MAGNETIC SCREENING IN ACCRETING NEUTRON STARS

ANDREW CUMMING, ELLEN ZWEBEL, and LARS BILDSTEN

Institute for Theoretical Physics, Kohn Hall, University of California, Santa Barbara, CA 93106; cumming@itp.ucsb.edu, zweibel@solarz.colorado.edu, bildsten@itp.ucsb.edu

Received 2001 February 8; accepted 2001 April 16

ABSTRACT

We investigate whether the magnetic field of an accreting neutron star may be diamagnetically screened by the accreted matter. We assume the freshly accumulated material is unmagnetized and calculate the rate at which the intrinsic stellar magnetic flux is transported into it via Ohmic diffusion from below. For very high accretion rates $M$ (larger than the Eddington rate $M_{\text{Edd}}$), Brown & Bildsten have shown that the liquid ocean and outer crust of the neutron star are built up on a timescale much shorter than the Ohmic penetration time. We extend their work to lower accretion rates and calculate the resulting screening of the magnetic field. We find that the Ohmic diffusion and accretion timescales are equal for $M \approx 0.1 M_{\text{Edd}}$. We calculate the one-dimensional steady state magnetic field profiles and show that the magnetic field strength decreases as one moves up through the outer crust and ocean by $n$ orders of magnitude, where $n \approx M/0.02 M_{\text{Edd}}$. We show that these profiles are unstable to buoyancy instabilities when $B \gtrsim 10^{10} - 10^{11}$ G in the ocean, providing a new limit on the strength of the buried field. Our results have interesting implications for the weakly magnetic neutron stars in low-mass X-ray binaries. We find that magnetic screening is ineffective for $M < 10^{-2} M_{\text{Edd}}$, so that, no matter how the accreted material joins onto the star, the underlying stellar field should always be evident. This is consistent with the fact that the only known persistently pulsing accreting X-ray millisecond pulsar, SAX J1808.4 – 3658, has an unusually low accretion rate of $M \approx 10^{-3} M_{\text{Edd}}$. Although the simplified magnetic model we adopt here does not allow us to definitively say so, we are led to suggest that perhaps most of the weakly magnetic neutron stars in low-mass X-ray binaries have a screened magnetic field, explaining the lack of persistent pulsations from these sources. If screened, then the underlying field will emerge after accretion halts, on a timescale of only 100–1000 yr, set by the Ohmic diffusion time across the outer crust.

Subject headings: accretion, accretion disks — magnetic fields — pulsars: individual (SAX J1808.4 – 3658) — stars: neutron — X-rays: binaries

1. INTRODUCTION

Accreting neutron stars fall generally into two classes (see White, Nagase, & Parmar 1995). Those in high-mass X-ray binaries (HMXBs) are X-ray pulsars, with dipole magnetic fields $B \sim 10^{12}$ G, large enough to disrupt the accretion flow and channel it onto the polar caps. Most of the neutron stars in low-mass X-ray binaries (LMXBs), however, show no direct evidence for a magnetic field ($B \lesssim 10^{10}$ G). The accretion disk is believed to extend almost to the surface of the neutron star, allowing slow accretion onto the stellar surface, allowing these neutron stars to be spun up to millisecond periods by accretion. Thus LMXB neutron stars are favored candidates for the progenitors of millisecond radio pulsars (Bhattacharya 1995) and, as such, are expected to have magnetic fields $B \sim 10^{8} - 10^{9}$ G.

Only one weakly magnetic neutron star so far has shown a pulse indicative of magnetically controlled accretion. The newly discovered transient SAX J1808.4 – 3658 shows persistent 401 Hz pulsations, making it the first accretion-powered millisecond X-ray pulsar (Wijnands & van der Klis 1998; Chakrabarty & Morgan 1998). Interpreting the pulsations as disruption of the accretion flow by a magneto-sphere leads to an estimated magnetic field strength of $10^{8} - 10^{9}$ G (Psaltis & Chakrabarty 1999), suggesting that this source will become a millisecond radio pulsar once accretion ceases (Bildsten & Chakrabarty 2001).

The origin of such low external magnetic fields $B \lesssim 10^{10}$ G is not understood but is believed to be directly connected with accretion of matter onto the neutron star. This suggestion is based on the observations that isolated neutron stars show no evidence for dramatic field decay and that low field radio pulsars occur almost exclusively in binaries (Bhattacharya & Srinivasan 1995). Utilizing binary evolution calculations, Taam & van den Heuvel (1986) studied a number of systems, finding that magnetic field strength decreases with the amount of accreted matter, although Wijers (1997) argues against this interpretation. A number of different mechanisms for achieving this reduction in magnetic field strength have been proposed (see Bhattacharya & Srinivasan 1995 for a review).

One possibility that could explain the low dipole fields and the lack of pulsations from these sources is that the external dipole magnetic field is “buried” or “screened” by the accreted matter. Bisnovatyi-Kogan & Komberg (1974) suggested that pulsars in binaries may have low magnetic fields because of this effect. Romani (1990, 1995) outlined a picture in which matter initially accretes onto the polar cap, eventually spreads under the weight of the accumulated matter, and advects the magnetic field inward. He proposed that the magnetic flux would be trapped in the crust and screened by further accretion of diamagnetic material.

In this paper, we test this possibility. We consider accretion of unmagnetized matter onto a neutron star with a buried magnetic field and ask whether the field remains buried or whether the newly accreted material becomes magnetized faster than the rate at which it is accreted.
allowing the intrinsic stellar field to “leak out.” We study the competition between downward compression and advection by accretion at the local rate, $\dot{m}$, and upward Ohmic diffusion. We carry out this calculation for the ocean and outer crust of the neutron star that are in steady state once accretion has been occurring for an extended period. We show that, in steady state, the magnetic field strength decreases through the outer crust and ocean by $n \approx \dot{m}/0.02 \dot{m}_{\text{Edd}}$ orders of magnitude, where $\dot{m}_{\text{Edd}}$ is the Eddington accretion rate. We show that these profiles are unstable to buoyancy instabilities in the ocean when $B \gtrsim 10^{10} - 10^{11}$ G, giving a new limit to the strength of any buried field.

Our results are not directly relevant to the high magnetic field accreting X-ray pulsars, in which accretion occurs along the magnetic field lines. However, our results have interesting implications for the weakly magnetic neutron stars. We show that, in steady state, the magnetic field and accretion geometry we adopt in this paper, cannot definitively state that screening occurs at higher accretion rates. However, it is interesting that consideration of both the observations and evolutionary state of SAX J1808.4−3658 (Chakrabarty & Morgan 1998; Bildsten & Chakrabarty 2001) makes it clear that the time-averaged accretion rate is unusually low, $\dot{m} \approx 10^{-3} \dot{m}_{\text{Edd}}$. That SAX J1808.4−3658 shows direct evidence of its magnetic field in its persistent emission is consistent with this low accretion rate, below the range of accretion rates where screening can occur. Further, the fact that it is the only weakly magnetic neutron star so far to show a persistent pulsation leads us to suggest that perhaps most neutron stars in LMXBs have a screened magnetic field, which is a new explanation for the lack of persistent pulsations from these sources.

