Bended reinforced concrete beams deformability under long-term and environmental influences

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Abstract. The article deals with the problem of determining the bended reinforced concrete beams deformability indices, which are subjected to a combined long-term load and aggressive environment influence during operation. It is noted that due to the concrete creep, the corrosion damages accumulation and cracks formation, the reinforced concrete structures deformability increases in time. In current methods of beams deflections calculating, a stepwise description of stiffness along the span is implemented, which in some cases can lead to significant errors and, as a consequence, to the limiting state. Therefore, during calculations, it is important to take into account the proper stiffness parameters. In this regard, on the basis of predetermined deformations method, a new methodology deformability indices calculation has been developed, in which the hardness in the span is distributed along the parabola. For its validation, special experimental studies of samples of reinforced concrete beams that were under a static load in sulfate and chloride-containing aggressive environments for a long time were carried out. After comparing the corresponding experimental and calculated samples deflections values, it was concluded that the developed method has sufficient accuracy and, thus, can be used for practical calculations of bended concrete elements deformability under long-term and environmental influences.

1. Introduction

The best building and structures operating practices show that with a combined long-term impact on reinforced concrete structures of a load and an aggressive environment, their deformability increases with time. It is directly related to the creep of concrete [1, 2], the formation of cracks, as well as the accumulation of corrosion damage, which reduces the bending stiffness of the sections [3–6]. With significant deformations, a limiting state may occur when reinforced concrete structures become unsuitable for further normal operation [7–10]. In this regard, in our opinion, studies on determining the deformability indicators of reinforced concrete beams that are exposed to long-term power and environmental influences during operation are relevant.

To date, fairly broad theoretical and experimental studies have been carried out on the long-term deformation of bent reinforced concrete elements under load in various aggressive environments [11–17]. It is established that the deformation process is non-linear, non-equilibrium and partially irreversible, while the duration and intensity of operational effects influences significantly the general stress-strain state of structures.

The development of corrosion damages depends on the sign and level of stresses acting in structural materials. When concrete is compressed to the stress level corresponding to the beginning of
structural microcrack formation, the permeability of concrete for an aggressive environment is reduced to a minimum. With a further increase in the level of compressive stresses to the limit of long-term compressive strength, the permeability of concrete increases. When tensile at any level, the permeability of concrete for an aggressive environment becomes higher. The presence of tensile or compressive stresses in steel reinforcement leads to an acceleration of the accumulation of corrosion damage due to an increase in the structural heterogeneity of steel, and, as a result, an increase in the number of galvanic couples. As a result of corrosion damages, there is a decrease in cross-sectional area and a deterioration in the deformation-strength characteristics of concrete and steel reinforcement [18]. In addition, disbonding between concrete and reinforcement is possible [19–21], which also leads to an increase in the deformability of structures.

In the existing methods of structural mechanics, when calculating the deflections of beams, the description of their curved axis is carried out by a second-order equation and since curvature is found as a second-order derivative, the curvature, and therefore the stiffness, will be described by a constant, which makes it impossible to take into account changes in stiffness along the span. To obtain an acceptable solution in this case, the beam is divided into sections and the stiffness is considered constant within each section. In this regard, these approaches [22–26], representing a stepwise description of the stiffness along the length, can in some cases lead to significant errors. Therefore, the correct accounting of stiffness parameters is important.

The purpose of this article is to develop and experimentally substantiate a new methodology for calculating the deformability of reinforced concrete beams exposed to prolonged force and environmental influences during operation.

To achieve this goal, the following tasks are solved:

- development of a methodology for calculating the deformability of reinforced concrete beams under prolonged force and environmental influences, taking into account the variable stiffness of the sections along the span;
- conducting experimental studies of the deformability of the images of reinforced concrete beams that have been under stress in aggressive environments for a long time;
- verification of the developed methodology by comparing theoretical and experimental data on the deformability of samples of reinforced concrete beams with corrosion damage.

### 2. Methods

#### 2.1. Calculation Procedure

The calculation procedure implements the method of predetermined deformation. In it, depending on the grip conditions, the curvature or the rotation angle of one of the bent reinforced concrete element sections is considered as a known parameter. And the unknown parameter is external load module $F$, determined from the expression

$$ F^i = F \cdot F^0, $$

where: $F$ is the vector of the external load acting on the element; $F^0$ is the unit load vector.

In order to consider more accurately non-linear deformation of concrete and uneven accumulation of damage along the span, we will approximate the curved axis of the beam with a fourth degree polynomial. Such a function will allow us to take into account in the calculation the influence on the deflection of the transverse force and the distribution of stiffness in the span along the parabola. In the end it will allow a more correct assessment of the deformability parameters.

