Charmed baryon strong decays in a chiral quark model

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Charmed baryon strong decays are studied in a chiral quark model. The data for the decays of $\Lambda_c^+(2593)$, $\Lambda_c^+(2625)$, $\Sigma_c^{++}(2520)$ and $\Sigma_c^{++}(2520)$, are accounted for successfully, which allows to fix the pseudoscalar-meson-quark couplings in an effective chiral Lagrangian. Extending this framework to analyze the strong decays of the newly observed charmed baryons, we classify that both $\Lambda_c(2880)$ and $\Lambda_c(2940)$ are $D$-wave states in the $N = 2$ shell; $\Lambda_c(2880)$ could be $|\Lambda_c |^2 D_{\lambda \frac{\rho}{2}}^{++} \rangle$ and $\Lambda_c(2940)$ could be $|\Lambda_c |^2 D_{\lambda \frac{\rho}{2}}^{++} \rangle$. Our calculation also suggests that $\Lambda_c(2765)$ is very likely a $\rho$-mode $P$-wave excited state in the $N = 1$ shell, and favors a $|\Lambda_c \rho_1 \rho_1 \rangle$ configuration. The $\Sigma_c(2800)$ favors being a $|\Sigma_c \rho_1 \rho_1 \rangle$ state. But its being $|\Sigma_c \rho_1 \rho_1 \rangle$ cannot be ruled out.

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\section{I. INTRODUCTION}

In the past years, some new charmed baryons, such as $\Lambda_c(2880)$, $\Lambda_c(2940)$, $\Lambda_c(2765)$ and $\Sigma_c^{++}(2800)$, were observed in Belle, BaBar and CLEO \cite{1,2,3,4}. It initiated great interests in the heavy flavor baryon spectrum in both experiment and theory. At present, the experimental information is still limited and nearly nothing is known for their spin-parity quantum numbers (some review of the charmed baryons can be found in \cite{5,6,7,8}). How to understand the properties of these new charmed baryons, e.g. their structures, and interactions with known particles in their production and decay, and how to establish the charmed baryon spectroscopy, have been hot topics in both experiment and theory.

By studying the transitions of their different decay modes, one expects to extract information about their structures and the underlying dynamics. Several classes of models have been developed to deal with the strong decays of baryons. One is the hadrodynamic model, in which all hadrons are treated as point-like objects. The heavy hadron chiral perturbation theory (HHCChPT) approach \cite{9,10} belongs to this class. The second class of models treats the baryons as a three quark system, while the meson behaves as a point-like particle emitted from active quarks when the initial baryon decays. Typical models of this class were review in Refs. \cite{10,11}. The third class of models is the pair creation model, in which both the baryon and meson have internal structures. The decay of a hadron is recognized by the creation of a quark-antiquark pair from vacuum, which combine with the initial quarks to form meson and baryon in the final state. Typical ways of treating the pair creations include the $3P_0$ model \cite{6,12}, string-breaking model \cite{13,14}, and flux-tube breaking models \cite{15,16,17,18}. Detailed review of these phenomenologies can be found in Ref. \cite{19}. A large number of recent papers have been contributed to the determination of the quantum numbers of these newly observed states \cite{3,4,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37}. An extension of this approach to describe the process of $\pi N$ scattering also turns out to be successful and inspiring \cite{38}. In this framework, the charmed baryon spatial wave functions are described by harmonic oscillators. An effective chiral Lagrangian is then introduced to account for the quark-meson coupling. Since the quark-meson coupling is invariant under the chiral transformation, some of the low-energy properties of QCD are retained \cite{38,39,40}. This approach is similar to that used in \cite{10,11}, the only difference is that two constants in the decay amplitudes in Refs. \cite{10,11} are replaced by two energy-dependent factors deduced from the chiral Lagrangian in our model.

In this work, we will analyze the strong decays of the well determined charmed baryons, $\Lambda_c^+(2593)$, $\Lambda_c^+(2625)$, $\Sigma_c^{++}(2520)$ and $\Sigma_c^{++}(2520)$. Using the measurement of $\Sigma_c^{++}(2520)$ as an input, we then determine the only free parameter $\delta$ in our model, with which we calculate the strong decays of $\Lambda_c^+(2593)$, $\Lambda_c^+(2625)$, $\Sigma_c^{++}(2520)$ and $\Sigma_c^{++}(2520)$ as a prediction. By comparing with the data we can extract information about these states, in particular, about these structures and quantum numbers \cite{39}.

Finally, we analyze the strong decays of the new observed charmed baryons $\Lambda_c(2880)$, $\Lambda_c(2940)$, $\Lambda_c(2765)$ and $\Sigma_c^{++}(2800)$. We predict that both $\Lambda_c(2880)$ and $\Lambda_c(2940)$ are $D$-wave states in $N = 2$ shell. $\Lambda_c(2880)$ could be a $|\Lambda_c |^2 D_{\lambda \frac{\rho}{2}}^{++} \rangle$ state and $\Lambda_c(2940)$ could be a $|\Lambda_c |^2 D_{\lambda \frac{\rho}{2}}^{++} \rangle$ state. We suggest that $\Lambda_c(2765)$ is most likely a $\rho$-mode
$P$-wave excitations charmed baryon in $N = 1$ shell. The most possible state is $|\Lambda_c^{+}P_{\rho}^{1^-}\rangle$. The calculations indicate that $\Sigma_c(2800)$ favors a $|\Sigma_c^{+}P_{\lambda}^{1^-}\rangle$ state over the other ones for its broad width in experiment.

The paper is organized as follows. In the subsequent section, the charmed baryons in the quark model is outlined. Then, the non-relativistic quark-meson couplings are given in Sec. III. The decay amplitudes are deduced in Sec. IV. We present our calculations and discussions in Sec. V. Finally, a summary is given in Sec. VI.

II. CHARMED BARYONS IN THE QUARK MODEL

A. oscillator states

For a $udc$ basis state, it contains two light quarks 1 and 2 with equal mass $m$, and a heavy charmed quark 3 with mass $m'$. The basis states are generated by the Hamiltonian [40]

$$\mathcal{H} = \frac{1}{2m} (p_1^2 + p_2^2) + \frac{1}{2m'} p_3^2 + \frac{1}{2} K \sum_{i<j} (r_i - r_j)^2.$$  \hspace{1cm} (1)

In the above non-relativistic expansions, vectors $r_j$ and $p_j$ are the coordinate and momentum for the $j$-th quark in the baryon rest frame. The quarks are confined in an oscillator potential with the potential parameter $K$ independent of the flavor quantum number. One defines the Jacobi coordinates to eliminate the c.m. variables:

$$\bar{\rho} = \frac{1}{\sqrt{2}} (r_1 - r_2),$$  \hspace{1cm} (2)

$$\bar{\lambda} = \frac{1}{\sqrt{6}} (r_1 + r_2 - 2 r_3),$$  \hspace{1cm} (3)

$$R_{c.m.} = \frac{m(r_1 + r_2) + m' r_3}{2m + m'}.$$  \hspace{1cm} (4)

With the above relations (2–4), the oscillator Hamiltonian (1) is reduced to

$$\mathcal{H} = \frac{p_{cm}^2}{2M} + \frac{1}{2m_\rho} p_\rho^2 + \frac{1}{2m_\lambda} p_\lambda^2 + \frac{3}{2} K (\rho^2 + \lambda^2).$$  \hspace{1cm} (5)

where

$$p_\rho = m_\rho \bar{\rho}, \quad p_\lambda = m_\lambda \bar{\lambda}, \quad p_{c.m.} = M \dot{R}_{c.m.},$$  \hspace{1cm} (6)

with

$$M = 2m + m', \quad m_\rho = m, \quad m_\lambda = \frac{3mm'}{2m + m'}.$$  \hspace{1cm} (7)

With Eqs. (2) and (3), the coordinate $r_j$ can be translated into functions of the Jacobi coordinates $\lambda$ and $\rho$:

$$r_1 = R_{c.m.} + \frac{3m'}{\sqrt{6}} \frac{2m + m'}{3m + m'} \bar{\lambda} + \frac{1}{\sqrt{2}} \bar{\rho},$$  \hspace{1cm} (8)

$$r_2 = R_{c.m.} + \frac{3m'}{\sqrt{6}} \frac{2m + m'}{3m + m'} \bar{\lambda} - \frac{1}{\sqrt{2}} \bar{\rho},$$  \hspace{1cm} (9)

$$r_3 = R_{c.m.} - \sqrt{\frac{2}{3}} \frac{3m}{2m + m'} \bar{\lambda},$$  \hspace{1cm} (10)

and the momentum $p_j$ is given by

$$p_1 = \frac{m}{M} p_{c.m.} + \frac{1}{\sqrt{6}} p_\lambda + \frac{1}{\sqrt{2}} p_\rho,$$  \hspace{1cm} (11)

$$p_2 = \frac{m}{M} p_{c.m.} + \frac{1}{\sqrt{6}} p_\lambda - \frac{1}{\sqrt{2}} p_\rho,$$  \hspace{1cm} (12)

$$p_3 = \frac{m'}{M} p_{c.m.} - \sqrt{\frac{2}{3}} p_\lambda.$$  \hspace{1cm} (13)
The spatial wave function is a product of the $\rho$-oscillator and the $\lambda$-oscillator states. With the standard notation, the principal quantum numbers of the $\rho$-oscillator and $\lambda$-oscillator are $N_\rho = (2n_\rho + l_\rho)$ and $N_\lambda = (2n_\lambda + l_\lambda)$, and the energy of a state is given by

$$E_N = (N_\rho + \frac{3}{2})\omega_\rho + (N_\lambda + \frac{3}{2})\omega_\lambda.$$  \hfill (14)

The total principal quantum number (i.e. shell number) $N$ is defined as

$$N = N_\rho + N_\lambda,$$  \hfill (15)

and the frequencies of the $\rho$-mode and $\lambda$-mode are

$$\omega_\rho = (3K/m_\rho)^{1/2}, \quad \omega_\lambda = (3K/m_\lambda)^{1/2}.$$  \hfill (16)

In the quark model two useful oscillator parameters, i.e. the potential strengths, are defined by

$$\alpha_\rho = (m_\rho \omega_\rho)^{1/2}, \quad \alpha_\lambda = (m_\lambda \omega_\lambda)^{1/2}.$$  \hfill (17)

Combining Eqs. (17) and (16) with (17), we obtain the relation between these two parameters:

$$\alpha_\lambda^2 = \sqrt{\frac{3m_\rho}{2m + m_\rho}}\alpha_\rho^2.$$  \hfill (18)

Then, the wave function of an oscillator is given by

$$\psi^{n_\rho l_\rho}_{l_\lambda m}(\sigma) = R_{n_\rho l_\rho}(\sigma)Y_{l_\lambda m}(\sigma),$$  \hfill (19)

where $\sigma = \rho, \lambda$. The total orbital angular momentum $L$ of a state is obtained by coupling $l_\rho$ to $l_\lambda$:

$$L = l_\rho + l_\lambda.$$  \hfill (20)

The total spatial wave function can then be constructed. All the functions with principal quantum number $N \leq 2$ are listed in Tab. I.

### B. Flavor and spin wave functions

For the $udc$ basis states which violate $SU(4)$ symmetry, as done in Ref. [11], we introduce

$$\phi_{\Lambda_c} = \frac{1}{\sqrt{2}}(ud - du)c,$$  \hfill (21)

and

$$\phi_{\Sigma_c} = \begin{cases} ddc & \text{for } \Sigma_c^0, \\ \frac{1}{\sqrt{2}}(ud + du)c & \text{for } \Sigma_c^+, \\ uuc & \text{for } \Sigma_c^{++}, \end{cases}$$  \hfill (22)

for the $\Lambda_c$- and $\Sigma_c$-type flavor wave functions, respectively.

