Absence of a charge renormalization in the Higgs model interacting with conformal two-dimensional gravity

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Abstract

We discuss $D$-dimensional scalar and electromagnetic fields interacting with a quantized metric. The gravitons depend solely on twodimensional coordinates. We consider a conformal field theory as a model for the metric tensor. We show that an interaction with gravity improves the short distance behaviour. As a result there is no charge renormalization in the four-dimensional Higgs model.

1 Introduction

We discuss $D$-dimensional scalar and electromagnetic fields interacting with a quantized scale invariant metric. The gravitons depend only on twodimensional coordinates. These coordinates could be considered either as additional coordinates (e.g., in the brane picture when gravitons can escape to extra dimensions [1]) or as a (stringy) perturbation of the physical fourdimensional space-time deforming the flat Minkowski metric.

In our earlier paper [2] we have shown that as a result of an interaction with a scale invariant quantum gravity the propagators of quantum matter fields become more regular than the ones in a classical gravitational field. In this paper we consider a specific model of a scale invariant quantum metric. We show that as a result of an interaction with a conformal invariant twodimensional gravity the fourdimensional Higgs model has no charge renormalization. This property can still hold true in five dimensions depending on the scale dimension of the twodimensional gravity. Higher dimensional models can still be renormalizable if we can treat properly more singular gravitational fields. We also show that after an interaction with twodimensional gravity the standard dispersion relation
A model of quantum gravity

We consider a metric tensor on the Riemannian manifold (Euclidean formulation)

\[ (G)^{AB} = g^{AB} \]  

as a twodimensional field \( G \) with values in a set of real symmetric positive definite \( D \times D \) matrices \( G \). We choose the metric in a block diagonal form \( G^{AB} = \delta^{AB} \) if \( A, B > D - 2 \) and for \( A, B \leq D - 2 \) the tensor \( g^{\mu\nu}(x_F) \) is a \((D - 2) \times (D - 2)\) matrix depending on \( x_F \in \mathbb{R}^2 \). The manifold of matrices is homeomorphic to \( \mathbb{R} \times SL(D - 2, \mathbb{R})/O(D - 2) \). The corresponding decomposition takes the form

\[ G = (\det G)^{D - 2} (\det G)^{-D - 2} G \equiv (\det G)^{\frac{1}{D - 2}} \tilde{G} \equiv (\exp 2\psi) \tilde{G} \]

We choose a conformal invariant action for \( G \) (this is an infinite dimensional group \([4]\), the model is not invariant under the whole group of diffeomorphisms of the metric, but such an invariance is anyway expected to be broken in a quantum theory)

\[ W(G) = \frac{\alpha}{2} Tr \int d^2 x_F G^{-1} \partial GG^{-1} \partial G + WZW \]

\[ = 2\alpha \int d^2 x_F \partial \psi \partial \psi + \frac{\alpha}{2} Tr \int d^2 x_F \tilde{G}^{-1} \partial \tilde{G} \tilde{G}^{-1} \partial \tilde{G} + WZW \]  

where \( \partial = \partial_1 - i\partial_2 \) is the holomorphic derivative and \( WZW \) denotes the Wess-Zumino-Witten term \([5]\). It is convenient to choose the following parametrization for \( G \) (\( T \) denotes the transposition)

\[ G = \mathcal{N} \Lambda \mathcal{N}^T \]

where \( \Lambda \) is a diagonal matrix, \( \mathcal{N} \) is a nilpotent matrix with 1 on the diagonal and the matrix elements below the diagonal are equal to zero. Then

\[ e = \mathcal{N} \sqrt{\Lambda} \]

can be chosen as the tetrad. Using the WZW cocycle condition

\[ W(GH^{-1}) = W(G) + W(H^{-1}) + 2\alpha Tr \int G^{-1} \partial GH^{-1} \partial H \]

we can express \( W \) explicitly in the coordinates (3)

\[ W(G) = \frac{\alpha}{2} Tr \int \Lambda^{-1} \partial \Lambda \Lambda^{-1} \partial \Lambda + \frac{\alpha}{2} Tr \int (\Lambda^{-1} \mathcal{N}^{-1} \partial \mathcal{N})^T \mathcal{N}^{-1} \partial \mathcal{N} \Lambda \]  

