Twist-3 fragmentation functions in a spectator model with gluon rescattering

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We study the twist-3 fragmentation functions $H$ and $\tilde{H}$, by applying a spectator model. In the calculation we consider the effect of the gluon rescattering at one loop level. We find that in this case the hard-vertex diagram, which gives zero contribution to the Collins function, does contribute to the fragmentation function $H$. The calculation shows that the twist-3 T-odd fragmentation functions are free of light-cone divergences. The parameters of the model are fitted from the known parametrization of the unpolarized fragmentation $D_1$ and the Collins function $H^+_{1T}$. We find our result for the favored fragmentation function is consistent with the recent extraction on $H$ and $\tilde{H}$ from pp data. We also check numerically the equation of motion relation for $H$, $\tilde{H}$ and find that relation holds fairly well in the spectator model.

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I. INTRODUCTION

The Collins effect \cite{Collins} has played an important role in the understanding of spin asymmetries in various high energy processes, such as semi-inclusive deep inelastic scattering (SIDIS), hadron production in pp collision, and $e^+e^-$ annihilation into hadron pairs. The mechanism can be traced back to the so called Collins fragmentation function \cite{Collins}, denoted by $H^+_{1T}$, which is a transverse momentum dependent (TMD) nonperturbative object entering the factorized description of hard processes. It originates from the correlation between the transverse momentum of the fragmenting hadron and the transverse spin of the parent quark. Different from the ordinary unpolarized fragmentation function $D_1$, the Collins function is time-reversal-odd and chiral-odd. The extraction of the Collins function has been performed in Ref. \cite{Collins90}, and in Ref. \cite{Collins95} by considering TMD evolution.

Recently, a phenomenological analysis \cite{Schmidt} on SSA of inclusive pion production in pp collision \cite{Collins90} within the collinear factorization, shows that the fragmentation contribution is crucial in describing the SSAs of different processes in a consistent manner. Three twist-3 fragmentation functions, $H(z)$, $H(z)$ and $\tilde{H}^2_{FU}(z,z_1)$, participate in those processes. The first one corresponds to the first moment of the TMD Collins function and has been applied to interpret the SSA in pp collisions in previous studies \cite{H16,H19}. The second one appears in subleading order of a $1/Q$ expansion of the quark-quark correlator, while its TMD version $H(z,k_T^2)$ is also a twist-3 function. The function $\tilde{H}^2_{FU}(z,z_1)$ is the imaginary part of $H_{FU}(z,z_1)$, which involves the F-type multiparton correlation \cite{Collins90,Collins95}. The three functions are not independent, as they are connected by the equation of motion relation

$$H(z) = -2z\tilde{H}(z) + 2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \PV \frac{1}{2 - \frac{1}{z_1}} \tilde{H}^2_{FU}(z,z_1) = -2z\tilde{H}(z) + \tilde{H}(z). \quad (1)$$

In the last equation we have used $\tilde{H}(z)$ to denote the “moment” of $H^2_{FU}(z,z_1)$. The function $\tilde{H}$ might also contribute to the sin $\phi_S$ SSA in SIDIS through the coupling with the transversity distribution \cite{Collins95}.

Except for $\tilde{H}$, currently the quantitative knowledge about the other twist-3 fragmentation functions mainly relies on the parametrization in Ref. \cite{Schmidt}. The purpose of this work is to study the those fragmentation functions from the model aspect. Particularly, we will perform a calculation on the function $H$ and $\tilde{H}$ for the first time, using a spectator model. This model has been applied to calculate the Collins function for pions \cite{Collins90} and kaons \cite{Collins99}, by considering the pion loop, or the gluon loop. In our calculation we will incorporate the effect of the gluon loop. We first calculate the TMD function $H(z,k_T^2)$ and $\tilde{H}(z,k_T^2)$. The corresponding collinear functions are obtained by integrating over the transverse momentum.

