Effective degrees of freedom and gluon condensation in the high temperature deconfined phase

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The Equation of State and the properties of matter in the high temperature deconfined phase are analyzed by a quasiparticle approach for $T > 1.2 T_c$. In order to fix the parameters of our model we employ the lattice QCD data of energy density and pressure. First we consider the pure SU(3) gluon plasma and it turns out that such a system can be described in terms of a gluon condensate and of gluonic quasiparticles whose effective number of degrees of freedom and mass decrease with increasing temperature. Then we analyze QCD with finite quark masses. In this case the numerical lattice data for energy density and pressure can be fitted assuming that the system consists of a mixture of gluon quasiparticles, fermion quasiparticles, boson correlated pairs (corresponding to in-medium mesonic states) and gluon condensate. We find that the effective number of boson degrees of freedom and the in-medium fermion masses decrease with increasing temperature. At $T \simeq 1.5 T_c$ only the correlated pairs corresponding to the mesonic nonet survive and they completely disappear at $T \simeq 2 T_c$. The temperature dependence of the velocity of sound of the various quasiparticles, the effects of the breaking of conformal invariance and the thermodynamic consistency are discussed in detail.

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I. INTRODUCTION

The state of matter at asymptotic high temperature is predicted unambiguously by Quantum Chromodynamics (QCD). Indeed, it is by now well-established that, at sufficiently high temperature $T$, the hadronic constituents form a deconfined and chirally symmetric Quark-Gluon Plasma (QGP). In this regime, due to the asymptotic freedom of QCD, the QGP is expected to be a weakly interacting gas of quark- and gluon-quasiparticles.

However, the various heavy-ion experiments at Brookhaven, GSI and CERN laboratories, devoted to the creation and detection of new forms of highly excited matter indicate that, at the reachable energy scales, the produced phase exhibits a strong collective behavior which is incompatible with a weakly interacting QGP.

In particular, standard perturbative QCD cross sections for quarks and gluons do not allow for a rapid thermalization and fail to reproduce, by hydrodynamical models, the observed phenomena (e.g. the so-called elliptic flow). Moreover, the results obtained at the Relativistic Heavy Ion Collider (RHIC) seem to show that, at large temperature and small baryon density, the new state of matter can be described as an almost perfect fluid of strongly interacting particles.

At present the best available tool to study the non-perturbative properties of QCD in these extreme conditions is the numerical simulation on the lattice. Lattice QCD (lQCD) \[1, 2\] gives precise indications on the deviations of the pressure, $p$, and of the energy density, $\epsilon$, of the system from the Stefan-Boltzman (SB) values of an ideal gas of quarks and gluons even at temperatures as high as $T \sim 4 \, T_c$. This suggests that remnants of the confining interaction play a non-trivial role in the deconfined phase \[3\].

In order to motivate our approach to the deconfined phase of QCD let us first consider how the properties of matter in the confined phase change with increasing temperature. It is well known that the hadronic content of matter and its dynamical properties change dramatically with increasing temperature \[4, 5, 6, 7\]. In particular, for $T = 100$ MeV, the largest fraction of particles ($\sim 82\%$) are pions, a smaller fraction, $\sim 12\%$, consists of strange mesons and only about 6% of particles are in excited states. The mean free path can be estimated to be $\sim 15$ fm and the volume available per particle is roughly $30$ fm$^3$. In these conditions hadrons do not overlap and the system can be described as a mixture of the various hadronic states. With increasing temperature the density increases and the hadronic content changes. As an example, for $T = 150$ MeV, the excited particles are $\sim 40\%$, the remaining $60\%$ of particles are pions and strange mesons; the mean free path is $\sim 2$ fm and the volume per particle is $4$ fm$^3$. Therefore in the confined phase, with increasing temperature, the percentage of particles in excited states increases and close to the transition temperature it is reasonable to expect that the density of particles is large with a large fraction in excited states. Since the mean free path can be of the order of the average distance between particles the system may have liquid like properties.

For temperature larger than the critical temperature, it is not clear which are the correct degrees of freedom. In any case since the lattice simulations show that the transition from the confined phase to the deconfined phase is a smooth crossover \[8, 9\], we expect that for $T \gtrsim T_c$ mesonic states may survive in the deconfined phase. This conclusion is confirmed by the surprising results of Refs. \[10, 11, 12\] where, by evaluating the static two point correlation function of a quark-antiquark pair, it has been shown that correlated $\bar{q}q$ states survive up to temperatures $\sim 2 T_c$. Moreover it seems that the dissociation temperature for the correlated states depends on the mass of the quarks, the higher the mass the higher the melting temperature \[13, 14\]. This implies that a detailed analysis is required to understand the particle content of matter or, more generally, the effective degrees of freedom above $T_c$. The presence of mesonic states in the deconfined phase, observed in lQCD, has also been discussed by various authors in the random-phase approximation \[15\] and by employing phenomenological models \[16, 17, 18, 19\] and has been shown \[20\] that may lead to a
rapid thermalization of heavy quarks in the QGP. 

Since correlated mesonic states may survive in the deconfined phase they have to be included in a consistent thermodynamical description of the system above $T_c$. In our previous paper [21], we have, indeed, shown that lQCD data of pressure, $p$, and energy density, $\epsilon$, in the range $T = 1.2 - 2 T_c$, are consistent with a description based on $\bar{q}, q, g$ quasiparticles, relatively light boson states and gluon condensate.

In the present paper we shall review in more details those results and analyze in a more general way the thermodynamics of the system and the role of the gluon condensate. Due to the short mean free path, a quasi-particle approach is probably not reliable for temperature very close to the transition point, therefore we expect our results to be valid for temperatures larger than $\sim 1.2 T_c$.

