Revisiting $B \to \pi K, \pi K^*$ and $\rho K$ Decays: CP Violations and Implication for New Physics

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Abstract

Combining the up-to-date experimental information on $B \to \pi K, \pi K^*$ and $\rho K$ decays, we revisit the decay rates and CP asymmetries of these decays within the framework of QCD factorization. Using an infrared finite gluon propagator of Cornwall prescription, we find that the time-like annihilation amplitude could contribute a large strong phase, while the space-like hard spectator scattering amplitude is real. Numerically, we find that all the branching ratios and most of the direct CP violations, except $A_{CP}(B^\pm \to K^\pm \pi^0)$, agree with the current experimental data with an effective gluon mass $m_g \simeq 0.5$ GeV. Taking the unmatched difference in direct CP violations between $B \to \pi^0 K^\pm$ and $\pi^\mp K^\pm$ decays as a hint of new physics, we perform a model-independent analysis of new physics contributions with a set of $s(1+\gamma_5)b \otimes \bar{q}(1+\gamma_5)q$ ($q=u,d$) operators. Detail analyses of the relative impacts of the operators are presented in five cases. Fitting the twelve decay modes, parameter spaces are found generally with nontrivial weak phases. Our results may indicate that both strong phase from annihilation amplitude and new weak phase from new physics are needed to resolve the $\pi K$ puzzle. To further test the new physics hypothesis, the mixing-induced CP violations in $B \to \pi^0 K_S$ and $\rho^0 K_S$ are discussed and good agreements with the recent experimental data are found.

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1 Introduction

With the fruitful running of BABAR and Belle in past decade, plenty of exciting results has been produced, which provides a very fertile testing ground for the Standard Model (SM) picture of flavor physics and CP violations. Although most of the measurements are in perfect agreement with the SM predictions, there still exist some unexplained mismatches. Especially, a combination of experimental data on a set of related decays will increase the tension between the SM predictions and experimental measurements. At present, there are discrepancies between the measurement of several observables in $B \rightarrow \pi K$ decays and the predictions of the SM, the so-called “$\pi K$ puzzle” [1], which have attracted extensive investigations in the SM [2 3 4 5 6 7], as well as with various specific New Physics (NP) scenarios [8].

Recently, Belle has measured the direct CP violations $B \rightarrow K\pi$ decays [9]

$$A_{CP}(\bar{B}^0 \rightarrow K^-\pi^+) \equiv \frac{\Gamma(\bar{B}^0 \rightarrow K^-\pi^+) - \Gamma(B^0 \rightarrow K^+\pi^-)}{\Gamma(\bar{B}^0 \rightarrow K^-\pi^+) + \Gamma(B^0 \rightarrow K^+\pi^-)} = -0.094 \pm 0.018 \pm 0.008. \quad (2)$$

The difference between direct CP violations in charged and neutral modes is

$$\Delta A \equiv A_{CP}(B^- \rightarrow K^-\pi^0) - A_{CP}(\bar{B}^0 \rightarrow K^-\pi^+) = 0.164 \pm 0.037. \quad (3)$$

The averages of the current experimental data of BABAR [10], Belle [9], CLEO [11] and CDF [12] by the Heavy Flavor Averaging Group (HFAG) [13] are

$$A_{CP}(B^- \rightarrow K^-\pi^0) = 0.050 \pm 0.025,$$

$$A_{CP}(\bar{B}^0 \rightarrow K^-\pi^+) = -0.097 \pm 0.012, \quad (4)$$

and the difference $\Delta A = 0.147 \pm 0.028$ is established at 5$\sigma$ level. However, within the SM, it is generally expected that $A_{CP}(\bar{B}_d^0 \rightarrow \pi^+K^-)$ and $A_{CP}(B_u^- \rightarrow \pi^0K^-)$ are close to each other. For example, the recent theoretical predictions for these two quantities based on the QCD factorization approach (QCDF) [14], the perturbative QCD approach (pQCD) [15] and the
soft-collinear effective theory (SCET) [16] read

\[
\begin{align*}
A_{CP}(B^-_u \to \pi^0 K^-)_{QCDF} &= -3.6\% , \\
A_{CP}(\bar{B}^0_d \to \pi^+ K^-)_{QCDF} &= -4.1\% ; \\
A_{CP}(B^-_u \to \pi^0 K^-)_{PQCD} &= (-1^{+3}_{-5})\% , \\
A_{CP}(\bar{B}^0_d \to \pi^+ K^-)_{PQCD} &= (-9^{+6}_{-8})\% ; \\
A_{CP}(B^-_u \to \pi^0 K^-)_{SCET} &= (-11 \pm 9 \pm 11 \pm 2)\% , \\
A_{CP}(\bar{B}^0_d \to \pi^+ K^-)_{SCET} &= (-6 \pm 5 \pm 6 \pm 2)\% .
\end{align*}
\]

We can see that the present theoretical estimations within the SM are confronted with the established \( \Delta A \). The mismatch may be due to our limited understanding of the strong dynamics in B decays which hinders precise estimations of the SM contributions, but equally possible due to new physics effects [17, 18].

As is known, the annihilation decay of B meson into two light mesons offers interesting probes for the dynamical mechanism governing these decays, as well as the exploration of CP violation. In most of B meson non-leptonic decays, the annihilation corrections could generate some strong phases, which are important for estimating CP violation. However, unlike the vertex-type correction amplitude, the calculation of annihilation amplitude always suffers from end-point divergence in collinear factorization approach. In the pQCD approach, such divergence is regulated by the parton transverse momentum \( k_T \) at expense of modeling additional \( k_T \) dependence of meson distribution functions [15], and a large strong phase is found. In the QCD factorization (QCDF) approach [14], to give a conservative estimation, the divergence is parameterized by complex parameters, \( X_A = \int^1_0 dy/y = \ln(m_b/\Lambda)(1 + \rho_A e^{i\phi_A}) \), with \( \rho_A \leq 1 \) and unrestricted \( \phi_A \), which will sometimes introduce large theoretical uncertainties in the final results. In Refs. [6, 19], annihilation diagram is studied with SCET and also parameterized by a complex amplitude. At present, the dynamical origin of these corrections still remains a theoretical challenge.

In this paper, we will revisit \( B \to \pi K, \pi K^* \) and \( \rho K \) decays within QCDF framework. However, we shall quote the infrared finite gluon propagator of Cornwall prescription [20] to regulate these divergences in hard-sepectator scattering and annihilation amplitudes. With this alternative scheme, we could evaluate both the strength and the strong phase of hard spectator and annihilation corrections at the expense of a dynamic gluon mass, which will be fitted in the
twelve decay modes. It is interesting to note that the infrared finite behavior of gluon propagator are not only obtained from solving the well known Schwinger-Dyson equation [20, 21, 22], but also supported by recent Lattice QCD simulations [23]. Numerically, a sizable strength and a large strong phase of annihilation corrections are found. Except $A_{CP}(B^\pm \to K^\pm \pi^0)$, our predictions for most of the branching ratios and the direct CP asymmetries of $B \to \pi K$, $\pi K^*$ and $\rho K$ agree with the current experimental data with an effective gluon mass $m_g = 0.45 \sim 0.55$ GeV. However, we get $A_{CP}(B^\pm \to K^\pm \pi^0) = -0.109 \pm 0.008$ which is still in sharp contrast to experimental data $0.050 \pm 0.025$. To resolve this mismatch, we perform a model-independent analysis of new physics contributions with a set of flavor-changing neutral current (FCNC) $\bar{s}(1+\gamma_5)b \otimes \bar{q}(1+\gamma_5)q$ (q=u,d) operators. To fit the twelve decay modes, parameter spaces are found generally with large weak phases. Our results indicate that both strong phase from annihilation amplitude and new weak phase from new physics are needed to account for the experimental data.

In Section 2, we revisit $B \to \pi K$, $\pi K^*$ and $\rho K$ decays in the SM with QCDF modified by an infrared finite gluon propagator for annihilation and spectator scattering kernels. After recalculating the hard-spectator scattering and the weak annihilation corrections, we present our numerical results and discussions. In Section 3, to find resolution to the CP violation difference $\Delta A$, we present analyses of NP operators. Then, using the constrained parameters for the operators, we discuss the mixing-induced CP violations in $B \to \pi^0 K_S$ and $\rho^0 K_S$. Section 4 contains our conclusions. Appendix A recapitulates the decay amplitudes for the twelve decay modes within the SM [3]. All the theoretical input parameters are summarized in Appendix B.

2 Revisiting $B \to \pi K$, $\pi K^*$ and $\rho K$ Decays in the SM

In the SM, the effective weak Hamiltonian responsible for $b \to s$ transitions is given as [24]

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{us}^* (C_1 O_1^u + C_2 O_2^u) + V_{cb} V_{cs}^* (C_1 O_1^c + C_2 O_2^c) - V_{tb} V_{ts}^* \left( \sum_{i=3}^{10} C_i O_i \right) + C_7 O_7 + C_8 g O_{8g} \right] + \text{h.c.,}
$$

(8)
where $V_{qb}V_{qs}^\ast$ ($q = u, c$ and $t$) are products of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [25], $C_i$ the Wilson coefficients, and $O_i$ the relevant four-quark operators whose explicit forms could be found, for example, in Refs. [2, 24].