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diagrams (and their hermitian conjugates) that may contribute to the correlator \( \Delta(z, k_T^2) \) in the spectator model. In the spectator model, the tree level diagrams lead to a vanishing result because of lack of the imaginary part of the propagator. Here we setup the notations adopted in our calculation. We use \( k \) and \( P_h \) to denote the momenta of the parent quark and the final hadron, respectively. We also apply the following kinematics:

\[
k = (k^-, k^+, k_T) = \left( k^-, \frac{k^2 + k_T^2}{2k^-}, k_T \right), \quad P_h = (P^+_h, P^-_h, 0_T) = \left( zk^-, \frac{M_h^2}{2zk^-}, 0_T \right),
\]

where the light-front coordinates \( a^+ = a \cdot n^\pm \) have been used, \( k_T \) denotes the momentum component of the quark transverse to the two light-like vectors \( n^\pm \), and \( z = P^-_h/k^- \) is the momentum fraction of the hadron. The transverse momentum of the hadron with respect to the parent quark direction is given by \( K_T = -zk_T \).

## II. Spectator Model Calculation of \( H \) and \( \tilde{H} \)

Here we set up the notations adopted in our calculation. We use \( k \) and \( P_h \) to denote the momenta of the parent quark and the final hadron, respectively. We also apply the following kinematics:

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k = (k^-, k^+, k_T) = \left( k^-, \frac{k^2 + k_T^2}{2k^-}, k_T \right), \quad P_h = (P^+_h, P^-_h, 0_T) = \left( zk^-, \frac{M_h^2}{2zk^-}, 0_T \right),
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### A. Calculation of \( H \) up to one gluon loop

The fragmentation function \( H(z, k_T^2) \) can be obtained from the following expression:

\[
\Delta(z, k_T) = \frac{1}{2z} \sum_X \frac{d^4 k - d^2 \xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle 0 |\mathcal{U}^{\alpha\beta}_{\infty} \mathcal{U}_{\infty}^{\beta}(\xi) \psi(h, X)\psi(0)\mathcal{U}^T_{0^+, \infty+} |0 \rangle_{\xi^+ = 0}.
\]

Here \( \mathcal{U}^{\alpha\beta}_{(a,b)} \) denotes the Wilson line running from \( a \) to \( b \) at the fixed position \( c \), to ensure the gauge invariance of the operator. In the spectator model, the tree level diagrams lead to a vanishing result because of lack of the imaginary phase. To obtain a nonzero result one has to go to the loop diagrams. In one-loop level there are four different diagrams (and their hermitian conjugates) that may contribute to the correlator \( \Delta(z, k_T^2) \), as shown in Fig. 1. These include the self-energy diagram (Fig. 1a), the vertex diagram (Fig. 1b), the hard vertex diagram (Fig. 1c), and the box diagram (Fig. 1d). They have also been applied to calculate the Collins function in Refs. \[18, 19\].

We will focus on the favored fragmentation function, i.e. the fragmentation of \( u \to \pi^- \). In this case the expressions for each diagram in Fig. 1 are as follows:

\[
\Delta_a(z, k_T) = i \frac{4C_F \alpha_s}{2(2\pi)^3(1 - z)P_h^-} \frac{(\not k + m)}{(k^2 - m^2)^2} g_{q_h} \gamma_5 (\not k - P^-_h + m_s) g_{q_h} \gamma_5 (\not k + m)
\]

\[
\int \frac{d^4 l}{(2\pi)^4} \gamma^\mu ((\not l - l + m) \gamma_\mu (\not k + m) \int \frac{d^4 l}{(2\pi)^4} \left( (l^2 - m^2 + i\varepsilon)l^2 + i\varepsilon \right),
\]

\[
\Delta_b(z, k_T) = i \frac{4C_F \alpha_s}{2(2\pi)^3(1 - z)P_h^-} \frac{(\not k + m)}{(k^2 - m^2)^2} g_{q_h} \gamma_5 (\not k - P^-_h + m_s) g_{q_h} \gamma_5 (\not k + m)
\]

\[
\int \frac{d^4 l}{(2\pi)^4} \gamma^\mu ((\not k - P^-_h + l + m_s) g_{q_h} \gamma_5 (\not k - l + m) \gamma_\mu ((\not k + m) \int \frac{d^4 l}{(2\pi)^4} \left( (l^2 - m^2 + i\varepsilon)l^2 + i\varepsilon \right),
\]