\[ = \frac{\alpha}{2} Tr \int \Lambda^{-1} \partial \Lambda \Lambda^{-1} \partial \Lambda + \frac{\alpha}{2} Tr \int (\Lambda^{-1} \mathcal{N}^{-1} \partial \mathcal{N})^T \mathcal{N}^{-1} \partial \mathcal{N} \Lambda \]
This is a model of an exactly soluble conformal field theory \[6\] \[7\] \[8\]. In the following we use the result that \(e = N \sqrt{\Lambda}\) is a conformal field. In the coordinates \(4\) the conformal fields are \(e_1 = \exp(\psi + \phi)\), \(e_2 = \exp(\psi - \phi)\), \(e_3 = \exp(2\psi + 2\phi)\), \(\Lambda_1 = \exp(2\psi - 2\phi)\) and \(\Lambda_2 = \exp(2\psi + 2\phi)\). The scale dimension of \(e\) is \(\gamma = -\alpha\) (\(\alpha\) must be chosen negative, this is inconsistent with a finiteness of the functional integral but we may work at the beginning with a positive \(\alpha\) and only at the level of correlation functions continue \(\alpha\) to negative values; the relation of the functional integral to the free field representation of conformal field theories is discussed in \[7\]). The conformal invariance implies that the random fields \(\lambda^\gamma e^\mu_a(\lambda x_F)\) and \(e^\mu_a(x_F)\) are equivalent, i.e., they have the same correlation functions.

3 The scalar propagator

We consider a complex scalar matter field \(\Phi\) in \(D\) dimensions interacting with gravitons depending only on a \(d\)-dimensional submanifold. We split the coordinates as \(x = (x_G, x_F)\) with \(x_F \in \mathbb{R}^d\). Without a self-interaction the \(\Phi\) correlation function is equal to an average

\[
h \int \mathcal{D}e \exp \left( \frac{1}{\hbar} W(G) \right) A^{-1}(x, y) \tag{6}
\]

over the gravitational field \(e\) of the Green's function of the operator

\[
A = \frac{1}{2} \sum_{\mu=1,\nu=1}^{D-d} g^{\mu\nu}(x_F) \partial_\mu \partial_\nu + \frac{1}{2} \sum_{k=D-d+1}^{D} \partial_k^2 \tag{7}
\]

We repeat some steps of ref.\[2\] (our case here is simpler and more explicit). We represent the Green’s function by means of the proper time method

\[
A^{-1}(x, y) = \int_0^\infty d\tau \left( \exp(\tau A) \right)(x, y) \tag{8}
\]

For a calculation of \(\left( \exp(\tau A) \right)(x, y)\) we apply the functional integral

\[
K_\tau(x, y) = \left( \exp(\tau A) \right)(x, y) = \int \mathcal{D}x \exp \left( -\frac{\tau}{2} \int \frac{dx_F}{dt} \frac{dx_F}{dt} - \frac{\tau}{2} \int g^{\mu\nu}(x_F) \frac{dx_F}{dt} \frac{dx_F}{dt} \right)
\]

\[
\delta(x(0) - x) \delta(x(\tau) - y) \tag{9}
\]

In the functional integral \(9\) we make a change of variables \((x \rightarrow b)\) determined by Stratonovitch stochastic differential equations \[3\]

\[
dx^\Omega(s) = e^\mu_A(x(s)) db^\lambda(s) \tag{10}
\]

where for \(\Omega = 1, 2, \ldots, D - d\)

\[
e^\mu_a e^\nu_a = g^{\mu\nu}
\]
and $e_{\alpha}^\mu = \delta_{\alpha}^\mu$ if $\Omega > D - d$.