In Section II we discuss the gluon plasma i.e., pure SU(3) gauge theory. The lattice results of the so-called interaction measure, indicate deviations of the system from the conformal symmetric behavior. We find that part of this deviation is due to the gluon condensate. However, the gluon condensate is not enough to fit the lattice values of the trace anomaly at finite temperature, that measures the breaking of the conformal symmetry. Therefore there must be other mechanisms that break the scale invariance of the system (dynamically) generating a new scale. In Section II A where we discuss a quasiparticle model for the gluon plasma, we assume that such a mechanism can be described in terms of massive gluon quasiparticle whose effective number of degrees of freedom is also temperature dependent. Since at asymptotic high temperatures QCD is approximately scale invariant we consistently find that the effective gluon mass decreases with increasing temperatures. In Section III we extend our analysis to the Quark-Gluon Plasma and show that a quasiparticle description can reproduce the lattice results of the energy density and of the pressure of the QGP. This in turn leads to a constraint on the number of effective boson degrees of freedom and on the mass of the fermion quasiparticles. Section IV is devoted to conclusions and outlooks.

II. ANALYSIS OF THE GLUON PLASMA

Lattice simulations of pressure and energy density provide useful information regarding the properties of QCD in a large range of temperatures (for a review see Ref. [22]). One interesting point is to understand whether the system can be approximately scale invariant. We know that QCD at zero temperature is not a conformal invariant theory: the coupling constant depends on a typical energy scale, $\Lambda_{QCD}$, and the stress energy tensor $\Theta^{\mu\nu}$ has a trace anomaly. In this Section we analyze the pure SU(3) gluon plasma, where it is assumed that the masses of the fermions are infinitely large, i.e., in the so-called quenched approximation; the analysis of full QCD with finite quark masses (unquenched approximation) will be developed in the next Section.

The trace of the average energy momentum tensor is related to the energy density, $\epsilon$, and the pressure, $p$, of the system by the relation:

$$\Theta^\mu_\mu = \epsilon - 3 p.$$  \hspace{1cm} (1)

At finite temperature, one can write [23],

$$\Theta^\mu_\mu(T) = \epsilon(T) - 3p(T) = < G^2 >_0 - < G^2 >_T,$$ \hspace{1cm} (2)

where $G^2 = -\frac{2}{g} G^a_{\mu\nu} G^a_{\mu\nu}$ and $< G^2 >_0$ and $< G^2 >_T$ are the gluon condensate at zero and at finite temperature respectively. Note that the energy density $\epsilon(T)$ and pressure $p(T)$ are consistently normalized to zero at $T = 0$.

Equation (2) states that the breaking of the conformal invariance at finite temperature is only due to the gluon condensate. We can check this relation by using different lattice results. Indeed
in Ref. [24, 25] the gauge invariant two-point correlation function of the gauge field strengths have been evaluated on the lattice by using the correlator method. In particular in Ref. [25] the gluon condensate has been evaluated at finite temperature in both pure gauge and full-QCD. It turns out that, at moderate temperatures \( T < \sim 2T_c \), the chromo-magnetic component \( < G^2 >_m^{ag} \) of the gluon condensate survives above the deconfining temperature and is (within the statistical errors) temperature independent:

\[
\frac{< G^2 >_T^m}{< G^2 >_0^m} \simeq 1 , \tag{3}
\]

whereas the ratio between the chromo-electric contributions \( < G^2 >^e \) at zero and at finite temperature,

\[
\frac{< G^2 >^e_T}{< G^2 >^e_0} = c_{eq}(T) \tag{4}
\]

rapidly decreases above the deconfining temperature.

Since a \( T = 0 \) the electric and magnetic terms are equal, defining,

\[
\Delta_1(T) = \frac{1}{2T^4} < G^2 >^m_0 [1 - c_{eq}^0(T)] , \tag{5}
\]

and assuming that in a pure gluon plasma the conformal invariance is only broken by the gluon condensate, one has

\[
\frac{\epsilon - 3p}{T^4} = \Delta_1(T) . \tag{6}
\]

Now considering the lattice results of energy density, \( \epsilon_L \), and pressure, \( p_L \), of Ref. [1, 2] we can obtain the “interaction measure” for a pure SU(3) gluon plasma,

\[
\Delta(T) = \frac{\epsilon_L(T) - 3p_L(T)}{T^4} \tag{7}
\]

and we can compare such expression with \( \Delta_1(T) \).

In Fig. 1 we show the plots of \( \Delta(T) \), full line (black online), and of the \( \Delta_1(T) \), dashed line (red online). In this plot we are assuming that the gluon condensate at zero temperature is given by, \( < G^2 >^{ag}_0 \sim 0.03 \text{ GeV}^4 \) \([26, 27, 28]\) and \( c_{eq}^0(T) \) is obtained by interpolation of the data obtained in lattice QCD simulations reported in Ref. [25].

It is clear from Fig. 1 that the gluon condensate is not able to describe the trace anomaly and other contributions must be present that stem from interactions and/or masses.

The gluon condensate plays an important dynamical role in QCD. Indeed, at zero temperature it contributes to a negative non-perturbative vacuum energy. In general, a bosonic condensate is a macroscopically populated state with zero momentum and the contribution of the modes with \( k \neq 0 \) is small. For these reasons, we will assume that at any temperature the gluon condensate does not significantly contribute to the pressure but does contribute to the energy density of the system.

