LMXBs may be important LIGO sources after all

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ABSTRACT

Andersson et al. and Bildsten proposed that the spin of accreting neutron stars is limited by removal of angular momentum by gravitational radiation which increases dramatically with the spin frequency of the star. Both Bildsten and Andersson et al. argued that the $r$-modes of the neutron star for sufficiently quickly rotating and hot neutron stars will grow due to the emission of gravitational radiation, thereby accounting for a time varying quadrupole component to the neutron star's mass distribution. However, Levin later argued that the equilibrium between spin-up due to accretion and spin-down due to gravitational radiation is unstable, because the growth rate of the $r$-modes and consequently the rate of gravitational wave emission is an increasing function of the core temperature of the star. The system executes a limit cycle, spinning up for several million years and spinning down in less than a year. However, the duration of the spin-down portion of the limit cycle depends sensitively on the amplitude at which the nonlinear coupling between different $r$-modes becomes important. As the duration of the spin-down portion increases the fraction of accreting neutron stars which may be emitting gravitational radiation increases while the peak flux in gravitational radiation decreases. Depending on the distribution of quickly rotating neutron stars in the Galaxy and beyond, the number of gravitational emitters detectable with LIGO may be large.

Subject headings: stars: neutron — gravitational waves — stars: oscillations

1. Introduction

Accretion onto the surface of a neutron star can in principle spin up the rotation of the neutron star until the spin frequency equals the Kepler frequency of the inner edge of the disk.

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In low-mass x-ray binaries, the disk is thought to extend to stellar surface so the maximal frequency that the neutron star can achieve exceeds 1 kHz. However, the observed and inferred spin frequencies of neutron stars in low-mass x-ray binaries (LMXBs) are clustered around 250–500 Hz (e.g Bildsten 1998); the millisecond X-ray pulsars SAX J1808.4 and XTE J1751 have slightly higher frequencies of 402 Hz and 435 Hz (Wijnands & van der Klis 1998; Markwardt & Swank 2002). Millisecond radio pulsars have been discovered with frequencies up to 640 Hz (Backer et al. 1982). All of these limits are well below the Keplerian limit on the spin frequency of a neutron star, so an alternative explanation for the maximal observed spin frequency of neutron stars is required. Andersson et al. (1999) and Bildsten (1998) proposed that inertial modes (specifically the $r$–modes) inside the neutron may grow while generating gravitational waves (GW). For sufficiently quickly rotating stars, GW carry away the angular momentum as quickly as it is deposited on the star by accretion.

Levin (1999) found that this proposed equilibrium between spin-up and spin-down is unstable. A neutron star will execute a limit cycle (see Andersson et al. 2000, for additional discussion). It spins up for several million years and then quickly spins down emitting GW in less than a year. Only a small fraction of neutron stars is spinning down at any time; it is unlikely that any neutron star within the galaxy is currently spinning down, so none would be detected by LIGO. However, the duration of the spin down depends sensitively on the assumed maximal amplitude of the $r$–mode. Levin (1999) assume that the $r$–modes saturate when their amplitude is of order unity. Sperhake et al. (2001), Wu et al. (2001) and Arras et al. (2002) found that the saturation amplitude may be two to three orders of magnitude smaller; this may increase the duration of spin down to be greater than several thousand years. This dramatically increases the number of neutron stars whose GW could be detected both by increasing the typical GW flux relative to the estimates of Andersson et al. (1999) and Bildsten (1998) and the number of currently emitting sources relative to Levin (1999).

This Letter explore the implications of saturation of $r$–modes in rapidly rotating neutron stars whose spins are accelerated by accretion. §2 will describe a series of straightforward calculations similar to those of Levin (1999) but with various values of the saturation amplitude (§2.1). Both Andersson et al. (2000) and Andersson & Kokkotas (2001) have estimated the number of observable sources if the duty cycle corresponds to an $r$–mode saturation amplitude of unity. The observed number of millisecond pulsars, the presumed descendents of LMXBS (e.g. Bhattacharya & van den Heuvel 1991), yields an estimate of the number of potential sources and their amplitudes (§§2.2-2.3) as a function of the duty cycle. §3 will outline some consequences of these results and the observational outlook.
2. Calculations

