Bounds on Neutrino Mass in Viscous Cosmology

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Abstract. Effective field theory of dark matter fluid on large scales predicts the presence of viscosity of the order of $10^{-6}H_0M_P^2$. It has been shown that this magnitude of viscosities can resolve the discordance between large scale structure observations and Planck CMB data in the $\sigma_8-\Omega_m^0$ and $H_0-\Omega_m^0$ parameters space. Massive neutrinos suppresses the matter power spectrum on the small length scales similar to the viscosities. We show that by including the effective viscosity, which arises from summing over non linear perturbations at small length scales, severely constrains the cosmological bound on neutrino masses. Under a joint analysis of Planck CMB and different large scale observation data, we find that upper bound on the sum of the neutrino masses at 2-$\sigma$ level, decreases from $\sum m_\nu \leq 0.396$ eV (normal hierarchy) and $\sum m_\nu \leq 0.378$ eV (inverted hierarchy) to $\sum m_\nu \leq 0.267$ eV (normal hierarchy) and $\sum m_\nu \leq 0.146$ eV (inverted hierarchy) when the effective viscosities are included.

Keywords: massive neutrinos, viscous dark matter, effective viscosity, large scale structures, CMB
1 Introduction

In the past few decades, neutrino oscillation experiments have established the fact that neutrinos are massive. Since the oscillation probability depends on the difference of squared masses of neutrinos, these experiments are insensitive to their absolute mass. Moreover, these experiments can not determine the mass ordering of three generation of neutrinos either. Thus, determination of the absolute masses of neutrinos and their ordering remains to be an open issue in particle physics and need to be settled. Apart from particle physics experiments, cosmological observations can also be used to extract these informations about neutrinos since they play a crucial role in the background evolution as well as formation of structures in the universe.

In the recent past, some discordances between cosmic microwave background (CMB) and large scale structure (LSS) surveys have been reported. In particular, the value of $\sigma_8$, the r.m.s. fluctuation of density perturbations at $8\, h^{-1}\text{Mpc}$ scale and $H_0$ the value of Hubble parameter observed today, inferred from CMB and LSS observations are not in agreement with each other\cite{1–7}. It was argued that the mismatch between the Planck CMB observations and LSS surveys could be a signature of nonzero neutrino masses and hence bound on sum of neutrino masses was obtained \cite{8}. However, these attempts could not resolve the discordance in the above mentioned cosmological observations simultaneously. Furthermore, those mismatches are shown to be resolved in a better way by using effective viscous description of cold dark matter (CDM) on large scales. In this framework, the above mentioned discrepancies can be lifted simultaneously. With the success of effective viscous framework in resolving the discordance between CMB and LSS experiments, we will move on to include massive neutrinos as well. In this paper we will demonstrate that in the effective viscous framework, we can
indeed constrain the absolute masses of the neutrinos in a stringent way and differentiate the mass ordering of neutrinos to some extent.

Ascribing the aforementioned discordance to some exotic physics, several attempts have been made to address the issue. For instance, the interaction between dark matter and dark energy [9–11] as well as dark matter and dark radiation [12–14] was studied to resolve this tension to some extent. Similarly, in other attempts neutrino sector has been modified [8, 15, 16]. Massive sterile neutrino in the system was reported to reduce the tension in $\sigma_8-\Omega_m^0$ plane but it fails to do that in $H_0-\Omega_m^0$ [8, 15]. Interestingly, in ref [17] it has been shown that viscosities in CDM on large scales solves both $\sigma_8$ and $H_0$ tension simultaneously. We, therefore, use the viscous dark matter description to the study of neutrinos.

In order to demonstrate the tension between LSS and Planck CMB observation quantitatively, we use the following data sets. To study the LSS sector we used Planck SZ survey [18], Planck lensing survey [19], Baryon Acoustic Oscillation data from BOSS [20, 21], South Pole Telescope (SPT) [22, 23] and CFHTLens [24, 25]. This combined data set will be referred as LSS data in this paper. However, by Planck data we mean only Planck CMB observation[26].

This paper is structured as follows: We start with a brief description of tensions between CMB and LSS observations in section 2. Further, we describe the massive neutrinos in section 3 followed by effective viscous frame in section 4. After describing the massive neutrinos and viscous cosmology, we move on to the linear perturbation theory in section 5. This section is divided further into three subsections. In 5.1 we give the perturbation equations for viscous CDM while in 5.2 we provide the set of equations for massive neutrinos used in CLASS code and in 5.3 we discuss the effect of them on matter power spectrum. In section 6, we demonstrate that the inclusion of massive neutrinos can ease the tension between cosmological observations but do not solve them completely. However, the effective viscous framework solves the tension completely. After demonstrating the success of viscous CDM model we perform similar Markov Chain Monte Carlo (MCMC) analyses in section 7 to constrain the neutrino parameter space and finally conclude in section 8.

### 2 Tensions in cosmological observations

Large scale structure observations through lensing and Sunyaev-Zeldovich (SZ) effect have been consistently reported some deficiency in the number of clusters from the expected value obtained from CMB-fitted parameters. The problem can be described in the following way.

