Tensor Structure Function $b_1(x)$

For Spin-One Hadrons

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Abstract
High-energy spin physics became a popular topic recently after the EMC finding for the proton’s spin content. There exist unmeasured spin-dependent structure functions ($b_1$, $b_2$, $b_3$, and $b_4$) for spin-one hadrons such as the deuteron. The tensor structure function $b_1(x)$ could be measured by the proposed 15 GeV European Electron Facility. The measurement provides important clues to physics of non-nucleonic components in spin-one nuclei and to tensor structures on the quark-parton level.

1. Introduction to $b_1$
Experimental results for the proton’s $g_1(x)$ by the European Muon Collaboration (EMC) [1] indicated that “none of the proton’s spin is carried by quarks”. Since then, parton-spin distributions in the nucleon have been a popular topic among particle and nuclear physicists. Because of the dramatic conclusion contrary to naive quark-model expectation, efforts have been made to interpret it in terms of small-$x$ contributions, gluon polarizations, sea-quark polarizations, orbital angular momenta, and so on [1].

Recently experimental data for $g_1(x)$ of “the neutron” have been taken by the Spin Muon Collaboration (SMC) [2] for testing the Bjorken sum rule. Because there exists no fixed neutron target, polarized deuterons (or $^3$He) have to be used as the target. However, the deuteron is interesting in its own right. The structure function $g_1$ exists for hadrons with spin more than 1/2. On the other hand, the deuteron spin is one so that other spin-dependent structure functions exist due to its electric-quadrupole structure. These are named $b_1$, $b_2$, $b_3$, and $b_4$ in Ref. [3]. In the Bjorken scaling limit, the only relevant structure function is $b_1$ or equivalently $b_2$. Detailed analyses using the operator product expansion and a parton model have been done in Ref. [3], and some examples of $b_1(x)$ are also discussed.

The structure function $b_1$ can be measured by using a target polarized parallel (and antiparallel) to the lepton beam direction. The lepton does not have to be polarized. From the polarized cross sections and unpolarized ones, we could obtain $b_1$. However, because $b_1$ for a nuclear target is considered to be very small compared with the unpolarized one ($F_1$), we need an intense electron-accelerator facility to measure it. The 15 GeV European Electron Facility provides a good opportunity of measuring this tensor structure function [4], which could provide clues to physics of non-nucleonic components in spin-one nuclei and to tensor structures on the quark-parton level.
components in spin-one nuclei and to tensor structures on the quark-parton level.

In section 2, we discuss structure functions for spin-one hadrons in general. These structure functions are expressed in terms of quark-spin distributions by using a quark-parton model. A phenomenological sum rule for the tensor structure function $b_1(x)$ based on a quark-parton model is discussed in section 3. Examples of $b_1(x)$ are given in section 4 and conclusions are in section 5.

2. Structure functions for spin-one hadrons

Structure functions for spin-one hadrons have been investigated in detail recently by Hoodbhoy, Jaffe, and Manohar [3]. Discussions in this section are based on their publication. We find earlier investigations of the tensor structure function by Pais (for real photons) and Frankfurt-Stirikman [5].

The cross section of deep-inelastic lepton scattering from a spin-one hadron is given by

$$d\sigma \propto k'^\mu k^\nu + k'^\nu k^\mu - g^{\mu\nu} k' \cdot k + i\epsilon^{\mu\nu\lambda\sigma} s^\lambda q^\sigma,$$

where $k$ and $k'$ are incident and scattered lepton momenta, $q$ is the momentum transfer, $s^e$ is the electron polarization, and $\epsilon^{\mu\nu\lambda\sigma}$ is an antisymmetric tensor with $\epsilon_{0123} = 1$. The hadron tensor is

$$W_{\mu\nu}(p, q, H_1, H_2) = \frac{1}{4\pi} \int d^4 \xi e^{i\xi q} \langle p, H_2 | [J_\mu(\xi), J_\nu(0)] | p, H_1 \rangle,$$

where $p$ is the hadron momentum, $H$ and $H'$ are the $z$ components of the hadron spin, and $J_\mu$ is the electromagnetic current. Using momentum conservation, parity invariance, and time-reversal invariance, we have eight independent amplitudes for

$$\gamma(H_1) + \text{target}(H_1) \rightarrow \gamma(H_2) + \text{target}(H_2).$$

