A New Mechanism for Leptogenesis

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(March 26, 2022)

Abstract

Unified theories containing a $U(1)_{B-L}$ gauge symmetry predict heavy Majorana right-handed neutrinos. In such theories, cosmic strings may form at the $B - L$ breaking scale. If the Higgs field forming the strings is also the Higgs field which gives mass to the right-handed neutrinos, there are right-handed neutrinos trapped as transverse zero modes in the core of the strings. When cosmic string loops decay, they release these neutrinos. This is an out-of-equilibrium process. The released neutrinos acquire heavy Majorana mass and decay into massless leptons and electroweak Higgs particles to produce a lepton asymmetry, which is converted into a baryon asymmetry via sphaleron transitions.

PACS Numbers: 98.80.Cq, 12.10.-g, 11.30.Fs
Grand unified theories (GUT’s) provide the standard scenario for baryogenesis, via the out-of-equilibrium decays of heavy gauge and Higgs bosons which violate baryon number \((B)\) and lepton number \((L)\). But it has been realized a decade ago that, unless the universe started with a non-vanishing \(B - L\) asymmetry, any \(B\) or \(L\) asymmetry generated at the GUT scale would be erased by sphaleron transitions \([1]\). An initial \(B - L\) asymmetry can be obtained in theories containing an extra gauge \(U(1)_{B - L}\) symmetry via the out-of-equilibrium decays of heavy Majorana right-handed neutrinos \([2, 3]\). This mechanism however requires either very heavy neutrinos or extreme fine tuning of the parameters in the neutrinos mass matrix \([4]\). Also, the masses of the new gauge bosons must be bigger than the smallest heavy neutrino mass \([5]\). Hence there is a wide range of parameters for which the mechanism does not produce enough baryon asymmetry.

In this letter, we show that in unified models involving an extra gauge \(U(1)_{B - L}\) symmetry, a primordial \(B - L\) asymmetry can be generated by the out-of-equilibrium decays of heavy Majorana right-handed neutrinos released by collapsing cosmic string loops. As a consequence of \(U(1)_{B - L}\) breaking, cosmic strings may form at the \(B - L\) breaking scale, according to the Kibble mechanism \([6]\). We call them \(B - L\) cosmic strings. The Higgs field mediating the breaking of \(B - L\) is the Higgs field forming the strings and it is the same Higgs field that gives heavy Majorana mass to the right-handed neutrinos. Hence, due to the winding of the Higgs field around the string, we expect right-handed neutrino zero modes \([7]\) trapped in the core of the strings. These zero modes are predicted by an index theorem \([8]\). There are also modes of higher energy bounded to the strings. We shall consider only the zero modes, which are the most favourable to be trapped. \(B - L\) cosmic string loops lose their energy by emitting gravitational radiation and rapidly shrink to a point, releasing these right-handed neutrinos. This is an out-of-equilibrium process. Right-handed neutrinos acquire heavy Majorana mass and decay into massless leptons and electroweak Higgs particles to produce a lepton asymmetry. This lepton asymmetry is converted into a baryon asymmetry via sphaleron transitions.

Topological \(B - L\) cosmic strings form when a gauge group \(G \supset U(1)_{B - L}\) breaks down to a subgroup \(H \not\supset U(1)_{B - L}\) of \(G\), if the vacuum manifold \(\frac{G}{H}\) is simply connected, that is if the first homotopy group \(\pi_1(\frac{G}{H})\) is non-trivial. If \(\pi_1(\frac{G}{H}) = I\) but string solutions still exist, then embedded strings \([9]\) form when \(G\) breaks down to \(H\). Embedded strings are stable for a wide range of parameters. In left-right models, embedded \(B - L\) strings usually form. In the simple \(U(1)\) extension of the standard model \(SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}\) where \(Y'\) is a linear combination of \(Y\) and \(B - L\) topological strings form \([10]\). In grand unified theories with rank greater than five, such as \(SO(10)\) or \(E(6)\), \(B - L\) cosmic strings may form, depending on the symmetry breaking pattern and on the set of Higgs fields used to do the breaking down to the standard model gauge group \([11, 10]\). There is a wide range of theories which contain both \(U(1)_{B - L}\) and \(B - L\) cosmic strings.

