Chiral vortical catalysis

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Abstract Gluon interaction introduces remarkable corrections to the magnetic polarization effects on the chiral fermions, which is known as the inverse magnetic catalysis. It is a natural speculation that the vorticity, which has many similar properties as magnetic field, would bring non-negligible contribution to the chiral rotational suppression. Using an intuitive semi-classical background field method we studied the rotation dependence of the effective strong interaction coupling constant. Contrary to the magnetic field case the rotation increases the effective coupling which would slow down the condensate melting with temperature. This could be named as the chiral vortical catalysis or inverse rotation suppression. Imposing such dependence on the 4-fermion coupling in the NJL model, we numerically checked this analysis qualitatively. The pseudo critical temperature is shown to rise with the rotation and approach saturation eventually which may be induced by the model cutoff.

1 Introduction

Emerging in chiral fermion systems, quantum effects induced by the strong magnetic and vortical fields have been widely studied in condensed matter [1–7], cold atom [8–13], astrophysics [14,15] and high energy nuclear physics [16,17] recently. In the laboratory several novel chiral effects have been observed in systems of cold atoms and condensed matter, such as chiral magnetic effect, magnetic catalysis and chiral vortical effect. Undoubtedly these experiments require strong field generators, delicate field controllers as well as novel material samples which could excite chiral collective modes. In high energy nuclear physics studies of magnetic and vortical effects are motivated by the extremely large (compare to light quark mass) background fields generated in relativistic heavy ion collisions and the hot system of fundamental chiral fermions – light flavor quarks in the produced quark–gluon plasma (QGP). As indicated by simulations of phenomenological models and measurements of hadron polarization at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), the peak magnetic field would exceed $10^{14}$ T and local rotation $10^{24}$ Hz in QGP [18–22]. And the remaining large amount of collision energy would heat up the QGP to hotter than 165MeV that is supposed to melt chiral condensate easily and turn light flavor dressed quarks to approximate chiral fermions. Therefore the chiral magnetic/vortical effects are expected to take place in the QGP doubtlessly. And corresponding measurable signals would be detected in the heavy ion experiments if strong fluctuations can be dealt with systematically and deliberately [23–26].

Theoretically these chiral effects can be roughly categorized into three groups. The first includes effects that can be understood by the polarization energy shift of chiral fermions in present of background field, such as the condensate catalysis or inhibition [17,27]. In the context of different systems, such effects usually involve energy modification of different fermion pairings, for example the fermion–antifermion pairing in the chiral magnetic catalysis, vortical inhibition and di-fermion pairing in superconductivity. The second is usually understood with the kinetic behavior of single chiral fermions by transport equations in the magnetic/vortical field [1,5,28–30]. They are all semi-classical expansions of the full quantum state in certain background fields and essentially induced by the non-trivial topological properties of chiral spinor’s wave function in the momentum space. This can be systematically studied with the relativistic quantum kinetic theory [31,32]. The famous chiral magnetic/vortical effects are in this group. They are straightforward physical results of single-particle equation of motion in a classical background field. Hence such effects are ubiquitous and expected to be almost the same in different systems, such as condensed matter, cold atom or QGP. Obviously in a strong interacting sys-

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tem these effects will be modified and more non-trivial phenomena may emerge. Go beyond the system of free fermions and consider more quantum corrections from interactions, we arrive at the last group. In the context of quantum chromodynamics (QCD) a typical example is the inverse magnetic catalysis (IMC) which was observed in lattice QCD simulation around a decade ago [33,34]. In such simulations the chiral condensate appears to melt more easily at larger background magnetic field. At the same time such field will also help to increase the condensate at low temperature, which is a purely single-particle result. Hence in IMC the chiral condensate behavior, especially around the crossover temperature, can not be completely understood by the magnetic polarization of chiral fermions (light quarks) any longer. The gluons, which do not interact directly with the background magnetic field, will bend the increasing curve of the critical temperature along magnetic field. Vortical and magnetic fields share many of characteristics [16,17]. Analogy to the magnetic catalysis and chiral magnetic effect in the group 1 and 2 we have similar effects known as chiral rotational suppression/inhibition and chiral vortical effects respectively. So what about the group 3? Will the gluons’ corrections behave non-trivially again in the rotational case? We will show some clues to this question in this work.

