Two-way relaying channel with lattice forwarding and microwave power transfer

Zhigang Wen, You Lv, Shan Li¹ and Dan Liu

Beijing Key Laboratory of Work Safety Intelligent Monitoring, School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing, China

¹Email: lish01@ehualu.com

Abstract. In this paper, we consider a two-way relaying channel with simultaneous wireless information and power transfer. The intermediate relay station forwards lattice codes under transmit power constraint. Meanwhile, it charges user terminals for static circuit consumption and uplink transmission power. We novelly propose an iterative algorithm to maximize the sum-rate with joint beam-forming and power splitting design in MPT assisted TWRC. The method achieves the highest achievable rate compared with separate optimization designs and time switching method. Moreover, an extremely small gap between the proposed method and the outer cut-set bound is observed.

1. Introduction

In energy-constrained networks, microwave power transfer (MPT) enables a power beacon to wirelessly charge mobile terminals. Based on the energy-harvesting technique, simultaneous wireless information and power transfer (SWIPT) refers to using the same emitted electromagnetic (EM) wave to transport power to the energy harvester (EH), and signals to the information decoder (ID) [1].

The application of SWIPT to multiple-input multiple-output (MIMO) relay networks recently becomes a topic of interest of the research communities. The beam-forming design with amplify-and-forward (AF) relay and EH is studied in [2]. The joint beamforming and power splitting ratio design for multi-antenna relay network to minimize relay transmit power is proposed in [3]. [4] considers a joint beamforming and time switching/power splitting scheme in MPT to maximize the network throughput. In MIMO relay networks, two-way relaying channels (TWRC) have received considerable attention, since it is a common building block of general wireless networks and can provide the best platform to show the benefit of network coding [5, 6]. [7] proposes a lattice codes for inter-pair interference cancellation in TWRC with amplify and compute relay. And then, many researchers have combined MPT with TWRC. [8] proposes a ping-pong scheme (time switching) with MPT. [9] investigates throughput maximization problem for TWRC in MPT. Considering the network coding mode in SWIPT TWRC communication scenario, [10] uses physical-layer network coding (PLNC) in SWIPT TWRC system for optimizing beam-forming to maximize the weighted sum energy. [11] considers beam-forming design for orthogonal-space–time-block-code (OSTBC) based AF TWRC system with SWIPT. However, very limited research has been focused on introducing lattice forwarding (LF) physical-layer network coding (PLNC) mode into MPT TWRC system.

The main contributions of this paper are as follows:

We propose a MPT assisted TWRC system with lattice forwarding (LF) physical-layer network coding (PLNC).
For the considered system, we propose an iterative algorithm based on semi-definite programming (SDP), Lagrange multiplier method and Karush-Kuhn-Tucker (KKT) condition to obtain a joint design of beam-forming and power splitting, in an effort to maximize the sum-rate. Numerical results validate our mathematical analysis.

2. System model

As is shown in Figure 1, a TWRC system consists of multi-antenna relay station $r$ and two single-antenna nodes 1, 2. The antenna array at the relay station is used to reduce the propagation loss by steering beams towards intended directions [1]. Two phases multiple access (MAC) and broadcast (BC) on a single sub-carrier can be modeled as:

$$\begin{align*}
T_1: & \quad y_i = g_i h_i \sqrt{P_i} x_i + h_i \sqrt{P_2} x_2 + n_i \\
T_2: & \quad y_i = h'_i T_i x_i + n_i, \quad i = 1, 2
\end{align*}$$

(1)

(2)

Where $T_1$ represents uplink MAC phase, $T_2$ represents the downlink BC phase, $E[|y_i|^2] = 1$ is the transmit symbol with unit power for node $i$, $P_i$ the transmission power for node $i$. $\mathbf{f}$ is the maximum transmission power at relay side. $h_i \in C^{n \times 1}$ is the uplink channel gains. $g, f \in C^{1 \times n}$ are combining and beam-forming vectors. The design of $g$ is beyond our discussion. Abbreviate $g_i = |g_i h_i|^2$ as effective downlink channel gains, $n_i = |gn_i|$ the effective noises. Additive white gaussian noises (AWGN) obeys $CN(0,\sigma^2)$.