In recent years, QCDF has been employed extensively to study the B meson non-leptonic decays. For example, all of the decay modes considered here have been studied comprehensively within the SM in Refs. [2, 3, 4, 26]. The relevant decay amplitudes for $B \to \pi K$, $\pi K^\ast$ and $\rho K$ decays within the QCDF formalism are shown in Appendix A. It is also noted that the framework contains estimates of some power-suppressed but numerically important contributions, such as the annihilation corrections. However, due to the appearance of endpoint divergence, these terms usually could not be computed rigorously. In Refs. [2, 3], to probe their possible effects conservatively, the endpoint divergent integrals are treated as signs of infrared sensitive contribution and phenomenological parameterized by

$$\int_0^1 \frac{dx}{x} \to X_A = (1 + \rho_A e^{i\phi_A}) \ln \frac{m_B}{\Lambda_h}, \quad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2} (X_A)^2$$

with $\rho_A \leq 1$ and $\phi_A$ unrestricted. The different scenarios corresponding to different choices of $\rho_A$ and $\phi_A$ have been thoroughly discussed in Ref. [3]. Although this way of parametrization seems reasonable, it is still very worthy to find some alternative schemes to regulate these endpoint divergences, as precise as possible, to estimate the strength and the associated strong phase in these power suppressed contributions.

It is interesting to note that recent theoretical and phenomenological studies are now accumulating supports for a softer infrared behavior of the gluon propagator [22, 27, 28]. Furthermore, an infrared finite dynamical gluon propagator, which is shown to be not divergent as fast as $\frac{1}{q^2}$, has been successfully applied to the B meson non-leptonic decays [29, 30]. Following these studies, in this paper we adopt the gluon propagator derived by Cornwall [20], to regulate the endpoint divergent integrals encountered within the QCDF formalism. The infrared finite gluon propagator is given by (in Minkowski space) [20]

$$D(q^2) = \frac{1}{q^2 - M_g^2(q^2) + i\epsilon},$$

where $q$ is the gluon momentum. The corresponding strong coupling constant reads

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln \left( \frac{q^2 + 4M_g^2(q^2)}{\Lambda_{QCD}^4} \right)},$$
where $\beta_0 = 11 - \frac{2}{3} n_f$ is the first coefficient of the beta function, and $n_f$ the number of active flavors. The dynamical gluon mass $M_g^2(q^2)$ is obtained as [20]

$$M_g^2(q^2) = m_g^2 \left[ \ln \left( \frac{q^2 + 4m_g^2}{\Lambda_{QCD}^2} \right) - \frac{11}{12} \right],$$

(12)

where $m_g$ is the effective gluon mass, with a typical value $m_g = 500 \pm 200$ MeV, and $\Lambda_{QCD} = 225$ MeV.

### 2.1 Recalculate the hard-spectator scattering and the annihilation contributions

The next-to-leading order penguin contractions and vertex-type corrections to these decays are known free of infrared divergence and well-defined in QCDF [2, 3, 4], for which we would not repeat the calculation and concentrate on the hard-spectator scattering and the annihilation contributions. With the infrared finite gluon propagator to deal with the endpoint divergences, we will re-calculate the hard spectator and the annihilation corrections in $B \to PP$ and $PV$ decays. The hard spectator scattering Feynman diagrams are shown in Fig. 1, where the spectator anti-quark goes from the $\bar{B}$ meson to the final-state $M_1$ meson and the $M_2$ meson is emitted from the weak vertex. The longitudinal momentum fraction of the constituent quark in the $M_2(1)$ meson is denoted by $x$ ($y$), and $\xi$ is the light-cone momentum fraction of the light anti-quark in the B meson. To leading power in $1/m_b$, the hard spectator scattering contributions can be expressed as (where $x, y \gg \xi$ is assumed)

$$H_i(M_1 M_2) = \frac{B_{M_1 M_2}}{A_{M_1 M_2}} \int_0^1 dx dy d\xi \frac{\alpha_s(q^2)}{\xi} \Phi_{B1}(\xi) \Phi_{M_2}(x) \left[ \frac{\Phi_{M_1}(y)}{x(y + \omega^2(q^2)/\xi)} + r_{x}^{M_1} \frac{\phi_{m_1}(y)}{x(y + \omega^2(q^2)/\xi)} \right],$$

(13)

Figure 1: Feynman diagrams of hard spectator-scattering contributions.
for the contributions of operators $Q_{i=1-4,9,10}$,

$$H_i(M_1M_2) = - \frac{B_{M_1M_2}}{A_{M_1M_2}} \int_0^1 dx dy d\xi \frac{\alpha_s(q^2)}{\xi} \Phi_{B1}(\xi) \Phi_{M_2}(x) \left[ \frac{\Phi_{M_1}(y)}{x(y + \omega^2(q^2)/\xi)} + r_{M_1} \frac{\phi_{m_1}(q)}{x(y + \omega^2(q^2)/\xi)} \right],$$

for $Q_{i=5,7}$, and $H_i(M_1M_2) = 0$ for $Q_{i=6,8}$.

In the above Eqs. (13) and (14), $\Phi_{B1}(\xi)$ is the B meson light-cone distribution amplitude(LCDA), $\Phi_{M_i}(x)$ and $\phi_{m_1}(y)$ are the twist-2 and the twist-3 LCDA s of light mesons, respectively, which are listed in Appendix B. $\omega^2(q^2) = M_B^2(q^2)/M_B^2$, $q^2 = -Q^2$ and $Q^2 \simeq -\xi y M_B^2$ is the space-like gluon momentum square in the scattering kernels. The quantities $A_{M_1M_2}$ and $B_{M_1M_2}$ collect relevant constants which can be found in Ref. [3].

The Feynman diagrams of the weak annihilation topologies are shown in Fig. 2. When both $M_1$ and $M_2$ are pseudoscalars, the final decay amplitudes can be expressed as

$$A_1^f = \pi \int_0^1 dx dy \alpha_s(q^2) \left\{ \frac{\bar{x}}{(\bar{y} - \omega^2(q^2) + i\epsilon)(1 - \bar{y})} + \frac{1}{(\bar{y} - \omega^2(q^2) + i\epsilon)\bar{x}} \right\} \Phi_{M_1}(y) \Phi_{M_2}(x),$$

$$A_2^f = A_2^f = 0,$$

$$A_2^f = \pi \int_0^1 dx dy \alpha_s(q^2) \left\{ \frac{y}{(\bar{y} - \omega^2(q^2) + i\epsilon)(1 - \bar{y})} + \frac{1}{(\bar{y} - \omega^2(q^2) + i\epsilon)y} \right\} \Phi_{M_1}(y) \Phi_{M_2}(x).$$

Figure 2: Feynman diagrams of weak annihilation contributions.
\[ A_3^i = \pi \int_0^1 dx dy \alpha_s(q^2) \left\{ \frac{2y}{(xy - \omega^2(q^2) + i\epsilon)(1 - xy)} r_{M_1}^{M_2}(x) \phi_{m_1}(y) \Phi_{M_2}(x) \right. \\
- \frac{2x}{(xy - \omega^2(q^2) + i\epsilon)(1 - xy)} r_{M_2}(x) \phi_{m_2}(x) \Phi_{M_1}(y) \right\}, \tag{18} \]

\[ A_3^f = \pi \int_0^1 dx dy \alpha_s(q^2) \left\{ \frac{2(1 + \bar{x})}{(xy - \omega^2(q^2) + i\epsilon)x} r_{M_1}^{M_2}(x) \phi_{m_1}(y) \Phi_{M_2}(x) \right. \\
+ \frac{2(1 + y)}{(xy - \omega^2(q^2) + i\epsilon)y} r_{M_2}(x) \phi_{m_2}(x) \Phi_{M_1}(y) \right\}, \tag{19} \]

where \( q^2 \simeq \bar{x}yM_B^2 \) is the time-like gluon momentum square. The “chirally-enhanced” factor \( r_{M}^{M_2} \) is presented in Appendix B. The superscript “\( i \)” and “\( f \)” refer to the gluon emission from initial- and final-state quarks, respectively. The subscript “1”, “2”, and “3” correspond to three possible Dirac structure, with “1” for \((V - A) \otimes (V - A)\), “2” for \((V - A) \otimes (V + A)\), and “3” for \((S - P) \otimes (S + P)\), respectively. When \( M_1 \) is a vector meson and \( M_2 \) a pseudoscalar, the sign of the second term in \( A_1^i \), the first term in \( A_2^i \), and the second terms in \( A_3^i \) and \( A_3^f \) are needed to be changed. When \( M_2 \) is a vector meson and \( M_1 \) a pseudoscalar, one only has to change the overall sign of \( A_2^i \).

Figure 3: The singularities in integral spaces (left figure) in annihilation contributions and the variations of strong coupling constant corresponding to different \( m_\alpha \) choices (in unit of GeV).

