An improved upper limit for the (muon based) neutrino mass

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Abstract

The width for the $\mu$ decay is calculated in the V-A theory leaving open the possibility of non zero neutrino masses. It is shown that not only the agreement with the experimental data is kept, but the smallness of the experimental error allows to improve the constraint of $\nu$ mass (muon based) down to 0.021 MeV, provided that $\nu$ mass (electron based) is as low as indicated by the $^3H$ beta decay. An analogous constraint for the $\nu$ mass (tau based) is not possible since in this case the decay width has a larger experimental error.

Although in the Standard Model (SM) the three neutrinos are assumed to be massless, the observation of their oscillation implies that at least two of them have a non zero mass. However a reliable determination of such masses up to now has not been possible and presently only upper limits can be given [1].

From the analysis of the $\beta$ decay of $^3H$ one can derive that the $\nu$ mass (electron based) is lower than 1.1eV [2], while for the $\nu$ mass (muon based) the upper limit is 0.19MeV [3]. In the case of the $\nu$ mass (tau based) the result, obtained from the data on the three- and five-prong decays, is 18.2 MeV [4].

The predictions of the V-A theory with massless neutrinos are in good agreement with the observed data, such as mean life and electron helicity, however their experimental uncertainties are compatible with non zero neutrino masses, provided that the latter are not too high [5].

This point can be reversed. As it will be shown below, the error in the determination of mean life for the $\mu$ decay

$$\mu^- \rightarrow e^- + \nu_e + \nu_\mu$$

is so small that a stringent limitation on the $\nu$ mass (muon based), an order of magnitude lower that the value quoted above, is obtained.

In fact, the $\mu$ lifetime is [1]

$$\tau_\mu = (2.1969811 \pm 0.0000022) \times 10^6 s$$

and the relative uncertainty is of the order of $10^{-6}$. Therefore a non zero neutrino mass is acceptable, provided that the modification in the theoretical
Table 1: The expressions of the Q-quantities in Eq. (4) as functions of $x = \frac{m_e^2}{m_\mu}$ and their numerical evaluations.

| Q values | analytical expression | approximation |
|----------|-----------------------|---------------|
| $Q_0$    | $1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$ | $\simeq 1$ |
| $Q_{\Sigma a}$ | $-\frac{22}{3} + 14x - 14x^2 + \frac{22}{3}x^3 + 8(1 - x)^3 \ln(1 - x) + 4x^2(x - 3) \ln x$ | $\simeq -22/3$ |
| $Q_{\Sigma b}$ | $-4(1 - x)^3$ | $\simeq -4$ |
| $Q_{\Delta a}$ | $-4(1 - x)^2 + 48(1 - x^2) \ln(1 - x) + 24x^2 \ln x$ | $\simeq -4$ |
| $Q_{\Delta b}$ | $-24(1 - x^2)$ | $\simeq -24$ |
| $Q_{\Delta c}$ | $-16(1 - x)^3$ | $\simeq -16$ |

The expression is lower than $10^{-6}$. To analyze this point it is necessary to calculate the $\mu$ decay width with non zero neutrino masses.

Starting from the standard V-A interaction, the amplitude $M(\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu)_{\alpha\beta}$ for the $\mu$ decay can be written

$$M(\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu)_{\alpha\beta} = \frac{G}{\sqrt{2}} [\bar{u}_e \gamma^\alpha (1 - \gamma_5) \nu_1] [\bar{\nu}_2 \gamma^\beta (1 - \gamma_5) u_\mu]$$  (3)

where $G$ is the fermi constant, $u_e$, $u_\mu$ are the electron and $\mu$ spinors (with masses $m_e$ and $m_\mu$), respectively and $\nu_1$, $\nu_2$ are the neutrino spinors with masses $m_1$ and $m_2$: $\gamma_5$ are Dirac matrices.

The decay width $\Gamma$ in the $\mu$ rest frame, after having performed the proper spin sum and averages and having integrated over the momenta of the electron and of both neutrinos, is given by

$$\Gamma = \frac{m_\mu^5 G^2}{192 \pi^3} [Q_0 + \frac{\Sigma}{2m_\mu^2}(Q_{\Sigma a} + Q_{\Sigma b} \ln Z_m) + \frac{\Delta^2}{4m_\mu^4}(Q_{\Delta a} + Q_{\Delta b} \ln Z_m + \frac{Q_{\Delta c}}{Z_m})]$$  (4)

