Bound on distributed entanglement

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Abstract

Using the convex-roof extended negativity and the negativity of assistance as quantifications of bipartite entanglement, we consider the possible remotely distributed entanglement. For two pure states $|\psi\rangle_{CD}$ and $|\phi\rangle_{AB}$ on bipartite systems $AB$ and $CD$, we first show that the possible amount of entanglement remotely distributed on the system $AC$ by joint measurement on the system $BD$ is not less than the product of two amounts of entanglement for the states $|\psi\rangle_{CD}$ and $|\phi\rangle_{AB}$ in two-qubit and two-qutrit systems. We also provide some sufficient conditions, for which the result can be generalized into higher-dimensional quantum systems.

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1. Introduction

Quantum entanglement plays a crucial role in various kinds of quantum information tasks such as quantum teleportation, dense coding and quantum cryptography [1–3]. Whereas shared entanglement between different parties is generally consumed as a resource in the tasks of quantum informational processing, generating entangled states is usually expensive in practice. Furthermore, due to the fragile nature of quantum entanglement, entanglement in quantum states (and thus its non-classical ability for quantum information tasks) may be gradually destroyed under the interaction with the environment, which is known as the decoherence. On this account, it must be an important and necessary task to provide an efficient way to create or distribute entanglement between desired parties, and hence it is surely meaningful to quantify the possible amount of entanglement distributed under given restricted conditions.

As a generalization of the entanglement swapping [4–6], the distribution of entanglement through quantum networks was characterized in qubit systems by using the concurrence [7] as a measure of bipartite entanglement. If we have two states $\rho_{AB}$ and $\rho_{CD}$ in two-qubit systems $AB$ and $CD$, respectively, the possible entanglement, which can be remotely distributed on
the system $AC$ by arbitrary measurement on the system $BD$ and classical communication, was shown to be always bounded above by the product of two amounts of entanglement for $\rho_{AB}$ and $\rho_{CD}$ [8]. Later, this bound was generalized to arbitrary qudit systems by using the G-concurrence [9] as another bipartite entanglement measure.

However, for a bipartite pure state in a $d \otimes d$ quantum system, the G-concurrence is defined as the $d$th root of the determinant of its reduced density matrix with a proper normalization. Although the G-concurrence has several good properties such as computability and multiplicativity [9], it can only detect genuine $d$-dimensional entanglement (that is, a bipartite pure state has a non-zero value of G-concurrence if and only if its reduced density matrix has full rank), and thus it has zero value for a lot of entangled states in two-qudit systems whose reduced density matrices are not of full rank. In other words, the G-concurrence cannot even give us a separability criterion, which is one of the requirements necessary for entanglement measures.

Even though we have an entangled state shared between desired parties by entanglement distribution schemes, it is also necessary to quantify the amount of distributed entanglement in a proper way otherwise it is still ambiguous if it contains enough entanglement for the quantum information tasks of our demands. Thus, the quantification of distributed entanglement must be performed in accordance with the criteria for good entanglement measures, namely separability and entanglement monotone. Furthermore, most quantum information tasks use maximally entangled states as their non-local quantum resource, and thus it would be practically important and efficient if the entanglement measure used to quantify the distributed entanglement can also quantify the possible distillation of maximal entanglement.

A well-known quantification of bipartite entanglement, which is strongly related with maximal entanglement distillation is the negativity [10], and it is based on the positive partial transposition (PPT) criterion [11, 12]. Besides its separability criterion in two-qubit systems, PPT is known as a sufficient condition for nondistillability of quantum states [13, 14]. In other words, if a quantum state has a vanishing negativity, it cannot be converted into a maximally entangled state even in asymptotic sense. In $2 \otimes d$ quantum systems, PPT also becomes a necessary condition for nondistillability. However, in higher-dimensional quantum systems rather than $2 \otimes 2$ and $2 \otimes 3$ systems, there exist mixed entangled states with PPT. These quantum states are known as bound entangled states [13, 15], and thus their negativity values are not positive even though they are entangled.

In order to overcome the lack of separability criterion for mixed states, the negativity can be modified by means of the convex-roof extension, which takes the minimal average of negativity values over all possible pure-state decompositions. This modified negativity for mixed states is called the convex-roof extended negativity (CREN) [16], and it gives a perfect discrimination between PPT bound entangled states and separable states in any bipartite quantum system. Moreover, it was also shown that CREN does not increase under local operations and classical communication (LOCC) [16]. In other words, CREN is a good entanglement measure satisfying separability criterion and entanglement monotone in any bipartite quantum systems.

