A Minimalist Model of Characteristic Earthquakes

Miguel Vázquez-Prada, Alvaro González, Javier B. Gómez and Amalio F. Pacheco
Faculty of Sciences, University of Zaragoza, 50009 Zaragoza, Spain.

In a spirit akin to the sandpile model of self-organized criticality, we present a simple statistical model of the cellular-automaton type which produces an avalanche spectrum similar to the characteristic-earthquake behavior of some seismic faults. This model, that has no parameter, is amenable to an algebraic description as a Markov Chain. This possibility illuminates some important results, obtained by Monte Carlo simulations, such as the earthquake size-frequency relation and the recurrence time of the characteristic earthquake.

I. INTRODUCTION

If there is a well-established fact about regional seismicity this is the relationship between the magnitude of an earthquake and its frequency, known as the Gutenberg-Richter (GR) law. This law is of the power-law type when magnitudes are expressed in terms of rupture area

\[ N \propto S^{-b}, \]  

(1.1)

where \( N \) is the number of observed earthquakes with rupture area greater than \( S \), and \( b \) is the so-called \( b \)-value, which is a “universal constant” in the range 0.8-1.2. The GR law implies that earthquakes are, on a regional or world-wide scale, a self-similar phenomenon lacking a characteristic scale (but see [3]).

It is important to notice, however, that the GR law is a property of regional seismicity, appearing when we average seismicity over big enough areas and long enough time intervals. Recently, a wealth of new data has been collected to extract statistics on individual systems of earthquake faults. Interestingly, it has been found that the distribution of earthquake magnitudes may vary substantially from one fault to another and that, in general, this type of size-frequency (SF) relationship is different from the GR law. Many single faults or fault zones display power-law distributions only for small events, which occur in the intervals between roughly quasi-periodic earthquakes of much larger size which rupture the entire fault. These earthquakes are termed “characteristic”, and the resulting SF relationship, characteristic earthquake (CE) distribution.

There is much debate about the origin of the CE distribution. Because of the short period of instrumental earthquake records and the scarcity of paleoseismic studies, the statistics of naturally occurring earthquakes in single faults are poor. This fact justifies the development of “synthetic seismicity” models, in which long catalogs of events are generated by computer models of seismogenesis. Such models can be tuned by requiring that they reproduce what is known of the statistics of past seismicity to a reasonable degree, and then use them to forecast statistical inferences about the behavior of seismicity using much longer and homogeneous catalogs of synthetic events.

Many different seismicity models have been presented in the past twenty years or so. Robinson and Benites classify these modeling approaches into five groups: (1) cellular automata models, (2) spring-block models, (3) models of single faults in which slip is discretized into patches and obey simplified friction laws, (4) continuum models that utilize realistic constitutive friction laws, and (5) actual physical models.

Cellular automata models have only recently appeared in seismological literature, hand in hand with the concept of self-organized criticality. These models are usually nondeterministic and represent faults as one- or two-dimensional features. They neglect the details of both elasticity and fault friction, substituting them by simple cellular automata rules. Despite their simplicity, they are able to reproduce various types of SF statistics, including GR and CE distributions.

The key ingredients of any of these models are: (1) the dimensionality of the fault (1D or 2D), (2) the number of faults included in the model (one, a few, or many faults), (3) the employed stress transfer mechanism (nearest-neighbors, long-range elasticity, mean-field), (4) the degree of incorporation of inertial effects (quasi-static, quasi-dynamic, or fully dynamic), (5) the assumed constitutive stress-slip law (experimental, static-dynamic, velocity-weakening, etc.), and (6) the degree of stress conservation (conservative versus dissipative models).

Various discrete models of seismicity are able to show a transition from GR to CE statistics. Among them, we want to cite here the models of Ceva and Perazzo, Carlson et al., Lomnitz-Adler, Rundle and Klein.
Consider a one dimensional vertical array of length \( N \). The ordered positions, or levels, in the array will be labeled by an integer index \( i \) varying from 1 to \( N \). This system performs two functions, it is loaded by receiving individual particles in the various positions of the array, and unloaded by emitting groups of particles through the first level, \( i = 1 \), which are called avalanches or earthquakes.

