Nonlinear mechanics with suspended nanomembranes

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Abstract – We study the nonlinear dynamics induced in suspended nanomembranes by their clappings. The nonlocal character of the nonlinearity is demonstrated via intermode couplings. We also monitor the resonator phase-space trajectory and characterize its mechanical response in the presence of a strong pump excitation. We observe a shift in the oscillation frequency and phase conjugation of the mechanical mode. Such nonlinear effects are inherent to any submicron-scale mechanical resonator and are expected to play a role in their quantum dynamics as well.

Optomechanical systems close to their quantum ground state (QGS) \cite{1,2} and nonlinear nanoelectromechanical systems (NEMS) are two hot topics of current physics research. Demonstrating the QGS allows to shed new light on quantum coherent effects in meso- and macroscopic systems, whereas NEMS operated in a nonlinear regime \cite{3} are used either to demonstrate the underlying physics of shear nonlinear effects \cite{4–6} or as RF amplifiers, with sensitivity improvement both at the classical \cite{7} or quantum noise level \cite{8,9}. Nonlinear effects could be taken advantage of in demonstrating quantum dynamics as well \cite{10}. As high reflectivity and low mass are crucial features to improve optomechanical coupling towards the QGS, we have designed, fabricated and characterized photonic-crystal (PhC) nanomembranes \cite{11}, at the crossroad of both topics. The system is a monolithic indium-phosphide PhC nanomembrane clamped by four tethers. A 95\% reflectivity at normal incidence is obtained with a square-lattice PhC, without any optical coating.

In this letter we investigate the nonlinear dynamics of this device induced by its clappings. We demonstrate its nonlocal character and we study its underlying dynamics, both by monitoring the phase-space trajectory of the resonator and by characterizing the mechanical response in the presence of a strong pump actuation. We show in particular the emergence of a phase conjugate mechanical response to a weaker probe actuation. Our results are relevant to understand the nonlinear features of the PhC membranes vibrations, and possibly to look for nonlinear signatures of their quantum dynamics \cite{10}.

The 10 × 20 μm\textsuperscript{2} InP membrane is 260 nm thick. It is clamped by four 210 nm × 500 nm cross-section tethers located at nodes of the membrane vibration modes in order to reduce clamping losses \cite{12}. Details about the mechanical and optical properties, as well as the fabrication process are described elsewhere \cite{11,13}. The mechanical modes under scrutiny here (see fig. 1) are flexural modes of the tethers, either symmetrical (mode 1) or anti-symmetrical (mode 2). Note the membrane itself is not warped, the stress being only located in the clappings. We use in the following different membranes with resonance frequencies Ω/2π for the modes of interest varying from 782 kHz to 1057 kHz, depending on the tether length, from 6 to 12 μm. The typical mass is of a hundred of picograms and the mechanical quality factor is $Q \approx 5000$, mainly limited by surface effects and phonon scattering on impurities from unintentional doping.

As routinely observed in NEMS, displacements induce a stress larger than the intrinsic one, yielding to a nonlinear behaviour with displacement amplitudes only at the nanometre level. To probe the mechanical response of the membrane, the sample is actuated by a piezoelectric stack, and the membrane displacement is monitored by a Michelson interferometre. The sample is operated in
a vacuum chamber at $10^{-2}$ mbar to prevent air viscous damping and squeezed film effects. A network analyser drives the sample and monitors the resulting displacement spectrum. Figure 2 (left) shows the forced oscillation amplitude $\tilde{x}_p[\Omega_p]$ of the membrane as a function of the piezoelectric actuation frequency $\Omega_p$. Linear to nonlinear transition is clearly visible on left curves a to e, obtained with an upward frequency sweep of the network analyser and for increasing actuation powers (10 dBm to 30 dBm with a 5 dB step). Curves on the right are swept either up or down with a frequency generator at 30 dBm, and exhibit a typical hysteresis cycle as expected from the theoretical fit (dashed curve).

Fig. 2: (Colour on-line) Oscillation amplitude $\tilde{x}_p[\Omega_p]$ of the membrane as a function of the piezoelectric actuation frequency $\Omega_p$. Linear to nonlinear transition is clearly visible on left curves a to e, obtained with an upward frequency sweep of the network analyser and for increasing actuation powers (10 dBm to 30 dBm with a 5 dB step). Curves on the right are swept either up or down with a frequency generator at 30 dBm and exhibit a typical hysteresis cycle as expected from the theoretical fit (dashed curve).

