A LECTURE ON NEUTRINO MASSES, MIXING AND OSCILLATIONS

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Abstract

Neutrino mixing and basics of neutrino oscillations are considered. Recent evidences in favour of neutrino oscillations, obtained in the solar and atmospheric neutrino experiments, are discussed. Neutrino oscillations in the solar and atmospheric ranges of \( \Delta m^2 \) are considered in the framework of the minimal scheme with the mixing of three massive neutrinos.

1 Introduction

There exist at present convincing evidences of neutrino oscillations obtained in experiments with neutrinos from natural sources: in the atmospheric \([1, 2, 3]\) and in the solar neutrino experiments \([4, 5, 6, 7, 8, 9, 10]\). The observation of neutrino oscillations give us first evidence for nonzero neutrino masses and neutrino mixing.

The investigation of neutrino oscillations is based on:

1. Interaction of neutrinos with other particles is given by the Standard Model. It was proved by numerous experiments, including very precise LEP experiments, that the Standard Model perfectly describes experimental data in the energy region up to a few hundreds GeV. The Standard Charged Current (CC) and Neutral Current (NC) Lagrangians are given by

\[
\mathcal{L}^{\text{CC}}_I = -\frac{g}{2\sqrt{2}} j^{\text{CC}}_{\alpha} W^\alpha + \text{h.c.}; \quad \mathcal{L}^{\text{NC}}_I = -\frac{g}{2 \cos \theta_W} j^{\text{NC}}_{\alpha} Z^\alpha. \quad (1)
\]

Here \( g \) is the SU(2) gauge coupling constant, \( \theta_W \) is the weak angle, \( W^\alpha \) and \( Z^\alpha \) are fields of charged \( W^\pm \) and neutral \( Z^0 \) vector bosons and for

\footnote{Report at the International School of Physics “Enrico Fermi”, Varenna, August 2002.}
the leptonic charged current \( j_{\alpha}^{CC} \) and neutrino neutral current \( j_{\alpha}^{NC} \) we have

\[
    j_{\alpha}^{CC} = \sum_l \bar{\nu}_l \gamma_{\alpha} l_L; \quad j_{\alpha}^{NC} = \sum_l \bar{\nu}_l \gamma_{\alpha} \nu_l L.
\]  

(2)

2. Three flavour neutrinos \( \nu_e, \nu_\mu \) and \( \nu_\tau \) exist in nature.

From the LEP experiments on the measurement of the width of the decay \( Z \to \nu_l + \bar{\nu}_l \) for the number of flavour neutrinos \( n_{\nu_f} \) it was obtained the value \( [11] \)

\[
    n_{\nu_f} = 3.00 \pm 0.06.
\]  

(3)

From the global fit of the LEP data for \( n_{\nu_f} \) it was found

\[
    n_{\nu_f} = 2.984 \pm 0.008.
\]  

(4)

2 Neutrino mixing

The hypothesis of neutrino mixing is based on the assumption that there is a neutrino mass term in the total Lagrangian. It was proposed several mechanisms of the generation of the neutrino mass term. Later we will discuss the most popular see-saw mechanism \( [12] \).

There are two types of possible neutrino mass terms (see \( [13, 14] \))

1. Dirac mass term

\[
    \mathcal{L}^D = -\bar{\nu}_R' M^D \nu'_L + \text{h.c.}
\]  

(5)

Here

\[
    \nu'_L = \begin{pmatrix}
        \nu_{eL} \\
        \nu_{\mu L} \\
        \nu_{\tau L} \\
        \vdots
    \end{pmatrix}; \quad \nu'_R = \begin{pmatrix}
        \nu_{eR} \\
        \nu_{\mu R} \\
        \nu_{\tau R} \\
        \vdots
    \end{pmatrix}
\]  

(6)

and \( M^D \) is a complex non-diagonal matrix.
In the case of the Dirac mass term the total Lagrangian is invariant under global gauge transformation

\[ \nu'_L \rightarrow e^{i\alpha} \nu'_L; \, \nu'_R \rightarrow e^{i\alpha} \nu'_R; \, l \rightarrow e^{i\alpha} l \]

This invariance means that the total lepton number \( L = \sum_l L_l \) is conserved.

2. Majorana mass term

\[ \mathcal{L}^{Mj} = -\frac{1}{2} (\nu'_L)^c M^{Mj} \nu'_L + \text{h.c.} \]  

(7)

Here \( M^{Mj} \) is a complex non-diagonal symmetrical matrix and

\[ (\nu'_L)^c = C \bar{\nu}'_L^T, \]

where \( C \) is the unitary matrix of the charge conjugation, which satisfies the conditions \( C \gamma^c \gamma^T = -\gamma^c \gamma, \quad C^T = -C \). It is obvious that in the case of the Majorana mass term there are no any conserved lepton numbers.

After the standard diagonalization of a neutrino mass term we have

\[ \nu'_{iL} = \sum_i U_{iL} \nu_i, \]  

(8)

where \( U \) is a unitary mixing matrix and \( \nu_i \) is the field of neutrino with mass \( m_i \).

In the case of the Dirac mass term \( \nu_i \) is the field of the Dirac neutrinos and antineutrinos which possess conserved lepton numbers \( L(\nu_i) = 1; \, L(\bar{\nu}_i) = -1 \). In the case of the Majorana mass term \( \nu_i \) is the field of truly neutral Majorana neutrinos. The field \( \nu_i \) satisfies the Majorana condition

\[ \nu_i = \nu_i^c = C \bar{\nu}'_i^T. \]  

(9)

If there are only flavour fields \( \nu_{iL} \) in the column \( \nu'_L \), the number of the massive neutrinos \( \nu_i \) is equal to three and \( U \) is a \( 3 \times 3 \) unitary matrix.

In the neutrino mass term it could be also fields, which do not enter into the standard CC and NC interactions. Such fields are called sterile. If in the
column $\nu'_L$ there are $n_s$ sterile fields $\nu_{s_L}$, the number of massive neutrinos $\nu_i$ is equal to $3 + n_s$ and $U$ is a $(3 + n_s) \times (3 + n_s)$ unitary matrix. In this case in addition to the mixing relation (8) we have

$$\nu_{s_L} = \sum_i U_{s_i L} \nu_{i L}.$$  \hspace{1cm} (10)

Sterile fields can be right-handed neutrino fields, SUSY fields etc. If more than three neutrino masses are small, transition of the flavour neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$ into sterile states become possible.

If sterile fields are right-handed neutrino fields $\nu_{i R}$, neutrino mass term has the form of the sum of the left-handed Majorana, Dirac and right-handed Majorana mass terms:

$$\mathcal{L}^{D+Mj} = -\frac{1}{2} (\bar{\nu}_L)^c M^M_L \nu_L - \bar{\nu}_R M^D \nu_L - \frac{1}{2} \bar{\nu}_R M^M_R (\nu_R)^c + \text{h.c.}$$  \hspace{1cm} (11)

Here

$$\nu_L = \begin{pmatrix} \nu_{e L} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad \nu_R = \begin{pmatrix} \nu_{e R} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix},$$  \hspace{1cm} (12)

$M^M_L$ and $M^M_R$ are complex non-diagonal symmetrical $3 \times 3$ Majorana matrices and $M^D$ is a complex non-diagonal $3 \times 3$ Dirac matrix. The mass term $\mathcal{L}^{D+Mj}$ is called the Dirac and Majorana mass term. After the diagonalization of the mass term (11) we have

$$\nu_{i L} = \sum_{i=1}^{6} U_{i L} \nu_{i L}; \quad (\nu_{i R})^c = \sum_{i=1}^{6} \bar{U}_{i R} \nu_{i L},$$  \hspace{1cm} (13)

where $U$ is the unitary $6 \times 6$ mixing matrix.

