MULTIPLE-PLANET SCATTERING AND THE ORIGIN OF HOT JUPITERS

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ABSTRACT

Doppler and transit observations of exoplanets show a pile-up of Jupiter-size planets in orbits with a 3 day period. A fraction of these hot Jupiters have retrograde orbits with respect to the parent star’s rotation, as evidenced by the measurements of the Rossiter–McLaughlin effect. To explain these observations we performed a series of numerical integrations of planet scattering followed by the tidal circularization and migration of planets that evolved into highly eccentric orbits. We considered planetary systems having three and four planets initially placed in successive mean-motion resonances, although the angles were taken randomly to ensure orbital instability in short timescales. The simulations included the tidal and relativistic effects, and precession due to stellar oblateness. Our results show the formation of two distinct populations of hot Jupiters. The inner population (Population I) is characterized by semimajor axis \( a < 0.03 \) AU and mainly formed in the systems where no planetary ejections occurred. Our follow-up integrations showed that this population was transient, with most planets falling inside the Roche radius of the star in \(<1\) Gyr. The outer population of hot Jupiters (Population II) formed in systems where at least one planet was ejected into interstellar space. This population survives the effects of tides over \( >1\) Gyr and fits nicely the observed 3 day pile-up. A comparison between our three-planet and four-planet runs shows that the formation of hot Jupiters is more likely in systems with more initial planets. Due to the large-scale chaoticity that dominates the evolution, high eccentricities and/or high inclinations are generated mainly by close encounters between the planets and not by secular perturbations (Kozai or otherwise). The relative proportion of retrograde planets seems of be dependent on the stellar age. Both the distribution of almost aligned systems and the simulated 3 day pile-up also fit observations better in our four-planet simulations. This may suggest that the planetary systems with observed hot Jupiters were originally rich in the number of planets, some of which were ejected. In a broad perspective, our work therefore hints on an unexpected link between the hot Jupiters and recently discovered free floating planets.

Key words: methods: numerical – planets and satellites: general – planet–star interactions

Online-only material: color figures

1. INTRODUCTION

As of 2012 February, the Rossiter–McLaughlin (RM) effect has been measured for 43 exoplanets (e.g., Moutou et al. 2011). Of these, nine planets (\(\sim 20\%\)) are in retrograde orbits, while in about half of the cases the planet orbit normal is probably aligned with the stellar spin vector \((|\lambda| < 30^\circ\), where \(\lambda\) is the usual projected spin–orbit misalignment angle). Interestingly, the known planets with \(|\lambda| > 50^\circ\) have masses \(M \lesssim 2M_{\text{Jup}}\), where \(M_{\text{Jup}}\) is the mass of Jupiter, while planets with \(M > 3M_{\text{Jup}}\) have \(|\lambda| < 50^\circ\) (Moutou et al. 2011). If this trend holds with the new data, it could provide an important hint on the dynamical origin of the misaligned population. For example, as we will discuss in detail in Section 3, planet scattering followed by tidal circularization and migration is expected to produce such a trend because the less massive planets generally evolve into more eccentric and inclined orbits than the more massive ones, and are therefore more likely to show misalignment.

There also seems to be an indication that almost circular orbits are accompanied by small values of \(|\lambda|\), while large values of \(|\lambda|\) appear less coupled with the eccentricity (Schlaufman 2010). This can be interpreted as the evidence for two distinct populations of close-in exoplanet systems. The former population can be consistent with the smooth planetary migration in a gaseous disk (e.g., Benítez-Llambay et al. 2011), while the later probably requires a more complex orbital history. However, the same data can also be interpreted in terms of an enhanced obliquity realignment in cold stars (Winn et al. 2010).

To produce large values of \(|\lambda|\), it is either necessary to tilt the spin axis of the star so that it ends up being misaligned with the original protoplanetary disk in which planets formed, or to tilt the planetary orbit. A tilt of the star’s axis could be produced by the cumulative effect of stellar flares, late anisotropic (Bondi–Hoyle) accretion on the star (Throop & Bally 2008; Moeckel & Throop 2009), or due to the interaction between the stellar magnetic field and the protoplanetary disk (Lai et al. 2011, and references therein). An orbital tilt can be produced by: (1) planetary scattering (e.g., Nagasawa et al. 2008); (2) Kozai migration produced by a planetary perturber in a distant and inclined orbit (Naoz et al. 2011); or (3) secular migration in well-spaced, eccentric, and inclined planetary systems (Wu & Lithwick 2011).

Here we concentrate on the orbital tilt theories because the observational evidence suggests that at least some planetary systems have large orbital inclinations (e.g., planets c and d of \(\upsilon\) Andromedae have mutual inclination \(I \sim 30^\circ\); McArthur et al. 2010). In addition, the observed large eccentricities of exoplanets can be best explained if the original packed planetary systems underwent a dynamical instability followed by planet formation.

\(^3\) Inclined stellar perturber could in principle also trigger Kozai migration, but no distant star companions are generally observed in systems with known hot Jupiters.
scattering (e.g., Weidenschilling & Marzari 1996; Rasio & Ford 1996). As planet scattering naturally leads to large orbital inclinations as well, the orbital tilt theories are therefore a logical extension of the generally accepted planet scattering model.

Theories (2) listed above invoke the so-called Kozai mechanism, or Lidov–Kozai resonance (Lidov 1961; Kozai 1962), to drive up the orbital inclination $I$ toward values larger than 90°. As the Lidov–Kozai resonance appears in the secular dynamics of the three-body problem for mutual inclinations $I \geq 40°$ (Libert & Henrard 2007), a question arises of how such a large inclination between planetary orbits can be achieved in the first place.

Wu & Lithwick (2011) argued that the retrograde orbits can be achieved in systems with a significant initial angular momentum deficit (AMD) and well-separated planetary orbits. They showed that the smaller planets in these systems can exchange angular momentum with the more massive planets and evolve into highly inclined, and possibly retrograde orbits, by the slow secular interaction between orbits. Still, it remains to be explained how the large AMD assumed by Wu & Lithwick (2011) arises in planetary systems as planets should form with a very low AMD.

The initial AMD can arise as a result of planet scattering, which brings us back to the statistical study of Nagasawa et al. (2008). It is possible then that the initial setups of Naoz et al. (2011) and Wu & Lithwick (2011) could therefore be traced to planet scattering (unless alternative explanations are offered for their initial conditions; e.g., Libert & Tsiganis 2011a, 2011b).

While many works considered planet scattering (e.g., Marzari & Weidenschilling 2002; Chatterjee et al. 2008; Juric & Tremaine 2008), Nagasawa et al. (2008) pioneered the statistical studies of planet scattering with tidal effects. The principal role of tides is the circularization of the planetary orbit while approximately preserving the angular momentum. This effect helps to stabilize the orbits of scattered planets reaching small pericenter distances ($q \lesssim 0.05$ AU) and, consequently, leads to the formation of hot Jupiters.

Nagasawa & Ida (2011) recently presented a new set of simulations, concentrating on the inclination and eccentricity distribution of the final planets. They find that almost 30% of initial systems form hot Jupiters and about 29% of these end up in retrograde orbits. This speaks of an extremely efficient formation mechanism, which, however, may be closely tied to the chosen tidal model and the lack of tidal follow-up for Gyr timescales.

For the tidal effects, Nagasawa et al. (2008) and Nagasawa & Ida (2011) adopted the dynamic tide model by Ivanov & Papaloizou (2004, 2007), which is applicable for fully convective planets with near-parabolic orbits, but is not well suited for low-to-moderate eccentricities. Conversely, Wu & Lithwick (2011) used the equilibrium tide model (e.g., Hut 1981; Mardling 2007), which is strictly valid only for low-to-moderate eccentricities. Since the real evolution of planets likely spans the whole range from near-parabolic to near-circular orbits, it is not clear whether any of the two approximations mentioned above is adequate. We explain how we deal with this problem in Section 2.

For the work described in this paper we assumed a physical scenario similar to Nagasawa et al. (2008) starting with multiple-planetary systems in unstable orbits and following their evolution through the stage of close encounters and planet scattering. Our main simulations employed an $N$-body code, incorporating the relativistic effects, stellar oblateness, and tidal precession.

In Section 3 we describe our $N$-body code and the results of scattering experiments with three-planet systems. An extension to four planets is reported, fittingly, in Section 4. The long-term tidal evolution of hot Jupiters is discussed in Section 5, while conclusions close the paper in Section 6.

