Regularities of motion of a particle in the spiral-screwed device

Yuri Isaev1,*, Nikolay Semashkin1, and Vadim Zlobin1
1Ulyanovsk State Agrarian University, Ulyanovsk, Russia

Abstract. The article describes the functional relationship between the geometric parameters of the spiral screw and the kinematic motion characteristics of the material being moved and its individual particles. The authors considered the movement of a particle in the cylindrical system of coordinates. Differential equations have been obtained, which describe the motion trajectory of a material particle along the inner surface of the casing of a spiral-helical device.

1 Introduction

To calculate and design a device with a spiral-helical working body, it is necessary to know the functional relationship between its parameters and the kinematic motion characteristics of the material being moved and its individual particles. Such a connection can be found for the case when a material particle, with the steady state of motion, moves along the device casing in the axial direction and makes a curved linear motion along the internal surface of the casing.

2 Materials and methods of research

Let us consider a spiral-helical device at the angle \( \delta \) to the horizon (Fig. 1.). The spiral screw rotates around its axis with the constant angular speed \( \omega \), with variable pitch of the helical line \( s_1 < s_2 < \ldots < s_n \).

The following forces have been applied to the moving particle: \( G=mg \) – gravity force, N; \( N_2 \) – normal reaction of the inner surface of the casing, N; \( N_1 \) – normal reaction of the spiral turn surface, N; \( f_2N_2 \) – friction force of particles on the inner surface of the casing; \( f_1N_1 \) – friction force of a particle on the surface of a spiral turn.

The angle \( \beta \) between normal reaction of the surface of a spiral turn and the axis, perpendicular to the helical line, depend on geometrical characteristics of a spiral screw, the cylindrical casing and size of the moved material particles in a device [1, 2, 3, 4]:

\[
\beta = \arcsin \left( \frac{(r_0 + r_3 - r_1 - r_2)}{(r_1 + r_3)} \right),
\]

(1)

* Corresponding author: isurma@yandex.ru

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where \( r_0 \) – the inner radius of the cylindrical casing, m; \( r_1 \) – the radius of a particle, m; \( r_2 \) – the radius of a spiral screw, m; \( r_3 \) – the radius of the spiral wire, m.

![Diagram: The scheme of the forces applied to a particle in a spiral-helical device.](image)

**Fig. 1.** The scheme of the forces applied to a particle in a spiral-helical device.

We will consider a particle being moved in the cylindrical system of coordinates \( O\rho, O\rhoz \) with the left-hand frame of reference. When a particle moves along the casing in a plane perpendicular to the axis \( O\rhoz \) of the spiral screw, the particle performs a relative angular motion at an angle \((\omega t - \varphi)\) in the specified plane (Fig. 1.). Accordingly, motion, velocity and acceleration of the particle in the axial direction will be expressed in the following way:

\[
\begin{align*}
z &= r_0(\omega t - \varphi) \tan \alpha, \\
\dot{z} &= r_0(\omega - \dot{\varphi}) \tan \alpha, \\
\ddot{z} &= -r_0 \ddot{\varphi} \tan \alpha. 
\end{align*}
\]

(2)

In this case, the differential equations of particle motion of a unit mass, \( m = 1 \), in projections on the coordinate axes will take the following form [5, 6, 7, 8]:

\[
\begin{align*}
\dot{r}_0 \dot{\varphi} &= N_1 (\sin \alpha \cos \beta + f_1 \cos \alpha) - g \cos \delta \cdot \sin \varphi - f_2 N_2 \frac{r_0 \ddot{\varphi}}{\sqrt{r_0^2 (\omega - \dot{\varphi})^2 \tan^2 \alpha + r_0^2 \dot{\varphi}^2}}; \\
-r_0 \dot{\varphi}^2 &= g \cos \delta \cdot \cos \alpha - N_2 + N_1 \sin \beta; \\
m \ddot{z} &= N_1 (\cos \alpha \cos \beta - f_1 \sin \alpha) - g \sin \delta - f_2 N_2 \frac{r(\omega - \dot{\varphi}) \tan \alpha}{\sqrt{r_0^2 (\omega - \dot{\varphi})^2 \tan^2 \alpha + r_0^2 \dot{\varphi}^2}};
\end{align*}
\]

(3)

where \( m \) – the particle mass, kg; \( \dot{r}_0 \), \( \ddot{r}_0 \) – the first and second derivative of the particle being moved on the axis \( O\rho \) respectively; \( \varphi \) – the angle of the particle movement on the inner surface of the casing in the direction perpendicular to the axis of the spiral screw, degrees \( \varphi \), \( \dot{\varphi} \) – the first and second derivative of the particle movements on the axis \( O\rho \) respectively; \( \alpha \) – the variable angle of helix of the spiral screw to the plane of its cross-section, depending
on its pitch, degrees; $f_1$ – the coefficient of friction of the particle on the surface of the spiral screw; $f_2$ – the coefficient of friction of the particle on the inner surface of the casing; $\lambda$ – the angle between the direction of the component of gravity along the line of the largest slope and the direction of the axis of the spiral screw, degrees; $\ddot{z}$, $\dot{z}$ – the first and second derivative of the particle motion on the axis Oz respectively; $r$ – the coordinate of the particle motion along the axis Or, m.

