Evaluation of CHAMP Satellite Orbit with SLR Measurements

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ABSTRACT  The technique of Evaluating CHAMP satellite orbit with SLR measurements is presented. As an independent evaluation of the orbit solution, SLR data observed from January 1 to 16, 2002 are processed to compute the residuals after fixing the GFZ's post science orbits solutions. The SLR residuals are computed as the differences of the SLR measurements minus the corresponding distances between the SLR station and the GPS-derived orbit positions. On the basis of the SLR residuals analysis, it is found that the accuracy of GFZ's post science orbits is better than 10 cm and that there is no systematic error in GFZ's post science orbits.

KEYWORDS  satellite laser ranging (SLR); CHAMP satellite; satellite orbit

CLC NUMBER  P207; P228.41

Introduction

The German geophysical research satellite CHAMP (CHAllenging Minisatellite Payload), which is designed for geosciences and atmospheric research, was launched on July 15, 2000. It carries 7 different instruments including the GPS receiver TRSR-2 and the Laser retro-reflector to fulfill the mission with scientific objectives. By determining the transmitting time of laser pulses between the satellite laser ranging (SLR) instrument on the ground and the laser retro-reflector on board of the satellite, SLR system provide highly accurate distance measurements. These measurements can not only be used for precise orbit determination in connection with GPS, but also be used to validate the quality of the GPS-derived orbit. GFZ (GeoForschungsZentrum) which is responsible for the science operation provides rapid science orbits and post science orbits of CHAMP\textsuperscript{14}. Many analysis centers including the TUM (Technische Universität München), JPL (Jet Propulsion Laboratory), CSR (Center for Space Research) and ESOC (European Space Operations Center) calculated CHAMP orbits\textsuperscript{24}. Because of different methods, different force modes and different resolving parameters, etc. the results of CHAMP orbits are different. By pairwise comparison between all solutions and independent computation of SLR tracking residuals to all orbits, ESOC finds that the accuracy of the best orbits (CSR, GFZ, TUM and JPL) generally are at a level of better than 10 cm\textsuperscript{17}.

The principle of evaluation of CHAMP Satellite Orbit with SLR measurements is comparing the difference between the observed range by SLR normal point measurements and the computed range between the SLR station and the CHAMP orbit positions. To obtain the computed range, it is necessary to consider the effects of the displacements of ground station and the corrections of SLR measurements. The displacements of a ground station are mainly caused by tidal and tectonic motion. Station displacements...
resulted from tidal effects can be divided into three parts: the displacement due to the Earth tide, the displacement due to the ocean tide and the displacement due to the pole tide. According to the relative plate motion model the effect of the tectonic plate movement can be replaced by the effect of the station velocities. The corrections of SLR measurements include the correction for tropospheric delay, the correction for general relativity, the correction for the offset of the satellite's center-of-mass from the laser retroreflector and the correction for the eccentricity of the station. The post science orbits of CHAMP, which are obtained from GFZ, are validated with SLR data observed from January 1 to 16, 2002.

1 Displacements of ground station due to tides

1.1 Displacement due to the Earth tide

The Earth's motion combined with solar and lunar attraction forces causes Earth tide, which results in the changes of ground station coordinates. In this paper, the theory of Wahr is used to compute the displacement caused by the Earth tide. In the theory, site displacements caused by the Earth tide are characterized by the Love number and the Shida number. Because the values of the Love number depend on tidal frequency, computation of the variations of station coordinates due to Earth tide is done most efficiently by the use of a two-step procedure, as follows.

Step 1: Computing site displacements by use of the tidal term of frequency \( K_i^{(2)} \):

\[
\Delta h_1 = -0.025 \frac{3}{2} \sin \varphi \cos \lambda \sin \left( \theta_s + \lambda \right) - \frac{1}{2} \sin \varphi
\]

where \( \varphi \) is the geocentric latitude of the station; \( \lambda \) is the longitude of the station; \( \theta_s \) is Greenwich mean sidereal time.

