Strange mesons in strong magnetic fields

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Abstract

The masses of the strange mesons ($K$, $K^*$ and $\phi$) are investigated in the presence of strong magnetic fields. The changes in the masses of these mesons arise from the mixing of the pseudoscalar and vector mesons in the presence of a magnetic field. For the charged mesons, these mass modifications are in addition to the contributions from the lowest Landau energy levels to their masses. The decay widths, $\phi \rightarrow K\bar{K}$ and $K^* \rightarrow K\pi$, in the presence of the magnetic field are studied using a field theoretic model of composite hadrons with constituent quarks/antiquarks. The model uses the free Dirac Hamiltonian in terms of the constituent quark fields as the light quark antiquark pair creation term and explicit constructions for the meson states in terms of the constituent quarks and antiquarks to study the decay processes. The study of the masses and decay widths of the strange mesons in strong magnetic fields can have observable consequences on the production of the open and hidden strange mesons in the peripheral ultra high energy collisions at LHC, where created magnetic fields can be huge.

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I. INTRODUCTION

The topic of the in-medium properties of strange hadrons is a subject of intense research in the context of the heavy ion collision experiments as well as in the study of the bulk matter in nuclear astrophysical objects, e.g., (proto) neutron stars. The topic is important due to its relevance to the experimental observables, e.g., the collective flow as well as the yield and spectra of these hadrons resulting from the heavy ion collision experiments. The strange baryons could exist in the high density bulk matter in neutron stars and there could be the possibility of the antikaon condensation [1] in the interior of the neutron stars. Recently, there have been a lot of studies on the properties of the hadrons in the presence of strong magnetic fields. This is due to the estimation of the magnetic fields produced in the peripheral ultra relativistic heavy ion collision experiments to be huge ($eB \sim 2m^2_\pi$ at RHIC and $eB \sim 15m^2_\pi$ at LHC) [2]. Also, strong magnetic fields exist in the astrophysical compact objects, e.g. magnetars, where the magnetic field is of the order of $10^{15} - 10^{16}$ Gauss at the surface and the magnitude of the magnetic field could be much larger in the interior of magnetars [3]. Recent studies of the heavy flavour mesons due to the mixing of the pseudoscalar and the vector mesons in the presence of a magnetic field [4–8] have shown to lead to dominant modifications to the masses of these mesons. The study of heavy flavour charmonium states as well as of $D$ mesons in the presence of a magnetic field within a QCD sum rule approach [4–6] as well as within an effective potential approach [7] show the effects of the mixing on the masses to be quite pronounced. A study of the mixing effects on the formation times of the vector and pseudoscalar charmonium states ($J/\psi - \eta_c$ and $\psi' - \eta'_c$ mixing effects) shows to lead to faster (slower) formation times for the pseudoscalar (vector) states [8]. Due to the mixing with the vector charmonium states, the pseudoscalar mesons, $\eta_c$ and $\eta'_c$ might show as peaks in the dilepton spectra, arising from anomalous decay modes $\eta_c, \eta'_c \rightarrow l^+l^-$ and can act as probe of a strong magnetic field at the early stage [8].

The changes in the masses of the charmonium states [9] and the open charm mesons [10] in the presence of strong magnetic fields due to the pseudoscalar-vector mixing effects, in addition to the Landau level contributions to the charged mesons, are observed to be quite appreciable. The effects of the magnetic field on the decay widths $\psi(3770) \rightarrow D\bar{D}$ [9] as well as $D^* \rightarrow D\pi$ [10] are calculated from their mass modifications in the presence of a magnetic field, using a
field theoretical model of composite hadrons with constituent quark (antiquark) \[11, 12\]. The model of composite hadrons uses the light quark-antiquark pair creation term of the free Dirac Hamiltonian in terms of the constituent quark fields and explicit constructions of the initial and final states to study the decay processes. In the absence of a magnetic field, the model has been used to calculate the decay widths of the charmonium states to $D\bar{D}$ and $D^* \rightarrow D\pi$ \[13\], as well as, to study the decay widths of bottomonium states to $B\bar{B}$ \[14\] in asymmetric strange hadronic matter. The charmonium decay widths to $D\bar{D}$ were also calculated using a light quark antiquark pair creation (in the $^3P_0$ state) model \[15, 16\], namely, the $^3P_0$ model \[17, 18\]. The modifications of the decay widths were computed from the mass modifications of the charmonium states as well as $D$ and $\bar{D}$ mesons calculated using a chiral effective model \[16\]. When the internal structure of the mesons in the initial and final states are taken into account, within the composite model of hadrons \[13\], as well as the $^3P_0$ model \[15\], the heavy quarkonium decay widths were observed to vanish at specific densities (so called nodes) \[15, 16\]. In both the models, the decay process is through the creation of a light quark-antiquark pair and the constituent quark (antiquark) of the decaying meson, combines with the light antiquark (quark) created to form the final state mesons. The study of the heavy quarkonium decay widths in asymmetric hyperonic matter using the field theoretical model of composite hadrons \[13, 14\] show that the effects of density are the dominant effects as compared to the effects from isospin asymmetry as well as strangeness. The mass modifications of the hidden and open charm mesons in the presence of strong magnetic fields, arising from the pseudoscalar-vector mesons mixings, in addition to the Landau contributions to the masses of the charged mesons, and their effects on the decay widths of charmonium states to $D\bar{D}$ and $D^* \rightarrow D\pi$ \[9, 10\] have been studied using the model of composite hadrons \[11, 12, 19\]. In the present work, we study the mass modifications of the strange $K$, $K^*$, $\phi$ mesons in the presence of a magnetic field due to the mixing of the pseudoscalar ($P$) and vector ($V$) mesons ($K - K^*$ and $\phi - \eta'$ mixing), including the Landau level contributions for the charged mesons, and their subsequent effects on the decay widths $K^* \rightarrow K\pi$ and $\phi \rightarrow K\bar{K}$.

