End-to-End Trajectory Distribution Prediction Based on Occupancy Grid Maps

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Abstract

In this paper, we aim to forecast a future trajectory distribution of a moving agent in the real world, given the social scene images and historical trajectories. Yet, it is a challenging task because the ground-truth distribution is unknown and unobservable, while only one of its samples can be applied for supervising model learning, which is prone to bias. Most recent works focus on predicting diverse trajectories in order to cover all modes of the real distribution, but they may despise the precision and thus give too much credit to unrealistic predictions. To address the issue, we learn the distribution with symmetric cross-entropy using occupancy grid maps as an explicit and scene-compliant approximation to the ground-truth distribution, which can effectively penalize unlikely predictions. In specific, we present an inverse reinforcement learning based multi-modal trajectory distribution forecasting framework that learns to plan by an approximate value iteration network in an end-to-end manner. Besides, based on the predicted distribution, we generate a small set of representative trajectories through a differentiable Transformer-based network, whose attention mechanism helps to model the relations of trajectories. In experiments, our method achieves state-of-the-art performance on the Stanford Drone Dataset and Intersection Drone Dataset.

1. Introduction

Trajectory prediction has gained increasing attention due to its emerging applications such as robot navigation and self-driving cars. Due to the inherent multimodal uncertainty from an agent’s intention or environment, a large number of works have been proposed to learn a multimodal distribution of the future trajectories. For example, in [20, 26], the multimodal distribution is explicitly modeled using the Gaussian mixture model, though it is hard to optimize and prone to overfitting. Others have attempted to model the trajectory distribution implicitly using generative models such as conditional variational autoencoder (CVAE) [6, 19, 28, 44], normalizing flow (NF) [34, 38], or generative adversarial network (GAN) [2, 8, 9, 11, 43].

However, most previous works focus on the diversity of the predicted trajectories rather than the more important precision, except a few works (e.g., [34, 38]). The issue is that if the model is only encouraged to cover all modes of real distribution, it may assign too many probabilities to unrealistic predictions and cannot accurately reflect the real probability density. One such example is shown in Fig. 1 where a large portion of the diverse trajectories predicted by P2T [6] turn and intersect with obstacles, which are certainly implausible and inconsistent with common knowledge that moving straight ahead is more likely than turning. In such circumstances, a navigation decision based on the predictions will overreact to less likely futures, while underestimating the more likely ones.

Specifically, to learn a diverse trajectory distribution, previous works usually minimize the variety loss [6, 9, 13] or the forward cross-entropy [23, 26, 44]. Yet, the variety loss does not penalize bad predictions as long as there exists one prediction close to the ground-truth, and it does not lead to ground-truth distribution but approximately its square root [46]. On the other hand, the forward cross-
entropy also fails to adequately penalize the unlikely predictions \[34,38\] and exhibits noise sensitivity \[48\]. To overcome the limitations of these losses, our solution is to learn a distribution minimizing the symmetric cross-entropy, i.e., the combination of forward and reverse cross-entropy between the predictive distribution and ground-truth distribution. Compared with the forward cross-entropy, the reverse cross-entropy can penalize the prediction with low likelihood, but it requires ground-truth distribution as a reference, which unfortunately is not available in many cases. An effective solution is to employ an occupancy grid map (OGM), which divides the social space into grid cells with an occupancy probability in each cell. Thus, the trajectory probability can be approximated as the product of all future position probabilities conditioned on the OGM. In \[38\], an OGM, parameterized as a cost map, is embedded from spatial scene features by a convolutional neural network (CNN) to assign proper probabilities to different social areas. However, representing all future position distributions with a single OGM is inaccurate, since it neglects the spatio-temporal correspondence of trajectories. Instead, we predict an OGM for each future position with a convolutional long short-term memory (ConvLSTM) \[51\] network based on our novel deconvolution parameterization of the position probability flow. The resulting dynamic OGMs can help not only the trajectory prediction \[23\] but also downstream planning tasks \[4,53\].

