New Physics Signatures in Dijets at Hadron Colliders

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Abstract

We show how to detect and disentangle at the upgraded Tevatron and at LHC, the effects of the three purely gluonic $\text{dim} = 6 \ SU(3) \times SU(2) \times U(1)$ CP-conserving and CP-violating gauge invariant operators $\overline{G}_D$, $G_\sigma$ and $\tilde{G}$. These operators are inevitably generated by New Physics (NP), if the heavy particles responsible for it are coloured. We establish the relations between their coupling constants and the corresponding NP scales defined through the unitarity relations. We then study the sensitivity and limits obtainable through production processes involving one or two jets, and express these limits in terms of the NP scales implied by unitarity. A detailed comparison with the results of the studies of the analogous electroweak operators, is also made.

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1 Introduction

The main experimental indication for the physics beyond the Standard Model (SM) is up to now only provided by non-accelerator observations, like the apparent predominance of the dark matter in the Universe and the non-vanishing of the neutrino masses. On the contrary, accelerator experiments at LEP and lower energies seem to be quite consistent with SM [1, 2]; except of an admittedly meager evidence for an enhancement compared to the QCD predictions, in deep inelastic e+ p scattering at HERA [3, 4], and the apparent excess of jet production at large transverse energy ET, recently observed by the CDF Collaboration at the Tevatron [5, 6].

These mean that any acceptable form of New Physics (NP), involving only enhancements or depressions of the number of events with respect to the standard QCD expectations, can at most only induce very slight modifications of SM. Of course it is almost certain that such NP effects cannot always have an unambiguous interpretation. Therefore, the search of such NP effects requires that, for any hint of an experimental signal, various possible interpretations are explored and ways of discriminating among them are identified.

One possible way to search for NP, is within the scenario that no new particles, apart from one scalar standard-like Higgs, will be reachable in the future colliders. Under such a philosophy, a classification of a rather wide class of NP models has been presented in [7, 8, 9]. At the energy range of the foreseeable colliders, NP is then assumed to be described by the set of all \( \text{dim} = 6 \text{ SU}(3) \times \text{SU}(2) \times U(1) \) gauge invariant operators involving an isodoublet Higgs field, the quarks of the third family, and of course the gauge bosons, inevitably introduced by the gauge principle whenever a derivative appears. The complete list of the CP conserving such operators has been given in [8, 9], while the CP violating ones have appeared in [10, 11].

Most of these operators, are either directly or indirectly constrained by LEP1 or lower energy experiments [12], or will be constrained by the direct production of weak gauge boson, Higgs and top or b-quark at \( e^- e^+ \) and \( \gamma \gamma \) colliders [13, 14], as well as at hadron colliders [15, 16, 17]. There exist two CP conserving operators though [14], which cannot be efficiently constrained through the processes just listed, namely

\[
\mathcal{O}_{DG} = 2 \left( D_\mu \vec{G}^{\mu\rho} \right) \left( D_\nu \vec{G}^{\nu\rho} \right) ,
\]

and

\[
\mathcal{O}_G = \frac{1}{3!} f_{ijk} G^{i\mu} G^{j}_\nu G^{k\lambda} G^{\nu\lambda}_{\mu} .
\]

These operators will inevitably appear at present energies, if the heavy new particles generating NP are coloured [17, 8, 18]. The second one, \( \mathcal{O}_G \), is a genuine purely gluonic operators; while the equations of motion of the QCD lagrangian implies that \( \mathcal{O}_{DG} \) is essentially equivalent to a four quark operator [16]. They are on the same footing as the operators \( \mathcal{O}_{DW} \) and \( \mathcal{O}_W \), responsible for inducing anomalous electroweak triple gauge couplings [13, 14].
In addition to these two CP conserving operators, there exist also the unique analogous CP violating \( \text{dim} = 6 \) operator \([13, 11]\)

\[
\tilde{O}_G = \frac{1}{3!} f_{ijk} \tilde{G}_{\mu \nu}^i \epsilon_{\nu \lambda} G_k^{\lambda \mu},
\]

(3)

where \( \tilde{G}_{\mu \nu}^i = 1/2 \epsilon_{\mu \nu \lambda \sigma} G_i^{\lambda \sigma} \). \( \tilde{O}_G \) caused considerable discussion some time ago, which started from Weinberg’s observation that the existing limits on the neutron electric dipole moment, (combined with reasonable assumptions on how to calculate such a low energy parameter), put an extremely strong constraint on the \( \tilde{O}_G \) coupling. Since indirect constraints are always submitted to ambiguities, it is worthwhile to also have a direct check of this operator at the Tevatron and LHC.

In order to complete the study of the aforementioned set of the \( \text{dim} = 6 \) \( SU(3) \times SU(2) \times U(1) \) gauge invariant operators, the sensitivity limits to the three operators \( \tilde{O}_{DG}, O_G \) and \( \tilde{O}_G \) must also be investigated. The best way to do this, is to look at single and two jet production at hadron colliders. Such studies have already appeared in \([16, 20]\), while an application to the Tevatron was performed in \([21]\). The identification of the NP operators was subsequently also discussed in \( Z \to 4 \text{ jets} \) \([22]\), in 3 jet production \([23]\), and more recently in top-antitop production \([24]\). We should also quote \([18]\) who study the dijet angular distribution, but choose to concentrate on a specific coherent combination of four CP-conserving \( \text{dim} = 6 \) operators, two of which coincide with \( \tilde{O}_{DG} \) and \( O_G \).

