On the hyperdeformed state of the $^{36}$Ar nucleus

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Abstract. The hyperdeformed state of the $^{36}$Ar nucleus is discussed, including its prediction from alpha-cluster and binary cluster studies, via its experimental verification from heavy-ion resonances up to its derivation from Nilsson-model calculations. Its possible clusterizations are also investigated both from the microscopic structure and from the energetic preference viewpoint.

1. Introduction

The investigation of extreme, e.g. superdeformed (SD) and hyperdeformed (HD) nuclear shapes is one of the exciting topics in the present day nuclear structure research. Much attention is focused on the $N = Z$ nuclei, in which the proton-neutron interactions, clusterization, etc. play especially important role. In these nuclei even more exotic shapes, like e.g. linear alpha-chains have been predicted. In spite of the considerable experimental and theoretical efforts, however, only very limited convincing evidence is available [1].

Apart from some very simple and well-known cluster states, like e.g. the ground state of the $^8$Be nucleus, the superdeformed (SD) states of these self-conjugate nuclei have been observed experimentally only during the last decade. The SD band of $^{36}$Ar was reported for the first time in [2], populated in an $^{24}$Mg($^{20}$Ne,2$\alpha$)$^{36}$Ar reaction, and detected by multiple (gamma-ray and charged-particle) coincidence techniques. Following the experimental observation a lot of theoretical effort was concentrated on this state, therefore, we have a fairly good theoretical understanding of it. According to the joint conclusion of large-scale shell model, Nilsson model, different Hartree-Fock, and various cluster-model calculations this state has 4 nucleon excitation (for a brief recent review and references see [3]).

The question of the hyperdeformed (HD) state of these nuclei was so far mainly a territory for theoretical predictions. For $^{36}$Ar it was first predicted by an alpha-cluster model calculation [4], giving the moment of inertia of the state. Then based on binary cluster studies it was predicted that it can be populated predominantly in $^{24}$Mg+$^{12}$C, and $^{20}$Ne+$^{16}$O reactions [5]. A recent study on the $^{24}$Mg+$^{12}$C elastic scattering, completed with some available $^{20}$Ne+$^{16}$O reaction data seems to verify this prediction [6]. The moment of inertia is in good agreement with the prediction of the alpha-cluster model. In [3, 5] the possible binary clusterizations have been
extensively discussed from the microscopic structure viewpoint, and here we present detailed double-folding calculations for the energetic preference of different configurations. Since the theoretical predictions on the HD state came from cluster models, it is highly desirable to study whether or not this state shows up also in the shell model. A recent calculation within the framework of the Nilsson-model resulted in a HD state, which has not only the same moment of inertia, as obtained from alpha-cluster model, and observed experimentally, but it has also the same $U(3)$ symmetry of the state [3], as obtained in [4].

In what follows we will discuss these aspects more in detail.

2. Theoretical predictions from cluster studies

In [4], and later on in [7] unconstrained Bloch-Brink alpha-cluster model calculations have been carried out. A hyperdeformed state with ratios of major axes 3:1:1 was found [4], and the corresponding shell-model configuration is given in Table 1 of [7]. (Please, note that a [400] part is missing there due to printing error [8].) The geometrical arrangement of the 9 $\alpha$-particles is a $\alpha - C - \alpha - C - \alpha$ linear chain, where $C$ stands for an oblate $^{12}$C nucleus, i.e. 3 $\alpha$’s in a triangular arrangement, and the carbon nuclei line up their symmetry axes with the molecular axis, as shown in Fig. 1. The corresponding shell-model configuration indicate a $[48,8,8]$ $U(3)$ symmetry for this state. The moment of inertia (corresponding to this symmetry) turns out to be 4.2 in units of $10^9$ fm$^2$ MeV/c$^2$.

The allowed binary clusterizations of the superdeformed and hyperdeformed states in $^{36}$Ar (together with those of the ground state) were investigated systematically in [5]. The clusters were supposed to be in their intrinsic ground state, with spherical or deformed (even triaxial) shapes, with no simplifying assumption for their relative orientation and all the stable isotopes were considered as possible clusters. The problem was investigated both from the viewpoint
of the microscopic structure and from the angle of energetic preference, calculated in terms of the criterion of maximum stability (combined with the no-dipole constraint) [9]. The structural aspect was treated by the combined application of the $U(3)$ selection rule and Harvey’s prescription (for a general discussion see [10]).

