Automatic control systems: optimization and sensitivity study

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Abstract. The paper discusses the optimization of a pulsed automatic control system (ACS) with constant parameters, which is used to control non-stationary objects. We obtained the dependences of the integral quality criterion on perturbations acting on the control object. The sensitivity of the ACS was studied by simulation using MATLAB/Simulink. The system was optimized based on the criterion constructed taking into account sensitivity to parameter variations. It is shown that the synthesis of the ACS insensitive to changes in characteristics is advisable only if the deviations of the parameters are small. In this case, the quality of automatic control will improve.

1. Introduction

One of the main engineering problems of creating automatic control systems (ACS) of technological processes is the problem of optimizing their quality indicators. In recent years, it is customary at the initial stage of ACS design to use simulation models of control objects for calculating the optimal characteristics of controllers (taken as initial parameters) [1-6]. However, the parameters of the mathematical model and the real object are always different, and these differences can be significant. In addition, during normal operation of systems, changes in the behavior of control objects often occur due to a shift in the operating point, for example, when the load changes [1, 2, 7-9]. Therefore, when creating an ACS, we need to proceed from the fact that, firstly, the adopted model does not accurately describe the behavior of the object, and secondly, that the characteristics of the control object change during its operation. The latter circumstance is usually known from operating experience and is decisive in the design and configuration of controllers [1-3]. In practice, for the control of technological processes, systems are widely used that have constant parameters (it is not possible to reconfigure them in real time) [8-11], and not adaptive controllers. In the event that it is known that the characteristics of the control object change during operation, and we intend to use a system with unchanged parameters for control, it is necessary to study the possibility of optimizing the ACS.

When creating a system implemented on controllers with constant parameters, designed to control a non-stationary object, it is advisable for us to use the apparatus of the theory of sensitivity. This will allow us to take into account the sensitivity of the system to variations of the parameters on which the controlled process depends [7, 8]. In order to reduce the influence of external disturbances on the quality of functioning of the control system, we study the possibility of optimizing the system parameters taking into account the sensitivity of the quality criterion.

2. The problem statement

Let us consider how the changes affect the control characteristics of the object to change the performance control system functioning. By a change in characteristics, we mean both the mismatch...
of the model with the object and changes in the parameters of the object with respect to the nominal value during the operation of the system.

In this paper we investigate the dependence of ACS performance indicators for small variations in process parameters from nominal values using controllers with constant parameters. We evaluate the sensitivity of the system to variations in the parameters of the object and based on this characteristic we calculate the parameters of the controllers. In this way, the configured controllers will improve the quality of control due to their low sensitivity to changes in object parameters.

Studies have been performed for a pulse control system for a liquid level that implements a widely used in practice automatic control method [2-5, 10, 11]. This method consists in the formation by the controller of a proportional-integral (PI) control law in conjunction with an actuating mechanism (AM), which has a constant speed. The controller is a pulse proportional differential (PD) converter, and the duration of its output pulses depends on the control error.

The control is carried out according to the so-called single-pulse circuit (we use a pulse by level) without inputting a feedback signal on the material balance (there are no pulses on the flow rates of the inflow and outflow from the tank), see Figure 1.

![Figure 1. Fluid level control system diagram.](image)

The controller changes the flow rate at the outflow from the tank by acting on the drain control valve (CV): when the level rises, the valve opens. The change in the inflow into the tank is not controlled and is considered as a disturbance to the control object.

To ensure stability, a rigid feedback is introduced into this system on the control valve position and thereby the P-law of control is implemented. Such control schemes are used when we do not have the ability to control disturbing influences. These systems are characterized by a static error (control unevenness) with respect to the disturbance in the steady state.

The main parameter for setting this pulse P-controller is the coefficient \( k_g \) of the feedback signal from the CV position (this parameter is inversely proportional to the gain of the control circuit). The remaining parameters of the controller (gain \( k_p \) and time constant \( T_i \)) have little effect on the characteristics of the process and are taken fixed from the allowable ranges of their values.

It is necessary to determine the value of the feedback coefficient \( k_g \) in such a way as to optimize the system in nominal mode and reduce the influence of disturbances.

We consider two types of disturbances: a change in the inflow into the tank \( G_f \) (1) and a gain \( k_{ob} \) for the control channel “water flow at the drain \( G_w \) – change in water level \( H_t \)” (2). We also evaluate the effect of these disturbances on the quality of control with respect to the output variable \( H_t \).

3. Theory

To optimize the control system, we choose a quality indicator that allows us to evaluate together the speed of the process damping and the deviation of the output variable from the set point. As such an indicator, we take a criterion that is an integral of the weighted square of the control error [9, 11]:

\[
I_k = \int_0^T |e(t)|^2 dt.
\] (1)
Here $t$ is the current time; $\varepsilon(t)$ is the control error, that is, the deviation of the control variable from the set value; $T$ is the upper limit of integration, which is recommended to choose at least the time of completion of the transition process. When calculating this quality assessment, we multiply the squared control error by the current time. This is done to reduce the contribution (to the generated indicator) of a large initial error and to take into account the error that appears during further operation of the system.