2.1. Thermal and spin evolution

The calculations presented here are essentially a recapitulation of those of Levin (1999); however, a brief summary of the equations governing the system is useful (Owen et al. 1998). The angular velocity of the star $\Omega$ is normalized by the dynamical frequency of the star, $\tilde{\Omega} = \Omega / (\pi G \bar{\rho})^{1/2}$ where $\bar{\rho}$ is the mean density of the star. If the dimensionless $r$–mode magnitude is less than the amplitude at which the mode saturates $\alpha_{\text{max}}$, the spin and $r$–mode amplitude are determined by

$$
\frac{d\tilde{\Omega}}{dt} = -\frac{2\alpha^2 Q \tilde{\Omega}}{1 + \alpha^2 Q \tau_v} + \sqrt{\frac{4}{3} \tilde{I} M p} 
$$

(1)

$$
\frac{d\alpha}{dt} = -\left( \frac{1}{\tau_{\text{grav}}} + \frac{1}{\tau_v (1 + \alpha^2 Q)} \right) \alpha 
$$

(2)

where $Q$ and $\tilde{I}$ are determined by the density profile of the star, a $n = 1$ polytrope (Lindblom et al. 1998). Following Levin (1999), $Q = 0.094$ and $\tilde{I} = 0.261$. $p$ depends on the angular velocity at the inner edge of the accretion disk relative to that of the star. The time for the star to spin up is inversely proportional to $p$, so it can be subsumed into the uncertain value of the accretion rate $\dot{M}$, i.e. $p = 1$. $\dot{M}$ is assumed to be $10^{-8} M_{\odot} \text{yr}^{-1}$ and $M$ is the mass of the star. The timescale for the growth and attenuation of the $r$–mode for a fluid neutron-star consisting of neutrons, protons and electrons are $\tau_{\text{grav}} = -3.26\tilde{\Omega}^{-6} s$ and $\tau_v^{-1} = (1.03 \times 10^5 s)^{-1} T_8^{-2} + (6.99 \times 10^{14} s)^{-1} T_8^6$ where $T_8 = T/(10^8 K)$ (Owen et al. 1998).

Both the growth and decay rates depend on the properties of the neutron star. The presence of a crust (Lindblom et al. 2000; Wu et al. 2001; Levin & Ushomirsky 2001), magnetic field (Rezzolla et al. 2000), hyperonic or superfluid core (Lindblom & Mendell 2000; Lindblom & Owen 2002) can dramatically affect the dynamics of $r$–modes in accreting neutron stars, shrinking the range of spin frequencies and temperatures where an $r$–mode will grow. On the other hand, a strange-quark-matter core may decrease the viscosity, shifting the instability region to cooler temperatures (Andersson et al. 2001). Consequently, the observation of GW from an accreting neutron star would provide a unique probe of the physics of its interior.

If the mode is saturated ($\alpha \geq \alpha_{\text{max}}$), $d\alpha/dt = 0$ and

$$
\frac{d\tilde{\Omega}}{dt} = \frac{2\alpha_{\text{max}}^2 Q}{\tau_{\text{grav}}} \frac{\tilde{\Omega}}{1 - \alpha_{\text{max}}^2 Q} + \sqrt{\frac{4}{3} \tilde{I} M p}. 
$$

(3)

Arras et al. (2002) provide a simple expression for $\alpha_{\text{max}}$ in terms of the total energy in the $r$–mode when nonlinear couplings become important, $\alpha_{\text{max}}^2 = A / (\tilde{\Omega} |\tau_{\text{grav}}| \tilde{J})$. The dimen-
A relation determining the thermal evolution of the system closes the system of equations. Neutrino emission by the modified URCA process (Shapiro & Teukolsky 1983) is taken to dominate the cooling while the core is heated through accretion and dissipation. The equilibrium temperature of the core under accretion alone is taken to be $T = 10^8$ K. These assumptions yield the following equation

$$\frac{dT}{dt} = \frac{1}{C_v} \left[ \frac{\alpha^2 Q^2 M R^2 \bar{J}}{\tau_v} - 7 \times 10^{31} (T^8 - \bar{T}^8) \text{ergs s}^{-1} \right]$$  \hspace{1cm} (4)$$

where $C_v$ is the heat capacity of the star taken to be $1.4 \times 10^8 T^8$ ergs K$^{-1}$ (Shapiro & Teukolsky 1983) and $\bar{J} = 1.635 \times 10^{-2}$ for the density profile considered (Lindblom et al. 1998). The mass of the star $M = 1.4 M_\odot$ and its radius $R = 12.53$ km.