2. TIDAL MODEL

2.1. Equilibrium versus Dynamical Tides

Current tidal models were constructed for two limit cases. If the orbital separation between the interacting bodies is roughly constant due to low orbital eccentricity, the tides vary slowly and generate an equilibrium figure in the extended bodies. Viscosity causes this tidal bulge to deviate from the instantaneous equipotential shape which, in turn, leads to an angular momentum exchange between the orbital and rotational motions. The dynamical evolution of the bodies in this approximation is described by the so-called equilibrium tide model, originally developed by Darwin (1879). In its simplest version (Mignard 1979, 1980), it is assumed that the equilibrium shape of each body at time $t$ is defined by the equipotential surface at time $t + \Delta t$. The individual time lags are assumed independent of the tidal frequencies and therefore all equal (e.g., Correia et al. 2003; Ferraz-Mello et al. 2008).

The opposite limit case occurs when $e \sim 1$. In this case the tidal distortion is generated only at the pericenter and is negligible during the rest of the orbit. Consequently, the bodies can no longer achieve equilibrium figures. Instead, they undergo forced oscillations, the most important being $f$-mode (or surface gravity) waves. The subsequent effects on the rotational and orbital motion are described by the dynamical tide model (e.g., Lai 1997; Ivanov & Papaloizou 2004, 2007, 2011). Dynamical tides are much more complex than their equilibrium counterparts, and their effect on the orbital and rotational evolution of the participating bodies is not so well understood. For example, up to date only planar or polar orbits have been studied, and there is no model for planets with arbitrary inclinations.

Unfortunately, the dynamical evolution that leads to formation of hot Jupiters covers both limit cases discussed above. Initially, the planet has an almost parabolic orbit caused by scattering and slow secular evolutions. Thus, the tidal interaction with the star should be treated in the frame of the dynamical tide model. As the orbit decays and circularizes, the system approaches the equilibrium tide regime, and the dynamical tide model ceases to be valid.

Since the two models are based on a consideration of two completely different physical phenomena (equipotential figures versus damped forced oscillations), it may be difficult to construct an unified physical tidal model that would be adequate for the entire orbital evolution of hot Jupiter. To deal with this problem we empirically modified the equations of the equilibrium tide model to mimic the effects of the dynamical tide model when the parabolic limit is approached (Section 2.3).

2.2. Equilibrium Tidal Model

We use the equilibrium tidal equations derived in Correia et al. (2011). These equations are based on the weak-friction Mignard model, and contain explicit expressions for the variations of the mutual inclination, argument of the pericenter, and obliquities, all in a consistent manner. They also include additional perturbations from gravitational interactions with
other planets, stellar oblateness, and general relativity (GR) in
the post-Newtonian approximation.

Tidal precession is usually neglected in tidal equations, since
the precessional period is much shorter than the timescale
associated with circularization and orbital decay. However, since
it is possible that Lidov–Kozai resonance may play an important
role in the orbital evolution of hot Jupiters, we retain these
 secular terms in our model.

We introduce three important changes with respect to
Correia et al. (2011). First, instead of limiting the gravitational
interactions between the planets to the quadrupole secular ap-
proximation, we extend it to the octupole level. Second, since we
expect large-scale orbital exchanges between the planets, we use
Poincaré astrocentric coordinates instead of Jacobi coordinates.

Finally, although we adopted the so-called constant time-lag
tidal model, recall that this invariance refers to the dependence
with respect to the tidal frequency terms and not with respect to
the orbital separation between the interacting bodies. Although
most authors also assume that the time lag \( \Delta_t \) is also constant
for any orbital separation (e.g., Hansen 2010), this is not the
only option. Matsumura et al. (2010) discuss different scalings
and both Mardling & Lin (2002) and Barker & Ogilvie (2009)
propose that \( \Delta_t \) scales inversely with the mean motion (i.e.,
\( \Delta_t \propto 1/n \)). This allows us to still use the “constant time-lag
tidal model” but define a tidal parameter \( Q_i^1 \) that does not change
with the semimajor axis but is only dependent on the physical
properties of the extended body. This is a very desirable property
since it also allows us to compare the values of \( Q_i^1 \) with other
systems (e.g., Jupiter).

Apart from simplifying the comparisons, the main advantage of
\( \Delta_t \propto 1/n \) is that it allows a much better fit with the results of
dynamical tide for quasi-parabolic orbits, thus allowing better
decision agreement during most of the tidal evolution of the system. For
these reasons, we adopted such a scaling for the present work.

However, once the hot planets were formed, and their eccen-
tricities were reduced below the parabolic limit, we found
little change in their evolution adopting either \( \Delta_t \) = const or
\( \Delta_t \propto 1/n \).

We assume three extended bodies: \( m_0 \) (star), \( m_1 \) (inner planet),
and \( m_2 \) (outer planet), with the planets having orbital elements
\( a_i, e_i, I_i \), and \( \omega_i \), where \( \omega_i \) are arguments of pericenters.
Tidal effects will only be felt by \( m_0 \) and \( m_1 \). The outer body is assumed
to be too far from the central star to generate or receive tidal
distortions, but will interact gravitationally with \( m_1 \).

We use the following variables for the orbital motion:

\[
G_i = \beta_i \sqrt{\mu_i a_i (1 - e_i^2)} \hat{k}_i, \tag{1}
\]

\[
e_i = \frac{r_2 \times G_2}{\beta_i \mu_i} - \frac{r_1}{r_1},
\]

where \( \beta_i \) and \( \mu_i \) are the Poincaré mass factors (e.g., Laskar &
Robutel 1995), \( G_i \) are the orbital angular momentum vectors,
and \( e_i \) are the Lenz vectors. The unit vector \( \hat{k}_i \) is perpendicular
to the orbital plane (the reference plane is arbitrary), while
the direction of the Lenz vector points toward the argument
of pericenter.

Additionally, for the tidally interacting bodies, we also define
the rotational frequencies by \( \Omega_0 \) and \( \Omega_1 \). The equations
of motion for the objects’ spins will be written in terms of the
rotational angular momenta:

\[
L_i = C_i \Omega_i \hat{s}_i, \tag{2}
\]

\[
ge_1 = 0.04, \quad e_1 = 0.05, \quad I_1 \approx 45^\circ
\]

\[
\theta_0 \quad \theta_1 \quad \theta_2
\]

\[
\begin{array}{c}
\text{rotational period [d]} \\text{inclination [deg]} \\text{obliquity [deg]}
\end{array}
\]

Figure 1. Tidal evolution of a Jovian-mass planet with initial semimajor axis
\( a_1 \) = 0.04 AU and \( e_1 \) = 0.05. \( P_0 \) is the stellar rotation period, while \( P_1 \) is
the planet’s rotation period. The dashed green line in the bottom-left panel shows
the planet’s orbital period. The stellar and planetary obliquities (\( \theta_0 \) and \( \theta_1 \),
respectively) are shown in the bottom-right panel and measured with respect to
the inner planet’s orbital plane. Both spin axes where initially chosen parallel to
the orbital angular momentum vector.

(A color version of this figure is available in the online journal.)

where \( C_i \) are the principal moments of inertia and \( \hat{s}_i \) is the spin
axis referred to the same reference plane. It is assumed that the
spin vectors always coincide with the orientation of the principal
moments of inertia.

Correia et al. (2011) constructed the variational equations for
the complete set of variables \( \{G_i, e_i, L_i\} \) that include all
the above-mentioned perturbations, from secular terms of the
mutual gravitational interactions between the planets to tidal
effects, GR, and stellar oblateness. At any given instant, the
eccentricities can be obtained from the modulus of the Lenz
vector, the semimajor axis from \( |G_i| \) and the rotational frequen-
cies \( \{L_i\} \). The mutual inclination of both orbits can be calculated from

\[
\cos I_{\text{mut}} = \hat{k}_1 \cdot \hat{k}_2. \tag{3}
\]

Finally, the obliquities can be determined from

\[
\cos \theta_1 = \hat{s}_1 \cdot \hat{k}_1; \quad \cos \theta_2 = \hat{s}_2 \cdot \hat{k}_2. \tag{4}
\]

where \( \theta_i \) is the obliquity of \( m_i \) with respect to the orbital plane
of the inner body, while \( \epsilon_i \) are the obliquities with respect to the
orbital plane of the outer companion. We will be most interested
in variables \( \theta_0 \) and \( \theta_1 \).

The orbital/rotational equations of motion yield a complete
description of the tidal and gravitational evolution of the
system, valid as long as there are no resonances or close
encounters between the planets. It then constitutes a semi-
analytical model that requires much less CPU time than a direct
N-body integration.

