Probabilistic Uncertainty Analysis of Reliability of Systems with Complex Interconnections

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Abstract. In the present age, modern technical systems, such as vehicles, should have reliability requirements. The sensors and their networks used in modern technical systems can be modelled as a System with Complex Interconnections (SwCI). Most of the real SwCIs in the automotive industry are sensory, as Bridge Structure System (BSS). The BSS is widely used as an example in the field of system reliability analysis. The aims of this paper are to apply the True Table Method (TTM) and the Linear Sensitivity Model of System Reliability (LSMoSR), to investigate uncertainties of reliability of BSS in different cases.

1. Introduction
Industrial safety, reliability and uncertainties of system reliability are common phenomena. The system having no simple interconnections is called System with Complex Interconnection (SwCI) or complex system [4]. System and model uncertainties play a key role in mathematical model-based investigations of reliability of SwCIs such as vehicle sensory networks. Most of the real SwCIs in the automotive industry are sensory, such as the Bridge Structure System (BSS), which is widely used as an example in the field of system reliability analysis.

The Institute of Mechatronics and Vehicle Engineering of Óbuda University is examining sensory networks and systems including their reliable and safe operation [2]. In paper [5] the sensor network of Nissan Leaf Z0 electric vehicle was explored by Nagy and Tuloki.

Boucerred et al. worked out a novel approach for optimizing quantitative and qualitative analysis used for dependability evaluation of intelligent systems [1].

The main application of mathematics in engineering is to mathematically model technical systems and their synthesis, analysis or model-based simulation. In the course of mathematical modeling the modelers can meet different model uncertainties. The reasons can be data inaccuracy or lack of knowledge by the modelers.

The parametric model of uncertainty can be described by interval and stochastic variables [6]. These describing methods are called interval and probabilistic uncertainty analysis [7]. Möller and Beer demonstrated different methods of uncertainty modeling [3].

Pokorádi reviewed literature and summarized mathematical modelling methods and their application to diagnose technical systems and actual questions of mathematical model uncertainties [8]. The paper [15] gives a short overview of mathematical models and tools for safety management, which were re-searched by the Author.

Pokorádi adapted linearized mathematical diagnostic methodology to determine system reliability sensitivity. He worked out the Linear Fault Tree Sensitivity Model (LFTSM) [9]; Linear Sensitivity
Model of System Reliability (LSMoSR) and Linear Sensitivity Model of System Unreliability (LSMoSU) [10].

By adaptation of the models mentioned above Pokorádi proposed a new modular approach method to determine sensitivity of failure probability [11] and reliability [12] of BSSs. Then the Author analyzed the uncertainty of SwCIs by interval method [13], [14] and by the Monte-Carlo Simulation-based probabilistic method [16]. Figure 1 demonstrates connection between references.

![Connection between References](image)

**Figure 1.** Connection between References

The present paper applies the approach of the works mentioned above. The main aim of this paper is to present the methodology of modular approach probabilistic analysis, in order to determine the probabilistic parameters of uncertainty of SwCI’s reliability.

The paper is organized as follows: Section 2 shows the uncertainties of mathematical models. Section 3 describes the reliability analysis of SwCI based on TT. Section 4 outlines methodology of linearized sensitivity modeling. Section 5 demonstrates a linearized LSMoSR-based probabilistic uncertainty analysis method Section 6 shows a case study to demonstrate possibilities of use of the proposed method.

2. Uncertainties of Mathematical Models

A mathematical model is the simplified description of an investigated system by mathematical equation or system of equations from the point of view of the given investigation. Real technical systems are complex. Additionally, a SwCI consists of a large number of units with interconnections. However, the mathematical model should be simplified, but doing so can also introduce imprecision.

A mathematical model can be characterized by

\[
M - \text{structure (e.g. stochastic, as reliability models of SwCIs)}; \\
p - \text{inner – independent – parameter(s) (in this study: number of elements)}; \\
u - \text{input – independent – parameter(s) (in this study: reliabilities of elements)};
\]

and responds by output – dependent – parameter(s) \( y \).
For demonstration of types of uncertainty and their investigation methods, let
\[ y = f(x). \]  
(1)
general mathematical model, where
\[ y \] - vector of dependent (output) parameters;
\[ x \] - vector of independent (input and inner) parameters.

