Could the pulsars actually be oscillators?

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Abstract. A simplified form of a nonsymmetric metric is used to develop the coupling of the electrostatic and gravitational fields, which only occurs in dynamic solutions. The coupling results in a resonance in an object near the Schwarzschild radius. The resonator is unique in that the frequency decreases with radius, reaching a lower limit near 0.4 Hz at the gravitational limit. The limiting frequency does not depend on the mass or any other parameter. It is a constant of physics. Gravitational contraction supplies energy to the resonator, causing it to break into oscillation. The solution is in reasonable agreement with the properties of the mainstream pulsars. An observational test to determine if the pulsars are oscillators has never been performed, and an observational technique is developed for distinguishing between the rotational and oscillatory cases.

Key words. pulsars: general – stars: neutron

1. Introduction

In the most general case mathematically possible the distance between two infinitesimally spaced points has both symmetric and antisymmetric components. It is the conventional conclusion that the antisymmetric terms do not occur in nature, but there is the consideration that the general theory is not a theory of electricity. The Maxwell equations are accommodated by the theory; they are not obtainable from the asymptotic limits of it.

There have been many investigations into the possibility that the antisymmetric terms play a role (Damour et al 1993), but none of them has led to an established theory. It is suggested that some of the difficulty is due to the lack of observational data of an appropriate form, and that pulsar observations may be able to supply it.

The existing nonsymmetric gravitational theories are not well enough developed that they can be used to obtain a persuasive pulsar solution, while without observational evidence that the pulsars are oscillators there is no compelling need to develop them. Progress in such areas has traditionally been made by a series of small steps, with observational data leading the way. The theoretical and observational consequences of discovering that the pulsars are oscillators are so enormous that a further analysis of the observational data seems justified.

The effort required in evaluating the data is minimal. It is simply that if the pulsars are oscillators then the bands of drifting subpulses will tend to synchronize with the main pulse, while there is no possibility of synchronism in an isolated rotating pulsar. The test is simple and effective in distinguishing between the two models. There are unfortunately no existing published data that are of an appropriate form for resolving the question, with that being due to the widespread but unvalidated belief that the pulsars are rapidly rotating neutron stars.

A fully developed derivation of the pulsar solution will be a formidable exercise, since the symmetric and antisymmetric terms uncouple in linearized solutions (Moffat 1995). There appears to be a simpler way of obtaining the essential characteristics of the solution. The method is not very accurate, but the solution obtained should be sufficient for the initial evaluations, and if the pulsars are found to be oscillators then a substantial amount of theoretical work in the literature will be brought to bear on the problem.

It might seem that after more than 30 years of pulsar observation that there could be no question of the outcome. The problem is harder than that, because point–like oscillating and rotating radiators are not distinguishable in any elementary way. One reason that oscillatory pulsar models have not received much attention is that the resonant frequency must decrease as the radius decreases if gravitational contraction is to supply the energy. The oscillator developed here is probably the only one in existence with that characteristic.

2. The electrical scaling relationships

The behavior of light rays in the Schwarzschild solution can be represented by a refractive index model (de Felice 1971). The solution is only valid from the perspective of a distant observer, which corresponds to the
The field limit of the Dirac equations is a likely identification. Then, with the sign of $\xi$ being undetermined,
\[
1/(E_m c) = \xi = \hbar q_e/(bc^4m^2_e) = b^{-1} \times 2.52 \times 10^{-27} \text{ s V}^{-1}.
\]
A field of displaced times can be transformed. One of the ways of deriving the Liénard–Wiechert retardation equations consists of transforming to the frame of reference of a point charge. The solution represents a static inverse square law field in that system, which is transformed back to the original frame of reference. Reviewing the derivation [Morse & Feshbach 1953] shows that if the scalar potential is replaced by a displacement in time the only effect is to change the system of units, with the solution being
\[
\Delta r = \xi q r/\left[4\pi\varepsilon_0(1 - \hat{r} \cdot v_r/c)\right] \\
\Delta t = \xi q/\left[4\pi\varepsilon_0 r(1 - \hat{r} \cdot v_r/c)\right].
\]

The solution is the Liénard–Wiechert retardation equations in a system of units in which the scalar potential represents a displacement in time and the vector potential represents a displacement in space. $v_r$ is the retarded velocity and $r_r$ is the vector from the particle at the retarded position to the field point. All solutions to the retardation equations are also solutions to the Maxwell equations in potential form, from which it follows that those equations are also compatible with this system of units. But does this system have a special physical significance? It is doubtful that the question can be answered by purely mathematical methods. Further, the electrostatic force is a strong force and the displacements are extremely small, making laboratory evaluation of the equations difficult or impossible. But if the potentials do represent metrical displacements then, while the Maxwell equations are linear, the displacements will affect light rays in ways not represented by the Maxwell equations, with the result being that the full system of equations becomes self-interacting and nonlinear. The full system is therefore not the Maxwell equations, but is a system that is not yet known. It is nevertheless sometimes possible to make meaningful calculations in an incompletely understood system from energy and symmetry considerations.

