BRST Invariance and Renormalisability of the SU(2)×U(1)
Electroweak Theory with Massive W Z Bosons

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Abstract

Since the SU(n) gauge theory with massive gauge bosons has been proven to be renormalisable we reinvestigate the renormalisability of the SU_L(2) × U_Y(1) electroweak theory with massive W Z bosons. We expound that with the constraint conditions caused by the W Z mass term and the additional condition chosen by us we can performed the quantization and construct the ghost action in a way similar to that used for the massive SU(n) theory. We also show that when the δ– functions appearing in the path integral of the Green functions and representing the constraint conditions are rewritten as Fourier integrals with Lagrange multipliers λ_α and λ_γ, the BRST invariance is kept in the total effective action consisting of the Lagrange multipliers, ghost fields and the original fields. Furthermore, by comparing with the massless theory and with the massive SU(n) theory we find the general form of the divergent part of the generating functional for the regular vertex functions and prove the renormalisability of the theory. It is also clarified that the renormalisability of the theory with the W Z mass term is ensured by that of the massless theory and the massive SU(n) theory.

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I. Introduction

Although the negative answer to the problem of renormalisability of a SU(n) theory with massive
gauge bosons is widely known, such theories continue to be studied (see for example Refs. [1-8]). However,
since the negative answer had not been voted down, it was naturally difficult to investigate the possibility
of directly adding a mass term to the SU_L(2) × U_Y(1) theory. Recently, the renormalisability of the
massive SU(n) gauge theory has been proven [1,2]. Therefore we will reinvestigate the SU_L(2) × U_Y(1)
theory of S.L.Glashow [9] with the mass term of the W Z fields. The study of the theory including the
mass term of the matter fields as well will be reported in Ref. [10].

In order to make appropriate the mass ratio, the W Z mass term must contain a product of the SU_L(2)
and U_Y(1) fields and thus cause constraint conditions containing products of such fields. Next, such a
mass term is invariant under an infinitesimal gauge transformation with δθ_1 and δθ_2 equal to zero and δθ_3
equal to δθ_y, where θ_a and θ_1 are the parameters of the gauge group. Therefore an additional constraint
condition should be properly chosen. We will expound that with the constraint conditions caused by
the W Z mass term and the additional condition chosen by us we can performed the quantization and
construct the ghost action in a way similar to that used for the massive SU(n) theory [1]. We will also
show that when the δ– functions appearing in the path integral of the Green functions and representing
the constraint conditions are rewritten as Fourier integrals with Lagrange multipliers λ_a and λ_y, the
BRST invariance is kept in the total effective action consisting of the Lagrange multipliers, ghost fields
and the original fields.

As the constraint conditions contain the products of the SU_L(2) and U_Y(1) fields, the divergent
part of the generating functional Γ for the regular vertex functions is dependent on the classical fields
of the Lagrange multipliers λ_a and λ_y when the generating functional for the Green functions contains
the sources of these Lagrange multipliers. The problem of whether such a generalized form of the theory
is renormalisable becomes complicated. However, we are not interested in using the Green functions
involving λ_a or λ_y. Thus we can avoid introducing the sources of these Lagrange multipliers to the
generating functional for the Green functions. An equivalent and convenient procedure is to derive the
Slavnov-Taylor identities and the additional identities for Γ with the help of the generalized form of the
theory and then let vanish the functional derivatives of Γ with respect to the classical fields of these
Lagrange multipliers. In this way the divergent part of Γ will be shown to satisfy the same equations
appearing in the massless theory. Furthermore, by comparing with the massless theory and with the
massive SU(n) theory we will be able to find the general form of the divergent part of \( \Gamma \) and prove the
renormalisability of the theory. Meanwhile it will be clarified that the renormalisability of the theory
with the W Z mass term is ensured by that of the massless theory and the massive SU(n) theory.

In section 2 we will find the constraint conditions caused by the W Z mass term. The additional
constraint condition will also be chosen. The method of quantization will be explained in section 3.
Section 4 is devoted to prove the renormalisability of the theory. Concluding remarks will be given in the
final section.

II. Original and Additional Constraint Conditions

For the sake of convenience we assume in the present work that the matter fields consist only of the
electron and electron-neutrino fields and are often denoted by \( \psi(x) \) and \( \overline{\psi}(x) \). The former stands for the
purely left-handed neutrino field \( \nu_L \), the left- and right-handed parts of the electron field namely \( e_L, e_R \),
and the latter stands for \( \nu_L, e_L \) and \( e_R \). Next let \( W_{a\mu}(x), W_{y\mu}(x) \) be the SU\(_L\)(2) and U\(_Y\)(1) gauge fields
and \( g, g_1 \) be the coupling constants. Thus the W Z mass term in the Lagrangian is

\[
\mathcal{L}_{WM} = \frac{1}{2} M^2 W_{a\mu} W^a_{\mu} + \frac{1}{2} M^2 \left( \frac{g_1}{g} \right)^2 W_{y\mu} W^y_{\mu} - M^2 \left( \frac{g}{g} \right) W_{3\mu} W^y_{\mu},
\]

or

\[
\mathcal{L}_{WM} = \frac{1}{2} M^2 W_{1\mu}(x) W^1_{\mu}(x) + \frac{1}{2} M^2 W_{2\mu}(x) W^2_{\mu}(x) + \frac{1}{2} M^2 Z_\mu(x) Z^\mu(x),
\]

where \( M^2_z \) stands for \( g^{-2}(g^2 + g_1^2)M^2 \), and \( Z_\mu(x), A_\mu(x) \) are the field functions of Z boson and photon,

\[
Z_\mu = \frac{1}{\sqrt{(g^2 + g_1^2)}} (g W_{3\mu} - g_1 W_{y\mu}),
\]

\[
A_\mu = \frac{1}{\sqrt{(g^2 + g_1^2)}} \varepsilon (g_1 W_{3\mu} + g W_{y\mu}),
\]

where \( \varepsilon \) is 1 or \(-1\).

The original Lagrangian of the SU\(_L\)(2) \( \times \) U\(_Y\)(1) electroweak theory with the mass term \( \mathcal{L}_{WM} \) is

\[
\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_\psi W + \mathcal{L}_{WM} + \mathcal{L}_{WL} + \mathcal{L}_{WY},
\]

3
where $L$ describe the pure matter fields, $L_W$ is the coupling term between the matter and gauge fields and

$$L_W = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu}^a,$$

$$L_W = -\frac{1}{4} B_{\mu\nu} B_{\mu\nu},$$

with

$$F_{\mu\nu} = \partial_{\mu} W_{\nu} - \partial_{\nu} W_{\mu} - g C_{abc} W_{b\mu} W_{c\nu},$$

$$B_{\mu\nu} = \partial_{\mu} W_{\nu} - \partial_{\nu} W_{\mu}.$$ 

$C_{abc}$ stands for the structure constants of SU(2) with $C_{123}$ equal to 1.

Denote by $\theta_a(x), \theta_b(x)$ the parameters of the gauge group. Thus, under an infinitesimal gauge transformation, the fields $W_{\mu}^a, W_{\mu}^b, \psi$ and $\overline{\psi}$ transform as

$$\delta W_{\mu}^a(x) = -\frac{1}{g} \partial^\mu \delta \theta_a(x) - C_{abc} W_{\nu}^b(x) \delta \theta_c(x),$$

$$\delta W_{\mu}^b(x) = -\frac{1}{g_1} \partial^\mu \delta \theta_b(x),$$

$$\delta \nu_L(x) = \frac{i}{2} \delta \theta_1(x) \epsilon_L(x) + \frac{1}{2} \delta \theta_2(x) \epsilon_L(x) + \frac{i}{2} \delta \theta_3(x) \epsilon_L(x) - \frac{i}{2} \delta \theta_1(x) \nu_L(x),$$

$$\delta e_L(x) = \frac{i}{2} \delta \theta_1(x) \nu_L(x) - \frac{1}{2} \delta \theta_2(x) \nu_L(x) - \frac{i}{2} \delta \theta_3(x) \epsilon_L(x) - \frac{i}{2} \delta \theta_1(x) e_L(x),$$

$$\delta e_R(x) = -i \delta \theta_1(x) e_R(x),$$

$$\delta \nu_L(x) = -\frac{i}{2} \delta \theta_1(x) \epsilon_L(x) + \frac{1}{2} \delta \theta_2(x) \epsilon_L(x) - \frac{i}{2} \delta \theta_3(x) \nu_L(x) + \frac{i}{2} \delta \theta_1(x) \nu_L(x),$$

$$\delta e_L(x) = -\frac{i}{2} \delta \theta_1(x) \epsilon_L(x) - \frac{1}{2} \delta \theta_2(x) \epsilon_L(x) + \frac{i}{2} \delta \theta_3(x) \epsilon_L(x) + \frac{i}{2} \delta \theta_1(x) e_L(x),$$

$$\delta e_R(x) = i \delta \theta_1(x) e_R(x).$$

Therefore the action transforms as

$$\delta \int d^4 x L(x) = \delta \int d^4 x L_{WM}(x)$$

$$= \int d^4 x \left\{ \left( \frac{M^2}{g} \partial_\mu W_{1\mu}^a(x) + \frac{M^2}{g} g_1 W_{2\mu}^a(x) W_{\nu}^a(x) \right) \delta \theta_1 \right. $$

$$+ \left( \frac{M^2}{g} \partial_\mu W_{2\mu}^a(x) - \frac{M^2}{g} g_1 W_{1\mu}^a(x) W_{\nu}^a(x) \right) \delta \theta_2$$

$$+ \left( \frac{M^2}{g} \partial_\mu W_{3\mu}^a(x) - \frac{M^2}{g^2} g_1 \partial_\mu W_{\nu}^a(x) \right) (\delta \theta_3 - \delta \theta_4) \right\}.$$ 