For the spin wave functions the usual ones are adopted [10, 11]:

$$\chi^{3/2}_+ = \uparrow\uparrow\uparrow, \quad \chi^{3/2}_- = \downarrow\downarrow\downarrow,$$

$$\chi^{1/2}_+ = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow),$$

$$\chi^{1/2}_- = \frac{1}{\sqrt{3}}(\downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow + \uparrow\downarrow\uparrow),$$  \hfill (23)

for the spin-3/2 states;

$$\chi^{1/2}_+ = \frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow),$$

$$\chi^{1/2}_- = \frac{1}{\sqrt{2}}(\downarrow\uparrow\downarrow - \uparrow\downarrow\uparrow),$$  \hfill (24)
for the spin-1/2 states, in which the first two quark spins are antisymmetric; and

\[ \chi_{1/2}^\Lambda = -\frac{1}{\sqrt{6}}(1\uparrow1 + 1\uparrow1 - 2\uparrow\downarrow), \]

\[ \chi_{1/2}^{\bar{\Lambda}} = -\frac{1}{\sqrt{6}}(1\downarrow1 + 1\downarrow1 - 2\downarrow\uparrow), \]

for the spin-1/2 states, in which the first two quark spins are symmetric.

C. The total wave functions

The spin-flavor and spatial wave functions of baryons must be symmetric since the color wave function is antisymmetric. The flavor wave functions of the \( \Lambda_c \)-type charmed baryons, \( \phi_{\Lambda_c} \), are antisymmetric under the interchange of the \( u \) and \( d \) quarks, thus, their spin-space wave functions must be symmetric. In contrast, the spin-space wave functions of \( \Sigma_c \)-type charmed baryons are required to be antisymmetric due to their symmetric flavor wave functions under the interchange of the \( u \) and \( d \) quarks. The wave functions of the \( \Lambda_c \)-type and \( \Sigma_c \)-type charmed baryons are listed in Tabs. II and III respectively.

III. THE QUARK-MESON COUPLINGS

In the chiral quark model, the low energy quark-meson interactions are described by an effective Lagrangian \[ 33, 35 \]

\[ \mathcal{L} = \bar{\psi} \left[ \gamma_\mu (i\partial^\mu + V^\mu + \gamma_5 A^\mu) - m \right] \psi + \cdots, \]

(26)

where \( V^\mu \) and \( A^\mu \) correspond to vector and axial currents, respectively. They are given by

\[ V^\mu = \frac{1}{2} (\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi), \]

\[ A^\mu = \frac{1}{2i} (\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi), \]

(27)

with \( \xi = \exp (i\phi_m/f_m) \), where \( f_m \) is the meson decay constant. In the flavor \( SU(3) \) sector, the pseudoscalar-meson octet \( \phi_m \) can be expressed as

\[ \phi_m = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ \pi^- \\ -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ K^- \\ K^0 \\ -\sqrt{\frac{2}{3}} \eta \end{pmatrix}, \]

(28)

and the quark field \( \psi \) is given by

\[ \psi = \begin{pmatrix} \psi(u) \\ \psi(d) \\ \psi(s) \end{pmatrix}. \]

(29)

The tree-level quark-meson pseudovector coupling is thus given by

\[ H_m = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu \gamma_5 \psi_j \partial^\mu \phi_m. \]

(30)

where \( \psi_j \) represents the \( j \)-th quark field in a baryon. This effective quark-meson pseudovector coupling can be used for \( D \)-mesons as well, if we extend the \( SU(3) \) case to the \( SU(4) \) case.

In the quark model, the non-relativistic form of Eq. (30) is written as \[ 33, 35, 38 \]

\[ H_{m}^{nr} = \sum_j \left\{ \frac{\omega_m}{E_f + M_f} \sigma_j \cdot P_f + \frac{\omega_m}{E_i + M_i} \sigma_j \cdot P_i - \sigma_j \cdot q + \frac{\omega_m}{2\mu_q} \sigma_j \cdot P_j \right\} I_j \psi_m, \]

(31)
where \( \sigma_j \) and \( \mu_q \) correspond to the Pauli spin vector and the reduced mass of the \( j \)-th quark in the initial and final baryons, respectively. For emitting a meson, we have \( \varphi_m = e^{-i\mathbf{q} \cdot \mathbf{r}_f} \), and for absorbing a meson we have \( \varphi_m = e^{i\mathbf{q} \cdot \mathbf{r}_i} \).

In the above non-relativistic expansions, \( \mathbf{p}'_j = \mathbf{p}_j - (m_j/M)\mathbf{P}_{c.m.} \) is the internal momentum for the \( j \)-th quark in the baryon rest frame. \( \omega_m \) and \( \mathbf{q} \) are the energy and three-vector momentum of the meson, respectively. The isospin operator \( I_j \) in Eq. (31) is expressed as

\[
I_j = \begin{cases} 
  a_j^\dagger(u)a_j(c) & \text{for } D^0 \\
  a_j^\dagger(u)a_j(d) & \text{for } \pi^- \\
  a_j^\dagger(d)a_j(u) & \text{for } \pi^+ \\
  \frac{1}{\sqrt{2}}[a_j^\dagger(u)a_j(u) - a_j^\dagger(d)a_j(d)] & \text{for } \pi^0 
\end{cases} \tag{32}
\]

where \( a_j^\dagger(u,d,c) \) and \( a_j(u,d,c) \) are the creation and annihilation operators for the \( u, d \) and \( c \) quarks.

**IV. THE DECAY OF CHARMED BARYON IN THE QUARK MODEL**

In the calculations, we select the initial-baryon-rest system for the decay processes. The energies and momenta of the initial charmed baryons are denoted by \( (E_i, \mathbf{P}_i) \), while those of the final state mesons and baryons are denoted by \( (\omega_f, \mathbf{q}) \) and \( (E_f, \mathbf{P}_f) \). Note that \( \mathbf{P}_i = 0 \) (\( E_i = M_i \)) and \( \mathbf{P}_f = -\mathbf{q} \).

**A. \( B_c \to B'_c\pi(q) \)**

Because the \( \pi \)-meson only couples to the light quark 1 or 2 in a \( udc \) basis state, the strong decay amplitudes for the process \( B_c \to B'_c\pi(q) \) can be written as

\[
\mathcal{M}[B_c \to B'_c\pi(q)] = 2 \langle B'_c|\{G\sigma_1 \cdot \mathbf{q} + h\sigma_1 \cdot \mathbf{p}'_1\}I_1 e^{-i\mathbf{q} \cdot \mathbf{r}_1}|B_c\rangle, \tag{33}
\]

with

\[
G \equiv -\frac{\omega_\pi}{E_f + M_f} - 1, \quad h \equiv \frac{\omega_\pi}{m}. \tag{34}
\]

where \( B_c \) and \( B'_c \) stand for the initial and final charmed baryon wave functions, which are listed in Tabs. III and III. Similar expressions were also derived in Refs. [10, 11]. By selecting \( q = q\hat{z} \), namely the meson moves along the \( z \) axial, we can simplify the amplitude to

\[
\mathcal{M}[B_c \to B'_c\pi(q)] = 2 \left\{ Gq - \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{3}}q_\lambda + q_\rho \right) h \right\} \langle B'_c|\sigma_1z\phi I_1|B_c\rangle \\
- i\sqrt{2}h\langle B'_c|\sigma_1 \cdot \vec{\nabla}_\lambda - \alpha_\rho^2\sigma_1 \cdot \vec{\nabla}_\rho \phi I_1|B_c\rangle.
\]

where \( \vec{\nabla}_\lambda \) and \( \vec{\nabla}_\rho \) are the derivative operators on the spatial wave function of the final baryon except the factor \( \exp[(-\alpha_\lambda^2q^2 - \alpha_\rho^2q^2)/2] \) which has been worked out, and

\[
q_\lambda = \frac{1}{\sqrt{6}}\frac{3m'}{2m + m'}q, \quad q_\rho = \frac{1}{\sqrt{2}}q. \tag{36}
\]

and

\[
\phi = \exp(-iq_\lambda \lambda_z)\exp(-iq_\rho \rho_z), \tag{37}
\]

In Eq. (33), the first term comes from the c.m. motion of the system, while the last two terms attribute to the \( \lambda \)- and \( \rho \)-mode orbital excitations of the charmed baryons, respectively.
For example, we calculate the decay process $|\Lambda_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Sigma_c \pi$. The initial and final charmed baryon wave functions are given by (see Tab. III)

$$|B_c\rangle = \left[ \sqrt{\frac{1}{3}} \Psi_{11} \chi_{-1/2}^\rho + \sqrt{\frac{2}{3}} \Psi_{10} \chi_{1/2}^\rho \right] \phi_{\Lambda_c},$$

$$|B'_c\rangle = \Psi_{00} \chi_{1/2}^\rho \phi_{\Sigma_c}. \quad (38)$$

Substituting into Eq. (35), we obtain the decay amplitude

$$\mathcal{M} = ig_1 g_I \left\{ \sqrt{\frac{2}{3}} \left[ Gq - \frac{h}{2\sqrt{3}} \left( \frac{1}{\sqrt{3}} \sigma_\lambda + q_\rho \right) \right] \frac{q_\lambda}{\alpha_\lambda} 
+ h\alpha_\lambda \right\} F(q_\lambda, q_\rho), \quad (40)$$

where the spin and isospin factors are

$$g_1 = \langle \chi_{1/2}^\rho | \sigma_{1z} | \chi_{1/2}^{\rho'} \rangle, \quad (41)$$

and

$$g_I = \langle \phi_{\Sigma_c} | \sigma_{1z} | \phi_{\Lambda_c} \rangle. \quad (42)$$

The spatial integral gives

$$F(q_\lambda, q_\rho) = \exp \left( -\frac{q_\lambda^2}{4\alpha_\lambda^2} - \frac{q_\rho^2}{4\alpha_\rho^2} \right), \quad (43)$$

which plays a role of form factor.

The corresponding spin factors are listed in Tab. [IV]. Some of the decay amplitudes for $|\Lambda_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Sigma_c \pi$, $|\Lambda_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Sigma_c(2520) \pi$, $|\Sigma_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Lambda_c \pi$, $|\Sigma_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Lambda_c \pi$ and $|\Sigma_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Sigma_c(2520) \pi$ are listed in Tabs. [V] [VI] [VII] [VIII] and [IX] respectively.

