Can the initial singularity be detected by cosmological tests?

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Abstract

In the presented paper we raised the question whether initial cosmological singularity can be proved by cosmological tests. The classical general relativity theory predicts the existence of singularity in the past if only some energy conditions are satisfied. On the other hand the latest quantum gravity applications to cosmology suggest the possibility of avoiding the singularity and replacing it with a bounce. Bounce is the moment in the evolution of the Universe when the Universe's size has minimum. Therefore the existence of observationally detected bounce in past of Universe could indicate the validity of the loop quantum gravity hypothesis and nonexistence of initial singularity which is present in the classical ΛCDM. We investigated the bouncing model described by the generalized Friedmann-Robertson-Walker (FRW) equation in the context of the observations of the currently accelerating universe. The distant type Ia supernovae data are used to constraint on bouncing evolutional scenario where square of the Hubble function $H^2$ is given by formulae $H^2 = H_0^2 [\Omega_m,0 (1 + z)^m - \Omega_n,0 (1 + z)^n]$, where $\Omega_m,0, \Omega_n,0 > 0$ are density parameters and $n > m > 0$. In this paper are showed that the on the base of the SNIa data standard bouncing models can be ruled out on the 4σ confidence level. After adding the cosmological constant to the standard bouncing model (the extended bouncing model) we obtained as the best-fit that the parameter $\Omega_n,0$ is equal zero which means that the SNIa data do not support the bouncing term in the model. The bounce term is statistically insignificant on the present epoch. We also demonstrated that BBN offers the possibility of obtaining stringent constraints of the extra term $\Omega_n,0$. The other observational test methods like CMB and the age of oldest objects in the Universe are also used. We use as well the Akaike informative criterion to select a model which fits data the best and we concluded that bouncing term should be ruled out by Occam’s razor, which makes the big bang scenario more favorable then the bouncing scenario.

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I. INTRODUCTION

We are living in an age of high precision cosmology which offers a possibility of testing exotic physics, which is obvious for the early Universe [1]. In this context the most important are BBN constraints because the present Universe opens only small windows on the exotic physics. The main aim of this paper is to discuss whether the initial singularity can be checked against the astronomical observations. The question of singularity cannot be answered directly, therefore we use two prototype models based on the classical and quantum gravity theory. The first is the ΛCDM which is a concordance model describing the evolution of the Universe from the initial singularity (the big bang) driven by the cold dark matter and the cosmological constant (dark energy). The second is a bouncing model which appears in the context of quantum cosmology and characterized by the lack of initial singularity. During its evolution, the expansion phase is proceeded by the contraction phase at the bounce where the scale factor assumes the minimum nonzero value.

We use some tests to discriminate between these two alternative models. One of the most important tests applies the SNIa data to fit the cosmological models. Recent measurements of type Ia supernovae observations suggest that the universe is presently accelerating [2, 3]. A dark energy component has usually been proposed as a source of acceleration mechanism [4]. Many theoretical propositions have been suggested about these components. However, the different effects arising from quantum fluctuation, spinning fluid, etc. can also mimic dynamically the role the dark energy which drives acceleration through an additional term in the Friedmann equation [5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. Some of them give rise to the bounce. In many cases they prevailed in the very early epoch but are very small in the present epoch. Therefore it is very difficult to detect the existence of this component in the present and those of the relatively close past (after CMB) observations of SNIa.

In the present work we investigate observational constraints on the evolutionary scenario of the standard bouncing cosmological models defined as a class of models for which the Hubble function \( H \) and the scale factor \( a \) are related by the formula

\[
H^2 = H_0^2 (\Omega_{m,0} x^{-m} - \Omega_{n,0} x^{-n})
\]

where \( n > m > 0 \) and \( x = \frac{a}{a_0} \) where the index zero denotes the quantities evaluated in the present epoch, the parameter \( \Omega_{n,0} \) is called the bouncing term; and the density parameters
satisfy the constraint relation
\[ \Omega_{m,0} - \Omega_{n,0} = 1. \]

While focus mainly on the constraints coming from SN Ia data and WMAP observations, the complementary constraints coming from BBN and the age of the oldest high-redshift objects are also considered. We use the maximum likelihood method to estimate the model parameters \( m, n \) and \( \Omega_{n,0} \). Similarly we analyze the models with the additional parameter— the cosmological constant. It is called the generalized bouncing model.

The proposition of the bounce type evolution of the early universe seems to be very attractive not only from the point of view of the quantum description of the early Universe because the expansion of the universe is accelerated automatically due to the presence of the bouncing term.

The standard bouncing scenario predicts the acceleration around the bounce with a transition to the deceleration epoch. The cosmological constant brings this deceleration epoch to the end and a new acceleration epoch begins.

Therefore, these models can be proposed as the models of our Universe, because they include the epoch of acceleration. However we show that the influence of the bouncing term is insignificant in the present epoch. Therefore, the data from the present epoch, such as the SNIa data, have not power to rsider the model with bouncing term statistically significant. So the ΛCDM model with big-bang scenario is strongly favored by data over the model with the bounce.

By the application of standard Akaike criterion of the model selection we can choose the ΛCDM model over the generalized bouncing model. We conclude that the data fail to support the existence of the bouncing term. The bouncing term in the present epoch is insignificant and it is not possible to detect its influence by the use the latest SNIa data.

This fact justifies certain scepticism about the existence of the SNIa window on exotic physics in the current epoch. However we cannot rule out other models by testing them against the current data. It is also possible to investigate the differences in the predictions of these models for some earlier epoch.

For example the BBN epoch is a well tested area of cosmology. From this analysis we gather that the extra term \( \Omega_{n,0}x^{-n} \) causing the bounce should be constrained to be sufficiently small during nucleosynthesis.

The organization of the text is the following. In section 2 the evolutionary scenario of
bounce FRW cosmologies is investigated by the use of dynamic system methods. We show that they are structurally unstable due to the presence of centers in the phase portraits. In section 3 we discuss the constraints from SNIa data on the standard bouncing models. In section 4 we extend the bouncing models by introducing the cosmological constant and then we study how these models fit the current supernovae and WMAP data. In section 5 we formulate conclusions.

II. THE BOUNCING MODELS: BASIC EQUATIONS

The idea of bounce in FRW cosmologies appeared in Tolman’s monograph devoted to cosmology [16]. This idea was strictly connected with oscillating models [17, 18, 19]. At present oscillating models play an important role in the brane cosmology [20, 21]. The FRW universe undergoing a bounce instead of the big-bang is also an appealing idea in the context of quantum cosmology [22]. The attractiveness of bouncing models comes from the fact that they have no horizon problem and they explain quantum origin of structures in the Universe [23, 24, 25]. Molina-Paris and Visser and later Tippett [26, 27] characterized the bouncing models by the minimal condition under which the present universe arises from a bounce from the previous collapse phase (the Tolman wormhole is different name for denoting such a type of evolution). The violation of strong energy condition (SEC) is in general a necessary (but not sufficient) condition for bounce to appear. For closed models it is sufficient condition and none of other energy condition need to be violated (like null energy condition (NEC): $\rho + p \geq 0$, weak energy condition (WEC): $\rho \geq 0$ and $\rho + p \geq 0$, dominant energy condition (DEC): $\rho \geq 0$ and $\rho \pm p \geq 0$ energy conditions can be satisfied).

