Driving-Induced Symmetry Breaking in the Spin-Boson System

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A symmetric dissipative two-state system is asymptotically completely delocalized independent of the initial state. We show that driving-induced localization at long times can take place when both the bias and tunneling coupling energy are harmonically modulated. Dynamical symmetry breaking on average occurs when the driving frequencies are odd multiples of some reference frequency. This effect is universal, as it is independent of the dissipative mechanism. Possible candidates for an experimental observation are flux tunneling in the variable barrier rf SQUID and magnetization tunneling in magnetic molecular clusters.

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The dissipative two-state system is the simplest model allowing the study of decoherence in a tunneling system \cite{1,2,3}. The two states may model, e.g., the localized states in a tight-binding double well. This model has found broad application to tunneling systems in physics and chemistry \cite{4,3}. It has been used, e.g., to study the tunneling of atoms between the tip of an atomic-force microscope and a surface \cite{5}. The dynamics of the magnetic flux in a superconducting interference device (SQUID) \cite{6}, or the relaxation behavior of tunneling centers in submicron Bi-wires \cite{7}. Also the low temperature dynamics of the magnetization of anisotropic magnetic molecular clusters can be described in terms of a two-state system \cite{8}. Tunneling of magnetization has been observed in molecular Mn$_{12}$ \cite{9} and Fe$_8$ \cite{10} nanomagnets.

Driven dissipative systems exhibit a variety of dynamics and opened new potentials of application \cite{11}. For example, the complete suppression of tunneling of a bare two-state system (TSS) induced by a suitably chosen monochromatic driving field \cite{12} can also persist in the presence of dissipation \cite{13,14}. A variety of different nonequilibrium initial preparations, relevant e.g. in electron transfer, can equivalently be described in terms of a particular time-dependent driving force \cite{15}. An applied dc-ac field can invert the population of donor and acceptor sites \cite{16}. Time-dependent modulation of the bias energy can be achieved, e.g., by an ac electric field coupled to the dipole moment of the TSS, or, in a SQUID ring device, by a time-dependent external magnetic field threading the ring. In contrast, “quadrupole”-like couplings induce a variation of barrier height and width and thus lead to a modulation of the tunneling matrix element (TME). The effects of monochromatic modulation of the TME have been studied in Ref. \cite{15}. Promising candidates to study the effects of concerted harmonic modulation of the bias and TME are the variable barrier rf SQUID ring \cite{13} and magnetic molecular clusters in longitudinal and transverse ac magnetic fields \cite{16}.

We consider the cooperative effects of two monochromatic fields modulating the bias energy $\hbar c$ and the TME $\hbar \Delta$. The dissipative TSS is described by the spin-boson Hamiltonian \cite{1,2,3} (we set $\hbar = k_B = 1$)

$$H(t) = -\frac{i}{\hbar}\left[\epsilon(t) \sigma_z + \Delta(t) \sigma_x\right] - \frac{1}{2}\sigma_x X + H_B,$$  \hspace{1cm} (1)

where $H_B = \sum_i [p_i^2/2m_i + m_i\omega_i^2 x_i^2/2]$ represents the bath of bosons, and the collective variable $X = \sum_i c_i x_i$ describes the bath polarization. All effects of the boson bath on the TSS are captured by the spectral density $J(\omega) = \frac{\omega}{\pi} \sum_i \frac{\omega_i^2}{m_i^2} \delta(\omega - \omega_i)$. A dipole interaction contributes to the bias energy, whereas a quadrupole modulation results in an exponential modification of the TME,

$$\Delta(t) = \Delta_0 \exp\left[\frac{\epsilon(t)}{\epsilon_1} \sin(\Omega_{\Delta} t)\right], \hspace{1cm} \epsilon(t) = \epsilon_1 \sin(\Omega_{\Delta} t).$$ \hspace{1cm} (2)

