To circumvent this difficulty (i), we propose an alternative design as shown in Fig. 2. The design is very similar to the previous one [Fig. 1], but the optical lattice is implemented so that its minima are situated at the space coordinates of NFFD trapped atoms from the outset. To implement a two-qubit gate operation, we decompose the procedure into five steps as shown in Fig. 3 and outlined in the following.

![FIG. 2: (Color online) Schematic representation of atoms trapped in an optical lattice with an NFFD potential. Thick and thin contours show the profiles of NFFD traps and an optical lattice, respectively. The blue (darker) arrow shows the trapping laser beam, while the red (lighter) arrow shows the control laser beam. Minima of the one-dimensional optical lattice along the x-axis are situated at the space coordinates of atoms trapped by NFFD.](image)

1. We choose two neighboring trapped atoms to be operated on by the gate. The NFFD laser beams of the two atoms are adiabatically turned off so that the two atoms remain in the optical lattice.

2. The polarizations of the pair of counterpropagating laser beams, with which the optical lattice is created, are rotated in opposite directions so that the qubit state \[|0\rangle\] is transferred toward the negative x-direction, while \[|1\rangle\] is transferred toward the positive x-direction \[18\]. Therefore, it is possible to collide \[|0\rangle\] of one atom and \[|1\rangle\] of the other atom. It is important to ensure that ‘spectator’ atoms other than the chosen atomic pair should be left in the ground state of the potential made by the combined NFFD and optical lattice during this process.

3. Now, component \[|0\rangle\] of one atom and \[|1\rangle\] of the other atom are trapped in the same potential well in the optical lattice. They interact with each other for a duration \(t_{\text{hold}}\) so that the particular state \(|0\rangle|1\rangle\) acquires an extra phase compared to other components \(|0\rangle|0\rangle\), \(|1\rangle|0\rangle\), and \(|1\rangle|1\rangle\). Here, \(U_{\text{int}} = (4\pi\hbar^2 a_s/m) \int |\psi|^4 \, dr\) is the on-site interaction energy with the atomic mass \(m\) and the s-wave scattering length \(a_s\). When \(t_{\text{hold}}\) satisfies the condition \(U_{\text{int}} t_{\text{hold}} = \pi\), we obtain a nonlocal two-qubit gate \(|00\rangle\langle 11| - |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|\).

4. After acquiring the phase, the two states are separated along the optical lattice by the inverse process of Step 2.

5. The NFFD laser beams are adiabatically turned on.
Difficulty (i) is circumvented in this new design, thereby making the method more adaptable for experimentalists, although in the present scheme the two-qubit gate operation can only be applied on selected nearest-neighbor trapped atoms. We should mention that one drawback of this proposal is that we need SWAP gates to apply two-qubit gates between two remote atoms. In relation to difficulty (ii), it is by now possible to fabricate high transmission, tapered optical micro- and nanofibers relatively routinely and they have found applications in several areas of cold atom physics [25]. A half-taper, as required in the proposed setup, can be made by pulling a tapered fiber until breaking point. To place the fiber tips close to the substrate, they would need to be guided through place holders since they are very fragile and sensitive to vibrations. In vacuum, the latter point is less of an issue since air currents are minimal. We should also mention that making an optical lattice very close to the surface of the array limits the size of the 2D lattice of trapped atoms. Rayleigh length, which is distance along the propagation direction of the laser beam from the most focused point to the point at which the beam size becomes $\sqrt{2}$ times larger, is $x_R = \frac{xw^2}{\lambda OL} \simeq 40 \mu m$. The focused light beam will spread, and the lower half of the beam will touch the substrate. Thus, effective range that can be used as qubit operation is $2x_R$, i.e., 80 $\mu m$. Then, we can have the order of 10 qubits in the array. It can be a small number of qubits to realize a quantum computer, but might still be good starting point to test our scheme.

IV. NUMERICAL CALCULATION

In the following, as an example, we take two hyperfine states $|0\rangle = |F = 1, m_F = 1\rangle$ and $|1\rangle = |F = 2, m_F = 1\rangle$ of $^{87}$Rb atoms and give detailed numerical evaluations corresponding to the above-mentioned processes. We analyze
forms an attractive potential which is given by [21] in Table I. A red-detuned laser beam passing through an aperture with a radius on the order of the laser wavelength and the fidelity of the two-qubit gate operation which is limited by the adiabatic condition. Lewies-Riesenefeld invariant [26] associated with the Hamiltonian for example [27, 28]. Application of nonadiabatic with shorter execution time compared to adiabatic transfer and improve the fidelity. This is realized by employing the difference between the ground state and the first excited state. We might also use nonadiabatic transport of an atom time though. The lower bound of the execution time for adiabatic transport of an atom is limited by the energy the error [1]. The fidelity may be improved by spending more time for each step, which leads to longer execution time. Therefore, we should turn off the NFFD trap potential gradually so that the atoms’ wave functions are transformed.

