A cosmological model with complex scalar field

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Abstract. In this paper, we wish to point out the possibility of using a complex scalar field to account for the inflationary stage and the current acceleration. By the analysis of the dynamical system and numerical work, we show that the amplitude of the complex scalar field plays the role of the inflaton whereas the phase is the quintessence field. The numerical solutions describe heteroclinic orbits, which interpolate between an unstable critical point and a late-time de Sitter attractor. Therefore, this model is more natural to explain the two stages of acceleration.

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1. Introduction

Astronomical observation on the cosmic microwave background (CMB) anisotropy \[1\], supernova type Ia (SNIa) \[2\] and SLOAN Digital Sky Survey (SDSS) \[3\] converge on that our Universe is spatially flat, with about 70% of the total density resulting from dark energy that has an equation of state \( w < -1/3 \) and drives the accelerating expansion of the Universe which began at a redshift of order one-half. The origin of the dark energy remains elusive from the point of view of general relativity and standard particle physics. To represent dark energy, several candidates have been suggested and confronted with observation: a cosmological constant, quintessence with a single field \[4\] or with \( N \) coupled field \[5\], a phantom field with a canonical \[6\] or Born-Infeld type Lagrangian \[7\], \( k \)-essence \[8\] and the generalized Chaplygin gas (GCG) \[9\]. Among these models, the most typical ones are cosmological constant and quintessence which has caught much attention ever since its invention.

The idea of inflation is legitimately regarded as a great advancement of modern cosmology: it solves the horizon, flatness and monopole problem, and it provides a mechanism for the generation of density perturbations needed to seed the formation of structures in the universe \[10\]. In standard inflationary models \[11\], the physics lies in the inflation potential. The underlying dynamics is simply that of a single scalar field rolling in its potential. This scenario is generically referred to as chaotic inflation in reference to its choice of initial conditions. This picture is widely favored because of its simplicity and has received by far the most attention to date. The properties of inflationary models are also tightly constraint by the recent result from the observation. The standard inflationary \( \Lambda \)CDM model provides a good fit to the observed cosmic microwave background (CMB) anisotropy. Peebles and Vilenkin proposed and quantitatively analyzed the intriguing idea \[12\] that a substantial fraction of the present cosmic energy density could reside in the vacuum potential energy of the scalar field responsible for inflation (quintessential inflation). After that, there were some models to be presented in succession.

In this paper, we wish to point out the possibility of using a complex scalar field to account for the inflationary stage and the current acceleration. We put the emphasis on the study of the dynamics. According to the phase space analysis and numerical calculation, we show that how the amplitude of complex field plays the role of the inflaton whereas the phase is the quintessence field. There are stable and unstable critical points for our model. In our case, one of the unstable critical point is corresponding to the inflationary stage at very first moments of the Universe and the stable critical point is corresponding to the second stage of accelerated expansion began at a redshift \( z \sim 1.5 \) and is still operative, which alleviates the fine tuning problem. The equation of state \( w \) varies with the cosmic evolution and approaches towards -1 asymptotically showing the existence of a cosmological constant at late times. The heteroclinic orbits \[15\] connect unstable and stable critical points. The numerical calculation shows that brought the Universe back to the usual Friedman-Robertson-Walker expansion, then the second stage of accelerated expansion began at \( z \sim 1.5 \).
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2. The basics

Since current observations favour a flat Universe, we will work in the spatially flat Robertson-Walker metric,

$$ds^2 = dt^2 - a^2(t)dx^2,$$

the Lagrangian density for the spatially homogeneous complex scalar field $\Phi$ is

$$L_\Phi = \frac{1}{2} g^{\mu\nu} \left( \partial_\mu \Phi^* \right) \left( \partial_\nu \Phi \right) - V(|\Phi|) - U(\theta),$$

when consider the presence of barotropic fluid, the action for the model is

$$S = \int d^4x \sqrt{g} \left( -\frac{1}{2\kappa^2} R - \rho_\gamma + L_\Phi \right),$$

where $\kappa^2 = 8\pi G$, $R$ is the Ricci scalar, and $\rho_\gamma$ is the density of the fluid with a barotropic equation of state $p_\gamma = (\gamma - 1)\rho_\gamma$, where $0 \leq \gamma \leq 2$ is a constant that relates to the equation of state by $w = \gamma - 1$.

