Steep Decay of GRB X-Ray Flares: The Results of Anisotropic Synchrotron Radiation

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Abstract

When an emitting spherical shell with a constant Lorentz factor turns off emission abruptly at some radii, its high-latitude emission would obey the relation of $\dot{\alpha}$ (the temporal index) = 2 + $\beta$ (the spectral index). However, this relation is violated by the X-ray flares in some gamma-ray bursts (GRBs), whose $\dot{\alpha}$ is much more steeper. We show that the synchrotron radiation should be anisotropic when the angular distribution of accelerated electrons has a preferable orientation, and this anisotropy would naturally lead to a steeper decay for the high-latitude emission if the intrinsic emission is limb-brightened. We use this simple toy model to reproduce the temporal and spectral evolution of X-ray flares. We show that our model can well interpret the steep decay of the X-ray flares in the three GRBs selected as an example. Recent simulations on particle acceleration may support the specific anisotropic distribution of the electrons adopted in our work. Reversely, confirmation of the anisotropy in the radiation would provide meaningful clues to the details of electron acceleration in the emitting region.

Key words: gamma-ray burst: general – radiation mechanisms: non-thermal – relativistic processes

1. Introduction

For relativistic astrophysical phenomena, such as gamma-ray bursts (GRBs), it is known that the relativistic bulk motion would induce two significant effects on the radiation from their emission site when they are being observed. First, due to relativistic boosting, the emission is beamed in the direction of motion. So, for a jet with a certain opening angle (e.g., jets in GRBs or active galactic nuclei), the emission from higher latitudes will have a smaller Doppler factor. Second, due to the curvature of the geometry, photons at higher latitudes will arrive later than those from the line of sight even though they are emitted simultaneously (Waxman 1997; Moderski et al. 2000; Granot 2005; Huang et al. 2007). The combination of these two effects is also called as the “curvature effect” for a relativistic spherical shell. In other words, for a long-lasting, spherical emitting jet, the photons received by the observer at specific observer time actually comes from a distorted ellipsoid, rather than a spherical surface.

If the relativistic spherical shell flashes only sometimes, its temporal and spectral evolution of the light curve will have been predicted by prior research. Assuming the flux spectrum is a power-law form as $F_{\nu} \propto \nu^{-\beta}$ in the co-moving frame of the shell and the bulk Lorentz factor of the shell $\Gamma$ is a constant, then the observed spectral flux would obey $F_{\nu_{\text{obs}}} \propto \nu_{\text{obs}}^{-\beta} \Gamma^{-\dot{\alpha}}$ and one can get the relation $\dot{\alpha} = \beta + 2$ together (e.g., Kumar & Panaitescu 2000; Dermer 2004; Liang et al. 2006; Kumar & Zhang 2015), where $\nu_{\text{obs}}$ is the observed frequency, and $\dot{\alpha}, \beta$ are the temporal index and spectral index, respectively. Hereafter, the superscript prime (') is used to denote the quantities in the co-moving frame and the letters “obs” are used to denote the quantities in the observer frame.

After the GRB trigger, X-ray flares are often observed, thanks to the the X-Ray Telescope (XRT; Burrows et al. 2005) on the Swift satellite (Gehrels et al. 2004). In general, X-ray flares show rapid rise and steep decay structures superposed on the underlying afterglow (Zhang et al. 2006). Several scenarios have been proposed for X-ray flares, including the clumpy accretion of the central engine (Perna et al. 2006), the reconnection-driven explosive event from a post-merger neutron star (Dai et al. 2006), the episodic accretion of the central black hole (Proga & Zhang 2006), or the delayed magnetic dissipation of the outflow (Giannios 2006), etc. (see Kumar & Zhang 2015 for a review). Although the proper model for X-ray flares is still uncertain, some studies suggest that X-ray flares and the gamma-ray prompt emission may share a common origin, which further indicates that X-ray flares come from relativistic jets (e.g., Chincarini et al. 2007; Lazzati & Perna 2007; Maxham & Zhang 2009; Margutti et al. 2010). The X-ray flares may be released either by the dissipation of the magnetic energy (Mészáros & Rees 1997; Zhang & Yan 2011) or the kinetic energy of the jet (e.g., Paczynski 1986; Shemi & Piran 1990; Rees & Mészáros 1992). On the other hand, the decay phase of the X-ray flare should be the consequence of the cease of the energy release at the emitting site. Therefore, X-ray flares may well be a benchmark to test the curvature effect for a relativistic spherical shell.

