Research Article

Analysis of the Bearing Characteristics of Single Piles under Vertical and Torsional Combined Loads

Shun-Wei Wang,1,2,3 Jie Jiang,1,2,3 Chen-Zhi Fu,1,2,3 Xiao-Duo Ou,1,2,3 and Juan Tang1,2,3

1College of Civil Engineering and Architecture, Guangxi University, Nanning 530004, China
2Key Laboratory of Disaster Prevention and Structural Safety of Ministry of Education, Guangxi University, Nanning 530004, China
3Guangxi Key Laboratory of Disaster Prevention and Engineering Safety, Guangxi University, Nanning 530004, China

Correspondence should be addressed to Jie Jiang; jie_jiang001@126.com

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To investigate the bearing characteristics of single piles under the combined action of vertical loads (V) and torque (T) on a pile head, the displacement governing equations of the pile shaft were proposed in consideration of the load transfer method, and the nonlinear solution of a single-loaded pile (vertical load or torque) was calculated by the finite difference method. According to the above mentioned investigation, a new numerical analysis model was proposed to modify the vertical and circumferential ultimate friction resistance of the pile side. Subsequently, the nonlinear solutions of single piles under the combined loads of V-T loading with different loading paths were obtained by MATLAB programming, and the bearing capacity envelopes were plotted. Based on the comparison with the existing research results, the correctness of the theoretical calculation method proposed in this article was verified. The results show that compared with a single loaded pile (V or T), with the action of the V-T combined loads, the bearing capacity of a single pile will decrease, and the single piles under the V⟶T loading path will have a higher bearing capacity than those under the T⟶V loading path. When the aspect ratio is comparatively small, the influence of the loading path is negligible; however, when the aspect ratio is comparatively large, it is necessary to avoid the preliminary application of the torque to a single pile to prevent the decrease in the ultimate bearing capacity on account of the formation of the T⟶V loading mode. The deformation of the pile body mainly occurs in the range from 0 to 0.6L, so effective deformation reduction can be achieved by reinforcing the shallow foundation. It is inadvisable for a single friction pile to be used to improve the bearing capacity by increasing the concrete grade.

1. Introduction

To ensure the safety of buildings (structures), pile foundations are often used in actual projects. Large infrastructure pile foundations, such as high-rise buildings, urban overpasses, and offshore wind power systems, will bear not only vertical loads (V) but also large torque (T) due to horizontal eccentric loads, lateral impact loads, and earthquakes. The conventional pile foundation design method merely considers a single vertical load and ignores the effect of torsional loading, which may cause substantial engineering losses in serious cases [1]. Therefore, research on the bearing characteristics of the V-T combined loaded pile is of great importance for the formulation of a reasonable engineering scheme.

Currently, the study on the bearing and deformation characteristics of single piles under a single vertical or torsional load has been relatively established by scholars. For a single vertically loaded pile, Seed and Reese [2] studied the load transfer mechanism at the pile-soil interface and the load transfer method was first proposed. Since then, several scholars have carried out a series of in-depth studies. Zhang et al. [3] utilized a hyperbolic curve model to explain the lateral friction of single piles and a polyline model to simulate the hardening of the pile tip. Based on this, an elastoplastic solution for single piles in layered soil was
obtained, which laid the foundation for the further study of different loading paths. The torsional and axial loading based on the simplified continuum model. The above-mentioned methods did not consider the effect of loading paths on bearing capacity of single piles. Preloading the pile top will inevitably change the side friction of the pile. For instance, under the \( T \rightarrow V \) loading path, the existence of circumferential friction at the pile-soil interface will inevitably cause a decrease in the vertical ultimate friction. For single piles under the \( T \rightarrow V \) loading path, the hoop friction resistance under the action of torque can be calculated first, and the vertical ultimate friction resistance can be determined by the following formula:

\[
\tau_{vf}(i) = \sqrt{\left[\tau_f(i)\right]^2 - \left[\tau_v(i)\right]^2}.
\]

Similarly, under the \( V \rightarrow T \) loading path, the vertical frictional resistance under the vertical load can be calculated first, and the circumferential ultimate frictional resistance of the single pile can be determined by the following formula:

\[
\tau_{vf}(i) = \sqrt{\left[\tau_f(i)\right]^2 - \left[\tau_v(i)\right]^2}.
\]
The number of pile units \((n)\) is increased from 5 to 1000 for sensitivity analysis. A large number of calculations show that when the value of \(n\) exceeds 120, more computer resources are required and the computational accuracy has not improved significantly. Therefore, analyses are conducted with \(n \leq 120\).

