Effect of coupled channels of the multi-channel pion-pion scattering in two-pion transitions of the \( \Upsilon \) mesons

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The 9th joint International HADRON STRUCTURE’15 Conference, GRAND HOTEL BELLEVUE, Horný Smokovec, Slovak Republic, 29. June 3. July, 2015
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Introduction

In the analysis of practically all available data on two-pion transitions of the $\Upsilon$ mesons from the ARGUS, CLEO, CUSB, Crystal Ball, Belle, and BaBar Collaborations —

$$\Upsilon(mS) \rightarrow \Upsilon(nS)\pi\pi \ (m > n, \ m = 2, 3, 4, 5, \ n = 1, 2, 3)$$ —

the contribution of multi-channel $\pi\pi$ scattering in the final-state interactions is considered. The analysis, which is aimed at studying the scalar mesons, is performed jointly considering the above bottomonia decays, the isoscalar S-wave processes $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$, which are described in our model-independent approach based on analyticity and unitarity and using an uniformization procedure, and the charmonium decay processes — $J/\psi \rightarrow \phi(\pi\pi, K\bar{K}), \psi(2S) \rightarrow J/\psi\pi\pi$ —

with data from the Crystal Ball, DM2, Mark II, Mark III, and BES II Collaborations.

Possibility of using two-pion transitions of heavy quarkonia as a good laboratory for studying the $f_0$ mesons is related to the expected fact that the dipion is produced in a $S$-wave whereas the final quarkonium is a spectator [D. Morgan, M. R. Pennington, PR D48 (1993) 1185].
Importance of studying properties of scalar mesons is related to the obvious fact that a comprehension of these states is necessary in principle for the most profound topics concerning the QCD vacuum, because these sectors affect each other especially strongly due to possible ”direct” transitions between them. However the problem of interpretation of the scalar mesons is faraway to be solved completely [K.A.Olive et al. (PDG), Chin.Phys. C38 (2014) 090001].

E.g., applying our model-independent method in the 3-channel analyses of processes $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta, \eta\eta'$ [Yu.S. Surovtsev et al., PR D81 (2010) 016001; PR D85 (2012) 036002] we have obtained parameters of the $f_0(500)$ and $f_0(1500)$ which differ considerably from results of analyses which utilize other methods (mainly those based on dispersion relations and Breit–Wigner approaches).

On the other hand, explanation of the dipion mass distributions of the $\Upsilon(mS)$ where $m > 2$ contains a number of surprises. E.g., a distinction of the $\Upsilon(3S)$ decays from the $\Upsilon(2S)$ ones consists in the fact that in former case a phase space cuts off, as if, possible contributions which can interfere destructively with the $\pi\pi$-scattering contribution giving a characteristic two-humped shape of the dipion mass distribution in $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$. 
In a number of works (see, e.g., Yu.A. Simonov and A.I. Veselov, PR D79 (2009) 034024 and the references therein, and our discussion in Yu.S. Surovtsev et al., PR D91 (2015) 037901) various (sometimes rather doubtful) assumptions were made to obtain the needed result. We have explained this effect on the basis of our previous conclusions without any additional assumptions. In (Yu.S. Surovtsev et al., PR D89 (2014) 036010; J.Phys.G: Nucl.Part.Phys. 41 (2014) 025006; PR D86 (2012) 116002) we have shown: If a wide resonance cannot decay into a channel which opens above its mass, but the resonance is strongly coupled to this channel (e.g. $f_0(500)$ and $K\bar{K}$ channel), then one should consider this resonance as a multi-channel state. The closed channel should be included while taking into account the Riemann-surface sheets related to the threshold branch-point of this channel and performing the combined analysis of the considered and coupled channels.

In one's turn, the $\Upsilon(4S)$ and $\Upsilon(5S)$ are distinguished from the lower $\Upsilon$-states by the fact that their masses are above the $B\bar{B}$ threshold. The dipion mass distributions of these decays have the additional mysteries, e.g. the sharp dips about 1 GeV in the two-pion transitions of these states to the basic ones.
These mesons predominantly decays into pairs of the $B$-meson family because these modes are not suppressed by the OZI rule: the $\Upsilon(4S)$ decays into the $B\bar{B}$ pairs form $> 96\%$ in the total width, the $\Upsilon(5S)$ decays into the pairs of the $B$-meson family in sum compose about $90\%$. In contrast, strongly reduced decay modes are $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi\pi$ and $\Upsilon(4S) \rightarrow \Upsilon(2S)\pi\pi$ of about $(8.1 \pm 0.6) \times 10^{-5}\%$ and $(8.6 \pm 1.3) \times 10^{-5}\%$, and $\Upsilon(5S) \rightarrow \Upsilon(1S,2S,3S)\pi\pi$ with $(5 \div 8) \times 10^{-3}\%$. The total widths of $\Upsilon(5S)$ and $\Upsilon(4S)$ are 110 MeV and 20.5 MeV, respectively, and the one of the $\Upsilon(3S)$ is 20.32 keV. The partial decay widths of $\Upsilon(5S) \rightarrow \Upsilon(1S,2S,3S)\pi\pi$ are almost of the same order as the ones of the decays $\Upsilon(3S) \rightarrow \Upsilon(1S,2S)\pi\pi$. The decay widths of $\Upsilon(4S) \rightarrow \Upsilon(1S,2S)\pi\pi$ are even smaller than the latter ones by about two orders of magnitude.

