Adaptive position observer for moving object

T.D. Hoang¹, A.A. Pyrkin²
¹ITMO University, Saint Petersburg
²ITMO University and Robotronica LLC, Saint Petersburg
a.pyrkin@gmail.com

Abstract. The paper presents a new adaptive observer of the local coordinates of a moving object based on measurements of linear velocity, yaw angle (heading), and distance to a single beacon with known position. The conditions under which the algorithm is asymptotically stable are shown, as well as the states for which a robust estimate is reasonable to be used.

1. Introduction

In this paper, we consider a problem of estimating the current coordinates of a moving object (for example, a mobile robot) by direction-finding of observed targets with known coordinates. The single-beacon navigation problem is presented in articles [1], [2], [3]. The authors propose a method for determining the position of an autonomous underwater vehicle (AUV) from measurements of a distance to the beacon and radial velocity, based on Extended Kalman Filter and Particle Filter. In contrast to the analogs, we consider a formulation in which the angular velocity of an object is considered unknown. Radio beacons are considered as landmarks, which can be recognized as points with known coordinates. The robot is equipped with sensors that provide information about the linear motion speed, the relative bearing (yaw angle), as well as the distance to a single beacon with the known position in the absolute earth-fixed coordinate system. It is assumed that the location of the beacon relative to the moving object (to the left or right of the robot, in front or behind) is known: in which quadrant the beacon is relative to the robot. The paper proposes a new adaptive beacon bearing observer which provides asymptotic convergence to zero of the estimation error. Based on the bearing value, estimates of the global coordinates of the mobile robot are formed.

2. Problem statement

Consider a moving object on a plane, the position of which can be described in polar (ρ, θ) and Cartesian (y₁, y₂) coordinates. The object moves at a longitudinal speed V(t) with a yaw angle K(t) near a stationary beacon with coordinates (y₁∗, y₂∗).

In order to simplify calculations, we assume that the beacon is located at the origin: y₁ = 0, y₂ = 0.

The object motion model is described by the equations:

\[ \dot{\rho} = V \sin(\theta + K) \]  
\[ \dot{\theta} = \frac{V}{\rho} \cos(\theta + K) \]

Knowing the values of the heading K(t), speed V(t), distance to the beacon ρ(t) and the location of the beacon (located to the left or right of the moving object), we need to:

a) synthesize the angle observer in polar coordinates \( \dot{\theta}(t) \)
\[
\lim_{t \to \infty} |\theta(t) - \hat{\theta}(t)| = 0. \tag{3}
\]

b) find the position in the Cartesian coordinate system \((y_1, y_2)\)

\[
\lim_{t \to \infty} |y_{1,2}(t) - \hat{y}_{1,2}(t)| = 0. \tag{4}
\]

Fig. 1. Robot in absolute coordinate system

3. Observer design
Consider an auxiliary variable \(\varphi = \theta + K\) for which the relation is valid:

\[
\dot{\varphi} = \dot{\theta} + K + \frac{v}{\rho} \cos \varphi. \tag{5}
\]

If the deviation of the yaw angle from the line connecting the object and the beacon is more than some given \(\delta\), then the variable \(\psi\) can be estimated from the known values of the yaw angle, the distance to the beacon, and the linear velocity of the object in the earth-fixed coordinate system.

4. Proposition. The \(\psi\) variable observer:
\[
\dot{\psi}(t) = \begin{cases} 
\ddot{x}, & \text{if the beacon is on the left: } \cos \psi > 0 \\
\pi - \ddot{x}, & \text{if the beacon is on the right: } \cos \psi < 0
\end{cases}
\]

\[
\ddot{x}(t) = K(t) \text{sign}(\cos \psi) + \gamma \rho(t) + \zeta(t),
\]

\[
\ddot{\zeta}(t) = \frac{\nu(t)}{\rho(t)} \cos \ddot{x}(t) - \gamma V(t) \sin \ddot{x}(t),
\]

ensures that the condition \(\lim_{t \to \infty} |\psi(t) - \hat{\psi}(t)| = 0\) is met, if the deviation of the relative bearing from the line connecting the object and the beacon is more than a certain specified value \(\delta\).

5. Proof of the statement. Consider the error model \(\ddot{\varphi} = \psi - \hat{\psi}\)

If the beacon is on the left and \(\cos \psi > 0\), then

\[
\begin{align*}
\dot{\psi} &= \psi - \ddot{x} = \dot{K} + \frac{V}{\rho} \cos \psi - \dot{K} - \gamma \dot{\rho}(t) - \dot{\zeta}(t) = \frac{V}{\rho} (\cos \psi - \cos \hat{\psi}) - \gamma V (\sin \psi - \sin \hat{\psi}) \\
&= -\frac{2V}{\rho} \sin \psi + \frac{V}{\rho} \sin \frac{\psi - \hat{\psi}}{2} - \frac{2\gamma V}{2} \left(\sin \psi + \sin \frac{\psi - \hat{\psi}}{2}\right) \\
&= -\frac{2V}{\rho} \left(\sin \frac{\psi + \hat{\psi}}{2} + \gamma \rho \cos \frac{\psi + \hat{\psi}}{2}\right) \sin \left(\frac{1}{2} \hat{\psi}\right) 
\end{align*}
\]

