Qubit Measurement by Multi-Channel Driving

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We theoretically propose and experimentally implement a method to measure a qubit by driving it close to the frequency of a dispersively coupled bosonic mode. The separation of the bosonic states corresponding to different qubit states begins essentially immediately at maximum rate, leading to a speedup in the measurement protocol. Also the bosonic mode can be simultaneously driven to optimize measurement speed and fidelity. We experimentally test this measurement protocol using a superconducting qubit coupled to a resonator mode. For a certain measurement time, we observe that the conventional dispersive readout yields above 100\% higher measurement error than our protocol. Finally, we use an additional resonator drive to leave the resonator state to vacuum if the qubit is in the excited state during the measurement protocol. This suggests that the proposed measurement technique may become useful in unconditionally resetting the resonator to a vacuum state after the measurement pulse.

Introduction—Since the birth of quantum mechanics, the quantum measurement and the related collapse of the wavefunction has puzzled scientists [1, 2], culminating in various interpretations of quantum mechanics such as that of many worlds [3]. With the recent rise of quantum technology [4–6], the quantum measurement has become one of the most important assets for practical applications. For example, measurements of single qubits are the key in reading out the results of quantum computations [7–10] and parity measurements in multi-qubit systems are frequently required in quantum error correction codes such as the surface and color codes [11–15]. Furthermore, single-qubit measurement and feedback can be used to reset qubits [16–18] or even solely provide the non-linearity needed to implement multi-qubit gates [19–21].

One of the most widespread ways to measure qubits is to couple them to one or several bosonic modes, such as those of the electromagnetic field, and to measure their effect on the radiation [22]. This method is currently used, for example, in quantum processors based on superconducting circuits [23–28], quantum dots [29–31], and trapped ions [32]. Especially with the rise of circuit quantum electrodynamics [33, 34], this measurement technique has become available to many different hybrid systems such as mechanical oscillators [35, 36] and magnons [37].

Theoretically, the interacting system of a qubit and a bosonic mode is surprisingly well described by the Jaynes–Cummings model [38, 39]. If the qubit frequency is far detuned from the mode frequency, i.e., we operate in the dispersive regime, the interaction term renders the mode frequency to depend on the qubit state. Consequently, a straightforward way to implement a non-demolition measurement on the qubit state is to drive the mode at a certain frequency close to the resonance and measure the phase shift of the output field with respect to the driving field. This kind of dispersive measurement has been extremely successful, for example, in superconducting qubits [40] with increasing accuracy and speed [24, 41–43] currently culminating to 99.2\% fidelity in 88 ns [26].

In the dispersive measurement, one of the key issues has been the ability to quickly populate the bosonic mode in the beginning of the measurement protocol [24] without surpassing the critical photon number, and to quickly evacuate the excitations from the mode after the measurement [17, 44]. These requirements point to the need of a fast, low-quality readout mode. However, this poses a trade-off on the qubit lifetime, which to some extent, can be answered using Purcell filters [24, 26, 45] with the cost of added circuit complexity. However, a simple and fast high-fidelity measurement scheme is of great interest not only to the field of superconducting qubits, but also to other quantum technology platforms utilizing bosonic modes as the measurement tool.

Inspired by our recent work [46] on quickly stabilizing resonator states by a qubit drive, we propose in this letter a qubit measurement protocol which is based on driving the qubit close to the mode frequency through a non-resonant channel. Owing to the dispersive coupling, the initial vacuum state of the resonator begins to rotate selectively on the qubit state about a point fully controlled by the strength and phase of the qubit drive. Importantly, this rotation begins immediately after the drive pulse arrives at the qubit with no bandwidth limitation imposed by the resonator. We demonstrate this non-demolition readout scheme in planar superconducting qubits and observe that it leads to a significantly reduced readout error. Furthermore, we discuss how this method can be used to unconditionally reset the resonator state into vacuum after the readout without...
FIG. 1. (a) Schematic presentation of the proposed readout scheme where the qubit is driven (blue color) at the frequency of a dispersively coupled bosonic mode. A compensation tone (brown color) on the resonator may be used to optimize the result. We consider the case where the detuning \( \Delta = \omega_q - \omega_r \) is much greater than the qubit–mode coupling strength \( g \).

Evolution of the mean of the resonator state in phase space for the conventional dispersive readout starting from a vacuum state (cross) provided that the qubit was prepared in \(|g\rangle\) (blue) or \(|e\rangle\) (red). (b) As (b) but the readout pulse is applied directly to the qubit. Thus, the resonator states start to rotate about a new virtual origin \( \alpha_{\text{vo}} \) (circle) leading to a faster separation. After measurement, we may reverse the sign of the virtual origin and wait for the resonator states corresponding to different qubit states to fully overlap (faint colors). A subsequent shift (brown color) finalizes an unconditional reset of the resonator.

The term \( \chi b^\dagger b \sigma_z \) corresponds to a rotation of the distribution \( Q(\alpha) \) in the complex plane \( \alpha \in \mathbb{C} \) about a virtual origin \( \alpha_{\text{vo}} \) with an angular frequency \( \chi \) in a direction determined by the qubit state. Thus driving the qubit at the resonator frequency \( \omega_r \) effectively shifts the origin of the resonator phase space to a point \( \alpha_{\text{vo}} \) in the rotating frame.

The virtual origin may be shifted very fast, causing the distributions \( Q_g \) and \( Q_e \) corresponding to the eigenstates \(|g\rangle\) and \(|e\rangle\) of the qubit to separate in the complex plane at least with the initial speed \( |\chi| \approx |\alpha_{\text{vo}}|/\Delta \), see Fig. 1(c). This minimum speed is achieved for an initial vacuum state in the resonator. For a finite-amplitude coherent state \( |\alpha_0\rangle \), the speed becomes \( |\chi| (|\alpha_{\text{vo}}| + |\alpha_0|) \) if we choose the direction of \( \alpha_{\text{vo}} \) opposite to that of \( \alpha_0 \). Since the Husimi Q distribution of any coherent state is Gaussian with width \( \sigma = 1/\sqrt{2} \), the overlap error related to our measurement scheme assuming no dissipation in the resonator and \( \tau \lesssim 1/|\chi| \) is approximately given by exp\((-2|t\chi\alpha_{\text{vo}}|^2)/(2\pi)\). This approximation suggests an error below \( 10^{-4} \) in \( 1/|\chi| \approx 100 \text{ ns} \) for \( |\alpha_{\text{vo}}| \sim 2 \).

The term \( \Omega_r \chi/g - \Omega_q \) \( \sigma_+ + \text{H.c.} \) in Eq. (2) shows that the drives slightly tilt the qubit Hamiltonian, while the driving term \( \Omega_r b^\dagger + \text{H.c.} \) acting on the resonator retains its form in comparison to Eq. (1). The tilt of the qubit Hamiltonian determines the speed at which

\[
\begin{align*}
H/\hbar &= \Delta \hat{\sigma}_+ \hat{\sigma}_- + (g \hat{a} \hat{\sigma}_+ + \Omega_q \hat{\sigma}_+ + \Omega_r \hat{a}^\dagger + \text{H.c.}),
\end{align*}
\]

where \( g \) denotes the qubit–resonator coupling strength, \( \hat{a}^\dagger \) and \( \hat{\sigma}_+ \equiv |e\rangle \langle g| \) are the creation operators of the resonator mode and of the qubit, respectively, and \( \hat{\sigma}_- = \hat{a} \). Above, we have introduced the rotating-wave approximation justified by \( g \ll \omega_q, \omega_r \).

