The paper is motivated by observation of a kind of branes in the vacuum state of the lattice SU(2) gluodynamics. The branes represent two-dimensional vortices whose total area scales in physical units while the non-Abelian action diverges in the ultraviolet. We consider the question whether effects of the branes can be accommodated into the continuum theory. We demonstrate that at least in case of the gluon condensate (plaquette action) and of the heavy quark potential the contribution of the branes corresponds to the ultraviolet renormalon. Thus, the vortices might represent a non-perturbative match of the ultraviolet renormalon. Such an identification constrains, in turn, properties of the branes.

1. Introduction

Recently it has been discovered that monopoles and central vortices–non-perturbative fluctuations commonly considered responsible for the confinement– have non-trivial structure in the ultraviolet. Namely both the monopoles, see $^1$ and references therein, and vortices $^2$ are associated with an excess of the non-Abelian action which is divergent in the ultraviolet:

$$\langle S_{\text{mon}} \rangle \sim \ln 7 \cdot \frac{L}{a}, \quad \langle S_{\text{vort}} \rangle \approx 0.54 \cdot \frac{A}{a^2},$$

where $L$ is the length of the monopole trajectory, $A$ is the area of the vortex, $a$ is the lattice spacing representing the ultraviolet cut off. In case of monopoles, the overall constant, which we quote as $\ln 7$ is known actually with rather poor accuracy but this is not crucial for our purposes.

Naively, one would expect that monopoles and vortices with action (1) propagate only very short distances, $L \sim a$, $A \sim a^2$. However, both monopoles and vortices form clusters which percolate through the whole
of the lattice volume $V_4$. Defining the corresponding densities as:

$$L_{\text{perc}} \equiv 4\rho_{\text{perc}}V_4 \quad \text{and} \quad A_{\text{vort}} \equiv 6\rho_{\text{vort}}V_4 \quad (2)$$

one finds that both the monopole density (see, e.g., \cite{3} and references therein) and the vortex density (for review and references see \cite{4}) scale in physical units and are independent on the lattice spacing. According to the latest measurements:

$$\rho_{\text{perc}} = 7.70(8) \text{ fm}^{-3} \quad \text{and} \quad A_{\text{vort}} \approx 4.0(2) \text{ fm}^{-2} \quad (3)$$

see \cite{5} and \cite{2}, respectively.

The data (1), (3) imply that the standard picture of vacuum fluctuations is to be adjusted to incorporate fine tuning, that is coexistence of the ultraviolet and infrared scales within the same fluctuations \cite{6,7}. A comprehensive theory of the monopoles and vortices is not yet in sight mainly because the monopoles and vortices are defined not in terms of the original Yang-Mills fields but rather in terms of projected fields, for review see, e.g., \cite{4}.

While lattice data on the monopoles and vortices have been accumulating since long, the discovery of the ultraviolet divergences (1) makes the challenge to the theory much more direct. Indeed, it is commonly believed that in an asymptotically free theory all the ultraviolet divergences can be understood from first principles. And it is worth emphasizing at this point that the brane properties we are considering, that is (1), (3) are perfectly $SU(2)$ invariant.

We will argue that, indeed, starting from the continuum theory it is possible to derive strong constraints on the ultraviolet properties of non-perturbative fluctuations \footnote{we briefly mentioned the constraints in the talks \cite{8}}. Moreover, the data can be confronted with the constraints without knowledge of the anatomy of the branes in terms of the original fields.

The idea of the derivation is as follows. The continuum theory is well defined at short distances, while the branes are extended objects. Thus, we will project the effect of the branes onto matrix elements calculable in the continuum theory. In particular, we will concentrate on the gluon condensate and heavy quark potential at short distances.

Because of the asymptotic freedom the parton-model predictions are a reliable zero-order approximation for observables of this kind. If one tries to derive a complete answer, then the outcome is an infinite perturbative
series plus power-like corrections. The power corrections are the price for the factorial growth of the coefficients of perturbative expansions \(^9\), for review see, e.g., \(^10\).

We will argue that the branes with the newly discovered properties do fit this scheme providing a match to the so called ultraviolet renormalon. It has never been foreseen, though, that a non-perturbative completion of the ultraviolet renormalon could be a brane with an action density divergent in the ultraviolet. Still, the new addition seems to fit well the phenomenology of the power corrections. In turn, identification of the branes with a non-perturbative counterpart of the ultraviolet renormalon puts constraints on properties of the branes.

Although our main interest is the effects of the branes, we begin in Sect. 2 with a rather detailed discussion of the \(Q^{-2}\), or ultraviolet-renormalon related power corrections. In particular, we argue that there are two complementary ways to account for these corrections. Either one confines oneself to low-order perturbation theory and adds the quadratic corrections ad hoc, or one is prepared to consider high orders in perturbation theory (sometimes, say, up to ten loops) and then the \(Q^{-2}\) corrections are implicitly contained in the perturbative sum. While we do not suggest any new explanation of a particular effect, our overall conclusions vary from what can be found in the literature. The point is that commonly it is assumed, explicitly or tacitly, that the two procedures outlined above are mutually inconsistent.

In Sect. 3 we review phenomenological evidence on the \(Q^{-2}\) corrections and argue that the data are consistent with the theoretical conclusions.

In Sect. 4 we argue that the branes provide a non-perturbative match to the ultraviolet renormalon. In view of the discussion above this means, in turn, that the branes—when projected onto local matrix elements—are dual to high orders of perturbation theory. In Sect. 5 we discuss also the corresponding constraints on properties of the vortices.

2. Perturbative expansions

2.1. Divergences of perturbative expansions

Standard analysis of divergences of perturbative expansions \(^b\) introduces hierarchy of three scales,

\(^b\)This subsection is a very sketchy review. For details, see, e.g., \(^10\).
\[ \Lambda_{QCD}^2 \ll Q^2 \ll \Lambda_{UV}^2 , \]

where \( \Lambda_{QCD} \) and \( \Lambda_{UV} \) are the standard infrared and ultraviolet scales while the intermediate scale \( Q \) is specific for a particular process considered. Then a generic perturbative expansion for an observable \( \langle O \rangle \) looks as:

\[
\langle O \rangle = (\text{parton model}) \cdot \left( 1 + \sum_{n=1}^{\infty} a_n \alpha_s^n(Q^2) \right),
\]

where for simplicity of presentation we normalized the anomalous dimension of the operator \( O \) to zero. Note also that \( \alpha_s(Q^2) \ll 1 \).