Figures 1 and 2 show two examples, both with \( m_0 = M_\odot \) and
\( m_1 = m_2 = M_{Jup} \). For these illustrations, we used \( Q_0^1 \to 10^4 \)
and \( Q_p = 5 \times 10^6 \) for the star and planet tides, respectively. The value for the stellar parameter \( Q_p \) was taken from Benítez-Llambay et al. (2011) as the value that best fits the semimajor axis distribution of short-period planets believed to be in circular orbits. The particular value chosen for \( Q_p \) will be explained later on.

In Figure 1, the initial semimajor axes and eccentricity are similar to that of the known hot Jupiters (\( a_1 = 0.04 \) AU and \( e_1 = 0.05 \)). In Figure 2, we assumed an initially quasi-parabolic orbit with larger semimajor axis (\( a_1 = 2 \) AU and \( e_1 = 0.985 \)) similar to those obtained from scattering experiments (e.g., Nagasawa et al. 2008). In the latter case, \( q_1 = a_1(1 - e_1) = 0.03 \) AU, thus placing the planet’s pericenter within the region affected by tides. In both cases \( m_r \) was placed in a distant circular orbit at \( a_2 = 2000 \) AU to make its gravitational perturbations on the inner planet insignificant. The role of the second planet was simply to help define the initial orbital invariant plane of the system and as a test for the full set of variational equations developed in the model by Correia et al. (2011).

The mutual inclination was chosen \( I_{mut} = 45^\circ \), and both initial spin vectors were set to be perpendicular to the invariant planetary plane. Finally, the initial rotational periods were \( P_0 = 2\pi/\Omega_0 = 28 \) days and \( P_1 = 2\pi/\Omega_1 = 0.4 \) days.

Both simulations show practically no change in the orbital inclination of the inner planet (i.e., \( I_1 \)) due to tides, except during the final stages just before the planet is engulfed by the star. For quasi-circular orbits, the orbital decay time \( \tau_o \) is longer than the circularization timescale \( \tau_c \), implying that the planet is circularized before becoming a hot Jupiter (Figure 1). The opposite occurs for the initially quasi-parabolic orbits, where \( \tau_c > \tau_o \) (Figure 2). This latter case is consistent with the results of Ivanov & Papaloizou for dynamical tides (see also Nagasawa et al. 2008). However, even in this case, once the eccentricity decreases to small values, the relation switches back and the final stage of the orbital evolution occurs as in Figure 1. Note that the results shown in Figure 2 were obtained with our \( \Delta_1 \propto 1/n \) tidal model. If this dependence were not introduced, the orbital decay and circularization timescales would increase by about two orders of magnitude, thus making the tidal trapping of such bodies practically impossible in timescales consistent with stellar ages.

Another difference in the high eccentricity case is that the synchronization of the planetary spin occurs with respect to the orbital frequency at pericenter \( \Omega_p \) (blue dashed line) and not with respect to the mean motion \( n \) (green dashed line). Once again, this is in good agreement with the predictions of Ivanov & Papaloizou for the dynamical tide model. Thus, it appears that several of the orbital evolutionary properties of the dynamical tide model can be fairly and qualitatively reproduced with our chosen version of the equilibrium tide approximation.

Finally, we discuss the behavior of the obliquities \( \theta_i \). Tides raised on the planet cause a relatively rapid alignment of the planet’s equator with respect to the orbital plane, thus leading to \( \theta_1 \sim 0 \). At least for the simulations discussed in this paper, we found no cases of trapping in other Cassini states. This occurred even for the retrograde orbits, where the initial obliquity was larger than 90\(^\circ\). However, the timescale of obliquity evolution is of the order of \( 10^5 \) yr. This is much larger than the synchronization timescale (\( \sim 10^4 \) yr), but much smaller than the orbital decay times. Thus, in the absence of additional perturbations, it is expected that \( \theta_1 \sim 0 \) during most of the planet’s evolution.

The stellar obliquity \( \theta_0 \) shows a much longer damping timescale, comparable to or even larger than that of orbital decay. This is not unexpected (e.g., Lai 2011): In order for the star to significant tilt its rotational axis, the planet must be extremely close and tidal interactions must be very large. Recall that \( \theta_0 \) measures the misalignment between the planetary orbit and the stellar equator and, projection considerations notwithstanding, it is the equivalent of \( \lambda \). Thus, unless we carry out extremely long-term numerical integrations, there will be very little difference between the \( \theta_0 \) and the orbital inclination of the planet, provided the initial stellar equator was aligned with the orbital reference plane.

### 2.3. Correction Terms in the Tidal Model

One of the most important differences between the equilibrium and dynamical tide models is their implication for the orbital decay and orbital circularization e-folding times (\( \tau_o \) and \( \tau_c \), respectively). Figure 3 shows the value of \( \tau_o \) obtained from the equilibrium tide model (same tidal parameters as used in the previous figures) and from the dynamical model of Ivanov & Papaloizou (2011, see their Figure 5). The decay times are plotted as a function of the initial pericentric distance \( q \). The dynamical model predicts shorter decay times below \( q \sim 0.022 \) AU and suggests longer timescales for larger separations.

A way to reproduce, at least qualitatively, the results of Ivanov & Papaloizou (2011) with the equilibrium tide model is to modify the values of the tidal parameters according to the following empirical recipe:

\[
Q'_i \rightarrow Q'_i \beta^{200e^2(q - 0.022)}.
\]

The factor \( e^2 \) guarantees that this change is only significant for highly eccentric orbits, since for the lower eccentricity values the equilibrium tide model works well. This expression is only weakly dependent on the semimajor axis. For other values...
of $Q_p$, the same expression may be used adopting a slightly different change-over value of $q$. Moreover, we implemented the correction term only for orbits with $a > 1$ AU. This is because the dynamical tide is expected to be relevant in our simulations only for large semimajor values.

The value of $\tau_a$ determined with this new tidal parameter is also plotted in Figure 3. The overall agreement is satisfactory. The correction in Equation (5) also affects the circularization timescale $\tau_e$, yielding values very close to those predicted by Ivanov & Papaloizou (2011).

Figure 4 shows the distribution of known exoplanets with measured $\lambda$. Left plot shows eccentricity as a function of the semimajor axis. The dashed line denotes constant pericentric distance $q = 0.03$ AU. Right plot shows $\lambda$ as a function of the semimajor axis.

2.4. Constraint on $Q_p^*$

Figure 4 shows the distribution of known exoplanets with measured $\lambda$. Interestingly, the eccentricities show a marked correlation with the semimajor axis, roughly along a line with $q = 0.03$ AU. This could potentially be used to constrain the value of $Q_p^*$.

To do that, we followed the tidal evolution of fictitious Jovian-mass planets. Their initial semimajor axis was taken from a uniform distribution in $\log(a)$ from 0.01 AU to 4 AU. The eccentricities were taken from a uniformly random distribution of $e$ between 0 and 1. A total of $10^4$ initial orbits were generated.

Each orbit was evolved for 1 Gyr. Results are shown in Figure 5 for four different values of the planetary tidal parameter $Q_p^*$, ranging from $10^5$ (top left) to $10^8$ (bottom right).

For $Q_p^* = 10^5$, the envelope of final orbits does not seem to reproduce the observed eccentricity distribution. The tidal effects are apparently too efficient, and all orbits with final semimajor axes below 0.06 AU become circularized. Better results are obtained with $Q_p^* = 10^6$ and $Q_p^* = 10^7$, although larger values seem to be too inefficient. If the real exoplanets are the outcome of scattering events and tidal capture from quasi-parabolic orbits, it is expected that they underwent a phase when their orbital eccentricities were very high. Figure 6 factors this assumption in the results of our tests. Once again the standard equilibrium tide model with $Q_p^* = 10^5$ does not yield a good agreement, but now it is clear that the modified model with $Q_p^* = 10^8$ is also not adequate, since practically all the real planets fall into the region with relatively small initial eccentricities.

From these tests it appears that the eccentricity–semimajor axis distribution of the real exoplanets can be reproduced by planetary tidal parameters of the order of $Q_p^* \sim 10^6–10^7$, at least for or adopted tidal model. Similar values have been proposed by other authors. Using similar analysis, Jackson et al. (2008) found that the distribution of all close-in planets shows good agreement assuming $Q_p^* \approx 3 \times 10^6$, while a value of $Q_p^* \approx 2.2 \times 10^6$ has recently been proposed to explain the current orbital characteristics of CoRoT-20b (Deleuil et al. 2012).

