Can electron and muon $g - 2$ anomalies be jointly explained in SUSY?

Song Li$^{1,2,a}$, Yang Xiao$^{1,2,b}$, Jin Min Yang$^{1,2,c}$

$^1$ CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
$^2$ School of Physics Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

1 Introduction

There has been a long-standing discrepancy between the standard model (SM) prediction and experiment for muon anomalous magnetic moment $a_{\mu} = (g - 2)_{\mu}/2$. The combined result of the FNAL E989 experiment [1] and the BNL experiment gives a value which is 4.2σ above the SM prediction [2]:

$$\Delta a_{\mu}^{\text{Exp}-\text{SM}} = a_{\mu}^{\text{Exp}} - a_{\mu}^{\text{SM}} = (2.51 \pm 0.59) \times 10^{-9}.$$  (1)

On the other hand, for electron anomalous magnetic moment $a_e$, the SM predicted value [3] derived from the measurement of the fine-structure constant $\alpha_{\text{em}}$ using $^{133}\text{Cs}$ atoms at Berkeley [4] is 2.4σ above the electron $g - 2$ experimental value [5]:

$$\Delta a_{e}^{\text{Exp}-\text{SM}} = a_{e}^{\text{Exp}} - a_{e}^{\text{SM}}(^{133}\text{Cs}) = (-8.8 \pm 3.6) \times 10^{-13}.$$  (2)

However, another experimental result of $\alpha_{\text{em}}$ measured with $^{87}\text{Rb}$ atoms at Laboratoire Kastler Brossel (LKB) [6] gives a value of $a_e$ [7] which agrees with the electron $g - 2$ experimental value [5]. So far, the cause of the discrepancy between the Berkeley and LKB results is not clear. Obviously, if the Berkeley result is correct, it may serve as a plausible hint for new physics; if the LKB result is correct, there will be no need for new physics to provide a contribution to $(g - 2)_e$.

Actually, neither the Berkeley result for electron $g - 2$ nor the FNAL + BNL result for muon $g - 2$ can serve as a robust evidence for new physics beyond the SM. Whereas, while waiting for more forthcoming independent experiments to make confirmation, many theorists have chosen to keep open minds and intensively studied the implications of these anomalies for new physics. In this work, we keep open minds and study the implications of these anomalies for low energy supersymmetry.

The $g - 2$ of a charged lepton relates to the new physics scale $\Lambda$ as [8]
\[ \frac{\delta a_{\ell}}{a_{\ell}} \propto \left( \frac{m_{\ell}}{\Lambda} \right)^2, \]  

(3)

where \( \Lambda \) is assumed to be much larger than the lepton mass \( m_{\ell} \). If the new physics is the same for different lepton flavors, we may expect

\[ \frac{\delta a_{e}}{a_{e}} \approx \left( \frac{m_{e}}{m_{\mu}} \right)^2 \approx \frac{1}{43000}. \]  

(4)

But from Eqs. (1) and (2) we obtain

\[ \frac{\Delta a_{e}^{\text{Exp-SM}}}{\Delta a_{\mu}^{\text{Exp-SM}}} \approx -15 \times \frac{1}{43000}, \]  

(5)

which implies that the new physics should not be flavor blind [9]. This brings us a serious challenge for building new physics models. Intuitively, in order to jointly explain the electron and muon \( g - 2 \) anomalies, new physics models must have different couplings for charged leptons of different flavors. This will suggest us to consider some new physics models with lepton flavor universality violation. However, in some models the manifest breaking of lepton flavor universality is not required [10-12].

There have been many studies trying to explain electron and muon \( g - 2 \) jointly: (i) using the two-Higgs-doublet model (2HDM) or its extended versions [13-19], among which the aligned 2HDM with right-handed neutrinos was used in [13], the 2-loop contribution in 2HDM with flavor conservation was considered in [14], while a neutral scalar \( H \) with mass satisfying \( \mathcal{O}(1) \) \( \text{MeV} < m(H) < \mathcal{O}(1) \) \( \text{GeV} \) was used in [16]; (ii) using leptoquarks [17,20,21] and axion-like particles [21]; (iii) using flavor models [15,22] which attempted to solve the fermion mass hierarchy problem while provide a joint explanation for the muon/electron \( g - 2 \) anomalies; (iv) using the inverse type-III seesaw model with a pair of vector-like leptons [23]; (v) using an abelian flavor symmetry [24] to make electron and muon decouple to circumvent the MEG bound of \( B(\mu^+ \rightarrow e^+ \gamma) \) [25] (note that \( Z' \)-model cannot explain the muon/electron \( g - 2 \) anomalies under the MEG bound, as shown in [26,27]); (vi) using a flavor-dependent global \( U(1) \) symmetry and a discrete \( \mathbb{Z}_2 \) symmetry with some additional fermion and scalar fields [28].

As a leading candidate for new physics models, the MSSM (minimal supersymmetric standard model) was also considered [29] to interpret \( \Delta a_{e, \mu} \) while escaping the \( B(\mu^+ \rightarrow e^+ \gamma) \) constraint by assuming 1-3 flavor violation. Another attempt in the MSSM without assumption of flavor violation but with flavor non-universality was given in [30], where the authors decouple the right-hand smuon and choose a negative soft mass parameter \( M_1 \) to achieve the goal. Furthermore, one can explain \( \Delta a_{e, \mu} \) jointly by using the SUSY threshold correction to change the selectron and smuon Yukawa couplings including size and sign [31]. In addition, the B-LSM and an extended NMSSM (next-to-minimal supersymmetric standard model) was used for a joint explanation of muon/electron \( g - 2 \) anomalies [32,33].

