On the Incremental Asymmetric Signatures

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ABSTRACT
The purpose of incremental cryptography is to allow the updating of cryptographic forms of documents undergoing modifications, more efficiently than if we had to recompute them from scratch. This paper defines a framework for using securely a variant of the incremental hash function designed by Bok-Min Goi et al. The condition of use of their hash function is somehow impractical since they assume that the blocks of the message are all distinct. In this paper we show how we can discard this strong assumption so as to construct the first practical incremental asymmetric signature scheme that keeps efficient update operations. Finally, as the proposed scheme has the defect to severely expand the signature size, we propose a solution which drastically reduces this drawback.

Keywords
Incremental cryptography, Obliviousness, Parallel cryptography

1. INTRODUCTION
Incremental cryptography, introduced by Bellare, Goldreich and Goldwasser in [1][2][3], is used to maintain up-to-date at low computational cost the outputs of cryptographic algorithms. If one has the signature of the current version of a file, it is preferable to avoid to recompute from scratch the signature algorithm applied to the entire file whenever a modification is performed. Such a low computational efficiency for update operations finds application in various situations, for example when we want to maintain constantly changing authenticated databases, editable documents or even when we have to sign similar letters addressed to different recipients. Nowadays, an area of application of greatest interest is its use in cloud storage systems, trends to outsource personal data make computational efficiency as well as space efficiency particularly important. Besides, an other interest of incremental cryptographic schemes is their inherent parallelism which allow performances to fit accordingly by using appropriate multi-processing platforms.

Typical operations supported by incremental cryptography are the replace, insert and delete operations. Authentication schemes based on static hash tree can support only both replacement and insert operations. Authentication schemes based on dynamic hash tree (or skip list) can discard this strong assumption about the blocks of a message when used in un-keyed hash function. This one is discarded in the PCIHF hash function in favour of a modular arithmetic operator to avoid this second type of attack. Unfortunately, the use of such an operator remains delicate to recent sub-exponential attacks [5][4].

Bok-Min Goi et al [10] have designed an efficient incremental hash function based on the randomize-then-combine paradigm [3]. Unfortunately, their construction is based on the impractical assumption that the blocks in the message to hash are all distinct. Besides, due to recent attacks described in the Generalized Birthday Problem [15][3], the security parameters of the PCHF hash function are no longer adapted. We show in this paper how one can discard this strong assumption about the blocks of a message to construct the first practical incremental asymmetric signature scheme that keeps update operations within a linear runtime worst case complexity. Since the first incremental signature scheme that we describe produces signatures of size twice the length of the messages, we propose a parametrizable version which reduces this overhead with the counterpart of less time efficient updates. Finally, we show that we can reduce the security of our schemes to the security of the underlying primitives involved, namely a set-collision resistant hash function, a traditional signature scheme and a simple combinatorial problem.

The paper is organized as follows: in the next section, we discuss pair-wise chaining and give some useful definitions. In Section 4 we introduce our incremental signature scheme for which an improvement is given in Section 5. Proofs of security are given in Section 5 and efficiency of our solution is discussed in Section 6. Finally we conclude in Section 7.

2. PRELIMINARIES
Cryptographic schemes take as input a document $D$ which is divided into a sequence of fixed-size blocks $D_1, D_2, \ldots, D_n$. Doc-
2.1 Operations on documents

We denote by $\mathcal{M}$ the space of modifications and define the main modification operations $M \in \mathcal{M}$ allowed on a document as follows: (i) $M = (\text{delete}, i, j)$ deletes data from the block $i$ to $j$ (included); (ii) $M = (\text{insert}, i, \sigma)$ inserts $\sigma$ between the $i^{th}$ and $(i+1)^{th}$ block, where $\sigma$ represents data of size a multiple of $b$; (iii) $M = (\text{replace}, i, \sigma)$ changes to $\sigma$ the data starting from block $i$ to block $(i+k-1)$ (included), where $\sigma$ is of size $kb$.

