Semiclassicality and Decoherence of Cosmological Perturbations

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Abstract

Transition to the semiclassical behaviour and the decoherence process for inhomogeneous perturbations generated from the vacuum state during an inflationary stage in the early Universe are considered both in the Heisenberg and the Schrödinger representations to show explicitly that both approaches lead to the same prediction: the equivalence of these quantum perturbations to classical perturbations having stochastic Gaussian amplitudes and belonging to the quasi-isotropic mode. This equivalence and the decoherence are achieved once the exponentially small (in terms of the squeezing parameter $r_k$) decaying mode is neglected. In the quasi-classical limit $|r_k| \to \infty$, the perturbation mode functions can be made real by a time-independent phase rotation, this is shown to be equivalent to a fixed relation between squeezing angle and phase for all modes in the squeezed-state formalism. Though the present state of the gravitational wave background is not a squeezed quantum state in the rigid sense and the squeezing parameters loose their direct meaning due to interaction with the environment and other processes, the standard predictions for the rms values of the perturbations generated during inflation are not affected by these mechanisms (at least, for scales of interest in cosmological applications). This stochastic background still occupies a small part of phase space.

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1 Introduction

A unique and remarkable property of the inflationary scenario of the early Universe (irrespective of its concrete realization) is that it opens an exciting possibility to directly observe the outcome of a genuine quantum-gravitational effect: generation of quasi-classical fluctuations of quantum fields including the gravitational one in strong external gravitational fields (in other words, by space-time curvature). Historically, this effect was known as "particle creation from the vacuum in a background gravitational field", but it is clear now that what can be measured at present are not "particles" but rather inhomogeneous fluctuations (perturbations) of the gravitational field. In non-inflationary cosmological models, the effect of particle creation is exceedingly small and does not lead to observable consequences. Just the opposite, not only can minimal perturbations of the gravitational potential generated in the simplest versions of the inflationary scenario (first quantitatively calculated in [1]) be sufficient to explain galaxy formation and the large-scale structure in the Universe, but also their predicted spectrum (approximately flat, $n \approx 1$) and statistics (Gaussian) have been successfully quantitatively confirmed by the COBE discovery of large-angle fluctuations $\Delta T$ of the cosmic microwave background temperature [2] 10 years after the prediction was made. Moreover, in the case of the inflationary scenario and in contrast with other cases, the corresponding "pure" quantum-gravitational effect - creation of gravitons by background gravitational fields - produces a large relic gravitational-wave background with frequencies $\ll 10^{10}$Hz in the Universe [3], and it is even possible that a modest, but still significant part of the observed large-angle $\Delta T$ fluctuations (at the level proposed in [4], but probably not larger) is due to these gravitational waves. Another possibility of observing creation of particles by gravitational fields might be through the Hawking radiation from primordial black holes (PBH) with masses $M \leq 10^{15}$g, but it follows from direct or indirect observational tests (see, e.g. [5]) that the number density of such PBH in the Universe is very small if they formed at all (and the inflationary scenario typically predicts their complete absence).

In spite of this definite success of the inflationary scenario and of the quantum theory in curved space-time, some confusion still seems to exist in the literature regarding how rigid the derivation of the perturbations is (see, e.g. polemics in [6, 7, 8]. The key problem here is that though the process of creation from the vacuum and the perturbations themselves are purely of a quantum-mechanical nature (at least initially), the observed temperature or density fluctuations in the Universe are certainly classical.

Thus a complete derivation should include some mechanism of quantum-to-classical transition and decoherence of the perturbations. Connected with this are fundamental questions about the wave function of the Universe being pure or mixed and its interpretation. An additional complication is the relation between the Heisenberg and the Schrödinger representations in quantum mechanics and quantum field theory. Of course, these representations are completely equivalent and contain the same physics, but they use a different language and different parameters for the description of a given state of a quantum field. Almost all initial studies of particle creation in
cosmology in general [9, 10, 11], and in the inflationary scenario in particular [1] were performed using the Heisenberg approach. This approach is more convenient for the purpose of renormalization [10, 12] and a description of the quantum-to-classical transition (we shall come back to the latter point below). The use of mode functions satisfying a classical wave equation and the Bogolubov transformation for creation and annihilation operators is a characteristic feature of this approach. Derivation of the perturbations in this approach usually ends up (like in [1, 3]) by taking these mode functions as classical variables with stochastic Gaussian amplitudes but satisfying a certain condition in the regime outside the Hubble radius ("non-decreasing modes"). On the other hand what naturally follows from quantum cosmology where a homogeneous isotropic background is quantized too, is the Schrödinger representation for the wave functions of perturbations (see e.g. [13]). This representation is usually used also in order to consider the decoherence process. Here one speaks about a two-mode squeezed state and describes it with the help of squeezing parameters. Though, of course, it is generally well-known that the Bogolubov transformation of annihilation and creation operators in the Heisenberg picture just corresponds to the evolution of the vacuum state into a squeezed one in the Schrödinger picture (see e.g. a mathematical analysis in [14, 15, 16]), there still exists a point of discussion about how the two approaches are related in the cosmological context and which of them is "better". Inspired by the impression that a squeezed state has a non-classical behaviour even for large values of the squeezing parameter \( r \), there were even claims that the squeezed state formalism gives observable predictions which are superior to the usual Heisenberg approach [3, 4] (actually it does not, see also [8]). It is clear that a deep understanding of the generation process of perturbations is of utmost importance both for further development of the theory of quantum gravity and quantum cosmology and for observational implications. That is why we reconsider this question here.

We will deal with the simplest case of a quantum real massless scalar field \( \phi \) in a Friedman-Robertson-Walker (FRW) background because it is sufficiently representative for our purposes. First, the equation for the time-dependent part of \( \phi \) coincides exactly with the equation for the time-dependent part of gravitational waves on a FRW background [17] under the condition that the non-diagonal components of the matter pressure tensor are zero in the first order of perturbation expansion. The latter condition is satisfied e.g. in the case of matter consisting of a mixture of an arbitrary number of hydrodynamical components with barotropic equations of state \( p_i = p_i(\epsilon_i) \) and scalar fields with arbitrary mutual interactions and minimal coupling to the gravitational field. Furthermore, this equation has a structure very similar (though generally not completely identical) to the equation satisfied by the gravitational potential.

