On AdS/QCD correspondence and the partonic picture of deep inelastic scattering

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Abstract

We critically examine the question of scaling of the Deep Inelastic Scattering process in the medium Bjorken $x$ region on a scalar boson in the framework of the AdS/QCD correspondence. To get the right polarization structure of the forward electroproduction amplitude, we show that one needs to add (at least) the scalar to scalar and scalar to vector hadronic amplitudes. This illustrates how the partonic picture may emerge from a simple scenario based on the AdS/QCD correspondence, provided one allows the conformal dimension of the hadronic field to equal 1 and use the concept of "hadron - parton duality".
1 Introduction

The near conformal symmetry of QCD has driven a number of theoretical attempts named as the AdS/QCD correspondence to obtain some information about scattering amplitudes of processes at high energies. The seminal paper of Polchinski and Strassler \[1\] showed how one may hope to recover some scaling laws in the realm of hard hadronic reactions. Exclusive amplitudes have also been studied in a number of cases \[2, 3\] and a partonic interpretation has been proposed by Brodsky and de Teramond \[2\] in the framework of the light-front dynamics. Various aspects of deep inelastic scattering have also been studied \[4, 5, 6, 7\].

In this paper we critically scrutinize the simplest case, analysed also in \[1\], where the partonic structure emerges from an OPE analysis of a scattering amplitude, namely the imaginary part of the forward amplitude for the process

\[\gamma^* H \rightarrow \gamma^* H\]

where \(H\) is a hadron, which is related to the total cross section for the deep inelastic electroproduction process

\[\gamma^* H \rightarrow X\]

by the usual optical theorem.

To begin with, let us outline the well-known two facts that seem to us essential to the understanding of the deeply inelastic scattering (DIS) process on a hadron in terms of the underlying QCD process where partons, namely quarks in the lowest order, respond to the electromagnetic current.

- The amplitude scales as \(Q^0\) (up to logarithmic modifications) at large \(Q^2\) and fixed Bjorken variable \((x_{Bj} = Q^2/s)\), where \(Q^2 = -q^2\) is the virtuality of the photon and \(s\) the squared energy of the process, and this scaling behaviour is the signal that the electromagnetic current scatters on pointlike particles inside the hadron.

- The leading amplitude corresponds to the case of a transversally polarized virtual photon, which is the signal that these pointlike constituents are fermions.

Let us stress that these features do not depend on the spin of the hadronic target, so that we feel free to restrict our study to the technically simplest case of a scalar boson (e.g. a \(f^0\) meson, for definiteness).

Our aim is to explore whether we can get these two features from an analysis of DIS in the framework of the AdS/QCD correspondence, in its simplest version as defined in the paper of Polchinski and Strassler \[8\], i.e. the hard-wall model (our discussion actually does not depend on this choice: see Appendix for the case of soft-wall models). The conclusions of Ref. \[8\] show that there is no obvious answer to this question, and one may even wonder whether the results of this approach establish a bridge between the partonic and AdS/QCD descriptions of DIS.

The plan of the paper is as follows. In section 2, we briefly remind the reader of the basics of the AdS/QCD correspondence strategy to analyse the amplitude of the process \(\gamma^* H \rightarrow X\) where \(X\) is a massive state that corresponds to the final state of the DIS process. Taking \(X\) as a scalar object, we recover the results of Ref. \[8\] and discuss their virtues and defects with respect to a partonic interpretation in the kinematical domain known as the Bjorken scaling region. Motivated by the defects discovered, we calculate in section 3 the corresponding process where \(X\) is now a spin 1 object. In section 4, we argue that the physically sensible amplitude is a weighted sum of the spin 0 and spin 1 amplitudes and show that indeed we can recover the needed features of the DIS amplitude, provided we allow the conformal dimension of the hadronic field to equal 1.

2 DIS on a scalar target with a scalar intermediate state

Let us recall the results obtained in Ref. \[8\] for the calculation of the amplitude of the process \(\gamma^*(q) S \rightarrow X\), where \(S\) and \(X\) are scalar states. The basic quantities are a massless vector field \(A^m(x, z)\) and massive complex scalar fields \(\Phi(x, z)\) which are treated as free modes propagating in a
five-dimensional Anti-de Sitter space (AdS) with curvature $R$. We shall use the Poincaré coordinates with metric $g_{mn}$ defined by

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) ,$$

where $\eta_{\mu\nu} = (-, +, +, +)$ is the Minkowski metric.

