Effect of Hairpin Diagram on Two-body Nonleptonic $B$ Decays and $CP$ Violation

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Abstract

A careful quark-diagram analysis shows that a number of two-body nonleptonic $B$ decays can occur through the so-called hairpin diagram, a QCD loop-induced graph different from penguin in final-state hadronization of valence quarks. Using the two-loop renormalization-group-improved effective Hamiltonian and the naive factorization approximation, we demonstrate the effect of the hairpin diagram on decay rates and $CP$ asymmetries for a few interesting channels such as $B^0_d, \bar{B}^0_d \to \psi K_S$ and $B^0_u, \bar{B}^0_u \to \phi K^\pm$. Branching ratios of some pure hairpin decay modes, e.g., $B^-_u \to \phi \pi^-, \phi \rho^-$ and $\bar{B}^0_d \to \phi \pi^0, \phi \rho^0, \phi \omega, \phi \eta$, etc., are estimated to be on the order of $10^{-7}$.
I Introduction

The study of nonleptonic weak decays of $B$ mesons is very useful for determining the quark mixing parameters, probing the origin of $CP$ violation, and investigating the nonperturbative final-state interactions. In practice, the exclusive two-body mesonic $B$ decay modes are of more immediate interest in experiments and can be predicted using approximate or empirical methods in theory [1]. To the lowest order of the weak interactions, the physical picture of a $B$ meson decaying into two light mesons may be described very well by a few distinct quark diagrams [2], and the decay amplitude can be calculated by using the effective low-energy Hamiltonian for $\Delta B = \pm 1$ transitions. In employing the recently-presented two-loop renormalization-group-improved effective Hamiltonian $\mathcal{H}_{\text{eff}}(\Delta B = \pm 1)$ [3] to analyze amplitudes for some two-body decays of $B$ mesons, we follow the spirit of the spectator approximation (i.e., neglect Okubo-Zweig-Iizuka (OZI) suppressed and annihilation terms) and find that there appears a new term which can not correspond to any of the conventional six quark diagrams classified by Chau [2,4]. A detailed topological reanalysis shows that this new term can be described by a color-matched loop diagram, the so-called hairpin diagram (see Fig.1(d) and Fig.2). In this letter, we are going to demonstrate the role of the hairpin diagram in a number of two-body mesonic $B$ decays. The case for three-body decays will be discussed elsewhere. By means of the approaches of the quark diagram analysis and the naive factorization approximation [5], we find a few pure hairpin channels such as $B^- \rightarrow \phi \pi^-$, $\phi \rho^-$ and $B^0 \rightarrow \phi \pi^0, \phi \rho^0$, whose branching ratios are on the order of $10^{-7}$. We also demonstrate the hairpin contributions to $B^0_d \rightarrow \psi \bar{K}^0$, $B^-_u \rightarrow \phi K^-$, etc., which are interesting for probing $CP$ violation in the $B$-meson system [6]. With the help of the future experimental data on nonleptonic $B$ decays, we are sure that the detailed analysis made here will be helpful for investigating the rare nonleptonic $B$ decays, testing the short-distance QCD calculations at the low-energy scale, and extracting the information of $CP$ violation.

II Quark diagram analysis

According to the topology of the lowest order weak interactions with all QCD strong interaction effects included, the inclusive nonleptonic decays of a heavy meson can be
generally described by six distinct quark diagrams as illustrated in Ref.[2]. For exclusive two-body mesonic decays one needs to combine final-state quarks into specific hadron states. In some graphs an extra quark-antiquark pair needs to be created in order to produce a pair of light mesons in the final state. Assuming that $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs are produced with equal probability from the vacuum, as implied by $SU(3)$ symmetry, the final-state quarks are then allowed to hadronize and arrange themselves in all possible ways. As a whole, ten distinct quark diagrams can be formed and contribute to two-body mesonic $B$ ($D$) decay modes, in which six of them are either OZI forbidden or formfactor suppressed. Following the spirit of the spectator model and neglecting those OZI suppressed and annihilation topologies, we find that there remains four quark diagrams, as depicted in Fig.1, that may contribute to a two-body mesonic mode. It is worthwhile to remark that Fig.1(d), the so-called hairpin diagram, has been ignored in the previous topological classification [2,4]. In calculations of decay amplitudes by using the traditional QCD-noncorrected penguin Hamiltonian [4,7], such hairpin contributions do not appear.