Model uncertainty is epistemic, if the modeler uses improper natural laws or reduction. This uncertainty may be comprised of substantial amounts of objectivity and subjectivity.

The epistemic uncertainty means the incorrect structure \( M \) of model (figure 2.a).

![Figure 2. Epistemic and Parametric Uncertainty](image)

Parametric uncertainties are an inseparable variation associated with the modelled system or its environment (see figure 2.b). Their engineering sources are:
- measuring noise;
- inaccurate measuring;
- imprecise digitalization;
- wrong statistical information.

Parametric uncertainty means anomalies of independent parameters.

Referring to equation (1), there are two primary ways to investigate parametric uncertainties. The first is the interval uncertainty analysis that characterizes a given uncertainty by
\[ i_y = f_i(i_x) \]  
(2)
general function, where
\[ i_y \] - vector of intervals of output parameters;
\[ i_x \] - vector of intervals of input parameters.

Another fundamental investigation method is the probabilistic analysis that describes uncertainty by probability distributions. In this case the
\[ d_y = f_{dd}(d_x) \]  
(3)
general function is used, where
\[ d_x \] - vector of distributions of independent parameters;
\[ d_y \] - vector of distributions of dependent parameters.

The main task of uncertainty analysis is the determining of functions \( f_i \) and \( f_{dd} \) by mathematical model. This study shows a linearized model-based method to determine the function \( f_{dd} \) of distributions of variables.
3. Reliability Model of System with Complex Interconnections

The investigated SwCI is named Bridge Structure system (BSS) that contains five blocks, A; B; C; D; E (see Figure 3). Their availabilities are characterized by reliability \( r_s \) and probability of failure \( p_s \).

The elements have only two states, – operable (designated as 1) and fault (designated by 0). The sum of the probabilities of the states mentioned above

\[
r_s + p_s = 1.
\]  

The Truth Table (TT) lists each possible system states and their probabilities \( Q_s \).

| \( i \) | A | B | C | D | E | System | \( Q_s \) |
|---|---|---|---|---|---|---------|---------|
| 1 | 0 | 0 | 0 | 0 | 0 | \( p_A p_B p_C p_D p_E \) |
| 2 | 1 | 0 | 0 | 0 | 0 | \( r_A p_B p_C p_D p_E \) |
| 3 | 0 | 1 | 0 | 0 | 0 | \( p_A r_B p_C p_D p_E \) |
| 4 | 1 | 1 | 0 | 0 | 0 | \( r_A r_B p_C p_D p_E \) |
| 5 | 0 | 0 | 1 | 0 | 0 | \( p_A p_B r_C p_D p_E \) |
| 6 | 1 | 0 | 1 | 0 | 0 | \( r_A p_B r_C p_D p_E \) |
| 7 | 0 | 1 | 1 | 0 | 0 | \( p_A r_B r_C p_D p_E \) |
| 8 | 1 | 1 | 1 | 0 | 0 | \( r_A r_B r_C p_D p_E \) |
| 9 | 0 | 0 | 0 | 1 | 0 | \( p_A p_B p_C r_D p_E \) |
| 10 | 1 | 0 | 0 | 1 | 0 | \( r_A p_B p_C r_D p_E \) |
| 11 | 0 | 1 | 0 | 1 | 0 | \( p_A r_B p_C r_D p_E \) |
| 12 | 1 | 1 | 0 | 1 | 0 | \( r_A r_B p_C r_D p_E \) |
| 13 | 0 | 0 | 1 | 1 | 0 | \( p_A p_B p_C r_D p_E \) |
| 14 | 1 | 0 | 1 | 1 | 0 | \( r_A p_B p_C r_D p_E \) |
| 15 | 0 | 1 | 1 | 1 | 0 | \( p_A r_B r_C r_D p_E \) |
| 16 | 1 | 1 | 1 | 1 | 0 | \( r_A r_B r_C r_D p_E \) |
| 17 | 0 | 0 | 0 | 0 | 1 | \( p_A p_B p_C p_D r_E \) |
| 18 | 1 | 0 | 0 | 0 | 1 | \( r_A p_B p_C p_D r_E \) |
| 19 | 0 | 1 | 0 | 0 | 1 | \( p_A r_B p_C p_D r_E \) |
| 20 | 1 | 1 | 0 | 0 | 1 | \( r_A r_B p_C p_D r_E \) |
| 21 | 0 | 0 | 1 | 0 | 1 | \( p_A p_B r_C p_D r_E \) |
| 22 | 1 | 0 | 1 | 0 | 1 | \( r_A p_B r_C p_D r_E \) |
| 23 | 0 | 1 | 1 | 0 | 1 | \( p_A r_B r_C p_D r_E \) |
| 24 | 1 | 1 | 1 | 0 | 1 | \( r_A r_B r_C p_D r_E \) |
| 25 | 0 | 0 | 0 | 1 | 1 | \( p_A p_B p_C r_D r_E \) |
| 26 | 1 | 0 | 0 | 1 | 1 | \( r_A p_B p_C r_D r_E \) |
| 27 | 0 | 1 | 0 | 1 | 1 | \( p_A r_B p_C r_D r_E \) |
| 28 | 1 | 1 | 0 | 1 | 1 | \( r_A r_B p_C r_D r_E \) |
| 29 | 0 | 0 | 1 | 1 | 1 | \( p_A p_B r_C r_D r_E \) |
| 30 | 1 | 0 | 1 | 1 | 1 | \( r_A p_B r_C r_D r_E \) |
| 31 | 0 | 1 | 1 | 1 | 1 | \( p_A r_B r_C r_D r_E \) |
| 32 | 1 | 1 | 1 | 1 | 1 | \( r_A r_B r_C r_D r_E \) |
3.1. General System Reliability

