GEOMETRIC SHRINKAGE PRIORS FOR KÄHLERIAN SIGNAL FILTERS

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Abstract. We construct geometric shrinkage priors for Kählerian signal filters. Based on the characteristics of Kähler manifold, an algorithm for finding the superharmonic priors is introduced. The algorithm is efficient and robust to obtain the Komaki priors. Several ansätze for the priors are also suggested. In particular, the ansätze related to Kähler potential are geometrically intrinsic priors to the information manifold because the geometry is derived from the potential. The implication to the ARFIMA model is also provided.

1. Introduction

Signal processing is one of the most important applications in information geometry. In particular, an information geometric approach to various time series models has been also well-known [1, 2, 3, 4, 5, 6, 7, 8, 9]. The geometric description of the time series models is not confined to the pursuit of mathematical beauty. Komaki’s work [4] is on the line of producing practical tools to time series analysis. Using the Kullback–Leibler divergence as a risk function for estimation, he found that superharmonic shrinkage priors outperform the Jeffreys prior in the viewpoint of information theory. The better prediction in time series analysis is attainable by the Komaki priors.

However, a difficulty in Komaki’s idea is that it is hard to verify whether or not a prior function is superharmonic. In particular, when the dimension of the statistical manifold is large enough, it is technically tricky to test the superharmonicity of the prior functions because Laplace–Beltrami operators on general manifolds are non-trivial. Although some superharmonic priors for the autoregressive (AR) models are obtained not only in the two dimensional case [7, 3] but also in arbitrary dimensions [8], there is no clue about the Bayesian shrinkage priors of more complicated models such as the autoregressive moving average (ARMA) models or the fractionally integrated ARMA (ARFIMA) models or any arbitrary signal filters. Additionally, systematic and generic algorithms for finding the information shrinkage priors are not known yet.

Recently, the mathematical correspondence between the Kähler manifold and the information geometry of signal filters is explicitly derived [3] while the connection between the Kähler manifold and the information geometry has been implied [2]. Under conditions on the transfer function, the information geometry of signal filters or time series models is described by the Kähler manifold. Moreover, many practical aspects of introducing the Kähler manifold to the information geometry for signal processing filters are also reported in the same literature [3]. One of the benefits
in the Kählerian information geometry is that the simpler form of the Laplace–Beltrami operator on the Kähler manifold enables to take advantage of finding the superharmonic priors.

In this paper, we construct Komaki-style shrinkage priors for Kähler-signal filters. By introducing an algorithm which is based on the characteristics of the Kähler manifold, the Bayesian predictive priors can be obtained in the more efficient and robust way. Several prior ansätze outperforming the Jeffreys prior are also suggested. In particular, the geometric shrinkage priors stemmed from the Kähler potential are intrinsic priors on the information manifold because the geometry is given by the Kähler potential. We also provide the geometric priors for the ARFIMA model where the Komaki priors have not been reported. The structure of this paper is the following. In next section, the theoretical backgrounds of the Kählerian information geometry and the superharmonic priors are introduced. In section 3 the algorithm and the ansätze of the geometric shrinkage priors are suggested. The implication of the algorithm to the ARFIMA model is given in section 4. We conclude the paper in the last section.

2. Theoretical backgrounds

2.1. Kählerian signal filter. A signal filter with parameters $\xi$ is characterized by a transfer function $h(w; \xi)$ with

$$y(w) = h(w; \xi)x(w; \xi)$$

where $y$ and $x$ are output and input signals, respectively. A spectral density function $S(w; \xi)$ is defined as the absolute square of the transfer function

$$S(w; \xi) = |h(w; \xi)|^2$$

and it is the real-valued measurable quantity. With $z$-transformation, the holomorphic transfer function can be written in the form of the series expansion

$$h(z; \xi) = \sum_{r=0}^{\infty} h_r(\xi) z^{-r}$$

where $h_r$ is an impulse response function. Similarly, the power spectrum is also expressed in $z$-transformation.