The calculation begins with $T_8 = 1$, $\bar{\Omega} = 0.1$ and $\alpha = 0$, whenever the system becomes unstable to growing $r-$modes, $\alpha$ is set to $10^{-8}$. Figure 1 depicts the limit cycle for several values of $A$ which determines the amplitude at which the mode saturates (for each value of $A$ the peak amplitude of the mode is typically $2 \times 10^{-3} A^{1/2}$). Three important trends are apparent. The duration of the GW emission increases dramatically as the saturation amplitude decreases. The time for the system to complete the circuit depends only weakly on $A$ (3.7–4.8 Myr), so the duty cycle for GW emission also increases. The range of spin frequencies over the cycle decreases with $A$ (Andersson et al. 2000). For this model the range in spin frequencies over the population of LMXBs also depends on the assumed equilibrium temperature of the core $\bar{T}$. Stars which accrete less on average will have smaller typical core temperatures, and therefore experience a wider range of spin frequencies over the limit cycle. Finally, if the $r-$mode saturates at a smaller amplitude, the temperature of the core during the epoch of GW emission decreases. However, for $A > 0.1$, the temperature of the core exceeds $5 \times 10^8$ K during spin down.

The relation is well fit by a power law with $\tau_{on} \approx 5000 A^{-0.8}$ yr, in agreement with the discussion after Eq. 91 of Arras et al. (2002). The fraction of the time that a source is emitting GW increases from $\sim 10^{-3}$ to $\sim 10^{-1}$ as $A$ decreases from unity to $10^{-3}$. To probe the possible consequences of a crust, superfluidity or other sources of additional shear viscosity, the strength of the shear viscosity is increased by a factor of sixty-four. Here the critical spin frequency where the $r-$mode begins to grow is doubled, the spin-down timescale decreases by a factor of 375, and the peak $r-$mode amplitude increases by a factor of 5.66. Decreasing the equilibrium temperature of the core by a factor of two (without changing the shear viscosity) has a less dramatic effect of halving the spin-down timescale. Depending on
Fig. 1.— The limit cycle in core temperature and spin frequency for an accreting neutron star with several values of the dimensionless matching constant, $A$. The star follows a clockwise trajectory in phase space; from the outermost loop inward the values of $A$ are 1, 0.1, 0.01 and $10^{-3}$. The curves are labelled with the duration of the GW emission during each circuit of the limit cycle. The bold curve marks the boundary of the instability region. Above this curve, the amplification of the $r$–modes due to gravitational wave emission dominates over their attenuation by viscosity.
the properties of the neutron star and how the $r$–mode amplitude saturates, the spin-down timescale may vary from 14 to $10^6$ years; 34,000 years is the result for a fluid star using $A = 0.1$ (the conservative estimate of Arras et al. 2002).

### 2.2. Distribution of Sources - Galactic LMXBs

Obtaining a better estimate of the number of detectable GW sources requires a estimate of the total number LMXBs in the Galaxy, their accretion rates and distances. Since the known LMXBs number only about one hundred (Liu et al. 2001), one would expect not to observe GW emission from any of them for duty cycles less than one percent. Since LMXBs are thought to be the progenitors of millisecond radio pulsars (MSPs), the demographics of MSPs can provide an estimate of the total number of Galactic LMXBs (Alpar et al. 1982; Bhattacharya & van den Heuvel 1991). Equating the rate of MSP formation to the rate of LMXB formation yields an estimate of the number of LMXBs that could be emitting GW at any given time,

$$N_{\text{LMXB, on}} = \frac{\tau_{\text{on}}}{\tau_{\text{cycle}}} N_{\text{LMXB}} = \frac{\tau_{\text{on}}}{\tau_{\text{cycle}}} (r_{\text{MSP}} t_{\text{LMXB}}) = \tau_{\text{on}} \frac{\tilde{\Omega}_{\text{MSP}}}{\Delta \tilde{\Omega}} r_{\text{MSP}} \sim \frac{\tau_{\text{on}}}{10^4 \text{yr}}$$