In [5] the superdeformed and hyperdeformed states were defined in two different ways: on the one side by their quadrupole deformation (from the experiment for the superdeformed state and by the usual $\beta \approx 0.86$ value for the hyperdeformed state, noted there as SD(b) and HD(b) states, respectively), or by corresponding simple shell model configurations noted as SD(a) and HD(a).

This study revealed that the ground state allows more asymmetric, while the hyperdeformed state more symmetric cluster-configurations, and the superdeformed state shows a picture in between. It also turned out that the $^{24}\text{Mg}+^{12}\text{C}$ clusterization is allowed in each of these three states, the difference lies in the relative orientations of the deformed clusters.

As for the hyperdeformed state is concerned, in addition to the $^{24}\text{Mg}+^{12}\text{C}$ binary configuration the $^{20}\text{Ne}+^{16}\text{O}$ turned out to be allowed from among the alpha-like cluster-configurations, which are energetically preferred.

It is remarkable, that these two binary cluster configurations were also selected on the basis of alpha-cluster calculation [4]. The difference in this respect between the two works is that in [5] all the possible binary configurations were studied systematically, and the forbidden or allowed nature of the clusterization was investigated not only from the viewpoint of Harvey’s prescription, but also on the basis of the $U(3)$ selection rule. These two methods are complementer to each other in a sense, therefore both of them should be investigated (for a detailed discussion see [10]), otherwise the result might be misleading.

We mention here that a similar study concerning the possible ternary clusterizations of the ground, superdeformed and hyperdeformed states have been performed in [11].
3. Experimental verification
In [6] some high-resolution experimental data [12] of the \( ^{24}\text{Mg}+^{12}\text{C} \) elastic scattering has been analyzed. In particular, fifteen angular distribution, measured near the Coulomb-barrier were investigated from the viewpoint of potential scattering, phase-shifts, Regge-poles, and Breit-Wigner terms in the S-matrix. The conclusion was that the fine structure of the cross-section curves can be explained only by the presence of resonances with spins 2-8. These quasi-bound states together with the \( ^{20}\text{Ne}+^{16}\text{O} \) resonances [13] determine a rotational band, as shown in Fig. 2. Its moment of inertia (4.4) is very close to the value, predicted by the alpha-cluster model calculations [4]. It is remarkable that independent experimental studies [14] also showed the importance of these two binary decay channels, although with less detailed analysis of the resonances.

4. HD state from Nilsson-model
After the experimental data seemed to verify the predictions of the cluster studies both on the moment of inertia, and on the reaction channels, it is of great interest to check if the hyperdeformed shape in question shows up also in shell model or mean-field calculations. The first investigation along this line has been carried out within the Nilsson-model in [3]. The stable shapes were obtained from symmetry-considerations, and the result is shown in Fig. 3. This study revealed that, i) in addition to the well-known ground and superdeformed states (which are in line with other results), two candidates are there for the hyperdeformed shape, which are very similar to the simple guess of [5]. One of them (HD\(_b\)) has a small triaxiality, while the other one (HD) is cylindrical, and very stable. Furthermore, ii) the HD state obtained in this way has not only similar value for the moment of inertia, than the one predicted from Bloch-Brink-model, and given by the experiment, [4], but also its \( U(3) \) symmetry is identical with the previous one.
5. Binary clusterizations

The possible binary cluster-configurations of the shape isomers in $^{36}$Ar have been investigated in [3, 5] by taking into account both the requirements of the exclusion principle, and the energy-minimum principle. Some simple clusterizations corresponding to the shape isomers of Fig. 3 are shown in Fig. 1.

As for energetic preference of the allowed binary clusterizations are concerned, it was obtained in [5] from binding energy arguments. The criterium of maximal stability [15], requires the largest value of the summed differences of the measured binding energies and the corresponding liquid drop values: $D(1, 2) = [B(1) - B_L(1)] + [B(2) - B_L(2)]$, where $B(i)$ is the experimental binding energy of the $i$th cluster, while $B_L(i)$ stands for liquid drop value. The alpha-like clusterizations turned out to be more stable than the others.