By re-grouping the second and third terms in Eq. (1) as \( g (\hat{a} + \Omega_q/g) \hat{\sigma}_+ + \text{H.c.} \), and noting that the unitary displacement operator \( \hat{D}(x) = \exp (x \hat{a}^\dagger - x^* \hat{a}) \) acts on \( \hat{a} \) as \( \hat{D}(x) \hat{a} \hat{D}(x)^\dagger = \hat{a} - x \), we may interpret this term as the Jaynes–Cummings interaction term in a frame displaced by \( \alpha_{\text{vo}} = -\Omega_q/g \). To justify this argument more precisely, we employ the standard dispersive approximation [47] in the regime \( g \ll |\Delta| \). We show in Ref. [48] that this yields, up to constant energy terms, the Hamiltonian

\[
\begin{align*}
H''/\hbar &\approx \tilde{\Delta} \hat{\sigma}_+ \hat{\sigma}_- - \chi b^\dagger b \sigma_z \\
&\quad + \left[ \Omega_r \hat{a}^\dagger + (\Omega_r \chi/g + \Omega_q) \hat{\sigma}_+ + \text{H.c.} \right],
\end{align*}
\]

where \( \tilde{\Delta} = |\Delta| + \chi (1-2|\alpha_{\text{vo}}|^2) \) denotes the detuning shifted by the dispersive interaction, \( \chi = g^2/\Delta \), and we have defined the displaced mode operator \( \hat{b} = \hat{a} - \alpha_{\text{vo}} \).

Let us describe the resonator state using the Husimi Q representation [49] \( Q(\alpha) = \langle \alpha | \rho_\text{Q} | \alpha \rangle /\pi \), where \(|\alpha\rangle\) is a coherent state such that \( \hat{a} |\alpha\rangle = \alpha |\alpha\rangle \) and \( \rho_\text{Q} \) is the reduced density operator of the resonator. The evolution arising from the term \( -\chi b^\dagger b \sigma_z \) corresponds to a rotation of the distribution \( Q(\alpha) \) in the complex plane \( \alpha \in \mathbb{C} \) about a virtual origin \( \alpha_{\text{vo}} \) with an angular frequency \( \chi \) in a direction determined by the qubit state. Thus driving the qubit at the resonator frequency \( \omega_r \) effectively shifts the origin of the resonator phase space to a point \( \alpha_{\text{vo}} \) in the rotating frame.
the drives can be turned on while maintaining adiabaticity, the lowest-order condition being approximately \( \Omega_{q} \ll \Delta / \sqrt{2} \). Since \( \Omega_{q} \ll \Delta \), the rise time of the qubit drive pulse can be negligibly short compared with the relevant dynamics of the resonator states. Thus the qubit-state-dependent separation dynamics of the resonator state starts to take place essentially instantly in this readout protocol. In contrast, the usual dispersive readout employs the Hamiltonian (2) with \( \alpha_{\text{env}} = 0 \), which implies that one needs to use the resonator drive to populate the resonator for the state separation to take place, see Figs. 1(b) and 1(c). The characteristic time scale for the population \( 1/\kappa \) is determined by the internal and external damping rates of the resonator \( \kappa_{i} \) and \( \kappa_{e} \), respectively, as \( \kappa = \kappa_{i} + \kappa_{e} \).

In addition to the potentially faster readout provided by our scheme, including also a resonator drive mode in the protocol offers more control over the evolution of the states. For example, we may continuously drive the resonator such that either \( Q_{e} \) or \( Q_{z} \) end in any desired position at the end of the readout or we may shift the distributions at the end using a fast drive pulse on the resonator. Interestingly, we may also reset the resonator to the vacuum state unconditionally on the qubit state and without feedback control. As illustrated in Fig. 1(e), one may shift the phase of \( \alpha_{\text{env}} \) by \( \pi \) after the actual measurement pulse and wait for both of the distributions \( Q_{e} \) and \( Q_{z} \) to rotate on top of each other. Subsequently, both distributions may be shifted to the vacuum state using a single pulse on the resonator.

Note that due to the finite bandwidth of the resonator \( 2\pi/\kappa \), the distributions will slowly saturate towards their respective steady states. We obtain the steady states by solving the standard Lindblad master equation \( \dot{\rho} = -i[H, \rho]/\hbar + \frac{\chi}{2} L[\hat{a}]\rho \), where \( L[\hat{a}] \) is the Lindblad superoperator and \( \rho \) is the density operator of the qubit-resonator system. Forcing the states to remain coherent, the steady states \( |\alpha_{g/e}\rangle \) are given by \( |\alpha_{g/e}\rangle = (-\Omega_{q} \pm \sqrt{\Omega_{q}^{2} - g^{2}}) / (\sqrt{\kappa} / 2 \mp \chi) \). Above, we have restricted our theory to the case of a two-level system. However, we show in Ref. [48] that the scheme also works in the case of many non-equidistant levels such as those of a superconducting transmon qubit [50] studied below. Here, the driving frequency needs to be slightly offset from that of the resonator and an additional resonator drive is needed to obtain essentially Eq. (2) for the transmon.

**Sample—** To implement our theoretical scheme we have fabricated [48] a superconducting Xmon qubit [51] shown in Fig. 2(a). It is coupled with strength \( g = 2\pi \times 130 \text{ MHz} \) to a coplanar waveguide resonator with frequency \( \omega_{r} / 2\pi = 6.02 \text{ GHz} \). The resonator has internal and external loss rates \( \kappa_{i} = 2\pi \times 0.5 \text{ MHz} \) and \( \kappa_{e} = 2\pi \times 1.5 \text{ MHz} \), respectively. We tune the qubit to the point of optimal phase coherence [48], \( \omega_{q} / 2\pi = 7.86 \text{ GHz} \), where it is characterized by the energy relaxation time \( T_{1} = 3.0\mu s \).

This leads to a dispersive shift \( \chi = -2\pi \times 1.6 \text{ MHz} \). We mount the sample to the base temperature stage, \( T = 20 \text{ mK} \), of a dilution refrigerator and extract the effective qubit temperature \( T_{\text{eff}} = 73 \text{ mK} \) from histograms of single-shot measurements [48]. For this purpose, we use a traveling-wave parametric amplifier [52] and a heterodyne detection setup to measure the two quadratures \( \text{Re} \langle \hat{a} \rangle \) and \( \text{Im} \langle \hat{a} \rangle \) of the resonator field.