We will discuss now the see-saw mechanism of neutrino mass generation \cite{12}. In order to explain an idea of the mechanism we will consider the simplest case of one type of neutrino. Let us assume that the standard Higgs
mechanism with one Higgs doublet, which is the mechanism of the generation of the masses of quarks and leptons, generates the Dirac neutrino mass term

\[ \mathcal{L}^{D} = -m \bar{\nu}_R \nu_L + \text{h.c.} \]  

(14)

It is natural to expect that the mass \( m \) is of the same order of magnitude as masses of the corresponding lepton or quark. We know, however, from experimental data that neutrino masses are much smaller than the masses of leptons and quarks. In order to “suppress” neutrino mass let us assume that there exists lepton number violating beyond the SM mechanism of the generation of the right-handed Majorana mass term

\[ \mathcal{L}^{Mj}_R = -M \bar{\nu}_R (\nu_R)^c + \text{h.c.}, \]  

(15)

with \( M \gg m \) (usually it is assumed that \( M \simeq M_{\text{GUT}} \simeq 10^{15} \text{GeV} \)).

The total mass term is the Dirac and Majorana one with

\[ M^{D+Mj} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}; \quad \nu'_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} \]  

(16)

After the diagonalization of the mass term we have

\[ \nu_L = i \cos \theta \nu_{1L} + \sin \theta \nu_{2L} \]  

\[ (\nu_R)^c = -i \sin \theta \nu_{1L} + \cos \theta \nu_{2L}, \]  

(17)

where \( \nu_1 \) and \( \nu_2 \) are fields of the neutrino Majorana with masses

\[ m_1 = -\frac{1}{2} + \frac{1}{2} \sqrt{M^2 + 4m^2} \simeq \frac{m^2}{M} \ll 1 \]  

(18)

and

\[ m_2 = \frac{1}{2} + \frac{1}{2} \sqrt{M^2 + 4m^2} \simeq M \]  

(19)

The mixing angle \( \theta \) is given by

\[ \tan 2 \theta = \frac{2m}{M} \ll 1 \]  

(20)

Thus, the see-saw mechanism is based on the assumption that in addition to the standard Higgs mechanism of the generation of the Dirac mass term
there exist a beyond the SM mechanism of the generation of the right-handed Majorana mass term, which change the lepton number by two and is characterised by a mass $M \gg m$. The Dirac mass term mixes left-handed field $\nu_L$, the component of doublet, and right-handed singlet field $\nu_R$. As a result of this mixing neutrino acquires small Majorana mass.

In the general case of three generation for neutrino masses we have

$$m_i \simeq \frac{(m_i^f)^2}{M_i} \ll m_i^f.$$  \hspace{1cm} (21)

Here $m_i^f$ is the mass of quark or lepton in $i$-th family.

Let us stress that if neutrino masses are of the see-saw origin in this case:

- Neutrino with definite masses are Majorana particles.
- There are three light neutrinos.
- Neutrino masses satisfy the hierarchy $m_1 \ll m_2 \ll m_3$.
- The heavy Majorana particles must exist.

The existence of the heavy Majorana particles, see-saw partners of neutrinos, could be a source of the barion asymmetry of the Universe (see [17]).

3 Neutrino oscillations

In the case of neutrino mixing it is important to distinguish flavour neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$ and neutrinos with definite masses $\nu_1$, $\nu_2$,... The flavour neutrinos are particles that take part in the standard weak interaction. For example, neutrino that is produced together with $\mu^+$ in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ is the muon neutrino $\nu_\mu$, electron antineutrino $\bar{\nu}_e$ produces $e^+$ in the process $\bar{\nu}_e + p \rightarrow e^+ + n$ etc.

In order to determine the states of the flavour neutrinos let us consider a decay

$$a \rightarrow b + l^+ + \nu_l,$$ \hspace{1cm} (22)

If there is neutrino mixing

$$\nu_{\alpha L} = \sum_i U_{\alpha i} \nu_{i L}$$

$^2$It is obvious that for charged particles such mechanism does not exist.
the state of the final particles is given

$$|f> = \sum_i |b> |l^+> |\nu_i> \langle i l^+ b|S| a>,$$

(23)

where $|\nu_i>$ is the state of neutrino with momentum $\vec{p}$ and energy

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}; \ (p^2 \gg m_i^2)$$

and $\langle i l^+ b|S| a>$ is the element of S-matrix.

We will assume that the mass- squared differences $\Delta m_{ik}^2 = m_i^2 - m_k^2$ are so small that emission of neutrinos with different masses cannot be resolved in the neutrino production (and detection) experiments. In this case we have

$$\langle i l^+ b|S| a> \simeq U_{li}^* \langle \nu_l l^+ b|S| a>_{SM},$$

(24)

where $\langle \nu_l l^+ b|S| a>_{SM}$ is the Standard Model matrix element of the process (22).

From (23) and (24) for the normalised state of the flavour neutrino $\nu_l$ we obtain

$$|\nu_l> = \sum_i U_{li}^* |\nu_i>.$$

(25)

Thus, in the case of the mixing of the fields of neutrinos with small neutrino mass-squared differences the state of flavour neutrino is a coherent superposition of the states of neutrinos with definite masses.$^3$

In the general case of active and sterile neutrinos we have

$$|\nu_\alpha> = \sum_i U_{\alpha i}^* |\nu_i>,$$

(26)

where index $\alpha$ takes the values $e, \mu, \tau, s_1, ....$. From the unitarity of the mixing matrix it follows that

$$\langle \nu_{\alpha'}|\nu_\alpha> = \delta_{\alpha'\alpha}.$$

(27)

The phenomenon of neutrino oscillations is based on the relation (26). Let us consider the evolution of the mixed neutrino states in vacuum. If at

$^3$The relation (25) is analogous to the relations that connects the states of $K^0$ and $\bar{K}^0$ mesons, particles with definite strangeness, with the states of $K^0_S$ and $K^0_L$ mesons, particles with definite masses and widths.
the initial time \( t = 0 \) flavour neutrino \( \nu_\alpha \) is produced, for the neutrino state at the time \( t \) we have

\[
|\nu_\alpha\rangle_t = e^{-iH_0t} |\nu_\alpha\rangle = \sum_i U_{\alpha i}^* e^{-iE_i t} |i\rangle. \tag{28}
\]

Because of different neutrino masses, phase factors in (28) are different. This means that the flavour content of the final state differs from the initial one. At macroscopic distances this effect can be large in spite of small differences of neutrino masses.

Neutrinos are detected through the observation of CC and NC processes. Developing the state \( |\nu_\alpha\rangle_t \) over the total system of the flavour (and sterile) neutrino states \( |\nu_\alpha\rangle \), we have

\[
|\nu_\alpha\rangle_t = \sum_{\alpha'} A(\nu_\alpha \rightarrow \nu_{\alpha'}) |\nu_{\alpha'}\rangle, \tag{29}
\]

where

\[
A(\nu_\alpha \rightarrow \nu_{\alpha'}) = \langle \nu_{\alpha'} | e^{-iH_0t} |\nu_\alpha\rangle = \sum_i U_{\alpha' i}^* e^{-iE_i t} U_{\alpha i}^*. \tag{30}
\]

is the amplitude of the transition \( \nu_\alpha \rightarrow \nu_{\alpha'} \) during the time \( t \).

