Prediction of the limiting redrawing ratio

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Abstract. Redrawing process is used for reducing the lateral dimensions and increasing the depth of the cylinder cup, which was formed by deep drawing. To determine the optimum values of the process parameters, it is necessary to know the redrawability. In this paper, a new mathematical model is established to predict the limiting redrawing ratio (LDR), which is based on Hill’s anisotropic criteria, tensile instability in the cup wall and sheet bending/unbending theory around the die arc radius. The maximum and minimum thickness ratio \((t_{\text{max}}/t_{\text{min}})\) of cup wall and friction effect were considered in the model. The calculation from the model is in good agreement with the results of finite element simulation, and also was confirmed by the experiments of commercially pure titanium (TA1) cup redrawing processing. The effects of various material properties, friction coefficient \(\mu\), radius of the die arc \(r_d\), half die cone angle \(\alpha\), half die opening \(r_o\), cup maximum thickness \(t_{\text{max}}\), cup thickness ratio \(t_{\text{max}}/t_{\text{min}}\) have been discussed to understand and control redrawing process for improving redrawability.

1. Introduction

Under a certain deep drawing condition, a material has its limiting drawing ratio (LDR). If the depth of product is so large that the drawing ratio exceeds its LDR, the product could not be drawn from a flat sheet once. It must be redrawn several times to reach the needed dimension. To decide a suitable redrawing processing parameter, it is necessary to know the redrawability of the semi-finished product. The LDR, being the ratio of the maximum possible initial blank diameter that can be drawn into a cup to the punch diameter, provides a measure of drawability of sheet metal. In the same way, the ratio of the first cup diameter to the second cup diameter is used as a measure of cup redrawability.

Many researchers have done a great deal of studies to find the relationship between sheet metal drawability (LDR) and material characteristic and processing parameters. Whiteley [1] tested twenty-two drawing materials (sixteen low carbon steels, two stainless steels, two aluminum alloys, commercial pure copper and 65/35 brass), in which the value of normal anisotropy (\(R\)) ranged from 0.6 to 1.6. The results showed that the LDR increased with the value of normal anisotropy. Mir and Hillier [2] found that the LDR is independent of the type of blank-holder. Shawki [3] showed the relationship between the LDR and the cup diameter/sheet thickness ratio, the LDR decreases with increases in the ratio. Taisuke AKAMATSU et al. [4] carried different types of lubrication to draw commercially pure titanium sheet, and showed that the lubrication type affected the LDR greatly. Daw Kwei Leu [5] derived an equation to predict LDR, which based on Hill’s anisotropic yield criteria, and considered the friction effect around the die arc radius. The effect of material properties and processing parameters to LDR was discussed too. Rahul K. Verma et al. [6] listed an more accurate equation, which considered the bending and unbending force in the die arc region and the blank holder force. Though there are many researches on the drawability of sheet metal, a few of studies have been done about the redrewability of cup. In
In this study, a new equation was derived to predict the redrawability of cylinder cup. The equation involves parameters of normal anisotropy $R$, strain hardening exponent $n$, friction coefficient $\mu$, die arc radii $r_d$, die half cone angle $\alpha$, half die opening $r_o$, cup maximum thickness $t_{\text{max}}$, cup thickness ratio $t_{\text{max}}/t_{\text{min}}$. The effect of these parameters on redrawability was analyzed theoretically. Experiments on commercial pure titanium cup were carried out to verify the predicted value.

2. Analysis

The process of cup redrawing was shown in Fig.1, in which a cup with original radius $R_o$, and different wall thickness is deep drawn by the punch through a conical die to gain a new cup with radius $r_o$. The die was designed with half cone angle $\alpha$ and arc radius $r_d$. The left part of figure represents the state before redrawing, and the right part shows the redrawing state. During analysis, the material is assumed with rigid-plastic strain hardening characteristic, also the normal anisotropy of material and the cup wall thickness diversity are considered. Plane strain deformation is assumed in the cone deforming area, where the thickness is not changed. Around the die arc radius, bending and unbending forces were calculated based on the theory of bending and straightening of a sheet pulled around the die arc, and the friction between cup and die is calculated.