[K.A.Olive et al.(PDG), Chin.Phys. C38 (2014) 090001].

Above comparison of decay widths implies that in the two-pion transitions of $\Upsilon(4S)$ and $\Upsilon(5S)$ the basic mechanism, which explains the dipion mass distributions, cannot be related to the $B\bar{B}$ transition dynamics. We show that the two-pion transitions both of bottomonia and of charmonia are explained by the unified mechanism which is based on our previous conclusions on the wide resonances [Yu.S.Surovtsev et al., J.Phys. G: Nucl.Part.Phys. 41 (2014) 025006; PR D89 (2014) 036010] and is related with interference of the contributions of multi-channel $\pi\pi$ scattering in the final-state interactions.
The model-independent amplitudes for multi-channel $\pi\pi$ scattering

Considering the multi-channel $\pi\pi$ scattering, we shall deal with the 3-channel case (namely with $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$) because it was shown [Yu.S. Surovtsev et al., PR D86 (2012) 116002; J.Phys. G: Nucl.Part.Phys. 41 (2014) 025006] that this is a minimal number of channels needed for obtaining correct values of scalar-isoscalar resonance parameters.

- **Resonance representations on the 8-sheeted Riemann surface**

The 3-channel $S$-matrix is determined on the 8-sheeted Riemann surface. The matrix elements $S_{ij}$, where $i, j = 1, 2, 3$ denote channels, have the right-hand cuts along the real axis of the $s$ complex plane ($s$ is the invariant total energy squared), starting with the channel thresholds $s_i$ ($i = 1, 2, 3$), and the left-hand cuts related to the crossed channels.
The Riemann-surface sheets are numbered according to the signs of analytic continuations of the square roots \( \sqrt{s - s_i} \) \( (i = 1, 2, 3) \) as follows:

|          | I | II | III | IV | V  | VI  | VII | VIII |
|----------|---|----|-----|----|----|-----|-----|------|
| \( \text{Im} \sqrt{s - s_1} \) | + | − | − | + | + | − | − | + |
| \( \text{Im} \sqrt{s - s_2} \) | + | + | − | − | − | − | + | + |
| \( \text{Im} \sqrt{s - s_3} \) | + | + | + | + | − | − | − | − |

An adequate allowance for the Riemann surface structure is performed taking the following uniformizing variable [Yu.S.Surovtsev, P.Bydžovský, V.E.Lyubovitskij, PR D85 (2012) 036002)]:

\[
w = \frac{\sqrt{(s - s_2)s_3} + \sqrt{(s - s_3)s_2}}{\sqrt{s(s_3 - s_2)}} \quad (s_2 = 4m_K^2 \text{ and } s_3 = 4m^2_\eta).
\]

where we have neglected the \( \pi\pi \)-threshold branch-point and taken into account the \( K\bar{K} \)- and \( \eta\eta \)-threshold branch-points and the left-hand branch-point at \( s = 0 \) related to the crossed channels.
Resonance representations on the Riemann surface are obtained using formulas from [D.Krupa, V.A.Meshcheryakov, Yu.S.Surovtsev, NC A109 (1996) 281], expressing analytic continuations of the $S$-matrix elements to all sheets in terms of those on the physical (I) sheet that have only the resonances zeros (beyond the real axis), at least, around the physical region. Then multi-channel resonances are classified. For analytic continuations the resonance poles on sheets II, IV and VIII, which are not shifted due to the coupling of channels, correspond to zeros on the physical sheet in $S_{11}$, $S_{22}$ and $S_{33}$, respectively. They are at the same points on the energy plane as the resonance poles. It is convenient to classify multi-channel resonances according to resonance zeros on sheet I.

In the 3-channel case, there are 7 types of resonances corresponding to 7 possible situations when there are resonance zeros on sheet I only in $S_{11}$ – (a); $S_{22}$ – (b); $S_{33}$ – (c); $S_{11}$ and $S_{22}$ – (d); $S_{22}$ and $S_{33}$ – (e); $S_{11}$ and $S_{33}$ – (f); $S_{11}$, $S_{22}$ and $S_{33}$ – (g). The resonance of every type is represented by the pair of complex-conjugate clusters (of poles and zeros on the Riemann surface).
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Yu.S. Surovtsev (BLTP JINR)
The $S$-matrix parametrization

The $S$-matrix elements $S_{ij}$ are parameterized using the Le Couteur-Newton relations \[ K.J.Le \ Couteur, \ Proc.R.London, \ Ser. \ A256 \ (1960) \ 115; \ R.G.Newton, \ J.Math.Phys. \ 2 \ (1961) \ 188; \ M.Kato, \ Ann.Phys. \ 31 \ (1965) \ 130 \]. On the $w$-plane, we have derived for them:

$$S_{11} = \frac{d^*(-w^*)}{d(w)}, \quad S_{22} = \frac{d(-w^{-1})}{d(w)}, \quad S_{33} = \frac{d(-1)}{d(w)},$$

$$S_{11}S_{22} - S_{12}^2 = \frac{d^*(w^{-1})}{d(w)}, \quad S_{11}S_{33} - S_{13}^2 = \frac{d^*(-w^{-1})}{d(w)}.$$

