A distributed modelling method of the transmission tower and transient response analysis of lightning wave process

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Abstract
Lightning transient response analysis is the key to determining the lightning resistance level of transmission lines and the performance of lightning protection measures. This paper proposes a distributed modelling method for a typical 110 kV transmission tower and further analyses the lightning transient response. The refined modelling of the nominal height part of the tower is carried out. Then the influence of the refined modelling on the refraction and reflection process of the lightning wave propagation is investigated. Based on the distributed tower model, the impact of the foot distance and the equivalent radius of the main body cylinder on the top overvoltage is studied. Field experiments indicate that the proposed distributed tower model can effectively reflect the transient lightning wave process. Moreover, it can provide technical support for the lightning resistance level of transmission lines and the lightning protection of narrow base towers.

1 | INTRODUCTION

Lightning is the core factor causing the trip accidents of overhead transmission lines, which seriously threatens the safety and stability of the power system [1–4]. Therefore, it is important to study the wave process of lightning propagation under the lightning impact, and further to conduct lightning protection design.

Usually, the transmission tower model is an effective tool on both the lightning protection level and the lightning protection design of the transmission line. Thus, in recent years, a large number of researchers have carried out modelling studies on transmission towers [5], which can be roughly divided into the single-wave impedance model [6–14], frequency-dependent model [15, 16] and multi-wave impedance model [17–25]. Among them, the multi-wave impedance models are mainly divided into the composite tower equivalent model [17, 18], multilayer transmission tower model (MTTM) [19–22] and Hara model [23–25]. Huangfu et al. considered the coupling between the ground conductors of the composite tower and established a non-uniform multi-conductor transmission line model combined with lumped circuit elements, and finally formed a composite transmission line tower model under lightning strike [17, 18]. Ishii et al. tested and evaluated lightning impulse response characteristics of UHV transmission towers, and proposed a reasonable value of wave impedance at each segment of the multi-layer transmission tower model [19]. Mota [20] used the finite element method to construct a three-dimensional geometric simulation model of the transmission tower, and obtains electromagnetic parameters through simulation, and establishes a multi-layer transmission tower model. Based on empirical formula obtained from measured data, Hara et al. proposed a generally representative tower model, which was most widely applied [23]. However, the nominal height part of the Hara model was represented by a lossless transmission line, which could not accurately reflect the lightning wave process of the nominal height part, especially for high towers. Moreover, the same problem also existed in the composite tower equivalent model and the multilayer transmission tower model.

Actually, the nominal height part is often expressed by only a section of transmission line. The problem is that this
expression way cannot reflect the propagation process of the lightning wave in the nominal height part. Besides, it will affect the research and analysis of the transient process of the lightning wave. Therefore, to address the aforementioned problems, this paper proposes a distributed modelling method for a typical 110 kV double-circuit transmission tower. Considering the actual structure and the size, the tower is subdivided into eight segments. Specifically, the nominal height part is set to be five segments for two reasons. One is that this setting can fully reflect the wave process of the lightning wave in the nominal height part. The other is that it is convenient to study the effect of the nominal height part on the transient process of the tower struck by lightning.

Based on the distributed tower model, the transient response of single tower and multiple transmission towers is studied. For a single tower, the electric potential distribution at different positions of the tower is analysed. The potential at the C-phase crossarm of the proposed model is given comparative analysis with the Hara model. Further, the paper investigates the effect of the equivalent radius of the main body cylinder and the foot distance on the top overvoltage of the tower. As the equivalent radius increases, the top overvoltage decreases. Additionally, the top overvoltage of the tower rises with the shortening of the tower foot distance. Therefore, in the lightning protection design, attention should be paid to the lightning protection of the narrow base tower. Finally, the proposed model in this paper is verified through EMTP simulation and field experiments.

2 DISTRIBUTED MODELLING OF TRANSMISSION TOWER

2.1 Modelling framework

A typical 110 kV transmission tower can be divided into three parts: main body, bracings and crossarm, as shown in Figure 1.

Firstly, the main body can be regarded as an integrated system of four inclined cylindrical conductors. The modelling analysis of the wave impedance of the main body is based on the wave impedance formula of a single cylindrical conductor microelement segment, as shown in Figure 2. This paper derives the wave impedance calculation formula of the main body based on the double-cone antenna theory. On this basis, the wave impedance calculation formula of the main body is further developed. The specific derivation process can be seen in Section 2.2.

Secondly, the bracings are inclined conductors laid on the basis of the main body structure, as shown in Figure 1. Aiming at this tower parts, genetic programming algorithm is introduced to identify the calculation formulas between the size parameters and the bracings wave impedance. It is worthy of noting that the algorithm is built on the results of three-dimensional finite element method simulation. The specific derivation process can be seen in Section 2.3.

Thirdly, the crossarm wave impedance can be computed by the empirical formula [23].

Distributed modelling of the three parts of the main body, the inclined material and the crossarm is the basis for completing the model construction of the tower. The detailed steps of distributed towers modelling construction are shown in Figure 3.
To make the demonstration more clearly, the modelling steps can be further described as follows:

S1: To carry out distributed modelling of transmission towers, a typical 110 kV transmission tower needs to be divided into three parts: main body, bracings and crossarm.

S2: Based on the biconical antenna theory, the wave impedance calculation formula of a single vertical cylindrical conductor is derived by the mirror image method.

S3: On the basis of the wave impedance calculation formula of a single vertical cylindrical conductor, the correction coefficient is introduced. Combined with electromagnetic field theory and finite element simulation, the main body wave impedance calculation formula is derived. The correction coefficient is divided into capacitance correction coefficient and inductance correction coefficient. The two parameters are solved by performing electrostatic field simulation and static magnetic field simulation on a single vertical cylindrical conductor and four inclined cylindrical conductors, respectively.

S4: Followed by the calculation formula of the main body wave impedance, the correction coefficient of the bracings is introduced to develop the calculation formula of the wave impedance of the bracings. is divided into magnetic field correction coefficient and electric field correction coefficient. Similarly, these parameters are determined by implementing electrostatic field simulation and static magnetic field simulation on the main body with bracings and the main body without bracings, respectively.

