Research Article

Dissipativity Analysis for a Class of Discrete-Time Neutral Stochastic Nonlinear Systems with Time Delay

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This paper focuses on the problem of dissipativity analysis for a class of discrete-time neutral stochastic nonlinear systems (DTNSNSs) with time delay and parameter uncertainties. Different from the existing results on this topic of neutral system, a kind of discretizing the neutral system is considered. Firstly, a sufficient condition of the dissipativity, which is dependent on the solution of the Lyapunov–Krasovskiitechnique and linear matrix inequalities (LMIs), is established. Moreover, the state-feedback controller is designed to guarantee the dissipative performance of the closed-loop system. The effectiveness of the theoretical results is finally demonstrated by a numerical example.

1. Introduction

Time delay is an ubiquitous phenomenon in nature, which is the main reason of oscillation, instability, and bad system performance of the control system. As a result, the problem of time delay is inevitably involved in many control systems [1–5]. At the same time, it is not only the time delay that has an impact on the system; random factors also exist objectively in the actual production, and the influence of random errors can hardly be ignored [1, 6–12]. Accordingly, the work of a stochastic time-delay system is particularly important.

In the research of stochastic time-delay systems, neutral stochastic time-delay systems are a special and crucial kind of control systems with time delay, which are widely used in natural science and engineering technology, such as economics, automatic control theory, medicine, biology, and chemistry. Neutral time-delay systems are those in which both the state and state derivative are delayed. Furthermore, the delay in general system state is called discrete delay, and the delay in system state derivative is called neutral delay, which can reflect the change law of things more profoundly and accurately and reveal the essence of things. Therefore, it is necessary to study the neutral stochastic delay systems, which have captured the attention of a large number of scholars [1, 8–10].

As we all know, most phenomena in nature are described by nonlinear systems [3, 13–16]. Although the research results of stochastic nonlinear systems are not as much as those of linear stochastic systems, there are also rich achievements [2, 17–22]. Based on fixed point theory, Wu et al. [17] investigated the asymptotic mean square stability of the nonlinear neutral stochastic differential equations with time-varying delays. Without assuming linear growth conditions, Shen et al. [19] studied the boundedness and exponential stability of the exact solution, whose results can be applied to practical applications.

The notion of dissipative in control systems has caused many scholars’ concern since it was introduced by Willems firstly [23], due to its wide applications in control theory and practical systems such as robotic systems, electromechanical systems, power systems, internal combustion engine engineering, and chemical processes [3, 11, 24–26]. Dissipativity is a very important concept in a control system, and as special cases, the concepts of $H_{\infty}$ performance and passivity exist widely in the fields of physics, applied mathematics, and mechanics, (see, for example, [26, 27]. By constructing
an appropriate Lyapunov–Krasovskii Functional (LKF), Caol et al. [3] focused on the problem of the global asymptotic stability and dissipativity problem for a class of neutral-type stochastic Markovian jump Static Neural Networks (NTSMJSSNNs) with time-varying delays. Yang [25] probed the robust control problem of nonlinear time-delay systems based on dissipative analysis. The problem of extended dissipativity analysis for delayed uncertain discrete-time singular neural networks (DTSNNs) with Markovian jump parameters and stochastic behavior was studied in [11]. By utilizing the linearization technique, Hanmei Wang and Jun Zhao have investigated passivity and $H_{\infty}$ control of switched discrete-time nonlinear systems [26].

In a discrete system, namely, the discrete time system, the variables of all or key components of the system exist in the form of discrete signals, and the state of the system changes at discrete time points. Discrete systems need to be described by difference equations, which are widely used in social, economic, and engineering systems, such as automata, impulse control, sampling regulation, and digital control [12, 27–30]. In practical control systems, some discrete dynamical systems described by difference equations are often encountered. The stability research of them in the sense of Lyapunov–Krasovskii has been poured enough attention into [31, 32]. However, due to the existence of the

unknown real time-varying matrix function with Lebesgue norm measurable elements satisfying

$$\Delta^T(s)\Delta(s) \leq I, \quad \forall s \geq 0. \quad (3)$$

The parameter uncertainties are said to be admissible if both (2) and (3) hold. When $\Delta(s) = 0$, the system is referred to as a nominal system.

Remark 1. Generally speaking, the structure of the parameter uncertainties with the form (2) and (3) has been widely considered in practical problems of control systems including both continuous and discrete time [1, 9, 18, 30]. Compared with the novel framework of discrete-time neutral systems [32], the stochastic and nonlinear perturbations are taken into this paper, which makes the model more practical.

For systems (1), we design the following state feedback controller:

$$u(s) = K x(s), \quad (4)$$

where $d$ is a time delay, $\tau$ denotes a neutral delay, $x(s) \in \mathbb{R}^n$ is the state variable, $v(s) \in \mathbb{R}^i$ is the exogenous disturbance input or a reference signal, $u(s) \in \mathbb{R}^n$ is the control input, $z(s) \in \mathbb{R}^p$ is the control output vector, $\varphi(s)$ is a continuous vector-valued initial function, $f(s, x(s)), \tilde{f}_d(s, x(s), d(s - d)) \in \mathbb{R}^n \times \mathbb{R}^n$ represent unknown nonlinear functions which describe the structure uncertainty of the system, and $w(s)$ is a scalar Brownian motion on the complete probability space $(\Omega, \mathbb{F}, P)$ with $E[w(s)] = 0, E[w^2(s)] = 1$. Here, $A, A_d, B, G, H, D_d, C, C_d$ are known real matrices. $\Delta A(s), \Delta A_d(s), \Delta H(s), \Delta H_d(s), \Delta C(s), \Delta C_d(s)$ are unknown time-varying matrices, which are the parameter uncertainties and satisfy the following condition:

$$\begin{bmatrix}
\Delta A(s) & \Delta A_d(s) \\
\Delta H(s) & \Delta H_d(s) \\
\Delta C(s) & \Delta C_d(s)
\end{bmatrix} = \begin{bmatrix}
\mathcal{M}_1 & \\
\mathcal{M}_2 & \Delta(s)[\mathcal{N}_1 & \mathcal{N}_2], \\
\mathcal{M}_3 & 
\end{bmatrix} \quad (2)$$

