TARGET SPACE INTERPRETATION OF NEW MODULI IN 2D STRING THEORY

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ABSTRACT

We analyze the new states that have recently been discovered in 2D string theory by E. Witten and B. Zwiebach. Since the Liouville direction is uncompactified, we show that the deformations by the new ghost number two states generate equivalent classical solutions of the string fields. We argue that the new ghost number one states are responsible for generating transformations which relate such equivalent solutions. We also discuss the possible interpretation of higher ghost number states of those kinds.

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1. Introduction

Recent investigations of critical string theory in two dimensional target spacetime have displayed a very rich and interesting structure. One of the most unexpected features of this theory is that it contains infinite number of degrees of freedom (apart from a massless boson) in its spectrum. It would seem from a naive light-cone gauge argument would indicate that, since there are no transverse dimensions, there are no physical excitations except for the centre-of-mass of the string. However, it turns out that besides this massless excitation, this model contains new non-trivial operators at discrete values of momenta and at various ghost numbers [1]. In a subsequent analysis [3], it was shown that among those, the ghost number zero BRST invariant operators are the members of a ring called the ground ring and the ghost number one states are associated with symmetries of the theory. These symmetries also appeared in a different analysis of this model [4] [5].

Recently, Witten and Zwiebach [6] carried out a detailed BRST analysis of the theory and, after combining the left movers and the right movers, they found more physical discrete states and symmetries than had usually been supposed. In this letter, we concentrate on the target space interpretations of some of these operators which we refer to as the new operators. Specifically, we consider the new operators of ghost number two and one and comment on the members of higher ghost numbers. We find, since the Liouville coordinate is uncompactified, the deformations by the ghost number two operators of these kinds (which corresponds to moduli) generate different equivalent solutions of string fields. We explicitly demonstrate this fact by taking the lowest member of the series. Following descent equation [6], we show that if we add the corresponding term in the sigma model action as perturbation, it can be removed by suitably redefining the matter and Liouville coordinates. In the corresponding string field theory of this model, these deformations can be understood as pure gauge deformations where gauge transformation parameters are singular in momentum space (i.e. large at the target

† In the context of matrix model it appeared in Ref.[2].
space boundaries). In this sense, we believe the new states are trivial. We also
discuss the new ghost number one states which can naturally be thought of as
gauge transformation parameters in string field theory. The charges corresponding
to these parameters rotate ghost number two operators into the new operators and
hence, in a sense act trivially on the space of gauge inequivalent solutions.

Since some of our analysis closely mimic the analysis of the marginal defor-
mations in usual critical string theory [7], we start with that example focusing on
relevant issues. In section 3, we discuss the new ghost number two and one opera-
tors in 2D string theory. This letter ends with discussion on higher ghost number
operators which can be identified with the antifields in the Batalin-Vilkovisky for-
malism [8].

2. Marginal perturbation in critical string theory

We start with the example of 26 dimensional critical string theory action in
flat background, where,

\[ S = \frac{1}{8\pi} \int \sqrt{h} h^{\alpha\beta} \eta_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu \]  

(1)

This action has a set of marginal deformations of the form \( \int \lambda_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu h^{\alpha\beta} \sqrt{h} \),
with \( \lambda_{\mu\nu} \) being the coupling constants. With this perturbation, the action is

\[ S_{\text{pert.}} = \frac{1}{8\pi} \int \sqrt{h} h^{\alpha\beta} \eta_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + \int \lambda_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu h^{\alpha\beta} \sqrt{h}. \]

(2)

Naturally, depending on whether \( X^\mu \) is compact or non-compact, the perturbed
theory will behave in different manner.

(a) \( X^\mu \) uncompactified:

Redefining \( X^\mu \) as

\[ X'^\mu = (\sqrt{1 + \lambda})_{\mu\nu} X^\nu \]

(3)

\[ = X\mu + 1/2 \lambda_{\mu\nu} X^\nu + O(\lambda^2) \]

the perturbed action reduces to the original form (1). Hence the partition function
and the correlation functions will remain unchanged under such perturbation as long as $X^\mu$ is un-compactified.

