A Design of Fuzzy Immune PID Controller for Six-rotor UAV Under Gyroscopic Effect

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Abstract: The six-rotor UAV has the characteristics of vertical take-off and landing, air hovering and large carrying capacity, which can accomplish tasks that cannot be completed by many fixed-wing aircrafts. In recent years, the research of dynamic modelling and control methods of the six-rotor UAV has gradually become hot topics at home and abroad. Considering the gyroscopic effect of the blade, a more accurate dynamic model of the six-rotor UAV is established, and then combined with the feedback mechanism of the biological immune system and the fuzzy control theory, a fuzzy immune PID controller is designed to optimize the parameters of the traditional PID controller, which can perform self-tuning of PID parameters. The position and attitude control of the UAV under hovering state is simulated by Matlab/Simulink software. The results show that compared with the conventional PID controller and fuzzy PID controller, the control system has the advantages of better dynamic and static performance such as shorter response time, smaller overshoot and improves the anti-interference ability and robustness of the system.

1. Introduction
The multi-rotor aircraft is a kind of aircraft capable of vertical take-off and landing, air hovering and autonomous flight. It has the characteristics of simple structure, easy operation and low noise, and is very suitable for performing short and medium distance missions. It has a wide range of application prospects in military and civilian applications, and has received extensive attention from research institutions at home and abroad.

The flight control system of the multi-rotor UAV is the key to ensure the stable flight of the UAV. Therefore, the research on the flight control system of the multi-rotor UAV is particularly important. Among them, the four-rotor UAV and the six-rotor UAV are the most studied. At present, many articles on the design and control theory of quadrotor UAV have been published internationally. Literature [1] introduced two rotorcraft modelling methods and simulated comparisons; McKerrow [2] gave the dynamic model of the quadrotor UAV; Jiawen Li [3] studied the application of Kalman filter and PID methods to control the attitude of the quadrotor UAV; in addition, foreign scholars have also been tried to use the modern control theory such as neural network algorithm, sliding mode control, adaptive control and robust nonlinear dynamic control to achieve the flight control. Compared with the quadrotor UAV, the six-rotor UAV comes out in recent years. There are relatively few studies on the six-rotor UAV at home and abroad, and the theoretical research on its dynamic model and control method is not mature enough.
In the force analysis of the six-rotor aircraft, the frictional forces and moments of the aircraft and the blades during the flight were considered, and a more accurate dynamic model was established to provide a basis for subsequent controller research. Because this paper studied the position and attitude control problems in hovering state or small angle motion, the model was simplified to reduce the complexity of the model. The control effect of the traditional PID control algorithm is often greatly affected by the uncertainty of the load and the change of the parameters, and even sometimes the control requirements cannot be met [4-5]. In the design of the six-rotor UAV controller, based on the traditional PID control method, a fuzzy immune PID controller that combines the advantages of the biological immune mechanism and fuzzy control theory was established. Then the simulation analysis was carried out by using MATLAB/SIMULINK tool and the results show that the controller has high control precision and good robustness, and it has high application value in the control performance of the multi-rotor UAVs.

2. Six-rotor aircraft dynamics modelling

2.1. The gyroscopic effect of the blade

During the flight of the six-rotor aircraft, the rotor rotates at a high speed. When the roll angle or pitch angle of the aircraft changes, the direction of the axis around which the rotor rotates changes, which needs to overcome the inertia of the rotor spin. This is impact of the gyroscopic effect on the rotorcraft. The amount of torque generated when each rotor rotates around the axis is as the follow:

\[
J_r \frac{d}{dt} \Omega_w
\]  

(1)

Where \( J_r \) is the moment of inertia of the rotor, \( l \) is the arm length of the six-rotor aircraft, \( \Omega \) is the angular velocity of the tilt around the axis, and \( w \) is the rotational speed of the rotor.

2.2. The force analysis of the six-rotor aircraft

The force analysis of the six-rotor aircraft is shown in Figure 1. As the figure shows, \( V \) is the flight speed vector of the aircraft; \( D_i \) is the air resistance received by the first rotor; \( D_w \) is the air resistance received by the body; \( W_i \) is the speed of the number \( i \) rotor; \( L_i \) is the lift generated by the number \( i \) rotor; \( T_i \) is the yaw moment generated by the number \( i \) rotor; \( I \) is the arm length of the rotor; \( h \) is the vertical distance from the center of the aircraft to the centroid of the aircraft; \( m \) is the total weight of the aircraft; \( g \) is the acceleration of gravity; \( \{ \theta \} \) represents the coordinate system of the aircraft body: \( x_b, y_b, z_b, o_b \) respectively represents the \( x \) axis, \( y \) axis, \( z \) axis and the coordinate origin in the coordinate system of the aircraft body; \( \{ E \} \) represents the inertial coordinate system: \( x, y, z, o \) respectively represents the \( x \) axis, \( y \) axis, \( z \) axis and the coordinate origin in the inertial coordinate system.