The massless vector field $A^m(x, z)$ is identified with the electromagnetic field and obeys the Maxwell equations in AdS with the boundary conditions

$$\lim_{z \to 0} A_\mu(x, z) = A_\mu(x)|_{4d} = n_\mu e^{iq \cdot x}, \quad \lim_{z \to 0} A_z(x, z) = 0 ,$$

where $n_\mu$ is an arbitrary polarization vector. With the Lorentz-like gauge condition $\eta_{\mu\nu} \partial^\mu A^\nu + z \partial_z (\frac{1}{z} A_z) = 0$, the solution for a spacelike photon, $Q^2 = q \cdot q > 0$, reads:

$$A_\mu(x, z) = Q z K_1(Q z) e^{iq \cdot x} n_\mu , \quad A_z(x, z) = i(q \cdot n) z K_0(Q z) e^{iq \cdot x} .$$

The electromagnetic current is written as

$$J_m(x, z) = i \sum_X \phi_0 \partial_m \phi_X^* - \phi_X^* \partial_m \phi_0 ,$$

where the scalar initial field $\phi_0(x, z)$ and final field $\phi_X(x, z)$ are normalizable classical solutions of the Laplacian in Anti-de Sitter space, which is a representation of the quadratic Casimir operator of the isometry group $SO(4, 2)$ of AdS. These solutions belong to the same irreducible representation of $SO(4, 2)$ labelled by the conformal dimension $\Delta_0$.

The constant $c_X$ is fixed by imposing a Dirichlet boundary condition at $z_\infty = 1/\Lambda$ and by normalizing to unity the charge density $J^0(x, z)$ of the field $\phi_X$ over each time slice,

$$i \int_0^{z_\infty} \frac{dz}{z^3} \left( \phi_X \partial_0 \phi_X^* - \phi_X^* \partial_0 \phi_X \right) = 2 E \quad \implies \quad c_X = \frac{\sqrt{2} \Lambda}{\sqrt{R^3}} J_{\Delta_0 - 1} \left( \frac{\sqrt{s}}{\Lambda} \right) \sim \Lambda^{1/2} s^{1/4} \; \Lambda \to 0 \frac{\Lambda^{1/2} s^{1/4}}{\sqrt{R^3}} .$$

$$\sqrt{s} = m_n = \zeta_{\Delta_0 - 2, n} \Lambda , \quad J_{\Delta_0 - 2} (\zeta_{\Delta_0 - 2, n}) = 0 .$$

The normalization integral is defined for $\Delta_0 > 1$.

The covariant interaction with the electromagnetic field $A_m(x, z)$ reads, to first order in the electromagnetic coupling $e$,

$$S_{int} = e \int d^4 x dz \sqrt{-g} \; g^{mn} A_m(x, z) J_n(x, z) .$$

In the low energy limit for the initial scalar field,

$$\phi_0(x, z) \sim c_0 z^{\Delta_0 - \frac{1}{2}} e^{ip_0 \cdot x} , \quad c_0 \propto \Lambda^{\Delta_0 - 1} \left( \frac{\Lambda \sqrt{s}}{\sqrt{R^3}} \right) .$$

The $\gamma^* S \to X$ amplitude is proportional to

$$M_\mu^0 = Q^{-\Delta_0} A(x) \left( p_{0\mu} + \frac{1}{2x} q_{0\mu} \right) , \quad A(x) = 2^{\Delta_0} \Gamma(\Delta_0) x^{\frac{2\Delta_0}{2} - 1} (1 - x)^{\frac{\Delta_0}{2} - 1} , \quad x = -\frac{Q^2}{2p_0 \cdot q} .$$

Plugging in the normalization constants one gets for the electromagnetic hadronic tensor,

$$W^0_{\mu\nu} = \frac{1}{2} (c_0 c_X R^3)^2 \sum_n \delta(s - m_n^2) Q^{-2\Delta_0} \left( p_{0\mu} + \frac{1}{2x} q_{0\mu} \right) \left( p_{0\nu} + \frac{1}{2x} q_{0\nu} \right) A^2(x) .$$

$^1$Indeed 4d gauge invariance implies that $\Delta_X = \Delta_0$.  

