Evaluating the RiskMetrics Methodology in Measuring Volatility and Value-at-Risk in Financial Markets

Szilárd Pafka\textsuperscript{a,1}, Imre Kondor\textsuperscript{a,b,2}

\textsuperscript{a}Department of Physics of Complex Systems, Eötvös University, Pázmány P. sétány 1/a, H-1117 Budapest, Hungary
\textsuperscript{b}Market Risk Research Department, Raiffeisen Bank, Akadémia u. 6, H-1054 Budapest, Hungary

Abstract

We analyze the performance of RiskMetrics, a widely used methodology for measuring market risk. Based on the assumption of normally distributed returns, the RiskMetrics model completely ignores the presence of fat tails in the distribution function, which is an important feature of financial data. Nevertheless, it was commonly found that RiskMetrics performs satisfactorily well, and therefore the technique has become widely used in the financial industry. We find, however, that the success of RiskMetrics is the artifact of the choice of the risk measure. First, the outstanding performance of volatility estimates is basically due to the choice of a very short (one-period ahead) forecasting horizon. Second, the satisfactory performance in obtaining Value-at-Risk by simply multiplying volatility with a constant factor is mainly due to the choice of the particular significance level.

Key words: RiskMetrics, market risk, risk measurement, volatility, Value-at-Risk

1 Introduction

Risk management is one of the top priorities in the financial industry today. A huge effort is being invested into developing reliable risk measurement methods and sound risk management techniques by academics and practitioners alike. J. P. Morgan was the first to develop a comprehensive market risk management methodology based on the Value-at-Risk (VaR) concept [1]. Their
product, RiskMetrics [2] has become extremely popular and widespread. It has greatly contributed to the dissemination of a number of basic statistical risk measurement methods and to the general acceptance of VaR as an industry standard. Although backtests performed first by J. P. Morgan and later by other market participants lent support to the RiskMetrics model, its basic assumptions were shown to be questionable from several points of view (see e.g. [3]). Moreover, the existence of fat tails in real market data (see [4,5] for a discussion) is in a clear conflict with RiskMetrics’ assumption of normally distributed returns, which can lead to a gross underestimation of risk. Furthermore, serious doubt has recently been raised as to the stability and information content of the empirical covariance matrices used by the model for calculating the risk of portfolios [6–8].

Accurate evaluation of risks in financial markets is crucial for the proper assessment and efficient mitigation of risk. Therefore, it is important to see to what extent widespread methodologies like RiskMetrics are reliable and what their possible limitations are. In particular, we try to understand why, despite the evident oversimplifications embedded in the model, it can perform satisfactorily. We will argue that the apparent success of RiskMetrics is due basically to the way risk is quantified in this framework, which does not necessarily mean that this particular risk measure is the most adequate one.

The paper is organized as follows. In Section 2 a sketchy overview of the RiskMetrics methodology will be presented. In Section 3 the reasons for the success of RiskMetrics will be discussed. Finally, Section 4 is a short summary.

2 Overview of the RiskMetrics Methodology

It is well-known that daily returns are uncorrelated whereas the squared returns are strongly autocorrelated. As a consequence, periods of persistent high volatility are followed by periods of persistent low volatility, a phenomenon known as “volatility clustering”. These features are incorporated in RiskMetrics by choosing a particular autoregressive moving average process to model the price process (see below). Furthermore, RiskMetrics makes the very strong assumption that returns are conditionally\[3\] normally distributed. Since the standard deviation of returns is usually much higher than the mean, the latter is neglected in the model\[4\]; and, as a consequence, the standard deviation

\[\text{standard deviation} \approx 0.95\%\]

3 Conditional here means conditional on the information set at time \(t\), which usually consists of the past return series available at time \(t\).

4 The typical yearly mean return in equity markets is 5%, while the typical standard deviation is 15%. For a time horizon of one day, however, the mean becomes \(5%/250=0.02\%\), while the standard deviation becomes \(15%/\sqrt{250} \approx 0.95\%\), i.e.
remains the only parameter of the conditional probability distribution function. In order to avoid the usual problems related to the uniformly weighted moving averages, Riskmetrics uses the so called exponentially weighted moving average (EWMA) method [2, pp. 78–84] which is meant to represent the finite memory of the market. Accordingly, the estimator for the volatility is chosen to be

$$\sigma_{t+1|t}^2 = \frac{\sum_{\tau=0}^{\infty} \lambda^\tau r_{t-\tau}^2}{\sum_{\tau=0}^{\infty} \lambda^\tau} = (1 - \lambda) \sum_{\tau=0}^{\infty} \lambda^\tau r_{t-\tau}^2, \quad (1)$$

