We evaluate energy levels of the $K\pi$ system in the $K^*$ channel in finite volume using chiral unitary theory. We use these energy levels to obtain $K\pi$ phase shifts, and then obtain the $K^*$ mass and its decay width. We investigate their dependence on the pion mass and compare this with Lattice QCD calculations. We also solve the inverse problem and obtain the $K\pi$ phase shifts from these “synthetic” lattice data.

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II. THE CHIRAL UNITARY APPROACH IN INFINITE AND FINITE BOX

The $K\pi$ scattering amplitude in $P$-wave has been studied in Refs. [18 19] by using the chiral unitary model. In this paper we shall follow the same approach and use the following Bethe-Salpeter equation in their on-shell factorized form [18 21] (for a quantitative study of off-shell effects in this context, see, e.g., Ref. [22]):

$$T(s) = \frac{V(s)}{1 - V(s)G(s)}.$$  \hspace{1cm} (1)

We note that there is only one channel, the $K\pi$ channel. The relevant $V$-matrix for the $K\pi$ scattering has been studied in Refs. [18 23]:

$$V(s) = -\frac{p^2}{2f^2} (1 + \frac{2G_V^2}{f^2} \frac{s}{M_{K^*}^2 - s}),$$  \hspace{1cm} (2)

where $M_{K^*}$ is the bare $K^*$ mass, $f$ is the $\pi/K$ decay constant, and $G_V$ is the coupling for a vector meson to two pseudoscalar mesons. We note that this potential $V(s)$ is a bit different from the one used in Ref. [19], where the factor $p^2$ is absorbed into their $G$-function so that $V(s)$ does not depend on the momentum. The $G$-function for the two-meson ($\pi-K$) propagator having masses $m_{\pi}$ and $m_K$ is defined as

$$G(p^2) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_{\pi}^2 + i\epsilon} \frac{1}{(p-q)^2 - m_K^2 + i\epsilon}.$$  \hspace{1cm} (3)

where $p$ is the four-momentum of the external meson-meson system. There are many methods to regularize this loop-function. In Ref. [19] the authors use the cut-off method, but in this paper we shall use the dimensional regularization which is more convenient when studying the $K\pi$ interaction in finite volume. We note that these two methods are equivalent up to certain energy level range, as proved in Ref. [21]. The dimensional regularization result is

$$G(s) = \frac{1}{(4\pi)^2} \left\{ a(\mu) + \log \frac{m_{\pi}^2}{\mu^2} + \frac{m_{K}^2 - m_{\pi}^2 + s}{2s} \log \frac{m_K^2}{m_{\pi}^2} 
+ \frac{Q(\sqrt{s})}{\sqrt{s}} \left[ \log(s - (m_K^2 - m_{\pi}^2)) + 2\sqrt{s}Q(\sqrt{s}) \right] + \log(s + (m_K^2 - m_{\pi}^2)) + 2\sqrt{s}Q(\sqrt{s}) 
- \log(-s - (m_K^2 - m_{\pi}^2)) + 2\sqrt{s}Q(\sqrt{s}) \right\},$$  \hspace{1cm} (4)

where $s = p^2$, $Q(\sqrt{s})$ is the on-shell momentum of the particles, $\mu$ is a regularization scale and $a(\mu)$ is a subtraction constant. In this paper we shall work in the center-of-mass frame, where the energy of the system is $E = \sqrt{s}$. The regularization parameters are chosen to be

$$a(\mu) = -1.0,$$  \hspace{1cm} (5)

$$\mu = M_{K^*}.$$  \hspace{1cm} (6)