In the present work we consider in detail the implications of \( \tilde{O}_{DG}, O_G \) and \( \tilde{O}_G \), for single and dijet production at the Tevatron and LHC. Thus, in Section 2 we first establish the unitarity constraints on their couplings, which allows us to check the self-consistency of our perturbative treatment, and to obtain an unambiguous connection between the sensitivity limit on the value of each NP coupling, and the NP scale at which the corresponding operator might be generated. In this same Section 2 we also present the observables studied, while the related technical details are put in the Appendix. In Section 3, we present the effects of each of the three operators on the inclusive single jet and the dijet production at the Tevatron (present and upgraded stages), and at the LHC. We compute the rapidity, transverse energy, invariant mass and angular distributions and we examine how they reflect the presence of NP contributions. We then establish the sensitivity limits in terms of couplings constants and new physics scales for each operator separately. In Section 4 we discuss ways to disentangle contributions from the three operators, and give special illustrations using the dijet angular distribution. Finally in Section 5 we compare with the results previously obtained for the analogous electroweak operators involving \( W \) bosons, and we draw the conclusion on the picture of NP which should come out of the whole set of \( \text{dim} = 6 \) \( SU(3) \times SU(2) \times U(1) \) gauge invariant operators.

2 Formalism.

The NP effects are described through the effective Lagrangian

\[
\mathcal{L}_{NP} = f_{DG} \tilde{O}_{DG} + f_{Gg_s} O_G + \tilde{f}_{Gg_s} \tilde{O}_G,
\]

(4)
where the dimensional NP couplings $f_j$ are conveniently measured in $\text{TeV}^{-2}$.

### 2.1 Unitarity constraints.

We first establish the relations between these dimensional coupling constants and the scales at which unitarity is saturated, following the same method as in \[25, \underline{8}\] and working separately for each operator. From the strongest partial wave unitarity constraint, we derive a relation between the dimensional coupling $f_j$ and the energy at which unitarity is saturated, which is denoted as $\Lambda_U$. It can be interpreted as a practical definition of the NP scale; (elsewhere denoted by $\Lambda_{NP}$ \[25, \underline{8}\]). This procedure allows us to unambiguously associate to each value of the NP coupling $f_j$, a corresponding value of the scale $\Lambda_U$.

Thus, for $\mathcal{O}_{DG}$, the strongest constraint arises from the two flavour- and colour-singlet $J = 0$ channels $|qq \pm \pm\rangle$ and is given by

$$|f_{DG}| = \left(\frac{3}{8\alpha_s}\right) \frac{1}{\Lambda_U^2} \simeq 3.7 \Lambda_U^2 .$$

(5)

For $\mathcal{O}_G$, the strongest constraint arises from the colour-singlet $J = 0$ channels $|gg \pm \pm\rangle$ and it is given by

$$f_G = \left(\frac{10}{\sqrt{g_s}} + g_s\right) \frac{1}{\Lambda_U^2} \simeq \frac{10.4}{\Lambda_U^2} \quad \text{for } f_G > 0 ,$$

$$f_G = \left(\frac{-10}{\sqrt{g_s}} + g_s\right) \frac{1}{\Lambda_U^2} \simeq \frac{-8}{\Lambda_U^2} \quad \text{for } f_G < 0 .$$

(6)

The same channels also give the strongest constrain for the $\tilde{\mathcal{O}}_G$ operator, which now leads to

$$|\tilde{f}_G| = \left(\frac{10}{\sqrt{g_s}}\right) \frac{1}{\Lambda_U^2} \simeq \frac{9.5}{\Lambda_U^2} .$$

(7)

These relations allow us to check the self-consistency of our perturbative results, in the sense that we can check that the values of the NP couplings used, are such that the associated $\Lambda_U$ scale is larger than the effective subprocess energy. In other words, $s \lesssim \Lambda_U^2$ is a necessary condition in order to guaranty that the description of NP in terms of $\text{dim} = 6$ operators is valid, for the process considered.

### 2.2 Jet production at hadron colliders.

The observables at a hadron collider that we consider are the inclusive one jet $pp \rightarrow j \ldots$, and the exclusive\[1\] two jet production $pp \rightarrow i j \ldots$ production, where $i$, $j$ denote the light quark or gluon jet produced. In the single jet study we consider the distribution

$$\frac{d\sigma(pp \rightarrow j \ldots)}{d\eta dx_T} ,$$

(8)

\[1\]As far as the number of jets is concerned.
where \((\eta, E_T)\) are the pseudorapidity and transverse energy of the observed jet. While, in the two jet case, we discuss the invariant mass \(M_{ij}\) and angular \(\chi\) distribution

\[
\frac{d\sigma(pp \to i j \ldots)}{dM_{ij}^2 d\chi}
\]

where \(\chi = (1 + |\cos \theta^*|)/(1 - |\cos \theta^*|)\), and \(\theta^*\) is the subprocess scattering angle in the parton c.m. frame. The relevant formulae expressing these distributions in terms of parton structure functions and subprocess cross sections, are given in Appendix A.

3 Results

a) General features.