In section 2, we remind the reader of explicit relations between mode functions of the field, coefficients of the Bogolubov transformation and squeezing parameters. Then, in section 3, we consider the quasi-classical limit in the Heisenberg and Schrödinger representations in parallel in order to emphasize the fact that a two-mode squeezed state for large absolute values of the squeezing parameter \(|r|\) is completely equivalent to a classical standing wave with a stochastic Gaussian amplitude. The
term "standing" means that there is a definite deterministic correlation between \( k \) and \(-k\) modes for each wave vector \( k \). Also, we show how this property is related to the fundamental fact that the field modes can be made real by a time-independent phase rotation in this limit. All this is illustrated with a specific but very important example, namely that of the de Sitter background. In section 4, we remind the physical process in the Universe leading to extreme squeezing (\( |r| \to \infty \)) and to the quantum-to-classical transition - the different behaviour of non-decreasing and decaying modes outside the Hubble radius. Then it follows that quantum-to-classical transition and decoherence in the Heisenberg representation (in contrast with the Schrödinger one) are achieved simply by omitting an exceedingly small part of the field operator (the decaying mode), without any need to consider some interaction of the mode with an "environment". This may be called, following J. A. Wheeler’s favourite way to put it, "decoherence without decoherence". After this omission, it becomes unimportant whether the field is in a pure or in a mixed state. As a result, we come to the conclusion that the Heisenberg (field mode) approach becomes more straightforward in real situations when very small interactions of the field with other fields take place. Namely, due to these interactions and the resulting decoherence process, the present quantum state of the field \( \phi \) is neither a pure squeezed state nor even can it be described by a squeezed density matrix, so the use of squeezing parameters looses sense. On the other hand, interactions practically do not change the field modes (at least for sufficiently large scales). Thus, all predictions about present-day perturbations remain unchanged. A possibility of having nevertheless some "quantum signature" in the present-day spectrum of perturbations is mentioned. Finally, we discuss the point that the omitted, exponentially small part of the field may be important for the calculation of the entropy of the perturbations.

## 2 Bogolubov transformation and two-mode squeezed state

We give here the essential about quantized fields on a flat FRW background. Let us consider a real massless scalar field \( \phi \). It is described by the following action \( S \):

\[
S = \frac{1}{2} \int d^4x \sqrt{-g} \partial^\mu \phi \partial_\mu \phi
\]

where \( \mu = 0, \ldots, 3 \), \( c = \hbar = 1 \) and the Landau-Lifshitz sign conventions are used. The space-time metric has the form

\[
ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j, \quad i, j = 1, 2, 3.
\]

Let us remind how the dynamics of this system will lead to the appearance of squeezed states. We first write down the classical Hamiltonian \( H \) in terms of the field \( y \equiv a\phi \) and the conformal time \( \eta = \int \frac{dt}{a(t)} \). The following result is then obtained

\[
H = \int d^3x \, \mathcal{H}(y, p, \partial_t y, t)
\]
\[
\frac{1}{2} \int d^3k [p(k)p^*(k) + k^2 y(k)y^*(k) + \frac{a'}{a} (y(k)p^*(k) + p(k)y^*(k))] \tag{3}
\]

where

\[
p \equiv \frac{\partial L(y, y')}{\partial y'} = y' - \frac{a'}{a} y \tag{4}
\]

and a prime stands for derivation with respect to the conformal time. Here the following Fourier transform convention is used: \( \Phi(k) \equiv (2\pi)^{3/2} \int \Phi(r)e^{-ikr}d^3r \) for functions as well as for operators. In order to avoid too heavy notations, we will often write simply \( y(k), a(k), ... \) instead of \( y(k, \eta), a(k, \eta), ... \) though the Fourier transforms are time-dependent c-functions or time-dependent operators in the Heisenberg representation. Due to reality of the field \( y \), we have that \( y(k) = y^*(-k), \; \text{resp.} \; y^\dagger(-k) \) for operators. Therefore, any classical field configuration is completely specified by giving the Fourier transforms in half Fourier space. This may be not true in the quantum case, and the full Fourier space has to be used if a quantum state of the field is not invariant under the reflection \( k \rightarrow -k \). However, for the vacuum initial state that we will use below, there is no such complication. The Fourier transforms appearing in (3) satisfy the equation

\[
y''(k) + \left(k^2 - \frac{a''}{a}\right)y(k) = 0. \tag{5}
\]

When the field \( y \) is quantized, the Hamiltonian becomes

\[
H = \int \frac{d^3k}{2} [k(a(k)a\dagger(k) + a\dagger(k)a(k)) + i\frac{a'}{a}(a\dagger(k)a\dagger(-k) - a(k)a(-k))]. \tag{6}
\]

The time-dependent (in the Heisenberg representation) operator \( a(k) \) appearing in (6) is defined as usual:

\[
a(k) = \frac{1}{\sqrt{2}} \left( \sqrt{k} y(k) + i\frac{1}{\sqrt{k}} p(k) \right), \tag{7}
\]

so that

\[
y(k) = \frac{a(k) + a\dagger(-k)}{\sqrt{2k}}, \; \; \; p(k) = -i\frac{\sqrt{k}}{2} \left( a(k) - a\dagger(-k) \right). \tag{8}
\]

The canonical commutation relations

\[
[y(x, \eta), \; p(x', \eta)] = i\delta^{(3)}(x - x') \tag{9}
\]

imply the following commutation relations

\[
[y(k, \eta), \; p\dagger(k', \eta)] = i\delta^{(3)}(k - k'), \quad [a(k, \eta), \; a\dagger(k', \eta)] = \delta^{(3)}(k - k'). \tag{10}
\]

The last piece in the integrand of (5) is responsible for the squeezing. Let us see first how it affects the time evolution of the system. We have

\[
\begin{pmatrix}
a'(k) \\
(a\dagger(-k))'
\end{pmatrix} = \begin{pmatrix}
-i k & \frac{a'}{a} \\
\frac{a'}{a} & i k
\end{pmatrix}
\begin{pmatrix}
a(k) \\
(a\dagger(-k))
\end{pmatrix}. \tag{11}
\]
Clearly, the general solution of these two coupled equations are
\[
a(k, \eta) = u_k(\eta)a(k, \eta_0) + v_k(\eta)a^\dagger(-k, \eta_0),
\]
\[
a^\dagger(-k, \eta) = u_k^*(\eta)a^\dagger(-k, \eta_0) + v_k^*(\eta)a(k, \eta_0).
\]
(12)