Previous authors have considered the competition between accretion and Ohmic diffusion but in different contexts. Brown & Bildsten (1998) compared the Ohmic diffusion and advection times in the atmosphere, ocean, and crust of the polar cap of an accreting X-ray pulsar. In this case, the accreted matter arrives on the star already threaded by the magnetic field and may spread laterally if Ohmic diffusion is rapid enough. Considering magnetic field evolution in accreting neutron stars, Kônar & Bhattacharya (1997) assumed that the currents supporting the magnetic field were confined to the crust and calculated the competing effects of Ohmic dissipation and advection of magnetic flux into denser regions with longer dissipation times (and eventually into the superconducting core).

Another process that acts to transport magnetic flux outward is thermomagnetic drift (e.g., Urpin & Yakovlev 1980; Geppert & Urpin 1994). In this paper, we consider the liquid ocean and solid crust of the neutron star, for which we find that thermomagnetic drift is unimportant; the magnetic field profile is set by the competition between Ohmic diffusion and accretion. However, in the H/He layer which overlies the ocean, we find that thermomagnetic drift dominates the transport of magnetic field. The evolution of the magnetic field in this layer, which periodically undergoes a thermonuclear instability giving rise to type I X-ray bursts, will be discussed in a separate paper.

We start by describing the thermal structure of the ocean and crust and what sets the electrical conductivity there (§ 2). In § 3, we discuss the evolution of the magnetic field under the joint action of accretion and Ohmic diffusion. In § 4, we evaluate the characteristic timescales for these processes, using simple analytic estimates to understand our detailed numerical results. In § 5, we compute steady state magnetic profiles. We show that the magnetic field decreases with height in the ocean by an amount that depends strongly on the local accretion rate, $\dot{m}$. In § 6, we consider the stability of the steady state profiles to magnetic buoyancy instabilities. We summarize our results and conclude in § 7 with the implications for steadily and transiently accreting neutron stars.

2. MICROPHYSICS AND STRUCTURE OF THE OCEAN AND CRUST

In this section, we outline the structure of the outer layers of the neutron star and describe how we calculate the electrical conductivity.

2.1. Thermal Structure

Most neutron stars in LMXBs accrete hydrogen and helium from their companions at rates $M \sim 10^{-11} - 10^{-8}$ $M_\odot$ yr$^{-1}$. Since we are concerned with the outermost layers of thickness $\lesssim 100$ m—small compared to the radius—we adopt plane-parallel coordinates and measure the accretion rate as a local rate $\dot{m}$ (units g cm$^{-2}$ s$^{-1}$). The local Eddington accretion rate is $\dot{m}_{\text{Edd}} = 2m_p c/(1 + X) R \sigma_T$, where $\sigma_T$ is the Thomson scattering cross section, $m_p$ is the proton mass, $c$ is the speed of light, $R$ is the stellar radius, and $X$ is the hydrogen mass fraction. We use the Eddington accretion rate for solar composition ($X = 0.71$) and $R = 10$ km, $\dot{m}_{\text{Edd}} = 8.8 \times 10^{14}$ g cm$^{-2}$ s$^{-1}$, as our basic unit for the local accretion rate. For a 10 km neutron star, this corresponds to a global rate $M_{\text{Edd}} = 1.7 \times 10^{-8}$ $M_\odot$ yr$^{-1}$.

The neutron star atmosphere is in hydrostatic balance as the accreted hydrogen and helium accumulate because the downward flow speed $v \approx \dot{m}/\rho$ is much less than the sound speed $c_s$. The pressure varies with height as $dp/dz = -\rho g$, where the gravitational acceleration $g$ is constant in the thin envelope. We assume in this paper that the magnetic field does not play a role in hydrostatic balance, i.e., $d(B^2 / 8\pi)/dz \approx dp/dz$ (we check this assumption in § 6 when we consider buoyancy instabilities). A useful variable is the column depth $y$ (units g cm$^{-2}$), defined by $dy \equiv -pdz$, giving $P = gy$. As the matter accumulates, a given fluid element moves to greater and greater column depth. We take $g_{14} \equiv g/10^{14}$ cm s$^{-2} = 1.9$, the Newtonian gravitational acceleration for a $M = 1.4$ $M_\odot$ and $R = 10$ km neutron star.

The accreted H and He burn at $\approx 10^5 - 10^9$ g cm$^{-2}$ and temperature $T \approx 2 - 5 \times 10^8$ K, producing heavy elements that form the underlying ocean and crust. The microphysics of these layers has been discussed by Bildsten & Cutler (1995) and Brown & Bildsten (1998). The pressure in the ocean and outer crust is provided mostly by degenerate electrons. For the ions, we incorporate Coulomb corrections to the equation of state using the fit of Farouki & Hamaguchi (1993). The crystallization point is determined by $\Gamma = (Ze^2 k_B T_a)/a$, where $a$ is the interion spacing given by $4\pi n_i a_n=1$, $Ze$ is the ionic charge, $n_i$ is the ion number density (we assume only one species of ion is present). For typical conditions at the top of the ocean, this is

$$\Gamma = 12 \rho_s^{1/3} \frac{2}{7a} \left( \frac{Z}{30} \right)^{5/3} \left( \frac{2Ze^2}{A} \right)^{1/3},$$  

where $\rho_s$ is the ion number density in the ocean, $a$ is the interion spacing, and $Z$ is the ionic charge.

where \( \rho_s \equiv \rho/10^5 \text{ g cm}^{-3} \), \( T_8 \equiv T/10^8 \text{ K} \), and the mass of each nucleus is \( Am_e \). The ions solidify when \( \Gamma = \Gamma_m \approx 173 \) (Farouki & Hamaguchi 1993), at a depth

\[
y_{ei} \approx 2 \times 10^{11} \text{ g cm}^{-2} \left( \frac{T_8}{5} \right)^4 \left( \frac{Z}{30} \right)^{-20/3} \left( \frac{\Gamma_m}{173} \right)^4 \times \left( \frac{1.9}{\delta_{14}} \right) \left( \frac{\Gamma_m}{173} \right)^4,
\]

where we assume fully relativistic electrons and use the \( T = 0 \) Fermi energy.

