Classical Gravity Coupled to Liouville Theory

Francisco D. Mazzitelli

Noureddine Mohammedi

International Centre for Theoretical Physics
34100 Trieste, Italy.

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Abstract

We consider the two dimensional Jackiw-Teitelboim model of gravity. We first couple the model to the Liouville action and scalar fields and show, treating the combined system as a non linear sigma model, that the resulting theory can be interpreted as a critical string moving in a target space of dimension \( D = c + 2 \). We then analyse perturbatively a generalised model containing a kinetic term and an arbitrary potential for the auxiliary field. We use the background field method and work with covariant gauges. We show that the renormalisability of the theory depends on the form of the potential. For a general potential, the theory can be renormalised as a non linear sigma model. In the particular case of a Liouville-like potential, the theory is renormalisable in the usual sense.
1. Introduction

There are two motivations behind the recent intense activity in two dimensional gravity. The first stream of thoughts sees in two dimensional gravity a toy model for tackling the more subtle problem of four dimensional gravity. The second considers two dimensional gravity as fundamental to the study of string theory where one has to sum over all two dimensional geometries. During the revival of string theory, however, the complication of summing over geometries was spared by a restriction to certain critical space-time dimensions and the Liouville mode was neglected [1]. Eventually, this mode was included and new features appeared in the quantisation of string theory [2-8]. Undoubtedly, the most striking one is the fact that two dimensional surfaces cannot be "embedded" in target spaces of dimensions between 1 and 25. This is rather unnatural if the two dimensional surfaces are to be thought of as world-sheets swept by the propagation and interactions of strings in space-time.

A major issue in the treatment of two dimensional gravity is in finding a locally covariant action at the classical level. In the pioneering work of Polyakov, this was a non-local functional which reproduces the well-know trace anomaly of the energy momentum tensor. How one analyses this action in a general gauge is yet still unknown. The natural analogue of the Einstein-Hilbert action is a topological invariant counting the number of handles of the manifold. Nevertheless, some sense can be made out of this action using perturbation theory and dimensional regularisation [9], at least up to leading order [10]. An other alternative for two dimensional gravity was proposed by Jackiw and Teitelboim [11]. This expresses the constancy of the scalar curvature through the introduction of
an extra field (the dilaton) which seems to spoil its geometric interpretation. It
turns out that it is this same field which makes the quantisation of this theory
much more interesting. Indeed, it was shown in ref.[12] that in this model the
restriction on the dimension of the target space is completely lifted.

In the present paper we analyse the Jackiw-Teitelboim model of two dimen-
sional gravity. Partial results of these analyses were reported earlier in a short
letter [13]. When coupled to $c$ scalar fields, this model behaves like a critical
string in the sense that it forces $c$ to be equal to 24. We then add a Liouville
term to this model and treat the resulting theory as a non-linear sigma model.
The vanishing of the different beta functions, up to linear terms in the tachyon
field, leads to the same results obtained in ref.[12], where the same model was
considered as a theory of non-critical strings. Therefore, this model can be in-
terpreted also as a theory of critical bosonic strings moving in a target space of
dimension $D = c + 2$, where the Liouville mode and the extra field are string
coordinates too.

Then we consider a more general model which contains a kinetic term and
an arbitrary potential for the dilaton. We analyse this model perturbatively
and discuss its renormalisability. We show that the theory is renormalisable in
the usual sense for Liouville (exponential) potentials. For other potentials, the
theory can be renormalised only in the sense of the non-linear sigma models,
that is, allowing for a change in the functional form of the potential.

The paper is organised as follows: In section two we study the Jackiw-
Teitelboim model together with the Liouville action using conformal field theory
techniques. We then add matter fields and treat the whole theory as a non-linear
sigma model. In the third section we consider the above mentioned generalised model. We expand the action up two second order in the quantum fields of the background field expansion and choose our gauge fixing terms and calculate their corresponding ghost action. In the fourth section we present our results for the one loop divergences using a generalised formula of the heat-kernel method. The proof of this formula is given in an appendix. Section five deals with the renormalisation procedure. Finally, we end our article with some concluding remarks.

2. The Jackiw-Teitelboim model coupled to Liouville

The classical gravity action is assumed to be given by

\[ S_{JT} = \frac{b}{\pi} \int d^2 x \sqrt{\hat{g}} \hat{N}(R + \Lambda), \]  

(2.1)

where \( b \) is a constant and \( \hat{N}(x) \) is an auxiliary field whose equation of motion yields the Einstein-like equation in two dimensions

\[ R + \Lambda = 0. \]  

(2.2)

This action was first proposed by Jackiw and Teitelboim [11] as an alternative to the usual Einstein-Hilbert action, \( \int d^2 x \sqrt{g} R \), which is trivial in two dimensions. Many interesting aspects of this model were considered in refs.[14-18].

In the background geometry specified by \( \hat{g}_{\alpha\beta} \), where

\[ g_{\alpha\beta} = \hat{g}_{\alpha\beta} e^{\gamma \sigma}, \quad \gamma = 1 \text{ or } 2, \]  

(2.3)

the above action becomes

\[ S_{JT} = \frac{b}{\pi} \int d^2 x \sqrt{\hat{g}} \left[ \gamma \hat{g}^{\mu\nu} \partial_{\mu} \partial_{\nu} \hat{N} + N(\hat{R} + \Lambda e^{\gamma \sigma}) \right], \]  

(2.4)

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where we have used the well-known result
\[ R = e^{-\gamma \sigma} (\hat{R} - \gamma \nabla^2 \sigma) \]
\[ \nabla^2_{\beta} = \frac{1}{\sqrt{g}} \partial_{\alpha}(\sqrt{g} g^{\alpha \beta} \partial_{\beta}) \] \hspace{1cm} (2.5)

The energy momentum tensor corresponding to \( S_{JT} \) with \( \Lambda = 0 \) is given by
\[ T_{\alpha \beta} = -4\pi \sqrt{g} \delta L \delta g^{\alpha \beta} = -4b[\gamma \partial_{(\alpha} \partial_{\beta)} N - \hat{\nabla}_{\alpha} \hat{\nabla}_{\beta} N + \hat{g}_{\alpha \beta}( -\frac{\gamma}{2} \hat{g}^{\mu \nu} \partial_{\mu} \sigma \partial_{\nu} N + \nabla^2 N)] \] \hspace{1cm} (2.6)

