Energy characteristics of acoustic emission under various modes of uniaxial loading

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Abstract. Acoustic emissions were recorded in the process of uniaxial compression of samples of various geomaterials. The experiments were carried out on a low-noise lever setup with water leakage; the maximum load on the sample did not exceed 250 kN. Some of the samples were tested at a continuously increasing load, the other at its stepwise change. The energy distribution of acoustic emission signals was investigated. The energy characteristic of acoustic emission was the square of the maximum signal amplitude. The flow of AE events is considered from the standpoint of nonequilibrium thermodynamics and Tsallis statistics. A decrease in the steepness of the linear part of the repeatability plots for a particular geomaterial was revealed when changing the loading mode from linear to stepwise, which means an increase in the proportion of higher-energy events with a stepwise change in load.

1. Introduction
When processing and analyzing the results of the experiments, the approaches and patterns developed in seismology were used. The basis for their application to AE signals is the concept of self-similarity of the processes of destruction of rocks at different scale levels [1]. In particular, these concepts imply the use of the AE method when studying the process of cracking in laboratory conditions under uniaxial compression of rocks. The energy indicators of the AE recorded in the process of deformation and fracture of homogeneous and inhomogeneous materials were considered earlier in [2-4]. Subsequent AE studies, carried out using more advanced technical means, also confirmed the effectiveness of laboratory modeling of conditions typical for seismogenic zones of the earth's crust [5-8]. In particular, they showed that the distribution function of AE signals by energy is similar to the distribution of seismic events described by the Gutenberg – Richter law [9]:

\[ P(E) \sim E^\gamma, \]

where \( \gamma \approx 5/3 \), \( E \) – earthquake energy.

For the total number of events with a magnitude \( M \) exceeding a certain threshold value \( m \) (cumulative distribution), the law of earthquake energy distribution takes the form:

\[ N(M > m) \sim 10^{-bm}, \]

where \( N \) denotes the number of events for a specified fixed time period in a given geographic region, \( b \) is a constant (\( b \)-value), in most cases takes a value of about 0.9 [10], and this ratio is fulfilled only for the linear portion of the earthquake energy distribution.
If we use the function of the distribution of earthquakes by energy, then the Gutenberg-Richter law is described by a linear function of the form:
\[ \log N(M) = a - \gamma M, \]
where \( N(M) \) is the number of earthquakes with magnitudes (or classes) not less than \( M \), and \( a \) and \( \gamma \) are constants of the equation. The parameter \( a \) (\( a \)-value) formally describes the seismic activity at \( M = 0 \), and \( \gamma \) is the slope of the linear part of the earthquake distribution density graph (recurrence graph), which determines the rate of decrease in the number of seismic events with an increase in their magnitude.

In [11], it was proposed to modify the model of stick-slip earthquakes - “discontinuous sliding” of two plates over each other along a fault in the presence of friction [12]. This model (Fragment-Asperity Interaction Model for Earthquakes, [11]), based on the principles of statistical physics, considers the interaction of fault sides and rigid fragments filling the gaps between the sides. This model underlies the analytical expression describing the generalized Gutenberg – Richter law, which relates the cumulative number of earthquakes with a magnitude \( M \) exceeding the threshold value \( M_{th} \) with the Tsallis parameter \( q \) [13]:
\[ \log \left( \frac{N(M > M_{th})}{N} \right) = \left( \frac{2 - q}{1 - q} \right) \log \left[ 1 - \left( \frac{1 - q}{2 - q} \right) \left( \frac{10^{M_{th}}}{a^{q/2}} \right) \right], \]
where \( N(M > M_{th}) \) is the number of earthquakes with energies greater than the threshold value \( M_{th} \), and \( M \sim \log (E) \), \( E \) is the earthquake energy, \( N \) is the total number of earthquakes, \( a \) is the proportionality coefficient between the earthquake energy \( E \) and the size of the fragment of blocks \( r^3 \) between faults, has the dimension of the volumetric energy density [13-16].

According to [15-25], the parameter \( q \) can be used as a measure of the stability of the active tectonic zone. A sharp increase in the parameter \( q \) indicates an increase in the interaction between fault blocks and their fragments and is associated with their deviation from the equilibrium state [16]. Examples of non-extensive analysis of AE are still few in number [26-27]. Again, empirical attempts are made to describe the distribution functions of AE events by power-law or exponential dependences at various stages of sample loading.

The aim of this work is to attempt to describe the energy distribution function of AE events from the point of view of nonequilibrium thermodynamics, using Tsallis statistics and the approach used in seismology to describe the flow of earthquakes as a system with memory and long-range correlations. This can also serve as another confirmation of the self-similarity of the destruction processes occurring at different scale levels.

