Coherent Perfect Absorption: an electromagnetic perspective

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We present a simple closed expression for determining the condition for Coherent Perfect Absorption derived through electromagnetic wave treatment and interface boundary conditions. Apart from providing physical insight this expression can be used to estimate the values of various parameters required for observation of coherent perfect absorption in a given medium characterized by a complex dielectric constant. The results of the theoretical expression are found to be in good agreement with those obtained through numerical simulations.

I. INTRODUCTION

The phenomenon of coherent perfect absorption[1–11] via symmetric illumination (at same incident angle from opposite sides) of artificially fabricated media by identical electromagnetic waves has been of tremendous interest in recent years. As the name indicates, coherent perfect absorption (CPA) is the process in which the incident coherent electromagnetic radiations are completely absorbed in a medium and appear as some other form of internal energy such as thermal or electrical energy. Such a medium is called a coherent perfect absorber or an anti lasing device as this function of the medium is exactly opposite to that of a laser (which converts some form of incoherent energy such as thermal or electrical energy into coherent light).

Accordingly several schemes [1, 2, 4, 6, 7, 9, 10] have been proposed that involve designing of novel CPA media and investigation of CPA in them. Almost all of these however, tend to be computational in nature. While computational techniques are very effective tools for investigating CPA in complicated structures such as metal-dielectric or meta-materials, they are incapable of providing insight into the nature of physical processes involved in the occurrence of CPA phenomenon.

For example, the essential requirement of a slightly absorbing medium (i.e., complex permittivity with positive imaginary part) for occurrence of CPA is a well known fact [1–11] but the actual mechanism involved is not apparent. Further given a medium characterized by a dielectric constant (permittivity) $\varepsilon$ it is not possible to assess or estimate the dependence of CPA on various field and medium parameters such as the incident angle, wavelength, medium composition etc. It is thus imperative to develop a theoretical formalism for the phenomenon of CPA. Because, a theoretical expression describing dependence of the CPA on various field and system parameters; apart from providing physical insight, would also be useful in determining the field parameters such as the angle of incidence, wavelength given the permittivity of the medium or vice-versa. This article deals with such a theoretical formulation of CPA for plane electromagnetic waves.

The organization of the article is as follows: Section II deals with derivation of reflection and transmission coefficients at the interfaces of a thin slab for oblique incidences of plane electromagnetic waves using Maxwell’s equations and boundary conditions on interfaces. Using these expressions, in section III we develop the theory of CPA through derivation of an analytical expression that leads to the condition for the occurrence of CPA. Thereafter we outline briefly a few schemes for observing CPA in various metal-dielectric composite media.

In section IV we present a comparison between the results obtained from the analytical expression derived in previous section with those obtained through computational means for various system field parameters. Summary and conclusions are presented in section V.

II. GEOMETRY AND FORMULATION OF CPA

In a typical CPA configuration, two identical electromagnetic waves are incident from opposite direction on the left (L) and the right (R) interfaces of the CPA medium henceforth called slab. Both electromagnetic waves have the same angle of incidence $\theta$. As shown in the fig. 1, the two air-slab interfaces are in the $x-y$ plane at $z = -d$ and $z = d$ so that the slab thickness is $2d$. Owing to the symmetry of the CPA configuration and superposition principle we can consider the reflection and transmission characteristics of each electromagnetic wave separately and superpose them later to obtain the resultant transmission or reflectivity at any interface and deduce the CPA condition. The two TE ($s$) polarized waves are incident at oblique angle $\theta$ upon the interfaces between air and CPA medium [12] at $z = -d$ and $z = d$. The incident light is partly reflected back in air, a part is transmitted through the CPA medium (to exit at the other interface) and the remaining part is reflected back and forth into the CPA medium (assuming no other losses). The amplitude of incident, reflected and transmitted waves are denoted $A_i$, $A_r$ and $A_t$ respectively. This process occurs for both the incident waves. Therefore, for incident TE plane wave on left hand side (LHS) of the interface (at $z=-d$), the wave equations in...
section I, II and III shown in the Fig. [I] as follows:

\[ E_{1y} = (A_{in} e^{ik_{1z}} + A_{r} e^{-ik_{1z}}) e^{i(k_{2}x-\omega t)} \text{ for } z < -d \]  
\[ E_{2y} = (P_{L} e^{ik_{2z}} + Q_{L} e^{-ik_{2z}}) e^{i(k_{2}x-\omega t)} \text{ for } -d \leq z \leq d \]  
\[ E_{3y} = A_{r} e^{ik_{1z}} e^{i(k_{2}x-\omega t)} \text{ for } z > d \]  