Here, we provide a new bound for remotely distributed entanglement (RDE) using CREN and its dual quantity, the negativity of assistance (NoA) [17]. Given a pair of pure states $|\phi\rangle_{AB}$ and $|\psi\rangle_{CD}$ in two bipartite systems $AB$ and $CD$, respectively, we first show that the possible RDE on the system $AC$ by joint measurement on the system $BD$ is not less than the product of two amounts of CREN for $|\phi\rangle_{AB}$ and $|\psi\rangle_{CD}$ in two-qubit and two-qutrit systems. For $d \otimes d$ quantum systems ($d \geq 4$), we provide some sufficient conditions of $|\phi\rangle_{AB}$ and $|\psi\rangle_{CD}$ for which the result in lower-dimensional systems can be generalized.
This paper is organized as follows. In section 2.1, we recall the definition of negativity, CREN and NoA. In section 2.2, we derive an analytic lower bound of possible RDE for arbitrary dimensional quantum systems. In section 3.1, we establish the inequality for the lower bound of the possible RDE in low-dimensional quantum systems with respect to CREN and NoA. In section 3.2, we provide some sufficient conditions, for which the result in low-dimensional systems can be generalized into arbitrary dimensional systems, and we summarize our results in section 4.

2. Convex-roof extended negativity and negativity of assistance

In this section, we recall the definitions of CREN and NoA for bipartite quantum states. We also provide an analytic lower bound of the distributed entanglement of $\rho_{AC}$, which can be remotely prepared from two bipartite states $|\phi\rangle_{AB}$ and $|\psi\rangle_{CD}$ by a joint measurement in the system $BD$.

2.1. Definitions

For a bipartite state $\rho_{AB}$ in a $d_A \otimes d_B$ quantum system, its negativity $N(\rho_{AB})$ is defined as

$$N(\rho_{AB}) = \frac{\|\rho_{TB}^A\|_1 - 1}{d - 1},$$

where $\|\cdot\|_1$ is the trace norm, $d = \min\{d_A, d_B\}$ and $\rho_{TB}^A$ is the partial transposition of $\rho_{AB}$ [10]. For the case when $\rho_{AB}$ is a pure state $|\psi\rangle_{AB}$ with the following Schmidt decomposition:

$$|\psi\rangle_{AB} = \sum_{i=0}^{d-1} \sqrt{\lambda_i} |ii\rangle, \quad \lambda_i \geq 0, \quad \sum_{i=0}^{d-1} \lambda_i = 1,$$

(without loss of generality, we may assume that the Schmidt basis is taken to be the standard basis) its negativity can also be expressed in terms of its Schmidt coefficients, that is,

$$N(|\psi\rangle_{AB}) = \frac{2}{d-1} \sum_{i<j} \sqrt{\lambda_i \lambda_j}.$$

In order to compensate for the lack of separability criterion in the negativity, CREN for a mixed state $\rho_{AB}$ is defined as

$$N_c(\rho_{AB}) = \min \sum_k p_k N(|\psi_k\rangle_{AB}),$$

where the minimum is taken over all possible pure state decompositions of $\rho_{AB} = \sum_k p_k |\psi_k\rangle_{AB} \langle \psi_k|$.

In addition to the separability criterion, CREN is known to be a good entanglement measure in bipartite quantum systems with the property of entanglement monotone: CREN does not increase under local operations and classical communication [16]. Furthermore, as in the definition of entanglement of formation [18], CREN can also be considered as the amount of entanglement needed to prepare the state $\rho_{AB}$ quantified by the negativity, that is, the concept of formation.

As a dual quantity to CREN, NoA is defined as

$$N^a(\rho_{AB}) = \max \sum_k p_k N(|\psi_k\rangle_{AB}),$$

where the maximum is taken over all possible pure-state decompositions of $\rho_{AB}$ [17].
For the case when $\rho_{AB}$ is a pure state, its CREN as well as its NoA coincides with the original negativity, that is,
\[
N_c(|\psi\rangle_{AB}) = N(|\psi\rangle_{AB}) = N^{\Omega}(|\psi\rangle_{AB})
\]
for any pure state $|\psi\rangle_{AB}$.

