Cross diffusion, radiation and chemical reaction effects on MHD combined convective flow towards a stagnation-point upon vertical plate with heat generation

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Abstract. The impacts of cross diffusion on stagnation point MHD combined convective flow approaching a plate which is upright in a porous surrounding subject to chemical reaction, heat generation and radiation is explored. Relations of similarity are employed for the conversion of flow relations as ordinary differential equations upon the application of relations of similarity, shooting method and Runge-Kutta algorithm are applied to find the solution. An analysis is made upon the graphical depictions on velocity of the liquid, its temperature and its concentration with respect to some physical entities and conclusions drawn. It is noticed that velocity shoots up across the boundary layer with a rise in heat generation, Dufour number and Biot number whereas it reduces for strong chemical reaction. For ascending values of Dufour number, chemical reaction parameter and Biot number, the temperature soars up and as a result, thickness of thermal boundary layer inflates.

Keywords: MHD; Radiation; combined convection; stagnation-point; porous medium; Soret/Dufour effect.

1 Introduction

The case of mass and energy transport of an electrically conducting liquid flowing on a porous plate upon a magnetic domain has attracted numerous investigators in view of reactors in nuclear power stations, the controlling of boundary layer in aerodynamics and plasma studies. Dufour and Soret effects on MHD mass energy transfer with chemical reaction and convective condition on boundary over a plate placed upright were reported in [1]. Radiation effects on the flowing of MHD electrically conducting liquid over a porous surrounding semi-infinite vertical plate with unsteady suction were dealt in [2]. Lie group analysis was employed in [3] to investigate the normal convection past a semi-infinite inclined plate. Many analytical as well as numerical considerations were performed on the flow upon the point of stagnation in a liquid saturated porous surrounding. In [4], the transfer of mass and heat by normal convection in the flow of a liquid upon the point of stagnating with Dufour and Soret effects, blowing/suction using model of Darcy-Boussinesq, was investigated. MHD combined convective flowing on an upright surface in porous neighborhood having convective condition on boundary was considered in [5]. A study on the similarity solution of the flow on stagnating-point in a stretching sheet with slip effects was carried out by the authors in [6]. This paper explores the behavior of the flow on MHD combined convection stagnating-point towards porous surrounding plate placed upright with mass and energy transfer influenced by radiation, heat generation, Soret and Dufour effects subject to convective boundary condition.

2 Flow Analysis

Two dimensional steady MHD flow of an incompressible electrically conducting viscous liquid in the vicinity of a point of stagnation at the surface $y = 0$ with the flow region as $y > 0$ is considered. By setting the origin fixed, we apply two forces of equal magnitude and opposite in direction along x-axis. The liquid arriving from y-axis make an impact on the wall at $y = 0$ and as a result, there are two streams leaving in upper and lower directions. In nearby surrounding of a stagnating point, flow velocity is taken as $U_0 = l \xi$, where $l$ is positive. A magnetic domain $B_0$, a constant, is employed in y direction. The electric field as well as the magnetic field is negligible because of polarization. The
concentration and the temperature of the liquid are $C_w$ and $T_w$ and that of the liquid at stretching wall are $C_s$ and $T_s$, respectively. We assume that the plate gets heated due to convection by a hot liquid having $T_s$ as temperature. Further, we assume that the electrical and viscous dissipation are negligibly small. The equations pertaining to this MHD stagnating-point flow of mass and energy transfer upon a hot vertical plate are

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0$$  \hspace{1cm} (1)

$$\tilde{u} \frac{\partial \tilde{u}}{\partial x} + \tilde{v} \frac{\partial \tilde{u}}{\partial y} = \nu \frac{\partial^2 \tilde{u}}{\partial y^2} + (T - T_s) g \beta + (\tilde{C} - \tilde{C}_s) g \beta^*$$  \hspace{1cm} (2)

$$+ \left( \frac{\sigma B_0^2}{\rho} + \frac{\nu}{K} \right) \left( U - \tilde{u} \right) + \frac{dU}{dx} U_x$$

$$\tilde{u} \frac{\partial \tilde{T}}{\partial x} + \tilde{v} \frac{\partial \tilde{T}}{\partial y} = \alpha \frac{\partial^2 \tilde{T}}{\partial y^2} + Q(T - T_s) + (D_s K_s \frac{\partial \tilde{C}}{\partial y}) (\tilde{C} - \tilde{C}_s)$$  \hspace{1cm} (3)