Consider a unified model with an extra \(U(1)_{B - L}\) gauge symmetry and stable \(B - L\) cosmic strings. The gauge and Higgs fields forming the strings will be the \(B - L\) associated gauge boson \(A'\) and the Higgs field \(\phi_{B - L}\) used to break \(U(1)_{B - L}\). Right-handed neutrinos acquire heavy Majorana mass via Yukawa couplings to \(\phi_{B - L}\). The \(U(1)_{B - L}\) part of the theory is described by the Lagrangian

\[
L = \frac{1}{4} f_{\mu \nu} f^{\mu \nu} + (D_\mu \phi_{B - L})^\dagger (D^\mu \phi_{B - L}) - V(\phi_{B - L})
\]
The covariant derivative \( D_\mu = \partial_\mu - ie_\mu \gamma^\nu A^\nu \) where \( e \) is the gauge coupling constant and \( Y' \) is a linear combination of \( B - L \) and \( Y \). \( \lambda \) is a Yukawa coupling constant, and \( V(\phi_{B-L}) \) is the Higgs potential. The spinor \( N = \nu_R + \nu^c_L \) is a Majorana spinor satisfying the Majorana condition \( N^c = C \gamma_0^L N^* = N \), where \( C \) is the charge conjugation matrix. Hence \( N \) has only two independent components, two degrees of freedom. \( L_f \) is the fermionic \( B - L \) part of the Lagrangian which does not contain neutrino fields.

For a straight infinite cosmic string lying along the \( z \)-axis, the Higgs field \( \phi_{B-L} \) and the \( Y' \) gauge field \( a \) in polar coordinates \((r, \theta)\) have the form

\[
\phi_{B-L} = f(r)e^{i\eta_{B-L}}
\]

\[
a_\theta = -n \tau \frac{g(r)}{er} \quad a_z = a_r = 0,
\]

where \( n \) is the winding number; it must be an integer. Most strings have winding number \( n = 1 \); strings with winding number \( |n| > 1 \) are unstable. \( \tau \) is the string’s generator; it is the normalised \( U(1)_{Y'} \) generator. It has different eigenvalues for different fermion fields. The functions \( f(r) \) and \( g(r) \) must satisfy the following boundary conditions

\[
f(0) = 0 \quad \text{and} \quad f \to \eta_{B-L} \quad \text{as} \quad r \to \infty,
\]

\[
g(0) = 0 \quad \text{and} \quad g \to 1 \quad \text{as} \quad r \to \infty,
\]

where \( \eta_{B-L} \) is the scale of \( B - L \) breaking. The exact forms of the functions \( f(r) \) and \( g(r) \) depend on the Higgs potential \( V(\phi_{B-L}) \).

From the Lagrangian (3) we derive the equation for the right-handed neutrino field:

\[
i\gamma^\mu D_\mu \nu^c_L - i\lambda \phi_{B-L}^* \gamma^0 \nu^c_L = 0
\]

where \( \nu^c_L = C \gamma_0^L \nu^c_R \). Solving (3), we find that Majorana neutrinos trapped as transverse zero modes in the core of \( B - L \) cosmic strings have only one independent component. For an \( n = 1 \) vortex it takes the form:

\[
N_1 = \beta(r, \theta) \alpha(z + t)
\]

where \( \beta(r, \theta) \) is a function peaked at \( r = 0 \) which exponentially vanishes outside the core of the string, so that the fermions effectively live on the strings. The \( z \) and \( t \) dependence of \( \alpha \) shows that the neutrinos travel at the speed of light in the \(-z\) direction, so that they are effectively massless. In an \( n = -1 \) vortex, the function \( \alpha = \alpha(t - z) \), so that the fermions travel at the speed of light in the \(+z\) direction. These fermions can be described by an effective theory in \( 1 + 1 \) dimensions. The usual energy to momentum relation

\[
E = P
\]

holds. We have no boundary conditions in the 1 spatial dimension, and the spectrum of states is continuous. In the ground state the Fermi momentum of the zero modes is \( p_F = 0 \).
The field solution (9) and the energy to momentum relation (10) have been derived for fermions on a straight infinite string. However, physical cosmic strings are very wiggly and are not straight. Hence, relations (9) and (10) do not hold in the physical case. Neither do they hold for cosmic string loops, even if the latter are assumed to be smooth. On a cosmic string loop, fermions are characterised by their angular momentum $L$. The energy relation becomes [12]

$$E = \frac{(L + \frac{1}{2})}{R} = P + \frac{1}{2R}$$

(11)

and hence the energy spectrum is

$$E = \frac{(n + \frac{1}{2})}{R}$$

(12)

where $n \in \mathbb{N}$. We see from Eqs. (11), (10), and (12) that, when $R$ is very large, the string looks locally like a straight string. We have an almost continuous spectrum of states. The fact that the string gets a finite curvature acts as a perturbation on the string bound states. The energy levels get quantised and the Fermi energy gets a non-vanishing value, $E_F = \frac{1}{2R}$. As the string loop shrinks, its radius $R$ decreases and we see from Eq. (12) that the Fermi energy increases and that the separation between energy levels gets wider.