We can find necessary and sufficient conditions for an evolutional path with a bounce by analyzing dynamics on the phase plane $(a, \dot{a})$, where $a$ is the scale factor and dot denotes differentiation with respect to cosmological time. We understand the bounce as in [26, 27], namely there must be some moment say $t = t_{\text{bounce}}$ in evolution of the universe at which the size of the universe has a minimum, $\dot{a}_{\text{bounce}} = 0$ and $\ddot{a} \geq 0$. This weak inequality $\ddot{a} \geq 0$ is enough for giving domains in the phase space occupied by trajectories with the bounce. Let us consider the dynamics of the FRW cosmological models filled by perfect fluid with energy density $\rho$ and pressure $p$ parameterized by the equation of state in the general form

$$p = w(a)\rho. \tag{2}$$
The basic dynamical system constitutes two equations

\[ \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) \]  
\[ \dot{\rho} = -3H(\rho + p) \]

Equation (3) is the Rauchaduri equation while equation (4) is the conservation condition. If the equation of state is postulated in the form (2) then from (4) we obtain

\[ \rho = \rho(a) = \rho_0 a^{-3} \exp \left(-3 \int \frac{w(a)}{a} da \right) \]

It is interesting that dynamics of the model under consideration can be represented in the analogous form to the Newtonian equation of motion

\[ \ddot{a} = -\frac{\partial V}{\partial a} \]

where \( V = -\frac{\rho a^2}{6} \) plays the role of potential function for the FRW system. Therefore different cosmological models are in a unique way characterized by the potential function \( V = V(a) \) and we can write down the Hamiltonian for the fictitious particle-universe moving in the one-dimensional potential as

\[ \mathcal{H} = \frac{p_a^2}{2} + V(a), \quad p_a = \dot{a} \]

It is useful to represent eq. (6) in the form of dynamical system

\[ \dot{x} = y, \quad \dot{y} = -\frac{\partial V}{\partial x}, \]

where we denote \( x = a, y = \dot{a} \) and system (8) has the first integral in the form

\[ \frac{y^2}{2} + V(x) = -\frac{k}{2} \]

where \( k \) is the curvature index.

The critical points of the system (8) if exist are: \( y_0 = 0, (\frac{\partial V}{\partial x})_{x_0} = 0 \), i.e. they are always static critical points located on \( x \)-axis. The form of first integral (9) defines the algebraic curves in the phase plane \((a, \dot{a})\) on which lies solutions of the system. This solutions are in two types: regular is represented by trajectories or singular is represented by singular solutions for which the right-hand side of (8) are null (or \( V(x_0) = -\frac{k}{2} \) for nonflat models). Note that the bouncing points are intersections points of trajectories situated in the region
of the configuration space in which $\frac{\partial V}{\partial a} \leq 0$, i.e. $V(a)$ is a decreasing function of $a$ or has extrema. It is well known that the systems in the form have only critical points of two admissible types: centres if $(\frac{d^2V}{dx^2})_{x_0} > 0$ or saddles in opposite case if $(\frac{d^2V}{dx^2})_{x_0} < 0$. Therefore all trajectories with bounce intersect an $x$-axis and then they are situated on the right side from the critical point at which $\dot{a} = 0$ and $\ddot{a} \geq 0$. The critical points are represented by points as well as by separatrices of the saddle point. In others words bouncing trajectories are represented by such trajectories in the phase plane which are passing through the $x$-axis in such a direction that they always belong to the accelerating region (in the neighborhood of bounce). Of course it is only possible if the SEC is violated.

Let us consider some prototype of bouncing models given by the Friedmann first integral in the form

$$H^2 = \frac{A}{a^m} - \frac{B}{a^n}, \quad (10)$$

where $A$, $B$ are positive constants and $n > m$, $H = (\ln a)\dot{}$ is the Hubble function and a dot denotes differentiation with respect to cosmological time $t$.

It is convenient to rewrite (10) to the new form

$$H^2 = H_0^2(\Omega_{m,0}x^{-m} - \Omega_{n,0}x^{-n}), \quad (11)$$

where $\Omega_{m,0}$, $\Omega_{n,0}$ are density parameters for noninteracting fluids which give some contributions to right-hand sides of eq. (10). We define density parameters $\Omega_{m,0} = \frac{3Aa^{-m}}{3H_0^2}$, $\Omega_{n,0} = \frac{3Ba^{-n}}{3H_0^2}$, where an index “0” means that corresponding quantities are evaluated at the present epoch, $x = \frac{a}{a_0}$ is the scale factor expressed in the units of its present value $a_0$.

After differentiation the both sides of (11) with respect to the reparameterized time variable $\tau: t \to \tau$, $|H_0|dt = d\tau$ we obtain

$$\ddot{x} = \frac{1}{2} \left( \Omega_{m,0} (2 - m) x^{-m} + \Omega_{n,0} (n - 2) x^{-n} \right) \quad (12)$$

If we consider the generalization of the bouncing models with the cosmological constant then in both equations (11) and (12) the parameter $\Omega_{\Lambda,0}$ should be added to their right-hand sides.

Note that the bouncing models can be treated as the standard FRW models with two noninteracting fluids with energy density and pressure in the form

$$\rho = \rho_m + \rho_n = 3H_0^2\Omega_{m,0}x^{-m} - 3H_0^2\Omega_{n,0}x^{-n}$$

$$p = \left( -1 + \frac{m}{3} \right) \rho_m + \left( -1 + \frac{n}{3} \right) \rho_n \quad \rho_m > 0, \rho_n < 0.$$
The curvature term as well as cosmological constant term can be obtained in an analogous way by putting $m = 2$ or $m = 0$, respectively.

If we postulate that the present universe is accelerating, i.e. $\ddot{x} > 0$ at $x = 1$ then in the general case with the cosmological constant we obtain the following condition

$$\Omega_{m,0} (2 - m) + \Omega_{n,0} (n - 2) + 2\Omega_{\Lambda,0} > 0 \quad (13)$$

Because relation (13) is validate any time, the substitution $H = H_0$ and $x = 1$ to (13) gives constraint

$$\Omega_{m,0} - \Omega_{n,0} + \Omega_{\Lambda,0} = 1 \quad (14)$$

Let us now concentrate on the standard bouncing models (SB) without the cosmological term. Then from (13) including constraints (14) we obtain the sufficient condition for acceleration at present

$$\Omega_{m,0} (n - m) > 2 - n, \quad (15)$$

where for the case of $n = 2$, $k = 1 \Omega_{m,0} > 0$ is only required for present acceleration.

If only $\Omega_{n,0}$ is larger then $\Omega_{m,0}$ the bouncing universe is presently accelerating for any $m$, $n$ parameters. It is worthy to mention that condition (15) is minimal qualitative information about acceleration and the rate of this acceleration is required for explanation SNIa data.