We find the thermal structure of the ocean and crust by integrating the heat equation,

\[
\frac{dT}{dy} = \frac{3kF}{4acT^3},
\]

where \( F \) is the outward heat flux and \( \kappa \) is the opacity. The heat flux is set by heat released from compression of matter by accretion and from pycnonuclear reactions and electron captures deep in the crust (Brown & Bildsten 1998). For our models, we assume a constant heat flux \( F = 150 \text{ keV per nucleon} \). The opacity \( \kappa \) is calculated as described by Schatz et al. (1999) and Brown (2000) for the crust (see discussion in § 2.2). The temperature and pressure at the top of the ocean are set by H/He burning in the upper atmosphere. For accretion rates \( \dot{m} < \dot{m}_\text{edd} \), the H/He burns unstably, and we take the temperature and column depth at the top of the ocean to be the conditions at He ignition, as calculated by Cumming & Bildsten (2000). For \( \dot{m} = \dot{m}_\text{edd} \), we use the steady state burning model of Schatz et al. (1999), giving \( y = 10^{14} \text{ g cm}^{-2} \) and \( T = 5 \times 10^9 \text{ K} \) at the top of the ocean. We take the composition of the ocean and crust to be nuclei with \( Z = 30 \) and \( A = 60 \). We arbitrarily stop our integrations at \( y = 10^{14} \text{ g cm}^{-2} \), thus we include only the outer crust, above neutron drip.

The upper panel of Figure 1 shows the temperature profiles for \( \dot{m} = 0.01, 0.1, \) and \( 1 \dot{m}_\text{edd} \). For the 0.01 and 0.1 \( \dot{m}_\text{edd} \) models, we mark the liquid-solid interface with a dot (the \( \dot{m}_\text{edd} \) model is still liquid at \( y = 10^{14} \text{ g cm}^{-2} \)). In the crust, we show two models for each accretion rate, each with a different choice for the thermal conductivity: phonon scattering or electron-ion scattering (see § 2.2 for discussion of conductivity in the crust).

### 2.2. Electrical Conductivity

In the relaxation time approximation, the electrical conductivity is

\[
\sigma = n_e e^2/m_e v_e,
\]

where \( n_e \) is the electron number density, \( m_e \) is the effective electron mass, \( \mu_e \) is the mean molecular weight per electron, and \( v_e \) is the electron collision frequency. The lower panel of Figure 1 shows the conductivity as a function of depth.

In the liquid ocean, electron-ion collisions set the electrical conductivity. We use the results of Yakovlev & Urpin (1980), who give

\[
\sigma = \frac{8.5 \times 10^{21} \text{ s}^{-1}}{ZA_{ei}} \frac{x^3}{1 + x^2},
\]  

where \( A_{ei} \approx 1 \) is the Coulomb logarithm, and \( x \equiv p_F/m_e c \) measures the electron Fermi momentum \( p_F \).

**Fig. 1.—** Temperature and conductivity profiles in the ocean and outer crust. We take the temperature and column depth at the top of the ocean from the X-ray burst ignition models of Cumming & Bildsten (2000) for \( \dot{m} = 0.01 \) (solid curves) and 0.1 \( \dot{m}_\text{edd} \) (dotted curves) and from the steadily burning model of Schatz et al. (1999) for \( \dot{m} = \dot{m}_\text{edd} \) (dashed curves). We assume a single species of nuclei with \( A = 60 \) and \( Z = 30 \) and assume a constant heat flux of \( F = 150 \text{ keV per nucleon} \). We indicate the ocean-crust boundary with a filled circle (the \( \dot{m}_\text{edd} \) model remains liquid for \( y < 10^{14} \text{ g cm}^{-2} \)). For 0.01 and 0.1 \( \dot{m}_\text{edd} \), the two different curves beyond the filled circle are for either electron-ion scattering or phonon scattering in the crust.

The conductivity in the solid crust depends on the level of impurity. Itoh & Kohyama (1993) give the electron-impurity collision frequency as

\[
v_{ei} = 1.75 \times 10^{16} \text{ s}^{-1} \frac{Q}{\langle X \rangle} \Lambda_{ei}(1 + x^2)^{1/2},
\]

where \( Q = \sum Y_i(Z_i - \langle Z \rangle)^2/\sum Y_i \) is the impurity parameter, \( \langle Z \rangle = \sum Y_i Z_i/\sum Y_i \) is the average charge (the number abundance of species \( i \) is \( Y_i = X_i/A_i \), where \( X_i \) is the mass fraction), and the Coulomb logarithm is of order unity. The other contribution to the conductivity is scattering with phonons, for which we adopt the results of Baiko & Yakovlev (1995). For temperatures much greater than the Debye temperature \( [T \gg \Theta_B = 2.4 \times 10^9 \text{ K} \times 3/2(2Z/A)] \), they find

\[
v_{ep} = \frac{2k_B T}{h} 13\Lambda_{ep} = 1.24 \times 10^{18} \text{ s}^{-1} T_8 \Lambda_{ep},
\]

where \( z = 1/137 \) is the fine-structure constant, and \( \Lambda_{ep} \) is of order unity. The ratio of the impurity and phonon scattering frequencies is

\[
\frac{v_{ei}}{v_{ep}} \approx 10^{-2} \frac{3}{T_8} \left( \frac{\Lambda_{ei}}{\Lambda_{ep}} \right),
\]
Previous studies have assumed a low level of impurity, \( Q \ll 1 \), as might be expected for a primordial crust. In this case, phonon scattering dominates the conductivity. However, recent calculations of the products of \( H/He \) burning in the atmosphere show that the impurity level is probably much greater. Schatz et al. (1999) calculated the ashes of steady state \( H/He \) burning at local accretion rates \( \dot{m} \gtrsim \dot{m}_{\text{edd}} \), and found \( \langle Z \rangle = 24 \) and \( Q \approx 100 \). At such high impurity factors, it is not clear whether equation (5) is correct. Brown (2000) suggests that when \( Q \approx \langle Z \rangle^2 \), the appropriate conductivity is that for electron-ion scattering in the liquid.

In this paper, we show results for two limiting cases for the crust: a pure crust with only phonon scattering and an impure crust with electron-ion scattering calculated as for the liquid. The conductivity in each case is shown in the lower panel of Figure 1. For each accretion rate, we plot the two solutions for the crust: for electron-ion scattering, the conductivity is continuous across the liquid-solid boundary; for phonon scattering, the conductivity changes discontinuously.\(^3\) The difference in conductivity between these two cases is roughly a factor of 5. In §4, we show that the impurity level has an important effect on magnetic field evolution in the outer crust.