#### $\sigma_8-\Omega_m^0$ tension

Lensing observations estimates the power spectrum of the lensing potential ($C^{\phi\phi}_\ell$). This $C^{\phi\phi}_\ell$ depends on two parameters, amplitude of primordial perturbations, $A_s$, and the scale corresponding to the matter-radiation equality, $k_{eq}$ [27, 28]. The amplitude of the matter power spectrum $P(k)$ increases if the value of $A_s$ goes up. However, when $k_{eq}$ changes, $P(k)$ gets shifted. Therefore there exist a degeneracy in $A_s$ and $k_{eq}$ for a fixed value of $P(k)$ at certain $k$. This effect gets manifested in $C^{\phi\phi}_\ell$ too. $\sigma_8$ is propositional to $A_s$ and depends $\Omega_m$ through the growth factor. Similarly $k_{eq} \equiv a_{eq} H_{eq} \propto \Omega_m h^2$. Therefore the degeneracy in $A_s$ and $k_{eq}$, when converted in terms of standard cosmological parameters, shows up as a degeneracy in $\sigma_8$ and $\Omega_m$.

In the case of SZ surveys what is measured is the number of clusters with given mass in a given volume along the line of sight. This number is also a combination of $\sigma_8$ and the growth factor which is characterized by $\Omega_m$. Therefore most of the LSS observations report
their likelihood in $\sigma_8$-$\Omega_m^0$ plane as

$$\sigma_8 \left( \frac{\Omega_m^0}{\Omega_{m, \text{ref}}} \right)^\alpha = \text{const}.$$  \hspace{1cm} (2.1)\]

The values of $\alpha$ and $\Omega_{m, \text{ref}}$ are fitted so that the above combination remain independent of $\Omega_m$. Therefore $\alpha$ and $\Omega_{m, \text{ref}}$ changes for different observations.

However, the bestfit value of $\Omega_{m}^0$ and $A_s$ obtained from the CMB experiments gives a value of $\sigma_8$ from the theoretically predicted matter power spectrum using $\Lambda$CDM cosmology. This value does not match with the $\sigma_8$-$\Omega_m^0$ degeneracy direction at 2-$\sigma$ level (see Fig. (1a)). Joint analyses by combining different LSS experiments remove the degeneracy in the $\sigma_8$-$\Omega_m^0$. But still a mismatch between the allowed region of $\sigma_8$-$\Omega_m^0$ from Planck CMB and LSS experiments persists.

$H_0$-$\Omega_{m}^0$ tension: Measurement of $H_0$ is done in an indirect way in CMB and LSS observations. The scales of baryon acoustic oscillation (BAO) at last scattering surface, $\theta_{\text{MC}}$ is actually observed in CMB. Similarly acoustic oscillation in the matter power spectrum is also observed by LSS surveys like SDSS. Since, the co-moving acoustic oscillation scale is taken as the standard ruler in cosmology we can determine the co-moving distance from BAO [29]. The comoving distance at a particular $z$ is

$$\chi(z) = \int_0^z \frac{dz'}{H(z')} ,$$  \hspace{1cm} (2.2)\]

where,

$$H(z)^2 = H_0^2(\Omega_{m}^0(1+z)^3 + \Omega_\Lambda) .$$  \hspace{1cm} (2.3)\]
Therefore BAO observations can provide the value of $H_0$ when the value of $\Omega_0^m$ is supplied. Joint analyses of LSS experiments give some best-fit value of $\Omega_0^m$ rather than a large range. This $\Omega_0^m$ is little less than the $\Omega_0^m$ obtained from Planck CMB observations, which makes the value of $H_0$ derived from LSS joint analysis little higher than that derived from Planck CMB observation as seen in Fig. (1b).

This mismatch in $\sigma_8$-$\Omega_0^m$ and $H_0 - \Omega_0$ is a problem which can be solved simultaneously if one can reduce $\sigma_8$ without adding some extra matter component in the theory. In literature this problem has been addressed in many different ways as discussed in the sec. 1. Adding massive neutrinos increase $\Omega_0$ while reducing $\sigma_8$. On the other hand effective viscous description of dark matter on large scales does not increase $\Omega_0^m$ for reducing $\sigma_8$. A comparison between these two different ways of easing tension will be discussed later in sec. 6. In next three sections we will discuss the available understanding of mass of neutrinos, effective viscosity of CDM and the modifications in standard perturbation theory due to the inclusion of these two non-standard parameters.

3 Massive neutrino in cosmology

Several experiments with solar, atmospheric and reactor neutrinos have established the phenomenon of neutrino oscillation, caused by non-zero neutrino mass and neutrino mixing, beyond doubt. Theoretically, neutrino oscillation can be explained successfully by assuming that the neutrino flavor eigenstates namely, $\nu_e$, $\nu_\mu$, $\nu_\tau$ are linear superposition of mass eigenstates $\nu_1$, $\nu_2$, $\nu_3$. The fundamental parameters which characterize the neutrino mixing are: three mixing angles, $\theta_{12}, \theta_{13}, \theta_{23}$, three neutrino masses, $m_1, m_2, m_3$ and the CP violating phase $\delta_{\text{CP}}$. For the purpose of this paper we will call the mass of the lightest neutrino as $m_0$.