As a result of the transformation $x \rightarrow b$ the functional integral becomes Gaussian with the covariance

$$E[b_a(t)b_c(s)] = \delta_{ac} \min(s,t)$$

In contradistinction to [2] eq.(10) can be solved explicitly. The solution $q_\tau$ of eq.(10) consists of two vectors $(q_G, q_F)$ where

$$q_F(\tau, x_F) = x_F + b_F(\tau)$$

and $q_G$ has the components (for $\mu = 1, \ldots, D - d$)

$$q_\mu(\tau, x) = x_\mu + \int_0^\tau e_\mu^a (q_F(s, x_F)) db^a(s)$$

The kernel is

$$K_\tau(x, y) = E[\delta(y - q_\tau(x))] =$$

$$= E[\delta(y_F - x_F - b_F(\tau)) \prod_\mu \delta(y_\mu - q_\mu(\tau, x))]$$

Using eq.(12) and the Fourier representation of the $\delta$-function we write eq.(13) in the form

$$K_\tau(x, y) = (2\pi)^{-D} \int d^p_G d^p_F$$

$$E[\exp(i p_F(y_F - x_F) + i p_G(y_G - x_G) - i \int p_\mu e_\mu^a (q(s, x_F)) db^a(s))]$$

As discussed in ref. [10] the expectation value in eq.(14) is finite if $\gamma < \frac{1}{2}$.

## 4 The scale invariant model

In general we cannot calculate the average over the metric explicitly. However, the scale invariance of the metric is sufficient for a derivation of the short distance behaviour of the scalar propagator.

Let us note that $\sqrt{\tau} b(s/\tau) \simeq \tilde{b}(s)$ where $\tilde{b}$ denotes an equivalent Brownian motion (the equivalence means that both random variables have the same correlation functions). Then, using the scale invariance of $e$ with the index $\gamma$ (discussed in sec.2) we can write

$$e(\sqrt{\tau} x_F) \simeq \tau^{-\frac{\gamma}{2}} e(x_F)$$

Hence, in eq.(13)

$$q_\mu(\tau, x) = x_\mu + \tau^{\frac{\gamma}{2} - \frac{1}{2}} \int_0^1 \tilde{e}_\mu^a \left( \tau^{-\frac{\gamma}{2}} x_F + \tilde{b}_F(s) \right) db^a(s)$$

4
The expectation value over e is

$$
\langle K_\tau(x,y) \rangle = \tau^{-\frac{\nu}{2}(1-\gamma)-1} E \left[ \delta \left( (y - x_F) \tau^{-\frac{1}{2}} - \tilde{b}_F (1) \right) \delta \left( \tau^{-\frac{1}{2}} (y - x) \right) \right]
$$

where

$$
\eta^\mu = \int_0^1 \hat{e}^\mu \left( \tau^{-\frac{1}{2}} x_F + \tilde{b}_F (s) \right) \hat{b}^\alpha (s)
$$

(17)

Let $P(u,v)$ be the joint distribution of $(\eta, \tilde{b}_F (1))$ ( $P$ does not depend on $x_F$ because of the translational invariance). Then, the propagator of the $\Phi$ field is

$$
\hbar \langle A^{-1}(x,y) \rangle = \hbar \int_0^\infty d\tau \tau^{-1-1-1} \left( (x_G - y_G) \tau^{-1+1/2}, (x_F - y_F) \tau^{-1/2} \right)
$$

(18)

Eq.(19) in momentum space has the representation

$$
\hbar \langle A^{-1}(k_G, k_F) \rangle = \hbar \int_0^\infty d\tau \tilde{P}(\tau^{-1/2} k_G, \sqrt{\tau} k_F)
$$

where $\tilde{P}$ denotes the Fourier transform of $P$. Using eq.(14) we may write explicitly

$$
\hbar \langle A^{-1}(k_G, k_F) \rangle = \hbar \int_0^\infty d\tau \langle \exp \left[ \sqrt{\tau} k_F \tilde{b}_F (1) + \tau^{1/2} k_G \eta_G \right] \rangle
$$