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Hence, performing the sum over \( n \), with a density of states
\[
\sum_n \delta(s - m_n^2) \approx \int \, dn \delta(n^2 \pi^2 \Lambda^2 - s) = (2\pi\sqrt{s}\Lambda)^{-1},
\]
the usual structure functions \( F_1(x, Q^2) \) and \( F_2(x, Q^2) \) can be identified as
\[
F_1(x, Q^2) = 0, \quad F_2(x, Q^2) = \frac{c}{4\pi} \left( \frac{\Lambda}{Q} \right)^{2\Delta_0-2} \frac{1}{2\pi} A^2(x) .
\]

where \( c \) is some dimensionless constant. One could wonder whether the \( Q^2 \) behavior depends upon the specific mechanism of conformal symmetry breaking of the hard-wall model. In the Appendix we show that this \( Q^2 \)-dependence is quite general since one obtains the same result in the soft-wall models where one introduces background dilaton fields [4].

We can now address the two questions emphasized in Section 1.

- Firstly, the \( Q^2 \) behaviour is controlled by the dimension \( \Delta_0 \) attached to the incoming field. If one identifies this dimension to the conformal dimension of a hadron seen as a bound state of elementary quarks and gluons (namely \( \Delta_0 = 2 \) for a meson) as proposed in the analysis of exclusive reactions [1, 2], we completely miss the scaling behaviour of DIS structure functions which is central to the parton picture [11] and to the recognition of QCD as the theory of strong interactions able to understand the experimental data. In order to make contact with reality, one is thus lead to consider the case where \( \Delta_0 = 1 \), which may be interpreted as the recognition that the incoming hadron fluctuates to an elementary field before scattering with the virtual photon. Most interestingly, it turns out that this value of \( \Delta_0 = 1 \) coincides with the unitarity bound on the dimension of scalar operator in four dimensions [12]. We thus propose to take seriously \( \Delta_0 = 1 \) as the starting point of a partonic interpretation of the AdS calculation of DIS structure functions.

- The fact that \( F_1 \) vanishes shows that the tensorial structure of the result is completely at odds with a sensible partonic interpretation.

We thus conclude that although a scaling behaviour may be marginally recognized in the results of the AdS/QCD calculation the tensorial structure of the amplitude prevents us from being able to make contact with the partonic interpretation. This is by no ways a surprise, since the spin structure is indeed very poor when one considers a scalar to scalar amplitude and indeed, the restriction to a scalar meson in the final state is completely unjustified since the kinematics considered emphasize large (4-dimensional) invariant masses \( M_X^2 = s \).

We thus advocate that it is very important to consider the case of higher spin final states and, as a concrete example, calculate the corresponding observables for a spin 1 final state.

### 3 DIS on a scalar target with a vector intermediate state

The missing ingredient for a spin 1 final state is the solution \( V_m(x, z) \) of the free massive Maxwell equations in AdS_5,
\[
G_{mn} = \partial_m V_n - \partial_n V_m, \quad (D_m G)^{mn} \equiv \frac{1}{\sqrt{-g}} \partial_m (\sqrt{-g} G^{mn}) = m_V^2 V^n, \quad \forall n .
\]

We look for a solution in the radial gauge such that
\[
V_\mu(x, z) = C e^{i k \cdot x} V(z), \quad V_z = 0, \quad e \cdot k = 0, \quad e^2 = 1 ,
\]
and which belongs to an irreducible representation of \( SO(4, 2) \) labelled by the conformal dimension \( \Delta_V \) and characterized by the asymptotic behavior
\[
\lim_{z \to 0} V_\mu(x, z) \propto z^{\Delta_V} e^{ik \cdot x} .
\]
The solution with the right boundary conditions reads

\[ V_\mu(x, z) = C_{\Delta V} e^{ik \cdot x} z J_{\Delta V - 1}(\sqrt{s} z), \quad \Delta V(\Delta V - 2) = (m_V R)^2, \quad k^2 = -s < 0. \] (19)