Here \( k_{jz} = \sqrt{k_{0}^{2} \epsilon_{j} - k_{0}^{2} \sin^{2} \theta} \). Because

optical frequencies the CPA medium is non-magnetic so that \( \mu = 1 \) and the corresponding magnetic field
\( \vec{H} = (\hat{z} H_{x} + \hat{y} H_{y} + \hat{z} H_{z}) e^{i(k_{z}x-\omega t)} \) can be evaluated from
the Maxwell’s equation \(-\frac{1}{c^{2}} \frac{\partial^{2} \vec{H}}{\partial t^{2}} = \nabla \times \vec{E} \) as \( ik_{0} \vec{H} = \nabla \times \vec{E} \).
So, \( ik_{0} \vec{H} = -\hat{z} \frac{\partial E_{x}}{\partial z} + \frac{\partial E_{y}}{\partial x} \) where \( k_{0} = \frac{\omega}{c} = \frac{2\pi}{\lambda} \). Now
equating the coefficients of \( \hat{x} \)-axis we obtain \( ik_{0} H_{1x} = -\frac{\partial E_{1y}}{\partial z} \). So, from eq. (1), (2) and (3), one can have

\[ H_{1x} = S_{1}(-A_{in} e^{ik_{1z}} + A_{r} e^{-ik_{1z}}) e^{i(k_{2}x-\omega t)} \text{ for } z < -d \]  
\[ H_{2x} = S_{2}(-P_{L} e^{ik_{2z}} + Q_{L} e^{-ik_{2z}}) e^{i(k_{2}x-\omega t)} \text{ for } -d \leq z \leq d \]  
\[ H_{3x} = S_{1}(-A_{r} e^{ik_{1z}}) e^{i(k_{2}x-\omega t)} \text{ for } z > d \]  

where \( S_{j} = \frac{k_{jz}}{k_{0}} \) and \( j = 1, 2 \). Therefore, in section I, II
and III of Fig. [I] the wave equations can be written in

FIG. 2. Top figure depicts CPA as a function of wavelength
for Gold-Silica composite and the bottom figure shows that at
CPA frequency, the real and imaginary parts of the function
\( \Psi \) become zero. Thickness of medium is \( 2d = 10 \mu m \) and
angles of incidence, \( \theta = 45^{\circ} \).

FIG. 3. Top figure depicts CPA as a function of volume
fraction \( (f_{m}) \) for Gold-Silica composite and the bottom figure
shows that at CPA frequency, the real and imaginary parts of
the function \( \Psi \) become zero. Thickness of medium is \( 2d = 10 \mu m \) and
angles of incidence, \( \theta = 45^{\circ} \).

FIG. 4. Top figures (a) and (c), depict \( \log_{10}|SI| \) as a function
of half width of the slab \( (d \text{ in } nm) \) and angle of incidence \( \theta^{\circ} \),
respectively. In the bottom figures (b) and (d), \( \Re \Psi \) and \( \Im \Psi \)
are plotted as a function of half width of the slab \( (d \text{ in } nm) \) and
angle of incidence \( \theta^{\circ} \) respectively.
matrix form as
\[
\begin{bmatrix}
E_{1y} \\
H_{1x}
\end{bmatrix}
= \begin{bmatrix}
e^{ik_{1}z} & e^{-ik_{1}z} \\
-S_{1}e^{ik_{1}z} & S_{1}e^{-ik_{1}z}
\end{bmatrix}
\begin{bmatrix}
A_{in} \\
A_{r}
\end{bmatrix}
\] for \( z < -d \) \( (7) \)

and
\[
\begin{bmatrix}
E_{2y} \\
H_{2x}
\end{bmatrix}
= \begin{bmatrix}
e^{ik_{2}z} & e^{-ik_{2}z} \\
-S_{2}e^{ik_{2}z} & S_{2}e^{-ik_{2}z}
\end{bmatrix}
\begin{bmatrix}
P_{L} \\
Q_{L}
\end{bmatrix}
\] for \(-d \leq z \leq d \)