Anyway, it is rather challenging for the MSSM to simultaneously explain electron and muon \( g - 2 \). Although previous studies found out a plausible part of parameter space in the MSSM, some relevant constraints were not considered (say from dark matter) or not fully considered (say from collider experiments). Given the popularity of the MSSM and the plausible new physics hints from the muon/electron \( g - 2 \) anomalies, we in this work revisit the MSSM to give a more comprehensive study. We will explore the MSSM parameter space to figure out the possibility to accommodate the muon/electron \( g - 2 \) anomalies under other relevant constraints from collider experiments and dark matter measurements.

This work is organized as follows. In Sect. 2, we describe the MSSM contributions to muon/electron \( g - 2 \). In Sect. 3, we explore parameter space to accommodate the muon/electron \( g - 2 \) anomalies. In Sect. 4, we show relevant constraints on the favored parameter space. Finally, we conclude in Sect. 5.

## 2 MSSM contributions to electron and muon \( g - 2 \)

The MSSM contributions to a charged lepton \( \ell \) (electron or muon) \( g - 2 \) mainly come from neutralino-slepton and chargino-sneutrino loops. The analytical expressions contributed by these two parts are given by [34]

\[ \delta a_{\ell}^{\text{SM}} = \frac{m_{\ell}}{16\pi^2} \sum_{i,m} \left\{ -\frac{m_{\ell}}{2m_{\ell}^2}(|n_{i,m}^L|^2 + |n_{i,m}^R|^2)F_1^{\ell}(x_{im}) + \frac{m_{\mu}}{3m_{\mu}^2}Re[n_{i,m}^L n_{i,m}^R F_2^{\ell}(x_{im})] \right\}, \]  

(6)

\[ \delta a_{\ell}^{\pm} = \frac{m_{\ell}}{16\pi^2} \sum_k \left\{ -\frac{m_{\ell}}{2m_{\ell}^2}(|c_k^L|^2 + |c_k^R|^2)F_1^{\ell}(x_k) + \frac{2m_{\mu}}{3m_{\mu}^2}Re[c_k^L c_k^R F_2^{\ell}(x_k)] \right\}, \]  

(7)

where \( x_{im} = m_{i,m}^2/m_{\ell}^2 \) and \( x_k = m_{k}^2/m_{\ell}^2 \). The definitions of \( n_{i,m}^{L,R} \), \( c_{k}^{L,R} \) and \( F_{1,2}^{\ell,N} \) can be found in appendix.

In this work, we mainly use Eqs. (6) and (7) to calculate the muon/electron \( g - 2 \) and some significant 2-loop corrections will also be included. After considering the 2-loop
corrections, we have
\[
\delta a^{\text{SUSY}}_\ell = \left( 1 - \frac{4\alpha}{\pi} \ln \frac{M^{\text{SUSY}}}{m_\ell} \right) \times \left( \frac{1}{1 + \Delta_\ell} \right) \delta a^{\text{SUSY},1L}_\ell, \tag{8}
\]

where \(\delta a^{\text{SUSY},1L}_\ell\) denotes one-loop SUSY contributions. The term in the first bracket arises from the leading-logarithmic QED correction [35]. This correction takes into account renormalization group evolution of effective operators from \(M^{\text{SUSY}}\) to \(m_\ell\) scale, which can lead to a reduction of about 7\% for \(\delta a_\mu\), and a reduction of about 11\% for \(\delta a_e\). The term in the second bracket of Eq. (8) arises from the tan \(\beta\)-enhanced loop diagrams that can correct the Yukawa couplings of sleptons, and a resummation has been made [36,37]. \(\Delta_\ell\) is given by
\[
\Delta_\ell = -\mu \tan \beta \frac{g^2 M_2}{16\pi^2} \left[ I(m^2_{\tilde{\chi}_1^0}, m^2_{\tilde{\chi}_2^0}, m^2_{\tilde{\tau}_e^0}) + \frac{1}{2} I(m^2_{\tilde{\chi}_1^0}, m^2_{\tilde{\chi}_2^0}, m^2_{\tilde{\tau}_\mu^0}) \right] \\
- \mu \tan \beta \frac{g^2 M_1}{16\pi^2} \left[ I(\mu^2, M^2_{\tilde{\tau}_e^0}, m^2_{\tilde{\tau}_e^0}) - \frac{1}{2} I(\mu^2, M^2_{\tilde{\tau}_\mu^0}, m^2_{\tilde{\tau}_e^0}) \right], \tag{9}
\]

where the loop function \(I(a, b, c)\) is given by
\[
I(a, b, c) = \frac{ab \ln(a/b) + bc \ln(b/c) + ca \ln(c/a)}{(a-b)(b-c)(c-a)}. \tag{10}
\]

We use some tricks to prevent enormous errors while avoid false singularities of \(I(a, b, c)\). We note that \(I(a, b, c)\) is fully symmetric to the three parameters, and therefore we assume \(a \leq b \leq c\) and define \(x = a/b, y = c/b\). Then we obtain
\[
I(a, b, c) = \frac{1}{c-a} \left( x \times \frac{\ln y}{y-1} - x \times \frac{\ln x}{x-1} \right). \tag{11}
\]