S5: The calculation of the wave impedance of the crossarm refers to the empirical formula.

S6: Considering the actual structure and size of the 110 kV transmission tower, the tower is subdivided into eight segments, of which the nominal height part is 5 segments to reflect the wave process of the lightning wave in the nominal height part.

S7: Import the wave impedance calculation results of the three core parts (i.e. the crossarm if it exists the corresponding segment), the main body and bracings) of each segment into the EMTP software. Then the distributed modelling of the tower can be finally obtained.

2.2 Wave impedance model of main body

2.2.1 Derivation of wave impedance formula of single vertical cylindrical conductor

Due to the small radius of the single vertical cylindrical conductor, the electromagnetic wave propagating along it will be close to a spherical shape. In reported studies about modelling the transmission towers, many literatures neglect the non-zero resistance and permittivity of the ground, i.e. the ground is regarded as a perfect conductor [10, 20, 26–28]. There are two reasons for this approach. First, treating the ground as a perfect conductor can greatly simplify calculations and improve calculation efficiency. Second, the characteristic impedance caused by a lossy ground plane has a small modulus, especially for high-frequency lightning currents. Consequently, in this paper, the assumption is also employed. The ground is regarded as a good conductor to simplify the calculation. In addition, the wave impedance of the single vertical cylindrical conductor can be derived by the mirror image method and the principle of the biconical antenna [29].

Take a micro-element $dB$ on a single vertical cylinder. Its equivalent radius is set to be $R$, while the height above the ground is $h$ and the cone apex angle is $\theta$. Obviously, the three parameters satisfy the formula $r = (h^2 + R^2)$. The specific micro segment is shown in Figure 4. Without considering the coupling relationship between vertical micro-elements, the mirror image method can be adopted to obtain the double-cone antenna model. Then the transient electromagnetic wave generated by it can be given by the following expression [29]:

$$
\begin{align*}
H_\varphi &= \frac{Ae^{-j\theta}}{r \sin \theta} \\
E_\theta &= \frac{Ae^{-j\theta}}{r \sin \theta} \sqrt{\frac{\mu_0}{\varepsilon_0}} \\
H_\theta &= H_i = E_i = E_\varphi = 0
\end{align*}
$$

where $A$ is the magnetic vector position; $\beta$ is the phase constant; $\mu_0$ is the permeability in vacuum; $\varepsilon_0$ is the dielectric constant of vacuum.

The voltage $U$ generated by the micro-element $dB$ relative to the ground can be acquired via Equation (2) (the ground is zero potential by default). That is to say, the electric field is integrated from 0 to $\pi/2$ to determine the ground potential of the micro-element. The current $I$ flowing through the micro-element $dB$
can be computed by using the ampere loop law, as shown in Equation (2):

\[
\begin{align*}
U &= \int_{0}^{\pi/2} E_\theta r d\theta \\
I &= \int_{0}^{2\pi} H_y R d\varphi
\end{align*}
\]

By substituting Equation (1) into Equation (2), the definite integral equation can be solved. The specific results can be formulated by Equation (3).

\[
\begin{align*}
U &= A e^{-i\beta r} \sqrt{\frac{\mu_0}{2\pi \varepsilon_0}} \ln \frac{R}{\sqrt{R^2 + b^2} - b} \\
I &= 2\pi A e^{-i\beta r}
\end{align*}
\]

Finally, combined with the definition of wave impedance \(Z = U/I\) \([23]\), the wave impedance \(Z\) of the micro-element can be developed:

\[
Z = \frac{U}{I} = \sqrt{\frac{\mu_0}{4\pi^2 \varepsilon_0}} \ln \frac{R}{\sqrt{R^2 + b^2} - b} = 60 \ln \frac{R}{\sqrt{R^2 + b^2} - b}
\]

The wave impedance of the micro-element segment can be utilized to replace the wave impedance of a single vertical cylindrical conductor of a certain length. It can be seen that the value of wave impedance changes with the height variation of the single vertical cylindrical conductor. And the more refined the modelling segment, the more accurate the simulation analysis result.

### 2.2.2 Modelling and analysis of tower main body wave impedance

As described in Section 2.1, the modelling analysis of the wave impedance of the main body is based on the wave impedance formula of a single cylindrical conductor micro-element segment. Hence, the inclination factor of the conductor system is considered, and the correction coefficient \(k_m\) is introduced. To solve the correction coefficient \(k_m\), simply and conveniently, the \(k_m\) is divided into the inductance correction coefficient \(K_l\) and the capacitance correction coefficient \(K_c\). The solution of \(K_c\) corresponds to Section 2.2.3, and the solution of \(K_l\) corresponds to Section 2.2.4. The simulation analysis is executed by ANSYS Maxwell software from the perspective of the energy of the electromagnetic field. Finally, the calculation formula of the system wave impedance \(Z_{\text{main}}\) of four inclined cylindrical conductors is derived:

\[
Z_{\text{main}} = \sqrt{\frac{I_{\text{main}}}{C_{\text{main}}}} = k_m Z = \sqrt{\frac{K_l}{K_c}} \sqrt{\frac{L}{C}}
\]

where \(L\) is the inductance of a single cylindrical conductor; \(C\) is the capacitance of a single cylindrical conductor; \(I_{\text{main}}\) is the equivalent inductance of four inclined cylindrical conductors; \(C_{\text{main}}\) is the equivalent capacitance of four inclined cylindrical conductors.

Then, followed by the theory of the electromagnetic field, the calculation of the total electric field energy \(W_E\) and the total magnetic field energy \(W_M\) of the conductor system is shown in the following formula \([20, 27, 29]\):

\[
\begin{align*}
W_E &= \int \frac{1}{2} \varepsilon E^2 dv = \frac{1}{2} L I^2 \\
W_M &= \int \frac{1}{2} \frac{1}{\mu} H^2 dv = \frac{1}{2} C U^2
\end{align*}
\]

where \(r\) is the volume of the region where the conductor system is located; \(H\) is magnetic field strength; \(\mu\) is the electrical permeability; \(E\) is the electric field intensity; \(\varepsilon\) is the electric permittivity.

