Automatic model order reduction for vibro-acoustic problems

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Numerical models of vibro-acoustic systems require a fine spatial discretization to be valid also for high frequent vibrations. This results in large models with high memory consumption and a large computational cost. Model order reduction (MOR) methods reduce the size of such problems, allowing an efficient design process. However, the assumptions made prior to the reduction process, e.g. desired size of the reduced model or distribution of expansion points, heavily influence the quality of the reduced models. In this contribution, we investigate interpolation based MOR methods to automatically generate reduced order models of vibro-acoustic systems valid in a limited frequency range not starting at zero.

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1 Introduction

Vibro-acoustic systems are characterized the interaction between a vibrating structure and the surrounding acoustic fluid. Many engineering problems in automotive or aviation design or building acoustics involve the solution of such systems. They exhibit a large modal density in higher frequency regions and frequency dependent damping mechanisms. If discretized with the finite element method (FEM), a fine spatial discretization is required in order to resolve the wave patterns correctly. This leads to very large and computationally demanding numerical models. A typical FEM discretization of a vibro-acoustic problem with one input and one output is given by

\[
\begin{align*}
(s^2M + sC + K)x(s) &= fu(s) \\
y(s) &= g^T x(s),
\end{align*}
\]

(1)

with mass, damping, and stiffness distribution matrices $M, C, K$, input and output vectors $f, g$, harmonic loading $u$, system state $x$, and system output $y$. $s = 2\pi i \omega$ is the complex frequency. The resulting mass and stiffness matrices are non-symmetric and the damping matrix can have zero entries on its diagonal, depending on the modeled damping mechanisms. The transfer function of the vibro-acoustic system is given by

\[
h(s) = g^T(s^2M + sC + K)^{-1}f.
\]

(2)

2 Model order reduction by moment matching

Model order reduction methods seek for an approximation $h_r(s)$ of the original system’s transfer function $h(s)$ for some frequency values $s_i$. The reduced system’s order $r$ is much lower than the original system’s order $n$, while the structure of $h(s)$ should be preserved. Moment matching methods seek a reduced transfer function $h_r(s)$ rationally interpolating the original transfer function $h(s)$ by Petrov-Galerkin projecting the original system matrices onto a lower dimensional subspace using $V, W \in \mathbb{R}^{n \times r}$. They are selected such that

\[
W^HV\left((s^2M + sC + K)Vx_r(s) - fu(s)\right) = 0
\]

(3)

and the reduced system quantities are thus given by

\[
M_r = W^HMV, C_r = W^HCV, K_r = W^HKV, f_r = W^Hf, g_r = gV.
\]

(4)

Different methods to find $V$ and $W$ are given in literature, in the following we will use the two-level orthogonal Arnoldi procedure (TOAR) \cite{1}.

3 Automatic model order reduction strategies

To automatically find a reduced model valid in a certain frequency range, two different approaches will be compared: First, a greedy method similar to the method proposed in \cite{2}. This algorithm iteratively increases the number of matched moments at an expansion point until a defined maximum order is reached or the approximation does not improve anymore. In this case, a new expansion point is chosen at the location with the highest approximation error. Since the true approximation error $\varepsilon$ is only available if the full system solution is known, an error estimator $\hat{\varepsilon}$ is employed, which is cheaper to evaluate. Following, two estimators will be employed:

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• \( \hat{\varepsilon}_c \) compares the transfer functions of two reduced models of different size [3, Sec. 5.1].

• The residual \( r_f(s) = f - (s^2M + sC + K) \mathbf{v}(s) \) is used to compute the error approximation \( \hat{\varepsilon}_r(s) = \frac{\|r_f(s)\|}{\|\hat{\varepsilon}_c\|} \) [4].

The second approach is an extension of the iterative rational Krylov algorithm (IRKA), which is used to create optimal reduced order models regarding the \( H_\infty \) norm. In the iterative algorithm, \( r \) linear systems of order \( n \) have to be solved each iteration, making the method rather expensive. Instead of performing the iterative optimization step on the full order model, CIRKA creates an intermediate reduced model, which is used for the optimization [5]. Using [6], CIRKA can be modified to find a local optimum for the expansion points in a certain frequency range, making the reduced model valid in this region.

4 Numerical example

To test the performance of the methods mentioned above, reduced order models of an acoustic cavity coupled to two parallel plates separated by an air gap are computed. Details on geometry and material parameters are given in [7]. The model is discretized according to (1) with \( n = 3300 \) and proportional damping with \( \beta = 1 \times 10^{-6} \) is applied to both plates. The strategies from Section 3 are now applied to create reduced models of the vibro-acoustic system valid for \( s = 2\pi i \{900, 1500\} \).

CIRKA’s initial order is 10, the greedy algorithm starts with order \( r_0 = 3 \) and increases it to \( r = 9 \) for each expansion point in steps of 2, until searching for a new expansion point. The convergence tolerance of all methods is set to \( \varepsilon_{tol} = 1 \times 10^{-5} \).

The transfer function of the original system, the reduced models, and the approximation errors of the different surrogate models are shown in Fig. 1. All methods succeeded in computing reduced models valid in the frequency range of interest. As expected, automatic CIRKA yields the smallest model with \( r = 10 \) but has the highest relative error. The models reduced with the greedy method and error estimators \( \hat{\varepsilon}_c \) and \( \hat{\varepsilon}_r \) are of comparable size and error. \( \hat{\varepsilon}_c \) can be computed faster than \( \hat{\varepsilon}_r \) (\( t_c = 1.70 \text{s}, \ t_r = 3.27 \text{s} \)). CIRKA’s computation time is longer (\( t_{CIRKA} = 4.47 \text{s} \)), because more matrix decompositions have to be performed to reach convergence.

![Diagram](image)

Fig. 1: Transfer function of the original system and the reduced models on the left; relative approximation error of the reduced models on the right. The convergence interval is highlighted in gray (\( \square \)).

5 Conclusion

All methods are able to automatically compute reduced models of vibro-acoustic systems valid in a defined frequency range given only this range as input parameter. Error estimation based on the comparison of two reduced models of different size is cheap to compute as well as sufficiently accurate. If the size of the reduced model should be minimal, IRKA can, in a similar way to CIRKA, be used on the reduced model created with the greedy algorithm to get a model with optimal size.

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