Studies on Nuclear Fuel Rod Thermal Performance

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Abstract

In this article we used ABAQUS software to study temperature and heat flux changes in a nuclear fuel rod. This software is based on finite element method. For this case, it divides nuclear fuel rod to series of elements and then investigates the changes in each element. During this study, we divide a nuclear fuel rod to 10 elements. Since the thermal conductivity depends on temperature and temperature is different in one nodal point to other point (in radial direction), each element has a different thermal conductivity. During this article, the temperature distribution is investigated in each nodal point and finally we represent the general expression for the temperature and heat flux changes in radial direction.

Keywords: ABAQUS code, Finite element analysis, Nuclear fuel rod, Thermal performance

1. Introduction

Energy is released by fission within the fuel rod and is transferred by heat conduction to the surface of the fuel and through the cladding [1]. From the surface of the cladding heat is transferred by convection to the coolant, which passes from the core to the external heat exchangers in which steam is generated to operate on a power cycle. A nuclear fuel rod is used as the source of nuclear energy in a reactor. Most nuclear reactors are powered by fuel rods that contain two isotopes of uranium: uranium-238 and uranium-235. The power generation process in a nuclear core is directly proportional to the fission rate of the fuel and the present thermal neutron flux. The thermal power produced by a reactor is directly related to the mass flow rate of the reactor coolant and the temperature difference across the core [2]. The fuel elements are usually long cylindrical rods or rectangular plates of uranium (or thorium) enclosed by cladding. The uranium may be in the pure metallic form, in the form of a compound such as uranium

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oxide, UO$_2$, or in the form of an alloy with another metal such as aluminum or zirconium [2]. The desirable properties of a fuel, which must be fissionable, are high thermal conductivity, good corrosion resistance, good mechanical strength at high temperatures and a high limiting temperature for operation. The numerical method of solution is used extensively in practical applications to determine the temperature distribution and heat flow in solids having complicated geometries, boundary conditions, and temperature-dependent thermal properties [3].

In this paper the finite-difference method is used. The problem is then discretized and the numerical method of solutions is played out using finite difference method. Finally, we simulate nuclear fuel rod using commercial finite element code ABAQUS/Standard 2009 and investigate thermal performance of nuclear fuel rod. Nuclear energy at a non-uniform rate of $q$ (w/m$^3$) is generated in the rod. Surrounding coolant temperature is $T_a$, and the heat transfer coefficient $h$ is large.

1.1. Internal heat generation

The heat generation due to fission within a nuclear fuel rod is not uniform, and for a cylindrical fuel rod the heat generation is generally given by [4]

$$q_g = q_0 \left(1 - \left(\frac{r}{R_a}\right)^2\right)$$  \hspace{1cm} (1)

Where $q_0$ is the heat generation rate per unit volume at the centre ($r = 0$) and $R_a$ is the outer radius of the solid fuel rod. Evidently $q_g$ is a function of position $r$, i.e., the radial distance from the axis of the rod [1]. For steady state one-dimensional heat conduction in the radial direction we have:

$$\frac{d}{dr} \left( r \cdot \frac{dT}{dr} \right) + q_g \frac{r}{K} = 0$$  \hspace{1cm} (2)

Substituting $q_g$ from Eq. (1) we have:

$$\frac{d}{dr} \left( r \cdot \frac{dT}{dr} \right) + q_0 \left(1 - \left(\frac{r}{R_a}\right)^2\right) \frac{r}{K} = 0$$  \hspace{1cm} (3)

Upon twice integration

$$T + \frac{q_0 \left(\frac{r^2}{4} - \left(\frac{r^4}{16R_a^2}\right)\right)}{K} = C_1 \ln(r) + C_2.$$  \hspace{1cm} (4)

Invoking the boundary conditions, at $r = 0$ $\left(\frac{dT}{dr}\right) = 0$, $T = T_{max}$, then $C_1 = 0$ & $C_2 = T_{max}$

Therefore Eq. (4) becomes,

$$T - T_{max} = -\frac{q_0 \left(\frac{r^2}{4} - \left(\frac{r^4}{16R_a^2}\right)\right)}{K}.$$  \hspace{1cm} (5)

