Neutron stars with dark energy

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Abstract. After a short review on the possible experimental observations to verify pseudo-complex General Relativity, neutron stars as a particular object of interest are investigated. Dark energy is added to the structure of a neutron star, while for the nuclear part the chiral SU(3) model is used. For the coupling of matter to dark energy a special assumption is made. The consequences are discussed. We show that neutron stars of up to six solar masses are obtained, which already behave similar to a black hole.

1. Introduction
Neutron stars are the results of supernova explosions leading to highly compact objects, whose densities are of the order or several times of the nuclear density. In order to understand these compact stars, the equation of state for nuclear matter has to be known. In [1] the chiral SU(3) model was developed which includes the contributions of hadronic resonances, scalar and vector mesons. The model predicts for the maximal mass of a neutron star 2.05 solar masses. Larger stars should contract to a black hole.

In [2, 3] a modified theory of General Relativity, the pseudo-complex General Relativity (pc-GR), was presented. The novel feature is that it includes a minimal length. But even when the limit of zero minimal length is considered, an additional contribution of a dark energy remains. This dark energy is repulsive and finally avoids the formation of a black hole. The event horizon also disappears and thus no black hole exists in this theory rather very high dense objects are possible, though for a distant observer they look similar to a black hole.

It is therefore interesting to describe these highly dense objects within the new theory. For that the properties of matter at very high densities have to be known, which we do not at this moment. Nevertheless, as a starting point one can assume that the chiral SU(3) model is still valid and take further assumptions on the coupling of mass to the dark energy. The important point is to see if this dark energy is repulsive enough to avoid the formation of a black hole, to investigate its properties and observational consequences and what is the highest mass possible. Of course, the results will depend on the knowledge of the properties of nuclear matter at high densities, which is not well known.

In what follows, we will first resume the main properties of the pc-GR and present some observational consequences. We show that standard GR has problems to describe Quasi Periodic
Objects (QPO), which form in accretion disks around galactic black holes, while pc-GR is able to explain the observations. After that, neutron stars are described and we will show that under certain assumptions neutron stars of up to six solar masses are produced. The larger the mass the more it resembles a black hole, but never it is a black hole.

2. Properties of pc-GR and observations

In pc-GR instead of real coordinates $X^{\mu} (\mu=0,1,2,3)$ pseudo-complex coordinates are introduced [2, 3]. Pseudo-complex means that the coordinate is given by $X^{\mu} = x^{\mu} + i y^{\mu}$ where $x^{\mu}$ is the standard real coordinate and $y^{\mu}$ the pseudo-imaginary component. The label pseudo-complex is due to $I^2 = 1$, which has important mathematical and physical consequences. The imaginary component is proportional to the four-velocity and due to dimensional reasons (that the unit of the pseudo-imaginary component is equal to the real component) a minimal length parameter appears as a factor, i.e. $y^{\mu} = \frac{c}{l} u^{\mu}$. This structure may have important consequences when one tries to quantize the theory, but for our purpose (large objects) one can neglect the contribution of the minimal length, i.e. it is set to zero. But there is another consequence of the extended theory: It requires the presence of a dark energy which is repulsive.

The origin of the dark energy is probably due to vacuum fluctuations, as was shown in [4]. There, a calculation within semi-classical quantum mechanics shows that mass produces quantum fluctuations in the neighborhood of a Schwarzschild black hole, which increases with its mass and falls off as $1/r^6$, with corrections. Semi-classical quantum mechanics means that the back ground metric is fixed, i.e. the event horizon remains. In [5] this result is used to speculate on the possibility that the collapse of a star is halted before the black hole is formed, when the collapse is "slow" enough. The disadvantage of this theory is that it can not determine the action of the dark energy on the metric. In pc-GR the dark energy is considered as a classical fluid and its back-reaction on the metric can be determined, with the result that the event horizon is avoided (the metric component $g_{00}$ is always positive).

From these observations a new principle is proposed: Mass not only curves the space around it but also changes the vacuum properties near a mass, which in turn changes the metric.

