Simulating the effect of weak measurements by a phase damping channel and extraction of bipartite correlations in nuclear magnetic resonance

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Quantum discord is a measure based on local projective measurements which captures quantum correlations that go beyond entanglement. A change in the measurement process, achieved by replacing rank-one projectors with a weak positive operator-valued measure (POVM), allows one to define a quantity termed as weak quantum discord. In this work, we experimentally simulate the effect of a weak POVM on a nuclear magnetic resonance quantum information processor. The two-qubit system under investigation is part of a three-qubit system, where one of the qubits is used as an ancillary to implement the phase damping channel. The strength of the weak POVM is controlled by varying the strength of the phase damping channel. We experimentally observed weak discord in two-qubit states and computed a cost function whose purpose is to optimally extract correlations representing the difference between weak and strong quantum discord. The resultant dynamics of the states is investigated as a function of the measurement strength.

PACS numbers: 03.65.Ud, 03.67.Mn

I. INTRODUCTION

Quantum correlations play an important role in quantum communication and quantum information processing.1 While quantum entanglement was discovered early on by Schrödinger2, Ollivier and Zurek3 and Henderson and Vedral4, independently pointed out that nonclassical correlations can be present even in mixed separable states, and therefore can go beyond entanglement. The measure to quantify such correlations is termed as quantum discord (QD)3,4. The evaluation of QD is computationally a hard task as it involves numerical optimization and hence, alternative measures have been proposed such as geometric quantum discord5, Gaussian geometric discord in continuous variable systems6 and relative entropy of discord7. The presence of nonclassical correlations in bipartite states was measured experimentally on a nuclear magnetic resonance (NMR) setup8, witnessed via a single-shot experiment9 and its preservation has also been explored using NMR10. Several recent studies have explored the advantage of nonclassical correlations in quantum information processing even if the state has almost null entanglement11.13.

In classical information theory, the mutual information between two random variables has two equivalent expressions which can be computed from the respective Shannon entropies. In contrast, for quantum mutual information, these two definitions give rise to different values as one of the expressions requires the von Neumann entropy conditioned by projective measurements. QD is defined as the difference between total mutual information and classical mutual information, wherein classical mutual information is found by extracting the information gain about one subsystem while measuring the other subsystem via projective measurements4. What if one replaces these projective measurements by weak measurements? The crux of a weak measurement lies in the weak interaction between the system to be measured and the measuring apparatus. While originally the weak measurements were used in the context of weak values14, it was later realized that any projective measurement can be obtained by a sequence of weak measurements15. These ideas have been used in several applications including the observation of spin Hall effect of light16, direct measurement of single photon wavefunction17, protection of quantum entanglement from decoherence18, feedback control of a quantum system19, to amplify small transverse deflections of an optical beam in a Sagnac interferometer20 and for quantum state tomography21,22.

Recently weak measurements have been utilized in a new definition of bipartite correlations through weak quantum discord (WQD)23,24, which is numerically greater than QD, indicating a greater preservation of the bipartite correlations. Since then WQD has attracted interest in various contexts in quantum information processing25,26. While projective measurements result in a loss of coherence, the precise relationship between decoherence and measurement needs to be clarified. Simulations of the phase damping (PD) channel have been
realized as projective measurements in NMR [31] and using this method quantum teleportation and the quantum Zeno effect have been implemented in an NMR setup [32].

In this work, we experimentally capture the excess correlations quantified by WQD, in two-qubit states. The extraction of WQD from a bipartite state requires a weak measurement on one subsystem instead of a projective measurement. The result of a weak measurement on a Bell-diagonal state and on a Werner state while finding WQD, affects the state in the same manner as if a PD channel was acting on it. The PD channel is a non-unitary operation used to model the decoherence process [1]. We mapped the weak measurement to a PD channel and the weak measurement strength is controlled by tuning the strength of the PD channel. We use three nuclear spins to encode three qubits, where the two qubits are used to prepare two-qubit states and the third qubit acts as an ancillary qubit used to simulate a PD channel acting on one of the qubit of two-qubit states [23]. This study reports the first experimental implementation of a controlled weak measurement on an NMR hardware, to extract the maximum intact correlation by evaluating WQD with the minimum disturbance to the overall two-qubit state as delineated by an appropriate cost function [25]. We have also investigated the experimental behavior of WQD and have successfully demonstrated that WQD approaches QD as the measurement strength is increased.

