Cosmological implications of supersymmetric axion models

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Abstract. We derive general constraints on the supersymmetric extension of axion models, in particular paying careful attention to the cosmological effects of saxion. It is found that, for every mass range of the saxion from keV to TeV, severe constraints on the energy density of the saxion are imposed. Together with constraints from axino we obtain stringent upper bounds on the reheating temperature.

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1. Introduction

One of the main problems of the standard model is the strong violation of CP invariance due to non-perturbative effects of quantum chromodynamics (QCD). In general, QCD effects generate terms in the Lagrangian such as

$$L_\theta = \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu},$$

where $g_s$ is the QCD gauge coupling constant, $G^a_{\mu\nu}$ is the field strength of the gluon and $\tilde{G}^{a\mu\nu} = \epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}/2$. Experimentally $\theta$ must be smaller than about $10^{-9}$, but in the standard model there seems to be no theoretical reasons that $\theta$ must be so small. This is the well-known strong CP problem. As a solution to the strong CP problem, Peccei and Quinn [1] introduced anomalous global $U(1)$ symmetry, which we denote $U(1)_{PQ}$. When this $U(1)_{PQ}$ is broken spontaneously, there appears a pseudo-Nambu–Goldstone boson called an axion [2]. The axion dynamically cancels the $\theta$ term effectively and no strong CP violation is observed in a true vacuum. In order not to contradict with terrestrial experiments, astrophysical [3] and cosmological arguments [4], the breaking scale of PQ
symmetry $F_a$ should lie in the range $10^{10}$ GeV $\lesssim F_a \lesssim 10^{12}$ GeV. If $F_a$ is close to this upper bound, the axion is an interesting candidate for cold dark matter.

On the other hand, another problem in the standard model is the quadratic divergence of the radiative correction to the Higgs mass. In the standard model, the mass of the Higgs is not protected by any symmetry, and hence naturally the Higgs boson is expected to obtain the mass of the cutoff scale. Thus to obtain a hierarchically small mass scale down to the weak scale requires unnatural fine-tuning. Supersymmetry (SUSY) [5] is the most motivated solution to this problem, since SUSY protects the mass of scalar fields and the weak scale becomes stable against radiative correction. From the cosmological points of view, SUSY also provides interesting candidates for the dark matter. Due to the $R$-parity conservation, under which standard model particles have the charge $+1$ and their superpartners have $-1$, the lightest SUSY particle (LSP) is stable. So if the LSP is a neutralino or gravitino, they are the dark matter candidates.

Therefore, it seems reasonable to combine these two paradigms. In fact, axion models are easily extended to implement SUSY. As we will see, in SUSY extensions of the axion models, many non-trivial cosmological consequences arise. In SUSY axion models, both the scalar partner of the axion, the saxion, and the fermionic superpartner of the axion, the axino, have significant effects on cosmology. But there have not been many studies which treat both of them in spite of their importance [6]–[9]. Moreover, these earlier works were based on specific models and only the restricted parameter regions were investigated. In this paper, we investigate all possible mass ranges of the saxion and corresponding various cosmological bounds. Our results are easily applied to any axion models with slight modifications, and hence provide general cosmological constraints. Furthermore, since the saxion and axino densities depend on the reheating temperature $T_R$ after inflation, we can obtain constraints on $T_R$. The important result is that the upper bound on $T_R$ becomes more stringent due to the late-decaying saxion and axino, than that from the usual gravitino problem.

This paper is organized as follows. In section 2 we explain the dynamics of the saxion and its properties. In section 3 we derive various cosmological constraints on SUSY axion models, especially an upper bound on the reheating temperature. Implications for the dark matter in the SUSY axion models are discussed in section 5. In section 6 we briefly comment on the ultra-light gravitino scenario, where the gravitino is $\mathcal{O}(10)$ eV. We conclude in section 7.

2. Supersymmetric axion models

The axion is the pseudo-Nambu–Goldstone boson which appears due to the spontaneous breaking of PQ symmetry. The axion obtains a mass from the effect of the quantum anomaly, but this contribution is very small. It is estimated as $m_{a0} \sim 6 \times 10^{-6}$ eV ($10^{12}$ GeV/$F_a$).

In SUSY extensions of axion models, the axion field forms a supermultiplet, which contains a scalar partner (saxion) and fermionic superpartner (axino) of the axion [10]. The saxion mass is expected to be of the order of the gravitino mass ($m_{3/2}$). On the other hand, the axino mass somewhat depends on the models, but generically can be as large as $m_{3/2}$. Although the interactions of both particles with standard model particles are
suppressed by the PQ scale $F_a$, they may cause significant effects on cosmology as will be seen in section 3.

### 2.1. Dynamics of the saxion

In SUSY models, superpotentials must satisfy the holomorphy. When the real $U(1)_{\text{PQ}}$ symmetry is combined with the holomorphic property of the superpotential, it is extended to complex $U(1)$ symmetry, which inevitably includes scale transformation \[\{1\]. The invariance under the scale transformation means the existence of a flat direction along which the scalar field does not feel the scalar potential. The saxion corresponds to just such a flat direction of the potential. But SUSY breaking effects lift the flat direction and the saxion receives a mass of the order of $m_{3/2}$.\[2\] The saxion field can develop to a large field value during inflation and begins to oscillate around its minimum when the Hubble parameter $H$ becomes comparable to the saxion mass $m_a \sim m_{3/2}$. In general such coherent oscillation of the saxion field has large energy density and hence its late decay may have significant effects on cosmology \[6,8,9,13\].

As an example, let us consider a model with the following superpotential:

$$W = \lambda X (\Phi \bar{\Phi} - F_a^2),$$  \tag{2}$$

where the superfields $X, \Phi$ and $\bar{\Phi}$ have the PQ charges 0, +1, −1 respectively. The saxion direction is easily identified as $\Phi \bar{\Phi} = F_a^2$ with $X = 0$.\[3\] Including SUSY breaking mass terms due to gravity-mediation effects, the scalar potential can be written as

$$V = m_{3/2}^2 \left( c_X |X|^2 + c_1 |\Phi|^2 + c_2 |\bar{\Phi}|^2 \right) + |\lambda|^2 \left| \Phi \bar{\Phi} - F_a^2 \right|^2 + |X|^2 (|\Phi|^2 + |\bar{\Phi}|^2),$$ \tag{3}$$

where $c_X, c_1$ and $c_2$ are $O(1)$ constants which are assumed to be positive. The potential minimum appears at $|\Phi| \sim |\bar{\Phi}| \sim F_a$, but the initial amplitude of the saxion remains undetermined. In general, if the saxion remains light during inflation, the field value may naturally take the value of the order of the reduced Planck scale $M_P$. But the initial amplitude can be suppressed by introducing the Hubble-induced mass terms, which are induced by the saxion coupling with inflaton field through the supergravity effect, given by

$$V_H = H^2 \left( c'_X |X|^2 + c'_1 |\Phi|^2 + c'_2 |\bar{\Phi}|^2 \right),$$ \tag{4}$$