The resulting document after a modification operation $M$ is denoted $D(M)$. Other frequent modifications are the cut and paste operations but, for the sake of simplicity, we do not deal with such composite modifications here.

2.2 Hash functions based on pair-block chaining

Pair-wise chaining roughly describes the following process: given the $n$-block string $D$, each block (except the last one) is paired up with the subsequent one, a pseudo-random function is evaluated at the resulting point. We obtain in this way a sequence of $n-1$ values which characterize a relation between the blocks of the document. The following paragraph gives a detailed description.

The PCHF hash function. This is the first really incremental hash function [10] implementing the randomize-then-combine paradigm introduced by Bellare et al. [1], as this one supports insert and delete operations. A document $D$ is divided into a sequence of blocks of size $b$. If the document length is not a multiple of $b$-bits, a standard padding method is applied at its tail with a bit “1” followed by the sufficient number of consecutive bits “0” so that the last block is of size $b$. The final message is of the form $D = D_1 || D_2 | \ldots | D_n$ with $D_i \in \{0,1\}^b$ for $i \in [1,n]$. Besides, a stronger assumption is made about the blocks of the document which have to be all distinct, meaning that $D_i \neq D_j$ for $i, j \in [1,n]$ and $i \neq j$. The hash value $\mu$ for PCHF is calculated as following:

$$\mu = \sum_{i=1}^{n-1} \mathcal{R}(D_i || D_{i+1}) \mod 2^{160}$$

where the randomize operation $\mathcal{R} : \{0,1\}^b \rightarrow \{0,1\}^{160}$ is a compression function, or a pseudo-random function. Note that the padding used is not secure, as shown with examples in [14]. Besides, the authors of PCHF advocate the use of a standard hash function for the randomize operation and in this case the padding is not required. This is because in standard hash functions a padding is enforced whatever the input length is.

To ensure integrity of content a hash function must ensure first and second pre-image resistance as well as collision resistance. The security of a signature scheme based on the hash-then-sign paradigm relies on the collision resistance property of the underlying hash function. A hash function is said to be collision resistant if it is hard to find two messages that hash to the same output value. The PCHF hash function allows insert operation with a running time proportional to the number of blocks to insert, and replace/delete operations with constant cost. Besides, this one has been proved to be collision resistant in the random oracle model [10].

The design problem. Let us denote $(X,Y)$ a subsequence of blocks starting with $X$ and ending with $Y$, and $(X,Y)$ a pair of consecutive blocks. We recall here the problems raised in [14] about pair-wise chaining used in hash functions. The authors found message patterns which lead to trivial collisions: (i) Palindromic messages of any block length, for instance the block message $B[A] || B$ which produces the same hash value than $A[B] || A$; (ii) Certain non-palindromic messages. As said explicitly in [14], “any two messages with the same block at both ends, and where all consecutively paired blocks follow the same order, would cause collisions”. The given examples are the messages $A[B] || C || A$ and $B || C || A$ which produce the same hash value; (iii) In the case where the XOR operator is used, messages with repetitive blocks could also produce collisions. This is the case of the messages $A[B] || B || C$ and $A || B || C$ for which a pair number of pair-wise links $B,B$ does not change the hash value.

Now, concerning the second case above we can observe the following two more general patterns leading to collisions: (i) If a message contains at least three identical blocks, for instance $B[X] || C[X] || D[X] || E$, then we have the interesting sequence of pairwise links $[X, C]$, $[C, X]$, $[X, D]$ and $[D, X]$. We see that we can permute the subsequence $(X,C)$, $(C,X)$ and $(X,D)$, $(D,X)$ so that the message $B[X] || D[X] || C[X] || E$ produces the same hash value; (ii) If a message contains at least two pairs of identical blocks, for instance the message $B[X] || C[Y] || D[X] || E[Y] || F$ for which the two subsequences $(X,X)$ and $(Y,Y)$ overlap, a permutation gives us the message $B[X] || E[Y] || D[X] || C[Y] || F$ which produces the same hash value.

