The Design and Simulation of a New Z-axis Resonant Micro-accelerometer Based on Electrostatic Stiffness

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Abstract. Resonant micro-accelerometers have good properties such as the large dynamic range, the high sensitivity, the strong anti-interference ability as well as the direct digital output. A new z-axis resonant micro-accelerometer based on electrostatic stiffness is researched. The new z-axis resonant micro-accelerometer consists of a torsional accelerometer and two plane resonators. The sensing movement of the accelerometer is decoupled with oscillation of the plane resonators by electrostatic stiffness, which will benefit to improve the performance of the new z-axis resonant micro-accelerometer. The new structure is designed. The sensitive theory of the acceleration is investigated and the equation of scale factor is deduced under ideal conditions. The simulation is implemented to verify the basic principle by the Ansys and Matlab. The structure simulation results prove that the effective frequency of the torsional accelerometer and the resonator are 0.66kHz and 13.3kHz separately. And the interference modes are isolated with the effective mode apparently. The system simulation results indicate that the scale factor is 37Hz/g and the system has excellent capabilities in locking and tracking natural frequency of resonators, which proves that the basic theory is feasible.

Introduction

Resonant micro-accelerometers which measure the external acceleration through the frequency variation of a resonator have good properties such as the large dynamic range, the high sensitivity, the strong anti-interference ability as well as the direct digital output. Therefore, a variety of resonant micro-accelerometers have been developed [1-3]. Due to the merits of the good linearity, the high accuracy and the process compatible with conventional silicon micromachining technology in the in-plane linear resonators, most of the resonant micro-accelerometers studied are currently used to measure the planar acceleration [3-4]. Only a few resonant accelerometers with out-of-plane resonator that can measure the acceleration in the vertical plane is investigated[5-8]. A resonant accelerometer with a single z-axis resonator is researched in the literature[5-6]. However, it is difficult to ensure stability of the resonant mode due to the sensing direction coincides with the oscillation direction. Furthermore, the out-of-plane resonators have a large nonlinearity and are susceptible to the influence of the electrostatic pull-in effect, which makes it difficult to achieve a better performance. An alternative proposal [7] makes use of a tilting proof mass and two torsional resonators. Nevertheless, the sensing movement is still coupled with the oscillation of torsional resonators. In addition, a resonant accelerometer based on the electromagnetically excitation [8] has a poor process compatibility and bulky volume.

This paper focuses on the design and analysis of a new out-of-plane resonant micro-accelerometer based on electrostatic stiffness. The basic principle on which this sensor works is similar to those of already existing resonant accelerometers [7]. But the main difference is that a linear plane resonator is adopted and the sensing movement is decoupled with the oscillation of a plane resonator.
### Structure Design

The structure of the new z-axis resonant micro-accelerometer is shown in Fig.1(a). The z-axis resonant micro-accelerometer, including the torsional proof mass, resonant proof mass, torsional beam, suspension beam, drive electrode, drive-sense electrode, electrostatic coupling comb, anchor1/2 and so on, consists of a torsional accelerometer and two plane resonators. Firstly, the resonant proof masses in the left and the right will be driven to vibrate in the natural frequency of resonators along the Y-axis. The resonant displacements can be detected by the drive-sense electrodes. By feeding back the resonant displacements to the drive electrodes, the closed-loop self-oscillation system is implemented and can track the natural frequency of the resonator. Secondly, the bias voltage is applied on the electrostatic coupling combs between the torsional proof mass and the resonant proof mass. The electrostatic force and electrostatic stiffness are generated. Finally, when the z-axis acceleration is input, due to the imbalance of the torsional proof mass in the left and the right, the torsional proof mass will be rotated through the torsional beam around the Y-axis. The overlapping area of the electrostatic coupling combs shown in the Fig.1(b) will be changed, which will cause the change of the electrostatic force and electrostatic stiffness. Therefore the natural frequency of the resonator is altered as a result of the change of the electrostatic stiffness. By measuring the frequency changes in the closed-loop self-oscillation system, the acceleration can be detected.

