Alpha-decay chains of $^{288}_{173}$115 and $^{287}_{172}$115 in the Relativistic Mean Field theory

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Abstract

In the recent experiments designed to synthesize the element 115 in the $^{243}$Am+$^{48}$Ca reaction at Dubna in Russia, three similar decay chains consisting of five consecutive $\alpha$-decays, and another different decay chain of four consecutive $\alpha$-decays are detected, and the decay properties of these synthesized nuclei are claimed to be consistent with consecutive $\alpha$-decays originating from the parent isotopes of the new element 115, $^{288}$115 and $^{287}$115, respectively$^{[1]}$. Here in the present work, the recently developed deformed RMF+BCS method with a density-independent delta-function interaction in the pairing channel is applied to the analysis of these newly synthesized superheavy nuclei $^{288}$115, $^{287}$115, and their $\alpha$-decay daughter nuclei. The calculated $\alpha$-decay energies and half-lives agree well with the experimental values and with those of the macroscopic-microscopic FRDM+FY and YPE+WS models. In the mean field Lagrangian, the TMA parameter set is used. Particular emphasis is paid on the influence to both the ground-state properties and energy surfaces introduced by different treatments of pairing. Two different effective interactions in the particle-particle channel, i.e., the constant pairing and the density-independent delta-function interaction, together with the blocking effect are discussed in detail.

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I. INTRODUCTION

Since the prediction of the existence of superheavy islands in 1960s [2, 3], the synthesis of superheavy elements has been a hot topic in nuclear physics. Following numerous ground breaking technical developments [4] in the last three decades, the process of synthesizing superheavy elements has been sped up dramatically. From 1995 to 1996, Hofmann et al. [4, 5, 6, 7] at GSI in Germany successfully produced the elements Z=110, 111, and 112 by using low-energy heavy-ion collisions. In January 1999, the new element Z=114 was reported at Dubna in Russia [8, 9]. Two years later, the element Z=116 was also reported at Dubna [10]. In August 2003, in the reaction $^{243}$Am+$^{48}$Ca held at Dubna [1], with a beam dose of $4.3 \times 10^{18}$ 248-MeV and 253-MeV $^{48}$Ca projectiles, nine new odd-Z nuclei originating from the isotopes of the new element 115, $^{288}$115 and $^{287}$115, were produced, respectively. So far, all elements with $110 \leq Z \leq 116$ have been produced successfully in laboratory. All these exciting discoveries have greatly extended our knowledge about superheavy nuclei around the predicted superheavy islands and drawn more and more attention from the theoretical side.

The experimental progress has led to a large-scale investigation of superheavy nuclei by both refined macroscopic-microscopic (MM) models such as the finite-range droplet model with folded-Yukawa single-particle potentials (FRDM+FY) [11] or the Yukawa-plus-exponential model with Woods-Saxon single-particle potentials (YPE+WS) [12], and microscopic models such as the Skyme-Hartree-Fock-Bogoliubov method [13] and the latest relativistic mean field model [14, 15, 16, 17, 18]. Exploration for the next so-called "superheavy element island", i.e, the next spherical doubly magic nucleus, is a dream for physicists for the past several decades. There are already many works in this respect (see Ref. [13, 14, 15] and references therein). Possible candidates predicted by different theories are $^{298}_{184}$114, $^{292}_{172}$120 or even $^{310}_{184}$126. However, due to the limit of proper projectiles, the superheavy elements synthesized are always neutron deficient and lie in the deformed region. The deformation effects are very important to understand the nuclear structures in superheavy nuclei [13, 16, 18].

It is known experimentally that the heavy nuclei of the actinum series (Z=93-103) are well deformed and Bohr and Mottelson [19] also pointed out that the deformation can increase the stability of the heavy nuclei. The microscopic and self-consistent relativistic mean field model due to its natural description of spin-orbit interaction [20, 21, 22], which is a purely
relativistic effect, has been proved to be a reliable method to describe exotic and superheavy nuclei [14, 15, 16, 17, 18].

