I. INTRODUCTION

In 1984 Witten [11] conjectures that stable and bound strange quark matter (SQM) may be formed under specific conditions (relevant early work in this idea can be found in [2–5]). The validation of this hypothesis still remains a challenge as the region of baryonic chemical potential where it is expected to occur is of difficult access in heavy ion experiments and on the lattice.

The possibility of windows for stable SQM has been the subject of a significant number of theoretical approaches. In [6] the properties of quark matter in beta equilibrium, with comparable densities of up, down and strange quarks, are studied using the Fermi gas model and the MIT Bag model [7, 8] with extensions to include surface tension and Coulomb effects.

The subject of quark pair formation at finite densities, see [9] for a review, was revived in more recent years with the understanding that at asymptotic high densities the ground state of cold matter is an electronless state with the understanding that at asymptotic high densities the ground state of cold matter is an electronless state with the understanding that at asymptotic high densities the ground state of cold matter is an electronless state with the understanding that at asymptotic high densities the ground state of cold matter is an electronless state with the understanding that at asymptotic high densities the ground state of cold matter is an electronless state with the understanding that at asymptotic high densities the ground state of cold matter is an electronless state with the understanding that at asymptotic high densities the ground state of cold matter is an electronless state. Away from this perturbative QCD regime a complex pattern of phases may occur, for reviews see [10–12].

In [13] a model is constructed for the interface between the electron rich nuclear matter regime and the CFL phase. The windows in Bag model parameter space for SQM are shown to be much larger with the CFL pairing than without pairing [14].

While bound properties of SQM can be very conveniently addressed within Bag model studies, the aspects of SQM formation can be naturally addressed starting from a detailed scenario of chiral symmetry ($\chi_S$) breaking. The seminal papers of Nambu [20, 21] introduce a model for dynamical $\chi_S$ breaking with the quark anti-quark condensate as order parameter in the chiral limit. Physical masses for the related pseudoscalar Goldstones are obtained through explicit $\chi_S$ breaking by QCD current quark mass terms.

NJL based investigations of SQM require the three quark flavors version of the model. Therefore one must consider known extensions of the original NJL Lagrangian. They include the $1/N_c$ suppressed next to leading order (NLO) terms with multi-quark interactions. If one is interested only in the $U(1)_A$ breaking effects the extension includes the six-quark $\Upsilon$ Hooft interactions (NJLH) [22–36]. A more detailed approach can also include an appropriate set of $SU(3)_L \times SU(3)_R$ symmetric eight-quark interactions (NJLH8) [37, 38] completing the number of vertices which are important in four dimensions for dynamical $\chi_S$ breaking [39, 40]. If one wants also to include at this order the effects of explicit $\chi_S$ symmetry breaking one should take advantage of the model presented in [41–43], the NJL8m model, where m indicates that the NLO terms in current quark masses are taken into account in the effective multi-quark Lagrangian.

Note that the role played by the current quark masses on phase transitions in hot and dense hadronic matter is presently a subject of intense study, as they are known to change the order of the transition or transform it into a smooth crossover [44].

In this work we address for the first time the problem of the formation of a SQM phase using a detailed picture of the $\chi_S$ breaking assembled in the NJL8m Lagrangian. We introduce the thermodynamic potential at finite temperature, T, and baryonic chemical potential, $\mu$, in Section II and study the chiral $T - \mu$ phase diagram in Subsection II A. The parameters of the model have been previously fixed at $T = \mu = 0$ in such a way that the NJL8m Lagrangian accurately describes the low-lying spectrum of the pseudoscalar and scalar mesons [41]. They additionally provide a good de-
scription of the radiative two photon decays of the pseudoscalars and of the strong two body decays of the scalars [42]. As we will see, this apparently quite natural extension (NJLH8→NJLH8m1) has surprising consequences. In Subsection III B we address the case study for cold dense matter in β-equilibrium. The conclusions and further discussion of the obtained results are presented in Section IV.