(5)

where $\tau_{\text{cycle}}$ is the time for the neutron star to traverse the limit cycle once. This is essentially equal to the time for the star to spin up by $\Delta \tilde{\Omega}$ from the bottom of the limit cycle to the top. $t_{\text{LMXB}}$ is the time it takes the neutron star to spin up as an accreting LMXB to the final spin rate $\Omega_{\text{MSP}}$, and $r_{\text{MSP}}$ is the rate of formation of MSPs in the Galaxy $\sim 3 \times 10^{-5} \text{ yr}^{-1}$ (Lorimer et al. 1995; Lyne et al. 1998). The ratio of $\tau_{\text{cycle}}$ to $t_{\text{LMXB}}$ is inferred from Eq. 1.

Estimating the typical strain amplitude that would be observed from one of these sources is also straightforward since the total angular momentum radiated is proportional to $\Delta \Omega$. This yields

$$h = \frac{\alpha R}{8 d} \left( \frac{10}{\tilde{I} \tilde{\Omega}} G M \frac{c^3}{\tilde{\Omega} > c^3} \right)^{1/2} \approx 7 \times 10^{-25} \left( \frac{M}{1.4 \text{M}_\odot} \frac{10^4 \text{yr}}{\tau_{\text{on}}} \right)^{1/2} \frac{R}{10 \text{km}} \frac{10 \text{kpc}}{d}$$

(6)

where $\alpha$ is a constant which depends on geometry – its mean value is 2.9 (Brady et al. 1998). To obtain the final approximation, $\Delta \tilde{\Omega} = 0.12$ and $< \tilde{\Omega} > = 0.2$ which is appropriate for $A = 0.1$. This is thirty times larger than the strain for the brightest source, Sco X-1, if the GW emission is continuous (Bildsten 1998) and should be easily detected with the initial LIGO (Brady et al. 1998). One could possibly detect such sources even in M31, so for $\tau_{\text{on}} \sim 10^4 \text{ yr}$, one could expect to find several sources in the Local Group.

If $\tau_{\text{on}}$ is greater than a few hundred years, the radiation from the surface will reflect the heightened core temperature (Tsuruta 1998). The X-ray radiation is powered by the
dissipation of the \( r \)–modes. The typical core temperature as the star is spinning down is \( 6 \times 10^8 \) K, yielding a effective temperature of \( 3 \times 10^6 \) K and a soft-X-ray luminosity of \( 10^{35} \) erg/s (Heyl & Hernquist 2001). Assuming that the source is found at the maximum possible distance for an enhanced LIGO with \( h_{\text{min}} \sim 10^{-27} \) (Brady et al. 1998),

\[
d_{\text{max}} = 7 \text{ Mpc} \frac{10^{-27}}{h_{\text{min}}} \left( \frac{10^4 \text{yr}}{\tau_{\text{on}}} \right)^{1/2} \left( \frac{M}{1.4M_\odot} \right)^{1/2} \frac{R}{10 \text{km}}. \tag{7}
\]

this yields an observed X-ray flux of

\[
F = 2 \times 10^{-17} \text{ erg cm}^{-2} \text{s}^{-1} \left( \frac{h_{\text{min}}}{10^{-27}} \right)^{2} \frac{\tau_{\text{on}}}{10^4 \text{yr}} \left( \frac{M}{1.4M_\odot} \right)^{-1} \left( \frac{R}{10 \text{km}} \right)^{-2} \tag{8}
\]
due to the dissipation of the \( r \)–mode energy alone. The typical Galactic source (at a distance of \( 10 \) kpc), discussed earlier would be 500,000 times brighter with a flux of about 3.2 mCrab. Brown & Ushomirsky (2000) discussed the X-ray emission produced if the GW emission is steady and derive constraints on either the accretion or the \( r \)–modes. If the GW emission is transient, the X-ray emission is significantly stronger while GW are being emitted but the vast majority of LMXBs would not be in this stage, thereby avoiding these constraints.