In order to consider the effect of the gluon condensate on the energy density we consider the quantity \( \epsilon_L - \Delta_1 \). In Fig. 2 we show the plots corresponding to the lattice data of pressure, \( p_L \), versus energy density, \( \epsilon_L \), dashed line (red online) and the data of pressure, \( p_L \), versus \( \epsilon_L - \Delta_1 \).
FIG. 1: (color online) Interaction measure $\Delta(T)$ defined in Eq. (7), full line (black online), and contributions of the gluon condensate $\Delta_1$, dashed line (red online), defined in Eq. (5) as a function of $T/T_c$ in the pure SU(3) Yang-Mills.

FIG. 2: (color online) Plots of pressure as a function of energy density. The diagonal line (green online) corresponds to the conformal solution with $p = \epsilon/3$. The dashed line (red online) corresponds to the lattice data of energy and pressure, whereas the dot-dashed (back online) line corresponds to the quasiparticle gluons, with pressure equal to $p_L$ and energy $\epsilon_L - \Delta_1$. The (blue) dot on the diagonal line corresponds to the ideal SU(3) gas solution.

The effect of subtracting the gluon condensate is of making the relation between energy density and pressure of the gluon system more similar to the conformal solution, that corresponds to the diagonal full line $p = \epsilon/3$. However, subtracting the gluon condensate is not enough for making the system conformal invariant. Note that for $p = 0$ and $\epsilon = 0$ the system seems to be conformal, but this happens because the pressure and energy density have been normalized to be zero in the confined phase. The (blue) dot on the diagonal line corresponds to the energy density and pressure of an ideal gluon gas.

Therefore one can conclude that the trace anomaly at finite temperature is not saturated by gluon condensation and that the description of the system requires other dynamical ingredients. The rigorous analysis in Ref. [29], where a gluon gas with physical state space reduced to the so-called Gribov fundamental modular region has been considered, gives essentially the same indication.

In the next sections we shall consider a more phenomenological approach, based on the introduction of massive gluon quasi-particle, to describe the difference between trace anomaly and gluon condensate shown in Figs. 1 and 2.
A. Quasiparticle model for the Gluon Plasma

Let us recall some thermodynamical relations. The free energy density of a system is given by

\[ f = -\frac{T}{V} \ln Z(T, V), \]  

(8)

where \( Z(T, V) \) is the partition function, and energy density and pressure are given by,

\[ \epsilon = \frac{T^2}{V} \frac{\partial \ln Z(T, V)}{\partial T}, \]  

(9)

\[ p = T \frac{\partial \ln Z(T, V)}{\partial V}. \]  

(10)

For homogeneous systems, in the thermodynamic limit,

\[ \frac{\ln Z(T, V)}{V} \simeq \frac{\partial \ln Z(T, V)}{\partial V}, \]  

(11)

and therefore the pressure can directly be obtained from the free energy density,

\[ p = -f. \]  

(12)

Using this relation one can express the entropy density \( s \) and the trace anomaly in terms of derivatives of the pressure with respect to temperature,

\[ s = \frac{\epsilon + p}{T} = \frac{\partial p}{\partial T}, \]  

(13)

\[ < \Theta^\mu_\mu >_T = \epsilon - 3p = T^5 \frac{\partial}{\partial T} \left( \frac{p}{T^4} \right). \]  

(14)

Note that for an ideal gas \( p \sim T^4 \) and the trace of the stress energy tensor is equal to zero. However, when interactions and/or masses are present the pressure is not proportional to \( T^4 \) and one has to include these effects in the trace anomaly.

An effective method to take into account the interaction is by considering temperature dependent gluon masses and temperature dependent gluonic degrees of freedom. Therefore in our analysis we work in a quasiparticle picture and all the in-medium effects are treated, at the mean field level, as an effective mass, \( M_g(T) \), and as an effective number of degrees of freedom, \( D_g(T) \), for the gluons. The values of these parameters will depend on the temperature and this dependence will be determined by fitting the lattice data of pressure and energy density where the gluon condensate contribution will be properly taken into account. However, temperature dependent parameters and also the presence of the gluon condensate require that the thermodynamical consistency must be carefully checked. This point is discussed in details in the Appendix A.

According to the previous discussion, let us rewrite Eq. (2) for a gluon plasma as follows:

\[ < \Theta^\mu_\mu >_T = \epsilon - 3p = < G^2 >_0 - < G^2 >_T + \Delta \Theta_g, \]  

(15)

where \( \Delta \Theta_g \) represents the contribution to the trace anomaly due to gluon quasiparticles.

As discussed, the gluon condensate is an important dynamical ingredient and we will assume that it contributes to the energy density but not to the pressure of the system. Therefore in our quasiparticle picture, where the contributions to the thermodynamical quantities come from gluon
condensation and gluons with in medium mass $M_g(T)$, the pressure and energy density of the system are given by,

$$
p = p_g \quad \quad \epsilon = \epsilon_g + \epsilon_{\text{con}}^{\text{qu}},
$$

(16)

where,

$$\epsilon_{\text{con}}^{\text{qu}} = T^4 \Delta_1(T) = \frac{1}{2} < G_2^2 >_0 \left[ 1 - e^{-\epsilon_e(T)} \right],
$$

(17)

is contribution to the energy density due to the gluon condensate. The quasiparticle contributions to pressure and energy density are respectively,

$$p_g(T) = D_g(T) \int \frac{d^3k}{(2\pi)^3} \frac{\omega_g}{\omega_g - 1} = D_g(T) \tilde{p}_g(T),
$$

$$\epsilon_g(T) = D_g(T) \int \frac{d^3k}{(2\pi)^3} \frac{\omega_g}{\omega_g - 1} = D_g(T) \tilde{\epsilon}_g(T),
$$

(18)

with $D_g(T)$ the temperature dependent number of gluon degrees of freedom, $\tilde{p}_g(T)$ and $\tilde{\epsilon}_g(T)$ the gluonic pressure and energy density per degree of freedom, and

$$\omega_g(k, T) = \sqrt{k^2 + M_g(T)^2},
$$

(19)

is the gluon dispersion law.

