How devious are deviations from quantum mechanics: the case of the $B^0\bar{B}^0$ system

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Abstract

Considering semileptonic decays of the entangled $B^0\bar{B}^0$ state which is generated in the decay of $\Upsilon(4S)$, we simply multiply the quantum-mechanical interference term by a factor $(1 - \zeta)$ and use the “decoherence parameter” $\zeta$ as a measure for deviations from quantum mechanics. We investigate several consequences of this modification of semileptonic $B^0\bar{B}^0$ decays. In particular, we show that when confronted with the experimental values of the ratio $R = (#$ like-sign dilepton events)/(# opposite-sign dilepton events) and of the $B_{H-L}$ mass difference, the ensuing one standard deviation range of the decoherence parameter depends strongly on the basis in the $B^0-\bar{B}^0$ space used to build the entangled $B^0\bar{B}^0$ state. On the other hand, in quantum mechanics physical quantities are, of course, independent of such arbitrary basis choices.

PACS: 13.25.Hw; 14.40.Nd; 03.65.Bz
Keywords: $B^0\bar{B}^0$ system; Dilepton events; Entangled state; EPR-correlations; Decoherence parameter
1 Introduction

In recent years there is increasing interest in testing quantum systems exhibiting Einstein–Podolsky–Rosen correlations. Such systems are suitable to discriminate between quantum mechanics and any local realistic (hidden variable) theory \cite{1} (see, e.g., Ref. \cite{2} for a short review). Usually such experiments are carried out by using photons (see, e.g., Refs. \cite{3,4}), however, we find it desirable to perform such tests also with massive particles. Appropriate quantum systems are given by $K^0\bar{K}^0$ \cite{5,6,7} and $B^0\bar{B}^0$ \cite{8,9}. In particular, the entangled $B^0\bar{B}^0$ system produced at the $\Upsilon(4S)$ offers the possibility to test quantum-mechanical interference over macroscopic distances of order $3 \times 10^{-2}$ mm.

In a previous work \cite{10} we have studied this entangled $B^0\bar{B}^0$ state by simply multiplying the quantum-mechanical interference term by a factor $(1-\zeta)$ and using the “decoherence parameter” $\zeta$ \cite{7} as a measure for deviations from quantum mechanics. Confronting the ratio $R = (# \text{ like-sign dilepton events})/(# \text{ opposite-sign dilepton events})$ with experimental values restricts the decoherence parameter to $\zeta \leq 0.53$ (90\% CL). This is a result which conforms nicely with quantum mechanics and shows that local realistic theories ($\zeta = 1$) are disfavoured.

Recently, Dass and Sarma \cite{11} have performed a similar analysis of the same entangled $B^0\bar{B}^0$ state, however, employing the basis provided by the mass eigenstates $B_H, B_L$ rather than the flavour states $B^0, \bar{B}^0$ which were in used in Ref. \cite{10}. When they confront the ratio $R$ with the data they obtain a result which is very close to the expectation of quantum mechanics, being nearly 8 standard deviations away from complete decoherence. Of course, there is no contradiction between the results of Ref. \cite{10} and \cite{11}. Evidently, different basis choices in the $B^0-\bar{B}^0$ space produce in general different quantum-mechanical interference terms, each with its own decoherence parameter. When different interference terms get modified by decoherence parameters their values inferred from experimental input will also be different in general.

In this paper we generalize the considerations of Refs. \cite{10,11} by representing the $B^0\bar{B}^0$ state with the help of a general basis in the $B^0-\bar{B}^0$ space. We will show that, given a basis, the ensuing range of the corresponding decoherence parameter depends strongly on that particular basis choice. In addition, arbitrary basis choices can also mimic CP violation in the $B^0\bar{B}^0$ system.