The outline of the paper is as follows. In section II, the mass modifications of the strange $K$, $K^*$ and $\phi$ mesons and their effects on the decay widths $K^* \rightarrow K\pi$ and $\phi \rightarrow K\bar{K}$, are investigated in the presence of a uniform magnetic field. The mass modifications arise from
the pseudoscalar-vector mesons mixing ($K\bar{K}$ and $\phi\eta'$) in the presence of a magnetic field.

For the charged $K$ and $K^*$ mesons, the Landau level contributions to these masses are also taken into account. The effects of the mass modifications on the decay widths of $K^* \to K\pi$ as well as of $\phi$ meson to $K\bar{K}$ ($K^+K^-$ and $K^0\bar{K}^0$) are investigated from the mass modifications of these mesons in the presence of a magnetic field. These decay widths are studied using a field theoretical model of composite hadrons with quark and antiquark constituents. We discuss the results of the masses and decay widths of the strange mesons in the presence of strong magnetic fields in section III. Section IV summarizes the results of the present study.

II. STRANGE MESONS IN PRESENCE OF A MAGNETIC FIELD

A. MASSES:

In this section, the mass modifications for the $K$ and $K^*$ mesons, as well as of $\phi$ and $\eta'$, are studied arising due to the pseudoscalar-vector meson mixing in the presence of a magnetic field. The effect of the mixing is considered through a phenomenological Lagrangian interaction 

$$\mathcal{L}_{PV\gamma} = \frac{g_{PV}}{m_{av}} e F_{\mu\nu}(\partial^\mu P)^V^\nu,$$

(1)

where $m_{av}$ is the average of the masses of the pseudoscalar and vector mesons. The coupling strengths of the radiative decay of the vector meson, $V$ to the pseudoscalar meson, $P$, as described by the interaction given by (1) are determined from the observed decay widths of $V \to P\gamma$ in vacuum.

For the charged $K$ and $K^*$ mesons, the contributions of the mixing on the masses of these mesons are in addition to the lowest level Landau level contributions to their masses. The masses of the charged pseudoscalar and vector mesons in their ground states in a background magnetic field are given as $m_P(B) = (m_P^2 + eB)^{1/2}$ and $m_V(B) = (m_V^2 - eB)^{1/2}$, with $m_{P,V}$ as the masses of these mesons at zero magnetic field. For the vector meson, the gyromagnetic ratio is taken to be 2 [6, 20]. For the neutral mesons, $m_{V,P}(B) = m_{V,P}$, as there are no Landau level contributions. In the presence of a magnetic field, there is mixing of the pseudoscalar and
the longitudinal component of the vector mesons, which modify their masses to

\[ m_{V||,p}^{(PV)} = \left[ \frac{1}{2} \left( M^2_{+} + \frac{c_{PV}^2}{m^2_{av}} \right) \pm \sqrt{M^4 + \frac{2c_{PV}^2M^2_{+}}{m^2_{av}} + \frac{c_{PV}^4}{m^4_{av}}} \right]^{1/2}, \]

where \( M^2_{\pm} = m^2_{V}(B) \pm m^2_{P}(B), \ c_{PV} = g_{PV}eB \) and \( m_{av} = (m_{V}(B) + m_{P}(B))/2. \)

**B. DECAY WIDTHS:**

The decay widths of \( K^* \rightarrow K\pi \) and \( \phi \rightarrow K\bar{K} \) are modified in the presence of a magnetic field due to the changes in the masses of these mesons arising from the pseudoscalar-vector mesons mixing effects, in addition to the Landau level contributions for the charged mesons. These decay widths are studied using a field theoretical model of composite hadrons with quark/antiquark constituents \[11, 12, 19\]. The matrix element of the quark antiquark pair creation term of the free Dirac Hamiltonian term for light quarks \( (q = u, d) \) is evaluated between the initial and final meson states to calculate the decay widths. The initial and final meson states are constructed explicitly in terms of the constituent quark fields, assuming the harmonic oscillator potential between the quark and antiquark constituents. Using a Lorentz boosting, the constituent quark field operators of the hadron in motion are obtained from the constituent quark field operators of the hadron at rest.

