THEORETICAL ASPECTS OF GRAVITATIONAL LENSING IN TeVeS

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1. INTRODUCTION

While Newtonian gravitational theory was applied to the extragalactic region, it no longer passed the trials as easily as it had done in the solar system. Within the Coma and Virgo Clusters the velocities of individual galaxies are so large that the total mass must exceed the sum of the masses of the shining galaxies if the clusters are gravitationally bound systems (Zwicky 1933; Smith 1936). The 21 cm emission line from $H_1$ of spiral galaxies also shows a need for extra masses besides visible stars (van Albada et al. 1985; Begeman 1989). Moreover, X-ray observations and strong gravitational lensing in early-type galaxies and clusters imply that there is $\sim 60\%$–$85\%$ of the mass in some undetected form (Fabbiano et al. 1989; Böhinger 1995; Loewenstein & White 1999). This so-called missing mass problem may be explained in two different ways: the existence of dark matter and a modification of Newton’s law and hence of general relativity (GR). The most famous representative of the modified gravity theory party is modified Newtonian dynamics (MOND), proposed by Milgrom in 1983 (Milgrom 1983a, 1983b, 1993): $\mu(a_0) = -\nabla \Phi_N$.

Here $\Phi_N$ is the usual Newtonian potential of the visible matter and $a_0 \approx 1 \times 10^{-10}$ m s$^{-2}$ from the empirical data, such as the Tully-Fisher relation and rotation curves (Sanders & Verheijen 1998); $\mu(x) \approx x$ for $x \ll 1$, and $\mu(x) \rightarrow 1$ for $x \gg 1$. In the solar system, where accelerations are strong compared to $a_0$, formula (1) is just Newton’s second law $a = -\nabla \Phi_N$.

From this simple formulation the Tully-Fisher relation and flat rotation curves follow trivially (Sanders & McGaugh 2002). The rotation curves of spiral galaxies, especially of low surface brightness (LSB) galaxies, can be explained extremely well (Begeman et al. 1991; Sanders 1996; McGaugh & de Blok 1998; Sanders & Verheijen 1998). Moreover, the dearth of dark matter in some ordinary elliptical galaxies can be intrinsically explained in the MOND paradigm (Milgrom & Sanders 2003). Even though many such phenomena in galaxy systems can be explained by this simple modification with an astonishing precision, the MOND paradigm lacked a flawless relativistic gravitational theory for two decades (Bekenstein & Milgrom 1984; Bekenstein 1988; Sanders 1988, 1997) and thus could not address the lensing and other relativistic gravitational observations. However, since the end of 2004 March, the embarrassment has been removed by Bekenstein (2004), who finally contrived a Lorentz covariant relativistic gravitational theory called TeVeS, which is based on three dynamical gravitational fields: an Einstein metric $g_{\mu\nu}$, a vector field $\mathbf{U}_\mu$, and a scalar field $\phi$ (the acronym TeVeS is for this tensor-vector-scalar content). In this scheme, TeVeS contains MOND for nonrelativistic dynamics, agrees with the solar system measurements for the $\beta$ and $\gamma$ parameterized post-Newtonian (PPN) coefficients, and avoids superluminal propagation of the metric, scalar, and vector waves. Moreover, concerning cosmology, it not only has cosmological models that are similar to that
of GR but also overcomes the Silk damping problem of the cold dark matter (CDM)-dearth universe with the help of the scalar field (Skordis et al. 2005). Now we can investigate the relativistic MOND paradigm with many observations and experiments (Giannios 2005; Skordis et al. 2005; Zhao et al. 2005).

On the other hand, the GR with dark matter (DM) paradigm, despite its success for the large-scale structure and the cosmic microwave background (CMB; see, e.g., Spergel et al. 2003), has some problems on galactic scales. For example, the dark matter halos, if they really exist, must have a shallower core than the universal Navarro-Frenk-White (NFW) density profile predicts (cuspier core problem; see, e.g., Navarro & Steinmetz 2000; Romanowsky et al. 2003); the disk size will be too small, considering the angular transformation from a baryonic to a dark matter halo (angular momentum catastrophe; see, e.g., Navarro et al. 1995; Moore et al. 1999; Maller & Dekel 2002; Kazantzidis et al. 2004); and the number of subhalos predicted by nonbaryonic dark matter halo (substructure problems; see, e.g., Kauffmann et al. 1993; Maller & Dekel 2002; Kazantzidis et al. 2004). Despite these problems, the GR with DM paradigm is less falsifiable and supported by a larger group because of its unlimited choice of models and parameters.