Therefore, the hadron tensor $W_{\mu\nu}$ can be written in terms of eight independent structure functions:

$$W_{\mu\nu} = -F_1 g_{\mu\nu} + \frac{F_2 p_{\mu} p_{\nu}}{\nu} + g_1 \frac{i}{\nu} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + g_2 \frac{i}{\nu^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma)
- b_1 r_{\mu\nu} + \frac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu}),$$

where $r_{\mu\nu}, s_{\mu\nu}, t_{\mu\nu}, u_{\mu\nu}, s^\sigma, \nu,$ and $\kappa$ are defined by

$$r_{\mu\nu} = \frac{1}{\nu^2} (q \cdot E^* q \cdot E - \frac{1}{3} \nu^2 \kappa) g_{\mu\nu},$$

$$s_{\mu\nu} = \frac{2}{\nu^2} (q \cdot E^* q \cdot E - \frac{1}{3} \nu^2 \kappa) \frac{p_{\mu} p_{\nu}}{\nu},$$

$$t_{\mu\nu} = \frac{1}{2\nu^2} (q \cdot E^* p_{\mu} E_{\nu} + q \cdot E^* p_{\nu} E_{\mu} + q \cdot E p_{\mu} E_{\nu} + q \cdot E p_{\mu} E_{\nu}^* - \frac{4}{3} \nu p_{\mu} p_{\nu}),$$

$$u_{\mu\nu} = \frac{1}{\nu} (E_{\mu}^* E_{\nu} + E_{\nu}^* E_{\mu} + \frac{2}{3} M^2 g_{\mu\nu} - \frac{2}{3} \nu p_{\mu} p_{\nu}).$$
\[ s^\sigma = -\frac{i}{M^2} \sigma^{\alpha\beta\tau} E^*_\alpha E_\beta p_\tau, \quad (4.5) \]
\[ \nu = p \cdot q, \quad \kappa = 1 + \frac{M^2 Q^2}{\nu^2}. \quad (4.6) \]

\( M \) is the target hadron mass and \( E \) is the target polarization, which satisfies \( p \cdot E = 0 \) and \( E^2 = -M^2 \). Considering current conservation, we dropped terms proportional to \( q_\mu \) and \( q_\nu \) for simplicity.

Structure functions \( F_1, F_2, g_1, \) and \( g_2 \) are defined in the same manner as the spin-1/2 case. The terms of new structure functions \( b_{1-4} \) are symmetric under \( \mu \leftrightarrow \nu \) and under \( E \leftrightarrow E^* \), and those of \( g_{1,2} \) are antisymmetric under \( \mu \leftrightarrow \nu \) and under \( E \leftrightarrow E^* \). The terms of \( b_{1-4} \) vanish upon target-spin averaging. From these symmetry properties and those for \( L^{\mu\nu} \) in Eq. (1), we find that the lepton beam does not have to be polarized for measuring \( b_{1-4} \) although both polarized beam and polarized target are necessary for \( g_{1,2} \).

The new structure functions are analyzed by using an operator product expansion [3]. The analysis shows that the twist-two contributes to \( b_1 \) and \( b_2 \). Because only higher orders in the twist expansion contribute to \( b_3 \) and \( b_4 \), we do not discuss them in this report. There exists a “Callan-Gross type” relation for \( b_1 \) and \( b_2 \):
\[ b_2(x) = 2xb_1(x), \quad (5) \]
which is valid only in the lowest order in QCD. The above equation is no longer satisfied in higher orders; however, the relations
\[ M_n(2xb_1)M_n(F_2) = M_n(b_2)M_n(2xF_1), \quad n = \text{odd}, \quad (6) \]
are still satisfied, where the \( n \)-th moment is defined by \( M_n(f) = \int x^{n-1}f(x)dx \). The analysis by the operator product expansion indicates that \( b_1 \) and \( b_2 \) obey the same scaling equations as \( F_1 \) and \( F_2 \).