Global analyses of neutrino data provide us the neutrino oscillation parameters namely, $\Delta m^2_{21}, \theta_{12}, |\Delta m^2_{31}|(|\Delta m^2_{32}|), \theta_{13}, \theta_{23}$ with high precision. Here,

$$\Delta m^2_{ij} = m_i^2 - m_j^2.$$  \hspace{1cm} (3.1)

However, the existing data is blind towards the sign of $\Delta m^2_{31}$ or $\Delta m^2_{32}$ and hence give rise to two types of neutrino mass spectrum. Conventionally, these two spectra are given as follows:

- the normal hierarchy:

$$m_1(m_0) < m_2 < m_3, \quad \Delta m^2_{31} > 0, \quad \Delta m^2_{21} > 0,$$  \hspace{1cm} (3.2)

$$m_2 = \sqrt{m_0^2 + \Delta m^2_{21}}, \quad m_3 = \sqrt{m_0^2 + \Delta m^2_{31}};$$  \hspace{1cm} (3.3)

- the inverted hierarchy:

$$m_3(m_0) < m_1 < m_2, \quad \Delta m^2_{32} < 0, \quad \Delta m^2_{21} > 0,$$  \hspace{1cm} (3.4)

$$m_2 = \sqrt{m_0^2 + \Delta m^2_{23}}, \quad m_1 = \sqrt{m_0^2 + \Delta m^2_{32} - \Delta m^2_{21}}.$$  \hspace{1cm} (3.5)

The solar neutrino data tells us that the best fit values of $\Delta m^2_{21} = 7.37 \times 10^{-5}$ eV$^2$. The values of $\Delta m^2_{32}$ and $\Delta m^2_{31}$ is obtained from the global fit of the atmospheric neutrino oscillation data. In the case of normal hierarchy the bestfit value of $|\Delta m^2_{31}|$ is $2.54 \times 10^{-3}$ eV$^2$ and for inverted hierarchy $|\Delta m^2_{32}|$ is $2.42 \times 10^{-3}$ eV$^2$ [30, 31]. We use these bestfit values in calculating the neutrino masses in CLASS code [32]. However, while providing 2-$\sigma$ upper bound on the the
sum of the neutrino masses we will be using the upper 2-σ value of these quantities from ref [30].

Massive neutrinos have important property that they are relativistic in the early universe and contributes to the radiation density. However, in the late time, when they turn non-relativistic they contribute to the total matter density. We would also like to highlight that the collisionless nature of the neutrinos, after it become non-relativistic, allow them to free-stream on scales $k > k_{fs}$, where $k_{fs}$ is wavenumber corresponding to the scale of neutrino free-streaming. Hence it will wash out the perturbations on length scales smaller than the characteristic scale $k_{fs}$. This leads to suppression of power on small scales in the matter power spectrum and modifies the gravitational lensing potential sourced by cosmological structures which in turn modifies the shape of CMB anisotropy.

The number density of the neutrinos is very high and even small mass in neutrino can change the matter or radiation density of the universe. The radiation density corresponding to the neutrinos in early universe, after it decouples from the electron-positron plasma, is given by [33, 34]

$$\rho_\nu = \left[ \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_\gamma, \quad (3.6)$$

where, $\rho_\gamma$ is the photon density and $N_{\text{eff}}$ is the effective number of relativistic neutrinos at early times. The value of $N_{\text{eff}}$ has been estimated to be 3.046 [35]. The slight change of the value of $N_{\text{eff}}$ from exact 3 is attributed to the effect of spectral distortion of the neutrinos just after the decoupling. At the late time, when these neutrinos turns into non-relativistic species, their energy density fraction in the present universe becomes

$$\Omega_\nu = \frac{\sum_i m_i}{93.1 h^2}. \quad (3.7)$$

This $\Omega_\nu$ contributes to the total matter density fraction $\Omega_m$.

## 4 Viscous cold dark matter

Viscosities in the cold dark matter can be sourced by two different ways. When viscosities are generated due to self interaction between the dark matter particles, they are called fundamental viscosities. There is a second type of viscosity which is known as effective viscosity. This kind of viscosities are expected to be generated on large scales as the integrated effect of the back-reaction of small scales non-linearities.

Bounds on the ratio of cross section to the mass of dark matter particles are extracted from bullet cluster observation [36]. Fundamental viscosities can be calculated from these bounds on self interactions between the dark matter particles if the underlying quantum field theory of dark matter is well formulated [37] or an energy distribution function of the dark matter particles is known [38, 39]. Nevertheless, the fundamental viscosities are expected to be small compared to the effective viscosities [40].

Therefore we will concentrate on the second kind of the viscosities, known as effective viscosity. This kind of viscosities are expected to play its role in the late time of the universe. Linear perturbation theory works quite well for describing matter power spectrum for a large range of scales. The reason behind this is the huge hierarchy between the virialization scale and the Hubble scale. However, in late time when the non-linearities have started growing
one can still compute an effective linearized perturbation theory on large cosmological scales taking into account the effect of small scales. It has been shown in the literature that the effect of these small scale dynamics when integrated out works as effective bulk and shear viscosities \[41, 42\]. Let us decompose Einstein equation in background (with bar), linear (L) and non-linear (NL) parts.

