Higher derivative corrections to $R$-charged AdS$_5$ black holes and field redefinitions

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ABSTRACT

We consider four-derivative corrections to the bosonic sector of five-dimensional $\mathcal{N} = 2$ gauged supergravity. Since this theory includes the $\mathcal{N} = 2$ graviphoton, we consider both curvature and graviphoton field-strength terms that show up at the four-derivative level. We construct, to linear order, the higher-derivative corrections to the non-rotating $R$-charged AdS$_5$ black hole and demonstrate how this solution transforms under field redefinitions.
1 Introduction

Higher derivative corrections to the Einstein-Hilbert action have received much notice in recent years, as such terms naturally show up in the $\alpha'$ expansion of effective actions derived from string theory. In general, the first non-trivial terms arise at the four derivative level, corresponding to curvature-squared corrections to classical Einstein theory of the form

$$e^{-1}\delta\mathcal{L} = \alpha_1 R^2 + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma},$$

where the coefficients $\alpha_1$, $\alpha_2$ and $\alpha_3$ are determined by the underlying theory. It was suggested in [1] that the natural form of such terms would be given by the Gauss-Bonnet combination

$$e^{-1}\delta\mathcal{L}_{\text{GB}} = \alpha (R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}),$$

as this is the unique combination that avoids introducing ghosts in the effective theory. It was subsequently argued, however, that in the absence of an off-shell formulation such as string field theory, the $\alpha_1$ and $\alpha_2$ coefficients are physically indeterminate as they may be eliminated by an on-shell field redefinition of the form $g_{\mu\nu} \rightarrow g_{\mu\nu} + aRg_{\mu\nu} + bR_{\mu\nu}$. In this sense, only the Riemann-squared term parameterized by $\alpha_3$ carries physical information from the underlying string theory.

The form of the higher derivative corrections are further constrained by supersymmetry. Explicit computations for the uncompactified closed superstring indicate that the first corrections enter at the $R^4$ order [2–4]. This is a feature of maximal supersymmetry, as curvature-squared terms are present in, for example, the uncompactified heterotic theory [5, 6]. An alternate route to obtaining supersymmetric higher derivative corrections is to make use of supersymmetry itself to construct higher derivative invariants that may show up in the action. This was applied in the heterotic supergravity by supersymmetrizing the Lorentz Chern-Simons form responsible for the modified Bianchi identity $dH = \alpha' \text{Tr} (F \wedge F - R \wedge R)$ [7]; the result agrees with the explicit calculations, once field redefinitions are properly taken into account [8]. More recently, the supersymmetric completion of the $A \wedge \text{Tr} R \wedge R$ term in five-dimensional $\mathcal{N} = 2$ supergravity (coupled to a number of vector multiplets) was obtained in [9]. This result has led to new progress in the study of black hole entropy and precision microstate counting in five dimensions (see e.g. [10] and references therein).

The supersymmetric four-derivative terms given in [9] were obtained using conformal supergravity methods. Thus it should be no surprise that they involve the square of the
five-dimensional Weyl tensor \([9]\)

\[
e^{-1} \delta \mathcal{L}_{\text{sugra}} = \frac{c_I}{24}[\frac{1}{8} M^I \mathcal{C}_{\mu\nu\rho\sigma} \mathcal{C}^{\mu\nu\rho\sigma} + \cdots] = \frac{c_I}{24}[\frac{1}{8} M^I (\frac{1}{6} R^2 \rightleftharpoons \frac{4}{3} R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}) + \cdots],
\]

(1.3)
as opposed to the Gauss-Bonnet combination, (1.2). In principle, an appropriate field redefinition may be performed to bring this into the Gauss-Bonnet form. However, this is usually not done, as it would obscure the overall supersymmetric structure of the theory. Thus in practice two somewhat complimentary approaches have been taken to investigating the curvature-squared corrections to the Einstein-Hilbert action. The first, which applies whether the underlying theory is supersymmetric or not, is to use a parameterized action of the form (1.1), with special emphasis on the Gauss-Bonnet combination. The second is to focus directly on supergravity theory, and hence to use explicitly supersymmetric higher-derivative actions of the form (1.3). In principle, these two approaches are related by appropriate field redefinitions. However, in practice this is complicated by the fact that additional matter fields (e.g. \([92x598] N = 2\) vector multiplets) as well as auxiliary fields may be present, thus making any field redefinition highly non-trivial.

In this letter, we investigate and clarify some of the issues surrounding field redefinitions in the presence of additional fields. In particular, we take the bosonic sector of five-dimensional \([92x598] \mathcal{N} = 2\) gauged supergravity and extend it with four-derivative terms built from the Riemann tensor \(R_{\mu\nu\rho\sigma}\) as well as the graviphoton field-strength tensor \(F_{\mu\nu}\). Although we introduce eight such terms, we demonstrate that only four independent combinations remain physical once field redefinitions are taken into account. To be explicit, we construct the higher-derivative corrections to the spherically symmetric \(R\)-charged AdS\(_5\) black holes of [11, 12], working to linear order in the higher-derivative terms, and then investigate the effect of field redefinitions on these black hole solutions.

To some extent, our solutions generalize the Gauss-Bonnet black holes originally constructed in [13, 14] and extended to Einstein-Maxwell theory in [15] and, with the inclusion of Born-Infeld terms, in [16]. One advantage that the Gauss-Bonnet combination has over the generic form of (1.1) is that it leaves the graviton propagator unmodified, and also yields a modified Einstein equation involving at most second derivatives of the metric. With an appropriate metric ansatz, the resulting Gauss-Bonnet black holes are then obtained by solving a simple quadratic equation. Furthermore, this feature of the Gauss-Bonnet term leads to a good boundary variation and natural generalization of the Gibbons-Hawking surface term [17]. This is a primary reason behind the popularity of applying Gauss-Bonnet
(and more generally Lovelock) extensions to braneworld physics (see e.g. [18]).

