Two Types of Magnetohydrodynamic Sheath Jets

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Abstract

Recent observations of astrophysical jets emanating from various galactic nuclei strongly suggest that a double layered structure, or a spine-sheath structure, is likely to be their common feature. We propose that such a sheath jet structure can be formed magnetohydrodynamically within a valley of the magnetic pressures, which is formed between the peaks due to the poloidal and toroidal components, with the centrifugal force acting on the rotating sheath plasma is balanced by the hoop stress of the toroidal field. The poloidal field concentrated near the polar axis is maintained by a converging plasma flow toward the jet region, and the toroidal field is developed outside the jet cone owing to the poloidal current circulating through the jet. Under such situations, the set of magnetohydrodynamic (MHD) equations allows two main types of solutions, at least, in the region far from the footpoint. The first type solution describes the jets of marginally bound nature. This type is realized when the jet temperature decreases like viral one, and neither the pressure-gradient nor the MHD forces, which are both determined consistently, cannot completely overcome the gravity even at infinity. The second type is realized under an isothermal situation, and the gravity is cancelled exactly by the pressure-gradient force. Hence, the jets of this type are accelerated purely by the MHD force. It is suggested also that these two types correspond, respectively, to the jets from type I and II radio galaxies in the Fanaroff-Riley classification.

Key words: galaxies: jets — galaxies: magnetic field — accretion, accretion disks

1. Introduction

In his penetrating review paper on the active galactic nuclei (AGN), Leahy (1999) stresses the importance of the dichotomy of radio loud and quiet AGN and suggests the essential difference between the respective central engines. In this context, our interest in the present paper is substantially restricted to the radio loud AGN and their power-down version, the low luminosity active galactic nuclei (LLAGN).

It is well known (see, e.g., Urry & Padovani 1995; de Gouveia Dal Pino 2005) that there are two kinds of astrophysical jets associated with the type I and II radio galaxies in the Fanaroff-Riley (or FR) classification (Fanaroff & Riley 1974). The radio emissions of the low-luminosity FR I objects peak near the nuclei and often show symmetric two-sided jets. The collimations of the jets are not so strong and the radial velocities show considerable deceleration at large distances. These objects are found in rich cluster environments. On the other hand, the emissions from the high-luminosity FR II objects have radio lobes with prominent hot spots and bright outer edges. The collimations of the jets are very sharp and strong asymmetries often exist between the jets and counter jets, with the latter being undetectable in many cases. The radial velocities do not seem to be decelerated until the end points are reached. These objects exist in more isolated environments than FR I’s.

Another feature we have to keep in mind is the double-layered structure, or spine-sheath structure, of the jets, which is now believed to be a common feature to the FR I and II jets (Giovannini et al. 1999; Giovannini et al. 2001). The limb-brightened structure observed in the jets of both types can be well interpreted by the presence of velocity shear associated with the double-layered structure (Giroletti et al. 2004), in which a spine region of highly relativistic speed is enclosed by a mildly relativistic or non-relativistic sheath jet. Owing to the relativistic beaming effect, the radiation from the spine becomes highly concentrated within a small angle around the jet direction, and hence becomes weak when seen from other directions. The possibility that the plasma in the spine region may be the electron-positron pair plasma has long been suggested in the literature (see, e.g., Begelman et al. 1987; Pelletier 2004).

Historically, the problem of jet formation has been discussed viably in the framework of the theory of force-free or ideal magnetohydrodynamic (MHD) magnetospheres (e.g., Sauty et al. 2002; Pelletier 2004) originally developed in relation to the discussions of the pulsar magnetospheres. Although such treatments may be mathematically sophisticated and hence beautiful, it seems unfortunately too formal to be able to quickly solve many practical problems associated with the actual astrophysical jets. Therefore, we adopt here a much more practical approach to this problem. The essence of the jet formation and acceleration is in their non-force-free nature and non-ideal MHD aspects of the electrodynamic current systems.

As stressed in our previous works (e.g., Kaburaki 2000; Kaburaki 2007), the jets can be formed by pinching and accelerating effects of the Ampère force acting on the current carrying plasma in the polar regions. The current flowing through the polar jet forms a returning portion of the poloidally circulating current system (see, figure 1). This current is originally
driven by the electromotive force caused in the accretion disc by the rotational motion of the disc plasma in a global, poloidal magnetic field. The mechanism of jet formation is thus directly coupled to the accretion process taking place in the same activity site. Although there are several streams of investigations that attribute the formation of jets or outflows to the action of magnetized accretion discs (e.g., Blandford & Payne 1982), no satisfactory theory from the view point of the current closure seems to exist so far in the literature. The importance of viewing the jet as a part of a globally circulating current system has been emphasized from early times by Benford (1978), and remarkable stability natures of such current-carrying systems have also been discussed repeatedly by the same author (e.g., Benford 2006).

Recent observations have also revealed that both types of jets in FR classification are likely to be enclosed within the X-ray cavities, as far as they are young. The cavities are probably maintained by the magnetic pressure of the toroidal field generated by the global current system circulating the disc-jet compounds (Kaburaki 2007). It is interesting to note that the shapes of such cavities may be different reflecting the different types of the current systems enclosed therein (see, figure 1 and also the discussion in the final section). It is theoretically probable that if a jet is being driven by the accretion disc the resulting cavity would take an hour-glass shape, whereas the cavity would become of spindle shape if the accretion has been switched off. Observationally, the cavities of FR I jets seem to be of hour-glass shape with narrow necks near the equator (e.g., for Cen A, see figure 2 and 3 of Kraft et al. 2002, and figure 4 of Kraft et al. 2007; for M87, figure 6 of Forman et al. 2007 ). On the other hand, the examples of the spindle shape are fairly rare. The only one plausible example hitherto known to the author is Cyg A (see figure 1 of Wilson et al. 2006), which belongs to FR II type.

Our aim in the present paper is to construct an analytic model of conical MHD jets with small opening angles, which can reproduce the main properties of the jets described above, at least, in the regions well beyond their launching site. The method of analysis adopted here is analogous to that in our earlier work (Kaburaki & Itoh 1987) addressed to the jets in star forming regions. The essential difference between the situations considered here and there is in the assumed external magnetic fields. Instead of a stellar dipole field in the former paper, we assume here a uniform field perpendicular to the equatorial plasma disc, in the context of galactic nuclei. The effects of relativity are not included for simplicity, since the jet velocities in the MHD sheaths are expected to be at most mildly relativistic. Preliminary results have been reported in Kaburaki (2004), though it contains the solution of only one out of two types obtained in the present paper.

In section 2 we first describe the global configuration considered in the present paper, and then specify the angular dependences of relevant physical quantities in spherical polar coordinates. With these expressions, we discuss in section 3 Ohm’s law and the equation of motion, which are essential to determine the jet velocities. In order to obtain the solutions to this set of equations, we proceed in section 4 from the static force balance to a plausible case of dynamic balance. In the following two sections, two types of solutions are derived and their properties are discussed in detail. Finally in section 7, the results are summarized and then a possible interpretation of these solutions is discussed in the light of the FR dichotomy of the galactic radio jets.

2. Separation of Variables

When the processes taking place in the central accretion disks are explicitly taken into account, the mechanisms of jet formation are expected to be as follows. In terms of the circuit theory, the accretion disc acts as a DC current generator and drives poloidally circulating current system (for more details, see section 2 of Kaburaki 2007 and figures therein). This is the physical cause of the toroidal magnetic component that is added as a twist to the originally vertical seed field. In the case of an upward polarity of the seed field, the radial current is driven outward in the disc, and dominant portion of this current is expected to close after circulating remote regions along the thin boundary layer separating the jet cavity (cocoon or hour glass) from surroundings. This means that the twisted field lines are confined exclusively within the cavity. After passing through the tops of the cavity wall, the currents return to the inner edge of the accretion disc along the polar axes.

Under such circumstances, a polar jet can be formed from plasma in the polar regions, because the Ampère force exerted
by the toroidal magnetic field on the return current always has the necessary components both for radial acceleration and for collimation as shown in figure 1 of Kaburaki (2007). This fact implies that there may always be a bipolar jet associated with such an accretion disc in a large-scale magnetic field, even if it might be strong or rather weak depending on the circumstances.

In an axisymmetric situation, the only relevant variables in spherical polar coordinates \((r, \theta, \varphi)\) are the radial distance from the center \(r\) and the polar angle \(\theta\). It is often convenient to introduce their normalized versions, \(z = r/r_j\) and \(\eta = \theta/\Theta\), where \(r_j\) is the footpoint radius and \(\Theta\) is the half-opening angle, respectively, of a conical jet. The angular variable \(\eta\) introduced here should not be confused with that used in our previous papers dealing with the problems of disc accretion (Kaburaki 2000; Kaburaki 2001; Kaburaki 2007); in these papers, \(\eta\) stands for \((\theta - \pi/2)/\Delta\), with \(\Delta\) being the half-opening angle of an accretion disc. The magnetic field is generally written as \(B_0 + \mathbf{b}\), where \(B_0\) is the seed field and \(\mathbf{b}\) is the deformation due to the action of plasma motion. However, the latter is sufficiently large (i.e., \(|\mathbf{b}| \gg |B_0|\)) in the accretion discs and jets. The influence of the seed field is taken into account only at the outer edge of an accretion disc (see Kaburaki 2000).