(b) $X^\mu$ compactified:

In this case the perturbation in (2) is no longer trivial and in fact it can be thought of as radius changing perturbation in the following sense. Suppose $X^\mu$ is compactified on a circle of radius $R^\mu$. Take $\lambda_{\mu\nu} = \lambda_\mu \eta_{\mu\nu}$ (no summation). The coordinate redefinition of the form (3) will then change the radius from $R^\mu$ to $R^\mu \sqrt{1 + \lambda_\mu}$ (no summation). Since the spectrum of the theory changes with compactification radius, physics of the perturbed action can not be reproduced by the original action by target space coordinate redefinition. Hence we conclude that the marginal deformations of the kind mentioned above are genuine perturbations for compactified coordinates.

For later convenience, it is useful to discuss this situation in BRST closed string field theory [9]. In what follows, we will show that this marginal perturbation can be understood as pure gauge deformation in string field theory, if only $X^\mu$ is uncompactified and is not the case otherwise. To linear order, the string field $|\Psi\rangle$ satisfies the equation of motion

$$Q_B|\Psi\rangle = 0. \quad (4)$$

The linearized gauge variation of the string field is

$$\delta|\Psi\rangle = Q_B|\Lambda\rangle. \quad (5)$$

Here $Q_B$ is the nilpotent BRST charge of the first quantized theory and $\Lambda$ is the gauge transformation parameter.*

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* We take the off-shell string fields to be of ghost number two and they are annihilated by $b^-_0$ and $L^-_0$. Similarly $\Lambda$ are the parameters with ghost number one and as before are annihilated by $b^-_0$ and $L^-_0$. 

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For our purpose we expand $|\Psi\rangle$ in component fields as

$$|\Psi\rangle = h_{\mu\nu}c_1 \bar{c}_1 \alpha_{-1}^\mu \bar{\alpha}_{-1}^\nu |0\rangle + d(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1}) |0\rangle + ....... (6)$$

Here $h_{\mu\nu}$ and $d$ are the string fields of tensorial rank two and zero respectively and .... contains all other fields which are not important for our present discussion. Now let us take a particular gauge transformation

$$|\Lambda\rangle = \lim_{p \to 0} \xi_{\mu\nu}(c_1 \alpha_{-1}^\mu - \bar{c}_1 \bar{\alpha}_{-1}^\mu)e^{ipX_\nu} - 1 |0\rangle. (7)$$

Notice that this is a valid gauge transformation parameter when $X^\mu$ is uncompactified if we work with finite $p$ and take the limit at the end of the calculations.

Upon acting $Q_B$, we get,

$$Q_B|\Lambda\rangle = -2i \xi_{\mu\nu}c_1 \bar{c}_1 \alpha_{-1}^\mu \bar{\alpha}_{-1}^\nu |0\rangle - i \xi_{\mu}^\mu(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1}) |0\rangle. (8)$$

Now the gauge transformed string field in component form looks like

$$|\Psi'\rangle = (h_{\mu\nu} - 2i \xi_{\mu\nu})c_1 \bar{c}_1 \alpha_{-1}^\mu \bar{\alpha}_{-1}^\nu |0\rangle + (d - i \xi_{\mu}^\mu)(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1}) |0\rangle + ....... (9)$$

Since $|\Psi'\rangle$ and $|\Psi\rangle$ differ by a pure gauge state, they are certainly equivalent string field configurations at least to linear order. On the other hand, adding (8) to string field $|\Psi\rangle$ would mean adding $\int \lambda_{\mu\nu}\partial X^\mu \bar{\partial} X^\nu$ in the corresponding sigma model ($X^\mu$ uncompactified) action with the identification $\lambda_{\mu\nu} = -2i \xi_{\mu\nu}$ and with required change in the dilaton coupling [10].

Now we turn to the case when $X^\mu$ is compactified. Notice that since in this case the vertex operators take discrete momenta depending upon the radius, we can not use the limiting procedure as in (7). Instead, we write $|\Lambda\rangle$ as

$$|\Lambda\rangle = \xi_{\mu\nu}(c_1 \alpha_{-1}^\mu - \bar{c}_1 \bar{\alpha}_{-1}^\mu)X^\nu |0\rangle. (10)$$

This corresponds to a coordinate transformation parameter $\xi_{\mu}(X)$ going as $\xi_{\mu\nu}X^\nu$, which is not a well defined gauge transformation parameter. Hence $Q_B|\Lambda\rangle$ should
not be considered as a gauge deformation. Consequently, $|\Psi\rangle$ and $|\Psi'\rangle$ will then be two inequivalent string field configurations, which amounts to saying that the corresponding perturbation in the sigma-model acts as a non-trivial deformation of the original theory.