The dispersion relation (relating the frequency to the wave number) is determined by (after an analytic continuation $k_0 \to ik_0$)

$$
\left( \langle A^{-1}(k_G, k_F) \rangle \right)^{-1} = 0
$$

It can be concluded from eq.(19) that in general the dispersion relation will be different from the standard one (resulting from a non-linear wave equation) $k_0 \sim |k|$. In particular, we can see that if $|k_F| \gg |k_G|$ then $\langle A^{-1}(k_G, k_F) \rangle \sim |k_F|^{-2}$ whereas if $|k_G| \gg |k_F|$ then $\langle A^{-1}(k_G, k_F) \rangle \sim |k_G|^{-2}$. In the configuration space, the propagator tends to infinity if both $|x_F - y_F|$ and $|x_G - y_G|$ tend to zero. However, the singularity depends in a rather complicated way on the approach to zero. It becomes simple if either $|x_F - y_F| = 0$ or $|x_G - y_G| = 0$. So, if $|x_F - y_F| = 0$ then we make a change of the time variable

$$
\tau = t|x_G - y_G|^{1/2}
$$

(20)

Using eq.(19) we obtain the factor depending on $|x_G - y_G|$ in front of the integral and a bounded function $A$ of coordinates, i.e.,

$$
\langle A^{-1}(x,y) \rangle = A|x_G - y_G|^{-D+2}
$$

(21)

5
If $|x_G - y_G| = 0$ then we change the time variable

$$\tau = t|x_F - y_F|^2$$

As a result

$$\langle A^{-1}(x, y) \rangle = A|x_F - y_F|^{-(D-2)(1-\gamma)}$$

with a certain bounded function $A$. We can see that in the $x_G$ coordinate the singularity remains unchanged but the propagator is more regular in the $x_F$ coordinate.

It is not easy to calculate the probability distribution $P$ exactly. Choosing as a first approximation $\eta \simeq b_G(1)$ we obtain

$$P(u, v) = (2\pi)^{-D} \exp\left(-\frac{u^2}{2} - \frac{v^2}{2}\right)$$

In this approximation

$$\hbar \langle A^{-1}(k_G, k_F) \rangle = \frac{\hbar}{2} \int_0^\infty d\tau \exp\left(-\frac{1}{2}\tau^{1-\gamma}|k_G|^2 - \frac{1}{2}\tau|k_F|^2\right)$$

If $D = 4$ then

$$\int dxF dxF|\langle A^{-1}(x, y) \rangle|^2 = \int dk_G dk_F|\langle A^{-1}(k) \rangle|^2 < \infty$$

for any $\gamma > 0$.

In $D$ dimensions the integral (23) takes the form

$$\int_0^\infty d\tau_1 \int_0^\infty d\tau_2 (\tau_1 + \tau_2)^{-1}(\tau_1^{1-\gamma} + \tau_2^{1-\gamma})^{-\frac{D-2}{2}}$$

It is finite if $(1-\gamma)(D-2) < 2$. Hence, it can be finite in $D = 5$ if $\gamma < \frac{1}{2}$ is large enough. In $D = 6$ power-law singularities appear if $\gamma < \frac{1}{2}$ and the model has logarithmic (presumably renormalizable) singularities for $\gamma = \frac{1}{2}$.

It can be shown that a singularity of the mean value of any power $n$ of $A^{-1}(x, y)$ is equal to the $n$-th power of the singularity of the mean value $\langle A^{-1} \rangle$. Then, it follows that there will be no coupling constant renormalization in the $\Phi^4$ model in four dimensions.