**B. $B_c \rightarrow D(q)p$**

For a charmed baryon decaying into $Dp$, since the $D$-meson only couples to the charm quark 3 in a $udc$ basis state, the strong decay amplitudes for the process $B_c \rightarrow D(q)p$ can be written as

$$\mathcal{M}[B_c \rightarrow D(q)p] = \langle p \mid \{G\sigma_3 \cdot q + h\sigma_3 \cdot \mathbf{p}'\} I_3 e^{-i\mathbf{q} \cdot \mathbf{r}_3} \mid B_c \rangle, \quad (44)$$

where the wave function of a proton in the quark model is expressed as

$$|p\rangle = \frac{1}{\sqrt{2}} (\Phi_\rho \chi^\rho + \Phi_\lambda \chi^\lambda) \Psi_{00}^S,$$ \quad (45)

with

$$\Phi_\rho = \frac{1}{\sqrt{2}} (ud - du)u, \quad (46)$$

$$\Phi_\lambda = -\frac{1}{\sqrt{6}} (udu + duu - 2uud). \quad (47)$$

We can also simplify the amplitude (44) to

$$\mathcal{M}[B_c \rightarrow D(q)p] = \left[ Gq - \sqrt{\frac{2}{3}} q_\lambda h \right] (p|\sigma_3 \cdot q_3 | B_c) 
+ i \sqrt{\frac{2}{3}} h (p|\sigma_3 \cdot \nabla_\lambda - \alpha_\lambda \sigma_3 \cdot \nabla_\lambda \phi' I_3 | B_c), \quad (48)$$

Substituting into Eq. (35), we obtain the decay amplitude

$$M = ig_1 g_I \left\{ \sqrt{\frac{2}{3}} \left[ Gq - \frac{h}{2\sqrt{3}} \left( \frac{1}{\sqrt{3}} \sigma_\lambda + q_\rho \right) \right] \frac{q_\lambda}{\alpha_\lambda} 
+ h\alpha_\lambda \right\} F(q_\lambda, q_\rho), \quad (40)$$

where the spin and isospin factors are

$$g_1 = \langle \chi_{1/2}^\rho | \sigma_{1z} | \chi_{1/2}^{\rho'} \rangle, \quad (41)$$

and

$$g_I = \langle \phi_{\Sigma_c} | \sigma_{1z} | \phi_{\Lambda_c} \rangle. \quad (42)$$

The spatial integral gives

$$F(q_\lambda, q_\rho) = \exp \left( -\frac{q_\lambda^2}{4\alpha_\lambda^2} - \frac{q_\rho^2}{4\alpha_\rho^2} \right), \quad (43)$$

which plays a role of form factor.

The corresponding spin factors are listed in Tab. [IV]. Some of the decay amplitudes for $|\Lambda_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Sigma_c \pi$, $|\Lambda_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Sigma_c(2520) \pi$, $|\Sigma_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Lambda_c \pi$, $|\Sigma_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Lambda_c \pi$ and $|\Sigma_c^{-} P_{\frac{3}{2}}^{1-}\rangle \rightarrow \Sigma_c(2520) \pi$ are listed in Tabs. [V] [VI] [VII] [VIII] and [IX] respectively.
with
\[ G = -\frac{\omega D}{E_f + M_f} - 1, \quad h = \frac{\omega D (m + m')}{2m' m}, \]

(49)

and
\[ q'_\lambda = \frac{2}{\sqrt{6}} \frac{3m}{2m + m'} q, \quad \phi' = \exp(iq'_\lambda \lambda_c). \]

(50)

In Eq. 48, the first term comes from the c.m. motion of the system, the last term attributes to the λ-mode orbital excitations of the charmed baryons, respectively.

There exist selection rules for the \( D^0 p \) decay channel of \( \Lambda_c \) excitations, in which only \(|\Lambda_c^{-2D_{\lambda\lambda}}_{\lambda+}^\pi\), \(|\Lambda_c^{-2D_{\lambda\lambda}}_{\lambda+}^\rho\) and \(|\Lambda_c^{-2S_{\lambda\lambda}}_{\lambda+}^\pi\) can decay into \( D^0 p \). Their decay amplitudes are listed in Tab. XI. States of \(|\Lambda_c^{-2P_{\lambda\lambda}}_{\lambda-}^\pi\) and \(|\Lambda_c^{-2P_{\lambda\lambda}}_{\lambda-}^\rho\) are likely below the \( D^0 p \) threshold, while others are forbidden by the spin-isospin selection rule.

V. CALCULATION AND ANALYSIS

With the resonance decay amplitudes, one can calculate the width
\[ \Gamma = \left( \frac{\delta}{f_m} \right)^2 \frac{(E_f + M_f)|q|}{\pi M_i} \sum_{J_z J_{f_z}} |M_{J_z, J_{f_z}}|^2, \]

(51)

where \( J_i \) and \( J_f \) are the total angular momenta of the initial and final baryons, respectively. A dimensionless constant, \( \delta \), is introduced to take into account uncertainties arising from the model and to be determined by experimental data. In the calculation, the standard parameters of the quark model are adopted. For the oscillator parameters, we use \( \alpha^2 = 0.16 \text{ GeV}^2 \). The \( u, d \) constituent quark masses are \( m = 350 \text{ MeV} \), and the charm quark mass is \( m' = 1700 \text{ MeV} \).

The decay constants for \( \pi^- \) and \( D \)-mesons are \( f_\pi = 132 \text{ MeV} \) and \( f_D = 226 \text{ MeV} \), which are taken from the Particle Data Group (PDG) [39]. All the charmed baryon masses are also adopted from the PDG [39].

A. \( \Sigma_c \) and \( \Sigma_c(2520) \)

\( \Sigma_c \) and \( \Sigma_c(2520) \) are the two lowest states in the \( \Sigma_c \)-type charmed baryons. They are assigned to the two \( S \)-wave states, \(|\Sigma_c^{-2S_{\lambda\lambda}}_{\lambda+}^\pi\) and \(|\Sigma_c^{-4S_{\lambda\lambda}}_{\lambda+}^\pi\), respectively [11, 39]. We use the measured width for \( \Sigma_c^{-+}(2520) \rightarrow \Lambda_c^{-+} \pi^+ \) as an input (i.e. \( \Gamma = 14.9 \text{ MeV} \)) to determine parameter \( \delta \) in Eq. (51), which gives
\[ \delta = 0.557. \]

(52)

Applying this value for \( \delta \), we can predict the other strong decay widths. In particular, the decay widths of \( \Sigma_c \rightarrow \Lambda_c \pi \), \( \Sigma_c^{-+}(2520) \rightarrow \Lambda_c^+ \pi^+ \) and \( \Sigma_c^0(2520) \rightarrow \Lambda_c^0 \pi^- \) are calculated. The results are listed in Tab. XI from which we find that our predictions are in a good agreement with the experimental data [39], and compatible with other theoretical predictions [5, 6, 23, 26, 27]. We also see that the decay width of \( \Sigma_c(2520) \) is larger than that of \( \Sigma_c \) by a factor of \( \sim 7 \) though their decay amplitudes have the same form (see Tab. VII). The reasons are due to: i) the spin factor \( g_{\Sigma}^\Sigma \) for \( \Sigma_c(2520) \) is larger than \( g_{\Sigma}^\Sigma \) for \( \Sigma_c \) by a factor \( \sqrt{2} \); ii) the three-momentum of the pion, \(|q|\) in the \( \Sigma_c(2520) \rightarrow \Lambda_c \pi \) are about two times larger than that in the \( \Sigma_c \rightarrow \Lambda_c \pi \). It leads to larger values for quantities \( G \) and \( h \). This feature was also mentioned in Ref. [5].

B. \( \Lambda_c(2593) \) and \( \Lambda_c(2625) \)

\( \Lambda_c(2593) \) and \( \Lambda_c(2625) \) have \( J^P = 1/2^- \) and \( J^P = 3/2^- \), respectively, and can be naturally assigned to \( N = 1 \) shell with one unit of orbital angular momentum excitation. They can be excited via either \( P_\lambda \)-mode or \( P_\rho \)-mode. For the former assignment, their spatial wave functions are \(|\Lambda_c^{-2P_{\lambda\lambda}}_{\lambda+}^\pi\) and \(|\Lambda_c^{-2P_{\lambda\lambda}}_{\lambda+}^\rho\), from which the decay widths can be calculated. As shown in Tab. XI the results are in a good agreement with the experimental data [39] and consistent with the classification of Ref. [11] in the quark model.
Assuming \( \Lambda_c(2593) \) and \( \Lambda_c(2625) \) are \( P_\rho \)-mode excitations, we also calculate their widths. In contrast with the \( P_\rho \)-mode, they turn out to be much broader than the \( P_\lambda \)-mode. For \( \Lambda_c(2593) \) it is possible that the physical state is a mixture of the \( P_\lambda \) and \( P_\rho \)-modes within the uncertainties of the present data though the determination of the mixing angle will also rely on the mass of the second state.

For \( \Lambda_c(2625) \) the \( P_\rho \)-mode excitation turns to overestimate the data significantly. The experimental upper limit is about two orders of magnitude smaller than the predictions from the \( P_\rho \)-mode excitation, while the \( P_\lambda \)-mode results are consistent with the data. This could be a sign that the mixing between the \( P_\lambda \) and \( P_\rho \)-mode in \( \Lambda_c(2625) \) should be small. Concerning the possible mixings between the \( P_\lambda \) and \( P_\rho \)-mode excitations, the search for the second heavier \( 1/2^- \) and \( 3/2^- \) states in experiment should be interesting.

Comparing \( \Lambda_c(2593) \) with \( \Lambda_c(2625) \), it shows that the decay width of \( \Lambda_c(2593) \) is much narrower than that of \( \Lambda_c(2625) \), which can be well understood in our model. In the decay amplitude of \( \Lambda_c(2593) \rightarrow \Sigma_c \pi \) (see Tab. X), only c.m. motion contributions are present, which leads to the small decay width. We should also emphasize that the partial decay width of \( \Lambda_c(2593) \rightarrow \Sigma_c \pi \) is sensitive to the mass of \( \pi \)-meson due to its mass close to the \( \Sigma_c \pi \) threshold. It leads to the decay width of \( \Sigma_c \pi^0 \) channel is about two times larger than those of \( \Sigma_c \pi^\pm \). Interestingly, experimental data for \( \Lambda_c(2593) \) and \( \Lambda_c(2625) \rightarrow \Sigma_c^+ \pi^0 \) are still not available.

Since the well-determined \( S \)- and \( P \)-wave charmed baryon strong decay widths are successfully described in our chiral quark model, we extend this approach in the next subsections to investigate the strong decays of other newly observed charmed baryons, such as \( \Lambda_c(2880) \) and \( \Lambda_c(2940) \).

\[ \Lambda_c(2880) \]

\( \Lambda_c(2880) \) was observed in \( \Lambda_c^+ \pi^- \) by CLEO \[4\], in \( D^0 p \) channel by BaBar \[1\], and in \( \Sigma_c \pi, \Sigma_c(2520) \pi \) by Belle \[2\]. It has a narrow decay width less than 8 MeV \[2, 39\], based on which it was proposed to be a \( \Lambda_c(2880) \) \[3\] state in Ref. 3. In the heavy hadron chiral perturbation theory, Cheng et al. made a conjecture that \( \Lambda_c(2880) \) is an admixture of \( \Lambda_c^+ \pi^- \) which are both \( L = 2 \) orbitally excited states. Chen et al. suggested that \( \Lambda_c(2880) \) favors \( \Lambda_c^+ \pi^- \) within the \( ^3P_1 \) model \[6\]. According to the quark model predictions, the mass for \( J^P = 3/2^+ \) is around 2.9 GeV, which indicates \( \Lambda_c(2880) \) maybe favor \( J^P = 3/2^+ \) as well \[21, 41\]. The other suggestions about its quantum numbers also can be found in Ref. \[24\].