In the present work, the recently developed deformed RMF+BCS method with a density-independent delta-function interaction in the pairing channel [23] is adopted to analyze properties of lately synthesized superheavy nuclei $^{288}_{115}$, $^{287}_{115}$ and their $\alpha$-decay daughter nuclei. The density-independent (or dependent) delta-function interaction has been proved to be very successful to take into account the continuum effect both in relativistic and non-relativistic self-consistent mean field models [23, 24, 25, 26, 27, 28]. In the mean field part, the TMA parameter set [29] is used, which has been proved to be very successful in describing superheavy nuclei [16, 17, 18] in the relativistic mean field model. Particular emphasis is put on the effective interactions used in the particle-particle channel and blocking effects. In what follows, we discuss in detail numerical details and results of our calculations.

II. NUMERICAL DETAILS AND RESULTS

The RMF calculations have been carried out using the model Lagrangian density with nonlinear terms both for the $\sigma$ and $\omega$ mesons as described in detail in Ref. [23, 29], which is given by

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - g_\sigma \bar{\psi}\sigma\psi$$

$$- \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} g_4 (\omega_\mu \omega^\mu)^2 - g_\omega \bar{\psi} \gamma^\mu \psi \omega_\mu$$

$$- \frac{1}{4} R_{\mu\nu} R^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^2 - g_\rho \bar{\psi} \gamma_\mu \tau^a \psi \rho^a$$

$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi} \gamma_\mu \frac{1}{2} - \frac{2}{3} A_\mu \psi,$$

where all symbols have their usual meanings. The corresponding Dirac equation for nucleons and Klein-Gordon equations for mesons obtained with the mean field approximation are solved by the expansion method on the widely-used axially-deformed Harmonic-Oscillator basis [23, 30]. The number of basis used for expansion is chosen as $N_f = N_b = 20$. More basis have been tested for convergence considerations. We use the parameter set, TMA [29], for the RMF Lagrangian.

Three kinds of approaches to take into account the pairing correlations have been adopted in the present work. The first is the usual RMF+BCS calculation with constant pairing interaction. The inputs of pairing gaps are $\Delta_n = \Delta_p = 11.2 / \sqrt{A}$ and the blocking effect is ignored. Such calculations have been performed extensively by Ren et al. [18]. The
second is the RMF+BCS calculation with a density-independent delta-function interaction, \( V = -V_0 \delta(\vec{r}_1 - \vec{r}_2) \) \[23\]. Here, the blocking effect is also ignored for comparison. The third is the same as the second one except that the blocking effect is taken into account by the usual blocking method \[23, 31, 32\]. The pairing force strengths \( V_0 \) are fixed by obtaining similar binding energy for \(^{288}_{115}\) as the first approach, i.e., \( V_0 = 280 \text{ MeV fm}^3 \) in the second and \( V_0 = 330 \text{ MeV fm}^3 \) in the third calculations respectively. The same \( V_0 \) has been used for both protons and neutrons. A slight change of the pairing strength, say ten percent, only changes the absolute binding energy less than 1.0 MeV and other results are hardly changed. Throughout the paper, the first, second, and third kind of calculations will be abbreviated by ”Const”, ”Delta1” and ”Delta2”.

A. \( \alpha \)-decay energies

In Table. \[ III \] a comparison for binding energies and \( \alpha \)-decay energies between our three calculations, Const, Delta1, Delta2, the results from the macroscopic-microscopic FRDM+FY model \[11\] and the experimental values for the \(^{288}_{115}\) chain and the \(^{287}_{115}\) chain is tabulated. More detailed properties obtained from the calculations Delta2 are shown in Table. \[ IV \]. The theoretical half-lives \( T_\alpha \) are calculated with the well-known Viola-Seaborg formula \[11, 33\]. The difference between the predicted \( Q_\alpha \) by Const, Delta1, Delta2, the FRDM+FY model \[11\], the YPE+WS model \[12\], and the experimental value, \( \Delta_\alpha(\text{theo.}) = Q_\alpha(\text{theo.}) - Q_\alpha(\text{expt.}) \), is plotted in Fig. 1 and Fig. 2 for the \(^{288}_{115}\) chain and the \(^{287}_{115}\) chain, respectively.

First, for the \(^{288}_{115}\) chain, we notice that the quality of agreement between our calculations ( particularly Delta2 ) and the experimental values is similar to those of MM models (FRDM+FY and YPE+WS). For the last two nuclei in the \(^{288}_{115}\) chain, \(^{272}_{107}\) and \(^{276}_{109}\), results of MM models are closer to experimental values. For \(^{280}_{111}\), our calculations are between the FRDM+FY model and the YPE+WS model. For \(^{284}_{113}\), predicted \( \alpha \)-decay energy by our calculations, similar to that of the YPE+WS model, is larger than the experimental value while the FRDM+FY model predicts a smaller value. The biggest difference, about 1.0 MeV, between theory and experiment is also found for this nucleus. For \(^{288}_{115}\), both our calculations and the FRDM+FY model predict similar values that are smaller than the experimental value, while the result from the YPE+WS model is larger than the
experimental value.