where $c'_X, c'_1$ and $c'_2$ are $O(1)$ constants. If either $c'_1$ or $c'_2$ are negative, the $\Phi$ or $\bar{\Phi}$ field roll away to $\sim M_P$ during inflation. But if both coefficients are positive, the potential minimum during inflation is also given by $|\Phi| \sim |\bar{\Phi}| \sim F_a$, which almost coincides with the low energy true minimum. Since there is \textit{a priori} no reason that we expect that $c_1/c_2$ is exactly equal to $c'_1/c'_2$, these two minima are separated by

$$s \sim \left[ \left( \frac{c_1}{c_2} \right)^{1/4} - \left( \frac{c'_1}{c'_2} \right)^{1/4} \right] F_a,$$ \tag{5}$$

\[2\] In gauge-mediated SUSY breaking models, the saxion receives a logarithmic potential from gauge-mediation effects. If the PQ scalar is stabilized by the balance between the logarithmic potential and the gravity-mediation effect, $m_a \sim m_{3/2}$ still holds \[12\].

\[3\] If the $A$-term contribution $V_A \sim m_{3/2} \lambda X F_a^2 + \text{h.c.}$ is included, the $X$ field can have the VEV $|X| \sim m_{3/2}$. But this does not modify the following arguments.
where $s$ denotes the saxion field (here we have assumed $c_1 > c_2$ and $c'_1 > c'_2$). This simple model provides one realization of the scenario for the initial saxion amplitude $s_i$ to be $\sim F_a$. For the case of $s_i \sim M_p$, the cosmological constraints become much more stringent than the case of $s_i \sim F_a$ and hence hereafter we consider only the latter case.

For this scenario to work, the Hubble parameter during inflation $H_I$ should be smaller than $\sim F_a$, since otherwise the large Hubble mass term during inflation takes all the fields to the origin. Once they are trapped at the origin, the saxion has unsuppressed interaction with particles in a thermal bath and gets a large thermal mass, which results in further trapping of the saxion at the origin. Thus, the oscillation epoch is significantly delayed \cite{9}.

Because delayed oscillation only makes the cosmological saxion problems worse, we do not consider such a case. On the other hand, the axion field has isocurvature perturbations with amplitude $\sim H_I/(\pi s_i)$ during inflation \cite{16,17}, which leads to a constraint on $H_I$ as

$$H_I \lesssim 2 \times 10^7 \text{ GeV } \theta_i^{-1} \left(\frac{\Omega_m h^2}{0.13}\right) \frac{s_i}{F_a} \left(\frac{F_a}{10^{12} \text{ GeV}}\right)^{-0.175},$$

where $\theta_i$ denotes the initial misalignment angle of the axion, $\Omega_m$ denotes the density parameter of the non-relativistic matter and $h$ is the present Hubble parameter in units of 100 km s$^{-1}$ Mpc$^{-1}$. Here we have used the observational constraint that the ratio of the isocurvature perturbation to the adiabatic one should be less than about 0.3 \cite{18}. Thus, as long as we stick to $s_i \sim F_a$, the requirement $H_I \lesssim F_a$ is, in fact, valid from a cosmological point of view\footnote{This is the case in the KSVZ (or hadronic axion) model \cite{14}. In the DFSZ model \cite{15} such a thermal mass may not arise because of the small coupling of the PQ scalar, but there arises another difficulty from domain wall formation.}

Although we have presented a specific model above, the dynamics of the saxion does not depend on axion models much, once the initial amplitude and the mass of the saxion are fixed. The saxion field starts oscillation at $H \sim m_s$ with initial amplitude $s_i$. As explained above, $m_s$ is likely of the order of $m_{3/2}$ and the natural expectation of the initial amplitude is $s_i \sim F_a$ or $s_i \sim M_p$.

Now let us estimate the saxion abundance. First we consider the saxion abundance in the form of coherent oscillation. It is independent of the reheating temperature when the reheating temperature is high, i.e. $\Gamma_f > m_s$ (where $\Gamma_f$ is the decay rate of the inflaton). The saxion-to-entropy ratio is fixed at the beginning of the saxion oscillation ($H \sim m_s$) and is given by

$$\left(\frac{\rho_s}{s}\right)^{(C)} = \frac{1}{8} T_{osc} \left(\frac{s_i}{M_p}\right)^2,$$

$$\simeq 1.5 \times 10^{-5} \text{ GeV} \left(\frac{m_s}{1 \text{ GeV}}\right)^{1/2} \left(\frac{F_a}{10^{12} \text{ GeV}}\right)^2 \left(\frac{s_i}{F_a}\right)^2,$$

where $T_{osc}$ denotes the temperature at the beginning of the saxion oscillation. On the other hand, if $\Gamma_f < m_s$, the ratio is fixed at the decay of inflaton $H \sim \Gamma_f$ and the

\footnote{The saxion does not give rise to isocurvature fluctuation because it has large Hubble mass and its quantum fluctuation is suppressed during inflation.}
The saxion is also produced by scatterings of particles in high-temperature plasma. For $T_R \gtrsim T_D \sim 10^9$ GeV ($F_a/10^{11}$ GeV)$^2$, the saxions are thermalized through these scattering processes and the abundance is determined as [10]

\[
\frac{\rho_s}{s} \sim 1.0 \times 10^{-9} \text{ GeV} \left( \frac{m_s}{1 \text{ GeV}} \right). \tag{11}
\]

For $T_R \lesssim T_D$, this ratio is suppressed by the factor $T_R/T_D$. The result is

\[
\left( \frac{\rho_s}{s} \right)^{\text{(TP)}} \sim 1.0 \times 10^{-9} \text{ GeV} \left( \frac{m_s}{1 \text{ GeV}} \right) \left( \frac{T_R}{10^5 \text{ GeV}} \right) \left( \frac{10^{12} \text{ GeV}}{F_a} \right)^2. \tag{12}
\]

Here we have assumed that thermally produced saxions become non-relativistic before they decay. This assumption is valid for the parameter regions we are interested in. We can see that the contribution from coherent oscillation is proportional to $F_a^2$ while that from thermal production is proportional to $F_a^{-2}$. Thus for small $F_a$ thermal production may be dominant. Note that this expression is valid for $T_R \gtrsim m_s$. Otherwise the saxion cannot be produced thermally. As a result, the total saxion abundance is the sum of these two contributions:

\[
\frac{\rho_s}{s} = \left( \frac{\rho_s}{s} \right)^{\text{(C)}} + \left( \frac{\rho_s}{s} \right)^{\text{(TP)}}. \tag{13}
\]

In figure 1 we show theoretical predictions for the saxion-to-entropy ratio with $F_a = 10^{10}$ GeV, $10^{12}$ GeV and $10^{14}$ GeV.\(^6\) In the figures the thick blue lines represent the contribution from the coherent oscillation $(\rho_s/s)^{(C)}$ with $s_i \sim F_a$ and the thin red ones represent the thermal contribution $(\rho_s/s)^{(TP)}$. Solid, dashed and dotted lines correspond to $T_R = 10^{10}$ GeV, $10^5$ GeV and 1 GeV, respectively. We can see that, for $F_a \lesssim 10^{12}$ GeV ($F_a \lesssim 10^{10}$ GeV), the contribution from thermal production dominates at $T_R \gtrsim 10^{10}$ GeV ($T_R \gtrsim 10^5$ GeV).