Based on Equations (5) and (6), the inductance correction coefficient \((K_l)\) and capacitance correction coefficient \((K_c)\) can be deduced as follows:

\[
\begin{align*}
K_l &= \frac{L_{\text{main}}}{L} = \frac{W_{m\text{main}}}{W_m} \\
K_c &= \frac{C_{\text{main}}}{C} = \frac{W_{e\text{main}}}{W_e}
\end{align*}
\]

where \(W_{m\text{main}}\) is the magnetic field energy of the four inclined cylindrical conductor system; \(W_{m}\) is the electric field energy of four inclined cylindrical conductor system; \(W_{m}\) is the magnetic field energy of a single vertical cylindrical conductor system; \(W_e\) is the electric field energy of a single vertical cylindrical conductor system.

In summary, the solution of \(K_c\) and \(K_l\) is the prerequisite to determine \(k_m\). To be more specific, \(K_c\) and \(K_l\) can be solved by performing electrostatic field simulation and static magnetic field simulation on a single vertical cylindrical conductor and four inclined cylindrical conductors, respectively. Therefore, simulation modelling of vertical cylindrical conductors and four inclined cylindrical conductors is necessary. Additionally, it is worth noting that \(K_c\) and \(K_l\) have nothing to do with the size of the excitation source when the following case is satisfied: the amplitude of the excitation source applied to a single vertical cylindrical conductor and four inclined cylindrical conductors is the same. Therefore, the voltage applied in two working conditions (i.e. the single cylindrical conductor and the four inclined cylindrical conductors) are equal in the simulation. And so as the applied current.

### 2.2.3 Solving the capacitance correction coefficient of the main body

To obtain the total energy storage of the electric field and then determine the capacitance correction coefficient, this section conducts electric field simulation analysis under two
working conditions (a single cylindrical conductor and four inclined cylindrical conductors). The basic steps of electric field simulation analysis can be elaborated as: (1) establish a simulation model; (2) assign materials to the model; (3) set the boundary of the solution domain; (4) add excitation; (5) mesh division, inspection and calculation. The material of the main body of the tower chooses iron, and the solution domain is considered to be vacuum [20, 27]. To improve the accuracy of the model calculation, the boundary of the solution domain is set according to the model size. More specifically, the boundary of all directions is expanded 1000 times except for the negative direction of the $Z$-axis. The length in the negative direction of the $Z$-axis is set as the height of the segment conductor to the ground. Besides, the excitation for the model is set to a voltage source and its voltage is 100 V. The ground plane and the ground. Besides, the excitation for the model is set to a

For meshing, select the adaptive meshing with excellent performance of ANSYS Maxwell software. In the solver setup, the maximum number of iterations is set to 10, and the percentage error of convergence is set to the default value of 1%.

To test the rationality of the boundary selection, an analysis line is randomly drawn along the $X$-axis in the single cylindrical conductor model. Then the size distribution of the energy density on the line was solved, as shown in Figure 5. It can be known from Figure 5 that after 15 m, the energy density has reached an order of magnitude of $10^{-13}$, approximately 0 J/m$^3$. This indicates that the energy in the solution domain can replace the energy of the entire field.

Figure 6 shows the energy density distribution of the single cylindrical conductor and the four inclined cylindrical conductors. In the case of four inclined cylindrical conductors, there exists an obvious shielding effect. The energy density in the centre is very small, while it increases gradually getting close to the conductor. When it is far from the conductor, the energy density begins to decrease again. On the contrary, in the case of the single cylindrical conductor, the maximum value occurs on the cylindrical surface, then it gradually decreases to the surroundings.

To embody the coupling relationship within the system of the four inclined cylindrical conductors, the coupling matrix of the self-capacitance and mutual capacitance of the conductors in the system is obtained when the equivalent radius $R = 0.0946$ m, the upper spacing $D_1 = 3.2$ m, the lower spacing $D_2 = 4$ m, and the segment length $l = 5$ m, as shown in Table 1. It can be seen from the table that the self-capacitance of each conductor, the mutual capacitance between any two adjacent conductors and the mutual capacitance between any two diagonal conductors are approximately equal respectively.

In this paper, three typical 110 kV double-circuit transmission towers are selected as simulation research, and the towers are subdivided into eight sections for processing. All the segmented size data and simulation calculation results are shown in Table 2. The segment size data includes the equivalent radius $R$ of the cylinder, the upper distance $D_1$, the lower distance $D_2$, and the segment length $l$. The simulation calculation data contains the electric field energy of a single cylindrical conductor and four inclined cylindrical conductors, and the capacitance correction coefficient $K_C$.

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Based on the segmented size data of the simulation calculation result, the genetic programming (GP) algorithm is applied to identify capacitance correction coefficient $K_C$, $R$, $D_1$, $D_2$, and $l$ [30]. The concrete expression can be demonstrated by Equation (8).

\[
K_C = 2.6663 + 0.1454D_2 - \frac{0.5621}{D_2} + \frac{0.007683D_1}{R} - 0.1029/ + 0.0005928/D_2^2 - 0.01037D_2^2 \tag{8}
\]

