Magnetic Stresses at the Inner Edges of Accretion Disks Around Black Holes

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Abstract. For the past twenty-five years, nearly all analyses of accretion disk dynamics have assumed that stress inside the disk is locally proportional to pressure (the “α-model”) and that this stress goes to zero at the marginally stable orbit. Recently, it has been demonstrated that MHD turbulence accounts for the bulk of internal disk stress. In contradiction with the traditional view, the stress from this MHD turbulence does not diminish near the marginally stable orbit, and the ratio of magnetic stress to pressure rises sharply there. Examples of the consequences include: an increase in accretion efficiency that may also be time- and circumstance-dependent; a decrease in the rate of black hole spin-up by accretion; and generation of disk luminosity fluctuations. Preliminary results from numerical simulations lend support to analytic estimates of these effects.

I INTRODUCTION

Ever since the seminal work of Novikov and Thorne [1] and Shakura and Sunyaev [2], our understanding of the physics of accretion disks around black holes has been built on two fundamental beliefs: the energy available for radiation is equal to the binding energy of the innermost stable orbit, and is therefore determined solely by the spin of the black hole; and the (vertically-integrated) stress responsible for transporting angular momentum outwards through the disk is proportional to the local pressure. Recent work involving magnetic forces in the innermost regions of relativistic accretion disks has undercut both of these claims. In this review I will summarize the reasons for this change of heart and explain the consequences that are likely to follow if the new point of view proves correct.

II THE TRADITIONAL VIEW OF THE INNER EDGES OF RELATIVISTIC ACCRETION DISKS

The basic framework for understanding the inner regions of relativistic accretion disks was laid out more than twenty-five years ago [1,3]. In the simplest possible
picture (i.e., one assuming that disks are time-steady, axi-symmetric, and geometrically thin), their radial structure may be defined in terms of conservation equations that appear almost Newtonian, as the general relativistic effects can be collapsed into multiplicative correction factors. Conservation of angular momentum, for example, may be expressed through a single first-order differential equation in radius.

Thus, to specify the entire solution, all that is necessary is to choose a boundary condition for this differential equation. This boundary condition may be physically interpreted as determining the conserved outward angular momentum flux through the disk, or, equivalently, as specifying \( T_{r\phi} \), the \( r-\phi \) component of the stress exerted on the disk, at its inner edge. This inner edge is conventionally taken to lie at the radius of the marginally stable orbit, \( r_{ms} \).

Selecting this boundary condition is the only part of the procedure in which there is any guesswork. Until very recently, it was almost universally assumed that the stress must be zero at \( r_{ms} \). Two reasons were commonly cited for this choice. The first, due to Thorne and collaborators [1,3], was that the radial speed inside \( r_{ms} \) is so much larger than the radial speed outside \( r_{ms} \) that, assuming a constant mass accretion rate, the inertia of mass inside \( r_{ms} \) must be much smaller than the inertia of the disk proper in the region of stable orbits. Given that, it appears difficult to see how the small amount of matter in the plunging region could exert any significant force on the much heavier disk. The second argument [4] begins with the assumption that the stress carrying angular momentum outward should always be proportional to the local pressure; i.e., \( T_{r\phi} = \alpha \rho \) [2]. If this is so, as material accelerates inward and expands, its pressure falls rapidly, so the stress must do likewise.

If there is no stress inside \( r_{ms} \), then no forces change the energy or angular momentum of material inside that point. Matter therefore arrives at the event horizon with the energy and angular momentum it had at \( r_{ms} \). The maximum energy per unit mass available to be radiated in the disk (the maximum radiative efficiency) is then defined simply by the binding energy of the marginally stable orbit, and the spin-up rate of the black hole is the mass accretion rate times the specific angular momentum of that orbit. Both the specific binding energy and the specific angular momentum at \( r_{ms} \) depend only on the black hole’s normalized angular momentum \( a/M \); thus, it has long been thought that both the radiative efficiency and the spin-up rate per unit accreted mass are functions only of \( a/M \).

