A Study of Over-modulation Impact on the MSI Series Micro-Grid Output Voltage

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Abstract. In order to analyse the over-modulation technique impact on the micro-source inverter (MSI) series micro grids (SMGs) output voltage, firstly Double Fourier method applied to derive the single inverter output voltage characteristics under over-modulation condition; Secondly, based on the characteristics of the MSI series micro-grids, a new formula of the system output voltage is derived. Thirdly, the impact on the system output voltage fundamental component and harmonic component is analysed in detail, the results show that the output voltage fundamental component increases with the increasing modulation while keeps constant after a certain over-modulation depth. Finally, a simulation was carried out and the simulation verified the theoretical analysis.

Key words. Micro Source Inverter; series connection; over-modulation; voltage characteristics; fundamental wave; harmonic wave

1. Introduction
The ac-type, dc-type and hybrid ac/dc type micro grids micro grid structure are nowadays commonly adopted by the scholars and engineers, however, these traditional power network structures results in problems like current circulation, harmonics, complicated control and low liability [1-4]. The MSI series micro-grids studied in this dissertation is a new type of power network structure, which is composed of several micro source inverters in series. This structure can completely solve the main problems existing in the common power network structures, but still there are some new problems needs to be resolved in this particular system[5].

In the MSI series micro-grids system, the total output power of the system equals the sum of the output power of each micro source. For the output power balance control of the system, the phase shifted and amplitude variation Carrier PWM modulation method is used to control the output voltage of each micro-source inverter (MSI) independently, and changing modulation degree is more suitable for controlling the amplitude of output voltage[6]. However, when the MSI series micro-grid system performs power balance control, some MSI may go into the over-modulation state. Then the amplitude of fundamental output voltage of the system is not linearly related to the modulation degree, and the harmonic distribution changes accordingly. This will cause certain difficulties for the control of the micro-grids system. This paper analyses the output voltage characteristics of the MSI series micro-grids system under over-modulation condition.

Literature [7-9] give the expansion of the output voltage of a single inverter in the modulation state, and analyse the simulation results of the output voltage under over-modulation, but the mathematical model of the output voltage is not provided. The mathematical expressions are determined for
BSPWM under over-modulation in the literature [10], but it may lead to high switching loss. Besides, since the modulations of the inverters are the same, which can’t realize the power balance control of the micro grids system.

2. MSI Output Voltage Model under Over-modulation Condition

By applying Double Fourier analysis, the general expression of the output waveform of any carrier-based PWM modulation strategy is [7]:

\[ u_o = \frac{A_{x0}}{2} + \sum_{n=1}^{\infty} \left[ A_{x_n} \cos(nx) + B_{y_n} \sin(ny) \right] + \sum_{m=1}^{\infty} \left[ A_{x_m} \cos(mx) + B_{y_m} \sin(my) \right] + \sum_{m=n=-1}^{\infty} \left[ A_{x_mn} \cos(mx + ny) + B_{y_mn} \sin(mx + ny) \right] \]  \hspace{1cm} (1)

Where:

\[ A_{x_mn} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_0^{2\pi} F(x, y) \cos(mx + ny) \, dx \, dy \]  \hspace{1cm} (2)

\[ B_{y_mn} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_0^{2\pi} F(x, y) \sin(mx + ny) \, dx \, dy \]  \hspace{1cm} (3)

\( m, n \) are defined as the coefficients of the carrier harmonics and baseband harmonics. The coefficients in formula (1) can be determined for particular PWM process through the solution of the Double Fourier integral:

\[ C_{x_m} = A_{x_m} + B_{y_m} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_0^{2\pi} F(x, y) e^{ixy} \, dx \, dy \]  \hspace{1cm} (4)

![Figure 1. Schematic Diagram of Single SPWM under over-modulation.](image)

A waveform of a unipolar carrier in one cycle is represented by a piecewise linear function, using the equation:

\[ u_c = \begin{cases} 
-\left( x + \alpha - 2k\pi \right) \frac{U_c}{\pi} + U_c, & 2k\pi \leq \omega t \leq 2k\pi + \pi \\
\left( x + \alpha - 2k\pi \right) \frac{U_c}{\pi}, & 2k\pi + \pi \leq \omega t \leq 2k\pi + 2\pi 
\end{cases} \]  \hspace{1cm} (5)

The expression of the sine wave is:

\[ u_s = U_s \sin(\omega t) \]  \hspace{1cm} (6)

Where, \( \omega_c \) is carrier-frequency, \( \omega_s \) is the frequency of modulation wave, \( U_c \) is amplitude of carrier wave, \( U_s \) is amplitude of modulation wave, \( x = \omega_s t \), \( y = \omega_c t \), \( \alpha \) is the shift angle.

