Joint Optimization of Coverage, Capacity and Load Balancing in Self-Organizing Networks

Qi Liao*, Daniyal Amir Awan*, Sławomir Stańczak†
*Technische Universität Berlin, 10587 Berlin, Germany
qi.liao.de@ieee.de, {daniyal.a.awan, slawomir.stanczak}@tu-berlin.de

Abstract—This paper develops an optimization framework for self-organizing networks (SON). The objective is to ensure efficient network operation by a joint optimization of different SON functionalities, which includes capacity, coverage and load balancing. Based on the axiomatic framework of monotone and strictly subhomogeneous function, we formulate an optimization problem for the uplink and propose a two-step optimization scheme using fixed point iterations: i) per base station antenna tilt optimization and power allocation, and ii) cluster-based base station assignment of users and power allocation. We then consider the downlink, which is more difficult to handle due to coupled variables, and show downlink-uplink duality relationship. As a result, a solution for the downlink is obtained by solving the uplink problem. Simulations show that our approach achieves a good trade-off between coverage, capacity and load balancing.

I. INTRODUCTION

A major challenge towards self-organizing networks (SON) is the joint optimization of multiple SON use cases by coordinate handling multiple configuration parameters. Widely studied SON use cases include coverage and capacity optimization (CCO), mobility load balancing (MLB) and mobility robustness optimization (MRO) [1]. However, most of these works study an isolated single use case and ignore the conflicts or interactions between the use cases [2], [3].

In contrast, this paper considers a joint optimization of two strongly coupled use cases: CCO and MLB. The objective is to achieve a good trade-off between coverage and capacity performance, while ensuring a load-balanced network. The SON functionalities are usually implemented at the network management layer and are designed to deal with “long-term” network performance. Short-term optimization of individual users is left to lower layers of the protocol stack. To capture long-term global changes in a network, we consider a cluster-based network scenario, where users served by the same base station (BS) with similar SINR distribution of the interference among the BSs.

Joint optimization of assignment parameters with the same objective functions, formulated either as a problem of per-cluster variables or as a problem of per-base variables. We propose a two-step optimization algorithm in the uplink to iteratively optimize the per BS variables (antenna tilts and BS power budgets) and the cluster-based variables (assignments and cluster power). Since both problems aim at optimizing the same objective function, the algorithm is shown to be convergent.

The decoupled property of antenna tilt and assignment in the uplink decomposes the high-dimensional optimization problem and enables more efficient optimization algorithm. We

See you later.
then analyze the uplink-downlink duality by using the Perron-Frobenius theory \cite{15}, and propose an efficient sub-optimal solution in the downlink by utilizing optimized variables in the dual uplink.

II. SYSTEM MODEL

We consider a multicell wireless network composed of a set of BSs $N := \{1, \ldots, N\}$ and a set of users $\mathcal{K} := \{1, \ldots, K\}$. Using fuzzy C-means clustering algorithm \cite{16}, we group users with similar SINR distributions $\alpha$ and served by the same BS into clusters. The clustering algorithm is beyond the scope of this paper. Let the set of user clusters be denoted by $\mathcal{C} := \{1, \ldots, C\}$, and let $A$ denote a $C \times K$ binary user/cluster assignment matrix whose columns sum to one. The BS/cluster assignment is defined by a $N \times C$ binary matrix $B$ whose columns also sum to one.

Throughout the paper, we assume a frequency flat channel. The average/long-term downlink path attenuation between $N$ BSs and $K$ users are collected in a channel gain matrix $H \in \mathbb{R}^{N \times K}$. We introduce the cross-link gain matrix $V \in \mathbb{R}^{K \times K}$, where the entry $v_{lk}(\theta_j)$ is the cross-link gain between user $l$ served by BS $j$, and user $k$ served by BS $i$, i.e., between the transmitter of the link $(j,l)$ and the receiver of the link $(i,k)$. Note that $v_{lk}(\theta_j)$ depends on the antenna downtilt $\theta_j$. Let the BS/user assignment matrix be denoted by $J$ so that we have $J := BA \in \{0,1\}^{N \times K}$, and $V := J^T H$. We denote by $r := [r_1, \ldots, r_K]^T$, $q := [q_1, \ldots, q_K]^T$ and $p := [p_1, \ldots, p_K]^T$ the BS transmission power budget, the cluster power allocation and the user power allocation, respectively.

A. Inter-cluster and intra-cluster power sharing factors

We introduce the inter-cluster and intra-cluster power sharing factors to enable the transformation between two power vectors with different dimensions. Let $b := [b_1, \ldots, b_C]^T$ denote the serving BSs of clusters $\{1, \ldots, C\}$. We define the vector of the inter-cluster power sharing factors to be $\beta := [\beta_1, \ldots, \beta_C]^T$, where $\beta_k := q_k/r_{\alpha_k}$. With the BS/cluster assignment matrix $B$, we have $q := B_{\beta} r$, where $B_{\beta} := B \text{diag}\{\beta\}$. Since users belonging to the same cluster have similar SINR, we allocate the cluster power uniformly to the users in the cluster. The intra-cluster sharing factors are represented by $\alpha := [\alpha_1, \ldots, \alpha_K]^T$ with $\alpha_k = 1/|X_{\alpha_k}|$ for $k \in \mathcal{K}$, where $X_{\alpha_k}$ denotes the set of users belonging to cluster $c_k$, while $c_k$ denotes the cluster with user $k$. We have $p := A_{\alpha} q$, where $A_{\alpha} := A \text{diag}\{\alpha\}$. The transformation between BS power $r$ and user power $p$ is then $p := Tr$ where the transformation matrix $T := A_{\alpha} B_{\beta}^T$.