2.3 Incremental signature schemes

Definition 1. An asymmetric incremental signature scheme is specified by a 4-tuple of algorithms $\Pi = (G, S, I, V)$ in which:

- $G$, the key generation algorithm, is a probabilistic polynomial time algorithm that takes as input a security parameter $k$ and returns a key pair $(sk, pk)$ where $sk$ is the private key and $pk$ the public key.
- $S$, the signature algorithm, is a probabilistic polynomial time algorithm that takes as input $sk$ and a document $D \in \Sigma^*$ and returns the signature $s = S_{sk}(D)$ where $s$ is a signature with appendix, that is, $s$ is of the form $(D,s')$ where $D$ is the document and $s'$ the appendix.
- $I$, the incremental update algorithm, is a probabilistic polynomial time algorithm that takes as input $sk$ and a document $D \in \Sigma^*$ and returns the signature $s = S_{sk}(D)$ where $s$ is a signature with appendix, that is, $s$ is of the form $(D,s')$ where $D$ is the document and $s'$ the appendix.
- $V$, the verification algorithm, is a deterministic polynomial time algorithm that takes as input a public key $pk$ and a signature $s = S_{sk}(D)$ and returns 1 if the signature is valid, 0 otherwise.

Considering a modified document $D' = D(M)$, the desired behaviours of an incremental signature scheme are the followings: (i) It is required that $V_{pk}(I_{sk}(S_{sk}(D), M)) = 1$. (ii) Optionally, an incremental signature scheme could be oblivious (or perfectly private) in the sense that the output of a signature $S_{sk}(D')$ is indistinguishable from the output of an incremental update $I_{sk}(S_{sk}(D), M)$. This property is particularly useful if we want to hide the modification history of a signed document, or even the fact that an update operation has been performed.
2.4 Unforgeability

For incremental signature schemes, existential unforgeability measures the unability for an adversary to generate a new pair \((D^*, S^*)\) where: (i) \(D^*\) is not a document that has been signed by the signing oracle; (ii) \(D^*\) is not a modified document obtained via the incremental update oracle; (iii) \(S^*\) is a signature on \(D^*\). More precisely, the notion of existential unforgeability under a chosen message attack is defined using the following game between the adversary \(\mathcal{A}\) and the challenger:

1. The challenger runs algorithm \(G\) to obtain a public key \(pk\) and a private key \(sk\). The adversary \(\mathcal{A}\) is given \(pk\).

2. \(\mathcal{A}\) chooses and requests signatures (adaptively) on at most \(q_s\) messages. Additionally, \(\mathcal{A}\) chooses and requests at most \(q_t\) valid incremental updates (adaptively) on signatures issued from the signing oracle (or from the updating oracle). The term “valid” means that the pair \((D, S)\) on which the update is requested satisfies \(\Pi.S^t.pk(D, S) = 1\). The challenger responds to each query.

3. Eventually, \(\mathcal{A}\) outputs a pair \((D^*, S^*)\) where \(D^*\) is not a document signed by the signing oracle \(\mathcal{O}^S\) or an updated document whose the signature has been obtained from the incremental update oracle \(\mathcal{O}^I\). We say that \(\mathcal{A}\) succeeds if \(\Pi.S^t.pk(D^*, S^*) = 1\).

**Definition 2.** Let \(\Pi = (G, S, I, \Psi)\) be an incremental signature scheme over a modification space \(\mathcal{M}\), and let \(\mathcal{A}\) be an adversary. Let

\[
\text{Adv}^\text{unf}_{\mathcal{A}, \Pi} = \mathbb{P}[sk \leftarrow G; \mathcal{S}^I, \mathcal{O}^S.I: \Pi.S^t.pk(S^*) = 1].
\]

We say that \(\Pi\) is \((t, q_s, q_t, \epsilon)\)-secure in the sense of existential unforgeability if, for any adversary \(\mathcal{A}\) which runs in time \(t\), making \(q_s\) queries to the signing \(\mathcal{O}^S\) oracle and \(q_t\) valid queries to the update \(\mathcal{O}^I\) oracle, \(\text{Adv}^\text{unf}_{\mathcal{A}, \Pi}\) is less than \(\epsilon\).