B. Determination of $M_g$ and $D_g$

In order to obtain the functions $D_g(T)$ and $M_g(T)$ we will employ the data of the pressure and energy density obtained in lattice simulations reported in Refs. [1][2]. We obtain $D_g(T)$ and $M_g(T)$ substituting

$$p_L(T) = p(T) \quad \quad \epsilon_L(T) = \epsilon(T),
$$

(20)

in the Equations (16) and (18), and solving for the effective mass of the gluons and the effective number of the degrees of freedom. In particular the gluonic mass can be obtained solving the equation,

$$\frac{\epsilon_L(T) - \epsilon_{\text{con}}^{\text{qu}}(T)}{p_L(T)} = \frac{\tilde{\epsilon}_g(T)}{\tilde{p}_g(T)},
$$

(21)

and the effective number of gluonic degrees of freedom can be obtained from the relation,

$$D_g(T) = \frac{p_L(T)}{\tilde{p}_g(T)},
$$

(22)

where the solution for the effective gluon mass obtained solving Eq. (21) has been plugged in $\tilde{p}_g(T)$.

Before presenting our results for $D_g(T)$ and $M_g(T)$, it is useful to clarify some points regarding the range of values of the temperature where our results are reliable. Let us consider the equation

$$\Delta(T) = \Delta_1(T) + \frac{D_g(T)(\tilde{\epsilon}_g - 3\tilde{p}_g)}{T^4},
$$

(23)
which at the critical temperature implies
\[
\frac{D_g(T_c)(\bar{c}_g - 3\bar{p}_g)}{T_c^4} = \Delta(T_c) - \Delta_1(T_c) = d_c.
\] (24)

From this equation and from Eq. (21) one can see that close to \( T_c \) the values of \( M_g(T) \) and \( D_g(T) \) will strongly depend on the values of the lattice data.

Indeed, for \( T \) close to \( T_c \) there are two solutions of Eq. (21): \( M_g(T_c) \to 0 \) (corresponding to the saturation of the trace anomaly with the gluon condensate) but also \( M_g(T_c) \to \infty \). Moreover, for \( T \to T_c \), we have that \( d_c \to 0 \) and the actual value for \( D_g(T) \) will depend on the ratio between two small quantities. In particular we find that when the gluon mass diverges also \( D_g(T_c) \) diverges, unless \( d_c \) is exactly zero at \( T_c \). In any case, as previously discussed, a quasi-particle approach is unreliable at the critical point and for these reasons our numerical analysis will be limited to the range \( \sim 1.2 - 4 \ T_c \).

From the solutions of Eqs. (21) and (22) we obtained the values of \( M_g(T) \) and \( D_g(T) \) reported in Fig. 3.

![Figure 3](image.png)

**FIG. 3:** Left panel: Effective mass of the gluonic degrees of freedom in the quenched approximation as a function of the temperature. Right panel: Effective number of degrees of freedom of the gluonic quasiparticles in the quenched approximation as a function of the temperature.

Both the effective number of degrees of freedom and the effective mass monotonically decrease with increasing temperature. Therefore, in our quasiparticle approach to the deconfined gluon plasma, gluons are propagating quasiparticles with a mass and an effective number of modes that decrease with increasing temperature. The reduction of the number of the gluonic modes with temperature is not new and is physically due to the fact that the spectrum of the gluons depends on the temperature. For momenta \( k \lesssim gT \) not only the transverse components are present, but also the longitudinal ones. On the other hand, at high momenta \( k >> gT \) the longitudinal modes are not relevant \([30]\) because the corresponding pole in the propagator becomes exponentially small. Since the equation of state is dominated by particles with momenta \( k \sim T \) one can expect that at low temperatures the longitudinal modes are excited, whereas in the high temperature regime the contribution of the longitudinal poles becomes negligible. In principle we should treat the longitudinal modes employing a different dispersion law, however for the sake of simplicity we have assumed that all modes are degenerate in mass. This is a first possible ingredient of a more complete dynamical description of the system that may lead also to avoid the divergent behavior of the effective number of degrees at the transition temperature. A second possible motivation for the sudden increase of the number of degrees of freedom close to \( T_c \) is that other quasiparticles, like glueballs, may be present (analogously to the fermionic correlated states that survive above
$T_c$). As a matter of fact it has recently been pointed out in Ref. [31] that the mass of scalar and pseudoscalar glueballs in the deconfined phase is much less than in the confined phase. Therefore one can expect that these states significantly contribute to the various thermodynamical quantities.

A third possibility is that the dispersion relation in Eq. (28) is strongly modified in the infrared region as in Ref. [29].

In any case, even with our very simple model, we obtain that at high temperatures the number of the gluonic modes is roughly 15, not far from the expected result 16, that corresponds to the number of transverse modes.

![Figure 4](image_url)

**FIG. 4:** Gluonic pressure (left panel) and energy density (right panel) per degree of freedom as a function of the temperature.

In Fig. 4 we show the plot of the pressure $p_g$ and energy density $\varepsilon_g$ per degree of freedom. Note that the energy density does not suddenly saturates at $T \approx 1.2 T_c$ as observed in the lattice simulations. The reason is that we have subtracted from the energy density the contribution due to the gluon condensate.