We investigate the decay processes, \( K^{*+} \rightarrow K\pi(K^+\pi^0, K^0\pi^+), K^{*0} \rightarrow K\pi(K^0\pi^0, K^+\pi^-) \) and \( \phi \rightarrow K\bar{K}(K^+K^-, K^0\bar{K}^0) \), within the composite model of hadrons in the present work. For the vector mesons, \( K^* \) and \( \phi \) decaying at rest, the outgoing pseudoscalar meson \( (K, \pi, K, \bar{K}) \) is with finite momentum. The explicit construction for the final state pseudoscalar meson, \( P \) with momentum \( p \), assuming harmonic oscillator wave function, is given as

\[ |P(p)\rangle = \frac{1}{\sqrt{6}} \left( \frac{R^3_p}{\pi} \right)^{3/4} \int dk \exp \left( -\frac{R^2_p|k|^2}{2} \right) Q_{1r}^i(k + \lambda_2p)^\dagger u^\dagger_r\bar{Q}_2^\dagger_s((-k + \lambda_1p)v_s)dkd|\text{vac}\rangle, \]

where, \( Q_{1r}^i(k)^\dagger(\bar{Q}_2^i(k)) \) is the quark (antiquark) creation operator of flavour \( Q_1(Q_2) \) with spin \( r \), color \( i \) and momentum \( k \), and \( u_r \) and \( v_s \) are the two component spinors. For the meson \( P(\equiv K^+, K^0, K^-, K^0, \pi^+, \pi^-, \pi^0) \), the quark-antiquark constituents are given as \( (Q_1, Q_2) \equiv (u, \bar{s}), (d, \bar{s}), (s, \bar{u}), (s, \bar{d}), (u, \bar{d}), (d, \bar{u}), \frac{1}{\sqrt{2}}[(u, \bar{u}) - (d, \bar{d})]. \)
The decaying vector meson, \( V(\equiv K^{*+}, K^{*0}, \phi) \) at rest, with polarisation \( m \), is constructed as [13]

\[
|V^m(0)\rangle = \frac{1}{\sqrt{6}} \left( \frac{R_V^2}{\pi} \right)^{3/4} \int d\mathbf{k} \exp \left( -\frac{R_V^2 k^2}{2} \right) Q_1^i(k)^\dagger u^\dagger \sigma^m \tilde{Q}_2^i(-\mathbf{k}) v_s d\mathbf{k} |\text{vac}\rangle,
\]

(4)

with the quark-antiquark constituents for the vector meson \( V(\equiv K^{*+}, K^{*0}, \phi) \), as given by \( (Q_1, \tilde{Q}_2) \equiv (u, \bar{s}), (d, \bar{s}), (s, \bar{s}) \). The parameters \( R_P \) and \( R_V \) in equations (3) and (4), correspond to the harmonic oscillator strengths for the pseudoscalar and vector mesons. In the pseudoscalar state, \( |P(p)\rangle \) given by equation (3), \( \lambda_1 \) and \( \lambda_2 \) are the fractions of the mass (energy) of the pseudoscalar meson at rest (in motion), carried by the constituent antiquark, \( \tilde{Q}_2 \) and the constituent quark, \( Q_1 \). The constituent quark (and antiquark) of the hadron occupying specific energy level is similar to as in the MIT bag model [21]. The fractions of the energy carried by the constituent quark (antiquark) are calculated by assuming the binding energy of the meson as shared by the quark (antiquark) to be inversely proportional to the quark (antiquark) mass [12]. For the pion states, \( \pi^+ \) and \( \pi^0 \), which are light \( (q = u, d) \) quark-antiquark bound states, the fractions of energy carried by the quark and antiquark are the same, i.e., \( \lambda_1 = \lambda_2 = 1/2 \), as the masses of the \( u \) and \( d \) quarks are assumed to be the same. The decay widths for the processes \( K^{*} \rightarrow K\pi \) and \( \phi \rightarrow K\bar{K} \), are obtained from the matrix element of the light quark antiquark pair creation term of the free Dirac Hamiltonian density, between the initial and the final states [13]. The matrix element is multiplied with a strength parameter, \( \gamma_{V=K^{*},\phi} \), which is fitted to the observed decay width of the vector meson in vacuum.

1. DECAW WIDTH OF \( K^{*} \rightarrow K\pi \)

In the absence of the pseudoscalar-vector mesons mixing, the decay width for \( K^{*} \rightarrow K\pi \) is obtained from the matrix element of the free Dirac Hamiltonian between the initial and final states as [13]

\[
\Gamma (K^{*}(0) \rightarrow K(p) + \pi(-p)) = \gamma_{K^{*}} g_s^2 \frac{8\pi^2 p_0^0}{3m_{K^{*}}} A_{K^{*}} (|p|)^2 |p|^3 ,
\]

(5)

where \( p_0^0 = (|p|^2 + m_{K}^2)^{1/2} \) and \( p_{\pi}^0 = (|p|^2 + m_{\pi}^2)^{1/2} \) are the energies of the outgoing \( K \) meson and pion respectively, in terms of the magnitude of the 3-momentum of \( K(\pi) \) meson, \( |p| \), given
as
\[
|p| = \left( \frac{m_{K^*}^2}{4} - \frac{m_K^2 + m_\pi^2}{2} + \frac{(m_{K^*}^2 - m_\pi^2)^2}{4m_{K^*}^2} \right)^{1/2}.
\] (6)