In general, the BSS is operating if the matter, sign or energy can “go across”. Figure 4 shows system reliabilities $R_{sys}$ plotted against different reliabilities of component $r_i$.

The state probabilities resulting in an operating system are included in the rows 6; 8; 11; 12; 14; 15; 16; 22; 23; 24; 26; 27; 28; 30; 31 and 32. The general system reliability can be determined by

$$R_{sys} = Q_6 + Q_8 + Q_{11} + Q_{12} + Q_{14} + Q_{15} + Q_{16} + Q_{17} + Q_{22} + Q_{23} + Q_{24} + Q_{26} + Q_{27} + Q_{28} + Q_{30} + Q_{31} + Q_{32}. \quad (5)$$

sum of the operable system state probabilities.

3.2. The Most Critical System Reliability

In the most critical case there are not any redundancies in the system (see Figure 5.).

There are four – 6; 11; 23 and 26 (see Table 1.) – most critical system states. The probability of the most critical system state can be determined by the following equation

$$R_{crit} = Q_6 + Q_{11} + Q_{23} + Q_{26}. \quad (6)$$
The figure 6 shows the most critical system reliabilities $R_{crd}$ depend on reliabilities of blocks $r_i$.

**Figure 5. The Most Critical System States**

**Figure 6. The Most Critical System Reliability depends on Reliabilities of Blocks**

### 4. Linearized Sensitivity Model

The general methodology of setting up Linear Sensitivity Model can be read in detail in references [8] and [10]. The probabilities of possible system states (see Table 1.) can be described by the following general form

$$Q_j = \prod_{i=A}^{E} u_j(r_i) .$$

If the state of the component is operable, the function is $u_j(r_i) = r_i$, and the sensitivity coefficient is:

$$K_{ij} = r_i .$$

If the state of the component is faulty, the function is $u_j(r_i) = 1 - r_i$, so the sensitivity coefficient is:

$$K_{ji} = -\frac{r_i}{Q_j} \prod_{k \neq i}^{E} u_k .$$
In case of functions determining system reliabilities directly – see equations (5) and (6) – the sensitivity coefficient is:

\[ K_i = \frac{Q_i}{R_{sys}}. \]  

(10)

The connection between relative changes of the output (dependent) and the independent variables can be described by

\[ \mathbf{A}\delta y = \mathbf{B}\delta x \]  

(11)

matrix equation, where \( \delta y \) and \( \delta x \) are vectors of relative changing of dependent and independent variables. \( \mathbf{A} \) and \( \mathbf{B} \) coefficient matrices are dependent and independent variables.

