Anomalous Lense-Thirring precession in Kerr-Taub-NUT spacetimes

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Exact Lense-Thirring (LT) precession in Kerr-Taub-NUT spacetime is reviewed. It is shown that the LT precession does not obey the general inverse cube law of distance at strong gravity regime in Kerr-Taub-NUT spacetime. Rather, it becomes maximum just near the horizon, falls sharply and becomes negligibly small near the horizon. The precession rate increases again and after that it falls obeying the general inverse cube law of distance. This anomaly is maximum at the polar region of this spacetime and it vanishes after crossing a certain ‘critical’ angle towards equator from pole. In addition, we show that if the Kerr-Taub-NUT spacetime rotates with the angular momentum $J = Mn$ (Mass×Dual Mass), one horizon disappears and only event horizon exists at the distance $r = 2M$. 

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I. INTRODUCTION

The Lense-Thirring (LT) precession[1] is a very important phenomena in General relativity as well as in Relativistic astrophysics. In this phenomena the locally inertial frames are dragged along the rotating spacetime due to the angular momentum of the stationary spacetime. LT precession rate is proportional to the curvature as well as the angular velocity of the rotating spacetime. Thus, the effect will be larger for massive and rapidly rotating spacetime around which the curvature effect is maximum. It is perhaps Majumdar and myself [2] first to motivate investigation on the LT precession in strong gravity situation. Without making any preliminary assumption we have derived the exact LT precession rate of a gyroscope in the Kerr and Kerr-Taub-NUT (KTN) spacetimes. Now, it can easily be obtained the frame-dragging rate of a gyroscope[3] just outside a Kerr spacetime[4] as well as a KTN spacetime[5, 6]. Kerr and KTN spacetimes both are the vacuum solution of Einstein equation. Kerr spacetime has two parameters: mass and Kerr parameter (angular momentum per unit mass) but it needs three parameters to describe the KTN spacetime. The parameters are: mass, Kerr parameter and NUT parameter. If NUT parameter vanishes KTN spacetime reduces to Kerr spacetime and if the Kerr parameter vanishes KTN spacetime reduces to Taub-NUT spacetime. In the absence of NUT parameter, Taub-NUT spacetime reduces to pure Schwarzschild spacetime which is non-rotating. Kerr spacetime is very well-known to us and it is also physically reliable. We can easily describe the exterior geometry of many rotating astrophysical objects by Kerr spacetime. So, Kerr spacetime is astrophysically relevant. But, KTN spacetime is quite different than Kerr geometry. As it holds an additional parameter (NUT), this spacetime does not physically relevant till now.

Lynden-Bell and Nouri-Zonoz [7] are the first to motivate investigation on the observational possibilities for NUT charges or (gravito)magnetic monopoles. They have claimed that the signatures of such spacetime might be found in the spectra of supernovae, quasars, or active galactic nuclei. It has been recently brought into focus by Kagamanova et. al [8] by a detail and careful analysis of geodesics in Taub-NUT spacetime. A rigorous analysis in extremal and non-extremal KTN spacetimes for timelike and spacelike geodesics have already been done by myself [9]. It should be noted that the (gravito)magnetic monopole spacetime with angular momentum (basically KTN spacetime) admit relativistic thin accretion disks of a black hole in a galaxy or quasars [10]. The accretion disks are basically formed just near the above mentioned astrophysical objects. In this sense the accretion phenomena takes place in a very strong gravity regime in where the frame-dragging effect is expected to be very high. Thus the frame-dragging effect should also has to be a greater impact on accretion disks phenomena. This provides us a strong motivation for studying the LT precession or frame-dragging effect in KTN spacetime more detail because it will affect accretion in such spacetimes from massive stars, and might offer novel observational prospects.