![Figure 1. System model of TWRC with MPT.](image)

Suppose $P_r$ is fixed while nodes 1, 2 are both powered only by $r$. The circuit power constraint in unit time slot is $P_c$. For PS scheme, the received signal in BC is divided by the power splitting factor $\beta$, into two branches, of which one stream to EH and the other $1-\beta$ to ID. For TS scheme, an MPT phase works before MAC and BC. The switch controller chooses EH or ID at one time.

Now assume that we have nested lattices $\Lambda_1 \subseteq \Lambda_2 \subseteq \Lambda$ with Voronoi Region $V_1, V_2$ shown in Figure 1 bottom. The nested lattice code $L_i$ should use $\Lambda$ as codewords and Voronoi Region $V_i$ as a shaping region:

$$L_i = \{ \Lambda \mod \Lambda_i \} = \{ \Lambda \cap V_i \}$$

(3)
Source codes $c_i, c_j$ are one-to-one mapped to $w_1 \in L_1, w_2 \in L_2$. The detailed encoding and decoding procedure are given in [12, 13].

3. Proposed PS-SWIPT with LF

For PS-SWIPT scheme, BC phase transmits information and energy simultaneously. In a stable process, the uplink transmission power is given by:

$$P_i = (1 - \beta_i) |h_i^f|^2 - 2P_i, \quad i = 1, 2$$

(4)

Assume $\sigma_i^2, \sigma_j^2$ are noise power before and after the splitter, satisfying $\sigma^2 = \sigma_i^2 + \sigma_j^2$. Based on the theorem in [12], we obtain the achievable rate region for PS-SWIPT as below:

$$R_{r,i} < \frac{1}{2} \log \left( 1 + \frac{|h_i^f|^2}{\sigma_i^2} \right)$$

(5)

$$R_{r,j} < \frac{1}{2} \log \left( 1 + \frac{|h_j^f|^2}{\sigma_j^2} \right)$$

(6)

where $\gamma_i = \frac{|h_i^f|^2}{2P_i}$.

The upper bound of TWRC is a special case when $\gamma_i = 1$. Applying cut-set bound theory, we formulate the optimization problem below:

$$\max_{f, \beta_i} \quad R = \min(R_{r,i}, R_{r,j}) + \min(R_{s,i}, R_{s,j})$$

$$s.t. : \quad \mathbf{f} \mathbf{f}^H \leq P_i$$

(7)

where $P_i = \sum_{i=1}^2 R_{r,i}$, the original problem can be approximately transformed into sum-rate maximization in BC phase. Let the relay transmit signals with maximum power, i.e. $\mathbf{f} \mathbf{f}^H = P_i$. We arrive at:

$$\sum_{i=1}^2 R_{r,i} = \log \left( \prod_{i=1}^2 \left( \frac{\mathbf{f} \mathbf{f}^H}{P_i} + \frac{\beta_i |h_i^f|^2}{\sigma_i^2} \right) \right) = \log \rho$$

(8)

Hence, we only need to find the maximum of $\rho$. Define $A_i = \frac{1}{P_i} I + \beta_i |h_i^f|^2 / \sigma_i^2$. Change the variables as: $\mathbf{F} = \mathbf{f} \mathbf{f}^H \in \mathbb{C}^{N \times N}$. $A_1, A_2$ are all Hermitian matrices. Do the following mathematical operation:

$$\rho = \text{Tr} \left\{ A_1 A_2 \mathbf{F} \right\} = \text{Tr} \left\{ A_1 A_2 \mathbf{F} \right\} = P_i \text{Tr} \left\{ A_1 A_2 \mathbf{F} \right\}$$

(9)

Thus, an equivalent optimization is given by:

$$\max_{\mathbf{F}, \beta_i} \quad P_i \text{Tr} \left\{ A_1 A_2 \mathbf{F} \right\}$$

$$s.t. : \quad \text{Tr} \left\{ \mathbf{F} \right\} = P_i$$

$$\text{Rank} \left( \mathbf{F} \right) = 1$$

(10)