This is just a Bogolubov transformation. Eq. (12) can be interpreted as giving the time evolution of the creation and annihilation operators in the Heisenberg representation, or as a definition of explicitly time-dependent operators in the Schrödinger representation. The commutation relations (10) are preserved under the unitary time evolution which yields the constraint
\[
|u_k(\eta)|^2 - |v_k(\eta)|^2 = 1,
\]
therefore, allowing the following standard parameterization of the functions \( u_k, v_k \):
\[
u_k(\eta) = e^{-i\theta_k(\eta)} \cosh r_k(\eta),
\]

\[
u_k^*(\eta) = e^{i(\theta_k(\eta)+2\varphi_k(\eta))} \sinh r_k(\eta).
\]
(14)

Here \( r_k \) is the squeezing parameter, \( \varphi_k \) is the squeezing angle and \( \theta_k \) is the phase.

The relation between these quantities and those introduced in the \( \alpha - \beta \) formalism [10] is the following (if \( \Omega_k \) in [10] is chosen to be equal to \( k \)):
\[
u_k = \alpha_k e^{-ik\eta}, \quad \nu_k^* = \beta_k e^{ik\eta}.
\]
(15)

Also, the parameters \( s_k, \tilde{\alpha}_k, \tau_k \) introduced in [10], that proved to be very useful to make the adiabatic expansion for large \( k \) and to obtain a finite average value of the energy-momentum tensor of the quantum field \( \phi \) either by the \( n \)-wave regularization method [10] or by the equivalent adiabatic regularization method [12], are expressed through the squeezing parameters as:
\[
s_k = |\beta_k|^2 = \sinh^2 r_k,
\]
\[
\tilde{\alpha}_k = \alpha_k \beta_k e^{-2ik\eta} + \alpha_k^* \beta_k^* e^{2ik\eta} = \cos 2\varphi_k \sinh 2r_k,
\]
\[
\tau_k = i(\alpha_k \beta_k^* e^{-2ik\eta} - \alpha_k^* \beta_k e^{2ik\eta}) = -\sin 2\varphi_k \sinh 2r_k
\]
(16)

and we use here the notation \( \tilde{\alpha}_k \) to avoid confusion with the quantity \( u_k \) used in the present paper. Note that \( s_k \) is equal to the average number of created particles with momentum \( k \) in the WKB regime (in particular, if \( a(\eta) \) becomes constant). We don’t intend here to introduce the notion of particles in a non-WKB regime because it is ambiguous and does not lead to interesting results.

Let us now define the field modes \( f_k(\eta) \) with \( \Re f_k \equiv f_{k1} \) and \( \Im f_k \equiv f_{k2} \), \( f_k(\eta_0) = 1/\sqrt{2k} \), we will adopt a similar notation for the quantities \( p \) and \( y \),
\[
y(k) \equiv f_k(\eta)a(k, \eta_0) + f_k^\dagger(\eta)a^\dagger(-k, \eta_0)
\]
\[
y = \sqrt{2k} f_{k1}(\eta)y(k, \eta_0) - \sqrt{2} f_{k2}(\eta)p(k, \eta_0)
\]
(17)
and the momentum modes \( g_k(\eta), \ g_k(\eta_0) = \sqrt{\frac{k}{2}} \),

\[
p(\mathbf{k}) = -i[g_k(\eta)a(\mathbf{k}, \eta_0) - g^*_k(\eta)a(\mathbf{k}, \eta_0)] = \sqrt{\frac{2}{k}}g_{k1}(\eta)p(\mathbf{k}, \eta_0) + \sqrt{2k}g_{k2}(\eta)y(\mathbf{k}, \eta_0).
\]

The modes \( f_k \) satisfy the Euler-Lagrange equation (5). Note that

\[
f_k = \frac{u_k + v^*_k}{\sqrt{2k}}, \quad |f_k|^2 = \frac{1}{2k}(\cosh 2r_k + \cos 2\varphi_k \sinh 2r_k),
\]

\[
g_k = \sqrt{\frac{k}{2}}(u_k - v^*_k) = i(f'_k - \frac{a'}{a}f_k).
\]

Eqs (14,19) give explicitly the relation between the modes \( f_k \) which are typically used in the Heisenberg approach and the squeezing parameters characteristic for the Schrödinger approach. Also, they can be used to obtain the dynamical equations satisfied by the squeezing parameters, see Eq. (39) below. The Wronskian condition for Eq.(5), as well as the commutation relations (10), yield the following equality

\[
g_k f'_k + g^*_k f_k = i(f'_k f^*_k - f'_k f_k) = 1.
\]

We will be interested, in particular, in the quantum state of the field \( y \) defined to be vacuum at some time \( \eta_0 \) in the following way

\[
\forall \mathbf{k} : a(\mathbf{k}, \eta_0)|0, \eta_0\rangle = 0.
\]

This state corresponds to a Gaussian state and time evolution preserves its Gaussianity. Indeed it follows from (17,18) that in the Heisenberg representation, the time independent state \( |0, \eta_0\rangle_H \) is an eigenstate of the operator \( y(\mathbf{k}) + i\gamma_k^{-1}(\eta)p(\mathbf{k}) \), namely

\[
\left\{ y(\mathbf{k}) + i\gamma_k^{-1}(\eta)p(\mathbf{k}) \right\} |0, \eta_0\rangle_H = 0,
\]

where the operators \( y(\mathbf{k}), p(\mathbf{k}) \) as well as the function \( \gamma_k \) depend on time,

\[
\gamma_k = k \frac{u_k^* - u_k}{u_k^* + u_k} = k \frac{1 - i \sin 2\varphi_k \sinh 2r_k}{\cosh 2r_k + \cos 2\varphi_k \sinh 2r_k} = \frac{1}{2} |f_k|^2 - \frac{F(k)}{|f_k|^2},
\]

\[
F(k) = \Im u_k v_k = \Im f^*_k g_k = \frac{1}{2} \sin 2\varphi_k \sinh 2r_k.
\]

