Multiparticle production in the model with antishadowing

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Abstract

We discuss the role of absorption and antishadowing in particle production. We reproduce power-like energy behavior of the mean multiplicity in the model with antishadowing and discuss physical implications of such behavior for the hadron structure.
Introduction

Multiparticle production and global observables such as mean multiplicity and its energy dependence alongside with total, elastic and inelastic cross–sections provide us a clue to the mechanisms of confinement and hadronization. General principles are very important in the nonperturbative sector of QCD and unitarity which regulates the relative strength of elastic and inelastic processes is the one of such principles. There is no universal, generally accepted method to implement unitarity in high energy scattering and as a result of this fact a related problem of the absorptive corrections role and their sign has a long history (cf. [1] and references therein). However, a choice of particular unitarization scheme is not just a matter of taste. Long time ago arguments based on analytical properties of the scattering amplitude were put forward [2] in favor of the rational form of unitarization. It was shown that this form of unitarization reproduced correct analytical properties of the scattering amplitude in the complex energy plane much easier compare to the exponential form.

Interest in unitarity limitations and the respective dependencies of the global observables was stimulated by the preparation of the experimental program for the LHC and the future plans to study soft interactions at the highest available energies. For example, correct account for unitarity is essential under theoretical estimates of the Higgs production cross-section via diffractive mechanisms. The region of the LHC energies is the one where new, so called antishadow scattering mode can be observed. Such a mode naturally appears when the rational form of unitarization being exploited [3]. It has been demonstrated that this mode can be revealed at the LHC directly measuring $\sigma_{el}(s)$ and $\sigma_{tot}(s)$ [4] (and not only by means of the analysis of impact parameter distributions). Antishadowing leads to self–damping of the inelastic channels and asymptotically dominating role of elastic scattering, i. e. $\sigma_{el}(s)/\sigma_{tot}(s) \rightarrow 1$ at $s \rightarrow \infty$. Immediate question arises on consistency of this mechanism with the growth of mean multiplicity in hadronic collisions with energy. Moreover, many models and experimental data suggest a power–ilke energy dependence of mean multiplicity\footnote{Recent discussions of power–like energy dependence of the mean hadronic multiplicity and list of references to the older papers can be found in [5]} and a priori the compatibility of such dependence with antishadowing is not evident.

In this note we apply a rational ($U$–matrix) unitarization method [6] to consider the global features of multiparticle dynamics such as mean multiplicity and role of absorptive corrections. We show that it is possible to reproduce power-like energy behavior of the mean multiplicity in the model with antishadowing and discuss some physical implications.
1 Multiparticle production in the $U$–matrix approach

The rational form of unitarization is based on the relativistic generalization of the Heitler equation of radiation dumping \[6\]. In this approach the elastic scattering amplitude satisfies unitarity since it is a solution of the following equation

$$ F = U + iUDF $$

presented here in the operator form. Eq.\[1\] allows one to fulfill the unitarity provided the inequality

$$ \text{Im}U(s, b) \geq 0 $$

is satisfied. The form of the amplitude in the impact parameter representation is

$$ f(s, b) = \frac{U(s, b)}{1 - iU(s, b)}, $$

where $U(s, b)$ is the generalized reaction matrix. It is considered as an input dynamical quantity similar to the eikonal function. It is to be noted that the analogous form for the scattering amplitude was obtained by Feynman in his parton model for diffractive scattering which he has never published (cf. \[7\]).

In the impact parameter representation the unitarity equation rewritten at high energies for the elastic amplitude $f(s, b)$ has the form

$$ \text{Im} f(s, b) = |f(s, b)|^2 + \eta(s, b) $$

where the inelastic overlap function

$$ \eta(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{inel}}{db^2} $$

is the sum of all inelastic channel contributions. It can be expressed as a sum of $n$–particle production cross–sections at the given impact parameter

$$ \eta(s, b) = \sum_n \sigma_n(s, b), $$

where

$$ \sigma_n(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_n}{db^2}, \quad \sigma_n(s) = 8\pi \int_0^\infty bdb\sigma_n(s, b). $$