The normalization of charged vector fields can be defined as for charged scalar fields by imposing a Dirichlet boundary condition at \( z_\infty = 1/\Lambda \) and by normalizing to unity the charge density \( J^0(x, z) \) of the field \( V_\mu \) over each time slice,

\[
i \int_0^{z_\infty} dz \frac{R}{z} (V_\mu G^{\mu \nu} - V_\nu G^{\mu \nu}) = 2 E \quad \implies \quad C_{\Delta V} = \frac{\sqrt{2\Lambda}}{\sqrt{R}} J_{\Delta V - 1}(\sqrt{s}) \Lambda^{-1/4} \frac{\sqrt{s}}{R} \] (20)

\[ \sqrt{s} = m_{V,n}, \quad \Delta V - 1, n \Lambda, \quad J_{\Delta V - 1}(\Delta V - 1, n) = 0. \] (21)

The normalization integral is defined for \( \Delta V > 0 \). We note that the \( s \) dependence of the normalization constant and of the sum over the intermediate states are the same for a massive scalar field and for a massive vector field (in this so-called “hard-wall” model).

The interaction between the electromagnetic field \( A_m \), a scalar field \( S \) and a vector field \( V_m \) can be described to first order in the electromagnetic coupling \( e \) by an effective gauge-invariant lagrangian with non-minimal coupling of the following form

\[ \mathcal{L}_{V \gamma S} = ie \frac{g_{V \gamma S}}{m_V} (\partial^i V^j \partial_i A_j - \partial^i V^j \partial_j A_i) S. \] (22)

The covariant interaction in the Anti-de Sitter space AdS\(_5\) reads

\[
S_{\text{int}} = ie \frac{g_{V \gamma S}}{m_V} \int d^4x dz \sqrt{-g} \Phi_0 \left( g^{\mu \nu} g^{\rho \sigma} \partial_\mu V^*_\nu F^\rho_{\mu \nu} \right) \] (23)

where \( F_{\mu \nu} \) is the electromagnetic tensor. In the low energy limit \( \Lambda \) for the initial scalar field, and with the massive vector field in the radial gauge \( \frac{1}{9} \), we get (with implicit summation over mute indices understood with the four-dimensional Minkowski metric)

\[
S_{\text{int}} = ie \frac{g_{V \gamma S}}{m_V} \phi_0 c_V \int d^4 x dz \frac{R}{z} z^{\Delta_0} \left( \partial^\mu V^{\mu \nu} F_{\mu \nu} + \partial_\nu V^{\mu \nu} F_{\nu \mu} \right) \\
= ie \frac{g_{V \gamma S}}{m_V} \phi_0 c_V R \delta^{(4)}(k - p_0 - q) \int dz z^{\Delta_0 - 1} \\
\times \left( Q ((k \cdot q)(n \cdot e) - (k \cdot n)(q \cdot e)) z^2 J_{\Delta V - 1}(\sqrt{s} z) K_1(Q z) + (e \cdot q)(q \cdot n) z K_0(Q z) \partial_z (z J_{\Delta V - 1}(\sqrt{s} z)) + (n \cdot e) \partial_z (Q z K_1(Q z)) \partial_z (z J_{\Delta V - 1}(\sqrt{s} z)) \right) 
\]

The matrix element \( M_\mu \) of the electromagnetic current is defined by

\[
S_{\text{int}} = ie \frac{g_{V \gamma S}}{m_V} \phi_0 c_V R \delta^{(4)}(k - p_0 - q) n^\mu M_\mu \] (24)

with

\[
M_\mu = Q e_\mu \int dz z^{\Delta_0 - 1} \times ((k \cdot q) z J_{\Delta V - 1}(\sqrt{s} z) K_1(Q z) + \partial_z (z K_1(Q z)) \partial_z (z J_{\Delta V - 1}(\sqrt{s} z))) \\
- Q k_\mu (q \cdot e) \int dz z^{\Delta_0 - 1} z^2 J_{\Delta V - 1}(\sqrt{s} z) K_1(Q z) + q_\mu (e \cdot q) \int dz z^{\Delta_0 - 1} z K_0(Q z) \partial_z (z J_{\Delta V - 1}(\sqrt{s} z)) 
\]