In equation (5), \(A_{K^*}(|p|)\) is given as
\[
A_{K^*}(|p|) = 6c_{K^*} \left( \frac{\pi}{a_{K^*}} \right)^{3/2} \exp \left[ \left( a_{K^*} - b_{K^*} \right) \left( \frac{1}{2} \right) \left( \frac{\lambda_2}{4} \right) \right] |p|^2 \left[ F_{0K^*} + \left( \frac{3F_{1K^*}}{2a_{K^*}} \right) \right],
\] (7)

where,
\[
F_{0K^*} = (b_{K^*} - 1) \left( 1 - \frac{1}{8M_q^2} |p|^2 \lambda_2 - \frac{1}{2} \right)
\]
\[= (b_{K^*} - \lambda_2) \left( 1 + \frac{1}{4M_q^2} |p|^2 \right) \left( \frac{3}{4} b_{K^*}^2 - \frac{5}{4} b_{K^*} + \frac{7}{16} \right)
\]
\[= (b_{K^*} - 1) \left[ 1 + \frac{1}{4M_q^2} |p|^2 \right] \left( \frac{3}{4} b_{K^*}^2 - (1 + \frac{1}{2} \lambda_2) b_{K^*} + \lambda_2 - \frac{1}{4} \lambda_2^2 \right)
\] (8)
\[
F_{1K^*} = -\frac{1}{4M_q^2} \left[ \frac{5}{2} b_{K^*} - \frac{9}{8} - \frac{11}{12} \lambda_2 \right].
\] (9)

The parameters \(a_{K^*}, b_{K^*}, c_{K^*}\) are given as
\[
a_{K^*} = \left( \frac{R_{2K^*}^2 + R_{2K^*}^2 + R_{2\pi}^2}{2} \right),
\]
\[b_{K^*} = \frac{1}{2a_{K^*}} \left( R_{2K} \lambda_2 + \frac{1}{2} R_{2\pi}^2 \right),
\]
\[c_{K^*} = \frac{1}{12 \sqrt{3}} \left( \frac{R_{K^*}^2 R_{2K^*} R_{2\pi}^2}{\pi^3} \right)^{3/4}.
\] (10)

In equations (8)–(9), \(M_q\) is the constituent light quark \((q = u, d)\) mass. As has already been mentioned, the expression for the decay width for \(K^* \rightarrow K\pi\) as given by equation (5) (modulo \(\gamma_{K^*}\)) is obtained from the matrix element of the free Dirac Hamiltonian between the initial and final states. For \(\pi_0, (\pi^\pm)\) in the final state, the value of \(g^2=1(2)\). The decay amplitude is multiplied by \(\gamma_{K^*}\), which is the production strength of \(K\pi\) from decay of \(K^*\) meson, and is fitted from its vacuum decay width. The decay width has the dependence on the masses of the decaying and outgoing mesons, through \(|p|\), and its dependence as given by equation (5), is mainly through a polynomial part multiplied by an exponential part.

The expression for the decay width of \(K^* \rightarrow K\pi\) given by equation (5) does not account for the mixing of the \(K\) and \(K^*\) mesons in the presence of the magnetic field. The mixing of the pseudoscalar mesons and the vector mesons leads to a drop (increase) in the mass of the \(K\) meson (longitudinal component of the \(K^*\) meson), given by equation (2). This leads to the expression for the decay width of \(K^* \rightarrow K\pi\) to be modified to
\[
\Gamma^{PV}(K^*(0) \rightarrow K(p)\pi(-p)) = \gamma_{K^*}^2 g^2 \frac{8\pi^2}{3} \left[ \left( \frac{2}{3} |p|^2 p_{K^*}^0(|p|)^2 \right) A_{K^*}(|p|)^2 \right].
\]
and \( \phi \) mesons and the longitudinal component of \( \phi \) in the absence of the pseudoscalar-vector mesons mixing effects. In equation (12) for the

\[
\Gamma(\phi(0) \to K(\mathbf{P})\bar{K}(-\mathbf{P})) = \frac{\gamma_{\phi}^2 8\pi^2}{3} \left| \frac{P_{0K}^0 P_{0K}}{m_\phi} A_\phi(|\mathbf{P}|)^2 \right| (\mathbf{P} \to |\mathbf{P}|(m_{K^*} = m_{K^*}, m_K = m_{K^*}))
\]

(11)
The first term corresponding to the transverse polarizations for the vector \( K^* \) meson are unaffected by the mixing of the \( K \) and \( K^* \) states, whereas the second term has the masses the \( K \) and the longitudinal \( K^* \) mesons modified due to the \( K - K^* \) mixing.

2. DECAY WIDTH OF \( \phi \to K\bar{K} \)

The decay width of \( \phi \to K\bar{K} \) is calculated to be

\[
\Gamma(\phi(0) \to K(\mathbf{P})\bar{K}(-\mathbf{P})) = \frac{\gamma_{\phi}^2 8\pi^2}{3} \left| \frac{P_{0K}^0 P_{0K}}{m_\phi} A_\phi(|\mathbf{P}|)^2 \right| (\mathbf{P} \to |\mathbf{P}|(m_{K^*} = m_{K^*}, m_K = m_{K^*}))
\]

(12)
in the absence of the pseudoscalar-vector mesons mixing effects. In equation (12) for the expression of decay width of \( \phi \) to \( K\bar{K} \), \( P_{0K(K)}^0 = (m_{K(K)}^2 + |\mathbf{P}|^2)^{1/2} \), and, \(|\mathbf{P}| \) is the magnitude of the momentum of the outgoing \( K(\bar{K}) \) meson, given as

\[
|\mathbf{P}| = \left( \frac{m_\phi^2}{4} - \frac{m_K^2 + m_{K^*}^2}{2} + \frac{(m_{K^*}^2 - m_K^2)^2}{4m_\phi^2} \right)^{1/2}.
\]