**Experimental Results—** Figures 2(b) and 2(c) present the experimentally measured temporal trajectories of ensemble-averaged expectation values \( \langle \hat{a} \rangle = \int z Q_{g/e}(z) d^{2}z \) for the conventional readout and our method, respectively. The trajectories show qualitative agreement with the theory: In the conventional readout, the distributions move along with the drive and tend to precess about the origin. In our
scheme, the distributions move to opposite direction owing to precession about the virtual origin lying in the first quadrant of the plane. We attribute the differences between Figs. 1(c) and 2(c) to parasitic cross-coupling between the qubit drive line and the resonator and to imperfect calibrations.

To characterize the performance of our method, we perform single-shot measurements \( S \), obtained by time integration \( S = \int_0^t [W_{\text{re}}(t)\Re \hat{a}(t) + iW_{\text{im}}(t)\Im \hat{a}(t)] \, dt \) of the readout signal. Here, the normalized weight functions are determined from the previously measured trajectories as \( W_{\text{re}}(t) \propto |\Re [\alpha_\text{re}(t) - \alpha_\text{re}(t)]| \) and \( W_{\text{im}}(t) \propto |\Im [\alpha_\text{im}(t) - \alpha_\text{im}(t)]| \). Thus the most weight is given to the signal when the state separation is known to be the largest. We also determine reference points \( \sigma^\text{ref}_j \) by averaging shots conditioned on the qubit being in state \( j \in \{ g, e \} \). For a single measurement shot \( S \), we infer that the qubit was in state \( |g\rangle \) if \( |S - \alpha_\text{re}^g| < |S - \alpha_\text{re}^e| \).

The error probability of assigning an incorrect label for the intended qubit state is calculated as \( \epsilon_\text{total} = |p(e | g) + p(g | e)|/2 \), where \( p(j | k) \) is the sampled probability to assign label \( j \) to a state supposedly prepared in state \( k \). To extract the error due to readout, we independently measure the state preparation errors caused by faulty gate operations, spontaneous decay, and thermal excitation. We estimate that these sources account for \( \epsilon_\text{prep} = 3.6\% \) of the total error, mainly limited by the short \( T_1 \) of our sample (see Ref. [48] for details).

We benchmark the speed and fidelity of our readout scheme against the conventional method in Fig. 3, which demonstrates that driving the qubit directly, with or without the compensation tone on the resonator, yields considerably lower errors for integration times below 250 ns. Thus, measuring the qubit state by direct or multi-channel driving results in a noticeable speedup over driving only the resonator. For each readout scheme, the drive power is independently maximized with the condition that the third level of the transmon is negligibly excited during readout, to ensure that the readout realizes a non-demolition measurement. For the multi-channel readout, the relative phase between the resonator and qubit drives \( \phi_r - \phi_q \) is also optimized to achieve the fastest decrease in error. We observe that for a given integration time, the conventional readout bears more than 100\% larger measurement error than the multi-channel driving scheme.

Combining the two drive channels allows for unique and versatile tools for controlling the state of the resonator. In Fig. 4(a), we show that as a function of the phase \( \phi_q \), the steady state draws a circle in the complex plane, the center of which is shifted depending on the qubit state. This behavior is in agreement with the above result \( \alpha_{g/e} = (\Omega_r \pm \Omega_q \chi/g)/(i\kappa/2 \mp \chi) \). It appears possible to choose the phase of \( \Omega_q \) such that only the resonator state corresponding to one of the qubit states remains in the vacuum state (grey arrows), a situation inaccessible by driving only the resonator. In Fig. 4(b), we show that with the multi-channel method we can instead leave the distribution \( Q_e \) to vacuum while significantly displacing \( Q_g \). The initial deviation from the vacuum state is attributed to a delay of the qubit pulse compared to the resonator pulse. As discussed above, a similar mecha-
nism may be utilized to unconditionally reset the resonator after the readout to further reduce the duration of the overall measurement protocol.

Conclusions—In conclusion, we have proposed and experimentally demonstrated a readout scheme for a qubit dispersively coupled to a bosonic mode. By driving both the qubit and the mode close to the mode frequency, the readout can be turned on much faster than any other relevant time scale in the system and the resonator can be unconditionally brought back to the vacuum state without the need for feedback control. Our experiments with a superconducting qubit demonstrate resonator control through the qubit. For a given readout time in our sample, we experimentally observe that the conventional readout may lead to more than 100% larger error than that of the proposed scheme.

In the future, we aim to extend the scheme to multiple driving frequencies and to optimize the sample design such that we improve on the present state of the art [26]. Furthermore, our proposal could be implemented in a variety of systems such as qubits coupled to nanomechanical resonators [35, 36]. We expect that in addition to qubit readout, an extension of our protocol may also be beneficial for resonator state control such as creation of cat states [53].