Jets are assumed to be narrow (i.e., \(\Theta \ll 1\)), and \(\Theta\) is regarded as a smallness parameter in ordering the physical quantities. Our main interest is not in obtaining a strict solution, but in demonstrating the existence of a physically reasonable configuration that satisfies the set of equations, at least in an approximate sense. We therefore expect that the solution to the set of resistive-MHD equations can be written in a variable-separating form. The concrete form of each physical quantity is determined here by taking its symmetry property into account, and we do not insist that such a choice is the unique possibility.

We adopt the following functional forms for the three components of the magnetic field:

\[
b_r(z, \eta) = \tilde{b}_r(z) \text{sech}^2\eta, \tag{1}
\]

\[
b_\theta(z, \eta) = -\Theta \tilde{b}_\theta(z) f(\eta), \tag{2}
\]

\[
b_\varphi(z, \eta) = -\tilde{b}_\varphi(z) \tanh \eta. \tag{3}
\]

Here we have introduced the functions carrying tildes over them to represent the radius-dependent parts of the corresponding quantities. Although they are written as functions of normalized radius \(z\), but they can also be regarded as functions of \(r\) (i.e., radius itself) whenever it is necessary. Hereafter, we discuss only one side of a bipolar-jet system, and hence the values of \(\theta\) are restricted to the range \(0 \leq \theta \leq \pi/2\) whereas the azimuthal angle varies in the whole range \(0 \leq \varphi \leq 2\pi\).

The \(\eta\)-dependence of \(b_r\) is assumed to represent a localized structure within the cone of \(\theta \sim \Theta\), which is expected to occur in this component owing to the balance between the sweeping up effect of the accreting flow at the footpoint of a jet and the outward diffusion caused by the presence of a finite electrical resistivity. The appearance of the smallness parameter \(\Theta\) in \(b_\theta\) indicates that this quantity is of order \(O(\Theta)\), and the concrete form of the function \(f(\eta)\) is specified below from the requirement of flux conservation. The presence of \(\tanh \eta\) in \(b_\varphi\) guarantees that \(b_\varphi\) should vanish on the polar axis and also remain finite outside the jet cone.

The sign convention on the right-hand sides of the above expressions is adopted for an upward seed field whose \(r\)-component on the polar axis is positive. When the seed field is directed downward, \(B_\theta\) in the resulting solutions should be regarded as having minus sign, and any other inclined case is not considered here. Although the seed field is amplified strongly by the inward motion in an accretion disc, its polarity remains the same. The rotational motion in the disc causes a large negative \(\varphi\)-component, which is assumed to be absent in the seed field. Further, the negative sign in the \(\theta\)-component means that a converging magnetic field in the jet structure is taken as standard.

For the expressions (1) and (2), the law of magnetic flux conservation \(\nabla \cdot \mathbf{b} = 0\) yields

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 b_r \right) \text{sech}^2\eta - \Theta \frac{\tilde{b}_\theta}{r \sin \theta} \frac{d}{d\theta} \left( f \sin \theta \right) = 0. \tag{4}
\]

As far as we remain within the jet cone (i.e., \(\theta \leq \Theta\)), the separation of variable is attained approximately by the choice

\[
f(\eta) = \tanh \eta - \frac{\Theta}{\sin \theta} \ln (\cosh \eta), \tag{5}
\]

where the second term on the right-hand side is of the first order in \(\Theta\) since \(\ln(\cosh \eta)/\sin \theta\) remains finite, even for very small \(\theta\). Then the conservation equation reduces to

\[
\frac{\tilde{b}_\theta}{b_r} = r \frac{d}{dr} \ln(r^2 \tilde{b}_r). \tag{6}
\]

In most cases, however, we need only the expression of \(f(\eta)\) that has the accuracy to the leading (i.e., zero-th) order in \(\theta\) as far as we remain in the jet-cone region stated above. In such cases, we can replace (5) by its approximate version,

\[
f(\eta) \simeq \tanh \eta. \tag{7}
\]

This procedure corresponds to regarding \(\sin \theta\) in the \(\eta\)-derivative in equation (4) as a slowly varying quantity (i.e., a constant) compared with a rapidly varying function \(\tanh \eta\). Then, we can reproduce equation (6) within this approximation. Exceptional cases in which the accurate expression (5) is needed are discussed in the next section.

Given the above expressions for the magnetic field components, we can derive the forms of the current density from Ampère’s law

\[
\mathbf{j} = \frac{c}{4\pi} \nabla \times \mathbf{b}, \tag{8}
\]

where \(c\) is the light velocity in a vacuum. Their components are

\[
\begin{align*}
  j_r(z, \eta) &= \frac{c}{4\pi r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta b_\varphi) \\
  &\simeq -\Theta^{-1} \tilde{j}_r(z) \text{sech}^2\eta, \tag{9}
\end{align*}
\]

\[
\begin{align*}
  j_\theta(z, \eta) &= -\frac{c}{4\pi r} \frac{\partial}{\partial r} (rb_\varphi) = \tilde{j}_\theta(z) \tanh \eta, \tag{10}
\end{align*}
\]

\[
\begin{align*}
  j_\varphi(z, \eta) &= \frac{c}{4\pi r} \left\{ \frac{\partial}{\partial r} (rb_\varphi) - \frac{\partial b_r}{\partial \theta} \right\} \\
  &\simeq \Theta^{-1} \tilde{j}_\varphi(z) \text{sech}^2\eta \tanh \eta, \tag{11}
\end{align*}
\]

where
The density is assumed to be
\[ \tilde{\rho}(z) = \frac{c}{4\pi r_3} \frac{\tilde{b}_\varphi(z)}{z}, \]

\[ \tilde{\rho}(z) = \frac{c}{4\pi r_3} \frac{1}{z} \frac{d}{dz}\left[ \tilde{b}_\varphi(z) \right], \]

\[ \tilde{\rho}(z) = \frac{c}{2\pi r_3} \frac{\tilde{b}_\varphi(z)}{z}. \]

The factor \( \Theta^{-1} \) expresses the order of magnitude of the relevant quantity has been made explicit in equations (9) and (11). In performing the \( \eta \)-derivative in equation (9), \( \sin \theta \) has been regarded also as a constant as noted above. In deriving the last expression of equation (11), the first term in the curly brackets has been omitted compared with the second term, because the former is of order \( O(\Theta) \) while the latter is \( \sim O(\Theta^{-1}) \) owing to the presence of the derivative \( \partial / \partial \theta = \Theta^{-1} \partial / \partial \eta \). The negative sign in front of \( \tilde{\rho}_r \) means that the incoming current is a standard when the seed field is directed upward. It is easy to confirm that the above expressions actually satisfy \( \nabla \cdot j = 0 \).

The functional forms of the velocity components are introduced as follows:
\[ v_r(z, \eta) = \tilde{v}_r(z) \text{sech}^2 \eta, \]
\[ v_\theta(z, \eta) = -\Theta \tilde{v}_\theta(z) \text{sech}^2 \eta, \]
\[ v_\varphi(z, \eta) = \tilde{v}_\varphi(z) \text{sech}^2 \eta \text{tanh} \eta. \]

Again, the hyperbolic functions and \( h(\eta) \) represent the existence of a localized structure, but the sign convention here is somewhat different from the case of magnetic field. The jet proceeds outward, of course, and is expected to rotate in the same direction as the accretion disc and to converge toward the polar axis. When \( \tilde{v}_r(z, \eta) \), \( \tilde{v}_\theta(z, \eta) \) and \( \tilde{v}_\varphi(z, \eta) \) are regarded as quantities of order unity, only \( v_\theta(z, \eta) \) is of order \( O(\Theta) \) owing to the presence of a factor \( \Theta \). Although \( v_\varphi \) remains finite even outside the jet cone, the plasma density vanishes there (see below).

Analogously to the case of the flux conservation, the equation of mass continuity \( \nabla \cdot (\rho \mathbf{v}) = 0 \) can be reduced to the form
\[ \tilde{\rho} = \frac{r \frac{d}{dr} \ln(r^2 \tilde{\rho} \tilde{v}_r)}{\tilde{v}_r}, \]
by fixing as
\[ h(\eta) = f(\eta). \]

Of course, however, some difference from the former case arises according to the appearance of an extra function \( \tilde{\rho} \) in the continuity equation. The angular dependence of the matter density is assumed to be
\[ \rho(z, \eta) = \tilde{\rho}(z) \text{sech}^2 \eta \text{tanh}^2 \eta, \]
reflecting a hollow-cone structure suggested by observations. Then, the density maximum is attained where \( d\rho / d\eta = 0 \), i.e. where \( \text{sech}^2 \eta = \text{tanh}^2 \eta \), and we identify this surface of maximum density as the middle surface of a hollow-cone jet (i.e., \( \theta \sim \Theta \)). In the above derivation of equation (18), therefore, the term that is proportional to \( \frac{d}{d\eta}(\text{sech}^2 \eta \text{tanh}^2 \eta) / d\eta \) has been omitted.

The remaining thermodynamic quantities are assumed to take the following forms:

\[ T(z, \eta) = \tilde{T}(z), \]
\[ p(z, \eta) = \tilde{\rho}(z) \text{sech}^2 \eta \text{tanh}^2 \eta. \]

Here, the variation of the temperature \( T \) across a jet is ignored since the jet is very thin. The \( \eta \)-dependence of the pressure represents also a hollow-cone structure. We should emphasize again that all of the functional forms introduced above are, by no means, insisted as a unique possibility for each relevant quantity, but that we are searching only for a plausible example.