The \( z-z \) component of this energy momentum tensor is written as
\[ T_{zz} = -4b(\gamma \partial_{z} \sigma \partial_{z} N - \partial_{z}^2 N) \] \hspace{1cm} (2.7)

The only propagator of this theory is
\[ \langle \sigma(z) N(w) \rangle = -\frac{1}{4\gamma b} \ln(z - w) \] \hspace{1cm} (2.8)

The operator product expansion of the energy momentum tensor produces a central charge for gravity given by
\[ c_{gravity} = 2 \] \hspace{1cm} (2.9)

If we introduce matter interactions through an action for \( c \) scalar fields \( X^i \)
\[ S_{matter} = \frac{1}{4\pi} \int d^2 x \sqrt{\hat{g}} \hat{g}^{\mu \nu} \partial_{\mu} X^i \partial_{\nu} X^i , \quad i = 2, \ldots, c + 1 \] \hspace{1cm} (2.10)
then their contribution to the total central charge, together with that of the ghosts, is \( c - 26 \). Hence requiring that the total central charge vanishes leads to \( c = 24 \) ! The obvious question to be asked now is whether it is possible to couple matter fields to our theory when \( c \neq 24 \).

The action \( S_{JT} \) can be modified in two ways: The first one consists in adding a kinetic term for the field \( N \). This, however, results in a change in equation
which is the main motivation for proposing (2.1) as an action for classical gravity in two dimensions. The other alternative which will be adopted in this paper is to add a Liouville action

\[ S_L = \frac{1}{\pi} \int d^2 x \sqrt{\hat{g}} (a \hat{g}^{\mu \nu} \partial_\mu \sigma \partial_\nu \sigma + Q \sigma \tilde{R}) \]  

(2.11)
to our action in (2.1), where \(a\) and \(Q\) are two constants. The resulting action \(S_{tot} = S_{JT} + S_L + S_{matter}\), is proportional to the action for two dimensional gravity interacting with matter fields proposed in ref.[12]. Standard analyses show that the energy momentum tensor corresponding to \(S_{JT} + S_L\) has a central charge given by

\[ c^{gravity} = 2 + 96 \left( \frac{Q}{\gamma} - \frac{a}{\gamma^2} \right) . \]  

(2.12)

Including the matter and ghost contributions, the vanishing of the total central charge leads to

\[ c - 96 \left( \frac{1}{\gamma} - \frac{Q}{\gamma^2} + \frac{a}{\gamma^2} \right) = 0 . \]  

(2.13)

Unlike the Liouville alone [2-4], there is no restriction on the matter central charge.

Notice that \(S_{tot}\) can be written as a non-linear sigma model in some special backgrounds. This is given by

\[ S_{tot} = \frac{1}{4\pi} \int d^2 x \sqrt{\hat{g}} [G_{ab}(\eta) \hat{g}^{\mu \nu} \partial_\mu \eta^a \partial_\nu \eta^b + \hat{R} \Phi(\eta) + 2 \mu T(\eta)] . \]  

(2.14)

Here \(\eta^0 = \sigma, \eta^1 = N\) and \(\eta^a = X^i\) for \(a = 2, ..., c + 1\). The target space metric is given by \(G_{00} = 4a, G_{01} = 2gb, G_{11} = 0, G_{ab} = \delta_{ij}\) for \(a, b = 2, ..., c + 1\) and \(G_{0i} = G_{1i} = 0\). The dilaton field is

\[ \Phi(\eta) = 4Q\sigma + 4bN . \]  

(2.15)
We have also included a cosmological term in the form of a tachyon

\[ T(\eta) = e^{\alpha \sigma} \]

(2.16)

where \( \alpha \) is a constant to be determined.

To linear terms in the tachyon \( T \), the vanishing of the \( \bar{\beta} \) functions leads to

\[ \bar{\beta}_{ab}^G = R_{ab} + 2 \nabla_a \nabla_b \Phi + ... = 0 \]

(2.17)

\[ \bar{\beta}^\Phi = \frac{1}{6} (D - 26) - \frac{1}{2} \nabla^2 \Phi + G^{ab} \nabla_a \Phi \nabla_b \Phi + ... = 0 \]

(2.18)

\[ \bar{\beta}^T = \frac{1}{2} G^{ab} \nabla_a \nabla_b T + G^{ab} \nabla_a \Phi \nabla_b T - 2T + ... = 0 \]

(2.19)

In \( \bar{\beta}^\Phi \), \( D = c + 2 \) is the dimension of the target space (or the number of fields). With the above backgrounds \( \bar{\beta}_{ab}^G = 0 \) is automatically satisfied, whereas the equations for \( \bar{\beta}^\Phi \) and \( \bar{\beta}^T \) lead respectively to

\[ \frac{1}{6} (c - 24) + 16 \left( \frac{Q}{\gamma} - \frac{a}{\gamma^2} \right) = 0 \]

(2.20)

\[ \alpha = \gamma \]

(2.21)

The last equation is just the requirement that \( T(\eta) \) is a (1,1) operator with respect to the energy momentum tensor corresponding to \( S_{JT} + S_L [12] \). Notice that Eq. (2.10) is exactly equivalent to Eq. (2.13) obtained by conformal field theory considerations. Therefore \( S_{tot} \), which is a theory of non-critical strings, can be interpreted as a theory of a critical bosonic string moving in a target space of dimension \( D = c + 2 \). In particular, for \( Q = a \) and \( \gamma = 2 \) we have

\[ a = \frac{1}{24} (24 - c) \]

(2.22)

This is also the result obtained in ref.[12] utilising conformal field theory.

\[ ^1 \text{Our conventions for the } \beta \text{-functions are those of ref.[19], and we have included only the terms that are relevant to our calculation.} \]
methods.

3. Perturbation theory

We will use the background field expansion and dimensional regularisation where \( d = 2 - \epsilon \). In this section we will find the expansion up to second order in the quantum fields of the classical action. Our starting point is

\[
S = \int d^d x \sqrt{g} \left( \bar{N} \bar{R} + V(\bar{N}) + \frac{G}{2} \partial_\mu \bar{N} \partial^\mu \bar{N} + \frac{1}{2} \partial_\mu \bar{X}^i \partial^\mu \bar{X}^i \right), \tag{3.1}
\]

where we have included a kinetic term and an arbitrary potential for the dilaton field. Here \( G \) is an arbitrary constant. For \( V(\bar{N}) = 0 \) the first two terms of the action are the local counterpart of the non-local Polyakov action \( S = \int d^d x \sqrt{g} R (\nabla^2)^{-1} R \). Similar models have been considered in ref. [22].