2 The formalism

The decay $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ generates the state

$$\Psi(t = 0) = \frac{1}{\sqrt{2}} \left( |B^0\rangle \otimes |\bar{B}^0\rangle - |\bar{B}^0\rangle \otimes |B^0\rangle \right)$$

(1)

with the charge conjugation quantum number $C = -1$. Eq. (1) is the point of departure in Ref. \cite{10}. On the other hand, the mass eigenstates of the neutral $B$ mesons are given by

$$|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle, \quad |B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

with $|p|^2 + |q|^2 = 1$. (2)
Starting with this basis, the state (1) is rewritten as

\[ \Psi(t = 0) = \frac{1}{\sqrt{2pq}} (|B_H\rangle \otimes |B_L\rangle - |B_L\rangle \otimes |B_H\rangle) . \]  

(3)

It has been stressed in Ref. [11] that taking Eq. (3) and modifying the interference terms by \((1 - \zeta)\) the results are different from those obtained in Ref. [10] with the starting point Eq. (1). Of course, with respect to quantum mechanics the states (1) and (3) are identical.

To generalize this consideration we take an arbitrary basis

\[ |b_j\rangle = S_{1j}|B^0\rangle + S_{2j}|\bar{B}^0\rangle \quad \text{with} \quad j = 1, 2 \]  

(4)

such that

\[ \Psi(t = 0) = \frac{1}{\sqrt{2|S|}} (|b_1\rangle \otimes |b_2\rangle - |b_2\rangle \otimes |b_1\rangle) . \]  

(5)

The discussions in Refs. [10] and [11] correspond to the special cases \(S = 1\) and \(S = M\) with

\[ M = \begin{pmatrix} p & p \\ q & -q \end{pmatrix} , \]  

(6)

respectively.

The time evolution of the basis vectors (4) is given by

\[ |b_j(t)\rangle = (M\hat{g}(t)M^{-1})_{1j}|B^0\rangle + (M\hat{g}(t)M^{-1})_{2j}|\bar{B}^0\rangle \]  

(7)

with \(\hat{g} = \text{diag}(g_H, g_L)\) where the functions

\[ g_{H,L}(t) = \exp \left( -i(m_{H,L} - \frac{i}{2}\Gamma_{H,L})t \right) \]  

(8)

correspond to the time evolution of \(|B_H\rangle\) and \(|B_L\rangle\), respectively. Eq. (7) can be simplified by

\[ M\hat{g}(t)M^{-1} = \begin{pmatrix} g_+(t) & \frac{p}{q}g_-(t) \\ \frac{q}{p}g_-(t) & g_+(t) \end{pmatrix} \]  

(9)

with

\[ g_\pm(t) = \frac{1}{2}(g_H(t) \pm g_L(t)) . \]  

(10)

Then we obtain the time evolution of the state (1) in terms of the basis (4) as [12]

\[ |\Psi; t, t'\rangle = \frac{1}{\sqrt{2|S|}} (|b_1(t)\rangle \otimes |b_2(t')\rangle - |b_1(t)\rangle \otimes |b_2(t')\rangle) \]  

(11)

or

\[ |\Psi; t, t'\rangle = \frac{1}{\sqrt{2|S|}} (T_{j1}(t)T_{k2}(t') - T_{j2}(t)T_{k1}(t')) |\beta_j\rangle \otimes |\beta_k\rangle \]  

(12)
where a summation over equal indices is understood and we have defined

$$|\beta_1\rangle \equiv |B^0\rangle, \quad |\beta_2\rangle \equiv |\bar{B}^0\rangle$$  \hspace{1cm} (13)

and

$$T(t) \equiv M\hat{g}(t)M^{-1}S.$$  \hspace{1cm} (14)

Eq. (12) follows from Eq. (11) by inserting Eq. (7).

We want to underline the invariance of quantum mechanics under the arbitrary basis choice (4) by formulating the following theorem.

**Theorem 1** The matrix element $\langle f_1 \otimes f_2 | \Psi; t, t' \rangle$ where $|f_1\rangle$, $|f_2\rangle$ are arbitrary states is independent of the matrix $S$ which characterizes the basis choice in the $B^0 - \bar{B}^0$ space.