We need to synthesize a control system that would not be sensitive to the changing parameters of the object, for example, to such disturbing influences as changes in the influx of water into the tank $\Delta G_f$, and variations in the transmission coefficient of the object $\Delta k_{ob}$.

For the control system, as a measure of sensitivity to changes in the parameter $k_g$ (relative to its nominal value), we will consider the parametric sensitivity of the quality criterion $I_k$ determined in accordance with expression (1).

We need to get a formula for calculating a new criterion (based on expression (1)), which we should optimize with respect to unknown parameters of the controller.

To implement an insensitive system to changes in the perturbation $\Delta G_f$, we need to find the derivatives: $\frac{\partial I_k}{\partial k_g}$ for $I_k = f(k_g)$ and $\frac{\partial k_g}{\partial G_f}$ for $k_g = f(G_f)$, and optimize a criterion of the form

$$I = I_k + \frac{\partial I_k}{\partial k_g} \frac{\partial k_g}{\partial G_f} = I_k + \frac{\partial I_k}{\partial G_f}.$$  \hspace{1cm} (2)

In order to calculate the derivative $\frac{\partial I_k}{\partial G_f}$, we obtained the dependences of the integral estimate $I_k$ on the coefficient $k_g$ (feedback gain coefficient for the CV position) and the coefficient $k_g$ on the perturbation (flow rate at the supply) $G_f$.

All necessary calculations are performed by MATLAB / Simulink software, suitable for performing engineering and scientific calculations, as well as simulation of dynamic systems [6, 12]. For the level control loop in the tank, the system simulation model is created, shown in Figure 2. This model was created in accordance with the system diagram shown in Figure 1.

![Figure 2. Control system simulation model in MATLAB / Simulink.](image)

In the system model, we presented a three-position relay, which is part of the pulse controller, with two on-position relays (for input signals of different polarity); also took into account local feedback.
The control valve equipped with AM is represented by an integrating unit with a restriction (to account for the range of movement of the CV from 0 to 100%). The control object is represented by an integrating unit, since the water level in the tank is an integral of the material balance between inflow and outflow. An error signal at the input to the controller is generated by a weighted summation of the setpoint signals, the measured level values, as well as a signal about the position of the control valve.

The necessary information about the parameters of the control object is obtained on the basis of data from a study of the dynamics of level control in the drainage tank of a power unit, which is a typical control object for thermal automation [2, 4, and 10].

4. Simulation studies
As a result of the simulation studies, we obtained the dependences of the quality indicator on the ACS parameters when the inflow into the tank changes in the range of (3–12) kg/%/s (this corresponds to a balanced drain when the valve is opened by (15–63) %, i.e. when it moves in the operating range). In order to calculate the derivative ∂Ik/∂Gf used in expression (2), we obtained the dependences of the integral estimate Ik on the coefficient kg (gain of feedback on the valve position) and the coefficient kg on perturbation (flow rate) Gf. The range of values of the coefficient kg was chosen taking into account the preservation of sufficient stability margins.

For the graphs constructed according to the results of system simulation, we performed the approximation of the experimental data by the corresponding polynomials to obtain analytical dependences (Figure 3). Analytical expressions for the dependences Ik=f(kg) and kg=f(Gf) are necessary for their subsequent differentiation in a symbolic form using MATLAB.

![Figure 3](image-url)

**Figure 3.** Dependencies: the criterion Ik on the coefficient kg (a); kg from disturbance Gf (b).

Figure 3a shows the dependence Ik on the coefficient kg when the inflow into the tank is Gf=6 kg/%, which corresponds to 32% of the valve opening. As can be seen, the optimal calculated value kg=5 mm/%. Figure 3b shows the dependence of the coefficient kg on the change in the inflow into the tank Gf (disturbance) in the considered range of the inflow variation.

The value of the coefficient kg is recommended to be selected in the range from 3 to 7 mm/%. This does not contradict the requirement to ensure the stability of the system: we recommend choosing kg>1.7 mm/% in order to maintain sufficient stability margins.
The MATLAB script performs a symbolic calculation of the derivatives of the obtained analytical expressions to take into account the sensitivity of the quality criterion to changes in the disturbance and the controller settings. Based on these calculations, we constructed graphs of criteria of the form (1) and (2), presented in Figure 4.

![Graphs showing the dependence of the criterion on the coefficient k_g in the nominal mode (G_f = 6 kg/%/s) and from the perturbation G_f.](image)

**Figure 4.** Dependence of the criterion: on the coefficient $k_g$ in the nominal mode ($G_f = 6 \text{ kg/}%/\text{s}$) (a); from the perturbation $G_f$ (b).

