Natural Hadronic Degrees of Freedom 
for an Effective QCD Action in the Front Form

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On the one hand any effective action of the QCD, \( S[\pi, \rho, \omega, \ldots, N, \overline{N}, \Delta^{3/2}, \ldots] \), would involve, at least in principle, a tower of hadronic fields of all spins; on the other hand the front form of the field theory has emerged as an appropriate framework to investigate the high energy behaviour of many systems. Motivated by these simple considerations we present a systematic formalism which explicitly provides the front form fields appropriate for constructing a \( S[\pi, \rho, \omega, \ldots, N, \overline{N}, \Delta^{3/2}, \ldots] \). Further, a transformation matrix \( \Omega(j) \) is constructed which determines how the front form hadronic fields superimpose to yield the more familiar instant form fields. Explicit results for the front form hadronic fields up to spin two are catalogued in appendixes. For spin one half, the \( \Omega(1/2) \) coincides with the celebrated “Melosh transformation.” The formalism, without explicitly invoking any wave equations, reproduces spin one half front-form results of Lepage and Brodsky, and Dziembowski.

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I. INTRODUCTION

As presently understood, the fundamental constituents of the observed hadrons are quarks and gluons. A great spectrum of nuclear phenomenon are at energy/length scales where the empirically observed hadrons appear to be the natural \[ \text{degrees of freedom} \]. As a new generation of nuclear physics facilities become available at least two aspects of nuclear matter will become more pronounced or observable. First, the \( j > \frac{1}{2} \) hadronic degrees of freedom will become increasingly important. Second, a transition between hadronic matter and the quark-gluon phase will offer new insights into nuclear matter and the physical content of the QCD action. The first of the most obvious question which one can ask is the relationship between these two aspects. That is, does the transition hadronic matter \( \leftrightarrow \) quark-gluon phase exploit the high-spin degrees of freedom provided by various hadronic resonances? The essential spirit of this point of view is contained in the work of Cahill and collaborators \[2\] where the fundamental defining action of QCD, \( S[\overline{\mathbf{q}}, \mathbf{q}, A^a_{\mu}] \), is related via a change of variables (using techniques of the functional integral calculus) to an effective action for hadronic physics, \( S[\pi, \rho, \omega, \ldots, N, \overline{N}, \Delta^{3/2}, \ldots] \). This paper will provide, what in our opinion, are one of the most natural objects for constructing \( S[\pi, \rho, \omega, \ldots, N, \overline{N}, \Delta^{3/2}, \ldots] \).

We begin with the observation that since the simplest description \[1\] of nucleons in nuclear matter requires the use of Dirac equation the nuclear phenomenon is **inherently** relativistic. As a result the front form of the quantum field theory is expected to play an important role in nuclear and hadronic physics. The high-spin hadronic degrees of freedom logically merge with this aspect — for **spin** emerges as a logical consequence of the Poincaré symmetry \[3\]. In addition to the well known advantages of the front form \[4,5,6,7\] of the quantum field theory we have recently gained some additional insights into the nature of front-form field theory by studying the evolution of a quantum system along an appropriately parameterised timelike \[8\] direction. By varying a single parameter the standard instant form and front form results can be obtained \[9\] in a continuous and well defined fashion for \( j \leq \frac{1}{2} \). Because of the absence of a unique wave equation and various problems which are encountered for wave equations with \( j > \frac{1}{2} \), these works do not easily extend beyond spin one half. The clue to a possible solution to this problem lies in a recent work where, following...
Weinberg [10] and Ryder [11], one of the present authors in collaboration with Ernst [12,13,14,15,16] has explicitly constructed instant-form Dirac-like spinors, Feynman-Dyson propagators, and wave equations for arbitrary spins; and discovered several interesting aspects of the Weinberg equations [10]. In contrast to the Rarita-Schwinger/Bargmann-Wigner [17,18] formalism, the instant-form work of Refs. [12,13] requires no constraints or auxiliary fields and in addition incorporates the physically correct number of particle and antiparticle spinorial degrees of freedom.

Here in this paper, following the logic of Refs. [10,6,12,13], we present a general procedure for constructing the front form covariant spinors for any spin without a direct reference to any wave equation. A new transformation, $\Omega(j)$, which relates the front form spinors to the instant form spinors will be constructed. Explicit examples for $j = \frac{1}{2}, 1, \frac{3}{2}, 2$ will be presented. For $j = \frac{1}{2}$, the formalism reproduces the front form spinors of Lepage and Brodsky [20] and $\Omega(\frac{1}{2})$ coincides with the celebrated “Melosh transformation” [21,22]. Our work thus opens a possibility to construct $S[\pi, \rho, \omega, \ldots, N, \bar{N}, \Delta^{3/2}, \ldots]$ in terms of the front form fields obtained here, and provide it’s connection with the QCD action $S[q, q, A_{\mu}^a]$. This approach is in the spirit of Dirac’s [23] original motivation for the high-spin wave equations for “approximate application to composite particles” and provides a very natural framework for constructing a truly QCD based effective field theory of hadronic resonances along the lines of recent work of Cahill et al. cited above. Here we combine Dirac’s original motivation just mentioned with another of his classic ideas of constructing the front form of field theory [4] for high energy physics. The present work also provides the unifying link between Weinberg’s high spin work [10] and his well known [24] paper on the infinite momentum frame.

Unless otherwise indicated we follow the notation of Refs. [11,13]. We use the notation $x^\mu = (x^+, x^1, x^2, x^-)$. In terms of the instant form variables $\underline{x}^\mu = (x^0, x^1, x^2, x^3)$, we have $x^\pm = x^0 \pm x^3$. The evolution of a system is studied along the coordinate $x^+$, and as such it plays the role of “time.” In the nuclear physics community the words “spinors” and “relativistic wave functions” are often used interchangeably. We will adhere to this usage here. For the more formally minded reader what we present are the front form objects, and their relation with the instant form counterparts, in the representation space on which the finite dimensional realisations of $SU^L(2)$ and
$SU^R(2)$ generated by $\vec{J} \pm i \vec{K}$ act.

