Application of the internal substructure method for seismic wave input in structural dynamic analysis considering SSI effect

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Abstract. A new internal substructure method for seismic wave input in soil-structure systems was proposed very recently. This method simplifies the calculation of equivalent input seismic loads, and avoids the participation of artificial boundaries in the process of seismic wave input. In this study, we introduce the application of the internal substructure method in the structural seismic analysis considering soil-structure interaction (SSI). A typical frame structure is taken as the example, and the effect of SSI on the structural dynamic response is studied.

1. Introduction
For large-scale structures such as high-rise buildings, nuclear power plants and dams, the interaction between soil and structure has a significant impact on the dynamic response of the structure itself. Meanwhile, the presence of the upper structures also changes the propagation of seismic waves in the unbounded foundation. As a result, the importance of dynamic soil-structure interaction (SSI) comes out prominently, and such research issues have drawn the researchers’ attention for decades [1-5].

In the earthquake engineering, we usually designate the local site containing the structures and the surrounding soil as the region of interest. In order to simulate the wave radiation effect of infinite foundation, an effective approach is to set artificial boundaries on the truncation boundaries. The commonly used artificial boundaries can be divided into the following types: the boundary element method (BEM) [6,7], the scaled boundary finite element method (SBFEM) [8], the perfectly matched layers (PML) [9,10], the transmitting boundaries [11,12], the viscous boundaries [13] and the viscoelastic boundaries [14,15]. The latter two are decoupled in space and can be simulated by basis mechanical components. Therefore they are also referred to as the stress-type artificial boundaries.

One of the key issues when using the artificial boundaries to simulate the external excitation problems, such as the far-field explosions and earthquakes, is how to input the waves into the near-field model without affecting the absorption of scattered waves by the artificial boundaries. To settle this issue, the current schemes generally convert the incident waves into the equivalent input loads on the truncation boundaries. The representative approaches include the domain reduction method (DRM) [16-18], the wave method [19-21] and the boundary substructure method (BSM) [22,23]. Both DRM and the wave method involve the calculation of equivalent loads and the determination of load directions, separately on each boundary node, which lead to complicated algorithm and heavy workload. While BSM is newly developed based on the theory of wave method, and it effectively simplifies the implementation procedures and reduces the calculation effort. However, similar to the wave method, BSM conducts the seismic wave input on the truncation boundaries, thus it also requires
the applied artificial boundaries to be the stress-type boundaries. In order to overcome the abovementioned problems, we proposed the internal substructure method (ISM) very recently [24]. In this method, an internal substructure model is intercepted directly from the original soil-structure model. And its size is much smaller, thus the effort for data management on the input boundaries gets effectively reduced. In addition, this method avoids the participation of the artificial boundaries in the process of inputting seismic waves. Therefore the restrictions of the traditional methods on the type of artificial boundaries can be completely released.

In this article, we study the applicability and accuracy of ISM in the dynamic SSI analysis. With the help of this method, the dynamic response of a typical soil-structure system under the obliquely incident plane waves is analyzed.

2. Methods

2.1. Internal substructure method
According to ISM, the incident seismic waves can be converted into the equivalent input loads around the region of interest, and the substructure for calculating the equivalent loads is named as the internal substructure. Meanwhile, these loads are only related to the free field model, the incident wave and the incident angle, and unaffected by the presence of the structures. Therefore, by imposing the free wave motions on the nodes of the internal substructure model and conducting a dynamic analysis, the equivalent input loads can be obtained, which are exactly the reaction forces on the internal substructure. In the following, we present the implementation procedures of ISM for seismic wave input in soil-structure system:

(1) Establish the soil-structure finite element model, and apply the appropriate artificial boundaries on the truncation boundaries, as shown in figure 1(a). The size and location of the internal substructure can be determined according to the spatial range of the soil in which we wish the seismic waves to be input, as shown by the region within the blue node box in figure 1(a). Then, the nodes in the SSI model can be classified into five types according to their positions, namely, the internal nodes (I), external nodes (E) and three layers of internal substructure nodes (A, B and C from the outside to the inside).