The mean square error (MSE) and mean absolute error (MAE) are $2.021 \times 10^{-5}$ and 0.003331, respectively, and the goodness of fit $R^2$ is 0.9933. Through Equation (8), $K_C$ can be obtained according to the size of $R$, $D_1$, $D_2$, and $l$ of the segments.
TABLE 2  Segmented size data and capacitance correction factor values

| R, cm | D1, m | D2, m | L, m | h, m | Wmain, J | Wc, J | Kc |
|-------|-------|-------|------|------|----------|-------|-----|
| 7     | 1.2   | 1.6   | 4    | 36.1 | 6.90499E-7 | 3.07248E-7 | 2.1981 |
| 7     | 1.6   | 2     | 4.1  | 32.1 | 7.50023E-7 | 3.12014E-7 | 2.3910 |
| 7     | 2     | 2.4   | 4    | 28   | 7.82672E-7 | 3.07248E-7 | 2.5567 |
| 8.1   | 2.4   | 3.43  | 4    | 24   | 8.73515E-7 | 3.20380E-7 | 2.5831 |
| 9.48  | 3.43  | 4.7   | 5    | 20   | 1.10789E-6 | 3.91630E-7 | 2.7343 |
| 10.84 | 4.7   | 6     | 5    | 15   | 1.22020E-6 | 4.07420E-7 | 2.8485 |
| 12.2  | 6     | 7.3   | 5    | 10   | 1.31587E-6 | 3.91420E-7 | 2.9488 |
| 13.56 | 7.3   | 8.6   | 5    | 5    | 1.39487E-6 | 4.35409E-7 | 2.8580 |
| 7     | 1.6   | 1.1   | 4    | 38.9 | 7.75812E-7 | 3.77762E-7 | 2.5831 |
| 7     | 2     | 1.5   | 4    | 34.9 | 7.34626E-7 | 3.01894E-7 | 2.4176 |
| 8.23  | 3.2   | 1.6   | 4    | 30.9 | 9.75812E-7 | 3.77762E-7 | 2.4654 |
| 9.46  | 4     | 3.2   | 5    | 22   | 1.07026E-6 | 3.91630E-7 | 2.6343 |
| 10.69 | 4.8   | 5     | 5    | 17   | 1.15462E-6 | 4.05341E-7 | 2.7504 |
| 11.93 | 5.6   | 4.8   | 5    | 12   | 1.23371E-6 | 4.18047E-7 | 2.8602 |
| 13.65 | 6.5   | 5.6   | 7    | 7    | 1.58037E-6 | 5.52966E-7 | 2.7827 |
| 7     | 1.2   | 0.818 | 4    | 38.9 | 6.25851E-7 | 3.07248E-7 | 2.0370 |
| 7     | 1.6   | 1.2   | 4    | 36.1 | 6.90499E-7 | 3.07248E-7 | 2.2474 |
| 7     | 2     | 1.6   | 3.9  | 30.9 | 7.29844E-7 | 3.01894E-7 | 2.4176 |
| 8.23  | 2.74  | 2     | 5    | 27   | 9.31315E-7 | 3.77762E-7 | 2.4654 |
| 9.463 | 3.48  | 2.74  | 5    | 22   | 1.03112E-6 | 3.91420E-7 | 2.6343 |
| 10.69 | 4.22  | 3.48  | 5    | 17   | 1.11487E-6 | 4.05341E-7 | 2.7504 |
| 11.927| 4.96  | 4.22  | 5    | 12   | 1.19568E-6 | 4.18047E-7 | 2.8602 |
| 13.65 | 6     | 4.96  | 7    | 7    | 1.53876E-6 | 5.52966E-7 | 2.7827 |

2.2.4  Solving the inductance correction coefficient of the main body

To solve the total energy storage of the magnetic field, three-dimensional static magnetic field simulation is performed in the finite element software. The simulation consists of two working conditions (i.e. a single cylindrical conductor and four inclined cylindrical conductors). According to Equation (7), the inductance correction coefficient $K_L$ can be gained.

The excitation of the model selects current source excitation, whose value is 1 A. By the way, the sum of injected current of the four inclined cylindrical conductors is also 1 A, but the current on one of them is 1/4 A. What is more, the single cylindrical conductor system and the four cylindrical conductors’ system are both disconnected conductors. To ensure the formation of a current loop, the $z$-direction value of the solution domain is set to 0, while the other directions are enlarged by 100 times.

It needs to be pointed out that the material of the model, the meshing, and the solver setup in Section 2.2.3 stay the same with those in this section, except for the applied field and excitation. Section 2.2.3 relates to the electrostatic field with a voltage source, while Section 2.2.4 is involved with the static magnetic field with a current source.

Besides, the inductance coefficient $K_L$ can also be solved by using another method. First, current $I_i$ is assumed to be injected into the four inclined cylindrical conductors. The total magnetic field energy of the whole system is quantified as $W'$. Then the current flowing through each cylindrical conductor can be approximately viewed as $0.25I_i$. Here, the total magnetic field energy stored by the single cylindrical conductor, any two adjacent cylindrical conductors and any two diagonal cylindrical conductors are set as $W'_1$, $W'_2$, and $W'_3$, respectively. According to the law of conservation of magnetic field energy, the relational expression in the system of four inclined cylindrical conductors can be demonstrated as Equation (9):

$$
\begin{align*}
\frac{1}{2}I_{\text{main}}I_i^2 &= W' = 4W'_1 + 8W'_2 + 4W'_3 \\
W'_1 &= \frac{1}{2}L_2 \left( \frac{1}{4}I_i \right)^2 \\
W'_2 &= \frac{1}{2}M_1 \left( \frac{1}{4}I_i \right)^2 \\
W'_3 &= \frac{1}{2}M_2 \left( \frac{1}{4}I_i \right)^2
\end{align*}
$$
TABLE 3 Self-inductance, mutual inductance and inductance correction factor values