3. THE PROPOSED INCREMENTAL ASYMMETRIC SIGNATURE SCHEME

We assume the use of a simpler version of the PCHF hash function, denoted \(H\), where input blocks of the random function are expressly unrelated. This incremental hash function has been proved to be set-collision resistant [6] in the random oracle model under the weighted knapsack assumption. From there we will use it as a black-box primitive to design secure incremental signature schemes. First we format the message correctly by enforcing the padding in the following way: if the message length is not a multiple of \(2b\)-bit, we pad the last block with a bit 1 followed by the sufficient number of bits 0, otherwise we add a new block whose content is \(1\{0\}{2^{b-1}}\). The final message is of the form \(M = M_1||M_2||...||M_{n-1}\) with \(M_i \in \{0, 1\}{2^{b}}\) for \(i \in [1, n-1]\). The hash value is computed such that \(\mu = H(D)\) where \(H(.\cdot)\) is a function taking a string of size a multiple of \(2b\)-bit and returning a value in \(\{0, 1\}{3200}\). This last one is defined as follows:

\[
H(M) = \sum_{i=1}^{n-1} R_i(M_i) \mod 2^{3200}
\]

where the randomize function \(R_i : \{0, 1\}{2^{b}} \rightarrow \{0, 1\}{3200}\) has an output size of \(3200\) bits and the addition is performed modulo \(2^{3200}\). Considering the problem of generating a second preimage, the output size of \(R_i\) in the original PCHF hash function is no longer secure due to the efficient (subexponential) generalized birthday attack [15]. This force us to increase the output size of \(R_i\) by considering for instance \(3200\) bits, a choice of parameter which allows us to guard against this attack by keeping an equivalent security of 112 bits.

Let us suppose a correctly padded \((n-1)\)-block message \(D = D_1||D_2||...||D_{n-1}\) with \(D_i \in \{0, 1\}{b}\) for \(i \in [1, n-1]\). We assume the use of a digital signature algorithm \(\Psi = (G, S, V)\) in which: (i) \(G\) takes as input a security parameter and returns a pair of keys \((sk, pk)\); (ii) \(S\) is a ppt signature algorithm taking as input the secret key \(sk\), a document \(D\) and returns a signature \(s\); (iii) \(V\) is a dpt algorithm taking as input the public key \(pk\), a document \(D\), a signature \(s\) and returns 1 if the signature is valid, \(\bot\) otherwise.

Now we can describe the 4-tuple of algorithm IncSIG = \((G, S, I, \Psi')\). The key generation of the incremental algorithm is simply IncSIG.\(G = \Psi.G\). The signature algorithm IncSIG.\(S\) taking as input the secret key \(sk\) and the document \(D\) is the following:

1. Pick uniformly at random \(n\) blocks of size \(b/2\) denoted \(R_1, R_2, ..., R_n\);
2. Compute the hash value
\[
\mu = \sum_{i=1}^{n-1} R_i(R_i)||R_{i+1}||D_i) \mod 2^{3200};
\]
3. Let \(l\) be the size of \(D\), compute a signature \(s = \Psi.S_{sk}(\mu, l)\) and return the incremental signature \((R_1, R_2, ..., R_n, \mu, s)\).

The verification algorithm IncSIG.\(V'\) taking as input the public key \(pk\), the document \(D\) and the signature \((R_1, R_2, ..., R_n, \mu, s')\) is the following:

1. Compute the hash value
\[
\mu' = \sum_{i=1}^{n-1} R_i(R_i)||R_{i+1}||D_i) \mod 2^{3200};
\]
2. Let \(l'\) be the size of \(D'\), run the verification \(b = \Psi.V_{sk}(s', \mu, l)\) and return \(b\).

