Grand unification and exotic fermions
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Grand Unification and Exotic Fermions

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We exploit the recently developed software package LieART to show that SU(N) grand unified theories with chiral fermions in mixed tensor irreducible representations can lead to standard model chiral fermions without additional light exotic chiral fermions, i.e., only standard model fermions are light in these models. Results are tabulated which may be of use to model builders in the future.

An SU(6) toy model is given and model searches are discussed.

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I. INTRODUCTION

In the past, building grand unified theories (GUTs) with SU(N) gauge groups has nearly always been carried out using fermions in totally antisymmetric tensor irreducible representations (irreps). Choosing a chiral anomaly free set of these SU(N) irreps guarantees all fermions will continue to be anomaly free and in totally antisymmetric irreps when decomposed into regular SU(N') subgroups with N' < N. We will typically choose N' = 5. Hence, under the decomposition

SU(N) → SU(5)

we have

asym anomaly free SU(N) irreps
→ n(\(\overline{5} + 10\)) + \(\bar{n}(5 + \overline{10})\) + singlets

so that \(n_F = n - \bar{n}\) gives the number of families. There are only a few cases of studies of SU(N) models where other than totally antisymmetric irreps have been used. For example, single complex anomaly free irreps of SU(N) that contain chiral fermions have been searched for [1], and models with fermions in \(6s\) and \(8s\) of SU(3) color have been studied [2]. Here we ask if there are SU(N) models that start with fermions in complex mixed tensor irreps that lead to models with only standard model (SM) chiral fermions being light. The simplest way to explore such SU(N) models is to require that the only chiral fermions at the SU(5) level are in standard \((\overline{5} + 10)\)s families which then lead to SU(3) × SU(2) × U(1) standard model families

\(\overline{5} + 10 \rightarrow (3, 2)_{\frac{1}{6}} + (\overline{3}, 1)_{\frac{1}{3}} + (\overline{3}, 1)_{-\frac{1}{3}} + (1, 2)_{-\frac{1}{6}} + (1, 1)_{1}\)

(2)

However, GUT models [3] and partial gauge unifica-

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1 LieART is hosted by Hepforge, IPPP Durham. The LieART project home page is http://lieart.hepforge.org and the LieART Mathematica application can be freely downloaded as a tar.gz archive from http://www.hepforge.org/downloads/lieart
and keep track of the chirality in going from SU(N) to SU(5). Our results are displayed in the tables in the next section and other possible searches are discussed. An SU(6) toy model is given in section IV before we conclude in section V. Checking any of these results by hand will clearly demonstrate the power and flexibility of LieART.

III. RESULTS

Let us begin with the simplest example we have found—an SU(6) model with only two-column tableaux as displayed in Table I. The non-conjugated, complex, two-column tableaux irreps of SU(6) decompose to SU(5) irreps as

\[
\begin{align*}
21 & \rightarrow 1 + 5 + 15 \\
70 & \rightarrow 5 + 10 + 15 + 40 \\
84 & \rightarrow 5 + 10 + 24 + 45 \\
105 & \rightarrow 10 + 10 + 40 + 45 \\
105' & \rightarrow 15 + 40 + 50 \\
210 & \rightarrow 40 + 45 + 50 + 75
\end{align*}
\]

and the complex conjugated irreps decompose analogously. One then just has to find linear combinations of SU(6) irreps with three families that are free from exotics at the SU(5) level, which for SU(6) delivers the single example

\[
6(\overline{21}) + 9(\overline{70}) + 6(\overline{84}) + 9(\overline{105}) + 3(\overline{105'}) + 3(\overline{210})
\]

which when decomposed into SU(5) irreps reduces to

\[
3(10 + \overline{5}) + 9(5 + \overline{5}) + 15(10 + \overline{10}) \\
+ 9(15 + \overline{15}) + 12(40 + \overline{40}) \\
+ 9(45 + \overline{45}) + 3(50 + \overline{50}) \\
+ 6(1) + 6(24) + 3(75)
\]

where all irreps not belonging to the three families come in conjugated pairs, thus being vector-like.