**Proof:** The theorem follows from the fact that $\ell^+$ tags $B^0$ whereas $\ell^-$ originates from a $\bar{B}^0$ decay. Defining

$$b_+ = \sum_X |A(B^0 \rightarrow X\ell^+\bar{\nu}_\ell)|^2 \quad \text{and} \quad b_- = \sum_X |A(B^0 \rightarrow X\ell^-\bar{\nu}_\ell)|^2$$  \hspace{1cm} (17)

we obtain the following expressions for the number of dilepton events [12, 13, 14]:

$$N_{++} = \frac{1}{2\Gamma} b_+^2 \frac{p^2}{q} I_+ + \frac{\zeta}{\det S^2} b_+^2 \int_0^\infty dt T_{11}^*(t)T_{12}(t) \bigg| \int_0^\infty dt' T_{22}^*(t')T_{21}(t') \bigg|,$$  \hspace{1cm} (18)

$$N_{--} = \frac{1}{2\Gamma} b_-^2 \frac{q^2}{p} I_- + \frac{\zeta}{\det S^2} b_-^2 \int_0^\infty dt T_{22}^*(t)T_{21}(t) \bigg| \int_0^\infty dt' T_{11}^*(t)T_{12}(t') \bigg|,$$  \hspace{1cm} (19)

$$N_{+-} = N_{-+} = \frac{1}{2\Gamma} b_+ b_- I_+ + \frac{\zeta}{\det S^2} b_+ b_- \times \Re \left( \int_0^\infty dt T_{11}^*(t)T_{12}(t) \int_0^\infty dt' T_{22}^*(t')T_{21}(t') \right).$$  \hspace{1cm} (20)

These equations show that deviations from quantum mechanics parameterized by the decoherence parameter $\zeta$ are all characterized by the following two integrals:

$$\int_0^\infty dt T_{11}^*(t)T_{12}(t) =$$

$$I_+S_{11}^*S_{12} + \left| \frac{p}{q} \right|^2 I_--S_{21}^*S_{22} + \frac{p}{q} I_--S_{11}^*S_{22} + \left( \frac{p}{q} \right)^* I_+S_{21}^*S_{12},$$  \hspace{1cm} (21)

$$\int_0^\infty dt T_{22}^*(t)T_{21}(t) =$$

$$I_+S_{22}^*S_{21} + \left| \frac{q}{p} \right|^2 I_--S_{12}^*S_{11} + \frac{q}{p} I_--S_{22}^*S_{11} + \left( \frac{q}{p} \right)^* I_+S_{12}^*S_{21},$$  \hspace{1cm} (22)
where

\[ I_\pm = \int_0^\infty dt \left| g_\pm(t) \right|^2 = \frac{1}{2\Gamma} \left( \frac{1}{1+y^2} \pm \frac{1}{1+x^2} \right), \]

\[ I_{+-} = (I_{-+})^* = \int_0^\infty dt g_+(t)g_-(t) = -\frac{1}{2\Gamma} \left( \frac{y}{1-y^2} + \frac{x}{1+x^2} \right), \]

and \( x \) and \( y \) are defined as

\[ x = \frac{\Delta m}{\Gamma} \quad \text{and} \quad y = \frac{\Delta \Gamma}{2\Gamma}. \]

CPT invariance leads to \( b_+ = b_- \) to a very good approximation.

As in our previous work Ref. [10], we have introduced the decoherence parameter \( \zeta \) in Eqs. (18), (19) and (20) by multiplying with it the interference terms which appear because \( \Psi(t, t') \) is the sum of two terms (see Eqs. (11) and (12)). We want to stress once more that actually one should consider cases with different \( S \) as different extensions of the quantum-mechanical case and thus label \( \zeta \) by \( S \). This is also manifest from the fact that for \( \zeta \neq 0 \) the expressions for the number of dilepton events depend on \( S \). We omit such a label for the sake of simplicity.