The $d(w)$ is the Jost matrix determinant. The 3-channel unitarity requires the following relations to hold for physical $w$-values:

$$|d(-w^*)| \leq |d(w)|, \quad |d(-w^{-1})| \leq |d(w)|, \quad |d(w^{-1})| \leq |d(w)|,$$

$$|d(w^{-1})| = |d(-w^{-1})| = |d(-w)| = |d(w)|.$$
The S-matrix elements in Le Couteur–Newton relations are taken as the products $S = S_B S_{\text{res}}$; the main (model-independent) contribution of resonances, given by the pole clusters, is included in the resonance part $S_{\text{res}}$; possible remaining small (model-dependent) contributions of resonances and influence of channels which are not taken explicitly into account in the uniformizing variable are included in the background part $S_B$. The d-function is:

For the resonance part

$$
\begin{align*}
    d_{\text{res}}(w) &= w^{-\frac{M}{2}} \prod_{r=1}^{M} (w + w_r^*) \\
    (M \text{ is the number of resonance zeros})
\end{align*}
$$

For the background part

$$
    d_B = \exp[-i \sum_{n=1}^{3} \frac{\sqrt{s-s_n}}{2m_n} (\alpha_n + i \beta_n)],
$$

where

$$
\begin{align*}
    \alpha_n &= a_{n1} + a_{n\sigma} \frac{s-s_\sigma}{s_\sigma} \theta(s-s_\sigma) + a_{nv} \frac{s-s_v}{s_v} \theta(s-s_v), \\
    \beta_n &= b_{n1} + b_{n\sigma} \frac{s-s_\sigma}{s_\sigma} \theta(s-s_\sigma) + b_{nv} \frac{s-s_v}{s_v} \theta(s-s_v)
\end{align*}
$$

$s_\sigma$ is the $\sigma\sigma$ threshold; $s_v$ is the combined threshold of the $\eta\eta'$, $\rho\rho$, $\omega\omega$ channels. The resonance zeros $w_r$ and the background parameters were fixed by fitting to data on processes $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta \eta$. 

Yu.S. Surovtsev (BLTP JINR)
Results of the analysis of data on $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$

For the data on multi-channel $\pi\pi$ scattering we used the results of phase analyses which are given for phase shifts of the amplitudes $\delta_{\alpha\beta}$ and for the modules of the $S$-matrix elements $\eta_{\alpha\beta} = |S_{\alpha\beta}|$ ($\alpha, \beta = 1, 2, 3$):

$$S_{\alpha\alpha} = \eta_{\alpha\alpha} e^{2i\delta_{\alpha\alpha}}, \quad S_{\alpha\beta} = i\eta_{\alpha\beta} e^{i\phi_{\alpha\beta}}.$$  

If below the third threshold there is the 2-channel unitarity then the relations

$$\eta_{11} = \eta_{22}, \quad \eta_{12} = (1 - \eta_{11}^2)^{1/2}, \quad \phi_{12} = \delta_{11} + \delta_{22}$$

are fulfilled in this energy region.

For the $\pi\pi$ scattering, the data are taken from the threshold to 1.89 GeV from [J.R.Batley et al, EPJ C54 (2008) 411; B.Hyams et al., NP B64 (1973) 134; 100 (1975) 205; A.Zylbersztejn et al., PL B38 (1972) 457; P.Sonderegger, P.Bonamy, in: Proc. 5th Intern. Conf. on Elem. Part., Lund, 1969, paper 372; J.R.Bensinger et al., PL B36 (1971) 134; J.P.Baton et al., PL B33 (1970) 525, 528; P.Baillon et al., PL B38 (1972) 555; L.Rosselet et al., PR D15 (1977) 574; A.A.Kartamyshev et al., Pis’ma v ZhETF 25 (1977) 68; A.A.Bel’kov et al., Pis’ma v ZhETF 29 (1979) 652].
For $\pi\pi \rightarrow K\bar{K}$, practically all the accessible data are used [W.Wetzel et al., NP B115 (1976) 208; V.A.Polychronakos et al., PR D19 (1979) 1317; P.Estabrooks, PR D19 (1979) 2678; D.Cohen et al., PR D22 (1980) 2595; G.Costa et al., NP B175 (1980) 402; A.Etkin et al., PR D25 (1982) 1786].

For $\pi\pi \rightarrow \eta\eta$, we used data for $|S_{13}|^2$ from the threshold to 1.72 GeV [F.Binon et al., NC A78 (1983) 313].

More preferable scenarios: the $f_0(500)$ is described by the cluster of type (a); the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, type (c) and $f_0'(1500)$, type (g); the $f_0(980)$ is represented only by the pole on sheet II and shifted pole on sheet III — this result is important for the interpretation of the $f_0(980)$ as neither a $q\bar{q}$ state nor the $K\bar{K}$ molecule [Yu.S.Surovtsev, P.Bydžovský, V.E.Lyubovitskij, PR D85 (2012) 036002].

