Cosmological constraints from the local X-ray luminosity function of the most X-ray luminous galaxy clusters

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ABSTRACT

We present precise constraints on the normalization of the power spectrum of mass fluctuations in the nearby universe, \( \sigma_8 \), as a function of the mean local matter density, \( \Omega_m \). Using the observed local X-ray luminosity function of galaxy clusters from the extended BCS and REFLEX studies, a mass-luminosity relation determined from Chandra and ROSAT X-ray data and weak gravitational lensing observations, and the mass function predicted by the Hubble Volume simulations of Evrard et al., we obtain \( \sigma_8 = (0.508 \pm 0.019) \Omega_m^{0.253 \pm 0.024} \) with \( \Omega_m < 0.34 \) at 68 per cent confidence. The degeneracy between \( \sigma_8 \) and \( \Omega_m \) can be broken using Chandra measurements of the X-ray gas masses in dynamically relaxed clusters. Using this information and including Gaussian priors on the mean baryon density of the universe and the Hubble constant, we obtain \( \sigma_8 = 0.695 \pm 0.042 \) and \( \Omega_m = 0.287 \pm 0.036 \), for an assumed flat \( \Lambda \) CDM cosmology (marginalized 68 per cent confidence limits). Our results are in good agreement with some recent studies based on the local X-ray temperature function of clusters, the redshift evolution of the X-ray luminosity and temperature functions of clusters, early results from the Sloan Digital Sky Survey, the most recent results from studies of cosmic shear, and combined analyses of the 2dF galaxy redshift survey and cosmic microwave background anisotropies.

Key words: cosmological parameters – X-rays: galaxies: clusters – gravitational lensing — large-scale structure of the universe — X-rays: galaxies: clusters

1 INTRODUCTION

The X-ray luminosity function of galaxy clusters in the nearby universe provides a powerful cosmological probe. The observed luminosity function, \( n(L) \), can be combined with a relation linking the observed X-ray luminosity and mass, and the mass function, \( n(M) \), predicted by simulations, to obtain tight constraints on the combination of cosmological parameters \( \Omega_m \) and \( \sigma_8 \), where \( \Omega_m \) is the mean matter density of the local universe and \( \sigma_8 \) is the root-mean-square (rms) variation of the density field smoothed by a top hat window function of size \( 8h^{-1}\text{Mpc} \). The degeneracy between \( \sigma_8 \) and \( \Omega_m \) can be broken using Chandra measurements of the X-ray gas masses in dynamically relaxed clusters. Observationally, the keys to such studies are precise determinations of the local X-ray luminosity function of clusters and the relation linking the observed X-ray luminosities and total masses. X-ray selection currently offers the best way to identify massive galaxy clusters, and the local X-ray luminosity function has now been precisely determined by the BCS (Ebeling et al. 1997; Ebeling et al. 2000) and REFLEX (Böhringer et al. 2002) studies. The flux-limited BCS and REFLEX samples, which are based on data from the ROSAT All-Sky Survey (RASS; Trümper 1993), together include \( \sim 750 \) clusters and cover approximately two thirds of the sky. Recently, significant effort has also been invested into measuring the local temperature function of clusters (e.g. Markevitch 1999; Nevalainen, Markevitch & Forman 2000; Finoguenov, Reiprich & Böhringer 2001; Allen, Schmidt & Fabian 2001b; Sanderson et al. 2003). In particular, the launch of the Chandra X-ray Observatory has permitted the first precise measurements of the temperature and mass profiles of relaxed clusters from X-ray data. Using a combination of Chandra...
and gravitational lensing data, Allen et al. (2001b) confirmed that luminous, relaxed galaxy clusters follow the simple scaling relations predicted by theory, but that the normalization of the observed mass-temperature relation measured within $r_{2500}$ (where the mean enclosed mass density is 2500 times the critical density of the universe at the redshifts of the clusters) is approximately 40 per cent lower than predicted by standard adiabatic simulations. This highlights the likely importance of additional physics such as cooling and pre-heating in the intracluster gas (see also Pearce et al. 2000; Thomas et al. 2002; Voit et al. 2002; Muanwong et al. 2002).

Theoretically, the primary requirement for cosmological studies using the observed luminosity and/or temperature functions of clusters is a precise prediction of the mass function. This has now been achieved for flat ΛCDM (and τCDM) cosmologies using the Hubble Volume simulations of Jenkins et al. (2001) and Evrard et al. (2002).

In this paper we present precise constraints on $σ_8$ and $Ω_m$ based on the observed local luminosity function of the most X-ray luminous clusters in the extended BCS (Ebeling et al. 2000) and REFLEX samples, and a new calibration, using pointed Chandra and ROSAT X-ray observations and weak gravitational lensing results, of the mass-luminosity relation linking the masses of clusters measured within $r_{200}$ to their total 0.1–2.4 keV ROSAT luminosities. Having determined our combined constraint on $σ_8$ and $Ω_m$, we show that the degeneracy between these parameters can be broken using Chandra results on the X-ray gas mass fractions in the most dynamically relaxed clusters. Including Gaussian priors on the mean baryon density of the universe ($Ω_b h^2 = 0.0205 \pm 0.0018$; O’Meara et al. 2001), the Hubble constant ($h = 0.72 \pm 0.08$; Freedman et al. 2001), and a theoretical bias factor ($b = 0.93 \pm 0.05$; Bialek, Evrard & Mohr 2001) relating the asymptotic baryon fraction in the most X-ray luminous clusters to the mean value for the universe as a whole, we obtain $σ_8 = 0.695 \pm 0.042$ and $Ω_m = 0.287 \pm 0.036$ (marginalized 68 per cent confidence limits for an assumed flat ΛCDM cosmology). We compare our results to other measurements based on the local number density of clusters, evolution of the X-ray luminosity and temperature functions, the 2dF galaxy redshift survey, cosmic microwave background anisotropies, and measurements of cosmic shear.

Throughout this paper, a flat ΛCDM cosmology with $Ω_λ = 1 − Ω_m$ is assumed. In order to facilitate a direct comparison with previous X-ray studies, results on the masses, X-ray luminosities and X-ray gas mass fractions of individual clusters are quoted for a Hubble parameter $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1} = 0.5$ or, equivalently, $h_{50} = H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1} = 1.0$.

### 2 THEORY: THE PREDICTED LUMINOUSITY FUNCTION OF GALAXY CLUSTERS

Jenkins et al. (2001) show that the predicted mass function of galaxy clusters of mass $M$ at redshift $z$ can be written as a function of $\ln σ^{-1}(M, z)$, where $σ^2(M, z)$ is the variance of the linearly evolved density field smoothed by a spherical top-hat filter of comoving radius $R$, enclosing a mass $M = 4πR^3ρ_0/3$. Here $ρ_0 = Ω_m(0)ρ_c(0)$ is the mean co-moving matter density of the universe, $Ω_m(0)$ is the mean, present matter density in units of the critical density, and $ρ_c(0) = 3H_0^2/8πG$ is the critical density at redshift zero.