Indeed, Ulhm & Zhang (2015) have shown that the relation $\dot{\alpha} = \beta + 2$ is invalid in the steep decay phase of some X-ray flares. Furthermore, they pointed out that this invalidation may be evidence that the X-ray flare emission region is undergoing rapid bulk acceleration (Jia et al. 2016; Ulhm & Zhang 2016a). However, except for the bulk acceleration, there is another potential effect—the anisotropy of the radiation\textsuperscript{3}—that can change the standard relation. A main assumption above is that the radiation in the co-moving frame is isotropic, which still remains unconfirmed. If the radiation in the co-moving frame is anisotropic, i.e., the radiation is latitude dependent, then the decay phase would be determined by both the curvature effect and the intrinsic anisotropic characteristics. In fact, the steep decay induced by the anisotropy have been revealed in the afterglow (Beloborodov et al. 2011) and the prompt emission

\textsuperscript{3}In this Letter, by saying the anisotropy of the radiation, we mean that the emissivity of the emitting electrons would have an average anisotropic angular distribution in the co-moving frame, rather than the anisotropic characteristics of the jet’s properties (e.g., Dai & Gou 2001).
One possible origin for the anisotropy is the preferable relative orientation between the direction of accelerated electrons and the magnetic field. Therefore, it is crucial to see whether the anisotropy can interpret the steep decay of the X-ray flares.

In our study, we select X-ray flares in three GRBs as an example, of which the flare structures are clear and the data are of high quality. We show that the steep decay can be well explained by considering the anisotropy in the radiation. Our Letter is organized as follows. In Section 2, we present the analytical derivation for the observed spectral flux in the case of anisotropic synchrotron emission. In Section 3, we develop a simple model to do numerical calculation and show how the anisotropy would reproduce the temporal and spectral evolution of these X-ray flares. The conclusions are summarized in Section 4.

2. Radiation from a Thin Shell

Like GRB’s prompt emission, X-ray flares are expected to be produced when the jet’s magnetic or kinetic energy is released. The emission region is far from the GRB central engine and can be treated as a part (limited opening angle) of an expanding spherical shell. Here, we take the synchrotron radiation as the main emission mechanism in X-ray flares and analytically derive the light curve from the shell. The spectral emissivity of a single electron of Lorentz factor γ at frequency ν′ in the fluid rest frame is given by (Rybicki & Lightman 1979)

$$P'_\gamma = \frac{3q_0^2B'_r\sin\alpha}{m_e c^2} F \left(\frac{\nu'}{\nu'_e \sin \alpha}\right) = P'_0 \sin \alpha F \left(\frac{\nu'}{\nu'_e \sin \alpha}\right),$$

(1)

where $q_0$ is electron charge, $m_e$ is electron mass, $c$ is the speed of light, α is the pitch angle between the direction of the electron’s velocity and the local rest-frame magnetic field $B'_r$, $F$ is the synchrotron spectrum function (Lyubarsky 2009; Bromberg & Tchekhovskoy 2016). Also, the radial expansion of the jet would suppress the longitudinal component of the magnetic fields. So, in our modeling, we assume the magnetic fields $B'_r$ are transverse to the jet direction and they are tangled in the local shock plane. On the other hand, the electron distribution may be anisotropic, i.e., the angular distribution of electron moving directions is assumed to be described by a function $f(\alpha)$, which gives

