Dynamics of a gas bubble during fluid decompression at a constant rate

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Abstract. The bubble growth process in a magmatic melt during decompression at a constant rate under various conditions is numerically studied. The effect of the formation of a narrow boundary layer with a high viscosity around the bubble, which significantly changes the dynamics of the bubble, is taken into account. It is shown that, at a low initial pressure in the medium, in the bubble high pressure is maintained throughout the calculation time. In the case of long decompression time, the final bubble radius is bigger than in the case with small decompression time. This effect can be associated with different dynamics of the medium viscosity in a narrow boundary layer around the bubble.

1. Introduction

Many natural and technical processes are associated with the growth of gas bubbles in a liquid viscous gas-saturated medium. In some cases, the dynamics of gas bubbles is an important feature that determines the entire process. Of the natural processes, the most obvious example can be the process of volcanic eruption, when gas bubbles form and grow during rapid decompression of a column of magmatic melt. Their rapid growth leads to foaming of magma, and this process largely determines the nature and type of eruption. Since experimental studies and field observations are time-consuming or even impossible, mathematical and numerical modeling of such processes is required.

The growth of gas bubbles in magmatic melts is particularly difficult, and a whole complex of phenomena arises, each of them affecting the growth process. The main feature of magmatic melts is high content of dissolved volatile components (mainly water, the concentration of which can reach several percent by mass), as well as high viscosity, which changes by orders of magnitude during degassing, significantly limiting the dynamics of bubbles. As a result, the currently prevailing equilibrium models do not allow one to describe the features of the dynamics of bubbles under essentially nonequilibrium conditions.

For the case of instant decompression and for the diffusion growth stage, an analytical solution was constructed in [1]. In [2], the growth mechanism of a single gas bubble in a highly viscous gas-saturated liquid during its rapid decompression was studied. In particular, an analytical solution to the problem of bubble growth was found taking into account the inhomogeneity of the fluid viscosity due to the formation of a diffusion boundary layer around the bubble during instant decompression.
In the case when decompression is not instantaneous, the construction of such analytical solutions is associated with significant difficulties [3]. Therefore, in this work, we conduct a numerical study of the bubble growth process under various decompression conditions.

2. Statement of problem
Let us consider a gas bubble with initial radii \( R_0 \) in a gas-saturated viscous medium; as an example, a magmatic melt is in an equilibrium state at the initial time. The pressure in the medium around the bubble during the decompression time \( t_d \) drops from the initial \( p_i \) to the final \( p_f \), which is taken as atmospheric \( p_0 = 1 \) atm, with a constant rate. Naturally, in the general case, in the process of decompression, all parameters characterizing the state of the medium are a function of time, and the final pressure may not be set (for example, under changing external conditions).

Initially, the medium contains dissolved gas in an equilibrium state, and in a magmatic melt it is water vapor whose concentration is in accordance with Henry's law (\( K_H \) - Henry's constant):

\[
C_s = \sqrt{K_H p}
\]

During decompression, the medium is already in a nonequilibrium state, and in it the bubble begins to grow. Under these conditions, two growth mechanisms are involved. On the one hand, growth is determined by the difference between pressure in the bubble and in the liquid; on the other hand, the onset of diffusion increases the mass of gas in the bubble, and therefore the pressure. To describe the distribution of the concentration of dissolved gas \( C \) around the bubble, the diffusion equation written in spherical symmetry is used:

\[
\frac{dC}{dt} + \frac{R}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial C}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 D \frac{\partial C}{\partial r} \right)
\]

where \( r \) is the radial coordinate, \( R \) is the radius of the spherical bubble, \( D \) is the diffusion coefficient, and the origin of coordinate system is located at the center of the bubble.

Degassing leads to a change in the gas mass in the bubble \( m_g \), which is described by the equation of mass balance:

\[
\frac{dm_g}{dt} = 4\pi R^2 \rho_l \left( \frac{\partial C}{\partial r} \right) \bigg|_{r=R}
\]

An important difference from equilibrium models is the use in the equation of a concentration gradient at the bubble boundary, rather than the difference in concentrations at the bubble boundary and at infinity.

The dynamics of the bubble is described by the Rayleigh equation with a viscous term, taking into account the distribution of viscosity around the bubble [4]:

\[
\rho_l \left( R \frac{\dot{R}}{R} + \frac{3}{2} \frac{\dot{R}^2}{R} \right) = p_g - p + \frac{2\sigma}{R} - 4\hat{\mu} \frac{\dot{R}}{R}, \quad \hat{\mu} = 3R^3 \int_0^{\infty} \frac{\eta(r)}{r^4} dr,
\]

where \( \hat{\mu} \) is the effective viscosity.

