Metrological analysis of measurements of elastic deformations in the contact zone of spherical corundum of the tips with the surface of the product

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Abstract. The article analyzes indirect measurements of elastic deformations of $l_d$ during video recording of the contact zone between the corundum spherical tip and the surface of the product. These measurements are due to the need to compensate for elastic deformations $l_d$ to improve the accuracy of contact measurements of the dimensions of the products characteristic of all measuring instruments: coordinate measuring machines, active control devices and other devices.

The method of measuring elastic deformations $l_d$ is similar to the measurement of Brinell hardness, but with a small clamping and immersion in the product of a spherical tip made of high-strength optically transparent material and the implementation of on-line visualization and video recording by a built-in measuring microscope of the formed control zone. Determined the sensitivity of this method, non-linear related to the radius of the spherical tip, $R_{tip}$ and elastic deformations $l_d$. The obtained expression for the measurement error of the deformations $\Delta l_d$ that largely depends on thermal expansion of the tip $\Delta R_{tip}(\Delta T)$ and the registration error of the zone control $\Delta l_{zc}$.

1. Introduction
Contact measurements are the easiest and most common way to determine the geometric dimensions of parts in most devices, ranging from the simplest, such as a caliper and up to complex systems: measuring heads [1-3] in coordinate measuring machines [4,5], active control devices [6-10], etc., including for the control of threads [11-14]. These measurements are based on the formation of a mechanical contact of the measuring tip (hereinafter referred to as the tip) with a small pressing force of $F_p$ (0.5...6 H) to the surface of the product, leading to the appearance of one of the components of errors due to contact elastic deformations (further - deformation) $l_d$, usually not exceeding 0.5...1.5 μm.

Improving the accuracy of measurements can be implemented with partial or full compensation of $l_{nc}$, which led to the emergence of various calculation methods. However, the results of such calculations are very different from each other with a difference of at least 16%, and from the experimentally obtained $l_d$ values with a difference of at least 25% [11,13,14]. Therefore, the use of calculation methods does not allow to achieve and the more complete compensation is almost impossible.

In this regard, it is proposed to implement a method of indirect measurements of $l_d$ deformation due to video measurement of the size of the $l_{zc}$ zone of contact (contact area) formed from the contact of the tip and the surface of the product (Figure 1a,b). The basis of this method is the joint use of an optically transparent tip made of high-strength material and a built-in microscope (video measuring system), providing visualization and video measurement of the control zone. At the same time, the determination of $l_d$ values as a result of such indirect measurements will make it possible to compensate the result of contact measurements with higher accuracy (Figure 1c). This approach can be implemented in different ways in different measuring devices, but the development of its common features is relevant for high-precision coordinate measurements.
2. Formulation of the problem
In connection with the above, the objectives of this work is to determine the metrological characteristics, namely, the sensitivity and measurement error of elastic deformations, their components and the relationship between them. The study of this in the open press in its entirety has not been previously presented and this article aims to fill this gap.

3. Theory
The tasks of the research involve consideration of the choice of tip material, measurement scheme, determination of sensitivity and calculation of the error of measurement of deformation $\Delta l_d$ with the construction of graphs of dependence on the values of its components, which are presented below.

3.1. The choice of material of the nozzle
Currently, the most suitable for the manufacture of high-strength tips and optically transparent materials can be the following materials (chemical formula and another name): diamond (C), borazon (BN - cubanit, kingmagic, kibrit), stishovite (SiO$_2$), silicon carbide (SiC - carborundum, moissanite), cubic zirconia (ZrO$_2$ - Zr$_{0.8}$Ca$_{0.2}$O$_{1.92}$), corundum (Al$_2$O$_3$ - sapphire, synthetic sapphire/leucosapphire, ruby), oxynitride aluminium (Al$_2$O$_2$N$_3$). By the combination of strength parameters, namely, elastic modulus $E$, Poisson's ratio $\nu$ and the microhardness of the $H_{\text{micro}}$, and based on manufacturability at the production and processing and the most affordable is a synthetic sapphire[15-17] - leucosapphire (further- the sapphire).

3.2. Measurement circuit
So, Figure 1b shows a sideview is presented in the zone of contact between the tip and the product, explaining that the tip with a radius $R_{\text{tip}}$ is pressed into the surface layer of the product to the depth of the segment [CD] corresponding to the deformations $l_d$ to form a contact zone of length $l_{zc}$ (seen from above). Such a scheme is similar to the Brinell hardness measurement scheme with the difference that $l_{zc}$ is measured in the measurement process, i.e. on-line behind the optically transparent tip and use the built-in microscope.