Writing the complex field $\Phi$ as (see [13])

$$\Phi(t) = \sqrt{2} \phi(t) e^{i\theta(t)/f},$$

Substituting (4) into (2) and varying the action, one can obtain the Einstein equations and the equations of motion for the scalar field as

$$H^2 = \frac{\kappa^2}{3} \left[ \rho_\gamma + \frac{1}{2} \left( \dot{\phi}^2 + \frac{\phi^2}{f^2} \dot{\theta}^2 \right) + V(\phi) + U(\theta) \right],$$

$$\dot{H} = \frac{\kappa^2}{2} \left[ \rho_\gamma + p_\gamma + \dot{\phi}^2 + \frac{\phi^2}{f^2} \dot{\theta}^2 \right],$$

$$\dot{\rho}_\gamma = -3H(\rho_\gamma + p_\gamma),$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\dot{\theta}^2}{f^2} \phi + V'(\phi) = 0,$$

$$\ddot{\theta} + \left( 3H + 2\frac{\dot{\phi}}{\phi} \right) \dot{\theta} + \frac{\dot{\theta}^2}{\phi^2} U'(\theta) = 0,$$

where $H$ is the Hubble parameter, dot and prime denote derivatives with respect to $t$ and $\phi$ (or $\theta$) respectively. We define $\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)$ and $\rho_\theta = \frac{1}{2} \frac{\dot{\theta}^2}{f^2} + U(\theta)$.

3. Phase space

In this section, we investigate the global structure of the dynamical system via phase plane analysis and compute the cosmological evolution by numerical analysis. Introduce the following dimensionless variables:

$$x_1 = \frac{\kappa}{\sqrt{6H}} \dot{\phi}, \quad y_1 = \frac{\kappa}{\sqrt{3H}} \sqrt{V(\phi)},$$

$$x_2 = \frac{\phi}{f} \frac{\kappa}{\sqrt{6H}} \dot{\theta}, \quad y_2 = \frac{\kappa}{\sqrt{3H}} \sqrt{U(\theta)}.$$
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\[
\begin{align*}
\lambda_1 &= -\frac{V'(\phi)}{\kappa V(\phi)}, \quad \Gamma_1 = \frac{V(\phi)V''(\phi)}{V'(\phi)}, \\
\lambda_2 &= -\frac{U'(\theta)}{\kappa U(\theta)}, \quad \Gamma_2 = \frac{U(\theta)U''(\theta)}{U'(\theta)}, \\
\end{align*}
\]

so we could have \(\xi = \mathcal{O}(1)\). It is useful when we do numerical calculation.

Now, the equations (5)-(9) become the following system:

\[
\begin{align*}
\frac{dx_1}{dN} &= \frac{3}{2} x_1 \left[ \gamma (1 - x_1^2 - x_2^2 - y_1^2 - y_2^2) + 2(x_1^2 + x_2^2) \right] - 3x_1 + x_2 z + \sqrt{\frac{3}{2}} \lambda_1 y_1, \\
\frac{dx_2}{dN} &= \frac{3}{2} x_2 \left[ \gamma (1 - x_1^2 - x_2^2 - y_1^2 - y_2^2) + 2(x_1^2 + x_2^2) \right] - 3x_2 - x_1 x_2 z + \frac{1}{2} \lambda_2 y_2^2 z, \\
\frac{dy_1}{dN} &= \frac{3}{2} y_1 \left[ \gamma (1 - x_1^2 - x_2^2 - y_1^2 - y_2^2) + 2(x_1^2 + x_2^2) \right] - \sqrt{\frac{3}{2}} \lambda_1 x_1 y_1, \\
\frac{dy_2}{dN} &= \frac{3}{2} y_2 \left[ \gamma (1 - x_1^2 - x_2^2 - y_1^2 - y_2^2) + 2(x_1^2 + x_2^2) \right] - \frac{1}{2} \lambda_2 x_2 y_2 z, \\
\frac{dz}{dN} &= -x_1 z^2, \\
\frac{d\lambda_1}{dN} &= -\sqrt{\frac{3}{2}} \lambda_1 x_1 (\Gamma_1 - 1), \\
\frac{d\lambda_2}{dN} &= -\lambda_2^2 x_2 (\Gamma_2 - 1). \\
\end{align*}
\]

and dimensionless constant

\[
\xi = f \kappa. \tag{11}
\]

Note that \(f = \mathcal{O}(M_{Pl}), \kappa = \sqrt{8\pi G}, G = m_{Pl}^2 \) (natural units) and \(m_{Pl} = \sqrt{8\pi G} M_{Pl}\), so we could have \(\xi = \mathcal{O}(1)\). It is useful when we do numerical calculation.