Assuming that a loop decays when its radius $R$ becomes comparable to its width $w \sim \eta_{B-L}$, we deduce that the Fermi energy level when the loop decays is $E_F \sim \frac{1}{2}\eta_{B-L}$, where the $B-L$ breaking scale $\eta_{B-L}$ is of the order of the right-handed neutrino mass. $E_F$ is lower than the energy needed by the neutrinos to escape the string [12]. Hence, when a cosmic loop decays, it releases at least $n_\nu = 1$ heavy Majorana neutrinos. Quantum fluctuations and finite temperature corrections may increase $n_\nu$. Part of the final burst of energy released by the decaying cosmic string loop is converted into mass energy for the gauge and Higgs particles released by the string, and into mass energy for the neutrinos. A decaying $B-L$ cosmic string loop releases heavy $B-L$ Higgs particles which can decay into right-handed neutrino pairs, and hence increase $n_\nu$. This is an out-of-equilibrium process. Due to angular momentum conservation, the massive Majorana neutrinos released by a decaying cosmic string loop which were trapped as transverse zero modes are spinning particles.

Heavy Majorana right-handed neutrinos interact with the standard model leptons via the Yukawa couplings

$$L_Y = h_{ij} l_l H_{ew} \nu_{Rj} + h.c.$$  

(13)

where $l$ is the usual lepton doublet; for the first family $l = (e, \nu)_L$. $H_{ew}$ is the standard model doublet of Higgs fields. Majorana right-handed neutrinos can decay via the diagrams shown in Fig. 1.a. and 1.b. CP is violated through the one loop radiative correction involving a Higgs particle as shown in figure 1.b. The right-handed neutrinos are out-of-equilibrium, and hence a lepton asymmetry can be generated. The lepton asymmetry is characterised by the CP violation parameter $\epsilon$ which, assuming that the neutrino Dirac masses fall into a hierarchical pattern qualitatively similar to that of the leptons and quarks, is estimated to be [3]

$$\epsilon \simeq \frac{m_{D_4}^2}{\pi v^2} \frac{M_{N1}}{M_{N2}} \sin \delta$$

(14)
where $m_{D_3}$ is the Dirac mass of the third lepton generation, $v$ is the vacuum expectation value of the electroweak Higgs field $v = < H_{ew} > = 174$ GeV, $M_{N_1}$ and $M_{N_2}$ are the right-handed neutrino Majorana masses of the first and second generation respectively and $\delta$ is the CP violating phase.

The corresponding $B-L$ asymmetry (we use $B-L$ instead of $L$ since the $(B+L)$-violating electroweak anomaly conserves $B-L$) must be calculated solving Boltzmann equations which take into account all $B$, $L$ and $B+L$ violating interactions and their inverse decay rates. We can however calculate the $B-L$ asymmetry produced taking into account only the out-of-equilibrium decays of right-handed neutrinos released by decaying cosmic string loops and assuming that the rates of inverse decays are negligible. Hence an upper limit on the baryon number per commoving volume at temperature $T$ is then given by 

$$B(T) = \frac{1}{2} \frac{N_\nu(t) \epsilon}{s},$$

(15)

where $s$ is the entropy at time $t$ and $N_\nu(t)$ is the number density of right-handed neutrinos which have been released by decaying cosmic string loops at time $t$. Recall that the temperature $T$ is related to the cosmic time $t$ via the relation $t = 0.3 g_*^{\frac{1}{2}} \frac{M_{pl}}{T^2}$, where $g_*$ counts the number of massless degrees of freedom in the corresponding phase and $M_{pl}$ is the Planck mass. $s$, the entropy at time $t$, is given by $s = \frac{2}{45} \pi^2 g_* T^3$. $N_\nu(t)$ is approximately $n_\nu$ times the number density of cosmic string loops which have shrunk to a point at temperature $T$. Assuming that sphaleron transitions are not in thermal equilibrium below $T_{ew}$, and neglecting any baryon number violating processes which might have occurred below $T_{ew}$, the baryon number of the universe at temperature $T \leq T_{ew}$ is then given by

$$B = B(T_{ew}),$$

(16)

which is also the baryon number of the universe today. If sphaleron transitions are also rapid below $T_{ew}$, we should include the neutrinos released below $T_{ew}$. However, below $T_{ew}$ the number density of decaying cosmic string loops is negligible, and hence this would not affect the result in any sense.