Analyzing these data, we have obtained two solutions which are distinguished mainly in the width of $f_0(500)$. Further we show the solution which has survived after adding to the analysis the data on decays $J/\psi \rightarrow \phi(\pi\pi, K\bar{K})$ from the Mark III, DM2 and BES II Collaborations.
Table: The pole clusters for resonances on the $\sqrt{s}$-plane. \( \sqrt{s_r} = E_r - i\Gamma_r / 2 \).

| Sheet | \( f_0(500) \) | \( f_0(980) \) | \( f_0(1370) \) | \( f_0(1500) \) | \( f'_0(1500) \) | \( f_0(1710) \) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|
| II    | \( E_r \)      | 521.6±12.4     | 1008.4±3.1     |                |                | 1512.4±4.9     |
|       | \( \Gamma_r/2 \) | 467.3±5.9      | 33.5±1.5       |                |                | 287.2±12.9     |
| III   | \( E_r \)      | 552.5±17.7     | 976.7±5.8      | 1387.2±24.4    | 1506.1±9.0     |
|       | \( \Gamma_r/2 \) | 467.3±5.9      | 53.2±2.6       | 167.2±41.8     | 127.8±10.6     |
| IV    | \( E_r \)      |                | 1387.2±24.4    | 1512.4±4.9     |
|       | \( \Gamma_r/2 \) |                | 178.2±37.2     | 215.0±17.6     |
| V     | \( E_r \)      |                | 1387.2±24.4    | 1493.9±3.1     | 1732.8±43.2    |
|       | \( \Gamma_r/2 \) |                | 261.0±73.7     | 72.8±3.9       | 142.3±6.0      | 114.8±61.5    |
| VI    | \( E_r \)      | 573.4±29.1     | 1387.2±24.4    | 1493.9±5.6     | 1732.8±43.2    |
|       | \( \Gamma_r/2 \) | 467.3±5.9      | 250.0±83.1     | 58.4±2.8       | 179.3±4.0      | 111.2±8.8    |
| VII   | \( E_r \)      | 542.5±25.5     |                | 1493.9±5.0     | 1732.8±43.2    |
|       | \( \Gamma_r/2 \) | 467.3±5.9      |                | 47.8±9.3       | 55.2±38.0      |
| VIII  | \( E_r \)      |                |                | 1493.9±3.2     | 1732.8±43.2    |
|       | \( \Gamma_r/2 \) |                |                | 62.2±9.2       | 58.8±16.4      |
The obtained background parameters are:

\[
\begin{align*}
& a_{11} = 0.0, \quad a_{1\sigma} = 0.0199, \quad a_{1\nu} = 0.0, \quad b_{11} = b_{1\sigma} = 0.0, \quad b_{1\nu} = 0.0338, \\
& a_{21} = -2.4649, \quad a_{2\sigma} = -2.3222, \quad a_{2\nu} = -6.611, \quad b_{21} = b_{2\sigma} = 0.0, \\
& b_{2\nu} = 7.073, \quad b_{31} = 0.6421, \quad b_{3\sigma} = 0.4851, \quad b_{3\nu} = 0; \quad s_\sigma = 1.6338 \text{ GeV}^2, \\
& s_\nu = 2.0857 \text{ GeV}^2.
\end{align*}
\]

The very simple description of the $\pi\pi$-scattering background confirms well our assumption $S = S_B S_{\text{res}}$ and also that representation of multi-channel resonances by the pole clusters on the uniformization plane is good and quite sufficient.

It is important that we have obtained practically zero background of the $\pi\pi$ scattering in the scalar-isoscalar channel because a reasonable and simple description of the background should be a criterion for the correctness of the approach. Furthermore, this shows that the consideration of the left-hand branch-point at $s = 0$ in the uniformizing variable solves partly a problem of some approaches (see, e.g., N.N. Achasov, G.N. Shestakov, PR D49 (1994) 5779) that the wide-resonance parameters are strongly controlled by the non-resonant background.
Figure: The phase shifts and modules of the $S$-matrix element in the $S$-wave

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Generally, wide multi-channel states are most adequately represented by pole clusters, because the pole clusters give the main model-independent effect of resonances. The pole positions are rather stable characteristics for various models, whereas masses and widths are very model-dependent for wide resonances.

However, mass values are needed in some cases, e.g., in mass relations for multiplets. Therefore, we stress that such parameters of the wide multi-channel states, as masses, total widths and coupling constants with channels, should be calculated using the poles on sheets II, IV and VIII, because only on these sheets the analytic continuations have the forms:

\[ \propto 1/S_{11}, \quad \propto 1/S_{22} \quad \text{and} \quad \propto 1/S_{33}, \]

respectively, i.e., the pole positions of resonances are at the same points of the complex-energy plane, as the resonance zeros on the physical sheet, and are not shifted due to the coupling of channels.
E.g., if the resonance part of amplitude is taken as

\[ T^{\text{res}} = \sqrt{s} \Gamma_{el}/(m_{\text{res}}^2 - s - i\sqrt{s} \Gamma_{\text{tot}}), \]

for the mass and total width, one obtains

\[ m_{\text{res}} = \sqrt{E_r^2 + (\Gamma_r/2)^2} \quad \text{and} \quad \Gamma_{\text{tot}} = \Gamma_r, \]

where the pole position \( \sqrt{s_r} = E_r - i\Gamma_r/2 \) must be taken on sheets II, IV, VIII, depending on the resonance classification.