II. MODEL THERMODYNAMIC POTENTIAL

The thermodynamic potential of the model Lagrangian (for the explicit expression of the latter see [41, 42]) is given in the mean field approximation by:

\[ \Omega = \mathcal{V}_{st} + \sum_i \frac{N_c}{8\pi^2} J_{-1}(M_i, T, \mu_i), \]  

(1)

where the index refers to the sum over flavors \( i = u, d, s \). The integrals \( J_{-1} \) stem from the fermionic path integral over the quark bilinears which appear after bosonization. Using a regularization kernel,

\[ \rho(\tau \Lambda^2) = 1 - (1 + \tau \Lambda^2) e^{-\tau \Lambda^2}, \]  

(2)

corresponding to two Pauli-Villars subtractions in the integrand [43, 44] (previously used for instance in [45, 46]), the Dirac and Fermi sea contributions, \( J^{vac}_{-1} \) and \( J^{med}_{-1} \), can be written as:

\[ J_{-1} = J^{vac}_{-1} + J^{med}_{-1}, \]

\[ J^{vac}_{-1} = \int \frac{d^4 p}{(2\pi)^4} \int_0^\infty \frac{d\tau}{\tau} \rho(\tau \Lambda^2) \left( \frac{1}{8\pi^2} e^{-(\sqrt{p^2 + \tau^2})} \right), \]

\[ J^{med}_{-1} = - \int \frac{d^3 p}{(2\pi)^3} 16\pi^2 T (Z^+ - Z^-) \left( \frac{M}{0} + C(T, \mu) \right), \]

\[ Z^\pm = \text{log} \left( 1 + e^{-\frac{E \pm \mu}{T}} \right) \text{- log} \left( 1 + e^{-\frac{E \mp \mu}{T}} \right) - \frac{\Lambda^2}{2TE^2} \left( e^{-\frac{E \pm \mu}{T}} \right), \]

\[ C(T, \mu) = \int \frac{d^3 p}{(2\pi)^3} 16\pi^2 T \text{ log} \left( \left( 1 + e^{-\frac{E^+ \mp \mu}{T}} \right) \left( 1 + e^{-\frac{E^- \pm \mu}{T}} \right) \right), \]

(3)

where \( E = \sqrt{M^2 + p^2} \) and \( E_\Lambda = \sqrt{E^2 + \Lambda^2} \). The \( \left| M \right| \) notation refers to the subtraction of the same quantity evaluated for \( M = 0 \), which is done so as to set the zero of the potential to a uniform gas of massless quarks (it amounts to a subtraction of a constant). The \( C(T, \mu) \) term is needed for thermodynamic consistency [47].

The stationary phase contribution coming from the integration over the auxiliary bosonic fields, \( \mathcal{V}_{st} \), was evaluated using standard techniques [48] and is given by:

\[ \mathcal{V}_{st} = \frac{1}{16} \left( 4G \left( h_i^2 \right) + 3g_1 \left( h_i^2 \right)^2 + 3g_2 \left( h_i \right)^4 + 4g_3 \left( h_i^2 \right) m_i \right) + 4g_4 \left( h_i^2 \right) \left( h_i m_j \right) + 2g_5 \left( h_i^2 m_j^2 \right) + 2g_6 \left( h_i^2 m_j^2 \right) + 4g_7 \left( h_i m_j \right)^2 + 8\kappa h_u h_d h_s + 8\kappa_2 \left( h_u h_d h_s + h_u m_i m_j + h_u m_d m_s \right) \right|_0^M, \]

where \( h_i \) \( (i = u, d, s) \) are twice the quark condensates and \( m_i \) the current masses (a summation over the \( i, j \) flavor indices in \( \mathcal{V}_{st} \) is implicit).

The stationary phase conditions which relate the dynamical masses to the condensates,

\[ \Delta_f = M_f - m_f \]

(4)

\[ = - Gh_f - \frac{g_1}{2} h_f (h_i^2) - \frac{g_2}{2} (h_i^2) + \frac{3g_3}{4} h_f^2 m_f \]

\[ - \frac{g_4}{4} \left( m_f (h_i^2) + 2h_f (m_i h_i) \right) - \frac{g_5 + g_6}{2} h_f m_j^2 \]

\[ - g_7 m_f (h_i m_j) - \frac{\kappa}{4} t_{fi} h_i h_j - \kappa_2 t_{fi} h_i m_j, \]

are solved self-consistently with the gap equations which correspond to the minimization of the thermodynamic potential (implicit summation over \( i, j \)).