B. Signal-to-interference-plus-noise ratio

Given the cross-link gain matrix $V$, the downlink SINR of the $k$th user depends on all powers and is given by

$$\text{SINR}_{k}^{(d)} := \frac{p_k \cdot v_{kk}(\theta_{nk})}{\sum_{l \in \mathcal{K} \setminus k} p_l \cdot v_{lk}(\theta_{nl}) + \sigma_k^2}, k \in \mathcal{K} \tag{1}$$

We assume the Kullback-Leibler divergence as the distance metric.

where $n_k$ denotes the serving BS of user $k$, $\sigma_k^2$ denotes the noise power received in user $k$. Likewise, the uplink SINR is

$$\text{SINR}_{k}^{(u)} := \frac{p_k \cdot v_{kk}(\theta_{nk})}{\sum_{l \in \mathcal{K} \setminus k} p_l \cdot v_{lk}(\theta_{nl}) + \sigma_k^2}, k \in \mathcal{K} \tag{2}$$

Assuming that there is no self-interference, the cross-talk terms can be collected in a matrix

$$[\tilde{V}]_{lk} := \begin{cases} v_{lk}(\theta_{nk}), & l \neq k, \\ 0, & l = k. \end{cases} \tag{3}$$

Thus the downlink interference received by user $k$ can be written as $I_{k}^{(d)} := [\tilde{V}^T p]_k$, while the uplink interference is given by $I_{k}^{(u)} := [\tilde{V}^r p]_k$.

A crucial property is that the uplink SINR of user $k$ depends on the BS assignment $n_k$ and the single antenna tilt $\theta_{nk}$ alone, while the downlink SINR depends on the BS assignment vector $n := [n_1, \ldots, n_K]^T$, and the antenna tilt vector $\theta := [\theta_1, \ldots, \theta_N]^T$. The decoupled property of uplink transmission has been widely exploited in the context of uplink and downlink multi-user beamforming \cite{11} and provides a basis for the optimization algorithm in this paper.

The notation used in this paper is summarized in Table I.



III. UTILITY DEFINITION AND PROBLEM FORMULATION

As mentioned, the objective is a joint optimization of coverage, capacity and load balancing. We capture coverage by the worst-case SINR, while the average SINR is used to represent capacity. A cluster-based utility $U_c(\theta, r, q, b)$ is introduced as the combined function of the worst-case SINR and average SINR, depending on BS power allocation $r$, antenna downtilt $\theta$, cluster power allocation $q$ and BS/cluster

| TABLE I |
| NOTATION SUMMARY |
| --- |
| $N$ | set of BSs |
| $K$ | set of users |
| $\mathcal{C}$ | set of user clusters |
| $A$ | cluster/user assignment matrix |
| $B$ | BS/cluster assignment matrix |
| $J$ | BS/user assignment matrix |
| $c_k$ | cluster that user $k$ is subordinated to |
| $\mathcal{X}_c$ | set of users subordinated to cluster $c$ |
| $H$ | channel gain matrix |
| $V$ | interference coupling matrix |
| $V_b$ | interference coupling matrix without intra-cluster interference |
| $V_\theta$ | interference coupling matrix depending on BS assignments $\theta$ |
| $r$ | BS power budget vector |
| $q$ | cluster power vector |
| $p$ | user power vector |
| $\alpha$ | intra-cluster power sharing factors |
| $\beta$ | inter-cluster power sharing factors |
| $A_{\alpha}$ | transformation from $q$ to $p$, $p := A_{\alpha}^T q$ |
| $B_{\beta}$ | transformation from $r$ to $q$, $q := B_{\beta}^T r$ |
| $T$ | BS antenna tilt vector |
| $b$ | serving BSs of clusters |
| $b_c$ | serving BS of cluster $c$ |
| $n$ | serving BSs of the users |
| $n_k$ | serving BS of user $k$ |
| $\sigma$ | noise power vector |
| $p_{\text{max}}$ | sum power constraint |
To achieve the load balancing by distributing the clusters to the BSs such that their utility targets can be achieved\(^3\) we formulate the following objective
\[
\max_{(r, \hat{\theta}, q, b)} \frac{U_c(r, \hat{\theta}, q, b)}{\gamma_c}
\]
where \(\gamma_c\) is the predefined utility target for cluster \(c\). The BS variables \((r, \hat{\theta})\) and cluster variables \((q, b)\) are optimized by iteratively solving
1) Cluster-based BS assignment and power allocation
\[
\max_{(q, b)} \max_{c \in C} U_c(r, q, b) = \frac{\gamma_c}{\gamma_c}
\]
2) BS-based antenna tilt optimization and power allocation
\[
\max_{r, \hat{\theta}} \frac{U_c(r, \hat{\theta}, q, b)}{\gamma_c} = \frac{\gamma_c}{\gamma_c}
\]

In the following we introduce the utility definition and problem formulation for the cluster-based and the BS-based problems respectively. We start with the problem statement and algorithmic approaches for the uplink. We then discuss the downlink in Section \(\text{V}\).