If $T_w$ is the temperature at the outer surface (wall) of the rod i.e., at $r = R_a$, then

$$T_w - T_{max} = -3 \frac{q_0 R_a^2}{16 K}.$$  \hspace{1cm} (6)

The heat flow at the surface of the fuel rod is [4],

$$Q = -KA \frac{dT}{dr} \hspace{1cm} \text{(at $r=R_a$)}$$

$$Q = q_0 A R_a \frac{4}{4}.$$  \hspace{1cm} (7)

Under steady state conditions, this heat would be converted from the outside surface of the rod.

$$\left(q_0 A R_a \frac{4}{4}\right) = hA \left(T_w - T_a\right)$$
Where \( h \) is the convective heat transfer coefficient and \( T_a \) is the ambient temperature. From Eqs. (6) and (8), we get

\[
T_{\text{max}} - T_a = \left( \frac{q_0 A R_a}{4} \right) \left( \frac{3R_a}{4} - \left( \frac{1}{h} \right) \right)
\]

The governing non-dimensional energy equation for the steady state one dimensional radial heat conduction with non-uniform internal heat generation for a cylindrical nuclear fuel rod is given as [5]

\[
\left( \frac{\partial^2 \theta}{\partial R^2} \right) + \left( \frac{\partial \theta}{\partial R} \right) + 1 = 0
\]

Where, \( \theta = \frac{(T - T_a)}{\left( \frac{q_0 A R_a}{4} \right) \left( \frac{3R_a}{4} - \left( \frac{1}{h} \right) \right)} \) and \( R = \left( \frac{r}{R_a} \right) \) are non-dimensional temperature and non-dimensional radius respectively.

1.2. Boundary conditions

The non-dimensional boundary conditions are at \( R = 0 \rightarrow \frac{\partial \theta}{\partial R} = 0 \) & at \( R = 1 \rightarrow \theta = 0 \).

Equation 10 is discretized using central difference for \( \left( \frac{\partial^2 \theta}{\partial R^2} \right) \) and \( \frac{\partial \theta}{\partial R} \) at any interior grid point ‘i’ as follows,

\[
\left( \theta_{i+1} - 2 \theta_i + \theta_{i-1} \right) (2R) + \left( \theta_{i+1} - \theta_{i-1} \right) \Delta R + 2R(\Delta R)^2 = 0
\]

The second term on the right hand side of the governing differential equation can be written as \( \frac{\partial \theta}{\partial R}/R \).

At \( R=0, \frac{\partial \theta}{\partial R} = 0 \) from the second boundary condition. Therefore the term \( \frac{\partial \theta}{\partial R}/R \) will give rise to \( 0/0 \) condition. However, this difficulty can be alleviated by making use of the L’Hopital’s rule. Then we have:

\[
2 \left[ \frac{\left( \theta_{i+1} - 2 \theta_i + \theta_{i-1} \right)}{(\Delta R)^2} \right] + 1 = 0
\]

At the centre, \( i = 1, i-1 = 0, i+1 = 2 \).

Using mirror-image technique (Fig. 1) at the centre \( R=0 \),

![Fig. 1: mirror image technique](image)

therefore Eq. (12) becomes,

\[
4(\theta_i - \theta_2) + (\Delta R)^2 = 0
\]

The outer boundary is maintained at temperature \( T_a \), the temperature of the surrounding fluid, assuming a large heat transfer coefficient \( h \) (i.e.) \( T_w = T_a \).

At \( R=1, \theta = 0 \), that is due to second boundary condition. Let us consider an example in which \( \Delta R = \frac{1}{9} \). Therefore, the number of nodal points is ten (i.e. from 1 to 10) and the numbers of unknown temperatures are nine (i.e. from 1 to 9). Since \( T = T_a \) at the outer boundary, the value of \( \theta \) is zero at the nodal point 10 [5].