and is gauge invariant by construction,

\[
q_\mu M^\mu = Q (e \cdot q) \int dz z^{\Delta_0 - 1} \partial_z (z J_{\Delta V - 1}(\sqrt{s} z)) (\partial_z (z K_1(Q z)) + Q z K_0(Q z)) \equiv 0. 
\]
Hence the matrix element \( M_\mu \) can be written as

\[
M_\mu = Q (e_\mu (k \cdot q) - k_\mu (e \cdot q)) \int dz \, z^{\Delta_0 + 1} J_{\Delta V - 1} (\sqrt{s} z) K_1 (Q z) + \left(-Q^2 e_\mu + q_\mu (e \cdot q)\right) \int dz \, z^{\Delta_0} K_0 (Q z) \partial_z (z J_{\Delta V - 1} (\sqrt{s} z)) .
\]

If \( \Delta_V + \Delta_0 > 0 \), the first integral equals

\[
\mathcal{I}_1 = Q^{-\Delta_0 - 2} \left( \frac{s}{Q^2} \right)^{\Delta_0 - 1} \Gamma \left( \frac{\Delta_V + \Delta_0}{2} \right) \frac{2^{\Delta_0 - 1}}{\Delta_0 \Gamma (\Delta_V)} \times 2F_1 \left( \frac{\Delta_V + \Delta_0}{2}, \frac{\Delta_V + \Delta_0}{2}; \Delta_V; -\frac{s}{Q^2} \right)
\]

and the second one is

\[
\mathcal{I}_0 = \Delta_V Q^{-\Delta_0 - 1} \left( \frac{s}{Q^2} \right)^{\Delta_0 - 1} \Gamma \left( \frac{\Delta_V + \Delta_0}{2} \right) \frac{2^{\Delta_0 - 1}}{\Delta_0 \Gamma (\Delta_V)} \times 2F_1 \left( \frac{\Delta_V + \Delta_0}{2}, \frac{\Delta_V + \Delta_0}{2}; \Delta_V; -\frac{s}{Q^2} \right)
\]

Setting

\[
x = \frac{Q^2}{s + Q^2} = \frac{1}{2} \frac{q^2}{p_0 \cdot q} , \quad k^2 = (p_0 + q)^2 = -s , \quad p_0^2 = 0 , \quad Q^2 = q \cdot q
\]

we get

\[
\mathcal{I}_1 = \left( \Delta_V + \Delta_0 \right) \frac{\Gamma^2 \left( \frac{\Delta_V + \Delta_0}{2} \right)}{2^{\Delta_0} \Gamma (\Delta_V)} \times Q^{-\Delta_0 - 2} \frac{\Delta_0 + 1}{x^{\Delta_0 + 1}} (1 - x)^{\Delta_0 - 1} 2F_1 \left( \frac{\Delta_V + \Delta_0}{2}, \frac{\Delta_V - \Delta_0}{2}; \Delta_V; 1 - x \right)
\]

\[
\mathcal{I}_0 = \frac{\Delta_V \Gamma^2 \left( \frac{\Delta_V + \Delta_0}{2} \right)}{2^{\Delta_0} \Gamma (\Delta_V)} Q^{-\Delta_0 - 1} \frac{\Delta_0 + 1}{x^{\Delta_0 + 1}} (1 - x)^{\Delta_0 - 1} \left( 2F_1 \left( \frac{\Delta_V + \Delta_0}{2}, \frac{\Delta_V - \Delta_0}{2}; \Delta_V; 1 - x \right) - \frac{(\Delta_V + \Delta_0)^2}{2\Delta_V} \times (1 - x)^2 2F_1 \left( \frac{\Delta_V + \Delta_0}{2}, \frac{\Delta_V - \Delta_0}{2}; \Delta_V; 1 - x \right) \right)
\]

Denoting \( \mathcal{I}_1 \equiv Q^{-\Delta_0 - 2} A_1 (x) , \quad \mathcal{I}_0 \equiv Q^{-\Delta_0 - 1} A_0 (x) \), the amplitude \( M_\mu \) reads

\[
M_\mu = Q^{-\Delta_0 - 1} \left( e_\mu (k \cdot q) - k_\mu (e \cdot q) \right) A_1 (x) + \left( -Q^2 e_\mu + q_\mu (e \cdot q) \right) A_0 (x) .
\]

