Gate controlled anomalous phase shift in Al/InAs Josephson junctions

William Mayer\textsuperscript{1}, Matthieu C. Dartialh\textsuperscript{1}, Joseph Yuan\textsuperscript{1}, Kaushini S. Wickramasinghe\textsuperscript{1}, Enrico Rossi\textsuperscript{2} & Javad Shabani\textsuperscript{1}\textsuperscript{*}

In a standard Josephson junction the current is zero when the phase difference between superconducting leads is zero. This condition is protected by parity and time-reversal symmetries. However, the combined presence of spin-orbit coupling and magnetic field breaks these symmetries and can lead to a finite supercurrent even when the phase difference is zero. This is the so called anomalous Josephson effect—the hallmark effect of superconducting spintronics—which can be characterized by the corresponding anomalous phase shift. Here we report the observation of a tunable anomalous Josephson effect in InAs/Al Josephson junctions measured via a superconducting quantum interference device. By gate controlling the density of InAs, we are able to tune the spin-orbit coupling in the Josephson junction. This gives us the ability to tune the anomalous phase, and opens new opportunities for superconducting spintronics, and new possibilities for realizing and characterizing topological superconductivity.
Superconductivity and magnetism have long been two of the main focuses of condensed matter physics. Interfacing materials with these two opposed types of electron order can lead to many new phenomena. Recently these systems have drawn renewed theoretical and experimental attention in the context of superconducting spintronics\(^1\) and in the search for Majorana fermions\(^2\)-\(^5\). Novel heterostructures can provide the ingredients that are typically needed: superconducting pairing, breaking of time-reversal symmetry, and strong spin–orbit coupling.

A basic property of superconducting systems is that we can introduce a relation between charge current and the superconductor's phase. In the canonical example of a Josephson junction (JJ), this is the current-phase relationship (CPR). Systems with nontrivial spin texture generally introduce a relationship between charge and spin. In the case of spin–orbit coupling this can manifest in many ways including the spin Hall effect and topological edge states\(^6\).

A hybrid system, combining spin–orbit coupling and superconductivity, results in a much richer physics where phase, charge current and high transparency can be measured as a static Zeeman gradient can generate a charge current with no current.

Recent years have seen a resurgence of interest in these systems, driven by the discovery of new ingredients that are typically needed: superconducting pairing, and Majorana fermions. These new ingredients can lead to many new phenomena. Recently these systems have drawn renewed theoretical and experimental attention in the context of superconducting spintronics\(^1\) and in the search for Majorana fermions\(^2\)-\(^5\). Novel heterostructures can provide the ingredients that are typically needed: superconducting pairing, breaking of time-reversal symmetry, and strong spin–orbit coupling.

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considered individually. If we assume a Gaussian distribution of transparencies we approximately recover the single channel result for the mean transparency. In a more realistic system finite disorder can mix channels and substantially alter the junction properties as will be discussed in the context of an anomalous phase shift below. To measure the CPR, we apply gate voltages to the junctions to create a highly asymmetric current configuration ($I^1_2 \approx 4I_2^2$). This effectively fixes the phase of the high current junction so we measure only the CPR of the lower current junction. Figure 2a shows resistance maps at $B_1 = 50 \text{ mT}$, $B_2 = 200 \text{ mT}$, and $B_3 = 350 \text{ mT}$ in the CPR regime. At $B_1 = 50 \text{ mT}$ the plot shows a forward skew indicating high JJ transparency. To fit the SQUID oscillations, we sum the contributions of each JJ with a phase difference due to applied $B_2$ and maximize the current with respect to the sum of the phases. The resulting fits are shown in Fig. 2a as orange overlays. The transparencies obtained from the fits are indicated in each plot. Measurement at $B_1 = 350 \text{ mT}$ reveals the oscillations are more sinusoidal, indicating reduced transparency. The dependence of transparency on $B_1$ for JJ2 is shown in Fig. 2b. We observe near unity transparency at low fields, with a rapid decline above 200 mT. Both junctions show similar dependence of transparency on $B_1$. The mechanism leading to the decreased transparency as a function of $B_1$ is not well understood. Note that these fits are based on the assumption that the JJ CPR is captured by Eq. (1).

