On the mass of the glueballonium

E. Trotti and F. Giacosa

\textsuperscript{a}Institute of Physics, Jan Kochanowski University, ul. Uniwersytecka 7, 25-406, Kielce, Poland.
\textsuperscript{b}Institute for Theoretical Physics, J. W. Goethe University, Max-von-Laue-Str. 1, 60438 Frankfurt, Germany.

Received 30 December 2021; accepted 20 January 2022

According to lattice simulations and other theoretical approaches, the scalar glueball is the lightest state in the Yang-Mills sector of QCD. Since within this sector the scalar glueball is stable, the scattering between two glueballs is a well-defined process. Moreover, a glueball-glueball bound state, called glueballonium, might exist if the attraction turns out to be large enough. In this work, we concentrate on the formation of the glueballonium in the context of the dilaton potential. In particular, we investigate the parameter values for which such a glueballonium emerges.

Keywords: Hadrons; glueballs; bound state.

DOI: https://doi.org/10.31349/SuplRevMexFis.3.0308014

1. Introduction

In the Yang-Mills (YM) sector of Quantum Chromodynamics (QCD), glueballs, i.e. bound states of gluons, have been predicted by a variety of approaches, which range from bag models \cite{1, 2, 3} to lattice QCD \cite{4, 5, 6}. According to lattice simulations and other theoretical approaches, the scalar glueball is the lightest state in the Yang-Mills sector of QCD. Thus, the scattering between two scalar glueballs is a well-defined process that can be investigated in models and on the lattice \cite{7, 8, 9}. Here, following the discussion of Ref. \cite{10}, we study the scattering of two glueballs by using the well-known dilaton potential \cite{11, 12}. Moreover, if the attraction is strong enough, a glueball-glueball bound state, referred to as the glueballonium, may form. In these proceedings, we study under which conditions the dilaton potential allows for the emergence of the glueballonium. To this end, one needs to introduce a unitarization of the scattering amplitude: a rather simple and often used unitarization within the so-called on-shell approximation is applied.

Quite interestingly, such a glueballonium could be searched for on lattice YM and, eventually, in experiments. For the latter, it is important to remind that in full QCD the scalar glueball (as any other glueball) is not stable and is therefore possible only if it turns to be narrow enough.

2. Tree-level scattering

A quite famous low-energy theory of the YM sector of QCD is given by the dilaton Lagrangian, which contains a single scalar dilaton/glueball field $G$ \cite{11, 12, 18}:

$$\mathcal{L}_{\text{dil}} = \frac{1}{2} (\partial_{\mu} G)^2 - V(G),$$  \hspace{1cm} (1)

with

$$V(G) = \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left( G^4 \ln \left| \frac{G}{\Lambda_G} \right| - \frac{G^4}{4} \right).$$ \hspace{1cm} (2)

The dilaton potential contains one dimensional parameter, the scale $\Lambda_G$, which embodies the trace anomaly at the composite confined level, see e.g. \cite{18}. Moreover, the dilaton potential contains the dimensionless quantity $m_G/\Lambda_G$. The quantity $m_G$ corresponds to the glueball mass (second derivative at the minimum $G = \Lambda_G$); its numerical value reads $m_G \approx 1.7 \text{ GeV}$ obtained in lattice QCD \cite{4, 5, 6}. The scale $\Lambda_G$ can be obtained by a comparison with the gluon condensate and takes the (approximate) value $\Lambda_G \approx 0.4 \text{ GeV}$ \cite{19}, but this determination is quite uncertain, see e.g. Ref. \cite{13}. Indeed, one of the goals of the glueball-glueball scattering would be an independent determination of this important quantity that affects the whole low-energy phenomenology \cite{13, 20, 21, 22}.

