Modeling of dialogue regimes of distance robot control

E V Larkin, A N Privalov

1 Tula State University, 92 “Lenina” prospect, Tula, 300012, Russia,
2 Tula State Pedagogical University named after L.N. Tolstoy, 125 “Lenina” prospect, Tula, 300026, Russia.

E-mail: elarkin@mail.ru

Abstract. Process of distance control of mobile robots is investigated. Petri-Markov net for modeling of dialogue regime is worked out. It is shown, that sequence of operations of next subjects: a human operator, a dialogue computer and an onboard computer may be simulated with use the theory of semi-Markov processes. From the semi-Markov process of the general form Markov process was obtained, which includes only states of transaction generation. It is shown, that a real transaction flow is the result of «concurrency» in states of Markov process. Iteration procedure for evaluation of transaction flow parameters, which takes into account effect of «concurrency», is proposed.

1. Introduction
At present mobile robots (MR) are rather of widely use in such domains of human activity as environment monitoring, anti-terror systems, intelligence activity etc. [1, 2]. The main feature of the current state of mobile robotics is a restriction of functions which MR can do absolutely independently. This feature deprives the robot of a real autonomy, and that is why, from existing natural levels of hierarchy of control systems (strategic, tactic and functional logic) only tasks of two lowest levels are realized in practice today [3], such as:
- tactic level, at which MR receives commands for solving tasks, modifies a genetic algorithms of their solving with taking into account of current states of onboard robot equipment and environment, and generates execution commands to onboard units;
- functional logic level, at which control commands are realized, feedbacks are closed and coordination of functioning of onboard equipment is executed.

Main features of tactic and functional logical levels tasks are the strong restrictions both for time intervals of sensors and actuator poll, and the lag time between sensor information reading and actuator command generation. Besides, there is the problem of synchronization of operation of control loops with the flow of commands from an outer human operator. That is why, the definition of mobile robot control time characteristics at present is rather actual, and not a properly solved task [4, 5].

2. Common model of distance control process
The principle of MR control [3] is shown on fig. 1 a.
MR is directed by a human operator, which is situated at the remote control point. An operator interacts with the dialogue computer and generates a flow of commands, which are transmitted through communication channel onto the onboard computer. The onboard computer decodes commands and activates the proper algorithm to execute the commands. The execution of commands leads to switching of MR state. The information about the current state of the robot and its environment is transmitted backward through the onboard computer, communication channel and dialogue computer to a human operator.

Due to the fact, that three subjects: a human operator, the dialogue computer and the onboard computer are in uninterrupted interaction, the primary model, which explains the control principle, was obtained with use of Petri-Markov nets apparatus [6]. Petri-Markov nets (fig. 1 b.), describing interactions in the system under investigation is as follows:

\[
\sigma = \{A, Z, \rho_{a\zeta}, \rho_{\zeta a}, A_p^1\}
\]

where \(A = \{\alpha_1, \alpha_2, \alpha_3\}\) - is the set of places, which describes algorithms of interacted subjects functioning; \(Z = \{\zeta_{11}, \zeta_{12}, \zeta_{21}, \zeta_{22}, \zeta_{31}, \zeta_{32}, \zeta_{33}\}\) - is the set of transitions, which describes the logics of subjects interaction; \(\rho_{a\zeta}\) - is the adjacency matrix of size 3×7, which reflects the set of places to the set of transitions; \(\rho_{\zeta a}\) - is the adjacency matrix of size 7×3, which reflects the set of transitions to the set of places; \(A\) - matrix of logical conditions of semi-steps execution from transitions to places:

\[
\rho_{a\zeta} = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 2 & 3 & 1 & 3 & 2 & 3 & 2 & 1 & 2 & 2
\end{pmatrix}
\]
\[
\rho_{\zeta} = \begin{pmatrix}
1 & 0 & 1 \\
1 & 0 & 0 \\
1 & 1 & 1 \\
0 & 0 & 1 \\
0 & 0 & 1 \\
0 & 1 & 1 \\
0 & 1 & 0 \\
\end{pmatrix};
\]

\[
A = \begin{pmatrix}
[a_1, \zeta_{11}] & 0 & [a_1, \zeta_{11}] \\
[a_1, \zeta_{12}] & 0 & 0 \\
[a_1, \zeta_{21}] & [a_3, \zeta_{31}] & [a_3, \zeta_{31}] \\
0 & 0 & [a_3, \zeta_{32}] \\
0 & 0 & [a_3, \zeta_{33}] \\
[0 & [a_2, \zeta_{21}] & [a_2, \zeta_{22}] \\
0 & [a_2, \zeta_{22}] & 0
\end{pmatrix},
\]

where vector \( [a_m, \zeta_n] \) describes the procedure of semi-steps execution from the place \( a_m \) to the transition \( \zeta_n \), \( m \in \{1, 2, 3\} \), \( n \in \{11, 12, 31, 32, 33, 21, 22\} \).

