Comments on “The Einstein-Hilbert Lagrangian Density in a 2-dimensional Spacetime is an Exact Differential” by R. da Rocha and W.A. Rodrigues, Jr.

N. Kiriushcheva and S.V. Kuzmin
Department of Applied Mathematics
University of Western Ontario, London, N6A 5B7 Canada

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Abstract

We argue that the recent result of da Rocha and Rodrigues that in two dimensional spacetime the Lagrangian of tetrad gravity is an exact differential [1], despite the claim of the authors, neither proves the Jackiw conjecture [2], nor contradicts the conclusion of [3]. This demonstrates that the tetrad formulation is different from the metric formulation of the Einstein-Hilbert action.

The Lagrangian density of two dimensional tetrad gravity has recently been demonstrated, using the “powerful and economic” methods of Clifford algebra by da Rocha and Rodrigues [1], to be an exact differential. Its explicit form is given in the last, fourteenth, section of [1]. This well-known fact can hardly be called a new result (see, e.g., [4]) and, in particular, cannot be a good illustration of the power of the method used by authors. We discuss the economy of this approach in the next section. The relevance of this result [1] to the Jackiw conjecture [2] and to the conclusion of [3] is the subject of the second section. In the third section we comment on some particular results in [1]. The last section is our conclusion.
We start by reviewing some results of the tetrad formulation, in order to establish notation. Tetrads are a set of orthogonal vectors $e^a_\mu$ erected at each point of $D$-dimensional spacetime ($\mu = 0, ..., D - 1$ are world coordinates indices and $a = 0, ..., D - 1$ are tetrad indices). The Lagrangian of tetrad gravity (TG) can be obtained by performing a direct substitution of the metric tensor $g_{\mu\nu}$ in terms of tetrads, $g_{\mu\nu} = e_{a\mu}e_{b\nu}$, into the Einstein-Hilbert (EH) Lagrangian (e.g., Eq. (9) of [5]). This can also be done by using particular combinations built from tetrads and their derivatives, such as Ricci rotational coefficients [6] ($\gamma_{abc} \equiv e_{a\mu\nu}e^\mu_b e^\nu_c$ which is a world scalar), spin connections

$$\omega_{\mu ab} \equiv -e_{a\nu,\mu}e^\nu_b = -e_{a\nu,\mu}e^\nu_b + \Gamma^\lambda_{\nu\mu}e_{a\lambda}e^\nu_b,$$  \hspace{1cm} (1)

which is a world vector), etc.; and then using properties of the commutator of covariant derivatives of a covariant vector in Riemannian space $V_{\alpha\beta\gamma\delta} = V_{\sigma} R^\sigma_{\alpha\beta\gamma\delta}$ (see, e.g., p. 291 of [7] or p. 49 of [8]). We are then left with

$$L(e) = eR(e)$$  \hspace{1cm} (2)

with

$$R(e) = R(g(e)) = R(g(e), \Gamma(e)) = R(e, \gamma(e)) = R(e, \omega(e))$$

where $e = \det |e^a_\mu|$ and semicolons “;” indicate covariant differentiation.

One particularly common form of the Lagrangian of tetrad gravity is (see, e.g. [10]; Eq. (11-12) of [5]; Eq. (2.1) of [9])

$$L(e) = ee_{a\mu}e_{b\nu}(\omega_{a\nu,\mu} - \omega_{a\mu,\nu} + \omega_{\muac}\omega^c_{\nu b} - \omega_{\nuac}\omega^c_{\mu b}).$$  \hspace{1cm} (3)

Using (1) and the expression for $\Gamma^\lambda_{\nu\mu} (g(e))$ one easily obtains (Eq. (7) of [5]; Eq. (2.4) of [13])

$$\omega_{\nu ab} = \frac{1}{2} [e^a_\mu (e_{b\rho,\nu} - e_{b\rho,\mu}) - e^b_\nu (e_{a\rho,\nu} - e_{a\rho,\mu}) + e^c_\rho (e_{c\rho,\mu} - e_{c\rho,\nu}) e^\nu_b].$$  \hspace{1cm} (4)