$$\frac{dN_e}{d\Omega_e} = \frac{N_{tot}}{4\pi} f(\alpha),$$

(2)

where $N_{tot}$ is the total number of electrons and $f(\alpha)$ is normalized by $\int f(\alpha) d\Omega'_e = 4\pi$. By averaging $P'_\gamma$ on random $B'_r$ in the shock plane, we can obtain the effective spectral emissivity per solid angle for a single electron as

$$\frac{dP'_{\gamma}}{d\Omega'} \simeq \frac{A(\theta')}{4\pi} P'_0 F \left(\frac{\nu'}{\nu'_e A(\theta')}\right),$$

(3)

with

$$A(\theta') = \frac{1}{2\pi} \times \int_{0}^{2\pi} (1 - \sin^2 \theta' \cos^2 \phi)^{1/2} f[\arccos(\sin \theta' \cos \phi)] d\phi,$$

(4)

where $\theta'$ is the angle between the direction of emitted photons and the local radial direction and the geometric relation $\cos \alpha = \sin \theta' \cos \phi$ has been used. For approximation, the term $A(\theta')$ emerges in the coefficient and the spectral function $F$ separately in Equation (3).

For a group of electrons that obey a spectrum of $dN_e/d\nu_e$, the total spectral power from them should be

$$\frac{dP'_{\gamma, tot}}{d\Omega'} = \int \frac{dP'_{\nu_e}}{d\Omega'} dN_e d\nu_e \approx \frac{N_{tot}}{4\pi} P'_0 G \left(\frac{\nu'}{\nu'_e A(\theta')}\right).$$

(5)

In the last equality of Equation (5), we introduce the function $G(x)$ to approximate the integral of electron spectrum and to simplify the calculation. In practice, $G(x)$ should have a prior form according to the observations, such as a “Band-function” shape (Band et al. 1993).

2.1. Light Curve

We assume electrons in a spherical shell of radius $r$ instantaneously emit photons in a very short time interval $\delta t$ bursted in the measurement time $\delta t \ll \delta t_{ring}$, $\delta t_{ring}$ is defined below). The shell is expanding with a bulk Lorentz factor $\Gamma$. An observer will first see photons emitted along the line of sight with $\theta = 0$ ($\theta$ denotes the latitude of the region on the shell). If we set the observer time $t_{obs}$ equal to zero when receiving the photons emitted from $\theta = 0$, then photons from a location of $\theta$ will be detected by the observer at observer time

$$t_{obs} = \frac{\Gamma}{c} (1 - \mu)(1 + z),$$

(6)

where $\mu = \cos \theta$, and $z$ is the redshift of the burst.

The number of electrons in the ring of $[\theta, \theta + d\theta]$ is $\delta \mu N_{tot}/2$ ($N_{tot}$ is the total number of electrons of the shell). In the local burst frame, the spectral energy $\delta E_{\nu_e}$ emitted into the solid angle $\delta \Omega$ in $\delta t$ can be related to the quantities in the co-moving frame by

$$\frac{\delta E_{\nu_e}}{\delta t \delta \Omega} = \frac{D^2}{\Gamma} \frac{\delta E'_{\nu_e}}{\delta t' \delta \Omega'},$$

(7)

where $D = \Gamma^{-1}(1 - \beta \mu)^{-1}$ is the Doppler factor. When the observer sees the ring, the corresponding energy $\delta E_{\nu, ring} = \delta E_{\nu_e} \delta \mu/2$ and the corresponding time duration is $\delta t_{ring} = r \delta \mu/c$. The spectral luminosity is thus $\delta L_{\nu} = \delta E_{\nu,ring} / \delta t_{ring}$. For an observer at distance $D_L$, the observed flux is $\delta F_{\nu,obs} = (1+z)D_L \delta E_{\nu,obs}/D^2 \delta \Omega$. 