II. FRONT FORM HADRONIC FIELDS

We will work under the assumption that the center of mass of a composite hadrons is best described by the $(j,0) \oplus (0,j)$ wave functions in the front form. These wave functions will be constructed from the right handed, $\phi^R(p^\mu)$, and left handed, $\phi^L(p^\mu)$, matter fields. We begin form transformation which takes a particle from rest, $\hat{p}^\mu = (m,0,0,0)$, to a particle moving with an arbitrary four momentum $p^\mu = (p^+,p^1,p^2,p^-)$ [Note: for massive particles $p^+ > 0$.]

In the \textit{instant form of field theory} the transformation which takes the rest momentum $\hat{p}^\mu = (m,0,0,0) \rightarrow p^\mu = (p^0,\vec{p})$ is constructed out of the boost operator $\vec{K}$ and is given by

$$ p^\mu = \Lambda^\mu_\nu \hat{p}^\nu, \quad (1) $$

with

$$ \Lambda = \exp \left( i \vec{\varphi} \cdot \vec{K} \right). \quad (2) $$

In Eq. (2) the boost parameter $\vec{\varphi}$ is defined as follows

$$ \varphi = \vec{p}/|\vec{p}|, \quad \cosh(\varphi) = E/m, \quad \sinh(\varphi) = |\vec{p}|/m. \quad (3) $$

Note that the stability group of the $x^\circ = 0$ plane consists of the six generators $\vec{J}$ and $\vec{P}$. The $\vec{K}$, along with the $P^\circ$, generate the instant-form dynamics.

In the \textit{front form of field theory} the transformation which takes $\hat{p}^\mu = (m,0,0,0) \rightarrow p^\mu = (p^0,\vec{p})$ is given \cite{3,4} by

$$ p^\mu = L^\mu_\nu \hat{p}^\nu, \quad (4) $$

with the matrix $L$ given by

$$ L = \exp \left( i \vec{\upsilon} \cdot \vec{G} \right) \exp \left( i \eta K_3 \right). \quad (5) $$
The parameters \( \eta \) and \( \vec{v}_\perp = (v_x, v_y) \) specify a given boost. The generators \( \vec{G}_\perp \), which along with \( K_3 \), take \( \vec{p}_\mu \to p_\mu \), are defined as follows

\[
G_1 = K_1 - J_2, \quad G_2 = K_2 + J_1, \tag{6}
\]

and together with

\[
P_\mu = P_0 - P_3, \quad P_1, \quad P_2, \quad J_3, \tag{7}
\]

form the seven generators of the stability group of the \( x^+ = 0 \) plane. (Note that \( P_- = P^+ \)). The algebra associated with the stability group is summarised in Table I. The generators \( D_1 = K_1 + J_2, \ D_2 = K_2 - J_1 \) and \( P_+ = P_0 + P_3 \) generate the front-form dynamics.

It is important to note that while the front-form transformation \( L \) is specified entirely in terms of the generators of the \( x^+ = 0 \) plane stability group, the instant-form transformation \( \Lambda \) involves dynamical generators associated with the \( x^0 = 0 \) plane.

Using the explicit matrix expressions for \( \vec{J} = (J_1, J_2, J_3) \) and \( \vec{K} = (K_1, K_2, K_3) \) given in Eqs. (2.65 - 2.67) of Ref. [11] we obtain an explicit expression for the boost \( L \) defined in Eq. (5)

\[
[L^\mu_{\nu}] = \begin{bmatrix}
\cosh(\eta) + \frac{1}{2} \vec{v}_\perp^2 \exp(\eta) & v_x & v_y & \sinh(\eta) + \frac{1}{2} \vec{v}_\perp^2 \exp(\eta) \\
v_x \exp(\eta) & 1 & 0 & v_x \exp(\eta) \\
v_y \exp(\eta) & 0 & 1 & v_y \exp(\eta) \\
\sinh(\eta) - \frac{1}{2} \vec{v}_\perp^2 \exp(\eta) & v_x & v_y & \cosh(\eta) - \frac{1}{2} \vec{v}_\perp^2 \exp(\eta)
\end{bmatrix}. \tag{8}
\]

Recalling that the components of the front form momentum \( p_\mu \) are defined as \( p^\pm = p^0 \pm p^3 \) this yields

\[
p^+ = m \exp(\eta), \quad \vec{p}_\perp = m \exp(\eta) \vec{v}_\perp, \quad p^- = m \exp(-\eta) + m \exp(\eta) \vec{v}_\perp^2. \tag{9}
\]

The variables \( \eta \) and \( \vec{v}_\perp \) are fixed by requiring \( p_\mu \) generated by Eq. (1) to be identical to the \( p_\mu \) produced by Eq. (4).

Given the transformation \( L \), Eq. (5), we now wish to construct the front form \((j, 0) \oplus (0, j)\) hadronic wave functions. To proceed in this direction we rewrite \( L \) by expanding the exponentials in Eq. (5), and using Table I to to arrive at [25]
\[ L = \exp \left[ i \left( a \bar{v} \cdot \vec{G} + \eta K_3 \right) \right], \quad (10) \]

with

\[ a = \frac{\eta}{1 - \exp(-\eta)}. \quad (11) \]

For the \((j,0)\) matter fields, \(\phi^R(p^\mu)\), we have \(\vec{K} = -i \vec{J}\). For the \((0,j)\) matter fields, \(\phi^L(p^\mu)\), \(\vec{K} = +i \vec{J}\). Using this observation, along with Eq. (10) and definitions (11), we obtain the transformation properties of the front form \((j,0)\) and \((0,j)\) hadronic fields

\[ \phi^R(p^\mu) = \exp \left( + \eta \hat{b} \cdot \vec{J} \right) \phi^R(\hat{p}^\mu), \quad (12) \]

and

\[ \phi^L(p^\mu) = \exp \left( - \eta \hat{b}^\ast \cdot \vec{J} \right) \phi^R(\hat{p}^\mu). \quad (13) \]

In Eqs. (12) and (13) we have introduced the unit vectors \(\hat{b}\) and its complex conjugate \(\hat{b}^\ast\) as follows

\[ \hat{b} = \eta^{-1} \left( a v_r, -i a v_r, \eta \right), \quad (14) \]

\[ \hat{b}^\ast = \eta^{-1} \left( a v_\ell, i a v_\ell, \eta \right); \quad \hat{b} \cdot \hat{b} = 1 = \hat{b}^\ast \cdot \hat{b}^\ast, \quad (15) \]

with \(v_r = v_x + i v_y\) and \(v_\ell = v_x - i v_y\).