There are already some observations that pc-GR may be a correct extension to GR, namely the QPO in galactic black holes [6, 7, 8]. These are binary systems which contain an assumed black hole with a stellar partner, which provides mass to the accretion disk. In the inner boarders of the accretion disk QPO may form, which are local eruptions. These QPO follow the rotation of the disk and emit Fe-Kα lines, from which the red-shift can be deduced. The GR and pc-GR predict the dependence of the orbital frequency and the red-shift on the distance to the center of the dense object. In a consistent theory, from both observations the same radial distance has to result. In [9] the dependence of the orbital frequency and the red-shift are determined for GR and pc-GR. In GR the orbital frequency always increases toward smaller distance while pc-GR exhibits a maximum from which on it decreases again. As a consequence, low orbital frequencies are possible for small radial distances. The red-shift, at the equator, shows in both theories a similar behavior, however in pc-GR the curve is shifted to smaller radial distances and it never reaches infinity, though it assumes very large values. The observations show [6, 7, 8] that assuming GR there is a striking difference in the deduced radial distance from the orbital frequency and the red-shift. Using the observed orbital frequency a distance between 8 to 16 Schwarzschild radii are observe for the system XTE J-1550-564, which corresponds to a compact star with $9.10\pm0.15$ solar masses [10], while from the observed red-shift a distance between 2-3 Schwarzschild radii is obtained! Similar for all other systems observed up to now. In pc-GR, however, the same radial distance is obtained. Assuming Keplerian Motion and that QPO’s and relativistic iron lines originate from the same region, this is the first observation which may confirm pc-GR! It also gives us an argument to continue the study neutron stars.
3. Neutron stars within pc-GR

In order to describe neutron stars, the modified Einstein equations are used, which include on the right hand side of these equations not only the baryonic energy-momentum tensor but also the contribution due to the dark energy. In the limit of zero minimal length, the equations are

$$\mathcal{R}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \mathcal{R} = -\frac{8\pi G}{c^4} T^{\mu\nu} + \frac{8\pi G}{c^4} T^{\mu\nu}_{\Lambda}$$

(1)

($i$ stands for the interior region of the neutron star) where $\mathcal{R}^{\mu\nu}$ denotes the real projection of the Ricci tensor, $\mathcal{R}$ the real projection of the Ricci scalar and $g^{\mu\nu}$ the corresponding projection of the metric.

For both, the matter and the dark energy, we assume an isotropic fluid, though it can be easily extended to an anisotropic fluid. With this assumption, the Einstein equations can be rewritten in terms of the Tolman-Oppenheimer-Volkov (TOV) equations [11] (for details, please consult [12])

$$\frac{dp_m}{dr} = -\left(\frac{\varepsilon_m(r) + p_m(r)}{r[r - 2m_m(r) + 2m_m(r)]}\right) \times \left[m_m(r) - m_m(r) + \frac{4\pi k}{c^4} r^3(p_m(r) + p_m(r))\right]$$

$$\frac{dp_{\Lambda_i}}{dr} = -\left(\frac{\varepsilon_{\Lambda_i}(r) + p_{\Lambda_i}(r)}{r[r - 2m_m(r) + 2m_m(r)]}\right) \times \left[m_m(r) - m_{\Lambda_i}(r) + \frac{4\pi k}{c^4} r^3(p_{\Lambda_i}(r) + p_m(r))\right]$$

(2)

$p_m$ is the matter pressure and $\varepsilon_m$ the matter density. The $p_{\Lambda_i}$ is the pressure of the dark energy density in the internal region of the neutron star and $\varepsilon_{\Lambda_i}$ its density. The $m_m$ and $m_{\Lambda_i}$ is the accumulated mass of the matter and dark energy as a function of radial distance. They are obtained by integrating over the distance.

These equations are not complete and the equation of state of the matter and the dark energy has to be known yet. For the matter the chiral-$SU(3)$ model [1] is used. It is a mean field model, which not only takes into account the nucleon fields (proton and neutron) but also baryon resonances ($\Lambda$, $\Sigma$ and $\Xi$) and their coupling to several scalar and vector mesons. The model includes several parameters which where fixed to properties of nuclear matter, as for example binding energies. The resulting equation of state is shown in Fig. 1. As can be seen the dependence of the pressure versus energy density is not linear in all the region.

Assumptions have to be made on the unknown coupling of the matter density to the dark energy density. We take the simplest assumption, as done in [13], which corresponds to a linear coupling

$$\varepsilon_{\Lambda_i} = \alpha \varepsilon_m$$

(3)

This is a crude approximation and may need modifications, as we will see later on. In difference to [13] the proportionality factor $\alpha$ is negative, i.e. the dark energy density is negative.

Having defined the equation of states, we proceed in solving numerically the TOV equations. A 4th order Runge-Kutta algorithm is applied, defining at the center of the star initial pressure and density values.

