ANNIHILATING DARK MATTER AND THE GALACTIC POSITRON EXCESS

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The possibility that the Galactic dark matter is composed of neutralinos that are just above half the $Z^0$ mass is examined, in the context of the Galactic positron excess. In particular, we check if the anomalous bump in the cosmic ray positron to electron ratio at 10 GeV can be explained with the “decay” of virtual $Z^0$ bosons produced when the neutralinos annihilate. We find that the low energy behaviour of our prediction fits well the existing data. Assuming the neutralinos annihilate primarily in the distant density concentration in the Galaxy and allowing combination of older, diffused positrons with young free-streaming ones, produces a fit which is not satisfactory on its own but is significantly better than the one obtained with homogeneous injection.

1. Introduction

The possibility that weakly interacting dark matter particles (WIMP’s) could annihilate into detectable cosmic radiation was suggested by Silk and Srednicki. Tylka and Eichler noted a reported positron excess, curiously localized near 10 GeV, and considered whether it could be due to the annihilation of photinos (as a simple example of neutralinos) in the tens of GeV mass range. The difficulty was that this process, given the laboratory constraints on the neutralinos, seemed to fall short of providing enough positrons, and the results were not published. Nevertheless, various papers on this excess eventually appeared, and noted that the potential for positron excess could be bolstered by clumpiness in the annihilating dark matter or by decay of weakly unstable dark matter particles.

The approach usually found in present literature is to try and fit the overall $e^+/(e^+ + e^-)$ ratio, without giving special attention to the curious behaviour at 10 GeV. Baltz and Edsjo considered a whole class of minimal standard supersymmetric models and failed to get any non-
monotonicity in the $e^+/(e^+ + e^-)$ ratio. They note that an enhancement factor, presumably due to clumping, of at least 30 to 100 is needed to account for the observed positron excess.

Eichler and Maor considered the possibility that the observed excess in positrons, and in particular the anomalous behaviour around $\sim 10$ GeV, is a result of annihilating dark matter particles with mass just above $m_Z/2$. A non-relativistic virtual $Z^0$ decay (i.e. when the rest mass of the annihilating dark matter particle is slightly above 1/2 the $Z^0$ mass) provides a remarkably good fit to the observed $e^+/(e^+ + e^-)$ ratio below 10 GeV. At higher energies, however, the predicted $e^+/(e^+ + e^-)$ ratio rises above the observed values, within conventional assumptions about the injection and propagation. In particular, it was assumed that the positrons and primary electrons are each injected homogeneously, with the same spatial profile, and that their propagation in the Galaxy is identical. Relaxing the assumption of homogeneous injection for the $Z^0$ decay products was explored in a consequent work, and the fit to the data at high energies improved considerably. A summary of the virtual $Z^0$ decay model is presented here.

2. The basic equation and its solutions

The fact that the observed positron excess appears at the characteristic energy range of positrons from non-relativistic $Z^0$ decay motivates our consideration of this origin. Such $Z^0$ particles could be produced by the annihilation of neutralinos that are just beyond half the $Z^0$ mass (so that they do not contribute to the $Z^0$ decay width). Alternatively, they could be produced via the slow decay of some other particle that is just higher than twice the $Z^0$ mass. One could even imagine that such a particle would couple to non-relativistic matter but not to relativistic matter, e.g. a massive dilaton coupled to the trace of $T_{\mu\nu}$, so that it would decay preferentially into $Z$'s, but not directly into the lighter decay products of the $Z$.

In order to achieve enough annihilation of the neutralinos, clumpiness needs to be invoked. This is a generic problem of models which make use of dark matter annihilation products. The fact that clumping is required, and that a likely place for this is near the Galactic centre, brings us to consider that the positrons in our neighborhood that are dark matter annihilation products would have a minimum age. The time needed to diffuse from the source to our neighborhood, and the age distribution of the positrons that make it to the Earth’s vicinity will result in fewer young positrons than
one associates with the standard leaky box model. We therefore consider, in addition to standard assumptions about propagation, that the positrons are injected by an effectively point source at a finite distance, and look at the diffused equation.

The steady state diffused equation for the particle number density is

$$\frac{\partial n}{\partial t} = 0 = \hat{D}n - Rn + \frac{1}{m_Z} \frac{\partial}{\partial x} \left( m_Z \frac{dx}{dt} n \right) + I(x)\delta(r)$$  \hspace{1cm} (1)

where $x = E/m_Z$. $\hat{D}$ is a diffusion operator, $R = Bx^{0.5}$ is the escape rate, $m_Z dx/dt = A x^2$ with $A = 8.5 \times 10^{-16}$ erg/s is the Compton loss rate, corresponding to an electromagnetic energy density in the Galaxy of $10^{-12}$ erg/cm$^3$ and $I(x)$ is the injected spectrum.