Using a spherical overdensity algorithm to measure the masses of clusters within radii $r_{200}$, where the mean enclosed mass density is 200 times the critical density of the universe at the redshift of interest, Evrard et al. (2002) show that for a flat ΛCDM cosmology, the mass fraction $f(σ^{-1})$ can be written as

$$f(σ^{-1}) = A \exp[- ln σ^{-1} + B |^a],$$

where, for $Ω_m = 0.3$ and $z = 0$, $A = 0.22$, $B = 0.73$ and $ε = 3.86$. Evrard et al. (2002) also provide simple interpolations for $A, B$ and $ε$ for other values of $Ω_m$ and $z$. The differential number density of clusters with mass $M$ at redshift $z$ is

$$\frac{dn(M, z)}{dln σ^{-1}} = \frac{f(σ^{-1})ρ(z)}{M},$$

where $ρ(z) = ρ_0(1 + z)^3$ is the mean mass density of the universe at redshift $z$. Following Viana & Liddle (1999), we write

$$σ(R, z) = σ_8(z) \left(\frac{R}{8h^{-1} \text{ Mpc}}\right)^{-γ(R)},$$

where

$$γ(R) = (0.3Γ + 0.2) \left[2.92 + \log_{10} \left(\frac{R}{8h^{-1} \text{ Mpc}}\right)\right]$$

and $Γ$ is the shape parameter of the cold dark matter transfer function. Following Sugiyama (1995), we set

$$Γ = Ω_m(0)h \left(\frac{2.7K}{T_0}\right)^2 \exp \left[-Ω_b(0) - \sqrt{\frac{h}{0.5Ω_m(0)}}\right],$$

where $T_0 = 2.726K$ is the temperature of the cosmic microwave background (Mather et al. 1994) and $Ω_b(0) = (0.0205 \pm 0.0018)h^2$ is the mean, present-day baryon density in units of the critical density (O’Meara et al. 2001). For this calculation, we set $h = 0.72$ and fix the primordial spectral index $n = 1$. The redshift-dependent quantity $σ_8(z)$ is related to its present value, $σ_8(0)$, by

$$σ_8(z) = σ_8(0) \frac{g(Ω_m(z))}{g(Ω_m(0))} \frac{1}{1 + z},$$

where, for a flat ΛCDM universe

$$g(Ω_m(z)) = \frac{5}{2}Ω_m(z) \left[\frac{1}{70} + \frac{209Ω_m(z)}{140} - \frac{Ω_m(z)^2}{140} + Ω_m(z)^{4/7}\right]^{-1},$$

and

$$Ω_m(z) = Ω_m(0) \left[1 - \frac{(1 + z)^3}{Ω_m(0)}\right].$$

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Cosmological constraints from the local X-ray luminosity function of galaxy clusters

3 OBSERVATIONS

3.1 The observed X-ray luminosity function of the most luminous galaxy clusters in the RASS

For this study, we concentrate on the local \( z \lesssim 0.3 \) X-ray luminosity function and restrict ourselves to the most luminous clusters, with \( L_X > 10^{44} \, h_{50}^{-2} \, \text{erg s}^{-1} \) in the 0.1–2.4 keV ROSAT band (for a flat \( \Lambda \)CDM cosmology with \( \Omega_m = 0.3 \) and \( \Omega_{\Lambda} = 0.7 \)). This selection is facilitated by the large sample size and well-determined selection functions of the BCS and REFLEX data sets.

The restriction to high luminosities reduces systematic uncertainties by matching the luminosity range of the luminosity function data to the range over which the mass-luminosity relation has been calibrated (Section 3.2.4). At lower luminosities, the effects of pre-heating and cooling in the intracluster gas are expected to become important and cause the mass-luminosity and mass-temperature relations to deviate from simple power-law forms (e.g. Cavaliere, Menci & Tozzi 1997). Since the most massive clusters

\[ \frac{d(n(M,z))}{dM} = \frac{\gamma \bar{\rho}(z)}{3M^2} \sigma^{-1} f(\sigma^{-1}), \]  

we obtain

\[ \frac{d(n(L,z))}{dL} = \frac{\gamma \bar{\rho}(z) \alpha}{3M_0} \left( \frac{E(z)}{L} \right)^{\alpha+1} f(\sigma^{-1}). \]  

The predicted differential luminosity function (i.e. the comoving number density of clusters at redshift \( z \) with luminosities in an interval \( dL \) around \( L \)) given by equation 12 can be compared with the observed X-ray luminosity function from the BCS and REFLEX studies to constrain the combination of cosmological parameters \( \Omega_m(0) \) and \( \sigma_8(0) \), hereafter referred to as \( \Omega_m \) and \( \sigma_8 \), respectively.

Table 1. The observed, binned X-ray luminosity function of the most X-ray luminous \( (L_X \gtrsim 10^{45} \, h_{50}^{-2} \, \text{erg s}^{-1}) \) galaxy clusters from the extended BCS and REFLEX studies. Column 2 gives the mean 0.1–2.4 keV X-ray luminosity for each bin, \( L_X \), in \( 10^{44} \, h_{50}^{-2} \, \text{erg s}^{-1} \). Error bars indicate the bin boundaries. Column 3 gives the number of clusters in each bin and column 4 the comoving space density in \( h_{50}^2 \, \text{Mpc}^{-3} \) \((10^{44} \, \text{erg s}^{-1})^{-1}\). A \( \Lambda \)CDM cosmology with \( \Omega_m = 0.3 \) and \( \Omega_{\Lambda} = 0.7 \) is assumed.

| \( L_X \) | \( n_{clus} \) | \( n(L) \) |
|---|---|---|
| BCS | 11.73^{+1.62}_{-1.73} | 17 | 1.32 \pm 0.32 \times 10^{-9} |
| 16.65^{+2.30}_{-2.30} | 17 | 7.45 \pm 1.81 \times 10^{-10} |
| 23.91^{+4.06}_{-4.06} | 17 | 8.10 \pm 1.97 \times 10^{-11} |
| REFLEX | 11.25^{+2.19}_{-2.19} | 20 | 1.56 \pm 0.34 \times 10^{-9} |
| 16.27^{+3.81}_{-3.81} | 20 | 4.44 \pm 0.98 \times 10^{-10} |
| 29.95^{+5.62}_{-5.62} | 20 | 1.77 \pm 0.39 \times 10^{-11} |

provide the most powerful constraints on cosmological parameters, relatively little information is lost by restricting ourselves to the largest systems.

The binned X-ray luminosity functions for clusters with \( L_X > 10^{45} \, h_{50}^{-2} \, \text{erg s}^{-1} \) from the northern extended BCS (Ebeling et al. 2000) and southern REFLEX (Böhringer et al. 2002) studies are summarized in Table 1. The mean redshift of the BCS clusters in this luminosity range is \( z = 0.21 \).

3.2 The observed mass-luminosity relation for the most X-ray luminous galaxy clusters

In determining the mass-luminosity relation we have used mass measurements obtained from Chandra X-ray observations of dynamically relaxed clusters and weak gravitational lensing results drawn from the literature. X-ray luminosities are determined from pointed ROSAT observations. In total, the sample used to define the mass-luminosity relation includes 17 clusters with precise mass estimates and X-ray luminosities \( L_X \gtrsim 10^{44} \, h_{50}^{-2} \, \text{erg s}^{-1} \), spanning the redshift range \( 0.08 < z < 0.47 \).

3.2.1 Chandra mass measurements

The Advanced CCD Imaging Spectrometer (ACIS) on Chandra permits direct, simultaneous measurements of the X-ray gas temperature and density profiles and, via the hydrostatic assumption, the total mass distributions in galaxy clusters. We have used Chandra to obtain precise mass measurements for a sample of ten of the most X-ray luminous, dynamically relaxed clusters identified from the RASS. The relaxed dynamical states of the clusters are demonstrated by their regular X-ray and optical morphologies, X-ray temperature maps and, in 6/10 cases, from the availability of consistent, independent mass measurements from gravitational lensing studies (see Section 4.4).

