Gravitational lensing probability for the Konus-Wind gamma-ray bursts detected in the triggered mode

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Abstract. Although half a century has passed since the discovery of Gamma-ray bursts (GRBs), there is no evidence for any strongly lensed event. Using GRB luminosity function and GRB formation rate estimates for the Konus-Wind burst sample with known redshifts, and a Singular Isothermal Sphere as a gravitational lens model, we predict the detection rate of both GRB images being \( \approx 0.02 - 0.05 \) events per year. The result is consistent with the non-detection of lensed bursts in the KW triggered GRB sample.

1. Introduction
Gravitational lensing occurs when light from a distant object is distorted by a massive object in the foreground. Commonly, there are three types of gravitational lensing: strong, weak and microlensing. Strong gravitational lensing produces distorted, magnified, multiple images of the background object. Gamma-ray bursts (GRBs) are the most luminous explosions in the Universe, which are detected out to redshifts \( \approx 9 \) [1], so they were considered as events with relatively high strong lensing probability. Estimates of the probability that a GRB is strongly gravitationally lensed vary from \( 10^{-2} - 10^{-4} \) [2, 3] up to 60% [4]. This implies that in a sample of several thousand bursts, we might expect several lensed sources, and possibly many more. Meanwhile, no candidates for the gravitationally lensed GRBs were detected by [5] in a search for gravitationally lensed events in a sample of 2301 GRBs, detected by Konus-Wind (KW; [6]) in the triggered mode between 1994 and 2017 and localized by the interplanetary network (IPN). Here, we present a calculation of the detection rate for the strongly-lensed events from the KW triggered burst sample using the SIS gravitational lens model. Throughout the paper, we assume the standard \( \Lambda \)CDM cosmological model: \( H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega_\Lambda = 0.685, \) and \( \Omega_M = 0.315 \) [7].

2. The SIS gravitational lens model
The point mass is the simplest model of a gravitational lens, while the simplest approximation for an extended gravitational lens is the Singular Isothermal Sphere (hereafter SIS). Although indeed there are more advanced models such as Non-Singular Isothermal Sphere (NSIS; e.g. [8]) or Navarro-Frenk-White (NFW; [9]) profiles. A model of a gravitational lensing system is presented in Figure 1.

This spherical mass distribution yields flat rotation curves, similar to the observed for the
Figure 1. Sketch of a typical gravitational lens system from [10], where $\alpha$ is the deflection angle, $\beta$ is the “true” source position, and $\theta$ is the visible source position.

Spiral galaxies ([10]). The SIS density profile is

$$\rho(r) = \frac{v^2}{2\pi G r^2}.$$ 

Physically this model corresponds to a distribution of self-gravitating particles where the velocity distribution at all radii $r$ is a Maxwellian with one-dimensional velocity dispersion $v$ (hence “isothermal”). The mass distribution has two pathological properties: the central density diverges as $\rho \propto r^{-2}$ (hence “singular”), and the total mass of this distribution diverges as $r \to \infty$. Within the framework of the SIS model two images are produced if the source position lies inside the Einstein radius ($\theta < \theta_E$), which is the critical radius (the outermost source radius for multiple images being produced) for this model:

$$\theta_{\text{crit}} = \theta_E = 4\pi \left(\frac{v}{c}\right)^2 \frac{D_a(z_l, z_s)}{D_a(0, z_s)},$$

where $v$ is the line-of-sight velocity dispersion of the lensing galaxy; $z_l$ and $z_s$ are the lens and source redshifts, respectively; and $D_a(0, z_s)$ and $D_a(z_l, z_s)$ are the angular diameter distances to the source and the lens-to-source, respectively. The magnification coefficients for the brighter and fainter images are, correspondingly,

$$\mu_+ = 1 + \frac{1}{y}$$

and

$$\mu_- = \frac{1}{y} - 1,$$

where $y = \beta/\theta_{\text{crit}}$. 
3. Gravitational lensing rate

The differential lensing rate [11] is given by

\[ \frac{dN_l}{dz_s} = \frac{dN}{dz_s} \sigma_{\text{tot}}(z_s) \frac{4\pi}{dz_s}, \]

where \( \sigma_{\text{tot}}(z_s) \) is the total cross section of all lenses in the sky to produce an observable multiple GRB from a source in the redshift interval \([z_s, z_s + dz_s]\). If we assume the gravitational lenses being galaxies, the total cross section will be

\[ \sigma_{\text{tot}}(z_s) = \int_0^{z_s} \frac{dV}{dz}(z_l)dz_l \int_0^\infty \Phi(L)\sigma(z_l, z_s, L) dL, \]

where \( \sigma(z_l, z_s, L) \) is the cross section for a single galaxy, that depends on the lens mass, which, in turn, is directly related to lens luminosity, and \( \frac{dV}{dz}(z) \) is the comoving volume. The inner integral in the relation above sums over all lensing masses at a single redshift \( z_l \) using the galaxy luminosity function \( \Phi(L) \) as a weight, while the outer integral sums over all lensing galaxies at each lens redshift.