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[1] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, Rev. Mod. Phys. 85, 471 (2013).
[2] M. Fuwa, S. Takeda, M. Zwierz, H. M. Wiseman, and A. Furusawa, Nat. Commun. 6, 6665 (2015).
[3] B. S. DeWitt and N. Graham, The many worlds interpretation of quantum mechanics (Princeton University Press, 2015).
[4] J. L. O’Brien, A. Furusawa, and J. Vukovi, Nat. Photon. 3, 687 (2009).
[5] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O’Brien, Nature 464, 45 (2010).
[6] G. Kurizki, P. Bertet, Y. Kubo, K. Molmer, D. Petrosyan, P. Rabl, and J. Schmiedmayer, PNAS , 8424 (2015).
[7] D. Ristè, S. Poletto, M.-Z. Huang, A. Bruno, V. Vedral, O.-P. Saira, and L. DiCarlo, Nat. Comm. 6, 6983 (2015).
[8] J. Kelly, R. Barends, A. G. Fowler, A. Megrant, E. Jeffrey, T. C. White, D. Sank, J. Y. Mutus, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, I.-C. Hoi, C. Neill, P. J. J. O’Malley, C. Quintana, P. Roushan, A. Vainsencher, J. Wenner, A. N. Cleland, and J. M. Martinis, Nature 519, 66 (2015).
[9] S. Hacohen-Gourary, L. S. Martin, E. Flurin, V. V. Ramasesh, K. B. Whaley, and I. Siddiqi, Nature 538, 491 (2016).
[10] M. Reagor, C. B. Osborn, N. Tezak, A. Staley, G. Prawiroatmodjo, M. Scheer, N. Alidoust, E. A. Sete, N. Didier, M. P. da Silva, E. Acala, J. Angeles, A. Bestwick, M. Block, B. Bloom, A. Bradley, C. Bui, S. Caldwell, L. Capelluto, R. Chilcott, J. Cordova, G. Crossman, M. Curtis, S. Deshpande, T. E. Bouayadi, D. Girvish, S. Hong, A. Hudson, P. Karalekas, K. Kuang, M. Lenihan, R. Manenti, T. Manning, J. Marshall, Y. Mohan, W. O’Brien, J. Otterbach, A. Papageorge, J.-P. Paquette, M. Pelstring, A. Polloreno, V. Rawat, C. A. Ryan, R. Renzas, N. Rubin, D. Russel, M. Rust, D. Scarcelli, M. Selvanayagam, R. Sinclair, R. Smith, M. Suska, T.-W. To, M. Vahidpour, N. Vodrahalii, T. Whyland, K. Yadav, W. Zeng, and C. T. Rigetti, Sci. Adv. 4, eaao3603 (2018).
[11] A. G. Fowler, A. M. Stephens, and P. Groszkowski, Phys. Rev. A 80, 052312 (2009).
[12] A. G. Fowler, Phys. Rev. A 83, 042310 (2011).
[13] D. Ristè, M. Dukalski, C. A. Watson, G. de Lange, M. J. Tiggelman, Y. M. Blanter, K. W. Lehnert, R. N. Schouten, and L. DiCarlo, Nature 502, 350 (2013).
[14] R. Barends, J. Kelly, A. Megrant, A. Veitia, D. Sank, E. Jeffrey, T. C. White, J. Mutus, A. G. Fowler, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, C. Neill, P. O’Malley, P. Roushan, A. Vainsencher, J. Wenner, A. N. Korotkov, A. N. Cleland, and J. M. Martinis, Nature 508, 500 (2014).
[15] D. Nigg, M. Müller, E. A. Martinez, P. Schindler, M. Hennrich, T. Monz, M. A. Martin-Delgado, and R. Blatt, Science , 302 (2014).
[16] K. Geerlings, Z. Leghtas, I. M. Pop, S. Shankar, L. Fruazio, R. J. Schoelkopf, M. Mirrahimi, and M. H. Devoret, Phys. Rev. Lett. 110, 120501 (2013).
[17] C. C. Bultink, M. A. Rol, T. E. O’Brien, X. Fu, B. C. S. Dikken, C. Dickel, R. F. L. Vermeulen, J. C. de Sterke, C. C. Neill, O.-P. Saira, and L. DiCarlo, Phys. Rev. Applied 6, 034008 (2016).
[18] P. Magnard, P. Kurpiers, B. Royer, T. Walter, J.-C. Besse, S. Gasparinetti, M. Pechal, J. Heinsoo, S. Storz, A. Blais, and A. Wallraff, Phys. Rev. Lett. 121, 060502 (2018).
[19] E. Knill, R. Laflamme, and G. J. Milburn, Nature 409, 46 (2001).
[20] M. A. Nielsen, Phys. Rev. Lett. 93, 040503 (2004).
[21] P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, Rev. Mod. Phys. 79, 135 (2007).
[22] J. Gambetta, W. A. Braff, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A 76, 012325 (2007).
[23] D. Ristè, C. C. Bultink, K. W. Lehnert, and L. DiCarlo, Phys. Rev. Lett. 109, 240502 (2012).
[24] E. Jeffrey, D. Sank, J. Y. Mutus, T. C. White, J. Kelly, R. Barends, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth,
A. Megrant, P. J. J. O’Malley, C. Neill, P. Roushan, A. Vainsencher, J. Wenner, A. N. Cleland, and J. M. Martinis, Phys. Rev. Lett. 112, 190504 (2014).

[25] J. Goetz, S. Pogorzalek, F. Deppe, K. G. Fedorov, P. Eder, M. Fischer, F. Wulschner, E. Xie, A. Marx, and R. Gross, Phys. Rev. Lett. 118, 103602 (2017).

[26] T. Walter, P. Kurpiers, S. Gasparinetti, P. Magnard, A. Potočnik, Y. Salathé, M. Pechal, M. Mondal, M. Oppigler, C. Eichler, and A. Wallraff, Phys. Rev. Applied 7, 054020 (2017).

[27] S. J. Weber, G. O. Samach, D. Hover, S. Gustavsson, D. K. Kim, A. Melville, D. Rosenberg, A. P. Sears, F. Yan, J. L. Yoder, W. D. Oliver, and A. J. Kerman, Phys. Rev. Applied 8, 014004 (2017).

[28] J. Bochmann, A. Vainsencher, D. D. Awschalom, and P. Scarlino, D. J. van Woerkom, A. Stockklauser, X. Mi, C. G. P´ eterfalvi, G. Burkard, and J. R. Petta, Phys. Rev. Lett. 111, 046805 (2013).

[29] J. I. Colless, A. C. Mahoney, J. M. Hornibrook, A. C. Doherty, H. Lu, A. C. Gossard, and D. J. Reilly, Phys. Rev. Lett. 110, 046805 (2013).

[30] P. Scarlino, D. J. van Woerkom, A. Stockklauser, X. Mi, C. G. P´ eterfalvi, G. Burkard, and J. R. Petta, ArXiv e-prints (2017), arXiv:1711.01906 [quant-ph].

[31] Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Science, 350, 307 (2015).

[32] B. Vlastakis, G. Kirchmair, Z. Leghtas, S. E. Nigg, L. Frunzio, S. M. Girvin, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, Science 342, 607 (2013).

[33] J. J. 110, 176803 (2017).

[34] R. Blatt and C. F. Roos, Nat. Phys. 8, 277 (2012).

[35] A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A 69, 062320 (2004).

[36] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature 431, 162 (2004).

[37] J. Goetz, F. Deppe, M. Haeberlein, F. Wulschner, C. W. Zollitsch, S. Meier, M. Fischer, P. Eder, E. Xie, K. G. Fedorov, E. P. Menzel, A. Marx, and R. Gross, Phys. Rev. A 89, 063915 (2014).

[38] J. Heinsoo, C. K. Andersen, A. Remm, S. Krinner, T. Walter, Y. Salathé, S. Gasparinetti, J.-C. Besse, A. Potočnik, A. Wallraff, and C. Eichler, Phys. Rev. Applied 10, 034040 (2018).

[39] D. T. McClure, H. Paik, L. S. Bishop, M. Steffen, J. M. Chow, and J. M. Gambetta, Phys. Rev. Applied 5, 011001 (2016).

[40] N. T. Bronn, Y. Liu, J. B. Hertzberg, A. D. Cicoles, A. A. Houck, J. M. Gambetta, and J. M. Chow, Appl. Phys. Lett. 107, 172601 (2015).

[41] J. Ikonen and M. Möttönen, New J. Phys., DOI: 10.1088/1367-2630/aae621 (2018).

[42] M. Boissonneault, J. M. Gambetta, and A. Blais, Phys. Rev. A 79, 013819 (2009).

[43] See Supplemental Material for experimental techniques and theoretical derivations, which include Refs. 54–60.

[44] D. Walls and G. Milburn, Quantum Optics (Springer Berlin Heidelberg, 2008) p. 133.

[45] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A 76, 042319 (2007).

[46] R. Barends, J. Kelly, A. Megrant, D. Sank, E. Jeffrey, Y. Chen, Y. Yin, B. Chiaro, J. Mutus, C. Neill, P. O’Malley, P. Roushan, J. Wenner, T. C. White, A. N. Cleland, and J. M. Martinis, Phys. Rev. Lett. 111, 080502 (2013).