### 2.2. Decay of the saxion

Since the interaction of the saxion with other particles is suppressed by the PQ scale $F_a$, it has a long lifetime and decays at cosmological timescales, which leads to several cosmological effects. First, let us consider the saxion decay into two axions, $s \rightarrow 2a$. If we parameterize PQ scalar fields $\Phi_i$ as

\[
\Phi_i = v_i \exp \left[ \frac{q_i \sigma}{\sqrt{2} F_a} \right], \tag{14}
\]

Axion overclosure bound ensures $\theta_i F_a \lesssim 10^{12}$ GeV. Thus by tuning $\theta_i$, $F_a \sim 10^{14}$ GeV is allowed. Late-time entropy production also makes such a large value of $F_a$ viable, although the low reheating temperature $T_R \lesssim 1$ GeV is needed [19]–[21].
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Figure 1. Theoretical predictions for the saxion-to-entropy ratio. Thick blue lines represent the contribution from the coherent oscillation \( \rho_s / s \) with \( s_i \sim F_a \) and the thin red ones represent the thermal contribution \( \rho_s / s \). Solid, dashed and dotted lines correspond to \( T_R = 10^{10} \) GeV, \( 10^5 \) GeV and 1 GeV, respectively.

where \( q_i \) is the PQ charge of the \( i \)th PQ field and \( F_a = \sqrt{\sum_i q_i^2 v_i^2} \), the saxion and axion are identified as \( s = \text{Re}[\sigma] \) and \( a = \text{Im}[\sigma] \). The kinetic term is expanded as

\[
\sum_i |\partial_\mu \Phi_i|^2 \sim \left( 1 + \frac{\sqrt{2} f}{F_a} \right) \left( \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{1}{2} \partial_\mu s \partial^\mu s \right) + \cdots ,
\]

where \( f = \sum_i q_i^3 v_i^2 / F_a^2 \). From this coupling, we can estimate the decay rate of the saxion into axions as

\[
\Gamma(s \rightarrow 2a) \simeq \frac{f^2 m_a^3}{64 \pi F_a^2} .
\]

If \( f \sim 1 \), as in many cases including the case with only one PQ scalar, this is the dominant decay mode of the saxion \[22\]. Then the lifetime is given by

\[
\tau_s \simeq 1.3 \times 10^2 \, f^{-2} \, s \left( \frac{1 \, \text{GeV}}{m_a} \right)^3 \left( \frac{F_a}{10^{12} \, \text{GeV}} \right)^2.
\]

But for the model with superpotential equation (2), \( f \) can be zero at tree level due to the cancellation if \( c_1 = c_2 \) in equation (3). It is crucial for cosmological arguments whether the dominant decay mode is into axions or not, because axions produced in the decay do not interact with other particles and cosmological constraints can be relaxed if this is the dominant decay mode. In this paper, we consider both possibilities \( f \sim 1 \) and \( f \sim 0 \) and derive cosmological constraints.

Next we consider other modes in which the saxion decays into standard model particles. Here we assume the saxion is lighter than SUSY particles and its decay into SUSY particles is kinematically forbidden. Implications of the saxion decay into SUSY particles are discussed in section 5. For the hadronic axion model, the leading contribution for \( m_s \gtrsim 1 \, \text{GeV} \) comes from the decay into two gluons. The decay rate is estimated as

\[
\Gamma(s \rightarrow 2g) \simeq \frac{\alpha_s^2 m_s^3}{64 \pi f^2} .
\]
where $\alpha_s$ denotes the $SU(3)_c$ gauge coupling constant. The emitted gluons produce hadron jets which may affect the big bang nucleosynthesis (BBN) as seen in section 3.

On the other hand, the decay into two photons is always possible, which has the decay rate

$$\Gamma(s \rightarrow 2\gamma) \approx \frac{\kappa^2 \alpha^2_{\text{EM}} m^3_s}{512 \pi^3 F^2_a}, \quad (19)$$

where $\alpha_{\text{EM}}$ denotes the $U(1)$ gauge coupling constant and $\kappa$ is a model-dependent constant of $O(1)$. These photon produced in the decay also bring about cosmological difficulty.

In the DFSZ axion model, the PQ scalar has tree level coupling with the ordinary quarks and leptons. For $m_s > 2m_{u_i}(2m_{d_i})$ where $u_i$ ($d_i$) denotes the up-type (down-type) quark in the $i$th generation ($i = 1, 2, 3$) the saxion decays into a fermion pair with the decay rate

$$\Gamma(s \rightarrow u_i \bar{u}_i) = \frac{3}{8\pi} \left( \frac{2x^{-1}}{x + x^{-1}} \right)^2 m_s \left( \frac{m_{u_i}}{F_a} \right)^2 \left( 1 - \frac{4m^2_{u_i}}{m^2_s} \right)^{3/2}, \quad (20)$$

$$\Gamma(s \rightarrow d_i \bar{d}_i) = \frac{3}{8\pi} \left( \frac{2x}{x + x^{-1}} \right)^2 m_s \left( \frac{m_{d_i}}{F_a} \right)^2 \left( 1 - \frac{4m^2_{d_i}}{m^2_s} \right)^{3/2}, \quad (21)$$

where $x = \tan \beta = \langle H_u \rangle / \langle H_d \rangle$ (through this paper, we set $x = 5$). Here it should be noticed that, for $m_s \lesssim 1$ GeV, the effective coupling of the saxion with hadrons (mesons) should be used. However, there is no parameter region where the saxion decay into mesons has the main effects on cosmology in the following discussion. Moreover, the decay into the muon pair (see below) gives the same order of the decay rate for $m_s \gtrsim 210$ MeV (decay into mesons are kinematically forbidden for $m_s \lesssim 270$ MeV). Therefore, for simplicity we neglect the effects of saxion decay into mesons for $m_s < 1$ GeV.

The saxion coupling with leptons is not suppressed for the DFSZ axion model, which gives the decay rate

$$\Gamma(s \rightarrow l_i \bar{l}_i) = \frac{1}{8\pi} \left( \frac{2x}{x + x^{-1}} \right)^2 m_s \left( \frac{m_{l_i}}{F_a} \right)^2 \left( 1 - \frac{4m^2_{l_i}}{m^2_s} \right)^{3/2}. \quad (22)$$

Thus we can see that the saxion decay into heavier fermions is enhanced, as long as it is kinematically allowed. In fact, the decay into fermions may be the dominant mode for some mass region even if $f = 1$.