We note that NoA in equation (5) is mathematically dual to CREN in equation (4) because one of them is the minimal average of entanglement over all possible pure-state decompositions whereas the other is defined as the maximal one. Moreover, if we introduce a reference system $C$ for a purification of $\rho_{AB}$, it can be easily shown that there is a one-to-one correspondence between the set of all possible pure-state decompositions of $\rho_{AB}$ and the set of all possible rank-1 measurements on the system $C$. In other words, $N^\Omega(\rho_{AB})$ is the maximal entanglement that can be distributed between the systems $A$ and $B$ with the assistance of the environment $C$, whereas $N_c(\rho_{AB})$ is the minimal amount of entanglement needed to prepare $\rho_{AB}$. Thus, NoA can also be considered as the quantity physically dual to CREN: the possible distribution versus the concept of formation.

2.2. Analytic lower bound of the distributed entanglement

Let us consider two-qudit pure states $|\phi\rangle_{AB} = \sum_{j=0}^{d-1} \sqrt{p_j} |j\rangle_{A} |j\rangle_{B}$ and $|\psi\rangle_{CD} = \sum_{j'=0}^{d-1} \sqrt{q_{j'}} |j'\rangle_{C} |j'\rangle_{D}$ in two bipartite quantum systems $AB$ and $CD$, respectively, and let
\[
|\Psi_{k,l}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \omega^{jl} |j, j+k\rangle
\]
be a maximally entangled state in $d \otimes d$ quantum systems where $\omega = \exp(2\pi i/d)$ is the $d$th root of unity. (Throughout this paper, all indices are considered to be integers modulo $d$.) The set $S = \{|\Psi_{k,l}\rangle : k, l = 0, \ldots, d-1\}$ forms an orthonormal basis for the $d \otimes d$ quantum system, and we furthermore have
\[
\frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} \omega^{-lj} |\Psi_{j'-j,l}\rangle = \frac{1}{d} \sum_{l,m=0}^{d-1} \omega^{lj(m-j)} |m, m+j'-j\rangle = \sum_{m=0}^{d-1} \delta_{m,j} |m, m+j'-j\rangle = |j, j'\rangle
\]
for each $j, j' \in \{0, \ldots, d-1\}$. Thus, it follows that
\[
|\phi\rangle_{ABCD} = |\phi\rangle_{AB} \otimes |\psi\rangle_{CD} = \sum_{j=0}^{d-1} \sum_{j'=0}^{d-1} \sqrt{p_j q_{j'}} |j, j'\rangle_{AC} |j, j'\rangle_{BD}
\]
\[
= \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} \sum_{j,j'=0}^{d-1} \sqrt{p_j q_{j'} \omega^{-lj}} |j, j'\rangle_{AC} |\Psi_{j'-j,l}\rangle_{BD}.
\]
Assume that the system $BD$ is measured in the basis $S$ and the measurement outcome is $|\Psi_{k,l}\rangle_{BD}$ for some $k$ and $l$. Then the resulting state in the system $AC$ is the normalized vector of
\[
|\tilde{\Psi}_{k,l}\rangle_{AC} = \frac{1}{\sqrt{r_{k,l}}} \sum_{j=0}^{d-1} \sqrt{p_j q_j \omega^{-lj}} |j, j+k\rangle_{AC} = \sqrt{r_{k,l}} |\psi_{k,l}\rangle_{AC},
\]
where $r_{k,l}$ is the probability of the measurement outcome, that is, $r_{k,l} = |\langle \tilde{\Psi}_{k,l} | \tilde{\Psi}_{k,l} \rangle|$. Because we have $N(|\tilde{\Psi}\rangle) = pN(|\tilde{\psi}\rangle)$ for any unnormalized state $|\tilde{\psi}\rangle = \sqrt{p} |\psi\rangle$ with $\langle \psi | \psi \rangle = 1$, the average negativity that can be distributed on the system $AC$ from the state
we have
\[ N^C(\rho_{AC}) \geq \frac{2}{d-1} \sum_{k=0}^{d-2} \sum_{j < j'} \sqrt{p_j p_{j'}} \sqrt{q_{j+k} q_{j'+k}}. \] (12)

Thus, from equation (11) together with the definition of NoA in equation (5), we have
\[ N^C(\rho_{AC}) \geq N_c(\langle \phi_{AB} \rangle) N_c(\langle \psi_{CD} \rangle). \] (13)

3. Bound on possible remotely distributed entanglement

In this section, using the analytic lower bound in (12), we show that the possible RDE on the system AC by joint measurement on the system BD is not less than the product of two amounts of entanglement for \(|\phi_{AB}\rangle\) and \(|\psi_{CD}\rangle\) for several cases, by employing CREN and NoA.