More generally we implemented an efficient determination of exotic-free combinations of mixed tensor irreps of SU(N) utilizing LieART. The requirement of three families and no chiral exotics at the SU(5) level leads to a system of linear equations which reduces the number of independent parameters being initially one per irrep type. To this end we introduce special multiplicities \( m_i \) coding the imbalance of complex-conjugated and non-conjugated irrep pairs, i.e., a positive multiplicity denotes an excess of non-conjugated irreps and a negative multiplicity an excess of conjugated irreps. For the SU(6) model with only two-column tableaux the ansatz for the determination of an exotic-free, three SM family model reads

\[
m_1 \cdot 21 + m_2 \cdot 70 + m_3 \cdot 84 + m_4 \cdot 105 + m_5 \cdot 105' + m_6 \cdot 210 \\
\rightarrow -3(5) + 3(10) + 0(15) + 0(40) + 0(45) + 0(50).
\]

Note that real irreps such as 1, 35, 189, 175 of SU(6) and 1, 24 and 75 of SU(5) do not contribute chiral fermions and are disregarded here. Decomposing the SU(N) two-column tableaux irreps to SU(5) using (3) we obtain an inhomogeneous system of linear equations for the multiplicities \( m_i \):

\[
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & \cdots & -3 \\
0 & 1 & 1 & 0 & 0 & 0 & \cdots & 3 \\
1 & 1 & 0 & -1 & 0 & 0 & \cdots & 0 \\
0 & -1 & 0 & 1 & 1 & 1 & \cdots & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & \cdots & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & \cdots & 0
\end{bmatrix}
\]

Since the coefficient matrix is quadratic and of full rank the system has the unique solution given by \( m_1 \rightarrow -6, m_2 \rightarrow 9, m_3 \rightarrow -6, m_4 \rightarrow -9, m_5 \rightarrow 3, m_6 \rightarrow -3 \) which translates to (4).

In SU(7) we have 9 complex, non-conjugated, two-column tableau irreps: 28, 112, 140, 196, 210, 224, 490, 490' and 588. The system of equations for the corresponding multiplicities \( m_i \), with \( i = 1, \ldots, 9 \), is underdetermined leading to solution sets with three independent coefficients, \( c_1, c_2 \) and \( c_3 \):

\[
\begin{align*}
m_1 & \rightarrow c_1, m_2 \rightarrow c_2, m_3 \rightarrow c_1 + 2c_3, \\
m_4 & \rightarrow 3c_1 + 2c_2 + 2c_3 + 6, \\
m_5 & \rightarrow -20c_1 - 8c_2 - 19c_3 - 51, \\
m_6 & \rightarrow -16c_1 - 7c_2 - 16c_3 - 36, \\
m_7 & \rightarrow 20c_1 + 8c_2 + 20c_3 + 51, \\
m_8 & \rightarrow -28c_1 - 12c_2 - 27c_3 - 69, \\
m_9 & \rightarrow 13c_1 + 6c_2 + 12c_3 + 30.
\end{align*}
\]
With these self imposed limitations, we find 9 solutions (7) displayed in Table III. A limited range of integers for the independent coefficients as displayed in Table III.