A strong background field will change the vacuum structure of a system drastically in both classical and quantum systems [17], e.g. in a uniformly rotating or magnetized system the eigen stats are Bessel or confluent hypergeometric functions [16]. The missing of translational symmetry will make the straightforward loop computations of the vertices much more difficult. In this work we adopt the very intuitive and novel method [35] to sidestep the loop integration involving complicated summations of special function series. The idea of this approach starts from computing the polarization energy of a certain system in present of semi-classical background color magnetic field by summing over all the eigen energy levels $E \sim \sum E_n$. Then the effective coupling constant will be obtained by extracting the effective permeability $\mu$ and the corresponding dielectric permittivity $\epsilon = \mu^{-1}$ by comparing the polarization energy to the standard form $E_{\text{vac}} = -4\pi \chi B^2/2$. This method has reproduced the famous asymptotic freedom formula successfully and been applied to study the coupling running at finite temperature and chemical potential [36–38]. Introducing such qualitative dependence into a effective model, the IMC has also been reproduced successfully [39]. In this work after a brief review of this method, we will first determine the rotation dependence of the effective coupling constant in a pure gluon system for simplicity, and then switch to a pure fermion system to study the chiral restoration at finite temperature with Nambu–Jona-Lasinio (NJL) model by imposing the same qualitative rotation dependence on the effective 4-fermion coupling constant $G$. It will be shown that the rotation will increase the coupling and thus make the condensate harder to melt with temperature increasing which appears to be completely contrary to the magnetic case.

2 An alternative derivation of the QCD running coupling

We first briefly review the calculation of Refs. [35–38]. In these works a background color magnetic field is introduced to the quark–gluon system to help to extract the coupling running behavior as a function of the field strength $B$. Taking the QED system for simplicity, first by noticing the non-relativistic potential $V(r)$ between two static charges is $V(r) = q_1 q_2 / 4 \pi \epsilon r = \alpha / r$, and the effective coupling constant is defined

$$a_{\text{eff}} = \alpha / \epsilon$$

where $\epsilon$ is the dielectric permittivity which is supposed to depend on the background field strength. And the $\alpha$ could be treated as the coupling at zero background field. Then the question is how to obtain a scale-dependent $\epsilon$. Here such dependence of $\epsilon$ is introduced by considering its relation with a background-field-dependent magnetic permeability $\mu$, that is $\mu \epsilon = 1$. In a background magnetic field the field-dependent permeability $\mu$ can be extracted by computing the polarization energy

$$E = \frac{1}{2} [4 \pi \chi] V B^2 = \sum_{n,i=\{\text{fermions}\}} E_n^i$$

where the magnetic susceptibility $\chi = \chi(B)$ is field-dependent and relates with the permeability as $\mu(B) = 1 + 4 \pi \chi(B)$. Once the summation over energy levels of charged fermions $E_n^i$ is completed the field-dependent $\chi(B)$ will be obtained. In the QCD case the procedure is almost the same by replacing the background field with the color magnetic field $A_8^\mu$ and summation degrees of freedom with quarks and gluons. In such a color magnetic field the eigen functions is the same as those in the QED case up to several color factors. In the original reference [35] these computations can reproduce the leading order results of QED and QCD running couplings $\alpha(k^2)$ by eventually replacing the field-dependence $2gB$ with the momentum scale $k^2$.

This idea appears quite different from the running coupling calculation in standard quantum field theory books which using the loop renormalization procedure. In the standard approach the coupling is modified by the quark–gluon and gluon–gluon interaction loops. As a result it depends on the scale of vertex momentum instead of some background fields. Why can the background field approach give the correct result? Here we could convince ourselves by answering two questions. First why can the approach give the correct
running form which depends on the \(2gB\). Second how to justify the replacement of the \(2gB\) with \(k^2\). The first one can be understood by noticing that only the leading order result can be obtained with the background field approach. It is not a systematic method as the renormalization procedure. So what is the main quantum correction to the physical quarks and gluons which interact with each other at a vertex by exchanging virtual/intermediate gluons? If we neglect the back action to the virtual gluons, the full quantum corrections to the physical particles are all encoded in their equations of motion. And these equations are the same as those in a background gauge field. As the eigen system is actually equivalent to the eigen equation, we can convince ourselves that by taking all the energy levels of quarks and gluons into account there is indeed a large possibility to obtain the leading order loop corrections to the vertex. For the same reason we should expect that two-loop and higher corrections can not be obtained because more complicated virtual processes are all lost in this method. And in order to translate the only scale here, the background field \(2gB\), to our familiar \(k^2\), we should firstly notice that the \(B \sim \partial A \sim kA\) and \(\tilde{\alpha} \sim gA\) in the equation of motion for the quark field. And secondly the background field characterizes the energy of quark and gluons as

\[
E_{nk3} = \sqrt{k_3^2 + 2gB(n + 1/2 + s_3)}. \tag{3}
\]