Taking into account the unitarity of the mixing matrix, for the probability of the transition \( \nu_\alpha \rightarrow \nu_{\alpha'} \) we obtain the following expression

\[
P(\nu_\alpha \rightarrow \nu_{\alpha'}) = |\delta_{\alpha'\alpha} + \sum_i U_{\alpha' i} U_{\alpha i}^* (e^{-i\Delta m_{1i}^2 \frac{L}{E}} - 1)|^2, \tag{31}
\]

where \( L \simeq t \) is the distance between neutrino source and neutrino detector and \( E \) is the neutrino energy.

Analogously, for the probability of the transition \( \bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'} \) we have

\[
P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}) = |\delta_{\alpha'\alpha} + \sum_i U_{\alpha' i} U_{\alpha i}^* (e^{-i\Delta m_{1i}^2 \frac{L}{E}} - 1)|^2. \tag{32}
\]

Let us notice the following general features of the transition probabilities (see, for example, [13, 14]):

- Transition probabilities depend on \( \frac{L}{E} \).
- Neutrino oscillations can be observed if the condition \( \Delta m_{1i}^2 \frac{L}{E} \gtrsim 1 \) is satisfied for at least one value of \( i \).

\[4\] We label neutrino masses in such a way that \( m_1 < m_2 < m_3 < ... \)
• From the comparison of (31) and (32) we conclude that the following relation holds

\[ P(\nu_\alpha \to \nu_{\alpha'}) = P(\bar{\nu}_{\alpha'} \to \bar{\nu}_\alpha). \]

This relation is the consequence of the CPT invariance intrinsic for any local field theory.

• In the case of the CP invariance in the lepton sector the mixing matrix \( U \) is real in the Dirac case. In the Majorana case the mixing matrix satisfies the condition

\[ U_{\alpha i} = U_{\alpha i}^* \eta_i, \]

(33)

where \( \eta_i = \pm i \) is the CP parity of the Majorana neutrino \( \nu_i \). From (31), (32) and (33) we conclude that in the case of the CP invariance in the lepton sector we have the following relation

\[ P(\nu_\alpha \to \nu_{\alpha'}) = P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}). \]

4 Oscillations between two types of neutrinos

We will consider here the simplest case of the transitions between two types of neutrinos (\( \nu_\mu \to \nu_\tau \) or \( \nu_\mu \to \nu_e \) etc). In this case the index \( i \) in Eq.(31) takes only one value \( i = 2 \) and for the transition probability we obtain expression

\[ P(\nu_\alpha \to \nu_{\alpha'}) = |\delta_{\alpha'\alpha} + U_{\alpha'2}U_{\alpha2}^* (e^{-i\Delta m^2 \frac{L}{2E}} - 1)|^2, \]

(34)

where \( \Delta m^2 = m_2^2 - m_1^2 \).

From this expression for the appearance probability (\( \alpha' \neq \alpha \)) we have

\[ P(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2} A_{\alpha'\alpha} (1 - \cos \Delta m^2 \frac{L}{2E}), \]

(35)

where the amplitude \( A_{\alpha'\alpha} \) is given by

\[ A_{\alpha'\alpha} = 4 |U_{\alpha'2}|^2 |U_{\alpha2}|^2 = A_{\alpha\alpha'}. \]

Let us introduce the mixing angle \( \theta \). We have

\[ |U_{\alpha2}|^2 = \sin^2 \theta; \quad |U_{\alpha'2}|^2 = 1 - |U_{\alpha2}|^2 = \cos^2 \theta. \]
and the amplitude $A_{\alpha';\alpha}$ is given by

$$A_{\alpha';\alpha} = \sin^2 2\theta.$$  

Hence, the two-neutrino transition probability takes the standard form

$$P(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2} \sin^2 2\theta \ (1 - \cos \Delta m^2 \frac{L}{2E}). \quad (36)$$

It is obvious that in the two-neutrino case the following relations are valid

$$P(\nu_\alpha \to \nu_{\alpha'}) = P(\nu_{\alpha'} \to \nu_{\alpha}); \quad (\alpha' \neq \alpha). \quad (37)$$

Thus, the CP violation in the lepton sector can not be revealed in the case of the transitions between two types of neutrinos.

The survival probability $P(\nu_\alpha \to \nu_\alpha)$ is determined by condition of the conservation of the probability. We have

$$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_{\alpha'}) = 1 - \frac{1}{2} \sin^2 2\theta \ (1 - \cos \Delta m^2 \frac{L}{2E}). \quad (38)$$

From (37) it follows that the two-neutrino survival probabilities satisfy the following relation

$$P(\nu_\alpha \to \nu_\alpha) = P(\nu_{\alpha'} \to \nu_{\alpha'}). \quad (39)$$

Thus, in the case of the transition between two types of neutrinos all transition probabilities are characterised by the two oscillation parameters: $\sin^2 2\theta$ and $\Delta m^2$.

The expressions (36) and (38) describe periodical transitions between two types of neutrinos (neutrino oscillations). They are widely used in the analysis of experimental data.

The expression (36) for the two-neutrino transition probability can be written in the form

$$P(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2} \sin^2 2\theta \ (1 - \cos \frac{2\pi L}{L_0}), \quad (40)$$

where

$$L_0 = 4\pi \frac{E}{\Delta m^2} \quad (41)$$

is the oscillation length.
Finally, the two-neutrino transition probability and the oscillation length can be written as

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos 2.53 \Delta m^2 \frac{L}{E}\right)$$  \hspace{1cm} (42)$$

and

$$L_0 \simeq 2.48 \frac{E}{\Delta m^2} \text{ m},$$  \hspace{1cm} (43)

where $E$ is the neutrino energy in MeV, $L$ is the distance in m and $\Delta m^2$ is neutrino mass-squared difference in eV$^2$.

## 5 Neutrino oscillation data

### 5.1 Evidence in favour of oscillations of atmospheric neutrinos

Atmospheric neutrinos are produced mainly in the decays of pions and muons

$$\pi \rightarrow \mu + \nu_\mu; \mu \rightarrow e + \nu_\mu + \nu_e.$$  \hspace{1cm} (44)

In the Super-Kamiokande (S-K) experiment [1] neutrinos are detected via the observation of the Cherenkov light emitted by electrons and muons in the large water Cherenkov detector (50 kt of H$_2$O).

At energies smaller than about 1 Gev practically all muons decay in the atmosphere and from (44) it follows that $R_{\mu/e} \simeq 2$, where $R_{\mu/e}$ is the ratio of the numbers of muon and electron events.

At higher energies the ratio $R_{\mu/e}$ is larger than two. It can be predicted, however, with an accuracy better than 5%.

The ratio ($R_{\mu/e}$), measured in the S-K [1] and SOUDAN 2 [2] atmospheric neutrino experiments, is significantly smaller than the predicted ratio ($R_{\mu/e}$)$_{MC}$. In the S-K experiment for the ratio of ratios in the Sub-GeV ($E_{vis} \leq 1.33 \text{ GeV}$) and Multi-GeV region ($E_{vis} > 1.33 \text{ GeV}$) regions it was obtained, respectively

$$\frac{(R_{\mu/e})_{\text{meas}}}{(R_{\mu/e})_{\text{MC}}} = 0.638 \pm 0.016 \pm 0.050; \quad \frac{(R_{\mu/e})_{\text{meas}}}{(R_{\mu/e})_{\text{MC}}} = 0.658 \pm 0.030 \pm 0.078$$
The fact that the ratio \( (R_{\mu/e})_{\text{meas}} \) is significantly smaller than the predicted ratio was known from the results of the previous atmospheric neutrino experiments Kamiokande \[15\] and IMB \[16\]. During many years this “atmospheric neutrino anomaly” was considered as an indication in favour of neutrino oscillations.

The compelling evidence in favour of neutrino oscillations was obtained recently by the S-K collaboration \[1\] from the observation of the large up-down asymmetry of the atmospheric high energy muon events.