### 3 Like-sign dilepton events and CP violation

If quantum mechanics is valid, then CP violation in \( B^0\bar{B}^0 \) mixing occurs for \( |p/q| \neq 1 \) which is equivalent to \( N_{++} \neq N_{--} \). However, because of the introduction of the matrix \( S \), which is arbitrary apart from \( \det S \neq 0 \), even for \( |p/q| = 1 \) we have \( N_{++} \neq N_{--} \) in general\footnote{This can, e.g., be seen by setting \( S_{11} = S_{22} = 1 \) and \( S_{21} = 0 \) and varying \( S_{12} \).}. This is one of the strange consequences of the simple modification of the quantum-mechanical expressions by the decoherence parameter: the matrix \( S \) mimics CP violation and the basis choice (4) which is unphysical at the level of quantum mechanics gets a physical meaning because \( N_{++} - N_{--} \) depends on \( S \).

On the other hand, the cases \( S = 1 \) and \( S = M \) discussed in Refs. [10] and [11] lead indeed to \( N_{++} = N_{--} \) for \( |p/q| = 1 \) as can easily be checked by inspecting Eqs. (21) and (22). It is difficult to give the general conditions for \( S \) such that in the limit \( |p/q| \to 1 \) the corresponding limit of \( N_{++} - N_{--} \) is zero.

However, apart from the two special cases mentioned above, there are conditions such that this requirement is fulfilled for a whole class of matrices \( S \).

**Theorem 2** If \( y = 0 \) and the matrix \( S \) is unitary then \( N_{++} = N_{--} \) for \( |p/q| = 1 \).

**Proof:** The first assumption can be justified by looking at the experimental branching ratios of the decay channels common to \( B^0 \) and \( \bar{B}^0 \). Taking \( y = 0 \) leads to \( I_{-+} = -I_{+-} \). Using the parameterization

\[ S = e^{i\phi} \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \]

\[ (25) \]
for a general unitary matrix and assuming \( p/q = e^{i\delta} \) we find from Eqs. (21) and (22)

\[
\int_0^\infty dt T_{11}^*(t)T_{12}(t) = -\left( \int_0^\infty dt T_{22}^*(t)T_{21}(t) \right)^* = (I_+ - I_-)a^*b + I_+e^{i\delta}(a^*)^2 + e^{-i\delta}b^2.
\]

(26)

Then Eqs. (18) and (19) provide for \( N_{++} = N_{--} \) under the above conditions. \( \square \)

In the following we will stick to the assumptions

\[
y = 0, \quad S \in U(2) \quad \text{and} \quad \left| \frac{p}{q} \right| = 1
\]

(27)

for two reasons. First of all, it has been shown experimentally that the ratio measuring CP violation in mixing [14] \( A_{CP} \equiv (N_{++} - N_{--})/(N_{++} + N_{--}) \) is small [13, 16]. Our assumptions comply with this fact through \( A_{CP} = 0 \). Secondly, the general case, i.e. without the assumptions (27) and with \( R \) and \( A_{CP} \) as experimental input, has more parameters and the conceptual difficulty that we fake CP violation by the matrix \( S \), which we would like to avoid. We will see in the next section that our simplified scenario is general enough to illustrate in a clear way the point we want to make.

4 How far is the \( B^0\bar{B}^0 \) system from total decoherence?

Under conditions Eq. (27) the formulas for the number of dilepton events are very simple. Defining \( B \equiv b_+ = b_- \) we obtain

\[
N_{++} = N_{--} = \frac{B^2}{4\Gamma^2} \left\{ \frac{x^2}{1 + x^2} + \frac{\zeta Z}{(1 + x^2)^2} \right\}
\]

(28)

\[
N_{+-} = N_{-+} = \frac{B^2}{4\Gamma^2} \left\{ \frac{2 + x^2}{1 + x^2} - \frac{\zeta Z}{(1 + x^2)^2} \right\}
\]