If \(x \approx 1\) and \(a \approx c\), one can perform Fourier expansion for \(\ln x/(x - 1)\) around \(x = 1\) for the calculation. The practice is similar when \(y \approx 1\) and \(a \approx c\). If \(a \approx c\), we demand the Fourier expansion of the binary function. Defining \(\delta x = x - 1\) and \(\delta y = y - 1\), then we have
\[
I(a, b, c) = \frac{1}{2b} \left[ 1 - \frac{\delta x + \delta y}{3} + \frac{g(\delta x, \delta y, 2)}{6} - \frac{g(\delta x, \delta y, 3)}{10} + \frac{g(\delta x, \delta y, 4)}{15} + \cdots \right], \tag{12}
\]

where
\[
g(\delta x, \delta y, n) = \sum_{i=0}^{n} \delta x^i \delta y^{n-i}. \tag{13}
\]

From Eq. (9) we know that this correction can be enhanced by a large \(\mu \tan \beta\). We calculate \(\delta a^{\text{SUSY}}_{e,\mu}\) using the above results. Our code is cross-checked with \textsc{CPsuperH 2.3} [38–40], and the difference between numerical results is less than 0.1\% in magnitude.

In order to find out the parameter space that can jointly explain \(\Delta a^{\text{Exp-SM}}_{e,\mu}\), we classify the SUSY contributions approximately as [41,42].
\[
\delta a_{\ell}(\tilde{W}, \tilde{H}, \tilde{\nu}_\ell) \simeq 15 \times 10^{-9} R \left( \frac{(100\text{GeV})^2}{\mu M_2} \right), \tag{14}
\]
\[
\delta a_{\ell}(\tilde{W}, \tilde{H}, \tilde{\nu}_L) \simeq -2.5 \times 10^{-9} R \left( \frac{(100\text{GeV})^2}{\mu M_2} \right), \tag{15}
\]
\[
\delta a_{\ell}(\tilde{B}, \tilde{H}, \tilde{\nu}_L) \simeq 0.76 \times 10^{-9} R \left( \frac{(100\text{GeV})^2}{\mu M_1} \right), \tag{16}
\]
\[
\delta a_{\ell}(\tilde{B}, \tilde{H}, \tilde{\nu}_R) \simeq -1.5 \times 10^{-9} R \left( \frac{(100\text{GeV})^2}{\mu M_1} \right), \tag{17}
\]
\[
\delta a_{\ell}(\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}) \simeq 1.5 \times 10^{-9} R \left( \frac{(100\text{GeV})^2}{\mu M_1} \right). \tag{18}
\]

where \(\ell = e, \mu, R = (m_\ell/m_\mu)^2 (\tan \beta/10)\) for brevity. Equations (14–18) are obtained through simplification under certain conditions, focusing on the order of magnitude. \(\delta a_{\ell}\) is derived from the loop correction. As the masses of the sleptons in the loops increase, the contribution of the loop diagrams becomes smaller. This dependency does not appear explicitly in Eqs. (14–17). We see that the MSSM contribution to \(g - 2\) of a charged lepton can be positive or negative. To have a negative \(\delta a^{\text{SUSY}}_{e,\mu}\) and a positive \(\delta a^{\text{SUSY}}_{e,\mu}\), we need to assume non-universality between smuon and selectron soft masses. We can find out two typical scenarios to have a negative \(\delta a^{\text{SUSY}}_{e,\mu}\) and a positive \(\delta a^{\text{SUSY}}_{e,\mu}\) for a joint explanation of muon/electron \(g - 2\) anomalies:

(i) Use the chargino-sneutrino loop in Eq. (14) to give a positive \(\delta a^{\text{SUSY}}_{e,\mu}\) assuming \(\mu M_2 > 0\); use the bino-selectron loop in Eq. (18) to give a negative \(\delta a^{\text{SUSY}}_{e,\mu}\) assuming \(\mu M_1 < 0\). This scenario was studied in [30] and will not be restudied in this work.

(ii) Use the chargino-sneutrino loop in Eq. (14) to give a negative \(\delta a^{\text{SUSY}}_{e,\mu}\) assuming \(\mu M_2 < 0\); use the bino-higgsino-smuon loop in Eq. (17) to give a positive \(\delta a^{\text{SUSY}}_{e,\mu}\) assuming \(\mu M_1 < 0\). This scenarios will be studied in detail in this work.
We would like to comment on the virtues of the second scenario. Since the SUSY contributions to $a_\mu$ are suppressed by $(m_\chi/m_{\tilde{\mu}})^2$ compared with the contributions to $a_\mu$, we need much larger SUSY loop effects for $a_\mu$. From the above formulas we see that the coefficient of the chargino-sneutrino loop in Eq. (14) is one order of magnitude higher than other four kinds of loops. Using it with assumption $\mu M_2 < 0$ to explain $\Delta a^\text{Exp-SM}_\mu$ in Eq. (5) is a wise choice. In this way, winos and higgsinos do not need to be very light. On the other hand, in order for $\delta a^\text{SUSY}_\mu$ not to get a sizable negative contribution from the chargino-sneutrino loop in Eq. (14), we assume the left-handed smuon $\tilde{\mu}_L$ to be very heavy (note that $\tilde{\nu}_\mu$ and $\tilde{\mu}_L$ are in a $SU(2)_L$ doublet and thus have the same soft mass). Therefore, in this scenario only the bino/higgsino-smuon loop in Eq. (14) is one order of magnitude higher than other SUSY loop effects for $a_\mu$.