Since the classical equations of motion make the action invariant under an arbitrary infinitesimal transformation of the field functions, they certainly make the $WZ$ mass term invariant under an arbitrary
infinitesimal gauge transformation. This means that when \( M \) is not equal to zero, the classical equations of motion leads to the following constraint conditions

\[
\frac{M^2}{g} \partial_\mu W_1^\mu(x) + \frac{M^2}{g} g_1 W_2^\mu(x) W_\gamma^\mu(x) = 0, 
\]

\[
\frac{M^2}{g} \partial_\mu W_2^\mu(x) - \frac{M^2}{g} g_1 W_1^\mu(x) W_\gamma^\mu(x) = 0, 
\]

\[
\frac{M^2}{g} \partial_\mu W_3^\mu(x) - \frac{M^2}{g^2} g_1 \partial_\mu W_\gamma^\mu(x) = 0. 
\]

These are the original constraint conditions. As it can be seen from (2.9) that the W Z mass term is invariant under an infinitesimal gauge transformation with \( \delta \theta_1 \) and \( \delta \theta_2 \) equal to zero and \( \delta \theta_3 \) equal to \( \delta \theta_\gamma \). For this reason, \( \partial_\mu W_3^\mu \) and \( \partial_\mu W_\gamma^\mu \) appear in one constraint. We now choose an additional condition and replace (2.12) with

\[
\frac{M^2}{g} \partial_\mu W_3^\mu(x) + \frac{M^2}{g} g_1 W_3^\mu(x) W_\gamma^\mu(x) = 0, 
\]

\[
\partial_\mu W_\gamma^\mu(x) + g W_3^\mu(x) W_\gamma^\mu(x) = 0. 
\]

III. Quantization and BRST Invariance

Write (2.10), (2.11) and (2.13),(2.14) as

\[
\Phi_a(x) = 0, \quad \Phi_\gamma(x) = 0, 
\]

with

\[
\Phi_1(x) = \partial_\mu W_1^\mu(x) + g_1 W_2^\mu(x) W_\gamma^\mu(x), 
\]

\[
\Phi_2(x) = \partial_\mu W_2^\mu(x) - g_1 W_1^\mu(x) W_\gamma^\mu(x), 
\]

\[
\Phi_3(x) = \partial_\mu W_3^\mu(x) + g_1 W_3^\mu(x) W_\gamma^\mu(x), 
\]

\[
\Phi_\gamma(x) = \partial_\mu W_\gamma^\mu(x) + g W_3^\mu(x) W_\gamma^\mu(x). 
\]

Taking the constraint conditions (3.1) into account one should write the path integral of the Green functions involving only the original fields as

\[
\frac{1}{N_0} \int \mathcal{D}[W, \bar{\psi}, \psi] \Delta[W, \bar{\psi}, \psi] \prod_{x', x} \delta (\Phi_\alpha(x')) \delta (\Phi_\gamma(x')) W_{a\mu}(x) W_{\delta \nu}(y) \cdots \exp \{iI\}, 
\]
where
\[
I = \int d^4x L(x),
\]
\[
N_0 = \int D[W, \overline{\psi}, \psi] \Delta[W, \overline{\psi}, \psi] \prod_{a', x'} \delta(\Phi_{a'}(x')) \delta(\Phi_y(x')) \exp\{iI\}.
\]

The weight factor $\Delta[W, \overline{\psi}, \psi]$ is to be determined. Since only the field functions which satisfy the constraint conditions can play roles in the integral (3.6), the value of the Lagrangian can be changed for the field functions which do not satisfy these conditions. In view of the fact that the conditions (3.1) make the action invariant with respect to the infinitesimal gauge transformation, we now imagine to replace the mass term $L_{WM}$ in (3.6) with a gauge invariant mass term which is equal to $L_{WM}$ when the conditions (3.1) are satisfied. Thus, analogous to the case in the Fadeev–Popov method [1,11-16], $\Delta[W, \overline{\psi}, \psi]$ should be gauge invariant and make the following equation valid for an arbitrary gauge invariant quantity $O(W, \overline{\psi}, \psi)$
\[
\int D[W, \overline{\psi}, \psi] \Delta[W, \overline{\psi}, \psi] \prod_{a', x'} \delta(\Phi_{a'}(x')) \delta(\Phi_y(x')) O(W, \overline{\psi}, \psi) \exp\{i\tilde{I}\} \propto \int D[W, \overline{\psi}, \psi] O(W, \overline{\psi}, \psi) \exp\{i\tilde{I}\}.
\]

where $\tilde{I}$ is a gauge invariant action constructed by replacing $L_{WM}$ with the imagined mass term. This means that the weight factor $\Delta[W, \overline{\psi}, \psi]$ can be determined according to the Fadeev–Popov equation of the following form
\[
\Delta[W, \overline{\psi}, \psi] \int \prod_z d\Omega(z) \prod_{\sigma, x} \delta(\Phi^\Omega_{\sigma}(x)) = 1.
\] (3.7)

where $\sigma$ stands for $1, 2, 3, y$, $\Phi^\Omega_{\sigma}(x)$ is the result of acting on $\Phi_{\sigma}(x)$ with a gauge transformation having the parameters of the element $\Omega(x)$ of the gauge group, $d\Omega(z)$ is the volume element of the group integral.

It follows that with the F–P ghost fields $C_a(x)$, $C_y(x)$, $\overline{C}_a(x)$, $\overline{C}_y(x)$ as new variables, one can express the ghost Lagrangian as
\[
L^{(C)}(x) = \overline{C}_a(x) \Delta\Phi_a(x) + \overline{C}_y(x) \Delta\Phi_y(x),
\] (3.8)

where $\Delta\Phi_a(x)$, $\Delta\Phi_y(x)$ are defined by the BRST transformation of $\Phi_a(x)$ and $\Phi_y(x)$ so that
\[
\delta_B \Phi_a(x) = \delta\zeta \Delta\Phi_a(x), \quad \delta_B \Phi_y(x) = \delta\zeta \Delta\Phi_y(x),
\] (3.9)

where $\delta\zeta$ is an infinitesimal fermionic parameter independent of $x$. The BRST transformation of the gauge fields or matter fields is nothing but the infinitesimal gauge transformation with $\delta\theta_a$ and $\delta\theta_y$ equal.
to \(-g\delta\zeta C_a\) and \(-g_1\delta\zeta C_y\) respectively. Namely
\begin{align}
\delta_B W^\mu_a(x) &= \delta\zeta \Delta W^\mu_a(x) = \delta\zeta D^\mu_{ab} C_b(x), \\
\delta_B W^\mu_y(x) &= \delta\zeta \Delta W^\mu_y(x) = \delta\zeta \partial^\mu C_y(x), \\
\delta_B \psi(x) &= \delta\zeta \Delta \psi(x), \quad \delta_B \overline{\psi}(x) = \delta\zeta \Delta \overline{\psi}(x),
\end{align}
(3.10) (3.11) (3.12)

where
\begin{align*}
D^\mu_{ab}(x) &= \delta_{ab} \partial^\mu + g f_{abc} A^\mu_c(x), \\
\Delta \nu_L(x) &= -\frac{i}{2} g C_1(x) e_L(x) - \frac{1}{2} g C_2(x) e_L(x) - \frac{i}{2} g C_3(x) \nu_L(x) + \frac{i}{2} g_1 C_y(x) \nu_L(x), \\
\Delta e_L(x) &= -\frac{i}{2} g C_1(x) \nu_L(x) + \frac{1}{2} g C_2(x) \nu_L(x) + \frac{i}{2} g C_3(x) e_L(x) + \frac{i}{2} g_1 C_y(x) e_L(x), \\
\Delta e_R(x) &= i g_1 C_y(x) e_R(x), \\
\Delta \overline{\nu}_L(x) &= \frac{i}{2} g C_1(x) \overline{\nu}_L(x) - \frac{1}{2} g C_2(x) \overline{\nu}_L(x) + \frac{i}{2} g C_3(x) \overline{\nu}_L(x) - \frac{i}{2} g_1 C_y(x) \overline{\nu}_L(x), \\
\Delta \overline{e}_L(x) &= \frac{i}{2} g C_1(x) \overline{e}_L(x) + \frac{1}{2} g C_2(x) \overline{e}_L(x) - \frac{i}{2} g C_3(x) \overline{e}_L(x) - \frac{i}{2} g_1 C_y(x) \overline{e}_L(x), \\
\Delta \overline{e}_R(x) &= -i g_1 C_y(x) \overline{e}_R(x).
\end{align*}

C_a(x) and C_y(x) are also transformed as usual
\begin{align*}
\delta_B C_a(x) &= \delta\zeta \Delta C_a(x) = \delta\zeta \frac{g}{2} C_{abc} C_b(x) C_c(x), \\
\delta_B C_y(x) &= 0.
\end{align*}

Now we can write \(\Delta \Phi_a(x)\), \(\Delta \Phi_y(x)\) as
\begin{align}
\Delta \Phi_1 &= \partial_\mu \Delta W^\mu_2(x) + g_1 \Delta W^\mu_3 W_{y\mu}(x) + g_1 W_{y\mu}(x) \Delta W^\mu_y(x), \\
\Delta \Phi_2 &= \partial_\mu \Delta W^\mu_3(x) - g_1 \Delta W^\mu_2 W_{y\mu}(x) - g_1 W_{y\mu}(x) \Delta W^\mu_y(x), \\
\Delta \Phi_3 &= \partial_\mu \Delta W^\mu_2(x) + g_1 \Delta W^\mu_3 W_{y\mu}(x) + g_1 W_{y\mu}(x) \Delta W^\mu_y(x), \\
\Delta \Phi_y &= \partial_\mu \Delta W^\mu_3(x) + g \Delta W^\mu_3 W_{y\mu}(x) + g W_{y\mu}(x) \Delta W^\mu_y(x),
\end{align}
(3.13) (3.14) (3.15) (3.16)

Since \(\Delta W^\mu_2\), \(\Delta W^\mu_3\), \(\Delta \psi(x)\), \(\Delta \overline{\psi}(x)\) and \(\Delta C_a(x)\) are BRST invariant, it is easy to see that \(\Delta \Phi_a(x)\) and \(\Delta \Phi_y(x)\) are also BRST invariant.