From the definition (11) one can obtain the domain admissible for motion of the bouncing models

$$D = \{x: x \geq x_b \quad \text{where} \quad x_b = \left(\frac{\Omega_{n,0}}{\Omega_{m,0}}\right)^{\frac{n-m}{m}}\} \quad (16)$$

From (11) the potential function $V(x)$ in the particle-like description can be determined

$$V(x) = -\frac{\rho_{eff} x^2}{6H_0^2} = -\frac{1}{2} \Omega_{eff} (x) x^2 \quad (17)$$

where effective density parameter $\Omega_{eff} = \Omega_{m,0} x^{-m} - \Omega_{n,0} x^{-n}$. The acceleration region in the phase plane can be determined in term of potential function, namely if

$$\frac{dV}{da} < 0 \quad (18)$$

then universe is accelerating.

From (17) we obtain result that at the bounce moment

$$\ddot{x} = - \left(\frac{dV}{dx}\right)_{x_b} = \frac{1}{2} \Omega_{m,0} \left(\frac{\Omega_{m,0}}{\Omega_{n,0}}\right)^{\frac{m+n}{m-n}} (n - m) \quad (19)$$
which indicate that bouncing models defined by equation (11) at the bounce are in accelerating phase for any ranges of model parameter $m, n, \Omega_{m,0}, \Omega_{n,0}$. Because $a_b$ is larger then $a_0: \left(\frac{\partial V}{\partial a}\right)_{a_0} = 0$ the bouncing universe stay still in the accelerating region.

From eq. (19) we obtain that the universe start to accelerate at the point $x = x_0$ such that

$$x_0 = \left(\frac{\Omega_{m,0}(m-2)}{\Omega_{n,0}(n-2)}\right)^{\frac{1}{m-n}}$$  

where a positive value of $(m - 2)(n - 2)$ is required. Note that in any case

$$x_0 < x_b$$  

i.e., the start of acceleration proceeds the bounce. The value of $x_0$ determine the location of the critical point of the dynamical system $\dot{x} = y, \dot{y} = -\frac{\partial V}{\partial x}$ on $x$-axis. The sign of the second derivative of potential function determines the type of critical points (centre or saddle) because the eigenvalues of the linearization matrix of the system satisfy the characteristic equation

$$\lambda^2 + \left(\frac{\partial^2 V}{\partial x^2}\right)(x_0) = 0.$$  

From (17) we obtain

$$\left(\frac{\partial^2 V}{\partial x^2}\right)(x_0) = -\frac{1}{2}\Omega_{n,0}x_0^{-n} [(2 - m)(1 - m) - (2 - n)(1 - n)].$$

Therefore if $(m, n)$ belong to the interval $(3/2, \infty)$ then we have centres, while if $(m, n)$ belong to interval $(-\infty, 3/2)$ we obtain saddles.

Let us concentrate, for example, let us concentrate on the case of $m = 3$ (dust matter). Then

$$\left(\frac{\partial^2 V}{\partial x^2}\right)(x_0) = \frac{1}{2}\Omega_{n,0}x_0^{-n}n(n - 3).$$

Hence, if $n > 3$ we obtain $\left(\frac{\partial^2 V}{\partial x^2}\right)(x_0)$ positive which corresponds to the centres in the phase plane. The presence of centres on the phase portraits means that all bouncing models are oscillating. Let us note, however that the corresponding systems are structurally unstable because of the presence of nonhyperbolic critical points on the phase portraits. Physically it means that the small perturbation of right-hand sides of the system under consideration disturbs a qualitative structure of the orbits (i.e., a phase portrait). On Fig. 1 and 2 the phase portraits and diagrams of the potential function are presented for the standard and generalized bouncing model. Fig. 1 describes all special cases listed in Table 1. The
TABLE I: Some special cases of bouncing models

| model                                                                 | dynamical equations (first integral)                                                                 |
|-----------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|
| FRW model dust filled universe with global rotation [14]              | $\dot{x} = yx^3$                                                                                     |
| or brane models with dark radiation [40]                              | $\dot{y} = \frac{1}{2} (-Ω_{m,0}x^{-2} + 2|Ω_{ω,0}|x^{-3})x^3$                                        |
| $\frac{\dot{y}^2}{2} = \frac{1}{2} (-Ω_{m,0}x^{-1} - |Ω_{ω,0}|x^{-2} + Ω_{k,0})x^6$ |                                                                                                     |
| FRW dust filled universe with spinning fluid [52]                     | $\dot{x} = yx^5$                                                                                     |
| or a class of MAG models [30]                                         | $\dot{y} = \frac{1}{2} (-Ω_{m,0}x^{-2} + 4|Ω_{s,0}|x^{-5})x^5$                                        |
| $\frac{\dot{y}^2}{2} = \frac{1}{2} (-Ω_{m,0}x^{-1} - |Ω_{s,0}|x^{-4} + Ω_{k,0})x^{10}$ |                                                                                                     |
| Stephani models                                                       | $\dot{x} = y$                                                                                       |
| filled by perfect fluid $p = γρ$ [13]                                 | $\dot{y} = \frac{1}{2} (-Ω_{γ,0}(1 + 3γ)x^{-2-3γ} + δΩ_{k,0}x^{δ-1})$                                |
| $\frac{\dot{y}^2}{2} = \frac{1}{2} (Ω_{γ,0}x^{-1-3γ} + Ω_{k,0}x^{δ})$ |                                                                                                     |

classically forbidden region for $a < a_0$ is shaded. The evolution of the model is represented in the configuration space by a Hamiltonian level

$$\mathcal{H} = E = \frac{1}{2}Ω_{k,0}.$$  

The trajectory of the flat model separates the regions occupied by both closed and open models. The decreasing of the potential function with respect to the scale factor determines the domain of phase space occupied by accelerating trajectories. The bounce is the intersection point of trajectory with the axis $a$. Note that around the bounce we have acceleration. On the phase plane of Hamiltonian dynamical systems only centres and saddle points are admissible. The centres are structurally unstable while saddles are structurally stable. Because the centre appears in the phase portraits of standard and generalized bouncing models, both models are structurally unstable. The critical points represent the static universes. Generalized bouncing model has two disjoint acceleration regions. The first is due to bouncing term while the second is forced by the cosmological constant term.

The full knowledge of the dynamics required its analysis at infinity, i.e. at the circle at infinity $x^2 + y^2 = \infty$. The standard procedure is to use projective maps on the plane and then analyze the system in a standard way. One can find the critical points at infinity as an a intersections of trajectory of a flat model with circle at infinity, i.e. $\{(x, y): Ω_{k,0} = 0\}$ and $\{z = 1/x, u = y/x, \quad x = \infty, v = 1/y, w = x/y, y = \infty\}$ – the trajectory of the flat model $\frac{\dot{x}^2}{2} = -V(x)$ with a circle at infinity.
FIG. 1: The phase portrait and the diagram of the potential function for BM model (all case from Table I). The minimum of the potential function corresponds to a centre on the phase plane. The acceleration region is located on the right from the $a_{\text{min}}$. 
FIG. 2: The phase portrait and the diagram of the potential function for ABCDM model. The minimum (maximum) of the potential function corresponds to a centre (saddle) on the phase plane. The system is structurally unstable because of the presence of nonhyperbolic critical point (a centre).
Some special cases of the bouncing systems contain Table I. Because we are dealing with autonomous dynamical systems their phase portrait is always given modulo diffeomorphism or equivalent modulo any time reparameterization following the rule: \( \tau \to \eta: d\tau = f(x) d\eta \), where \( f(x) \) is diffeomorphism, \( x \) is point which belong to phase plane.