3. The electrograv resonance

3.1. The space–time coupling

The analysis will utilize the quasi–static form of the Maxwell equations, which is often satisfactory for low frequency problems. For the oldest mainstream pulsars the wavelength exceeds the diameter by a factor of $10^5$, justifying the simplification. The coupling of the electrostatic and gravitational fields varies with frequency as $\omega^1$, and the time–varying part of the magnetic field also varies as $\omega^1$ in these solutions, so the magnetic–gravitational coupling varies as $\omega^2$, and it can be neglected at low frequencies.
Begin with two conductive masses connected by a wire. Mount a clock on each mass. Also place a reference clock near the midpoint. Measure time by reading the clock faces. The magnitude of a potential is not locally detectable, so an observer riding with the mass on its journey in time will conclude that the local system behaves in an entirely conventional way. $\partial \psi / \partial t$ behaves like the expansion factor of the space part, and the expansion factor is not locally detectable either. The mass emits a gravitational flux which moves radially outward at $c$. As the voltage increases with time (with an undetermined polarity) the clock moves into the future relative to the reference clock. The time interval of the moving clock maps into a shorter time interval at the reference clock. An observer at the reference point therefore perceives a greater time-integrated flux from the source. So, if the scalar potential does represent a displacement in time, then a changing electrostatic field modulates the intensity of the gravitational field.

The voltage on one mass is $\frac{1}{2} V_0 \sin \omega t$. The voltage on the other mass is $-\frac{1}{2} V_0 \sin \omega t$. The distance between the masses is $d$, and their static gravitational binding energy is $-Gm_1 m_2 / d$. The magnitude of the one way modulation is $\xi V/dt$, and the modulated field strength enters into the energy equation in the same way as the magnitude of either mass, so from the preceding considerations the energy modulation is

$$\frac{1}{2} (1 - V_0 \xi \omega \cos \omega t)(1 + V_0 \xi \omega \cos \omega t).$$

In being proportional to $dV/dt$ the coupling seems to be linear, and in some sense it is, but in the overall system the coupling is associated with nonlinearity. That is more clear if the same method of analysis is applied to two charges, where the interactions produce fields quadratic in source strength. The significance is that in a fully developed derivation the electrostatic–gravitational coupling will be lost if the equations are linearized, because the coupling is a space–time cross term that is in the same order as the quadratic terms. The full system of equations is nonlinear, and it will be difficult to obtain the complete solution.

Proceeding similarly, if the gravitational flux from a parent body is enhanced in propagating to a test mass then the flux from the test mass propagates to the parent body along a path where the $E$ field is of the opposite sign. The acceleration at the outer surface of a thin spherical mass shell is $Gm/r^2$. The acceleration at the inner surface is zero, so the average acceleration is $\frac{1}{2} Gm/r^2$. Beginning with a shell of infinite radius and integrating force times distance leads to the gravitational potential energy, $-\frac{1}{2} Gm^2 / r_0$. The gravitational energy can also be computed by integrating $-a^2 / (8 \pi G)$ over all space, where $a$ is the acceleration. The following calculations proceed by multiplying the energy density by the modulation factor, then integrating. The pulsar is assumed to be a good electrical conductor, so the $E$ field does not penetrate to the interior region.

Without supposing that the appropriate pulsar spherical harmonics are yet known, begin with the lowest order Legendre polynomial that conserves charge, $P_1$. The spherical harmonics represent the charge density on the surface of the sphere. The radial dependency must be computed. The static solution is multiplied by $\sin \omega t$. The effects of retardation are not considered so the result is not an exact solution to the Maxwell equations, but it is quite close when the frequency is low. Then

$$\psi = V_0 r_0^2 \sin \omega t \cos \theta / r^2.$$

Neglecting the vector potential $A$ at low frequencies, the $E$ field is $-\nabla \psi$.

$$E_\rho = 2 V_0 r_0^2 \sin \omega t \cos \theta / r^3$$
$$E_\theta = V_0 r_0^2 \sin \omega t \sin \theta / r^3$$
$$E_\phi = 0$$

The energy density is $\frac{1}{2} \epsilon_0 E \cdot E$,

$$2 V_0^2 \epsilon_0 r_0^4 \sin^2 \omega t / r^6 - \frac{3}{2} V_0^2 \epsilon_0 r_0^4 \sin^2 \theta \sin^2 \omega t / r^6$$
Integrating over the exterior region,

$$U_e = \frac{2}{3} \pi V_0^2 \epsilon_0 r_0 (1 - \cos 2 \omega t).$$

The fringing capacitance between the northern and southern hemispheres forms a capacitor. The equation gives the energy stored in the capacitor. The gravitational inductance of the conductive sphere prevents the capacitor from being short-circuited.