Then, the ideal gas law \( p = K \rho T \) is separated completely to yield
\[ \tilde{\rho} = K \tilde{\rho} \tilde{T}, \]
where \( K = R / \mu \), with \( R \) and \( \mu \) being the gas constant and the mean molecular weight of the jet plasma, respectively.

3. Ohm’s Law and Equation of Motion

The remaining fundamental equations are Ohm’s law and equation of motion. Although the electric field is usually eliminated from the scheme of MHD, especially in its ideal version, we treat it explicitly here. This is because it turns out that Ohm’s law, rather than the equation of motion, plays an essential role in determining the radial velocity of a jet, at least in the region far from the footpoint. It turns out unfortunately that some components of Ohm’s law and the equation of motion (EOM) cannot be variable-separated completely. In such cases, we approximately replace the remaining angular dependences by their representative values evaluated at the middle surface of a jet cone, \( \theta \sim \Theta \), where most of the jet matter is concentrated. For example, whenever the dependences on \( \text{sech}^2 \eta \) and \( \text{tanh}^2 \eta \) appear, we evaluate their representative values as \( \langle \text{sech}^2 \eta \rangle = \langle \text{tanh}^2 \eta \rangle = 1 / 2 \), because these satisfy the condition for the maximum density, \( \text{sech}^2 \eta = \text{tanh}^2 \eta \), and the identity \( \text{sech}^2 \eta + \text{tanh}^2 \eta = 1 \), simultaneously.

When the system of current carrying jet satisfies a strict stationarity condition, the electric field should be irrotational and hence can be written as the gradient of a scalar potential \( \tilde{U}(z, \eta) \), i.e., \( \mathbf{E} = -\nabla \tilde{U} \). Reflecting the localized structure associated with the jet, we assume for the potential as
\[ U(z, \eta) = \Theta \tilde{U}(z) \text{sech}^2 \eta, \]
and obtain
\[ E_r(z, \eta) = -\frac{1}{r \sin \theta} \frac{\partial U}{\partial r} = -\Theta \tilde{E}_r(z) \text{sech}^2 \eta, \]
\[ E_\theta(z, \eta) = -\frac{1}{r} \frac{\partial U}{\partial \theta} = \Theta \tilde{E}_\theta(z) \text{sech}^2 \eta \text{tanh} \eta, \]
\[ E_\varphi(z, \eta) = -\frac{1}{r \sin \theta} \frac{\partial U}{\partial \varphi} = 0, \]
where
\[ \tilde{E}_r(z) = \frac{1}{r J} \frac{d \tilde{U}(z)}{dz}, \]
\[ \tilde{E}_\theta(z) = \frac{2}{r J} \frac{\tilde{U}(z)}{z}. \]
The signs of $E_r$ and $E_\theta$ here have been chosen in accordance with those of $j_r$ and $j_\theta$, respectively.

If the situation we are considering requires even a slow change of the magnetic field in time $t$, however, the electric field becomes non-irrotational according to Faraday’s law, $\nabla \times E = -(1/c) \partial \phi / \partial t$. Assuming that even in such a case the functional forms of each component of $E$ given above is not altered, we can show that the only non-vanishing value remains in its $\varphi$-component,

$$\langle \nabla \times E \rangle_{\varphi} = \frac{1}{c} \left\{ \frac{d}{dr} \left[ r \tilde{E}_\varphi \right] - 2 \tilde{E}_r \right\} \text{sech}^2 \eta \tanh \eta. \tag{30}$$

Then, the $\varphi$-component of Faraday’s law reduces to

$$\frac{\partial \tilde{b}_\varphi}{\partial t} = - \frac{c}{r} \left\{ \tilde{E}_r - \frac{1}{2} \frac{d}{dr} \left[ r \tilde{E}_\theta \right] \right\}, \tag{31}$$

after the remaining $\eta$-dependence has been replaced by its representative value at the surface of the middle plane, $\theta \sim \Theta$. This equation can be used to judge whether the electric field obtained from Ohm’s law satisfies a strict stationarity condition or not. Even if the electric field is non-irrotational, the rate of change for $b_\varphi$ is expected to be very small since the self-inductance of the whole current system is very large (Benford 2006). This point will be discussed after the explicit solutions are obtained.

Ohm’s law in its vector form is

$$\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{b} = \frac{\mathbf{j}}{\sigma}, \tag{32}$$

where $\sigma$ is the electrical conductivity, and is assumed to be a constant for simplicity. This law yields

$$\tilde{E}_r = \frac{1}{c} \left( \tilde{v}_\varphi \tilde{b}_\varphi + \tilde{v}_r \tilde{b}_r \right) + \frac{\tilde{j}_r}{\sigma \Theta^2}, \tag{33}$$

$$\tilde{E}_\theta = - \frac{1}{c} \left( \tilde{v}_r \tilde{b}_r + \tilde{v}_\varphi \tilde{b}_\varphi \right), \tag{34}$$

$$\tilde{E}_\varphi = 0 = \frac{1}{c} \left( \tilde{v}_\varphi \tilde{b}_\theta - \tilde{v}_\theta \tilde{b}_r \right) + \frac{\tilde{j}_\varphi}{\sigma \Theta^2}, \tag{35}$$

when the remaining $\eta$-dependence are replaced by their representative values. In deriving the above expressions, $\sigma$ has been regarded as a quantity of $O(\Theta^{-2})$. This is because the existence of the term $\tilde{j}_r / \sigma$ in the $\varphi$-component equation is essential for taking the non-ideal MHD aspects into account. As a consequence, the term $\tilde{j}_\theta / \sigma$ has been omitted as small quantity of $O(\Theta^2)$ in equation (34). The origin of the resistivity may be classical or turbulent, but we do not need to specify it here because $\sigma$ appears in the final results only through the magnetic Reynolds number, which we may treat as a parameter.

The MHD equation of motion in a stationary state is written as

$$(\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{\rho} \nabla p + \frac{1}{4\pi \rho} [\mathbf{b} \times (\nabla \times \mathbf{b})] - \mathbf{g} = 0, \tag{36}$$

where $\mathbf{g}$ is the gravitational acceleration, and the contribution from the seed field $\mathbf{E}_0$ has been neglected in the magnetic field term compared with that from the deformed part $\mathbf{b}$.

We first discuss the $\theta$-component of EOM. After writing out the full terms in this component, we keep only the leading order terms in the smallness parameter $\Theta$, i.e., the terms of $O(\Theta^{-1})$:

$$\frac{\tilde{v}_\theta^2}{4} \left( \frac{d \ln \tilde{v}_r}{dr} + 2 \tilde{v}_r \right) \tilde{v}_\theta + \frac{1}{\rho} \frac{d \tilde{p}}{dr} + \frac{G M}{r^2} + \frac{\tilde{b}_\varphi^2}{2\pi \rho r} \left( \frac{d \ln \tilde{b}_\varphi}{dr} - \tilde{b}_\theta \frac{\partial \tilde{b}_r}{\partial \theta} \right) = 0, \tag{42}$$

The function $f(\eta)$ in equation (5) is conceived in the definition of the directional derivative $(\mathbf{v} \cdot \nabla)$. Then, after substituting the functional forms of relevant quantities and evaluating the remaining $\eta$-dependences at $\theta \sim \Theta$, we obtain

$$\left\{ \frac{\tilde{v}_\varphi^2}{4\pi} + \frac{\tilde{b}_\varphi^2}{4\pi} \right\} \cot \theta = \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial \theta} \left( p + \frac{1}{8 \pi} (b_r^2 + b_\varphi^2) \right). \tag{37}$$

This equation states that if we expect the magnetic pressure confinement of the sheath plasma (i.e., the vanishing of the right-hand side), then the centrifugal force caused by the rotational motion of a jet plasma should be balanced by the hoop stress of the toroidal field lines. Although we do not insist that this is the only possibility to obtain a consistent set of solutions, we examine in this paper only the cases in which this condition is satisfied. It is easy to suppose that such a configuration would be realized in the magnetic fields developed in the galactic nuclear regions. Indeed, the radial component $b_r$ is expected to be packed into the spine region if a converging flow toward the polar axis is maintained, while the toroidal component $b_\varphi$ should remain weak near the polar axis as far as the poloidal current density is non-singular on the polar axis.

Then, we obtain two separated equations:

$$\tilde{v}_\varphi = \frac{b_\varphi^2}{4\pi \rho}, \quad p + \frac{1}{8 \pi} (b_r^2 + b_\varphi^2) = \bar{p}, \tag{38}$$

where $\bar{p}$ is a function of $r$ only. The former equation yields

$$\tilde{v}_\varphi = \frac{\tilde{b}_\varphi^2}{\pi \rho} \tag{39}$$

after substitution of the representative value for $\theta$, and the latter can be confirmed to hold if the condition

$$\bar{p} = \frac{\tilde{b}_\varphi^2}{8\pi} = \frac{\tilde{b}_\varphi^2}{8\pi} \tag{40}$$

is satisfied.

Similarly, retaining only the leading order terms in $\Theta$ in the $r$-component of EOM, we have

$$r v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_r^2}{r} + \frac{1}{\rho} \frac{\partial p}{\partial r} - g + \frac{1}{4\pi \rho r} \left\{ b_r \frac{\partial}{\partial r} (r b_r) - b_\theta \frac{\partial b_r}{\partial \theta} \right\} = 0. \tag{41}$$

In the above expression, the term $v_\varphi / r$ can be eliminated with the aid of the former one of equations (38). Although the gravity is usually neglected in the discussions of jet structures at large distances from the central object, here it is kept in the equation as a term of $O(1)$. Indeed, it turns out in the next section that the gravity plays an essential role in determining the structures of the actual jets, since the most fundamental thing that every jet should do is to overcome the gravity and to reach almost infinity.

