PURE AND LOADED FIREBALLS IN SOFT GAMMA-RAY REPEATER GIANT FLARES

EHUD NAKAR1, TSVI PIRAN1,2 AND RE'EM SARI1

1THEORETICAL ASTROPHYSICS, CALTECH 130-33, PASADENA, CA, USA
2 RACAH INSTITUTE FOR PHYSICS, THE HEBREW UNIVERSITY, JERUSALEM 91904, ISRAEL

ABSTRACT

On December 27, 2004, a giant flare from SGR 1806–20 was detected on earth. Its thermal spectrum and temperature suggest that the flare resulted from an energy release of about $10^{46}$ erg close to the surface of a neutron star in the form of radiation and/or pairs. This plasma expanded under its own pressure producing a fireball and the observed gamma-rays escaped once the fireball became optically thin. The giant flare was followed by a bright radio afterglow, with an observable extended size, implying an energetic relativistic outflow. We revisit here the evolution of relativistic fireballs and we calculate the Lorentz factor and energy remaining in relativistic outflow once the radiation escapes. We show that pairs that arise naturally in a pure pairs-radiation fireball do not carry enough energy to account for the observed afterglow. We consider various alternatives and we show that if the relativistic outflow that causes the afterglow is related directly to the prompt flare, then the initial fireball must be loaded by baryons or Poynting flux. While we focus on parameters applicable to the giant flare and the radio afterglow of SGR 1806–20 the calculations presented here might be also applicable to GRBs.

1. INTRODUCTION

The giant flare from SGR 1806–20 was the most powerful flare of gamma-rays ever measured on earth. It lasted about 0.2sec. Its fluence of $\approx 1$ erg cm$^{-2}$ (Hurley et al. 2005; Palmer et al. 2005), correspond to energy of $3 \times 10^{46}$ erg released at a distance of $1$. Our conclusions do not depend on the exact distance. 15kpc (Corbel & Eikenberry 2004). This energy exceeds the energies of the giant flares from SGR 0526–66 (the famous March 5th 1979 event) and from SGR 1900+14 (August 27th 1998) by a factor of a hundred (Mazets et al. 1999). The spectrum of the flare is consistent with that of a cooling blackbody spectrum with an average temperature of $175 \pm 25$keV (Hurley et al. 2005). The reflection from the moon, as detected by Helicon-Corona-F (Golenetskii et al. 2004) with a fluence of $7.5 \times 10^{-7}$erg cm$^{-2}$, provides an alternative mean of estimating the fluence at the energy range 25–400keV. The albedo of the moon in this energy range is around 0.25 (Nakar et al. 2005), resulting in an isotropic energy release of about $10^{46}$ erg. This value is consistent with the fluence and the spectrum measured by RHESSI, since for a 170 keV black body, only a quarter of the energy is radiated in the 25–400 keV range.

Like the two other giant flares this flare was followed by a pulsed softer X-ray emission that lasted more than 380sec (Mazets et al. 2004). Radio afterglow was detected from VLA observations (Cameron et al. 2005; Gaensler et al. 2005$^2$). After one week the radio source was extended with a size of $\approx 15$kpc even though SGR 1806-20 might be closer by a factor of $\approx$2 (Figer et al. 2004; Nakar et al. 2005; Cameron et al. 2005) see however McClure-Griffiths & Gaensler (2005). We express the dependence of our calculations on the flare energy as implied from this distance

via annihilation of the magnetic field of a highly magnetized neutron star. This annihilation deposits energy at the form of photons and pairs near the surface of the neutron star. The pair-radiation plasma evolves as an accelerating fireball (Goodman 1986; Paczynski 1986; Shemi & Piran 1990; Piran et al. 1993; Meszaros et al. 1993; Grimsrud & Wasserman 1998) resulting in a thermal radiation burst carrying the bulk of the initial energy with roughly the original temperature and a fraction of the energy in the form of relativistic pairs. The thermal spectrum of the flare and its temperature support this picture.

Here we compare the energy required to produce the radio afterglow (calculated in §2) with a new simple calculation (§3) of the energy of the pairs outflow. As the available pairs energy is short by at least two orders of magnitude we consider (§4) baryonic or electromagnetically loaded fireballs, again providing new simple estimates for these cases. We compare our calculations of the fireball evolution to previous works in §5 and find that our simple estimates correct previous works. Finally, we discuss the implications of our results to the giant flare of Dec 27th flare from SGR 1806–20 and of August 27th from SGR 1900+14 in §6.