These indicate that we can replace the \(2gB\) with \(k^2\) in our final results if we are only interest in the qualitative running behavior of the coupling. Here we list some key equations of the complete calculation in Refs. [35,36]. As mentioned in above the polarization energy is computed by summing all the energy levels at zero temperature:

\[
E = \left(-1\right)^{2s} \sum_{nk3} \sqrt{k_3^2 + 2gB(n + 1/2 + s_3)}. \tag{4}
\]

Making use of the density of states of the Landau levels the summation can be rewritten as

\[
\sum_{k3} \frac{V}{4\pi^2 (gB)} \int dk_3. \tag{5}
\]

With the help of a Euler sum rule

\[
\sum_{n1}^{n2} f(n + 1/2) = \int_{n1}^{n2} f(x)dx - \frac{1}{24} f'(x)\bigg|_{n1}^{n2} \tag{6}
\]

and introducing a cutoff \(\Lambda\) to the integration of \(k_3\) the polarization energy can be obtained as

\[
E = -\frac{V(gB)^2}{2\pi^2} \left[\frac{-1}{24} \sum_{s_3} \left(\frac{s_3^2}{2} + \frac{1}{24}\right) \ln\left(\frac{\Lambda^2}{2gB}\right)\right]. \tag{7}
\]

In the QCD case collecting all the contributions of quarks and gluons gives the result for the susceptibility as

\[
4\pi \xi = -g^2 \frac{11N_c - 3N_f}{128\pi^2} \ln\left(\frac{2gB}{\Lambda^2}\right). \tag{8}
\]

Substituting it into the \(\alpha_{\text{eff}}\) we will find that the usual leading order renormalization result is reproduced as expected. As our arguments in above this background field treatment is not a systematic method to calculate the quantum corrections perturbatively. The leading order is the best result it can produce according to our understanding. However it is a very good approach to study the running coupling when the space-time translational symmetry is broken. In such cases different special functions will involve into the standard perturbation loop integration which may make the analytical calculation almost impossible [45] to complete. And the rotating system is one of such cases. Here we will adopt this background field method to study the qualitative behavior of the running coupling and furthermore explore its impact on the chiral phase transition at finite temperature and rotation speed.

### 3 Rotating vector field in color magnetic field

In IMC, although the chiral condensate computation is highly non-perturbative, it could be qualitatively understood by noticing that light quark loops in the soft gluon polarization function would also be polarized by the magnetic field and quantitatively confirmed by the functional renormalization group computation [40,41] non-perturbatively. With effective models, such as NJL model, it has also been reproduced by introducing a magnetic-dependent effective coupling constant. It is a more intuitive approach and the coupling running behavior has also been confirmed qualitatively by the straightforward computations of loop corrections. In the approach of effective models the IMC is induced by the coupling weakening by the background magnetic field. Intuitively it means the condensate is easier to melt when temperature increases because the coupling is smaller at a larger background magnetic field, although the field can enhance the pairing probability and thus increase the amplitude of condensate at a given temperature [39] at the same time. This picture is confirmed explicitly by the study with the NJL model.

For simplicity we will follow the effective model approach which requires the running behavior of the coupling constant as a function of rotation firstly. As mentioned above, we will extract it in a pure gluon system in this work. The equations of motion (EOM) of the vector particle under rotation has been studied in [42–45]. The general principle of relativity suggests in the rotating frame the Lagrangian density should be almost the same as the free one except replacing the usual derivative with the covariant one which includes extra terms.
of the connections from the curved metrics in the local rest frame. And also because of working in a local inertial frame we can continue using $\mu e = 1$ in the next section. The EOM of the massive vector field reads

$$\partial_t f_{\mu \nu} - m^2 A_\nu = \Delta A_\mu - m^2 A_\mu = 0 \quad (9)$$

which gives $A_\mu = 0$ for the transverse polarization components $A_{TE}$ and $A_{TM}$. And

$$\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - 2 \nu \partial_{\mu} \partial_{\nu} A_{\lambda} + (\partial_{\mu} \nu - \partial_{\nu} \nu) \partial_{\lambda} A_{\nu} + v_j \partial_j (\nu \partial_n A_{\lambda} + 2 \epsilon_{jmn} \omega_m A_n) - (\omega^2 A_{\nu} - \omega_n \omega_{\nu} A_n) + m^2 A_{\nu} = 0 \quad (10)$$

for the $\vec{A}$ part, where $f_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$ and $\vec{v} = \vec{\omega} \times \vec{r}$ with $\vec{\omega}$ the angular velocity vector and $m$ the mass of the vector field. By considering the uniform rotation around the z-axis $\vec{\omega} = \omega \vec{e}_z$, the EOM indicates in cylindrical coordinates $\vec{A} = A_\rho \vec{e}_\rho + A_\phi \vec{e}_\phi + A_z \vec{e}_z$ should satisfy

