Multiparty simultaneous quantum identity authentication based on entanglement swapping

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We present a multiparty simultaneous quantum identity authentication protocol based on entanglement swapping. In our protocol, the multi-user can be authenticated by a trusted third party simultaneously.

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Quantum cryptography has been one of the most remarkable applications of quantum mechanics in quantum information science. Quantum key distribution (QKD), which provides a way of exchanging a private key with unconditional security, has progressed rapidly since the first QKD protocol was proposed by Bennett and Brassard in 1984 [1]. A good many of other quantum communication schemes have also been proposed and pursued, such as quantum secret sharing (QSS) [2, 3, 4, 5, 6, 7], quantum secure direct communication (QSDC) [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25], and quantum identity authentication (QIA) [22, 23, 24, 25]. QSS is the generalization of classical secret sharing to quantum scenario and can share both classical and quantum messages among sharers. QSDC’s object is to transmit the secret message directly without first establishing a key to encrypt it. Authentication is a well-studied area of classical cryptography, including identity and message authentication. QIA aims to generalize classical identity authentication to quantum scenario for providing unconditional security. Duek et al. [22] proposed a secure quantum identification system combining a classical identification procedure and quantum key distribution. Zeng and Zhang [23] put forward a quantum key verification scheme which can simultaneously distribute the quantum secret key and verify the communicators’ identity. T. Mihara [24] presented three quantum identification schemes by using entangled state and unitary operation. Lee et al. [25] presented two QSDC protocols with user authentication.

In this paper, we present a multiparty simultaneous quantum identity authentication protocol based on entanglement swapping, which combines the idea in Ref. [21] with that in Ref. [23]. In our protocol, We suppose a trusted third party, Trent, authenticates r legal users, {Alice$_1$, Alice$_2$, · · · , Alice$_r$} simultaneously. Similar to Ref. [23], Trent shares a secret identity number $ID_i$ ($i = 1, 2, · · · , r$) and a secret hash function $h_i$ ($i = 1, 2, · · · , r$) with each user. Here the hash function is defined as

$$h : \{0,1\}^l \times \{0,1\}^m \to \{0,1\}^n,$$

where $l$, $m$ and $n$ denote the length of the identity number, the length of a counter and the length of authentication key, respectively. Thus the user’s authentication key can be expressed as $AK = h(ID, C)$, where $C$ is the counter of calls on the user’s hash function. When the length of the authentication key is not enough to satisfy the requirement of cryptographic task. The parties can increase the counter and then generates a new authentication key. We denote the authentication keys of Alice$_1$, Alice$_2$, · · · , Alice, as $AK_{A_1} = h_{A_1}(ID_{A_1}, C_{A_1})$, $AK_{A_2} = h_{A_2}(ID_{A_2}, C_{A_2})$, · · · , $AK_{A_r} = h_{A_r}(ID_{A_r}, C_{A_r})$, respectively.

Entanglement swapping can entangle two quantum systems that do not have direct interaction with each other [20]. It plays an important role in quantum information. We first describe entanglement swapping simply.

The four Bell states are

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle),$$
$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).$$

Suppose two distant parties, Alice and Bob, share $|\phi_{12}\rangle$ and $|\phi_{34}\rangle$ where Alice has qubits 1 and 4, and Bob possesses 2 and 3. Note that

$$|\phi_{12}^+\rangle \otimes |\phi_{34}^+\rangle = \frac{1}{2}(|\psi_{14}^+\rangle|\phi_{23}^+\rangle + |\phi_{14}^+\rangle|\phi_{23}^+\rangle + |\phi_{14}^+\rangle|\phi_{23}^−\rangle + |\phi_{14}^−\rangle|\phi_{23}^−\rangle).$$

After Bell basis measurement on qubits 1 and 4, the state of the qubits 1, 2, 3, 4 collapses to $|\phi_{14}^+\rangle|\phi_{23}^+\rangle$, $|\phi_{14}^+\rangle|\phi_{23}^−\rangle$, $|\psi_{14}^+\rangle|\psi_{23}^+\rangle$ and $|\psi_{14}^−\rangle|\psi_{23}^−\rangle$ each with probability 1/4. If Alice and Bob share other Bell states, similar results can be achieved.