On the other hand, in the Schrödinger representation the time-evolved state \( |0, \eta\rangle_S \equiv S|0, \eta_0\rangle \), where \( S \) is the \( S \)-matrix, satisfies the equation

\[
S a(\mathbf{k}, \eta_0) S^{-1} |0, \eta\rangle_S = 0
\]

or equivalently

\[
\left\{ y(\mathbf{k}, \eta_0) + i\gamma_k^{-1}(\eta)p(\mathbf{k}, \eta_0) \right\} |0, \eta\rangle_S = 0.
\]
Note the similar structure of Eqs (22, 24). In the coordinate Schrödinger representation, \( p(k, \eta_0) = -i \partial / \partial y(-k, \eta_0) \). Hence the state \( |0, \eta_0\rangle \) has a Gaussian wave functional in this representation consisting of the product of

\[
\Psi[y(k, \eta_0), y(-k, \eta_0)] = N_k \exp \left( -\frac{y(k, \eta_0)y(-k, \eta_0)}{2|f_k|^2} \{1 - i2F(k)\} \right)
\]

for each pair \( k, -k \) where \( N_k \) is a normalization coefficient. The time dependence of \( \Psi \) is through \( f_k, F(k) \), and \( N_k \). This structure of the wave functional just reflects the fact that we get a two-mode squeezed state. The corresponding probability \( P[y(k, \eta_0), y(-k, \eta_0)] \) is given by

\[
P[y(k, \eta_0), y(-k, \eta_0)] \propto \exp \left( -\frac{|y(k, \eta_0)|^2}{|f_k|^2} \right).
\] (27)

At \( \eta = \eta_0 \), we have \( \gamma_k(\eta_0) = k \) or equivalently \( F(k) = 0 \), in other words we have a minimum uncertainty wave function.

### 3 Transition to semiclassical behaviour

Let us first consider the transition in the Heisenberg approach and take the formal limit "\( h \to 0 \)" keeping the rms amplitude \( |f_k| \), when expressed in physical units, fixed. Since the right-hand sides of the commutation relation (13) and of Eqs (13, 20) are proportional to \( h \) in physical units and do not depend on \( |f_k| \), they may be approximately replaced by 0 in this limit. In other words, \( |u_k| \approx |v_k| \gg 1, |f_k| \gg 1/\sqrt{2K}, |g_k| \gg \sqrt{2} \) in natural units in the quasi-classical limit. Then it follows from Eq.(20) with 0 in the right-hand side that \( f_k^* = c_k f_k \), where \( c_k \) is a time-independent constant. As a result, it is possible to make \( f_k \) real for all times by a time-independent phase rotation, viz. \( f_k \to f_k \exp(-\frac{i}{2} \arg c_k) \). On the other hand, \( g_k \) is purely imaginary in this limit.

A further consequence is that all variables \( y(k) \) and \( p(k) \) become mutually commuting. However, we still cannot ascribe any definite numerical values to them, in contrast with coherent states in the quasi-classical limit; there is no Bose condensate. The correct way to put it is that field modes become equivalent to stochastic c-number functions of time with some probability distribution \( \rho(y(k)y(-k)) \equiv \rho(|y(k)|^2) \) for each pair of modes \( k, -k \) in the following sense

\[
H(0, \eta_0|G(y(k))G^\dagger(y(k))|0, \eta_0) = \int \int dy_1(k)dy_2(k) \rho(|y(k)|)|G(y(k))|^2
\] (28)

for any arbitrary function \( G(y(k)) \) and \( k \neq 0 \), (for \( k = 0 \) the proof is a little bit different). Here we assume for simplicity of notation the \( k \) spectrum to be discrete, this can be achieved e.g. by formally making the \( T^3 \) identification of space with three topological comoving scales much larger than all scales of interest. By considering
average values of arbitrary powers of \( y(k) \) and \( y^\dagger(k) \) and using the Wick theorem, it is straightforward to show that the two-dimensional probability distribution \( \rho(|y(k)|) \) is Gaussian with the dispersion \( \langle y_1^2 \rangle + \langle y_2^2 \rangle = |f_k|^2 \). Indeed, if

\[
G(y) = \sum_{m=0}^{\infty} q_m y^m, \quad \rho(|y|) = \frac{1}{\pi |f_k|^2} \exp\left(-\frac{|y|^2}{|f_k|^2}\right)
\]  

(29)

where the argument \( k \) of \( y \) is omitted for brevity and \( f_k \) has been made real, then

\[
H \langle 0, \eta_0 | G(y) G^\dagger(y) | 0, \eta_0 \rangle_H = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} q_m q_n^* f_k^{m+n} H \langle 0, \eta_0 | (a(k, \eta_0) + a^\dagger(-k, \eta_0))^m
\]

\[
\left(a^\dagger(k, \eta_0) + a(-k, \eta_0)\right)^n | 0, \eta_0 \rangle_H = \sum_{m=0}^{\infty} m! |q_m|^2 f_k^{2m} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} q_m q_n^* \int dy_1 dy_2 \rho(|y|) y^m y^{*n} = \int \int dy_1 dy_2 \rho(|y|) |G(y)|^2.
\]

(30)

Thus, in the continuous limit, the equivalent classical stochastic field can be written as \( y(k, \eta) = f_k(\eta) e(k) \) where the quantities \( e(k) \) are time-independent \( \delta \)-correlated Gaussian variables with zero average and unit dispersion: \( \langle e(k) e^*(k') \rangle = \delta^{(3)}(k - k') \), \( e^*(k) = e(-k) \). The crucial property here is that the time-dependent part \( f_k(\eta) \) factorizes both in the operator part \( a(k, \eta_0) + a^\dagger(-k, \eta_0) \) of the field mode \( y(k, \eta) \) and in the stochastic variable \( e(k) \) of the equivalent c-number function in the limit involved. As a result, after some realization of the stochastic amplitude of the field mode has occurred, further evolution of the mode is deterministic and is not affected by quantum noise. This kind of quantum-to-classical transition is similar to that used in the stochastic approach to inflation [18, 19].