\[
\bar{G}_{\mu\nu} + G^{L}_{\mu\nu} + G^{NL}_{\mu\nu} = 8\pi G (\bar{T}_{\mu\nu} + T^{L}_{\mu\nu} + T^{NL}_{\mu\nu}), \tag{4.1}
\]

where, \(G_{\mu\nu}\) is the Einstein tensor and \(T^{\mu\nu}\) is the stress-energy tensor. After equating \(\bar{G}_{\mu\nu}\) and \(8\pi G \bar{T}_{\mu\nu}\) we can write

\[
G^{L}_{\mu\nu} = 8\pi G (T^{L}_{\mu\nu} + T^{NL}_{\mu\nu} - \frac{G^{NL}_{\mu\nu}}{8\pi G}) \equiv T^{v}_{\mu\nu} - \bar{T}^{v}_{\mu\nu}. \tag{4.2}
\]

Here superscript \(v\) stands for viscous fluid. In this formulation all the components of energy density like baryonic matter and CDM are expected to generate effective viscosities on large scales. For the sake of simplicity we will consider viscosities only in dark matter fluid and treat baryonic matter as ideal fluid. We write the stress-energy tensor for non-ideal CDM fluid as \[43\]

\[
T_{\mu\nu}^{cdm} = \rho_{cdm} u^\mu u^\nu + (p + p_b) \Delta^{\mu\nu} + \pi^{\mu\nu}, \tag{4.3}
\]

where \(\rho_{cdm}\) is the energy density of the CDM and \(p\) is the pressure which will be taken to be zero for further calculations. Here, \(u^\mu\) is fluid flow vector and \(p_b = -\zeta \nabla^\mu u^\mu\) is the bulk pressure with \(\zeta\) being the coefficient of bulk viscosity. \(\pi^{\mu\nu}\) is the anisotropic stress tensor and has the following form

\[
\pi^{\mu\nu} = -2\eta \sigma^{\mu\nu} = -2\eta \left[ \frac{1}{2} \left( \Delta^{\mu\alpha} \nabla_\alpha u^\nu + \Delta^{\nu\alpha} \nabla_\alpha u^\mu \right) - \frac{1}{3} \Delta^{\mu\nu} (\nabla^\alpha u_\alpha) \right], \tag{4.4}
\]

where \(\eta\) is the coefficient of shear viscosity. For baryonic matter stress-energy tensor will same as eq. (4.3) but the coefficients of viscosities will vanish, since we are considering the baryonic matter to be ideal fluid.

The effects of viscosities are manifested in the cosmological parameters like, equation of state \(w\) and sound speed \(c^2_s\) and viscous sound speed \(c^2_{vis}\).

\[
w = -\frac{3\zeta H}{a\rho_{cdm}}, \quad c^2_s = -\frac{\zeta \theta}{a\rho_{cdm} \delta}, \quad c^2_{vis} = \left( \frac{4}{3} \eta + \zeta \right) \frac{H}{\rho_{cdm}} \tag{4.5}
\]

Here \(H\) is the comoving Hubble parameter, \(\delta = \delta \rho/\rho\) is the density perturbation and \(\theta = \nabla_i v^i\) is called the velocity perturbation. This velocity \(v^i\) is the perturbation in spatial parts of fluid vector \(u^\mu\).

For having an estimation of the viscous co-efficients we follow ref. [40]. Let us take \(k_m\) to be the scale beyond which non-linearities become important and we will restrict our linear perturbation theory only up to that scale. Then, as assumed in ref. [40], the viscosity parameters vary in the following way,

\[
c^2_s = \alpha_s \left( \frac{H}{k_m} \right)^2, \quad c^2_{vis} = \alpha_v (1 + w) a \left( \frac{H}{k_m} \right)^2. \tag{4.6}
\]
If we assume \( k_m \) to be order of 1 Mpc and take the value of all other quantities at to be of the present time, then \( \frac{4}{3} \eta + \zeta \) turns out to be of the order of \( 10^{-6} M_{\text{P}}^2 H_0 \). Here, \( \alpha_s \) and \( \alpha_\nu \) are kept to be order one for matching the results with N-body simulation [40, 42]. Therefore, for \( k_m \sim 1 \) Mpc, \( c_s^2 \) also turn out to be of the order of \( 10^{-6} \).

As described in ref. [17] the effect of viscosities are mostly visible in the very late time of the growth of perturbations. In this period the value of the effective viscosities doesn’t change that much; at least the order remains same [42]. Therefore we will consider constant viscosity for the time being in this paper. Moreover, although there are two types of viscosities, bulk and shear, only a combination of them acts as a single parameter in the theory. So, we use only non-zero shear viscosity which mimics this combination.

5 Perturbation theory

After a brief discussion of viscous cold dark matter and massive neutrinos, we would like to discuss the linear perturbation theory for these two components. Perturbation theory for these two components is dealt in two different ways. Although the massive neutrinos cannot travel freely beyond the free streaming scale \( k_{fs} \), the mean-free path in between the neutrinos is infinity, since neutrinos don’t have self interaction in standard picture. This forbids us to treat neutrino as an fluid.

Therefore we will treat CDM as viscous fluid and derive its perturbation theory by using conservation equation, but for the neutrinos we will discuss the Boltzmann equations which describe the evolution of density and pressure perturbations of neutrinos.