For a homogeneous injection, $\hat{D}n = 0$,

$$n(x) = \frac{m_Z}{Ax^2} \exp \left[ -\frac{2m_Z B}{A\sqrt{x}} \right] \int_x^\infty I(x') \exp \left[ \frac{2m_Z B}{A\sqrt{x'}} \right] dx'$$ \hspace{1cm} (2)

A one-dimensional diffusion operator with coefficient $\mathcal{D}$, $\hat{D} = \mathcal{D} \frac{\partial^2}{\partial x^2}$, and with boundary conditions such that $\frac{\partial n}{\partial r}|_{r=L} = 0$ (conserving the number of particles except for the escape term), gives

$$n(x,r) = \frac{m_Z}{Ax^2} \exp \left[ -\frac{2m_Z B}{A\sqrt{x}} \right] \times$$

$$\int_x^\infty I(x') \exp \left[ \frac{2m_Z B}{A\sqrt{x'}} \right] \sum_{\pi n r} \cos \left[ \frac{\pi n L}{L} \right] \exp \left[ -\left( \frac{\pi n L}{L} \right)^2 \frac{m_Z \mathcal{D}}{A} \left( \frac{1}{x'} - \frac{1}{x} \right) \right] dx'$$ \hspace{1cm} (3)

While $\mathcal{D}$ (the diffusion coefficient) and $L$ (the size of the leaky box) are free parameters, we took $r = 8$ Kpc, the distance to the galactic centre. $K \equiv \mathcal{D}/r^2$ gives the inverse time for diffusion.

3. Injected spectrums

3.1. Backgrounds

The injected spectrum of primary electrons is taken to be $I(x) = C x^{-2}$. For the positron background we take $Dx^{-2.8}$, presumed to come from cosmic ray collisions.
3.2. **Z⁰ Products**

We now calculate the spectrum of positrons resulting from the annihilation of the neutralinos (χ) via the virtual Z⁰ channel assuming that m_χ is just above m_Z/2. It is essentially Z⁰ decay. We have taken three generations of particle families, except for the top quark since it is much heavier than the mass of the Z⁰. All other particles were considered massless, and the calculation is in zeroth order. The positrons’ as well as the electrons’ injected spectrum has contributions from the following channels:

- **Z → e\bar{e}**
  The branching ratio of direct decay to electron-positron pair is \(\Gamma(Z \rightarrow e\bar{e})/\Gamma(Z \rightarrow \text{all}) = 0.0344\). The energy spectrum of these positrons is \(\delta(x-1/2)\).

- **Z → μ\bar{μ}**
  All the produced muons eventually decay into ν_μν_e, in an exchange of a W boson. The resulting spectrum energy in the muon rest frame is:
  \[
  I(\epsilon) = \frac{16}{m_μ^4} \left(3m_μ\epsilon^2 - \epsilon^4\right), \quad 0 < \epsilon < \frac{m_μ}{2}
  \]
  where \(\epsilon\) is the positron’s energy in the muon’s rest frame. To boost the spectrum into the observer’s frame, we have taken \(\beta \approx 1\) and \(\gamma = m_Z/(2m_μ)\). Assuming the muon decay is isotropic:
  \[
  I(x) = \frac{2}{3} [5 - 36x^2 + 32x^3]
  \]
  with a similar branching ratio, 0.0344.

- **Z⁰ → τ\bar{τ}**
  The tau can decay into 3 colors of ud pairs, into electrons directly, or into muons which then decay into electrons. For the 20% of taus which decay directly into electrons, the calculation is the same as the above, with a branching ratio of 0.0344 × 0.2.

  For the 20% of the \(\tau \rightarrow μ \rightarrow e\) channel, the resulting contribution is:
  \[
  I(x) = \frac{2}{9} \left[ -\frac{95}{3} - 108x^2 + \frac{1408}{3}x^3 - (25 + 324x^2 + 128x^3) \ln(2x) \right]
  \]
  with a branching ratio of 0.0344 × 0.2.