We turn now to a description of the same transition in the Schrödinger representation. We see from (23) that semiclassicality is implied if the following condition is satisfied

\[
|F(k)| \gg 1.
\]

(31)

It is clear from (31) that this requires the quantum state to be extremely squeezed, namely \( |\gamma_k| \gg 1 \). Note that, in this limit, we cannot omit the number 1 appearing inside the figure brackets of (20) because it would make the wave function non-normalizable. The classicality is to be understood in the following sense: if we assign to each point in phase-space \( (y(k), p(k) \approx \frac{F(k)}{|f_k|^2} y(k)) \) the probability given by (27), it will move with time according to the classical Hamiltonian equations. Writing \( y(k) = |y(k)| e^{i\vartheta} \), it follows from (27) that \( |y(k)| \) obeys a Rayleigh distribution while the phase \( \vartheta \) becomes a stochastic variable uniformly distributed \((0 \leq \vartheta \leq 2\pi)\). Condition (31) can also be expressed in the following way

\[
\frac{|\gamma_{k2}|}{|\gamma_{k1}|} \gg 1, \quad \gamma_k \equiv \gamma_{k1} + i\gamma_{k2}.
\]

(32)

This can also be seen from the following vacuum expectation values

\[
\langle \Delta y(k, \eta) \Delta y^\dagger(k', \eta) \rangle \equiv \Delta y^2(k) \delta^{(3)}(k - k') = |f_k|^2 \delta^{(3)}(k - k'),
\]

\[
\langle \Delta p(k, \eta) \Delta p^\dagger(k', \eta) \rangle \equiv \Delta p^2(k) \delta^{(3)}(k - k') = |g_k|^2 \delta^{(3)}(k - k').
\]

(33)
where $\Delta \Phi \equiv \Phi - \langle \Phi \rangle$ and we have further adopted the notation $\langle \Phi(k, \eta)\Phi^\dagger(k', \eta) \rangle \equiv \Phi^2(k)\delta^{(3)}(k-k')$, where the quantity $\Phi^2(k)$, the power spectrum of the quantity $\Phi$ depends only on $k$ if the state is invariant under spatial translations and rotations. Note further that $\langle y \rangle = \langle p \rangle = 0$. The following identities can then be shown to hold, in complete accordance with the wave functional representation (32):

$$
\Delta y^2(k) \Delta p^2(k) = |f_k|^2 |g_k|^2
= (f_{k1}g_{k2} - f_{k2}g_{k1})^2 + \frac{1}{4}
= \frac{1}{4} (\sin^2 2\varphi_k \sinh^2 2r_k + 1) = F^2(k) + \frac{1}{4} .
$$

(34)

Then the condition of semiclassicality (31) corresponds to an uncertainty which is much bigger than the minimal one allowed by the rules of quantum mechanics. This shows once more that we are dealing in this limit with stochasticity of a classical type. It is interesting to note that we also have the following vacuum expectation values

$$
\langle \Delta y_1(k, \eta) \Delta y_1(k', \eta) \rangle = \left| \frac{f_{k1}}{2} \left( \delta^{(3)}(k - k') + \delta^{(3)}(k + k') \right) \right| ,
$$

(35)

$$
\langle \Delta y_2(k, \eta) \Delta y_2(k', \eta) \rangle = \left| \frac{f_{k2}}{2} \left( \delta^{(3)}(k - k') - \delta^{(3)}(k + k') \right) \right| ,
$$

(36)

$$
\langle \Delta p_1(k, \eta) \Delta p_1(k', \eta) \rangle = \left| \frac{g_{k1}}{2} \left( \delta^{(3)}(k - k') + \delta^{(3)}(k + k') \right) \right| ,
$$

(37)

$$
\langle \Delta p_2(k, \eta) \Delta p_2(k', \eta) \rangle = \left| \frac{g_{k2}}{2} \left( \delta^{(3)}(k - k') - \delta^{(3)}(k + k') \right) \right| .
$$

(38)

The equalities (35)-(38) express the correlation existing between $k$ and $-k$ modes. They are in complete agreement with the fact that for any real quantity $f$ with $f(k) \equiv f_1(k) + if_2(k)$, we have $f_1(k) = f_1(-k)$, $f_2(k) = -f_2(-k)$.

Let us prove now that $f_k$ can be made real for $|r_k| \to \infty$ using the squeezed state formalism. When we write the equation of motions for $\theta_k$ and $\varphi_k$, the following equations are obtained

$$
r'_k = \frac{a'}{a} \cos 2\varphi_k ,
$$

$$
\varphi'_k = -k - \frac{a'}{a} \coth 2r_k \sin 2\varphi_k ,
$$

$$
\theta'_k = k + \frac{a'}{a} \tanh r_k \sin 2\varphi_k .
$$

(39)

The first two equations are coupled and their solutions can then be substituted in the last equation. However, it is interesting to combine the last two equations, yielding the following property

$$
\lim_{|r_k| \to \infty} (\theta_k + \varphi_k)' = 0 .
$$

(40)

This means that $\theta_k + \varphi_k \to \delta_k$, where $\delta_k$ is some constant phase. This in turn implies that the field modes $f_k$ have the following asymptotic behaviour for $r_k \to \infty$

$$
\sqrt{2k}f_k \to e^{-i\delta_k} e^{r_k} \cos \varphi_k ,
$$

(41)
and analogously for the modes $g_k$

$$\sqrt{2\frac{g_k}{k}} \to ie^{-i\delta k}e^{r_k} \sin \varphi_k . \quad (42)$$

The quantity $\delta k$ can be made zero by a time-independent phase rotation in which case $f_k$ becomes real. It is interesting to note that (31) implies

$$\lim_{|r_k| \to \infty} \sin 2\varphi_k \sinh 2r_k = \infty \quad (43)$$

even if $\sin 2\varphi_k \to 0$. This is also in accordance with the fact $\langle n_k \rangle = \sinh^2 r_k \delta^{(3)}(k-k')$ so that large $r_k$ alone is enough in order to have semi-classicality.