[47] C. Macklin, K. O’Brien, D.Hover, M. E. Schwartz, V. Bolkhovtsky, X. Zhang, W. D. Oliver, and I. Siddiqi, Science 350, 307 (2015).

[48] B. Vlastakis, G. Kirchmair, Z. Leghtas, S. E. Nigg, L. Frunzio, S. M. Girvin, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, Science 342, 607 (2013).

[49] G. D. Dolan, Appl. Phys. Lett. 31, 337 (1977).

[50] J. M. Sage, V. Bolkhovtsky, W. D. Oliver, B. Turek, and P. B. Wender, J. Appl. Phys. 109, 063915 (2011).

[51] E. Magesan, J. M. Gambetta, and J. Emerson, Phys. Rev. A 85, 042311 (2012).

[52] A. Megrant, C. Neill, R. Barends, B. Chiaro, Y. Chen, L. Feigl, J. Kelly, E. Lucero, M. Mariantoni, P. O’Malley, D. Sank, A. Vainsencher, J. Wenner, T. White, Y. Yin, J. Zhao, C. Palmström, J. M. Martinis, and A. Cleland, Appl. Phys. Lett. 100, 113510 (2012).

[53] J. M. Epstein, A. W. Cross, E. Magesan, and J. M. Gambetta, Phys. Rev. A 89, 062321 (2014).

[54] J. Goetz, F. Deppe, M. Haeberlein, F. Wulschner, C. W. Zollitsch, S. Meier, M. Fischer, P. Eder, E. Xie, K. G. Fedorov, E. P. Menzel, A. Marx, and R. Gross, J. Appl. Phys. 119, 015304 (2016).

[55] J. Goetz, F. Deppe, P. Eder, M. Fischer, M. Mütting, J. Puertas Martínez, S. Pogorzalek, F. Wulschner, E. Xie, K. G. Fedorov, A. Marx, and R. Gross, Quant. Sci. Tech. 2, 025002 (2017).
Supplemental Materials: Qubit Measurement by Multi-Channel Driving

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SAMPLE FABRICATION

We fabricate the sample shown in Fig. S1 on a high-resistivity (> 10 kΩ cm) silicon substrate. The substrate has lateral dimensions of 10 mm × 10 mm and a thickness of 525 µm. To reduce losses due to two-level fluctuators [1], we remove the native oxide with argon ion milling before niobium metalization. By sputter deposition, we add 200 nm of niobium, which we use for defining the co-planar waveguide structures. They are patterned with optical lithography and reactive ion etching. We cover all relevant sample areas with circular 6 µm wide holes to trap flux vortices. In a next step, we define the loops for the superconducting quantum interference devices (SQUIDs) and the Josephson junctions for the transmon qubits [2]. A double-layer Polymethyl methacrylate (PMMA) resist is patterned with electron beam lithography. The junctions have a size of 100 nm × 150 nm and are fabricated with aluminum shadow evaporation [3]. To reduce the loss at the aluminum–niobium interfaces [4], we clean the niobium surface with argon ion milling before we start the aluminum evaporation. The junctions have a resistance of 8 kΩ each, resulting in a Josephson energy $E_J = h \times 34$ GHz. The transmon qubit is made in the Xmon design [5] and has a charging energy $E_C = h \times 264$ MHz. We mount the sample chip in a gold plated sample box made from copper and connect the feedlines to a printed circuit board (PCB) with aluminum wire bonds.

Figure S1: Photograph of the sample that we connect to a printed circuit board (PCB) with aluminum bonds. The sample contains six qubit-resonator systems and we use the highlighted qubit in our experiments. The meandering quarter-wavelength resonators are coupled to the Xmon qubits through a horseshoe-shaped capacitor. The resonators operate at different frequencies separated by 200 MHz to enable multiplexed readout from a common transmission line. There is a voltage gate line for each qubit to control the qubit states individually. At the corner of the sample, we use test pads to characterize the resistance and size of test Josephson junctions. The grey area in the center of the sample is covered with holes in the ground plane, whereas the outer area is completely covered with niobium. The inset shows a typical Josephson junction used for the transmon qubits.
**MEASUREMENT SETUP**

Cryogenic setup—For our experiments, we mount the sample to the base temperature of 20 mK of a dilution refrigerator (see Fig. S2). Our low-temperature setup employs multistage shielding against magnetic flux noise and thermal radiation containing a μ-metal shield and an aluminum shield at the sample stage. In addition, we use a μ-metal shield to protect a Josephson traveling-wave parametric amplifier (TWPA) [6] against stray magnetic fields. This amplifier is required for the single-shot measurements and has been fabricated at MIT Lincoln Laboratory. We operate the TWPA with a coherent drive that we optimize for a minimum noise temperature of the TWPA. Using an automated optimization protocol, we find a TWPA noise temperature of approximately 380 mK for a gain of 21.4 dB at 6.2 GHz. To obtain the TWPA noise temperature, we have used an in-situ power calibration based on the photon-number-dependent frequency shift of the qubits. In addition to the TWPA, we use a high-electron-mobility transistor (HEMT) amplifier at the 4-K stage of the cryostat. The readout and the qubit gate lines are heavily attenuated and we mount microwave filters at the sample stage to reduce the influence of thermal noise [7]. To apply a static magnetic flux to the SQUID loops of the qubits, we use a custom-made superconducting coil with approximately 200 windings.

Readout setup—We use a sophisticated readout setup for qubit measurements depicted as the blue components in Fig. S2. Our measurements are carried out with a heterodyne microwave detection setup that is synchronized with the qubit control hardware. We generate coherent readout pulses by up-conversion of a local oscillator signal using a double-balanced I/Q mixer. The temporal envelopes of the readout pulses applied to the mixer are generated by a field-programmable gate array (FPGA). The FPGA logic controls the output of two digital-to-analog converters (DACs) running at 250 MS/s. They generate a 62.5 MHz intermediate frequency superimposed to the temporal envelopes of the in-phase and quadrature components, I and Q, of the readout signal. The pulses are sent through the cryostat, amplified, and down-converted using another I/Q mixer. Finally, we digitize the transmitted I and Q components using two analog-to-digital converters (ADCs). These ADCs run at 500 MS/s and are controlled by the same FPGA board as the DACs. Our FPGA code allows us to either analyze single measurement events or ensemble averages to reproduce the time traces in phase space. Currently, the speed of our detection setup is limited by the 40 MHz bandwidth of the band-pass filters in the amplification chain. In addition to the cryogenic amplifiers, this chain contains two room temperature RF amplifiers and a single amplifier for each quadrature component after down-conversion.

Multi-channel readout—In our experiments, we can apply a readout pulse either to the resonator through the common transmission line and simultaneously to the qubit through a voltage gate line. To ensure the required phase coherence between the two drives, we use a single local oscillator and split the signal after it has passed the I/Q mixer. We obtain full control over the absolute amplitudes of each signal part by using tunable attenuators and digital phase shifters.