![Figure 1. Force analysis diagram of the six-rotor aircraft.](image-url)
2.3. Dynamics model and simplification

According to the force analysis of the six-rotor aircraft, the equations of force and torque in the coordinate system of the aircraft body can be obtained:

\[
F_b = mg_b - D_w - \sum_{i=1}^{4} (L_i + D_i) \\
M_b = M_L - M_\Omega + \sum_{i=1}^{4} (T_i + D_i \omega + M_{r,i}) + \tau_g
\]

(2) (3)

According to the actual situation of the six-rotor aircraft, the following reasonable assumptions are made:

(1) The body of the six-rotor aircraft is a rigid structure with no elastic deformation;
(2) The structure and mass of the six-rotor aircraft are symmetrical, and the moment of inertia is zero except for the main moment of inertia;
(3) The center of mass of the six-rotor aircraft coincides with the center of gravity of the structure, therefore the equation is a reality: \( h = 0 \);

Some of them are small in quantity relative to other quantities and can be ignored. Therefore, the model needs to be simplified appropriately, and further assumed as follows:

(1) The resistance of the wind received by the rotor itself is small, so the resistance and torsional friction torque it receives are negligible;
(2) Since the mass of the rotor blade and the motor rotor is small compared with the body, and the relative rotor speeds of the six rotors are opposite in opposite directions, the yaw torque generated when the rotor rotates can be neglected, and the rotation of the rotor inertia is also negligible;
(3) This paper studies the six-rotor position and attitude control problem in hovering or small-angle motion, so the Euler angular rate \((\phi, \theta, \psi)^T\) is approximately equal to the angular rate of the body \((\phi, \theta, \psi)^T\).

Based on the above assumptions, a simplified model of the six-rotor aircraft can be obtained:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = 
\begin{bmatrix}
-(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \frac{b_r}{m} \sum_{i=1}^{6} w_i^2 \\
-(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \frac{b_r}{m} \sum_{i=1}^{6} w_i^2 \\
g - (\cos \phi \cos \theta) \frac{b_z}{m} \sum_{i=1}^{6} w_i^2
\end{bmatrix}
\]

\[\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = 
\begin{bmatrix}
\phi \psi (\frac{I_x - I_z}{I_x}) - J_x \phi \dot{\omega} + \frac{b_1 l_1}{l_x} \frac{1}{2} (w_1^2 + w_2^2 - w_3^2 - w_4^2) + (w_5^2 - w_6^2) \\
\phi \psi (\frac{I_y - I_z}{I_y}) + J_y \phi \dot{\omega} + \frac{\sqrt{3} b_1 l_1}{2 l_y} (w_1^2 + w_2^2 - w_3^2 - w_4^2) \\
\phi \psi (\frac{I_z - I_x}{I_z}) + \frac{b_2 l_1}{l_z} (w_1^2 + w_2^2 + w_3^2 - w_4^2 - w_5^2 - w_6^2)
\end{bmatrix}
\]

(4) (5)

This model assumes \((x, y, z, \phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi})\) as the state, with the square of the six rotor speeds \((w_1^2, w_2^2, w_3^2, w_4^2, w_5^2, w_6^2)\) as the input to the system, which is the final model used in this paper.

3. The design of fuzzy immune PID controller

3.1. Immune system feedback theory
According to biological knowledge, immune function is closely related to T cells. Such cells can be divided into two classes of inhibitory and helper T cells, which the function of is the opposite. After the antigen invades, it stimulates the T cells on a certain basis of digestion, and then stimulates the secretion of antibodies, and the antigen is cleared. In the case of an increase in the number of antigens, the number of helper T cells also increases, and the amount of B cells is increased, and more antibodies are secreted. In the case of a decrease in the amount of antigen, B cells are reduced. On the basis of certain feedback adjustment, a dynamic immune balance state is formed. In this paper, a new control model is designed by combining the above immune mechanism and PID algorithm in the research process. In this design process, the following assumptions are made for the convenience of analysis [6]:

(1) It is hypothesized that during the immunization process, T inhibitory cells simply inhibit B cells, regardless of the killing effect associated there with.

(2) It is assumed that the number of antigens of the kth generation is \( \varepsilon(k) \), the output of the helper T cells is \( T_h(k) \), and the regulation of the other cells by the cells is \( T_s(k) \).

Under these conditions, the immune algorithm formula is obtained:

\[
S(k) = (k_1 - k_2 f(\Delta S(k))) \varepsilon(k) = k_1 (1 - K f(\Delta S(k))) \varepsilon(k)
\]

Where \( K = k_2 / k_1 \).

