Microlensing by brown dwarfs: the case of dark clusters

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Abstract

Experiments are looking for diffuse brown dwarfs in the dark galactic halo through the gravitational lens effect. If brown dwarfs are clumped in dark clusters, the event rate is not changed, but events are spatially clustered, and stars nearby a micro-lensed one are likely to be micro-lensed in a near future. Therefore, an intensive survey of the region where a micro-lensing occurred should reveal many other events.

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1 Introduction

The least exotic dark matter candidate is baryonic matter, under the form of compact objects such as very massive black holes or brown dwarfs (Carr, Bond & Arnett 1984). Two teams reported the possible detection of brown dwarfs in our galactic halo through gravitational micro-lensing (Alcock et al. 1993, Aubourg et al. 1993): the light of a star is amplified in a characteristic way when a brown dwarf crosses its line of sight (Paczyński 1986). Brown dwarfs could be clumped in dark clusters similar to globular clusters (Carr & Lacey 1987) and we explore the observational consequences of such dark clusters. We conclude that:

i) the event rate is not changed.

ii) micro-lensing events should cluster on a few spots on the sky.

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iii) stars which are near a lensed star are likely to be lensed in a near future by a brown dwarf of the same cluster.

2 Event rate

The gravitational lens effect is a consequence of the deflection of light rays by massive objects. When a brown dwarf comes at a distance $R$ to the line of sight to a star, the light from this star is amplified by a factor $A(R)$:

$$A(R) = \frac{R^2 + 2R_E^2}{R \sqrt{R^2 + 4R_E^2}}$$

(1)

where the Einstein radius $R_E$ is defined by:

$$R_E^2 = \frac{4GM_{bd} D_{bd}(D_{star} - D_{bd})}{c^2 D_{star}}$$

(2)

$M_{bd}$ is the mass of the brown dwarf, $D_{bd}$ its distance from the observer, and $D_{star}$ the distance of the star. The dimensionless impact parameter $u$ is defined as:

$$u(A) = \frac{R}{R_E} = \left[ \frac{2A}{\sqrt{A^2 - 1}} - 2 \right]^{1/2}$$

(3)

A micro-lensing event is detected if the amplification is larger than some threshold $A$. The event rate is the number $\Gamma(A)$ of detectable events per star and per unit time, and is given by the number of brown dwarfs which enter per unit time the micro-lensing “tube” of radius $R(A, D_{bd}) = u(A) R_E(D_{bd})$ around the line of sight to the star (Griest 1991):

$$\Gamma(A) = \int 2u(A) R_E(D_{bd}) V_{\perp} n_{bd}(D_{bd}) dD_{bd}$$

(4)

where $V_{\perp}$ is the transverse velocity of brown dwarfs, and $n_{bd}(D_{bd})$ is their number density at distance $D_{bd}$. The number density $n_{bd}$ is the same whether brown dwarfs are clustered or not. Therefore the event rate does not depend on the clustering.

We assume that the mass density profile of the halo is well described by an approximate isothermal distribution with a core radius $a = 5 \pm 3$ kpc (Bahcall & Soneira 1980, Caldwell & Ostriker 1981):

$$\rho(r) = \rho_\odot \frac{r^2 + a^2}{r^2 + a^2}$$

(5)

The halo mass density in the solar neighbourhood is estimated to be $\rho_\odot = 0.008 M_\odot/pc^3$ (Flores 1988), and the distance between the Sun and the galactic center is $r_\odot = 8.5$ kpc.
The possible mass \( M_{\text{bd}} \) of a brown dwarf ranges from \( 10^{-7} \, M_\odot \) (evaporation limit, De Rújula et al. 1992) to \( 10^{-1} \, M_\odot \) (hydrogen burning limit). Then:

\[
\Gamma(A) = \Gamma_0 \, u(A) \, \frac{V_\perp}{200 \text{ km/s}} \left[ \frac{0.1 \, M_\odot}{M_{\text{bd}}} \right]^{1/2}
\]  