Because $\text{Rank} \left( A_1, A_2 \right) = 1$, $\text{Rank} \left( \mathbf{F} \right) = 1$ always hold. The semi-definite program (SDP) problem can be solved by Matlab CVX [14] [15]. The singular value decomposition (SVD) of $\mathbf{F}$ leads to:

$$\mathbf{F} = \mathbf{U} \mathbf{Q} \mathbf{U}^H = \text{diag}(\lambda_1, \lambda_2, ..., \lambda_N), \quad \mathbf{U} = [u_1, u_2, ..., u_N]$$

(11)
where the eigenvalues in $Q$ are ordered as $\lambda_1 > \ldots > \lambda_N$, with $u_i$ as corresponding eigenvectors. Then the projection is proposed as $f^* = \lambda_1 u_1 f = \lambda_2 u_2$.

Next, consider the design of power splitter given certain $r$. The increasing of $\beta$ enables a faster transmission in BC, but leads to lower power and achievable rate in MAC. Abbreviate uplink channel gain as $h_i = |h_i^*|^2$.

For simplicity, define a linear function:

$$s_i(\beta_1, \beta_2) = l_i \beta_1 + m_i \beta_2 + n_i$$

where $l_i = h_i g_i P_r m_i = h_3 \sigma^2 (1 - \gamma) + g_i (2P_r - h_i P_r)$.

Using $\min(A, B) = \begin{cases} A & A \leq B \\ B & A > B \end{cases}$, the feasible region is divided into four feasible sections $S_1, S_2, S_3, S_4$ by lines $s_1 = (\beta_1, \beta_2) = 0, s_2 = (\beta_1, \beta_2) = 0$. For each part we have a quadratic programming (QP) problem with linear constraint quality (LCQ), which is solvable through Lagrange multiplier method and KKT condition.

We illustrate an example when $\{S_i : s_i(\beta_1, \beta_2) \leq 0, s_i(\beta_1, \beta_2) \leq 0\}$. Form the Lagrange dual function below [14]:

$$L = \sum_{i=1}^{2} \left( 1 + \frac{\beta_i P_i h_i}{\sigma^2} \right) - \sum_{i=1}^{2} \mu_i s_i(\beta_1, \beta_2)$$

$$- \sum_{i=1}^{2} \lambda_i \left( \beta_i - 1 + 2P_i h_i P_r \right) + \sum_{i=1}^{2} \eta_i \beta_i$$

where $\mu_i, \lambda_i, \eta_i$ are Lagrange multipliers. From the equation $\frac{\partial L}{\partial \beta_i} = 0$, the saddle points in $S_i$ are given by:

$$\beta_{i,1} = \begin{cases} -P_i h_i / \sigma^2 + h_i P_r (\mu_{i,1} + \mu_{i,2} g_i) + \lambda_{i,1} - \eta_{i,1} \sigma^2, i = 1, 2 \\ \end{cases}$$

$$\beta_{i,2} = \begin{cases} 0, i = 1, 2 \end{cases}$$

where Lagrange multipliers should satisfy KKT condition:

$$\begin{aligned}
\mu_i &\geq 0, \quad \lambda_i ^* \geq 0, \quad \eta_i ^* \geq 0 \\
s_i(\beta_1, \beta_2) &\leq 0, \quad \beta_i ^* - 1 + 2P_i h_i P_r \leq 0, \quad \beta_i ^* \geq 0 \\
\mu_i s_i(\beta_1, \beta_2) & = 0, \quad \lambda_i ^*(\beta_i ^* - 1 + 2P_i h_i P_r) = 0, \quad \eta_i ^* \beta_i ^* = 0
\end{aligned}$$

Hence the optimal solution $\beta_{i,1} ^*$ in $S_i$ is selected among KKT points $\beta_{i,S_i} ^*$. The optimal solution is obtained as:

$$\beta_i ^* = \arg \max (R), \quad \Phi = \left\{ \beta_i ^* \mid j \in \{1, 2, 3, 4\} \right\}$$

The final calculation procedure is given in Algorithm 1. The algorithm is fast-convergent, requiring few iterations, i.e. 2-3 times.

