Tuning the Mobility of a Driven Bose-Einstein Condensate via Diabatic Floquet Bands

Tobias Salger,1 Sebastian Kling,1 Sergey Denisov,2 Alexey V. Ponomarev,2 Peter Hänggi,2 and Martin Weitz1
1Institut für Angewandte Physik der Universität Bonn, Wegelerstrasse 8, 53115 Bonn, Germany
2Institute of Physics, University of Augsburg, Universitätsstrasse 1, 86159 Augsburg, Germany

We study the response of ultracold atoms to a weak force in the presence of a temporally strongly modulated optical lattice potential. It is experimentally demonstrated that the strong ac driving allows for a tailoring of the mobility of a dilute atomic Bose-Einstein condensate with the atoms moving ballistically either along or against the direction of the applied force. Our results are in agreement with a theoretical analysis of the Floquet spectrum of a model system, thus revealing the existence of diabatic Floquet bands in the atoms’ band spectra and highlighting their role in the nonequilibrium transport of the atoms.

DOI: 10.1103/PhysRevLett.110.135302 PACS numbers: 67.85.Hj, 03.75.Kk, 05.60.Gg

Experiments with ultracold atomic gases in periodically modulated lattice potentials have shown that these systems represent an attractive testing ground to explore nonequilibrium quantum states [1,2]. Along these lines, the suppression of tunneling in ac-driven optical lattices was experimentally observed, both in the single-particle works [3–5], are well explained by assuming that the temporal modulations do not push the particle outside the corresponding region of the classical phase space. Evidently, this recipe no longer applies for the systems operating in the deep quantum limit.

Here we show that in this limit diabatic Floquet states reveal their presence via a strong ballisticlike response of a driven quantum particle to a weak net force. Upon changing the modulation parameters, we populated different diabatic bands and switched between regimes of positive and negative responses; i.e., the particle then moves against the applied bias. We measured the mobility [24] of a dilute atomic rubidium Bose-Einstein condensate (BEC) in an ac-driven optical potential. Good agreement between the experimental results and our theoretical model is obtained.

Model.—Consider a particle with mass M moving in a time- and space-periodic potential $U(\xi, t) = V(\xi) A(t)$, where the periodic functions $V(x + L) = V(x)$ and $A(t + T) = A(t)$ possess the periods $L$ and $T = 2\pi/\omega$, respectively. In addition, let the particle be exposed to a tunable net bias $F$. The corresponding Hamiltonian then reads

$$\hat{H} = \hat{p}^2/2M + V(\xi)A(t) - F\xi.$$ (1)

The Hamiltonian of the nondriven system, i.e., when $A(t) = 1$ and $F = 0$, is a spatially periodic operator and its reciprocal space is spanned by the Bloch bands, $E_n(\kappa)$, $\kappa \in [-\pi/L, \pi/L]$, $n = 1, 2, \ldots$. By assuming that the initial state is localized at the point with quasimomentum $\kappa = 0$, the action of a bias $F$ can be considered as a linear ramp of the quasimomentum, $\kappa(t) = Ft/\hbar$. On a time scale $t \ll T_B$, the response of the system to a weak bias is determined by
the band curvature $\hbar^2 \partial^2 E_p(k)/\partial \kappa^2$, i.e., by the effective mass [25]. The curvature of the ground-state band is small and positive near the center of the Brillouin zone. One could, in principle, obtain a variety of mobility responses by placing the particle into different excited bands; however, the exponentially small splitting between neighboring bands makes this idea less practical. It is possible to prepare an initial atomic sample outside the center of the Brillouin zone, where the slope of the ground band is nonzero [26]; this would mean that the atoms were already set into a slow motion in the absence of any bias.