In the KSVZ model, the decay of the saxion into quarks and leptons is suppressed because the saxion does not directly couple with them.

Hereafter, we consider the following four typical cases labeled as model (a)–(d). Model (a) denotes the KSVZ model with $f = 1$ and model (b) denotes the KSVZ model with $f = 0$. Model (c) denotes the DFSZ model with $f = 1$ and model (d) denotes the DFSZ model with $f = 0$.

### 3. Cosmological constraints from saxion

Given the decay modes of the saxion, we can derive generic constraints on the saxion density depending on its lifetime and mass. As we will see, for almost all the mass range
(1 keV \lesssim m_s \lesssim 1 \text{ TeV}) the saxion density is bounded from above, although the upper bounds depend on the cosmological scenario such as the reheating temperature and the initial displacement of the saxion field.

3.1. Effective number of neutrinos

Relativistic particles produced by decaying particles would contribute to the additional radiation energy density, parameterized by the increase of the effective number of neutrinos, \( \Delta N_\nu \). The definition of \( N_\nu \) is given through the relation

\[
\rho_{\text{rad}}(T) = \left[ 1 + \frac{7}{8} N_\nu \left( \frac{T_\nu}{T_\gamma} \right)^4 \right] \rho_{\gamma}(T_\gamma),
\]

(23)

where \( \rho_{\text{rad}} \) denotes the total relativistic energy density, and \( T_\gamma \) and \( T_\nu \) denote the temperature of the photon and neutrino. In the standard model with three species of light neutrinos, \( N_\nu \approx 3.046 \). But if there exists another species of relativistic particle, either thermally or non-thermally, it contributes to the total radiation energy density parameterized by \( N_\nu \). Then the additional contribution \( \Delta N_\nu \) is given as \( \Delta N_\nu = 3(\rho_{\text{rad}} - \rho_\gamma - \rho_\nu) / \rho_\nu \). The increase of \( N_\nu \) speeds the Hubble expansion up and causes earlier freeze-out of the weak interaction, which results in \(^4\text{He}\) overproduction. The recent analyses of primordial \(^4\text{He}\) abundance [23]–[25] are almost consistent with \( N_\nu \sim 3 \). Thus we conservatively adopt \( \Delta N_\nu \leq 1 \) as the BBN constraint. Note that this constraint applies to the saxion whose lifetime is shorter than 1 s and whose main decay mode is \( s \to 2a \) [26].

The increase of \( \Delta N_\nu \) changes the epoch of the matter–radiation equality and affects the structure formation of the universe. Thus, \( \Delta N_\nu \) is also constrained from the cosmic microwave background (CMB), galaxy clustering and Lyman-\( \alpha \) forest. According to the recent analyses [27]–[31], \( \Delta N_\nu \gg 1 \) is not favored\(^7\). This constraint applies to the saxion with lifetime \( \tau_s \lesssim 10^{13} \text{ s} \).

If the saxion decays mainly into axions, \( \Delta N_\nu \) is determined from the relation

\[
\frac{\rho_s}{s} \sim 0.34 g_{ss}(T_s)^{-1} \Delta N_\nu T_s,
\]

(24)

where \( T_s \) is the temperature at the decay of the saxion and \( g_{ss} \) counts the relativistic degrees of freedom. Thus the requirement \( \Delta N_\nu \leq 1 \) constrains the saxion abundance as

\[
\frac{\rho_s}{s} \lesssim 3.4 \times 10^{-5} \text{ GeV} \left( \frac{10}{g_{ss}(T_s)} \right) \left( \frac{T_s}{1 \text{ MeV}} \right).
\]

(25)

Note that almost the same constraint is applied even if the saxion decay into axions is suppressed, if its lifetime is longer than \( \sim 1 \text{ s} \). The reason is as follows. Roughly speaking, the constraint \( \Delta N_\nu \lesssim 1 \) means that the saxion should not dominate the universe before its decay. If it dominates the universe before its decay, a substantial amount of entropy is released by its decay. But entropy production after BBN is severely constrained because the baryon-to-entropy ratio should be unchanged between BBN and the recombination epoch, as CMB anisotropy measurements and observed abundances of the light elements.

\(^7\) It is pointed out that including Lyman-\( \alpha \) forest data raises the best-fit value of \( N_\nu \) [31], but \( \Delta N_\nu \gg 1 \) is still disfavored. If \( \Delta N_\nu \) is close to 1 and \( \tau_s > 1 \text{ s} \), this may solve the observational discrepancy of \( N_\nu \) at BBN and structure formation [32].
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Figure 2. Various cosmological constraints on the saxion abundance for $F_a = 10^{10} \text{ GeV}$. Thin dotted black lines represent theoretical prediction $\rho_s/s = (\rho_s/s)_{(C)} + (\rho_s/s)_{(TP)}$ for $T_R = 10^{10} \text{ GeV}$ (upper) and $T_R = 1 \text{ GeV}$ (lower) with $s_i = F_a$. Four panels correspond to different models. Model (a) KSVZ with $f = 1$, model (b) KSVZ with $f = 0$, model (c) DFSZ with $f = 1$ and model (d) DFSZ with $f = 0$.

indicate [33]. Thus in this case the constraint (25) is applied. On the other hand, entropy production before BBN is possible\(^8\). No constraint is imposed in this case if the branching ratio into axions is suppressed (see the cases (b) and (d) of figures 2 and 3). The following analyses and the resulting constraints on the saxion abundance do not depend on whether the saxion dominates or not.

3.2. Big bang nucleosynthesis

The saxion with its lifetime $\gtrsim 10^{-2} \text{ s}$ may affect BBN [34]. The saxion decays into ordinary particles either radiatively or hadronically. If the hadronic decay occurs at an early epoch ($\tau_s \gtrsim 10^2 \text{ s}$), the main effect on BBN is $p \leftrightarrow n$ conversion caused by injected pions, which results in helium overproduction. At a later epoch, photo- and hadro-dissociation processes of light elements take place efficiently. When $s \rightarrow 2a$ is the dominant decay

\(^{8}\) See [26, 35] for the case of thermal inflation driven by the saxion field trapped at the origin.
mode, the branching ratios into radiation or hadrons are small. Nevertheless, even a small fraction of the energy density of the saxion which goes into radiation or hadrons may have impacts on the BBN. In particular, if hadronic decay modes are open, the constraint is very stringent.