It should be noted that the accurate version of the function $f(\eta)$ in equation (5) is conceived in the definition of the directional derivative $(\mathbf{v} \cdot \nabla)$. Then, after substituting the functional forms of relevant quantities and evaluating the remaining $\eta$-dependences at $\theta \sim \Theta$, we obtain

$$\tilde{v}_\theta^2 - \frac{\tilde{b}_\varphi^2}{4\pi} \cot \theta = \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial \theta} \left( p + \frac{1}{8 \pi} (b_r^2 + b_\varphi^2) \right). \tag{37}$$
where $A$ and $D$ are newly introduced representative values, i.e., $A \equiv \langle f(\eta) \tanh \eta \rangle$ and $D \equiv \langle f(\eta)/\tanh \eta \rangle$. We can specify the range of these values as $0 < A < 1/2$ and $0 < D < 1$, since $f(\eta) < \tanh \eta$. However, as it is hard to specify these values more exactly, we treat them as parameters in this paper.

In the $\varphi$-component of EOM, all the terms survive since they are all quantities of $O(1)$. After some manipulations, we can reach the form

$$\left( v_r \frac{\partial}{\partial r} + \frac{v_\varphi}{r} \frac{\partial}{\partial \varphi} \right) l - \frac{1}{4\pi \rho} \left( b_r \frac{\partial}{\partial r} + \frac{b_\varphi}{r} \frac{\partial}{\partial \varphi} \right) m = 0, \quad (43)$$

where we have introduced the definitions

$$l \equiv rv_r \sin \eta = \ell \sin \eta \tanh \eta, \quad \ell \equiv rv_\ell, \quad m \equiv rb_\varphi \sin \theta = -\tilde{m} \sin \theta \tanh \eta, \quad \tilde{m} \equiv rb_\varphi. \quad (44)$$

Within the approximation that $d(\sin \theta \tanh \eta)/d\eta \approx \sin \theta \sech^2 \eta$, we have

$$\tilde{v}_r \tilde{l} \left( \frac{d \ln \tilde{l}}{d\varphi} - \frac{D \tilde{b}_\varphi}{v_\varphi} \right) + \frac{\tilde{b}_r \tilde{m}}{\pi \rho} \left( \frac{d \ln \tilde{m}}{d\varphi} - \frac{D \tilde{b}_\varphi}{b_\varphi} \right) = 0, \quad (46)$$

again after substituting representative values for the remaining angular dependences.

4. From Static to Dynamic Equilibrium

In considering the problem of jet formation, we have to carefully chose the starting point from which our chain of reasoning to begin. As such a point, we select the static balance of a gas sphere around the center of gravity, apart from the transversal equilibrium attained in a conical jet. When neglected the magnetic and inertial forces, equation (42) reduces to

$$\frac{1}{\rho} \frac{d \tilde{p}}{d\varphi} + \frac{GM}{r^2} = 0, \quad (47)$$

and further with the aid of equation (23), to

$$K \tilde{T} \frac{d \ln \tilde{p}}{d\varphi} + \frac{GM}{r^2} = 0. \quad (48)$$

If the temperature varies with the radius according to a power law,

$$\tilde{T}(z) = T_0 z^{-\alpha} \quad (\alpha \geq 0), \quad (49)$$

then we can readily obtain the solution

$$\tilde{p}(z) = \begin{cases} p_0 \exp \left[ -\frac{\kappa}{\alpha - 1} \left\{ z^{\alpha - 1} - 1 \right\} \right], & (\alpha > 1) \\ p_0 z^{-\alpha}, & (\alpha = 1) \\ p_0 \exp \left[ -\frac{\kappa}{1 - \alpha} \left\{ 1 - z^{-(1-\alpha)} \right\} \right], & (1 > \alpha \geq 0) \end{cases} \quad (50)$$

where $\kappa \equiv GM/KT_0 r_3$. It can be seen from this solution that the static equilibrium is realized (i.e., $\tilde{p} \to 0$ as $z \to \infty$) only for $\alpha \geq 1$, and that the appearance of outflowing plasma is suggested for $1 > \alpha \geq 0$ since then $\tilde{p}$ remains finite even at $z \to \infty$.

Extending this consideration to the case of a dynamical force balance realized in a jet under the presence of other forces than the gravity and pressure gradient, we assume that $\tilde{p}(z)$ takes a form like

$$\tilde{p}(z) = p_0 z^{-\beta} \exp \left[ -\frac{\kappa}{\beta - 1} \left\{ 1 - z^{-(1-\beta)} \right\} \right], \quad (51)$$

e.g., a mixture of marginal and outflow types in equation (50). Here, $\alpha$ and $\beta$ are both constants whose values are restricted in the ranges, $1 > \alpha \geq 0$ and $\beta \geq 0$. For this choice, we can show easily that

$$\frac{1}{\tilde{p}} \frac{d\tilde{p}}{d\varphi} + \frac{GM}{r^2} = \beta \frac{\tilde{T}}{r}. \quad (52)$$

This relation indicates that, when the parameter $\beta$ does not vanish, a part of the pressure gradient force survives after cancellation with the gravity. Speaking in other way, the functional form of the gravity remains in equation (42) through the form of the miss-cancelled pressure force. On the other hand, when $\beta$ exactly vanishes, nothing that can trace the gravity is left in equation (42), and only the MHD forces control the jets.

Since the functional form of the pressure has been specified, it follows immediately from the equation of state (23) that

$$\tilde{p}(z) = p_0 z^{-\alpha-\beta} \exp \left[ -\frac{\kappa}{\beta - 1 - \alpha} \left\{ 1 - z^{-(1-\alpha)} \right\} \right], \quad (53)$$

where $\rho_0 \equiv \rho_0/KT_0$, and further from equation (40) that

$$\tilde{b}_r(z) = \tilde{b}_r(z) = b_0 z^{-\beta/2} \exp \left[ -\frac{\kappa}{2(1 - \alpha)} \left\{ 1 - z^{-(1-\alpha)} \right\} \right], \quad (54)$$

where $b_0 \equiv (8\pi p_0)^{1/2}$. Substituting the above expressions into equation (39), we have

$$\tilde{v}_\varphi(z) = (8KT_0)^{1/2} z^{-\alpha/2}. \quad (55)$$

The radial velocity of the jet is essentially controlled by the $\varphi$-component of Ohm’s law. Substituting the expression for $\tilde{v}_\varphi$ from equation (14), we can rewrite equation (35) in the form

$$\frac{\tilde{b}_r}{\tilde{v}_r} - \frac{\tilde{v}_\varphi}{\tilde{v}_r} + \frac{c^2}{2\pi \sigma \Theta^2 r_1 v_\infty} \frac{v_\infty}{\tilde{v}_r} z^{-1} = 0. \quad (56)$$

Further inserting the results from the flux and mass conservations, we finally obtain the expression

$$\frac{d d \ln \left( \frac{\tilde{p}}{\tilde{p}_0} \right)}{d \varphi} = \frac{R_j^{-1} v_\infty}{\tilde{v}_r} z^{-2}, \quad (57)$$

where $R_j \equiv 2\pi \sigma \Theta^2 r_1 v_\infty/c^2$ is the magnetic Reynolds number characterizing the jet structures. We have to note here that the assumption of infinite conductivity (i.e., $\sigma \to \infty$, i.e., the limit of ideal MHD) restricts the result obtained above too strictly (see also equation (62) below).

Among the asymptotic solutions of the above equation (57), only the one that tends to a constant velocity $v_\infty$ at large distances (i.e., at $z \gg 1$) is of our interest here. Putting $\tilde{v}_r = v_\infty$ on the right hand side of equation (57), we obtain by integration

$$\frac{\tilde{p}}{b_r/b_0} = \exp \left[ R_j^{-1} (1 - z^{-1}) \right]. \quad (58)$$

Further substituting expressions (53) and (54), we have

$$\tilde{v}_r(z) = v_0 z^{\beta/2-\alpha} \times \exp \left[ \frac{\kappa}{2(1 - \alpha)} \left\{ 1 - z^{-(1-\alpha)} \right\} + R_j^{-1} (1 - z^{-1}) \right]. \quad (59)$$
In order to guarantee that this $\hat{v}_r(z)$ actually tends to a constant value $v_\infty$ when $z \to \infty$, its power law dependence on $z$ should vanish. Here, we have to identify this condition very carefully. Since the power law dependence in an arbitrary function $F(z)$ causes a $z^{-1}$ term in the derivative $d \ln F/dz$, we discuss here

$$
\frac{d}{dz} \ln \hat{v}_r = \left( \frac{\beta}{2} - \alpha \right) z^{-1} + \frac{\kappa}{2} z^{-(2-\alpha)} + \Re_j^{-1} z^{-2}. \quad (60)
$$

Within the allowed values for $\alpha$ (0 $\leq$ $\alpha$ $<$ 1), there are two possibilities as for whether the second term on the right-hand side belongs to the first or third term: 1) when $\alpha = 1$ (or more exactly, $\alpha \to 1$) the vanishing of $z^{-1}$ term is attained if $\beta = 2 - \kappa$, and 2) when $\alpha = 0$ the condition yields $\beta = 0$. These cases may be called the “virial-jet” and “isothermal-jet” solutions, respectively, because of the behaviors in temperature. The detailed discussions of these solutions are given in the following two sections.