In the next section we shall analyze the effect of the hairpin diagram on some two-body nonleptonic $B$ decays by means of the two-loop renormalization-group-improved effective Hamiltonian for $\Delta B = \pm 1$ transitions [3] and the naive factorization approximation [5].

Now we classify a number of two-body mesonic $B$ decays which can get contributions from the hairpin diagram. Using the valence-quark notation the two-body hairpin transition

$$B(b\bar{q}_s) \rightarrow X(q_v\bar{q}_v) + Y(q\bar{q}_s)$$

is illustrated in Fig.2 where $q_s = u$ or $d$ denotes the spectator quark; $q_v = u, d, s$ or $c$ denotes the quark from the vacuum; and $q = d$ or $s$ is from the flavor changing transition $b \rightarrow q$. We take the wave functions for some $SU(3)$ mesons as follows [8,4]:

$$|\pi^0 > = \frac{1}{\sqrt{2}}|\bar{u}u - \bar{d}d > , \quad |\omega > = \frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d > ,$$

$$|\rho^0 > = \frac{1}{\sqrt{2}}|\bar{u}u - \bar{d}d > , \quad |\eta > = \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d - \bar{s}s > ,$$

$$|\eta^0 > = \frac{1}{\sqrt{2}}|\bar{u}u - \bar{d}d > , \quad |\eta' > = \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d + 2\bar{s}s > ,$$

and $|\phi > = |\bar{s}s >$. According to the four spectator quark diagrams illustrated in Fig.1, we find four groups of two-body mesonic channels in which the hairpin diagram is involved.
(i) Channels occurring only through the hairpin diagram (Fig.1(d)):

\[
\begin{align*}
B_u^- & \rightarrow \phi\pi^- , \phi\rho^- , \phi a_1^- ; \\
\bar{B}_d^0 & \rightarrow \phi\pi^0 , \phi\rho^0 , \phi a_1^0 , \phi \omega , \phi \eta , \phi \eta'.
\end{align*}
\]  

(3)

It should be noted that these nine rare decay modes have not been discussed before [1].

(ii) Channels occurring through both the penguin diagram (Fig.1(c)) and the hairpin diagram (Fig.1(d)):

\[
\begin{align*}
B_u^- & \rightarrow \phi K^- , \phi K^*^- ; \\
\bar{B}_d^0 & \rightarrow \phi \bar{K}^0 , \phi \bar{K}^{*0} .
\end{align*}
\]  

(4)

In Ref. [9], these four channels were regarded as the pure penguin channels and could be used to probe \(CP\) violation in the decay amplitude. We shall see later on that the hairpin contribution to them is significant in the large \(N_c\) approximation.

(iii) Channels occurring through both the color-mismatched tree-level diagram (Fig.1(b)) and the hairpin diagram (Fig.1(d)):

\[
\begin{align*}
B_u^- & \rightarrow \psi\pi^- , \psi\rho^- , \psi a_1^- , \psi K^- , \psi K^*^- ; \\
\bar{B}_d^0 & \rightarrow \psi\pi^0 , \psi\rho^0 , \psi a_1^0 , \psi \bar{K}^0 , \psi \bar{K}^{*0} , \psi \omega , \psi \eta , \psi \eta'.
\end{align*}
\]  

(5)

Some of these decay modes, e.g., \(\bar{B}_d^0 \rightarrow \psi \bar{K}^0\), are worthwhile to look at because \(B_d^0\) versus \(\bar{B}_d^0 \rightarrow \psi K_S\) are very interesting for extracting \(CP\)-violating signals in the \(B\) system.

(iv) Channels occurring through the tree-level diagram(s) (Fig.1(a) or Fig.1(b)), the penguin diagram (Fig.1(c)) and the hairpin diagram (Fig.1(d)):

\[
\begin{align*}
B_u^- & \rightarrow \omega\pi^- , \eta\pi^- , \eta'\pi^- , \omega\rho^- , \eta\rho^- , \eta'\rho^- , \omega a^-_1 , \eta a^-_1 , \eta' a^-_1 , \\
& \quad \omega K^- , \eta K^- , \eta' K^- , \omega K^{*^-} , \eta K^{*^-} , \eta' K^{*^-} ; \\
B_d^0 & \rightarrow \omega\pi^0 , \eta\pi^0 , \eta'\pi^0 , \omega\rho^0 , \eta\rho^0 , \eta'\rho^0 , \omega a^0_1 , \eta a^0_1 , \eta' a^0_1 , \\
& \quad \omega K^0 , \eta K^0 , \eta' K^0 , \omega K^{*0} , \eta K^{*0} , \eta' K^{*0} , \\
& \quad \omega \omega , \omega \eta , \omega \eta' , \eta \eta , \eta' \eta' .
\end{align*}
\]  

(6)

For our purpose these modes are not as interesting as the above three groups of modes, since it is difficult to distinguish the hairpin contribution from the penguin and tree-level
ones in them.