We now make two observations. First, under the operation of parity

\[ \text{Parity : } \phi^R(p^\mu) \leftrightarrow \phi^L(p^\mu). \quad (16) \]

Second, because of the isotropy of the null vector \(\vec{p} = \vec{0}\) (As argued in Ref. [11] and discussed in more detail in Ref. [26]) the concept of handedness looses its physical significance for \(p^\mu = \hat{p}^\mu\), and this in turn yields the relation

\[ \phi^R(\hat{p}^\mu) = \pm \phi^L(\hat{p}^\mu). \quad (17) \]

To exploit these observations we now introduce the spin-j hadronic wave functions

\[ \psi(p^\mu) = \begin{bmatrix} \phi^R(p^\mu) + \phi^L(p^\mu) \\ 0 \\ \phi^R(p^\mu) - \phi^L(p^\mu) \end{bmatrix}, \quad (18) \]
The plus (minus) sign in Eq. (17) yields hadronic wave functions with even (odd) intrinsic parity. We will denote the even intrinsic parity wave functions by $U(p^\mu)$; and the odd intrinsic parity wave functions by $V(p^\mu)$.

The transformation property for these hadronic wave functions under the boost (5) is now readily obtained by using Eqs. (12) and (13). The result is

$$\psi(p^\mu) = M(L) \psi(\hat{p}^\mu),$$

with the hadronic-wave-function boost operator, $M(L)$, given by

$$M(L) = \begin{bmatrix} \exp(\eta \hat{b} \cdot \vec{J}) + \exp(-\eta \hat{b}^* \cdot \vec{J}) & \exp(\eta \hat{b} \cdot \vec{J}) - \exp(-\eta \hat{b}^* \cdot \vec{J}) \\ \exp(\eta \hat{b} \cdot \vec{J}) - \exp(-\eta \hat{b}^* \cdot \vec{J}) & \exp(\eta \hat{b} \cdot \vec{J}) + \exp(-\eta \hat{b}^* \cdot \vec{J}) \end{bmatrix}. \quad (20)$$

The construction of the hadronic wave functions $U(p^\mu)$ and $V(p^\mu)$, introduced above, for a specific spin requires: (i) Explicit evaluation of the $2(2j + 1) \times 2(2j + 1)$ matrix $M(L)$ defined via Eq. (20); and (ii) Some other necessary technical details (see below). We establish this procedure in the next section by explicitly constructing $U(p^\mu)$ and $V(p^\mu)$ for $j = \frac{1}{2}, 1, \frac{3}{2}, 2$.

**III. CONSTRUCTION OF $U(p^\mu)$ AND $V(p^\mu)$ AND THEIR PROPERTIES**

For obtaining specific explicit expressions for the hadronic wave functions $U(p^\mu)$ and $V(p^\mu)$, the technical detail which still needs to be taken care of is to fix the representation for the generators of rotation $\vec{J}$. Towards this end we note that the front form helicity operator

$$\mathcal{J}_3 \equiv J_3 + \frac{1}{P^-} (G_1 P_2 - G_2 P_1),$$

introduced by Soper [4] and discussed by Leutwyler and Stern [7], commutes with all generators of the stability group associated with the $x^+ = 0$ plane. The front form helicity operator associated with the $(j, 0) \oplus (0, j)$ hadronic fields constructed above is then readily defined to be

$$\Theta = \begin{bmatrix} \mathcal{J}_3 & 0 \\ 0 & \mathcal{J}_3 \end{bmatrix}. \quad (22)$$
Taking a matrix representation of the $\vec{J}$ operators with $J^3$ diagonal (in the standard convention of Ref. [27]), the $2(2j + 1)$-element basis spinors for the hadronic wave functions which correspond to $p^\mu = \bar{p}^\mu$ have then the general form

$$U_{+j}(\bar{p}^\mu) = \begin{bmatrix} \mathcal{N}(j) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad U_{j-1}(\bar{p}^\mu) = \begin{bmatrix} 0 \\ \mathcal{N}(j) \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \ldots \ldots, \quad V_{-j}(\bar{p}^\mu) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \mathcal{N}(j) \end{bmatrix}. \quad (23)$$

The index $h = j, j-1, \ldots, -j$ on the $U_h(p^\mu)$ and $V_h(p^\mu)$ corresponds to the eigenvalues of the the front form helicity operator $\Theta$. The spin-dependent normalisation constant $\mathcal{N}(j)$ is to be so chosen that for the massless particles the $U_h(p^\mu)$ and $V_h(p^\mu)$ vanish (There can be no massless particles at rest!); and only the $U_{h=\pm j}(p^\mu)$ and $V_{h=\pm j}(p^\mu)$ survive in the high energy limit. As will be shortly verified the simplest choice satisfying these requirements is

$$\mathcal{N}(j) = m^j. \quad (24)$$

We now have all the details needed to construct $U_h(p^\mu)$ and $V_h(p^\mu)$ for any hadronic field. In this paragraph we summarise the algebraic construction used for $j = \frac{1}{2}, 1, \frac{3}{2}, 2$. Using Eqs. (A27,A28) and (A31,A32) of Ref. [10] along with Eqs. (9,14,15) above we obtain the the expansions for the $\exp(\eta \hat{b} \cdot \vec{J})$ and $\exp(-\eta \hat{b}^* \cdot \vec{J})$ which appear in the hadronic-wave-function boost matrix $M(L)$, Eq. (20). These expansions are presented in Appendix A. Using the results of Appendix A, explicit expressions for $M(L)$, Eq. (20), are then calculated as a simple, but somewhat lengthy, algebraic exercise. The $M(L)$ so obtained in conjunction with Eqs. (19) and (23), then yield the hadronic wave functions presented in Appendix B. The generality of the procedure for any spin is now obvious, and the procedure reduces to a well defined algebraic manipulations.

We now introduce the following useful matrices,

$$\Gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}, \quad \Gamma^5 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \quad (25)$$
with \( I = (2j + 1) \times (2j + 1) \) identity matrix.