When minimizing the symmetric cross-entropy, previous approaches \[34,38\] usually make use of the normalizing flow, which transforms a simple Gaussian distribution into the target trajectory distribution through a sequence of auto-regressive mappings. These mappings are required to be invertible, differentiable, and easy for computing Jacobian determinants, which are difficult to be satisfied in practice. In addition, the latent variable sampled from the Gaussian distribution is hard to interpret. To address these issues, we develop an end-to-end interpretable model to back-propagate the symmetric cross-entropy loss. In particular, we construct a CVAE model using a coarse future trajectory plan within neighboring grids as the interpretable latent variable, similar to P2T \[6\]. However, P2T cannot be trained in an end-to-end manner, because it learns the planning policy using the maximum-entropy inverse reinforcement learning (MaxEnt IRL) \[50,58\] by matching feature expectation. Instead, we implement value iteration in IRL by differentiable value iteration network (VIN) \[45\] and incorporate Gumbel-Softmax \[15\] into the discrete planning policy sampling. In our VIN-based IRL, planning and trajectory generation policy can be learned simultaneously by maximizing the data likelihood.

Even though a large number of possible future trajectories can be sampled from the learned distribution, many downstream applications often demand a small set of representative predictions. This requirement is traditionally accomplished by learning the distribution model with the variety loss \[5,9,13\] or post-processing with heuristic methods like greedy approximation \[36\] or K-means \[6,7\]. Motivated by the insight that clustering like K-means can be regarded as paying different attention to different samples, we propose a Transformer-based refinement network, whose attention mechanism can also ensure sampling diversity, to attentively obtain a small set of representative samples from the over-sampled outcomes of our prediction model. The representative properties can be conveniently adjusted by its loss, e.g., the variety loss for diversity. In experiments, we compare our method with a set of state-of-the-art approaches on the Stanford Drone Dataset \[40\] and Intersection Drone Dataset \[3\] and demonstrate the superiority of our method in both trajectory diversity and quality.

In summary, the main contributions are as follows.

- We propose a VIN-based IRL method, simplifying the learning process while allowing the gradients from trajectory generation to flow back to the planning module.
- We improve the approximation of ground-truth with OGMs in learning trajectory distribution using symmetric cross-entropy;
- We introduce a Transformer-based refinement network for sampling from trajectory distribution to obtain representative and realistic trajectories;
- We demonstrate the state-of-the-art performance of our framework on two real-world datasets: Stanford Drone dataset \[40\] and Intersection Drone dataset \[3\].

2. Related Work

2.1. Trajectory Distribution Prediction

We focus on trajectory distribution prediction approaches based on deep learning. Refer to \[41\] for a survey of more classical approaches. In the early literature, the trajectory distribution is usually modeled as a simple unimodal distribution, e.g., a bivariate Gaussian distribution \[1,25,54\]. However, the unimodal models tend to predict the average of all possible modes, which may be not valid.

Recently, various generative models such as GAN, NF and CVAE, have been proposed to address the multimodality, which capture the stochasticity with a latent variable. GAN-based methods \[2,8,9,11,43\] use a discriminator to generate diverse realistic trajectories but are difficult to train and suffer from the mode collapse. NF methods \[34,38\] sample the latent variable from a standard Gaussian distribution and map it to the target trajectory through a sequence of transformations. Some CVAE approaches such as DESIRE \[19\], Trajectron++ \[44\], learn a Gaussian or categorical latent distribution using the constraint between prior and posterior distribution. Others leverage an interpretable latent variable to incorporate prior knowledge. For
example, PECNet [28], TNT [55] consider the destination position as the latent variable. Further, LB-EBM [33] takes positions at several future steps as the latent variable which is sampled from an energy-based model. P2T [6] samples a coarse grid plan from a policy learned by deep MaxEnt IRL [50] as the latent variable. Even though our model also leverages the plan as the latent variable, we learn the plan and trajectory distributions in a unified framework.

2.2. Occupancy Grid Maps Prediction

OGMs prediction works aim at predicting the categorical occupancy distribution over a grid at each future time step. Even though there are extensive studies forecasting OGMs of a crowd [21,31] or all objects [24,32] in a scene, we focus on reviewing the literature predicting one agent’s OGMs like our work.

Kim et al. [16] directly outputs the future probability of each grid cell using the LSTM network. Y-net [27] yields the OGM at each future step directly from different channels of the feature map output by a CNN. Similarly, in MP3 [4], the feature map at each channel is embedded into the temporal motion fields at each future step, which obtains the probability transition flow between consecutive OGMs by bilinear interpolation of motion vectors on the field. To take advantage of the temporal and spatial patterns in the sequential OGMs, ConvLSTM [51] is widely applied. In [39], they directly derive an OGM from the hidden map of ConvLSTM at each time step. To increase time consistency, DRT-NET [14] learns the residual provability flow between the consecutive OGMs. To incorporate the prior knowledge of local movement, Multiverse [23] uses a graph attention network to aggregate neighborhood information on the hidden map of ConvLSTM. Similarly, SA-GNN [24] considers the interactions with neighbors by graph neural networks. Based on the ConvLSTM and deconvolution parameterization, our method is not only computationally efficient but also explicitly models the local transition probability.