2. A LOWER LIMIT ON THE RADIO AFTERGLOW’S ENERGY

The energy emitted in the radio during the second week after the burst is about $10^{38}$erg (Cameron et al. 2005; Gaensler et al. 2005). This is clearly an absolute lower limit to the energy of the relativistic ejecta. A stronger limit can be obtained if we assume synchrotron emission as suggested by the optically thin spectrum, $F_\nu \propto \nu^{-0.7}$ and the observed linear polarization (Gaensler et al. 2005). We employ here the familiar equipartition method (Pacholczyk 1970; Scott & Readhead 1977). To emphasize the robustness of this method, which depends only on the assumption of synchrotron radiation, we sketch the derivation here. We characterize the emitting region by the number of its pairs $N$, the magnetic field, $B$, and the typical thermal Lorentz factor of the pairs, $\gamma_t$. We ignore the mildly

$^1$ Through out the paper we consider a distance of 15kpc even though SGR 1806-20 might be closer by a factor of $\approx$2 (Figer et al. 2004; Nakar et al. 2005; Cameron et al. 2005) see however McClure-Griffiths & Gaensler (2005). We express the dependence of our calculations on the flare energy as implied from this distance

$^2$ A radio afterglow was detected also after the August giant flare from SGR 1900+14 (Frail et al. 1999)
relativistic motion. The most conservative assumption that all the electrons emit at a synchrotron frequency of \( \nu_R = 8.5 \text{GHz} \) requires that the pairs have a Lorentz factor of:

\[
\gamma_e \approx \left( \frac{2 \pi m_e c^2 \nu_R}{eB} \right)^{1/2}.
\]

While to obtain the observed flux the number of pairs must satisfy:

\[
N \approx \frac{12 \pi d^2 eF_\nu R}{\sigma_T m_e c^2 B}.
\]

The sum of the energies of the pairs and of the magnetic field, \(E = R^3 B^2/6 + N m_e c^2 \gamma_e\), is minimized once the former is 3/4 of the latter:

\[
E_{\text{min}} \approx (5 \times 10^{42} \text{ergs}) \left( \frac{0.003}{\nu_\text{G}} \right)^\frac{12}{7} \times \left( \frac{F_\nu}{\text{mJy/Hz}} \right)^{\frac{12}{7}}.
\]

We assumed here, conservatively, that all the emission is radiated at \( \nu_R \). As the observed afterglow shows a spectral power law over more than an order of magnitude in frequency our estimate for \( E_{\text{min}} \) should be larger by a factor of 5 (Pacholczyk 1970; Cameron et al. 2005; Gaensler et al. 2005) bringing it to \( \approx 3 \times 10^{43} \text{erg} \).

If the radio afterglow of this SGR arises, like in GRBs or SNRs, from shocks going into the surrounding medium, farther constraint exists. The observed shock size determines its velocity and therefore the typical electron Lorentz factor. The external density and the Lorentz factor dictate the magnetic field. These additional constraints will force \( \gamma_e \) and \( N \) to deviate from equations 1 & 2 and therefore result in a higher energy estimate.

3. PAIRS-RADIATION FIREBALL

Consider an energy \( E = 10^{46} \text{erg} \) that is deposited in the vicinity of a neutron star \( R_0 \approx 10\text{cm} \) as photons with a typical energy \( e_\gamma \approx 500 \text{keV} \) (with a black body temperature of 170 keV, more than half of the energy is in photons with \( e > e_\gamma \)). If the energy is released instantaneously then the duration of the observed emission would be \( R_0/c \approx 0.1 \text{ms} R_{0.6} \). To be compatible with the observed duration of the flare, the source must have been active for a time comparable to the observed duration, \( t \approx 0.1 \text{sec} \). The optical depth for pair production would be:

\[
\tau_{\gamma \gamma} \gtrsim \frac{E \sigma_T}{4 \pi e^2 R_0 c t} \approx 2 \times 10^{11} \frac{E_{46} R_{0.6}}{L_1}.
\]

where \( \sigma_T \) is the Thompson cross-section and \( N_i \) denotes \( N/10^5 \) in cgs units. The large magnetic fields of the magnetar, decrease the effective cross-section (Herold 1979) but not sufficiently to make this radiation optically thin. With such a large optical depth the radiation forms a radiation-pairs plasma at a thermal equilibrium with an initial temperature of

\[
T_0 \approx \left( \frac{E}{4 \pi R_0^2 \sigma_T} \right)^{1/4} \approx 300 \text{keV} E_46^{1/4} R_{0.6}^{-1/2} L_1^{-1/4},
\]

where \( \sigma \) is the Stephan-Boltzmann constant. This radiation-pairs plasma expands relativistically (Goodman 1986; Paczynski 1986; Shemi & Piran 1990; Piran et al. 1993; Meszaros et al. 1993; Katz 1996; Thompson & Duncan 1996) with \( \Gamma \propto R \) and \( T \propto R^{3} \) until at \( R_\pm \) the pairs stop annihilating and their number freezes. The number of remaining pairs, \( N_{\pm} \), is determined by the condition that their annihilation rate \( n_{\pm} \sigma (\beta_{\pm}) \beta_{\pm} c \) equals the local expansion rate \( c/(R/\gamma) \approx c/R_0 \):

\[
N_{\pm} = \frac{4 \pi R_0 c t_3^3}{\sigma_T F^2_{\nu}} \approx 2.7 \times 10^{44} E_{46}^{3/4} R_{0.6}^{-1/2} L_1^{-1/4},
\]

