Threshold Corrections to the Bottom Quark Mass Revisited

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Abstract

Threshold corrections to the bottom quark mass are often estimated under the approximation that \(\tan \beta\) enhanced contributions are the most dominant. In this work we revisit this common approximation made to the estimation of the supersymmetric threshold corrections to the bottom quark mass. We calculate the full one-loop supersymmetric corrections to the bottom quark mass and survey a large part of the phenomenological MSSM parameter space to study the validity of considering only the \(\tan \beta\) enhanced corrections. Our analysis demonstrates that this approximation severely breaks down in parts of the parameter space. The size of the threshold corrections has significant consequences for the estimation of fits to the bottom quark mass, couplings to Higgses, and flavor observables, and therefore the approximate expressions must be replaced with the full contributions for accurate estimations.

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1 Introduction

The supersymmetric (SUSY) threshold corrections to the bottom quark mass in the large tan$\beta$ regime are often expressed as an approximation of the dominant gluino-sbottom and chargino-stop loop contributions [1–3],

$$(\Delta m_b/m_b)_{\text{app}} = \frac{8}{3} \frac{g_3^2}{16\pi^2} M_{\tilde{g}} (\mu \tan \beta - A_b) I(M_{\tilde{g}}, m_{b_1}, m_{b_2}) + \frac{\lambda^2}{16\pi^2} \mu (A_t \tan \beta - \mu) I(\mu^2, m_{t_1}, m_{t_2}). \tag{1}$$

In order to fit the bottom quark mass, $m_b(M_Z)^{\text{SM}} = m_b(M_Z)^{\text{MSSM}} (1 + (\Delta m_b/m_b))$, where $m_b(M_Z)^{\text{MSSM}}$ is obtained from the evolution of the bottom Yukawa coupling from a UV scale (such as the GUT scale) to the $M_Z$ scale. The effects of these supersymmetric threshold corrections are important especially in the era of precision Higgs couplings and flavor physics and has been a part of many analysis. For some recent work, see [4–11].

Let us first summarize some of the well-known consequences of the above expression for a common type of model that has large tan$\beta$ such as models with third family Yukawa unification. In such models, the threshold corrections typically need to be $O(\text{few\%})$ and negative. These corrections can often be large thus the two terms in Eq. (1) must either nearly cancel or both be suppressed. For $\mu > 0$ and tan$\beta \approx 50$, $A_t$ must be large and negative in order for the two contributions to approximately cancel and yield a negative value. This in turn has consequences for flavor physics. The branching ratio for $B_s \to \mu^+ \mu^-$ receives large tan$\beta$-enhanced contributions from Higgs-mediated neutral currents that are proportional to $A_t^2 (\tan \beta)^6/M_A^4$ [12, 13]. In order to be in agreement with the experimental value which is $O(10^{-9})$, $M_A$ must be large if $A_t$ is large. An important constraint to then consider is the inclusive decay $B_s \to X_s \gamma$ to which the dominant SUSY contributions are a chargino-stop loop and a top-charged Higgs loop [14–16]. The chargino contribution is tan$\beta$-enhanced and, with large and negative $A_t$, adds destructively to the SM branching ratio. The charged Higgs contribution, on the other hand, adds constructively to the SM branching ratio, but is suppressed by the heavy Higgs masses required to be consistent with $B_s \to \mu^+ \mu^-$. Since the SM prediction is in good agreement with the data, these two contributions must nearly cancel. Such a cancellation is difficult to obtain in the given region of parameter space and one is then led to consider heavy scalars [17].

The situation is different for $\mu < 0$ since the gluino contribution, which is the dominant contribution, already has the needed sign. In this case, the parameters need not be large in order to obtain a small threshold correction. This region of parameter space however was initially disfavored due to conflicts with flavor physics. When $\mu < 0$, the chargino contributions add constructively with the SM contributions to the $B(B_s \to X_s \gamma)$ observable and hence yield enhanced values [16, 18–20]. Additional complications also arise due to tensions with the $(g - 2)_\mu$ observable in this regime, where the theoretical prediction is too small to match the experimental value. More recently, viable models with $\mu < 0$ have been constructed but they typically have squark masses greater than 1 TeV [21–23].

Fitting the bottom quark mass and satisfying current experimental constraints from flavor physics has therefore pushed Yukawa unified models into the territory of heavy scalars. Other models may
of course be constructed that evade such restrictions but the absence of the detection of any new physics at the LHC generically requires one to consider heavy scalar masses. The current limits on the colored superpartner masses are already approaching the TeV range \[24, 25\]. As we transition into the TeV region of the SUSY parameter space, a re-evaluation of the approximations of SUSY threshold corrections to the bottom quark is warranted. This is especially important in the era of precision physics since the approximation is often invoked in studies of bottom quark mass and couplings.

In order to understand the size and behavior of the threshold corrections to the bottom quark, we survey a large part of the parameter space of interest and choose to scan over the parameters of the pMSSM instead of restricting ourselves to a particular model. For each point, we calculate both the full, exact one-loop radiative corrections to the bottom quark and compare with the value obtained from the approximate form of the corrections as given in Eq. (1). For each point in the pMSSM scan, we additionally check the Higgs mass and constraints from \(B_s \rightarrow X_s \gamma\) and \(B \rightarrow \mu^+ \mu^-\).

This paper is organized as follows. The details of the parameter scan are presented in Section 2. In Section 3, we present the full, exact one-loop corrections compared to the approximate form of the contributions and motivate the need for a scrutiny of this approximation. We then consider in turn each approximation made to the individual contributions to the threshold correction in Section 4. We return to the comparison of the full, exact one-loop expression and the approximate form in Section 5 to discuss the regions of parameter space in which the approximation breaks down. Section 6 surveys the consequences of using the full expression of the threshold corrections to the bottom quark. Finally, we conclude in Section 7.