The NP contributions to the squared amplitudes of the available subprocesses with massless partons, averaged (summed) over the initial (final) spins and colours, are given for completeness in Table 1, where we only keep terms up to quadratic in the NP couplings. The results for \(O_{DG}, O_G\) have been first derived in [16, 21], where it has also been observed that the \(O_G\) contribution does not interfere with the QCD amplitudes for massless partons; while for the \(\tilde{O}_{DG}\) case a non vanishing interference with QCD exists. This is reflected by the fact that in the results in Table 1, there exist linear contributions in \(f_{DG}\), but only quadratic in \(f_G\). Because of this, the operator \(\tilde{O}_{DG}\) produces strong effects on quark-(anti)quark subprocesses at high energies\(^2\) and consequently the sensitivity to its coupling \(f_{DG}\) is quite large. Such a greater sensitivity is also confirmed by the value of the corresponding unitarity scale, which is found to be much higher than the average energy of the subprocess, (see below).

The operators \(O_G\) and \(\tilde{O}_G\), give identical contributions to the spin averaged (summed) squared amplitudes for all \(2 \rightarrow 2\) subprocess involving massless partons; see Table 1. For both operators, but for different basic reasons (in the \(O_G\) case due to the helicity dependence \([10]\), and in the \(\tilde{O}_G\) one due to the CP violation), there is no interference between NP and the QCD amplitudes, so that the dominant contributions only arise at the quadratic level, \(f_G^2\) or \(\tilde{f}_G^2\) level. Such contributions are in general on the same footing as the contributions induced by possible \(\text{dim}=8\) operators, which happen to have a non vanishing interference with the standard QCD results\(^3\). As a consequence the sensitivity is weaker than in the \(\tilde{O}_{DG}\) case. Thus, if some indications of a non vanishing contribution from these operators is found, then a detail program for disentangling them from possible \(\text{dim}=8\) contributions should be pursued. The disentangling of these two operators among themselves also requires a difficult study of CP-odd observables. We will come back to this point in the conclusion.

In spite of these difficulties, we have found that observable effects due to the operators \(O_G\) or \(\tilde{O}_G\), can be obtained for values of the coupling constants for which our perturbative

\(^2\)This is essentially due to the fact that it induces a kind of renormalization of the gluon propagator, which is so strong that it cancels its \(1/q^2\) behaviour, and forces the four quarks to interact locally.

\(^3\)Contributions linear to \(f_G\) first appear in three jet effects; see end of Section 3 below [23].
treatment is acceptable; i.e. for NP couplings which are below (but close to) the unitarity limit for all relevant subprocess energies. Thus, acceptable values of the couplings exist, for which the effect of $\mathcal{O}_G$ or $\tilde{\mathcal{O}}_G$ on single jet production is similar to the one due to $\overline{\mathcal{O}}_{DG}$. This raises the problem of disentangling the contributions arising from the $\overline{\mathcal{O}}_{DG}$ on the one hand, and the sum of the squares of the couplings of the operators $\mathcal{O}_G$ and $\tilde{\mathcal{O}}_G$ on the other hand. We discuss it in the next section and in the conclusion.

b) Applications to Tevatron and LHC.

We next turn to the discussion of the results presented in Fig. 1. For any physical quantity, we always plot the relative variation induced by NP with respect to the QCD prediction. Such a presentation is very convenient since it can be adequately calculated using the leading log QCD formalism. In Fig. 1a we give the relative NP effect for the $E_T$ distribution at the Tevatron. The results are averaged over $\eta$ in the region $0.1 < |\eta| < 0.7$, and the CDF [5] and the D0 [26] data are also shown [27]. In Fig. 1b the same results are given for LHC.

As mentioned already, the $\mathcal{O}_G$ ($\tilde{\mathcal{O}}_G$) prediction is independent of the sign of the NP coupling and it always enhances the QCD expectation. On the contrary, if the source of NP is $\overline{\mathcal{O}}_{DG}$, then an enhancement appears essentially only for negative $f_{DG}$ values. In the examples given in the figures, the magnitudes of the $f_G$, $f_{DG}$ (for $f_{DG} < 0$) were chosen, so that they induce effects of similar magnitudes for the $E_T$ distributions. For the Tevatron case in particular, the NP effects are chosen so that they are roughly consistent with an NP enhancement like the one tolerated by the CDF data. In Fig. 1a,b we also give the scales $\Lambda_U$ where the NP interactions would saturate unitarity. We have checked that the relevant subenergies of all partonic processes, are sufficiently below the corresponding $\Lambda_U$ scales, so that our perturbative treatment is safe. Thus, an attempt to understand an NP effect of e.g. the order suggested by the Tevatron data in terms of $\overline{\mathcal{O}}_{DG}$, necessitates couplings very much below the unitarity limit. On the contrary, if we try to understand the same kind of effect in terms of $\mathcal{O}_G$ (or $\tilde{\mathcal{O}}_G$), we need an NP coupling close to (but still below) the unitarity limit.