In reference to the hadronic wave functions presented in Appendix B, we define

\[
\overline{\psi}(p^\mu) \equiv \psi_\dagger(p^\mu) \Gamma^\circ.
\]  

(26)

Using the explicit expressions for \( \mathcal{U}(p^\mu) \) and \( \mathcal{V}(p^\mu) \), Eqs. (B1-B7), we verify that

\[
\overline{\mathcal{U}}_h(p^\mu) \mathcal{U}_{h'}(p^\mu) = m^{2j} \delta_{hh'}, \tag{27}
\]

\[
\overline{\mathcal{V}}_h(p^\mu) \mathcal{V}_{h'}(p^\mu) = -m^{2j} \delta_{hh'}, \tag{28}
\]

\[
\overline{\mathcal{U}}_h(p^\mu) \mathcal{V}_{h'}(p^\mu) = 0 = \overline{\mathcal{V}}_h(p^\mu) \mathcal{U}_{h'}(p^\mu). \tag{29}
\]

One of the most noteworthy features of the \( \mathcal{U}_h(p^\mu) \) and \( \mathcal{V}_h(p^\mu) \) constructed here in the \((j, 0) \oplus (0, j)\) representation is the observation [28, Sec. III] that the high energy limit, \( p^+ \gg m \), is equivalent to the massless case. This observation is consistent with the explicit results presented in Appendix B: \( \mathcal{U}_h(p^\mu) \) and \( \mathcal{V}_h(p^\mu) \) identically vanish in this limit unless the associated front form helicity \( h = \pm j \).

This is precisely the type of behaviour that a theory suited for the high energy phenomenon is expected to have. An examination of Eqs. (B3), (B4) and (B5-B7) further suggests that in the high energy limit the \( \mathcal{U}_h(p^\mu) \) and \( \mathcal{V}_h(p^\mu) \) fall off as \( \sim (m/p^+)^j - |h| \).

Finally, we note that for spin one half the result given by Eq. (B1) is identical to that given by Lepage and Brodsky [19, Eq.A3]. Note however a slightly different normalisation and their choice of phase for the odd intrinsic parity wave functions.

**IV. THE CONNECTION WITH THE INSTANT FORM**

In a representation appropriate for comparison with the front form spinors \( \mathcal{U}_h(p^\mu) \) and \( \mathcal{V}_h(p^\mu) \), the instant form hadronic wave functions \( u_\sigma(p^\mu) \) and \( v_\sigma(p^\mu) \), \( \sigma = j, j - 1, \ldots, -j \), were recently constructed (following Weinberg [10] and Ryder [11]) explicitly in Refs. [12,13,19]. A brief report, sufficient for the present discussion, can be found in Ref. [13]. Here we only remark that the construction of instant form spinors follows the steps outlined in Eqs. (10-20) above, with the only difference that one starts with transformation \( \Lambda \) of Eq.(2) rather than \( L \) of Eq.(10).
The instant form hadronic wave functions of Refs. [12,13,19] satisfy the following normalisation properties

\[
\begin{align*}
\bar{u}_\sigma(p^\mu) u_{\sigma'}(p^\mu) &= m^2 j \delta_{\sigma\sigma'} \quad (30) \\
\bar{v}_h(p^\mu) v_{\sigma'}(p^\mu) &= -m^2 j \delta_{\sigma\sigma'} \quad (31) \\
\bar{u}_h(p^\mu) v_{h'}(p^\mu) &= 0 = \bar{v}_h(p^\mu) u_{h'}(p^\mu) \quad (32)
\end{align*}
\]

where

\[
\bar{u}_\sigma(p^\mu) = [u_\sigma(p^\mu)]^\dagger \gamma^\circ, \quad \bar{v}_\sigma(p^\mu) = [v_\sigma(p^\mu)]^\dagger \gamma^\circ \quad (33)
\]

with \(\gamma^\circ = \) having the form identical to \(\Gamma^\circ\) of Eq. (25). In what follows we assume that \(p^\mu\) and \(p^\mu\) correspond to the same physical momentum.

The connection between the front form and instant form is established by noting that on general algebraic grounds, we can express the instant form hadronic wave functions as linear combination of the front form hadronic wave functions. That is

\[
\begin{align*}
\bar{u}_\sigma(p^\mu) &= \Omega^{(uU)} \Omega_{\sigma h} U_h(p^\mu) + \Omega^{(uV)} \Omega_{\sigma h} V_h(p^\mu), \\
\bar{v}_\sigma(p^\mu) &= \Omega^{(vU)} \Omega_{\sigma h} U_h(p^\mu) + \Omega^{(vV)} \Omega_{\sigma h} V_h(p^\mu),
\end{align*}
\]

where the sum on the repeated indices is implicit.

We now multiply Eq. (34) by \(\bar{U}_{h'}(p^\mu)\) from the left, and using the orthonormality relations, Eqs. (27-29), we get

\[
\Omega^{(uU)} \Omega_{\sigma h} = \frac{1}{m^2 j} \left[ \bar{U}_h(p^\mu) u_\sigma(p^\mu) \right]. \quad (36)
\]

Similarly by multiplying Eq. (35) from the left by \(\bar{V}_{h'}(p^\mu)\) and again using the orthonormality relations, Eqs. (27-29), we obtain

\[
\Omega^{(uV)} \Omega_{\sigma h} = \frac{1}{m^2 j} \left[ \bar{V}_h(p^\mu) u_\sigma(p^\mu) \right]. \quad (37)
\]

Further it is readily verified, e.g. by using the results presented in Appendix B here and explicit expressions for \(u_\sigma(p^\mu)\) and \(v_\sigma(p^\mu)\) found in Refs. [12,13,19], that
\[ \nabla_h(p^\mu) u_\sigma(p^\mu) = 0 = \overline{U}_h(p^\mu) v_\sigma(p^\mu); \] (38)

which yields
\[ \Omega^{(u\nu)}_{\sigma h} = 0 = \Omega^{(vU)}_{\sigma h}. \] (39)