Geng, Huang, & Dai

THE ASTROPHYSICAL JOURNAL LETTERS, 841:L15 (6pp), 2015 May 20

(Barniol Duran et al. 2016; Beniamini & Granot 2016; Granot 2016). One possible origin for the anisotropy is the preferable relative orientation between the direction of accelerated electrons and the magnetic field. Therefore, it is crucial to see whether the anisotropy can interpret the steep decay of the X-ray flares.
which can be further expressed as (also see Uhm & Zhang 2015)

$$
\delta F_{\text{obs}}^{\text{iso}} = \frac{1 + z}{4 \pi D_L^2} \delta t f
\times A(\theta') N_{\text{rad}} P_d' G((1 + z) D^{-1} A(\theta')^{-1} \nu_{\text{obs}} / \nu') \Gamma (1 - \beta \nu_{\text{obs}})^2 \Gamma (1 - \beta \nu_{\text{obs}})^2
$$

(8)

by combining Equations (5)–(7). Note that Equation (8) is only valid for the shell that flashes once (within $\delta t$), while it is analytically useful since we only focus on the steep decay phase here. When calculating the observed flux from a continually emitting shell, one should integrate the differential flux over the equal-arrival-time surface (Waxman 1997; Granot et al. 1999; Huang et al. 2000), or equally $\delta t$ in Equation (8), which will be done in Section 3.

For $G(x) \propto x^{-\beta}$, we can obtain

$$
\delta F_{\text{obs}}^{\text{iso}} = A(\theta') \gamma_{\text{ch}} \Gamma (1 - \beta \nu_{\text{obs}})^2
$$

where $A(\theta') = A(\theta' (t_{\text{obs}}))$ can be calculated by considering the relation $\cos \theta' = \cos \theta - \beta \cos \theta$ and Equation (6). One can find that the flux is affected by the factor $A(\theta' (t_{\text{obs}}))$, which is further determined by the function $f$.

2.2. Anisotropic Case

In the isotropic case, $A(\theta') = 1$, Equation (9) can recover the relation $\delta = 2 + \beta$. However, for the anisotropic case, the relation will not hold. Here, for instance, we consider the limb-brightened case, i.e., the emission is strong at $\theta' = \pi/2$ and weak near $\theta' = 0$. For photons emitted at $\theta' = \pi/2$, the angle between these photons and the radial direction in the observer frame is $\sim 1/T$. Therefore, most of the emission comes from a ring of angle $\theta = \Gamma^{-1}$ and the peak will be delayed compared with the isotropic source (Barniol Duran et al. 2016).

On the other hand, if we assume $A(\theta') \propto \sin(\theta')^n$, $n > 0$, then, using $\cos \theta' \simeq (\tau - t_{\text{obs}})/(\tau + t_{\text{obs}})$, we have

$$
\delta F_{\text{obs}}^{\text{iso}} = \left[ \frac{4 \pi r_{\text{obs}}^2}{(\tau + t_{\text{obs}})^2} \right]^{1.5 + \beta} \Gamma (1 - \beta \nu_{\text{obs}})^2
$$

(10)

where $\tau = r (1 + z)/(2T^2 c)$, and $\beta \simeq 1 - \frac{1}{2T^2}$ is used. We will have the temporal decay index as

$$
\lambda = -\frac{d \ln F_{\text{obs}}}{d \ln t_{\text{obs}}} \simeq (1 + \beta) n/2 + (2 + \beta)
$$

(11)

where $t_{\text{obs}} \gg \tau$ is used. According to Equation (11), the anisotropy of the radiation will lead to a steep decay.

3. Application to X-Ray Flares

Based on the analysis above, we now propose a simple model and perform numerical calculations to reproduce the temporal and spectral behaviors of the X-ray flares. In order to achieve the steep decay, we choose a function $f$ that corresponds to a limb-brightened case, i.e., $f(\alpha) \propto (a^2 + \sin^2 \alpha)^{-3}$, where $a$ is the characteristic beaming angle of the electron distribution. This expression can be achieved when the electrons are preferentially moving along $\mathbf{B}$ and the resulting expression for $A(\theta')$ is limb-brightened.