5.1 Perturbation theory with viscous cold dark matter

We introduce perturbations in the CDM fluid in following way.

\[
\rho_{\text{cdm}}(\tau, \vec{x}) = \rho_{\text{cdm}}(\tau) + \delta \rho_{\text{cdm}}(\tau, \vec{x}).
\]  

(5.1)

In order to preserve homogeniety and isotropy, the background quantities are functions of time only whereas the perturbations are both space-time dependent. Perturbations in the FRW metric, taken in conformal Newtonian gauge, can be written as

\[
ds^2 = a^2(\tau) \left[ -(1 + 2 \psi(\tau, \vec{x})) d\tau^2 + (1 - 2\phi(\tau, \vec{x})) dx_i dx^i \right],
\]

(5.2)

where \( \psi(\tau, \vec{x}) \) and \( \phi(\tau, \vec{x}) \) depends on space-time. This allows us to introduce first order perturbation in the fluid flow \( u^\mu \) as

\[
u^\mu = (1 - \psi, v^i),
\]

(5.3)

while \( u^\mu u_\mu = -1 \) is preserved in first order limit.

The Friedmann equation for this system reads as

\[
\mathcal{H}^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} (\rho_m + \Lambda) a^2,
\]

(5.4)

where \( \rho_m = \rho_b + \rho_{\text{cdm}} + \rho_\nu \), \( \Lambda \) is the sum of baryonic matter density, CDM density and neutrino density. Here dot denotes the derivative with respect to conformal time \( \tau \). The continuity equation of each species in the background has the following form

\[
\dot{\rho}_i + 3 \mathcal{H} (\rho_i + p_i) = 0,
\]

(5.5)
In the case of neutrinos, when they are relativistic their pressure is $p_\nu = \frac{1}{3} \rho_\nu$. However, when neutrino temperature drops below its mass, its pressure drops down to small value and neutrinos become non-relativistic. Therefore, depending on the neutrino temperature at a particular time, different neutrino species with different mass might obey separate continuity equations.

The perturbed part of continuity equation for the cold dark matter, $T^\mu_\nu = 0$, provides us two basis sets of equations, known as density perturbation equations and velocity perturbation equations [17].

The perturbed part of continuity equation for the cold dark matter, $T^\mu_\nu = 0$, provides us two basis sets of equations, known as density perturbation equations and velocity perturbation equations [17].

$$\dot{\delta} = - \left( 1 - \frac{\zeta a}{\Omega_{cdm} H} \right) (\theta - 3\dot{\phi}) + \left( \frac{\zeta a}{\Omega_{cdm} H} \right) \theta - \left( \frac{3 H \zeta a}{\Omega_{cdm} H} \right) \delta$$

and,

$$\dot{\theta} = -\mathcal{H} \theta + k^2 \psi - \frac{k^2 a \theta}{3 \mathcal{H} (\Omega_{cdm} H - \zeta a)} \left( \frac{4 \eta}{3} \right) - 6 \mathcal{H} \theta \left( 1 - \frac{\Omega_{cdm}}{4} \right) \left( \frac{\zeta a}{\Omega_{cdm} H} \right),$$

where $\eta = \frac{8\pi G \eta}{\mathcal{H}_0}$ and $\zeta = \frac{8\pi G \zeta}{\mathcal{H}_0}$ are the dimensionless parameters constructed from the viscosity coefficients. After outlining the evolution equations for density and velocity perturbations in viscous CDM, we outline the same for massive neutrinos in the next sub-section.

### 5.2 Perturbation theory with massive neutrinos

In this section, we will closely follow Ma & Bertschinger [44] to describe the perturbation equations for massive neutrinos. In terms of distribution function and the 4-momentum component, the energy momentum tensor is given by

$$T^\mu_\nu = \int dP_1 dP_2 dP_3 \left( -g \right)^{-1/2} \frac{P_\mu P_\nu}{P_0} f(x^i, P_j, \tau),$$

where $g$ and $P_\mu$ are the determinant of metric $g_{\mu\nu}$ and 4-momentum respectively and $f(x^i, P_j, \tau)$ is the phase space distribution function, which can be expressed as

$$f(x^i, P_j, \tau) = f_0(q)[1 + \Psi(x^i, P_j, \tau)].$$

$f_0$ is the zeroth-order phase space distribution which is Fermi-Dirac for the case of neutrinos and $\Psi$ is the perturbation in it. Massive neutrinos also obey the collisionless Boltzmann equation. However, non-zero mass complicates the evolution of distribution function. The unperturbed energy density and pressure of massive neutrinos are given by

$$\bar{\rho}_h = 4\pi a^{-4} \int q^2 dq \epsilon f_0(q), \quad \bar{P}_h = \frac{4\pi a^{-4}}{3} \int q^2 dq \frac{q^2}{\epsilon} f_0(q)$$

where $\epsilon = \epsilon(q, \tau) = \sqrt{q^2 + m_\nu^2 a^2}$. Unlike massless neutrinos, the $\epsilon$ depends on both the momentum and time. As a result, we can not simplify our calculation by integrating out the momentum dependence in the distribution function. To proceed further, the perturbation $\Psi$ is expanded in a Legendre series as

$$\Psi(k, \hat{n}, q, \tau) = \sum_{l=0}^{\infty} (-i)^l (2l + 1) \Psi_l(k, q, \tau) P_l(k, \hat{n}).$$
Using the above mentioned Legendre series expansion of $\Psi$, the perturbed energy density, pressure, energy flux and shear stress for the massive neutrinos in $k$ space can be given as

