Two-phase pressure drop study for cryosurgical probes using one-dimensional homogeneous model

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Abstract. Cryosurgery is an effective medical treatment approach for treating solid cancerous malignancy, as it is a minimally invasive treatment therapy. The main aim of cryosurgery is to use extreme cold to destroy abnormal tissues while minimizing damage to healthy surrounding tissue. Recent technological advancements in cryogenics have attracted the attention of many researchers to study the low-temperature cryosurgical devices, generally termed cryoprobes. Saturated Liquid Nitrogen (LN2) from pressurized dewar is fed to the cryoprobe through a transfer hose. The fluid evaporates as it flows downstream because of various heat loads and experiences pressure drop. In this paper, a two-phase pressure drop solver is presented following a simplified one-dimensional analysis to estimate pressure losses occurring throughout the system. A qualitative discussion on the contribution of various pressure drop components is provided. It is observed that the major pressure loss occurs along the length of the cryoprobe tubes. The effect of the cryoprobe tube dimensions on the pressure drop is also studied. It is concluded that the miniaturized probe design demands higher system inlet pressure.

1. Introduction
Two-phase liquid-gas flow is encountered in the liquid nitrogen (LN2) cooled low-pressure cryoprobes. Low pressure LN2 cooled cryoprobes operate at much lower pressure, i.e., less than 5 barg. Their counterpart, Argon cooled cryoprobes, required much higher operating pressure which is about 300 to 400 barg. However, the flow boiling characteristics of LN2 flow through the smaller diameter tubes of the cryoprobes often leads to high pressure drop. In order to achieve the required cooling, it is essential to estimate the pressure drop inside the cryoprobe tubes. In general, tubes used in cryoprobes are categorized as mini-tubes (i.e., < 6 mm). Various authors have studied the two-phase pressure drop characteristics of cryogenic LN2 flows. Chen et al. [1] studied the two-phase flow boiling characteristics of LN2 in horizontal circular mini-tubes. Their results indicated that the pressure drop increases with the increasing heat flux and LN2 mass flow rate but decreases with the increasing inlet pressure. Qi et al. [2] observed that a homogeneous model yields better prediction than three separated flow models at high LN2 mass flux. The frictional pressure drop of two-phase LN2 in the circular horizontal channel is investigated by Chen et al. [3] using two models, the homogeneous model and the separated flow model. It was concluded that the homogeneous model predicts better estimation than the other correlations.

In the present study, a homogeneous model is used to estimate the two-phase pressure drop through the cryosurgical probe. Effects of LN2 mass flow rate, tank operating pressure, and cryoprobe dimensions on the pressure drop are investigated.
2. Mathematical modeling

Steady flow and the mean value of the velocity and density of each phase are assumed to exist across a given plane normal to the flow. It is further assumed that the pressure across any plane normal to the channel axis is uniform. The forces acting on each phase can be equated to the rate of change of momentum of that phase. The total static pressure gradient can be expressed in terms of the separate components of friction, acceleration, and static head as:

$$\Delta p = \Delta p_f + \Delta p_a + \Delta p_z$$  \hspace{1cm} (1)

Where, $\Delta p_f$, $\Delta p_a$, and $\Delta p_z$ represent part of the overall static pressure gradient required to overcome frictional, acceleration, and gravitational pressure drop, respectively.

The homogeneous model considers the two phases to flow as a single-phase possessing mean fluid properties. In fact, such a situation arises in very high or small vapor quality. Despite this limitation, the homogeneous model has been used extensively for analyzing the pressure drop due to its simplicity. Assumptions for the homogeneous model are [4],

a. Equal vapor and liquid velocities,

b. The attainment of thermodynamic equilibrium between the phases and

c. The use of a suitably defined single-phase friction factor for two-phase flow.