Meanwhile, the Belle measurement \[2\] shows contributions from intermediate \( \Sigma_c^+ \) states in \( \Lambda_c^+ \) \( \rightarrow \Sigma_c^+ \pi \rightarrow \Lambda_c^+ \pi^+ \pi^- \), and the ratio of the partial decay widths for the intermediate \( \Sigma_c^+(2520) \) and \( \Sigma_c^+ \) is extracted:

\[
\mathcal{R} = \frac{\Gamma(\Sigma_c(2520) \pi)}{\Gamma(\Sigma_c^+ \pi)} = 0.225 \pm 0.062 \pm 0.025. \tag{53}
\]

With the analysis of the angular distributions in \( \Lambda_c(2880) \rightarrow \Sigma_c^0 \pi^+ \pi^- \) decays, the \( \Lambda_c(2880) \) spin-parity assignment is favored to be \( J^P = 5/2^+ \) over the others.

In the quark model the masses of \( N = 1 \) shell \( \Lambda_c \) excitations are at the order of 2.5-2.6 GeV, which is much less than 2.88 GeV. We hence only consider the possible assignment of \( \Lambda_c(2880) \) in the \( N = 2 \) shell. As shown by Tab. XII only \( |\Lambda_c^2 D_{\lambda \lambda} \frac{1}{2}^{++} \rangle \) can produce results that fit in the three experimental observations: i) with a narrow decay width; ii) decaying into \( D^0 p \); iii) and with the ratio \( \mathcal{R} = \Gamma(\Sigma_c(2520) \pi)/\Gamma(\Sigma_c^+ \pi) \sim 0.25 \). This is an orbital excitation with \( l_\lambda = 2 \) and \( l_\rho = 0 \). Note that the Capstick-Isgur quark model \[41\] and the relativistic quark model \[21\] predict the lowest \( J^P = 3/2^+ \) \( \Lambda_c \) excitation at 2910 MeV and 2874 MeV, respectively, which are consistent with the experimental value within the model accuracies. In this sense the assignment of \( \Lambda_c(2880) \) as \( |\Lambda_c^2 D_{\lambda \lambda} \frac{5}{2}^{++} \rangle \) turns to be possible.

Interestingly, the experimental analysis of the decay angular distribution \[2\] indicates a preference of \( J^P = 5/2^+ \) over \( 3/2^+ \) at a level of more than 4.5 standard deviations. By assigning 5/2+ to the \( \Lambda_c(2880) \), we find that only the state \( |\Lambda_c^2 D_{\lambda \lambda} \frac{5}{2}^{++} \rangle \) is close to the experimental measurements. However, its \( D^0 p \) decay channel is forbidden and the ratio \( \mathcal{R} = 0.5 \) turns to be too large compared with the Belle data. This controversy may suggest that \( \Lambda_c(2880) \) is neither a pure \( |\Lambda_c^2 D_{\lambda \lambda} \frac{5}{2}^{++} \rangle \) nor \( |\Lambda_c^2 D_{\lambda \lambda} \frac{5}{2}^{++} \rangle \). We expect that more accurate measurements of the decay angular distributions will clarify its nature. In contrast, the calculations of Refs. \[3, 6\] seem to agree with the data. The details of our model calculations are listed in Tab. XII.

\[ \Lambda_c(2940) \]

\( \Lambda_c(2940) \) was first seen in its decay into \( D^0 p \) by BaBar \[1\], and then confirmed by Belle in \( \Sigma_c^0 \pi^+ \pi^- \) \[2\]. Its spin-parity has not yet been determined. In this mass region, it can be \( J^P = 5/2^+ \), \( J^P = 3/2^+ \), \( J^P = 1/2^+ \) or...
$J^P = 5/2^-$ as suggested by the quark model \cite{41}. The $^3P_0$ model \cite{42} suggests that its configuration favors $\bar{\Lambda}^\nu_1(3^+)$ or $\bar{\Lambda}^\nu_0(1^+)$, while a molecular state with $J^P = 1/2^-$ is also proposed \cite{42}.

In our analysis it shows that only three states, $|\Lambda_c^+ 2D\lambda_\frac{1}{2}^+\rangle$, $|\Lambda_c^+ 2D\lambda_\frac{3}{2}^+\rangle$ and $|\Lambda_c^- 2S\lambda_\frac{1}{2}^+\rangle$, can decay into $D^0p$. In case that we have assigned $\Lambda_c(2880)$ to be $|\Lambda_c^+ 2D\lambda_\frac{3}{2}^+\rangle$, the $\Lambda_c(2940)$ could thus be either $|\Lambda_c^+ 2D\lambda_\frac{1}{2}^+\rangle$ or $|\Lambda_c^- 2S\lambda_\frac{1}{2}^+\rangle$. In the quark model, the mass of $|\Lambda_c^+ 2S\lambda_\frac{1}{2}^+\rangle$ should be less than that of $|\Lambda_c^- 2D\lambda_\frac{3}{2}^+\rangle$ [i.e. $\Lambda_c(2880)$]. This leaves $\Lambda_c(2940)$ to be assigned as $|\Lambda_c^+ 2D\lambda_\frac{3}{2}^+\rangle$.

In Tab. \textbf{XIII} the calculation results are listed. The vanishing $D^0p$ channel will eliminate most of those states, especially, with anti-symmetric spatial wavefunctions and mixed $\rho$-$\rho$-type. The states which have nonvanishing decays into $\Sigma_c\pi$, $\Sigma_c(2520)\pi$ and $D^0p$ are $|\Lambda_c^+ 2D\lambda_\frac{1}{2}^+\rangle$ and $|\Lambda_c^+ 2D\lambda_\frac{3}{2}^+\rangle$. Based on the argument made in the last paragraph, we see that it is natural to assign the $\Lambda_c(2940)$ as $|\Lambda_c^+ 2D\lambda_\frac{3}{2}^+\rangle$. Note that the Capstick-Isgur quark model predicts the lowest $J^P = 5/2^+$ state at 2910 MeV \cite{41}, which will enhance the above assignment.

It should be noted that there are no $\Lambda_c$ excitation states around 2940 MeV in the relativistic quark model predictions \cite{21}. There, $\Lambda_c(2940)$ was assigned to be the first radial excited state with $J^P = 3/2^+$, of which the predicted mass was slightly below the experimental value. As the decay of the radial excited state into the $D^0p$ channel is forbidden in the non-relativistic limit, more elaborate estimate of the relativistic corrections should be necessary.

\section{E. $\Lambda_c(2765)$}

Experimental information about the $\Lambda_c(2765)$ is much poorer than $\Lambda_c(2880)$ and $\Lambda_c(2940)$. Thus, we leave it to be discussed as the last $\Lambda_c$ excitation state.

$\Lambda_c(2765)$ was first observed in $\Lambda_c\pi\pi$ by CLEO Collaboration \cite{4,39} with a rather broad width of about 50 MeV, and appeared to resonate through $\Sigma_c\pi$ and probably also $\Sigma_c(2520)\pi$. At Belle, its broad structure stands out clearly in the $\Lambda_c\pi\pi$ invariant mass spectrum \cite{2}. However, almost nothing about its quantum numbers is known, including whether it is a $\Lambda_c$ or a $\Sigma_c$ excitation. Cheng et al. suggest that $\Lambda_c(2765)$ could be the first excited state of $\Lambda_c$ with positive-parity according to the predictions of Skyrme model \cite{43} and the quark model \cite{41}. It was also proposed that the $\Lambda_c(2765)$ could be either the first radial $(1S)$ excitation of the $\Lambda_c$ ($J^P = 1/2^+$) with a light scalar diquark component, or the first orbital excitation $(1P)$ of the $\Sigma_c$ ($J^P = 3/2^-$) with a light axial vector diquark \cite{21}.

Interestingly, our calculation shows that $\Lambda_c(2765)$ is very likely to be a $P_\rho$-mode excitation of $\Lambda_c$. The reason is that, the masses of the two $^2P_\lambda$-mode excitations, $\Lambda_c(2593)$ and $\Lambda_c(2625)$, are about 2600 MeV, and according to the quark model the energies of $P_\rho$-mode are $\sim 140$ MeV higher than those of $P_\rho$-mode \cite{11}. An implication from this is that the mass of $P_\rho$-mode excitation is around 2740 MeV, which seems to fit into the mass spectrum well. We calculate the widths of all possible configurations, and the results are listed in Tab. \textbf{XIV}. Comparing with the experiment data, we find that $\Lambda_c(2765)$ is a $|\Lambda_c^+ 2P_\frac{1}{2}^-\rangle$ or $|\Lambda_c^- 2P_\frac{5}{2}^-\rangle$ state is excluded due to the much broader widths. The first radial excitation of the $\Lambda_c$ with $J^P = 1/2^+$ is also excluded for its extremely narrow width.

Note that the $\Lambda_c(2765)$ was observed in a similar decay channel as $\Lambda_c(2880)$, i.e. in $\Lambda_c\pi\pi$, and via $\Sigma_c\pi/\Sigma_c(2520)\pi$. We hence assume that the decay modes of $\Lambda_c(2765)$ have a similar behavior as those of $\Lambda_c(2880)$, except that the $D^0p$ channel is forbidden since the $\Lambda_c(2765)$ is below the $D^0p$ threshold. One thus expects that $[\Gamma(\Sigma_c\pi) + \Gamma(\Sigma_c(2520)\pi)]/\Gamma(\Lambda_c\pi\pi) \sim 0.4$ \cite{39}, which is similar to the experimental value of $\Lambda_c(2880)$. We can then calculate the partial decay width of $\Lambda_c(2765)$ to $\Sigma_c\pi$ and $\Sigma_c(2520)\pi$ for different configurations, and predict its total width into $\Lambda_c\pi\pi$. For $\Lambda_c(2765)$ being a $|\Lambda_c^+ 4P_{\frac{1}{2}}\rangle$ state, we obtain $[\Gamma(\Sigma_c\pi) + \Gamma(\Sigma_c(2520)\pi)] \simeq 21.6$ MeV. Thus, $\Gamma_{\text{total}} \simeq 21.6/0.4 = 53.4$ MeV is obtained and agrees well with the experimental value $\Gamma < 73$ MeV \cite{2,4}.

For $|\Lambda_c^+ 4P_{\frac{5}{2}}\rangle$, it shows that the partial decay width for $\Lambda_c(2765) \rightarrow \Sigma_c(2520)\pi$ is much larger than for $\Sigma_c\pi$ by about a factor of 50. If this is the case, one would expect that $\Sigma_c(2520)\pi$ be the dominant decay channel which however is not consistent with the data. For $|\Lambda_c^- 4P_{\frac{5}{2}}\rangle$, the extracted decay widths are rather small to compare with its total width. The above results make the $\Lambda_c(2765)$ a good candidate for $|\Lambda_c^- 4P_{\frac{5}{2}}\rangle$ state, which also agrees with the quark model prediction.

We also check the possibility of the $\Lambda_c(2765)$ being a $\Sigma_c\pi$-type state. As the masses of the $D$-wave $\Sigma_c\pi$-type states in the $N=2$ shell are generally larger than 2.8 GeV in the quark model \cite{11,41}, and the decay channel $\Lambda_c\pi$ of $P$-wave states in the $N=2$ shell is forbidden due to the quark model selection rules (see Tab. \textbf{VIII}), only the $P$-wave states in the $N=1$ shell and radial excitations are possible.