As shown by Eqs. (13) and (14) of the hard-spectator scattering contributions, the endpoint divergences are regulated by the infrared finite form of the gluon propagator. It is easy to observe from Eqs. (13) and (14) that hard-spectator scattering contributions are real. For the annihilation contributions shown by Eqs. (15)–(19), singularities of the time-like gluon propagators at the end-point of integrations (end-point divergence) are moved into integral
Table 1: The $CP$-averaged branching ratios (in units of $10^{-6}$) of $B \to \pi K, \pi K^*$ and $\rho K$ decays in SM with different $m_g$ (in unit of GeV) are presented in QCDF columns.

| Decay Mode       | QCDF          | Experiment   |
|------------------|---------------|--------------|
|                  | $m_g = 0.3$   |              | $m_g = 0.7$   |              | $m_g = 0.45 \sim 0.55$ | data     |
| $B_u^- \to \pi^- \overline{K}^0$ | 44.4          | 16.8         | 23.17 ± 3.28 | 23.1 ± 1.0 |
| $B_u^- \to \pi^0 K^-$         | 23.4          | 9.3          | 12.50 ± 1.65 | 12.9 ± 0.6  |
| $\overline{B}_d^0 \to \pi^+ K^-$ | 44.7          | 16.3         | 22.71 ± 3.27 | 19.4 ± 0.6  |
| $\overline{B}_d^0 \to \pi^0 \overline{K}^0$ | 21.2          | 7.3          | 10.50 ± 1.63 | 9.9 ± 0.6   |
| $B_u^- \to \pi^- \overline{K}^*^0$ | 28.3          | 5.2          | 8.90 ± 1.59  | 10.0 ± 0.8  |
| $B_u^- \to \pi^0 K^*$          | 15.2          | 3.4          | 5.25 ± 0.83  | 6.9 ± 2.3   |
| $\overline{B}_d^0 \to \pi^+ K^*$ | 28.7          | 5.3          | 9.13 ± 1.68  | 10.6 ± 0.9  |
| $\overline{B}_d^0 \to \pi^0 \overline{K}^*$ | 13.4          | 1.9          | 3.89 ± 0.82  | 2.4 ± 0.7   |
| $B_u^- \to \rho^- \overline{K}^0$ | 31.8          | 5.6          | 10.27 ± 1.96 | 8.0^{+1.5}_{-1.4} |
| $B_u^- \to \rho^0 K^-$         | 14.9          | 2.5          | 4.81 ± 0.94  | 3.81^{+0.48}_{-0.46} |
| $\overline{B}_d^0 \to \rho^+ K^-$ | 38.6          | 8.0          | 13.42 ± 2.31 | 8.6^{+0.9}_{-1.1} |
| $\overline{B}_d^0 \to \rho^0 \overline{K}^*$ | 21.0          | 4.8          | 7.53 ± 1.25  | 5.4^{+0.9}_{-1.0} |

Intervals with the infrared finite form of the gluon propagator. Singularities in the integral intervals and variations of the effective strong coupling constant are shown in Fig. 3. It is noted that effective strong coupling constant is finite, but rather large in the small $q^2$ region. However, there is strong cancellations among the contributions of the small $q^2$ region nearby $m_g^2$, which renders the annihilation contribution dominated by $q^2 > m_g^2$ region associated with a large imaginary part. This situation is quite similar to pQCD [15] where the large imaginary part from propagator regulated by $k_T$

$$\frac{1}{xym_B^2 - k_T^2 + i\epsilon} = P\left(\frac{1}{xym_B^2 - k_T^2}\right) - i\pi\delta(xym_B^2 - k_T^2),$$

and it is also found the power suppression of these terms relative to the leading contributions was not very significant, and important to account for CP violations in $B \to \pi K$ decays.
Table 2: The direct CP asymmetries (in unit of $10^{-2}$) of $B \to \pi K$, $\pi K^*$ and $\rho K$ decays in SM with different $m_g$ (in unit of GeV). Other captions are the same as Table 1.

| Decay Mode | QCDF | | | Experiment |
|------------|------|-------------------------------------------------|---------------------------------|------------------|
| $B_{u}^{-} \to \pi^{-}K^{0}$ | 0.06 | 0.19 | 0.10 ± 0.08 | 0.9 ± 2.5 |
| $B_{u}^{-} \to \pi^{0}K^{-}$ | −11.6 | −8.3 | −10.85 ± 0.84 | 5.0 ± 2.5 |
| $\overline{B}_{d}^{0} \to \pi^{+}K^{-}$ | −11.0 | −11.4 | −12.38 ± 0.69 | −9.7 ± 1.2 |
| $\overline{B}_{d}^{0} \to \pi^{0}K^{0}$ | 2.5 | 0.1 | 1.39 ± 0.35 | −14 ± 11 |
| $B_{u}^{-} \to \pi^{-}K^{*0}$ | 0.3 | −0.0 | 0.16 ± 0.16 | −11.4 ± 6.1 |
| $B_{u}^{-} \to \pi^{0}K^{*+}$ | −27.0 | −34.1 | −41.20 ± 6.69 | 4 ± 29 |
| $\overline{B}_{d}^{0} \to \pi^{+}K^{*+}$ | −27.2 | −47.6 | −47.58 ± 8.42 | −10 ± 11 |
| $\overline{B}_{d}^{0} \to \pi^{0}K^{*0}$ | 3.9 | 2.1 | 4.67 ± 1.14 | −9^{+32}_{-23} |
| $B_{u}^{-} \to \rho^{-}K^{0}$ | 0.1 | 1.2 | 0.53 ± 0.21 | −12 ± 17 |
| $B_{u}^{-} \to \rho^{0}K^{-}$ | 28.1 | 49.7 | 46.27 ± 5.94 | 37 ± 11 |
| $\overline{B}_{d}^{0} \to \rho^{+}K^{-}$ | 19.3 | 31.5 | 31.40 ± 4.63 | 15 ± 13 |
| $\overline{B}_{d}^{0} \to \rho^{0}K^{0}$ | −4.2 | 0.2 | −3.26 ± 1.29 | −2 ± 29 |

### 2.2 The branching ratios and direct CP asymmetries in the SM

With the prescriptions for the endpoint divergences, we will present our numerical results of branching ratios and CP violations in these decays. Decay amplitudes and input parameters are listed in Appendices A and B, respectively. Our results are summarized in Table 1 and Table 2 where the relevant experimental data are also tabled for comparison.

In Table 1 (2), the experimental data column is the up-to-date averages for these branching ratios (direct CP violations) by HFAG [13]. It is shown that all the results are in good agreements with the experimental data with $m_g = 0.45 \sim 0.55$ GeV. It is also noted that the dynamical gluon mass $m_g = 0.45 \sim 0.55$ GeV are also consistent with findings in other phenomenal studies of B decays [29, 30] and the different solutions of SDE [20, 21, 22]. The phenomenology successes may indicate that the gluon mass, although not a directly measurable
quantity, furnishes a regulator for infrared divergences of QCD scattering processes.

From the CP averaged branching ratios in the fourth column of Table I we get

\[ R_c \equiv 2 \left[ \frac{Br(B^+ \to \pi^0 K^-)}{Br(B^+ \to \pi^- K^0)} \right] = 1.08 \pm 0.30, \]
\[ R_n \equiv \frac{1}{2} \left[ \frac{Br(B^0 \to \pi^+ K^-)}{Br(B^0 \to \pi^0 K^0)} \right] = 1.08 \pm 0.32, \] (21)

which agree with the experimental data \( R_c = 1.12 \pm 0.10 \) and \( R_n = 0.98 \pm 0.09 \) [13].

Table 2 is our results for direct CP violations. The fourth column is the results estimated with \( m_g = 0.45 \sim 0.55 \) GeV fixed by branching ratios, where the error-bars are simply due to the \( m_g \) variations. Compared with the experimental data, our results, except \( A_{CP}(B^+_u \to \pi^0 K^-) \), agree with the measurements. For the most significant experimental result among the measurements of direct CP violations in the twelve decay modes \( A_{CP}(\bar{B}^0_d \to \pi^+ K^-) = -0.097 \pm 0.012 \) [13], our result \( A_{CP}(\bar{B}^0_d \to \pi^+ K^-) = -0.124 \pm 0.007 \) is in good agreement with it. As expected in the SM, we find again \( A_{CP}(B^-_u \to \pi^0 K^-) = -0.108 \pm 0.008 \) very close to \( A_{CP}(\bar{B}^0_d \to \pi^+ K^-) \), which are generally in agreement with the results of Refs. [3, 5, 6] listed in Eq. (5)–(7). So, it is very hard to accommodate the measured large difference between \( A_{CP}(B^-_u \to \pi^0 K^-) \) and \( A_{CP}(\bar{B}^0_d \to \pi^+ K^-) \) in the SM with the available approaches for hadron-dynamics in B decays.

Although the problem could be due to hadronic effects unknown so far, the difference between \( A_{CP}(B^-_u \to \pi^0 K^-) \) and \( A_{CP}(\bar{B}^0_d \to \pi^+ K^-) \) could be an indication of new sources of CP violation beyond the SM [18, 31, 32].

### 3 Possible resolution with new \((S+P) \otimes (S+P)\) operators

In this Section we will pursue possible NP solutions model-independently with a set of FCNC \((S+P) \otimes (S+P)\) operators. The effects of anomalous tensor and (pseudo-)scalar operators on hadronic B decays have attracted many attentions recently [31, 33, 34, 35, 36, 37]. For example, it is shown that they could help to resolve the abnormally large transverse polarizations observed in \( B \to \phi K^* \) decay, as well as the large \( Br(B \to \eta K^*) \) [36].