3.1. Low-dimensional systems: qubits and qutrits

We first consider the case that \(d = 2\). Then we have \(|\phi_{AB}\rangle = \sqrt{p_0}|00\rangle_{AB} + \sqrt{p_1}|11\rangle_{AB}\) and \(|\psi_{CD}\rangle = \sqrt{q_0}|00\rangle_{CD} + \sqrt{q_1}|11\rangle_{CD}\). In this case, the right-hand side of the inequality (12) becomes
\[ 4\sqrt{p_0 p_1} \sqrt{q_0 q_1} = N_c(\langle \phi_{AB} \rangle) N_c(\langle \psi_{CD} \rangle), \] (13)
and we have
\[ N^C(\rho_{AC}) \geq N_c(\langle \phi_{AB} \rangle) N_c(\langle \psi_{CD} \rangle). \] (14)

We now take account of the case that \(d = 3\). Then we have \(|\phi_{AB}\rangle = \sqrt{p_0}|00\rangle_{AB} + \sqrt{p_1}|11\rangle_{AB} + \sqrt{p_2}|22\rangle_{AB}\) and \(|\psi_{CD}\rangle = \sqrt{q_0}|00\rangle_{CD} + \sqrt{q_1}|11\rangle_{CD} + \sqrt{q_2}|22\rangle_{CD}\). As in the case that \(d = 2\), the right-hand side of the inequality (12) becomes
\[ \left( \sqrt{p_0 p_1} + \sqrt{p_0 p_2} + \sqrt{p_1 p_2} \right) \left( \sqrt{q_0 q_1} + \sqrt{q_0 q_2} + \sqrt{q_1 q_2} \right), \] (15)
which is equal to \(N_c(\langle \phi_{AB} \rangle) N_c(\langle \psi_{CD} \rangle)\). Therefore, we are ready to have the following theorem.

**Theorem 1.** For any states \(|\phi_{AB}\rangle\) and \(|\psi_{CD}\rangle\) in \(d \otimes d\) quantum systems AB and CD with \(d = 2, 3\), the possible RDE onto the system AC by joint measurement of the systems B and D is always bounded below by the product of two CREN values of \(|\phi_{AB}\rangle\) and \(|\psi_{CD}\rangle\), that is,
\[ N^C(\rho_{AC}) \geq N_c(\langle \phi_{AB} \rangle) N_c(\langle \psi_{CD} \rangle). \] (16)
3.2. General quantum systems

While there is a simple equality between the right-hand side of the inequality (12) and \( \mathcal{N}(\vert \phi \rangle_{AB}) \cdot \mathcal{N}(\vert \psi \rangle_{CD}) \) in low-dimensional quantum systems, it can be easily checked that such a direct equality does not hold for the general case of higher-dimensional systems when \( d \geq 4 \). Here, we provide two sufficient conditions for general states of \( \vert \phi \rangle_{AB} \) and \( \vert \psi \rangle_{CD} \) to have the same relation as in the inequality (16).

One simple sufficient condition is that \( \vert \phi \rangle_{AB} \) or \( \vert \psi \rangle_{CD} \) is a \( d \)-dimensional maximally entangled state. Suppose \( \vert \phi \rangle_{AB} \) is a maximally entangled state, that is,

\[
\vert \phi \rangle_{AB} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \vert jj \rangle_{AB}.
\]  

Because \( \mathcal{N}(\vert \phi \rangle_{AB}) = 1 \), it can be easily checked that the right-hand side of the inequality (12) becomes

\[
\frac{2}{d-1} \sum_{j<j'} \sqrt{q_{jj'}} = \mathcal{N}(\vert \psi \rangle_{AB}) = \mathcal{N}(\vert \phi \rangle_{AB}) \cdot \mathcal{N}(\vert \psi \rangle_{AB}).
\]  

Hence, in this case, we can readily obtain the same inequality as in (16).

Now, let us consider another sufficient condition that the states \( \vert \psi \rangle_{AB} \) and \( \vert \phi \rangle_{CD} \) have the same Schmidt coefficients, that is, \( p_j = q_j \) for each \( j = 0, \ldots, d-1 \).