in which $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{N}_1, \mathcal{N}_2$ are determinate constant matrices with appropriate dimensions and $\Delta(s)$ is an
in which the matrix $K$ is the gain matrix, which is to be designed. Therefore, the closed-loop DTNSNSs with time delay and parameter uncertainties can be rewritten as

$$
\begin{align*}
\dot{x}(s + 1) &= \tilde{A}x(s) + \tilde{A}_d x(s - d) + Gv(s) + \tilde{f} (s, x(s)) \\
&+ \tilde{f}_d (s, x(s - d)) + \mu (x(s + 1 - \tau)) + \left[ \tilde{H}x(s) + \tilde{H}_d x(s - d) \right]w(s), \\
z(s) &= \tilde{C}x(s) + \tilde{C}_d x(s - d), \\
x(s) &= \varphi(s), \quad \forall s \in [-\max\{d, \tau\}, 0],
\end{align*}
$$

where $\tilde{A} = A + BK + \Delta A(s), \quad \tilde{A}_d = A_d + \Delta A_d(s), \quad \tilde{H} = H + \Delta H(s), \quad \tilde{H}_d = H_d + \Delta H_d(s), \quad C = C + \Delta C(s), \quad \text{and} \quad \tilde{C}_d = C_d + \Delta C_d(s)$.

In the first place, we make the following assumptions.

**Assumption 1.** The unknown nonlinear perturbations $\tilde{f}(s, x(s)), \tilde{f}_d (s, x(s))$ satisfy the following condition:

$$
\begin{align*}
\tilde{f}^T (s, x(s)) R \tilde{f} (s, x(s)) &\leq \alpha^2 x^T(s) R x(s), \\
\tilde{f}_d^T (s, x(s-d)) R \tilde{f}_d (s, x(s-d)) &\leq \beta^2 x^T(s-d) R x(s-d),
\end{align*}
$$

where $\alpha, \beta$ are the given positive real constants and $R$ is a matrix with appropriate dimensions.

As a matter of fact, Assumption 1 is more general and has significant implications for dealing with the nonlinear function of the system [25].

**Assumption 2.** Without loss of generality, for the neutral term $\mu (x(s + 1 - \tau))$, it satisfies the following condition:

$$
\| \mu (x(s + 1 - \tau)) \| \leq \eta \| x(s + 1 - \tau) \|, \quad \eta \in (0, 1).
$$

In fact, the abovementioned inequality is equivalent to

$$
\eta^2 x^T (s + 1 - \tau) x(s + 1 - \tau) - \mu^T (x(s + 1 - \tau)) \mu (x(s + 1 - \tau)) \geq 0.
$$

Before presenting the main results of this paper, two definitions for the DTNSNSs (1) with time delay are introduced, which will be necessary for the derivations of following theorems.

**Definition 1** (see [17]). $u(s) = 0$ and $v(s) = 0$ of systems (1) are called stochastic mean square stable if for any $\varepsilon > 0$, there exists constant $\delta(\varepsilon) > 0$ such that

$$
E \| x(s) \|^2 < \varepsilon, \quad s > 0,
$$

when $\sup_{t \in [0, \infty)} E \| x(s) \|^2 < \delta(\varepsilon)$, $h = \max\{d, \tau\}$.

If, additionally, $\lim_{s \to -\infty} E \| x(s) \|^2 = 0$ for any initial conditions, then systems (1) with $u(s) = 0$ and $v(s) = 0$ are said to be stochastic mean square asymptotically stable.

For systems (1), the energy supply function is defined as

$$
\Phi (v, z, N) = \langle z, \mathcal{X}z \rangle_N + 2 \langle v, \mathcal{Y}z \rangle_N + \langle v, \mathcal{Z}v \rangle_N, \quad \forall N \geq 0,
$$

where the matrices $\mathcal{X}^T = \mathcal{X} \in \mathcal{X}_{\rho^q} \leq 0$, $\mathcal{Y}^T = \mathcal{Y} \in \mathcal{Y}_{\rho^q} > 0$, and $\mathcal{Z}^T = \mathcal{Z} \in \mathcal{Z}_{\rho^q} \geq 0$, and

$$
\langle h, \mathcal{A} \rangle_N \triangleq \sum_{s=0}^{N} h^T(s) \mathcal{A}(s).
$$

**Definition 2** (see [4]). Given supply rates function $\Phi (v, z, N)$, the DTNSNSs (1) with time delay are called strictly $(\mathcal{X}, \mathcal{Y}, \mathcal{Z})$-dissipative if under zero initial condition, the energy supply function satisfies

$$
E [\Phi (v, z, N)] \geq 0, \quad \forall N \geq 0.
$$

Furthermore, if there exists a small enough scalar $\rho > 0$,

$$
E [\Phi (v, z, N)] \geq \rho E (\langle v, v \rangle_N), \quad \forall N \geq 0.
$$

The DTNSNSs (1) with time delay are called strictly $(\mathcal{X}, \mathcal{Y}, \mathcal{Z})$-dissipative.

**Remark 2.** The abovementioned performance of strict $(\mathcal{X}, \mathcal{Y}, \mathcal{Z})$-dissipativity includes the $H_\infty$ performance and passivity as special cases, which are listed as follows:

(1) When $\mathcal{X} = -I$, $\mathcal{Y} = 0$, and $\mathcal{Z} = \gamma^2 I$, strictly $(\mathcal{X}, \mathcal{Y}, \mathcal{Z})$-dissipative, (13) reduces to an $H_\infty$ performance requirement

(2) When $\mathcal{X} = 0$, $\mathcal{Y} = I$, and $\mathcal{Z} = 0$, (1) corresponds to a strict passive problem

In addition, the following lemmas are used for proving our desired results.

