Low energy effective theory for axion strings

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Abstract. We consider aspects of the low energy (classical) effective field theory description of global cosmic strings. In the non-relativistic limit, we study the extent to which these are described by the combination of the Nambu-Gotto and Kalb-Ramond actions. While in a formal sense this is the case for infinitely long strings, for more realistic situations, the collective coordinates of the system do not provide a complete description for more than brief intervals; as time evolves it becomes necessary to include low momentum goldstone excitations as well.

Keywords: axions, dark matter theory

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1 Introduction

There has been ongoing discussion for almost four decades of the possible role of axion strings in axion dark matter production (see, e.g., [1–6]). There are a range of analyses, some analytical, many numerical simulations, with the latter quite large and sophisticated in recent years [7–13]. In some cases, these simulations appear to point to a parameterically enhanced production of axion dark matter due to cosmic strings; in other cases, not so much so. There is a large parameter in the problem, the infrared sensitive value of the string tension divided by $f_a^2$. Assuming that the Hubble constant provides the infrared cutoff, this parameter is typically taken to be:

$$\xi \equiv \log(f_a/H_{qcd}) \sim 70.$$  \hspace{1cm} (1.1)

If there is of order one string per horizon, and if most of the energy in the axion field surrounding axion strings were converted into very low momentum axions, there would be an enhancement of the dark matter axion density by such a factor over naive, textbook [14] considerations. This could have significant consequences for cosmology and for the ability of experiments such as ADMX to detect the dark matter. In [15], qualitative arguments were put forth that there is no such enhancement. This work did enumerate a number of challenges for analytic dark matter estimates at the order one level, which simulations have the potential to resolve.

In this paper, we will look more closely at the effective theory of axion strings, asking whether one might expect enhancements of axion emission at low momentum by powers of $\xi$. In effect, we are revisiting the older literature on these objects. Crucial will be the role of the infrared in axion production. Simulations, when sufficiently accurate, generally consider all degrees of freedom simultaneously. They do not exploit low energy actions for collective coordinates and low momentum axions of the type we will consider here. It would be interesting to perform simulations in the low energy effective theory, and/or to match the results of simulations to these theories.

As opposed to Nielsen-Olesen, gauge strings [16], strings associated with the breaking of a global symmetry have some problematic features. In particular, if one tries to define a tension, one finds that it is infrared divergent. As with all infrared divergences, this is a signal,
as stressed in [15], that one must include an enlarged set of degrees of freedom in describing
the system.\footnote{It is perhaps worth recalling how this is realized in QED. There, the three point function is infrared
divergent. It cannot be used in an effective field theory by itself to compute scattering amplitudes; it is
necessary to consider emission of extremely low energy photons as well.} In the case of global cosmic strings, this means that one can’t consider the system
of string collective coordinates by themselves. Instead, it is necessary to consider an effective
action involving these coordinates and axions with momentum below some cutoff scale. For
an infinitely long string, the cutoff should be chosen so as to avoid problems of causality.
Implementing such a cutoff might be complicated. For a closed string of circumference of
order $R$, $R$ itself cuts off infrared divergences, and ideally one should choose the (momentum
space) cutoff large compared to $R^{-1}$. For long, parallel string anti-string segments separated
by a distance $b$, $b$ acts as a cutoff. Such configurations are likely more representative than the
infinite string of the actual cosmological situation. In this note, we will look at the effective
action for this system, particularly in the non-relativistic limit in which the description of
solitons becomes simple. We will see that for a long straight string, with little excitation of
non-zero modes, the Nambu-Goto action, supplemented with the Kalb-Ramond coupling [14]
of the axion to the string (NGKR), in a certain formal sense, captures the dynamics. But for
closed strings or string-antistring segments the NGKR description, by itself, fails to correctly
capture the dynamics of the system. The problem is that while, at one instant, the system
might be well described by its collective coordinates without axion excitations, this will
typically not be true a short time later. This is in sharp contrast to infrared insensitive
systems like magnetic monopoles and gauge strings in four dimensions. Said another way,
for most of its history, the string, as an entity, is not the simple object considered in many
analyses. Note, here, we are speaking of the classical (as opposed to quantum) theory of these
strings. This will be our focus throughout this paper.