These two functions proceed with the following four rules:

(i) The incoming particles arrive at the system at a constant time rate. Thus, the time interval between each two successive particles will be considered the basic time unit in the evolution of the system.

(ii) All the positions in the array, from \( i = 1 \) to \( i = N \), have the same probability of receiving a new particle. When a position receives a particle we say that it is occupied.

(iii) If a new particle comes to a level which is already occupied, this particle disappears from the system, or in other words, this particle assignment is wasted. Thus, a given position \( i \) can only be either non-occupied when no particle has come to it, or occupied when one or more particles have come to it.

(iv) The level \( i = 1 \) is special. When a particle goes to this first position an avalanche occurs. Then, if all the successive levels from \( i = 1 \) up to \( i = k \) are occupied, and the position \( k + 1 \) is unoccupied, the effect of the avalanche is to unload all the levels from \( i = 1 \) up to \( i = k \). Hence, the size of this avalanche is \( k \), and the remaining levels \( i > k \) remain unaltered in their occupancy.

Thus, from what has been mentioned above, this model has no parameter; the size \( N \) is the unique specification to be made, and the spatial correlation is induced by the \( i^{th} \) rule. Now, the state of the system is given by stating which of the \( (i > 1) \) \( N - 1 \) levels are occupied. Each of these states corresponds to a stable configuration, and therefore the total number of possible configurations is \( 2^{(N-1)} \). We use the term “total occupancy” for the configuration in which all but the first level are occupied.

After the occurrence of an avalanche the system is left in a stable configuration; the following particle additions go progressively loading the system, and when a particle is again assigned to the first level a new avalanche is triggered. Each avalanche empties the lower levels of the system as explained in rule (iv), and the system is left in another stable configuration. The size of the avalanches can range from 1 up to \( N \) and the avalanche of maximum size, \( k = N \), will be called the characteristic one.

From these evolution rules we deduce that after a time unit, i.e., after a new incoming particle assignment, we will have an avalanche if the new particle goes to \( i = 1 \), and this occurs with a probability \( 1/N \). Conversely, with a probability of \( (N-1)/N \) there will be no avalanche. In this case the system will advance one unit in its level of occupation when the new particle is assigned to a non-occupied level, and it will remain at the same configuration if the assigned level was already occupied.

As this model is 1-dimensional, extensive Monte Carlo simulations can be performed to accurately explore its properties. In this paper we focus on two important properties, the avalanche size-frequency relation and the statistics of the time of return of the maximum-size avalanche.
The results for the avalanche size-frequency relation, \( p_k \), are drawn in Fig. 1 and written in Table II. In Fig. 1 we have superposed \( p_k \) for \( N = 10, N = 100 \) and \( N = 1000 \). This has been partly included in Table II as well. In Fig. 1 there are three notable properties to be commented on. First and most important, we see that the characteristic avalanche, \( k = N \), has a much higher probability of occurrence than the avalanches of big size but with \( k < N \). In fact, for \( N = 10, 100 \) and \( 1000 \), the probability of their respective characteristic avalanches does not differ much, and is near 10%. We can express this fact by saying that in this model, \textit{grosso modo}, one would likely only observe very small avalanches and the characteristic one. Secondly, forgetting for a moment the case \( k = N \), we observe an approximate power law behavior, \textit{a la} Gutenberg-Richter, for the rest of avalanches. The exponent \( b \) of this differential distribution is roughly 1.6. And thirdly, we observe the perfect coincidence of these curves of probability for systems of different size \( N \). This is also appreciated in the numbers collected in Table II. This, in a sense, unexpected size-invariance will be discussed in detail in the next Section.