To appreciate the sensitivity of the future hadron colliders to the $E_T$ distribution shown in Fig. 1a,b, we define the signal $S$ and background $B$ by the number of events for

$$S = (NP + QCD)^2 - (QCD)^2, \quad B = (QCD)^2.$$  \hfill (10)

For the upgraded 2 TeV Tevatron with luminosity $\mathcal{L} = 10^4 pb^{-1}$, we consider the events in the range $0.1 < |\eta| < 0.7$ and $0.06 < E_T < 0.65$, which then give

$$\overline{\mathcal{O}}_{DG} \quad \Rightarrow \quad \frac{S}{\sqrt{B}} = \{-3 f_{DG} + 0.31 f^2_{DG}\} \sqrt{\mathcal{L}}, \quad \hfill (11)$$

$$\mathcal{O}_G, \tilde{\mathcal{O}}_G \quad \Rightarrow \quad \frac{S}{\sqrt{B}} = 0.011 f^2_G \sqrt{\mathcal{L}} \quad \hfill (12)$$

where the NP couplings are measured in $TeV^{-2}$, and we only keep terms up to quadratic in the NP couplings. The meaning of (12) is that an NP signal at the upgraded Tevatron, like the one shown by the curves in Fig. 1a for $f_{DG} = -0.3$, would imply an enhancement of
about $S/\sqrt{B} = 92$ standard deviations, with respect to the QCD expectations. Similarly, for the $\mathcal{O}_G$ ($\bar{\mathcal{O}}_G$) case with $f_G = \pm 6$ ($\bar{f}_G = \pm 6$), the results would imply effects of $S/\sqrt{B} = 39.6$ standard deviations. This suggests that the upgraded Tevatron could improve present Tevatron sensitivity limits by roughly an order of magnitude.

Correspondingly for LHC, integrating over all events in the range $0.1 \leq |\eta| \leq 0.7$ and $0.1 < E_T < 3.5 \text{ TeV}$, we get

$$\mathcal{O}_{DG} \implies \frac{S}{\sqrt{B}} = \left\{-11.2 f_{DG} + 4.16 f_{DG}^2\right\} \sqrt{\mathcal{L}}, \quad (13)$$

$$\mathcal{O}_G \implies \frac{S}{\sqrt{B}} = 0.44 f_G^2 \sqrt{\mathcal{L}}. \quad (14)$$

Eqs. (13, 14) indicate that LHC should be sensitive to effects of the order of magnitude shown in Fig.1b. This is inferred from the fact that using $f_{DG} = -0.005$ in (13), gives $S/\sqrt{B} = 5$; while from $f_G = \pm 0.2$ or $\bar{f}_G = \pm 0.2$ we get $S/\sqrt{B} = 1.8$.

Summarizing the $E_T$ discussion we conclude that, if we assume that NP observability demands $S/\sqrt{B} \gtrsim 10$, then an enhancement due to an $\mathcal{O}_{DG}$ contribution will be observable at the upgraded Tevatron, if $-f_{DG} \gtrsim 0.03 \text{ TeV}^{-2}$ ($\Lambda_U \lesssim 11 \text{ TeV}$). Correspondingly at LHC, $-f_{DG}$ can go down to $0.009 \text{ TeV}^{-2}$, which means that the highest $\mathcal{O}_{DG}$ scale to which LHC is sensitive, is $\Lambda_U \simeq 20 \text{ TeV}$. In both cases, the $\mathcal{O}_{DG}$ couplings are much below the unitarity limits for the suitable average energies of the various subprocesses. We also note that in this case, if $f_{DG} > 0$, then we could even end up in a situation where no NP enhancement is actually realized at the present Tevatron with $\sqrt{s} = 1.8 \text{ TeV}$, while at LHC we might even have a depression; see Figs.1a,b.

For the $\mathcal{O}_G$ and $\bar{\mathcal{O}}_G$ cases on the other hand, the corresponding sensitivity limits to the NP coupling are at $f_G \simeq 3.0 \text{ TeV}^{-2}$ ($\Lambda_U \simeq 1.7 \text{ TeV}$) for the upgraded Tevatron, and $0.48 \text{ TeV}^{-2}$ ($\Lambda_U \simeq 4.3 \text{ TeV}$) for LHC. These limits (arising from quadratic NP contributions) are in fact as large as they can possibly be, in consistency with our perturbative treatment.

Further information on the $\mathcal{O}_G$ couplings could also be obtained [23] by looking at 3-jet final states, in which the $\mathcal{O}_G$ contribution is enhanced by its appearance at the linear level, but at the same time it is suppressed by an extra power of $\alpha_s$. In [23] an application was made for the Tevatron. It was shown that in the domain where 2 jets are collinear, the azimuthal distribution ($\cos 2\phi$) is especially sensitive to the $\mathcal{O}_G$ contribution. Observability limits taking into account all experimental constraints have not yet been obtained using this method, but we feel that they are more or less of the same size as those in the dijet studies, for which the expected sensitivity limits lie close to the unitarity bounds. The 2-jet and 3-jet final states give therefore complementary information and should both be studied at future machines in the search of NP interactions.


4 Disentangling of the three operators

We have seen above that the NP effects in the $E_T$ distribution, induced by $\mathcal{O}_{DG}$ for $f_{DG} < 0$, could be very similar to those implied by $\mathcal{O}_G$ or $\tilde{\mathcal{O}}_G$. In \cite{16} it was already noticed that the angular distribution (see the $t$ and $u$ dependence in Table 1) should allow distinguishing $\mathcal{O}_{DG}$ from $\mathcal{O}_G$. The interest in the angular distribution is further enhanced by remarking that it depends on different features of the structure functions than the $E_T$ distribution, and can therefore be used to test whether a possible $E_T$ effect really necessitates an NP explanation \cite{27}. To study this in more detail, we present in Fig.2 the relative variation to the $\chi$ distribution induced by NP. Notice, that if the NP contribution had exactly the same $\chi$ shape as in QCD, then the NP curves in this figure, would had been completely flat. In order to increase the NP sensitivity as much as possible, we have selected the highest range of dijet invariant mass, compatible with the requirement to have an acceptable number of events. In Fig.2 we use the same values of the NP couplings as in Fig.1 and we also give the CDF \cite{28} and D0 \cite{29} data. We also note that because of $M_{jj} = E_T(\chi + 1)/\sqrt{\chi}$, we get $E_T \gtrsim 80$ GeV, for all points in Fig.2; which means that the chosen NP couplings are always below the unitarity limit and the perturbative treatment is safe.