The Kerr-Taub-NUT (KTN) spacetime is a stationary and axisymmetric vacuum solution of Einstein equation. This spacetime consists the Kerr parameter and NUT parameter. The Kerr parameter is responsible for the rotation of the spacetime. In the general sense NUT charge should not be responsible explicitly for the rotation of the spacetime but implicitly this NUT charge can add a “rotational sense” in a non-rotating spacetime. The NUT charge is also called as ‘dual mass’ whose properties have been investigated in detail by Ramaswamy and Sen [11]. They also called the NUT parameter as the “angular momentum monopole” [12] which is quite sound in this sense that it can give a “rotational sense” of Taub-NUT spacetime even when the Kerr parameter vanishes. In this regards, though the Kerr parameter vanishes in Kerr-Taub-NUT spacetime, the Taub-NUT spacetime retains the rotational sense due to the NUT parameter. Due to the presence of the NUT parameter the spacetime still remains stationary and violates the time reflection symmetry. Time reflection changes the direction of rotation and thus does not restore one to its original configuration[13]. Thus, the failure of the hypersurface orthogonality (it also means that the spacetime preserves the time translation symmetry but violates the time reflection symmetry) condition implies that the neighbouring orbit of $\xi^a$ (the timelike Killing vector which must exist in any stationary spacetime) “twist” around each other. In the Kerr spacetime the presence of Kerr parameter makes the spacetime stationary instead of static. Similarly, in the case of Taub-NUT spacetime the NUT parameter compels the spacetime stationary instead of static. So, the Kerr parameter and NUT parameter both are responsible to make the spacetime in rotation. Thus, it is needless to say that the KTN spacetime must be stationary.

The strong gravity LT precession in KTN spacetime has already been highlighted in [2] by Majumdar and myself but we could not studied it in detail. Though our target was to investigate LT precession in Kerr and KTN spacetimes, it had taken a turn into the investigation of LT precession in Taub-NUT spacetime which was really very interesting in that situation. We were busy to shown that the LT precession could not vanish even in non-rotating (as Kerr parameter vanishes) Taub-NUT spacetime. Later, Modak, Bandyopadhyay and myself [14] recently have discovered that frame-dragging curves are not smooth along the equator and its surroundings inside the rotating neutron star. Rather, frame-dragging effect shows an interesting anomaly along the equator inside the pulsars. Frame-dragging rate is maximum at the center and decreases initially away from the center, tends to zero (not exactly zero but very less) before the surface of the neutron star, rises again and finally approaches small value on the surface as well as outside of the pulsars. We think that this
may not be the only case in which we see this anomaly. After that we start to hunt for this type of feature in other spacetimes which are the vacuum solutions of Einstein equations and we get the almost similar anomaly in KTN spacetime (we note that there are many differences between KTN spacetime and the spacetime of a neutron star, they are not same). Previously, the strong gravity LT precession in Plebański-Demiański (PD) spacetimes (most general axisymmetric and stationary spacetime till now) has been investigated by Pradhan and myself [15]. As this is not astrophysically relevant, we are not more interested about this spacetime. But our close observation says that due to the presence of NUT charge this anomaly in frame-dragging can also arise in the PD spacetime. In this present paper, we are now investigating only for LT precession in KTN spacetime as it may be astrophysically sound in near future.

The paper is organized accordingly as follows: in section II, we review the LT precession in KTN spacetime. We also discuss on a very special case in the KTN spacetime in a subsection of the section II. We discuss our result in section III and finally, we conclude in section IV.

II. LENSE-THIRRING PRECESSION IN KERR-TAUB-NUT SPACETIME

The KTN spacetime is a geometrically stationary and axisymmetric vacuum solution of Einstein equation. This spacetime consists mainly three parameters: mass \((M)\), angular momentum \((J)\) per unit mass or Kerr parameter \((a = J/M)\) and NUT charge \((n)\) or dual mass. The metric of the KTN spacetime can be written as

\[
ds^2 = -\frac{\Delta}{p^2} (dt - A d\phi)^2 + \frac{p^2}{\Delta} dr^2 + p^2 d\theta^2 + \frac{1}{p^2} \sin^2 \theta (dt - B d\phi)^2
\]

(1)

With

\[
\Delta = r^2 - 2Mr + a^2 - n^2, p^2 = r^2 + (n + a \cos \theta)^2, \quad A = a \sin^2 \theta - 2n \cos \theta, B = r^2 + a^2 + n^2.
\]

