Hall Voltage Fluctuations as a Diagnostic of Internal Magnetic Field Fluctuations in High Temperature Superconductors and the Half-filled Landau Level

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Fluctuations of the Hall voltage reveal information about long wavelength magnetic field fluctuations. If gauge theories of strongly correlated electrons are correct, such fluctuations are particularly large in the half-filled Landau level and in high \( T_c \) superconductors. We present estimates for the magnitude, system size and frequency dependence of these fluctuations. The frequency dependence contains information about instantons in the gauge field.

It is widely believed that the anomalous properties of high \( T_c \) superconductors imply that a soft mode exists, but there is as yet no consensus on the origin of this mode. One possibility is an internal gauge field associated with spin-charge separation \( [4,5] \). In this communication we show that measurements of the fluctuations in the Hall voltage at zero applied magnetic field and non-zero applied current can reveal the presence of internal gauge fluctuations. The idea is simply that the internal gauge fluctuations produce an effective magnetic field which affects the motion of electrons in the same way as a conventional magnetic field. In the presence of a current, fluctuations in the effective field will therefore lead to fluctuations in the voltage transverse to the current. Experimental detection of the fluctuations would confirm the validity of the gauge field description while their frequency dependence contains information about dynamics of the gauge field which is difficult to obtain by other means.

The internal gauge field is associated with the spin-charge separation: within the gauge theory approach it occurs at \( T > T_c \) for optimally doped and underdoped materials. For overdoped materials spin-charge separation may exist at high \( T \), but there is a crossover to Fermi liquid like behavior with smaller gauge field fluctuations at \( T_{FL} > T_c \) \( [3] \). As we shall discuss below, the fluctuations should be most easily observable in moderately underdoped materials at temperatures near \( T_c \), but sufficiently far above it that conventional superconducting fluctuations are not observable.

If spin-orbit coupling is important, long wavelength fluctuations of the electron spin degrees of freedom will lead to similar effects which we show are much smaller in magnitude.

Rather similar considerations apply to the half-filled Landau level, where a Chern-Simons gauge theory has been proposed to describe the low energy physics \( [6,7] \), but one must measure fluctuations relative to the background Hall voltage caused by the magnetic field which created the \( \nu = 1/2 \) state. Also the dynamics of the gauge field is somewhat different as discussed below.

We suppose that there exists a dimensionless field \( \phi \) which has significant long wavelength fluctuations and which induces an internal field

\[
B_{int}(\vec{r}, t) \equiv f \phi(\vec{r}, t)
\]  

which we assume affects the motion of electrons in the same way as does a magnetic field. Because we are interested in long wavelength fluctuations we take the relation between \( \phi \) and \( B \) to be local and specified by a constant, \( f \), which has the dimension of magnetic field.

In the U(1) gauge theory of high \( T_c \) materials or of the half-filled Landau level, \( \phi \) is essentially the internal magnetic field \( h = \nabla \times a \) induced by the gauge field \( a \). Because \( h \) has dimension \([\text{length}]^{-2}\) we make it dimensionless by multiplying by two factors of a microscopic length \( l_0 \). Different choices of \( l_0 \) correspond to different values of the coupling constant \( f \). For high \( T_c \) materials it is natural to take \( l_0 \) to be the lattice constant, 4\( \AA \); for the half-filled Landau level the natural choice is the interparticle spacing, which is of order 300\( \AA \). In both cases \( f = \hbar c/(e l_0^2) \). For high \( T_c \) materials (\( l_0 = 4\AA \)) \( f \approx 4 \times 10^7 \text{gauss} \); for the half-filled Landau level (\( l_0 = 300\AA \)) \( f \approx 8 \times 10^3 \text{gauss} \).

The electron spin degrees of freedom are an additional source of magnetic field fluctuations. In this case \( \phi \) is the spin polarization per unit cell and \( f = \mu_B/\lambda^3 \) where \( \lambda \) is a length. If the spin polarization were spread uniformly over the unit cell one would take \( \lambda \sim l_0 \), leading to \( f \sim 125 \text{ gauss} \) for high \( T_c \) materials. However, in high \( T_c \) materials the conduction-band states involve Cu d-orbitals and the associated large spin-orbit interaction will increase the coupling of the spin fluctuations to the orbital motion of the electrons by a factor which we estimate as follows. The Cu d-orbitals are of size...
\[ r_{Cu} \sim 0.5 \AA \sim l_0/10. \] Because the only d-orbital with appreciable weight at the fermi surface is the \( d_{x^2-y^2} \) orbital, the effect on the motion of the electrons is controlled by the flux per unit cell, which for a dipole of size \( r_{Cu} \) is \( \mu_B/r_{Cu} \sim 10^5 \text{gauss} \). Thus, for spin effects in high \( T_c \) materials \( f \sim 10^3 \text{gauss} \), four orders of magnitude smaller than the \( f \) for gauge fields.