There are \( 4 + 10 = 14 \) low-energy constants at leading and NLO of the effective 1/\( N_c \) expansion. Let us note that the increase in the number of coupling constants is a common feature of any effective theory approach. The model parameters are fully controlled on the theoretical side by symmetry arguments, completing the number of vertices which contribute at the NLO in 1/\( N_c \) (the same order as the ‘t Hooft determinant), and on the experimental side by the characteristics of the low lying pseudoscalars and scalars at T=0. The number of observables described until now by far surpasses the number of parameters [42]. The multiquark interaction couplings which only break the SU(3) spontaneously are \( G, \kappa, g_1, g_2 \) and refer to the four quark \( (g) \), the ‘t Hooft 6q and two 8q strengths respectively. Of these \( \kappa, g_1 \) are OZI-violating. In the explicit SU(3)-broken sector, the parameters \( \kappa_1, \kappa_2, g_4, g_7, g_8, g_{10} \) are OZI-violating, whereas \( g_3, g_5, g_6, g_9 \) are not. The parameters \( \kappa_1, g_9, g_{10} \) are related to the Kaplan-Manohar ambiguity [49] of this model and are set to zero without loss of generality.

III. RESULTS AND DISCUSSION

A. T – \( \mu \) phase diagram

Fitting the model parameters to properties of the low lying scalars and pseudoscalars (as seen in Table I) the most remarkable feature of the \( \mu – T \) phase diagram, when compared to NJLH and NJLH8 [49, 47, 50], is
the appearance of a second 1st order transition line as seen in Fig. 4. A realistic fit to the spectra appears to imply this feature as it also occurs when using the parameter sets previously reported in [22] (see Table 1). The appearance of this additional line is somewhat surprising as usually an increase in finite current mass terms has the effect of smoothing out the transition behaviour.

We trace back the existence of two critical lines mainly to the ordering \(m_{K^*} < m_{a_0} \approx m_{f_0}\) in the low lying scalar meson spectrum [3]. It has been shown analytically that the parameter \(g_3\) plays a pivotal role in the assignment leading to the empirical masses of these mesons [31, 32]. The coupling \(g_3\) is associated to a non OZI-violating term which counterbalances the flavour mixing 't Hooft interaction: we note, for instance, that by fitting the meson mass spectrum and weak decay constants to the values shown in Table I, but relaxing the constraint for \(m_{K^*}\), a second critical endpoint (CEP) is still present for \(m_{K^*} \approx 953\) MeV (obtained by fixing \(g_3 = -1600\), but an increase to values closer to \(m_{a_0} = 980\) MeV such as \(m_{K^*} \approx 972\) MeV (\(g_3 = -800\) MeV) leads to its disappearance as the additional first order transition changes into a crossover.

As the light and strange sectors are coupled by the OZI-violating interactions, all the quark masses are affected simultaneously by the two transitions (henceforth we will use the subscript \(I/III\) when referring to the one occurring at lower/higher \(\mu\)). Nevertheless a correspondence can be made between the transitions \(I/III\) and light/strange quarks as the chemical potential at which they occur at \(T = 0\) is relatively close to \(M_I(0)\) and \(M_{III}(\mu_{crit})\), respectively, and the jump is highly unequal in intensity (see Figs. 2).

At \(T = 0\) the first order transitions occur at \(\mu_I = 0.319\) GeV and \(\mu_{III} = 0.426\) GeV connecting the phases with baryon number densities, \(\rho = (\rho_u + \rho_d + \rho_s)/3\), given by: \(\{\rho_I^ -, \rho_I^ +\} = \{0, 1.664\}\) \(\rho_0\) and \(\{\rho_{III}^ -, \rho_{III}^ +\} = \{3.984, 5.643\}\) \(\rho_0\) (for the nuclear saturation density we use \(\rho_0 = 0.17\) fm\(^{-3}\); the subscript \(-/+\) refers to low/high density).

For intermediate densities the system can be described by a mixture with a volume fraction, \(0 < \alpha < 1\), occupied by the higher density phase. For densities in the range \(\rho_I^- < \rho < \rho_I^+\), for instance, we expect a partial occupation of the phase by the zero pressure \(\rho_I^+\) phase (with an energy per baryon of \(E/A = 958\) MeV) in equilibrium with the vacuum. Pressure and chemical potential remain constant in this mixed phase regime.

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1 In the NJLH model, two critical lines occur only for unphysically small values of \(\kappa\) and \(m_a \neq m_s\) [3]; otherwise a crossover behaviour for the 2nd transition prevails.