The general four-quark tensor operators can be expressed as

\[ O^q_T = \bar{s}\sigma_{\mu\nu}(1 + \gamma_5)b \otimes \bar{q}\sigma^{\mu\nu}(1 + \gamma_5)q, \quad O'^q_T = \bar{s}_i\sigma_{\mu\nu}(1 + \gamma_5)b_j \otimes \bar{q}_j\sigma^{\mu\nu}(1 + \gamma_5)q_i, \] (22)
which could be expressed, through the Fierz transformations, as linear combinations of the 
(pseudo-)scalar operators. In our present case, however, we find that the tensor operators with 
$q = u, d$ give the same contributions to the $B_u^- \rightarrow \pi^0 K^-$ and $B_d^0 \rightarrow \pi^+ K^-$ decays, so that they are 
hardly possible to resolve the direct CP violation difference, because after Fierz transformations, 
$O_T^q$ and $O_T'^q$ with $q = u, d$ will give operators like $\bar{q}(1 + \gamma_5)b \otimes \bar{s}(1 + \gamma_5)q$ which are different 
from $\bar{s}(1 + \gamma_5)b \otimes \bar{s}(1 + \gamma_5)s$ of the Fierz transforming $O_T = \bar{s}\sigma_{\mu\nu}(1 + \gamma_5)b \otimes \bar{s}\sigma^{\mu\nu}(1 + \gamma_5)s$ 
for $B \rightarrow \phi K^*$ decays. On the other hand, the new operators like $\bar{s}(1 + \gamma_5)b \otimes \bar{q}(1 + \gamma_5)q$ may 
give a possible solution to $\Delta A$ because of their different contributions to the $B^- \rightarrow \pi^0 K^-$ and 
$\bar{B}^0 \rightarrow \pi^+ K^-$ decays.

We write the NP effective Hamiltonian for $b \rightarrow s$ transitions as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,d} |V_{tb}V_{ts}|^2 e^{i\delta_q} \left[ C_{S1}^q O_{S1}^q + C_{S8}^q O_{S8}^q \right] + \text{h.c.} ,$$

with $O_{S1}^q$ and $O_{S8}^q$ defined by

$$O_{S1}^u = \bar{s}(1 + \gamma_5)b \otimes \bar{u}(1 + \gamma_5)u , \quad O_{S1}^d = \bar{s}(1 + \gamma_5)b \otimes \bar{d}(1 + \gamma_5)d ,$$

$$O_{S8}^u = \bar{s}_i(1 + \gamma_5)b_j \otimes \bar{u}_j(1 + \gamma_5)u_i , \quad O_{S8}^d = \bar{s}_i(1 + \gamma_5)b_j \otimes \bar{d}_j(1 + \gamma_5)d_i ,$$

where $i$ and $j$ are color indices. The coefficient $C_{S1(S8)}^q$ describes the relative interaction strength 
of the operator $O_{S1(S8)}^q$, and $\delta_q$ is their possible NP weak phase. Since both the coefficients and 
the weak phase are unknown parameters, for simplicity, we shall only consider their leading 
contributions with the naive factorization(NF) approximation.

![Feynman diagrams](image)

Figure 4: Feynman diagrams contributing to the amplitudes of $B \rightarrow \pi K, \pi K^*$ and $\rho K$ decays 
due to the $(S + P) \otimes (S + P)$ operators.
The relevant Feynman diagrams of the NP operators are shown in Fig. 4 with $q = u, d$. With the NF approximation, it is easy to see that, for the $B \to \pi^0 K^{*-}$ and $\pi^0 K^{*0}$ decay modes, only Fig. 4 (a) contributes, for the $B \to \pi^- K^0$, $\pi^+ K^-$ and $\rho K$ decay modes, only Fig. 4 (b) contributes, while both topology structures contribute to the $B \to \pi^0 K^-$ and $\pi^0 K^{*0}$ decay modes. However, none of them contributes to $B \to \pi^- K^{*0}$ and $\pi^+ K^{*-}$ decays. After some simple calculations, these NP contributions to the decay amplitudes of the $B \to \pi K$, $\pi K^*$ and $\rho K$ decays are obtained as

$$\mathcal{A}_{B^0 \to \pi^- K^0}^{NP} = i \frac{G_F}{\sqrt{2}} \frac{1}{4} |V_{tb} V_{ts}^*| m_{B_u}^2 e^{i \delta_S} g_S^u g^d r^K r^K_{F_0} (m^2_K) f_K,$$

$$\mathcal{A}_{B^0 \to \pi^0 K^-}^{NP} = i \frac{G_F}{\sqrt{2}} \frac{1}{4\sqrt{2}} |V_{tb} V_{ts}^*| m_{B_u}^2 [e^{i \delta_S} g_S^u g^d r^K r^K_{F_0} (m^2_K) f_K - 2 (e^{i \delta_S} g_S^u - e^{i \delta_S} g^d) r^K r^K_{F_0} (m^2_K) f_K],$$

$$\mathcal{A}_{B^0 \to \pi^+ K^-}^{NP} = i \frac{G_F}{\sqrt{2}} \frac{1}{4\sqrt{2}} |V_{tb} V_{ts}^*| m_{B_u}^2 e^{i \delta_S} g_S^u g^d r^K r^K_{F_0} (m^2_K) f_K,$$

$$\mathcal{A}_{B^0 \to \pi^0 K^+}^{NP} = i \frac{G_F}{\sqrt{2}} \frac{1}{4\sqrt{2}} |V_{tb} V_{ts}^*| m_{B_u}^2 [- e^{i \delta_S} g_S^d g^u r^K r^K_{F_0} (m^2_K) f_K - 2 (e^{i \delta_S} g_S^d - e^{i \delta_S} g^u) r^K r^K_{F_0} (m^2_K) f_K],$$

$$\mathcal{A}_{B^0 \to \pi^- K^0}^{NP} = 0,$$

$$\mathcal{A}_{B^0 \to \pi^0 K^-}^{NP} = i \frac{G_F}{\sqrt{2}} \frac{1}{2\sqrt{2}} |V_{tb} V_{ts}^*| m_{B_u}^2 [e^{i \delta_S} g_S^u g^d - e^{i \delta_S} g^d g_S^u] r^K r^K_{F_0} (m^2_K) f_K,$$

$$\mathcal{A}_{B^0 \to \pi^+ K^-}^{NP} = 0,$$

$$\mathcal{A}_{B^0 \to \pi^0 K^+}^{NP} = i \frac{G_F}{\sqrt{2}} \frac{1}{2\sqrt{2}} |V_{tb} V_{ts}^*| m_{B_u}^2 [e^{i \delta_S} g_S^u g^d - e^{i \delta_S} g^d g_S^u] r^K r^K_{F_0} (m^2_K) f_K,$$

$$\mathcal{A}_{B^0 \to \rho^- K^0}^{NP} = - i \frac{G_F}{\sqrt{2}} \frac{1}{4\sqrt{2}} |V_{tb} V_{ts}^*| m_{B_u}^2 e^{i \delta_S} g_S^u g^d r^K r^K_{F_0} (m^2_K) f_K,$$

$$\mathcal{A}_{B^0 \to \rho^0 K^-}^{NP} = - i \frac{G_F}{\sqrt{2}} \frac{1}{4\sqrt{2}} |V_{tb} V_{ts}^*| m_{B_u}^2 e^{i \delta_S} g_S^u g^d r^K r^K_{F_0} (m^2_K) f_K,$$

$$\mathcal{A}_{B^0 \to \rho^+ K^-}^{NP} = - i \frac{G_F}{\sqrt{2}} \frac{1}{4\sqrt{2}} |V_{tb} V_{ts}^*| m_{B_u}^2 e^{i \delta_S} g_S^u g^d r^K r^K_{F_0} (m^2_K) f_K,$$

$$\mathcal{A}_{B^0 \to \rho^0 K^+}^{NP} = i \frac{G_F}{\sqrt{2}} \frac{1}{4\sqrt{2}} |V_{tb} V_{ts}^*| m_{B_u}^2 e^{i \delta_S} g_S^d g^u r^K r^K_{F_0} (m^2_K) f_K,$$

where

$$g_S^u = C_{S1}^u + \frac{1}{N_c} C_{SS}^u,$$

$$g_S^d = C_{S1}^d + \frac{1}{N_c} C_{SS}^d.$$
Comparing the NP amplitudes Eq. (26) with Eq. (27), we expect that these new (pseudo-)scalar operators might provide a possible resolution to the direct CP violation difference, which is realized in the following numerical analyses.

3.1 Numerical analyses and discussions of new pseudo-scalar operators

Our analysis consists of five cases with different assumptions for dominance of NP operators, namely,

- Case I: $b \rightarrow s u \bar{u}$ operators $O^u_{S1}$ and $O^u_{S8}$,
- Case II: $b \rightarrow s d \bar{d}$ operators $O^d_{S1}$ and $O^d_{S8}$,
- Case III: $b \rightarrow s d \bar{d}$ operator $O^d_{S1}$ solely,
- Case IV: only color singlet operators $O^u_{S1}$ and $O^d_{S1}$,
- Case V: all the operators $O^u_{S1}$, $O^u_{S8}$, $O^d_{S1}$ and $O^d_{S8}$.