Qubit control—All components required for qubit control are depicted in pink in Fig. S2. We use a timing module to synchronize the readout FPGA with another FPGA board that generates the qubit control pulses. All these boards are PXI-based and installed in a common controller chassis. The FPGA board for qubit control feeds Gaussian envelopes to the two outputs of a DAC operating at 1.25 GS/s. These envelopes have an intermediate frequency of 350 MHz and are upconverted to the qubit frequency using a local oscillator and an I/Q mixer. To optimize the spectral distribution of the qubit control and also of the readout pulses, they can be routed to a spectrum analyzer through a microwave multiplexer. For our experiments we use typical pulse lengths of 100 ns to perform a π rotation of the qubit state on the Bloch sphere. The FPGA code for the qubit pulses can generate arbitrary waveforms, which allows us to optimize the gate fidelities using randomized benchmarking.

**SAMPLE CHARACTERIZATION**

We use spectroscopy measurements to characterize the relevant sample parameters summarized in Table I. We obtain the resonator frequency and the resonator loss rates from spectroscopy measurements as shown in Fig. S3(a). More precisely, we fit an input-output model [8] to the complex transmission coefficient $S_{21}$ after adjusting the magnetic flux to a value corresponding to the maximum qubit frequency. We additionally extract the qubit–resonator coupling strength $g$ from the anticrossings shown in Fig. S3(a). We apply a two-tone measurement protocol to determine the flux-dependent qubit transition frequency $\omega_q(\Phi) = (\sqrt{8E_1E_c} \cos \pi \Phi/\Phi_0 - E_c)/\hbar$, where $\Phi_0$ denotes the flux quantum. As shown in Fig. S3(b), our high-power drive also excites the two photon process of the $|g\rangle \leftrightarrow |f\rangle$ transition,
Figure S2: Measurement setup: We use a heterodyne detection scheme (upper part of the figure) to characterize the qubit-state-dependent shift of superconducting resonators mounted in a dilution refrigerator (lower part of the figure). Details are described in the text.
which is detuned from the $|g\rangle \leftrightarrow |e\rangle$ transition by $E_c/2h$. Depending on the qubit state, the resonator experiences a frequency shift $\chi/2\pi = -g^2E_c/\Delta(\Delta - E_c) = -1.5$ MHz. This predicted value fits nicely to our experimental result $\chi/2\pi = -1.6$ MHz, which we extract from measuring the resonator frequency after preparing the qubit either in $|g\rangle$ or in $|e\rangle$.

Table I: Overview of the most important sample parameters.

| Qubit parameters                  |                           |
|-----------------------------------|---------------------------|
| transition frequency              | $\omega_q/2\pi = 7.86$ GHz|
| charging energy (anharmonicity)   | $E_c = h \times 264$ MHz  |
| Josephson energy                  | $E_J = h \times 34$ GHz   |
| energy decay rate                 | $\gamma_1 = 1/(3.5 \mu s)$|
| Ramsey decay rate                 | $\gamma_{I,R} = 1/(3.0 \mu s)$|
| effective qubit temperature       | $T_{\text{eff}} = 73$ mK  |
| qubit–resonator coupling strength | $g/2\pi = 130$ MHz        |

| Resonator parameters              |                           |
|-----------------------------------|---------------------------|
| resonance frequency               | $\omega_r/2\pi = 6.02$ GHz|
| external loss rate                | $\kappa_x = 2\pi \times 1.5$ MHz |
| internal loss rate                | $\kappa_i = 2\pi \times 0.5$ MHz |
| dispersive shift                  | $\chi = -2\pi \times 1.6$ MHz |

GATE ERRORS AND THERMAL EXCITATIONS

Gate error—We implement the randomized benchmarking protocol [9] to estimate the error caused by non-ideal gate operations when preparing the qubit in the excited state. The protocol amounts to applying a series of quantum gates to the qubit, followed by a standard dispersive readout. The sequence of $L$ gates is built from randomly selected Clifford gates and is terminated with a single gate such that the total operator corresponding to the whole sequence is the identity operator. To estimate the error of a particular gate, in our case the $\pi$ pulse, the selected gate is inserted to the sequence after each random gate. The benchmarking results are shown in Fig. S4. We fit a function of the form $A p^L + B$ to the decaying curves of non-interleaved and $\pi$-interleaved sequences to extract the parameters $p_{\text{ref}}$ and $p_{\pi}$, respectively. The gate error of the $\pi$ pulse is found to be $\epsilon_{\text{gate}} = (1 - p_{\pi}/p_{\text{ref}})/2 = (3.0 \pm 0.5)\%$. Following Ref. [10], we further estimate that spontaneous decay during the 200 ns gate time contributes with 2.2% to $\epsilon_{\text{gate}}$. 

Figure S3: (a) Resonator spectroscopy: Phase response of a readout tone as a function of the probe frequency and the magnetic flux generated by a coil current. We observe clear anticrossings due to the strong coupling between the resonator mode and the transmon qubit. (b) Two-tone qubit spectroscopy: The phase response of the readout tone as a function of the magnetic flux and the frequency of a drive tone applied to the qubit. Due to the strong driving, we observe signatures of the two photon process of the $|g\rangle \leftrightarrow |f\rangle$ transition at frequencies slightly below the qubit $|g\rangle \leftrightarrow |e\rangle$ transition.
Thermal excitations—Another source of state preparation error is the thermal excitation of the qubit, the probability of which we extract from the single-shot histogram shown in Fig. S4(b). We model the single-shot distribution corresponding to the qubit ground state as sum of two Gaussians,

\[ Q_g(z) = (1 - \epsilon_{th}) \exp\left(-\frac{(z - \alpha_g)^2}{2\sigma_{th}^2}\right) + \epsilon_{th} \exp\left(-\frac{(z - \alpha_e)^2}{2\sigma_{th}^2}\right), \tag{S1} \]

where \( \epsilon_{th} \) denotes the excitation probability and \( \sigma_{th}^2 \) is the variance. By fitting this model to the data, we obtain \( \epsilon_{th} = 0.6\% \), which is used to calculate the effective qubit temperature as \( T_{eff} = \hbar \omega_q / [k_B \log(1/\epsilon_{th})] \approx 73\, \text{mK} \). Thus the overall state preparation error is \( \epsilon_{prep} = \epsilon_{gate} + \epsilon_{th} = 3.6\% \), as stated in the main text. We also use the fitted variance \( \sigma_{th}^2 \) to obtain a conversion factor from Volts to units of photons for the readout signal. This follows from the fact that the vacuum state is a Husimi distribution with a variance of 1/2 photons.

SYSTEM HAMILTONIAN

Here, we mathematically derive the complete dispersive system Hamiltonian, starting from the system Hamiltonian in the lab frame, we first switch to a rotating frame, in which the results above were simulated. Second, we make the dispersive approximation, which gives a more intuitive understanding of the system and allows for solving the trajectories analytically.