Our interest in studying the higher order corrections to $R$-charged AdS$_5$ black holes is also motivated by our desire to explore finite ’t Hooft coupling corrections in AdS/CFT. Using the relation $\alpha' = L^2/\sqrt{\lambda}$, we see that each additional factor of $\alpha' R_{\mu\nu\rho\sigma}$ in the string effective action gives rise to a $1/\sqrt{\lambda}$ factor in the strong coupling expansion of the dual gauge theory. Since supersymmetry ensures that the leading correction terms in IIB theory are of order $\alpha^3$, this indicates that the $\mathcal{N} = 4$ super-Yang Mills theory dual to AdS$_5 \times S^5$ will first receive such corrections at the $\lambda^{-3/2}$ order. The effect of these finite ’t Hooft coupling corrections on both the thermodynamics [19, 20] and hydrodynamics [21–26] of the $\mathcal{N} = 4$ plasma have received much attention in the context of extrapolations between the strong and weak coupling limits of the $\mathcal{N} = 4$ theory.

In principle, it would be greatly desirable to extend the finite coupling analysis to $\mathcal{N} = 1$ gauge theories dual to AdS$_5 \times Y^5$ where $Y^5$ is Sasaki-Einstein. This is of particular interest in resolving conjectures on the nature of the shear viscosity bound $\eta/s$ [27, 28, 21, 29–32]. One difficulty in doing so, however, lies in the fact that the higher derivative corrections involving the Ramond-Ramond five-form have not yet been fully explored (but see [33]). While it may be argued that these terms will not contribute in the maximally supersymmetric case, there is no reason to expect this to continue to hold for the reduced supersymmetric backgrounds dual to $\mathcal{N} = 1$ super-Yang Mills. For this reason, recent investigations of the shear viscosity [30–32] (and drag force [34, 35]) have assumed a parameterized set of curvature-squared corrections of the form indicated above in (1.1). Our present construction of higher-derivative corrected $R$-charged black holes allows for a generalization of the finite coupling shear viscosity calculation to backgrounds dual to turning on a chemical potential [36].

We start with the two-derivative bosonic action of $\mathcal{N} = 2$ gauged supergravity and in Section 2 we introduce a parameterized set of four derivative terms involving both curvature and graviphoton field strengths. Then, in Section 3, we obtain the linearized corrections to the spherically symmetric $R$-charged AdS$_5$ black holes. As one of the aims of this letter is to clarify the use of field redefinitions, we take a closer look at this in Section 4. Finally, we conclude with a discussion of our results in Section 5.
2 The higher-derivative theory

Our starting point is the bosonic sector of pure $N = 2$ gauged supergravity in five dimensions, with Lagrangian given by

$$e^{-1}L_0 = R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + 12g^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}A_\lambda. \quad (2.1)$$

Although the Chern-Simons term is important from a supergravity point of view, it will not play any role in the electrically charged solutions that are investigated below.

In general, higher-derivative corrections to $L_0$ may be expanded in the number of derivatives. We are mainly interested in the first non-trivial corrections, which arise at the four-derivative level. In a pure gravity theory, this would correspond to the addition of $R^2$ terms to the Lagrangian. However, for the Einstein-Maxwell system, we may also consider higher-order terms in the Maxwell field, such as $F^4$ and $RF^2$ terms. We thus introduce the higher-derivative Lagrangian

$$L = L_0 + L_{R^2} + L_{F^4} + L_{RF^2}, \quad (2.2)$$

where $L_0$ is given in (2.1), while the additional terms are

$$e^{-1}L_{R^2} = \alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma},$$

$$e^{-1}L_{F^4} = \beta_1 (F_{\mu\nu}F^{\mu\nu})^2 + \beta_2 F_{\mu\nu}F_{\rho\sigma}F^{\mu\nu}F^{\rho\sigma},$$

$$e^{-1}L_{RF^2} = \gamma_1 RF_{\mu\nu}F^{\mu\nu} + \gamma_2 R_{\mu\nu}F^{\mu\nu}F_{\rho\sigma} + \gamma_3 R_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}. \quad (2.3)$$

Note that we have not considered terms such as $F_{\mu\nu}\Box F^{\mu\nu}$ that would in principle enter at the same order. Although we are not complete in this regard, the terms that enter in $L_{F^4}$ are nevertheless sufficient for capturing the expansion of the Born-Infeld action.

2.1 Equations of Motion

Both the Maxwell and Einstein equations pick up corrections from the higher-derivative terms in (2.2). The modified Maxwell equation is straightforward

$$\nabla_\mu F^{\mu\nu} + \frac{1}{4\sqrt{3}}\epsilon^{\nu\rho\lambda\delta}F_{\rho\lambda}F_{\sigma\delta} = \nabla_\mu (8\beta_1 F^2F^{\mu\nu} - 8\beta_2 F_{\mu\lambda}F^{\rho\sigma}F^{\sigma\nu}$$

$$+ 4\gamma_1 RF^{\mu\nu} + 4\gamma_2 (R^{[\mu\lambda}F_{\nu]}) + 4\gamma_3 R_{\mu\nu\lambda\sigma}F_{\lambda\sigma}) \quad (2.4)$$
The Einstein equation is somewhat cumbersome, but can be expressed in Ricci form as

\[ R_{\mu\nu} + 4g^2 g_{\mu\nu} - \frac{1}{2} F_{\mu\lambda} F_{\nu}^{\lambda} + \frac{1}{12} g_{\mu\nu} F^2 = \]