3. Models of algorithms

Places \( a_1, a_2, a_3 \) describe sequences of operations of the next subjects: \( a_1 \) - a human operator; \( a_2 \) - the dialogue computer; \( a_3 \) - the onboard computer. A human operator realizes the sequence of operations as a decision-making creative process, based on qualification, intuition, instructions, description of hardware/software, etc. The dialogue computer and the onboard computer realize sequences of their operations as interpretation of some algorithms. Anyway sequences of operations develop in time. After completion of current operation algorithm switches to one of possible next operation in the stochastic way, from the point of view of an external observer. Due to random time of execution of operation and stochastic way of switching, the natural model of sequences of operations in a system under investigation is the semi-Markov processes \([7,8]\)

\[
\alpha_i = \{A_i, r_i, h_i(t)\}, i \in \{1, 2, 3\},
\]

where \( A_i = \{a_i, ..., a_j, ..., a_{j_i}\} \) - is the set of states; \( r_i = (r_{j,m}) \) and \( h_i(t) = [h_{j,m}(t)] \) - are the adjacency and the semi-Markov matrices correspondingly, both matrices of size \( J_i \times J_i \).

In most common case \( \forall r_{j,m} = 1 \) (full graph with loops). The elements of semi-Markov matrix satisfy the following conditions:

\[
\sum_{m=1}^{J_i} \int_0^{\infty} h_{i,j,m}(t) dt = 1; \quad \lim_{t \to \infty} \frac{1}{t} \int_0^t h_{i,j,m}(t) dt \neq \delta(t - \tau),
\]

where \( \delta(t - \tau) \) - is shifted Dirac \( \delta \)-function.

Restrictions (6) point that processes \( \alpha_1, \alpha_2, \alpha_3 \) are the ergodic ones.

In accordance with Petri-Markov net (1), in every semi-Markov process \( \alpha_1, \alpha_2, \alpha_3 \) with sets of states \( A_1, A_2, A_3 \), there are subsets of states simulating generation of transactions to the adjacent semi-Markov process. Processes \( \alpha_1, \alpha_2 \) have one such subset. Without loss of generality one can assume, that states under consideration have numbers from the first till \( S_i \)-th, \( S_i \leq J_i \), \( i \in \{1, 2\} \). So, sets \( A_1, A_2 \) are as follows:
In semi-Markov process \( \alpha_3 \) there are two subsets of states, which simulate generation of transactions. States of first subset generate transactions to \( \alpha_1 \). States of second subset generate transactions to \( \alpha_2 \). The named states have numbers from first till \( S_1 \)-th, and from \((S_1+1)\)-th till \( U_3 \)-th. Thus set \( A_3 \) is as follows:

\[ A_3 = \{ a_{i_1}, \ldots, a_{i_n}, a_{S_1+1}, \ldots, a_{u_1}, a_{U_1+1}, \ldots, a_{j_1}, \ldots, a_{j_n} \}, \]

(7)

For evaluation of parameters of the transactions flow one should simplify semi-Markov processes \( \alpha_1, \alpha_2, \alpha_3 \) till processes which contain states of generation of transactions only. For this purpose in processes \( \alpha_1, \alpha_2 \) one should split every state \( a_{s_i} \) onto two states: starting one \( b_{a_{s_i}} \) and absorbing one \( e_{a_{s_i}} \), \( 1 \leq s_i \leq S_1, i \in \{1, 2\} \). In semi-Markov process \( \alpha_3 \) states \( a_{s_i}, a_{u_i}, 1 \leq s_i \leq S_3, 1 \leq u_i \leq U_3 \) are split onto starting states \( b_{a_{s_i}}, b_{a_{u_i}} \) and absorbing states \( e_{a_{s_i}}, e_{a_{u_i}} \).