In information geometry, it is well-known by Amari and Nagaoka that the metric tensor for the geometry of the signal filter is represented with the spectral density function $S(w; \xi)$ by

$$g_{\mu\nu}(\xi) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \partial_\mu \log S \partial_\nu \log S \, dw$$

where the partial derivative is with respect to the model parameters $\xi$. The metric tensor can be written in terms of a complexified coordinate system and the transfer function in $z$-transformation:

(1) $$g_{ij}(\xi) = \frac{1}{2\pi i} \oint_{|z|=1} \partial_i \log h(z; \xi) \partial_j \log h(z; \xi) \frac{dz}{z}$$

and

(2) $$g_{ij}(\xi) = \frac{1}{2\pi i} \oint_{|z|=1} \partial_i \log h(z; \xi) \partial_j \log \bar{h}(\bar{z}; \bar{\xi}) \frac{dz}{z}$$
where \( i, j \) run from 1 to \( n \) and \( g_{ij}, g_{i\bar{j}} \) are the complex conjugates of \( g_{ij} \) and \( g_{i\bar{j}} \), respectively.

After plugging the \( z \)-transformed transfer function into the metric tensor expressions, eq. (1) and eq. (2), the metric tensor is expressed with the series expansion coefficients in \( z \) of the log-transfer function by

\[
\begin{align*}
\eta_r &= \frac{\partial}{\partial \log h_0} \partial_{\bar{j}} g_{i\bar{j}} \\
&= \frac{1}{2} \left( \partial_i \log h_0 \partial_j \log \bar{h}_0 + \sum_{r=1}^{\infty} \partial_i \eta_r \partial_j \bar{\eta}_r \right)
\end{align*}
\]

where \( \eta_r \) is the coefficient of \( z^{-r} \) in the series expansion of the log-transfer function.

Recently, it is found that the information geometry of a signal filter is the Kähler manifold if and only if the impulse response function with the highest degree in \( z \), i.e. \( h_0 \) in this case, is a constant with respect to the model parameters \( \xi \). It is easily verified with the definition of the Kähler manifold that is the Hermitian manifold with the closed Kähler form: \( g_{ij} = g_{i\bar{j}} = 0 \) for the Hermitian manifold and \( \partial_i g_{j\bar{k}} = \partial_j g_{i\bar{k}}, \partial_i g_{k\bar{j}} = \partial_j g_{i\bar{k}} \) for the closed Kähler form. In this paper, for simplicity, we only consider the unilateral transfer function with non-zero \( h_0 \). In this case, the necessary and sufficient condition for the Kähler manifold is that \( h_0(\xi) \) is a constant in \( \xi \).

According to Choi and Mullhaupt \( \textit{[3]} \), the benefits of the Kählerian description are the followings. First of all, geometric objects and connections are easily computed on the Kähler manifold. The non-vanishing metric tensor is simply derived from the following formula

\[
(3) \quad g_{ij} = \partial_i \partial_j \mathcal{K}
\]

where \( \mathcal{K} \) is the Kähler potential of the manifold. The Kähler potential of the signal filter geometry is the square of the Hardy norm of the log-transfer function

\[
(4) \quad \mathcal{K} = \oint_{|z|=1} |\log h(z; \xi)|^2 \frac{dz}{z}
\]

and the details of the derivation are given in the literature \( \textit{[3]} \). The Levi-Civita connection is expressed by

\[
(5) \quad \Gamma_{ij,k} = \partial_i g_{j\bar{k}} = \partial_j g_{i\bar{k}} \partial_k \mathcal{K}
\]

and other connections are all vanishing. Moreover, it is much simpler than the connection on the non-Kähler manifold defined by

\[
\Gamma_{ij,k} = \frac{1}{2} \left( \partial_i g_{jk} + \partial_j g_{ik} - \partial_k g_{ij} \right)
\]

and it is clear that the number of calculation steps is reduced in the Kähler case. The Riemann curvature tensor is represented in the simpler form given in the paper by Choi and Mullhaupt \( \textit{[3]} \). The Ricci tensor is obtained by

\[
(6) \quad R_{ij} = -\partial_i \partial_j \log \det g_{m\bar{n}}
\]

and it is evident that we can skip the calculation of the Riemann tensors in order to get the sectional curvature.