The vector field polarizations must be summed in the square of the transition amplitude. The terms linear in \( e \) vanish whereas the quadratic terms are written as

\[
e_\mu e_\nu = \eta_{\mu \nu} + \frac{k_\mu k_\nu}{s} , \quad e_\mu (e \cdot q) = q_\mu + \frac{k_\mu}{s} k_\nu , \quad (e \cdot q)(e \cdot q) = q^2 + \frac{(k \cdot q)^2}{s} .
\]

Hence, with \( p_0 \cdot q = -y Q^2 \equiv -\frac{Q^2}{2x} \), we get:

\[
M_\mu M_\nu = Q^{-2\Delta_0 + 2} \left( \eta_{\mu \nu} - \frac{q_\mu q_\nu}{Q^2} \right) (A_0 (x) + (y - 1) A_1 (x))^2 + \frac{Q^{-2\Delta_0 + 2}}{s} \left( p_0 \mu p_0 \nu + y (p_0 \mu q_\nu + p_0 q_\nu p_\mu) + y^2 q_\mu q_\nu \right) A_0^2 (x) + \frac{s}{Q^2} A_1^2 (x) .
\]
The electromagnetic hadronic tensor thus reads

\[
W_{\mu\nu} = \frac{1}{2} \left( \frac{g_{\nu S}^V}{m_V} c_0 c_V R \right)^2 \sum_n M_{\mu} M_{\nu} \frac{1}{2E} \delta \left( E - \sqrt{s + \tilde{k}^2} \right) \]

\[
= \frac{c}{2} \frac{g_{\nu S}^V}{M_V^2 R^2} \Lambda^{2\Delta_0 - 2} \sqrt{s} \sum_n \delta(s - m_n^2) M_{\mu} M_{\nu} = \frac{c}{4\pi} \frac{g_{\nu S}^V}{M_V^2 R^2} \Lambda^{2\Delta_0 - 2} M_{\mu} M_{\nu} .
\]

where \( c \) is a dimensionless constant. The structure functions then read

\[
F_1(x, Q^2) = \frac{c}{4\pi} \frac{g_{\nu S}^V}{M_V^2 R^2} \left( \frac{\Lambda}{Q} \right)^{2\Delta_0 - 2} \left( A_0(x) + \left( \frac{1}{2x} - 1 \right) A_1(x) \right)^2,
\]

\[
F_2(x, Q^2) = \frac{c}{4\pi} \frac{g_{\nu S}^V}{M_V^2 R^2} \left( \frac{\Lambda}{Q} \right)^{2\Delta_0 - 2} \frac{1}{2x} \left( \frac{x}{1-x} A_0^2(x) + A_1^2(x) \right).
\]

As may have been anticipated from dimensional arguments, the \( Q^2 \) dependence of the structure functions induced by vector intermediate states is the same as for the scalar intermediate states. It is controlled only by the conformal dimension \( \Delta_0 \) of the scalar initial state and does not depend on the conformal dimension \( \Delta_V \) of the vector intermediate states. We note however that \( \Delta_V \) is not restricted by gauge invariance.

4 Scaling behaviour and interpretation

We can thus repeat our argument and advocate that the case \( \Delta_0 = 1 \) indeed dominates the large \( Q^2 \) behaviour of the DIS cross-section. The tensorial behaviour is now sufficiently rich to consider a meaningful correspondence between the AdS/QCD results and real physics, namely the partonic description of the structure functions. To do this, let us recall the reader that the cornerstone of the interpretation of the DIS amplitude as a scattering of a hard probe on pointlike fermions inside the hadron, is the Callan-Gross relation \[9\]

\[
F_2(x, Q^2) = 2xF_1(x, Q^2),
\]

which states that the longitudinal cross section is small with respect to the transverse cross section \( \sigma_L/\sigma_T \sim 1/Q^2 \). We note that by a superposition of scalar and vector final states the Callan-Gross relation can now hold.