The rest of this paper is organized as follows. In the next section, we briefly review well
known aspects of the collective coordinate method for solitons, both particles and strings.
For systems such as monopoles and Nielsen-Olsen strings, we explain the sense in which the
collective coordinates provide a complete description of the system, in uniform motion, for all
times. We consider the collective coordinate analysis for an infinitely long, straight global
string in section 3. We derive, formally, the non-relativistic Nambu-Gotto action for the
collective coordinates, with its infrared divergent tension, and in section 4 we will see that the
Kalb-Ramond coupling does formally describe the axion couplings to the collective modes.
We then consider, in section 5, the description of closed string systems. We note that, related
to the infrared divergence of the infinite string, there is not a unique definition of collective
coordinates. It is then not surprising, as we elucidate, that the NGKR description of such
systems is inadequate. In section 6, we summarize and describe our expectations for the
spectrum of axion radiation emitted by the sorts of closed string systems one might expect to
encounter in early universe cosmology and in simulations of such cosmologies.

2 Collective coordinate review

The utility of the collective coordinate method follows from elementary considerations. Con-
sider, for example, a kink in a translationally invariant theory in two dimensions, described
by a classical solution, $\Phi_{cl}(x)$

$$\Phi(x,t) = \Phi_{cl}(x - X(t)) + \delta \Phi.$$  \hspace{1cm} (2.1)
δΦ describes the fluctuations about the classical solution. We can obtain an action for the
degrees of freedom X and δΦ by substituting (2.1) into the field theory action. There is a
kinetic term for X of the form
\[ L(X, δΦ) = \frac{1}{2} M \dot{X}^2 + \ldots \] (2.2)
and higher order terms in \( \dot{X} \). Because of translational invariance, there are no terms involving
X without derivatives. In addition, precisely because Φcl is a solution of the classical equations,
when we work out the action for X, there are no terms in the action linear in δΦ for a kink in
uniform motion. So if one has a soliton in uniform motion at time \( t = t_0 \), say, with \( ΔΦ = 0 \),
δΦ remains zero for all time.

The same is true for other collective coordinates associated with symmetries of solutions,
such as translational and dyonic excitations of monopoles. Infinite gauge strings in four dimen-
sions have two collective coordinates, the transverse coordinates of the string solution, \( \vec{X}_⊥ \).
Calling \( z \) the coordinate along the string, and writing the field in the presence of the string as
\[ Φ(\vec{x}_⊥, z, t) = Φ(\vec{x}_⊥ - \vec{X}(z, t)) + δΦ, \] (2.3)
the action for these collective coordinates has the form:
\[ \int d^2σ L(X, δΦ) = \int d^2σ \frac{1}{2} T \left( \frac{∂\vec{X}}{∂σ_α} \right)^2 + \ldots \] (2.4)
Here \( σ = (z, t) \); this is the non-relativistic limit of the usual Nambu-Gotto action. Again,
there is no term linear in δΦ. So, for a string executing uniform motion, starting with δΦ = 0
at some time, δΦ remains zero. All of this follows from the translational invariance of the
theory. The Nambu-Gotto description remains good if the string is curved, with curvature
small compared to the characteristic scales of the system.

This simple observation is the origin of the power of the collective coordinate method for
such systems. It will not be the case for strings arising from the breaking of global symmetries
in four dimensions (or vortices in \( 2 + 1 \) dimensions). This is precisely because of the infrared
divergences. The effects which cut off the divergence, such as finite circumference, will spoil
the collective coordinate story, in the sense that the system as described by the collective
coordinates alone does not obey the equations of motion. This is closely tied to the need
to include other degrees of freedom in the effective action. The infrared divergence yields a
dependence on the breaking scale of the symmetry. There will be a potential for the collective
coordinates and coupling of the collective coordinates to δΦ.