In order to judge which side better accounts for the phenomena of the natural world, the disparity between the TeVeS and GR with DM paradigms has to be closely investigated. This distinction will be important for the physics of MOND and the “missing mass” problem. Gravitational lensing (GL) phenomena provide us an opportunity, because they dominate on scales that even surpass the size of dark matter halos (Kochanek 2005).

For a long time, light bending has been a great challenge for modified gravity. Any scalar field jointly coupling with the Einstein metric via a conformal transformation would yield the same optical phenomena as in the standard Einstein metric. Thus, in the context of MOND, where little dark matter exists, it is hard to produce the anomalously strong deflection of photons (Bekenstein & Sanders 1994; Soussa & Woodard 2003, 2004). However, this problem can be solved by postulating a disformal relation between the physical metric and the Einstein metric (Sanders 1997). This is what Sanders’s stratified theory and then TeVeS take.

In this paper we discuss the theoretical aspects of gravitational lensing (GL) systems within the framework of TeVeS. In §2 we summarize the characteristics of TeVeS and introduce a cosmological model within which gravitational lensing phenomena are discussed. In §3 we derive the deflection angle in the general form and its two particular limits: the Newtonian and the MOND regimes. From this we obtain lens equations, which are the foundation of the other GL phenomena. In §4 we apply the standard time delay formalism in TeVeS. Finally, §5 summarizes the presuppositions of this paper and discusses some possible conclusions of our results in §§3 and 4.

2. FUNDAMENTALS OF TeVeS
AND THE COSMOLOGICAL MODEL

Before introducing the action of TeVeS, we should explain some important features of the disformal metric relation. In Bekenstein’s tensor-vector-scalar theory (TeVeS), the role of Einstein’s metric $g_{\alpha\beta}$ is replaced by the physical metric (Bekenstein 2004)

$$\tilde{g}_{\alpha\beta} = e^{-2\phi}(g_{\alpha\beta} + \mathbf{U}_\alpha \mathbf{U}_\beta) - e^{2\phi} \mathbf{U}_\alpha \mathbf{U}_\beta,$$

with a normalization condition on the vector field

$$g^\alpha{}{}^\beta \mathbf{U}_\alpha \mathbf{U}_\beta = -1. \tag{3}$$

The physical metric, which governs the dynamical behavior of matter (the equivalence principle, i.e., minimal coupling) is simultaneously influenced by a 4-vector field $\mathbf{U}_\alpha$, a scalar field $\phi$, and the Einstein’s gravity $g_{\alpha\beta}$.

From the theoretical viewpoint, if DM does not exist in galaxies on a cluster scale, the concept of geodesics must be different from that of GR. Although there is no deeper theoretical basis for the disformal relation (2), empirical tests like lensing effects will be important to justify this choice (Sanders 1997).

The cosmological model, which determines the background spacetime structure, is an essential ingredient in gravitational lensing (Schneider et al. 1992; Petters et al. 2001). On the one hand, the distance in the gravitational lensing scenario would vary under different cosmological models. On the other hand, gravitational lensing provides a tool to determine $H_0$ and to constrain the cosmological parameters (Blandford & Narayan 1986; Schneider 1996). Since we have used an alternative gravitational theory, it is important to discuss its effects on cosmology, which is solely influenced by gravity. In this section, we introduce Bekenstein’s relativistic MOND theory and then describe the revised Friedman model.

2.1. Actions of TeVeS

The conventions in this paper are the metric signature +2 and units with $c = 1$.