When splitting semi-Markov processes \( \alpha_1, \alpha_2 \) are converted to processes

\[ \alpha'_i = \{ A'_i, r'_i, h'_i(t) \}, i \in \{1, 2, 3\}, \]

(9)

where

\[ A'_1 = \{ b_{a_{i_1}}, \ldots, b_{a_{i_n}}, b_{a_{S_1+1}}, \ldots, b_{a_{U_1+1}}, b_{a_{U_1+1}}, \ldots, b_{a_{j_1}}, \ldots, b_{a_{j_n}} \}; \]

\[ A'_2 = \{ e_{a_{i_1}}, \ldots, e_{a_{i_n}}, e_{a_{S_1+1}}, \ldots, e_{a_{U_1+1}}, e_{a_{U_1+1}}, \ldots, e_{a_{j_1}}, \ldots, e_{a_{j_n}} \}; \]

\[ A'_3 = \{ b_{a_{i_1}}, \ldots, b_{a_{i_n}}, b_{a_{S_1+1}}, \ldots, b_{a_{U_1+1}}, b_{a_{U_1+1}}, \ldots, b_{a_{j_1}}, \ldots, b_{a_{j_n}} \}; \]

(10)

Both the adjacency matrix \( r' \) and semi-Markov matrix \( h'_i(t) \) in (9) are of size \((J_i+S_i)\times(J_i+S_i)\), when \( i \in \{1, 2\} \) and \((J_3+U_3)\times(J_3+U_3)\), when \( i = 3 \). Matrices are formed as follows:

- columns with numbers from \( l_i \) till \( S_i \), if \( i \in \{1, 2\} \), and from \( 1_3 \) till \( U_3 \), if \( i = 3 \), should be transferred to columns with numbers from \( J_i+1 \) till \( J_i+S_i \), if \( i \in \{1, 2\} \), and from \( J_3+1_3 \) till \( J_3+U_3 \), if \( i = 3 \);  
- columns with numbers from \( l_i \) till \( S_i \), if \( i \in \{1, 2\} \), and from \( 1_3 \) till \( U_3 \), if \( i = 3 \), should be fulfilled with zeros;
- rows with numbers from \( J_i+1 \) till \( J_i+S_i \), if \( i \in \{1, 2\} \), and from \( J_3+1 \) till \( J_3+U_3 \), if \( i = 3 \), should be fulfilled with zeros.

Semi-Markov processes (9) with divided states cease to be ergodic ones. For them probabilities and densities of time of wandering from state \( b_{a_{m_i}} \) to state \( e_{a_{m_i}} \) may be found, where \( 1 \leq m_i \leq S_i \), if \( i \in \{1, 2\} \), and \( 1 \leq m_i \leq U_3 \), if \( i = 3 \); \( J_i+1 \leq n_i \leq J_3+S_i \), if \( i \in \{1, 2\} \), and \( J_3+1 \leq n_i \leq J_3+U_3 \), if \( i = 3 \). Probabilities, densities and expectations of time of wandering are as follows:

\[ h_{m_i,n_i}^*(t) = L \left[ \int \sum_{k=1}^{\infty} [L[h'_i(t)]]^k \cdot I_m \right]; \]

(11)

\[ p_{m_i,n_i} = \int_0^\infty h_{m_i,n_i}^*(t)dt; \]

(12)

\[ f_{m_i,n_i}(t) = \frac{h_{m_i,n_i}^*(t)}{p_{m_i,n_i}}; \]

(13)

\[ T_{m_i,n_i} = \int_0^\infty f_{m_i,n_i}(t)dt, \]

(14)
where \( L \) and \( L^{-1} \) - are direct and inverse Laplace transforms correspondingly; \( \mathbf{I}_{n_i} \) - is the row vector of size \( J_i + S_i \), if \( i \in \{1, 2\} \), and of size \( J_3 + U_3 \), if \( i = 3 \), \( n_i \)-th element of which is equal to one, and all other elements are equal to zero; \( \mathbf{I}_{m_i} \) - is the column vector of size \( J_i + S_i \), if \( i \in \{1, 2\} \), and of size \( J_3 + U_3 \), if \( i = 3 \), \( m_i \)-th element of which is equal to one, and all other elements are equal to zero.