$$(-E^2 + \Omega^2 + m^2 - 2E \omega n - \omega^2 n^2) A_{\rho, \phi, z} = 0 \quad (11)$$

where we have already assumed the angle and z-axis dependence of the eigen solutions are separated as $A_{\rho, \phi, z} = e^{i n \phi} e^{i k_z z} a_{\rho, \phi, z}(\rho)$. The $\Omega$ is the eigen value of the $\Delta = \nabla^2$ operator, i.e., $-\Delta \vec{A} = \Omega^2 \vec{A}$. Obviously the $\omega$ and $\omega^2$ terms are rotation relevant ones. The eigen energies could be read as $E_0 = \pm \sqrt{\Omega^2 + m^2} - n \omega$. This equation shows that no matter what the radial solutions in static($\omega = 0$) are like, the rotation terms will only introduce a $n \omega$ to energy levels if the angle dependence of the solutions are chosen as $e^{i n \phi}$ in cylindrical coordinates. In [45] we have already proven that when $m = 0$ and $\omega = 0$ the propagator of the vector field could be reduced to the usual gauge field propagator by discarding the $m^{-\sigma}$ terms and the longitudinal polarization component $A_z$. In the following we will set $m = 0$ and focus on the $\vec{A}$ part to extract the eigen energies of the gauge field.

We further introduce the constant background magnetic field $\vec{B} = B \vec{e}_z$ as [35] which does not couple with the rotation. The solutions are well-known Landau levels in the asymmetric background magnetic field case, i.e., $\vec{A} = (0, Bx, 0)$. However we should work out eigen solutions in the symmetric scheme($\vec{A} = \frac{1}{2}(Bx, Bx, 0)$) which could permit us to choose azimuthal eigen functions as $e^{i n \phi}$ in cylindrical coordinates. The explicit form of the equation is

$$\left[ \frac{1}{\rho} \partial_\rho + \frac{1}{4} \rho \partial_\rho - \frac{s^2 B^2}{\rho^2} - \frac{(n \pm 1)^2}{\rho^2} + (n \omega B \pm 3 g B) \right] A_{\pm} = \left[ -(E + n \omega)^2 + k_z^2 \right] A_{\pm} \quad (12)$$

where $A_{\pm} = A_{\rho, \pm} i A_{\phi}$ and the $\pm$ terms corresponding to the two spin components of gauge field. This solution has also been studied in [16] for a fermionic system under rotation and the real magnetic background field. Here as the original idea [35] suggests, the introduced magnetic field is actually the background color field $A_{\mu}^c$ for the SUc(2) gauge field and $A_{\mu}^c$ for the SUc(3) case. That is why the background field has interaction with the dynamic gauge fields which carry no electric charges. The solutions are the Laguerre functions $L_{l+1,±1}(g B \rho^2/2)$ or the general confluent hypergeometric functions and the eigen energies are labelled by the $l + 1/2 \pm 1 = \frac{E^2 - m^2 - k_z^2}{2 g B}$ where $E_n = E + n \omega$. In static case($\omega = 0$) the system transverse size could be infinite and the boundary condition $A(r \to +\infty) = 0$ requires the solutions to be Laguerre polynomials and thus $l$ non-negative integers. As the result eigen energies become the well-known Landau levels $E = \sqrt{2g B(l + 1/2 \pm 1) + k_z^2}$.

It appears that in a rotating system the rotational polarization makes Landau levels $E = \sqrt{2g B(l + 1/2 \pm 1) + k_z^2} - n \omega$ lose the lower bound. It is actually not a serious problem because the system should be strictly bounded by the light speed limit if the rotation is uniform. Otherwise if the system size is too large comparing to the background rotation, one would expect the uniform rotation field will corrupt into several small ones with the same or lower angular velocity, which appears as a lattice of vorticity. By the way the boundless energy will actually bring no serious issues to the chiral condensate computation because of the Bessel functions’ asymptotic behaviors in finite temperature cases. For the zero temperature case the boundary will screen the chiral condensate from all the rotational effects which is known as no rotation of the vacuum. In order to make the discussion more rigorous, we consider the boundary explicitly in this work.

In the uniformly rotating system we set the largest radius $\rho = R$ and the corresponding largest vorticity is $\omega = R^{-1}$ because of the light speed limit. At the boundary the wave function should be zero $L^m_{l+1/2 \pm 1}(g B R^2/2) = 0$. Thus at given $n$, the $l$ should be chosen to make the $g B R^2/2$ locate at one of the zeros of Laguerre function $L^m_{l+1/2 \pm 1}(z)$. This means with fixed $n$ and $R$ there are infinite number of zeros $l_m, m = 1, 2, \ldots, \text{ and each } l_m$ is not necessary an integer but an implicit function of $n$ and $R$. However the eigen energies are in the same form as the usual Landau levels $E = \sqrt{2g B(l(n, R) + 1/2 \pm 1) + k_z^2} - n \omega$ with $\omega < R^{-1}$. Numerically we can find the energies are always positive at any $n$.