The dynamics of such a time-modulated system (1), $A(t) \neq \text{const}$, is more diverse. The Hamiltonian is periodic in time and the solution of the corresponding Schrödinger equation for a given value of the quasimomentum $\kappa$, i.e.,

$$[\hat{H}(\kappa, t) - i\hbar \partial_t] \Psi_\kappa(t) = 0, \quad \hat{H}(\kappa, t + T) = \hat{H}(\kappa, t),$$

(2)

can be obtained by solving the eigenvalue problem for the operator that propagates the system over one period of driving [21],

$$|\psi_{n, \kappa}(T)\rangle = \hat{U}(T)|\psi_{n, \kappa}(0)\rangle = \exp[-iE_n(\kappa)t/\hbar]|\phi_{n, \kappa}(0)\rangle.$$  

(3)

The eigenfunctions obey the Floquet theorem, $|\psi_{n, \kappa}(t)\rangle = \exp[-iE_n(\kappa)t/\hbar]|\phi_{n, \kappa}(t)\rangle$, where Floquet states are time periodic, $|\phi_{n, \kappa}(t + T)\rangle = |\phi_{n, \kappa}(t + T)\rangle$. The quasienergies $E_n(\kappa)$ are conventionally restricted to the interval $[-\hbar \pi/T, \hbar \pi/T]$. Figure 1 depicts Floquet band spectra of the temporally periodically driven optical lattice system, for typical experimental parameters and two different drive frequencies. The spectra exhibit a complex, weblike structure, which can be tailored by varying the modulation $A(t)$. The finiteness of the quasienergy range creates a certain problem with the ordering of Floquet bands. The concept of adiabatic following of a band loses its mathematical rigor here because the set of avoided crossing points is dense everywhere [13,27]. However, there exists no problem with the concept of diabatic following Ref. [22]. A diabatic band can be obtained by moving through the parameter space and connecting segments of different bands by ignoring all avoided crossings met on the way when the gap width of the upcoming avoided crossing is below certain threshold $\varepsilon$; see Refs. [22,23] for details. The threshold is related to the velocity of the excursion through the parameter space, which here is given by the strength of the bias $F$ [28]. The corresponding diabatic bands assume straight lines, running across the Brillouin zone [23,29]; cf. Fig. 1(b).

Transport properties of the $n$th Floquet state are characterized by the average velocity [29], $\bar{v}_{n, \kappa} = \langle v_{n, \kappa}(t) \rangle_T$, where $v_{n, \kappa}(t)$ is the instantaneous expectation value of the velocity operator, $\hat{v} = (-i\hbar/M)\partial_x$, and $(\ldots)_T$ denotes a time average over one period of the driving [30].

Because diabatic bands have near constant velocities, they are entitled to be termed ballistic bands. The issue of symmetry is significant here: When the system Hamiltonian equation (1) is invariant under time or space reversal, all Floquet bands are flat at the center of the Brillouin zone, thus exhibiting zero average velocities [16,17]. Formally, the presence of a symmetry implies that the potential function $U(x, t) = A(t)V(x)$ remains invariant either under the transformation $\hat{S}; t \rightarrow -t + \tau$, $x \rightarrow x + \chi$ or $\hat{S}; t \rightarrow -t + \tau$, $x \rightarrow -x + \chi$, where $\tau$ and $\chi$ are appropriate shift constants; see in Ref. [17]. In contrast to genuine Floquet bands, a diabatic band does need to be periodic in quasimomentum $\kappa$ space [16] and sometimes can wrap the Brillouin zone several times before meeting a broad avoided crossing and losing the diabaticity property. This fact does not contradict the above symmetry statement because the presence of a symmetry only means that two ballistic bands with opposite velocities cross each other at the point $\kappa = 0$. Note, however, that
such a crossing mimics a narrow avoided crossing between the corresponding pair of genuine Floquet bands.