Let us split a single-span beam along the length into $n$ equal sections (figure 1), on each of them the deformability parameters vary according to the following functions

$$ y = \sum_{i=1}^{5} A_i x^{i-1}; $$

(2)
\[
\varphi = \frac{dy}{dx} = \sum_{i=1}^{5} (i-1) A_i x^{i-2};
\]

\[
\kappa = \frac{d^2 y}{dx^2} = \sum_{i=1}^{5} (i-1)(i-2) A_i x^{i-3};
\]

\[
\frac{Q}{B} = \frac{d^3 y}{dx^3} = \sum_{i=1}^{5} (i-1)(i-2)(i-3) A_i x^{i-4},
\]

where: \(y, \varphi\) is respectively, the deflection and angle of rotation of the curved axis of the beam at a distance \(x\) from the beginning of the section; \(\kappa\) is the curvature in this section; \(Q\) is the transverse force in cross section; \(B\) is section stiffness; \(A_i\) is coefficients polynomials determined by the values of the initial parameters.

Consider the first section of the beam. In section 1 \(x = 0\), then from equations (2-5) we find:

\[
A_1 = y_1, \quad A_2 = \varphi_1, \quad A_3 = \frac{\kappa}{2}, \quad A_4 = \frac{Q}{6B_1}.
\]

In section 2 \(x = \Delta\), then from equation (5) we obtain

\[
A_2 = \frac{1}{24\Delta} \left( \frac{Q}{B_2} - \frac{Q}{B_1} \right).
\]

We define equations (2-4) for cross section 2 of zone 1.

\[
y_2 = y_1 + \varphi_1 \Delta + \frac{\kappa}{2} + \frac{Q^8 \Delta^3}{8B_1} + \frac{Q^2 \Delta^3}{24B_2};
\]

\[
\varphi_2 = \varphi_1 + \kappa \Delta + \frac{Q^8 \Delta^2}{3B_1} + \frac{Q^2 \Delta^2}{6B_2};
\]

\[
\kappa_2 = \kappa_1 + \frac{Q^8 \Delta}{2B_1} + \frac{Q^2 \Delta}{2B_2}.
\]

We define equations (2-4) for cross section 3 of zone 2 taking into account (6-8) using the property of equality of derivatives.

\[
y_3 = y_1 + 2\varphi_1 \Delta + \frac{3\kappa}{2} + \frac{\kappa}{2} + \frac{11Q^8 \Delta^3}{24B_1} + \frac{5Q^2 \Delta^3}{24B_2} + \frac{5Q^8 \Delta^3}{8B_2} + \frac{Q^2 \Delta^3}{24B_3};
\]

\[
\varphi_3 = \varphi_1 + \kappa_1 \Delta + \frac{Q^8 \Delta^2}{3B_1} + \frac{Q^2 \Delta^2}{3B_2} + \frac{Q^8 \Delta^2}{6B_2} + \frac{Q^2 \Delta^2}{6B_3};
\]

\[
\kappa_3 = \kappa_1 + \frac{Q^8 \Delta}{2B_1} + \frac{Q^2 \Delta}{2B_2} + \frac{Q^8 \Delta}{2B_3}.
\]

Writing similar to (6-8) equations and equations (9-11) for the following sections, we obtain formulas for determining the deflection, rotation angle, and curvature in a certain section \(j\).

\[
y_j = y_1 + (j-1) \varphi_1 \Delta + \sum_{k=1}^{j-1} \frac{(j-2k-1)\kappa}{2} + \sum_{m=2}^{j-1} \frac{(4j-4m+1)Q^8 \Delta^3}{24B_m} + \sum_{s=1}^{j-1} \frac{(8j-8s-5)Q^8 \Delta^3}{24B_s};
\]
\[ \varphi_j = \varphi_i + \sum_{k=i}^{j-1} \frac{N_j}{B_k} \Delta^2 + \sum_{m=2}^{j} \frac{Q_L^2 \Delta^2}{B_m} + \sum_{k=1}^{j-1} \frac{Q_R^2 \Delta^2}{B_j}. \] (13)

\[ N_j = N_i + \Delta \left( \frac{Q_L^2}{2B_i} + \frac{Q_R^2}{2B_j} + \sum_{k=2}^{j-1} \frac{Q_L^2 + Q_R^2}{2B_k} \right). \] (14)

We write equation (12) for the section with serial number \( n+1 \) according to figure 1.

\[ y_{n+1} = y_1 + n \varphi \Delta + \sum_{k=2}^{n} \frac{(2n-2k+1)N_k \Delta^2}{2} + \sum_{m=2}^{n+1} \frac{(4n-4m+5)Q_{m}^L \Delta^3}{24B_m} + \sum_{k=1}^{n} \left( \frac{8n-8s+3}{24B_s} \right). \] (15)

The following boundary conditions are valid for a single-span pin-ended beam:

\[ y_1 = y_{n+1} = 0; \] (16)
\[ \varphi_1 = -\varphi_{n+1}; \] (17)
\[ N_1 = N_{n+1} = 0. \] (18)

It should be noted that (17) is valid only for symmetric force and environmental influences relative to the cross section in the middle of the beam span.