The nonlinearity can be accounted for by the Duffing model in which the resonance frequency has a quadratic dependence with the displacement $x_p(t)$:

$$\ddot{x}_p(t) + \Gamma \dot{x}_p(t) + \Omega_0^2 [1 + \beta x_p^2(t)] x_p(t) = \alpha_p(t),$$

where $\alpha_p(t) = 2\alpha_p \cos(\Omega_0 t)$ is the driving force, $\Gamma = \Omega_0/Q$ the mechanical damping, and $\beta$ the nonlinearity strength.

At first order the forced displacement is mainly monochromatic at the actuation frequency $\Omega_p$, with a Fourier component given by (assuming $Q \gg 1$)

$$\tilde{x}_p[\Omega_p] = \frac{\tilde{\alpha}_p}{\Omega_0^2 (1 + \varepsilon) - \Omega_p^2 - i\Gamma \Omega_0}. \quad (2)$$

This expression corresponds to the usual Lorentzian resonance, except for a shift of the resonance frequency proportional to the mean square displacement through the term

$$\varepsilon = 3\beta |\tilde{x}_p[\Omega_p]|^2. \quad (3)$$

This shift is responsible for the spectra observed in fig. 2, as shown by the theoretical fit on the right (dashed curve). Although higher-order effects must be considered to accurately fit the displacement, values of $\beta$ can nonetheless be evaluated in the range of $10^{13}$ m$^{-2}$.

Also note that such a more complete derivation would lead to the emergence of odd harmonics of motion with an amplitude at least $\varepsilon \lessgtr 10^{-3}$ times smaller than the fundamental oscillation at $\Omega_p$.

Stable solutions of eqs. (2) and (3) correspond to the upper and lower branches of a bistable hysteresis cycle. As the network analyser only sweeps frequencies upward, curves in fig. 2 (left) explore the upper branch upward until they suddenly fall down to the lower branch. Both upward and downward frequency sweeps are shown in fig. 2 (right) for the 30 dBm actuation level obtained with a frequency generator and clearly exhibit a hysteresis cycle.

The nonlinearity arises from additional stress induced by large displacements and we expect nonlocal effects as intrinsic elastic parameters such as the Young modulus may be affected [14]. We have thus investigated the influence of one mode (mode 1, here at 782 kHz) on the natural frequency of another one (mode 2 at 837.5 kHz), using both the signal generator and the network analyser.

An upward frequency sweep is first produced by the signal generator with a 20 dBm power to activate the first mode in its nonlinear regime. In order to explore different points on the upper branch of the nonlinearity, the sweep is stopped at different frequencies $\Omega_p$, represented by the arrows in fig. 3 (left). While the drive of mode 1 remains at that frequency, we monitor the linear response of mode 2 using the network analyser with a 0 dBm actuation power. The central curves in fig. 3, obtained for increasing actuation frequencies $\Omega_p$, clearly show that the resonance frequency $\Omega_n$ of mode 2 is shifted as the displacement amplitude of mode 1 gets larger. Considering that mode 2 has a quality factor of 6000, its resonance shift is actually up to 170 times its intrinsic width. Also note that since the effective Young modulus increases, tethers get stiffer and the amplitude of mode 2 decreases. We also explore
Nonlinear mechanics with suspended nanomembranes

Fig. 4: (Colour on-line) Time evolution in phase space for the driven (blue) and free (red) regimes. Three cases are investigated, as sketched on top of each column, from left to right: linear regime actuated at resonance (5 dBm at 1057 kHz), out of resonance (9 dBm at 1056 kHz), and nonlinear regime (29 dBm at 1058 kHz). Curves from top to bottom represent the phase-space trajectories and the amplitude and frequency evolutions.

The frequency behaviour drastically changes between the linear and nonlinear regimes. In the former case, even for a nonresonant actuation, the frequency instantaneously jumps back to its natural value when the drive is released, as can also be inferred from the straight line trajectories in phase space. This result is actually expected for a linear harmonic oscillator, as its free evolution for $t > 0$ only depends on the initial position and speed at $t = 0$, and not on the previous forced actuation frequency.

In the nonlinear case, the frequency hops towards the top of the upper bistability branch and then slowly relaxes down to the natural frequency as the nonlinearity fades out. This can be understood from eq. (2) which shows that for a given oscillation amplitude $\tilde{x}_p$, the preferred oscillation frequency is $\Omega_0 (1 + \varepsilon/2)$. As the oscillation amplitude and then the nonlinear coefficient $\varepsilon$ (eq. (3)) exponentially decrease with time, one thus expects the same behaviour for the free-oscillation frequency.