We calculate the decay widths for those possible states, and the results are listed in Tab. \textbf{XV}. It shows that the radial excitation should be excluded since the decay width is extremely narrow. The negative parity states, except $|\Sigma_c^+ 2P_{\frac{3}{2}}\rangle$, can produce widths at the same order of magnitude as the data when sum all the decay channel together.
However, note that the dominant channel of \( \Sigma_c \)-type charmed states is \( \Lambda_c \pi \). The assignment of \( \Lambda_c(2765) \) to a \( \Sigma_c \) excitation will lead to apparent contradictions to the experimental observations, thus can be ruled out.

\[ F. \quad \Sigma_c(2800) \]

The observation of \( \Sigma_c^{++} \to 3(2800) \) by Belle in \( \Lambda_c \pi \) channel enriches the spectrum of \( \Sigma_c \) excitation states. However, the present experimental information still cannot determine its quantum numbers. Theoretical studies appear strongly model-dependent where its spin-parity of \( J^P = 1/2^- \) or \( 3/2^- \), or \( 5/2^- \), seems possible.

Almost all the recent theoretical predictions suggest that \( \Sigma_c(2800) \) could be the first orbital excitation, however, its quantum numbers are different in different models. Its spin-parity could be \( J^P = 3/2^- \) in the heavy hadron chiral perturbation theory predictions, \( J^P = 3/2^- \) or \( J^P = 5/2^- \) in the \( ^3P_0 \) model, \( J^P = 5/2^- \) in the relativistic quark model, \( J^P = 1/2^- \) or \( 3/2^- \) in the Faddeev studies, and \( J^P = 1/2^- \), \( 3/2^- \) or \( 5/2^- \) the latest calculations with the relativistic quark model.

Again, taking the quark model guidance that the masses of the \( D \)-wave \( \Sigma_c \) excitations in the \( N = 2 \) shell are much larger than \( 2800 \) MeV, while the decay channel \( \Lambda_c \pi \) of \( P \)-wave states in \( N = 2 \) shell is forbidden (see Tab. VII), we classify the \( \Sigma_c(2800) \) as a \( P \)-wave state in either the \( N = 1 \) shell (i.e., the first orbital excitation) or the radial excitation. The decay widths of \( \Lambda_c \pi, \Sigma_c \pi \) and \( \Sigma_c(2520) \pi \) are calculated, and the results are listed in Tab. XVI.

The radial excitations can be excluded easily due to the extremely small predictions of the widths compared with the experimental data. Furthermore, the \( \Lambda_c \pi \) channel may dominate over other channels since \( \Sigma_c(2800) \) was only seen there. Thus, \( |\Sigma_c^{++} 2P_{\lambda \frac{3}{2}^-} \rangle, |\Sigma_c^{4}P_{\lambda \frac{3}{2}^-} \rangle \) and \( |\Sigma_c^{++} 4P_{\lambda \frac{3}{2}^-} \rangle \) should be ruled out due to the dominance of either \( \Sigma_c(2520) \pi \) or \( \Sigma_c \pi \). After this, it leaves two possible states, \( |\Sigma_c^{++} 2P_{\lambda \frac{5}{2}^-} \rangle \) and \( |\Sigma_c^{++} 4P_{\lambda \frac{5}{2}^-} \rangle \), to be assigned to \( \Sigma_c(2800) \). This comes to the same starting point as other works, and indicates how poor we know about this state.

In these two states, \( \Sigma_c(2800) \) as a \( |\Sigma_c 2P_{\lambda \frac{1}{2}^-} \rangle \) state (i.e. a first \( P \)-wave orbital \( \Sigma_c \) excitation) is favored if there are no other decay channels to contribute significantly to the total width. Considering there might exist other decay channels and the uncertainties of the model, \( |\Sigma_c^{++} 2P_{\lambda \frac{3}{2}^-} \rangle \) is favored since its decays into \( \Lambda_c \pi \) are the dominant channel, while into \( \Sigma_c \pi \) and \( \Sigma_c(2520) \pi \) are relatively small. The sum of these three channels, though smaller than the experimental total width, is acceptable taking into account the uncertainties. To determine the quantum number of \( \Sigma_c(2800) \), a measurement of the ratio of \( \Lambda_c \pi/\Sigma_c(2520) \pi \) or \( \Sigma_c \pi/\Sigma_c(2520) \pi \), or the \( \Lambda_c \pi \) angular distributions should be useful.

VI. SUMMARY

In the framework of the non-relativistic quark model, the strong decays of charmed baryons are analyzed with an effective chiral Lagrangian for the pseudoscalar-meson-quark coupling. This framework is successful in reproducing the strong decay widths of \( \Sigma_c \to \Lambda_c \pi \), \( \Lambda_c(2530) \to \Sigma_c \pi \) and \( \Lambda_c(2625) \to \Sigma_c \pi \). It allows us to fix an additional parameter \( \delta \) which is introduced to account for model uncertainties arising from the pseudoscalar-meson-quark coupling constants. We then carry out calculations for those newly observed states by assuming their possible configurations in the quark model. By comparing the theoretical results with the experimental measurement, we extract information about the classification of those states and their possible quantum numbers.

To be more specific, our results show that both \( \Lambda_c(2800) \) and \( \Lambda_c(2940) \) are consistent with being internal \( D \)-wave states. For the \( \Lambda_c(2800) \), its narrow widths, visible decays into \( D^0 \bar{p} \) and the measured ratio \( R = \Gamma_c(2520) \pi/\Gamma_c(2455) \) suggest a favored configuration \( |\Lambda_c 2D_{\lambda \frac{1}{2}^-} \rangle \) with \( l_\lambda = 2 \) and \( l_\rho = 0 \). Considering the decay width and decay channel of \( \Lambda_c(2940) \), our results indicate that \( \Lambda_c(2940) \) could be a \( |\Lambda_c 2D_{\lambda \frac{5}{2}^-} \rangle \) state. Our predictions are different from the suggestions of Ref. [2, 6] that \( \Lambda_c(2800) \) is a \( l_\lambda = l_\rho = 1 \) orbital excitation state with \( J^P = 5/2^+ \). Although the angular distribution fit for \( \Lambda_c(2800) \to \Sigma_c \pi \) favors \( J = 5/2^- \), the data still possess large uncertainties and more precise measurements are desired.

We propose that \( \Lambda_c(2765) \) is most likely a \( \rho \)-mode \( P \)-wave excitation in the \( N = 1 \) shell. In those multiplets, the most possible state is \( |\Lambda_c 4P_{\lambda \frac{3}{2}^-} \rangle \), which also turns to be consistent with the quark model predictions.

For the \( \Sigma_c(2800) \), the present experimental information seems not sufficient for its classification in our approach. Assuming that no other sizeable decay channels apart from \( \Lambda_c \pi, \Sigma_c \pi \) and \( \Sigma_c(2520) \pi \), to contribute to its total width, it is most likely a \( |\Sigma_c 2P_{\lambda \frac{1}{2}^-} \rangle \) state. Otherwise, the possibility of its being a \( |\Sigma_c^{++} 4P_{\lambda \frac{3}{2}^-} \rangle \) state cannot be excluded. Measurements of the ratio of \( \Lambda_c \pi/\Sigma_c(2520) \pi \) and/or \( \Sigma_c \pi/\Sigma_c(2520) \pi \) are recommended to clarify its spin-parity.
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**TABLE 1:** The spatial wave functions with principal quantum number $N \leq 2$, denoted by $^{N}\Psi_{L_{s}}^{s,\rho,\lambda}$, where $s, \lambda, \rho, A, \rho\rho, \lambda\lambda$ stands for different excitation modes in quark model.

| $N$ | $L$ | $l_{\rho}$ | $l_{\lambda}$ | $n_{\rho}$ | $n_{\lambda}$ | $L_{s}$ | wave function |
|-----|-----|------------|-------------|-----------|-------------|-------|--------------|
| 0   | 0   | 0          | 0           | 0         | 0           | 0     | $^{0}\Psi_{00}^{s} = \psi_{00}(\rho)\psi_{00}(\lambda)$ |
| 1   | 1   | 1          | 0           | 0         | m           | 1     | $^{1}\Psi_{1m}^{s} = \psi_{0m}(\rho)\psi_{1m}(\lambda)$ |
| 1   | 1   | 1          | 1           | 0         | m           | 1     | $^{1}\Psi_{1m}^{\rho}\rho = \psi_{1m}(\rho)\psi_{00}(\lambda)$ |
| 2   | 2   | 0          | 1           | 1         | 1           | 2     | $^{2}\Psi_{01}^{s} = \sqrt{2}R_{01}(\rho)R_{01}(\lambda)\{Y_{11}(\lambda)Y_{11}(\rho) - Y_{10}(\lambda)Y_{10}(\rho) + Y_{1-1}(\lambda)Y_{1-1}(\rho)\}$ |
| 2   | 2   | 1          | 1           | 1         | 0           | 1     | $^{2}\Psi_{11}^{s} = \sqrt{2}R_{11}(\rho)R_{11}(\lambda)\{Y_{11}(\lambda)Y_{10}(\rho) - Y_{10}(\lambda)Y_{10}(\rho)\}$ |
| 2   | 2   | 1          | 1           | 1         | 0           | 0     | $^{2}\Psi_{10}^{s} = \sqrt{2}R_{01}(\rho)R_{01}(\lambda)\{Y_{11}(\lambda)Y_{1-1}(\rho) - Y_{1-1}(\lambda)Y_{11}(\rho)\}$ |
| 2   | 2   | 1          | 1           | 1         | 0           | -1    | $^{2}\Psi_{1-1}^{s} = \sqrt{2}R_{01}(\rho)R_{01}(\lambda)\{Y_{10}(\lambda)Y_{1-1}(\rho) - Y_{1-1}(\lambda)Y_{10}(\rho)\}$ |
| 2   | 2   | 1          | 1           | 1         | 0           | 1     | $^{2}\Psi_{21}^{s} = \sqrt{2}R_{21}(\rho)R_{21}(\lambda)\{Y_{11}(\lambda)Y_{10}(\rho) + Y_{10}(\lambda)Y_{11}(\rho)\}$ |
| 2   | 2   | 1          | 1           | 1         | 0           | -1    | $^{2}\Psi_{2-1}^{s} = \sqrt{2}R_{01}(\rho)R_{01}(\lambda)\{Y_{10}(\lambda)Y_{1-1}(\rho) + Y_{1-1}(\lambda)Y_{10}(\rho)\}$ |
| 2   | 2   | 1          | 1           | 1         | 0           | -2    | $^{2}\Psi_{2-2}^{s} = \sqrt{2}R_{01}(\rho)R_{01}(\lambda)\{Y_{10}(\lambda)Y_{1-1}(\rho) - Y_{1-1}(\lambda)Y_{10}(\rho)\}$ |
| 2   | 2   | 0          | 0           | 0         | m           | 2     | $^{2}\Psi_{00}^{s} = \psi_{00}(\rho)\psi_{00}(\lambda)$ |
| 2   | 2   | 0          | 0           | 0         | m           | 2     | $^{2}\Psi_{00}^{\rho\rho} = \psi_{00}(\rho)\psi_{00}(\lambda)$ |
| 2   | 0   | 0          | 0           | 1         | 0           | 2     | $^{2}\Psi_{01}^{s} = \psi_{01}(\rho)\psi_{01}(\lambda)$ |
| 2   | 0   | 0          | 0           | 0         | 1           | 2     | $^{2}\Psi_{01}^{\rho\rho} = \psi_{01}(\rho)\psi_{01}(\lambda)$ |
TABLE II: The total wave functions of $\Lambda_c$-type charmed baryons, denoted by $|\Lambda_c \, ^{2S+1}L_{\alpha} \, J^P \rangle$ as used in Ref.\textsuperscript{11}. The Clebsch-Gordan series for the spin and angular-momentum addition $|\Lambda_c \, ^{2S+1}L_{\alpha} \, J^P \rangle \rightarrow \sum_{L_z+S_z=J_z} (LL_z,SS_z|JJ_z)^S \, \Psi_{LL_z}^{S} \, \chi_{S_z} \phi_{\Lambda_c}$ has been omitted.

