Research on robust optimization of emergency logistics network considering the time dependence characteristic

Qingrong WANG¹, Changfeng ZHU², Ying LI², Zhengkun ZHANG²

¹School of Electronic and Information Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China
²School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou,730070, China

Email: wangqr003@163.com, cfzhu003@163.com

Abstract. Considering the time dependence of emergency logistic network and complexity of the environment that the network exists in, in this paper the time dependent network optimization theory and robust discrete optimization theory are combined, and the emergency logistics dynamic network optimization model with characteristics of robustness is built to maximize the timeliness of emergency logistics. On this basis, considering the complexity of dynamic network and the time dependence of edge weight, an improved ant colony algorithm is proposed to realize the coupling of the optimization algorithm and the network time dependence and robustness. Finally, a case study has been carried out in order to testify validity of this robustness optimization model and its algorithm, and the value of different regulation factors was analyzed considering the importance of the value of the control factor in solving the optimal path. Analysis results show that this model and its algorithm above-mentioned have good timeliness and strong robustness.

1 Introduction

In recent years, especially, with large-scale sudden events taking place frequently, the research about emergency logistic network has become a much-talk-about topic.

He et al. build a mixed integer programming model based on the time [1]. Zhu et al. come up with a super network optimization model based on resource allocation and risk [2]. Chen et al. make conclusion that network is much easier damaged under random attacking than target attacking, besides [3]. Wu et al. research it’s resilience both in flexibility and reliability [4]. Xiong et al. give a new way to measure robustness. Xiong et al. comes up with some methods to improve the robustness of emergency logistic network [5].

By analyzing many related literatures, we can find that most of those researches are about path and network. There is no doubt that they are useful for researching emergency logistic network, however, some shortcomings are still reflected. Firstly, time-varying and indeterminacy are generally existed in realistic emergency system, many parameters in this network are variable usually. But, during the research, most of literature regards it as constant values, which is not reflect realistic condition well. Secondly, there are some researches considering the complexity of environment to emergency logistic network, but, they optimize the network without from view of path. Most of them are just to improve resistance of network. Besides, some paper optimize path according to the property of emergency supplies, however, they neglect the complexity of environment. Thus, the best path they finally get has poor robustness.
Based on those shortcomings that has been listed above, considering the property of time-varying and indeterminacy to emergency logistic network, this paper combines the theory of time-dependent network with discrete robustness optimization to build a optimization model of dynamic network with property of robustness.

2 Building Model

2.1 Basic theory
Dynamic network, which is also called as time-dependent network, can describe time dependency very well, and it can be expressed as $TDN = (V, A, G)$, where, $V = \{v_1, v_2, \ldots, v_n\}$ is a set for all the nodes in network, $A = \{a_1, a_2, \ldots, a_n\}$ is a set for all the incident edges and $G = \{g_{ij}(t)\}$ is a set about $g_{ij}(t)$. $g_{ij}(t) \geq 0$ is a function of time as the independent variable. In this paper, the function $g_{ij}(t)$ reflects the cost-time to emergency supplies that starts from a node $v_i$ at time $t$ to next node $v_j$.

Besides, considering the complexity about emergency logistic network, the value of $g_{ij}(t)$ may change in an interval after the network is affected by an uncertain factors. According to the theory of discrete robustness optimization, this interval values can be expressed as $g_{ij}(t) = [\bar{g}_{ij}(t) + d_{ij}(t)]$, where, $\bar{g}_{ij}(t)$ is a time function to $a_{ij} = (v_i, v_j)$ in normal condition, $d_{ij}(t) \geq 0$ is a deviation value to $\bar{g}_{ij}(t)$ after $a_{ij}$ is affected by an uncertain factors.

2.2 Definition and assumption
(1) We definite $x_{ij}$ as a binary-variable to describe the relevance between $v_i$ and $v_j$, where

$$ x_{ij} = \begin{cases} 0, & (v_i, v_j) \not\in A \\ 1, & (v_i, v_j) \in A \end{cases} $$

(2) The emergency logistic network in this paper is assumed as a single source and single sink network. The starting point $v_0$ is a supply centre, the end point $v_n$ is a supplies demand point, and other points in the network are transit point for emergency supplies.

$$ \sum_{j=0}^{n} x_{i0} = \sum_{j=0}^{n} x_{nj} = 0 $$

(3) According to timeliness to emergency supplies, we define an ordered pair $(v_i, t_i)$ to describe state for supplies on each point, and the ordered pair means that a batch of supplies arrives at point $v_i$ when the time is $t_i$.