\textsuperscript{1}The different sign of (3) in some articles (e.g., [5]) is, probably, the result of using different from [7], [11], [8] convention for Riemannian tensor as, for example, in [12].
In two dimensions (2D), because of the antisymmetry of $\omega$ in the tetrad indices ($\omega_{\mu ab} = -\omega_{\mu ba}$) it immediately follows that the part of (3) that is quadratic in $\omega$,

$$L_{\omega\omega} = ee^{a\mu}e^{b\nu}(\omega_{\mu ac}\omega_{\nu}^c - \omega_{\nu ac}\omega_{\mu}^c),$$

(5)

is zero, and the only non-zero part of (3) is

$$L(e) = 2\omega_{0(0)(1),1} - 2\omega_{1(0)(1),0}.$$  

(6)

(We use () brackets to distinguish explicit values of tetrad indices.) Equation (6) follows from the antisymmetry of $\omega$ in the tetrad indices and the 2D relation

$$e\left(e^{(0)0}e^{(1)1} - e^{(1)0}e^{(0)1}\right) = -1$$

(7)

since $e = e_0^{(0)}e_1^{(1)} - e_1^{(1)}e_0^{(0)}$ (we use $\eta_{ab} = (+)$ for lowering and rising tetrad indices). Equation (6) can also be derived using differential forms.

To express the 2D Lagrangian of TG as a total divergence in a manifestly covariant (tensorial) form, we can use a trick similar to what was employed in the metric formulation (see p. 269 [7] or p. 98 of [11] and for its use in the Hamiltonian formulation of the EH Lagrangian [14], [15], [3]) in order to express terms with second order derivatives of the tetrads as a total derivative. This can be done directly in terms of tetrads or, simpler, by working with $\omega_{\mu ab}$.

This converts (3) into

$$L(e) = -\left[e\left(e^{a\mu}e^{b\nu} - e^{a\nu}e^{b\mu}\right)\right]_{,\mu} \omega_{\nu ab} + V_{,\mu} + L_{\omega\omega},$$

(8)

where

$$V_{\mu} = \left[e\left(e^{a\mu}e^{b\nu} - e^{a\nu}e^{b\mu}\right)\right]_{,\mu} \omega_{\nu ab} = 2ee^{a\mu}e^{b\nu}\omega_{\nu ab}.$$  

(9)

In 2D $L_{\omega\omega} = 0$ and the first term also vanishes because all non-zero contributions to the first term, when written in terms of components, give a constant value to the term in square brackets, as can be seen from (7).

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2 Another similarity between $\Gamma$ and $\omega$ is the equivalence of the first and second order formulations for the metric tensor and tetrad fields in higher than two dimensions, when $g_{\mu\nu}$ and $\Gamma$, and, respectively, $e_a^\mu$ and $\omega$, are treated as independent fields. Note, that for both of them the first and second order formulations are not equivalent in 2D.
Hence, in 2D the Lagrangian of TG is a total derivative of a vector $V^\mu$ ($L_{2D}(e) = V^\mu_{;\mu}$), and this expression is equivalent to (6). In 2D, $V^\mu$ can be written in a very simple form

\[ V^\mu = 2\varepsilon^{ab}\varepsilon^{\nu\rho}e^\mu_a e_{b\nu} \]  

(10)

where $\varepsilon$ is the totally antisymmetric tensor ($\varepsilon^{01} = \varepsilon^{(0)(1)} = 1$). Note, that (9) was obtained using just one step by rearranging the terms in (3) leading to (8). One more manipulation is needed; we must substitute (4) into (9) which in 2D gives (10). This is a compact form of the main result of [1] (see Eqs. (35-39) of section fourteen). This result also is given by Eq. (1.55) of [4], where it is obtained using a different method. Equation (1.55) of [4] is equivalent to (10).