The exact LT precession rate in Kerr-Taub-NUT spacetime is (Eq.(20) of [2])

\[
\vec{\Omega}_{LT} = \sqrt{\Delta} \left[ \frac{a \cos \theta}{p^2 - 2Mr - n^2} - \frac{a \cos \theta + n}{p^2} \right] \hat{\rho} + \frac{a \sin \theta}{p} \left[ \frac{r - M}{p^2 - 2Mr - n^2} - \frac{r}{p^2} \right] \hat{\theta}
\]

(3)

where, \(\rho^2 = r^2 + a^2 \cos^2 \theta\). The modulus of the above LT precession rate is

\[
\Omega_{LT} = |\vec{\Omega}_{LT}| = \frac{1}{p} \left[ \Delta \left( \frac{a \cos \theta}{p^2 - 2Mr - n^2} - \frac{a \cos \theta + n}{p^2} \right)^2 + a^2 \sin^2 \theta \left( \frac{r - M}{p^2 - 2Mr - n^2} - \frac{r}{p^2} \right)^2 \right]^{\frac{1}{2}}
\]

(4)

It could be easily seen that the above equation is valid only in timelike region, we mean, outside of the ergosphere which is located at \(r_+ = M + \sqrt{M^2 + n^2 - a^2 \cos^2 \theta}\).

If we plot \(r \) vs \(\Omega_{LT}\) for \(a < n\) (Fig. 1) and \(a > n\) (Fig. 2) we see that the LT precession rate curve is smooth along the equator (panel (b)) but it is not smooth along the pole (panel (a)). The LT precession rate is very high just outside the ergosphere and falls sharply and negligibly small (tends to zero but not exactly zero), rises again and finally approaches a small value after crossing the very strong gravity regime. We will now discuss an interesting situation in which the Kerr parameter \(a\) is equal to the NUT parameter \(n\).

![Plot of Ω_LT vs r in KTN spacetime for a = 0.1 m, n = 1 m & M = 1 m](image-url)
The horizons of the KTN spacetime are located at \( r_+ = M \pm \sqrt{M^2 + n^2 - a^2} \) [9]. One horizon is located at \( r_+ > 0 \) and another is located at \( r_- < 0 \) (if \( n > a \))[8]. The Kerr parameter \( a \) takes any value but less than or equal to \( \sqrt{M^2 + N^2} \) in case of KTN spacetime whereas \( a \) takes its highest value as \( M \) in case of Kerr spacetime. Without this restriction (if \( a^2 > M^2 + n^2 \)) the both spacetimes lead to show the naked singularities. There are two special cases in KTN spacetimes for which \( a \) can take the value only \( M \) and for the second case \( a \) can take the value \( n \). For the first case the angular momentum of the KTN spacetime would be \( J = M^2 \) which is similar to the case of extremal Kerr spacetime. In this case the horizons will be located at the distances \( r_+ = M + n \) and \( r_- = M - n \). If the mass of the spacetime is greater than the dual mass of the spacetime \((M > n)\), the both horizons could be located at the positive distances \((r_+ > 0)\) but if the dual mass is greater than the mass of the spacetime \((M < n)\) \( r_- \) will be located at the negative distance \((r_- < 0)\).

For the second case \((a = n)\) the angular momentum of the KTN spacetime would be \( J = Mn \). It is a very interesting situation. In this case the line element of the KTN spacetime would be

\[
ds_n^2 = \frac{\Delta_n}{p_n^2}(dt - A_n d\phi)^2 + \frac{p_n^2}{\Delta_n} dr^2 + p_n^2 d\theta^2 + \frac{1}{p_n^2} \sin^2 \theta (n dt - B_n d\phi)^2
\]

With

\[
\Delta_n = r(r - 2M), p_n^2 = r^2 + n^2(1 + \cos \theta)^2, \\
A_n = n(\sin^2 \theta - 2 \cos \theta), B_n = r^2 + 2n^2.
\]