In the deep redrawing process, the plastic behaviour also varies with direction. It is assumed that there is planer isotropy, and the average value of the normal anisotropy is given as,

$$ R = \frac{R_0 + 2R_{45} + R_{90}}{4} $$

Where $R_0$, $R_{45}$ and $R_{90}$ denote the normal anisotropy at 0°, 45° and 90° respectively.

In order to simplify calculation, material is assumed to be follow Hill’s anisotropic theory [7]. The equivalent stress $\sigma_e$ and the equivalent incremental strain $d\varepsilon_e$ can be written with the function of directional stresses, strains and average value of normal anisotropy.

2.1. The critical load in the cup wall

In the redrawing process, similarly with simple tension, if the load exceeds the necking point, the cup wall would be instable and failure. Assuming there is no contact between the cup and the punch in redrawing, so there is no frictional force to assist in reducing the redrawing load. The critical load for LDR can be obtained by maximizing the redrawing force without the onset of instability.

Since the original cup was deep drawn, the cup wall thickness varies along center axis. The minimum thickness ($t_{\text{min}}$) is around the bottom of the cup, and the maximum thickness ($t_{\text{max}}$) is at the top brim. The redrawing instability occurs at the part of minimum thickness first, so the critical load expression should be revised as:
2.2. Drawing in the die arc region

As cup wall moving round the die arc radius, there exist two types of force. The first one is bending/unbending force, and the other is friction at die-cup wall interface. If assuming the bending force is equal to straightening force, and denoted by \( F \). The drawing stress after bending and before straightening is written with equation \[6\]. After substituting, the drawing stress at \( r_1 \) can be gained by:

\[
P_e = 2\pi r_o t_{\text{min}} \left( \frac{1 + \frac{R}{1 + 2R}}{\sqrt{1 + 2R}} \right)^{1+n} \left( n^* e^{-n K} \right) \tag{2}\]

\[\sigma_t = K \left( \frac{1 + \frac{R}{1 + 2R}}{\sqrt{1 + 2R}} \right)^{1+n} \left( \frac{n}{e} \right)^n \left( \frac{r_o}{r_i} \right)^{e^{(\alpha - \sigma)\mu} - \frac{F}{2\pi r_i t_o} \left( e^{(\alpha - \sigma)\mu} + 1 \right)} \tag{3}\]

Using the model of sheet metal pulled around the die arc, the bending and straightening force is given by the principle of the equation between sheet bending work and internal work. The bending force can be written as:

\[
F = 2\pi r_o K n^* \frac{t^2}{2r_o + t} \tag{4}\]

2.3. Drawing in the cone cup region

Taking out a small element in the conical deformation area, as shown in Fig.1. Based on the plane strain deformation condition, the relationship between the meridional stress \( \sigma_\rho \) and the circumferential stress \( \sigma_\theta \) could be gotten. According the rigid-plastic strain-hardening material, the equivalent stress \( \sigma_e \) is the function of the equivalent strain \( \varepsilon_e \). Considering the condition of incompressibility and plane strain deformation, the equivalent strain can be calculated. Finally, the radial drawing stress at the position \( r_1 \) can be obtained as:

\[
\sigma_{r_1} = -\frac{1}{2} \frac{K (A + 1)}{A} \left[ \ln \left( \frac{r_o}{R_o} \right) \right]^* \left[ 1 - \left( \frac{r_o}{R_o} \right)^d \right] \tag{5}\]

Where \( A = \mu \cot \alpha \). Since the conical die is adopted in cup redrawing process, the radii \( r_1 \) is close to cup radii \( r_o \), \( r_i \approx r_o \). Combining Eq. (3), Eq. (4) and Eq. (5), the expression predicting the limiting redrawing ratio (LDR) can be written:

\[
f(LDR) = K \frac{t_{\text{min}}}{t_{\text{max}}} \left( \frac{1 + \frac{R}{1 + 2R}}{\sqrt{1 + 2R}} \right)^{1+n} \left( \frac{n}{e} \right)^n \left( \frac{r_o}{r_i} \right)^{e^{(\alpha - \sigma)\mu} - \frac{F}{2\pi r_i t_{\text{max}}} \left( e^{(\alpha - \sigma)\mu} + 1 \right)} + \frac{1}{2} K \frac{(A + 1)}{A} \left[ \ln \left( \frac{1}{LDR} \right) \right]^* \left[ 1 - \left( \frac{1}{LDR} \right)^d \right] = 0 \tag{6}\]

Where LDR is denoted by: \( LDR = \frac{R_o}{r_o} \).