\section{Parameter Scan}

The pMSSM parameter space is defined by \[\{m_{Q_i}, m_{u_i}, m_{d_i}, m_{L_i}, m_{e_i}, A_i, M_i, M_A, \mu, \tan \beta\}\] with the family index \(i = 1-3\). We consider the inter-generational mixing to be negligible and that the masses of the first two family scalars are large relative to the third family scalar masses. We therefore ignore contributions to the bottom quark mass from the first two families. For most of this analysis, we fix \(\tan \beta = 50\) in which region the SUSY threshold corrections are dominant.\(^1\) We also briefly study the size of the threshold corrections for smaller values of \(\tan \beta\). The ranges for the remaining SUSY parameters are given in Tab. 1. With these parameter bounds, we randomly generate 50,000 points. We then use micrOMEGAs [26] to calculate the quantities \(m_h\), \(B(B_s \rightarrow \mu^+ \mu^-)\), and \(B(B_s \rightarrow X_s \gamma)\). Only points for which these quantities satisfy current experimental bounds are retained.

For the Standard Model parameters, we use the measured values of the top quark, \(W\), \(Z\), and Higgs masses. Note that we use \(m_h = 125.3\) GeV for all points when calculating threshold corrections. After running through micrOMEGAs, the points that survive all have a Higgs mass within 3 GeV of this value. This is at most a \(\sim 2\%\) difference. Furthermore, the Higgs mass only occurs in the calculation of the neutral Higgs contribution. The error in this approximation is therefore negligible and the results remain unaffected. For the bottom quark mass, we use the RunDec package [27] to run \(m_b(m_b)\) to \(m_b(M_Z)\).

\(^1\)With \(\tan \beta = 50\), third family Yukawa unification can also be satisfied.
\[ g_1 = 0.46 \quad g_2 = 0.64 \quad g_3 = 1.2 \]
\[ M_t = 173.36 \quad m_b = 2.69 \quad V_{tb} = 1 \]
\[ M_Z = 91.1876 \quad M_W = 80.385 \quad m_h = 125.3 \]
\[ v = 246 \quad \tan \beta = 50 \]

\[
\begin{align*}
1000 &< \{m_{Q_3}, m_{u_3}, m_{d_3}\} < 5000 \\
100 &< \{m_{L_3}, m_{e_3}\} < 5000 \\
-15000 &< \{A_1, A_b\} < 15000 \\
-1000 &< \{M_1, M_2\} < 1000 \\
500 &< M_3 < 2000 \\
1000 &< M_A < 2000 \\
-2000 &< \mu < 2000 \\
\end{align*}
\]

Table 1: Parameter values and ranges at \( M_Z \).
All masses in GeV.

3 Exact vs. Approximation

![Plot](image)

Figure 1: The plot shows the full, exact one-loop threshold corrections to the bottom quark mass vs. the approximate form of the correction given in Eq. (1). Darker shades of blue represent increasing squark masses from 1 TeV to \( \geq 4 \) TeV. The diagonal line represents where the exact and approximate forms would be equal. As the squark masses increase, the approximations begin to worsen. There are also two outer “islands” where the approximate form seems to break down drastically.

The complete set of one loop corrections to the bottom quark mass is given by [28]

\[
\Delta m_b(M_Z) = \Delta m_{\tilde{g}} + \Delta m_{\tilde{t}^\pm_b} + \Delta m_{\tilde{\chi}_0^\pm_b} + \Delta m_{H^\pm_b} + \Delta m_{A_b} + \Delta m_{h_b} + \Delta m_{W_b} + \Delta m_{Z_b},
\]

(2)
with the tree level mass given by \( \lambda_b(M_Z) \frac{v}{\sqrt{2}} \cos \beta \).

In Fig. 1, we present the results of the parameter scan by plotting the full, exact one-loop threshold corrections to the bottom quark mass against the approximate form of the corrections given in Eq. (1). The color gradient represents squark masses from 1 TeV at the lightest to \( \geq 4 \) TeV at the darkest. The diagonal line represents the points for which the approximate form would be equal to the exact result. As the squark masses increase, the points draw near to the vertical line, showcasing the worsening of the approximate form in this limit. Also, there are two outer “islands” where the approximate form seems to break down drastically.

4 Individual contributions

4.1 Gluino-Sbottom

We look first at the approximation made to the gluino-sbottom contribution. Gluinos couple to the down-type squarks and quarks proportional to the SU(3) gauge coupling \( g_3 \) and hence contribute large corrections to the bottom quark mass. The corrections are dominant when the squarks belong to the third family since the inter-generational mixings between the squarks are typically (and by assumption in this study) small. The detailed calculation can be found in the appendix. We quote the final, exact form here \[28\].

\[
\Delta m_\tilde{b}^g = \frac{8}{3} \frac{g_3^2}{16\pi^2} \left[ \sin 2\theta_b M_\tilde{g} \left( B_0 \left( p, M_\tilde{g}, m_\tilde{b}_1 \right) - B_0 \left( p, M_\tilde{g}, m_\tilde{b}_2 \right) \right) - \frac{m_b}{2} \left( B_1 \left( p, M_\tilde{g}, m_\tilde{b}_1 \right) + B_1 \left( p, M_\tilde{g}, m_\tilde{b}_2 \right) \right) \right], \quad (3)
\]

where the momentum of the bottom quark is given by \( p \). In the limit \( p \to 0 \) (which is a good assumption here since \( p^2 = m_b^2 \)), the Passarino-Veltman functions can be written as

\[
B_0(0, M_\tilde{g}, m_\tilde{b}) = -\ln \left( \frac{m_b^2}{Q^2} \right) + 1 + \left( \frac{1}{1 - x} \right) \ln x \quad (4)
\]