There are three dynamical fields $g_{\alpha\beta}, \mathbf{U}_\alpha$, and $\phi$ built in TeVeS. In addition to these fields, there is another nondynamical scalar field $\sigma$, which is introduced into the scalar field action to eliminate the superluminal problem of $\phi$ (Bekenstein 2004).

By varying the actions for these fields, the respective equations of motion for the scalar fields, vector fields, and the Einstein metric can be arrived at; therefore, the physical metric is determined.

The action in TeVeS can be divided into four parts, $S = S_g + S_\sigma + S_\phi + S_m$. The geometric action,

$$S_g = (16\pi G)^{-1} \int g^{\alpha\beta} R_{\alpha\beta}(-g)^{1/2} d^4x, \tag{4}$$

is identical to the Einstein-Hilbert action in GR and is used to produce the Einstein metric $g_{\alpha\beta}$. The matter action coupling to the scalar field $\phi$ and vector field $\mathbf{U}_\alpha$ through $\tilde{g}_{\alpha\beta}$ is taken to be

$$S_m = \int \mathcal{L}(-\tilde{g})^{1/2} d^4x, \tag{5}$$

where $\mathcal{L}$ is a Lagrangian density for the fields under consideration and should be considered as a functional of the physical metric and its derivatives.

The vector action with the form

$$S_\mathbf{U} = -\frac{K}{32\pi G} \int \left[ g^{\alpha\beta} g^{\mu\nu} \mathbf{U}_{(\alpha\beta)} \mathbf{U}_{(\mu\nu)} - 2 \left(\frac{\lambda}{K}\right) (g^{\alpha\nu} \mathbf{U}_\alpha \mathbf{U}_\mu + 1) \right] (-g)^{1/2} d^4x \tag{6}$$

yields the vector field equation. Here, $K$ is a dimensionless parameter, and $\lambda$ is a Lagrange multiplier. Moreover, $\mathbf{U}_{(\alpha\beta)}$ means $\mathbf{U}^\kappa_{(\alpha} \mathbf{U}_{\beta)}$.

The action of the two scalar fields is taken to have the form

$$S_\sigma = -\frac{1}{2} \int \left[ \sigma^2 h^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + \frac{1}{2} G \sigma^2 F(kG\sigma^2) \right] (-g)^{1/2} d^4x, \tag{7}$$
where \( \eta^{\alpha \beta} \equiv g^{\alpha \beta} - \mathbf{U} \mathbf{U}^T \); \( F \) is a free function in order to produce the dynamical behaviors of MOND, which needs to be constrained empirically from the cosmological model; \( l \) is a constant length; and \( k \) is another dimensionless parameter in this theory. In the FRW-like cosmological models, \( k \) is smaller than \( 10^{-2} \) (Bekenstein 2004; Skordis et al. 2005). Variation of \( \sigma \) of the action gives

\[
-\mu F(\mu) - \frac{1}{2} \mu^2 F'(\mu) = y,
\]

where \( y = kl^2 h^\phi \phi, \phi, \phi \) is a free function of \( \mu \) equivalent to \( F \). However, in later discussions, it is more convenient to consider \( \mu(y) \) as a function of \( y \). Variation of \( \phi \) of the action then gives

\[
(\mu(y) h^{\alpha \beta})_{\gamma \beta} = kG \left[ (1 + e^{-4\phi}) \mathbf{U} \mathbf{U}^T \right] \delta_{\alpha \beta}.
\]

As \( \sigma \), through \( \mu = kG\sigma^2 \), can be expressed in terms of \( \phi \) explicitly, equation (9) involves \( \phi \) only.

To illustrate our ideas, in this work we adopt the free function suggested by Bekenstein (2004),

\[
y = \frac{3}{4} \mu(x^2 - 2)^2 (1 - \mu^{-1}),
\]

whence \( y \) approaches zero asymptotically as \( y \sim 3\mu^2 \) when \( \mu \rightarrow 0 \) and \( y \rightarrow \infty \) when \( \mu \rightarrow 1 \).