**Lemma 1** (see [2]). Let $a \in \mathbb{R}^n$, $b \in \mathbb{R}^n$, and $\varepsilon > 0$; then, we have

$$
a^T b + b^T a \leq \varepsilon a a^T + \varepsilon^{-1} b b^T.
$$

**Lemma 2** (see [14]). For the given symmetric matrix $\Sigma$ and matrices $\mathcal{D}$ and $\mathcal{L}$ with appropriate dimensions, $\Sigma + \mathcal{D} \mathcal{F}(s) \mathcal{L} + \mathcal{D}^T \mathcal{F}^T(s) \mathcal{L} \leq 0$ with $\mathcal{F}(s)$ satisfying $\mathcal{F}^T(s) \mathcal{F}(s) \leq 1$ holds if and only if $\Sigma + \varepsilon \mathcal{D} \mathcal{F}^T + \varepsilon^{-1} \mathcal{L} \mathcal{F} \leq 0$ holds for any $\varepsilon > 0$. 
Lemma 3 (see [21]) (Schur Complement). Given one positive definite matrix $G^2 > 0$ and constant matrices $G_1$ and $G_3$, where $G_1 = G_1^T$, $G_1 + G_3 G_2^{-1} G_3 < 0$ if and only if

$$
\begin{bmatrix}
G_1 & G_3 \\
G_3 & -G_2
\end{bmatrix} < 0
$$

or

$$
\begin{bmatrix}
-G_2 & -G_3 \\
G_3^T & G_1
\end{bmatrix} < 0.
$$

The aim is to study the stability and dissipativity analysis for DTNSNSs with time delay and parameter uncertainties.

3. Main Results and Proofs

3.1. Dissipativity Analysis for DTNSNSs with Time Delay

\textbf{Theorem 1.} We consider $u(s) = 0$ of the DTNSNSs (5) with time delay and parameter uncertainties. The nonlinear functions $f(s, x(s))$ and $f_d(s, x(s - d))$ satisfy (6) and (7). If there exist matrices $P > 0$, $Q_i > 0$ ($i = 1, 2, 3, 4, 5$), $Y \geq 0$, $Z \geq 0$, $X \leq 0$, and positive scalars $\lambda_i$ ($i = 1, 2, 3, 4, 5$) such that

$$
E =
\begin{bmatrix}
E_{11} & E_{12} & E_{13} & E_{14} & E_{15} \\
* & E_{22} & 0 & 0 & 0 \\
* & * & E_{33} & 0 & 0 \\
* & * & * & E_{44} & 0 \\
* & * & * & 0 & E_{55}
\end{bmatrix}
$$

is strictly negative definite, where

\hspace{1cm}$\beta_T$ he aim is to study the stability and dissipativity analysis for DTNSNSs with time delay and parameter uncertainties.
\[ V_1(s) = [x(s) - \mu x(s - \tau)]^T \mathcal{P} [x(s) - \mu x(s - \tau)], \]
\[ V_2(s) = \sum_{i=\tau-d}^{\tau-1} x^T(i) \mathcal{C}_1 x(i), \]
\[ V_3(s) = \sum_{\kappa=\tau+1}^{\tau+\tau-1} x^T(\kappa) \mathcal{C}_2 x(\kappa). \]

Calculating the forward difference of \( V_j(s) \) as \( \Delta V_j(s) = V_j(s + 1) - V_j(s) \) and taking the mathematical expectation, we obtain

\begin{equation}
E[\Delta V_1(s)] = x^T(s + 1) \mathcal{P} x(s + 1) - x^T(s + 1) \mathcal{P} x(s) - x^T(s) \mathcal{P} x(s) - \mu^T x(s - \tau) \mathcal{P} x(s) - \mu^T x(s - \tau),
\end{equation}

\begin{equation}
E[\Delta V_2(s)] = x^T(s) \mathcal{C}_1 x(s) - x^T(s + 1 - \tau) \mathcal{C}_1 x(s + 1 - \tau),
\end{equation}

\begin{equation}
E[\Delta V_3(s)] = x^T(s + 1) \mathcal{C}_2 x(s + 1) - x^T(s + 1 - \tau) \mathcal{C}_2 x(s + 1 - \tau).
\end{equation}

Accordingly,

\begin{equation}
E[\Delta V(s)] = \left[ \begin{array}{c} \tilde{A} x(s) + \tilde{A}_d x(s - d) + \tilde{f}(s, x(s)) + \tilde{f}_d(s, x(s - d)) + Gv(s) \\ \tilde{A} x(s) + \tilde{A}_d x(s - d) + \tilde{f}(s, x(s)) + \tilde{f}_d(s, x(s - d)) + Gv(s) \\ \tilde{H} x(s) + \tilde{H}_d x(s - d) + \mu^T x(s + 1 - \tau) \end{array} \right]^T (\mathcal{P} + \mathcal{G}_2)
\end{equation}

\begin{equation}
= \left[ \begin{array}{c} \tilde{H} x(s) + \tilde{H}_d x(s - d) + \mu^T x(s + 1 - \tau) \end{array} \right]^T \mathcal{C}_2
\end{equation}

\begin{equation}
+ \mu^T (x(s + 1 - \tau)) \mathcal{C}_2 x(s + 1 - \tau) - x^T(s) \mathcal{P} x(s) + x^T(s) \mathcal{P} x(s) - \mu^T (x(s - \tau)) \mathcal{P} x(s) + x^T(s) \mathcal{C}_1 x(s) - x^T(s - d) \mathcal{C}_1 x(s - d) - x^T(s + 1 - \tau) \mathcal{C}_2 x(s + 1 - \tau).
\end{equation}
By virtue of Lemma 1 and Assumption 1, we proceed to express

\begin{align*}
&x^T(s)\bar{A}^T\bar{P}\bar{\bar{f}}(s,x(s)) + \bar{f}^T(s,x(s))\bar{A}\bar{x}(s) \\
&\quad \leq \varepsilon_1 x^T(s)\bar{A}^T\bar{A}\bar{x}(s) + \varepsilon^{-1}_1 \alpha^2 x^T(s)\bar{P}^T\bar{P}x(s), \\
&\quad \leq \varepsilon_2 x^T(s)\bar{A}^T\bar{A}\bar{x}(s) + \varepsilon^{-1}_2 \beta^2 x^T(s-d)\bar{P}^T\bar{P}x(s-d), \\
&\quad \leq \varepsilon_3 x^T(s-d)\bar{A}\bar{A}x(s-d) + \varepsilon^{-1}_3 \alpha^2 x^T(s-d)\bar{P}^T\bar{P}x(s-d), \\
&\quad \leq \varepsilon_4 x^T(s-d)\bar{A}\bar{A}x(s-d) + \varepsilon^{-1}_4 \beta^2 x^T(s-d)\bar{P}^T\bar{P}x(s-d), \\
&\quad \leq \varepsilon_5 x^T(s-d)\bar{A}\bar{A}x(s-d) + \varepsilon^{-1}_5 \alpha^2 x^T(s-d)\bar{P}^T\bar{P}x(s-d), \\
&\quad \leq \varepsilon_6 x^T(s-d)\bar{A}\bar{A}x(s-d) + \varepsilon^{-1}_6 \beta^2 x^T(s-d)\bar{P}^T\bar{P}x(s-d).
\end{align*}