$$\tilde{C} \frac{\partial \tilde{C}}{\partial x} + \tilde{v} \frac{\partial \tilde{C}}{\partial y} = D_s \frac{\partial^2 \tilde{C}}{\partial y^2} + \frac{D_s K_s}{T_s} \frac{\partial \tilde{T}}{\partial y}$$  \hspace{1cm} (4)

with $\tilde{u} = 0, \tilde{v} = 0, K \frac{\partial \tilde{T}}{\partial y} = h(T - T_s), \tilde{C} = C_s$ when $y = 0$

$$\tilde{u} \to \tilde{U}_w = lx, \tilde{T} \to \tilde{T}_w, \tilde{C} \to \tilde{C}_w$$ when $y \to \infty$$$

where, with usual notations, $q = -4\alpha' / 3K'' \frac{\partial \tilde{T}^4}{\partial y}$. On the assumption that the temperature variations are too minimal, we can take $\tilde{T}$ as

$$\tilde{T}^4 \approx 4\tilde{T}_w^3 \tilde{T} - 3\tilde{T}_w^4.$$  \hspace{1cm} (6)

The dimensionless quantities are

$$Gr = \frac{\beta \theta(T_s - T_w) x^3}{\nu^2}, \hspace{0.5cm} Gc = \frac{\beta \tilde{C}(C_s - \tilde{C}_s) x^3}{\nu^2}, \hspace{0.5cm} S = \frac{Q \nu}{\alpha' l}, \hspace{0.5cm} Re = \frac{4\alpha' \tilde{T}_w^3}{kK''}$$

$$Df = \frac{K_s D_s (\tilde{C} - \tilde{C}_s)}{c \alpha^2 (T_w - T_s)}, \hspace{0.5cm} Pr = \frac{\nu}{\alpha'}, \hspace{0.5cm} Sc = \frac{\nu}{D_s}, \hspace{0.5cm} M = \frac{\sigma B_0^2}{\nu}, \hspace{0.5cm} \nu = \frac{\mu}{\rho}$$

$$Sr = \frac{K_s (\tilde{T}_w - T_s)}{T_s (\tilde{C}_s - \tilde{C}_w)}, \hspace{0.5cm} Re_s = \frac{U_w \nu}{D_s}, \hspace{0.5cm} Re_c = \frac{Gc}{Re_s};$$  \hspace{1cm} (7)

We introduce the following similarity relations

$$\eta = \sqrt{\frac{v}{u}}, \eta(x, y) = \sqrt{\frac{1}{u}} f' \tilde{\eta}, \hspace{0.5cm} \theta(\eta) = \frac{T - T_s}{T_w - T_s}, \hspace{0.5cm} \phi(\eta) = \frac{\tilde{C} - \tilde{C}_s}{C_s - \tilde{C}_s}$$  \hspace{1cm} (8)

where $\psi(x, y)$ is given by $\tilde{u} = \frac{\partial \psi}{\partial y}$ and $\tilde{v} = -\frac{\partial \psi}{\partial x}$ so that Eq.(1) satisfied identically. Eqs.(2)-(5), on the application of Eqs.(6)-(8), yield the following equivalent set of equations.

$$\tilde{f}^* + \dot{f}^* - \dot{f} + \tilde{f} = -1, (1 + 4R(3)) \theta + Pr f \theta' + S \theta + Df \phi'^* = 0,$$

$$\phi'^* + Sc \theta \theta' + Scf \phi' - K \frac{1}{C} Sc \phi = 0$$

subject to $\dot{f}^* = 0$, $\hat{f} = 0$, $\theta' = B_s (\theta - 1)$, $\phi = 1$, when $\eta = 0$, $\hat{f} = 1$, $\theta = 0$, $\phi = 0$ when $\eta \to \infty$  \hspace{1cm} (12)
Here $B = \frac{h}{k} \sqrt{\frac{c}{\rho}}$, the convective heat transfer parameter.