Establishing the relative sensitivity coefficient matrix of the system reliabilities, the equation

\[ \delta y = \mathbf{D}\delta x \]  

(13)

can be depicted for relative sensitivity investigations [11]. The independent variables vector consists of reliabilities of components:

\[ \mathbf{x}^T = [r_A, r_B, r_C, r_D, r_E] \]  

(14)

4.1. General System Reliability

By equation (5), the vector of dependent parameters has the following probability of system reliability and operating system states probabilities:

\[ \mathbf{y}_{sys} = \begin{bmatrix} R_{sys}, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, Q_9, Q_{10}, Q_{11}, Q_{12} \end{bmatrix} \]  

(15)

The coefficient matrix of the dependent parameters:

\[ \mathbf{A}_{sys} = \begin{bmatrix} 1 & -K_8 & -K_9 & -K_{11} & -K_{12} & -K_{13} & -K_{14} & -K_{15} & -K_{16} & -K_{17} & -K_{18} & -K_{19} & -K_{20} & -K_{21} & -K_{22} & -K_{23} & -K_{24} & -K_{25} & -K_{26} & -K_{27} & -K_{28} & -K_{29} & -K_{30} & -K_{31} & -K_{32} \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]  

(16)
The coefficient matrix of independent variables:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & K_{6B} & 1 & K_{6D} & K_{6E} \\
1 & 1 & 1 & K_{6D} & K_{6E} \\
K_{11A} & 1 & K_{11C} & 1 & K_{11E} \\
1 & 1 & K_{12C} & 1 & K_{12E} \\
1 & K_{14B} & 1 & 1 & K_{14E} \\
K_{15A} & 1 & 1 & 1 & K_{15E} \\
1 & 1 & 1 & 1 & K_{16E} \\
1 & K_{22B} & 1 & K_{22D} & 1 \\
K_{23A} & 1 & 1 & K_{23D} & 1 \\
1 & 1 & 1 & K_{24D} & 1 \\
1 & K_{26B} & K_{26C} & 1 & 1 \\
K_{27A} & 1 & K_{27C} & 1 & 1 \\
1 & K_{28C} & 1 & 1 & 1 \\
1 & K_{30B} & 1 & 1 & 1 \\
K_{31A} & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\]

\[B_{sys} = \begin{bmatrix}
1 & K_{11A} & 1 & K_{11C} & 1 & K_{11E} \\
1 & K_{12C} & 1 & K_{12E} \\
1 & K_{14B} & 1 & 1 & K_{14E} \\
K_{15A} & 1 & 1 & 1 & K_{15E} \\
1 & 1 & 1 & 1 & K_{16E} \\
1 & K_{22B} & 1 & K_{22D} & 1 \\
K_{23A} & 1 & 1 & K_{23D} & 1 \\
1 & 1 & 1 & K_{24D} & 1 \\
1 & K_{26B} & K_{26C} & 1 & 1 \\
K_{27A} & 1 & K_{27C} & 1 & 1 \\
1 & K_{28C} & 1 & 1 & 1 \\
1 & K_{30B} & 1 & 1 & 1 \\
K_{31A} & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}\]

(17)

4.2. The Most Critical System Reliability

The vector of dependent parameters has the probability of the most critical system reliability and its system states – 6; 11; 23 and 26 in Table I. – see equation (6):

\[
y_{\text{crit}}^T = [R_{\text{crit}} \quad Q_6 \quad Q_{11} \quad Q_{23} \quad Q_{26}] \]

(18)

The coefficient matrix of the dependent variables:

\[
A_{\text{crit}} = \begin{bmatrix}
1 & -K_6 & -K_{11} & -K_{23} & -K_{26} \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(19)