If the beacon is on the right and \(\cos \psi > 0\), then

\[
\begin{align*}
\dot{\psi} &= \psi - \pi + \ddot{x} = \dot{K} + \frac{V}{\rho} \cos \psi - \dot{K} + \gamma \dot{\rho}(t) + \dot{\zeta}(t)
\end{align*}
\]

\[
\begin{align*}
\dot{\psi} &= \psi - \pi + \ddot{x} = \dot{K} + \frac{V}{\rho} \cos \psi - \dot{K} + \gamma \dot{\rho}(t) + \dot{\zeta}(t)
\end{align*}
\]

\[
\begin{align*}
\dot{\psi} &= \psi - \pi + \ddot{x} = \dot{K} + \frac{V}{\rho} \cos \psi - \dot{K} + \gamma \dot{\rho}(t) + \dot{\zeta}(t)
\end{align*}
\]

\[
\begin{align*}
\dot{\psi} &= \psi - \pi + \ddot{x} = \dot{K} + \frac{V}{\rho} \cos \psi - \dot{K} + \gamma \dot{\rho}(t) + \dot{\zeta}(t)
\end{align*}
\]
\[ \frac{V}{\rho} (\cos \psi + \cos[\pi - \hat{\psi}]) + \gamma V (\sin \psi - \sin[\pi - \hat{\psi}]) \]
\[ = -2V \sin \frac{\psi + \hat{\psi}}{2} \sin \frac{\psi - \hat{\psi}}{2} + 2\gamma V \cos \frac{\psi + \hat{\psi}}{2} \sin \frac{\psi - \hat{\psi}}{2} \]
\[ = -2V \rho \left( \sin \frac{\psi + \hat{\psi}}{2} - \gamma \rho \cos \frac{\psi + \hat{\psi}}{2} \right) \sin \left( \frac{1}{2} \hat{\psi} \right) \]

(7)

To guarantee the asymptotic stability of the equilibrium positions \( \hat{\psi} = 0 \), it is necessary to require the fulfillment of the double inequality
\[-\gamma \rho < \tan \frac{\psi + \hat{\psi}}{2} < \gamma \rho, \]
which is true almost on the entire interval of \( \psi \) determination, except for the neighborhoods near \( \psi = \frac{\pi}{2} \) and \( \psi = \frac{3\pi}{2} \). It can be shown that the size of this neighborhood \( \delta \) decreases with the increase in the tuning parameter \( \gamma \).

5.1. Remark.

If it is impossible to reliably determine whether the beacon is on the left or on the right, this means that the deviation of the relative bearing from the direction to the beacon is in some small neighborhood \( \delta \) (almost co-directional), and the \( \psi \) variable estimate should be taken equal to
\[ \hat{\psi}(t) = \begin{cases} \frac{\pi}{2}, & \text{if the beacon is behind (sin} \psi > 0) \\ \frac{3\pi}{2}, & \text{if the beacon is ahead (sin} \psi < 0) \end{cases} \]

Estimates of the object coordinates can be calculated by the formula
\[ \hat{y}_{1,2} = \rho(t) \left[ \frac{\cos \hat{\theta}}{\sin \hat{\theta}} \right] \]
where \( \hat{\theta} \) is the estimate of the angle \( \theta \) related to the bearing, determined by the following algorithm:
\[ \hat{\theta} = (\hat{\psi} - K) \mod 2\pi, \]

6. Simulation

The numerical example is considered to illustrate the efficiency of the proposed observer. Two scenarios are investigated: motion along a line and circular motion. The simulation results are shown in Figs. 2-5.
Acknowledgements
This paper was financially supported by the grant of Saint Petersburg in the field of scientific and technical activities in the form of subsidies in 2020 (project “Development of algorithms for sensorless sensing of mobile technical objects”).

Conclusion
The article proposes a new method for synthesizing an observer of the coordinates of a moving object on a plane based on the available measurements of the heading, linear speed and distance to a stationary beacon with precisely known coordinates. In situations where the object is moving along the direction to the beacon, it is advisable to use coarse estimates. In the future, it is planned to continue studying the problem of observability of the considered mathematical model near singular points in order to obtain asymptotic and exponential convergence of the estimation errors to zero for all values of the relative bearing and bearing to the beacon.

References
[1] Stepanov, O.A., Vasiliev, V.A., Toropov, A.B., Loparev, A.V., Basin, M.V., Efficiency analysis of a filtering algorithm for discrete-time linear stochastic systems with polynomial measurements, Journal of the Franklin Institute, 2019, vol. 356, no. 10, pp. 5573–5591.
[2] Ferreira, B., Matos, A., Cruz, N., Single Beacon Navigation: Localization and Control of the MARES AUV, Proceedings of OCEANS ‘10 MTS/IEEE, September 20–23, 2010. Seattle, WA, USA.
[3] Koshchev, D.A., Multiple Model Algorithm for Single-Beacon Navigation of Autonomous Underwater Vehicle without Its A Priori Position. Part 1. Mathematical Formulation. Gyroscopy Navig., 2020, no. 11, 230–243.