(6)

where \( \Gamma_0 = 16 \times 10^{-14} \, \text{s}^{-1} \) for stars in the LMC. Table 1 shows the variations of \( \Gamma_0 \) with the core radius \( a \), for stars in the LMC and in the galactic center. We assumed that all brown dwarfs had the same transverse velocity \( V_\perp = 200 \, \text{km/s} \). It is possible to include the actual velocity distribution of the halo and to take into account the velocities of the star and of the Earth (Griest 1991), but this is an unnecessary complication at this level. The approximation of identical velocities is better for brown dwarfs inside a cluster than for diffuse ones, because the velocity dispersion \( V_{\text{bd}} \) inside a cluster of radius \( R_{\text{tidal}} \) is small compared to the transverse velocity \( V_\perp \simeq 200 \, \text{km/s} \) of the cluster:

\[
V_{\text{bd}} < \left[ \frac{GM_{\text{cluster}}}{R_{\text{tidal}}} \right]^{1/2} \simeq 21 \, \text{km/s} \left[ \frac{M_{\text{cluster}}}{10^6 \, M_\odot} \frac{10 \, \text{pc}}{R_{\text{tidal}}} \right]^{1/2}
\]  

(7)

The expected number of events for \( N_{\text{stars}} \) stars monitored during a time \( t_{\text{obs}} \) is:

\[
N_{\text{events}} = \Gamma(A) \, N_{\text{stars}} \, t_{\text{obs}}
\]

\[
\simeq 13 \, u(A) \left[ \frac{0.1 \, M_\odot}{M_{\text{bd}}} \right]^{1/2} \frac{N_{\text{stars}}}{5 \times 10^6 \, \text{stars}} \frac{t_{\text{obs}}}{180 \, \text{nights}} \left( \times \text{Detection efficiency} \right)
\]  

(9)

The EROS team uses a CCD camera with a field of view of about 0.4 square degree to monitor about \( 8 \times 10^4 \) stars every half hour in the LMC bar, to be sensitive to lighter brown dwarfs \( (M_{\text{bd}} = 10^{-6\pm1} \, M_\odot) \), and a Schmidt telescope with a field of view of 25 square degrees to monitor \( 5 \times 10^5 \) stars every night to be sensitive to heavy brown dwarfs \( (M_{\text{bd}} = 10^{-2\pm1} \, M_\odot) \). The MACHO team gets the same number of stars using a CCD camera to covers about 60 fields (0.5 square degree each) once per night.

3 Space clustering

Lensing events will obviously be more concentrated on a few spots on the sky if brown dwarfs are clustered than if they are diffuse. How many clusters lie in the area surveyed by experiments? The mean number \( N_{\text{clusters}} \) of dark clusters in a narrow cone of solid angle \( d\Omega \) is just the integral along the line of sight of the number density of dark clusters:

\[
N_{\text{clusters}} = \int \frac{\rho(D)}{M_{\text{cluster}}} \, D^2 \, dD \, d\Omega
\]

(10)

\[
= 9.3 \, \text{clusters} \left( \frac{10^6 \, M_\odot}{M_{\text{cluster}}} \right) \frac{d\Omega}{1 \, \text{square degree}}
\]  

(11)
Searches for heavy brown dwarfs scan an area of several square degrees where about 230 dark clusters of $10^6 M_\odot$ lie, and more if they are lighter. It is not surprising that the few claimed events (Alcock et al. 1993, Aubourg et al. 1993) do not appear to be concentrated. The eventual presence of dark clusters will not be apparent before statistics are increased by one order of magnitude. On the other hand, light brown dwarf searches scan a smaller area of 0.4 square degree, where 3 to 5 clusters of $10^6 M_\odot$ are expected. There might even be no cluster in the field, although the probability of such a disaster is low (between 5% and 1%, from Poisson statistics). The good point of having few clusters is that even a small number of events will be very clustered on the sky. Such a pattern will be a clear signature of the clustering of brown dwarfs. Let us stress that the probability to find at least 2 events at the same spot is not negligible, even for few events and many clusters: there is a 42% probability for instance that at least 2 events out of 5 are at the same spot if there are 20 clusters in the field (see Table 1).