One more way to study the quantum-to-classical transition is through the Wigner function formalism. The Wigner function for the two-mode squeezed state $k, -k$ is defined as

$$W(y(k), y(-k), p(k), p(-k)) = \frac{1}{(2\pi)^2} \int \int dx_1 dx_2 e^{-i(p_1 x_1 + p_2 x_2)} \langle y(k) - \frac{x(k)}{2} | \hat{\rho} | y(k) + \frac{x(k)}{2} \rangle$$

where $\hat{\rho}$ is the density matrix of the state and $y_1$ and $y_2$ are the real and imaginary parts of $y(k)$ in the Schrödinger coordinate representation, and the same convention applies to $x(k)$ and $p(k)$. Using $\hat{\rho} = |\Psi\rangle \langle \Psi|$ with $\Psi$ given by (24), we obtain

$$W(k, -k) = \frac{1}{(2\pi)^2} \int \int dx_1 dx_2 e^{-i(p_1 x_1 + p_2 x_2)} \Psi^* \left( y(k) - \frac{x(k)}{2} \right) \Psi \left( y(k) + \frac{x(k)}{2} \right) =$$

$$N_k^2 \frac{|f_k|^2}{\pi} \exp \left( -\frac{|y(k)|^2}{|f_k|^2} \right) \exp \left( -\frac{|f_k|^2 |p(k) - F(k)|^2 y(k)^2}{|f_k|^2} \right). (45)$$

$W > 0$ in this case, so there is no problem with its interpretation as a probability distribution in phase space. As explained above, the quantum-to-classical transition is achieved by taking the formal limit $\hbar \to 0$ keeping the physical amplitude $|f_k|$ fixed. Then the Wigner function becomes

$$W(k, -k) = N_k^2 \exp \left( -\frac{|y(k)|^2}{|f_k|^2} \right) \delta \left( p_1(k) - \frac{F(k)}{|f_k|^2} y_1(k) \right) \delta \left( p_2(k) - \frac{F(k)}{|f_k|^2} y_2(k) \right) =$$

$$\mathcal{P}[y(k), y(-k)] \delta \left( p(k) - \frac{F(k)}{|f_k|^2} y(k) \right). (46)$$

This just describes the deterministic motion in phase space of a bunch of trajectories with stochastic Gaussian initial amplitude.

The Wigner function (10) coincides with the probability distribution in phase space that was obtained in [20] for a toy model of an upside-down harmonic oscillator. We see that this behaviour of the Wigner function is general in the large-squeezing limit and does not rely neither on assumptions made in [20], nor on the existence of the inflationary stage. In the paper [21] an objection was raised against this way of making the quantum-to-classical transition based on the observation that it is always
possible to find new canonical variables $\tilde{y}(k), \tilde{p}(k)$ for which $\langle \tilde{y}^2 \rangle \langle \tilde{p}^2 \rangle = \frac{1}{4}$ (even in the limit $|r_k| \to \infty$). However, it is clear from the previous discussion that this objection is not relevant because the requirement $\langle \tilde{y}^2 \rangle \langle \tilde{p}^2 \rangle \gg 1$ for all possible canonically conjugate variables is not the necessary condition for the semi-classical behaviour. The only necessary condition is $|r_k| \gg 1$, or Eq. (56) below for the physical amplitude of gravitational waves.

In inflationary theories, primordial perturbations are generated by vacuum quantum fluctuations of a real scalar field where the power spectrum of the quantum field fluctuations is given by $|f(k)|^2$. Let us consider the very important example of a massless real scalar field on a (quasi) de Sitter space. In that case we have

$$\sqrt{2}k f_k = e^{-ik\eta}(1 - \frac{i}{k\eta}), \quad \sqrt{2}k g_k = e^{-ik\eta}, \quad \eta \equiv -\frac{1}{aH} < 0.$$ (47)

The modes (47) give also a very accurate description for slowly varying Hubble parameter $H$, namely when $|\dot{H}| \ll 3H^2$. After some straightforward calculation we get

$$u_k = e^{-i(k\eta + \delta_k)} \cosh r_k \quad v_k = e^{i(k\eta + \frac{\pi}{2})} \sinh r_k$$ (48)

and $\sinh r_k = \frac{1}{2k\eta} \to -\infty$ when $k\eta \to 0$. The crucial point in (48) is that $\tan \delta_k = \frac{1}{2k\eta}$, therefore $\delta_k$ will tend to the constant value $-\frac{\pi}{2}$. Hence in the limit $k\eta \to 0$, the modes $f_k$ are purely real up to a constant phase transformation. Let us give for completeness the solution for $\varphi_k$ and $\theta_k$

$$\varphi_k = \frac{\pi}{4} - \frac{1}{2} \arctan \frac{1}{2k\eta},$$
$$\theta_k = k\eta + \arctan \frac{1}{2k\eta}.$$ (49)

We have finally

$$F(k) = \frac{1}{2} \sin 2\varphi_k \sinh 2r_k \sim (2k\eta)^{-1} \to -\infty$$ (50)

for $k\eta \to 0$ though $\sin 2\varphi_k \to 0$.

### 4 Long-wave mode behaviour and decoherence

The physical mechanism producing the Bogolubov transformation and the extreme squeezing in the Universe is, as well known, the expansion of the Universe (not necessarily inflation) and the existence of the Hubble radius $R_H \equiv H^{-1} = a^2 \omega^2$. For modes with $kR_H \ll a$, i.e. with wavelengths outside the Hubble radius, the general solution of Eq. (3) has the following form in terms of the mode functions $f_k$, with $y(k)$ expressed through $f_k$ using Eq. (2):

$$f_k = C_1(k)a + C_2(k)a \int_{\infty}^{\eta} d\eta' \frac{y'(\eta')}{a^2(\eta')}, \quad g_k = O(iC_1(k)k^2a\eta) + i\frac{C_2(k)}{a}.$$ (51)
$C_1(k)$ can be made real by a phase rotation. Then the normalization condition leads to the relation

$$C_1 \Im C_2 = -\frac{1}{2}. \quad (52)$$

The first term in (51) is the quasi-isotropic mode, also called the growing mode. It corresponds to a constant value of the field $\phi$. The same behaviour is shared by the leading term of scalar (adiabatic) metric perturbations in the synchronous gauge for an arbitrary scale factor $a(\eta)$ as well as by the gravitational potential $\Phi$ during stages of power-law expansion. The name quasi-isotropic means that this mode does not spoil the isotropic expansion of the Universe at early stages, furthermore it is contained in the linear expansion of the Lifshits-Khalatnikov quasi-isotropic solution [22]. The second term in (51) is the decaying mode.