For each case, the corresponding effective Hamiltonian could be read from Eq. (23). It could be expected that a collection of related decay modes could constrain the relevant NP parameter spaces restrictively.

Our fitting is performed with the experimental data varying randomly within their $2\sigma$ error-bars, while the theoretical uncertainties are obtained by varying the input parameters within the regions specified in Appendix B. Our numerical results are summarized in Table 3–5 where the assigned uncertainties of our fitting results should be understood at $2\sigma$ statistical level. Illustratively, the constrained NP parameter spaces are shown in Figs. 5–9, respectively. It is noted that, to leading order approximation, both $B^-_u \rightarrow \pi^- K^{*0}$ and $\bar{B}^0_d \rightarrow \pi^+ K^{*-}$ decays do not receive these NP contributions, so we perform fitting for the remained ten decay modes. In the following, we present numerical analyses subdivided into five cases.

**Case I: $b \rightarrow s u \bar{u}$ operators $O^u_{S1}$ and $O^u_{S8}$**

We just take into account the contributions of $O^u_{S1}$ and $O^u_{S8}$ in Eq. (23), i.e. $C^u_{S1} = C^u_{S8} = 0$. In this case, we take the branching ratios of the seven relevant decays $B^-_u \rightarrow \pi^0 K^-$, $\pi^0 K^{*-}$,
Table 3: The $CP$-averaged branching ratios (in units of $10^{-6}$) in different NP Cases with $m_g = 0.5 GeV$. The dash means (pseudo-)scalar operators of the Case irrelevant to the corresponding decay mode.

| Decay Mode | Experiment | NP |
|------------|------------|----|
| $B_u^- \rightarrow \pi^- K^0$ | 23.1 ± 1.0 | 23.0 ± 1.0 | 22.9 ± 0.9 | 21.5 ± 0.3 | 22.4 ± 0.9 |
| $B_u^- \rightarrow \pi^0 K^-$ | 12.9 ± 0.6 | 12.1 ± 0.4 | 12.8 ± 0.7 | 12.7 ± 0.6 | 12.1 ± 0.3 | 12.1 ± 0.4 |
| $\bar{B}_d^0 \rightarrow \pi^+ K^-$ | 19.4 ± 0.6 | 20.2 ± 0.3 | — | — | 20.4 ± 0.2 | 20.1 ± 0.4 |
| $\bar{B}_d^0 \rightarrow \pi^0 K^0$ | 9.9 ± 0.6 | 9.0 ± 0.3 | 9.9 ± 0.6 | 10.0 ± 0.7 | 9.0 ± 0.2 | 9.1 ± 0.4 |
| $B_u^- \rightarrow \pi^0 K^*$ | 6.9 ± 2.3 | 4.2 ± 0.2 | 4.4 ± 0.4 | 4.4 ± 0.4 | 4.3 ± 0.3 | 4.3 ± 0.3 |
| $\bar{B}_d^0 \rightarrow \pi^0 K^0$ | 2.4 ± 0.7 | 3.4 ± 0.3 | 3.5 ± 0.2 | 3.5 ± 0.2 | 3.1 ± 0.3 | 2.9 ± 0.2 |
| $B_u^- \rightarrow \rho^- K^0$ | 8.0^{+1.5}_{-1.4} | — | 8.6 ± 0.7 | 8.6 ± 0.7 | 7.4 ± 0.4 | 7.1 ± 0.4 |
| $B_u^- \rightarrow \rho^0 K^-$ | 3.81^{+0.48}_{-0.46} | 3.4 ± 0.2 | — | — | 3.4 ± 0.2 | 3.4 ± 0.2 |
| $\bar{B}_d^0 \rightarrow \rho^+ K^-$ | 8.6^{+0.9}_{-1.1} | 9.7 ± 0.5 | — | — | 9.7 ± 0.5 | 9.8 ± 0.5 |
| $\bar{B}_d^0 \rightarrow \rho^0 K^0$ | 5.4^{+0.9}_{-1.0} | — | 6.5 ± 0.4 | 6.5 ± 0.4 | 5.5 ± 0.3 | 5.4 ± 0.4 |

$\rho^0 K^-$ and $\bar{B}_d^0 \rightarrow \pi^+ K^-$, $\pi^0 K^0$, $\pi^0 K^*$, $\rho^+ K^-$ as constraints and leave the direct CP asymmetries as our predictions. The allowed regions of the NP parameters $C^u_{S1}$, $C^u_{SS}$ and $\delta^u_S$ are shown in Fig. 5. From which, we find the spaces of $C^u_{S1}$ and $\delta^u_S$ consist of two parts (dark and gray). However, with the gray part, we get $A_{CP}(B_u^- \rightarrow \pi^0 K^-) = -0.154 \pm 0.038$ which conflicts with experimental data 0.050 ± 0.025. So, the gray region should be excluded. With the dark part of parameter spaces, our prediction $A_{CP}(B_u^- \rightarrow \pi^0 K^-) = 0.088 \pm 0.064$ is consistent with experimental data. Furthermore, the branching ratios and direct CP asymmetries of the other decay modes, listed in the third column of Table 3 and 4 agree with experimental data within error bars. The constrained parameter space $C^u_{S1}$, $C^u_{SS}$ and $\delta^u_S$ are listed in the second column of Table 4. We note that $C^u_{S1} \approx -C^u_{SS} \approx -0.04$ with $\delta^u_S \approx 100^\circ$, it means the strength of color-singlet and color-octet operators are similar, however, such a situation may be hard to be generated with a realistic available NP model.
Table 4: The direct CP asymmetries (in unit of $10^{-2}$) of $B \to \pi K$, $\pi K^*$ and $\rho K$ decays. Other captions are the same as Table 3.

| Decay Mode       | Experiment data | Case I  | Case II | Case III | Case IV  | Case V  |
|------------------|-----------------|---------|---------|----------|----------|---------|
| $B^+_u \to \pi^- K^0$ | $0.9 \pm 2.5$   | —       | $1.7 \pm 2.9$ | $2.0 \pm 0.2$ | $3.9 \pm 1.0$ | $3.2 \pm 1.3$ |
| $B^+_u \to \pi^0 K^-$ | $5.0 \pm 2.5$   | $8.8 \pm 6.4$ | $1.1 \pm 0.9$ | $1.2 \pm 0.9$ | $2.8 \pm 5.5$ | $1.8 \pm 1.3$ |
| $B^+_d \to \pi^+ K^-$ | $-9.7 \pm 1.2$  | $-5.7 \pm 4.4$ | —       | —       | $-10.0 \pm 0.8$ | $-9.2 \pm 1.3$ |
| $B^+_d \to \pi^0 K^0$ | $-14 \pm 11$    | $-18.6 \pm 7.5$ | $-12.8 \pm 3.9$ | $-12.6 \pm 1.6$ | $-10.2 \pm 7.0$ | $-8.2 \pm 2.8$ |

Table 5: The numerical results for the parameters $C^{u}_{S1}$, $C^{u}_{S1}$, $\delta^{u}_S$, $C^{d}_{S1}$, $C^{d}_{S8}$ and $\delta^{d}_S$ in different NP Cases. The dashes mean the corresponding operators are neglected in the Case.

| NP para.                      | Case I       | Case II      | Case III     | Case IV      | Case V       |
|-------------------------------|--------------|--------------|--------------|--------------|--------------|
| $C^{u}_{S1} \times 10^{-3}$   | $-41.6 \pm 13.4$ | —           | —            | $25.8 \pm 8.4$ | $-6.7 \pm 10.5$ |
| $C^{u}_{S8} \times 10^{-3}$   | $38.7 \pm 18.2$ | —           | —            | —            | $16.0 \pm 7.1$ |
| $\delta^{u}_S$               | $99.5^\circ \pm 6.1^\circ$ | —            | —            | $107.0^\circ \pm 11.5^\circ$ | $73.0^\circ \pm 23.8^\circ$ |
| $C^{d}_{S1} \times 10^{-3}$   | —            | $23.0 \pm 5.1$ | $22.8 \pm 2.3$ | $50.3 \pm 12.8$ | $17.5 \pm 10.1$ |
| $C^{d}_{S8} \times 10^{-3}$   | —            | $-0.8 \pm 13.7$ | —            | —            | $10.5 \pm 9.4$ |
| $\delta^{d}_S$               | —            | $100.0^\circ \pm 8.7^\circ$ | $99.3^\circ \pm 9.2^\circ$ | $106.6^\circ \pm 7.3^\circ$ | $114.7^\circ \pm 18.6^\circ$ |

**Case II:** $b \to s d \bar{d}$ operators $O^{d}_{S1}$ and $O^{d}_{S8}$

In a large category of NP scenarios with scalar interactions, for example, two-Higgs doublets model II, down type fermion Yukawa couplings are enhanced. So, in this case, we evaluate the
effects of $O_{S1}^d$ and $O_{S8}^d$ and neglect $O_{S1}^u$ and $O_{S8}^u$.