The two parameters $f$ and $G_V$ are taken from Ref. [19]:

$$G_V = 53.81 \text{ MeV},$$  \hspace{1cm} (7)

$$f = 86.22 \text{ MeV},$$  \hspace{1cm} (8)

but the parameter $M_{K^*}$ is a bit different from the one used in Ref. [19], because we are using the dimensional regularization other than the cut-off method used in Ref. [19]. To fix $M_{K^*}$, we use the experimental data of the $K\pi P$-wave phase shifts, which are related to the $T(s)$ through:

$$T(E) = \frac{-8\pi E}{p\cot \delta(p) - ip},$$  \hspace{1cm} (9)

where $p$ is the center-of-mass momentum. We use the experimental data of Refs. [24 25], and evaluate $M_{K^*}$. The fitting results are shown in Fig. 11 where $M_{K^*}$ is fitted to be:

$$M_{K^*} = 919.03 \text{ MeV}.$$  \hspace{1cm} (10)

All the above formulae are defined in the infinite space. To study the $K^*$ meson in the finite volume, we simply change the $G$-function of dimensional regularization (Eq. (4)) by the one which is defined in the finite box of side.
FIG. 1: The solid curve shows \( K\pi \) scattering \( P \)-wave phase shifts obtained using Eq. (1) and Eq. (9), and the dot-dashed curve the results from Ref. [19]. The experimental data are taken from Ref. [24] and Ref. [25], shown using solid circles and triangles, respectively.

\[ L \in [5, 26], \text{i.e., we simply change the integration over momenta by a sum over the discrete values of the momenta allowed by the periodic conditions in the box. We denote the latter one by } \tilde{G}(s, L), \text{ and it can be obtained through:} \]

\[
\tilde{G}(s, L) - G(s) = \lim_{q_{\text{max}} \to \infty} \left( \frac{1}{L^3} \sum_{q_i} I(q_i) - \int_{q < q_{\text{max}}} \frac{d^3 q}{(2\pi)^3} I(q) \right). \tag{11}
\]

In this equation the discrete momenta in the sum are given by \( \vec{q} = \frac{2\pi}{L} \vec{n} \) (\( \vec{n} \in \mathbb{Z}^3 \)) and the function \( I(q_i) \) is

\[
I(q_i) = \frac{1}{2 \omega_1(q) \omega_2(q)} \frac{\omega_1(q) + \omega_2(q)}{E^2 - (\omega_1(q) + \omega_2(q))^2}, \tag{12}
\]

where \( \omega_{1,2}(q) = \sqrt{m_{1,2}^2 + q^2} \). We show the real part of \( \tilde{G}(s, L) - G(s) \) in Fig. 2 as a function of \( q_{\text{max}} \), where \( L \) is fixed to be 2.5 \( m^{-1}_\pi \) and \( E \) to be 800 MeV. Its convergence is good when \( q_{\text{max}} \) is larger than 3000 MeV. However, we shall still make an average of this quantity for smaller values of \( q_{\text{max}} \) in order to save the computational time [5, 26].

### III. THE ENERGY LEVELS IN THE CHIRAL UNITARY APPROACH

To calculate the energy levels of the \( K\pi \) scattering amplitude in \( P \)-wave, we need to find the poles of the \( T(s) \) matrix, which are just solutions of the following equation

\[ 1 - V(s)\tilde{G}(s, L) = 0. \tag{13} \]

Here \( \tilde{G}(s, L) \) is defined in the finite volume and can be obtained through Eq. (11). From this equation we can clearly see that the energy levels for \( K\pi \) \( P \)-wave scattering are functions of the cubic box size \( L \), as well as the pion mass \( m_\pi \). In the following sections we shall study their dependence on these variables. In this section we study the volume dependence and in the next section we shall study the pion mass dependence. We note again that our procedures follow closely the method used in Refs. [5, 10–13, 26].

On the left panel of Fig. 3 we show the energy levels as functions of the cubic box size \( L \), which are obtained after performing an average for different \( q_{\text{max}} \) values between 1200 MeV and 2000 MeV. On the right panel we show these energy levels separately for \( q_{\text{max}} \) values 1300, 1500, 1700 and 1900 MeV. We clearly see that results for different \( q_{\text{max}} \) values are almost the same.