We discuss gravitational lensing of a point mass in §§ 3 and 4. Thus, we are interested in static and spherically symmetric geometry. In this case, \( \mu \) runs from 1 (the Newtonian regime) to 0 (the deep MOND regime). In order to satisfy the behavior of the potential at these limits, we require \( \mu \rightarrow \mu^2 \rightarrow \infty \) when \( \mu \rightarrow 0 \) and \( y \rightarrow \infty \) as \( \mu \rightarrow 1 \). In the intermediate regime, \( y(\mu) \) is arbitrary. Nonetheless, the exact functional form does not affect our results, because we do not consider the intermediate regime.

2.2. The Cosmological Model

As pointed out by Bekenstein (2004), the physical metric of FRW cosmology in TeVeS is obtained by replacing \( dt \rightarrow e^{\phi} dt \) and \( a \rightarrow e^{-2\phi} \) in the primitive Robertson-Walker metric. Here, the Friedmann equation becomes

\[
\frac{\dot{a}}{a} = e^{-\phi} \left( \frac{\dot{a}}{a} - \phi \right),
\]

where \( \dot{a}/a \) and \( a/\dot{a} \) are essentially equal in the whole history; because \( e^{-\phi} \sim 1 \) and \( \phi \sim 0 \) (Bekenstein 2004; Hao & Akhoury 2005).

Since we have the additional scalar fields and vector fields in Bekenstein’s picture, the Einstein equation, which tells us the geometric properties, is not only determined by the energy-momentum tensor as before, but also by the scalar and vector fields. Here, \( \phi, \sigma, \) and \( \mathbf{U} \), are assumed to partake of the symmetry of the spacetime. The vector field is considered to be parallel with the cosmological time, i.e., \( \mathbf{U} = \delta^{\mu}_{\nu} \), because there should be no preferred spatial direction (Bekenstein 2004). Moreover, if we apply Weyl’s postulate, which states that galaxies move along nonintersecting world lines and therefore the spatial components in the energy-momentum tensor can be neglected (Weyl 1923), then only the \( tt \) component of Einstein’s equation remains:

\[
G_{\phi} = 8\pi G \left( \rho e^{-2\phi} + \sigma^2 \phi^2 \right) + \frac{2\pi}{k^2 t^2} \mu^2 F(\mu) + \Lambda
\]

\[
= 8\pi G \rho e^{-2\phi} + \frac{4\pi}{k^2 t^2} \left[ \frac{3}{2} \mu^2 F(\mu) + \frac{1}{2} \mu^2 F'(\mu) \right] + \Lambda.
\]

The appearance of \( \Lambda \) in equation (12) results from the choice of the free function \( F(\mu) \), which is decided by equations (8) and (10). It is also possible to construct another form of \( F(\mu) \) that makes the dark energy an effective phenomenon of the scalar field \( \phi \) (Hao & Akhoury 2005). However, before having any theoretical reason for the choice of the free function \( F(\mu) \), we would like to leave the interpretation alone and only treat it as a parameter in the cosmological model.

By the Robertson-Walker metric, which determines the \( tt \) component of the Einstein tensor in equation (12), we can get the modified Friedmann equation in a flat universe in the form

\[
\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho_M + \frac{8\pi G}{3} \rho_\phi + \frac{\Lambda}{3},
\]

where \( \rho_M \) is the matter density in Einstein’s picture and

\[
\rho_\phi = \frac{3}{2Gk^2t^2} \left[ \frac{3}{2} \mu^2 F(\mu) + \frac{1}{2} \mu^2 F'(\mu) \right]
\]

is the density due to the scalar field. For convenience, we can rewrite equation (13) in terms of \( \Omega \) density as

\[
H^2 = H_0^2 \left[ \Omega_m(z) + \Omega_\phi(z) + \Omega_\Lambda(z) \right].
\]

Since gravitational lensing must be observed within the framework of galaxies or clusters of galaxies, we only need to consider the cosmological model in the matter- and \( \Lambda \)-dominated era, so \( \Omega_m(z) \simeq \Omega_m(1 + z)^3 \). Furthermore, in order to avoid a significant effect of the integrated Sachs-Wolfe term on the CMB power spectrum, \( \Omega_\phi(z) \) must be extremely small compared with the matter density (Skordis et al. 2005). Thus, even if we do not know exactly how \( \Omega_\Lambda(z) \) evolves with \( z \), we can neglect it, because its value is overwhelmed by the uncertainty of \( \Omega_m \) and \( \Omega_\Lambda \) for a non-CDM universe (McGaugh 1999, 2004; Skordis et al. 2005).