2 In contradiction with this empirical observation the calculated scalar spectrum in the absence of the explicit \(\chi_s\)-breaking interactions usually displays the ordering \(m_{a_0} < m_{K^*} < m_{f_0}\) [24, 33].

### B. \(\beta\)-equilibrium case study

To have a better understanding of the consequences of these two critical lines on the formation of SQM we now focus on charge neutral quark matter at \(T = 0\) subject to \(\beta\)-equilibrium, a case study of relevance for compact stars. Including charge neutrality this results in the conditions:

\[
\begin{align*}
\mu_u &= \mu - \frac{2}{3} \mu_e \\
\mu_d &= \mu_s = \mu + \frac{1}{3} \mu_e \\
0 &= \frac{2}{3} \rho_u - \frac{1}{3} (\rho_d + \rho_s) - \rho_e
\end{align*}
\]

where the average quark chemical potential is given by \(\mu = \frac{1}{3} \sum_i \mu_i\), \(\mu_e\) is the electron chemical potential and \(\rho_i\) denotes number densities (which depend on the chemical potentials). The neutrino chemical potential is discarded as they are considered to escape the system. Muons need not be considered since the electron chemical potential remains below the muon mass in the present study. At \(T = 0\) the total energy density of the system is \(\epsilon = \Omega + \sum_f \mu_f \rho_f\) (where \(f = u, d, s, e\)).

As can be seen in Fig. 3(a) for chemical potential values lower than the dynamical masses the densities are trivially zero but as one increases \(\mu\) the process of chiral restoration is initiated. After the lowest 1st order transition \((\mu_I = 0.325\) MeV) the Fermi sea gets populated by a finite density of up and down quarks, in chemical equilibrium with the electrons, while the density of strange quarks remains null until the highest 1st order transition \((\mu_{III} = 0.409\) MeV). For higher chemical potential the quark densities become comparable (a necessary condition to obtain SQM) while the electron density vanishes.

If we restrict ourselves to the consideration of pure phases these 1st order chiral transitions will give rise to jumps in the densities as a function of \(\mu\): \(\{\rho_I^ -, \rho_I^ +\} = \{0, 1.628\}\) \(\rho_0\) and \(\{\rho_{III}^ -, \rho_{III}^ +\} = \{3.292, 4.958\}\) \(\rho_0\).

The consideration of a mixed phase can however lower the total thermodynamic potential. This happens in the intervals of density: \(0 < \rho/\rho_0 < 1.629\) and \(2.998 < \rho/\rho_0 < 5.476\) (note that these intervals contain the ones corresponding to the jumps when considering pure phases). The main difference is that, with the inclusion of \(\beta\)-equilibrium, the chemical potential and pressure are no longer constant throughout the mixed phase [51] and the discontinuity of \(\rho/\mu\) disappears. One should note, nonetheless, that for each total density (or volume fraction) the pressure and chemical potentials of both phases must be equal for the system to be in mechanical and chemical equilibrium. Global charge neutrality is assured by imposing (note that \(\rho_e\) is constant throughout the vol-
Table I. Model parameters (first row) given in the following units: $[\Lambda] = \text{MeV}$, $[G] = \text{GeV}^{-2}$, $[\kappa_2] = \text{GeV}^{-3}$, $[g_5] = [g_6] = [g_7] = [g_8] = \text{GeV}^{-4}$, $[\kappa] = \text{GeV}^{-5}$ and $[g_3] = [g_4] = \text{GeV}^{-6}$, $[g_1] = [g_2] = \text{GeV}^{-8}$. For their fitting we used the current quark masses values $m_u = 4$ MeV, $m_s = 100$ MeV and the empirical input presented in the second row (meson masses and weak decays in MeV, pseudoscalar and scalar mixing angles, $\theta_{ps}$ and $\theta_{s}$, in degrees). From the self-consistent resolution of the gap equations we obtain $M_u = 375$ MeV, $M_s = 546$ MeV for the constituent quark masses.

$$
\begin{array}{cccccccccccc}
G & \kappa & \kappa_2 & g_1 & g_2 & g_3 & g_4 & g_5 & g_6 & g_7 & g_8 & \Lambda \\
9.834 & -122.9 & 6.189 & 443.6 & 211.0 & -6647 & 1529 & 215.4 & -1666 & 29.81 & -63.20 & 0.8275 \\
\end{array}
$$