For the present work we adopted $Q_p^* = 5 \times 10^6$ as a half-way suitable value, and $Q_p^* = 10^7$ for the stellar tide. As long as the planet has a non-negligible orbital eccentricity, the stellar tidal parameter $Q_*$ is of secondary importance because the orbital dynamics is mainly controlled by the planetary tide. However, the stellar tide is important once the planet reaches a circular orbit.
3. N-BODY SIMULATIONS OF THREE-PLANET SCATTERING

To study the formation of hot planets, we must first follow the orbital evolution of a planetary system through the instability phase, when planets scatter off of each other. The code must also be able to track the evolution in the late stage when hot Jupiters become circularized and migrate by tides, as discussed in Section 2. Here we describe our N-body integrator that is able to handle close encounters between planets, near-parabolic orbits that may result from encounters, and tides. To start with, we will apply this code to three-planet systems, a case similar to that studied by Nagasawa et al. (2008) and Nagasawa & Ida (2011).

3.1. Explicit Expressions for the Forces

In an astrocentric reference frame, the differential equation affecting the position vector \( \mathbf{r} \) of the planet is

\[
\dot{\mathbf{r}} = \mathbf{f}_0 + \mathbf{f}_{\text{TD}} + \mathbf{f}_{\text{TD}} + \mathbf{f}_{\text{GR}} + \mathbf{f}_{\text{SO}},
\]

where \( \mathbf{f}_0 \) is the gravitational acceleration from the central star (point-mass approximation) and \( \mathbf{f}_{\text{TD}} \) that from the other planets.

The tidal distortion that generates the apsidal precession (see Hut 1981) is given by

\[
\mathbf{f}_{\text{TP}} = -3 \frac{\mu}{r^5} \left[ k_{20} \left( \frac{m_1}{m_0} \right) R_0^2 + k_{21} \left( \frac{m_0}{m_1} \right) R_1^2 \right] \mathbf{r},
\]

with \( \mu = G(m_0 + m_1) \) and \( G \) is the gravitational constant. The corresponding tidal dissipation term is

\[
\mathbf{f}_{\text{TD}} = \sum_{i=0,1} \left( \frac{m_1}{m_1} \right) k_{2i} \Delta_i R_i^5 (2 \mathbf{r} \cdot \dot{\mathbf{r}} + r^2 (\mathbf{r} \times \dot{\mathbf{r}} + \dot{\mathbf{r}})).
\]

Here \( \Omega_i \) are the spin vectors of both bodies with respect to the reference frame that does not have to coincide with the Laplace plane. The factor \( k_{2i} \Delta_i \) is related to the tidal parameter through

\[
k_{2i} \Delta_i = \frac{3}{2 Q_i^2 n},
\]

where \( n = \sqrt{\mu / a^3} \) is the mean motion of the planet. We used the hybrid equilibrium tidal model discussed in the previous section with fixed values of \( Q_e = 10^7 \) and \( Q_p = 5 \times 10^6 \).

Finally,

\[
\mathbf{f}_{\text{GR}} = -\frac{\mu a (1 - e^2)}{c^2 r^5} \mathbf{r},
\]

where \( c \) is the speed of light, is the post-Newtonian radial term that approximates the general relativity effects, and

\[
\mathbf{f}_{\text{SO}} = \nabla \left( \frac{\mu J_2 R_0^2}{r^3} \left( \frac{3}{2} \sin \delta - \frac{1}{2} \right) \right)
\]

is the acceleration from stellar oblateness (e.g., Beutler 2005), where \( R_0 \) is the stellar radius, \( \delta \) is the declination angle of the orbit with respect to the stellar equator, and \( J_2 \) is the quadrupole coefficient.

From the complete expression for the acceleration we can calculate its effect on the spins, assuming a conservation of the complete (orbital + rotational) angular momentum (see Correia et al. 2011 for more details). We disregard any variation of the stellar rotational frequency due to momentum loss driven by stellar winds (e.g., Skumanich 1972), and assume that it is only affected by tides.

3.2. The Code

Our original plan was to implement these forces into SyMBA (Duncan et al. 1998), which is an efficient symplectic N-body code that is capable of tracking close encounters between massive bodies. As in Mercury (Chambers 1999), SyMBA uses the Poincaré variables to be able to handle encounters. The symplectic algorithms written in Poincaré variables, however, have troubles in following orbits with very high eccentricities. This is a problem because the orbits of hot Jupiters are expected to have \( e \sim 1 \) before they can become circularized by tides.

To solve this problem, Levison & Duncan (2000) proposed a hybrid integration scheme in which the outer part of the eccentric planetary orbit is integrated with the usual SyMBA algorithm. The code then symplectically switches to the Bulirsch–Stoer (BS) algorithm to follow the planet’s evolution near the inner part of its orbit. The switch radius is set to a fixed apocentric distance, usually of order of 0.1 AU.

We tested the hybrid algorithm in the extreme case when \( e \sim 1 \) and found, perhaps not surprisingly, that the time step needs to be set to a very small fraction of the orbital period. This slows down the symplectic algorithm so much that the BS integrator, if used to follow the full evolution of the system (i.e., also outside the switch radius), is actually faster and more accurate. For this reason, we constructed a new N-body code based on the BS method and used it in this study.

The code follows the interaction of \( N \) massive planets orbiting a central star of mass \( m_0 \). It tracks both the orbital and spin dynamics according to the equations given in Section 3.1. As in Nagasawa et al. (2008), the integrations were stopped when reaching \( 10^8 \) yr, or if:
1. A hot Jupiter formed with \( e < 0.01 \) and stable orbit.
2. One planet was ejected and the other two remained in stable orbits. In this case, the evolution of the system was continued using the secular model described in Section 2. The use of the semi-analytical model helped to speed up the simulation, and proved adequate for two-planet systems where no additional close encounters between planets occurred. This was tested comparing the model with direct \( N \)-body simulations.
3. Two planets were ejected and the pericentric distance of the survivor was larger than 0.1 AU.

To determine whether a surviving pair of planets attained stable orbits we used the Hill criterion of Marchal & Bozis (1982; see also Gladman 1993).

3.3. Initial Conditions

Planetary migration produced by planet–gas-disk interactions is an important evolutionary process during the early history of planetary systems. As we do not model these early stages here, we will need, in an uncertain leap of faith, to adopt some initial conditions for our simulations. These condition should be at least broadly consistent with the state of the planetary systems just after the gas disk dispersal.

While classical hydrodynamical simulations of disk–planet interactions and resonance trapping have focused on two-planet systems (e.g., Snellgrove et al. 2001; Kley 2003), recent studied have been extended to multiple planetary systems (e.g., Morbidelli et al. 2007; Libert & Tsiganis 2011a, 2011b). It appears that multiple-resonance trapping is a common outcome, although not all the resonant configurations are long-term stable. Thommes et al. (2008) also suggested that stable configurations within the gas disk may become unstable after disk dispersal and subsequent planetary scattering may occur.

We used several different mass ratios between planets. The masses in the units of \( M_{\text{Jup}} \) were

\[
m_i = 1 \quad \text{and} \quad m_{i+1} = H m_i \quad (i = 2, 3),
\]

where \( H = 0.5, 1, \) or 2. After the mass values were specified (using a random generator for the \( H \) values), we shuffled the radial order of the planets, such that the body identified as \( m_1 \) was not necessarily the one closest to the star.

The semimajor axis of the inner planet was chosen randomly between 1 and 5 AU. The other planets were placed in successively mean-motion resonances. We used the 2/1, 3/2, or 4/3 resonant ratios for different planet pairs, with the specific choice depending on the individual masses of the two planets. To select the resonant ratio that should apply to a specific mass ratio, we adopted the results of Pierens & Nelson (2008), who studied the resonant capture with a hydrocode. Using the same disk parameters as Pierens & Nelson (2008), we also performed additional hydrocode simulations with FARGO (Masset 2000) to confirm and extend these results to multi-planet systems.

Orbital eccentricities were chosen randomly between 0 and 0.1 and inclinations between 0° and 1°. Although this seems arbitrary, we performed tests with different distributions (e.g., lower eccentricities for more massive planets) and found that the results were largely independent of these assumptions. The angular variables were randomly changed from their resonant values to mimic the situation at the onset of instability. All initial conditions were consequently dynamically unstable and none remained in the initial resonant configuration.

To test the dependence on initial conditions, we performed additional simulations adopting slightly different semimajor axes. Both the formation frequency of hot planets and their orbital distribution did not show any significant change. We believe this is due to the strong orbital instability of the system that, on a very short timescale, erases all memory of its initial state. This implies that our results should be pretty much independent on the initial conditions.

The initial rotational period of the star was chosen as \( P_0 = 2\pi/\omega_0 = 28 \) days, while for the planets we adopted a value of 0.4 days. The initial spin axes were taken perpendicular to the orbital invariant plane, so at the beginning of our simulations \( \theta_i = I_i \).