According to Figs.2a,b, $\mathcal{O}_{DG}$ for $f_{DG} < 0$, and $(\mathcal{O}_G, \tilde{\mathcal{O}}_G)$, always enhance the number of central events; with the $(\mathcal{O}_G, \tilde{\mathcal{O}}_G)$ enhancement being the strongest one. On the other hand for positive $f_{DG}$ (e.g. $f_{DG} = 0.3$ TeV$^{-2}$), we could even have a depression in the number of events at the upgraded Tevatron and LHC, (with the depression being somewhat stronger in the central region); while no effect would actually arise at the present Tevatron with $\sqrt{s} = 1.8$ TeV.

The difference of the $\chi$-shapes does not look observable at the present Tevatron because of the large errors of existing data; compare Figs.2a,b. But at the upgraded Tevatron and the LHC, because of the larger energy and the expected luminosity $\mathcal{L} = 10^4 pb^{-1}$, the statistical errors should decrease sufficiently to allow discrimination of the two shapes. This is suggested by the comparison of Figs.2c,d with Table 2, where the signal over background ratio (see (10)) is given for the indicated $\chi$-bins and sufficiently high dijet mass. Table 2 implies that for the upgraded Tevatron and the $\chi$ bins indicated there, we get $S/\sqrt{B} = (48.7, 21, 13)$ for $f_{DG} = -0.3$, and $S/\sqrt{B} = (39.6, 9, 2.8)$ for $f_G = \pm 6$ (or $\tilde{f}_G = \pm 6$). This difference in the $\chi$ shape, combined with the similarity of the $E_T$ distributions, should allow disentangling between $\mathcal{O}_{DG}$ and $(\mathcal{O}_G, \tilde{\mathcal{O}}_G)$, at least for couplings of the order of those indicated in the figures. In the LHC case, the results in Table 2 for all events in the range $1 < \chi < 5$, give $S/\sqrt{B} = 10.6$ for $f_{DG} = -0.005$, and $S/\sqrt{B} = 15.5$ for $f_G = \pm 0.2$ or $\tilde{f}_G = \pm 0.2$. Thus, in the predictions shown in Fig.2d, there is a $\sim 5$ standard deviation difference between the $\mathcal{O}_{DG}$ and the $(\mathcal{O}_G, \tilde{\mathcal{O}}_G)$ results for $1 < \chi < 5$, and essentially no difference for higher $\chi$. Therefore, it should be possible to discriminate between these two alternatives at LHC, at the level of the NP couplings used in the figure.

A useful quantity for describing the dependence of the $\chi$-shape on the dijet mass, is
$R_\chi$ defined by the ratio \cite{28, 29}

$$R_\chi = \frac{N(1 < \chi < \chi_1)}{N(\chi_1 < \chi < \chi_{\text{max}})} . \quad (15)$$

In Fig. 3 we plot the relative variation of $R_\chi$, with respect to the QCD prediction. The same NP couplings are used, as in the previous figures. The results in Figs.3a correspond to the present Tevatron and are compared with the CDF data using $\chi_1 = 2.5$, $\chi_{\text{max}} = 5$ \cite{28}; while in Fig.3b the D0 data for four experimental points with $\chi_1 = 4$ and $(\chi_{\text{max}} = 20, \ 20, \ 13, \ 11)$ \cite{29}, are compared with the corresponding NP prediction. Finally Fig.3c,d examples of NP effects are given using the same couplings as before.

Concerning the CDF data, it is amusing to remark that although the $O_G$, $\tilde{O}_G$ prediction for the $\chi$ distribution in Fig.2a is not particularly close to these data, the $R_\chi$ prediction seems to follow their central values. This simply emphasizes again the point made in the introduction that the discovery of NP through studies of enhancement or depression effects, necessitates that all possible interpretations and ways of analysing the data should be tried.

An independent way of disentangling the effects of $O_G$ was recently discussed in \cite{24} through top+antitop quark production. For heavy quarks, a contribution to the cross section linear in $f_G$ and proportional to $m_t^2$ appears. It is for example found that this operator contributes more strongly to the transverse momentum distribution, than the other gluonic operators and the $\text{dim} = 6$ $SU(3) \times SU(2) \times U(1)$ gauge invariant operators involving directly the top quark. So this leads us to expect that a global study of light quark and gluon jets and heavy quark production might allow a good disentangling of the various operators.

5 Conclusion

In this paper we have studied the 3 purely gluonic operators $\overline{O}_{DG}$ and $O_G, \tilde{O}_G$. We have followed the same method that we used for the other $\text{dim} = 6$ $SU(3) \times SU(2) \times U(1)$ gauge invariant operators involving gauge bosons, Higgs bosons and heavy quarks \cite{30}.