In fact, expansions (4) are only formal since the coefficients \( a_n \) grow factorially at large \( n \):

\[
|a_n| \sim c_i^n \cdot n! ,
\]

where \( c_i \) are constants. Moreover, there are a few sources of the growth (5) and, respectively, \( c_i \) can take on various values. First, the growth of the coefficients \( a_n \) may reflect the growth of the number of perturbative graphs in high orders, which is combinatorial in nature. The value of \( c_i \) is determined then by the classical action of an instanton solution. On the other hand, the divergence (5) can be triggered by a single graph of \( n \)-th order. This is a renormalon-type graph which contains \( n \) insertions of vacuum polarization into a gauge boson propagator.

The factorial growth of \( a_n \) implies that the expansion (4) is asymptotic at best. Which means, in turn, that (4) cannot approximate a physical quantity to accuracy better than

\[
\Delta \sim \exp \left( -1/c_i \alpha_s(Q^2) \right) \sim \left( \frac{\Lambda_{QCD}^2}{Q^2} \right)^{b_0/c_i} ,
\]

where \( b_0 \) is the first coefficient in the \( \beta \)-function,

\[
\frac{d}{dQ^2} \alpha_s(Q^2) = -b_0 \alpha_s^2(Q^2) - b_1 \alpha_s^3(Q^2) - ... ,
\]

and we accounted for the fact that \( \alpha_s(Q^2) \) is logarithmically small at large \( Q^2 \).

To compensate for these intrinsic uncertainties one modifies the original expansion (4) by adding power corrections with unknown coefficients:

\[
\langle O \rangle = (\text{parton model}) \cdot \left( 1 + \sum_{n=1}^{\infty} \tilde{a}_n \alpha_s^n(Q^2) + \sum_k b_k (\Lambda_{QCD}/Q)^k \right),
\]
where powers $k$ are determined in terms of $c_k$ entering Eq. (5) and the factorial divergences are removed from the modified perturbative coefficients $\tilde{a}_n$, for further discussion see, e.g., \textsuperscript{11}.

\subsection*{2.2. Borel summation}

So far, we discussed only behavior of absolute values of the coefficients $a_n$. Now, if there is sign oscillation,

$$a_n \sim (-1)^n n! c^n_i ,$$

then the sum is Borel summable while if $a_n \sim (+1)^n n! c^n_i$ there is no way at all to define the sum. In case of an asymptotically free theory the sign oscillation (8) is characteristic for the ultraviolet renormalons. For the first ultraviolet renormalon,

$$(a_n)_{uv} \sim n! b_0^n (-1)^n ,$$

where $b_0$ is the first coefficient in the $\beta$-function. Usually one does not reserve for any uncertainty in case of a Borel summable series.

The criterion of Borel summability might look too formal. Let us mention, therefore, that there exist also intuitive reasons to believe that there is no intrinsic uncertainty of perturbative series due to the ultraviolet renormalons. Indeed, one can readily check that the physical meaning of the power terms is that they correspond to contributions of non-typical virtual momenta $k^2_{\text{virt}}$ which are either small or large compared to $Q^2$. The ultraviolet renormalons are associated with

$$k^2_{\text{virt}} \gg Q^2,$$

while infrared renormalons correspond to

$$k^2_{\text{virt}} \ll Q^2.$$

Furthermore, it is rather obvious that the contribution of large momenta in asymptotically free theories is calculable exactly. Then, there could be no intrinsic uncertainty due to the ultraviolet renormalon.

On a more technical level, the argument runs as follows \textsuperscript{12}. If one changes normalization of the coupling used in the expansion (4) from $g^2(Q^2)$ to $g^2(\mu^2)$ then the uncertainty (6) associated with the ultraviolet renormalon changes to:

$$\Delta(\mu^2) \sim \frac{\Lambda^2_{QCD}}{\mu^2} \cdot \frac{Q^2}{\mu^2}.$$  

(10)
Thus using a normalization point $\mu^2 \gg Q^2$ one can get rid of the corresponding power-like uncertainty. There is no similar trick in case of the infrared renormalons, signaling that the corresponding power terms is a genuine uncertainty of the perturbative expansion.

### 2.3. Non-perturbative match to infrared renormalon

In phenomenological applications, we will concentrate mostly on two particular examples, that is, gluon condensate $\langle \alpha_s(G^a_{\mu\nu})^2 \rangle$, where $G^a_{\mu\nu}$ is the non-Abelian field-strength tensor, and the heavy quark potential $V(r)$ at short distances, $r \to 0$. Keeping only the leading power corrections we have

$$
\langle (G^a_{\mu\nu})^2 \rangle \approx (N_c^2 - 1) \cdot a^{-4} \left( 1 + \sum_n a_n \alpha_s^n(a^2) + c_4(\Lambda_{QCD} \cdot a)^4 \right), \quad (11)
$$

where $N_c$ is the number of colors and $a$ is the lattice spacing, and $\alpha_s$:

$$
\lim_{r \to 0} V(r) \approx -\left( \frac{N_c^2 - 1}{2N_c} \right) \frac{\alpha_s(r)}{r} \left( 1 + \sum_n a_n \alpha_s^n(r) + c_3\Lambda_{QCD}^3 r^3 \right). \quad (12)
$$

A salient feature of the predictions (11) and (12) is the absence of quadratic corrections of order $(\Lambda_{QCD} \cdot a)^2$, $(\Lambda_{QCD} \cdot r)^2$.

It is worth emphasizing that although the power corrections are so to say detected through pure perturbative graphs the actual vacuum fluctuations which dominate the power corrections in (11), (12) can well be genuinely non-perturbative. In particular, the leading power corrections both in (11) and (12) correspond to the so called infrared renormalon, a perturbative graph with many iterations of a loop insertion into the gluon propagator. However, the same dependence on $\Lambda_{QCD}$ is provided by instantons:

$$
\langle \alpha_s(G^a_{\mu\nu})^2 \rangle_{\text{inst}} \approx (\text{const})\Lambda_{QCD}^4. \quad (13)
$$

Moreover, there are well developed and, in many respect, successful models of vacuum which assume the instanton dominance, see \cite{17} and references therein.