To our knowledge there is no analytical results of $l_m$ for equations $L^m_{l+1/2 \pm 1}(g B R^2/2) = 0$ with $m = 1, 2, \ldots$. In order to obtain the energy levels in present of classical color magnetic field, we choose the smallest $l_m=1$ (lowest level) for each $n$ at different sizes $R$ and fit them to obtain a analytical expression as $a(R) + b(R)n + c(R)n^2$. Although it appears a brute approximation numerically, this is mainly motivated by the Boltzmann factor $\exp(-E/T)$ suppression in the finite temperature case. Hence if we study the phase struc-
three typical curves at
able. Technically a more reliable way is to solve out the zeros

\[ Fitting\ coefficients \]

\[ Fig.\ 1 \]

\[ Fig.\ 2 \]

\[ ture\ along\ temperature,\ this\ approximation\ is\ more\ acceptable.\ Technically\ a\ more\ reliable\ way\ is\ to\ solve\ out\ the\ zeros \]

\[ l(R) = 0.2^{-1}, 1.0^{-1}, 1.8^{-1}(gB)^{-1/2}, n \] with \( l = a(R) + b(R)n + c(R)n^2 \)

\[ a, b\ and\ c \]

\[ R^{-1} \]

\[ 0.01a(R) \]

\[ 0.02b(R) \]

\[ c(R) \]

\[ Noting\ that\ at\ large\ n\ the\ Landau-level-like\ part \]

\[ \sqrt{2gB(l(n, R) + 1/2 \pm 1) + k_z^2} \geq \sqrt{1.08nR^{-1}} \] which is always larger than the rotational polarization part \( n\omega \) where \( \omega \leq R^{-1} \) because of light speed limit. This preserves the energy levels are always lower-bounded in the case of finite transverse size.

As an intuitive method we introduced several approximations in this work in this and following sections. And it would be helpful to summarize them here. First mainly for technical problem we only keep the smallest eigen energy corresponding to \( l_1(R, n) \) at given \( n \) and \( R \). We expect the chiral condensate in finite temperature (in next section) will suffer from no remarkable impact of this. Second we solve the chiral condensate (in next section) in local potential approximation, i.e. \( \delta_r \mathcal{M}(r) = 0 \), which will not changed its qualitative behavior in finite temperature case. Finally as the most chiral condensate works with NJL model we adopt the mean field approximation.

\[ 4\ Rotation\ dependence\ of\ the\ running\ coupling \]

In principle the coupling running is computed straightforwardly by the one-loop correction of the \( \bar{\psi}\psi A \) vertex. However technically it is almost impossible to complete such computation with propagators in the cylindrical coordinates. In this case it is difficult to complete the loop integrals/summations analytically and further extract the \( \gamma^\mu \) coefficient as the vertex correction because of no good enough summations results for multi-Bessel or Laguerre functions’ products involving in the computation. Alternatively as a preliminary study for a qualitative result we follow the approach in Sect. 3 to extract the rotation dependence of the coupling by summing all the energy levels of single gluon. Adopting the fitting result in Eq. 13 and including its density of state \( \frac{2}{gBR^2}(15.5 + 1.08n) \) we completed the same summation over energy levels in Sect. 3 to yield the polarization energy density of a rotating gluon system in a background color field as

\[ E_{vac} = \frac{1}{8\gamma^2} \int dk_z \sum_n \frac{15.5 + 1.08n}{gBR^2} \delta(\Lambda - E) \times \left( \sqrt{2gB(l(n, R) + 1/2 \pm 1) + k_z^2 - n\omega} \right) \]

where the \( \Lambda \) is the ultra-violate cutoff. The longitudinal momentum \( k_z \) can be integrated out analytically. And we further used the Euler summation formula to complete the summation over \( n \). And we follow the same regularization procedure in above to remove the \( \Lambda \) and large \( n \) divergent terms. Up to the \( R^{-2} \) terms we obtain the color magnetic polarization energy density as
Temperature and choose a strong interacting system of many chiral fermions at finite temperature. In Refs. [33,36–38] this method has been extended to finite temperature, density and magnetic field systems. In the following we consider the coupling constant. For more technical details please refer to Ref. [33]. In this case one should consider both the background color magnetic and real magnetic field with this method. Because the real magnetic field only interact with charged fermions, one can explicitly separate these two fields in the running coupling calculation and eventually obtain both the QCD non-trivial running behavior (from the color magnetic field) and the real magnetic field modification to the QCD non-trivial running behavior (from the color magnetic field). In [33,36–38] this method has been extended to finite temperature, density and magnetic field systems. In the following we consider the coupling constant increases with angular velocity. It shows in a fixed transverse size system the coupling constant increases with angular velocity. Because the real magnetic field only interact with charged fermions, one can explicitly separate these two fields in the running coupling calculation and eventually obtain both the QCD non-trivial running behavior (from the color magnetic field) and the real magnetic field modification to the coupling constant. For more technical details please refer to Refs. [33,36–38]. In the following we consider a strong interacting system of many chiral fermions at finite temperature and choose \((k^2) R^2 \sim 1\). With effective model we will study the crossover temperature of chiral condensate as a function of temperature and rotation speed.