Furthermore, some of these works attempt to obtain trajectories by sampling the OGMs. But the positions independently sampled from each OGM suffer from discretization errors and lack spatio-temporal correspondence in trajectories. To address this problem, [39] leverages the OGMs as input to another ConvLSTM which outputs the coordinates of a fixed number of future trajectories. Multiverse [23] predicts a continuous offset at each cell to mitigate the discretization error and applies a diverse beam search to generate multiple distinct trajectories. Y-net [27] samples intermediate positions conditioned on the sampled goal and waypoints. Instead of sampling OGMs like all previous works, we use the OGMs as auxiliary information in training loss to generate more feasible trajectories.

2.3. Trajectory Sample Refinement

Trajectories sampled from the predicted trajectory distribution usually do not satisfy the downstream requirement. The most common two requirements are precision and diversity to cover all future scenarios accurately [34,38]. To improve accuracy, previous works [19,29,55] usually score the samples using a neural network and refine the top samples. For diversity, the relation between samples needs to be considered. Most literature [5,9,13] directly use a variety loss to improve diversity. In addition, P2T [6], PGP [7] and Y-net [27] use K-means to cluster samples while CoverNet [36] employs a greedy approximation algorithm to create a diverse set. To capture both diversity and quality, DSF [52] learns a diverse sampling function to sample the latent variable of CVAE using a diversity loss based on a determinantal point process at test time while DiversityGAN [12] samples distinct latent semantic variables to predict diverse trajectories. Different from previous work, our sample refinement network based on Transformer is an independent and differentiable module and is flexible with the downstream requirement and trajectory sample number.

3. Background

3.1. Problem Formulation

Given an observation \( \Omega \) including a context and history trajectory \( X = \{X_t \in \mathbb{R}^2 \mid t = -T_p + 1, \ldots, 0\} \) of a target agent, our objective is to predict the distribution \( p(Y|\Omega) \) of its future trajectory \( Y = \{Y_t \in \mathbb{R}^2 \mid t = 1, \ldots, T_f\} \). The context consists of neighbors’ history trajectories and an image \( I \), which is a bird’s eye view (BEV) perception of the local scene centered at the agent’s current position.

We assume that an agent has a grid-based plan on which its future trajectory is conditioned. An agent’s planning process is modeled using a Markov decision process (MDP) \( \mathcal{M} = \{S, A, T, r\} \), with a time horizon \( N \). A state set \( S \) consists of all cells over a 2D grid and an absorbing end state of zero value. An action set \( A \) includes 4 adjacent movements \( up, down, left, right \) and an \( end \) action leading to the absorbing state. A deterministic transition function \( T : S \times A \rightarrow S \) describes system dynamics. A non-stationary reward function \( r^s : S \times A \rightarrow \mathbb{R} \) determines a reward for each state and action per step \( n \). We assume that the agent uses a non-stationary stochastic policy \( \pi^s(a|s) \) to determine the probability of selecting an action \( a \) at a state \( s \) at MDP step \( n \), and finally make a plan in terms of the state sequence \( S = \{s^n \in S \mid n = 1, \ldots, N\} \). Note that here we are using the superscript \( n \) as the MDP step \( n \), to distinguish with the time step \( t \) as subscript.

To relieve the difficulty of modeling the multi-modal future trajectory distribution \( p(Y|\Omega) \), we introduce the plau-
sible plan as the latent variable and decompose it as:

\[ p(Y | \Omega) = \int_{S \in \mathbb{S}(\Omega)} p(S | \Omega) p(Y | S, \Omega) \, dS, \]

where \( \mathbb{S}(\Omega) \) is the space of plausible plans conditioned on the observation. In this way, since the plan uncertainty can well capture the multimodality, trajectory conditioned on a plan can be well approximated as a unimodal distribution.