2. We will now show that this well-known result (derived using a longer method in [1]) neither proves the Jackiw conjecture [2], nor contradicts the conclusion of [3].

The Jackiw conjecture (p. 353 of [2], Ref. 1 of [1]), is that “…in two dimensions it [Einstein theory] cannot even be formulated since $G_{\mu\nu}$ [Einstein tensor] vanishes identically. Correspondingly, the Hilbert-Einstein action is a surface term...”. More generally, it appears that according to this conjecture, from triviality of the equations of motion, it follows that the action is a surface term. This is the converse of the statement that Lagrangians which are pure divergences give trivial equations of motion.

This conjecture can be shown to be incorrect by finding a counter example, while, at the same time, finding a single example (such as 2D tetrad gravity) is not sufficient for establishing its proof. One particular counter example is the Einstein-Hilbert Lagrangian density when expressed in terms of the metric. This observation was made by Deser and Jackiw (see the second sentence after Eq. (2.8) on p. 1504 of [16] (Ref. 2 of [1])): “$R^\mu$ [such as $\partial_\mu R^\mu = \sqrt{-g}R$ cannot be presented explicitly and locally in terms of the generic metric $g_{\mu\nu}$ and its derivatives $\partial_\alpha g_{\mu\nu}$]”. See also p.18 of Strobl [17] “…it does not seem to be possible to express $\sqrt{-g}R$ explicitly as a covariant total differential of $g$ itself”. An explicit demonstration of this is provided in [3]. How can this be reconciled with the results for 2D tetrad gravity?

The result of [3] was obtained for the EH (metric) Lagrangian and confirmed the above statements. This was used to find a Hamiltonian formulation of non-divergent part of the metric Lagrangian in a way similar to
the treatment [14] of the second order form of the EH Lagrangian in higher dimensions. In 2D, use of the ADM formulation [18] leads to the unphysical result that there are negative degrees of freedom [19]; i.e. it is an overconstrained system. In contrast, the results of [3] are completely self-consistent.

It appears that the source of confusion about apparent contradiction between the results contained in [3] (so as the statements quoted above) and the well-known result for tetrad gravity (rederived in [1]) is based on the commonly held belief that the EH Lagrangian, when expressed in terms of the metric and in terms of the tetrad, are equivalent, and, moreover, since the two Lagrangians equivalent, then if one of them is a total divergence the second one also must be a total divergence (this would be the logical conclusion if the Jackiw conjecture could be proven). It is important to note though that in [1] it is not established that the two forms of the EH action are equivalent, and hence we cannot conclude that since the tetrad form of this Lagrangian is a total divergence, then so is the metric form.

We now will consider the question of whether the two Lagrangians are really equivalent.

Even though both Lagrangians lead to the trivial equations of motion we cannot call these Lagrangians equivalent. The part of the EH Lagrangian (when expressed in terms of the metric) that is not a surface term (the TT-part only) is not generally covariant [14] and is not a true scalar [11]. Though the equations of motion are the same both with and without the surface term, the part of Lagrangian without a surface term is not generally covariant. To consider question of equivalence, then all terms have to be taken into account since in the presence of a surface contribution, we cannot rely only on the equations of motion being equivalent. The role of a surface term can be analyzed using a generalization of the Hamiltonian procedure (e.g., see [20]). Recently, the peculiar features of surface terms has been reconsidered from quite a different point of view in [21].

Further, it was stated a long time ago by Einstein in his first article on tetrad (n-bein) gravity [22]: “The n-bein field is determined by n^2 functions e^a_\mu [tetrads], whereas the Riemannian metric is determined just by n^{(n+1)}/2 quantities. According to (3) [g_{\mu \nu} = e_{a \mu} e^a_\nu], the metric is determined by the n-bein field but not vice versa”. This inequivalence was his main reason for introducing tetrads; by modifying the EH Lagrangian a unification of gravity with electrodynamics might be possible. Different models considered include
also modifications of Riemannian space itself to Riemann-Cartan, Weyl, etc. spaces. A list of the different spaces examined is contained in Eqs. (4.103)-(4.107) on p. 105 of [23].