The constraints from photo-(hadro-)dissociation are approximately written as

$$B_r \left( \frac{\rho_s}{s} \right) \lesssim \begin{cases} 10^{-6} - 10^{-14} \text{ GeV} & \text{for } 10^4 \lesssim \tau_s \lesssim 10^7 \text{ s} \\ 10^{-14} \text{ GeV} & \text{for } 10^7 \lesssim \tau_s \lesssim 10^{12} \text{ s} \end{cases}$$

for radiative decay and

$$B_h \left( \frac{\rho_s}{s} \right) \lesssim \begin{cases} 10^{-9} - 10^{-13} \text{ GeV} & \text{for } 1 \lesssim \tau_s \lesssim 10^4 \text{ s} \\ 10^{-13} - 10^{-14} \text{ GeV} & \text{for } 10^4 \lesssim \tau_s \lesssim 10^{12} \text{ s} \end{cases}$$

for hadronic decay, where $B_r$ and $B_h$ denote the radiative and hadronic branching ratios, respectively (here $B_r$ includes the hadronic decay modes). Note that if the injected photon energy (which is equal to half of the saxion mass) is smaller than the threshold energy to destroy the light elements especially $^4\text{He}$, which is typically $\mathcal{O}(10)$ MeV, the photo- and hadro-dissociation constraints are much weakened. In particular BBN constraints are neglected for $m_s \lesssim 4.5$ MeV, which corresponds to the threshold energy for the process $D + \gamma \rightarrow n + p$. 

Figure 3. Same as figure 2, but for $F_a = 10^{12}$ GeV.
3.3. Cosmic microwave background

The saxion with lifetime $10^6 \lesssim \tau_s \lesssim 10^{13}$ s may affect the blackbody spectrum of CMB. Since preserving the blackbody spectrum requires the photon number-violating processes such as double-Compton scattering, to maintain thermal equilibrium between photons and electrons, photons injected in the decay distort the CMB spectrum at $t \gtrsim 10^6$ s when the double-Compton scattering becomes inefficient. The distortion is characterized by the chemical potential $\mu$ at $t \lesssim 10^9$ s when the energy transfer by the Compton scattering is efficient, and the Compton $y$ parameter at a later epoch, which characterizes the deviation of the CMB spectrum from thermal distribution due to the inverse Compton scattering by high energy electrons. They are constrained from COBE FIRAS measurement as $|\mu| \lesssim 9 \times 10^{-5}$ and $y \lesssim 1.2 \times 10^{-5}$ \cite{36}. $\mu$ and $y$ are related to the injected photon energy $\delta \rho_\gamma$ as \cite{37,38}

$$\frac{\delta \rho_\gamma}{\rho_\gamma} \sim 0.714 \mu, \quad (28)$$

for $10^6 \lesssim \tau_s \lesssim 10^9$ s and

$$\frac{\delta \rho_\gamma}{\rho_\gamma} \sim 4y, \quad (29)$$

for $10^9 \lesssim \tau_s \lesssim 10^{13}$ s. This in turn constrains the saxion energy density depending on its branching ratio into radiation $B_r$ as

$$B_r \left( \frac{\rho_s}{s} \right) \leq \begin{cases} 
9.0 \times 10^{-13} \text{ GeV} \left( \frac{10^9 \text{ s}}{\tau_s} \right)^{1/2} & (10^6 \lesssim \tau_s \lesssim 10^9 \text{ s}) \\
6.7 \times 10^{-13} \text{ GeV} \left( \frac{10^9 \text{ s}}{\tau_s} \right)^{1/2} & (10^9 \lesssim \tau_s \lesssim 10^{13} \text{ s}). 
\end{cases} \quad (30)$$

3.4. Diffuse $x(\gamma)$-ray background

The two-photon decay of the saxion with lifetime longer than $\sim 10^{13}$ s may contribute to the diffuse $x(\gamma)$-ray background. The mass of the saxion which has such a long lifetime is typically smaller than 1 GeV. The photon with energy $1 \text{ keV} \lesssim E_\gamma \lesssim 1 \text{ TeV}$ is transparent against the scattering with cosmic background photons and intergalactic medium, and hence such decay-produced photons freely propagate through the universe and can be observed as diffuse background photons \cite{39}.

The flux of the photons from the decay of the saxion is calculated as \cite{40}

$$F_\gamma(E) = \frac{E}{4\pi} \int_0^{t_0} dt \frac{B_\gamma n_s(z)}{\tau_s} (1 + z)^{-3} \frac{dE'}{dE} 2\delta \left( E' - \frac{m_s}{2} \right), \quad (31)$$

where $B_\gamma$ denotes the branching ratio into two photons and $n_s(z)$ is the number density of the saxion at the redshift $z$. $E'$ is the energy of the photon at the instant of production and $E$ is the present redshifted energy; the relation between them is given by $E' = (1 + z)E$. Under the assumption of the flat universe ($\Omega_\Lambda + \Omega_m = 1$), this expression can be integrated
yielding
\[
F_\gamma(E) = \frac{B_\gamma n_{s0}}{2\pi \tau_s H_0} g \left( \frac{m_s}{2E} \right) \exp \left[ -\frac{1}{3H_0 \tau_s \sqrt{\Omega_\Lambda}} \ln \left( \frac{\sqrt{\Omega_\Lambda} g (m_s/2E) - 1}{\sqrt{\Omega_\Lambda} g (m_s/2E) + 1} \right) \right],
\]
where \( n_{s0} \) denotes the present number density of the saxion, \( H_0 \) denotes the present Hubble constant and
\[
g(x) = \left[ \Omega_\Lambda + \Omega_m x^2 \right]^{-1/2}.
\]

On the other hand, the observed photon flux in the range \( 1 \text{ keV} \lesssim E \lesssim 100 \text{ GeV} \) is roughly given as
\[
F_{\gamma\text{obs}}(E) \sim \begin{cases} 
8 \left( \frac{E}{\text{keV}} \right)^{-0.4} & (0.2 \text{ keV} \lesssim E \lesssim 25 \text{ keV}) \\
57 \times 10^{-4} \left( \frac{E}{\text{MeV}} \right)^{-1.6} & (25 \text{ keV} \lesssim E \lesssim 4 \text{ MeV}) \\
17 \times 10^{-6} \left( \frac{E}{100 \text{ MeV}} \right)^{-1.1} & (4 \text{ MeV} \lesssim E \lesssim 120 \text{ GeV}),
\end{cases}
\]
in units of \( \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \), from the observations of ASCA [41], HEAO1 [42], COMPTEL [43] and EGRET [44]. Thus, from the requirement \( F_\gamma < F_{\gamma\text{obs}} \), the tight constraint on the saxion density is derived. Note that, even for the saxion lifetime longer than the age of the universe, the small fraction of the decayed saxion at \( t < t_0 \) contributes to the diffuse background and its abundance is limited.