where \( n_{\pm} \) is the pairs density and \( T_3 \), the temperature in the fireball rest frame at \( R_\pm \), is 18 keV (Goodman 1986; Paczynski 1986; Shemi & Piran 1990). We have used here the fact that at low energy the cross section for annihilation is \( \sigma (\beta) \approx \sigma_T \) with \( \beta_{\pm} \approx 1 \) the thermal velocity of the pairs in the local frame. The photons decouple from the pairs around the same time that the pairs freeze out. While at this stage the thermal velocity of the pairs \( v_{\pm} \) is much smaller than \( c \), the cross section for annihilation is larger than the cross section for photon scattering by the same factor. The escaping photons, which carry most of the energy, have a quasi-thermal spectrum (Goodman 1986; Grimusrud & Wasserman 1998) with \( T_{obs} = \Gamma R_{\pm} = T_0 \), roughly the initial temperature.

Even after the pairs stop annihilating and the photons decouple from the pair the pairs do not decouple from the photons. The huge photon flux continues to accelerate the pairs (Meszaros et al. 1993; Grimusrud & Wasserman 1998). The acceleration continues as long as the force that the photon field applies on an electron (positron) is sufficient to accelerate the electron so that it remains at the same Lorentz factor as the bulk of the photon field. The condition for effective acceleration is that during the time that the radius doubles the work done by the photon field on an electron \( E \sigma_T /4 \pi c R_0 c t^2 \) is larger than \( \gamma m_e c^2 \), the energy that the electron needs to gain during this period in order to keep up with the accelerating flow. This implies that the pairs accelerate until they reach a bulk Lorentz factor of:

\[
\Gamma_{\pm} = \left( \frac{E \sigma_T}{4 \pi c^2 \gamma m_e R_0} \right)^{1/4} \approx 680 E_{46}^{1/4} R_{0.6}^{-1/4} L_1^{-1/4},
\]

and their kinetic energy is:

\[
E_{\pm} = N_{\pm} m_e c^2 \Gamma_{\pm} \approx 1.4 \times 10^{41} \text{erg} E_{46} R_{0.6}^{3/4}.
\]

This kinetic energy is smaller by two orders of magnitude than the minimal energy required to produce the observed radio flux (Eq. 3). Therefore, the energy source of the radio afterglow cannot be the kinetic energy that remain in the pairs outflow.

It is important to note that while strong magnetic fields, that might be dragged from the magnetar into the fireball, may influence the interaction between the photons and the pairs they do not change the conclusion. Strong magnetic fields \( (B \gtrsim 10^{13} \text{G}) \) would decrease the cross-section for photon-electron (positron) scattering (Herold 1979) resulting in a smaller \( \Gamma_{\pm} \). In addition, a strong magnetic field would suppress the cross-section for pair annihilation into two photons, but it would also open a new channel of pair annihilation into a single photon (Wunner 1979; Harding 1986). The latter becomes the dominant annihilation process and its cross-section is larger than the cross-section with no magnetic field that was used in Eq. 6. The overall result is a lower \( N_{\pm} \). Thus, strong magnetic field would only decrease the energy that remain in the pairs. The energy of the magnetic field itself may, however, contribute a significant component to the energy of the relativistic outflow. We address this contribution in section §4.

\footnote{This result corrects the one presented in Grimusrud & Wasserman (1998) that find no dependence of the final pairs energy on the initial radius. We show here that \( E_{\pm} \propto R_{0.6}^{3/4} \). The value of \( E_{\pm} \) for \( R_0 = 10^6 \text{cm} \) is similar in both works (see §5).}
3.1. Interaction between the radiation and the circum burst medium

The energy needed to power the afterglow is only a small fraction of the total prompt \( \gamma \)-ray energy. It is therefore worthwhile exploring whether the interaction of the prompt radiation with the circum-flare medium can give rise to a relativistic outflow with the required energy. The optical depth is given by \( \tau = \sigma_T n R \), where \( n \) is the ambient density and \( R \approx 10^{16} \text{ cm} \) is the observed size of the radio emitting region. The energy acquired by the ambient electrons within \( R \) is therefore

\[
\frac{E_{\text{in}}} {E} = \sigma_T n R = 5 \times 10^{-3} n R_{16} \tag{9}
\]

Tapping \( 10^{-3} \) of the energy requires an unreasonably large average density of \( n \geq 10^{16} \text{ cm}^{-3} \). This is not expected around the magnetar given possible previous bursts. Moreover, giving this amount of energy to such a large mass would result in a sub-relativistic velocity (\( v \approx 0.1c \)).