\(\text{A. Cluster-Based BS Assignment and Power Allocation}\)

Assume the per-BS variables \((\hat{r}, \hat{\theta})\) are fixed, let the interference coupling matrix depending on BS assignment \(b\) in \(\text{[5]}\) be denoted by \(V_0\). We first define two utility functions indicating capacity and coverage per cluster respectively, then we introduce the joint utility as a combination of the capacity and coverage utility. After that we define the cluster-based max-min utility balancing problem based on the joint utility.

1) Average SINR Utility (Capacity): With the intra-cluster power sharing factor introduced in Section \(\text{II-A}\) we have \(p := A^T q\). Define the noise vector \(\sigma := [\sigma_1^T, \ldots, \sigma_K^T]^T\), the average SINR of all users in cluster \(c\) is written as
\[
\tilde{U}_c^{(1)}(q, b) := \frac{1}{|\mathcal{X}_c|} \sum_{k \in \mathcal{X}_c} \max_{c \in C} \Phi_{V_b} A^T q + \sigma_k^T
\]
\[
\geq \frac{1}{|\mathcal{X}_c|} \sum_{k \in \mathcal{X}_c} \max_{c \in C} \frac{q_c \kappa_k v_{kk}}{\sigma_k^T}
\]
\[
= U_c^{(1)}(q, b)
\]

The uplink capacity utility of cluster \(c\) denoted by \(U_c^{(u,1)}\) is measured by the ratio between the total useful power and the total interference power received in the uplink in the cluster. Utility \(U_c^{(u,1)}\) is used instead of \(U_c^{(1)}\) because of two reasons: First, it is a lower bound for the average SINR. Second, it has certain monotonicity properties (introduced in Section \(\text{IV}\)) which are useful for optimization.

Introducing the cluster coupling term \(\Phi_{V_b} := \Psi A V_b A^T\), where \(\Psi := \text{diag}\{g_1, \ldots, g_C\}\) and \(g_c := \sum_{k \in \mathcal{X}_c} \kappa_k v_{kk}\) for \(c \in \mathcal{C}\); and the noise term \(\Xi := \Psi A \sigma\), the capacity utility is simplified as
\[
U_c^{(u,1)}(q, b) := \frac{q_c}{\beta_c^{(u,1)}(q, b)}
\]
where \(\beta_c^{(u,1)}(q, b) := \frac{\Phi_{V_b} A^T q + \sigma_k^T}{c}\).

2) Worst-Case SINR Utility (Coverage): Roughly speaking, the coverage problem arises when a certain number of the SINRs are lower than the predefined SINR threshold. Thus, improving the coverage performance is equivalent to maximizing the worst-case SINR such that the worst-case SINR achieves the desired SINR target. We then define the uplink coverage utility for each cluster as
\[
U_c^{(u,2)}(q, b) := \min_{k \in \mathcal{X}_c} \max_{c \in C} \Phi_{V_b} A^T q + \sigma_k^T
\]
where \(\Phi := \text{diag}\{1/\alpha_1 v_{11}, \ldots, 1/\alpha_K v_{KK}\}\). We define a \(C \times K\) matrix \(X := [x_1, \ldots, x_C]^T\), where \(x_c := e^T_k\) and \(e^T_k\) denotes an \(i\)-dimensional binary vector which has exact one entry (the \(j\)-th entry) equal to 1. Introducing the term \(\mathcal{C}_b := \Phi V_b A^T\), and the noise term \(\Xi := \Phi \sigma\), the coverage utility is given by
\[
U_c^{(u,2)}(q, b) := \frac{q_c}{\beta_c^{(u,2)}(q, b)}
\]
where \(\beta_c^{(u,2)}(q, b) := \max_{k \in \mathcal{X}_c} \mathcal{C}_b q + X \Xi\).

3) Joint Utility and Cluster-Based Max-Min Utility Balancing: The joint utility \(U_c^{(u)}(q, b)\) is defined as
\[
U_c^{(u)}(q, b) := \frac{q_c}{\beta_c^{(u)}(q, b)}
\]
where \(\beta_c^{(u)}(q, b) := \mu q_c^{(u,1)} + (1 - \mu) q_c^{(u,2)}(q, b)\).

In other words, the joint interference function \(\beta_c^{(u)}\) is a convex combination of \(\beta_c^{(u,1)}\) in \(\text{[6]}\) and \(\beta_c^{(u,2)}\) in \(\text{[9]}\).

The cluster-based power-constrained max-min utility balancing problem in the uplink is then provided by

\(\text{Problem 1 (Cluster-Based Utility Balancing):}\)

\[
\max_{q \geq 0, b \in \mathcal{B}} \min_{c \in C} \frac{U_c^{(u)}(q, b)}{\gamma_c}, \text{ s.t. } \|q\| \leq P_{\text{max}}
\]

Here, \(\| \cdot \|\) is an arbitrary monotone norm, i.e., \(q \leq q'\) implies \(\|q\| \leq \|q'\|\). \(P_{\text{max}}\) denotes the total power constraint.