The total tree-level amplitude for the scattering $G(p_1)G(p_2) \rightarrow G(p_3)G(p_4)$ can be obtained from the 3- and 4-leg vertices obtained in the expansion of the potential around its minimum. As function of the Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_2 - p_3)^2$ the amplitude reads:

$$A(s, t, u) = -\frac{11}{4} \frac{m_G^2}{\Lambda_G^2} - \left( \frac{m_G^2}{\Lambda_G^2} \right)^2 \frac{1}{s - m_G^2} - \left( \frac{m_G^2}{\Lambda_G^2} \right)^2 \frac{1}{t - m_G^2} \times \frac{1}{u - m_G^2}.$$ \hspace{1cm} (3)

Since $s + t + u = 4m_G^2$, only two of them are independent. Moreover, upon introducing the scattering angle $\theta$ and expanding in partial waves, we rewrite Eq. \eqref{eq:amplitude} as:

$$A(s, \cos \theta) = \sum_{\ell=0}^{\infty} (2\ell + 1) A_{\ell}(s) P_{\ell}(\cos \theta),$$ \hspace{1cm} (4)

where $P_{\ell}(\cos \theta)$ is the $\ell$-th Legendre polynomial. Conversely, the $\ell$-th amplitude reads:

$$A_{\ell}(s) = \frac{1}{2} \int_{-1}^{1} d \cos \theta A(s, \cos \theta) P_{\ell}(\cos \theta).$$ \hspace{1cm} (5)
Odd waves vanish because $A(s, \cos \theta)$ is symmetric in $\cos \theta$ for Bose symmetry. In these proceedings, we shall concentrate only on the S-wave ($l = 0$), which is the channel in which the glueballonium eventually forms (for the D-wave and G-wave, see Ref. [10]). The corresponding S-wave amplitude is given by:

$$A_0(s) = -\frac{11}{2} \frac{m_G^2}{\Lambda_G^2} - \frac{25}{4} \frac{m_G^4}{\Lambda_G^4} \left(\frac{1}{s_1} - \frac{\log \left(\frac{s_3}{s_2}\right)}{s_4}\right),$$

where $s_N = s - Nm_G^2$.

The tree-level scattering length is calculated as:

$$a_0 = \frac{A_0(4m_G^2)}{32\pi m_G} = \frac{1}{32\pi m_G} \frac{92m_G^2}{3\Lambda_G^2},$$

which is inversely proportional to the energy scale $\Lambda_G^2$. A future determination of the scattering length determined on the lattice would be very useful: since $m_G$ is known, one could use that determination in order to extract $\Lambda_G$. Yet, care is needed with our derivation: a unitarization is necessary since the tree-level result is not sufficient. In particular, it cannot access the eventual existence of a bound state.

The unitarization process that we introduce below is constructed in such a way to leave unchanged the singularities of the tree-level amplitude at $s = m_G^2$ and $s = 3m_G^2$. Namely, the pole in $m_G^2$ corresponds to the single glueball state, so it is reasonable to require that it is left unchanged. Moreover, the $t$- and $u$-channels projected onto the S-wave give rise to the left hand cut and, consequently, to a logarithmic singularity that we shall require to not be modified by the employed unitarization process.

3. Unitarization and mass of the glueballonium

Loop contributions have to be taken into account in order to produce poles representing bound states. The inclusion of these quantum fluctuations can be done through a proper unitarization of the tree-level partial wave amplitude. Among the various schemes proposed for chiral Lagrangians [23–29], the one we choose here is the so-called on-shell approximation [30, 31] leading to:

$$A_U^0(s) = \left[A_0^{-1}(s) - \Sigma(s)\right]^{-1},$$

where $\Sigma(s)$ is the glueball-glueball self-energy loop function. Note, $A_0 \simeq A_U^0$ whenever $A_0$ is small, but sizable loop contributions may occur in general. The imaginary part of $\Sigma(s)$ is fixed by the relativistic 2-body phase space of two identical particles:

$$\text{Im}\Sigma(s) = \frac{1}{2} \frac{1}{16\pi} \sqrt{1 - \frac{4m_G^2}{s}}.$$

The imaginary part of the loop can be used to reconstruct the real part using the dispersion relation by imposing certain constraints. The previously discussed requirements of pre-serving both the pole at $s = m_G^2$ and the logarithmic divergence at $s = 3m_G^2$ can be expressed by requiring that $\Sigma(s = m_G^2) = \Sigma(s = 3m_G^2) = 0$. Including these subtractions, the loop function takes the form:

$$\Sigma(s) = \int_{4m_G^2}^{\infty} \frac{(s-m_G^2)(s-3m_G^2)}{\pi(s-s-i\epsilon)(s'-m_G^2)(s'-3m_G^2)} \text{Im}\Sigma(s') ds'.$$