The gravitational energy density is $-a^2 / (8 \pi G)$, with $a = Gm/r^2$. Multiplying by the modulation factor leads to

$$-Gm^2 / (8 \pi r^4) + GV_0^2 m^2 r_0^4 \xi^2 \cos^2 \theta \cos^2 \omega t / (8 \pi r^8),$$
which is integrated over the exterior region.

$$U_k = -Gm^2 / (2 r_0) + (1 + \cos 2 \omega t) G \xi^2 V_0^2 m^2 \omega^2 / (60 r_0)$$

At time $t = 0$ the voltage is zero but it is changing at its maximum rate. Viewing the equation for the total energy as representing the sum of a constant negative binding energy and a positive periodic energy flow shows that the positive energy is at a maximum when the electrical potential difference is zero.

The energy stored in an inductor is $\frac{1}{2} L I^2$, where $I$ is the current. When resonated with a capacitor the energy stored in the inductor is also at a maximum when the voltage across it is zero. Thus, at any given frequency, the coupling of the electrostatic and gravitational fields looks like an inductor. The frequency dependence is that of a capacitor, but it is the phase relationships that matter in computing the resonant frequency. For dense objects the gravitational inductance is far greater than the Maxwellian inductance, so the latter will be neglected.

Now connect an oscillator between the polar regions of the conductive mass. The energy equation shows that,
when averaged over many cycles, the magnitude of the gravitational potential energy is less than before. The modulation weakens the gravitational field. The object is not in equilibrium, so its resilience will increase the radius. When the oscillator is turned off the object will return to its original radius, and the electrical energy transferred to the system could be recovered during the return.

Next, allow the radius to reach equilibrium with the oscillator on, then slowly compress the mass. The oscillator did work on the system in increasing the radius, so the system must do work on the oscillator when the mass is compressed. Gravitational contraction can power the oscillator.

At time $t = 0$ all of the resonator energy is stored in the inductor. At the 90° point on the cycle all of the energy has been transferred to the capacitor, so the time-dependent portion of $U_E$ at the first time must be equal to $U_C$ at the second time. The two equations are solved for $\omega$, then divided by $2\pi$ to obtain the frequency in Hz. The solution is then reparameterized by the substitution $r_0 = 2kGm/c^2$.

$$f = (40\varepsilon_0 G/\pi)^{1/2}k/(c^2\xi)$$

The parameter $k$ represents the ratio of the actual radius to the Schwarzschild radius. The solution evaluates numerically to 0.383$kb$ Hz. Proceeding similarly, the $P_2$ solution is 0.555$kb$. The $P_3$ solution is 0.726$kb$ and the spherical harmonic $\sin 2\phi \sin \theta$ comes in at 0.557$kb$.

3.2. Limitations

The gravitational redshift will significantly lower the computed frequency of the oldest pulsars. The calculation is Newtonian, so there are several other inaccuracies near the Schwarzschild radius.

Self-initiated oscillation modes probably cannot evolve on the surface of a sphere without the preferred direction specified by a pre-existing magnetic field. The nonlinear coupling to the static field was not included in these calculations, but its stabilizing influence is probably essential.

The millisecond pulsars are outside the range of validity of the low frequency approximations utilized if they are of normal mass, and observational data indicate that they are {Callanan et al 1998}. There may be a second solution representing the coupling of the gravitational and magnetic fields, with the $E$ field playing only an auxiliary role. This kind of duality exists in the Maxwell equations, and it could carry over into the gravitational solutions. The difficulty in accommodating the millisecond pulsars with this solution encourages attempts to find a second solution. The millisecond pulsars are not just ordinary pulsars that operate at a higher frequency. There are essential differences between the two pulsar classes.

The oscillation amplitude will build to the point where some nonlinear limiting process occurs, and that point is probably much less than $E_m$. For example, a field of only $10^5$Vm$^{-1}$ would be sufficient to levitate a proton. This field is weak even by laboratory standards, and at some point it may become possible for surface charges to be swept away by the $E$ field. The mass ejection would become so large at that point that further increases in oscillation amplitude would not be possible. If that happens then the pulsar pulses would be associated with the peaks of the sine wave, which reach some critical threshold of charged particle ejection.

3.3. The spindown power

The gravitational potential energy of a solid sphere, including the internal energy, is $U = -\frac{2}{3}Gm^2/r_0$. The internal energy is 1/6 of the total. Substituting $r_0 = k(t)2Gm/c^2$ to parameterize by the Schwarzschild radius yields $U = -\frac{5}{18}mc^2/k(t)$. The outbound energy flow is $-dU/dt = -\frac{5}{18}mc^2(2kdt/k)^2$. The frequency is proportional to $k$, so the fractional frequency change per second is $(2kdt/k) = (2df/dt)/f = -(dP/dt)/P$, where $P$ is the pulsar period. Then, neglecting the mass lost by the equation $E = mc^2$, the energy flow is

$$p = 3mc^2/(10kP)\frac{dP}{dt}.$$  

This calculation includes neutrino radiation and the kinetic energy of the ejected mass, except that the equation is not valid when the pulsar ejects its own mass. It overestimates the power output in that case.

