Three generations mixing and $\tau$ decay puzzle

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We consider the possibility that the $\tau$ decay puzzle is a consequence of the Kobayashi-Maskawa mixing in the leptonic sector.

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The physics of the $\tau$ lepton will provide in the near future evidence concerning the question if this lepton, with its neutrino partner, is a sequential lepton or not. So far, as was stressed in Ref. [1], all experiments are internally consistent with the Standard Model. Notwithstanding, it is well known that the accuracy of the $\tau$ data are still poor and it should be possible that new physics will come up when the proposed $\tau$-Charm factory gives new and more accurate data about $\tau$ decays and properties [2].

For example, new data could confirm the so called “$\tau$ decay puzzle” [3], which is a difference between the measured world average $\tau$ lifetime, $\tau$ and the theoretically expected value within two standard deviations. Explicitly [5]

$$\tau_{exp} - \tau_{th} = (0.16 \pm 0.09) \times 10^{-13} \text{s.}$$

although this discrepancy is not yet statistically significant, it can be translated into discrepancies in particular branching ratios, implying that the expected leptonic branching ratios are about $2.3\sigma$ higher than the average measurements [3]. The branching ratio conflicts suggest that a shift in $\tau$ and/or $\tau$ mass, $m_\tau$, should occur when more precise measurements become available. In fact, preliminary results from BES in Beijing point to a down shift of $2\sigma$ in $m_\tau$ in relation to the world average value [6]. Notwithstanding, in order to solve the discrepancy a down shift of $6.4\sigma$ is required as pointed out in Ref. [3].

Other possibility is that no such a shift on $\tau$ or $m_\tau$ is needed but the problem with the branching ratios would still exist. This would imply a new physics. The relationship of Eq. (1) with a possible deviation from universality can be see as follows. It is well known that the decay diagram of the muon into electron being similar to that of the $\tau$ decay into electron or muon. For example, implies the following relationship

$$\left(\frac{G_\tau}{G_\mu}\right)^2 = \left(\frac{\tau_\mu}{\tau_\tau}\right) \left(\frac{m_\mu}{m_\tau}\right)^5 B.R.(\tau \rightarrow e\bar{\nu}\nu),$$

where $G_\tau$ and $G_\mu$ are the coupling constant of the $\tau$ and $\mu$ to the charged weak current respectively. Assuming $e\mu$ universality an average leptonic branching ratio can be defined as in Ref. [6]:

$$2$$
\[ \langle B_\tau^e \rangle = \frac{B_\tau^e + \frac{B_\mu^e}{0.973}}{2} = 17.88 \pm 0.26\%. \] (3)

where the factor 0.973 is due to the mass of the muon.

Using the world average value for the branching ratio Eq. (3) in Eq. (2) it follows that \( G_\tau/G_\mu = 0.975 \pm 0.010 \) using non-LEP and LEP data or \( G_\tau/G_\mu = 0.985 \pm 0.0009 \) using the BEPC value for the \( \tau \) mass [3]. Of course in the Standard Model universality implies \( G_\tau = G_\mu \).

There have been some speculations about this possible deviation from the Standard Model. Some examples, usually discussed in the literature, of the new physics needed to solve the puzzle are:

1. the introduction of new gauge bosons [7],

2. four-generation leptons, mixing mainly with the third generation [8,9],

3. scalar particles which interfere destructively with the \( W \)-exchange amplitude in the \( \tau \) decay [10].

Of course, each of the above possibilities, and their variants, have their own difficulties. The first one implies a drastic modification of the Standard Model; the second implies also a modification of the quark sector, necessary in order to avoid anomalies; finally, the third one needs a larger Higgs sector (two triplets) in the Standard Model.

On the other hand, the simplest solution has not been even mentioned in the literature. It is possible, if the neutrinos are massive, that a mixing similar to the Kobayashi-Maskawa one [11] occurs with three lepton generations. Here we will consider the analysis of the experimental data with five free parameters, three angles and two neutrino mass differences from Ref. [12]. The effects of the Kobayashi-Maskawa mixing in the leptonic sector were considered in Ref. [13]. In particular the effects of such a mixing for the case of leptonic decays of the \( \tau \)-lepton were explicitly considered in Ref. [14]. Here we will consider this scenario in the context of a possible “\( \tau \) decay puzzle”.

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The Kobayashi-Maskawa lepton mixing matrix is

\[
\begin{pmatrix}
\nu'_e \\
\nu'_\mu \\
\nu'_\tau
\end{pmatrix} =
\begin{pmatrix}
V_{ee} & V_{e\mu} & V_{e\tau} \\
V_{\mu e} & V_{\mu\mu} & V_{\mu\tau} \\
V_{\tau e} & V_{\tau\mu} & V_{\tau\tau}
\end{pmatrix}
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}.
\] (4)

The unprimed fields are mass eigenstates and we will not consider the hierarchical mixing \[15\]. In Ref. \[12\] the Maiani parameterization \[16\] of the mixing matrix was chosen and two solutions were found for the oscillation parameters which imply the following ranges for the diagonal matrix elements \((V_{ee}, V_{\mu\mu}, V_{\tau\tau})\)

**solution a)** \(1.00—0.98, 1.00—0.99, 1.00—0.98\)

**solution b)** \(1.00—0.97, 1.00—0.98, 1.00—0.98\)

In the theoretical predictions of the \(\tau\) partial widths we will neglect neutrino masses. In fact the neutrino mass is only important if \(m_{\nu_\tau} > 100\,\text{MeV}\) for \(\tau \to e\nu_\tau\bar{\nu}_e\) decay, or if \(m_{\nu_\tau} > 50\,\text{MeV}\) for \(\tau \to \mu\nu_\tau\bar{\nu}_\mu\) decay \[14\] while the present current limit is \(m_{\tau \nu} < 35\,\text{MeV}\) \[4\].

