Free vibrations and flutter analysis of cylindrical shells

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Abstract. The paper present the free vibration and the flutter of multi-layered cylindrical shells using the Rayleigh-Ritz method. The optimal design with constraints imposed on the eigenfrequencies is particularly useful in the selection of dynamic characteristics of structures. For composite cylindrical shells, the fiber orientation variations in individual layers results in an increase (decrease) the critical value of free vibrations. Optimal fibre orientations depends on the shell support and on the geometric parameters of a structure. In the case of the optimal laminate configuration finding, the optimization task cannot be solved using standard FEM software the application and genetic algorithms are required.

1. Introduction
Vibrations of the composite structures and their optimization constitute a number of separate issues important from the point of view of engineering practice as well as the possibility of applying many approaches, descriptions and methods used in static problems to the vibration problems. In the case of vibrations of the composite structures, the basic advantage of formulating basic relationships is the possibility of using relations for the static problems. In addition, all optimization algorithms used in the static analysis are easily used in the analysis of the construction vibrations. Another very important feature is the similarity of many issues regarding optimization in the case of flexural vibrations for the buckling tasks. In the mathematical sense, both problems boil down to looking for values and their own functions.

Optimal design of the composite structures with boundary conditions on the values of natural frequencies is particularly beneficial in the case of selection of the dynamic characteristics of the construction. For issues concerning the analysis of the constructions working in the low frequency range, the structure's response to the dynamic forcing depends primarily on the value of the fundamental frequency, understood as the lowest natural frequency of the structure and the form of vibrations for this frequency. Similarly, in the aerodynamic problems of aircraft wings, deformations depend on the elastic and torsional properties of the wing, and the best method of testing is to determine the lowest forms of the torsional and flexural vibrations. In airplanes, space rockets and space ferries, it is important not only to determine the natural frequency, but also to prevent self-excited vibrations hereinafter called the flutter and possible resonances of the navigation and the control systems.

In the work [1] Muc presents the methods of optimization of composite structures placed in the gas flow. They draws attention to the fact that, in flutter analysis, take part not only a major concerns like the effects of physical materials, geometrical model, and boundary conditions on flutter but also the factors and complex physical environments like aerodynamic uncertainties, new composite materials, fluid–thermal–structural coupling, boundary layer, and shear deformation. In order solve more common aeroelastic problems, the analysis actually starts with finite element modeling and free vibration studies.
The influence of the flutter on the cylindrical structures also, was presented by Karagiozisa at work [2], in which concerned with the nonlinear dynamics and stability of thin circular cylindrical shells clamped at both ends and subjected to axial fluid flow. The novel Legendre polynomials to investigate the dynamic behavior of an isotropic thin cylindrical shell by using asymptotic approach under cylindrical symmetry was described by Shaw at [3], where he found, that the motion in n-th mode of the cylinder does not depend on higher or lower modes and variants of each mode satisfies same set of equations. In work [4] Kormanikova and Kotrasova presented a sizing optimization analysis of laminated circular cylindrical shell, used the classical shell theory. Formal comparison among various theories and an influence of nonlinear vibrations on laminated circular cylindrical shells we find in work Amabili [5] or Tornabene [6].

For composite panels/shells, in order solve more common aeroelastic problems (divergence, flutter and aeroelastic response at subsonic and supersonic speeds), the analysis actually starts with the finite element modeling and free vibration studies. Then, eigenmode shapes are interpolated from the structural grid points to aerodynamic control points to formulate modal equations of motion and to investigate critical divergence velocities, flutter speed, flutter frequency etc. When the aerodynamic damping is negligible, the panel flutter analysis is normally made using frequency coalescence method. For zero dynamic pressure parameter, the eigenvalues of the system are real and positive. They are basically the natural frequencies of the system in vacuum. As the dynamic pressure parameter is increased from zero the frequencies keep changing some with decrease in value and some with increase in value. For some value of dynamic pressure parameter, known as critical dynamic pressure parameter two of the eigenvalues come very close to each other and become complex conjugate. The merging of the frequencies is known as frequency coalescence.