The phase shift can be extracted from these energy levels. To do this we follow the procedure used in Ref. [3], and use Eq. (9) to calculate the \( K\pi \) \( P \)-wave phase shifts, where the scattering amplitudes \( T(E, L) \) are obtained using the
FIG. 2: The real part of Eq. (11). Here we choose $L = 2.5 \ m_{\pi}^{-1}$ and $E = 800 \ MeV$.

FIG. 3: Energy levels as functions of the cubic box size $L$, derived using $\tilde{G}(s, L)$ from Eq. (11). We perform an average for different $q_{\text{max}}$ values between 1200 MeV and 2000 MeV on the left panel, while show them separately for $q_{\text{max}} = 1300, 1500, 1700$ and 1900 MeV on the right panel.

The lowest energy level shown in Fig. 3:

$$T(E, L) = \frac{V(E)}{1 - V(E)G(E)} = \frac{\tilde{G}(E, L)^{-1}}{1 - \tilde{G}(E, L)^{-1}G(E)}.$$  \hfill (14)

Here we have used Eq. (13), i.e., $V(s)^{-1} = \tilde{G}(s, L)$. These procedures can be done for all energy levels. However, the lowest energy level should be the best one, because we are using the chiral unitary approach which is an effective theory for low energies. Therefore, we use the lowest energy level to evaluate phase shifts, and the result is shown in Fig. 4.

Using the phase shift $\delta(E)$ we can fit the physical quantities for the $K^*$ meson, and evaluate $m_{K^*}$, $g_{K^* \pi K}$ and $\Gamma_{K^*}$. We note that $m_{K^*}$ is the $K^*$ mass we obtained, i.e., one of our outputs; while $M_{K^*}$ is the bare $K^*$ mass, i.e., one of our inputs. To do that, we use the following two equations in Refs. [13, 27] to extract the $K^*$ properties:

$$\cot \delta(s) = \frac{m_{K^*}^2 - s}{\sqrt{s} I_{K^*}(s)}, \quad \text{and} \quad \Gamma_{K^*}(s) = \frac{g_{K^* \pi K}^2}{s} \frac{8\pi}{\sqrt{s}}.$$  \hfill (15)

We note that the factor $8\pi$ in the second equation is our normalization, while in Ref. [10] the authors use $6\pi$. The
FIG. 4: The solid curve is the $K\pi$ scattering $P$-wave phase shifts extracted from the lowest energy level of the left panel of Fig. 3, the dashed curve is the phase shifts calculated in the infinite volume, and the experimental data: solid circles [24]; triangles [25].

The results from fitting the phase shifts calculated using the lowest $K\pi$ energy level are

$$m_{K^*} = 894.89^{+9.74}_{-9.65} \text{ MeV}, \quad g_{K^*\pi K} = 6.48^{+0.04}_{-0.03}, \quad \Gamma_{K^*} = 50.74^{+1.99}_{-1.96} \text{ MeV}. \quad (16)$$

In these results the theoretical uncertainties are estimated following Ref. 13, where we assume that the uncertainty of the three parameters $G_V, M_{K^*}$ and $f$ in Eq. (2) is about 1%. The uncertainty of the energy levels and phase shifts are not large, as shown in Fig. 5. Particularly, if we fix $L = 2.0\ m^{-1}$, the uncertainty of the energy levels is less than 1%, and the corresponding uncertainty of the phase shifts is about 5% around $E = 900\ \text{MeV}$. These results compare favorably with the experimental results [28]:

$$m_{K^*} = 891.66 \pm 0.26\ \text{MeV}, \quad g_{K^*\pi K} = 6.7 \pm 1.2, \quad \Gamma_{K^*} = 50.8 \pm 0.9\ \text{MeV}. \quad (17)$$

FIG. 5: The uncertainty of energy levels and phase shifts.