3 The effective action for the collective coordinates for a long global string

Consider a theory of a complex scalar, Φ, with action symmetric under Φ → e^{iα}Φ:
\[ S = \int d^4x \left( |∂_μΦ|^2 + m^2 |Φ|^2 - \frac{λ}{2} |Φ|^4 \right). \] (3.1)

For this action,
\[ ⟨|Φ|⟩ ≡ f_a ≡ v = \frac{m^2}{λ}. \] (3.2)
This action admits static string solutions. Taking the string to lie along the $z$ axis, with coordinates $(x, y, z) \equiv (\vec{x}_\perp, z) = (\rho, \theta, z)$,
\[
\Phi(t, z, \vec{x}_\perp) = \Phi_{\text{cl}}(\vec{x}_\perp) = f(\rho) e^{i\theta}.
\] (3.3)  

The function $f(\rho)$ has the properties:
\[
f(\rho) \to 0 \text{ as } \rho \to 0; \quad f(\rho) \to f_a \left(1 - \frac{1}{f_a \rho^2}\right) \text{ as } \rho \to \infty.
\] (3.4)  

We first obtain the effective action for the collective coordinates for the string. Note, first that if we calculate the tension, thought of as the energy per unit length of the string, the result is infrared divergent:
\[
T = \int d^2 x_\perp (|\vec{\nabla} \Phi|^2 + V(\Phi) - V_0) \approx 2\pi f_a^2 \int d\rho \frac{1}{\rho^2} = 2\pi \log(\Lambda/\mu).
\] (3.5)  

Here $\Lambda$ is an ultraviolet cutoff, of order the core size; we’ll take $\Lambda = f_a$. $\mu$ is an infrared cutoff.  

If we have a closed string with radius $R$, $\mu = R^{-1}$; if we have a parallel string and antistring separated by a distance $b$, $\mu = b^{-1}$.  

We can see more directly that $T$ is the tension of the string by allowing slow variation in the transverse directions, introducing collective coordinates $X_i(z, t)$, $i = 1, 2$. We also allow variation of the phase of $\Phi$ from $i\theta$, i.e. a spatially and time-dependent axion field:
\[
\Phi(t, z, \vec{x}_\perp) = \Phi_{\text{cl}}(\vec{X}(t, z) - \vec{x}_\perp) e^{i\delta a(t, z, \vec{x}_\perp)}.
\] (3.6)  

Now plug this expression into the terms in the action:
\[
S = \int dt dz d^2 x_\perp \left( |\partial_t \Phi|^2 - |\partial_z \Phi|^2 \right)
\]
\[
\approx \int dt dz \left( (\partial_i X^i)^2 - (\partial_z X^i)^2 \right) \int d^2 x_\perp |\vec{\nabla} \Phi_{\text{cl}}(x_\perp)|^2
\]
\[
+ \int d^4 x f_a^2 \left( \partial_i X^i \partial_t \theta \partial_t \delta a - \partial_z X^i \partial_t \theta \partial_z \delta a \right).
\] (3.7)  

\[= I_{\text{transverse}} + I_{\delta a}
\]

This is valid for distances from the string small compared to the wavelength of the disturbance. In this regime, the variation of the coordinate is roughly constant, and the light transit time can be neglected. The first term can be rewritten as
\[
I_{\text{transverse}} = T \int dz dt \left( \left( \frac{\partial \vec{X}_\perp(t, z)}{\partial t} \right)^2 - \left( \frac{\partial \vec{X}_\perp(t, z)}{\partial z} \right)^2 \right)^2
\] (3.8)  

with $T$ given by our earlier expression. The infrared divergence appears again. In the second term, $\partial_t \theta = \frac{x_j \epsilon_{ij}}{|\vec{x}_\perp|^2}$. so
\[
I_{\delta a} = \int d^4 x \epsilon_{ij} \frac{x_j}{|\vec{x}_\perp|^2} f_a^2 \left( \partial_i X^i \partial_t \delta a - \partial_z X^i \theta \partial_z \delta a \right).
\] (3.9)  