**Table:** The masses and total widths of the \( f_0 \) resonances.

|                  | \( f_0(600) \)     | \( f_0(980) \)     | \( f_0(1370) \)    | \( f_0(1500) \)    | \( f_0'(1500) \)   | \( f_0(1710) \)    |
|------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|
| \( m_{\text{res}} \) [MeV] | 693.9±10.0        | 1008.1±3.1        | 1399.0±24.7        | 1495.2±3.2         | 1539.5±5.4        | 1733.8±43.2        |
| \( \Gamma_{\text{tot}} \) [MeV] | 931.2±11.8        | 64.0±3.0          | 357.0±74.4         | 124.4±18.4        | 571.6±25.8        | 117.6±32.8         |
The contribution of multi-channel $\pi\pi$ scattering in the final states of decays of $\Psi$- and $\Upsilon$-meson families

For $J/\psi \to \phi\pi\pi, \phi K\bar{K}$ we have taken data from [W.Lockman, Proc.Hadron’89 Conf., ed. F.Binon et al.(Mark III), (Editions Frontières, Gif-sur-Yvette,1989) p.109; A.Falvard et al.(DM2), PR D38 (1988) 2706; M.Ablikim et al.(BES II), PL B607 (2005) 243]; for $\psi(2S) \to J/\psi(\pi^+\pi^-)$ from [G.Gidal et al.(Mark II), PL B107 (1981) 153]; for $\psi(2S) \to J/\psi(\pi^0\pi^0)$ from [M.Oreglia et al.(Crystal Ball(80)), PRL 45 (1980) 959]; for $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$ from [H.Albrecht et al.(Argus), PL B134 (1984) 137; D.Besson et al.(CLEO), PR D30 (1984) 1433; V.Fonseca et al.(CUSB), NP B242 (1984) 31; D.Gelphman et al.(Crystal Ball(85)), PR D32 (1985) 2893 (1985)]; for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$ and $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$ from [D.Cronin-Hennessy et al.(CLEO(07)), PR D76 (2007) 072001; F.Butler et al.(CLEO(94)), PR D49 (1994) 40]; finally, for $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ from [B.Aubert et al.(BaBar(06)), PRL 96 (2006) 232001; A.Sokolov et al.(Belle(07)), PR D75 (2007) 071103; A.Bondar et al.(Belle(12)), PRL 108 (2012) 122001].

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Formalism for calculating di-meson mass distributions of decays $J/\psi \to \phi(\pi\pi, K\bar{K})$ and $V' \to V\pi\pi$ ($V = \psi, \Upsilon$) can be found in [D.Morgan, M.R.Pennington, PR D48 (1993) 1185]. There is assumed that pairs of pseudo-scalar mesons of final states have $I = J = 0$ and only they undergo strong interactions, whereas a final vector meson ($\phi, \psi, \Upsilon$) acts as a spectator. The amplitudes of decays are related with the scattering amplitudes $T_{ij}$ ($i, j = 1 − \pi\pi, 2 − K\bar{K}$) as follows

\[
F(J/\psi \to \phi\pi\pi) = \sqrt{2/3} [c_1(s) T_{11} + c_2(s) T_{21}],
\]
\[
F(J/\psi \to \phi K\bar{K}) = \sqrt{1/2} [c_1(s) T_{12} + c_2(s) T_{22}],
\]
\[
F(\psi(2S) \to \psi(1S)\pi\pi) = [d_1(s) T_{11} + d_2(s) T_{21}],
\]
\[
F(\Upsilon(mS) \to \Upsilon(nS)\pi\pi) = [e_{1}^{(mn)} T_{11} + e_{2}^{(mn)} T_{21}],
\]

$m > n$, $m = 2, 3, 4, 5$, $n = 1, 2, 3$

where $c_1 = \gamma_{10} + \gamma_{11}s$, $c_2 = \alpha_2/(s - \beta_2) + \gamma_{20} + \gamma_{21}s$, $d_i = \delta_{i0} + \delta_{i1}s$ and $e_i^{(mn)} = \rho_{i0}^{(mn)} + \rho_{i1}^{(mn)}s$ are functions of couplings of the $J/\psi$, $\psi(2S)$ and $\Upsilon(mS)$ to channel $i$; $\alpha_2, \beta_2, \gamma_{i0}, \gamma_{i1}, \delta_{i0}, \delta_{i1}, \rho_{i0}^{(mn)}, \rho_{i1}^{(mn)}$ are free parameters.
The pole term in $c_2$ is an approximation of possible $\phi K$ states, not forbidden by OZI rules when considering quark diagrams of these processes. Obviously this pole should be situated on the real $s$-axis below the $\pi\pi$ threshold.