In the absence of driving, $\Delta_0 = 0, \epsilon_1 = 0$, the equilibrium state reached at long times has the localized states $|R\rangle$ and $|L\rangle$ occupied with equal probability. If either the bias or the coupling energy is modulated, the left-right symmetry is dynamically broken. However, on average over a period, one finds asymptotically again equal occupation of both states. When both parameters are modulated with commensurable frequencies ($n, m$ integer)

$$\Omega_{\epsilon} = m \Omega \hspace{1cm} \text{and} \hspace{1cm} \Omega_{\Delta} = n \Omega,$$ \hspace{1cm} (3)

the Hamiltonian \cite{1} is periodic (we assume $T = 2\pi/\Omega$ to be the smallest period in common). We find that equal occupation on average is reached at long times when $n$ or $m$ is even. However, when both $n$ and $m$ are odd, the right-left symmetry is broken even on average. The symmetry breaking is maximal when $n = m = 1$. The phenomenon that the occupation of one of the two states is preferred on average at long times is schematically sketched and explained in Fig. \textsuperscript{1}. In the following, we give a quantitative study of this effect.

Suppose that the TSS has been prepared at time zero in the state $|R\rangle$ ($\sigma_z = +1$) with the bath in thermal equilibrium. The dynamical quantity of interest is then the population difference $\langle \sigma_z \rangle_t = P(t) = P_R(t) - P_L(t)$ for this factorizing initial condition. Since the bath is harmonic, it can be traced out exactly. Then, $P(t)$ is expressed as a double path integral over the forward and backward spin paths $\sigma(\tau)$ and $\sigma'(\tau)$ which are piecewise
constant with values ±1. The effects of the environment are in an influence functional which introduces nonlocal correlations between different path segments. It is convenient to switch to the path combinations η(τ) = [σ(τ) + σ′(τ)]/2 and ξ(τ) = [σ(τ) − σ′(τ)]/2. Then, P(t) is expressed in terms of the double path sum

\[ P(t) = \int Dξ Dη A[ξ, η] \exp \left[ \Phi'_F V[ξ] + i \Phi''_F V[ξ, η] \right], \]

where A is the path weight in the absence of the bath coupling, and the influence function is

\[ \Phi'_{F V}[ξ, η] = \int_0^t dt_2 \int_0^{t_2} dt_1 \xi(t_2) \times \left[ Q'(t_2 - t_1)\xi(t_1) + iQ''(t_2 - t_1)\eta(t_1) \right]. \]

For the Ohmic spectral density J(ω) = 2παω e^{−ω/ω_c}, where α is the dimensionless Ohmic coupling, the kernel takes for times ω_cτ ≫ 1 the so-called scaling form

\[ Q(τ) = 2\alpha \ln \left[ (\betaω_c/\pi) \sinh (\piτ/\beta) \right] + i\piα \text{sgn} τ. \]  

It is convenient to write P(t) = P_s(t) + P_a(t), where P_s(t) and P_a(t) are the symmetric and antisymmetric parts under inversion of the bias ε → −ε. Since \( \lim_{t→∞} P_s(t) = 0 \), the asymptotic value is determined by the antisymmetric part P_a(t). In the absence of the biasing force, ε = 0, we have P_a(t) = 0 for all t. For any initial preparation, we then have \( \lim_{t→∞} P(t) = 0 \), i.e., the TSS is completely delocalized at long times. Similarly, if we choose A = 0 and ε ≠ 0, the time average \( \langle \sin ξ(t_2, t_1) \rangle_T \) over a period 2π/Ω_s of the external field vanishes. Thus, on average, the TSS is again delocalized, i.e., \( P_{∞} ≡ \lim_{t→∞} \langle P(t) \rangle_T = 0 \). However, the constructive interference between diagonal and off-diagonal driving, i.e. A ≠ 0 and ε ≠ 0, can lead to driving-induced symmetry breaking, as qualitatively illustrated in Fig. 1. We find \( P_{∞} ≠ 0 \) for Ω_s = Ω_s', and the tight-binding site preferred is determined by the relative sign of A and ε.