Second, for the $^{287}\text{115}$ chain, similar things happen. For $^{271}\text{107}$, Delta2 and the FRDM+FY model predict similar $Q_\alpha$, while the YPE+WS model predicts a larger value. Because no experimental value is observed for this nucleus, prediction of Delta2 is taken as the experimental value for comparison. For $^{275}\text{109}$, predictions of all our three calculations deviate from the experimental value more than those of the MM models. While for $^{279}\text{111}$ and $^{287}\text{115}$, our calculations are closer to experimental values than the MM models. For $^{283}\text{113}$, just like the case of $^{284}\text{114}$, the difference between theory and experiment is relatively large. Our calculations and the YPE+WS model predict different trend for this nucleus from the FRDM+FY model also.

Third, we note that all our three calculations predict similar $\alpha$-decay energies for both the $^{288}\text{115}$ chain and the $^{287}\text{115}$ chain. The calculations Const and Delta1 give essentially the same results for both decay chains. While the calculations Delta2 are generally better than the other two calculations. This is more obvious for the odd-odd $^{288}\text{115}$ chain than for the odd-even $^{287}\text{115}$ chain. Since the main difference between the second and the third calculations is the blocking effect, we could safely conclude that a proper blocking treatment can improve the calculated observables for odd-even or odd-odd nuclei. We also can say that the RMF+BCS calculations with constant pairing can describe even-even superheavy nuclei reasonably well. For odd-odd and odd-even nuclei, the inclusion of the blocking effect becomes necessary. We will see this point more clearly in the following discussions.

Last, we would like to point out the difference between the MM models and the relativistic mean field theory used here. As we have seen in the above discussions (see also Table. 1 and Fig. 1-2), predictions of the MM models are closer to the experimental values for the elements 109 and 107, while for the other three elements, our calculations are better than those of the MM models. The reason could be that the MM models depend more on the knowledge of known nuclei. While the RMF model, due to its natural description of spin-orbit interaction, after including deformation, pairing interaction and blocking effect properly, could be more powerful in predicting the properties of unknown nuclei.
B. Energy surfaces and ground-state deformations

Now let us discuss a bit more about the differences between our three different kinds of calculations. We have performed the constrained quadrupole calculations \[23, 34\] for both the $^{288}\text{115}$ chain and the $^{287}\text{115}$ chain in all the three calculations. The corresponding energy curves are shown in Fig. 3 and Fig. 4. We should mention that such calculations are very time-consuming. First thing we see is that Const and Delta1 give quite similar energy curves for both decay chains. In fact they also give almost the same results for all calculated quantities (see also Table. I and Fig. 1-2), except for the $\alpha$-decay energies where Delta1 is better. Another noticeable difference is that the height of the barrier between different minima can differ a little bit. In most cases, Delta1 give shallower barriers than Const.

Second we could see the difference between calculations with and without blocking, Delta2 and Delta1, is relatively large, despite that the ground-state properties are quite similar. This shows that proper treatment of blocking effect is definitely necessary for odd-odd or odd-even nuclei (see also Fig. [11,2]). For the $^{287}\text{115}$ chain, due to the way that we fixed the pairing strength $V_0$, the absolute binding energies from calculations with and without blocking differ around 1.0 MeV for some nuclei.