We have first established the unitarity constraints relating the coupling constants to the energy scale at which unitarity is saturated. This allows to associate unambiguously to the coupling constant of each operator, an effective NP scale where this operator is generated. We have then proceeded to a phenomenological analysis of the effects of these operators in hadron collisions at present and future machines. More precisely we have concentrated on single jet and dijet production at the Tevatron and at the LHC. On the basis of these, we determined the observability limits for each operator, which we describe below in terms of the NP scale $\Lambda_U$ that can be possibly probed.

The operator $\overline{O}_{DG}$ (whose effect is essentially a renormalization of the gluon propagator) produces strong effects on quark (anti)-quark subprocesses. The sensitivity limits correspond to an NP scale

$$\Lambda_U = 11, \ 20 \ TeV \quad (16)$$
at the upgraded Tevatron and LHC respectively. This operator is the gluonic analogue of the operator $\bar{O}_{DW}$, which also produces direct modifications of the W boson propagator and which is already strongly constrained at LEP1/SLC by $\Lambda_U \approx 10 \text{ TeV}$, and should be further studied at LEP2 and NLC with the expected sensitivities of 17 and 38 TeV respectively, see [31].

The two other operators, $O_G$ and its CP-violating partner $\tilde{O}_G$, lead to identical effects in all two-body processes involving massless unpolarized quarks and gluons. They contribute through diagrams involving their anomalous three-gluon coupling. Due to a different helicity dependence, the part of the amplitude which is linear in this anomalous term does not interfere with the SM amplitude. The anomalous contribution to the cross section only starts with quadratic terms [10]. The sensitivity to these processes is then weaker than the one to $O_{DG}$. In terms of NP scales one now finds

$$\Lambda_U = 1.7, \quad 4.3 \text{ TeV}$$

for the upgraded Tevatron and LHC respectively. These operators are the analogues of the operators $O_W$ and $\bar{O}_W$ which have been extensively studied in $e^+e^- \rightarrow W^+W^-$ and $\gamma\gamma \rightarrow W^+W^-$, and whose effects are also much less constrained than the $\bar{O}_{DW}$ ones[1]. Thus, the expected sensitivity scales for $O_W$ and $\bar{O}_W$ are 1.5 TeV at LEP2 and 10 TeV at NLC, see [32].

So there is an interesting parallelism between the situation for gluonic and for electroweak operators.

For disentangling the effects of the three gluonic operators, it is difficult to pursue the analogy with W-boson operators. In the electroweak case, the disentangling of $\bar{O}_{DW}$ from $O_W$ and $\bar{O}_W$ is obvious, because only $\bar{O}_{DW}$ contributes directly to 4-fermion processes, whereas the other two operators contribute directly to W pair production. For the gluonic operators the situation is the same, replacing fermions by quarks and W bosons by gluons; but it is now very difficult (or even impossible for light quarks), to identify the respective contributions of quarks and gluons in the initial and final states. In the case of heavy quarks $\bar{O}_{DG}$ contributes to $q\bar{q} \rightarrow t\bar{t}$ but not to $gg \rightarrow t\bar{t}$, whereas $O_G$ and $\tilde{O}_G$ contribute to $gg \rightarrow t\bar{t}$ but not to $q\bar{q} \rightarrow t\bar{t}$; but the game becomes more complex because genuine operators involving directly the top quark may contribute [24].

In the present paper we have found that the disentangling of $\bar{O}_{DG}$ effects from those of $O_G$ and $\tilde{O}_G$ can be achieved by looking at the dijet angular distribution. The $\bar{O}_{DG}$ contribution is closer to the SM one (particularly for $f_{DG} > 0$), whereas the $O_G$ and $\tilde{O}_G$ contributions are more concentrated in the central region. The use of the parameter $R_X$, essentially the ratio of the number of events in the central and in the peripheral regions, allows to clearly separate these two types of distributions. This study however requires an important luminosity which will only be available at upgraded Tevatron and LHC.

The disentangling of the CP-conserving $O_G$ from the CP-violating $\tilde{O}_G$ is much more difficult. In the $W$ case, methods were proposed using asymmetries in $W^+$ and $W^-$ decay distributions in the process $e^+e^- \rightarrow W^+W^-$ [33], and $\gamma\gamma \rightarrow W^+W^-$ [34] for polarized

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4The reason is that they contribute only indirectly at Z peak.
initial states. There is no analogue possibility here. The only solution seems to be a direct study of CP violation in multijet production processes. One should then extend the analysis done in \[23\] and consider CP-odd observables in order to reach the effect due to $\tilde{O}_C$, for example along the lines of the study done for $Z$ decay into three jets $\[35\].

When all these informations will be available from experiments, one should have at our disposal a clear panorama of residual NP effects expressed in terms of $\text{dim} = 6$ $SU(3) \times SU(2) \times U(1)$ gauge invariant operators. We have emphasized the parallelism between the operators involving gluons and the ones involving $W$ bosons. This should tell us about the role of colour in the underlying NP dynamics. More generally, a comparison of the possible effects or of the upper limits in the various sectors (electroweak gauge sector, Higgs sector, heavy quark sector, gluonic sector) should give valuable informations on the nature and the origin of the NP dynamics.