2. Results

Uniaxial compression of rock samples in the experiments was carried out on a low-noise lever unit with water inlet [28]. The maximum load at which the sample was destroyed did not exceed 250 kN. The tested samples had the shape of a rectangular parallelepiped, sandstone with a square section, dimensions 25 x 25 x 60 mm, granite and marble with a rectangular section, dimensions 40 x 20 x 80 mm.

The square of the maximum amplitude of the AE waveform was taken as the energy characteristic for these signals. To obtain an analogy between the catalog of AE signals with the catalog of earthquakes, in which the magnitude is \( M \sim \log (E) \), further, in the calculations, the decimal logarithm of the square of the maximum amplitude of the AE waveform is used as the “magnitude” of a single AE signal.

In the experiments, some of the samples were tested at a continuously increasing load, the other at its stepwise change. As an example, consider the typical test results of sandstone samples, which are shown in Figure 1 by the time dependences of changes in the load and deformation values under continuous and stepwise loading, respectively.
Figure 1. Loading (a) and deformation (b) diagrams. Blue - for linear, red - step loading.

As you can see, the maximum load at which the failure occurred is approximately the same in both cases. Despite this, the deformation value of the specimen under step loading is more than twice that for continuous loading. In both cases, an increase in the AE activity (Fig. 3, 4) was observed in the second half of the experiment, which indicates intense cracking in the material, despite the absence of obvious signs of impending fracture on the load and longitudinal deformation diagrams of the samples. But, with step loading after the last loading of the sample, additional deformation steps are noted, obviously associated with the formation of rather large cracks in it.

The results of constructions using expressions (2) and (3) for linear and step loading are shown in Figure 2.

As can be seen, the slope of the linear part of the repeatability plots decreases when changing the loading mode from linear to stepwise. That is, in the latter case, the AE recurrence graphs become flatter, which means an increase in the proportion of higher-energy events under step loading. In other words, the probability of the formation of a coarse defect in rocks at a step load is higher than at its continuous growth. This may be due to the fact that the fraction of small defects that takes part in the enlargement of cracks under active loading is less than the same fraction under step loading. Therefore, the rate of accumulation of enlarged defects in a stepwise loaded sample is higher than in a continuously loaded one. This promotes the localization of defects and increases the likelihood of brittle fracture of the material under step loading.

Figure 2. a) Density of distribution of AE signals by energy (expression (3)), b) cumulative distribution (expression (2)), at uniaxial compression of sandstone. Blue - for linear, red - step loading.

The event energy distribution function, its approximation according to expression (4), the values of the Tsallis parameters \( q \) and \( a \), and the errors in determining these values for sandstone are shown in Fig. 3. This figure shows in black the approximation of the linear portion of the cumulative Gutenberg-Richter dependence, according to expression (2).
Figure 3. Tsallis parameters. Sandstone, (a) - linear, (b) - step loading).

The time dependences of the accumulation of AE events, AE activity, Tsallis parameters \(q\) and \(a\) and \(b\)-value and \(\gamma\)-value parameters during linear and step loading of sandstone are shown in Figs. 5 and 6. Values of Tsallis parameters \(q\) and \(a\) and parameters \(b\)-value and \(\gamma\)-value were calculated in a sliding window of 500 events with a shift by 1 event.

Judging by the AE loading and accumulation diagrams, the sample was elastically deformed up to almost \(1.3 \times 10^5\) s, when a local fracture site was formed, which manifested itself as a characteristic step on the AE accumulation curve and a sharp increase in the AE activity. The second step on the cumulative curve and an increase in the AE activity were observed immediately before the final destruction of the sample. It can be seen that the formation of a local focus of destruction and the final destruction of the sample are preceded by a bay-like change in the values of both slope coefficients, density graphs, and the event energy distribution function. Such a behavior of these parameters indicates an active coarsening of defects in the loaded material on the eve of the destruction of the sample. A similar change in the slope of the recurrence plots, as a characteristic flattening before strong earthquakes, was noted in various regions of the Earth [29].

As can be seen, in both cases, the parameter \(q\) before local and final fracture demonstrates a steady growth, which can be interpreted as an increase in the mutual correlations of AE events caused by the spatio-temporal localization of the crack formation process in the sample, leading to the formation of foci, and, ultimately, the final destruction [30].