The values of \( h_2 \) and \( l_2 \) described above will cause site permanent displacement. The radial component and transverse component of the permanent displacement are as follows:

\[
\Delta h_2 = -0.120\, 83 \left( \frac{3}{2} \sin^2 \varphi - \frac{1}{2} \right)
\]

\[
\Delta N = -0.050\, 71 \sin \varphi \cos \varphi
\]

1.2 Displacement due to the ocean tide

Ocean tide causes a temporal variation of the ocean mass distribution and the associated load on the crust and produces time-varying deformations of the Earth. The site displacements due to ocean tide can be decomposed into vertical component and horizontal components, and the vertical displacement is approximately three times larger than the horizontal ones. In this paper, only vertical displacement due to ocean tide is taken into account.

\[
\Delta h_3 = \sum_{i=1} A_{mp}(i) \cos(\text{Arg}(i,t) - \text{phase}(i))
\]

where \( A_{mp}(i) \) is the amplitudes of partial tides \( i \) on the site; \( \text{Arg}(i,t) \) is the astronomical argument of partial tides \( i \) at time \( t \) on the site; phase \( i \) is the phases lag of partial tides \( i \) on the site.

1.3 Displacement due to the pole tide

The pole tide is generated by the centrifugal effect of polar motion, and the variation of station coordinates can amount to a couple of centimeters. So the displacements due to the pole tide also need to be taken into account. The site displacements due to pole tide can also be decomposed into vertical and horizontal components. In this paper, only vertical component is taken into account.
2 Corrections of SLR measurements

2.1 Correction for tropospheric delay

The troposphere is the lower atmosphere extending from sea surface to approximately 30 km\(^{[6]}\). The refraction takes place when laser pulses transmit across the troposphere. The refractive index depends on the temperature, the atmospheric pressure, the pressure of water vapor, and the frequency of the laser. The correction for tropospheric delay is computed by use of the model of Marini and Murray\(^{[7]}\):

\[ \Delta \rho_{\text{TR}} = \alpha \cdot \frac{f(\lambda)}{f(\phi, h)} \left( \frac{A + B}{\sin E} + \frac{B/(A+B)}{\sin E + 0.01} \right) \]

where \( f(\lambda) = 0.965 + 0.016 4 + 0.000 228 \frac{1}{\lambda^4} \), \( f(\phi, h) = 1 - 0.002 6 \cos 2\phi - 3.1 \times 10^{-7} h \), \( A = 0.002 357 P + 0.001 41 P_w \), \( B = 1.084 \times 10^{-8} \times P \times T \times K \times 2 \times 4.734 \times 10^{-8} \times P \times T \times (3 - 1/K) \), \( P_w = \frac{W}{100} \times 0.611 \times (10^{7.5}(T+273.15) / (Pw+273.15)) \), \( K = 1.163 - 0.009 68 \cos 2\phi - 0.001 04 T + 0.000 014 35 P \),

where \( E \) is elevation of satellite; \( \lambda \) is laser wavelength(\( \mu \text{m} \)); \( \phi \) is station geodetic latitude; \( h \) is station height; \( P \) is atmospheric pressure at station(mb); \( T \) is atmospheric temperature at station(\( K \)); \( W \) is relative humidity(\%); \( P_w \) is water vapor pressure at station; \( a \) is the factor of tropospheric refraction, \( a = 1 \).

2.2 Correction for general relativity

On the theory of Einstein’s General Relativity, the speed of light is reduced when light passes near a massive body causing a time delay. When the corrections due to gravitational potential of the Sun and the Earth are taken into account, the corrections can be modeled as\(^{[8]}\):

\[ \Delta \rho_{\text{GR}} = (1 + \gamma) \cdot \frac{GM_i}{C^2} \cdot \ln \left( \frac{r_1 + r_2 + \rho}{r_1 + r_2 - \rho} \right) \]

where \( GM_i \) is gravitational constant for the Sun \((i=1)\) or the Earth \((i=2)\); \( \gamma \) is the parameterized post-Newtonian parameter \((\gamma = 1 \text{ for general relativity})\); \( r_1 \) is the distance of the Sun \((i=1)\) or the Earth \((i=2)\) to satellite; \( r_2 \) is the distance of the Sun \((i=1)\) or the Earth \((i=2)\) to the considered station.