\* We make the comparison between the observed and predicted luminosity functions at \( z = 0.21 \), the mean redshift of the BCS clusters with \( L_X \gtrsim 10^{45} \, h_{50}^{-2} \, \text{erg s}^{-1} \). Shifting this redshift by \( \pm 0.05 \) does not significantly change the results.
The Chandra observations were made using the ACIS and the back-illuminated S3 detector between 1999 August 30 and 2001 November 3. We have used the level-2 event lists provided by the standard Chandra pipeline processing. These lists were cleaned for periods of background flaring using the CIAO software package, resulting in the net exposure times summarized in Table 2.

The data have been analysed using the methods described by Allen et al. (2001a, 2002b) and Schmidt et al. (2001; these papers present detailed mass analyses of Abell 2390, RXJ1347-1145 and Abell 1835, respectively). In brief, concentric annular spectra were extracted from the cleaned event lists, centred on the peaks of the X-ray emission from the clusters. (For RXJ1347-1145, the data from the southeast quadrant of the cluster were excluded due to ongoing merger activity in that region; Allen et al. 2002b.) The spectra were analysed using XSPEC (version 11.0: Arnaud 1996), the MEKAL plasma emission code (Kaastra & Mewe 1993; incorporating the Fe-L calculations of Liedhal, Osterheld & Goldstein 1995), and the photoelectric absorption models of Bahcall-Church & McCammon (1992; the absorbing column density was included as a free parameter in the fits, alleviating problems associated with uncertainties in the quantum efficiency of the detectors at low energies). Only data in the 0.5–7.0 keV energy range were used. The spectra for all annuli were modelled simultaneously in order to determine the deprojected X-ray gas temperature profiles, under the assumption of spherical symmetry.

For the mass modelling, azimuthally averaged surface brightness profiles were constructed from background subtracted, flat-fielded images with a 0.984×0.984 arcsec$^2$ pixel scale (2×2 raw detector pixels). When combined with the deprojected spectral temperature profiles, the surface brightness profiles can be used to determine the X-ray gas mass and total mass profiles in the clusters. For this analysis,

\[ E(z) \ 

Table 3. Summary of the Chandra mass measurements. Column 2 gives the evolution parameter, $E(z)$, appropriate for each cluster. Columns 3 and 4 summarize the best-fitting NFW model parameters: the scale radius, $r_s$ (in $h^{-1}$ comoving Mpc) and concentration parameter, $c$. Columns 5 and 6 give the virial radii, $r_{200}$ (in $h^{-1}$ comoving Mpc) and masses, $M_{200}$ (in $10^{14} h^{-1} M_\odot$). Error bars are 68 per cent confidence limits for a single interesting parameter, determined from the $\chi^2$ grids. A flat ΛCDM cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ is assumed.

| Cluster    | $E(z)$  | $r_s$  | $c$  | $r_{200}$ | $M_{200}$ |
|------------|---------|--------|------|----------|-----------|
| Abell 478  | 1.042   | 0.94+0.18 | 0.11 | 3.67+0.31 | 3.45+0.27 | 25.8+6.7  |
| PKS0745-191| 1.050   | 0.90+0.11 | 0.17 | 3.83+0.52 | 3.44+0.20 | 26.0+6.9  |
| Abell 963  | 1.107   | 0.49+0.07 | 0.27 | 3.02+0.74 | 2.96+0.38 | 19.0+4.9  |
| Abell 2390 | 1.122   | 1.06+0.52 | 0.23 | 3.20+1.79 | 3.40+0.82 | 28.9+8.6  |
| Abell 2667 | 1.124   | 0.95+0.24 | 0.25 | 4.21+0.53 | 3.24+0.44 | 25.5+4.2  |
| Abell 1835 | 1.135   | 0.77+0.12 | 0.25 | 4.58+2.06 | 2.56+1.04 | 13.1+2.3  |
| Abell 611  | 1.158   | 0.56+0.97 | 0.25 | 8.71+1.22 | 1.95+0.31 | 5.95+1.17 |
| MS2137-2353| 1.174   | 0.29+0.04 | 0.22 | 6.34+0.61 | 3.28+0.56 | 33.2+19.9 |
| RXJ1347-1145| 1.271   | 0.52+0.25 | 0.22 | 6.34+0.61 | 3.28+0.56 | 33.2+19.9 |
| 3C295      | 1.279   | 0.23+0.10 | 0.06 | 7.90+1.72 | 1.77+0.22 | 5.27+1.43 |

Table 4. Summary of the mass results based on the Dahle et al. (2002) weak lensing observations. Columns 2 and 3 summarize the redshifts and evolution parameters for the clusters. Column 4 lists the virial masses, $M_{200}$ (in $10^{14} h^{-1} M_\odot$), determined from fits to the observed tangential shear profiles using NFW models with a fixed concentration parameter, $c = 5$. No correction for the effects of correlated substructure has been applied. Error bars are 68 per cent confidence limits for a single interesting parameter. A ΛCDM cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ is assumed.

| Cluster    | $z$    | $E(z)$  | $M_{200}$ |
|------------|--------|---------|-----------|
| Abell 520  | 0.203  | 1.106   | 14.3+5.0  |
| Abell 299  | 0.206  | 1.107   | 4.5+3.4  |
| Abell 963  | 0.206  | 1.107   | 7.0+4.4  |
| Abell 141  | 0.230  | 1.122   | 13.6+6.2  |
| Abell 267  | 0.230  | 1.122   | 15.5+4.4  |
| Abell 1576 | 0.299  | 1.165   | 17.5+4.3  |
| Abell 1995 | 0.320  | 1.179   | 20.1+4.6  |
| Abell 1351 | 0.328  | 1.184   | 42.3+7.8  |

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we have used an enhanced version of the image deprojection code described by White, Jones & Forman (1997) with distances calculated using the code of Kayser, Helbig & Schramm (1997).

We have parameterized the cluster mass (luminous plus dark matter) profiles using a Navarro, Frenk & White (1997; hereafter NFW) model with

$$\rho(r) = \frac{\rho_c(z) \delta_c}{(r/r_s)(1 + r/r_s)^2},$$

where \(\rho(r)\) is the mass density, \(\rho_c(z) = 3H(z)^2/8\pi G\) is the critical density for closure at redshift \(z\), \(r_s\) is the scale radius, \(c\) is the concentration parameter \(c = r_{200}/r_s\) and \(\delta_c = 200c^3/3[\ln(1 + c) - c/(1 + c)]\). The normalizations of the mass profiles can also be expressed in terms of an effective velocity dispersion, \(\sigma = \sqrt{50r_s(cH(z))}\) (with \(r_s\) in units of Mpc and \(H(z)\) in km s\(^{-1}\) Mpc\(^{-1}\)).

The best-fit NFW parameter values and 68 per cent confidence limits are summarized in Table 3. This table also lists the ‘virial’ radii, \(r_{200}\), where the mean enclosed density is 200 times the critical density of the universe at the redshifts of the clusters, and the masses within these radii, \(M_{200}\). (The uncertainties on parameters are determined directly from the \(\chi^2\) grids.) Note that the Chandra data only cover the central regions of the clusters out to radii 0.2 – 0.5 \(r_{200}\) and thus some extrapolation of the models, assuming that the NFW parameterization remains valid to \(r_{200}\), is required in calculating the virial masses.