The above eq. \( (10) \) yields the solutions for \( P_{L} \) and \( Q_{L} \) in terms of \( A_{i} \) as follows:
\[
\begin{bmatrix}
P_{L} \\
Q_{L}
\end{bmatrix}
= \begin{bmatrix}
(1 + \frac{S_{1}}{S_{2}})e^{ik_{1z}} \frac{A_{i}}{2} \\
(1 - \frac{S_{1}}{S_{2}})e^{ik_{1z}} \frac{A_{i}}{2}
\end{bmatrix}
\] \( (11) \)

Again, the electromagnetic boundary condition at the interface \( z = -d \) yields:
\[
\begin{bmatrix}
e^{-ik_{1z}} & e^{ik_{1z}} \\
-S_{1}e^{-ik_{1z}} & S_{1}e^{ik_{1z}}
\end{bmatrix}
\begin{bmatrix}
A_{in} \\
A_{r}
\end{bmatrix}
= \begin{bmatrix}
e^{-ik_{2z}} & e^{ik_{2z}} \\
-S_{2}e^{-ik_{2z}} & S_{2}e^{ik_{2z}}
\end{bmatrix}
\begin{bmatrix}
P_{L} \\
Q_{L}
\end{bmatrix}
\] \( (12) \)

Substituting the value of \( P_{L} \) and \( Q_{L} \) from eq. \( (11) \) in the above eq. \( (12) \) we obtain
\[
\begin{bmatrix}
e^{ik_{1z}} \frac{A_{i}}{2} \\
\frac{A_{i}}{2}
\end{bmatrix}
= \begin{bmatrix}
e^{-ik_{1z}} \\
\frac{A_{i}}{2}e^{-ik_{2z}}
\end{bmatrix}
\]

Here, \( t_{L} = \frac{A_{i}}{A_{in}} \) and \( r_{L} = \frac{A_{i}}{A_{r}} \) respectively, are the transmission \( (t_{L}) \) and reflection \( (r_{L}) \) coefficient that can be obtained from the solution of the following matrix equation
\[
\begin{bmatrix}
t_{L} \\
r_{L}
\end{bmatrix}
= D^{-1} \begin{bmatrix}
e^{ik_{1z}} \\
\frac{A_{i}}{2}e^{-ik_{2z}}
\end{bmatrix}, \text{ where}
\]

\[
D = \begin{bmatrix}
\frac{S_{2}}{S_{1}}e^{-ik_{1z}} \\
\frac{S_{2}}{S_{1}}e^{ik_{1z}}
\end{bmatrix}
\begin{bmatrix}
(1 + \frac{S_{1}}{S_{2}})e^{-2ik_{2z}} + (1 - \frac{S_{1}}{S_{2}})e^{2ik_{2z}} \\
(1 + \frac{S_{1}}{S_{2}})e^{-2ik_{2z}} - (1 - \frac{S_{1}}{S_{2}})e^{2ik_{2z}}
\end{bmatrix}
\] \( (14) \)