We describe hereafter the incremental algorithm IncSIG.\(I\) that takes as input the secret key \(sk\), the document \(D\), the signature \((R_1, R_2, ..., R_n, \mu, s)\) and an insert operation \(M = (\text{insert}, i, \sigma)\) that changes \(D\) in \(D'\) where \(\sigma\) is only one block (for the sake of simplicity):

1. Draw a new random block of size \(b/2\) denoted \(R\);
2. Compute the hash value
\[
\mu' = \mu - R_i(R_i)||D_i) \mod 2^{3200} + R_i(R_i)||D_{i+1}||\sigma) \mod 2^{3200};
\]
3. Let \(l'\) be the size of \(D'\), compute a signature \(s' = \Psi.S_{sk}(\mu', l')\) and return the updated incremental signature \((R_1, R_2, ..., R_i, R, R_{i+1}, ..., R_n, \mu', s')\).
We do not describe the other operations which can be deduced from the previous one.

Remark. We could change the verification algorithm and reject signatures with non distinct \( R’_i \)'s. Checking that all the random blocks are distinct would simplify the security analysis, but there are several reasons for not doing this: (i) This verification is a costly operation; (ii) The owner of the secret key is not considered as an adversary in the standard definition of existential unforgeability. The signing oracle is then implemented exactly as specified.

4. A SIMPLE IMPROVEMENT INC SIG*

By considering the same previous notations, let us also define two fixed integers \((k, d) \in \mathbb{N}^2\) such that \(b = kd\) with \(d \geq 2\). We describe an improvement of IncSIG in which we use a \(d\)-wise chaining of random \(b\)-bit blocks. This parametrization will allow a user to find a time/space efficiency trade-off without sacrificing security.

Now we describe the 4-tuple of algorithm IncSIG* = \((G, S, I, \Psi)\). The key generation of the incremental algorithm is simply IncSIG*; \(G = \Psi G\). The signature algorithm IncSIG*; \(\Psi’\) taking as input the secret key \(sk\) and the document \(D\) is the following:

1. Pick uniformly at random \(n\) blocks of size \(b\) denoted \(R_1, R_2, \ldots, R_n, R_{n+1}, \ldots, R_{n+d-2}\);
2. Compute the hash value
   \[
   \mu = \sum_{i=1}^{n-1} R_i\|R_{i+1}\| \ldots \|R_{i+d-2}\|\|R_{i+d-1}\||D_i) \mod 2^{3200};
   \]
3. Let \(l’\) be the size of \(D’\), compute a signature \(s’ = \Psi S_{sk}(\mu’\|l’))\) and return the updated incremental signature \((R_1, R_2, \ldots, R_{n+d-2}, \mu’, s’))\).

The verification algorithm IncSIG*; \(\Psi’\) taking as input the public key pk, the document \(D\) and the signature \((R’_1, R’_2, \ldots, R’_{n+d-2}\|\mu’, s’))\) is the following:

1. Compute the hash value
   \[
   \mu’ = \mu - \sum_{j=i-1}^{d-2} R_j\|R_{j+1}\| \ldots \|R_{j+d-2}\|\|R_{j+d-1}\||D_j)
   \]
2. Compute the hash value
   \[
   \mu’ = \mu - \sum_{j=i}^{d-2} R_j\|R_{j+1}\| \ldots \|R_{j+d-2}\|\|R_{j+d-1}\||D_j)
   \]
3. Let \(l’\) be the size of \(D’\), compute a signature \(s’ = \Psi S_{sk}(\mu’\|l’))\) and return the updated incremental signature \((R_1, R_2, \ldots, R_{n+d-2}, \mu’, s’))\).

Let us now describe this incremental algorithm in the case of a replace operation \(M = (\text{replace}, i, \sigma)\) where \(\sigma\) is only one block (for the sake of simplicity):

1. Draw a new random block of size \(b \| \mu\) denoted \(R’\);
2. Compute the hash value
   \[
   \mu’ = \mu - \sum_{i=1}^{d-2} R_i\|R_{i+1}\| \ldots \|R_{i+d-2}\|\|R_{i+d-1}\||D_j)
   \]
3. Let \(l’\) be the size of \(D’\), compute a signature \(s’ = \Psi S_{sk}(\mu’\|l’))\) and return the updated incremental signature \((R_1, R_2, \ldots, R_{n+d-2}, \mu’, s’))\).