The coefficient matrix of independent variables:

\[
B_{\text{crit}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & K_{6B} & 1 & K_{6D} & K_{6E} \\
K_{11A} & 1 & K_{11C} & 1 & K_{11E} \\
K_{23A} & 1 & 1 & K_{23D} & 1 \\
1 & K_{26B} & K_{26C} & 1 & 1 \\
\end{bmatrix}
\]

(20)
5. Probabilistic Uncertainty Analysis based on Linearized Uncertainty Model

For a general uncertainty model, the following matrices and vectors should be introduced:

The nominal value matrix of the independent parameters:

\[ \mathbf{X} = \begin{bmatrix} x_{1\text{nom}} & x_{2\text{nom}} & \cdots & x_{z\text{nom}} \end{bmatrix} \]  

(21)

The nominal value matrix of the dependent variables:

\[ \mathbf{Y} = \begin{bmatrix} y_{1\text{nom}} & y_{2\text{nom}} & \cdots & y_{unom} \end{bmatrix} \]  

(22)

Vector of measured variances of independent parameters:

\[ \mathbf{x}^T = \begin{bmatrix} \hat{x}_1 & \hat{x}_2 & \cdots & \hat{x}_z \end{bmatrix} \]  

(23)

Vector of measured expected values of independent parameters:

\[ \mathbf{x}^T = \begin{bmatrix} \bar{x}_1 & \bar{x}_2 & \cdots & \bar{x}_z \end{bmatrix} \]  

(24)

Let us suppose that manufacturing anomalies of the internal parameters are interdependent random variables with a normal distribution.

5.1. Determination of Variances

The variances of reliabilities of elements can be determined by their statistical analysis. If there are not enough statistical data, variances should be determined as a sixth parts of tolerance zones due to the so-called "3σ-rule":

\[ \hat{x}_i = \frac{x_{i\text{max}} - x_{i\text{min}}}{6}. \]  

(25)

Because the random variables of normal distribution with expected value \( m \) and variance \( \sigma \) will fall "practically certainly" in the \((m-3\sigma, m+3\sigma)\) interval – its probability in fact is 0.9973 [8].

To determine the variances of dependent parameters, the vector of relative variances of independent parameters should be determined by the equation

\[ \delta \mathbf{x} = \mathbf{X}^{-1} \mathbf{x} \]  

(26)

Using the sensitivity matrix of the investigated system, the vector of relative variances of external parameters is:

\[ \delta \mathbf{y} = \mathbf{D} \mathbf{X}^{-1} \mathbf{x} \]  

(27)

Knowing the nominal values of the external parameters, the vector of their measured variances should be determined by following equation:

\[ \mathbf{y} = \mathbf{Y} \mathbf{D} \mathbf{X}^{-1} \mathbf{x} \]  

(28)

By introducing the "measured sensitivity coefficient matrix":

\[ \mathbf{S} = \mathbf{Y} \mathbf{D} \mathbf{X}^{-1} \]  

(29)

the equation (28) can be simplified:

\[ \mathbf{y} = \mathbf{S} \mathbf{x} \]  

(30)

5.2. Determination of Expected Values

To determine expected values of dependent parameters, initially the vector of relative expected values of independent parameters should be determined. The expected values of independent parameters are
the means of their tolerance zones. It is important to mention, if the tolerance zones are asymmetric, the expected value will not be equal to the nominal value of the given parameters. This vector should show the relative values of difference between measured expected and nominal values to nominal ones

\[ \delta \bar{x}_i = \frac{\bar{x}_i - x_{\text{nom}}}{x_{\text{nom}}} . \] (31)

Applying the measured diagnostic coefficient matrix \( S \) – see equation (29):

\[ \bar{y} = S(\bar{x} - x_{\text{nom}}) + y_{\text{nom}} . \] (32)

The equations (30) and (32) mean, the function of probabilistic uncertainties of BSS’s reliability is generally depicted by the equation (3).