First, we give the form of the trapping potential and optical lattice used in our calculations. Parameters are given in Table I. A red-detuned laser beam passing through an aperture with a radius on the order of the laser wavelength forms an attractive potential which is given by [21]

\[ U_F(r) = -U_0 \frac{|\mathcal{E}(r)|^2}{E_0^2}, \]  

(2)

where

\[ \mathcal{E}(r) = \frac{E_0}{2\pi} \int \int \frac{e^{ik_Fr}}{r} \left( \frac{1}{r} - i k_F \right) dx' dy', \]  

(3)

and

\[ U_0 = \frac{3}{8} \frac{\Gamma_x}{|\Delta_{eg}|} \frac{E_0^2}{k_F^2}, \]  

(4)

with the distance \( r \) between \( r = (x, y, z) \) and \( r' = (x', y', 0) \) in the plane. Here, \( \Gamma_x \) is half of the spontaneous decay rate, \( E_0 \) and \( k_F = 2\pi/\lambda_F \) are the amplitude and wave number of the incident plane wave to the aperture from the \( -z \)-axis. We use the \( D_1 \) transition of the \( ^{85}\text{Rb} \) atom for trapping, so the detuning is

\[ \Delta_{eg} = \omega_L - \omega_0 = 2\pi c \left( \frac{1}{\lambda_F} - \frac{1}{\lambda_0} \right) = -4.1 \times 10^{11} \text{ Hz}. \]  

(5)

which is negative for red detuning. \( U_0 \) is the potential depth at \( t = 0 \); we have to choose its value so that the states of trapped spectator atoms are unchanged while the two-qubit gate operation is implemented. The integral is over the aperture region \( (x'^2 + y'^2 \leq a^2) \). The NFFD trap laser intensity \( I_0 \) is related to the amplitude of the electric field \( E_0 \) as \( I_0 = cE_0^2/8\pi \). The power of each NFFD trap can be obtained by multiplying \( I_0 \) and the area of the aperture, that is expected to be approximately \( 1.1 \times 10^{-2} \text{ W} \). The optical lattice potential, which is made by a pair of counterpropagating laser beams with the same frequency, amplitude, and polarization along the \( x \)-axis, is introduced as

\[ V_{OL}(r) = -V_0 \cos^2(k_{OL}x) e^{-2(y^2+(z-z_m)^2)/w^2}. \]  

(6)

Here, \( k_{OL} \) and \( \lambda_{OL} \) are the wave number and wavelength of the lasers, \( w = 4\lambda_{OL} \) the waist size, \( z_m \) the \( z \)-coordinate corresponding to the minimum of the NFFD potential, and \( V_0 \) the potential depth at \( t = 0 \). The lattice constant is always \( \lambda_{OL}/2 \), whose value multiplied by integers should be matched to the separation between the neighboring apertures.

\[ \text{A. Step 1} \]

In this step we want to leave two neighboring atoms in an optical lattice to apply the gate operation on them. Therefore, we should turn off the NFFD trap potential gradually so that the atoms’ wave functions are transformed.
TABLE I: Parameters used in calculations.

| Parameter                                      | Value                           |
|------------------------------------------------|---------------------------------|
| Radius of an aperture                          | \(a = 1.5 \lambda_F\)          |
| \(\lambda_F\)                                 | \(795.118\) nm                 |
| \(I_0\)                                       | \(\simeq 2.5 \times 10^5\) W/cm² |
| \(\lambda_0\)                                 | \(794.979\) nm                 |
| \(z_m\)                                       | \(\simeq 1.7\) µm              |
| \(\lambda_{OL}\)                              | \(785\) nm                     |
| \(U_0\)                                       | \(h \times 1.03 \times 10^6\) Hz |
| \(V_0\)                                       | \(h \times 1.47 \times 10^5\) Hz |

adiabatically from the ground state of the combined potentials \(V_{OL} + U_F\) to that of the optical lattice only. The NFFD potential is switched off as

