Hole spin dephasing in p-type semiconductor quantum wells

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Hole spin dephasing time due to the D’yakonov-Perel’ mechanism in p-type GaAs (100) quantum wells with well separated light-hole and heavy-hole bands is studied by constructing and numerically solving the kinetic spin Bloch equations. We include all the spin-conserving scattering such as the hole-phonon and the hole-nonmagnetic impurity as well as the hole-hole Coulomb scattering in our calculation. Different effects such as the temperature, the hole density, the impurity density and the Rashba coefficient on the spin dephasing are investigated in detail. We also show that the Coulomb scattering makes marked contribution to the spin dephasing. The spin dephasing time can either increase or decrease with temperature, hole/impurity density or the inclusion of the Coulomb scattering depending on the relative importance of the spin-orbit coupling and the scattering.

I. INTRODUCTION

Much attention has been devoted to the spin degree of freedom of carriers in Zinc-blende semiconductors recently due to the possible application to the spintronic devices.1–3,4 Understanding spin dephasing/relaxation of carriers in semiconductor quantum wells (QWs) is one of the most important prerequisites for the realization of such devices. There are many studies of spin dephasing/relaxation of electrons in n-type QWs where the spin dephasing is determined by the D’yakonov and Perel’ (DP) mechanism.5 Most studies are within the framework of single-particle approximation of carriers in semiconductor quantum wells (QWs) at high temperatures, including the carrier-carrier Coulomb scattering (beyond the Hartree-Fock self-energy from the Coulomb interaction), can cause irreversible dephasing. This many-body approach takes account of the inhomogeneous broadening not only from different directions of Ω(k) (not only −|Ω(k)| and +|Ω(k)|), but also from the modulus of the DP effective field, i.e., |Ω(k)|. Moreover, this approach also takes full account of the counter effect of the scattering to the inhomogeneous broadening instead of only the lowest-order elastic scattering: The scattering tends to drive carriers to more homogeneous states and therefore suppresses the inhomogeneous broadening induced by the DP term. Finally, this approach is valid even when |Ω(k)|τp ≫ 1 and is applicable to systems far away from equilibrium (e.g., systems with high spin polarization and/or in the presence of high electric field parallel to QWs).6,7 Using this method, Weng and Wu performed a systemic studies of spin dephasing in n-type GaAs QWs at high temperatures and showed that the effects beyond the single-particle approach Eq. (1) are dominant even for systems near equilibrium.19 These effects include the many-body effects, the inhomogeneous broadening induced spin dephasing and the counter effect of the scattering to the inhomogeneous broadening. For small well width, the accidental small rotations.6 This approach captures the lowest (first) order of the anisotropy due to the fact that Ω(−k) = −Ω(k).

It is shown recently by Wu et al. from a full many-body microscopic approach16,17,18,19,20,21,22,23 that the single-particle approach is inadequate in accounting for the spin dephasing/relaxation. The momentum dependence of the effective magnetic field (the DP term), and even the momentum dependence of the spin diffusion rate along the spatial gradient22 or the random spin-orbit coupling,24 serve as inhomogeneous broadenings.17,18 In the presence of the inhomogeneous broadening, any scattering, including the carrier-carrier Coulomb scattering (beyond the Hartree-Fock self-energy from the Coulomb interaction), can cause irreversible dephasing. This many-body approach takes account of the inhomogeneous broadening not only from different directions of Ω(k) (not only −|Ω(k)| and +|Ω(k)|), but also from the modulus of the DP effective field, i.e., |Ω(k)|. Moreover, this approach also takes full account of the counter effect of the scattering to the inhomogeneous broadening instead of only the lowest-order elastic scattering: The scattering tends to drive carriers to more homogeneous states and therefore suppresses the inhomogeneous broadening induced by the DP term. Finally, this approach is valid even when |Ω(k)|τp ≫ 1 and is applicable to systems far away from equilibrium (e.g., systems with high spin polarization and/or in the presence of high electric field parallel to QWs).6,7 Using this method, Weng and Wu performed a systemic studies of spin dephasing in n-type GaAs QWs at high temperatures and showed that the effects beyond the single-particle approach Eq. (1) are dominant even for systems near equilibrium.19 These effects include the many-body effects, the inhomogeneous broadening induced spin dephasing and the counter effect of the scattering to the inhomogeneous broadening. For small well width, the
calculated electron SDTs using this microscopic many-body theory increase with temperature and are in agreement with the experiment both qualitatively and quantitatively, while the SDTs of earlier simplified treatment drop dramatically with temperature and are one order of magnitude larger than the experiment data.\textsuperscript{19} For larger well width, the SDT may first increase then decrease with temperature.\textsuperscript{21} These properties come from the competing effects between the DP term and the scattering.

Although there are extensive investigations on the spin relaxation/dephasing of electrons, investigations on the spin relaxation/dephasing of holes in p-type semiconductor QWs are relatively limited.\textsuperscript{26,27} Nevertheless, knowledge of the spin relaxation/dephasing of holes in p-type QWs is very important to the assessment of the feasibility of hole-based spintronic devices. This is because a possible way to achieve high electronic spin injection without the conductance mismatch\textsuperscript{32} is to use magnetic semiconductors as spin source and most magnetic semiconductors are p-type at high temperature.\textsuperscript{33} Vey recently there are some reports on the hole spin relaxation/dephasing.\textsuperscript{28,29,30,31} All the theoretical calculations in these works are within the framework of the single-particle approximation Eq. (1).\textsuperscript{30,31}

It has been shown in electron systems that Eq. (1) is inadequate in accounting for the spin dephasing. Moreover, the electronic states and spin-orbit coupling of holes are very different from those of electrons.\textsuperscript{34,35,36} In bulk material, the Γ-point degeneracy of the heavy hole (HH) and the light hole (LH) makes the hole spin relaxation in the same order of the momentum relaxation (100 fs).\textsuperscript{28} This degeneracy is lifted in QWs. Under the parabolic approximation, the HH and LH bands can be treated independently for QWs of small well width. Unlike the conduction band where the DP term mainly comes from the BIA contribution in GaAs QWs, in p-type GaAs QWs, the SIA contribution is usually the dominant one. It is noted that in hole system the relation $\tau_p \ll 1$ is usually unsatisfied due to the strong spin-orbit coupling and consequently the validity of Eq. (1) is even more questionable. Therefore, in this paper we investigate the hole spin dephasing using our full many-body microscopic approach. We calculate the SDT of the HH and LH by numerically solving the many-body spin kinetic Bloch equations with all the scattering explicitly included. Then we discuss how the temperature, the hole density, the Coulomb scattering, the Rashba coefficient and the impurity density affect the SDT. We show that the earlier treatment based on the single particle approximation is not valid in hole systems and unlike the case of electrons where the scattering “always” raises the SDT at low-spin polarization, the scattering can either enhance or suppress the SDT of holes based on the relative importance of the Rashba term and the scattering.