where $\lambda$ is a parameter of the model ($0 < \lambda < 1$). The notation $\sigma_{t+1|t}$ emphasizes that the volatility estimated on a given day ($t$) is actually used as a predictor for the volatility of the next day ($t+1$). The daily VaR at confidence level $p$ (e.g. 95%) can then be calculated (under the normality assumption) by multiplying $\sigma_{t+1|t}$ with the $1 - p$ quantile of the standard normal distribution. Moreover, this technique can be used to measure the risk of individual assets and portfolios of assets as well. For linear portfolios (i.e. containing no options) the usual method to obtain the volatility is to estimate the covariance matrix of asset returns, element-by-element, using the EWMA technique and then calculate the portfolio volatility as $\sigma_p^2 = \sum_{i,j} w_i w_j \sigma_{ij}$, where $w_i$ is the weight of asset $i$ in the portfolio. Equivalently, however, one can calculate the return on the portfolio first, and then apply the EWMA technique directly to the whole portfolio [9]. Finally, the value of the parameter $\lambda$ is determined by an optimization procedure. On a widely diversified international portfolio, RiskMetrics found that the value $\lambda = 0.94$ produces the best backtesting results [2, pp. 97–101].

3 Why Does RiskMetrics Work?

In order to explain the reasonably successful performance of RiskMetrics, first we recall the work by Nelson [10] who showed that even misspecified models can estimate volatility rather accurately. More explicitly, in [10] it is shown that if the return generating process is well approximated by a diffusion, a broad class of even misspecified ARCH models can provide consistent estimates of the conditional volatility. Since RiskMetrics can be considered as an IGARCH(1,1) model the results of [10] offer a natural explanation for the success of RiskMetrics in estimating volatility. Actually, in the RiskMetrics

---

5 See [11] for a survey on ARCH type models (e.g. ARCH, GARCH, IGARCH, etc.).

6 In the IGARCH(1,1) model, returns are generated by the following stochastic process: $r_t = \sigma_t \varepsilon_t$, where $\varepsilon_t \sim$ i.i.d.$(0,1)$, $\sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2$ and $\alpha_1 + \beta_1 = 1$. 

---
framework, this estimate is used as a one-day ahead volatility forecast, nevertheless it seems that this does not significantly worsen its accuracy. However, if one uses this estimate to calculate (as often required by regulators) a multi-period forecast using the simple “square-root-of-time” rule\(^7\), the quality of the forecast is bound to decline with the number of periods. A comparative study of the rate of deterioration of these forecasts with time within the RiskMetrics model resp. other, more sophisticated volatility models is an interesting topic for further research.

Let us turn to the apparent success of RiskMetrics in estimating the VaR now. The model sets the confidence level at 95%. The prescription to obtain this 5% quantile is to simply multiply the volatility estimate by 1.65 (as if returns were conditionally normally distributed). Why such a recipe can pass the backtests can be understood if one analyzes numerically the 5% quantiles of a few leptokurtic (fat tailed) distributions. It is very often found that despite the presence of fat tails, for many distributions the 5% quantile is roughly -1.65 times the standard deviation. For example, the 5% quantile of the Student \(t\) distribution with 7 degrees of freedom (which is leptokurtic and has a kurtosis of 5 similar to the typical kurtosis of returns in financial markets) is -1.60, very close to -1.65, or, conversely, the -1.65 percentile is 4.6%. This is illustrated in Fig. 1, where the PDF’s of the Student and the normal distributions are shown. It can be seen from the figure that the areas under the two PDF’s to the left of the -1.65 line are roughly equal. We also analyzed the empirical frequencies of RiskMetrics 95% VaR violations which correspond to returns \(r_{t+1} < -1.65 \sigma_{t+1|t}\), where \(\sigma_{t+1|t}\) is the RiskMetrics volatility estimate obtained

Since from Eq. (1) \(\sigma^2_{t+1|t} = (1 - \lambda) \left( r_t^2 + \lambda \frac{\sigma^2_{t+1|t-1}}{1-\lambda} \right)\), it follows that \(\sigma^2_{t+1|t} = \lambda \sigma^2_{t|t-1} + (1 - \lambda) r_t^2\) just as in the IGARCH(1,1) model (\(\omega = 0, \beta_1 = \lambda, \alpha_1 = 1 - \lambda\)).

\(^7\) The rule says that \(\sigma_{t+h|t} = \sqrt{h} \cdot \sigma_{t+1|t}\), which is derived on the basis of the assumption of uncorrelated returns [2, pp. 84–88].
on day $t$. For the 30 stocks of the Dow Jones Industrial Average (DJIA), which are among the most liquid stocks traded on the New York Stock Exchange, it was found\footnote{The sample consisted of the daily closing prices of the 30 stocks of the DJIA from August 1996 to August 2000, about 1000 data points for each stock.} that the VaR violations frequency was $4.7 \pm 0.6\%$, i.e. close to 5%. This explains why RiskMetrics is usually found so successful in evaluating risk (which it \textit{defines} as the VaR at 95% confidence).