$$\delta \rho_h = 4\pi a^{-4} \int q^2 dq f_0(q) \Psi_0,$$

$$\delta P_h = \frac{4\pi}{3} a^{-4} \int q^2 dq \frac{q^2}{\epsilon} f_0(q) \Psi_0,$$

$$\langle \rho_h + P_h \rangle \theta_h = 4\pi ka^{-4} \int q^2 dq q f_0(q) \Psi_1,$$

$$\langle \rho_h + P_h \rangle \sigma_h = \frac{8\pi}{3} a^{-4} \int q^2 dq \frac{q^2}{\epsilon} f_0(q) \Psi_2. \quad (5.12)$$

Boltzmann equations for different moments of distribution function take the following forms in conformal Newtonian gauge:

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon} \Psi_1 - \phi \frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon} (\Psi_0 - 2\Psi_2) - \frac{e_k}{3q} \psi \frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_l = \frac{qk}{(2l + 1)\epsilon} [l\Psi_{l-1} - (l + 1)\Psi_{l+1}], \quad \text{for, } l \geq 2. \quad (5.13)$$

The perturbations $\delta \rho_h$, $\delta P_h$, $\delta \theta_h$ and $\delta \sigma_h$ which comes from the energy momentum tensor are equated with the perturbed part of the Einstein equations. These perturbations in the energy momentum tensor are evaluated from eq. (5.12) by numerically integrating the $\Psi$s over $q$.

5.3 Effect on matter power spectrum

We have numerically solved the above mentioned Boltzmann hierarchy equations for neutrino, eq. (5.13) and density-velocity perturbation equations for viscous CDM, eq. (5.6) and eq. (5.7) using publicly available CLASS code [32, 45]. The value of lightest massive neutrino has been taken to be 0.2 eV and masses of the other neutrinos have been taken as functions of it as described in sec. 3. Effect of the shear viscosity and massive neutrinos on matter power spectrum is plotted in Fig. (2)

The effect of viscous CDM and massive neutrinos have some similarities. The combination of $\zeta$ and $\eta$ in the third term of the right hand side of eq. (5.7) provides leading order contribution of the viscous effect. This term reduces the velocity perturbations and feeds that to the eq. (5.6) in which a reduced $\theta$ suppresses the growth of $\delta$. Therefore combination of these viscosities acts as a damping term in evolution of $\delta$. Since viscosity coefficients always come with $k$ in the perturbation equations their effect is larger in large $k$ values (see Fig. (2)).

Since bulk and shear viscosity behaves in an indistinguishable way (see ref. [17]) we use only non-zero value of shear viscosity.

In the case of neutrinos we see that Newtonian potentials $\psi$, created by the CDM perturbations, feeds in to the second moment of Boltzmann hierarchy equations, eq. (5.13). Iteratively it leaves its effect on all the $\Psi_l$s. Then the solution of Boltzmann equations changes the neutrino density perturbations through eq. (5.12). Power-spectrum generated from these massive neutrinos then adds up to the $P(k)$. However, this way of modifying neutrino density perturbations through the potentials of CDM is only possible on those scales which are larger than the free streaming scale corresponding to $k_{fs}$. In general $k_{fs}$ is a $z$ dependent quantity.
It reaches its minimum value $k_{nr}$, where $k_{nr}$ is the scale which reenters horizon at the time when neutrinos becomes non-relativistic. This scale depends on the neutrino masses as [46]

$$k_{nr} = 0.018 \left( \Omega_{m0}^0 \right)^{1/2} \left( \frac{m_i}{1 \text{ eV}} \right)^{1/2} h \text{ Mpc}^{-1}$$

(5.14)

For the particular mass $m_0 = 0.2 \text{ eV}$, $k_{nr}$ turns out to be $4.5 \times 10^{-3} h \text{ Mpc}^{-1}$. Therefore, for the modes with $k$ smaller than this, neutrinos will fall in the potentials of CDM and for larger $k$s they will freely steam without contributing in $P(k)$. That means in late time these neutrinos, on scales larger than $k_{nr}$, will behave as cold dark matter but won’t form any structure. This effect is visible in the suppression of $P(k)$ above that scale in Fig. (2).

6 Massive neutrino and viscous dark matter as solutions

We have already seen the effect of massive neutrinos and effective viscosity of CDM on matter power spectra (see Fig. (2)). Both the components suppresses $P(k)$ on small scales, though the effect of viscosity on the smaller scales are more prominent than massive neutrinos. Moreover, massive neutrinos increase the matter density at late time while effective viscosity keep the matter density unaffected. To quantify their effect we proceed through the following steps.

First we run a MCMC analysis using MontePython [47] of Planck $TT$ data with six cosmological parameters, lightest neutrino mass $m_0$ and $\sigma_8$, $H_0$ as derived parameters. For normal hierarchy of neutrinos other two mass values are set following eq. (3.3) and similarly for inverted hierarchy following eq. (3.5). Then the 2-$\sigma$ range obtained for two early universe parameters $A_s$ and $n_s$ are kept as bound with flat prior on these two parameters in our next run with LSS data. In both the runs $m_0$ was varied from range zero to 0.5 eV.
Figure 3: (a) Inclusion of massive neutrinos helps to resolve the $H_0$ tension (lower panel), but cannot solve the $\sigma_8$ tension simultaneously (upper panel). (b) Presence of any of the two viscosities resolves both the tensions simultaneously.