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Appendix A: Kinematics for one jet and two jet studies
When the hadrons $A$ and $B$ collide inelastically through their respective partons $a$ and $b$, the transverse energy $E_T$ and rapidity distributions of an inclusively produced jet $j$ associated with the light parton $c$, are expressed as $\[30\]

$$
\frac{d\sigma(AB \to j \ldots)}{d\eta dx_T} = \frac{\pi e^\eta \alpha_s(\mu)^2}{s} \sum_{abcd} \int_{x_{1\min}}^1 \frac{dx_1}{x_1} f_{a/A}(x_1, \mu) f_{b/B}(x_2, \mu) |M(ab \to cd)|^2 \frac{1}{1 + \delta_{cd}} \quad (A.1)
$$

Here, all parton masses are neglected,

$$
x_T = \frac{2E_T}{\sqrt{s}} , \quad x_{1\min} = \frac{x_T e^\eta}{2 - x_T e^{-\eta}} , \quad x_2 = \frac{x_1 x_T e^{-\eta}}{2x_1 - x_T e^{\eta}} , \quad (A.2)
$$

$(\eta, E_T)$ are the pseudorapidity and transverse energy of the observed jet, and the QCD scale $\mu$ is usually taken equal to $E_T/2$, or $E_T$. In $\[A.1\],$

$$
F(ab \to cd) \equiv g_s^2 M(ab \to cd) \quad (A.3)
$$

denotes the invariant amplitude of the partonic subprocess $(ab \to cd)$. $|M(ab \to cd)|^2$ is the squared reduced amplitude averaged (summed) over the initial (final) spins and colours and the Mandelstam parameters are written as

$$
\hat{s} = sx_1 x_2 , \quad \hat{t} = -x_1 \sqrt{s} E_T e^{-\eta} , \quad \hat{u} = -\hat{s} - \hat{t} \quad , \quad (A.4)
$$
where $s$ is the c.m. energy-squared of the Collider.

The $\chi \equiv (1 + |\cos\theta^*|)/(1 - |\cos\theta^*|)$ and invariant mass $M_{ij}^2 \equiv \hat{s}$ distributions for the dijet are similarly given by [36]

$$
\frac{d\sigma(AB \to i j \ldots)}{dM_{ij}^2 d\chi} = \frac{2\pi\alpha_s^2(\mu)}{sM_{ij}^2(1 + \chi)^2} \cdot \sum_{abcd} \int_{\eta_{lim}}^{\eta_{lim}} d\bar{\eta} f_{a/A}(x_1, \mu) f_{b/B}(x_2, \mu) |\mathcal{M}(ab \to cd)|^2 \frac{1}{1 + \delta_{cd}},
$$

(A.5)

where $\bar{\eta}$ is the rapidity of the c.m. of the pair of the two produced jets and

$$
x_1 = \frac{M_{jj}}{\sqrt{s}} e^{\bar{\eta}}, \quad x_2 = \frac{M_{jj}}{\sqrt{s}} e^{-\bar{\eta}},
$$

(A.6)

$$
|\bar{\eta}| \leq \eta_{lim} \equiv \min \left\{ \ln \left( \frac{\sqrt{s}}{M_{jj}} \right), \eta_c - \frac{1}{2} \ln \chi \right\}.
$$

(A.7)

In (A.7), $\eta_c$ denotes the imposed cut in the pseudorapidities of each of the two jets; i.e. $|\eta_1| \leq \eta_c$, $|\eta_2| \leq \eta_c$. In all numerical applications here we take $\eta_c = 2$.

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Table 1: $\mathcal{O}_{DG}$ and $\mathcal{O}_G$, $\tilde{\mathcal{O}}_G$ corrections to $\sum |F|^2 / g^4$ for the indicated parton subprocesses with massless partons. The colour and spin indices are averaged (summed) over final (initial) states; $n_f$ is the number of final light quark flavours.

| Process                          | $\mathcal{O}_{DG}$                                                                 |
|----------------------------------|------------------------------------------------------------------------------------|
| $qq \rightarrow qq$              | $- (\frac{88 f_{DG}}{27 t u}) (s^2 + 9t^2 + 9u^2) + \left(\frac{32 f_{DG}^2}{9}\right) (\frac{4s^2}{3} + \hat{u}^2 + \hat{t}^2)$ |
| $q q'(\bar{q}') \rightarrow q q'(\bar{q}')$ | $- (\frac{16s f_{DG}}{9u t}) (s^2 + 3t^2 + 3u^2) + \left(\frac{64 f_{DG}^2}{9}\right) (2s^2 + \hat{u}^2 + \hat{t}^2)$ |
| $q\bar{q} \rightarrow q\bar{q}$  | $\left(\frac{32 f_{DG}}{9}\right) \left(\frac{2s^3}{tu} + 3s + \hat{u}^2 + \hat{t}^2 + 2n_f \frac{\hat{t}^2 + \hat{u}^2}{s}\right) + \left(\frac{64 f_{DG}^2}{9}\right) \left[2s^2 + \frac{6n_f + 1}{3}(\hat{u}^2 + \hat{t}^2)\right]$ |

| Process                          | $\mathcal{O}_G$ or $\tilde{\mathcal{O}}_G$                                      |
|----------------------------------|------------------------------------------------------------------------------------|
| $q\bar{q} \rightarrow gg$       | $\frac{4}{3} \bar{u} t f_G^2$                                                     |
| $g q(\bar{q}) \rightarrow g q(\bar{q})$ | $\frac{1}{2} s \bar{u} f_G^2$                                                     |
| $gg \rightarrow q\bar{q}$       | $\frac{3}{16} \bar{u} t f_G^2$                                                    |
| $gg \rightarrow gg$             | $\frac{171}{32} [s^2 - \hat{u}] f_G^2$                                           |