If there are no neutrino oscillations, for the number of the electron (muon) events we have the following relation

\[
N_l(\cos \theta_z) = N_l(-\cos \theta_z) \quad (l = e, \mu),
\]

where \( \theta_z \) is the zenith angle.

For electron events a good agreement with this relation was obtained in the S-K experiment. For the Multi-GeV muon events the significant violation of the relation (14) was observed. For the ratio of the total number of the up-going muons \( U_\mu (\pi/2 \leq \theta_z \leq \pi) \) to the total number of the down-going muons \( D_\mu (0 \leq \theta_z \leq \pi/2) \) it was found the value

\[
\left( \frac{U}{D} \right)_\mu = 0.54 \pm 0.04 \pm 0.01.
\]

At high energies leptons are emitted practically in the direction of neutrinos. Thus, up-going muons are produced by neutrinos which travel distances from \( \simeq 500 \text{ km} \) to \( \simeq 13000 \text{ km} \) and the down-going muons are produced by neutrinos which travel distances from \( \simeq 20 \text{ km} \) to \( \simeq 500 \text{ km} \). The observation of the up-down asymmetry clearly demonstrates the dependence of the number of the muon neutrinos on the distance which they travel from the production point in the atmosphere to the detector.

The S-K data \[1\] and data of other atmospheric neutrino experiments (SOUDAN 2 \[2\], MACRO \[3\] ) are well described, if we assume that the two-neutrino oscillations \( \nu_\mu \rightarrow \nu_\tau \) take place. From the analysis of the S-K data it was found that at 90 \% CL neutrino oscillation parameters \( \Delta m^2_{\text{atm}} \) and \( \sin^2 2\theta_{\text{atm}} \) are in the range

\[
1.6 \times 10^{-3} \leq \Delta m^2_{\text{atm}} \leq 3.9 \times 10^{-3} \text{ eV}^2; \quad \sin^2 2\theta_{\text{atm}} > 0.92; \quad \chi^2_{\text{min}} = 163.2/170 \text{ d.o.f.}
\]

The best-fit values of the parameters are equal

\[
\Delta m^2_{\text{atm}} = 2.5 \times 10^{-3} \text{ eV}^2; \quad \sin^2 2\theta_{\text{atm}} = 1.0 \quad (\chi^2_{\text{min}} = 163.2/170 \text{ d.o.f.})
\]
5.2 Evidence in favour of transitions of solar $\nu_e$ into $\nu_{\mu,\tau}$

The energy of the sun is produced in the reactions of the thermonuclear pp and CNO cycles in which protons and electrons are converted into helium and electron neutrinos

$$4p + 2e^- \rightarrow^4 \text{He} + 2\nu_e.$$ 

The most important for the solar neutrino experiments reactions are listed in the Table I.

Table I

The main sources of the solar neutrinos. The maximum neutrino energies and SSM BP00 fluxes are also given.

| Reaction | Neutrino energy | SSM BP00 flux |
|----------|-----------------|---------------|
| $p p \rightarrow d e^+ \nu_e$ | $\leq 0.42 \text{ MeV}$ | $5.95 \cdot 10^{10} \text{ cm}^{-2} \text{ s}^{-1}$ |
| $e^- + ^7 \text{Be} \rightarrow \nu_e \ ^7 \text{Li}$ | $0.86 \text{ MeV}$ | $4.77 \cdot 10^{9} \text{ cm}^{-2} \text{ s}^{-1}$ |
| $^8\text{B} \rightarrow^8 \text{Be}^* e^+ \nu_e$ | $\leq 15 \text{ MeV}$ | $5.05 \cdot 10^{6} \text{ cm}^{-2} \text{ s}^{-1}$ |

As it is seen from the Table I, the major part of the solar neutrino flux constitute the small energy pp neutrinos. According to the SSM BP00 [18] the medium energy monoenergetic $^7\text{Be}$ neutrinos make up about 10 % of the total flux. The high energy $^8\text{B}$ neutrinos constitute only about $10^{-2}$ % of the total flux. However, in the S-K [7] and SNO [8, 9, 10] experiments due to high energy thresholds practically only neutrinos from $^8\text{B}$-decay can be detected. $^8\text{B}$ neutrinos give dominant contribution to the event rate measured in the Homestake experiment [4] experiment.

The event rates measured in all solar neutrino experiments are significantly smaller than the event rates, predicted by the Standard Solar models. For the ratio $R$ of the observed and the predicted by SSM BP00 [18] rates in the Homestake [4], GALLEX-GNO [3], SAGE [3] and S-K [7] experiments

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5 According to the SSM BP00 the flux of the high energy hep neutrinos, produced in the reaction $^3\text{He} + p \rightarrow^4 \text{He} + e^+ + \nu_e$, is about three order of magnitude smaller than the flux of the $^8\text{B}$ neutrinos.
the following values were obtained:

\[
R = 0.34 \pm 0.03 \quad \text{(Homestake)} \\
R = 0.58 \pm 0.05 \quad \text{(GALLEX \text{-} GNO)} \\
R = 0.60 \pm 0.05 \quad \text{(SAGE)} \\
R = 0.465 \pm 0.018 \quad \text{(S \text{-} K)}
\]

If there is neutrino mixing, original solar $\nu_e$'s due to neutrino oscillations or matter MSW transitions are transferred into another types of neutrinos, which cannot be detected by the radiochemical Homestake, GALLEX-GNO and SAGE experiments. In the S-K experiment mainly $\nu_e$ are detected: the sensitivity of the experiment to $\nu_\mu$ and $\nu_\tau$ is about six times smaller than the sensitivity to $\nu_e$. Thus, neutrino oscillations or MSW transition in matter provide the natural explanations of depletion of the fluxes of solar $\nu_e$.

Recently strong model independent evidence in favour of the transition of the solar $\nu_e$ into $\nu_\mu$ and $\nu_\tau$ was obtained in the SNO experiment [8, 10, 9]. The detector in the SNO experiment is a heavy water Cherenkov detector (1 kton of D$_2$O). Neutrinos from the sun are detected via the observation of the following three reactions:

1. CC reaction
   \[
   \nu_e + d \rightarrow e^- + p + p ,
   \]
2. NC reaction
   \[
   \nu_x + d \rightarrow \nu_x + n + p ,
   \]
3. ES process
   \[
   \nu_x + e \rightarrow \nu_x + e
   \]

During 306.4 days of running 1967$_{-60.9}^{+61.9}$ CC events, 576.5$_{-48.9}^{+49.5}$ NC events, and 263.6$_{-25.6}^{+26.4}$ ES events were recorded in the SNO experiment. The kinetic energy threshold for the detection of electrons was equal to 5 MeV. The NC threshold is 2.2 MeV. Thus, practically only neutrinos from $^8$B-decay are detected in the SNO experiment. The initial spectrum of electron neutrinos from the decay $^8$B $\rightarrow$ $^8$Be + e$^+$ + $\nu_e$ is known [19].

The total CC event rate is given by

$^6\nu_x$ stand for any flavour neutrino
\[ R^{CC}_{\nu_e} = \sigma^{CC}_{\nu_e} \Phi^{CC}_{\nu_e}, \] (50)

where \( \sigma^{CC}_{\nu_e} \) is cross section of the process (47), averaged over known initial spectrum of \(^8\)B neutrinos, and \( \Phi^{CC}_{\nu_e} \) is the flux of \( \nu_e \) on the earth. The flux \( \Phi^{CC}_{\nu_e} \) is given by the relation

\[ \Phi^{CC}_{\nu_e} = \langle P(\nu_e \rightarrow \nu_e) >_{CC} \Phi^0_{\nu_e}, \] (51)

where \( \Phi^0_{\nu_e} \) is the total (unknown) initial flux of \( \nu_e \) and \( \langle P(\nu_e \rightarrow \nu_e) >_{CC} \) is the averaged \( \nu_e \) survival probability.