3.2.2 Weak lensing mass measurements

In order to expand the sample of clusters used to construct the mass-luminosity relation, and allow certain tests of the fairness of this relation (Section 4.4), we have also included data for clusters with precise mass measurements from wide field weak gravitational lensing studies. In particular, we have included data from the study of Dahle et al. (2002), who present aperture mass profiles for eight clusters with \(L_{X,0.1-2.4} > 10^{44} h_{200}^{-2}\) erg s\(^{-1}\) (Section 2.3.2) obtained from wide-field imaging with the University of Hawaii (UH) 2.24m telescope and UH8K camera. One of these clusters, Abell 963, is also in our Chandra sample. Since the Chandra and lensing data for this cluster give consistent \(M_{200}\) results, but the Chandra data provide tighter constraints, we use the Chandra result in our default analysis. (The Dahle et al. 2002 mass measurement for Abell 963 is, however, used in our analysis of the weak lensing subsample, discussed in Section 4.4.) The weak lensing mass measurements made with the UH8K camera used a control annulus of 550 arcsec, which corresponds to \(3.4 h_{200}^{-1}\) Mpc for a cluster at \(z = 0.3\).

Unlike X-ray mass measurements, which are based on the hydrostatic assumption, lensing mass measurements are independent of the dynamical state of the gravitating matter. As a result, there are no restrictions in the Dahle et al. (2002) sample on the dynamical states of the clusters; several of the systems appear to be undergoing major merger events. The inclusion of the Dahle et al. (2002) clusters allows us both to examine the effects of dynamical activity on the mass-luminosity relation and assess whether the use of Chandra data for dynamically relaxed clusters is likely to bias our determination of cosmological parameters (see Section 4.4).

We have used the aperture mass profiles presented by Dahle et al. (2002) to recover the mean tangential shear profiles for the clusters and have fitted these with NFW models. In general, the lensing data are unable to constrain both the concentration parameter and scale radius of the NFW models and so we have fixed \(c = 5\) for this analysis, a typical value for such massive clusters inferred from simulations (e.g. Navarro et al. 1997), and consistent with the Chandra results listed in Table 3. The masses of the clusters determined from the Dahle et al. (2002) data are summarized in Table 4.

In addition to the Dahle et al. (2002) UH8K data, accurate weak lensing mass measurements are also available for Abell 2390 (Squires et al. 1996) and RXJ1347-1145 (Fischer & Tyson 1997). In both cases the lensing mass measurements at \(r_{200}\) are in good agreement with the Chandra results (Allen et al. 2001a, 2002b). Dahle et al. (2002) also present a weak lensing mass measurement for Abell 1835 using a smaller camera, which is consistent with the Chandra result in Table 3.

On the basis of numerical simulations, Metzler, White & Loken (2001) argue that large scale structure in the environments \((r \sim 10 – 20 h^{-1} \text{ Mpc})\) of galaxy clusters are likely cause measurements of \(M_{200}\) from weak lensing to overestimate the true masses of clusters by, on average, \(\sim 30\) per cent. Previous work by Cen (1997) and Reblinksy & Bartelmann (1999) had argued for smaller effects, of the order of \(\sim 10\) per cent. Based on these studies, we have included a statistical correction of 20 per cent to the lensing masses in our determination of cosmological parameters i.e. we multiply the masses in Table 4 by 0.83. We note that the effects of more distant, uncorrelated structure along the lines of sight to the clusters are not expected to bias the lensing mass measurements (e.g. Metzler et al. 2001, Hoekstra 2002).

3.2.3 ROSAT luminosity measurements

In constructing the mass-luminosity relation, we have used pointed ROSAT observations to determine the total, intrinsic 0.1 – 2.4 keV luminosities of the clusters. This minimizes systematic uncertainties by matching the observing band and (as far as possible) detector technology to that used for the RASS observations. The agreement between the BCS and REFLEX fluxes, which are based on RASS data, and the fluxes determined from deep, pointed ROSAT observations is good (Ebeling et al. 1998; Böhringer et al. 2002). The details of the pointed ROSAT observations are summarized in Table 5. The data were analysed using the XSELECT pack-
Table 5. Summary of the pointed ROSAT observations. Columns 2 and 3 list the date of observation and the detector used. Column 4 gives the exposure times in ks. Column 5 lists the intrinsic $0.1 - 2.4$ luminosities, $L_{0.1-2.4}$ (in $10^{44} h_{50}^{-2}$ erg s$^{-1}$). Error bars are 68 per cent confidence limits. A $\Lambda$CDM cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ is assumed.

| Date         | Detector | Exposure | $L_{0.1-2.4}$ |
|--------------|----------|----------|---------------|
| Abell 478    | 1991 Aug 31 | PSPC      | 21.4          | 22.7 ± 0.2   |
| PKS0745-191  | 1993 Oct 15 | PSPC      | 10.5          | 28.2 ± 0.6   |
| Abell 520    | 1998 Mar 09 | HRI       | 27.7          | 18.0 ± 0.8   |
| Abell 209    | 1996 Jul 01 | HRI       | 10.6          | 15.2 ± 1.0   |
| Abell 963    | 1991 Apr 20 | HRI       | 10.5          | 13.4 ± 1.0   |
| Abell 141    | 1996 Dec 10 | HRI       | 16.2          | 12.6 ± 0.7   |
| Abell 267    | 1996 Jan 03 | HRI       | 15.7          | 11.1 ± 0.9   |
| Abell 2390   | 1993 Nov 13 | PSPC      | 10.3          | 31.7 ± 0.5   |
| Abell 2667   | 1994 Dec 14 | HRI       | 21.3          | 29.2 ± 1.1   |
| Abell 1835   | 1993 Jul 03 | PSPC      | 6.2           | 38.3 ± 0.9   |
| Abell 611    | 1996 Apr 04 | HRI       | 17.3          | 11.4 ± 1.1   |
| Abell 1576   | 1993 Nov 07 | PSPC      | 16.3          | 13.4 ± 0.4   |
| MS2137-2353  | 1993 Nov 07 | PSPC      | 10.5          | 19.1 ± 0.9   |
| Abell 1995   | 1995 Nov 13 | HRI       | 16.5          | 13.7 ± 1.4   |
| Abell 1351   | 1995 Apr 29 | HRI       | 31.9          | 16.5 ± 3.0   |
| RXJ1347-1145 | 1995 Jan 28 | HRI       | 15.8          | 81.1 ± 3.1   |
| 3C295        | 1995 Jun 19 | HRI       | 22.2          | 12.2 ± 1.3   |

Figure 1. The observed mass-luminosity relation. Chandra mass measurements for dynamically relaxed clusters are indicated by filled circles. Weak lensing mass measurements are indicated by open squares. The best-fitting power law model from Section 3.2.4 is shown as the dashed curve. The two most significant outliers above (Abell 1351) and below (Abell 209) the best-fit curve appear to be undergoing major merger events (see Section 4.4). A $\Lambda$CDM cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ is assumed.

3.2.4 The observed mass-luminosity relation

The mass-luminosity relation for the 17 X-ray luminous clusters in our sample, measured within radii $r_{200}$ corresponding to a density contrast $\Delta = 200$ with respect to the critical density of the universe at the redshifts of the clusters, is shown in Fig. 1. Those clusters with mass measurements from Chandra X-ray data are indicated by filled circles. Clusters with weak lensing mass measurements are marked with open squares. The four clusters with Chandra mass measurements and consistent weak lensing results are indicated by filled circles surrounded by open squares. A flat $\Lambda$CDM cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ is assumed.