One can find different equivalent formulations of the EH action by going from one set of field variables (fields) to another, provided the functional Jacobian of such a transformation is non-singular. For example, we recently used the particular linear combinations \( \xi^\lambda_{\alpha\beta} = \Gamma^\lambda_{\alpha\beta} - \frac{1}{2} \left( \delta^\lambda_{\alpha} \Gamma^\sigma_{\beta\sigma} + \delta^\lambda_{\beta} \Gamma^\sigma_{\alpha\sigma} \right) \) in Hamiltonian formulation of the first order EH action beyond two dimensions [24], [25]. The Jacobian of the transformation between \( \Gamma \) and \( \xi \) is non-singular and field independent. Elimination of the fields \( \xi \) using their equations of motion leads back to the EH action, providing a proof of the equivalence of the first and second order formalisms. However, for a singular change of field variables (it is not, probably, correct even to call this “a change of variables”) without being able to invert it (“not vice versa”), we instead generate a new model which, though it might be an interesting, is not equivalent to the original one, at least mathematically. We obviously are dealing with a singular case when the number of fields in the two sets of variables is different (e.g., \( n^2 \) of tetrad components and \( \frac{n(n+1)}{2} \) components of the metric tensor).

Moreover, even if these (EH and TG) mathematically non-equivalent Lagrangians give physically equivalent results (such as, for example, the same number of degrees of freedom, the same observables, etc.) it would not be enough to draw the conclusion that if the 2D tetrad Lagrangian is a total divergence, then the EH (metric) Lagrangian is also a total divergence.

Some authors define physical equivalence in very broad ways. For example, according to [26]: “All experimental knowledge about gravitation existing at present is therefore compatible with any theory that coincides for weak fields with the linearized version of Einstein’s theory. In particular, a vierbein field theory of gravitation is sufficiently supported by experiment, provided the field equations written in terms of the variables \( e^k_{\alpha} \), defined by \( e^k_{\alpha} = \delta^k_{\alpha} + \eta^k_{\alpha} \), reproduce in an approximation linear in these variables the physical content of Einstein’s linearized theory”. This definition of equivalence is hard to accept.

We note that the tetrad formulation was introduced to couple fermionic fields to gravity because, as was shown by Cartan, one cannot couple them directly to the metric (see [5], [13] and references therein). Another motivation to use tetrads is to have more similarities with ordinary gauge theories than the metric formulation has [27]. Can these facts serve as an additional indication that, at least, in some sense the EH and TG Lagrangians are
different?

3. Now we would like to provide some additional and less general comments on particular results and conclusions (statements) of [1].

3.1. According to the authors of [1] (Sect. 8), their Eq. (17) “...are first order Lagrangian densities (first introduced by Einstein)...”.

First of all, we would like to mention that though Einstein, and not Palatini [28] (as is generally believed), was the first who introduced first order formalism using the affine-metric formulation [29], he never discussed the first order formulation of tetrad gravity (Riemannian space), Eq. (17) of [1], and was interested only in the theory of distant teleparallelism (which is an example of non-Riemannian space, see, e.g., Definition 236 of [23], Ref. 9 of [1]).

Secondly, the first order tetrad-spin connection formulation in 2D is not equivalent to the second order (tetrad) formulation [31]. This is because the equation of motion for spin connection in 2D does not result in (4). This is similar to the EH Lagrangian, where in 2D the first (affine-metric) and second order (metric) formulations are also not equivalent [32], [31] as the equation of motion for the affine connection does not yield the Christoffel symbol (App. A of [24]). Consequently, in 2D ω must be considered to be a dependent field, denoting the function of tetrads given by (4).