A quantity that is relevant for the dynamical properties of the system and gives a different measure of the deviation from the conformal behavior is the velocity of sound, defined as,

\[
\varepsilon_s^2 = \frac{\partial p}{\partial \varepsilon}.
\] (25)

For a conformal symmetric system the velocity of sound squared is equal to $1/3$ and therefore deviation from this numerical value indicate a breaking of conformal symmetry.

In our case we can define the velocity of sound of the gluon quasiparticles by,

\[
\varepsilon_{s,g}^2 = \frac{\partial p_g}{\partial \varepsilon_g},
\] (26)

which is clearly different from the total velocity of sound in Eq. (25). The two results are shown in Fig. 4: the dashed line (blue online) corresponds to Eq. (25), whereas the full line (red online) corresponds to Eq. (26).

The two velocity of sound are quite similar except for $T \approx T_c$. Unfortunately this is the region where the errors in lattice data are larger and one can assume that our result have at least a 20% error bar (for recent more accurate lattice data regarding the velocity of sound in pure SU(3) see [32]). Note that the velocity of sound drives the hydrodynamical evolution of the system produced in heavy-ion collisions and in particular of the elliptic flow. Therefore a more detailed knowledge of the sound velocity would be helpful in understanding the dynamical properties of the system.
FIG. 5: (color online) The velocity of sound squared of the gluon plasma as a function of the temperature. The full line (red online) corresponds to the velocity of sound for the gluon quasiparticles. The dashed line (blue online) corresponds to the velocity of sound for the total system.

III. ANALYSIS OF THE QUARK GLUON PLASMA

Let us now consider the case of the Quark-Gluon Plasma where gluons as well as quarks are dynamical degrees of freedom. As in the previous Section we first consider the result of lattice simulations and show that the gluon condensate is not sufficient to explain the trace anomaly of the system. In analogy with Eq. (4) let us define $c^{m_q}(T)$ the ratio between the chromo-electric gluon condensates at zero and finite temperature for finite quark mass $m_q$. The pure gauge results, $c^{m_q}(T)$ corresponds to the limit $m_q \to \infty$.

It turns out that in full QCD with a quark mass $m_q = 0.1 \, T$, $c^{m_q}(T) < c^{m_q}(T)$ for any $T \ [25, 33]$. In the following we will refer to $c^{m_q}(T)$ at $m_q = 0.1 \, T$ as $c^{m_q}(T)$ and we will assume that it is not strongly dependent on the actual value of the quark mass.

Following the same approach of Sec. II, in Fig. 6 we compare the unquenched results for $(\epsilon - 3p)/T^4$ with the gluon condensate contribution, given by the Equation (6) with $<G^2> \simeq 0.02$ GeV$^4$ and $c^{m_q}(T)$ obtained by interpolating the lattice QCD results of Ref. [25].

In general, the trace anomaly has a fermionic contribution proportional to $<\bar{q}q>$. However, above the chiral symmetry restoration temperature, that we consider coincident with the deconfinement temperature, one has $<\bar{q}q> = 0$.

FIG. 6: (Color online) Interaction measure (black) and gluon condensate (red) as a function of $T/T_c$.

In full QCD we find a result similar to the one obtained in the previous Section for the gluon plasma: the gluon condensate is not the only contribution to the trace anomaly. Therefore we rewrite Eq. (2) as follows:
\[ <\Theta^\mu_\mu>_T = \epsilon - 3P = <G^2>_0 - <G^2>_T + \Delta\Theta, \quad (27) \]

where \(\Delta\Theta\) represents the contribution to the trace anomaly from gluonic, fermionic as well as bosonic degrees of freedom.

### A. Quasiparticle model for the Quark-Gluon Plasma

We shall assume that above the confining temperature the system consists of fermionic quasiparticles with the quantum numbers of \(u, d\) and \(s\) quarks, gluon quasiparticles and in-medium mesons (or correlated pairs).

Concerning the fermionic sector, we assume that the number of quark (antiquark) degrees of freedom \(D_q (D_{\bar{q}})\) is independent of the temperature and \(D_q = D_{\bar{q}} = 18\). This is motivated by the fact that the baryon chemical potential is close to zero and charm quarks are too massive to play a role in the temperature range that we are studying. The general form of the dispersion law for in-medium fermions can be written as,

\[ \omega_{\bar{q}}(k, T) = \omega_q(k, T) = \sqrt{k^2 + m(k, T)^2 + \Sigma_R(k, T)}, \quad (28) \]

and has been studied in \([19, 34]\), where \(m\) and the self-energy \(\Sigma_R\) have been evaluated taking into account the interaction of the quasiparticles in the medium.

Here we will assume that for the relevant momenta, of the order of the thermal momentum, we can neglect \(m/k\) (see e.g. \([19]\)) and treat the term \(\Sigma_R\) as a chiral invariant and temperature dependent effective mass \(M(T)\). Quarks are considered degenerate in mass because \(M(T)\) is generated by the interaction of the quarks with the medium and for light quarks this is independent of the value of the bare mass. From this assumptions the dispersion relation for \(u, d\) and \(s\) quasiparticles can be written as,

\[ \omega_{\bar{q}}(k, T) = \omega_q(k, T) = k + M(T). \quad (29) \]

The structure of the in-medium correlated states as a function of temperature is not easily to valuate. These string like states may describe \(qq\) states as well as more exotic states \([35]\). We will consider that there are \(D_b(T)\) bosonic degrees of freedom and we expect that \(D_b(T)\) increases close to the \(T_c\) and decreases with increasing \(T\), because for asymptotic values of the temperature the system is made up of quarks and gluons and \(D_b(T)\) must vanish.