2.3 Correction for the offset

SLR measurement is the range of station to the laser retro-reflector on the satellite, and the laser retro-reflector is offset from the satellite’s center of mass. To compute the range of station to satellite’s center of mass which is the reference point of the orbits of GFZ, the correction for the offset of the satellite’s center of mass and the laser retro-reflector must be taken into account. The corrections can be modeled as:

\[ \Delta \rho_{\text{OF}} = |C| \left( \frac{(R - r) \cdot S_p}{|R - r|} \right) \]

where \([C]\) is the transformation matrix of the J2000 reference frame to the satellite body-fixed reference frame; \( R \) is the position vector of station in the J2000 reference frame; \( r \) is the position vector of satellite in the J2000 reference frame; \( S_p \) is position vector of the laser retro-reflector in the satellite body-fixed reference frame; \( S_p = (0.00 \text{ m} 0.00 \text{ m} 0.25 \text{ m})^T \). It needs to point out that; the laser retro-reflector of CHAMP consists of 4 cube corner prisms, and the reference point \( S_p \) given in Reference \(^{[9]}\) is defined as the intersection of the optical axes of all 4 cube corner prisms. Only one prism contributes to the reflected signal in generally. This causes a difference of \( 4 \pm 2 \text{ mm} \) in the value of the offset\(^{[10]}\).

2.4 Correction for the eccentricity

The position of the SLR instrument on the ground may be different to the coordinate of the station. The difference is called station eccentricity and needs to be corrected. The corrections can be modeled as\(^{[4]}\):

\[ \Delta \rho_{\text{EC}} = \]
where \((p_x, p_y, p_z)\) is the position vector of satellite in local topocentric system; \((\Delta x, \Delta y, \Delta z)\) is the difference between the coordinate of SLR instrument and the coordinate of the station.

After above corrections, there are still systematic errors in the direct difference between observed range and computed range. These systematic errors are station dependent systemic errors which depend on every pass of each station. The effect of station dependent systematic errors can be divided into two parts: one is the ranging bias which is a constant, and the other is time bias which is relative to the velocity in radial direction.

The functional models used in analysis of station dependent systematic errors can be described as follows:

\[
v = \Delta \rho - (b + \hat{\rho})
\]

where \(\Delta \rho\) is the direct residuals of SLR measurements minus the corresponding distances between the SLR station and the orbit positions; \(b\) is the estimates of ranging bias and \(\hat{\rho}\) is the estimates of time bias; \(\hat{\rho}\) is the velocity in radial direction; \(v\) is the residual of \(\Delta \rho\) due to high frequency and random measurement noise.

### 3 Test computation and analysis

SLR data observed from January 1 to 16, 2002 are processed to compute the residuals after fixing the GFZ's post science orbits solutions. SLR data is the normal point obtained from NASA crustal dynamics data Information system (CDDIS), and the normal point is given in 5-second epochs coincident with the UTC timescale. The GFZ's post science orbits are given in 10-second epochs coincident with the TT timescale. In the computation, we first convert the CHAMP orbits from TT timescale to UTC timescale. Then we get CHAMP orbits at UTC epochs corresponding to SLR observations by use of Lagrangian interpolation. Finally we obtain the direct residuals of SLR measurements minus the corresponding distances between the SLR station and GFZ's post science orbits positions.

The SLR observations are a few because the altitude of the CHAMP is low and the speed of the satellite is high. Over the 15-day period of data processing, there are only 1 600 normal points are obtained by 11 SLR tracking stations. After a 30 cm rejection window is applied to reject individual bad point, there are 1 576 normal points. Two different elevation cut-off angles, 10 and 65, were tested. High elevation SLR residuals usually represent the radial component of orbit accuracy. After data editing, there are 1551 good observations left for 10 cut-off, while there were only 23 observations for 65 cut-off. Table 1 summarizes the direct residual of SLR for CHAMP orbit. Fig. 1 gives the residual of SLR for CHAMP orbit in which the station dependent systemic errors have been removed and not taken the elevation cut-off angles into account.