| state | N | J | L | S | $J^P$ | wave function |
|-------|---|---|---|---|---|----------------|
| $|\Lambda_c \, ^2S_{1/2}\rangle$ | 0 | 1/2 | 0 | 1/2 | 1$^+$ | $^0\Psi_{00}^S \chi^S_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^2P_{1/2}\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1$^+$ | $^1\Psi_{1L_z}^0 \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^2P_{1/2}^*\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1$^+$ | $^1\Psi_{1L_z}^0 \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^4P_{1/2}\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1$^+$ | $^1\Psi_{1L_z}^0 \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^4P_{1/2}^*\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1$^+$ | $^1\Psi_{1L_z}^0 \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^2S_A^{1/2}\rangle$ | 2 | 1/2 | 0 | 1/2 | 1$^+$ | $^2\Psi_{10}^S \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^4S_A^{1/2}\rangle$ | 2 | 1/2 | 0 | 1/2 | 1$^+$ | $^2\Psi_{10}^S \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^2P_A^{1/2}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1$^+$ | $^2\Psi_{10}^S \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^2P_A^{1/2}^*\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1$^+$ | $^2\Psi_{10}^S \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^4P_A^{1/2}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1$^+$ | $^2\Psi_{10}^S \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^4P_A^{1/2}^*\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1$^+$ | $^2\Psi_{10}^S \chi^0_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^2D_A^{3/2}\rangle$ | 2 | 3/2 | 2 | 3/2 | 3$^+$ | $^2\Psi_{20}^S \chi^3_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^2D_A^{3/2}^*\rangle$ | 2 | 3/2 | 2 | 3/2 | 3$^+$ | $^2\Psi_{20}^S \chi^3_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^4D_A^{3/2}\rangle$ | 2 | 3/2 | 2 | 3/2 | 3$^+$ | $^2\Psi_{20}^S \chi^3_S \phi_{\Lambda_c}$ |
| $|\Lambda_c \, ^4D_A^{3/2}^*\rangle$ | 2 | 3/2 | 2 | 3/2 | 3$^+$ | $^2\Psi_{20}^S \chi^3_S \phi_{\Lambda_c}$ |
TABLE III: The total wave functions of $\Sigma_c$-type charmed baryons, denoted by $|\Sigma_c^{-2S+1}L_S J S J F⟩$. The Clebsch-Gordan series for the spin and angular-momentum addition $|\Sigma_c^{-2S+1}L_S J S J F⟩ = \sum_{L_z, S_z} |LL_z, SS_z JJ_z⟩^N \Psi^L_{\Sigma L_z} \chi_{S_z} \phi_{\Sigma_c}$ has been omitted.

| State | N | J | L | S | J' | Wave Function |
|-------|---|---|---|---|----|---------------|
| $|\Sigma_c^{-2S+1}\rangle$ | 0 | 1/2 | 0 | 1/2 | 1/2 | $0\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-4S+3}\rangle$ | 0 | 1/2 | 0 | 1/2 | 3/2 | $0\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2P_\lambda \frac{1}{2}}\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1/2 | $1\Psi^\lambda_{1L_z} \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2P_\lambda \frac{3}{2}}\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1/2 | $1\Psi^\lambda_{1L_z} \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-4P_\lambda \frac{1}{2}}\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1/2 | $1\Psi^\lambda_{1L_z} \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-4P_\lambda \frac{3}{2}}\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1/2 | $1\Psi^\lambda_{1L_z} \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2P_\rho \frac{1}{2}}\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1/2 | $1\Psi^\rho_{1L_z} \chi_0^\rho \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2P_\rho \frac{3}{2}}\rangle$ | 1 | 1/2 | 1/2 | 1/2 | 1/2 | $1\Psi^\rho_{1L_z} \chi_0^\rho \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2S_A \frac{1}{2}}\rangle$ | 2 | 1/2 | 0 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2P_A \frac{1}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2P_A \frac{3}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2D_A \frac{1}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2D_A \frac{3}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2D_{\mu\nu} \frac{1}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2D_{\mu\nu} \frac{3}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-4D_{\mu\nu} \frac{1}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-4D_{\mu\nu} \frac{3}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-4D_{\mu\nu} \frac{1}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-4D_{\mu\nu} \frac{3}{2}}\rangle$ | 2 | 1/2 | 1/2 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2S_{\mu\nu} \frac{1}{2}}\rangle$ | 2 | 1/2 | 0 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-2S_{\mu\nu} \frac{3}{2}}\rangle$ | 2 | 1/2 | 0 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-4S_{\mu\nu} \frac{1}{2}}\rangle$ | 2 | 1/2 | 0 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $|\Sigma_c^{-4S_{\mu\nu} \frac{3}{2}}\rangle$ | 2 | 1/2 | 0 | 1/2 | 1/2 | $2\Psi_{00}^\rho \chi_0^\lambda \phi_{\Sigma_c}$ |
| $g_1 = \langle x_{1/2}^\lambda | \sigma_{1z} | x_{1/2}^\lambda \rangle$ | $= -\frac{1}{\sqrt{2}}$ | $g_7 = \langle x_{1/2}^\lambda | \sigma_{1z} | x_{1/2}^\lambda \rangle$ | $= 0$ |
| $g_2 = \langle x_{1/2}^\lambda | \sigma_{1z} | x_{-1/2}^\lambda \rangle$ | $= -\frac{1}{\sqrt{2}}$ | $g_8 = \langle x_{1/2}^\lambda | \sigma_{1z} | x_{1/2}^\lambda \rangle$ | $= 0$ |
| $g_3 = \langle x_{1/2}^\lambda | \sigma_{1z} | x_{1/2}^\lambda \rangle$ | $= \frac{1}{2}$ | $g_9 = \langle x_{1/2}^\lambda | \sigma_{1z} | x_{1/2}^\lambda \rangle$ | $= 0$ |
| $g_4 = \langle x_{1/2}^\lambda | \sigma_{1z} | x_{-1/2}^\lambda \rangle$ | $= \frac{1}{2}$ | $g_{10} = \langle x_{1/2}^\lambda | \sigma_{1z} | x_{1/2}^\lambda \rangle$ | $= -\frac{1}{\sqrt{2}}$ |

**TABLE IV:** The spin-factors used in this work.
| initial state | amplitude |
|---------------|-----------|
| \(|\Lambda_+^2 S_{1/2}^+\)| forbidden by the kinematics |
| \(|\Lambda_+^2 P_{3/2}^-\)| \(i\frac{\sqrt{5}}{6}g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F + i\frac{\sqrt{5}}{3} g_1 g_i \hbar \alpha \lambda F \) |
| \(|\Lambda_+^2 P_{1/2}^-\)| \(-i\frac{\sqrt{5}}{6} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F \) |
| \(|\Lambda_+^4 P_{3/2}^-\)| \(-i\frac{\sqrt{5}}{6} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F - i\frac{\sqrt{5}}{3} g_4 g_{ij} \sqrt{3} g_3 - g_4 \right) g_i h \alpha \lambda F \) |
| \(|\Lambda_+^4 P_{1/2}^-\)| \(-i\frac{\sqrt{5}}{6} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F - i\frac{\sqrt{5}}{3} g_4 g_{ij} \sqrt{3} g_3 - g_4 \right) g_i h \alpha \lambda F \) |
| \(|\Lambda_+^2 S_{1/2}^+\)| \(\frac{\sqrt{5}}{6} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F + \frac{\sqrt{5}}{3} g_1 g_i h \left( \alpha_\lambda \alpha_\sigma + \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^4 S_{1/2}^+\)| \(\frac{\sqrt{5}}{6} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F + \frac{\sqrt{5}}{3} g_4 g_{ij} h \left( \alpha_\lambda \alpha_\sigma + \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^2 P_{1/2}^-\)| \(\frac{1}{15} g_4 g_{ij} h \left( \alpha_\lambda \alpha_\sigma - \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^2 P_{3/2}^-\)| \(\frac{1}{15} g_4 g_{ij} h \left( \alpha_\lambda \alpha_\sigma - \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^4 P_{1/2}^-\)| \(\frac{1}{15} g_4 g_{ij} h \left( \alpha_\lambda \alpha_\sigma - \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^4 P_{3/2}^-\)| \(\frac{1}{15} g_4 g_{ij} h \left( \alpha_\lambda \alpha_\sigma - \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^2 D_{1/2}^+\)| \(\frac{\sqrt{5}}{30} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F + \frac{\sqrt{5}}{6} (2 g_3 + 3 g_4) g_i h \left( \alpha_\lambda \alpha_\sigma + \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^2 D_{1/2}^+\)| \(-\frac{\sqrt{5}}{30} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F - \frac{\sqrt{5}}{6} (g_3 - g_4) g_i h \left( \alpha_\lambda \alpha_\sigma + \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^4 D_{1/2}^+\)| \(-\frac{\sqrt{5}}{30} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F - \frac{\sqrt{5}}{6} (\sqrt{3} g_3 - g_4) g_i h \left( \alpha_\lambda \alpha_\sigma + \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^4 D_{1/2}^+\)| \(-\frac{\sqrt{5}}{30} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F - \frac{\sqrt{5}}{6} (g_3 - 3 g_4) g_i h \left( \alpha_\lambda \alpha_\sigma + \sqrt{3} \alpha_\rho \alpha_\lambda \right) F \) |
| \(|\Lambda_+^2 D_{3/2}^+\)| \(\frac{\sqrt{5}}{30} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F + \frac{\sqrt{5}}{6} g_i h q_4 F \) |
| \(|\Lambda_+^2 D_{3/2}^+\)| \(-\frac{\sqrt{5}}{30} g_4 g_{ij} \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F - \frac{\sqrt{5}}{6} g_i h q_4 F \) |
| \(|\Lambda_+^2 D_{3/2}^+\)| \(\sqrt{5} g_1 g_i \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F + \frac{\sqrt{5}}{6} g_i h q_4 F \) |
| \(|\Lambda_+^2 D_{3/2}^+\)| \(-\sqrt{5} g_1 g_i \left\{ Gq - \left( \frac{\sqrt{5}}{12} q_3 + \frac{\sqrt{4}}{4} q_4 \right) h \right\} \frac{\alpha_\lambda}{\alpha_\sigma} F - \frac{\sqrt{5}}{6} g_i h q_4 F \) |
TABLE VI: The decay amplitudes for all the states $|\Lambda_c^{2S+1}L_s J^P\rangle$ up to $N = 2$ shell in $\Sigma_c(2520)\pi$ channel (a factor 2 is omitted). $F$, as the decay form factor, is defined in Eq. (39).
Table VII: The decay amplitudes for all the states $|\Sigma_e \, {}^{2S+1}L_\nu J^F \rangle$ up to $N = 2$ shell in $\Lambda_c \pi$ channel (a factor 2 is omitted). $F$, as the decay form factor, is defined in Eq. (13).