\[
\Delta_c(z, k_T) = i \frac{4C_F \alpha_s}{2(2\pi)^3(1 - z)P_h^-} \frac{(\not k + m)}{k^2 - m^2} g_{q_h} \gamma_5 (\not k - P^-_h + m_s) g_{q_h} \gamma_5 (\not k + m)
\]

\[
\int \frac{d^4 l}{(2\pi)^4} \left( (l^2 - m^2 + i\varepsilon)(-l^-\pm i\varepsilon)l^2 + i\varepsilon \right),
\]

\[
\Delta_d(z, k_T) = i \frac{4C_F \alpha_s}{2(2\pi)^3(1 - z)P_h^-} \frac{(\not k + m)}{(k^2 - m^2)^2} g_{q_h} \gamma_5 (\not k - P^-_h + m_s) g_{q_h} \gamma_5 (\not k + m)
\]

\[
\int \frac{d^4 l}{(2\pi)^4} \left( (l^2 - m^2 + i\varepsilon)(l^-\pm i\varepsilon)l^2 + i\varepsilon \right),
\]
In the literature two different approaches have been applied to regularize this divergence. One strategy is to adopt a Gaussian form factor

\[ g_{qh} \rightarrow g_{qh} e^{-\frac{\Lambda^2}{z}} \quad \text{(11)} \]

where \( \Lambda^2 \) has the general form \( \Lambda^2 = \lambda^2/(z^\alpha(1-z)^\beta) \). The \( \lambda, \alpha, \) and \( \beta \) are the parameters of the form factor that will be determined in the next section. The advantage of the choice in Eq. (11) is that it can also reasonably reproduce the unpolarized fragmentation function.

In Eqs. (10) or (8), in principle one of the form factors should depend on the loop momentum \( l \). Here we will drop this dependence and merely use \( k^2 \) instead of \( (k-l)^2 \) in that form factor to simplify the integration. The same choice has also been adopted to calculate the Collins function \( H_1 \), which is a leading-twist fragmentation function. For the subleading-twist T-odd functions the situation is more involved. As shown in Ref. \cite{24}, the calculation of T-odd twist-3 TMD distributions suffers from a light-cone divergence. In phenomenological studies the divergence has to be regularized by introducing form factors, explicitly depending on loop momentum. However, as we will show later, we find that in the case of twist-3 fragmentation functions, the calculation is free of this light-cone divergence. The reason behind this distinction is that the kinematical configuration contributing to T-odd fragmentation functions is different from that to the T-odd distribution functions.

After performing the integration over \( l \) using the cuts in Eq. (9) we organize the expression for \( H(z, k_T^2) \) as follows

\[ H(z, k_T^2) = \frac{2\alpha_s g_{\pi T}^2 C_F}{(2\pi)^4} \frac{1}{z^2(1-z)} \frac{1}{M_h(k^2 - m^2)} \left( H(a)(z, k_T^2) + H(b)(z, k_T^2) + H(c)(z, k_T^2) + H(d)(z, k_T^2) \right) . \quad \text{(12)} \]

The four terms in the bracket of the right hand side of (12) have the forms

\[ H_{(a)}(z, k_T^2) = \frac{-m}{2(k^2 - m^2)} (3 - \frac{m^2}{k^2}) (k^2 - m^2 + (1 - 2/z)m_h^2) I_1 , \quad \text{(13)} \]

\[ H_{(b)}(z, k_T^2) = \left( \frac{k^2 - m_h^2 + m_s^2}{\lambda(m_h, m_s)} I_1 - m_s I_2 \right) (k^2 - m_s^2 + (1 - 2/z)m_h^2) , \quad \text{(14)} \]

\[ H_{(c)}(z, k_T^2) = -((m_s - m)(k^2 - m m_h) + m m_h I_1/(k^2 - m^2) - (m_s - m + z m) I_3 k^- , \quad \text{(15)} \]

\[ H_{(d)}(z, k_T^2) = \frac{I_2}{2z k_T^2} \left( (m_s - m + z m)(\lambda(m_s, m_h) + ((1 - 2/z) k^2 + m_h^2 - m_s^2)(k^2 - m^2 + (1 - 2/z)m_h^2) + z m (k^2 - m_s^2 + (1 - 2/z)m_h^2) I_2 - I_2 (m_s - m)(k^2 - m m_h + m_h^2) + (m_s - m + z m) I_3 k^- . \quad \text{(16)} \]
The functions $I_i$ represent the results of the following integrals