For analysis of bouncing models in term of dynamical systems, it is useful to reparameterized original time variable in order to obtain nondegenerate critical points at infinity. Then we obtain \( \frac{dx}{x^{\beta/2}} \) and \( y^2 = \frac{1}{2}(\Omega_m,0 x^{2-m} - \Omega_n,0 x^{2-n}) \) is now representing the trajectory of the flat model, \( \beta \) should be chosen in the suitable way to regularize critical points. Let \( \beta = n - 2 \) then as \( x \) goes to infinity then \( y \) also goes to infinity. Only the sign of the parameter \( m \) (if \( m < 0 \)) decides whether the future of the system is the type of a big rip singularity. If \( m = 0 \) then the case of cosmological constant can be recovered.

It is convenient to regularize the system by multiplication both sides of the system \( x^3 \) in the first case and \( x^4 \) in the second one respectively. It is equivalent to reparameterized time variable following rule \( \tau \to \eta: \frac{dx}{x^{\beta/2}} = d\eta \), where \( \beta = 3 \) and (5) for the system from Table I. For both systems from the table we have an additional term in the generalized FRW equation. In the first case the effects of global rotation produce contribution corresponding to the negative energy scaling like radiation. The same contribution appears in the brane models on the charged brane. It is known as the dark radiation [10]. Please note, that analogous term appeared if we include the Casimir effect coming for example from quantum effects of massless scalar fields [22, 28, 29]. In the second case (Table I) it is the model with spinning dust fluid. It can be also recovered as a class of MAG models [7, 30].

In both cases we can find centre at finite domain and periodic orbits. At infinity we have unstable and stable nodes at \( x = +\infty, y = \mp\infty \). The trajectory of the flat model separates the regions occupied by closed and open models. All models have bounce but some from them are oscillating models without the initial and final singularities. For our future investigations of observational constraints on bouncing models it is convenient to derive crucial formulae for \( H(z) \) where \( z \) is redshift \( z: 1 + z = x^{-1} \). We obtain from (10) that \( H^2 = H_0^2 [\Omega_m,0(1 + z)^m - \Omega_n,0(1 + z)^n] \). It is useful to represent it in the corresponding bouncing parameters.

For this aim we find \( x_b \) corresponding to the bounce and value of redshift which identify this moment during the evolution: \( x_b = \left( \frac{\Omega_m,0}{\Omega_n,0} \right)^{\frac{1}{m-n}}, z_b = -1 + \left( \frac{\Omega_m,0}{\Omega_n,0} \right)^{\frac{1}{m-n}} \). Finally we obtain independent model parameters characterizing its role in evolution (modulo present
value of $H_0$), namely

$$H = H_0 \sqrt{(1 + z_b)^{n-m}} (1 + z)^{m/2} \sqrt{1 + \left(\frac{z + 1}{z_b + 1}\right)^{n-m}}.$$  

(25)

If $\Omega_{m,0}$ is fixed, for example from independent galaxy observations then the evolutionary scenario is parameterized by single $n$ parameter

$$H = H_0 \sqrt{\Omega_{3,0}} (1 + z)^{3/2} \sqrt{1 + \left(1 - \frac{1}{\Omega_{3,0}}\right) (1 + z)^n},$$

(26)

where we put $\Omega_{m,0} = \Omega_{3,0}$, i.e. dust filled universe.

In the case of generalized bouncing models the potential function takes the following form

$$V(x) = \frac{1}{2} \Omega_{n,0} x^{2-n} - \frac{1}{2} \Omega_{m,0} x^{2-m} - \frac{1}{2} \Omega_{\Lambda,0} x^2.$$  

It means that if only $n, m > 0$ then we obtain the de Sitter solution as a global attractor in the future. In the opposite case if $m > 0$ the big-rip singularities are generic future of the model. Note that in the class of generalized bouncing models only trajectories around point $(x_0, 0)$ represent oscillating models without a singularity and there is admissible large class of closed, open and flat models which evolve to infinity.

It is interesting that the characteristic bounce can be defined in terms of geometry of potential function only. By bouncing cosmology we can understand all cosmological models for which the potential function has at some point a minimum.

III. BOUNCING MODEL AND DISTANT SUPERNOVAE OBSERVATIONS.

In this section we confront the bouncing cosmological models with observations of distant SNIa. These observations in the framework of the FRW model indicate that present acceleration of our Universe is due to an unknown form of matter with negative pressure called dark energy [3]. Apart from the cosmological constant there are also other candidates for dark energy which were tested from SNIa observations [31, 32, 33]. We use the SNIa data to test the acceleration in the bouncing models. Moreover these models are attractive because they have no horizon and initial singularity, and they yield an explanation of structures which originated in the quantum epoch [22].

We consider the flat FRW model since there is a very strong evidence that the Universe is flat in the light of recent WMAP data [34]. We confront the two “bouncing” models (with
and without extra $\Lambda$ fluid) with SNIa data. For this purpose we calculate the luminosity distance in a standard way

$$d_L(z) = (1 + z) \int_0^z \frac{dz}{H(z)}$$  \hspace{1cm} (27)$$

To proceed with fitting models to SNIa data we need the magnitude-redshift relation

$$m(z, \mathcal{M}, \Omega_{m,0}, \Omega_{\Lambda,0}, n, m) = \mathcal{M} + 5 \log_{10} D_L(z, \Omega_{m,0}, \Omega_{\Lambda,0}, m, n)$$ \hspace{1cm} (28)$$

where $M$ being the absolute magnitude of SNIa and

$$D_L(z, \Omega_{m,0}, \Omega_{\Lambda,0}, m, n) = H_0 d_L(z, H_0, \Omega_{m,0}, \Omega_{\Lambda,0}, m, n)$$ \hspace{1cm} (29)$$

is the luminosity distance with $H_0$ factored out, so that marginalization over the parameter $\mathcal{M}$

$$\mathcal{M} = -5 \log_{10} H_0 + 25$$ \hspace{1cm} (30)$$

reads actually marginalization over $H_0$.

The parameter $\mathcal{M}$ is actually determined from the low-redshift part of the Hubble diagram which should be linear in all realistic cosmologies. It lead to value of $H_0 \simeq 65$ km/s Mpc, i.e., $\mathcal{M} \simeq 15.955$. In further analysis we estimate the models with this value of $\mathcal{M}$ and without any prior assumption on $H_0$.

Then we can obtain the best fit model minimizing the function $\chi^2$

$$\chi^2 = \sum_i \frac{(\mu_i^{\text{theor}} - \mu_i^{\text{obs}})^2}{\sigma_i^2}$$ \hspace{1cm} (31)$$

where the sum is over the SNIa sample and $\sigma_i$ denote the (full) statistical error of magnitude determination and $\mu_i = m_i - M_i$.