Similar to the derivation given above, we denote

\begin{align*}
&x^T(s)\bar{A}^T\bar{G}\bar{\bar{f}}(s,x(s)) + \bar{f}^T(s,x(s))\bar{G}\bar{x}(s) \\
&\quad \leq \varepsilon_7 x^T(s)\bar{A}^T\bar{A}\bar{x}(s) + \varepsilon^{-1}_7 \alpha^2 x^T(s)\bar{G}^T\bar{G}x(s), \\
&\quad \leq \varepsilon_8 x^T(s)\bar{A}^T\bar{A}\bar{x}(s) + \varepsilon^{-1}_8 \beta^2 x^T(s-d)\bar{G}^T\bar{G}x(s-d), \\
&\quad \leq \varepsilon_9 x^T(s-d)\bar{A}\bar{A}x(s-d) + \varepsilon^{-1}_9 \alpha^2 x^T(s-d)\bar{G}^T\bar{G}x(s-d), \\
&\quad \leq \varepsilon_{10} x^T(s-d)\bar{A}\bar{A}x(s-d) + \varepsilon^{-1}_{10} \beta^2 x^T(s-d)\bar{G}^T\bar{G}x(s-d), \\
&\quad \leq \varepsilon_{11} x^T(s-d)\bar{A}\bar{A}x(s-d) + \varepsilon^{-1}_{11} \alpha^2 x^T(s-d)\bar{G}^T\bar{G}x(s-d), \\
&\quad \leq \varepsilon_{12} x^T(s-d)\bar{A}\bar{A}x(s-d) + \varepsilon^{-1}_{12} \beta^2 x^T(s-d)\bar{G}^T\bar{G}x(s-d),
\end{align*}
Together with Lemma 1 and Assumptions 1 and 2, we get

\[
v^T(s)G^T \mathcal{Q}_2 \mathcal{f}(s, x(s)) + v^T(s)G^T \mathcal{Q}_2 \mathcal{f}(s, x(s)) \mathcal{Q}_2 Gv(s) \leq \epsilon_{12} v^T(s)G^T Gv(s) + \epsilon_{12}^2 \alpha^T(s) \mathcal{Q}_2^T \mathcal{Q}_2 x(s),
\]

\[
\tilde{v}^T(s)G^T \mathcal{Q}_2 \mathcal{f}(s, x(s)) + \tilde{v}^T(s, x(s)) \mathcal{Q}_2 Gv(s) \leq \epsilon_{13} v^T(s)G^T Gv(s) + \epsilon_{13}^2 \beta^T(s) \mathcal{Q}_2^T \mathcal{Q}_2 x(s - d),
\]

\[
\tilde{f}^T(s, x(s)) \mathcal{Q}_2 \mathcal{f}(s, x(s)) + f^T_d(s, x(s)) \mathcal{Q}_2 Gv(s) \leq \epsilon_{14}^2 \alpha^T x(s) + \epsilon_{14}^2 \beta^T(s) \mathcal{Q}_2^T \mathcal{Q}_2 x(s - d),
\]

\[
\tilde{f}^T_d(s, x(s)) \mathcal{Q}_2 \tilde{f}(s, x(s)) + \tilde{f}^T_d(s, x(s)) \mathcal{Q}_2 x(s) \leq \alpha^T x(s) \mathcal{Q}_2 x(s),
\]

\[
\tilde{f}^T_d(s, x(s)) \mathcal{Q}_2 \tilde{f}(s, x(s)) \leq \alpha^T x(s) \mathcal{Q}_2 x(s),
\]

Substituting \((23)\)–\((27)\) into \((22)\), it can be derived that

\[
E[\Delta V(s)] \leq \zeta^T(s) \Lambda(s) \zeta(s),
\]
\[
\zeta^T(s) = \begin{bmatrix} x^T(s), x^T(s-d), x^T(s+1-\tau), v^T(s), \mu^T(x(s-\tau)) \end{bmatrix},
\]

\[
A(s) = \begin{bmatrix}
\gamma_{11} & \eta A^T \ eta A_2 & 0 & P \\
\gamma_{22} & \eta A_d^T \ eta A_2 & 0 & 0 \\
\gamma_{33} & \eta A_d^T G_2 & 0 & 0 \\
\gamma_{44} & 0 & 0 & 0
\end{bmatrix} + \begin{bmatrix}
A^T & A^T & H^T & H^T \\
A_d^T & A_d^T & H_d & H_d \\
G^T & G^T & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} + \begin{bmatrix}
\gamma_{11} & \eta A_d^T \ eta A_2 & 0 & P \\
\gamma_{22} & \eta A_d^T G_2 & 0 & 0 \\
\gamma_{33} & \eta A_d^T G_2 & 0 & 0 \\
\gamma_{44} & 0 & 0 & 0
\end{bmatrix}
\]

\[
\gamma_{11} = (\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4) A + (\epsilon_1^{-1} + \epsilon_3^{-1} + \epsilon_5^{-1}) \\
\alpha^2 P^T \rho + \left( \epsilon_1^{-1} + \epsilon_3^{-1} + \epsilon_5^{-1} \right) \alpha^2 \eta \ G_2 \ G_2 + \\
\epsilon_7 + \epsilon_1 \alpha^2 \eta \ G_2 \ G_2 + \alpha^2 \eta \ G_2 \ G_2 + G_2 - \rho,
\]

\[
\gamma_{22} = (\epsilon_2 + \epsilon_4 + \epsilon_6 + \epsilon_8) A_d + \left( \epsilon_2^{-1} + \epsilon_4^{-1} + \epsilon_6^{-1} + \epsilon_8^{-1} \right) \beta^2 \rho^T \rho + \\
\left( \epsilon_9^{-1} + \epsilon_1^{-1} + \epsilon_3^{-1} + \epsilon_5^{-1} \right) \beta^2 \eta \ G_2 \ G_2 + \beta^2 \eta \ G_2 \ G_2 + \beta^2 \eta \ G_2 \ G_2 - \eta \ G_2 \\
\gamma_{33} = (\epsilon_9 + \epsilon_{11}) \eta \ G_2 \ G_2 + (\epsilon_9 + \epsilon_{11}) \eta \ G_2 \ G_2 + (\epsilon_9 + \epsilon_{11}) \eta \ G_2 \ G_2 \\
\gamma_{44} = (\epsilon_9 + \epsilon_{11} + \epsilon_{13} + \epsilon_{14} + \epsilon_{16}) \eta \ G_2 \ G_2.
\]