Inelastic overlap function is related to $U(s, b)$ according to Eqs. \[3\] and \[4\] as follows

$$ \eta(s, b) = \frac{\text{Im}U(s, b)}{|1 - iU(s, b)|^2}. $$

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Then the unitarity Eq. 4 points out that the elastic scattering amplitude at given impact parameter value is determined by the inelastic processes when the amplitude is a pure imaginary one. Eq. 4 imply the constraint \( |f(s, b)| \leq 1 \) while the “black disk” limit presumes inequality \( |f(s, b)| \leq 1/2 \) and the elastic amplitude satisfying the latter condition is a shadow of inelastic processes. In its turn the imaginary part of the generalized reaction matrix is the sum of inelastic channel contributions:

\[
ImU(s, b) = \sum_n \bar{U}_n(s, b),
\]

where \( n \) runs over all the inelastic states and

\[
\bar{U}_n(s, b) = \int d\Gamma_n |U_n(s, b, \{\xi_n\})|^2.
\]

In Eq. (8) \( d\Gamma_n \) is the \( n \)-particle element of the phase space volume. The functions \( U_n(s, b, \{\xi_n\}) \) are determined by dynamics of \( h_1 + h_2 \rightarrow X_n \) processes, where \( \{\xi_n\} \) stands for the full set of respective kinematical variables. Thus, the quantity \( ImU(s, b) \) itself is a shadow of the inelastic processes. However, unitarity leads to self–damping of the inelastic channels [8] in the sense that increase of the function \( ImU(s, b) \) results in decrease of the inelastic overlap function \( \eta(s, b) \) when \( ImU(s, b) \) exceeds unity (cf. Fig. 1).

\[\text{Figure 1: Shadow and antishadow scattering regions}\]

Respective inclusive cross–section [9, 10] which takes into account unitarity in the direct channel has the form

\[
\frac{d\sigma}{d\xi} = 8\pi \int_0^\infty bdb \frac{I(s, b, \xi)}{|1-iU(s, b)|^2}.
\]

The function \( I(s, b, \xi) \) in Eq. (9) is expressed via the functions \( U_n(s, b, \xi, \{\xi_{n-1}\}) \) determined by the dynamics of the processes \( h_1 + h_2 \rightarrow h_3 + X_{n-1} \):

\[
I(s, b, \xi) = \sum_{n \geq 3} n \int d\Gamma_n |U_n(s, b, \xi, \{\xi_{n-1}\})|^2
\]
and
\[ \int I(s, b, \xi) d\xi = \bar{n}(s, b) \text{Im} U(s, b). \]  

(11)

The kinematical variables \( \xi \) (\( x \) and \( p_\perp \), for example) refer to the produced particle \( h_3 \) and the set of variables \( \{\xi_{n-1}\} \) describe the system \( X_{n-1} \) of \( n-1 \) particles.

Now we turn to the mean multiplicity and consider first the corresponding quantity in the impact parameter representation. As it follows from the above the \( n \)–particle production cross–section \( \sigma_n(s, b) \)
\[ \sigma_n(s, b) = \frac{\bar{U}_n(s, b)}{|1 - iU(s, b)|^2} \]  

(12)

Then the probability
\[ P_n(s, b) \equiv \frac{\sigma_n(s, b)}{\sigma_{inel}(s, b)} = \frac{\bar{U}_n(s, b)}{\text{Im} U(s, b)}. \]  

(13)

Thus, we observe the cancellation of unitarity corrections in the ratio of the cross-sections \( \sigma_n(s, b) \) and \( \sigma_{inel}(s, b) \). Therefore the mean multiplicity in the impact parameter representation
\[ \bar{n}(s, b) = \sum_n nP_n(s, b) \]

is not affected by unitarity corrections and therefore cannot be proportional to \( \eta(s, b) \). This conclusion is consistent with Eq. (11). The above mentioned proportionality is a rather natural assumption in the framework of the geometrical models, but it is in conflict with the unitarization. Because of that the results [11] based on such assumption should be taken with precaution. However, the above cancellation of unitarity corrections does not take place for the quantity \( \bar{n}(s) \) which we address in the next section.