All flavour neutrinos \( \nu_e, \nu_\mu \) and \( \nu_\tau \) are recorded via the detection of the NC process (48). Taking into account \( \nu_e - \nu_\mu - \nu_\tau \) universality of the NC for the total NC event rate we have

\[ R^{NC}_\nu = \sigma^{NC}_{\nu d} \Phi^{NC}_{\nu}, \] (52)

where \( \sigma^{NC}_{\nu d} \) is the cross section of the process (48), averaged over the initial spectrum of the \(^8\)B neutrinos, and \( \Phi^{NC}_{\nu} \) is the total flux of all flavour neutrinos on the earth. We have

\[ \Phi^{NC}_\nu = \sum_{l=e,\mu,\tau} \Phi^{NC}_{\nu_l}. \] (53)

Here

\[ \Phi^{NC}_{\nu_l} = \langle P(\nu_e \rightarrow \nu_l) >_{NC} \Phi^0_{\nu_e}, \] (54)

where \( \langle P(\nu_e \rightarrow \nu_l) >_{NC} \) is the averaged probability of the transition \( \nu_e \rightarrow \nu_l \).

All flavour neutrinos are detected also via the observation of the ES process (49). However, the cross section of the (NC) \( \nu_{\mu,\tau} + e \rightarrow \nu_{\mu,\tau} + e \) scattering is about six times smaller than the cross section of the (CC + NC) \( \nu_e + e \rightarrow \nu_e + e \) scattering.

The total ES event rate can be presented in the form

\[ R^{ES}_\nu = \sigma_{\nu_e} \Phi^{ES}_\nu. \] (55)

Here \( \sigma_{\nu_e} \) is the cross section of the process \( \nu_e e \rightarrow \nu_e e \), averaged over initial spectrum of the \(^8\)B neutrinos and

\[ \Phi^{ES}_\nu = \Phi^{ES}_{\nu_e} + \frac{\langle \sigma_{\nu_{\mu,e}} \rangle}{\langle \sigma_{\nu_{e}} \rangle} \Phi^{ES}_{\nu_{\mu,\tau}}, \] (56)
where $\Phi_{\nu_e}^{ES}$ is the flux of $\nu_e$, $\Phi_{\nu_{\mu,\tau}}^{ES}$ is the flux of $\nu_\mu$ and $\nu_\tau$ and

$$\frac{\langle \sigma_{\nu_e} \rangle}{< \sigma_{\nu_e} >} \simeq 0.154.$$  (57)

We have

$$\Phi_{\nu_e}^{ES} = \langle P(\nu_e \to \nu_l) >_{ES} \Phi_{\nu_e}^0,$$  (58)

where $\langle P(\nu_e \to \nu_l) >_{ES}$ is the averaged probability of the transition $\nu_e \to \nu_l$.

In the SNO experiment it was obtained [10]

$$\left(\Phi_{\nu_e}^{ES}\right)_{SNO} = (2.39^{+0.24}_{-0.23} \text{ (stat.)} \pm 0.12 \text{ (syst.)}) \cdot 10^6 \text{ cm}^{-2} \text{s}^{-1},$$  (59)

This value is in a good agreement with the S-K value. In the S-K experiment [7] solar neutrinos are detected via the observation of the ES process $\nu_x e \to \nu_x e$. During 1496 days of running a large number $22400 \pm 800$ solar neutrino events with recoil total energy threshold 5 MeV were recorded. From the data of the S-K experiment it was obtained

$$\left(\Phi_{\nu_e}^{ES}\right)_{S-K} = (2.35 \pm 0.02 \text{ (stat.)} \pm 0.08 \text{ (syst.)}) \cdot 10^6 \text{ cm}^{-2} \text{s}^{-1}. $$  (60)

In the S-K experiment the spectrum of the recoil electrons was measured. No sizable distortion of the spectrum with respect to the expected spectrum was observed. The spectrum of electrons, produced in the CC process [17], was measured in the SNO experiment [10]. No distortion of the electron spectrum was observed also in this experiment.

Thus, the data of the S-K and SNO experiments are compatible with the assumption that in the high-energy $^8B$ region the probability of the solar neutrinos to survive is a constant:

$$P(\nu_e \to \nu_e) \simeq \text{const.}$$  (61)

From (61) it follows that

$$\langle P(\nu_e \to \nu_e) >_{CC} \simeq < P(\nu_e \to \nu_e) >_{NC} \simeq < P(\nu_e \to \nu_e) >_{ES}.$$  

Taking into account these relations, from (51), (54) and (58) it follows that in the high energy $^8B$ region the fluxes of electron neutrinos, detected via the observation of CC, NC and ES processes, are the same:

$$\Phi_{\nu_e}^{CC} \simeq \Phi_{\nu_e}^{NC} \simeq \Phi_{\nu_e}^{ES}. $$  (62)

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7Expected spectra were calculated under the assumption that the shape of the spectrum of $\nu_e$ on the earth is given by known initial $^8B$ spectrum
From the data of the SNO experiment [9, 10] it was obtained that the flux of $\nu_e$ on the earth is equal to

$$(\Phi_{\nu_e}^{CC})_{\text{SNO}} = (1.76^{+0.06}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}(\text{syst.})) \cdot 10^6 \, \text{cm}^{-2}\text{s}^{-1}. \quad (63)$$

For the flux of all flavour neutrinos $\Phi_{\nu}^{NC}$ it was found the value

$$(\Phi_{\nu}^{NC})_{\text{SNO}} = (5.09^{+0.44}_{-0.43}(\text{stat.})^{+0.46}_{-0.43}(\text{syst.})) \cdot 10^6 \, \text{cm}^{-2}\text{s}^{-1}, \quad (64)$$

which is about three times larger than the value of the flux of electron neutrinos.

It is obvious that the NC flux $\Phi_{\nu}^{NC}$ is given by

$$\Phi_{\nu}^{NC} = \Phi_{\nu_e}^{NC} + \Phi_{\nu_{\mu,\tau}}^{NC} \quad (65)$$

where $\Phi_{\nu_e}^{NC}$ is the flux of $\nu_e$ and $\Phi_{\nu_{\mu,\tau}}^{NC}$ is the flux of $\nu_\mu$ and $\nu_\tau$.

Combining CC and NC fluxes and using the relation (62), we can determine now the flux $\Phi_{\nu_{\mu,\tau}}^{NC}$. Taking into account also the value (59) of the ES flux, in [8] it was obtained

$$(\Phi_{\nu_{\mu,\tau}})_{\text{SNO}} = (3.41^{+0.45}_{-0.45}(\text{stat.})^{+0.48}_{-0.45}(\text{syst.})) \cdot 10^6 \, \text{cm}^{-2}\text{s}^{-1}. \quad (66)$$

Thus, detection of the solar neutrinos via the simultaneous observation of CC, NC and ES processes allowed the SNO collaboration to obtain the direct model independent $5.3 \sigma$ evidence of the presence of $\nu_\mu$ and $\nu_\tau$ in the flux of the solar neutrinos on the earth.

The total flux of the $^8B$ neutrinos, predicted by SSM BP00 [18], is given

$$(\Phi_{\nu_e}^0)_{\text{SSM BP}} = (5.05^{+1.01}_{-0.81}) \cdot 10^6 \, \text{cm}^{-2}\text{s}^{-1} \quad (67)$$

This flux is compatible with the total flux of all flavour neutrinos (64), measured in the SNO experiment.