Solutions of Eqs.(8)-(11) are arrived by the application of shooting method combined with Runge-Kutta fourth-order algorithm. Localized Nusselt and Sherwood numbers and then the coefficient of skin-friction are given by

$$Nu = \frac{xq_s}{(T_s - \hat{T}_s)k}, \quad Sh = \frac{xq_s}{(\hat{C}_s - \hat{C}_c)D_n}, \quad \hat{C}_s = \frac{2\tau_s}{\rho U^*_z},$$

Here

$$\tau_s = \mu \frac{\partial \hat{C}}{\partial y} \bigg|_{(x=0)}, \quad q_s = -k \frac{\partial \hat{T}}{\partial y} \bigg|_{(x=0)} = -\frac{4\sigma \gamma \partial \hat{T}}{3K^* \partial y} \bigg|_{(x=0)}, \quad \dot{q}_s = -D \frac{\partial \hat{C}}{\partial y} \bigg|_{(x=0)}$$

The relations for Localized Nusselt and Sherwood numbers and then the coefficient of skin-friction are derived as

$$Re^{1/2} \cdot Nu = -(1 + 4R/3)\phi'(0), \quad Re^{1/2} \cdot Sh = -\phi'(0), \quad Re^{1/2} \cdot C_j = f^*(0)$$

3 Results and Discussion

Eqs. (9)-(11) with its associated conditions on the boundary (12) were dealt with shooting method along with fourth order Runge–Kutta algorithm for the solution. By applying shooting method, we first break down the non-linear differential equations of higher order into first order linear simultaneous differential equations and further we transform these into initial value problem. The resulting equation is evaluated by incorporating Runge-Kutta algorithm of order four. The numerical iteration procedure was performed repeatedly until the level of tolerance of error is attained.

### Table 1. Computations of $-\phi'(0)$, $-\phi'(0)$ and $f^*(0)$ for varying governing parameters

| $K$ | $M$ | $S$ | $R$ | $Cr$ | $Sr$ | $Df$ | $f^*(0)$ | $-\phi'(0)$ | $-\phi'(0)$ |
|-----|-----|-----|-----|-----|-----|-----|---------|-------------|-------------|
| 0.1 | 0.5 | 1.0 | 0.1 | 0.1 | 0.2 | 0.2 | 2.246971 | 0.0978520 | 0.5451982 |
|     |     |     |     |     |     |     | 2.319502 | 0.1008649 | 0.5458624 |
|     |     |     |     |     |     |     | 2.415747 | 0.1037373 | 0.5465281 |
| 1.0 | 0    | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 2.396287 | 0.1032492 | 0.5463949 |
|     |     |     |     |     |     |     | 3.245195 | 0.1212068 | 0.5505579 |
|     |     |     |     |     |     |     | 4.999324 | 0.09244894 | 0.5562972 |
| 1.0 | 0.1  | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 2.415747 | 0.1037379 | 0.5465281 |
|     |     |     |     |     |     |     | 2.968114 | 0.09464353 | 0.5712622 |
|     |     |     |     |     |     |     | 3.507120 | 0.08798717 | 0.5951335 |
| 1.0 | 0.1  | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 2.632905 | 0.1021410 | 0.5552750 |
|     |     |     |     |     |     |     | 3.054221 | 0.1009250 | 0.5715686 |
|     |     |     |     |     |     |     | 3.462389 | 0.1021722 | 0.5863139 |
| 1.0 | 0.1  | -0.5| 0.1 | 0.1 | 0.2 | 0.2 | 2.336319 | 0.5544120 | 0.4726206 |
|     |     |     |     |     |     |     | 2.399971 | 0.1909342 | 0.5327260 |
|     |     |     |     |     |     |     | 2.491805 | -0.3105458 | 0.6091604 |
| 1.0 | 0.1  | 0.1 | 0.1 | 0.5 | 0.2 | 0.2 | 2.416824 | 0.0901831 | 0.5486991 |
|     |     |     |     |     |     |     | 2.411396 | 0.1356563 | 0.5411857 |
|     |     |     |     |     |     |     | 2.410166 | 0.3011181 | 0.5600268 |
| 1.0 | 0.1  | 0.1 | 0.1 | -1.0| 0.2 | 0.2 | 2.448736 | 0.1746870 | 0.3498008 |
|     |     |     |     |     |     |     | 2.418123 | 0.1094058 | 0.5075595 |
|     |     |     |     |     |     |     | 2.398602 | 0.05824485 | 0.8481207 |
| 1.0 | 0.1  | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 2.415747 | 0.1037379 | 0.5465281 |
|     |     |     |     |     |     |     | 2.415123 | 0.09949238 | 0.5708835 |
|     |     |     |     |     |     |     | 2.414324 | 0.09286911 | 0.6136006 |
| 1.0 | 0.1  | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 2.407324 | 0.1385104 | 0.5409370 |
|     |     |     |     |     |     |     | 2.428890 | 0.04820966 | 0.5555990 |
|     |     |     |     |     |     |     | 2.452206 | -0.05507862 | 0.5728061 |
The coefficient of skin-friction, Nusselt and the Sherwood numbers respectively vary with \( f'(0) \), \(- \theta'(0)\) and \(- \phi'(0)\). These are computed for some values of the parameters \( K, M, Ri_x, Ri_c, S, R, Cr, Sr \) and \( Df \) when \( Pr = 1.0, Sc = 0.5 \) and their numerical values are presented in Table 1. We observe that, at the plate surface, the skin-friction become high when there is a rise in \( K, M, Ri_x, Ri_c, S \) and \( Df \) and lowers for raising values of \( R, Cr \) and \( Sr \). It can thus be understood that the influence of magnetic field, buoyancy forces, permeability of porous medium, thermal radiation and Dufour effect tend to accelerate skin-friction. The rise in the localized Nusselt number is seen when \( K \) and \( R \) rise. But it reduces for soaring values of \( Ri_x, S, Cr \) and \( Sr \). Further, we notice that, except for the thermal radiation parameter \( R \), the localized Sherwood number goes up for raising values of all other parameters.