Here \( \Lambda \) is essentially the susceptibility of the field conjugate to \( \phi \), it has the dimension of \((\text{length})^2/\text{energy}\).

The thermal and quantum fluctuations of \( \phi \) are given as usual by \( \langle \phi(\omega) \phi(\omega') \rangle_{q,\omega} = \text{cosh} q\omega/\gamma \). Knowing the fluctuations of \( \phi \) we may calculate the fluctuations of \( B \) and thus the fluctuations of the Hall voltage. For definiteness we discuss the situation shown in the inset to Fig. 1. The fundamental equation is

\[ \vec{j} = \mathbf{\sigma} \vec{E} \]  

where the current density \( \vec{j} \) is the sum of the imposed current density \( j_x = I/L_y \) and a transverse part \( j_T \) whose \( y \) component vanishes at the upper and lower edges of the strip; the electric field \( \vec{E} \) is longitudinal and because of the fluctuating magnetic field \( \mathbf{\sigma} \) is not diagonal. Because \( B_{int} \) is always small, we use the linear approximation

\[ \mathbf{\sigma}^{-1} = \rho_{xx} \mathbf{1} - \epsilon \rho_H B_{int} \]  

Here \( \epsilon \) is the antisymmetric tensor, \( \rho_{xx} \) is the usual dc resistivity, and \( \rho_H \) the usual Hall resistivity.

Equations (3) and (4) may be solved to determine the potential difference \( V_H(x) \) between the upper and lower edges of the strip at position \( x \). It is more convenient to write a formula for \( V_H = \int dx \, e^{iqx} V_H(x) \):

\[ V_H = \frac{\rho_H I}{L_y} \int_{-L_y/2}^{L_y/2} dy dy' \left[ \delta(y-y') - q^2 K(y,y') \right] B_{int}(q,y) \]  

The kernel \( K \) expresses the effect of the boundary condition \( \vec{j}(y = \pm L_y/2) = 0 \) and may be written

\[ 2q K(y,y') = \exp \left[ -qy - q'y \right] - \exp \left[ -qL_y \right] \left[ \frac{\sinh qy \sinh q'y}{\sinh qL_y/2} + \cosh qy \cosh q'y/2 \right] \]  

The important values of \( q \) are \( q \sim 1/L_x \). \( K(y,y') \) suppresses the contributions of wavevectors \( q > 1/L_y \); if the experimental system has \( L_x \gtrsim L_y \) \( K \) may be neglected in Eq. (6).

These arguments must be slightly modified for the gauge field case. Here one has “spinons” (s) and “holons” (h) coupled by the internal gauge field. The dynamical equations are

\[ \vec{j}_s = \mathbf{\sigma}_s \vec{e} \]  

\[ \vec{j}_h = \mathbf{\sigma}_h (\vec{e} + \vec{E}) \]  

The internal electric field \( \vec{e} \) and the physical electric field \( \vec{E} \) are longitudinal, and fixed by the requirements \( \vec{j}_h = I/L_x + \vec{j}_{h,T} \), \( \vec{j}_s = -I/L_x + \vec{j}_{s,T} \), and \( \vec{j}_{h,s}(y = \pm L/2) = 0 \). Solving these equations leads to Eq. (6) with \( \rho_H \) replaced by \( \rho_H' + \rho_H^2 \). As discussed in Ref. [1], the quantity \( \rho_H' + \rho_H^2 \) is not the physical Hall resistivity, but is of the same order of magnitude.

