Design of additional excitation controller based on \( H \infty \)

Qiuhua Hou\(^1\), Fanmin Meng\(^1\*\), Xiaoyu Liu\(^1\), Xiaoyan Qi\(^1\), Xuju Chen\(^1\), Huifang Ding\(^1\) and Qing Zhang\(^2\)

\(^1\)State Grid Shandong Electric Power Company Laiwu Power Supply Company., Laiwu City, Shandong Province, 271100, China
\(^2\)Qingdao Ruinengda Electrical Technology Co. Ltd., Qingdao City, Shandong Province, 266400, China

*Corresponding author’s e-mail: lk@8.upc.edu.cn

Abstract. This paper presents an additional excitation controller using wide area information without delay. Firstly, the mathematical model of the system is established according to the state equation of the system, and then the parameters are calculated by using the \( H \infty \) theory and the relevant knowledge of linear matrix inequality. Finally, the control effect is verified by simulation in Matlab/Simulink, and the transmission of wide area information is simulated by truetime2.0 toolbox.

1. Introduction

Low frequency oscillation\([1]\) is a common problem in large power grid, especially in power system with weak electrical connection or heavy load, the traditional additional excitation controller\([2]\) only takes the local signal, which can not be suppressed. With the development of network technology and synchronous vector measurement technology, wide area measurement system\([3]\) is also a research hotspot. The wide area measurement system can obtain the information of the whole power system in a certain delay range, and it can transmit these information to the distributed controllers in real time.

In this paper, a wide area additional excitation controller without time delay is designed. The design of the controller is mainly to calculate the corresponding coefficient matrix by establishing the mathematical model of the controller, and then use the \( H \infty \) theory\([4]\) and the matrix inequality theory to calculate the parameters of the controller. The matrix inequality can be solved by LMI toolbox, and then verify the controller by Simulink simulation.

2. Steady state modeling of power system considering excitation

In the analysis of power system stability, the effect of generator damping winding is ignored, and each generator is regarded as an agent. The equation of state of the \( i \) th agent is as follows:

\[
\begin{align*}
\frac{d\Delta \delta_i}{dt} &= \Delta \omega_i \\
\frac{d\Delta \omega_i}{dt} &= -\frac{\omega_c}{T_i} \Delta P_{ei} + w_i \\
\frac{d\Delta E_{qi}}{dt} &= \frac{\Delta E_{q0} - \Delta E_{q0i}}{T_{d0}}
\end{align*}
\]

(1)
Where, $\delta$ is power angle, $w_i$ is the sum of the linearization errors of disturbance and electromagnetic power variation, $w$ is rotor speed, $P_{el}$ is the electromagnetic power of the generator, $E_{q0}$ is forced no load potential, $T_i$ is the inertia time constant of the generator, $E_{q1}$ is transient potential.

The incremental model is applied: $\delta_j = \delta_{j0} + \Delta \delta_j$, the fluctuation of generator output and terminal voltage are

$$\Delta P_{el} = \sum_{j=1} E_j E_f (B_j \cos \delta_{j0} - G_j \sin \delta_{j0}) \Delta \delta_j$$

$$= \sum_{j=1} K_j (\Delta \delta_i - \Delta \delta_j)$$

$$\Delta E_{q1} = (\Delta E_{q1} + (x_{q1} - x_{q1}^*) \sum_{j=1} E_{qj} (G_j \cos \delta_{j0} + B_j \sin \delta_{j0}) \Delta \delta_j)$$

$$= \Delta E_{q1} + \sum_{j=1} M_j (\Delta \delta_i - \Delta \delta_j)$$

The following formula can be obtained by taking formula (2) into formula (1)

$$\begin{cases}
\frac{d\Delta \delta_i}{dt} = \Delta \omega_i \\
\frac{d\Delta \omega_i}{dt} = -\sum_{j=1} K_j (\Delta \delta_i - \Delta \delta_j) + w_i \\
\frac{d\Delta E_{q1}}{dt} = u_i - \frac{\Delta E_{q1}}{T_{d0}} - \sum_{j=1} M_j (\Delta \delta_i - \Delta \delta_j)
\end{cases}$$

Equation (3) is written as equation of state as follows:

$$\begin{bmatrix}
\dot{x}_i \\
y_i
\end{bmatrix} = 
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
x_i \\
y_i
\end{bmatrix} + 
\begin{bmatrix}
u_i \\
0
\end{bmatrix}$$

Where, $u_i = \Delta E_{q1} / T_{d0}$, $K_j = \frac{\omega_{ni}}{T_i}$, $M_j = \frac{M_{ni}}{T_{d0}}$.