| initial state | amplitude |
|---------------|------------|
| $|\Sigma_e \, {}^{2S+1}_1 J^3 \rangle$ | $g_1^q g_1 \left\{ Gq - \left( \frac{\sqrt{7}}{2} q_\lambda + \frac{\sqrt{7}}{2} q_\rho \right) \right\} F$ |
| $|\Sigma_e \, {}^{2S+1}_1 J^0 \rangle$ | $g_3^q g_1 \left\{ Gq - \left( \frac{\sqrt{7}}{2} q_\lambda + \frac{\sqrt{7}}{2} q_\rho \right) \right\} F$ |
| $|\Sigma_e \, {}^{2S+1}_1 J^\pm \rangle$ | $i \frac{\sqrt{7}}{2} g_3^q g_1 \left\{ Gq - \left( \frac{\sqrt{7}}{2} q_\lambda + \frac{\sqrt{7}}{2} q_\rho \right) \right\} \frac{q_\pm}{m_\chi} F + i \frac{\sqrt{7}}{2} (g_3^q + g_4^q) g_1 h_\alpha \lambda F$ |
| $|\Sigma_e \, {}^{2S+1}_3 J^3 \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_3 J^0 \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_3 J^\pm \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_5 J^3 \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_5 J^0 \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_5 J^\pm \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_7 J^3 \rangle$ | $\frac{\sqrt{70}}{30} g_3^q g_1 \left\{ Gq - \left( \frac{\sqrt{7}}{12} q_\lambda + \frac{\sqrt{7}}{4} q_\rho \right) \right\} \frac{q_6}{m_\alpha} F + \frac{\sqrt{70}}{30} (2g_1^q + 3g_2^q) g_1 h_\alpha q F$ |
| $|\Sigma_e \, {}^{2S+1}_7 J^0 \rangle$ | $\frac{\sqrt{70}}{30} g_3^q g_1 \left\{ Gq - \left( \frac{\sqrt{7}}{12} q_\lambda + \frac{\sqrt{7}}{4} q_\rho \right) \right\} \frac{q_6}{m_\alpha} F - \frac{\sqrt{70}}{30} (g_3^q + g_4^q) g_1 h_\alpha q F$ |
| $|\Sigma_e \, {}^{2S+1}_7 J^\pm \rangle$ | $\frac{\sqrt{70}}{30} g_3^q g_1 \left\{ Gq - \left( \frac{\sqrt{7}}{12} q_\lambda + \frac{\sqrt{7}}{4} q_\rho \right) \right\} \frac{q_6}{m_\alpha} F - \frac{\sqrt{70}}{30} (g_3^q + g_4^q) g_1 h_\alpha q F$ |
| $|\Sigma_e \, {}^{2S+1}_9 J^3 \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_9 J^0 \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_9 J^\pm \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_{11} J^3 \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_{11} J^0 \rangle$ | forbidden |
| $|\Sigma_e \, {}^{2S+1}_{11} J^\pm \rangle$ | forbidden |
TABLE VIII: The decay amplitudes for the first orbital and radial excitations $|\Sigma_c^ {2S+1}L_o f^p\rangle$ in $\Sigma_c\pi$ channel (a factor 2 is omitted). $F$, as the decay form factor, is defined in Eq. [33].

| initial state | amplitude |
|---------------|-----------|
| $|\Sigma_c^2 P^{3}_{\frac{1}{2}}\rangle$ | $\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F + i\frac{1}{6}(g^2_2 + 2g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^2 P^{3}_{\frac{3}{2}}\rangle$ | $-\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F - i\frac{\sqrt{5}}{14}(g^2_2 - g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^4 P^{3}_{\frac{1}{2}}\rangle$ | $\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F + i\frac{1}{6}(g^2_2 + 2g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^4 P^{3}_{\frac{3}{2}}\rangle$ | $-\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F + i\frac{\sqrt{5}}{14}(g^2_2 - g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^2 S^{\frac{1}{2}}_{\frac{1}{2}}\rangle$ | $\frac{i\sqrt{5}}{14}g_2 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F + i\frac{1}{6}(g^2_2 + 2g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^4 S^{\frac{1}{2}}_{\frac{1}{2}}\rangle$ | $-\frac{i\sqrt{5}}{14}g_2 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F - i\frac{\sqrt{5}}{14}(g^2_2 - g_1^2)ig_1h_{\alpha\lambda}F$ |

TABLE IX: The decay amplitudes for the first orbital and radial excitations $|\Sigma_c^ {2S+1}L_o f^p\rangle$ in $\Sigma_c(2520)\pi$ (a factor 2 is omitted). $F$, as the decay form factor, is defined in Eq. [33].

| initial state | amplitude |
|---------------|-----------|
| $|\Sigma_c^2 P^{1}_{\frac{1}{2}}\rangle$ | $\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F + i\frac{1}{6}(g^2_2 + 2g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^2 P^{1}_{\frac{3}{2}}\rangle$ | $-\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F - i\frac{\sqrt{5}}{14}(g^2_2 - g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^4 P^{1}_{\frac{1}{2}}\rangle$ | $\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F + i\frac{1}{6}(g^2_2 + 2g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^4 P^{1}_{\frac{3}{2}}\rangle$ | $-\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F - i\frac{\sqrt{5}}{14}(g^2_2 - g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^2 S^{\frac{1}{2}}_{\frac{1}{2}}\rangle$ | $\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F + i\frac{1}{6}(g^2_2 + 2g_1^2)ig_1h_{\alpha\lambda}F$ |
| $|\Sigma_c^4 S^{\frac{1}{2}}_{\frac{1}{2}}\rangle$ | $-\frac{i\sqrt{5}}{14}g_2g_1 \{Gq - \left(\frac{i\sqrt{5}}{14}q_\lambda + \frac{\sqrt{5}}{4}q_\rho\right) h\} \frac{x_{\alpha}}{\alpha_{\pi}} F - i\frac{\sqrt{5}}{14}(g^2_2 - g_1^2)ig_1h_{\alpha\lambda}F$ |

TABLE X: The amplitudes of $\Lambda_c$-type charmed baryons decay into $D^0 p$. $F(q_{\lambda}^2) = \exp(q_{\lambda}^2/4\alpha_{\lambda}^2)$ is the form factor.

| initial state | amplitude |
|---------------|-----------|
| $|\Lambda_c^2 D^{0}_{\frac{3}{2}}\rangle$ | $\frac{\sqrt{\alpha_{\lambda}}}{\sqrt{12}}(Gq - \frac{\sqrt{5}}{2}h_{\lambda}q_\lambda) \left(\frac{q_{\lambda}}{\alpha_{\lambda}}\right)^2 F(q_{\lambda}^2) + \frac{\sqrt{\alpha_{\lambda}}}{\sqrt{6}}h_{\lambda}q_\lambda F(q_{\lambda}^2)$ |
| $|\Lambda_c^2 D^{0}_{\frac{5}{2}}\rangle$ | $-\frac{\sqrt{\alpha_{\lambda}}}{\sqrt{12}}(Gq - \frac{\sqrt{5}}{2}h_{\lambda}q_\lambda) \left(\frac{q_{\lambda}}{\alpha_{\lambda}}\right)^2 F(q_{\lambda}^2)$ |
| $|\Lambda_c^2 S^{\frac{1}{2}}_{\frac{1}{2}}\rangle$ | $\frac{\sqrt{\alpha_{\lambda}}}{\sqrt{12}}(Gq - \frac{\sqrt{5}}{6}h_{\lambda}q_\lambda) \left(\frac{q_{\lambda}}{\alpha_{\lambda}}\right)^2 F(q_{\lambda}^2) + \frac{\sqrt{\alpha_{\lambda}}}{\sqrt{6}}h_{\lambda}q_\lambda F(q_{\lambda}^2)$ |
TABLE XI: The decay widths for the low-lying charmed baryons. $\Lambda_c(2593)$ and $\Lambda_c(2625)$ assigned as both $P_\lambda$- and $P_\rho$-mode excitations are listed. The partial decay widths for $\Sigma_c$ and $\Lambda_c(2520) \to \Lambda_c\pi$ are also listed. They serve as experimental input for the determination of parameter $\delta$ in this approach.

| notation | channel | $\Gamma_{\text{exp}}$ (MeV) | $\Gamma_{\text{th}}$ (MeV) |
|-----------|----------|-----------------------------|-----------------------------|
| $\Lambda_c(2593)$ | $|\Lambda_c \to 2P_\lambda\frac{1}{2}^+\rangle$ | $\Sigma_c^0\pi^+$ | 0.65$\pm$0.041 $0.73$ |
| | | $\Sigma_c^+\pi^-$ | 0.37 |
| | | $|\Sigma_c\pi^+$ | 0.67$\pm$0.041 $0.40$ |
| $\Lambda_c(2625)$ | $|\Lambda_c \to 2P_\rho\frac{1}{2}^+\rangle$ | $\Sigma_c^0\pi^+$ | $< 0.10$ $1.47 \times 10^{-2}$ |
| | | $\Sigma_c^+\pi^-$ | 1.50$\times$10$^{-2}$ |
| $\Lambda_c(2593)$ | $|\Lambda_c \to 2P_\lambda\frac{1}{2}^+\rangle$ | $\Sigma_c^0\pi^+$ | 0.64$\pm$0.041 |
| | | $\Sigma_c^+\pi^-$ | 1.02 |
| | | $|\Sigma_c\pi^+$ | 2.08 |
| $\Lambda_c(2625)$ | $|\Lambda_c \to 2P_\rho\frac{1}{2}^+\rangle$ | $\Sigma_c^0\pi^+$ | $< 0.10$ $10.00$ |
| | | $\Sigma_c^+\pi^-$ | $10.50$ |
| $\Sigma_c(2455)$ | $|\Sigma_c \to 2S_{\frac{1}{2}}^+\rangle$ | $\Lambda_c\pi^+$ | 2.23$\pm$0.30 $1.89$ |
| | | $\Lambda_c\pi^0$ | $< 4.6$ $2.18$ |
| | | $\Lambda_c\pi^-$ | 2.2$\pm$0.4 $1.86$ |
| $\Sigma_c(2520)$ | $|\Sigma_c \to 4S_{\frac{1}{2}}^+\rangle$ | $\Lambda_c\pi^+$ | 14.9$\pm$1.9 input |
| | | $\Lambda_c\pi^0$ | $< 17$ $15.53$ |
| | | $\Lambda_c\pi^-$ | 16.1$\pm$2.1 $14.92$ |