(2) By deleting the nodes E and I, as well as the finite elements connected to these nodes, the internal substructure model can be established, as shown in figure 2(b). According to the incident seismic waves, the free wave motions \( \mathbf{u}^0 \) can be obtained, through the free field analysis algorithms [25,26]. Then, by fixing the nodes A and applying the corresponding free wave motions \( \mathbf{u}_B^0 \) and \( \mathbf{u}_C^0 \) on nodes B and C respectively, as shown in figure 2(b), we conduct a dynamic analysis on the internal substructure. The reaction forces on nodes A and B can be gained, which are exactly the equivalent input seismic loads \( \mathbf{F}_A \) and \( \mathbf{F}_B \).

(3) By applying the equivalent input seismic loads \( \mathbf{F}_A \) and \( \mathbf{F}_B \) on the corresponding nodes of the original soil-structure model, as shown in figure 2(c), and conducting a dynamic analysis on the original soil-structure model, then the seismic response of the soil-structure system can be obtained.

In extreme cases, the internal substructures can be adjacently intercepted beneath the structures. Thereby the size of substructure model gets much smaller compared to that of BSM. This size reduction feature effectively reduces the computational burden, especially the tremendous data management on the wave input locations.
2.2. Consistent viscous-spring artificial boundary elements
In this study, the consistent viscous-spring artificial boundary element [27] is adopted to simulate the wave radiation effect of the semi-infinite foundation. The core idea of this method is using a layer of equivalent elements, which work equally as the viscoelastic artificial boundaries, to absorb the scattered waves. For two-dimensional application, the shear modulus $E$, Young's modulus $E$ and elemental material damping coefficient $\eta$ of the equivalent elements are given by:

\[
\begin{align*}
\tilde{G} &= \alpha_t h \frac{G}{R}, \\
\bar{E} &= \alpha_s h \frac{E}{\rho} \frac{(1+\tilde{\gamma})(1-2\tilde{\gamma})}{(1-\tilde{\gamma})}, \\
\tilde{\eta} &= \frac{\rho R}{2G} \left( \frac{c_s}{\alpha_t} + \frac{c_p}{\alpha_\nu} \right)
\end{align*}
\]

where $h$ is the thickness of the equivalent element layer, and $R$ is the distance from the wave source to the artificial boundary node. For the lateral and bottom equivalent elements in our model, the values of $R$ are the distances from the midpoint of the structure bottom to the lateral and bottom boundaries, respectively. $\alpha_\nu$ and $\alpha_t$ are the artificial boundary coefficients, whose recommended values are 1 and 0.5, respectively. $c_s$ and $c_p$ are the velocities of the shear and compression waves, respectively. $G$ and $\rho$ are the shear modulus and mass density of the solid medium, respectively. $\tilde{\eta}$ is the equivalent Poisson's ratio, defined by:

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**Figure 1.** Implementation procedures of ISM: (a) SSI system model, (b) internal substructure model and calculation of equivalent input loads, (c) seismic wave input in SSI system.
\[
\bar{y} = \begin{cases} 
\frac{\alpha - 2}{2(\alpha - 1)} & \alpha \geq 2 \\
0 & \alpha < 2 
\end{cases}
\]  

(2)

where \( \alpha = \alpha_\gamma / \alpha_t \).

### 3. Seismic analysis of frame structure considering SSI effect

In this section, we apply ISM for the seismic analysis of a typical frame structure considering SSI effect. The computational model contains a two-span, five-floor concrete frame structure and a near-field soil domain, as shown in figure 2. The floor height is 4 m and the span length is 8 m. The dimensions of the beam sections and column sections are all 0.5 m \times 1.0 m. The foundation of the structure is simplified as a foundation beam, and its section dimension is 0.8 m \times 1.0 m. The dimension of the near-field soil domain is 200 m \times 100 m. \( \theta \) denotes the incident angle of the seismic wave. The material parameters of concrete and soil are given in table 1. The consistent viscous-spring artificial boundary elements \[27\] are set on the truncation boundaries. The dashed boxes in figure 2 denote the three internal substructures, which are denoted as ISM 1, ISM 2 and ISM 3, respectively. The dimensions of these three internal substructures are 18 m \times 2 m, 50 m \times 25 m and 80 m \times 40 m. Meanwhile, we also apply BSM as a comparison. The observation point A is the top point of the center column, and point B is 10 m beneath the free surface. A plane SV impulse wave with the pulse duration of 0.2 s is selected as the incident wave, and its displacement time history is plotted in figure 3. The incident angle is \( \theta = 20^\circ \).