The expressions for decays $J/\psi \rightarrow \phi(\pi\pi, K\bar{K})$

$$N|F|^2 \sqrt{(s - s_i)[m_\psi^2 - (\sqrt{s} - m_\phi)^2][m_\psi^2 - (\sqrt{s} + m_\phi)^2]}$$

and the analogues relations for $\psi(2S) \rightarrow \psi(1S)\pi\pi$ and $\Upsilon(mS) \rightarrow \Upsilon(nS)\pi\pi$ give the di-meson mass distributions.

$N$ (normalization to experiment) is: for $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^-$, 4.3439 for ARGUS, 2.1776 for CLEO(94), 1.2011 for CUSB; for $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^0\pi^0$, 0.0788 for Crystal Ball(85); for $\Upsilon(3S) \rightarrow \Upsilon(1S)(\pi^+\pi^- \text{ and } \pi^0\pi^0)$, 0.5096 and 0.2235 for CLEO(07); for $\Upsilon(3S) \rightarrow \Upsilon(2S)(\pi^+\pi^- \text{ and } \pi^0\pi^0)$, 7.7397 and 3.8587 for CLEO(94); for $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi\pi$, 7.1476 for BaBar(06) and 0.5553 for Belle(07); for $\Upsilon(4S) \rightarrow \Upsilon(2S)\pi\pi$, 58.143 for BaBar(06); for $\Upsilon(5S) \rightarrow \Upsilon(1S)\pi\pi$, $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi\pi$ and $\Upsilon(5S) \rightarrow \Upsilon(3S)\pi\pi$ respectively 0.1626, 4.8355 and 10.858 for Belle(12).
Parameters of the coupling functions of the decay particles ($J/\psi$, $\psi(2S)$, $\Upsilon(mS)$ ($m = 2, \ldots, 5$)) to channel $i$, obtained in the analysis, are:

$(\alpha_2, \beta_2) = (0.0843, 0.0385)$,
$(\gamma_{10}, \gamma_{11}, \gamma_{20}, \gamma_{21}) = (1.1826, 1.2798, -1.9393, -0.9808)$,
$(\delta_{10}, \delta_{11}, \delta_{20}, \delta_{21}) = (-0.1270, 16.621, 5.983, -57.653)$,
$(\rho_{10}^{(21)}, \rho_{11}^{(21)}, \rho_{20}^{(21)}, \rho_{21}^{(21)}) = (0.4050, 47.0963, 1.3352, -21.4343)$,
$(\rho_{10}^{(31)}, \rho_{11}^{(31)}, \rho_{20}^{(31)}, \rho_{21}^{(31)}) = (1.1619, -2.915, 0.7841, 1.0179)$,
$(\rho_{10}^{(32)}, \rho_{11}^{(32)}, \rho_{20}^{(32)}, \rho_{21}^{(32)}) = (7.2842, -2.5599, 0.0, 0.0)$,
$(\rho_{10}^{(41)}, \rho_{11}^{(41)}, \rho_{20}^{(41)}, \rho_{21}^{(41)}) = (0.6162, -2.5715, -0.8467, 0.2128)$,
$(\rho_{10}^{(42)}, \rho_{11}^{(42)}, \rho_{20}^{(42)}, \rho_{21}^{(42)}) = (2.329, -7.3511, 1.8096, -10.1477)$,
$(\rho_{10}^{(51)}, \rho_{11}^{(51)}, \rho_{20}^{(51)}, \rho_{21}^{(51)}) = (0.7078, 4.0132, 4.838, -3.9091)$,
$(\rho_{10}^{(52)}, \rho_{11}^{(52)}, \rho_{20}^{(52)}, \rho_{21}^{(52)}) = (0.8133, 2.2061, -0.7973, 0.3247)$,
$(\rho_{10}^{(53)}, \rho_{11}^{(53)}, \rho_{20}^{(53)}, \rho_{21}^{(53)}) = (0.8946, 2.538, 0.627, -0.0483)$. 
Satisfactory combined description of all considered processes is obtained with the total $\chi^2/\text{ndf} = 824.236/(714 - 91) \approx 1.32$; for the $\pi\pi$ scattering, $\chi^2/\text{ndf} \approx 1.15$; for $\pi\pi \rightarrow K\overline{K}$, $\chi^2/\text{ndf} \approx 1.65$; for $\pi\pi \rightarrow \eta\eta$, $\chi^2/\text{ndf} \approx 0.87$; for decays $J/\psi \rightarrow \phi(\pi^+\pi^-, K^+K^-)$, $\chi^2/\text{ndp} \approx 1.36$; for $\psi(2S) \rightarrow J/\psi(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndp} \approx 2.43$; for $\Upsilon(2S) \rightarrow \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndp} \approx 1.01$; for $\Upsilon(3S) \rightarrow \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndp} \approx 0.67$; for $\Upsilon(3S) \rightarrow \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndp} \approx 0.61$; for $\Upsilon(4S) \rightarrow \Upsilon(1S)(\pi^+\pi^-)$, $\chi^2/\text{ndp} \approx 0.27$; for $\Upsilon(4S) \rightarrow \Upsilon(2S)(\pi^+\pi^-)$, $\chi^2/\text{ndp} \approx 0.27$; for $\Upsilon(5S) \rightarrow \Upsilon(1S)(\pi^+\pi^-)$, $\chi^2/\text{ndp} \approx 1.80$; for $\Upsilon(5S) \rightarrow \Upsilon(2S)(\pi^+\pi^-)$, $\chi^2/\text{ndp} \approx 1.08$; for $\Upsilon(5S) \rightarrow \Upsilon(3S)(\pi^+\pi^-)$, $\chi^2/\text{ndp} \approx 0.81$. 
Figure: The $J/\psi \rightarrow \phi \pi\pi$ and $J/\psi \rightarrow \phi K\bar{K}$ decays.
Figure: The $J/\psi \to \phi \pi\pi$ decay; the data of BES II Collaboration.