![Fig.1](image)

**Fig.1** The scheme of z-axis resonant micro-accelerometer (a) The structure of micro-accelerometer (b) Partial sectional view of electrostatic coupling comb

### Sensitive Theory of the Acceleration

The torsional proof mass, the torsional beam and the anchor1 constitute the basic torsional accelerometer. The equation of motion is

\[
J \frac{d^2 \theta(t)}{dt^2} + b \frac{d \theta(t)}{dt} + k_o \theta(t) = k_a a(t) .
\]  

(1)

Where \( J \) is the moment of inertia, \( b \) is the viscous damping coefficient, \( k_o \) is the torsional stiffness, \( k_a \) is torque coefficient, \( \theta \) is the angle displacement.

In the static state, the output angel displacement is

\[
\theta(t) = \frac{k_a}{k_o} a(t) .
\]  

(2)
The displacement of electrostatic coupling comb along the z-axis is

\[ z = BS\sin(\theta(t)) \approx B \frac{k}{k_0} a(t). \]  

(3)

Where B is the equivalent distance from the electrostatic coupling combs to Y-axis. The equation of motion of the resonator is

\[ m \frac{d^2y(t)}{dt^2} + c \frac{dy(t)}{dt} + ky(t) = F_{d}(t) + F_{e}(t) \]  

(4)

Where m is the resonant proof mass, c is the viscous damping coefficient, k is the stiffness coefficient of the suspension beam, \( F_d(t) \) is the drive force, \( F_e(t) \) is the coupling electrostatic force of electrostatic coupling comb.

The scheme of electrostatic coupling theory is shown in Fig.2. The electrostatic coupling capacitances in the upper and the down separately are

\[ C_1 = n \frac{\varepsilon L(h+z)}{d-y} \quad C_2 = n \frac{\varepsilon L(h+z)}{d+y} \]

The coupling electrostatic force is

\[ F_e(t) = \frac{1}{2} \frac{\partial C}{\partial y} V^2 = \frac{1}{2} n \varepsilon L(h+z) V^2 - \frac{1}{2} n \varepsilon L(h+z) V^2 \approx \frac{2 n \varepsilon L(h+z) V^2}{d^3} y = k_e (h+z) y. \]  

(5)

Where V is the bias voltage of the electrostatic coupling beam, \( k_e = 2 n \varepsilon L V^2 / d^3 \).

Substituting Eq.(5) into Eq.(4), the natural frequency is

\[ f = \frac{1}{2\pi} \sqrt{\left(k-k_e(h+z)\right) / m}. \]  

(6)

Expanding Eq.(6) with Taylor method

\[ f \approx f_0 + f_0' \Delta. \]  

(7)

Where \( f_0 = \sqrt{\left(k-k_e(h+z)\right) / m} \), \( f_0' = n \varepsilon L V^2 / (4\pi^2 f_0 d^3 m) \).

Substituting Eq.(3) into Eq.(7)

\[ f \approx f_0 + S a(t). \]  

(8)

Where S is the scale factor, and \( S = n \varepsilon L V^2 B k_0 / (4\pi^2 f_0 d^3 m k_0) \). It is evident that the scale factor can be increased by reducing the \( f_0, d, m \) and \( k_0 \) or adding the \( n, L, V, B \) and \( k_a \).
Simulation

In order to verify the basic principle, the simulation is implemented by the Ansys and Matlab. The resonators are coupled with the torsional accelerometer by electrostatic stiffness. It is difficult to simulate directly. Therefore, the resonators and the torsional accelerometer are simulated separately. The mode simulation of the resonator is shown in Fig.3. The first mode is the effective resonant mode along the Y-axis and the frequency is 13.3kHz. Theoretically the frequency of the first mode should be selected as small as possible. However, the electrostatic negative stiffness will lead to the decrease of the effective resonant frequency. The suspension beams should be maintained at a certain stiffness. The other three are the interference mode. The interference modes shown in the Table.1 are isolated with effective mode apparently. The resonator is driven to vibrate in the first mode. And the closed-loop self-oscillation system is locked in the first mode. The mode simulation of the torsional accelerometer is shown in Fig.4. The first mode is the effective torsional resonant mode around the Y-axis and the frequency is 0.66kHz. Similarly, the frequency of the first mode should be selected as small as possible in order to increase the sensitivity. However, the lower resonant frequency will also result in stiffness decrease along the z-axis direction, which will cause the asymmetry of displacement in the left and the right. The other three are the interference mode. The interference modes shown in the Table.1 are isolated with effective mode.