III Factorization approximation

In this section we use the two-loop low-energy effective Hamiltonian $H_{eff}(\Delta B = \pm 1)$ and the factorization approach to estimate the contribution of the hairpin diagram to some of the aforelisted $B$ decay modes. At the physical scale $\mu = O(m_b)$, $H_{eff}(\Delta B = -1)$ is given by [3]

$$H_{eff}(\Delta B = -1) = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{uq}^* \left( \sum_{i=1}^{2} c_i Q_i^u \right) + V_{cb} V_{cq}^* \left( \sum_{i=1}^{2} c_i Q_i^c \right) - V_{tb} V_{tq}^* \left( \sum_{i=3}^{6} c_i Q_i \right) \right],$$

where $V_{jb} V_{jq}^*$ ($j = u, c, t$; $q = d, s$) are the Cabibbo-Kobayashi-Maskawa (CKM) factors corresponding to $b \rightarrow q$ transition; $c_i (i = 1, ..., 6)$ are the QCD-corrected Wilson coefficients at the physical scale $\mu = O(m_b)$; $Q_i^u$ ($i = 1, 2$) and $Q_i^c$ ($i = 3, ..., 6$) are the current-current and penguin operators respectively. In the notation of Ref.[3], we have

$$Q_1^u = (\bar{q}_\alpha u_\beta)_{V-A} (\bar{u}_\beta b_\alpha)_{V-A}, \quad Q_2^u = (\bar{q} u)_{V-A} (\bar{u} b)_{V-A},$$

$$Q_3 = (\bar{q} b)_{V-A} \sum_{q'} (\bar{q'} q')_{V-A}, \quad Q_4 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q'} q'_\alpha)_{V-A},$$

$$Q_5 = (\bar{q} b)_{V-A} \sum_{q'} (\bar{q'} q')_{V+A}, \quad Q_6 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q'} q'_\alpha)_{V+A};$$

and $Q_i^c$ can be obtained from $Q_i^u$ through the replacement $u \rightarrow c$. For the purpose of numerical illustration, we shall use the following values of $c_i$:

$$c_1 = -0.324, \quad c_2 = 1.151, \quad c_3 = 0.017,$$

$$c_4 = -0.038, \quad c_5 = 0.011, \quad c_6 = -0.047,$$

which are obtained in [3] by taking $\Lambda_{MS}^{(4)} = 350$ MeV, $m_b = 4.8$ GeV and $m_t = 150$ GeV.

Let us estimate branching ratios of a few pure hairpin channels by taking $B_u^- \rightarrow \phi\pi^-$ for example. In the naive factorization scheme, we neglect all the OZI forbidden or annihilation terms and obtain

$$<\phi\pi^- | H_{eff}(\Delta B = -1) | B_u^- >$$

$$= -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* (a_4 + a_6) M_{s\pi}^{\phi\pi^-},$$

(10)
where $a_4$ and $a_6$ are the short-distance QCD coefficients defined by

$$a_{2i-1} \equiv \frac{c_{2i-1}}{N_c} + c_{2i}, \quad a_{2i} \equiv c_{2i-1} + \frac{c_{2i}}{N_c}$$

with $i = 1, 2, 3$; $N_c$ is the number of colors; and $M_{ssd}^{\phi \pi^-}$ is the hadronic matrix elements given as

$$M_{ssd}^{\phi \pi^-} \equiv \langle \phi | (\bar{s}s)_{V-A} | 0 \rangle < \pi^- | (\bar{d}b)_{V-A} | B^-_u \rangle .$$

One can evaluate $|M_{ssd}^{\phi \pi^-}|$ with the help of the successful empirical approach of Bauer, Stech and Wirbel (BSW) [10]. Considering the approximate rule of discarding $1/N_c$ corrections in exclusive nonleptonic $B$ decays [5], we shall take both $N_c = 3$ and $N_c = \infty$ to give one a feeling of $N_c$ dependence in this factorization approach. Our numerical results of branching ratios for some pure hairpin channels are listed in Table 1. We observe that in the case of $N_c = 3$ both $a_4$ and $a_6$ are very small so that the values of branching ratios are vanishingly small. In the case of $N_c = \infty$, however, these branching ratios are on the order of $10^{-7}$.