The results from our exploration of parameter space are shown in Fig. 1. Because $\delta a^\text{SUSY}_\mu (\tilde{B}, \tilde{H}, \tilde{\mu}_R)$ is the dominant contribution to $\delta a^\text{SUSY}_\mu$, the value of $\delta a^\text{SUSY}_\mu$ (especially its $1\sigma$ range required by the explanation of $\Delta a^\text{Exp-SM}_\mu$) is sensitive to $M_1$. With respect to the case with $|M_1| = 80$ GeV, the $1\sigma$ range of $|M_1| = 120$ GeV moves down significantly. For $|M_1| > 120$ GeV, the value of $\delta a^\text{SUSY}_\mu$ will decrease rapidly, which is not shown in the figures. As for $\delta a^\text{SUSY}_\mu$, the dominant loop contribution is not sensitive to $M_1$. Hence even if $|M_1|$ is very large, $\delta a^\text{SUSY}_\mu$ does not change drastically. Therefore, in order to satisfy the experimental value of $a_\mu$, $|M_1|$ cannot be greater than $\sim 120$ GeV. This makes the tree-level mass of $\chi_1^0$ lower than 120 GeV.

From Fig. 1 we can also see that $\mu$ and $M_2$ have an inverse relationship when $\delta a^\text{SUSY}_\mu$ has a fixed value. This is easy to understand because of the appearance of $\mu M_2$ in Eq. (14). However, $\delta a^\text{SUSY}_\mu$ unexpectedly increases with the increase of $|M_2|$. This is because $\delta a^\text{SUSY}_\mu (\tilde{W}, \tilde{H}, \tilde{\nu}_\mu)$ is not suppressed enough by assuming $m_{L2} = 10 m_{L1}$, which is getting suppressed by increasing $|\mu| M_2$. If we set the value of $m_{L2}$ bigger than $10 m_{L1}$, we can further reduce the dependence of $\delta a^\text{SUSY}_\mu$ on $M_2$. In that way, the range of $M_2$ allowed by the experimental constraints will be extended.

From Fig. 1 we note that for tan $\beta = 40$ the two $1\sigma$-ranges for the explanations of $\Delta a^\text{Exp-SM}_{e,\mu}$ do not overlap. Therefore, a joint explanation of $\Delta a^\text{Exp-SM}_{e,\mu}$ at $1\sigma$ level needs a larger tan $\beta$. For tan $\beta = 60$ the two $1\sigma$-ranges of $\Delta a^\text{Exp-SM}_{e,\mu}$ overlap, which, however, requires $\mu < 200$ GeV and $|M_2| < 300$ GeV. For a joint explanation of $\Delta a^\text{Exp-SM}_{e,\mu}$ at $2\sigma$ level, the ranges of the relevant parameters are relaxed significantly.

4 Dark matter and collider constraints

4.1 Dark matter constraints

We use FlexibleSUSY 2.6.0 [45,46] to calculate the mass spectrum of supersymmetric particles, and then use MicrOMEGAs 5.2.7 [47–50] to calculate the dark matter relic density $\Omega h^2$ and the LSP-nucleon scattering cross section. For the parameters that are not directly related to $\delta a^\text{SUSY}_\mu$, we generally take 2 TeV. In addition, the mass parameters of stau are taken as $10 m_{L1}$ and the $A$-parameters that describe the trilinear soft-breaking terms are set to be 0.

We first assume the lightest neutralino $\chi_1^0$ is the dark matter candidate and search for the parameter space that meets the following three requirements:

1. The SUSY contributions $\delta a^\text{SUSY}_{e,\mu}$ in the $2\sigma$ ranges of $\Delta a^\text{Exp-SM}_{e,\mu}$;
Fig. 1 Contours of $\delta a^{\text{SUSY}}_e$ and $\delta a^{\text{SUSY}}_\mu$ on the plane of $\mu$ versus $M_2$ for $\tan\beta = 40$ (left) and $\tan\beta = 60$ (right). The colored regions correspond to $M_1 = -80$ GeV while the colored curves correspond to $M_1 = -120$ GeV. The 1$\sigma$ and 2$\sigma$ mean to explain $\Delta a^{\text{Exp}}_{e,\mu} - \Delta a^{\text{SM}}_{e,\mu}$ at 1$\sigma$ and 2$\sigma$ levels, respectively.

Fig. 2 The parameter regions survived the dark matter relic density constraint and the XENON1T exclusion limits on the plane of $\mu$ versus $M_2$ with $M_1 = -100$ GeV and $\tan\beta = 40, 60$. The 2$\sigma$ regions for explaining the muon and electron $g - 2$ anomalies are also shown. For the curves of limits, the regions indicated by the arrows are the allowed regions.

2. The dark matter thermal freeze-out relic density under the upper bound, i.e., $\Omega h^2 < 0.12$ [51];
3. The XENON1T limits from the dark matter direct detection [52].

We scan over the parameter space and the results are shown in Fig. 2. In the regions shown in this figure, the dominant component of the lightest neutralino is bino, whose thermal freeze-out relic density can easily give an over-abundance.