Here we present the results of a double-folding calculations carried out within the Dinuclear System Model [16]. In this approach effective nucleon-nucleon interactions are applied, which are suitable for structure studies [17]. For the details of this method we refer to the work [18]. The potential energy was determined for the geometrical arrangement obtained from the microscopic considerations. The results are listed in Table 1. For some states different (but close-lying) theoretical candidates are given. In particular e refers to effective symmetry from the Nilsson-calculation, c stands for a cylindrical, while tri for a triaxial shape.

In the energetic preference of the microscopically allowed clusterizations one can observe a general tendency in favour of the lighter clusters: alpha-particle has deeper binding than $^8$Be, etc. But the relative orientation turned out to be very important, too. For clusterizations, which are present in different shape isomers, like e.g. $^{24}$Mg+$^{12}$C and $^{28}$Si+$^8$Be the largest deviation between different geometrical locations is 5.4 MeV (in both cases), which is not smaller than the difference between the averages of the two different cluster-configurations (4.0 MeV).

In [3] the relation of the moments of inertia of the shell model and the corresponding cluster states were discussed. Both of them were calculated classically for rigid bodies, corresponding to the appropriate $U(3)$ symmetries. For the clusterizations a touching configuration was supposed for the (generally deformed) clusters. The conclusion (in agreement with other similar studies) was that this cluster configuration gives larger moment of inertia, than the shell model, and two different ways were discussed for obtaining closer values. With the Dinuclear Molecular System calculation now we are in a position of being able to check a different method. Namely, this consideration provides us with the intercluster distance ($R_D$) through the minimum of the energy. We have compared these values for the binary clusterizations of Fig. 1 with those of the touching configurations ($R_T$). It turned out that these values are very close to each other, and consequently so are the corresponding moments of inertia ($I_D$, and $I_T$), as well. In particular for the $^{24}$Mg+$^{12}$C clusterization they are $R_D$ ($R_T$) = 5.2 (4.6), 6.2 (6.6), 6.7 (6.6), 7.9 (7.9), in units of fm, respectively for the states with increasing deformation. Correspondingly: $I_D$ ($I_T$) = 3.1-3.6 (2.7-3.2), 4.4-4.7 (4.9-5.0), 5.0-5.2 (4.9-5.0), 6.4-6.5 (6.4-6.5) in units of $10^5 fm^2 MeV/c^2$, and the two values coupled by the sign of "-" appear due to the non-cylindric shape of the configuration. A similar situation was found for the other binary configurations of Fig. 1 as well. For the $^{32}$S+$^4$He $R_D$ ($R_T$) = 6.4 (6.4) and $I_D$ ($I_T$) = 3.7-4.1 (3.7-4.1). For the $^{20}$S+$^{16}$O $R_D$ ($R_T$) = 5.8 (5.8) and $I_D$ ($I_T$) = 3.9 (3.9). For the $^{28}$Si(p)+$^8$Be $R_D$ ($R_T$) = 6.7 (6.8), 8.4 (8.7) and $I_D$ ($I_T$) = 5.2 (5.3), 7.3 (7.7), respectively, with increasing deformation of the states. In short, the intercluster distances and moments of inertia obtained from the energy-calculation show a very similar behaviour to those for the touching configuration.

6. Summary and conclusions

In this paper we have reviewed some investigations related to a possible hyperdeformed state of the $^{36}$Ar nucleus. It was first predicted from an alpha-cluster model calculation [4], where also a correspondence to the $^{24}$Mg+$^{12}$C and $^{20}$Ne+$^{16}$O resonances was put forward. Then a
Table 1. Orientation and binding energy of binary alpha-like cluster-configurations in the ground and deformed sates of $^{36}$Ar. Here Sh stands for shape, while DNS denotes potential energy from the Dinuclear System Model (in MeV). The molecular axis is chosen to be z. s means spherical, p: prolate, o: oblate, t: triaxial, a: symmetry axis. The explanation for the specific orientations are as follows 0*: $^{28}$Si symmetry (short) axis in yz plane (closer to z); 1*: $^{24}$Mg the largest axis in x direction; 2*: $^{28}$Si symmetry (large) axis in xz plane (closer to x); 3*: $^{32}$S the largest axis in z direction; 4*: $^{20}$Ne symmetry (large) axis in xz plane at 45°; 5*: $^{24}$Mg the largest axis in z direction; 6*: $^{24}$Mg the largest axis in yz plane (closer to y);