The value of the Tsallis parameter \(q = 1.489\) for the energy distribution functions of AE events characterize the behavior of sandstone under uniaxial loading as a system with memory and long-range correlations. The values of the Tsallis parameters \(q\) and \(a\), the errors in determining these quantities, and the number of AE signals for different experimental sessions are given in the table. According to the data presented, the values of \(q \sim 1.5\) do not depend on the scenario of changing the load (Figs 3-5), or on the size or material of the sample, or on the number of events (see table).
Figure 4. Sandstone, stepwise loading: a - accumulation and activity of AE events, b - b-value, expression (2), γ-value, expression (3), с - Tsallis parameters q and a, expression (4).

Figure 5. Sandstone, loading is linear: a - accumulation and activity of AE events, b - b-value, expression (2), γ-value, expression (3), с - Tsallis parameters q and a, expression (4)
Table 1. Tsallis parameters q and a and b-value for tested rocks.

| Material  | Tsallis parameters | Parameters of the Gutenberg-Richter law | N      | Load type |
|-----------|-------------------|----------------------------------------|--------|-----------|
|           | q                 | a                                      | b-value | γ-value   | N        |
| Granite 19| 1.523             | 11743258.081                          | 1.5875  | 0.7471    | 16365    |
| Granite 12| 1.552             | 13880630.842                          | 1.5672  | 0.6779    | 150621   |
| Granite 25| 1.468             | 353639549.444                         | 1.2779  | 0.7480    | 161062   |
| Granite 26| 1.453             | 351432302.842                         | 1.3997  | 0.7096    | 65960    |
| Marble 82 | 1.495             | 2900797.719                           | 1.9906  | 0.9455    | 699      |
| Marble 83 | 1.476             | 22805293.283                          | 1.7462  | 0.8461    | 76776    |
| Sandstone 3| 1.490             | 262988606.462                         | 1.1682  | 0.6528    | 18060    |
| Sandstone 4| 1.489             | 234244102.256                         | 1.2159  | 0.7074    | 24650    |
| Sandstone 5| 1.493             | 258073894.356                         | 1.1766  | 0.6270    | 24345    |
| Sandstone 10| 1.476             | 404050405.071                         | 1.3296  | 0.6919    | 9072     |
| Sandstone 11| 1.489             | 310878514.676                         | 1.2302  | 0.6514    | 60780    |
| Sandstone 12| 1.522             | 47507330.291                          | 1.2519  | 0.6513    | 101304   |
| Sandstone 13| 1.522             | 64183575.890                          | 1.2724  | 0.6189    | 119073   |
| Sandstone 14| 1.526             | 37180815.761                          | 1.2607  | 0.6486    | 44730    |
| Sandstone 15| 1.537             | 58360511.739                          | 1.0526  | 0.5423    | 60084    |
| Sandstone 16| 1.539             | 34807190.44                           | 0.50356 | 0.57014   | 91011    |

The values presented in the table practically coincide with the values of the Tsallis parameter q for the magnitudes of seismic events in various seismically active regions of the Earth [15-25, 31-34]. This testifies to the unified nature of the processes of destruction of geomaterials at such different scales as the earth's crust and laboratory specimen.

3. Conclusion

The paper considers AE signals recorded under uniaxial compression by a continuously increasing and stepwise load of rocks of various genesis: sandstone, marble, granite. The square of the maximum amplitude of the waveform of the recorded AE signal is conventionally taken as the energy characteristic of the AE.

A decrease in the steepness of the linear part of the repeatability plots for a specific geomaterial was revealed when changing the loading mode from linear to stepwise, which means an increase in the proportion of higher-energy events with a stepwise change in load. This is explained by the increased probability of the formation of an enlarged defect during stepwise loading of rocks compared with its continuous growth. Consequently, with stepwise loading in rocks, the localization of defects is facilitated and, thereby, the tendency to brittle fracture of the material and the energy of the AE increase. An additional role under a step load can be played by the strain hardening of the material, which increases the material's resistance to fracture and, thereby, contributes to an increase in the AE energy. According to the results obtained, according to the degree of tendency to brittle fracture, the tested rocks can be presented in the following order: sandstone, granite, marble. AE. The b-value and γ-value are different for different geomaterials: the highest ~ 2 for marble samples, the lowest ~ 1.0-1.3 for sandstone samples, and intermediate ~ 1.2-1.6 for granite samples.

To describe the energy distribution function of AE signals, we used the provisions of nonequilibrium thermodynamics with the use of Tsallis statistics, which generalizes classical thermodynamics to the case of nonextensive systems. The calculated values of the Tsallis parameter q ~ 1.5 practically coincide with the values obtained for the magnitudes of catalogs of various seismically active regions. Thus, the values of the Tsallis parameters q for field observations and for simulating processes in laboratory conditions demonstrate self-similarity in terms of the nonextensiveness parameter. It is shown that the set of structural defects and, accordingly, the AE pulse flux is a system with memory and long-range spatial correlations.
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