The lens population can be described by the Schecter luminosity function

\[ \Phi(L) dL = \Phi^* e^{-L/L_*} \left( \frac{L}{L_*} \right)^\gamma dL, \]

where parameters are \( \Phi^* = 1.56 \times 10^{-2} \left( \frac{H_0}{100 \text{km s}^{-1} \text{Mpc}^{-1}} \right)^3 \text{Mpc}^{-3} \) and \( \gamma = -1.1 \) [12]. The galaxy luminosity is related to the galaxy velocity dispersion via the Faber-Jackson or Tully-Fisher relations

\[ \frac{L}{L_*} = \left( \frac{\nu}{\nu_*} \right)^n. \] (1)

Following [12] the lens population is assumed to be comprised of E (12%), S0 (19%), and S (69%) galaxies. For the simplicity, for the galaxy luminosity function we adopt the parameters of the dominating fraction of S galaxies [12]: \( \nu_* = 144 \text{ km s}^{-1} \) and \( n = 2.6 \).

Thus the galaxy luminosity function can be rewritten as

\[ \Phi(u) du = n\Phi^* e^{-u^n} u^{n(\gamma+1)-1} du, \]

where \( u = \frac{\nu}{\nu_*} \) is a dimensionless galaxy velocity dispersion.

The cross section to produce an observable multiple GRB by a gravitational lensing by a single foreground galaxy of luminosity \( L \) is

\[ \sigma(z_l, z_s, L) = \int_0^{\theta_{\text{crit}}} 2\pi \beta \epsilon(\beta, z_s) d\beta = \int_0^{1} 2\pi y\theta_{\text{crit}}^2 \epsilon(y, z_s) dy, \]

where \( \epsilon(y, z_s) \) is an efficiency of triggering KW by a GRB from the redshift \( z_s \) depending on the GRB source position relative to the lens axis:

\[ \epsilon(y, z_s) = \int_0^\infty \Theta(L_{\text{GRB}} - L_{0-}(y))\Psi(L_{\text{GRB}}, z_s) dL_{\text{GRB}}, \]
where $L_{\text{GRB}}$ is the burst isotropic-equivalent luminosity.

We adopt the Heaviside step function as the efficiency of KW trigger ($\Theta$) depending on the GRB source position via the threshold luminosity:

$$L_0(y) = \frac{4\pi D_L^2(y, z_s) F_0}{\mu(y)},$$

where $F_0 = 1 \times 10^6$ erg cm$^{-2}$ s$^{-1}$ is the effective KW trigger threshold flux [13]. Here the magnification coefficient for the fainter image $\mu_-$ is used to calculate the detection rate of both lensed images. When estimating the detection rate of only bright images, the $\mu_+$ coefficient should be used instead.

The efficiency is weighted with the GRB cumulative luminosity function, which we adopt as

$$\Psi(L_{\text{GRB}}, z) \propto \begin{cases} 0, & L_{\text{GRB}} < L_{\min}, \\ L_{\text{GRB}}(1 + z)\delta, & L_{\min} \leq L_{\text{GRB}} < L_{\text{break}}, \\ L_{\text{break}}^{-\eta_1}L_{\text{GRB}}^\eta_1(1 + z)\delta, & L_{\text{break}} \leq L_{\text{GRB}} \leq L_{\max}, \\ 1, & L_{\text{GRB}} > L_{\max}, \end{cases}$$

where $L_{\min} = 2.94 \times 10^{50}$ erg s$^{-1}$, $L_{\max} = 4.65 \times 10^{54}$ erg s$^{-1}$, and $\delta = -1.7$, $\eta_1 = -0.44$, $\eta_2 = -1.03$, $L_{\text{break}} = 1.84 \times 10^{52}$ erg s$^{-1}$ for the case of the cosmological luminosity evolution and $\delta = 0$, $\eta_1 = -0.44$, $\eta_2 = -0.98$, $L_{\text{break}} = 9.25 \times 10^{52}$ erg s$^{-1}$ for the case of the non-evolving luminosity function (see [13] for details).