When performing a deletion operation, in order to maintain consistency in the \(d\)-wise chain, the contribution of \(d\) hash values of the non updated chain has to be deducted from \(\mu’\) while \(d-1\) new values have to be added. This algorithm can be deduced from the insertion one.

Remark. We could change the verification algorithm and reject signatures with non distinct \((d-1)\)-tuples of random blocks. Checking that all these \((d-1)\)-tuples are distinct would simplify the security analysis, but there are several reasons for not doing this: (i) This verification is a costly operation; (ii) The signer is not considered as an adversary in the standard definition of existential unforgeability.

5. SECURITY ANALYSIS

The property of obliviousness of the schemes IncSIG and IncSIG* is obvious and we focus only on the security analysis of unforgeability.

Theorem 1. Let suppose that \(\mathcal{F}\) is a \((t, q_1, q_2, \varepsilon)\)-forger against our incremental signature scheme IncSIG, then there exists a \((t’, q_3, \varepsilon)\)-collision finder \(\mathcal{C}^F\) against the underlying hash function \(H\) and a \((t^*, q_4 + q_5, \varepsilon^*)\)-forger \(\mathcal{F}^*\) against the underlying
signature scheme $\Psi$, where the quantities are related by

$$
\varepsilon \leq \frac{q^2 - q}{2^{2l/2 + 1}} + \epsilon' + \epsilon''; \quad t \geq \max\{t' - (q_s + q_t)\text{sign}(t')\} - q_{\text{stop}},
$$

where $q = q_s(n_{\max} + 1) + q_t, q_h = q_s n_{\max} + 3q_t,$ and $\text{sign}$ and $\text{top}$ are, respectively, the maximum running times to perform a signature with $\Psi$ and an operation in $(F_{2^{2l/2}}, +)$.

PROOF. For the sake of simplicity in the sketch of proof, we only deal with insert operations of one block and suppose that a document can have a maximum length of $n_{\max}$ blocks. Let us suppose that there exists a $(t, q_s, q_t, \varepsilon)$-forger $F$ against our incremental signature scheme IncSIG. By interacting adaptively with the challenger in the game defined Section 3.4 the forger eventually outputs a pair $(D^*, S^*)$. The forger can win the game according to both following possibilities:

- case 1: The value $\mu^*$ contained in the successful forgery has already been retrieved in a response to a query.
- case 2: The value $\mu^*$ contained in the successful forgery has never been encountered in the responses to the queries.

By assuming a forger against our incremental signature scheme IncSIG, we show that the case 1 allows the construction of a collision finder $C^*$ for the hash function $H$ and the case 2 a forger $F'$ against the signature scheme $\Psi$. Indeed, we build a collision finder $C^*$ for $H$ and a forger $F'$ for the underlying signature algorithm in the following way:

- case 1: The collision finder $C^*$ uses $F$ as a subroutine and simulates an incremental signing oracle as follows: first of all, $C^*$ executes $\Psi.G$ to obtain a pair of keys $(sk, pk)$ and conveys $pk$ to the forger $F$. Whenever $F$ queries a signature for a document, $C^*$ forms correctly the document with an enforced padding to obtain a $n$-block document $D = D_1 \| D_2 \| \cdots \| D_{n-1}$, then he (she) generates $n$ random blocks $R_1, R_2, \ldots, R_n$ and computes