Because the best-fit values alone are not sufficient, the statistical analysis is supplemented with the confidence levels for the parameters. We performed the estimation of model parameters using the minimization procedure, based on the maximum likelihood method. We assume that supernovae measurements came with uncorrelated Gaussian errors and the likelihood function $L$ could be determined from the chi-square statistic $L \propto \exp(-\chi^2/2)$.

The first published large samples of SNIa appeared at the end of the 90s, later other data sets have been made either by correcting errors or by adding new supernovae. The latest compilation of SNIa was prepared by Riess et al. and became de facto a standard data set. It should be noted that this compilation encloses the largest number of
TABLE II: Results of the statistical analysis of the bouncing model without dust (BM) and bouncing cold dark matter model (BCDM) obtained for SNIa data from the best fit with minimum $\chi^2$ (denoted as BF) and from the likelihood method (denoted as L). The case of a fixed value of the parameter $\mathcal{M}$ is denoted as F. If in BF method we obtain $\Omega_{n,0} = 0$ than $n$ could be taken arbitrary (marked as A).

| model        | $\Omega_{m,0}$ | $\Omega_{n,0}$ | $n$ | $\mathcal{M}$ | $\chi^2$ | method |
|--------------|-----------------|-----------------|-----|---------------|----------|--------|
| BM model     | 1.00 1.4 0.00   | A               | 15.975 | 181.6   | BF       |
|              | 1.00 1.5 0.00 1.7| 15.975 | —       | L         |
|              | 1.54 1.4 0.54 1.5| F               | 15.955 | 182.3   | BF       |
|              | 1.00 1.4 0.00 1.6| F               | 15.955 | —       | L         |
| BCDM model   | 1.86 — 0.86 3.7 | 16.085          | 217.4 | BF       |
| (dust matter $m = 3$) | 1.86 — 0.86 3.7 | 16.095 | —       | L         |
|              | 1.86 — 0.86 3.7 | F               | 15.955 | 273.7   | BF       |
|              | 1.86 — 0.86 3.7 | F               | 15.955 | —       | L         |

high-redshift $z > 1.25$ objects in compare to older compilations. From this compilation we take the “Silver” sample which contains all 186 SNIa, and the restricted “Gold” sample of 157 SNIa (with higher quality of the spectroscopic and photometric records).

In order to test a cosmological model we calculate the best fit with minimum $\chi^2$ as well as estimate the model parameters using the maximum likelihood method [2]. For both statistical methods we take the parameters $m$ and $n$ in the interval $[0, 10]$, $n > m$. We test separately the models with and without the cosmological constant term. We also assume priors about $\Omega_{m,0}$ and we estimate it or take $\Omega_{m,0} = 0.3$ (baryonic plus dark matter in galactic halos) [36].

The results of two fitting procedures performed on the “Gold” sample for the cosmological bouncing models with different prior assumptions are presented in (Table II and III). These tables refer both to the $\chi^2$ (best fit) and results from marginalized probability of density functions.

At first we analyzed bouncing model without any priors for $m$ parameter (BM). We obtain the value $\chi^2 = 181.6$ what means that this model is acceptable on the $2\sigma$ level with degree of freedom $df = 153$. However, the estimated value of $m = 1.4$ in the model is unrealistic.
TABLE III: The results of statistical analysis of BCDM models \((m = 3)\) obtained for SNIa data from the best fit with minimum \(\chi^2\) (denoted as BF) and from the likelihood method (denoted as L). The case of a fixed value of \(\mathcal{M}\) is denoted as F.

| model          | \(\Omega_{m,0}\) | \(\Omega_{n,0}\) | \(\mathcal{M}\) | \(\chi^2\) | method |
|----------------|-------------------|-------------------|-----------------|-----------|--------|
| BCDM model     | 1.50              | 0.50              | 16.105          | 226.6     | BF     |
| with \(n = 4\) | 1.50              | 0.50              | 16.095          | —         | L      |
|                | 1.50 0.50         | F15.955           | 296.4           | BF        |        |
|                | 1.50 0.50         | F15.955           | —               | L         |        |
| BCDM model     | 1.03              | 0.03              | 16.175          | 291.2     | BF     |
| with \(n = 6\) | 1.03              | 0.03              | 16.175          | —         | L      |
|                | 1.03 0.03         | F15.955           | 443.4           | BF        |        |
|                | 1.03 0.03         | F15.955           | —               | L         |        |

because the dust matter is present in the universe \((m = 3)\). With the prior \(m = 3\) we obtain \(\chi^2 = 217.4\) with the value of the parameter \(n = 3.7\). For the more realistic model with \(m = 3\) and \(n = 4\) (because of the presence of radiation matter in the Universe) (Table III) we obtain \(\chi^2 = 226.6\). While the bouncing model with dust (BCDM) is better fitted than the Einstein-De Sitter model it is rejected at least on the 4\(\sigma\) level. With priors \(\mathcal{M} \simeq 15.955\) the model is rejected on the 8\(\sigma\) level.

In Fig. 3 we present of residuals plots of the \(m-z\) relation for considered models with respect to the Einstein-de Sitter (CDM) model. Apart the CDM model (the zero line) the three models \(\Lambda\)CDM, BM and BCDM are shown. The diagrams for bouncing models intersect the \(\Lambda\)CDM diagram in such a way that the supernovae on intermediate distances are brighter than expected in the \(\Lambda\)CDM model, while very high redshift supernovae should be fainter than they are expected in the \(\Lambda\)CDM model. Note that this effects are more stronger for the BCDM model than for the BM model.

Similarly we analyze the bouncing models with the additional parameter—the cosmological constant. We fixed value of \(m = 3\) (dust matter) and it is called the extended bouncing model (\(\Lambda\)BCDM). This model with \(\text{df} = 153\) is statistically admissible on the 2\(\sigma\) level (Table IV), but we obtain \(\Omega_{n,0} = 0\) (no bounce term) as a most probable value. This result is similar also for models with fixed values \(n = 4\) and \(n = 6\) (Table V), as well as it is inde-
FIG. 3: Residuals (in mag) between the Einstein-de Sitter model and the Einstein-de Sitter itself (zero line), the ΛCDM flat model (upper curve) the best-fitted BM model, (upper-middle curve) and the best-fitted BCDM model with $m = 3$ (lower-middle curve) (with assumed $M = 15.955$).

Independent from the assumption on $\Omega_{m,0}$. In this way ΛBCDM reduces to “classical” ΛCDM model.

The confidence levels in the $(\Omega_{n,0}, n)$ plane are presented in Fig. 4. In order to complete the picture we have also derived one-dimensional probability distribution functions (PDF) for $\Omega_{n,0}$ (Fig. 5) and $n$ (Fig. 6) obtained from the joint marginalization over remaining model parameters. The maximum value of such a PDF informs us about the most probable value of $\Omega_{n,0}$, supported by supernovae data within the extended bouncing dust model.