We discuss twist-two structure functions, \( F_1, F_2, g_1, b_1, \) and \( b_2 \), for spin-1 hadrons in a parton model [3]. The hadron tensor \( W_{\mu\nu} \) is obtained for a lepton scattering from free quarks. Comparing the results with Eq. (2), we write \( F_1(x), g_1(x), \) and \( b_1(x) \) in terms of quark (spin) distributions in the hadron as
\[ F_1(x) = \frac{1}{2} \sum_i e_i^2 \left[ q_i(x) + \bar{q}_i(x) \right], \quad (7.1) \]
\[ g_1(x) = \sum_i e_i^2 \left[ \Delta q_i(x) + \Delta \bar{q}_i(x) \right], \quad (7.2) \]
\[ \Delta q_i(x) = \frac{1}{2}(q_{i1}^{+1}(x) - q_{i1}^{-1}(x)), \quad (7.3) \]
\[ b_1(x) = \sum_i e_i^2 \left[ \delta q_i(x) + \delta \bar{q}_i(x) \right], \quad (7.4) \]
\[ \delta q_i(x) = q_{i1}^{0}(x) - \frac{1}{2}[q_{i1}^{+1}(x) + q_{i1}^{-1}(x)] = \frac{1}{2}[q_{i1}^{0}(x) - q_{i1}^{-1}(x)], \quad (7.5) \]
The $F_2$ structure function is given by the Callan-Gross relation $F_2(x) = 2xF_1(x)$ and $b_2$ is in the similar equation $b_2(x) = 2xb_1(x)$. As it is shown above, the $b_1(x)$ does not depend on the quark spin but on the hadron one. It is very different from the $g_1$ structure function, hence it probes different spin structures.

3. Sum rule for $b_1(x)$ in a parton model

We discuss a sum rule for the $b_1$ structure function in a parton model. The following discussions are based on the derivation by Close and Kumano [6]. It should be noted that the sum rule is not a "strict" one such as those derived by current algebra [7]. It is rather a phenomenological sum rule based on a naive parton model. This is because an assumption for sea-quark tensor polarizations needs to be introduced in order to reach the sum rule. The situation is very similar to the Gottfried sum rule, which became an interesting topic recently due to the results obtained by the New Muon Collaboration (NMC) [8]. The $SU(2)_{flavor}$ symmetric sea has to be assumed in order to get the Gottfried sum rule. Therefore, it is also not a "strict" sum rule, but it is the one based on a naive parton model. Nevertheless, as the Gottfried sum rule became an important topic for investigating the $SU(2)_{f}$ breaking in antiquarks, the $b_1$ sum rule could become useful for studying tensor polarizations in sea quarks.

Integrating Eq. (7.4) for the deuteron over $x$, we have

$$I(b_1^D) \equiv \int dx b_1^D(x) = \int dx \left[ \frac{4}{9}(\delta u^D + \delta \bar{u}^D) + \frac{1}{9}(\delta d^D + \delta \bar{d}^D) + \frac{1}{9}(\delta s^D + \delta \bar{s}^D) \right] .$$

(8)

The valence distribution in the deuteron is defined by $q_v^D = q^D - \bar{q}^D$, which obviously comes from the valence quarks in the proton and neutron: $u_v^D = u_v^p + u_v^n = u_v + d_v$, $d_v^D = d_v^p + d_v^n = d_v + u_v$. Then, the above equation becomes

$$I(b_1^D) = \frac{5}{9} \int dx [\delta u_v(x) + \delta d_v(x)] + \frac{1}{9} \delta Q^D_{sea} .$$

(9)

where

$$\delta Q^D_{sea} = \int dx [8\delta \bar{u}(x) + 2\delta \bar{d}(x) + \delta s(x) + \delta \bar{s}(x)]^D .$$

(10)

In a naive parton model, there is no tensor polarization in sea quarks: $\delta Q_{sea}=0$.