The classical equations of motion are given by

\[
R + V' - G \nabla^2 N = 0 \tag{3.2}
\]

\[
\nabla^2 N g_{\mu\nu} - N g_{\mu\nu} + N [R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}] - \frac{1}{2} V g_{\mu\nu} =
\]

\[
\frac{G}{2} [g_{\mu\nu} N, N] + N [R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}] + \frac{1}{2} [g_{\mu\nu} X^i X^j, X^i X^j] - X^i X^j
\]

\[
\nabla^2 X^i = 0. \tag{3.3}
\]

Note that eq. (3.3) implies that \( \nabla^2 N = V + O(\epsilon) \).

To compute the one loop effective action we use the background field method, expanding the full fields \( \bar{g}_{\mu\nu}, \bar{N} \) and \( \bar{X}^i \) around the classical configurations.

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\(^2\)After completion of this work we learned that the sigma-model interpretation of the action \( S_{\text{tot}} \) has been treated by Chamseddine in Ref. [20]

\(^3\)This non-local action has been also studied perturbatively in ref.[21]
\( g_{\mu \nu}, N \) and \( X^i \), that is

\[
\bar{g}_{\mu \nu} = g_{\mu \nu} + h_{\mu \nu} \\
\bar{N} = N + \varphi \\
\bar{X}^i = X^i + \xi^i ,
\]

(3.5)

where \( h_{\mu \nu}, \varphi \) and \( \xi^i \) are quantum fluctuations. Dropping the linear terms proportional to the classical equations of motion we get

\[
S(\bar{g}_{\mu \nu}, \bar{N}, \bar{X}^i) - S(g_{\mu \nu}, N, X^i) = S^{(2)} = \\
= \frac{1}{2} \int d^d x \sqrt{g} N \left( \frac{1}{2} h_{\mu \nu} h^{\mu \nu} - h^{\mu \nu} h^\nu_{\mu \nu} - \frac{1}{2} h_{\mu \nu \rho \sigma} h^{\mu \nu \rho \sigma} + h_{\mu \nu} h^\nu_{\mu \nu} \right) \\
+ \frac{1}{2} \int d^d x \sqrt{g} N \left( \frac{1}{2} R h_{\mu \nu} h^{\mu \nu} + \frac{1}{4} R h^2 - h h^{\mu \nu} R_{\mu \nu} + 2 h^{\mu \nu} h^\nu_{\rho \sigma} R_{\rho \sigma} \right) \\
+ \int d^d x \sqrt{g} \varphi \left( \frac{1}{2} R h - R_{\mu \nu} h^{\mu \nu} - \nabla^2 h + h_{\mu \nu}^{\mu \nu} \right) \\
+ \int d^d x \sqrt{g} \left[ \frac{1}{2} \nabla^2 N (h_{\mu \nu} h^{\mu \nu} - \frac{1}{2} h^2) + N_{\mu \nu} (h h^{\mu \nu} - h^\mu_{\mu} h^{\mu \nu}) + \frac{1}{2} N_{\mu \nu} h h^{\mu \nu} \right] \\
+ \int d^d x \sqrt{g} \left[ \frac{1}{2} h^2 - h_{\mu \nu} h^{\mu \nu} \right] + \frac{1}{2} \left( \frac{V'}{V} \varphi h + \frac{1}{2} V'' \varphi^2 \right) \\
+ \frac{G}{2} \int d^d x \sqrt{g} \left[ \varphi_{;\mu} \varphi^{;\mu} + N_{;\mu} N_{;\nu} (- \frac{1}{2} h h^{\mu \nu} + h^{\mu \nu} h^\nu_{\rho \sigma}) \right] \\
+ \frac{1}{4} g^{\mu \nu} (\frac{h^2}{2} - h_{\rho \sigma} h^{\rho \sigma}) + N_{\mu \nu} \varphi_{;\mu} (h g^{\mu \nu} - 2 h^{\mu \nu}) \\
+ \frac{1}{2} \int d^d x \sqrt{g} \left[ \xi^i_{;\mu} \xi^i_{;\mu} + X^i_{;\mu} \xi^i_{;\mu} (h g^{\mu \nu} - 2 h^{\mu \nu}) \right] \\
+ X^i_{;\mu} X^i_{;\nu} \left[ \frac{1}{4} g^{\mu \nu} (\frac{h^2}{2} - h_{\rho \sigma} h^{\rho \sigma}) - \frac{1}{2} h h^{\mu \nu} + h^{\mu \nu} h^\nu_{\rho \sigma} \right] ,
\]

(3.6)

where \( h \equiv h^\mu_{\mu} \).

To proceed, we must add a gauge fixing Lagrangian. We will choose a gauge in such a way that the differential operator in the kinetic term is always \( \nabla^2 = \nabla_\mu \nabla^\mu \). Having a kinetic term proportional to the Laplacian, the evaluation of the one loop divergences becomes simpler, as one has to compute the
determinant of a minimal operator. Standard heat-kernel techniques are then applicable [23]. In usual gravity the DeWitt gauge does the job [24]. A suitable generalisation of this gauge to our model is found to be

\[ S_{gf} = -\frac{1}{2} \int d^d x \sqrt{g} N [h^\mu_{\nu,\mu} - \frac{1}{2} h_{\nu,\nu} - \frac{1}{N} \phi_{,\nu} - \beta(N) N^{\mu}_{\nu} h_{\mu\nu}]^2 , \quad (3.7) \]

where \( \beta(N) \) is an arbitrary function of the classical field \( N \).