$$I_1 = \int d^4l \delta(l^2)\delta((k-l)^2 - m^2) = \frac{\pi}{2k^2} (k^2 - m^2) ,$$

$$I_2 = \int d^4l \delta(l^2)\delta((k-l)^2 - m^2) \frac{(k - P_h - l)^2 - m^2}{2k^2} = - \frac{\pi}{2\lambda(m_h, m_s)} \ln \left(1 + \frac{2\sqrt{\lambda(m_h, m_s)}}{k^2 - m_h^2 + m_s^2 + \sqrt{\lambda(m_h, m_s)}}\right) ,$$

$$I_3 = \int d^4l \delta(l^2)\delta((k-l)^2 - m^2) \left(\frac{1}{k^2 - m^2} - 1 + \frac{1}{\xi}\right) ,$$

with $\lambda(m_h, m_s) = (k^2 - (m_h + m_s)^2)(k^2 - (m_h - m_s)^2)$.

We would like to point out that the quark-photon hard-vertex diagram gives nonzero contribution to $H(z, k_T^2)$, as shown in Eq. [15]. This is different from the calculation of the Collins function $H_T^c$, in which case the contribution from the hard-vertex diagram vanishes [19]. We note that this is because the Dirac structure of $H(z, k_T^2)$ appearing in the decomposition of the correlation function $\Delta(z, k_T)$ is different from that of the Collins function. The sum of $H(c)(z, k_T^2)$ and $H(d)(z, k_T^2)$ can be cast into

$$H_{(c+d)}(z, k_T^2) = \frac{I_2}{2k_T^2} \left((m_s - m + zm) \left(\lambda(m_s, m_h) + \frac{1}{2} (1 - 2z) k^2 + m_h^2 - m_s^2 \right) \left(k^2 - m_h^2 + (1 - 2z) m_h^2\right) \right)$$

$$- zm \left(k^2 - m_s^2 + (1 - 2z) m_h^2\right) I_2 - \left(\frac{I_1}{k^2 - m^2} + I_2\right) \left((m_s - m)(k^2 - mm_s) + mm_h^2\right) ,$$

where the terms containing $I_3$ cancel out. As we can see, the final result of $H(z, k_T^2)$ in Eq. [12] is free of the light-cone divergence.

### B. Calculation of $\hat{H}$ with gluon rescattering

The fragmentation function $\hat{H}(z, k_T^2)$ originates from the quark-gluon-quark (qgq) correlation [13, 20]:

$$\hat{\Delta}_A^\alpha(z, k_T^2) = \sum_X \frac{1}{2zN_c} \int d\xi^+ d^2\xi_T (2\pi)^3 \int e^{ik_\perp \xi_T} \int_{\pm \infty} d\eta^+ \mathcal{U}_{(\infty^+, \eta^+)}^{\xi_T}$$

$$\cdot gF_\perp^\alpha(\eta) \mathcal{U}_{(\eta^+, \xi^+)}^{\eta^+} \psi(\xi) |P_h; X\rangle \langle P_h; X|\bar{\psi}(0) \mathcal{U}_{(0^+, \xi^+)}^{\bar{\eta}^+} \mathcal{U}_{(0^+, \xi^+)}^{\eta^+} \rangle_{\eta_T = \xi_T = 0} ,$$

where $F_{\mu\nu}$ is the antisymmetric field strength tensor of the gluon. For later convenience in order to perform the calculation, we rewrite the qgq correlator as

$$\tilde{\Delta}_A^\alpha(z, k_T^2) = \sum_X \frac{1}{2zN_c} \int d\xi^+ d^2\xi_T d\eta^+ (2\pi)^3 \int d \left(\frac{1}{z} - \frac{1}{z_1}\right) e^{\left(\frac{\xi^+}{z} - \frac{\eta^+}{z_1}\right)} e^{\frac{\eta^+}{z} - \frac{\bar{\eta}^+}{z_1}} e^{-ik_\perp \xi_T}$$