For the bino-like dark matter to give a correct relic density, higgsinos or wino must be mixed into it, which, however, will be not allowed by the XENON1T limits. As shown in this figure, only a very large $\tan\beta = 60$ can give a corner of parameter space to satisfy both the dark matter relic density and the XENON1T limits, which, however, is not allowed by the LHC constraints [44].

So we conclude that for the SUSY contributions $\delta a^{\text{SUSY}}_{e,\mu}$ in the 2$\sigma$ ranges of $\Delta a^{\text{Exp}}_{e,\mu} - \Delta a^{\text{SM}}_{e,\mu}$, the assumption of the lightest
neutralino as the dark matter candidate cannot satisfy the dark matter constraints under the LHC search bounds. Note that in the framework of SUSY, the lightest neutralino is not the only candidate for cosmic dark matter. Instead, the lightest super particle (LSP) as the dark matter candidate can be a super-WIMP (super-weakly interacting massive particle) like the gravitino [53,54] (its goldstino component has a relatively stronger interaction than gravity) or pseudo-goldstino [55] in multi-sector SUSY breaking with gauge mediation. As discussed in detail in [53], in such superWIMP dark matter scenarios, the superWIMPs produced thermally before inflation are diluted by inflation and the superWIMP dark matter is produced from the late decay of the lightest neutralinos which are produced from the thermal freeze-out after reheating (the reheating temperature is not high enough to thermally produce superWIMPs). Since the decay is one neutralino $\chi^0_1$ to one superWIMP $\tilde{G}$, i.e., $\chi^0_1 \rightarrow \tilde{G} + X$ ($X = \gamma, Z, h$), the superWIMP dark matter inherits the number density of the parent neutralinos and hence its relic density is suppressed by a factor $m_{\tilde{G}}/m_{\chi^0_1}$, where $m_{\chi^0_1}$ is $\mathcal{O}(100)$ GeV and $m_{\tilde{G}}$ can be lighter than $\mathcal{O}(1)$ GeV [55,56]. So the relic density upper bound can be easily satisfied by such light superWIMP dark matter. Of course, a superWIMP scatters with a nucleon super-weakly and the direct detection limits can also be easily satisfied.

A possible problem caused by such superWIMP dark matter produced from the late decay of the lightest neutralinos is that the decay may release much energy to affect BBN if the decay happens after BBN. Such a problem and its constraints have been discussed in detail in [53,57]. Recently, it was found [58] that late decay of the freeze-out neutralinos to very light gravitino dark matter can ameliorate the tension of Hubble constant. At a future lepton collider the decay of a bino-like neutralino to gravitino plus a photon may be tested [59].

4.2 LHC constraints

Now we consider constraints from colliders. We use SPheno 4.0.4 [60,61] to calculate the decay branching ratios of the relevant super particles. We consider two cases:

\begin{align}
\text{case 1} & \quad \begin{cases} M_1 = -100 \text{ GeV}, & \tan \beta = 40, \\
 m_{L_1} = m_{E_2} = \frac{m_{E_1}}{5} = \frac{m_{L_2}}{10} = 130 \text{ GeV}; \end{cases} \\
\text{case 2} & \quad \begin{cases} M_1 = -100 \text{ GeV}, & \tan \beta = 60, \\
 m_{L_1} = m_{E_2} = \frac{m_{E_1}}{5} = \frac{m_{L_2}}{10} = 145 \text{ GeV}; \end{cases}
\end{align}

where the 45 GeV increment of $M_{L_1}$ and $M_{E_2}$ in Eq. (21) is to avoid the decay $\chi^0_2 \rightarrow \ell \tilde{\chi}_{\pm}^0$ for escaping the CMS constraints [62]. At the same time, the 45 GeV increment of $M_{L_1}$ and $M_{E_2}$ will not make $\delta a_{\mu, \text{SUSY}}$ too small.

For these two cases we plot $\delta a_{\mu, \text{SUSY}}$ in Fig. 3. We see that a joint explanation of $\Delta a_{\mu, \text{SM}}$ may need a relatively a compressed spectrum for the bino-like $\chi^0_1$, the higgsino-like $\chi^0_2$ and $\chi^\pm_1$. For such compressed electroweakinos, the LHC performed the searches and gave the bounds [44]. Together with other searches for electroweakinos and sleptons at the LHC because they give similar results or have been considered in our scan (for an extensive recasting of LHC constraints, see, e.g., [66]). For the ATLAS1911 limits, the regions indicated by the arrows are the allowed regions

![Fig. 3](image-url)
[62, 63] and LEP [43], the relevant experimental constraints are displayed in Fig. 3.

From Fig. 3 we see that in both cases the results of the CMS Collaboration give strong limits on \( \mu : \mu < 130 \text{ GeV} \) for \( \tan \beta = 40 \) and \( \mu < 145 \text{ GeV} \) for \( \tan \beta = 60 \). For \( \tan \beta = 40, 470 \text{ GeV} < M_2 < 900 \text{ GeV} \) is required for \( \delta_{\mu_{Z}}^{\text{SUSY}} \) within the 1\( \sigma \) range. However, the value of \( M_2 \) has a much wider range for \( \tan \beta = 60 \). Of course, the constraints plotted in Fig. 3 can be relaxed if the relevant decay branch ratios are not assumed to be 100\%. So from Fig. 3 we see that there indeed exist a MSSM parameter space for a joint explanation of muon/electron \( g-2 \) anomalies.