\[
(2\alpha_1 + \alpha_2 + 2\alpha_3) \nabla_\mu \nabla_\nu R - (\alpha_2 + 4\alpha_3) \Box R_{\mu\nu} \\
-2\alpha_1 R_{\mu\nu} + 4\alpha_3 R_{\mu\lambda} R_{\nu}^{\lambda} - 2(\alpha_2 + 2\alpha_3) R_{\mu\lambda\nu\sigma} R_{\rho\lambda\sigma} - 2\alpha_3 R_{\mu\rho\lambda\sigma} R_{\nu}^{\rho\lambda\sigma} \\
+\frac{1}{2} g_{\mu\nu} [(2\alpha_1 + \alpha_2 + 2\alpha_3) \Box R + \alpha_1 R^2 + \alpha_2 R_{\lambda\sigma}^2 + \alpha_3 R_{\rho\lambda\sigma}^2] \\
-4\beta_1 F_{\mu\lambda} F_{\nu}^{\lambda} - 4\beta_2 F_{\mu\rho} F_{\sigma\lambda} F_{\lambda\sigma} F_{\mu\nu} + g_{\mu\nu} [\beta_1 (F^2)^2 + \beta_2 F^4] \\
+\gamma_1 (\nabla_\mu \nabla_\nu F^2 - R_{\mu\nu} F^2 - 2RF_{\mu\lambda} F_{\nu}^{\lambda}) \\
+\gamma_2 (\nabla_\mu \nabla_\nu F_{\rho} F^{\lambda\rho} + \frac{1}{2} \Box F_{\mu\lambda} F_{\nu}^{\lambda} + 2R_{(\mu\rho} F_{\sigma\nu)}^{\lambda} F_{\rho\sigma} + R_{\lambda\sigma} F_{\mu\nu}^{\lambda} F_{\nu}^{\sigma}) \\
-\gamma_3 (2\nabla_\mu \nabla_\nu F_{\mu\lambda} F_{\nu}^{\lambda} + 3R_{\mu\lambda\sigma} F_{\nu}^{\rho\lambda\sigma} F_{\rho\lambda}) \\
+\frac{1}{2} g_{\mu\nu} [(\gamma_1 - \frac{1}{2} \gamma_2) \Box F^2 + 2\gamma_3 \nabla_\mu \nabla_\nu F^{\rho\lambda\sigma} F_{\rho\lambda\nu\sigma} + 2\gamma_3 R_{\rho\lambda\sigma} \Box F_{\rho\lambda\nu\sigma}] \cdot (2.5)
\]

Since we are mainly interested in obtaining corrections \textit{linear} in the parameters (\(\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \gamma_1, \gamma_2, \gamma_3\)) of the higher derivative terms, we may substitute the lowest order equations of motion, given by the left-hand-sides of (2.3) and (2.5) into the right-hand-side of (2.5) to obtain a slightly simpler form of the Einstein equation

\[ R_{\mu\nu} + 4g^2 g_{\mu\nu} - \frac{1}{2} F_{\mu\lambda} F_{\nu}^{\lambda} + \frac{1}{12} g_{\mu\nu} F^2 = \]

\[
4g^2 (5\alpha_1 + \alpha_2 - 2\alpha_3 + 10\gamma_1 - 2\gamma_2) F_{\mu\lambda} F_{\nu}^{\lambda} \\
-2\alpha_3 R_{\mu\rho\lambda\sigma} F_{\nu}^{\rho\lambda\sigma} - (\alpha_2 + 2\alpha_3 - \gamma_2) R_{\mu\lambda\nu\sigma} F^{\lambda\rho} F_{\rho\sigma} - 3\gamma_3 R_{(\mu\rho\lambda\sigma} F_{\nu)}^{\rho\lambda\sigma} F_{\rho\lambda} \\
+\frac{1}{12} (2\alpha_1 + \alpha_2 + 2\alpha_3 + 12\gamma_1 - 3\gamma_2) \nabla_\mu \nabla_\nu F^2 - \frac{1}{2} (2\alpha_1 + 4\alpha_3 - \gamma_2) \Box F_{\mu\lambda} F_{\nu}^{\lambda} \\
-2\gamma_3 \nabla_\mu \nabla_\nu F_{\mu\lambda} F_{\nu}^{\lambda} - \frac{1}{12} (\alpha_1 + \alpha_2 - 2\alpha_3 + 3\beta_1 + 12\gamma_1 + 2\gamma_2) F^2 F_{\mu\lambda} F_{\nu}^{\lambda} \\
+(\alpha_3 - 4\beta_2 + \gamma_2) F_{\mu\rho} F_{\nu}^{\rho\lambda\sigma} F_{\lambda\sigma} F_{\nu}^{\rho} \\
+\frac{1}{2} g_{\mu\nu} [-16g^4 (5\alpha_1 + \alpha_2) - \frac{2}{3} g^2 (17\alpha_1 + 7\alpha_2 + 42\gamma_1 - 12\gamma_2) F^2 \\
+\frac{1}{2} (\alpha_1 + 2\alpha_2 + 7\alpha_3 + 6\gamma_1 - 3\gamma_2 + 3\gamma_3) \Box F^2 \\
+\frac{1}{12} (7\alpha_1 - 13\alpha_2 + 432\beta_1 + 60\gamma_1 + 24\gamma_2) (F^2)^2 \\
+\frac{1}{2} (\alpha_2 + 12\beta_2 - 4\gamma_2) F^4 + \alpha_3 R_{\rho\lambda\sigma\delta}^2 + 2\gamma_3 R_{\rho\lambda\sigma\delta} F_{\rho\lambda\nu\sigma} F_{\nu\sigma}] \\
+ \cdots . \quad (2.6)
\]

This is valid to first order in the four-derivative corrections.