The integration starts at the center and going toward larger $r$. The initial values for the baryon density varies from one to several times the nuclear density. The one for the dark energy
Figure 1. Equation of state, pressure versus energy density, for nuclear matter. Two further curves are shown for comparison, one for radiation and the other with $p_m = \varepsilon_m$.

is fixed by the proportionality relation assumed. The integration stops when the baryonic pressure is zero (for numerical reasons it is set to be less than $10^{-11}$ in units of the pressure). This defines the surface of the star! The results vary with the change of the central baryonic density and pressure, but the general structure keeps the same. of course, the central baryonic density can not be arbitrarily large. The integration is not performed from the outside to the inside, because this would require the knowledge of the position of the surface. This is the procedure as explained in more detail in the book by N. K. Glendenning [14].

One of the results is shown in Fig. 2, where the total mass of neutron stars is plotted versus their radii. The stable branch is given by the negative slope part of the curve. As is seen, stars with up to six solar masses are obtained. The larger the absolute value of the proportionality factor $\alpha$ (stronger repulsion) the larger the masses possible for neutron stars.

In Fig. 3 the baryonic density is plotted versus the radial distance. The larger the absolute value of $\alpha$ the further the density distribution reaches outside, i.e. the mass has to increase because the surface under the curve increases. The radius of the star is obtained when the pressure lowers below a fixed but very small value.

As a last result, we plot the so-called compactness in Fig. 4. The compactness is defined as $\frac{2M_m}{R}$, where $M_m$ is the total baryonic mass and $R$ the radius of the star. The definition originates in standard GR where the $g_{00}$ component of the Schwarzschild solution is $\left(1 - \frac{2M_m}{r}\right)$. When $r = R$ reaches the Schwarzschild radius, thus values is zero, i.e. a value near to 1 indicates the onset of the forming of a black hole. In pc-GR it gives an idea on how near the star resembles a black hole. If the value is near to 1, the red-shift acquires large values and for a distant observer the object resembles a black hole, though it is not a black hole.

Fig. 4 demonstrates that the neutron stars near six solar masses resemble more black holes than normal neutron stars.

Larger masses of neutron stars can not be obtained with the assumption of a linear coupling between baryonic and dark energy density. When $|\alpha|$ is greater than 1, the fall off of the dark energy density toward the radius of the star is not quick enough and the repulsion is so large that it starts to evaporate the surface, avoiding the accumulation of more mass. In order to resolve this problem, one has to use a different coupling between the baryonic and dark energy
Figure 2. Baryonic total mass vs. total radius for different values of the coefficient $\alpha$. The central $\Lambda$-term pressure $p_{\Lambda c}$ has been fixed to $1\varepsilon_0$.

Figure 3. Baryonic energy density profile for different values of the coefficient $\alpha$. The central $\Lambda$-term pressure $p_{\Lambda c}$ has been fixed to $1\varepsilon_0$.

density, depending on the radial distance. One possibility is to model such a dependence but it would be more attractive to derive the dependence of the vacuum fluctuation as a function of the baryonic energy density. This can only be done, up to now, using the semi-classical quantum mechanics, i.e. a fixed metric background. Work in this direction is currently done.

4. Conclusions
In this contribution we applied an extension of the General Relativity, which includes the contribution of dark energy, to neutron stars. At the beginning we shortly resumed the main
Figure 4. Baryonic compactness vs. total radius for different values of the coefficient $\alpha$. The central $\Lambda$-term pressure $p_{\Lambda c}$ has been fixed to $1\varepsilon_0$.

characteristics of the pc-GR, relevant for the description of pc-neutron stars. Some recent observations of galactic black holes were shown, which indicate a possible inconsistency of GR, namely that the distances deduced from the orbital frequency of QPO and from the measured red-shift are different. In contrast, pc-GR can reproduce the same radial distance.

The interior region of a neutron star was investigated, including the presence of baryonic and dark energy densities. The equation of state was deduced within the chiral $SU(3)$ model, while the equation of state of the dark energy part is determined through a linear coupling between the dark energy and baryonic density.

Neutron stars with up to six solar masses where obtained, resembling toward larger masses more and more black holes. Larger neutron stars than six solar masses could not be obtained due to the too strong coupling of the energy densities near the surface. Nevertheless, the current study shows that large masses of neutron stars can be obtained and probably no upper limit for stable stars exist.

Details of the pc-GR can be consulted in [2, 3, 9] and the complete investigation on neutron stars will be published elsewhere [12].

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