**Anomalous phase shift.** If we consider a single JJ with an anomalous phase, a typical current-biased measurement will show no measurable signature. When a JJ is current biased, the CPR dictates that the phase will change so the critical current is maximized. This means that any phase shift applied to such a system will be invisible once the current is maximized. A simple alternative which has been employed in previous studies of $\phi_0$ is to use a SQUID geometry, whose primary property is phase sensitivity. Even in a SQUID, any single scan generally has an phase offset obscuring the effect of $\phi_0$. In order to experimentally measure $\phi_0$, a phase reference is necessary. To this end we compare scans taken consecutively at the same field but changing $V_g$ of one JJ. The gate voltage varies both the density and strength of spin–orbit coupling which should change $\phi_0$. Figure 3 shows resistance maps taken at different $B_1$ for three $V_g^2$. By finding the phase shift between these different gate voltages we can measure the variation of $\phi_0$. This shift is most easily seen by comparing the positions of SQUID oscillation maxima at different $V_g^2$. To extract the phase difference we fit the data using a similar procedure as applied to the CPR of Fig. 2. The only adjustment is that we include $\phi_1 = \phi + \phi_0$ in each CPR relation. In the case of a varying transparency, one could observe an apparent phase shift unrelated to $\phi_0$. However this shift would have the opposite sign on the positive and negative bias branches of the measurement. The data presented in Fig. 3 are symmetric in bias, which allows us to definitively separate the effects of transparency and a $\phi_0$ shift. A
As high as 180 meV higher, the presence of an in-plane orbit coupling can be tuned from close to zero to \( \pi \) orbit coupling. Previous work on InAs indicates that \( \phi_0 \) is much smaller than what we observe. This is not surprising considering that in our devices \( \xi \approx 770 \text{ nm} \). In addition, both expressions are obtained in the limit of weak proximitized superconductivity, obtained by imposing a finite contact resistance at the interface. In addition, theoretical work in the short junction limit is generally restricted to nanowire systems with only a few conduction channels. This leads to a geometry that is still drastically opposed to the current situation where \( W \gg L \), which cannot be achieved in nanowires.

To understand the large value of \( \phi_0 \) in our devices it is important to first understand the effect of having a very large number of transverse modes. For a few of these modes \( V_\parallel \) is very small and therefore \( L/\xi > 1 \). Consequently these few modes can be described in a long diffusive limit, greatly increasing their contribution to \( \phi_0 \). Coupled with the fact that the proximity effect is strong in this system, this provides a qualitative explanation for the larger than expected values of \( \phi_0 \).

Figure 4b shows the dependence of \( \Delta \phi_0 \) on \( B_y \) at a range of gate voltages. The strong agreement with linear fits confirms that \( \Delta \phi_0 \) is proportional to the Zeeman energy in agreement with theory. With a more complete theoretical understanding in the limit of strong proximity effect, it should be possible to estimate the strength of spin–orbit coupling from the slope of the anomalous phase dependence. At the largest \( B_y \) and \( V_g \) measured we observe \( \Delta \phi_0 \approx 2 \pi \) setting a lower bound on \( \phi_0 \). It is possible to optimize both \( L \) and \( W \) of each JJ to increase \( \Delta \phi_0 \), and consequently \( \phi_0 \).

Discussion

In summary, we have shown the capability to tune the anomalous phase shift of JJs formed by InAs and Al. This tunability results from the ability to vary the strength of the spin–orbit coupling via an external gate. The observation of a finite \( \phi_0 \) indicates a coupling of the superconductors phase, charge current, and spin in these heterostructures. We find \( \phi_0 \) to be proportional to the Zeeman energy, as expected, and its magnitude to be much larger than the currently available theoretical scalings. This is most likely due to the presence of a large number of conductions channels and the strong proximity effect in our system.