2.2. Solution under a Single Vertical Load. The pile unit with length \(dz\) at depth \(z\) was taken for analysis. Under the action of vertical loads and according to the static equilibrium condition, the following equation can be obtained [3]:

\[
\frac{dP(z)}{dz} = -U_p r_v(z),
\]

where \(U_p\) is the perimeter of the pile shaft, \(P(z)\) is the axial force of the single pile at any depth \(z\), and \(r_v(z)\) is the vertical frictional resistance of the pile side at depth \(z\).

The elastic compression of the pile microunit is expressed as follows:

\[
dw(z) = \frac{P(z)dz}{E_pA_p}.
\]

where \(w(z)\) is the vertical displacement of the pile body, \(E_p\) is the elastic modulus, and \(A_p\) is the cross-sectional area of the pile.

Solving equations (5) and (6) can obtain the vertical governing equation of the pile body as follows:

\[
\frac{d^2w(z)}{dz^2} - \frac{U_p}{E_pA_p}r_v(z) = 0.
\]

The load transfer function of the pile-soil interface adopts the function form proposed by Kraft et al. as follows [18]:

\[
\tau_v(z) = \frac{G_s w(z)}{r_0 \ln(r_m/r_0 - \psi/1 - \psi)}.
\]

The expression of tangent stiffness of the soil on the pile side can be written as follows:

\[
k_v = \frac{U_p \tau_v(z)}{w(z)}.
\]

The initial tangent stiffness is as follows:

\[
k_v' = U_p \frac{\partial \tau_v(z)}{\partial w(z)} \bigg|_{w(z)=0} = \frac{2\pi G_s}{\ln(r_m/r_0)}.
\]

The governing equation of the pile is expressed as follows:

\[
\frac{d^2w(z)}{dz^2} - \lambda^2 w(z) = 0,
\]

where \(\lambda = \sqrt{k_v/E_pA_p}\).

The marginal conditions of the pile top and tip are as follows.

For the pile top, the displacement is expressed as follows:

\[
w(0) = w_0,
\]

where \(w_0\) is the known reloading displacement.

For the pile tip, the vertical bearing capacity can be written as

\[
P(l) = \frac{w(l)}{1/K_{bt} + w(l)/q_{ult}}.
\]
where \( w(l) \) is the vertical displacement of the pile tip and \( K_{bz} \) is the initial stiffness of the soil at the pile tip. The initial stiffness can be determined by the following equation [19, 20]:

\[
K_{bz} = \frac{4G_Ir_0}{(1 - v_j)}
\]

where \( v_j \) is the Poisson’s ratio of the soil. When the soil at the pile tip is sand, silt, or clay, the ultimate resistance of the pile tip \( q_{ult} \) can be expressed as follows [21]:

\[
q_{ult} = cN_c + qN_q,
\]

where \( N_c \) and \( N_q \) are the dimensionless bearing capacity constants related to the internal friction angle, which can be determined by referring to the table, \( c \) is the cohesion of the soil, and \( q \) is the average vertical pressure on the side of the plane at pile tip, which can be obtained as follows:

\[
q = \frac{(1 + 2k_0)\gamma L}{3}.
\]

As shown in Figure 2, the finite difference method is utilized to solve the pile, which divides the pile into \( n \) equal parts along the length and adds a virtual equalization node \( n + 1 \) to the bottom of the pile.

The governing equations and boundary conditions of the pile can be differentially discretized to form a system of equations, which is written as a matrix as follows:

\[
[K_z'] [w] = [F_z'],
\]

where \([w]\) is the vertical displacement vector generated along the pile node when single piles are subjected to vertical loads, \([w] = [w_0, w_1, \ldots, w_i, \ldots, w_{n-1}, w_n]^T\); \([K_z']\) is the vertical stiffness matrix of the pile; \([F_z']\) is the vertical load vector of the pile node.