\[
B_1(0, M_\tilde{g}, m_\tilde{b}) = \frac{1}{2} \left[ -\ln \left( \frac{m_b^2}{Q^2} \right) + \frac{1}{2} + \frac{1}{1 - x} + \frac{\ln x}{(1 - x)^2} - \theta(1 - x) \ln x \right] \quad (5)
\]

where \( x = m_b^2/M_\tilde{g}^2 \). The first term in the above expression simplifies to

\[
\frac{\sin 2\theta_b M_\tilde{g}}{2} \left[ B_0 \left( p, M_\tilde{g}, m_\tilde{b}_1 \right) - B_0 \left( p, M_\tilde{g}, m_\tilde{b}_2 \right) \right] = \frac{\sin 2\theta_b M_\tilde{g}}{2} \left[ \ln \left( \frac{m_b^2}{m_{\tilde{b}_1}^2} \right) + M_\tilde{g}^2 \left( \frac{1}{M_\tilde{g}^2 - m_{\tilde{b}_1}^2} \ln \left( \frac{m_{\tilde{b}_2}^2}{m_{\tilde{b}_1}^2} \right) \right) - \frac{1}{M_\tilde{g}^2 - m_{\tilde{b}_2}^2} \ln \left( \frac{m_{\tilde{b}_2}^2}{M_\tilde{g}^2} \right) \right]. \quad (6)
\]

The angle \( \sin 2\theta_b \) can be determined to be

\[
\sin 2\theta_b = \frac{2m_b(\mu \tan \beta - A_b)}{\sqrt{\left( m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2 \right)^2 + (2m_b(\mu \tan \beta - A_b))^2}} = \frac{2m_b(\mu \tan \beta - A_b)}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2}, \quad (7)
\]
where we have ignored terms proportional to $M_Z$ or $m_b$. The trilinear coupling $A_b$ is often ignored since $\mu$ is enhanced by $\tan \beta$. Similarly, the second term in Eq. (3) is also neglected. Collecting terms, we arrive at the form in Eq. (1),

$$\frac{\Delta m_{\tilde{g}}}{m_b} \approx \frac{8}{3} \frac{g_s^2}{16\pi^2} M_{\tilde{g}} (\mu \tan \beta - A_b) I(M_{\tilde{g}}^2, m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2),$$

where

$$I(a, b, c) = \frac{ab \ln \left(\frac{a}{b}\right) + bc \ln \left(\frac{b}{c}\right) + ac \ln \left(\frac{c}{a}\right)}{(a - b)(b - c)(c - a)}.$$  (9)

This is the expression that is typically used in most of the literature with large $\tan \beta$ models. It is therefore necessary to discuss the validity of ignoring the second term in Eq. (3).

We refer to the term containing the $B_{0(1)}$ Passarino-Veltman functions and its prefactor as the “$B_{0(1)}$” term. In Fig. 2, the $B_0$ term is plotted against the $B_1$ term. The color gradient from light to dark represents increasing sbottom masses from 1 TeV to $\geq 4$ TeV. As the sbottom masses get pushed toward more than a few TeV, the two terms are nearly the same magnitude. Thus, in this regime it is important that the $B_1$ term not be ignored. Furthermore, the points along the vertical line have $(\mu \tan \beta - A_b) \approx 0$ and so one must be careful to check the size of $A_b$ relative to $\mu \tan \beta$.

Figure 2: We plot the $B_0$ term against the $B_1$ term ($B_0$ and $B_1$ are defined in the text). Darker shades represent increasing sbottom masses. As the sbottom masses increase from 1 TeV to $\geq 4$ TeV, the $B_0$ terms becomes smaller and the two terms are nearly the same magnitude. Furthermore, the points along the vertical line have $(\mu \tan \beta - A_b) \simeq 0$.

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\footnote{We keep $A_b$ here in order to be consistent with the definitions of the squark masses.}
4.2 Chargino-Stop

We turn now to the approximation made to the chargino-stop contribution. The charginos couple to the up-type squarks and down-type quarks proportional to the SU(2) coupling $g_2$ and the Yukawa couplings $\lambda_{t,b}$ with strength depending upon their respective wino-higgsino composition. The corrections dominate when the squarks are from the third family due to CKM suppression of the contributions from the first two families of squarks.

The chargino-stop contribution is calculated in detail in the appendix. The exact closed form cannot be put into a simplified form as was the case for the gluino-sbottom contribution. This is due to the non-trivial convolution of the elements of the stop mixing matrix, the elements of the chargino mixing matrices, and the weak and Yukawa coupling constants obtained by summing over the left and right stops and the two charginos. We list the exact results from the appendix and discuss the approximations made to obtain the form in Eq. (1).

The full expression is [28]

$$\Delta m_b^{\pm} = \sum_{i=1}^{2} \sum_{x=1}^{2} B_{LR_i}^x + \frac{m_0}{2} (A_{L_i}^x + A_{R_i}^x)$$

with

$$B_{LR_i}^x = -\frac{\Phi_i^x \Phi_i^x M_{\tilde{\chi}_{i^x}}}{16\pi^2} B_0(p, M_{\tilde{\chi}_{i^x}}, m_{\tilde{t}_x})$$

$$A_{L_i}^x = -\frac{(\Phi_i^x)^\dagger \Phi_i^x}{16\pi^2} B_1(p, M_{\tilde{\chi}_{i^x}}, m_{\tilde{t}_x})$$

$$A_{R_i}^x = -\frac{(\Phi_i^x)^\dagger \Phi_i^x}{16\pi^2} B_1(p, M_{\tilde{\chi}_{i^x}}, m_{\tilde{t}_x}) .$$

(11)

Here $i = 1, 2$ is the chargino index and $x = 1, 2$ is the stop index.