The factorized decay amplitudes of $B^-_u \rightarrow \phi K^-, \phi K^{*-}$ and $\bar{B}^0_d \rightarrow \phi \bar{K}^0, \phi \bar{K}^{*0}$ are composed of two terms, e.g.,

$$\langle \phi \bar{K}^0 | \mathcal{H}_{eff}(\Delta B = -1) | \bar{B}^0_d \rangle = - \frac{G_F}{\sqrt{2}} V_{td} V_{ts}^* [a_3 + (a_4 + a_6)] M_{sss}^{\phi \bar{K}^0} ,$$

which get contributions from the penguin and hairpin diagrams, respectively. Using the values of $c_i$ given in Eq.(9), we obtain the ratio of the hairpin amplitude to the penguin one for these four modes:

$$\left| \frac{a_4 + a_6}{a_3} \right| \approx \begin{cases} 1.2\% & N_c = 3 \\ 73.7\% & N_c = \infty \end{cases} .$$

Obviously, the hairpin contribution to decay amplitudes is significant if one discards those $1/N_c$ terms. Although the above estimate is based on the naive factorization approximation and can not be strictly justified, it shows that in these penguin-hairpin mixed rare decays the latter contributions may be important and should be considered seriously.

Conventionally a two-body decay mode of $B^-_u$ (or $\bar{B}^0_d$) into $\psi$ and a $SU(3)$ meson is analyzed at the tree level [4,7,9]. Employing $\mathcal{H}_{eff}(\Delta B = -1)$ in Eq.(7), here we examine
the influence of the hairpin diagram on the overall decay amplitudes of such channels. Taking \( B_d^0 \to \psi K^0 \) for example, we obtain

\[
\langle \psi \bar{K}^0 | \mathcal{H}_{\text{eff}}(\Delta B = -1) | \bar{B}_d^0 \rangle = \frac{G_F}{\sqrt{2}} [V_{cb}V_{cs}^* a_2 - V_{tb}V_{ts}^*(a_4 + a_6)] M_{\psi\bar{K}^0}^{\psi\bar{K}^0}. \tag{15}
\]

The ratio of the hairpin amplitude to the tree-level one is

\[
\left| \frac{V_{tb}V_{ts}^*}{V_{ub}V_{ud}^*} \right| \left| \frac{a_4 + a_6}{a_2} \right| \approx \left( 1 + \frac{\lambda^2}{2} \right) \left| \frac{a_4 + a_6}{a_2} \right| \approx \begin{cases} 0.7\% & N_c = 3 \\ 8.8\% & N_c = \infty \end{cases}, \tag{16}
\]

where \( \lambda = 0.22 \) is a CKM parameter [11]. We observe that in the case of \( N_c = \infty \) the hairpin amplitude is not too small compared with the tree-level amplitude, and this enhancement merely comes from the short-distance QCD.

As for the charmless two-body decay modes listed in Eq.(6), the charged \( B \) channels can occur through all the four quark diagrams of Fig.1, while the neutral \( B \) transitions get contributions from Fig.1(b-d). Considering the fact that

\[
\left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right| \sim 1, \quad \left| \frac{V_{tb}V_{ts}^*}{V_{ub}V_{us}^*} \right| \sim \frac{1}{\lambda^2}, \tag{17}
\]

the effect of the penguin and hairpin amplitudes are expected to be very large for those modes corresponding to \( b \to (u\bar{u})s \) such as \( B_u^- \to \omega K^- \) and \( \bar{B}_d^0 \to \eta \bar{K}^*0 \), and to be comparable to the tree-level ones in those modes corresponding to \( b \to (u\bar{u})d \) such as \( B_u^- \to \eta \rho^- \) and \( \bar{B}_d^0 \to \omega a_1^0 \). Since these channels are tree-loop mixed, they are not very interesting for probing the loop-induced effects in branching ratios and \( CP \) asymmetries.