(4)In order to describe the state of each path, we need to define a set $p_{l,a} = \{v_i', t_0, (v_i', t_1), \ldots, (v_i', t_k)\}$ to record the related information for each path, where, $0 \leq l \leq k$, and $k$ is the number of path that connects staring point $v_0$ and ending point $v_n$.

(5) We make an assumption that those uncertain nodes and arcs in emergency logistic network are broken by uncertain factors randomly, and definite an uncertain set $J$ to contain those nodes and arcs. We use $|J|$ to express the number of elements in the set $J$.

(6) Giving the controlling parameter $\Gamma$, where $\Gamma \in [0, |J|]$, $\Gamma \in N$. By controlling the value of $\Gamma$ we can control the robustness and conservative property to the network.

2.3 Robustness optimization model for dynamic network
According to the definitions and assumptions above, using the theories related in literature [6] and literature [7], we give the model as follows:

$$ Z = \min \{T^1, T^2, \ldots, T^i, \ldots, T^k\} $$
In this model, (1) is related time dependency and robustness, it’s purpose is to search a path with robustness in time. Both (2) and (3) are time dependency functions, especially, (2) reflect the time dependency under normal situation and (3) reflects under abnormal situation. (4), (5) and (6) are the other constrains conditions.

3. Design of Algorithms

Dynamic network is essentially different from general network. The main reason is that the value to each arc is dependent on the starting time strictly.

\[
\begin{align*}
\text{FIG 1: an example for dynamic network}
\end{align*}
\]

\[
\begin{align*}
g_{1,2}(10) &= 20 \\
g_{1,2}(5) &= 35 \\
g_{2,3}(40) &= 30 \\
g_{2,3}(30) &= 35
\end{align*}
\]

According to the model, in order to select a better choice for each artificial ant, the visibility \( \eta \) should be improved as (7).

\[
\eta_{ij}(t) = \frac{\left| \hat{t}_{i,i+1} - t_{i,i+1} \right|}{(t_{i,i+1})^2}
\]

In (7), by introducing \( \xi \in [0,1] \), we adjust the importance between \( \left| \hat{t}_{i,i+1} - t_{i,i+1} \right| \) and \( t_{i,i+1} \). There is no doubt, if \( \xi = 0 \), the choice may have a good timeliness, but with poor robustness, if \( \xi = 1 \), it may have a good robustness, but with poor timeliness. So, if we give a reasonable value to \( \xi \), the choice will have a good result both timeliness and robustness.

Step1: Beginning and initialization
Step2: Give in uncertain set \( J \) and controlling parameter \( \Gamma \).
Step3: Attack \( \Gamma \) elements in \( J \) randomly, and generate \( d_{i,j} \).
Step4: Begin for ant colony algorithm.
Step4.1: set iteration \( N \), \( n = 1 \), \( T_0 = \omega \), \( k = 1 \).
Step4.2: Let all the artificial ants distribute at starting point, and generate battle \( Tabu \) for each ant.
Step 4.3: Ant k starts to search.
Step 4.3.1: Search next point $v_j$ and update $Tabu_k$.
Step 4.3.2: According to (2) and (3), calculate $t_{ij}$ and $\hat{t}_{ij}$.
Step 4.3.3: If $v_j$ is the ending point, do $k \leftarrow k + 1$, else, return to step 4.3.
Step 4.3.4: If $k < m$, return to step 4.3, else, go to next step.
Step 4.4: Update global pheromone.
Step 4.5: Choose out the best path according to pheromone and calculate $T_n$ on this path.
Step 4.6: If $T_n \leq T_{n-1}$, mark this path as $P_n$ and do $n \leftarrow n + 1$, go to step 5, else, return to step 4.3.
Step 5: If $n = N + 1$, do $n \leftarrow n - 1$ and go to step 6, else, return to step 4.2.
Step 6: Output $P_n$ and $T_n$, end.

4 Example of Verification
In order to verify the reasonability to the model and algorithm the paper presents, we build an emergency logistics dynamic network model as Figure 2, where $v_0$ is the point that offers emergency supplies, $v_j$ is the point that accepts the supplies, and other points are translating facility points.