The couplings are given by

$$\Phi_i^x = \frac{\lambda_t \sqrt{2}}{v_2} \Gamma_{L_i}^x \dagger - g_2 V_{i1}^\dagger (\Gamma_{R_i}^x)^\dagger$$

$$\bar{\Phi}_i^x = \frac{\lambda_b \sqrt{2}}{v_2} U_{i2}^\dagger \Gamma_{L_i}^x ,$$

(12)

where $U, V$ are the chargino mixing matrices and $\Gamma_{L,R}$ are the columns of the stop mixing matrix. The momentum of the top quark is given by $p$.

The $B_1$ terms are often neglected and so we focus on the $B_{LR_i}^x$ contributions. Setting $p = 0$ and expanding these terms,

$$B_{LR_i}^x = -\frac{\Phi_i^x \Phi_i^x M_{\tilde{\chi}_{i^x}}}{16\pi^2} B_0(0, M_{\tilde{\chi}_{i^x}}, m_{\tilde{t}_x})$$

$$= -\frac{M_{\tilde{\chi}_{i^x}}}{16\pi^2} \left[ \frac{\lambda_b \sqrt{2}}{v_2} U_{i2}^\dagger \Gamma_{L_i}^x \right] \left[ \frac{\lambda_t \sqrt{2}}{v_2} V_{i2}^\dagger (\Gamma_{R_i}^x)^\dagger - g_2 V_{i1}^\dagger (\Gamma_{L_i}^x)^\dagger \right] B_0(0, M_{\tilde{\chi}_{i^x}}, m_{\tilde{t}_x}) .$$

(13)
Neglecting terms proportional to $g_2$ and summing over the stops and charginos yields

\[
\sum_{i=1}^{2} \sum_{x=1}^{2} B_{LR_i}^x \simeq -\frac{M_{\tilde{b}_i}}{16\pi^2} \left[ \lambda_b \lambda_t U_{12}^T V_{12}^T \frac{\sin 2\theta_t}{2} \right] \left[ B_0(0, M_{\tilde{\chi}^+_1}, m_{\tilde{t}_1}) - B_0(0, M_{\tilde{\chi}^+_1}, m_{\tilde{t}_2}) \right] + -\frac{M_{\tilde{b}_x}}{16\pi^2} \left[ \lambda_b \lambda_t U_{22}^T V_{22}^T \frac{\sin 2\theta_t}{2} \right] \left[ B_0(0, M_{\tilde{\chi}^+_2}, m_{\tilde{t}_1}) - B_0(0, M_{\tilde{\chi}^+_2}, m_{\tilde{t}_2}) \right].
\]

Again ignoring terms proportional to $g_2$, one finds that $U_{12}^T V_{12}^T \simeq 0$ and $U_{22}^T V_{22}^T \simeq 1$. Furthermore, $\sin 2\theta_t = 2\lambda_t v_d \tan \beta (A_t - \mu \tan \beta)/(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)$ so that\(^3\)

\[
\Delta m_{\tilde{b}_i} \simeq -\frac{M_{\tilde{\chi}^+_1}}{16\pi^2} \left[ m_b \lambda^2_t (A_t \tan \beta - \mu) \right] \left[ B_0(0, M_{\tilde{\chi}^+_1}, m_{\tilde{t}_1}) - B_0(0, M_{\tilde{\chi}^+_1}, m_{\tilde{t}_2}) \right] \Rightarrow \frac{\Delta m_{\tilde{b}_i}}{m_b} \simeq \frac{\lambda^2_t}{16\pi^2} M_{\tilde{\chi}^+_1} (A_t \tan \beta - \mu) I(M_{\tilde{\chi}^+_1}, m_{\tilde{t}_1}, m_{\tilde{t}_2}). \tag{15}
\]

Finally, $M_{\tilde{\chi}^+_1}$ is often approximated as $\mu$ and we arrive at the form of Eq. (1). We must then address the validity of ignoring terms proportional to $g_2$ and also taking $M_{\tilde{\chi}^+_1}$ to be $\mu$. These approximations are in essence assumptions about the wino-higgsino composition of the charginos. Namely, they are assumptions about the relative size of the higgsino mass parameter $\mu$ compared to the wino mass parameter $M_2$. We can therefore determine the validity of the approximate form for the chargino contribution by comparing the exact form to the common approximate expression in different regions of the $\mu$-$M_2$ parameter subspace.

We present first the comparison of the exact and approximate forms without setting a relation between $\mu$ and $M_2$. Fig. 3 shows that this comparison yields three “islands”. The center island along the diagonal line contains points for which the approximate form is nearly exact. The approximate form is clearly poor for points in the outer two islands. The task at hand is then to determine what parts of the $\mu$-$M_2$ parameter subspace are represented by each island.

The two cases $|\mu| < |M_2|$ and $|\mu| \gg |M_2|$ are shown in Fig. 4. For $|\mu| < |M_2|$, only the outer islands remain. This is to be expected since the approximation above assumes $M_{\tilde{\chi}^+_1} = \mu$, i.e., $|M_2| < |\mu|$. When $|\mu|$ is greater than about $2|M_2|$, the lightest charginos are sufficiently wino-like and only the center island along the diagonal remains. Thus the approximation for the chargino contribution becomes increasingly accurate as the relation $M_{\tilde{\chi}^+_1} = \mu$ becomes increasingly true. In other words, the larger $|\mu|$ is than $|M_2|$, the closer the approximate value of the chargino contribution is to the exact value.

