New features of the thermal Casimir force at small separations

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The difference of the thermal Casimir forces at different temperatures between real metals is shown to increase with a decrease of the separation distance. This opens new opportunities for the demonstration of the thermal dependence of the Casimir force. Both configurations of two parallel plates and a sphere above a plate are considered. Different approaches to the theoretical description of the thermal Casimir force are shown to lead to different measurable predictions.

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The Casimir effect [1] which is a rare, direct manifestation of the zero-point oscillations of electromagnetic field has attracted a great deal of interest in the last few years. This is in part due to the resurgence of interest in the fundamental problems connected with the physical vacuum. In the simplest situation, the Casimir effect is the attractive force arising between two parallel uncharged metallic plates placed in vacuum at zero temperature. This force is unique as it depends on only the fundamental constants $\hbar$ and $c$, and on the separation between the plates. No charges or other interaction constants are involved, which are usually present with other forces. The Casimir force arises from the difference of the zero-point oscillation spectrum in the absence and in the presence of plates (extensive literature on the subject can be found in monographs [2–4] and in reviews [5,6]).

After the first, order of magnitude, report of observation of the Casimir force [7], a lot of precision measurements have been performed [8–15]. As a result, the Casimir effect has acquired much broader impact, being successfully used in nanotechnological applications [15,16] and for constraining of hypothetical forces predicted by the Kaluza-Klein supergravity and other promising extensions to the standard model [17–19]. Theoretically, a lot of different boundaries were considered and the Casimir force between realistic materials was calculated including effects of surface roughness, finite conductivity of the boundary metal and nonzero temperature (see [6] for a review). Note, that although the surface roughness and finite conductivity corrections have been already demonstrated experimentally (see [9–12,14]), the temperature effect on the Casimir force is yet to be measured. Even the theoretical treatment of the temperature effect of the Casimir force is fraught with serious problems in the case of real metals [20].

It is common knowledge that at $T = 300$ K the relative thermal correction to the Casimir force (ratio of the thermal correction to the result at $T = 0$ K) is an increasing function of the separation distance and achieves substantial values only at sufficiently large separations (for the configuration of two plates made of ideal metal it contributes 0.16% and 2.5% of the result at $T = 0$ K at separations 1 $\mu$m and 2 $\mu$m, respectively; for the configuration of a plate and a sphere the respective contributions are 2.7% and 18.2% [20]). With the increase of a separation, however, the total force decreases so rapidly that thermal corrections to the Casimir force have not yet been measured.

In the present paper we consider the more sensitive “difference force measurements” where the difference in the thermal Casimir forces $\Delta F$ at two different temperatures rather than the absolute value of the thermal Casimir force is measured. As is proved below for the case of real metals, this difference of the thermal Casimir forces (in contrast with the relative thermal correction) does not increase but decreases with increasing separation distance. This opens up new opportunities for the observation of the thermal effect on the Casimir force at the relatively small separations of about 0.5 $\mu$m where the relative thermal correction itself is negligible. Both configurations of two parallel plates and a sphere (spherical lens) above a plate are considered. In conclusion a realistic experiment is proposed which could help resolve differences between the various contradictory theoretical approaches to the description of the thermal Casimir force between real metals.

We consider first the configuration of two parallel plates made of real metals at a separation distance $a$ in thermal equilibrium at a temperature $T_1$. For separations $\lambda_p \leq a \leq 2 \mu$m (where $\lambda_p$ is the plasma wavelength) and for not too high temperatures ($\leq 350$ K) the characteristic frequency $c/(2a)$ of the Casimir effect between good metals, Au for instance, belongs to the region of infrared optics. The first Matsubara frequency $2\pi k_B T/\hbar$, characterizing the thermal effect, also belongs to the same frequency region ($k_B$ is the Boltzmann constant). For such frequencies the plasma model for the dielectric function is applicable.
\[ \varepsilon(\omega) = 1 - \frac{\omega^2}{\omega_p^2}, \]  

(1)

where \( \omega_p = 2\pi c/\lambda_p \) is the plasma frequency.