Table II. CEPs (position given as $\{\mu, T\}$) and the critical chemical potential, $\mu_{crit}$, at $T = 0$ (sets a, b, c and d from [12]; set e from Table I) for both transitions. All values are given in MeV.

$$
\begin{array}{|c|c|c|c|c|}
\hline
\text{Set} & \text{CEP}_I & \mu_{\text{crit}}^{I}|_{T=0} & \text{CEP}_{II} & \mu_{\text{crit}}^{II}|_{T=0} \\
\hline
a & \{234.5, \ 96.1 \} & 332.0 & \{282.2, \ 109.4 \} & 410.6 \\
b & \{233.4, \ 96.7 \} & 332.0 & \{279.5, \ 110.4 \} & 410.0 \\
c & \{272.1, \ 85.5 \} & 343.6 & \{300.4, \ 109.3 \} & 423.6 \\
d & \{246.0, \ 90.7 \} & 332.8 & \{319.0, \ 107.0 \} & 434.8 \\
e & \{194.2, \ 104.7 \} & 319.1 & \{307.8, \ 106.2 \} & 425.5 \\
\hline
\end{array}
$$

During the mixed phase associated with transition $II$ we see that the density of down quarks almost doubles that of up quarks and we enter the regime seen in the intermediate interval of Fig. 3(a).

In Figs. 3(b) and 3(c) the chemical potential dependence of the number densities of quarks and electrons and of the volume fraction is shown for both transitions. The mixed phase transitions span very different ranges: for transition $I$ it spans 6 MeV (with $\mu_I$ almost as its upper limit) and 25 MeV for transition $II$ (roughly centered in $\mu_{II}$). Furthermore, in the second transition the rise of $\alpha$ is approximately linear along the chemical potential interval whereas in the first transition the rise from 1/100 to 1 is achieved in the last 0.646 MeV of the interval.

In the mixed phase corresponding to transition $I$ we see that in the limit of vanishing $\alpha$ the densities of up and down quarks are very close as one would expect from the $\beta$-equilibrium condition. The difference $\mu_d - \mu_u = \mu_e$ must go to the electron mass for the electron density to become increasingly small (see Eq. [6]): for small $\alpha$ the charge imbalance from a small portion of quark matter is being compensated by a large volume of electron gas (in the low density phase we have $\rho_{e,i} = 0$ for $i = u, d, s$). As $\alpha$ approaches unity the density of down quarks almost doubles that of up quarks and we enter the regime seen in the intermediate interval of Fig. 3(a).

As in the present work we include no description of the interface between phases nor the electrostatic contribution, the Gibbs construction is energetically favourable, nevertheless for completeness we include both approaches in our study.
Figure 1. Phase diagram in the $\mu-T$ plane using the parameter set from Table I, displaying the two transition lines (first order/crossover corresponds to the thick/thin lines) associated with the light and strange quarks (outer refers to the strange) in 1(a). Dynamical mass profiles as a function of temperature at the two chemical potentials indicated in 1(a) by the vertical lines: $\mu = 0.300$ GeV in Fig. 1(b) and $\mu = 0.400$ GeV in Fig. 1(c). The upper two lines are for $M_s$, the lower ones for $M_l$; thick lines represent physical solutions, thin and dashed lines show relative minima and maxima of the thermodynamic potential, respectively; first-order transitions are represented as dashed lines connecting two circles. In these figures, $\mu_u = \mu_d = \mu_s = \mu$.

Figure 2. Dynamical mass of the light quarks in 2(a) and the strange in 2(b) as a function of the chemical potential at the transitions. For the light quarks a much larger jump occurs at the lowest $\mu$ transition whereas for the strange quarks the situation is reversed. Thin lines correspond to the mass at the crossover and the thick lines to the masses in the first order transition (parameter set from Table I with $\mu_u = \mu_d = \mu_s = \mu$). The corresponding temperatures can be extracted from Fig. 1(a).