For simplicity, let
\[
\epsilon_1 = \epsilon_3 = \epsilon_5 = 3 \lambda_1, \\
\epsilon_8 = \epsilon_{10} = \epsilon_{12} = \epsilon_{15} = 4 \lambda_2, \\
\epsilon_2 = \epsilon_4 = \epsilon_6 = \epsilon_7 = 4 \lambda_3, \\
\epsilon_9 = \epsilon_{11} = \epsilon_{13} = \epsilon_{14} = \epsilon_{16} = 5 \lambda_4, \\
3 \lambda_1 + 4 \lambda_2 + 4 \lambda_3 + 5 \lambda_4 = \lambda_5.
\]

Therefore, \( \gamma_{11}, \gamma_{22}, \gamma_{33}, \text{ and } \gamma_{44} \) can be rewritten as follows:

\[
\gamma_{11} = \lambda_5 A + \lambda_1 \alpha^2 \rho^T \rho + \lambda_2 \alpha^2 \eta \ G_2 \ G_2 + \left( 4 \lambda_3 + 5 \lambda_4 \right) \alpha^2 \eta \ G_2 \ G_2 + \alpha^2 \eta \ G_2 \ G_2 + \alpha^2 \eta \ G_2 \ G_2 - \rho,
\]

\[
\gamma_{22} = \lambda_5 A_d A_d + \lambda_3 \beta^2 \rho^T \rho + \lambda_4 \beta^2 \eta \ G_2 \ G_2 + \beta^2 \rho^T \rho + \beta^2 \eta \ G_2 \ G_2 - \eta \ G_2,
\]

\[
\gamma_{33} = (4 \lambda_2 + 5 \lambda_4) \eta \ G_2 \ G_2 + (4 \lambda_2 + 5 \lambda_4) \eta \ G_2 \ G_2 + (4 \lambda_2 + 5 \lambda_4) \eta \ G_2 \ G_2 \\
\gamma_{44} = \lambda_5 \eta \ G_2 \ G_2.
\]
\( \eta^T(s) = [x^T(s), x^T(s-d), x^T(s+1-t), \mu^T(x(s-t))] \),

\[
\Xi(s) = \begin{bmatrix}
\mathbb{L}_{21} & 0 & \eta A & \partial_2 & \mathcal{P} \\
\ast & \mathbb{L}_{22} & \eta A_d & \partial_2 & 0 \\
\ast & \ast & \mathbb{L}_{33} & 0 & 0 \\
\ast & \ast & \ast & -\mathcal{P} & 0 \\
\end{bmatrix} + \begin{bmatrix}
\mathbb{L}_{A}^T & \mathbb{L}_{A_d}^T & \mathbb{L}_{A_d}^T & \mathbb{L}_{A_d}^T \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(33)

Using Schur complement lemma, it can be written that

\[
\psi_{11} = \begin{bmatrix}
\mathcal{L}_1 - \mathcal{P} & 0 & \mathbb{L}_{A_d}^T & \partial_2 \\
\ast & -\mathcal{L}_1 & \eta A_d & \partial_2 & 0 \\
\ast & \ast & -\partial_2 & 0 \\
\ast & \ast & \ast & -\mathcal{P} \\
\end{bmatrix}
\]

\[
\psi_{12} = \begin{bmatrix}
\mathbb{L}_{A}^T & \mathbb{L}_{A}^T & \mathbb{L}_{A}^T & \mathbb{L}_{A}^T \\
\mathbb{L}_{A_d}^T & \mathbb{L}_{A_d}^T & \mathbb{L}_{A_d}^T & \mathbb{L}_{A_d}^T \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\psi_{13} = \begin{bmatrix}
\alpha I & \alpha I & 0 & 0 & 0 \\
0 & \beta I & \beta I & 0 \\
0 & 0 & 0 & \eta I \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\psi_{14} = \begin{bmatrix}
\mathbb{L}_{A}^T & 0 & \alpha I & \alpha I & 0 & 0 \\
0 & \mathbb{L}_{A_d}^T & 0 & 0 & 0 \\
0 & 0 & 0 & \eta I & \eta I \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

where

\[
\psi_{11} \psi_{12} \psi_{13} \psi_{14} \psi_{15}
\]

\[
\ast \psi_{22} \ 0 \ 0 \ 0 \\
\ast \ast \ \psi_{33} \ 0 \ 0 \\
\ast \ast \ast \ \psi_{44} \ 0 \\
\ast \ast \ast \ast \ \psi_{55}
\]

(34)
\[\psi_{15} = (\pi_1, \pi_2, \pi_3, \pi_4),\]
\[\pi_1 = (a\bar{\rho}, 0, 0, 0)^T,\]
\[\pi_2 = (a\bar{\sigma}_2, 0, 0, 0)^T,\]
\[\pi_3 = (0, \beta\bar{\rho}, 0, 0),\]
\[\pi_4 = (0, \beta\bar{\sigma}_2, 0, 0),\]
\[\psi_{22} = \text{diag}\left(-\rho^{-1}, -\rho^{-1}, -\sigma_2^{-1}, -\sigma_2^{-1}\right),\]
\[\psi_{33} = \text{diag}\left(-\rho^{-1}, -\sigma_2^{-1}, -\rho^{-1}, -\sigma_2^{-1}\right),\]
\[\psi_{44} = \text{diag}\left(-\lambda_3^{-1}I, -\lambda_3^{-1}I, -\frac{1}{4}\lambda_3^{-1}I, -\frac{1}{4}\lambda_3^{-1}I, -\frac{1}{5}\lambda_4^{-1}I, -\frac{1}{5}\lambda_4^{-1}I\right),\]
\[\psi_{55} = \text{diag}\left(-\lambda_1I, -\lambda_2I, -\lambda_3I, -\lambda_4I\right).\]

Combined with (17), we find that \(\Xi(s) < 0 \Rightarrow E[\Delta V(s)] < 0\). Therefore, there must exist a positive scalar \(\lambda > 0\) such that
\[\Delta V(s) \leq -\lambda \|x(s)\|^2, \quad \forall x(s) \neq 0.\]  \hspace{1cm} (36)

From Definition 1, the DTNSNs (5) with time delay and \(v(s) = 0\) are asymptotically stochastically stable in the mean square.