This article is organized as follows: Sec. [II A] describes nonclassical correlation as quantified by QD. Sec. [II B] describes weak measurement and its application in finding the WQD in two-qubit states and Sec. [II C] introduces a cost function based on WQD and a fidelity measure to estimate the disturbance to a weakly measured state and the extracted correlations. Sec. [II D] details the mapping of weak measurement to the PD channel and Sec. [II E] contains experimental results of implementing a PD channel to observe WQD, as well as the computation of the cost function in two-qubit states for varied measurement strengths. Sec. [II F] contains some concluding remarks.

II. OPTIMAL EXTRACTION OF BIPARTITE CORRELATION

A. Quantum Discord

It was independently noted by Ollivier and Zurek [3] and by Henderson and Vedral [4] that certain types of mixed separable states have zero entanglement, yet contain nonclassical correlations. These correlations thus are more general and go beyond entanglement. Mathematically, QD captures these correlations and is the difference between the total correlation $I(A : B)$ and the classical correlation $J(A|B)$ where

$$I(A : B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

$$J(A|B) = S(\rho_A) - S(\rho_{A|B})$$

where $S(\rho_{AB}) = -\text{Tr}(\rho_{AB} \log_2 \rho_{AB})$ is the von Neumann entropy of the quantum state $\rho_{AB}$ shared between two parties $A$ and $B$, $\rho_{A|B} = \text{Tr}_{B,A}(\rho_{AB})$ is the reduced density operator of a subsystem and $S(\rho_{A|B})$ is the conditional von Neumann entropy of subsystem $A$ when $B$ has already been measured. For a two-qubit system, the state of the subsystem $A$ after a projective measurement on subsystem $B$ can be written as

$$\rho_{A|B} = \frac{\text{Tr}_B \left[ (I \otimes \Pi_j^{\theta,\phi}) \rho_{AB} (I \otimes \Pi_j^{\theta,\phi}) \right]}{p_j}$$

where $p_j$ is the probability of the measurement outcome corresponding to the projectors $\Pi_j^{\theta,\phi}$ and is given by $p_j = \text{Tr}_{AB} \left[ (I \otimes \Pi_j^{\theta,\phi}) \rho_{AB} (I \otimes \Pi_j^{\theta,\phi}) \right]$. The expression to find conditional von Neumann entropy required in Eq. (1) is

$$S(\rho_{A|B}) = \sum_{j=1}^2 p_j S(\rho_{A|\Pi_j^{\theta,\phi}})$$

Therefore, evaluating classical correlations is a non-trivial task as it requires numerical optimization over all projectors to find the maximum classical correlation. Projectors $\Pi_1^{\theta,\phi}$ and $\Pi_2^{\theta,\phi}$ can be constructed utilizing two orthogonal vectors as follows:

$$|\psi_1^{\theta,\phi}\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$$

$$|\psi_2^{\theta,\phi}\rangle = -\sin \frac{\theta}{2} |0\rangle + e^{i\phi} \cos \frac{\theta}{2} |1\rangle$$

$$\Pi_1^{\theta,\phi} = |\psi_1^{\theta,\phi}\rangle \langle \psi_1^{\theta,\phi}| \quad \text{and} \quad \Pi_2^{\theta,\phi} = |\psi_2^{\theta,\phi}\rangle \langle \psi_2^{\theta,\phi}|$$

QD can be computed by utilizing the above defined projectors in Eq. (5), followed by substituting the parameterized conditional entropy into Eq. (1):

$$QD(A : B) = I(A : B) - \max_{\{\theta,\phi\}} J(A|B)$$

Thus one can compute QD, by maximizing the $J(A|B)$ by varying $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$, and substituting $\theta$ and $\phi$ back into Eq. (5).