In the following we will neglect, as a first approximation, the effect on the thermodynamics quantities of the width of the bosonic states. Therefore we employ the dispersion law

\[ \omega_b(k, T) = \sqrt{k^2 + M_b(T)^2}, \quad (30) \]

where \(M_b(T)\) is the in medium mass of the bosons. To evaluate the dependence of the numerical results on this parameter we have changed the mesons masses in the range \(M(T) - 3M(T)\) obtaining variation of less than 15% of our results.

Concerning the gluonic sector we will employ the results obtained in Section \([1A]\). Assuming that the effective number of degrees of freedom and mass of gluons is given by the result obtained in the quenched case is clearly a rough approximation, because it assumes that the effect of dynamical quarks on the gluon quasiparticles is much smaller that the effect of the gluon medium. Nonetheless we expect that in the unquenched case the effective number of gluon degrees of freedom has a behavior similar to the one obtained in the quenched case. Indeed we expect that \(D_g(T)\) decreases
with increasing temperature going from 24 close to \( T_c \) to 16 at high temperatures, for the reasons explained in Section \( \text{II B} \). Therefore we will assume that \( D_g(T) \) is given by the same expression obtained in the quenched case.

Regarding the temperature dependence of the in-medium mass of the gluons we expect in both the quenched and unquenched cases a similar behavior. The reason is that at asymptotic high temperatures the system has to become scale invariant and therefore the gluon mass must decrease with increasing temperature. In the following we will assume that the gluon mass in the unquenched case, \( \hat{M}_g(T) \), is proportional to the gluon mass in the quenched case, \( M_g(T) \), that is shown in the left panel of Fig. 3. However, since the numerical value of the gluon mass in the unquenched case can be different from the numerical value in the quenched case, we write

\[
\hat{M}_g(T) = c M_g(T), \tag{31}
\]

where \( c \) is a coefficient which parameterizes the variation of the gluon mass in the medium when dynamical quarks are present. In the numerical study we will consider the values \( c = 0.5, 1, 2 \).

The corresponding dispersion law for gluon quasiparticles turns out to be

\[
\hat{\omega}_g(k, T) = \sqrt{k^2 + \hat{M}_g(T)^2}. \tag{32}
\]

Within these approximations, the expressions for the pressure, \( p \), and energy density, \( \epsilon \), are given by

\[
p = p_f + p_b + \hat{p}_g, \tag{33}
\]
\[
\epsilon = \epsilon_f + \epsilon_b + \hat{\epsilon}_g + \epsilon_{\text{con}}^{\text{un}}, \tag{34}
\]

where the subscripts \( f, b \) and \( g \) refer to fermionic, bosonic (in-medium mesons) and gluonic degrees of freedom respectively and where

\[
p_i(T) = TD_i \int \frac{d^3k}{(2\pi)^3} \log(1 \pm e^{-\omega_i/T}) \pm 1, \tag{35}
\]

\[
\epsilon_i(T) = D_i \int \frac{d^3k}{(2\pi)^3} \frac{\omega_i}{e^{\omega_i/T} \pm 1}, \tag{35}
\]

with \( i = f, b \); the sign + (−) refers to fermions (bosons), whereas the gluon contributions are given by

\[
\hat{p}_g(T) = TD_g(T) \int \frac{d^3k}{(2\pi)^3} \log(1 - e^{-\hat{\omega}_g/T})^{-1}
\]
\[
\hat{\epsilon}_g(T) = D_g(T) \int \frac{d^3k}{(2\pi)^3} \frac{\hat{\omega}_g}{e^{\hat{\omega}_g/T} - 1}, \tag{36}
\]

and the contribution of the gluon condensate to the energy density is given by,

\[
\epsilon_{\text{con}}^{\text{un}} = \frac{1}{2} < G^2 >_0 [1 - \epsilon_{\text{con}}^{\text{un}}(T)]. \tag{37}
\]

In order to evaluate \( M(T) \) and \( D_b(T) \) we perform a simultaneous fit of the lattice data of pressure and energy density of 3 flavors quark matter with bare quark masses \( m = 0.4 T \) of Refs. [10, 36] as a function of the temperature employing Eqs. (33) and (31). For each value of the temperature we consider the central value of pressure and energy density of the lattice data. We will discuss
the dependence of our results on the numerical values of the lattice data in the following. In [21] we evaluated with an analogous method $M(T)$ and $D_b(T)$ in the range $1.2 - 2. T_c$, keeping the mass of the gluons and their effective number fixed; here we extend such a study to a larger temperature range and considering temperature dependent gluon mass and effective number of degrees of freedom.

In Fig. 7 we show the quasiparticle chiral mass (left panel) and the effective number of bosonic degrees of freedom (right panel) as a function of the temperature for $T ≃ 1. - 4. T_c$. The various line correspond to different values of the parameter $c$ in Eq. (31) that is the ratio between the gluon mass in the quenched and unquenched cases. In the three cases considered the chiral mass rapidly decreases with increasing temperature. This behavior suggests that the breaking of scale invariance is mainly due to the gluon mass and the gluon condensate.

Also the effective number of bosonic degrees of freedom rapidly decrease with increasing temperature. At temperature larger than $1.5 T_c$, we find that less than 20 bosonic modes are excited. The divergence of $D_b(T)$ for temperature close to $T_c$ is due to the same numerical problem that we have discussed in the previous Section when we have presented the results regarding the temperature dependence of $D_g(T)$. However in this case the exponential growth of $D_b$ begins at a value of the temperature $\sim 1.2 T_c$. We have checked, that the numerical behavior below $\sim 1.2 T_c$ is strongly dependent on the numerical value of the gluon condensate and of the energy density and pressure. On the other hand for temperatures larger than $\sim 1.2 T_c$ our results are basically almost independent of the value of the lattice data. Considering different values of energy density, pressure or gluon condensate within the statistical error bars determines a variation of less than 10% of our results.