\( M = \frac{U_s}{U_c} \) is the modulation depth, the DC side voltages of the MSI is \( u_{dc} \).

According to the principle of SPWM modulation, the triangular wave and the sine wave are compared to generate a switching signal. When \( u_s > u_c \), the value of the switching function is \( F(x, y) = u_{dc} \); When \( u_s < u_c \), \( F(x, y) = -u_{dc} \).
Figure 1. shows schematic diagram of single SPWM under over-modulation, it can be seen that when \( U_s \sin(\omega t) = U_c, \ \omega t \) has 4 solutions, defined \( \gamma = \arcsin \frac{1}{M} \) as overshoot critical angle, which also reflects the depth of over-modulation. When the 6 integral regions from I-VI showed in Figure1 are projected into the coordinate system, the harmonic coefficients of the switching function in the effective integration region can be calculated and the expression of the output voltage of the inverter in the over-modulated state is:

\[
u_o = \frac{2}{\pi} \left( \frac{\gamma}{\sin \gamma} + \cos \gamma \right) u_{dc} \sin \gamma + \sum_{n=3,5,7}^{\infty} \frac{2M}{\pi} \left[ \frac{\sin[(n-1)\gamma]}{n-1} - \frac{\sin[(n+1)\gamma]}{n+1} \right] + \sum_{m=1}^{\infty} \sum_{n=1,3,5}^{\infty} B_{mn} \cdot \sin(mx + ny)
\]

(7)

Where

\[
B_{mn} = \frac{4u_{dc} \cos(m\pi)e^{jmA}}{m\pi} J_\alpha(m\pi) \left[ \gamma - \frac{\sin(2n\gamma)}{2n} \right]
\]

(8)

From the formula (7,8), under the over-modulation state, the output voltage harmonic distribution characteristics are as follows: Both in the over-modulation state and the modulation state, the inverter output voltage has the carrier and carrier sideband harmonics, but the amplitude of the harmonic is different from the modulation state; The output voltage add the odd multiples of the fundamental harmonics under the over-modulation state.

3. Output Voltage model of MSI Series Micro-Grid System under Over-modulation Condition

![Figure 2. Equivalent circuit model of N H-Bridge inverters connected in series.](image)

Figure 2. is a sketch map of MSI series micro-grids [11], the DC side voltages of the MSI are all \( u_{dc} \), and the phase shift angles \( \alpha \) between the micro-sources inverters are equal, \( \alpha = (i - 1) \frac{2\pi}{N} \) \((i = 1, 2, \cdots, N)\), then the output voltage expression is:

\[
u_{cil}(x, y) = u_{cil} \left( x + (i - 1) \frac{2\pi}{N}, y \right)
\]

(9)

![Figure 3. Schematic Diagram of phase shifted and amplitude variation Carrier PWM modulation](image)
The total output voltage of the system is:

\[
u_{AN} = \sum_{i=1}^{N} \left[ \frac{2}{\pi} \left( \frac{
abla_i}{\sin \nabla_i} + \cos \nabla_i \right) \right] u_{dc} \sin(\omega_i t) + \sum_{n=3,5,7}^{\infty} \sum_{i=1}^{N} \left\{ \frac{4}{\pi n} \cos(n\nabla_i) + \frac{2M_i}{\pi} \left[ \sin[(n-1)\nabla_i] \right] \right.\left. \right. \frac{n-1}{n-1} \right] u_{dc} \sin(n\omega_i t) + \sum_{m=1}^{\infty} \sum_{n=1,3,5,7}^{\infty} B_{mn} \sin[m\omega_i t + n\omega_i t] \]

When the frequency of carrier and carrier sideband harmonics is \(m\omega_i \ (m = 1, 2, 3, \ldots)\):

\[
\sum_{i=1}^{N} B_{mn} \sin[m\omega_i t + n\omega_i t] = \sum_{i=1}^{N} \left\{ \frac{4u_{dc}}{m\pi^2} \cos(m\pi e^{j(\nabla_i - \frac{1}{2})} \frac{2\pi}{N} \cdot J_i(m\pi M_i) \left[ \gamma_i - \frac{\sin(2\nabla_i)}{2n} \right] \right\} \sin[m\omega_i t + n\omega_i t] \]