First assume that \( d \geq 3 \) is odd. Then, there exists a positive integer \( n \) such that \( d = 2n + 1 \). Let us define a set \( S \) as

\[
S = \{ \frac{\sqrt{p_i p_j}}{d} : 0 \leq i < j \leq d-1 \},
\]  

and, for each \( l = 1, \ldots, n \), its subset \( P_l \) as

\[
P_l = \{ \sqrt{p_{0l}}, \sqrt{p_{1l}}, \ldots, \sqrt{p_{2n+l}} \},
\]  

where all indices are integers modulo \( d \) as mentioned before. It is then straightforward to check that the subsets \( P_l \) and \( P_{l'} \) do not intersect with each other if \( l \neq l' \), and \( S = \bigcup_{l=1}^{n} P_l \). In other words, \( P_l \)'s form a partition of the set \( S \).

Now, for each \( l = 1, \ldots, n \), let

\[
s_l = \sqrt{p_{0l}} + \sqrt{p_{1l}} + \ldots + \sqrt{p_{2n+l}},
\]  

which is the sum of all elements in \( P_l \). Then, the right-hand side of the inequality (12) becomes

\[
\frac{2}{d-1} \sum_{k=0}^{d-1} \sum_{j<j'} \sqrt{p_{jk} p_{j'k}} = \frac{1}{n} \sum_{l=1}^{n} s_l^2.
\]  

Furthermore, we obtain

\[
\mathcal{N}(\vert \phi \rangle_{AB}) \cdot \mathcal{N}(\vert \psi \rangle_{CD}) = (\mathcal{N}(\vert \phi \rangle_{AB}))^2 = \left( \frac{1}{n} \sum_{l=1}^{n} s_l \right)^2.
\]  

By letting \( \mathcal{K} = \sum_{l=1}^{n} s_l^2 / n \) and \( \mathcal{L} = \left( \sum_{l=1}^{n} s_l / n \right)^2 \), we have the following equalities:

\[
n^2(\mathcal{K} - \mathcal{L}) = (n-1) \sum_{l=1}^{n} s_l^2 - 2 \sum_{l<l'} s_l s_{l'} = \sum_{l<l'} (s_l - s_{l'})^2,
\]  

which is clearly nonnegative. Hence, it follows that

\[
\mathcal{N}(\rho_{AC}) \geq \mathcal{K} \geq \mathcal{L} = \mathcal{N}(\vert \phi \rangle_{AB}) \cdot \mathcal{N}(\vert \psi \rangle_{CD}).
\]
We now assume that \( d \) is even such that \( d = 2m \) for some positive integer \( m \). Similarly, for each \( l = 1, \ldots, m - 1 \), let us define the subset \( Q_l \) of the set \( S \) in equation (19) as
\[
Q_l = \{ \sqrt{P_0 P_l}, \sqrt{P_1 P_{1+l}}, \ldots, \sqrt{P_{2m-1} P_{2m-1+l}} \}
\]
and the subset \( Q_m \) as
\[
Q_m = \{ \sqrt{P_0 P_m}, \sqrt{P_1 P_{1+m}}, \ldots, \sqrt{P_{m-1} P_{2m-1}} \}.
\]
Then it is also straightforward to check that \( Q_l \cap Q_{l'} \) is the empty set for \( l \neq l' \) and \( S = \bigcup_{l=1}^{m} Q_l \).

Let us define
\[
t_l = \sqrt{P_0 P_l} + \sqrt{P_1 P_{1+l}} + \cdots + \sqrt{P_{2m-1} P_{2m-1+l}}
\]
for each \( l = 1, \ldots, m - 1 \), and
\[
t_m = \sqrt{P_0 P_m} + \sqrt{P_1 P_{1+m}} + \cdots + \sqrt{P_{m-1} P_{2m-1}}
\]
which are the sums of all elements in \( Q_l \) for \( l = 1, \ldots, m - 1 \) and \( Q_m \), respectively.