Consider next the situation when at time $t = 0$ a dilute and delocalized cloud of ultracold atoms is loaded into the temporarily driven lattice potential. The initial atomic wave function $|\varphi_0\rangle$ has the form of a wave packet, well localized in quasimomentum at the point $\kappa = 0$. For our calculations, we assume that the initial atomic wave function is of the form of the Bloch ground state of the nondriven lattice potential, which is experimentally reasonable. We can order the Floquet states, $FS[n]$, according to their overlaps with the initial state, i.e., $c_n = \langle \phi_{n,\kappa=0}(0)|\varphi_0\rangle^2$, $c_1 > c_2 > \ldots$. We find that, for our experimental parameters, the overlap with the first Floquet band is $c_1 \approx 0.85$, and it is natural to expect that the system mobility is determined mainly by the properties of this band. We are interested in the situation where this state transforms itself into a ballistic band outside the center of the Brillouin zone, for example, due to a broad avoided crossing with another Floquet state, as is the case for the parameters used in Fig. 1. A motion through the $\kappa$ space, induced by a weak bias force, is slow enough for the system to remain on the band when passing through this ac. We expect that the nonvanishing slope of the band beyond this point will determine the mean particle velocity. In general, the described scenario assumes that (i) a significant population of the relevant band can be achieved and (ii) the band group velocity is high. There is no general recipe how to manage such a situation. There are, however, several prerequisites serving as a guide. First, the system should operate in the deep quantum regime, ideally with only few Floquet states within the potential range. Otherwise, the initial wave function will be distributed among several Floquet states of different group velocities. Their joint contributions then yield a inconclusive asymptotic response, while interference effects will blur the finite-time response even more. Second, the modulation frequency should be chosen close to the frequency of oscillations at the bottom of the potential well, $\omega_R = 4\pi^2\hbar/L^2M$. In numerical studies we have observed that the resonant modulation typically produces at the vicinity of the initially populated $FS[1]$ state at $\kappa = 0$ two well-separated Floquet states that are ballistic, with opposite velocities just outside the center of the zone. By slightly adjusting the frequency of the driving, one can then map the initially populated Floquet state (which has a vanishing group velocity) into a ballistic state by means of a broad avoided crossing between the $FS[1]$ state and one of the states of the ballistic pair.

**Experiment.**—In our experiments we used an optical potential, $U(x, t) = V(x)A(t)$, where $V(x)$ and $A(t)$ are of the form [17,18,23]

\[
V(x) = (V_1/2) \cos(2kx) + (V_2/2) \sin(4kx),
\]

\[
A(t) = A_1 \sin^2(\omega t/2) + A_2 \cos^2(\omega t). \tag{6}
\]

Here, $\lambda$ is the wavelength of the driving laser field and $k = 2\pi/\lambda$. Using the spatial lattice period, $L = \lambda/2$, we arrive at a resonance frequency of $\omega_R = 8\omega_\epsilon$, where $\omega_\epsilon = \hbar^2/2M_Rb$ is the recoil frequency, and $E_r = \hbar\omega_r$ denotes the recoil energy. Note that this setup, although possessing a ratchetlike spatial profile [31], is perfectly time symmetric since $A(-t) = A(t)$ [32]. Therefore, in the absence of an external bias force, $F = 0$, no transport occurs [17,34]; i.e., the average current vanishes for any initial atomic wave packet with zero average velocity. We performed the experiments by loading a Bose-Einstein condensate of rubidium atoms into a periodically time-modulated optical potential, of the form given by Eq. (4), to an external dc bias force.

We start the experimental sequence by preparing a Bose-Einstein condensate of rubidium ($^{87}$Rb) atoms in a far detuned optical dipole trap. During the next step the atoms are allowed to freely expand ballistically for 2.5 ms, which reduces the atomic density. This leaves the atomic cloud with essentially kinetic energy only, while the interaction energy is strongly diminished. After then loading the atoms into an optical lattice potential, a dc bias is realized by moving the lattice potential with a constant acceleration. This acceleration emulates a constant force in the comoving frame (see the Supplemental Material [33]). The measured mean velocity of the atomic cloud versus the bias strength $F$ is depicted in Fig. 2(b). For small values of $F > 0$, a negative response is clearly detectable; i.e., the average atomic velocity is opposite to the applied bias force. We attribute this to the population of the Floquet state marked by the heavy line in the left panel of Fig. 1, which for positive quasimomentum values has a negative group velocity. For larger absolute values of the dc force, above roughly $|F| = 0.02E_r/\lambda$, the mobility again becomes positive. This is attributed to the population of other Floquet states when moving beyond the quasimomentum region indicated by the thick (magenta) lines in Fig. 1. We have also investigated the dependence of the atomic transport on the drive frequency; see Fig. 3(a). By changing the frequency to slightly higher values, the atomic mobility can be tuned to a normal, i.e., positive, response. This response is attributed to the overlap of the $FS[1]$ state with the symmetry-related counterpart of the previously involved ballistic band, as shown in the right panel of Fig. 1.