One can further generalize the theory by regarding as new variables the Lagrange multipliers \(\lambda_a(x)\) and \(\lambda_y(x)\) associated with the constraint conditions. Thus the total effective Lagrangian and action consist of these Lagrange multipliers, ghosts and the original variables, namely
\begin{align}
\mathcal{L}_{\text{eff}}(x) = \mathcal{L}(x) + \mathcal{L}^{(C)}(x) + \lambda_a(x) \Phi_a(x) + \lambda_y(x) \Phi_y(x),
\end{align}
(3.17)
\[ I_{\text{eff}} = \int d^4x L_{\text{eff}}(x). \] (3.18)

Correspondingly, the path integral of the generating functional for the Green functions is
\[
Z[\eta, \bar{\eta}, \chi, \bar{\chi}, J, j] = \frac{1}{N_\lambda} \int \mathcal{D}[\bar{\psi}, \psi, W, \bar{C}, C, \lambda] \exp \left\{ i(I_{\text{eff}} + I_s) \right\},
\] (3.19)

where \( N_\lambda \) is a constant, \( I_s \) is the source term in the action. They are defined by
\[
N_\lambda = \int \mathcal{D}[\psi, \bar{\psi}, W, \bar{C}, C, \lambda] \exp \left\{ iI_{\text{eff}} \right\},
\]
\[
I_s = \int d^4x \left\{ \bar{\eta}(x) \psi(x) + \bar{\psi}(x) \eta(x) + \bar{\chi}\alpha(x)C_\alpha(x) + \bar{\psi}_a(x)\alpha_a(x) + \bar{\chi}_y(x)C_y(x) + \bar{\psi}_y(x)\lambda_y(x) + \bar{\chi}_y(x)\lambda_y(x) \right\},
\] (3.20)

where \( \bar{\eta}(x), \eta(x) \cdots \) stand for the sources. In particular, \( j_a(x), j_y(x) \) are the sources of \( \lambda_a(x), \lambda_y(x), \) respectively.

We now check the BRST invariance of the effective action \( I_{\text{eff}} \) defined by (3.17) and (3.18). With \( \bar{C}_a(x), \bar{C}_y(x) \) transforming as
\[
\delta_B \bar{C}_a(x) = -\delta \zeta \lambda_a(x), \quad \delta_B \bar{C}_y(x) = -\delta \zeta \lambda_y(x),
\]
and noticing the invariance of \( \Delta \Phi_a, \Delta \Phi_y, \) one has
\[
\delta_B \int d^4x L(C)(x) = \int d^4x \left\{ -\lambda_a(x)\delta_B \Phi_a(x) - \lambda_y(x)\delta_B \Phi_y(x) \right\}.
\]

Therefore
\[
\delta_B I_{\text{eff}} = \delta_B I_{WM} + \int d^4x \left\{ (\delta_B \lambda_a(x))\Phi_a(x) + (\delta_B \lambda_y(x))\Phi_y(x) \right\}.
\]

From this and the expression of \( \delta_B I_{WM}, \) it can be shown that the effective action is invariant, when the transformation of \( \lambda_a(x) \) and \( \lambda_y(x) \) are defined as
\[
\delta_B \lambda_1(x) = \delta \zeta M^2 C_1(x),
\]
\[
\delta_B \lambda_2(x) = \delta \zeta M^2 C_2(x),
\]
\[
\delta_B \lambda_3(x) = \delta \zeta M^2 C_3(x) - \delta \zeta \frac{g_1}{g} M^2 C_y(x),
\]
\[
\delta_B \lambda_y(x) = \delta \zeta \frac{g_2}{g^2} M^2 C_y(x) - \delta \zeta \frac{g_1}{g} M^2 C_3(x).
\]
IV. Renormalisability

Let \( W_{\mu}(x), W_{\nu}(x), C_{a}(x), C_{b}(x), \ldots \) stand for the renormalized field functions, \( g, g_{1} \) and \( M \) be renormalized parameters. By introducing the source terms of the composite field functions \( \Delta W_{\mu}^{a}(x), \Delta W_{\nu}^{a}(x), \Delta C_{a}(x), \Delta \psi(x), \Delta \overline{\psi}(x) \) and the sources \( K_{\mu}^{a}(x), K_{\nu}^{a}(x), L_{a}(x), n_{a}(x), l_{a}(x), p_{a}(x), n'_{a}(x), l'_{a}(x) \) and \( p'_{a}(x) \), the effective Lagrangian without counterterm becomes

\[
\mathcal{L}_{\text{eff}}^{[0]}(x) = \lambda_{\alpha}(x)\Phi_{\alpha}(x) + \lambda_{\gamma}(x)\Phi_{\gamma}(x) + \mathcal{L}_{WL}(x) + \mathcal{L}_{WY}(x) \\
+ \mathcal{L}_{W\mu}(x) + \mathcal{L}_{W\nu}(x) \\
+ \mathcal{L}_{\psi}(x) + \mathcal{L}_{\bar{\psi}}(x) \\
+ K_{\mu}^{a}(x)\Delta W_{\mu}^{a}(x) + K_{\nu}^{a}(x)\Delta W_{\nu}^{a}(x) + L_{a}(x)\Delta C_{a}(x) \\
+ n_{a}(x)\Delta \nu_{La}(x) + l_{a}(x)\Delta e_{La}(x) + p_{a}(x)\Delta e_{Ra}(x) \\
+ n'_{a}(x)\Delta \nu_{La}(x) + l'_{a}(x)\Delta e_{La}(x) + p'_{a}(x)\Delta e_{Ra}(x). \quad (4.1)
\]

The complete effective Lagrangian is the sum of \( \mathcal{L}_{\text{eff}}^{[0]} \) and the counterterm \( \mathcal{L}_{\text{count}} \)

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^{[0]} + \mathcal{L}_{\text{count}}. \quad (4.2)
\]

With (4.1), the generating functional for Green functions is defined as

\[
Z^{[0]}[\overline{\psi}, \eta, \overline{\chi}, \chi, J, j, K, L, n, l, p, n', l', p'] = \frac{1}{N} \int \mathcal{D}[\overline{\psi}, \psi, W, \overline{C}, C, \lambda] \exp \left\{ i \mathcal{I}_{\text{eff}}^{[0]} + I_{s} \right\}, \quad (4.3)
\]

\( I_{\text{eff}}^{[0]} \) is the effective action \( \int d^{4}x \mathcal{L}_{\text{eff}}^{[0]}(x) \), \( N \) is a constant to make \( Z^{[0]} \) equal to 1 in the absence of

\[
I_{s} = \int d^{4}x \left\{ \overline{\psi}(x)\psi(x) + \overline{\chi}(x)\chi(x) + \overline{\alpha}(x) C_{a}(x) + \overline{C}_{a}(x) \alpha(x) + \overline{\gamma}(x) C_{b}(x) + \overline{\gamma}(x) C_{b}(x) \right\},
\]

where \( \overline{\psi} \) and \( \overline{\psi} \) stand for

\[
\overline{\psi} = \overline{\alpha}(x) \nu + \overline{\eta}(x) e_{Ra},
\]

\[
\overline{\psi} = \overline{\alpha}(x) \nu + \overline{\eta}(x) e_{Ra}.
\]

Denoting by \( \mathcal{W}^{[0]} \) and \( \Gamma^{[0]} \) the generating functionals for connected Green functions and regular vertex functions respectively, one has

\[
Z^{[0]} = \exp \left\{ i \mathcal{W}^{[0]}[\overline{\psi}, \eta, \overline{\chi}, \chi, J, j, K, L, n, l, p, n', l', p'] \right\}, \quad (4.4)
\]
Besides, for where

\[
\begin{align*}
\Gamma^{[0]}[\bar{\psi}, \psi, \bar{\omega}, \bar{C}, \bar{C}, \bar{\lambda}, K, L, n, l, p, n', l', p'] \\
= \mathcal{W}^{[0]} - \int d^4x \left[ J_\mu \mathcal{W}_\alpha + J_y \mathcal{W}_{y\mu} + j_a \mathcal{\bar{\lambda}}_a + j_y \lambda_y + \bar{\chi}_a \bar{C}_a + \mathcal{\bar{C}}_a \lambda_a + \bar{\chi}_y \mathcal{C}_y \\
+ \mathcal{\bar{C}}_y \lambda_y \right. & + \mathcal{\bar{\eta}}^{(r)} \bar{e}_L + \mathcal{\bar{\eta}}^{(r)} \bar{e}_L + \mathcal{\bar{\eta}}^{(r)} \bar{e}_R + \mathcal{\bar{\eta}}^{(r)} \bar{e}_R + \mathcal{\bar{\eta}}^{(r)} \bar{e}_R + \left. + \mathcal{\bar{\eta}}^{(r)} \bar{e}_R \right],
\end{align*}
\]