The paper is organized as follows: In Sec. II we set up our model and kinetic equations. Then in Sec. III we present our numerical results. We first show the time evolution of the spin signal in Sec. III A. In Sec. III B we investigate how the temperature affects the spin dephasing. The Coulomb scattering, the impurity density and hole density dependence of the SDT are discussed separately in Sec. III B, C and D. We conclude in Sec. IV. In Appendix A we show the effect of the scattering to spin dephasing when it is much weaker than the spin-orbit coupling strength. In Appendix B we present a simplified analytical analysis of the SDT and show the different effects of the scattering to the spin dephasing at strong/weak scattering regime.

II. KINETIC SPIN BLOCH EQUATIONS

We start our investigation from a p-doped (100) GaAs QW of well width $a$. The growth direction is assumed to be along the $z$ axis. A moderate magnetic field $B$ is applied along the $x$ axis (in the Voigt configuration). Here we assume only the lowest subband is populated. It is noted that for two-dimensional hole system, the lowest subband is HH-like. By applying a suitable strain, the lowest subband can be LH-like. We assume the confinement is large enough so that the HH and LH bands are well separated and we may consider the HHs and LHS separately. With the DP term included, the Hamiltonian of the holes can be written as:

$$H_{\lambda} = \sum_{k, \sigma, \sigma'} \{ \varepsilon_{k\lambda} \delta_{\sigma\sigma'} + [g\lambda \mu_B B + \Omega_x^{(\lambda)}(k)] \cdot \frac{\sigma_{\sigma\sigma'}}{2} \} c_{k\lambda\sigma}^\dagger c_{k\lambda\sigma'} + H_I.$$  \hspace{1cm} (3)

Here $\lambda = LH, HH$ denotes the LH or HH state, $\sigma = +, -$ stands for the spin. $\varepsilon_{k\lambda} = k^2 / 2m^*_\lambda$ is the energy of hole with wave vector $k$ and effective mass $m^*_\lambda$. $\sigma$ are the Pauli matrices. The DP term is mainly from the Rashba term. For (100) GaAs QWs, we have

$$\Omega^{HH}_x(k) = 2E_z[\gamma_{53}^6 \gamma_{7}^6 k_x^2 k_y + \gamma_{54}^7 k_y (k_x^2 - 3k_z^2)] , \hspace{1cm} (4)$$

$$\Omega^{HH}_y(k) = -2E_z[\gamma_{53}^6 \gamma_{7}^6 k_x^2 k_z + \gamma_{54}^7 k_z (k_x^2 - 3k_y^2)] , \hspace{1cm} (5)$$

and

$$\Omega^{HH}_z(k) = 0 , \hspace{1cm} (6)$$

for HHs and

$$\Omega^{LH}_x(k) = 2E_z[\gamma_{62}^6 \gamma_{53}^6 k_x^2 k_y + \gamma_{62}^6 \gamma_{53}^6 k_x^2 k_y + \gamma_{54}^7 k_y (k_x^2 - 3k_z^2)] , \hspace{1cm} (7)$$

$$\Omega^{LH}_y(k) = -2E_z[\gamma_{62}^6 \gamma_{53}^6 k_x^2 k_z + \gamma_{62}^6 k_z (k_x^2 - 3k_y^2)] , \hspace{1cm} (8)$$

and

$$\Omega^{LH}_z(k) = 0 \hspace{1cm} (9)$$

for LHS.\textsuperscript{35} It is seen from these equations that the magnitude of the Rashba term can be tuned by means of an external gate voltage which changes the electric field $E_z$ in the sample.\textsuperscript{38,39,40,41} $\gamma_{53}^6$, $\gamma_{54}^7$, $\gamma_{52}^6$, $\gamma_{53}^6$ and $\gamma_{54}^7$ in Eqs. (4-9) are the Rashba coefficients;\textsuperscript{35} $\gamma_{53}^6 = \frac{3e\hbar}{m^*_\lambda} \gamma_3 (\gamma_2 + \gamma_3) (\frac{1}{\Delta_{h1}} - \frac{1}{\Delta_{h2}})$, $\gamma_{54}^7 = \frac{3e\hbar}{m^*_\lambda} \gamma_3 (\gamma_2 + \gamma_3) (\frac{1}{\Delta_{l1}} - \frac{1}{\Delta_{l2}})$, $\gamma_{52}^6 = -\frac{3e\hbar}{m^*_\lambda} \gamma_3 \Delta_{l2}$.
\[ \gamma_{66}^{6l} = \frac{3}{4} \frac{e^4}{m_0} \gamma_3 \left[ \frac{1}{2 \Delta_{h}} + \frac{1}{2 \Delta_{s}} \right] \gamma_2 + \frac{7 \gamma_3}{2 \Delta_{h}} \] and \[ \gamma_{54}^{6l} = \frac{3}{4} \frac{e^4}{m_0} \gamma_3 (\gamma_2 - \gamma_3) \Delta_{h} \] in which \( \Delta_{h} \), \( \Delta_{s} \) and \( \Delta_{ls} \) present the energy gaps between the HH and the LH bands, the HH and the split-off bands, the LH and the split-off bands respectively:

\[ \Delta_{h} = 4 \frac{\gamma_3 k^2}{\gamma_2 - \gamma_3}, \quad (10) \]

\[ \Delta_{h} = \Delta_{0} - (\gamma_1 - 2 \gamma_2) \frac{h^2 k^2}{2 m_0}, \quad (11) \]

\[ \Delta_{ls} = \Delta_{0} - (\gamma_1 + 2 \gamma_2) \frac{h^2 k^2}{2 m_0}, \quad (12) \]

with \( \Delta_{0} \) representing the energy gap of the split-off band (from the \( \Gamma \)-point of the valence band). \( \gamma_1 \), \( \gamma_2 \) and \( \gamma_3 \) are the Luttinger parameters. From Eqs. (4-9) one can see that the HH Rashba terms include only the cubic terms whereas the LH ones include both the cubic and the linear terms. The ratio of the cubic and the linear terms depends on the well width \( a \) : \( k^2 \) in the linear terms decreases with \( a^2 \). Furthermore, one can see from Eqs. (10) and (12) that \( \Delta_{h} \) decreases faster with \( a \) than \( \Delta_{ls} \), which makes \( \gamma_{66}^{54} \) and \( \gamma_{54}^{6l} \) increase faster with \( a \) than \( \gamma_{54}^{6l} \). Therefore, the cubic terms weighted by \( \gamma_{66}^{6l} \) and \( \gamma_{54}^{6l} \) increase faster with well width than the linear terms weighted by \( \gamma_{54}^{6l} \). In brief, when \( a \) is small, both the linear and the cubic terms are important; When \( a \) gets larger, the cubic terms are the leading terms.