From the PDFs the most probable value of $\Omega_{n,0}$ is also equal 0, however non-zero value of $\Omega_{n,0}$ cannot be excluded. In this way, it is crucial to determine which combination of parameters give the preferred fit to data. This is the statistical problem of model selection \[37\]. The problem is usually the elimination of parameters which play insufficient role in improving the fit data available. Important role in this area plays especially the Akaike information criterion (AIC) \[38\]. This criterion is defined as

$$AIC = -2 \ln L + 2k$$  \hspace{1cm} (32)
TABLE IV: Results of the statistical analysis of the extended bouncing models \((m = 3)\), obtained for SNIa data from the best fit with minimum \(\chi^2\) (denoted as BF) and from the likelihood method (denoted as L). The case of a fixed value of parameter \(\Omega_{m,0}\) is denoted as F. If in BF method we obtain \(\Omega_{n,0} = 0\) than \(n\) could be taken arbitrary (marked as A).

| model                   | \(\Omega_{m,0}\) | \(\Omega_{n,0}\) | \(n\) | \(\Omega_{\Lambda,0}\) | \(M\)   | \(\chi^2\) | method |
|-------------------------|-------------------|-------------------|------|----------------------|--------|-----------|--------|
| ABCDM model             | 0.31              | 0.00              | A    | 0.69                 | 15.955 | 175.9     | BF     |
|                         | 0.34              | 0.00              | 3.0  | 0.67                 | 15.965 | —         | L      |
|                         | F0.30             | 0.00              | A    | 0.70                 | 15.955 | 175.9     | BF     |
|                         | F0.30             | 0.00              | 3.0  | 0.70                 | 15.945 | —         | L      |
| ABCDM model with \(M = 15.955\) | 0.31              | 0.00              | A    | 0.69                 | —      | 175.9     | BF     |
|                         | 0.31              | 0.00              | 3.0  | 0.68                 | —      | —         | L      |
|                         | F0.30             | 0.00              | A    | 0.70                 | —      | 175.9     | BF     |
|                         | F0.30             | 0.00              | 3.0  | 0.70                 | —      | —         | L      |

where \(L\) is the maximum likelihood and \(k\) is the number of the parameter of the model. The best model is the model which minimizes the AIC. The AIC for the models under consideration is presented in the (Table VI). It is clear that model which minimizing AIC is \(\Lambda\)CDM. Therefore it is no reason to introduce a model with bouncing terms and such model should be ruled out by Occam’s razor. Because the extended bouncing dust model is statistically admissible from SNIa data it can be reconsidered only if the firm theoretical reason appears. Only this situation can justify consideration of the model with a small, but non-zero bouncing term.

The existence of the oldest high-redshift extragalactic (OHReG) objects could be used as a test of the cosmological models (Table VII). The globular cluster analysis indicated that the age of the Universe is 13.4 Gyr [39]. We demonstrate that the age of OHReG objects restricts the model parameter. As a criterion we take that the age of the Universe in a given redshift should be bigger than, at least equal, to the age of its oldest objects. With the assumption of \(H_0 = 65\) km/s MPc, the age of the universe on particular \(z\) for three class of models is calculated (Fig. 7). This test admits the \(\Lambda\)CDM model with \(\Omega_{m,0} = 0.3\). In this model, the age of the Universe is 14.496 Gyr. The BM model seems to be allowed from this test, however, that model predicts much longer age of the Universe (more than 20 Gyr).
TABLE V: Results of comparison of ΛCDM model with the extended bouncing models \((m = 3)\) with fixed values \(n = 4\) and \(n = 6\). The result of statistical analysis for SNIa data from the best fit with minimum \(\chi^2\) (denoted as BF) and from the likelihood method (denoted as L). The case of a fixed value of \(\Omega_{m,0}\) is denoted as F.

| model            | \(\Omega_{m,0}\) | \(\Omega_{n,0}\) | \(\Omega_{\Lambda,0}\) | \(\mathcal{M}\) | \(\chi^2\) | method |
|------------------|-------------------|------------------|-------------------|---------------|-----------|--------|
| ΛCDM model       | 0.31              | —                | 0.69              | 15.955        | 175.9     | BF     |
|                  | 0.34              | —                | 0.67              | 15.965        | —         | L      |
|                  | F0.30             | —                | 0.70              | 15.955        | 175.9     | BF     |
|                  | F0.30             | —                | 0.70              | 15.945        | —         | L      |
| ABCDM model with \(n = 4\) | 0.31 0.00         | 0.69              | 15.955            | 175.9         | BF        |
|                  | 0.37 0.00         | 0.65              | 15.965            | —             | L         |
|                  | F0.30 0.00        | 0.70              | 15.955            | 175.9         | BF        |
|                  | F0.30 0.00        | 0.70              | 15.945            | —             | L         |
| ABCDM model with \(n = 6\) | 0.31 0.00         | 0.69              | 15.955            | 175.9         | BF        |
|                  | 0.34 0.00         | 0.66              | 15.965            | —             | L         |
|                  | F0.30 0.00        | 0.70              | 15.955            | 175.9         | BF        |
|                  | F0.30 0.00        | 0.70              | 15.945            | —             | L         |

then ΛCDM. In turn the BCDM model must be rejected because its age is 11.5 Gyr.

IV. CMB PEAKS IN THE EXTENDED BOUNCING MODEL

Acoustic oscillations in the primeval plasma during the last scattering give rise to the temperature map of cosmic microwave background (CMB). Peaks in the power spectrum correspond to maximum density of the wave. In the Legendre multipole space these peaks correspond to the angle subtended by the sound horizon at the last scattering. Further peaks answer to higher harmonics of the principal oscillations.

The locations of these peaks depend on the variations in the model parameters. Therefore, they can be used to constrain the parameters of cosmological models.
FIG. 4: For the extended bouncing model with $\mathcal{M} = 15.955$ there are shown the confidence levels on the plane $(\Omega_{n,0}, n)$ minimized over parameter $\Omega_{m,0}$.

The acoustic scale $\ell_A$ which puts the locations of the peaks is defined as

$$
\ell_A = \pi \int_{z_{\text{dec}}}^{\infty} \frac{dz'}{c_s H(z')} \int_{z_{\text{dec}}}^{z'} \frac{dz''}{H(z'')}
$$

where

$$
H(z) = H_0 \sqrt{\Omega_{m,0}(1 + z)^3 + \Omega_{c,0}(1 + z)^4 - \Omega_{n,0}(1 + z)^n + \Omega_{\Lambda,0}}
$$

and $c_s$ is the speed of sound in the plasma given by

$$
c_s^2 \equiv \frac{dp_{\text{eff}}}{d\rho_{\text{eff}}} = \frac{4}{3}\Omega_{\gamma,0}(1 + z) - \frac{n-3}{3}n\Omega_{n,0}(1 + z)^{n-3} 
$$

and

$$
3\Omega_{b,0} + 4\Omega_{\gamma,0}(1 + z) - n\Omega_{n,0}(1 + z)^{n-3}.
$$
FIG. 5: Extended Bouncing model with M=15.955. The density distribution for Ω_{n,0}. Confidence level 68.3% and 95.4% are also marked on the figure.