Using \( D^{-1} \) and from eqs. \( (14) \) the solution for \( t_{L} \) and \( r_{L} \) are
\[
t_{L} = \frac{4S_{1}}{S_{2}}e^{-2ik_{1z}},
\]

\[
r_{L} = \frac{4S_{1}}{S_{2}}e^{-2ik_{1z}} - \frac{(1 + \frac{S_{1}}{S_{2}})e^{-2ik_{2z}} + (1 - \frac{S_{1}}{S_{2}})e^{2ik_{2z}}}{(1 + \frac{S_{1}}{S_{2}})e^{-2ik_{2z}} - (1 - \frac{S_{1}}{S_{2}})e^{2ik_{2z}}},
\]

respectively. Similarly one can obtain the solutions for an electromagnetic wave incident from RHS on interface at \( z = d \) and these are found to be similar as \( t_{R} = t_{L} \) and \( r_{R} = r_{L} \) shown in Eq. \( (15) \) and Eq. \( (16) \). Now we have all reflection \( (r_{L} \text{ and } r_{R}) \) and transmission \( (t_{L} \text{ and } t_{R}) \) coefficients for waves incident on both left and right side interfaces of the slab. In the next section we determine the condition for CPA.

III. CPA CONDITION

As shown in Fig.1 the field exiting the medium at either interface has contributions from both forward (wave on LHS) and the backward (wave on RHS). Thus for CPA to occur, the coefficient of the field at the interface \( z = -d \) (or \( z = d \)) given by \( r_{L} + t_{R} \) (or \( r_{R} + t_{L} \)) must vanish, that is:
\[
e^{-2ik_{1z}}(1 - \frac{S_{1}}{S_{2}})e^{-2ik_{2z}} + e^{2ik_{2z}} - e^{-2ik_{2z}} = 0
\]

Using the relations \( S_{1} = k_{1z}/k_{0}, S_{2} = k_{2z}/k_{0} \) and \( e^{2ik_{2z}} - e^{-2ik_{2z}} = 2i\sin(2k_{2z}d) \) in above Eq. \( (17) \) and

FIG. 5. (a) Log_{10}|ST| (or CPA) plotted as functions of wavelength \( \lambda \) and volume fraction \( (f_{m}) \) for Gold-Silica composite medium. (b) Real(\Psi) plotted as functions of wavelength \( \lambda \) and volume fraction \( (f_{m}) \). (c) Imag(\Psi) plotted as functions of wavelength \( \lambda \) and volume fraction \( (f_{m}) \). Half width of medium \( d = 5 \mu m \) and angle of incidences \( \theta = 45^\circ \).
upon simplification, we arrive at the final expression governing the occurrence of CPA in the medium II
\[
2k_{1z}k_{2z} + i(k_{2z}^2 - k_{1z}^2)\sin(2k_{2z}d) = 0. \tag{18}
\]

The above expression Eq.\,(18) is a function (through \(k_{1z}, k_{2z}\)), of the permittivities \(\epsilon_j, j=1, 2, 3\) of medium I, II and III (medium I and III are air in the present case), angle of incidence \(\theta\) and wavelength \(\lambda\) of the incident waves and slab thickness, \(2d\). Thus given the above parameters, it is possible to determine the permittivity of the medium II under CPA condition. Here it should also be noted that though all the given parameters are real quantities in the present case, the solution of above Eq.\,(18) for propagation constant \(k_{2z}\) (or \(\sqrt{\epsilon_2}\)) in the medium will be complex. In other words, the expression Eq.\,(18) demonstrates that an essential condition for CPA to occur is: the medium must definitely be dissipative. Thus if we write the expression Eq.\,(18) as a complex function \(\Psi = 2k_{1z}k_{2z} + i(k_{2z}^2 - k_{1z}^2)\sin(2k_{2z}d)\) it is observed that in order to fulfill the CPA condition stipulated in Eq.\,(18), both \((\Re)\) and imaginary \((\Im)\) part of \(\Psi\) have to be simultaneously zero for given \(d, \theta, \lambda\) and \(\epsilon_j\) \(j=1, 2, 3\).