(29)

with

\[
Z = \left| 2ab^* + ix(e^{-i\delta}a^2 + e^{i\delta}(b^*)^2) \right|^2.
\]

(30)

For the ratio of the the number of like-sign dilepton events to opposite-sign dilepton events we find

\[
R = \frac{N_{++} + N_{--}}{N_{+-} + N_{-+}} = \frac{x^2 + \zeta Z}{2 + x^2 - \zeta Z}.
\]

(31)

For \( \zeta = 0 \) we have the familiar quantum-mechanical result neglecting \( y \) and CP violation in \( B^0\bar{B}^0 \) mixing.

The relevant properties of \( Z \) are summarized in the following lemma.
Lemma:

1. $Z$ is a function of $S$ and depends on two variables which can be taken as $|a| = \sqrt{1 - |b|^2}$ and the phase of $e^{-i\delta}ab$. Thus, from the four parameters of a general unitary $2 \times 2$ matrix only two are relevant in the problem under discussion.

2. The range of $Z$ is given by the interval $0 \leq Z \leq 1 + x^2$.

Proof: We parameterize $a$ and $b$ by

$$a = \cos \omega e^{i(\delta + \gamma)}, \quad b = \sin \omega e^{i(\rho - \gamma)}$$   \hspace{1cm} (32)

and obtain

$$Z = (\sin 2\omega - x \cos 2\omega \sin \rho)^2 + x^2 \cos^2 \rho.$$   \hspace{1cm} (33)

Thus $Z$ is a function of $\omega$ and $\rho = \arg(e^{-i\delta}ab)$, which demonstrates the first statement. With Eq. (33) we easily check that

$$1 + x^2 - Z = (\cos 2\omega + x \sin 2\omega \sin \rho)^2.$$   \hspace{1cm} (34)

As a consequence we obtain $Z \leq 1 + x^2$. The right-hand side of Eq. (34) is a continuous function of $\omega$ and $\rho$. Clearly, its minimum is 0 and one can easily show that the maximum is given by $1 + x^2$. Therefore, the range of $Z$ is specified by the interval in the second statement of the lemma. $\square$

The analysis in our previous work Ref. [10] corresponds to $Z = x^2$ whereas the case discussed in Ref. [11] corresponds to the maximal value $Z = 1 + x^2$. Eqs. (31) and (33) represent the main result of this work. The latter formula shows that $R$ depends only on the product $\zeta Z$ which is uniquely determined by measurements of $R$ and $x$. On the other hand, varying the matrix $S$, though only in the set of unitary matrices, the function $Z$ varies in the interval given by the lemma. Clearly, using $R$ of Eq. (31) and the experimental values $\bar{R}$ and $\bar{x}$ of $R$ and $x$, respectively, for a determination of $\zeta$, the result will strongly depend on the basis chosen in the $B^0 - \bar{B}^0$ space represented by the matrix $S$.

In the following we take the values $\bar{R} = 0.189 \pm 0.044$ and $\bar{x} = 0.74 \pm 0.05$ as given in Ref. [10]. $\bar{R}$ has been determined from the results of the CLEO [16] and ARGUS [17] experiments by simply adding the squares of the statistical and systematic errors for each experiment and using the law of combination of errors. The same method has been applied to determine $\bar{x}$ from the $\Delta m$ results of the four LEP experiments [18] and the $B^0$ lifetime which is found in Ref. [19].