These results depend weakly on the halo core radius $a$ and on the target direction (see Table 2) but they depend strongly on the mass of dark clusters. Since we know nothing about eventual dark clusters, we just assume that they are similar to globular clusters (Harris & Racine 1979, Djorgovski 1988). These have masses $M_{\text{cluster}}$ ranging between $10^4$ and $10^6 M_\odot$, an external (tidal) radius $R_{\text{tidal}} \simeq 10$ pc, and a core radius $R_{\text{core}} \simeq 0.5$ pc (the core radius is defined as the radius where the surface density drops to half its central value (King 1966), and more massive clusters tend to have smaller cores). The core contains nearly half the cluster mass. The angular diameter of such clusters is small:

$$\theta_{\text{core}} = 20 \text{ arcsec} \frac{R_{\text{core}}}{0.5 \text{ pc}} \frac{10 \text{ kpc}}{D_{\text{cluster}}}$$  \hspace{1cm} (12)

$$\theta_{\text{tidal}} = 400 \text{ arcsec} \frac{R_{\text{tidal}}}{10 \text{ pc}} \frac{10 \text{ kpc}}{D_{\text{cluster}}}$$  \hspace{1cm} (13)

Clusters do not overlap on the sky: the mean angular distance between 2 clusters is 20 arcmin ($= 1/\sqrt{N_{\text{clusters}}}$ degree), whereas the angular diameter of the cluster core is 20 arcsec. The mean angular separation of monitored stars is about 8 arcsec ($2 \times 10^5$ stars are monitored per square degree in the LMC). Therefore, there is a high probability that at least one monitored star lies behind each dark cluster core.

If $N_{\text{events}}$ lensing events are observed, the mean angular separation between them is expected to be:

$$\theta_{\text{diffuse}} = \left[ \frac{\text{Total area}}{N_{\text{events}}} \right]^{1/2} \simeq 95 \text{ arcmin} \left[ \frac{10}{N_{\text{events}}} \right]^{1/2}$$  \hspace{1cm} (14)
for diffuse events, and:

$$\theta_{\text{clustered}} = \frac{\theta_{\text{core}}}{\left[N_{\text{events}}/N_{\text{clusters}}\right]^{1/2}} \simeq 1.6 \text{arcmin} \left[\frac{10}{N_{\text{events}}}\right]^{1/2}$$

(15)

for events due to the same dark cluster. Note that the ratio of these angles:

$$\frac{\theta_{\text{diffuse}}}{\theta_{\text{clustered}}} = \frac{1}{\theta_{\text{core}} \sqrt{dN_{\text{clusters}}/d\Omega}} \simeq 60$$

(16)

is quite large and independent of the number $N_{\text{events}}$ of events and of the size $d\Omega$ of the surveyed area.

### 4 Time clustering

Once a micro-lensing event is detected, the strategy should change if we assume that it is due to a dark cluster. This cluster is at a fixed distance from us, and there is a negligible probability that another cluster overlaps on the same line of sight, as we just saw. The rate of subsequent events in the same direction is then different, because we no longer integrate over the line of sight. We now expect more events near the cluster, and less events away from it. Two kinds of repetitive events can be expected:

i) the lensing of a nearby star by the same brown dwarf.

ii) the lensing of a nearby star by another brown dwarf.

The first case happens whether brown dwarfs are clustered or not, and the minimum time $t_{\text{wait}}$ to wait between successive lensings is:

$$t_{\text{wait}} = \frac{\theta_{\text{star}}}{V_{\perp}/D_{\text{bd}}} = 2000 \text{years} \frac{\theta_{\text{star}}}{8 \text{arcsec}} \frac{D_{\text{bd}}}{10 \text{kpc}} \frac{200 \text{km/s}}{V_{\perp}}$$

(17)

where $\theta_{\text{star}}$ is the angular distance between monitored stars. This is a minimum time since there is no reason that the nearest monitored star lie in the direction of the brown dwarf movement. Note that this time is about the same if brown dwarfs belong to a thick disk instead of a halo (the decrease in $D_{\text{bd}}$ being compensated by the decrease in $V_{\perp}$).

The second case, the lensing of a nearby star by another brown dwarf, is specific of dark clusters. The mean time $t_{\text{wait}}$ to wait between two micro-lensings of the same star (or between the micro-lensings of two given stars) in the diffuse case is $\Gamma^{-1}$ by definition. Hence:

$$t_{\text{wait}} = 200000 \text{years} \frac{1}{u(A)} \frac{200 \text{km/s}}{V_{\perp}} \left[\frac{M_{\text{bd}}}{0.1 M_{\odot}}\right]^{1/2}$$