3.2. The sentences in the end of Sect. 8 and the beginning of Sect. 9 of [1] seem to contradict each other: “…the statement that in general coordinate chart \( L_g \neq 0 \) in a 2-dimensional spacetime is correct...” and in almost the next line “…let us first show that \( L_g = 0 \) in a 2-dimensional spacetime” and further “which implies also that corresponding \( L_\Gamma = 0 \)”. Let us comment on the last conjecture that from \( L_g = 0 \) follows \( L_\Gamma = 0 \).

We note that, in addition to the fact that the metric and tetrad formulations are different and do not have a very simple correspondence with each other, it is also true for PARTS of these different Lagrangians. In particular, the part \( L^o_g \) (Eq. (22)) and Eq. (23) of [1] are not equivalent. One way to demonstrate this is to use the definition of \( \omega \) in (1). Solving (1) for \( \Gamma \), we see that

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3Einstein used this formulation in 1928-1930 papers in his second version of the unified theory. In 1931 he wrote [30] “…we reached the conclusion that we were striving in the wrong direction...”. He abandoned this approach and never returned to it again.
\[ \Gamma^\alpha_{\beta \mu} = \omega_{\mu ab} e^{a \alpha} e^b_{\beta} + e_{a \beta, \mu} e^{a \alpha}, \]  

and then substituting this into the \( \Gamma \Gamma \)-part of the EH action

\[ L_{\Gamma \Gamma} = \sqrt{-g} g^{\alpha \beta} \left( \Gamma_{\sigma \lambda} \Gamma_{\alpha \beta} - \Gamma_{\sigma \beta} \Gamma_{\alpha \lambda} \right), \]  

one obtains

\[ L_{\Gamma \Gamma} (g (e), \Gamma (e, \omega)) = L_{\omega \omega} + L_{\text{extra}} \]  

with

\[ L_{\text{extra}} = e e^f \left( e^a_{[\beta} e_{\alpha_\lambda]} + e^{a \lambda} \omega^{b}_{[\lambda a b} e^b_{\alpha, \beta]} - e^{b}_{a \beta a e} e^{a \lambda} \right), \]  

where square brackets indicate antisymmetrization

\[ h_{\ldots [\beta ..., g_{\ldots \lambda]}...} = h_{\ldots \beta ..., g_{\ldots \lambda}...} - h_{\ldots \lambda ..., g_{\ldots \beta}...}. \]

This manipulation is valid in any dimension but, what is more interesting is the fact that \( L_{\text{extra}} \neq 0 \) even in 2D, making the conjecture that \( L_{\Gamma \Gamma} = L_{\omega \omega} \) in [1] to be incorrect. \( L_{\text{extra}} \) is absent in the total Lagrangian of TG because the substitution of (11) into the part of the EH Lagrangian containing derivatives of \( \Gamma \) gives

\[ \sqrt{-g} g^{\mu \nu} \left( \Gamma^\lambda_{\mu \nu, \lambda} - \Gamma^\lambda_{\mu \lambda, \nu} \right) = e e^a_{\mu e} c^{b \nu} (\omega^{v a b, \mu} - \omega_{\mu a b, \nu}) - L_{\text{extra}}. \]  

so, in the total Lagrangian \( L_{\text{extra}} \) cancels.

3.3. Equation (29) (Sect. 11 of [1]) is a total differential for the tetrad gravity, not for the EH Lagrangian (metric), and authors confirmed that “...(29) NEEDS the introduction of tetrad field to be written”. Does it contradict the result of [3] and the above mentioned statements of Deser and Jackiw [16] and Strobl [17]? Is it the proof that the EH (metric) Lagrangian is an exact differential in 2D, as stated in the title of [1], if one finds the introduction of tetrads unavoidable to write the final result?