2.1. Pressure drop components

The individual pressure drop components in the section under consideration are explained as:

2.1.1. Frictional pressure drop. The frictional pressure drop component can be expressed as,

$$-\Delta p_f = -2f_{TP}G^2 v_f \frac{dx}{dx} \left[1 + x \frac{v_{fg}}{v_f} \right], f_{TP} = 0.079 \left(\frac{GD}{\mu} \right)^{\frac{1}{4}} \frac{1}{ \overline{\mu}} = x \frac{\mu_g}{\mu_f} + (1-x)$$  \hspace{1cm} (2)

Where, $G$ is the mass velocity, $f_{TP}$ is two-phase friction factor, $v_g$ and $v_f$ are specific volume for gaseous and liquid phases, $D$ is tube diameter and $\mu_g$ and $\mu_f$ are viscosity for gaseous and liquid phases, and $x$ is quality. Assuming that the friction factor can be expressed in terms of the Reynolds number by the Blasius equation. The $f_{TP}$ has been evaluated using a mean two-phase viscosity, $\overline{\mu}$. The form of the relationship between $\overline{\mu}$ and quality $x$ is chosen as given by McAdams et al. [4].

2.1.2. Acceleration pressure drop

$$-\Delta p_a = G^2 \left[ v_{fg} \frac{dx}{dz} + x \frac{dv_{fg}}{dp} (\frac{dp}{dz}) \right]$$  \hspace{1cm} (3)

2.1.3. Gravitational pressure drop

$$-\Delta p_z = \frac{gsin\theta}{\overline{v}} = \frac{gsin\theta}{v_f + xv_{fg}}$$  \hspace{1cm} (4)

Where $\theta$ is the orientation of the test section. For horizontal flows, $\theta$ is 0º, so the flow will not experience any gravitational pressure drop. For vertically downward flow, $\theta$ is 90º, and gravitational pressure drop will be negative in nature, and in that case, it will reduce net pressure drop will. Whereas for vertically upward flow, $\theta$ is –90º, and hence gravitational pressure drop will add up in net pressure drop. Substituting equations (2), (3) and (4) into equation (1) and rearranging we get,

$$-\Delta p = \frac{-2f_{TP}G^2 v_f \frac{dx}{dx} \left[1 + x \frac{v_{fg}}{v_f} \right] + G^2 \left[v_{fg} \frac{dx}{dz} + x \frac{dv_{fg}}{dp} \right] + \frac{gsin\theta}{v_f + xv_{fg}}}{1 + G^2 x \left(\frac{dv_{fg}}{dp}\right)}$$  \hspace{1cm} (5)
2.2. Evaluation of heat flux for various sections

In this section, all the heat loads needed for the calculation of pressure drop are calculated. Consider the test section subjected to uniform heat flux $q$ as shown in Figure 1.

![Figure 1. Energy balance for control volume in the two-phase situation.](image)

Following assumptions were considered for the given study:

a. Steady-state with no volumetric heat generation

b. Constant heat flux, $q$. Constant mass flow rate, $\dot{m}$, and constant cross-sectional area.

c. Negligible changes in kinetic and potential energy.

d. Homogeneous model is considered, i.e., same vapor and liquid linear velocities ($u_f = u_g$)

The energy balance for the given control volume gives the following expression for quality change,

$$\frac{dx}{dz} = \frac{qP}{\dot{m}H_{fg}}$$

(6)

Where $P$ is the perimeter, $\dot{m}$ is LN$_2$ mass flow rate, and $H_{fg}$ is the latent heat of vaporization. For the two-phase situation subjected to constant heat flux, the change in quality can be obtained by integrating equation (1) over the given length. This expression will be used to evaluate how quality changes through the entire system. All the parts of the system are discussed in the next section.

2.2.1. A system under consideration. In order to deliver the LN$_2$ to the probe, we require the connecting hose. As soon as the LN$_2$ enters the hose and travels through the system, it will be subjected to different heat loads depending upon the location and surrounding conditions. All these heat loads are estimated in the subsequent section. The simplified system consists of an inlet hose, cryoprobe, and outlet hose. A complete system along with LN$_2$ flow is shown in Figure 2.