In inflationary models, comparing (47) with (51), we get

$$C_1 = \frac{H_k}{\sqrt{2k^3}}, \quad C_2 = -\frac{ik^{3/2}}{\sqrt{2H_k}} \quad (53)$$

in agreement with (52) where $H_k$ is the value of the slowly varying Hubble parameter $H$ at the moment of the first Hubble radius crossing $\eta_1$ ($\eta_1 < 0, |\eta_1(k)| = k^{-1}$). Here we have multiplied $f_k$ in (47) by $-i$ to make it both real and positive. Then both terms in (51) are of the same order at $\eta \sim \eta_1$. After that, the decaying mode quickly becomes exceedingly small. For example, for scales of interest for cosmological applications that crossed the Hubble horizon about $60 - 70$ e-folds before the end of inflation, the rms of the decaying mode is $e^{-2r_k} \sim 10^{-80}$ or less from that of the quasi-isotropic mode at the end of inflation, and it becomes $< 10^{-95}$ up to the present moment. It is clear that we should neglect the decaying mode completely. One more formal reason for this is that one cannot keep small terms of the relative order of $e^{-r_k}$ as far as renormalization contributions to each mode, which are of the order of $e^{-r_k}$, are not taken into account.

But once the second term in the expression for $f_k$ in (51) is omitted, we obtain immediately decoherence because quantum coherence is described by the correlation (52) between the non-decaying and decaying modes. Therefore, when working with the field modes $f_k(\eta)$ in the Heisenberg representation, there is no need to consider any interaction with an ”environment” and trace over its degrees of freedom in order to get decoherence. Moreover, after neglecting the decaying mode, it becomes unimportant whether the quantum state of a given mode $k$ (or the Universe as a whole) is pure of mixed since the difference between the three following field configurations, viz. classical stochastic field with modes $f_k$ given by (51), pure squeezed quantum state satisfying (51, 52) and mixed squeezed quantum state with the relation (52) understood in the sense of rms values for $C_1(k)$ and $C_2(k)$, is exponentially small ($\sim e^{-2r_k}$) and disappears after this omission. Summarizing, in the peculiar case of quantum cosmological perturbations generated during inflation, the decoherence can be obtained without consideration of any concrete decoherence process, that is why we may call it ”decoherence without decoherence”.

This property makes the Heisenberg approach more straightforward and convenient in real physical situations when one takes into account small interactions of
the perturbations with matter in the Universe. For example, it is known that the interaction of scalar perturbations and gravitational waves with a background matter having shear viscosity does not change the quasi-isotropic mode but yields an additional exponential decay of the decaying mode. On the other hand, new perturbations belonging to both modes may be generated by local physical processes, especially after the second Hubble radius crossing during the radiation- or matter-dominated FRW stages. Their amplitude is much more than the fantastically small amplitude of the decaying mode remaining from the inflationary stage but much less (at least for scales exceeding \( R_{eq} \approx 15h^{-2}\text{Mpc} \)) than the amplitude of the quasi-isotropic mode. As a result, the present state of the perturbations is (of course) neither a pure squeezed state, nor does it make sense to call the corresponding density matrix "squeezed" because one has no reasons to expect to find any direction in phase space where the noise (uncertainty) would be less or even comparable to the quantum limit \( \hbar \frac{h}{2} \) for the vacuum state. A much more adequate description of this state is that it consists of the classical stochastic part described by the quasi-isotropic modes \( f_k \) (the first term in Eq.\((51)\)) for \( k|\eta| \ll 1 \) or its continuation to the regime \( k\eta \geq 1 \), see below) plus some small noise of indefinite structure. Then the squeezing parameters loose their original sense. One may still formally introduce them through the mode functions using Eqs \((14, 19)\) as their definitions (as is done, e.g. in Eq.\((54)\) below), but they will have little if any relation to the squeezed quantum state in a narrow, rigid sense.

Note that our method of omitting the decaying mode or, equivalently, taking the "\( \hbar \to 0 \)" limit with \(|f_k|\) being fixed in physical units may be also thought of as coarse graining, or smoothing, in phase space. Namely, the radius of the coarse graining should be chosen in such a way as to smooth completely the "subfluctuant" variable \( p(k) - \frac{f_k}{|f_k|}y(k) \) (using the terminology of \([14])\) but still without affecting the "superfluctuant" variable \( y(k) \) significantly. After that, this radius may be put zero in all problems concerning dynamical evolution and stochastic properties of perturbations (except for the calculation of their entropy). However, it is clear from the previous discussion that final results do not depend on any concrete type of coarse graining as far as the leading quasi-isotropic mode remains unaffected by it.

Let us now consider what happens to the quasi-isotropic mode after the second Hubble radius crossing. Then one has \( f_k = C_1(k)f_q(k, \eta) \) where \( f_q \) is the exact solution of Eq.\((5)\) with the asymptotic behaviour \( f_q(k, \eta) = a(\eta) \) for \( k\eta \ll 1 \), in particular, \( f_q(k, \eta) = \frac{a_0}{k} \sin k\eta \) during the radiation-dominated regime \( a = a_0\eta \). Note that \( f_q = A(k)\sin(k\eta + \xi_k) \) for \( k\eta \gg 1 \) and an arbitrary behaviour of \( a(\eta) \). Here \( \xi_k \) is a constant and \( A(k) \sim a(\eta_2) \) where \( \eta_2 \) is the moment of second Hubble radius crossing, \( \eta_2 \sim k^{-1} \). It is this constant phase \( \xi_k \), or the phase \( \theta_k \) linearly growing with conformal time \( \eta \) (see \((53)\)) that one usually has in mind when saying that the gravitational waves generated during inflation have stochastic amplitudes but fixed phases. The fact that \( y(k) \) becomes zero at some moments of time shows that these waves are standing ones. The adiabatic perturbations do not have such an oscillating behaviour at the matter-dominated stage. Hence, it is more general to speak not
about the fixed phase of perturbations but about the type of the mode (the quasi-isotropic one).