5 The photon propagator

Let us consider the $D$-dimensional electromagnetic Lagrangian

$$W = \frac{1}{4} \int d^D x \sqrt{g} R^{BCRS} F_{BC} F_{RS}$$

(24)
We define
\[ \hat{g}^{BC} = \sqrt{g} g^{BC} \] (25)
and
\[ \hat{\epsilon}^{BC} = \hat{e}^B_S \hat{e}^C_S \] (26)
We consider also the inverse matrix \( l \)
\[ \hat{l}^{RS} \hat{e}^B_R = \delta^S_R \] (27)
In terms of the tetrad we define fields in a local Euclidean frame
\[ \hat{A}_R = \hat{e}^B_R A_B \] (28)
We choose the Feynman gauge. This means an addition to the action of the term
\[ W_0 = \frac{1}{2} \int d^D x (g^{BC} \partial_B \sqrt{g} A_C)^2 \] (29)
We express the action by the hat-fields. We divide \( W = W_1 + W_2 + W_3 + W_0 \) as follows
\[ W_1 = \frac{1}{2} \int d^D x \hat{A}_a (-g^{\rho\sigma} \partial_\rho \partial_\sigma) \hat{A}_a \] (30)
where \( \hat{A}_a = \hat{e}_a^c A_c \) and
\[ W_2 = \frac{1}{2} \int d^D x \hat{A}_k ((- \partial_j + \hat{\Gamma}_j)(\partial_j + \hat{\Gamma}_j) - g^{\rho\sigma} \partial_\rho \partial_\sigma) \hat{A}_k \] (31)
where
\[ (\hat{A})_k = g^{\frac{1}{2}} \hat{A}_k \] (32)
is a vector and
\[ (\hat{\Gamma}_j)_k^l = g^{\frac{1}{2}} \partial_j g^{\frac{1}{2}} \delta^l_k \] (33)
is a diagonal matrix. \( \hat{\Gamma} \) is a connection defined in eq. (33) for the last \((D-1, D)\) components and
\[ \hat{\Gamma}^c_{j\mu} = \hat{e}_a^c \partial_j \hat{\epsilon}^a_{\mu} \] (34)
for the first \( D - 2 \) components. After the transformation (28) \( W_3 \) reads
\[ W_3 = \frac{1}{2} \int (\partial_j \delta_{ca} + \hat{\Gamma}_j^a) \hat{A}_a (\partial_j \delta_{cb} + \hat{\Gamma}_j^b) \hat{A}_b \equiv \frac{1}{2} \int \hat{A}_a [(- \partial_j + \hat{\Gamma}_j)(\partial_j + \hat{\Gamma}_j)]_{ab} \hat{A}_b \] (35)
We can write down the whole action in the form
\[ W = \frac{1}{2} \int d^D x \hat{A}_R g^{BC} (-\partial_B + \hat{\Gamma}_B)(\partial_C + \hat{\Gamma}_C) \hat{A}_R \hat{A}_S \equiv \frac{1}{2} \int \hat{A}_R (-\Delta_{EM})_{RS} \hat{A}_S \] (36)
where \( \hat{\Gamma}_j \) has a block form with components defined in eqs.(33) and (34) \( (\hat{\Gamma}_\mu = 0) \).
For a calculation of the correlation functions in QFT expressed by $\triangle^{-1}_{EM}$ we are interested in the heat equation

$$\partial_\tau \hat{A} = \frac{1}{2} \Delta_{EM} \hat{A}$$

(37)

The solution can be expressed in a general form

$$\hat{A}(\tau, x) \equiv (T_\tau \hat{A})(x) = E[T_\tau \hat{A}(q_\tau(x))]$$

(38)

where $\hat{A}_B$ is a $D$-dimensional vector and the matrix $T$ is a solution of the equation

$$dT = \hat{\Gamma}_j T dB^j$$

(39)

where $b^j$ are independent Brownian motions.

We need the kernel $K$ of the operator $T_\tau$ defined as the solution of the equation

$$\partial_\tau T_\tau = \frac{1}{2} \Delta_{EM} T_\tau$$

with the initial condition $(T_\tau)_{\tau=0} = 1$. In components, eq.(40) reads

$$\partial_\tau K^S_R(\tau; x, y) = \frac{1}{2} \int d^D z \Delta_{EM}(x, z) K^S_R(\tau; z, y)$$

(41)

with the initial condition $K(\tau = 0; z, y)^S_D = \delta^S_D \delta(z - y)$. Note that

$$\hat{\Gamma}_j = e \partial_j e^{-1}$$

(42)

is of the form of a pure gauge. Then, eq.(39) can be solved exactly. Inserting the solution of eq.(39) into eq.(38) we obtain the solution of the heat equation in the form

$$\hat{A}_R(\tau, x) = E[e^C_R(x)q_\tau(x)]\hat{A}_S(q_\tau(x))$$

(43)

Hence, the kernel is

$$K^S_R(\tau; x, y) = e^D_R(xy)q_\tau(x)$$

(44)

We must take an average over the tetrad in order to compute correlations in a random metric field. It is not easy to calculate expectation values of the inverse of $e$. So, we restrict ourselves to some special correlations.