Optimization of natural frequencies is also a central concept in the design of plated/shell composite structures. Design parameters (denoted by the vector \( s \)) such as layer thicknesses and ply angles can be employed to achieve an optimized structure. Genetic algorithms (GA) has been highly successful as one of evolutionary computation techniques in searching for a broad class of stacking sequence, size, topology optimization problems for composite structures (see Muc [7]). However, the application of that optimization technique is limited to a very few inequality constraints only. Now, our attention will be particularly focused on the formulation of the alternative to GA evolutionary computation optimization technique. It is based on the idea of the evolution strategy so that the proposed algorithm allows us to eliminate the problems associated with a large number of constraints – Muc [8].

Optimal material design covers the vast area of optimization problems strongly dependent on the formulation of objectives, forms of design variables and used optimization techniques (algorithms) that should be appropriate to solved problems. Possible directions of the research in this area are presented by Bendsøe, Siegmund [9] and Muc [10, 11]. It is worth to mention also herein the work [12] dealing mainly with one particular optimization problem, i.e. stacking sequence optimization of plated structures.

The aim of the paper, was focused not only on general relations used in the description of free vibrations but also the self-excited vibrations for the structures located in the gas flow (flutter) as well as on the optimization problems encountered in this area of engineering.

2. Equation of motion of the composite structures

Studies [13-16] provide a detailed overview of the work on the dynamics of the composite shells, including the emphasis on numerical methods, while in [17], attention was paid to non-linear effects in flexural vibrations of the cylindrical shells. The main directions of the research in the field of the numerical analysis in review articles by Noor et al. [18] and Reddy, Robbins [19] are presented.

The fundamental equations are formulated with the use of the Hamilton principle (the detailed discussion of the problem is presented by Muc, Flis [20]):
The reduced stiffness constants of the materials \( Q \) and the transformation matrix to the global coordinate system \( T \) are defined as:

\[
[Q] = [T^{(l)}]^T [C] [T^{(l)}], [C] = \begin{bmatrix}
E_1/(1 - \nu_{12}\nu_{21}) & \nu_{21}E_1/(1 - \nu_{12}\nu_{21}) & 0 \\
\nu_{21}E_1/(1 - \nu_{12}\nu_{21}) & E_2/(1 - \nu_{12}\nu_{21}) & 0 \\
0 & 0 & G_{12}
\end{bmatrix}
\]

\[
[T^{(l)}] = \begin{bmatrix}
\frac{c^2}{s^2} & \frac{s^2}{sc} & \frac{sc}{-sc} \\
s^2 & -sc & 2sc \\
-2sc & sc & c^2 - s^2
\end{bmatrix}, s = \sin(\theta^{(l)}), c = \cos(\theta^{(l)})
\]

\[
A_{ij} = \sum_{t=1}^{N} \int_{z_{t-1}}^{z_t} Q_{ik} dz, D_{ij} = \sum_{t=1}^{N} \int_{z_{t-1}}^{z_t} Q_{ij} z^2 dz, i, j = 1, 2, 3
\]

where \( t \) denotes time, \( h \) is the shell thickness, \( R \) is the radius of the cylindrical shell, and \( u_1 \) – the longitudinal \((x_1)\) displacements, \( u_2 \) – the circumferential \((x_2)\) displacements and \( u_3 \) – the normal (radial \( x_3 \)) displacements. \( \Lambda \) denotes the aerodynamic pressure and \( \rho \) is the density of the composite material. \( N \) is the total number of layers, and the layer thickness \( t_i \) is \( z_i - z_{i-1} \).

3. Optimization of the cylindrical shell laminate configuration

3.1. Dynamic behaviour of the cylindrical shell

The first works in the field of the optimization natural frequencies of the multi-layered cylindrical shells [21, 22] concerned the selection of a laminate composed of the individual layers with 0°, ±45°, 90° orientations, in such a way, that the shell’s weight was minimal, with an existing limitation to the value of the minimum basic frequency. In work [23], this task was solved with regard to attenuation in the form of a layer made of material with viscoelastic properties. The influence of the laminate configuration on the critical frequencies was also investigated by Kayran [24], but not using optimization methods to determine the best orientation of fibers in the individual layers.

For any type of boundary conditions, Fischer [25] proposed analytical relations to determine the natural frequency. These compounds correctly reflect the order of the magnitude, but they are only an estimation of the upper correct values, because they are based on the Kirchhoff-Love shell theory. It should be noted, that the quoted relations are only introduced to determine of the circumferential forms of vibrations, and in the longitudinal direction (in the sense of the assumed form of the vibration) the shell is treated as a beam.