The corresponding ghost action for this gauge condition is

\[
S_{gh} = \int d^d x \sqrt{g} C^\mu \Delta^{gh}_{\mu \nu} C^\nu
\]

\[
\Delta^{gh}_{\mu \nu} = N [g_{\mu\nu} \nabla^2 - \frac{1}{N} N_{,\nu} \nabla_{\mu} - \beta(N_{,\nu} \nabla_{\mu} + g_{\mu\nu} N^{,\rho} \nabla_{\rho}) - \frac{1}{N} N_{,\nu} + R_{\mu\nu}]. \quad (3.8)
\]

Adding the gauge fixing term (3.7) to the quadratic Lagrangian (3.6) we get, in a condensed notation,

\[
S^{(2)}_T = S^{(2)} + S_{gf} = \frac{1}{2} \int d^d x \sqrt{g} h_{mn} (-\Delta^{mn,pq} \nabla^2 + Y^{mn,pq}_{,\mu} \nabla^\mu + X^{mn,pq} - \frac{1}{2} \nabla^\mu S^{mn,pq}_{,\mu} h_{pq} . \quad (3.9)
\]

Here the indices \( mn, pq \) run from 1 to \( d+1+c \). The field \( h_{mn} \) coincides with \( h_{\mu\nu} \) for \( m, n = 1, \ldots d \), while \( h_{d+1, d+1} \equiv h_{\varphi\varphi} \) is defined to be equal to \( \varphi \) and \( h_{d+1+i, d+1+i} \equiv h_{ii} \) denotes the matter field \( X^i \). The pairs \( (mn) \) and \( (pq) \) take the values \( (\mu \nu) \), \( (d+1 d+1) \equiv (\varphi \varphi) \) or \( (d+1+i d+1+i) \equiv (ii) \) but crossed pairs like \( (\mu \varphi), (\mu i) \) and \( (i \varphi) \) must not be included. The matrices \( \Delta, Y_\mu, S_\mu \) and \( X \) are given by

\[
\Delta^{mn,pq} = \begin{cases} 
\Delta^{ii,ii} = 1 \\
\Delta^{\varphi \varphi, \varphi \varphi} = G - \frac{1}{N} \\
\Delta^{\varphi \varphi, \mu \nu} = \Delta^{\mu \nu, \varphi \varphi} = \frac{1}{2} g^{\mu \nu} \\
\Delta^{\mu \nu, \rho \tau} = N \delta^{\mu \nu, \rho \tau} \end{cases} \quad (3.10)
\]
\[ X_{mn,\rho\sigma} = \begin{cases} 
X^{ii,ii} = X^{\phi,\phi;\phi} = 0 \\
X^{\mu\nu,\phi\phi} = X^{\phi,\phi,\mu\nu} = -R_{\mu\nu} + \frac{1}{2}Rg_{\mu\nu} + \frac{1}{2}g^{\mu\nu} \\
X^{\mu\nu,\rho\sigma} = NT^{\mu\nu,\rho\sigma} + G\left[ \frac{1}{2}N_{\tau}N^{\tau}P^{\mu\nu,\rho\sigma} - \frac{1}{4}(g^{\mu\nu}N^{\tau\rho}N^{\sigma\sigma} + g^{\rho\sigma}N^{\tau\mu}N^{\nu\nu}) \right] \\
&+ \frac{1}{2}(G - \beta^2 N)[N^{i}(\mu N^{\rho}g^{\nu\rho} + N^{i}(\rho N^{\nu}g^{\mu\rho})] \\
&- 2P^{\mu\nu,\rho\sigma}\nabla^2 N + \frac{1}{2}(N^{i\mu}g^{\rho\sigma} + N^{i\rho}g^{\mu\sigma}) \\
+ \frac{1}{2}g_{\alpha X}^{\mu\nu}X^i\nabla^\alpha X^jP^{\mu\nu,\rho\sigma} - \frac{1}{2}(g^{\mu\nu}\partial^\rho X^i\partial^\sigma X^j + g^{\rho\sigma}\partial^\mu X^i\partial^\nu X^j) \\
+ \frac{1}{2}[g^{\mu(\nu}\partial^\rho X^i\partial^\sigma X^j + g^{\rho(\mu}\partial^\nu X^i\partial^\sigma X^j)] \\
\end{cases} \quad (3.11) \]

\[ S_{\tau mn,pq} = \begin{cases} 
S^{\phi,\phi,\phi} = -\frac{1}{2\tau}N_{\tau} \\
S^{\phi,\phi,\rho\sigma} = S^{\rho\sigma,\phi,\phi} = 2GN_{\nu}P^{\rho\sigma,\nu} - \frac{1}{2}\beta(N^{i\mu}g_{\nu}^{\rho} + N^{\sigma\sigma}g_{\tau}^{\mu}) \\
S^{\mu\nu,\rho\sigma}_{\tau} = -N_{\tau}P^{\mu\nu,\rho\sigma} - (N\beta + 2)(N^{i\mu}P^{\rho\sigma,\nu}_{\tau} \\
&+ N^{\nu\nu}P^{\rho\sigma,\mu}_{\tau} + N^{\sigma\sigma}P^{\mu\nu,\rho}_{\tau}) \\
S^{\rho\sigma,ii}_{\tau} = S^{ii,\rho\sigma}_{\tau} = 2X^{i\nu}P^{\rho\sigma,\nu}_{\tau} \\
\end{cases} \quad (3.12) \]

\[ Y_{\tau mn,\rho\sigma} = \begin{cases} 
Y^{\phi,\phi,\phi} = 0 \\
Y^{\rho\sigma,\phi,\phi}_{\tau} = Y^{\phi,\phi,\rho\sigma}_{\tau} = S^{\rho\sigma,\phi,\phi}_{\tau} \\
Y^{\rho\sigma,ii}_{\tau} = Y^{ii,\rho\sigma}_{\tau} = S^{\rho\sigma,ii}_{\tau} \\
Y^{\mu\nu,\rho\sigma}_{\tau} = (\beta N + 1)(N^{i\mu}P^{\rho\sigma,\nu}_{\tau} + N^{\sigma\sigma}P^{\mu\nu,\rho}_{\tau} - N^{\nu\nu}P^{\rho\sigma,\mu}_{\tau} - N^{\mu\mu}P^{\sigma\sigma,\nu}_{\tau}) \\
\end{cases} \quad (3.13) \]

where

\[ P^{\mu\nu,\rho\sigma} = \frac{1}{4}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \quad (3.14) \]

\[ F^{\mu\nu,\rho\sigma} = \frac{1}{2}(g^{\mu\nu}g^{\sigma\rho} + g^{\mu\sigma}g^{\nu\rho}) \quad (3.15) \]

and

\[ T^{\mu\nu,\rho\sigma} = \frac{1}{4}(R + \frac{V}{N})(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\
+ \frac{1}{4}(g^{\mu\rho}R^{\nu\sigma} + g^{\mu\sigma}R^{\nu\rho} + g^{\nu\rho}R^{\mu\sigma} + g^{\nu\sigma}R^{\mu\rho}) \\
- \frac{1}{2}(g^{\mu\nu}R^{\rho\sigma} + g^{\rho\sigma}R^{\mu\nu}) + \frac{1}{2}(R^{\nu\mu\rho} + R^{\nu\sigma\mu}) \quad , \quad (3.16) \]
The components of $\Delta^{-1}$ are given by

$$
(\Delta^{-1})_{pq,rs} = \begin{cases}
\Delta^{-1}_{ii,ii} = 1 \\
\Delta^{-1}_{\varphi\varphi,\varphi\varphi} = 0 \\
\Delta_{\varphi\varphi,\mu\nu} = \Delta_{\mu\nu,\varphi\varphi} = g_{\mu\nu} \\
\Delta_{\mu\nu,\rho\sigma} = \frac{2}{N}[-1_{\mu\nu,\rho\sigma} + g_{\mu\nu}g_{\rho\sigma}(1 - \frac{1}{2}NG)]
\end{cases}
$$