Now, the right-hand side of the inequality (12) becomes
\[
\frac{2}{d-1} \sum_{k=0}^{d-1} \sum_{j<k} \sqrt{P_j P_k} \sqrt{P_{j+k} P_{j+k}} = \frac{2}{2m-1} \left( \sum_{l=0}^{m-1} t_l^2 + 2t_m^2 \right).
\]
Moreover, we have
\[
\mathcal{N}_c(\langle \phi \rangle_{AB}) \mathcal{N}_c(\langle \psi \rangle_{CD}) = (\mathcal{N}_c(\langle \phi \rangle_{AB}))^2 = \left( \frac{2}{2m-1} \right)^2 \left( \sum_{l=0}^{m-1} t_l + t_m \right).
\]
By letting \( U = 2 \left( \sum_{l=0}^{m-1} t_l^2 + 2t_m^2 \right)/(2m - 1) \) and \( V = 4 \left( \sum_{l=0}^{m-1} t_l + t_m \right)^2/(2m - 1)^2 \), we obtain
\[
\frac{(2m - 1)^2}{2} (U - V) = (2m - 1) \left( \sum_{l=0}^{m-1} t_l^2 + 2t_m^2 \right) - 2 \left( \sum_{l=0}^{m-1} t_l + t_m \right)^2
\]
\[
= (2m - 1) \sum_{l=0}^{m-1} t_l^2 + 2(2m - 1)t_m^2 - 2 \left[ \left( \sum_{l=0}^{m-1} t_l \right)^2 - t_m^2 - 2t_m \sum_{l=0}^{m-1} t_l \right]
\]
\[
= 2(m - 2) \sum_{l=0}^{m-1} t_l^2 - 4 \sum_{l=0}^{m-1} t_l t_m + \sum_{l=0}^{m-1} t_l^2 - 4t_m \sum_{l=0}^{m-1} t_l + 4(m - 1)t_m^2
\]
\[
= 2 \sum_{l \neq l'} (t_l - t_{l'})^2 + \sum_{l=0}^{m-1} (t_l - 2t_m)^2.
\]
which is also clearly nonnegative. Therefore, it follows that
\[
\mathcal{N}_c^\nu(\rho_{AC}) \geq U \geq V = \mathcal{N}_c(\langle \phi \rangle_{AB}) \mathcal{N}_c(\langle \psi \rangle_{CD}).
\]

Now, we are ready to have the following theorem.

**Theorem 2.** Let \( |\phi\rangle_{AB} \) and \( |\psi\rangle_{CD} \) be any pure states in two \( d \otimes d \) quantum systems \( AB \) and \( CD \), respectively, and assume that they have the same Schmidt coefficients or one of them is maximally entangled. Then the possible RDE onto the system \( AC \) by joint measurement on the systems \( B \) and \( D \) is always bounded below by the product of two CREN values for \( |\phi\rangle_{AB} \) and \( |\psi\rangle_{CD} \), that is,
\[
\mathcal{N}_c^\nu(\rho_{AC}) \geq \mathcal{N}_c(\langle \phi \rangle_{AB}) \mathcal{N}_c(\langle \psi \rangle_{CD}).
\]

Due to the continuity of the inequality (34) with respect to the Schmidt coefficients of \( |\phi\rangle_{AB} \) and \( |\psi\rangle_{CD} \), we note here that theorem 2 also holds if \( |\phi\rangle_{AB} \) and \( |\psi\rangle_{CD} \) have similar Schmidt coefficients to each other or one of them is nearly maximally entangled.
4. Summary

We have provided the bound for RDE using CREN and NoA. For a pair of pure states $|\phi\rangle_{AB}$ and $|\psi\rangle_{CD}$ in quantum systems $AB$ and $CD$, respectively, we have shown that the possible RDE on $AC$ by joint measurement of $BD$, $N_e(\rho_{AC})$, is bounded below by the product of two amounts of entanglement in terms of CREN or the negativity, $N_e(|\phi\rangle_{AB})N_e(|\psi\rangle_{CD})$ for the case of low-dimensional quantum systems. We have also presented some sufficient conditions of the states $|\phi\rangle_{AB}$ and $|\psi\rangle_{CD}$, for which the result of low-dimensional systems can be generalized into higher-dimensional quantum systems.

By considering the pure states $|\phi\rangle_{AB}$ and $|\psi\rangle_{CD}$ as the resource, our bound implies a quantitative relation between the possible amount of distributed entanglement and the entanglement of resource. The possible amount of entanglement that can be remotely distributed between $A$ and $C$ can always be made to exceed the entanglement of the resources. Furthermore, the proof we provided here to show the bound is constructive. In other words, we can always find a measurement on subsystems $BD$ that distributes entanglement exceeding the entanglement of the resource, and this is specially important and efficient for practical RDE schemes in quantum network. Our result also provides an operational interpretation of NoA as the capacity of possible RDE.

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