According to the Pythagorean theorem, the equality of the lengths of the segments $[AB]$, $[AD]$ with the tip radius: $|AB| = |AD| = R_{\text{tip}}$ and taking into account the aspect ratio in the ABC triangle we have: elastic deformations $l_{\text{def}} = l_{zc}$, chord (for the form from the side) or the length of the control zone (for the top view, frontal to the control zone) $|BE| = l_{zc}$, and $|AC| = |AD| - |CD| = R_{\text{tip}} - l_d$.

So, we can write next formula $R_{\text{tip}}^2 = (R_{\text{tip}} - l_d)^2 + l_{zc}^2$ and after conversion to receive $R_{\text{tip}}^2 - l_{\text{def}}^2 = (R_{\text{tip}} - l_d)^2$ and then, respectively, in the following form $\sqrt{R_{\text{tip}}^2 - l_{\text{def}}^2} = R_{\text{tip}} - l_d$. And then we get the desired formula for calculating the deformations $l_{\text{def}}$.
from what for the contact zone, which is a chord of $l_{zc}$, we get

$$l_{zc} = 2\sqrt{2 \cdot R_{tip} \cdot l_d - l_d^2}. \tag{2}$$

By using the usually satisfied condition $R_{tip} >> l_d$, the resulting expression can be simplified

$$l_{zc} \approx 2\sqrt{2 \cdot R_{tip} \cdot l_d}. \tag{3}$$

### 3.3. Determination of sensitivity

The sensitivity $S$ of this method, defined as the ratio of the size of the contact zone $l_{zc}$ to the deformations forming it $-l_d$, taking into account the expression (4), can be written by the formula

$$S = \frac{l_{zc}}{l_d} \approx \frac{2\sqrt{2 \cdot R_{tip} \cdot l_d}}{l_d} \approx 2.8 \frac{R_{tip}}{l_d}. \tag{4}$$

The resulting expression (4) indicates a nonlinear dependence of the sensitivity of measurements $S$ with the tip radius $R_{tip}$ and deformations $l_d$, three graphs of which are shown in Figure 2 for $R_{tip}: 2.0$ mm, $2.5$ mm and $3.0$ mm. The graphs show that in the range of small values of $d$, ranging from 0 to $\approx 2…4 \mu m$, the sensitivity of measurements $S$ is weakly dependent on $R_{tip}$ and much more strongly dependent on $l_d$.

![Figure 2. Graphs of sensitivity S for the tip radius Rtip and deformations l_d](image)

### 3.4. The calculation of the error of measurements of deformations $\Delta l_d$

Deformations $l_d$ occurring in the contact zone of corundum spherical tips with the surface of the product are determined by an indirect method using the formula (1), so the measurement error $\Delta l_d$, determined according to the rules for calculating the error of indirect measurements by the differential method will take the form:

$$\Delta l_d = \sqrt{(a \Delta R_{tip})^2 + (b \Delta l_d)^2}, \tag{5}$$

where $a$ and $b$ are coefficients determined by partial derivatives of formula (1): $a = \frac{\partial l_d}{\partial R_{tip}}$ and $b = \frac{\partial l_d}{\partial l_{zc}}$. 