![Figure 2: An example of model](image)

According to Figure 2, we regard 24 hours as a cycle. Evenly divide this cycle into 4 period of time. we give the values to each points and arcs during different period as following.

| point | 0-6 | 7-12 | 13-18 | 19-24 |
|-------|-----|------|-------|-------|
| $v_0$ | 2.3 | 1.7  | 2.2   | 1.4   |
| $v_1$ | 3.7 | 3.4  | 2.1   | 4.2   |
| $v_2$ | 2.2 | 1.1  | 1.7   | 3.9   |
| $v_3$ | 2.2 | 4.8  | 4.4   | 2.1   |
| $v_4$ | 4.5 | 3.1  | 2.0   | 3.4   |
| $v_5$ | 3.7 | 2.7  | 4.8   | 1.0   |
| $v_6$ | 2.2 | 3.6  | 3.8   | 1.8   |
| $v_7$ | 3.0 | 1.6  | 2.9   | 2.1   |
| $v_8$ | 2.9 | 3.7  | 2.9   | 4.7   |
| $v_9$ | 3.5 | 3.7  | 3.8   | 2.3   |
Table 2: time (h) for each arc

| arc       | 0-6 | 7-12 | 13-18 | 19-24 |
|-----------|-----|------|-------|-------|
| (v_1,v_2) | 11.7| 21.9 | 19.4  | 19.4  |
| (v_2,v_3) | 12.6| 10.4 | 14.0  | 20.4  |
| (v_3,v_4) | 14.6| 17.1 | 17.8  | 13.0  |
| (v_4,v_5) | 12.3| 9.2  | 11.9  | 12.5  |
| (v_5,v_6) | 12.7| 15.3 | 16.1  | 15.6  |
| (v_6,v_7) | 10.7| 16.0 | 8.4   | 10.8  |
| (v_7,v_8) | 21.5| 20.5 | 8.5   | 15.3  |
| (v_8,v_9) | 9.3 | 16.8 | 20.4  | 19.7  |
| (v_9,v_10)| 21.5| 14.2 | 18.3  | 18.0  |

We make an assumption that uncertain point set is \( J_v = \{ v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8 \} \), uncertain arc set is \( J_a = \{ a_{1,2}, a_{2,3}, a_{3,4}, a_{4,5}, a_{5,6}, a_{6,7}, a_{7,8}, a_{8,9} \} \), where \( J_v \cup J_a = J \). According to \( J \), we give value to each parameter in ant colony algorithm as research by literature (Zhan et al., 2003), where \( \Gamma = 10, \alpha = 3, \beta = 3.5, \rho = 0.7, Q = 100, \tau_0(0) = 0 \).

Besides, we give a reasonable value to \( \xi \) by simulation. As has mentioned that \( \xi \in [0, 1] \), we can give \( \xi \) different values to compute average number of results to 40 paths. The final result as Figure 3 shows.

![Figure 3: Influence to average number by different value to \( \xi \)](image)

From the Figure 3 we can find that the average number is best when \( \xi \in [0.35, 0.75] \). It illustrates that algorithm has a good search capability in this condition. Based on this conclusion, we can assign 0.55 (the median for this interval) to \( \xi \).

Using the parameters have assigned above, we use the algorithm to search the best path in Figure 2 for different \( \Gamma \) when starting time \( t=0 \). The algorithm convergence figure as Figure 4 shows.
Figure 4: convergence figure for different values to $\Gamma$

From the Figure 4, we can find that the target value to the best path increases with $\Gamma$ increasing. The reason is that $\Gamma$ reflects the number of affected points and arcs, and the $d_{ij}$ for those points and arcs lead to the change for the target value. In order to research the best paths further, we give each best path and suboptimal path for different $\Gamma$ in Table 3.

Table 3: when $\xi = 0.55$, the better paths for different $\Gamma$

| $\Gamma$ | Mark | Path | Target Value $T(h)$ |
|---------|------|------|---------------------|
| 0       | a    | 0->2->3->7->9 | 45.0                |
| 10      | c    | 0->3->8->9    | 54.0                |
| 20      | e    | 0->2->3->7->9 | 64.2                |

As the Table 3 shows, although $\Gamma$ is different values, the path marked a, d and e are the same path. It shows that this path has the best timeliness and robustness under this assumption the paper makes.

5 Conclusions
As the timeliness is very important to emergency supplies, it is necessary to research time dependency for emergency logistic network, besides, the environment for the network is complexity and it also effects this network in return. So, we should consider the time dependency and complexity together. In an emergency logistic network, a path with better timeliness and robustness is vitally important both in transporting emergency supplies to disaster rapid and timely and reducing the whole disaster loss.

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