An alternative mechanism can be pair enrichment by collisions of the outgoing radiation with photons that are backscattered by the ambient medium (Thompson & Madau 2000; Beloborodov 2002; Mészáros et al. 2001). These pairs would in turn scatter more photons that would create more pairs and so forth. This process will take place up to a radius where each ambient electron scatters at least one photon:

\[
R_{\text{enrich}} = \left( \frac{E \sigma_T} {4 \pi \epsilon_\gamma} \right)^{1/2} = 2.5 \times 10^{13} \text{ cm} E_{46}^{1/2}. \tag{10}
\]

At smaller radii the number of pairs will grow exponentially until there will be about \( m_p / m_e \) pairs for each ambient medium electron. At this point there are enough pairs to accelerate the ambient medium to a relativistic velocity. Once this happens the temperature of the radiation in the rest frame of the accelerated medium drops significantly. Since the spectrum of the radiation is thermal there are no photons with an energy larger than \( m_e c^2 \) at this frame. The scattered photons stop creating additional pairs and the exponential process stops. Thus, up to \( R_{\text{enrich}} \) the density of pairs is expected to be about \( m_p / m_e \) times larger than the external medium density. The fraction of energy acquired by this pair enriched region is

\[
\frac{E_{\text{enrich}}} {E} = \frac{m_p} {m_e} \sigma_T R_{\text{enrich}} n \approx 10^{-8} E_{46}^{1/2} R_{16}. \tag{11}
\]

Again, this enrichment is insufficient to tap \( 10^{-3} \) of the initial energy unless the external medium average density is \( \sim 10^{5} \text{ cm}^{-3} \). Even if one sets such large density around the neutron star, the high density region must be truncated shortly after \( 3 \times 10^{11} \text{ cm} \), to allow for the observed mildly relativistic motion. Such a configuration seems to be too contrived.

We conclude that the interaction between the prompt radiation and the external medium is unlikely to be the source of the afterglow energy.

4. LOADED FIREBALLS

4.1. Baryonic Load

The processes that govern the evolution of a fireball loaded with protons are similar to those that govern a pure pairs-radiation fireball. However, the electrons that accompany the protons contribute to the opacity while the protons contribute to the inertia. Both effects can be taken into account by generalizing Eqs. 7-8. To do so we replace \( m_e \) with the mean mass per particle:

\[
\bar{m} = m_p \frac{(m_p/m_e) N_+ + N_p} {N_+ + N_p}, \tag{12}
\]

and we replace \( N \pm \) with the total electrons and positrons density:

\[
N = N_p + N_\pm, \tag{13}
\]

where \( N_p = E / (m_p c^2 \eta) \) is the number of protons and \( \eta \equiv E / M c^2 \) characterize the baryonic load. The generalized equations are valid as long as the baryons load is small enough and the radiation escapes before most of its energy is converted to the baryonic kinetic energy.

The behavior of \( \bar{m} \) and \( N_p / N_\pm \) as a function of the baryon load, \( \eta \), depends on the ratio \( N_p / N_\pm \). There are two critical values of this ratio: \( N_p / N_\pm = m_e / m_p \) and \( N_p / N_\pm = 1 \). The former marks equal mass for baryons and pairs, the latter marks equal contribution to the Thompson scattering. These values correspond respectively to the following critical values of \( \eta \):

\[
\eta_1 = \frac{E \sigma_T} {4 \pi R_6 \epsilon_{\text{cm}} c^2} \left( \frac{T_\gamma} {T_0} \right)^3 = 4.5 \times 10^7 E_{46}^{1/4} R_6^{-1/2} T_0^{-1/4}, \tag{14}
\]

\[
\eta_2 = \frac{E \sigma_T} {4 \pi R_6 \epsilon_{\text{cm}} c^2} \left( \frac{T_\gamma} {T_0} \right)^3 = 2.5 \times 10^4 E_{46}^{1/4} R_6^{-1/2} T_0^{-1/4}. \tag{15}
\]

An additional critical value of \( \eta \) is defined by the condition that the photons have transferred effectively all their energy to the baryons:

\[
\eta_3 = \left( \frac{E \sigma_T} {4 \pi R_6 \epsilon_{\text{cm}} c^2} \right)^{1/4} \approx 100 E_{46}^{1/4} R_6^{-1/4} T_0^{-1/4}. \tag{16}
\]

For this critical \( \eta \) the photons decouple from the baryons just at the moment that the baryons stop being accelerated by the photons.

Figure 1 illustrates the dependence of the energy that remain in the plasma, \( E_{p, \pm} \), and its final Lorentz factor \( \Gamma_{p, \pm} \) on \( \eta \). When \( \eta_1 < \eta \) the baryons do not affect the evolution, \( \bar{m} \approx m_e \) and \( N_p \approx N_\pm \), and therefore the fireball evolves as a pure pairs-plasma fireball. The bulk Lorentz factor and energy are given by equations 7 and 8. When \( \eta_2 < \eta \), the baryons carry most of the inertia while the pairs are still responsible for most of the opacity. Therefore, \( \bar{m} \approx m_e \eta / \eta_1 \) while \( N_p \approx N_\pm \) and \( E_{p, \pm} \approx \eta^{3/4} \) while its final Lorentz factor \( \Gamma_{p, \pm} \approx \eta^{1/4} \). In this case the pairs become transparent for the radiation at \( R_\pm \). For \( \eta_1 < \eta < \eta_2 \) the baryons provide both the inertia and the opacity. The final Lorentz factor is constant and \( E_{p, \pm} \propto \eta^{-3/4} \). The radiation decouples from the electrons at radius larger than \( R_\pm \), but the radiation still carry most of the energy and its temperature is \( \approx T_0 \). For \( \eta < \eta_1 \) the radiation decouples from the baryons only after it transferred most of its energy to the baryons. The baryons energy is the total energy \( E \) and their Lorentz factor is \( \eta \). The radiation energy however is much smaller than \( E \) and its temperature is much smaller than \( T_0 \). We call this \( (\eta < \eta_1) \) a heavy load.