The evolution of P(t) cannot be given in analytic form apart from special limits. Numerical methods cover short to intermediate times, but they become costly or even inadequate if one is interested in the asymptotic behavior.
\[ L_{a/\alpha}(t, t') = \Delta(t)\Delta(t')B_{a/\alpha}[\zeta(t, t')] \, . \] 

The equilibrium state for a TSS with static bias \( \epsilon \) is \( P_\infty = [k_+(\epsilon) - k_-(\epsilon)]/[k_+(\epsilon) + k_-(\epsilon)] \), where \( k_+(\epsilon) \) is the forward/backward rate for the static case. One has \( k_{\pm}(\epsilon) = (\Delta_0^2/4) \int_{-\infty}^{\infty} dt \exp[\pm i\epsilon t - Q(t)] \), obeying detailed balance, \( k_-(\epsilon) = \exp(-\beta\epsilon)k_+(\epsilon) \). In the scaling limit, the Ohmic forward rate reads

\[ k_+ = \frac{\Delta_0^2}{4}\omega_\epsilon (\beta\omega_\epsilon^2/2\pi)^{1/2} \left[ \frac{\Gamma(\alpha+i\beta\epsilon/2\pi)^2}{\Gamma(2\alpha)} \right] e^{\beta\epsilon/2} \, . \] 

For the driven TSS, Eqs. (1) – (3), the average rate is

\[ \bar{k}_\pm = \frac{\Omega}{2\pi} \int_{0}^{2\pi/\Omega} dt \int_{0}^{\infty} d\tau \frac{1}{2} \left[ K_s(t, t-\tau) \pm K_a(t, t-\tau) \right] \, . \]

Next, we expand the bias phase terms \( e^{\mp i\epsilon(t/\Omega)} \cos(\Omega t, t) \) and \( e^{\pm i\epsilon(t/\Omega)} \cos[\Omega(t-\tau)] \) in series in \( J \)-Bessel functions, and the oscillatory factors of the tunneling coupling \( e^{\pm \Delta \sin(\Omega t, \tau)} \) and \( e^{\pm \Delta \sin(\Omega(t-\tau))} \) into series in \( I \)-Bessel functions. We then find the average rate \( \bar{k}_\pm \) in the form

\[ \bar{k}_\pm = \sum_{l,m,s=-\infty}^{\infty} e^{i\pi(n/m-1)(r+s)/2} k_{s} \left[ (m\ell - ns)\Omega \right] \]

\[ \times I_{\ell}(A_{\Delta})I_{s}(A_{\Delta})J_{l}\left(\frac{\ell_1}{m\Omega}\right)J_{\ell-\left(r+s\right)n/m}\left(\frac{\ell_1}{m\Omega}\right) \, . \] 

The individual channels in (14) describe tunneling under emission and absorption of a fixed number of quanta with frequencies which are multiples of the frequency \( \Omega \). The channel weights are given in terms of \( I \) and \( J \) Bessel functions. The exclusive channel rates are expressed in terms of the rates \( k_{\pm} \) for a static bias \( \epsilon = (m\ell - ns)\Omega \).

The mean asymptotic position is then found as

\[ \bar{P}_\infty = (\bar{k}_+ - \bar{k}_-)/(\bar{k}_+ + \bar{k}_-) \, . \] 

Substituting Eq. (11) and the symmetry relation \( k_-(\epsilon) = k_+(\epsilon) \), and using the properties \( J_{-\ell}(z) = (-1)^\ell J_{\ell}(z) \) and \( I_{-\ell}(z) = I_{\ell}(z) \), we find \( \bar{P}_\infty = 0 \) when \( n \) or \( m \) is even, whereas \( \bar{P}_\infty \neq 0 \) when both \( n \) and \( m \) are odd.