In Fig. 2 we represent the probability curve for \textit{“the time of recurrence of the characteristic avalanche”}. This curve, obtained by Monte Carlo simulations, corresponds to a system of \( N = 10 \). In the abscessas axis, time (denoted by \( n \)) starts at 0 just after the occurrence of a \( k = N \) avalanche. It is clear in this model that only after a minimum time lapse of size \( N \) the probability of occurrence of a new \( k = N \) avalanche can be non null. We observe in Fig. 2 that after this minimum time lapse, \( P(n) \) grows to a maximum and then decays. For the size \( N = 10 \) analyzed in this figure, the maximum of probability corresponds to a time interval \( n = 34 \).

### III. The Model as a Markov Chain

It is easy to become convinced that, for a given \( N \), the \( 2^{(N-1)} \) stable configurations of our model can be considered as the states of a finite, irreducible and aperiodic Markov chain with a unique stationary distribution [19]. These configurations are classified in groups according to its \textit{occupation number} (number of occupied levels); the number of configurations with \( j \) occupied levels is \( C \left( \frac{N-1}{j} \right) \). One step in the chain corresponds to the result of adding a new particle to the system. Up to approximately \( N = 10 \), the transition matrix, \( M \), can be easily obtained using Mathematica as well as the corresponding stationary probabilities for each configuration, which correspond to the components of the eigenvector of \( M \) with eigenvalue, \( \lambda \), equal to unity.

For small \( N \), \( M \) and its eigenvectors are obtained by inspection. Let us then start reproducing the first numbers quoted in Table II for the probabilities of occurrence of avalanches in systems of small size. With this aim, let us consider Fig. 3. There, in (A), (B) and (C) there appear all the stable configurations, ordered in an increasing state of occupation, for \( N = 2, N = 3 \), and \( N = 4 \), respectively. For the moment, the black level in the top position of the configurations has no meaning.

For \( N = 2 \), using the same order and notation for the configurations as in Fig. 3A, the transition probabilities are \( M_{a,a} = 1/2, M_{a,b} = 1/2 \), \( M_{b,a} = 1/2 \), and \( M_{b,b} = 1/2 \). Thus

\[
M = \begin{pmatrix}
1/2 & 1/2 \\
1/2 & 1/2
\end{pmatrix}, \quad (N = 2).
\]

This \( M \) is a doubly stochastic matrix and hence the two stationary probabilities are equal.

\[
p_a = 1/2, \quad p_b = 1/2, \quad (N = 2).
\]

For \( N = 3 \), the non-null transition probabilities are: \( M_{a,a} = 1/3, M_{a,b} = 1/3, M_{a,c} = 1/3; M_{b,b} = 2/3, M_{b,d} = 1/3; M_{c,a} = 1/3, M_{c,c} = 1/3, M_{c,d} = 1/3; M_{d,a} = 1/3, \) and \( M_{d,d} = 2/3 \). Thus

\[
M = \begin{pmatrix}
1/3 & 1/3 & 1/3 & 0 \\
0 & 2/3 & 0 & 1/3 \\
1/3 & 0 & 1/3 & 1/3 \\
1/3 & 0 & 0 & 2/3
\end{pmatrix}, \quad (N = 3).
\]

And the components of the eigenvector corresponding to \( \lambda = 1 \) are:

\[
p_a = 1/4, \quad p_b = 1/8, \quad p_c = 1/4, \quad p_d = 3/8, \quad (N = 3).
\]

Finally, for \( N = 4 \) the non-null transition probabilities are \( M_{a,a} = 1/4, M_{a,b} = 1/4, M_{a,c} = 1/4, M_{a,d} = 1/4; M_{b,b} = 2/4, M_{b,e} = 1/4, M_{b,f} = 1/4; M_{c,c} = 2/4, M_{c,e} = 1/4, M_{c,g} = 1/4, M_{d,a} = 1/4, M_{d,d} = 1/4, M_{d,f} = 1/4, M_{d,g} =
\]
1/4; \( M_{a,c} = 3/4, M_{b,h} = 1/4; M_{f,b} = 1/4, M_{f,f} = 2/4, M_{f,h} = 1/4; M_{g,a} = 1/4, M_{g,g} = 2/4, M_{g,h} = 1/4; M_{h,a} = 1/4, M_{h,h} = 3/4 \). Thus