Knowing the variances and the expected values of external parameters; their "manufacturing tolerance zones," that are result of manufacturing anomalies of units and elements of the system, can be determined. These intervals should be determined using the „3σ-rule” mentioned above, that is the vectors of their maximum and minimum values:

\[ y_{\text{max}} = \bar{y} + 3\hat{y} \]
\[ y_{\text{min}} = \bar{y} - 3\hat{y} . \] (32)

The equation (32) depicts the function of interval uncertainties of BSS’s reliability that is generally shown by equation (3).

5.3. Theoretical Investigations

To demonstrate possibility of use of the proposed method, variances of system reliabilities were determined in case of different reliabilities of elements and \( \hat{p}_i = 0.0667 \).

![Figure 7. Variances of System Reliabilities depend on Reliabilities of Components (\( \hat{p}_i = 0.0667 \))](image)

The variances are shown by figure 7 (general case) and figure 8 (the most critical case). The expected values in cases of different reliabilities of elements can be seen in figures 4 and 6.

The system reliability \( R_{\text{sys}} \) was determined by equation (2) and the above-mentioned method was used. The results are shown by figure 6, by figure 9 (general case), and by figure 10 (the most critical case). The expected values in cases of different reliabilities of elements can be seen in figures 4 and 6.
Figure 8. Variances of Critical System Reliabilities depend on Reliabilities of Components

\[ \hat{p}_i = 0.0667 \]

Figure 9. Intervals of System Reliabilities depend on Reliabilities of Components

\[ \hat{p}_i = 0.0667 \]

Figure 10. Intervals of Critical System Reliabilities depend on Reliabilities of Components

\[ \hat{p}_i = 0.0667 \]
6. Investigation of Manufacturing Unit’s Reliability (Practical Case Study)

A manufacturing factory has two production lines that are connected by a buffer unit to balance fluctuation of their productivities. The reliability of these two parallel production lines and the buffer store system can be investigated as a BSS.

During determination of the reliabilities of a factory statistically, relatively large-scale uncertainties were observed. The normality tests shown can be estimated as normal distribution data. The result of statistical tests are shown by Table 2.

| Table 2. Statistical Data of Reliability of Blocks |
|---------------------------------|
| i   | A    | B    | C    | D    | E    |
| r_{imax} | 0.885 | 0.874 | 0.911 | 0.877 | 0.958 |
| r_{inom}  | 0.871 | 0.872 | 0.901 | 0.881 | 0.955 |
| r_{imin}  | 0.861 | 0.870 | 0.899 | 0.884 | 0.951 |
| \hat{r}_i | 0.873 | 0.872 | 0.905 | 0.8755 | 0.9545 |
| \hat{\hat{r}}_i | 0.0041 | 0.0006 | 0.0019 | 0.0005 | 0.0012 |

6.1. General Manufacturing Unit’s Reliability

Initially the proposed method was used to determine uncertainty of reliability of a manufacturing unit in a general case. Table 3 shows the results of probabilistic uncertainty analysis of the manufacturing unit reliabilities and their comparison with results of interval uncertainty analysis shown by publication [14]. The relative differences between results of the two methods are acceptable (less than $1.0464 \times 10^{-03}$). They are observable as numerical errors.

| Table 3. Comparison Results of Probabilistic and Interval Uncertainty Analyses |
|---------------------------------|
|                                   | Probabilistic | Interval [14] | Difference | Rel. Difference |
| R_{sysmax}                      | 0.97387       | 0.97387       | 0.0000     | 0.0000         |
| R_{sysnom}                      | 0.97110       | 0.97092       | -1.7505 \times 10^{-04} | -1.8029 \times 10^{-04} |
| R_{sysmin}                      | 0.96832       | 0.96933       | 1.0143 \times 10^{-03} | 1.0464 \times 10^{-03} |
| \bar{R}_{sys}                   | 0.97110       |               |            |                |
| \bar{\bar{R}}_{sys}            | 9.2547 \times 10^{-4} |           |            |                |

6.2. The Most Critical Manufacturing Unit’s Reliability

Using proposed method, the probabilistic and interval uncertainty analysis of the given production unit was done too. Table 4 shows the results of uncertainty analysis.