**Algorithm 1**: Proposed PS-SWIPT with LF.

1. Initialize $f = \frac{P}{N} I, \beta_i = 0.5, \forall i$ and calculate $\gamma_i, \forall i$
2. Repeat.
3. Calculate the optimal $f$ maximizing capacity of BC, w.r.t. $\gamma_i, \beta_i$.
4. Calculate the optimal solution of $\beta_i$ w.r.t. $\gamma_i, \beta_i$.
5: Update $\gamma$, w.r.t. $\beta, f$
6: Until convergence.

4. Numerical results
In this section we provide numerical results and comparisons. With optimized components in both the transmitter and rectenna, the beamed transmission achieves high dc to dc power transfer efficiency $45\%$ [16]. The uplink channel gain $h_i$ is defined as

$$h_i = \sqrt{\eta} \times \sqrt{0.5 \times \left( \text{randn}(N,1) + \sqrt{-1} \times \text{randn}(N,1) \right)} \ (17)$$

where $\eta = 0.3$ is the power transfer efficiency.

And the downlink channel gain $g_i$ is defined as

$$g_i = \frac{1}{N} \times \left| \sum_{n=1}^{N} h_i \right|^2 \ (18)$$

The relay is equipped with $N = 4$ antennas and serves terminals with power $P_r = 10W$. 1000 random channels are generated in total for Monte Carlo simulation.

The compared schemes are:

1. Proposed joint beam-forming and power splitting design with LF, which means joint beam-forming and power splitting design with LF; 2. Outer cut-set bound [17],[18] scheme, which is the theoretical optimal upper bound; 3. Optimal PS without BF scheme, which means the optimal PS factor design with LF but without beamforming. 4. Fixed PS ($\beta_r, \beta_p = (0.8, 0.8)$) scheme, which means we select ($\beta_r, \beta_p = (0.8, 0.8)$) for fixed PS-SWIPT and do not consider beamforming optimization. 5. TS without BF scheme, which means we consider time-switching (TS) MPT with LF but without beamforming. 6. TS with BF scheme, which means we consider time-switching (TS) MPT with LF and with beamforming.

Figure 2 provides the achievable rate versus SNR, when $P_r = 20$dBm. As SNR increases, all the curves experience sharp increases. Proposed joint BF-PS design has the largest incremental gradient, and the highest achievable rate. From the figure, it is concluded that appropriate beamforming brings significant gains. Besides, adaptive PS outperforms TS scheme, but fixed PS could not change the power proportion, leading to relative low performances. The proposed method with lattice codes is near the upper bound, and the gap vanishes when $\text{SNR} \to \infty$.

Figure 3 provides the achievable rate versus circuit power constraint, when signal to noise ratio $\text{SNR} = 20dB$. In high $\text{SNR}$ region, joint BF-PS design greatly exceeds all the others by over 20% gain. As the circuit consumption increases, the sum-rates of all the schemes degrade, and down to zero around $P_c = 35$dBm where the circuit consumption starts to exceed the maximum collected power.
Figure 2. Achievable rate versus SNR. $P_c = 20$ dBm.

Figure 3. Achievable rate versus circuit power consumption. SNR = 20 dB.

5. Conclusions
In this paper, we consider a TWRC system with SWIPT and LF. Observed from the Monte-Carlo simulations, our proposed joint beam-forming and power splitting design outperforms all the other schemes. Separate optimization designs result in average achievable rates. Generally, power splitting has advantage over time switching. Future work may include other PLNC implementations and consideration of power allocation in OFDM systems.

