Seismic response of Heitiejiao tailings dam by the finite element method

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Abstract. In this study, seismic response of a tailings dam is investigated by finite element method. The Heitiejiao dam constructed in She county, China is chosen as a numerical example. The interaction of the tailings dam with the reservoir is neglected, but not the foundation rock. The properties of the dam materials were taken from the dam project and assumed to be linearly elastic, homogenous and isotropic in the analysis. A stationary and ergodicity assumption is made for seismic analysis. The E–W component of the Tianjin Earthquake is chosen as a ground motion since it occurred nearby the dam site. The component considered is applied to the dam in the horizontal direction. The seismic response of the Heitiejiao dam subjected to the Tianjin Earthquake is also obtained by the deterministic method. The results obtained from stochastic and deterministic analysis are compared to each other. It is seen that the results obtained from the time-history analysis are smaller than those from the deterministic analysis.

1. Introduction

Dams are constructed for various purposes such as irrigation, energy production, flood control and recreation. A serious damage on these dams has been not recorded in the literature due to an earthquake ground motion [1]. Accordingly, it can be said that tailings dams are highly resistant to seismic load. The satisfactory seismic behavior of these dams is due to the capacity of the tailings body. Ozkan [1] and Gazetas and Dakoulas [2] have presented comprehensive reviews on theoretical methods for estimating the dynamic response and the performance of earth and tailings dams subjected to strong earthquake ground motions. Several factors such as liquefaction effects, non-linear material behavior, and permanent deformations affect the dynamic response of earth and tailings dams during the earthquakes. Linear and non-linear earthquake responses of earth and tailings dams including these factors were carried out by many researchers [3]. These studies were performed by using the deterministic methods. In recent years, the stochastic seismic responses of earth dams have also been investigated by only a limited number of researchers [4]. However, it can be seen from the literature review that a few works on stochastic response of tailings dams to ground motion have been studied. Therefore, the objective of this study is to determine the stochastic seismic response of a tailings dam using the finite element method.
2. Analysis formulation

An acceleration–time history of ground motion recorded at one point is used as seismic input in the deterministic method. In the stochastic method, however, recorded ground motions appropriate to the site are characterized as statistically. Since the ground motion caused by seismic disturbance is random, the best way to characterize the random excitation statistically is to employ a power density function and autocorrelation function. Once, the power spectral density function or the autocorrelation function of the seismic input is known, the cross power spectral density function can be determined freely.

In this study, only the final expression for the cross power spectral density function will be given. Detailed expressions for this function are explained elsewhere [5]. If a single ground acceleration record is used for the input, cross power spectral density function, $S_{ij}(\omega)$ can be determined by using the equation of motion of the system as [6]

$$S_{ij}(\omega) = S_{in}(\omega) \sum_{r=1}^{N} \sum_{s=1}^{N} \psi_{ir} \psi_{js} H_{ir}(\omega) H_{js}^*(\omega)$$

(1)

Where $\omega$ is the frequency, $H(\omega)$ is the frequency response function, $S_{in}(\omega)$ is the power spectral density function of the ground motion, $N$ is the number of modes which are considered to contribute to the response, $^*$ denotes the complex conjugate.

2.1. Spectral moments

Statistics related to the structural behavior for a stationary process can be determined by using the zeroth, the first and the second spectral moments of the output process. Spectral moments, which can be expressed in terms of power spectral density function and frequency, may be calculated as follows [6]

$$\lambda_{m,ij} = 2 \int_0^{\omega} \omega^m S_{ij}(\omega) d\omega$$

(2)

Where $m=0,1,2,3$ is the zeroth, the first and the second spectral moments, respectively. These parameters will then be used while obtaining the mean of maximum value, variance and frequency of occurrence [6].

2.2. Expected maximum value

The expected maximum value is considered to be the most important parameter in the stochastic analysis of structures affected by seismic loads. In the stochastic analysis the expected maximum value $\mu$ is the mean value of all maximum values. The expected maximum value, which depends on the peak factor and the root-mean-square response, can be expressed as

$$\mu = p \sqrt{\lambda_0}$$

(3)

Standard deviation of $\mu$ is expressed as

$$\sigma = q \sqrt{\lambda_0}$$

(4)

Where $\lambda_0$ is the zeroth spectral moment defined by Eq. (2), $p$ and $q$ are the peak factors, which are the functions of the duration of the motion and the mean zero crossing rate, respectively [7].

2.3. Occurrence frequency

Frequency of occurrence is described as the average number of times that the line ($y(t) = 0$) is crossed by the response in a unit of time. For Gaussian process of zero average, the average number of times in the zero level crossed by the process in a unit of time is expressed as
Because the zero level is crossed two times for each cycle, frequency of occurrence for the response process will be equal to $\nu/2$ as

$$f_0 = \frac{\nu}{2} = \frac{1}{2\pi} \sqrt{\frac{\lambda_2}{\lambda_0}}$$  \hspace{1cm} (6)

Where $\lambda_2$ is the second spectral moment defined by Eq. (2).