It is important to note that in problems of bubble growth in magmatic melts, effective viscosity must be considered [2]; this is due to a strong increase in viscosity during degassing. In the work, viscosity is described by the dependence of the Arrhenius type with a linear dependence of the activation energy on the concentration of the dissolved gas:

\[
\eta(C) = \eta^* \exp \left( \frac{E_\eta^0 (1 - k\eta C)}{9RT} \right)
\]

Equations (2) and (3) are solved numerically using an explicit scheme in time and the second order spatial discretization. For equation (4), the fourth-order Runge-Kutta-Merson scheme with automatic time step correction, which has proved itself well in problems of bubble dynamics, is chosen.
3. Numerical results

Below are the results of numerical study of decompression for two initial pressures $p_i = 500$ atm and $p_i = 1800$ atm. In each case rate decompression is constant for the three times $t_d = 0.1s$ (almost instantaneous decompression), $t_d = 1s$ and $t_d = 10s$ (rather slow decompression). The total calculation time in all cases is 1000 s, the initial bubble radius $R_0 = 10^{-8}$m.

![Figure 1. Distribution of the gas phase concentration and the viscosity of the medium around the bubble in absolute ($r$) and relative ($r/R$) coordinates at different times for the case $p_i = 1800$ atm.](image)

In all cases, a narrow layer is formed around the bubble in which degassing and a corresponding increase in viscosity occur. The distribution of concentration and viscosity around the bubble for the case $p_i = 1800$ atm and decompression time $t_d = 10s$ is shown in Figure 1. Dependences are presented both in absolute and in relative coordinates for four times, the calculation area around the bubble being $50R$. It is clearly seen that the boundary of the layer moves with the growth of the bubble and is at a distance of approximately $10R$. At the bubble boundary, especially at the initial moment of time, the concentration gradient is quite substantial, and in the future it becomes smaller. This result illustrates the importance of considering the conditions at the bubble boundary. The viscosity in this layer increases by orders of magnitude, from the initial $10^3$, to $10^{10}$ Pa s. Calculations show that it is the value of viscosity at the bubble boundary that determines, in large part, the value of effective viscosity.

The curves in relative coordinates are very close to each other, which indicates that the dynamics of the layer is close to self-similar. This correlates with the results obtained previously for the case of instant decompression.

The dynamics of pressure in the medium and bubble for different initial pressures is shown in Figure 2. When decompression occurs from an initial pressure of 1800 atm to approx 500 atm, the situation in the medium and bubble remains equilibrium and the pressures in the liquid and bubble are the same. And then, despite the continued drop in external pressure, the pressure dynamics in the bubble changes and a rapid drop is replaced by a slow decrease in pressure. This is naturally associated with an increase in viscosity near the bubble, which greatly slows down its growth. This corresponds to the transition from the exponential stage of bubble growth to the slow diffusion one; this transition is also visible in the graph for the radius of the bubble.
Figure 2. Dynamics of pressure in the medium and bubble for two initial values \(p_i = 1800\) atm, \(p_i = 500\) atm and decompression time \(t_d = 10\)s.

In the case of a lower initial pressure \(p_i = 500\) atm, the medium is initially more viscous. This significantly changes the pattern of bubble growth. The pressure drop in the bubble begins only when the decompression of the medium has already ended, and no rapid pressure drop in the bubble occurs. Slow diffusion growth leads to the fact that by the time the simulation is complete, the pressure in the bubble is still more than 150 atm, that is, about a third of the initial one.

Figure 3. Bubble dynamics for two initial values \(p_i = 1800\) atm and \(p_i = 500\) atm.

The features indicated above are clearly visible in Figure 3, where the bubble dynamics is shown for all cases. Since the full-scale features of the initial stage are difficult to distinguish, the initial and final periods are separately shown. During decompression from the initial \(p_i = 1800\) atm for 10s, the exponential growth stage and its transition to the diffusion stage are clearly visible. Naturally, a long decompression time leads to a lag in the bubble radius growth compared with other cases. Dependence of bubble radius on decompression time is obtained in numerical simulation. In case of a long decompression time the final radius of the bubble is bigger than in the case of fast decompression. However, in the initial period the situation is exactly the opposite.

For the case of initial pressure of 500 atm, there is no exponential growth stage, which is clearly noticeable in the first case. The curves for different decompression times look similar, except for a time shift. A high initial viscosity leads to the fact that the bubble immediately appears at the diffusion growth stage. The dependence on the decompression time in this case is not so pronounced as for the
case of higher pressure. By the end of the calculation, the curves almost converge, and we can assume that in the future a longer decompression time will lead to an increase in the radius of the bubble.

**Conclusions**
The dynamics of a gas bubble in viscous gas-saturated magma during decompression at a constant speed has been numerically studied, and the distribution of the concentration of dissolved gas and the viscosity of the medium around the bubble has been obtained. Due to this, the concentration gradient at the bubble boundary is correctly considered and the effective viscosity is used for the equation of bubble dynamics. The simulation shows that with slow decompression, the final radius of the bubble is higher than in the case of instant or fast decompression. The effect obtained as a result of a numerical study is most likely due to the fact that the conditions at the bubble boundary are assumed to be equilibrium and, in the case of rapid decompression, the bubble quickly finds itself in a very viscous environment. Thus, the dynamics of diffusion of the dissolved gas from the medium changes; and the mass of gas in the bubble is less than in other cases. Slow decompression, on the contrary, increases the formation time of a layer with a very high viscosity around the bubble, thereby ensuring the maximum bubble radius.

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