B. Weak Quantum Discord

If one replaces the projective measurement used to compute QD by a weak measurement, one obtains WQD. The value of WQD depends upon the strength of the measurement and is numerically greater than QD.

The positive operator valued measure (POVM) corresponding to the weak measurement is defined through operators

$$P(x) = \sqrt{\frac{1 - \tanh x}{2} \Pi_{\psi_1}^{\theta,\phi} + \frac{1 + \tanh x}{2} \Pi_{\psi_2}^{\theta,\phi}}$$

$$P(-x) = \sqrt{\frac{1 + \tanh x}{2} \Pi_{\psi_1}^{\theta,\phi} + \frac{1 - \tanh x}{2} \Pi_{\psi_2}^{\theta,\phi}}$$

where $\Pi_{\psi_j}^{\theta,\phi} = |\psi_j^{\theta,\phi}\rangle \langle \psi_j^{\theta,\phi}|$ are the projectors corresponding to the weak measurement.
where the strength of the weak measurement is parameterized by the real parameter $x \geq 0$. The POVM operators satisfy $P(x) = P(x) + P(-x) = I$. For $x = 0$, $P(0)$ reduces to $I$ i.e. no measurement at all and in the case of $x \to \infty$, $P(x)$ and $P(-x)$ reduces to the projectors $\Pi_{\psi}^{\phi}$ and $\Pi_{\psi}^{\phi}$ respectively (as defined in Eq.(4)).

After the weak measurement, the state of the combined system becomes

$$\rho'_{AB} = p(x) (I \otimes P^B(x)) \rho_{AB} (I \otimes P^B(x)) + p(-x) (I \otimes P^B(-x)) \rho_{AB} (I \otimes P^B(-x)),$$

(7)

with $p(\pm x)$ being the probabilities for $P(\pm x)$. The post-measurement state of subsystem $A$ can be written as

$$\rho_A |p_B(\pm x) = \text{Tr}_B(\rho'_{AB})$$

(8)

The conditional entropy, classical information and WQD can therefore be written as

$$S_w(\rho_{AB}) = p(x)S_w(\rho_A |P_B(x)) + p(-x)S_w(\rho_A |P_B(-x))$$

$$J_w(A : B) = S(\rho_A) - S_w(\rho_{AB})$$

$$WQD(A : B) = I(A : B) - \max_{\{\theta, \phi\}} J_w(A |B)$$

(9)

all of which depend on the measurement strength $x$. The value of WQD is greater than QD because $S_w(\rho_{AB})$ is always larger than $S(\rho_{AB})$, as the weak measurement performed on subsystem $B$ while disturbing the state weakly, reveals less information about the subsystem $A$. In the limiting case of no measurement performed on system $B$ at all, $S_w(\rho_{AB}) \to S(\rho_A)$ and WQD discord will be equal to the total correlation.

C. Cost Function

While defining QD, we consider the correlations which can be obtained via projective measurements to be ‘classical’ [34]. Weak measurements capture less correlations, therefore, the excess correlations quantified in WQD must contain both unextracted classical correlations as well as quantum correlations [25, 31]. Another viewpoint maintains that the inherent definition of classical correlations are those that are obtained via a measurement, be it weak or projective. This view therefore leads to the conclusion that weak (or ‘super’) discord captures greater quantum correlations than that accounted for by QD [24]. While the question of whether these excess correlations are classical or quantum currently remains open, we can quantify these correlations through the difference between WQD and QD:

$$\Delta D = WQD(A : B) - QD(A : B)$$

$$= J(A |B) - J_w(A |B)$$

(10)