It is important to realize that the instanton dominance in no way contradicts (11), (12). Indeed, analysis of the $n$-dependence of the coefficients $a_n$ fixes the position of a singularity in the Borel plane and predicts in this way that the leading power correction is proportional to $\Lambda_{QCD}^4$. This prediction is confirmed by evaluating the instanton contribution. Instantons might enhance the correction numerically and make the whole analysis more tractable.
2.4. Status of the ultraviolet renormalon

Turning back to the ultraviolet renormalon, – which is most relevant to the present paper, – one might feel that the summary presented above is in fact self-contradictory. Indeed, if we start with expansion in $\alpha_s(Q^2)$, see (4) then we would reserve for a $(\Lambda^{2}_{QCD}/Q^2)$ correction corresponding to the ultraviolet renormalon. Which would be the leading power correction. Even if we apply summation a la Borel, a $(\Lambda^{2}_{QCD}/Q^2)$ contribution would arise as a result of the summation of the divergent tail of the perturbative series. On the other hand, if we start with expanding in $\alpha_s(\mu^2)$, $\mu^2 \gg Q^2$, there is no intrinsic uncertainty to the perturbative series which would correspond to the ultraviolet renormalon.

The contradiction is superficial, however. Namely, if one uses expansion in $\alpha_s(\mu^2)$, then the $(\Lambda^{2}_{QCD}/Q^2)$ terms are to emerge as a result of explicit summation of high orders of perturbation theory. Indeed, in terms of the original expansion (4) the ultraviolet renormalon corresponds to terms of order

$$N_{uv} \approx \frac{1}{b_0\alpha_s(Q^2)} .$$

In terms of the $\alpha_s(\mu^2)$ expansion one needs to keep even higher orders.

It is worth emphasizing that an explicit calculation of the ultraviolet renormalon in gluodynamics is not straightforward at all. The point is that the ultraviolet-renormalon divergence (9) is in fact related not only to the simplest chain graph but to a whole class of graphs, and no explicit calculation is possible.

To summarize, one expects that the quadratic corrections are present but they are hidden either in the postulated procedure of summation a la Borel or in explicit summation of higher orders. It is worth emphasizing that numerical enhancement of the $(\Lambda^{2}_{QCD}/Q^2)$ terms is not ruled out by this analysis. However, such an enhancement is not suggested either.

3. Status of $Q^{-2}$ corrections

3.1. Power corrections from the infrared

There exists huge literature on power corrections, for review see, e.g.,. Here, we will mention only a few points relevant to our discussion.

The most standard sum rules apply to the two-point functions

$$\Pi_j(Q^2) = i \int \exp(iqx)\langle 0|T\{j(x), j(0)\}|0 \rangle , \quad (q^2 \equiv -Q^2) ,$$

(14)
where \( j(x) \) is a current constructed on the quark and gluon fields and for simplicity we omitted the Lorenz indices. Moreover, one usually considers the Borel transform of (14),

\[
\Pi_j(M^2) = [Q^{2n}/(n-1)!](-d/dQ^2)^n \Pi_j(Q^2),
\]

where \( n \to \infty, Q^2 \to \infty \) with \( Q^2/n \equiv M^2 \) fixed. Somewhat symbolically, the sum rules then read:

\[
\Pi_j(M^2) \approx \text{(parton model)} \left( 1 + \frac{a_j}{\ln(M^2/\Lambda_{QCD}^2)} + \frac{c_j}{M^4} + O(\ln(M^2/\Lambda_{QCD}^2)^{-2}, M^{-6}) \right),
\]

(15)

where the constants \( a_j, c_j \) depend on the channel, i.e. on the current \( j \). Terms of order \( 1/\ln M^2 \) and \( M^{-4} \) are the first perturbative correction and contribution of the gluon condensate, respectively. Note that we kept only first order correction in \( \alpha(M^2) \). We could have included, say, one more perturbative term, but the sum-rule approach is still based on the assumption that the effect of the power corrections is most important numerically.

The physical meaning of the power corrections in (15) is that they parameterize contribution of large distances of order \( \Lambda_{QCD}^{-1} \) in terms of local operators, like \((G_{\mu\nu})^2\). The technical means behind is the operator product expansion (OPE) and there the procedure is well defined and unambiguous.

While there is no doubt that (15) retains the leading power corrections if \( M^2 \) is large enough, it was noticed quite early \(^{21}\) that in some channels there could exist terms which start as sub-leading power corrections at large \( M^2 \) but numerically dominate at moderate \( M^2 \sim \text{few GeV}^2 \). Moreover, realistically such corrections are associated with so called direct instantons \(^{17}\), for a very recent and impressive example see \(^{22}\).

### 3.2. The case for large \( Q^{-2} \) corrections

While sum rules (15) claim many successes, there has also been accumulating evidence suggesting introduction of \( Q^{-2} \) corrections which apparently go beyond the standard picture summarized in the preceding subsection.

To begin with, the instanton-dominated model of vacuum does not reproduce the confining potential for heavy external quarks \(^{23}\). On the other hand, lattice studies indicate that the so called Cornell potential for heavy quarks,

\[
V_{Q\bar{Q}}(r) \approx -\frac{c_V}{r} + \sigma \cdot r,
\]

(16)
describes the data at all the distances \( r \), for a review see \(^{24}\). At large distances, the linear potential provides confinement. For our purposes here it is crucial, however, that the fit \(^{16}\) suggests also that if we start with short distances the leading power correction to the Coulomb-like interaction at ‘moderate’ distances \( r \) is of order \( \sigma \cdot r^2 \). This observation is one of the motivations to modify the r.h.s. of Eq. (12) by adding a \((\Lambda_{QCD} \cdot r)^2\) term (or \(\delta V(r) \sim \Lambda_{QCD}^2 r^2\)) \(^{16,25}\).