In order to assess the validity of quantum mechanics, there are two obvious measures associated with the parameter $\zeta$. With $\bar{R} = \bar{R}_0 \pm \Delta \bar{R}$ and $\bar{x} = \bar{x}_0 \pm \Delta \bar{x}$ as experimental input we can calculate $\bar{\zeta} = \bar{\zeta}_0 \pm \Delta \bar{\zeta}$. Then the two measures are given by the distance of $\bar{\zeta}_0$ from 0 and from 1, respectively, each expressed in units of the one standard deviation $\Delta \bar{\zeta}$. It is easy to check numerically that 0 is within the one standard deviation interval of $\bar{\zeta}$ for the whole range of $Z$ (see also Fig. 1). The distance of $\bar{\zeta}_0$ from 1 shows how far the $B^0 \bar{B}^0$
system is from complete decoherence. In this context we have two extreme cases. If we chose $S$ such that $Z$ is very close or equal to 0 then in this picture complete decoherence is not excluded. On the other hand, for $Z = 1 + x^2$ chosen in Ref. [11] complete decoherence is excluded at around 8 standard deviations. Thus, as was noticed in Ref. [11], the question how far the $B^0\bar{B}^0$ system is from total decoherence has no unique answer. It depends on the basis choice represented by the matrix $S$. What is significant, however, is the existence of a basis where the $B^0\bar{B}^0$ system is far away from total decoherence and where the corresponding $\zeta$ is close to 0 in agreement with quantum mechanics. We have shown that the “best basis” in this respect is given by the mass eigenstates $B_H$, $B_L$ which was chosen in Ref. [11].

In Fig. 1 we have plotted $R$ (Eq. (31)) as a function of $\zeta$. The three thin solid lines which are not so steep represent $R$ with $Z = x^2$ taking into account the three values $\bar{x}_0 + \Delta \bar{x}$ (upper curve), $\bar{x}_0$ (middle curve) and $\bar{x}_0 - \Delta \bar{x}$ (lower curve). The three steep thin lines are the analogous curves for $Z = 1 + x^2$. The thick horizontal lines depict $\bar{R}$ with its one standard deviations. For each $Z$ the mean value $\bar{\zeta}_0$ is found at the intersections of the middle curves and the middle horizontal line, whereas $\Delta \bar{\zeta}$ is approximately given by the distance of $\bar{\zeta}_0$ from the $\zeta$ where the lower (upper) curves cut the upper (lower) horizontal line. The figure nicely illustrates Eq. (31). As a function of $\zeta$ the ratio $R$ is steeper if $Z$ is larger. Therefore, for larger $Z$ the allowed range of $\bar{\zeta}$ is smaller and closer to 0. We can read off from the figure that the mean values $\bar{\zeta}_0$ of $\bar{\zeta}$ are less than a standard deviation away from 0 for $Z = x^2$ and $Z = 1 + x^2$. We can also check from the figure that the distance of $\bar{\zeta}_0$ from 1 is nearly 8 standard deviations for the maximal $Z = 1 + x^2$.

5 Conclusions

In this paper we have used the ratio $R = (\# \text{ like-sign dilepton events})/(\# \text{ opposite-sign dilepton events})$ as an observable testing for interference effects in the $B^0\bar{B}^0$ state generated by the $\Upsilon(4S)$ decay. Quantum-mechanically, this $B^0\bar{B}^0$ state is entangled and extends over macroscopic distances. On average, $B^0$ and $\bar{B}^0$ are separated by $3 \times 10^{-2}$ mm at the time when one of the two particles decays. We have shown that already with present data for $R$ and the $B_H$–$B_L$ mass difference interesting conclusions can be drawn on the decoherence parameter $\zeta$ introduced to parameterize deviations from the interference term given by quantum mechanics. Depending on the basis choice in the $B^0$–$\bar{B}^0$ space and under certain simplifying assumptions, the distance of the experimentally determined mean value $\bar{\zeta}_0$ from $\zeta = 1$, which represents total decoherence, ranges from 0 to 8 standard deviations. The maximal distance is obtained by using the basis $|B_H\rangle$, $|B_L\rangle$ of mass eigenstates.