### IV Discussion

Now we are in a position to discuss the effect of the hairpin diagram on \( CP \) asymmetries for a few interesting \( B \) decay modes such as \( B^0_d, \bar{B}_d^0 \to \psi K_S \) and \( \phi K_S \). To a high degree of accuracy in the standard model, the time-integrated partial rate asymmetry of \( B^0_{\text{phys}} \) and \( \bar{B}^0_{\text{phys}} \) decaying into a \( CP \) eigenstate \( f \) is given by [12]

\[
A_{\text{CP}} = \frac{(1 - |\xi_f|^2) - 2x\text{Im} \left( \frac{q}{p} \xi_f \right)}{(1 + x^2)(1 + |\xi_f|^2)}, \tag{18}
\]
where $x \equiv \Delta m/\Gamma$ is the $B^0 - \bar{B}^0$ mixing parameter; and

$$\xi_f \equiv \frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)} \cdot \frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}}$$

(19)
denote the ratio of transition amplitudes and mixing phase, respectively. If all the contributing components of $A(B^0 \to f)$ have the same weak phase or the same strong phase shift, then $\xi_f$ will be a pure weak phase and a nonzero $A_{CP}$ will arise only from $B^0 - \bar{B}^0$ mixing [13].

In contrast with the previous discussions in which $B^0_d \to \psi K_S$ was assumed to occur through only the tree-level diagram [6,13], our analysis induces the hairpin contribution to this mode (see Eq.(15)). As a result,

$$\xi_{\psi K_S} = \frac{V_{cb}^* V_{cs} a_2 - V_{tb}^* V_{ts} (a_4 + a_6)}{V_{cb} V_{cs} a_2 - V_{tb} V_{ts} (a_4 + a_6)} \cdot \frac{V_{cs}^* V_{cd}}{V_{cs} V_{cd}}$$

(20)

where we have taken into account the $K^0 - \bar{K}^0$ mixing phase, and the minus sign comes from a $CP$ transformation for the $CP$-odd state $\psi K_S$. In terms of the CKM parameters [11], we obtain

$$\frac{V_{tb} V_{ts}^*}{V_{cb} V_{cs}^*} \approx (1 + \frac{\lambda^2}{2}) (1 - i\eta \lambda^2), \quad \frac{V_{cs} V_{cd}^*}{V_{cb} V_{cs}^*} \approx -\frac{1}{A\lambda} (1 - i\eta \lambda^2).$$

(21)

The above equation shows that the relative phase shifts among $V_{cb} V_{cs}^*, V_{tb} V_{ts}^*$ and $V_{cs} V_{cd}^*$ are $O(\eta \lambda^2)$ suppressed. Accordingly $\xi_{\psi K_S} \approx 1 + iO(\eta \lambda^2)$, and $\text{Im} (\frac{q}{p} \xi_{\psi K_S}) \approx \text{Im} (\frac{q}{p})$ remains to a good accuracy. This means that the effect of the hairpin diagram on the $CP$ asymmetry of $B^0_d$ versus $\bar{B}^0_d \to \psi K_S$ is unimportant, although its influence on the decay rates of these two modes are not too small in the large $N_c$ approximation. Note that the penguin and hairpin transitions have different isospin structure from the tree-level one, thus the above estimates of the hairpin contribution to decay rates and $CP$ asymmetries can only serve as an illustration. Anyway $B^0_d, \bar{B}^0_d \to \psi K_S$ remain the most interesting decay modes for probing $CP$ violation with little hadronic uncertainties. In general, however, we can not expect a little influence of the hairpin diagram on $CP$ asymmetries in the charmless nonleptonic $B$ decays listed in Eq.(6).

In Ref.[9], the decay modes $B^0_u \to \phi K^\pm$ were predicted by using the QCD-noncorrected penguin Hamiltonian and expected to manifest large $CP$ violation in the decay amplitude. Taking into account the hairpin contribution, here we employ the two-loop
renormalization-group-improved effective Hamiltonian in estimating decay rates for such charmless channels. As we have seen in Eq.(7), however, the penguin operators $Q_3,\ldots,6$ are only present in the $u - t$ sector and are thus all proportional to the same CKM factor $V_{tb}V^*_{tq}$. As a result, the partial decay rate difference of $B^+_u$ and $B^-_u$ vanishes in principle. This puzzle arises from the form of effective Hamiltonian we take in calculations, and is of course unreasonable. In Ref.[14], an attempt was made to generate penguin phases in both the $u - t$ and $u - c$ sectors by inducing the one-loop penguin matrix elements of $Q_2$ operators and removing the renormalization scheme dependence of the Wilson coefficient functions including next-to-leading QCD corrections. Here we do not want to follow such an argument to recover $CP$ asymmetries for the aforementioned charmless channels, since it is unjustified and untrustworthy. For our purpose we want to show that the hairpin contributions are significant in these rare decay modes and may induce $CP$ violation in the decay amplitude. A deeper investigation of this problem will be made elsewhere.