\[
E = -\frac{(gB)^2}{8\pi^2} \left[ \ln(R\Lambda) + \frac{1.08}{12gBR^2} \frac{\omega}{\Lambda} \right]
\]

\[
= -\frac{1}{2} \frac{4\pi}{1+4\pi} \chi B^2
\]

where \(\chi\) is the effective magnetic susceptibility. Hence the effective dielectric permittivity is obtained

\[
\epsilon = \frac{1}{1+4\pi} \chi = \left[ 1 + \frac{g^2}{4\pi} \left( \ln(R\Lambda) + \frac{1.08}{12gBR^2} \frac{\omega}{\Lambda} \right) \right]^{-1}
\]

Substituting it into the definition of effective coupling in Eq. \ref{eq:1} we get

\[
\alpha_{\text{eff}} = \alpha \left[ 1 + \frac{\omega}{\pi} \ln(R\Lambda) + \frac{1.08}{6(k^2)R^2} \frac{\omega}{\Lambda} \right].
\]

Here we have replaced the background field \(2gB\) with the average momentum scale of the system \((k^2)\) according to the argument in Sect. 3. It shows in a fixed transverse size system the coupling constant increases with angular velocity. And as a byproduct it is clear that in static case the coupling decreases with \(R\), which is consistent with the famous asymptotic freedom. In [33,36–38] this method has been extended to finite temperature, density and magnetic field systems. In the following the chiral condensate behavior will be studied at finite temperature. And we only take the qualitatively rotation dependence in Eq. \ref{eq:18} for a preliminary study and explore the existence of a non-trivial impact on the chiral transition. For the interesting IMC topic one can refer to the short review [33]. In this case one should consider both the background color magnetic and real magnetic field with this method. Because the real magnetic field only interact with charged fermions, one can explicitly separate these two fields in the running coupling calculation and eventually obtain both the QCD non-trivial running behavior (from the color magnetic field) and the real magnetic field modification to the coupling constant. For more technical details please refer to works in Refs. [33,36–38]. In the following we consider a strong interacting system of many chiral fermions at finite temperature and choose \((k^2) R^2 \sim 1\). With effective model we will study the crossover temperature of chiral condensate as a function of temperature and rotation speed.

5 Chiral rotational catalysis in NJL model

The coupling enhancement by rotation suggests that the chiral condensate would melt more slowly with temperature. Equivalently it is expected that the critical temperature of chiral restoration will increase under rotation. Comparing to the IMC we name this phenomenon as chiral vortical catalysis. In order to show this numerically we choose the NJL model with traditional parameters \(\Lambda = 0.65\) GeV, \(m = 0.005\) GeV and set the 4-fermion coupling running as

\[
G(\omega) = G_0 \left( 1 + 0.32 \frac{\omega}{\Lambda_{\text{NJL}}} \right)
\]

where \(G_0 = 4.93\) GeV\(^{-2}\). Adopting the usual mean field \(M = m - 2\langle \bar{\psi}\psi \rangle\) and local potential approximation (LPA), the gap equation for the chiral condensate has already been studied in [27]. We write it down directly as

\[
M = m - 4G M \sum_{nk,ks} \frac{2n_f(E_{kn}) - 1}{E_k} \times (J^2_m(k_t r) + J^2_{m+1}(k_t r))
\]

where \(J^2_m = 1/8\pi \int k_t dk_t dk_z \sum_{n=-\infty}^{\infty}\) and \(E_{kn} = \sqrt{k_t^2 + k_z^2 + M^2 - (n+1/2)\omega}\). Going beyond LPA the self-consistent position dependence of the condensate can be studied as [46]. If we only focus on the position averaged condensate, it shows that the LPA’s results qualitatively agree with the self-consistent results. Here we also consider the finite size case where the integral over \(k_t\) should be replaced by summation over the zeros of \(J_m(k_t R) = 0\). And we choose \(R = 2\) GeV\(^{-1}\) and compute the condensate at \(r = 0.1\) GeV\(^{-1}\).