The flux of $\nu_\mu$ and $\nu_\tau$ on the earth can be also obtained from the SNO CC data and the S-K ES data. In first SNO publication [8] it was found the value

$$(\Phi_{\nu_{\mu,\tau}})_{\text{S-K SNO}} = (3.69 \pm 1.13) \cdot 10^6 \, \text{cm}^{-2}\text{s}^{-1}, \quad (68)$$

which is in a good agreement with the value (66).

The data of all solar neutrino experiments can be described if we assume that there are transitions of the solar $\nu_e$ into $\nu_{\mu,\tau}$ and $\nu_e$ survival probability
has two-neutrino form, which is characterized by two oscillation parameters $\Delta m^2_{\text{sol}}$ and $\tan^2 \theta_{\text{sol}}$. From the global $\chi^2$ fit of the total event rates measured in all solar neutrino experiments several allowed regions in the plane of the oscillation parameters were obtained (see, for example, [20]): large mixing angle MSW LMA and LOW regions, small mixing angle MSW SMA region, vacuum oscillations VO region and others. The situation changed after the day and night recoil electron spectra were measured in the S-K experiment [7] and SNO data [8, 10, 9] were obtained. From all analysis of the existing solar neutrino data it follows that the most plausible allowed region is the MSW LMA region (see [21] and references therein).

In [10] as a free variable parameters $\Delta m^2_{\text{sol}}$, $\tan^2 \theta_{\text{sol}}$ and the initial flux of the $^8B$ neutrinos $\Phi^0_{\nu_e}$ were used. From the analysis of all solar neutrino data the following best-fit values of the parameters were found ($\chi^2_{\text{min}} = 57/72$ d.o.f.):

$$\Delta m^2_{\text{sol}} = 5 \cdot 10^{-5} \text{eV}^2; \tan^2 \theta_{\text{sol}} = 0.34 \quad \Phi^0_{\nu_e} = 5.89 \cdot 10^6 \text{cm}^{-2} \text{s}^{-1} \quad (69)$$

If neutrino oscillation parameters are in the LMA region, neutrino oscillations in the solar range of $\Delta m^2$ can be explored in experiments with reactor $\bar{\nu}_e$'s if a distance between reactors and a detector is about 100 km. In the experiment KamLAND [22], which started in January 2002, $\bar{\nu}_e$ from several Japanese reactors are recorded by a large liquid scintillator detector (1 kt of liquid scintillator). The distance between reactors and the detector is $175 \pm 35$ km. The average energy of $\bar{\nu}_e$ from a reactor is about 3 MeV. Thus, at large mixing angles the KamLAND experiment is sensitive to the solar LMA range of neutrino mass-squared difference ($\Delta m^2 \simeq E^L \simeq 10^{-5}\text{eV}^2$).

### 5.3 Reactor experiments CHOOZ and Palo Verde

The results of the long baseline reactor experiments CHOOZ [23] and Palo Verde [24] are very important for the neutrino mixing. In these experiments the disappearance of the reactor $\bar{\nu}_e$'s in the atmospheric range of $\Delta m^2$ were searched for.

In the CHOOZ experiment $\bar{\nu}_e$ from two reactors at the distance of about 1 km from the detector were detected via the observation of the process

$$\bar{\nu}_e + p \rightarrow e^+ + n.$$
No indications in favour of disappearance of $\bar{\nu}_e$ were found in the experiment. For the ratio $R$ of the total number of the detected $\bar{\nu}_e$ events to the expected number it was found the value

$$R = 1.01 \pm 2.8\% \text{ (stat)} \pm 2.7\% \text{ (syst)} \quad \text{(CHOOZ)}$$

In the similar Palo Verde experiment it was found:

$$R = 1.01 \pm 2.4\% \text{ (stat)} \pm 5.3\% \text{ (syst)} \quad \text{(PaloVerde)}$$

The data of the experiments were analysed in [23, 24] in the framework of two-neutrino oscillations and exclusion plots in the plane of the oscillation parameters $\Delta m^2$ and $\sin^2 2 \theta$ were obtained. From the CHOOZ exclusion plot at $\Delta m^2 = 2.5 \cdot 10^{-3}$ eV$^2$ (the S-K best-fit value) we have

$$\sin^2 2 \theta \lesssim 1.5 \cdot 10^{-1}.$$  

6 Neutrino oscillations in the framework of three-neutrino mixing

6.1 Neutrino oscillations in the atmospheric range of $\Delta m^2$

We have discussed evidences in favour of neutrino oscillations that were obtained in the solar and atmospheric neutrino experiments. There exist at present also an indication in favour of the transitions $\bar{\nu}_\mu \to \bar{\nu}_e$, that was obtained in the single accelerator experiment LSND [25]. The LSND data can be explained by neutrino oscillations. From analysis of the data for the values of the oscillation parameters it was obtained the ranges

$$2 \cdot 10^{-1} \lesssim \Delta m^2 \lesssim 1 \text{ eV}^2; \quad 3 \cdot 10^{-3} \lesssim \sin^2 2 \theta \lesssim 4 \cdot 10^{-2}$$

In order to describe the data of the solar, atmospheric and LSND experiments, which requires three different values of neutrino mass-squared differences, it is necessary to assume mixing of (at least) four massive neutrinos (see, for example, [14]).

The result of the LSND experiment requires, however, confirmation. MiniBooNE experiment at Fermilab [26], that started in 2002, is aimed to check the LSND result.
We will consider here the *minimal* scheme of three neutrino mixing

\[ \nu_{\alpha L} = \sum_{i=1}^{3} U_{\alpha i} \nu_{iL}, \]  

(70)

which provide two independent \( \Delta m^2 \) and allow to describe solar and atmospheric neutrino oscillation data. In (70) \( U \) is the unitary 3×3 PMNS mixing matrix [27, 28].

Let us consider first neutrino oscillations in the atmospheric range of \( \Delta m^2 \), which can be explored in the atmospheric and long baseline accelerator and reactor neutrino experiments. In the framework of the three-neutrino mixing with \( m_1 < m_2 < m_3 \) there are two possibilities:

I. Hierarchy of neutrino mass-squared differences

\[ \Delta m^2_{21} \simeq \Delta m^2_{\text{sol}}; \quad \Delta m^2_{32} \simeq \Delta m^2_{\text{atm}}; \quad \Delta m^2_{21} \ll \Delta m^2_{32}. \]  

(71)

II. Inverted hierarchy of neutrino mass-squared differences

\[ \Delta m^2_{32} \simeq \Delta m^2_{\text{sol}}; \quad \Delta m^2_{21} \simeq \Delta m^2_{\text{atm}}; \quad \Delta m^2_{32} \ll \Delta m^2_{21}. \]  

(72)

We will assume that neutrino mass spectrum is of the type I. For the values \( L \) relevant for neutrino oscillations in the atmospheric range of \( \Delta m^2 \) (\( \Delta m^2_{32} \frac{L}{E} \geq 1 \)) we have

\[ \Delta m^2_{21} \left( \frac{L}{E} \right) \ll 1. \]

Hence we can neglect the contribution of \( \Delta m^2_{21} \) to the transition probability Eq.(71). For the probability of the transition \( \nu_{\alpha} \rightarrow \nu_{\alpha'} \) we obtain in this case the following expression

\[ P(\nu_{\alpha} \rightarrow \nu_{\alpha'}) \simeq |\delta_{\alpha'\alpha} + U_{\alpha'3}U_{\alpha3}^* (e^{-i\Delta m^2_{32} \frac{L}{2E}} - 1)|^2 \]  

(73)

Thus, in the leading approximation transition probabilities in the atmospheric range of \( \Delta m^2 \) are determined by the largest neutrino mass-squared difference \( \Delta m^2_{32} \) and the elements of the third column of the neutrino mixing matrix, which connect flavour neutrino fields \( \nu_{\alpha L} \) with the field of the heaviest neutrino \( \nu_{3L} \).