### CPA medium:
For verification of theoretical expression, we now choose metal-dielectric composite medium (CM) which is a homogeneous mixture of Gold (\(Au\)) or Silver (\(Ag\)) with Silica (\(SiO_2\)). To calculate CM permittivity we use the Bruggeman effective medium theory \cite{6 9 15 17} \((BEMT)\), which is as follows
\[
\epsilon_{CM} = \frac{1}{2}\left\{3f_1 - 1)\epsilon_1 + (3f_2 - 1)\epsilon_2 \pm \sqrt{[(3f_1 - 1)\epsilon_1 + (3f_2 - 1)\epsilon_2]^2 + 8\epsilon_1\epsilon_2} \right\}. \tag{19}
\]
Here, \(\epsilon_1, (f_1)\) and \(\epsilon_2, (f_2 = 1-f_1)\) are permittivity (filling factor) of metal and dielectric respectively. Experimental parameters of Jhionson and Christy for \(Au\) or \(Ag\) \cite{14} are used in Eq.\,(19) to estimate permittivity of metal-dielectric composite (Gold composite, \(GC = Au + SiO_2\) or Silver composite, \(SC = Ag + SiO_2\)) which is useful for calculation of CPA medium (as given in refs. \cite{6 9 10}). Here, width of CPA medium is \(2d = 10\ \mu m\) and angles of incidence, \(\theta = 45^\circ\) are fixed for all CPA examples. The various scattering intensities (\(SI\)) given by square of scattering amplitudes (\(SA\)) are denoted by \(SA_{\text{LHS}} = r_L + t_R\) at \(z = -d\) and \(SA_{\text{RHS}} = r_R + t_L\) at \(z = d\) in Fig. 1. These are numerically calculated from Fresnel’s formulas of reflection and transmission coefficients. As is well known, at CPA frequency the \(SI\) reduces to zero. To determine it’s magnitude and frequency at which CPA occurs, we calculate \(\log_{10}|SI|\) \cite{11 11}. Now it is time to verify the results obtained from theoretical expression using (function \(\Psi\)) with those obtained through numerical simulations of CPA. Under the condition of CPA the real part \((\Re(\Psi))\) and imaginary \((\Im(\Psi))\) of the function \(\Psi\) given by Eq.\,(18) should simultaneously reduce to zero.

### IV. RESULTS AND DISCUSSION
We now verify the results of function \(\Psi\) (eq.\,(18)) with CPA results obtained through numerical computation and depicted in fig.\,(2) and\,(3).

In fig.\,(2a) \(\log_{10}|SI|\) is plotted as function of volume fractions (\(f_m\)) and incident wavelength (\(\lambda\)) is 591.34 nm; which gives CPA at (metal) volume fraction \(f_m = 0.005594\). Now in fig.\,(2b), \(\Re\Psi\) and \(\Im\Psi\) are plotted as a function of volume fraction and it is observed that at the wavelength where CPA occurs, both are simultaneously zero as they cross the x axis. In fig.\,(3a) at \(f_m = 0.005594\) on GC, \(\log_{10}|SI|\) is plotted as a function of wavelength (\(\lambda\)) and CPA is observed at \(\lambda = 591.34\) nm. In fig.\,(3b) \(\Re\Psi\) and \(\Im\Psi\) are plotted as a function of wavelength and it is found they simultaneously cross zero line exactly at 591.34 nm. We can see characteristics of \(\Re\Psi\) and \(\Im\Psi\) for GC while both volume fraction and wavelength simultaneously vary in the fig.\,(5)\,(5c) respectively where as fig.\,(5)\,(a) shows \(\log_{10}|SI|\) under the same condition.

It is observed that at every CPA point \(\Re\Psi\) and \(\Im\Psi\) goes to zero simultaneously. An obvious question that arises is that if at some point \(\Re\Psi\) and \(\Im\Psi\) value is zero simultaneously for any arbitrary element then can we claim CPA at that point?

So, we choose \(SC\) a different composite medium for this test. We tested three different wavelengths (537.74 nm, 538.00 nm and 621.54 nm) as shown in the fig.\,(5)\,(a),

\[\text{FIG. 6. Top row shows CPA for different frequencies as a function of volume fraction (\(f_m\)) for Silver-Silica composite and the bottom row shows that at CPA frequency, the real and imaginary parts of the function \(\Psi\) become zero. Here, half thickness of the medium is, \(d = 5\mu m\) and angle of incidences, \(\theta_i = 45^\circ\).} \]
FIG. 7. Top row shows CPA for different volume fraction as a function of wavelengths for Silver-Silica composite and the bottom row shows that at CPA frequency, real and imaginary part of the function $\Psi$ become zero.