In Figs. 4(a) the energy per baryon $E/A$ as function of density is displayed. In the first transition mixed phase, as $\alpha$ goes to zero the energy per baryon changes from the $\beta$-equilibrium minimum value (when only considering pure phases the minimum is $E/A = 975$ MeV) to a value close to that without $\beta$-equilibrium ($E/A = 950$ MeV) as one expects from the fact that the densities of up and down quarks become very close (see discussion above). The second mixed phase connects smoothly the points: $\{\rho, E/A\} = \{2.998 \rho_0, 1.027 \text{ GeV}\}$ and $\{\rho, E/A\} = \{5.476 \rho_0, 1.119 \text{ GeV}\}$.

At densities close to $\rho_0$ we have at most a metastable solution (at 1.051 $\rho_0$ and 1.017 $\rho_0$, with and without $\beta$-equilibrium, respectively). This inability of the model to describe bound nuclear matter at the nuclear saturation density is a well known property for the NJL model [53] and can be related to the lack of confinement.

We call however attention to the fact that the onset of stable solutions, after the chiral transition of light quarks, occurs at densities closer to the nuclear density as compared to the values reported in [53], $\rho = 2.8 \rho_0$ in the chiral limit and $\rho = 2.25 \rho_0$ (with $m_u = m_d = 5.5$ MeV, $m_s = 140.7$ MeV of [54]). Also at this point $E/A \approx 975$ MeV, closer to the value of nuclear stability of iron, 930 MeV as compared to the corresponding value in [54], $E/A \approx 1100$ MeV.

A similar situation occurs at the 1st order transition leading to the onset of SQM solutions at values of $E/A$ much closer to nuclear matter stability energies as compared to the values obtained for the NJLH model in [53], where the smooth crossover of $M_s$ leads to values too large to support SQM. By extending the NJLH to in-
Figure 3. Number densities for quarks (u, d and s) and electrons (scaled by a factor of 100) as a function of the chemical potential, $\mu$, are shown in Fig. 3(a) for pure phases. The mixed-phase constructions for both transitions using Gibbs conditions are described in Figs. 3(b) and 3(c) where besides the number densities of the species involved the volume fraction occupied by the emerging phases is displayed (the $-/+\superscript{+}$ superscript refers to the lower/higher density phases). The thin vertical dashed lines refer to the critical chemical potentials of the first order transitions when only pure phases are considered. Note that on 3(b) both the last portion of the line referring to the denser phase volume fraction and the critical chemical potential are extremely close to the right-hand side axis and are as such barely visible.

Figure 4. In Fig. 4(a) the energy per baryon number, $E/A = \epsilon/\rho$, as a function of baryon number density. Below the first order transition occurring at lower chemical potential there are two small regions of local minima solutions for $0 < \rho/\rho_0 < 0.040$ and $1.051 < \rho/\rho_0 < 1.628$ (thin full lines), the first of which is barely visible in the plot, connected by a zone of unstable solutions. The dot-dashed lines refer to the mixed phase constructions, a zoom of which can be seen in Fig. 4(b). The thicker dot-dashed line refers to the Gibbs construction whereas the thinner refers to the Maxwell construction (which only exists in the grey areas). In the interval where they are both defined these constructions lead to lines which almost coincide (apart from the small density limit $\rho \approx 0.5\rho_0$). The state equation (pressure as a function of energy density) is shown in Fig. 4(c). The grey areas correspond to the jumps in density when we restrict the system to pure phases. The line notation (thin, thick and dashed) is the same as in previous figures.

include diquarks, a 1st order transition in $M_s$ also occurs [13], in connection to the 2SC/CFL transition. For the latter case the $E/A$ values have about the same magnitude in the region $\rho > 6\rho_0$ as the ones obtained in the present study.

Despite the fact that certain pertinent aspects of compact stars, such as the effects of strong magnetic fields and rotational effects, have not been taken into account in the present study, one can use the obtained equation of state, EoS (pressure as function of the energy density, $p(\epsilon)$, see Fig. 4(c)), in the integration of the Tolman–Oppenheimer–Volkoff equations [55, 56] to obtain...
the mass and radius of a neutron star as a function of its central energy density (or pressure). The obtained maximum star mass falls short of the recently observed values of about two solar masses [57, 58] but that should not be surprising due to these simplifications (furthermore the presence of more than one first order transition tends to soften the equation of state thus lowering the maximum mass). For this simplified scenario we obtain for the maximal star the following characteristics: a mass and radius of $M_{\text{max}} = 1.521 \, M_\odot$, $R_{\text{max}} = 10.261 \, \text{km}$ (the values reported in the literature using the NJLH derived EoS range from $1.55 \, M_\odot$ to $1.45 \, M_\odot$ with a central pressure of $p_{\text{central}} = 0.128 \, \text{GeV fm}^{-3}$ (which corresponds to a baryonic density $\rho_{\text{central}} = 5.144 \, \rho_0$). This central pressure leads to a core in the mixed phase with a SQM volume occupation fraction of $\alpha = 0.867$. The mixed phase core has a radius and mass of: $R_{\text{core}} = 4.029 \, \text{km}$, $M_{\text{core}} = 0.166 \, M_\odot$ (of which about one third are in the SQM phase, $M_{\text{SQM}} = 0.055 \, M_\odot$).