The Eq. (6) for calculating redrawing LDR is solved by the graphing method in the paper.

3. Experiment, results and discussion

In the present study, experiments on the redrawing of commercially pure titanium (TA1) cup were carried to verify the prediction of the Eq. (6). The experimental TA1 cups (Fig.2.a) were deep drawn from a circular flat-plate with the radii of 85mm.

The mechanical properties of TA1 sheet are shown in Table 1. Test pieces were taken from Longitudinal (L), Diagonal (X) and Transverse (C) directions. The original radii of TA1 deep drawn cup is 82mm(Fig.2a), and the wall thickness changes from centre to the edge. The minimum thickness
is 2.85mm, and the maximum thickness is 3.85mm. Conical dies were adopted in redrawing experiments with the half die cone angle 30°, opening radii 28.3mm and 33mm, and die arc radius 8mm. The machine oil lubricant was used with the friction coefficient about 0.15.

### Table 1 Mechanical properties of TA1, Average value=(L+2X+C)/4

| Sample direction | σ₀.₂ (MPa) | σ₆₅ (MPa) | Total Elongation (%) | n value | K value | r value | Strain energy density |
|------------------|------------|------------|----------------------|---------|---------|---------|-----------------------|
| Longitudinal (L) | 238.22     | 345.35     | 51                   | 0.281   | 658     | 1.407   | 1.44×10⁸              |
| Diagonal (X)     | 265.77     | 340.06     | 47.5                 | 0.147   | 516     | 1.95    | 3.28×10⁷              |
| Transverse (C)   | 276.75     | 344.34     | 42                   | 0.112   | 486     | 2.17    | 3.19×10⁷              |
| Average value    | 261.63     | 342.45     | 47                   | 0.172   | 544     | 1.87    | 6.03×10⁷              |

The prediction from Eq. (6) for the experiment is that redrawing LDR=1.365, and the experimental redrawing ratio (DR) are 1.2 and 1.44. The drawn products are shown in Fig.2b and Fig.2c. The product drawn with DR=1.44 was broken at the bottom, while the product drawn with DR=1.2 was undamaged. So the Eq. (6) is appropriate to estimate cup redrawability, and the correlation amongst the LDR and the various process parameters of redrawing can be well studied.

![Fig.2. The drawn cups: (a) original cup, (b) redrawn with DR=1.44, (c) redrawn with DR=1.2](image)

#### 3.1. Finite element simulation

Using The Comsol multiphysics finite element software, the simulation parameters are the same as the test parameters are set, and the pull-depth ratio is established in the simulation two axis symmetry models, which are DR 1.2 and DR 1.44, respectively. The pull-deep part is set as a catapult material, the plastic stage hardening model is isometric, and the flow stress and the equivalent plastic strain conform to the linear relationship, and the material performance parameters in the simulation process are shown in Table 2.

### Table 2 Drawn cup material and properties

| Material and properties | Material and properties |
|-------------------------|-------------------------|
| Material                | TA1 commercial pure titanium |
| Young’s modulus         | 107 GPa |
| Poisson ratio           | 0.33      |
| Yield strength          | 260 MPa |
| Isotropic tangent modulus | 0.48 GPa |