3.2. Trajectory Distribution Learning

We predict the future trajectory distribution by minimizing the discrepancy between the distribution \( q_\theta(\hat{Y} | \Omega) \) of the predicted trajectory \( \hat{Y} \) and the ground-truth distribution \( p(Y | \Omega) \). As a straightforward distance metric between these two distributions, forward cross-entropy (a.k.a., negative log-likelihood (NLL)) is computed as:

\[ \mathcal{H}(q_\theta, p) = - \mathbb{E}_{\Omega \sim \Psi, Y \sim p(Y | \Omega)} \log q_\theta(S | \Omega) q_\theta(Y | S, \Omega), \]

where \( \Psi \) denotes the ground-truth observations' distribution and \( S(Y) \) is the space containing the ground-truth plan \( S \), i.e., the grid state sequence the trajectory \( Y \) goes through.

Although the NLL loss encourages the predicted distribution to cover all plausible modes of the ground-truth distribution, it assigns a low penalty to the implausible predictions which are less likely to take place under the ground-truth distribution [34, 38]. The reverse cross-entropy \( \mathcal{H}(q_\theta, p) \) can evaluate the likelihood of the prediction under the ground-truth distribution and penalize unlikely predictions, but the ground-truth distribution \( p \) is unknown in the real world with only one sample observed. To address this issue, we approximate the continuous joint distribution \( p(Y | \Omega) \) of future trajectory as a product of future positions' categorical marginal distributions \( O = \{ O_t \mid t = 1, \ldots, t_f \} \), represented as OGMs:

\[ p(Y | \Omega) \approx p(O | \Omega) \prod_{t=1}^{t_f} O_t(Y_t), \]

where \( O_t(Y_t) \) denotes the agent’s location probability at \( Y_t \) at time \( t \), which is bilinearly interpolated from nearby probabilities on \( O_t \) and \( p(O | \Omega) \) is assumed to be deterministic and parameterized by neural networks \( O = o_{\alpha}(\Omega) \). Thus, the reverse cross-entropy \( \mathcal{H}(q_\theta, p) \) can be approximated as:

\[ \mathcal{H}(q_\theta, O) = - \mathbb{E}_{\Omega \sim \Psi, Y \sim q_\theta(\cdot | \Omega)} \log p(O | \Omega) \prod_{t=1}^{t_f} O_t(\hat{Y}_t). \]

4. Approach

As shown in Fig. 2, our model is composed of five modules that can be learned in an end-to-end manner: an Observation Encoder, a Policy Network, an Occupancy Grid Maps Decoder (OGMs Decoder), a Trajectory Decoder and a Refinement Network.

4.1. Observation Encoder

The first component of our approach is an observation encoder composed of a motion encoder to extract motion features from the past trajectories of the target and its neighbors and a scene encoder to extract scene features from the BEV image of the surrounding environment.

**Motion encoder:** The motion encoder is designed to embed the past trajectories of the target agent and its neighbors into a feature vector and a feature map. To represent the neighbors’ state succinctly, we leverage a directional pooling grid from [18], where each cell contains the relative velocity of a neighbor located in that cell with respect to the target agent. At each past time step \( t \), we first flatten the grid into a vector \( d_t \) and then concatenate the vector with the agent velocity \( X_t - X_{t-1} \) as input to an RNN. The hidden state of the RNN at time \( t \) is given by:

\[ m_t = \text{RNN}_m(m_{t-1}, \phi[d_t, X_t - X_{t-1}]), \]

where \( \phi \) is a linear embedding layer and the brackets indicate concatenation. The first hidden state \( m_{-t+1} \) is set to zero and the last hidden state \( m_0 \) is regarded as the motion feature. The \( m_0 \) is duplicated over all cells in the scene and then is concatenated with each cell’s agent-centered, world-aligned coordinate to construct a motion feature map \( M \):

\[ M(x, y) = [m_0, x, y]. \]

**Scene encoder:** We apply a CNN to extract a scene feature map from the BEV image \( I \) of the neighborhood:

\[ F = \text{CNN}_f(I), \]

where the spatial dimensions of the scene feature map \( F \) are the same as that of the MDP grid for simplicity.

4.2. Policy Network

We generate a policy in two steps end-to-end: mapping the observation features into rewards and then computing a policy with a value iteration network.

We adopt non-stationary rewards to capture the dynamic agent-to-scene and agent-to-agent interaction. Based on the scene and motion feature maps, a ConvLSTM architecture is applied to yield the reward map at each step. The ConvLSTM hidden map and the reward map at MDP step \( n \) are:

\[ H^n = \text{ConvLSTM}_c(H^{n-1}, F), \quad r^n = \Phi(H^n), \]

where \( \Phi \) is a fully connected convolutional layer. The initial hidden map \( H^0 \) is the embedded motion feature map \( \Phi(M) \).