The gluon condensate, also, contains large quadratic corrections if the perturbative series is truncated at first terms. We will discuss this example in detail in the next subsection.

Another dramatic example of large quadratic corrections is found in Ref. \(^{26}\) and concerns instanton density as function of the instanton size \( \rho \). Namely, according to the standard operator product expansion, underlying also the sum rules (15), one has \(^{27}\):

\[
ln(\rho) \approx ln_{\text{pert}}(\rho) \left(1 + \frac{\pi^2}{12g^4}(0)\langle G_{\mu\nu}^a(0)^2\rangle \right),
\]

where \(g(\rho)\) is the gauge coupling and the salient feature of (17) is, again, absence of a quadratic correction. However, it is demonstrated in \(^{26}\) that the data on the instanton density, to the contrary, unequivocally require a quadratic term. The data on the instanton density are described by an effective action:

\[
S_{\text{eff}} \approx \frac{8\pi^2}{g^2(\rho^2)} + c_\rho \sigma \rho^2, \quad c_\rho \approx 2\pi,
\]

where the first term is the standard action in the quasiclassical approximation and \( \sigma \) is the string tension. The expression (18) describes the variation of the instanton density \( dn(\rho) \) by about four orders of magnitude.

There exist further examples when quadratic corrections improve phenomenological fits, see, e.g., \(^{28}\). However, the \(Q^{-2}\) corrections are associated with short distances and there is no substitution for the OPE which would allow to relate quadratic corrections to various quantities in a model-independent way. One of the ideas put forward in different contents, see, e.g., \(^{29}\) is a universal change of the running coupling by a \(Q^{-2}\) term. In particular, one argues sometimes \(^{29}\) that analyticity requires removal of the Landau pole from the running coupling. The use of the modified, or ‘analytical’ coupling,

\[
\alpha_s(Q^2)_{\text{anal}} \approx \frac{1}{b_0} \left( \frac{1}{\ln(Q^2/\Lambda_{QCD}^2)} + \frac{\Lambda_{QCD}^2}{\Lambda_{QCD}^2 - Q^2} \right),
\]
then introduces a $1/Q^2$ correction at large $Q^2$. An apparent weak point is that imposing analyticity on higher orders in $\alpha_s$ introduces further $Q^{-2}$ corrections which are not suppressed.

One can try to generalize (19) and think in terms of a universal coupling which includes a $Q^{-2}$ term. Let us note that the quadratic corrections in (16) and (18) differ numerically by a factor of 1.5 and agree in sign if the correction is ascribed to a universal coupling. This can be considered as a success of the model. Anyhow, till now there was no more convincing theoretical estimates of the quadratic term in (18).

Quantitatively, the model with a short-distance gluon mass turns to be most successful. According to the model, the sum rules (15) are modified as

$$\Pi_j(M^2) \approx (\text{parton model}) \left(1 + \frac{a_j}{\ln M^2/\Lambda_{QCD}^2} + \frac{b_j}{M^2} + \frac{c_j}{M^4} + \ldots\right),$$

where the coefficients $b_j$ are proportional to the gluon mass squared and are calculable for any channel. The model cleared all the hurdles known. For example, one finds:

$$b_\pi \approx 4b_\rho,$$

and this resolves a long standing puzzle of the analysis of the sum rules in the $\pi$-meson channel.

To summarize, it seems reasonable, – as far as phenomenology of the confinement-related effects is concerned, – to replace the standard free gluon propagator by a propagator which reproduces the whole of the potential (16) already in the zero-order approximation of perturbation theory. On the theoretical side, such a program is a refinement of the variational approach and is outlined in

Practically, full higher order calculations within such a scheme are known to be very cumbersome. As far as the power corrections are concerned, this approach amounts to introducing a tachyonic gluon mass at short distances already in the lowest order of perturbation theory. Which is easy to implement and successful phenomenologically.

### 3.3. Perturbative - non-perturbative ‘ambiguity’

Note that all the phenomenological successes mentioned in the previous section refer to the fits with only first order perturbative contribution retained, along with a $Q^{-2}$ term. The successes claimed refer to moderate $Q^2$ where the power correction becomes sizable. However, one could argue that if we start with large $Q^2$ then to extract a small power-like corrections
one needs to subtract as many orders of perturbation theory as possible. Then, the question is how this would affect the fits to the $Q^{-2}$ correction at small distances.

Numerically, this problem was studied in greatest detail in case of the gluon condensate on the lattice \textsuperscript{35,36}. In terms of the lattice formulation the gluon condensate is nothing else but the average plaquette action \textsuperscript{37}. The result of the calculations can be summarized in the following way. Represent the plaquette action $\langle P \rangle$ as:

$$\langle P \rangle \approx P^N_{\text{pert}} + b^N a^2 \Lambda_{\text{QCD}}^2 + c^N a^4 \Lambda_{\text{QCD}}^4,$$

where the average plaquette action $\langle P \rangle$ is measurable directly on the lattice and is known to high accuracy, $P^N_{\text{pert}}$ is the perturbative contribution calculated up to order $N$:

$$P^N_{\text{pert}} \equiv 1 - \sum_{n=1}^{n=N} p_n g^{2n},$$

and, finally coefficients $b^N, c^N$ are fitting parameters whose value depends on the number of loops $N$. Moreover, the form of the fitting function (22) is rather suggested by the data than imposed because of theoretical considerations.

The conclusion is that up to ten loops, $N = 10$ it is the quadratic correction which is seen on the plots while $c^N$ are consistent with zero. However, the value of $b^N$ decreases monotonically with growing $N$ \textsuperscript{35} and somewhere after 10 loops becomes consistent with zero while the $\Lambda_{\text{QCD}}^4$ term finally shows up \textsuperscript{36}. Moreover, the emerging value of the $\Lambda_{\text{QCD}}^4$ term is consistent with current phenomenological estimates from the sum rules \textsuperscript{36}.