\[
\begin{align*}
28 & \quad 112 & \quad 140 & \quad 196 & \quad 210 & \quad 224 & \quad 490 & \quad 490' & \quad 588 \\
-2 & \quad 1 & \quad -4 & \quad 0 & \quad 0 & \quad 5 & \quad -1 & \quad 2 & \quad -2 \\
-1 & \quad 1 & \quad -5 & \quad 1 & \quad -1 & \quad 5 & \quad -1 & \quad 1 & \quad -1 \\
0 & \quad 1 & \quad -6 & \quad 2 & \quad -2 & \quad 5 & \quad -1 & \quad 0 & \quad 0 \\
-7 & \quad 4 & \quad -1 & \quad 0 & \quad 0 & \quad 3 & \quad -2 & \quad -1 \\
-6 & \quad 4 & \quad -2 & \quad 0 & \quad -1 & \quad 0 & \quad 3 & \quad -3 & \quad 0 \\
-3 & \quad -1 & \quad -1 & \quad -3 & \quad -2 & \quad 3 & \quad 0 & \quad 0 & \quad -3 \\
2 & \quad -1 & \quad -2 & \quad -3 & \quad 3 & \quad 3 & \quad -3 & \quad 1 & \quad -2 \\
-1 & \quad -1 & \quad -3 & \quad -1 & \quad -4 & \quad 3 & \quad 3 & \quad -0 & \quad 2 \\
0 & \quad -1 & \quad -4 & \quad 0 & \quad -5 & \quad 3 & \quad 3 & \quad -3 & \quad 0
\end{align*}
\]

Table III. Three family solutions for two-column tableau SU(8) irreps

Finally, for SU(9) we obtain solution sets with 10 independent coefficients \(c_j\) for the multiplicities of the 16 complex, non-conjugated, two-column tableau irreps 45, 240, 315, 540, 630, 720, 1008, 1050, 1890, 2520, 2700, 3402, 3780, 5292, 6048 and 7560:

\[
m_1 \to c_1, m_2 \to c_2, m_3 \to c_3, m_4 \to c_4, m_5 \to c_5, \\
m_6 \to 2c_1+3c_2+2c_3+6c_6, m_7 \to 2c_3+2c_5+3c_7, \\
m_8 \to 3c_1+3c_3+3c_5+3c_8, m_9 \to c_9, \\
m_{10} \to 31c_1+33c_2+8c_3+43c_5+44c_6+19c_7 \\
+10c_8+30c_9+57c_{10}+54, \\
m_{11} \to 29c_1+27c_2+3c_3-4c_4+45c_5+38c_6 \\
+21c_7+11c_8+36c_9+63c_{10}+56, \\
m_{12} \to -263c_1-270c_2-58c_3-20c_4+378c_5-372c_6 \\
-178c_7-86c_8-291c_9-518c_{10}-483, \\
m_{13} \to -185c_1-185c_2-41c_3+15c_4-263c_5-258c_6 \\
-119c_7-65c_8-200c_9-357c_{10}-329, \\
m_{14} \to -773c_1-790c_2-167c_3+60c_4-1107c_5+1092c_6 \\
-514c_7-256c_8-851c_9-1518c_{10}+1411, \\
m_{15} \to 485c_1+495c_2+103c_3-40c_4+698c_5+685c_6 \\
+327c_7+160c_8+540c_9+960c_{10}+892, \\
m_{16} \to 220c_1+224c_2+51c_3-15c_4+310c_5+310c_6 \\
+140c_7+75c_8+234c_9+420c_{10}+390
\]

We find 11 solutions for a maximum of 19 two-column tableau irreps and \(c_j = -1, \ldots, 1\), with \(j = 1, \ldots, 10\) displayed in Table IV.

We do not need to limit ourselves to two-column tableaux. For instance we can search for three column sets that are exotic free, anomaly free and have three families. Here we conclude with the two- and three-column SU(6) case, where we find solution sets with six independent coefficients \(c_j\) for the multiplicities of the 22 complex, non-conjugated, two- and three-column tableau irreps 21, 56, 70, 84, 105, 105', 120, 210, 210', 280, 336, 384, 420, 490, 560, 680, 840, 840', 896, 1050, 1176.
handled with LieART [10].