To estimate this constraint, let us consider the case with \( \tau_s > t_0 \). In this case, the \( \gamma(\gamma) \)-ray spectrum of photons from the saxion decays has the maximum at \( E_{\text{max}} = m_s/2 \), where the flux is simplified as \( F_\gamma(E_{\text{max}}) = B_\gamma n_{s0}/(2\pi \tau_s H_0) \). Then from the condition \( F_\gamma(E_{\text{max}}) < F_{\gamma\text{obs}}(E_{\text{max}}) \), we obtain a constraint
\[
B_\gamma \left( \frac{\rho_\gamma}{s} \right) \lesssim 2\pi \frac{m_s \tau_s H_0}{s_0} F_{\gamma\text{obs}} \left( \frac{m_s}{2} \right)
\approx 2.4 h \times 10^{-18} \text{ GeV} \left( \frac{m_s}{1 \text{ MeV}} \right) \left( \frac{\tau_s}{10^{18} \text{ s}} \right) \left( \frac{F_{\gamma\text{obs}}(m_s/2)}{10^{-2} \text{ cm}^{-2} \text{ s}^{-1}} \right),
\]
where \( s_0 \) denotes the present entropy density. For the case with \( \tau_s < t_0 \), the photon energy which gives the flux maximum deviates from \( E = m_s/2 \) due to the Hubble expansion. For \( \tau_s < t_0 \), it is given by \( E_{\text{max}} = (m_s/2)(3H_0 \tau_s \sqrt{\Omega_m/2})^{2/3} \). This leads to the constraint
\[
B_\gamma \left( \frac{\rho_\gamma}{s} \right) \lesssim \frac{4\pi m_s}{3 s_0} F_{\gamma\text{obs}} \left( E_{\text{max}} \right)
\approx 4.8 \times 10^{-19} \text{ GeV} \left( \frac{m_s}{1 \text{ MeV}} \right) \left( \frac{F_{\gamma\text{obs}}(E_{\text{max}})}{10^{-2} \text{ cm}^{-2} \text{ s}^{-1}} \right).
\]

### 3.5. Reionization

If the saxion decays after the recombination era and the injected photon energy is relatively small \( (m_s \lesssim O(1) \text{ keV} - O(1) \text{ MeV}) \) and \( 10^{13} \text{ s} \lesssim \tau_s \), redshifted photons may leave the transparency window until the present epoch [39]. Then, emitted photons interact with
and ionize the intergalactic medium (IGM), and they contribute as an additional source of reionization. If this contribution is too large, the optical depth to the last scattering surface is too large to be consistent with the WMAP data [33]. Here we apply the results from [39, 45], simply assuming that if the decay-produced photon leaves the transparency window, one-third of the photon energy is converted to the ionization of the IGM (the remaining goes to the excitation and heating of the IGM). According to [39, 45], this is a good approximation when the decay occurs before the reionization due to astrophysical objects takes place and most of the hydrogen atoms exist in the form of neutral state. (The Gunn–Peterson test indicates that the reionization occurred at \( z \approx 6 \) [46].) The constraint on the saxion density can be written as

\[
B \left( \frac{\rho_s}{s} \right) \lesssim \left( \frac{\rho_s}{s} \right)_{\text{bound}},
\]

where \( (\rho_s/s)_{\text{bound}} \) can be read off from figure 2 of [45]. For example, for \( \tau_s \gtrsim t_0 \), it is given by

\[
\left( \frac{\rho_s}{s} \right)_{\text{bound}} \approx 4.3 \times 10^{-17} \text{ GeV} \left( \frac{\tau_s}{10^{18} \text{ s}} \right) \left( \frac{\Omega_B h^2}{0.022} \right),
\]

where \( \Omega_B \) denotes the density parameter of the baryonic matter. This constraint is complementary to the diffuse x(\( \gamma \))-ray limit.

3.6. Present matter density limit

For the saxion with its lifetime \( \tau_s > t_0 \), its energy density contributes to the dark matter of the universe and hence the saxion density should be less than the observed matter density, \( \Omega_s h^2 \lesssim \Omega_m h^2 \). In terms of the saxion-to-entropy ratio, this is written as

\[
\frac{\rho_s}{s} \lesssim 4.7 \times 10^{-10} \text{ GeV} \left( \frac{\Omega_m h^2}{0.13} \right). \tag{39}
\]

3.7. LSP overproduction

If the saxion mass is larger than about 1 TeV, the saxion can decay into SUSY particles. Here we suppose that the LSP is the lightest neutralino. The decay into SUSY particles was investigated in detail in [47] and it was found that decay into gauginos has roughly the same branching ratio as that into gauge bosons. Thus we should be careful about LSP overproduction from the saxion decay. The resultant abundance of the LSP depends on \( T_s \), and for \( T_s \gtrsim m_{\text{LSP}}/20 \) LSPs produced from the saxion decay are thermalized and have the same abundance as that expected in the standard thermal relic scenario of the LSP dark matter. In this case no upper bound on the saxion abundance is imposed. On the other hand, if \( T_s \lesssim m_{\text{LSP}}/20 \), the abundance of the LSP is given by

\[
\frac{\rho_{\text{LSP}}}{s} \sim \begin{cases} 
B_s \frac{2m_{\text{LSP}} \rho_s}{m_s} + \frac{\rho_{\text{thermal}}}{s} & \text{for } n_{\text{LSP}}(T_{\text{s}}) \langle \sigma v \rangle < H(T_{\text{s}}), \\
\sqrt{\frac{45}{8\pi^2 g_*(T_s) \langle \sigma v \rangle T_s M_P}} m_{\text{LSP}} & \text{for } n_{\text{LSP}}(T_{\text{s}}) \langle \sigma v \rangle > H(T_{\text{s}}),
\end{cases} \tag{40}
\]

where \( B_s \) denotes the branching ratio of the saxion into SUSY particles, \( \langle \sigma v \rangle \) denotes the thermally averaged annihilation cross section of the LSP and \( \rho_{\text{thermal}} \) denotes the
contribution from thermal relic LSPs taking account of the dilution from the saxion decay. The LSP number density immediately after the saxion decay $n_{\text{LSP}}(T_s)$ is defined as $n_{\text{LSP}} = 2B_s \rho_s(T_s)/m_s$. For deriving the constraint, we ignored the contribution to the LSP production from thermal scattering processes. Moreover, the second line of equation (40) always results in overproduction of LSPs with the annihilation cross section for ordinary neutralino dark matter. Thus, for deriving the constraint, we consider only the first term of the first line of the right-hand side of equation (40). The bound can be written in the form

$$\rho_s / s \lesssim 2.4 \times 10^{-10} \text{ GeV} \left( \frac{m_s}{m_{\text{LSP}}} \right) \left( \frac{\Omega_{\chi h^2}}{0.13} \right),$$

for $m_s \gtrsim 1$ TeV and $T_s \lesssim m_{\text{LSP}}/20$. This constraint can be relaxed if the annihilation cross section of the LSP is significantly large. We will revisit this issue in section 5. Hereafter we set $m_{\text{LSP}} = 500$ GeV as a reference value.

Including all of these constraints, we can derive general upper bounds on the saxion-to-entropy ratio as a function of the saxion mass $m_s$ for models (a)–(d). In figures 2–4, we show the results with $F_a = 10^{10}, 10^{12}$ and $10^{14}$ GeV, respectively. In each panel, the orange line represents the bound from $\Delta N_e \lesssim 1$, the thick solid brown line represents the bound from BBN, the thick dotted purple line represents the bound from CMB, the thick dotted–dashed green line represents the bound from diffuse x($\gamma$)-ray background,
the thin dotted–dashed blue line represents the bound from reionization, the thin dashed red line represents the limit from the present matter density and the thick dashed gray line represents the LSP overproduction limit from the saxion decay. We also show the theoretical prediction for the saxion energy density in the figures for $T_R = 10^{10}$ GeV and 1 GeV by thin dotted black lines.