Then the state equation of the system composed of N generators is as follows:

$$\begin{bmatrix}
\dot{x} \\
y
\end{bmatrix} = 
\begin{bmatrix}
(I_n \otimes A)x + (I_n \otimes B_2)u + (K \otimes D_1)x + (M \otimes D_2)x + (I_n \otimes B_1)w_i + \text{diag}(\Delta A)x \\
(I_n \otimes C)x
\end{bmatrix}$$

3. Design of additional excitation controller

The distributed output feedback controller designed for the model is as follows:

$$\begin{bmatrix}
\dot{v}_i \\
u_i
\end{bmatrix} = 
\begin{bmatrix}
A & B_k \\
C & D_k
\end{bmatrix}
\begin{bmatrix}
v_i \\
y_i - y_j
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0
\end{bmatrix}$$

$$\begin{bmatrix}
\dot{v}_i \\
u_i
\end{bmatrix} = 
\begin{bmatrix}
A & B_k \\
C & D_k
\end{bmatrix}
\begin{bmatrix}
v_i \\
y_i - y_j
\end{bmatrix}$$
The controller can be abbreviated as:
\[ K_c = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \]
\[(7)\]

The global controller can be expressed as:
\[ v = (I_n \otimes A_k) v + (L \otimes B_k) (I_n \otimes C) x \\
u = (I_n \otimes C_k) v + (L \otimes D_k) (I_n \otimes C) x \]
\[(8)\]

The system equation obtained from equation (9) is as follows:
\[ \dot{x} = [(I_n \otimes A) + \text{diag}(\Delta A)] x + (I_n \otimes B) v + (L_n \otimes B_2) (I_n \otimes C) x \\
+ (K \otimes D_1) x + (M \otimes D_2) x + (I_n \otimes B_i) w_i \\
y = (I_n \otimes C) x \]
\[(9)\]

According to equation (9), it can be concluded that:
\[ x = I_n \otimes C \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} x + L \otimes \begin{bmatrix} B_2 C_k \\ B_k C_k \end{bmatrix} v + K \otimes \begin{bmatrix} D_1 \\ 0 \end{bmatrix} x + M \otimes \begin{bmatrix} D_2 \\ 0 \end{bmatrix} x + I_n \otimes \begin{bmatrix} B_i \end{bmatrix} w_i \]
\[ y = (I_n \otimes C, 0_n \otimes 0_{m,n}) x \]
\[(10)\]

Rewrite equation (10) as follows:
\[ \dot{\xi} = (\hat{A} + \Delta A) \xi + \hat{B} w_i \]
\[ \dot{\hat{y}} = C \xi \]
\[(11)\]

The objective function is defined as follows:
\[ J = \int_{0}^{\infty} (\dot{y}^T \dot{y} - \gamma^2 w_i^T w_i) dt \]
\[(12)\]

When the controller \( K_c \) satisfies the condition \( \|\xi\|_2 \rightarrow 0 \) of the asymptotic stability of the system, \( \|y\|_2 \rightarrow 0 \) can be obtained, that is, each synchronizer realizes synchronization, and makes \( J < 0 \).

Lyapunov function is defined as:
\[ V = \xi^T (I \otimes \hat{P}) \xi \]
\[(13)\]

The time derivative of the function is obtained:
\[ \dot{V} = \xi^T (I \otimes \Lambda^T \hat{P} + I \otimes \hat{P} \Lambda) \xi + 2 \xi^T (I \otimes \hat{P} \Delta \Lambda) \xi + 2 \xi^T (L \otimes \hat{P} \Lambda) \xi \\
+ 2 \xi^T (K \otimes \hat{P} \Lambda) \xi + 2 \xi^T (M \otimes \hat{P} \Lambda) \xi + 2 \xi^T (I \otimes \hat{P} \Lambda) w_i \]
\[(14)\]

For any non-zero \( w_i \), the formula can be written as follows:
\[ J = \int_{0}^{\infty} (\dot{y}^T \dot{y} - \gamma^2 w_i^T w_i + \dot{V}) dt - V(\infty) + V(0) \]
\[(15)\]

In equation (15), \( V(0) = 0, V(\infty) \geq 0 \). If \( \dot{y}^T \dot{y} - \gamma^2 w_i^T w_i + \dot{V} < 0 \), we can get \( J < 0 \).

\[ J_t \leq (4 + \delta) \xi^T \xi + \left[ \xi^T (I \otimes \Lambda^T \hat{P} + I \otimes \hat{P} \Lambda) \xi + \left[ \xi^T (I \otimes \hat{P} \Delta \Lambda) \xi \right] + \left[ \gamma^2 (I \otimes \hat{P} \Lambda) \xi \right] \right] + \left[ \sigma^2 (L) \xi^T (I \otimes \hat{P} \Lambda) \xi \right] + \left[ \sigma^2 (M) \xi^T (I \otimes \hat{P} \Lambda \Lambda^T \hat{P}) \xi \right] + \left[ \sigma^2 (K) \xi^T (I \otimes \hat{P} \Lambda) \xi \right] \]
\[(16)\]

Let \( \eta = \hat{P} \xi, \hat{P} = \hat{P}^{-1} \), according to the properties of Kronecker product, we can get the following results
\( J_f \leq \eta^T \left[ I \otimes (\tilde{P}^T \Lambda_1^T + \Lambda_1 \tilde{P} + (4 + \delta) \tilde{P}^T \tilde{P} + I + \sigma^2 (L) \Lambda_2 \Lambda_2^T + \sigma^2 (K) \Lambda_3 \Lambda_3^T + \sigma^2 (M) \Lambda_4 \Lambda_4^T) \right] \eta \) \hspace{1cm} (17)