### 2.3. Distribution of Sources - Extragalactic LMXBs

For much smaller values of \( \tau_{\text{on}} \), the expected number of active sources in the Galaxy vanishes; however, any sources that are active will be visible well beyond the Local Group. The total number of sources brighter than \( h_{\text{min}} \) is given by

\[
N_{\text{LMXB, on}} = \frac{4}{3} \pi \left( \frac{\alpha}{8 h_{\text{min}}} \right)^3 \left( \frac{10 \hat{I} \Delta \tilde{\Omega}}{\Omega > e^3} \frac{GM}{c^3} \tau_{\text{on}}^{-1/2} \frac{\tilde{\Omega}_{\text{MSP}}}{\Delta \Omega} R_{\text{MSP}} \right)^{3/2} \tag{9}
\]

where \( R_{\text{MSP}} \) is the formation rate of MSPs per unit volume averaged over the local region of the universe, \( R_{\text{MSP}} \approx r_{\text{MSP}} \mathcal{L}/L_{\text{Milky Way}} \) where \( \mathcal{L} \) is the local luminosity density, about \( 2 \times 10^{-2} h_{100} L_{\text{Milky Way}} \) Mpc\(^{-3}\) (Heyl et al. 1997) where \( h \) is the Hubble Constant in units of 100 km/s/Mpc. Using the values for \( \alpha_{\text{max}} = 1 \), the case examined by Levin (1999), yields,

\[
N_{\text{LMXB, on}} = 3 h_{100} \left( \frac{10^{-27}}{h_{\text{min}}} \right)^3 \left( \frac{1 \text{yr}}{\tau_{\text{on}}} \right)^{-1/2} \left( \frac{M}{1.4M_\odot} \right)^{3/2} \left( \frac{R}{10 \text{km}} \right)^3. \tag{10}
\]

The typical distance of the sources detected at this sensitivity with \( \tau_{\text{on}} \sim 1 \) yr would be nearly 1 Gpc. Even if \( N_{\text{LMXB, on}} \) is less than one, one would expect to discover one source during a period of \( \tau_{\text{on}}/N_{\text{LMXB, on}} \).
3. Discussion

If the spin of accreting neutron stars is indeed limited by the emission of gravitational radiation (Andersson et al. 1999; Bildsten 1998), low-mass X-ray binaries may be an important source for LIGO. Although neutron stars may execute a duty cycle (Levin 1999) of spin-up and spin-down which reduces the number of active sources at a given time, the sources that are active are typically much brighter than in a model where they emit constantly. Unfortunately, the number of LMXBs that have been discovered actively accreting is too small (∼100) to determine which effect dominates for duty cycles less than ten percent.

However, if low-mass X-ray binaries are assumed to be the exclusive progenitors of millisecond pulsars (see van den Heuvel 1984, for an alternative), the demographics of the millisecond pulsars in the Galaxy and beyond provides an estimate of the number of sources. Specifically, if the duration of the epochs when the neutron star is emitting gravitational radiation is greater than 10,000 years, several objects in the Galaxy will be above the detection thresholds of LIGO. For $\tau_{\text{on}} \sim 10,000$ year, these sources would be detectable throughout the Local Group. Those objects in the Galaxy could also be detected from their X-ray emission which would be powered by gravitational radiation reaction; they would be GW-powered neutron stars.

For $\tau_{\text{on}}$ much less than 10,000 years and much greater than one year, no sources are likely to be detectable even with an enhanced LIGO detector. However, if the duration of gravitational wave emission per cycle is less than several years, several sources could be detected by an enhanced LIGO. In this case, the sources will be located at a typical distance of 1 Gpc.

Depending on the nature of the $r-$mode instability in the cores of quickly spinning neutron stars, LMXBs may provide gravitational-wave beacons throughout the Galaxy and the Local Group or at cosmologically significant distances.

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