In our protocol, the eight three-particle GHZ states are
defined as
\[ |\Psi_1\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle), |\Psi_2\rangle = \frac{1}{\sqrt{2}} (|000\rangle - |111\rangle), \]
\[ |\Psi_3\rangle = \frac{1}{\sqrt{2}} (|100\rangle + |011\rangle), |\Psi_4\rangle = \frac{1}{\sqrt{2}} (|100\rangle - |011\rangle), \]
\[ |\Psi_5\rangle = \frac{1}{\sqrt{2}} (|010\rangle + |101\rangle), |\Psi_6\rangle = \frac{1}{\sqrt{2}} (|010\rangle - |101\rangle), \]
\[ |\Psi_7\rangle = \frac{1}{\sqrt{2}} (|110\rangle + |001\rangle), |\Psi_8\rangle = \frac{1}{\sqrt{2}} (|110\rangle - |001\rangle), \] (4)

which form a complete orthonormal basis. The parties agree that the two unitary operations
\[ I = |0\rangle\langle 0| + |1\rangle\langle 1|, \]
\[ i\sigma_y = |0\rangle\langle 1| - |1\rangle\langle 0|, \] (5)
can be encoded into one bit classical information as
\[ I \to 0, i\sigma_y \to 1. \] (6)

We first present our QIA protocol with two users (Alice1, Alice2) and then generalize it to the case with many users (Alice1, Alice2, · · · , AliceT). Each user shares a authentication key with Trent, as we have described above.

(S1) Trent prepares an ordered N three-particle GHZ states, each of which is in the state \[ |\Psi_1\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)_{T,A_1,A_2}, \] where the subscripts T, A1 and A2 represent the three particles of each GHZ state. Trent takes particle T (A1, A2) for each state to form an ordered particle sequence, called T (A1, A2) sequence. He then sends A1 and A2 sequences to Alice1 and Alice2, respectively and keeps T sequence.

(S2) To ensure the security of the quantum channel, the parties check eavesdropping as follows: (a) After hearing from the users, Trent selects randomly a sufficiently large subset from the ordered N GHZ states. (b) He measures the sampling particles in T sequence, in a random measuring basis, Z-basis(|0⟩,|1⟩) or X-basis (|+⟩=1/√2(|0⟩ + |1⟩), |−⟩=1/√2(|0⟩ − |1⟩)). (c) Trent announces publicly the positions of the sampling particles and the measurement basis for each of the sampling particles. Alice1 (Alice2) measures the sampling particles in A1 (A2) sequence, in the same measuring basis as Trent. After measurements, the users publishes their measurement results. (d) Trent can then check the existence of eavesdropper by comparing their measurement results. If the channel is safe, their results must be completely correlated. When Trent performs Z-basis measurement on his particle, Alice1’s result should be |0⟩ (|1⟩) if Trent’s result is |0⟩ (|1⟩). On the contrary, Alice1’s result should be |+⟩ (|−⟩) (|−⟩ (|+⟩) if Trent performs X-basis measurement on his particle and gets the result |+⟩ (|−⟩). (e) If Trent confirms that their results are completely correlated, he announces publicly his measurement results of the sampling particles. The users can make certain whether they share a sequence of GHZ states with Trent. If the users confirms that there is no eavesdropping, they continue to execute the next step. Otherwise, they inform Trent and abort the communication.

(S3) After hearing from the users, Trent divides randomly the remaining GHZ states into M ordered groups, \{P(1)T,A_1,A_2, Q(1)T′,A_1′, A_2′\}, \{P(2)T,A_1,A_2, Q(2)T′,A_1′, A_2′\}, · · · , \{P(M)T,A_1,A_2, Q(M)T′,A_1′, A_2′\}, where 1, 2, · · · , M represent the order of the group and the subscripts T and T′ (A1, A1′ and A2, A2′) denote the particles belonging to Trent (Alice1’s and Alice2’s).

(S4) For each of the groups, Alice1 (Alice2) performs one of the two operations \{I, i\sigma_y\} on particle A1 (A2) according to her authentication key, AKA1 (AKA2). For example, if the ith value of AKA1 is 0 (1), Alice1 executes I (i\sigma_y) operation on particle A1. As we have described above, here AKA1 = h(IDA_1,C_A1), AKA2 = h(IDA_2,C_A2). If the length of AK is not long enough to M, new AK can be generated by increasing the counter until the length of AK is no less than M. They inform Trent that they have transformed their qubit by using unitary operation according to their authentication keys.