(13)
In the above, \( A_\phi(|\mathbf{P}|) \) is given as

\[
A_\phi(|\mathbf{P}|) = 6c_\phi \exp[(a_\phi b_\phi^2 - R_{K^*}^2 \lambda_2^2)|\mathbf{P}|^2] \cdot \left( \frac{\pi}{a_\phi} \right)^{\frac{5}{2}} \left[ F_{0\phi} + F_{1\phi} \frac{3}{2a_\phi} \right],
\]

(14)
with

\[
F_{0\phi} = (\lambda_2 - 1) - \frac{1}{2M_\phi^2}|\mathbf{P}|^2(b_\phi - \lambda_2) \left( \frac{3}{4} b_\phi^2 - (1 + \frac{1}{2} \lambda_2) b_\phi + \lambda_2 - \frac{1}{4} \lambda_2^2 \right),
\]

\[
F_{1\phi} = \frac{1}{4M_\phi^2} \left[ -\frac{5}{2} b_\phi + \frac{2}{3} + \frac{11}{6} \lambda_2 \right].
\]

(15)
and the parameters \( a_\phi, b_\phi \) and \( c_\phi \) given as

\[
a_\phi = \frac{1}{2} R_{K^*}^2 + R_{K}^2, \quad b_\phi = R_{K^*}^2 \lambda_2/a_\phi, \quad c_\phi = \frac{1}{6\sqrt{6}} \cdot \left( \frac{R_{K^*}^2}{\pi} \right)^{3/4} \cdot \left( \frac{R_K^2}{\pi} \right)^{3/2},
\]

(16)

When the pseudoscalar meson–vector meson mixing is included, the masses of the \( K \) and \( \bar{K} \) mesons and the longitudinal component of the \( \phi \) meson are modified due to \( K - K^*, \bar{K} - \bar{K}^* \) and \( \phi - \eta' \) mixings. The modified expression for the decay width \( \phi \to K\bar{K} \) is given as

\[
\Gamma^{PV}(\phi(0) \to K(\mathbf{P})\bar{K}(-\mathbf{P})) = \frac{\gamma_{\phi}^2 8\pi^2}{3} \left| \frac{P_{0K}^0 P_{0\bar{K}}^0}{m_\phi} A_\phi(|\mathbf{P}|)^2 \right| (\mathbf{P} \to |\mathbf{P}|(m_{\phi} = m_{\phi}^{PV}, m_{K(K)} = m_{K(K)}^{PV}))
\]

(17)
III. RESULTS AND DISCUSSIONS

In the presence of a uniform magnetic field, the modifications of the masses of the $K, K^*, \phi$ and $\eta'$ due to the mixing of the pseudoscalar and vector mesons are investigated. These are in addition to the Landau level contributions for the charged mesons. The mixing is taken into account through a phenomenological Lagrangian interaction given by equation (1). The coupling strength parameter $g_{PV}$ for the radiative decay of the vector meson, $V$, to the pseudoscalar meson, $P$, which is described by the interaction Lagrangian (1) is determined from the observed decay width of $V \rightarrow P\gamma$ in vacuum. For the processes $K^{*+} \rightarrow K^+\gamma$, $K^{*0} \rightarrow K^0\gamma$ and $\phi \rightarrow \eta'\gamma$, the observed decay widths of 50.292 keV, 46.827 keV and 0.26429 keV [22] determine the coupling parameters, $g_{K^{*+}K^+}$, $g_{K^{*0}K^0}$, and $g_{\eta'\phi}$ to be 0.5793, 0.5611 and 0.7043 respectively. The vacuum values for the masses (in MeV) of these mesons are taken to be $m_{K^{*+}} = 891.66$, $m_{K^{*0}} = 895.55$, $m_{K^+} = 493.677$, $m_{K^0} = 497.61$, $m_\phi = 1019.461$, $m_{\eta'} = 957.78$ [22]. The presence of a magnetic field leads to the mixing of the pseudoscalar meson and the longitudinal component of the vector meson, with their modified masses given by equation (2) due to the mixing effect. As has already been mentioned, for the charged mesons, these modifications in the masses are in addition to the contribution arising from the lowest Landau levels, due to the direct interactions of the charged mesons with the external magnetic field. The masses of the charged and neutral open strange mesons are plotted in figure 1. The mixing of the pseudoscalar and vector mesons (indicated as PV) are observed to be a monotonic drop (rise) in the mass of the $K^+(K^0)$ (longitudinal component of the vector $K^{*+}(K^{*0})$) meson. The modifications of the masses of the charged as well as neutral $K$ and $K^*$ mesons are observed to be quite small due to the (PV) mixing effects, with modified values of for $K^{*+}$, $K^+$, $K^{*0}$, $K^0$ masses as 897 (912.46), 490.75 (482.4), 900.49 (914.87) and 494.88 (487.1) mesons respectively, at $eB = 5(10)m_\pi^2$. For the maximum value of the magnetic field considered in the present work, $eB = 10m_\pi^2$, the mass shifts (in MeV) for $K^{*+}$, $K^+$, $K^{*0}$ and $K^0$ mesons as thus observed to be around 21, 11, 19 and 10.5 respectively. The Landau level contributions to the masses of the $K^{*+}$ ($K^+$) are observed to lead to monotonic drop (rise) with increase in magnetic field. From panel (a) of figure 1 it is observed that there is a drop (increase) in the mass of the $K^{*+}(K^+)$ meson with increase in the magnetic field, upto a value of $eB \sim 5m_\pi^2$, when the Landau contributions dominate over the mixing effects. As the mass difference of these
FIG. 1: (Color online) The masses of the $K$ and the longitudinal components of the $K^*$ mesons are plotted as functions of $eB/m_\pi^2$. The effects of the pseudoscalar–vector (PV) mixing on these masses are shown for the charged and neutral mesons in panels (a) and (b) respectively. The Landau contributions to the masses of the charged $K^+$ and $K^{*+}$ mesons are also shown as the dot-dashed lines.
FIG. 2: (Color online) The decay widths for $K^* \rightarrow K\pi$ for the charged $K^{*+}$ and neutral $K^{*0}$ are plotted as functions of $eB/m^2_\pi$ in panels (a) and (b) respectively.