The formula

$$\langle \hat{A}_R(x)\hat{A}_P(y) \rangle = \langle (\triangle_{EM})^{-1}_{RP}(x, y) \rangle \equiv \langle D_{RP}(x, y) \rangle$$

(45)

follows from the action (36). We can use this result to compute

$$\langle D^C_S(x, y) \rangle \equiv \langle A_B(x)A^C(y) \rangle = \langle A_B(x)g^{CD}(y)A_D(y) \rangle = \langle \hat{R}_B(x)\hat{A}_R(x)e^C_S(y)e^D_P(y)\hat{A}_P(y) \rangle = \int_0^\infty d\tau \langle \hat{R}_B(x)e^C_S(y)e^D_P(y)K^P_R(\tau; x, y) \rangle$$

$$= \delta_B \int_0^\infty d\tau \langle E[\delta(y - q_\tau(x))] \rangle$$

(46)
where the r.h.s. is equal to the scalar propagator. We can conclude that the short distance behaviour of the scalar as well as photon propagators are the same. It is less singular than the canonical one and determined by the scale dimension of $e$.

Finally, we consider the Abelian Higgs model. Its gauge invariant perturbation expansion in the $(\Phi\Phi^*)^2$ interaction is expressed by the propagators

$$
\langle \Phi(x)\Phi^*(y) \rangle = \langle \int_0^\infty d\tau E[\delta(q(\tau,x)-y) \exp\left(i \int_0^\tau dq B(s)A_B(q(s))\right)] \rangle
$$

$$
= \langle \int_0^\infty d\tau E[\delta(q(\tau,x)-y) \exp\left(-\frac{1}{2} \int_0^\tau dq B(s)dq_C(s')D_B^C(q(s)-q(s'))\right)] \rangle \nonumber
$$

where $D$ has been calculated in eq.(46). In the approximation $\langle \exp(-D) \rangle \simeq \exp(-\langle D \rangle)$ we obtain

$$
\langle \Phi(x)\Phi^*(y) \rangle 

\simeq \int_0^\infty d\tau E[\delta(q(\tau,x)-y)] \exp\left(\frac{1}{2} \langle \int_0^\tau dq B(s)dq_C(s')D_B^C(q(s)-q(s'))\right)] \nonumber
$$

(47)

The double line integral in the exponential (48) is more regular than the canonical one and may be finite depending on the dimension $D$ and the value of $\gamma$. In particular, it is finite if $D = 4$. The expression for higher order correlation functions is similar to eq.(47). So, for the fourth order correlation function we have

$$
\langle \Phi(x)\Phi(x')\Phi^*(y)\Phi^*(y') \rangle = \langle \int_0^\infty d\tau d\tau' E[\delta(q(\tau,x)-y) \delta(q'(\tau',x')-y')] \exp\left(i \int_0^\tau dq_B A_B + i \int_0^\tau dq_B A_B^2\right) \rangle \nonumber
$$

$$
= \langle \int_0^\infty \int_0^\infty d\tau d\tau' E[\delta(q(\tau,x)-y) \delta(q'(\tau',x')-y')] \exp\left(-\frac{1}{2} \int_0^\tau dq_B A_B^2 dC_C(s')D_B^C - \int_0^\tau dq_B A_B^2 dC_C(s')D_B^C\right) \rangle \nonumber
$$

$$
+ \langle \Phi(x)\Phi(x')\Phi^*(y)\Phi^*(y') \rangle \nonumber
$$

(49)

where the last term means the same expression with $x$ replaced by $x'$. We can obtain similar formulae for higher order correlation functions (including the correlations with the gauge potential). It follows that as a result of the more regular short distance behaviour of the scalar propagator (19) and the electromagnetic propagator (45) there will be no charge renormalization in the fourdimensional Higgs field interacting with a scale invariant two-dimensional quantum gravity. In higher dimensions $D$ the charge renormalization depends on the values of $D$ and $\gamma$. There still will be no charge renormalization if $D = 5$ and $\gamma$ is sufficiently close to $\frac{1}{2}$.

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