According to the joint utility in \(\text{[10] - [11]}\), the algorithm optimizes the performance of capacity when we set the tuning parameter \(\mu = 1\) (utility is equivalent to the capacity utility in \(\text{[5]}\), while with \(\mu = 0\) it optimizes the performance of coverage (utility equals to the coverage utility in \(\text{[8]}\)). By tuning \(\mu\) properly, we can achieve a good trade-off between the performance of coverage and capacity.
B. BS-Based Antenna Tilt Optimization and Power Allocation

Given the fixed \((q, b)\), we compute the intra-cluster power allocation factor \(\beta\), given by \(\beta_c := \frac{q_c}{\sum_{c \in \mathcal{C}} q_c}\) for \(c \in \mathcal{C}\). We denote the cross-link coupling matrix depending on \(\theta\) by \(V_\theta\). In the following we formulate the BS-based max-min utility balancing problem such that it has the same physical meaning as the problem stated in (12). We then introduce the BS-based joint utility interpreted by \((r, \theta)\).

1) BS-Based Max-Min Utility Balancing: To be consistent with our objective function \(C^\theta(P_{\max})\) in (12), we transform the cluster-based optimization problem to the BS-based optimization problem:

\[
C^{(u)}(P_{\max}) = \max_{r \geq 0, \theta \in \Theta} \min_{c \in \mathcal{C}} \frac{U_c^{(u)}(r, \theta)}{\gamma_c} = \max_{r \geq 0, \theta \in \Theta} \min_{n \in \mathcal{N}} \frac{U_n^{(u)}(r, \theta)}{\gamma_c} = \max_{r \geq 0, \theta \in \Theta} \min_{n \in \mathcal{N}} \tilde{U}_n^{(u)}(r, \theta), \text{ s.t. } \|r\| \leq P_{\max}
\]

\[(13)\]

where \(\Theta\) denotes the predefined space for antenna tilt configuration.

2) BS-Based Joint Utility: It is shown in (13) that the cluster-based problem is transformed to the BS-based problem by defining

\[
\tilde{U}_n^{(u)}(r, \theta) := \min_{c \in \mathcal{C}} \frac{U_c^{(u)}(r, \theta)}{\gamma_c} = \frac{r_n}{\beta_c^{(u)}(r, \theta)}
\]

\[(14)\]

\[
\beta_c^{(u)}(r, \theta) := \max_{c \in \mathcal{C}} \frac{\gamma_c}{\beta_c}
\]

\[(15)\]

where \(\beta_c^{(u)}(r, \theta)\) is obtained from \(\beta_c^{(u)}(q, b)\) in (11) by substituting \(q := B_\theta^c r\) and \(V_c\) with \(V_\theta\). Note that (14) is derived by applying the inter-cluster sharing factor such that \(r_n := q_n/\beta_c\) for \(n = b_c\). Due to lack of space we omit the details of the individual per BS capacity and coverage utilities corresponding to the cluster-based utilities (4) and (7).

IV. OPTIMIZATION ALGORITHM

We developed our optimization algorithm based on the fixed-point iteration algorithm proposed by Yates [5], by exploiting the properties of the monotone and strictly sub-homogeneous functions.

A. MSS function and Fixed-Point Iteration

The vector function \(f : \mathbb{R}^K_+ \rightarrow \mathbb{R}^K_+\) of interest has the following two properties:

- Monotonicity: \(a \leq y\) implies \(f(x) \leq f(y)\).
- Strict subhomogeneity: for each \(\alpha > 1\), \(f(\alpha x) < \alpha f(x)\).

A function satisfying the above two properties is referred to be monotonic and strictly subhomogeneous (MSS). When the strict inequality is relaxed to weak inequality, the function is said to be monotonic and subhomogeneous (MS).

**Theorem 1:** [7] Suppose that \(f : \mathbb{R}^K_+ \rightarrow \mathbb{R}^K_+\) is MSS and that \(h = x/l(x)\), where \(l : \mathbb{R}^K_+ \rightarrow \mathbb{R}_+\) is MS. For each \(\theta > 0\), there is exactly one eigenvector \(v\) and the associated eigenvalue \(\lambda\) of \(f\) such that \(l(v) = \theta\). Given an arbitrary \(\theta\), the repeated iterations of the function

\[
g(x) = \theta f(x)/l(f(x))
\]

\[(16)\]

converge to a unique fixed point such that \(l(v) = \theta\). The fixed point iteration in (16) is used to obtain the solution of the following max-min utility balancing problem

\[
\max_{p} \min_{k \in \mathcal{K}} U_k(p), \text{ s.t. } \|p\| \leq P_{\max}
\]

\[(17)\]

where the utility function can be defined as \(U_k(p) := p_k/f_\theta^k(p)\).

B. Joint Optimization Algorithm

We aim on jointly optimizing both problems, by optimizing \((q, b)\) and \((r, \theta)\) in Problem 1 and \((r, \theta)\) in Problem 2 iteratively with the fixed-point iteration. In the following we present some properties that are required to solve the problem efficiently and to guarantee the convergence of the algorithm.