V Conclusions

We have made a brief quark-diagram analysis to show that, generally, the QCD loop-induced hairpin diagram can contribute to two-body nonleptonic $B$ decays( of course, also to multi-body nonleptonic $B$ decays). In the naive factorization calculations for decay amplitudes, the hairpin component appears if one employs the two-loop renormalization-group-improved effective Hamiltonian instead of the traditional QCD-noncorrected penguin Hamiltonian. We find a number of exclusive channels occurring only through the hairpin diagrams, whose branching ratios are estimated to be on the order of $10^{-7}$ in the large $N_c$ approximation. These rare decay modes, if they were measured in the near future, could be used to test the short-distance QCD calculations for $\Delta B = \pm 1$ decays. Although the effect of the hairpin diagram on decay rates for $B^0_d, \bar{B}^0_d \rightarrow \psi K_S, \phi K_S$ and $B^\pm_u \rightarrow \phi K^\pm$ are not too small, its influence on $CP$ asymmetries for these interesting channels is still expected to be very small in our rough estimates. At present, the decay modes $B^0_d$ versus $\bar{B}^0_d \rightarrow \psi K_S$ remain the cleanest nonleptonic channel for probing $CP$ violation in the $B$ system, since their partial rate asymmetry is approximately independent of hadronic uncertainties. For a number of charmless decay modes, however, our
theoretical techniques for calculating their decay rates are still problematic, especially in treating hadronic matrix elements.

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Figure Captions

Fig.1  Four distinct spectator quark diagrams for two-body nonleptonic $B$ decays at the scale $\mu = O(M_W)$:

(a) the color-matched tree-level diagram;

(b) the color-mismatched tree-level diagram;

(c) the penguin diagram;

(d) the hairpin diagram.

Fig.2  The hairpin diagram for $B(b\bar{q}_s) \rightarrow X(q_v\bar{q}_v) + Y(q\bar{q}_s)$ transitions at the scale $\mu = O(m_b)$. 
Table Caption

Table 1 Branching ratios for a few pure hairpin decay modes of $B$ mesons. Here the CKM parameters [11] are taken as $\lambda = 0.22$, $A = 1.0$, $\rho = -0.4$ and $\eta = 0.25$ [15]; and the relevant values of decay constants and formfactors are quoted from [10].

| Decay mode          | Branching ratio |
|---------------------|-----------------|
|                     | $N_c = 3$       | $N_c = \infty$ |
| $B_a^- \rightarrow \phi\pi^-$ | $5.4 \times 10^{-11}$ | $3.8 \times 10^{-7}$ |
| $B_a^- \rightarrow \phi\rho^-$ | $7.6 \times 10^{-11}$ | $5.3 \times 10^{-7}$ |
| $B_a^- \rightarrow \phi\eta_1$ | $6.6 \times 10^{-11}$ | $4.6 \times 10^{-7}$ |
| $B_\eta^0 \rightarrow \phi\pi^0$ | $2.7 \times 10^{-11}$ | $1.9 \times 10^{-7}$ |
| $B_\eta^0 \rightarrow \phi\rho^0$ | $3.8 \times 10^{-11}$ | $2.7 \times 10^{-7}$ |
| $B_\eta^0 \rightarrow \phi\eta_1$ | $3.3 \times 10^{-11}$ | $2.3 \times 10^{-7}$ |
| $B_\eta^0 \rightarrow \phi\omega$ | $3.7 \times 10^{-11}$ | $2.6 \times 10^{-7}$ |
| $B_\eta^0 \rightarrow \phi\eta$ | $1.5 \times 10^{-11}$ | $1.0 \times 10^{-7}$ |
| $B_\eta^0 \rightarrow \phi\eta'$ | $1.9 \times 10^{-11}$ | $1.3 \times 10^{-7}$ |