4.3 $W$, $Z$, Higgses, and Neutralinos

Due to weaker coupling strengths compared to $g_3$ and $\lambda_t$, the contributions to the threshold correction of the bottom quark mass from $W$, $Z$, Higgses, and neutralinos are often neglected. It is

\(^3\)The $\mu/\tan \beta$ term is often neglected. We keep it here however in order to be consistent with the definitions of the squark masses.
Figure 3: The plot shows that the comparison between the exact and the approximate chargino expressions yields three “islands”. The center island along the diagonal line contains points for which the approximate form is nearly exact. In contrast, the approximate form for points in the outer two islands is quite poor.

Figure 4: The chargino contributions in two cases: $|\mu| < |M_2|$ and $|\mu| \gg |M_2|$. For $|\mu| < |M_2|$, only the outer islands remain. When $|\mu|$ is greater than about 2$|M_2|$, the lightest charginos are sufficiently wino-like and only the center island remains.

possible that while the gluino and chargino contributions may each be of much greater magnitude than these other contributions, a cancellation occurs such that their sum is of the same magnitude as the other contributions. Since these terms are dropped altogether, the validity of this approxi-
mation is simply based on the magnitude of their contribution compared to the total approximate correction as given in Eq. (1). Fig. 5 shows the relative size of these quantities. The correction from the collective contribution of the $W$, $Z$, Higgses, and neutralinos is fairly constant at $\sim$few% while the total approximate correction ranges from $\sim$-40% to $\sim$40%. As squark masses increase, the points gravitate toward the vertical line at which the total approximate correction is 0. Thus the validity of neglecting these other contributions begins to break down as squark masses get pushed to a few TeV.

![Figure 5](image.png)

Figure 5: The plot shows the relative size of the total approximate correction in comparison with the correction from the collective contribution of the $W$, $Z$, Higgses, and neutralinos. The correction of the collective contribution is fairly constant at $\sim$few% while the total approximate correction ranges from $\sim$-40% to $\sim$40%.

5 Exact vs. Approximation Revisited

Now that we have determined in what parts of parameter space the various approximations break down we can revisit the comparison of the full, exact one-loop corrections to the total approximate corrections. This comparison in Fig. 1 resembles the comparison of the exact chargino correction to the approximate chargino correction shown in Fig. 3 with the points spread out more diagonally and shifted up vertically. Fig. 1 can be “built” by starting with Fig. 3 and adding the missing contributions. The diagonal broadening is due to adding the $B_0$ part of the gluino contributions to both axes of Fig. 3 and the vertical shift is due to adding the contributions from the $B_1$ part of the exact gluino contribution and the collective contribution from the $W$, $Z$, Higgses, and neutralinos to the vertical axis. We thus expect Fig. 1 to behave in the same way as Fig. 3 when restricting to the regions of $|\mu| < |M_2|$ and $|\mu| \gg |M_2|$. This is affirmed in Fig. 6. For $|\mu| < |M_2|$ the points are restricted to the outer islands while for $|\mu| \gg |M_2|$ the points are restricted to the center island.
Figure 6: The full, exact one-loop correction vs. the total approximate correction in two cases: $|\mu| < |M_2|$ and $|\mu| \gg |M_2|$. For $|\mu| < |M_2|$, only the outer islands remain. When $|\mu|$ is greater than about 2$|M_2|$, the lightest charginos are sufficiently wino-like and only the center island remains.

Furthermore, as shown in Fig. 2, the $B_0$ term of the gluino contribution shrinks in magnitude as the sbottom masses increase (and also when $(\mu \tan \beta - A_b) \to 0$). A decrease in the $B_0$ term of the gluino contribution would result in a reduction of the diagonal breadth of the points in Fig. 1 (Fig. 6). This is reflected by the color gradient shift from light to dark as points move toward the vertical line.

We emphasize here the disparity between the size of the corrections given by the full, exact one-loop corrections and the total approximate corrections. In the outer islands, the approximate corrections given by the model points are almost all within $\sim 10\%$ while the exact corrections from these same models can be $\sim 100\%$. Finally, in contrast to the chargino comparison in Fig. 4, the total approximate correction starts to breakdown with increasing squark masses for points in the center island due to the vertical shift. As squark masses increase, the right plot of Fig. 6 shows that points move toward the vertical line but are not on the diagonal line as was the case for the right plot of Fig. 4. The results of this analysis are summarized in Tab. 2 where we list the regions of parameter space in which the gluino, chargino, and total approximate forms are valid or invalid.

**Small $\tan \beta$**

We briefly consider the situation for smaller $\tan \beta$. In Fig. 7 we present the full, exact one-loop threshold corrections versus the total approximate corrections for $\tan \beta = 10$. We again see three islands with the outer islands representing the region of parameter space in which $|\mu| < |M_2|$. The approximate corrections are small ($\lesssim 10\%$) for most points and may typically be ignored. The exact corrections however can be around 10-30% and thus it is important to consider the full, exact one-loop corrections even in the small $\tan \beta$ regime.
Table 2: The regions of parameter space in which the gluino, chargino, and total approximate forms are valid or invalid are summarized. The valid region entry for the total approximate correction \((\Delta m_{\tilde{g},\tilde{\chi}^\pm}^{app} / m_b)\) states squark instead of sbottom because, due to the vertical shift in Fig. 1 (see text), the approximation becomes invalid close to the vertical line towards which points with heavy squarks gravitate. This is in contrast to the left plot in Fig. 4 where points near the vertical were still along the diagonal.