The interaction Hamiltonian \( H_I \) in Eq. (3) is composed of the hole-hole Coulomb interaction and hole-phonon scattering, as well as hole-impurity scattering. Their expressions can be found in textbooks.\(^{32,43}\)

We construct the many-body kinetic spin Bloch equations by the non-equilibrium Green function method as follows:

\[ \rho_{k\lambda,\sigma\sigma'} = \rho_{k\lambda,\sigma\sigma'}^{\text{coh}} + \rho_{k\lambda,\sigma\sigma'}^{\text{scatt}} \]

with \( \rho_{k\lambda,\sigma\sigma'}^{\text{coh}} \) representing the single-particle density matrix elements. The diagonal elements \( \rho_{k\lambda,\sigma\sigma} \equiv f_{k\lambda,\sigma} \) describe the hole distribution functions of wavevector \( k \), state \( \lambda \) and spin \( \sigma \). The off-diagonal elements \( \rho_{k\lambda,\lambda,\pm} \) describe the inter-spin-band correlations for the spin coherence. \( \rho_{k\lambda,\sigma\sigma'}^{\text{coh}} \) describe the coherent spin precessions around the applied magnetic field \( B \) in the Voigt configuration, the effective magnetic field \( \Omega(k) \) as well as the effective magnetic field from the hole-hole Coulomb interaction in the Hartree-Fock approximation and can be written as:

\[ \partial f_{k\lambda,\sigma} \partial t \bigg|_{\text{coh}} = -2 \sigma \{ g_{\lambda,\mu} B + \Omega_{\lambda}(k) \} \text{Im} \rho_{k\lambda,\sigma} + \Omega_{\lambda}(k) \text{Re} \rho_{k\lambda,\sigma} + 4 \sigma \text{Im} \sum_{q} V_{\lambda}^{*} \rho_{k+q,\lambda,\sigma} \rho_{k\lambda} \]

\[ \rho_{k\lambda,\sigma\sigma'}^{\text{scatt}} \] in Eq. (13) denote the hole-hole Coulomb, hole-phonon and hole-impurity scattering. The expressions of these scattering terms and the details of solving these many-body kinetic spin Bloch equations can be found in Ref. [20].

### III. NUMERICAL RESULTS

We numerically solve the kinetic spin Bloch equations and obtain temporal evolution of the hole distribution \( f_{k\lambda,\sigma}(t) \) and the spin coherence \( \rho_{k\lambda}(t) \). We include the hole-phonon and the hole-hole scattering throughout our computation. As we concentrate on the relatively high-temperature regime \( (T \geq 120 \text{K}) \), we only include the hole-LO-phonon scattering. The hole-impurity scattering is included when stated. As discussed in the previous papers,\(^{16,17,42,44}\) the irreversible spin dephasing can be well defined by the slope of the envelope of the incoherently summed spin coherence

\[ \rho_{\lambda} = \sum_{k} |\rho_{k\lambda}(t)|. \]

The material parameters of GaAs in our calculation are tabulated in Table I where \( \Omega_{LO} \) represents the LO phonon frequency and \( \kappa_\infty (\kappa_0) \) is the optical (static) dielectric constant.\(^{35,45}\) Our main results are plotted in Figs. 1-6. In these calculations the width of the QW is chosen to be 5 nm unless otherwise specified; the initial spin polarization \( P_{\lambda} = (N_{\lambda,+} - N_{\lambda,-})/(N_{\lambda,+} + N_{\lambda,-}) \) is 2.5 % with

\[ N_{\lambda,\sigma} = \sum_{k} f_{k\lambda,\sigma}, \quad (17) \]

representing the hole density of \( \sigma \)-spin band; the magnetic field \( B = 4 \text{ T} \), and the Rashba coefficient \( \gamma_3 E_{z} m_0 \) is taken to 0.5 nm when \( a = 5 \text{ nm} \).
The typical evolution of the HH densities in the spin-up and -down bands together with the incoherently summed spin coherence $\rho_{HH}$ is shown for the case of the HHs. Note that the scale of the spin coherence is on the right side of the figure. The dotted line represents the slope of the envelope of $\rho_{HH}$.

A. Temporal evolution of the spin signal

We first study the temporal evolution of the spin signal in a GaAs QW at $T = 300$ K. In Fig. 1 we show the typical evolution of the HH densities in the spin-up and -down bands together with the incoherently summed spin coherence for the total HH density $N_{HH, \uparrow} + N_{HH, \downarrow} = 4 \times 10^{11}$ cm$^{-2}$ and impurity density $N_i = 0$. It is seen from the figure that the hole densities in the spin-up and -down bands and the incoherently summed spin coherence oscillate due to the presence of the magnetic field. From the slope of the envelope of the incoherently summed spin coherence, one is able to deduce the SDT.

B. Temperature dependence of the SDT

We now turn to study the temperature dependence of the SDT at different impurity densities $N_i$. We plot the SDTs of the LH and the HH in Fig. 2(a) and (b) as functions of temperature. The total LH and HH densities $N_{LH}$ and $N_{HH}$ are taken to be $N_h = 4 \times 10^{11}$ cm$^{-2}$. One finds from Fig. 2(a) that for the LH, when there are no impurity $N_i = 0$ or low impurities $N_i < 0.1N_h$, the SDT first decreases then increases with temperature. The minimum occurs at smaller temperature for higher impurity densities: 140 K when $N_i = 0.1N_h$ and 200 K when $N_i = 0$. When the impurity density $N_i = N_h$, the SDT increases with temperature monotonically. These temperature dependences are quite different from those of electrons in QWs with the same electron density and initial spin polarization where the SDT increases monotonically with temperature.\(^{19}\)

It is noted that the property of spin dephasing is quite different when $|\Omega|\tau_p \gtrsim 1$ and $|\Omega|\tau_p \ll 1$. When $|\Omega|\tau_p \gtrsim 1$, the scattering is weak in comparison to the DP effective field (inhomogeneous broadening) and the counter effect of the scattering to the inhomogeneous broadening is unimportant or can be ignored. In the presence of inhomogeneous broadening, adding a new scattering provides an additional dephasing channel.$^{17,42,46}$ This effect has been revealed in detail in Appendix A. Therefore, the scattering in this regime provides a spin dephasing channel and the increase of the temperature leads to a stronger scattering and consequently a faster spin dephasing. Moreover, the increase of the temperature drives holes to a higher $k$-state, and holes experience a larger $|\Omega(k)|$, i.e., a stronger inhomogeneous broadening. This tends to reduce the SDT too. Therefore, the SDT decreases with temperature when $|\Omega|\tau_p \gtrsim 1$. When $|\Omega|\tau_p \ll 1$, the scattering is strong in comparison to the DP term. Hence the counter effect of the scattering to the inhomogeneous broadening cannot be ignored any more. As the scattering tends to drive carriers to more homogeneous states in $k$-space, it tends to increase the SDT. Therefore, whether the SDT increases or decreases with temperature depends on the competition between the scattering and the DP term. It will be shown later that when the linear part in the DP term is dominant, the increase of the inhomogeneous broadening with temperature is relatively slower than that of the scattering and the SDT increases with temperature. Nonetheless, when the cubic part in the DP term is dominant, the increase of the inhomogeneous broadening with temperature turns out to be faster than the increase of the scattering and the SDT decreases with temperature.