To summarize, in TeVeS the angular distance at any redshift can be calculated by

\[
d_A \simeq \frac{(1 + z)^{-1}}{H_0} \int_0^z dz \left[ \Omega_m(1 + z)^3 + \Omega_\Lambda \right]^{-1/2}.
\]

We conclude that if there is any difference in the angular distance between GR and TeVeS, it arises from the low matter density universe in TeVeS rather than the influence of the scalar field \( \phi \).

Finally, it is worth noting that although it is not trivial to consider an inhomogeneous universe on the gravitational lens scale (Dyer & Roder 1973), we do not consider that case because we only deal with a single point-mass lens embedded in a background Friedmann universe in this paper.

3. GRAVITATIONAL LENSING IN TeVeS

When building gravitational lensing models, people usually use some approximations, such as the static and thin lens approximations. We also adopt the weak-field assumption, since all known gravitational lens phenomena are influenced by weak gravitational systems. Although the Schwarzschild lens (point-mass model) is not sufficient for gravitational lensing effects, we consider this for its simplicity. Moreover, it gives results of the same order of magnitude as those for more realistic lenses (Schneider et al. 1992).
3.1. Deflection Angle in Terms of Mass

In the weak-field limit, the physical metric of a static spherically symmetric system in TeVeS can be expressed in the isotropic form

\[ g_{\alpha \beta} \, dx^\alpha \, dx^\beta = -(1 + 2\Phi) \, dt^2 + (1 - 2\Phi) \left( \frac{\partial \Phi}{\partial \theta} + \dot{\theta}^2 \right) d\theta^2 + \sin^2 \theta \, d\phi^2 \],

(17)

where \( \Phi = \Xi \Phi_N + \phi \) and \( \Xi = e^{-\phi_c} (1 + K/2)^{-1} \), where \( \phi_c \) is the asymptotic boundary value of \( \phi \) and \( K < 10^{-5} \) from the constraint of PPN parameters (Bekenstein 2004).

Considering a quasi-static system with a perfect fluid, the scalar equation (9) can be reduced to

\[ \nabla \cdot \left[ \mu(y) \nabla \phi \right] = kG \dot{p}, \]

(18)

because \( \mu_\Phi \) has only a time-independent temporal component and \( \phi \ll 1 \) in this case. Here \( y = kl^2 (\nabla \phi)^2 \), which is obtained from the general form \( y = kl^2 \phi^2 \Phi_\mu \phi^2 \phi \) in spherical symmetric geometry. From equation (18), we can find a relation between \( \Phi_N \) and \( \phi \),

\[ \nabla \phi = \left( k/4\pi \mu \right) \nabla \Phi_N. \]

Therefore, \( \phi \) can be determined whenever \( \mu \) is known.

It is also worth stressing that \( \ddot{\Phi}_{\mu} = -(1 + 2\Phi) \Phi \) follows the empirical definition of the gravitational mass \( m_\mu \) in the solar system when \( \mu \rightarrow 1 \),

\[ g = \frac{1}{2} \frac{\partial \ddot{\phi}_{\mu}}{\partial \theta} = \left( \Xi + \frac{k}{4\pi} \right) \frac{Gm_\mu}{\dot{\theta}^2} = G\Phi m_\mu \frac{Gm_\mu}{\dot{\theta}^2}, \]

(20)

and that the spatial components \( \ddot{\Phi}_{\mu} = -(1 + 2\Phi) \) guarantee that the theory passes the classical PPN tests of gravity (Bekenstein 2004).