6 Consequences

In the previous section, we compared the magnitude of the SUSY threshold corrections to the bottom quark mass. Particularly, we have shown that the approximate formula for the threshold corrections breaks down in many regions of the parameter space. In this section, we will highlight some of the consequences of the full, exact one-loop corrections to the bottom quark mass.
Fits to the bottom quark mass

A good choice of scale to integrate out the massive SUSY particles is the \(M_Z\) scale. At the \(M_Z\) threshold one then has to match the value of \(m_b\) before and after integrating out the massive states. This leads to the relation

\[
m_b(M_Z)^{\text{SM}} = m_b(M_Z)^{\text{MSSM}} (1 + \Delta m_b/m_b) .
\] (16)

\(m_b(M_Z)^{\text{below}}\) can be determined by taking the value of \(m_b(m_b) = 4.19\) GeV and running it to the \(M_Z\) scale. This is evaluated using the RunDec package to be \(m_b(M_Z)^{\text{below}} = 2.69\) GeV. The hope then is that the right choice of bottom Yukawa coupling and the appropriate set of SUSY boundary conditions at some UV scale will give rise to the necessary \(m_b(M_Z)^{\text{above}}\) and \(\Delta m_b/m_b\) to satisfy Eq. (16).

Let us consider an exercise for the scenario when the Yukawa couplings are unified at the GUT scale. Then, to a good approximation,

\[
m_b(M_Z)^{\text{above}} = \lambda_b(M_Z) v_d
\]
\[
= \frac{v}{\tan \beta} \lambda_t(M_Z)
\]
\[
= 3.46\ \text{GeV}
\] (17)

This yields a \(\Delta m_b/m_b = -0.22\). In principle, \(\lambda_b(M_Z)\) is smaller than \(\lambda_t(M_Z)(\sim 1)\) and therefore \(\Delta m_b/m_b\) is also similarly smaller. This concurs with the usual observation that the bottom quark threshold corrections in Yukawa unified GUTs have to be \(\mathcal{O}(\text{few})\%\). With our full analysis of the threshold corrections, we see that fitting the bottom quark mass in Yukawa unified models clearly requires \(|\mu| >> |M_2|\). This agrees with the results obtained in Ref. \([29, 30]\). We point out that most numerical spectrum calculators, such as SOFTSUSY \([5]\) and SPheno \([31]\) use the full expression to calculate the threshold corrections to the bottom quark mass/Yukawa coupling.

Higgs couplings to the bottom quark

The MSSM predicts four new physical Higgs states in addition to the light CP-even (SM like) Higgs boson. The coupling of the Higgs bosons to the bottom quark depends on the MSSM parameters, particularly, \(\tan \beta\). In addition, the couplings also depend on the bottom quark threshold corrections and the effect of these corrections have been the subject of many works especially in the large \(\tan \beta\) regime \([32–35]\). The couplings between the bottom quark and the Higgs bosons in the MSSM, relative to the SM Higgs can be written as

\[
g_b^h = -\frac{\sin \alpha}{\cos \beta (1 + \Delta_b)} \left( 1 - \frac{\Delta_b}{\tan \alpha \tan \beta} \right)
\] (19)

\[
g_b^H = \frac{\cos \alpha}{\cos \beta (1 + \Delta_b)} \left( 1 - \frac{\Delta_b}{\cot \alpha \tan \beta} \right)
\] (20)

\[
g_b^A = \frac{\tan \alpha}{\cos \beta (1 + \Delta_b)} \left( 1 - \frac{\Delta_b}{\tan^2 \beta} \right)
\] (21)
where $\Delta_b \equiv \Delta m_b/m_b$. As discussed in Section 5, models that obtain a negligible value for the SUSY threshold correction to the bottom quark mass using the approximate form in Eq. (1) may in fact obtain corrections as large as $\sim 100\%$ when considering the exact corrections. The factor $(1 + \Delta_b)^{-1}$ present in each of the couplings above could therefore change by up to a factor of 2. Such an alteration could significantly affect the predicted bottom-Higgs phenomenology. Note in the MSSM decoupling limit, where $M_A \gg M_Z$,

$$
\begin{align*}
g_b^h & \rightarrow 1 \\
g_b^H & \rightarrow \frac{\tan \beta}{(1 + \Delta_b)} \left( 1 + \frac{\Delta_b}{\tan^2 \beta} \right) \\
g_b^A & \rightarrow \frac{-1}{\sin \beta (1 + \Delta_b)} \left( 1 - \frac{\Delta_b}{\tan^2 \beta} \right).
\end{align*}
$$

(22)

Thus one must still consider the full expression for the SUSY threshold corrections to the bottom quark mass in this limit.

We have shown in our analysis that even in the large $\tan \beta$ regime, the effect of the Higgses and neutralinos could play an important role in the SUSY threshold corrections to the bottom quark mass. In SUSY models where large bottom threshold corrections can be accommodated, the Higgs couplings can be an interesting probe for new physics [36, 37].

**Flavor physics**

It is well known that in the large $\tan \beta$ regime of SUSY models, there are interesting consequences to $B$-physics observables such as $B(B \to X_s \gamma)$, $B_s^0 \bar{B}_s^0$ mixing, $B(B_{s,d} \to l^+l^-)$, and $B(B_u \to \tau \nu)$ [38–40]. Each of these observables has significant contributions from SUSY particles in loops. For example, tree-level charged Higgs contributions to the observable $B(B_u \to \tau \nu)$ interfere destructively with the SM contribution and lead to a suppression in the total $B(B_u \to \tau \nu)$ rate. The observable $B(B_s \to \mu^+\mu^-)$ is also enhanced at large $\tan \beta$. The effect of the SUSY threshold corrections is known to be important in the estimation of the above mentioned flavor observables. Our analysis highlights the need for a careful and full calculation of these effects.