Acknowledgments
This work was supported by the National Key R&D Program of China under Grant No. 2019YFF0302601

References
[1] Huang K B and Larsson E 2013 Simultaneous Information, and Power Transfer for Broadband Wireless Systems. IEEE Trans. Signal Process. vol. 61, no. 23, pp. 5972-5986, Dec. 2013
[2] Huang J L, Li Q Z, Zhang Q, Zhang G C and Qin J Y 2014 Relay Beamforming for
Amplify-and-Forward Multi-Antenna Relay Networks with Energy Harvesting Constraint. *IEEE Signal Process. Lett.* vol. 21, no. 4, pp. 454-458, Apr. 2014

[3] Yuan Y, Chu Z, Ding Z G 2014 Kanapathippillai Cumanan, and Martin Johnston, Joint Relay Beamforming and Power Splitting Ratio Optimization in a Multi-Antenna Relay Network. *2014 Sixth International Conference on Wireless Communications and Signal Processing (WCSP)*, Hefei, China, Oct. 2014

[4] Hu G J, Cai Y M, Ao L and Wang X D 2019 Joint Design of Beamforming and Time Switching/Power Splitting for Wireless-Powered Multi-Antenna Dual-Relay Network. *J. Wireless Com. Network*, https://doi.org/10.1186/s13638-019-1580-4

[5] Ding Z G, Wang T, Peng M G, Wang W B and Leung K 2011 On the Design of Network Coding for Multiple Two-Way Relaying Channels. *IEEE Trans. Wireless Commun.* vol. 10, no. 6, pp. 1820 -1832, April 2011

[6] Sagar Y T, Yang J, Kwon H M and Nam W 2014 Achievable Rate of a Two-Way Relay Channel with Structured Code under Rayleigh Fading. *2014 International Conference on Computing, Networking and Communications (ICNC)*, Honolulu, HI, USA, Feb. 2014

[7] Islam S M, Durrani S and Sadeghi P 2016 Multi-Pair Two-way Relay Networks: Interference Management Using Lattice Codes and Amplify and Compute Relaying. *2016 Australian Communications Theory Workshop (AusCTW)*, Melbourne, VIC, Australia, Jan. 2016

[8] Li D D, Shen C and Qiu Z D 2013 Two-Way Relay Beamforming for Sum-Rate Maximization and Energy Harvesting. *IEEE ICC’13*, pp. 3155-3120, Budapest, Hungary, Jun. 2013

[9] Tutuncuoglu K, Varan B and Yener A 2015 Throughput Maximization for Two-Way Relay Channels With Energy Harvesting Nodes: The Impact of Relaying Strategies. *IEEE Trans. Commun.* vol. 63, no. 6, Jun. 2015

[10] Wang W, Wang R, Mehrpouyan H, Zhao N and Zhang G A 2017 Beamforming for Simultaneous Wireless Information and Power Transfer in Two-Way Relay Channels. *IEEE Access*, vol. 5, May 2017

[11] Zhang Q, Li Q Z and Qin J Y 2012 Beamforming Design for OSTBC-Based AF-MIMO Two-Way Relay Networks With Simultaneous Wireless Information and Power Transfer. *IEEE Trans. Veh. Technol.* vol. 65, no. 9, pp. 7285-7296, Dec. 2012

[12] Tian Y F, Wu D, Yang C Y and Molisch A F 2012 Asymmetric Two-Way Relay with Doubly Nested Lattice Codes. *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, Feb. 2012

[13] Wang S, Wen Z G, Chen D J and Xiang W D 2014 Network Coding with Nested Lattice for Interference Coordination of Relay Heterogeneous Networks. *IEEE ICC’14 Workshop*, Sydney, Australia, June. 2014

[14] Boyd S and Vandenberghe L 2004 *Convex Optimization*, U.K.: Cambridge, 2004

[15] Luo Z, Ma W, So A M, Ye Y and Zhang S 2010 Semidefinite Relaxation of Quadratic Optimization Problems. *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20-34, May 2010

[16] McSpadden J O, Mankins J C 2012 Space Solar Power Programs and Microwave Wireless Power Transmission Technology. *IEEE Microwave Magazine*, vol. 3, no. 4, pp. 46-57, December 2012

[17] Kramer G and Savari S A 2004 Cut sets and information flow in networks of two-way channels. *International Symposium on Information Theory, 2004. ISIT 2004. Proceedings*, Chicago, IL, USA, July 2004

[18] Fong S L and Yeung R W 2015 Cut-Set Bounds for Networks With Zero-Delay Nodes. *IEEE Trans. on Information Theory*, vol. 61, no. 7, pp. 3837-3850, July 2015