In Fig. 8 we show the contribution to the pressure and energy density due to the fermions (full black line), gluons (dashed red line) and correlated bosons (dot-dashed blue line) quasiparticles employing a gluonic mass given by Eq. (31) with different values of $c$.

In all the considered cases we find that at high temperature the dominant contribution to energy density and pressure is due to the fermionic modes. However at sufficiently low temperature the contributions of gluons and correlated pairs become relevant. In particular, the correlated states give a non negligible contribution to energy density and pressure for temperature $\lesssim 1.5 T_c$. Unfortunately, as we have already stressed, for temperatures smaller than $1.2 T_c$ our results for the
FIG. 8: Contributions to the pressure (left panels) and energy density (right panels) of fermionic (full black line), correlated states (dot-dashed blue line) and gluonic (dashed red line) states. The in-medium gluonic mass is given by Eq.(31) where we have employed different values of $c$. From top to bottom we have taken $c = 0.5$, $c = 1$ and $c = 2$ respectively.

boson modes are not reliable. However, it is reasonable that the qualitative behavior that we find, with a peak in energy density and in the pressure at a certain value of the temperature, does not depend on our approximations and is not an artifact of the numerical errors. The reason is that close to $T_c$ energy density and pressure must be zero, because of the normalization of the lattice data. On the other hand at large temperatures we expect that the system consists of quarks and gluons, the correlated pairs being melted and therefore cannot contribute to the thermodynamical quantities. Then $p_b(T)$ and $\epsilon_b(T)$ must have a maximum for a certain value of the temperature.

In Fig. 9 we show the squared sound speed of the various quasiparticles for three different values of the parameter $c$ defined in Eq. (31). At high temperatures the fermionic quasiparticle have the largest velocity of sound, however at moderate temperature it is not clear which component is dominant. Note also that fermionic and bosonic quasiparticles seem to be “approximately conformal” for any value of $c$, in the sense that the corresponding velocity of sound approaches the conformal values already at temperatures of the order of $2T_c$. In the case $c = 2$ and $c = 1$,
gluons seem to be more scale dependent. However when we reduce the gluon mass taking \( c = 0.5 \) also gluons seem to approach the conformal limit at a temperature close to \( 2T_c \). Regarding the correlated pairs, we have shown their behavior up to the temperature where \( D_b \) is different from zero.

![Graphs showing sound velocity of various components.](image)

**FIG. 9:** (color online) Sound velocity of the various components. Full line (black online) correspond to fermions, dashed line (red online) corresponds to gluons and dot-dashed line (blue online) corresponds to correlated pairs. From left to right the three panels correspond to the values \( c = 2, 1, 0.5 \) respectively for the in-medium gluonic mass given by Eq. (31).

### IV. CONCLUSIONS AND OUTLOOKS

In the present paper we have described the lattice QCD data of pressure and energy density in terms of a quasiparticle model. Such description of lQCD data is not new. However our results show that there are two dynamical ingredients, not taken into account in previous analyses, which play an important role: the difference between trace anomaly and gluon condensate and the survival of correlated, string like, pairs for \( T > T_c \), both suggested by different lattice simulations.

The pure SU(3) gluon plasma lattice results can be described in terms of gluon condensate and gluonic quasiparticles whose mass decreases with increasing temperature and whose effective number of degrees of freedom tends to the expected value, 16, for large \( T \).

Analogously, numerical simulations results for QCD with finite quark masses can be fitted by the gluon condensate plus a mixture of gluonic and fermionic quasiparticles and bosonic correlated pairs.

Also in this case the effective number of bosonic degrees of freedom and the in-medium masses decrease with increasing temperature. At \( T \simeq 1.5T_c \) only the correlated pairs corresponding to the mesonic nonet survive and they completely disappear at \( T \simeq 2T_c \).

The temperature dependence of the sound velocity has been studied to give indications for the hydrodynamical models of relativistic heavy ion collisions.

The present analysis can give partial answers to important problems. For example, lattice data show a deviations of the pressure and of the energy density of the system from the SB values of an ideal gas of quarks and gluons even at temperatures \( T \geq 3T_c \). At \( T = 3T_c \simeq 500 \text{ MeV} \) the running coupling constant is large and the deviations from the SB behavior can only be partially understood with improved perturbative methods \cite{37,38}. Our results show that at large \( T \) there are still effects due to quasiparticle masses that determine variation from the SB values, whereas the contributions of gluon condensate and of the correlated pairs turns out to be small. The dominant contribution to the deviation from the ideal gas results seems to be due to the mass of the gluons.

Our conclusions follow by considering a temperature independent chromo-magnetic gluon condensate that has been evaluated in lattice simulations only up to \( T \simeq 2T_c \). A gluon condensate growing as \( T^4 \) for large \( T \) suggested in Ref. \cite{39}, would modify the whole picture of the system.
for $T >> T_c$. However it is not clear at which temperature the results of Ref. [39] would be relevant. For this reason we have assumed a constant value of the chromo-magnetic gluon condensate, extrapolating lattice data up to $4T_c$.

Another limitation of the present study is the zero width approximation of the correlated pair. The effect of the width would be to modify the expression of energy density and pressure of the correlated pairs, but also to change in a self-consistent way the dispersion law of fermions [18, 19, 40]. These aspects and a more refined treatment of the gluon condensate will be analyzed in a forthcoming paper.