We take the angle of rotation in section 1 (on the left support) as a given parameter, then (15) taking into account (16) and (18) will take the form

\[ n \varphi \Delta + \sum_{k=2}^{n} \frac{(2n-2k+1)N_k \Delta^2}{2} + \sum_{m=2}^{n+1} \frac{(4n-4m+5)Q_{m}^L \Delta^3}{24B_m} + \sum_{k=1}^{n} \left( \frac{8n-8s+3}{24B_s} \right) = 0. \] (19)

Having expressed the curvature as \( \kappa = \frac{M}{B} \), we rewrite (19).

\[ n \varphi \Delta + \sum_{k=2}^{n} \frac{(2n-2k+1)M_k \Delta^2}{2B_k} + \sum_{m=2}^{n+1} \frac{(4n-4m+5)Q_{m}^L \Delta^3}{24B_m} + \sum_{k=1}^{n} \left( \frac{8n-8s+3}{24B_s} \right) = 0. \] (20)

Represent the bending moment and shear force in various sections as

\[ M_j = F \cdot f_{M_j}; \] (21)
\[ Q_j = F \cdot f_{Q_j}; \] (22)

where \( f_{M_j} \) and \( f_{Q_j} \), respectively, are functions for describing the nature of the epure of the bending moment and shear force, which are set depending on the load application scheme.

In view of (21) and (22), equation (20) takes the form

\[ n \varphi \Delta + \sum_{k=2}^{n} \frac{(2n-2k+1)F \cdot f_{M_k} \Delta^2}{2B_k} + \sum_{m=2}^{n+1} \frac{(4n-4m+5)F \cdot f_{Q_m}^L \Delta^3}{24B_m} + \sum_{k=1}^{n} \left( \frac{8n-8s+3}{24B_s} \right) = 0. \] (23)

from here

\[ F = \frac{n \Delta \varphi_i}{\sum_{k=2}^{n} \frac{(2k-2n+11)M_k \Delta^2}{2B_k} + \sum_{m=2}^{n+1} \frac{(4m-4n-5)Q_{m}^L \Delta^3}{24B_m} + \sum_{k=1}^{n} \left( \frac{8s-8n-3}{24B_s} \right) \}. \] (24)

Thus, based on the functions describing the nature of the epures of the bending moment and the shear force, the value of the external load is determined from the given values of the angle of rotation on the support. Further, by the formula (12), the value of the deflection in the required section is established.

This technique allows you to perform calculations of the parameters of reinforced concrete elements bending deformability, taking into account different corrosion damage of normal sections along the span. For this, different stiffnesses are assigned to the corresponding sections.
2.2. Methods of experimental research

In order to obtain experimental data for assessing the accuracy of the developed calculation procedure, special experimental studies were carried out on the deformability of reinforced concrete beams samples that were under a static load for a long time in various aggressive environments.

Experimental studies were carried out in accordance with the developed program, which provides for testing reinforced concrete beams samples in an amount of 27 pcs. The samples were made of a rectangular cross-section of 60x100 (h) mm with a calculated span of \( l_0 = 1400 \) mm.

After manufacturing, all samples were stored for 28 days in one laboratory with normal temperature and humidity conditions, after which they were divided into one control (11 pcs.) and two main (8 pcs.) groups of twin beams. Then, the first series of tests of reinforced concrete beams with a short-term load on static bending before failure was carried out, which included three samples from control group, the results of which determined the actual value of the design breaking load \( P_u^{BK:0} \). The remaining 24 samples were loaded and placed for a certain period of time in various environmental conditions.

The samples of the control group were in the same laboratory as during strength gain. Samples of the first and second main groups were respectively in sulfate and chloride containing aggressive media, artificially created in the room to accumulate corrosion damages of concrete and steel reinforcement.

The subsequent series of tests of reinforced concrete beams with a static bending load before failure included two samples from each group. Thus, four more test series of six samples of reinforced concrete beams were formed. In these experimental studies, the main variable parameter was the duration of power and environmental impacts on the samples, equal to 180, 360, 720 and 1080 days from the moment they gained design strength. The accepted samples marking of reinforced concrete beams and series numbers of their tests with load before failure are given in table 1.

| Sample group | Type of environment | Reinforced concrete beams samples marking, taking into account the impact duration and the number of the test series of the load before failure |
|--------------|---------------------|-------------------------------------------------------------------------------------------------------------------------------------|
| Control (BK) | Non-aggressive       | BK-0-1  0 days, series no. 1  BK-180-1  180 days, series no. 2  BK-360-1  360 days, series no. 3  BK-720-1  720 days, series no. 4  BK-1080-1  1080 days, series no. 5 |
|              | environment         |                                                                                                                                     |
|              | BK-0-2              | BK-180-2  0 days, series no. 1  BK-360-2  180 days, series no. 2  BK-720-2  360 days, series no. 3  BK-1080-2  720 days, series no. 4  BK-1080-2  1080 days, series no. 5 |
|              | BK-0-3              |                                                                                                                                     |
| Main no. 1  | Sulphate            | BOS-180-1  0 days, series no. 1  BOS-360-1  180 days, series no. 2  BOS-720-1  360 days, series no. 3  BOS-1080-1  720 days, series no. 4  BOS-1080-2  1080 days, series no. 5 |
| (BOS)       | aggressive          |                                                                                                                                     |
|              | environment         | BOS-180-2  0 days, series no. 1  BOS-360-2  180 days, series no. 2  BOS-720-2  360 days, series no. 3  BOS-1080-2  720 days, series no. 4  BOS-1080-2  1080 days, series no. 5 |
| Main no. 2  | Chloride            | BOH-180-1  0 days, series no. 1  BOH-360-1  180 days, series no. 2  BOH-720-1  360 days, series no. 3  BOH-1080-1  720 days, series no. 4  BOH-1080-2  1080 days, series no. 5 |
| (BOH)       | aggressive          |                                                                                                                                     |
|              | environment         | BOH-180-2  0 days, series no. 1  BOH-360-2  180 days, series no. 2  BOH-720-2  360 days, series no. 3  BOH-1080-2  720 days, series no. 4  BOH-1080-2  1080 days, series no. 5 |