![Figure 2. System components and LN2 flow direction.](image)

For better clarity, Figure 2 also shows the enlarged view of the cryoprobe internal structure. Dimensions considered for each component of the system are given in Table 1.
### Table 1. Dimensions of various parts in the system.

| Parameter | Stainless Steel hose | Cryoprobe tubes |
|-----------|----------------------|-----------------|
| Internal Diameter, mm | 8.60 | 2.0 |
| Outside Diameter, mm | 16.0 | 4.0 |
| Length, mm | 3000 | 100 |

2.2.2. **Heat load estimation for system components.** In order to evaluate the pressure drop at various locations, we need to estimate how quality changes through these locations. For this calculation, the value of heat flux at respective locations needs to be estimated beforehand. Figure 3 shows the schematic representation of the inlet/outlet hose cross-sectional view. Various dimensions and material details have been given in. From this data, heat flux is determined for the inlet hose and outlet hose.

![Figure 3. Schematic representation of hose cross-section.](image)

For the case under consideration wherein air travels in crossflow over hose and factor Re.Pr ≥0.2, we can use the Churchill–Bernstein equation to estimate the surface averaged Nusselt number. For the inner side, we have used Dittus–Boelter relation. The heat flux is calculated using a steady-state one-dimensional heat transfer relation in cylindrical coordinates. For this purpose, the core temperature through which LN\(_2\) flows is assumed to be maintained at 77 K, and the ambient is assumed to be maintained at 300 K. The heat load on the inner and outer hose is calculated, which comes out to be equal to 2533 W/m\(^2\).

Further, the heat load for cryoprobe is estimated. The innermost tube of the cryoprobe is always in contact with flowing LN2 on either side; for this reason, it is assumed to be adiabatic, i.e., zero heat load from outside. Return tube is insulated to length \(L_1\) and length \(L_2\) is considered to be a Cryogenic length i.e. non insulated. The heat fluxes for \(L_1\) and \(L_2\) are estimated independently depending on their boundary conditions. As the length \(L_1\) of the return tube is insulated, it is assumed that it is subjected to radiation heat load only. Considering the emissivity of the S.S tube as 0.5, radiation heat flux is determined which comes out to be equal to 163.4 W/m\(^2\). Further, length \(L_2\) is the cryogenic length that will be inserted inside the tumor. Typical heat load in this section can be estimated considering the freezing capacity required to reduce the tumor temperature to –50°C. Tumor diameter is taken as 10 mm. The calculated heat load is equal to 20 W. Cooling capacity in a similar range was estimated by Rabin and Shitzer [5] in their numerical model. All these calculated heat fluxes are used to get the pressure drop across the whole system. The numerical model for the same is explained in the next section.
2.3. Development of two-phase pressure drop solver

The numerical solver is developed to calculate the pressure drop as given by equation (5) for various heat flux conditions estimated in the previous section. The solver calculates the relative contribution of each component of pressure drop towards the total pressure drop. The numerical algorithm is better understood by referring to the flow chart, as shown in Figure 4. The whole system is divided into a number of small elements, and pressure drop is calculated at each segment. The procedure shown in Figure 4 is executed for each element of the system.

As shown in Figure 4, the solver is initialized by loading all relevant thermophysical properties of LN2 for suitable pressure. Then relevant parameters of all three parts of the system, viz. inlet hose, cryoprobe, and outlet hose, are declared, and other flow parameters such as mass flow rates and input pressure are declared. The quality of LN2 at the input to the inlet hose is assumed as zero. At input pressure, all properties are obtained by linear interpolation at the very first node. Then for the given heat flux of the input hose, change in quality is calculated for the given node by solving equation (6). Based on this quality, two-phase viscosity is calculated. Further Reynold number and then two-phase friction factor is evaluated for the same node. With all these values, equation (5) is solved to obtain individual pressure drop components and total pressure drop at that node. The pressure is updated at this node by negating the pressure drop from system pressure. Then the counter is moved to the next node and new properties are calculated, and the same procedure is followed till all the nodes of the input hose are evaluated for total pressure drop. After that cryoprobe inner tube is evaluated, followed by a Cryoprobe return tube, and finally, pressure drop across the outlet hose is obtained.