\[
M = \begin{pmatrix}
1/4 & 1/4 & 1/4 & 1/4 & 0 & 0 & 0 & 0 \\
0 & 2/4 & 0 & 0 & 1/4 & 1/4 & 0 & 0 \\
0 & 0 & 2/4 & 0 & 1/4 & 0 & 1/4 & 0 \\
1/4 & 0 & 0 & 1/4 & 0 & 1/4 & 1/4 & 0 \\
0 & 0 & 0 & 0 & 3/4 & 0 & 0 & 1/4 \\
0 & 1/4 & 0 & 0 & 0 & 2/4 & 0 & 1/4 \\
1/4 & 0 & 0 & 0 & 0 & 0 & 2/4 & 1/4 \\
1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 3/4
\end{pmatrix}, \quad (N = 4).
\]

(3.5)

And, after its diagonalization, one finds

\[
p_a = 9/64, \quad p_b = 7/64, \quad p_c = 9/128, \quad p_d = 3/64, \quad p_e = 23/128, \quad p_f = 5/64, \quad p_g = 15/256, \quad p_h = 81/256, \quad (N = 4).
\]

(3.6)

From these numbers, one deduces that in a system with \( N = 2 \) levels one should expect avalanches of size \( k = 1 \) with a probability \( p_1 = p_0 = 1/2 \), and of size \( k = 2 \) with \( p_2 = p_b = 1/2 \). In \( N = 3 \), \( p_1 = p_0 + p_b = 1/2, p_2 = p_c = 1/8 \), and \( p_3 = p_d = 3/8 \). And in \( N = 4 \), \( p_1 = p_a + p_b + p_c + p_e = 1/2, p_2 = p_d + p_f = 1/8, p_3 = p_g = 15/256 = 0.05859.., and \( p_4 = p_h = 81/256 \).

Thus, we have observed that in systems with \( N = 2, N = 3 \) and \( N = 4 \) levels, the value of \( p_1 \) is a constant equal to 1/2 and this result coincides with Table I. We have also observed that in \( N = 3 \) and \( N = 4 \) the value of \( p_2 \) is a constant equal to 1/8, and in \( N = 4 \) we have deduced that \( p_3 = 0.05859. \) All this agrees with Table I.

A conclusive argument proving that in this model the value of \( p_k \) is a constant independent on the size \( N \) is obtained by re-analyzing Fig. 3A from a wider perspective. Now we consider that in Fig. 3A, the configuration labeled by \( a \) represents, in a system of size \( N \), all the configurations in which the level \( i = 2 \) is not-occupied; the rest of the levels \( i > 2 \) are in any possible state of occupation (this is represented by the black level at the top of the configuration). And in Fig. 3A, configuration \( b \), we consider a system of size \( N \) which has its second level occupied. The probabilities of these two cases must add to unity. Then defining a Markov chain for these two excluding states, and using the same notation as before, we find \( M_{a,a} = (N - 1)/N, M_{a,b} = 1/N, M_{b,a} = 1/N, \) and \( M_{b,b} = (N - 1)/N \). The diagonalization of this matrix leads for \( p_a \) and \( p_b \) to the same results written in Eq. (3.3), \( p_a = p_b = 1/2 \); however, its interpretation now is different. Here, \( p_a = 1/2 \) implies that for any value of \( N \), the probability of having an avalanche \( k = 1 \) is 1/2; and \( p_b = 1/2 \) simply means that the probability of having avalanches \( k > 1 \) is 1/2. Using the same line of reasoning, and referring to Fig. 3B, configuration \( a \) represents all the configurations, in a system of size \( N \), where levels \( i = 2 \) and \( i = 3 \) are not occupied. And configuration \( b \) represents all the configurations where the level \( i = 2 \) is free and the level \( i = 3 \) is occupied, etc. Then, the non null transition probabilities are \( M_{a,a} = (N - 2)/N, M_{a,b} = 1/N, M_{b,a} = 1/N, \) and \( M_{b,b} = (N - 1)/N \). The diagonalization of this matrix provides the same stationary probabilities quoted in Eq. (3.3). Here \( p_c = 1/8 \) means that for an arbitrary \( N \), the probability of occurrence of avalanches of size 2, \( p_2 = 1/8 \). The fact that \( p_a + p_b = 1/2 \) confirms that for any \( N \), \( p_1 = 1/2 \).