As discussed in Ref. [9], we try to relate the right hand sides of Eq. (9) to the following elastic amplitude

$$\Gamma_{H,H} = \langle p, H \mid J_0(0) \mid p, H \rangle .$$

(11)

We calculate the above amplitude in the infinite momentum frame in order to use a quark-parton picture. The amplitudes is then described in terms of quark distributions in the hadron as

$$\Gamma_{H,H} = \sum_i e_i \int dx [q^H_{vi}(x) + q^H_{vi}(x) - \bar{q}^H_{vi}(x) - \bar{q}^H_{vi}(x)] .$$

(12)
The tensor combination of the amplitudes is expressed by $\delta q^D_i(x) - \delta \bar{q}^D_i(x)$. Because $q_i^D - \bar{q}_i^D$ is the valence quark in the deuteron and $q_v^D = q_v^p + q_v^n$, we obtain

$$\frac{1}{2} \left[ \Gamma_{00} - \frac{1}{2}(\Gamma_{11} + \Gamma_{-1-1}) \right] = \frac{1}{3} \int dx \left[ \delta u_v(x) + \delta d_v(x) \right].$$

(13)

The right-hand side is identical to the first term in Eq. (9), so that the integral of $b_1$ is written by the elastic amplitudes as

$$\frac{5}{6} \left[ \Gamma_{00} - \frac{1}{2}(\Gamma_{11} + \Gamma_{-1-1}) \right] + \frac{1}{9} \delta Q_{\text{sea}}.$$

(14)

Macroscopically, these amplitudes can be expressed in terms of charge and quadrupole form factors of the deuteron [10]:

$$\Gamma_{00} = \lim_{t \to 0} \left[ F_C(t) - \frac{t}{3M^2} F_Q(t) \right],$$

(15)

$$\Gamma_{11} = \Gamma_{-1-1} = \lim_{t \to 0} \left[ F_C(t) + \frac{t}{6M^2} F_Q(t) \right],$$

(16)

where $F_C$ and $F_Q$ are measured in the units of $e$ and $e/M^2$. If the tensor combination of the amplitudes is taken, the first terms cancel out and we obtain the quadrupole term as $[\Gamma_{00} - \frac{1}{2}(\Gamma_{11} + \Gamma_{-1-1})]/2 = \lim_{t \to 0} -t/(4M^2) F_Q(t)$. Using this equation, we finally obtain the integral as

$$\int dx \ b_1^D(x) = \lim_{t \to 0} \left[ \frac{5}{6} - \frac{t}{3} \frac{F_Q(t)}{4M^2} + \frac{1}{9} \delta Q_{\text{sea}} \right].$$

(17)

This equation is very similar to the Gottfried sum rule. If the sea is not $SU(2)_f$ symmetric, the Gottfried sum rule is modified as

$$\int dx \ [F_2^p(x) - F_2^n(x)] = \frac{1}{3} + \frac{2}{3} \int dx [\bar{u}(x) - \bar{d}(x)].$$

(18)

As we have the $SU(2)_f$ symmetric sea ($\bar{U} - \bar{D} = 0$) in a naive parton model, the tensor polarization for sea quarks should vanish ($\delta Q_{\text{sea}} = 0$) in the parton-model case. Hence, we call the following equation as a sum rule on the same level with the Gottfried sum rule:

$$\int dx \ b_1(x) = 0.$$

(19)

If the sea quarks are tensor polarized, we obtain a nonzero value

$$\int dx \ b_1(x) = \frac{1}{9} \delta Q_{\text{sea}}.$$

(20)

All the results for $b_1(x)$ in Refs. 3, 12, and 13 satisfy the above sum rule in Eq. (19). As the breaking of the Gottfried sum rule became an interesting topic recently, it
is worth while investigating a possible mechanism to produce the tensor polarization $\delta Q_{\text{sea}}$, which breaks the sum rule.

Even though the sum-rule value is expected to be zero for the $b_1$, it does not mean that $b_1$ itself is zero. In fact, it is shown in the next section that $b_1(x)$ can be negative in a certain $x$ region.

4. Examples of $b_1(x)$

Some calculations for $b_1(x)$ are presented in Refs. 3, 5, 11, 12, and 13. We first discuss some examples based on Ref. 3. We consider the simplest case: a spin-1 system with two spin-1/2 nucleons at rest. This system obviously does not have a tensor structure. Hence, we have $b_1(x) = 0$.