(S5) After hearing from the users, Trent performs randomly I or i\sigma_y operation on particles T in each group. After the three-party’s operations, |Ψ1⟩ can be transformed into one of the eight three-particle GHZ states \{|Ψ1⟩, |Ψ1⟩, · · · , |Ψ8⟩\}, as shown in Table 1.

| TABLE I: The transformation relations of GHZ states |
| --- |
| \| | unitary operations performed on the three particles |
| | \| |
| | I \otimes I \otimes I |
| | i\sigma_y \otimes i\sigma_y \otimes i\sigma_y |
| | I \otimes i\sigma_y \otimes i\sigma_y |
| | i\sigma_y \otimes I \otimes I |
| | i\sigma_y \otimes I \otimes i\sigma_y |
| | I \otimes i\sigma_y \otimes I |
| | i\sigma_y \otimes i\sigma_y \otimes I |
| | I \otimes I \otimes i\sigma_y |

(S6) Trent lets Alice1 (Alice2) measure particles A1 and A2′ (A2 and A2′) of each group in Bell basis. After measurements, Alice1 and Alice2 publish their measurement results. Trent performs Bell basis measurement on particles T and T′ of each group and authenticates the users according to their measurement results. We then explain it in detail. The state of a group can be written
as

\[
|\Psi_1\rangle_{TA_1A_2} \otimes |\Psi_1\rangle_{T'A_1'A_2} = \frac{1}{2\sqrt{2}} \left( (\phi^+_T) |\phi_{A_1}^+, \phi_{A_2}^+\rangle + (\phi^-_T) |\phi_{A_1}^-, \phi_{A_2}^-\rangle \\
+ (\phi^+_T) |\phi_{A_1}^+\rangle |\phi_{A_2}^+\rangle + (\phi^-_T) |\phi_{A_1}^-, \phi_{A_2}^-\rangle \\
+ (\phi^+_T) |\phi_{A_1}^+\rangle |\phi_{A_2}^-\rangle + (\phi^-_T) |\phi_{A_1}^-, \phi_{A_2}^+\rangle \right). 
\]

(7)

If Trent’s random operation is $i\sigma_y$, Alice1’s ith value of her authentication key is 1 which corresponds to operation $i\sigma_y$ and Alice2’s ith value of her authentication key is 0 corresponding to operation $I$. $|\Psi_1\rangle_{TA_1A_2}$ is then transformed to $|\Psi_2\rangle_{TA_1A_2}$ and the state of the group becomes

\[
|\Psi_2\rangle_{TA_1A_2} \otimes |\Psi_2\rangle_{T'A_1'A_2} = \frac{1}{2\sqrt{2}} \left( |\psi^+_T\rangle |\psi_{A_1}^+, \psi_{A_2}^+\rangle - |\psi^-_T\rangle |\psi_{A_1}^-, \psi_{A_2}^-\rangle \\
- |\psi^-_T\rangle |\psi_{A_1}^-, \psi_{A_2}^+\rangle + |\psi^+_T\rangle |\psi_{A_1}^+, \psi_{A_2}^-\rangle \\
+ |\phi^+_T\rangle |\phi_{A_1}^+, \phi_{A_2}^+\rangle - |\phi^-_T\rangle |\phi_{A_1}^-, \phi_{A_2}^-\rangle \\
- |\phi^-_T\rangle |\phi_{A_1}^-, \phi_{A_2}^+\rangle + |\phi^+_T\rangle |\phi_{A_1}^+, \phi_{A_2}^-\rangle \right). 
\]

(8)

From the published results of Alice1 and Alice2 and his measurement results, Trent can obtain the users’ operation information and then authenticates the users because the three parties’ results correspond to an exclusive state. For example, the results of Trent, Alice1 and Alice2 are each $|\psi^+_T\rangle$, $|\psi_{A_1}^+, \phi_{A_2}^+\rangle$ and $|\phi_{A_1}^+\rangle$. According to Eq. 8, the state of the group must be $|\Psi_2\rangle_{TA_1A_2} \otimes |\Psi_2\rangle_{T'A_1'A_2}$. Trent then knows the ith value of Alice1’s and Alice2’s authentication keys are 1 and 0 because only the operation $i\sigma_y$ applied on particles $T$, $A_1$ and $A_2$ can change the state $|\Psi_2\rangle$ into $|\Psi_1\rangle$. Trent compares his deduced result with the authentication key they shared and then authenticates Alice1 and Alice2.

Now let us discuss the security for the present protocol. An eavesdropper, Eve, has little chance to eavesdrop the users’ operation information because it is unnecessary for the users to resend their particles on which each of users has performed their corresponding operations according to their authentication keys, to the trusted third party. Moreover, from the published results of the users, Eve also cannot obtain any information of the users because she has not Trent’s result. Suppose the published results of Alice1 and Alice2 are each $|\psi_{A_1}^+, \psi_{A_2}^+\rangle$ and $|\phi_{A_1}^+, \phi_{A_2}^+\rangle$. Without Trent’s result, Eve can only know that the state of the group is one of the four state $\{ |\Psi_2\rangle \otimes |\Psi_1\rangle, |\Psi_2\rangle \otimes |\Psi_1\rangle, |\Psi_1\rangle \otimes |\Psi_2\rangle, |\Psi_2\rangle \otimes |\Psi_1\rangle \}$. The eavesdropping check aims to prevent Eve from impersonating attack and let the legal users share a safe quantum channel with Trent. Suppose Eve prepares $N$ ordered three-particles GHZ states, each of which is $|\Psi_1\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)_{E_1E_2}$. Eve intercepts particles $A_1$ and $A_2$ and resends particles $E_1$ and $E_2$ to each Alice1 and Alice2. Eve attempts to personate Trent for acquiring the users’ authentication key. However, during the eavesdropping check, Eve’s attack will be detected by the parties because Eve cannot tamper with the classical message published by the trusted third party, Trent. Thus the users’ results have no correlation with the result published by Trent.