(4.5)

where \( \mathcal{\bar{\omega}}_\alpha, \bar{\nu}_L, \cdots \) are the so-called classical fields defined by

\[
\begin{align*}
\tilde{\mathcal{W}}_\alpha(x) &= \frac{\delta \mathcal{W}^{[0]}}{\delta J_\alpha(x)}, \\
\tilde{\lambda}_a(x) &= \frac{\delta \mathcal{W}^{[0]}}{\delta j_a(x)}, \\
\tilde{\mathcal{C}}_a(x) &= \frac{\delta \mathcal{W}^{[0]}}{\delta \bar{\lambda}_a(x)}, \\
\tilde{\mathcal{\bar{\lambda}}}_y(x) &= \frac{\delta \mathcal{W}^{[0]}}{\delta \lambda_y(x)}, \\
\tilde{\bar{\mathcal{C}}}_y(x) &= \frac{\delta \mathcal{W}^{[0]}}{\delta \bar{\lambda}_y(x)}, \\
\tilde{\bar{\nu}}_{La}(x) &= \frac{\delta \mathcal{W}^{[0]}}{\delta \bar{\lambda}_a(x)}, \\
\tilde{\bar{\nu}}_{Ra}(x) &= \frac{\delta \mathcal{W}^{[0]}}{\delta \bar{\lambda}_a(x)}. \\
\end{align*}
\]

Therefore

\[
\begin{align*}
J_\alpha(x) &= - \frac{\delta \Gamma^{[0]}}{\delta \mathcal{W}_\alpha(x)}, \\
J_y(x) &= - \frac{\delta \Gamma^{[0]}}{\delta \mathcal{W}_{y\mu}(x)}, \\
\bar{\lambda}_a(x) &= - \frac{\delta \mathcal{\bar{\lambda}}_a(x)}{\delta \mathcal{C}_a(x)}, \\
\lambda_y(x) &= - \frac{\delta \lambda_y(x)}{\delta \bar{\mathcal{C}}_y(x)}, \\
\bar{\nu}_{La}(x) &= - \frac{\delta \lambda_y(x)}{\delta \bar{\mathcal{C}}_y(x)}, \\
\mathcal{\bar{\nu}}_{La}(x) &= - \frac{\delta \lambda_y(x)}{\delta \bar{\mathcal{C}}_y(x)}. \\
\end{align*}
\]

Besides, for \( K_\mu, L_a \cdots \), the spectators in the Legendre tranformation, one has

\[
\begin{align*}
\frac{\delta \mathcal{W}^{[0]}}{\delta K_\mu(x)} &= \frac{\delta K_\mu^{[0]}}{\delta \mathcal{W}^{[0]}}, \\
\frac{\delta \mathcal{W}^{[0]}}{\delta K_\mu^{[0]}} &= \frac{\delta K_\mu^{[0]}}{\delta \mathcal{W}^{[0]}}, \\
\frac{\delta \mathcal{W}^{[0]}}{\delta L_a(x)} &= \frac{\delta L_a^{[0]}}{\delta \mathcal{W}^{[0]}}, \\
\frac{\delta L_a^{[0]}}{\delta \mathcal{W}^{[0]}} &= \frac{\delta L_a^{[0]}}{\delta \mathcal{W}^{[0]}}, \\
\frac{\delta \mathcal{W}^{[0]}}{\delta \mathcal{\bar{\lambda}}_a(x)} &= \frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}}, \\
\frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}} &= \frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}}, \\
\frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}} &= \frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}}, \\
\frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}} &= \frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}}, \\
\frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}} &= \frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}}, \\
\frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}} &= \frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}}, \\
\frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}} &= \frac{\delta \mathcal{\bar{\lambda}}_a^{(r)}(x)}{\delta \mathcal{\bar{\lambda}}_a^{[0]}}.
\end{align*}
\]

In order to find the Slavnov–Taylor identity satisfied by the generating functional for the regular vertex functions, we change the variables in the path integral of \( Z^{[0]} \) as follows

\[
\begin{align*}
W_\mu^{[0]}(x) &\to W_\mu^{[0]}(x) + \delta \zeta \Delta W_\mu^{[0]}(x), \\
W_y^{[0]}(x) &\to W_y^{[0]}(x) + \delta \zeta \Delta W_y^{[0]}(x),
\end{align*}
\]
\[ C_a(x) \rightarrow C_a(x) + \delta \zeta \Delta C_a(x), \quad C_y(x) \rightarrow C_y(x), \]
\[ \overline{C}_a(x) \rightarrow \overline{C}_a(x) - \delta \zeta \lambda_a(x), \quad \overline{C}_y(x) \rightarrow \overline{C}_y(x) - \delta \zeta \lambda_y(x), \]
\[ \psi(x) \rightarrow \psi(x) + \delta \zeta \Delta \psi(x), \quad \overline{\psi}(x) \rightarrow \overline{\psi}(x) + \delta \zeta \Delta \overline{\psi}(x), \]
\[ \lambda_a(x) \rightarrow \lambda_a(x), \quad \lambda_y(x) \rightarrow \lambda_y(x). \]

The volume element of the path integral does not change and the changes in \( I_s \) and \( \mathcal{L}_{WM} \) lead to

\[
\int d^4x \left\{ \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} \frac{\delta \Gamma^{[0]}}{\delta \bar{\psi}_\mu(x)} + \frac{\delta \Gamma^{[0]}}{\delta K_\mu^y(x)} \frac{\delta \Gamma^{[0]}}{\delta \bar{\psi}_\mu(x)} + \frac{\delta \Gamma^{[0]}}{\delta L_a(x)} \frac{\delta \Gamma^{[0]}}{\delta \bar{C}_a(x)} + \frac{\delta \Gamma^{[0]}}{\delta L_y(x)} \frac{\delta \Gamma^{[0]}}{\delta \bar{C}_y(x)} + \frac{\delta \Gamma^{[0]}}{\delta \bar{\psi}_\mu(x)} \frac{\delta \Gamma^{[0]}}{\delta \bar{\psi}_\mu(x)} + \frac{\delta \Gamma^{[0]}}{\delta \bar{\psi}_\mu(x)} \frac{\delta \Gamma^{[0]}}{\delta \bar{\psi}_\mu(x)} \right\} = 0,
\]

where

\[
\langle \Delta \mathcal{L}_{WM}(x) \rangle^{[0]} = \frac{1}{N \mathcal{Z}^{[0]}} \int \mathcal{D}[\overline{\psi}, \psi, W, \overline{C}, C] \Delta \mathcal{L}_{WM}(x) \exp \left\{ i (I^{[0]}_\text{eff} + I_s) \right\}.
\]

With the definition of \( \Delta \mathcal{L}_{WM}(x) \)

\[
\delta_B \mathcal{L}_{WM}(x) = \delta \zeta \Delta \mathcal{L}_{WM}(x),
\]

one can write

\[
\langle \Delta \mathcal{L}_{WM}(x) \rangle^{[0]} = M^2 \bar{W}_{\mu \nu}(x) \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} + M^2 \left( \frac{g_1}{g} \right)^2 \bar{W}_{\mu \nu}(x) \frac{\delta \Gamma^{[0]}}{\delta K_\mu^y(x)} - M^2 \frac{g_1}{g} \bar{W}_{\mu \nu}(x) \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} - M^2 \frac{g_1}{g} \bar{W}_{\mu \nu}(x) \frac{\delta \Gamma^{[0]}}{\delta K_\mu^y(x)}.
\]

Next, from the invariance of the path integral of \( \mathcal{Z}^{[0]} \) with respect to the translation of the integration variables \( \overline{C}_a(x), \overline{C}_y(x), \lambda_a(x) \) and \( \lambda_y(x) \), one can get a set of auxiliary identities

\[
\frac{\delta \Gamma^{[0]}}{\delta \overline{C}_1(x)} - \frac{\partial_\mu \delta \Gamma^{[0]}_1(x)}{\delta K_\mu^a(x)} - g_1 \bar{W}_{\mu \nu} \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} - g_1 \bar{W}_{\nu \mu} \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} = 0,
\]

\[
\frac{\delta \Gamma^{[0]}}{\delta \overline{C}_2(x)} - \frac{\partial_\nu \delta \Gamma^{[0]}_2(x)}{\delta K_\mu^a(x)} + g_1 \bar{W}_{\mu \nu} \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} + g_1 \bar{W}_{\nu \mu} \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} = 0,
\]

\[
\frac{\delta \Gamma^{[0]}}{\delta \overline{C}_3(x)} - \frac{\partial_\mu \delta \Gamma^{[0]}_3(x)}{\delta K_\mu^a(x)} - g_1 \bar{W}_{\mu \nu} \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} - g_1 \bar{W}_{\nu \mu} \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} = 0,
\]

\[
\frac{\delta \Gamma^{[0]}}{\delta \overline{C}_4(x)} - \frac{\partial_\nu \delta \Gamma^{[0]}_4(x)}{\delta K_\mu^a(x)} + g_1 \bar{W}_{\mu \nu} \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} + g_1 \bar{W}_{\nu \mu} \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)} = 0.
\]
and
\[ \frac{\delta \Gamma^{[0]}}{\delta \lambda_\alpha(x)} = \langle \Phi_\alpha(x) \rangle^{[0]}, \quad \frac{\delta \Gamma^{[0]}}{\delta \lambda_\nu(x)} = \langle \Phi_\nu(x) \rangle^{[0]} \] (4.11)

where
\[ \langle \Phi_\alpha(x) \rangle^{[0]} = \frac{1}{N Z^{[0]}} \int \mathcal{D}[\overline{\psi}, \psi, W, C, \lambda] \Phi_\alpha(x) \exp \left\{ i \langle I^{[0]}_{\text{eff}} + I_0 \rangle \right\} \] (4.12)
\[ \langle \Phi_\nu(x) \rangle^{[0]} = \frac{1}{N Z^{[0]}} \int \mathcal{D}[\overline{\psi}, \psi, W, C, \lambda] \Phi_\nu(x) \exp \left\{ i \langle I^{[0]}_{\text{eff}} + I_0 \rangle \right\} \] (4.13)