Though this indefiniteness may seem confusing at the moment it is merely the consequence of the freedom in quantum mechanics to choose a basis in the $B^0$–$\bar{B}^0$ space, which eventually leads to different interference terms for different basis choices. Parameters $\zeta$ modifying different interference terms are intrinsically different and adopt therefore in general different values when confronted with experimental data. However, with respect
to testing quantum mechanics it is essential that there exist bases such that the decoherence parameter is significantly distant from 1 and at the same time close to 0 \[10, 11\]. From this point of view the presence of an interference term in semileptonic $B^0\bar{B}^0$ decays according to quantum mechanics is quite well confirmed and future improvement of the measurements of $R$ and the $B_H-B_L$ mass difference allow to expect even more precise confirmation of interference effects over macroscopic distances for massive particles.

At present no consistent theory extending quantum mechanics is known and the introduction of the decoherence parameter might represent a devious path leading away from consistent extensions. In any case, this procedure is rather arbitrary and the analysis of this work corroborates this impression through the strong basis dependence of the modification of the quantum-mechanical expressions. Thereby even CP violation in $B^0\bar{B}^0$ mixing can be simulated. These features certainly exist in $K^0\bar{K}^0$ and similar systems as well if they are analogously modified.
References

[1] J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).

[2] R.A. Bertlmann, Found. Phys. 20 (1990) 1191.

[3] A. Aspect, J. Dalibard and G. Roger, Phys. Rev. Lett. 49 (1982) 1840.

[4] P.G. Kwiat, K. Mattle, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. 75 (1995) 4337.

[5] J. Six, Phys. Lett. B 114 (1982) 200.

[6] F. Selleri, Lett. Nuovo Cim. 36 (1983) 521.

[7] P.H. Eberhard, in *The Second DaΦne Physics Handbook*, Vol I, p. 99, edited by L. Maiani, G. Pancheri and N. Paver (SIS–Pubblicazioni dei Laboratori di Frascati, Italy, 1995).

[8] A. Datta and D. Home, Phys. Lett. A 119 (1986) 3.

[9] B. Kayser and L. Stodolsky, Phys. Lett. B 359 (1996) 343.

[10] R.A. Bertlmann and W. Grimus, Phys. Lett. B 392 (1997) 426.

[11] G.V. Dass and K.V.L. Sarma, preprint TIFR/TH/97-37 ([hep-ph/9709249](https://arxiv.org/abs/hep-ph/9709249)).

[12] A.B. Carter and A.I. Sanda, Phys. Rev. D 23 (1981) 1567; I.I. Bigi and A.I. Sanda, Nucl. Phys. B 193 (1981) 85.

[13] A. Pais and S.B. Treiman, Phys. Rev. D 12 (1975) 2744.

[14] L.B. Okun, V.I. Zakharov and B.M. Pontecorvo, Lett. Nuovo Cim. 13 (1975) 218.

[15] F. Abe et al. (CDF Collaboration), Phys. Rev. D 55 (1997) 2546.

[16] J. Bartelt et al. (CLEO Collaboration), Phys. Rev. Lett. 71 (1993) 1680.

[17] H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. B 324 (1994) 249.

[18] D. Busculic et al. (ALEPH Collaboration), Z. Phys. C 75 (1997) 397; P. Abreu et al. (DELPHI Collaboration), Z. Phys. C 72 (1996) 17; M. Acciarri et al. (L3 Collaboration), Phys. Lett. B 383 (1996) 487; G. Alexander et al. (OPAL Collaboration), Z. Phys. C 72 (1996) 377.

[19] Review of Particle Physics, Phys. Rev. D 54 (1996) 1.
Figure 1: The ratio $R$ (Eq. (31)) as a function of the decoherence parameter $\zeta$ for $Z = 1 + x^2$ [11], the maximal $Z$, and $Z = x^2$ [10]. For each $Z$ the three thin solid lines correspond to three different values of $x$ given by $\bar{x} = 0.74 \pm 0.05$, derived from the experimental results in Ref. [18] and the $B^0$ lifetime taken from Ref. [19]. The value of the dilepton ratio $\bar{R} = 0.189 \pm 0.044$ obtained by the combined experimental results of Refs. [16] and [17] is shown by the thick horizontal lines.