Laboratory frame—We treat the qubit as an anharmonic oscillator with eigenfrequencies \( \omega_k = k\omega_r + \Delta_k \), where \( \Delta_k \) denotes the detuning between the \( k \)th energy level of the qubit from the resonator angular frequency \( \omega_r \). Namely, we define \( \Delta_0 = 0 \) for the ground state, \( \Delta_1 = \Delta \) for the first excited state, and \( \Delta_2 = 2\Delta + \alpha \), where \( \alpha \) is the anharmonicity, for the second excited state. In the dispersive regime, the detuning is larger than the qubit–resonator coupling strength \( g \), i.e., \( |\Delta| \gg g \). The Hamiltonian that describes the system can be written as

\[ \hat{H}_{total} = \hat{H}_0 + \hat{H}_{int} + \hat{H}_{QD} + \hat{H}_{RD}, \tag{S2} \]
Figure S5: (a) Simulated expectation values $\alpha_{g/e} = \langle \hat{a} \rangle_{g/e}$ of the resonator states during a 400-ns-long measurement of the state of a two-level system by driving the resonator. The numerical values used for the parameters correspond to those of the experimental sample given in Table I. The markers are spaced every 40 ns. The brown arrow indicates the direction of the applied drive $\Omega_r$. (b) As (a), but driving is applied to the qubit instead of the resonator. The virtual origin $\alpha_{vo} = -\Omega_q/g$ is denoted by the green circle. (c) Driving is simultaneously applied to the resonator and the qubit, with the relative phase chosen as indicated in the figure to lock $\alpha_e$ to the origin.

where the free, interaction, qubit-driving, and resonator-driving Hamiltonians are, respectively, given by

\[ \hat{H}_0/\hbar = \omega_r \hat{a}^\dagger \hat{a} + \sum_{k=0} \omega_k |k\rangle \langle k|, \]  
\[ \hat{H}_{\text{int}}/\hbar = \sum_{k=0} g_k (\hat{a}^\dagger + \hat{a}) (|k\rangle \langle k + 1| + |k + 1\rangle \langle k|), \]  
\[ \hat{H}_{\text{QD}}/\hbar = 2\tilde{\Omega}_{qd}(t) \sum_{k=0} \lambda_k (|k\rangle \langle k + 1| + |k + 1\rangle \langle k|), \]  
\[ \hat{H}_{\text{RD}}/\hbar = 2\tilde{\Omega}_{rd}(t) (\hat{a}^\dagger + \hat{a}), \]  

where $\hat{a}$ denotes the annihilation operator of the resonator mode, and $|k\rangle$ refers to the $k$th eigenstate of the qubit. For a transmon, the coupling constants for different transmon levels are typically assumed to be of the form $g_k = g\sqrt{k+1}$, $\lambda_k = \sqrt{k+1}$. The real driving waveforms $\Omega_{r/q}(t)$ at drive frequency $\omega_d$ are constructed from the real and imaginary
parts (i.e., $I$ and $Q$ quadratures) of the complex amplitudes as
\[ \tilde{\Omega}_{r/q}(t) = \text{Re}(\Omega_{r/q}) \cos(\omega_d t) + \text{Im}(\Omega_{r/q}) \sin(\omega_d t). \]

**Rotating frame**—We transform $\hat{H}_{\text{total}}$ into the frame rotating at the angular frequency $\omega_d$. Applying the unitary operator
\[ \hat{U}_1 = \exp \left[ i t \left( \omega_d \hat{a}^\dagger \hat{a} + \sum_k k \omega_d |k\rangle \langle k| \right) \right] = e^{i t \omega_d \hat{a}^\dagger \hat{a}} \sum_k e^{i t \omega_d |k\rangle \langle k|}, \]
and employing the rotating-wave approximations, justified by $g \ll |2\omega_i|$ and $|\omega_i - \omega_d| \ll |\omega_i + \omega_d|$, yields
\[ \hat{H}'_{\text{total}} = \hat{U}_1 \hat{H}_{\text{total}} \hat{U}_1^\dagger = (\omega_i - \omega_d) \hat{a}^\dagger \hat{a} + \sum_k \tilde{\Delta}_k |k\rangle \langle k|, \]
\[ \hat{H}'_{\text{int}} = \tilde{\Delta}_k \sum_k g_k |k\rangle \langle k + 1| + \text{H.c.}, \]
\[ \hat{H}'_{\text{QD}} = \Omega_q \sum_k \lambda_k |k + 1\rangle \langle k| + \text{H.c.}, \]
\[ \hat{H}'_{\text{RD}} = \Omega_d \hat{a}^\dagger + \text{H.c.}, \]
where $\tilde{\Delta}_k = \Delta_k + k \omega_i - k \omega_d$ denotes the shifted detunings. Ignoring $\hat{H}'_{\text{RD}}$ for a moment, the total transformed Hamiltonian is given by
\[ \hat{H}'_{\text{total}} \approx (\omega_i - \omega_d) \hat{a}^\dagger \hat{a} + \sum_k \tilde{\Delta}_k |k\rangle \langle k| + \sum_k \left\{ \left[ g_k \hat{a}^\dagger + \Omega_q \lambda_k \right] |k + 1\rangle \langle k| + \text{H.c.} \right\}. \]

Here, we make the key observation that the frame is displaced by $\alpha_{vo} \equiv -\Omega_q \lambda_k / g_k = -\Omega_d / g$. Thus, in the non-shifted frame, the phase space distribution of the resonator should rotate about the point $\alpha_{vo}$. The location of $\alpha_{vo}$ is fully controllable by the pulse, $\Omega_q$ and $\omega_d$.

To account for the decay of the resonator state, we use the Lindblad master equation
\[ \dot{\rho} = -i[\hat{H}'_{\text{total}}, \rho] / \hbar + \frac{\kappa}{2} \mathcal{L}[\hat{a}] \rho, \]
where $\rho$ is the reduced density operator of the resonator, $\kappa$ denotes the resonator energy decay rate, and $\mathcal{L}[\hat{a}] \rho = \hat{a} \rho \hat{a}^\dagger - \frac{1}{2} \{ \hat{a}^\dagger \hat{a} \rho + \rho \hat{a}^\dagger \hat{a} \}$.

We simulate single- and multi-channel readout processes based on driving the resonator and/or the qubit by numerically solving the master equation with the experimentally obtained parameter values listed in Table I. The results are shown in Fig. S5. Notably, the figure showcases the different rates at which the states separate in the two readout schemes before saturating towards steady states due to the dissipation. Figure S5 demonstrates that given the experimental parameters and proper calibration of the delays, frequencies, powers, and phases of the drive tones, it is possible to obtain trajectories closer to the ideal case of Figs. 1(b) and 1(c) of the main article than we demonstrate in Figs. 2(b) and 2(c) of the main article. Such fine calibrations are left for future work since the aim of this work was to propose the readout scheme and to experimentally demonstrate its main working principles.