Numerous previous studies higher-derivative corrections in five dimensions have concentrated on the purely gravitational sector of the theory. In this case, the first order
Einstein equation simplifies to
\[
R_{\mu\nu} + 4g^2 g_{\mu\nu} = -2\alpha_3 R_{\mu\rho\lambda\sigma} R_{\nu}^{\rho\lambda\sigma} + \frac{1}{3} g_{\mu\nu} [-16g^4(5\alpha_1 + \alpha_2) + \alpha_3 R_{\rho\lambda\sigma\delta}^2].
\] (2.7)

Working to this same order, we may define an effective cosmological constant
\[
g_{\text{eff}}^2 = g^2 [1 + \frac{2}{3}(10\alpha_1 + 2\alpha_2 + \alpha_3)g^2],
\] (2.8)

so that
\[
R_{\mu\nu} + 4g_{\text{eff}}^2 g_{\mu\nu} = \alpha_3 (-2C_{\mu\rho\lambda\sigma} C_{\nu}^{\rho\lambda\sigma} + \frac{1}{3} g_{\mu\nu} C_{\rho\lambda\sigma\delta}^2),
\] (2.9)

where we made the substitution \(R_{\mu\nu\lambda\sigma} = C_{\mu\rho\lambda\sigma} - g^2(g_{\mu\lambda} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\lambda}) + \cdots\) which is a consequence of the zeroth order Einstein equation, \(R_{\mu\nu} = -4g^2 g_{\mu\nu} + \cdots\). We see that the coefficients \(\alpha_1\) and \(\alpha_2\) of \(R^2\) and \(R_{\mu\nu}^2\), respectively, do not enter at linear order, so long as we use the effective cosmological constant given by \(g_{\text{eff}}\). This is related to the fact that these two terms may be removed by a field redefinition of the form \(g_{\mu\nu} \rightarrow g_{\mu\nu} + ag_{\mu\nu} R + bR_{\mu\nu}\) with appropriate constants \(a\) and \(b\).

Although neutral black hole solutions may be obtained directly from (2.9), we are mainly interested in \(R\)-charged solutions which may be obtained from the full equations (2.4) and (2.6). We turn to this in the next section.

3 R-charged black holes

The two-derivative Lagrangian, (2.1), admits a well-known two-parameter family of static, stationary AdS\(_5\) black hole solutions, given by [11,12]
\[
ds^2 = -H^{-2} f dt^2 + H(f^{-1} dr^2 + r^2 d\Omega_3^2),
\]
\[
A = \sqrt{3} \coth \beta \left( \frac{1}{H} - 1 \right) dt,
\] (3.1)

where the functions \(H\) and \(f\) are
\[
f = 1 - \frac{\mu}{r^2} + g^2 r^2 H^3,
\]
\[
H = 1 + \frac{\mu \sinh^2 \beta}{r^2}.
\] (3.2)

The parameter \(\mu\) is a non-extremality parameter, while \(\beta\) is related to the electric charge of the black hole. The extremal (BPS) limit is obtained by taking \(\mu \rightarrow \infty\) and \(\beta \rightarrow 0\)
with $Q \equiv \mu \sinh^2 \beta$ fixed, so that $f = 1 + g^2 r^2 H^3$ with $H = 1 + Q/r^2$. These extremal solutions are naked singularities, and may be interpreted as ‘superstars’ [37]. In the absence of higher-derivative corrections, the BPS solutions may be smoothed out by turning on angular momentum to form true black holes [38–41]

### 3.1 The first order solution

We wish to find the first order corrections to the $R$-charged black hole solution given by (3.1). To do so, we treat the coefficients ($\alpha_1, \alpha_2, \ldots, \gamma_3$) of the four-derivative terms in (2.3) as small parameters, and make the ansatz

$$ds^2 = -H^{-2} f dt^2 + H(f^{-1} dr^2 + r^2 d\Omega_3^2),$$

$$A = \sqrt{3} \coth \beta \left( \frac{1 + a_1}{H} - 1 \right) dt,$$

where

$$H = 1 + \frac{\mu \sinh^2 \beta}{r^2} + h_1,$$

$$f = 1 - \frac{\mu}{r^2} + g^2 r^2 H^3 + f_1.$$  \hspace{1cm} (3.4)

Here, we treat $h_1, f_1$ and $a_1$ as small corrections, and will solve for them to linear order in the parameters of the higher-derivative Lagrangian. Note that this ansatz was designed so that the zeroth order equations are automatically satisfied in the absence of $h_1, f_1$ and $a_1$.

Even after linearization in the small parameters, the individual equations of motion, (2.4) and (2.6), yield complicated coupled equations for the first order corrections. However, the use of certain symmetries of these equations yields tractable equations. In particular, the difference between the $tt$ and $rr$ components of the Einstein equation, $R^t_t - R^r_r$, gives a second order equation involving only $h_1$, which is easily solved. The solution for $h_1$ can then be inserted into the Maxwell equation, (2.4), to obtain a solution for $a_1$. Finally, the remaining components of the Einstein equation can be solved for $f_1$, thus yielding the full
solution. The result is

\[ h_1 = \frac{\mu^2 \sinh^2 2\beta}{6H_0^3 r^6} \left(7\alpha_1 + 5\alpha_2 + 13\alpha_3 + 42\gamma_1 - 12\gamma_2 + 12\gamma_3\right), \]  
\( \text{(3.5)} \)

\[ a_1 = \frac{\mu^2 \sinh^2 2\beta}{6H_0^3 r^6} \left[(7\alpha_1 + 5\alpha_2 + 13\alpha_3 + 42\gamma_1 - 12\gamma_2 - 12\gamma_3 \tanh^2 \beta)\right. \]
\[ + \frac{\mu \sinh^2 \beta}{2r^2} 
\left. (7\alpha_1 + 5\alpha_2 + 13\alpha_3 + 42\gamma_1 - 12\gamma_2 - 12\gamma_3)\right] \]
\[ + 24(6\beta_1 + 3\beta_2 + 2\gamma_1 - \gamma_2 + \gamma_3(1 + \text{sech}^2 \beta)) \right), \]  
\( \text{(3.6)} \)