Reinforced concrete beams reinforcement was performed by flat single frames with working stretched reinforcement in the form of rods Ø8 mm of A240 class. To ensure anchorage, the ends of the rods are brought out beyond the supporting section of the beams by 90 mm with a bend length of 70 mm. As compressed reinforcement, Ø4 mm rods made of class B500 wire are structurally installed. The concrete protective layer for stretched and compressed reinforcement was adopted 10 mm. The transverse reinforcement is made in the form of clamps from rods Ø3 mm of class A240, installed with a constant pitch of 40 mm. The reinforcement scheme of laboratory beams samples is shown in figure 2.

For all samples manufacture, concrete of class compressive strength B20 is used. At the same time, fine grinding sodium chloride (NaCl) in a ratio up to 5% from cement weight was introduced into the concrete mixture intended for beams samples of the main group no. 2 (BOH). This did not lead to a significant decrease in the strength of concrete, but under wet conditions it contributed to steel reinforcement corrosion initiation.
After the first series of tests, based on the condition of the same rigidity, the remaining beams with the help of hydraulic jack and cross beam were pairwise spanned together by a force equal to $0.6P_{0.86}$ (figure 3, a), which was further supported by bands (figure 3, b). The force control transmitted to the samples was carried out according to the clock-type indicator readings of the master proving ring of pressure.

The design models for reinforced concrete beams during long-term tests and short-term load tests before failure are assumed to be the same: a single-span hinged-supported beam, loaded with two concentrated forces with the formation in clean bending zone of $0.164l_0$ a span (figure 3, c).

In order to initiate the development of corrosion processes in concrete and steel reinforcement, a special stand for long-term testing was made (figure 4). Its basic element was corrosion-resistant polymer baths with liquid electrolyte. A 2% aqueous solution of sulphuric acid ($H_2SO_4$) was in one bath. It was intended to simulate a sulfate aggressive medium effect on the samples of BOS group, the other bath with distilled water was intended to simulate the effect of a chloride aggressive environment on the samples of BOH group containing an additive of sodium chloride ($NaCl$) in concrete. The samples were sink every day and at the same time for 15-20 minutes into the electrolyte baths for moistening, after which they were lifted and allowed to dry using conditions under which the...
control group samples were stored. The densities of electrolytes during the experiment were constant and were monitored by a hydrometer. All metal elements on reinforced concrete beams, ensuring the constancy of the design model and the power load magnitude, were coated with anticorrosive compounds. The number of damping-drying cycles was determined by the number of days from the long-time tests beginning to the reinforced concrete beams removal for destruction by a short-time load.

At the end of long-time tests and samples unloading, the residual values of the deformation characteristics, mass, section sizes were measured, concrete strength was determined by non-destructive methods on all faces.

For reinforced concrete beams tests with a short-time bending load before failure, a test bench was made, the spatial rigidity and stability of which is many times higher than the prototypes rigidity. The general tests view is shown in figure 5.

**Figure 4.** Stand for long-time samples tests.

**Figure 5.** General view of reinforced concrete beams test before failure.
To measure the deflections of reinforced concrete beams at different points of span, dial gauges ICh-10 and a deflection meter 6PAO with a scale division value of 0.01 mm were installed. Tests were carried out in accordance with the provisions of Russian State Standard GOST 8829 with step loading. After bringing the beams to failure, various parameters of corrosion damage to structural materials were established. By the indicator method, according to the saturation of the color of the cross section with a 1% phenolphthalein solution, at all its faces the depth at which the concrete properties changed, was determined. Next, reinforcing bars, which determined the depth of corrosion, weight loss, and the change in the deformation-strength characteristics of steel reinforcement, were removed from the beams.

3. Results and Discussion
During long-time tests, under influence of aggressive sulfate environment, steel reinforcement in concrete reinforced beams was in a passive state.

The initiation of corrosion of steel reinforcement in concrete of reinforced concrete beams occurred during prolonged exposure to a corrosive chloride environment.

The biggest steel reinforcement damages were found in sections with operational cracks in concrete. A graph-based mapping of development rate of corrosion damages to the accepted steel reinforcement during chloride aggression, taking into account the existing stresses, is presented in figure 6.

![Figure 6. General view of reinforced concrete beams test before failure.](image)

Note: in figure 6 markers indicate the points, obtained during experimental studies, and the solid lines - calculated curves.

![Figure 7. The change nature in the deformation diagrams of steel reinforcement.](image)
The change nature in deformation diagrams of steel reinforcement obtained by tests of rod samples on axial tension is shown in figure 7. After a long-time influence to reinforced concrete beams of an external loading and a chloride aggressive environment, the deformation-strength characteristics of steel reinforcement remained within the limits of variations of the corresponding characteristics of control samples. In this case, a decrease in the length of the yield area is observed.