Two derived parameters $\sigma_8$ and $H_0$ have been plotted against $\Omega_m^0$ in Fig. (3). Since massive neutrino increases $\Omega_m^0$ while decreasing the $\sigma_8$ we can see that the allowed regions for two set of experiments in $\sigma_8$-$\Omega_m^0$ plane moves side by side but never overlap. But in the $H_0$-$\Omega_m^0$ plane the varying neutrino mass enlarges the range of $H_0$ for LSS as well as Planck. This enlargement ultimately allows LSS value to be accommodated within the limits of Planck.

For demonstrating the effect of viscosity we repeat the same steps, but this time keeping all the neutrinos massless and fixing the effective shear viscosity parameter $\tilde{\eta}$ at $6 \times 10^{-6}$. This value of shear viscosity has been reported to be the bestfit value in a joint Planck-LSS
analysis in ref. [17]. In the same paper it has been shown that bulk and shear viscosity plays indistinguishable role. Therefore we restricts our study to only one type of the viscosities.

Effect of effective shear viscosity on $\sigma_8 - \Omega_0^m$ and $H_0 - \Omega_m$ plane has been demonstrated in Fig. (3). As discussed earlier shear viscosity damps the growth on small scales and reduces the value of $\sigma_8$ without adding any extra contribution to $\Omega_0^m$. Since there exists a degeneracy in $\sigma_8$ and $\Omega_0^m$ for LSS data, as discussed in sec. 2, reduction in the value of $\sigma_8$ drives $\Omega_0^m$ to higher values. On the other hand, for CMB data the derived $\sigma_8$ gets reduced due to inclusion of viscosity and $\Omega_0^m$ remains unchanged. This helps to overlap the allowed regions in $\sigma_8$-$\Omega_0^m$ from two different sets of observations. Similarly, since $\Omega_0^m$ remains unchanged the derived values of $H_0$ from CMB data also does not get modified. However, the increase in the allowed values of $\Omega_0^m$ from LSS data reduces the derived values of $H_0$. In this way effective viscosity helps to ease the tension at two different fronts, while neutrino can resolve only one of them.

These analyses motivate us to incorporate neutrino mass in the effective viscous CDM scenario. Therefore, we do a joint MCMC analysis with combined Planck and LSS data with extra two parameters namely mass of lightest neutrino $m_0$ and viscosity coefficient $\tilde{\eta}$. This provides us a bestfit value of $\tilde{\eta}$, which is sufficient to ease the tension between LSS and Planck data, and a maximum allowed value of $m_0$. We discuss this effect in details in the next section.

7 Parameter space of neutrino mass

As discussed earlier, inclusion of massive neutrinos in cosmology changes the shape of $P(k)$ which leads to reduction in the value of $\sigma_8$. Therefore it is expected that better the measurement of $P(k)$ from large scale observations more stringent will be the constraint on the neutrino mass. However, precise measurements of $P(k)$ and CMB spectrum in recent years has shown a discordance in between this two sets of observations which cannot be removed by inclusion of neutrino mass as it was thought earlier. Rather a better description of cold dark

Figure 4: Planck CMB bound on the lowest neutrino mass $m_0$ drastically improves over inclusion of LSS data along with viscous cold dark matter.
matter fluid on large scale which includes the effect of small scale nonlinearities as effective viscosity in linear regime, has been found sufficient to reconcile this discordance. However, neutrino oscillation experiments indicate that neutrinos are massive. Consequently in the viscous fluid description it is expected that we will get more stringent constraint on lowest neutrino mass. In the joint MCMC analyses of Planck and LSS data in this frame work we found the maximum allowed value of $m_0$ at 2-$\sigma$ level to be 0.084 eV for normal hierarchy and 0.03 eV for inverted hierarchy.

We have also performed a joint MCMC analysis of Planck and LSS data with only massive neutrinos (without viscosities) as it has been argued to provide the evidence of nonzero mass of lightest neutrino[8, 15]. We found that in the case of normal hierarchy the 2-$\sigma$ lower bound of $m_0$ is 0.012 eV, whereas for inverted hierarchy the 2-$\sigma$ bound accommodates zero. However, as discussed above this preference of non-zero neutrino mass is nothing but an effect of not including effective viscosity in the analyses. From this same analysis we can also see that the upper bound on the $m_0$ goes up to 0.126 eV for normal and 0.119 eV for inverted hierarchy. That provides the sum of the neutrino masses $\sum m_\nu$ to be 0.396 eV and 0.378 eV respectively (taking all the other parameters at 2-$\sigma$ upper level).