For electrons in GaAs QWs, the spin-orbit coupling is not very strong. $|\Omega|\tau_p$ is usually much smaller than 1 (typically $|\Omega|\tau_p = 0.016$ at $T = 100$ K, $a = 15$ nm, and the total electron density $N_{e,+} + N_{e,-} = 4 \times 10^{11}$ cm$^{-2}$). Therefore when the linear (cubic) term in Eq. (2) is dominant, the SDT of electrons increases (decreases) with temperature.$^{19,21}$

| Temperature (K) | $\Delta_0$ | $m_{LH}$ | $m_{HH}$ | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ |
|----------------|---------|---------|---------|---------|---------|---------|---------|
| 100            | 0.341   | 0.067   | 0.596   | 2.1     | 2.1     | 5.6     | 2.9     |
| 200            | 0.341   | 0.067   | 0.596   | 2.1     | 2.1     | 5.6     | 2.9     |

**TABLE I:** Parameters used in the calculation.

| Parameter | Value |
|-----------|-------|
| $\kappa_0$ | 12.9  |
| $\kappa_0$ | 12.9  |
| $\gamma_0$ | 2.1   |
| $\gamma_1$ | 2.1   |
| $\gamma_2$ | 2.1   |
| $\gamma_3$ | 2.9   |

**TABLE II:** Rashba coefficients [unit: nm/(E_g m_0)].

| Parameter | Value |
|-----------|-------|
| $a = 5$ nm | $-0.193$ |
| $a = 7$ nm | $-0.156$ |

Situations are more complicated for hole system due to the strong spin-orbit coupling. The Rashba coefficients of the coupling in Eqs. (4-9) are listed in Table II. For LHs, when $a = 5$ nm, $\xi_{20}(E_g m_0 k_2^2)$ changes from $-3.36$ nm to $-1.46$ nm when the temperature changes.
from 100 K to 300 K. Here \( \langle k^2 \rangle \) represents the average of \( k^2 \). It is seen from Table II that both \( \gamma_{65}^{66} \) and \( \gamma_{54}^{66} \) are smaller than \( \gamma_{65}^{66} \). Therefore the linear terms in Eqs. (7) and (8) are dominant. From these coefficients, one may further find that when there is no impurity, the spin-orbit coupling for LHs is one or two orders of magnitude larger than that of electrons. Consequently neither \( \Omega_L^{a1} \) nor \( \Omega_L^{\tau} \) is usually slightly smaller than 1. In this regime, both the competing effects of the scattering addressed above can not be neglected. Therefore the temperature dependence of the SDT depends on the competition between the effect of the increase of the spin dephasing due to the increase of the inhomogeneous broadening and the increase of the scattering with temperature (The latter provides additional spin dephasing channel) (Effect I) and the effect of the decrease of the spin dephasing due to the counter effect from the increase of scattering which suppresses the inhomogeneous broadening (Effect II). The results for impurity free case shown in Fig. 2(a) indicate that when \( T < 200 \) K and the total scattering is not so strong, Effect I is more important and hence the SDT decreases with \( T \). When temperature keeps increasing and the total scattering is further enhanced, the counter effect of the scattering to the inhomogeneous broadening (Effect II) becomes more important and the SDT increases with \( T \). Comparing with our previous works, one further finds that the absolute value of the SDT of LHs is one or two orders of magnitude smaller than the SDT of electrons. This can be easily understood from the fact that the Rashba coefficients here are larger.

Now we include the hole-impurity scattering with the impurity density \( N_i = 0.1N_h \). As expected, when the total scattering becomes stronger, the counter effect of the scattering to the inhomogeneous broadening takes the leading place easier and the SDT starts to increase with temperature earlier than the impurity-free case. When \( N_i = 1.0N_h \), the total scattering is further enhanced. Now if one uses the hole-impurity scattering and the hole-phonon scattering to calculate the momentum relaxation time, and takes the lowest order of \( \tau_{\tau}^{-1}(k) \) after expanding them over the function \( A_l(\theta_k) \) defined in Eq. (B1), one gets the typical value of \( |\Omega_L^{a1}(k)|\tau_{\tau}(k) \) at the average of \( k \) to be 0.11 at \( T = 100 \) K. It has already entered the regime of strong scattering, and similar to the case of electrons when the linear part of the DP term is dominant, the SDT increases monotonically with temperature.

When the well width becomes larger, the cubic term becomes more important. For example, when \( a = 7 \) nm, \( \gamma_{52}^{66} \) changes from \(-1.39\) nm to \(-0.60\) nm when the temperature changes from 100 K to 300 K. One can see from Table II that the cubic terms weighted by \( \gamma_{52}^{66} \) are dominant. The SDT in this case is plotted in Fig. 2(a) as dashed curves for comparison. One can see that now the SDT decreases monotonically with temperature. It is because the increase of the inhomogeneous broadening with temperature is much faster when the cubic term in the DP term is dominant. Therefore Effect I always surpasses Effect II with the increase of temperature and the SDT decreases monotonically with \( T \). The same situation happens in the case of HHs where there is only cubic term in the DP term. It is seen from Fig. 2(b) that the SDT decreases monotonically with temperature even when \( a = 5 \) nm. This is consistent to the electron case when the cubic term is dominant or the only term (bulk case) where the SDT also decreases monotonically with temperature.