The concrete corrosion, related with the aggressive effect modeling of chloride on steel reinforcement, proceeded according to the first type, in which salt spots of dissolved cement stone components appeared on the samples surface. Damages accumulation according to the scheme without complete concrete destruction and during the long-time tests was insignificant.

Under the influence of a sulfate-containing aggressive environment, concrete corrosion took place in according to the third form, in which the resulting chemical reaction products settled in the concrete structure, which eventually led to its complete destruction. The corrosion damage accumulation corresponded to the design scheme, which has three zones with different destruction degree. At the same time, in the course of experimental studies it was found that for this scheme the function of the concrete damage coefficient $K^c(z)$ in the transition zone, where the composite resistance is partially preserved, does not take values equal to zero.

The experimental studies results of the concrete state in reinforced concrete beams and unloaded auxiliary samples during various operating environments are presented in tables 2 and 3.

According to the data obtained, the change in the strength and deformation characteristics of concrete and corrosion damage development in it with time depends on the operating environment type, the sign and level of acting stresses. The concrete properties greatest degradation is observed under the sulfate aggressive environment influence. The maximum and minimum depth of concrete corrosion damage is determined respectively in the stretched and compressed zones of the sections of reinforced concrete beams. The concrete damages in unloaded auxiliary samples and in beams at the neutral axis level was approximately equal. The advance of the corrosion front deep into the concrete was uniform at all faces, with the exception of the side faces of the cross sections of the bent elements, where the sign and level of stress changes in height.

### Table 2. The concrete strength and deformation characteristics in various operating environments.

| Concrete characteristics | Operating environment | Duration of power and environmental influences $t$, days |
|--------------------------|-----------------------|-------------------------------------------------------|
|                          |                       | 0  | 180 | 360 | 720 | 1080 |
| Cube compressive strength $R$, MPa | Non-aggressive | 31.0 | 39.2 | 39.6 | 39.9 | 40.1 |
|                          | Sulfate               |    | 29.6 | 28.4 | 26.9 | 25.8 |
|                          | Chloride              |    | 38.4 | 38.9 | 39.1 | 38.9 |
| Prism compressive strength $R_p$, MPa | Non-aggressive | 23.8 | 30.4 | 30.7 | 30.9 | 31.0 |
|                          | Sulfate               |    | 23.7 | 22.7 | 21.5 | 20.6 |
|                          | Chloride              |    | 29.6 | 29.9 | 30.0 | 29.7 |
| Tensile strength $R_t$, MPa | Non-aggressive | 2.3 | 2.9 | 3.0 | 0.00180 |
|                          | Sulfate               |    | 2.4 | 2.3 | 2.2 | 2.2 |
|                          | Chloride              |    | 2.9 | 2.9 | 3.0 | 2.9 |
| Initial modulus of elasticity $E$, MPa | Non-aggressive | 30000 | 35500 | 35000 | 36000 | 36000 |
|                          | Sulfate               | | 30500 | 30000 | 29500 | 29000 |
|                          | Chloride              | | 34000 | 34000 | 34500 | 34100 |
| Deformations $\varepsilon_{bR}$ | Non-aggressive | 0.00190 | 0.00193 | 0.00195 | 0.00195 |
|                          | Sulfate               | | 0.00190 | 0.00193 | 0.00200 | 0.00205 |
|                          | Chloride              | | 0.00195 | 0.00200 | 0.00205 | 0.00210 |
| Creep measure $C(t, 28) \cdot 10^6$, MPa$^{-1}$ | Non-aggressive | 77 | 90 | 94 | 94 |
|                          | Sulfate               | | 91 | 106 | 110 | 112 |
|                          | Chloride              | | 79 | 92 | 96 | 96 |
| Shrinkage deformations $\varepsilon_{pR}(t, 28)$ | Non-aggressive | 0.00018 | 0.00019 | 0.00019 | 0.00019 |
|                          | Sulfate               | | 0.00017 | 0.00018 | 0.00018 | 0.00018 |
|                          | Chloride              | | 0.00017 | 0.00018 | 0.00018 | 0.00018 |
Table 3. Kinetics characteristics of concrete corrosion damages in various operating environments.

| Concrete damages characteristics | Aggressive environment | Concrete stress $\sigma_b$ | Duration of power and environmental influences $t$, days |
|----------------------------------|------------------------|----------------------------|--------------------------------------------------------|
| Corrosion damage depth $\delta$, mm (including completely destroyed concrete layer thickness) | Sulfate | $\sigma_{bt} \approx R_{bt}$ | 6.5 (1.8) 10.5 (2.4) 12.5 (2.8) 13 (3.1) |
|                                  |chloride | $\sigma_{bt} = 0$ | 5 (1.6) 7.5 (2.1) 9 (2.4) 9.5 (2.6) |
|                                  |          | $\sigma_{bt} = 0.55R_{bt}$ | 3.5 (1.4) 5 (1.9) 5.5 (2.2) 5.5 (2.4) |
| Damage coefficient $K_{\text{min}}$ | Sulfate | – | 0.78 0.74 0.70 0.67 |
|                                  | chloride | – | 0.97 0.97 0.97 0.96 |

A graphic representation of the corrosion damages development rate in concrete under various operating environments is shown in figure 8. It should be noted that concrete damages accumulated under sulfate-containing environment influence at a lower development depth have a higher destruction degree than damages received in a chloride-containing environment. Herewith, both types of concrete damages have a damping in time character.