The final Lorentz factor can be approximated as (see Fig 1):

\[
\Gamma_{p, \pm} \approx \begin{cases} 
680 E_{46}^{1/4} R_6^{-1/4} \eta_1^{-1/4} & \eta_1 < \eta \\
680 (\eta_1 / \eta)^{1/4} E_{46}^{1/4} R_6^{-1/4} \eta_1^{-1/4} & \eta_1 < \eta < \eta_2 \\
100 E_{46}^{1/4} R_6^{-1/4} \eta_2^{-1/4} & \eta_2 < \eta < \eta_3 \\
& \eta < \eta_3
\end{cases} \tag{17}
\]

While the energy that remain in the ejecta is:

\[
\frac{E_{p, \pm}} {E} \approx \begin{cases} 
1.4 \times 10^{-5} R_6^{-3/4} & \eta_1 < \eta \\
4 \times 10^{-3} (\eta_1 / \eta_2)^{-3/4} R_6^{-3/4} & \eta_1 < \eta < \eta_2 \\
\eta_1 / \eta & \eta_2 < \eta < \eta_3 \\
1 & \eta < \eta_3
\end{cases} \tag{18}
\]
So far we have only considered protonic loading. The evolution of a neutron rich fireball was explored in the past by several authors (Derishev et al. 1999; Beloborodov 2003; Rossi et al. 2004; Vlahakis et al. 2003). The neutrons are coupled to the plasma only through collisions with protons. Initially the protons drag the neutrons efficiently. A neutron that collides with a proton receives in such a collision at most 1 GeV. Therefore, the dragging takes place as long as each neutron collides with at least one proton during the time that the protons double their Lorentz factor. This happens when:

$$R_{ph} \sigma_0 / \Gamma \geq 1$$

(19)

where \(n_p\) is the protons density and \(\sigma_0 = \sigma_{np} \beta_{rel} \approx 3 \cdot 10^{-26} \text{cm}^2\) where \(\sigma_{np}\) is the neutron-proton cross section and \(\beta_{rel}\) is the relative velocity between neutrons and protons.

With \(\sigma_0\) about an order of magnitude lower than \(\sigma_T\), the neutrons always decouple from the protons before the radiation decouples from the plasma. If the neutrons do not decouple before the acceleration ends the load is essentially heavy and the photons transfer most of their energy to the baryons. The final Lorentz factor in this case is \(\eta\), corresponding to the total baryonic load. If the neutrons decouple before the acceleration ends then during the decoupling phase (when the neutrons begin to lag after the protons) the neutrons become relativistically hot and the inelastic n-n collisions convert neutrons to protons while producing pions. If initially \(N_p / N_n \gg 1\), a significant fraction of the neutrons will be converted to protons and this ratio will become of order unity (Derishev et al. 1999) or somewhat larger (Fuller et al. 2000) after decoupling. Thus the final protons load, that carries most of the energy, is comparable to the initial total baryonic load. The final Lorentz factor and the energy of the plasma can be approximated by Eqs. 17 & 18 if \(\eta\) is defined according to the total baryonic load.

4.2. Heavy Loading

For completeness we also consider here the energy and the temperature of the thermal radiation that escapes from a heavily loaded fireball, namely a fireball with \(\eta \leq \eta_0\). This situation is clearly inapplicable to this giant flare, but it may be relevant to GRBs, possibly as an explanation of precursors which are less energetic than the burst itself. In this case the Lorentz factor saturates at \(R_\eta = R_0 \eta\), and the fireball enters its coasting phase. The fireball is coasting without spreading (both in the local and the observer frame) until \(R_{spread} = c t \eta^2\). At larger radii the fireball spreads to a width of \(\sim R / \eta^2\) in the observer frame. The radiation in this case (\(\eta < \eta_0\)) decouples from the matter long after \(R_\eta\) at:

$$R_{ph} = \begin{cases} R_{ph} \left( \frac{E_{\gamma EM} \beta_{ph}}{4 \pi m_\eta c \eta} \right)^{1/2} & R_{ph} < R_{spread} \\ R_{ph} > R_{spread} \end{cases}$$

(20)

During the acceleration phase \((R < R_p)\) the photons cool in their rest frame. The acceleration compensates for this cooling keeping the temperature and the overall radiation energy in the observer frame, \(E_{ph}\), constant. During the coasting phase, with no acceleration, the photons cool and lose energy in the observer frame as well. Since the ratio of the photon number to the proton number, \(\sim m_p c^2 / \eta T_0 \gg 1\), is roughly constant at all times, the photons govern the cooling with an adiabatic index of \(4/3\) \((TV^{1/3} = \text{const})\, \text{where} \, V\) is the volume.