As shown by Eqs. (25)–(36), $O_{S1(8)}^d$ contributes to the decays $B\rightarrow\pi^-K^0$, $\pi^0K^-$, $\pi^0K^{*-}$, $\rho^-K^0$, $\pi^0K^0$, $\pi^0K^{*0}$, and $\rho^0K^0$. From Table 1 one can find that the SM predictions for their branching ratios are consistent with the experimental data. So, in this Case, NP weak phase $\delta_{S1}^d$ would be arbitrary for very small strengths of $C_{S1}^d$ and $C_{S8}^d$, we thus have to take into account both branching ratios and direct CP violations as constraints. The allowed region of $C_{S1}^d$, $C_{S8}^d$ and $\delta_{S1}^d$ are shown in Fig. 6. The fitted results are shown in the fourth column of Table 3, 4 and the third column of Table 5. Interestingly, we note that $C_{S1}^d = 0.023 \pm 0.005$, $C_{S8}^d = -0.001 \pm 0.013$ (consistent with zero) with $\delta_{S1}^d \approx 100^\circ$. It indicates that color-singlet operator $O_{S1}^d$ dominates the NP $b\rightarrow sd\bar{d}$ contributions. Actually, with $O_{S1}^d$ only, we could find a solution to the “$\pi K$ puzzle” which will be discussed in next Case.

Compared with Case I, it is found that $|C_{S1}^d| < |C_{S1}^u| \approx |C_{S8}^u|$. However, we can’t conclude that $O_{S1(8)}^u$ dominates the NP contribution until we consider the two operators simultaneously, which will be discussed in coming Case IV and Case V.

**Case III: $b\rightarrow sd\bar{d}$ operator $O_{S1}^d$ solely**

As the former Case, both branching ratio and direct CP violation are taken as constraints. With $O_{S1}^d$ solely, we find a solution to the “$\pi K$ puzzle” with the $C_{S1}^d$ and $\delta_{S1}^d$ allowed region shown in Fig. 7. The numerical results are listed in fifth column of Table 3, 4 and fourth column of Table 5 respectively. $C_{S1}^d$ and $\delta_{S1}^d$ are found similar to the ones of Case II. It confirms our findings in Case II that $O_{S1}^d$ dominates the NP contributions and the contribution of $O_{S8}^d$ is
Case IV: only color-singlet operators $O^u_{S1}$ and $O^d_{S1}$

In order to compare the relative strength of two color singlet operators $O^d_{S1}$ and $O^u_{S1}$, we take them into account at the same time and neglect the other two color-octet ones. Taking the branching ratios of the relevant decays as constraints, we find the allowed regions for the NP parameters $C^u_{S1}$, $\delta^u_{S}$, $C^d_{S1}$ and $\delta^d_{S}$, which are shown in Fig. 8. All our predictions for the direct CP violations, listed in sixth column of Table 4, agree with experimental data. Especially, we note our predictions $A_{CP}(B^- \to \pi^0 K^-) = 0.028 \pm 0.055$ and $\Delta A = 0.128 \pm 0.056$ agree with
experimental data very well.

The fifth column of Table 5 is the parameter space obtained for the present Case. We find that strength of \( C_{dS1} \) in Case IV is larger than the ones in Case II and Case III, because the terms of \( C_{dS1} \) and \( C_{uS1} \) always have opposite sign in Eqs. (26), (28), (30) and (32), but only one of them exists in the other decay modes. It is found that \( C_{dS1} \approx 2 \times C_{uS1} \approx 0.05 \) with \( \delta_d \approx \delta_u \approx 107^\circ \), which shows \( O_{dS1} \) dominance.

![Figure 8: The allowed regions for the parameters \( C_{uS1}^u, \delta_u, C_{dS1}^d \) and \( \delta_d \) of Case IV.](image)

**Case V: all the operators** \( O_{uS1}^u, O_{uS8}^u, O_{dS1}^d \) and \( O_{S8}^d \)

At last, we fit the measured branching ratios and the direct CP violations of all the relevant ten decay models with the four operators in Eq. (24). Generally the ten CP averaged branching ratios are measured with high significants, however, only \( A_{CP}(B^0 \rightarrow \pi^\pm K^\mp) \) is well established at 8\( \sigma \) level and \( A_{CP}(B^- \rightarrow \pi^0 K^-) \) by itself is at 2\( \sigma \) level only.

From the fit, the allowed regions for the six NP parameters \( C_{uS1}^u, C_{uS8}^u, \delta_u, C_{dS1}^d, C_{S8}^d \) and \( \delta_d \) shown in Fig. 9. The fitted branching ratios and CP violations are listed in the seventh column of Table 3 and 4, and the fitted values of the NP parameters are presented in the last column of Table 5, respectively. Since the experimental data are allowed varying randomly within their 2\( \sigma \) error-bars, the uncertainties of our fitting results are turned to be quite large.

We find \( C_{uS1}^u = (-6.7 \pm 10.5) \times 10^{-3} \) and \( C_{S8}^u = (16.0 \pm 7.1) \times 10^{-3} \) with \( \delta_u = 73.0^\circ \pm 23.8^\circ \), which shift our predication \( A_{CP}(B^0 \rightarrow \pi^\pm K^\mp) \approx -0.124 \) in the SM more closer to the experimental data \(-0.097\). However, it does not indicate that the \( b \rightarrow su\bar{u} \) operators are important for resolving CP violation difference \( \Delta A \), since the sum of their contributions to \( B^- \rightarrow \pi^0 K^- \) is quite small due to cancellation among them. For the \( b \rightarrow sd\bar{d} \) operators, we
get $C_{S1}^d = (17.5 \pm 10.1) \times 10^{-3}$ and $C_{S8}^d = (10.5 \pm 9.4) \times 10^{-3}$ with $\delta_S^d = 114.7^\circ \pm 18.6^\circ$. The results are consistent with those of Case II and Case III as shown in Table 5, however, due to interferences with $b \to s u \bar{u}$ contributions, the uncertainties are much larger than the two former Cases where $b \to s u \bar{u}$ operators are dropped. Moreover, as shown by Eq. (26), $C_{S8}^d$ is suppressed by $1/N_c$ in the amplitude of $B^- \to \pi^0 K^-$, thus, the dominate status of $O_{S1}^d$ for resolving $\Delta A$ is remained.

### 3.2 The mixing-induced CP asymmetries in $B \to \pi^0 K_S$ and $B \to \rho^0 K_S$

So far we have discussed the direct CP asymmetries in these decays with five NP scenarios. However, it is naturally to question if we can account for the mixing-induced CP asymmetries in $B^0 \to \pi^0 K_S$ and $\rho^0 K_S$ decays with these constrained parameter spaces obtained in the former subsection. As known, the mixing-induced asymmetries are more suitable for probing new physics effects entered via $b \to sq\bar{q}$ parton processes than the direct ones, since the former ones...
Table 6: The mixing-induced CP asymmetries (in unit of $10^{-2}$) of $\bar{B}^0 \to \pi^0 K_S, \rho^0 K_S$ decays. Other captions are the same as Table 3.

| Decay Mode | Experiment | SM | NP |
|------------|------------|----|----|
| $\bar{B}^0_d \to \pi^0 K_S$ | 38 ± 19 | 77 ± 4 | 45 ± 11 | 56 ± 5 | 57 ± 3 | 59 ± 9 | 62 ± 8 |
| $\bar{B}^0_d \to \rho^0 K_S$ | 61$^{+25}_{-27}$ | 66 ± 3 | — | 61 ± 6 | 61 ± 3 | 56 ± 3 | 57 ± 4 |

could be predicted more accurately in QCDF. Detail discussions for the interesting feature could be found in Ref. [38]. Recently, the measured relative small mixing-induced asymmetry (with large error-bar) in $\bar{B}^0 \to \pi^0 K_S$ has attracted much attention in the literature [38, 39, 40, 18, 41].

The time-dependent CP asymmetries in $\bar{B}^0 \to \pi^0 K_S$ and $\rho^0 K_S$ decays could be written as

$$A_f(t) = S_f \sin(\Delta m_d t) - C_f \cos(\Delta m_d t), \quad (38)$$

where $-C_f \equiv A_{CP}$ is the direct CP violation already discussed in former subsection. $S_f = A_{CP}^{mix}$ is the mixing-induced asymmetry

$$A_{CP}^{mix}(\bar{B}^0 \to f) = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2} \quad (f = \pi^0 K_S, \rho^0 K_S, \eta_f = -1) \quad (39)$$

where $\lambda_f = -e^{-2i\beta} \tilde{A}^0/\tilde{A}^0$ and $\sin(2\beta) = \sin(2\beta)\Psi_{K_S} = 0.68 \pm 0.03$ [13], since the NP operators are irrelevant to $B^0 - \bar{B}^0$ mixing amplitude.

Using the constrained parameters of the NP operators in Table 5 and taking $m_g = 0.5$ GeV, our numerical results are listed in Table 6 for the SM and the five Cases of NP operators. The experimental data column is the averages by HFAG [13]. In the SM, up to doubly Cabibbo suppressed amplitudes, one can expect

$$A_{CP} \approx 0, \quad A_{CP}^{mix} = S \approx \sin(2\beta)\Psi_{K_S} = 0.68 \pm 0.03 \quad (40)$$

for the two decay modes. We get $A_{CP}^{mix}(\pi^0 K_S) = 0.77 \pm 0.04$ and $A_{CP}^{mix}(\rho^0 K_S) = 0.66 \pm 0.03$.