1) Decoupled Variables in Uplink: In uplink the variables \(b\) and \(\theta\) are decoupled in the interference functions (11) and (15), i.e., \(\beta_c^{(u)}(q, b) := \beta_c^{(u)}(q, b_c) \) and \(\beta_n^{(u)}(r, \theta) := \beta_n^{(u)}(r, \theta_n)\). Thus, we can decompose the BS assignment (or tilt optimization) problem into sub-problems that can be independently solved in each cluster (or BS), and the interference functions can be modified as functions of the power allocation only:

\[
\beta_c^{(u)}(q) := \min_{b_c \in \mathcal{B}} \beta_c^{(u)}(q, b_c)
\]

\[(18)\]

\[
\beta_n^{(u)}(r) := \min_{\theta_n \in \Theta} \beta_n^{(u)}(r, \theta_n)
\]

\[(19)\]

2) Standard Interference Function: The modified interference function (18) and (19) are standard. Using the following three properties: 1) an affine function \(I(p) := Vp + \sigma\) is standard, 2) if \(I(p)\) and \(J(p)\) are standard, then \(\beta I(p) + (1-\beta)J(p)\) are standard, and 3) If \(I(p)\) and \(J(p)\) are standard, then \(I_{\min}(p)\) and \(I_{\max}(p)\) are standard, where \(I_{\min}(p)\) and \(I_{\max}(p)\) are defined as \(I_{\min}(p) := \min\{I(p), J(p)\}\) and \(I_{\max}(p) := \max\{I(p), J(p)\}\) respectively [5], we can easily prove that (18) and (19) are standard interference functions.

Substituting (18) and (19) in Problem 1 and Problem 2 define \(U_n^{(u)}(q) := q_n/\beta_c^{(u)}(q)\) and \(U_n^{(u)}(r) := r_n/\beta_n^{(u)}(r)\), we can write both problems in the general framework of the max-min fairness problem (17):

Problem 1. \(\max_{q \geq 0} \min_{c \in \mathcal{C}} U_c^{(u)}(q)/\gamma_c, \|q\| \leq P_{\max}\).

Problem 2. \(\max_{r \geq 0} \min_{n \in \mathcal{N}} U_n^{(u)}(r), \|r\| \leq P_{\max}\).

The property of the decoupled variables in uplink and the property of utilities based on the standard interference functions enable us to solve each problem efficiently with two iterative steps: 1) find optimum variable \(b_c\) (or \(\theta_n\)) for each cluster \(c\) (or each BS \(n\)) independently, 2) solve the max-min balancing power allocation problem with fixed-point iteration.
3) Connections between The Two Problems: Problem 1 and Problem 2 have the same objective $C^{(u)}(P_{\text{max}})$ as stated in (12) and (13), i.e., given the same variables $\{q, b, r, \theta\}$, using (14), we have $\min_{c \in \mathcal{C}} U_c^{(u)} / \gamma_c = \min_{r_n \in \mathcal{R}} U_n^{(u)}$. Both problems are under the same sum power constraint. However, the convergence of the two-step iteration requires two more properties: 1) the BS power budget $r$ derived by solving Problem 2 at the previous step should not be violated by the cluster power allocation $q$ found by optimizing Problem 1 and 2) when optimizing Problem 2, the inter-cluster power sharing factor $\beta$ should be consistent with the derived cluster power allocation $q$ in Problem 1.

To fulfill the first requirement, we introduce the per BS power constraint $P_{\text{max}}$ for Problem 2 equivalent to the BS power budget $r_n$ in Problem 1. We also propose a scaled version of fixed point iteration similar to the one proposed in [7] to iteratively scale the cluster power vector and achieve the max-min utility boundary under per BS power budget constraints, as stated below.

$$q_c^{(t+1)} = \frac{\gamma_c c^{(u)}(q^{(t)})}{\|B(\gamma_c q^{(t)}) \otimes P_{\text{max}(c)}\|_\infty} \tag{20}$$

where $\otimes$ denote the element-wise division of vectors, $\| \cdot \|_\infty$ denotes the maximum norm, $P_{\text{max}(c)} := r^{(t)}$. To fulfill the second requirement, once $q^{(n+1)}$ is derived, the power sharing factors $\beta$ need to be updated for solving Problem 2 at the next step, given by

$$\beta^{(n+1)} := Q^{-1} B^T r^{(n)}$$ where $Q = \text{diag}\{q^{(n+1)}\}$ \tag{21}

The scaled fixed-point iteration to optimize Problem 2 is provided by

$$r_n^{(t+1)} = \frac{P_{\text{max}}}{\|\gamma_to_n^{(u)}(q^{(t)})\|_\infty} \cdot \theta_n^{(u)}(r^{(t)}) \tag{22}$$

The joint optimization algorithm is given in Algorithm 1.

V. UPLINK-DOWNLINK DUALITY

We state the joint optimization problem in uplink in Section IV and propose an efficient solution in Section V by exploiting the decoupled property of $V$ over the variables $\theta$ and $b$. The downlink problem, due to the coupled structure of $V^T$, is more difficult to solve. As extended discussion we want to address the relationship between the uplink and the downlink problem, and to propose a sub-optimal solution for downlink which can be possibly found through the uplink solution.

Let us consider cluster-based max-min capacity utility balancing problem in Section III-A1 as an example. In the downlink the optimization problem is written as

$$\max_{q, b} \min_{c} U_c^{(d)}(q, b), \text{s.t.} \|q\|_1 \leq P_{\text{max}},$$

where $U_c^{(d)} := \frac{q_c}{\Psi A b^T A_b^T q + \Psi z^{(d)}}$ \tag{23}

The cluster-based received noise is written as $z^{(d)} := A \sigma^{(d)}$.