| $M_i$, mH | $L_i$, mH | $M_{dig}$, mH | $K_L$ | Computed value | Error |
|----------|----------|--------------|------|----------------|-------|
| 0.003932 | 0.8084   | 0.003651     | 0.2548 | 0.2536         | 0.001236982 |
| 0.003991 | 0.8287   | 0.003705     | 0.2548 | 0.2535         | 0.001257905 |
| 0.003870 | 0.8088   | 0.003591     | 0.2549 | 0.2535         | 0.001363741 |
| 0.003923 | 0.8198   | 0.003644     | 0.2558 | 0.2535         | 0.002304524 |
| 0.004878 | 1.0243   | 0.004531     | 0.2542 | 0.2535         | 0.000724259 |
| 0.004846 | 1.0253   | 0.004499     | 0.2548 | 0.2535         | 0.001320141 |
| 0.003774 | 0.7886   | 0.003502     | 0.2552 | 0.2535         | 0.001661174 |
| 0.004877 | 1.0150   | 0.004530     | 0.2559 | 0.2535         | 0.002344979 |
| 0.004847 | 1.0150   | 0.004498     | 0.2558 | 0.2535         | 0.00232708 |
| 0.004826 | 1.0148   | 0.004478     | 0.2558 | 0.2535         | 0.002295731 |
| 0.004812 | 1.0112   | 0.004464     | 0.2558 | 0.2535         | 0.002339755 |
| 0.006730 | 1.4178   | 0.006243     | 0.2553 | 0.2535         | 0.001779457 |
| 0.003989 | 0.8085   | 0.003662     | 0.2548 | 0.2535         | 0.001240878 |
| 0.003943 | 0.7885   | 0.003524     | 0.2548 | 0.2535         | 0.001346674 |
| 0.004899 | 1.0138   | 0.004550     | 0.2548 | 0.2535         | 0.001323077 |
| 0.004858 | 1.0137   | 0.004510     | 0.2548 | 0.2535         | 0.002048273 |
| 0.004838 | 1.0140   | 0.004489     | 0.2548 | 0.2535         | 0.002092222 |
| 0.004820 | 1.0139   | 0.004472     | 0.2548 | 0.2535         | 0.002071469 |
| 0.006760 | 1.4200   | 0.006274     | 0.2557 | 0.2535         | 0.002188196 |

where $L_i$ denotes the self-inductance of each inclined cylindrical conductor; $M$ means the mutual inductance between any two adjacent cylindrical conductors; $M_{dig}$ is the mutual inductance between any two diagonal cylindrical conductors. Referring to Equation (9), the inductance $L_{main}$ of the four inclined cylindrical conductors can be further derived as:

$$L_{main} = \frac{1}{4} (L_i + 2M + M_{dig})$$  \hspace{1cm} (10)

Taking the inductance correction coefficient $K_L$ as the ratio of $L_{main}$ to the inductance $L_i$ of a single vertical cylindrical pillar, which can be expressed as Equation (11).

$$K_L = \frac{L_{main}}{L_i} = \frac{L_i + 2M + M_{dig}}{4L_i}$$  \hspace{1cm} (11)

To obtain the values of $L_i$, $M$ and $M_{dig}$, simulations are performed through the established magnetic field models of four inclined cylindrical conductors.

The final simulation and calculation results of all segment size data are shown in Table 3. All the original segmented size data of the tower is consistent with Table 2. In this way, Table 3 will not be repeated. It can be deduced from simulation results that the inductance correction coefficient $K_L$ is almost a constant distributed between 0.2542 and 0.2566. From the calculation results, the inductance correction coefficient $K_L$ also fluctuates between 0.2534 and 0.2536, which can also be regarded as a constant. The error of the two calculation methods stays on the order of $10^{-3}$. It has been verified that an error of this order of magnitude has little effect on the final main body wave impedance, so the inductance correction coefficient in this paper is 0.2543, which is the average value of the simulated value and the calculated value.

Thus, combining $K_C$, $K_L$, and Equation (7), the wave impedance calculation formula of the main body can be organized as follows:

$$Z_{main} = \sqrt{\frac{0.2543 Z_2^2}{2.6663 + 0.1454 D_2 - \frac{0.5621}{D_2} + \frac{0.007683 D_1}{D_2} + 0.0005928/D_2^2 - 0.1029I - 0.01037 D_2^2}}$$  \hspace{1cm} (12)
2.3 Wave impedance modelling of bracings

2.3.1 Derivation of wave impedance coefficient of the bracings

To establish an accurate wave impedance model of bracings of the transmission tower, a three-dimensional simulation model of the main body with bracings is built. In the model, the structure and size of the transmission tower is restored to the greatest extent, as shown in Figure 7.

Technically, the electromagnetic field theory and the three-dimensional simulation model can be combined. According to Equation (7), the wave impedance correction coefficient of bracing $k_z$ can be introduced. This can enable to analyse the relationship between the wave impedance of the main body with bracings and the main body without bracings:

$$k_z = Z_z Z_{main} \sqrt{L_z C_z} \sqrt{L_{main} C_{main}} = \frac{k_1 k_2}{W_{main}} W_{mz} W_{cz}$$

(13)

Among them, $Z_z$ is the wave impedance of the main body with bracings; $L_z$ is the equivalent inductance of the main body with bracings; $C_z$ is the equivalent capacitance of the main body with bracings; $W_{mz}$ is the magnetic energy of the main body with bracings; $W_{cz}$ is the electric field energy of the main body with bracings; $k_1$ is the magnetic field correction coefficient; $k_2$ is the electric field correction coefficient.

From the simulation results below, it can be seen that the wave impedance of the main body with bracings is smaller than that of the main body without bracings. That is to say, $k_z$ is always less than 1. This point is consistent with the result of the literature [23]. Therefore, the wave impedance of the main body with bracings is equivalent to the wave impedance of bracings in parallel with the wave impedance of the main body without the bracings, as shown in Equation (14). And the specific expression of the wave impedance $Z_s$ of the bracings is shown in Equation (15):

$$Z_z = Z_{main} Z_s Z_{main} + Z_s = k_z Z_{main}$$

(14)

$$Z_s = \frac{k_z}{1 - k_z Z_{main}} = k Z_{main}$$

(15)

Among them, $k$ is the wave impedance coefficient of the bracings.

2.3.2 Simulation modelling solution

Based on the finite element software, the three-dimensional structure simulation model of the main body with bracings is shown in Figure 7. The actual size of the tower is considered for analysis, including the equivalent radius $R$ of the cylindrical conductor, the upper spacing $D_1$, the lower spacing $D_2$, and the segment length $l$. The bracings are approximated by cylindrical conductors, and each connection between the bracings and the main body is smoothly connected with a sphere, which reduces the influence of the surrounding electromagnetic field on the uneven structure. The energy density distribution of the main body with bracings under the three-dimensional electrostatic field is shown in Figure 8.