Assuming $1.4M_\odot$ (2.78 $\times 10^{30}$ kg), a spindown rate of one part in $10^{15}$ per second, and $k = 3$, the equation evaluates to $2.5 \times 10^{31}$ watts. For $b = 1$ and the $P_1$ mode the frequency is 1.2 Hz and the radius is 12 km.

4. Observational tests

4.1. Aligned rotors

Charged particles can only escape through the magnetic field in the polar regions. Recent Chandra X-ray images of the Crab [Weisskopf et al 2000] and Vela [Helfand & Gotthelf] pulsars show jets that appear to be parallel to both the magnetic and spin axes, since if the axes were not aligned the jets would take the form of cones. A rotating pulsar becomes disabled when the axes are aligned.

The magnetic force between the poles of a magnetized sphere is attractive. The force at the equator is repulsive, so the magnetic field causes an equatorial bulge. Centrifugal force also causes an equatorial bulge. The energetically preferred orientation of the magnetic field is therefore parallel to the spin axis.

4.2. The low frequency cutoff

The oldest radio pulsars drop out at about 0.25 Hz. Baring & Harding (1998) have proposed that the pulsars become radio-quiet because of photon splitting in the intense magnetic field. The cutoff point is the same as the...
equation for $B_m$ with $b = 1$. There are several isolated X-ray pulsars in the 0.1 to 0.2 Hz range, leaving open the possibility that the pulsars continue to oscillate after they become radio-quiet. The gamma ray bursts sometimes exhibit a light curve resembling the exponential decay of a resonant system with a frequency of 0.1 to 0.2 Hz. In the case of the 1998 August 27 burst from SGR 1900+14 the period of the decaying light curve was the same as the pulsar period [Hurley et al 1999], tending to establish a connection between the two phenomena. The lowest frequency isolated pulsar known has a frequency of 0.085 Hz [Vasisht & Gotthelf 1997]. In view of the errors inherent in a Newtonian solution near the Schwarzschild radius, and also considering the neglect of the gravitational redshift, these observed limiting relationships are in reasonable agreement with the predicted cutoff frequency. The pulse frequency could be at the second harmonic of the oscillation frequency, in which case the discrepancy is greater.

The accretion–powered X-ray pulsars, which are found in binary systems, have periods up to 1400 seconds [Bildsten et al 1997], and are known to be routinely spun up by accretion. Pulsars necessarily do rotate, and the oscillatory model does not apply to the long period accretion–powered pulsars, except that detecting sustained high frequency oscillation in one of these systems would constitute an excellent observational test.

4.3. The energy deficit

When applied to the 7.47 second isolated X-ray pulsar SGR 1806-20 the oscillatory model overestimates the observed power output of the full system, including the nebula, by a factor of $10^5$. The rotational pulsar model fares no better, as it underestimates the total power output by a factor of $10^5$ if the pulsar radius is 10 km. The most commonly accepted explanation for the energy deficit is that the pulsar is powered by the decay of an unusually intense magnetic field rather than by rotational kinetic energy [Kouveliotou et al 1998]. The magnetar field estimates exceed the critical field of the Dirac equations by up to a factor of 20 [Gotthelf et al 1999]. That may be possible, but the field strength estimates do not appear to be reliable. The spindown rate of SGR 1900+14 doubled for 80 days following the August 1998 outburst, suggesting that magnetic braking is not the spindown mechanism for this pulsar, since the field strength would have had to increase by a factor of two in a short time, which is not plausible [Marsden et al 1999].

The oscillatory power calculation is based only on basic energy relationships, so the energy discrepancy in this model may be due to mass ejection. In this model the pulsar has an intense $E$ field so when infalling material becomes ionized half the particles are ejected from the system. The other half collides with the pulsar, creating more charged particles. At high energies a relatively small mass flow could account for the computed energy transfer, but a more likely explanation is that the equation badly overestimates the actual power output if the pulsar ejects its own mass, which can happen in an electrostatic field. The computed power output must therefore be taken as only an upper limit until observational data on the pulsar’s mass flow become available. Infrared observations show that the pulsar is surrounded by a dust cloud [Smith et al 1997], which may be relevant to the energy flow equation.

The 0.085 Hz pulsar 1E1841-045 radiates $3.5 \times 10^{28}$ watts in X-rays alone. The 0.091 Hz pulsar 1RXS J170849.0-400910 is similar, with a steady X-ray output of $1.2 \times 10^{29}$ watts [Israel et al 1999]. These power levels cannot be supplied by the rotational kinetic energy of the pulsar, and it is likely that there are other and still-unmeasured energy flows to be considered.