Algorithm 1: Joint Optimization of Problem 1 and 2

1. broadcast the information required for computing $V$, predefined constraint $P_{\text{max}}$, and thresholds $\epsilon_1, \epsilon_2, \epsilon_3$
2. arbitrary initial power vector $q^{(1)} > 0$ and iteration step $t := 0$
3. repeat \{joint optimization of Problem 1 and 2\}
4. repeat \{fixed-point iteration for every cluster $c \in C\}
5. broadcast $q^{(t)}$ to all base stations
6. for all assignment options $b_c \in \mathcal{B}$ do
7. compute $\gamma_c^{(u)}(q^{(t)}, b_c)$ with (11)
8. end for
9. compute $\gamma_c^{(u)}(q^{(t)})$ with (18) and update $b_c^{(t+1)}$
10. update $q_c^{(t+1)}$ with (20)
11. $t := t + 1$
12. until convergence: $\|q_c^{(t+1)} - q_c^{(t)}\|_q q_c^{(t)} \leq \epsilon_1$
13. update $\beta^{(t)}$ with (21)
14. repeat \{fixed-point iteration for every BS $n \in \mathcal{N}\}
15. broadcast $r_n^{(t)}$ to all base stations
16. for all antenna tilt options $\theta_n \in \Theta$ do
17. compute $\theta_n^{(t)}(r_n^{(t)}, b_n)$ with (15)
18. end for
19. compute $\gamma_n^{(t)}(r_n^{(t)})$ with (19) and update $\theta_n^{(t+1)}$
20. update $r_n^{(t+1)}$ with (22)
21. $t := t + 1$
22. until convergence: $\|r_n^{(t+1)} - r_n^{(t)}\|_r / r_n^{(t)} \leq \epsilon_2$
23. update $P_{\text{max}(n)} := r_n^{(t)}$
24. compute $l^{(t+1)} := \min_{n \in \mathcal{N}} \tilde{U}_n^{(u)}(r^{(n+1)})$
25. until convergence: $\|l^{(t+1)} - l^{(t)}\|_l / l^{(t)} \leq \epsilon_3$

In the following we present a virtual dual uplink network in terms of the feasible utility region for the downlink network in (23) via Perron-Frobenius theory, such that the solution of problem (23) can be derived by solving the uplink problem (24) with the algorithm introduced in Section V.

Proposition 1: Define a virtual uplink network where the link gain matrix is modified as $W_b := \text{diag}\{\alpha\} V_b \text{diag}\{-\{\alpha\}\}$, i.e., $w_{bk} := w_{bk} \alpha_k$, and the received uplink noise is denoted by $\sigma^{(u)}$, where $\sigma^{(u)} := \{\sigma_1^{(u)}(a), \ldots, \sigma_K^{(u)}(a)\}_a$, and $\Sigma_{\text{tot}} := \|\sigma^{(u)}\|_1 = \|\sigma^{(d)}\|_1$ (which means, the sum noise is equally distributed in clusters, while in each cluster the noise is equally distributed in the subordinate users). The dual uplink problem of (23) is given by

$$\max_{q, b} U_c^{(u)}(q, b), \text{s.t.} \|q\|_1 \leq P_{\text{max}},$$

where $U_c^{(u)} := \frac{q_c}{\Psi A W_b A_b^T q + \Psi z^{(d)}}$ \tag{24}

where $z^{(d)} := A \sigma^{(d)}$.

Proof: The proof is given in the Appendix. \hfill $\blacksquare$

Note that the optimizer $b^*$ for BS assignment in downlink can be equivalently found by minimizing the spectral radius $\Lambda^{(u)}(b)$ in the uplink. Once $b^*$ is found, the associate
optimizer for uplink power $q^{(u)}$ is given as the dominant right-hand eigenvector of matrix $\Lambda^{(u)}(b^*)$, while the associate optimizer for downlink power $q^{(d)}$ is given as the dominant right-hand eigenvector of matrix $\Lambda^{(d)}(b^*)$. Proposition 1 provides an efficient approach to solve the downlink problem with two iterative steps (as the one proposed in [11]): 1) for a fixed power allocation $\tilde{q}$ solve the uplink problem and derive the assignment $b^*$ that associated with the spectral radius of extend coupling matrix $\Lambda^{(u)}$, and 2) for a fixed assignment $\hat{b}$, update the power $q^*$ as the solution of (25).

Although we are able to find a dual uplink problem for the downlink problem in [24] with our proposed utility functions under sum power constraints, we are not able to construct a dual network with decoupled properties for the modified problem under per BS power constraints. However, numerical experiments show that our approach to the downlink based on the proposed uplink solution does improve the network performance, although the duality does not exactly hold between the downlink problem and our proposed uplink problem under the per BS power constraints.

VI. NUMERICAL RESULTS

We consider a real-world urban scenario based on a pixel-based mobility model of realistic collection of BS locations and pathloss model for the city of Berlin. The data was assembled within the EU project MOMENTUM and is available at [17]. We select 15 tri-sectored BS in the downtown area. Users are uniformly distributed and are clustered based on their SINR distributions as shown in Fig. 1. (UEs assigned to each sector are clustered into groups and are depicted in distinct colors). The SINR threshold is defined as -6.5 dB and the power constraint per BS is 46dBm. The 3GPP antenna model defined in [18] is applied.