\[
[K_z'] = \begin{bmatrix}
-2 & 1 & & & & \\
1 & -2 & 1 & & & \\
& \ddots & \ddots & \ddots & & \\
& & 1 & -2 & 1 & \\
& & & \ddots & \ddots & \ddots \\
& & & & 1 & -2 & 1 \\
& & & & & 2 & -2
\end{bmatrix}_{n \times n}
\]

\[
[F_z'] = \begin{bmatrix}
\lambda_1^2 h^2 w_1 - w_0 \\
\lambda_2^2 h^2 w_2 \\
\vdots \\
\lambda_i^2 h^2 w_i \\
\vdots \\
\lambda_{n-1}^2 h^2 w_{n-1} \\
\lambda_n^2 h^2 w_n + \frac{2h(q_{ult} + w_nK_{bz}) - w_n}{E_pA_pK_{bz}q_{ult}} w_n
\end{bmatrix}.
\]

By solving formula (17), the vertical displacement along the pile body under vertical loading may be written as

\[
[w] = [K_z']^{-1} [F_z'].
\]

The solution process is as follows:

(i) Given an arbitrary nonzero matrix \([w]\), \([w]\) can be set to be a random array between 0 and 1.

(ii) Calculate tangent stiffness of the soil on the pile side, then the matrices \([F_z']\) and \([K_z']\) can be solved.

(iii) Place \([F_z']\) and \([K_z']\) back into equation (19), then the vertical displacement vector \([w]^k\) is obtained.

(iv) Based on the vertical displacement vector \([w]^k\), repeat steps (b) and (c), then the vertical displacement vector update to be \([w]^{k+1}\).
(v) Set $|w|^{k} - |w|^{k+1}$ as iterative control value. If the value is greater than the specific tolerance, repeat steps (b)–(d) until the value is less than the tolerance.

For the pile top load $P_0$, its magnitude is equal to the axial force of the second pile node as follows:

$$P_0 = \frac{E_p A_p (w_0 - w_2)}{2h}.$$  \hfill (20)

### 2.3 Calculation of a Single Pile under Pure Torsional Loading

According to the force balance condition of the pile unit with length $dz$ under the action of torque, the equilibrium condition can be expressed as follows:

$$\frac{dT(z)}{dz} = 0.5 \pi \tau_i(z) D^2,$$  \hfill (21)

where $T(z)$ is the torque at depth $z$, $\tau_i(z)$ is the circumferential friction of the pile at depth $z$, and $D$ is the diameter of the pile.

Based on the deformation conditions of the pile element with a length of $dz$ under the action of torque, the equilibrium condition can be expressed as follows:

$$\frac{d\theta(z)}{dz} = -\frac{T(z)}{J_p G_p},$$  \hfill (22)

where $\theta(z)$ is the twist angle at depth $z$, $G_p$ is the shear modulus of the pile, and $J_p$ is the polar moment of inertia of pile section.

The twist angle of the pile body at any depth $z$ can be written as follows [8]:

$$\theta(z) = \frac{2s_t(z)}{D}.$$  \hfill (23)

Combining equations (21)–(23), the torsion governing equation of the pile can be obtained as follows:

$$\frac{d^2 \theta(z)}{dz^2} = \alpha^2 \theta(z),$$  \hfill (24)

where $\alpha = \sqrt{k_0 \pi D^3 / 4 G_p J_p}$, and the torsional tangent stiffness $k_0(z)$ can be expressed as follows:

$$k_0(z) = \frac{\tau_i(z)}{s_t(z)},$$  \hfill (25)

where $s_t(z)$ is the circumferential displacement for pile shaft at depth $z$.

The load transfer function of the pile-soil interface is given as follows.

$$\tau_i(z) = \frac{G_s s_t(z)}{r_0 \ln(r_m/r_0 - \psi/1 - \psi)}.$$  \hfill (26)

The marginal conditions of the pile top and tip are as follows:

The pile top should satisfy the following equation:

$$T(0) = T.$$  \hfill (27)

For the pile tip, assuming that the torsion angle of the pile body is linearly distributed, and combining the boundary conditions of the pile tip proposed by Poulos [5], the equilibrium can be obtained as follows:

$$\theta_n = \frac{(\theta_{n+1} + \theta_{n-1})}{2} = \frac{3T_b}{16G_z r_0^3},$$  \hfill (28)

where $G_z$ is the shear modulus of the soil at the bottom of the pile, and $T_b$ is the torque at the pile tip.

Solving equation (21), the torque at the bottom of the pile can be expressed as follows:

$$T_b = T - \sum_{i=0}^{n-1} 0.5 \pi \tau_i (i) D^2 h,$$  \hfill (29)

where $T$ is the torque at the pile top.