Extending this line of reasoning to the 8 configurations drawn in Fig. 3C, one concludes that, for any \( N, p_3 = p_g = 15/256 = .058593.. \) and one verifies the previous conclusions \( p_1 = 1/2 \) and \( p_2 = 1/8 \).

Therefore, in this model, if for the system of size \( N \) one knows all the \( p_k \) from \( k = 1 \) to \( k = N \), then for the system of size \( N + 1 \) the \( p_k \) are identical , with the exception of the last two, and these fulfill

\[
p_N(N) = p_{N+1}(N + 1) + p_N(N + 1)
\]

(3.7)

the recursive way in which \( p_N(N) \) divides into \( p_{N+1}(N + 1) \) and \( p_N(N + 1) \) is, however, non trivial.

Let us analyze now, from the Markov-chains point of view, the results for the time of return of the characteristic earthquake shown in Fig. 3. After a \( k = N \) avalanche, the system is left in the configuration of no occupancy (for the present discussion we will refer to this configuration as \( a_1 \)). A new characteristic avalanche will occur when, starting from the configuration \( a_1 \), the system reaches the configuration of total occupancy (which will henceforth be denoted by \( a_N \)), and then the next particle is assigned to the \( i = 1 \) level. The number of time steps elapsed between \( a_1 \) and \( a_N \), plus 1, will be denoted by \( n \). And our purpose is to compute the probability of occurrence of a \( k = N \) avalanche as a function of \( n \). It must be understood that between \( a_1 \) and the occurrence of the next \( k = N \) avalanche on time \( n \), the system may have visited \( a_N \) an arbitrary number of times but without triggering any \( k = N \) avalanche. In other words, in those visits there has been no transition from \( a_N \) to \( a_1 \).
In Markov chains, the transition matrix $M$ gives the probability of going from one configuration to another in one step, and the $m$ step transition probability is the $m$-th power of $M$. Thus a simple way to compute $P(n)$ is the following:

1. Take $M$, point to the element in the last row and the first column, and substitute it by 0. We will denote the new matrix $M'$.
2. Compute $T_n = M'^{(n-1)}$.
3. Take the element of the first row and the last column of $T_n$, and multiply it by $1/N$. This result is $P(n)$.

This is clear, because $M'$ does not permit transitions from $a_N$ to $a_1$. Thus, in $T_n$ we have all the probabilities of transitions, in $n - 1$ steps, between all the configurations, with the restriction that from $a_N$ to $a_1$ this transit is forbidden. Hence, in $T_n$ the matrix element of the first row and the last column corresponds to the transition from $a_1$ to $a_N$, in $n - 1$ steps and with the mode $a_N \rightarrow a_1$ locked. Finally, as the $1/N$ factor is the probability of the transition $a_N \rightarrow a_1$, we actually have built $P(n)$. In Fig. 2 for $N = 10$, we see the perfect match between the Monte Carlo simulations and the results coming the theory of Markov, calculated using Mathematica.