For \(R_{ph} < R < \min(R_{ph}, R_{spread})\), i.e during the coasting phase while the fireball is still opaque, \(V \propto R^2\) and therefore \(T \propto R^{-2/3}\) and \(E_{ph} \propto R^{-2/3}\). If the fireball is still opaque at \(R_{spread}\) then for \(R_{spread} < R < R_{ph}\) the radiation evolves with \(T \propto R^{-1}\) and \(E_{ph} \propto R^{-1}\). Thus as far as the radiation is concerned there is another critical \(\eta\) for which \(R_{ph} = R_{spread}\):

$$\eta_4 = 8E_{46}^{1/5} L_{-1}^{-2/5}$$

(21)

For \(\eta_4 < \eta < \eta_3\):

$$R_{ph} \frac{\eta}{\eta_0} = \left( \frac{m_p}{\eta} \right)^4$$

(22)

$$\frac{E_{ph}}{R_{obs}} = \frac{T_{ph}}{R_0} = \left( \frac{m_p}{\eta} \right)^{4/3} \left( \frac{\eta_0}{\eta} \right)^{2/3} E_{46}^{2/3} R_{0.6}^{2/3} L_{-1}$$

(23)

In GRBs the final Lorentz factor of the relativistic ejecta is \(\gtrsim 300\), as indicated by opacity considerations (Lithwick & Sari 2001), and therefore \(\eta \lesssim 300\). On the other hand, the non-thermal spectrum of the prompt emission implies \(\eta \lesssim \eta_3 \approx 10^3\) for GRBs, making the range of allowed \(\eta\) very narrow. It implies also that the ratio between the energy in the thermal radiation that escapes the fireball and its kinetic energy is \((\eta_3 / \eta)^{8/3} \gtrsim (1000 / 300)^{8/3} \approx 5\%\). Taking into account the efficiency in which this kinetic energy is converted into a non-thermal radiation, it indicates that there should be a non-negligible thermal component in almost any GRB. This result may be supported by the observations, as suggested by Ryde (2005). The idea that a thermal component in the prompt emission is indeed the radiation that escapes from the fireball can be tested simply by comparing the energy and the temperature in this component:

$$T_{obs} = 1\text{MeV} E_{ph.51} E_{c2}^{3/4} R_{0.6}^{1/2} L_{-1}^{-1/4}$$

(24)

Where \(E_{ph}\) is the energy in the thermal component and \(E\) is the total observed energy (note that we use here the typical values for GRBs). This relation should be tested within a given burst pulse by pulse.

4.3. Electromagnetic load

An alternative loading of the fireball is an electromagnetic load. Consider a magnetic field that is confined to the fireball and that is accelerated at it. The magnetic field carries an energy \(\gamma E_{EM}\) where \(E_{EM}\) is the electromagnetic energy in the fireball’s rest frame at the time that the pairs decouple from the radiation. The electromagnetic loading can be treated similarly to the baryonic loading where the magnetic field contribution to the inertial mass is:

$$m = m_e + \frac{E}{\eta_{EM} N_\pm c^2}$$

(25)

and \(\eta_{EM} = E/E_{EM}\) is a measure of the electromagnetic loading. The total number of pairs, and therefore the opacity, is similar to those of a pure pairs-radiation fireball:

$$N = N_\pm$$

(26)

\(^4\)This is true as long as the magnetic field is much smaller than \(10^{12}\) G at \(R_\pm\), so the cross-section for Compton scattering is not affected. In our case where \(E \approx 10^{46}\) erg, \(R_\pm \approx 10^3\) cm and if the relevant loading is \(\eta_{EM} \gtrsim 10\) this condition is satisfied.
Therefore, there are only two critical values of $\eta_{EM}$: $\eta_{EM,1} = \eta_1$ and $\eta_{EM,2} = T_0/T_{\pm}$. When $\eta_1 < \eta_{EM}$ the fireball evolves as a pure pairs-plasma fireball. When $\eta_{EM,2} < \eta_{EM} < \eta_{EM,1}$ the radiation still carries most of the energy and its observed temperature is $T_0$. The total energy that is left in the magnetized pairs only after it has transferred to them most of the energy, for $\eta_{EM} < \eta_{EM,2}$ the radiation decouples from the magnetized pairs only after it has transferred to them most of the energy. In this case $E_{EM,\pm} = E$ while the radiation energy is $E_{\gamma\phi} = E(\eta_{EM,2}/\eta_1)^{8/3}$ and its temperature $T_{\gamma\phi} = T_0(\eta_{EM,2}/\eta_1)^{8/3}$. Note that when the electromagnetic loading is significant ($\eta_{EM} < \eta_1$) the Lorentz factor of the pairs at the time that they decouple from the radiation is only a lower bound on their final Lorentz factor. The reason is that energy transfer between the electromagnetic field and the pairs can still take place at larger radii.