An important issue is the stability of the mobility response. Let us first discuss this issue in two different limits, classical and quantum ones. In the classical dissipationless limit, a stationary motion against a constant bias is possible [35] due to the existence of invariant manifolds, i.e. transporting regular islands, periodic orbits, and cantori [36], in the phase space of an ac-driven Hamiltonian system [29]. Diabatic bands can be viewed as the quantum counterpart of classical ballistic manifolds. However, this analogy is not exact. In contrast to classical manifolds, diabatic bands are not completely
the modulation frequency to\textsuperscript{3} \(=\) \(R\) approaching zero at modulation periods, and then starts to decrease again, reaching a broad maximum value of mean momentum of the atomic cloud initially increases, the observed atomic momentum increases to roughly \(T\) times. This finding is in agreement with the fact that the structure of Floquet bands is very sensitive to variations of the system parameters and, correspondingly, the system response can be controlled over a wide range.

Conclusions.—The experimental detection of diabatic Floquet states in a time-dependent driven Bose-Einstein condensate opens several directions that are worth exploring further. For example, there is the issue relating to the constructive role that decoherence may play for the stabilization of the asymptotic response, similar to what has been observed with stochastic models operating in the classical regime \textsuperscript{38}. Another promising research avenue is the implementation of the tunable dispersion relation, stemming from the Floquet spectrum of temporally modulated optical lattices, for simulations of relativistic physics effects with ultracold atoms \textsuperscript{39,40}. Finally, the inclusion of interaction between atoms of a dense condensate \textsuperscript{41,42} may open a way to the detection of many-body diabatic Floquet states.

This work was supported by the DFG Grants No. We1748/7 (M. W.) and No. HA1517/31-2 (S. D. and P. H.) and the German Excellence Initiative “Nanosystems Initiative Munich (NIM)” (A. V. P., S. D., and P. H.).

\begin{thebibliography}{42}
\bibitem{1} K. W. Madison, M. C. Fischwer, R. B. Diener, Q. Niu, and M. G. Raizen, \textit{Phys. Rev. Lett.} \textbf{81}, 5093 (1998).
\bibitem{2} N. Gemelke, E. Sarajlic, Y. Bidel, S. Hong, and S. Chu, \textit{Phys. Rev. Lett.} \textbf{95}, 170404 (2005).
\bibitem{3} H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo, \textit{Phys. Rev. Lett.} \textbf{99}, 220403 (2007); C. Sias, H. Lignier, Y. Singh, A. Zenesini, D. Ciampini, O. Morsch, and E. Arimondo, \textit{ibid.} \textbf{100}, 040404 (2008); A. Zenesini, H. Lignier, D. Ciampini, O. Morsch, and E. Arimondo, \textit{ibid.} \textbf{102}, 100403 (2009).
\bibitem{4} Y.-A. Chen, S. Nascimbène, M. Aidelsburger, M. Atala, S. Trotzky, and I. Bloch, \textit{Phys. Rev. Lett.} \textbf{107}, 210405 (2011).
\bibitem{5} P. Hauke, \textit{Phys. Rev. Lett.} \textbf{109}, 145301 (2012); J. Struck, C. Ölschläger, M. Weinberg, P. Hauke, J. Simonet,
it may happen that the new Floquet eigenspectrum appears as an exact renormalized copy of the eigenspectrum of the undriven system; see Ref. [15]. However, this noteworthy effect is a feature of the employed model rather than being generic.