Next, we will prove the strictly \((\mathcal{X}, \mathcal{Y}, \mathcal{Z})\)-dissipativeness of systems (5). When \(x(s_0) = 0\), let \(v(s) \neq 0\), and combined with (26), we can get
\[
E[\Delta V(s)] - E\left[v^T(s)\mathcal{Z}^T(s)\mathcal{Z}(s) + 2v^T(s)\mathcal{Y}z(s) + v^T(s)\mathcal{Z}v(s)\right] + qE\{v^T(s)v(s)\} \\
\leq \zeta^T(s)\Lambda(s)\zeta(s) - \left[\tilde{C}x(s) + \tilde{C}_dx(s-d)\right]^T\mathcal{X}\left[\tilde{C}x(s) + \tilde{C}_dx(s-d)\right] - 2\left[\tilde{C}x(s) + \tilde{C}_d x(s-d)\right]^T\mathcal{Y}v(s) \\
+ v^T(s)(qI - \mathcal{Z})v(s) = \zeta^T(s)\tilde{\Lambda}(s)\zeta(s),
\]

where
\[
\tilde{\Lambda}(s) = \begin{bmatrix}
\lambda_{11} & 0 & -\tilde{C}^T & \mathcal{Y} \\
\tilde{C} & -\tilde{C}_d^T & 0 & 0 \\
0 & 0 & \tilde{C}_d^T & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\mathcal{X} = \begin{bmatrix}
\mathcal{A}^T & 0 & 0 & 0 \\
0 & \mathcal{A}^T & 0 & 0 \\
0 & 0 & \mathcal{A}^T & 0 \\
0 & 0 & 0 & \mathcal{A}^T \\
\end{bmatrix}
\]

\[
\mathcal{Y} = \begin{bmatrix}
\mathcal{H}^T & 0 & 0 & 0 \\
0 & \mathcal{H}^T & 0 & 0 \\
0 & 0 & \mathcal{H}^T & 0 \\
0 & 0 & 0 & \mathcal{H}^T \\
\end{bmatrix}
\]

\[
\mathcal{Z} = \begin{bmatrix}
\mathcal{Q}_2^T & 0 & 0 & 0 \\
0 & \mathcal{Q}_2^T & 0 & 0 \\
0 & 0 & \mathcal{Q}_2^T & 0 \\
0 & 0 & 0 & \mathcal{Q}_2^T \\
\end{bmatrix}
\]

\[
\tilde{\Sigma}_{44} = \lambda_5G^T G + qI - \mathcal{Z}.
\]
By the Schur complement, (17) gives rise to
\[ \Lambda(s) < 0. \quad (39) \]

By means of (37), we have
\[
E\{x^T(s)X(z(s) + 2v) + v^T(s)Y(z(s) + v)Z(s)\} - qE\{v^T(s)v(s)\} - E[V(s+1)] + E[V(s)] > 0.
\] (40)

As a consequence, for any integer \( N > 0 \), summing up both sides of (40), from \( s = 0 \) to \( s = N - 1 \), and taking the mathematical expectation, it follows that
\[
E[V(0)] + E[\Phi(v, z, N)] > qE\langle v, v \rangle_N + E[V(N)].
\] (41)

On account of the zero initial condition, (41) means that
\[
E[\Phi(v, z, N)] > qE\langle v, v \rangle_N.
\] (42)

Then, by Definition 2, the DTNSNSs (5) with time delay are strictly \((\mathcal{X}, \mathcal{Y}, \mathcal{Z})\)-dissipative. Thus, the proof is completed.

Remark 3. In the proof of Theorem 1, we consider the existence of nonlinear perturbations \( \tilde{f}(s, x(s)) \) and \( \tilde{f}_d(s, x(s - d)) \) and the neutral term \( \mu(x(s + 1 - r)) \) of DTNSNSs (5) with time delay and parameter uncertainties, which are more complicated and laboured than that of general time-delay systems \([12, 30, 32]\), and the inequality techniques are used. In order to solve them by LMI technology, we make the following: (1) the items containing matrix \( \mathcal{P} + \mathcal{Q}_2 \) in (22) are separated into the items containing matrix \( \mathcal{P} \) and matrix \( \mathcal{Q}_2 \), respectively, processed separately, and then, added, so that they can be converted into linear matrices; (2) by analyzing the relationship between \( \varepsilon_i \) (i = 1, 2, ..., 16), \( \lambda_i (i = 1, 2, 3, 4, 5) \) are introduced to reduce the dimensions of the matrices (17).

3.2. State-Feedback Controller Design for DTNSNSs with Time Delay. In this part, the state-feedback controller for DTNSNSs (5) with time delay and parameter uncertainties is designed, and the following theorem is given in terms of LMI.