**7 Conclusions**

We have examined the validity of common approximations of the SUSY threshold corrections to the bottom quark mass. To avoid model dependency, we chose to work in the context of the pMSSM and performed a parameter scan to survey a large region of parameter space. In particular we considered large $\tan \beta$ and squark masses of $O(\text{few})$ TeV. For large regions of parameter space, the approximate form of the bottom quark threshold correction indeed breaks down. There are three main regions in which the approximate form is invalid: (1) $|\mu| < |M_2|$, (2) heavy squark masses ($\sim 3, 4$ TeV), and (3) $X_{b(t)} \to 0$, where $m_{b(t)}X_{b(t)}$ are the off-diagonal terms in the sbottom (stop) mass-squared matrices. For $|\mu| < |M_2|$, the break down is quite significant and worsens
with increasing squark masses. This is of particular concern for models with light higgsinos. The consequences of an invalid approximation for the bottom quark threshold corrections may also be considerable for fits to the bottom quark mass, the Higgs couplings to the bottom quark, and flavor physics. We stress the need to consider the full, exact one-loop contributions from SUSY particles to the bottom quark mass. Finally, we note that even for smaller tan $\beta$ the threshold corrections can be significant and should be included in calculations involving the bottom quark mass.

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Appendices

A Gluino-sbottom

Gluinos couple with the down-type squarks and quarks proportional to the SU(3) gauge coupling $g_3$ and hence contribute large corrections to the bottom quark mass. The corrections are dominant when the squarks belong to the third family since the inter-generational mixings between the squarks are typically (and by assumption in this study) small. We will now calculate the individual diagrams shown in Fig. 8 considering the contributions from the two bottom squarks.

The three diagrams correct the inverse propagator

$$S(p) = \frac{i}{p - m - \Sigma(p)},$$

where $-i\Sigma$ is the sum of the three diagrams in Fig. 8:

$$-i\Sigma(p) = -iB_{LR} - ip \cdot A_L - ip \cdot A_R.$$

The Lagrangian after including the corrections from the diagrams can be written as

$$\mathcal{L} = b^* i\gamma\sigma b(1 - A_L) + \tilde{b}^* i\gamma\sigma \tilde{b}(1 - A_R) + \tilde{b}b(m_{\tilde{b}b} + B_{LR}) .$$

By rescaling $b$ and $\tilde{b}$ by $\frac{1}{\sqrt{1-A_L}}$ and $\frac{1}{\sqrt{1-A_R}}$, respectively, the corrected bottom quark mass can be
written as

\[
m_b = \frac{m_{b0} + B_{LR}}{\sqrt{1 - A_L} \sqrt{1 - A_R}} \\
\simeq m_{b0} + B_{LR} + \frac{m_{b0}}{2} (A_L + A_R)
\]

\[
\Rightarrow \Delta m_b = m_b - m_{b0} = B_{LR} + \frac{m_{b0}}{2} (A_L + A_R).
\] (26)

We evaluate the loop integrals in each of the diagrams in Fig. 8:

\[
-iB_{LR}^x = \left( i\sqrt{2} g_3 \Gamma_R^x T_i^{a_j} \right) \int \frac{d^4k}{(2\pi)^4} \left[ \frac{iM_{\bar{g}}}{k^2 - M_{\bar{g}}^2} \right] \left[ -i\sqrt{2} g_3 (\Gamma_L^x)^\dagger T_j^{a_k} \right] \left[ \frac{i}{(p - k)^2 - m_{b_s}^2} \right]
\]

\[
= -\frac{8}{3} g_3^2 \Gamma_L^x (\Gamma_L^x)^\dagger \int \frac{d^4k}{(2\pi)^4} \left[ \frac{ik \cdot \bar{\sigma}}{k^2 - M_{\bar{g}}^2} \right] \left[ -i\sqrt{2} g_3 (\Gamma_L^x)^\dagger T_j^{a_k} \right] \left[ \frac{i}{(p - k)^2 - m_{b_s}^2} \right]
\]

\[
-i \cdot \bar{\sigma} A_{LR}^x = \left( i\sqrt{2} g_3 \Gamma_R^x T_i^{a_j} \right) \int \frac{d^4k}{(2\pi)^4} \left[ \frac{iM_{\bar{g}}}{k^2 - M_{\bar{g}}^2} \right] \left[ -i\sqrt{2} g_3 (\Gamma_R^x)^\dagger T_j^{a_k} \right] \left[ \frac{i}{(p - k)^2 - m_{b_s}^2} \right]
\]

\[
= -\frac{8}{3} g_3^2 \Gamma_R^x (\Gamma_R^x)^\dagger \int \frac{d^4k}{(2\pi)^4} \left[ \frac{ik \cdot \bar{\sigma}}{k^2 - M_{\bar{g}}^2} \right] \left[ -i\sqrt{2} g_3 (\Gamma_R^x)^\dagger T_j^{a_k} \right] \left[ \frac{i}{(p - k)^2 - m_{b_s}^2} \right].
\] (27)

Using the standard definition of the Passarino-Veltman functions,

\[
B_0(p, m_1, m_2) = 16\pi^2 \int \frac{d^4k}{i(2\pi)^4} \frac{1}{(k^2 - m_1^2)((k - p)^2 - m_2^2)}
\]

\[
p_\mu B_1(p, m_1, m_2) = 16\pi^2 \int \frac{d^4k}{i(2\pi)^4} \frac{k_\mu}{(k^2 - m_1^2)((k - p)^2 - m_2^2)},
\] (28)

we get

\[
B_{LR}^x = \frac{8}{3} \frac{g_3^3}{16\pi^2} \Gamma_R^x (\Gamma_L^x)^\dagger M_{\bar{g}} B_0 \left( p, M_{\bar{g}}, m_{b_s} \right)
\]

\[
A_{LR}^x = -\frac{8}{3} \frac{g_3^3}{16\pi^2} \Gamma_R^x (\Gamma_L^x)^\dagger B_1 \left( p, M_{\bar{g}}, m_{b_s} \right)
\]

\[
A_{LR}^x = -\frac{8}{3} \frac{g_3^3}{16\pi^2} \Gamma_R^x (\Gamma_R^x)^\dagger B_1 \left( p, M_{\bar{g}}, m_{b_s} \right).
\] (29)