[15] A. Eckardt, C. Weiss, and M. Holthaus, Phys. Rev. Lett. 95, 260404 (2005).

[16] H. Schanz, M.-F. Otto, R. Ketzmerick, and T. Dittrich, Phys. Rev. Lett. 87, 070601 (2001).

[17] S. Denisov, L. Morales-Molina, S. Flach, and P. Hänggi, Phys. Rev. A 75, 063424 (2007).

[18] T. Salger, S. Kling, T. Hecking, C. Geckeler, L. Morales-Molina, and M. Weitz, Science 326, 1241 (2009).

[19] N. H. Lindner, G. Rafael, and V. Galitski, Nat. Phys. 7, 490 (2011).

[20] H. Sambe, Phys. Rev. A 7, 2203 (1973).

[21] M. Grifoni and P. Hänggi, Phys. Rep. 304, 229 (1998).

[22] T. Takami, Phys. Rev. Lett. 68, 3371 (1992).

[23] S. Miyazaki and A. R. Kolovsky, Phys. Rev. E 50, 910 (1994).

[24] The term “mobility” is intended to denote the dynamical reaction of a quantum particle to an applied external static force. Here it does not bear the strict meaning of the ratio between the applied force and the asymptotic velocity of the particle, as typically assigned in solid-state and kinetic theories [25].

[25] C. Kittel, Introduction to Solid State Physics (Wiley, New York, 1996), 7th ed.

[26] E. Peik, M. BenDahan, I. Bouchoule, Y. Castin, and C. Salomon, Phys. Rev. A 55, 2989 (1997).

[27] D. W. Hone, R. Ketzmerick, and W. Kohn, Phys. Rev. A 56, 4045 (1997).

[28] This rather vague definition gives rise to an immediate question: How to define a relation $\varepsilon(F_{\text{min}})$ such that when the applied bias is larger than $F_{\text{min}}$, all the avoided crossings narrower than $\varepsilon$ are ignored? The resolution of this challenging issue, however, is beyond the scope of this work.

[29] H. Schanz, T. Dittrich, and R. Ketzmerick, Phys. Rev. E 71, 026228 (2005).

[30] This expression defines the velocity in the comoving frame, i.e., in the frame moving with the speed $\dot{\mathbf{r}}$. The transformation to the laboratory frame corresponds to the subtraction of $\mathbf{F} t$ from the instantaneous velocity value.

[31] T. Salger, C. Geckeler, S. Kling, and M. Weitz, Phys. Rev. Lett. 99, 190405 (2007).

[32] We have also performed experimental studies by using a single-harmonic setup in Eq. (5) with $V_2 = 0$. We detected for the BEC cloud a similar mobility response; see the Supplemental Material [33].

[33] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.110.135302 for details.

[34] P. Hänggi and F. Marchesoni, Rev. Mod. Phys. 81, 387 (2009).

[35] S. Denisov, S. Flach, and P. Hänggi, Europhys. Lett. 74, 588 (2006).

[36] D. F. Escande, Phys. Rep. 121, 165 (1985); G. M. Zaslavsky, The Physics of Chaos in Hamiltonian Systems (Imperial College, London, 2007), 2nd ed.

[37] H. P. Breuer and M. Holthaus, Phys. Lett. A 140, 507 (1989).

[38] L. Machura, M. Kostur, P. Talkner, J. Luczka, and P. Hanggi, Phys. Rev. Lett. 98, 040601 (2007).

[39] T. Salger, C. Grossert, S. Kling, and M. Weitz, Phys. Rev. Lett. 107, 240401 (2011).

[40] L. Tarruell, D. Greif, T. Uehlinger, G. Jotzu, and T. Esslinger, Nature (London) 483, 302 (2012).

[41] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008).

[42] R. Ma, M. E. Tai, P. M. Preiss, W. S. Bakr, J. Simon, and M. Greiner, Phys. Rev. Lett. 107, 095301 (2011).