Unlike medium or light nuclei where generally only two minima (one oblate minimum and one prolate minimum) or one spherical minimum are observed, the energy curves of superheavy nuclei are more complicated as we can see in Fig. [11,3]. This is not surprising. As there are more levels in heavy nuclei, level crossing is more frequent to happen and lots of local minima may appear. Except for $^{288}\text{115}$ and $^{284}\text{113}$ in the $^{288}\text{115}$ chain, $^{287}\text{115}$ and $^{283}\text{113}$ in the $^{287}\text{115}$ chain, the ground state of other nuclei can be determined without ambiguity, i.e., around $\beta_2 \sim 0.2$. Similar results have been obtained by the macroscopic-microscopic YPE+WS model \[12\], more specifically, $\beta_2 = 0.200, 0.211$ and 0.224 for $^{280}\text{111}$, $^{276}\text{109}$ and $^{272}\text{107}$; $\beta_2 = 0.202, 0.215$ and 0.228 for $^{279}\text{111}$, $^{275}\text{109}$ and $^{271}\text{107}$. The YPE+WS model predicts $\beta_2 = 0.138$ and $\beta_2 = 0.149$ for $^{284}\text{113}$ and $^{283}\text{113}$, which are also close to our calculations $\beta_2 = 0.17$ and $\beta_2 = 0.18$. While for $^{288}\text{115}$ and $^{287}\text{115}$, the YPE+WS model predicts $\beta_2 = 0.072$ and $\beta_2 = 0.066$ respectively, which are quite different from our calculations, $\beta_2 \sim 0.5$ for both these two nuclei. This difference can be understood easily because these MM models predict $^{288}\text{184}$ to be the next spherical doubly magic nucleus, while most self-consistent models shift this property to the more proton-rich side \[14\]. Further
experiments are needed to clarify these discrepancies between different theoretical models and different parameter sets in the same model. In our calculations, two other configurations $\beta_2 \sim -0.2$ and $\beta_2 \sim 0.3$ are also possible for $^{288}115$ under certain conditions such as the case of $\alpha$-decay. That is to say, decay from these two configurations to $^{283}113$ are also possible based on the calculated $\alpha$-decay energy. For $^{283}113$, we find that the minima around $\beta_2 \sim 0.2$ and $\beta_2 \sim 0.5$ are close to each other.

Since we see that isotopes of the element 115 are very deformed in our calculations, we would like to have a closer look at this element and the element 117, the mother element of the element 115 in the $\alpha$-decay chain. The corresponding energy surfaces from all the three calculations are plotted in Fig. 5 for $^{292}117$, $^{288}115$, $^{291}117$ and $^{287}115$. It is clearly seen that the configuration around $\beta_2 \sim 0.5$ is still stable against fission even for the element 117, but the barrier is lowered greatly for the calculation Delta2 than the other two calculations. Such an influence to the fission barrier introduced by the blocking effect has been demonstrated by Rutz et al. [35] in the RMF model. Here, we notice that the adoption of the density-independent delta-function interaction instead of the constant pairing in the pairing channel further reduces the fission barrier. Further calculations by Delta2 show that $\alpha$-decay energies of $^{292}117$ and $^{291}117$ are, respectively, 10.71 MeV ($B=2076.62$ MeV) and 10.83 MeV ($B=2053.36$ MeV), with $T_{\alpha} = 2.23$ s and $T_{\alpha} = 0.49$ s. It would be very interesting to synthesize the nuclei $^{292, 291}117$ and measure the $\alpha$-decay chains, since our calculations predict that these nuclei would make $\alpha$ decays.

To summarize, the constrained calculations show that the energy curves are relatively complicated and the predicted ground-state deformations by our calculations are close to those by the YPE+WS model. Further comparisons show that the fission barriers are quite different for our three calculations, especially for calculations with and without blocking. This suggests that to study superheavy nuclei more reliably one needs to use a more realistic effective interaction in the pairing channel other than the constant pairing interaction and at the same time include the blocking effect properly.

III. CONCLUSION

With the recently developed deformed RMF+BCS method with a density-independent delta-function interaction in the pairing channel, properties of the lately synthesized su-
perheavy nuclei $^{288}_{115}$, $^{287}_{115}$ and their $\alpha$-decay daughter nuclei in Dubna \cite{1} have been studied. The TMA parameter set is used for the mean field Lagrangian. In the particle-particle channel, three different treatments are introduced, i.e., the usual constant pairing without blocking, the delta-function interaction with and without blocking. Constrained quadrupole calculations have been performed also for all these three calculations. Relatively complicated energy curves are observed for these superheavy nuclei, especially those nuclei with proton number larger than 111. The calculated $\alpha$-decay energies, $Q_\alpha$, are found to agree well with the experimentally observed values and also are close to those of macroscopic-microscopic FRDM+FY model and YPE+WS model. Predicted ground-state deformations agree well with those of macroscopic-microscopic YPE+WS model. For odd-odd and odd-even superheavy nuclei, which we have studied here, the inclusion of the blocking effect can improve the overall performance and is thought to be necessary based on our calculations. For purposes other than studying the ground-state properties, a more realistic interaction in the pairing channel, such as the density-independent delta-function interaction used in the present work, would be necessary.