\[ f_1 = \frac{2}{5} g^4 (10\alpha_1 + 2\alpha_2 + \alpha_3) r^2 H_0^3 \]
\[ + g^2 \frac{\mu^2 \sinh^2 2\beta}{r^4} (10\alpha_1 - \alpha_2 - 13\alpha_3 + 20\gamma_1 - \gamma_2 - 6\gamma_3) \]
\[ + \frac{\mu^2}{r^6 H_0} \left[ \sinh^2 2\beta (3\alpha_1 - 3\alpha_3 + 18\gamma_1 - 3\gamma_2) + 2\alpha_3 \right] \]
\[ - \frac{\mu^3 \sinh^2 2\beta \cosh^2 2\beta}{2r^8 H_0^5} (5\alpha_1 + \alpha_2 + \alpha_3 + 30\gamma_1 - 6\gamma_2) \]
\[ + \frac{\mu^4 \sinh^4 2\beta}{96r^{10} H_0^6} \left(47\alpha_1 + 13\alpha_2 + 17\alpha_3 - 144\beta_1 - 72\beta_2 + 276\gamma_1 - 48\gamma_2 - 24\gamma_3\right), \]  
\( \text{(3.7)} \)

where \( H_0 = 1 + \mu \sinh^2 \beta / r^2 \) is the zeroth order solution for \( H \). (Since \( h_1, a_1 \) and \( f_1 \) are already linear in the parameters of the higher order corrections, we may use \( H \) and \( H_0 \) interchangeably in the above expressions.) Note that the first line in \( f_1 \) reproduces the shift of the cosmological constant \( g^2 \rightarrow g_{\text{eff}}^2 \) given in \( (2.8) \). This allows us to write

\[ f = 1 - \frac{\mu}{r^2} + g_{\text{eff}}^2 r^2 H^3 + \bar{f}_1, \]  
\( \text{(3.8)} \)

where \( \bar{f}_1 \) is given by the remaining terms in \( (3.7) \).

In obtaining the above solution, we have imposed the boundary conditions that \( h_1 \) and \( a_1 \) both fall off faster than \( 1/r^2 \) as \( r \rightarrow \infty \) so that the \( R \)-charge is not modified from its zeroth order value. For \( f_1 \), the boundary condition is taken as \( (3.8) \), with \( \bar{f}_1 \) falling off faster than \( 1/r^2 \).


4 Field Redefinitions

As given in (2.3), we have parameterized the four-derivative terms in the Lagrangian in terms of the eight coefficients \((\alpha_1, \alpha_2, \ldots, \gamma_3)\). However, not all of these coefficients are physical. This is because some of the terms in the higher derivative Lagrangian can be removed by field redefinition.

To proceed, we consider transformations of the form

\[
\begin{align*}
    g_{\mu\nu} &\to g_{\mu\nu} + a(R + 20g^2)g_{\mu\nu} + b(R_{\mu\nu} + 4g^2g_{\mu\nu}) + cF_{\mu\lambda}F^{\lambda}_{\nu} + dF^2g_{\mu\nu}, \\
    A_\mu &\to (1 + g^2(25a + 5b - 12c + 60d))A_\mu.
\end{align*}
\] (4.1)

Note that the first two terms in the metric shift incorporate the cosmological constant; this corresponds to the zeroth order Einstein equation in the absence of gauge excitations. While this shift by the cosmological constant is not strictly speaking necessary in performing the field redefinition, we nevertheless find it convenient, as this avoids a shift in the effective cosmological constant \(g_{\text{eff}}\) after the field redefinition. In addition, the scaling of the gauge field is chosen so that it will remain canonically normalized after the shift of the metric.

The result of this transformation is to shift the original Lagrangian (2.2) into

\[
e^{-1} \mathcal{L} = (1 + 12g^2(5a + b)) \left[ R - \frac{1}{4} F_{\mu\nu}^2 + 12g^2(1 - 2g^2(5a + b)) \right.
\]
\[
+ \frac{1}{12\sqrt{3}} \left( 1 + 3g^2(5a + b - 12c + 60d) \right) \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} A_\sigma \\
+ \left( \alpha_1 + \frac{1}{2}(3a + b) \right) R^2 + (\alpha_2 - b) R_{\mu\nu} R^{\mu\nu} + \alpha_3 R_{\mu\rho\sigma\tau} R^{\mu\rho\sigma\tau} \\
+ \left( \beta_1 + \frac{1}{8}(c - d) \right) (F_{\mu\nu} F^{\mu\nu})^2 + (\beta_2 - \frac{1}{4}c) F_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} F_{\mu\nu} \\
+ \left( \gamma_1 - \frac{1}{8}(a + b + 4c - 12d) \right) R F^2 \\
+ \left( \gamma_2 - \frac{1}{2}(b + 2c) \right) R_{\mu\nu} F_{\rho\sigma} F_{\nu}^{\rho} F_{\sigma}^{\rho} + \gamma_3 R_{\mu\rho\sigma\tau} F^{\mu\nu} F_{\rho\sigma} F_{\mu\nu} \right],
\] (4.2)

where, as usual, we only work to linear order in the shift parameters \((a, b, c, d)\).

Up to an overall rescaling, this new Lagrangian can almost be brought back to the original form, provided we shift the various coefficients as follows:

\[
\begin{align*}
    g^2 &\to g^2(1 + 2g^2(5a + b)), \\
    \alpha_1 &\to \alpha_1 - \frac{1}{2}(3a + b), \\
    \alpha_2 &\to \alpha_2 + b, \\
    \alpha_3 &\to \alpha_3, \\
    \beta_1 &\to \beta_1 - \frac{1}{8}(c - d), \\
    \beta_2 &\to \beta_2 + \frac{1}{4}c, \\
    \gamma_1 &\to \gamma_1 + \frac{1}{8}(a + b + 4c - 12d), \\
    \gamma_2 &\to \gamma_2 + \frac{1}{8}(b + 2c), \\
    \gamma_3 &\to \gamma_3.
\end{align*}
\] (4.3)
One difference remains, however, and that is the coefficient of the $F \wedge F \wedge A$ Chern-Simons term. This suggests that, when considering higher derivative corrections in gauged supergravity, there is in fact a preferred field redefinition frame where this Chern-Simons term remains uncorrected. (Such a preferred frame also shows up when considering the supersymmetric completion of the mixed Tr $R \wedge R \wedge A$ term [9].) This $F \wedge F \wedge A$ term is unimportant, however, for the spherically symmetric $R$-charged black holes considered above in Section 3.