In the next section we analyze the ‘new’ states in $c = 1$ matter coupled to Liouville theory. We show that since the Liouville direction is uncompactified, situation is very similar to the first case. In particular, deformations by those type of states can be identified as pure gauge deformations in the corresponding string field theory.
3. Analysis of ‘new’ states in d=2 string theory

The action for $d = 2$ string at $SU(2)$ point is\footnote{Throughout this section we follow the notations and conventions of [6]}

$$S = \frac{1}{8\pi} \int d^2x \sqrt{h} (h^{ij} \partial_i X \partial_j X + h^{ij} \partial_i \phi \partial_j \phi) - \frac{1}{2\sqrt{2}\pi} \int d^2x \sqrt{h} R^{(2)}. \quad (11)$$

Here $h$ and $R^{(2)}$ are the world sheet metric and Ricci scalar respectively and $\phi$ is the Liouville coordinate with background charge $2\sqrt{2}$.

3.1. States of Ghost number two:

Among the discrete states, annihilated by $Q_B$ and $b_0^-$, at ghost number two, there are states of the form:

$$(a + \bar{a}) Y_{s,n}^+ O_{s-1,n'}^+; \quad (a + \bar{a}) O_{s-1,n} Y_{s,n'}^+$$

where

$$Y_{s,n}^+ = c V_{s,n} e^{\sqrt{2}(1-s) \phi}$$

with $s = 0, 1/2, 1, ... ; n = s, s - 1, ..., -s$ and $V_{s,n}$ is a primary field constructed from the matter sector $X$. On the other hand, $O_{s,n}$ are the ground ring operators of ghost number zero such that

$$O_{s,n} = x^{s+n} y^{s-n}$$

with

$$x = O_{\frac{1}{2},\frac{1}{2}} = (cb + \frac{i}{\sqrt{2}} (\partial X - i \partial \phi)) e^{i(X+i\phi)/\sqrt{2}}$$
$$y = O_{\frac{1}{2},-\frac{1}{2}} = (cb - \frac{i}{\sqrt{2}} (\partial X + i \partial \phi)) e^{-i(X-i\phi)/\sqrt{2}}$$

The operator $a$ which plays a crucial role in constructing these new moduli is

$$a = [Q_B^L, \phi] = c \partial \phi + \sqrt{2} \partial c.$$
The first non-trivial example of these kind of new operators are states with \( s = 1, n = n' = 0 \). We take the combination

\[
(a + \bar{a})Y_{1,0}^+ \bar{O}_{0,0} - (a + \bar{a})O_{0,0}Y_{1,0}^+
= -c\bar{c}(\partial X \bar{\phi} + \partial \phi \bar{\phi}) - \sqrt{2}(c\partial c\partial X + c\bar{c}\partial\bar{c}\partial\bar{X} - \bar{c}\partial\bar{c}\partial\bar{X} - \bar{c}\partial\bar{c}\partial\bar{X})
\]

(12)

The 2-form corresponding to this state that can be added to the sigma-model action is \((\partial X \bar{\phi} + \partial \phi \bar{\phi})\). This comes from the first term in the right hand side of (12) using descent equation. As we will see soon the second term in r.h.s. of (12) corresponds to auxiliary field, which is not needed to be added to the action. Hence the action now takes the form

\[
S_{\text{pert}} = \frac{1}{8\pi} \int d^2 z (\partial X \bar{\phi} + \partial \phi \bar{\phi}) - \frac{1}{2\sqrt{2\pi}} \int R\phi d^2 z
+ \lambda \int (\partial X \bar{\phi} + \partial \phi \bar{\phi})
\]

(13)