Turn now to another example of a large $Q^{-2}$ correction, that is the heavy quark potential, see (16). The first attempt to analyze the effect of higher perturbative corrections on the linear term at short distances was undertaken in Ref. \textsuperscript{38}. Defining the potential as

$$V_{Q\bar{Q}}(q) \equiv -C_F \frac{4 \pi \alpha_V(q)}{q^2},$$

and using first terms in the perturbative expansion of $\alpha_V$ which are known explicitly,

$$\alpha_V(q) = \alpha_{\overline{MS}}(q)(1 + a_1 \alpha_{\overline{MS}}(q) + a_2 \alpha_{\overline{MS}}^2(q)),$$

one finds that the linear piece in the potential at short distances is 5 times larger than it would follow from (16). In other words, the result depends
crucially on the subtraction procedure. To proceed further, one is to invoke a model.

In particular, it was suggested to saturate higher orders by the leading infrared renormalon evaluated in the large-$b_0$ approximation:

$$\alpha_V(q) \approx \alpha_{\overline{MS}}(q) \left(1 + a_1\alpha_{\overline{MS}} + a_2\alpha_{\overline{MS}}^2 + \sum_{n=3}^{N} a_n^{\text{ren}} \alpha_{\overline{MS}}^n\right)$$

(26)

where $N \sim 3/2b_0\alpha_{\overline{MS}}$. The potential observed on the lattice is reproduced then by (26) including $\delta V(r) \sim \sigma r$ at short distances. Moreover, the result is stable against reasonable variations in $N$. Note that no explicit $Q^{-2}$ terms are to be introduced within this procedure since high orders of perturbation theory are presumably accounted for explicitly.

### 3.4. Complementary ways of describing the $Q^{-2}$ terms

Thus, the lesson from calculations of the perturbative gluon condensate is that one can either approximate total matrix elements by a low-order perturbative contribution plus a quadratic correction, or by high-order perturbative contributions plus a quartic correction.

Our central point, which motivated us to review the evidence in favor and against non-standard $Q^{-2}$ corrections, is that there is no contradiction between the two approaches. Moreover, existence of the dual descriptions of the $Q^{-2}$ terms is expected in fact on pure theoretical grounds, see discussion of the status of the ultraviolet renormalon in Sect. 2.4.

Theoretically, it is known that no ad hoc $Q^{-2}$ terms are allowed to be added to untruncated perturbative series. However, then one is to be prepared to calculate high orders indeed. In the only case when such a calculation turns possible (that is, the gluon condensate\cite{35,36}) theoretical expectations are fully confirmed. Namely, lowest orders plus a quadratic correction give reasonable fits\cite{35,36}. On the other hand, if many loops are accounted for, then there is no need for an ad hoc power correction any longer. What remains unanswered at this point is why the $Q^{-2}$ terms, – described in one or the other way, – are important phenomenologically. As we will argue later, observation of the branes seems to provide us with a key to answer this question.
4. Branes and power corrections

4.1. Quadratic correction to the gluon condensate

Monopoles and vortices are detected, for a given configuration of the vacuum fields, for the whole of the lattice. Thus, they are seen as a nonlocal structure. Moreover, they are manifestly non-perturbative. Indeed, the probability $\theta(\text{plaq})$ for a particular plaquette to belong to a brane has been found to be proportional to:

$$\theta(\text{plaq}) \approx (\text{const}) \exp(-1/b_0 g^2(a)) \sim (a \cdot \Lambda_{QCD})^2.$$  

On the other hand, the branes have an ultraviolet divergent tension which assumes a kind of locality.

To make contact with the continuum theory it is useful to evaluate contribution of the branes into local or quasi-local matrix elements. The gluon condensate (11) turns to be the easiest case. Indeed, combining Eqs (3) and (1) one gets for the contribution of the vortices to the gluon condensate:

$$\langle (G_{\mu\nu}^a)^2 \rangle_{\text{vort}} \approx 0.3 \text{ GeV}^2 a^{-2},$$

which matches the ultraviolet renormalon. The beauty of this result is that all the quantities considered are manifestly gauge invariant.

4.2. Quadratic correction to the heavy quark potential

In case of the heavy quark potential, one can also argue that the branes match the ultraviolet renormalon, or the $Q^{-2}$ correction. Indeed, it is well known that both monopoles and vortices generate the linear potential for heavy quarks. In particular, in case of the monopoles:

$$\langle V_{QQ}(r) \rangle_{\text{mon}} \approx \sigma_{\text{mon}} \cdot r,$$

where $\sigma$ is close numerically to the string tension in the full potential (16). Moreover, the potential (29) is linear at all distances tested, beginning with $r = a$. The reservation is that the observation (29) refers to the Abelian projected potential. However, the approximation is known to be valid for the quarks in the fundamental representation (for detailed discussion see, e.g., 3).

4.3. Probing properties of the branes

To produce a linear potential (29) at short distances, vacuum fluctuations are to satisfy highly non-trivial constraints. First, the size of fluctuations is
to be small\textsuperscript{16,41}. Indeed consider again for simplicity the Abelian projection. Then the potential can be represented as an integral over the overlap of electric fields associated with the external charges charges $\pm Q_{el}$:

$$V(r) = \frac{1}{4\pi} \int \mathbf{E}(\mathbf{r}') \cdot \mathbf{E}(\mathbf{r} + \mathbf{r}') d^3 r' ,$$

(30)

where $\mathbf{r}$ is the radius-vector connecting the charges. Imagine that there is a change in the field at distances of order $R$. In the applications, $R \sim (\Lambda_{QCD})^{-1}$. At distances $R \gg r$ the field of the charges is a dipole field:

$$|\mathbf{E}_{dip}| \sim Q r / R^3 .$$

Respectively, the change in the potential is of the order:

$$\delta V(r) \sim Q^2 r^2 \int_0^\infty d^3 r' / (r')^6 \sim \frac{Q^2 r^2}{R^3} ,$$

(31)

and we would conclude that the non-perturbative potential at small distances is proportional to $r^2$, see also Eq. (12).

Therefore, the standard estimate might not work only in case that fluctuations responsible for the confinement have size comparable or less than the distance between the quarks, $r$. Monopoles and vortices with the action (1) do satisfy this constraint since the ultraviolet divergence in the action implies size of order $a$.