Let \( \tilde{\Phi}_\alpha(x), \tilde{\Phi}_\nu(x), \tilde{L}_{WM} \) be the results obtained from \( \Phi_\alpha(x), \Phi_\nu(x), \mathcal{L}_{WM} \) by replacing the field functions with the classical field functions and define
\[ \Gamma^{[0]} = \Gamma^{[0]} - \int d^4x \left\{ \overline{\lambda}_\alpha(x) \tilde{\Phi}_\alpha(x) + \overline{\lambda}_\nu(x) \tilde{\Phi}_\nu(x) + \tilde{L}_{WM} \right\} \] (4.14)

Thus, from (4.6)–(4.11), one gets
\[ \int d^4x \left\{ \frac{\delta \Gamma^{[0]}}{\delta K^{\mu}_\alpha(x)} \frac{\delta \Gamma^{[0]}}{\delta W^{\mu}_\nu(x)} + \frac{\delta \Gamma^{[0]}}{\delta K^{\mu}_\nu(x)} \frac{\delta \Gamma^{[0]}}{\delta W^{\mu}_\nu(x)} + \frac{\delta \Gamma^{[0]}}{\delta L_\alpha(x)} \frac{\delta \Gamma^{[0]}}{\delta C_\alpha(x)} + \frac{\delta \Gamma^{[0]}}{\delta L_\nu(x)} \frac{\delta \Gamma^{[0]}}{\delta C_\nu(x)} + \frac{\delta \Gamma^{[0]}}{\delta \nu_{\alpha}(x)} \frac{\delta \Gamma^{[0]}}{\delta \nu_{\nu}(x)} + \frac{\delta \Gamma^{[0]}}{\delta \nu_{\nu}(x)} \frac{\delta \Gamma^{[0]}}{\delta \nu_{\alpha}(x)} \right\} = 0 \] (4.15)

and
\[ \frac{\delta \Gamma^{[0]}}{\delta \lambda_\alpha(x)} = \langle \Phi_\alpha(x) \rangle^{[0]} - \tilde{\Phi}_\alpha(x), \quad \frac{\delta \Gamma^{[0]}}{\delta \lambda_\nu(x)} = \langle \Phi_\nu(x) \rangle^{[0]} - \tilde{\Phi}_\nu(x) \] (4.16)

As \( \Phi_\alpha(x), \Phi_\nu(x) \) contain the products of the \( SU_L(2) \) and \( U_Y(1) \) fields, (4.16) is complicated unless the generating functional for the Green functions does not contain the sources of the Lagrange multipliers \( \lambda_\alpha \) and \( \lambda_\nu \). Actually we are not interested in using the Green functions involving \( \lambda_\alpha \) or \( \lambda_\nu \). Our intention to
use the generalized form of the theory containing the sources of these Lagrange multipliers to study the
Renormalisability of the theory for which such sources are absent from the generating functional for the
Green functions and therefore \( \langle \Phi_a(x) \rangle^{[0]} \) and \( \langle \Phi_y(x) \rangle^{[0]} \) are equal to zero. We now, according to (4.11),
let vanish \( \frac{\delta I^{[0]}}{\delta \lambda_a(x)} \) and \( \frac{\delta I^{[0]}}{\delta \lambda_y(x)} \) to make \( \langle \Phi_a(x) \rangle^{[0]} \) and \( \langle \Phi_y(x) \rangle^{[0]} \) equal to zero. This means
\[
\Phi_a(x) = 0, \quad \Phi_y(x) = 0, \tag{4.21}
\]
and
\[
\frac{\delta \Gamma^{[0]}}{\delta \lambda_a(x)} = 0, \quad \frac{\delta \Gamma^{[0]}}{\delta \lambda_y(x)} = 0, \tag{4.22}
\]
In the following we will denote by \( \Gamma^{[0]} \) the functional that is obtained from \( \Gamma^{[0]} \) by replacing the classical field functions with the usual field functions. Assume that the dimensional regularization method is used and the Slavnov–Taylor identity
\[
\delta \Gamma^{[0]} = 0, \quad \delta \Gamma^{[0]} = 0, \tag{4.24}
\]
and
\[
\Lambda_{op} \Gamma^{[0]} = 0, \tag{4.25}
\]
\[\text{and} \]
\[
\Gamma_0^{[0]} \ast \Gamma_1^{[0]} + \Gamma_1^{[0]} \ast \Gamma_0^{[0]} = \Lambda_{op} \Gamma^{[0]} = 0, \tag{4.26}
\]
where \( \Lambda_{op}, \Sigma_a(x) \) and \( \Sigma_y(x) \) are defined by
\[
\Lambda_{op} = \int d^4x \left\{ \frac{\delta I^{[0]}}{\delta K^a_\mu(x)} \frac{\delta}{\delta W^a_\mu(x)} + \frac{\delta I^{[0]}}{\delta W^a_\mu(x)} \frac{\delta}{\delta K^a_\mu(x)} + \frac{\delta I^{[0]}}{\delta L^a_\mu(x)} \frac{\delta}{\delta W^a_\mu(x)} + \frac{\delta I^{[0]}}{\delta W^a_\mu(x)} \frac{\delta}{\delta L^a_\mu(x)} \right\} \delta
\]
\[\text{and} \]
\[
\Lambda_{op} = \int d^4x \left\{ \frac{\delta I^{[0]}}{\delta K^a_\mu(x)} \frac{\delta}{\delta W^a_\mu(x)} + \frac{\delta I^{[0]}}{\delta W^a_\mu(x)} \frac{\delta}{\delta K^a_\mu(x)} + \frac{\delta I^{[0]}}{\delta L^a_\mu(x)} \frac{\delta}{\delta W^a_\mu(x)} + \frac{\delta I^{[0]}}{\delta W^a_\mu(x)} \frac{\delta}{\delta L^a_\mu(x)} \right\} \delta
\]
\[+ \frac{\delta \Gamma_1^{[0]}}{\delta \rho_{\alpha}(x)} \delta \rho_{\alpha}(x) + \frac{\delta \Gamma_2^{[0]}}{\delta \rho_{\alpha}(x)} \delta \rho_{\alpha}(x) \}, \quad (4.27)\]

\[
\Sigma_1(x) = \frac{\delta}{\delta C_1(x)} - \partial_\mu \frac{\delta}{\delta K^1_\mu(x)} - g_1 W_{\mu \nu} \frac{\delta}{\delta K^2_\mu(x)} - g_1 W_{2\mu} \frac{\delta}{\delta K^3_\mu(x)}, \quad (4.28)\]

\[
\Sigma_2(x) = \frac{\delta}{\delta C_2(x)} - \partial_\mu \frac{\delta}{\delta K^2_\mu(x)} + g_1 W_{\mu \nu} \frac{\delta}{\delta K^3_\mu(x)} + g_1 W_{1\mu} \frac{\delta}{\delta K^3_\mu(x)}, \quad (4.29)\]

\[
\Sigma_3(x) = \frac{\delta}{\delta C_3(x)} - \partial_\mu \frac{\delta}{\delta K^3_\mu(x)} - g_1 W_{\mu \nu} \frac{\delta}{\delta K^3_\mu(x)} - g_1 W_{3\mu} \frac{\delta}{\delta K^3_\mu(x)}, \quad (4.30)\]

\[
\Sigma_y(x) = \frac{\delta}{\delta C_y(x)} - \partial_\mu \frac{\delta}{\delta K^y_\mu(x)} - g W_{\mu \nu} \frac{\delta}{\delta K^y_\mu(x)} - g W_{3\mu} \frac{\delta}{\delta K^y_\mu(x)}. \quad (4.31)\]

The meaning of the notation \( A \ast B \) is the same as in the common use, namely

\[
A \ast B = \int d^4 x \{ \frac{\delta A}{\delta K_\mu^\alpha(x)} \frac{\delta B}{\delta W_{\alpha}(x)} + \frac{\delta A}{\delta K_\mu^\beta(x)} \frac{\delta B}{\delta W_{\beta}(x)} + \frac{\delta A}{\delta L_\alpha(x)} \frac{\delta B}{\delta C_\alpha(x)} + \frac{\delta A}{\delta \nu_{\lambda}(x)} \frac{\delta B}{\delta \nu_{\lambda}(x)} + \frac{\delta A}{\delta e_{\lambda}(x)} \frac{\delta B}{\delta e_{\lambda}(x)} + \frac{\delta A}{\delta \rho_{\alpha}(x)} \frac{\delta B}{\delta \rho_{\alpha}(x)} \}, \quad (4.32)\]

\((4.24) - (4.26)\) are of course satisfied by the finite part and the pole part of \( \Gamma_1^{[0]} \). Thus the equations of the pole part \( \Gamma_{1, \text{div}}^{[0]} \) are

\[
\frac{\delta \Gamma_{1, \text{div}}^{[0]}}{\delta \lambda_\alpha(x)} = 0, \quad \frac{\delta \Gamma_{1, \text{div}}^{[0]}}{\delta \lambda_\gamma(x)} = 0, \quad (4.33)\]

\[
\Lambda_{\text{op}} \Gamma_{1, \text{div}}^{[0]} = 0, \quad (4.34)\]

\[
\Sigma_\alpha(x) \Gamma_{1, \text{div}}^{[0]} = 0, \quad \Sigma_\gamma(x) \Gamma_{1, \text{div}}^{[0]} = 0. \quad (4.35)\]

Obviously, the same equations should be found for a SU \( L(2) \times U_Y(1) \) theory without the mass term if the same constraint conditions are chosen.