$$
\mu = \sum_{i=1}^{n-1} R_i \| R_{i+1} \| D_i \mod 2^{32000},
$$

by resorting to the random oracle for the function $\mathcal{R}$. After that, he (she) computes a signature $s$ on $\mu$ using the secret key $sk$ and responds to $F$ with the incremental signature $S = (R_1, R_2, \ldots, R_n, s)$. The updating oracle is simulated as follows: whenever $F$ queries an insertion of one block right after index $i$ in a signed document $(D, R_1, R_2, \ldots, R_n, s)$, $C^*$ generates a new random block $R$ and updates the value of $\mu$ (random oracle accesses to $\mathcal{R}$ are needed) by first removing the contribution of the pair-wise link $[R_i, R_{i+1}]$ and then adding the contributions of the two new links $[R_i, R]$ and $[R, R_{i+1}]$. Then he (she) computes a signature $s'$ on the updated hash $\mu'$ and sends the updated incremental signature $S' = (R_1, R_2, \ldots, R_i, R_{i+1}, \ldots, R_n, s')$ to $F$. At the end, $F$ comes with a new pair $(D^*, S^*)$ where $S^* = (R_1^*, \ldots, R_m^*, s')$ and $D^* = D_1^* \| \cdots \| D_{m-1}^*$. As we supposed that $s'$ was obtained from the signing oracle $\Psi_{sk}, s$, the set $\{R_i^* \| R_{i+1}^* \| D_i^* \| ... \| D_m^*\}$ corresponds to a set-collision for the hash function $H$ if a certain condition is fulfilled: the document corresponding to the forged signature is not simply a reordering of the blocks of a queried document.

Let us denote by $g$ the total number of random blocks used as input to the random function such that $g = q_s(n_{\max} + 1) + q_t$.

We consider the list $L$ of these $g$ blocks reindexed for the occasion so that $L = [\{R_i^*\}_{i=1}^{g}]$. Let $AD$ be the event that the blocks of $L$ are distinct, or in other words, $R_i^* \neq R_j^*$ for all $i, j \in [1, q]$ with $i \neq j$.

When the event $AD$ occurs the $b$-bit blocks of a signed message can not be permuted without re-evaluating $\mathcal{R}$ to the new appearing points, changing the final value of $H$. Let us denote simply by $S_{\text{casel}}$ the event of success of $F$ in the current case that we are describing. Then,

$$
\Pr(S_{\text{casel}}) = \Pr(AD) \Pr(\mathcal{AD}) + \Pr(S_{\text{casel}} | AD) \Pr(AD) \leq \frac{q^2 - q}{2^{2l/2 + 1}} + \epsilon' + \epsilon'',
$$

where $\Pr(AD)$ is the advantage of $C^*$ for breaking the hash function $H$ in the sense of set-collision resistance.

- case 2: The forger $F'$ uses $F$ as a subroutine and simulates its environment as done by $C^*$ in the case 1 except for the following differences: $F'$ simulates the random oracle for $\mathcal{R}$ by generating on the fly a table mapping input values to random output strings and uses its own signing oracle to obtain a signature on a hash value $\mu$. Eventually, $F'$ comes with a new pair $(D^*, S^*)$ and we have supposed that $s'$ has not been obtained from the signing oracle, the output of $F'$ is setted to $(\mu^*, s')$ which corresponds to a valid forgery.

□

Now we can focus on the security of IncSIG'. A $d$-wise chain staying a $d$-wise chain after an update, the obliviousness property is obvious. Then we give only the interesting details of the proof of unforgeability concerning the following theorem since this one is very similar to the above.

**THEOREM 2.** Let suppose that $F$ is a $(t, q_s, q_t, \varepsilon)$-forger against our incremental signature scheme IncSIG', then there exists a $(t', q_{s'}, q_{e'}, \varepsilon')$-collision finder $C^*$ against the hash function $H$ and a $(t', q_s + q_t, q_{e'})$-forger $F'$ against the underlying signature scheme $\Psi$, where the quantities are related by

$$
\varepsilon \leq \frac{(q_s + q_t)(n_{\max} + 1)^2}{2^{(d-1)/2l/2 + 1}} + \epsilon' + \epsilon'';
$$

$$
t \geq \max\{t' - (q_s + q_t)\text{sign}(t')\} - q_{\text{stop}},
$$

where $q_s = q_s n_{\max} + (2d - 1)q_t,$ and $\text{sign}$ and $\text{top}$ are, respectively, the maximum running times to produce a signature with $\Psi$ and to perform an operation in $(F_{2^{2l/2}}, +)$.