It is noted that the former is slight larger than $\sin(2\beta)\Psi_{K_S}$ which is due to corrections of the suppressed amplitudes proportional to $V_{ub}V_{us}^*$ as discussed in Ref. [38]. As shown in Table 6, if the old data $\sin(2\beta)\Psi_{K_S} = 0.725 \pm 0.037$ used, we get $\Delta S_{\pi^0 K_S} = S_{\pi^0 K_S} - \sin(2\beta)\Psi_{K_S} = 0.05 \pm 0.08$ and

1 If the old data $\sin(2\beta)\Psi_{K_S} = 0.725 \pm 0.037$ used, we get $\Delta S_{\pi^0 K_S} = S_{\pi^0 K_S} - \sin(2\beta)\Psi_{K_S} = 0.05 \pm 0.08$ and
the NP pseudoscalar operators decrease $S_{\pi^0K_S}$ and $S_{\rho^0K_S}$ (weaker than former), which seems to be favored by the experimental data.

We note that HFAG has not included the following data yet

\[ A_{CP}^{mix}(\bar{B}^0 \rightarrow \pi^0K_S) = 0.55 \pm 0.20 \pm 0.03 \quad \text{BABAR} \quad (41) \]
\[ A_{CP}^{mix}(\bar{B}^0 \rightarrow \pi^0K_S) = 0.67 \pm 0.31 \pm 0.08 \quad \text{Belle} \quad (42) \]

which are reported very recently at ICHEP08. The average reads $A_{CP}^{mix}(\bar{B}^0 \rightarrow \pi^0K_S) = 0.58 \pm 0.17$. Again from Table 6 we find the outputs of all the five Cases with their fitted parameter spaces are in good agreements with the new experimental results since the error-bar are still large. Taking Case II as example, i.e., assuming NP from $b \rightarrow sdd$, we present the correlations of the direct and the mixing-induced CP asymmetries for $\bar{B}^0 \rightarrow \pi^0K_S$ and $\bar{B}^0 \rightarrow \rho^0K_S$ decays in Fig. 10, where the constrained parameters listed in Table 5 are used. Although all points fall in the present experimental error-bars, Fig. 10 shows interesting correlations between $A_{CP}^{dir}$ and $A_{CP}^{mix}(S)$. If the experimental $S_{\pi^0K_S}$ shrank to be much lower than $\sin(2\beta)\Psi_{K_S}$, the NP Case II would give large negative direct CP asymmetry. Similar implication also applies to $\rho^0K_S$ final states.

\[ \Delta S_{\rho^0K_S} = -0.05 \pm 0.07, \] which agree well with the results in the paper. Considering our different treatments of the end-point divergences, the agreement numerically confirms the observation that the mixing induced CP violations are insensitive to strong phases in the decay amplitudes.
In summary, assuming NP effects entering $B \to \pi K, \pi K^*$ and $\rho K$ decays via $\bar{s}(S + P)b \otimes \bar{q}(S + P)q$ operators, we have performed fittings for the observables in these decays with a model-independent approach. It's found that all the experimental data, especially the direct CP violation difference $\Delta A$, could be accommodated by new $b \to su\bar{u}$ or $b \to sd\bar{d}$ contributions, of course by their combination. Assuming the dominance of new $b \to su\bar{u}$ operators (Case I), we find the color-octet operator has the similar strength as the color-singlet one, which is rather exotic for electro-weak NP models. However, taking the new $b \to sd\bar{d}$ operators dominant (Cases II and III), we have shown that color-singlet operator $\bar{s}(S + P)b \otimes \bar{d}(S + P)d$ solely can provide a resolution to the derivations with a strength about half of $b \to su\bar{u}$ operators. We also have performed fits (Cases IV and V) with both $b \to su\bar{u}$ and $b \to sd\bar{d}$ contributions to infer the their relative size in these decays. It is found that the strength of $b \to sd\bar{d}$ is stronger than that of $b \to su\bar{u}$. In all cases, to account for the experimental deviations from the SM predictions for direct CP violations, especially for $A_{CP}(B^- \to \pi^0 K^-)$, new electro-weak phase about 100$^\circ$ relative to the SM $b \to sq\bar{q}$ penguin amplitude is always required. With the fitted parameters, we present results for the mixing induced CP asymmetries in $\bar{B}^0 \to \pi^0 K_S$ and $\rho^0 K_S$ decays. It is found the NP effects generally reduce $S_{\pi^0 K_S}$ and $S_{\rho^0 K_S}$. However, due to the large error-bars, the present experimental data do not further reduce the parameter spaces of the NP operators.

4 Conclusions

At present, the successful running of the B factories with their detectors BABAR (SLAC) and BELLE (KEK) have already taken about $10^9$ data together at $\Upsilon(4S)$ resonance, and have produced plenty of exciting results. Tensions between the experimental data and the SM predictions based on different approaches for strong dynamics are accumulated, which may be due to our limited understanding of the strong dynamics, but equally possible due to NP effects. Motivated by the recent observed $\Delta A$ of the difference in direct CP violation between $A_{CP}(B^+ \to \pi^0 K^\mp)$ and $A_{CP}(B^0 \to K^\mp \pi^\mp)$ and theoretical issues of endpoint divergences, strong phases and annihilation contributions in charmless hadronic B decays, we have revisited the $B \to \pi K, \pi K^*$ and $\rho K$ decays with an infrared finite form of the
gluon propagator supplemented to the QCDF approach. In this way, we can get large strong phases from the annihilation contributions, while the hard spectator-scattering amplitudes are real. From our numerical analyses, we find that the contributions of the annihilation and the hard-spectator topologies are sensitive to the value of the effective gluon mass \( m_g \). With \( m_g = 500 \pm 50 \) MeV, our predictions in the SM agree with the current experimental data well, except \( A_{\text{CP}}(B^\pm \to K^\pm\pi^0) \). Actually with \( m_g \) varying from 300 MeV to 700 MeV, we always get \( A_{\text{CP}}(B^+ \to \pi^0K^+) \approx A_{\text{CP}}(B^0 \to K^\pm\pi^\mp) \) as shown in Table 2, which also agree with the results in the literature. We conclude that NP effects is required, at least can not be excluded, to resolve the discrepancies between the observed \( \Delta A \) and the SM expectations.

With four effective NP \( b \to su\bar{u} \) and \( b \to sd\bar{d} \) operators, we have performed a model-independent approach to the discrepancies. Our main conclusions are summarized as:

- Assuming dominance of \( b \to su\bar{u} \) operators, the fit gives a quite small center value for \( A_{\text{CP}}(B^0 \to K^\pm\pi^\mp) \) although consistent with the data within its large error-bar. Moreover, the strength of color-octet operator \( O^{u}_{S8} \) is comparable with color-singlet \( O^u_{S1} \) which may be rather exotic for most NP models.

- With the \( b \to sd\bar{d} \) operator \( O^d_{S1} \) solely, the observables in \( B \to \pi K, \pi K^* \) and \( \rho K \) decays could be well accommodated, since \( \bar{B}^0 \to K^-\pi^+ \) is irrelevant to the \( b \to sd\bar{d} \) operator and it’s branching ratio and CP violation agree with the SM prediction very well.

- Assuming dominance of color-singlet operators \( O^u_{S1} \) and \( O^d_{S1} \), it is found that the two operators have the similar weak phase with \( C^u_{S1} \approx \frac{1}{2} C^d_{S1} \).

- For all Cases, to account for the experimental deviations from the SM predictions for direct CP violations, especially for \( A_{\text{CP}}(B^- \to \pi^0K^-) \), new electro-weak phase about 100° relative to the SM \( b \to sq\bar{q} \) penguin amplitude is always required.

- With the fitted parameter spaces, the NP operators decrease the mixing-induced CP violations in \( B^0 \to \pi^0K_S \) and \( \rho^0K_S \) decays, especially that of \( \pi^0K_S \) final states.

It is reminded that both direct and mixing-induced CP violations have not been well established in most of charmless nonleptonic B decays. Although the difference in direct CP asymmetries between \( A_{\text{CP}}(B^+ \to \pi^0K^+) \) and \( A_{\text{CP}}(B^0 \to K^\pm\pi^\mp) \) shows some hints of new
physics activities, we still need refined measurements of the mixing-induced CP asymmetries in the related decays $B^0 \to \pi^0 K_s$ and $\rho^0 K_s$ to confirm or refute the NP hints, since the former strongly depends on strong phases in the decay amplitudes while the latter not so much and can be predicted more precisely. In the coming years, the precision of experimental measurement of the observables in these decays will be improved much with LHCb at CERN, which will shrink the parameter space and reveal the relative importance of the five Cases studied in this paper. Then, the favored Case will deserve detail studies with particular NP models.