One can find from the discussion above that the spin dephasing is a combined effect from the scattering and the inhomogeneous broadening due to the DP term. The inhomogeneous broadening induced spin dephasing and the counter effect of the scattering to the inhomogeneous broadening, are both very important and neither can be neglected. Nevertheless, these effects are either not or not fully accounted in the simplified model.
which is based on the single-particle approximation Eq. (1). Furthermore, one should notice that the simplified model is based on the assumption of $|\Omega \tau_p| \ll 1$, which is not always satisfied for holes. To show the differences between the many-body approach and the earlier treatment, we also compare our results with those given by the simplified model which now reads:

$$\frac{1}{\tau_\lambda} = \frac{1}{2}\int_0^\infty dE_k (f_{k\lambda,+} - f_{k\lambda,-}) \Gamma_\lambda (k),$$

in which

$$\Gamma_{LH} (k) = k^2 [\tau_{1.1,H}(\gamma_{53}^2 + \gamma_{52}^2 \pi^2 \alpha^2)^2 + \tau_{3.H}(\gamma_{54}^2)]^2,$$

$$\Gamma_{HH} (k) = k^6 [\tau_{1.1,H}(\gamma_{53}^2)^2 + \tau_{3.H}(\gamma_{54}^2)^2],$$

with

$$\tau_{1.1}^{-1} = \int_0^{2\pi} \sigma_\lambda (E_k, \theta) [1 - \cos(\theta)] d\theta .$$

$\sigma_\lambda (E_k, \theta)$ stands for the scattering cross section of the hole-phonon and the hole-impurity scattering, and the expressions can be found in Eq. (B8).

In the insets of Figs. 2(a) and (b), we plot the corresponding SDTs of LHs and HHs from the simplified treatments in solid curves. SDTs from our many-body approach are plotted in dashed curves. One can see that both the curvatures and the absolute values are markedly different between the two treatments. The simplified treatment shows that the SDT of both HHs and LHs decreases monotonically with temperature regardless of the impurity densities. Moreover, when the impurity density increases, the SDT increases very fast. This is because the single-particle treatment totally ignores the fact that in the presence of the inhomogeneous broadening, adding a new scattering means adding a new spin dephasing channel. It also does not treat the counter effect of the scattering to the inhomogeneous broadening sufficiently. Moreover, it does not include the Coulomb scattering which we will show in the next subsection to be very important. By comparing the SDTs predicted by the two models, one can see that it is important to study the SDT of holes from the many-body approach.

C. Effect of Coulomb scattering on SDT

Now we turn to study the effect of the Coulomb scattering in $\hbar k_{\lambda,\sigma} |_{\text{scatt}}$ to the SDT. It has been shown recently by Wu et al. that unlike the common belief that the Coulomb scattering cannot cause spin dephasing, in the presence of inhomogeneous broadening, it can also lead to spin dephasing and for electrons in GaAs QWs, the Coulomb scattering is very important and can markedly increase the SDT. Glazov and Ivchenko have also drawn the similar conclusion. Since the spin-orbit coupling of hole system is much larger than that of electron one, it would be interesting to see how the Coulomb scattering can affect the SDT. Unlike the case of electron system, here we find that the hole-hole scattering markedly reduces the SDT. This is consistent to the optical dephasing of semiconductors where the Coulomb scattering gives rise to a stronger optical dephasing.

In order to understand the difference between the hole system and the previous electron one, we plot in Fig. 3 the SDT of the LH and the HH as function of a dimensionless scale coefficient of the DP term $\chi$ at $T = 120$ K and 300 K. Here $\chi$ is introduced by hand in front of the DP term, i.e., $\chi \Omega^\lambda (k)$ with $\chi = 1$ corresponding to the case of the original DP term. The solid curves are for the case with both the hole-hole scattering and the hole-phonon scattering and the dashed ones are for the case with the hole-phonon scattering only. It is pointed out here that notwithstanding the fact we sweep $\chi$ through two orders of magnitude, experimentally the value of the Rashba coefficients can only be tuned within a small range by applying an external electric field, and can be determined by analyzing the Shubnikov-de Haas oscillations. It is seen from
the figure that when $\chi = 1$, the SDTs of both the HH and the LH decrease when the Coulomb scattering is included. However, when one decreases the spin-orbit coupling by decreasing the scale coefficient $\chi$, one enters a regime where the Coulomb scattering increases the SDT. This is consistent with our previous observation that the competing effects of the inhomogeneous broadening and the scattering in different regimes. In the regime where the inhomogeneous broadening is weak ($|\Omega^h|\tau_p \ll 1$), the hole-hole scattering mainly suppresses the inhomogeneous broadening and consequently raises the SDT. In the regime of weak inhomogeneous broadening and consequently raises the SDT. When the scale coefficient $\chi$ increases monotonically with the impurity density, one enters the strong scattering regime ($|\Omega^LH(k)|\tau_p(k)|_{k_\perp(k)} = 0.16$, the SDT reaches a minimum. Further increasing the impurity density, one enters the strong scattering regime ($|\Omega^LH(k)|\tau_p(k)|_{k_\perp(k)} = 0.049 \ll 1$ at $N_i = 1.0 N_h$), and the SDT keeps increasing with $N_i$ in this regime. When the scale coefficient $\chi = 1.024$, $|\Omega^LH(k)|$ becomes even larger and the minimum of the SDT occurs at larger $N_i$. The similar is also true for the case of HHs in Fig. 4(b). It is noted that in reality $\chi$ is around 1 and the SDT will first decrease then increase with the impurity density. This is totally different from the electron case and also different from the prediction of the single-particle approach where the SDT always increases with the impurity density.

We further investigate the impurity density dependence of the SDT at different temperatures. Here the scale coefficient $\chi$ is fixed to be 1. In Fig. 5 one finds that when the temperature is low, the SDT first decreases with $N_i$ as then the total scattering is weak, and then increases with it after the SDT reaches a minimum. When the temperature gets higher, the SDT always increases with $N_i$ as then $|\Omega^h|\tau_p \ll 1$ can always be satisfied and the hole-impurity scattering mainly suppresses the inhomogeneous broadening.

D. Impurity density dependence of SDT

Now we turn to study how impurities affect the hole SDT. In Fig. 4 we plot the SDT of LHs as function of the impurity density $N_i$. It is seen from Fig. 4(a) that for LHs when $\chi = 0.128$ or $\chi = 0.256$, the SDT increases monotonically with the impurity density. Again the spin-orbit coupling here is in the regime where the inhomogeneous broadening is weak and the hole-impurity scattering mainly suppresses the inhomogeneous broadening and raises the SDT. When the scale coefficient $\chi = 0.512$ and $N_i = 0$, $|\Omega^{LH}(k)|\tau_p(k)$ is close to 1 at the average of $k$: $|\Omega^{LH}(k)|\tau_p(k)|_{k_\perp(k)} = 0.32$. The effect that adding a new scattering provides a new spin dephasing channel becomes dominant and the SDT first decreases with the impurity density. This is similar to the effect of the Coulomb scattering discussed above while the spin-orbit coupling is large. However, with the increase of the impurity density, $\tau_p$ gets smaller and $|\Omega^{LH}(k)|\tau_p(k)$ gets smaller again. When $N_i = 0.1 N_h$, $|\Omega^{LH}(k)|\tau_p(k)|_{k_\perp(k)} = 0.16$, the SDT reaches a minimum. Further increasing the impurity density, one enters the strong scattering regime ($|\Omega^{LH}(k)|\tau_p(k)|_{k_\perp(k)} = 0.049 \ll 1$ at $N_i = 1.0 N_h$), and the SDT keeps increasing with $N_i$ in this regime. When the scale coefficient $\chi = 1.024$, $|\Omega^L(k)|$ becomes even larger and the minimum of the SDT occurs at larger $N_i$. The similar is also true for the case of HHs in Fig. 4(b). It is noted that in reality $\chi$ is around 1 and the SDT will first decrease then increase with the impurity density. This is totally different from the electron case and also different from the prediction of the single-particle approach where the SDT always increases with the impurity density.