For the appearance probability from (73) we obtain

\[ P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \frac{1}{2} \delta_{\alpha'\alpha} (1 - \cos \Delta m^2_{32} \frac{L}{2E}) \quad (\alpha \neq \alpha'), \]

(74)
where the oscillation amplitude is given by the expression

\[ A_{\alpha'\alpha} = 4 \left| U_{\alpha'3} \right|^2 \left| U_{\alpha3} \right|^2 \]  

(75)

The survival probability can be obtained from the condition of the conservation of probability and Eq. (74). We have

\[ P(\nu_\alpha \to \nu_\alpha) = 1 - \frac{1}{2} B_{\alpha;\alpha} \left( 1 - \cos \Delta m_{32}^2 \frac{L}{2E} \right). \]  

(76)

Taking into account the unitarity of the mixing matrix, for the oscillation amplitude \( B_{\alpha;\alpha} \) we have

\[ B_{\alpha;\alpha} = \sum_{\alpha' \neq \alpha} A_{\alpha'\alpha} = 4 \left| U_{\alpha3} \right|^2 \left( 1 - \left| U_{\alpha3} \right|^2 \right). \]  

(77)

Let us notice that in the case of the inverted hierarchy of the neutrino mass squared differences transition probabilities can be obtained from (74)-(77) by the change \( \Delta m_{32}^2 \to \Delta m_{21}^2 \) and \( \left| U_{\alpha3} \right|^2 \to \left| U_{\alpha1} \right|^2 \).

Transition probabilities Eq.(74) and Eq.(75) depend only on \( \left| U_{\alpha3} \right|^2 \) and \( \Delta m_{32}^2 \). The CP phase does not enter into expressions for the transition probabilities. This means that in the leading approximation the relation

\[ P(\nu_\alpha \to \nu_{\alpha'}) = P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) \]  

(78)

is satisfied.

Thus, investigation of effects of the CP violation in the lepton sector in the long baseline neutrino oscillation experiments will be a difficult problem: possible effects are suppressed due to the smallness of the parameter \( \frac{\Delta m_{32}^2}{\Delta m_{21}^2} \).

High precision experiments on the search for effects of the CP-violation in the lepton sector are planned for future Neutrino Superbeam facilities \cite{29} and Neutrino Factories \cite{30, 31}.

Transition probabilities (74)-(77) have \textit{two-neutrino form} in every channel. This is obvious consequence of the fact that only the largest mass-squared difference \( \Delta m_{32}^2 \) contributes to the transition probabilities. The elements \( \left| U_{\alpha3} \right|^2 \), which determine the oscillation amplitudes, satisfy the unitarity condition \( \sum_\alpha \left| U_{\alpha3} \right|^2 = 1 \). Hence, in the leading approximation transition probabilities are characterised by three parameters. In the standard parametrisation of the neutrino mixing matrix (see \cite{11}) we have
\[ U_{\mu 3} = \sqrt{1 - |U_{e3}|^2} \sin \theta_{23}; \quad U_{\tau 3} = \sqrt{1 - |U_{e3}|^2} \cos \theta_{23}, \]  

(79)

where \( \theta_{23} \) is the mixing angle.

From (75), and (79) for the amplitude of the transition \( \nu_\mu \rightarrow \nu_{\tau} \) and \( \nu_\mu \rightarrow \nu_e \) we will obtain, respectively

\[ A_{\tau;\mu} = (1 - |U_{e3}|^2)^2 \sin^2 2 \theta_{23}; \quad A_{e;\mu} = 4 |U_{e3}|^2 (1 - |U_{e3}|^2) \sin^2 \theta_{23}. \]  

(80)

For the amplitude \( B_{e;e} \), we have\[ B_{e;e} = 4 |U_{e3}|^2 (1 - |U_{e3}|^2). \]  

(81)

In the S-K atmospheric neutrino experiment \[1\] no any indications in favour of \( \nu_\mu \rightarrow \nu_e \) transitions were obtained. The data of the experiment are well described under the assumption \( |U_{e3}|^2 \approx 0 \). In this approximation oscillations in the atmospheric range of \( \Delta m^2 \) are pure \( \nu_\mu \rightarrow \nu_{\tau} \) two-neutrino oscillations. The values of the two-neutrino oscillation parameters \( \Delta m^2_{atm} \approx \Delta m^2_{32} \) and \( \sin^2 \theta_{atm} = \sin^2 \theta_{23} \), obtained from the analysis of the S-K data, are given in \[40\].

### 6.2 Oscillations in the solar range of \( \Delta m^2 \)

Let us consider now in the framework of the tree-neutrino mixing neutrino oscillations in the solar range of \( \Delta m^2 \). The \( \nu_e \) survival probability in vacuum can be written in the form

\[ P(\nu_e \rightarrow \nu_e) = \left| \sum_{i=1,2} |U_{ei}|^2 e^{-i \Delta m^2_{3i} \frac{L}{E}} + |U_{e3}|^2 e^{-i \Delta m^2_{31} \frac{L}{E}} \right|^2 \]  

(82)

We are interested in the survival probability averaged over the region where neutrinos are produced, neutrino energy spectrum etc. Because \( \Delta m^2_{32} \) is much larger than \( \Delta m^2_{31} \), in the averaged survival probability the interference between the first and the second terms in (82) disappears. The averaged survival probability can be presented in the form

\[ A_{e;\mu} = B_{e;e} \sin^2 \theta_{23}. \]
\[ P(\nu_e \rightarrow \nu_e) = |U_{e3}|^4 + (1 - |U_{e3}|^2)^2 \ P^{(1,2)}(\nu_e \rightarrow \nu_e). \]  

(83)

Here \( P^{(1,2)}(\nu_e \rightarrow \nu_e) \) is given by the expression

\[ P^{(1,2)}(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} \ A^{(1,2)} (1 - \cos \Delta m_{21}^2 \frac{L}{2E}), \]  

(84)

where

\[ A^{(1,2)} = 4 \ \frac{|U_{e1}|^2 |U_{e2}|^2}{(1 - |U_{e3}|^2)^2}. \]  

(85)

In the standard parametrisation of the neutrino mixing matrix we have

\[ U_{e1} = \sqrt{1 - |U_{e3}|^2} \ \cos \theta_{12}; \ U_{e2} = \sqrt{1 - |U_{e3}|^2} \ \sin \theta_{12}, \]  

(86)

where \( \theta_{12} \) is the mixing angle. From (83) and (86) for the amplitude \( A^{(1,2)} \) we obtain the following expression

\[ A^{(1,2)} = \sin^2 2 \theta_{12} \]  

(87)

Thus, the probability \( P^{(1,2)}(\nu_e \rightarrow \nu_e) \) is characterised by two parameters and have the standard two-neutrino form.

The expression (83) is also valid in the case of matter [32, 14]. In this case \( P^{(1,2)}(\nu_e \rightarrow \nu_e) \) is the two-neutrino \( \nu_e \) survival probability in matter. In the calculation of this quantity the density of electrons \( \rho_e(x) \) in the effective Hamiltonian of the interaction of neutrino with matter must be changed by \((1 - |U_{e3}|^2) \rho_e(x)\).