We plot both the chiral susceptibility (Fig. 3) and the corresponding pseudo critical temperatures (Fig. 4). Because of the small current quark mass chiral restoration is a crossover and the pseudo critical temperature is defined as the peak location of chiral susceptibility. As expected, from the Fig. 3 one can find that the peak moves towards right when rotation becomes faster. By plotting the pseudo critical temperature as the function of rotation speed the Fig. 4 shows the catalysis more clearly. It means that as the rotation becomes faster the stronger coupling make the condensate more and more...
difficult to melt at finite temperature. While in the constant coupling case $T_{pc}$ decreases monotonously. It is the usual result because the rotation will reduce the condensate amplitude. However one should treat the $T_{pc}$ saturation in Fig. 4 carefully because the rotation speed has already approached the momentum cutoff of NJL there.

6 Summary and discussion

Using the background field method we studied the rotation dependence of the effective strong interaction coupling. It is shown that the effective coupling will become larger when the rotation becomes faster which is contrary to the magnetic field case. As the physical result the stronger coupling will slow down the condensate melting along temperature. This could be named as the chiral vortical catalysis or inverse rotational suppression. Introducing the running behavior to the coupling of the NJL model, we numerically computed the chiral condensate and showed this catalysis effect in the mean field approximation. The pseudo critical temperature is confirmed to rise with rotation and approaches saturation eventually which may be induced by the model cutoff.

In the context of QCD strong interaction coupling is crucial in different systems, from the phase structure in thermalized systems to the hadron production during the heavy ion collision. The rotation dependence study should be further extended to different number of colors and light quark flavors as well as different thermal conditions, such as finite temperature, density and magnetic field cases. One potential measurable signal could be the vector $\phi$ and $K^{*0}$ polarization in heavy ion collisions [47–51]. In these surprising polarization measurements of vector hadrons with different strangeness number, the rotation and the flavor dependence of coupling may play an important role in the hadronization process. We will extend the study to these problems with both this framework and other systematic non-perturbative approaches, such as functional renormalization group, in future. Apart from the high energy topics, the non-abelian gauge interaction has also been realized in the cold atom and condensed matter systems. We expect the non-trivial coupling running behavior would produce more interesting physics in such systems both theoretically and experimentally because of much better maneuverability of them. Finally one should note that the rotating QCD system has no sign problem although the rotation relevant terms appear as a effective chemical potential in the dispersion relation. This means it could be simulated and checked with lattice QCD as the magnetic case. We are very glad to be informed that a similar trend has already been observed in recent lattice QCD researches [52,53].

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References

1. K. Fukushima, D.E. Kharzeev, H.J. Warringa, Phys. Rev. D 78, 074033 (2008). https://doi.org/10.1103/PhysRevD.78.074033. arXiv:0808.3382 [hep-ph]
2. A.A. Zyuzin, A.A. Burkov, Phys. Rev. B 86, 115133 (2012). https://doi.org/10.1103/PhysRevB.86.115133
3. D.T. Son, N. Yamamoto, Phys. Rev. Lett. 109, 181602 (2012). https://doi.org/10.1103/PhysRevLett.109.181602. arXiv:1203.2697 [cond-mat.mes-hall]
4. M.M. Vazifeh, M. Franz, Phys. Rev. Lett. 111, 027201 (2013). https://doi.org/10.1103/PhysRevLett.111.027201
5. D.E. Kharzeev, Prog. Part. Nucl. Phys. 75, 133–151 (2014). https://doi.org/10.1016/j.ppnp.2014.01.002. arXiv:1312.3348 [hep-ph]
6. C. Rylands, A. Parhizkar, A.A. Burkov, V. Galitski, Phys. Rev. Lett. 126(18), 185303 (2021). https://doi.org/10.1103/PhysRevLett.126.185303. arXiv:2102.04371 [cond-mat.str-el]
28. A.V. Sadofyev, M.V. Isachenkov, Phys. Lett. B 697, 404–406 (2011). 
https://doi.org/10.1016/j.physletb.2011.02.041. arXiv:1010.1550 [hep-th]

29. A. Avkhadiev, A.V. Sadofyev, Phys. Rev. D 96(4), 045015 (2017). 
https://doi.org/10.1103/PhysRevD.96.045015. arXiv:1702.07340 [hep-th]

30. X.G. Huang, A.V. Sadofyev, JHEP 03, 084 (2019). 
https://doi.org/10.1007/JHEP03(2019)084. arXiv:1805.08779 [hep-th]

31. N. Weickgenannt, X.L. Sheng, E. Speranza, Q. Wang, D.H. Rischke, Phys. Rev. D 100(5), 056018 (2019). 
https://doi.org/10.1103/PhysRevD.100.056018. arXiv:1902.06153 [hep-ph]