3.4. In Sect. 14 of [1] the “divergence term” is calculated (Eq. (35-39)) by using the “powerful and economic” formalism. The resulting vector is again expressed in terms of tetrads, so, as in our previous comment, the introduction of tetrads is NEEDED again. Moreover, for the tetrad gravity the same result can be easily obtained directly (as we demonstrated, see Eq.
The necessity of using tetrads to construct an expression of a vector $V^\mu$ such that $L_{2D}(e) = V^\mu_{,\mu}$ was emphasized before in Ref. 11 of [16], where it was also stated that this vector "depends on the essentially non-metric part". Indeed, part of it cannot be expressed in terms of a metric field and hence, it is not possible to construct such a vector for the 2D EH (metric) Lagrangian.

3.5. The last paragraph in the end of Sect. 14 (which is a kind of “Conclusion”) is even difficult to call a conjecture. Here any reader of [1] can only guess about the relation of the 2D tetrad gravity to the Hamiltonian formulation of the EH Lagrangian considered in [3], and about even more mysterious connections with different 2D MODELS of [34], [35] (Refs. 5,6 of [1]) and, probably, with all the existing (e.g., [4]) and 2D models not yet devised. Even if, according to the authors of [1], there is no consistent Hamiltonian formulation for pure divergent Lagrangians, it is not clear why 2D models [4], [34], [35] with non-divergent Lagrangians also do not have a consistent Hamiltonian formulation.

The consistency of the Hamiltonian formulation is based on the closure of the Dirac procedure [36], [37], [38] (closure of the constraint algebra) as well as possibility of finding gauge transformations from a knowledge of the first class constraints. These transformations can be verified directly to be a symmetry of the Lagrangian. Such transformations leave the Lagrangian invariant, and follow from the consistent Hamiltonian treatment of the EH action in 2D.

It is not clear how the fact, that the TG Lagrangian is a total divergence in 2D implies that the Hamiltonian formulation of different 2D models is inconsistent as is claimed in [1]. Possibly the authors of [1] found some inconsistencies in the Hamiltonian formulation of 2D tetrad gravity, and again extrapolated this result onto the EH Lagrangian and all 2D models. In principle, one can try to find a Hamiltonian formulation even for models (such as 2D tetrad gravity) which are total divergences but, for example, consist of terms linear in second order derivatives and quadratic in first order derivatives. In such cases, one can try to treat these Lagrangians as higher order

\footnote{What can, probably, illustrate the power of the method used in [1] (or authors) is the ability in two days to obtain two absolutely different solutions (see V.1 on December 14 and V.2 on December 16 of [1]). However, the method of [1] does not seem very powerful compare with methods of the string theory that in twenty years reach much larger number ($10^{500}$) of possible solutions [33] that makes it the theory of “more than everything”.}
derivative theories, and apply the appropriate formulation for such models, due to Ostrogradsky [39], or its modification [38] for the singular (gauge invariant) cases.\footnote{The first attempt to consider Hamiltonian formulation of EH Lagrangian as higher order theory with a purpose to preserve general covariance is due to Dutt and Dresden [40].} For the 2D EH (metric) Lagrangian, the application of generalized Ostrogradsky method gives a consistent Hamiltonian formulation [41], which supports the result obtained in [3].

4. In conclusion we would like to emphasize that in order to discuss how the Einstein-Hilbert (metric) and the tetrad gravity Lagrangians are equivalent, one has to take into account the effect of all terms in these Lagrangians, including surface terms, because all terms are needed to preserve general covariance. In dimensions higher than two, we can avoid “the surface term problem” by using first order formulation, but this is not possible in 2D. However, generalization of the Hamiltonian formulation for higher order derivative Lagrangians or some other modifications (e.g., see p. 54 of [42] and references therein) may provide a means of doing so. Dirac himself conjectured [36], “I [Dirac] feel that there will always be something missing from them [non-Hamiltonian methods] which we can only get by working from a Hamiltonian”.