Finally, we exploit the facts
\[ \{ \Gamma^5, \Gamma^\circ \} = 0, \quad \Gamma^{5\dagger} = \Gamma^5, \quad \text{and} \quad (\Gamma^5)^2 = I, \] (40)

to conclude that \( \nabla_h(p^\mu) v_\sigma(p^\mu) = -\overline{U}_h(p^\mu) u_\sigma(p^\mu) \). Thus defining a \((2j + 1) \times (2j + 1)\) matrix \( B(j) \) such that \( B_{\sigma h} = \overline{U}_h(p^\mu) u_\sigma(p^\mu) = \Omega^{(u\nu)}_{\sigma h} = \Omega^{(v\nu)}_{\sigma h} \) we get the matrix which connects the instant form hadronic wave functions with the front form wave functions. It reads
\[ \Omega(j) = \begin{bmatrix} B(j) & 0 \\ 0 & B(j) \end{bmatrix}. \] (41)

The explicit expressions for \( \Omega(j) \) are presented in Appendix C. For spin one half the transformation matrix \( \Omega(1/2) \) computed by us coincides with the celebrated “Melosh transformation” given by Melosh in Ref. [20, Eq. 26] and by Dziembowski in Ref. [21, Eq. A8]. As formally demonstrated by Kondratyuk and Terent’ev [29], the transformation matrix \( \Omega(j) \) represents a pure rotation of the spin basis. However, since \( \Omega(j) \) has block zeros off-diagonal, what manifestly emerges here is that this rotation does not mix the even and odd intrinsic parity wave functions.

V. SUMMARY AND CONCLUDING REMARKS

We begin with a summary of the results obtained. In this paper we have presented a relativistic formalism to construct hadronic fields of arbitrary spin in the front form, and established their connection with the instant form fields. For a given a spin, \( j \), there are \((2j + 1)\) hadronic wave functions with even intrinsic parity, \( U_h(p^\mu) \), and \((2j + 1)\) hadronic wave functions with odd intrinsic parity, \( V_h(p^\mu) \). From a formal point of view, the fundamental object required to construct these wave functions is the hadronic-wave-function boost matrix \( M(L) \) in the front form. The matrix \( M(L) \) is
The explicit construction of the front form \((j, 0) \oplus (0, j)\) hadronic spinors requires the introduction of the front form helicity operator \(J_3\), Eq. (21), introduced by Soper \[6\] and Leutwyler and Stern \[7\]. Further, the normalisation of these wave functions should be so chosen that for the massless particles the \(U_h(p^\mu)\) and \(V_h(p^\mu)\) identically vanish; and only \(U_h = \pm j(p^\mu)\) and \(V_h = \pm j(p^\mu)\) survive in the high energy limit. The simplest choice of this normalisation is given by Eq. (24). Next we constructed a matrix \(\Omega(j)\) which provides the connection between the front form hadronic wave functions with the more familiar \[i.e. more “familiar” at least for spin one half case\] instant form hadronic wave functions. We discovered that the transformation matrix \(\Omega(1/2)\) coincides with the well known “Melosh transformation” \[21,22\], and the spin one half wave functions are in agreement with the previous results of Lepage and Brodsky \[20\]. Explicit results for \(U_h(p^\mu)\), \(V_h(p^\mu)\) and \(\Omega(j)\) up to spin two are found in Appendixes B and C here.

Having provided a brief summary of the main results of this work, we now take liberty of making some concluding remarks. While, in principle, the effective action of the QCD, \(S[\pi, \rho, \omega, \ldots, N, \bar{N}, \Delta^{3/2}, \ldots]\), contains arbitrarily high spins, an effective theory wishing to describe high energy phenomenon up to a certain energy scale can comfortably truncate the expansion at a certain physically determined value of spin. This physical value may be determined, for example, by the usual partial wave analysis of the cross sections. Even though the details of this procedure and the actual connecting link with QCD does not yet exist in detail, there is little doubt that the hadronic fields constructed here are one of the most natural objects in terms of which to express a phenomenological and effective action of medium and high energy hadronic and nuclear physics. For perturbative calculations we can construct front form field operators \(\Psi[x^\mu, j]\) as expansions in terms of the front form fields \(U_h(p^\mu)\) and \(V_h(p^\mu)\) presented here, and then evaluate the vacuum expectation value of the \(x^+\) ordered product \(\langle x^+ | X^+ \left[ \Psi [x^\mu, j] \bar{\Psi} [y^\mu, j] \right] | \rangle\) to obtain the front form Feynman-Dyson propagators. The \(x^+\) ordered product of a set of arbitrary front form field operators \(\Psi_{(\alpha)}[x^\mu_{(\alpha)}] \Psi_{(\beta)}[x^\mu_{(\beta)}] \ldots \Psi_{(\epsilon)}[x^\mu_{(\epsilon)}]\) is defined as follows

\[
X^+ \left[ \Psi_{(\alpha)}[x^\mu_{(\alpha)}] \Psi_{(\beta)}[x^\mu_{(\beta)}] \ldots \Psi_{(\epsilon)}[x^\mu_{(\epsilon)}]\right] = (-1)^f \Psi_{(\omega)}[x^\mu_{(\omega)}] \Psi_{(\lambda)}[x^\mu_{(\lambda)}] \ldots \Psi_{(\rho)}[x^\mu_{(\rho)}],
\]  

where on the right hand side, the field operators are the same ones as on the left but are ordered
in such an order that

\[ x^+_{(\omega)} \geq x^+_{(\lambda)} \geq \ldots \geq x^+_{(\rho)}, \] (43)

and \( f \) = the number of necessary transpositions among the fermion field operators that are needed to achieve the ordering. These \( x^\mu \)-space propagators can be Fourier transformed to obtain the \( p^\mu \)-space front form Feynman-Dyson propagators. At this stage one can either further develop a front form formalism similar to Weinberg’s equations in the instant form to guide oneself to define a class of relativistic phenomenologies characterised by a specific choice of \( S[\pi, \rho, \omega, \ldots, N, \nabla, \Delta^{3/2}, \ldots] \), or invert the \( p^\mu \)-space front form Feynman-Dyson propagator as recently done in Ref. [14] to obtain a class of non-local (for \( j > 1/2 \); for the \( j = 1/2 \) the equation is identical to the Dirac equation) wave equations in the instant form. Even though it has been recently shown that Weinberg equations for arbitrary spin suffer from certain kinematically spurious [13,16] solutions, we have been able to develop [30] a Lorentz covariant procedure which prevents the interactions to induce transitions between the kinematically acceptable solutions (which are the counterpart of the front form wave functions in the instant form) and kinematically unacceptable solutions.