The properties of the bouncing term Ω_{n,0} are unknown. In particular, we do not know whether it influences the sound velocity. But we assume that sound can propagate in it as well as in baryonic matter and photons. Let us note that with the lack of the bouncing term (i.e. Ω_{n,0} = 0) and/or when sound does not propagate in the bouncing fluid, we obtain the standard formula for $c_s^2$ [40].

Knowing the acoustic scale we can determine the location of $m$-th peak

$$\ell_m \sim \ell_{A}(m - \phi_m) \quad (36)$$

where $\phi_m$ is the phase shift caused by the plasma driving effect. Assuming that $\Omega_{m,0} = 0.3$, on the surface of last scattering $z_{dec}$ it is given by

$$\phi_m \sim 0.267 \left[ \frac{r(z_{dec})}{0.3} \right]^{0.1} = 0.267 \left[ \frac{1}{0.3 \rho_m(z_{dec})} \right]^{0.1} = 0.267 \left[ \frac{1}{0.3} \frac{\Omega_{r,0}(1+z_{dec})}{0.3} \right]^{0.1} \quad (37)$$
FIG. 6: Extended Bouncing model with M=15.955. The density distribution for \( n \). Confidence level 68.3\% and 95.4\% are also marked on the figure.

where \( \Omega_{b,0}h^2 = 0.02 \), \( r(z_{\text{dec}}) \equiv \rho_r(z_{\text{dec}})/\rho_m(z_{\text{dec}}) = \Omega_{r,0}(1 + z_{\text{dec}})/\Omega_{m,0} \) is the ratio of the radiation to matter densities at the surface of the last scattering.

The locations of the first two peaks are taken from the CMB temperature angular power spectrum [41, 42], while the location of the third peak is from the BOOMERANG measurements [43]. They values with uncertainties on the level 1\( \sigma \) are the following

\[
\ell_1 = 220.1^{+0.8}_{-0.8}, \quad \ell_2 = 546^{+10}_{-10}, \quad \ell_3 = 845^{+12}_{-25}.
\]

From the WMAP data, only the Hubble constant is \( H_0 = 72 \) km/s MPc (or the parameter \( h = 0.72 \)), the baryonic matter density \( \Omega_{b,0} = 0.024h^{-2} \), and the matter density \( \Omega_{m,0} = 0.14h^{-2} \) [41], which give a good agreement with the observation of the position of the first peak.

In the analysis of the constraints on the bouncing cosmological model parameters we fix
TABLE VI: The Akaike information criterion (AIC) for models under consideration: Einstein-de Sitter model (CDM), ΛCDM model (ΛCDM), bouncing model (BM), bouncing model with dust \( m = 3 \) (BCDM) and extended bouncing model with dust \( m = 3 \) (ΛBCDM).

| model                | no. of parameters | AIC  |
|----------------------|-------------------|------|
| CDM                  | 1                 | 325.5|
| ΛCDM                 | 2                 | 179.9|
| BM                   | 4                 | 189.6|
| BCDM                 | 3                 | 223.4|
| BCDM with \( n = 4 \)| 2                 | 230.6|
| BCDM with \( n = 6 \)| 2                 | 295.2|
| ΛBCDM                | 4                 | 183.9|
| ΛBCDM with \( n = 4 \)| 3                 | 181.9|
| ΛBCDM with \( n = 6 \)| 3                 | 181.9|

TABLE VII: The age of extragalactic objects.

| No | object          | \( z \) | age in Gys |
|----|-----------------|--------|-----------|
| 1  | globular cluster| 0.     | 13 – 15   |
| 2  | 3C65 quasar     | 1.175  | 4.0       |
| 3  | LBDS 53W069     | 1.43   | 4.0       |
| 4  | LBDS 53W091     | 1.55   | 3.5       |

the baryonic matter density \( \Omega_{b,0} = 0.05 \), the spectral index for initial density perturbations \( n = 1 \), and the radiation density parameter \ref{eq:40}

\[
\Omega_{r,0} = \Omega_{\gamma,0} + \Omega_{\nu,0} = 2.48h^{-2} \times 10^{-5} + 1.7h^{-2} \times 10^{-5} = 4.18h^{-2} \times 10^{-5} \tag{38}
\]

which is a sum of the photon \( \Omega_{\gamma,0} \) and neutrino \( \Omega_{\nu,0} \) densities.

Assuming \( \Omega_{m,0} = 0.3 \) and \( h = 0.72 \) we obtain for the standard ΛCDM cosmological model the following positions of peaks

\[
\ell_1 = 220, \quad \ell_2 = 521, \quad \ell_3 = 821
\]

with the phase shift \( \phi_m \) given by \ref{eq:57}.
FIG. 7: The age of the universe on particular \( z \) for three class of models: ΛCDM (middle curve), bouncing model BM (upper curve) and bouncing model with dust matter BCDM (lower curve). We marked age of 4 extragalactic object by star (Table VII).

From the SNIa data analysis, it was found that the Hubble constant has a lower value. Assuming that \( H_0 = 65 \text{ km/s MPc} \) (or \( h = 0.65 \)), we have \( \Omega_{r,0} = 9.89 \times 10^{-5} \) from eq. (38). For further calculations we take \( \Omega_{r,0} = 0.0001 \). If we consider the standard ΛCDM model, with \( \Omega_{m,0} = 0.3, \Omega_{b,0} = 0.05 \), the spectral index for the initial density perturbations \( n = 1 \), and \( h = 0.65 \), where sound can propagate in baryonic matter and photons, we obtain the following locations of first three peaks

\[
\ell_1 = 225, \quad \ell_2 = 535, \quad \ell_3 = 847.
\]

We note the difference between the observational and theoretical values in this case. We check whether the presence of the bouncing term \( \Omega_{n,0} \) moves the locations of the peaks. We do not know whether it influences the sound velocity, but we assume that sound can
To obtain the bounce, \( n > 4 \) is necessary because the presence of the radiation term is required by the physics of primordial plasma in the recombination epoch. From the location of the first peak we obtain, the limit for \( \Omega_{n,0} \) term. In the case \( n = 5 \), with \( H_0 = 72 \) km/s MPc, we obtain that \( \Omega_{n,0} < 2 \times 10^{-11} \) while for \( n = 6 \) we have that \( \Omega_{n,0} < 2 \times 10^{-17} \).

Please note that the special case \( n = 6 \) was analyzed for both values \( H_0 = 65 \) km/s MPc and \( H_0 = 72 \) km/s MPc and the agreement with the observation of the location of the first peak was obtained, also for the non-zero values of the parameter \( \Omega_{n,0} \) [30]. It means that both values of \( H_0 \) are allowed from the CMB constraints for the case \( n = 6 \). However, this value is of order \( 10^{-10} \). The results of calculations of the peak locations and the values of the parameter \( \Omega_{n,0} \) are presented in Table VIII. In the special case \( n = 4 \), the bounce term scale like radiation, the existing of the bounce requires \( \Omega_{n,0} > \Omega_{r,0} \). In this case, we also obtain the agreement with the observation of the location of the first peak for the non-zero values of the parameter \( \Omega_{n,0} \) (in order to \( 3 \times 10^{-4} \)).