IV. ACKNOWLEDGMENTS

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TABLE I: The binding energies, $B$, and $\alpha$-decay energies, $Q_\alpha$, of decay chains of $^{288}_{\text{115}}$ and $^{287}_{\text{115}}$. Listed are the RMF+BCS calculations with constant pairing, Const, with the delta-function interaction without blocking, Delta1, and with Blocking, Delta2. FRDM+FY are results from the finite-range droplet model with folded Yukawa single-particle potentials \cite{11}. The last column is the experimental $Q_\alpha$ from Dubna \cite{1}. All energies are in units of MeV.

| Nuclei | Const | Delta1 | Delta2 | FRDM+FY | Experiment |
|--------|-------|--------|--------|----------|------------|
|        | $B$   | $Q_\alpha$ | $B$   | $Q_\alpha$ | $B$   | $Q_\alpha$ | $Q_\alpha$ |
| $^{288}_{\text{115}}$ | 2059.10 | 9.78 | 2058.80 | 9.91 | 2059.03 | 10.30 | 2059.12 | 10.12 | 10.61 ± 0.06 |
| $^{284}_{\text{113}}$ | 2040.58 | 11.22 | 2040.41 | 11.04 | 2041.03 | 10.74 | 2040.95 | 9.15 | 10.15 ± 0.06 |
| $^{280}_{\text{111}}$ | 2023.50 | 10.50 | 2023.15 | 10.45 | 2023.47 | 10.49 | 2021.81 | 10.13 | 9.87 ± 0.06 |
| $^{276}_{\text{109}}$ | 2005.70 | 9.75 | 2005.30 | 9.73 | 2005.66 | 9.42 | 2003.64 | 9.93 | 9.85 ± 0.06 |
| $^{272}_{\text{107}}$ | 1987.15 | 8.16 | 1986.73 | 8.27 | 1986.78 | 8.60 | 1985.27 | 8.88 | 9.15 ± 0.06 |
| $^{268}_{\text{105}}$ | 1967.01 | 8.16 | 1966.70 | 8.27 | 1967.08 | 8.60 | 1965.86 | 9.15 ± 0.06 |
| $^{287}_{\text{115}}$ | 2051.88 | 10.96 | 2051.72 | 10.82 | 2053.36 | 10.90 | 2052.72 | 10.25 | 10.74 ± 0.09 |
| $^{283}_{\text{113}}$ | 2034.54 | 11.31 | 2034.24 | 11.19 | 2035.96 | 10.98 | 2034.68 | 9.35 | 10.26 ± 0.09 |
| $^{279}_{\text{111}}$ | 2017.55 | 10.55 | 2017.13 | 10.55 | 2018.64 | 10.33 | 2015.73 | 10.92 | 10.52 ± 0.16 |
| $^{275}_{\text{109}}$ | 1999.80 | 9.67 | 1999.38 | 9.67 | 2000.67 | 9.52 | 1998.36 | 10.06 | 10.48 ± 0.09 |
| $^{271}_{\text{107}}$ | 1981.17 | 8.18 | 1980.75 | 8.29 | 1981.89 | 8.65 | 1980.13 | 8.66 | 10.48 ± 0.09 |
| $^{267}_{\text{105}}$ | 1961.05 | 8.18 | 1960.74 | 8.29 | 1962.24 | 8.65 | 1960.49 | 9.15 ± 0.06 |
TABLE II: The binding energies, $B$, neutron and proton quadrupole deformations, $\beta_{2n}$ and $\beta_{2p}$, neutron and proton rms radii, $R_n$ and $R_p$, the calculated alpha-decay energies and life-lives, $Q_\alpha$ and $T_\alpha$, of superheavy nuclei on the alpha-decay chains of $^{288}\text{115}$ and $^{287}\text{115}$ from the calculations Delta2. The last two columns are experimental decay energies and lifetimes. All energies are in units of MeV and all radii in units of Fermi.