4.4. The second resonance

There are an infinity of spherical resonance modes, and it may be possible for more than one mode to oscillate simultaneously. Even if the other modes are not oscillatory, the main pulse is narrow and the harmonic content high, so it can excite the high frequency modes that are almost harmonically related, and the effect is about the same.

The 0.54 Hz pulsar PSR 0826-34 exhibits drift bands over a wider region than most, having 5 bands spaced over nearly 200° of the pulsar cycle [Biggs et al 1985]. The uniformity of the spacing implies that there are other hidden bands. The autocorrelation function shows the band spacing to be 29° ± 2, so there must be a total of 12 bands in 360°, implying the existence of a resonance at the 12th harmonic of $F_p$. The bands wander about in unison, but they do not drift systematically, implying that the resonance is almost exactly at a harmonic. But in general the spherical modes are not harmonically related, and in most drifters the drift rate from pulse to pulse evidently gives an indication of the extent that the resonances differ from an integer ratio. Such high order spherical resonances are densely spaced, and a mode that has a frequency which is spaced as closely as possible to an integer times the pulsar frequency will automatically be selected. This coupling would not occur when the higher frequency is not an exact harmonic if the main pulse always occurred at the same position, but it doesn’t. The high frequency resonance has enough reactive energy flow that it can either dump the main pulse prematurely or defer it slightly, depending on the phasing requirements, which is illustrated in figure 2 of [Vivekanand & Joshi 1997]. In a variation of this scheme the higher frequency is at approximately $n + \frac{1}{2}$ times the pulsar frequency and the crests of the waves coincide on every other pulsar cycle.

It is surprising that after 30 years of study the first Fourier transform of the subpulse drift pattern extending to frequencies higher than 0.5 of pulsar frequency will not be published until the year 2001 [Deshpande & Rankin]. It is unobvious that such high frequencies should be of
interest, but the data for the 0.91 Hz pulsar B0943+10 imply that much is to be learned of that region.

Figure 3 of the reference shows the spectrum of 256-pulse sequences. There is a broad series of peaks at around 37 times the pulsar frequency $F_p$, suggesting that there is a resonance at that frequency. The signal is only visible for about 20° out of the 360° of the pulsar cycle, which causes an extensive sideband system. The modulation index is so high that it impossible to determine which peak is the resonance itself. However, each peak is narrow, meaning that the phase coherence from one pulsar cycle to the next is excellent. The spectrum is interpretable as representing a highly monochromatic signal, $F_2$, which is modulated by the pulsar frequency.

The peaks are at frequencies of $n + 0.5355 \pm 0.0003 F_p$. The second harmonic is at $2(n + 0.5355) = (2n + 1 + 0.071)F_p$, and these components are also present in the figure, although the $2n$ value is aliased by ±1. The intermediate products of $F_2$ and $F_p$ produce signals with periods much longer than the pulsar period. There are $n + 0.5355$ cycles of the higher frequency in one pulsar cycle, and in 28 cycles there are $28n + 14.994 \pm 0.008$ cycles, with $n$ being about 36. That is approximately an integral number of cycles for both frequencies, so the system repeats then. The pulsar sometimes switches modes, with the drift pattern becoming disorganized in the other mode.

The Fourier transform also shows a peak at 0.0697 ± 0.0005 $F_p$, implying a repeating pattern of about 14 cycles. But since the resonance is at approximately $(n + \frac{1}{2})F_p$, an even pulse nearly matches odd pulses that are displaced by 13 and 15 cycles, causing the peak to be at the second harmonic of the actual repeat interval. If even pulses are matched with even pulses then the repeat interval is 28.7 ± 0.2 cycles, which is not in agreement with the preceding analysis. Another estimate from the same paper places the peak at 0.0710 ± 0.0007, and a repeat interval of 28 cycles is within the error bound in this case. The uncertainty is probably best resolved by folding on various intervals.

The data also show two other weaker sidebands at about 0.46 ± 0.027 $F_p$, and they imply a repeat interval in the vicinity of 37$C_p$. The autocorrelation function shows a broad peak in the 35–40 cycle range, again implying that the repeat interval is not 28 cycles. However, a similar autocorrelation peak is still there in the “Q” mode of the pulsar, where the 36$F_p$ spectral feature is missing, so it is evidently not associated with the 28-cycle sequence. Further, the “B” mode autocorrelation shows a local maximum at a lag of 28 cycles, and the even–odd pattern causes the correlation to be negative at 27 and 29 cycles, supporting the interpretation that the broad peak is unrelated to the basic repeat interval. The data in the graph were smoothed on an interval of 5 cycles, which, because of its even–odd pattern, discriminates against the true amplitude of the 28-cycle peak. More than one spherical mode can be excited, and it is unlikely that any given drift sequence is completely pure. Figure 13 of the reference shows some weak sidebands at around 24$F_p$ in the Q mode, suggesting that other resonances do play a role.