Here \( \lambda \) is the infinitesimal coupling associated with the new state. Clearly, the action (13) is not coformally invariant since the total central charge differs from 26 by order \( O(\lambda^2) \). To preserve the conformal invariance to this order, we need to add a term in the action of the form \( \frac{\lambda^2}{4\pi\sqrt{2}} \int R\phi \). With such changes we get,

\[
S_{\text{pert}} = \frac{1}{8\pi} \int d^2 z (\partial X \bar{\phi} + \partial \phi \bar{\phi}) - \frac{\sqrt{1 - \lambda^2}}{2\sqrt{2\pi}} \int R\phi d^2 z
+ \lambda \int (\partial X \bar{\phi} + \partial \phi \bar{\phi})
\]

(14)

We now redefine the target space coordinates as follows:

\[
X' = X + \lambda \phi; \quad \phi' = \sqrt{1 - \lambda^2} \phi.
\]

(15)

Hence the perturbed action in terms of these coordinates can be written as,

\[
S_{\text{pert}} = \frac{1}{8\pi} \int d^2 x \sqrt{h} (h^{ij} \partial_i X' \partial_j X' + h^{ij} \partial_i \phi' \partial_j \phi') - \frac{1}{2\sqrt{2\pi}} \int d^2 x \sqrt{h} \phi' R^{(2)}.
\]

(16)

This is exactly same as (11) with \( \phi \) and \( X \) replaced by \( \phi' \) and \( X' \) respectively. Notice that since the Liouville direction is uncompactified, the coordinate choice
(15) is globally possible. We therefore conclude that this new moduli will act as trivial perturbation to the original theory. The situation is quite similar to the case of marginal deformation in critical string theory as discussed before.

Now we pass on to the corresponding BRST string field theory where we identify this deformation to be a pure gauge deformation with singular gauge transformation parameter. We expand the string field $|\Psi\rangle$ as

$$
|\Psi\rangle = \tilde{g} c_1 \tilde{c}_1 (\alpha_{-1} \tilde{\phi}_{-1} + \phi_{-1} \tilde{\alpha}_{-1}) |0\rangle + i \tilde{s} c_0^+ (c_1 \alpha_{-1} - \tilde{c}_1 \tilde{\alpha}_{-1}) |0\rangle + \ldots
$$

(17)

Here $\tilde{g}$ and $\tilde{s}$ are the component string fields and $\alpha_n, \phi_n$ are the matter and Liouville oscillators respectively. We choose the gauge transformation parameter to be

$$
|\Lambda\rangle = \lim_{p \to 0} i \lambda (\tilde{c}_1 \tilde{\alpha}_{-1} - c_1 \alpha_{-1}) e^{p \phi_{-1}} - \frac{1}{p} |0\rangle.
$$

(18)

With this choice of $|\Lambda\rangle$, we find,

$$
Q_B |\Lambda\rangle = \lambda c_1 \tilde{c}_1 (\alpha_{-1} \tilde{\phi}_{-1} + \phi_{-1} \tilde{\alpha}_{-1}) |0\rangle - 2i \lambda c_0^+ (c_1 \alpha_{-1} - \tilde{c}_1 \tilde{\alpha}_{-1}) |0\rangle
$$

(19)

Adding this to the string field configuration (17), we get the changes in the component fields as

$$
\delta \tilde{g} = \lambda ; \quad \delta \tilde{s} = -2 \lambda.
$$

(20)

If we redefine the fields as

$$
g = \tilde{g} ; \quad s = \tilde{s} + 2 \tilde{g},
$$

then the corresponding transformations are

$$
\delta g = \lambda ; \quad \delta s = 0.
$$

Since, to linear order, the string field theory equation of motion is $Q_B |\Psi\rangle = 0$, it is easy to see that the equation of motion involving $s$ is: $s = 0$, showing that it is an auxiliary field to this order and can be set to zero using its equation of motion.
In sigma model language, this would correspond to adding a term $\lambda \int (\partial \phi \bar{\partial} X + \bar{\partial} \phi \partial X)$ to the action at least to lowest order in $\lambda$. Recall that in order to preserve the conformal invariance, we had to add higher order $\lambda$ dependent curvature term in the action. In string field theory, we believe, it will be taken care of automatically. There are higher order $\lambda$ dependent corrections to gauge transformation in (5). In particular, the $\lambda^2$ order corrections will have non-zero contribution along the dilaton direction, hence will guarantee the conformal invariance to this order.