Table IV. Three family solutions for two-column tableau SU(9) irreps

| 45 | 240 | 315 | 540 | 630 | 720 | 1008 | 1050 | 1890 | 2520 | 2700 | 3402 | 3780 | 5292 | 6048 | 7560 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | -1 | 1 | -1 | 0 | -1 | 2 | 3 | -1 | -1 | 0 | 0 | 0 | 3 | 0 | -2 |
| -1 | 1 | -1 | 0 | 0 | -1 | -2 | -2 | 0 | 1 | -1 | 0 | 4 | 1 | -1 | -2 |
| 1 | -1 | 1 | -1 | 0 | 1 | -1 | -2 | -1 | 1 | -3 | 1 | -1 | 0 | -2 | 3 |
| 1 | -1 | 1 | -1 | -1 | 1 | -3 | 3 | 0 | -2 | -1 | 2 | -3 | 0 | 0 | 2 |
| 0 | 0 | 0 | 1 | 0 | -6 | -3 | 1 | 0 | 1 | 4 | 1 | -2 | -1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | -6 | -1 | 3 | -1 | 4 | 2 | 0 | 0 | 1 | -2 | 1 |
| 0 | 0 | -1 | 1 | 0 | -2 | -5 | -3 | 1 | 0 | 1 | 0 | 3 | -3 | 2 | -2 |
| 0 | 1 | -1 | 0 | -1 | 1 | -7 | -2 | 1 | 0 | -2 | 2 | 1 | -2 | -1 | 2 |
| 1 | 0 | -1 | 0 | 1 | -6 | -3 | 3 | 0 | 0 | 5 | 2 | -2 | 0 | 0 | -1 |
| 0 | -1 | 1 | -1 | -1 | -1 | 0 | 4 | 0 | -4 | 2 | 1 | -2 | 3 | 2 | -3 |
| 0 | 0 | -1 | 1 | 1 | -2 | -3 | -4 | 0 | 3 | -1 | -1 | 5 | -3 | 0 | -1 |

1176' and 1470:

| 21 | 19 | 17 | 13 | 10 | 6 | 2 |
|---|---|---|---|---|---|---|
| 1470 | + | 3 | 70 | + | 105 | + | 105' | + | 210 |

While this example may not be simple enough to be a useful physical model, it is still instructive to examine it further. For instance, if we break the symmetry along the path SU(6) → SU(5) × U(1)' then as long as the extra U(1)' is unbroken, some of the SU(5) conjugate pair exotics (as well as some (5 + 5') and (10 + 10) pairs) stay light, as long as their U(1)' charges are imbalanced. This remains true even if we break to SU(3) × SU(2) × U(1) gauge group all the exotics can finally acquire mass.

If we were to keep U(1)' unbroken until ~ 1 TeV, then we would predict very many light (TeV scale) exotic fermions. Since keeping the extra U(1)' does not directly lead to proton decay it is probably allowed to be unbroken down nearly to the electroweak scale. However, since this model leads to so many exotics, a low energy U(1)' would undoubtedly upset the renormalization group running and spoil unification. So we conjecture that the best we can do is bring the U(1)' scale down a few orders of magnitude from the GUT scale. This model is by no means compelling, but it is still interesting, as it is the first example of a type of model with exotic fermions that can exist well below the GUT scale. As we noted above, better would be a model with only a few light exotics and a low energy U(1)' where the exotics could even be within reach of the LHC.

Other one-family exotic models can be found directly with our algorithm by requiring the decomposition to only one set of 5 + 10 and all other fermions to be vector-like. In Table VI we list the one-family model equation systems and some solutions for two-column tableaux for SU(7), SU(8) and SU(9). We have three column examples but they are complicated and not very enlightening, so we have chosen not to display them.

V. CONCLUSIONS

We have explored SU(N) gauge theory examples that start with mixed tensor fermionic irreps that none the less have only three standard families of chiral fermions...
at the SU(5) level. These results have been obtained with LieART, which is a programmable group theory software package capable of handling such complicated tasks. If we relax the constraint of starting with 20 irreps and a limited scan range for the independent coefficients, then there is an arbitrarily large class of models that start with chiral exotic fermions (i.e., fermions in multicolour tableaux) at the SU(N) level, but where there are only standard chiral families at the SU(6) and SM level. While so far none of these models are particularly compelling, the results do demonstrate a new avenue for model building. It is conceivable that a model like one of these could describe the UV completion of the SM. Although at present we do not have an example, that such models could arise remains a logical possibility. We plan to search for such models.