3. Numerical application and discussions

In this study, the Heitiejiao dam constructed in She County, Hebei (Figure 1) is chosen as a numerical example to investigate the stochastic seismic response of a tailings dam by the finite element method. Figure 2 shows the vertical cross-section of the Heitiejiao dam. The finite element of the dam is meshed. The Heitiejiao dam is 115 m high from riverbed. The crest has a length of 916 m. The main purpose of the dam is to regulate river flow and supply energy. In the finite element mesh of the dam, there are 1326 nodes and 2286 quadrilateral elements. The dam is treated as a plane strain problem. The interaction of the tailings dams with the reservoir has generally neglected. Therefore, the interaction with the reservoir is accordingly ignored, but not the foundation rock.

Materials in the dam section can be grouped in six main categories: compacted tailings placed at various lifts, the impervious clay core flanked by transition filters and a concrete core at the bottom of the dam. The properties of these materials taken from the dam project are as follows [8]:

![Figure 1. The Heijiaotie dam constructed in She County, Hebei](image)

![Figure 2. The calculated profile of the dam](image)
A stationary assumption here the statistical parameters are independent of time is made for the stochastic analysis. Besides, while calculating the statistics to represent the random process, like ensemble averages, some difficulties are encountered. The ergodicity assumption is made to overcome these difficulties and only one earthquake record is used in this study. Although earthquake motions are not completely stationary, these motions can be considered as stationary processes under certain conditions because of its analytical simplicity. Perhaps an earthquake ground motion is not stationary along the whole motion, but it is an acceptable approximation to consider the ground motion stationary along the duration's where maximum structural responses take place. A stationary model simplifies the computations and gives satisfying results. The response of structural systems to stationary excitations is of wide engineering interest [9].

| Strata title     | Natural specific weight (KN/m³) | Saturated specific weight (KN/m³) | Shearing strength target |
|------------------|---------------------------------|-----------------------------------|--------------------------|
|                  |                                 |                                   | Cohesive force (KPa)     |
|                  |                                 |                                   | Water (up) | Water (down) |
|                  |                                 |                                   | Angle of internal friction (°) |
|                  |                                 |                                   | Water (up) | Water (down) |
| Initial period dam ① | 19.4                             | 20.0                             | 2.0 | 0.5 | 38 | 38 |
| Stack dam ②       | 19.2                             | 20.7                             | 6.0 | 2.0 | 29 | 26 |
| Stack dam ③       | 19.0                             | 21.5                             | 5.0 | 1.0 | 28 | 25 |
| Stack dam ④       | 19.0                             | 21.5                             | 5.0 | 1.0 | 28 | 25 |
| Soil layer ⑤      | 17.7                             | 18.6                             | 8.0 | 3.5 | 22 | 19 |
| Foundation rock ⑥ | 20.0                             | 22.1                             | 2005.0 | 2000.0 | 58 | 55 |

The E–W component of the Tianjin Earthquake is chosen as ground motion and given in Figure 4 since it occurred nearby the dam site. The component considered is applied to the dam in the upstream–downstream direction. The power spectral density (PSD) function of the Tianjin Earthquake is determined with the Fourier transforms of the autocorrelation function as shown in Figure 5.

In this paper, the dynamic response of the Heitiejiao dam subjected to the Tianjin Earthquake is also obtained by the deterministic method. The results obtained from stochastic and deterministic analysis are compared to each other. The seismic responses of the Heitiejiao dam are obtained for a time interval of 0.00225s.

![Figure 3](image)

**Figure 3.** The E–W component of the Tianjin Earthquake

### 3.1. Displacements

Mean of maximum values of displacements are calculated from stochastic seismic analysis. The absolute maximum values of displacements are obtained from deterministic dynamic analysis. Horizontal displacements along the core of the tailings dam at the marked nodes on line 1 (Figure 3) obtained from stochastic and deterministic seismic analyses of the Heitiejiao dam are plotted in Figure 6. Vertical displacements along the horizontal length of the dam at the marked nodes on line 2 and line 3 (Figure 3) obtained from stochastic and deterministic seismic analyses of the dam are also plotted in Figs. 7 and 8, respectively. It is seen from these figures that the expected maximum values of horizontal and vertical displacements obtained from stochastic dynamic analysis are smaller than the absolute maximum horizontal and vertical displacements obtained from deterministic dynamic analysis. It is also seen from
Figs. 7 and 8 that the vertical displacements at the marked nodes on line 2 and line 3 decrease towards the core of the rock-fill dam.