Note that, unlike the pure gravity case [9], the inverse matrix $\Delta^{-1}$ is well defined and has no pole in the limit $\epsilon \to 0$. This is of course due to the fact that the action (3.1) is non trivial in exactly two dimensions. This fact is also crucial for the absence of any renormalisation for the anomalous dimensions of operators [9]. We also stress that the structure of the propagator here is different from that of the conformal gauge analysis in section 2. There, after adding the Liouville action, one has $<NN> \neq 0$, $<N\sigma> \neq 0$ and $<\sigma\sigma> = 0$ (see also ref. [20]). This result does not appear in Eq. (3.17) in the limit $G = 0$. The reason for this apparent discrepancy is that the gauge fixing Lagrangian induces new kinetic terms for the field $\varphi$ and thus modifies the structure of the propagator of the theory.

In deriving Eqs.(3.9)-(3.16) we performed some integrations by parts in such a way that the matrices $\Delta$, $X$ and $S_\mu$ are symmetric and $Y_\mu$ is antisymmetric under the interchange of the pairs $(mn)$ and $(pq)$. This property will be important when computing the divergences.

4. The one loop divergences

To compute the one loop effective action, we need to evaluate the functional determinant of the (symmetrized) differential operators that appear in Eqs.(3.8) and (3.9). Using heat-kernel techniques [23], it can be shown that given the
operator
\[
[-\nabla^2 I_{ij} + A^\mu_{ij} \nabla_\mu + M_{ij}]
\]
(4.1)

the logarithm of its determinant is given by
\[
\ln \det[-\nabla^2 I_{ij} + A^\mu_{ij} \nabla_\mu + M_{ij}] = \frac{1}{2\pi \epsilon} Tr \left(-\frac{1}{6}RI + \frac{1}{4}A^\mu A_\mu + M\right) = \frac{1}{2\pi \epsilon} \int d^d x \sqrt{g} \left(-\frac{1}{6}RI_{ii} + \frac{1}{4}A_{\mu i j} A^\mu_{ji} + M_{ii}\right),
\]
(4.2)

where \( \epsilon = d - 2 \) and \( I_{ij} \) is the identity operator. The proof of this formula is presented in the Appendix A.

In our case, there is an additional complication because the Laplacian in the kinetic term is multiplied by a space-time dependent matrix which does not commute with the covariant derivative. To get rid of this we use the doubling trick of t’Hooft and Veltman [25]. We consider the action (3.9) and add the same expression but with different fields \( h'_{mn} \) instead of \( h_{mn} \). The ‘doubled’ effective action will be two times the original effective action. In terms of the complex fields
\[
\lambda_{mn} = h_{mn} + i h'_{mn}
\]
\[
\bar{\lambda}_{mn} = h_{mn} - i h'_{mn}
\]
we can write the doubled action as
\[
\mathcal{S}^{(2)}_T = \frac{1}{2} \int d^d x \sqrt{g} \lambda_{mn} (-\Delta_{mn,pq} \nabla^2 + (Y_{\mu}^{mn,pq} - \nabla_\mu \Delta_{mn,pq}) \nabla^\mu 
+ X^{mn,pq} - \frac{1}{2} \nabla_\mu S^{mn,pq}_\mu + \frac{1}{2} \nabla_\mu Y^{mn,pq}_\mu - \frac{1}{2} \nabla^2 \Delta_{mn,pq}) \lambda_{pq}
\]
(4.4)

where we have used the symmetry and antisymmetry properties of \( \Delta, X, S_\mu \) and \( Y_\mu \) respectively. As we are now dealing with complex fields, the one loop divergences remain unchanged under the replacement [25]
\[
\bar{\lambda}_{rs} \rightarrow \Delta^{-1}_{rs,mn} \bar{\lambda}^{mn}_{\bar{\lambda}}.
\]
(4.5)
As a consequence, the effective action associated with the action (3.9) is

\[ W^{(1)} = -\frac{1}{2} \ln \det \left[ -\nabla^2 I_{rs}^{pq} + \Delta_{rs,mn}^{-1} (Y_{\mu}^{mn,pq} - \nabla_\mu \Delta^{mn,pq}) \nabla^\mu \right] + \Delta_{rs,mn}^{-1} (X^{mn,pq} - \frac{1}{2} \nabla_\mu S_{\mu}^{mn,pq} + \frac{1}{2} \nabla_\mu Y_{\mu}^{mn,pq} - \frac{1}{2} \nabla_2 \Delta^{mn,pq}) \]  

where we have included a factor \( \frac{1}{2} \) to take into account the doubling of the degrees of freedom.

A replacement analogous toEq.(4.5) can be done for the ghost fields in order to get rid of the \( N g_{\mu\nu} \) factor appearing in the kinetic term. We have then

\[ W^{(1)}_{gh} = \ln \det N^{-1} g^{\rho\sigma} \Delta_{\rho\sigma}^{gh} \]  

where now the coefficient in front of \( \ln \det \) is +1 due to the fact that the ghosts are complex anticommuting fields.