The capability to realize a large value of \( \phi_0 \) and to tune it is of great importance for applications in superconducting spintronics where large spin gradients can be used to realize phase batteries, and opens the possibility to generate, in a controllable way, spin gradients through Josephson currents or a phase bias. In addition, the observation that a significant \( \phi_0 \) can be present in InAs/Al heterostructures, and the fact that it strongly depends on the density of InAs, are directly relevant to efforts to realize topological superconducting states. In particular, the knowledge that an intrinsic phase difference \( \phi_0 \) can be present in InAs/Al JJs is of great importance for recent proposals to realize topological superconductivity in phase-controlled JJs.
Methods

Growth and fabrication. The structure is grown on semi-insulating InP (100) substrate. This is followed by a graded buffer layer. The quantum well consists of a 4 nm layer of InAs grown on a 4 nm layer of In0.81Ga0.19As and finally a 10 nm In0.81Ga0.19As layer on the InAs which has been found to produce an optimal interface while maintaining high 2DEG mobility. This is followed by in situ growth of epitaxial Al (111). Molecular beam epitaxy allows growth of thin films of Al where the in-plane critical field can exceed ~2T. Devices are patterned by electron beam lithography using PMMA resist. Transene type D is used for wet etching of Al and a III-V wet etch (H2O:C6H12O: H3PO4:H2O) is used to define deep semiconductor mesas. We deposit 50 nm of AlOx using atomic layer deposition to isolate gate electrodes. Top gate electrodes consisting of 5 nm Ti and 70 nm Au are deposited by electron beam deposition.

Measurements. All measurements are performed in an Oxford dilution refrigerator with a base temperature of 7 mK. The system is equipped with a 6:3:1.5 T vector magnet. All transport measurements are performed using standard dc and lock-in techniques at low frequencies and excitation current $I_{dc} = 10 nA$. Measurements are taken in a current-based configuration by measuring $R = dV/dI$ with $I_{dc}$, while sweeping $I_{ac}$. This allows us to find the critical current at which the junction or SQUID switches from the superconducting to resistive state. It should be noted we directly measure the switching current, which can be lower than the critical current due to effects of noise. For the purposes of this study we assume they are equivalent.

Fitting procedure. As illustrated in Fig. 2, the junctions forming the SQUID display a saw-tooth like CPR characteristic of junctions with high transparencies, and this even at low gate. We hence model the CPR using Eq. 1 in which we neglect the temperature dependence which would only induce minor corrections. To model the SQUID pattern, we sum the contributions of two JJs with a phase difference and maximize (minimize for negative bias current) the current with respect to the sum of the phases. This requires the use of six parameters: the out-of-plane magnetic field to phase conversion factor, the transparency of each junction, the critical current of each junction (defined as independent of the transparency) and a phase. This represents a large number of parameters for fitting a single trace. To improve the accuracy of our procedure we consider multiple traces and reduce the number of parameters based on physical arguments.

Since we cannot experimentally access a reliable phase reference, we always compare measurements taken within a single magnetic field sweep, for different values of the gate voltage applied to one of the junction (referred to as the active junction). The second junction (idler) stays at a constant gate voltage. We can hence fix the amplitude of the idler current for a given parallel field.

Changes in the transparency of a junction can cause an apparent phase shift when considering only the positive bias current branch of the SQUID oscillation. However this apparent shift would have the opposite sign for the negative bias current branch. We have checked, as illustrated in Fig. 3, that the phase shift we observe is present with the same sign on both branches. As a consequence we can reasonably assume that the transparency of the junctions is constant over the gate.

Fig. 3 Resistance of the device as a function of the phase bias applied on the SQUID and the bias current at three different values of the in-plane field $B_y$ and three different values of $V_g^2$. In all scans $V_g^2$ is set to $-2 V$. The dashed orange line indicates the position of the maximum of the oscillation at $V_g^2 = -4 V$. Orange stars indicate the position of the maximum at each field.

Fig. 4 Tuning anomalous phase shift using gate voltage and in-plane magnetic field. Evolution of the phase shift in JJ2 as a function of the gate voltage (a) and of the applied in-plane field along $y$ (b). The phase shift $\Delta \phi$ is measured between the oscillations at a given value of $V_g^2$ and the ones at $-4 V$ used as reference. In b the solid lines corresponds to linear fits to the measured phase shifts.
voltage range considered. This assumption allows us to use one transparency value per junction at a given field. The transparency value is better constrained in a CPR-like measurement and this is why, to have a well constrained problem, we combine data sets taken in both configurations: JJ1 as active junction and JJ2 as idler and JJ2 as the active junction and JJ1 as idler.