**Theorem 2.** Given a negative semidefinite matrix \( \mathcal{X} \leq 0 \), a real matrix \( \mathcal{Y} \), and the positive matrix \( \mathcal{Z} \geq 0 \), if there exist matrices \( X_1 > 0, X_2 > 0, Y, \) and \( Z_1 (i = 1, 2) > 0 \) and positive scalars \( \lambda_i (i = 1, 2, 3, 4, 5) \) satisfying the following \( \Theta \) which is strictly negative definite, such that

\[
\Theta = [J_{11} J_{12} J_{13} J_{14} J_{15} J_{16} J_{22}] \begin{bmatrix} Z_1 - X_1 & 0 & -X_1 C^T Y & X_1 \\ * & -Z_1 & \eta X_1 A_d^T & -X_1 C_d^T Y \\ * & * & -X_2 & \eta G \\ * & * & * & \diamond_2 \\ * & * & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix} > 0,
\] (43)

where

\[
J_{11} = \begin{bmatrix} \diamond_3 & X_1 H_d^T & \diamond_4 & X_1 H_d^T & X_1 C^T \\ X_1 A_d^T & X_1 A_d^T & X_1 H_d^T & X_1 C_d^T \end{bmatrix},
\]

\[
J_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ G^T & G^T & 0 & 0 \end{bmatrix},
\]

\[
J_{13} = \begin{bmatrix} \alpha X_1 & \alpha X_1 & 0 & 0 & 0 \\ 0 & 0 & \beta X_1 & \beta X_1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
J_{14} = \begin{bmatrix} \diamond_5 & 0 & \alpha \lambda_1 X_1 & \alpha \lambda_4 X_1 & 0 & 0 \\ 0 & \lambda_4 X_1 A_d^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
J_{15} = \begin{bmatrix} \alpha I & \alpha Z_2 & 0 & 0 \\ 0 & 0 & \beta I & \beta Z_2 \\ 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
J_{16} = \begin{bmatrix} 0 & X_1 N_1^T & 0 & X_1 N_1^T & 0 & 0 \\ 0 & X_1 N_2^T & 0 & 0 & 0 & X_1 N_2^T \\ \varepsilon_1 \eta \mathcal{M}_1 & 0 & 0 & 0 & 0 & 0 \\ -\varepsilon_1 \mathcal{Y} \mathcal{M}_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
J_{22} = \text{diag} \left( -X_1, -X_1, -X_2, -X_3, \mathcal{X}^{-1} \right).
\]
\[ J_{26} = (\theta_1, 0, 0, 0, 0), \]
\[ J_{33} = \text{diag} (-X_1, -X_2, -X_3, -X_4, -X_5), \]
\[ J_{44} = \text{diag} (-\lambda_2 I, -\lambda_3 I, -\frac{1}{4} \lambda_4 I, -\frac{1}{5} \lambda_5 I, -\frac{1}{5} \lambda_6 I), \]
\[ J_{46} = (\theta_2, \theta_3, 0, 0, 0), \]
\[ J_{55} = \text{diag} (-\lambda_1 I, -\lambda_2 I, -\lambda_3 I, -\lambda_4 I), \]
\[ J_{66} = \text{diag} (-\epsilon_1 I, -\epsilon_2 I, -\epsilon_3 I, -\epsilon_4 I, -\epsilon_5 I), \]
\[ \theta_1 = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5), \]
\[ \theta_2 = (0, 0, 0, 0, 0), \]
\[ \theta_3 = (0, 0, 0, 0, 0), \]
\[ \Delta_1 = \eta X_1 A^T + \eta Y^T B^T, \]
\[ \Delta_2 = \lambda_2 G^T + \theta I - Z, \]
\[ \Delta_3 = \lambda_2 X_1 A^T + Y^T B^T, \]
\[ \Delta_5 = \lambda_2 X_1 A^T + \lambda_2 Y^T B^T, \]

then the systems (5) are asymptotically stable in the mean square and strictly \((\mathcal{X}, \mathcal{Y}, \mathcal{Z})\)-dissipative, and the state feedback gain is \(K = X_1^{-1}\).

**Proof.** Applying Lemma 2, the matrix inequality (17) can be written as

\[ (\bar{E} + Y_1^T \Delta^T (s) Y_2 + Y_2^T \Delta (s) Y_1 + Y_3^T \Delta^T (s) Y_4 + Y_4^T \Delta (s) Y_3 + Y_5^T \Delta^T (s) Y_6 + Y_6^T \Delta (s) Y_5 < 0, \]  

(45)

where

\[ \bar{E} = \begin{bmatrix} \bar{E}_{11} & \bar{E}_{12} & \bar{E}_{13} & \bar{E}_{14} & \bar{E}_{15} \\ * & \bar{E}_{22} & 0 & 0 & 0 \\ * & * & \bar{E}_{33} & 0 & 0 \\ * & * & * & \bar{E}_{44} & 0 \\ * & * & * & * & \bar{E}_{55} \end{bmatrix}, \]

\[ \bar{E}_{11} = \begin{bmatrix} \mathcal{Q}_1 - \mathcal{P} & 0 & \eta (A + BK)^T \mathcal{Q}_1 - C^T \mathcal{Y} \\ * & -\mathcal{Q}_1 & \eta A^T \mathcal{Q}_1 - C_d^T \mathcal{Y} & 0 \\ * & * & -\mathcal{Q}_2 & \eta \mathcal{Q}_2 G & 0 \\ * & * & * & -\mathcal{Q}_3 & 0 \end{bmatrix}, \]

\[ \bar{E}_{12} = \begin{bmatrix} (A + BK)^T H^T & (A + BK)^T H^T C^T \\ A_d^T & H_d^T & A_d^T & H_d^T C_d^T \end{bmatrix}, \]

\[ \bar{E}_{13} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \]

\[ \bar{E}_{14} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, \]

\[ \bar{E}_{15} = \begin{bmatrix} 0 & 0 \end{bmatrix}. \]
\[
\tilde{E}_{13} = \begin{bmatrix}
al & al & 0 & 0 & 0 \\
0 & 0 & \beta l & \beta l & 0 \\
0 & 0 & 0 & 0 & \eta l \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
\tilde{E}_{14} = \begin{bmatrix}
(A + BK)^T & al & al & 0 & 0 \\
0 & A_d^T & 0 & 0 & 0 \\
0 & 0 & 0 & \eta l & \eta l \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[\tilde{E}_1 = \lambda_5 G^T G + d I - \mathcal{X},\]

\[
\tilde{E}_{15} = (\Phi_1, \Phi_2, \Phi_3, \Phi_4),
\]

\[
\Phi_1 = (\alpha P, 0, 0, 0, 0)^T,
\]

\[
\Phi_2 = (\alpha Q, 0, 0, 0, 0)^T,
\]

\[
\Phi_3 = (0, \beta P, 0, 0, 0)^T,
\]

\[
\Phi_4 = (0, \beta Q, 0, 0, 0)^T,
\]