Also, we have a constraint equation

\[
\Omega_\phi + \Omega_\theta + \frac{\kappa^2 \rho_1}{3H^2} = 1, \tag{13}
\]

where

\[
\begin{align*}
\Omega_\phi &= \frac{\kappa^2 \rho_\phi}{3H^2} = x_1^2 + y_1^2, \tag{14} \\
\Omega_\theta &= \frac{\kappa^2 \rho_\theta}{3H^2} = x_2^2 + y_2^2. \tag{15}
\end{align*}
\]

The equation of state for the complex scalar field could be expressed in terms of the new variables as

\[
w_\phi = \frac{x_1^2 + x_2^2 - y_1^2 - y_2^2}{x_1^2 + x_2^2 + y_1^2 + y_2^2}. \tag{16}
\]

According to the definitions, the parameters \(\lambda_1, \Gamma_1, \lambda_2, \Gamma_2\) become constants and are equal to \(\lambda_{1(0)} \), \(1, \lambda_{3(0)}\) and \(1\), respectively. Under the circumstance, the equations constitute an autonomous system as follows,

\[
\begin{align*}
\frac{dx_1}{dN} &= \frac{3}{2} x_1 \left[ \gamma (1 - x_1^2 - x_2^2 - y_1^2 - y_2^2) + 2(x_1^2 + x_2^2) \right] - 3x_1 + x_2 z + \sqrt{\frac{3}{2}} \lambda_{1(0)} y_1, \\
\frac{dx_2}{dN} &= \frac{3}{2} x_2 \left[ \gamma (1 - x_1^2 - x_2^2 - y_1^2 - y_2^2) + 2(x_1^2 + x_2^2) \right] - 3x_2 - x_1 x_2 z + \frac{1}{2} \lambda_{2(0)} \xi y_2^2 z, \\
\end{align*}
\]
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\[
\frac{dy_1}{dN} = \frac{3}{2}y_1 \left[ \gamma (1 - x_1^2 - x_2^2 - y_1^2 - y_2^2) + 2(x_1^2 + x_2^2) \right] - \sqrt{\frac{3}{2} \lambda_1(0)} x_1 y_1, \\
\frac{dy_2}{dN} = \frac{3}{2}y_2 \left[ \gamma (1 - x_1^2 - x_2^2 - y_1^2 - y_2^2) + 2(x_1^2 + x_2^2) \right] - \frac{1}{2} \lambda_2(0) \xi x_2 y_2 z, \\
\frac{dz}{dN} = -x_1 z^2. 
\] (17)

When barotropic matter is under consideration or the equations of motion are too difficult to solve analytically, phase space methods become particularly useful, because numerical solutions with random initial conditions usually do not expose all the interesting properties. In table 1, we list the critical points and the cosmological parameters there. To gain some insight into the property of the critical points, similarly as in [14], we investigate the stability of these critical points. For the critical points listed in table 1, we find the eigenvalues of the linear perturbation matrix, see table 2. For stability we require the all 5 eigenvalues to be negative.

The corresponding conclusions of numerical calculation are shown in Fig.1 and Fig.2. This numerical solutions describe heteroclinic orbits, which interpolate between an unstable critical point (case i) and a late-time de Sitter attractor (case iii). Therefore, this model is more natural to explain the two stages of acceleration. A point worth emphasizing is that the ordinary matter (radiation and dust) affect the evolution of the scalar field via their contribution to the general expansion of the Universe because of the couples can be neglected in the model. In Fig.1 the behavior of the equation of state parameter is shown. At initial time, \( w \simeq -1 \) (we choose \( \lambda_1(0) = 0.5 \), then \( w \simeq 0.916 \)) corresponds the inflationary phase, then it increases and becomes positive. After arriving at the value \( 1/3 \), the Universe comes to the radiation dominated epoch and \( w \) stays on a broad platform. Next, \( w \) drops to zero and stays on a narrow platform. Finally, \( w \) drops below zero and approaches to \(-1\), which corresponds second stage of accelerated expansion. The evolution of cosmic density parameters are shown in Fig.2. The phase part contribution \( \Omega_\phi \) stays at \( \Omega = 0 \) at the very first moments of the Universe and becomes 1 in late-time, which plays the role of quintessence field. On the other hand, the amplitude part contribution \( \Omega_\theta \) plays the role of inflaton. During the whole evolution, the radiation energy density and the dust matter energy density become dominate respectively. Therefore, we see that the constraints arising from cosmological nucleosynthesis and structure formation are satisfied.