![Flow chart of the algorithm for two-phase pressure drop solver.](image)

The sample calculation is performed considering a system inlet pressure as 4 barg with an LN2 mass flow rate of 1 liter per minute. For this case, the magnitudes of frictional, acceleration, and gravitational pressure drops are plotted in Figure 5. As shown in Figure 5(a), the frictional pressure drop magnitude is very high in the cryoprobe inner and return tube as compared to the inlet and outlet hose. This is due to the fact that cryoprobe tubes are very small in size as compared to the size of inlet and exit hose. Further, it is observed from Figure 5(b) that the magnitudes of acceleration pressure drop in cryoprobe tubes are also very high compared to other parts. This is due to the fact that the phase change occurring in this section is very high due to the high heat load.

Here we have considered that the inlet and outlet hose are horizontal, so there will not be any gravitational pressure drop occurring in these two parts. However, there will be a small gravitational pressure drop inside the cryoprobe due to its vertical location, refer to Figure 5(c). Figure 5(d) shows the pressure variation throughout the system. From Figure 5(d), it is clear that significant pressure loss is associated with the cryoprobe only.
Figure 5. Magnitudes of pressure for (a-c) Frictional, (d-f) Acceleration, (g) Gravitational, and (h) Total pressure drop

3. Results and discussion

Process parameters such as mass flow rate of LN2, and tank pressure, govern the effectiveness of cryoprobe. This section studies the effect of these process parameters on pressure drop. Along with this, the effect of cryoprobe dimensions on the system inlet pressure is also presented.

3.1. Effect of LN2 mass flow rate on pressure-drop

In this section effect of the mass flow rate of LN2 is investigated for the given system configuration. Figure 6 (a-c) shows the effect of LN2 mass flow rate on system pressure drop and quality. It is to be noted that for some cases, the value of exit pressure is above the atmospheric pressure. This simply means that the assumed value of inlet pressure is sufficient to overcome all the pressure losses and the flow is possible. This approach is adopted as we are aiming to qualitatively analyze the contribution of pressure drop in each component. It can be observed that for a relatively high mass flow rate, the pressure drop is also high. However, due to increased mass flow rate, quality at cryoprobe and at exit gets lowered. As the mass flow rate is lowered, the pressure drop inside the system is also lowered but quality
goes on increasing. It is important to note that the present numerical model is based on the homogeneous model, considering equal vapor and liquid velocities. Gas-phase gets separated in real-life situations and hence corresponding gas phase velocity is expected to be underestimated by the homogeneous model. Increased gas phase velocities will lead to more frictional pressure drops. Frictional pressure-drop predicted using separated flow model for the water-steam case is approximately 50% higher than frictional pressure-drop predicted using homogeneous model [4]. Considering this, it is worth noting that the homogeneous model under-predicts the total pressure drop. Hence for the given case, it is safe to assume a mass flow rate of more than 15 g/s.

![Diagram](image1.png)

**Figure 6.** Variation in pressure and quality for a tank inlet pressure of (a) 2 barg, (b) 3 barg, (c) 4 barg, (d) Effect of cryoprobe dimensions on pressure-drop.