3.4. Results

In total, we followed the evolution of 2891 initial systems, a number sufficiently large for a detailed statistical analysis. Figure 7 shows the orbital distribution of all planetary systems at the end of simulations. A total of 288 hot Jupiters formed, which is approximately 10% of the number of initial systems. This fraction is about three times smaller than what was reported in Nagasawa et al. (2008) and Nagasawa & Ida (2011). Most hot Jupiters acquired circular orbits due to tidal damping. Some hot Jupiters, particularly those with \( a \geq 0.03 \) AU, retained high eccentricities.

The left-hand panels in Figure 7 show the results of the \( N \)-body simulation for a total integration time of \( T = 10^8 \) yr. Since most of the known hot Jupiters with measured \( \lambda \) have stellar ages of the order of 1–3 Gyr (e.g., Triaud 2011), we extended the simulations to see how the population of hot Jupiter can be modified by tidal effects over Gyr-long timescales. First, we selected the planetary systems, where the \( N \) body integrations described above led to the formation of a hot Jupiter.
We disregarded the outer planets in each system because their interaction with the hot Jupiter is weak. We then used the semi-analytical code described in Section 2 to follow the tidal evolution of hot Jupiters up to $T = 1$ Gyr. The results are shown in the right-hand plots of Figure 7.

Note that we stopped the simulation if a hot Jupiter evolved into a nearly circular orbit (defined as $e < 0.01$). The results shown in Figure 7 therefore include all hot Jupiters that formed in our simulations at any time. As we will discuss in Sections 3.6 and 4, many of these hot Jupiters continue to evolve by tides and are dynamically short lived.

As for the eccentricity distribution, we find two types of final orbits. The first type consists of planets in almost circular orbits. They were either tidally trapped very close to the star ($q < 0.01$ AU) and underwent rapid orbital circularization, or captured at larger pericentric distances but early in the simulation. The second orbit type shows moderate-to-large eccentricities (in some cases as high as $e \sim 0.8$), and a clear correlation between $a$ and $e$. Such a correlation is expected for a population of planets evolving from quasi-parabolic orbits while maintaining almost constant pericentric distance. These planets were tidally trapped at larger distances and still maintain finite eccentricity even after 1 Gyr.

As for the inclination distribution, we note two distinct populations of hot Jupiters that can be conveniently classified as having $a \leq 0.03$ AU (Population I or Pop-I) and $a > 0.03$ AU (Population II or Pop-II). Nagasawa & Ida (2011) found similar segregation, referring to both populations as being originated by three-body circularization (our Pop-I) or two-body circularization (our Pop-II). Practically all orbits with $a > 0.03$ AU have $I < 90^\circ$, while about 22% of those with $a < 0.03$ AU are retrograde. For this timescale, there is practically no difference between the orbital inclination of the inner planet and the stellar obliquity $\theta_0$, so the lower plots in Figure 7 can also be considered representative of the spin–orbit misalignment of the hot planets.

A second difference between Pop-I and Pop-II is the behavior of planet’s obliquity $\theta_1$. Practically all Pop-I planets have zero obliquities (relative to planet’s orbit), as expected from a sustained tidal evolution. In contrast, many Pop-II planets retained relatively large values of $\theta_1$. These planets continue to evolve by tides even 1 Gyr after their parent system’s formation.

Next we studied the effect of planet masses and starting semimajor axes on the results (Figure 8). We find that Pop-I mainly contains planets that formed through scattering in systems in which no planet was ejected. The Pop-II planets, on the other hand, formed in systems where one planet was ejected. We discuss this interesting result in more detail in Section 3.5.

The middle frame in Figure 8 shows the $a$ and $I$ distribution sorted according to the planetary mass. This plot shows that the more massive planets remain near their original orbits at $a > 1$ AU. The hot Jupiters, on the other hand, tend to be the least massive planets in the original systems. This is easy to understand because the lighter planets are easier to scatter, and more likely to become hot Jupiters. Although there seems to be no significant difference in the inclination distribution between $m = 0.5 M_{\text{Jup}}$ and $m = 1 M_{\text{Jup}}$, the inclinations attained by the more massive planets are noticeably smaller.

The bottom panel in Figure 8 separates the final planets according to the planet mass ratio in the initial systems. The results indicate that the Pop-I planets tend to form in planetary systems with a large mass ratio, while the outer Pop-II planets primarily evolve from the systems with planets more similar in mass.

### 3.5. Orbit Evolution

To understand the dynamical mechanism responsible for the formation of hot Jupiters, we now discuss the dynamical evolution of individual systems. We illustrate things on three different planetary systems.

Figure 9 shows the first case. In this case, none of the planets escape, which is characteristic for the systems in which Pop-I hot Jupiters formed in our simulations. The initially inner planet ($m_1$ shown in black) is quickly scattered to an exterior orbit. At $t \sim 7000$ yr, it suffers a sequence of close encounters with the other two (more massive) planets, a period that lasts several thousand years. In consequence, the lighter planet is scattered into an inner orbit, its eccentricity raises to $\sim 1$, and orbital inclination very rapidly reaches values larger than $90^\circ$.

In the process, the pericentric distance drops and the planet decouples from other bodies in the system to evolve solely by tides over the rest of its lifetime. Both the semimajor axis
and eccentricity decrease due to tides while maintaining similar values of the pericentric distance (see lower black dashed curve in top panel of Figure 9). During this late stage, the inclination remains nearly constant and close the value excited by close encounters between planets.

A second case of orbit evolution is shown in Figure 10, and corresponds to a Pop-I hot Jupiter on a prograde orbit. In this case, the planet that became hot Jupiter was originally the middle body in the system (denoted by $m_2$).

The orbital evolution of all planets is characterized by a prolonged phase of chaotic evolution, when planets move on crossing orbits and interact with each other. During the later stages, the inclinations of $m_2$ and $m_3$ are periodically excited to values close to 50$^\circ$, returning to lower values each time the eccentricity suffers a jump. This anti-correlation of $e$ and $I$ is characteristic of the Lidov–Kozai resonance, but is also observed in the circulation domain of $\omega$ in the secular three-body problem.

The evolution of $\omega$ shows some evidence of temporary capture in the Lidov–Kozai resonance (when either $\omega_2 \approx 90^\circ$ or $\omega_3 \approx 270^\circ$; see the bottom panel in Figure 10), but overall this seems to have only a minor cumulative effect on the final inclination. This is not surprising since the Kozai mechanism is only expected to be more regular in almost hierarchical systems and not here, where strong chaos and orbital crossings are frequent. The pericenter argument rapidly precesses during the very late stage (after $t \sim 1.4 \times 10^5$ yr) due to the combined effects of tides, relativity, and stellar oblateness.

Finally, Figure 11 shows an example of a system that produced the Pop-II hot Jupiter. Here the outer planet ($m_3$) is ejected from the system at $t \approx 10^5$ yr. The system evolves more regularly after this time. Planet $m_1$ becomes trapped in the Lidov–Kozai resonance, and remains in the resonance until the semimajor axis drops to very small values. The orbital inclination shows no significant long-term evolution in the Lidov–Kozai resonance, remaining close to the value attained immediately after the escape of $m_3$.

A similar analysis of a large sample of our simulations indicates an important difference between the runs resulting in Pop-I planets, and those generating Pop-II planets. In the first case, due to the fact that all planets remain in the system for the duration of the run, the dynamical evolution of systems with no escapes shows a prolonged phase of strong interaction between planets, leading to the excitation of their eccentricities and inclinations. Since planetary orbits suffer very strong perturbations, a hot Jupiter can form only if the orbital pericenter drops to very small values, where it can be tidally trapped. Consequently, practically all hot Jupiters that formed in these systems belong to Pop-I. Also, the system’s strong chaoticity produces a larger spread in the inclination distribution, including numerous retrograde orbits in Pop-I.
Another consequence of strong interaction in these systems is that the Lidov–Kozai resonance can be sustained only for short intervals of time, and does not seem to be important for the formation of hot Jupiters (retrograde or not). Indeed, in systems with no planetary ejections, we found no clear examples of the standard Kozai migration (Wu & Murray 2003). Again, this is expected since the extreme chaoticity dominates over smoother secular evolutionary tracks.