Apart from these practical aspects, the formalism has certain in-built elegance and exhibits great beauty of the front form of field theory. The work presented here, as already indicated, provides the unifying link between the early works of Weinberg [10] and Dirac [23] on high spin fields and their later works on infinite momentum frame [24] and front form [4] of field theory.

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a visiting faculty at the Department of Physics of the Texas A&M University. He would like to gratefully thank Professor David J. Ernst and members of the Department of Physics for warm hospitality extended to him during his stay. The financial support of the NSF under the Grant PHY-8907852 is gratefully acknowledged.
APPENDIX A: EXPANSIONS FOR THE $\exp(\eta \hat{b} \cdot \vec{J})$ AND $\exp(-\eta \hat{b}^* \cdot \vec{J})$ UP TO SPIN TWO

This appendix provides the expansions for the $\exp(\eta \hat{b} \cdot \vec{J})$ and $\exp(-\eta \hat{b}^* \cdot \vec{J})$, up to $j = 2$, which appear in the hadronic-wave-function boost matrix $M(L)$, Eq. (20).

\[ j = \frac{1}{2} \]
\[
\exp(\eta \hat{b} \cdot \vec{J}) = \cosh(\eta/2) I + \left(\hat{b} \cdot \vec{\sigma}\right) \sinh(\eta/2),
\]
\[
\exp(-\eta \hat{b}^* \cdot \vec{J}) = \cosh(\eta/2) I - \left(\hat{b}^* \cdot \vec{\sigma}\right) \sinh(\eta/2).
\]

\[ j = 1 \]
\[
\exp(\eta \hat{b} \cdot \vec{J}) = I + 2 \left(\hat{b} \cdot \vec{J}\right)^2 \sinh^2(\eta/2) + 2 \left(\hat{b} \cdot \vec{J}\right) \cosh(\eta/2) \sinh(\eta/2),
\]
\[
\exp(-\eta \hat{b}^* \cdot \vec{J}) = I + 2 \left(\hat{b}^* \cdot \vec{J}\right)^2 \sinh^2(\eta/2) - 2 \left(\hat{b}^* \cdot \vec{J}\right) \cosh(\eta/2) \sinh(\eta/2).
\]

\[ j = \frac{3}{2} \]
\[
\exp(\eta \hat{b} \cdot \vec{J}) = \cosh(\eta/2) \left[ I + \frac{1}{2} \left\{ \left(2\hat{b} \cdot \vec{J}\right)^2 - I \right\} \sinh^2(\eta/2) \right]
\]
\[
+ \left(2\hat{b} \cdot \vec{J}\right) \sinh(\eta/2) \left[ I + \frac{1}{6} \left\{ \left(2\hat{b} \cdot \vec{J}\right)^2 - I \right\} \sinh^2(\eta/2) \right],
\]
\[
\exp(-\eta \hat{b}^* \cdot \vec{J}) = \cosh(\eta/2) \left[ I + \frac{1}{2} \left\{ \left(2\hat{b}^* \cdot \vec{J}\right)^2 - I \right\} \sinh^2(\eta/2) \right]
\]
\[
- \left(2\hat{b}^* \cdot \vec{J}\right) \sinh(\eta/2) \left[ I + \frac{1}{6} \left\{ \left(2\hat{b}^* \cdot \vec{J}\right)^2 - I \right\} \sinh^2(\eta/2) \right].
\]

\[ j = 2 \]
\[
\exp(\eta \hat{b} \cdot \vec{J}) = I + 2 \left(\hat{b} \cdot \vec{J}\right)^2 \sinh^2(\eta/2) + \frac{2}{3} \left(\hat{b} \cdot \vec{J}\right)^2 \left\{ \left(\hat{b} \cdot \vec{J}\right)^2 - I \right\} \sinh^4(\eta/2)
\]
\[
+ 2 \left(\hat{b} \cdot \vec{J}\right) \cosh(\eta/2) \sinh(\eta/2) + \frac{4}{3} \left(\hat{b} \cdot \vec{J}\right) \left\{ \left(\hat{b} \cdot \vec{J}\right)^2 - I \right\} \cosh(\eta/2) \sinh^3(\eta/2),
\]
\[
\exp(-\eta \hat{b}^* \cdot \vec{J}) = I + 2 \left(\hat{b}^* \cdot \vec{J}\right)^2 \sinh^2(\eta/2) + \frac{2}{3} \left(\hat{b}^* \cdot \vec{J}\right)^2 \left\{ \left(\hat{b}^* \cdot \vec{J}\right)^2 - I \right\} \sinh^4(\eta/2)
\]
\[
- 2 \left(\hat{b}^* \cdot \vec{J}\right) \cosh(\eta/2) \sinh(\eta/2) - \frac{4}{3} \left(\hat{b}^* \cdot \vec{J}\right) \left\{ \left(\hat{b}^* \cdot \vec{J}\right)^2 - I \right\} \cosh(\eta/2) \sinh^3(\eta/2).
\]

In Eqs. (A1)-(A2) the $\vec{\sigma}$ are the standard Pauli matrices. In this appendix, $I$ are the $(2j + 1) \times (2j + 1)$ identity matrices.
APPENDIX B: FRONT FORM HADRONIC WAVE FUNCTIONS UP TO SPIN TWO

We begin with collecting together front form hadronic wave functions up to spin 2 for even intrinsic parity, first. In what follows we use the notation \( p_r = p_x + i p_y \) and \( p_\ell = p_x - i p_y \), c.f. Eq.(15).