Table 1. The first six modes of the resonator and the torsional accelerometer

| Mode# | 1    | 2    | 3    | 4    | 5    | 6    |
|-------|------|------|------|------|------|------|
| Frequency of the resonator[kHz] | 13.3 | 19.4 | 20.7 | 33.2 | 51.6 | 67.0 |
| Frequency of the torsional accelerometer[kHz] | 0.66 | 4.16 | 4.20 | 11.3 | 11.6 | 14.9 |

According to the simulation parameters in Table.2, the system simulation model shown in Fig.5 is constructed to validate the system theory. The simulation model consists of two parts. The first part is used to achieve the conversion from the input of acceleration to the output of coupling electrostatic force, including the transfer function of accelerometer, the transform from the angle to the displacement, the generation of coupling electrostatic force. The second part is used to achieve the closed-loop self-oscillation, including the transfer function of resonator, the transform from the displacement to capacitance, pre-amplifier and Auto Gain Control.
Two differential system models are simulated simultaneously. The one displacement of acceleration $z$ is increased, while the other displacement of acceleration is reduced. The results of the scale factor simulation are shown in Fig.6. Obviously, the output frequency of resonator 1 is increased, while the output frequency of resonator 2 is decreased under the same input acceleration, which is shown in Fig.6(a). The output frequencies of the resonant micro-accelerometer can be obtained by subtracting two output frequencies of resonators, shown in Fig.6(b). The scale factor is $37\text{Hz/g}$, which is consistent with the theoretical calculation. The step response of resonator shown in Fig.7 is simulated in 1g input acceleration. The step signal of 1g acceleration is input in the five seconds. It is evident that the frequency response of resonator can be stable immediately under the step input, while the output of pre-amplifier need to go through the transition process, which indicates that the system has excellent capabilities in locking and tracking natural frequency of system.

### Table 2. The simulation parameters

| Parameter | Value       | Parameter | Value       |
|-----------|-------------|-----------|-------------|
| $m$ [kg]  | $3.3 \times 10^{-8}$ | $V$ [V]   | 12          |
| $c$ [N·s/m] | $2.8 \times 10^{-8}$ | $B$ [um]  | 1152        |
| $k$ [N/m]  | 232.5       | $k_o$ [N·m/rad] | $5.2 \times 10^{-5}$ |
| $n$        | 242         | $k_a$ [N·m]  | $9.2 \times 10^{-10}$ |
| $L$ [um]   | 35          | $J$ [kg·m$^2$]       | $3.0 \times 10^{-12}$ |
| $h$ [um]   | 25          | $b$ [N·m·s/rad]  | $2.3 \times 10^{-8}$ |
| $d$ [um]   | 2           | $S$ [Hz/g](Theory) | $2 \times 18.52$ |

![Fig.5 The system simulation model](image1)

![Fig.6 The scale factor simulation](image2)
Conclusion

In this work, a new z-axis resonant micro-accelerometer based on electrostatic stiffness is researched. The structure of the new z-axis resonant micro-accelerometer which consists of a torsional accelerometer and two plane resonators is designed. The sensing movement is decoupled with oscillation of the plane resonators by electrostatic stiffness, which will benefit to improve the performance of the new z-axis resonant micro-accelerometer. The sensitive theory of acceleration is investigated and the equation of scale factor is deduced under ideal conditions. In order to verify the basic principle, the simulation is implemented by the Ansys and Matlab. The structure simulation results prove that the effective frequency of the torsional accelerometer and the resonator are 0.66kHz and 13.3kHz separately. At the same time, the interference modes are isolated with effective mode apparently. The system simulation results indicate that the basic theory is feasible and the system has excellent capabilities in locking and tracking natural frequency of system. The scale factor which is consistent with the theoretical calculation is 37Hz/g. Future work includes the structure fabrication, the vacuum package and experiments.

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