$$\cdot \langle 0|igF_\perp^\alpha(\eta) \psi(0) |P_h; X\rangle \langle P_h; X|\bar{\psi}(0)\rangle \rangle_{\eta_T = \xi_T = 0} ,$$

where we have suppressed the Wilson lines for brevity. The fragmentation function $\hat{H}$ can be extracted from the correlator $\tilde{\Delta}_A^\alpha(z, k_T^2)$ by the following projection:

$$\frac{1}{2} \text{Tr}[\tilde{\Delta}_A^\alpha(z, k_T^2) \sigma_\alpha] = \hat{H}(z, k_T^2) + i\hat{E}(z, k_T^2) .$$

The integrated fragmentation function $\hat{H}(z) = z^2 \int d^2k_T \hat{H}(z, k_T^2)$ is related to the collinear twist-3 fragmentation function $H_{FU}^3(z, z_1)$ by

$$\hat{H}(z) = 2z^3 \int \frac{dz_1}{z_1} \text{PV} \frac{1}{z_1} \hat{H}_{FU}^3(z, z_1) ,$$

where $\text{PV}$ denotes the principal value of the integral.
Therefore the factor \( \frac{1}{H} \) has the form represents a qgq correlation. The circle at the end of the gluon line represents the field strength tensor correlator which appears in the integration where we have used the replacement here \( A \) indices of the gluon line. The qgq correlator thus has the form:

\[
\tilde{\Delta}_A(z, k_T) = i \frac{4C_F\alpha_s}{2(2\pi)^2(1-z)P_h^2} \frac{1}{k^2 - m^2} \int \frac{d^4l}{(2\pi)^4} \frac{\left( \gamma_\mu \right)(k - P_h - l + m_s)\gamma_\lambda (k + P_h + m_s)}{(-l^+ \pm i\epsilon)((k - l)^2 - m^2 - l^-)^2 - m_s^2 - i\epsilon)},
\]

where we have used the replacement

\[
\left( \frac{1}{z} - \frac{1}{z_1} \right) P_h^- \rightarrow l^-.
\]

To obtain a nonzero \( \tilde{H} \), again we should apply the cut rules given in Eq. (9) to perform the integration over \( l \). Therefore the factor \( 1/(1/z - 1/z_2 \pm i\epsilon) \) will take the principal value, as shown in Refs. [27, 28]. The final result for \( \tilde{H} \) has the form

\[
\tilde{H}(z, k_T^2) = \alpha_s g_f^2 \frac{\pi}{(2\pi)^4} C_F \frac{e^{-\frac{2k^2}{\Lambda^2}}}{z^2} \frac{1}{(1-z)M_h(k^2 - m^2)} \left\{ -\mathcal{A}(m_s - m)k_T^2 \\
+ (m_s - m + zm) \left[ \mathcal{A}(k^2 + k_T^2) + \mathcal{B}M^2 / z - I_1 / z - (k^2 - m)^2 / z^2 \right] \\
+ \left[ (k^2 - mm_s)(m_s - m) + mm_s^2 \right] \left[ I_1 / (2zk^2) + \mathcal{A} / z + \mathcal{B} \right] \right\}. \tag{28}
\]

Here \( \mathcal{A} \) and \( \mathcal{B} \) denote the following functions

\[
\mathcal{A} = \frac{I_1}{\lambda(m_h, m_s)} \left( 2k^2 - m^2 - m_s^2 \right) I_2 / \pi + (k^2 + m_s^2 - m^2) \right), \tag{29}
\]

\[
\mathcal{B} = -\frac{2k^2}{\lambda(m_h, m_s)} I_1 \left( 1 + \frac{k^2 + m^2 - m_s^2}{\pi} I_2 \right), \tag{30}
\]

which appears in the integration

\[
\int d^4l^{\mu} \frac{\delta(l^2 - m_s^2) \delta((k - l)^2 - m^2)}{(k - P_h - l)^2 - m^2} = \mathcal{A}k^\mu + \mathcal{B}P_h^\mu. \tag{32}
\]
| $m_s$ (GeV) | $\lambda$ (GeV) | $g_{av}$ | $m$ (GeV) | $\alpha$ | $\beta$ |
|-----------|-----------|--------|--------|--------|--------|
| 0.53      | 2.18      | 5.09   | 0.3 (fixed) | 0.5 (fixed) | 0 (fixed) |

TABLE I: Fitted values of the parameters in the spectator model. The values of the last three parameters are fixed in the fit.