Based on the reward maps, we use the approximate value iteration to generate a policy map \( \pi^n \) at each step \( n \). To back-propagate the loss through the value iteration, we take advantage of the value iteration network as [35, 37, 45],
Algorithm 1 Approximate Value Iteration Network

Input: \( r^n(s, a) \)

Output: \( \pi(n)(a|s) \)

1: \( V^N(s) = 0, \forall s \in \mathcal{S}; \)
2: for \( n = N, \ldots, 2, 1 \) do
3: \( Q^n(s, a) = r^n(s, a) + V^n_{s'=T(s, a)}(s'), \forall s \in \mathcal{S}, \forall a \in \mathcal{A}; \)
4: \( V^{n-1}(s) = \text{logsumexp}_a Q^n(s, a), \forall s \in \mathcal{S}; \)
5: \( \pi(n)(a|s) = \text{softmax}_a Q^n(s, a), \forall s \in \mathcal{S}; \)
6: end for

which recursively computes the next value map by a convolution of the current value map with transition filters. To improve the value iteration network’s performance, we utilize an approximate value iteration in the MaxEnt IRL formulation \([50, 58]\) with non-stationary rewards. Algo. 1 describes the overall computation process of this network.

### 4.3. OGMs Decoder

To provide an explicit approximation of the ground-truth trajectory distribution, we predict a sequence of dynamic OGMs based on the observation features using a ConvLSTM network. With the scene feature map as input, the hidden map of the ConvLSTM network at time \( t \) is:

\[
H_t = \text{ConvLSTM}_0 \left( H_{t-1}, F \right),
\]

where the initial OGM \( O_0 \) is a probability matrix to be learned. Our deconvolution method can directly model the probability density transition process. Besides, the limited size of the normalized deconvolution kernel ensures that the probability mass diffuses into nearby grid cells in a conservative manner, reflecting the prior knowledge that agents do not suddenly disappear or jump between distant locations.

### 4.4. Trajectory Decoder

Conditioned on a plan from the policy roll-out or the data, an RNN decoder is applied to generate the future position distribution recursively based on local features.

**Plan sampling:** We generate a plan \( \hat{S} = \{ s^n \in \mathbb{R}^2 | n = 1, \ldots, N \} \) by sampling the non-stationary policy outputted by the policy network. However, directly sampling the policy with discrete state and action spaces will introduce difficulty in loss back-propagation. To overcome this difficulty, we sample the policy with the Gumbel-Softmax trick \([15]\), resulting in continuous action and state. Besides, we obtain the policy at continuous state \( s^n \) by bilinear interpolation.

**Plan encoder:** Given a ground-truth plan \( S \) (or a sampled plan \( \hat{S} \)), we first collect the local scene feature from scene feature map \( F \) and non-stationary feature from the corresponding hidden map of ConvLSTM, at each plan state. Then we concatenate these features with the state’s coordinates as input to an RNN, whose hidden state at step \( n \) is:

\[
h^n = \text{RNN}_0 \left( h^{n-1}, \phi(s^n, F(s^n), H^n(s^n)) \right)
\]

Since the sampled plan’s state \( s^n \) is on the continuous plane, the local features like \( F(s^n) \) are gathered by bilinear interpolation at the spatial dimensions of the feature map \( F \) corresponding to the physical position \( s^n \). Fig. 3 illustrates how the plan encoder extracts the plan features \( h^{1:N} = \{ h^n | n = 1, \ldots, N \} \).

**Multi-head attention based decoder:** Since different dimensions of the plan features at different steps may have different impacts on current hidden state \([30]\), we utilize a multi-head scaled dot product attention module \([47]\) to ag-
Figure 3. The local scene and non-stationary features at each plan state are concatenated with its location coordinates and then fed into an RNN to obtain all plan features.