Now we are ready to compute the total one loop effective action

\[ W^{(1)}_T = W^{(1)} + W^{(1)}_{gh} \]  

All we have to do is to apply the heat-kernel formula (4.2) to evaluate the determinants. We begin with the ghosts contribution. According to Eqs.(3.8), (4.2) and (4.7) we have

\[ W^{(1)}_{gh} = \ln \det [g^\rho_\mu \nabla^2 - A^\tau_\mu \nabla_\tau + R^\rho_\mu - \frac{1}{N} N;_{\rho}] = \frac{1}{2\pi \epsilon} \int d^4 x \sqrt{g} \left[ -\frac{4}{3} R + \frac{1}{4} A^\tau_\mu A_\tau^\mu + \frac{1}{N} \nabla^2 N \right] \]  

where

\[ A^\tau_\mu = \frac{1}{N} N;^\tau_\rho g^\rho_\mu + \beta (N;\rho^\tau g^\rho_\mu + N;^\tau g^\rho_\mu) \]  

A simple calculation gives

\[ A^\tau_\mu A_\tau^\mu = \frac{1}{N^2} N;_\mu N;^\rho (1 + 4N \beta + 5N^2 \beta^2) \]
so, after integrations by parts we obtain

\[ W_{gh}^{(1)} = \frac{1}{4\pi \epsilon} \left[ -\frac{8}{3} R + \frac{1}{2} \frac{N_{\mu\nu} N^{\mu\nu}}{N^2} (5 + 4N\beta + 5N^2\beta^2) \right] \]  

(4.12)

Now we consider the calculation of \( W^{(1)} \). From Eqs.(4.2) and (4.6) we have

\[
W^{(1)} = -\frac{1}{2} \ln \det[-\nabla^2 I_{rs}^{pq} + \tilde{Y}_r^{\tau pq} \nabla_\tau + \tilde{X}_{rs}^{pq}]
\]

\[
= \frac{1}{4\pi \epsilon} \int d^d x \sqrt{g} \left[ \left( c + 4 \right) \frac{1}{6} R - \frac{1}{4} \tilde{Y}_r^{\tau pq} \tilde{Y}_r^{\mu pq} - \tilde{X}_{rs}^{pq} \right] \]

(4.13)

where

\[
\tilde{X}_{rs}^{pq} = \Delta^{-1}_{rs,mn} (X^{mn,pq} + \frac{1}{2} \nabla^\mu Y^{mn,pq}_\mu - \frac{1}{2} \nabla^\mu S^{mn,pq}_\mu - \frac{1}{2} \nabla^2 \Delta^{mn,pq}(q) \]

\[
\tilde{Y}_r^{\tau pq} = \Delta^{-1}_{rs,mn} (Y^{mn,pq}_\tau - \nabla_\tau \Delta^{mn,pq})
\]

(4.14)

The trace of the \( X \)-term is given by

\[
\tilde{X}_{rs}^{rs} = 2R + 2V' + \frac{2V}{N} + \frac{N \tau N^{\tau}}{N^2} ((\beta N + 1)^2 + GN(\beta^2 N^2 - 1))
\]

\[
- \frac{1}{N} X^{i\tau} X_i^\tau
\]

(4.16)

The next step is to compute the \( \tilde{Y}^2 \)-trace in Eq.(4.13). The following identities are useful in this calculation

\[
P^{\mu\nu}_{\rho\sigma} = 0 \quad P^{\mu\nu}_{\mu\nu} = -1 \quad P^{\mu\nu}_{\nu\sigma} = -\frac{1}{4} g^{\mu\sigma}
\]

\[
I_{\mu\nu,\rho} = g_{\mu\nu} \quad I_{\mu\nu,\nu} = \frac{3}{2} g^\sigma
\]

\[
P^{\mu\nu,\tau}_{\rho\sigma} = 4 g^\rho \quad I^{\mu\nu,\tau}_{\rho} = \frac{3}{2} g^{\nu\tau}
\]

(4.17)

We are not including \( O(\epsilon) \) terms because they produce finite contributions to the effective action. According to Eq.(4.15) we have that

\[
tr \tilde{Y}^2 = \Delta^{-1}_{rs,mn} (Y^{mn,pq}_\tau - \nabla_\tau \Delta^{mn,pq}) \Delta^{-1}_{pq, tu} (Y^{tu,rs}_\tau - \nabla_\tau \Delta^{tu,rs})
\]

\[
= \Delta^{-1}_{rs,mn} \nabla_\tau \Delta^{mn,pq} \Delta^{-1}_{pq, tu} \nabla_\tau \Delta^{tu,rs} + \Delta^{-1}_{rs,mn} Y^{mn,pq}_\tau \Delta^{-1}_{pq, tu} Y^{tu,rs}_\tau
\]

(4.18)
Note that due to the symmetry of $\Delta^{-1}$ and the antisymmetry of $Y$ the crossed terms do not contribute. Using the cyclic property of the trace an neglecting total derivatives we get

$$tr \Delta^{-1}(\nabla_\tau \Delta)\Delta^{-1}(\nabla^\tau \Delta) = 2\frac{N \rho N}{N^2}$$ (4.19)

The result for the second term in (4.18) is

$$tr \Delta^{-1}Y\Delta^{-1}Y = \frac{4}{N}X^i_{\tau}X^{i\tau} + \frac{N \rho N}{N^2} \times$$

$$\times [2\beta^2 N^2 + 8N(\beta N + 1)(\beta + G) - 4(\beta N + 1)^2(1 + GN)]$$ (4.20)

Combining the above equations we find, finally,

$$W^{(1)}_T = \frac{1}{2\pi \epsilon} \int d^4x \sqrt{g} \left[ \frac{(c - 24)}{12} R - V' - \frac{V}{N} + \frac{\nabla^2 N}{N} \right],$$ (4.21)

where we dropped a boundary term.

Several comments are in order. First of all, we see that the divergences are not of the form of the classical action. As usual in quantum gravity, a field redefinition [25,26] will be necessary to renormalise the theory. However, as we will see in the next section, the theory is renormalisable in the usual sense only for a particular class of potentials. It is also worth noting that the arbitrary function $\beta(N)$ introduced by the gauge fixing term has disappeared from our final answer, as well as the constant $G$ and the classical matter fields $X^i$. Other interesting feature is that the coefficient of $R$ in the one loop divergence reproduces exactly the result for the coefficient $a$ appearing in section 2 (see eq. (2.22)).
5. Renormalisation

The usual renormalisation procedure would be to absorb the infinities into the bare constant $G$ and the bare constants appearing in the potential $V(N)$, allowing for a field redefinition of $N$ and $g_{\mu\nu}$. We will follow here a generalised procedure, allowing also for a change of the functional form of the potential $V(N)$. This is similar to what is done when renormalising a non-linear sigma model in two spacetime dimensions.