The sidebands at $n + 0.5355 F_p$ extend to negative values of $n$, with the negative frequencies being folded into the positive region. That results in another sideband system at $m + 0.4645 F_p$. The folded sidebands are weak, and are only significant at low frequencies. They are visible if the e-print version of the figure is electronically magnified. The folded and unfolded sidebands combine to form low frequency sideband pairs at $(n \pm 0.5355)F_p$, which represents a modulation with a period of $1/0.5355 = 1.867 C_p$. At higher frequencies only the upper sideband is significant, but that still represents a modulation. Figure 8 of the reference is folded on this interval, confirming that a modulation exists. The upper sidebands at $n + 0.5355 F_p$ can be viewed equally well as being lower sidebands at $n - 0.4645 F_p$, so another periodicity at 2.153$C_p$ is indicated. Within observational accuracy, there are an integral number of both the 1.867 and 2.153$C_p$ periods in 28 pulsar cycles.

If the region 0 to 0.5$F_p$ is transformed, as is commonly done, the lowest frequency unfolded sideband at 0.5355$F_p$ is above the Nyquist frequency, and is aliased to $1 - 0.5355 = 0.4645 F_p$, which is the same as one of the negative frequency sidebands. Thus the peak at this frequency is the sum of aliased and folded sidebands. The high order harmonics dominate the aliasing in this case, and the folded sidebands are weak in that region, so the feature is mostly an alias.

The 1975 data for the same pulsar place the aliased/folded peak at 0.473 ± 0.002$F_p$, which then corresponds to a resonance at 36.527$C_p$ in that epoch. In the figure showing the drifting pulses the dotted line marked “A” is one of the drift lines for even pulses. The resonance is at approximately $(n + \frac{1}{2})F_p$, so the pulses at a given location occur on every other cycle. Assuming that the frequency of the second resonance is the 1975 value, in two pulsar cycles there are 73.054 cycles, and the nearest even–even or odd–odd peak occurs 0.054 of a cycle early. In one pulsar cycle the value is half that. The figure shows 18 pulses, so the phase should advance by $17 \times 0.027 = 0.459$ cycle in that time. From this value the location of the drift band for odd pulses, spaced by 0.5 cycle, can be computed. Carrying through the construction shows that the computed band spacing is 68 percent greater than the actual spacing, meaning that the calculation contains a systematic error. One possibility is that nonlinear interactions systematically change the phase of the reactive energy flow at the second resonance each time a pulse is emitted, and there is evidence that missing pulses do upset the regularity of the drift pattern in other pulsars.

The drifters are not well understood in any theory, and the full mechanism appears to be fairly elaborate. The mechanism is also difficult to discern, since in most cases only a few degrees of the full 360° cycle are visible in the data, and a single high order resonance, combined with a narrow main pulse, can produce a repeating pattern with a period of many pulsar cycles. But regardless of the mechanisms, there will be a tendency for all the
components to progress in a syncopated lockstep if the pulsar is an oscillator, while there is no known mechanism that could account for synchronism in an isolated rotating pulsar.

4.5. Subharmonic phase lock

For many pulsars the drift rate varies across the window, which is interpretable as a “phase pulling” effect. Such effects are universal in free running oscillators, and they can lead to phase lock when two coupled oscillators are almost harmonically related. If phase lock tends to occur then there will be preferred phase relationships between the repeating pattern and the mean pulse profile, and these preferred locations can be discovered by statistical techniques.

The full repeat interval is usually not the same as the $P_3$ interval of the literature. (In this model $P_3$ is the time required for the signal at the second resonance to drift through one full cycle.) When $P_3$ is not an integral multiple of the basic pulsar period the repeat interval is a multiple of $P_3$, with the constraint being that the total interval must span an integral number of pulsar cycles. This relationship has no meaning for rotating pulsars, but it is a vital consideration in developing statistical tests to determine if there is a tendency for the pattern to phase lock. The full repeat interval must be reasonably short if there is to be any possibility of detecting lock.

If the tendency exists then for a full repeat interval of $n$ pulsar cycles there will be $n$ preferred phases of the repeating pattern, with the preferred phases being separated by multiples of $C_p$. This test will be difficult to apply, since disturbances will cause cycle slips, and the slips will destroy the long term coherence at the repeat interval. Further, even if the pulses do not have preferred positions, but are more visible at some phases of the pulsar cycle (which is known to be the case), then in any statistical process they will seem to have preferred positions. These relationships will make it difficult to determine if preferred positions do exist.