The orbit evolution of Pop-II Jupiters is qualitatively different from that of Pop-I Jupiters, but the mechanisms that operate to produce large eccentricities/inclinations are the same. After the third planet’s ejection in the systems that produce Pop-II, the outer perturber generally has a large semimajor axis. This causes smaller perturbations on and slower evolution of the inner planet orbit, and permits tidal capture at larger pericentric distance. Since the evolution is less chaotic and close encounters less frequent, the excitation of the inclination is modest. The retrograde orbits are therefore more rare in Pop-II than in Pop-I. Similar results were also found by Nagasawa & Ida (2011) comparing their two-body and three-body circularizations. However, even in Pop-II systems, cases of Kozai trapping were rare and we found no case in which this mechanism was responsible for generating a retrograde orbit.

We tested how the results discussed above depend on the tidal model. For example, we switched off the modifications of the equilibrium tidal model described in Section 2 and/or used different values of $Q'$. We then conducted a set of simulations starting from the same initial conditions as above. Although the results of individual runs were sensitive the assumed tidal model, which is expected from the stochasticity of planet scattering, the overall statistical results were very similar to those discussed above. This shows that the origin of hot Jupiters is insensitive of details of the tidal interaction, at least for the adopted model in which the tidal time lag scales inversely with the mean motion.

### 3.6. Comparison with Observed Hot Jupiters

To compare our results with observations, we should ideally follow each planetary system for the estimated age of its host star. This is not practical, however, because the analysis of thousands of synthetic systems and dozens of different times would be complicated. We opted for a simplified comparison instead. First we continued the orbits of hot Jupiters, using our semi-analytical method, to 1 Gyr. This is the average age of stars with known hot Jupiters in retrograde orbits (e.g., Triaud 2011). Unlike in Sections 3.4 and 3.5., however, the hot Jupiters that reached the Roche radius of the star before 1 Gyr were removed. The final distribution that we obtained here should therefore be characteristic of “aged” planetary systems.

Results are shown in the left-hand plots of Figure 12, where gray open circles reproduce the data shown in Figure 7, while the black filled circles show only those planets that survive at 1 Gyr. Notably, most hot Jupiters in Pop-I disappear as they evolve by tides and are engulfed by the star. Conversely, the orbit distribution of Pop-II Jupiters does not change much. While

![Figure 11](image-url)  
**Figure 11.** Rare example of a planetary system showing clear effects of the Lidov–Kozai resonance. A Pop-II hot Jupiter formed in this simulation. Planetary masses were $m_1 = 1 \, M_{\text{Jup}}$, $m_2 = 2 \, M_{\text{Jup}}$, and $m_3 = 1 \, M_{\text{Jup}}$. (A color version of this figure is available in the online journal.)

![Figure 12](image-url)  
**Figure 12.** Left: orbit distribution of hot Jupiters that we obtained starting from the three-planet systems. Gray open circles show the transient population of orbits at $10^9$ yr, or the moment when the orbit of the hot Jupiter became circularized by tides (defined as $e < 0.01$). Black circles show orbits of planets that survived the tidal decay for $T = 10^9$ yr, regardless of their eccentricity. Right: comparison between the synthetic population (black circles) and known hot Jupiters with measured $|\lambda|$ (orange squares). (A color version of this figure is available in the online journal.)

4 Recall that previously the orbital evolution was stopped when the eccentricity reached $e = 0.01$, independently of the timescale when this occurred.
Pop-I hot Jupiters should therefore be expected only around young stars. Pop-II hot Jupiters are more permanent.

The right-hand plots in Figure 12 show the orbit distribution of planets at 1 Gyr (open black circles), while the orange squares show that of the hot Jupiters with measured $\lambda$ as of 2012 February. The distribution of semimajor axes and eccentricities of hot Jupiters produced in our simulations closely matches observations. There is a clear trend, both in our results and observations, that hot Jupiters with larger semimajor axes tend to have larger eccentricities. We discussed this in Section 2.4.

In our simulations, the correlation between final semimajor axes and eccentricities appear as a well-defined curve, while the real hot Jupiters seem to be more spread on both sides. This spread may be produced by uncertainties in the estimation of planetary eccentricities, or by a spread of $Q_p$ values of real exoplanets. If this were indeed the case, then Figure 6 seems to indicate possible values of $Q_p$ between $10^5$ and $10^7$. Finally, with the Pop-I now completely removed, our simulations show no hot Jupiters with $a < 0.015$ AU, exactly as observed.

For real planets we assumed that the measured $\lambda$ can be interpreted as the orbital tilt, and plot it together with the orbital inclinations of hot Jupiters obtained in our simulations. Note $\lambda$ is the projected misalignment angle; a small value of $\lambda$ therefore does not guarantee that star’s spin vector and planet’s orbit normal are actually aligned.\(^5\)

The distributions of both $I$ and $|\lambda|$ are broad, covering the whole range from 0° to 180° (Figure 12, bottom-right panel), which is encouraging. There are several, potentially important differences between these two distributions as well. For example, the inclination distribution of surviving hot Jupiters that we obtain in our simulations shows the vast majority of prograde orbits, while the measured values of $\lambda$ indicate that a relatively large fraction ($\sim 20\%$) of hot Jupiters are retrograde. Second, our simulations show a substantial lack of hot planets with very low values of $\theta_0$. Interestingly, however, this problem may be resolved when longer evolution timescales are considered (Section 5).

4. FOUR-PLANET SYSTEMS

Here we consider the planetary systems with four planets initially. We used the method described in Section 3.3 to set up the initial orbits and masses of four planets. Specifically, the masses of planets were chosen according to Equation (12), with an additional restraint that the lightest planet has mass $>0.4 M_{\text{Jup}}$. In total, 2195 planetary systems were followed. Of these, 498 produced hot Jupiters (as defined by final $a < 0.1$ AU), including both Pop-I and Pop-II. This is 23%, a fraction twice as high as the one obtained with the three-planet systems.

The eccentricity and inclination distribution of exoplanets is shown in Figure 13, where we have already separated cases according to the final number of planets in each system. These results were obtained by numerically integrating the systems for $T = 10^8$ yr, and extending these simulations to $T = 10^9$ yr using our semi-analytical model. We show the final orbits of all planets at $T = 10^9$ yr or, if a hot Jupiter reached $e < 0.01$, we show the orbits at that time instant.

As in the three-planet simulations discussed above, we find two distinct populations of hot Jupiters: Pop-I with $a \leq 0.03$ AU, and Pop-II with $a > 0.03$ AU. As before, the Pop-II planets form in the systems that end up with two planets (two planets are ejected in this case), while planet ejection was less common in those systems that produced Pop-I. Interestingly, however, the inclination distribution of Pop-II hot Jupiters is now broader and includes a larger fraction of retrograde orbits (per original system) than in the three-planet case.

A detailed analysis of individual four-planet simulations shows similar mechanisms at work as for the three-planet systems (see Section 3.5). The orbit evolutions are complex, show a dominant effect of planetary encounters and slow secular interactions that are typically not related to the Lidov–Kozai resonance.

Finally, the left-hand panels in Figure 14 show the final orbits of hot Jupiters at 1 Gyr. This result confirms those obtained with the three-planet systems. The Pop-I hot Jupiters do not generally survive, while the orbits in Pop-II do not change much at late stages. Both the semimajor axis and eccentricity distribution of final orbits show a good match to observations. The match to the observed 3 day period pile-up of hot Jupiters is particularly good.

Unlike in the three-planet case, the inclination (and stellar obliquity) distribution obtained with four initial planets seem to show a larger dispersion, even though at 1 Gyr the ratio of retrograde over total hot planets is about the same ($\sim 10\%$). Also, some of the retrograde orbits now have $I > 150^\circ$, as required to match several known systems with $|\lambda| \gtrsim 150^\circ$. For

\(^5\) To compare simulations and observations more precisely, we would need to generate orbit normal vectors having an inclination distribution that our model predicts, and random orientation of nodes. This distribution should then be projected to the observer plane, and compared with observed distributions of $\lambda$ (e.g., Fabrycky & Winn 2009).
comparison, observations indicate that \( \sim 20\% \) of hot Jupiters have \( |\lambda| > 90^\circ \). It is not clear at this point whether the difference between simulated 10% and observed 20% is significant, mainly because the observational statistics is still pretty low.

5. LONG-TERM EVOLUTION

Here we discuss how the orbital distribution of hot Jupiters changes over Gyr-long timescales. This is relevant because the stars with known hot Jupiters typically have ages > 1 Gyr. If we want to compare the results of our simulations with observations we must therefore understand how the simulated distributions evolve on long timescales.

Using the semi-analytical model described in Section 2.2 we followed the evolution of each planetary system for up to 10 Gyr. This revealed two principal effects. First, the semimajor axis distribution of hot Jupiters changes because as we extend our simulations to longer timescales the tides have more time to act, which leads to a progressive depletion of orbits with \( a < 0.03 \) AU (Figure 15).