On the other hand, the disturbance caused to the overall state due to a weak measurement on subsystem $B$ can be quantified in terms of fidelity between the state $\rho'_{AB}$ before and the state $\rho_{AB}$ after a weak measurement on subsystem $B$ defined as [37, 38]

$$\Delta F = 1 - F(\rho_{AB}, \rho'_{AB})$$

$$F(\rho_{AB}, \rho'_{AB}) = \text{Tr} \left( \sqrt{\rho_{AB} \rho'_{AB} \rho_{AB}^\dagger} \right)^2$$

(11)

Combining Eq.(11) and Eq.(10) leads to the definition of the cost function $C$ [25]:

$$C = \Delta F + \Delta D$$

$$= 1 - F(\rho_{AB}, \rho'_{AB}) + WQD(A : B) - QD(A : B)$$

(12)

The cost function $C$ depends on the measurement strength $x$, and has at the most one minimum when it is plotted by varying measurement strength $x$ as depicted in the case of two-qubit Werner and Bell state in Fig. 5. More discussion on the experimental results shown in Fig. 5 is given in the results section. The minimum point is a result of the competition between $\Delta F$ which is a monotonically increasing function of $x$ and $\Delta D$ which is a monotonically decreasing function of $x$. Thus, the existence of a minimum in $C$ at a particular measurement strength, say $x_m$, corresponds to the maximum intact correlation between two parts of the state, with minimum disturbance to the overall state. A definitive answer to the question of whether this excess correlation is classical or quantum, lies in an interpretation of the information gain from measurement operators for WQD, as well as the cost function [35, 36].

III. SIMULATING WEAK POVM VIA A PHASE DAMPING CHANNEL

A projective measurement collapses the state, thereby killing all the off-diagonal terms (coherences) in the density matrix, in the measurement basis [32]. The weak measurement formalism, as described by Aharonov, Albert and Vaidmann (AAV) [14] utilizes the weak interaction [39]. The weak interaction couples the system weakly with the measuring device, and therefore the state of the system retains its coherence partially, even after the measurement. Later, Oreshkov and Bruun [15] showed that any generalized measurement can be modeled by a sequence of weak POVMs and for the two-qubit case are given in Eq.(6).

We consider two types of states to investigate the behavior of WQD with respect to measurement strength and to compare it with QD, namely, the Werner states and Bell-type states. The two-qubit Werner states are defined as

$$\rho_{W}^{\omega} = z|\psi^-(\psi^-)\rangle + \frac{1}{4}(1 - z)I$$

(13)

where $z$ quantifies the amount of mixedness, $0 \leq z \leq 1$ and $|\psi^-(\psi^-)\rangle = \frac{1}{\sqrt{2}}((01) - |10\rangle)$. The two-qubit Bell-type states [40] are defined as

$$\rho_{AB}^{bs} = \frac{1}{4} \left[ I \otimes I + \sum_{i=1}^{3} c_i (\sigma_i \otimes \sigma_i) \right]$$

(14)
where \((\sigma_1, \sigma_2, \sigma_3)\) are the Pauli matrices and \(-1 \leq c_1, c_2, c_3 \leq 1\).

The evaluation of WQD in both the states involves an optimization over all possible projectors by varying \(\theta \in [0, \pi]\) and \(\phi \in [0, 2\pi]\) as given in Eq. (9). The optimization gives the highest possible classical correlations at \(\theta = \pi\) and \(\phi = \pi\) for the Werner states as well for the Bell-type states. On substituting the optimal values of \(\theta\) and \(\phi\) into Eq. (6), the weak POVMs get simplified to

\[
P(x) = \sqrt{\frac{1 - \tanh(x)}{2}} |0\rangle\langle 0| + \sqrt{\frac{1 + \tanh(x)}{2}} |1\rangle\langle 1|
\]

\[
P(-x) = \sqrt{\frac{1 + \tanh(x)}{2}} |0\rangle\langle 0| + \sqrt{\frac{1 - \tanh(x)}{2}} |1\rangle\langle 1|
\]