Interestingly, at threshold the loop takes the value $\Sigma(s = 4m_G^2) = 0.028715$ (independent on $m_G$). As a consequence, the S-wave unitarized scattering length, called $a_U^0$, takes the form:

$$a_U^0 = \frac{1}{32\pi m_G} A_U^0(s) = \frac{1}{32\pi m_G} \frac{3\Lambda_G^2}{92m_G} = 0.028715.$$ 

One can easily observe that for:

$$\frac{3\Lambda_G^2}{92m_G} = 0.0028715 = 0,$$

the scattering length diverges. Numerically, the critical value for $\Lambda_G$ reads:

$$\Lambda_{G,\text{crit}} = m_G \sqrt{\frac{92}{3}} \cdot 0.0028715 = 0.2967m_G.$$ 

For $m_G = 1.7$ GeV, the critical value reads $\Lambda_G = 0.504$ GeV. In Fig. 1 the critical value $\Lambda_G$ is plotted as function of $m_G$. The divergence of the scattering length signals the emergence of a glueball-glueball bound state. Such a glueballonium forms whenever the attraction is strong enough, that is when $\Lambda_G < \Lambda_{G,\text{crit}}$.

In general, the mass of the glueballonium can be found as a solution of the equation:

$$(A_U^0)^{-1} - A_0^{-1}(s) - \Sigma(s) = 0.$$ 

We refer to Fig. 2, where the unitarized inverse amplitude is plotted as function of $s$ for three values of $\Lambda_G$. A zero of this
The mass of the glueballonium ranges between 1.5 and 1.7 GeV, respectively. Note, the glueballonium mass decreases together with \( \Lambda_G \) (thus for increasing attraction). When using the specific value \( \Lambda_G \approx 0.4 \) GeV obtained in lattice YM, the mass of the glueballonium reads \( m_B \approx 3.37 \) GeV (for \( m_G = 1.7 \) GeV).

4. Conclusions

In this paper, we used the dilaton potential to study the scattering process between two scalar glueballs. We calculated the scattering length, first at tree level and then in the unitarized version of the theory: this quantity can be evaluated in future Lattice QCD. In this way, the scale \( \Lambda_G \), which is important in many effective model of QCD at low energy, could be also fixed by lattice results.

We find that the emergence of a glueball-glueball bound state, that we called glueballonium, is possible. Such a glueballonium is formed when the value of the scale \( \Lambda_G \) is below a certain critical value: in the case of \( m_G = 1.7 \) GeV, the critical value of \( \Lambda_G \) is around 0.504 GeV. The existence of the glueballonium could be also investigated on the Lattice. An eventual experimental search is also conceivable, if this state turns out to be not too broad. For instance, the planned PANDA experiment [32] could search for the existence of this bound state, as it covers an energy range that includes the expected mass of the glueballonium (at about 3 GeV). Several ongoing experiments are investigating glueballs [33–38]. As a recent example, the TOTEM and the DØ collaborations together recently announced the discovery the odderon [39], which in turn implies that C-odd-parity glueballs exist. On the other hand, the pomeron implies that C-even parity glueballs (such as the scalar one) exist, see e.g. Refs. [40–43] and Refs. therein.

Finally, it would be interesting to repeat our work using different unitarization schemes in order to check the dependence of the unitarization on the results. Moreover, one may study the scattering of other glueballs as well and extend the study at nonzero temperature following the procedure of Refs. [44, 45].

Acknowledgments

The authors thank A. Pilloni with whom Ref. [10] was written. E.T. acknowledges financial support through the project AKCELERATOR ROZWOJU Uniwersytetu Jana Kochanowskiego w Kielcach (Development Accelerator of the Jan Kochanowski University of Kielce), co-financed by the European Union under the European Social Fund, with no. POW.03.05.00-00-Z212/18. F.G. acknowledges financial support from the Polish National Science Centre NCN through the OPUS projects no. 2019/33/B/ST2/00613.