The thermal Casimir force was found [21,22] by the substitution of Eq. (1) into the Lifshitz formula [23] written in the form of a discrete sum over the Matsubara frequencies. For our purposes, the perturbation result obtained in [22] (see also [20]) is the most convenient. It is given by

\[ F_{pp}(a,T) = F_{pp}^{(0)}(a) \left[ 1 + \frac{1}{3} \left( \frac{T}{T_{eff}} \right)^4 - \frac{16}{3} \frac{\delta}{a} \left[ 1 - \frac{45\zeta(3)}{8\pi^3} \left( \frac{T}{T_{eff}} \right)^3 \right] + \sum_{i=2}^{6} c_i \frac{\delta^i}{a^i} \right], \]  

(2)

where \( k_B T_{eff} = \hbar c/(2a) \), \( \delta = \lambda_p/(2\pi) \), \( F_{pp}^{(0)}(a) = -\pi^2 \hbar c/(240a^4) \), \( \zeta(z) \) is zeta function, and coefficients \( c_i \) \( (2 \leq i \leq 6) \) are explicitly calculated in [22] (their exact values are not needed for our present purposes). In fact Eq. (2) is the perturbation expansion in powers of two parameters \( \delta/a \), where \( \delta \) is the skin depth of electromagnetic oscillations in the metal, and \( T/T_{eff} \), which are small in the above separation range. One of these parameters takes into account the finite conductivity of a metal and the other one the nonzero temperature. It should be noted that there are no thermal corrections up to \( (T/T_{eff})^4 \) in the higher order conductivity correction terms from the second up to the sixth order. If one would wish consider \( a < \lambda_p \), our parameter \( \delta/a \) is not small and it is necessary to use the optical tabulated data for the complex refractive index to compute the thermal Casimir force [6]. At \( a > 2 \mu m \) the low-temperature asymptotic (2) is not applicable and numerical computations of Ref. [22] should be used. The regions \( a < \lambda_p \) and \( a > 2 \mu m \) are not of our interest here as the first one is not achievable experimentally for the test bodies of \( > 1 \) mm size, and within the second the total Casimir force is too small.

Now let us suppose that the equilibrium temperature is rapidly changed from \( T_1 \) to a new temperature \( T_2 \) such that \( T_2 > T_1 \). The subject of our interest is the difference of the two thermal Casimir forces

\[ \Delta F_{pp} = \Delta F_{pp}(a,T_1,T_2) = F_{pp}(a,T_2) - F_{pp}(a,T_1). \]  

(3)

Substituting Eq. (2) into Eq. (3), we arrive to

\[ \Delta F_{pp} = -\Delta^{(1)} F_{pp}(a,T_1,T_2) \Delta^{(2)} F_{pp}(a,T_1,T_2), \]  

(4)

where

\[ \Delta^{(1)} F_{pp}(T_1,T_2) = \frac{\pi^2 k_B^4 (T_2^4 - T_1^4)}{45 \hbar^2 c^3}, \]  

(5)

\[ \Delta^{(2)} F_{pp}(a,T_1,T_2) = 1 + \frac{90\zeta(3)}{\pi^3} \frac{\delta}{a} \frac{T_{eff}}{T_1 + T_2} \left( 1 + \frac{T_1 T_2}{T_1^2 + T_2^2} \right). \]

It is clearly seen that the quantity (4) is negative (i.e. has the same sign as an attractive Casimir force) because with an increase of temperature the magnitude of the force increases. It should be particularly emphasized that the magnitude of \( \Delta F_{pp} \) decreases with an increase of separation distance. This leads to the conclusion that in difference force measurements the thermal effect of the Casimir force can be measured more likely at small separations than at large ones where the relative thermal correction itself is greater.

In Fig. 1 the dependence of \( \Delta F_{pp} \) on separation distance is plotted for \( Au \) (\( \lambda_p = 136 \) nm), \( T_1 = 300 \) K, and \( T_2 = 350 \) K (solid line). In the same figure the result for ideal metal (infinite conductivity) is shown as a dashed line. It is seen that at the smallest separation where the above computations are applicable \( (a = 0.15 \mu m) \) the difference thermal effect is more than 9 times stronger than at the largest separation \( (a = 2 \mu m) \). The low-temperature result for the ideal metal is obtained from Eqs. (4), (5) by putting \( \delta = 0 \). It does not depend on the separation distance.