To confirm the nonlinear shift of the preferred oscillation frequency, we have studied the mechanical response to a weak probe while the mode is simultaneously set in the nonlinear regime by a strong pump drive. The setup is similar to the one used for the two-mode actuation (fig. 3), except that both pump and probe are driving the same mechanical mode, close to its natural frequency.

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appear: when actuated at a frequency \( \Omega_s \), the pump-probe combination leads to the concomitant generation of a mechanical response at frequencies \( \Omega_p \) and \( 2\Omega_p - \Omega_s \), symmetrically disposed on both sides of the pump frequency.

This nontrivial behaviour can be understood from the Duffing equation (1), adding a probe source term \( \alpha_s(t) = 2\alpha_s \cos(\Omega_s t) \). The cubic term in the Duffing equation is then responsible for the mixing of pump and probe signals, in a way similar to a \( \chi^{(3)} \) nonlinearity in optics. Indeed, writing the total displacement in the presence of the probe as the sum of the un perturbed displacement \( x_p(t) \) due to the pump only (eq. (2)) and an additional displacement \( x_s(t) \), we obtain the following equation (assuming its amplitude to be much smaller than the one of the unperturbed motion \( x_p(t) \)):

\[
\ddot{x}_s(t) + \Gamma \dot{x}_s(t) + \Omega_s^2 x_s(t) + [1 + 3\beta x_s^2(t)] x_s(t) = \alpha_s(t). \tag{5}
\]

As \( x_p(t) \) oscillates at frequency \( \Omega_p \), the nonlinear term \( x_s^2(t) \) contains both a static term and an additional fast oscillation at \( 2\Omega_p \). The former is responsible for a modification of the linear mechanical response at frequency \( \Omega_s \) whereas the latter is formally similar to phase conjugation in optics and generates a response at \( 2\Omega_p - \Omega_s \). From (5) one gets the following coupled equations for the two Fourier components of \( x_s \) at frequencies \( \Omega_s \) and \( 2\Omega_p - \Omega_s \):

\[
\zeta[\Omega_s] \ddot{x}_s[\Omega_s] + \varepsilon \Omega_s^2 x_s[\Omega_s] = \alpha_s, \tag{6}
\]

\[
\zeta[2\Omega_p - \Omega_s] \ddot{x}_s[2\Omega_p - \Omega_s] + \varepsilon \Omega_s^2 x_s[\Omega_s] = 0, \tag{7}
\]

where \( \zeta[\Omega] = \Omega_s^2 (1 + 2\varepsilon) - \Omega_s^2 - i\Gamma \Omega_p \) is a Lorentzian denominator and we have assumed for simplicity \( \varepsilon K \) to be real. The resonances of \( \zeta[\Omega] \) lead to two phase conjugate responses at frequencies:

\[
\Omega_{s+} = \Omega_p (1 + \varepsilon), \quad \Omega_{s-} = 2\Omega_p - \Omega_s. \tag{8}
\]

Note the difference between \( \Omega_{s+} \) and the resonance frequency \( \Omega_p (1 + \varepsilon/2) \) in response to a single actuation (eq. (2)), equivalent to the difference obtained between self- and cross-phase modulations in Kerr media [15].

Dots in fig. 5 (right) show frequencies \( \Omega_{s+} \) and \( \Omega_{s-} \) deduced from spectra similar to the ones displayed in fig. 5 (left), as a function of the pump frequency \( \Omega_p \). Curves \( d \) and \( e \) are theoretical predictions obtained using eqs. (8) with \( \varepsilon \) estimated from the experimental nonlinear behaviour (eq. (2) and fig. 2). At low pump frequency \( \Omega_p < \Omega_0 \), the membrane is almost in a linear regime and we only observe one peak close to the natural frequency \( \Omega_0 \). At higher frequency, the two resonances are symmetrically located around \( \Omega_p \) (dashed line), in good agreement with theoretical predictions.

We have demonstrated nonlinear effects in the dynamics of PhC nanomembranes: bistability, intermodal tuning and the generation of phase conjugate modes which can be used to transfer energy from one mode to its conjugate. Pump-probe experiments have in particular allowed us to underline the nonlocal character of the Young modulus modification. Our results are crucial to understand the full nonlinear features of the PhC membranes. Such nonlinear behaviours are intrinsic to nano- and microscale systems [3], and may have dramatic consequences not only in the fields of optomechanics and nonlinear dynamics, with possible nonlinear signatures of the quantum dynamics [10], but in quantum optics as well, as intermode coupling can be used to perform QND measurements [16–18]. Optomechanical systems can also be used for classical or quantum information processing, using either their long mechanical coherence times to store information in a mechanical excitation [19], or their nonlinear character for nonvolatile memories [20].

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