When \(m = mN \ (m = 1, 2, 3, \ldots)\), the frequency of carrier and carrier sideband harmonics is:

\[
\sum_{i=1}^{N} B_{mn} \sin[m\omega_i t + n\omega_i t] = \frac{4u_{dc}}{m\pi^2} \cos(m\pi N\pi J_i(m\pi NM_i) \left[ \gamma_i - \frac{\sin(2\nabla_i)}{2n} \right] \sin[m\omega_i t + n\omega_i t] \]

At this time, the carrier harmonics of the output voltage in the MSI series micro grids system cancel each other out, and only the odd multiples of the baseband harmonics and the carrier sideband harmonics whose frequency equals \(m\omega_i\) are contained.

Comparing equations (11-13) with the output voltage of the system in the modulation state in [12], it can be seen that the fundamental amplitude of the output voltage of each micro-source inverter and the amplitude of the carrier harmonic are both the real part of the general formula calculated in this paper, when \(M \leq 1\), the fundamental amplitude is:

\[
U_{Ai} = M_i u_{dc} = \text{Re} \left\{ \frac{2}{\pi} \left( \frac{\nabla_i}{\sin \nabla_i} + \cos \nabla_i \right) u_{dc} \right\} \]

The amplitude of the carrier harmonics is:

\[
B_{mn} = \text{Re} \left\{ \frac{4}{m\pi^2} \cos(m\pi e^{j(\nabla_i - \frac{1}{2})} \frac{2\pi}{N} \cdot J_i(m\pi M_i) \left[ \nabla_i - \frac{\sin(2\nabla_i)}{2n} \right] u_{dc} \right\}
\]

3.1. Fundamental Analysis

Performing Taylor's expansion on the equation (14), assuming that \(\nabla_i = \gamma_i^2\), the number of micro source inverters is \(N\). Thus the fundamental amplitude of the system output voltage is:

\[
\sum_{i=1}^{N} U_{Ai} = U_{AN} = \text{Re} \left\{ \frac{2}{3\pi} \left[ 6N + \sum_{i=1}^{N} \nabla_i \left( \frac{11}{60} \nabla_i - 1 \right) \right] \right\} u_{dc}
\]

Making \(\delta = \sum_{i=1}^{N} \nabla_i \left( \frac{11}{60} \nabla_i - 1 \right)\) as total modulation value, then (16) can be expressed as:
\[
\sum_{i=1}^{N} U_{oi} = \text{Re} \left[ \frac{2}{3\pi} \left( 6N + \delta \right) \right] u_{dc} 
\]  

(17)

When all the MSIs are in the critical overshoot state, that is to say all the modulation of MSIs satisfy \( M = 1 \), it is easy to calculate that \( \delta = -1.3N \), making \( \eta = \delta + 1.3N \) as the total over-modulation value of the MSI series micro-grids. Then the amplitude of the fundamental of the system output voltage in the critical overshoot state can be expressed as:

\[
U_{AN} = NU_o = \frac{2}{3\pi} (6N - 1.3N) u_{dc} = Nu_{dc} 
\]  

(18)

In non-critical overshoot state, system output voltage fundamental amplitude is:

\[
\sum_{i=1}^{N} U_{oi} = \text{Re} \left[ \frac{2}{3\pi} (\eta + 4.7N) \right] u_{dc} 
\]  

(19)

However, the \( M \) of each micro-source inverter can be large or small, the total over-modulation value also changes. When \( \eta > 0 \), the system is in an over-modulated state and when \( \eta < 0 \) the system is in modulated state. The influence of each micro-source on the total overshoot of the system can cancel each other out. When the modulation of all micro-sources tends to infinity, \( \delta \to 0 \), \( \eta \to 1.3N \), then \( U_{AN} = 1.28Nu_{dc} \), so the voltage fluctuation of the total system is less than 28%.

Figure 4. Relationship between the fundamental amplitude and total overshoot of the system.

It can be seen from Figure 4. that when the number of micro-sources is constant, the smaller the \( \eta \), the smaller the system output voltage is. When \( \eta \) is constant, the larger \( N \) is, the larger the system output voltage is. Therefore, under the premise that the total output power of the system meets the load demand, each micro-source can distribute the output power according to the principle of “can do more work” to achieve power balance control.