In the deuteron, a pion exchange produces a tensor force between nucleons and gives rise to the D-state admixture. We use a convolution picture for calculating the helicity amplitudes. Namely, the helicity amplitude is given by a helicity amplitude for the nucleon convoluted with the light-like momentum distribution of the nucleon. $b_1$ for the deuteron is calculated as

$$b_1(x) = \sum_{k=p,n} \int dydz \sin^2(\alpha) \Delta f_{dd}(y) - \frac{4\sqrt{2}}{\sqrt{3}} \sin \alpha \cos \alpha \Delta f_{sd}(y) F_k^1(z), \quad (21)$$

where $\sin \alpha$ is the D-state admixture. $\Delta f(y)$ is the light-cone momentum distribution of a nucleon in the tensor polarized deuteron [$\Delta f(y) = f^0(y) - (f^+(y) + f^-(y))/2$]. The first term $\Delta f_{dd}(y)$ is due to the D-state and the second $\Delta f_{sd}(y)$ to the S-D interference. Because of the small D-state admixture, the above $b_1(x)$ is much smaller than the unpolarized $F_1(x)$. (An extension of this work is done by Khan and Hoodb-hoy [11].) The dynamics of producing the tensor structure contributes to the nonzero $b_1(x)$. However, it is interesting to find that its integral still satisfies the sum rule $\int dx b_1(x) = 0$ in Eq. (19) by explicitly integrating Eq. (21).

Miller studied an exchanged-pion contribution to $b_1(x)$ [12]. Pions are associated with the tensor force, so that it is natural to have large contributions to $b_1$ from the pions. The contribution is roughly parametrized as $b_1(x)/F_1(x) \approx 0.02(x - 0.3)$. If we integrate his pionic contribution (not the above approximate equation), we find that the sum rule $\int dx b_1(x) = 0$ is still fulfilled.

Mankiewicz [13] studied $b_1(x)$ for the $\rho$ meson by using light-cone wave-functions for constituent quarks. Calculated $b_1(x)$ shape is very similar to the one in Fig. 1. $b_1(x)$ needs not be small in relativistic systems.
In order to illustrate how \( b_1(x) \) looks like as a function of \( x \), we show the following example \([3]\). We consider a relativistic system with a quark with \( j = 3/2 \) which couples to another quark with \( j = 1/2 \) to form a \( j = 1 \) state. In this case, \( b_1 \) is as large as \( F_1 \) as shown in Fig. 1. The \( b_1 \) oscillates as a function of \( x \) and has negative values in the medium-\( x \) region. Integrating \( b_1(x) \) over \( x \), we find that this example again satisfies the sum rule \( \int dx b_1(x) = 0 \).

We learned the following from the above examples. Static nucleons alone do not contribute to \( b_1 \). The dynamics of a pion exchange produces nonzero \( b_1 \). \( b_1(x) \) has very different \( x \)-dependence from that of \( F_1(x) \) or \( g_1(x) \), and it satisfies the sum rule \( \int dx b_1(x) = 0 \) in all the cases considered in this section. \( b_1 \) is suitable for studying non-nucleonic degrees of freedom in nuclei such as nuclear pions, rhos, and perhaps nucleon substructures if we find an experimental deviation from conventional theoretical predictions. Much physics could be studied in the near future, for example, details of \( b_1(x) \) for \( \text{D, } ^6\text{Li, } ^{14}\text{N} \) and possible mechanisms of breaking the sum rule.

5. Conclusions

We discussed the tensor structure function \( b_1 \) based on recent publications. Although much more theoretical efforts have to be made to understand details of \( b_1(x) \), we expect that \( b_1 \) provides important clues to physics of non-nucleonic components in nuclei and to new tensor structures on the quark-parton level. Because \( b_1 \) for a nuclear target is considered to be small, we need an intense electron accelerator for measuring it. The proposed 15 GeV European Electron Facility is an appropriate one for measuring \( b_1 \).

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