To obtain the pile-side circumferential frictional resistance of each pile unit, as shown in Figure 3, the centre difference method is used to discretize the pile length into $n$ equal units, and a virtual equalization node is added at the top and tip of the pile. By substituting the boundary conditions into equation (24) for differential discretization, the following system of equations can be obtained as follows:

$$[K'_i] \{\theta\} = \{T'_i\},$$  \hfill (30)

where $\{\theta\}$ is the torsion angle vector of the pile node, $\{\theta\} = \{\theta_0, \theta_1, ..., \theta_{n-1}, \theta_n\}^T$; $\{T'_i\}$ is the torsional load vector of the pile, where $\{T'_i\} = [-2T h / (G_z J_p), 0, 0, 0, 0, 0, 0, -3T_b h / (8G_z r_0^3)]^T$, and $[K'_i]$ is the torsional stiffness matrix of the pile.

$$[K'_i] = \begin{bmatrix} B_0 & 2 & & & & & & & \\ 1 & B_1 & 1 & & & & & & \\ & \ddots & \ddots & \ddots & & & & & \\ & & \ddots & \ddots & \ddots & \ddots & & & \\ & & & 1 & B_{n-1} & 1 & & & \\ & & & & & & \vdots & \ddots & \ddots & \ddots \\ & & & & & & & 1 & B_n & 1 \end{bmatrix},$$  \hfill (31)

where $B_i = -((\alpha^2 h^2 + 2)$. Solving equation (30) can provide the torsional angle along the pile body as follows:

$$\{\theta\} = \{K'_i\}^{-1} \{T'_i\}.$$  \hfill (32)

The solution process of equation (29) is as follows:

(i) Assuming that the torsion angle $\theta$ along the pile node is an arbitrary nonzero value.

(ii) The circumferential displacement $s_t(z)$ at depth $z$ is obtained from equation (23), then the matrices $\{T'_i\}$ and $K'_i$ can be solved.

(iii) Place $\{T'_i\}$ and $K'_i$ back into equation (19), then the $\theta^k$ at each node of the pile are obtained.

(iv) Based on $\theta^k$ along the pile node, repeat steps (b) and (c), then the torsion angle of the pile body update to be $\theta^{k+1}$. 

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Figure 3: Schematic diagram of the differential dispersion of pure torsion piles.

(v) Set $|\theta^{i+1} - \theta^i|$ as iterative control value. If the value is greater than the specific tolerance, repeat steps (b)–(d) until the value is less than the tolerance.

2.4. Solution Process. The solution process of $V$-$T$ combined loaded piles under different loading paths is as follows. Taking the $V$-$T$ loading path as an example, the vertical displacement $w$ of a single vertical loaded pile can be obtained according to Section 2.2. Solving equation (8), the vertical frictional resistance of the pile body $\tau_w(i)$ is obtained. Subsequently, the ultimate circumferential frictional resistance of the pile side $\tau_f(i)$ is easily obtained by solving equation (3). Finally, substituting $\tau_f(i)$ into equation (26), the $T_\theta$ curve under the $V$-$T$ loading path can be achieved by the method in Section 2.3. Moreover, the similar method can be used to obtain the $V$-$s$ curve under the $T$-$V$ loading path.

2.5. Bearing Capacity Envelope under Different Loading Paths. The drawing method of the single pile bearing capacity envelope under different loading paths is as follows: taking the $V$-$T$ loading path as an example, the $T$-$\theta$ curves corresponding to different vertical loads $V_i$ is obtained according to the method in Section 2.4. The torque $T_{uw}$ corresponding to the reverse bending point of the curve is the limit torque under the $V$-$T$ loading path. Draw the points with $V_i$ as the ordinate and $T_{uw}$ as the abscissa, and the single pile bearing capacity envelope under the $V$-$T$ loading path is plotted by connecting the above points in sequence. A similar method can be used to obtain the single pile bearing capacity envelope under the $T$-$V$ loading path.

3. Method Validation

Since the calculation theory proposed in this article is based on the calculation approach of a single-loaded pile; to confirm the correctness, the calculation results of the single-loaded pile are verified first, and then the calculation method of the $V$-$T$ combined loaded pile is verified.