III. NUMERICAL RESULT

In this section we present the numerical result for the fragmentation functions $H$ and $\tilde{H}$. To this end the values of the parameters in the model have to be specified. In Ref. [19] the parameters of the model were determined by fitting the model result of unpolarized fragmentation function $D_1(z)$ with the Kreitzer parameterization [29] of $D_1(z)$. The parameters were then used to make prediction on the Collins function. In this paper we will obtain the parameters by fitting simultaneously the model calculations of the unpolarized fragmentation function and the Collins function with the known parameterizations of them, since the Collins functions have been extracted and are well constrained by the $e^+e^-$ annihilation data and the SIDIS data. Specifically, we will use the half-$k_T$ moment of the Collins function

$$H^{(1/2)}_1(z) = z^2 \int d^2 k_T \frac{|k_T|}{2m_h} H_1(z,k_T^2)$$

in the fit.

For the theoretical expressions of $D_1$ and $H^{+}_1$, we use the calculation in the same model, which has already been done in Ref. [19]. For the parameterization of $D_1$, we will adopt the DSS leading order set [30]. For the parameterization of the Collins function, we apply the recent extraction By Anselmino et.al. [2]. We note that in Ref. [2], the DSS fragmentation function is also used to extract the Collins function.

Our model calculation is valid at the hadronic scale which is rather low, while the standard parametrization of $D_1$ is usually given at $Q^2 > 1$ GeV$^2$. Therefore we extrapolate the DSS $D_1$ fragmentation to that at the model scale $Q^2 = 0.4$ GeV$^2$ in order to perform the fit. For the same reason, the Collins function should be evolved at that scale for comparison. However, the evolution of the Collins function is rather complicated [10, 31, 32]. In the extraction of the Collins function in Ref. [2], the authors used the assumption that the Collins function evolves in the same way of $D_1(z)$. The same assumption has also used in Ref. [19]. For consistency we will use this assumption since in the fit we use the parameterization of Collins function from Ref. [2].

In Table. I we list the fitted values of the parameters in the model. In the left panel of Fig. 3 the curve (the solid line) vs $z$ for the unpolarized fragmentation function $D_1(z)$ at the model scale $Q^2 = 0.4$ GeV$^2$ is compared with

FIG. 3: Unpolarized fragmentation function $D_1(z)$ (left panel) and the half moment of the Collins function (right panel) vs $z$ for the fragmentation $u \rightarrow \pi^+$ at the model scale $Q^2 = 0.4$ GeV$^2$. The parameters are fitted to the parameterizations in Refs. [30] and [2]. The result in Ref. [10] (dashed lines) is also shown for comparison.

1 We recalculate the Collins function and find that our result does not exactly agree with the result in Ref. [10]. For completeness we present our result for $H^{+}_1$ in the Appendix.
FIG. 4: Left panel: The twist-3 fragmentation functions $H(z)$ and $\tilde{H}(z)$ vs $z$, plotted by the solid line and the dashed line, respectively. Right panel: $H(z)$ compared with $-2z\tilde{H}(z) + \tilde{H}(z)$ in the spectator model.

the curve (dotted line) from the DSS parameterization. We also show the result (dashed line) calculated from the parameters fitted in Ref. [19]. In the right panel of Fig. 4, we display the fitted curve for $H^{(1/2)}_1(z)$ and compare it with the parametrization of Ref. [2].

In the left panel of Fig. 4 we plot our prediction on $H(z)$ and $\tilde{H}(z)$ using the parameters in Table I. We present the result at the model scale $Q^2 = 0.4\text{GeV}^2$, We find that the sign of the favored $H(z)$ is negative and its magnitude is sizable. This is consistent with the extraction in Ref. [4], where a negative $H(z)$ for the favored fragmentation is given. For the function $\tilde{H}(z)$, we find that the result is nonzero and has a minus sign. In Ref. [4], a similar result is also hinted by the fit on $H^U_1(z, z_1)$, which contribute substantially to $\tilde{H}(z)$ through Eq. [1].