Considering measurements at N parallel fields with M different gate values in both configuration (JJ1 active/JJ2 active), we fit for each junction N transparencies, N x M amplitudes as idler, N x M amplitudes as active. Furthermore we extract 2 x N x M phases. Because the field to phase conversion factor depends only on geometrical considerations we use a single value for each configuration (We observed that for data sets taken several weeks apart we could see small changes in the field to phase conversion factor, that we attribute to the magnet. As a consequence we use different factors for data taken when tuning JJ1 or JJ2). For the most extensive dataset, presented in Fig. 4, N = 7 and M = 6. Similarly, we can also take into account the Fraunhofer envelope of the oscillation using two global parameters: a period and a phase.

By comparing the transparencies from independent measurements of JJ1 and JJ2 at a given magnetic field, we find that the junction transparencies are very similar. Hence, the data for Figs. 2a and 3 have been fitted using the equal transparencies assumption. The data for Figs. 2b and 4 have been fitted using the full method presented above but we focused on JJ2 results.

Data availability
All data are available from the corresponding author upon reasonable request.

Received: 18 June 2019; Accepted: 11 December 2019;
Published online: 10 January 2020

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Supplementary Information
The application of an in-plane magnetic field on the sample leads to a reduction of the critical current of the Josephson and a distortion of the Fraunhoffer pattern as illustrated in Supplementary Figure 1.

The change in the critical current of the junction appears to strongly depend on the direction of the applied in-plane field. In Supplementary Figure 1, the amplitude of the critical current is similar in both plots but the magnitude of the applied magnetic field is twice as large in the y direction compared to the x direction.

For both directions of the field, the Fraunhofer pattern appears asymmetric which is not the case in the absence of the in-plane as illustrated in the main text. The observed distortions are similar for both orientations of the field. Despite these distortions a clear central peak remains at all magnetic fields below $B_c$. Additionally, as stated in the main text, the period of Fraunhofer oscillations is unchanged. This indicates there are not large deviations from a uniform current distribution even in the presence of large in-plane magnetic fields.

When comparing those data to the ones presented in the main text, one can notice that the width of the first node has been divided by about two. We attribute this effect, which is also visible in the SQUID oscillations, to the transition out of the superconducting state of the indium layer at the back of the sample. The transition occurs around 30 mT and does not impact our study otherwise.

To alleviate any concern of the reader may have regarding the fact that we plot most of our data as a function of the phase of the SQUID, we plot in Supplementary Figure 2 the data of the middle panel of Fig. 3 as a function of the out-of-plane magnetic field. We would like however to underline here that when fitting our data a single frequency is used for all the data presented together and as a consequence the relationship between the SQUID phase and the magnetic field is linear. Furthermore since the data at different gates are acquired within a single magnetic field field there cannot be arbitrary phase offsets in the SQUID from one gate voltage to the next.

The current phase relationship (CPR) of a Josephson junction with a high transparency present a notable sawtooth like profile which leads to distortions of the typical SQUID oscillations. In the following we discuss how this affects our measurements.

In Supplementary Figure 3, we present calculations performed for two junction of varying critical currents and transparencies. For junctions with different transparencies, it appears that changing the relative amplitude of the CPR only depends on the shape of the CPR. This validates our method of extraction of the phase shift under the assumption that the applied gate voltage does not affect the junction transparency.

In Supplementary Figure 4 we illustrate the artificial phase-shift that can be induced by varying the transparency of one junction while the other is kept at a fixed transparency (0.5). We consider equal current in each arm, but as mentioned above this has no consequence on the phase-shift. As the transparency is varied between 0 and 0.99, the oscillations are shifted by about 0.25π which is about half of the largest phase-shift we measured. Furthermore that shift has the opposite sign on the positive and negative branches of the SQUID critical current, which allows us to rule out this effects as being the dominant mechanism in our experiment as illustrated.
in Fig. 3 of the main text.