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Appendix A: Decay amplitudes in the SM with QCDF

The amplitudes for $B \to \pi K$, $\pi K^*$ and $\rho K$ are recapitulated from Ref. [3]:

$$A_{SM}^{B^+ \to \pi^+ K^-} = \sum_{p=u,c} V_{pb} V_{ps}^* A_{\pi K} \left[ \delta_{pu} \beta_2 + \alpha_{4,EW}^p - \frac{1}{2} \alpha_{3,EW}^p + \frac{1}{2} \beta_{3,EW}^p \right],$$  \hspace{1cm} (43)

$$\sqrt{2} A_{SM}^{B^+ \to \pi^0 K^-} = \sum_{p=u,c} V_{pb} V_{ps}^* \left\{ A_{\pi^0 K^0} \left[ \delta_{pu} \alpha_1 + \alpha_{4,EW}^p + \frac{3}{2} \alpha_{3,EW}^p \right] + A_{\pi^0 K^-} \delta_{pu} \alpha_2 \right\},$$ \hspace{1cm} (44)

$$A_{SM}^{\bar{B}^0 \to \pi^+ K^-} = \sum_{p=u,c} V_{pb} V_{ps}^* A_{\pi K^0} \left[ \delta_{pu} \alpha_1 + \alpha_{4,EW}^p + \frac{3}{2} \alpha_{3,EW}^p \right],$$ \hspace{1cm} (45)

$$\sqrt{2} A_{SM}^{\bar{B}^0 \to \pi^0 K^-} = \sum_{p=u,c} V_{pb} V_{ps}^* \left\{ A_{\pi^0 K^0} \left[ - \alpha_{4,EW}^p + \frac{1}{2} \beta_{3,EW}^p - \frac{1}{2} \beta_{3,EW}^p \right] + A_{\pi^0 K^-} \delta_{pu} \alpha_2 \right\},$$ \hspace{1cm} (46)

where the explicit expressions for the coefficients $\alpha_{i}^p \equiv \alpha_{i}^p (M_1 M_2)$ and $\beta_{i}^p \equiv \beta_{i}^p (M_1 M_2)$ can also be found in Ref. [3]. Note that expressions of the hard spectator terms $H_i$ appearing in $\alpha_{i}^p$ and
the weak annihilation terms appearing in $\beta_i^p$ should be replaced with our recalculated ones. The amplitudes of $B \to \pi K^*$ and $B \to \rho K$ decays could be obtained by setting $(\pi K) \to (\pi K^*)$ and $(\pi K) \to (\rho K)$, respectively.

**Appendix B: Theoretical input parameters**

**B1. Wilson coefficients and CKM matrix elements**

The Wilson coefficients $C_i(\mu)$ have been evaluated reliably to next-to-leading logarithmic order [24, 44]. Their numerical results in the naive dimensional regularization scheme at the scale $\mu = m_b$ ($\mu_h = \sqrt{\Lambda_h m_b}$) are given by

\[
\begin{align*}
C_1 & = 1.074 \ (1.166), \quad C_2 = -0.170 \ (-0.336), \quad C_3 = 0.013 \ (0.025), \\
C_4 & = -0.033 \ (-0.057), \quad C_5 = 0.008 \ (0.011), \quad C_6 = -0.038 \ (-0.076), \\
C_7/\alpha_{e.m.} & = -0.016 \ (-0.037), \quad C_8/\alpha_{e.m.} = 0.048 \ (0.095), \quad C_9/\alpha_{e.m.} = -1.204 \ (-1.321), \\
C_{10}/\alpha_{e.m.} & = 0.204 \ (0.383), \quad C_{7\gamma} = -0.297 \ (-0.360), \quad C_{8g} = -0.143 \ (-0.168).
\end{align*}
\]

(47)

The values at the scale $\mu_h$, with $m_b = 4.80 \text{ GeV}$ and $\Lambda_h = 500 \text{ MeV}$, should be used in the calculation of hard-spectator and weak annihilation contributions.

For the CKM matrix elements, we adopt the Wolfenstein parameterization [45] and choose the four parameters $A$, $\lambda$, $\rho$, and $\eta$ as [46]

\[
A = 0.807 \pm 0.018, \quad \lambda = 0.2265 \pm 0.0008, \quad \overline{\rho} = 0.141^{+0.020}_{-0.017}, \quad \overline{\eta} = 0.343 \pm 0.016,
\]

(48)

with $\overline{\rho} = \rho (1 - \frac{\lambda^2}{2})$ and $\overline{\eta} = \eta (1 - \frac{\lambda^2}{2})$.

**B2. Quark masses and lifetimes**

As for the quark mass, there are two different classes appearing in our calculation. One type is the pole quark mass appearing in the evaluation of penguin loop corrections, and denoted by $m_q$. In this paper, we take

\[
m_u = m_d = m_s = 0, \quad m_c = 1.64 \pm 0.09 \text{ GeV}, \quad m_b = 4.80 \pm 0.08 \text{ GeV}.
\]

(49)
The other one is the current quark mass which appears in the factor $r^M_χ$ through the equation of motion for quarks. This type of quark mass is scale dependent and denoted by $\bar{m}_q$. Here we take \[47, 48\]

$$\begin{align*}
\bar{m}_u(\mu)/\bar{m}_d(\mu) &= 27.4 \pm 0.4 \ [48], \\
\bar{m}_u(2 \text{ GeV}) &= 87 \pm 6 \text{ MeV} \ [48], \\
\bar{m}_b(\mu) &= 4.20 \pm 0.07 \text{GeV} \ [47],
\end{align*}$$

(50)

where $\bar{m}_q(\mu) = (\bar{m}_u + \bar{m}_d)(\mu)/2$, and the difference between $u$ and $d$ quark is not distinguished.

As for the lifetimes of B mesons, we take \[47\] $\tau_{B_u} = 1.638 \text{ ps}$ and $\tau_{B_d} = 1.530 \text{ ps}$ as our default input values.

**B3. The decay constants and form factors**

In this paper, we take the decay constants

$$\begin{align*}
f_B &= (216 \pm 22) \text{ MeV} \ [50], \\
f_{B_s} &= (259 \pm 32) \text{ MeV} \ [50], \\
f_\pi &= (130.7 \pm 0.4) \text{ MeV} \ [47], \\
f_K &= (159.8 \pm 1.5) \text{ MeV} \ [47] \\
f_{K^*} &= (217 \pm 5) \text{ MeV} \ [49], \\
f_\rho &= (209 \pm 2) \text{ MeV} \ [47],
\end{align*}$$

(51)

and the form factors \[49\]

$$\begin{align*}
F_0^{B\to\pi}(0) &= 0.258 \pm 0.031, \\
F_0^{B\to K}(0) &= 0.331 \pm 0.041, \\
V_0^{B\to K^*}(0) &= 0.411 \pm 0.033, \\
A_0^{B\to K^*}(0) &= 0.374 \pm 0.034, \\
A_1^{B\to K^*}(0) &= 0.292 \pm 0.028, \\
V_0^{B\to \rho}(0) &= 0.323 \pm 0.030, \\
A_0^{B\to \rho}(0) &= 0.303 \pm 0.029, \\
A_1^{B\to \rho}(0) &= 0.242 \pm 0.023.
\end{align*}$$

(52)

**B4. The LCDAs of mesons and light-cone projector operators.**

The light-cone projector operators of light pseudoscalar and vector meson in momentum space read \[51, 3\]

$$\begin{align*}
M^P_{\alpha\beta} &= \frac{i f_P}{4} \left( \bar{\phi}_P \gamma_5 \Phi_P(x) - \mu_P \gamma_5 \frac{k_2}{k_1} \phi_p(x) \right)_{\alpha\beta}, \\
(M^V_{\parallel})_{\alpha\beta} &= -\frac{i f_V}{4} \left( \bar{\phi}_V \Phi_V(x) - \frac{m_V f_V}{f_V} \frac{k_2}{k_1} \phi_v(x) \right)_{\alpha\beta},
\end{align*}$$

(53)
where $\mu_P$ is defined as $m_b r_\pi^P/2$, and $f_P(V)$ is the decay constant. The chirally-enhanced factor appearing in this paper is defined as

$$r_\pi^\chi(\mu) = \frac{2m_\pi^2}{m_b(\mu)2m_q(\mu)}, \quad r_K^\chi(\mu) = \frac{2m_K^2}{m_b(\mu)(m_q + m_s)(\mu)},$$

$$r_V^\chi(\mu) = \frac{2m_V f_V}{m_b(\mu) f_V},$$

(54)

where the quark masses are all running masses defined in the $\overline{\text{MS}}$ scheme which we have given in Appendix B2. For the LCDAs of mesons, we use their asymptotic forms $[52, 53]$

$$\Phi_P(x) = \Phi_V(x) = 6x(1-x), \quad \phi_p(x) = 1, \quad \phi_v(x) = 3(2x-1).$$

(55)

As for the B meson wave function, we take the form $[54]$

$$\Phi_B(\xi) = N_B \xi(1-\xi) \exp\left[- \left(\frac{M_B}{M_B - m_b}\right)^2 (\xi - \xi_B)^2\right],$$

(56)

where $\xi_B \equiv 1-m_b/M_B$, and $N_B$ is the normalization constant to make sure that $\int_0^1 d\xi \Phi_B(\xi) = 1$.

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