Here, thickness of the medium $2d = 10 \mu m$ and angle of incidences $\theta_i = 45^\circ$.

V. CONCLUSION

In this article we have developed a theoretical formulation of CPA in order to gain insight into the complicated process of CPA and to explain the numerical results of CPA in composite mixtures of metals and dielectric. Excellent agreement between the results obtained from the theoretical expression and the numerically computed results of CPA for various composite medium is observed. From theoretical expression, it is clear that at CPA the real ($\Re\Psi$) and imaginary ($\Im\Psi$) part would always simultaneously reduce to zero. Therefore, on the other hand one can remark that CPA could be achieve when $\Re\Psi = 0$ and $\Im\Psi = 0$ simultaneously. The function $\Psi$ is very useful for both theoretical and experimental studies, because it gives all parameter details required for observation of CPA phenomenon in a medium.

[1] Y. D. Chong, L. Ge, H. Cao and A. D. Stone, Phys. Rev. Lett. 105, 053901 (2010).
[2] C. F. Gmachl, Nature 467, 37-39 (2010).
[3] W. Wan, Y. D. Chong, L. Ge, H. Noh, A. D. Stone and H. Cao, Science 331, 889-892 (2011).
[4] H. Noh, Y. Chong, A. D. Stone and H. Cao, Phys. Rev. Lett. 108, 186805 (2012).
[5] S. Longhi, Phy. Rev. A, 83, 055804 (2011).
[6] S. D. Gupta, O. J. F. Martin, S. Duttagupta and G. S. Agarwal,Opt.Exp. 20, 1330 (2012).
[7] J. Zhang, C. Guo, K. Liu, Z. Zhu, W. ye, X. Yuan and S. Qin, Opt. Expt. 22, 12524 (2014).
[8] Y. Fan, Z. Liu, F. Zhang, Q. Zhao, Z. Wei, Q. Fu, J. Li, C. Gu and H. Li, Sci. Rep., 5, 13936 (2015).
[9] S. Dey, Opt. Com. 356, 515-521 (2015).
[10] S. Dey and S. Singh, IIT-Delhi-WRAP- doi:10.10.13111 (2013).
[11] P. Ma and L. Gao, Opt. Exp. 25, 9, 9676 (2017).
[12] J. R. Tischler, M. S. Bradley and V. Bulovic, Opt. Lett. vol. 31, 2045-2047 (2006).
[13] H. Noh, Y. Chong, A. D. Stone, and H. Cao
[14] P. B. Johnson and R. W. Christy, Phys. Rev. B, Vol.6, pp. 4370-4379(1972).
[15] W. Cai and V. Shalaev, “Optical Metamaterials:Fundamentals and Applications”, Springer, New York(2010), pp. 25.
[16] C. F. Bohren and D. R. Huffman, “Absorption and Scattering of Light by Small Particles”, John Wiley & Sons, New York (1983) pp.77.
[17] G. L. Fischer, R. W. Boyd, R. J. Gehr, S. A. Jenekhe, J. A. Osaheni, J. E. Sipe, and L. A. Weller-Brophy, Phys. Rev. Lett. Vol.74, pp. 1871 (1995).
[18] P. Yeh, “Optical waves in layered media”, (John wiley and sons, New York (1988)) pp. 83.
[19] G. S. Agarwal, and S. Dutta Gupta, Phys. Rev. A, Vol.38, pp. 5678 (1988).
[20] S. Dutta Gupta, “Nonlinear optics of Stratified media”, Progress in Optics, E.Wolf, ed.(Elsevier Science, 1998), Vol. 38, pp.1-84.
[21] M. Born and E. Wolf, “Principle of Optics”, 7th ed., Cambridge University Press, New York(2005), pp.75.
[22] L. Novotny and B. Hecht, “Principle of Nano-Optics”, Cambridge University Press, New York(2006), pp.45.