**IV. DEPENDENCE ON THE PION MASS**

As we know, due to the computer limitation, the Lattice QCD calculations usually use a non-physical pion mass. Therefore, in this section, we also use non-physical pion masses to study the mass and decay width of the $K^*$ meson,
in order to compare with the Lattice QCD result. We define \( m_0 \) to be the physical \( \pi \) mass, and now \( m_\pi \) is a free parameter. We change it from \( m_0 \) to \( 3m_0 \). At the same time other parameters can also change with \( m_\pi \). We follow the same approach of Refs. \[14, 15, 32\], where the variation of \( f \) as a function of \( m_\pi \) is
\[
\frac{f(m_\pi)}{f(m_0)} = 1 + 0.048((\frac{m_\pi}{m_0})^2 - 1),
\]
with \( f(m_0) = 86.22 \text{ MeV} \). The coupling \( G_V \) is related to \( f \) \[33, 36\], as \( G_V = f/\sqrt{2} \) valid to leading order \[36\], consequently, we keep \( G_V \) unchanged. The kaon mass \( m_K \) can also change with the pion mass \( m_\pi \), and we use the following relation \[37\]:
\[
m_K^2 = a + bm_\pi^2,
\]
where \( a = 0.291751 \text{ GeV}^2 \), and \( b = 0.670652 \). We note that the Lattice calculations also use non-physical kaon masses \[14, 15\], but all these values are not much different from the physical one. Accordingly, in this paper we shall first keep it unchanged, then use the kaon mass in Eq. \[19\], and finally use the same values of \( m_K \) as the Lattice ones \[14, 15\], in order to compare our results with theirs. On the other hand, the bare \( K^* \) mass, \( M_{K^*} \) in Eq. \[2\], provides the link of the theory to a genuine component of the \( K^* \) meson, not related to the pion cloud, and we assume it to be \( m_\pi \) independent.

To calculate the energy levels we follow the same procedures which have been used in the previous section. The result is shown in Fig. \[6\] where we have used \( m_\pi = 1.5 \ m_0 \) (left), \( m_\pi = 2.0 \ m_0 \) (middle) and \( m_\pi = 2.5 \ m_0 \) (right). The solid curves are obtained using the physical kaon mass \( m_K = 496 \text{ MeV} \), and the dotted curves are obtained using the non-physical kaon mass evaluated using Eq. \[19\]. We can see that the results obtained using these different kaon masses do not differ much. Here, we note that the \( x \)-coordinate is expressed in units of \( m_\pi^{-1} \), not \( (m_0)^{-1} \).

These energy levels can be used to calculate the phase shifts again following our previous procedures. The results are shown in Fig. \[7\] where again we have used \( m_\pi = 1.5 \ m_0 \) (left), \( m_\pi = 2.0 \ m_0 \) (middle) and \( m_\pi = 2.5 \ m_0 \) (right). The solid curves are obtained using the physical kaon mass \( m_K = 496 \text{ MeV} \), and the dashed curves are obtained using the non-physical kaon mass evaluated using Eq. \[19\]. We note that the dashed curve on the right panel of Fig. \[7\] vanishes, because the sum of \( 2.5 \ m_0 \) and non-physical kaon mass \( m_K = 610 \text{ MeV} \) is already above the \( K^* \) threshold.

FIG. 6: Energy levels as functions of the cubic box size \( L \). The left, middle and right figures correspond to \( m_\pi = 1.5 \ m_0 \), \( m_\pi = 2.0 \ m_0 \) and \( m_\pi = 2.5 \ m_0 \), respectively. The solid curves are obtained using the physical kaon mass \( m_K = 496 \text{ MeV} \), and the dotted curves are obtained using the non-physical kaon mass evaluated using Eq. \[19\].