To appreciate another constraint on the properties of the vortices, consider the $Z_2$ projection, (for a review see\textsuperscript{4}). Then the area law for the Wilson loop is derived from the assumption that the probabilities for plaquettes in the plane of the Wilson loop to belong to the vortex are uncorrelated\textsuperscript{50}.

Indeed, in this case the Wilson loop on average is given by:

$$\langle W(C) \rangle = \Pi_{\text{plaq}} \langle (1 - f) + f(-1) \rangle \langle W_0(C) \rangle ,$$

(32)

where the product is taken over all plaquettes belonging to the minimal area $A$ spanned on the contour $C$, $f$ is the probability for a plaquette to belong to the vortex, and $\langle W_0(C) \rangle$ is the value of the loop with the constraint that no vortices pierce the minimal area. Then,

$$\langle W(C) \rangle \approx \exp(-\sigma A / a^2) \langle W_0(C) \rangle ,$$

(33)

where the string tension $\sigma = -\ln(1 - 2f)$ and $A$ is the minimal area in the lattice units. Note that $f \sim (a\Lambda_{QCD})^2$, see Eq. (27).

Now, we are discussing the case when the contour $C$ has one of dimensions very small, $A \sim T \cdot r$ where $T$ is the time extension and $r \to 0$. Eq.
(32) applies only if the vortex remains random even at distances comparable to the lattice spacing $a$. Which implies, in turn, that the entropy factor for the vortex grows exponentially with its area:

$$ (\text{Entropy})_{\text{vort}} \sim \exp\left( cA/a^2 \right). $$

This is indeed true for the branes observed on the lattice, see 2.

So far, we have not discussed corrections to the instanton density, see (18) since no quantitative framework is known in this case. Let us only note that the physics behind the standard prediction (17) is similar to the case of the heavy quark potential. Indeed, in case of instantons we deal with color dipoles in $d=4$. Again, the OPE prediction (17) follows from the assumption that the confining fields are soft and modify the original instanton field at distances of order $(\Lambda_{\text{QCD}})^{-1}$. The validity of (18) implies, therefore, that there exist small-size vacuum fluctuations. There are no other (non-perturbative) candidates but the branes.

### 4.4. Branes and perturbative series

The original ultraviolet renormalon sequence of the perturbative coefficients is given by (9). The $n!$ factor arises then from integrals over virtual momenta of the type:

$$ \int_{\sim Q^2}^{\Lambda_{UV}^2} \frac{dk^2}{k^4} \left( \ln \frac{k^2}{Q^2} \right)^n. $$

Such a contribution is related to a single graph with $n$ insertions of the vacuum polarization into a gauge boson propagator. However, this is not a single source of contributions of order (9) $^{18}$. Namely, one can reserve, say, for one loop insertion less because then one gains in terms of number of graphs. In this way there arise contributions of order

$$ a_n \sim (n-1)! \cdot l \cdot l \sim n, $$

where the first factor is due the mechanism (35) and the factor $l \sim n$ is combinatorial. Thus, one gradually switches to evaluating both the number of graphs and powers of the logs. This two-parameter problem becomes practically intractable after a few steps.

It is worth emphasizing that in case of the power corrections to the gluon condensate there is no mechanism (35) at all. Indeed the expansion is in terms of $g^2(\Lambda_{\text{UV}}^2)$ and there are no virtual momenta larger than the normalization point.
Thus, the only mechanism left for having a large quadratic correction is a large number of graphs. Moreover, large power-like terms cannot be associated now with sign oscillation. Indeed, sign oscillation plus expansion in $g^2(A_{UV}^2)$ would result in a negligible contribution. Thus, we come to conclusion that the series associated with the quadratic correction should not oscillate in sign. Only then we can expect that the series corresponds to a sizable quadratic correction.

Let us check the expectations against the only known example of an explicit calculation of a ‘long’ perturbative series, that is, turn to the case of the gluon condensate in the lattice $SU(3)$ gluodynamics. According to \cite{36}:

$$\frac{r_n}{p_n} \approx u \left( 1 - \frac{1 + q}{n + s} \right),$$  \hspace{1cm} (36)

where the coefficients $p_n$ are introduced in (23) the numerical values of the parameters are:

$$u = 0.961(9), \quad q = 0.99(7), \quad s = 0.44(10).$$

The quadratic correction is affected by $n \approx 10$.

We see, indeed, that the perturbative series has no sign oscillation.

So far, two regular mechanisms for generating same-sign perturbative expansions were discussed \cite{10}. Namely, infrared renormalons and perturbative-vacuum instability due to instantons. In both cases the ratio $r_n$ grows at large $n$ linearly with $n$. In case of the leading infrared renormalon:

$$r_n^{ren} \approx \frac{nb_0}{8\pi}.$$  \hspace{1cm} (37)

Remarkably enough, in the crucial region of $n \sim 10$ this ratio is still smaller than (36) and the infrared renormalon catches up with (36) only around $n \sim 25$ \cite{36}. In case of instanton-related divergence the ratio $r_n$ is even smaller than in case (37):

$$r_n^{inst} \approx \frac{2nb_0}{44\pi}.$$  \hspace{1cm} (38)

From the theoretical point of view, both the instantons and infrared renormalons are irrelevant to the $Q^{-2}$ corrections. Thus, it is gratifying that the perturbative series (36) looks indeed different. The series (36) is convergent for $|g^2| < u$ and seems to exhibit a novel mechanism of generating same-sign perturbative series.
4.5. Crossover

The non-analyticity in $g^2$ indicated by (36) might be a reflection of the crossover transition $^{36}$ Indeed, the irregular behavior of the specific heat associated with the crossover is known since long $^{42}$. Note also that at the crossover the branes, with the properties known in the weak-coupling region, disappear. Indeed, in the strong-coupling region, that is beyond the crossover there is no scale $\Lambda_{QCD}$ at all and the basic properties of the branes, like (27) make no sense.