To perform the calculations, we need to notice the “timing” of the data of X-ray flares. Looking at the X-ray light curve, one may find one point ($t_{\text{obs}} = T_0$) after which the rising phase of the flare emerges. If we reset the reference time ($t_{\text{obs}} = 0$) at time $T_0$, then the burst-frame time $t$ can be connected to the observer-frame time $t_{\text{obs}}$ by

$$
t_{\text{obs}} = \frac{1}{c} [r_s + c (t - t_s) - r \cos \theta] (1 + z),
$$

(12)

where the initial photons of the flare are emitted at $r_s$ at $t_s$ and $t = t_s + \int_{r_s}^r \frac{dr}{(c/\beta)}$. However, the true value of $T_0$ may be obscured by the prompt emission, and its precision is limited by the timing resolution of the observation. In practice, we use another parameter $\Delta T$ to describe the missed portion as is done in Uhm & Zhang (2015), so that the observer time of the flare is

$$
t_{\text{obs}} = \frac{1}{c} [r_s + c (t - t_s) - r \cos \theta] (1 + z) - \Delta T.
$$

(13)

Moreover, the shape of the spectrum $G(x)$ should be given since we do not focus on detailed radiation mechanism in this paper. The spectrum of prompt emission is usually described as a “Band” function (Band et al. 1993). However, the rapid softening of the spectrum during the decay phase of the X-ray flare indicates that the spectrum may be a power-law with an exponential cutoff (also see Uhm & Zhang 2016a). We thus take $G(x) = x^{-7} e^{-x}$ in the following calculations.

In the co-moving frame, the total number of radiating electrons in the shell is $N_{\text{shell}} = 0$ at the starting radius $r_s$ and is assumed to increase at a rate $R_{\text{inj}}$ before the turn-off radius $r_{\text{off}}$. In our calculations, we model the evolution of the characteristic Lorentz factor $\gamma_{\text{ch}}$ and $R_{\text{inj}}$ as

$$
\gamma_{\text{ch}}(r) = \gamma_{\text{ch}}^0 \left( \frac{r}{r_s} \right)^g,
$$

(14)

$$
R_{\text{inj}}(r) = R_{\text{inj}}^0 \left( \frac{r}{r_s} \right)^n,
$$

(15)

where $\gamma_{\text{ch}}^0$ and $R_{\text{inj}}^0$ are the initial value of $\gamma_{\text{ch}}$ and $R_{\text{inj}}$ at $r_s$. The indices $g$, $n$ describe how the characteristic Lorentz factor and the injection rate evolve with radius $r$, respectively. They are essential to model the rapid rise of the X-ray. In addition, $r_s = 10^{14}$ cm is commonly adopted in all our calculations. We then integrate the flux from a series of rings of which the emitted photons reach the observer at the same time to obtain the light curve of the X-ray flare. In our calculations, the redshifts of selected GRBs are assumed to be $z = 1$ and the standard $\Lambda$CDM universe with $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.27$, and $\Omega_{\Lambda} = 0.73$ is adopted.

Three GRBs are selected as examples, i.e., GRB 090621A, GRB 121108A, and GRB 140108A, of which the X-ray data show significant rapid rise and steep decay. The numerical results are shown in Figure 1, in comparison with the observed light curve and photon index $\Gamma$ ($\Gamma = \beta + 1$). The corresponding parameters in each case are listed in Table 1. Our results from numerical calculations are in good agreement with the observations. Note that the numerical light curves become steeper than the observed ones at the late stage of the decay phase. However, this deviation does not change our main conclusion. The deviation can be understood in two aspects. There is an obvious turning point.
Figure 1. Modeling the X-ray flares in GRB 090621A, GRB 121108A, and GRB 140108A by using the anisotropic radiation scenario. For each GRB, in the upper panel, the original observed light curve at 10 keV (the orange points) is shown. The black points are the “shifting” version (this approach was presented in Uhm & Zhang 2016a to model the data for the first time) of the original data by setting the reference time at $T_0$ (see details in Section 3), and the model-calculated light curve is presented as a blue line. The lower panel is similar to the upper panel, but presents the corresponding XRT band (0.3–10 keV) photon index.
in the evolution curve of the observed photon index at the late stage of the decay phase, which strongly indicates that another flare component (or some intrinsic variabilities) should emerge and dominate at that moment. We are only modeling one component of the X-ray light curve, while the observational data do not come purely from one single component. On the other hand, our model is a toy model with some simplifications, such as Equations (14)–(15), which makes us unable to model additional variabilities shown in the observational data. When we calculate the flux for the other component and add it to the result, the new total flux would fit the data better.