3.2. Effect of system inlet pressure on pressure-drop

It is important to quantify the tank pressure required for operating the cryoprobe. For this purpose, the effect of inlet pressure is investigated for different mass flow rates. Figure 6(a-c) shows the pressure drop and quality change for a system inlet pressure of 2, 3, and 4 barg, respectively, for different mass flows. From Figure 6(a-c), it is observed that as system pressure is lowered from 4 barg to 2 barg, the magnitude of pressure-drop increases for all mass flow rates. For a mass flow rate of 10 g/s, the pressure drop is 1.2 bar, 0.76 bar and 0.67 bar for a system inlet pressure of 2, 3, and 4 barg respectively. This observation is in agreement with the experimental observations reported by Chen et al. [1]. The major contributor to total pressure drop is frictional pressure drop and acceleration pressure drop occurring...
inside the cryoprobe. Both these pressure components are directly proportional to the difference between saturated gas specific volume and saturated liquid specific volume. This difference increases with a decrease in pressure causing a larger magnitude of pressure-drop at reduced inlet system pressure. Also, it can be observed that at 2 barg inlet pressure and with a mass flow rate equal to 15 g/s, we get exit pressure very close to atmospheric pressure, which is also the case in a real scenario.

3.3. Effect of cryoprobe tube diameter on pressure-drop
From the above discussion, it is observed that the majority of pressure loss takes place in cryoprobe tubes only. This is because of the fact that the flow area of the tube suddenly reduces to very narrow dimensions of the cryoprobe, which results in more significant pressure losses. Hence, it is essential to study the effect of tube diameters on pressure drop. For this study, three different cases were taken and were evaluated for total pressure drop and LN$_2$ mass flow rate of 10 g/s. Table 2 below shows different cases under consideration with corresponding fluid flow areas.

| Parameters                  | Case-1 | Case-2 | Case-3 |
|-----------------------------|--------|--------|--------|
| Inner tube ID and OD, mm    | 1.6    | 2.0    | 2.0, 2.4 |
| Return tube ID and OD, mm   | 3.0, 3.4 | 3.4, 4.0 | 3.6, 4.0 |
| Outer tube ID and OD, mm    | 4.4, 4.8 | 5.0, 5.5 | 5.0, 5.5 |

The effect of changing tube dimensions on system pressure is shown in Figure 6(d). The probe with case-1 is the most miniature but can only work with a system inlet pressure of more than 4 barg. On the other hand, if larger dimensions are acceptable, then a probe with case-3 is more attractive as it can operate at a system inlet pressure of 2 barg.

4. Conclusions
A homogeneous numerical model is used to investigate the effect of process parameters, such as tank inlet pressure and LN2 mass flow rate, on the pressure drop occurring inside the cryoprobe. The pressure drop calculation suggests that the majority of the pressure loss takes place across the cryoprobe tubes due to their smaller size. It is observed that for the given system configuration, an LN2 mass flow rate of around 10 g/s is required with a system inlet pressure of 2 barg. Under these conditions, the effect of the cryoprobe, tube dimensions were evaluated against pressure drop. As far as cryoprobe dimensions are concerned, the probe with Case-1 is found to be most miniaturized but demands higher system pressure of more than 4 barg. However, system inlet pressure could be lowered to 2 barg if cryoprobe dimensions are compromised, and a slightly larger size probe (i.e., Probe in Case-3) is chosen.

5. References
[1] Chen X, Chen S, Chen J, Li J, Liu X, Chen L and Hou Y 2017 Two-phase flow boiling frictional pressure drop of liquid nitrogen in horizontal circular mini-tubes: Experimental investigation with comparison with correlations Cryogenics (Guild) 83 85–94
[2] Qi S L, Zhang P, Wang R Z and Xu L X 2007 Flow boiling of liquid nitrogen in micro-tubes: Part I - The onset of nucleate boiling, two-phase flow instability, and two-phase flow pressure drop Int J Heat Mass Transf 50 4999–5016
[3] Chen X, Zhang Q, Xue R, Lai T and Hou Y 2016 An experimental study of two-phase pressure drop of liquid nitrogen in circular horizontal channels Refrigeration Science and Technology vol 22-25-June pp 12–8
[4] Collier J 1972 Convective Boiling and Condensation (McGraw-Hill Book Company (UK) Limited)
[5] Rabin Y and Shitzer A 1998 Numerical solution of the multidimensional freezing problem during cryosurgery J Biomech Eng 120 32–7