Interestingly, the very first work on this subject [3] acknowledged that strong magnetic fields could upset the arguments that the stress must go to zero at \( r_{ms} \), but chose (reasonably, given the state of knowledge at the time) to ignore this possibility. However, work of the past decade (summarized in [5]) has shown that magnetic fields are essential to angular momentum transfer in accretion disks. We must therefore re-open the question of the appropriate boundary condition on \( T_{r\phi} \) at \( r_{ms} \).
III  MAGNETIC FIELDS IN ACCRETION DISKS

This recent work has shown that, in the main body of the disk, MHD turbulence creates a stress that, when vertically-averaged, is roughly proportional to the pressure, with an $\alpha \sim 0.01 - 0.1$ [5–7]. The magnetic stress accounts for the bulk of the angular momentum flux so long as two conditions are met: there must be some seed magnetic field in the accreting gas, but in the disk midplane the energy density of any imposed magnetic field must be smaller than the total pressure; and the matter must have high enough conductivity that the MHD approximation is valid. In the conditions surrounding black holes, these are all almost certain to apply. Weak magnetic fields are virtually ubiquitous in astrophysical environments; and near a black hole the gas temperature will very nearly always be high enough to maintain the gas in an ionized, and therefore highly conducting, state.

This turbulence is generated by an MHD instability driven by the orbital shear [8]. Its growth rate is always comparable to the orbital frequency, so it grows roughly as rapidly (relative to an orbital period) in the relativistic part of the disk as in the Newtonian part. The nonlinear amplitude of the turbulence in the Newtonian part of the disk is determined by a cascade of energy toward shorter wavelengths, where it can ultimately be dissipated. This process, too, should operate more or less in the same fashion (as viewed in the fluid frame) in the relativistic part of the disk as in the non-relativistic part, provided the local inflow time is long enough for equilibrium to be established. We may therefore reasonably conclude that the ratio $B^2/(8\pi p)$ doesn’t change dramatically at radii a few times $r_{\text{ms}}$. Because orbital shear automatically stretches field lines in the sense that produces outward angular momentum flow ($\langle B_r B_\phi \rangle < 0$), the effective $\alpha$-parameter ($=\langle B_r B_\phi \rangle/(4\pi p)$) is also unlikely to change much. But if this is so, why should the stress diminish as the marginally stable orbit is approached?

The only aspect of this process that changes in any qualitative way as $r_{\text{ms}}$ is approached from the outside is the ordering of four timescales: the inflow time $t_{\text{in}}$, the thermal time $t_{\text{th}}$, the turbulent dissipation time $t_{\text{diss}}$, and the dynamical time $t_{\text{dyn}}$. In the disk body, $t_{\text{in}} \sim \alpha^{-1}(r/h)^2 t_{\text{dyn}}$ and $\beta t_{\text{diss}} \sim t_{\text{th}} \sim \alpha^{-1} t_{\text{dyn}}$, where $\beta$ is the ratio of gas (+ radiation) pressure to magnetic energy density. Thus, $t_{\text{in}} \gg t_{\text{th}} > t_{\text{diss}} > t_{\text{dyn}}$. However, just outside $r_{\text{ms}}$ the effective potential flattens (this is, of course, what it means for $r_{\text{ms}}$ to be the location of the innermost marginally stable orbit), so $t_{\text{in}}$ diminishes toward $t_{\text{dyn}}$. The concomitant decline in surface density likewise reduces $t_{\text{th}}$. However, $t_{\text{diss}}$ (crudely $\sim h/v_A$ for disk thickness $h$ and Alfven speed $v_A$) declines more slowly than $t_{\text{th}}$. As a result, in the vicinity of $r_{\text{ms}}$, the interplay of plasma dynamics and flux-freezing should be at least as important to determining the field intensity as the balance between the turbulent dynamo and turbulent dissipation.

If flux-freezing really does determine the evolution of the field as the matter plunges inside $r_{\text{ms}}$, the magnetic field strength in the fluid frame stays roughly constant or increases somewhat even while the fluid pressure decreases dramatically [9]. It follows that the effective $\alpha$ rises sharply as the inflow accelerates near and
inside $r_{ms}$. In fact, when the radial component of the velocity becomes relativistic, it is easy to show that these assumptions imply $B^2/(8\pi) \sim \rho c^2$. That is, when the inflow is relativistic, magnetic forces become competitive with gravity, and the Alfvén speed becomes relativistic. Matter may then retain a causal coupling with the disk it left behind even when it has fallen well inside $r_{ms}$, allowing significant transfer of energy and angular momentum [9,10].