Note that here we used the constraints from the LHC searches in which the LSP is assumed to be the lightest neutralino. If the LSP is a superWIMP, the signals of the searched processes could be different, depending on the lifetime of the lightest neutralino. If the lightest neutralino has a relatively long lifetime and decays outside the detector [53], the above LHC search constraints are applicable. If the lightest neutralino has a relatively short lifetime and decays inside the detector [67, 68], the signals of the relevant processes will be different. In the latter case, the signals may be more difficult to detect, for example, if the decay is dominated by \( \chi_1^0 \rightarrow G + h \) rather than by \( \chi_1^0 \rightarrow G + Z/\gamma \) [67, 68].

Finally we should remark that in SUSY only the low energy effective MSSM has enough free parameters to possibly allow for a joint explanation of muon/electron \( g-2 \) anomalies (we do not go beyond the minimal framework of SUSY, albeit some extensions like NMSSM has the virtue of smaller fine-tuning extent confronting with the requirement of a 125 GeV Higgs boson, see, e.g., [69], which can explain muon \( g-2 \) plus the AMS-02 anti-proton excess [70]). The GUT-constrained models like mSUGRA cannot even explain the single anomaly of muon \( g-2 \) (the MSSM can readily explain the single anomaly of the muon \( g-2 \), see, e.g., [71–75]). For this end, some extensions have been proposed for these models, e.g., in [76–84].

5 Conclusions

Given the FNAL+BNL measurements for muon \( g-2 \) and the Berkeley \(^{133}\)Cs measurement for electron \( g-2 \), we explored the parameter space for a joint explanation, which requires a positive contribution to muon \( g-2 \) and a negative contribution to electron \( g-2 \). Assuming no universality between smuon and selectron soft masses, we found out a part of parameter space for such a joint explanation at 2\( \sigma \) level, i.e., \( \mu M_1, \mu M_2 < 0 \), the masses of left selectron and right smuon below 200 GeV, \( m_{1,2} \) much larger than the soft masses of other sleptons, \( |M_1| < 125 \text{ GeV} \) and \( \mu < 400 \text{ GeV} \) (\( |M_2| \) is not subject to strict restrictions). This part of parameter space can survive the LHC and LEP constraints, but gives an over-abundance for the dark matter if the bino-like lightest neutralino is assumed to be the dark matter candidate. Then with the assumption that the dark matter candidate is a superWIMP (such as a pseudo-goldstino in multi-sector SUSY breaking scenarios, whose mass can be as light as GeV and produced from the late-decay of the thermally freeze-out lightest neutralinos), the dark matter problem can be avoided. So, we conclude that the MSSM may give a joint explanation for the muon and electron \( g-2 \) anomalies at 2\( \sigma \) level (the muon \( g-2 \) anomaly can be ameliorated to 1\( \sigma \)).

Acknowledgements We thank Yang Zhang for helpful discussions. This work was supported by the National Natural Science Foundation of China (NNSFC) under Grant nos. 11821505 and 12075300, by Peng-Huan-Wu Theoretical Physics Innovation Center (12047503), by the CAS Center for Excellence in Particle Physics (CCEPP), by the CAS Key Research Program of Frontier Sciences, and by a Key R&D Program of Ministry of Science and Technology of China under number 2017YFA0402204, and by the Key Research Program of the Chinese Academy of Sciences, Grant no. XDBP15.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The data in this article is kept by authors and available upon request.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

Funded by SCOAP3.

Appendix

The definitions of \( n^{L-R}_{im}, c^{L-R} \) and the kinematic loop functions \( F^{(i)}_{1,2,N} \) used in Eqs. (6) and (7) are given by [34]

\[
\begin{align*}
n^{R}_{im} &= \sqrt{2} g_1 N_{i1} X^{(i)}_{m2} + y_{t} N_{i3} X^{(i)}_{m1}, \\
n^{L}_{im} &= \frac{1}{\sqrt{2}} (g_2 N_{i2} + g_1 N_{i1}) X^{(i)*}_{m1} - y_{t} N_{i3} X^{(i)*}_{m2}, \\
c^{R}_{i} &= y_{t} U_{i2}, \\
c^{L}_{i} &= -g_2 V_{i1}, \\
F^{N}_{1}(x) &= \frac{2}{(1-x)^4} \left( 1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x \right), \\
F^{N}_{2}(x) &= \frac{3}{(1-x)^3} \left( 1 - x^2 + 2x \ln x \right).
\end{align*}
\]
\[ F_1^C(x) = \frac{2}{(1 - x)^4} \left( 2 + 3x - 6x^2 + 3x^3 + 6x \ln x \right), \]  
\[ F_2^C(x) = -\frac{3}{2(1 - x)^3} \left( 3 - 4x + x^2 + 2 \ln x \right), \]

where \( y_i = g_2m_i^2/(\sqrt{2}m_W \cos \beta) \). \( N, (U, V) \) and \( X^{(i)} \) are the mixing matrices for the neutralinos, charginos and sleptons, respectively. In other words, these matrices satisfy

\[ N^* M_{\chi_0^0} N^T = \text{diag}(m_{\tilde{e}_1^0}, m_{\tilde{e}_2^0}, m_{\tilde{\mu}_1^0}, m_{\tilde{\mu}_2^0}), \]  
\[ U^* M_\chi V^T = \text{diag}(m_{\tilde{e}_1^\pm}, m_{\tilde{\mu}_1^\pm}), \]  
\[ (X^{(i)})^2 (M_\ell^2 X^{(i)}) = \text{diag}(m_{\tilde{e}_1^0}, m_{\tilde{\mu}_2^0}). \]