A single-qubit mixed state \(\rho\) on the Bloch sphere can be expressed as

\[
\rho = \frac{1}{2} \left( I + r_x \sigma_1 + r_y \sigma_2 + r_z \sigma_3 \right)
\]

where \(r_x, r_y\) and \(r_z\) are the coordinates of the Bloch vector and \(I\) is the \(2 \times 2\) identity matrix. The effect of the simplified weak POVM given in Eq. (15) on the single-qubit state \(\rho\) can be readily computed and in the matrix form is written as:

\[
\rho = \frac{1}{2} \left[ \begin{array}{cc} 1 + r_x & r_x - ir_y \\ r_x + ir_y & 1 - r_x \end{array} \right]
\]

\[
\rho'_{\text{wm}} = \frac{1}{2} \left[ \begin{array}{cc} 1 + r_z & (r_x - ir_y) \sech x \\ (r_x + ir_y) \sech x & 1 - r_z \end{array} \right]
\]

It is clear from the post weak-measurement state \(\rho'_{\text{wm}}\) that the off-diagonal terms are a monotonically decreasing function of the measurement strength \(x\), leading to decoherence. The extent to which the weak measurement decoheres the state \(\rho\) depends on the measurement strength \(x\).

We now turn to the PD channel, which causes loss of coherence and leads to the decay of the off-diagonal terms of the density matrix and can be described by a completely positive trace preserving map described through the Kraus operators \[43\]:

\[
\rho_{PD} = E_0 \rho E_0^\dagger + E_1 \rho E_1^\dagger
\]

\[
E_0 = \frac{1 + \sqrt{1 - \lambda}}{2} I + \frac{1 - \sqrt{1 - \lambda}}{2} \sigma_3
\]

\[
E_1 = \frac{\sqrt{\lambda}}{2} I - \frac{\sqrt{1 - \lambda}}{2} \sigma_3
\]

where the parameter \(\lambda \in [0, 1]\) represents the strength of the PD channel. The action of the PD channel on a general one-qubit state in the matrix form is given as:

\[
\rho = \frac{1}{2} \left[ \begin{array}{cc} 1 + r_x & r_x - ir_y \\ r_x + ir_y & 1 - r_x \end{array} \right]
\]

\[
\rho'_{PD} = \frac{1}{2} \left[ \begin{array}{cc} 1 + r_z & (r_x - ir_y) \sqrt{1 - \lambda} \\ (r_x + ir_y) \sqrt{1 - \lambda} & 1 - r_z \end{array} \right]
\]

The effect of the PD channel is similar to the weak POVM on a single-qubit state, wherein the off-diagonal terms are diminished. Since both \(\sech x\) and \(\sqrt{1 - \lambda}\) are monotonically decreasing functions, they can be mapped onto each other with an appropriate scaling factor. Therefore, the action of the PD channel is in one-to-one correspondence with weak POVM described in Eq. (6).

In order to implement the PD channel we follow an indirect approach \[33\], wherein non-unitary operators can be thought of as unitary operations on an extended quantum system built upon the Duality Quantum Computing (DQC) framework \[42\]. This framework requires an ancillary qubit. It has been demonstrated \[33\] that the Kraus operator \(E_k\) describing the non-unitary transformation corresponding to the PD channel can be efficiently implemented on a qubit if the unitary operators \(V, W, U_0\) and \(U_1\) can be found in the two-qubit space, where \(V\) and \(W\) act on the ancilla qubit, and \(U_0\) and \(U_1\) act on the target qubit controlled by the ancilla qubit.
FIG. 2. (a) Molecular structure of $^{13}$C-labeled diethylfluoromalonate with the three qubits labeled as $^1$H, $^{19}$F and $^{13}$C. NMR parameters i.e. the chemical shift $\nu_i$ (in Hz) of each nuclear spin, spin-spin coupling between them $J_{ij}$ (in Hz), spin-lattice relaxation times $T_1^i$ and spin-spin relaxation times $T_2^i$ (in seconds). NMR spectrum of (b) thermal equilibrium state obtained after a $\pi/2$ readout pulse and (c) pseudopure state. The resonance lines of each qubit in the spectra are labeled by the corresponding logical states of the other qubits.