## 2 Growth of mean multiplicity

We use a model for the hadron scattering described in [12]. It is based on the ideas of chiral quark models. The picture of a hadron consisting of constituent quarks embedded into quark condensate implies that overlapping and interaction of peripheral clouds occur at the first stage of hadron interaction (Fig. 2). Non-linear field couplings could transform then the kinetic energy to internal energy and mechanism of such transformations was discussed by Heisenberg [13] and
Carruthers [14]. As a result massive virtual quarks appear in the overlapping region and some effective field is generated. Valence constituent quarks located in the central part of hadrons are supposed to scatter simultaneously in a quasi-independent way by this effective field.

$$\tilde{N}(s, b) \propto \frac{(1 - \langle k_Q \rangle) \sqrt{s}}{m_Q} D^{h_1}_c \otimes D^{h_2}_c \equiv N_0(s) D_C(b), \quad (14)$$

where $m_Q$ – constituent quark mass, $\langle k_Q \rangle$ – average fraction of hadron energy carried by the constituent valence quarks. Function $D^h_c$ describes condensate distribution inside the hadron $h$, and $b$ is an impact parameter of the colliding hadrons.

Thus, $\tilde{N}(s, b)$ quarks appear in addition to $N = n_{h_1} + n_{h_2}$ valence quarks. In elastic scattering those quarks are transient ones: they are transformed back into the condensates of the final hadrons. Calculation of elastic scattering amplitude has been performed in [12]. However, valence quarks can excite a part of the cloud of the virtual massive quarks and these virtual massive quarks will subsequently fragment into the multiparticle final states. Such mechanism is responsible for the particle production in the fragmentation region and should lead to strong correlations between secondary particles. It means that correlations exist between particles from the same (short–range correlations) and different clusters (long–range correlations) and, in particular, the forward–backward multiplicity correlations should be observed. This mechanism can be called as a correlated cluster production mechanism. Evidently, similar mechanism should be signif-
icantly reduced in $e^+e^-$–annihilation processes and therefore large correlations are not to be expected there.

As it was already mentioned simple (not induced by interactions with valence quarks) hadronization of massive $\tilde{N}(s,b)$ quarks leads to formation of the multiparticle final states, i.e. production of the secondary particles in the central region. The latter should not provide any correlations in the multiplicity distribution.

Remarkably, existence of the massive quark-antiquark matter in the stage preceding hadronization seems to be supported by the experimental data obtained at CERN SPS and RHIC (see [15] and references therein).

Since the quarks are constituent, it is natural to expect direct proportionality between a secondary particles multiplicity and number of virtual massive quarks appeared (due to both mechanisms of multiparticle production) in collision of the hadrons with given impact parameter:

$$\bar{n}(s,b) = \alpha(n_{h_1} + n_{h_2})N_0(s)D_F(b) + \beta N_0(s)D_C(b),$$

with constant factors $\alpha$ and $\beta$ and

$$D_F(b) \equiv D_Q \otimes D_C,$$

where the function $D_Q(b)$ is the probability amplitude of the interaction of valence quark with the excitation of the effective field, which is in fact related to the quark matter distribution in this hadron-like object called the valence constituent quark [12]. The mean multiplicity $\bar{n}(s)$ can be calculated according to the formula

$$\bar{n}(s) = \int_0^\infty \frac{\bar{n}(s,b)\eta(s,b)bdb}{\int_0^\infty \eta(s,b)bdb}.$$  \hspace{1cm} (16)

It is evident from Eq. (16) and Fig. 1 that the antishadow mode with the peripheral profile of $\eta(s,b)$ suppresses the region of small impact parameters the main contribution to the mean multiplicity is due to peripheral region of $b \sim R(s)$.

To make explicit calculations we model for simplicity the condensate distribution $D_C(b)$ and the impact parameter dependence of the probability amplitude $D_Q(b)$ of the interaction of valence quark with the excitation of the effective field by the exponential forms, and thus we use exponential dependencies for the functions $D_F(b)$ and $D_C(b)$ with the different radii. Then the mean multiplicity

$$\bar{n}(s,b) = \bar{\alpha}N_0(s)\exp(-b/R_F) + \bar{\beta}N_0(s)\exp(-b/R_C).$$  \hspace{1cm} (17)

The function $U(s,b)$ is chosen as a product of the averaged quark amplitudes

$$U(s,b) = \prod_{Q=1}^{N} \langle f_Q(s,b) \rangle$$

(18)
Figure 3: Energy dependence of mean multiplicity, theoretical curve is given by the equation $\bar{n}(s) = as^\delta$ ($a = 2.328, \delta = 0.201$); experimental data from the Refs. [16].