The predictions following from (13) are shown in Fig. 2 (horizontal lines) in comparison with the exact \( P(t) \) (oscillating curves). We have chosen \( \alpha = 1/2 \) to allow for a direct comparison with the predictions of the exact solution (14). Evidently, the asymptotic position value is very accurately reproduced by the NIBA.

In Fig. 3 we show \( \bar{P}_\infty \) as a function of the bias amplitude \( \epsilon_1 \) for different coupling strengths. The full curve (\( \alpha = 0.1 \)) exhibits oscillations. The oscillations are already damped out in the curves for the higher damping values displayed. Interestingly, the symmetry breaking effect grows with increasing \( \alpha \) and increasing \( \epsilon_1 \). For large bias amplitude, in the investigated bias regime, \( \bar{P}_\infty \) saturates. The inset depicts \( \bar{P}_\infty \) versus \( \alpha \) for \( \epsilon_1 = 10\Delta_0 \) and different temperatures. For weak coupling \( \alpha \ll 1 \), the symmetry breaking becomes more effective with increasing temperature. For stronger coupling, on the contrary, it is more pronounced the lower the temperature.

In Fig. 4 we show \( \bar{P}_\infty \) for different ratios \( \Omega_\epsilon/\Omega_\Delta \). The full and long-dashed curves represent the cases \( \Omega_\epsilon = \Omega_\Delta \) and \( \Omega_\epsilon = 3\Omega_\Delta \). In both cases, the symmetry breaking is clearly evident. When the bias frequency is three times the barrier modulation frequency, the symmetry breaking effect is strongly reduced (short-dashed curve). The different behavior is due to the oscillatory and smooth behavior of the \( J_{\ell}(z) \) and \( I_{\ell}(z) \) functions, respectively.

We believe that the phenomenon of dynamical symmetry breaking can be seen, e.g., in Josephson systems. A promising candidate is a superconducting quantum interference device in which the tunneling coupling and bias energy can be varied independently [4]. In the variable

**FIG. 3.** The mean asymptotic value \( \bar{P}_\infty \) is shown as a function of the bias amplitude \( \epsilon_1 \) for Ohmic couplings \( \alpha = 0.1 \) (full curve), \( \alpha = 1/4 \) (long-dashed curve), and \( \alpha = 1/2 \) (short-dashed curve). The parameters are \( A_\Delta = 0.5 \), \( \Omega_\epsilon = \Omega_\Delta = \Delta_0/2 \), and \( \alpha = 0.1 \). The inset shows \( \bar{P}_\infty \) versus \( \alpha \) for temperatures \( T = 0.1\Delta_0 \) (short-dashed curve), \( T = \Delta_0 \) (dashed curve), and \( T = 10\Delta_0 \) (full curve). The bias amplitude is \( \epsilon_1 = 10\Delta_0 \).

**FIG. 4.** Plots of \( \bar{P}_\infty \) versus bias \( \epsilon_1 \) for \( \Omega_\epsilon = \Omega_\Delta = \Delta_0/2 \) (full curve), \( \Omega_\epsilon = 3\Omega_\Delta = \Delta_0/2 \) (long-dashed curve), and \( \Omega_\epsilon = 3\Omega_\Delta = \Delta_0/2 \) (short-dashed curve). The parameters are \( \alpha = 1/4 \), \( A_\Delta = 0.5 \), and \( \alpha = 0.1 \).
barrier rf SQUID, the single Josephson junction in a standard rf SQUID ring with inductance $L_0$ is replaced by a superconducting loop with inductance $L$, interrupted by two junctions. In the limit $L_0/L \to 0$, the 2D Josephson potential $V(\Phi, \Phi_{dc})$ goes over into the 1D potential of a standard rf SQUID with adjustable critical current $I_c(\Phi_{dc})$. Within the RSJ model, the deterministic equation of motion of the rf SQUID for the flux variable $\phi = 2\pi \Phi/\Phi_0 = 2\pi \Phi$ is $C \dot{\phi} + \phi/R + 4e^2(\partial/\partial \phi)U(\phi) = 0$.