  We have neglected the 60% of the \(\tau \rightarrow q\).
The hadronic channel of the $Z$ decay eventually contributes some electrons and positrons through the production of pions. We have estimated the energy spectrum of the pion production from $^{11}$:

$$P(E_\pi) = 10^{a_k - b_k \bar{x}}$$

$a_1 = 3$, $b_1 = 10$ \hspace{1cm} $0 < \bar{x} < 0.1$

$a_2 = 2$, $b_2 = 4$ \hspace{1cm} $0.1 < \bar{x} < 1$

with $\bar{x} \equiv 2E_\pi/m_Z$.

In the pion’s rest frame, the energy spectrum of the electrons is close to a delta function, with an energy of $\sim 45$ MeV. In transferring back to the lab frame, we have used $\beta \approx 1$ and $\gamma = E_\pi/m_\pi$.

The resulting contribution is:

$$I_h(x) = \frac{14}{9} \int_{\frac{1}{2}}^{1} \frac{d\bar{x}}{\bar{x}} 10^{a_k - b_k \bar{x}}$$

The branching ratio of of the $Z^0$ decay into hadrons is 0.6916.

After taking the right weight of each of these channels, the injected spectrums are:

$$I(x) = 0.6916 N I_h(x) + 0.0344 N \times$$

$$\left[ \delta \left( x - \frac{1}{2} \right) + \frac{70}{27} - \frac{168}{5} x^2 + \frac{6272}{135} x^3 - \left( \frac{10}{9} + \frac{72}{5} x^2 + \frac{256}{45} x^3 \right) \ln(2x) \right]$$

where $N$ is the annihilation rate density. The needed injection rate to supply the observed positrons is about $1 \times 10^{-29} \text{cm}^{-3} \text{s}^{-1}$ above 10 GeV. The actual annihilation rate density $N$ therefore needs to be about $(2.2 \times 0.034)^{-1}$ times that, $N = 1.3 \times 10^{-29} \text{cm}^{-3} \text{s}^{-1}$. Here the factor of 2.2 assumes that $Z$ decay into electrons, muons, and 20 percent of the taus results in positrons above 10 GeV. The need for significant dark matter clumping has not been avoided in the present scenario.

4. Results

We first consider a homogeneous injection of both backgrounds and the $Z^0$ products, see Figure 1. A striking feature of this figure is the good fit below 10 GeV, which is mainly due to the positrons that emerge from decaying muons. Above 10 GeV, however, the fit grows very quickly, which is not compatible with the plateau that the data suggests. The reason for this
rise is that some \( Z^0 \)'s decay directly into high energy \( e^+e^- \) pairs so that the \( e^+ \) energy is half the \( Z^0 \) mass, and this gives rise to a high energy bump in the \( e^+/(e^++e^-) \) ratio at about 50 GeV. Even though the branching ratio into direct \( e^+e^- \) is rather small (0.0344), it is dominant over the power-law background contribution at this energy. While this bump can be partially washed out by losses and escape, it was found that the high energy \( e^+/(e^++e^-) \) ratio is nevertheless apparently too high to fit the observations. As discussed in Eichler (1989) this is a generic problem for any positron source that is significantly harder than the primary electrons above 10 GeV.

Next we consider an inhomogeneous injection for the \( Z^0 \) products, while still keeping the backgrounds homogeneous. Figure 2 shows a fit with a single point source of \( Z^0 \) decay and a single diffusion coefficient. The good fit to the low energies from Figure 1 is still present, but the excess in energies toward \( x = 1/2 \) is now is suppressed by the finite age effect. As the figure shows, we are now facing a scenario which is opposite to the homogeneous injection case; the finite age effect tends to suppress the high energy excess at the price of killing it off altogether.

However, there are several possibilities that dispel this problem: There may be more than one source, and there may be more than one route by which the particles diffuse or free-stream from the source to our vicinity. High energy particles diffuse much less than low energy ones because they are fewer in number and create less waves. So their self-generated scattering is less efficient. Thus, the fraction of free streaming particles should be higher at higher energy. Figure 3 shows a combination of two \( Z^0 \) decay components, an older, larger one that arrives via diffusion, and a younger, smaller component that has managed more free streaming. This figure illustrates that if one takes an age distribution into account, the flexibility in adjusting the high energy spectrum becomes much larger, and can be fitted to the data.

For sake of comparison, we also include a power law injected spectrum. Figure 4 shows various power laws, and Figure 5 shows two components with different ages. We find that as long as the injected power law is hard enough, one can produce a low energy dip. While we find that the goodness of fit provided by \( Z^0 \) decay and by power law are qualitatively the same, we have deliberately not quantified this with the standard statistical measures. The GeV dip which we are trying to address formally has less statistical weight compared with the overall shape of the data, but it is the more striking observational result and the one that seems hardest to explain.
to get the best parameters (for either power law or $Z^0$ decay as injected spectrums) would wash out the low energy behavior that we are focusing on. As we are interested in exploring the possibility of connecting the GeV behavior to properties of dark matter more than constraining parameters, we settle for the time being in showing characteristic examples.