In order to compare the hole system with electron one, we add a scale coefficient $\chi$ in front of the electron Dresselhaus term Eq. (2) and calculate the SDT of electrons in a QW ($a = 5$ nm) as function of $\chi$ with and without the electron-electron Coulomb scattering. The results are plotted in the same figure for comparison. Similar to the case of holes, one finds that when the spin-orbit coupling is strong, the Coulomb scattering reduces the SDT also. It happens that for hole system, the spin-orbit coupling here is in the regime where the inhomogeneous broadening is weak ($|\Omega^h|\tau_p \ll 1$), the hole-phonon scattering mainly suppresses the inhomogeneous broadening and consequently raises the SDT. When the scale coefficient $\chi$ increases, $|\Omega^h|\tau_p \ll 1$ can always be satisfied and the hole-impurity scattering mainly suppresses the inhomogeneous broadening.

In the regime where the inhomogeneous broadening is weak ($|\Omega^h|\tau_p \ll 1$), the hole-hole scattering mainly suppresses the inhomogeneous broadening and consequently raises the SDT. It happens that for hole system, the spin-orbit coupling here is in the regime where the inhomogeneous broadening is weak ($|\Omega^h|\tau_p \ll 1$), the hole-phonon scattering mainly suppresses the inhomogeneous broadening and consequently raises the SDT. When the scale coefficient $\chi$ increases, $|\Omega^h|\tau_p \ll 1$ can always be satisfied and the hole-impurity scattering mainly suppresses the inhomogeneous broadening.
Finally we investigate the hole density dependence of the SDT at different temperatures and well widths. Here the hole-impurity scattering is excluded and $\chi \equiv 1$. In Fig. 6(a) we plot the SDT of LHs as function of the LH density with $a = 5$ nm. The SDT decreases with the hole density when the temperature is low but increases with it when the temperature is high enough. To understand this result, we first analyze the Rashba term [Eqs. (4-9)]. When $T = 300$ K, one finds $\gamma_{601}^{LH} E_z m_0 \langle k_z^2 \rangle \langle k_\parallel \rangle$ changes from $-2.58$ nm to $-1.46$ nm when $N_h$ changes from $5 \times 10^{10}$ cm$^{-2}$ to $4 \times 10^{11}$ cm$^{-2}$, and the absolute value becomes even larger when $T = 100$ K. Therefore, it can be seen from Table II that both $\gamma_{753}^{LH}$ and $\gamma_{754}^{LH}$ are smaller than $-\gamma_{601}^{LH} \langle k_z^2 \rangle \langle k_\parallel \rangle$, and the linear terms in Eqs. (7) and (8) are dominant. Moreover, again $|\Omega^{LH}|\tau_p$ is slightly smaller than 1. Similar to the case in Sec. III B when the linear Rashba term is dominant, the hole density $N_h$ influences the spin dephasing through two competing effects: Effect I: The increase of the spin dephasing due to the increase of the inhomogeneous broadening with $N_h$ as holes are populated at higher $k$-states at high hole density; and due to the increase of the scattering which provides additional spin dephasing channel. Effect II: The decrease of the spin dephasing due to the counter effect of the increased scattering which suppresses the inhomogeneous broadening. The results shown in Fig. 5(a) indicate that when $T \leq 220$ K and the total scattering is not so strong, Effect I is more important and the SDT decreases with $N_h$. When the hole density keeps increasing and the total scattering is further enhanced, Effect II becomes more important and the SDT increases with $N_h$.

We further plot the SDT of LHs with $a = 7$ nm in Fig. 6(b) where the cubic terms become more important. For
example, when \( T = 300 \) K, \( g_{52} \frac{866}{27} E_z m_0 \frac{k^2}{(k_f^2)} \) changes from \(-1.06 \) nm to \(-0.60 \) nm when \( N_h \) changes from \( 5 \times 10^{10} \) cm\(^{-2}\) to \( 4 \times 10^{11} \) cm\(^{-2}\). One can see from Table II that the cubic terms weighted by \( \gamma_{53} \) are dominant. Similar to the case in Sec. III B, when the cubic Rashba term is dominant, the increase of the inhomogeneous broadening with hole density is much faster than the counter effect of the scattering and consequently Effect I always surpasses Effect II with the increase of hole density. As expected, one finds that the SDT decreases monotonically with \( N_h \). The same is true for HHs in Fig. 6(c) where the Rashba term [Eqs. (4-6)] includes only the cubic one.

**IV. CONCLUSION**

In conclusion, we have performed a systematic microscopic many-body investigation on the hole spin dephasing of \( p \)-type GaAs QWs of small well width where the HH and LH bands are well separated, by constructing a set of kinetic spin Bloch equations based on the nonequilibrium Green function method. We included the magnetic field, the Rashba spin-orbit coupling and all spin conserving scattering such as the hole-phonon, the hole-nomagnetic impurity and the hole-hole scattering. By numerically solving the kinetic equations, we obtained the time evolution of the distribution functions and the spin coherence of holes. The SDT is calculated from the slope of the envelope of the incoherently summed spin coherence. Differing from earlier studies on spin dephasing based on the single-particle approach which only includes the lowest-order elastic scattering and the anisotropy from \(-|\Omega(k)|\) and \(+|\Omega(k)|\), this approach takes full account of the inhomogeneous broadening from different \( k \)-states of the Rashba term as well as the effect of all the scattering. Furthermore, this approach is valid regardless of the strength of scattering whereas the earlier single-particle approach is valid only when the scattering is strong enough, *i.e.*, \( |\Omega| \tau_p \ll 1 \). Using this many-body approach, we studied in detail how the hole spin dephasing are affected by temperature, the hole-hole Coulomb scattering, the impurity and the hole densities.