If \( M = 0 \), then it is known from the renormalisability of the theory that \( \Gamma_{1, \text{div}}^{[0]} \) is a combination of the following terms

\[
T_{GL} = g \frac{\partial \Gamma_{0}^{[0]}}{\partial g}, \quad T_{GY} = g_1 \frac{\partial \Gamma_{0}^{[0]}}{\partial g_1}, \quad (\text{4.36})\]

\[
T_{WL} = \int d^4 x \left\{ \frac{W_{\mu}(x)}{\delta W_{\mu}(x)} + I_{\alpha}(x) \frac{\delta \Gamma_{0}^{[0]}}{\delta L_\alpha(x)} \right\}, \quad (\text{4.37})\]

\[
T_{WY} = \int d^4 x W_{\mu}(x) \frac{\delta \Gamma_{0}^{[0]}}{\delta W_{\mu}(x)} \}, \quad (\text{4.38})\]

\[
T_{\text{CK}} = \int d^4 x \left\{ C_\alpha(x) \frac{\delta \Gamma_{0}^{[0]}}{\delta C_\alpha(x)} + C_\alpha(x) \frac{\delta \Gamma_{0}^{[0]}}{\delta C_\alpha(x)} + K_{\mu}(x) \frac{\delta \Gamma_{0}^{[0]}}{\delta K_{\mu}(x)} \right\}. \quad (\text{4.39})\]
\[ T_{CKY} = \int d^4x \left\{ C_g(x) \frac{\delta \Gamma_0^{[0]}}{\delta C(x)} + C_g(x) \frac{\delta \Gamma_0^{[0]}}{\delta C_g(x)} + K_{\mu}^{[0]}(x) \frac{\delta \Gamma_0^{[0]}}{\delta K_{\mu}^{[0]}(x)} \right\}, \]

\[ T_{\nu L} = \int d^4x \left\{ \nu_{\nu L}(x) \frac{\delta \Gamma_0^{[0]}}{\delta \nu_{\nu L}(x)} + \nu_{\nu L}(x) \frac{\delta \Gamma_0^{[0]}}{\delta \nu_{\nu L}(x)} \right\}, \]

\[ T_{\nu L} = \int d^4x \left\{ \epsilon_{\nu L}(x) \frac{\delta \Gamma_0^{[0]}}{\delta \epsilon_{\nu L}(x)} + \epsilon_{\nu L}(x) \frac{\delta \Gamma_0^{[0]}}{\delta \epsilon_{\nu L}(x)} \right\}, \]

\[ T_{\epsilon R} = \int d^4x \left\{ \epsilon_{\epsilon R}(x) \frac{\delta \Gamma_0^{[0]}}{\delta \epsilon_{\epsilon R}(x)} + \epsilon_{\epsilon R}(x) \frac{\delta \Gamma_0^{[0]}}{\delta \epsilon_{\epsilon R}(x)} \right\}, \]

\[ T_{\alpha'} = \int d^4x \left\{ n_\alpha(x) \frac{\delta \Gamma_0^{[0]}}{\delta n_\alpha(x)} + n_\alpha^\prime(x) \frac{\delta \Gamma_0^{[0]}}{\delta n_\alpha^\prime(x)} \right\}, \]

\[ T_{\mu} = \int d^4x \left\{ l_\alpha(x) \frac{\delta \Gamma_0^{[0]}}{\delta l_\alpha(x)} + l_\alpha^\prime(x) \frac{\delta \Gamma_0^{[0]}}{\delta l_\alpha^\prime(x)} \right\}, \]

\[ T_{\nu'} = \int d^4x \left\{ n_\alpha(x) \frac{\delta \Gamma_0^{[0]}}{\delta n_\alpha(x)} + n_\alpha^\prime(x) \frac{\delta \Gamma_0^{[0]}}{\delta n_\alpha^\prime(x)} \right\}. \]

With these terms one can form five solutions of equations (4.33) – (4.35), which can be chosen as

\[ T_1 = T_{WL} - T_{GL} - T_{CK}, \quad (4.36) \]

\[ T_2 = T_{WY} - T_{GY} - T_{CKY}, \quad (4.37) \]

\[ T_3 = T_{CK} + T_{CKY} + T_{\alpha'}, + T_{\mu'} + T_{\nu'}, \quad (4.38) \]

\[ T_4 = T_{\nu L} + T_{e L} - T_{\alpha'} - T_{\mu'}, \quad (4.39) \]

\[ T_5 = T_{e R} - T_{\nu'}. \quad (4.40) \]

Note that \( T_3 \) is \( 2(\Gamma_0^{[0]} - I_{WL} - I_{WY} - I_\psi - I_\psi W) \). \( T_1 \) is a combination of \( I_{WL} \), \( T_3 \) and \( \int d^4x C_g(x) \frac{\delta \Gamma_0^{[0]}}{\delta C_g(x)} \). \( T_2 \) is a combination of \( I_{WY} \) and \( \int d^4x C_g(x) \frac{\delta \Gamma_0^{[0]}}{\delta C_g(x)} \). The sum of \( T_4 \) and \( T_5 \) is \( 2(I_\psi + I_\psi W) \). \( \int d^4x C_g(x) \frac{\delta \Gamma_0^{[0]}}{\delta C_g(x)} \) and \( T_5 \) can be easily checked to satisfy (4.34) – (4.35). In addition to (4.36) – (4.40), a new term appearing in \( \Gamma_{1,\text{div}}^{[0]} \) when \( M \neq 0 \) should include \( M^2 \) as a factor and also satisfies (4.34) – (4.35). Only \( I_{WM} \) can be a candidate. Is such a term can really appear? Imagine a limiting case that the matter fields and the \( U_Y(1) \) fields are absent. Thus the constraint conditions become Lorentz conditions and the above five solutions become two, namely, \( (T_{WL} - T_{GL}) \) and \( T_{CK} \). This combination of \( T_{WL} \) and \( T_{GL} \) are due to the restriction of the constraint condition containing \( \partial_{\mu}W_{\mu \nu} \) and therefore should be decomposed into two independent terms when the \( U_Y(1) \) fields are absent. In fact, it is known that a \( SU(n) \) theory with massive gauge Bosons is renormalisability [1] and that when the matter fields are absent \( \Gamma_{n+1,\text{div}}^{[n]} \), such a theory is a combination of three independent terms \( T_{WL}, T_{GL} \) and \( T_{CK} \). It follows that for the present theory \( \Gamma_{1,\text{div}}^{[0]} \) does not contain the mass term \( I_{MW} \) neither and can be
expressed as
\[
\mathbf{T}^{[0]}_{1,\text{div}} = \alpha_1^{(1)}T_1 + \alpha_2^{(1)}T_2 + \alpha_3^{(1)}T_3 + \alpha_4^{(1)}T_4 + \alpha_5^{(1)}T_5,
\]
(4.41)
where, \(\alpha_1^{(1)}, \ldots, \alpha_5^{(1)}\) are constants of order \((\hbar)^1\) and are divergent when the space-time dimension tends to 4.