Mesons becomes smaller for larger values of the magnetic field, the mixing effect is observed to become more appreciable and this dominates over the contributions from the Landau levels. This is observed to lead to quite dominant rise (drop) of the mass of the $K^{*+}$ ($K^+$) meson for $eB$ greater than $\sim 6m^2_\pi$. The mass modifications of the charged $K$ and $K^*$ mesons are thus observed to be much more pronounced as compared to the neutral mesons, due to the
FIG. 3: (Color online) The mass modifications of the $\phi$ and $\eta'$ mesons arising from the pseudoscalar–vector meson mixing are illustrated as functions of $eB/m_{\pi}^2$ in panel (a). Panel (b) shows the effects of the magnetic field on the partial decay widths of $\phi$ to (I) $K^+K^-$ and (II) $K^0\bar{K}^0$ respectively. The Landau contributions to the masses of the charged mesons $K^\pm$ are taken into account and these are compared to the case of when these effects are not considered.
additional effects from the Landau level contributions. These mass modifications of the charged open strange mesons are observed to modify appreciably the decay widths of $K^{*+} \to K\pi$, as can be seen from figure 2. The mass modifications of the neutral mesons, $K^{*0}$ and $K^0$ due to the magnetic field, which arise only due to the pseudoscalar-vector mesons mixing, are shown in panel (b) of figure 1. These mass changes are observed to be moderate due to the small mixing coupling parameter, $g_{K^0K^{*0}}=0.5611$. It might be worthwhile to compare the effects of the mixing on the masses of the open strange mesons as studied in the present work, with the mass modifications of the open charm mesons due to the mixing effects [10]. The total width of the neutral $D^*$ meson is not yet measured experimentally, but the branching raito of the two modes $D^{*0} \to D^0\pi^0$ and $D^{*0} \to D^0\gamma$ is measured to be 64.7:35.3 [22]. The decay width of $D^{*0} \to D^0\pi^0$ (and hence of $D^{*0} \to D^0\gamma$) is obtained [6, 10] by taking the coupling strengths of the decays $D^{*0} \to D^0\pi^0$ and $D^{*+} \to D^+\pi^0$ are taken to be the same. This is observed to lead to the coupling parameter, $g_{D^0D^{*0}}$ to be quite large, about 4 times larger than the mixing parameter for $D^{*+}D^+$ mixing [6, 10]. The mass modifications for the neutral open charm mesons due to the mixing effects are thus observed to be much more pronounced compared to the mass modifications of the charged $D$ and $D^*$ mesons. On the other hand, in the present investigation, the mass modifications of the neutral open strange mesons are moderate. The much larger value of the mixing parameter in the neutral open charm sector could be due to the availability of a single channel for $D^* \to D\pi$, which is $D^{*0} \to D^0\pi$, in addition to the radiative decay channel $D^{*0} \to D^0\gamma$. The radiative decay width is comparable to the decay width of $D^* \to D\pi$ for the neutral $D^*$. On the other hand, for the open strange sector, there are two channels $K^{*0} \to K^0\pi^0$ and $K^{*0} \to K^0\pi^-$, and the decay width of $K^{*0} \to (K\pi)^0$ is about 99.754 % of the total width of $K^{*0}$ meson. The radiative decay width of $K^{*0} \to K^0\gamma$ is about 0.246 % of its total width, which makes the mixing of $K^{*0} - K^0$ to be quite small. Also, for the charged $K^{*+}$ meson, the decay is dominated by $K^{*+} \to (K\pi)^+$ and the decay to $K^{+}\gamma$ is around 0.099% of its total width. These lead to the mass modifications of the $K$ and $K^*$ (both the charged and neutral mesons) due to $K - K^*$ mixings to be quite moderate, as can be seen in figure 1. The masses of the charged open strange mesons, $K^{*\pm}$ and $K^{\pm}$ mesons, as modified due to the Landau level contributions are $(m_{K^{*\pm}}^2 - eB)^{1/2}$ and $(m_{K^{\pm}}^2 + eB)^{1/2}$ respectively [20]. The mass of the charged vector meson, $V$, becoming negative above a critical magnetic field,
\((eB)_{\text{crit}} = m_V^2\) leads to condensation of the charged vector meson [20]. For magnetic fields higher than the critical magnetic field, \((eB)_{\text{crit}} = m_\rho^2 \sim 30m_\pi^2\), there is condensation of the charged \(\rho\) mesons \((ud\bar{d}(d\bar{u})\) bound states), arising from the gluon mediated attractive interaction of the quark and antiquark of different flavours in spin 1 state [20]. For still stronger magnetic fields, \(eB\) greater than \((eB)_{\text{crit}} = m_{K^*}^2 \simeq 40m_\pi^2\), there should be condensation of the charged \(K^*\) mesons \((u\bar{s}(s\bar{u})\) bound states). The condensation of the charged vector mesons \((\rho, K^*)\) can have implications in ultra-peripheral ultra-relativisitcs collisions at LHC, where it is estimated that magnetic fields up to the order of \(eB \sim 100m_\pi^2\) can be achieved [23].