Additionally, the \( \alpha \)-generalization of the geometric objects is linear in \( \alpha \) on the Kähler manifold. Since the Riemann curvature tensor on the Kähler manifold is linear in the \( \alpha \)-connection which is \( \alpha \)-linear, the Riemann tensor is also linear in \( \alpha \) and the \( \alpha \)-linearity leads to the \( \alpha \)-linear Ricci tensor and scalar curvature. In
addition to these advantages, the Kählerian information geometry is also useful to find the superharmonic priors because of the simpler Laplace–Beltrami operator on the manifold. We will cover the details in the next subsection.

2.2. Superharmonic priors. First of all, we need to introduce the superharmonic priors suggested by Komaki [4]. In his paper, the superharmonic priors $\pi_I$ are derived from the difference between two risk functions: One from the Jeffreys prior and another from the superharmonic prior,

$$E(D_{KL}(p(y|\xi)||p_{\pi_J}(y|x^{(N)}))) - E(D_{KL}(p(y|\xi)||p_{\pi_I}(y|x^{(N)})))\xi) = \frac{1}{2N^2}g^{ij}\partial_i\log\left(\frac{\pi_I}{\pi_J}\right)\partial_j\log\left(\frac{\pi_I}{\pi_J}\right) - \frac{1}{N^2}\pi_I\Delta\left(\frac{\pi_I}{\pi_J}\right) + o(N^{-2})$$

where $D_{KL}$ is the Kullback–Leibler divergence and $\pi_J$ is the Jeffreys prior which is the volume form of the statistical manifold. Since the first term on the right-hand side is naturally positive, the risk function of the superharmonic prior is decreased with respect to the risk function of the Jeffreys prior if the prior function $\psi = \pi_I/\pi_J$ is superharmonic. If the superharmonic prior function $\psi$ can be found, it is possible to do better Bayesian predictive inference in the viewpoint of information theory. Komaki also pointed out that the information shrinkage prior beats the Jeffreys prior if and only if the square root of the prior function is superharmonic.

Several superharmonic priors for the AR models have been found [7, 8, 3] since Komaki’s paper [4]. The Komaki prior for the AR(2) model in the pole coordinates [7] is given by

$$\psi = 1 - \xi_1\xi_2$$

where $\xi_i$ is the pole of the transfer function. Tanaka [8] generalized the two-dimensional case to the superharmonic prior function for the AR model in an arbitrary dimension $p$. The shrinkage prior function for the AR$(p)$ model is in the form of

$$\psi = \prod_{i<j}(1 - \xi_i\xi_j)$$

where $\xi_i$ is the pole of the transfer function.

As mentioned before, one of the advantages in the Kählerian description is that finding the superharmonic prior functions becomes more efficient than the non-Kähler case because the Laplace–Beltrami operator on the Kähler manifold is in the simpler expression. The Laplace operator on the Kähler geometry is represented by

$$\Delta \psi = 2g^{ij}\partial_i\partial_j\psi.$$ 

Meanwhile, the Laplace–Beltrami operator on the non-Kähler manifold is given by

$$\Delta \psi = \frac{1}{\sqrt{g}}g^{ij}\partial_i\left(\sqrt{g}\partial_j\psi\right) = g^{ij}\partial_i\partial_j\psi + \frac{1}{2}g^{ij}\partial_i\log g\partial_j\psi$$

where $g$ is the determinant of the metric tensor. It is obvious that additional calculation steps for the latter term in the r.h.s. are indispensable in the non-Kähler case.
With the benefit in computation, the superharmonic prior for the Kähler–AR(2) model [3] is expressed by

$$\psi = (1 - |\xi|^2)(1 - \xi \bar{\xi})(1 - |\xi|^2)$$

where $\xi$ is the $i$-th pole of the AR transfer function and $\bar{\xi}$ is the complex conjugate of $\xi$. However, its generalization to any arbitrary dimensions is not known yet. Moreover, the Komaki priors for the ARMA models and the ARFIMA models are not reported either.

3. Geometric shrinkage priors

As shown in the previous section, the Kähler manifold as information geometry is useful when the superharmonic priors need to be obtained. In this section, we introduce an algorithm to find the geometric shrinkage priors by using the properties of the Kähler geometry. Moreover, several ansätze for the priors are suggested.