4.6. Training Process

To achieve different goals including a good OGM, distribution and representative sets at different steps, our training process has the following four steps:

1. OGMs learning: The observation encoder and OGMs decoder are trained to predict OGMs by minimizing the NLL loss:

   \[ \mathcal{H}(p, O) = -\mathbb{E}_{\Omega \sim p, Y \sim p(\Omega) | O = o_{\alpha}(\Omega)} \log \prod_{t=1}^{T} O_t(Y_t). \]

2. Trajectory distribution learning: Based on the learned observation encoder and OGMs decoder, we train the policy network and trajectory decoder to induce a trajectory distribution that minimizes the approximated symmetric cross-entropy loss:

   \[ \mathcal{L}_{scce} = \mathcal{H}(p, q_{\theta}) + \beta \mathcal{H}(q_{\theta}, O). \]

3. Representative trajectories learning: Using the trajectories sampled from the learned distribution, we train the refinement network to generate representative trajectories with the variety (MoN) loss [9]:

   \[ \mathcal{L}_{variety} = \min_{k \in \{1, \ldots, K\}} \| Y - \hat{Y}^{(k)} \|_2. \]

4. End-to-end fine-tuning: We fine-tune the whole network in an end-to-end manner with the variety loss.

Only the first two steps are required for learning a trajectory distribution while all four steps are for obtaining a compact set of representative trajectories.

5. Experimental Results

5.1. Implementation Details

We augment all trajectories and scene images in the training data by 90° rotations and flipping. All data fed into and generated from our model are in the world coordinates rather than the pixel coordinates as in previous works [6, 28, 42]. The BEV image fed to the scene encoder is a 200 × 200 crop of the RGB image around the agent’s location. The scene encoder CNN \( \psi \) consists of the first two layers of ResNet34 [10] and a convolutional layer with kernel size 2 and stride 2, which outputs scene feature map of 32 channels and size 25 × 25 as the MDP grid. The RNNs
are implemented by gated recurrent units (GRU) with hidden size 64. ConvLSTMr and ConvLSTMd have 1 and 2 layers each, with kernel size 3 and 32 hidden states respectively, and the deconvolution kernel size is 5. The multi-head attention module in the trajectory decoder has 4 heads of 16 dimensions. The Transformer encoder and decoder consist of 3 layers with hidden size 64 and 8 self-attention heads and a dropout rate of 0.1. We train using Adam optimizer with a learning rate of 0.001 in the first three steps and 0.0001 in the last step. We have released our code at https://github.com/Kguo-cs/TDOR.

5.2. Datasets and Metrics

Datasets. We evaluate our method on two datasets. Most of our tests are conducted on the Stanford Drone Dataset (SDD) [40] provides top-down RGB videos captured on the Stanford University campus by drones at 60 different scenes, containing annotated trajectories of more than 20,000 targets such as pedestrians, bicyclists and cars. Early works [5, 23, 43] consider all trajectories in SDD and subsequent works [27–29, 56] focus on pedestrian trajectories using the TrajNet benchmark [42]. On these two splits, we report the results of predicting the 12-step future with the 8-step history with 0.4 seconds step interval. Besides, we report our long-term prediction results on the Intersection Drone Dataset (inD) [3], which consists of longer drone recorded trajectories of road users than SDD collected at four German intersections. To evaluate our method’s long-term forecasting performance, we use data in [27], including 1222 training and 174 test trajectories with 5-second history and 30-second future with the 1Hz sampling rate.

Metrics. We evaluate our performance of representative samples with three metrics. The first two are commonly used sample-based diversity metrics [9]: minADEK, i.e., the minimum average, and minFDEK, i.e., the final displacement errors between K predictions and ground-truth trajectory in pixels. Following P2T [6], we also report results on the quality metric, Offroad Rate, which measures the fraction of the predicted positions falling outside road while ground-truth positions inside road.

5.3. Performance Evaluation

We benchmark against the following state-of-the-arts. Social GAN [9] proposes a GAN-based method to predict diverse and socially acceptable trajectories. Desire [19] uses CVAE to generate trajectory samples, which are then recursively ranked and refined. Multiverse [23] selects multiple coarse trajectories from predicted OGMs by beam search and then refines them with continuous displacement vectors. SimAug [22] improves Multiverse [23]’s robustness by utilizing simulated multi-view data. P2T [6] predicts the future trajectory conditioned on a plan generated by the deep MaxEnt IRL. PECNet [28] is a goal-conditioned model dividing the task into goal estimation and trajectory prediction. LB-EBM [33] infers intermediate waypoints using latent vector sampled from a cost-based history. Y-Net [27] models the future position’s multimodality with heatmaps and samples a trajectory from the heatmap conditioned on sampled goal and waypoints. V [49] is a concurrent method proposing a two-stage Transformer network to model the trajectory and its Fourier spectrum in the keypoints and interactions levels, respectively.