During the simulation, the part was divided into 3083 mesh, and the mesh density of the contact area plate is 2 times the density of the die mesh. The fracture in the redrawing process is a toughness fracture, which is related to the stress and strain. In this section of finite element simulation, the strain energy density was set as the criterion for determining fracture, with the one-way stretch test to determine the material just occurred in the case of local instability strain energy density as the limit value. If the strain energy is greater than the limit value, a break would happen.
Through the simulation of finite elements, we can learn the deformation process of the cylinder part, as shown in Fig. 3a, in the process of redrawing, the strain energy density of the cylinder piece is the largest at the bottom part, most likely to break. As the travel of the punch increases in Fig. 4b, the strain can have a negative value at a radius of 20mm, indicating that the material in this area is under compressive stress. As can be seen from Fig. 3b-c, in order to pull the depth ratio of DR=1.44, when the punch at h=30mm, the strain energy density at the bottom part is greater than the limit value, rupture occurs, and the pull depth ratio of DR=1.22, even if the punch at h=90mm, the strain energy density of each part is much less than the limit value. This is the same as the test result, when the bottom break occurs when DR is 1.44, and the redrawing process run smoothly when the DR is 1.22. Therefore, through finite element simulation, the crack criterion can be determined by strain energy density, and the cup redrawing forming performance can be effectively simulated.

3.2. Discussion of the parameters on cup redrawability

In the Eq. (6), the cup redrawability measure LDR is a function of material properties and various process parameters. Based on the material properties of TA1, the effects of these parameters are studied theoretically.
Fig. 4. Parameter’s effect on cup redrawability: (a) normal anisotropy, (b) strain hardening exponent, (c) coefficient of friction, (d) half die cone anger, (e) die arc radius, (f) cup thickness ratio, (g) half die opening $r_0$, (h) cup maximum thickness.
Fig 4a shows the effect of the normal anisotropy value R on the LDR. As the relationship between first deep drawability and R [5,6], the LDR increases linearly with the R-value increasing. The increment rate is different with different value of friction coefficient. The high the friction coefficient, the high increment rate.

Fig.4b shows the effect of strain hardening exponent n on the redrawability. It can be seen that the LDR decreases slowly with the n increasing, and n has relatively little effect on the LDR.

Fig.4c shows the effect of the friction coefficient on the LDR. With the μ increasing from 0.05 to 0.25, the LDR decreases almost linearly, which is the same as the relationship between the first deep drawability and μ [5, 6].

Fig.4d shows the effect of the half die cone angle α on LDR. It can be observed, from the curves, that the LDR decreases rapidly with the α range from 10° to 20°, and decreases slowly with the α from 20°to 50°. So the small die cone angle α can improve the LDR for different R.

Fig.4e shows the effect of the die arc radius rd on the LDR. In the figure, the LDR increases rapidly with the rd range from 5 and 15, and raises slowly with the rd from 15 to 40 for various value of R. So the rd must be large in order to increase the LDR.

Fig.4f shows the effect of the cup thickness ratio tmax/tmin on the LDR. On the contrary, the effect of the R on LDR, it can be seen that the LDR decreases linearly with the ratio increasing. So the uniform thickness cup is the optimal semi-product for redrawing.

Fig.4g shows the half die opening r0 on the LDR. In the figure, the curve is almost a horizon line for various of R. It is concluded that the r0 has little effect on the LDR.

Fig.4h shows the effect of the thickness tmax on the LDR. It can be seen that the LDR reduces linearly with the increasing tmax. This may be because of that the bending/unbending force increases with the thickness increasing.

4. Conclusions
The equation for predicting the cup redrawability (LDR) is established, which is the function of the normal anisotropy R, strain hardening exponent n, friction coefficient μ, die arc radii rd, die half cone angle α, half die opening r0, cup maximum thickness tmax, cup thickness ration tmax/tmin. The estimation from the present equation agrees well with the experimental results. The equation can be used to optimize the tool design and determine the process parameters.

It is explained in the influencing parameters discussion that all the parameters have great effects to the LDR except for the strain hardening exponent n and the half die opening r0. The LDR increases linearly with the R-value, and decreases almost linearly with the μ-value, which are similar trends for the first deep drawability. Alike the effect of the μ-value, the LDR decrease with the tmax and tmax/tmin increase. The LDR reduces rapidly for little α-value and reduces slowly for large α-value. Reversely, the LDR grows rapidly for little rd-value, and grows slowly for large rd-value.

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