For further discussion, let us set

$$\tau = u^* - \kappa(\xi, \bar{\xi})$$

where $u^*$ is a constant with respect to $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ and $\bar{\xi}$. With the setting, the following lemma is worthwhile when the algorithm for the prior functions is constructed.

**Lemma 1.** On the Kähler manifold, a function $\psi(\xi, \bar{\xi})$ is superharmonic if $\psi(\xi, \bar{\xi})$ is in the form of

$$\psi = \Psi(u^* - \kappa(\xi, \bar{\xi}))$$

where $\kappa$ is subharmonic, $\Psi'(\tau) > 0$, and $\Psi''(\tau) \leq 0$.

**Proof.** The action of the Laplace–Beltrami operator on $\psi$ is given by

$$\Delta \psi = 2g^{ij} \partial_i \partial_j \psi = 2g^{ij} \partial_i \left( - \partial_j \kappa \right) \Psi'$$

$$= 2\Psi'' g^{ij} \partial_i \kappa \partial_j \kappa - 2\Psi' g^{ij} \partial_i \partial_j \kappa$$

$$= 2\Psi'' ||\partial \kappa||^2 - 2\Psi' \Delta \kappa$$

where the derivative on $\Psi$ is with respect to $\tau$. It is obvious that if $\kappa$ is subharmonic and if $\Psi'(\tau) > 0$, $\Psi''(\tau) \leq 0$, then the r.h.s. is negative, i.e. $\psi$ is a superharmonic function. \hfill \Box

According to Lemma 1, superharmonic functions are easily obtained from subharmonic functions. To acquire the superharmonic function, the subharmonic function is simply plugged as $\kappa$ into Lemma 1.

Considering that the prior function should be positive, it is able to utilize Lemma 1 for obtaining the superharmonic prior functions. Let us confine the function $\psi$ in Lemma 1 to be positive.

**Theorem 1.** On the Kähler manifold, a positive function $\psi = \Psi(u^* - \kappa)$ is a superharmonic prior function if $\kappa$ is subharmonic and $\Psi'(\tau) > 0$, $\Psi''(\tau) \leq 0$.

**Proof.** Since this is a special case in Lemma 1, the proof is obvious. \hfill \Box

Although any subharmonic function $\kappa$ can be used for the superharmonic priors, the restriction on $\kappa$ makes finding the ansätze for the geometric priors easier. From now on, the upper-bounded functions are only our concerns. Additionally, we assume that $\kappa$ and $u^*$ are real. In this case, it is possible to set $u^*$ as a constant greater than the upper bound of $\kappa$ in order to make $\tau$ positive.
**Example 1.** Given positive \( \tau \), the following functions are candidates for \( \Psi \)

\[
\Psi_1(\tau) = \tau^a \\
\Psi_2(\tau) = \log (1 + \tau^a)
\]

where \( 0 < a \leq 1 \).

**Proof.** First of all, \( \Psi_1 \) and \( \Psi_2 \) are all positive. For \( \Psi_1 \), it is easy to verify the followings:

\[
\Psi_1'(\tau) = a \tau^{a-1} > 0 \\
\Psi_1''(\tau) = a(a-1) \tau^{a-2} \leq 0
\]

for \( 0 < a \leq 1 \). The similar calculation is repeated for \( \Psi_2 \)

\[
\Psi_2'(\tau) = \frac{a \tau^{a-1}}{1 + \tau^a} > 0 \\
\Psi_2''(\tau) = \frac{a \tau^{a-2}(a - (1 + \tau^a))}{(1 + \tau^a)^2} \leq 0
\]

for \( 0 < a \leq 1 \).

Both functions satisfy the conditions for \( \Psi \) in Lemma 1. \( \square \)

It is also possible to find the ansätze for \( \kappa \) which is subharmonic and upper-bounded.

**Example 2.** The following functions are candidates for \( \kappa \):

\[
\kappa_1 = \mathcal{K} \\
\kappa_2 = \sum_{r=0}^{\infty} a_r |h_r(\xi)|^2 \\
\kappa_3 = \sum_{i=1}^{n} b_i |\xi^i|^2
\]

where \( a_r \) and \( b_i \) are positive real numbers.