Important role of the BES II data: Namely this di-pion mass distribution rejects the solution with the narrower $f_0(500)$. The corresponding curve lies considerably below the data from the threshold to about 850 MeV.
Figure: The $\psi(2S) \rightarrow J/\psi \pi^+\pi^-$ decay.

Mark II

$\psi(2S) \rightarrow J/\psi \pi^0\pi^0$

Cristal Ball (80)
Figure: The $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi$ decay.
Figure: The decays $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi\pi$. 

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Effect of coupled channels of the multi-channel pion-pion scattering in two-pion transitions 

CLEO (2007)

CLEO (94)

CLEO (94)
Figure: The decays $\Upsilon(4S) \rightarrow \Upsilon(1S,2S)\pi^+\pi^-$. The solid lines correspond to contribution of all relevant resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$. 
Figure: The decays $\Upsilon(5S) \rightarrow \Upsilon(ns)\pi^+\pi^-$ ($n = 1, 2, 3$). The solid lines correspond to contribution of all relevant resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f'_0(1500)$; the dashed, of the $f_0(980)$ and $f'_0(1500)$. 
The curves demonstrate interesting behavior — a bell-shaped form in the near-\(\pi\pi\)-threshold region (especially for the \(\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-\) and \(\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi\)), smooth dips about 0.6 GeV in the \(\Upsilon(4S, 5S) \rightarrow \Upsilon(1S)\pi^+\pi^-\), about 0.44 GeV in the \(\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-\), and about 0.7 GeV in the \(\Upsilon(3S) \rightarrow \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)\), and also sharp dips about 1 GeV in the \(\Upsilon(4S, 5S) \rightarrow \Upsilon(1S)\pi^+\pi^-\). Obviously, this shape of dipion mass distributions is explained by the interference between the \(\pi\pi\) scattering and \(K\bar{K} \rightarrow \pi\pi\) contributions to the final states of these decays — by the constructive one in the near-\(\pi\pi\)-threshold region and by the destructive one in the dip regions.

However, whereas the data on \(\Upsilon(5S) \rightarrow \Upsilon(1S)\pi^+\pi^-\) confirms the sharp dips about 1 GeV, the scarce data on \(\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-\) do not permit to do that conclusion. Therefore, further we investigate a role of the individual \(f_0\) resonances in making up a shape of the dipion mass distributions in the decays \(\Upsilon(4S, 5S) \rightarrow \Upsilon(ns)\pi^+\pi^-\) \((n = 1, 2, 3)\). In this case we switched off only those resonances \((f_0(500), f_0(1370), f_0(1500)\) and \(f_0(1710)\)), removal of which can be somehow compensated by correcting the background to have the more-or-less acceptable description of the multi-channel \(\pi\pi\) scattering.
When switching off resonances, a minimal set of the $f_0$ mesons, when the description of multi-channel $\pi\pi$-scattering does not change in principle with the total $\chi^2/\text{ndf} \approx 1.20$, is the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$. The obtained background parameters are: $a_{11} = 0.0$, $a_{1\sigma} = 0.0321$, $a_{1\nu} = 0.0$, $b_{11} = -0.0051$, $b_{1\sigma} = 0.0$, $b_{1\nu} = 0.04$; $a_{21} = -1.6425$, $a_{2\sigma} = -0.3907$, $a_{2\nu} = -7.274$, $b_{21} = 0.1189$, $b_{2\sigma} = 0.2741$, $b_{2\nu} = 5.823$; $b_{31} = 0.7711$, $b_{3\sigma} = 0.505$, $b_{3\nu} = 0.0$.

Only the $f_0(500)$ can be switched off when obtaining the reasonable description of multi-channel $\pi\pi$-scattering with the total $\chi^2/\text{ndf} \approx 1.43$ and with the corrected background parameters: $a_{11} = 0.3513$, $a_{1\sigma} = -0.2055$, $a_{1\nu} = 0.207$, $b_{11} = -0.0077$, $b_{1\sigma} = 0.0$, $b_{1\nu} = 0.0378$; $a_{21} = -1.8597$, $a_{2\sigma} = 0.1688$, $a_{2\nu} = -7.519$, $b_{21} = 0.161$, $b_{2\sigma} = 0.0$, $b_{2\nu} = 6.94$; $b_{31} = 0.7758$, $b_{3\sigma} = 0.4985$, $b_{3\nu} = 0.0$.