5. Virial-Jet Solution

5.1. asymptotic behavior

Although we have set $\alpha = 1$ in the virial jet case, actually we need a more careful treatment of the argument in the exponential functions. Since $\alpha$ should be smaller than unity as far as an outflow exists at infinity, we have to write $\alpha = 1 - \epsilon$ ($\epsilon > 0$) and take the limit of $\epsilon \to 0$. Then, the resulting expressions for the radial velocity is

$$
\hat{v}_r(z) = v_0 z^{-\kappa/2} \exp \left[ \frac{\kappa}{2\epsilon} (1 - z^{-\epsilon}) + \Re_j^{-1} (1 - z^{-1}) \right] = v_\infty z^{-\kappa/2} \exp \left[ \frac{\kappa}{2\epsilon} (1 - z^{-\epsilon}) - \Re_j^{-1} z^{-1} \right], \quad (61)
$$

where

$$
v_\infty \equiv v_0 \exp \left( \Re_j^{-1} \right). \quad (62)
$$

Since $v_\infty$ represent the terminal velocity of a jet, the jet acceleration becomes the more effective the smaller the magnetic Reynolds number (or the conductivity) is. Especially, in the limit of infinite conductivity (i.e., $\Re_j \to \infty$) no acceleration can be obtained.

The apparent power law dependence, $z^{-\kappa/2}$, is actually cancelled by the exponential factor containing $\epsilon$, in the limit of $\epsilon \to 0$. This can be confirmed with the aid of the identity

$$
\lim_{\epsilon \to 0} \left[ z^{-\alpha} \exp \left( \frac{\alpha}{\epsilon} (1 - z^{-\epsilon}) \right) \right] = 1, \quad (63)
$$

which holds for an arbitrary real number $\alpha$. The constancy of the left-hand side of (63) is guaranteed by the vanishing of its derivative with respect to $z$, and its value is fixed as 1 by evaluating it at $z = 1$.

There is further subtlety to be considered, in this type of outflows. In the actual situations, every jet cannot extend to infinity but terminates at a large but finite radius owing to the interaction with the surrounding galactic or inter-galactic matter. It is therefore obvious that the jet solution too has the end point $r_{out}$, beyond which it cannot be applied. In such non-ideal cases, $\epsilon$ can remain finite (i.e., the out flow at infinity does not exist) even if it may be very small. When this is the case, we cannot replace any exponential factors that contain $\epsilon$ by a power law expression, by using the identity (63).

However, as far as the jet is well developed, we may consider that $z_{out} \equiv r_{out}/r_J \gg 1$ and the radial velocity already approaches a terminal velocity $v_\infty$ at $z = z_{out}$. In this case, we have

$$
\exp \left[ -\frac{\kappa}{2\epsilon} z^{-\kappa/2} - \Re_j^{-1} z^{-1} \right] \approx 1,
$$

$$
\exp \left( \frac{\kappa}{2\epsilon} \right) = 1, \quad (64)
$$

from equation (61). The former relation means that the limit of $z_{out} \to \infty$ should be taken while keeping $\epsilon$ finite, and the latter equation yields

$$
\epsilon = (\ln z_{out})^{-1}. \quad (65)
$$

In order to discuss the behaviors in the asymptotic regions ($z \gg 1$) of the present kind of jets, it is convenient to introducing a new non-dimensional radius $\zeta \equiv r/r_{out} = z/z_{out}$ in which the top of a jet corresponds to $\zeta = 1$. Then we obtain from equation (61)

$$
\hat{v}_r(z) = v_\infty \left( \frac{z}{z_{out}} \right)^{-\kappa/2} \exp \left[ -\frac{\kappa}{2\epsilon} z^{-\epsilon} - \Re_j^{-1} z^{-1} \right] \rightarrow v_\infty \zeta^{-\kappa/2} \equiv \hat{v}_r(\zeta),
$$

where $\hat{v}_r(\zeta)$ represents the asymptotic behavior of $\hat{v}_r(z)$ in the limit of $z \to z_{out}$. This implies that the actual jets (with only a finite length) of virial type experience rather deceleration in their asymptotic regions toward their terminal values $\hat{v}_r(1) = v_\infty$. Such a behavior is strongly reminiscent of the jets in FR I radio galaxies (see section 1 and references therein).

For other quantities of our interest, the original $z$-dependences and their asymptotic forms are summarized as follows:

$$
\tilde{T}(z) = T_0 z^{-1} = T_\infty \zeta^{-1} \equiv \tilde{T}(\zeta), \quad (67)
$$

$$
\tilde{p}(z) = p_0 z^{-(2-\kappa)} \exp \left[ -\frac{\kappa}{\epsilon} (1 - z^{-\epsilon}) \right] \rightarrow p_\infty \zeta^{-(2-\kappa)} \equiv \tilde{p}(\zeta), \quad (68)
$$

$$
\tilde{\rho}(z) = \rho_0 z^{-(1-\kappa)} \exp \left[ -\frac{\kappa}{\epsilon} (1 - z^{-\epsilon}) \right] \rightarrow \rho_\infty \zeta^{-(1-\kappa)} \equiv \tilde{\rho}(\zeta), \quad (69)
$$

$$
\tilde{\beta}_r(z) = \tilde{\beta}_\varphi(z) = b_0 z^{-(1-\kappa/2)} \exp \left[ -\frac{\kappa}{2\epsilon} (1 - z^{-\epsilon}) \right] \rightarrow \beta_\infty \zeta^{-(1-\kappa/2)} \equiv \tilde{\beta}_r(\zeta), \quad (70)
$$

$$
\tilde{\beta}_\varphi(z) = (8KT_0)^{1/2} z^{-1/2} \rightarrow (8KT_\infty)^{1/2} \zeta^{-1/2} \equiv \tilde{\beta}_\varphi(\zeta), \quad (71)
$$

$$
\tilde{\rho}_r(z) = j_0 z^{-(2-\kappa/2)} \exp \left[ -\frac{\kappa}{2\epsilon} (1 - z^{-\epsilon}) \right] \rightarrow j_\infty \zeta^{-(2-\kappa/2)} \equiv \tilde{\rho}_r(\zeta), \quad (72)
$$

$$
\tilde{\rho}_\varphi(z) = 2j_0 z^{-(2-\kappa/2)} \exp \left[ -\frac{\kappa}{2\epsilon} (1 - z^{-\epsilon}) \right] \rightarrow 2j_\infty \zeta^{-(2-\kappa/2)} \equiv \tilde{\rho}_\varphi(\zeta), \quad (73)
$$

5.2. collapse
where \( T_\infty = \frac{To}{z_{out}}, \rho_\infty = \rho_0/z_{out}^2, \rho_\infty = \rho_0/z_{out}, b_\infty = \frac{b_0}{z_{out}}, j_0 = cB_0/\pi r_0, \) and \( j_\infty = j_0/z_{out}. \) Since it is natural to expect that all the above quantities should decrease with radius \( \zeta, \) we obtain the restriction \( 0 < \kappa < 1. \)

In the asymptotic region of an actual jet of the virial type (i.e., \( \epsilon \neq 0 \)), the continuity equations for mass and magnetic flux yield

\[
\tilde{v}_0(\zeta) = v_\infty \left( 1 + \frac{\kappa}{2} \right) \zeta^{-1} - \zeta^{-1/2},
\]
and

\[
\tilde{\theta}_0(\zeta) = b_\infty \left( 1 + \frac{\kappa}{2} \right) \zeta^{-\kappa/2},
\]
respectively, since \( d\ln \tilde{v}_r/d\zeta = -(\kappa/2)\zeta^{-1} \) and \( d\ln \tilde{\theta}_r/d\zeta = -(1-\kappa/2)\zeta^{-1}. \) The \( \theta \)-component of the current density is obtained, from the relation (13), as

\[
\tilde{\gamma}_0(\zeta) = \frac{\kappa}{2} j_\infty \zeta^{-1} - \zeta^{-\kappa/2}. \quad (76)
\]

Within the jets of virial type, both sound and Alfvén velocities, \( C_S \) and \( V_A, \) vary as \( \zeta^{-1/2}: \)

\[
C_S^2 = \frac{\tilde{p}}{\tilde{\rho}} = KT_0 z^{-1} = KT_\infty \zeta^{-1}, \quad (77)
\]
and

\[
V_A^2 = \frac{\tilde{b}_0^2}{\pi \tilde{\rho}} = \frac{b_\infty^2}{\pi \rho} = 8 KT_0 z^{-1} = 8 KT_\infty \zeta^{-1}. \quad (78)
\]

5.2. consistency

Before the above obtained set of physical quantities are accepted for an actual solution to the full set of MHD equations, some more equations are remaining to be discussed. They are the \( r- \) and \( \varphi \)-components of EOM, and the stationarity condition expressed in terms of the electric field. These equations are checked below only in the asymptotic region.

We begin with the \( r \)-component of EOM. In the asymptotic region, the \( r \)-dependence of the miss-canceled pressure gradient force and the MHD-force are the same, i.e., \(-\beta KT/r \propto r^{-2} \) and \( b_0^2/2\pi \tilde{\rho} \propto r^{-2}. \) This means that the MHD-force cannot dominate over the gravity even in the asymptotic region, and is consistent with the anticipation based on the idealized situation (see the previous section) that the out flow tends to the marginally bound case in the limit of \( \alpha \to 1. \) It turns out that the inertial terms dominate over other forces since \( \tilde{v}_r^2/r \propto r^{-(1+\kappa)} \) and \( 1 < 1 + \kappa < 2. \) Thus, the \( r \)-component of EMO reduces to the vanishing of these terms,

\[
\zeta \frac{d\ln \tilde{v}_r}{d\zeta} + 2A \frac{\tilde{v}_0}{\tilde{v}_r} = 0 \quad (79)
\]
within the accuracy to the leading order in \( r \)-dependences, which yields

\[
A = \frac{\kappa}{2(2+\kappa)}. \quad (80)
\]
For a \( \kappa \) in the range \( 0 < \kappa < 1, \) we have \( 0 < A < 1/6 \) which satisfies the general requirement \( 0 < A < 1/2 \) discussed in section 3.