Now we consider the configuration of a sphere (spherical lens) above a plate. This configuration was used in the most precise experiments on measuring the Casimir force by means of an atomic force microscope [9–12]. Here the free energy is given by the Lifshitz formula. The Casimir force acting between a plate and a lens can be obtained by means of the so-called proximity force theorem. The relative error introduced by this theorem is of order \( a/R \) [24], where \( R \) is a sphere radius, i.e. it is a fraction of a percent for separations under consideration, given the large spheres with \( R \sim 1 \) mm to be used in experiment. The perturbation result obtained in analogy with Eq. (2) is [20,22]

\[ F_{ps}(a,T) = F_{ps}^{(0)}(a) \left[ 1 + \frac{45\zeta(3)}{\pi^3} \left( \frac{T}{T_{eff}} \right)^3 - \frac{T}{T_{eff}} \right] - \frac{4}{3} \frac{\delta}{a} \left[ 1 - \frac{45\zeta(3)}{2\pi^3} \left( \frac{T}{T_{eff}} \right)^3 + \left( \frac{T}{T_{eff}} \right)^4 \right] + \sum_{i=2}^{6} \frac{c_i}{a^i} \delta^i, \]  

(6)
where \( F_{ps}^{(0)} = -\pi^3 \hbar c R/(360 \alpha^3) \), and \( \bar{c}_i = 3c_i/(3 + i) \).

For the two thermal Casimir forces at temperatures \( T_1 \) and \( T_2 \) for the configuration of a sphere above a plate we consider the difference quantity \( \Delta F_{ps} \) defined as in Eq. (3). By the use of Eq. (6) the following result is obtained

\[
\Delta F_{ps} = -R \Delta^{(1)} F_{ps}(T_1, T_2) \Delta^{(2)} F_{ps}(a, T_1, T_2),
\]

where

\[
\Delta^{(1)} F_{ps}(T_1, T_2) = \frac{\zeta(3)}{\sqrt{c^2}} (T_2 - T_1)(T_1^2 + T_2^2),
\]

\[
\Delta^{(2)} F_{ps}(a, T_1, T_2) = \left( 1 + \frac{T_1 T_2}{T_1^2 + T_2^2} \right) \left( \frac{1 + 2 \delta a}{a} \right) - \frac{\pi^2}{45 \zeta(3)} \frac{T_1 + T_2}{T_{ef}} \left( 1 + 4 \frac{\delta a}{a} \right).
\]

In analogy with the case of two parallel plates, it is seen that at low temperatures \( (T_1, T_2 \ll T_{ef}) \), where the present theory is applicable, \( \Delta F_{ps} \) is negative as expected. It should again be noted that the magnitude of \( \Delta F_{ps} \) is a decreasing function of the separation distance. Thus the difference force measurements of the thermal effect on the Casimir force can be done at small separations rather than at large separations.

In Fig. 2 the dependence of \( \Delta F_{ps}/R \) versus separation distance is plotted for Au with \( T_1 = 300 \text{ K} \), and \( T_2 = 350 \text{ K} \). The case for the ideal metal with infinite conductivity is shown as a dashed line. At the smallest separation \( a = 0.15 \mu \text{m} \) the difference thermal effect is more than 2 times stronger than at \( a = 2 \mu \text{m} \). In contrast to the case of two parallel plates, for ideal metals, the quantity \( \Delta F_{ps}/R \) decreases with increasing separation distance.

The difference thermal forces considered above are well adapted to resolve the contradictions between alternative theoretical approaches to the calculation of the Casimir force at nonzero temperature. In the approach used here (see also [20–22,25]) the plasma dielectric function, valid at the characteristic frequencies \( c/(2a) \) and \( 2\pi k_BT/\hbar \), is extended for all frequencies. In particular, it was used to calculate the zero-frequency term of the Lifshitz formula for the force and free energy. The alternative approach [26,27] uses the physically correct behavior \( \varepsilon \sim \omega^{-1} \) at small frequencies at nonzero temperature. As a result, the contribution of the zero-frequency term in the two approaches is different and this has given rise to extensive discussion [28–33] in the recent literature. In Ref. [32] it was shown that the dependence \( \varepsilon \sim \omega^{-1} \) leads to surprises such as negative values of entropy of the fluctuating electromagnetic field between the plates and the violation of the Nernst heat theorem. To avoid this problem, it was suggested in Ref. [32] that for the surface separations of order \( 1 \mu \text{m} \) the behavior of \( \varepsilon \) at characteristic frequencies should be extended for all frequencies, including the zero frequency. This is in fact embedded in the use of the plasma model, as above, for the dielectric function [21,22]. It must be also emphasized that the physical results obtained in this manner coincide with those obtained by means of the Leontovich surface impedance [33].