| Nuclei  | $B$    | $\beta_{2n}$ | $\beta_{2p}$ | $R_n$ | $R_p$ | $Q_\alpha$ | $T_\alpha$ | $Q_\alpha$(expt) | $T_\alpha$(expt) |
|---------|--------|--------------|--------------|-------|-------|------------|------------|----------------|----------------|
| $^{288}\text{115}$ | 2059.03 | 0.48         | 0.50         | 6.58  | 6.41  | 10.30      | 6.86 s      | 10.61 ± 0.06   | $87^{+105}_{-30}$ ms |
| $^{284}\text{113}$ | 2041.03 | 0.17         | 0.17         | 6.37  | 6.18  | 10.74      | 111.96 ms   | 10.15 ± 0.06   | 0.48$^{+0.58}_{-0.17}$ s |
| $^{280}\text{111}$ | 2023.47 | 0.18         | 0.19         | 6.34  | 6.15  | 10.49      | 118.97 ms   | 9.87 ± 0.06    | 3.6$^{+4.3}_{-1.3}$ s |
| $^{276}\text{109}$ | 2005.66 | 0.20         | 0.20         | 6.32  | 6.12  | 9.42       | 25.08 s     | 9.85 ± 0.06    | 0.72$^{+0.87}_{-0.25}$ s |
| $^{272}\text{107}$ | 1986.78 | 0.20         | 0.21         | 6.30  | 6.09  | 8.60       | 1953.31 s    | 9.15 ± 0.06    | 9.8$^{+11.7}_{-3.5}$ s |
| $^{268}\text{105}$ | 1967.08 | 0.21         | 0.22         | 6.28  | 6.06  |            |            | $16^{+19}_{-6}$ h |
| $^{287}\text{115}$ | 2053.36 | 0.48         | 0.50         | 6.56  | 6.41  | 10.90      | 80.57 ms     | 10.74 ± 0.09   | $32^{+155}_{-14}$ ms |
| $^{283}\text{113}$ | 2035.96 | 0.18         | 0.18         | 6.36  | 6.18  | 10.98      | 12.76 ms     | 10.26 ± 0.09   | 100$^{+490}_{-45}$ ms |
| $^{279}\text{111}$ | 2018.64 | 0.20         | 0.20         | 6.34  | 6.15  | 10.33      | 142.07 ms    | 10.52 ± 0.16   | 170$^{+810}_{-80}$ ms |
| $^{275}\text{109}$ | 2000.67 | 0.21         | 0.21         | 6.32  | 6.12  | 9.52       | 5.77 s       | 10.48 ± 0.09   | 9.7$^{+4.6}_{-4.4}$ s |
| $^{271}\text{107}$ | 1981.89 | 0.21         | 0.21         | 6.29  | 6.09  | 8.65       | 604.91 s     |                  |                |
| $^{267}\text{105}$ | 1962.24 | 0.22         | 0.22         | 6.27  | 6.06  |            |            | $73^{+350}_{-33}$ min |                |
FIG. 1: The difference between calculated $Q_\alpha$(theo.) and experimental $Q_\alpha$(expt.), $\Delta_\alpha$(theo.) = $Q_\alpha$(theo.) - $Q_\alpha$(expt.), for the $^{288}\text{115}$ chain is plotted against mass number A.
FIG. 2: The difference between calculated $Q_\alpha$(theo.) and experimental $Q_\alpha$(expt.), $\Delta_\alpha$(theo.) = $Q_\alpha$(theo.) − $Q_\alpha$(expt.), for the $^{287}115$ chain is plotted against mass number A.
FIG. 3: The energy surfaces for $^{288}115$, $^{284}113$, $^{280}111$, $^{276}109$, $^{272}107$ and $^{268}105$ are plotted as a function of mass quadrupole deformation, $\beta_2$, for three calculations: Delta2 (solid line), Delta1 (dashed line) and Const (dotted line).
FIG. 4: The energy surfaces for $^{287}\text{115}$, $^{283}\text{113}$, $^{279}\text{111}$, $^{275}\text{109}$, $^{271}\text{107}$ and $^{267}\text{105}$ are plotted as a function of mass quadrupole deformation, $\beta_2$, for three calculations: Delta2 (solid line), Delta1 (dashed line) and Const (dotted line).
FIG. 5: The energy surfaces for $^{292\,117}$, $^{288\,115}$, $^{291\,117}$, and $^{287\,115}$ are plotted as a function of mass quadrupole deformation, $\beta_2$, for three calculations: Delta2 (solid line), Delta1 (dashed line) and Const (dotted line).