The research method of this section is consistent with that of the main body. Three typical 110 kV double-circuit transmission towers are selected and divided into eight segments. All the segment size data is shown in Table 4, including the equivalent radius $R$ of the cylindrical conductor, the upper spacing $D_1$, the lower spacing $D_2$, and the segment length $l$, $k_1$, $k_2$, the bracings wave impedance correction coefficient $k_z$, and the bracings wave impedance coefficient $k$. 

FIGURE 7 3D simulation model of the main body with bracings: (a) top view, (b) front view

FIGURE 8 The energy density distribution of a three-dimensional electrostatic field: (a) top view, (b) front view
TABLE 4  Sectional size and correction coefficient values

| R, cm | D1, m | D2, m | L, m | k1       | k2       | k3       | k   |
|-------|-------|-------|------|----------|----------|----------|-----|
| 0.07  | 1.2   | 1.6   | 4    | 0.83198  | 0.97508  | 0.81125  | 4.2980 |
| 0.07  | 1.6   | 2     | 4    | 0.76367  | 0.97037  | 0.74104  | 2.8616 |
| 0.07  | 2     | 2.4   | 4    | 0.81265  | 0.96644  | 0.78554  | 3.6628 |
| 0.081 | 2.4   | 3.43  | 4    | 0.78933  | 0.96498  | 0.76169  | 3.1962 |
| 0.0948| 3.43  | 4     | 5    | 0.76206  | 0.96156  | 0.73276  | 2.7420 |
| 0.1084| 4.7   | 6     | 5    | 0.79139  | 0.96048  | 0.76012  | 3.1687 |
| 0.122 | 6     | 7.3   | 5    | 0.73931  | 0.96014  | 0.79236  | 3.8160 |
| 0.1356| 7.3   | 8.6   | 5    | 0.78933  | 0.96156  | 0.76169  | 3.1962 |

By the application of the GP algorithm, the display expressions of inclined material wave impedance coefficients k, R, D1, D2 and l are identified, and the identification results are shown in the following formula.

\[
k = 3.419 - \frac{4.9}{D_2} + 297.93R - \frac{6D_2^{5.352\cos(5.9476D_2)}}{1000 l \cdot D_2} - 0.0581D_1 - 0.5572\cos(13870.582/ \cdot D_2) \quad (16)
\]

After calculation, the MSE is 0.01226, the MAE is 0.0745, and the goodness of fit R^2 reaches 0.9491. Combined with the size of the specific segment size data R, D1, D2 and l, the wave impedance coefficient k of the bracings and the wave impedance of the bracings can be gained in turn.

Therefore, combining Equations (15) and (16), the wave impedance calculation formula of the bracings can be obtained as:

\[
Z_s = (3.419 - \frac{4.9}{D_2} + 297.93R - \frac{6D_2^{5.352\cos(5.9476D_2)}}{1000 l \cdot D_2} - 0.0581D_1 - 0.5572\cos(13870.582/ \cdot D_2))Z_{main} \quad (17)
\]

### 3 | ANALYSIS OF TRANSIENT WAVE PROCESS

#### 3.1 | Tower model parameter calculation and simulation setting

In this section, a 110 kV double-circuit transmission tower SZ2-30 is selected as a case. It is divided into eight sections for modelling analysis. Apart from that, it has also been given a comparison with the Hara model. The size parameters are shown in Table 5. The calculation results of the proposed model and the Hara model are shown in Tables 6 and 7 respectively. The wave impedance calculation method of the Hara model can refer to the literature [23].

\[
Z_A = 60 \ln \frac{2h}{r_A} \quad (18)
\]

where h is the height of the corresponding crossarm from the ground, r_A is the equivalent radius of the corresponding crossarm. Generally, r_A is 0.25 times the width of the crossarm at the junction of the corresponding crossarm and the tower.
According to the calculation data in Tables 6 and 7, the transient simulation model of the lightning strike tower is built in EMTP. Heidler source is selected as the lightning current source. The parameter settings of the Heidler’s function are as follows: the amplitude of lightning current $I_0$ is 20 kA. The front duration time $T_f$ and the stroke duration time $T_{st}$ is set as 2.6 and 50 $\mu$s, respectively. The factor $n$ which influencing the rate of rise of the function keeps the default value 2 of the EMTP software. In addition, the lightning channel impedance is set to 400 $\Omega$. The wave impedance parameter information of each segment of the transmission tower is set through the distributed parameters line component in EMTP, and its wave speed is set to $2.1 \times 10^8$ m/s [31]. The footing impedance of the grounding device is modelled as a linear resistance, which is generally 10 $\Omega$ [16, 19, 21].

### 3.2 Analysis of simulation results of a single tower

#### 3.2.1 Transient potential distribution of the tower

The transient potential simulation results of the proposed model are shown in Figure 9. It can be seen that a whole tendency of attenuation and oscillation occurs to the lightning voltage waveform. This is due to the mismatch between the wave impedance of the tower and the impedance of the footing impedance. The mismatch can cause refraction and reflection. Some part of the lightning wave is refracted and penetrates into the earth, while the other part is reversed and reflected back to the point where the lightning strike is injected. The potential...
3.3 Wave process analysis of the nominal height part

To study the influence of the subdivision modelling of nominal height on the propagation process of lightning waves, the model proposed in this paper and the Hara model are compared and analysed. The potential comparison result of the C-phase crossarm (lower-phase crossarm) is shown in Figure 10. The model proposed and the Hara model have almost the same potential on the crossarm of phase C, with a peak difference of 9.6%. Besides, the voltage peak time of the proposed model is ahead of the Hara model voltage peak time, and the lead time $\Delta t$ is about 0.023 $\mu$s.