According to Eq. [11] the three twist-3 fragmentation function should satisfy the equation of motion relation, which is a model independent result derived from QCD. However, From Eqs. (12), (28) and (34), one cannot find an obvious relation among them since in the spectator model they are calculated from different diagrams. Thus we numerically check the relation (11) and show the the comparison between $H(z)$ (solid line) and $-2z\tilde{H}(z) + \tilde{H}(z)$ (dashed-dotted line) on the right panel of Fig. 4. We find that the two curves are close, which indicates that the relation holds approximately in the model, therefore it provide a crosscheck on the validity of our calculation.

IV. CONCLUSION

In this work, we studied the twist-3 fragmentation function for $H$ and $\tilde{H}$ in a spectator model. We first calculated the TMD functions $H(z, k_T^2)$ and $\tilde{H}(z, k_T^2)$, and then we obtained the corresponding collinear functions by integrating over the transverse momentum. In our study we considered the gluon rescattering effect and found that the hard-vertex diagram gives nonzero contribution to $H$. Using the parameters fitted to the known parameterizations of $D_1$ and $H^T_1$ simultaneously, we presented numerical results of $H$ and $\tilde{H}$. We found that our results agree with the recent extraction from the SSA in pp collision. We also tested the equation of motion relation among $\tilde{H}(z)$, $H(z)$ and $H(z)$, the numeric result shows that the relation approximately holds in our calculation. Our study may provide useful information on the twist-3 fragmentation function complementary to phenomenological analysis.

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Appendix A: Results of the Collins function

Here we present the model result of the Collins function \[19\]

\[
H^+_1(z, k_T^2) = -\frac{2\alpha_s g^2 \sigma_T}{(2\pi)^4} e^{-\frac{2k^2}{\lambda^2}} \frac{M_h}{z^2(1-z)} \frac{M_h}{k^2-m^2} \left( H^+_{1(a)}(z, k_T^2) + H^+_{1(b)}(z, k_T^2) + H^+_{1(d)}(z, k_T^2) \right) \tag{34}
\]

The three terms in the brackets correspond to the results from Fig. 1a, Fig. 1b, and Fig. 1c, respectively. In our calculation we find that those terms have the form

\[
H^+_{1(a)}(z, k_T^2) = \frac{m^2}{(k^2-m^2)} \left( 3 - \frac{m^2}{k^2} \right) I_1 \tag{35}
\]

\[
H^+_{1(b)}(z, k_T^2) = 2m_s I_2 - 2(m_s - m) \left( \frac{m^2 - m_s^2 - k^2}{\lambda(m_h, m_s)} I_1 - \frac{4k^2 m^2}{\lambda(m_h, m_s) \pi} I_1 I_2 \right) \tag{36}
\]

\[
H^+_{1(d)}(z, k_T^2) = \frac{1}{2z^2k^2} \left\{ -I_{34}(2zm + 2m_s - m) + 2zm (k^2 - m^2 + M_h^2(1-2/z)) \right. \\
\left. + 2(m_s - m) \left( (2-z)k^2 - M_h^2 + m_s^2 - zm(m + m_s) \right) \right\} \tag{37}
\]

Here \( I_{34} \) is the combination of two integrals

\[
I_{34} = k^- (I_3 + (1-z)(k^2-m^2)I_4) = \pi \ln \left( \frac{\sqrt{k^2(1-z)}}{m_s} \right) \tag{38}
\]

with

\[
I_4 = \int d^4l \frac{\delta(l^2)\delta((k-l)^2-m^2)}{(-l^2+i\varepsilon)(k-p-l)^2-m_s^2} \tag{39}
\]

We find that in \([20]\) there is a new term proportional to \(m_s - m\) that was not contained in Eq. (29) of Ref. \([19]\). Also in Eq. (37) the coefficient of certain terms containing \(m_s - m\) has a factor of 2 compared to Eq. (30) of Ref. \([19]\). But our calculation returns to the results in Ref. \([18]\) in the case \(m_s = m\) and by setting the form factor to 1.

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