**Dispersive approximation**—To make this observation more evident, we introduce the standard dispersive approximation. We begin by applying another transformation using
\[ \hat{U}_2 = \exp \left[ \sum_k \frac{g_k}{\Delta_{k+1} - \Delta_k} \left( \hat{a} |k + 1\rangle \langle k| - \hat{a}^\dagger |k\rangle \langle k + 1| \right) \right]. \]
We compute $\hat{H}''_{i} = \hat{U}_2 \hat{H}'_{i} \hat{U}_2^\dagger$ up to second order in $g_k / \Delta_{k+1}$ under the assumption $g_k \ll \Delta_{k+1}, \forall k$. For clarity, we restrict the following equations to only the first three levels of the transmon ($\{|g\rangle, |e\rangle, |f\rangle\} \equiv \{|0\rangle, |1\rangle, |2\rangle\}$). The
Hamiltonians become
\[ \left( \hat{H}_{\text{0}}'' + \hat{H}_{\text{int}}'' \right)/\hbar \approx \left( \tilde{\Delta} + \chi \right) |e\rangle \langle e| + \left( \tilde{\Delta} + \chi \right) |f\rangle \langle f| + \left( \tilde{\Delta} + \chi \right) |g\rangle \langle g| \]
where we have defined the dispersive constants \( \chi_0 = g_0^2/\tilde{\Delta} \) and \( \chi_1 = (g_1^2/\tilde{\Delta}_2 - \tilde{\Delta}_1) \). Finally, introducing the displaced operator \( \hat{b} = \hat{a} - \alpha_v \), the total Hamiltonian reads
\[ \hat{H}''_{\text{total}}/\hbar \approx \chi_0 |\alpha_v| g_0^2 |g\rangle \langle g| + \left( \tilde{\Delta} + \chi_0 - |\alpha_v| g_0^2 \right) |e\rangle \langle e| + \left( \tilde{\Delta} + \chi_0 - |\alpha_v| g_0^2 \right) |f\rangle \langle f| + \left( \tilde{\Delta} + \chi_0 - |\alpha_v| g_0^2 \right) |g\rangle \langle g| \]
\[ + \left( -z g_0 + \Omega \right) |g\rangle \langle g| + \left( -z g_0 + \Omega \right) |f\rangle \langle f| + \left( -z g_0 + \Omega \right) |e\rangle \langle e| \]
\[ + \left( z g_0 - \Omega \right) |f\rangle \langle f| + \left( z g_0 - \Omega \right) |e\rangle \langle e| \]
\[ + \left( z g_0 + \Omega \right) |g\rangle \langle g| + \left( z g_0 + \Omega \right) |f\rangle \langle f| + \left( z g_0 + \Omega \right) |e\rangle \langle e| \]
\[ + \left( -z g_0 + \Omega \right) |f\rangle \langle f| + \left( z g_0 - \Omega \right) \right] \hat{b}^\dagger \hat{b} + \text{H.c.} \]
\[ \text{(19)} \]
\[ \text{(20)} \]
\[ \text{(21)} \]
\[ \text{(22)} \]
Line (19) describes the constant frequency shifts caused by the coupling and the driving. Line (20) shows that driving from the qubit side tilts the qubit Hamiltonian. Importantly, line (21) predicts that any coherent state will rotate about point \( \alpha_v \). The angular frequencies of these rotations may be set to be equal to \( +\chi \equiv \chi_1/2 - \chi_0 \) and \( -\chi \) for \( \alpha_e \) and \( \alpha_b \), respectively, by choosing \( \omega_r - \omega_d = \chi_1/2 \). Line (22) shows that the transformation has an effect on the amplitude of the resonator drive that may be compensated by changing \( \Omega_r \). Note that the Hamiltonian of a conventional dispersive system is obtained by setting \( \alpha_v = 0 \).

The Hamiltonian given in Eq. (2) in the manuscript is produced from Eqs. (S19)–(S22) by assuming that the third level of the transmon is never populated, and making the above-discussed choice \( \omega_d = \omega_r - \chi_1/2 \). For Eq. (2), we have also defined the shifted detuning as \( \Delta = \tilde{\Delta}_1 + \chi_0 - |\alpha_v| g_0^2 \) and re-labeled \( g_0 \to g \).

**Trajectories**—Using Eq. (S14) with the approximate Hamiltonian \( \hat{H}''_{\text{total}} \), we obtain an analytical equation for the expectation value \( \alpha_{e/j} = \langle \hat{a}/\rangle_{j} \), as
\[ \frac{\partial \alpha_{e/j}(t)}{\partial t} = i \Omega_r^j \pm i \chi \left[ \alpha_{e/j}(t) - \alpha_v \right] - \frac{\kappa}{2} \alpha_{e/j}(t), \]
\[ \text{(23)} \]
Choosing constant control pulses, \( \frac{\partial \alpha_{e/j}}{\partial \omega} = 0 \), and assuming that the resonator is initially in the vacuum state, the solution is given by
\[ \alpha_{e/j}(t) = -\frac{\Omega_r^j}{\kappa} \left( 1 - e^{-\frac{\kappa}{2} t \mp i \chi} \right) \]
\[ \text{(24)} \]
Setting \( t \to \infty \) results in the steady state formulae given in the manuscript.

[1] J. M. Sage, V. Bolkhovsky, W. D. Oliver, B. Turek, and P. B. Welander, J. Appl. Phys. 109, 063915 (2011).
[2] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A 76, 042319 (2007).
[3] L. J. Dolan, Appl. Phys. Lett. 31, 337 (1977).
[4] J. Goetz, F. Deppe, M. Haebereklin, F. Wulschner, C. W. Zollitsch, S. Meier, M. Fischer, P. Eder, E. Xie, K. G. Fedorov, E. P. Menzel, A. Marx, and R. Gross, J. Appl. Phys. 119, 015304 (2016).
[5] R. Barends, J. Kelly, A. Megrant, D. Sank, E. Jeffrey, Y. Chen, Y. Yin, B. Chiaro, J. Mutus, C. Neill, P. O’Malley, P. Roushan, J. Wenner, T. C. White, A. N. Cleland, and J. M. Martinis, Phys. Rev. Lett. 111, 080502 (2013).
[6] C. Macklin, K. O’Brien, D. Hover, M. E. Schwartz, V. Bolkhovsky, X. Zhang, W. D. Oliver, and I. Siddiqi, Science 350, 307 (2015).
[7] J. Goetz, F. Deppe, P. Eder, M. Fischer, M. Müting, J. Puertas Martín, S. Pogorzalek, F. Wulschner, E. Xie, K. G. Fedorov, A. Marx, and R. Gross, Quant. Sci. Tech. 2, 025002 (2017).
[8] A. Megrant, C. Neill, R. Barends, B. Chiaro, Y. Chen, L. Feigl, J. Kelly, E. Lucero, M. Mariantoni, P. O’Malley, D. Sank, A. Vainsencher, J. Wenner, T. White, Y. Yin, J. Zhao, C. Palmström, J. M. Martinis, and A. Cleland, Appl. Phys. Lett. 100, 113510 (2012).

[9] E. Magesan, J. M. Gambetta, and J. Emerson, Phys. Rev. A 85, 042311 (2012).

[10] J. M. Epstein, A. W. Cross, E. Magesan, and J. M. Gambetta, Phys. Rev. A 89, 062321 (2014).