Formulae (11) ÷ (14) should be applied for all possible combinations of indexes of pairs \((m_i, n_i)\). As a result semi-Markov processes \( \alpha \alpha \alpha' \alpha' \alpha' \alpha' \), which include states of transaction generation only, are formed. Every switching of processes formed born one transaction. Processes are as follows:

\[
\alpha_i^n = \{ A_i^n, \mathbf{r}_i^n, \mathbf{h}_i^n(t) \}, i \in \{1, 2, 3\},
\]

where \( A_i^n = \{ a_{i_i}^n, ..., a_{m_i}^n, ..., a_{S_i}^n \} \), if \( i \in \{1, 2\} \), and \( A_i^n = \{ a_{i_i}^n, ..., a_{m_i}^n, ..., a_{U_i}^n \} \), if \( i = 3 \); \( \mathbf{r}_i^n = \{ r_{m_i, n_i} \} \) - is the adjacency matrix of size \( S_i \times S_i \), if \( i \in \{1, 2\} \) and \( U_3 \times U_3 \), if \( i = 3 \); \( \mathbf{h}_i^n(t) = \{ h_{m_i, n_i}^n(t) \} \) - is the semi-Markov matrix of the same size, as adjacency matrix.

It should be noted, that semi-Markov processes (15) were obtained from ergodic processes (5) by means of execution of equivalent transforms, so processes (15) are ergodic too. For the ergodic semi-Markov processes rightly are the next restrictions

\[
\sum_{n=1}^{S_i} p_{m, n_i} = 1, \text{ if } i \in \{1, 2\}, \text{ and } \sum_{n=1}^{U_i} p_{m, n_i} = 1, \text{ if } i = 3.
\]

For evaluation of parameters of transactions flow it is necessary to transform semi-Markov processes (15) to more convenient abstract models. Due to the fact, that every switching in (15) born a transaction, structures of such models are as it is shown on fig. 2 a, b with solid lines.

![Figure 2. Structures of abstract semi-Markov processes of transactions generation.](image)

States \( a_0^n \) into abstract models were inserted artificially. Density of time of residence in this state is described by Dirac \( \delta \)-function. Insertion into the structures the state \( a_0^n \) is caused by the necessity of re-switching the process in cycle to obtain a flow of transactions.

Abstract semi-Markov process is as follows:

\[
\alpha_i^n = \{ A_i^n, \mathbf{r}_i^n, \mathbf{h}_i^n(t) \}, i \in \{1, 2, 3\},
\]

5
where \(A^0 = \{a^0_1, a^0_2, \ldots, a^0_i, \ldots, a^0_n\}\), if \(i \in \{1, 2\}\), and \(A^3 = \{a^3_1, a^3_2, \ldots, a^3_i, \ldots, a^3_n\}\), if \(i = 3\), are the sets of states; \(M^i = (r_{m,n})\) - is the adjacency matrix of size \((S_i + 1) \times (S_i + 1)\), if \(i \in \{1, 2\}\) and \((U_3 + 1) \times (U_3 + 1)\), if \(i = 3\); \(h^i(t) = [h^i_{m,n}(t)]\) - is the semi-Markov matrix of the same size.

\[
M^i = \begin{cases} S_i, & \text{when } i \in \{1, 2\}; \\ U_3, & \text{when } i = 3. \end{cases}
\]

Probabilities of switching from state 0 into other states of semi-Markov process are as follows

\[
\pi_{m_i} = \frac{T_{m_i}}{\tau_{m_i}},
\]

where \(T_{m_i}\) - is the expectation of time of residence of ergodic semi-Markov process (15) in the state \(a^m_{m_i}\); \(\tau_{m_i}\) - is the expectation of time of returning to state \(a^m_{m_i}\).

Density and expectation of time of residence of process (15) in the state \(a^m_{m_i}\) are as follows:

\[
f^m_{m_i}(t) = \sum_{m=1}^{M} h^m_{m_i}(t);
\]

\[
T_{m_i} = \int_0^\infty t \cdot f^m_{m_i}(t) dt,
\]

\[
M^i = \begin{cases} S_i, & \text{when } i \in \{1, 2\}; \\ U_3, & \text{when } i = 3. \end{cases}
\]

For evaluation of the returning time expectation to the state \(a^m_{m_i}\) one should to split the state \(a^m_{m_i}\) to \(b^m_{m_i}\) and \(c^m_{m_i}\). This may be done by means of transferring of column of matrix \(h^i(t)\) with number \(m_i\) to column with number \(S_i\), if \(i \in \{1, 2\}\), and to column with number \(U_3\), if \(i = 3\). The column with number \(m_i\), and the row with number \(S_i\), if \(i \in \{1, 2\}\), and \(U_3\), if \(i = 3\), should be fulfilled with zeros. As a result matrix \(\tilde{h}^i(t)\) of size \((S_i + 1) \times (S_i + 1)\), if \(i \in \{1, 2\}\), and of size \((U_3 + 1) \times (U_3 + 1)\), if \(i = 3\), is formed. Expectation of time of returning is as follows:

