CHIRAL SOLITON MODEL PREDICTIONS FOR PENTAQUARKS

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We briefly describe chiral soliton model description of baryons and predictions for exotic antidecuplet. We discuss successful phenomenology which triggered experimental searches and problems which arise in the formal limit of large $N_c$.

1 Do we see $\Theta^+$ at all?

After almost two year excitement that the exotic antidecuplet has been discovered\(^1\) the results from high statistics G11 experiment at CLAS were presented in April at the APS meeting with negative result for the photoproduction of $\Theta^+$ on proton\(^2\). The sighting of the heaviest members of $\Xi^{10}$ that were seen only by NA49 experiment at CERN\(^3\) is even more problematic. Nevertheless the positive evidence of 11 experiments that reported the existence of $\Theta^+$ cannot be simply ignored. The reasons why some experiments see $\Theta^+$ while the others do not maybe either of experimental nature or a peculiar production mechanism or both. Therefore the present confusion concerning exotics calls for a new high precision $KN$ experiment in the interesting energy range.

2 Chiral models

Light antidecuplet was predicted within the chiral soliton models ($\chi$SM)\(^4\)\(^-\)\(^8\). Early estimate $\Delta M_{\Xi^{10}-\Xi^8} \sim 600$ MeV was obtained already in 1984 in a specific modification of the Skyrme model\(^5\). The estimates of both $\Theta^+$ and $\Xi^{10}$ masses obtained in the Skyrme model in 1987 are in a surprising agreement with present experimental findings\(^6\).

In this Section we will demonstrate that chiral models are deeply rooted in QCD and take into account quark degrees of freedom maybe even in a more complete way than the quark models.
themselves. The low energy effective theory of QCD could be in principle obtained by integrating out gluons. The resulting quark lagrangian would preserve chiral symmetry, whose spontaneous breakdown would produce nonzero constituent quark mass $M$ and the massless pseudoscalar Goldstone bosons, being at the same time $\bar{\psi}\psi$ pairs, would be present. A convenient model of such a lagrangian is provided by a semibosonized Nambu–Jona-Lasinio model:

$$\mathcal{L} = \bar{\psi} (i \gamma \partial - MU^\gamma [\varphi]) \psi$$

which looks like a Dirac Lagrangian density for a massive fermion $\psi$ if not for matrix $U$. In fact $\psi$ is a 3-vector in flavor space and also in color. Matrix

$$U^\gamma = \exp \left\{ \frac{i}{F_\varphi} \vec{\lambda} \cdot \vec{\varphi} \gamma_5 \right\}$$

parameterized by a set of eight pseudoscalar fields $\vec{\varphi}$ guarantees chiral symmetry of $\mathcal{L}$, given by a global multiplication of the fermion field by a phase factor

$$\psi \rightarrow e^{i\vec{x} \cdot \vec{\alpha} \gamma_5} \psi$$

provided we also transform meson fields

$$U^\gamma [\varphi] \rightarrow e^{-i\vec{x} \cdot \vec{\alpha} \gamma_5} U^\gamma [\varphi] e^{-i\vec{x} \cdot \vec{\alpha} \gamma_5}.$$  

Note that the color indices produce simply an overall factor $N_c$ in front of (1).

Since the vacuum state corresponds to $U^\gamma = 1$, spontaneous chiral symmetry breaking indeed takes place. Moreover the massless Goldstone bosons appear when we integrate out the quark fields. Then the resulting effective action contains only meson fields and can be organized in terms of a derivative expansion

$$S_{\text{eff}}[\varphi] = \frac{F_\varphi^2}{4} \int \text{Tr} \left( \partial_\mu U \partial^\mu U^\dagger \right) + \frac{1}{32e^2} \int \text{Tr} \left( \left[ \partial_\mu U U^\dagger, \partial_\nu U U^\dagger \right]^2 \right) + \Gamma_{\text{WZ}} + \ldots$$

where constants $F_\varphi$ and $e$ can be calculated from (1) with an appropriate cut-off. $\Gamma_{\text{WZ}}$ is the Witten Wess-Zumino term which takes into account axial anomaly. Perhaps the most important part are the ellipses which encode an infinite set of terms that are effectively summed up by the fermionic model of Eq. (1). The truncated series of Eq. (5) is the basis of the Skyrme model. Hence the Skyrme model is (a somewhat arbitrary, because it does not include another possible 4 derivative term) approximation to (1).