What we have noticed so far can be generalized to any of the members of this kind in ghost number two sector. Though it is hard to obtain explicit target space redefinitions for a general state in that tower, nevertheless, it is easy to understand their effect from string field theory. Since a general member of the series can be written as

$$(a + \bar{a}) Y_{s,n}^+ \bar{O}_{s-1,n'} = Q_B (\phi Y_{s,n}^+ \bar{O}_{s-1,n'})$$

(22)

following our earlier argument, the effect of (22) can be thought of as linearized pure gauge deformations when added to the string field configuration. As a result, the two classical solutions differing by (22) should be regarded as two equivalent solutions. Note that in the case where $X$ is a noncompact scalar field many more marginal operators become pure gauge in this sense. For example in the first multiplet for $s = 1$, $n = n' = 0$, the operator $\partial X \bar{\partial} X$ gave radius changing perturbation in the compact case. In the noncompact case, this is pure gauge. This is true for $Y_{s,n}^+ \bar{Y}_{s,n}^+$ in general. It is nontrivial for compact $X$ only.

Given two world sheet actions differing from each other by a marginal perturbation, one knows [11] how to construct solutions of equation of motion in corresponding string field theory. If the perturbation is non-trivial (i.e. can not be removed by suitable coordinate redefinition), the two solutions will certainly be inequivalent since the starting actions are different. Now, suppose we know the moduli corresponding to (22) that can be added as a marginal perturbation to the sigma model action. In the sense of above argument it would rather correspond to trivial deformation of the theory because the classical solutions of original and
perturbed actions are equivalent. Since the new moduli will have higher tensorial rank, the coupling will also be the same. Thus in general in the original action we need to keep higher non-zero background couplings. Moreover, the target space redefinition will no longer be the same as (15). It will include derivative dependence among the coordinates. Given a particular gauge transformation in the string field theory, there is no straightforward way to see the corresponding target space redefinition in sigma-model, but, a rather indirect method of identification has been discussed in [12].

3.2. States of Ghost Number One

Here we discuss about the discrete states with ghost number one satisfying \((b_0 - \bar{b}_0)\) condition. Notice that these states can be thought of as gauge transformation parameters in string field theory. As we mentioned before, the gauge transformation parameters in string field theory are of ghost number one and are annihilated by \((b_0 - \bar{b}_0)\). Under infinitesimal gauge transformation, string field changes as

\[
\delta(\Psi) = Q_B(\Lambda) + g[\Psi \Lambda] + \mathcal{O}(\Psi^2)
\]  

(23)

If \(\Lambda\) happens to be a BRST closed state, then the first term in the right hand side drops out. This is the case for global transformation, where fields transform homogeneously. More over, if \(\Lambda\) is BRST exact, i.e.

\[
|\Lambda\rangle = Q_B|\Theta\rangle
\]

(24)

then

\[
\delta(|\Psi\rangle) = g[\Psi Q_B|\Theta\rangle] + \mathcal{O}(\Psi^2).
\]

(25)

Now, if \(|\Psi\rangle\) satisfies the equation of motion, then,

\[
\delta(|\Psi\rangle) = Q_B g[\Psi |\Theta\rangle] + \mathcal{O}(\Psi^2).
\]

(26)

\* Here we use \(X^\mu\) as both matter and Liouville coordinates. Hence, in this notation, the metric \(G_{\mu\nu}\) is of tensorial rank two.
If we allow $[\Psi \Theta]$ to be a valid gauge transformation parameter, then $|\Psi\rangle$ and $|\Psi\rangle + \delta|\Psi\rangle$ are equivalent solutions. So the nontrivial global part of the gauge symmetries are those for which $\Lambda$ belongs to BRST cohomology. This is precisely the situation that occurs in $d = 2$ string theory. Among all the ghost number one states, some are of the form $(a + \bar{a})O_{u,n}\bar{O}_{u,n'}$ with

$$
(a + \bar{a})O_{u,n}\bar{O}_{u,n'} = Q_B(\phi O_{u,n}\bar{O}_{u,n'})
$$

(27)

Since, we allow the states $\phi O\bar{O}$ (in the limiting sense as defined in eqn. (18)) in the string field configuration, we can identify $\Theta$ with these states. As a consequence, $\Lambda = Q_B\Theta$ should be considered as trivial global symmetry generator in this string field theory.