\[
\tau_{m_i} = \int_0^\infty t \cdot L^{-1} \left( I_{S_i+1} \cdot \sum_{k=1}^{\infty} \left[ L \tilde{h}^i(t) \right]^k \cdot I_{m_i} \right) dt \quad \text{when } i \in \{1, 2\}; \]

\[
\int_0^\infty t \cdot L^{-1} \left( I_{U_3+1} \cdot \sum_{k=1}^{\infty} \left[ L \tilde{h}^i(t) \right]^k \cdot I_{m_i} \right) dt \quad \text{when } i = 3;
\]
where \( \mathbf{I}_m \) is the column vector of size \( S_i + 1 \), \( m_i \)-th element is equal to one, and other elements are zeros; \( \mathbf{I}_{S_i+1} \) \((\mathbf{I}_{U_i+1})\) - is the row vector of size \( S_i + 1 \) \((U_i + 1)\) element of which is equal to one, and all other elements are equal to zeros.

As the transactions are generated by means of wandering through different trajectories, train of transactions, formed when wandering on separate trajectory may be considered as a separate flow. Generation of transactions when wandering on the set of trajectories may be considered as a stochastic combination of flows. In accordance with results obtained in [9] which are verified with Monte-Carlo method [10] such flow may be considered as Poisson one. Thus densities of time between transactions are as follows:

\[
f_m(t) = \lambda_m \exp(-\lambda_m t),
\]

where \( \lambda_m = \frac{1}{T_m} \) is the parameter of density of flow.

So the process (17) may be considered as strongly Markov one.

4. Models of interaction as «concurrency» of subjects

Let us return back to Petri-Markov net (1). Interactions on the transitions \( \zeta_{11}, \zeta_{21}, \zeta_{31} \) are as follows. The execution of semi-steps \([\alpha_i, \zeta_{11}], [\alpha_2, \zeta_{21}] \) and \([\alpha_3, \zeta_{31}] \) means, that Markov process (17) after natural completion of residence in states \( a^*_{m_i} \) return to the state \( a^*_{0_i} \) and then follow the switching into states \( a^*_{11}, ..., a^*_{m_i}, ..., a^*_{U_i} \) with probabilities \( \pi_1, ..., \pi_{m_i}, ..., \pi_{U_i} \), if \( i \in \{1, 2\} \), and into states \( a^*_{11}, ..., a^*_{m_i}, ..., a^*_{U_i} \) with probabilities \( \pi_1, ..., \pi_{m_i}, ..., \pi_{U_{3i}} \), if \( i = 3 \). Besides, switching of any process into state \( a^*_{0_i} \) leads to a generation of transaction to the adjacent Markov process, herewith the process, which receives transaction, restarts. For modeling of such situation onto structures of processes additional states \( a^*_{S_i+1}, ..., a^*_{m_i}, ..., a^*_{U_i+1} \) and \( a^*_{U_i+2} \), if \( i = 3 \) are inserted. Switching from states inserted to states \( a^*_{11}, ..., a^*_{m_i}, ..., a^*_{S_i} \) is done in time, which is defined by \( \delta \)-function with probabilities \( q_{11}, ..., q_{m_i}, ..., q_{S_i} \), if \( i \in \{1, 2\} \), and to states \( a^*_{11}, ..., a^*_{m_i}, ..., a^*_{U_{3i}} \) with probabilities \( q_{11}, ..., q_{m_i}, ..., q_{U_{3i}} \), if \( i = 3 \), and transaction was received from \( a^*_{11} \), and probabilities \( q_{11}, ..., q_{m_i}, ..., q_{U_{3i}} \), if it is \( a^*_{21} \) transaction (fig. 2 a, b, dash and dash-dot lines, correspondingly).