As was previously mentioned the way the interface between phases in the mixed regime is considered is radically different in the Gibbs and Maxwell constructions. In the latter pressure is constant throughout the mixed phase regime and therefore that part of the EoS does not enter the integration of the TOV equation (a layer in the mixed phase would be squashed to vanishing thickness as no pressure gradient is present). Using the Maxwell construction the largest compact star is the one with a central pressure corresponding to transition II ($R_{\text{max}} = 10.412 \, \text{km}$ and $M_{\text{max}} = 1.544 \, M_\odot$) and as such no stable stars with a SQM core exist in this case.

Regarding the steepness of the EoS, a possible extension of the model, in line with the tower of relevant multi-quark interactions at NLO in $N_c$ counting, consists in the inclusion of the set of spin 1 interactions. It is known that four quark vector interactions stiffen the equation of state, however its onset is delayed, making hybrid stars with a quark core unstable; it was shown in a recent analysis of the SU(2) NJL model with four and eight quark spin 0 and spin 1 interactions that the higher order interactions allow to control stiffness without delaying the onset [01]. The SU(3) extension would provide information on the possibility of achieving the necessary stiffness with this model, while still allowing for SQM at the core of largest mass stars.

IV. CONCLUSIONS

We conclude highlighting two main results:

By including all non-derivative terms relevant at the scale of chiral symmetry breaking (NJLH8m) which are of the same order in $N_c$ counting as the ’t Hooft flavour determinant considered in the 3 flavour extension of the NJL model (NJLH), and using sets of parameters which describe the meson spectra of low-lying pseudoscalar and scalar meson nonets and the weak decay constants to great accuracy, we have obtained the chiral $T - \mu$ phase diagram of the model, which displays two critical lines and respectively two CEP. The second critical line is associated with the strange quark mass which undergoes a first order transition, in which it changes abruptly to values close to its current quark mass in a moderate chemical potential region, $\mu \approx 410 \, \text{MeV}$ at $T = 0$, with strong consequences for SQM.

When compared to previous studies based in the NJLH model the density at which SQM emerges using NJLH8m is lowered to: $\rho \approx 4.0 \rho_0$ in the case of equal quark chemical potentials, $\rho \approx 3.3 \rho_0$ in the case of pure phases in $\beta$-equilibrium and $\rho \approx 3.0 \rho_0$ if we consider a Gibbs constructed mixed phase in $\beta$-equilibrium.

The energy per particle ratio of SQM is much lower than in the NJLH; values of similar magnitude as the ones obtained in NJLH8m are only reached if the NJLH model is enhanced with diquark interactions, as a consequence of a first order transition from the 2SC to the CFL phase.

Our study can be refined to include the diquark interactions, although they cannot affect our central result, i.e. the model’s ability to describe SQM. Diquarks will increase the number of critical points, opening the door for new phases in the region of relatively cold and dense quark matter. One may hope that the combined effect of explicit symmetry breaking interactions and diquarks enhances further the SQM formation, a subject certainly worth studying, but beyond the scope of the present work.

The second main result is that the region for the minimum of quark matter stability ($\rho \approx 1.7 \rho_0$, $E/A = 958 \, \text{MeV}$ for the case with equal quark chemical potentials, $\rho \approx 1.6 \rho_0$, $E/A = 975 \, \text{MeV}$ for the case of pure phases in $\beta$-equilibrium and $\rho \approx 1.7 \rho_0$, $E/A = 959 \, \text{MeV}$ for the denser phase in a Gibbs constructed mixed phase in $\beta$-equilibrium) gets pushed much closer to the point of nuclear matter stability in comparison with other related model calculations.

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