In Figure 9, the displacements obtained from the stochastic dynamic analysis can be obtained by calculating the time-history of the horizontal displacements at the crest point (node 1) of the rock-fill dam from the deterministic dynamic analysis. Taking the average of 15 maximum horizontal displacements shown in Figure 9, the mean of maximum horizontal displacements can be calculated as 3.98 cm for the deterministic analysis. The expected maximum horizontal displacement obtained from stochastic dynamic analysis is 4.01 cm as shown in Figure 5. The maximum displacement obtained by averaging, which is 3.98 cm, is very close to 4.01 cm obtained from the stochastic analysis. This shows the correctness of the displacements obtained from the analysis.

**Figure 4.** Horizontal displacements along the height at the marked nodes on line 1 of the tailings dam.

**Figure 5.** Vertical displacements along the horizontal length at the marked nodes on line 2 of the tailings dam.
Figure 6. Vertical displacements along the horizontal length at the marked nodes on line 3 of the tailings dam.

Figure 7. Time-history of horizontal displacements at the crest point (node 1).

Figure 8. Frequencies of occurrence of horizontal displacements along the core of the tailings dam.
3.1.1. Frequencies of occurrence of displacements. The frequency of occurrence of horizontal displacements along the core of the rock-fill dam at the marked nodes on line 1 of the rock-fill dam (Figure 3) are calculated using Eq. (6) and depicted in Figure 10. The values of frequency of occurrence along the line 1 vary between 1.67 and 1.74. It can be seen by comparison of Figs. 6 and 10 that the frequency of occurrence decreases with increasing displacement values. Frequency of occurrence of the displacements obtained from the stochastic seismic analysis can be verified. For this purpose, a time interval between 3.6 and 4.3 s of the time-history of the crest displacements in Figure 9 where maximum horizontal displacement is occurred is zoomed as shown in Figure 11. Since the period is defined as the time required completing one cycle, it is taken the time interval between $t_1 = 3.68775$ and $t_2 = 4.27725$ shown in Figure 11 in order to compute the period of the maximum horizontal displacement. Difference between $t_1$ and $t_2$ is 0.5895 s. So, frequency of the maximum horizontal displacement can be calculated as 1.70 Hz from the deterministic seismic analysis. The value of frequency of occurrence of the horizontal displacement at the crest point (node 1) from stochastic dynamic analysis is obtained as 1.67 Hz. This also shows good accuracy of the results obtained from the dynamic analysis.

![Figure 9. Determination of value of occurrence period of the maximum horizontal displacement](image)

(a) Horizontal Stresses  
(b) Vertical stresses
The stress components, which are obtained from stochastic and deterministic dynamic analyses, are also compared with each other. The stress values are calculated at the middle points of the elements with a time interval of 0.0025s. To verify the stresses obtained from the stochastic dynamic analysis, time-history of the horizontal stresses at the element A selected from the dam body is calculated from the deterministic dynamic analysis The expected maximum horizontal stress obtained from stochastic dynamic analysis is 355.37 KN/m² as shown in Figure 12(a). The maximum stress obtained by averaging, which is 350.37KN/m², is very close to 355.37 KN/m² obtained from the stochastic analysis.

3.1.2. Frequencies of occurrence of stresses. Values of frequencies of occurrence of horizontal stresses at section I–I of the rock-fill dam are depicted. It can be seen that the value of frequency of occurrence of stochastic horizontal stress at the element A (Figure 3) is obtained as 3.31 Hz. To verify the frequency of occurrence of the horizontal stress obtained from the stochastic dynamic analysis, a time interval between 3.7 s and 4.2 s of the time-history of horizontal stresses in Figure 13. It is taken the time interval between $t_1 = 3.8205$ and $t_2 = 4.1130$ shown in Figure 13 in order to compute the period of the maximum horizontal stress. Difference between $t_1$ and $t_2$ is 0.2925s. So, frequency of the maximum horizontal stress can be calculated as 3.42 Hz. These also show good accuracy of the results obtained from the stochastic dynamic analysis.

Figure 11. Determination of occurrence period of the maximum horizontal stress at the element A.
4. Conclusion
Seismic response of a rock-fill dam by finite element method is investigated in this paper. The project values of the Heitiejiao dam were considered in the analyses. Frequencies of occurrence of expected maximum values of displacements and stresses are compatible with the results of the deterministic seismic analysis. It is observed that the displacement and stress results obtained from the deterministic dynamic analysis are greater than the mean of maximum values obtained from the stochastic dynamic analysis. Because the mean of maximum values obtained from the stochastic analysis is calculated by averaging all the maximum response values, it should be expected that the absolute maximum values obtained from the deterministic dynamic analysis would be greater than the mean of maximum values. The horizontal displacements obtained from both analyses increase along the dam height. The vertical displacements obtained along the horizontal length of the dam significantly decrease towards the core of the tailings dam for both analyses. The frequency of occurrences of horizontal displacements at the clay core decrease with increasing the values of displacements. The all stress components decrease towards the dam crest. The shear stresses obtained along the clay core are greater than the horizontal and vertical stresses. To generalize these results, solutions must be obtained using many earthquake inputs and different rock-fill dam models.

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