Thus in every state of processes \( \alpha^* \), \( i \in \{1, 2\} \) develops «concurrency» [11] between natural switching and switching through interruption. In every state of \( \alpha^* \) develops «concurrency» between natural switching, interruption from first process and interruption from second process. «Concurrency» of two and three subjects is modeled with Petri-Markov nets, shown on fig. 3 a and b correspondingly.
Petri-Markov net which describes «concurrency» of two subjects, includes two places \( \alpha_m, \alpha_3 \), and transitions \( \zeta_{i2} \), \( i \in \{1, 2\} \). Places \( \alpha_m \) are the models of states \( a^*_m \) of Markov process. Places \( \alpha_{31}, \alpha_{32} \) are the models of transactions received. Semi-steps \( (\zeta_{i1}, \alpha_m) \) and \( (\zeta_{i1}, \alpha_3) \) are executed simultaneously. «Winner» is that place, from which semi-step will be executed the first, \( (\alpha_m, \zeta_{i2}) \) or \( (\alpha_{31}, \zeta_{i2}) \). Petri-Markov net which describes «concurrency» of three subjects, includes three places \( \alpha_m, \alpha_{i3}, \alpha_{23} \), and transitions \( \zeta_{31}, \zeta_{32} \). Place \( \alpha_m \) is the model of states \( a^*_m \) of Markov process. States \( \alpha_{i3}, \alpha_{23} \) are the models of transactions received from the first and the second process, correspondingly. Semi-steps \( (\zeta_{31}, \alpha_m), (\zeta_{31}, \alpha_{i3}) \) and \( (\zeta_{31}, \alpha_{23}) \) are executed simultaneously. «Winner» is that place, from which semi-step will be executed first, \( (\alpha_m, \zeta_{32}), (\alpha_{i3}, \zeta_{32}) \) or \( (\alpha_{23}, \zeta_{32}) \).

Let us denote the density of flow of transactions from \( \alpha_3 \) to \( \alpha_1 \) as \( \lambda_{31} \), and from \( \alpha_3 \) to \( \alpha_2 \) as \( \lambda_{32} \). Then density of time of execution of semi-step to \( \zeta_{i2} \) the first, is defined as

\[
 f_{\zeta_{i2}}(t) = \lambda_{m_{i}} \exp\left[-t(\lambda_{m_{i}} + \lambda_{3i})\right] + \lambda_{3i} \exp\left[-t(\lambda_{m_{i}} + \lambda_{3i})\right], \quad i \in \{1, 2\}, \tag{24}
\]

where \( \lambda_{m_{i}} \) - are the parameters of densities of flows of transactions from states \( a^*_m \) to the process \( \alpha_3 \); \( \lambda_{3i} \) - are the parameters of densities of flows of transactions from the process \( \alpha_3 \) to the process \( \alpha_i \), \( i \in \{1, 2\} \).

Weighted conditional density of the time of execution of semi-steps \( (\alpha_m, \zeta_{i2}) \) and \( (\alpha_{3i}, \zeta_{i2}) \), the first, are as follows

\[
\begin{bmatrix}
\frac{h_{m, \zeta_{i2}} (t)}{h_{m, 0} (t)} \\
\frac{h_{m, \alpha_{i}} (t)}{h_{m, 0} (t)}
\end{bmatrix} = \begin{bmatrix}
\frac{\lambda_{3i}}{\lambda_{3i} + \lambda_{m_{i}}} \exp\left[-t(\lambda_{m_{i}} + \lambda_{3i})\right] \\
\frac{\lambda_{m_{i}}}{\lambda_{m_{i}} + \lambda_{3i}} \exp\left[-t(\lambda_{m_{i}} + \lambda_{3i})\right]
\end{bmatrix}, \quad i \in \{1, 2\}. \tag{25}
\]

Let us denote density of flow of transactions from \( \alpha_1 \) to \( \alpha_3 \) as \( \lambda_{13} \), from \( \alpha_2 \) to \( \alpha_3 \) as \( \lambda_{23} \). Then density of time of execution of semi-step to \( \zeta_{i2} \) the first, is as follows

\[
 f_{\zeta_{i2}}(t) = \lambda_{13} \exp\left[-t(\lambda_{m_{i}} + \lambda_{13} + \lambda_{23})\right] + \lambda_{23} \exp\left[-t(\lambda_{m_{i}} + \lambda_{13} + \lambda_{23})\right] + \\
+ \lambda_{m_{i}} \exp\left[-t(\lambda_{m_{i}} + \lambda_{13} + \lambda_{23})\right]. \tag{26}
\]