\[
U_F(r, t) = \cos^2 \left( \frac{\pi}{2T_F} t \right) U_F(r)
\]  

with the operation time \(T_F\). To estimate the time required for this process, we need to solve numerically the time-dependent Schrödinger equation Eq. (1) with \(V_{ext}(r, t) = V_{OL}(r, t) + U_F(r, t)\). We have taken \(V_0/E_r = 40, U_0/E_r = 280\) and \(w/\lambda_{OL} = 4\) with the scale of length \(\lambda_{OL} = 785\) nm, energy \(E_r = \hbar^2 k_{OL}^2/2m = h \times 3.7 \times 10^3\) Hz, and time \(\tau = h/E_r = 43\) µs. Figure 4 shows the fidelity as a function of dimensionless operation time \(T_F/\tau\), which indicates that a time \(T_F = 1.8\) ms is required to attain a fidelity of 0.99.

![Figure 4](image)

**FIG. 4:** (Color online) Fidelity as a function of dimensionless operation time when the NFFD potential is adiabatically turned off. Fidelity of 0.99 is attained when \(T_F/\tau = 42.5\).

**B. Step 2**

Let us consider two atoms in the optical lattice on which the gate acts. We must put the \(|0\rangle\) state of one atom and \(|1\rangle\) of the other atom in a single potential well so that the component \(|0\rangle|1\rangle\) acquires an extra phase factor.

Suppose the polarizations of laser beams propagating along the \(x\)-axis and \(-x\)-axis are tilted by angles \(\theta\) and \(-\theta\) from the \(z\)-axis, respectively. The time-dependent polarization in the counterpropagating laser beams is introduced as

\[
E_{+}(r) \propto e^{ik_{OL}x}(\hat{\mathbf{z}} \cos \theta + \hat{\mathbf{y}} \sin \theta),
\]

\[
E_{-}(r) \propto e^{-ik_{OL}x}(\hat{\mathbf{z}} \cos \theta - \hat{\mathbf{y}} \sin \theta).
\]

Superposition of these counterpropagating laser beams produces an optical potential of the form

\[
E_{+}(r) + E_{-}(r) \propto \sigma^+ \cos(k_{OL}x - \theta) + \sigma^- \cos(k_{OL}x + \theta),
\]
where \( \sigma^+ (\sigma^-) \) denotes a counterclockwise (clockwise) circular polarization vector. This means that the first component of Eq. (10) moves along the \( x \)-axis as \( \theta \) is increased, while the second component moves along the \(-x\)-axis under this change. The component \( \sigma^+ \) introduces transitions between fine structures; \( nS_{1/2} (m_J = -1/2) \rightarrow nP_{1/2} (m_J = 1/2) \), \( nS_{1/2} (m_J = -1/2) \rightarrow nP_{3/2} (m_J = 1/2) \), and \( nS_{1/2} (m_J = 1/2) \rightarrow nP_{3/2} (m_J = 3/2) \). The transitions from \( nS_{1/2} \) to \( nP_{3/2} \) are red-detuned, while the transition from \( nS_{1/2} \) to \( nP_{1/2} \) is blue-detuned if \( \omega_k \) is chosen between the transition frequencies of \( nS_{1/2} (m_J = -1/2) \rightarrow nP_{3/2} (m_J = 1/2) \) and \( nS_{1/2} (m_J = -1/2) \rightarrow nP_{1/2} (m_J = 1/2) \). Then by adjusting \( \omega_k \) properly, it is possible to cancel the attractive potential and the repulsive potential associated with these transitions. The net contribution of the \( \sigma^+ \) laser beam, in this case, is an attractive potential \( V_+ (r) \propto \cos^2 (k_{OL} x - \theta) \) for an atom in the state \( nS_{1/2} (m_J = 1/2) \). Similarly, the \( \sigma^- \) component introduces a net attractive potential \( V_- (r) \propto \cos^2 (k_{OL} x + \theta) \), through the transition \( nS_{1/2} (m_J = -1/2) \rightarrow nP_{3/2} (m_J = -3/2) \) on an atom in the state \( nS_{1/2} (m_J = -1/2) \).