4. Constraints from axino

So far, we have ignored the cosmological effects of the axino, the fermionic superpartner of the axion. The mass of the axino is model-dependent, but it can be as heavy as the gravitino mass, $m_{3/2}$ [48, 22, 49]. For example, in the model of equation (2), the axino mass is estimated as $m_{\tilde{a}} \sim |\lambda X|$. As noted earlier, taking into account the $A$-term potential like $V_A \sim m_{3/2} \lambda X F_{\tilde{a}}^2 + \text{h.c.}$, $X$ can have the VEV of the order of $m_{3/2}$. Thus in this model the axino mass is naturally expected to be $m_{3/2}$. Hereafter for simplicity we assume $m_{\tilde{a}} \sim m_{3/2}$.

Axinos and gravitinos are produced through scatterings of particles in a thermal bath. First, we assume either of them is the LSP, and hence their thermally produced abundance must not exceed the present abundance of the dark matter [50]. The abundance of thermally produced axinos is calculated as [52]

$$\frac{\rho_{\tilde{a}}}{s} \simeq 2.0 \times 10^{-7} g_s^6 \text{GeV} \left( \frac{m_{\tilde{a}}}{1 \text{ GeV}} \right) \left( \frac{10^{12} \text{ GeV}}{F_{\tilde{a}}} \right)^2 \left( \frac{T_R}{10^6 \text{ GeV}} \right),$$

where $g_s$ is the QCD gauge coupling constant. Thus for large $m_{\tilde{a}}(\sim m_s)$, the constraint on the reheating temperature becomes stringent. Note that axino thermal production for $T_R < 1$ TeV is negligible, because SUSY particles are not produced efficiently at such a low reheating temperature.

Next, if the axino mass is larger than about 1 TeV, the axino can decay into SUSY particles. This LSP abundance depends on the temperature at the decay of the axino, $T_{\tilde{a}}$, similar to the case of the saxion decay. If $T_{\tilde{a}} \gtrsim m_{\text{LSP}}/20$, LSPs produced from the axino decay are thermalized and the standard thermal relic scenario of the LSP dark matter is maintained. If $T_{\tilde{a}} \lesssim m_{\text{LSP}}/20$, the LSP abundance is determined by equation (40) after replacing $T_s$ and $\rho_s/s$ with $T_{\tilde{a}}$ and $\rho_{\tilde{a}}/s$.

On the other hand, the thermally produced gravitinos have the abundance as [53]

$$\frac{\rho_{3/2}^{TP}}{s} \simeq 6.3 \times 10^{-11} \text{ GeV} \left( \frac{1 \text{ GeV}}{m_{3/2}} \right) \left( \frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2 \left( \frac{T_R}{10^6 \text{ GeV}} \right)$$

for $m_{3/2} \ll m_{\tilde{g}}$, where $m_{\tilde{g}}$ denotes the mass of the gluino (here the logarithmic dependence on $T_R$ is omitted). Contrary to the axino, the constraint becomes more severe when $m_{3/2}(\sim m_s)$ becomes smaller. Note that for $m_{3/2} \lesssim 1$ keV, gravitinos get thermalized and their abundance becomes independent of the reheating temperature. But for 16 eV $\lesssim$...
Cosmological implications of supersymmetric axion models

Figure 5. Upper bounds on the reheating temperature $T_R$ for each model with $F_a = 10^{10}$ GeV. The initial amplitude of the saxion is assumed to be $s_i \sim F_a$. The thin short-dashed light blue line represents the bound from axino thermal production and the thin dotted black line represents the bound from gravitino thermal production. The shaded region contradicts with the lowest possible reheating temperature \[55\]. The other lines are the same as in figure 2. Four panels correspond to different models. Model (a) KSVZ with $f = 1$, model (b) KSVZ with $f = 0$, model (c) DFSZ with $f = 1$ and model (d) DFSZ with $f = 0$.

$m_{3/2} \lesssim 1$ keV they contribute to the dark matter density as a hot component due to their long free-streaming length, and such a contribution is constrained from cosmological observations, in particular Lyman-$\alpha$ forest data \[54\]. Thus the gravitino mass in this region is strongly disfavored. (The case of ultra-light gravitino $m_{3/2} \lesssim 16$ eV will be mentioned later.) In addition to the constraint from the present matter density, there may be another constraint coming from the late decay of SUSY particles into gravitinos or axinos, which may affect BBN. But the constraint is quite model-dependent, and hence we do not consider it here.

Including those constraints from the gravitino and axino, we derive the upper bounds on the reheating temperature for each saxion mass and show them in figures 5–7. In these figures, we have assumed that the initial amplitude of the saxion is given by $s_i \sim F_a$. The thin short-dashed light blue line represents the bound from axino thermal production and
the thin dotted black line represents the bound from gravitino thermal production. The other lines are the same as in figure 2. Because the saxion-to-entropy ratio \( \left( \frac{\rho_s}{s} \right)^{(C)} \) is proportional to \( \left( T_R s_i^2 \right) \), the constraints on \( T_R \) scale as \( s_i^{-2} \). However, it should be noted that, although the axino constraint is stringent for relatively large \( m_s \) as can be seen from these figures, it may be significantly relaxed if the axino is much lighter than the gravitino.

5. Dark matter candidates

We have seen that the very stringent bound on the reheating temperature is imposed for a wide range of saxion masses. It is typically stronger than the usual upper bound from the gravitino overproduction. It has some implications to the dark matter candidates. For example, it invalidates the gravitino dark matter for wide parameter regions. In this section we summarize the dark matter candidate in SUSY axion models.

5.1. \( m_s \lesssim 1 \) TeV

First consider the case where the gravitino (axino) is the LSP. In our model, we assume the axino has a mass comparable to the gravitino, and hence the axino (gravitino) is the NLSP. Which of them is the lighter is not important because both have similar properties. The saxion also has the mass comparable to the gravitino. The stability of the saxion...
Cosmological implications of supersymmetric axion models

Figure 7. Same as figure 5, but for $F_a = 10^{14}$ GeV.

is not ensured by the $R$ parity, but since its decay rate is suppressed by the PQ scale $F_a$, the saxion lifetime can exceed the present age of the universe. If this is the case, the saxion can be the dark matter. In addition, if $\theta_1^7 F_a \sim 10^{12}$ GeV, the saxion can also play the role of the dark matter. Therefore, we have four candidates for the dark matter, i.e. gravitino, axino, axion and saxion. However, the saxion dark matter is possible only when $F_a \sim 10^{14}$ GeV and $1 \text{ keV} \lesssim m_s \lesssim 10$ keV, as can be seen from figure 7, and hence is a less attractive candidate. For $10^{10}$ GeV $\lesssim F_a \lesssim 10^{12}$ GeV, the axino dark matter is allowed for wide parameter regions, but the gravitino dark matter is excluded except for $m_s \lesssim 100$ keV, as can be seen from figures 5 and 6. On the other hand, for $F_a \gtrsim 10^{14}$ GeV, the axino or gravitino dark matter is almost impossible and the axion is the most viable dark matter candidate, although other cosmological constraints are severe (figure 7).