3. EVOLUTION OF THE MAGNETIC FIELD

We consider a simplified geometry for the magnetic field, a plane-parallel model in which the magnetic field \( B \) depends only on depth. In this case, the vertical component of the magnetic field must be constant since \( \mathbf{V} \cdot \mathbf{B} = 0 \), and we set it equal to zero since we assume the accreted matter is unmagnetized. This leaves us with a purely horizontal field, \( \mathbf{B}(z) = B(z) \hat{e}_z \) (we align our \( x \)-axis with the horizontal component of the field).

The magnetic field evolves according to Faraday’s law

\[
\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{\nabla} \times \mathbf{E},
\]

where the electric field \( \mathbf{E} \) is given by Ohm’s law,

\[
\mathbf{E} = -\frac{\mathbf{v} \times \mathbf{B}}{c} + \frac{\mathbf{J}}{\sigma}.
\]

The electric current is

\[
\mathbf{J} = \frac{c}{4\pi} \mathbf{\nabla} \times \mathbf{B} = \hat{e}_x \frac{\partial B}{\partial z},
\]

for a given profile \( B(z) \), and the downward flow velocity is given by continuity as \( \mathbf{v} = -v(z) \hat{e}_z = -(\eta/\rho) \hat{e}_z \) (we define \( v \) so that it is a positive quantity for the downward flow). Thus \( \mathbf{E} = E_y \hat{e}_y \), where

\[
E_y = \frac{vB}{c} + \frac{c}{4\pi \sigma} \frac{\partial B}{\partial z},
\]

and equation (8) reduces to its \( \hat{x} \) component

\[
\frac{\partial B}{\partial t} = \frac{\partial}{\partial z} \left[ vB + \eta \frac{\partial B}{\partial z} \right],
\]

where \( \eta = c^2/4\pi \sigma \) is the magnetic diffusivity.

It is instructive to rewrite equation (12) as

\[
\frac{\rho D}{\partial t} \frac{B}{\rho} = \frac{\partial}{\partial z} \left( \eta \frac{\partial B}{\partial z} \right),
\]

where \( D/\partial t = \partial/\partial t + \mathbf{v} \cdot \nabla \) is the advective derivative and we have used the continuity equation \( \eta \partial P/\partial r = -\mathbf{v} \cdot \nabla \). Equation (13) clearly illustrates the different processes acting to change the magnitude of \( B \). The left hand side describes advection and compression by the downward flow; the right hand side describes Ohmic diffusion. Without diffusion, \( B \) in a given fluid element would grow \( \propto \rho \) as it was compressed.

In this paper, we calculate only steady state solutions of equation (12). Since we assume that the incoming material is unmagnetized, we set \( E_y = 0 \), and equation (12) reduces to

\[
\eta \frac{dB}{dz} = -vB.
\]

We rewrite this so that pressure is the independent variable; this makes the competition between advection and diffusion more transparent. Using hydrostatic balance, and introducing the pressure scale height \( H \equiv y/\rho = -(\ln P/\partial z)^{-1} \), equation (14) becomes

\[
\frac{d \ln B}{d \ln P} = \left( \frac{H^2}{\eta} \right) \left( \frac{\partial P}{\partial y} \right) = \frac{t_{\text{diff}}}{t_{\text{accr}}},
\]

where we have defined

\[
t_{\text{diff}} \equiv \frac{H^2}{\eta},
\]

the characteristic Ohmic diffusion time across a scale height, and

\[
t_{\text{accr}} \equiv \frac{y}{\dot{m}},
\]

the characteristic accretion flow time across a pressure scale height. Equation (15) shows that, in steady state, \( t_{\text{diff}}/t_{\text{accr}} \) measures the ratio of pressure scale height to magnetic scale height. For example, if \( t_{\text{diff}} > t_{\text{accr}} \), a small magnetic scale height is required for Ohmic diffusion to balance accretion.

The magnetic profile is given by integrating equation (15) since all the quantities on the right-hand side are known functions of pressure (or height). We find

\[
B(P) = B(\rho_0) S,
\]

where \( \rho_0 \) is a reference pressure, and

\[
S \equiv \exp \left[ -\int_P^{P_0} d\ln P \left( \frac{t_{\text{diff}}}{t_{\text{accr}}} \right) \right]
\]

is the amount by which the magnetic field changes because of screening from the overlying matter. We shall refer to the quantity \( S \) as the magnetic screening factor.

The Ohmic diffusion time is independent of \( B \) (see Goldreich & Reisenegger 1992 for an interesting discussion). Thus equation (15) is linear in \( B \), giving the profile of the magnetic field independent of its overall amplitude. In the field burial picture, the amplitude of \( B \) in the crust is fixed by conserving the magnetic flux in the original dipole field. We do not calculate the details of the burial process in this paper; thus, we treat the overall normalization of the magnetic field in the crust as a free parameter.
For our assumed field geometry, the Hall term in Ohm’s law exactly vanishes. More generally, the magnitude of the Hall term depends on the ratio of the electron cyclotron frequency, \( \omega_c = B e / m_e c \), to the electron collision frequency, \( v_e = \tau^{-1} \). For a typical value of \( v_e \), this is

\[
\omega_c \tau = \left( \frac{B}{5.6 \times 10^8 G} \right) \left( \frac{10^{16} \, s^{-1}}{v_e} \right) \left( \frac{m_e}{m_e} \right),
\]

where \( m_e = E_e / c^2 \) is the effective mass of the electron. Typically, \( \omega_c \tau > 1 \) requires \( B \gtrsim 10^{11} \) G in the ocean and crust; we show in § 6 that such large fields are buoyantly unstable. In addition, we neglect terms in Ohm’s law caused by thermomagnetic effects (e.g., Urpin & Yakovlev 1980); these are unimportant in the ocean and outer crust.

4. ACCRETION VERSUS OHMIC DIFFUSION

In § 3, we showed that the steady state magnetic profile depends directly on the ratio of accretion and Ohmic diffusion timescales. In this section, we evaluate this ratio before presenting the steady state profiles in § 5. Figure 2 shows the results of our detailed calculations of the accretion and diffusion timescales as a function of depth, and Figure 3 shows \( t_{\text{diff}} / t_{\text{accr}} \). We find that the diffusion time is longer than accretion time for \( m \gtrsim 0.1 \, m_{\text{Edd}} \). We now use simple analytic estimates to understand these results, discussing the ocean (§ 4.1) and crust (§ 4.2) in turn.