The performance of our model in the short-term trajectory prediction compared with state-of-the-art methods on the SDD is reported in Tab. 1. minADE20 and minFDE20 values follow the original papers, while the offroad rates are computed using the released codes and models of different approaches. On both data splits, our model achieves the best performance according to the metrics of minADE20 and minADE20 and offroad rate. Notably, our results are achieved without manually labeled semantic maps in Y-Net [27] or simulation data in SimAug [22].

We also report our long-term prediction results on the inD in Tab. 2. Our results are again achieved without the manually annotated semantic maps in Y-Net [27]. A set of qualitative examples are presented in the supplementary material, which demonstrates that our models are able to learn a diverse and feasible distribution and predict diverse representative trajectories.

5.4. Ablation Study

Ablation experiments on TrajNet split are used to expose the significance of different components of our model:
OGMs Prediction Model | $\mathcal{H}(p, O)$
---|---
CNN [38] | 17.52
ConvLSTM [39] | 10.52
ConvLSTM+DiscreteResidualFlow [14] | 10.64
ConvLSTM+GraphAttentionNetwork [34] | 10.40
ConvLSTM+Deconvolution (Ours) | **10.31**

Table 3. Comparison of four baselines and our method with close parameter numbers in predicting OGMs.

**OGMs decoder:** First, we consider the ground-truth distribution approximation using one OGM obtained by a CNN acting on scene and motion features as R2P2 [38]. Then, we study how effective our deconvolution parameterization in the ConvLSTM is for OGMs prediction. We implement three baseline OGMs prediction networks inspired by [14, 34, 39]: ConvLSTM directly outputs OGMs from hidden maps; ConvLSTM+DiscreteResidualFlow outputs the log-probability residual between OGMs from hidden maps; ConvLSTM+GraphAttentionNetwork processes hidden maps with a graph attention network at each step. We train these models as our first training step. Results in Tab. 3 about the OGM decoding loss measured by the NLL loss show that our approximation with different OGMs using deconvolution parameterization is the most effective.

**Hyperparameter $\beta$:** To investigate how the hyperparameter $\beta$ in the symmetric cross-entropy loss affects learning trajectory distribution, we train the policy network and the trajectory decoder with various $\beta$ values. To measure the learned distribution diversity, we leverage the RF$_K$ metric from [34], i.e. the ratio of the average FDE to the minimum FDE among $K$ predictions (avgFDE$_K$/minFDE$_K$). A large avgFDE$_K$ implies that predictions spread out while a small minFDE$_K$ ensures predictions not being arbitrarily stochastic. The off-road rate metric is also applied to evaluate the distribution’s precision. As shown in Tab. 4, with increasing $\beta$ values, the off-road rate and reverse cross-entropy decrease, implying a more precise distribution model, while the forward cross-entropy increases and RF$_{20}$ decreases, meaning the distribution is getting less diverse. It shows that the hyperparameter $\beta$ can balance the predicted distribution’s diversity and accuracy while a distribution only minimizing the forward cross-entropy can cover the data well but will produce implausible samples.

**Reward layers:** First, we study how the IRL learning method is beneficial compared to behavior cloning (BC). In the BC method, we ablate the value iteration network and directly output the non-stationary policy in replace of the non-stationary reward. Then, our non-stationary reward is compared with the stationary reward (SR) used in previous works [6, 35]. The SR method is implemented by mapping the concatenation of the motion and scene feature maps into one reward map through two fully connected convolutional layers. Results in Tab. 4 show that a non-stationary reward outperforms no reward or a stationary one in terms of forward and reverse cross-entropy and off-road rate.

**Refinement network:** We consider two models without the refinement network. One removes the refinement network from our method. The other replaces our refinement network with K-means in [6] and outputs $K$ cluster centers of trajectory samples as representatives. Both models are trained end-to-end using the variety loss based on the pretrained distribution model. The comparison between Tab. 5 and Tab. 1 bottom part shows that the refinement network is indispensable and more effective than K-means while increasing the offroad rate due to the diversity loss.

**Training process:** Firstly, we show the result of only completing the first three training processes without the end-to-end fine-tuning process. Besides, we also consider two other training processes. One is to train the network with the sum of all losses like multi-task. The other one is with the variety loss only. Tab. 5 demonstrates that our training process with end-to-end fine-tuning can improve performance in predicting representative trajectories. We find that training only with variety loss is unstable and may not converge.