In our divergence (4.21), the combination of the last two terms vanishes on shell because it is the trace of the classical equation (3.3). This means that these terms can be absorbed into a conformal rescaling of the metric. The term proportional to the curvature $R$ can be absorbed into a constant shift of the scalar field $N$. As a consequence, one way of absorbing the infinities is

$$
N \rightarrow N - \frac{1}{24\pi\epsilon}(c - 24)
g_{\mu\nu} \rightarrow g_{\mu\nu} \exp\left[\frac{1}{2\pi\epsilon N}\right]
V(N) \rightarrow V(N) + \Delta V(N)
X^i \rightarrow X^i
$$

where

$$
\Delta V(N) = \frac{V'}{2\pi\epsilon}[1 + \frac{1}{12}(c - 24)]
$$

From the above equation we see that the theory is renormalisable in the usual way when $\Delta V$ is proportional to $V$. This is the case for Liouville potentials of the form $V(N) = \mu \exp[\alpha N]$. More generally, if the potential depends on a set of bare constants $\alpha_i$, the theory is renormalisable whenever $V'$ can be written as a linear combination of $\frac{\partial V}{\partial \alpha_i}$. In this situation, the divergence can be absorbed.
into the bare constants.

For other potentials, the theory is renormalisable only in a generalised sense. For example, in the Jackiw Teitelboim model the linear potential $V(N) = \Lambda N$ gets an $N$-independent renormalisation, and the infinities cannot be absorbed into the bare constant $\Lambda$. However, if one considers $V(N) = \Lambda N + \mu$, the divergence can be absorbed into the constant $\mu$.

6. Conclusions

In this paper we showed that the Jackiw-Teitelboim model coupled to Liouville theory and $c$ scalar fields can be interpreted as a theory of critical strings as well as a theory of non-critical strings. The target space in which the critical string propagates has dimension $D = 2 + c$.

We then considered a more general dilaton-gravity theory and analysed it from a perturbative point of view. Some of the results of the conformal field theory were reproduced at the one loop level. In particular, the anomalous dimensions of certain operators do not get renormalised owing to the absence of any poles in the graviton propagator. Furthermore, the coefficient of the anomaly term (the term proportional to $R$) in the one loop divergence is exactly equal to the coefficient appearing in front of the Liouville action of the non-critical string.

We have also analysed the renormalisability of the dilaton-gravity theories. We showed that the Liouville potentials are privileged because they produce theories which are on shell renormalisable, i.e., the infinities can be absorbed
into the bare constants of the theory and field redefinitions. For other potentials, the renormalisation of the theory implies a change in the functional form of the potential, as the tachyon renormalisation in the non-linear sigma model. These results agree with previous ones obtained in the conformal gauge [27].

Let us now comment on previous calculations in background gauges done by us and other authors [21,28,29]. The results we obtained here correct our claims in an earlier version of this work about the $\beta$-dependence of the one loop divergence. The one loop counterterm has also been computed in ref. [28]. Unfortunately, the results obtained there are different and seem not to imply renormalisability for Liouville potentials. A possible source of disagreement may be the fact that the doubling trick is lacking in that calculation. The same problem appears in refs. [21,29], where the one loop divergences were computed for the non-local Polyakov action.

Finally, we mention that interesting developments would be to compute the Vilkovisky DeWitt effective action [30], as well as the analysis of the supersymmetric version of the Jackiw Teitelboim model.

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7. Appendix

In this appendix we will prove the formula (4.2). Consider the Green function
\[ G(m^2, x, x') \]
defined by
\[
[-\nabla^2 + A^\mu \nabla_\mu + M + m^2]G(m^2, x, x') = \delta^d(x, x')
\]  
(7.1)

where \( m \) is a mass which will disappear at the end of the calculation. We want to compute the divergent part of
\[
\ln \det [-\nabla^2 + A^\mu \nabla_\mu + M + m^2] = -\ln \det G = -\text{Tr } \ln G
\]  
(7.2)
in the limit \( d \to 2 \).

Inserting the proper time representation
\[
G(m^2, x, x') = \int_0^\infty ds K(x, x', s)
\]  
(7.3)

into Eq.(7.1) we find the following Schroedinger equation for the kernel \( K(x, x', s) \)
\[
\partial_s K = -[-\nabla^2 + A^\mu \nabla_\mu + M + m^2]K
\]  
(7.4)

with the boundary condition \( K(x, x', 0) = \delta^d(x, x') \).

According to the standard Schwinger DeWitt technique we write [23]
\[
K(x, x', s) = \frac{1}{(4\pi s)^{d/2}} \exp \left[ -\frac{\sigma(x, x')}{2s} + m^2 s \right] \Delta^{1/2}(x, x')\Omega(x, x')
\]  
(7.5)

where \( \sigma(x, x') \) is one half the squared geodesic distance between \( x \) and \( x' \) and
\( \Delta(x, x')g^{1/2}(x)g^{1/2}(x') \) is the Van Vleck-Morette determinant. The Schroedinger
equation for the kernel then becomes

$$\partial_s \Omega + \frac{\sigma^\mu \Omega_{,\mu}}{s} - \frac{A_\mu \sigma^\mu}{2s} \Omega + \Delta^{-1/2}[-\nabla^2 + A_\mu \nabla^\mu + M](\Delta^{1/2} \Omega) = 0 \quad (7.6)$$

Expanding $\Omega(x, x', s)$ in powers of $s$

$$\Omega(x, x', s) = \sum_{k \geq 0} s^k a_k(x, x') \quad , \quad (7.7)$$

we get a set of recursion relations for the functions $a_k(x, x')$. We will need only the first two which are

$$0 = \sigma^\mu a_{0;\mu} - \frac{1}{2} a_0 A_\mu \sigma^{\mu} \quad (7.8)$$

$$0 = a_1 + \sigma^\mu a_{1;\mu} - \frac{1}{2} a_1 A_\mu \sigma^{\mu} + \Delta^{-1/2}[-\nabla^2 + A_\mu \nabla^\mu + M](\Delta^{1/2} a_0) \quad (7.9)$$

with the boundary condition $[a_0] = 1$ (the brackets denote the coincidence limit).