It could be easily seen that this special rotating spacetime has one and only one horizon which is at the distance \( r_+ = 2M \) and there is no any other horizon because its radius is \( r_- = 0 \). It is similar to the Schwarzschild spacetime in where the event horizon is located at \( r = 2M \). Thus, there is no any second horizon in this special KTN spacetime. This spacetime can be treated as the rotating spacetime with the event horizon at \( r = 2M \) and its angular momentum will be

\[
J = Mn
\]

(7)

In the opposite sense it could be said that the KTN spacetime rotating with the angular momentum \( J = Mn \) could not possessed any second horizon other than event horizon which is located at \( r = 2M \). There is an apparent similarity in between Eq. (40) of [17] with us but it is completely different situation. Anyway, there should be an ergoregion in this special KTN spacetime. For this special case \((a = n)\), the radius of the ergosphere for the KTN spacetime will be \( M + \sqrt{M^2 + n^2 \sin^2 \theta} \). The LT precession rate in this special spacetime will be

\[
\Omega_{LT}\big|_{a=n} = \frac{n}{p_n} \left[ \Delta_n \left( \frac{\cos \theta}{r^2 - 2Mr - n^2 \sin^2 \theta} - \frac{1 + \cos \theta}{p_n^2} \right)^2 + \sin^2 \theta \left( \frac{r - M}{r^2 - 2Mr - n^2 \sin^2 \theta} - \frac{r}{p_n^2} \right)^2 \right]^{\frac{1}{2}}
\]

(8)

The above expression is valid outside the ergosphere as it diverges on the ergosphere and we also know that the LT precession is not defined in the spacelike surface. If we plot \( r \) vs \( \Omega_{LT} \) for \( a = n = 1 \) (Fig. 3) we see that the curve is smooth along the equator but it is not smooth along the pole (same as Fig.1 & Fig.2). The LT precession rate along the pole is very high just outside the ergosphere and falls sharply and negligibly small (tends to zero but not exactly zero), rises again and finally approaches a small value after crossing the strong gravity regime. This is really very absurd and this is not seen any other spacetimes (which are the vacuum solution of Einstein equation) previously.

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**FIG. 2.** Plot of \( \Omega_{LT} \) vs \( r \) in KTN spacetime for \( a = 0.7 \text{ m} \), \( n = 0.3 \text{ m} \) & \( M = 1 \text{ m} \)
III. RESULTS

We know that the LT precession varies as $1/r^3$ in weak gravity regime (‘weak’ Kerr metric) by the famous relation (Eq. 14.34 of [18])

$$\Omega_{LT} = \frac{1}{r^3} \left[ 3(\vec{J}.\hat{r})\hat{r} - \vec{J} \right]$$

where, $\hat{r}$ is the unit vector along $r$ direction. If we plot $\Omega_{LT}$ vs $r$ in strong gravity situation (see Eq.(42) of [2]) for maximally rotated Kerr spacetime along the pole and the equator we can obtain the plot like Fig.4. Close observation reveals that the LT precession rates at the same distances (for a fixed $r$) along the equator and the pole are not the same. In the strong gravity regime $\Omega_{LT}$ is higher than $\Omega_{LT}^p$ as the ratio ($\eta$) of the LT precession rate along the pole ($\Omega_{LT}^p$) to the equator ($\Omega_{LT}^e$) in strong gravity regime is

$$\eta_{K}^{strong} = \frac{\Omega_{LT}^p}{\Omega_{LT}^e} = \frac{2r^3(r - 2M)}{(r^2 + a^2)^{\frac{3}{2}}(r^2 - 2Mr + a^2)^{\frac{1}{2}}}$$

but in weak gravity regime it is just (from Eq.(9))

$$\eta_{K}^{weak} = \frac{\Omega_{LT}^p}{\Omega_{LT}^e} = \frac{2J}{r^3} = 2$$

which is a constant. If we look for the same in the case of KTN spacetime we see that

$$\eta_{KTN}^{strong} = \frac{\Omega_{LT}^p}{\Omega_{LT}^e} < 1$$

It is hold for ever we mean LT precession rate along the equator ($\Omega_{LT}^e$) is always higher than the LT precession rate along the pole ($\Omega_{LT}^p$). In weak gravity regime the ratio will be only

$$\eta_{KTN}^{weak} = 1$$
We can plot the above things for the clear scenario: The above plot (Fig. 5) for Kerr spacetime shows that $\Omega_{LT}^P$ and $\Omega_{LT}^E$ are same at the distance $r_0 = 3.324$ m. For $r < r_0$, $\Omega_{LT}^P < \Omega_{LT}^E$, and for $r > r_0$, $\Omega_{LT}^P > \Omega_{LT}^E$.