\[
\tilde{E}_{22} = \text{diag}\left(-\mathcal{P}^{-1}, -\mathcal{P}^{-1}, -\mathcal{Q}^{-1}, -\beta^{-1}, \mathcal{X}^{-1}\right),
\]

\[
\tilde{E}_{33} = \text{diag}\left(-\mathcal{P}^{-1}, -\mathcal{Q}^{-1}, -\mathcal{Q}^{-1}, -\beta^{-1}\right),
\]

\[
\tilde{E}_{44} = \text{diag}\left(-\lambda_5^{-1} I, -\lambda_3^{-1} I, -1 - \frac{1}{2} \lambda_1^{-1} I, -\frac{1}{2} \lambda_4^{-1} I, -\frac{1}{2} \lambda_4^{-1} I\right),
\]

\[
\tilde{E}_{55} = \text{diag}\left(-\lambda_1 I, -\lambda_2 I, -\lambda_3 I, -\lambda_4 I\right),
\]

\[
Y_1 = (N_1, N_2, 0_{1 \times 23}),
\]

\[
Y_2 = (0, \eta, M_1^T, 0, M_1^T, M_2^T, M_3^T, 0_{1 \times 15}),
\]

\[
Y_3 = (0, N_1, 0_{1 \times 24}),
\]

\[
Y_4 = (0_{1 \times 15}, M_1^T, 0_{1 \times 9}),
\]

\[
Y_5 = (0, N_2, 0_{1 \times 23}),
\]

\[
Y_6 = (0_{1 \times 16}, M_1^T, 0_{1 \times 8}).
\]

4. Illustrative Examples

In the following part, we use the previous conclusions to investigate a numerical simulation example, through which...
we indicate the usefulness of the provided controller and the correctness of the conclusions.

For DTNSNs (5) with time delay and parameter uncertainties, the corresponding parameters are listed as follows:

\[
A = \begin{bmatrix} -0.3 & 1 \\ 0 & 0.4 \end{bmatrix},
\]

\[
A_d = \begin{bmatrix} -0.5 & 0 \\ -0.5 & -0.3 \end{bmatrix},
\]

\[
B = \begin{bmatrix} -0.1 & 0.2 \\ 0.4 & -0.2 \end{bmatrix},
\]

\[
G = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix},
\]

\[
H = \begin{bmatrix} -0.5 & -0.1 \\ -0.1 & -0.8 \end{bmatrix},
\]

\[
H_d = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.1 \end{bmatrix},
\]

\[
C = \begin{bmatrix} -0.1 & 0.2 \\ 0.1 & -0.1 \end{bmatrix},
\]

\[
C_d = \begin{bmatrix} -0.2 & -0.1 \\ -0.1 & -0.1 \end{bmatrix},
\]

\[
M_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},
\]

\[
M_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},
\]

\[
M_3 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},
\]

\[
N_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},
\]

\[
N_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},
\]

\[
X = \begin{bmatrix} -3 \\ 0 \end{bmatrix},
\]

\[
Y = \begin{bmatrix} 150 \\ 0 \end{bmatrix},
\]

\[
Z = \begin{bmatrix} 1.45 \\ 0 \end{bmatrix}.
\]

Given \(d = 5, \tau = 6, \alpha = 0.003, \beta = 0.001, \eta = 0.8, \varrho = 0.03, \lambda_1 = 1/3, \lambda_2 = 1/4, \lambda_3 = 1/4, \lambda_4 = 1/5, \lambda_5 = 4 \), the exogenous disturbance input is

\[
\nu(s) = \begin{bmatrix} e^{-s} \sin(s) \\ e^{-0.8s} \cos(2s) \end{bmatrix}.
\]

Without consideration of the delay-feedback control, the simulation of state trajectories for the open-loop system is shown in Figure 1, and we find that the open DTNSNs (5) with time delay are not stable or not dissipative. Therefore, we need to design the state-feedback controller as (4) for the DTNSNs (5) with time delay such that dissipativity is satisfied. According to (43), the feasible solutions are obtained as follows:

\[
X_1 = 10^{-4} \times \begin{bmatrix} 0.3682 & 0.1429 \\ 0.1429 & 0.5446 \end{bmatrix},
\]

\[
X_2 = \begin{bmatrix} 0.2439 & -0.0015 \\ -0.0015 & 0.2437 \end{bmatrix},
\]

\[
Y = 10^{-3} \times \begin{bmatrix} -0.0935 & -0.1159 \\ -0.1159 & -0.2293 \end{bmatrix},
\]

\[
Z_1 = 10^{-4} \times \begin{bmatrix} 0.1197 & 0.0869 \\ 0.0869 & 0.0081 \end{bmatrix},
\]

\[
Z_2 = \begin{bmatrix} 0.3624 & -0.0392 \\ -0.0392 & 0.4074 \end{bmatrix},
\]

\[
\epsilon_1 = 1.8336 \times 10^{-4},
\]

\[
\epsilon_2 = 4.1671,
\]

\[
\epsilon_3 = 4.1959.
\]

Therefore, the controller gain can be designed as follows:

\[
K = \begin{bmatrix} -1.9071 & -1.6270 \\ -1.6847 & -3.7679 \end{bmatrix}.
\]

Hence, the simulation of the closed-loop DTNSNs (5) with time delay is given in Figure 2. Obviously, systems (5) are asymptotically stochastically stable in the mean square and dissipative. From Figure 3, we can see...
more clearly that the closed-loop DTNSNSs (5) with time delay are strictly \((X, Y, Z)\)-dissipative, and Figure 4 shows the variation of \(\varrho\) in \(E[\Phi(v, z, N)] \geq \varrho E\langle v, v \rangle_N, \forall N \geq 0\).

5. Conclusions

The problem of dissipativity analysis for discrete-time neutral stochastic nonlinear systems (DTNSNSs) with time delay and parameter uncertainties has been investigated in this paper. A sufficient condition has been proposed by constructing a discrete-time Lyapunov–Krasovskii functional (LKF), which guarantees the DTNSNSs to be asymptotically stochastically stable in the mean square and strictly \((X, Y, Z)\)-dissipative. In light of this condition, a feedback controller of DTNSNSs with time delay has been designed to ensure the stability and strictly dissipative performance of the resulting closed-loop system. At last, a numerical example has been demonstrated to validate the proposed results.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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