Weighted conditional density of the time of execution of semi-steps \( (\alpha_m, \zeta_{32}) \), and \( (\alpha_{13}, \zeta_{32}) \), (\( \alpha_{23}, \zeta_{32} \)) are as follows:

\[
\begin{bmatrix}
\frac{h_{m, \zeta_{32}} (t)}{h_{m, 0} (t)} \\
\frac{h_{m, \alpha_{13}} (t)}{h_{m, 0} (t)} \\
\frac{h_{m, \alpha_{23}} (t)}{h_{m, 0} (t)}
\end{bmatrix} = \begin{bmatrix}
\frac{\lambda_{13}}{\lambda_{13} + \lambda_{23} + \lambda_{m_{i}}} \exp\left[-t(\lambda_{m_{i}} + \lambda_{13} + \lambda_{23})\right] \\
\frac{\lambda_{23}}{\lambda_{13} + \lambda_{23} + \lambda_{m_{i}}} \exp\left[-t(\lambda_{m_{i}} + \lambda_{13} + \lambda_{23})\right] \\
\frac{\lambda_{m_{i}}}{\lambda_{13} + \lambda_{23} + \lambda_{m_{i}}} \exp\left[-t(\lambda_{m_{i}} + \lambda_{13} + \lambda_{23})\right]
\end{bmatrix}. \tag{27}
\]

Parameters (25) and (27) present parameters of Markov process with the structure, which is shown on fig 2 a, b with solid, dash and dash-dot lines. Process is as follows

\[
 \overline{a^*_m} = \left\{ \overline{A}^i, \overline{r}^i, \overline{h}^i (t) \right\}, \quad i \in \{1, 2, 3\}. \tag{28}
\]
where \( \overline{A}_i = \{ a_{0i}^*, a_{1i}^*, ..., a_{ni}^*, a_{Si}^* \} \), if \( i \in \{1, 2\} \), and \( \overline{A}_3 = \{ a_{03}^*, a_{13}^*, ..., a_{m3}^*, a_{U3+1}, a_{U3+2}^* \} \), if \( i = 3 \); \( \overline{r}_i = (r_{mi}) \) - is the adjacency matrix of size \((S_i + 1) \times (S_i + 1)\), if \( i \in \{1, 2\} \) and \((U_3 + 2) \times (U_3 + 2)\), if \( i = 3 \); \( \overline{r}_i = [\overline{r}_{mi}(t)] \) Markov matrix of the same size, as adjacency matrix;

\[
\overline{r}_i = \begin{pmatrix}
0 & 1 & \ldots & 1 & \ldots & 1 & 0 \\
1 & 0 & \ldots & 0 & \ldots & 0 & 1 \\
& & \ddots & & & & \\
& & & 1 & 0 & \ldots & 0 & 1 \\
& & & & 1 & 0 & \ldots & 0 \\
& & & & & 1 & \ldots & 1 & 0 \\
0 & 1 & \ldots & 1 & \ldots & 1 & 0
\end{pmatrix}, \quad i \in \{1, 2\}; \quad (28)
\]

\[
\overline{h}_i(t) = \begin{pmatrix}
\pi_{1i}\delta(t) & \pi_{2i}\delta(t) & \ldots & \pi_{ni}\delta(t) & 0 \\
0 & \ldots & 0 & \ldots & 0 & \overline{r}_{i0}(t) \\
& & \ddots & & & & \\
& & & 0 & \ldots & 0 & \overline{r}_{i0}(t) \\
& & & & \overline{r}_{i0}(t) & \overline{r}_{i0}(t) & \overline{r}_{i0}(t) \\
0 & q_{1i}\delta(t) & \ldots & q_{ni}\delta(t) & \ldots & q_{Si}\delta(t) & 0 \\
0 & 0 & \ldots & 0 & \ldots & 0 & \overline{r}_{i0}(t)
\end{pmatrix}, \quad i \in \{1, 2\}; \quad (29)
\]

\[
\overline{r}_3 = \begin{pmatrix}
0 & 1 & \ldots & 1 & \ldots & 1 & 0 & 0 \\
1 & 0 & \ldots & 0 & \ldots & 0 & 1 & 1 \\
& & \ddots & & & & & \\
& & & 1 & 0 & \ldots & 0 & 1 & 1 \\
& & & & 1 & 0 & \ldots & 0 & 1 & 1 \\
0 & 1 & \ldots & 1 & \ldots & 1 & 0 & 0 \\
0 & 1 & \ldots & 1 & \ldots & 1 & 0 & 0
\end{pmatrix}, \quad i = 3; \quad (30)
\]