We have already seen that the plots of $\Omega_{LT}$ vs $r$ along the pole and along the equator both are smooth for Kerr spacetime but the case is not same for the KTN spacetime. In the KTN spacetime though the curve of $\Omega_{LT}$ vs $r$ along the equator is smooth but it is not smooth along the pole. We have studied here basically three cases. These are the following:

(i) $a = n$: For the first case we take the Kerr parameter $a$ is equal to the NUT parameter $n$ ($a = n = 1$ m) and mass of the spacetime $M$ is unity. Thus, the radius of horizon is $r_h \sim 2$ m. The LT precession rate along the pole is tremendously high just outside the horizon. Then it falls sharply and negligibly small (local minima) at $r_{min} \sim 4.8$ m. It rises again and give a local maxima at $r_{max} \sim 7$ m. After that the curve of LT precession rate follows the general inverse cube law and falls accordingly. We cannot see the same thing along the equator. We plot a 3-D picture (Fig. 6) in where the $Y$ axis represents the cosine of colatitude ($\cos \theta$) and $X$ axis represents the distance ($r$) of the LT precession rate measurement form the centre of the spacetime. The colors represent the value of the LT precession rate and the values of the same precession rates are also separated by the isocurves. It shows that there is a local maxima and a local minima along the pole but it disappears after crossing a certain ‘critical’ angle. Here, it is around $\cos \theta \sim 0.6$.

(ii) $a > n$: In the second case the Kerr parameter $a = 0.7$ m and NUT parameter $n = 0.3$ m. Mass of the spacetime is $M = 1$ m. Radius of the horizon $r_h \sim 1.8$ m, distance of local minima is $r_{min} \sim 7$ and distance of local maxima is $r_{max} \sim 10$ m (Fig. 7). The ‘critical’ angle is around $\cos \theta \sim 0.8$.

(iii) $a < n$: For the third case the Kerr parameter $a = 0.1$ m and NUT parameter $n = 1$ m. Mass of the spacetime is $M = 1$ m. Radius of the horizon $r_h \sim 2.4$ m, distance of local minima is $r_{min} \sim 2.6$ and distance of local maxima is $r_{max} \sim 3.5$ m (Fig. 8). The ‘critical’ angle is around $\cos \theta \sim 0.4$. 

![Fig. 5. Plot of $\eta$ vs $r$ in Kerr and Kerr-Taub-NUT spacetimes for $a = n = 1$ m $\& M = 1$ m](image)

![Fig. 6. 3-D plot of $\Omega_{LT}(r, \theta)$ in KTN spacetime for $a = n = 1$ m $\& M = 1$ m](image)
Above all the three cases, the all plots hold the same feature but the numerical values are different depending on the values of $a$ and $n$. For a fixed value of $n$, if $a$ decreases the value of LT precession rate at local maxima increases and also the distance of local minima and maxima shifted towards the horizon of the spacetime. If the NUT parameter vanishes (for Kerr spacetime) there will be no any local maxima and minima (see Fig. 4 and 9). The local maxima and minima is arising only due to the NUT parameter. It is cleared from the plots (Fig. 10) which is valid for Taub-NUT spacetime in where the Kerr parameter vanishes but the NUT parameter does not vanish. Without the Kerr parameter the LT precession rate at ‘local maxima’ in Taub-NUT spacetime is higher than the LT precession rate at ‘local maxima’ in KTN spacetime. The presence of the Kerr parameter (presence or increasing value of the intrinsic angular momentum of spacetime) shifts the local maxima & minima towards
Fig. 10. Plot of $\Omega_{LT}$ in Taub-NUT spacetime for $n = 1$ and $M = 1$.