Finally, we analyze the models in which we assume that sound can propagate only in baryonic matter and photons. With \( H_0 = 72 \) km/s MPc, in the case of \( n = 5 \), we obtain that \( \Omega_{n,0} < 2.2 \times 10^{-8} \) while for \( n = 6 \) we have that \( \Omega_{n,0} < 5 \times 10^{-14} \). For the special case \( n = 4 \) we have that \( \Omega_{n,0} = 2.3 \times 10^{-4} \).

We have also calculated the age of the Universe in the \( \Lambda \)CDM model. We find that the difference in the age of the Universe is smaller than 10 mln years for all values of \( \Omega_{n,0} \) admissible by the CMB peaks location. So this model is admissible by the test of the age of the OHReG objects.
V. CONSTRAINT FROM THE BBN

The observations of abundance of light elements is in good agreement with the prediction of the standard big-bang nucleosynthesis (BBN). It means that the BBN does not allow for any significant divergence from the standard expansion law, apart from the beginning of BBN to the present epoch. Therefore, any nonstandard terms included in the Friedmann equation should give only a negligible small change during the BBN epoch to render the nucleosynthesis process unchanged.

It is crucial for the bouncing models to be consistent with BBN. These models have the nonstandard term $\Omega_n$ which scales like $a^{-n}$ where $n > 4$. For example we analyze the cases $n = 5$ and $n = 6$. This additional term scales like $(1 + z)^n$. It is clear that such a term gives rise to the accelerated Universe expansion if $\Omega_{n,0} > 0$. Going backwards in time, this term would become dominant at some redshift. If it happened before the BBN epoch then the radiation domination would never occur and the all BBN predictions would be lost.

The domination of the bouncing term $\Omega_n$ should end before the BBN epoch starts and we assume that the BBN results are preserved in the bouncing models. In this way we obtain another constraint on the value of $\Omega_{n,0}$. Let the model modification be negligible small during the BBN epoch and the nucleosynthesis process be unchanged. It means that the contribution of the bouncing term $\Omega_{n,0}$ cannot dominate over the radiation term $\Omega_{r,0} \approx 10^{-4}$ before the BBN ($z \simeq 10^8$)

$$|\Omega_{n,0}|(1 + z)^n < \Omega_{r,0}(1 + z)^4.$$ 

It means that $|\Omega_{n,0}| < 10^{-20}$ for the case $n = 6$ while $|\Omega_{n,0}| < 10^{-12}$ for the case $n = 5$ respectively. Of course, the case $n = 4$ is excluded because the existence of bouncing requires in this case $|\Omega_{n,0}| > \Omega_{r,0}$, while BBN constraints require $|\Omega_{n,0}| < \Omega_{r,0}$. Let us note that inequality $x_b \leq \left( \frac{|\Omega_{n,0}|}{\Omega_{r,0}} \right)^{\frac{1}{n-4}}$ constrains the minimal size of the universe. The general conclusion from BBN constraints is that in the present epoch, the bouncing term, if it exists, is insignificant in comparison to the matter term.

Table IX gives the value of $z_{\text{bounce}}$ calculated for the best-fitted model parameters. Because the bounce should take place before BBN epoch so $z_{\text{bounce}} > z_{\text{BBN}} \simeq 10^8$. Comparing with the $z_{\text{bounce}}$ presented in Table IX we obtain that only two classes of models $\Lambda$CDM and $\Lambda$BCDM are admissible.
TABLE IX: The value of $z_{\text{bounce}}$ for the models under consideration: Einstein-de Sitter model (CDM), $\Lambda$CDM model ($\Lambda$CDM), bouncing model (BM), bouncing model with dust $m = 3$ (BCDM) and extended bouncing model with dust $m = 3$ (ABCDM).

| model                   | $z_{\text{bounce}}$ |
|-------------------------|----------------------|
| CDM                     | —                    |
| $\Lambda$CDM            | —                    |
| BM                      | $3.54 \times 10^4$  |
| BCDM                    | 2.98                 |
| BCDM with $n = 4$       | 3                    |
| BCDM with $n = 6$       | $4.05 \times 10^4$  |
| ABCDM                   | $\infty$            |
| ABCDM with $n = 4$      | $\infty$            |
| ABCDM with $n = 6$      | $\infty$            |

VI. CONCLUSION

In this paper we confront the bouncing model with astronomical observations. We use the constraints from SNIa data, CMB analysis, and BBN and the age of the oldest high-redshift objects.

The standard bouncing model is excluded statistically at the $4\sigma$ level. If we take the extended bouncing model (with extra $\Omega_{\Lambda,0}$ term) then we obtain, as the best-fit, that the parameter $\Omega_{n,0}$ is equal to zero which means that the SNIa data do not support the existence of the bouncing term in the model. We also demonstrate that BBN gives stringent constraints on the extra term $\Omega_{n,0}$ and show that the bounce term is insignificant in the present epoch.

It is interesting that such bouncing models with extra inflationary expansion are presently favored in the loop quantum approach [44, 45, 46, 47]. The theory of loop quantum gravity predicts that there is no initial singularity because of the quantum effects in the Planck scale. It is due to the continuum break and granularity of spacetime. Therefore, we consider the model where we assume a small positive value of $\Omega_{n,0}$ and estimate the rest of the parameters. This model is statistically admissible. However, when we compare this model
with the standard ΛCDM model applying the Akaike criterion, the latter is preferred.

If the energy density is so large then quantum gravity corrections are important at both the big-bang and big-rip. It is interesting that the classical theory reveals its own boundaries (i.e. classical singularities). The account of quantum effect avoids not only an initial singularity but allows also to escape from a future singularity \[48, 49, 50\].

The avoidance of the initial singularity arises only on the quantum ground because the classical theory of gravity according to the Hawking-Penrose theorems states that these singularities are essential if only some reasonable conditions on the matter content are fulfilled.

If we assume the classical gravity is obvious during the whole evolution of the Universe than there is no reason to introduce the bouncing era. The ΛCDM model with the big-bang is a simpler model, while the bouncing model requires to the admittance of observationally unconfirmed assumptions. In this way Occam’s razor methodology rules out the generalized bouncing model. The general conclusion is that the present astronomical data does not support the bouncing cosmology.

We also adopt the methods of dynamic systems for investigating dynamics in the phase space. The advantages of these methods are that they offer the possibility of the investigation of all evolitional paths for all initial conditions. We show that the dynamics can be reduced to a two-dimensional Hamiltonian system. We also show structural instability of both the standard and generalized bouncing models. Let us note that the concordance ΛCDM models are structurally stable \[51\]. The structural stability is a reasonable condition which should be satisfied by models of real physical processes. From the dynamic investigation we obtain that all models with the bounce are rather fragile. It means that any small perturbation of the right-hand sides of the dynamic equations of the model changes the topological structure of the phase space. The bouncing models in the space of all dynamic system on the plane form non-dense (zero measure) subset of this plane following the Peixoto theorem. Therefore, the bouncing models are untypical while ΛCDM models are generic from the point of view structural stability.
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