| Case | Critical points \((x_1, x_2, y_1, y_2, z)\) | \(\Omega_\phi\) | \(\Omega_\theta\) | \(w\) |
|------|---------------------------------|----------------|----------------|-----|
| (i)  | \(\sqrt{\frac{\lambda_1(0)}{6}}, 0, \sqrt{1 - \frac{\lambda_1(0)}{6}}, 0, 0\) | 1 | 0 | \(-1 + \frac{\lambda_1^2(0)}{3}\) |
| (ii) | \(\sqrt{\frac{3}{2}} \gamma, 0, \sqrt{\frac{3}{2}} \sqrt{\frac{\lambda_1(0)}{\lambda_1(0)}}, 0, 0\) | \(\frac{3\gamma}{\lambda_1(0)}\) | 1 | \(\gamma - 1\) |
| (iii) | 0, 0, 1, 0, 0 | 0 | 1 | \(-1\) |
| (iv)  | \(\pm 1, 0, 0, 0, 0\) | 1 | 0 | 1 |
| (v)   | 0, \(\pm 1, 0, 0, 0\) | 0 | 1 | 1 |
| (vi)  | 0, 0, 0, 0, any | – | – | – |

Table 1. The properties of the critical points.
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Table 2. The eigenvalues of the critical points.

| case       | corresponding eigenvalues                                                                 | stability  |
|------------|------------------------------------------------------------------------------------------|------------|
| (i)        | $0, \frac{\lambda_{1(0)}}{2}, -3 + \frac{\lambda_{1(0)}}{2}, -3 + \frac{\lambda_{1(0)}}{2}, -3\gamma + \lambda_{1(0)}^2$ | unstable   |
| (ii)       | $0, \frac{\lambda_{1(0)}}{2}, \frac{3\gamma}{2}(\gamma - 2), 3\{\lambda_{1(0)}^2(\gamma - 2) - \sqrt{\lambda_{1(0)}^2(\gamma - 2)}\}$ | unstable   |
|            | $3\{\lambda_{1(0)}^2(\gamma - 2) + \sqrt{\lambda_{1(0)}^2(\gamma - 2)}\}$ |            |
| (iii)      | $-3, 0, 0, 0, -3\gamma$                                                                  | stable     |
| (iv)       | $3, 0, 0, 6 - 3\gamma, 3 \mp \sqrt{\frac{3}{2}\lambda_{1(0)}}$                          | unstable   |
| (v)        | $3, 3, 0, 0, 6 - 3\gamma$                                                                | unstable   |
| (vi)       | $0, \frac{3\gamma}{2}, \frac{3\gamma}{2}, -3 + \frac{3\gamma}{2}, -3 + \frac{3\gamma}{2}$ | unstable   |

Figure 1. Evolution of the equation of state at the presence of radiation and matter. The curves correspond to the different initial conditions.

Figure 2. Evolution of cosmic density parameter of quintessence energy density $\Omega_\theta$, inflaton energy density $\Omega_\varphi$, radiation energy density $\Omega_r$ and matter energy density $\Omega_m$. The curves correspond to the different initial conditions.
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4. Conclusions

In this paper, we have discussed the cosmological implication of a complex scalar model of quintessence inflation with a barotropic fluid. Using the numerical calculation for this model, we show that the phase contribution is negligible at the early epoch of the universe while it becomes dominate with the time evolving and the evolution of the amplitude part is the opposite, which is a viable way to unify the description of the inflationary stage and the current accelerated expansion. Analysis to the dynamical evolution of the complex scalar model indicates that it admits a late time attractor solution, at which the field behaves as a cosmological constant. Obviously, attractor and heteroclinic orbit are both alleviate the fine tuning problem.

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