![Figure 8](image-url) The corrosion damages development rate in concrete of class B20 with liquid sulfate (left) and chloride (right) aggression.

Note: in figure 8, the markers indicate the points obtained during the experimental studies, and the solid lines - calculated curves.

![Figure 9](image-url) Deformations in tensile concrete and reinforcement.

The deformations development in concrete and tensile reinforcement in the area between cracks at different levels of beam loading is shown as an example in figure 9, from which it can be seen that until the ratio $P / P_u \approx 0.85$, it proceeded at approximately the same rate (deviations do not exceed 25%). At the stage preceding the samples destruction, a partial breaking in the concrete and...
reinforcement deformation can be neglected due to many factors affecting the readings of tensiometers, and structural materials deformation as a whole can be considered the same with an acceptable error.

The value of the operational load on the beams during long-term tests was equal to 3.768 kN, which amounted to 60% of the breaking load of control samples at 28 days $P_u^{BK-0}$. At this load, at the beginning of the tests, the deflection of the beams in the middle of the span averaged 4.38 mm. The deflection values at the end of lengthy tests are given in table 4.

| Duration of power and environmental influences before failure testing, days | Control group (Non-aggressive environment) | Main group no. 1 (Sulphate-aggressive environment) | Main group no. 2 (Chloride-aggressive environment) |
|---|---|---|---|
| Sample marking | Deflection at the end of long tests | Sample marking | Deflection at the end of long tests | Sample marking | Deflection at the end of long tests |
| 0 | BK-0-1 | 4.54 | $P = 3.768\text{ kN}$ | BOH-0-1 | 6.05 | 3.40 |
| | BK-0-2 | 4.17 | | | | |
| | BK-0-3 | 4.44 | | | | |
| 180 | BK-180-1 | 5.43 | $P = 0\text{ kN (residual deflection)}$ | BOH-180-1 | 5.94 | 3.54 |
| | BK-180-2 | 5.55 | BOS-180-1 | 6.26 | 4.02 | |
| | BK-180-2 | 5.80 | BOS-180-2 | 5.39 | 3.96 | |
| 360 | BK-360-1 | 5.68 | BOS-360-1 | 6.14 | 3.96 | |
| | BK-360-2 | 5.63 | BOS-360-2 | 6.14 | 3.96 | |
| 720 | BK-720-1 | 5.79 | BOS-720-1 | 6.25 | 4.16 | |
| | BK-720-2 | 5.75 | BOS-720-2 | 6.57 | 4.20 | |
| 1080 | BK-1080-1 | 5.86 | BOS-1080-1 | 6.36 | 4.52 | |
| | BK-1080-2 | 5.79 | BOS-1080-2 | 6.67 | 4.54 | |

According to the data obtained, the most intense increase in the deflections of the beams occurred in the first year of testing, which is explained by the active development of creep deformations, crack formation and the accumulation of corrosion damage to concrete and steel reinforcement. In the future, the increase in deflections is of a decaying nature.

The deflections of the control beams from the long-term load when compared with the deflections of the beams of the main groups were less throughout the entire observation period. The difference with the deflections of the beams located in sulfate and chloride-containing aggressive media was respectively: 4.4% and 2.9% in 180 days; 9.5% and 6.0% at 360 days; 11.1% and 6.0% at 360 days; 11.8% and 11.7% in 1080 days. Lower deflections of the control beams are explained by the absence of corrosion damage to concrete and reinforcement.

After removing a long load ($P = 0\text{ kN}$), the deflections were not completely restored to their original values, therefore, the unloaded beams before the short-term loading before failure had a residual deflection.

The destruction of all samples of reinforced concrete beams occurred in the zone of clean bending along normal sections. Based on the test results, experimental diagrams of short-term beam deformation were obtained, examples of which are shown in figure 10.

As can be seen from the experimental diagrams, at the beginning of loading, all samples after prolonged exposure have a linear character of deformation. It is due to the presence of unclosed cracks and a selection of plastic deformations of concrete. After reaching the load value corresponding to the value of the operational load during long-term tests, between the steps of loading during the exposure of the beams, an increase in deflections is observed, which is explained by the creep of concrete,
corrosion damage, the development of cracks and, as a consequence, a decrease in the rigidity of normal sections. Along with that, the angle of climb of the line diagrams decreases.

![Figure 10. Deformations in tensile concrete and reinforcement.](image)

When the load is close to the breaking stress in the reinforcement, the yield strength is reached, after which there is a significant increase in deflections with a slight increase in the load. The line of diagrams in this section is almost linear with a small angle of elevation.