A change in the orientation of fibers in the individual layers results in a rise (decrease) in the fundamental frequency of the critical vibrations. An example of numerical solutions for crossed laminates ±θ is shown in figure 1. The NKTP 32 elements was used – quadrangular, shear and the first-order transverse shear deformation theory. An angle \( \theta = 0° \) means the direction parallel to the main
direction in the shell. The location of the optimal orientation of the fibers depends on the method of supporting the shell and the geometrical parameters of the cylinder.

![Diagram](image)

**Figure 1.** The value of the natural frequencies for the cross-ply laminates ($L/D=1$, $t/R=2/15$, $E_2/E_1 = 0.1$).

### 3.2. The free and self-excited vibrations of the cylindrical shells

For any method of supporting the edges of the shell and using continuous decision variables, the search for the optimal fiber orientation in individual layers in the case of the objective function defined by relationship: the maximum of frequency $\omega$ ($\text{Max }\omega$), can be made using the optimization package in the FEM program, e.g. NISAOPT. During solving this task numerically, one should remember about the appropriate modeling of the problem. The basic issue is to avoid the so-called twin forms of the critical vibration. For this purpose, only one half of the cylinder should be considered in the circumferential direction with the circumferential conditions of symmetry (ant symmetry) on the edges. Therefore, the form of the critical vibration depends on the orientation of the laminate. The change in the form of the basic vibrations can also be seen in figure 1. This corresponds to the approach of the curves characterizing the first two forms of the vibrations. Consequently, the solution to the optimization problem ($\text{Max }[\omega_l - \omega_{l-1}]$, $l = 2, 3, \ldots$) always exists, although the application of the specific numerical values requires the use of optimization algorithms.

In the case of searching for the optimal laminate configuration for a given set of orientation of the individual layers ($0^\circ$, $\pm 45^\circ$, $90^\circ$) the optimization task cannot be solved using standard FEM software and it is necessary to introduce the external optimization procedures (genetic algorithms). The function of the target has also been changed here, because we are currently looking for the orientation of the laminate corresponding to the minimum natural frequencies. This formulation is very often found in optimization tasks in this field [26, 27]. The analysis assumes that the laminate is composed of 64 individual layers, so the variable chain (chromosome) contains 16 elements (decision variables).
The issue was solved by looking for the minimum frequency for specific forms of vibrations \( m, n \). The issue was solved by looking for the minimum frequency for specific forms of vibrations \( m, n \). The numerical determination of the objective function (FEM) was used. In the calculations, 1/8 cylinder was discretized using the four node quadrilateral shell elements (NKTP 32). Free support of the cylinder edges was accepted.

The distributions presented in figure 2 demonstrate, that the minimum for the value of \( m=n=1 \) is achieved and corresponds to the string in the form: \([3333222222111111]\). The form of the optimal chromosome clearly shows, that in this case is follow a connection of the flexural effects (terms of the \([D]\) matrix) with the membrane effects (terms of the \([A]\) matrix). It is easy to check, that in the case of only flexural effects, the problem of searching for an optimum is reduced to the analysis of the free vibrations. In the case of the domination of flexural effects (high value \( n \) - peripheral direction) for cylindrical shells, the optimal configuration is to arrange the fibers at an angle of \( 0^\circ \) – the chain of ones.

In the case of the cylindrical shell in the flow of the gas stream, the critical vibrations using the Rayleigh-Ritz method were determined. The characteristic equation (complex due to the occurrence of friction) on the natural frequency \( \omega \) is obtained directly from the dependence (1):

\[
\omega^2 - i\omega(2\mu + \gamma) - \Omega^2 + i\lambda U \gamma = 0, \tag{3}
\]

where:

\[
h_0 \Omega^2 = \left[ D_{11} \lambda^4 + 2(D_{12} + 2D_{66}) \lambda^2 \left( \frac{n}{R} \right)^2 + D_{22} \left( \frac{n}{R} \right)^4 + \frac{AA^2}{A_{11} \lambda^4 + (A/A_{66} - 2A_{12}) \lambda^2 \left( \frac{n}{R} \right)^2 + A_{22} \left( \frac{n}{R} \right)^4} \right]. \tag{4}
\]

\[
\lambda = \frac{m \pi}{L}, \quad \gamma = \frac{v_p a_0}{\rho a_\infty}, \quad A = A_{11} A_{22} - A_{12}^2.
\]