Now we are ready to calculate the corrections to the bottom quark mass from the three diagrams as estimated in Eq. (26):
\[ \Delta m_b^\tilde{g} = \sum_{x=1,2} B_{LR}^x + \frac{m_{b_0}}{2} (A_L^x + A_R^x) \]

\[ = \frac{8}{3} \frac{g_3^2}{16\pi^2} \sum_{x=1,2} \{ \Gamma_R^x (\Gamma_L^x)^\dagger M_\tilde{g} B_0 \left( p, M_\tilde{g}, m_{\tilde{b}_x} \right) - \frac{m_{b_0}}{2} B_1 \left( p, M_\tilde{g}, m_{\tilde{b}_x} \right) (\Gamma_L^x (\Gamma_L^x)^\dagger + \Gamma_R^x (\Gamma_R^x)^\dagger) \} . \]

(30)

This is the exact expression for the one-loop threshold corrections to the bottom quark mass coming from the gluino-sbottom loops. In a full three family model, the \( \Gamma_{L,R} \) are the 6 \times 3 squark mixing matrices, and all the down-type squarks give rise to corrections to the bottom mass. Ignoring the off-diagonal elements that introduce the inter-generational mixing, we can consider a 2 \times 2 block that mixes the two bottom squarks. The sbottom mixing matrix can be written as

\[ \Gamma = \begin{pmatrix} \Gamma^L_L & \Gamma^L_R \\ \Gamma^R_L & \Gamma^R_R \end{pmatrix} = \begin{pmatrix} \cos \theta_b & \sin \theta_b \\ -\sin \theta_b & \cos \theta_b \end{pmatrix} , \]

such that

\[ \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} = \Gamma \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix} . \]

(31)

Then, \( \Delta m_b^\tilde{g} \) simplifies to

\[ \Delta m_b^\tilde{g} = \frac{8}{3} \frac{g_3^2}{16\pi^2} \left[ \sin 2\theta_b M_\tilde{g} \left( B_0 \left( p, M_\tilde{g}, m_{\tilde{b}_1} \right) - B_0 \left( p, M_\tilde{g}, m_{\tilde{b}_2} \right) \right) \right] 

\[ - \frac{m_{b_0}}{2} \left( B_1 \left( p, M_\tilde{g}, m_{\tilde{b}_1} \right) + B_1 \left( p, M_\tilde{g}, m_{\tilde{b}_2} \right) \right) \] . \]

(33)

B \quad Chargino-stop

The charginos couple to the up-type squarks and down-type quarks proportional to the SU(2) coupling \( g_2 \) and the Yukawa couplings \( \lambda_{t,b} \) with strength depending upon their respective wino-higgsino composition. The corrections dominate when the squarks are from the third family due to CKM suppression of the contributions from the first two families of squarks. We calculate here the individual diagrams shown in Fig. 9 considering the contributions from the two stop squarks. The calculation of the chargino-stop diagrams is similar to the calculation of the gluino-sbottom diagrams and yields

17
\[ -i \mathcal{B}^{\ell}_{LR} = \int \frac{d^4k}{(2\pi)^4} \left[ i \Phi^x \right] \frac{i M_{\tilde{\chi}^\pm}}{k^2 - (M_{\tilde{\chi}^\pm})^2} \left[ i \Phi^x \right] \frac{i}{(p-k)^2 - m_{\tilde{t}_x}^2} \]

\[ -i \sigma A^{\ell}_{L} = \int \frac{d^4k}{(2\pi)^4} \left[ i (\Phi^x)^\dagger \right] \frac{i k \cdot \bar{\sigma}}{k^2 - (M_{\tilde{\chi}^\pm})^2} \left[ i \Phi^x \right] \frac{i}{(p-k)^2 - m_{\tilde{t}_x}^2} \]

\[ -i \sigma A^{\ell}_{R} = \int \frac{d^4k}{(2\pi)^4} \left[ i (\Phi^x)^\dagger \right] \frac{i k \cdot \bar{\sigma}}{k^2 - (M_{\tilde{\chi}^\pm})^2} \left[ i \Phi^x \right] \frac{i}{(p-k)^2 - m_{\tilde{t}_x}^2} \]

where \( \Phi \) and \( \bar{\Phi} \) are the effective couplings of the bottom quark to a chargino mass eigenstate and a top squark. The gaugino fraction of the chargino couples proportional to the SU(2) gauge coupling \( g_2 \) and does not couple to the right-handed squarks. The Higgsino fraction of the charginos couples proportional to the Yukawa coupling of the top quark, \( \lambda_t \). Once again, using the standard definition of the Passarino-Veltman function defined in Eq. (28), we get,

\[ B^{\ell}_{LR} = -\frac{\Phi^x \Phi^x_{\tilde{\chi}^\pm}}{16\pi^2} B_0(p, M_{\tilde{\chi}^\pm}, m_{\tilde{t}_x}) \]

\[ A^{\ell}_{L} = -\frac{(\Phi^x)^\dagger \Phi^x_{\tilde{\chi}^\pm}}{16\pi^2} B_1(p, M_{\tilde{\chi}^\pm}, m_{\tilde{t}_x}) \]

\[ A^{\ell}_{R} = -\frac{(\Phi^x)^\dagger \Phi^x_{\tilde{\chi}^\pm}}{16\pi^2} B_1(p, M_{\tilde{\chi}^\pm}, m_{\tilde{t}_x}) \]
The corrections to the bottom quark mass from the three diagrams in Fig. 9 are then

$$
\Delta m^\pm_b = \sum_{i=1}^{2} \sum_{x=1}^{2} B_{LRi}^x + \frac{m_0^b}{2} (A_{L_b}^x + A_{R_b}^x),
$$

(36)

where the sum runs over the two chargino mass eigenstates and the two stop eigenstates.

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