4.1. Ocean

First, we evaluate \( t_{\text{diff}} = 4 \pi \sigma H^2 / e^2 \). The conductivity is given by equation (4), in which we take \( \Lambda_{a} \) to be constant. The degenerate electron pressure is \( P_e = (m_e c^2)^2 f(x) / \)

\[
24\pi^2 (hc)^3 \quad \text{Clayton 1983} \]

or, in terms of column depth \( y = P / g \),

\[
y_{\text{s}} = 3.2 f(x) \left( \frac{1.9}{g_{14}} \right),
\]

where

\[
f(x) = x(2x^2 - 3)(1 + x^2)^{1/2} + 3 \sinh^{-1} x \quad \text{(Clayton 1983)}
\]

and \( y_{\text{s}} \equiv y / 10^8 \) g cm\(^{-2} \). We write the density as \( \rho = \mu_e n_e m_p \), giving \( \rho_s = 9.7 \mu_e x^3 \). The scale height \( H = y / \rho \) is thus

\[
H = 170 \, \text{cm} \quad f(x) \left( \frac{1.9}{g_{14}} \right) \left( \frac{2}{\mu_e} \right),
\]

giving

\[
t_{\text{diff}} = 10^5 \, \text{s} \quad f(x) \left( \frac{1}{x^3} \right) \left( \frac{30}{Z} \right) \left( \frac{1.9}{g_{14}} \right)^2 \left( \frac{2}{\mu_e} \right)^2.
\]

The accretion timescale is

\[
t_{\text{accr}} = 10^4 \, \text{s} \quad y_{\text{s}} \left( \frac{m}{0.1 m_{\text{Edd}}} \right)^{-1},
\]

giving

\[
\frac{t_{\text{diff}}}{t_{\text{accr}}} = 3 g(x) \lambda_{a1}^{-1} \left( \frac{m}{0.1 m_{\text{Edd}}} \right)^2 \left( \frac{2}{\mu_e} \right)^2 \times \left( \frac{30}{Z} \right) \left( \frac{1.9}{g_{14}} \right),
\]

where \( g(x) \equiv f(x) / x^3(1 + x^2) \). Thus, \( t_{\text{diff}} \approx t_{\text{accr}} \) for \( m \approx 0.1 m_{\text{Edd}} \) in agreement with Figure 3.

In the nonrelativistic and relativistic limits, the functions \( f(x) \) and \( g(x) \) take the limiting values

\[
f(x) = \begin{cases} 8x^5 / 5 & x \ll 1 \\ 2x^4 & x \gg 1 \end{cases},
\]
and
\[ g(x) = \begin{cases} 8x^2/5 & x < 1 \\ 2x & x > 1 \end{cases}. \] (28)

Thus
\[
\left( \frac{t_{\text{diff}}}{t_{\text{accr}}} \right)_{NR} \approx 3y_8^{2/5} \Lambda_\text{el}^{-1} \left( \frac{\dot{m}}{0.1 \dot{m}_{\text{Edd}}} \right) \left( \frac{2}{\mu_e} \right)^2 \times \left( \frac{30}{Z} \right) \left( \frac{1.9}{g_{14}} \right)^{3/5},
\]
for \( x \ll 1 \), and
\[
\left( \frac{t_{\text{diff}}}{t_{\text{accr}}} \right)_R \approx y_1^{1/4} \Lambda^{-1} \left( \frac{\dot{m}}{0.1 \dot{m}_{\text{Edd}}} \right) \left( \frac{2}{\mu_e} \right)^2 \times \left( \frac{30}{Z} \right) \left( \frac{1.9}{g_{14}} \right)^{5/4},
\]
for \( x >> 1 \). These analytic estimates roughly agree with our numerical results shown in Figures 2 and 3 and explain the scaling of \( t_{\text{diff}}/t_{\text{accr}} \) with depth; for small \( x \), \( t_{\text{diff}}/t_{\text{accr}} \) increases with depth, whereas for large \( x \), \( t_{\text{diff}}/t_{\text{accr}} \) decreases with depth. The total screening factor \( S \) through the ocean is thus insensitive to the upper and lower boundaries. The maximum value of \( t_{\text{diff}}/t_{\text{accr}} \) occurs for intermediate values of \( y \), for which the electrons are partially relativistic \((x \sim 1, \rho \approx 10^6 - 10^7 \text{ g cm}^{-3})\).

### 4.2. Crust

We now turn to the solid crust. The behavior of \( t_{\text{diff}}/t_{\text{accr}} \) there is quite different depending on whether we choose electron-ion scattering or phonon scattering. For electron-ion scattering, \( t_{\text{diff}}/t_{\text{accr}} \propto g(x) \ll 1/x \), decreasing with depth, as shown in Figure 3. For phonon scattering,
\[ t_{\text{diff}} = 2 \times 10^5 s \left( \frac{x}{T_9} \right)^{13f_{\text{ep}}/\Lambda_{\text{ep}}} \left( \frac{1.9}{g_{14}} \right)^2 \left( \frac{2}{\mu_e} \right)^2, \] (31)
giving
\[
\left( \frac{t_{\text{diff}}}{t_{\text{accr}}} \right)_R = \frac{3}{T_9} \left( \frac{\dot{m}}{0.1 \dot{m}_{\text{Edd}}} \right) \left( \frac{1.9}{g_{14}} \right) \left( \frac{2}{\mu_e} \right)^2 \left( \frac{13f_{\text{ep}}}{\Lambda_{\text{ep}}} \right). \] (32)

Thus \( t_{\text{diff}}/t_{\text{accr}} \) is constant with depth in this case, in agreement with Figure 3. We see that, as suggested by Schatz et al. (1999), \( t_{\text{diff}} \) is much smaller in an impure crust (electron-ion scattering) than in a primordial crust (phonon scattering).

We do not calculate the conductivity of the inner crust, below neutron drip at \( y \approx 10^{15} - 10^{16} \text{ g cm}^{-2} \). At these depths, Brown & Bildsten (1998) found that \( t_{\text{diff}} < t_{\text{accr}} \) for \( \dot{m} < 5 \dot{m}_{\text{Edd}} \); see their Fig. 9 for a continuation of our Fig. 3 into the inner crust. Thus the inner crust does not give a significant contribution to the screening factor.

### 5. STEADY STATE PROFILES

In § 4, we compared the timescale for Ohmic diffusion with that for accretion. We now use the results of § 3 to compute the steady state magnetic profiles by integrating equation (15).

Figure 4 shows the result for a range of accretion rates. The solid lines are for models with electron-ion scattering in the crust; for 0.01 and 0.1 \( \dot{m}_{\text{Edd}} \), the dotted lines show models with phonon scattering. Rather than fix the overall magnitude of the magnetic field, we plot the ratio of the magnetic field at each depth to the magnetic field at the base \((y = 10^{14} \text{ g cm}^{-2})\). As expected from inspection of equation (15), we find that the magnetic and pressure scale heights are equal for \( \dot{m} \approx 0.1 \dot{m}_{\text{Edd}} \), the accretion rate at which \( t_{\text{diff}} \approx t_{\text{accr}} \) (§ 4.1).