(18)
from Equation 6. If brown dwarfs are clustered, the event rate now is the rate at which brown dwarfs enter the area of radius \( u(A) R_E \) around the star times the surface density \( \Sigma \) of brown dwarfs in the cluster:

\[
\Gamma = 2 u(A) R_E V_\perp \Sigma = 2 u(A) R_E V_\perp \frac{M_{\text{cluster}}}{M_{\text{bd}}} \frac{1}{\pi R_{\text{core}}^2}
\]  

(19)

The time \( t_{\text{wait}} \) between lensings is then much shorter:

\[
t_{\text{wait}} = \frac{1}{\Gamma} = \frac{\pi c R_{\text{core}}^2}{4 u(A) V_\perp M_{\text{cluster}}} \left[ \frac{M_{\text{bd}} D_{\text{star}}}{G D_{\text{bd}} (D_{\text{star}} - D_{\text{bd}})} \right]^{1/2}
\]  

(20)

\[
\simeq 32 \text{ years} \frac{1}{u(A)} \left[ \frac{M_{\text{bd}}}{0.1 M_\odot} \right]^{1/2} \left[ \frac{10^6 M_\odot}{M_{\text{cluster}}} \right]^{1/2} \left[ \frac{D_{\text{star}} - D_{\text{bd}}}{10 \text{ kpc}} \right] \left[ \frac{D_{\text{bd}}}{0.5 \text{ pc}} \right]^2
\]  

(21)

and it depends sensitively on the cluster parameters \( D_{\text{bd}}, R_{\text{core}} \) and \( M_{\text{cluster}} \), and on the brown dwarf parameters \( V_\perp \) and \( M_{\text{bd}} \). An essential information on the latter will be given by the duration \( t_{\text{event}} \) of the lensing event, which is the mean time elapsed from detection (threshold amplification \( A \)) to peak amplification and back to amplification \( A \) again. From simple geometry, it is:

\[
t_{\text{event}} = \frac{\pi}{2} \frac{u(A) R_E}{V_\perp}
\]  

(22)

for a brown dwarf of relative transverse velocity \( V_\perp \) (typically about 200 km/s). For a star in the Large Magellanic Cloud (LMC) at \( D_{\text{star}} \simeq 50 \text{ kpc} \), a lensing event lasts between a few hours and a few weeks, depending on the minimal amplification \( A \), and on the mass and distance of the brown dwarf:

\[
t_{\text{event}} \simeq 35 \text{ days} u(A) \left[ \frac{200 \text{ km/s}}{V_\perp} \right] \left[ \frac{M_{\text{bd}}}{0.1 M_\odot} \right] \left[ \frac{D_{\text{bd}}}{10 \text{ kpc}} \right] \left[ \frac{D_{\text{star}} - D_{\text{bd}}}{D_{\text{star}}} \right]^{1/2}
\]  

(23)

The ratio \( t_{\text{wait}} / t_{\text{event}} \):

\[
\frac{t_{\text{wait}}}{t_{\text{event}}} = \frac{1}{\pi u^2 R_E^2 \Sigma} = 320 \left[ \frac{10 \text{ kpc}}{D_{\text{bd}}} \right] \left[ \frac{D_{\text{star}}}{D_{\text{star}} - D_{\text{bd}}} \right] \left[ \frac{10^6 M_\odot}{M_{\text{cluster}}} \right] \left[ \frac{R_{\text{core}}}{0.5 \text{ pc}} \right]^2
\]  

(24)

is independent of the brown dwarf mass and velocity, but depends strongly on the core radius of the cluster. Thus, in principle, if one is lucky enough to observe a micro-lensing lasting for one day (as expected from a \( 10^{-4} M_\odot \) brown dwarf), the same star (or any nearby one) might well be micro-lensed every year for 5000 years (the time needed to travel 1 pc at 200 km/s). It is therefore worthwhile to follow a star the light of which has been amplified (even if there is some doubt that the event was actually a micro-lensing), because another micro-lensing would be very unlikely in the case of a diffuse distribution.
of brown dwarfs. A star which undergoes such successive “flashes” cannot be confused with a genuinely variable star (like a cepheid), because the time delay between flashes is irregular, and long compared to the flash duration. It cannot be confused with a flare star, because the amplification is time symmetric and achromatic.