\[
\overline{h}_3(t) = \begin{pmatrix}
0 & \pi_{13}\delta(t) & \ldots & \pi_{23}\delta(t) & \ldots & \pi_{ni}\delta(t) & 0 & 0 \\
0 & \ldots & 0 & \ldots & 0 & \overline{r}_{i0}(t) & \overline{r}_{i0}(t) & \overline{r}_{i0}(t) \\
& & \ddots & & & & & \\
& & & 0 & \ldots & 0 & \overline{r}_{i0}(t) & \overline{r}_{i0}(t) & \overline{r}_{i0}(t) \\
& & & & \overline{r}_{i0}(t) & \overline{r}_{i0}(t) & \overline{r}_{i0}(t) & \overline{r}_{i0}(t) \\
0 & q_{1i,1}\delta(t) & \ldots & q_{mi,1}\delta(t) & \ldots & q_{Si,1}\delta(t) & 0 & 0 \\
0 & q_{1i,2}\delta(t) & \ldots & q_{mi,2}\delta(t) & \ldots & q_{Si,2}\delta(t) & 0 & 0
\end{pmatrix}, \quad i = 3. \quad (31)
\]

Let us denote probabilities and densities, which are obtained when preliminary evaluation as \( \overline{0\pi}_m = \pi_m \), \( \overline{0\pi}_m(t) = \overline{h}_i(t) \), on the \( l \)-th stage of evaluation as \( \overline{l\pi}_m \), \( \overline{h}_i(t) \). Then parameters of flows of transactions may be obtained with the use of the following iteration procedure.

1. Parameters \( \overline{0\pi}_m \), \( \overline{h}_i(t) \) are calculated excluding interaction, as it was described above.
2. Recalculation of parameters \( \pi_{m_i}^{l+1} = \varphi_{z} \left[ \pi_{m_i}^{l}, \frac{1}{h_{i}(t)} \right], \quad h_{i}(t) = \varphi_{b} \left[ \pi_{m_i}^{l}, \frac{1}{h_{i}(t)} \right] \), where \( \varphi_{z} \) and \( \varphi_{b} \) - are the functions, which are defined by dependencies, obtained above.

3. Iterations should be repeated until satisfactory difference between previous and current meanings of parameters will be obtained.

5. Conclusion

The analytical model of mobile robot dialogue regimes has been developed. The model describes the interaction of three subjects: a human operator, the dialogue computer and the onboard computer. The operation of every subject is dissected into elementary operations for which time of execution and probabilities of transition to the next possible operation may be simply evaluated. This permit to tune a mobile robot control system to solving of the concrete tasks with concrete conditions of their execution, and concrete condition of environment. The result obtained may be used for working out other dialogue systems, for example for industry robots.

Further continuation of investigations in this domain may be directed to an improvement of iteration procedure, to optimization of dialogue algorithms and its adaptation to characteristics of both a human operator and a mobile robot, to optimization the transaction flows in concrete systems etc.

References

[1] Kahar S, Sulaiman R, Prbuwono S, Ahmad N and Abu Hassan M 2012 A review of wireless technology usage for mobile robot controller International conference on system engineering and modeling (ICSEM 2012) 34 pp 7 - 12

[2] Ivutin A, Larkin E and Kotov V 2015 Established routine of swarm monitoring systems functioning Advances in Swarm and Computational Intelligence pp 415–422

[3] Tzafestas S 2014 Introduction to Mobile Robot Control 750 p

[4] Larkin E and Ivutin A 2014 Estimation of Latency in Embedded Real-Time Systems 3-rd Mediterranean Conference on Embedded Computing (MECO-2014) pp 236 – 239

[5] Li Y, Malik S and Wolf A 2016 Efficient microarchitecture modeling and path analysis for real time software Proceedings of 16-th Real time systems symposium IEEE pp 298 - 307

[6] Ivutin A, Larkin E, Lutskov Y and Novikov A 2014 Simulation of concurrent process with Petri-Markov nets Life Science Journal 11(11) pp 506 – 511

[7] Korolyuk V and Swishchuk A 1995 Semi-Markov Random Evolutions pp 59 – 91

[8] Larkin E, Ivutin A, Kotov V and Privalov A 2016 Semi-Markov modeling of commands execution by mobile robot Lecture notes in artificial intelligence: Proceedings of first International conference «Interactive collaborated robotics ICR 2016» pp 189 – 198

[9] Grigelionis B 1963 On the convergence of sums of random step processes to a Poisson process Theory of Probability & Its Applications 8 pp 177 - 182

[10] Ivutin A and Larkin E 2015 Simulation of Concurrent Games Bulletin of the South Ural State University. Series: Mathematical Modelling 8 pp 43 – 54

[11] Cleaveland R and Smolka S 1996 Strategic directions in concurrency research CSUR 28 pp 607 – 625