An analytic estimate of the screening factor for the ocean follows straightforwardly from the results of § 4. The ratio \( t_{\text{diff}}/t_{\text{accr}} \) is given by equation (26). We then substitute \( d \ln P = (d \ln f/dx) \, dx \) in equation (19), where \( f(x) \) is given by equation (22) and \( df/dx = 8x^2(1 + x^2)^{1/2} \). As we discussed in § 4.1, most of the contribution to \( S \) comes from \( x \sim 1 \). Integrating, we find
\[
\ln S = 2.4 \left( \frac{\dot{m}}{0.01 \dot{m}_{\text{Edd}}} \right) \left( \frac{30}{Z} \right) \left( \frac{1.9}{g_{14}} \right) \left( 1 + x_7^2 \right)^{1/2} - \left( \frac{1}{1 + x_7^{1/2}} \right), \] (33)
where \( x_7 \) is the value of \( x \) at the top (bottom), and we take \( \mu_e = 2 \) and \( \Lambda_\text{el} = 1 \). Note that equation (33) gives \( \ln S \); the factor by which the magnetic field changes through the atmosphere is the exponential of this value.

Using the scalings of equation (33), and adopting the prefactor from our numerical results, we find that the magnetic field decreases by \( n \) orders of magnitude through the
ocean, where
\[ n \approx \frac{\dot{m}}{0.02 \, m_{edd}} \frac{Z}{30} \frac{g_{14}}{1.9}. \] (34)

The major contribution to \( n \) comes from the ocean. Thus the uncertainty in the conductivity of the crust (electron ion or phonon scattering) changes \( n \) by only a small amount, as shown by comparing the solid and dotted lines in Figure 4.

6. BUOYANCY INSTABILITY

We now investigate the stability of the steady state magnetic profiles to buoyancy instabilities. We first consider interchange and Parker instabilities in § 6.1 before including the effects of thermal diffusion in § 6.2.

6.1. Interchange and Parker Instabilities

The simplest case to consider is the interchange instability in which a magnetic field line and associated fluid is lifted vertically, maintaining pressure balance with its surroundings. If the new density is less than that of the surrounding fluid, it is buoyantly unstable. Newcomb (1961) showed that the criterion for instability is

\[ \mathcal{A} \equiv \frac{d \ln \rho}{dz} + \frac{\rho g}{\Gamma_1 P_g} > 0, \] (35)

where \( P_g \) is the gas pressure. Using the equation of hydrostatic balance, \( dP/dz = d(P_g + B^2/8\pi)/dz = -\rho g \), to substitute for \(-\rho g\), we find instability if (see also Acheson 1979)

\[ \left( \frac{2}{\Gamma_1} \right) \frac{B^2}{8\pi P_g} \left( -\frac{d \ln B}{dz} \right) > N^2/g, \] (36)

where

\[ \frac{N^2}{g} = \frac{d \ln \rho}{dz} - \frac{1}{\Gamma_1} \frac{d \ln P_g}{dz} \] (37)

defines the Brunt Väisälä frequency \( N^2 \) (e.g., Hansen & Kawaler 1994). Both thermal buoyancy and composition gradients may contribute to \( N^2 \) (e.g., Bildsten & Cumming 1998). We assume a uniform composition in the ocean so that \( N^2 \) has only a thermal piece,

\[ N^2 = \frac{g}{H} \frac{\chi_T}{\chi_p} \left( \frac{\nu_{ad}}{\nu} - \frac{d \ln T}{d \ln y} \right), \] (38)

where \( \chi_0 \equiv \partial \ln P/\partial \ln Q \) with the other independent thermodynamic variables held constant.

We see from equation (36) that the instability criterion depends on both how quickly the magnetic field decreases with height and the ratio of magnetic to gas pressure, \( B^2/8\pi P_g \). To understand this, first consider a magnetic field strength of which changes discontinuously with height. If the magnetic field above the discontinuity is smaller than that below, pressure continuity implies heavy fluid overlying light and a Rayleigh-Taylor unstable density jump no matter how small \( B^2/8\pi P_g \). On the other hand, if magnetic pressure dominates gas pressure, even a small magnetic field gradient may create an unstable density profile.

For the steady state magnetic profile, the gradient \( d \ln B/dz \) is given by \( d \ln B/d \ln y = \nu_{diff}/\nu_{accr} \) (eq. [15]). Inserting this gradient into equation (36), we find the critical magnetic field strength \( B_c \) required for instability of the steady state profile is

\[ B_c^2 = 4\pi \Gamma_1 \frac{\nu_{accr}}{\nu_{diff}} \frac{N^2 H}{g}. \] (39)

We plot \( B_c \) as a function of depth in Figure 5 (solid line). We assume \( P \approx P_g \) in making this plot. The dashed line in Figure 5 shows the steady state magnetic profile, arbitrarily scaled to \( B = 10^{10} \) G at the base. For clarity, we show only models with electron-ion scattering in the crust.

In the degenerate ocean, Bildsten & Cumming (1998) estimated the thermal buoyancy as

\[ N^2 H^2 \approx \frac{3k_B T}{8\mu m_p}, \] (40)

allowing us to make an analytic estimate of \( B_c \). We use equation (26) for \( \nu_{diff}/\nu_{accr} \), which gives

\[ B_c = \left( 3 \times 10^9 \right)^{1/2} \left( \frac{x^3}{g(x)} \right)^{1/2} \left( \frac{\dot{m}}{0.01 \, m_{edd}} \right)^{-1/2}, \] (41)

where we take \( \Lambda_{ad} = 1, g_{14} = 1.9, \Gamma_1 = 4/3, \) and \( \mu_p = 2. \) For \( x \ll 1 \), we find \( B_c \) is weakly dependent on depth, \( B_c \propto \)
\( x^{1/2} \propto y^{1/10} \). For \( x \gg 1 \), \( B_c \propto x^2 \), giving
\[
B_c = 4 \times 10^{11} \ G \ 1^{1/2} \left( \frac{m}{0.01 \ m_{\text{Edd}}} \right)^{-1/2},
\]
which is in good agreement with our numerical results.

### 6.2. Doubly Diffusive Instability

On small enough wavelengths, thermal diffusion acts to reduce thermal gradients during the perturbation, allowing instability even for \( B < B_c \). If the ratio of magnetic to thermal diffusivities, \( \eta/\mathcal{K} \), is small, the fluid elements retain their destabilizing magnetic buoyancy, but the stabilizing thermal buoyancy is washed out (e.g., Acheson 1979). The stability criterion is as given by equation (36), except that the thermal buoyancy term is reduced by a factor \( \eta/\mathcal{K} \).

When there is no composition gradient, the critical magnetic field \( B_c \) becomes \( B'_c = (\eta/\mathcal{K})^{1/2} B_c \). We plot \( B'_c \) as a dotted line in Figure 5.