Supl. Rev. Mex. Fis. 3 0308014
1. A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, A New Extended Model of Hadrons, Phys. Rev. D 9 (1974) 3471. [https://10.1013/PhysRevD.9.3471]

2. R. L. Jaffe and K. Johnson, Unconventional States of Confined Quarks and Gluons, Phys. Lett. B 60 (1976) 201. [https://10.1016/0370-2693(76)90035-7]

3. R. L. Jaffe, K. Johnson and Z. Ryzak, Qualitative Features of the Glueball Spectrum, Annals Phys. 168 (1986) 344. [https://10.1016/0003-4916(86)90357-7]

4. C. J. Morningstar and M. J. Peardon, The Glueball spectrum from an anisotropic lattice study, Phys. Rev. D 60 (1999) 034509. [https://10.1103/PhysRevD.60.034509] [arXiv:hep-lat9901004 [hep-lat]].

5. Y. Chen et al., Glueball spectrum and matrix elements on anisotropic lattices, Phys. Rev. D 73 (2006) 014516. [https://10.1103/PhysRevD.73.014516] [arXiv:hep-lat0510074 [hep-lat]].

6. M. Caselle, M. Hasenbusch, P. Provero and K. Zarembo, Bound states and glueballs in three-dimensional Ising systems, Nucl. Phys. B 623 (2002) 474. [https://10.1016/S0550-3213(01)00644-7] [arXiv:hep-th/0103130 [hep-th]].

7. N. Yamanaka, H. Iida, A. Nakamura and M. Wakayama, Dark matter scattering cross section and dynamics in dark Yang-Mills theory, Phys. Lett. B 813 (2021) 136056. [https://10.1016/j.physletb.2020.136056] [arXiv:1910.01440 [hep-ph]].

8. N. Yamanaka, H. Iida, A. Nakamura and M. Wakayama, Glueball scattering cross section in lattice SU(2) Yang-Mills theory, Phys. Rev. D 102 (2020) 054507. [https://10.1103/PhysRevD.102.054507] [arXiv:1910.07756 [hep-lat]].

9. N. Yamanaka, A. Nakamura and M. Wakayama, Interglueball potential in lattice SU(N) gauge theories, [arXiv:2110.04521 [hep-lat]].

10. F. Giacosa, A. Pilloni and E. Trotti, “Glueball-glueball scattering and the glueballonium”, [arXiv:2110.05582 [hep-ph]].

11. A. A. Migdal and M. A. Shifman, Dilaton Effective Lagrangian in Gluodynamics, Phys. Lett. B 114 (1982) 445. [https://10.1016/0370-2693(82)90089-2]

12. A. Salomone, J. Schechter and T. Tudron, Properties of Scalar Gluonium, Phys. Rev. D 23 (1981) 1143. [https://10.1103/PhysRevD.23.1143]

13. S. Janowski, F. Giacosa and D. H. Rischke, Is f0(1710) a glueball?, Phys. Rev. D 90 (2014) 114005. [https://10.1103/PhysRevD.90.114005] [arXiv:1408.4921 [hep-ph]].

14. F. E. Close and A. Kirk, Scalar glueball q anti-q mixing above 1-GeV and implications for lattice QCD, Eur. Phys. J. C 21 (2001) 531. [https://10.1007/s1005200100748] [arXiv:hep-ph/0103173 [hep-ph]].

15. F. Brünner, D. Parganlija and A. Rebhan, Glueball Decay Rates in the Witten-Sakai-Sugimoto Model, Phys. Rev. D 91 (2015) 106002, [erratum: Phys. Rev. D 93 (2016) 109903]. [https://10.1103/PhysRevD.91.106002] [arXiv:1501.07906 [hep-ph]].

16. A. Rodas et al. [JPAC], Scalar and tensor resonances in J/ψ radiative decays, [arXiv:2110.00027 [hep-ph]].

17. F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Scalar meson and glueball decays within a effective chiral approach, Phys. Lett. B 622 (2005), 277. [https://10.1016/j.physletb.2005.07.016] [arXiv:hep-ph/0504033 [hep-ph]].