To contrast the behavior of the difference thermal Casimir force between the two theoretical approaches, let us consider the experimentally preferred configuration of a sphere (spherical lens) above a plate. The difference of the thermal Casimir forces calculated in the approach of Refs. [20–22,25] is given by Eqs. (7), (8). We now fix a value of separation, say, \( a = 0.5 \mu \text{m} \), fix \( T_1 = 300 \text{ K} \), and consider \( \Delta F_{ps} \) as a function of \( T_2 \), where \( T_1 \leq T_2 \leq 350 \text{ K} \). In the approach of Refs. [26,27], the perpendicular polarization does not contribute to the zero-frequency term of the Lifshitz formula whereas the contribution of the parallel polarization is the same as the one given by the plasma model. All other terms of the Lifshitz formula are the same in both approaches in the low temperature limit under consideration here. The zero-frequency contribution of the perpendicular polarized modes into the Casimir force \( F_{ps}(a, T) \) calculated in the framework of the Lifshitz formula and plasma model is given by [20]

\[
\frac{k_BT R}{8a^2} \int_0^{\infty} dy \ln \left[ 1 - \left( \frac{y - \omega^2 + y^2}{y + \sqrt{\omega_p^2 + y^2}} \right)^2 e^{-y} \right] \approx -\frac{k_BT \zeta(3) R}{8a^2} \left( 1 - 4\frac{\delta a}{a} + 12 \frac{\delta^2 a}{a^2} \right)
\]

(higher order terms do not contribute at separations \( a \geq 0.5 \mu \text{m} \)). Then, the low-temperature thermal force in the approach of Refs. [26,27] is obtained by the subtraction of Eq. (9) from Eq. (6). As a result, the difference force in the approach of [26,27] is given by

\[
\Delta F_{ps} = -R \Delta^{(1)} F_{ps}(T_1, T_2) \Delta^{(2)} F_{ps}(a, T_1, T_2) + \frac{k_BT \zeta(3) R}{8a^2} (T_2 - T_1) \left( 1 - 4\frac{\delta a}{a} + 12 \frac{\delta^2 a}{a^2} \right),
\]

where \( \Delta^{(1)} F_{ps} \) and \( \Delta^{(2)} F_{ps} \) are defined in Eq. (8).
In Fig. 3, the quantity $\Delta F_{ps}/R$ is plotted versus $T_2$ at a separation $a = 0.5 \mu m$ and $T_1 = 300K$ using the approach of [20–22,25] (solid line) and using the alternative approach of [26,27] (dotted line). The result for ideal metal boundaries practically coincides with the solid line for the scale used in the figure. Three significant differences emerge for the $|\Delta F_{ps}/R|$ as a function of $T_2$ for the two approaches considered. First, it is seen in Fig. 3 that at $T_2 = 350K$, the value of $|\Delta F_{ps}/R|$, given by the dotted line, is more than 6 times larger than that given by the solid line, a difference which should be measurable experimentally. Second, it should be noted that in the approach of [26,27] the sign of $\Delta F_{ps}$ is positive, i.e. the magnitude of the thermal Casimir force decreases with an increase of $T$. Third, the quantity $|\Delta F_{ps}/R|$, given by the dotted line, changes rapidly with the change in $T_2$.

In conclusion we would like to note that the changes of the force amplitude predicted above are of order $10^{-13}N$ for the sphere of radius $R = 2 \text{mm}$ and $T_2 - T_1 = 50K$. This difference of temperature can be achieved by the illumination of the sphere and plate surfaces with laser pulses. If laser pulse durations of $10^{-2}s$ are chosen, calculations show that equilibrium temperatures of $T_1$ and $T_2$ can be achieved for sufficient duration allowing Casimir force measurements by means of an atomic force microscope (note that force oscillations of order $10^{-15}N$ were demonstrated with a relative error of about 20% at a 95% confidence level in the recent measurement of the lateral Casimir force [14]). The proposed experiment having the same accuracy would also help in resolving the differences in the alternative theoretical approaches to the description of the thermal Casimir force between real metals.

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FIG. 1. Difference of the thermal Casimir forces per unit area for two parallel plates versus their separation distance. The case of the plates made of real metal and that for ideal metal is shown as solid and dashed line, respectively.
FIG. 2. Difference of the thermal Casimir forces per unit radius between a sphere and a plate versus separation distance. The case of the bodies made of real metal and that for ideal metal is shown as solid and dashed line, respectively.
FIG. 3. Difference of the thermal Casimir forces per unit radius between a sphere and a plate versus the higher temperature for the two different theoretical approaches considered.