At this point both models, chiral quark model of Eq. (1) and Skyrme model of Eq. (5) (without the "dots"), look like mesonic theories describing only meson-meson scattering. Baryons are introduced in two steps, following large $N_c$ strategy described by Witten. First, one constructs a soliton solution, i.e. solution to the classical equations of motion that corresponds to matrix $U_0$ which cannot be expanded in a power series around unity. Second, since the classical soliton has no quantum numbers (except baryon number), one has to quantize the system. Perhaps this quantization procedure, which reduces both models to the nonrelativistic quantum system analogous to the symmetric top with two moments of inertia $I_{1,2}$, makes chiral-soliton models look odd and counterintuitive.

In chiral quark soliton models stabilization of the soliton occurs due to the valence quark level which also provides the baryon number. In the Skyrme model where no quarks are present the soliton is stable due to the specific choice of the 4-derivative term in (5) and the baryon number is given as a charge of the conserved topological current. The quantization on the other hand proceeds in both models almost identically, the only difference being that some model parameters dominated by the valence level in the quark soliton model are exactly zero in the Skyrme model.
3 Exotics in chiral models

Chiral soliton models predict that positive parity baryons fall into SU(3) representations that contain hypercharge $Y = N_c/3$ which is 1 in the real world. Therefore the lowest lying multiples are octet and decuplet, exactly as in the quark models. Moreover chiral models predict a tower of exotic rotational states starting with $\frac{10}{2}, \frac{27}{3}/2, \frac{35}{5}/2, \frac{3}{2}$ (subscripts refer to spin) etc. The splittings between the centers of the lowest-lying octet, decuplet and antidecuplet baryons are given in the $\chi$SM by
\[
\Delta M_{10-8} = 3/(2I_1), \quad \Delta M_{10-8} = N_c/(2I_2) = 3/(2I_2)
\]
where $I_{1,2}$ are two soliton moments of inertia that depend on details of the chiral Lagrangian. Since $I_1, I_2 \sim \mathcal{O}(N_c)$, this means that $\Delta M_{10-8} \sim \mathcal{O}(N_c^0)$, whereas $\Delta M_{10-8}$ is $\mathcal{O}(1/N_c)$. This has triggered some arguments $^{11,12}$ and counter-arguments $^{13}$ regarding the applicability of collective coordinate quantization to the $\bar{10}$.

We have already mentioned early estimates of the antidecuplet mass that have been recently reviewed in $^{14}$. The bottom line is that antidecuplet is much lighter than in the quark models. Therefore $\chi$SM’s predict light exotic baryons belonging to antidecuplet of positive parity.

Perhaps the most striking prediction of $\chi$SM is the small width of the antidecuplet states. The decay width is calculated by means of the formula for the decay width for $B \rightarrow B' + \varphi$:
\[
\Gamma_{B \rightarrow B' + \varphi} = \frac{1}{8\pi M M'} \frac{p_\varphi}{M^2} = \frac{1}{8\pi M M'} A^2
\]
up to linear order in $m_\varphi$. The “bar”over the amplitude squared denotes averaging over initial and summing over final spin (and, if explicitly indicated, over isospin). Anticipating linear momentum dependence of the decay amplitude $\mathcal{M}$ we have introduced reduced amplitude $A$ which does not depend on the meson momentum $p_\varphi$.

Soliton models can be used to calculate the matrix element $\mathcal{M}$. Explicitly
\[
\Gamma_{B \rightarrow B' + \varphi} = \frac{3}{8\pi M M'} \frac{G_R^B}{G_R^{B'}} \frac{C_R^B}{C_R^{B'}} p_\varphi^2.
\]
For antidecuplet decays ($R = \bar{10}$):
\[
G_{\bar{10}}^0 = G_0 - G_1 - 1/2 G_2, \quad C_{\bar{10}}^{0+1}_{\varphi} = 1/5,
\]
whereas for decuplet ($R = 10$):
\[
G_{10} = G_0 + 1/2 G_2, \quad C_{10}^{0+1}_{\varphi} = 1/5.
\]
In the nonrelativistic small soliton limit $^{15}$ in which chiral quark soliton model reproduces many results of the nonrelativistic quark model $G_1/G_0 = 4/5, G_2/G_0 = 2/5$ and $G_{\bar{10}} = 0!$ This nonintuitive cancellation $^{7}$ explains the small width of antidecuplet as compared to the one of 10 for example.