With an increase in the values of corrosion damage in the main samples, an increase in the deflections occurs in comparison with the control samples. For beams with steel reinforcement damage, this occurs due to the earlier yield stresses in the reinforcement due to a decrease in the cross-sectional area and partial loss of adhesion between concrete and reinforcement, and for beams with concrete damage due to a decrease in the total bending stiffness [3, 4, 6, 15–17].

**Table 5.** Comparison of experimental and calculated values of reinforced concrete sample beams deflections in the middle of the span.

| Samples series                  | Observation value $f_{mm}^{obs}$ / $f_{mm}^{obs}$ | Calculated value $f_{mm}^{cal}$ / $f_{mm}^{cal}$ | $f_{mm}^{cal}$ / $f_{mm}^{cal}$ | $f_{mm}^{cal}$ / $f_{mm}^{cal}$ |
|---------------------------------|--------------------------------------------------|-------------------------------------------------|--------------------------------|--------------------------------|
| Non-aggressive operating environment (the control group samples) |                                                   |                                                 |                                |                                |
| BK-0                            | 4.38 / 5.45                                      | 4.44 / 5.38                                     | 1.014 / 0.987                  |                                |
| BK-180                          | 5.49 / 6.22                                      | 5.41 / 6.31                                     | 0.983 / 1.014                  |                                |
| BK-360                          | 5.66 / 6.35                                      | 5.60 / 6.54                                     | 0.989 / 1.030                  |                                |
| BK-720                          | 5.77 / 6.45                                      | 5.74 / 6.64                                     | 0.995 / 1.029                  |                                |
| BK-1080                         | 5.83 / 6.51                                      | 5.79 / 6.70                                     | 0.993 / 1.029                  |                                |
| Samples group average           |                                                   |                                                 | 0.995 / 1.018                  |                                |
| Samples group root-mean-square deviation | 0.0117 / 0.0185                               |                                                 |                                |                                |
| Sulphate-containing aggressive environment (samples of main group no. 1) |                                                   |                                                 |                                |                                |
| BOS-180                         | 5.73 / 6.34                                      | 5.82 / 6.75                                     | 1.016 / 1.064                  |                                |
| BOS-360                         | 6.20 / 6.94                                      | 6.34 / 7.32                                     | 1.023 / 1.055                  |                                |
| BOS-720                         | 6.41 / 7.13                                      | 6.64 / 7.60                                     | 1.036 / 1.066                  |                                |
| BOS-1080                        | 6.52 / 7.23                                      | 6.77 / 7.73                                     | 1.038 / 1.069                  |                                |
| Samples group average           |                                                   |                                                 | 1.028 / 1.064                  |                                |
| Samples group root-mean-square deviation | 0.0105 / 0.0060                               |                                                 |                                |                                |
| Chloride-containing aggressive environment (samples of main group no. 2) |                                                   |                                                 |                                |                                |
| BOH-180                         | 5.65 / 6.17                                      | 5.52 / 6.42                                     | 0.977 / 1.041                  |                                |
| BOH-360                         | 6.00 / 6.57                                      | 5.93 / 6.85                                     | 0.988 / 1.043                  |                                |
| BOH-720                         | 6.23 / 6.91                                      | 6.26 / 7.22                                     | 1.005 / 1.045                  |                                |
| BOH-1080                        | 6.52 / 7.09                                      | 6.46 / 7.46                                     | 0.991 / 1.052                  |                                |
| Samples group average           |                                                   |                                                 | 0.990 / 1.045                  |                                |
| Samples group root-mean-square deviation | 0.0115 / 0.0048                               |                                                 |                                |                                |
| Average of all samples          |                                                   |                                                 | 1.004 / 1.040                  |                                |
| Root-mean-square deviation of all samples     | 0.0200 / 0.0230                               |                                                 |                                |                                |

In order to implement numerically the developed calculation methodology, a calculation algorithm was compiled and a computer program was developed. Using them, taking into account the resulting
corrosion damage, the calculated values of the deflections of reinforced concrete beams at various points of passage at all loading levels are determined.

To assess the accuracy of the proposed calculation method, an analytical comparison was made of the average experimental and calculated values of deflections in the middle of the span at loading levels equal to 0.6 and 0.7 of the breaking load of the $P_u^{BK,0}$ control samples. These data are given in table 5.

As it can be seen from the table, the average deviation between the corresponding experimental and calculated values of the deflections is about 4%, the maximum deviation does not exceed 7%. The standard deviation does not exceed 2.5%.

4. Conclusions
On the basis of the method of predetermined deformations, a new methodology has been developed for calculating the deformability of reinforced concrete beams, which are exposed to long-term force and environmental influences during operation, allowing to take into account the variable stiffness of the sections within the span.

New experimental data on the deformability of samples of reinforced concrete beams that have been under stress in sulfate and chloride-containing aggressive media for a long time have been obtained. The most intense increase in the deflections of the beams occurred in the first year of testing, which is associated with a more active development of creep deformations and the accumulation of corrosion damage to concrete and reinforcement during this period. In the future, the increase in deflections has a decaying nature.

After comparing the experimental and calculated values of the deflections of the samples, it should be concluded that the calculation method proposed in the work has sufficient accuracy and can be used for practical calculations of bent reinforced concrete elements operated under prolonged power and environmental influences.