In summary, optical potentials for the states \( S_{1/2} \) with \( m_J = \pm 1/2 \) are introduced as

\[
V_\pm (r) = -V_0 \cos^2 (k_{OL} x \pm \theta) e^{-2(w^2 + (z - z_0)^2)/w^2}
\]

(10)

and the effective potentials acting on \( |0\rangle \) and \( |1\rangle \) are evaluated as

\[
V_{|0\rangle} (r) = \frac{1}{4} V_+ (r) + \frac{3}{4} V_- (r),
\]

\[
V_{|1\rangle} (r) = \frac{3}{4} V_+ (r) + \frac{1}{4} V_- (r),
\]

(11)

which have been obtained by making use of the decompositions

\[
|0\rangle = |F = 1, m_F = 1\rangle = \frac{1}{2} \left| \begin{array}{c} 3 \ 1/2 \\ 2 \ 1/2 \\ 1 \ 1/2 \\ \end{array} \right| + \frac{\sqrt{3}}{2} \left| \begin{array}{c} 3 \ 3/2 \\ 2 \ 1/2 \\ 1 \ -1/2 \\ \end{array} \right|,
\]

\[
|1\rangle = |F = 2, m_F = 1\rangle = \frac{\sqrt{3}}{2} \left| \begin{array}{c} 3 \ 1/2 \\ 2 \ 1/2 \\ 1 \ -1/2 \\ \end{array} \right| + \frac{1}{2} \left| \begin{array}{c} 3 \ 3/2 \\ 2 \ 3/2 \\ 1 \ -1/2 \\ \end{array} \right|,
\]

(12)

where \( |3/2, 1/2\rangle \) \( |1/2, 1/2\rangle \) in the right hand side denotes a vector with nuclear spin \( (I = 3/2, I_z = 1/2) \) and electron spin \( (S = 1/2, S_z = 1/2) \), for example. We note that \( V_{|0\rangle} (x) (V_{|1\rangle} (x)) \) moves left (right) if \( \theta \) is increased, which implies that the components \( |0\rangle \) and \( |1\rangle \) move in opposite directions as \( \theta \) is changed. Therefore, it is possible to collide \( |0\rangle \) of one atom and \( |1\rangle \) of the other atom by changing \( \theta \).

We consider motion of the \( |1\rangle \)-state and solve the Schrödinger equation Eq. (1) with the potential \( V_{ext} = V_{|1\rangle} (r, t) \). The angle \( \theta \) is changed as

\[
\theta (t) = n \pi \sin^2 \left( \frac{\pi}{2T_{OL}} t \right),
\]

(13)

in which a state of an atom is transferred in the optical lattice by \( n \) wavelengths. In this calculation we have taken \( n = 6 \), which may be a minimal value for our design based on the comparison with the aperture size. Figure 4 shows the fidelity as a function of dimensionless time \( T_{OL}/\tau \) for \( n = 6 \). The fidelity is an oscillating function due to the fact that the lattice potentials \( V_{|0\rangle} \) and \( V_{|1\rangle} \) are superpositions of two counterpropagating plane waves \( V_\pm \) and there is interference between them. We have found that the operation time \( T_{OL} = 1.28 \) ms, yielding a fidelity of 0.99.

We also consider the fidelity of spectator atoms to check whether they are left in their ground states of the combined potential \( V_{OL} + U_F \) during the above described two-qubit gate operation. Figure 5 shows that the fidelity oscillates with a small amplitude and eventually goes to unity when the atoms undergoing the two-qubit operation are moved back to the initial potential wells. Observe that there are twelve dips in the fidelity corresponding to twelve peaks of the potential of the moving optical lattice kicking the spectator atom.

C. Step 3

Let \( \psi_0 (r) \) be the ground state wave function of an atom in one of the optical lattice potential wells and let \( a_s = 5.19 \) nm be the s-wave scattering length between two \( ^{87}\text{Rb} \) atoms in \( |F = 1, m_F = 1\rangle \) and \( |F = 2, m_F = 1\rangle \). If two atoms in the ground state are put in the same potential well, the interaction energy is given by

\[
\frac{E_{int}}{E_r} = \frac{2a_s}{\pi \lambda_{OL}} \int \psi_0^2 (r) dr = 0.055.
\]

(14)
FIG. 5: (Color online) Fidelity as a function of dimensionless time when the state $|1\rangle$ of an atom is transferred in the optical lattice by six wavelengths. Fidelity of 0.99 is attained when $T_{OL}/\tau = 29.7$.

FIG. 6: (Color online) Fidelity as a function of dimensionless time for a spectator atom during the procedure (Step 2) with $T_{OL} = 1.28$ ms of the two-qubit gate operation.