Figure 3b of [Hankins & Wolszan 1987] shows the two–dimensional autocorrelation function for pulsar B1918+19. It shows that the $P_3$ interval is very close to 4$C_p$. Most of the features shown in the figure are explainable by a resonance at $18.25 F_p$, which causes the phase to advance by 0.25 cycle per pulsar cycle. One cycle of $F_2$ corresponds to $360/18.25 = 19.7^\circ$ of the pulsar cycle. With these values it is possible to compute the coordinates in the figure of a grid of horizontal, vertical, and diagonal lines. The diagonal lines are the drift lines. The intersections of the three sets of lines agree quite well with the correlation peaks. The construction is not accurate enough to reliably determine that $n$ is 18, but the Fourier transform will show a sharp peak at the correct value.

Each of the four pulses that make up a cycle is visible in most of the drift bands, but that does not of itself mean that the pulses have preferred positions. Suppose that the pulse train is not coherent with the main pulse. Then for any arbitrarily selected reference phase in the autocorrelation calculation the pulses will occasionally drift to that position. In a long record (which this isn’t) there will be regions where the pulses coincide with any selected reference phase, and a systematic pattern will exist that looks essentially the same. If the pattern of seemingly preferred positions follows shifts in the reference phase then they are not preferred positions at all. But if preferred positions do exist then the intensity of the pattern will depend on the reference phase, with the result being that the high density points do not follow shifts in the reference phase. Thus if phase lock were exact and the reference phase were midway between two preferred positions then the pattern would vanish. It is therefore not possible to determine if phase lock is a tendency with any one autocorrelation calculation.

The figure also shows horizontal bands where the pulses do not drift. They coincide with pulse number 2 of the 4-pulse sequence. Pulse 2 is special by being precisely at the peak of the mean profile. The interactions are plausibly stronger at that point, resulting a ringing of the second resonance, and with two cycles of the ringing being visible after each occurrence of pulse 2. Pulse 2 always occurs in the same position, so the ringing bands do not drift. Observe also that when pulse 2 is early/late the non–drifting bands are also early/late. One problem with the ringing hypothesis is that pulse 2 also tends to be preceded by a weaker pulse. Other mechanisms may be at work in the system, perhaps a third and weaker resonance. The high frequency Fourier transform would answer that question, and the low frequency transform does show that the periodicities are different for the preceding pulses.

There is another way of checking for phase lock in this system. Simply Fourier transform the entire observing session as a single record, without regard for mode changes. If phase lock occurs then each time the pulsar returns to the mode shown in the figure pulse number 2 will still coincide with the peak of the profile, but, relative to the prior data, the pulse can be slipped by 0 to 3 pulsar cycles, which will destroy the coherence of the 18.25$F_p$ frequency. However, the pulse is narrow and the harmonic content high, and the fourth harmonic is at 73$F_p$, so the phase at that frequency is unaffected by cycle slips. Then if phase lock tends to occur there will be a relatively strong spectral feature at 88.91 Hz, and the central peak will have a spectral purity approaching that of the pulsar itself.

The pulsar has two other drift modes, as well as a mode where the drift pattern is disorganized. It occasionally switches between the these modes. In mode A the Fourier transform shows the repeat interval to be about 5.9$C_p$, which places the second resonance at 18.17$F_p$. In mode C the repeat interval is about 2.5$C_p$, with the resonance being at 18.4$F_p$. The autocorrelation for all three modes is shown in the reference, and a second resonance near 18$F_p$ is commensurate with all three modes.

A repeat interval of 2.5$C_p$ results in an even–odd pattern in the correlation bands, since in one band the pulse
phases are at 0, 0.4, and 0.8 cycle of the $F_2$ frequency, while in the next band they are at phases 0.2, 0.6, and 0 or 1.0. The result is two sets of drift lines, with the phase advancing by 0.4 cycle each $C_p$ in one set, and being retarded by 0.2 cycle in the other set. Both sets of lines are visible in the autocorrelation.

Then all four modes are explainable by an oscillation that wanders between 18.17 and 18.4$F_p$, with the random pattern occurring when the frequency is not near any of the three frequencies with drifting subpulses. The drifting of the second resonance could be tracked by Fourier transforming short sections of the data record. If phase lock tends to occur then certain frequencies will be preferred.

Figure 4 of the reference shows that the mean profile has a mode–dependent fine structure, suggesting that the subpulses have preferred positions. It can also be that the subpulses are incoherent but are just more visible at some phases. If they do have preferred positions then a coherent subharmonic relationship exists. One way of seeing the impossibility of a coherent subharmonic process in an isolated rotationally powered pulsar is to apply the equivalence principle to the problem. The principle also seems appropriate in computing the effective $E$ field near a rotating pulsar, since any plasma in the immediate vicinity will be dragged along by the rotating magnetic field. There are no absolute points of reference in the local system.

### 4.6. The rotational signature

If the pulsars are oscillators then any low frequency and highly periodic change in the pulsar signature might represent the rotational period. The rotation would cause a small variation in the pulse arrival time, but it is probably better technique is to search for periodic changes in the pulse profile. Several precessional effects are known in orbiting pulsars, so only isolated pulsars can be considered in looking for the rotational signature unless the rotation rate is so high that precession can be ruled out.