3.1. Verification of the Calculation Results of a Single Loaded Pile. Harris and Mayne [22] conducted a single pile axial compression test on the Georgia Institute of Technology campus in Atlanta. The soils are primarily the product of the in-place weathering of schists, gneisses, and granites. The stratum was composed of 1.6 m fill and 16.9 m silt sand and the water level was 17.0 m below the surface, as shown in Figure 4 [23]. Consolidation undrained triaxial compression tests were carried out on samples of different depths, and the results show that the value of $c$ and $\phi$ was relatively constant within the range of pile length, which were 0 and 36.1°, respectively; and the bulk density of the fill and silt sand were both assumed to be 21 kN/m$^3$ by Zhu and Chang [23]. The pile length and pile diameters were 16.8 m and 0.76 m, respectively, and the elastic modulus of the pile was assumed 20 GPa by Harris and Mayne [22].

A torsion test on the backfill concrete in the steel pipe pile was conducted by Stoll [24]. Pile-3 had an outer diameter of 0.273 m with a wall thickness of 6.3 mm, a length of 17.4 m, and a pile body torsional rigidity of 12.8 MN·m$^2$.

Zou et al. [25] assumed that the shear modulus of soil was distributed in an exponential function ($\tau_f = G_t e^{\mu z}$) as a function of the depth and obtained the relevant calculation parameters ($G_t = 5.07 \text{kPa}$, $m = 0.1$) for determining the soil limit friction resistance based on the parametric inversion method and established the theory of this article to obtain the comparison curve of the pile top torque $T$ and torsion angle ($T$-$\theta$ curve) by substituting the above parameters into the calculation theory in this article, as shown in Figure 6. The calculated result of Zou et al. [25] is less than the measured value, while the torsion angle of the pile top calculated in this study is close to Basack and Sen [9], Guo and Randolph [26], and all are larger than the measured value.

Guo and Randolph [26] pointed out that with the increase of the torque load, the cracking of the concrete causes the change of the pile body torsional stiffness, which is a possible reason for the inconsistency between the theoretical value and the observed value; in addition, the inability of the load transfer function to fully reflect the characteristics of the foundation is another possible reason for this result.

3.2. Verification of the Calculation Results under the $T$-$V$ Loading Path. Georgiadis and Saflekou [11] used aluminium alloy pipe piles to conduct four groups of single pile model tests under the $T$-$V$ loading path in clay soil. The pile length was 0.5 m, the pile diameter was 0.019 m, and the undrained shear strength of the soil was 8 kPa. Figure 7 shows the calculation results in this study of the $T$-$V$ loading path compared with experimental data and those
calculated by Georgiadis and Saflekou [11] and Zou et al. [14], and the four sets of experimental data have a large dispersion. In comparison, the calculation results in this study are within the two predicted results and closer to the test results, indicating that the calculation method in this study is more accurate.

3.3. Verification of the Calculation Results under the V ⟷ T Loading Path. Georgiadis [10] simulated the relationship of the interface between the pile and soil by utilizing interacting nonlinear axial and torsional springs, compiled a calculation program on the strength of the transfer matrix method, and obtained the nonlinear solution under single pile stress of the V ⟷ T loading path by combining with an engineering instance. The calculation parameters of the pile are $D = 1.52 \text{ m}$, $L = 50 \text{ m}$, $E_p = 20 \text{ GPa}$, $\tau_f = 50 \text{ kPa}$, and the shear modulus of the clay is $5 \text{ MN/m}^2$. The vertical load of 100 t was applied to the pile head at first; however, the torque was applied step by step. Comparing the $T-\theta$ curve calculated with those of Georgiadis [10] and Zou et al. [15], as shown in Figure 8, there is a certain error when the load is less than 6000 kN due to different load transfer functions, while the trend of the curve is consistent with the torque limit, which further verifies the correctness of the calculation method proposed in this article.

4. Parameter Analysis

To discuss the main load-bearing characteristics, deformation law and affecting factors of V-T combined loaded piles, combined with the theoretical calculation method proposed in this article, the load-bearing characteristics, aspect ratio, and elastic modulus of the pile were analyzed.