Looking back at the old argument that the small inertia of matter in the plunging region cannot do much to the main body of the disk, we now see that magnetic fields in effect turn this argument on its head. Magnetic connections between the disk and plunging matter act as “tethers” by which the massive disk restrains angular acceleration of the low-inertia streams inside $r_{ms}$. In so doing, stress can be exerted on the disk itself.

**IV CONSEQUENCES**

The existence of significant stress at the marginally stable orbit has major consequences for the most fundamental properties of accretion onto black holes. Energy taken from plunging matter and delivered to the disk is potentially available for radiation; removal of angular momentum from this matter retards the rate of black hole spin-up [11]. In fact, when the black hole is rapidly rotating, the rotational energy of the black hole itself can be tapped: frame-dragging gives plunging matter a high orbital frequency; magnetic connections from this region to the disk exert a torque; the end-result is energy drawn from black hole rotation given to matter in the disk [10,12]. In other words, the amount of energy drawn from accreting matter is no longer a quantity fixed by orbital mechanics: it is a dynamical quantity that is the product of complicated MHD dynamics, and may even be time-variable.

The fate of the work done by magnetic fields at $r \geq r_{ms}$ depends critically on the ratio between the dissipation and inflow timescales in the place where the energy is brought. If the energy can be converted into heat and radiated before the matter finds its way into the plunging region inside $r_{ms}$, it adds to the radiative efficiency; if, on the other hand, dissipation is slow, so that the energy stays in the gas either as organized kinetic energy or as magnetic field energy, it may end up being taken into the black hole. Given the arguments of the previous section, it is as yet unclear how this balance works out.

If additional heat is deposited in the disk, there can be substantial alterations to the disk spectrum [11]. Additional flux is radiated at high frequencies, and relativistic effects beam it strongly into the equatorial plane. In addition, because so much more radiation is produced in the most relativistic portion of the disk, the returning radiation fraction is greatly enhanced. This latter effect has implications for phenomenology as diverse as polarization of the emitted light and the synchronization of fluctuations.

However, there is an additional implication of stress at the inner edge of the disk that may be important even if the radiative efficiency is hardly altered by
these mechanisms: dynamics at the marginally stable orbit can be a powerful “noise” source to the entire dynamical system. In the long run, these dynamical fluctuations may be reflected in luminosity fluctuations, which are, of course, a hallmark of all known accreting black holes. The origin of this “noise” may be seen from a simple thought experiment: Consider a small magnetized fluid element orbiting just outside $r_{ms}$. Imagine that half this fluid element receives a negative angular momentum perturbation from the turbulence and begins to fall inward. As it does so, its orbital frequency increases simply due to its decreasing orbital radius. The resulting shear between the two halves of the fluid element stretches the magnetic flux tube connecting them, and a magnetic tension force creates a torque that transfers angular momentum from the falling half to the half that is still orbiting stably. The stably orbiting half must then move outward, launching a compressive wave into the disk. Thus, the basic mechanics of accretion create disk “noise” when there are magnetic connections across the marginally stable orbit.

V SIMULATIONS

All the arguments presented so far have been qualitative. Whether these effects are quantitatively important can only be ascertained as a result of genuine calculation. MHD turbulent dynamics are sufficiently complicated that (almost) all credible calculations are numerical simulations. Although much work remains to be done before simulations can be run with a requisite level of realism, some preliminary results have been achieved [13–15].

To date, all simulations touching on these issues have been limited in a number of regards, both physical and numerical. All have assumed Newtonian physics; general relativistic dynamics is mimicked by the use of the Paczyński-Wiita potential $U = -GM/(r - 2GM/c^2)$. This potential qualitatively reproduces motion in a Schwarzschild metric by creating a marginally stable orbit at $r = 6GM/c^2$. In addition, all simulations so far have substituted an assumed equation of state for a real energy equation. This assumption has two deleterious consequences: magnetosonic waves don’t propagate at the right speed because the pressure isn’t correctly computed; and, more importantly, it is impossible to use these simulations to estimate the radiative efficiency because energy taken from the plunging region to the disk falls right back in. There are also aspects of the numerical methods used in these simulations that are less than optimal. Both kinetic energy and magnetic energy can disappear if fluid from two adjacent cells with oppositely directed velocity or magnetic field components is combined. It also appears likely that no simulation to date has used a grid fine enough to resolve the magnetic field structure.