It is evident, however, that for higher significance levels (e.g. 99%) the effect of fat tails becomes much stronger, and therefore the VaR will be seriously underestimated if one assumes normality. For example, the 1\% quantile of the Student $t$ distribution considered above is -2.54, significantly larger than under the normality assumption (-2.33), while the percentile corresponding to -2.33 is 1.43\%. This can also be seen from Fig. 1. Furthermore, for the DJIA data considered above, the rejection frequencies were $1.4 \pm 0.3\%$, significantly larger than 1\%. These effects are even more pronounced for a truncated Lévy distribution (TLD). For example, a Koponen-like \cite{12} TLD with Lévy exponent $\nu = 1.50$, scale parameter $c = 0.50$ and truncation parameter $\lambda = 0.17$, which provides an excellent fit to the Budapest Stock Exchange Index (BUX) data, has a 5\% quantile equal to $1.76\sigma$ whereas the 1\% quantile is already $3.75\sigma$! \cite{13}

Therefore, it can be concluded that the satisfactory performance of RiskMetrics in estimating VaR is mainly the artifact of the choice of the significance level of 95\%. However, existing capital adequacy regulations require 99\% confidence, and at this level RiskMetrics systematically underestimates risk.

### 4 Summary

In this paper we analyzed the performance of RiskMetrics, perhaps the most widely used methodology for measuring market risk. The RiskMetrics model is based on the unrealistic assumption of normally distributed returns, and completely ignores the presence of fat tails in the probability distribution, a most important feature of financial data. For this reason, one would expect the model to seriously underestimate risk. However, it was commonly found by market participants that RiskMetrics performed satisfactorily well and this helped the method to quickly become a standard in risk measurement. Nevertheless, we found that the success of RiskMetrics is actually the artifact of the choice of the risk measure: the effect of fat tails is minor when one calculates Value-at-Risk at 95\%, however, for higher significance levels fat tails in the distribution of returns will make the simple RiskMetrics rule of calculating VaR to underestimate risk.
RiskMetrics has played and continues to play an extremely useful role in disseminating risk management ideas and techniques, even if oversimplified. It is available free of charge and, coming with a careful documentation, it is completely transparent and amenable for a study like the present one: its limitations can be explored and, given sufficient resources, overcome. This is far from being the case with the overwhelming majority of the commercially available risk management systems which incorporate at least as strong simplifications as RiskMetrics, but coming in the the form of “black boxes,” are completely impossible to modify. The continuing dominance of the Gaussian paradigm in risk management software packages represents an industry-wide model risk.

Acknowledgements

It is a pleasure to thank B. Janecskó for useful interactions. This work has been supported by the Hungarian National Science Found OTKA Grant No. T 034835.

References

[1] Jorion P. (1997). Value at Risk: The New Benchmark for Controlling Market Risk, Chicago: Irwin

[2] RiskMetrics Group (1996). RiskMetrics – Technical Document, New York: J.P. Morgan/Reuters

[3] Alexander C. (1996). “Evaluating the Use of RiskMetrics as a Risk Measurement Tool for Your Operation: What Are Its Advantages and Limitations?” Derivatives: Use Trading and Regulation, 2, 277–285

[4] Mantegna R. N., H. E. Stanley (1999). An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge: Cambridge UP

[5] Bouchaud J.-P., M. Potters (2000). The Theory of Financial Risk: From Statistical Physics to Risk Management, Cambridge: Cambridge UP

[6] Laloux L., P. Cizeau, J.-P. Bouchaud, M. Potters (1999). “Noise Dressing of Financial Correlation Matrices,” Physical Review Letters, 83, 1467–1471

[7] Plerou V., P. Gopikrishnan, B. Rosenow, L. A. N. Amaral, H. E. Stanley (1999). “Universal and Nonuniversal Properties of Cross Correlations in Financial Time Series,” Physical Review Letters, 83, 1471–1474

[8] Galluccio S., J.-P. Bouchaud, M. Potters (1998). “Rational Decisions, Random Matrices and Spin Glasses,” Physica A, 259, 449–458
[9] Zangari P. (1997). “A General Approach to Calculating VaR without Volatilities and Correlations,” RiskMetrics Monitor, 2nd Quarter, 19–23

[10] Nelson D. (1992) “Filtering and Forecasting with Misspecified ARCH Models: Getting the Right Variance with the Wrong Model,” Journal of Econometrics, 52, 61–90

[11] Bollerslev T., R. F. Engle, D. B. Nelson (1994). “ARCH models,” in Engle R. F., D. L. McFadden (eds.), Handbook of Econometrics IV, New York: North-Holland, 2959–3038

[12] Koponen I. (1995). “Analytic Approach to the Problem of Convergence of Truncated Lévy Flights towards the Gaussian Stochastic Process,” Physical Review E, 52, 1197–1199

[13] Janecskó B. private communication