The decay widths, \(K^* \rightarrow K\pi\) and \(\phi \rightarrow K\bar{K}\) as modified due to the mass modifications of these mesons in the presence of a magnetic field are studied using a field theoretical model of composite hadrons with constituent quarks and antiquarks. These decay processes are studied using a light quark-antiquark pair creation term, which is the quark-antiquark creation term of the free Dirac Hamiltonian in terms of the constituent quark fields. The matrix element of this term between the initial and final states of the decay process is calculated to compute the decay widths of the processes \(K^* \rightarrow K\pi\) and \(\phi \rightarrow K\bar{K}\). As has already been mentioned, \(\lambda_1\) and \(\lambda_2\) in the state \(K\) meson \((q\bar{s} \) bound state, \(q = (u, d)\)) given by equation [3], are the fractions of the mass (energy) of the \(K\) meson at rest (in motion) carried by the constituent strange antiquark and the constituent light quark \((u, d)\), and \(\lambda_1 + \lambda_2 = 1\). These are calculated by assuming the binding energy of the hadron as shared by the quark (antiquark) to be inversely proportional to the quark (antiquark) mass [12]. With the vacuum mass (in MeV) of \(K^+ (K^0)\) meson to be given as 493.677 (497.61), \(M_{u,d} = 330\) MeV [13], and \(M_s = 480\) MeV, the value of \(\lambda_2\) is obtained as 0.71.

The harmonic oscillator strengths of the pseudoscalar mesons \((K, \pi)\) and vector mesons \((\phi\) and \(K^*)\) are needed to calculate the decay widths of \(K^* \rightarrow K\pi\) and \(\phi \rightarrow K\bar{K}\). The value of \(R_\pi = (211\) MeV\)^{-1} [12, 13] was fitted from the value of the charge radius squared of pion given as 0.4fm\(^2\). We determine the harmonic oscillator strength parameter for the \(K\) meson, \(R_K\) assuming the ratio \(R_K/R_\pi\) to be same as the ratio of their charge radii, \((r_{ch})_K/(r_{ch})_\pi\). Taking \((r_{ch})_K = 0.56\) fm [22], the value of \(R_K\) is obtained as \((238.3\) MeV\)^{-1}. The value of \(R_\phi\) is obtained from the observed decay width of \(\phi \rightarrow e^+e^-\) of 1.26377 keV. The expression of the decay width
is given as
\[
\Gamma(\phi \to e^+ e^-) = \frac{16\pi \alpha^2}{9m_\phi^2} |\psi(r = 0)|^2, \tag{18}
\]
with \( \alpha = 1/137 \), yields the value if \( R_\phi \) to be \((290.7 \text{ MeV})^{-1} \). The value of \( R_{K^*} \) is assumed to be same as \( R_K \) in the present work. The values of \( \gamma_{K^*} \) for the decays \( K^{*+} \to (K\pi)^+ \) and \( K^{*0} \to (K\pi)^0 \), as fitted to their observed vacuum decay widths of 50.75 and 47.18 MeV \[22\], are obtained as 2.4742 and 2.347 respectively. These yield the values of the decay widths for the subchannels of the \( K^{*+} \) meson to \( K^+\pi^0 \) and \( K^0\pi^+ \) to be 16.98 and 33.77 MeV and of decay widths of \( K^{*0} \) to \( K^0\pi^0 \) and \( K^+\pi^- \) to be 15.87 and 31.31 MeV respectively. The effects of the magnetic field on the decay widths of the charged and neutral \( K^* \to K\pi \) are plotted in figure 2.

The decay widths for \( K^{*+} \) to \( K^0\pi^+ \) as well as to \( K^+\pi^0 \) are observed to increase monotonically with the magnetic field, when the mass modifications are taken into account only due to mixing of the \( K \) and \( K^* \) mesons (indicated as PV in figure 2), and these modifications are seen to be moderate. The vacuum values of these decay widths of 33.77 and 16.98 are observed to be modified to 40.63 and 20.34 at \( eB = 10m_\pi^2 \) for \( K^{*+} \to K^0\pi^+ \) and \( K^{*+} \to K^+\pi^0 \) respectively. When the Landau level contributions are also taken into account for the charged mesons, \( K^{*+} \) and \( \pi^+ \), there is observed to be a drastic drop in the decay width of \( K^{*+} \to K^0\pi^+ \), mainly due to the drop (rise) in mass the charged vector (pseudoscalar) meson from Landau level contributions. The value of the decay width vanishes at around \( eB = 5m_\pi^2 \) and remains zero upto around \( eB = 7m_\pi^2 \). As the magnetic field is further increased, the dominant increase in mass of \( K^{*+} \) due to mixing effects, lead to an increase in the decay width of \( K^{*+} \to K^0\pi^+ \).