4. Discussion

In this study, we consider the anisotropy of the synchrotron radiation in the high-latitude emission and apply it to the observed X-ray flares in three GRBs, i.e., GRB 090621A, GRB 121108A, and GRB 140108A. The steep decay phase can be well interpreted by our scenario in which the intrinsic radiation is limb-brightened. Also, the entire temporal, spectral behavior is consistent with the function $f(x)$ used in our work. Thus, the identification of the role the anisotropy plays in the steep decay of X-ray flares would give useful clues for the details of particle acceleration in the emitting region.

It has been proposed that the steep decay of the X-ray flares in three GRBs may be evidence of the emission site accelerating. However, in our work, we attribute the steep decay to the anisotropy of the radiation, rather than the acceleration of the jet. Both mechanisms can explain some current observational features. Observations on the polarization of X-ray flares may help to identify the prior model since the preferable relative orientation between the $B'$ field and the electrons moving direction in our model may lead to a polarization degree different from the other model (being prepared). On the other hand, since both mechanisms can coexist naturally in the same frame work (the jet is Poynting-flux dominated), the possibility that they will work together within the sample selected cannot be neglected. In Uhlm & Zhang (2016b), they found that the acceleration of the emission region is needed to interpret spectral lags in the prompt emission, but a shallower acceleration index is required, which suggests that the X-ray flare decay may have a contribution from the anisotropic effect. Beniamini & Kumar (2016) suggest that the material producing the X-ray flare may be confined to a jet that is narrow compared to $1/T$; this provides another possible solution to the steep decay.

An anisotropic minijets model has been invoked to explain the short-timescale variability of the GRB prompt emission (Barniol Duran et al. 2016), in which the radiation is also anisotropic in the co-moving frame. Relevant works show that minijets are essential in defining light curves of the prompt emission (Zhang & Zhang 2014; Deng et al. 2015, 2017). The anisotropic characteristics in the GRB emission mechanism thus seem to be common and need more research.

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Table 1

| Parameters Used in the Modeling of the X-Ray Flares of the Three GRBs |
|-------------------------|------------------|------------------|
| $\alpha$ | GRB 090621A | GRB 121108A | GRB 140108A |
| $\beta$ | $-0.58$ | $-0.35$ | $-0.48$ |
| $\dot{\gamma}$ | 100.0 | 100.0 | 100.0 |
| $\Gamma$ | 30.0 | 23.0 | 22.0 |
| $\dot{\gamma}$ | 2.4 | 0.6 | 1.2 |
| $\dot{f}$ | 0.0 | 1.35 | 0.7 |
| $\eta$ | 0.0 | 0.7 | 0.0 |
| $\Delta T$ (s) | 10.0 | 2.0 | 6.5 |
| $R_{\text{mag}}$ (10^{18} \text{ s}^{-1}) | 3.2 | 3.6 | 22.0 |
| $\dot{\epsilon}_{\text{eff}}$ (10^{14} \text{ cm}) | 10.0 | 3.65 | 3.3 |

Note.

$B_0$ is the strength of the rest-frame magnetic field $B'$. The anisotropic characteristics in the GRB emission mechanism thus seem to be common and need more research.
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