In order to get a quantitative insight on $P(n)$, let us apply this method to $N = 2$. In this case, $M$ is given by Eq. (3.1). Hence,

$$M' = \begin{pmatrix} 1/2 & 1/2 \\ 0 & 1/2 \end{pmatrix}, \quad M'^{(n-1)} = \left( \frac{1}{2} \right)^{(n-1)} \begin{pmatrix} 1 & n - 1 \\ 0 & 1 \end{pmatrix},$$

and, therefore,

$$P(n) = (n-1) \cdot (1/2)^{(n-1)} \cdot (1/2) = \frac{n-1}{2^n}, \quad (N = 2).$$

Thus, we observe that the asymptotic fall-off behavior of $P(n)$ is of the type

$$P(t) \propto t \exp(-t), \quad (N = 2).$$

It is important to recall that in aperiodic, irreducible and finite Markov Chains such as that of our model, the mean waiting time for a configuration is the inverse of the stationary probability of that configuration. Then, the mean time between characteristic avalanches in this model is

$$\langle n \rangle = \frac{1}{P_N} = \frac{N}{a_N}. \quad (3.11)$$

As an example, for $N = 4$, $\langle n \rangle = 4 \cdot (256/81) = 12.4$ time units.

IV. CONCLUSIONS

We have presented a one-dimensional discrete model of seismicity that displays a size-frequency spectrum similar to that expected from a characteristic-earthquake behavior. Although this stochastic model obviously lacks the explicit physics of other detailed and complex dynamical models of earthquakes, its basic hypotheses and implications are clear, phenomenologically reasonable and coherent. The size invariance of this model, also surprisingly manifests itself in predicting an identical probability of occurrence for earthquakes of the same magnitude independently of the size $N$ of the system. In this universal rule the characteristic earthquake is excluded. This model has the additional bonus that several important predictions can be algebraically derived by using the theoretical framework of the Markov chains. Specifically, the statistics of the time of return of the characteristic earthquake are neatly predicted by this formalism.

Dahmen et al. [6] report a model able to transit from the GR to the CE behavior and, what is more interesting, these authors define a “configurational entropy” as an appropriate concept that reflects in which of these two extreme behaviors the system is actually operating. In qualitative terms, the GR behavior corresponds to a high entropy mode of operation while the CE behavior corresponds to a low entropy mode. Therefore, we would like to make an assessment of our model from this configurational entropy point of view and check the concordance, or not, with the conclusions of Ref. [6].
In our model, the configurations are classified in groups according to the number of levels, \( j \), that are occupied. The statistical weight of each \( j \) is \( C \left( \frac{N - 1}{j} \right) \), which has its maximum values for \( j \) around \( N/2 \). Conversely, the statistical weight is minimum on the extrema: for \( j \) around 1 and for \( j \) near \( N - 1 \). Then, we need to find out the values of the stationary probabilities of each occupation number \( j \). This is shown in Fig. 4 for a system of size \( N = 100 \). There we observe how in our model the system resides most of the time in the configurations of maximum occupancy, that is, where the configurational entropy is a minimum, agreeing with the interpretation of Ref. [6].

As a final minor remark, note that in Fig. 4 \( p(j=N) \) and \( p(j=N-1) \) are identical. This is a property that holds in this model for any value of \( N \) and which can be easily proved. For brevity reasons, we omit this proof.