We showed that the spin dephasing is mainly affected by two effects: The inhomogeneous broadening and the scattering. Any effect that increases the inhomogeneous broadening tends to reduce the SDT. However, the effect of scattering on the spin dephasing is different when \( |\Omega| \tau_p \ll 1 \) and \( |\Omega| \tau_p \gg 1 \): When \( |\Omega| \tau_p \ll 1 \) and therefore the scattering is strong in comparison to the DP term, the counter effect of the scattering to the inhomogeneous broadening is important. In this regime, the scattering tends to drive carriers to a more homogeneous state in \( k \)-space and consequently reduces the inhomogeneous broadening. This tends to increase the SDT. When \( |\Omega| \tau_p \gg 1 \), the scattering is weak in comparison to the DP term (inhomogeneous broadening) and the counter effect can be neglected, adding a new scattering provides an additional dephasing channel. In this regime, the counter effect of the scattering to the inhomogeneous broadening can be ignored and the scattering reduces the SDT. All the factors, such as temperature, well width, impurity density and hole density, can affect the inhomogeneous broadening and the scattering and therefore influence the SDT.

The temperature affects the SDT in two ways: On the one hand, the increase of the temperature drives holes to higher \( k \)-states, and leads to a stronger inhomogeneous broadening. On the other hand, the scattering is enhanced with the increase of the temperature. When the linear Rashba term is dominant, such as LHs with \( a = 5 \) nm at the hole density in our investigation, it is shown that the SDT decreases with \( T \) when the temperature is low and the impurity density is small. This can be understood as it is in the regime where \( |\Omega^{LH}| \) and \( 1/\tau_p \) are comparable and the increase of the spin dephasing due to the increase of the inhomogeneous broadening and the increase of the spin dephasing channel by the increase of the scattering with temperature are dominant. When the temperature keeps increasing so that the scattering becomes stronger or when the impurity density is high, the SDT increases with \( T \) when the system enters the regime where \( |\Omega^{LH}| \tau_p \ll 1 \) and the counter effect of the scattering to the inhomogeneous broadening becomes dominant. When the cubic Rashba term is dominant (such as LHs with \( a = 7 \) nm in our investigation) or is the only term (such as HHs), the SDT decreases monotonically with temperature as the increase of the inhomogeneous broadening with temperature is much faster than the increase of scattering. These results are quite different from the case of electrons where the spin-orbit coupling is within the regime of weak inhomogeneous broadening (\( |\Omega| \tau_p \ll 1 \)) and the SDT *increases* monotonically with temperature when the linear DP term is dominant.

We also compared the SDTs predicted by our many-body approach with the results of the earlier simplified treatment, and showed that the simplified treatment is inadequate in studying the hole spin dephasing.

The hole density also influences the inhomogeneous broadening and the scattering simultaneously. Similar to the case of temperature dependence, it is shown that for the LHs with \( a = 5 \) nm where the linear Rashba term is dominant, the SDT decreases with \( N_h \) when the temperature is low because it is in the regime of strong inhomogeneous broadening, and increases with \( N_h \) when the temperature is higher and the inhomogeneous broadening is weak. For LHs with \( a = 7 \) nm or HHs where the cubic Rashba term is the leading/only term, the SDT decreases monotonically with \( N_h \).

We further showed that the Coulomb scattering contributes markedly to the SDT. When the inhomogeneous broadening is stronger than the scattering, the Coulomb scattering enhances the spin dephasing. Otherwise, it reduces the spin dephasing. In the earlier single-particle treatment, the Coulomb scattering was considered to be unable to cause spin dephasing.
In the calculation, the magnetic field in the Voigt configuration is taken to be 4 T. We found that for hole system, the magnetic field dependence is marginal as the Rashba term is very large. In this investigation, the Elliott-Yafet mechanism is not included. A full microscopic many-body treatment of the this mechanism is much more complicated than the DP mechanism and will be published elsewhere. Up till now there is no experimental investigation on the SDT for holes in (001) QWs. Experiments such as spin-echo measurements and time-resolved Faraday rotation measurements can be used to measure the SDT.

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APPENDIX A: EFFECT OF SCATTERING TO SDT

In Sec. III B we pointed out that the ratio of the DP term to the scattering rate determines the way how the scattering affects the spin dephasing. To reveal part of this effect analytically, we now study a much simplified case with only the hole-impurity scattering and the dominant part of the DP term, i.e., for LHs we include only the linear Rashba term weighted by $\gamma_{\mathrm{LH}}$ and for HHs only the cubic Rashba term weighted by $\gamma_{\mathrm{HH}}$. Furthermore, we will also neglect the inhomogeneous broadening later.

First we consider the HH case. We expand $2 \times 2$ density matrix $\rho_{\mathrm{HH}}$ as follows:

$$\rho_{\mathrm{HH}} = \sum_i \rho_{\mathrm{HH},i}(k) A_i(\theta_k)$$

with $A_i(\theta_k) = \frac{1}{\sqrt{2\pi}} e^{\theta_k}$. The coherent terms of the kinetic spin Bloch equations [Eqs. (14) and (15)] can be written into the matrix commutator as $\dot{\rho}_{\mathrm{HH}}|_{\mathrm{coh}} = i[H_{\mathrm{HH}}^s(k),\rho_{\mathrm{HH}}]$ with $H_{\mathrm{HH}}^s(k) = \frac{i}{2} \sigma \cdot \Omega_{\mathrm{HH}}^s(k)$. It is noted that here we neglected the Coulomb Hartree-Fock term. Furthermore, we expand $H_{\mathrm{HH}}^s$ as $H_{\mathrm{HH}}^s(k) = \sum_i H_{\mathrm{HH},i}(k) A_i(\theta_k)$. Therefore the coherent term in the matrix form can be written as:

$$\dot{\rho}_{\mathrm{HH}}|_{\mathrm{coh}} = \sum_{i,l} \frac{i}{2\pi} [H_{\mathrm{HH}}^s(k),\rho_{\mathrm{HH},i,l}(k)] A_i(\theta_k) .$$

With only the dominant part of the DP term (term weighted by $\gamma_{\mathrm{HH}}^7$) included, $H_{\mathrm{HH}}^s(k)$ is expanded as:

$$H_{\mathrm{HH}}^s = i S^z \gamma_{\mathrm{HH}}^7 E_z k^3 ,$$

$$H_{\mathrm{HH}}^s = -i S^z \gamma_{\mathrm{HH}}^7 E_z k^3 ,$$

$$H_{\mathrm{HH}}^s = 0 .$$
in which $S = \frac{1}{2}(\sigma_x - i\sigma_y)$. Substituting Eqs. (B3-B5) into Eq. (B2), one obtains

$$\dot{\rho}_{kH}^{coh} = \frac{1}{\sqrt{2\pi}} \sum_{l} S_{\|}^{kH} E_z k_3 \left( \{S, \rho_{HH,l+3}(k)\} - \{S^T, \rho_{HH,l-3}(k)\} \right) A_l(\theta_k).$$