Further, in the gauge field problem the gauge field propagator \( D \) is affected by the finite geometry. This may be seen from the fact that the inverse gauge-field propagator is the sum of two terms, one coming from the diamagnetic susceptibility energy density \( \chi_{diam} h^2 \), and the other from the fact that the gauge fluctuations induce particle currents, which dissipate. This latter term is affected by boundary conditions and in the hydrodynamic limit may be calculated as \( V_H(q) \) above. We find the formal expression...
Here $\hat{K}^{-1}$ is the inverse of the operator $\hat{K}$ given above, $\Lambda = l_0^2/\chi_{dia}$ and $D = \chi_{dia}/\tilde{\sigma}$, $\tilde{\sigma} = \rho_{df}/(2\pi)$ with $l_0$ the mean free path and $p_0$ the curvature of the Fermi surface. Note that if $L_y \rightarrow \infty$ $\hat{K}^{-1} \rightarrow q^2$. If also $\gamma \rightarrow 0$ then $\langle h(q)h(q)\rangle$ tends to the familiar form $\text{coth}^{1/2}(\frac{\omega}{2T})\Lambda\frac{\omega(D\hat{K}^{-1} + \gamma)}{\omega^2 + (D\hat{K}^{-1} + \gamma)^2}$ (9)

Using this value and a mean free path of 100 $\AA$ at $T=100K$ we find $D \sim 1 \times 10^{-2}$ cm$^2$/sec. In Eq. (8) we have added the cutoff $\gamma$ by hand; the formula is correct in the two limits $\gamma \gg D/L_y^2$ and in $\gamma \ll D/L_y$ but may not be quantitatively accurate in the crossover regime $L_y^2 \sim D/\gamma$.

In the gauge field case, $\gamma$ is due to instantons; these are tunneling processes in which the flux due to the gauge magnetic field changes by $2\pi$. The action of an instanton was shown to diverge as $\omega^{-1/3}$ at $T = 0$, so quantum fluctuations of instantons are completely suppressed. Thermally assisted tunneling is allowed; the rate per unit area $l_0^2$ may be roughly evaluated by using $T$ to cut off the divergence found in the quantum regime, yielding $\gamma \sim T(TD^2/\chi_{dia})^{2/3}\exp[-\alpha(\chi_{dia}l_0^2/\pi^2T)^{1/3}]$ with $\alpha \sim 3.5$. Thus one expects that $\gamma$ is small enough that it is not important for calculations of energetics or microscopic properties such as resistivities, but it may well be significant on the length and time scales relevant for noise experiments. The dynamics of instantons is “telegraph” like causing a change of $2\pi$ in the flux over a time scale $\sim 1/T$, while the time between instantons in the same plaquette is $\sim \gamma^{-1} \gg 1/T$. Each instanton changes the average internal field by $2\pi/L_yL_y$ which is small compared to the typical value of the internal field for reasonable sample sizes.

The above estimates for $\chi_{dia}$ and $\gamma$ were obtained on the basis of the uniform RVB model which is expected to describe the normal state of optimally doped materials [3]. In the overdoped materials a crossover to a Fermi liquid regime occurs at $T_{FL} > T_c$ [3]; in the present formalism this manifests itself as a divergence of $\chi_{dia}$ which implies $\Lambda \rightarrow 0$. In underdoped materials, a pseudogap is formed at a temperature $T_{PG} > T_c$. Within the spin-liquid approach this is due to a BCS-like pairing of spinons [3] which produces an increase in $\chi_{dia}$ and $\tilde{\sigma}$ thereby reducing $\Lambda$ and $\gamma$. In the underdoped materials the formation of the pseudogap changes bulk properties such as the spin susceptibility or $d\rho/dT$ only modestly so we expect that $\chi_{dia}$ and $\tilde{\sigma}$ and thus the total noise power do not decrease much. However, $\gamma$ depends exponentially on parameters, so the initial effect of the underdoping will be to dramatically decrease the characteristic noise frequency.

To obtain analytical formulas for the potential difference fluctuations we use Eq. (8) to write an equation for $V^2$ and then use Eqs. (11) and (12) to average over the field fluctuations. The resulting expressions are cumbersome in general but simplify for relatively large samples, $L^2 \gg D/\gamma$. In this case the local field is effectively $\delta$-correlated in space and we get for the potential averaged over a distance $L$:

$$\langle V^2 \rangle_\omega = 2\pi T\gamma\Delta l^2/\omega^2 + \gamma^2 (\rhoHl/\omega) Y(2L_x/L_y)$$ (10)

where $Y(x)$ is a dimensionless function which depends on the shape of the contact pads. For square pads shown in Fig. 1 we find the function shown in Fig. 1 with the following limiting values: $Y(x) \approx 1/(2\pi)\ln(1/x)x^2$ at $x \ll 1$, $Y(1) = 0.4316$, $Y(x) = x$ at $x \gg 1$.