Thus, there arises the following tentative picture of the mechanism behind the $Q^{-2}$ corrections. Because of the crossover transition, there appear corrections to the standard running of the coupling at mass scale of about $GeV$. This change in the running is manifested phenomenologically in various ways. Thus, there is evidence for a ‘freezing’ of the effective coupling, for review see, e.g., $^{43}$. In particular, the following effective coupling:

$$\alpha_{eff}(r) \approx \frac{1}{b_0 l_{mod}} \left( 1 + \frac{b_1 \ln l_{mod}}{b_0^2 l_{mod}} \right)^{-1},$$

where

$$l_{mod} \approx \ln \frac{1 + r^2 \cdot (GeV)^2}{r^2 \cdot \Lambda_{QCD}^2},$$

fits well various pieces of data $^{43}$.

On the non-perturbative side, the branes with properties (1), (3) emerge around the crossover (if one moves towards weak coupling). The simplest and qualitative fit is therefore lowest order in perturbation theory plus effect of the branes. This fit is justified in the region close to the crossover – and mostly the lattice measurements are performed for $\beta$ not far from the crossover. On the other hand, the same effect is to be described by perturbation theory since the measurements are already in the weak-coupling domain and there is no singularity which would block the perturbative expansion from being valid in the region close to the crossover (approached from the weak-coupling side). The price is that, the closer is the crossover, the more terms in the perturbative expansion are to be kept. Evaluating so many perturbative terms is practically impossible except for the case of the gluon condensate $^{36}$.

The perturbative series corresponding to the $Q^{-2}$ correction exhibits sign coherence, similar to the case of a classical solution. The series is not divergent in large orders, however. That is, branes are no classical solution. Numerically, the perturbative expansion coefficients in the crucial region of
are even larger than for known divergent series (infrared renormalon, instantons). This is correlated with the fact that the branes apparently represent strongly aligned vacuum fluctuations.

5. Constraints on the branes

5.1. Consistency of the branes with asymptotic freedom

In the preceding section we could convince ourselves that to match the ultraviolet renormalon the branes are to have highly non-trivial properties like (1), (27), (34). All these properties, crucial to fit the ultraviolet renormalon, reveal a point-like facet of the branes. Now we are reversing the question and ask whether this point-likeness is consistent with the asymptotic freedom. In particular, the action for the lattice monopoles is proportional to $a^{-1}$, the same as for point-like particles. At first sight, it is unavoidable that accepting (1) in the limit $a \to 0$ is equivalent to introducing new particles at short distances. Appearance of such particles would be inconsistent with the asymptotic freedom.

To have a closer look at the problem it is useful to translate the data on the monopole trajectories into a conventional field theoretic language. To describe monopoles one then introduces a magnetically charged field $\phi_M$. Moreover, since the monopoles are observed as trajectories, it is natural to use the polymer representation of field theory, see, in particular, 44. Proceeding in this way one can derive (see 6, 7 and references therein):

$$\langle 0 | \phi_M | 0 \rangle = \frac{\alpha}{8} (\rho_{perc} + \rho_{fin}) , \quad (40)$$

where density of the percolating cluster, $\rho_{perc}$ is defined in (2) while $\rho_{fin}$ is related in a similar way to the length of the finite monopole clusters.

The total density of the monopole clusters can be measured directly 3, 5.

It is crucial that the total monopole density diverges as $a^{-1}$ at small $a$: $\rho_{perc} + \rho_{fin} \approx (const) \Lambda^3_{QCD} + (const') \Lambda^2_{QCD} a$. \quad (41)

Finally, we get:

$$\langle 0 | \phi_M | 0 \rangle \approx (const') \Lambda^2_{QCD}/8 \approx 0.8 (fm)^{-2}. \quad (42)$$

Thus, $\langle 0 | \phi_M | 0 \rangle$ contains no ultraviolet divergence and is, therefore, perfectly consistent with the asymptotic freedom which does not allow to add new particles.

To appreciate the geometrical meaning of the observation (41) turn to the simplest case of uncorrelated percolation. One introduces then a
probability $p$ for a link to be “open”, that is to belong to a monopole trajectory in our notations. Then at a critical value $p_{cr}$ there arises an infinite, or percolating cluster. The density of this percolating cluster is vanishing at the point of the phase transition to the percolation. In the supercritical phase where $p > p_{cr}$ this density

$$\rho_{perc} \sim (p - p_{cr})^\alpha,$$

where the critical exponent $\alpha > 0$.

While the density (43) is vanishing at $p = p_{cr}$, no non-analyticity can happen in the total density. At the point of the phase transition, it is given simply by:

$$\rho_{tot} = \frac{p_{cr}}{a^{d-1}},$$

where $d$ is the number of dimensions.

Substituting (44) into (40) we recover in case of $d = 4$ the standard quadratic divergence,

$$\langle 0 | \phi_M^2 | 0 \rangle \sim a^{-2}$$

which is so familiar from the case of Higgs particles. However, the data (41) indicate only $a^{-1}$ behavior of the total monopole density. Which geometrically means that the monopoles spread over a $d = 2$ subspace of the total $d = 4$ space. (Of course, the $d = 2$ subspace is not a plane but rather a surface percolating itself through the $d = 4$ space).

Thus, we come to an amusing conclusion that it is the existence of the branes which eliminates a potential quadratic divergence in (40). Note that the fact that the monopoles are associated with the vortices (whose total area scales) was observed first in Ref. 45. Later the phenomenon was confirmed for various values of $a$ 46. In view of the ultraviolet divergence in the monopole action, see (1), this association of the monopoles with vortices becomes absolutely crucial for the consistency with the asymptotic freedom.