The following conclusions can be drawn from the Table 1 and Fig. 1.

1) The RMS of direct residual is usually smaller than 10 cm except the stations 7 403 and 7 810. This indicates that there are no large station dependent systemic errors in the SLR observations (because the SLR observations have been preprocessed by NASA, and the large station dependent systemic errors have been deleted) and the accuracy of GFZ's post science orbits is high. From the Table 1, we can see that there are still small station dependent systemic errors in the SLR observations.

![Fig. 1 SLR residual for CHAMP orbit](image-url)
Table 1: Result of CHAMP satellite orbit validated with SLR measurements

| ID of station | Number of observation | Direct residual of SLR/cm | Residual of SLR after removing the systematic error/cm |
|--------------|-----------------------|---------------------------|-------------------------------------------------------|
|              |                       | Mean | RMS | MIN | MAX | Mean | RMS | MIN | MAX |
| 1884         | 37                    | 4.16 | 9.79 | -18.92 | 28.64 | 0.00 | 7.60 | -18.42 | 15.41 |
| 7090         | 605                   | -1.92 | 8.74 | -27.39 | 20.77 | 0.00 | 6.70 | -21.40 | 19.53 |
| 7103         | 96                    | -0.10 | 7.24 | -20.54 | 19.39 | 0.00 | 6.46 | -17.32 | 18.27 |
| 7110         | 94                    | 1.41 | 8.45 | -20.92 | 19.45 | 0.00 | 5.17 | -14.27 | 11.58 |
| 7403         | 22                    | 8.72 | 14.47 | -14.64 | 27.69 | 0.00 | 4.65 | -8.12 | 6.27 |
| 7810         | 55                    | 1.45 | 11.31 | -19.72 | 29.39 | 0.00 | 6.36 | -14.03 | 15.29 |
| 7835         | 192                   | 0.69 | 7.69 | -22.42 | 26.32 | 0.10 | 6.74 | -21.23 | 27.36 |
| 7838         | 142                   | -0.77 | 7.07 | -16.95 | 19.33 | 0.00 | 6.19 | -15.32 | 22.29 |
| 7839         | 223                   | -1.88 | 7.45 | -21.62 | 20.05 | 0.00 | 6.26 | -16.92 | 17.74 |
| 7840         | 33                    | -3.28 | 4.39 | -9.37 | 8.62 | 0.00 | 3.28 | -8.44 | 7.86 |
| 7849         | 77                    | -3.34 | 6.68 | -27.38 | 11.89 | 0.00 | 5.77 | -17.13 | 17.73 |
| total        | 1576                  | -0.87 | 8.47 | -27.39 | 29.39 | 0.01 | 6.37 | -21.40 | 27.36 |
| No cut-off   |                       |      |     |      |      |      |      |      |      |
| 10° cut-off  | 1551                  | -0.92 | 8.40 | -27.39 | 29.39 | 0.00 | 6.26 | -21.40 | 22.29 |
| 65° cut-off  | 23                    | 2.59 | 3.62 | -5.08 | 8.66 | 0.12 | 2.45 | -6.27 | 4.00 |

3) By checking the residuals of the SLR measurements directly, we can judge whether the CHAMP satellite orbit has systematic errors or not. It is the fact that the station dependent systematic errors in each pass of each station are shown of random characteristics in the large number of measurements. Thus the average of the systematic errors could be applied in judging the orbit systematic errors. In order to evaluate the accuracy of the satellite orbit, we fit and delete the station dependent systematic errors of SLR measurements at first. After that the differences between the clean SLR measurements and the calculated distances from the stations to the satellite reflect the orbit errors mainly, because the clean SLR measurements arrive the accuracy of centimeter

4) The RMS of SLR residuals is smaller than the actual 3D orbit error because the direction of SLR observation is the line of sight between station and satellite. By pairwise orbit comparisons between all solutions and independent computation of SLR tracking residuals to all orbits, ESOC finds that the ratio of orbit error to SLR residual is 1.65

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