We will designate:

$$A(\phi) = \frac{r\dot{\phi}}{\sqrt{r_0^2(\omega-\phi)^2\tan^2\alpha + r_0^2\dot{\phi}^2}}; \quad B(\phi) = \frac{r(\omega-\phi)\tan\alpha}{\sqrt{r_0^2(\omega-\phi)^2\tan^2\alpha + r_0^2\dot{\phi}^2}};$$

$$C = \sin \alpha \cos \beta + f_1 \cos \alpha; \quad D = \cos \alpha \cos \beta - f_1 \sin \alpha.$$

Then

$$\begin{align*}
\begin{cases}
\dot{\phi} = CN_1 - g \cos \delta \sin \varphi - f_2 N_2 A(\phi); \\
-r_0^2 \dot{\phi}^2 = g \cos \delta \cos \alpha - N_2 + N_1 \sin \beta; \\
-rtg\alpha \dot{\phi} = DN_1 - g \sin \delta - f_2 N_2 \cdot B(\phi).
\end{cases}
\end{align*} \quad (4)$$

With the exclusion of normal reactions $N_1$ and $N_2$ from the system of equations (4), we obtain one expression with one unknown relative to the coordinate $\varphi$.

$$\begin{align*}
r_0 \dot{\phi} + g \cos \delta \sin \varphi + f_2 \left(g \cos \delta \cos \varphi + r_0 \dot{\phi}^2\right)\frac{r_0 \dot{\alpha}}{\sqrt{r_0^2(\omega-\alpha)^2\tan^2\lambda + r_0^2\dot{\alpha}^2}} &= \\
= \begin{pmatrix} U \\ V \end{pmatrix} & (5)
\end{align*}$$

and

$$\varphi = \frac{f_1 \left(g \cos \delta \cos \varphi + r_0 \dot{\varphi}^2\right) \left( \frac{U(\varphi)}{V(\varphi)} - A(\varphi) \right) - g \cos \delta \sin \varphi + g \frac{U(\varphi)}{V(\varphi)} \sin \delta}{1 + \tan\alpha \frac{U(\varphi)}{V(\varphi)}} \quad (6)$$

where $U = U(\varphi) = C - f_2 A(\varphi) \sin \beta; V = V(\varphi) = D - f_2 B(\varphi) \sin \beta$.

Differential equations obtained as a result of calculations describe the trajectory of moving a material particle along the inner surface of the casing of a spiral-helical device. The presented equations were solved with numerical methods using a computer. Having substituted the obtained time dependences into the first and second equations (6), we can find expressions to determine the displacement and velocity of the moving particle, taking into account the parameters of the design of a spiral screw [9, 10, 11, 12]. The initial conditions for the motion of particles and the numerical values of the constants included in the equations to solve the obtained equations for each particular case are different. The dependences obtained can be used in the calculation and design of spiral-helical devices with variable pitch of the helix, used in the transportation of small seeds in the seeding unit of seeders.
The use of devices with a working body in the form of a spiral screw with variable pitch allows one to increase or decrease the speed of a particle of material along the length of the working body without changing the frequency of rotation of the spiral screw.

3 Results of the research

In figures 2, 3 and 4 graphic interpretations of the calculation results are given for a spiral-helical device with the following characteristics: $f_1 = 0.5; f_2 = 0.3; \omega = 2 \text{ s}^{-1}; \delta = 5^\circ; d = 0.003 \text{ m}; r_1 = 0.004 \text{ m}; r_0 = 0.045 \text{ m}; r_2 = 0.02 \text{ m}; s = 0.006...0.012 \text{ m} –$ variable pitch of the helical line of a helix.

The damped oscillations of the particle predominate around the casing forming at angles of inclination to the horizon $\delta$ less than $15^\circ$ and rotational speeds determined by the values of the criterion $k = \omega^2 r / g \approx 1$. They are characterized by a phase trajectory shown in Figure 2.

![Fig. 2. Phase trajectory of angular velocity and moving particles.](image)

With increasing screw pitch over time, the speed of the particle movement increases, as shown in Figure 3.

![Fig. 3. Change in the axial velocity of a particle $\dot{z}$ depending on time $t$.](image)
When using a spiral screw with variable pitch, at the initial unsteady moment of time some oscillation of the axial velocity of the particle is observed. After the first second and further, the axial velocity of the particle increases continuously.

Figure 4 shows the results of calculations of the change in the lag coefficient $\varepsilon$, equal to the ratio of the axial velocity of a particle to the axial velocity of the spiral screw, depending on time.

![Graph showing the change in the lag coefficient $\varepsilon$]

**Fig. 4.** Dependence of the material particle lag coefficient $\varepsilon$ from time $t$

Changes in the coefficient $\varepsilon$ are observed at the initial time moment and stabilize rather quickly.

### 4 Conclusions

Analytical expressions have been obtained that describe the process of moving seeds with a spiral screw from the seed box to the seed tube. Equilibrium equations of a particle are obtained when moving along the surface of a spiral screw and expressions for determining the displacement, velocity, and acceleration of a particle in the axial direction. The dependences have been obtained for the calculation and design of spiral-helical devices with variable pitch of the helix used in the transportation of seeds of small-seed crops in seeding units of seeding machines.

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