References

1. B. Abi, T. Albahri, S. Al-Kilani et al., Measurement of the positive muon anomalous magnetic moment to 0.46 ppm. Phys. Rev. Lett. 126, 141801 (2021)
2. T. Aoyama et al., The anomalous magnetic moment of the muon in the Standard Model. Phys. Rep. 887, 1–166 (2020)
3. T. Aoyama, T. Kinoshita, M. Nio, Theory of the anomalous magnetic moment of the electron. Atoms 7(1), 28 (2019)
4. R.H. Parker, Yu. Chenghui, W. Zhong, B. Estey, H. Müller, Measurement of the fine-structure constant as a test of the Standard Model. Science 360, 191 (2018)
5. D. Hanneke, S. Fogwell, G. Gabrielse, New measurement of the electromagnetic moment and the fine structure constant. Phys. Rev. Lett. 100, 120801 (2008)
6. L. Morel, Z. Yao, P. Cladé, S. Guellati-Khélifa, Determination of the fine-structure constant with an accuracy of 1 parts per trillion. Nature 588(7836), 61–65 (2020)
7. T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, Tenth-order QED correction to the electron g-2 and an improved value of the fine structure constant. Phys. Rev. Lett. 109, 111807 (2012)
8. J. Fred, The Anomalous Magnetic Moment of the Muon. Springer Tracts in Modern Physics, vol. 274, 2nd edn. (Springer, Berlin, 2017)
9. G.F. Giudice, P. Paradisi, M. Passera, Testing new physics with the electron g-2. JHEP 11, 113 (2012)
10. G. Hiller, C. Hormigos-Feliu, D.F. Litim, T. Steudtner, Anomalous magnetic moments from asymptotic safety. Phys. Rev. D 102(7), 071901 (2020)
11. G. Hiller, C. Hormigos-Feliu, D.F. Litim, T. Steudtner, Model building from asymptotic safety with Higgs and flavor portals. Phys. Rev. D 102(9), 095023 (2020)
12. S. Bißmann, G. Hiller, C. Hormigos-Feliu, D.F. Litim, Multi-lepton signatures of vector-like leptons with flavor. Eur. Phys. J. C 81(2), 101 (2021)
13. L.D. Rose, S. Khalil, S. Moretti, Explaining electron and muon g - 2 anomalies in an Analigned 2-Higgs Doublet Model with right-handed neutrinos. Phys. Lett. B 816, 136216 (2021)
14. F.J. Botella, F. Cornet-Gomez, M. Nebot, Electron and muon g - 2 anomalies in general flavour conserving two Higgs doublet models. Phys. Rev. D 102(3), 035023 (2020)
15. A.E.C. Hernández, S.F. King, H. Lee. Fermion mass hierarchies from vector-like families with an extended 2HDM and a possible explanation for the electron and muon anomalous magnetic moments. I (2021)
16. S. Jana, P.K. Vishnu, S. Saad, Resolving electron and muon g - 2 within the 2HDM. Phys. Rev. D 101(11), 115037 (2020)
17. W.-Y. Keung, D. Marfatia, P.-Y. Tseng, Axion-like particles, two-Higgs-doublet models, leptoquarks, and the electron and muon g - 2 (2021)
18. S.-P. Li, X.-Q. Li, Y.-Y. Li, Y.-D. Yang, X. Zhang. Power-aligned 2HDM: a correlative perspective on \( (g - 2)_{\mu, e} \). JHEP 01, 034 (2021)
19. X.-F. Han, T. Li, L. Wang, Y. Zhang. Simple interpretations of lepton anomalies in the lepton-specific inert two-Higgs-doublet model. Phys. Rev. D 99(9), 095034 (2019)
20. I. Doršner, S. Fajfer, S. Saad, \( \mu \to e\gamma \) selecting scalar leptoquark solutions for the \( (g - 2)_{\mu, e} \) puzzles. Phys. Rev. D 102(7), 075007 (2020)
21. C. Cornella, P. Paradisi, O. Sumensari, Hunting for ALPs with lepton flavor violation. JHEP 01, 158 (2020)
22. L. Calibbi, M.L. López-Ibáñez, A. Melis, O. Vives. Muon and electron g - 2 and lepton masses in flavor models. JHEP 06, 087 (2020)
23. P. Escribano, J. Terol-Calvo, A. Vicente, \( (g - 2)_{\mu, e} \) in an extended inverse type-III seesaw model. Phys. Rev. D 103(11), 115018 (2021)
24. A. Crivellin, M. Hoferichter, P. Schmidt-Wellenburg. Combined explanations of \( (g - 2)_{\mu, e} \) and implications for a large muon EDM. Phys. Rev. D 98(11), 113002 (2018)
25. A.M. Baldini et al., Search for the lepton flavour violating decay \( \mu \to e\gamma \) with the full dataset of the MEG experiment. Eur. Phys. J. C 76(8), 434 (2016)
26. A.E.C. Hernández, S.F. King, H. Lee, S.J. Rowley, Is it possible to explain the muon and electron g - 2 in a Z' model? Phys. Rev. D 101(11), 115016 (2020)
27. A. Boda, R. Coy, S.J.D. King. Solving the electron and muon g - 2 anomalies in Z' models. 2 (2021)
28. K.-F. Chen, C.-W. Chiang, K. Yagyu. An explanation for the muon and electron g - 2 anomalies and dark matter. JHEP 09, 119 (2020)
29. B. Dutta, Y. Mimura, Electron g - 2 with flavor violation in MSSM. Phys. Lett. B 790, 563–567 (2019)
30. M. Badziak, K. Sakurai, Explanation of electron and muon g - 2 anomalies in the MSSM. JHEP 10, 024 (2019)
31. M. Endo, W. Yin, Explaining electron and muon g - 2 anomaly in SUSY without lepton-flavor mixings. JHEP 08, 122 (2019)
32. J.-L. Yang, T.-F. Feng, H.-B. Zhang, Electron and muon g - 2 (2021) in the B-LSSM. J. Phys. G 47(5), 055004 (2020)
33. J. Cao, Y. He, J. Lian, D. Zhang, P. Zhu, Electron and muon anomalous magnetic moments in the inverse seesaw extended NMSSM. 2 (2021)
34. S.P. Martin, J.D. Wells, Muon anomalous magnetic dipole moment in supersymmetric theories. Phys. Rev. D 64, 035003 (2001)
35. G. Degrassi, G.F. Giudice, QED logarithms in the electroweak corrections to the muon anomalous magnetic moment. Phys. Rev. D 58, 053007 (1998)
36. S. Marchetti, S. Mertens, U. Nierste, D. Stockinger, Tan(beta)-enhanced supersymmetric corrections to the anomalous magnetic moment of the muon. Phys. Rev. D 79, 013010 (2009)
37. M. Carena, D. Garcia, U. Nierste, C.E.M. Wagner, Effective Lagrangian for the \( ibH \) interaction in the MSSM and charged Higgs phenomenology. Nucl. Phys. B 577, 88–120 (2000)
38. J.S. Lee, A. Pilaftsis, M. Carena, S.Y. Choi, M. Drees, J.R. Ellis, C.E.M. Wagner, CP SuperH: a computational tool for Higgs phenomenology in the minimal supersymmetric standard model with explicit CP violation. Comput. Phys. Commun. 156, 283–317 (2004)
39. J.S. Lee, M. Carena, J. Ellis, A. Pilaftsis, C.E.M. Wagner, CP SuperH2.0: an improved computational tool for Higgs phenomenology in the MSSM with explicit CP Violation. Comput. Phys. Commun. 180, 312–331 (2009)
40. J.S. Lee, M. Carena, J. Ellis, A. Pilaftsis, C.E.M. Wagner, CP SuperH2.3: An Updated Tool for Phenomenology in the MSSM with
Explicit CP Violation. Comput. Phys. Commun. 184, 1220–1233 (2013)