Is there a risk that a star, already lensed by one brown dwarf, be simultaneously lensed by another one? A double lensing is detected when a brown dwarf gets within \( u(A) R_E \) distance to the line of sight to the star, and another one is already inside. If brown dwarfs are not correlated within the cluster (e.g. no binaries), the probability \( P_{dl} \) of such an event is just the area of the disk of radius \( u(A) R_E \), times the surface density \( \Sigma \):

\[
P_{dl} = \pi u^2 R_E^2 \Sigma \]

\[
= \pi u^2 \frac{4GM_{bd}D_{bd}(D_{\text{star}} - D_{bd})}{c^2} \frac{M_{\text{cluster}}}{M_{bd}} \frac{1}{\pi R_{\text{core}}^2} \quad (25)
\]

\[
= 3.1 \times 10^{-3} u^2 \frac{D_{bd}}{10 \text{kpc}} \frac{D_{\text{star}} - D_{bd}}{D_{\text{star}}} \frac{M_{\text{cluster}}}{10^6 M_\odot} \left[ \frac{0.5 \text{pc}}{R_{\text{core}}} \right]^2 \quad (26)
\]

This probability is less than 1 %. This is most welcome, since otherwise the light curve would be asymmetrical, and rejected as noise by the scanning algorithms of the brown dwarf experiments. Note that this probability does not depend on the mass of the brown dwarf, but only the the cluster parameters (mass, distance and radius).

5 Unresolved stars

The number of monitored stars behind a cluster is small because the angular diameter of the core (20 arcsec) is similar to the angular separation between monitored stars (8 arcsec). The density of brown dwarfs is large near the micro-lensed star, which increases the probability of lensing nearby stars, but we cannot use this advantage if there is no other monitored star nearby! One could monitor more stars in the same area by an increase of the sensitivity of the camera, but the stars then start to overlap and the confusion limit is quickly reached when their distance becomes less than the seeing (about one arcsec).

We suggested (Baillon et al. 1993) a more efficient way to detect micro-lensings. In a crowded field of unresolved stars (e.g. the Andromeda galaxy M31 or the bar of the LMC), one could detect the luminosity increase on one pixel of the image due to the micro-lensing of one of the unresolved stars on this pixel. There are typically about 10 stars on each pixel of a CCD picture of the LMC, but the amplification required to detect faint ones is so large that the lensing probability becomes negligible \( (\Gamma(A) \propto 1/A \text{ when } A \gg 1) \). It turns out that about one star only per pixel can be sufficiently amplified to become detectable. Monitoring pixels instead of stars has several advantages if brown dwarfs are
clustered:
i) the number of pixels is much larger than the number of monitored stars, hence a gain in detection efficiency.
ii) CCD pixels are contiguous, and there is one of them in front of any cluster.
iii) CCD pixels are often less than 1 arcsec wide, smaller than the angular size of a dark cluster which is then covered by several pixels. As a by-product, such an experiment would then allow a study of (dark) cluster cores without being hampered by the usual seeing problem.

Note added: Eyal Maoz recently presented similar but more optimistic conclusions because he takes the number of monitored stars to be $10^6$ per square degree (Maoz 1994). He also remarks that the durations of micro-lensing events due to the same dark cluster should be strongly correlated, a point that we overlooked.

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Tables

| Number of clusters | 3   | 10  | 20  | 100 |
|--------------------|-----|-----|-----|-----|
| Number of events   |     |     |     |     |
| 3                  | 78% | 28% | 14% | 3%  |
| 5                  | 100%| 70% | 42% | 10% |
| 10                 | 100%| 99.96%| 93% | 37% |
| 20                 | 100%| 100%| 100%| 73% |

**Table 1:** Probability to find at least two micro-lensing events at the same spot (within less than 1 arcmin) as a function of the number of clusters and of the number of events.

| Halo core radius $a$ (kpc) | 0   | 2   | 5   | 8   |
|-----------------------------|-----|-----|-----|-----|
| $\Gamma_0 \times 10^{14}$ in the LMC | 14  | 15  | 16  | 19  |
| $\Gamma_0 \times 10^{14}$ in the Galactic Center | $\infty$ | 24  | 10  | 7   |
| Number of $10^6 M_\odot$ clusters/square degree towards LMC | 7.2 | 7.6 | 9.3 | 12.1 |

**Table 2:** Event rate $\Gamma_0$ per star as a function of the halo core radius $a$, for stars in the LMC (at $D_{\text{star}} = 50$ kpc) and in the Galactic Center (at 8.5 kpc). Also shown is the number of $10^6 M_\odot$ clusters per square degree in the direction of the LMC.