Nonetheless, putting aside these qualms, the simulational results may still be used as an indication of what may appear in more realistic simulations. In particular, angular momentum transport should be more reliably treated than energy transport because it is not subject to the problems described in the previous paragraph. The result shown in Figure 1 [14] is particularly noteworthy. Two distributions of $T_{r\phi}$
FIGURE 1. The solid curve shows the time- and azimuthal-average of the vertically-integrated $r$-$\phi$ component of the magnetic stress in a simulation by Hawley and Krolik [14]. The dashed curve is the prediction of the Novikov-Thorne model for the mean mass accretion rate found in that simulation. Note that the unit of distance is $2GM/c^2$, so the marginally stable orbit is at $R = 3$.

integrated vertically and averaged over both azimuth and time are shown: the prediction of the Novikov-Thorne model for the mean mass accretion rate found in the simulation; and the magnetic contribution to the stress. The two curves coincide very closely over the radial range from $\simeq 8 - 24GM/c^2$, demonstrating that magnetic stresses account for very nearly all the angular momentum transport in the main part of the disk. They depart in an uninteresting way at large radius—the deviation here is due to the fact that the disk in the simulation had an outer edge, whereas the Novikov-Thorne model refers to a disk that extends to very large radius. At small radius, however, the contrast is very significant—as expected on the basis of the qualitative arguments presented earlier, magnetic stress remains important across the marginally stable orbit and throughout the region where gas plunges toward the event horizon. Indeed, as a result of this continuing stress, the mean specific angular momentum of matter crossing the inner edge of the simulation (at $r = 3GM/c^2$) is about 5% smaller than the specific angular momentum at $r_{ms}$. Also as expected, the effective $\alpha$ parameter increases sharply in the innermost part of the accretion flow: between $r = 8GM/c^2$ and $r = 4GM/c^2$, it increases by an order of magnitude.
VI FUTURE PROSPECTS

The results of these first simulations are encouraging, but they are far from the final answer. Fortunately, significant improvements are feasible. Improved resolution can be obtained both by cleverer gridding schemes and, of course, by a few years’ technological development. Real energy equations can be computed by explicitly incorporating phenomenological viscosity and resistivity and a radiative cooling function (following time-dependent radiation transfer is also possible [16], but will require somewhat greater computer power before it is feasible for this kind of simulation). Likewise, there is no fundamental impediment to writing MHD codes that work in a fixed relativistic metric. Thus, in a few years, we should be able to attach quantitative values to the effects pointed out qualitatively here.

REFERENCES

1. Novikov, I.D., and Thorne, K.S., in Black Holes, C. De Witt and B. De Witt, eds., New York: Gordon & Breach, 1973, p. 343
2. Shakura, N. and Sunyaev, R., Astron. Astrop. 24, 337 (1973)
3. Page, D. and Thorne, K.S., Ap. J. 191, 499 (1974)
4. Abramowicz, M.A. and Kato, S., Ap. J. 336, 304 (1989)
5. Balbus, S.A. and Hawley, J.F., Revs. Mod. Phys. 70, 1 (1998)
6. Stone, J.M., Hawley, J.F., Gammie, C.F., and Balbus, S.A. Ap. J. 463, 656 (1996)
7. Brandenburg, A., Stein, R.F., Nordlund, A., and Torkelsson, U., Ap. J. Letts. 458, L45 (1996)
8. Balbus, S.A. and Hawley, J.F., Ap. J. 376, 214 (1991)
9. Krolnik, J.H., Ap. J. Letts. 515, L73 (1999)
10. Gammie, C.F., Ap. J. Letts. 522, L57 (1999)
11. Agol, E. and Krolik, J.H., Ap. J. 528, 161 (2000)
12. Krolik, J.H., in Explosive Phenomena in Astrophysical Compact Objects, Proceedings of the 1st KIAS Astrophysics Workshop, I. Yi and M. Rho, eds., New York: AIP, in press (2001)
13. Hawley, J.F., Ap. J. 528, 462 (2000)
14. Hawley, J.F. and Krolnik, J.H., Ap. J. in press (2001)
15. Armitage, P.J., Reynolds, C.S. and Chiang, J., Ap. J. in press (2001)
16. Turner, N.J. and Stone, J.M., Ap. J. Suppl. in press (2001)