Our final comment concerning (42) is that the vacuum expectation value (42) is perfectly gauge invariant. Gauge invariant condensates of dimension two were widely discussed recently, see, e.g. 47. The beauty of the relation (42) is that it does not contain ultraviolet divergences which plague local condensates formulated in terms of the gluon fields. Thus, the vacuum expectation value (42) could be related directly to the non-perturbative part 48 of the gluonic dimension two condensates. No explicit relation of this kind is known, however.
5.2. Matching of the branes with ultraviolet renormalon

The lattice data on the branes, see, in particular (1), (3) are compelling but pure numerical. Since structure of the branes in terms of the original gluon fields is not known, it is very difficult, for example, to explain (27) theoretically. Matching of the branes with the ultraviolet renormalon brings constraints on the branes which turn to be quite restrictive. In particular, to match the ultraviolet renormalon in case of the gluon condensate the branes are to obviously satisfy the following constraint:

\[ \theta_{\text{plaq}} \cdot \frac{\text{(thickness)}}{a} \cdot \left( \text{(plaquette action)} \cdot a^4 \right) \sim \Lambda_{QCD}^2 a^2, \quad (45) \]

where \( \theta_{\text{plaq}} \) is the probability for a given plaquette to belong to the vortex (see (27)) and the thickness is understood in terms of the distribution of the excess of the \( SU(2) \) action. The presently available data indicate that \( \theta_{\text{plaq}} \approx (\Lambda_{QCD} \cdot a)^2 \), the plaquette action is of order \( a^{-4} \) and the thickness is equal to the resolution, that is \( a \). (All the quantities are on the average.) Such a regime is fully consistent with (45) and can, therefore, persist in the limit \( a \to 0 \) as well.

There is another and subtler point. The non-Abelian action, albeit ultraviolet divergent, is known to be much smaller than the projected one. In case of monopoles this was emphasized, in particular, in (49):

\[ S_{\text{non-Ab}}^{\text{mon}} \sim \frac{L}{a} \ll S_{\text{Ab}}^{\text{mon}} \sim \frac{1}{g^2} \frac{L}{a}, \quad (46) \]

where \( S_{\text{Ab}}^{\text{mon}} \) is the action in the Abelian projection, i.e. the same as for the Dirac monopole. Similar inequality holds in case of the vortices.

Now, we can derive (46). Indeed, the branes are dual to high orders of perturbation theory. Which means that the action associated with the branes is not allowed to be parametrically more singular than the perturbative fluctuations. This rules out the monopole action of order \( S_{\text{mon}} \sim g^{-2} \cdot L/a \), see Eq. (46). In other words, branes cannot be removed from the vacuum without affecting perturbative fields as well.

5.3. Casimir scaling

The lattice data on the quark potential (for review see, e.g., (24)) reveal a strikingly simple picture. Lack of a same simple theoretical explanation looks as a puzzle. In fact there exist rather a few puzzles to be explained:

a) at relatively short distances the potential is well approximated by a Coulomb like piece plus a linear potential;
b) the linear piece continues, with the same slope to large distances;
c) Casimir scaling.

The phenomenon of the Casimir scaling \(^{50}\) is that at intermediate distances static potentials for quarks belonging to various representations \(D\) of the color group are proportional to each other:

\[
V_D(r) \approx \frac{C_D}{C_F} V_F(r) ,
\]

where \(C_D\) is the eigenvalue of the quadratic Casimir operator

\[
C_D = \text{Tr} T_a^D T_a^D
\]

of the representation, \(C_F\) is the value of \(C_D\) for the fundamental representation and \(V_F(r)\) is the static quark potential in case of heavy quarks in the fundamental representation.

Explanation of the observed Casimir scaling is a challenge to theory, for review and models see \(^{50,51}\). It is not obvious either that observation of the branes brings a transparent solution to the problem. We still feel that the consideration of the branes does introduce new elements into understanding of the Casimir scaling.

As far as the observation b) above is concerned, a new point is that the branes look the same random at all scales tested. Namely, the properties (1), (3), (34) reiterate themselves for all the lattice spacings. Since there is no scale involved, except for \(\Lambda_{QCD}\), see Eq. (27) the same slope \(\sigma \sim \Lambda_{QCD}^2\) governs the brane-induced potential at all the distances. An explicit realization of this idea is provided by Eq. (32).

Thus, the branes allow to shift the emphasis in explaining the Casimir scaling from large to short distances. Concerning short distances, the linear potential is calculable, in principle, perturbatively. In the two-loop approximation (that is, three terms in the expansion of \(V(r)\) in \(\alpha_s\)) the Casimir scaling is checked by explicit calculations \(^{52}\):

\[
V_{D\,\text{two loop}}(r) = \frac{C_D}{C_F} V_{F\,\text{two loop}}(r) .
\]

However, the linear term at short distances is sensitive to even higher orders (for a discussion see Sect 3.3) and one has to turn to models. In particular, the model with a tachyonic gluon mass \(^{31}\) immediately gives:

\[
\delta V_D(r) = \frac{C_D \alpha_s}{2} \frac{m_g^2}{\Lambda_{QCD}} r ,
\]

reproducing the Casimir scaling.
More generally, the branes are to be dual to the ultraviolet renormalons. In turn, the factorization properties of the ultraviolet renormalon, see and references therein, result in the Casimir scaling. However, within such a framework it is difficult to expect that the Casimir scaling would hold to high accuracy.

To summarize, consideration of branes allows to reduce the problem of the Casimir scaling to the problem of evaluating the linear potential at short distances. The latter problem is perturbative in nature and the first two loop corrections are known to exhibit the Casimir scaling. Higher orders can be estimated only within models.

6. Conclusions
We have demonstrated that the monopoles and vortices are responsible for the ultraviolet-renormalon type corrections at short distances. In turn, the \( Q^{-2} \) corrections are dual to high orders of perturbation theory. Phenomenologically, such corrections are known to be welcome. Combination of the ultraviolet and infrared factors typical for the ultraviolet renormalon appears to be a reflection of the fine tuning for the monopoles and vortices as non-local objects. Vortices and monopoles appear, in this context, as non-perturbative counterpart of the ultraviolet renormalon. Highly non-trivial constraints on the properties of the branes implied by this identification are satisfied by the lattice data.

The overall conclusion is that the fine tuning observed on presently available lattices, see in particular (1), (3), could be the true asymptotic in the limit \( a \to 0 \).

In a broader context, fluctuations appearing as topological and suppressed in one formulation of a theory can become fundamental entities in a dual formulation of the same theory. If it is true in case of \( SU(2) \) Yang-Mills theory, then the hint is that the branes could appear in the dual formulation. Where by branes we understand (see above) \( d = 2 \) surfaces populated by the monopoles (tachyonic mode) and living on a \( d = 4 \) Euclidean space. Generically, this observation agrees with recent and well known proposals on theories dual to YM theories. To the best of our knowledge, no direct comparison of the two approaches is possible, however.

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