We use the following notation: \(\bar{B}_\tau^l = \bar{\Gamma}_\tau^l / \Gamma_{\tau\text{tot}}, \bar{\Gamma}_\tau^l\) being the \(\tau\) partial decay width into the \(i\) charged particle \((e^-, \mu^-, \pi^-, K^-)\) considering the mixing and \(\Gamma_{\tau\text{tot}}\) the total width.

The widths with this kind of mixing are given by

\[
\bar{\Gamma}_l^\tau = \frac{|V_{\tau\tau}|^2 |V_{ll}|^2}{|V_{\mu\mu}|^2 |V_{ee}|^2} \Gamma_l^\tau, \tag{5}
\]

and

\[
\bar{\Gamma}_h^\tau = \frac{|V_{\tau\tau}|^2}{|V_{\mu\mu}|^2 |V_{ee}|^2} \Gamma_h^\tau, \tag{6}
\]

where \(l = e, \mu\) and \(h = \pi, K\) and \(\Gamma_{l,h}^\tau\) are the \(\tau\) partial widths without mixing which have appeared in the literature \[3\]. In Eqs. (5) and (6) the denominator comes from the definition of the \(G_\mu\) constant in the \(\mu\) decay. Numerically we will consider the hadronic partial decay width as \(\Gamma_h^\tau = \Gamma_{\pi}^\tau + \Gamma_{K}^\tau\).

Let us start by the theoretical result for the partial width from Refs. \[3,17\], which includes radiative corrections, using the current data from Ref. \[4\] and presented with the range implied by the solution a) above:
\[ \bar{\Gamma}(\tau \to e^- \nu_\tau \bar{\nu}_e) = (3.95^{+0.03}_{-0.04} - 4.19^{+0.03}_{-0.04}) \times 10^{-13}\text{GeV}, \]  
(7)

\[ \bar{\Gamma}(\tau \to \mu^- \nu_\tau \bar{\nu}_\mu) = (3.84^{+0.03}_{-0.04} - 4.17^{+0.03}_{-0.04}) \times 10^{-13}\text{GeV}, \]  
(8)

\[ \bar{\Gamma}(\tau \to \pi \nu_\tau) = (2.45 \pm 0.06 - 2.71 \pm 0.07) \times 10^{-13}\text{GeV}, \]  
(9)

\[ \bar{\Gamma}(\tau \to K \nu_\tau) = (1.60 \pm 0.04 - 1.76 \pm 0.04) \times 10^{-14}\text{GeV}, \]  
(10)

\[ \bar{\Gamma}(\tau \to h \nu_\tau) = (2.61 \pm 0.06 - 2.88 \pm 0.07) \times 10^{-13}\text{GeV}. \]  
(11)

With this mixing we have instead of Eq. (2)
\[ (G_\tau G_\mu)^2 = |V_{\mu\mu}|^2 |V_{\tau\tau}|^2 \frac{m_\mu}{m_\tau} \frac{\Gamma_{\mu\tau}}{\Gamma_{e\tau}} \text{B.R.}(\tau \to e\bar{\nu}_e), \]  
(12)

and using the current \( \tau \) lifetime and mass [6]
\[ m_\tau = 1776.9 \pm 0.4 \pm 0.3 \text{, MeV} \quad \tau_\tau = (3.00 \pm 0.05) \times 10^{-13}\text{s}. \]  
(13)

\[ \left( \frac{G_\tau}{G_\mu} \right)^2 = \frac{|V_{\mu\mu}|^2}{|V_{\tau\tau}|^2} \left( \frac{\tau_\mu}{\tau_\tau} \right) \left( \frac{m_\mu}{m_\tau} \right)^5 \text{B.R.}(\tau \to e\bar{\nu}_e), \]  
(14)

With the values of the diagonal matrix elements given in solution a) we obtain \( |V_{\mu\mu}|^2 / |V_{ee}|^2 = 0.98 - 1.04 \). We see that there is consistency with the value \( G_\tau / G_\mu = 0.975 \pm 0.010 \) [3].

Eqs. (6)-(10) are also compatible with the respective experimental branching ratios. Similar results arise using the matrix elements given in (b).

We can also verify that ratios of partial widths are consistent with leptonic mixing
\[ \frac{\Gamma_\mu}{\Gamma_\tau} = \frac{|V_{\mu\mu}|^2 \Gamma_\tau}{|V_{ee}|^2 \Gamma_\mu} = 0.95 \pm 0.01 - 1.01^{+0.01}_{-0.02}, \]  
(15)

\[ \frac{\Gamma_h}{\Gamma_\tau} = \frac{1}{|V_{\mu\mu}|^2} \frac{\Gamma_\tau}{\Gamma_\mu} = 0.68 \pm 0.02 - 0.69 \pm 0.02, \]  
(16)

\[ \frac{\Gamma_h}{\Gamma_\tau} = \frac{1}{|V_{ee}|^2} \frac{\Gamma_\tau}{\Gamma_h} = 0.66 \pm 0.02 - 0.69 \pm 0.02. \]  
(17)

As before, we show in Eqs. (15)-(17) the range of the ratios of the partial widths taking into account the range of the matrix elements. Again it is possible to verify that there is consistency with experimental data.
We have shown in this comment that a possible deviation from $\mu - \tau$ universality if confirmed by future experiments is sufficiently small to be accounted by three generations mixing in the leptonic sector.

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