One series of observations of the Crab pulsar showed a weak 60-second periodic intensity modulation in the optical region [Cadede & Galicic 1996], but the result was not repeatable [Golden et al 2000]. The best example of a second period in an isolated pulsar is found in the radio pulsar PSR B1828-11, which exhibits a persistent 1000-day variation in both the pulse arrival time and the pulse shape. The second period has been interpreted as being due to free precession [Stairs et al 2000]. Free precession requires that the mass distribution be skewed with respect to the axis of rotation [Pandey 1996]. Current pulsar models assume that the interior is a superfluid, with a neutron crust. With a surface gravity of $10^{11}$ g it may be difficult to justify enough deviation from isopotential stratification to account for the precession. If the 1000-day period represents the rotational period then that would mean that the pulsars preferentially shed angular momentum at birth.

It has been suggested that a radio lobe associated with SGR 1806-20 is rotating with a period of 10 years [Kouveliotou et al 1998]. Further, the light curves shown in the reference appear to have undergone a phase inversion in 5 years. It will be difficult to distinguish between the two models if the period of rotation actually is measured in years, which is unexpectedly long. Another way of distinguishing between the oscillatory and rotational models is to look for phase jumps. A phase jump is possible in a rotating pulsar, but it requires that the angular velocity first change with one sign, then after a short time change again with the other sign, which is improbable. On the other hand for an oscillating pulsar any major disturbance would be expected to affect the frequency, and the frequency shift will integrate to a phase change after the event. High frequency oscillators are more susceptible to phase jumps.

### 4.7. Glitches

The radius decreases steadily as the pulsar ages, so the neutron crust must rupture and buckle at times. The buckling will increase the effective radius, increasing the frequency of resonance. The signatures of the oscillatory and rotational model glitches are very similar in this respect.

There is another factor to be considered in evaluating the glitch characteristics. The pulse profile of SGR 1900+14 changed at the time of the August 1998 outburst. The change persisted for 1.5 years and showed no signs of recovery [Woods et al 2000]. Some radio pulsars with drifting pulses undergo similar profile changes. They presumably represent a change in the preferred scheme of phase locked frequency ratios, and occasional readjustments will be necessary due to random disturbances, and systematically so as the Schwarzschild radius is approached. The readjustment will affect both the basic pulsar frequency and the spin down rate. The possibility can be considered that a major and sudden reshuffling of the frequency ratios is the cause of the glitches, in which case they will be accompanied by profile changes.

### 5. Discussion

These equations predict that young pulsars are much larger than is currently thought, and observational or theoretical data may exist that will not accommodate the larger size. The pulsar radii are difficult to infer, and there are very few observational data on the size, but one estimate of the 4.2 Hz Geminga pulsar’s radius concludes that it is less than 9.5 km [Shearer et al 1999] if the thermal emissivity is 1, which is not consistent with the oscillatory model. The estimate is based on an upper limit for the unpulsed blackbody optical brightness. No unpulsed optical emission was detected, and a demonstration that the unpulsed optical component actually is blackbody would be helpful in establishing the result. An even more essential consideration is that if the surface is highly conductive
then it will be specular, which translates into a low thermal emissivity.

At still higher frequencies there is the constraint that the oscillatory calculations become so inaccurate as to be useless when the diameter is comparable to a quarter wavelength, and, depending on the value of $b$, that problem can arise with the youngest conventional pulsars. The neglect of the magnetic field in the calculations can also cause the size to be overestimated when the frequency is relatively high. The current flow on the surface allows the external magnetic field to oscillate, but the internal magnetic field of a highly conductive sphere cannot change quickly. It may nevertheless be possible for mechanical deformations to generate internal $E$ fields, which cannot exist either, but if the $E$ field due to the rate of change of the magnetic field is equal and opposite to the $E$ field due to mechanical deformation then the interior magnetic field can oscillate at the pulsar frequency. Such an effect, if it exists, would significantly affect the higher frequency solutions. The low frequency solutions would not be much affected.

While the gravitational inductance looks like an inductor at any given frequency, the frequency dependence is quite different. In particular, the equations illustrate that if the vacuum surrounding the pulsar is replaced by a material with a higher dielectric constant then the frequency increases. Thus if there are additional interior energy terms that are in phase with the energy of the $E$ field then the frequency will increase, which will result in a smaller pulsar for a given frequency and mass.

There is then the possibility that a more refined derivation (along with some new theory) will predict smaller young pulsars, and by avoiding the regime of questionable accuracy at first the equations as given should be satisfactory for observationally discriminating between the rotary and oscillatory pulsar models.

Supplemental material, including detailed derivations of all the above equations, is available on the Internet at www.s-4.com/pulsar. Section 5 of the web page includes images with the drift lines drawn on them for some of the drifting subpulse bands analyzed here.

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