The peak error and peak potential difference between the proposed model and the Hara model can be attributed to the refined segmented modelling of the nominal height part. It further reflects the refraction and reflection process of lightning waves in the tower. This reflection makes the research and analysis results more authentic. In the nominal height part of the transmission tower, as the height decreases and the equivalent radius increases, the wave impedance values of a single vertical conductor, main body, and bracings all decrease. Compared with the Hara model, in the proposed model, before the lightning wave reaches the bottom of the transmission tower, part of the lightning reflection wave has been reflected back to the lightning injection point. This part of the reflected wave has two functions: (1) It slows down the potential rise speed of both the lightning strike point and each crossarm, so that they reach the peak earlier; (2) It reduces the peak. In summary, it shows the correctness of the model established in this article.

3.4 Simulation analysis of multiple transmission towers

Since the size of the tower has a great influence on the wave impedance, this paper further studies the influence of the tower structure on the lightning overvoltage based on the proposed model. The paper mainly analyses the influencing factors of the foot distance and the equivalent radius of the main body cylinder. The results enable to provide technical reference for selecting suitable size towers in lightning protection design.

3.4.1 Analysis the influence of the foot distance

To study the impact of foot distance $R_B$ on overvoltage, the height of the tower and the structure of the bracings should be kept unchanged. The foot distance is changed to 1.3 times, 1.1 times, 0.9 times and 0.8 times of the $R_B$, and the simulation analysis is carried out again based on the proposed model. The simulation model involves two transmission towers (SZ2-30). Besides, the line-span (300 m), the horizontal distance between the conductor and the central axis of the tower (HD), and some other factors are also taken into account, as seen in Table 8. The other simulation parameter settings are consistent with Section 3.1. The simulation result is displayed in Figure 11.

From the simulation results, the tower top voltage increases by 2.13% and 5.41% when the foot distance decreases to 0.9$R_B$ and 0.8$R_B$, respectively. When the foot distance rises to 1.1$R_B$ and 1.3$R_B$, the tower top voltage declines by 1.34% and 4.65%, respectively. As the foot distance of the tower decreases, the wave impedance of the corresponding segment increases, so the overvoltage at the top of the tower gradually increases. For urban areas where land construction resources are extremely deficient, the foot distance of the towers are usually small (e.g. 2–3 m). However, it can be seen from the above that the size variation of the foot distance has a greater impact on the overvoltage at the top of the tower. Hence in the lightning protection design of transmission line corridor, special attention should be paid to the lightning overvoltage protection of the smaller foot distance. In this way, the safety and reliability of the power system can be greatly enhanced.
### TABLE 8  The electrical and structural parameters of the transmission line

|                          | Phase A                  | Phase B                  | Phase C                  | Lightning protecting wire |
|--------------------------|--------------------------|--------------------------|--------------------------|---------------------------|
| Type                     | LGJ-300/25               | LGJ-300/25               | LGJ-300/25               | GJ-50                      |
| Section area             | 333.3 mm²                | 333.3 mm²                | 333.3 mm²                | 49.46 mm²                 |
| Outer diameter           | 23.01 mm                 | 23.01 mm                 | 23.01 mm                 | 9 mm                      |
| Hanging height           | 38.8 m                   | 34.2 m                   | 30 m                     | 41.8 m                    |
| HD                       | 3.2 m                    | 3.7 m                    | 2.9 m                    | 2.4 m                     |
| DC resistance            | 0.0944 (Ω/km)            | 0.0944 (Ω/km)            | 0.0944 (Ω/km)            | 0.4513 (Ω/km)             |

#### 3.4.2  Analysis the influence of the equivalent radius of the main body cylinder

To investigate the effect of the equivalent radius $R_{eq}$ of the main body cylinder on the overvoltage, the remaining size parameters of the tower remained fixed. The equivalent radius is changed to $1.4R_{eq}$, $1.2R_{eq}$, $0.8R_{eq}$ and $0.6R_{eq}$, and the simulation analysis is performed again based on the proposed model.

It can be seen from Figure 12 that when the equivalent radius $R_{eq}$ decreases to $0.8R_{eq}$ and $0.6R_{eq}$, the tower top voltage increases by 2.54% and 4.87%, respectively. When the equivalent radius $R_{eq}$ rises to the original $1.2R_{eq}$ and $1.4R_{eq}$, the tower top voltage declines by 1.97% and 4.31% respectively.

As the equivalent radius of the main body cylinder of the tower diminishes, the wave impedance of the corresponding segment gets raised. Meanwhile the overvoltage at the top of the tower gradually also increases. Therefore, in the lightning protection design, the equivalent radius of the main body cylinder can be increased as much as possible. This way enables to reduce the lightning overvoltage and improve the lightning resistance level of the line. But it should be noted that the radius increment is conducted under the premise of ensuring economy.

#### 4  FIELD EXPERIMENT AND VERIFICATION

##### 4.1  Experiment field and measuring equipment

For further model verification, a field experiment is conducted in the 110 kV transmission line in cooperation with the Foshan Power Supply Bureau, Guangdong Power Grid Company Limited. The experimental field contains three transmission towers and a test substation, which are only used for the induction training of grid employees and operate without electricity, as seen in Figure 13. The size parameters of the tower (type: 5F1W5-J1) are shown in Table 9. The role of the experimental field is special. Therefore, the transmission line covers 110 kV and 500 kV, specifically: A phase is 110 kV analogue transmission line, and B phase and C phase is 500 kV analogue transmission line. The specific transmission line parameters of the experimental field are shown in Table 10. The gear spans of the three base towers are 62 and 30 m respectively. The grounding device of the tower is connected to the ground grid of the test substation, and the grounding resistance is tested to be 0.4 Ω by the earth-ρ tester (ETCR3000B). The input pulse is generated by a pulse generator, whose name is Agilent 33220A with 20 MHz. Before the experiment, a voltage reference wire is set to act as the reference zero potential.