Similarly one can expand the scattering term as:

$$\dot{\rho}_{kH}^{scat} = \sum_{l} \rho_{HH,l}(k) U_l^2(k) A_l(\theta_k),$$

for the elastic scattering with

$$U_l^2(k) = \frac{2\pi N_l m^*}{\hbar^2} \int_0^{2\pi} \frac{d\theta}{(2\pi)^2} U_l^2(\theta) (1 - \cos l\theta).$$

Now we can expand the spin Bloch equations Eq. (13) into $\rho_{HH,l}(k)$ as follows:

$$\dot{\rho}_{HH,l}(k) + \frac{\gamma_7 E_z}{\hbar^2} k_3 \left( \{S, \rho_{HH,l+3}(k)\} - \{S^T, \rho_{HH,l-3}(k)\} \right) = -U_l^2(k) \rho_{HH,l}(k).$$

In order to find the solution, we multiply $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ to both sides of this equation and calculate the trace. By defining the HH “spin” vector to be $\text{Tr}(\rho_{HH,l}(k)\sigma) = S_{HH,l}(k)$, one can rewrite Eq. (B9) into

$$\dot{S}_{HH,l}(k) + \frac{\gamma_7 E_z}{\hbar^2} k_3 \left( \{S, \rho_{HH,l+3}(k)\} - \{S^T, \rho_{HH,l-3}(k)\} \right) = -U_l^2(k) S_{HH,l}(k).$$

thanks to the relation $\text{Tr}(\rho_{HH,l-1}(k)\sigma) = \text{Tr}(\rho_{HH,l-1}(k)[\sigma, S])$. In Eq. (B10) the tensor $\mathcal{F}$ reads

$$\mathcal{F} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -i \\ -1 & i & 0 \end{pmatrix}. $$

One finds that $S_{HH,l}(k)$ is only related to $S_{HH,l+3}(k)$. By considering the lowest orders with $l = 0$, and $\pm 3$ and defining $S_{HH}(k) = (S_{HH,-3}(k), S_{HH,0}(k), S_{HH,3}(k))^T$, Eq. (B10) can be written as:

$$\dot{S}_{HH}(k) + \frac{\gamma_7 E_z}{\hbar^2} k_3 \frac{1}{\sqrt{2\pi}} \mathcal{F} S_{HH}(k) = 0,$$

in which

$$G = \begin{pmatrix} 0 & \mathcal{F} & 0 \\ -\mathcal{F}^T & 0 & \mathcal{F} \\ 0 & -\mathcal{F}^T & 0 \end{pmatrix},$$

$$U = U_0^2(k) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. $$

In Eq. (B14) we have used the relations $U_0^2(k) = U_0^2(3k) = 0$. Now one can solve Eq. (B12) analytically. To reveal the main characteristic analytically, we make the assumption that the spin relaxation/dephasing occurs mainly around the Fermi surface and $\theta_{k_f} = 0$. By doing so, one throws away the interference between different k-states [except for the states with $\theta_{k_f} (= 0$ here), and $\theta_{k_f} \pm 2\pi/3$], and therefore the inhomogeneous broadening. Then the HH spin $S_{HH} = \sum_{l=-3,0,3} (S_{HH,l}, S_{HH,l+3}, S_{HH,l+3}, S_{HH,l})$ has the form:

$$S_{HH} = 0,$$

$$S_{HH} = \frac{2e^{-\frac{x_1}{2}(x+\sqrt{x^2-16})}}{\sqrt{x^2-16}} (e^{\frac{x_1}{2}(x+\sqrt{x^2-16})} - 1) S_0$$

$$S_{HH} = \frac{2e^{-\frac{x_1}{2}(x+\sqrt{x^2-16})}}{\sqrt{x^2-16}} \frac{[x(e^{\frac{x_1}{2}(x+\sqrt{x^2-16})} - 1)]}{16} S_0,$$

in which $x = \frac{U_0^2(k_f)\sqrt{2\pi}}{\gamma_7 E_z k_3}$ is proportional to the ratio of the scattering rate to the DP term, $t_1 = \frac{\gamma_7 E_z k_3}{\sqrt{2\pi}}$ and $S_0$ represents the initial spin polarization along the z-axis. One can see from Eq. (B16) that when $x < 4$, the SDT is proportional to $1/x$ and decreases with $x$ whereas when $x > 4$, the SDT is proportional to $1/(x - \sqrt{x^2-16})$ and increases with $x$. This result indicates that the scattering reduces the SDT when the scattering is weak in comparison to the DP effective field but increases the SDT when the scattering is strong in comparison to the DP effective field. Moreover, when $U_3 = 0$, i.e., there is no scattering, $x = 0$ and consequently there is no spin dephasing. This is consistent with the numerical result presented in Appendix A.

Similarly one can derive the equation for spin of LHs $S_{LH}$ with only the linear part of the Rashba term included. One gets the same equations Eqs. (B15-B17) but with $x = \frac{U_0^2(k_f)\sqrt{2\pi}}{\gamma_7 E_z k_3}$ and $t_1 = \frac{\gamma_7 E_z k_3}{\sqrt{2\pi}}$.

These results coincide qualitatively with the results shown in Fig. 4: The SDT first decreases then increases with the hole-impurity scattering when the spin-orbit coupling is strong; but increases monotonically with the scattering when the spin-orbit coupling is weak. Furthermore, by making the approximation that the hole-phonon scattering is also an elastic scattering and by including the hole-phonon scattering in the scattering term, $U_0^2$ in
Eq. (B8) can be modified as:

$$U_q^2(k) = 2\pi m^* k^2 \int_0^{2\pi} d\theta \left[ N_q U_q^2 + (1 + 2N_q)q^2 \right] \left( 1 - \cos \theta \right). \quad (B18)$$

Here $q^2 = \sum_q \{2\pi e^2 \Omega_{LO}/(q^2 + q^2)\} (1/\kappa_n - 1/\kappa_0) |I(q_z)|^2$ is the hole-phonon interaction matrix element, and $N_q = 1/|\exp(\Omega_{LO}/k_BT) - 1|$ is the Bose distribution of the LO phonon. Then the results also coincide qualitatively with those in Fig. 5(b): the SDT first decreases then increases with the hole-impurity scattering when the total scattering is weak, but always increases with scattering when the total scattering is strong. Finally we point out that as we do not include the inhomogeneous broadening and all the scattering in this simplified model, many other features predicted in the text cannot be obtained here.

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It is noted that differing from our many-body treatment, they only considered the electron-electron scattering to the lowest order (2nd order) and used perturbation approach to calculate the SDT. So the Coulomb scattering is only important for temperature lower than 100 K. In our many-body approach, we have taken account of the Coulomb scattering to all the orders and it has been shown the Coulomb scattering is important to temperature much higher than 100 K.\textsuperscript{20}