Equation (3.8) is, again, the non-relativistic limit of the Nambu-Goto action, but with an infrared divergent tension. Before considering how this divergence may be tamed, we can ask about the coupling to $\delta a$. As we now demonstrate (and was remarked in [15]), equation (3.9) is of the form of the Kalb-Ramond action [14].
4 Appearance of the KR term for a long, slowly moving string

In this section, we observe formally that for an infinitely long, straight string, the KR action does describe the coupling of the string collective coordinates to axion radiation for a slowly moving string. The $a$ configuration defining the string changes as $X^i$, the string collective coordinates, change. If the collective coordinates change slowly enough, we might expect $a$ to be, at each instant, nearly in its lowest energy (the $\nabla^2 a = 0$) configuration. The slight differences will correspond to radiation. This is encoded in our collective coordinate analysis, and we will see it is reproduced by the KR action.

4.1 The circulating axion field

We first express the axion field surrounding a static string in terms of the antisymmetric tensor field. Consider an infinite static string along the $z$ direction, i.e. $X^0 = t, X^z = z$. The KR action leads to the equation:

$$\Box B_{0z} = f_{\text{KR}}. \quad (4.1)$$

(We will fix the constant, $f_{\text{KR}}$ in a moment.) So, taking $B_{0z}$ to be $z$ and $t$ independent,

$$B_{0z} = G(\vec{x}_\perp)f_{\text{KR}} \quad (4.2)$$

where $G(\vec{x}_\perp)$ is the two dimensional Green’s function,

$$G(\vec{x}_\perp) = \frac{1}{2\pi} \log(|\vec{x}_\perp|). \quad (4.3)$$

From this, we can construct

$$H_{0zi} = \partial_i B_{0z} = \epsilon_{0zij} \partial_j a \quad (4.4)$$

so

$$\partial_j a = \epsilon_{ij} \partial_i f_{\text{KR}} G = \epsilon_{ij} \partial_i f_{\text{KR}} \frac{x^i}{|\vec{x}_\perp|^2} \quad (4.5)$$

which is $f_{\text{KR}} \partial_j \theta$, the polar angle in cylindrical coordinates. So the static configuration is as expected. Note that this is the minimal energy configuration for fixed $B (a)$, with the boundary condition of unit winding.

4.2 Axion emission from the string

Now consider axion emission from a moving string. Consider the term in the action

$$\int d^4x \mathcal{L} \sim \int d^4x \left( H_{zij}^2 - H_{0ij}^2 \right). \quad (4.6)$$

Noting, for example, that

$$H_{zij}(t, z, \vec{x}_\perp) = \partial_0 a + \epsilon_{ij} \partial_i G^{(2)}(\vec{x}_\perp) \frac{\partial X^j}{\partial t} \quad (4.7)$$

yields for the action a sum of terms, $\mathcal{L} = \mathcal{L}_{X^i} + \mathcal{L}_{a - X^i}$, where:

$$\mathcal{L}_{X^i} = \int d^4x \mathcal{L} = \int d^2x \left( \frac{dX^i}{dt} \right)^2 - \left( \frac{dX^i}{dz} \right)^2 \int d^2x_\perp \left( \frac{1}{x_\perp^2} \right) \quad (4.8)$$
and

$$L_{a\rightarrow X^i} = \int dz dt \, d^2 x⊥ \left( \partial_0 a \frac{dX^i}{dt} - \partial_z a \frac{dX^i}{dz} \right) \epsilon_{ij} \frac{x^j}{x⊥^2}. \quad (4.9)$$

The first expression is identical to the action for the transverse fluctuations of the string found in the field theory; the second is the axion-string collective coordinate coupling found there.