According to Stinespring dilation theorem, Eve’s action can be realized by a unitary operation $\hat{E}$ on a large Hilbert space, $H_{A_1A_2} \otimes H_{E}$. Then the state of Trent, Alice1, Alice1 and Eve is

\[
|\Phi\rangle = \sum_{T,A_1,A_2 \in \{0,1\}} |\epsilon_{T,A_1,A_2}\rangle |T\rangle |A_1A_2\rangle , 
\]

(9)

where $|\epsilon\rangle$ denotes Eve’s probe state and $|T\rangle$ and $|A_1A_2\rangle$ are states shared by Trent and the users. The condition on the states of Eve’s probe is

\[
\sum_{T,A_1,A_2 \in \{0,1\}} \langle\epsilon_{T,A_1,A_2}|\epsilon_{T,A_1,A_2}\rangle = 1. 
\]

(10)

As Eve can eavesdrop particle $A_1$ and $A_2$, Eve’s action on the system can be written as

\[
|\Phi\rangle = \frac{1}{\sqrt{2}} |0\rangle (|\alpha_1| |000\rangle + |\beta_1| |011\rangle + |\gamma_1| |100\rangle + |\delta_1| |111\rangle) + |1\rangle (|\delta_2| |000\rangle + |\gamma_2| |101\rangle + |\beta_2| |011\rangle + |\alpha_2| |110\rangle) + |0\rangle (|\epsilon_{001}\rangle + |\epsilon_{010}\rangle + |\epsilon_{011}\rangle + |\epsilon_{100}\rangle + |\epsilon_{101}\rangle + |\epsilon_{110}\rangle + |\epsilon_{111}\rangle). 
\]

(11)

The error rate introduced by Eve is $\epsilon = 1 - |\alpha_1|^2 = 1 - |\delta_2|^2$. Here the complex numbers $\alpha$, $\beta$, $\gamma$ and $\delta$ must satisfy $\hat{E}\hat{E}^\dagger = I$.

We then generalize our three-party QIA protocol to a multiparty one (more than three parties) (MQIA). In MQIA protocol, Trent can authenticate many users, $\{\text{Alice}_1, \text{Alice}_2, \cdots, \text{Alice}_r\}$ $(r > 2)$ simultaneously. Trent prepares an ordered $N$ $(r+1)$-particle GHZ states

\[
\frac{1}{\sqrt{2}} (|00\cdots0\rangle + |11\cdots1\rangle)_{TA_1, \cdots, A_r}. 
\]

(12)

The details of MQIA is very similar to those of three-party one. Trent sends $A_1, A_2, \cdots, A_r$ sequences to each Alice1, Alice2, $\cdots, \text{Alice}_r$. Similar to step (S2), Trent and the users check eavesdropping. If they confirm the quantum channel is safe, they continue to the next step. Otherwise, they abort the protocol. Trent divides the remaining GHZ states into $M$ ordered groups, $\{(\text{P}1)_{TA_1, \cdots, A_r}, (\text{Q}1)_{T'A_1', \cdots, A_r'}, \cdots , (\text{P}M)_{TA_1, \cdots, A_r}, (\text{Q}M)_{T'A_1', \cdots, A_r'}\}$. Alice1, Alice2, $\cdots, \text{Alice}_{(r-1)}$ each perform one of the two operations $\{I, i\sigma_y\}$ on their particles according to their authentication keys. Trent then performs randomly $I$ or $i\sigma_y$ operation on particle $T$ in each group. Each user measures particles $A_i$ and $A_i'$ $(i = 1, 2, \cdots, r)$ of each group in Bell basis. After measurements, Alice1, Alice2, $\cdots, \text{Alice}_r$
publish their measurement results. Trent performs Bell basis measurement on particles $T$ and $T'$ of each group and authenticates the users according to their measurement results.

In summary, we have presented a multiparty simultaneous quantum identity authentication protocol based on entanglement swapping. The trusted third party can authenticate many users simultaneously. If there are many users waiting for being authenticated by the system, the efficiency for identity authentication can be improved greatly.

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