Spin one half hadronic wave functions with even intrinsic parity:

\[
\mathcal{U}_{+ \frac{1}{2}}(p^\mu) = \frac{1}{2} \sqrt{\frac{1}{p^+}} \begin{pmatrix}
  p^+ + m \\
p_r \\
p^+ - m \\
p_r
\end{pmatrix}, \quad \mathcal{U}_{- \frac{1}{2}}(p^\mu) = \frac{1}{2} \sqrt{\frac{1}{p^+}} \begin{pmatrix}
  -p_\ell \\
p^+ + m \\
p_\ell \\
-p^+ + m
\end{pmatrix}.
\] (B1)

Spin one hadronic wave functions with even intrinsic parity:

\[
\mathcal{U}_{+1}(p^\mu) = \frac{1}{2} \begin{pmatrix}
  p^+ + (m^2/p^+) \\
\sqrt{2} p_r \\
p_r^2/p^+ \\
p^+ - (m^2/p^+)
\end{pmatrix}, \quad \mathcal{U}_0(p^\mu) = m \sqrt{\frac{1}{2}} \begin{pmatrix}
  -p_\ell/p^+ \\
\sqrt{2} \\
p_\ell/p^+ \\
0
\end{pmatrix}, \quad \mathcal{U}_{-1}(p^\mu) = \frac{1}{2} \begin{pmatrix}
  p_\ell/p^+ \\
p^+ + (m^2/p^+) \\
-p_\ell^2/p^+ \\
-p^+ + (m^2/p^+)
\end{pmatrix}.
\] (B2)

Spin three half hadronic wave functions with even intrinsic parity:
\[ \mathcal{U}_{+\pm}(p^\mu) = \frac{1}{2} \sqrt{1 \over p^+} \begin{bmatrix} p^+ - (m^2/p^+) \\ \sqrt{3} p_r p^+ \\ \sqrt{3} p_r^2 \\ p_r^3/p^+ \end{bmatrix} \quad \text{and} \quad \mathcal{U}_{+\pm}(p^\mu) = \frac{m}{2} \sqrt{1 \over p^+} \begin{bmatrix} -\sqrt{3} m p^+ \\ p^+ + m \\ 2 p_r \\ \sqrt{3} p_r^2/p^+ \\ \sqrt{3} m p^+ \\ p^+ - m \\ 2 p_r \\ \sqrt{3} p^2_r/p^+ \end{bmatrix}, \quad (B3) \]
\[
\mathbf{U}_{-\frac{1}{2}}(p^\mu) = \frac{m}{2} \sqrt{\frac{1}{p^+}} \begin{pmatrix}
\sqrt{3} p_t^2 / p^+ \\
-2 p_t \\
p^+ + m \\
\sqrt{3} m p_r / p^+ \\
-\sqrt{3} p_t^2 / p^+ \\
2 p_t \\
-p^+ + m \\
\sqrt{3} m p_r / p^+
\end{pmatrix} \quad \text{and} \quad \mathbf{U}_{-\frac{3}{2}}(p^\mu) = \frac{1}{2} \sqrt{\frac{1}{p^+}} \begin{pmatrix}
-p_t^3 / p^+ \\
\sqrt{3} p_t^2 \\
-\sqrt{3} p_t^2 p^+ \\
p^+^2 + (m^3 / p^+) \\
p_t^3 / p^+ \\
-\sqrt{3} p_t^2 \\
\sqrt{3} p_t p^+ \\
- p^+^2 + (m^3 / p^+)
\end{pmatrix}.
\] (B4)

Spin two hadronic wave functions with even intrinsic parity:
\[
\mathcal{U}_{+2}(p^\mu) = \frac{1}{2} \begin{pmatrix}
p^{+2} + \left(\frac{m^4}{p^{+2}}\right) \\
2p_r p^+ \\
\sqrt{6} p_r^2 \\
2p_r^3/p^+ \\
p_r^4/p^{+2}
\end{pmatrix}, \quad \mathcal{U}_{+1}(p^\mu) = \frac{m}{2} \begin{pmatrix}
-2m^2 p_r/p^{+2} \\
p^{+} + \left(\frac{m^2}{p^{+}}\right) \\
\sqrt{6} p_r \\
3p_r^2/p^+ \\
2p_r^3/p^{+2}
\end{pmatrix}
\]

\text{(B5)}
\[ U_0(p^\mu) = \frac{m^2}{2} \begin{pmatrix} \sqrt{6} p^2 / p^+^2 \\ - \sqrt{6} p_T / p^+ \\ 2 \\ \sqrt{6} p_r / p^+ \\ \sqrt{6} p_T^2 / p^+^2 \\ - \sqrt{6} p_T / p^+^2 \\ \sqrt{6} p_T / p^+ \\ 0 \\ \sqrt{6} p_r / p^+ \\ \sqrt{6} p_r^2 / p^+^2 \end{pmatrix} , \quad U_{-1}(p^\mu) = \frac{m}{2} \begin{pmatrix} - 2 p_T^3 / p^+^2 \\ 3 p_T^2 / p^+ \\ - \sqrt{6} p_T \\ p^+ + (m^2 / p^+) \\ 2 m^2 p_r / p^+^2 \\ 2 p_T^3 / p^+^2 \\ - 3 p_T^2 / p^+ \\ \sqrt{6} p_T \\ - p^+ + (m^2 / p^+) \\ 2 m^2 p_r / p^+^2 \end{pmatrix} \] 

(B6)
An examination of the hadronic-wave-function boost matrix $M(L)$, Eq.(20), implies that the odd intrinsic-parity hadronic wave can be obtained from the hadronic wave functions of the even intrinsic parity via the following simple relation

$$V_h(p^\mu) = \Gamma^5 U_h(p^\mu) ,$$

where the matrix $\Gamma^5$ defined in Eq.(25) interchanges the top $(2j + 1)$ elements with the bottom $(2j + 1)$ elements of the hadronic wave functions.
APPENDIX C: THE EXPLICIT EXPRESSIONS FOR $\Omega(j)$ UP TO SPIN TWO

In this appendix we present explicit expressions for the matrix $\Omega(j)$, Eq. (41), which connects the front form hadronic wave functions with the hadronic wave functions via Eqs. (34,35). As in Appendix B, $p_r = p_x + i p_y$ and $p_\ell = p_x - i p_y$, in what follows.