Ignoring the $F \wedge F \wedge A$ term, the freedom to perform field redefinitions of the form (4.1) indicates that at most four of the eight coefficients of the higher derivative terms will be physical. Clearly $\alpha_3$ and $\gamma_3$ are physical, as they cannot be removed by the transformation of (4.3). The additional two physical coefficients can be taken to be a linear combination of

\[
\hat{\beta}_1 \equiv \beta_1 + \frac{1}{144}(\alpha_1 - 7\alpha_2) + \frac{1}{12}(\gamma_1 + \gamma_2) \quad \text{and} \quad \hat{\beta}_2 \equiv \beta_2 + \frac{1}{12}\alpha_2 - \frac{1}{2}\gamma_2. \quad (4.4)
\]

In addition, although $g^2$ is shifted by the field redefinition, the physical cosmological constant, $g_{\text{eff}}^2$, as defined in (2.8), remains invariant.

The use of field redefinitions allows us to rewrite the four-derivative Lagrangian in various forms. A common choice would be to use the Gauss-Bonnet combination $R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\lambda\sigma}^2$ for the curvature-squared terms. This system has been extensively studied in the absence of higher-derivative gauge field corrections, and has the feature that it admits exact spherically symmetric black hole solutions, both without [13, 14] and with [15] $R$-charge. An alternate choice, which is perhaps more natural from a supersymmetric point of view [9], would be to use the Weyl-squared combination $C_{\mu\nu\lambda\sigma}^2 = \frac{1}{6}R^2 - \frac{2}{3}R_{\mu\nu}^2 + R_{\mu\nu\lambda\sigma}^2$. Either one of these choices would fix two of the coefficients (i.e. $\alpha_1$ and $\alpha_2$ in terms of $\alpha_3$). The additional freedom to perform field redefinitions may then be used to eliminate the mixed $RF^2$ and $R_{\mu\nu}F^{\mu\lambda}F^{\lambda\nu}$ terms parameterized by $\gamma_1$ and $\gamma_2$.

### 4.1 Field redefinitions and the first order solution

Given the above field redefinition, it is instructive to examine its effect on the first order black hole solution of (3.5), (3.6) and (3.7). In this case, it is straightforward to see that
the coefficient shift of (4.3) results in

\[ h_1 \rightarrow \tilde{h}_1 = h_1 + \frac{\mu^2 \sinh^2 2\beta}{8H_0^2 r^6} (-7a + b + 12c - 84d), \]

\[ a_1 \rightarrow \tilde{a}_1 = a_1 + \frac{\mu^2 \sinh^2 2\beta}{8H_0^2 r^6} \left[ (-7a + b + 12c - 84d) - \frac{3\mu \sinh^2 2\beta}{r^2}(a + b - 4c + 12d) \right], \]

\[ f_1 \rightarrow \tilde{f}_1 = f_1 - 2g^4(5a + b)r^2 H_0^3 - \frac{g^2 \mu^2 \sinh^2 2\beta}{2r^4} (25a + 8b - 18c + 60d) \]

\[ + \frac{3\mu^2 \sinh^2 2\beta}{8r^6 H_0} \left[ -2(3a + b - 8c + 36d) + \frac{\mu \cosh^2 2\beta}{r^2 H_0}(5a + b - 12c + 60d) \right. \]

\[ \left. - \frac{\mu^2 \sinh^2 2\beta}{r^4 H_0^2} (a - 2c + 12d) \right]. \quad (4.5) \]

At first, this result may appear somewhat surprising. After all, this field redefinition is supposed to be ‘unphysical’, and yet the form of the solution has changed. The resolution of this puzzle lies in the fact that the we have shifted the metric by terms that are not necessarily proportional to the lowest order equations of motion. (While we have taken care to incorporate the cosmological constant in (4.1), we have omitted the gauge field stress tensor in the shift.) In this sense, while the original and shifted metrics both solve the equations of motion, they nevertheless correspond to physically distinct solutions. The field redefinition of (4.1) is then more naturally thought of as a mapping between solutions.

More explicitly, we note that the shift of the metric given in (4.1) takes the black hole solution away from the form of the initial ansatz given by (3.3). In particular, shifting the metric by (4.1) and using the zeroth order solution gives

\[ g_{tt} \rightarrow \tilde{g}_{tt} = g_{tt} \left[ 1 - \frac{\mu^2 \sinh^2 2\beta}{2r^6 H_0^3} (a + 2b - 6c + 12d) \right], \]

\[ g_{rr} \rightarrow \tilde{g}_{rr} = g_{rr} \left[ 1 - \frac{\mu^2 \sinh^2 2\beta}{2r^6 H_0^3} (a + 2b - 6c + 12d) \right], \]

\[ g_{\alpha\beta} \rightarrow \tilde{g}_{\alpha\beta} = g_{\alpha\beta} \left[ 1 - \frac{\mu^2 \sinh^2 2\beta}{2r^6 H_0^3} (a - b + 12d) \right], \quad (4.6) \]

where \( \alpha \) and \( \beta \) refer to coordinates on \( S^3 \). It is now possible to see that a coordinate transformation \( r \rightarrow \tilde{r} \) is necessary in order to restore the canonical form of the shifted metric. By identifying

\[ ds^2 = \tilde{g}_{tt} dt^2 + \tilde{g}_{rr} dr^2 + \tilde{g}_{\theta\theta} d\Omega_3^2 \]

\[ = -\tilde{H}^{-2} \tilde{f} dt^2 + \tilde{H} (\tilde{f}^{-1} dr^2 + \tilde{r}^2 d\Omega_3^2), \quad (4.7) \]
we end up with expressions for \( \tilde{H} \) and \( \tilde{f} \)

\[
\tilde{H} = \frac{\tilde{g}_{\theta\theta}}{\tilde{r}^2}, \quad \tilde{f} = -\tilde{g}_{tt}\tilde{g}_{\theta\theta}\tilde{r}^4,
\]