This demonstrates that the loss of quantum coherence does not preclude the existence of strong quasi-classical correlations, c.f. comparison of quantum coherence and classical correlation in [23] (see also [24] in this connection). If we formally introduce the parameters \( r_k, \theta_k, \varphi_k \) using (14,19), then

\[
e^{2r_k} = 2kC_1^2 \left[ f_q^2 + \frac{a^2}{k^2} \left( \left( f_q/a \right)' \right)^2 \right], \quad r_k \gg 1,
\]

\[
\varphi_k = -\theta_k, \quad \sin \theta_k = -\frac{a_k \left( \frac{f_q}{a} \right)'}{\sqrt{f_q^2 + \frac{a^2}{k^2} \left( \left( \frac{f_q}{a} \right)' \right)^2}}
\]

for the quasi-isotropic mode. Note the important persisting correlation between \( \varphi_k \) and \( \theta_k \). Deep inside the Hubble radius, \( \eta \gg \eta_2(k) \),

\[
e^{2r_k} \sim 2kC_1^2 A^2 = \text{constant}, \quad \theta_k = k\eta + \xi_k - \frac{\pi}{2}.
\]

It is in this regime that it becomes possible to introduce the number of created particles (gravitons) \( n(k) = n(-k) \sim e^{2r_k}. \) The condition for the semiclassical behaviour of gravitational waves inside the Hubble radius expressed directly in terms of their amplitude \( h_{ij} = -\frac{\delta g_{ij}}{a} \sim \sqrt{G\phi} \) looks like

\[
k^3 h_g^2(k) \equiv \langle k^3 h_{ij}(k) h^{ij}(k) \rangle \sim e^{2r_k} \frac{l_p^2}{\lambda^2} \gg \frac{l_p^2}{\lambda^2}
\]

where \( l_p \) is the Planck length and \( \lambda = 2\pi a_k^{-1} \) is the wavelength of the perturbations. The corresponding condition for adiabatic perturbations is the same at the radiation-dominated stage inside the Hubble radius, and it is even less restrictive at the matter-dominated stage. Thus, if primordial perturbations are measurable at all, they are always classical; their quantum origin is reflected in their power spectrum and statistics only. It is important to emphasize here that if primordial perturbations are quasi-isotropic at present, this does not necessarily imply their quantum origin from a squeezed state. It is just the opposite: any classical or quantum process that does not spoil the isotropy of the Universe at sufficiently early times results in the dominance of the quasi-isotropic mode nowaday. The role of the inflationary scenario here is to provide a causal mechanism for the generation of the perturbations, while in a non-inflationary cosmology assuming FRW behaviour of the early Universe, power spectrum and statistics of the quasi-isotropic mode may be chosen arbitrarily by hand. On the contrary, if the present GW background was generated at late times, after the second Hubble radius crossing (e.g. by cosmic strings), then both modes would be present with equal probability and the phase \( \xi_k \) would be uniformly distributed. This, as well known, gives a possibility to discriminate between primordial and non-primordial GW.

Is it possible to verify experimentally the predicted quasi-isotropic character of the primordial gravitational wave background on sufficiently large scales, i.e. the standing wave behaviour of \( f_k \), or that of \( \theta_k \) in (55)? A direct LIGO-type experiment
is clearly hopeless because it would require a frequency resolution at the level of the Hubble constant, i.e. $10^{-18}$Hz. It is remarkable, however, that this prediction is already proved by observations in the following sense. Let us assume that the present gravitational-wave background is of primordial origin and its spectral energy density is at least

$$\frac{\lambda}{\epsilon_c} \frac{d\epsilon_g(\lambda)}{d\lambda} \sim 10^{-10} \left(\frac{\lambda}{R_H}\right)^2, \quad R_{eq} < \lambda < R_H,$$

$$\sim 10^{-14} \quad \lambda_\gamma \ll \lambda < R_{eq} \quad (57)$$

($\epsilon_c = 3H_0^2/8\pi G$ is the critical energy density, $R_H = H_0^{-1} = 3000h^{-1}$Mpc, $\lambda_\gamma = 2\pi T_\gamma^{-1} \sim 1$cm), as expected in the simplest versions of the inflationary scenario, or larger (the latter is possible for $\lambda < 100h^{-1}$Mpc). This background corresponds to the value $k^{3/2}h_g(k) \sim 10^{-5}$ at the second Hubble radius crossing. Then it follows from the CMB $\Delta T/T$ data that this background should be dominated by the quasi-isotropic modes $f_q(k)$ for scales $\lambda > 100h^{-1}$Mpc (corresponding to a multipole number $l < 40$). Furthermore, the success of the primordial nucleosynthesis theory proves convincingly that the Universe was isotropic beginning from $t \sim 1s$ ($T_\gamma \sim 1$MeV).

As a consequence, the present background (57) should be dominated by the quasi-isotropic modes for $\lambda > h^2R_{eq} \sim 15$Mpc ($t(\eta_2) \sim 10^{10}s$). This region can be further expanded using observational limits on the PBH number density [5]. Of course, the inflationary scenario predicts the dominance of the quasi-isotropic mode for all scales $\gg 1$cm for the primordial background. As explained above, this quasi-isotropic behaviour in itself cannot be interpreted as a proof for the quantum origin of the perturbations. However, in more complicated inflationary scenarios it is possible to find an observable, in principle, feature of the perturbation spectrum $|f_k|^2$ (namely the absence of zeroes of $|f_k|^2$ as a function of $k$) that might be interpreted as a "quantum signature" though perturbations themselves are classical nowadays as usual [25].

In addition, the standing wave behaviour of $f_k$ yields the appearance of a component in the $\Delta T/T$ polarization multipoles produced by the primordial GW background which is periodic in $l$. The period of oscillations is $T_l = \pi(\eta_0 - \eta_{rec})/\eta_{rec}$ in the spatially flat Universe where $\eta_0$ and $\eta_{rec}$ are the present conformal time and the recombination time respectively. The amplitude of this component is, however, on the level $\Delta T/T \sim 10^{-6}$ or less, the corresponding effect in the $\Delta T/T_l$ itself is even smaller.

The background consisting of waves $\propto f_q(k)$ belonging to the quasi-isotropic mode only, despite its stochasticity, occupies a volume of measure zero in phase space. Thus, one might expect that its entropy is low, if any. To determine the volume occupied in phase space, one has to restore the exponentially small decaying mode, or to consider other physical processes leading to the generation of both modes. If the quasi-isotropic mode remains unchanged, the resulting effect may be described as the appearance of a small stochastic correction to the phases $\xi_k$ and $\theta_k$. There were a number of proposals about how to calculate the entropy of the perturbations [26, 27, 28], all of them leading to the same expression $\Delta S_k \simeq 2r_k (r_k \gg 1)$ for each mode $k$ appearing as a result of coarse graining. It remains unclear, however, if this entropy is the minimal possible one that has to be ascribed to the perturbations irrespective of the choice of coarse graining. We hope to return to this question elsewhere.
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