### 3.1 Velocity Profiles

Figures 1(a-d) illustrate the consequences of physical parameters \( S, Df, Cr \) and \( Bi \) upon the velocity profile. Generally, at the plate surface, the velocity is zero and it accelerates to the free stream velocity which is away from the surface of the plate for all parameters. A rise in the liquid velocity in the case of increasing \( S \) values as depicted in Fig. 1(a). The action of Dufour number \( Df \) on velocity is shown in Fig. 1(b). From this plot, it is observed that a rise in the Dufour number \( Df \) enhances the velocity inside the boundary layer. From Fig. 1(c), we observe an opposite trend in the velocity profile. Here we see that the velocity diminishes as the chemical reaction parameter \( Cr \) goes up. The velocity profiles with respect to the Biot number \( B_i \) are plotted in Fig. 1(d). Further, when Biot number takes high value, the velocity rises.

![Velocity Profiles](image1.png)

**Fig. 1 (a-d)** Velocity profiles for different values of \( S, Df, Cr \) and \( Bi \).

### 3.2 Temperature Profiles

Variations on temperature by the influence of \( Df, Sr, Cr \) and \( Bi \) are represented in Figures 2 (a-d). The impact of Dufour number \( Df \) is depicted in Fig. 2(a). Temperature inside the boundary layer soars up
for increasing Dufour number $Df$. Profiles of temperature for varying Soret number $Sr$ is illustrated in Fig. 2(b). We see that there is a significant rise in temperature as the Soret number becomes larger. Fig. 2(c) represents the variations on temperature due to the influence of chemical reaction $Cr$. It is noted that the temperature shoots up for higher values of $Cr$. The temperature depictions for varying Biot number $Bi$ are illustrated in Fig. 2(d). It is observed that thermal boundary layer inflates and the temperature of the plate surface rises as the parameter $Bi$ takes high values.

3.3 Concentration Profiles

The graphs plotted in the Figs. 3 (a-d) depict the variations of concentration profiles with respect to the physical parameters $S$, $Df$, $Sr$ and $Cr$. For the case of different values of the heat source parameter $S$, the profiles of concentration are depicted in Fig. 3(a). We notice that the concentration declines for soaring values of $S$. We notice a similar trend in the concentration profile for the cases of $Df$, $Sr$ and $Cr$ which are respectively plotted in Figures 3(b-d). Ascending values of these parameters diminish the species concentration at the boundary layer.
4 Conclusions

Flow of MHD combined convection stagnating-point upon a plate placed upright in a porous surrounding with mass and heat transfer in the influence of internal heat generation, radiation, Dufour and Soret effects subject to convective boundary condition is studied. Shooting method with Runge-Kutta algorithm is applied for finding the solution. Conclusions of the study are as follows.

The velocity shoots up across the boundary layer with a rise in heat generation, Dufour number and Biot number whereas it reduces for strong chemical reaction. For ascending values of Dufour number, chemical reaction parameter and Biot number, the temperature soars up and as a result, thickness of thermal boundary layer inflates. A fall in liquid temperature in the boundary layer occurs for a rise in Soret number. The concentration of species diminishes when the values of S, Df, Sr and Cr ascend.

The coefficient of skin-friction and the localized Sherwood number rise whereas the localized Nusselt number diminishes when Dufour number and internal heat generation rise. When the radiation becomes high, the localized Nusselt and Sherwood numbers rise whereas the skin-friction coefficient diminishes. The coefficient of skin-friction and the localized Sherwood number rise with accelerating magnetic parameter.

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