5. Discussion

We have considered a scenario for positron excess via non-relativistic $Z^0$ decay in which the energies of the decay products are as low as possible and the neutralino annihilation rate that would give rise to the $Z^0$’s as high as possible. We considered homogeneous injection, as well as injection from a point source at a finite distance.

Homogeneous injection fits the data remarkably well for energies below 10 $GeV$, but produces too many high energy positrons (originating from the direct $Z^0 \rightarrow e^+e^-$ channel). A single source with a single diffusion coefficient does not provide a good fit to the high energy either, and the most promising scenario is to allow a small fraction of the positrons to free stream and arrive at the Earth’s vicinity much younger than the rest.

Although we can reproduce the 7 $GeV$ dip, the peak at $E \sim 15$ $GeV$ is still too big for the HEAT data. This seems to be a generic feature of our results, regardless of whether the injection source is virtual $Z^0$ decay or a power law. The problem would be worse if the virtual $Z^0$ had an energy well above $m_Z$. Knowing that the virtual $Z^0$ must be close to its mass shell if it is to provide a decent fit suggests that its loop corrections would be large and it might be discernable or falsifiable with particle collider data. Further possible observational consequences will be considered in a forthcoming publication 9.

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Figure 1. $f \equiv e^+/ (e^+ + e^-)$ as a function of $x \equiv E/m_Z$. Our model compared with HEAT 94+95 data. The data and its error bars are marked with the crosses. The parameters of this example are $A = 8.5 \times 10^{-16}$ erg/s, $B = 7.6 \times 10^{-15}$ 1/s, $C = 9.1 \times 10^{-28}$ $e/(cm^3 \cdot s)$, and $D = 1.3 \times 10^{-31}$ $p/(cm^3 \cdot s)$. 
Figure 2. The $e^+/(e^+ + e^-)$ as a function of $x = E/m_Z$, for a single source $Z$ decay injected spectrum. $A = 8.5 \times 10^{-16}$ erg s, $B = 7.1 \times 10^{-15}$ s, $C = 4.0 \times 10^{-29}$ cm$^3$ s, $D = 1.3 \times 10^{-31}$ cm$^3$ s, and $K = 1.9 \times 10^{-16}$ s.

Figure 3. The $e^+/(e^+ + e^-)$ as a function of $x = E/m_Z$, for a combination of 2 sources of $Z$ decay injected spectrum. $A = 8.5 \times 10^{-16}$ erg s, $B = 7.6 \times 10^{-15}$ s, $C = 4.9 \times 10^{-29}$ cm$^3$ s, $D = 1.3 \times 10^{-31}$ cm$^3$ s, $K_1 = 2.8 \times 10^{-14}$ s, and $K_2 = 2.8 \times 10^{-16}$ s. The ratio between the two components is $1:5$. 
Figure 4. The $\frac{e^+}{(e^++e^-)}$ as a function of $x = E/m_Z$, for various power laws, $N x^w$, as the injected spectrum. $A = 8.5 \times 10^{-16}$ \text{erg s}^{-1}$, $B = 4.4 \times 10^{-15}$ \text{erg s}^{-1}$, $C = 1.7 \times 10^{-29}$ \text{cm}^{-3} \text{s}^{-1}$, $D = 1.1 \times 10^{-31}$ \text{cm}^{-3} \text{s}^{-1}$ and $K = 6.6 \times 10^{-17}$ \text{s}^{-1}$. The ratio between the two components is 10 : 1.

Figure 5. The $\frac{e^+}{(e^++e^-)}$ as a function of $x = E/m_Z$, for a combination of 2 sources of power law ($w = -0.3$) injected spectrum. $A = 8.5 \times 10^{-16}$ \text{erg s}^{-1}$, $B = 4.4 \times 10^{-15}$ \text{erg s}^{-1}$, $C = 1.7 \times 10^{-29}$ \text{cm}^{-3} \text{s}^{-1}$, $D = 1.1 \times 10^{-31}$ \text{cm}^{-3} \text{s}^{-1}$, $K_1 = 7.2 \times 10^{-17}$ \text{s}^{-1}$ and $K_2 = 1.2 \times 10^{-16}$ \text{s}^{-1}$. The ratio between the two components is 10 : 1.