So formally, the collective coordinates and the axion perturbation of the solitonic string in field theory are described by the Nambu-Gotto-Kalb-Ramond (NGKR) action. We say formally both because of the infrared divergent tension and because of issues of causality: the axion field changes everywhere in response to transverse string displacements. These issues should be addressed by systems of closed strings, such as long, parallel strings and antistrings or circular strings, where string separations and/or radii will act as an infrared cutoff. We consider these issues in the next section. As we anticipated earlier, the fact that these configurations are not static solutions of the equations of motion limits the applicability of the collective coordinate description.

5 Systems of closed strings

In [15], two limits of string motion were considered, referred to as “adiabatic” and “sudden”. These were considered in a rather heuristic way. The adiabatic limit corresponds to slow motion of the string, and, in this limit, as the strings moves and the axion field surrounding them changes, the energy change in the axion field is compensated by the change in the kinetic energy of the collective coordinate. The sudden limit corresponds to fast motion of the string. In this case, most of the energy stored in the axion field is converted into freely moving axions. If this picture is correct, than in the case of adiabatic motion, the system should be well described by its collective coordinates; this is not true of the sudden case. In this section we will understand this picture in more detail in terms of the equations of motion of the system.

Because of the infrared issues associated with the axion field, equation (3.6) is not really sensible for an infinite string; it also implies an instantaneous change in the axion field throughout space. For a closed system, e.g. a closed, roughly circular string, or alternatively for a long, parallel string and antistring, this potentially makes more sense, as the infrared divergences cancel. In this case, however, we have to consider what we mean by $\Phi_{cl}$, since there are not static solutions of the equations of motion of this type. We might take this to be, for example, the minimal energy static (instantaneously) configuration with specified location of the string core. Then we can make the substitution of equation (3.6) and obtain an action for these coordinates. But the question of what configuration our collective coordinates describe does not have a unique answer. We have no reason to expect that as the system evolves, even if it starts in one such configuration, say, again, the axion configuration of lowest energy, it will remain in the corresponding lowest energy configuration.

For a long string-antistring pair, separated by a distance $b$ and moving slowly, the field configuration of lowest energy, far from the string core, is just the superposition of two static solutions. The corresponding field configuration is, far from the string core:

$$\Phi = f_a e^{ia(x,t)\frac{\pm a}{f_a}} \quad (5.1)$$

where

$$a(x,t) = a_0 \left( x⊥ - \frac{b(t)\dot{y}}{2} \right) - a_0 \left( x⊥ + \frac{b(t)\dot{y}}{2} \right). \quad (5.2)$$
Here \( a_0(\vec{x}_0) \) is the axion configuration around a single string. We now show that with initial conditions
\[
b(0) = b_0; \quad \dot{b}(0) = 0
\]  
this is \emph{almost} a solution of the equations of motion, for short times.

Substituting \( \Phi \) from equation (5.1) into the field theory action, yields an action for \( b \):
\[
L \approx f_a^2 \ell (\dot{b}^2 \log(f_a b) + \log(f_a b)).
\]  
as well as a coupling of the collective mode, \( b \), to the axion, \( \delta a \):
\[
\int d^3x \ dt \left( \dot{b} \frac{\partial a}{\partial b} \frac{\partial}{\partial \vec{x}} \frac{\partial \delta a}{\partial \vec{x'}} + \frac{\partial a}{\partial \vec{x}} \frac{\partial \delta a}{\partial \vec{x'}} \frac{\vec{\nabla}}{\partial \vec{x}} \right)
\]  
We can integrate by parts in the first term with respect to time. Provided that \( \dot{b} \) is small, the source for \( \delta a \) is then proportional to \( \ddot{b} = \frac{1}{b \log(f_a b)} \cdot \) The \textbf{energy} stored in \( \delta a \) is suppressed, at early times, by \( \frac{1}{\log(f_a b)} \) relative to the kinetic energy in \( b \). But we can make a stronger statement. The system can be described as approximately adiabatic [15]. Note that the source for the mode
\[
\delta a = C(t) \frac{\partial a}{\partial b}
\]  
vanishes at early times as a consequence of the equation of motion for \( b \). To see this, note that, the kinetic term, after integration by parts, neglecting \( \dot{b} \), yields:
\[
\dot{b} C \int d^2x_{\perp} (\frac{\partial a}{\partial b})^2
\]  
which is just \( C \) times the second derivative term in the equation of motion for \( b \), while from the integration over the \( \vec{\nabla}^2 \) term one has:
\[
C \frac{\partial}{\partial b} \int d^2x_{\perp} (\vec{\nabla} a)^2 = C \frac{\partial V(b)}{\partial b}.
\]  
So, provided \( \dot{b} \approx 0 \), the field of equation (5.1) is \emph{almost} a solution of the equations of motion. But as \( \dot{b} \) grows, \( C \) is significantly sourced. In particular, the \( \dot{b}^2 \) terms dominate once \( \dot{b} > \log(f_a b)^{-1/2} \). At this point the adiabatic approximation is breaking down.