32. J.H. Gao, Z.T. Liang, Phys. Rev. D 100(5), 056021 (2019). 
https://doi.org/10.1103/PhysRevD.100.056021. arXiv:1902.06510 [hep-ph]

33. A. Bandyopadhyay, R.L.S. Farias, Eur. Phys. J. ST 230(3), 719–728 (2021). 
https://doi.org/10.1140/epjs/s11734-021-00023-1. arXiv:2003.11054 [hep-ph]

34. G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fedor, S.D. Katz, A. Schafer, Phys. Rev. D 86, 071502 (2012). 
https://doi.org/10.1103/PhysRevD.86.071502. arXiv:1206.4205 [hep-lat]

35. N.K. Nielsen, Am. J. Phys. 49, 1171 (1981). 
https://doi.org/10.1119/1.12565

36. R.A. Schneider, Phys. Rev. D 66, 036003 (2002). 
https://doi.org/10.1103/PhysRevD.66.036003. arXiv:hep-ph/0202026

37. R.A. Schneider, Phys. Rev. D 67, 057901 (2003). 
https://doi.org/10.1103/PhysRevD.67.057901. arXiv:hep-ph/0210281

38. R.A. Schneider, arXiv:hep-ph/0303104

39. R.L.S. Farias, K.P. Gomes, G.I. Krein, M.B. Pinto, Phys. Rev. C 90(2), 025203 (2014). 
https://doi.org/10.1103/PhysRevC.90.025203. arXiv:1404.3931 [hep-ph]

40. N. Mueller, J.M. Pawlowski, Phys. Rev. D 91(11), 116010 (2015). 
https://doi.org/10.1103/PhysRevD.91.116010. arXiv:1502.08011 [hep-ph]

41. J.O. Andersen, Eur. Phys. J. A 57(6), 189 (2021). 
https://doi.org/10.1140/epja/s10050-021-00049-y. arXiv:2102.13165 [hep-ph]

42. L. Dai, M. Kaminkowski, D. Jeong, Phys. Rev. D 86, 125013 (2012). 
https://doi.org/10.1103/PhysRevD.86.125013. arXiv:1209.0761 [astro-ph.CO]

43. X.G. Huang, P. Mitkin, A.V. Sadofyev, E. Speranza, JHEP 10, 117 (2020). 
https://doi.org/10.1007/JHEP10(2020)117. arXiv:2006.03591 [hep-th]

44. M.N. Chernodub, A. Cortijo, K. Landsteiner, Phys. Rev. D 98(6), 065016 (2018). 
https://doi.org/10.1103/PhysRevD.98.065016. arXiv:1807.10705 [hep-th]

45. T. Xu, Y. Jiang, Chin. Phys. C 45, 114102 (2021). 
https://doi.org/10.1088/1674-1137/114102. arXiv:2106.04851 [hep-th]

46. J.R. Cho, Y. Jiang, Phys. Rev. D 100(10), 105020 (2019). 
https://doi.org/10.1103/PhysRevD.100.105020. arXiv:1901.00804 [nucl-th]

47. B. Mohanty (ALICE), EPJ Web Conf. 171, 16008 (2018). 
https://doi.org/10.1051/epjconf/201817116008. arXiv:1711.02018 [nucl-ex]

48. C. Zhou, Nucl. Phys. A 982, 559–626 (2019). 
https://doi.org/10.1016/j.nuclphysa.2018.09.009

49. B. Mohanty (ALICE), PoS LHCHEP2020, 555 (2021). 
https://doi.org/10.22323/1.390.0555. arXiv:2012.04167 [nucl-ex]

50. V.M. Shapoval, P. Braun-Munzinger, Y.M. Sinyukov, Nucl. Phys. A 968, 391–402 (2017). 
https://doi.org/10.1016/j.nuclphysa.2017.09.002. arXiv:1707.06753 [hep-ph]

51. X.L. Sheng, L. Oliva, Q. Wang, Phys. Rev. D 101(9), 096005 (2020). 
https://doi.org/10.1103/PhysRevD.101.096005. arXiv:1910.13684 [nucl-th]

52. V. Braguta, A. Y. Kotov, D. Kuznedelev, A. Roenko, PoS LATTICE2021, 125 (2022). 
https://doi.org/10.22323/1.396.0125. arXiv:2110.12302 [hep-lat]

53. V.V. Braguta, A.Y. Kotov, D.D. Kuznedelev, A.A. Roenko, Phys. Rev. D 103(9), 094515 (2021). 
https://doi.org/10.1103/PhysRevD.103.094515. arXiv:2102.05084 [hep-lat]