![Figure 2. Computational model.](image)

![Figure 3. Displacement time history of the incident impulse wave.](image)
Table 1. Material parameters of concrete and soil.

| Material | Parameter             | Magnitude |
|----------|-----------------------|-----------|
| Concrete | Mass density $\rho$ (kg/m$^3$) | 2500      |
|          | Yang’s modulus $E$ (GPa) | 35        |
|          | Poisson’s ratio $\gamma$ | 0.2       |
| Soil     | Mass density $\rho$ (kg/m$^3$) | 2000      |
|          | Velocity of shear wave $v_s$ (m/s) | 200       |
|          | Poisson’s ratio $\gamma$ | 0.25      |

Figure 4(a) shows the structural displacements in $x$ and $y$ directions calculated by ISM, which are well consistent with the results of BSM. In addition, by establishing a structure model of the same geometric dimensions and material properties, but without the soil foundation, and then conducting a dynamic analysis with the free field motions on the free surface as the excitations, the dynamic response of the structure can be obtained, as shown in figure 4(a). It can be seen that the absence of soil foundation significantly magnifies the seismic response of the structure. The relative error of the peak displacement is approximately 26%. Besides, when the SSI effect is not considered in the model, the seismic energy in the structure cannot be transmitted into the unbounded foundation. Therefore the structural vibration hardly decays with time. Figure 4(b) shows the displacement time history on point B. The numerical results calculated by ISM 2, ISM 3 and BSM agree well with each other. However, according to the theory of ISM, in the region outside the internal substructure, there only exists the scattered wave field, as illustrated by the result of ISM 1. Furthermore, the total wave field can be divided into the free wave field and the scattered wave field. Therefore the summation of ISM 1 result and the free field motion is consistent with the total wave motions calculated by other models.

Figure 5 shows the vertical distributions of the peak relative displacement along the center and side columns. The results of ISM and BSM are all consistent with each other, and significantly smaller than those of the model neglecting SSI effect. In addition, in the $x$-direction, the dynamic response of the center column is very close to that of the side column. As for the $y$-direction, however, the results of the side column get much larger than those of the center one, especially when the height increases.

Figure 6 shows the snapshots of displacement wave field. The dashed boxes denote the internal substructures, of which the red one is adopted to input the seismic waves. It can be seen that the wave field distributions within the internal substructures calculated by ISM are consistent with those of...
BSM. While when the outgoing waves reach the internal substructure, the free field motions are neutralized by the equivalent loads, and the scattered waves pass through and propagate outward until they are absorbed by the artificial boundaries.

Figure 5. Vertical distributions of the peak relative displacement.
4. Summary
ISM is a newly proposed seismic wave input method for SSI problems. This method has the superiority of model size reduction, and it effectively releases the restrictions of traditional methods on the artificial boundaries in seismic wave input.

In this article, we apply ISM in the dynamic analysis of a typical frame structure considering SSI effect, under the obliquely incident plane SV wave. The result comparison with BSM validates the applicability of ISM in such research issues. In addition, it is demonstrated that SSI effect has significant influence on the structures on the ground. For the frame structure established in this study, the maximum relative error of the peak displacement can exceed 25%. Therefore it is necessary to consider the SSI effect when conducting structural seismic analysis, and ISM provides an efficient numerical approach for seismic wave input in such soil-structure systems.

Acknowledgement
This study is supported by the National Natural Science Foundation of China (Grant No.51878384, U1839201) and National Key Research and Development Program of China (Grant No.2018YFC1504305). Financial support from these organizations is gratefully acknowledged.
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