The operators need to satisfy

$$E_k(k=0,1) = \sum_{i=0}^{1} W_{ki} V_0 U_i \quad (19)$$

Here $W_{ki}$ is $k$th element of $W$ operator, $V_0$ is an element of the first column of $V$ operator and $U_i$ is the controlled operator. A comparison with the decomposition of Kraus operators of the PD channel is given in Eq. (18). The unitary operators $V, W, U_0$ and $U_1$ can be evaluated as

$$U_0 = I, \quad U_1 = \sigma_3, \quad V = W = \frac{1}{2} \begin{bmatrix} 1 + \sqrt{1 - \alpha^2} & 1 - \sqrt{1 - \alpha^2} \\ 1 - \sqrt{1 - \alpha^2} & 1 + \sqrt{1 - \alpha^2} \end{bmatrix} \quad (20)$$

The quantum circuit to implement the PD channel is shown in Fig. 3(a), where Werner states and Bell-type states (Eq. (13) [14]) states are created on qubits 2 and 3, qubit 1 acts as an ancilla, and the the PD channel acts on qubit 3. The strength of the PD channel is controlled by the V gate. The effect of the PD channel can be extracted by tracing out the ancillary qubit. Physically this was achieved by performing the measurements on the state of the two-qubit subsystem consisting of qubits 2 and 3, while ignoring the qubit 1, as is shown in Fig. 1.

We are now ready to experimentally investigate the behavior of WQD and the cost function by varying the measurement strength. It is important to mention here that in an NMR set up the measurement is already weak (termed as an ensemble weak measurement) since the interaction of the measuring radiofrequency (rf) coil with the nuclear spins is weak [31]. However, we are not using that weak measurement here. Our weak measurement is simulated in a controlled way by the PD channel, which is implemented with the help of the ancilla qubit.

IV. EXPERIMENTAL MEASUREMENT OF WQD AND COST FUNCTION

As discussed earlier in Sec. [17], weak measurements can be mapped onto a PD channel and the strength of the weak measurement can be varied by tuning the strength of the PD channel. For the experimental realization on an NMR quantum processor, we realize the three qubits as the three spin-1/2 nuclei of $^{13}$C-labeled diethyl fluoromalonate dissolved in acetone-D6. The $^1$H, $^{19}$F and $^{13}$C nuclear spins are labeled as the first, second and third qubit, respectively. It should be noted here that two-qubit system was simulated by $^{19}$F and $^{13}$C nuclear spins while $^1$H spin was utilized as the ancillary qubit. The molecular structure along with relevant experimental parameters and corresponding NMR spectrum of the thermal equilibrium state are shown in Figs. 3(a) and (b) respectively. The Hamiltonian for a three-qubit system in a rotating frame under the weak approximation [43] is
given by

$$H = - \sum_{i=1}^{3} \nu_i I_z^i + \sum_{i>j; i=1}^3 J_{ij} I_z^i I_z^j$$

(21)

where \(i, j = 1, 2\) and \(3\) labels the qubit, \(\nu_i\) represents the chemical shift of the respective nuclei, \(J_{ij}\) is the scalar coupling constant between the \(i^{th}\) and \(j^{th}\) nuclear spins and \(I_z^i\) denotes the \(z\) component of the spin angular momentum operators for the \(i^{th}\) nucleus. We used the spatial averaging technique \[24, 45\] to achieve the initial three-qubit pseudopure state (PPS) \(|000\rangle\) from the thermal equilibrium state, with the density operator \(\rho_{000}\) given by

$$\rho_{000} = \frac{(1 - \epsilon)}{8} I + \epsilon |000\rangle \langle 000|$$

(22)