As we will see in the next subsection, from the data of the reactor CHOOZ and Palo Verde experiments it follows that element \( |U_{e3}|^2 \) is small. If we neglect \( |U_{e3}|^2 \) in Eq. (83) we come to the conclusion that in the framework of the three-neutrino mixing \( \nu_e \) survival probability in the solar range of neutrino mass-squared difference has two-neutrino form

\[ P(\nu_e \rightarrow \nu_e) \simeq P^{(1,2)}(\nu_e \rightarrow \nu_e). \]  

(88)

The values of the parameters \( \Delta m_{\text{sol}}^2 \simeq \Delta m_{21}^2 \) and \( \tan^2 \theta_{\text{sol}} \simeq \tan^2 \theta_{12} \), obtained from the analysis of the solar neutrino data, are given in (89).

Thus, due to the smallness of the parameter \( |U_{e3}|^2 \) and hierarchy of neutrino mass squared differences \( \Delta m_{12}^2 \ll \Delta m_{32}^2 \) neutrino oscillations in the atmospheric and solar ranges of \( \Delta m^2 \) in the leading approximation are decoupled [32] and are described by two-neutrino formulas, which are characterised by the parameters \( \Delta m_{32}^2, \sin^2 2 \theta_{23} \) and \( \Delta m_{21}^2, \tan^2 \theta_{12} \), respectively.
6.3 The upper bound of $|U_{e3}|^2$ from the data of the CHOOZ experiment

The reactor long baseline CHOOZ [23] and Palo Verde [24] experiments are sensitive to the atmospheric range of $\Delta m^2$. No any indications in favour of disappearance of reactor $\bar{\nu}_e$ were obtained in these experiments. From the analysis of the data of the CHOOZ and Palo Verde experiments the best bound on the parameter $|U_{e3}|^2$ can be obtained.

In the framework of the three-neutrino mixing the probability of $\bar{\nu}_e$ to survive is given by the expression

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2} B_{e;e} \left( 1 - \cos \Delta m^2_{32} \frac{L}{2E} \right), \quad (89)$$

where the amplitude $B_{e;e}$ is given by Eq. (81).

In [23, 24] exclusion plots in the plane of the parameters $\Delta m^2 \equiv \Delta m^2_{32}$ and $\sin^2 2\theta \equiv B_{e;e}$ were obtained. From these exclusion plots we have

$$B_{e;e} \leq B^0_{e;e}, \quad (90)$$

where the upper bound $B^0_{e;e}$ depends on $\Delta m^2$. For the S-K [1] allowed values of $\Delta m^2_{32}$ from the CHOOZ exclusion plot we find

$$1 \cdot 10^{-1} \leq B^0_{e;e} \leq 2.4 \cdot 10^{-1}. \quad (91)$$

Using (81) and (90), for the parameter $|U_{e3}|^2$ we have the bounds

$$|U_{e3}|^2 \leq \frac{1}{2} \left( 1 - \sqrt{1 - B^0_{e;e}} \right) \approx \frac{1}{4} B^0_{e;e} \quad (92)$$

or

$$|U_{e3}|^2 \geq \frac{1}{2} \left( 1 + \sqrt{1 - B^0_{e;e}} \right) \geq 1 - \frac{1}{4} B^0_{e;e} \quad (93)$$

Thus, parameter $|U_{e3}|^2$ can be small or large (close to one). This last possibility is excluded by the solar neutrino data. In fact, if $|U_{e3}|^2$ is large, from Eq. (83) it follows that in the whole range of the solar neutrino energies the probability of $\nu_e$ to survive is close to one in obvious contradiction with the solar neutrino data. Thus, the upper bound of the parameter $|U_{e3}|^2$ is given by (92). At the S-K best-fit point $\Delta m^2_{32} = 2.5 \cdot 10^{-3} eV^2$ we have

$$|U_{e3}|^2 \leq 4 \cdot 10^{-2} \quad (95\% \text{ CL}). \quad (94)$$
7 Conclusion

Compelling evidences in favour of neutrino oscillations, driven by small neutrino masses and neutrino mixing, were obtained in recent years in the S-K, SNO and other atmospheric and solar neutrino experiments. These findings opened a new field of research in the particle physics and astrophysics: physics of massive and mixed neutrinos.

From the results of the experiments it follows that neutrino masses are many orders of magnitude smaller than the masses of other fundamental fermions (leptons and quarks). There is a general consensus that tiny neutrino masses are of a beyond the Standard Model origin.

There are many unsolved problems in the physics of massive and mixed neutrinos. In the nearest years LMA solution of the solar neutrino problem will be tested by the KamLAND and the BOREXINO experiments. If neutrino oscillation parameters $\Delta m_{\text{sol}}^2$ and $\tan^2 \theta_{\text{sol}}$ are in the LMA region, neutrino oscillations in the solar range of $\Delta m^2$ can be studied in details in terrestrial experiments with well known antineutrino spectrum.

Another problem which will be probably solved in the nearest years is the problem of LSND. If LSND result will be confirmed by the MiniBOONE experiment it will mean that the number of light neutrinos is more than three and in addition to the three flavour neutrinos sterile neutrino(s) must exist. If LSND result will be not confirmed, the minimal scheme with three massive and mixed neutrinos will be very plausible possibility.

The problem of the nature of massive neutrinos (Dirac or Majorana?) is one of the most fundamental one. This problem can be solved by the experiments on the search for neutrinoless double $\beta$- decay ($(\beta\beta)_{0\nu}$ -decay). If massive neutrinos are Majorana particles the matrix element of this process is proportional to the effective Majorana mass $< m > = \sum_i U_{ei}^2 m_i$. The most stringent lower bounds for the time of life of the $(\beta\beta)_{0\nu}$ -decay were obtained in the $^{76}\text{Ge}$ experiments. Taking into account different calculations of the nuclear matrix elements, for the effective Majorana mass from the results of these experiments the following upper bounds was obtained

$$| < m > | \leq (0.3 - 1.3) \text{ eV}.$$  

Many new experiments on the search for $(\beta\beta)_{0\nu}$ -decay are in preparation at present. In these experiments the sensitivities

$$| < m > | \simeq (1.5 \cdot 10^{-2} - 1 \cdot 10^{-1}) \text{ eV}$$
is planned to be achieved. Existing neutrino oscillation data allow to obtain some constraint on the value of the effective Majorana mass (see, for example [38]). If the number of massive neutrinos is equal to three and there is see-saw inspired neutrino mass hierarchy $m_1 \ll m_2 \ll m_3$, the upper bound of the effective Majorana mass is presumably lower than the sensitivity of the $(\beta\beta)_{0\nu}$-experiments of the next generation.

One of the very important problem of neutrino mixing is the problem of $|U_{e3}|^2$. In order to see effects of the three-neutrino mixing in future long baseline neutrino experiments and, in particular, effects of the CP-violation in the lepton sector, it necessary that the parameter $\Delta m_{21}^2$ was in LMA region and the parameter $|U_{e3}|^2$ was larger than $10^{-4} - 10^{-3}$ (see [30]). Best limit on $|U_{e3}|^2$ was found from the data of the reactor experiment CHOOZ [23]. New information on $|U_{e3}|^2$ will be obtained in the nearest years in the MINOS [39], ICARUS [40] and JHF [41] experiments.

It is obvious that in order to reveal the real origin of the newly discovered phenomenon of small neutrino masses and neutrino mixing a lot of work have to be done. We would like to notice that it is not for the first time that breakthrough to a new physics was connected with neutrinos. The first Fermi theory of the $\beta$-decay was based on the Pauli hypothesis of neutrino. The phenomenological V-A theory of the weak interactions started with Landau, Lee and Young and Salam two-component neutrino theory. The first evidence for the Glashow, Weinberg and Salam Standard Model of the electroweak interaction was obtained on neutrino beam in CERN (discovery of the NC).

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