Equation (17) is written as LMI inequality equation as follows:
\[
\begin{bmatrix}
\tilde{P}^T \Lambda_1^T + \Lambda_1 \tilde{P} + I + \sigma^2 (L) \Lambda_2 \Lambda_2^T + \sigma^2 (K) \Lambda_3 \Lambda_3^T + \sigma^2 (M) \Lambda_4 \Lambda_4^T \\
\tilde{P}^T \Lambda_5 \\
\Lambda_5^T \\
-(4 + \delta)^{-1} I \\
-\gamma^2 I
\end{bmatrix} < 0 \hspace{1cm} (18)
\]

By substituting \( \Lambda_1 = F_0 + F_1 K_c F_2 \) and \( \Lambda_2 = F_3 K_c F_4 \) into formula (18), we can get the following results:
\[
\begin{bmatrix}
\tilde{F}^T P + P \tilde{F} + F \tilde{P}^T Q \\
+ Q \tilde{F}_2 + (5 + \delta) V^{-1} V^{-1} \\
(QF_3)^T \\
(P \tilde{\Lambda}_3)^T \\
(P \tilde{\Lambda}_4)^T \\
(P \tilde{\Lambda}_5)^T
\end{bmatrix}
\begin{bmatrix}
QF_3 \\
P \tilde{\Lambda}_3 \\
P \tilde{\Lambda}_4 \\
P \tilde{\Lambda}_5
\end{bmatrix} < 0 \hspace{1cm} (19)
\]

Where, \( \tilde{F}_0 = VF_0 V^{-1}, \tilde{F}_1 = VF_1, \tilde{F}_2 = F_2 V^{-1}, Q = P \tilde{F} K_c, V = (V^T)^{-1}, \tilde{\Lambda}_3 = V \Lambda_3, \tilde{\Lambda}_4 = V \Lambda_4, \tilde{\Lambda}_5 = V \Lambda_5 \)

4. Case Study

![Figure 1. Three machine nine node system.](image)

In this paper, the simplified three machine nine node system of WSCC system is adopted, and the detailed parameters are shown in reference [5]. From the controller (8), it can be obtained that:
\[
\dot{V} = A_i \cdot V + LB_i \cdot \Delta X
\]  
\[U_i = C \cdot V + LD_i \cdot \Delta X\]

Taking generator G1 as an example, it can be concluded as follows:

The controller is simulated and analysed. Figure 2 is the power angle difference between \(G_1\) and \(G_2\), and Figure 3 is the power angle difference between \(G_1\) and \(G_3\). The curve in the figure is measured under the condition of adding disturbance in 20 seconds. The solid line is the curve without cutting off the additional excitation controller, and the dotted line is the curve with cutting off the additional excitation controller. It can be seen from the figure that the waveform can be more stable after adding the designed additional excitation controller. So the design of additional excitation controller plays a certain role.

![Figure 2. Power angle difference between \(G_1\) and \(G_2\).](image1)

![Figure 3. Power angle difference between \(G_1\) and \(G_3\).](image2)
Figure 4 is the terminal voltage variation curve of $G_1$. The curve in the figure is measured under the condition of adding disturbance for 20 seconds. The solid line is the curve without cutting off the additional excitation controller, and the dotted line is the curve with cutting off the additional excitation controller. It can be seen from the figure that the designed additional excitation controller has a good control effect.

5. Conclusions
In this paper, an additional excitation controller without time delay is designed. This controller is a distributed feedback controller. In parameter calculation, $H_\infty$ performance index and linear matrix inequality theory are used. Finally, the LMI is solved by the solver in LMI toolbox. In order to verify its effect, taking the three machine nine node model as an example, the relevant controller parameters are calculated, and the simulation analysis is carried out. The simulation results show that the additional excitation controller can effectively maintain the stability of generator terminal voltage and generator power angle.

Acknowledgments
This work was supported by State Grid Shandong Electric Power Company Science and Technology Project (Project No.52061219006Q).

References
[1] Yu Y., Grijalva S. (2016) Oscillation Energy Analysis of Inter-Area Low-Frequency Oscillations in Power Systems. IEEE Transactions on Power Systems, 31:1195-1203.
[2] Kim K., Schaefer R. C. (2005) Tuning a PID controller for a digital excitation control system. IEEE Transactions on Industry Applications, 41:485-492.
[3] Li M., Chen Y. (2018) A Wide-Area Dynamic Damping Controller Based on Robust $H_\infty$ Control for Wide-Area Power Systems With Random Delay and Packet Dropout. IEEE Transactions on Power Systems, 33: 4026-4037.
[4] Li P., Zhang J. (2015) The $H_\infty$ Control Method of Grid-Tied Photovoltaic Generation. IEEE Transactions on Smart Grid, 6:1670-1677.
[5] Wan Y., Milano F. (2018) Nonlinear Adaptive Excitation Control for Structure Preserving Power Systems. IEEE Transactions on Power Systems, 33: 3107-3117.