[1] Gutenber, B. and Richter, C.F., Ann. Geofis., 9, 1 (1956).
[2] Kanamori, H. and Anderson, D.L., Bull. Seismol. Soc. Am., 65, 1073 (1975).
[3] Knopoff, L., Proc. Natl. Acad. Sci. USA, 97, 11880 (2000).
[4] Wesnosuky, S.G., Bull. Seismol. Soc. Am., 84, 1940 (1994); Sieh, K., Proc. Natl. Acad. Sci. USA, 93, 3764 (1996).
[5] Schwartz, D.P. and Coppersmith, K.J., J. Geophys. Res., 89, 5681 (1984).
[6] Dahmen, K., Ertas, D., and Ben-Zion, Y., Phys. Rev. E, 58, 1494, (1994).
[7] Robinson, R. and Benites, R., J. Geophys. Res., 100, 18229 (1995).
[8] Wolfram, S., Cellular automata: collected papers, Addison-Wesley (1994).
[9] Bak, P. and Tang, C. J. Geophys. Res., 94, 15635 (1989); Ito, K. and Matsuzaki, M., J. Geophys. Res., 95, 6853 (1990); Chen, K., Bak, P. and Obukhov, S.P., Phys. Rev. A, 43, 625 (1991); Matsuzaki, M. and Takayasu, H. J. Geophys. Res., 96, 19925 (1991); Olami, Z, Feder, J.S. and Christensen, K., Phys. Rev. Lett., 68, 1244 (1992).
[10] Lomnitz-Adler, J., Knopoff, L., and Martínez-Mekler, G., Phys. Rev. A, 45, 2211 (1992); Barriere, B. and Turcotte, D.L., Phys. Rev. E, 49, 1151 (1994).
[11] Main, I., Rev. Geophys., 34, 433 (1996).
[12] Ceva, H. and Perazzo, R.P.J., Phys. Rev. E, 48, (1993).
[13] Carlson, J.M., Grannan, E.R., Singh, C., and Swindle, G.H., Phys. Rev. E, 48, (1993).
[14] Lomnitz-Adler, J., J. Geophys. Res., 98, 17745 (1993).
[15] Ben-Zion, Y. and Rice, J.R., J. Geophys. Res., 98, 14109 (1993); ibid, 100, 12959 (1995).
[16] Rundle, J.B. and Klein, W., J. Stat. Phys., 72, 405 (1993).
[17] Moreno, Y., Gómez, J.B., Pacheco, A.F., Physica A, 274, 400 (1999).
[18] Heinzel, S., Zöller, G., and Kurths, J., J. Geophys. Res., 104, 7243 (1999); Heinzel, S. and Zöller, G., Physica A, 294, 67 (2001).
[19] See, for example: R.Durrett, Essentials of Stochastic Processes. Chapter 1. Springer (1999)

**ACKNOWLEDGMENTS**

A.F.P. thanks Jesús Asín, Jorge Ojeda and Carmen Sangüesa for many fruitful discussions. This work was supported by the Spanish DGICYT (Project PB98-1594).
FIG. 1. Probability of occurrence of earthquakes of magnitude $k$. The results for $N = 10, 100, 1000$ are superposed.

FIG. 2. Probability of return of the characteristic earthquake as a function of time.
FIG. 3. Explicit configurations for: A) $N = 2$, B) $N = 3$ and C) $N = 4$.

FIG. 4. Stationary probabilities as a function of the number of levels occupied, $j$. 
| $k$ | $N = 2$ | $N = 3$ | $N = 4$ | $N = 10$ | $N = 100$ |
|----|--------|--------|--------|--------|--------|
| 1  | 0.5    | 0.5    | 0.5    | 0.500022 | 0.49992 |
| 2  | 0.5    | 0.125  | 0.125  | 0.124974 | 0.12513 |
| 3  | 0.375  | 0.0585938 | 0.058599 | 0.058599 | 0.0586 |
| 4  | 0.316406 | 0.316406 | 0.316406 | 0.316406 | 0.316406 |
| 5  | 0.023499 | 0.023499 | 0.023499 | 0.023499 | 0.023499 |
| 6  | 0.017126 | 0.017126 | 0.017126 | 0.017126 | 0.017126 |
| 7  | 0.013151 | 0.013151 | 0.013151 | 0.013151 | 0.013151 |
| 8  | 0.010506 | 0.010506 | 0.010506 | 0.010506 | 0.010506 |
| 9  | 0.008627 | 0.008627 | 0.008627 | 0.008627 | 0.008627 |
| 10 | 0.208636 | 0.208636 | 0.208636 | 0.208636 | 0.208636 |
| 99 |        |        |        | 2.28507 $\cdot 10^{-4}$ | 0.11132 |
| 100|        |        |        |        | 0.11132 |

TABLE I. Probability of occurrence of the earthquake of magnitude $k$ for a different system size, $N$. 