Next, we check the \( \varphi \)-component (46) of EOM. In the asymptotic region, \( \tilde{\varphi} \) and \( \tilde{m} \) behave like \( \tilde{\varphi} \sim r^{1/2} \) and \( \tilde{m} \sim r^{-\kappa/2}, \) respectively. Therefore, the inertial forces behave like \( \tilde{v}_r(d\tilde{v}_r/d\zeta) - \tilde{v}_0 \tilde{\varphi}_\varphi / \zeta \sim r^{-(1-\kappa)/2}, \) and the MHD forces, like \( \tilde{b}_r/\pi \tilde{\rho} (d\tilde{m}/d\zeta) - \tilde{b}_\varphi \tilde{m} / \pi \tilde{c} \tilde{\varphi} \sim r^{-1}. \) In the leading order, this equation also requires the vanishing of the inertial terms, which reduces to

\[
D = \frac{1}{\kappa + 2}. \quad (81)
\]
For a \( \kappa \) in the range \( 0 < \kappa < 1, \) we have \( 1/3 < D < 1/2 \) which is again consistent with the general requirement \( 0 < D < 1. \) Thus, in this solution, the values of the parameters, \( A \) and \( D, \) have been fixed consistently in terms of \( \kappa. \)

The poloidal components of the electric field are calculated from the corresponding components of Ohm’s law. The \( \theta \)-component is calculated from equation (34) as

\[
\tilde{E}_\theta(\zeta) = \frac{\tilde{\gamma}_0}{c} \tilde{v}_r \tilde{b}_\varphi \left( \frac{\tilde{v}_0}{\tilde{v}_r} + \frac{\tilde{\varphi}_r}{\tilde{v}_r} \tilde{\theta}_r + \frac{\zeta}{\pi \sigma \Theta^2 v_\infty^2} \right) \zeta^{-1}, \quad (82)
\]
where the second and third terms in the parentheses has been neglected since it is proportional to \( r^{-(3-\kappa)/2}. \) From equations (33) and (12) we obtain for the \( r \)-component

\[
\tilde{E}_r(\zeta) = \frac{1}{c} \tilde{\varphi}_r \tilde{b}_\varphi \left( \frac{\tilde{v}_0}{\tilde{v}_r} + \frac{\tilde{\varphi}_r}{\tilde{v}_r} \tilde{\theta}_r + \frac{\zeta}{\pi \sigma \Theta^2 v_\infty^2} \right) \zeta^{-1}, \quad (83)
\]
where the second and third terms in the curly brackets have been neglected because they are proportional to \( r^{-(3-\kappa)/2} \) and \( r^{-(1-\kappa)/2}, \) respectively, while the first term remains finite.

Substituting the above obtained results for \( \tilde{E}_r \) and \( \tilde{E}_\theta \) into equation (31), we have

\[
\frac{db_\varphi}{dt} = -\left( 1 + \frac{\kappa}{2} \right) b_\infty v_\infty \zeta^{-2}, \quad (84)
\]
which means that the strength of the toroidal magnetic field decreases with time, at least in the asymptotic region. This fact seems rather natural, however, because \( b_\infty = b_0/z_{out} \) decreases gradually with increasing \( z_{out} \) as the top of a jet proceeds into the ambient plasma. Even if this is the case, the characteristic time for this change is estimated to be very large: \( t_{mag} \sim |d\ln b_\varphi/|dt|^{-1} \sim r_{out}/v_\infty, \) which gives \( t_{mag} \sim 10^7 \) yr for \( r_{out} \sim 100 \) kpc and \( v_\infty \sim 10^9 \) cm s\(^{-1}\). Therefore, unless we are interested in the evolutional aspects of the jets, such a small change is negligible as usually done in the current jet theories.

5.3. global aspects

In relation to the virial-jet solution, it should be emphasized that the solution can be reconciled naturally with RIAF (i.e., radiatively-inefficient accretion flow)-type accretion disks in the external magnetic field (Kaburaki 2000; Kaburaki 2001; Kaburaki 2007). This type of accretion model has a virial-type high temperature and too tenuous plasma density to be able to radiate efficiently. In contrast to the familiar RIAF models (e.g., Narayan & Yi 1994; Abramowicz et al. 1995; Narayan & Yi 1995), however, this model explicitly takes into account the presence of an ordered magnetic field in the galactic nuclear regions. Therefore, the extraction of angular momentum from the accretion flow is governed by the magnetic stress caused by the twisted magnetic field, instead of the
viscous stress. It has also been clarified that a transport of heat energy (i.e., a non-adiabaticity) through the non-radiating disc causes a wind from the disc surface (Kaburaki 2001; Maruta & Kaburaki 2003), but this wind should be distinguished from jets that emanate from the inner edges of such accretion discs.

Another feature to be mentioned is that the disc has a geometrically thin structure in spite of its high temperature. This is because the disc plasma is vertically compressed toward the equatorial plane by the magnetic pressure due to the toroidal field developed outside (i.e., above and below) the disc. The addition of this component is due to the twisting-up of the seed field by a rotational motion of the accreting plasma, and the degree of which is determined by the balance between the twisting motion and the slip-out (or magnetic diffusion) of the poloidal field under the presence of a finite electrical resistivity. In terms of the circuit theory, the origin of this twist can be attributed to the existence of a poloidally circulating electric current. As already mentioned in section 2, such a current is actually driven by the accretion disc operating as a current generator and should close finally through the polar jet regions.

The $\varphi$-component of the magnetic field developed above and below an RIAF-type accretion disc is given (Kaburaki 2001) by

$$b_\varphi(\xi) = R_D |B_0| \xi^{-(1-n)},$$  (85)

where $\xi \equiv r/r_A$ is the radius normalized by the size of disc’s outer edge, $r_A$. Further, $R_D$ is the magnetic Reynolds number characterizing the accretion disc, and the parameter $n$ specifies the strength of winds, which is a measure of the strength of the heat transport (or the non-adiabaticity) in the non-radiating disc plasma. Comparing the above expression with the asymptotic form $b_\varphi(\xi)$ in (70), we realize that these expressions coincide strictly under the identification of

$$n = \frac{\kappa}{2},$$  (86)

except for the difference in the normalizing factors for the radius. Therefore, the inequality $0 < \kappa < 1$ reduces to $0 < n < 1/2$, which means that there is no downward wind ($n < 0$) as far as an accretion disc is driving a jet. Substituting the expression for $\kappa$, we can specify the location of the footpoint of a jet in terms of the wind parameter as

$$r_j = \frac{GM}{2nKT_0} > \frac{GM}{KT_0}.$$  (87)

Since $r_j \to \infty$ as $n \to 0$, a nearly adiabatic accretion disc ($n \approx 0$) seems to be able to drive only a weak, and therefore undetectable, jet.

The coincidence of the toroidal magnetic fields generated by an accretion disc and by a jet implies that the origin of this field is one and the same current circulating through both disc and jet. Equating the field strength of both solutions at $r = r_A$ (i.e., $\xi = 1$, or $\xi = r_A/r_{out} < 1$), we obtain

$$b_0 = R_D z_{out} \xi_A^{-n} |B_0| = R_D z_A \xi_A^{-n} |B_0|,$$  (88)

which depends on $r_{out}$ unless $n = 0$. This specifies the central field strength $b_0$ in terms of the external field $|B_0|$. From this result, we can confirm that $b_\infty = b_0/z_{out} = R_D \xi_A^{-n} |B_0|$ decreases with increasing $r_{out}$ since $1 - n > 0$.

For one side of a pair of jets, mass ejection rate and integrated kinetic energy flux in the asymptotic region are calculated, respectively, as

$$M_j(\zeta) \equiv \int_0^{\pi/2} (\rho \nu_\rho) 2\pi r^2 \sin \theta d\theta = \frac{4\pi}{27} \Theta^2 r_J^3 \rho_0 v_{\infty}^3 z_{out} \zeta^{1+\kappa/2},$$  (89)

$$F(\zeta) \equiv \int_0^{\pi/2} \left(\frac{1}{2} \rho \nu_\rho^2\right) 2\pi r^2 \sin \theta d\theta \approx \frac{\pi}{20} \Theta^2 r_J^3 \rho_0 v_{\infty}^3 z_{out} \zeta^{1-\kappa/2}. $$  (90)

Although the range of integration on $\theta$ is extended to its full range ($0 \leq \theta \leq \pi/2$), for simplicity, this may be allowed since the matter density falls off rapidly to zero outside the jet cone. These quantities are both increasing functions of $\zeta$, reflecting that the velocity field has a converging component toward the polar axis (i.e., $v_\theta < 0$). Of course, however, these values remain finite with their terminal values at the top of a jet being $M_j(1)$ and $F(1)$.