(4.8)
as well as a differential equation relating \( \tilde{r}^2 \) with \( r^2 \)

\[
\frac{d(\tilde{r}^2)}{d(r^2)} = \frac{\tilde{g}_{tt}\tilde{g}_{rr}\tilde{g}_{\theta\theta}}{r^2}.
\]

(4.9)

Note that, in defining the angular coordinate \( \theta \), we have taken the metric on the unit \( S^3 \) to be of the form

\[
d\Omega_3^2 = d\theta^2 + \cdots.
\]

The equation for \( \tilde{r}^2 \) is easily solved, and yields the relation

\[
\tilde{r}^2 = r^2 \left[ 1 + \frac{3\mu^2\sinh^2\frac{2\beta}{\tilde{r}^2}}{8r^6\tilde{H}_0^2}(3a + 3b - 12c + 36d) \right],
\]

(4.10)

where we have set a possible integration constant to zero to preserve the \( r \to \infty \) asymptotics.

We are now able to explicitly compute the shifted metric functions \( \tilde{h}_1 \) and \( \tilde{f}_1 \) as well as the shifted gauge potential \( \tilde{a}_1 \). For \( \tilde{h}_1 \), we use the definition

\[
\tilde{H} = 1 + \frac{\mu\sinh^2\beta}{\tilde{r}^2} + \tilde{h}_1,
\]

(4.11)

along with (4.8) and (4.10) to obtain

\[
\tilde{h}_1 = h_1 + \frac{\mu^2\sinh^2\frac{2\beta}{\tilde{r}^2}}{8H_0^2}\left[-7a + b + 12c - 84d\right],
\]

(4.12)

which is in perfect agreement with (4.5). For \( \tilde{f}_1 \), on the other hand, we find

\[
\tilde{f}_1 = f_1 - 2g^4(5a + b)r^2H_0^3 - \frac{3g^2\mu^2\sinh^2\frac{2\beta}{2r^4}(b - 2c)}{r^4} + \frac{3\mu^2\sinh^2\frac{2\beta}{8r^6H_0^2}}{r^4}\left[-2(3a + b - 8c + 36d) + \frac{\mu\cosh\frac{2\beta}{r^2H_0^2}}{r^2H_0^2}(5a + b - 12c + 60d) - \frac{\mu^2\sinh^2\frac{2\beta}{r^4H_0^2}}{r^2H_0^2}(a - 2c + 12d)\right].
\]

(4.13)

Note that we have defined \( \tilde{f}_1 \) by

\[
\tilde{f} = 1 - \frac{\mu}{\tilde{r}^2} + g^2\tilde{r}^2\tilde{H}^3 + \tilde{f}_1,
\]

(4.14)

where \( \tilde{g}^2 = g^2(1 + 2g^2(5a + b)) \) is the shifted cosmological constant given in (4.3).
Comparison of (4.13) with (4.15) clearly demonstrates a difference in the $O(g^2)$ term. The origin of this difference is somewhat subtle, and is related to the choice of boundary conditions for the shifted and unshifted solutions. To see this, we recall that the gauge potential $A_\mu$ is also shifted by the field redefinition (4.1) so that it maintains canonical normalization. The implication of this shift on the black hole solution is that

$$A_t \to (1 + g^2(25a + 5b - 12c + 60d))A_t,$$

(4.15)

where

$$A_t = \sqrt{3} \coth \beta \left( \frac{1 + a_1}{H} - 1 \right), \quad H = 1 + \frac{\mu \sinh^2 \beta}{r^2} + h_1.$$  

(4.16)

In order to rescale the potential without adding any $O(1/r^2)$ terms to $H_0$, $h_1$ or $a_1$, we must instead shift the two parameters $\mu$ and $\beta$ of the black hole according to

$$\coth \beta \to \coth \beta(1 + g^2(25a + 5b - 12c + 60d)), \quad \mu \sinh^2 \beta \to \mu \sinh^2 \beta.$$  

(4.17)

This corresponds to a rescaling of the nonextremality parameter $\mu$

$$\mu \to \tilde{\mu} = \mu(1 + 2g^2 \cosh^2 \beta(25a + 5b - 12c + 60d)).$$  

(4.18)

In this case, the shifted metric function $\tilde{f}$, given in (4.14), ought to more properly be written as

$$\tilde{f} = 1 - \frac{\tilde{\mu}}{r^2} + g^2 r^2 \tilde{H}^3 + \hat{f}_1,$$

(4.19)

where

$$\hat{f}_1 = \tilde{f}_1 + \frac{2g^2 \mu \cosh^2 \beta}{r^2}(25a + 5b - 12c + 60d)$$

$$= f_1 + \frac{H_0}{r^2} - 2g^4(5a + b)r^2 H_3^3 - \frac{g^2 \mu^2 \sinh^2 2\beta}{2r^4}(25a + 8b - 18c + 60d) + \cdots.$$  

(4.20)

This now agrees with $\tilde{f}_1$ of (4.5) up to a solution $\lambda H_0/r^2$ to the homogeneous differential equation for $f_1$, where

$$\lambda = 2g^2 \mu \cosh^2 \beta(25a + 5b - 12c + 60d).$$  

(4.21)

This is a modification of the $O(1/r^2)$ term in $f_1$, which, however, is subdominant in $f$, as the leading behavior of $f$ is given by $f \sim g_{\text{eff}}^2 r^2$ for an asymptotically Anti-de Sitter background.
Finally, we may follow the effect of the field redefinition (4.1) on the gauge potential term \( a_1 \). Given the \( \mu \) and \( \beta \) rescaling of (4.17), we obtain

\[
\tilde{a}_1 = (1 + a_1) \frac{\tilde{H}}{H} - 1.
\]  

(4.22)

Working out the right hand side of this expression, we find that it agrees with (4.5). We have thus seen that the first order solution for the spherically symmetric \( R \)-charged black hole indeed transforms as expected under field redefinitions.