6.3. Discussions

These results can be used for the estimation of uncertainties of working expenditures and maintenance costs of the modelled factory; therefore, maintenance management can get supporting data to make correct decisions. Maintenance management can determine the required number of spare parts to ensure continuous operation of investigated manufacturing units.
Table 4. Results of the Most Critical Reliability of a Manufacturing Unit

| Reliability | Value   |
|-------------|---------|
| $R_{\text{critmax}}$ | 0.022623 |
| $R_{\text{critnom}}$ | 0.02133  |
| $R_{\text{critmin}}$ | 0.02003  |
| $\bar{R}_{\text{crit}}$ | 0.02133  |
| $\hat{R}_{\text{crit}}$ | $4.3217 \times 10^{-4}$ |

The following conclusions can be deduced from the comparison of uncertainty analyses:

a) Interval analysis “only” gives expected minimum and maximum values of reliability of a manufacturing unit.

b) Probabilistic analysis shows the parameters of distribution of reliability of a manufacturing unit. So, its results can be used for estimating required number for spare part depending on the required estimating uncertainty.

c) The drawback of probabilistic uncertainty analysis lies in the fact, that its result can be impossible from an engineering point of view. In the case of reliability modelling, one may find probability, which is greater than 1 or less than 0, because probability distributions generally have infinite domains from a mathematical point of view. This disadvantage can be eliminated by Probability–Bounds Analysis.

7. Closing Remarks

This paper proposed a new probabilistic uncertainty analysis method of SwCIs’ reliability. Its possibility of use was shown by way of a theoretical and a practical case study of BSS reliabilities.

The Author’s prospective future research direction is the study of uncertainty analysis methodologies of network structure systems (such as the sensor and commutation networks of vehicles) reliability using Monte-Carlo Simulation.

References

[1] Boucerredj L and Debbache N 2018, Qualitative and quantitative optimization for dependability analysis Informatica (Slovenia) 42 439-450
[2] Lázár-Fülep T 2018 Few Words about Reliability of Vehicle Systems with Complex Interconnections, Proc. of the CINTI 2018 273-276
[3] Möller B and Beer M 2008 Engineering computation under uncertainty - Capabilities of non-traditional models Computers and Structures 86 1024-1041
[4] Myers A 2010 Complex System Reliability (London: Springer)
[5] Nagy I and Tuloki Sz 2018 Fault Analysis and System Modelling in Vehicle Engineering, Proc of the CINTI 2018 313-319
[6] Oberkampf WL, DeLand SM, Rutherford BM, Diegert, KV and Alvin KF. 2002 Error and uncertainty in modeling and simulation Reliability Engineering and System Safety 75 333-357
[7] Oberkampf WL, Helton JC, Joslyn CA, Wojtkiewicz SF and Ferson S 2004 Challenge problems: Uncertainty in system response given uncertain parameters Reliability Engineering and System Safety 85 11-19
[8] Pokorádi L 2008 Rendszerek és folyamatok modellezése (Debrecen: Campus Kiadó)
[9] Pokorádi L. 2011 Sensitivity Investigation of Fault Tree Analysis with Matrix-Algebraic Method Theory and Applications of Mathematics & Computer Science 1 34-44.
[10] Pokorádi L 2014 Sensitivity analysis of reliability of Systems with Complex Interconnections *Journal of Loss Prevention in the Process Industries*, **32** 436-442

[11] Pokorádi L 2015 Failure probability analysis of bridge structure systems Proc. of the SACI 2015 147-150

[12] Pokorádi L and Seebauer M 2019 Sensitivity Analysis of Bridge Structure Systems' Reliability *Proc of the SACI 2019* 370-375

[13] Pokorádi L and Felker P 2019 Interval uncertainty analysis of bridge structure systems' reliability *Proc of the SISY 2019* 235-239

[14] Pokorádi L 2019a Interval Uncertainty Analysis of Reliability of Systems with Complex Interconnections *GRADUS* **6** 145-155

[15] Pokorádi L 2019b Models in Safety Management *Machine Design* **11** 85-94.

[16] Pokorádi L 2020 Probabilistic Uncertainty Analysis of Bridge Structure Systems’ Reliability *Acta Polytechnica Hungarica* – under publishing