Fig. 2 illustrates the convergence of the algorithm. Our algorithm achieves the max-min utility balancing, and improves the feasibility level $C^{(u)}(P_{\max})$ by each iteration step.

In Fig. 3 we show that the trade-off between coverage and capacity can be adjusted by tuning parameter $\mu$. By increasing $\mu$ we give higher priority to capacity utility (which is proportional to the ratio between total useful power and total interference power), while for better coverage utility (defined as minimum of SINRs) we can use a small value of $\mu$ instead.

Fig. 4, 5 and 6 illustrate the improvement of coverage and capacity performance and decreasing of the energy consumption in both uplink and downlink systems by applying the proposed algorithm, when the average number of the users per BS is chosen from the set $\{15, 20, 25, 30, 35\}$. In Fig. 5 we show that the actual average SINR is also improved, although the capacity utility is defined as a lower bound of the average SINR. Fig. 6 illustrate that our algorithm is more energy efficient when comparing with the fixed BS power budget scenario. Compared to the near-optimal uplink solutions, less improvements are observed for the downlink solutions as shown in Fig. 4, 5 and 6. This is because we derive the downlink solution by exploiting an uplink problem which is not exactly its dual due to the individual power constraints (as described in Section V). However, the sub-optimal solutions still provide significant performance improvements.

VII. CONCLUSIONS AND FURTHER RESEARCH

We present an efficient and robust algorithmic optimization framework build on the utility model for joint optimization of the SON use cases coverage and capacity optimization and load balancing. The max-min utility balancing formulation is employed to enforce the fairness across clusters. We propose a two-step optimization algorithm in the uplink based on fixed-point iteration to iteratively optimize the per base station antenna tilt and power allocation as well as the cluster-based BS assignment and power allocation. We then analyze the network duality via Perron-Frobenius theory, and propose a sub-optimal solution in the downlink by exploiting the solution in the uplink. Simulation results show significant improvements in performance of coverage, capacity and load balancing in a power-efficient way, in both uplink and downlink. In our follow-up papers we will further propose a more complex interference coupling model and the optimization framework where frequency band assignment is taken into account. We will also examine the suboptimality under more general form of power constraints.

APPENDIX

Proof: Proposition 1 For any fixed BS assignment $\hat{b}$, denote $W := W_{\hat{b}}$ and $V := V_{\hat{b}}$ for convenience, the optimal downlink power solution $q^{(d)}$ for problem (23) satisfies [9]

$$
\Lambda^{(d)} q^{(d)} = \frac{1}{C^{(d)}(\hat{b}, P_{\max})} q^{(d)}, \quad q^{(d)} \in \mathbb{R}^C
$$

where $\Lambda^{(d)} \in \mathbb{R}^{C \times C}$ is defined as

$$
\Lambda^{(d)} := \Gamma \Psi \left[ A V^T A^T + \frac{1}{P_{\max}} z^{(d)} 1_c^T \right].
$$

we denote $\Gamma := \text{diag}(\gamma_1, \ldots, \gamma_C)$, $C^{(d)}(\hat{b}, P_{\max}) = \max_{q \geq 0} \min_c u_c^{(d)} / \gamma_c$ subject to $\|q\|_1 \leq P_{\max}$, and $1_c$ is a C-dimensional all-one vector. (25) and (26) are derived by
writing the utility fairness $U_{C}^{(d,1)} / \gamma_{c} = C^{(d)}(\hat{b}, P_{\text{max}})$ for all $c \in \mathcal{C}$ and the power constraint $\|q^{(d)}\|_{1} = P_{\text{max}}$ with matrix notation. Targets $\gamma$ is feasible if and only if $C^{(d)}(\hat{b}, P_{\text{max}}) > 1$.

Similarly, the optimal uplink power solution $\hat{q}^{(u)}$ for uplink problem (24) needs to satisfy

$$\Lambda^{(u)} q^{(u)} = \frac{1}{C^{(u)}(\hat{b}, P_{\text{max}})} q^{(u)}, \quad q^{(u)} \in \mathbb{R}_{+}^{C}$$

(27)

where $\Lambda^{(u)} \in \mathbb{R}_{+}^{C \times C}$ is defined as

$$\Lambda^{(u)} := \Gamma \Psi \left[ AW A_{\alpha}^{T} + \frac{1}{P_{\text{max}}} z^{(u)} 1_{C}^{T} \right].$$

(28)

where $z^{(u)} := A \sigma^{(u)}$, i.e., $z^{(u)}_{c} = \sum_{k \in \mathcal{C}} / C$ for all $c \in \mathcal{C}$.

The balanced level $C^{(d)}(\hat{b}, P_{\text{max}})$ and $C^{(u)}(\hat{b}, P_{\text{max}})$ are the reciprocal spectral radius of the nonnegative extended coupling matrix $\Lambda^{(d)}$ and $\Lambda^{(u)}$. Moreover, according to Perron-Frobenius theorem, if both $\Lambda^{(d)}$ and $\Lambda^{(u)}$ are irreducible, they have unique real spectral radius and their corresponding eigenvectors (power allocation) have strictly positive components. By comparing the interference terms in (26) and (28), we have $(A V^{T} A_{\alpha}^{T})^{T} = A_{\alpha} V A^{T} = A \text{diag}(\alpha) V I A_{\alpha}^{T} = A \text{diag}(\alpha) V I \text{diag}^{-1}(\alpha) \text{diag}(\alpha) A^{T} = AW^{T} A_{\alpha}^{T}$.