**Proof.** For \( \kappa_1 \), it is easy to show that the Kähler potential \( \mathcal{K} \) is subharmonic.

\[
\Delta \kappa_1 = \Delta \mathcal{K} = 2g^{ij} \partial_i \partial_j \mathcal{K} = 2g^{ij} g_{ij} = 2n > 0.
\]

It is obvious that the Kähler potential is finite by definition. Moreover, we assume that the transfer function is the \( L^2 \) function for the stationarity and the log-transfer function is also the \( L^2 \) function to be the Kähler manifold.

For \( \kappa_2 \), the proof for the subharmonicity is the following:

\[
\Delta \kappa_2 = \Delta \sum_{r=0}^{\infty} a_r |h_r(\xi)|^2 = 2g^{ij} \partial_i \partial_j \sum_{r=0}^{\infty} a_r |h_r(\xi)|^2 \\
= \sum_{r=0}^{\infty} 2a_r g^{ij} \partial_i h_r \partial_j h_r g_{ij} = \sum_{r=0}^{\infty} 2a_r ||\partial h_r||_2^2 > 0.
\]

Since the transfer function of the stationary filter is the \( L^2 \)-function, \( \kappa_2 \) is upper-bounded.
The subharmonicity of $\kappa_3$ is checked out by

$$
\Delta \kappa_3 = \Delta \sum_{i=1}^{n} b_i |\xi^i|^2 = 2g^{ij} \partial_i \partial_j \sum_{r=0}^{\infty} b_i |\xi^i(\xi)|^2 \\
= \sum_{i=1}^{n} 2b_i g^{ii} > 0.
$$

It is easy to make $\kappa_3$ finite because of the following reasons. If $\xi^i$ is finite, any positive and finite $b_i$ can be chosen. In the case of infinite $\xi^i$, we can control it by assigning zero to $b_i$ or considering the finite domains of $\xi^i$.

The superharmonic prior functions are efficiently constructed from the following algorithm which exploits Theorem 1 and the candidates for $\Psi$ and $\kappa$. When we find the positive and superharmonic functions, it is automatically the Komaki-style prior functions as usual. If the upper-bounded and subharmonic functions are found, it is plugged into Theorem 1 in order to obtain the superharmonic prior functions. After then, we finally acquire the geometric shrinkage priors by multiplying the Jeffreys prior, which is the volume form of the information manifold, to the superharmonic prior functions. Additionally, since the ansätze are also provided, there is no extra cost to find the Komaki prior function except for verifying whether or not the information geometry is the Kähler manifold: simply checking if the impulse response function of degree zero is constant with respect to the model parameters. Comparing with the literature on the Komaki priors of the time series models \[7, 8, 3\], it is more efficient and robust to obtain the geometric priors.

4. Example: ARFIMA models

The ARFIMA model is the generalization of the ARMA model with a fractional differencing parameter in order to model the long memory process. The transfer function of the ARFIMA($p, d, q$) model with parameters $\xi = (\xi^{-1}, \xi^0, \xi^1, \ldots, \xi^{p+q}) = (\sigma, d, \lambda_1, \ldots, \lambda_p, \mu_1, \ldots, \mu_q)$ is given by

$$
h(z; \xi) = \frac{\sigma^2}{2\pi} \frac{(1 - \mu_1 z^{-1})(1 - \mu_2 z^{-1}) \ldots (1 - \mu_q z^{-1})}{(1 - \lambda_1 z^{-1})(1 - \lambda_2 z^{-1}) \ldots (1 - \lambda_p z^{-1})} (1 - z^{-1})^d
$$

where $d$ is the differencing parameter and $\mu_i, \lambda_i, \sigma$ are the pole, root, and gain in the ARMA model, respectively. Additionally, all poles and roots are located inside the unit disk, i.e. $|\lambda_i| < 1$ for $i = 1, \ldots, p$ and $|\mu_i| < 1$ for $i = 1, \ldots, q$. Simply speaking, the transfer function of the ARFIMA model is decomposed into the ARMA model part and the fractionally integrated part.

Similar to the ARMA case \[8\], the full geometry of the ARFIMA model is not the Kähler manifold but the submanifold of a constant standard deviation is the Kähler geometry. It is easy to check that the information geometry of the ARFIMA model is the Kähler manifold because $h_0 = 1$ up to the gain of the signal filter. We will work on this submanifold.