where \(\epsilon \approx 10^{-5}\) represents the spin polarization at room temperature and \(I\) is the \(8 \times 8\) identity operator. The identity operator does not evolve and the measurable NMR signal can be attributed to the deviation density matrix. The NMR spectrum of the three-qubit PPS is shown in Fig. 2(c). The experimentally prepared PPS was tomographed using full quantum state tomography \[46\]. The state fidelity was found to be 0.981±0.006 and was computed using the Uhlmann-Jozsa measure \[37, 38\] used in Eq. (11) where \(\rho_{AB}^{th}\) is replaced by \(\rho_{ex}\) representing experimentally prepared density operators and \(\rho_{AB}^{th}\) is replaced by \(\rho_{ex}\) representing theoretically expected density operators. All the experimental density matrices were reconstructed by performing full quantum state tomography \[46\] using a set of seven preparatory pulses \(\{III, XXX, JYY, XYX, YII, XXX, JYY\}\), where \(I\) represents ‘no-operation’ and \(X(Y)\) denotes local \(\frac{\pi}{2}\) unitary rotation with phase \(x(y)\) which is implemented by applying a spin-selective \(\frac{\pi}{2}\) pulse. We performed all the experiments at room temperature 293K on a Bruker Avance-III 600 MHz FT-NMR spectrometer equipped with a 5 mm triple-resonance probe.
with a QXI probe.

The quantum circuit to implement a weak measurement, simulated by a PD channel, is shown in Fig. 3(a). The left block in the circuit creates a Werner or a Bell-type state $|\rho_0 \otimes \rho_{\text{anc}}\rangle$ on two qubits $|\rho_{\text{anc}}\rangle$ with the third qubit $|\rho_0\rangle$ acting as ancillary: R is a NOT gate in the case of the Werner state and a ‘no-operation’ for a Bell-type state. The right block in the circuit depicts a PD channel acting on the one of the qubits of two-qubit state using an ancillary qubit, and the strength of the PD channel being controlled by a local rotation achieved by the V gate acting on the ancillary qubit.

Experimentally, we prepared a Werner state with mixedness strength $z = 0.8$. The second term on the RHS of Eq. (13) is a singlet state and was created experimentally, followed by QST. To obtain the Werner state of a desired $z$ value the QST generated singlet state was numerically added to the identity and thus, a Werner state with fidelity $0.990 \pm 0.001$ was created. Next the Bell-type state with $c_1 = 1$, $c_2 = -1$ and $c_3 = 1$ was prepared experimentally with fidelity $0.980 \pm 0.002$. The next step is to perform a weak measurement on the second qubit (treated as a subsystem of the two-qubit system). As described in Sec. III, the weak measurement is simulated by the PD channel and the strength of PD channel is controlled by the real parameter $\lambda$ which is related to weak measurement strength $x$ as $\lambda = 1 - \text{sech}^2 x$. We implemented the PD channel on the one of the qubits of the prepared Werner state using the ancillary qubit and the strength of the PD channel was increased corresponding to weak measurement strength $x$ as shown in Table I.

| Theory $x$ | Exp $x$ (Werner State) | Exp $x$ (Bell State) |
|------------|------------------------|----------------------|
| 0.00       | 0.091 ± 0.047          | 0.069 ± 0.036        |
| 0.34       | 0.503 ± 0.018          | 0.373 ± 0.002        |
| 0.55       | 0.666 ± 0.015          | 0.548 ± 0.005        |
| 0.75       | 0.831 ± 0.016          | 0.721 ± 0.005        |
| 0.95       | 1.016 ± 0.016          | 0.907 ± 0.004        |
| 1.20       | 1.215 ± 0.020          | 1.103 ± 0.007        |
| 1.50       | 1.479 ± 0.024          | 1.350 ± 0.007        |
| 1.75       | 1.819 ± 0.045          | 1.667 ± 0.018        |
| 2.00       | 2.122 ± 0.046          | 1.937 ± 0.018        |
| 2.50       | 2.454 ± 0.062          | 2.213 ± 0.026        |
| 3.00       | 3.242 ± 0.151          | 2.795 ± 0.042        |
| 3.50       | 3.758 ± 0.095          | 3.448 ± 0.069        |
| 4.00       | 4.107 ± 0.167          | 4.259 ± 0.123        |
| 4.50       | 4.402 ± 0.062          | 4.575 ± 0.069        |
| 5.00       | 4.839 ± 0.137          | 5.030 ± 0.180        |