Second, tides drive \( \theta_0 \) toward 0, such that the older systems are expected to have lower \( \theta_0 \) than the younger systems. This effect can explain the RM observations of hot Jupiters that indicate that older systems generally have \( \theta_0 < 20^\circ \) (Triaud 2011). We find here, however, that the realignment process is long and, even for \( t_{\text{age}} = T > 3 \) Gyr, many stars should still be misaligned, with some fraction still retrograde. This problem is not new and not restricted to the present study. Recently, Lai (2011) proposed that perhaps the tidal time lag \( \Delta t \) should not be independent of the tidal frequency, but be more effective in the spin than in the orbital evolution.

Figure 16 compares the expected distribution of \( \theta_0 \) with that of the projected misalignment angle, \( |\lambda| \), obtained from the RM measurements. The three-planet case shows the distribution of \( \theta_0 \) that peaks between \( 20^\circ \) and \( 40^\circ \), while the observed \( \lambda \) distribution peaks at \(<20^\circ \). While the projection effects can be responsible for this mismatch, it is also possible that the population of hot Jupiters with \( \lambda < 20^\circ \) have a different origin. A detailed investigation of this problem goes beyond the scope of this paper.

The four-planet case shows a better correspondence to observations (Figure 16, right panel), especially for \( T = 3 \) Gyr (red curve). In this case, the distribution of \( \theta_0 \) peaks below \( 20^\circ \) and shows a tail that extends to \( \theta_0 > 90^\circ \), as observed in the measured distribution of \( \lambda \). We therefore find that the four-planet case with long \( T \) may provide a better fit to observations than the three-planet case and/or shorter \( T \). This tentative conclusion will need to be tested in more detail, including a model for the projection effects, and the dependence on the initial conditions and tidal model.

Figure 17 shows our expectations for the semimajor axis distribution changes with the system’s age. Interestingly, the distribution in the three-planet case shows important changes with \( T \) because hot Jupiters with orbital periods shorter than...
model in which hot Jupiters form by planetary scattering and up that closely fits the observations. This gives support to our model is capable of producing a 3 day pile-up especially well.

We modified the standard equilibrium tidal model to include an ad hoc factor in the tidal parameters dependence of the time lag with respect to the orbital distance are mainly important for orbits with small pericenter distance. Tides (both conservative and dissipative terms), all of which are mainly important for orbits with small pericenter distance. We modified the standard equilibrium tidal model to include a dependence of the time lag with respect to the orbital distance plus an ad hoc factor in the tidal parameters $Q'$ to mimic the effects of dynamical tides for $e \sim 1$.

We found that scattering and subsequent slow secular interaction between planets generate a number of planets in quasi-parabolic orbits that tidally evolve. A fraction of these planets can survive over Gyr timescales. Several aspects of the surviving population provide a good match to observations, including the 3 day period pile-up, nearly circular orbits for $a<0.03$ AU, correlation between $a$ and $e$ for orbits with larger $a$, and the existence of a retrograde population. Other aspects, such as the ratio between retrograde over prograde planets, or the timescale for spin–orbit realignment are not so satisfactory and require further work.

In a scenario dominated by global instability, the dominant evolution path leading to the formation of hot Jupiters appears to be close encounters between the planets. The vast majority of hot Jupiters in our simulations acquire small pericentric distances (and in some cases retrograde orbits) by being scattered by other planets and/or during the subsequent phase of slow secular evolution (typically unrelated to the Kozai resonance).

We find that $\approx 10\%$ of planetary systems starting with three planets produce hot Jupiters, while this ratio increases to $\approx 23\%$ if four planets are considered. However, most of these are eliminated by subsequent tidal decay for timescales of the order of 1 Gyr. The proportion of surviving hot planets drop to $\approx 2\%$ for three-planet systems and $\approx 5\%$ for our four-planet runs.

In both cases, we find that hot Jupiters can be divided into two populations. The transient Pop-I hot Jupiters typically form in the systems where no (in the three-planet case) or up to one planet (in the four-planet case) is ejected, have $a<0.03$ AU, and a very broad inclination distribution, including a large fraction of retrograde orbits. These planets continue to evolve tidally and generally do not survive 1 Gyr. They are expected to be present only around younger stars.

The Pop-II hot Jupiters form in the systems where one (in the three-planet case) or two planets are ejected (in the four-planet case), have $a>0.03$ AU, and a smaller fraction of retrograde orbits. The Pop-II hot Jupiters are far enough from the central star to have only minimal tidal evolution, and generally survive on Gyr-long timescales. They provide the best match to the observed population of hot Jupiters.

Our simulations also show that the formation efficiency of a hot Jupiter is $2.5\%$–$5\%$ per one unstable system with three or four giant planets. The other $>95\%$ of systems that go unstable either do not form a hot Jupiter or form a hot Jupiter that tidally evolves and is swallowed by the host star. We also found that, independently of what exactly happens in the simulations, there is always at least one giant planet in each system in the end with $a<5$ AU. Assuming that such a planet would be detected by the radial velocity (RV) measurements, we can make several interesting predictions about the formation of giant planets and their evolution.

First of all, RV observations indicate that $\sim 10\%$ of solar-type stars have a giant planet within the detectability limits (conservatively, $a<5$ AU for a Jupiter-mass planet, e.g., Mayor et al. 2011). The giant planets commonly have large eccentricities, which is thought to be the result of instabilities and planetary scattering that happened in the giant planet systems (Weidenschilling & Marzari 1996; Rasio & Ford 1996; Chatterjee et al. 2008; Juric & Tremaine 2008). To explain the eccentricity distribution it must be assumed that most systems go unstable because if the unstable systems were in minority the eccentricity distribution should be strongly skewed toward zero (as planets form on nearly circular orbits).

Together, our simulations and observations therefore indicate that the systems with three or more giant planets should form around $\sim 10\%$ of solar-type stars (Mayor et al. 2011) and that most of these systems should evolve through a dynamical instability. This applies only to the systems of giant planets where at least one giant planet starts with $a<5$ AU (as in our simulations; close systems in the following), because we know from the simulations that at least one planet will also end up with $a<5$ AU. We do not have constraints on the formation

3 days get eliminated by tides. The agreement with observations improves with increasing $T$.

The simulated four-planet distributions fit data better than those obtained in the three-planet case, but the difference is minor and will need further scrutiny. In any case, Figure 17 illustrates that our model is capable of producing a 3 day pile-up that closely fits the observations. This gives support to our model in which hot Jupiters form by planetary scattering and tides.

6. DISCUSSIONS AND CONCLUSIONS

We report a series of simulations in which we followed the orbital and spin evolution of planetary systems starting with three and four planets. The planets were initially placed in resonant orbits as expected from their formation and migration in the protoplanetary gas disk. The instability was triggered in each of these systems by breaking the resonant locks. Soon after the onset of instability, planets scatter each other and typically obtain large orbital eccentricities and inclinations.

In addition to the gravitational interactions between planets, we also included the effects of relativity, stellar oblateness, and tides (both conservative and dissipative terms), all of which are mainly important for orbits with small pericenter distance. We modified the standard equilibrium tidal model to include a dependence of the time lag with respect to the orbital distance plus an ad hoc factor in the tidal parameters $Q'$ to mimic the effects of dynamical tides for $e \sim 1$.

We found that scattering and subsequent slow secular interaction between planets generate a number of planets in quasi-parabolic orbits that tidally evolve. A fraction of these planets can survive over Gyr timescales. Several aspects of the surviving population provide a good match to observations, including the 3 day period pile-up, nearly circular orbits for $a<0.03$ AU, correlation between $a$ and $e$ for orbits with larger $a$, and the existence of a retrograde population. Other aspects, such as the ratio between retrograde over prograde planets, or the timescale for spin–orbit realignment are not so satisfactory and require further work.

In a scenario dominated by global instability, the dominant evolution path leading to the formation of hot Jupiters appears to be close encounters between the planets. The vast majority of hot Jupiters in our simulations acquire small pericentric distances (and in some cases retrograde orbits) by being scattered by other planets and/or during the subsequent phase of slow secular evolution (typically unrelated to the Kozai resonance).

We find that $\approx 10\%$ of planetary systems starting with three planets produce hot Jupiters, while this ratio increases to $\approx 23\%$ if four planets are considered. However, most of these are eliminated by subsequent tidal decay for timescales of the order of 1 Gyr. The proportion of surviving hot planets drop to $\approx 2\%$ for three-planet systems and $\approx 5\%$ for our four-planet runs.