Three components of the Poynting flux in the asymptotic region are

$$P_r(\zeta, \eta) = \tilde{P}_r(\zeta) \sech^2 \eta \tanh^2 \eta,$$

$$\tilde{P}_r(\zeta) = -\frac{c}{4\pi} \tilde{E}_\rho \tilde{b}_\varphi = \frac{b_\infty^2 v_{\infty}}{4\pi} \zeta^{-(2-\kappa/2)},$$  (91)

$$P_\theta(\zeta, \eta) = -\Theta \tilde{P}_\theta(\zeta) \sech^2 \eta \tanh \eta,$$

$$\tilde{P}_\theta(\zeta) = \frac{c}{4\pi} \tilde{E}_\varphi \tilde{b}_r = \frac{b_\infty^2 v_{\infty}}{4\pi} \left(1 + \frac{\kappa}{2}\right) \zeta^{-(2-\kappa/2)},$$  (92)

$$P_\varphi(\zeta, \eta) = \tilde{P}_\varphi(\zeta) \sech^4 \eta \tanh \eta,$$

$$\tilde{P}_\varphi(\zeta) = -\frac{c}{4\pi} \tilde{E}_\theta \tilde{b}_r = \frac{b_\infty^2 v_{\infty}}{4\pi} \zeta^{-(2-\kappa/2)}. $$  (93)

The radial flow of electromagnetic energy through one side of a jet pair is defined as

$$S(\zeta) \equiv \int_0^{\pi/2} 2\pi r^2 P_r \sin \theta d\theta = \frac{1}{6} \Theta^2 r_J^3 b_\infty^3 v_{\infty}^3 \zeta^{-1-\kappa/2}. $$  (94)

This quantity is again an increasing function of $\zeta$, reflecting the convergence of the magnetic field (i.e., $b_\theta < 0$).

The ratio of the Poynting flux to the kinetic flux,

$$S(\zeta)/F(\zeta) = \frac{10}{3} \frac{8KT_0}{v_{\infty}^2 z_{out}} \zeta^{-(1-\kappa)}, $$  (95)

is a decreasing function of $\zeta$, and its terminal value is $S(1)/F(1) = (80/3)(KT_0/v_{\infty}^2 z_{out})$. If we assume roughly that the dominance of Poynting flux should be kept in the virial jets all the way to their end points, the terminal velocity can be estimated as

$$v_{\infty}^2 = \frac{80}{3} \frac{KT_0}{z_{out}}, $$  (96)

by setting $S(1)/F(1) \sim 1$. In this case, the terminal velocity decreases with the jet length according to $z_{out}^{-1/2}$, which also reflects the marginally trapped nature of the jets of this type.
6. Isothermal-Jet Solution

6.1. Consistency

The isothermal-jet solution is specified by the set of parameters, $\alpha = 0$ and $\beta = 0$. In this case, the relevant quantities take the following forms:

$$\tilde{T}(z) = T_0 = \text{const.},$$

$$\tilde{p}(z) = p_0 \exp \left\{ -\kappa \left( 1 - z^{-1} \right) \right\} = p_\infty \exp \left( \kappa z^{-1} \right),$$

$$\tilde{\rho}(z) = \rho_0 \exp \left\{ -\kappa \left( 1 - z^{-1} \right) \right\} = \rho_\infty \exp \left( \kappa z^{-1} \right),$$

$$\tilde{b}_r(z) = \tilde{b}_\theta(z) = b_0 \exp \left\{ -\frac{\kappa}{2} \left( 1 - z^{-1} \right) \right\} = b_\infty \exp \left( \frac{\kappa}{2} z^{-1} \right),$$

$$\tilde{v}_r(z) = v_0 \exp \left\{ \left( \frac{\kappa}{2} - \frac{\kappa}{2} z^{-1} \right) \right\} = v_\infty \exp \left\{ -\left( \frac{\kappa}{2} + \frac{\kappa}{2} z^{-1} \right) \right\},$$

$$\tilde{\nu}_\varphi(z) = (8KT_0)^{1/2} = \text{const.},$$

where $p_\infty = p_0 \exp(-\kappa)$, $\rho_\infty = \rho_0 \exp(-\kappa)$, $b_\infty = b_0 \exp(-\kappa/2)$ and $v_\infty = v_0 \exp(\kappa/2 + \kappa/2 z^{-1})$.

Since

$$\frac{d \ln \tilde{b}_r}{dz} = \frac{d \ln \tilde{b}_\varphi}{dz} = -\frac{\kappa}{2} z^{-2},$$

$$\frac{d \ln \tilde{v}_r}{dz} = \frac{(\kappa + \kappa)}{2} z^{-2},$$

$$\frac{d \ln \tilde{\nu}_\varphi}{dz} = \left\{ 2 - \left( \frac{\kappa}{2} - \frac{\kappa}{2} z^{-1} \right) \right\} z^{-1},$$

we obtain

$$\tilde{b}_\theta(z) = b_0 \left( 2 - \frac{\kappa}{2} z^{-1} \right) \exp \left\{ -\frac{\kappa}{2} \left( 1 - z^{-1} \right) \right\} = b_\infty \left( 2 - \frac{\kappa}{2} z^{-1} \right) \exp \left( \frac{\kappa}{2} z^{-1} \right)$$

and

$$\tilde{\nu}_\varphi(z) = v_0 \left\{ 2 - \left( \frac{\kappa}{2} - \frac{\kappa}{2} z^{-1} \right) \right\} \exp \left\{ \left( \frac{\kappa}{2} + \frac{\kappa}{2} \right) \left( 1 - z^{-1} \right) \right\} = v_\infty \left\{ 2 - \left( \frac{\kappa}{2} - \frac{\kappa}{2} z^{-1} \right) \right\} \exp \left\{ - \left( \frac{\kappa}{2} + \frac{\kappa}{2} z^{-1} \right) \right\}.$$

Further, for the characteristic velocities, we have

$$C_S^2 \equiv \frac{\tilde{b}_r}{\tilde{\rho}} = KT_0 = \text{const.},$$

$$V_\infty^2 \equiv \frac{\tilde{b}_\theta^2}{\pi \tilde{\rho}} = \frac{\tilde{b}_\theta^2}{\pi \tilde{\rho}} = 8KT_0 = \text{const.}$$

In contrast to the virial-jet solution, the isothermal-jet solution has an essentially exponential structure, and the quantities remain finite even at large distances ($z \to \infty$). As in the case of the viral-jet solution, we have next to check the consistency of the isothermal-jet solution. In the r-component of EOM, the inertial term tends to $Av_\infty^2$ since $\hat{v}_r \to v_\infty$ as $z \to \infty$, while the MHD-force term tends to $-8KT_0 D$. Therefore, in the leading order, the equation reduces to

$$v_\infty^2 A - 8KT_0 D = 0.$$ 

As for the $\varphi$-component of EOM, we find $\tilde{l} \equiv r\tilde{v}_\varphi \propto r$ and $\tilde{m} \equiv \tilde{b}_\theta r \propto r$ in the asymptotic region ($z \gg 1$). Since the inertial forces and MHD forces become of the same order of magnitude, i.e., $(b_\theta/\pi \tilde{\rho})(d\tilde{m}/dz) \sim \tilde{b}_\theta \tilde{m}/\pi \tilde{\rho} \sim z^0$ and $\tilde{v}_\varphi (d\tilde{l}/dz) \sim \tilde{v}_\varphi /z \sim z^0$, equation (46) is held only when

$$\frac{d \ln \tilde{l}}{dz} = \frac{D \tilde{\nu}_\varphi}{\tilde{v}_r} = \frac{d \ln \tilde{m}}{dz} = -\frac{D \tilde{b}_\theta}{\tilde{b}_r} = 0,$$

and both of them are satisfied if

$$D = \frac{1}{2}.$$ (111)

Further, if the approximation

$$D = \left( \frac{f(\eta)}{\tanh \eta} \right) \sim \left( \frac{f(\eta) \tanh \eta}{\tanh^2 \eta} \right) = 2A$$ (112)

is allowed, we can fix as $A = 1/4$ and $D = 1/2$, and the terminal velocity of a jet of this type is specified as

$$v_\infty = 4(KT_0)^{1/2}.$$ (113)

Speaking more generally, it can be concluded from equations (109) and (108) that the flow becomes super-Alfvénic as far as $D > A$.

The poloidal components of the electric field are obtained from Ohm’s law. In the asymptotic region ($z \gg 1$), they are

$$\tilde{E}_\theta = -\frac{\tilde{b}_r \tilde{v}_\varphi}{c} \left( 1 + \frac{\tilde{v}_r}{\tilde{v}_\varphi} \right) = -\frac{b_\infty v_\infty}{c} \left( 1 + \sqrt{\frac{A}{D}} \right)$$ (114)

and

$$\tilde{E}_r(z) = \frac{\tilde{b}_\theta}{c \tilde{b}_r} \tilde{v}_\varphi \tilde{v}_r + \frac{\tilde{b}_\theta}{\tilde{b}_r} \tilde{v}_r + \frac{c^2}{4\pi \sigma} \Theta^2 v_\infty \frac{v_\infty}{\tilde{b}_r} z^{-1} \approx \frac{2}{c} b_\infty v_\infty \left( 1 + \sqrt{\frac{A}{D}} \right),$$ (115)

where the third term in the curly brackets has been neglected.

As in the case of the viral jets, these field components do not satisfy the stationarity condition. The associated change rate of the magnetic field is given by

$$\frac{\partial b_r}{\partial t} = -\frac{2}{5} b_\infty v_\infty \left( 1 + \sqrt{\frac{A}{D}} \right) z^{-1}.$$ (116)

The characteristic time for the changing magnetic field is given by $t_{\text{mag}} \sim r_{\text{out}}/v_\infty$, which is also large as in the case of the viral jets. In both cases, the long time scales can be understood as a consequence of large inductances of the relevant current systems (Benford 2006). In the equations above and in the next subsection, we may substitute the value in (111) for $D$. 