The control algorithm of the traditional proportional controller is:

\[
u(k) = k_p e(k)\]

Under this condition, the immune control system is controlled based on the proportional model. The proportional gain is:

\[
k_p = k_1 [1 - K f(u(k), \Delta u(k))]\]

This gain \( k_p \) depends not only on the selection of \( k_1 \), \( f(\cdot) \) and \( K \), but also on the control output.

3.2. Fuzzy reasoning of the nonlinear function \( f(\cdot) \)

In complex control systems, the determined nonlinear function \( f(\cdot) \) is difficult to find [11]. In the search for this function, a fuzzy control algorithm is needed. In this algorithm, the corresponding input variable is the output \( u(k) \) and its variation \( \Delta u(k) \), and the fuzzy sets \( P(+) \) and \( N(-) \) are obtained on the basis of fuzzy. The output variable is the function \( f(u(k), \Delta u(k)) \) related to this, and after the fuzzification process the fuzzy set respectively is \( P(+) \), \( Z(0) \), \( N(-) \). The fuzzy rules are obtained according to the principle of the immune mechanism of "stimulation of cells and inhibitory negative correlation":

(1) If \( u \) is \( P \) and \( \Delta u(k) \) is \( P \) then \( f(\cdot) \) is \( N \); (2) If \( u \) is \( P \) and \( \Delta u(k) \) is \( N \) then \( f(\cdot) \) is \( Z \);

(3) If \( u \) is \( N \) and \( \Delta u(k) \) is \( P \) then \( f(\cdot) \) is \( Z \); (4) If \( u \) is \( N \) and \( \Delta u(k) \) is \( N \) then \( f(\cdot) \) is \( P \).

After performing the above analysis and hypothesis, the fuzzy inference Mamdani and the defuzzing algorithms centroid are applied to process, and the nonlinear function \( f(u(k), \Delta u(k)) \) is obtained[7]. The \( k_p \) is obtained by formula (12).

3.3. Self-tuning of controller parameters \( k_i, k_d \)

The six-rotor controller design can be divided into an attitude ring design and a position loop design. The attitude ring controls the motion posture of the aircraft, that is, the rolling motion, the pitch motion and the yaw motion, which are the key links to maintain the stable flight of the aircraft; the position loop controls the spatial position of the aircraft, which is the key link for performing the mission. Only when the flight attitude is stable, the safe flight and mission completion of the aircraft can be guaranteed. Therefore, the attitude ring is also called the inner ring, and the position ring is called the outer ring.
In the design process, the principle of PID and immune feedback is integrated. Based on this, a fuzzy immune PID control algorithm is established, and then the control system is designed accordingly. In the control, to adjust the size of the three parameters of proportional $k_p$, integral $k_i$ and differential $k_d$ are mainly used to achieve the purpose of control. The control block diagram is shown in Figure 2. Among them, the immune mechanism is used in the control process of $k_p$, and the adjustments of $k_i$, $k_d$ are based on fuzzy reasoning.

First design a conventional PID control algorithm as follow:

$$u_{PID}(k) = [k_p + \frac{k_i}{1-z^{-1}} + k_d(1-z^{-1})]e(k)$$

(9)

In this control, the amount of antigen is treated as a control deviation, in which case the corresponding control output is expressed as:

$$u(k) = k_i[1-Kf(u(k), \Delta u(k))]e(k) + \frac{k_d}{1-z^{-1}} + k_d(1-z^{-1})]e(k)$$

(10)

In the control process, the corresponding input parameters include error $e$ and $e_c$, and the output variables are $\Delta k_i, \Delta k_d$. In the process of fuzzification, it is determined that the fuzzy set associated with this is \{Positive Big(PB), Positive Medium(PM), Positive Small (PS), Negative Small(NS), Negative Medium(NM), Negative Big(NB)\}. The fuzzy control rules are shown in the following tables.

Table 1. Fuzzy control rule of $\Delta k_i$.

| $e$   | $e_i$ is NB | $e_i$ is NM | $e_i$ is NS | $e_i$ is Z0 | $e_i$ is PS | $e_i$ is PM | $e_i$ is PB |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| NB    | NB          | NB          | NM          | NM          | NS          | Z0          | Z0          |
| NM    | NB          | NB          | NM          | NS          | Z0          | Z0          | Z0          |
| NS    | NB          | NM          | NS          | Z0          | PS          | PS          | PS          |
| Z0    | NM          | NM          | NS          | Z0          | PS          | PM          | PM          |
| PS    | NM          | NS          | Z0          | PS          | PS          | PM          | PB          |
| PM    | Z0          | Z0          | PS          | PM          | PB          | PB          | PB          |
| PB    | Z0          | Z0          | PS          | PM          | PB          | PB          | PB          |