41. T. Moroi, The Muon anomalous magnetic dipole moment in the minimal supersymmetric standard model. Phys. Rev. D 53, 6565–6575 (1996) (Erratum: Phys. Rev. D 56, 4424 (1997))

42. D. Stöckinger, The muon magnetic moment and supersymmetry. J. Phys. G 34, R45–R92 (2007)

43. Combined LEP Selectron/Smuon/Stau Results, 183–208 GeV. http://lepsusy.web.cern.ch/lepsusy/www/sleptons_summer02/slep_2002.html (2002)

44. G. Aad et al., Searches for electroweak production of supersymmetric particles with compressed mass spectra in $\sqrt{s} = 13$ TeV $pp$ collisions with the ATLAS detector. Phys. Rev. D 101(7), 072005 (2020)

45. G. Aad et al., Search for electroweak production of charginos and sleptons decaying into final states with two leptons and missing transverse momentum in $\sqrt{s} = 13$ TeV $pp$ collisions using the ATLAS detector. Eur. Phys. J. C 80(2), 123 (2020)

46. G. Aad et al., Search for electroweak production of charginos and neutralinos in proton–proton collisions at $\sqrt{s} = 13$ TeV $pp$ collisions with the ATLAS detector. Phys. Rev. D 101(7), 072005 (2020)

47. G. Aad et al., Searches for electroweak production of supersymmetric particles with compressed mass spectra in $\sqrt{s} = 13$ TeV $pp$ collisions with the ATLAS detector. Phys. Rev. D 101(7), 072005 (2020)

48. G. Aad et al., Searches for electroweak production of supersymmetric particles with compressed mass spectra in $\sqrt{s} = 13$ TeV $pp$ collisions with the ATLAS detector. Phys. Rev. D 101(7), 072005 (2020)

49. G. Aad et al., Searches for electroweak production of supersymmetric particles with compressed mass spectra in $\sqrt{s} = 13$ TeV $pp$ collisions with the ATLAS detector. Phys. Rev. D 101(7), 072005 (2020)

50. G. Aad et al., Searches for electroweak production of supersymmetric particles with compressed mass spectra in $\sqrt{s} = 13$ TeV $pp$ collisions with the ATLAS detector. Phys. Rev. D 101(7), 072005 (2020)