Using the representation

$$\ln \det G = Tr \ln G = \int_0^\infty \frac{ds}{s} Tr K \quad (7.10)$$

we find that the divergent part is independent of $m$ and given by

$$\ln \det G|_{div} = \frac{1}{2\pi \epsilon} \int d^d x \sqrt{g}[tr a_1] \quad (7.11)$$

It is then necessary to extract $[a_1]$ from the equations (7.8) and (7.9). As usual, this can be done taking derivatives and coincidence limits of these equations. The following identities are useful [31]

$$[\sigma] = 0 \quad [\sigma_{\alpha_1 \ldots \alpha_{2k+1}}] = 0 \quad [\sigma_{\alpha\beta}] = g_{\alpha\beta}$$

$$[\Delta^{1/2}] = 1 \quad [\Delta^{1/2}_{\alpha}] = 0 \quad [\Delta^{1/2}_{,\alpha\beta}] = \frac{1}{6} R \quad (7.12)$$
The coincidence limit of Eq.(7.9) gives

\[ [a_1] = -M - A_\mu[a_0^\mu] + \frac{1}{6}R + [\nabla^2a_0] \]  

Equation (7.13)

Taking the covariant derivative of Eq.(7.8) and then the coincidence limit we find

\[ [a_0^\mu] = \frac{1}{2}A^\mu \]  

Equation (7.14)

Doing the same with the second covariant derivative we obtain

\[ [\nabla^2a_0] = \frac{1}{4}A_\mu A^\mu + \frac{1}{2}\nabla_\mu A^\mu \]  

Equation (7.15)

Combining Eqs.(7.13) to (7.15) we get, up to total derivatives,

\[ [a_1] = \frac{1}{6}R - \frac{1}{4}A_\mu A^\mu - M \]  

Equation (7.16)

Replacing Eq.(7.16) into Eqs.(7.11) and (7.2) we obtain the desired result Eq.(4.2).
Bibliography

[1] A. M. Polyakov, Mod. Phys. Lett. A2 (1987) 893.

[2] V. G. Knizhnik, A. M. Polyakov and A. B. Zamolodchikov, Mod. Phys. Lett. A3 (1988) 819.

[3] J. Distler and H. Kawai, Nucl. Phys. B321 (1989) 509.

[4] F. David, Mod. Phys. Lett. A3 (1988) 1651.

[5] V. Kazakov, Phys. Lett. B150 (1985) 282; Mod. Phys. Lett. A4 (1989) 2125; A. Ambjorn, B. Durhuus and J. Fröhlich, Nucl. Phys. B257 (1985) 433; V. Kazakov, I. Kostov and A. A. Migdal, Phys. Lett. B157 (1985) 295; D. Boulatov, V. Kazakov, I. Kostov and A. A. Migdal, Nucl. Phys. B275 (1986) 641; J. Ambjorn, B. Durhuus, J. Fröhlich and P. Orland, Nucl. Phys. B270 (1986) 457.

[6] E. Brézin and V. Kazakov, Phys. Lett. B236 (1990) 144.

[7] M. Douglas and S. Shenker, Nucl. Phys. B325 (1990) 635.

[8] D. J. Gross and A. A. Migdal, Phys. Rev. Lett. 64 (1990) 127; Nucl. Phys. B340 (1990) 333.
[9] H. Kawai and M. Ninomiya, Nucl. Phys. B336 (1990) 115.

[10] I. Jack and D. R. T. Jones, Nucl. Phys B358 (1991) 695.

[11] R. Jackiw, in Quantum Theory of Gravity, ed. S. Christensen (Hilger, 1984);
C. Teitelboim, in Quantum Theory of Gravity, ed. S. Christensen (Hilger, 1984); Phys. Lett. B126 (1983) 41.

[12] A. H. Chamseddine, Phys. Lett. B256 (1991) 379.

[13] F. D. Mazzitelli and N. Mohammedi, Phys. Lett. B268 (1991) 12.

[14] M. Henneaux, Phys. Rev. Lett. 54 (1985) 959.

[15] J.D. Brown, M. Henneaux and C. Teitelboim, Phys. Rev. D33 (1986) 319.

[16] T. Fukuyama and K. Kamimura, Phys. Lett. B160 (1985) 259.

[17] A. H. Chamseddine and D. Wyler, Phys. Lett. B228 (1989); Nucl. Phys. B340 (1990) 595;
K. Isler and C. A. Trugenberger, Phys. Rev. Lett. 63 (1989) 834.

[18] N. Mohammedi, Mod. Phys. Lett. A5 (1990) 1251.

[19] A. A. Tseytlin, "On the tachyonic terms in the string effective action”,
    preprint JHU-TIPAC-91004.

[20] A. H. Chamseddine, "A study of non critical strings in arbitrary dimensions”,
    preprint ZH-TH-13/1991

[21] S. Ichinose, Phys. Lett. B251 (1990) 49.

[22] T. Banks and O’Laughlin, Nucl. Phys. B362 (1991), 649;
    C. Callan, S. Giddings, J. Harvey and A. Strominger, Phys. Rev. D45,
R1005 (1992)
T. Banks, A. Dabholkar, M. Douglas and M. O’Loughlin, Phys. Rev D45, 3607 (1992);
S. Hawking, ”Evaporation of two dimensional black holes”, preprint CALT 68, 1774 (February 1992);
J.G. Russo, L. Susskind and L. Thorlacius, ”Black hole evaporation in 1+1 dimensions”, preprint SU-ITP-92-4 (January 1992);
L. Susskind and L. Thorlacius, ”Hawking radiation and back-reaction”, preprint SU-ITP-92-12 (March 1992).
[23] B. S. DeWitt, Dynamical Theory of Groups and Fields, (Gordon and Breach, 1965).
[24] B.S. DeWitt, Phys. Rev. 162 (1967) 1195.
[25] G. ’t Hooft and M. Veltman, Ann. Inst. H. Poincaré 20,1 (1974) 69.
[26] G. ’t Hooft, in Recent Developments in Gravitation, Cargese 1978, eds. M. Levy and S. Deser (Plenum Press, New York and London 1979).
[27] J. G. Russo and A.A. Tseytlin, ”Scalar-tensor quantum gravity in two dimensions”, preprint SU-ITP-92-2, DAMTP-1-1992.
[28] S. D. Odintsov and I. L. Shapiro, Phys. Lett. B263 (1991) 183;
preprint FTUAM 91-33 (October 1991).
[29] S. Ichinose, preprint YITP/K-876 (August 1990).
[30] R. Kantowski and C. Marzban, in preparation
[31] A. O. Barvinsky and G. A. Vilkovisky, Phys. Rep. 119 (1985) 1.

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