4.1. Influence of the Length-Diameter Ratio. The undrained shear strength of the clay is 39.6 kPa and the internal friction angle and elastic moduli are 18° and 15 MPa, respectively, and the elastic modulus of the pile body is 25 GPa. Table 1 shows the calculation parameters of pile length and depth.
diameter. With different pile lengths and diameters, the envelopment of the bearing capacity of a single pile under two loading paths are shown in Figures 9(a) and 9(b). By comparison, it can be found that with increasing pile length and diameter, the bearing capacity envelope of single piles will expand outward, and when the length-diameter ratio is relatively small, two different kinds of bearing capacity envelopes almost coincide. However, with increasing slenderness ratio, the bearing capacity envelope gradually separated, but the $V \rightarrow T$ bearing capacity envelope was always outside the $T \rightarrow V$ bearing capacity envelope. Therefore, when the slenderness ratio is relatively small, the effect of the loading path on the bearing capacity of single piles can be ignored. Inversely, when the length-diameter ratio is relatively large, it is necessary to try to avoid the single piles subjected to a torque in advance to prevent the reduction in the ultimate bearing capacity induced by the formation of the $T \rightarrow V$ loading mode.

### 4.2. Characteristics of the Pile Displacement

To investigate the displacement of the pile shaft, it is rational to take $T/T_p = 1/2$ and $V/V_p = 2/3$ as the $T \rightarrow V$ loading path and $V/V_p = 1/2$ and $T/T_p = 2/3$ as the $V \rightarrow T$ loading path. The lengths of the pile are 48 m, 32 m, 24 m, and 16 m, the diameter of the pile is 0.8 m, and the elastic modulus of the pile body is 25 GPa, and the calculation parameters of the soil are the same to those shown in Section 4.1. The displacement of the pile shaft is shown in Figures 10(a) and 10(b). As illustrated in the figure, when $V/V_p$ is a constant value under the $T \rightarrow V$ loading path and $T/T_p$ is a constant value under the $V \rightarrow T$ loading path, the displacement of the pile shaft is larger when $z/L$ is less than 0.6, and the displacement is basically stable below a depth of $0.6L$. Therefore, the displacement of the $V-T$ combination loaded pile mainly occurs above the pile body at a depth of $0-0.6L$. It is reasonable to pay attention to the protection of the shallow foundation to avoid the damage caused by the disturbance. In addition, strengthening the shallow foundation is an effective way to prevent excessive deformation of the pile foundation.

### 4.3. Influence of the Elastic Modulus of the Pile

To study the influence of the elastic modulus of the pile body on the bearing capacity of the $V-T$ combined loaded pile, take the pile length is 16 m, the pile diameter is 0.8 m, and the calculation parameters of the soil are the same as those shown in Section 4.1. In addition, the elastic modulus of the pile body is 2.5 GPa, 10 GPa, 17.5 GPa, and 25 GPa. The torque is 630 kN·m under the $T \rightarrow V$ loading path, and the vertical load is 1600 kN under the $V \rightarrow T$ loading path. The load-displacement curve of a single pile is shown in Figure 11. The ultimate bearing capacity of a single pile under the $T \rightarrow V$ loading path increases with an increasing elastic modulus of the pile body, as shown in Figure 11(a). However, taking $E_p = 2.5$ GPa and $E_p = 25$ GPa as examples, when the elastic modulus of the pile body increases by 10 times, the vertical bearing capacity barely increases from 1425 kN to 1720 kN at a rate of 17.5%. It is obvious from Figure 11(b) that with increasing elastic modulus of the pile body, the bearing capacity of the single pile under the $V \rightarrow T$ loading path hardly changes. Consequently, in the actual engineering under $V-T$ combined loads, the means is not to increase the concrete grade but to adjust the length-diameter ratio or strengthen the foundation to improve the bearing capacity of the pile-soil system.
Table 1: Table of the calculation parameters for Figures 9(a) and 9(b).

| Diameter (m) | Length (m) | Length-diameter ratio | Length (m) | Diameter (m) | Length-diameter ratio |
|--------------|------------|-----------------------|------------|--------------|-----------------------|
| 0.8          | 48         | 60                    | 0.5        | 32           | 60                    |
|              | 32         | 40                    |            | 0.8          | 40                    |
|              | 24         | 30                    | 16         | 1.2          | 20                    |
|              | 16         | 20                    |            | 1.6          | 1.2                   |

(a) $L = 48 \text{ m}$
(b) $D = 1.6 \text{ m}$

Figure 9: Effect of the pile length and diameter on the envelope of the single pile bearing capacity. (a) Influence of the pile length. (b) Influence of the pile diameter.