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APPENDIX A: THERMODYNAMICS CONSISTENCY

In order to describe the pressure and energy density of the Gluon Plasma in Section III A and of the Quark-Gluon Plasma in Section III A we have employed a quasiparticle expression for the various thermodynamical quantities. Since we have assumed that some parameters are temperature dependent the thermodynamics consistency of our results must be carefully checked. In general we can write the relation between pressure and energy density as follows,

$$ T \frac{dp}{dT} = p + \epsilon + C_r, \quad (A1) $$

where the correction $C_r$ depends on the temperature. The thermodynamics consistency requires that

$$ C_r = 0. \quad (A2) $$

The correction to the thermodynamical relation for the Gluon Plasma is due to the temperature dependence of the parameters $M_g(T)$ and $D_g(T)$ and to the introduction of the gluon condensate and is given by,

$$ C_{r}^{qu}(T) = TM_g \frac{dM_g}{dT} \frac{D_g}{2\pi^2} \int \frac{1}{dk} \frac{1}{1 - \exp(\omega_g/T)} \frac{1}{\omega_g} + \frac{T}{D_g} \frac{dD_g}{dT} p_g - \epsilon^{qu}_{con}. \quad (A3) $$

Requiring that $C_{r}^{qu}(T) = 0$, is equivalent to requiring that $D_g$ satisfies the differential equation

$$ A \frac{dD_g}{dT} + BD_g + C = 0, \quad (A4) $$

where the coefficients can be obtained from Eq. (A3) and read

$$ A = T\bar{p}_g \quad (A5) $$

$$ B = TM_g \frac{dM_g}{dT} \frac{1}{2\pi^2} \int \frac{1}{dk} \frac{1}{1 - \exp(\omega_g/T)} \frac{1}{\omega_g} \quad (A6) $$

$$ C = -\epsilon^{qu}_{con}. \quad (A7) $$

Substituting the values of $D_g(T)$ and of $M_g(T)$, obtained fitting the lattice data of energy and pressure, in the previous equations, represents a self-consistent check of our result. We find that
the differential equation (A4) is satisfied with a good accuracy and the corresponding correction $C_{\text{un}}^q$ is of the same order of the error in the lattice data. Therefore in our method the differential equation (A4) turns out to be selfconsistently satisfied within the range of values of the statistical errors in lattice data.

We can treat the correction to the thermodynamical equation for the Quark-Gluon Plasma in a similar way. In this case such a correction is due to the temperature dependence of the parameters $M(T), D_b(T), M_g(T), D_g(T)$ and to the introduction of the gluon condensate. In this case we obtain,

$$C_{\text{un}}^q(T) = - T \frac{dM}{dT} \frac{2D_b}{2\pi^2} \int \frac{dk k^2}{1 + \exp\left(\frac{\omega_q}{T}\right)} + T \frac{dM}{dT} \frac{D_b}{2\pi^2} \int \frac{dk k^2}{1 - \exp\left(\frac{\omega_b}{T}\right)} + \frac{1}{\omega_b} + T \frac{dD_b}{dT} \frac{p_b}{\pi^2} \int \frac{dk k^2}{1 + \exp\left(\frac{\omega_g}{T}\right)} + \frac{1}{\omega_g} + T \frac{dD_g}{dT} \frac{\tilde{p}_g}{\pi^2} \int \frac{dk k^2}{1 - \exp\left(\frac{\tilde{\omega}_g}{T}\right)} + \frac{1}{\tilde{\omega}_g} + T \frac{d\tilde{\omega}_g}{dT} \frac{\tilde{\omega}_g - \epsilon_{\text{con}}^\text{un}}. \tag{A8}$$

Also in this case the thermodynamical consistency can be cast as a differential equation for the effective number of degrees of freedom. Assuming that the dependency of $D_g$ on the temperature is the same as in the quenched case we obtain a differential equation for $D_b$ of the form

$$A \frac{dD_b}{dT} + B D_b + C = 0, \tag{A9}$$

where the coefficients $A, B$ and $C$ are given by

$$A = - T \frac{dM}{dT} \frac{2D_b}{2\pi^2} \int \frac{dk k^2}{1 + \exp\left(\frac{\omega_q}{T}\right)} \log\left[1 - \exp\left(-\frac{\omega_b}{T}\right)\right] \tag{A10}$$

$$B = - T \frac{dM}{dT} \frac{D_b}{2\pi^2} \int \frac{dk k^2}{-1 + \exp\left(\frac{\omega_b}{T}\right)} \frac{1}{\omega_b} \frac{dM}{dT} \tag{A11}$$

$$C = - T \frac{dM}{dT} \frac{2D_g}{2\pi^2} \int \frac{dk k^2}{1 + \exp\left(\frac{\omega_g}{T}\right)} \frac{dM}{dT} + T \frac{dD_g}{dT} \frac{\tilde{p}_g}{\pi^2} \int \frac{dk k^2}{1 - \exp\left(\frac{\tilde{\omega}_g}{T}\right)} \frac{1}{\tilde{\omega}_g} + \frac{1}{\tilde{\omega}_g} + T \frac{d\tilde{\omega}_g}{dT} \frac{\tilde{\omega}_g - \epsilon_{\text{con}}^\text{un}}. \tag{A12}$$

Substituting the values of $D_g(T), M_g(T), D_b(T)$ and $M(T)$ obtained fitting the lattice data of energy and pressure, we find that the differential equation (A9) is satisfied with a good accuracy. Also in this case the correction $C_{\text{un}}^q$ is of the same order of the error in the lattice data.

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