PROOF. Keeping the previous notations, first notice that in order for the adversary to permute two blocks of a message, say $R_i \| R_{i+1} \| \cdots \| R_{i+d-2} \| R_{i+d-1} \| D_j$ and $R_j \| R_{j+1} \| \cdots \| R_{j+d-2} \| R_{j+d-1} \| D_j$, with $j > i$, there are two possibilities: (i) The message is of length two blocks. In this case, $R_1 \| \cdots \| R_{i+d-3} \| R_{i+d-2}$ must be equal to $R_2 \| \cdots \| R_{j+d-2} \| R_{j+d-1}$. (ii) The message has a length greater than two blocks. In this case, having $R_{j+1} \| \cdots \| R_{j+d-2} \| R_{j+d-1}$ equal to $R_{j+1} \| \cdots \| R_{i+d-2} \| R_{i+d-1}$ is a prerequisite. Without this we can not ensure the consistency with the $(i + 1)$-$th$ $d$-tuple in input to $\mathcal{R}$.

Continuing, it remains to notice that a message of length $n$ contains $n + 1$ $(d - 1)$-tuples of random blocks. For each signature or update query providing a signature for a document $D'$ we consider the list $L'$ of the $(d - 1)$-tuples involved in the produced signature.
Let $AD^i$ be the event that the elements of $L^i$ are distinct for all $i \in [1,q_t+q_i]$. It follows that:

$$\Pr(AD) \leq \frac{(q_t+q_i)(n_{\text{max}}+1)^2}{2^{(d-1)b}b+1}.$$ 

6. EFFICIENCY

As we can see in Table 1 whatever the parametrization used the cost to make a signature does not change. The operations in $\mathbb{Z}/2^{3200}Z$ are much more expensive than the hash operations. Therefore, a solution to decrease the number of arithmetic operations could be to use larger blocks for the message at the counterpart of less efficient updates. Obviously, such a choice is not interesting in incremental cryptography, for which we would prefer to suffer higher costs for the signature generation and perform efficient updates.

When $d$ increases exponentially (and $k$ decreases exponentially), the size of the signature decreases in the same way. For instance, by choosing the triple $(b,k,d) = (256, 1, 256)$ the overhead for the signature size is only $n + 255$ bits, that is to say, an overhead of approximately $\frac{1}{100}$th the size of the message. On the other hand we notice in Table 2 that this is accompanied by larger update costs, showing that this is a question of compromise.

To effectively improve performances, the choice of the underlying primitives is of great importance. Concerning the underlying randomize function, we need a hash function capable of generating outputs of size 3200 bits. The new standard SHA-3, Keccak [5], allows the output size to be parametrized. Besides, it can be used with a hash tree mode in order to increase the degree of parallelism. However, if we can process the input message in parallel, it will be interesting to do the same for the output. Nevertheless, Keccak is based on the sponge construction and consequently the blocks which appear in this variable-sized output can not be generated in parallel, that is why we could prefer to use a solution based on a counter mode or a GGM technique [11] to generate them. A good choice could be the Skein hash function [5] which proposes a hash tree mode and permits to generate the output string in parallel as well. Concerning the underlying signature scheme $\Psi$, one can choose any signature scheme based on the hash-then-sign paradigm or a signature scheme providing message recovery. In this latter case, the hash value $\mu$ could be removed from the incremental signature since it can be retrieved during the verification process.

7. CONCLUSION

In this paper, we have described a method to construct an incremental asymmetric signature scheme which ensures the perfect privacy property. We have shown that we can discard the stronger assumption done about the blocks of the message and still use securely and in a practical way an incremental hash function based on pair-wise chaining. To the best of our knowledge this is the first incremental asymmetric signature whose the update algorithm has a linear time worst case complexity.

Besides, we have shown how we can reduce the size of the signature, but at the expense of greater number of hash operations and additions/subtractions. Such a signature scheme is interesting for many applications in which we have to authenticate a lot of documents that continuously undergo modifications, this is the case of the virus protections, the authentication of files systems and databases. More generally, this is particularly welcome for ensuring efficiently a secure handling of files in cloud storage systems.

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