In order to cancel the one loop divergence the counterterm of order \((\hbar)^1\) in the action should be chosen as
\[
\delta I^{[1]}_{\text{count}} = -\mathbf{T}^{[0]}_{1,\text{div}},
\]
(4.42)
Since
\[
\mathbf{T}^{[0]}_{\text{eff}} = \mathbf{T}^{[0]}_0,
\]
(4.43)
it is known from (4.41) that the sum of \(\mathbf{T}^{[0]}_{\text{eff}}\) and \(\delta I^{[1]}_{\text{count}}\), to order of \((\hbar)^1\), can be written as
\[
\begin{align*}
\mathbf{T}^{[1]}_{\text{eff}} &= [\psi, \overline{\psi}, W, C, \overline{C}, K, L, n, l, p, n', l', p', g, g_1] \\
&= T^{[0]}_{\text{eff}}[\psi, \overline{\psi}, W^{[0]}, C^{[0]}, \overline{C}^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, l^{[0]}, \ldots, g^{[0]}, g_1^{[0]}],
\end{align*}
\]
(4.44)
where the bare fields and the bare parameters (to order \((\hbar)^1\)) are defined as
\[
\begin{align*}
W_{a\mu}^{[0]} &= (Z_3^{[1]})^{1/2}W_{a\mu} = (1 - \alpha_1^{(1)})W_{a\mu}, \quad L_a^{[0]} = (Z_3^{[1]})^{1/2}L_a, \\
W_{g\mu}^{[0]} &= (Z_3^{[1]})^{1/2}W_{g\mu} = (1 - \alpha_2^{(1)})W_{g\mu}, \\
C_a^{[0]} &= (\bar{Z}_3^{[1]})^{1/2}C_a = (1 - \alpha_3^{(1)} + \alpha_1^{(1)})C_a, \\
\overline{C}_a^{[0]} &= (Z_3^{[1]})^{1/2}\overline{C}_a, \quad \overline{K}_\mu^{[0]} = (\bar{Z}_3^{[1]})^{1/2}\overline{K}_\mu, \\
C_y^{[0]} &= (\bar{Z}_3^{[1]})^{1/2}C_y = (1 - \alpha_3^{(1)} + \alpha_2^{(1)})C_y, \\
\overline{C}_y^{[0]} &= (Z_3^{[1]})^{1/2}\overline{C}_y, \quad K_\mu^{[0]} = (Z_3^{[1]})^{1/2}K_\mu, \\
\nu_L^{[0]} &= (Z_{\nu L}^{[1]})^{1/2}\nu_L = (1 - \alpha_4^{(1)})\nu_L, \quad \overline{\nu}_L^{[0]} = (Z_{\nu L}^{[1]})^{1/2}\overline{\nu}_L, \\
e_L^{[0]} &= (Z_{e L}^{[1]})^{1/2}e_L = (Z_{e L}^{[1]})^{1/2}e_L, \quad \overline{e}_L^{[0]} = (Z_{e L}^{[1]})^{1/2}\overline{e}_L, \\
e_R^{[0]} &= (Z_{e R}^{[1]})^{1/2}e_R = (1 - \alpha_5^{(1)})e_R, \quad \overline{e}_R^{[0]} = (Z_{e R}^{[1]})^{1/2}\overline{e}_R, \\
n^{[0]}_n &= (Z_n^{[1]})^{1/2}n = (1 - \alpha_3^{(1)} + \alpha_4^{(1)})n, \quad n^{[0]}_n = (Z_n^{[1]})^{1/2}n', \\
l^{[0]}_l &= (Z_l^{[1]})^{1/2}l = (Z_l^{[1]})^{1/2}l, \quad l^{[0]}_l = (Z_l^{[1]})^{1/2}l', \\
p^{[0]}_p &= (Z_p^{[1]})^{1/2}p = (1 - \alpha_3^{(1)} + \alpha_5^{(1)})p, \quad p^{[0]}_p = (Z_p^{[1]})^{1/2}p', \\
g^{[0]}_g &= (Z_g^{[1]})^{1/2}g = (Z_g^{[1]})^{1/2}g, \quad g^{[0]}_g = (Z_g^{[1]})^{1/2}g_1 = (Z_g^{[1]})^{1/2}g_1.
\end{align*}
\]
Next, defined
\[ \Phi_{1}^{[0]} = \partial_{\mu}W_{1\mu}^{[0]} + g_{1}^{[0]}W_{1\mu}^{[0]}W_{y}^{\mu[0]}; \]
\[ \Phi_{2}^{[0]} = \partial_{\mu}W_{2\mu}^{[0]} - g_{1}^{[0]}W_{1\mu}^{[0]}W_{y}^{\mu[0]}; \]
\[ \Phi_{3}^{[0]} = \partial_{\mu}W_{3\mu}^{[0]} + g_{1}^{[0]}W_{1\mu}^{[0]}W_{y}^{\mu[0]}; \]
\[ \Phi_{y}^{[0]} = \partial_{\mu}W_{y\mu}^{[0]} + g_{1}^{[0]}W_{3\mu}^{[0]}W_{y}^{\mu[0]}. \]

From (4.45), (4.46) and (4.57) one has
\[ g_{1}^{[0]}W_{a\mu}^{[0]} = g_{1}^{[0]}W_{a\mu}, \quad g_{1}^{[0]}W_{y\mu}^{[0]} = g_{1}^{[0]}W_{y\mu}, \]
and
\[ \Phi_{a}^{[0]} = (Z_{3}^{[1]})^{1/2} \Phi_{a}, \quad \Phi_{y}^{[0]} = (Z_{3}^{[1]})^{1/2} \Phi_{y}. \] (4.58)

Thus by adding \( I_{WM} \) and the \( \lambda \) terms into \( I_{eff}^{[1]} \) and forming
\[ I_{eff}^{[1]} = I_{eff}^{[1]} + I_{WM} + \int d^{4}x \left\{ \lambda_{a}(x)\Phi_{a}(x) + \lambda_{y}(x)\Phi_{y}(x) \right\}, \] (4.59)
one gets
\[ I_{eff}^{[1]} = I_{eff}^{[0]}[\psi, \bar{\psi}, W, C, \bar{C}, \lambda, K, L, n, l, p, n', l', p', g, g_{1}, M], \]
\[ = I_{eff}^{[0]}[\psi^{[0]}, \bar{\psi}^{[0]}, W^{[0]}, C^{[0]}, \bar{C}^{[0]}, \lambda^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, n^{[0]}, \ldots, g^{[0]}, g_{1}^{[0]}, M^{[0]}], \] (4.60)
where
\[ M^{[0]} = (Z_{3}^{[1]})^{-1/2} M, \quad \lambda_{a}^{[0]} = (Z_{3}^{[1]})^{-1/2} \lambda_{a}, \quad \lambda_{y}^{[0]} = (Z_{3}^{[1]})^{-1/2} \lambda_{y}. \] (4.61)

Obviously, if the action \( I_{eff}^{[1]} \) is used to replace \( I_{eff}^{[0]} \) in (4.3) and define \( Z^{[1]}, \Gamma^{[1]} \) as well as
\[ \Gamma^{[1]} = \Gamma^{[1]} - I_{WM} - \int d^{4}x \left\{ \lambda_{a}(x)\Phi_{a}(x) + \lambda_{y}(x)\Phi_{y}(x) + L_{WM} \right\}, \] (4.62)
then one has
\[ \Gamma^{[1]}[\psi, \bar{\psi}, W, C, \bar{C}, \lambda, K, L, n, l, p, n', l', p', g, g_{1}, M], \]
\[ = \Gamma^{[0]}[\psi^{[0]}, \bar{\psi}^{[0]}, W^{[0]}, C^{[0]}, \bar{C}^{[0]}, \lambda^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, n^{[0]}, \ldots, g^{[0]}, g_{1}^{[0]}, M^{[0]}]. \] (4.63)

From this it is easy to check that, to order \( \hbar^{1} \), \( \Gamma^{[1]} \) is finite. Moreover, by changing into bare fields and bare parameters the fields and parameters in (4.15)–(4.22) and then transforming them back into
the renormalized fields and renormalized parameters according to (4.45)–(4.59), one can see that, under condition (4.23), $\Gamma^{[1]}$ also satisfies

$$\Lambda_{op} \Gamma^{[1]} = 0, \quad (4.64)$$

$$\frac{\delta \Gamma^{[1]}}{\delta \lambda_a(x)} = 0, \quad \frac{\delta \Gamma^{[1]}}{\delta \lambda_y(x)} = 0, \quad (4.65)$$

$$\Sigma_a(x) \Gamma^{[1]} = 0, \quad \Sigma_y(x) \Gamma^{[1]} = 0. \quad (4.66)$$

It is now clear that the renormalisability of the theory can be verified by the inductive method. The following is an outline of the proof. Assume that up to $n$ loop the theory has been proved to be renormalisable by introducing the counterterm

$$I_{\text{count}}^{[n]} = \sum_{l=1}^{n} \delta I_{\text{count}}^{[l]},$$

where $\delta I_{\text{count}}^{[l]}$ is the counterterm of order $\hbar^l$ has the form of (4.41),(4.42). Therefore the modified generating functional $\Gamma^{[n]}$ for the regular vertex, defined by the action

$$I_{\text{eff}}^{[n]} = I_{\text{eff}}^{[0]} + I_{\text{count}}^{[n]}$$

satisfied equations (4.64) – (4.66) (under (4.23)) and, to order $\hbar^n$, is finite. This also means that the fields or parameters in each of the following brackets have the same renormalization factor:

$$(W_{a\mu}^{[0]}, L_a), (C_a, \overline{C}_a, K_a^\mu), (C_y, \overline{C}_y, K_y^\mu), (\nu_L, \overline{\nu}_L, e_L, \overline{e}_L), (\nu_R, \overline{\nu}_R, e_R), (n, n', l, l'), (p, p'), (\lambda, M, g),$$

and that

$$Z_{\gamma}^{[n]}(Z_3^{[n]})^{1/2} = 1, \quad Z_\nu^{[n]}(Z_{\gamma}^{[n]})^{1/2} = 1,$$

$$Z_3^{[n]} Z_3^{[n]} = Z_{\nu L}^{[n]} Z_{(n)}^{[n]} = Z_{\nu R}^{[n]} Z_{(n)}^{[n]} = Z_{\nu L}^{[n]} Z_{(n)}^{[n]}.$$

We have to proved that by using a counterterm of order $\hbar^{n+1}$ which also has the form of (4.41),(4.42), $\Gamma^{[n+1]}$ can be make satisfy (4.64) – (4.66) and finite to order $\hbar^{n+1}$, where $\Gamma^{[n+1]}$ is the modified generating functional for the regular vertex, determined by the action

$$I_{\text{eff}}^{[n+1]} = I_{\text{eff}}^{[n]} + \delta I_{\text{count}}^{[n+1]}.$$