The GRB formation rate is

$$\frac{dN}{dz_s} = R_{\text{GRB}}(z_s) \frac{dV}{dz}(z_s),$$

while $R_{\text{GRB}}(z)$ is obtained by fitting the $\rho(z)$ function from [13], i.e. $\frac{dN}{dz}$ is the $\frac{dv}{dz} function from [13]. For the case of the luminosity evolution the GRB formation rate can be approximated as

$$R_{\text{GRB}} \propto \begin{cases} (1 + z)^\zeta_1, & z < z_{\text{break},1}, \\ (1 + z_{\text{break},1})^{-\zeta_1} - (1 + z)^{\zeta_2}, & z_{\text{break},1} \leq z < z_{\text{break},2}, \\ (1 + z_{\text{break},1})^{-\zeta_1} - (1 + z)^{\zeta_2} - (1 + z)^{\zeta_3}, & z \leq z_{\text{break},2}, \end{cases}$$

where $\zeta_1 = 2.0$, $\zeta_2 = -0.081$, $\zeta_3 = -2.6$, $z_{\text{break},1} = 0.78$, and $z_{\text{break},2} = 2.47$. For the case of the non-evolving GRB luminosity $R_{\text{GRB}}$ can be approximated as

$$R_{\text{GRB}} \propto \begin{cases} (1 + z)^\zeta_1, & z < z_{\text{break}}, \\ (1 + z_{\text{break}})^{-\zeta_1} - (1 + z)^{\zeta_2}, & z \geq z_{\text{break}}, \end{cases}$$

where $\zeta_1 = 1.81$, $\zeta_2 = -0.34$, and $z_{\text{break}} = 1.61$.

The total luminosity function was normalized to the KW GRB detection rate which is 125 triggered GRBs per year:

$$\int_0^\infty \frac{dN}{dz_s} \int_{L_{\min}}^{L_{\max}} \Psi(L_{\text{GRB}}, z_s) dL_{\text{GRB}} = 125.$$
where the differential cross section is

\[
\frac{\partial \sigma(z_l, z_s, L)}{\partial \Delta t} = \frac{2\pi \beta d\beta}{(\partial \Delta t / \partial \beta) d\beta}.
\]

Within the framework of the SIS model the time delay of a pair of images for the region, where \( y < 1 \), is

\[
\Delta t(u, y, z_l, z_s) = \frac{32\pi^2}{c} \left( \frac{v}{c} \right)^4 \frac{D_a(z_l, z_s)D_a(0, z_l)}{D_a(0, z_s)} (1 + z_l) y = \frac{32\pi^2}{c} \left( \frac{v_s}{c} \right)^4 \frac{D_a(z_l, z_s)D_a(0, z_l)}{D_a(0, z_s)} (1 + z_l) y.
\]

Therefore

\[
u_{\text{min}}(y, \Delta t, z_l, z_s) = \frac{\nu_{\text{min}}}{v_s} = \left( \frac{c}{32\pi^2 \left( \frac{v_s}{c} \right)^4} \frac{\Delta t}{y} \frac{D_a(0, z_s)}{(1 + z_l) D_a(0, z_l) D_a(z_l, z_s)} \right)^{\frac{1}{4}}, \tag{3}\]

and

\[
\frac{\partial \sigma(\Delta t, z_l, z_s, u)}{\partial \Delta t} = \frac{\pi c^2}{2} \Delta t \frac{D_a^2(z_l, z_s)}{E(1 + z_l)^2 D_a^2(0, z_l) D_a^2(0, z_s)}. \tag{4}\]

Then we should substitute 3 and 4 in 2 via 1.

The arbitrarily normalized lensing rate as a function of time delay between two images is presented in Figure 2. One could note that the longer time delays correspond to the lower lensing rates.

![Figure 2](image)

**Figure 2.** The lensing rate as a function of time delay between two images.

4. Results and conclusions

Applying the described methodology we estimated the rate of detection of both images of a gravitationally lensed GRB in the KW triggered mode. Assuming the absence of cosmological
evolution of GRB luminosity function, we estimate the detection rate of both GRB images as \( \approx 0.05 \) events per year, or \( \approx 1 \) pair of GRB images during the 22 yr long time span of the KW observations. For the GRB luminosity function and the GRB formation rate corrected for the cosmological evolution, we estimate the detection rate of both GRB images as \( \approx 0.02 \) events per year, or \( \approx 0.5 \) pair in 22 years. These results are in agreement with the non-detection of gravitationally lensed GRBs in the sample of 2301 bursts, detected by KW in the triggered mode between 1994 and 2017 and localized by the interplanetary network (IPN) [5].

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