By comparing the noise terms we have $z^{(u)} = \frac{1}{C} 1_{C} z^{(u)} 1_{C}^{T}$ (by using $z^{(u)}_{c} = \sum_{k \in \mathcal{C}} / C$ for all $c \in \mathcal{C}$), thus $z^{(u)} 1_{C}^{T} = \frac{1}{C} 1_{C} z^{(d)} 1_{C}^{T} = 1_{C} z^{(d)} 1_{C}^{T} = (z^{(d)} 1_{C}^{T})^{T}$. By using the properties of spectral radius $\rho(X) = \rho(X^{T})$ and $\rho(XY) = \rho(YX)$ we have that $\rho(\Lambda^{(d)}) = \rho(\Lambda^{(u)})$ and thus $C^{(d)}(\hat{b}, P_{\text{max}}) = C^{(u)}(\hat{b}, P_{\text{max}})$. Notice that the network duality holds for any given BS assignment $\hat{b}$, the achievable utility regions are the same for both the downlink problem (23) and uplink problem (24).

**Acknowledgements**

We would like to thank Dr. Martin Schubert and Dr. Carl J. Nuzman for their expert advice.
REFERENCES

[1] 3GPP, “Self-configuring and self-optimizing network (SON) use cases and solutions, TR 36.902,” http://www.3gpp.org, Jan 2011.

[2] A. Giovanidis, Q. Liao, and S. Stanczak, “A distributed interference-aware load balancing algorithm for LTE multi-cell networks,” in Smart Antennas (WSA), 2012 International ITG Workshop on. IEEE, 2012, pp. 28–35.

[3] R. Razavi, S. Klein, and H. Claussen, “Self-optimization of capacity and coverage in LTE networks using a fuzzy reinforcement learning approach,” in Personal Indoor and Mobile Radio Communications (PIMRC), 2010 IEEE 21st International Symposium on, 2010, pp. 1865–1870.

[4] H. Klessig, A. Fehske, G. Fettweis, and J. Voigt, “Improving coverage and load conditions through joint adaptation of antenna tilts and cell selection rules in mobile networks,” in Wireless Communication Systems (ISWCS), 2012 International Symposium on. IEEE, 2012, pp. 21–25.

[5] R. D. Yates, “A framework for uplink power control in cellular radio systems,” IEEE J. Select. Areas Commun., vol. 13, no. 7, pp. 1341–1348, Sep. 1995.

[6] K. K. Leung, C. W. Sung, W. S. Wong, and T.-M. Lok, “Convergence theorem for a general class of power-control algorithms,” Communications, IEEE Transactions on, vol. 52, no. 9, pp. 1566–1574, 2004.

[7] C. J. Nuzman, “Contraction approach to power control, with non-monotonic applications,” in Global Telecommunications Conference, 2007. GLOBECOM ’07. IEEE. IEEE, 2007, pp. 5283–5287.

[8] N. Vucic and M. Schubert, “Fixed point iteration for max-min SIR balancing with general interference functions,” in Acoustics, Speech and Signal Processing (ICASSP), 2011 IEEE International Conference on. IEEE, 2011, pp. 3456–3459.

[9] S. Stanczak, M. Wiczanowski, and H. Boche, Fundamentals of resource allocation in wireless networks: theory and algorithms. Springer, 2009, vol. 3.

[10] M. Schubert and H. Boche, Interference Calculus. Springer, 2012.

[11] H. Boche and M. Schubert, Smart Antennas: State of the Art. Hindawi Publishing Corporation, 2006, ch. Duality theory for uplink downlink multiuser beamforming.

[12] M. Schubert and H. Boche, “Iterative multiuser uplink and downlink beamforming under SINR constraints,” Signal Processing, IEEE Transactions on, vol. 53, no. 7, pp. 2324–2334, 2005.

[13] Y. Huang, C. W. Tan, and B. D. Rao, “Joint beamforming and power control in coordinated multicell: max-min duality, effective network and large system transition,” IEEE Transactions on Wireless Communications, vol. 12, no. 6, pp. 2730–2742, 2013.

[14] S. He, Y. Huang, L. Yang, A. Nallanathan, and P. Liu, “A multicell beamforming design by uplink-downlink max-min SINR duality,” Wireless Communications, IEEE Transactions on, vol. 11, no. 8, pp. 2858–2867, 2012.

[15] C. D. Meyer, Matrix analysis and applied linear algebra. Siam, 2000.

[16] J. C. Bezdek, R. Ehrlich, and W. Full, “FCM: The fuzzy C-means clustering algorithm,” Computers & Geosciences, vol. 10, no. 2, pp. 191–203, 1984.

[17] MOMENTUM, “Models and simulations for network planning and control of UMTS,” http://momentum.zib.de] 2004.

[18] 3GPP, “TR 36.942 radio frequency (RF) system scenarios (release 10),” http://www.3gpp.org, 2010.