The compatibility system of equations is straightforwardly deduced from Eqs. \[14\], \[33\] and reads (after introducing an effective scalar coupling \( C' \) and redefining in Eq. \[33\] the constants \( K_{\Delta_V} = C' \frac{g_{\nu S}^V}{M_V^2 R^2} \))

\[
C' \frac{A_2(x)}{2x} + \sum_{\Delta_V} K_{\Delta_V} \frac{1}{2x} \left[ \frac{x}{1-x} A_0^2(x) + A_1^2(x) \right] = \sum_{\Delta_V} 2xK_{\Delta_V} \left[ A_0(x) + \left( \frac{1}{2x} - 1 \right) A_1(x) \right]^2 .
\]

A series expansion of the hypergeometric functions in Eqs. \[26\], \[27\] and some straightforward but tedious algebra show that one can fix recursively the parameters \( K_{\Delta_V} \) to fulfill this equation.

We can now interpret our results. We are able to obtain a non empty intersection between the QCD description of deep inelastic scattering on a scalar hadron at medium \( x \) and the simple AdS/QCD picture in the supergravity approximation proposed by Polchinski and Strassler, provided that

- we accept some fine tuning relating the scalar \( \rightarrow \) scalar and scalar \( \rightarrow \) vector amplitudes, enabling us to mimic the scattering on a spinor constituent through the vanishing of the longitudinal structure function \( F_L(x) = F_2(x) - 2xF_1(x) \).
- when considering medium \( x \) physics in the normal scaling region, we use the minimal value, \( \Delta_0 = 1 \) for the conformal dimension of the hadronic field. The choice \( \Delta_0 = 1 \) may look - and indeed is - contradictory with many recent works in the framework of the AdS/CFT.
correspondence, which take for granted that the value of $\Delta$ is to be fixed by the dimension of the interpolating current able to create the hadron from the vacuum. In terms of quark fields, this yields values such as $\Delta = 2$ for a scalar meson. Our point is that such an assignment is obviously completely incompatible with the scaling property of DIS on a meson at medium $x$, which is the most basic result in favor of the validity of quantum chromodynamics as the theory of strong interactions. We thus propose to take seriously the fact that the value, $\Delta_0 = 1$ describing a "partonic" fluctuation of the hadron is the right choice if we want to consider the AdS/QCD correspondence as a useful tool to describe strong interactions in a regime where a hard probe distinguishes the inner content of the hadrons. Let us note that the analysis of Ref. [7] shows that in the small $x$ region, with saturation effects taken into account, the quite different scaling behaviour - known as geometric scaling - may result from another choice of conformal dimension, namely $\Delta_0 = 2$.

5 Conclusions

By performing a sum on the final states of deep inelastic scattering on a scalar target, we derived a sensible expression for the structure functions in the regime of Bjorken scaling. Our strategy has been motivated by the well known - but quite badly understood - success of the concept of quark-hadron duality [10]. We refer here to the "global" Bloom-Gilman parton hadron duality which states, for instance in the analysis of the total cross section in electron positron annihilation, that the sum of hadronic amplitudes over the many resonances which may be produced equals the sum of the contributions of the quark amplitudes. This does not preclude that "local" parton hadron duality [13] survives at strong coupling, as has recently been shown not to be the case by some recent work [14] in the framework of N=4 SYM.

Our calculation is of course simplistic and one should not take seriously the $x-$dependence of the resulting structure functions. We however think that it is exemplary in the sense that it demonstrates that the quark content of the hadron, as it emerges from a partonic interpretation of the DIS cross section, can be borne out of a computation where only hadrons live in the 5-dimensional bulk. Although the right tensorial structure of the hadronic part is obtained at the price of some fine tuning of the parameters controlling the relative contributions of scalar and vector final states, this result may be viewed as a positive sign of the validity of some AdS/QCD approach to strong interaction physics in the domain where a partonic description has been proven to be very effective.

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Appendix

We explicitly show in this appendix that the $Q^2$-dependence of the structure functions is the same in the soft-wall and the hard-wall models.