For spin one half, the matrix connecting the instant form hadronic wave functions with the front form wave functions is

$$\Omega(1/2) = \frac{1}{[2(E + m) p^+]^{1/2}} \begin{bmatrix} \beta(1/2) & 0 \\ 0 & \beta(1/2) \end{bmatrix}, \quad (C1)$$

where the the $2 \times 2$ block matrix $\beta(1/2)$ is defined as

$$\beta(1/2) = \begin{bmatrix} p^+ + m & -p_r \\ p_\ell & p^+ + m \end{bmatrix}. \quad (C2)$$

For spin one, the matrix connecting the instant form hadronic wave functions with the front form wave functions is

$$\Omega(1) = \frac{1}{[2(E + m) p^+]^{1/2}} \begin{bmatrix} \beta(1) & 0 \\ 0 & \beta(1) \end{bmatrix}, \quad (C3)$$

where the the $3 \times 3$ block matrix $\beta(1)$ is defined as

$$\beta(1) = \begin{bmatrix} (p^+ + m)^2 & -\sqrt{2} (p^+ + m) p_r & p_\ell^2 \\ \sqrt{2} (p^+ + m) p_\ell & 2 [(E + m) p^+ - p_r p_\ell] & -\sqrt{2} (p^+ + m) p_r \\ p_\ell^2 & \sqrt{2} (p^+ + m) p_\ell & (p^+ + m)^2 \end{bmatrix}. \quad (C4)$$

For spin three half, the matrix connecting the instant form hadronic wave functions with the front form wave functions is
\[ \Omega \left( \frac{3}{2} \right) = \frac{1}{[2 (E + m) p^+]^{3/2}} \begin{bmatrix} \beta \left( \frac{3}{2} \right) & 0 \\ 0 & \beta \left( \frac{3}{2} \right) \end{bmatrix}, \]  

(C5)

where the the \( 4 \times 4 \) block matrix \( \beta \left( \frac{3}{2} \right) \) is defined as

\[
\beta \left( \frac{3}{2} \right) = \begin{bmatrix} 
(p^+ + m)^3 & -\sqrt{3} (p^+ + m)^2 p_r & \sqrt{3} (p^+ + m) p_r^2 & -p_r^3 \\
\sqrt{3} (p^+ + m)^2 p_\ell & \left( (p^+ + m)^2 - 2 p_r p_\ell \right) (p^+ + m) & -2 (p^+ + m)^2 - p_r p_\ell & \sqrt{3} (p^+ + m) p_\ell^2 \\
\sqrt{3} (p^+ + m) p_\ell^2 & \left[ 2 (p^+ + m)^2 - p_r p_\ell \right] p_\ell & \left( (p^+ + m)^2 - 2 p_r p_\ell \right) (p^+ + m) & -\sqrt{3} (p^+ + m)^2 p_\ell \\
p_\ell^3 & \sqrt{3} (p^+ + m) p_\ell^2 & \sqrt{3} (p^+ + m) p_\ell & (p^+ + m)^3
\end{bmatrix}.
\]

(C6)

For spin two, the matrix connecting the instant form hadronic wave functions with the front form wave functions is

\[ \Omega \left( 2 \right) = \frac{1}{[2 (E + m) p^+]^2} \begin{bmatrix} \beta \left( 2 \right) & 0 \\ 0 & \beta \left( 2 \right) \end{bmatrix}, \]  

(C7)

where the the \( 5 \times 5 \) block matrix \( \beta \left( 2 \right) \) is defined via the following five columns

\[
\beta_{\alpha,1} = \begin{bmatrix} 
(p^+ + m)^4 \\
2 (p^+ + m)^3 p_\ell \\
\sqrt{6} (p^+ + m)^2 p_\ell^2 \\
2 (p^+ + m) p_\ell^3 \\
p_\ell^4
\end{bmatrix}, \quad \beta_{\alpha,2} = \begin{bmatrix} 
-2 (p^+ + m)^3 p_r \\
2 [(E + m) p^+ - 2 p_r p_\ell] (p^+ + m)^2 \\
\sqrt{6} [(E + m) p^+ - p_r p_\ell] (p^+ + m) p_\ell \\
[6 p^+ (E + m) - 4 p_r p_\ell] p_\ell^2 \\
2 (p^+ + m) p_\ell^3
\end{bmatrix}.
\]
\[ \beta(2)_{\alpha,3} = \begin{bmatrix} \sqrt{6} (p^+ + m)^2 \ p_r^2 \\ \sqrt{6} \ (- (E + m) \ p^+ + 2 \ p_r \ p_t) \ (p^+ + m) \ p_r \\ \sqrt{6} \ [E + m] \ p^+ - 2 \ p_r \ p_t \ (p^+ + m) \ p_t \\ \sqrt{6} (p^+ + m)^2 \ p_t^2 \\ - 2 \ (p^+ + m) \ p_r^3 \\ [6 \ p^+ \ (E + m) - 4 \ p_r \ p_t] \ p_r^2 \end{bmatrix}, \]

\[ \beta(2)_{\alpha,4} = \begin{bmatrix} \sqrt{6} \ (- (E + m) \ p^+ + p_r \ p_t) \ (p^+ + m) \ p_r \\ 2 \ [E + m] \ p^+ - 2 \ p_r \ p_t \ (p^+ + m)^2 \\ 2 \ (p^+ + m)^3 \ p_t \end{bmatrix}, \quad \beta(2)_{\alpha,5} = \begin{bmatrix} p_r^4 \\ - 2 \ (p^+ + m) \ p_r^3 \\ - 2 \ (p^+ + m)^3 \ p_r \\ (p^+ + m)^4 \end{bmatrix}. \]
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TABLES

TABLE I. Algebra associated with the stability group of the $x^+ = 0$ plane. The Commutator 
[Element in the \textit{first} column, Element in the \textit{first} row] = The element at the intersection of the 
row and column.

|     | $P_1$ | $P_2$ | $J_3$ | $K_3$ | $P_-$ | $G_1$ | $G_2$ |
|-----|-------|-------|-------|-------|-------|-------|-------|
| $P_1$ | 0     | 0     | $-iP_2$ | 0     | 0     | $iP_-$ | 0     |
| $P_2$ | 0     | 0     | $iP_1$  | 0     | 0     | 0     | $iP_-$ |
| $J_3$ | $iP_2$ | $-iP_1$ | 0     | 0     | 0     | $iG_2$ | $-iG_1$ |
| $K_3$ | 0     | 0     | 0     | 0     | $iP_-$ | $iG_1$ | $iG_2$ |
| $P_-$ | 0     | 0     | 0     | $-iP_-$ | 0     | 0     | 0     |
| $G_1$ | $-iP_-$ | 0     | $-iG_2$ | $-iG_1$ | 0     | 0     | 0     |
| $G_2$ | 0     | $-iP_-$ | $iG_1$ | $-iG_2$ | 0     | 0     | 0     |