### 6. Conclusion

We have proposed an end-to-end interpretable trajectory distribution prediction model based on a grid plan. Our model can learn to produce a diverse and admissible trajectory distribution by minimizing the symmetric cross-entropy loss. We also design a flexible refinement network to generate a small set of representative trajectories. Finally, we demonstrate the effectiveness of our approach in two real-world datasets with state-of-the-art performance.
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1. Occupancy Grid Maps

In this section, we show examples of occupancy grid maps (OGMs) prediction results on the two datasets: Stanford Drone Dataset (SDD) and Intersection Drone Dataset (inD). Our temporal OGMs represent each future position distribution with an OGM, so we show the predicted OGMs from our model at 6 time steps. Fig. 1 and Fig. 2 show that our temporal OGMs can capture the scene compliant future position distribution while reflecting the multimodality.

![OGMs prediction results on the SDD](image1)

![OGMs prediction results on the inD](image2)

Figure 1. OGMs prediction results on the SDD. Warmer color indicates higher occupancy probability, while colder color represents lower occupancy probability.

Figure 2. OGMs prediction results on the inD.

2. Reward Maps

Our model learn non-stationary rewards dependent on the history trajectories and neighboring scene for producing non-stationary policy. Our non-stationary rewards \( r^n : S \times A \rightarrow \mathbb{R} \) are dependent on the action and state, where the action set includes 4 adjacent movements up, down, left, right and an end action leading to the absorbing state. For simplicity, we show the reward maps of taking four moving actions at the fixed MDP steps \( n = 10 \) on the SDD (see Fig. 3) while the end action at four differ-
ent MDP steps $n = 5, 10, 15, 20$ on the inD (see Fig. 4). As shown in Fig. 3, the regions with the highest rewards (i.e., in red color) are generally consistent with the action direction. Fig. 4 illustrates that the roadside or crosswalk regions have higher end rewards than the car lane and impassable regions, which is reasonable since the used training data of inD are pedestrian trajectories instead of vehicle tracks.

3. Policy Maps

We generate a non-stationary policy using an approximate value iteration network based on the non-stationary rewards. Similarly, we show the policy maps of taking four moving actions at the fixed MDP steps $n = 10$ on the SDD (see Fig. 5) while the end action at four different MDP steps $n = 5, 10, 15, 20$ on the inD (see Fig. 6).

Fig. 5 shows that our model learns to take an action to move on the road consistent with the action direction and avoid the terrains. Fig. 6 demonstrates that it learns to end at the roadsides in the last few MDP steps, or the impassable regions, where the pedestrian trajectories seen in the training data usually end in 30 seconds future.

3.1. Plan-conditioned Trajectory

The grid-based plans are sampled from the non-stationary policy by Gumbel-Softmax trick and bilinearly interpolation. Then, we recursively generate a trajectory using an RNN with multi-head attention on the sampled plan.

Figure 3. Reward maps of taking the 4 adjacent movements up, down, left, right at MDP step $n = 10$ on the SDD. Warmer color represents higher reward value, while colder color implies lower reward value.

Figure 4. Reward maps of taking the end action on the inD.

Figure 5. Policy maps of taking four movements up, down, left, right at MDP step $n = 10$ on the SDD. Warmer color means higher action probability, while colder color means lower action probability.

We show four examples of an sampled plan and its corresponding trajectory on SDD in Fig. 7 and inD in Fig. 8. We can observe that the sampled plans are scene compliant and the trajectory follows its corresponding plan while keeping smooth.
4. Trajectory Distribution

By minimizing the approximate symmetric cross-entropy loss, we learn a trajectory distribution close to the ground-truth. Examples of the predictive trajectory distribution from SDD and inD are shown in Fig. 9 and Fig. 10. We can see that our predicted trajectory distributions are diverse and feasible.

5. Representative Trajectories

We design a Transformer-based refinement network to generate a small set of representative trajectories based on a large number of trajectories sampled from the trajectory distribution. Fig. 11 and Fig. 12 show qualitative examples from SDD and inD. We note that the representative trajectories are diverse and capable to cover the ground-truth future trajectory due to the adopted variety loss.
Figure 10. Predicted trajectory distributions on the inD.

Figure 11. Representative trajectories predicted on the SDD.

Figure 12. Representative trajectories predicted on the inD.