**SU(N) Equation system**

| SU(7) | One-family model solutions |
|-------|----------------------------|
| 2 4 3 -2 -2 2 -1 0 1 -1 | \(4 \overline{27} + 3(112) + 210 + 224 + 490 + 490′ \) |
| 0 2 2 -1 -2 2 -1 0 1 1 | \(2(112) + 2(210) + 2(224) + 490 + 490′ + 2(588) \) |
| 1 2 0 -3 -1 0 -2 0 2 0 | \(2(211) + 2(110) + 2(210) + 2(224) + 490 + 588 \) |
| 0 -1 0 2 1 4 2 0 0 0 | \(3(112) + 3(196) + 196 + 224 + 490 + 588 \) |
| 0 0 1 0 1 2 1 3 0 0 | \(2(211) + 2(110) + 2(210) + 2(224) + 490 + 490′ \) |
| 0 0 0 1 0 0 2 1 1 0 | \(5(27) + 3(112) + 140 + 196 + 224 + 588 \) |

| SU(8) | \(36 + 216 + 336 + 378 + 504 + 2(1008) + 1716 + 1344 + 2(1512) + 2352′ \) |
|-------|------------------------------------------|
| 3 9 6 8 9 8 0 9 -3 6 0 0 -1 | \(36 + 2(168) + 336 + 378 + 2(504) + 1176 + 2(1512) + 2(2352′) \) |
| 0 3 3 6 6 6 0 -6 -2 5 0 0 1 | \(4(36) + 2(168) + 216 + 336 + 420 + 504 + 1008 + 1344 + 1512 \) |
| 1 3 0 6 3 0 -1 -8 -6 6 3 -3 3 | \(3(36) + 2(216) + 336 + 2(420) + 504 + 1008 + 1344 + 1512 \) |
| 0 -1 0 -3 -3 1 3 9 8 3 9 8 0 | \(3(78) + 2(276) + 4(504) + 1008 + 2(1176) + 1344 + 1512 + 1512 \) |
| 0 1 0 -1 1 3 3 3 6 6 6 0 | \(2(165) + 2(216) + 2(276) + 2(420) + 1008 + 1344 + 1512 \) |
| 0 0 0 0 0 0 0 3 5 1 3 5 0 | \(5(27) + 3(112) + 140 + 196 + 224 + 588 \) |

| SU(9) | \(45 + 210 + 315 + 720 + 1790 + 2520 + 3402 + 3780 + 6048 + 7560 \) |
|-------|-----------------------------------------------|
| 4 16 10 20 24 20 -12 11 -35 -20 20 -18 14 -3 -12 | \(45 + 210 + 315 + 720 + 1790 + 2520 + 3402 + 3780 + 6048 + 7560 \) |
| 0 4 4 6 12 12 -8 8 -18 -11 14 -12 11 -2 -8 5 3 | \(2(112) + 2(110) + 2(210) + 2(224) + 490 + 588 \) |
| 1 4 0 10 6 0 -4 -1 -20 -20 0 -15 -4 -9 -20 -6 0 | \(112 + 2(1008) + 1050 + 2(2200) + 3780 + 3780 \) |
| 0 0 1 0 -1 0 4 4 6 4 6 10 15 20 9 20 20 | \(3(36) + 2(216) + 2(420) + 1008 + 1344 + 1512 \) |
| 0 0 0 0 -1 0 0 0 0 4 9 1 6 4 10 16 9 0 | \(2(78) + (504) + 1008 + 2(1176) + 1344 + 1512 + 1512 \) |

**Table V. Three family solutions for two- and three-column tableau SU(6) irreps**

**Table VI. One family equation systems and solutions for two-column tableau irreps**

**NOTE ADDED IN PROOF**

The chirality and fermionic particle content of the SM coming from grand unified theories has been investigated from a somewhat different point of view in [12]. Where results overlap with our work they agree.

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