18. J. R. Ellis and J. Lanik, IS SCALAR GLUONIUM OBSERVABLE?, Phys. Lett. B 150 (1985) 289. [https://10.1016/0370-2693(85)91013-5]

19. A. Di Giacomo, H. G. Dosch, V. I. Shevchenko and Y. A. Simonyov, Field correlators in QCD: Theory and applications, Phys. Rept. 372 (2002) 319. [https://10.1016/S0370-1573(02)00140-0] [arXiv:hep-ph/0007223 [hep-ph]].

20. D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Meson vacuum phenomenology in a three-flavor linear sigma model with (axial-)vector mesons, Phys. Rev. D 87 (2013) 014011. [https://10.1103/PhysRevD.87.014011] [arXiv:1208.0855 [hep-ph]].

21. G. W. Carter, P. J. Ellis and S. Rudaz, An Effective Lagrangian with broken scale and chiral symmetry: 2. Pion phenomenology, Nucl. Phys. A 603 (1996) 367, [erratum: Nucl. Phys. A 608 (1996) 514]. [https://10.1016/0375-9474(96)80007-8] [arXiv:nucl-th/9512033 [nucl-th]].

22. D. Parganlija, F. Giacosa and D. H. Rischke, Vacuum Properties of Mesons in a Linear Sigma Model with Vector Mesons and Global Chiral Invariance, Phys. Rev. D 82 (2010) 054024. [https://10.1103/PhysRevD.82.054024] [arXiv:1003.4934 [hep-ph]].

23. T. N. Truong, Remarks on the unitarization methods, Phys. Rev. Lett. 67 (1991) 2260. [https://10.1103/PhysRevLett.67.2260]

24. A. Gomez Nicola and J. R. Pelaez, Meson meson scattering within one loop chiral perturbation theory and its unitarization, Phys. Rev. D 65 (2002) 054009. [https://10.1103/PhysRevD.65.054009] [arXiv:hep-ph/0109056 [hep-ph]].

25. O. V. Selyugin, J. R. Cudell and E. Predazzi, Analytic properties of different unitarization schemes, Eur. Phys. J. ST 162 (2008) 37. [https://10.1140/epjst/e2008-00773-0] [arXiv:0712.0621 [hep-ph]].

26. J. R. Cudell, E. Predazzi and O. V. Selyugin, New analytic unitarisation schemes, Phys. Rev. D 79 (2009), 034033. [https://10.1103/PhysRevD.79.034033] [arXiv:0812.0735 [hep-ph]].

27. J. Nebreda, J. R. Pelaez and G. Rios, Chiral extrapolation of pion-pion scattering phase shifts within standard and unitarized Chiral Perturbation Theory, Phys. Rev. D 83 (2011) 094011. [https://10.1103/PhysRevD.83.094011] [arXiv:1101.2171 [hep-ph]].

28. R. L. Delgado, A. Dobado and F. J. Llanes-Estrada, Unitarity, analyticity, dispersion relations, and resonances in strongly interacting W_3,W_L,Z_2L, and hh scattering, Phys. Rev. D 91 (2015) 075017. [https://10.1103/PhysRevD.91.075017] [arXiv:1502.04841 [hep-ph]].

29. J. A. Oller, Unitarization Technics in Hadron Physics with Historical Remarks, Symmetry 12 (2020) 1114. [https://10.3390/sym12071114] [arXiv:2005.14417 [hep-ph]].

Supl. Rev. Mex. Fis. 3 0308014
30. A. Dobado and J. R. Pelaez, A Global fit of pi pi and pi K elastic scattering in ChPT with dispersion relations, *Phys. Rev. D* **47** (1993) 4883, [https://10.1103/PhysRevD.47.4883](https://10.1103/PhysRevD.47.4883) [arXiv:hep-ph/9301276 [hep-ph]].