(a) Plot of $\Omega_{LT}$ vs $r$ (basically, the expression of $\Omega_{LT}$ (see Eq.(25) of [2]) is independent of $\theta$, thus the value (colour) of $\Omega_{LT}$ does not change with $\cos \theta$ in panel(b))

(b) 3-D plot

The away from horizon and reduce the LT precession rate at the local maxima.

We note that Taub-NUT spacetime is a stationary and spherically symmetric spacetime and the expression of $\Omega_{LT}$ (see Eq.(25) of [2]) is also independent of $\theta$. Thus the value (colour) of $\Omega_{LT}$ does not change with $\cos \theta$. It means that the LT precession rates are same everywhere in that spacetime for a fixed distance $r$ (no matter whether it is pole or equator) and LT precession rate curve always shows a ‘peak’ (see Fig.10(a)) near the horizon. But, if this Taub-NUT spacetime starts to rotate with an angular momentum $J(= aM, a$ is Kerr parameter), it turns to be a KTN spacetime. In this case, LT precession rate curve shows a ‘peak’ (or ‘local maxima’) along the pole but disappears after crossing the ‘critical’ angle and we cannot see any ‘peak’ in the LT precession rate curve along the equator. The ‘intrinsic’ angular momentum of the spacetime $(J)$ is fully responsible for disappearance of ‘local maxima’ along the equator. The Kerr parameter is also responsible for reducing the LT precession rate at the ‘local maxima’ which has already been discussed in the previous paragraph. Thus, the ‘dual mass’ or the ‘angular momentum monopole’ $n$ is only responsible for the ‘anomaly’ (appearance of local maxima and local minima in the LT precession rate) and Kerr parameter or the rotation of the spacetime tries to reduce this ‘anomaly’ as far as possible. Kerr parameter is fully successful to reduce this effect along the equator but slowly it loses its power of reduction of this anomaly towards the pole from equator.

For the consistency check of the appearance of ‘local maxima’ and ‘local minima’ if we take the derivative of Eq.(4) with respect to $r$ and plot $\frac{d}{dr} \Omega_{LT} |_{(r=R, \theta=\pi/2)}$ vs $r$ we cannot find any positive real root in the region $r_h < r < \infty$ but the plot $\frac{d}{dr} \Omega_{LT} |_{(r=R, \theta=0)}$ vs $r$ shows two positive real roots (which are basically local maximum $R_1 = r_{max}$ and local minimum $R_2 = r_{min}$) in the region $r_h < r < \infty$.

IV. DISCUSSION

We have shown that the LT precession is quite different than the LT precession in other spacetimes. Other vacuum solutions of Einstein equation do not show this type of strange feature in LT precession or frame-dragging effect. It has been discussed that this strangeness in KTN spacetime is due to only the presence of NUT parameter or (gravito)magnetic monopoles. Remarkably, it (frame-dragging effect in KTN spacetime) has an apparent similarity with the frame-dragging effect inside the rotating neutron star. Exact frame-dragging effect inside the rotating neutron star has recently been derived and discussed in detail by Modak, Bandyopadhyay and myself [14] but this is the interior solution of the Einstein equation, not the vacuum solution. In the case of the interior of a pulsar the LT precession shows the same ‘anomaly’ like KTN spacetime but there is also another basic difference: the anomaly appears in the LT precession rate in KTN spacetime along the pole but it appears along the equator in the case of pulsar. The basic features of the plots are same for both the cases. We do not know if there is any connection or not in these two spacetimes. We have already stated that Lynden-Bell and Nouri-Zonoz [7] first highlighted about the observational possibilities for NUT charges or (gravito)magnetic monopoles and they claimed that the signatures of such spacetime might be found in the spectra of supernovae, quasars, or active galactic nuclei. Is there any possibility to find the (gravito)magnetic monopoles inside the rotating neutron star or there are some other reasons for this anomaly? We are now trying to find the answer of this and it will be published elsewhere in the future.

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