Following [4] a background dilaton field $\chi(z)$ is introduced in the AdS metric $g_{ij}$. Then the action which describes 5d scalar fields propagating on this background reads

$$S = \int d^4x \sqrt{-g} e^{-\chi} \left( g^{ij} \partial_i \phi \partial_j \phi + m_S^2 \phi^2 \right), \quad (36)$$

and the Laplacian reads

$$\frac{e^\chi}{\sqrt{-g}} \partial_i \left( e^{-\chi} \sqrt{-g} g^{ij} \frac{\partial \phi}{\partial x^j} \right) = m_S^2 \phi. \quad (37)$$

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In Poincaré coordinates, the Laplacian becomes
\[ z^2 \Box \phi + z^5 e^x \partial_z (z^{-3} e^{-x} \partial_z \phi) = (m_S R)^2 \phi. \] (38)

Looking again for a solution that is a plane-wave in Minkowski space, and setting
\[ \phi(x,z) = e^{ip \cdot x} e^{\chi(z)/2} z^{3/2} \psi(z), \] (39)
the Laplacian equation becomes a Schrödinger-like equation
\[ \frac{d^2 \psi}{dz^2} - V(z) \psi = p^2 \psi \] (40)
\[ V(z) = \frac{(m_S R)^2 + 15/4}{z^2} + \frac{3}{2z} \partial_z \chi + \frac{1}{4} (\partial_z \chi)^2 - \frac{1}{2} \partial_z^2 \chi, \] (41)
which we solve in the WKB approximation, since we are interested in the highly excited states. The classically allowed region corresponds to \( s = -p^2 > V(z) \). Let \( z_0 < z_\infty \) be the classical turning points, \( V(z_0) = V(z_\infty) = s \), where the semi-classical approximation breaks down. The solution is
\[ \psi(z) = \frac{C}{\sqrt{K(z)}} \sin \left( \frac{\pi}{4} + \int_{z_0}^{z} K(t) dt \right), \quad z_0 < z < z_\infty, \] (42)
\[ K(z) = \sqrt{s - V(z)}, \quad s = -p^2, \] (43)
with the Bohr-Sommerfeld quantization rule
\[ \int_{z_0}^{z_\infty} K(z) dz = \pi \left( n + \frac{1}{2} \right). \] (44)
The normalization integral and the density of states read
\[ C^2 R^3 \int_{z_0}^{z_\infty} dz \frac{e^{\chi(z)}}{K(z)} \sin^2 \left( \frac{\pi}{4} + \int_{z_0}^{z_\infty} K(t) dt \right) = 1 \] (45)
\[ \frac{dn}{ds} = \frac{1}{2} \int_{z_0}^{z_\infty} \frac{dz}{\sqrt{s - V(z)}}. \] (46)
In the limit \( s \gg 1 \), the argument of the sine is a function of \( z \) which varies much faster than the other functions in the integrand. Hence one can replace in this limit the sine square by its average value \( 1/2 \). Then we get an approximate expression for the normalization constant,
\[ C^{-2} \approx \frac{R^3}{2} \int_{z_0}^{z_\infty} dz \frac{e^{\chi(z)}}{K(z)}. \] (47)
We can calculate the integrals approximately in the limit \( s \gg 1 \). Indeed inside the classically allowed regions, far from the turning points, we have then \( s \gg V(z) \) and we can expand the integrand with respect to \( s \). Hence
\[ \frac{dn}{ds} \approx \frac{1}{2 \sqrt{s}} \int_{z_0}^{z_\infty} dz \left( 1 + O \left( \frac{1}{s} \right) \right) \approx \frac{1}{2 \pi \sqrt{s}}, \quad z_\infty = \frac{1}{\Lambda} \] (48)
\[ C^{-2} \approx \frac{R^3}{2 \sqrt{s}} \int_{z_0}^{z_\infty} dz e^{\chi(z)} \left( 1 + O \left( \frac{1}{s} \right) \right) \approx \frac{R^3}{2 \sqrt{s}} \int_{z_0}^{z_\infty} dz e^{\chi(z)} = c_\chi \frac{R^3}{2 \Lambda \sqrt{s}}, \] (49)
where \( c_\chi \) is a dimensionless constant. We find that the densities of states (43) and (13), and the normalizations (49) and (7), have indeed the same dimensional structure in the soft-wall and hard-wall models. This generalizes the result of [5] in the specific soft-wall model with linear Regge trajectories.
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