31. J. A. Oller, E. Oset and J. R. Pelaez, Nonperturbative approach to effective chiral Lagrangians and meson interactions, *Phys. Rev. Lett.* **80** (1998) 3452, [https://10.1103/PhysRevLett.80.3452](https://10.1103/PhysRevLett.80.3452) [arXiv:hep-ph/9803242 [hep-ph]].

32. M. F. M. Lutz et al., Physics Performance Report for PANDA: Strong Interaction Studies with Antiprotons, [arXiv:0903.3905 [hep-ex]].

33. A. Hamdi, Search for exotic states in photoproduction at GlueX, *J. Phys. Conf. Ser.* **1667** (2020) 012012, [https://10.1088/1742-6596/1667/1/012012](https://10.1088/1742-6596/1667/1/012012) [arXiv:1908.11786 [nucl-ex]].

34. T. Gutsche, S. Kuleshov, V. E. Lyubovitskij and I. T. Obukhovsky, Search for the glueball content of hadrons in γp interactions at GlueX, *Phys. Rev. D* **94** (2016) 034010, [https://10.1103/PhysRevD.94.034010](https://10.1103/PhysRevD.94.034010) [arXiv:1605.01035 [hep-ph]].

35. D. Ryabchikov, Meson spectroscopy at VES and COMPASS, *EPJ Web Conf.* **212** (2019) 03010, [https://10.1051/epjconf/201921203010](https://10.1051/epjconf/201921203010).

36. S. Marcello [BESIII], Hadron Physics from BESIII, *JPS Conf. Proc.* **10** (2016) 010009, [https://10.7566/JPSCP.10.010009](https://10.7566/JPSCP.10.010009).

37. R. Aaij et al., Physics case for an LHCb Upgrade II - Opportunities in flavour physics, and beyond, in the HL-LHC era, [arXiv:1808.08865 [hep-ex]].

38. E. Kou et al., The Belle II Physics Book, *PTEP 2019* (2019) 123C01, [erratum: PTEP 2020 (2020) 029201] [https://10.1093/ptep/ptz106](https://10.1093/ptep/ptz106) [arXiv:1808.10567 [hep-ex]].

39. V. M. Abazov et al., Odderon Exchange from Elastic Scattering Differences between pp and p\bar{p} Data at 1.96 TeV and from pp Forward Scattering Measurements, *Phys. Rev. Lett.* **127** (2021) 062003, [https://10.1103/PhysRevLett.127.062003](https://10.1103/PhysRevLett.127.062003) [arXiv:2012.03981 [hep-ex]].

40. T. Csörgő, T. Novak, R. Pasechnik, A. Ster and I. Szanyi, Evidence of Odderon-exchange from scaling properties of elastic scattering at TeV energies, *Eur. Phys. J. C* **81** (2021) 180, [https://10.1140/epjc/s10052-021-08867-6](https://10.1140/epjc/s10052-021-08867-6) [arXiv:1912.11968 [hep-ph]].

41. M. Broilo, D. A. Fagundes, E. G. S. Luna and M. Peláez, Soft Pomeron in light of the LHC correlated data, *Phys. Rev. D* **103** (2021) 014019, [https://10.1103/PhysRevD.103.014019](https://10.1103/PhysRevD.103.014019) [arXiv:2012.08664 [hep-ph]].

42. A. Kirk and O. Villalobos Baillie, A Study of double pomeron exchange in ALICE, [arXiv:hep-ph/9811230 [hep-ph]].

43. M. Albrow, Hadron Spectroscopy in Double Pomeron Exchange Experiments, *AIP Conf. Proc.* **1819** (2017) no.1, 040008 [https://10.1063/1.4977138](https://10.1063/1.4977138) [arXiv:1701.09092 [hep-ex]].

44. S. Samanta and F. Giacosa, QFT treatment of a bound state in a thermal gas, *Phys. Rev. D* **102** (2020) 116023 [https://10.1103/PhysRevD.102.116023](https://10.1103/PhysRevD.102.116023) [arXiv:2009.13547 [hep-ph]].

45. S. Samanta and F. Giacosa, Thermal role of bound states and resonances in scalar QFT, [arXiv:2110.14752 [hep-ph]].