We have fitted the data using a model of the form

$$
\log_{10} \left( \frac{E(z) M_{200}}{h_{50}^{-2} M_{\odot}} \right) = \alpha \log_{10} \left( \frac{L}{E(z) 10^{44} h_{50}^{-2} \text{erg s}^{-1}} \right) + \log_{10} \left( \frac{M_0}{h_{50}^{-2} M_{\odot}} \right),
$$

(14)

age (version 2.0) and XSPEC (version 11.0). The emission-weighted temperatures and metallicities of the clusters were set to the values measured with Chandra. Where Chandra data were not available, the metallicity was set to 0.3 solar and the temperature was determined iteratively from the luminosity-temperature relation of Allen & Fabian (1998). Note, however, that the precise settings of the temperatures and metallicities have little effect on the measured $0.1 - 2.4$ keV luminosities for such hot, massive clusters. The absorbing column densities were set to the Galactic values determined by Dickey & Lockman (1990).

The intrinsic $0.1 - 2.4$ keV luminosities for a $\Lambda$CDM cosmology with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $h = 0.5$ (the same cosmology assumed in the BCS and REFLEX luminosity functions in Table 1) are listed in Table 5.
Using the BCES($Y|X$) estimator of Akritas & Bershady (1996), which accounts for errors in both axes and the presence of possible intrinsic scatter, we obtain best-fitting values and 68 per cent confidence limits from $10^6$ bootstrap simulations of $\log_{10}(M_0/h_{50}^{-1} M_\odot) = 14.29^{+0.20}_{-0.23}$ and $\alpha = 0.76^{+0.15}_{-0.13}$. The distribution of $M_0$ and $\alpha$ values are shown in Fig. 2. The observed slope is in good agreement with the expected slope of the mass–bolometric luminosity relation of $\alpha = 0.75$, from models of simple gravitational collapse.

The rms scatter of the observed $\log [E(z) M_{200}]$ values about the best fitting curve is 0.22 for the full sample of 17 clusters, 0.15 for the 10 dynamically relaxed clusters studied with Chandra, and 0.29 for the 8 clusters with weak lensing measurements from Dahle et al. (2002). The scatter in the full sample is similar to that measured by Reiprich & Böhringer (2001) using ASCA and ROSAT X-ray data. Note, however, that our best-fitting curve implies a slightly higher X-ray luminosity for a given mass.

4 DETERMINATION OF COSMOLOGICAL PARAMETERS

4.1 Monte Carlo method

Using the theoretical prescription for the mass function of galaxy clusters described in Section 2, the observed BCS and REFLEX X-ray luminosity functions summarized in Table 1, and the mass-luminosity data discussed in Section 3.2.4, we can determine $\sigma_8$ as a function of $\Omega_m$.

In determining our results on cosmological parameters, we have used a Monte Carlo approach. For each iteration of the code, we construct a random bootstrap sample of the $M_{200}$ and $L_{0.1–2.4}$ values listed in Tables 3–5 and examine a grid of $(96 \times 131)$ $\Omega_m$ and $\sigma_8$ values, covering the plane $0.05 < \Omega_m < 1.0$ and $0.20 < \sigma_8 < 1.50$. For each value of $\Omega_m$, we scale the mass and luminosity values appropriately and determine the best-fitting mass-luminosity relation. For each $\Omega_m$, $\sigma_8$ parameter pair, we construct a model X-ray luminosity function, including the effects of random scatter in the mass-luminosity relation, which we characterize using a log-normal distribution. The model X-ray luminosity function is then compared with the observed BCS and REFLEX data, also scaled to the appropriate cosmology, and the $\chi^2$ difference between the two is calculated. In this way, the best fitting $\Omega_m$, $\sigma_8$ pair for the grid is determined, which provides us with a single sample result. The whole process is repeated for $10^6$ iterations to produce the final results, discussed below.

4.2 Results on $\sigma_8$ as a function of $\Omega_m$

Fig. 3 shows the 68.3 and 95.4 per cent confidence contours in the $\sigma_8 – \Omega_m$ plane from one million iterations of the Monte Carlo code. The results exhibit the well known degeneracy between $\sigma_8$ and $\Omega_m$, which can be approximated (for $0.1 < \Omega_m < 0.4$) by the simple fitting formula $\sigma_8 = (0.510 \pm 0.019) \Omega_m^{0.253 \pm 0.024}$. Our analysis favours low values for $\Omega_m$: marginalizing over $\sigma_8$, we find $\Omega_m < 0.34$ at 68 per cent confidence.

Note that when neglecting the effects of scatter and uncertainties in the normalization and slope of the mass-luminosity relation, we obtain the best fit ($\chi^2 = 5.1$ for 4 degrees of freedom) for $\Omega_m = 0.23$ and $\sigma_8 = 0.74$. 
4.3 Breaking the $\sigma_8 - \Omega_m$ degeneracy using the Chandra $f_{\text{gas}}(z)$ data

The usual approach, discussed in the literature, to break the degeneracy between $\sigma_8$ and $\Omega_m$ (Fig. 3) is to use the redshift evolution of the luminosity and/or temperature function of clusters, which depends strongly on the mean mass density of the universe (see references in Section 5.2). However, this approach is both observationally challenging in terms of identifying complete, high-redshift cluster samples, and prone to systematic uncertainties due to potentially increased levels of dynamical activity and contaminating AGN emission at high redshifts, which introduce additional scatter into the mass-luminosity and mass-temperature relations.

Fortunately, the Chandra data offer a powerful alternative method to break the degeneracy between $\sigma_8$ and $\Omega_m$ using the observed X-ray gas mass fractions, $f_{\text{gas}}$, in the clusters and their apparent redshift dependence (e.g. White & Frenk 1991; White et al. 1993; Sasaki 1996; Pen 1997; Ettori & Fabian 1999; Allen et al. 2002a; Ergoddu, Ettori & Lahav 2002; Ettori, Tozzi & Rosati 2003). The matter content of rich clusters of galaxies is thought to provide an almost fair test of the baryon fraction in clusters is slightly depressed with respect to the universe as a whole (e.g. White et al. 1993). The observed ratio of baryonic to total mass in clusters is therefore expected to closely match the ratio of the cosmological parameters $\Omega_b/\Omega_m$, where $\Omega_b$ is the mean baryon density of the universe. The apparent redshift dependence of the $f_{\text{gas}}$ measurements arises from the fact that the measured $f_{\text{gas}}$ values depend upon the assumed angular diameter distances to the sources as $f_{\text{gas}} \propto D_{A}^{1.5}$. Thus, although we expect the measured $f_{\text{gas}}$ values to be invariant with redshift, this will only appear to be the case when the assumed cosmology matches the true, underlying cosmology.

The observed $f_{\text{gas}}$ profiles for the ten relaxed clusters studied with Chandra, for an assumed $h = 0.5$ standard cold dark matter (SCDM) cosmology with $\Omega_m = 1.0$ and $\Omega_{\Lambda} = 0.0$, are shown in Fig. 4. (The six clusters previously studied by Allen et al. 2002a are shown in a lighter shading.) With the possible exception of Abell 963, we see that the $f_{\text{gas}}$ profiles appear to have converged, or be close to converging, within $r_{2500}$. We note that the data for the two nearest clusters, Abell 478 and PKS0745-191, do not extend to $r_{2500}$. However, their $f_{\text{gas}}$ profiles appear to be close to converging within $r \sim 0.6 r_{2500}$.