We also note that this equivalence cannot be discussed at the expense of giving up general covariance. For example, if in 2D we set \( e_0^{(1)} = e_1^{(0)} = 0 \), so as \( g_{01} = 0 \), and consider only subset of coordinate transformations that preserve this initial restrictions, these two formulations may be even become equivalent, because when \( g_{01} = 0 \) the Einstein-Hilbert (metric) Lagrangian in fact becomes a total divergence [3] and the number of independent fields (those which are left) are equal for both Lagrangians (\( g_{00} \) and \( g_{11} \) for the metric, \( e_0^{(0)} \) and \( e_1^{(1)} \) for the tetrads). There is one-to-one correspondence among them: \( g_{00} = \left( e_0^{(0)} \right)^2 \), \( g_{11} = - \left( e_1^{(1)} \right)^2 \) and \( L_{\text{extra}} \) (14) is zero. However, true equivalence between the tetrad and metric formulations cannot be established only by fixing some field variables and imposing a restriction on the general coordinate transformations.

We would like also to add that the question of equivalence between different formulations of gravity is not purely a 2D problem, but it is also related, for example, to the new variables [43] which, according to [44], are equivalent to tetrads. These new variables are exclusively used in attempts to quantize
GR and, in particular, they are variables of loop quantum gravity (for a recent review see [45]) which is now the best developed [46] proposal for the theory of quantum gravity. It is not evident that a formulation of gravity in terms of these variables is equivalent to a formulation in terms of the metric.

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References

[1] R. da Rocha and W.A. Rodrigues, Jr., hep-th/0512168 (version 2)
[2] R. Jackiw, Nucl. Phys. B252 (1985) 343
[3] N. Kiriushcheva and S.V. Kuzmin, hep-th/0510260
[4] D. Grumiller, W. Kummer and D.V. Vassilevich, Phys. Rep. 369 (2002) 327
[5] S. Deser and P. van Nieuwenhuizen, Phys. Rev. D10 (1974) 411
[6] L.P. Eisenhart, Riemannian Geometry (Princeton Univ. Press, 1925)
[7] L.D. Landau and E.M. Lifshitz, The Classical Theory of Fields, 4th edition (Pergamonn Press, 1975)
[8] R.M. Wald, General Relativity (Univ. Chicago Press, 1984)
[9] L. Castellani, P. van Nieuwenhuizen and M. Pilati, Phys. Rev. D26 (1982) 352
[10] J. Schwinger, Phys. Rev. 130 (1963) 1253
[11] M. Carmeli, *Classical Fields: General Relativity and Gauge Theory* (Wiley, New-York, 1982)

[12] S. Weinberg, *Gravitation and Cosmology* (Wiley, New-York, 1972)

[13] S. Deser and C.J. Isham, Phys. Rev. D14 (1976) 2505

[14] F.A.E. Pirani, A. Schild and R. Skinner, Phys. Rev. 87 (1952) 452

[15] P.A.M. Dirac, Proc. Royal Soc. London A246 (1958) 333

[16] S. Deser and R. Jackiw, Int. J. Mod. Phys. B10 (1996) 1499; hep-th/9510145

[17] T. Strobl, hep-th/0011240

[18] R. Arnowitt, S. Deser and C.W. Misner, The Dynamics of General Relativity, in: *Gravitation: An Introduction to Current Research*, ed. by L. Witten (Wiley, New York, 1962) 227; gr-qc/0405109

[19] E. Martinec, Phys. Rev. D30 (1984) 1198

[20] T. Regge and C. Teitelboim, Ann. Phys. 88 (1974) 286; V.O. Soloviev, Phys. Lett. B292 (1992) 30

[21] T. Padmanabhan, gr-qc/0409089

[22] A. Einstein, Sitzungsber. preuss. Akad. Wiss., phys.-math. KL, (1928) 217 and *The complete collection of scientific papers* (Nauka, Moscow, 1966), v.2, p.223 (English translations of this and a few more Einstein’s works on the theory of distant teleparallelism are available on http://www.lrz-muenchen.de/~aunzicker/ae1930.html)

[23] W.A. Rodrigues, Jr. and E.C. de Oliveira, *The many Faces of Maxwell, Dirac and Einstein Equations. A Clifford Bundle Approach*, http://www.ime.unicamp.br/rel-pesg/2005/rp56-05.html

[24] N. Kiriushcheva and S.V. Kuzmin, hep-th/0507074 (to appear in Ann. Phys.)