To estimate \( \eta/\mathcal{K} \) in the ocean, we use the heat equation (eq. [3]) to define a thermal time \( t_{\text{term}} = \varepsilon_p y' \rho K = 3\kappa \varepsilon_p y' / 4acT^3 \) so that \( \eta/\mathcal{K} = t_{\text{term}} / t_{\text{diff}} \). The heat transport in the ocean is by electron conduction with \( K = \pi \nu_k k^2 B T / 3m_e \), and we use the results of § 4 for \( t_{\text{diff}} \), giving
\[
\frac{\eta}{\mathcal{K}} = \frac{2 \times 10^{-4} \ (1 + x^2)}{T_8 \ x^3} \ \Lambda_{\text{el}}^2 \left( \frac{Z}{30} \right). \tag{43}
\]

Taking \( \Lambda_{\text{el}} = 1 \), \( \varphi_{t4} = 1.9 \), \( \Gamma_1 = 4/3 \) and \( \mu_e = 2 \), the critical magnetic field is
\[
B'_c = 10^9 \ G \left[ \frac{1 + x^2}{\varphi(x)^{1/2}} \right] \left( \frac{m}{0.01 \ m_{\text{Edd}}} \right)^{-1/2} \left( \frac{Z}{30} \right)^{1/2}. \tag{44}
\]

When \( x \gg 1 \), we find
\[
B'_c = 5 \times 10^9 \ G \ y^{5/8} \left( \frac{m}{0.01 \ m_{\text{Edd}}} \right)^{-1/2} \left( \frac{Z}{30} \right)^{1/2}, \tag{45}
\]
a similar scaling with depth to \( B_c \).

To summarize, we find that instability requires \( B \gtrsim 10^{10} \ G \) at the base of the ocean for \( m = 0.01 \ m_{\text{Edd}} \) or \( B \gtrsim 10^{11} \ G \) for \( m = 0.1 \ m_{\text{Edd}} \). We do not expect buoyancy instability to operate in the crust, as long as the rigidity of the crust is able to counteract the buoyancy force. The detailed outcome of the instability will depend on its nonlinear development. However, it seems reasonable to take the critical magnetic field of equation (45) as an upper limit on the strength of any buried field at the boundary between the ocean and crust.

### 7. DISCUSSION AND CONCLUSIONS

We have investigated whether the magnetic field of a neutron star may be diamagnetically screened by accreted matter. Our approach has been to assume the accreted matter is unmagnetized and to ask whether there is sufficient time for the stellar field to penetrate the freshly accreted material. The magnetic profile in the outer layers of the neutron star is set by the competition between downward advection and compression by accretion and upward transport by Ohmic diffusion. In steady state, we have shown that the magnetic field strength decreases through the outer crust and ocean by \( n \) orders of magnitude, where \( n \approx m / 0.02 \ m_{\text{Edd}} \) (see § 5). The steady state profiles are stable to buoyancy instabilities provided the magnetic field in the ocean and crust is \( \lesssim 10^{10} \)–\( 10^{11} \) G.

Figure 6 summarizes these results. Arbitrarily choosing \( B = 10^9 \ G \) at the crust/ocean boundary, we plot the magnetic field at the surface of the ocean for different accretion rates. At large accretion rates, the magnetic field strength decreases dramatically through the ocean, and the stellar field is screened by the accreted layer. At low accretion rates, the magnetic field penetrates into the accreted layer, and screening is ineffective. The dashed and dotted lines in Figure 6 show, with and without thermal diffusion (see § 6 for the physics of this difference), the critical magnetic field needed at the top of the crust for magnetic buoyancy instability to occur in the ocean. This provides an upper limit to the strength of any buried field since, for larger field strengths, we expect additional upward transport of magnetic flux by buoyancy instabilities.

We have shown that for accretion rates \( \lesssim 0.02 \ m_{\text{Edd}} \), microscopic Ohmic diffusion prevents screening of the neutron star magnetic field by the accreted matter. At larger accretion rates, field burial may be possible; however, we stress that we have considered only a plane-parallel model with a simple field geometry. For example, consideration of a more complex field geometry in a spherical shell may indicate that additional modes of magnetic instability are available. In addition, we have not discussed the burial process itself, which is not understood, although some calculations of the spreading of matter initially confined to the polar cap of a strongly magnetized neutron star with \( B \sim 10^{12} \ G \) have been carried out (Hameury et al. 1983; Brown & Bildsten 1998; Litwin, Brown, & Rosner 2001).

Our results are not directly relevant to the high magnetic field accreting X-ray pulsars, in which accretion occurs at the polar cap of a strongly magnetized neutron star with \( B \sim 10^{12} \ G \) have been carried out (Hameury et al. 1983; Brown & Bildsten 1998; Litwin, Brown, & Rosner 2001).

Our results have interesting implications for the weakly magnetic neutron...
stars in LMXBs. Only one system so far, the accreting millisecond pulsar SAX J1808.4—3658, shows direct evidence for its magnetic field. No other weakly magnetic LMXB shows pulsations in its persistent emission (upper limits to the pulsed fraction are typically ~1%; Leahy, Elsner, & Weisskopf 1983; Mereghetti & Grindlay 1987; Wood et al. 1991; Vaughan et al. 1994). SAX J1808.4—3658 has similar X-ray spectral and variability properties to other LMXBs (van der Klis 2000, and references therein); however, consideration of both the observations and evolutionary state of the binary (Chakrabarty & Morgan 1998; Bildsten & Chakrabarty 2001) makes it clear that this source has an unusually low accretion rate. The time-averaged accretion rate is $\approx 10^{-11} M_\odot \text{ yr}^{-1} \approx 10^{-5} \dot{m}_{\text{Edd}}$ (Chakrabarty & Morgan 1998; Bildsten & Chakrabarty 2001) and at the peak of outburst reaches $\lesssim 0.03 \dot{m}_{\text{Edd}}$, which is below the range of accretion rates for which screening could be effective. This leads us to suggest that perhaps most LMXBs have their dipole magnetic fields screened by the accreted matter. SAX J1808.4—3658, on the other hand, accretes slowly enough that the magnetic field is able to rapidly penetrate the freshly accreted material. The other faint Galactic center transients discovered with BeppoSAX (e.g., King 2000) may therefore be promising sources to search for persistent pulsations.