This is quite general. For circular motion, one instead studies a classical solution with a circular core of radius \( R \). One can again start with a static configuration, minimizing the axion field energy as a function of \( R \). Again the mode:
\[
\delta a = C(t) \frac{\partial a}{\partial R}
\]  
is not sourced initially as a consequence of the equations of motion for \( R \). Now, however, the system quickly becomes relativistic, and \( C \) is driven away from zero. Correspondingly, much of the field energy is converted into radiation, rather than kinetic energy of \( R \).

Loosely speaking, treatments of radiation utilizing the Kalb-Ramond action require a careful treatment of “back reaction”. Once one enters the non-adiabatic regime, most of the energy of the field is converted to radiation rather than kinetic energy of \( b \). To fully account for this requires solving the full set of equations. It is reassuring, at least, that both regimes
can be analyzed, for the string-antistring case, in a non-relativistic limit. Circular strings become relativistic more quickly, and these back-reaction effects will be pronounced early on.

We conclude from this that initially for a long parallel string and antistring, if the system starts at rest, the bulk of the energy in the axion field is converted into kinetic energy of $b$. Little radiation is produced. As $b$ decreases and the velocity becomes relativistic, production of axions increases. Note, in particular, that one produces the mode $C$. This corresponds to the fact that, as the velocity increases, this motion is less and less an approximate solution of the equations of motion. All of this is consistent with the expectation that the energy in the axion field at a length scale $b(t)$ is converted into radiation of wavelength $b(t)$ except, possibly, at early stages of the motion, where long wavelength axions are suppressed if the velocity, $\dot{b}$, is small.

6 Conclusions

Global strings are peculiar objects. In isolation, their tensions are infrared divergent. This indicates that they are not really objects at all, in that they cannot, except possibly for brief periods, be described simply in terms of their collective coordinates. We have seen this explicitly for long, parallel strings and antistrings, and for closed strings. For these systems, the infrared divergences cancel, but static configurations are not solutions of the equations of motion. We have given a plausible definition of the collective coordinates for such systems. But as these systems move, even if initially the collective coordinates provide a good description, the description quickly becomes incomplete, with a substantial (typically order one or larger) fraction of the energy in excitations. In this sense, the Nambu-Goto-Kalb-Ramond description is inadequate. As signaled by the infrared divergence, it is necessary to consider degrees of freedom beyond the string collective coordinates.