The coefficient $a$ can be calculated as, $a = \frac{\partial l_a}{\partial R_{op}} = \left[ R_{op} - \sqrt{R_{op}^2 - \frac{l_{zc}^2}{4}} \right]^\gamma = \left[ 1 - \left( R_{op} - \frac{l_{zc}^2}{4} \right)^{\frac{\gamma}{2}} \right]$ then after the transformations we get $a = 1 - \frac{1}{2} \left( R_{op} - \frac{l_{zc}^2}{4} \right)^{\frac{\gamma}{2}} \left( R_{op} - \frac{l_{zc}^2}{4} \right)^{\frac{\gamma}{2}} = 1 - \frac{2R_{op}}{2} \sqrt{R_{op} - \frac{l_{zc}^2}{4}}$ and as a result $a = 1 - \frac{1}{\sqrt{1 - \left( \frac{l_{zc}}{2R_{op}} \right)^2}}$. In next step we can calculate for coefficient $b$: $b = \frac{\partial l_b}{\partial l_{zc}}$ and further $b = \frac{1}{2} \left( R_{op} - \frac{l_{zc}^2}{4} \right)^{\frac{\gamma}{2}} \left( R_{op} - \frac{l_{zc}^2}{4} \right)^{\frac{\gamma}{2}} = \frac{1}{2} \left( R_{op} - \frac{l_{zc}^2}{4} \right)^{\frac{\gamma}{2}} \left( R_{op} - \frac{l_{zc}^2}{4} \right)^{\frac{\gamma}{2}}$. Then the first $(a\Delta R_{op})^2$ and second $(b\Delta l_{zc})^2$ terms of expression (5) have the form $(a\Delta R_{op})^2 = \left[ \Delta R_{op} - \Delta R_{op} \right] = \Delta R_{op}^2 \left( 1 - \frac{1}{\left( R_{op} - \frac{l_{zc}^2}{4} \right)^2} \right)$ and $(b\Delta l_{zc})^2 = \left[ \frac{\Delta l_{zc}}{4 \left( \frac{R_{op}^2}{l_{zc}} - \frac{l_{zc}^2}{4} \right)} \right] = \frac{\Delta l_{zc}^2}{4 \left( \frac{R_{op}^2}{l_{zc}} - \frac{l_{zc}^2}{4} \right)}$ respectively. And with this in mind, the expression (5) takes the form: $a = \frac{\partial l_a}{\partial R_{op}} = \left[ R_{op} - \sqrt{R_{op}^2 - \frac{l_{zc}^2}{4}} \right]^\gamma = \left[ 1 - \left( R_{op} - \frac{l_{zc}^2}{4} \right)^{\frac{\gamma}{2}} \right] \left( R_{op} - \frac{l_{zc}^2}{4} \right)^{\frac{\gamma}{2}}$ (6)

If we assume that the shape of the tip is calibrated and its asphericity can be neglected, the error $\Delta R_{op}$ may have a predominantly temperature dependence. So the value of the linear expansion of the sapphire tip when heated $\Delta T$ at maximum for the worst conditions at 10°C from 15°C to 25°C, can be calculated by the next formula $\Delta R_{op} = \beta \cdot R_{opp} \cdot \Delta T$, where $\beta$ coefficient of linear expansion of sapphire, $R_{opp}$ - initial radius of the sapphire tip, taken for further calculations 2 mm.

Although the coefficient of linear expansion of sapphire $\beta$ is a variable $\beta(T)$, but this dependence is very small and taking into account [18], as well as a small simplification, it can be considered a constant $\beta=5.58 \cdot 10^{-6}$, obtaining the desired expression for $\Delta R_{op}$: $\Delta R_{op} = 5.58 \cdot 10^{-6} \cdot R_{opp} \cdot \Delta T$ and, respectively for next formula $\Delta R_{tip} = 31.14 \cdot 10^{-12} \cdot R_{opp} \cdot \Delta T^2$. 


According to the formula (7) calculations were carried out for $R_{tip}=2$ mm and for the control zone size $l_{zc}=300 \mu m$ with an error $\Delta l_{zc}$ determined mainly by the resolution of the camera, during its change in the range $1...10 \mu m$. The results of the calculations the two families of dependencies: the first $\Delta l_{a}$ changes $\Delta T$ in the range of $1^\circ C...10^\circ C$...and five values $\Delta l_{zc}$ = 2, 4, 6, 8, 10 mm (Figure 3a), the second - $\Delta l_{a}$ changes $\Delta l_{zc}$ in the range $1...10 \mu m$ for five values of $\Delta T=2^\circ C, 4^\circ C, 6^\circ C, 8^\circ C, 10^\circ C$ (Figure 3b) respectively. As can be seen from Figure 3a in all five dependency manifested very small dependence $\Delta l_{a}$ from $\Delta T$ a big step $\Delta l_{a}$ between the curves for different values $l_{zc}$. All the dependencies, arranged very closely and shown in Figure 3b also confirms the regularity that $\Delta l_{a}$ almost linearly dependent on $\Delta l_{zc}$ with small deviations for different values of $\Delta T$. As seen for the previously selected conditions of calculations the dependence $\Delta l_{a}$ to changes $\Delta l_{zc}$ prevails over $\Delta l_{a}$ dependence of $\Delta T$, the changes $l_{zc}$ in the range $1...10 \mu m$ results at different temperatures to increase the error of measurement of the deformations $l_{a}$ from 0.04...0.1 $\mu m$ to $\approx 0.39 \mu m$.