5. COMPARISON WITH PREVIOUS WORKS

The evolution of pure and loaded fireballs was explored by several authors in the past. Paczynski (1986) and Goodman (1986) considered only the properties of the radiation that emerges from a pure fireball. Shemi & Piran (1990), Piran et al. (1993) and Meszaros et al. (1993) considered the evolution of a loaded fireball. These papers miscalculated the radius that the baryons decouple from the radiation, resulting in an overestimate of $\eta_3$ and of the final Lorentz factor and the ejecta energy, for $\eta > \eta_3$. In the context of GRBs our result ($\eta_3 \sim 10^3$) is significantly lower than in those papers, and combined with the lower limits on the Lorentz factor, $\eta_3 \sim 500$ is narrowly constrained. More recently, Grimsrud & Wasserman (1998) carried out a detailed numerical and analytical calculation of the final Lorentz factor and energy of pure and loaded fireballs. Our much simpler estimates generally agree with their results with the exception of the final energy that remain in the pairs, $E_{\pm}$ and their opacity for the radiation once the Lorentz factor saturates $\tau_{\pm}$. While Grimsrud & Wasserman (1998) find that both quantities do not depend on the initial radius, we find that $E_{\pm} \propto R_0^{-3/4}$ and that $\tau_{\pm} \propto R_0^{-3/4}$ as well. However, the value that these authors obtain for $R_0 = 10^6$ cm are similar to the values that we present here.

6. DISCUSSION

We presented here a simple, yet comprehensive, derivation of the evolution of pure and loaded fireballs. Our results correct previous works (see §5), enabling us to compare the energy that remains in the plasma once the radiation escapes to the observations of the radio afterglow of the giant flare from SGR 1806-20.

The relativistic ejecta that produced this radio afterglow contained at least 0.1% of the energy emitted in $\gamma$-rays. The thermal spectrum of the flare and its temperature indicate that the flare resulted from an energy deposition near the neutron star surface. Such energy deposition must create an opaque accelerating fireball. An elegant explanation for the afterglow energy source could have been the inevitable energy of the remaining pairs after they decouple from the radiation. However, we find that this energy is short by at least two orders of magnitude, excluding this possibility. We considered also the possibility that the energy of the afterglow is obtained by the interaction of the outgoing radiation with the external medium. We find this scenario to be unlikely since it requires a rather high external density with a contrived profile.

We conclude, therefore, that if the relativistic ejecta that produces the afterglow is directly related to the prompt flare (rather than e.g. to the following confined fireball), then the fireball must have been loaded with either baryons or Magnetic field in

**Fig. 1.** The baryons’ energy (thick line, left y-axis) and the baryons’ final Lorentz factor (thin line, right y-axis) as functions of $\eta$ (lower x-axis). For clarity the total baryons mass is depicted on the upper x-axis. Also marked (short dashed horizontal lines) are the lower limit on the energy inferred from the radio afterglow and the energy that remains in the pairs in a pure radiation-pairs fireball. The vertical dashed lines mark the allowed loading in order to produce a thermal flare at the observed temperature. The parameters in this plot are $E = 3.5 \times 10^{39}$ erg, $t = 0.1$ sec and $R_0 = 10^6$ cm.
order to enhance the energy that remains in this ejecta. This loading should be however fine tuned in order to obtain the right amount of energy. Even if the afterglow energy is comparable to that of the ejecta (we have only lower limit on this energy) the loading cannot be too high. If \( \eta < \eta_1 \) the energy in the flare drops significantly and so does the temperature. The agreement of the observed temperature and the black body temperature estimated in Eq. 5 suggest that this is not the case. For the December 27\textsuperscript{th} giant flare the range of the allowed baryonic loading is \( 100 < \eta < 10^4 \), which corresponds to a baryons mass of \( 10^{20} \text{gr} \lesssim m_b \lesssim 10^{21} \text{gr} \) (see fig. 1).

Interestingly similar lower limit on the ratio of the afterglow energy to the flare energy was observed in the August giant flare from SGR 1900+14 (Frail et al., 1999), although this flare was hundred times dimmer than the December 27\textsuperscript{th} giant flare. With the caveats of small number statistics, and both afterglow energies being lower limit, this similarity suggests a common origin for the two afterglows and a linear relation between the observed afterglow energy and the flare energy. The similarity between the fractional energy in the afterglows of the two events, is not accounted for in our model. The most natural energy source, the pairs energy in a baryon free fireball, contain a constant fraction of the fireball energy (see Eq. 8) but this fraction is far too short. If this similarity is confirmed by future events it may indicate that some process regulates the baryonic load. For example it is possible that a constant number of baryons per unit energy are torn apart from the surface of the neutron star and are mixed with the fireball. Alternatively it is possible that a fixed fraction of the energy is ejected as a Poynting flux. A different solution might be that the fireball is heavily loaded aspherically, such that only a small portion of the \( 4\pi \) solid angle is loaded, while the rest of the fireball is pure. In this case \( E_{\text{p}} / E \) is simply the ratio of loaded portion solid angle to the pure portion solid angle, and this ratio might be similar between different flares. It is, of course, also possible that the afterglow is not related directly to the prompt flare and that it is created by an independent mechanism and its energy is not extracted of the initial fireball. Once more the similarity between the ratios of afterglow to flare energies in both cases poses a puzzle for this last explanation as well.