To summarize, for $m_s \lesssim 1$ TeV, the axino is a good dark matter candidate for $F_a \lesssim 10^{12}$ GeV and the axion may be a dark matter candidate for $F_a \gtrsim 10^{12}$ GeV.

5.2. $m_s \gtrsim 1$ TeV

For larger $m_s(\sim m_3/2)$, the saxion decay mode into SUSY particles opens. Then the LSP is assumed to be the lightest neutralino. As discussed in section 3.7, the abundance is given by equation (40) if the decay occurs after the freeze-out of the LSP. For the neutralino which has small annihilation cross section, a very low reheating temperature is
needed to obtain the correct abundance of the dark matter as can be seen from figures 5–7. However, for the neutralino with larger annihilation cross section such as wino- or higgsino-like LSP, the abundance may be significantly reduced and they can become the dark matter independent of the reheating temperature \[47\]. This is easily seen from equation (40), which describes that the resulting abundance is bounded from above. Thus for the saxion mass larger than 1 TeV, these non-thermally produced neutralinos may be dark matter. On the other hand, if \(T_s\) is larger than the freeze-out temperature of the LSP \(T_f \sim m_{\text{LSP}}/20\), the standard thermal relic scenario holds and the saxion has no impact on cosmology unless it dominates the universe before the decay. If the saxion once dominates the universe, it must not decay into axions mainly, since otherwise \(\Delta N_\nu \gg 1\) holds at BBN and contradicts with observation.

Therefore, for \(m_s \gtrsim 1\) TeV, the lightest neutralino produced either thermally or non-thermally is likely dark matter depending on \(m_s\) and \(F_a\). Of course, the axion is also a good candidate for the dark matter for \(F_a \gtrsim 10^{12}\) GeV.

### 6. Ultra-light gravitino scenario

We have seen that, for almost all the mass range of the gravitino, the reheating temperature is severely constrained. However, for an ultra-light gravitino with mass of the order of 1–10 eV, gravitinos are thermalized and their abundance is sufficiently lower than the dark matter abundance, and hence the reheating temperature is not constrained from gravitino overproduction. Thus thermal leptogenesis using a right-handed neutrino \[56\], which requires \(T_R \gtrsim 10^9\) GeV, may be possible.

In the SUSY axion model, the saxion oscillation also contributes to the present dark matter density as given in equations (8) and (10). We should ensure that this contribution does not exceed the present matter density of the universe. From equation (8), \(\rho_s/s\) is bounded as

\[
\left(\frac{\rho_s}{s}\right)^{(C)} \lesssim 1.5 \times 10^{-11} \text{ GeV} \left(\frac{m_s}{10 \text{ eV}}\right)^{1/2} \left(\frac{F_a}{10^{11} \text{ GeV}}\right)^2 \left(\frac{s_i}{F_a}\right)^2.
\]

Thus for \(F_a \lesssim 10^{11}\) GeV, the saxion abundance from coherent oscillation is smaller than that of the dark matter. However, we should be aware that thermally produced saxion and axino abundances are comparable to that of the gravitino for \(T_R \gtrsim 10^9\) GeV(\(F_a/10^{11}\) GeV)² if \(m_s = m_a = m_3/2\), since both are thermalized in the early universe. Thus the constraint on the gravitino mass may become more stringent by a factor of two, if thermal leptogenesis is assumed to work. In figure 8, we show allowed parameter regions on the \(F_a-T_R\) plane for \(m_s = 10\) eV (indicated by (a)) and for \(m_s = 1\) keV (indicated by (b)) with an assumption \(s_i = F_a\). It can be seen that, for \(m_s \lesssim 10\) eV, thermal leptogenesis is still possible as long as the PQ scale satisfies \(F_a \lesssim 5 \times 10^{11}\) GeV.

### 7. Conclusions and discussion

We have investigated the cosmological constraints on supersymmetric axion models which are motivated from the particle physics point of view. It is found that the presence of the saxion and the axino makes it rather difficult to construct a viable cosmological scenario which is free from any contradiction with observations. In particular, in almost all of the
range of the gravitino mass (which is assumed to be the same order as the saxion mass), the strict upper bound on the reheating temperature is imposed, and it is more stringent than the bound from the usual gravitino problem (figures 5–7). The constraint depends on whether the main decay mode of the saxion is into axions or not. It should be noted that, although the axino constraint is stringent for relatively large $m_s$, as can be seen from our results, it may be significantly relaxed if the axino is much lighter than the gravitino. The axion is a good candidate for the cold dark matter, although the axino or gravitino dark matter is also viable for some parameter regions.

The obtained stringent bound on the reheating temperature has some implications on the baryogenesis scenarios. As is well known, the standard thermal leptogenesis scenario using a right-handed neutrino [56] is incompatible with the gravitino problem except for $m_{3/2} \lesssim 10$ eV or $m_{3/2} \gtrsim 10$ TeV. Although the gravitino mass around $m_{3/2} \sim 10$ GeV may also be compatible with thermal leptogenesis, in this region the NLSP decay into gravitinos may cause another difficulty. The presence of the saxion makes this situation worse, and hence the standard thermal leptogenesis does not seem to work in supersymmetric axion models. The Affleck–Dine baryogenesis scenario [57] can work well even for such a low reheating temperature [58], except for the case of gauge-mediated SUSY breaking models for small $m_{3/2}$, where the Affleck–Dine mechanism may suffer from Q-ball formation [59]. On the other hand, for an ultra-light gravitino mass $m_{3/2} \lesssim 10$ eV, thermal leptogenesis is still possible. Here it should be noticed that these constraints from the saxion strongly depend on the initial amplitude of the saxion $s_i$ and we have assumed $s_i$ is roughly given by the PQ scale $F_a$. A concrete example which gives such an initial amplitude is given in section 2.1. Perhaps this is the smallest value expected from naturalness, and hence our bounds presented in this paper should be regarded as conservative, which means our constraints cannot be relaxed without significant changes in the cosmological scenario, such as additional late-time entropy production.
Finally we comment on the detectable signature of the SUSY axion models. If the saxion mass is around a few MeV, its decay into electron–positron pairs and their annihilation may be observed as the 511 keV line from the galactic center. Gamma photons from such a line are actually observed [60]. However, in order to explain the observed flux, a huge entropy production that dilutes the saxion density is needed [61]. Also it may be possible that the nature of the axion sector is determined from collider experiments if the axino is the LSP and a charged particle such as the stau is the NLSP [62].

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