In the opposite limit, $L^2 \ll D/\gamma$, we make estimates. We assume $L_x \sim L_y = L$ and neglect the details of the boundary effects, set $K^{-1} \sim \omega_0^2 + 1/L^2$ and cut off $q_y$ integrals at $q_y \sim \pi/L$. Because we are interested in length scales long compared to mean free paths we write

$$E_y(x, y) = \rhoHl/L B_{int}(x, y)$$ (11)

so for the fluctuations of $V_H = \int dq_y E_y$, averaged over the contact region $L_x$ and transformed into the frequency domain we have

$$\langle V_H(\omega) \rangle^2/\rho_Hl^2 = \frac{2\pi\Delta T}{D} Y(\Omega)$$ (12)

Here $\Omega = \omega L^2/(4D)$ and $Y$ is a dimensionless function with the following limits:

$$Y = \left\{ \begin{array}{ll}
\frac{1}{\ln[1/\Omega]} & \Omega \ll 1 \\
\frac{1}{\Omega^2} & \Omega \gg 1
\end{array} \right.$$

Finally, for $\omega < \gamma$ one should replace $\omega$ by $\gamma$ in (13). These estimates apply to one CuO$_2$ layer. Real systems have many layers and the gauge field fluctuations are uncorrelated from layer to layer. The voltages in the different layers add incoherently, as may most easily be seen by considering an experimental configuration in which the transverse voltage difference is held fixed at zero and the current fluctuation is measured. Thus in a system of $N$ layers $\langle V^2 \rangle$ is reduced by a factor of $1/N$.

We now estimate the size of a typical value of the instantaneous voltage difference, $V_{inst}$, in a physically realizable sample. We express this in terms of the uniform applied field, $B_{int}$, which would lead to the same voltage. Assuming that all relevant frequencies are less than $T$ we find

$$B_{inst}^2 = \frac{V_{inst}^2}{\rho_H l^2} = \int d\omega \frac{\langle V_\omega^2 \rangle}{2\pi \rho_H l^2}$$

$$= \frac{f^2 \pi AT}{NL_x^2} Y(2L_x/L_y)$$ (14)
The final equality follows because integrating Eq. (13) over frequency produces a field correlator with short-range spatial correlations which is moreover independent of \( \gamma \). Using the estimates previously given and setting \( L_x = L_y = 1 \, \mu m \), \( N = 200 \) and \( T = 100K \) we find \( B_{\text{inst}} \sim 0.03 \, T \). The sample-size and cutoff \( \gamma \) of course determine the frequency range in which the noise power is concentrated; this range is set by the largest of \( D/L^2 \) (which, using \( L \sim 1 \, \mu m \) and \( D \sim 10^{-2} \, \text{cm}^2/\text{sec} \), is \( 1 \, \text{MHz} \)) and \( D \) which is unfortunately difficult to determine reliably because it is an exponential function of parameters \( (p_0, \chi_{\text{dia}}) \) which are not well known. The situation is further complicated because in realistic models \( p_0 \) varies significantly along the spinon Fermi surface. The range of plausible values of these parameters leads to estimates of \( \gamma \) ranging from 100 \( kHz \) to 1 \( GHz \). The effects due to spin fluctuations are four orders of magnitude smaller because of the difference in \( f \) and, also more difficult to observe because \( \gamma \), due physically to spin orbit scattering, is at least 1 \( THz \).