Denote by $\Gamma^{[n]}_k$ the part of order $\hbar^k$ in $\Gamma^{[n]}$. For $k \leq n$, $\Gamma^{[n]}_k$ is equal to $\Gamma^{[k]}_k$, because it can not contain the contribution of a counterterm of order $\hbar^{k+1}$ or higher. Thus on expanding $\Gamma^{[n]}$ to order $\hbar^{n+1}$ one has

$$\Gamma^{[n]} = \sum_{k=0}^{n} \Gamma^{[n]}_k + \Gamma^{[n]}_{n+1} + \cdots.$$
Using this and extracting the terms of order $\hbar^{n+1}$ from the equations satisfied by $\Gamma^{[n]}$, namely (4.64) – (4.66), one finds

$$
\Lambda_{op} \Gamma^{[n]}_{n+1} = 0, \\
\frac{\delta \Gamma^{[n]}_{n+1}}{\delta \lambda_a(x)} = 0, \quad \frac{\delta \Gamma^{[n]}_{n+1}}{\delta \lambda_y(x)} = 0, \\
\Sigma_a(x) \Gamma^{[n]}_{n+1} = 0, \quad \Sigma_y(x) \Gamma^{[n]}_{n+1} = 0,
$$

(4.67)

(4.68)

(4.69)

Let $\Gamma^{[n]}_{n+1,\text{div}}$ stand for the pole part of $\Gamma^{[n]}_{n+1}$. By repeating the steps going from (4.33) to (4.41), one can arrive at

$$
\Gamma^{[n]}_{n+1,\text{div}} = \alpha_1^{(n+1)} T_1^{(1)} + \alpha_2^{(n+1)} T_2^{(2)} + \alpha_3^{(n+1)} T_3^{(3)} + \alpha_4^{(n+1)} T_4^{(4)} + \alpha_5^{(n+1)} T_5^{(5)},
$$

(4.70)

where $\alpha_1^{(n+1)}, \ldots, \alpha_5^{(n+1)}$ are constants of order $(\hbar)^{n+1}$. Therefore, in order to cancel the $n + 1$ loop divergence the counterterm of order $\hbar^{n+1}$ should be chosen as

$$
\delta I_{\text{count}}^{[n+1]} = -\Gamma^{[n]}_{n+1,\text{div}}[\bar{\psi}, \psi, W, C, \bar{C}] .
$$

(4.71)

Adding this counterterm, the mass term and the $\lambda$ terms to $\Gamma^{[n]}_{\text{eff}}$, one can express the effective action of order $\hbar^{n+1}$ as

$$
I^{[n+1]}_{\text{eff}}[\bar{\psi}, \psi, W, C, \bar{C}, \lambda, K, L, n, l, p, n', l', p', g, g_1, M] = I^{[0]}_{\text{eff}}[\bar{\psi}^{[0]}, \psi^{[0]}, W^{[0]}, C^{[0]}, \bar{C}^{[0]}, \lambda^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, l^{[0]}, \ldots, g^{[0]}, g_1^{[0]}, M^{[0]}],
$$

(4.72)

where the bare fields and the bare parameters (to order $(\hbar)^{n+1}$) are defined as

$$
W^{[0]}_{\alpha \mu} = (Z_3^{[n+1]})^{1/2} W_{\alpha \mu} = ((Z_3^{[n]})^{1/2} - \alpha_1^{(n+1)}) W_{\alpha \mu}, \quad L^{[0]}_a = (Z_3^{[n+1]})^{1/2} L_a ,
$$

(4.73)

$$
W^{[0]}_{\gamma \mu} = (Z_3^{[n+1]})^{1/2} W_{\gamma \mu} = ((Z_3^{[n]})^{1/2} - \alpha_2^{(n+1)}) W_{\gamma \mu},
$$

(4.74)

$$
C^{[0]}_a = (Z_3^{[n+1]})^{1/2} C_a = ((Z_3^{[n]})^{1/2} + (-\alpha_3^{(n+1)} + \alpha_1^{(n+1)})) C_a ,
$$

(4.75)

$$
\bar{C}^{[0]}_a = (Z_3^{[n+1]})^{1/2} \bar{C}_a , \quad K^{[0]}_\mu = (Z_3^{[n+1]})^{1/2} K_\mu ,
$$

(4.76)

$$
C^{[0]}_y = (Z_3^{[n+1]})^{1/2} C_y = ((Z_3^{[n]})^{1/2} + (-\alpha_3^{(n+1)} + \alpha_2^{(n+1)})) C_y ,
$$

(4.77)

$$
\bar{C}^{[0]}_y = (Z_3^{[n+1]})^{1/2} \bar{C}_y , \quad K^{[0]}_\mu = (Z_3^{[n+1]})^{1/2} K_\mu ,
$$

(4.78)

$$
\nu^{[0]}_L = (Z_\nu^{[n+1]})^{1/2} \nu_L = ((Z_\nu^{[n]})^{1/2} - \alpha_4^{(n+1)}) \nu_L , \quad \bar{\nu}^{[0]}_L = (Z_\nu^{[n+1]})^{1/2} \bar{\nu}_L ,
$$

(4.79)

$$
e^{[0]}_L = (Z_e^{[n+1]})^{1/2} e_L = ((Z_e^{[n]})^{1/2} - \alpha_5^{(n+1)}) e_L , \quad \bar{\tau}^{[0]}_L = (Z_e^{[n+1]})^{1/2} \bar{\tau}_L ,
$$

(4.80)

$$
e^{[0]}_R = (Z_e^{[n+1]})^{1/2} e_R = ((Z_e^{[n]})^{1/2} - \alpha_5^{(n+1)}) e_R , \quad \bar{\tau}^{[0]}_R = (Z_e^{[n+1]})^{1/2} \bar{\tau}_R ,
$$

(4.81)
\[ n^{[0]} = (Z_{(n)}^{[n+1]})^{1/2} n = ((Z_{(n)}^{[n]})^{1/2} + (-\alpha_3^{(n+1)} + \alpha_4^{(n+1)})) n, \quad n'^{[0]} = (Z_{(n')}^{[n+1]})^{1/2} n', \quad (4.82) \]

\[ l^{[0]} = (Z_{(l)}^{[n+1]})^{1/2} l = (Z_{(l)}^{[n]})^{1/2} l, \quad l'^{[0]} = (Z_{(l')}^{[n+1]})^{1/2} l', \quad (4.83) \]

\[ p^{[0]} = (Z_{(p)}^{[n+1]})^{1/2} p = ((Z_{(p)}^{[n]})^{1/2} - \alpha_3^{(n+1)} + \alpha_5^{(n+1)}) p, \quad p'^{[0]} = (Z_{(p')}^{[n+1]})^{1/2} p', \quad (4.84) \]

\[ g^{[0]} = Z_{g}^{[n+1]} g = (Z_{3}^{[n+1]})^{-1/2} g, \quad g_1^{[0]} = Z_{g}^{[n+1]} g_1 = (Z_{3}^{[n+1]})^{-1/2} g_1, \quad (4.85) \]

\[ g'^{[0]} = Z_{g}^{[n+1]} g' = (Z_{3}^{[n+1]})^{-1/2} g', \quad g_1'^{[0]} = Z_{g}^{[n+1]} g_1' = (Z_{3}^{[n+1]})^{-1/2} g_1, \quad (4.86) \]

\[ M^{[0]} = Z_{M}^{[n+1]} M = (Z_{3}^{[n+1]})^{-1/2} M, \quad (4.87) \]

and \( \lambda_a^{[0]}, \lambda_g^{[0]} \) are

\[ \lambda_a^{[0]} = (Z_{3}^{[n+1]})^{-1/2} \lambda_a, \quad \lambda_g^{[0]} = (Z_{3}^{[n+1]})^{-1/2} \lambda_g. \quad (4.88) \]

Therefore, in terms of such bare fields and bare parameters, \( \Gamma^{[n+1]} \) can be expressed as

\[ \Gamma^{[n+1]} = [W, C, \overline{C}, \psi, \overline{\psi}, K, L, n, \ell, p, n', \ell', p', g, g_1, M] \]

\[ = \Gamma^{[0]} [W^{[0]}, C^{[0]}, \overline{C}^{[0]}, \psi^{[0]}, \overline{\psi}^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, \ell^{[0]}, p^{[0]}, g^{[0]}, g_1^{[0]}, M^{[0]}]. \quad (4.89) \]

From this one can conclude that \( \Gamma^{[n+1]} \), under (4.23), satisfies (4.64)-(4.66) and is finite to order \( h^{n+1} \).

Since the theory can be renormalized to one loop the renormalisability has been proven.

\[ \text{V. Concluding Remarks} \]

By taking into account the original constraint conditions and the additional condition we have carried out the quantization of the \( SU_L(2) \times U_Y(1) \) electroweak theory with the W Z mass term and construct the ghost action in a way similar to that used for the massive SU(n) theory [1]. We have also shown that when the \( \delta- \) functions appearing in the path integral of the Green functions and representing the constraint conditions are rewritten as Fourier integrals with Lagrange multipliers \( \lambda_a \) and \( \lambda_g \), the total effective action consisting of the Lagrange multipliers, ghost fields and the original fields is BRST invariant. Furthermore, by comparing with the massless theory and with the massive SU(n) theory we have found the general form of the divergent part of the generating functional for the regular vertex functions and proven the renormalisability of the theory. It has also been clarified that the renormalisability of the theory with the W Z mass term is ensured by the renormalisability of the massless theory and the massive SU(n) theory.
If the harmlessness of the W Z mass term had been proven at the beginning of 1960s, the SU_L(2) \times U_Y(1) electroweak theory without the Higgs mechanism would have been deeply studied and tested. Today, the standard model of the electroweak theory has achieved great successes and the whereabouts of the Higgs Bosons is still unknown. It is therefore reasonable to ask if such successes really depends on the Higgs mechanism and to pay attention to the theory without the Higgs mechanism.

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