Since the information geometry of the ARFIMA model is the Kähler manifold, the Kähler potential of the ARFIMA geometry is obtained from the square of the
Hardy norm of the log-transfer function, eq. (4), represented by

\[
\mathcal{K} = \sum_{n=1}^{\infty} \left| \frac{d + (\mu_1^n + \cdots + \mu_q^n)}{n} - \frac{\lambda_1^n + \cdots + \lambda_p^n}{n} \right|^2.
\]

It is obvious that the Kähler potential is reduced to the potential for the ARMA model by setting \(d = 0\). It is easy to verify that the Kähler potential is upper-bounded by \((d + q + p)^2 \pi^2/6\).

The metric tensor of the Kähler geometry is easily derived from the Kähler potential by using eq. (3). The metric tensor of the Kähler–ARFIMA geometry is given by

\[
\begin{pmatrix}
\frac{\pi^2}{6} & \frac{1}{\lambda_j} \log (1 - \lambda_i) & -\frac{1}{\mu_j} \log (1 - \mu_j) \\
-\frac{1}{\lambda_i} \log (1 - \lambda_j) & \frac{1}{1 - \lambda_i \lambda_j} & -\frac{1}{1 - \mu_i \mu_j} \\
\frac{1}{\mu_i} \log (1 - \mu_j) & -\frac{1}{1 - \mu_i \mu_j} & \frac{1}{1 - \mu_i \mu_j}
\end{pmatrix}
\]

where \(g_{0\bar{0}} = \frac{\pi^2}{6}\). It is easy to show that the metric tensor contains the pure ARMA piece. This indicates that the ARMA geometry is embedded in the ARFIMA manifold and corresponds to the submanifold of the ARFIMA geometry. In addition to that, the submanifold of the ARFIMA geometry is also the Kähler geometry because the submanifold of the Kähler manifold is Kähler.

Other geometric objects can be derived from the metric tensor. For example, the 0-connection is given by eq. (5). It is noteworthy that the vanishing Levi-Civita connection is \(\Gamma_{\bar{a},k}^i\) for \(i = 0\) and others are all non-vanishing. Similar to the 0-connection, the Ricci tensor along the fractionally integrated direction is also zero because there is no dependence on \(d\) in the metric tensor.

It is the time to be back to the geometric shrinkage prior. Since the Kähler potential of the given ARFIMA model is upper-bounded by a constant \(u^* = (d + p + q)^2 \pi^2/6\), it is obvious that the intrinsic priors on the Kähler manifold can be found as it is proven in the previous section.

By using the algorithm and the ansätze related to the Kähler potential, the geometric shrinkage prior functions for the ARFIMA model are given by

\[
\begin{align*}
\psi_1 &= (u^* - \mathcal{K})^a \\
\psi_2 &= \log (1 + (u^* - \mathcal{K})^a)
\end{align*}
\]

where \(0 < a \leq 1\). It is also noteworthy that when \(d = 0\) in the Kähler potential, the superharmonic priors for the ARMA (or AR/MA) model are acquired and finding the priors becomes much simpler than the literature on the Komaki priors of the AR model [7, 8, 3]. Similarly, \(\kappa_2\) and \(\kappa_3\) are also utilized in the superharmonic prior function ansätze for the ARFIMA model. Moreover, if we set \(d = 0\) for \(\kappa_2\) or \(b_0 = 0\) for \(\kappa_3\), the ansätze for the ARFIMA model are reduced to the Komaki priors of the ARMA model.

5. Conclusion

In this paper, we build up the algorithm and ansätze for the geometric shrinkage priors of Kählerian signal filters. Using the properties of the Kähler manifold, the algorithm enables to find the Komaki priors in the more efficient and robust way. Additionally, some ansätze associated with the Kähler potential are geometrically
intrinsic to the statistical manifold because the geometry is derived from the Kähler potential of the signal filter.

Comparing with the literature on the Komaki priors of the time series models, the verification of the geometric priors on the Kähler manifold is much easier and it is also possible to acquire the geometric shrinkage priors for more complicated models. As an example, the information priors for the ARFIMA model are obtained from the algorithm and the prior function ansätze. The geometric shrinkage priors are straightforwardly reduced to the ARMA cases.

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