Figure 4a shows that the minimum $I_k$ corresponds to $I_1$ for $k_{g1}$ and the minimum $I=I_k+\partial I_k/\partial G_f$ corresponds to $I_2$ for $k_{g2}$. Less sensitive controls are characterized by worse quality: $I_2>I_1$. Figure 4b shows the obtained (as a result of the system simulation) dependences of the control quality criterion $I_k$ on the perturbation $G_f$ for feedback coefficients $k_g = k_{g1}$ and $k_g = k_{g2}$.

It can be seen from the graphs that the choice of the $k_g$ coefficient without taking into account the sensitivity provides the best control quality in the middle of the considered range of disturbances. The selection of the $k_g$ coefficient, taking into account the sensitivity, provides the best quality control at the beginning of the range.

Let us consider the second type of disturbances – a change in the transfer coefficient $k_{ob}$ for the control channel “water flow at the drain $G_w$ – change in water level $H_f$”. We also examine the effect of this disturbance on the quality of process control. We perform the same procedures (described above), on the basis of which we determined the parameter $k_g$ when we investigated the sensitivity of the system to the perturbation $\Delta G_f$. The research result is presented in Figure 5.

For the graph in Figure 5a, which we constructed according to the results of system simulation, an approximation was performed by the corresponding polynomial to obtain an analytical expression. The analytical dependence $k_g=f(k_{ob})$ was used by us further, for subsequent differentiation using MATLAB. Based on these calculations, we plotted for criterion $I_k$ (in accordance with expression (1)) and for a criterion that takes into account the sensitivity of the system $I=I_k+\partial I_k/\partial k_{ob} = I_k + 5 \frac{\partial I_k}{\partial k_{ob}}$ (similar to expression (2)). These graphs are presented in Figure 5b.

It can be seen from Figure 5b that the minimum $I_k$ corresponds to $I_1$ for $k_{g1}$, and the minimum $I=I_k+5\partial I_k/\partial k_{ob}$ corresponds to $I_2$ for $k_{g2}$. Moreover, the obtained values differ very little. Yes, less sensitive control is characterized by worse quality: $I_2>I_1$, but this difference is not significant. The
choice of the coefficient $k_g$ taking into account the sensitivity provides a not significant improvement in the quality of control for small perturbations $\Delta k_{ob}$ in the direction of increasing $k_{ob}$.

Figure 5. Dependencies: coefficient $k_g$ from perturbation with a change in $k_{ob}$ (a); the quality criterion from the coefficient $k_g$ in the nominal mode ($k_{ob} = 5.4 \text{ mm/(kg/s)}$) (b).

An increase in $k_{ob}$ (due to the fact that the control object has integrating properties) corresponds to a decrease in the time constant of the object, and as a result, a decrease in the number of controller operations to ensure the required control accuracy.

Figure 6 shows the graphs of transients in the control system at the obtained optimal value of the feedback parameter $k_g = 5\text{ mm/}%$ (perturbation by the task of the level). The nominal values of the inflow into the tank ($G_I = 6 \text{ kg/}%$/s) and the gain of the object ($k_{ob} = 5.4 \text{ mm/(kg/s)}$) were established. Taking into account the sensitivity to disturbance by the inflow will reduce the static error, and taking into account the sensitivity to variations of the object parameter $\Delta k_{ob}$ will reduce the number of controller operations. There is no static error on the graphs; the number of pulses from the controller (AM responses) is small.
5. The discussion of the results
We have received that automatic control of non-stationary objects with small deviations of parameters, on which the controlled process depends, can be implemented using controllers with constant parameters. Given the sensitivity of the optimality criterion to parameter variations, we can reduce the influence of external disturbances on the quality of the system.

For the considered pulse control system, the selection of the feedback coefficient by the position of the control valve, taking into account the sensitivity of the system, should be carried out if it is known that the magnitude of the most significant disturbance - the inflow into the tank, decreases. It should be noted that taking into account the sensitivity to disturbance by the inflow will reduce the static error (while maintaining the required water level), and taking into account the sensitivity to variations in the gain of the object will reduce the number of controller operations. But all this is true only for small changes in the parameters of the control object.

6. Conclusion
We investigated how changes in the characteristics of the control object affect the change in the quality control criterion. Such characteristics may include model parameters that differ from the actual parameters of the object, as well as variations in the parameters of the object with respect to the nominal value during the operation of the system. We analyzed the dependence of the performance indicators on the deviations of the object parameters, and also evaluated the possibility of improving the system.

It should be noted that taking into account small variations in the parameters of the object carried out by methods of the theory of sensitivity (when we use controllers with constant parameters) can improve the quality of control. But to ensure optimal operation in the entire range of possible disturbances, it is necessary to provide for the adjustment of parameters during operation, that is, the creation of an adaptive system.

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