This research was partially funded by a US-Israel BSF grant and NASA ATP NNG05GF58G grant. E. N. was supported at Caltech by a senior research fellowship from the Sherman Fairchild Foundation. R. S. is a Packard Fellow and an Alfred P. Sloan Research Fellow. We thank Brian Cameron, Kevin Hurley, Jonathan Katz, Arieh Konigl and Shri Kulkarni for helpful discussions.

REFERENCES

Beloborodov, A. M. 2002, ApJ, 565, 808  
—. 2003, ApJ, 585, L19  
Cameron, P. B. et al. 2005, ArXiv Astrophysics e-prints  
Corbel, S., & Eikenberry, S. S. 2004, A&A, 419, 191  
Derishev, E. V., Kocharovsky, V. V., & Kocharovsky, V. V. 1999, ApJ, 521, 640  
Duncan, R. C., & Thompson, C. 1992, ApJ, 392, L47  
Eichler, D. 2002, MNRAS, 335, 883  
Figer, D. F., Najarro, F., & Kudritzki, R. P. 2004, ApJ, 610, L109  
Frail, D. A., Kulkarni, S. R., & Bloom, J. S. 1999, Nature, 398, 127  
Fuller, G. M., Pruet, J., & Abazajian, K. 2000, Physical Review Letters, 85, 2673  
Gaensler, B. M. et al. 2005, ArXiv Astrophysics e-prints  
Golenetskii, S., Aptekar, R., Mazets, E., Pal’Shin, V., Frederiks, D., & Cline, T. 2004, GRB Circular Network, 2923, 1  
Goodman, J. 1986, ApJ, 308, L47  
Grimsrud, O. M., & Wasserman, I. 1998, MNRAS, 300, 1158  
Harding, A. K. 1986, ApJ, 300, 167  
Herold, H. 1979, Phys. Rev. D, 19, 2868  
Hurley, K. et al. 2005, ArXiv Astrophysics e-prints  
Katz, J. I. 1982, ApJ, 260, 371  
—. 1994, ApJ, 422, 248  
—. 1996, ApJ, 463, 305  
Lithwick, Y., & Sari, R. 2001, ApJ, 555, 540  
Mészáros, P., Ramirez-Ruiz, E., & Rees, M. J. 2001, ApJ, 554, 660  
Mazets, E., Golenetskii, S., Aptekar, R., Frederiks, D., Pal’Shin, V., & Cline, T. 2004, GRB Circular Network, 2922, 1  
Mazets, E. P., Cline, T. L., Aptekar’, R. L., Butterworth, P. S., Frederiks, D. D., Golenetskii, S. V., Il’Inskii, V. N., & Pal’Shin, V. D. 1999, Astronomy Letters, 25, 635  
McClure-Griffiths, N. M., & Gaensler, B. M. 2005, ArXiv Astrophysics e-prints  
Meszaros, P., Laguna, P., & Rees, M. J. 1993, ApJ, 415, 181  
Nakar, E., Gal-Yam, A., Piran, T., & Fox, D. B. 2005, ArXiv Astrophysics e-prints  
Pachočczyk, A. G. 1970, Radio astrophysics. Nonthermal processes in galactic and extragalactic sources (Series of Books in Astronomy and Astrophysics, San Francisco: Freeman, 1970)  
Paczynski, B. 1986, ApJ, 308, L43  
—. 1992, Acta Astronomica, 42, 1  
Palmer, D. M. et al. 2005, ArXiv Astrophysics e-prints  
Piran, T., Shemi, A., & Narayan, R. 1993, MNRAS, 263, 861  
Rößl, E. M., Beloborodov, A. M., & Rees, M. J. 2004, in AIP Conf. Proc. 727: Gamma-Ray Bursts: 30 Years of Discovery, 198–202  
Ryde, F. 2005, ArXiv Astrophysics e-prints  
Scott, M. A., & Readhead, A. C. S. 1977, MNRAS, 180, 539  
Shemi, A., & Piran, T. 1990, ApJ, 365, L55  
Thompson, C., & Duncan, R. C. 1995, MNRAS, 275, 255  
—. 1996, ApJ, 473, 322  
Thompson, C., & Madau, P. 2000, ApJ, 538, 105  
Vlahakis, N., Peng, F., & Königl, A. 2003, ApJ, 594, L23  
Woods, P. M., Kouveliotou, C., Güçlüs, E., Finger, M. H., Swank, J., Smith, D. A., Hurley, K., & Thompson, C. 2001, ApJ, 552, 748  
Wunner, G. 1979, Physical Review Letters, 42, 79