References
[1] Petschke T, Corres H, Ezeberry J I, Perez A and Recupero A 2013 Expanding the classic moment-curvature relation by a new perspective onto its axial strain Comput. Concr. 11 515–29
[2] Hamed E 2014 Modelling of creep in continuous RC beams under high levels of sustained loading Mech. Time-Dependent Mater. 18 589–609
[3] Sun J, Huang Q and Ren Y 2015 Performance deterioration of corroded RC beams and reinforcing bars under repeated loading Constr. Build. Mater. 96 404–15
[4] Hariche L, Ballim Y, Bouhicha M and Kenai S 2012 Effects of reinforcement configuration and sustained load on the behaviour of reinforced concrete beams affected by reinforcing steel corrosion Cem. Concr. Compos. 34 1202–9
[5] Bichara L, Saad G and Slika W 2019 Probabilistic identification of the effects of corrosion propagation on reinforced concrete structures via deflection and crack width measurements Mater. Struct. 52 89
[6] Dong J, Zhao Y, Wang K and Jin W 2017 Crack propagation and flexural behaviour of RC beams under simultaneous sustained loading and steel corrosion Constr. Build. Mater. 151 208–19
[7] Val D V, and Chernin L 2009 Serviceability Reliability of Reinforced Concrete Beams with Corroded Reinforcement J. Struct. Eng. 135 896–905
[8] Karpenko N I, Karpenko S N, Yarmakovskiy V N and Yerofeyev V T 2015 The modern methods for ensuring of the reinforced concrete structures durability Acad. Archit. Constr. 1 96–102
[9] Bondarenko V M and Kolchunov V I 2013 The concept and directions of development of the theory of structural safety of buildings and structures under the influence of force and environmental factors Ind. Civ. Eng. 2 28–31
[10] Travush V, Emelianov S, Kolchunov V and Bulgakov A 2016 Mechanical Safety and Survivability of Buildings and Building Structures under Different Loading Types and Impacts Procedia Eng. 164 416–24
[11] Wei A, Wang Y and Tan M Y J 2015 Monitoring corrosion of reinforced concrete beams in a chloride containing environment under different loading levels Struct. Monit. Maint. 2 253–67
[12] Park H-G, Hwang H-J, Hong G-H, Kim Y-N and Kim J-Y 2012 Immediate and Long-Term Deflections of Reinforced Concrete Slabs Affected by Early-Age Loading and Low Temperature ACI Struct. J. 109 413–22
[13] Shariq M, Abbas H and Prasad J 2019 Effect of magnitude of sustained loading on the long-term deflection of RC beams Arch. Civ. Mech. Eng. 19 779–91
[14] Morenon P, Multon S, Sellier A, Grimal E, Hamon F and Kolmayer P 2019 Flexural performance of reinforced concrete beams damaged by Alkali-Silica Reaction Cem. Concr. Compos. 104 103412
[15] El Maaddawy T, Soudki K and Topper T 2005 Long-term performance of corrosion-damaged reinforced concrete beams ACI Struct. J. 102 649–56
[16] Jung J-S, Lee B Y and Lee K-S 2019 Experimental Study on the Structural Performance Degradation of Corrosion-Damaged Reinforced Concrete Beams Adv. Civ. Eng. 2019 1–14
[17] Liu Y, Jiang N, Deng Y, Ma Y, Zhang H and Li M 2016 Flexural experiment and stiffness investigation of reinforcing concrete beam under chloride penetration and sustained loading Constr. Build. Mater. 117 302–10
[18] Shi J, Ming J, Sun W and Zhang Y 2017 Corrosion performance of reinforcing steel in concrete under simultaneous flexural load and chlorides attack Constr. Build. Mater. 149 315–26
[19] Lundgren K, Kettit P, Hanjari K Z, Schlune H and Roman A S S 2012 Analytical model for the bond-slip behaviour of corroded ribbed reinforcement Struct. Infrastruct. Eng. 8 157–69
[20] Khan I, François R and Castel A 2014 Experimental and analytical study of corroded shear-critical reinforced concrete beams Mater. Struct. 47 1467–81
[21] Kearsley E P and Joyce A 2014 Effect of corrosion products on bond strength and flexural behaviour of reinforced concrete slabs J. South African Inst. Civ. Eng. 56
[22] Mari A R, Bairán J M and Duarte N 2010 Long-term deflections in cracked reinforced concrete flexural members Eng. Struct. 32 829–42
[23] Zhou W and Kokai T 2010 Deflection calculation and control for reinforced concrete flexural members Can. J. Civ. Eng. 37 131–4
[24] Gribniak V, Bacinskas D, Kacianauskas R, Kaklauskas G and Torres L 2013 Long-term deflections of reinforced concrete elements: accuracy analysis of predictions by different methods Mech. Time-Dependent Mater. 17 297–313
[25] Gribniak V, Cervenka V and Kaklauskas G 2013 Deflection prediction of reinforced concrete beams by design codes and computer simulation Eng. Struct. 56 2175–86
[26] Castel A and François R 2013 Calculation of the Overall Stiffness and Irreversible Deflection of Cracked Reinforced Concrete Beams Adv. Struct. Eng. 16 2035–42