Now we can compare our results with the Lattice results of Refs. \[14, 15\], where \( m_\pi \) is chosen to be \( 240 \text{ MeV} \) and \( m_K \) is chosen to be \( 548 \text{ MeV} \) in Ref. \[14\], and \( m_\pi \) is chosen to be \( 266 \text{ MeV} \) and \( m_K \) is chosen to be \( 552 \text{ MeV} \) in Ref. \[15\]. We show their comparison in Table. \[II\] and Table. \[III\] where \( E_1 \) and \( E_2 \) are on the lowest and the second energy levels, \( \delta_1 \) and \( \delta_2 \) are extracted from \( E_1 \) and \( E_2 \), respectively. We find that the energy levels and the extracted phase shifts are similar, and so our results compare favorably with those lattice results obtained in Refs. \[14, 15\].

Again the theoretical errors are obtained by assuming that the uncertainties of the three parameters \( G_V, M_{K^*} \) and \( f \) in Eq. \[2\] are about 1%.

Finally, we use Eq. \[15\] to fit the phase shifts shown in Fig. \[7\] to obtain the \( K^* \) mass (left), the coupling constant \( g_{K^*\pi K} \) (middle) and the decay width \( \Gamma_{K^*} \) (right), which are shown in Fig. \[8\] as functions of \( m_\pi \). We can see that the results of \( g_{K^*\pi K} \) obtained using the physical kaon mass and non-physical kaon masses in Eq. \[19\] are very similar, while the results of \( M_{K^*} \) and \( \Gamma_{K^*} \) are not so similar. This may be because the phase spaces differ much, although the kaon masses do not differ much. We also note that when using Eq. \[19\], the physical kaon mass \( m_K = 496 \text{ MeV} \) can not be reached at the physical pion mass \( m_\pi = 138 \text{ MeV} \). Therefore, the two curves do not cross. Again we can compare our results with the lattice results in Ref. \[16\], where \( m_\pi = 266\text{MeV}, m_K = 552\text{MeV} \) and \( L = 1.98\text{fm} \). Their
FIG. 7: The $K\pi$ phase shifts with different pion masses. The left, middle and right figures correspond to $m_\pi = 1.5$ $m_0^\pi$, $m_\pi = 2.0$ $m_0^\pi$ and $m_\pi = 2.5$ $m_0^\pi$, respectively. The solid curves are obtained using the physical kaon mass $m_K = 496$ MeV, and the dashed curves are obtained using the non-physical kaon mass evaluated using Eq. (19).

TABLE I: Comparison with Ref. [14], where $m_\pi = 240$ MeV, $m_K = 548$ MeV and $L = 3$ fm.

| $E_1$ | $E_2$ |
|-------|-------|
| Our Results 912.6 $\pm$ 8.4 MeV | 1166.7 $\pm$ 1.3 MeV |
| Lattice Results 926.9 $^{+2.5}_{-10.0}$ MeV | 1171.7 $^{+0.7}_{-22.5}$ MeV |

TABLE II: Comparison with Ref. [12], where $m_\pi = 266$ MeV, $m_K = 552$ MeV and $L = 1.98$ fm.

| $E_1$ | $E_2$ | $\delta_1$ | $\delta_2$ |
|-------|-------|------------|------------|
| Our Results 926.2 $\pm$ 9.1 MeV | 1511.0 $^{+2.4}_{-2.0}$ MeV | 158.04 $\pm$ 2.13$^\circ$ | 175.52 $^{+0.76}_{-0.42}$ |
| Lattice Results 915.6 $\pm$ 3.0 MeV | 1522.3 $\pm$ 7.0 MeV | 160.61 $\pm$ 0.73$^\circ$ | 177.0$^\circ$ $\pm$ 2.6$^\circ$ |

results are $m_{K^*} = 891 \pm 14$ MeV and $g_{K^*\pi K} = 5.7 \pm 1.6$, which change to $m_{K^*} = 891 \pm 14$ MeV and $g_{K^*\pi K} = 6.6 \pm 1.9$ in our normalization after taking into account the factor $\frac{\delta m}{\delta \pi}$. These results are in agreement, within errors, with our result $m_{K^*} = 910.6 \pm 8.4$ MeV and $g_{K^*\pi K} = 5.54^{+0.16}_{-0.03}$.