in accordance since in the model valence quarks scatter in effective field simultaneously and quasi-independently. The $b$–dependence of $\langle f_Q \rangle$ related to the quark formfactor $F_Q(q)$ has a simple form $\langle f_Q \rangle \propto \exp(-m_Qb/\xi)$. Thus, the generalized reaction matrix (in a pure imaginary case) gets the following form [12]

$$U(s, b) = ig \left[ 1 + \alpha \sqrt{\frac{s}{m_Q}} \right]^N \exp(-Mb/\xi), \quad (19)$$

where $M = \sum_{q=1}^N m_Q$. At sufficiently high energies where increase of the total cross–sections is quite prominent we can neglect the energy independent term and rewrite the expression for $U(s, b)$ as

$$U(s, b) = ig \left( \frac{s}{m_Q^2} \right)^{N/2} \exp(-Mb/\xi). \quad (20)$$

After calculation of the integrals [16] we arrive to the power-like dependence of the mean multiplicity $\bar{n}(s)$ at high energies

$$\bar{n}(s) = as^\delta_F + bs^\delta_C, \quad (21)$$

where

$$\delta_F = \frac{1}{2} \left( 1 - \frac{\xi}{m_Q R_F} \right) \quad \text{and} \quad \delta_C = \frac{1}{2} \left( 1 - \frac{\xi}{m_Q R_C} \right).$$

There are four free parameters in the model, $\tilde{\alpha}, \tilde{\beta}$ and $R_F, R_C$, and the freedom in their choice is translated to $a$, $b$ and $\delta_F, \delta_C$. The value of $\xi = 2$ is fixed from
the data on angular distributions \([12]\) and for the mass of constituent quark the standard value \(m_Q = 0.35\) GeV was taken. However, fit to experimental data on the mean multiplicity leads to approximate equality \(\delta_F \simeq \delta_C\), and actually Eq. (21) is reduced to the two-parametric power-like energy dependence of mean multiplicity
\[
\bar{n} = a s^\delta,
\]
which is in good agreement with the experimental data (Fig. 3). Equality \(\delta_F \simeq \delta_C\) means that variation of the correlation strength with energy is weaker than the power dependence and could be, e.g. a logarithmic one. From the comparison with the data on mean multiplicity we obtain that \(\delta \simeq 0.2\), which corresponds to the effective masses, which are determined by the respective radii \((M = 1/R)\), \(M_C \simeq M_F \simeq 0.3m_Q\), i.e. \(M_F \simeq M_C \simeq m_\pi\).

The value of mean multiplicity expected at the LHC energy \((\sqrt{s} = 14\) TeV\) is about 110. It is not surprising that it is impossible to differentiate contributions from the two mechanisms of particle production at the level of mean multiplicity. The studies of correlations are necessary for that purpose.

**Conclusion**

It was shown that the model based on accounting unitarity [12] and extended to multiparticle production provides a reasonable description of the energy dependence of mean multiplicity leading to its power-like growth with a small exponent. This result is a combined effect of unitarity and existence of the phase preceding hadronization when massive quark–antiquark pairs are generated. It is worth noting again that power–like energy dependence of mean multiplicity appears in various models and is in a good agreement with heavy–ion experimental data too\(^2\).

Multiplicity distribution \(P_n(s, b)\) and mean multiplicity \(\bar{n}(s, b)\) in the impact parameter representation have no absorptive corrections, but since antishadowing leads to suppression of particle production at small impact parameters and the main contribution to the integral multiplicity \(\bar{n}(s)\) comes from the region of \(b \sim R(s)\). Of course, this prediction is to be valid for the energy range where antishadow scattering mode starts to develop (the quantitative analysis of the experimental data [18] gives the value: \(\sqrt{s_0} \simeq 2\) TeV\) and is therefore consistent with the “centrality” dependence of the mean multiplicity observed at RHIC [19].

It is also worth noting that no limitations follow from the general principles for the mean multiplicity, besides the well known one based on the energy conservation law. Having in mind relation (17), we could say that the obtained power–like

\(^2\)Recent analysis of mean multiplicity with power–low growth in Au+Au collisions at RHIC is given in [17].
dependence which takes into account unitarity effects could be considered as a kind of a saturated upper bound for the mean multiplicity growth.

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