To reduce the measurement time, we have often worked with only the positive branch of the SQUID critical current and assumed a constant transparency of the junction as a function of the gate. This can lead to errors in the determination of the phase-shift obviously but as discussed above we have checked that a varying transparency cannot alone explain all our results.

The application of a gate voltage on the junctions may alter the current distribution and hence the effective area of the SQUID. We examine here this possibility to ascertain it cannot explain our results.

Let’s consider an initial situation with a out of plane field $B$ applied to the SQUID of surface $S$ such that the enclosed flux is $n\phi_0$, where $\phi_0$ is the quantum of flux. When applying the gate let’s assume that the surface enclosed becomes $S + \Delta S$, such that the flux becomes $(n + x)\phi_0$. From this simple argument we can conclude that $x/n = \Delta S/S$. If we consider the case of the largest phase-shift we observed $\sim \pi/2$, which corresponds to a quarter of flux and since we always work close to the maximum of the Fraunhofer pattern let’s take $n = 5$.

To explain our observation, the surface of the SQUID would have to change by 5% which given the the surface of our SQUID ($25 \mu m^2$) and the surface of our junctions ($100 nm \times 1 \mu m$) is not possible even taking into account flux focusing. Flux focusing increases the effective surface of the junction by concentrating the magnetic flux lines inside the junction. However based on the comparison of the expected Fraunhofer frequency to the measured one, its impact doubles at most the effective area of the junction.

The phase-shift of JJ2 as a function of the applied field presented in Fig. 4 of the main text has been extracted by fitting the SQUID oscillations of both JJ1 and JJ2 in a constrained manner as described in the Methods section of the main text. We present in Supplementary Figure 5, the data and fits obtained at three different values of magnetic field. As in the main text, we mark the position of the maximum at $V_g = -4 \, V$ using a dashed line and the position of the maximum at each field using a star.

One can observe that the phase-shift observed for JJ1 is of the same order of magnitude than the one for JJ2 but of the opposite sign as expected from the SQUID equation.

According to most theoretical predictions, in the absence of Dresselhaus spin-orbit coupling applying a magnetic field along the $x$ axis should not give rise to an anomalous phase. In InAs, the spin-orbit interaction is expected to be mostly of the Rashba type and we hence expect a reduction of the phase shift by rotating the field.

We present in Supplementary Figure 6, data taken in the presence of a 300 mT field at $45^\circ$ (a) and along the $x$-axis (b) along with the extracted phase-shift as the function of the angle $\theta$ defined in Figure 1 c of the main text.

The phase-shift appears to diminish as we rotate the field away from the $y$-axis but remains finite as illustrated in (a) and (b). The error bars on the determination of the phase-shift are large due to fluctuations of the SQUID period inside the dataset (up to maximum of 10%) that forced us to treat it in two separate subsets.

**Supplementary Figure 3.** (Color online) SQUID critical current for highly transparent junction. The critical current of one of the junction is fixed to 1 and its transparency is set to 0.5. The values used for the other junction are the ones indicated on the figure. The method of calculation of the plotted current is the same one used to fit the experimental data. The dashed lines indicate the position of the maximum of the oscillation.

**Supplementary Figure 4.** (Color online) SQUID critical current (positive/negative) for varying transparency of one junction. The transparency of the other junction is fixed at 0.5 and the current in both amplitudes are taken equal. The dashed lines indicate the position of the maximum/minimum of the oscillation.
Supplementary Figure 5. (Color online) Fits performed simultaneously (see Methods) on JJ1 and JJ2 data to extract the phase shift. When working on JJ1, $V_{g2}$ is set to 0 V, when working on JJ2, $V_{g1}$ is set to -2 V.

Supplementary Figure 6. (Color online) JJ2 data and fits performed with an in-plane field of 300 mT applied at $\theta = 45(a)/90(b)$ with respect to the y-axis. (c) Phase-shift extracted from the fits as a function of $\theta$. Error bars indicate uncertainty due to fluctuations of SQUID period.