We have concentrated on the steady state magnetic field profiles. However, there may be important time-dependent evolution of the magnetic field. In particular, if the magnetic field is screened, then we expect it to emerge once accretion ceases. The timescale for this depends on the depth of the screening currents (Young & Channumag 1995). For the inner crust, the Ohmic diffusion time is $10^5$ yr, always less than the accretion time (see Fig. 9 of Brown & Bildsten 1998). Thus we expect the relevant timescale to be the Ohmic time across the outer crust, which is extremely short, only 100–1000 yr. It therefore seems unlikely that screening alone can explain the low magnetic fields of the millisecond radio pulsars. Potentially observable time-dependent effects may occur in those transiently accreting systems for which the accretion rate during outburst is large enough for screening to be important. After an outburst, we would expect the magnetic field profile to relax by Ohmic diffusion on a timescale comparable to the outburst duration.

Our results show that evolution of the magnetic field in the outer crust is sensitive to the level of impurity there, as suggested by Schatz et al. (1999). For a primordial crust with a low level of impurity, as assumed by previous authors, phonon scattering dominates the conductivity. However, as shown by recent calculations of nucleosynthesis during H/He burning in the atmosphere (Schatz et al. 1998, 1999; Koike et al. 1999), the crust is likely very impure. In this case, the conductivity is set by scattering from ions, leading to shorter Ohmic dissipation times than previously assumed.

We have not discussed the layer of H/He that lies above the ocean. Because of the different composition, the Ohmic diffusion time in this layer is longer than that in the ocean. However, we find that the magnetic profile in this layer is determined by a different process, thermomagnetic drift (Urpin & Yakovlev 1980) driven by the heat flux from nuclear burning. The evolution of the magnetic field in this layer is important for understanding type I X-ray bursts. In particular, the drifting oscillations during bursts observed with the Rossi X-Ray Timing Explorer (see Strohmayer 1999 and van der Klis 2000 for reviews) have been interpreted as being caused by a differentially rotating atmosphere. However, Cumming & Bildsten (2000) pointed out that a poloidal field $\gtrsim 10^7$ G is large enough to prevent differential rotation. Type I X-ray bursts are thus promising probes of the magnetic field in the surface layers. We address magnetic field evolution in the H/He layers in a separate paper.

We would like to thank Phil Arras, Steve Cowley, Ramesh Narayan, Andreas Reissenegger, Chris Thompson, and Dmitrii Yakovlev for useful conversations. This research was supported in part by the National Science Foundation under Grant PHY99-07949 and Grant AST 98-00616 and by NASA via grant NAG 5-8658. L. B. is a Cottrell Scholar of the Research Corporation.

REFERENCES

Acheson, D. J. 1979, Sol. Phys., 62, 23
Baiko, D. A., & Yakovlev, D. G. 1995, Astrophys. Lett., 21, 702
Bhattacharya, D. 1995, in X-Ray Binaries, ed. W. H. G. Lewin, J. van Paradijs, & E. P. J. van den Heuvel (Cambridge: Cambridge Univ. Press), 233
Bhattacharya, D., & Srinivasan, G. 1995, in X-Ray Binaries, ed. W. H. G. Lewin, J. van Paradijs, & E. P. J. van den Heuvel (Cambridge: Cambridge Univ. Press), 495
Bildsten, L., & Chakrabarty, D. 2001, ApJ, 557, 292
Bildsten, L., & Cumming, A. 1998, ApJ, 506, 342
Bildsten, L., & Cutler, C. 1995, ApJ, 449, 800
Bisnovatyi-Kogan, G. S., & Komberg, B. V. 1974, Soviet Astron., 18, 217
Brown, E. F. 2000, ApJ, 531, 988
Brown, E. F., & Bildsten, L. 1998, ApJ, 496, 915
Chakrabarty, D., & Morgan, E. H. 1998, Nature, 394, 346
Clayton, D. D. 1983, Principles of Stellar Evolution and Nucleosynthesis (Chicago: Univ. of Chicago Press)
Cumming, A., & Bildsten, L. 2000, ApJ, 544, 453
Farouki, R. T., & Hamaguchi, S. 1993, Phys. Rev. E, 47, 4130
Geppert, U., & Urpin, V. 1994, MNras, 271, 490
Goldreich, P., & Reisenegger, A. 1992, ApJ, 395, 250
Hameury, J. M., Bonazzola, S., Heyvaerts, J., & Lasota, J. P. 1983, A&A, 128, 369
Itoh, N., & Kohyama, Y. 1993, ApJ, 404, 268
Hansen, C. J., & Kawaler, S. D. 1994, Stellar Interiors: Physical Principles, Structure and Evolution (New York: Springer)
King, A. R. 2000, MNras, 315, L33
Koike, O., Hashimoto, M., Arai, K., & Wanao, S. 1999, A&A, 342, 464
Konar, S., & Bhattacharya, D. 1997, MNras, 284, 311

Leahy, D. A., Elsner, R. F., & Weisskopf, M. C. 1983, ApJ, 272, 256
Litwin, C., Brown, E. F., & Rosner, R. 2001, ApJ, 553, 788
Mereghetti, S., & Grindlay, J. E. 1987, ApJ, 312, 727
Newcomb, W. A. 1961, Phys. Fluids, 4, 391
Potekhin, A. Y., Baiko, D. A., Haensel, P., & Yakovlev, D. G. 1999, A&A, 346, 345
Psaltis, D., & Chakrabarty, D. 1999, ApJ, 521, 332
Romani, R. W. 1990, Nature, 347, 741
— 1995, in ASP Conf. Ser. 72, Millisecond Pulsars: A Decade of Surprise, ed. A. S. Fruchter, M. Tavani, & D. C. Backer (San Francisco: ASP), 288
Schatz, H., et al. 1998, Phys. Rep., 294, 167
Schatz, H., Bildsten, L., Cumming, A., & Wiescher, M. 1999, ApJ, 524, 1014
Strohmayer, T. E. 1999, preprint (astro-ph/9911338)
Taam, R. E., & van den Heuvel, E. P. J. 1986, ApJ, 305, 235
Urpin, V. A., & Yakovlev, D. G. 1980, Soviet Astron., 24, 425
van der Klis, M. 2000, ARA&A, 38, 717
Vaughan, B. A., et al. 1994, ApJ, 435, 362
White, N. E., Nagase, F., & Parmar, A. N. 1995, in X-Ray Binaries, eds. W. H. G. Lewin, J. van Paradijs, & E. P. J. van den Heuvel (Cambridge: Cambridge Univ. Press), 233
Wijers, R. A. M. J. 1999, ApJ, 527, 607
Wijnands, R., & van der Klis, M. 1998, Nature, 394, 344
Wood, K. S., et al. 1991, ApJ, 379, 295
Yakovlev, D. G., & Urpin, V. A. 1980, Soviet Astron., 24, 303
Young, E. J., & Channumag, G. 1995, ApJ, 442, L53
Ziman, J. M. 1964, Principles of the Theory of Solids (Cambridge: Cambridge Univ. Press)