Fig. 5 shows the $f_{\text{gas}}$ measurements at $r_{2500}$ for the ten clusters with convergent $f_{\text{gas}}$ profiles. Following Allen et al. (2002a), we have fitted these data with the model

$$f_{\text{gas}}^{\text{mod}}(z) = \frac{b \Omega_b}{(1 + 0.19 \sqrt{h}) \Omega_m} \left[ \frac{h}{0.5} \right]^{1.5} \left[ \frac{D_{\Lambda}^{1.0} - 1}{D_{\Lambda}^{1.0} - 1} \right]^{1.5} \left( \Omega_{\Lambda} = 0 \right) \left( \Omega_{\Lambda} = 1 \right) \left( \Omega_{\Lambda} = 0 \right) \left( \Omega_{\Lambda} = 1 \right) \left( \Omega_{\Lambda} = 0 \right) \left( \Omega_{\Lambda} = 1 \right) \left( \Omega_{\Lambda} = 0 \right) \left( \Omega_{\Lambda} = 1 \right)(15)$$

and determined the best-fitting value of $\Omega_m$, for an assumed flat $\Lambda$CDM cosmology. The parameter $b$ is a bias factor that is motivated by gasdynamical simulations, which suggest that the baryon fraction in clusters is slightly depressed with respect to the universe as a whole (e.g. Cen & Ostriker 1994; Eke, Navarro & Frenk 1998a; Frenk et al. 1999; Bialek et al. 2001). We include a Gaussian prior on the bias factor, $b = 0.93 \pm 0.05$, a value appropriate for hot ($kT > 5$ keV), massive clusters in the redshift range $0 < z < 0.5$ from the simulations of Bialek et al. (2001). We also include Gaussian priors on the Hubble constant, $h = 0.72 \pm 0.08$, the final result from the Hubble Key Project reported by Freedman et al. (2001), and $\Omega_m h^2 = 0.0205 \pm 0.0018$ (O’Meara et al. 2001), from cosmic nucleosynthesis calculations constrained by the observed abundances of light elements at high redshifts. The constraints on $\Omega_m$ determined from this analysis are shown as the dark, solid curve in Fig. 6. We obtain $\Omega_m = 0.291^{+0.040}_{-0.036}$ at 68 per cent confidence. Also shown (dashed curve) are the results obtained when fixing the bias parameter $b = 1.0$, for which $\Omega_m = 0.314^{+0.038}_{-0.035}$.

We can now combine (by multiplying the relevant probability densities) the constraints on $\Omega_m$ from Fig. 6 with the joint constraints on $\sigma_8$ and $\Omega_m$, shown in Fig. 3, to obtain our final results on $\sigma_8$ and $\Omega_m$. These are shown in Fig. 7. Using the Gaussian prior on the bias parameter, we obtain $\sigma_8 = 0.695 \pm 0.042$ and $\Omega_m = 0.287 \pm 0.036$ (marginalized 68 per cent confidence limits). With $b = 1.0$ fixed, we obtain $\sigma_8 = 0.683 \pm 0.041$ and $\Omega_m = 0.309 \pm 0.035$.

4.4 Systematic uncertainties and the effects of merger events

An important aspect of the present work is the reduced level of systematic uncertainty with respect to most previous studies based on the local abundance of galaxy clusters. In the first case, the independent BCS and REFLEX studies have well determined selection functions (Ebeling et al. 1998, 2000; Böhringer et al. 2002) and provide precise, consistent results on the local X-ray luminosity function. With the large size of the combined BCS-plus-REFLEX data set (which covers two thirds of the sky) we have been able to limit our analysis to the most luminous clusters, with...
Figure 5. The apparent redshift variation of the X-ray gas mass fraction measured at $r_{2500}$ (with root-mean-square 1σ errors) for the nine clusters with convergent $f_{\text{gas}}$ profiles in Fig. 4 (see text). The results for the six clusters previously discussed by Allen et al. (2002a) are shown in lighter shading. The solid curve shows the predicted $f_{\text{gas}}(z)$ behaviour for a flat $\Lambda$CDM cosmology with $\Omega_m = 0.291$.

Figure 6. The constraints on $\Omega_m$ from the Chandra $f_{\text{gas}}(z)$ data in Fig. 5. A flat $\Lambda$CDM cosmology and Gaussian priors of $h = 0.72 \pm 0.08$ and $\Omega_b h^2 = 0.0205 \pm 0.0018$ are assumed. The solid curve shows the results obtained using a Gaussian prior on the bias factor, $b = 0.93 \pm 0.05$. The dashed curves show the results for $b = 1$ (fixed). The 1, 2 and 3 sigma confidence limits are marked as dotted lines.

Figure 7. The 68.3, 95.4 and 99.7 per cent confidence contours on $\sigma_8$ and $\Omega_m$ from the combined analysis of the BCS+REFLEX luminosity function and Chandra $f_{\text{gas}}(z)$ data. A flat $\Lambda$CDM cosmology is assumed.

$L_{X,0.1-2.4} > 10^{45} h^{-2} \text{erg s}^{-1}$ (for an $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, cosmology), for which the mass-luminosity relation has been calibrated using Chandra and ROSAT X-ray data and weak lensing observations (Section 3.2.4). The complicating effects of cooling and pre-heating are minimized for such massive clusters and a simple power-law model provides a reasonable description of the mass-luminosity relation (Fig. 1), albeit with some residual intrinsic scatter that is in part related to the different dynamical histories of the clusters and residual systematic effects in the mass measurements (Sections 3.2.2, 3.2.4; see also below). Repeating the analysis using only the BCS luminosity function data, or only the REFLEX data, or using a higher luminosity cut in the luminosity function (e.g. $L_{X,0.1-2.4} > 1.3 \times 10^{45} h_{50}^{-2} \text{erg s}^{-1}$), leads to consistent results (although with larger statistical uncertainties). Consistent results on cosmological parameters are also obtained if we replace the Evrard et al. (2002) critical spherical overdensity mass function (Section 2) with the Jenkins et al. (2001) mean spherical overdensity (SO324) prescription, scaling the cluster masses accordingly.

The dominant statistical and systematic uncertainties in the analysis are associated with the mass-luminosity relation. However, care has been taken to minimize the systematic uncertainties. In particular, we have limited our study of the mass-luminosity relation to clusters with precise mass measurements from Chandra or wide-field gravitational lensing studies. The clusters studied with Chandra have been selected from the RASS as the most luminous, dynamically relaxed (in terms of their X-ray and optical morphologies) clusters known. The relaxed natures of these systems means that they are the clusters for which X-ray mass measurements are most reliable. For Abell 963,
1835, 2390 and RXJ1347-1145, independent confirmation of the Chandra mass measurements at $r_{200}$ is available from weak gravitational lensing studies (Dahle et al. 2002; Squires et al. 1996; Fischer & Tyson 1997; Allen et al. 2001a, 2002b). A programme of weak lensing measurements for the other clusters in our Chandra sample is underway (Gray et al., in preparation). In addition, consistent strong lensing mass measurements are available for Abell 1835, 2390, RXJ1347.5-1145, MS2137.3-2353 and PKS0745-191 (Schmidt et al. 2001, Allen et al. 2001a, 2002b, Schmidt et al., in preparation). The presence of significant non-thermal pressure support on large spatial scales in these clusters can therefore be excluded.