Similar considerations apply to the half-filled Landau level. Because of the Coulomb interaction the \( \langle \phi \phi \rangle \) correlator is

\[
D(q, \omega) = \frac{\omega \sigma k^2}{\omega^2 + \frac{k^2}{\sigma(k\epsilon + \kappa)}},
\]

(15)

Here \( \sigma = p_0 L/(4\pi) \) is the conductivity, \( U = e^2/(8\pi\epsilon) \) and \( \kappa^{-1} \) is the screening length of the Coulomb interaction. For typical samples, \( \kappa^{-1} \) is comparable to the device size, but it may be possible to introduce screening, e.g. via a metal gate, so \( \kappa^{-1} \sim l_0 \). Our previous arguments then imply that

\[
B_{\text{typ}} \sim f \left( \frac{T_{l0}}{U} \right)^{1/2} \left( \frac{L_0}{l_0} \right) (l_0 \kappa)^{1/2}
\]

(16)

Using \( \epsilon \sim 13 \) and \( l_0 = 300 \, A \) yields \( U/l_0 \sim 2K \), so for a device of size 1 \( \mu m \) the effective field \( B_{\text{typ}}[\text{gauss}] \sim 200(T[l]^1/2)(l_0 \kappa)^{1/2} \). Thus for unscreened samples \( \kappa^{-1} \sim 1 \, \mu m \) so at \( T \sim 1 \, K \), a typical field is less than 100 gauss while for the screened samples \( B_{\text{typ}} \sim 200 \, gauss \).

There are two contributions to the \( \nu = 1/2 \) Hall noise: usual particle number fluctuations and statistical gauge field fluctuations at fixed particle number; up to a numerical factor they give identical contribution to Eq. (16). The statistical flux contributions can be isolated by working at fixed particle number (or, equivalently, by studying fluctuations of \( \tan(\theta_H) = \sigma_{xy}/\sigma_{xx} \) or from the frequency dependence. Both contributions involve the diffusion timescale which is \( \nu \sim U/(\kappa L^2) \); the statistical contribution involves instanton time scale \( \gamma \) which dominates for large samples. Observation of fluctuations on a time scale faster than the diffusion time scale would be clear evidence for statistical gauge field effects. For an unscreened sample \( kL \sim 1 \) so for a 1 \( \mu m \) sample \( \nu \sim 150 \, MHz \), while for a sample with \( kL \sim 1 \) it would be \( \nu \sim 5 \, MHz \). Although instanton effects have been addressed \[10\] we believe that the consequences of the absence of an underlying lattice have not been adequately explored so there is no reliable estimate of \( \gamma \).

To summarize: fluctuations of internal gauge fields lead to fluctuations in the zero-applied-field Hall voltage of high \( T \) superconductors and in the Hall voltage of half-filled Landau levels. Measurement of these fluctuations would be an important test of the validity of the gauge theories proposed for these systems. Their frequency dependence contains information on the dynamics of the gauge field and in particular on instanton effects.

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\[1\] G. Baskaran, Z. Zou and P. W. Anderson, Solid State Comm. 63, 973 (1987);
\[2\] L. B. Ioffe and A. I. Larkin, Phys. Rev. B39, 8988 (1989).
\[3\] see, e.g. P. A. Lee, p. 96 in High Temperature Superconductivity: Proceedings, eds. K. S. Bedell, D. Coffey, D. E. Meltzer, D. Pines, and J. R. Schreiffer, Addison-Wesley (Reading, MA: 1990);
\[4\] V. Kalmeyer and S.-C. Zhang, Phys. Rev. B46, 9889 (1992).
\[5\] B. Halperin, P. A. Lee and N. Read, Phys. Rev. B 47, 7312 (1993)
\[6\] A. M. Polyakov, “Gauge Field and Strings”, Harwood (Chur 1987), Section 4.3. Polyakov shows that in compact QED instantons provide a mechanism for non-conservation of flux, and therefore lead to a mass term in the field correlator. Our problem differs from compact QED because it has a Fermi surface which causes the action of the instanton to diverge at \( T = 0 \), see L. B. Ioffe and A. I. Larkin, op. cit.
\[7\] L. B. Ioffe and B. Kotliar, Phys. Rev. B 42, 10348 (1990).
\[8\] T. Tanamoto, K. Kohno and H. Fukuyama, J. Phys. Soc. Jpn. 61 1886 (1992); B. L. Altshuler and L. B. Ioffe, Solid State Comm. 82, 253 (1992); M. Ubbens and P. A. Lee, Phys. B 50, 438 (1994); B. L. Altshuler, L. B. Ioffe and A. J. Millis, Phys. Rev. B 53, 423 (1996).
\[9\] B. L. Altshuler, L. B. Ioffe and A. J. Millis, Phys. Rev. B50 14048, (1994).
\[10\] Y. B. Kim and X-G. Wen, Phys. Rev. 50, 8078 (1994).