### Table 9  The size parameters of the tower

| Section number | \( R_c \), cm | \( D_1 \), m | \( D_2 \), m | \( l_1 \), m | \( h_1 \), m | \( r_A \), m | Length of crossarm, m |
|----------------|----------------|-------------|-------------|-------------|-------------|-------------|---------------------|
| 1              | 7              | 0.8         | 1.8         | 5.5         | 36.5        | 0.8          | 5.5                 |
| 2              | 7              | 1.8         | 2.5         | 6           | 31          | —            | —                   |
| 3              | 7.78           | 2.5         | 3.5         | 5           | 25          | 2.4          | 6.755               |
| 4              | 8.56           | 3.5         | 4           | 5           | 20          | —            | —                   |
| 5              | 9.54           | 4           | 4.7         | 5           | 15          | —            | —                   |
| 6              | 10.52          | 4.7         | 5.3         | 5           | 10          | —            | —                   |
| 7              | 11.5           | 5.3         | 6           | 5           | 5           | —            | —                   |

### Table 10  Transmission line parameters at the experimental filed

|                | Phase A                      | Phase B                      | Phase C                      | Lightning protecting wire |
|----------------|------------------------------|------------------------------|------------------------------|----------------------------|
| Type           | JL/LB1A-300/40               | 4\times JL/LB1A-400/35      | 4\times JL/LB1A-400/35      | JLB40-120                  |
| Section area   | 338.99 mm²                   | 425.24 mm²                   | 425.24 mm²                   | 121.21 mm²                 |
| Outer diameter | 23.94 mm                     | 26.82 mm                     | 26.82 mm                     | 14.25 mm                   |
| Hanging height | 25 m                         | 25 m                         | 25 m                         | 36.5 m                     |
| HD             | −6.755 m                     | −1.25 m                      | −6.755 m                     | 5.5 m                      |
| DC resistance  | 0.09209 (Ω/km)               | 0.07177 (Ω/km)               | 0.07177 (Ω/km)               | 0.3606 (Ω/km)              |

length is approximately 150 m, and its end is grounded using a matching resistance of approximately 500 Ω [9, 10, 27]. To clarify the surge response of the real tower on the earth surface, it is necessary to ensure that the voltage reference wire and the current injection wire are perpendicular to eliminate the influence of magnetic coupling between the two [8]. The currents are measured by Tektronix TCPA300 and TCPA312A, which has a passband from DC to 100 MHz. High-resistance and low-capacitance probes are applied to measure the voltages of the tower. The voltage is defined as the potential difference between the tower and the voltage reference wire. Finally, the injection current and tower voltage are digitized using a Rigol DS1102E oscilloscope, with 100 MHz, 1 GSa/s, and 1 Mpts of record length.

### 4.2 Experimental results

The measured injection current waveform has little oscillations on the wavefront of the ramp impulse waveform. Its rising period rise time is about 15 μs, as shown in Figure 14. Besides, the experimental measurements and simulation results of the tower top voltages are shown in Figure 15.

In this case, the results between measurements and simulations are highly consistent. The results error between the proposed model and the actual measurement is about 4.1%. Similarly, the results error between the Hara model and the measured results is about 12.1%. In addition, in terms of the measurement of the real tower on earth surface (such as the literature [32, 33] cited in literature [16]), the average error between the actual measurement and the Hara model is 8.5%, and the maximum error is 11.1%. The error between the measurement results in this paper and the simulation results of the Hara model is 12.1%, which is basically consistent with the results of the literature [32, 33].
It can be seen that the voltage waveform of the proposed model and the actual measurement reached the peak almost at the same time, while the Hara model lags slightly. The voltage peak time of the Hara model lags that of the measured data. The specific lagging period $t_4$ is $0.032 \mu s$. Similarly, the lagging period $t_3$ between the proposed model and the measurement is $0.027 \mu s$. The reason is that, as mentioned above, the proposed model performs refined segmented modelling on the nominal height part of the tower. The more segments, the smaller the time difference, and the better fit the measured results.

5 | CONCLUSION

In order to explore the influence of the nominal height part of the tower on the lightning wave process, and to carry out the structural optimization research of the tower, this paper proposes a distributed modelling method for a typical 110 kV transmission tower. Then further analysis of the transient response of the transmission tower is carried out through EMTP software and field experiment research. The following conclusions are drawn:

1. The proposed model is verified to be effective through field experiments of multi-base transmission towers, and the error between field measurement results and model calculation results is only 4.2%.
2. In the experimental comparison case in this paper, compared with the Hara mode, the potential peak value of the C-phase crossarm and the top of the tower is lower than the Hara model, and the peak time is ahead of the Hara model, which is more consistent with the measured results.
3. The reduction of the foot distance of the tower can give rise to the increase of top overvoltage. Therefore, in the lightning protection design, special attention should be paid to the lightning protection of the narrow base tower.

The distributed model proposed in this paper can provide a simulation research basis for transient lightning wave process analysis. Meanwhile, it is also guidable for the design transformation and protection optimization of the transmission tower in lightning protection design.

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APPENDIX A
The method to obtain the energy density distribution of a single vertical cylindrical conductor
The energy density curve is obtained by post-processing of ANSYS Maxwell software. The specific process is described as follows:
1. Select ‘Results’ and right-click → Select ‘Create Files Reports’ → ’Rectangular Reports’.
2. Select ‘polyline2’ in the ‘Geometry’ column. Then continue to select ‘Energy’ in the ‘Quantity’ column, and click ‘New Report’.

Some explanations of three typical power transmission towers
These three-base transmission towers are all typical 110 kV transmission towers, which are used. In terms of the three-base transmission, the 1F5-SJ1-27 m and the 1F5-SJ2-27 m are SJ strain towers, while the 1F5-SDJ-24 m belongs to SDJ terminal towers. In Table 2, the first 8 groups are the segmented size data of the tower 1F5-SJ2-27 m. And the 9 to 16 groups are the segmented size data of the tower 1F5-SDJ-24 m. Then the 17 to 24 groups are the segmented size data of the tower 1F5-SJ1-27 m. There are some differences in the foot distance and heights of typical three-base towers, but the equivalent radius of a single cylindrical conductor corresponding to the segment is similar, which constitutes a simulation database.