As well as increasing the size of our calibration sample, the inclusion of the Dahle et al. (2002) weak lensing data is important in that it allows us to examine the distribution of less dynamically relaxed systems in the mass-luminosity plane. Whereas X-ray data can only be used to obtain precise mass measurements for dynamically relaxed clusters (the X-ray mass measurements are based on the assumption of hydrostatic equilibrium in the X-ray gas), lensing mass measurements are essentially independent of the dynamical state of the gravitating matter and so can be extended to systems undergoing merger events. The Dahle et al. (2002) sample includes clusters with a range of dynamical states. Inspection of ROSAT and Chandra images of these systems shows that many exhibit significant dynamical activity, with Abell 141, 209, 520, 1351 and 1576 undergoing major merger events. (Similar conclusions are drawn by Dahle et al. 2002 from their optical data). The BCS and REFLEX samples, from which the X-ray luminosity functions have been constructed (Section 3.1), include all clusters above the respective X-ray flux limits, independent of their dynamical states. It is therefore important that the mass-luminosity relation provides a fair translation between luminosity and mass for all clusters included in the samples. Although the 17 clusters in our mass-luminosity relation do not represent a complete subsample of the BCS and REFLEX data sets, inspection of Fig. 1 suggests that the Dahle et al. (2002) clusters exhibit a similar mean mass per unit X-ray luminosity to the relaxed systems studied with Chandra, albeit with two significant outliers from the best-fit curve (see below) and, therefore, that the application of the mass-luminosity relation determined here to the BCS and REFLEX samples is reasonable.

We have carried out a more rigorous test of the validity of the mass-luminosity relation by repeating the determination of cosmological parameters using other mass-luminosity relations constructed from subsamples of the calibration data. In the first case, a mass-luminosity relation based on the Chandra mass measurements (10 clusters) was examined. In the second case, only the weak lensing measurements were used (8 clusters). These results were then compared with those obtained from the full sample (17 clusters, excluding the lensing result for Abell 963). The normalized, marginalized probability distributions for $\sigma_8$ are shown in Fig. 8. For the Chandra subsample, we obtain $\sigma_8 = 0.69 \pm 0.04$. For the lensing subsample, we find $\sigma_8 = 0.71^{+0.17}_{-0.12}$ (68 per cent confidence limits). Both results are consistent with those obtained for the whole sample, $\sigma_8 = 0.70 \pm 0.04$, although for the lensing subsample the increased scatter in the mass-luminosity relation leads to larger error bars. It appears, then, that the inclusion of Chandra mass measurements for dynamically relaxed clusters does not bias the best-fit result on $\sigma_8$ significantly, but does reduce the formal statistical uncertainties. Assessing whether this leads us to underestimate the true uncertainties in $\sigma_8$ will require more data and the ability to disentangle whether the scatter in the lensing results arises primarily from the effects of correlated substructure (or similar systematic problems in the lensing analysis), or the effects of dynamical activity on the X-ray gas (see below).

Ricker & Sarazin (2001), Ritchie & Thomas (2002) and Randall, Sarazin & Ricker (2002) present simulations of the effects of mergers on the X-ray properties of clusters for a variety of mass ratios, impact parameters and central gas densities. These authors suggest that major mergers can lead to significant short-term increases in the luminosities of clusters during the periods of closest approach, which are then followed by dips in luminosity as the merging dark matter cores move apart, before the cluster returns to equilibrium. From Fig. 1 we see that Abell 209 appears to have an unusually high X-ray luminosity/mass ratio, which may have been boosted by the ongoing merger activity in this cluster. (We note, however, that MS2137.3-2353, which is a highly relaxed cluster with no obvious merger activity and a sharp central density peak, also appears to have a high luminosity/mass ratio and lies below the best-fitting curve. In this case, the offset may indicate a relatively early for-
mation epoch for the cluster.) In contrast Abell 1351, which is also undergoing a major merger event, has an unusually low X-ray luminosity/mass ratio and lies above the best-fit curve in Fig. 1. Detailed simulations and further observations are required to improve our understanding of the effects of mergers on the X-ray properties of clusters. However, the indications from the present study are that the effects of mergers on the mean mass-luminosity relation for the most luminous clusters are relatively small.

Chandra measurements of the X-ray gas mass fraction, f_{gas}, in dynamically relaxed clusters provide one of the most simple and robust methods by which to measure Ω_m (see e.g. Allen et al. 2002a and references therein). The largest systematic uncertainties in the analysis lie in how well the measured baryonic mass fractions in the clusters approximate the universal mean. Lowering the bias factor, 6, by ~ 10 percent would cause the best-fitting value of Ω_m to rise by a similar amount. Similarly, although the f_{gas} profiles in Fig. 4 appear to have converged, or be close to converging, at the outer measurement radii, any rise in the f_{gas} values beyond these points would cause a corresponding reduction in Ω_m. Any change in the best-fitting value of Ω_m would then lead to a change in σ_8 as shown in Fig. 3.

Finally, we note that our Monte Carlo analysis (Section 4.1) takes account of the intrinsic scatter and uncertainties in the normalization and slope of the mass-luminosity relation. (The scatter is modelled as a log-normal distribution about the best-fitting curve, which provides a good description of the measured offsets.) Neglecting such effects would cause to the best-fitting value of σ_8 to rise slightly, and significantly reduce the formal statistical uncertainties on the measured parameters.

5 COMPARISON WITH OTHER RESULTS

5.1 Other local cluster abundance studies

Fig. 9 shows a comparison of the results on σ_8 as a function of Ω_m from the present study (thick, solid curves) with the findings from five other recent studies based on the observed local number density of galaxy clusters. The dot-dashed curve in Fig. 9 shows the result of Seljak (2002), σ_8 = (0.44 ± 0.04)Ω_m^{-0.44}(Γ/0.2)^{0.08}, using the observed local X-ray temperature function of rich clusters (Pierpaoli, Scott & White 2002) and a mass-temperature relation determined from ASCA and ROSAT observations (Finoguenov et al. 2001). The result of Seljak is consistent with ours for Ω_m ~ 0.3, although the present study leads to tighter constraints. The result of Pierpaoli et al. (2001), σ_8 = (0.50 ± 0.04)Ω_m^{-0.60} (dotted curve), based on the same local temperature function data but normalized by a theoretical mass-temperature relation, lies significantly above ours. The result of Reiprich & Böhringer (2001), σ_8 = 0.43Ω_m^{-0.38} (short dashes), based on ASCA and ROSAT observations of RASS selected clusters, is in good agreement with the present study for Ω_m ~ 0.3. The result of Viana, Nichol & Liddle (2002), σ_8 = 0.38Ω_m^{-0.48}±0.27Ω_m (long dashes), using the local REFLEX X-ray luminosity function and a mass-luminosity relation determined from ROSAT X-ray observations and a stacked weak lensing analysis of relatively low-mass clusters identified in Sloan Digitized Sky Survey (SDSS) commissioning data, is consistent with the present study for Ω_m ~ 0.3. The result of Bahcall et al. (2002), σ_8 = 0.35Ω_m^{-0.60}, obtained by combining the number density of optically-selected, relatively low-mass clusters observed in SDSS commissioning data with a mass-optical richness correlation, is in good agreement with this work for Ω_m ~ 0.3. Finally, our results are in good agreement with the recent findings of Schuecker et al. (2003; not plotted), σ_8 = 0.711^{+0.071}_{−0.031}, Ω_m = 0.341^{+0.031}_{−0.029}, from a combined analysis of the X-ray luminosity function and large-scale clustering in the REFLEX sample.