In Eq.(4), the $Q-$quantities are simple functions of $x = \frac{m_e^2}{m_\mu}$ (see Table 1), which is practically negligible. The dependence on the neutrino masses is contained in the expressions

$$\Sigma = m_1^2 + m_2^2 \quad \Delta = m_2^2 - m_1^2 \quad Z_m = \frac{(m_1 + m_2)^2}{m_\mu^2}$$  (5)
The relative modification of $\Gamma$ induced by the presence of massive neutrinos is

$$\frac{\Delta \Gamma}{\Gamma} = 1 \frac{1}{Q_0} \sum m^2 \mu \left[ (Q_{\Sigma 1} + Q_{\Sigma 2}) \ln Z_m + \frac{\Delta^2}{4m^4_\mu} (Q_{\Delta 1} + Q_{\Delta 2}) \ln Z_m + \frac{Q_{\Delta 3}}{Z_m} \right] (6)$$

$$\cong \left[ \sum m^2 \mu \left( -\frac{22}{3} - 4 \ln Z_m \right) + \frac{\Delta^2}{4m^4_\mu} (-4 - 24 \ln Z_m - \frac{16}{Z_m} \right] (7)$$

We can assume for $m_1$ its upper limit of 1 eV, in this way the theoretical relative uncertainty is a function of $m_2$ as shown in Fig. (1). Its intercept with the the experimental value of $10^{-6}$ indicates that $\nu$-mass (muon based) is lower than 0.021 MeV, an upper value an order of magnitude more stringent than the one reported in [1]. The curve calculated with $m_1 = 0$ is not distinguishable from the one reported in Fig. (??) and therefore the above limitation is valid for the entire range of $m_1$.

![Figure 1](image.png)

**Figure 1:**

Furthermore, if the limitation for the $\tau$ mass (electron based) is assumed to be valid also for the neutrino, one can perform an analogous calculation for the decay

$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$$

in this way one can surmise that the limitation of 0.021 MeV is valid also for the $\overline{\nu}_\mu$-mass (muon based).

The above considerations can in principle be applied also to the $\tau$ lepton decay

$$\tau^+ \rightarrow \mu^+ + \nu_\mu + \overline{\nu}_\tau$$

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Here the relevant phenomenological quantities are the mean life $\tau_{tot}$ and the branching ratio for the $\tau$ leptonic decay $B_l$, with values

$$\tau_{tot} = (2.903 \pm 0.005) \times 10^{-3} \, s \quad B_l = (17.39 \pm 0.04)\%$$

which is largely dominated by the error on $B_l$.

The calculations made in order to arrive at Eq. (6) can be repeated also for the lepton decay of the $\tau$ meson, provided that the following substitutions are performed

$$m_e \rightarrow m_\mu \quad m_\mu \rightarrow m_{\tau} \quad m_1 \rightarrow m_2 \quad m_2 \rightarrow m_3$$

The result is again given by Eq. (6), with slightly different values of the $Q$-quantities, because the ratio $(m_{\mu}/m_e)^2$ is no longer completely negligible.

$$\frac{\Delta \Gamma_l}{\Gamma_l} = \frac{\Delta B_l}{B_l} + \frac{\Delta \Gamma_{tot}}{\Gamma_{tot}} = 1.55 \times 10^{-2}$$

which is largely dominated by the error on $B_l$.

The relative error for the leptonic width $\Gamma_l = B_l \Gamma_{tot}$ is then

$$\frac{\Delta \Gamma_l}{\Gamma_l} = \frac{\Delta B_l}{B_l} + \frac{\Delta \Gamma_{tot}}{\Gamma_{tot}} = 1.55 \times 10^{-2}$$

The plot of $\Delta \Gamma_l/\Gamma_l$ as a function of $m_3$, for $m_2 = 0.021 \text{MeV}$, is shown in Fig. 2. Here again the curve with $m_2 = 0$ is not distinguishable from the one with non zero $m_2$ value. It is clear that the large uncertainty of the lepton decay width allows a much wider range of $\nu$ mass ($\tau$ based). In particular the upper
limit is one order of magnitude higher than the one quoted in \[1\]. In order to get a similar determination one should improve the measure of \(\frac{\Delta \tau}{\tau}\) of at least an order of magnitude.

To conclude, the agreement of the V-A theory with the experimental value of the \(\mu\) meson mean life is valid also in presence of massive neutrinos. The small error on \(\tau_\mu\) is able on the contrary to constrain the possible value of the \(\nu\)-mass (muon based), leading to an upper limit much lower than the one presently quoted \[1\]. Unfortunately, a similar limitation of the \(\nu\)-mass (tau based) cannot be obtained because of the large experimental error on the \(\tau\) width for the lepton decay; hopefully future more accurate experimental determinations of this quantity will modify the situation.

References

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