[25] N. Kiriushcheva, S.V. Kuzmin and D.G.C. McKeon, Int. J. Mod. Phys.A (to be published)
[26] F.A. Kaempffer, Phys. Rev. 165 (1968) 1420

[27] R. Utiyama, Phys. Rev. 101 (1956) 1597; T.W.B. Kibble, J. Math. Phys. 2 (1961) 212

[28] A. Palatini, Rendiconti del Circolo Matematico di Palermo, 43 (1919) 203 and in Cosmology and Gravitation, ed. by P.G. Bergmann and V. De Sabbata (Plenum press, New York, 1979), p.477; M. Ferraris, M. Francaviglia and C. Reina, Gen. Rel. and Grav. 14 (1982) 243

[29] A. Einstein, Sitzungsber. preuss.Akad.Wiss., phys.-math. K1, (1925) 414 and The complete collection of scientific papers (Nauka, Moscow, 1966), v.2, p.171 (see also www. in [22])

[30] A. Einstein, Science, 74 (1931) 438

[31] U. Lindström and M. Roček, Class. Quant. Grav. 4 (1987) L79

[32] J. Gegenberg, P.F. Kelly, R.B. Mann and D. Vincent, Phys. Rev. D37 (1988) 3463; S. Deser, J. McCarthy and Z. Yang, Phys. Lett. B222 (1989) 61; S. Deser, gr-qc/9512022

[33] G. Brumfiel, Nature 439 (2006) 10

[34] N. Kiriushcheva, S.V. Kuzmin and D.G.C. McKeon, Mod. Phys. Lett. A20 (2005) 1895

[35] N. Kiriushcheva, S.V. Kuzmin and D.G.C. McKeon, Mod. Phys. Lett. A20 (2005) 1961

[36] P.A.M. Dirac, Lectures on Quantum Mechanics (Yeshiva University, 1964)

[37] K. Sundermayer, Constrained Dynamics, Lecture Notes in Physics, 169 (Springer-Verlag, Berlin, 1982)

[38] D.M. Gitman and I.V. Tyutin, Quantization of Fields with Constraints (Springer-Verlag, Berlin, 1990)

[39] M. Ostrogradsky, Memoires de l’Academie Imperiale des Science de Saint-Petersbourg, IV (1850) 385
[40] S.K. Dutt and M. Dresden, Pure gravity as a constrained second-order system, Preprint ITP-SB-86-32, 1986

[41] N. Kiriushcheva and S.V. Kuzmin (in preparation)

[42] T. Thiemann, Lectures on Loop Quantum Gravity, in Quantum Gravity, ed. by D. Guilini, C. Kiefer and C. Lämmerzahl, Lecture Notes in Physics (Springer, Berlin, 2003) 41

[43] A. Ashtekar, Phys. Rev. Lett. 57 (1984) 2244; Phys. Rev. D36 (1987) 1587

[44] M. Henneaux, J.E. Nelson and C. Schomblond, Phys. Rev. D39 (1989) 434

[45] H. Nicolai, K. Peeters and M. Zamaklar, Class. Quant. Grav. 22 (2005) R193

[46] L. Smolin, Loop Quantum Gravity and Planck Scale Phenomenology, in Planck Scale Effects in Astrophysics and Cosmology, ed. by G. Amelino-Camelia, J. Kowalski-Glikman, Lecture Notes in Physics, 669 (Springer-Verlag, 2005) 363