V. THE INVERSE PROBLEM OF GETTING PHASE SHIFT FROM LATTICE DATA

In this section we study the inverse process of getting phase shifts from Lattice Data using two energy levels and a parametrized potential. This has also been done in Refs. [3, 5, 10–13], showing that this method is rather efficient. To do this we assume that the first and second energy levels shown in Fig. 3 are “Lattice” inputs, or “synthetic” data. We shall use them to inversely evaluate the $V$-matrix and then calculate phase shifts. At the same time we shall give these “lattice data” some error bars which can be used to evaluate the uncertainties of the phase shifts.

Our procedures follow Refs. [3, 5, 10–13]. We take five energies from the first level and five more from the second one, and associate to them an error of 10 MeV. Then we use the following function which accounts for a CDD pole [39] to do the one-channel fitting:

$$V = -ap^2(1 + \frac{bs}{c^2 - s}).$$

where $a$, $b$ and $c$ are three free parameters which we shall fit with the “Lattice” data shown in Fig. 3. The results are shown Fig. 4 where the energy levels are calculated from all the possible sets of parameters having $\chi^2 < \chi^2_{\text{min}} + 1$. Here $\chi^2_{\text{min}} = 0.064$ is the best fitting we obtained, where the three parameters are:

$$a = 6.50 \times 10^{-5} \text{ MeV}^{-2}, \quad b = 0.79, \quad c = 918.90 \text{ MeV},$$

we find that errors in the phase shift are large at small energies, but they become smaller as the energy increases.

As mentioned in Ref. [3] the result of this inverse analysis does not depend on which cut off, or subtraction constant one uses in the analysis, as far as one uses the same ones to induce $V$ from the lattice data and then later on to get phase shifts in the infinite volume from Eq. (1). The method proves to be practical and efficient.
FIG. 8: The $K^*$ mass (left), the coupling constant $g_{K^*\pi K}$ (middle) and the decay width $\Gamma_{K^*}$ (right) as functions of $m_\pi$. The solid curves are obtained using the physical kaon mass $m_K = 496$ MeV, and the dashed curves are obtained using the non-physical kaon mass evaluated using Eq. (19).

FIG. 9: The dashed curves are the best fitted results $\chi^2_{\text{min}} = 0.064$ and the bands are fits that fulfill the $\chi^2 < \chi^2_{\text{min}} + 1$ criterion. The discrete points on the right figure are the results of the direct determination from each “data” points on the left figure.

VI. CONCLUSION

In this paper we tried to find an efficient strategy to obtain $K\pi$ phase shifts, and thus the $K^*$ meson properties from energy levels obtained in lattice calculations. To do this we studied the $K\pi$ interaction in $P$-wave in a finite box using the chiral unitary approach which has been very successful to provide $K\pi$ phase shifts in infinite space. We evaluated energy levels which are functions of the cubic box size $L$ and the pion mass $m_\pi$. Then we use these energy levels to obtain $K\pi$ phase shifts. Finally we use these phase shifts to fit the physical quantities for the $K^*$ meson: $m_{K^*} = 894.89^{+9.74}_{-9.65}$ MeV, $g_{K^*\pi K} = 6.48^{+0.04}_{-0.03}$, $\Gamma_{K^*} = 50.74^{+1.99}_{-1.96}$ MeV. The results compare favorably with those listed in PDG [28].

To compare our results with the Lattice QCD calculations, we also used non-physical pion masses and redid the same calculations. We note that other parameters can also change with $m_\pi$, and we have considered these effects. The comparison of our results with the Lattice QCD results are shown in Table. I and Table. II where we can see our results compare favorably with those lattice results obtained in Refs. [14, 15]. We note that in these calculations we have estimated the theoretical uncertainties.

We also studied the inverse process of getting phase shifts from our “synthetic” lattice data using two energy levels and a parametrized potential. This procedure provides a global fit that allows one to get phase shifts for all energies, and produces phase shifts in a large range of energies with good accuracy.

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