The number density of decaying cosmic string loops can be estimated from the three scales model of reference [15]. The model is based on the assumption that the cosmic string network evolution is characterised by three length scales $\xi(t)$, $\bar{\xi}(t)$ and $\chi(t)$ related to the long string density, the persistence length along the long strings (which is related to the fact that the typical loop size is much smaller than $\xi$), and the small scale structure along the strings respectively.

Cosmic string loops lose their energy by emitting gravitational radiation at a rate $\dot{E} = -\Gamma_{\text{loops}} G \mu^2$ [15], where $\mu \sim T_c^2$ is the string mass-per-unit-length and $T_c = \eta_{B-L}$ is the critical temperature of the phase transition leading to the string network formation. $G$ is Newton’s constant. The numerical factor $\Gamma_{\text{loops}} \sim 50 - 100$ depends on the loop’s shape and trajectory, but is independent of its length. The mean size of a loop born at $t_b$ is assumed to be $(k - 1) \Gamma_{\text{loops}} G \mu t_b$. At a later time $t$, it is then $\Gamma_{\text{loops}} G \mu (kt_b - t)$. The loop finally disappears at a time $t = kt_b$. Numerically, $k$ is found to lie between 2 and 10.

The rate at which the string loops form in a volume $V$ is given by [15]

$$\dot{N}(t_b) = \frac{\nu V}{(k - 1) \Gamma_{\text{loops}} G \mu t_b^4},$$

(17)
where the parameter $\nu$ can be expressed in terms of the various length scales of the model which vary with time. We start with $\xi \sim \zeta \sim \chi$. Then $\xi$ and later $\zeta$ will start to grow and will evolve to the scaling regime characterised by $\xi(t)$ and $\zeta(t) \sim t$. The length scale $\chi$ grows much less rapidly. Therefore $\nu$ varies with time. In the scaling regime, $\nu$ is estimated to lie in the range $0.1 < \nu < 10$ [13].

Note that it has recently been shown that cosmic string networks reach the scaling solution at a time $t_*$ much smaller than previously estimated [14]. The authors of ref. [14] find that in the radiation dominated, for minimal GUT strings, $t_* = 8 \times 10^2 t_c$ [14], where $t_c$ is the time at which the strings form, and the associated temperature is $T_* \simeq 10^{14.5}$ GeV. Hence our approximation for the rate of cosmic string loop formation is suitable; it leads to a lower bound on the number of cosmic string loops. Only numerical simulations could lead us to a better estimate, but it is beyond the scope of this paper.

Since it has been shown that most of the loops formed have relatively small size, we shall assume that the number of loops rejoining the network is negligible and thus that the number of decaying loops is equal to the number of forming loops. Hence the number density of right-handed neutrinos which have been released by decaying cosmic string loops at time $t$ is given by

$$N_{\nu}(t) = n_{\nu} \int_{k t_c}^{t} \frac{N(t_b)}{r(t_b)} \frac{r(t_b)}{r(t)}^3 dt_b$$

(18)

where $r(t)$ is the cosmic scale factor and $n_{\nu}$ is the mean number of right-handed neutrinos released by a single decaying loop. In the radiation dominated era $r(t) \sim t^{1/2}$. After integration we obtain

$$N_{\nu}(t) = \frac{2}{3} n_{\nu} \frac{\nu}{(k-1) \Gamma_{\text{loops}} G\mu} \frac{1}{(0.3)^3 g_*^{1/4}} \left[ \frac{1}{k^{3/2}} \left( \frac{T_c}{M_{\text{pl}}} \right)^3 - \left( \frac{T}{M_{\text{pl}}} \right)^3 \right] T^3.$$  

(19)

Hence the baryon number per comoving volume today produced by the decays of heavy Majorana right-handed neutrinos released by decaying cosmic string loops given in Eq.(15) becomes

$$B \approx \frac{7.5 g_*^{1/2}}{(0.3 \pi)^3} \frac{\nu}{(k-1) \Gamma_{\text{loops}} G\mu} \frac{T_c}{M_{\text{pl}}} \frac{m_{D3}^2}{v^2} \frac{M_{N1}}{M_{N2}} \sin \delta,$$

(20)

where we have used Eq.(14) for the CP violation parameter $\epsilon$. The produced $B$ asymmetry depends on the cosmic string scenario parameters, on the neutrino mass matrix and on the strength of CP violation.