| State       | $C_2$ | Sh | Ratio | Position  | $C_2$ | Sh | Ratio | Position  | DNS |
|-------------|-------|----|-------|-----------|-------|----|-------|-----------|-----|
| GS(e) [20, 18, 14] | $^{24}$Mg | t | 1.7 : 1.2 : 1 | $x > y > z$ | $^{12}$C | o | 1.7 : 1 | a : z | 4.3 |
| | $^{28}$Si | p | 1.6:1 | $x = y > z$ | $^{8}$Be | p | 2.1 | a : x | 1.0 |
| | $^{32}$S | o | 1.7:1 | $x = y > z$ | $^{8}$Be | p | 2.1 | a : x | -1.7 |
| GS(c) [20, 20, 12] | $^{24}$Mg | t | 1.7 : 1.2 : 1 | $x = y > z$ | $^{3}$He | s | 1 | - | -3.7 |
| | $^{28}$Si | p | 1.6:1 | $x > y > z$ | $^{8}$Be | p | 2.1 | a : x | 1.0 |
| | $^{32}$S | o | 1.7:1 | 0* | $^{8}$Be | p | 2.1 | a : x | -1.6 |
| SD(e) [32, 14, 10] | $^{24}$Mg | t | 1.7 : 1.2 : 1 | $x = y > z$ | $^{12}$C | o | 1.7 : 1 | a : y | 7.2 |
| | $^{28}$Si | p | 1.6:1 | $x > z > y$ | $^{8}$Be | p | 2.1 | a : z | 3.7 |
| SD(c) [32, 12, 12] | $^{24}$Mg | t | 1.7 : 1.2 : 1 | 6* | $^{12}$C | o | 1.7 : 1 | a : y | 6.2 |
| | $^{28}$Si | p | 1.6:1 | $x = y > z$ | $^{8}$Be | p | 2.1 | a : z | 3.3 |
| SD(tri) [32, 16, 8] | $^{20}$Ne | p | 1.6:1 | a : x | $^{16}$O | s | 1 | - | 8.1 |
| | $^{24}$Mg | t | 1.7 : 1.2 : 1 | 1* | $^{12}$C | o | 1.7 : 1 | a : x | 6.2 |
| | $^{28}$Si | p | 1.6:1 | 2* | $^{8}$Be | p | 2.1 | a : z | 2.8 |
| | $^{32}$S | t | 1.5 : 1.3 : 1 | 3* | $^{4}$He | s | 1 | - | -4.3 |
| HD$_b$ [40, 12, 8] | $^{20}$Ne | p | 1.6:1 | 4* | $^{16}$O | s | 1 | - | 9.4 |
| | $^{24}$Mg | t | 1.7 : 1.2 : 1 | 5* | $^{12}$C | o | 1.7 : 1 | a : z | 3.4 |
| | $^{28}$Si | p | 1.6:1 | a : z | $^{8}$Be | p | 2.1 | a : x | -0.6 |
| HD [48, 8, 8] | $^{20}$Ne | p | 1.6:1 | a : z | $^{16}$O | s | 1 | - | 8.9 |
| | $^{24}$Mg | t | 1.7 : 1.2 : 1 | 5* | $^{12}$C | o | 1.7 : 1 | a : x | 6.0 |
| | $^{28}$Si | p | 1.6:1 | a : z | $^{8}$Be | p | 2.1 | a : z | 2.5 |

systematic study on the possible binary cluster-configurations [5] showed that these two channels are really the preferred ones for its population. A recent careful analysis of the $^{24}$Mg$+^{12}$C elastic scattering data revealed the existence of low-spin resonances [6], and it turned out, that they form a rotational band together with the $^{20}$Ne$+^{16}$O data (Fig. 2). The moment of inertia is very close to the prediction of the alpha-cluster model. Then from a Nilsson-model calculation a hyperdeformed state was found [3] (Fig. 3), which has a very similar moment of inertia, like that of the experiment and alpha-cluster model. Furthermore, it has an $U(3)$ symmetry, which coincides with that of [4].

All these circumstances make the rotational band of Fig. 2 a very good candidate for the hyperdeformed state of the $^{36}$Ar nucleus. For the final conclusion and better understanding we need, of course, more experimental evidence, and theoretical studies. Nevertheless, it seems that Fig. 2 is one of the very few candidates at present for showing evidence for the presence of ground, superdeformed and hyperdeformed shapes in a single nucleus.
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