In both cases, we find that hot Jupiters can be divided into two populations. The transient Pop-I hot Jupiters typically form in the systems where no (in the three-planet case) or up to one planet (in the four-planet case) is ejected, have $a<0.03$ AU, and a very broad inclination distribution, including a large fraction of retrograde orbits. These planets continue to evolve tidally and generally do not survive 1 Gyr. They are expected to be present only around younger stars.

The Pop-II hot Jupiters form in the systems where one (in the three-planet case) or two planets are ejected (in the four-planet case), have $a>0.03$ AU, and a smaller fraction of retrograde orbits. The Pop-II hot Jupiters are far enough from the central star to have only minimal tidal evolution, and generally survive on Gyr-long timescales. They provide the best match to the observed population of hot Jupiters.

Our simulations also show that the formation efficiency of a hot Jupiter is $2.5\%$–$5\%$ per one unstable system with three or four giant planets. The other $>95\%$ of systems that go unstable either do not form a hot Jupiter or form a hot Jupiter that tidally evolves and is swallowed by the host star. We also found that, independently of what exactly happens in the simulations, there is always at least one giant planet in each system in the end with $a<5$ AU. Assuming that such a planet would be detected by the radial velocity (RV) measurements, we can make several interesting predictions about the formation of giant planets and their evolution.

First of all, RV observations indicate that $\sim 10\%$ of solar-type stars have a giant planet within the detectability limits (conservatively, $a<5$ AU for a Jupiter-mass planet, e.g., Mayor et al. 2011). The giant planets commonly have large eccentricities, which is thought to be the result of instabilities and planetary scattering that happened in the giant planet systems (Weidenschilling & Marzari 1996; Rasio & Ford 1996; Chatterjee et al. 2008; Juric & Tremaine 2008). To explain the eccentricity distribution it must be assumed that most systems go unstable because if the unstable systems were in minority the eccentricity distribution should be strongly skewed toward zero (as planets form on nearly circular orbits).

Together, our simulations and observations therefore indicate that the systems with three or more giant planets should form around $\sim 10\%$ of solar-type stars (Mayor et al. 2011) and that most of these systems should evolve through a dynamical instability. This applies only to the systems of giant planets where at least one giant planet starts with $a<5$ AU (as in our simulations; close systems in the following), because we know from the simulations that at least one planet will also end up with $a<5$ AU. We do not have constraints on the formation
efficiencies and evolution histories of giant planet systems in which all planets start with $a > 5$ AU (distant systems).

Interestingly, this also implies that the initially close systems with one or two giant planets should be relatively uncommon because these systems cannot evolve in such a way to explain the observed large eccentricities. Therefore, the formation of gas-giant planets may be regulated by some sort of a step process where most protoplanetary disk do not form giant planets, but those that do, typically form many. This unexpected feature needs to be explained by planet formation models.

The observations indicate that about 0.5%–1% of solar-type stars have hot Jupiters. Therefore, the ratio of all Jupiters to hot Jupiters that can be inferred from observations is $r_o \sim 10–20$. From our simulations, we find that $r_s \sim 20–40$. Therefore, $r_o/r_s \sim 1–4$. While this is a relatively reasonable agreement, the comparison indicates that the formation efficiency of Jupiters in our simulations may be a factor of few lower than needed. Alternatively, some part (perhaps as much as $\sim 50\%$) of the observed hot-Jupiter population can be related to the migration of giant planets in the protoplanetary gas nebula (Benítez-Llambay et al. 2011).

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REFERENCES

Barker, A. J., & Ogilvie, G. I. 2009, MNRAS, 395, 2268
Benítez-Llambay, P., Masset, F., & Beaugé, C. 2011, A&A, 528, A2
Beutler, G. 2005, Methods of Celestial Mechanics (Berlin: Springer)
Chambers, J. E. 1999, MNRAS, 304, 793
Chatterjee, S., Ford, E. B., Matsumura, S., & Rasio, F. A. 2008, ApJ, 686, 580
Correia, A. C. M., Laskar, J., Farago, F., & Boué, G. 2011, Celest. Mech. Dyn. Astron., 111, 105
Correia, A. C. M., Laskar, J., & Néron de Surgy, O. 2003, Icarus, 163, 1
Darwin, G. H. 1879, Observatory, 3, 79
Deleuil, M., Bonomo, A. S., Ferraz-Mello, S., et al. 2012, A&A, 538, A145
Duncan, M. J., Levison, H. F., & Lee, M. H. 1998, AJ, 116, 2067
Fabrycky, D. C., & Winn, J. N. 2009, ApJ, 696, 1230
Ferraz-Mello, S., Rodríguez, A., & Hussman, H. 2008, CeMDA, 101, 171
Gladman, B. 1993, Icarus, 106, 247
Hansen, B. M. S. 2010, ApJ, 723, 285
Hut, P. 1981, A&A, 99, 126
Ivanov, P. B., & Papaloizou, J. C. B. 2004, MNRAS, 347, 437
Ivanov, P. B., & Papaloizou, J. C. B. 2007, MNRAS, 376, 682
Ivanov, P. B., & Papaloizou, J. C. B. 2011, Celest. Mech. Dyn. Astron., 111, 51
Jackson, B., Greenberg, R., & Barnes, R. 2008, ApJ, 678, 1396
Juric, M., & Tremaine, S. 2008, ApJ, 686, 603
Kley, W. 2003, Celest. Mech. Dyn. Astron., 87, 85
Kozai, Y. 1962, AJ, 67, 591
Lai, D. 1997, ApJ, 490, 847
Lai, D. 2011, MNRAS, submitted (arXiv:1109.4703v1)
Lai, D., Foucart, F., & Lin, D. N. C. 2011, MNRAS, 412, 2790
Laskar, J., & Robutel, P. 1995, Celest. Mech. Dyn. Astron., 62, 193
Levison, H. F., & Duncan, M. J. 2000, ApJ, 120, 2117
Libert, A.-S., & Henrand, J. 2007, Icarus, 191, 469
Libert, A.-S., & Tsiganis, K. 2011a, MNRAS, 412, 2353
Libert, A.-S., & Tsiganis, K. 2011b, Celest. Mech. Dyn. Astron., 111, 201
Lidov, M. L. 1961, Isk. Sput. Zemli, 8, 119
Marchal, C., & Bozis, G. 1982, Celest. Mech. Dyn. Astron., 26, 311
Mardling, R. A. 2007, MNRAS, 382, 1768
Mardling, R. A., & Lin, D. N. C. 2002, ApJ, 573, 829
Marzari, F., & Weidenschilling, S. J. 2002, Icarus, 156, 570
Masset, F. 2000, A&AS, 141, 165
Matsumura, S., Peale, S. J., & Rasio, F. A. 2010, ApJ, 725, 1995
Mayor, M., Marmier, M., Lovis, C., et al. 2011, A&A, submitted (arXiv:1109.2497v1)
McArthur, B. E., Benedict, G. F., Barnes, R., et al. 2010, ApJ, 715, 1203
Mignard, F. 1979, Moon Planets, 20, 301
Mignard, F. 1980, Moon Planets, 23, 185
Moeckel, N., & Throop, H. B. 2009, ApJ, 707, 268
Morbidelli, A., Tsiganis, K., Crida, A., Levison, H. F., & Gomes, R. 2007, AJ, 134, 1790
Moutou, C., Díaz, R. F., Udry, S., et al. 2011, A&A, 533, A113
Nagasawa, M., & Ida, S. 2011, ApJ, 742, 72
Nagasawa, M., Ida, S., & Bessho, T. 2008, ApJ, 678, 498
Naoz, S., Farr, W. M., Lithwick, Y., Rasio, F. A., & Tenissender, J. 2011, Nature, 473, 187
Pierens, A., & Nelson, R. P. 2008, A&A, 482, 333
Rasio, F. A., & Ford, E. B. 1996, Science, 274, 954
Schlaufman, K. C. 2010, ApJ, 719, 602
Skumanich, A. 1972, ApJ, 171, 565
Snellgrove, M. D., Papaloizou, J. C. B., & Nelson, R. P. 2001, A&A, 374, 1092
Thommes, E. W., Bryden, G., Wu, Y., & Rasio, F. A. 2008, ApJ, 675, 1538
Throop, H. B., & Bally, J. 2008, AJ, 135, 2380
Triaud, A. H. M. J. 2011, A&A, 534, L6
Weidenschilling, S. J., & Marzari, F. 1996, Nature, 384, 619
Winn, J. N., Fabrycky, D., Albrecht, S., & Johnson, J. A. 2010, ApJ, 718, L145
Wu, Y., & Lithwick, Y. 2011, ApJ, 735, 109
Wu, Y., & Murray, N. 2003, ApJ, 589, 605