3.2. Harmonic Analysis

Assuming each overshoot value of the micro-source inverter is \( M - 1 \), according to (10), when \( M \to \infty, \cos(n\gamma) \to 1 \), then \( \frac{2M}{\pi} \left[ \frac{\sin(n-1)\gamma}{n-1} - \frac{\sin(n+1)\gamma}{n+1} \right] \to 0 \), the switch waveform becomes a square-wave, and the harmonics can be expressed as follow:

\[
u_o = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} u_{dc} \sin(n\omega t) 
\]  

(20)

The variation of amplitude of the baseband harmonics and carrier harmonics in the series system with frequency and overshoot modulation is showed in Figure 5:
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1.

2.) = 50Hz , (b) = 2500Hz , Carrier ratio  = 50 , its harmonic distribution is showed in Figure 6. In Figure 6(a), each micro-source overshoot value is 0.2,-0.2,-0.2,-0.2,-0.2. In Figure 6(b), each micro-source overshoot value is 0.2,0.2,0.2,0.2,0.2. In Figure 6(c), each micro-source overshoot value is 99,199,149,99,249.

Figure 6. The output voltage spectrum of the five inverters connected in series

From Figure 6(a), when the over-modulation condition of the MSI is random, it can be seen from simulation that the harmonics distribution is consistent with the formula (10-12), which consist of carrier sideband harmonics and the odd multiples of the fundamental harmonics. From Figure 6(b), it is showed that when the overshoot values are same, the MSI series micro grid only contain odd multiples of the baseband harmonics and the carrier sideband harmonics whose frequency equals to
When the overshoot values are large enough, the output voltage become a square-wave, which can be seen from Figure 6(c) and Figure 5, both figures showed that as the multiple become larger, the amplitude of baseband harmonics and the amplitude of the carrier harmonics damping to 0, which proved that the theoretical calculation is correct.

In the MSI series micro-grids, changing the modulation degree is more suitable for controlling the amplitude of the output voltage. In the simulation, the DC side voltage of the micro-source inverter is 60V. Since the phase voltage of the micro-grid is 220V, it is necessary to ensure that the amplitude of the output voltage of the system holds on 310V.

the value of total over-modulation degree is: \( \eta = 0.8366 \).

It can be seen that the output voltage of the system is decided by the DC side voltage and total over-modulation value of the micro-source inverter. Since the sequence of MSI has a little effect on the voltage output of the series inverter when overshoot occurs, this simulation only analyses the number of overshots. From (13), it can be seen that the inverters who has the same modulation can offset some of their harmonics, and decrease distortion rate to a certain degree. In order to ensure the lowest harmonic distortion rate, fixing one of the micro-source inverters' over modulation degree, then take the other micro-source inverters have the same over-modulation degree, the output voltage and harmonic distortion rate of the MSI series micro grids system are showed in Table 1.

| Overshoot number | U_{d5} | FFT% |
|------------------|--------|------|
| \( \eta = 0.5048 \) |        |      |
| 1                | 311.4  | 10.76|
| 2                | 311.3  | 11.95|
| 3                | 311.5  | 13.09|
| 4                | 311.9  | 14.26|
| 5                | 331    | 11.36|

It can be seen from the above table that when the number of micro-sources and total over-modulation degree are constant. The more the number of overshoot MSI, the greater the harmonic distortion rate, when all the overshoot values are the same, the distortion rate will decrease to a certain extent.

5. Conclusion

In this paper, the expression of the MSI series micro-grids system output voltage in the over-modulation is derived by using the phase shifted and amplitude variation Carrier PWM modulation strategy. It is analysed that the output voltage fundamental component increases with the increasing modulation while keeps constant after a certain over-modulation depth, but the fluctuation is smaller than 28%; the carrier harmonic amplitude tends to zero; the system output voltage harmonics are mainly determined by the baseband harmonics as the overshoot modulation value increase. When the over-modulation depths of MSI are equal, the system only contain carrier and carrier sideband harmonics which only appear at the frequency of \( mN_0 \). When the number of MSI and the total overshoot value of the system is constant, the more the number of the overshoot MSI, the higher the harmonic distortion rate of the system output voltage is. Based on the above conclusions, the effect of overshoot on system output voltage can be accurately known. According to the total overshoot of the system, each micro-source modulation can be reasonably allocated, thus the power balance control can be better carried out.

This paper only analyses the influence of over-modulation on the output voltage characteristics of the MSI series micro-grid system, but its impact on system stability and control has not been involved, this need to be further explored.

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