We found that effective description of viscosity not only removes the notion of finding non-zero mass of the lightest neutrino from cosmological observations but also tightens up the bound on $m_0$ than that provided by the Planck+LSS joint analysis (see Fig. (5a) and Fig. (5b)). The 2-$\sigma$ upper bound on $m_0$ for normal hierarchy results in the sum of the neutrino

| Data sets      | Hierarchy | Without Viscosity | With viscosity |
|----------------|-----------|-------------------|----------------|
| Planck + LSS NH| 0.396 eV  | 0.267 eV          |                |
| Planck + LSS IH| 0.378 eV  | 0.146 eV          |                |
mass ($\sum m_\nu$) to be 0.267 eV and for inverted hierarchy it goes up to 0.146 eV (taking all the other parameters at 2-σ upper level). The value of $\sum m_\nu$ for NH is quite close to that of the Planck(TT + BAO + HST) joint analyses in ref. [26] with degenerate neutrinos. However we find that the bound obtained for $\sum m_\nu$ in inverted case is much lower (see Fig. (6a)).

### 7.1 Comparison with other experiments

Except the cosmological observations there are two types of experiments which try to resolve the absolute scale of mass of the neutrinos. The first type of experiments measure the $\beta$ decay spectrum and tries to find the cutoff of the spectrum at the tail which is definite signature of neutrino mass. The expected sensitivity from this kind of experiments is best from KATRIN which is expected to probe $m_0$ up to 0.2 eV [48]. Our bound on $m_0$ form normal hierarchy as well as inverted hierarchy goes below this sensitivity. Therefore if viscous cosmology is the right solution to the tension between Planck and LSS observations, we are expected to find no massive neutrino signal in KATRIN.

There are second types of experiments which try to observe the neutrinoless double beta decay which is not only a signature of neutrino mass but also the property of the neutrino, i.e. Dirac fermion or a Majorana fermion. Probability of neutrinoless double beta decay depends on electron neutrino mass ($m_{ee}$) [49], where

$$ m_{ee} = \sum_i m_i |U_{ei}|^2. \tag{7.1} $$

Here, $U_{ei}$ is the elements of PMNS mixing matrix. The present status of neutrinoless double beta decay by these experiments is as follows: The GERmanium Detector Array (GERDA) which uses germanium detector enriched in $^{76}$Ge achieved an upper limit on $m_{ee} < 0.2$ eV in

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**Figure 6:** a) Sum of the neutrino masses as a function of $m_0$ are shown. The width of the of the function corresponds to 2-σ uncertainties in $\Delta m^2_{ij}$. b) Mass of electron Majorana neutrino, $m_{ee}$ as a function of $m_0$ has been plotted where the values of other parameters are taken at 2-σ level.
phase I of this experiment [50]. In phase II of this experiments, GERDA achieved an improved upper limit of 0.15 eV [51] on $m_{ee}$. Another experiment, Enriched Xenon Observatory (EXO) provides an upper limit, $m_{ee} < 0.19$ eV in the first run [52]. The second phase of this experiments is expected to bring down the upper limit to 0.09 eV [53]. The combined analysis of KamLAND-Zen-I and EXO-I puts an upper limit of $m_{ee} < 0.12$ eV [54]. Second phase of KamLAND-Zen run achieved an upper limit of $m_{ee} < 0.06$ eV [55]. This is the best limit achieved so far. All these results mentioned above are at 90 % C.L. We calculate the $m_{ee}$ using our $m_{0}$ and the other parameters from oscillation data [30] in 2-σ level too. Our results shows that (see Fig. (6b)) some part of the allowed range of $m_{0}$ for normal hierarchy is already ruled out by KamLAND second run result, but for inverted hierarchy allowed $m_{0}$ is still below the sensitivity of the experiment.

8 Discussion and Conclusion

The tensions between the results of LSS experiment and Planck CMB observation are well studied in the literature. These tensions were believed to be the signature of some unknown, interesting and exotic physics. However, due to relevance of the current investigation, we have discussed only two such solutions namely effective viscosity on large scales and massive neutrinos throughout this paper. We have shown that the former one is a comparatively better solution to the problem than the later one. Nonetheless, from theoretical point of view effective viscous theory is just a better description of dark matter fluid on large scale and it is something very standard that should be naturally incorporated in the analysis. For simplicity, we have kept the viscosity coefficients constant for our analyses. We would like to stress that time varying viscous coefficient won’t change the conclusion drawn in the paper, but it might only change the required value of the viscosity parameter for resolving the tension between LSS and Planck data.

The amount of constant effective shear viscosity ($\eta$) required is $6 \times 10^{-6} M_{P}^{2}H_{0}$. This result is of the same order in $1h$ Mpc$^{-1}$ scale if we estimate the value of constant effective viscous coefficients using the assumption made in the two loop calculation of matter power spectrum [40] or the N-body simulation result in ref [42]. Therefore we can safely claim that this viscosity required for resolving LSS-Planck tension is of effective in nature.

Therefore, under this improved version of dark matter fluid on large scales, the scopes of introducing exotic physics in cosmological scenario is reduced. LSS-Planck tension has been advocated as a signature of massive neutrinos in cosmology many times in past [8, 15]. But in the effective viscous description of dark matter fluid, provision of accommodating massive neutrinos becomes more constrained. Moreover, the lower bound on lightest neutrino mass again touches zero value, making existence of one massless neutrino absolutely viable.

This approach of comparing different proposed solutions to some particular cosmological observations and then comparing them to constrain one using another is the central theme of this paper. We expect in future other exotic physics, like dark-matter neutrino interaction, presence of dark radiation, two component dark matter, dark energy-neutrino interaction etc., will be constrained in a better way using this approach.

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