TABLE XII: The decay widths of $\Lambda_c(2880)$ for all the possible states in $N = 2$ shell (in MeV). Ratio $R$ is defined as $R = \Gamma(\Sigma_c(2520)\pi^\pm)/\Gamma(\Sigma_c\pi^\pm)$

| assignment | $\Sigma_c^0\pi^+$ | $\Sigma_c^0\pi^0$ | $\Sigma_c^0\pi^+$, $\Sigma_c(2520)\pi^\pm$ | $R$ | $D_0^+$ |
|------------|-----------------|-----------------|--------------------------------|-----|----------|
| $|\Lambda_c \to 2S_{\frac{1}{2}}^+\rangle$ | 0.45 | 0.47 | 0.29 $0.62$ | 0 |
| $|\Lambda_c \to 4S_{\frac{1}{2}}^+\rangle$ | 0.11 | 0.12 | 0.40 $3.33$ | 0 |
| $|\Lambda_c \to 2P_\lambda\frac{1}{2}^+\rangle$ | 0.41 | 0.40 | 0.03 $0.08$ | 0 |
| $|\Lambda_c \to 2P_\rho\frac{1}{2}^+\rangle$ | 0.10 | 0.10 | 0.06 $0.60$ | 0 |
| $|\Lambda_c \to 4P_{\frac{1}{2}}^\pm\rangle$ | 0.21 | 0.20 | 0.01 $0.05$ | 0 |
| $|\Lambda_c \to 4P_{\frac{3}{2}}^\pm\rangle$ | 0.13 | 0.13 | 0.05 $0.38$ | 0 |
| $|\Lambda_c \to 4P_{\frac{5}{2}}^\pm\rangle$ | 0 | 0 | 0.12 | 0 |
| $|\Lambda_c \to 2D_{\frac{3}{2}}^\pm\rangle$ | 4.46 | 4.32 | 0.90 $0.21$ | 0 |
| $|\Lambda_c \to 2D_{\frac{5}{2}}^\pm\rangle$ | 3.85 | 3.84 | 1.93 $0.50$ | 0 |
| $|\Lambda_c \to 4D_{\frac{3}{2}}^\pm\rangle$ | 3.35 | 3.33 | 1.06 $0.32$ | 0 |
| $|\Lambda_c \to 4D_{\frac{5}{2}}^\pm\rangle$ | 3.86 | 3.85 | 3.12 $1.69$ | 0 |
| $|\Lambda_c \to 4D_{\frac{5}{2}}^\pm\rangle$ | 0.11 | 0.11 | 4.86 $44.18$ | 0 |
| $|\Lambda_c \to 4D_{\frac{7}{2}}^\pm\rangle$ | 0.40 | 0.38 | 0.36 $0.95$ | 0 |
| $|\Lambda_c \to 2D_{\rho_{\frac{1}{2}}}^\pm\rangle$ | 4.31 | 4.27 | 0.75 $0.18$ | 0 |
| $|\Lambda_c \to 2D_{\rho_{\frac{3}{2}}}^\pm\rangle$ | 0.45 | 0.43 | 5.18 $12.05$ | 0 |
| $|\Lambda_c \to 2S_{\rho_{\frac{1}{2}}}^\pm\rangle$ | 1.58 | 1.58 | 0.46 $0.29$ | 1.77 |
| $|\Lambda_c \to 2S_{\rho_{\frac{3}{2}}}^\pm\rangle$ | 0.48 | 0.46 | 1.33 $2.89$ | 1.44 |
| $|\Lambda_c \to 2S_{\lambda_{\frac{1}{2}}}^\pm\rangle$ | 0.46 | 0.47 | 0.86 $1.83$ | 0 |
| $|\Lambda_c \to 2S_{\lambda_{\frac{3}{2}}}^\pm\rangle$ | 0.02 | 0.02 | 0.15 $7.50$ | 0.65 |
TABLE XIII: The decay widths of $\Lambda_c(2940)$ for all the possible states in $N = 2$ shell (in MeV). Ratio $\mathcal{R}$ is defined as $\mathcal{R} = \Gamma(\Sigma_c(2520)\pi^\pm)/\Gamma(\Sigma_c\pi^\pm)$.

| assignment | $\Sigma_c^0\pi^0$ | $\Sigma_c^0\pi^0 + \pi^\pm$ | $\Sigma_c(2520)\pi^\pm$ | $\mathcal{R}$ | $D^0\rho$ |
|------------|-----------------|-----------------|-----------------|----------|----------|
| $|\Lambda_c^2 S_A^L\pi^L\rangle$ | 0.16 | 0.18 | 0.25 | 1.39 | 0 |
| $|\Lambda_c^4 S_A^L\pi^L\rangle$ | 0.04 | 0.05 | 0.35 | 7.00 | 0 |
| $|\Lambda_c^2 P_A^L\pi^L\rangle$ | 0.65 | 0.64 | 0.47 | 0.73 | 0 |
| $|\Lambda_c^2 P_A^L\pi^L\rangle$ | 0.16 | 0.16 | 0.12 | 0.75 | 0 |
| $|\Lambda_c^4 P_A^L\pi^L\rangle$ | 0.32 | 0.32 | 0.02 | 0.06 | 0 |
| $|\Lambda_c^4 P_A^L\pi^L\rangle$ | 0.20 | 0.20 | 0.09 | 0.45 | 0 |
| $|\Lambda_c^4 P_A^L\pi^L\rangle$ | 0.00 | 0.00 | 0.21 | 0 | 0 |
| $|\Lambda_c^2 D_A^L\pi^L\rangle$ | 8.95 | 8.73 | 2.04 | 0.23 | 0 |
| $|\Lambda_c^2 D_A^L\pi^L\rangle$ | 4.79 | 4.80 | 3.13 | 0.65 | 0 |
| $|\Lambda_c^4 D_A^L\pi^L\rangle$ | 4.27 | 4.27 | 1.61 | 0.38 | 0 |
| $|\Lambda_c^4 D_A^L\pi^L\rangle$ | 2.49 | 2.48 | 4.64 | 3.15 | 0 |
| $|\Lambda_c^4 D_A^L\pi^L\rangle$ | 0.26 | 0.25 | 7.98 | 31.92 | 0 |
| $|\Lambda_c^4 D_A^L\pi^L\rangle$ | 0.90 | 0.88 | 1.02 | 1.16 | 0 |
| $|\Lambda_c^2 D_{\rho}^L\pi^L\rangle$ | 5.97 | 5.94 | 1.51 | 0.25 | 0 |
| $|\Lambda_c^2 D_{\rho}^L\pi^L\rangle$ | 1.02 | 0.99 | 8.61 | 8.70 | 0 |
| $|\Lambda_c^2 D_{\lambda}^L\pi^L\rangle$ | 1.98 | 1.98 | 1.04 | 0.53 | 10.40 |
| $|\Lambda_c^2 D_{\lambda}^L\pi^L\rangle$ | 1.09 | 1.06 | 2.15 | 2.03 | 10.80 |
| $|\Lambda_c^2 S_{\rho}^L\pi^L\rangle$ | 0.46 | 0.36 | 0.95 | 2.64 | 0 |
| $|\Lambda_c^2 S_{\lambda}^L\pi^L\rangle$ | 0.01 | 0.008 | 0.06 | 7.50 | 1.38 |

TABLE XIV: The decay widths (in MeV) of $\Lambda_c(2765)$ for the possible excitation modes. $\Gamma_{\text{sum}} = \Gamma_{\Sigma_c\pi} + \Gamma_{\Sigma_c^*\pi}$, where $\Sigma_c^*$ stands for $\Sigma_c(2520)$.

| assignment | $\Sigma_c^0\pi^0$ | $\Sigma_c^0\pi^0 + \pi^\pm$ | $\Sigma_c^*\pi^0$ | $\Sigma_c^0\pi^0 + \pi^\pm$ | $\Gamma_{\text{sum}}$ | $\Gamma_{\text{exp}}$ |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $|\Lambda_c^4 P_{\rho}^L\pi^L\rangle$ | 7.0 | 0.2 | 21.6 | 50 - 73 |
| $|\Lambda_c^4 P_{\rho}^L\pi^L\rangle$ | 0.4 | 20.6 | 63 |
| $|\Lambda_c^4 P_{\rho}^L\pi^L\rangle$ | 2.2 | 0.6 | 8.4 |
| $|\Lambda_c^4 P_{\rho}^L\pi^L\rangle$ | 41.4 | 3.0 | 133.2 |
| $|\Lambda_c^4 P_{\rho}^L\pi^L\rangle$ | 29.8 | 11.4 | 123.6 |
| $|\Lambda_c^2 S_{\lambda}^L\pi^L\rangle$ | 0.08 | 0.1 | 0.5 |
| $|\Lambda_c^2 S_{\lambda}^L\pi^L\rangle$ | 0.3 | 0.3 | 1.8 |

TABLE XV: The decay widths (in MeV) of $\Lambda_c(2765)$ as a $\Sigma_c$-type excitation. $\Gamma_{\text{sum}} = \Gamma_{\Sigma_c\pi} + \Gamma_{\Sigma_c^*\pi}$, where $\Sigma_c^*$ stands for $\Sigma_c(2520)$.

| assignment | $\Sigma_c^0\pi^0$ | $\Sigma_c^0\pi^0 + \pi^\pm$ | $\Sigma_c^*\pi^0$ | $\Sigma_c^0\pi^0 + \pi^\pm$ | $\Gamma_{\text{sum}}$ | $\Gamma_{\text{exp}}$ |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $|\Sigma_c^+ P_{\lambda}^L\pi^L\rangle$ | 0.02 | 1.98 | 1.40 | 6.78 | $\sim 50 - 73$ |
| $|\Sigma_c^+ P_{\lambda}^L\pi^L\rangle$ | 36.9 | 9.66 | 1.06 | 58.34 |
| $|\Sigma_c^+ P_{\lambda}^L\pi^L\rangle$ | 6.86 | 4.20 | 0.03 | 15.14 |
| $|\Sigma_c^+ P_{\lambda}^L\pi^L\rangle$ | 0.13 | 8.67 | 20.96 |
| $|\Sigma_c^+ P_{\lambda}^L\pi^L\rangle$ | 20.16 | 0.75 | 0.60 | 22.86 |
| $|\Sigma_c^+ S_{\rho}^L\pi^L\rangle$ | 0.37 | 0.41 | 0.08 | 1.35 |
| $|\Sigma_c^+ S_{\rho}^L\pi^L\rangle$ | 0.37 | 0.33 | 0.22 | 1.47 |
| $|\Sigma_c^+ S_{\lambda}^L\pi^L\rangle$ | 0.003 | 0.11 | 0.03 | 0.283 |
| $|\Sigma_c^+ S_{\lambda}^L\pi^L\rangle$ | 0.003 | 0.03 | 0.07 | 0.203 |
TABLE XVI: The decay widths (in MeV) of $\Sigma_c(2800)$ for the possible excitation modes. $\Gamma_{\text{sum}} = \Gamma_{\Lambda_c\pi} + \Gamma_{\Sigma_c\pi} + \Gamma_{\Sigma^*_c\pi}$, where $\Sigma^*_c$ stands for $\Sigma_c(2520)$.

| assignment | $\Lambda_c\pi$ | $\Sigma^{++} + \pi^0$ | $\Sigma^{++} + \pi^+$ | $\Gamma_{\text{sum}}$ | $\Gamma_{\exp}^{\text{total}}$ |
|------------|--------------|----------------|----------------|----------------|----------------|
| $[\Sigma^+_c + ^2P_{\lambda} \frac{3}{2}^{-}]$ | 0.36 | 1.56 | 2.07 | 7.62 | 75 ± 22 |
| $[\Sigma^+_c + ^2P_{\lambda} \frac{1}{2}^{-}]$ | 46.59 | 14.66 | 1.05 | 78.01 |
| $[\Sigma^+_c + ^4P_{\lambda} \frac{3}{2}^{-}]$ | 4.36 | 4.23 | 0.34 | 13.5 |
| $[\Sigma^+_c + ^4P_{\lambda} \frac{1}{2}^{-}]$ | 4.5 | 0.22 | 10.79 | 26.53 |
| $[\Sigma^+_c + ^4P_{\lambda} \frac{1}{2}^{-}]$ | 27.08 | 1.34 | 1.41 | 32.58 |
| $[\Sigma^+_c + ^2S_{\rho\rho} \frac{1}{2}^{-}]$ | 0.26 | 0.52 | 0.13 | 1.56 |
| $[\Sigma^+_c + ^4S_{\rho\rho} \frac{1}{2}^{-}]$ | 0.26 | 0.42 | 0.37 | 1.84 |
| $[\Sigma^+_c + ^2S_{\lambda\lambda} \frac{1}{2}^{-}]$ | 0.06 | 0.11 | 0.05 | 0.38 |
| $[\Sigma^+_c + ^4S_{\lambda\lambda} \frac{1}{2}^{-}]$ | 0.06 | 0.03 | 0.11 | 0.34 |