$$
\Delta l_{a} = \sqrt{\frac{31.14 \cdot 10^{-12} \cdot R_{tip}^{3} \cdot \Delta T^{2}}{1 - \left(\frac{R_{tip}}{2R_{sp}}\right)^{2}} - 4 \left(\frac{R_{tip}}{l_{w}}\right)^{2}} 
$$

Figure 3. Family of five graphs for the two dependencies: $\Delta l_{a}$ of $\Delta T$ (a) and $\Delta l_{a}$ from $\Delta l_{zc}$ (b)

4. Experimental result

During experiments on visualization and video recording of the control zone, a video recording scheme was used (Figure 4a) with incoherent external lighting (by lamp) during mechanical contact of elastic bodies with a flat optically transparent glass plate, realizing the conditions of $R_{tip}>>R_{prod}$, where $R_{prod}$ is the radius of the product. Images were obtained with clearly distinguishable control zone (Figure 4b) outlined by a lighter border. However, the clarity of the formed boundaries leaves much to be desired and leads to more correct experiments, including with coherent external lighting. It seems promising to search for conditions for the registration of images of control zones with boundaries containing the so-called Newton's rings.

Increasing the accuracy of registration of the contact zone boundaries causes the use of error minimization methods, such as the least squares method, as well as image correction to reduce its blurring. Finding the position of the center of the contact area allows you to determine the direction of movement of the tip during contact with the product.
5. Discussion of results

1. The expression for measurement sensitivity has the form \( S \approx 2.8 \frac{R_{\text{tip}}}{l_d} \) and is non-linearly related to the radius of the spherical tip \( R_{\text{tip}} \) and \( l_d \) deformations. In the range of small values ranging from 0 to \( \approx 2...3 \) μm the sensitivity of \( S \) measurements is small dependent on \( R_{\text{tip}} \) and much more strongly dependent on \( l_d \).

2. Error of measurement of the deformations \( \Delta l_d \) depends on the thermal expansion of the sapphire tip and resolution \( l_z \) at registration zones of control. The dependence \( \Delta l_d \) changes \( \Delta l_{z_0} \) higher than that of \( \Delta T \), therefore, the use of sapphire tips reducing measurement error \( \Delta l_d \) basically limited to a resolution of the camera. At the same time, changes in \( \Delta l_{z_0} \) in the range of 1...10 μm leads to an increase in the measurement error of \( \Delta l_d \) deformations from 0.04...0.1 μm to \( \approx 0.39 \) μm at different temperatures.

3. The images of the registered contact zones between the elastic body and the flat optically transparent plate are outlined by a lighter border. However, the clarity of the formed boundaries leaves much to be desired and leads to more correct experiments, including with coherent external lighting. It seems promising to search for conditions for the registration of images of control zones with boundaries containing the so-called Newton's rings. Increasing the accuracy of registration of the contact zone boundaries causes the use of error minimization methods, such as the least squares method, as well as image correction to reduce the blurring of its boundaries. Finding the position of the center of the contact area allows you to determine the direction of movement of the tip during contact with the product.

6. Conclusion

1. With the combined use of high-strength and optically transparent corundum tips and built-in microscope through visualization, video measurement of the size of the contact zone resulting from mechanical contact tip with the part it is possible to compensate for deformations \( l_d \) and improve the accuracy of contact measurements.

2. One of the directions of improvement of this method can be considered the using of methods and means of video recording of the boundaries of the contact zone with the use of methods of high-precision determination of its position and center, as well as image correction to reduce the blurring of its boundaries.

3. The appearance and registration of the contact zone means the beginning of mechanical contact of the tip with the product and can be used to form a measuring circuit of the output electrical signal \( 1(l - l_0) \) at a certain value of \( l_0 \), the corresponding, so-called measuring head touch. An alternative would be to use the measured values of deformations \( l_d \) as a proportional output signal \( F_{\text{out}}(l_d) \) for a range of elastic deformation and the implementation of the so-called measuring head deviation. This can mean the possibility of transition from the electromechanical method of signal generation to optoelectric, potentially more high-precision with a lower measurement error.

7. References
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