performed a similar experiment for the Bell type states, which was directly prepared from the left block in the circuit as depicted in Fig. 3(a) and the PD channel was implemented with increasing weak measurement strength $x$ as shown in Table I. All the experimentally prepared three-qubit states were tomographed before and after implementing PD channel. Both the two-qubit Werner and Bell type states were reconstructed utilizing QST and tracing out the ancillary qubit. The tomograph of one such experimentally reconstructed density matrix of both initially prepared states is shown on the LHS of Fig. 3(a) and Fig. 3(b), respectively. The tomographs on the RHS of Fig. 3(a) and (b) depicts the states after the action of the PD channel corresponding to the weak measurement strength $x = 1.2$.

We investigated theoretically and experimentally, the behavior of WQD and QD present in both states while increasing the measurement strength $x$, and the results are plotted in Fig. 3. Our results show that in both types of states, WQD is maximum at zero measurement strength, which implies that the measured state is undisturbed. WQD approaches QD as the measurement strength increases. The disturbance caused by weak measurement to the state is quantified by computing the fidelity between the state before and after the measurement. The theoretically computed and experimentally obtained cost function from the experimental results of fidelity, QD and WQD (as given in Eq. (12)) is plotted for both states with increasing measurement strength $x$. Both the plots of the cost function from Fig. 3(a) and (b) show the curve which has minimum point and this minimum point emerges from the optimization of the tradeoff between the monotonically increasing function $DF$ and the decreasing function $\Delta D$. Therefore, we get a particular measurement strength $x_m$ from the minimum point such that maximum intact correlations can be extracted with minimum disturbance to the overall state. Experimentally, we obtained a minimum cost function $0.232 \pm 0.013$ at measurement strength $x_m = 1.819 \pm 0.045$ for the Werner state where theoretically, we get a minimum cost function $0.200$ at $x_m = 2.000$. Similarly, we experimentally obtained a minimum cost function $0.564 \pm 0.008$ at measurement strength $x_m = 2.794 \pm 0.042$ for the Bell-type state where theoretically, we obtained a minimum cost function $0.475$ at measurement strength $x_m = 3.000$. The experimentally obtained and theoretically computed values match well, within experimental error.

V. CONCLUDING REMARKS

We have implemented a weak POVM, exploiting its relationship with the PD channel, on an NMR quantum information processor. The noise induced by the PD channel has been exploited to mimic the disturbance introduced by a weak measurement process. The weak POVM was experimentally applied to find the WQD in two classes of bipartite quantum states: the Bell-diagonal state and the Werner state. The WQD was contrasted against QD and it was observed that these two converge as the strength of the measurement increases. The mono-
tonicity of the WQD was also confirmed. By utilizing weak measurement strength which the universal POVM allows, we demonstrate the method to preserve maximum possible correlations within these states at the cost of least possible disturbance to the overall state. In particular, the cost function which optimizes the trade-off shows a minimum as a function of the POVM measurement strength. This could potentially be useful for iterative experimental information processing protocols which seek to disturb the state only slightly. Although the interpretation of WQD remains elusive, this work opens up the possibility of further experimental investigations on WQD, which could potentially exploit correlations beyond QD [36, 47].

ACKNOWLEDGMENTS

All the experiments were performed on a Bruker Avance-III 600 MHz FT-NMR spectrometer at the NMR Research Facility of IISER Mohali.

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