5 Discussion

While we have considered general field redefinitions given by four parameters \((a, b, c, d)\), a preferred subset of this would be to shift the metric by the full zeroth order equation of motion

\[
R_{\mu\nu} + 4g^2g_{\mu\nu} - \frac{1}{2}F_{\mu\lambda}F_{\nu}^{\lambda} + \frac{1}{12}g_{\mu\nu}F^2.
\]

(5.1)

In the above notation, this corresponds to taking

\[
c = \frac{1}{2}b, \quad d = -\frac{1}{12}(a - b).
\]

(5.2)

In this case, we may redefine the coefficients of the higher derivative terms according to

\[
\beta_1 = \hat{\beta}_1 - \frac{1}{12}(\hat{\gamma}_1 + \hat{\gamma}_2) + \frac{1}{144}(\alpha_1 - 7\alpha_2),
\]

\[
\beta_2 = \hat{\beta}_2 + \frac{1}{2}\hat{\gamma}_2 + \frac{1}{4}\alpha_2,
\]

\[
\gamma_1 = \hat{\gamma}_1 - \frac{1}{6}(\alpha_1 - \alpha_2),
\]

\[
\gamma_2 = \hat{\gamma}_2 + \alpha_2,
\]

(5.3)

so that the set \((\alpha_3, \hat{\beta}_1, \hat{\beta}_2, \hat{\gamma}_1, \hat{\gamma}_2, \gamma_3)\) are invariant under the restricted field redefinitions. Note that \(\hat{\beta}_1\) and \(\hat{\beta}_2\) are the physical coefficients previously defined in (4.4).

It is illuminating to rewrite the higher derivative Lagrangian (2.2) in terms of the new parameters. Ignoring the Chern-Simons term, the result is

\[
e^{-1}\mathcal{L} = \left(1 - 8g^2(5\alpha_1 + \alpha_2)\right) \left[ R - \frac{1}{4}\tilde{F}^2 + 12g_{\text{eff}}^2 + \alpha_1\mathcal{E}^2 + \alpha_2\mathcal{E}_{\mu\nu}^2 + \alpha_3(R_{\mu\nu}^2) - 8g^4 \right] \\
+ \hat{\beta}_1(\tilde{F}^2)^2 + \hat{\beta}_2\tilde{F}^4 + \hat{\gamma}_1\tilde{E}\tilde{F}^2 - \hat{\gamma}_2\mathcal{E}_{\mu\nu}\tilde{F}^{\mu\sigma}\tilde{F}^{\nu\sigma} + \gamma_3R_{\mu\nu}^2\tilde{F}_{\mu\nu}\tilde{F}_{\lambda\sigma},
\]

(5.4)
where
\[ \mathcal{E}_{\mu \nu} \equiv R_{\mu \nu} + 4 g^2 g_{\mu \nu} - \frac{1}{2} \hat{F}_{\mu \lambda} \hat{F}^{\lambda}_{\nu} + \frac{1}{12} g_{\mu \nu} \hat{F}^2, \quad \mathcal{E} = \mathcal{E}^\mu_{\mu} \] (5.5)
is the zeroth order equation of motion. Note that we have worked to linear order in pulling out the overall factor \(1 - 8g^2(5\alpha_1 + \alpha_2)\) renormalizing Newton’s constant. Furthermore, \(\hat{F} = d\hat{A}\) is a rescaled field strength defined by
\[ \hat{A}_{\mu} = [1 + 8g^2 \left( \frac{1}{3}(5\alpha_1 + \alpha_2) + 5\hat{\gamma}_1 - \hat{\gamma}_2 \right)] A_{\mu}, \] (5.6)
so that \(\hat{A}_{\mu}\) remains invariant under the field redefinition of (4.1). The structure of (5.4) now clearly demonstrates that, of the four-derivative terms, only those parameterized by \((\alpha_3, \hat{\beta}_1, \hat{\beta}_2, \gamma_3)\) are physical, as the remaining terms are manifestly proportional to the zeroth order equation of motion.

In principle, the choice of field redefinitions allows us to go back and forth between the Gauss-Bonnet and Weyl-squared parameterizations of the higher-derivative terms in the Lagrangian. In this sense, it is perhaps not a complete surprise to see that in some cases both parameterizations yield the same results for the entropy of BPS black holes [42–44], even though the bare Gauss-Bonnet correction is not supersymmetric in itself. (Of course, the bare Weyl-squared term is not supersymmetric by itself either.) What this suggests is that the Riemann-squared term parameterized by \(\alpha_3\) plays a crucial and perhaps dominant role in the geometry of higher-derivative black holes, and that the additional matter and auxiliary field terms may contribute only indirectly through their effects on the geometry, at least in the BPS case where there is additional symmetry at the horizon.

Finally, given the general higher-derivative corrected \(R\)-charged black holes, it would be interesting to study their thermodynamics and hydrodynamics. One outcome of this study ought to be a clear identification of physical versus unphysical parameters of the theory. In particular, in the parameterization of (5.4), we would expect all dependence on \((\alpha_1, \alpha_2, \hat{\gamma}_1, \hat{\gamma}_2)\) to drop out of the thermodynamical quantities. One difficulty in exploring the higher-derivative theory is that some care must be taken in generalizing the Gibbons-Hawking surface term (which we have ignored throughout this letter). This is because the general \((i.e.\ non\ Gauss-Bonnet)\) combination of \(R^2\) terms leads to higher than second-derivative terms in the equations of motion, and hence necessitates specifying additional boundary data [17]. As demonstrated in [22], one way around this is to perturb in the higher-derivative terms and to demand that the undesired boundary variations vanish when the lowest-order equations of motion are imposed. We are currently applying this
procedure to the general parameterized four-derivative Lagrangian with a goal of exploring higher-derivative black hole thermodynamics using holographic renormalization.

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