The decay widths of \( K^{*+} \to K^+\pi^0 \) is observed to have a similar behaviour when the Landau level contributions as well as mixing effects are taken into account.

The masses of the \( \phi \) and \( \eta' \) due to their mixing on the presence of a magnetic field are shown in panel (a) in figure 3. These is observed to be an increase (drop) in \( \phi(\eta') \) meson mass due to this mixing. There are no Landau level contributions to their masses as these are electrically neutral. The values of \( \gamma_\phi^2 \) for the decay of \( \phi \to K^+K^- \) and \( \phi \to K^0\bar{K}^0 \) are obtained as 2.38 and 2.41 are obtained, as fitted from their observed vacuum decay widths of 2.0905 and 1.53 MeV \[22\]. The decay width of \( \phi \) to \( K^+K^- \) as well as to the neutral \( K^0\bar{K}^0 \) are shown in panel (b) in figure 3. The effects of the magnetic field on the decay width to the neutral \( K\bar{K} \) pair arises from the mass modifications of the \( \phi \) meson as well as \( K^0 \) (and \( \bar{K}^0 \)) due to the \( \phi - \eta', \ K^{*0} - K^0 \)
(and $K^* - K^0$), which leads to a rise of $\phi$ mass and drop in the pseudoscalar $K^0$ and $\bar{K}^0$ meson masses. This is observed as a monotonic increase in the decay width of $\phi \to K^0\bar{K}^0$ with increase in the magnetic field. The decay width of $\phi \to K^+K^-$ also shows a similar trend when the Landau contributions to the masses of these charged kaons and antikaons are neglected. The Landau contributions lead to increase in the masses of the charged $K\bar{K}$, due to which it is observed that the decay width vanishes at around $eB = m^2_{\pi}$. However, as the magnetic field is increased further, at around $eB = 8.1m^2_{\pi}$, the decay to $K^+K^-$ again becomes kinematically possible, as the mass of the charged $K(\bar{K})$ increases at high magnetic field due to the mixing effects becoming more important (as can be seen from figure for the $K^{*+} - K^+$ mixing in the presence of a magnetic field). The effects of the magnetic field on the netral kaon as well as $\phi$ meson in hadronic matter was studied using a chiral SU(3) model, which showed a critical field above which the decay of $\phi \to K^+K^-$ vanishes, due to the positive Landau level contributions to the masses of the charged kaons and antikaons.

IV. SUMMARY

In the present work, we have studied the modifications of the masses of the vector mesons ($\phi$ and $K^*$) and the pseudoscalar mesons ($\eta'$, $K$) in the presence of strong magnetic fields, due to the mixing of the $\phi - \eta'$, $K^* - K$ mixings. The modifications in masses of the charged mesons in their ground states are in addition to the contributions from the lowest Landau level. The effects of the mass modifications due to the magnetic field on the decay widths of $K^* \to K\pi$ and $\phi \to K\bar{K}$ are investigated using a field theoretic model of composite hadrons. For the neutral open strange mesons, $K^0$ and $K^{*0}$, as well as charged mesons $K^+$ and $K^{*+}$, there is observed to be marginal modifications of the masses leading to a drop (increase) in the mass of the pseudoscalar (longitudinal component of the vector) meson. The contributions of the Landau levels for the charged mesons are observed to lead to a rise (drop) in the mass of $K^+(K^{*+})$ meson, which gives rise to a smaller mass difference of the masses of $K^+$ and $K^{*+}$ with increase of the magnetic field. As the magnetic field is further increased, the mixing effects become more important, leading to a dominant increase (drop) in the $K^{*+}(K^+)$ mass. This leads to the effect of the magnetic field on the decay width of the charged $K^*$ meson to $K\pi$ ($K^{*+} \to K^0\pi^+$ as well as $K^{*+} \to K^+\pi^0$) to have an initial drop followed by moderate...
change and then a sharp rise when $eB$ is further increased. The decay width $K^{*+} \rightarrow K^0\pi^+$ is observed to be zero for values of $eB/m^2_\pi$ between 5 and 7, due to the increase in the mass of the charged pion in the final state. The decay width of $K^{0*} \rightarrow K^+\pi^-$ is observed to have a sharp drop with increase in the magnetic field and becomes zero at around $eB = 4.2m^2_\pi$. This is due to the positive contributions from the Landau levels to the masses of both the charged mesons in the final state.

For the decay width of $\phi \rightarrow K\bar{K}$, the Landau contributions are observed to lead to vanishing of the decay to the charged $K^+K^-$ pair above the value of $m^2_\pi$ for $eB$. The value of the decay width for the charged $K\bar{K}$ remains zero up to around $8m^2_\pi$. On the other hand, the decay width of $\phi \rightarrow K^0\bar{K}^0$ is observed to increase monotonically with the increase in the magnetic field. The contrasting behaviour in the decay channels of $\phi$ to the charged and the neutral $K\bar{K}$ should lead to suppression of the charged $K^{\pm}$ mesons as compared to the $K^0$ and $\bar{K}^0$ mesons.

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