Table 2: $S/\sqrt{B}$ ratio for various $\chi$ bins; ($\mathcal{L} = 10^4 pb^{-1}$).

| $\chi$ | $\mathcal{O}_{DG}$ | $\mathcal{O}_G$, $\tilde{\mathcal{O}}_G$ |
|---------|---------------------|------------------------------------------|
| $1 < \chi < 1.5$ | $- (1.25 f_{DG} + 1.24 f_{DG}^2) \sqrt{\mathcal{L}}$ | $0.011 f_G^2 \sqrt{\mathcal{L}}$ |
| $4.5 < \chi < 5$ | $- (0.62 f_{DG} + 0.28 f_{DG}^2) \sqrt{\mathcal{L}}$ | $0.0025 f_G^2 \sqrt{\mathcal{L}}$ |
| $9.5 < \chi < 10$ | $- (0.4 f_{DG} + 0.095 f_{DG}^2) \sqrt{\mathcal{L}}$ | $0.00079 f_G^2 \sqrt{\mathcal{L}}$ |

| $\chi$ | $\mathcal{O}_{DG}$ | $\mathcal{O}_G$, $\tilde{\mathcal{O}}_G$ |
|---------|---------------------|------------------------------------------|
| $1 < \chi < 1.5$ | $- (9 f_{DG} + 151 f_{DG}^2) \sqrt{\mathcal{L}}$ | $1.95 f_G^2 \sqrt{\mathcal{L}}$ |
| $4.5 < \chi < 5$ | $- (3.59 f_{DG} + 30.53 f_{DG}^2) \sqrt{\mathcal{L}}$ | $0.48 f_G^2 \sqrt{\mathcal{L}}$ |
| $9.5 < \chi < 10$ | $- (1.98 f_{DG} + 9.4 f_{DG}^2) \sqrt{\mathcal{L}}$ | $0.15 f_G^2 \sqrt{\mathcal{L}}$ |
| $1 < \chi < 5$ | $- (19.8 f_{DG} + 303 f_{DG}^2) \sqrt{\mathcal{L}}$ | $3.9 f_G^2 \sqrt{\mathcal{L}}$ |
Figure 1: Possible NP contribution to the $E_T$ distribution for the inclusive jet production averaged over $0.1 \leq |\eta| \leq 0.7$, from $\bar{\sigma}_{DG}$ or $O_G$, $\bar{O}_G$; compared with the CDF and D0 data at the Tevatron [5, 26, 27] (a), and a possible LHC signal (b). In the $\bar{O}_G$ case the results correspond to $f_G$ equal to the indicated $f_G$. 
Figure 2: Possible NP contribution from $\bar{O}_{DG}$ or $O_G$, $\bar{O}_G$ to the dijet angular distribution compared with the CDF [28] (a) and D0 [29] (b) data, and a possible signal at the upgraded Tevatron (c) and LHC (d). In the $\bar{O}_G$ case the results correspond to $f_G$ equal to the indicated $f_G$. 
Figure 3: Possible NP contribution from $\mathcal{O}_{DG}$ or $\mathcal{O}_G$, $\tilde{\mathcal{O}}_G$ to the $R_\chi$ dependence on the dijet-mass compared with the CDF [28] (a) and D0 [29] (b) data; the upgraded Tevatron with $\chi_1 = 3.5$, $\chi_{\text{max}} = 7$ (c); and the LHC with $\chi_1 = 5$, $\chi_{\text{max}} = 10$ (d); (see text). In (b) the theoretical points are offset in mass, to allow distinction from the experimental ones. In the $\tilde{\mathcal{O}}_G$ case the results correspond to $\tilde{f}_G$ equal to the indicated $f_G$. 
**Figure captions**

**Fig.1** Possible NP contribution to the $E_T$ distribution for the inclusive jet production averaged over $0.1 \leq |\eta| \leq 0.7$, from $\mathcal{O}_{DG}$ or $\mathcal{O}_G$, $\tilde{\mathcal{O}}_G$; compared with the CDF and D0 data at the Tevatron $[3, 26, 27]$ (a), and a possible LHC signal (b). In the $\tilde{\mathcal{O}}_G$ case the results correspond to $\tilde{f}_G$ equal to the indicated $f_G$.

**Fig.2** Possible NP contribution from $\mathcal{O}_{DG}$ or $\mathcal{O}_G$, $\tilde{\mathcal{O}}_G$ to the dijet angular distribution compared with the CDF $[28]$ (a) and D0 $[29]$ (b) data, and a possible signal at the upgraded Tevatron (c) and LHC (d). In the $\tilde{\mathcal{O}}_G$ case the results correspond to $\tilde{f}_G$ equal to the indicated $f_G$.

**Fig.3** Possible NP contribution from $\mathcal{O}_{DG}$ or $\mathcal{O}_G$, $\tilde{\mathcal{O}}_G$ to the $R_\chi$ dependence on the dijet-mass compared with the CDF $[28]$ (a) and D0 $[29]$ (b) data; the upgraded Tevatron with $\chi_1 = 3.5$, $\chi_{max} = 7$ (c); and the LHC with $\chi_1 = 5$, $\chi_{max} = 10$ (d); (see text). In (b) the theoretical points are offset in mass, to allow distinction from the experimental ones. In the $\tilde{\mathcal{O}}_G$ case the results correspond to $\tilde{f}_G$ equal to the indicated $f_G$. 