If the imaginary part \( \omega \) in equation (6) is negative, then the unconstrained motion of the shell is stable. Stability or lack of it depends on the value of gas velocity and the critical value is value:

\[
U_{kr} = \Omega (1 + 2\mu/\gamma)/\lambda. \tag{5}
\]

As in the previous issue of the own frequencies’ optimization, we are looking for a minimum value of \( U_{kr} \) by choosing a laminate configuration. The examples to be solved concern the analysis of the shell with identical geometry and made of the same material as before.
Figure 3. The optimal critical frequencies values of the self-excited vibrations.

Figure 3 shows distributions of the dimensionless, optimal critical frequencies. The minimum, due to the number of half-waves, is achieved for identical values as in the case of the analysis of natural frequencies, i.e. for $m=n=1$. However, in the case under consideration, the optimal orientation is fundamentally different than before – $[3131111111111111]$, because the form of the objective function is different.

4. Evaluation of the efficiency of the numerical algorithms in the optimization tasks of the laminate configuration

The set of the optimization problems allows comparing the effectiveness of a developed version of the optimization algorithms. As the assessment criterion, the number of population generations required to complete the optimization process was adopted (for simplification in the drawings marked as iterations). It should be emphasized, that in each of the numerical examples analyzed in chapter 3, the convergence of the process was achieved. However, it was always conditioned by the adoption of the appropriate values of the indicators regulating the convergence process. It was noted, that:

- in the genetic algorithms (GA) the values of the mutation probabilities must be low, close to zero and the crossover probability may not be too large (the best results were obtained for values 0.4-0.6),
- in the simulated annealing (SA) algorithms it is advantageous to adopt a moderate cooling rate,
- in the genetic algorithms, increasing the number of the crossbreeds points (above one) does not improve the convergence of the process, but even makes it worse,
- both in the genetic algorithms and the simulated annealing algorithms, increased population size improves process convergence, but in the case of using FEM packets, this requires extending the calculation time (in the examples studied the maximum number of individuals was 150)

After the adoption of the appropriate parameters ensuring the convergence, comparison and assessment was made:

- the impact of the type of the stochastic algorithm on the number of iterations; three methods were used: classical genetic algorithm (CGA), genetic algorithm with elitism (EGA) and simulated annealing algorithm (SA) figure 4; the results of the numerical experiment show that the convergence of the genetic algorithms with elitism was achieved the fastest, a little significant deviations observed during the process, and the slowest for the algorithm of the simulated annealing, although after about 100 iterations results were obtained close to the optimum.
Figure 4. Comparison of the convergence of the stochastic algorithm (number of the individuals in the population of 20).

In the analyzed issues, the repeatability of the search results of the extreme in the last five iteration steps was assumed as the criterion of the end of the process. At the same time, it should be added that at the end of the optimization process, the achievement of the extreme was always noted by the majority of individuals in the population created in the last steps (generations). In some cases, the number was even higher than 90%.

Some of the results presented in Muc work [7] have been used in the results cited in this article.

5. Conclusions
The paper presents general dependencies for describing free vibrations of cylindrical shells in terms of the shear theory of the first order, treating the basic compounds used in the Kirchhoff-Love theory as a special case. The issue of self-excited vibrations for structures located in gas flow is also described.

In the case of searching for the optimal laminate configuration for a given set of orientation of the individual layers (0°, ±45°, 90°) the optimization task cannot be solved using standard FEM software and it is necessary to introduce the external optimization procedures (genetic algorithms).

The optimal orientation of fibers depends on the shell support and the geometric parameters of a cylinder.

For the arbitrary boundary conditions of the shell edges and with the use continuous design variables the optimization of the fiber orientation can be provided by finite element method optimization packages like NISAOPT. The natural frequencies for clamped cylindrical shells was presented in the paper.

In the case of the cylindrical shell in the ultrasonic gas flow the Rayleigh-Ritz method has been used to determine the critical frequency value ω in terms of the Kirchhoff–Love theory.

In the case of the optimal laminate configuration finding the optimization task cannot be solved using standard FEM software and external optimization procedures (genetic algorithms) are required.

At the end evaluation of the efficiency of the numerical algorithms in the optimization tasks of the laminate configuration was presented.

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