Variants of calculations with contributions from the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$ and from the $f_0(980)$, and $f_0'(1500)$ are shown by the dotted and dashed lines, respectively, for the $\Upsilon(4S, 5S)$ decays.

The sharp dips about 1 GeV are related with the $f_0(500)$ contribution to the interfering amplitudes of $\pi\pi$ scattering and $KK \rightarrow \pi\pi$ process.

Note the considerable contribution of the $f_0(1370)$ to the bell-shaped form in the near-$\pi\pi$-threshold region.
Conclusions

The combined analysis was performed for data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\bar{K}, \eta\eta$ and on the decays of the charmonia — $J/\psi \to \phi(\pi\pi, K\bar{K}), \psi(2S) \to J/\psi \pi\pi$ — and of the bottomonia — $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n, m = 2, 3, 4, 5, n = 1, 2, 3$) from the ARGUS, Crystal Ball, CLEO, CUSB, DM2, Mark II, Mark III, BES II, BaBar, and Belle Collaborations.

It is shown that the dipion mass spectra in the above-indicated decays of charmonia and bottomonia are explained by the unified mechanism which is based on our previous conclusions on wide resonances [Yu.S.Surovtsev et al., J.Phys. G: Nucl.Part.Phys. 41 (2014) 025006; PR D89 (2014) 036010] and is related to contributions of the $\pi\pi$ and $K\bar{K}$ coupled channels and their interference.

It is shown that in the final states of these decays (except $\pi\pi$ scattering) the contribution of coupled processes, e.g., $K\bar{K} \to \pi\pi$, is important even if these processes are energetically forbidden. This is in accordance with our previous conclusions on the wide resonances [Yu.S.Surovtsev et al., J.Phys. G: Nucl.Part.Phys. 41 (2014) 025006; PR D89 (2014) 036010]: If a wide resonance cannot decay into a channel which opens above its mass but the resonance is strongly connected with this channel (e.g. the $f_0(500)$ and the $K\bar{K}$ channel), one should consider this resonance as a multi-channel state.
The role of the individual $f_0$ resonances in making up the shape of the dipion mass distributions in the bottomonia decays is considered.

Since describing the bottomonia decays, we did not change resonance parameters in comparison with the ones obtained in the combined analysis of the processes $\pi\pi \rightarrow \pi\pi, K \overline{K}, \eta\eta$ and charmonia decays, the results of this analysis confirm all of our earlier conclusions on the scalar mesons, main of which are:

1) Confirmation of the $f_0(500)$ with a mass of about 700 MeV and a width of 930 MeV (the pole on sheet II is $521.6 \pm 12.4 - i(467.3 \pm 5.9)$ MeV). This mass value is in line with prediction ($m_\sigma \approx m_\rho$) on the basis of mended symmetry by S.Weinberg [PRL 65 (1990) 1177] and with an analysis using the large-$N_c$ consistency conditions between the unitarization and resonance saturation suggesting $m_\rho - m_\sigma = O(N_c^{-1})$ [J.Nieves, E.Ruiz Arriola, PR D80 (2009) 045023]. Also the prediction of a soft-wall AdS/QCD approach [T.Gutsche et al., PR D87 (2013) 056001] for the mass of the lowest $f_0$ meson – 721 MeV – practically coincides with the value obtained in our work.

2) Indication for the $f_0(980)$ (the pole on sheet II is $1008.1 \pm 3.1 - i(32.0 \pm 1.5)$ MeV) to be neither a $q\bar{q}$ state nor the $K\overline{K}$ molecule, but possibly the bound $\eta\eta$ state.
3) Indication for the $f_0(1370)$ and $f_0(1710)$ to have a dominant $s\bar{s}$ component. This is in agreement with a number of experiments: Conclusion about the $f_0(1370)$ quite agrees with the one of work of Crystal Barrel Collaboration [C.Amsler et al., PL B355 (1995) 425] where the $f_0(1370)$ is identified as $\eta\eta$ resonance in the $\pi^0\eta\eta$ final state of the $\bar{p}p$ annihilation. This explains also quite well why one did not find this state considering only the $\pi\pi$ scattering [W.Ochs, arXiv:1001.4486v1 [hep-ph]; P.Minkowski, W.Ochs, EPJ C9 (1999) 283; arXiv: hep-ph/0209223; hep-ph/0209225]. Conclusion about the $f_0(1710)$ is consistent with the experimental facts that this state is observed in $\gamma\gamma \rightarrow K_S K_S$ [S.Braccini, Frascati Phys. Series XV (1999) 53] and not observed in $\gamma\gamma \rightarrow \pi^+\pi^-$ [R.Barate et al., PL B472 (2000) 189].

4) Indication for two states in the 1500-MeV region: the $f_0(1500)$ ($m_{res} \approx 1495$ MeV, $\Gamma_{tot} \approx 124$ MeV) and the $f_0'(1500)$ ($m_{res} \approx 1539$ MeV, $\Gamma_{tot} \approx 574$ MeV). The $f_0'(1500)$ is interpreted as a glueball taking into account its biggest width among the enclosing states [V.V.Anisovich et al., NP Proc.Suppl. A56 (1997) 270].