5.2 Evolution in the X-ray luminosity and temperature functions

Borgani et al. (2001) present constraints on σ_8 and Ω_m determined from an analysis of evolution in the X-ray luminosity function of clusters in the ROSAT Deep Cluster Survey, which spans the redshift range z < 1.3. Their results of σ_8 = 0.66^{+0.06}_{−0.07} and Ω_m = 0.35^{+0.14}_{−0.10} for a flat ΛCDM cosmology are consistent with those reported here.

Donahue & Voit (1999) report results from an analysis of evolution in the temperature function of clusters within z < 0.8, using the low-redshift cluster sample of Markevitch (1998) and a high redshift sample identified from the Einstein Observatory Extended Medium Sensitivity Survey (EMSS). Their results, for an assumed flat ΛCDM cosmology, of σ_8 = 0.73^{+0.03}_{−0.03} and Ω_m = 0.27 ± 0.10 are in good agreement with ours. Eke et al. (1998b) obtain σ_8 = 0.75 ± 0.15 and Ω_m = 0.36 ± 0.25 (flat ΛCDM) from an analysis of the temperature function within z < 0.4, which is consistent with the present work. Our results are marginally consistent with the values of σ_8 = 0.72 ± 0.10 and Ω_m = 0.49 ± 0.12 reported by Henry (2000) from an analysis combining the local temperature function of clusters with the properties of EMSS clusters within z < 0.6.

5.3 Cosmic shear measurements

Fig. 10 shows a comparison of the results on σ_8 as a function of Ω_m determined from the present study (dark, solid curves: as in Fig. 3) together with the findings from seven recent studies based on measurements of weak gravitational lensing due to large scale structure (cosmic shear). The upper dotted curve in Fig. 10 shows the result of Van Waerbeke et al. (2002), σ_8 = (0.57 ± 0.04)Ω_m^{(0.24±0.18)Ω_m^{−0.49. The upper short-dashed curve shows the result of Refregier, Rhodes & Groth (2002), σ_8 = (0.55 ± 0.08)Ω_m^{−0.44. The long-dashed curve shows the result of Bacon et al. (2002), σ_8 = (0.43 ± 0.06)Ω_m^{−0.68. The dot-dashed curve shows the result of Hoekstra, Yee & Gladders (2002), σ_8 = (0.45 ± 0.05)Ω_m^{−0.55. The results on σ_8 from these four studies lie 20 – 35 per cent above the present work for Ω_m ~ 0.3. (Note that in order to obtain agreement with these results, the mean mass per unit X-ray luminosity for the clusters included in the present study would need to be raised by a factor ~ 2.5, well beyond the systematic uncertainties, which are < 20 per cent.)

The lower three curves in Fig. 10 show the results from the three most recent cosmic shear studies, which have appeared on preprint servers after this paper was submitted. The lower dotted curve shows the result of Hamana et al. (2002), σ_8 = (0.41 ± 0.09)Ω_m^{−0.43. The lower dashed curve
shows the result of Brown et al. (2002), $\sigma_8 = (0.41 \pm 0.05) \Omega_m^{0.50}$. Finally, the solid curve shows the result of Jarvis et al. (2002), $\sigma_8 = (0.36 \pm 0.04) \Omega_m^{-0.57}$. Importantly, the Brown et al. (2002) and Jarvis et al. (2002) studies include improved estimates of the redshifts of the lensed, background sources. (A flat $\Lambda$CDM cosmology is assumed in all cases.) The results from the three most recent cosmic shear studies are in good agreement with the present work.

### 5.4 Cosmic microwave background anisotropies and the 2dF galaxy redshift survey

Fig. 11 shows a comparison of the 68.3 and 95.4 per cent confidence constraints on $\sigma_8$ and $\Omega_m$ obtained from the present study (inner, shaded contours), including the Chandra $f_{\rm xrb}(z)$ data and Gaussian priors on the bias parameter ($b = 0.93 \pm 0.05$), Hubble constant ($h = 0.72 \pm 0.08$) and $\Omega_b$ ($\Omega_b h^2 = 0.0205 \pm 0.0018$), with the results from analyses of cosmic microwave background (CMB) anisotropies and 2dF galaxy redshift survey data. For this comparison, we have used the published Markov Chain Monte Carlo samples of Lewis & Bridle (2002). The CMB data consist of a combination of COBE (Bennett et al. 1996), Boomerang (Netterfield et al. 2002), Maxima (Hanany et al. 2000), DASI (Halverson et al. 2002), Cosmic Background Imager (Pearson et al. 2002) and Very Small Array (Scott et al. 2002) data. The 2dF galaxy redshift survey constraints are from Percival et al. (2002). The results obtained from the CMB data alone, using the 6 parameter model of Lewis & Bridle (2002) and fixing the optical depth to reionization $\tau = 0.04$ and $\Omega_b h^2 = 0.0205 \pm 0.0018$. A flat $\Lambda$CDM cosmology is assumed in all cases.
marginalizing over \( h, \Omega_m h^2 \) and \( n \), are shown as the outer, shaded contours. The results obtained from the combined CMB+2dF data set, marginalizing over the same parameters, are shown as solid lines. A flat ΛCDM cosmology is assumed throughout. We see that the results from all three data sets are consistent at the 68 per cent confidence level. Our results are also in good agreement with the independent analyses of CMB+2dF data reported by Lahav et al. (2002), Percival et al. (2002) and Melchiorri & Silk (2002), the results from CMB data and Type Ia supernovae presented by Jaffe et al. (2001), and the analysis of the IRAS Point Source Catalogue Redshift Survey presented by Plionis & Basilakos (2001).

6 IMPLICATIONS FOR OTHER WORK

The constraints on cosmological parameters reported here \( (\sigma_8 = 0.695 \pm 0.042 \) and \( \Omega_m = 0.287 \pm 0.036 \) for an assumed flat ΛCDM cosmology\) are consistent with, though (in most cases) tighter than, those obtained from a number of other, recent studies based on the observed X-ray temperature and luminosity functions of galaxy clusters. (Schuecker et al. 2003 report results with similar precision to those presented here). Our results are also consistent with current findings from studies of anisotropies in the CMB, the distribution of galaxies in the 2dF galaxy redshift survey, the properties of type Ia supernovae, early results on large scale structure from the SDSS, and the most recent results from studies of cosmic shear.

Our results have a number of implications for other cosmological work. Firstly, the agreement between the cluster, CMB and 2dF results in Fig. 11 suggests that the optical depth to reionization is not large (\( \tau \lesssim 0.2 \); this issue will be examined in more detail in future work). Secondly, our result on \( \sigma_8 \) implies that the possible excess power detected at high multipoles \( (l \sim 2000 - 3000) \) in the CMB anisotropy power spectrum with the Cosmic Background Imager (Mason et al. 2002; Bond et al. 2002) is unlikely to be due to the Sunyaev-Zeldovich (SZ) effect. Our results also have important implications for future X-ray and SZ clusters surveys, since the number of clusters detected at high redshifts with high X-ray luminosities, temperatures and SZ fluxes will be much lower in a \( \sigma_8 \sim 0.7 \) universe than predicted by simulations with \( \sigma_8 \sim 1 \).

Finally, we suggest that given the precision of the constraints on \( \Omega_m \) available from current Chandra \( f_{\text{gas}}(z) \) data for relaxed clusters, and the relatively small systematic uncertainties involved, the \( f_{\text{gas}}(z) \) data should be considered as a powerful probe in future cosmological work, complementary to measurements of redshift evolution in the X-ray temperature and luminosity functions of galaxy clusters.

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