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6.2. global aspects

Since we do not know any analytic solution of the accretion disk in a large-scale magnetic field, which is naturally connected to the isothermal-jet solution, we cannot specify the strength of the magnetic field $b_0$, in relation to the accretion process. The mass and kinetic energy flux through one side of a jet are calculated in the asymptotic region as

$$\dot{M}_z(z) = \frac{4\pi}{27} \Theta^2 r_3^2 \rho \nu_\infty v_\infty z^2, \quad (117)$$

$$F(z) = \frac{\pi}{20} \Theta^2 r_3^2 \nu_\infty^2 v_\infty^2 z^2. \quad (118)$$

These are increasing functions of $z$, since the jet flow is again a converging one. Although these quantities diverge as $z \to \infty$, practically there is no jet that extends to infinity. If we want to know the exact nature of a jet expanding into real vacuum, then we must trace its entire evolution. However, such a problem is beyond the scope of the present paper.

The three components of the Poynting flux in the asymptotic region are

$$\tilde{P}_r(z) = - \frac{c}{4\pi} \tilde{E}_\theta \tilde{b}_r = \frac{b^2_\theta \nu_\infty}{4\pi} \left( 1 + \sqrt{\frac{A}{D}} \right), \quad (119)$$

$$\tilde{P}_\theta(z) = \frac{c}{4\pi} \tilde{E}_r \tilde{b}_\theta = \frac{b^2_\theta \nu_\infty}{2\pi} \left( 1 + \sqrt{\frac{A}{D}} \right), \quad (120)$$

$$\tilde{P}_\varphi(z) = - \frac{c}{4\pi} \tilde{E}_\varphi \tilde{b}_\varphi = \frac{b^2_\theta \nu_\infty}{4\pi} \left( 1 + \sqrt{\frac{A}{D}} \right). \quad (121)$$

Then, the integrated flux becomes

$$S_r(z) = \frac{1}{6} \left( 1 + \sqrt{\frac{A}{D}} \right) \Theta^2 r_3^2 b_0 \nu_\infty v_\infty z^2. \quad (122)$$

This is also an increasing function of $z$, reflecting the convergence of the magnetic field (i.e., $b_\theta < 0$). The ratio of the Poynting flux to the kinetic flux is given by

$$\frac{S(z)}{F(z)} = \frac{10}{3} \left( 1 + \sqrt{\frac{A}{D}} \right) \frac{A}{D}, \quad (123)$$

which is independent of $z$. Thus, the isothermal jets seem to remain always Poynting-flux dominated.

7. Summary and Discussion

We have found two types of jet solutions to the set of MHD equations, which are valid in the region except very close to the footpoint, under the restriction that the jet plasma is confined within a thin sheath structure centered on a conical surface of small opening angle. Commonly to both types of jet, such a density enhancement of jet plasma (assumed to be a normal plasma consisting of electrons and ions) is maintained in the valley of the magnetic pressures that is formed between the region of poloidal-component dominance near the polar axis and that of azimuthal-component dominance outside the cone. In this configuration, further, the centrifugal force due to jet rotation is always balanced by the hoop stress of the toroidal magnetic field. Although we have not discussed explicitly for it, the spine region is very likely to be filled with highly relativistic electron-positron pair plasma as various observations seem to suggest. The terminal velocity of a jet of both types of solution does not exceed by far the thermal velocity at its footpoint, and so remains to be at most mildly relativistic.

The virial-jet solution is characterized by a virial-like temperature profile, and it seems that the jets are marginally bound by the gravity. Namely, although the radial velocity of a jet increases toward a terminal velocity at infinity in the case of free expansion, when the jet terminates at a large but finite distance, as is the case in every actual situations, the velocity decreases according to a power law in the asymptotic region to reach a terminal value. All other quantities also vary according to power laws. This type of solution can be matched quite naturally with the RIAF type solution of accretion discs in a large-scale magnetic field (Kaburaki 2000; Kaburaki 2001; Kaburaki 2007). This fact strongly suggests that the virial jets are driven in the actual situations by such accretion discs. The ratio of the Poynting flux to kinetic flux decreases with the radius, and may decrease to a value smaller than unity.

On the other hand, the isothermal-jet solution is characterized by a temperature independent of the radius, and the jets are accelerated purely magnetohydrodynamically, since the gravity is exactly cancelled out by the pressure gradient force. The radial velocity is gradually accelerated and finally seems to reach a super-Alfvénic value. In this solution, however, the quantities other than the temperature vary as exponentials of inverse radius, and the changes in their values from the footpoint to the top remain rather small. The ratio of Poynting to kinetic flux is a constant in radius and remains always Poynting-flux dominated. Further, there is no accretion disk solution hitherto known to us, which can be matched naturally with this type of jet solution.

The correspondence of the virial jets to FR I jets may be already evident from the above description. The FR I jets are known from observations to have well defined opening angles and decreasing radial velocities at large distances. The latter fact is explained here by their marginally bound nature. The opinion that the matter entrainment is the cause of deceleration (as often suggested in the literature, e.g., Bicknell 1994) is not confirmed in our solutions. Although the virial-jet solution indeed has the flow and magnetic field that converge toward the jet axis, and hence the so-called entrainment of surrounding matter is realized there, such a configuration is also found in our isothermal-jet solution in which the radial velocity does not experience a deceleration.

The host of FR I jets are ellipticals often in rich cluster environments, and their accretion discs are believed to be radiatively inefficient. The smooth matching of the virial-jet solution to the RIAF-type accretion disc solution seems to be a strong support of the idea that a RIAF in a global magnetic field always drive a virial jet. The strength of such a jet is very likely to be controlled by the parameter $\kappa$, which specifies the height of jet’s footpoint. Since this value has been determined as twice of the wind parameter $n$ (defined in Kaburaki 2001), which is a measure of non-adiabaticity in the accreting matter, the strength of a virial jet may also be controlled by the strength of such a radial energy transport taking place in the accretion disc.
Although the connection of the isothermal-jet solution to FR II jets may be not so clear as the connection of the virial-jet solution to FR I jets, it seems nevertheless to be possible. The overall trend that the radial changes of various quantities are rather mild in this solution seems to be favorable to FR II jets. In contrast to the virial-jet solution, the jet velocity in this solution increases gradually until the end point is reached, in spite of the presence of an entrainment (i.e., a convergent flow).

Finally, we refer to a rather speculative possibility of interpreting FR II jets, i.e., the idea that FR II jets are dying systems, in which their central engines have been switched off. Originally, this idea was proposed with a support found in the trend seen in the power-size (P-D) diagram of FR II sources, which seems to suggest an decrease in the jet luminosities with their ages (Baldwin 1982). If this is the case, a jet that has been driven once as a RF I jet by the gas supply owing to some past event of galaxy encounter is now switched off, for the lack of matter supply, and is dying as a FR II jet. Also, we may see recurrences of the jet phenomena in rich clusters, because there may be periods of inefficient matter supply between successive galaxy encounters. In fact, examples of such recurrences can be seen in some examples as the successive bubbles seen in X-rays (for M87, see e.g., Forman et al. 2007).

Other kind of radio sources, such as compact symmetric objects (or CSO, see e.g., Readhead et al. 1996) that resemble FR II sources but are much smaller and hence believed to be young, have also been interpreted as dying systems. The problem of over population that may arise when we assume an evolutionary track from CSOs into FR II sources can thus be avoided. Most of the young sources will die young as CSOs (e.g., Giroletti 2000). Only a small fraction of young systems can evolve as FR I sources to a big size, and then die as well-developed FR II sources when their central engines (or more properly speaking, dynamos) are switched off. Therefore, large FR II sources such as Cyg A seem to be very rare cases.

If we take this interpretation of FR II sources seriously, it becomes likely that the morphology of their jet cavities, which are revealed by recent X-ray observations, are different from those of FR I sources. Jet cavities of RF I sources seem to be typically of hour-glass type that have narrow necks near the equatorial plane. This fact can easily been understood as reflecting the presence of infalling matter associated with their accretion discs (see figure 1a). On the other hand, the shape of the jet cavity of a switched off accretion system is likely to have a spindle shape with no neck in the equatorial plane (see figure 1b). This is understood as follows. The poloidal current flowing through a switched-off jet is maintained by the self-inductance of the system. The current flowing in the jet closes its circuit through the equatorial plasma disc and through the walls of the X-ray cavity in the northern and southern hemispheres. Different from the accretion-driven cases, however, the inductive electric field appears in the equatorial disc having the same sign as the electric current, trying to keep the current constant. Since the magnitude of the electric current cannot increase in spite of this inductive field, the rotational motion of the disc is forced to increase in order to satisfy Ohm’s law, equation (33). The unbalanced centrifugal force caused by this super-Keplerian rotation drives the disc plasma outward, resulting in a secretion disc (an inverse of the accretion disc).

In this respect, it is interesting to note that Cyg A, a typical of FR II sources, seems to have a spindle shaped X-ray cavity (Wilson et al. 2006) although further observations are needed for confirmation. The equatorial rings of hot gasses seen in their X-ray images might be reflecting the existence of outflows. Anyway, if the above quoted interpretation of FR II jets is correct, similar spindle shaped X-ray cavities are expected also for other FR II sources.

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