Table 2. Fuzzy control rule of $\Delta k_d$.

| $e_c$   | $e_c$ is NB | $e_c$ is NM | $e_c$ is NS | $e_c$ is Z0 | $e_c$ is PS | $e_c$ is PM | $e_c$ is PB |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| NB      | PS          | NS          | NB          | NB          | NM          | PS          |
| NM      | PS          | NS          | NB          | NM          | NS          | Z0          |
| NS      | Z0          | NS          | NM          | NS          | NS          | Z0          |
| Z0      | Z0          | Z0          | NS          | NS          | NS          | Z0          |
Based on the same fuzzy and defuzzification processing, the related parameters $\Delta k_i$ and $\Delta k_d$ are obtained and then are used to control the system accordingly.

Let $k_i$ and $k_d$ respectively have initial values of $k_i(0)$ and $k_d(0)$, and then:

$$
\begin{align*}
    k_i &= k_i(0) + \Delta k_i \\
    k_d &= k_d(0) + \Delta k_d
\end{align*}
$$

(11)

4. Simulation and analysis

All the six-rotor aircraft parameters used in the simulation are reference [8], and the specific parameters are shown in Table 3.

Table 3. The six-rotor aircraft parameters.

| Parameter | Unit | Value |
|-----------|------|-------|
| m         | kg   | 1.6265|
| I_x       | kg·m$^2$ | 0.031517|
| I_z       | kg·m$^2$ | 0.04975|
| J_r       | kg·m$^2$ | 0.00008|
| b_T       |      | 1.55e-05|
| b_Q       |      | 2.82e-07|

This paper intends to control the fixed-point hovering of the six-rotor aircraft under MATLAB/SIMULINK conditions. The initial state and target state of the aircraft are as follows, the position unit is m, and the angle unit is rad.

Initial state: $(x, y, z, \phi, \theta, \psi)^T = (0, 0, 0, 0, 0, 0)^T$

Target status: $(x, y, z, \phi, \theta, \psi)^T = (10, 5, 10, 0, 0, 0.06)^T$

The PID controller parameters used in the numerical simulation are shown in Table 4.

Table 4. PID controller parameters.

| State variable | P | I | D |
|----------------|---|---|---|
| $x$            | 1 | 0 | 2 |
| $y$            | 1 | 0 | 2 |
| $z$            | 1 | 0 | 2 |
| $\phi$        | 80| 0 | 10|
| $\theta$      | 80| 0 | 10|
| $\psi$        | 50| 0.2| 8 |

Note: The meanings of the axis symbols in all the following figures are as follows:
Displacement (unit: m): height z, front and rear direction x, left and right direction y;
Attitude angle (unit: rad): yaw angle $\psi$, roll angle $\phi$, pitch angle $\theta$.

Figure 3. Simulation effect comparison of $x$.

Figure 4. Simulation effect comparison of $y$.

Figure 5. Simulation effect comparison of $z$.

Figure 6. Simulation effect comparison of $\phi$.

Figure 7. Simulation effect comparison of $\theta$.

Figure 8. Simulation effect comparison of $\psi$.

The uncertainty of modelling is mainly reflected in the uncertainty of model parameters and aerodynamic parameters. The quality of the model is increased by 15% and a pulse function with a value of 5 is added to the system at the 5th second simulation time as the disturbance. The tracking of the $z$-coordinate is taken as an example to verify the robustness of the designed control system.

Figure 9. Simulation effect comparison under uncertainties.
It can be seen from the simulation results that the fuzzy immune PID algorithm can effectively control the hovering of the six-rotor UAV in fixed-point flight. It can be seen from Fig. 3-8 that the response time of the fuzzy immune PID control system is smaller than that of the fuzzy PID control system and the conventional PID control system. It can be seen from Fig. 9 that in the case of uncertainty, the controller designed in this paper can accurately track the instructions, while the fuzzy PID and the conventional PID control system both have tracking errors, and the overshoot is bigger than the fuzzy immune PID controller. This indicates that the controller designed in this paper has more advantages in dynamic performance, such as smaller overshoot, faster response and is more robust to model parameter perturbation and external disturbances.

5. Conclusion
In this paper, the six-rotor UAV is modelled and analysed considering the gyroscopic effect of the blade. Combined with the good robustness of fuzzy control, the adaptive activity of immune control and the wide-ranging characteristics of conventional PID control, the fuzzy immune PID controller is designed to optimize the parameters of the PID controller based on the double-loop control structure, through which the parameter self-tuning problem of the PID controller is well solved. The response speed of the system is significantly improved, and the overshoot is also largely reduced. Under the consideration of modelling uncertainty and external disturbances, the controller designed in this paper can still accurately track the instructions, showing great advantages in terms of speed, stability and robustness. Therefore, it has quite high application value.

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