We now calculate the lower and upper bounds on $B$ which correspond to different values of the parameters in the cosmic string scenario. We fix the neutrino mass matrix parameters and assume maximum CP violation, i.e. $\sin \delta = 1$. We assume that the Dirac mass of the third generation fermions lies in the range $m_{D3} = 1 - 100$ GeV, and that the ratio of the right-handed neutrino masses of the first and second generation $\frac{M_{N1}}{M_{N2}} = 0.1$. With the above assumptions, we find $B$ to lie in the range

$$B \simeq (1 \times 10^{-10} - 5 \times 10^{-2}) g_*^{1/2} \left( \frac{T_c}{M_{\text{pl}}} \right),$$

(21)
and we see that $B$ strongly depends on the cosmic string scenario parameters. Recall that the baryon number-to-photon ratio $n_B/n_\gamma$ is related to the baryon number of the universe $B$ by $n_B/n_\gamma = 1.80 g_* B$. Hence, if $g_* \simeq 10^2$, our mechanism alone can explain the baryon-number-to-photon ratio predicted by nucleosynthesis, $n_B/n_\gamma = (2 - 7) \times 10^{-10}$, with the $B - L$ breaking scale in the range

$$\eta_{B-L} = (2 \times 10^7 - 3 \times 10^{16}) \text{ GeV}. \quad (22)$$

We point out that the result could be better calculated solving Boltzmann equations, which take into account all $B$, $L$, and $B + L$ violating interactions and do not neglect the inverse decay rates. Furthermore, the rate of decaying cosmic string loops can be calculated via numerical simulations, which would have to take into account the different regimes of the network evolution which occur during and after the friction dominated era. This may change the allowed value for the $B - L$ breaking scale by a few orders of magnitude. Note also that if CP is not maximally violated, the $B - L$ breaking scale will be shifted towards higher values. Finally, we recall that when a cosmic string loop decays, it also releases massive Higgs bosons $\phi_{B-L}$ and massive gauge bosons $a$ which can decay into right-handed neutrinos. This process is not taken into account here because the masses of the Higgs and gauge bosons and of the neutrinos are very close to each other and the Higgs and gauge fields can also decay into other particles.

**ACKNOWLEDGMENTS**

The author would like to thank Brandon Carter, Nick Manton and Mark Hindmarsh for discussions. She would like to thank Newnham College and PPARC for financial support.
REFERENCES

[1] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. 155B, 36 (1985).
[2] M. Fukugita and T. Yanagida, Phys. Lett. 174B, 45 (1986).
[3] M. A. Luty, Phys. Rev. D 45, 455 (1992).
[4] W. Buchmüller and Y. Yanagida, Phys. Lett. 302B, 240 (1993).
[5] K. Enqvist and I. Vilja, Phys. Lett. 295 B, 281 (1993).
[6] T.W.B. Kibble, J. Phys. A 9, 387 (1976).
[7] R. Jackiw and P. Rossi, Nucl. Phys. B 190, 681 (1981).
[8] E. Weinberg, Phys. Rev. D 24, 2669 (1981).
[9] T. Vachaspati, Phys. Rev. Lett. 68, 1977 (1992).
[10] R. Jeannerot, Phys. Rev. D53, 5426 (1996).
[11] T.W.B. Kibble, G. Lazarides and Q. Shafi, Phys. Lett. B 113, 237 (1982); E. Witten, Nucl. Phys. B249, 557 (1985); R. Jeannerot and A.C. Davis, Phys. Rev. D 52, 7220 (1995).
[12] S.M. Barr and A.M. Matheson, Phys. Lett. B 198, 146 (1987).
[13] E.W. Kolb and M.S. Turner, The early universe, Redwood City, USA: Addison-Wesley (1990), (Frontiers in physics, 69).
[14] C.J.A.P. Martins and E.P.S. Shellard, Phys. Rev. D 53, 575 (1996).
[15] D. Austin, E.J. Copeland and T.W.B. Kibble, Phys. Rev. D 51, 2499 (1993).
FIG. 1a. The tree-level diagram for right-handed neutrino decay.

FIG. 1b. The one loop diagram for right-handed neutrino decay.