For the first multiplet, $\Theta = \phi; \Lambda = a + \bar{a}$. This corresponds to a constant gauge parameter for the antisymmetric tensor field. The corresponding charge measures the Liouville winding mode. This acts trivially on all the states. The charges from the higher multiplets do not act trivially but transform ghost number two states into the new ghost number two states as we saw.

4. Comments on higher ghost number states.

We could take the cohomology at higher ghost numbers and apply the same argument. For example, ghost number three states can be interpreted as antifield modes. If we take free string field action $S_0$ and carry out Batalin-Vilkovisky quantization procedure for the action, we would get $S = \langle \Psi | c_0^{-} Q_B | \Psi \rangle$ with $|\Psi\rangle$ having components from all ghost numbers, with $L_0^{-} |\Psi\rangle = b_0^{-} |\Psi\rangle = 0$. The components from ghost number one would correspond to target space ghost and those from ghost number three would correspond to target space antifields and so on.

* The original action $S_0$ is obtained by restricting $|\Psi\rangle$ to ghost number two.
Note that this action has a gauge invariance \( \delta |\Psi\rangle = Q_B |\Lambda\rangle \), where \( L_0^- |\Lambda\rangle = b_0^- |\Lambda\rangle = 0 \), \(|\Lambda\rangle\) has components from all ghost numbers. Now if we want to solve the equations of motion coming from \( S \) and get solutions upto this gauge transformation, we end up getting the BRST cohomology for all ghost numbers.

For a generic action of the form

\[
S_0 = \Psi^i A_{ij} \Psi^j, \tag{28}
\]

we have gauge invariance of the form \( \delta \Psi^i = R_{\alpha}^i \Lambda^\alpha \), if \( A_{ij} R_{\alpha}^i = 0 \). Corresponding Batalin-Vilkovisky action [13] would then be

\[
S = \Psi^i A_{ij} \Psi^j + \Psi^*_i R_{\alpha}^i C^\alpha + \ldots. \tag{29}
\]

where \( C^\alpha \) are the target space ghosts and \( \Psi^* \) are the antifields. The equations of motion of the antifields are \( \Psi^*_i R_{\alpha}^i = 0 \). If the action ended with these two terms, the gauge transformations would have been

\[
\delta \Psi^i = R_{\alpha}^i \Lambda^\alpha, \quad \delta \Psi^*_i = A_{ij} \chi^j, \quad \delta C^\alpha = 0. \tag{30}
\]

Hence the gauge variation of the antifields would have corresponded to off-shell field configuration. Its equations of motion makes it ‘orthogonal’ to the gauge transformations of the fields (since \( \Psi^*_i \delta \Psi^i = 0 \)). Hence the independent antifields solutions, modulo gauge transformations, would have been in one-to-one correspondence with the genuinely independent physical solutions.

In string field theory, the quadratic action does go beyond the first two terms. However, as long as we stick to the space of conformal fields, the conclusion presented here holds. Namely there is a one-to-one correspondence between fields and antifields, pure gauge and off-shell states. However as one comes out of the space of conformal fields, these correspondances break down. Consider a ghost number two state of the form \( Q_B \phi |\Lambda\rangle \), where \(|\Lambda\rangle\) is a member of the ghost number one
cohomology. It is a pure gauge field mode for a gauge parameter going as \( \phi \). This has a nontrivial overlap with some states in the ghost number three BRST cohomology. From the target space point of view, this is caused by the presence of the boundary term when we do integration by parts in an expression of the form \( \int \Psi^* R \Lambda \), \( R \) being generically some differential operator. Since equations for the antifields are \( R^\dagger \Psi^* = 0 \), we will get a boundary contribution after integrating by parts when \( \Lambda \sim \phi \). Similarly, pure gauge antifield modes will have overlap with physical fields.

Hence if we want the conjugates of the fields with ghost number less than three, which is not of the form \( Q_B(\phi O) \), we might have to keep the states of the form \( Q_B(\phi O) \) for the ghost numbers greater than or equal to three. In fact, it turns out only those are the ones that are to be kept. The complete meaning of this is not clear to us.

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