One problem concerning this cancellation is that formally
\[
G_0 \sim \mathcal{O}(N_c^{3/2}) + \mathcal{O}(N_c^{1/2}), \quad G_{1,2} \sim \mathcal{O}(N_c^{1/2})
\]
and it looks as if the cancellation were accidental as it occurs between terms of different order in $N_c$. For arbitrary $N_c$ antidecuplet $\bar{10} = (0, 3)$ generalizes to “$\bar{10}$” = (0, $\frac{N_c+3}{2}$), decuplet ”$10$” = (3, $\frac{N_c-3}{2}$) and octet ”$8$” = (1, $\frac{N_c-1}{2}$) $^{16}$ and the pertinent Clebsch-Gordan coefficients in fact depend on $N_c$:
\[
G_{\bar{10}} = G_0 - (N_c + 1)/4 G_1 - 1/2 G_2.
\]
So the subleading $G_1$–term is enhanced by additional factor of $N_c$ and the cancellation is consistent with $N_c$ counting\textsuperscript{17}.

Unfortunately there is another problem concerning the $N_c$ counting of the decay width. Because of\textsuperscript{6}

$$p_\pi \sim \mathcal{O}(1/N_c), \quad p_K \sim \mathcal{O}(1)$$

(12)

and consequently

$$\Gamma_{\Delta \to N^+\pi} \sim \mathcal{O}(1/N_c^2), \quad \Gamma_{\Theta^+ \to N^+K} \sim \mathcal{O}(1)$$

(13)

in the chiral limit. This $N_c$ counting contradicts experimental findings which suggest $\Gamma_{\Theta^+ \to N^+K} \ll \Gamma_{\Delta \to N^+\pi}$.

4 Closing remarks

Let us finish by summarizing and by adding some remarks.

Experimental situation concerning the existence of exotic baryons is unclear. The new data on photoproduction on deuteron from LEPS with positive evidence have been presented on various conferences but not published. Soon the similar data from G10 experiment at JLab will be released, however the decisive experiment would be certainly – if ever completed – high resolution KN scattering experiment.

Little is known about the production mechanism of exotics. Ironically this is an important factor in understanding present experimental situation.

Most models agree that spin of antidecuplet is $1/2$ in agreement with $\chi$SM prediction. On the contrary parity is a distinguishing feature. Were the parity of $\Theta^+$ positive as in the $\chi$SM some other models and some lattice calculations would require revision.

The smallness of the width is very unnatural, although $\chi$SM provides formal explanation. If the primary decay coupling of $\Xi_{10} \to 8$ is indeed very small, the SU(3) relations between the decay rates of different members of antidecuplet will not hold due to the flavor representation mixing.

Other members of antidecuplet should be found. This concerns not only $\Xi_{10}$ but also five quark cryptoexotic $\Sigma^-$ and $N$–like states. Recent data on photooexcitation of nucleon resonances from GRAAL\textsuperscript{19} may be interpreted as a new narrow antidecuplet $N^*$ resonance at 1680 MeV. GRAAL sees resonant structure only on neutron but not on proton. This can be understood in terms of magnetic transition moments\textsuperscript{19}$\mu_{8\to10}$ which is proportional to $Q-1$. Similarly modified PWA\textsuperscript{20} of $\pi N$ scattering indicates that such a state might exist, also STAR data show some structure in the same energy range.

There is no strong theoretical argument against pentaquarks except its unnaturally small width. But – as recent plethora of theoretical papers shows – theoretical explanation may be found in many different models. So if high precision experiments will not find $\Theta^+$ and its partners, this may be even more difficult to understand than the small widths and the small mass.

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