The corresponding frequency is $\nu_{int} = 0.0589 \times E_r \simeq 218$ Hz. To acquire a phase $\pi$ as necessary for the controlled-$Z$ gate, for example, these atoms must be kept in the same potential well for

$$t_{\text{hold}} = \frac{\pi}{2\nu_{int}} \simeq 2.29 \text{ ms}. \quad (15)$$

This results in a nonlocal two-qubit gate $U = |00\rangle\langle 00| - |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|$. The gate $U$ is locally equivalent to the CNOT and the CZ gates; application of further one-qubit gates maps $U$ to these gates.

This step does not involve any nonadiabatic process and we believe the operation can be executed with a high fidelity by fine tuning the holding time $t_{\text{hold}}$.

D. Steps 4 and 5

After Step 3 is completed, the atoms are returned to their initial positions in the optical lattice (reverse of Step 2) and then the NFFD potentials are switched on (inverse of Step 1). Since Steps 4 and 5 are the inverse processes of Steps 2 and 1, respectively, the required time and the fidelity of the inverse process are the same as those for a forward process.
E. Execution Time and Fidelity

We have estimated the time required for each step to give a fidelity of 0.99. Now, let us estimate the overall execution time and fidelity. The overall execution time of a two qubit gate is given by

$$T_{\text{overall}} = 2(T_F + T_{\text{OL}}) + t_{\text{hold}} \simeq 8.45 \text{ ms}.$$  \hfill (16)

The overall fidelity is estimated as follows. Each of Steps 1, 2, 4 and 5 involves two independent processes. Since these steps involve two atoms, the fidelity associated with each step must be 0.99. Thus, the overall fidelity is

$$0.99^8 \sim 0.923,$$

where we assumed that the interaction time is tunable so that Step 3 gives a fidelity arbitrarily close to 1. Compared to our previous proposal, the execution time has not changed considerably, but the fidelity has been improved since the number of steps required for the qubit operation is reduced. The fidelity can be improved further by spending more time on each step, yielding a longer overall execution time, however.

V. SUMMARY AND DISCUSSION

We have discussed a proposal to demonstrate a selective two-qubit gate operation, which is applied on nearest-neighbor, trapped, neutral atoms. The present setup is less demanding than the previous one proposed in Ref. [20] because it is no longer necessary to control the size of individual apertures for the NFFD traps. For this new setup, we have obtained an upper bound of the entangled gate operation time $\simeq 8.45 \text{ ms}$ with a corresponding fidelity of 0.923 bounded by the adiabaticity requirement. Since the two-qubit gate operation consists of a smaller number of processes than that in our previous proposal, the fidelity is expected to be improved. We believe that this proposal can be demonstrated using current technology, which would be a starting point toward the realization of a fully-controlled, selective, two-qubit gate operation using neutral atoms.

For the parameters considered in this proposal, with a wavelength of $\simeq 795 \text{ nm}$, an aperture radius of $\simeq 1.2 \mu m$ and aperture separation of $\simeq 10 \mu m$, fibers with tips of $\simeq 2.4 - 3 \mu m$ should be feasible. For fibers with diameters of $\simeq 3 \mu m$, more than 99% of light is confined within the fiber and cross-talk between neighboring fibres, separated by $\simeq 7 \mu m$, is negligible [30]. In fact, cross-talk between fibers should only be appreciable for subwavelength diameter fibers in which the light is weakly confined and when the fibers are in a strongly coupled configuration (i.e. the fibers are separated by approximately wavelength/10), two conditions which are not relevant in the proposed setup.

Our present proposal still maintains scalability in a sense that there are many qubits. If we concentrate only on the compatibility check of an NFFD trap [21] and a two-qubit operation [18, 19], it is possible to simplify the design further in order to circumvent difficulties (i) and (ii), as shown in Fig. 7. This experimental setup is even less demanding with only two apertures attached to a tapered optical fiber bundle. We can even remove the fibers completely, since operations on individual atoms are not necessary. In this case, only difficulty (iii) remains as a technical challenge.

FIG. 7: (Color online) Thin apertures shown here may be fabricated by using standard microfabrication methods, wherein the apertures are formed using an anisotropic wet etching technique [31]. The aperture separation is of the order of 10 $\mu m$ and, thus, the diameters of individual fibers in the bundle must be less than the separation. Such a fiber bundle is reported in Ref. [32].
An alternative to using half-taper fibers would be the integration of optical waveguides, with the aperture array at the end face (output) of the waveguide array. While this would require more sophisticated fabrication techniques, the fragility of the system would be reduced and alignment issues would be eased.

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