The rf SQUID potential is given by $U(\phi) = U_0 \left[ \frac{1}{2}(\phi - \phi_x)^2 - \beta_L(\phi_{dc} \cos(\phi)) \right]$, \(13\)
where $\beta_L(\phi_{dc}) = 2eLL_c \cos(\frac{1}{2}\phi_{dc})$. For an applied flux $\phi_x = \pi$ and $\beta_L > 1$, the potential is symmetric about $\phi = \pi$ with two degenerate minima at $\phi = \pi \pm \phi_m$, where $\phi_m$ is the smallest positive root of $\phi_m - \beta_L \sin \phi_m = 0$. The barrier height is $U_b = U_0[\beta_L - \frac{1}{2}\phi_m^2 - \beta_L \cos \phi_m]$, and the squared well frequency is $\omega_b^2 = (1 - \beta_L \cos \phi_m)/LC$.

With the modulation $\phi_x = \pi + \phi_1 \sin(\Omega t)$, the system is effectively described at low $T$ by Eq. (2), with the bias variation (2), and amplitude $\phi_1 = \phi_m e^{2\beta L}$. The parameter $v \equiv U_b/\omega_0$ can be varied independently of the bias by varying the flux $\phi_{dc}$ applied to the small loop. For $\beta_L \gg 1$, the tunneling length $2\phi_m$, and hence the damping parameter $\alpha = (\phi/m/\pi)^2 R_Q/R$ ($R_Q = \pi/2e^2 = 6.5$ $k\Omega$), is approximately constant under variation of $\phi_{dc}$. Harmonic modulation of the WKB exponent $v$ of the TME $\Delta \propto \exp(-cv)$, where $c$ depends on the barrier shape, can now be achieved by driving $\phi_x$. Thus one can experimentally reach the situation described by Eq. (2). Typical realizable junction and loop parameters are $L \approx 200$ $\mu$H, $C \approx 100$ $\Omega F$, $\beta_L \approx 5$, and $R/R_Q$ ranging from 0.5 to 10. This results in values for $\alpha$ ranging from 0.1 to 2, barrier height $U_b \approx 1 \ldots 10$ K, well frequency $\omega_0 \approx 10^{10} \ldots 10^{11}$ s$^{-1}$, and the tunneling matrix element in the MHz regime. We expect that the symmetry breaking effect can be observed at temperatures above 10 mK and driving frequencies $\Omega_\Delta$ and $\Omega_\pi$ in the 1 $\ldots$ 10 MHz regime.

A giant spin or nanomagnet is another possible candidate for the observation of the above symmetry breaking phenomenon. In these systems, the bias can be tuned by a magnetic field along the easy axis. It has been demonstrated that the TME of the Fe$_8$ molecular system can be tuned by a transverse magnetic field as theoretically predicted. Therefore, it seems feasible to operate nanomagnets in the appropriate parameter regime.

In summary, we have found that dynamical symmetry breaking can be induced by a concerted modulation of the bias and tunneling energy of a symmetric TSS. Because of constructive interference, dwelling in one of the localized states is preferred on average. This effect is robust since it is largely independent of the dissipative mechanism. Also, the dependence on coupling strength and temperature is interesting. For weak coupling, the asymptotic mean position increases with temperature, whereas for strong coupling it shows opposite behavior.

Readily, the results can be extended to quantum transport in a tight-binding (TB) lattice. In the incoherent tunneling regime, the asymptotic current is proportional to the asymptotic mean position of the TB system. Upon imposing concerted driving of bias and TME, it is possible to extract a finite current out of an (on average) unidirectional force. Thus, the system constitutes a novel realization of a discrete quantum Brownian rectifier.

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