As we have recalled, in [15], at a heuristic level two limiting cases of axion radiation by strings were considered for a long, parallel string and antistring. One, referred to as adiabatic, involved slow motion of strings, with the axion field adjusting at each instant to the configuration of the string core. In this limit, it was argued, the energy of the axion field would largely be converted to kinetic energy of the collective coordinate $b$. In this paper, we have seen how this is realized explicitly at the level of the equations of the effective field theory, in the limit that $\dot{b}$ is small. A second limit, referred to as sudden, involved rapid motion of the strings, with most of the energy in the axion field of the strings converted to axions. Again, we have seen this at the level of the field theory equations when $\dot{b}$ is not small. In the small $\dot{b}$, adiabatic case, few axions are produced. In the large $\dot{b}$ limit, most of the energy in the axion field surrounding the strings is converted to axions, but these have a spread in wavelength, with typical wavelengths at time $t$ of order the string separation at time $t$. From this, we see that in the sudden limit, the system is not well described by the collective coordinates alone. We also see that there is no reason to expect a lowlogarithmic enhancement of the low energy axion density; rather the energy should be distributed uniformly (on a log scale) over wave numbers between the Hubble scale and the scale $f_a$. These observations support the picture, put forward in [15], that cosmic strings are unlikely to yield a parameterically enhanced contribution to the population of low momentum axions, and thus to an enhanced axion dark matter density. Recent simulations with substantial reach in $\xi$ [13] appear consistent with these expectations.
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References

[1] R.L. Davis, *Cosmic Axions from Cosmic Strings*, Phys. Lett. B 180 (1986) 225 [inSPIRE].

[2] R.L. Davis and E.P.S. Shellard, *Do Axions Need Inflation?*, Nucl. Phys. B 324 (1989) 167 [inSPIRE].

[3] R.A. Battye and E.P.S. Shellard, *Global string radiation*, Nucl. Phys. B 423 (1994) 260 [astro-ph/9311017] [inSPIRE].

[4] R.A. Battye and E.P.S. Shellard, *Axion string constraints*, Phys. Rev. Lett. 73 (1994) 2954 [Erratum ibid. 76 (1996) 2203] [astro-ph/9403018] [inSPIRE].

[5] A. Dabholkar and J.M. Quashnock, *Pinning Down the Axion*, Nucl. Phys. B 333 (1990) 815 [inSPIRE].

[6] A. Vilenkin, *Cosmic Strings and Domain Walls*, Phys. Rept. 121 (1985) 263 [inSPIRE].

[7] M. Gorghetto, E. Hardy and G. Villadoro, *Axions from Strings: the Attractive Solution*, JHEP 07 (2018) 151 [arXiv:1806.04677] [inSPIRE].

[8] V.B. Klaer and G.D. Moore, *The dark-matter axion mass*, JCAP 11 (2017) 049 [arXiv:1708.07521] [inSPIRE].

[9] L. Fleury and G.D. Moore, *Axion dark matter: strings and their cores*, JCAP 01 (2016) 004 [arXiv:1509.00026] [inSPIRE].

[10] S. Chang, C. Hagmann and P. Sikivie, *Studies of the motion and decay of axion walls bounded by strings*, Phys. Rev. D 59 (1999) 023505 [hep-ph/9807374] [inSPIRE].

[11] C. Hagmann, S. Chang and P. Sikivie, *Axion radiation from strings*, Phys. Rev. D 63 (2001) 125018 [hep-ph/0012361] [inSPIRE].

[12] C. Hagmann, S. Chang and P. Sikivie, *Axions from string decay*, Nucl. Phys. B Proc. Suppl. 72 (1999) 81 [hep-ph/9807428] [inSPIRE].

[13] M. Buschmann et al., *Dark matter from axion strings with adaptive mesh refinement*, Nature Commun. 13 (2022) 1049 [arXiv:2108.05368] [inSPIRE].

[14] M. Kalb and P. Ramond, *Classical direct interstring action*, Phys. Rev. D 9 (1974) 2273 [inSPIRE].

[15] M. Dine, N. Fernandez, A. Ghalsasi and H.H. Patel, *Comments on axions, domain walls, and cosmic strings*, JCAP 11 (2021) 041 [arXiv:2012.13065] [inSPIRE].

[16] H.B. Nielsen and P. Olesen, *Vortex Line Models for Dual Strings*, Nucl. Phys. B 61 (1973) 45 [inSPIRE].