Figure 10: Influence of $L/D$ on the pile shaft displacement. (a) Settlement of the pile body under the $T \rightarrow V$ loading path. (b) Torsion angle of the pile body under the $V \rightarrow T$ loading path.
5. Conclusion

Considering different loading paths, the pile bearing characteristics of the V-T composite loaded pile were explored through theoretical analysis. The main conclusions are summarized as follows:

(1) The numerical solution of a single loaded pile was obtained by the finite difference method, and considering the influence of different loading paths, a new numerical analysis model was proposed to achieve the numerical solution of single piles under V-T combined loading. Compared with the existing research results, the correctness of the calculation model was verified.

(2) Under the combination of V-T loading, the bearing capacity of a single pile is lower than that of a single loaded (V or T) pile, and the bearing capacity envelope of the V→T loading path is outside the T→V loading path, hence, the former can bear a larger combined V-T load.

(3) Increasing the length and diameter can effectively improve the ultimate bearing capacity of a single pile; when the length-diameter ratio is relatively small, the influence of the loading path can be ignored. However, when the length-diameter is relatively large, the scenario that a single pile is subjected to the pretorque should be avoided to prevent the decrease of the ultimate bearing capacity induced by the formation of the T→V loading mode; the pile body has a larger displacement at a depth of 0–0.6L, so it is effective to protect or strengthen the shallow foundation to reduce deformation.

(4) Under the V→T loading path, the influence of the pile elastic modulus on the ultimate torque is minimal; as the elastic modulus increases, the vertical ultimate bearing capacity increases under the loading path of T→V. However, when the elastic modulus increases by 10 times, the vertical ultimate bearing capacity only increases by 17.5%. Therefore, it is not appropriate to enhance the ultimate bearing capacity of the pile-soil system by increasing the concrete grade.

Abbreviations

- $A_p$: Cross-sectional area of the pile
- $B_i$: Calculation parameter
- $c$: Cohesion of the soil
- $D$: Diameter of the pile
- $E_p$: Elastic modulus of the pile
- $E_p'$: Vertical load vector of the pile node
- $E_p$, $G_{11}$, $G_{p}$, and $G_{s}$: Shear modulus
- $h$: Length of pile unit
- $J_p$: Polar moment of inertia of pile section
- $k_{0}$: Coefficient of the active Earth pressure
- $K_{b0}$: Initial stiffness of the soil at the pile tip
- $[K'_T]$: Torsional stiffness matrix of the pile
- $[K'_z]$: Vertical stiffness matrix of the pile
- $k_v$ and $k_v'$: Vertical tangent stiffness
- $k_\theta(z)$: Torsional tangent stiffness
- $N_c$ and $N_q$: Dimensionless bearing capacity constants
- $L$: Pile length
- $n$: Number of pile elements
- $P_0$: Vertical load on pile top
- $P(z)$: Axial force of the single pile

![Figure 11: Influence of the pile elastic modulus on the bearing capacity of a single pile. (a) V-$\omega$ curve of a single pile under the V→T loading path. (b) T-$\theta$ curve of a single pile under the V→T loading path.](Image)
θ: Twist angle at depth
ρ_a: Ultimate resistance of the pile tip
r: Radius of the section of the pile
R: Stress-strain curve fitting constant
r_m: Effective influence radius
s_i(z): Circumferential displacement for pile shaft
T_i: Torque at pile tip
{T_i}: Torsional load vector of the pile
T(z): Torque of pile shaft at depth z
U_p: Perimeter of pile shaft
v_s: Poisson’s ratio of the soil
{w_i}: Vertical displacement vector along the pile node
w_i: Known vertical reloading displacement
w(z): Vertical displacement of the pile body
w_i: Vertical displacement of the pile unit
i (i = 1, 2, ..., n)
z: Depth
α: Calculation parameter
γ: Gravity of the soil
θ: Twist angle of the pile unit
{θ_i}: Twist angle at depth z
[θ_i]: Torsion angle vector of the pile node
τ_f(z): Circumferential friction of the pile at depth z
τ_s(z): Vertical frictional resistance of the pile at depth z
τ_f(i), τ_s(i), and τ_f(i): Frictional resistance
τ_f(i), τ_f(i), and τ_f(i): Ultimate friction force
φ: Effective internal friction angle of the soil
φ": Friction angle of the pile-soil interface.

Data Availability

The data, models, or codes generated or used during the study are available from the corresponding author by request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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