**At Nodal Point 1**

\( R=0 \) & \( i = 1 \)

\[
4(\theta_1 - \theta_2) = \left( \frac{1}{9} \right)^2
\]
At Nodal Point 2  
\[ R = \left(\frac{1}{9}\right) \& i = 2 \]  
\[ 2\theta_2 - \frac{1}{2}\theta_1 - \frac{3}{2}\theta_3 = \left(\frac{1}{9}\right)^2 \]

At Nodal Point 3  
\[ R = \left(\frac{2}{9}\right) \& i = 3 \]  
\[ 2\theta_3 - \frac{3}{4}\theta_2 - \frac{5}{4}\theta_4 = \left(\frac{1}{9}\right)^2 \]

At Nodal Point 4  
\[ R = \left(\frac{3}{9}\right) \& i = 4 \]  
\[ 2\theta_4 - \frac{5}{6}\theta_3 - \frac{7}{6}\theta_5 = \left(\frac{1}{9}\right)^2 \]

At Nodal Point 5  
\[ R = \left(\frac{4}{9}\right) \& i = 5 \]  
\[ 2\theta_5 - \frac{7}{8}\theta_4 - \frac{9}{8}\theta_6 = \left(\frac{1}{9}\right)^2 \]

At Nodal Point 6  
\[ R = \left(\frac{5}{9}\right) \& i = 6 \]  
\[ 2\theta_6 - \frac{9}{10}\theta_5 - \frac{11}{10}\theta_7 = \left(\frac{1}{9}\right)^2 \]

At Nodal Point 7  
\[ R = \left(\frac{6}{9}\right) \& i = 7 \]  
\[ 2\theta_7 - \frac{11}{12}\theta_6 - \frac{13}{12}\theta_8 = \left(\frac{1}{9}\right)^2 \]

At Nodal Point 8  
\[ R = \left(\frac{7}{9}\right) \& i = 8 \]  
\[ 2\theta_8 - \frac{13}{14}\theta_7 - \frac{15}{14}\theta_9 = \left(\frac{1}{9}\right)^2 \]

At Nodal Point 9  
\[ R = \left(\frac{8}{9}\right) \& i = 9 \]  
\[ 2\theta_9 - \frac{15}{16}\theta_8 - \frac{17}{16}\theta_{10} = \left(\frac{1}{9}\right)^2 \]

At Nodal Point 10  
\[ R = \left(\frac{9}{9}\right) = 1 \& i = 10 \]  
Recalling the second boundary condition  
At \( R = 1 \rightarrow \theta = 0 \)  
\( \theta_{10} = 0. \)

These are the 9 equations to be solved to find the 9 unknown temperatures. These 9 equations can be written in the matrix form and can be solved through various methods like Gaussian elimination, Gauss-Seidel iteration, etc. As those analytical approaches are too lengthy if the nodal points are more, computer programming is mostly preferred. Here matlab program is used to solve the above matrix. The results thus obtained are shown in Fig. 2.
1.3. Temperature changes in a nuclear fuel rod

Meshing of nuclear fuel rod is shown in Fig. 3. In this model we also consider the pellet-cladding gap which has different thermal conductivity from nuclear fuel rod. The parameters are used in this analysis are: distance of nodal points from each other, thermal conductivity of each element, thermal conductivity of pellet-cladding gap, temperature of coolant. Obtained results from simulation of nuclear fuel rod using ABAQUS/Standard 2009 are shown in Fig. 4.

1.4. Heat flux changes in a nuclear fuel rod

Heat flux can be expressed according to Eq. 7. Obtained results indicate a linear increase in heat flux through the outer surface of fuel rod during the time. Attained result is shown in Fig. 5.
1.5. Conclusions

The graphical representation between the non-dimensional temperature and the radial distance from the center of the nuclear fuel rod unveils that as expected the temperature is maximum at the center and minimum at the outer diameter. This means that temperature reduces over distance from the centerline and our calculations has shown that the temperature in each nodal point increase linearly over the time. This shows the heat is transferred from the nuclear fuel rod to the surrounding coolant according to the second law of thermodynamics. Thus from the surface of the nuclear fuel rod, heat is transferred by convection to the coolant, which passes from the core to the external heat exchangers in which steam is generated to operate on a power cycle. In this paper the temperature distribution studies for the nuclear fuel rod with cladding has been performed.

Since we should consider the direction of heat flux flow, and in the ABAQUS defaults, the input flux to the medium is positive and output flux is negative, the negative sign of heat flux can be explained. As the net heat flux is emerging from the outer surface of rod, its sign should be negative but its value increase over the time because the heat flux in nodal point i+1 is the sum of heat flux from the point i and the heat flux from that point itself.

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