We now apply the static spherically symmetric metric (see eq. [17]) to the GL systems. Considering a light ray that propagates on a null geodesic and moves in the equatorial plane, i.e., \( \theta = \pi/2 \), we can arrive at three constants of motion from the physical metric in equation (17) with some proper affine parameter \( \tau \),

\[ (1 + 2\Phi) \dot{r} = E, \]

(21)

\[ (1 - 2\Phi) \dot{\theta}^2 \dot{\phi} = L, \]

(22)

\[ (1 - 2\Phi) \dot{\theta}^2 + (1 + 2\Phi) \dot{\theta}^2 L^2 - (1 - 2\Phi) E^2 = 0, \]

(23)

where the dot denotes the derivative with respect to \( \tau \). Eliminating \( E \) and \( L \) in equation (23) from equations (21) and (22), and recalling the fact that \( \dot{\phi} \) vanishes at the closest approach \( \theta = \theta_0 \), yields

\[ \dot{\theta}^2 = \frac{L^2}{E^2} = \frac{\dot{\theta}_0^2(1 - 2\Phi_0)}{(1 + 2\Phi_0)}, \]

(24)

where \( \Phi_0 \equiv \Phi(\theta_0) \). Combining equation (24) and the three constants of motion gives the equation that describes the shape of the photon orbit,

\[-(1 - 4\Phi) + (1 - 4\Phi_0) (\dot{\theta}/\dot{\theta}_0)^2 \left[ \dot{\theta}^2 (d\phi/d\dot{\theta})^2 + 1 \right] = 0. \]

(25)

The solution of this equation in quadrature is

\[ \phi = \int^{\theta} \left\{ \left( \frac{\dot{\theta}}{\dot{\theta}_0} \right)^2 [1 - 4(\Phi - \Phi_0)] - 1 \right\}^{-1/2} \frac{d\theta}{\dot{\theta}}, \]

(26)

If we take the Taylor expansion of equation (26) to the first order of \( \Phi \), then it is easy to see that the integral is a combination of the angle of a straight line feeling no gravity and the angle of the deviation due to the gravitational field. Hence, the deflection angle can be approximated from the first-order terms of this Taylor expansion as

\[ \Delta \phi = \theta_0 \int_0^\theta \frac{2\rho(\Phi - \Phi_0)}{(\theta^2 - \theta_0^2)^{3/2}} \, d\theta. \]

(27)

Furthermore, after some manipulations (see, e.g., Bekenstein 2004), the deflection angle is

\[ \Delta \phi = \theta_0 \int_0^\theta \frac{4\Phi'}{(\theta^2 - \theta_0^2)^{1/2}} \, d\theta - \theta_0 \frac{4\Phi}{(\theta^2 - \theta_0^2)^{1/2}}. \]

(28)

It is obvious that the deflection angle between the closest approach and any position is always positive. First, gravity is always attractive (therefore, \( \Phi' > 0 \)), so the value of the integral is positive because every term in it is positive. Second, the second term of the right-hand side is negligible even in the MOND regime, in which \( \Phi \) behaves as \( \ln \dot{\phi} \), if \( \phi \) is large.

3.2. Two Limits of the Deflection Angle

Even though we can make sure that the deflection angle can be enhanced in TeVeS by the simple argument above, we need the exact form of the potential to reach the real value of a deflection angle. Unfortunately, the form of potential depends on the free function \( F(\mu) \), which can only be constrained empirically. However, there are two limits, the Newtonian and the MOND regimes, in which the dynamical behavior is clearly known, and so too the potential form. Actually, any choice of \( F(\mu) \) cannot contradict the empirical potential form in these two limits. Here, we discuss the deflection angles in the two regimes (see Fig. 1).
can obtain \( \mu \approx (k/3)^{1/2}/\sqrt{\nabla \phi} \) when \( \mu \ll 1 \) (i.e., in the MOND regime). It has to be mentioned that although this relation results from the freedom of an arbitrary function, \( \mu \sim \nabla \phi \) should always persist in order to guarantee that \( \nabla \phi \) varies as \( 1/\varrho \) in the deep MOND regime, a condition that follows from the MOND paradigm. Using this relation in equation (19) to eliminate \( \nabla \phi \) and replacing \( \nabla \Phi_N \) with \( \nabla \Phi \) gives

\[
\mu = \left( \frac{k}{8\pi \varrho} \right) \left( -1 + \sqrt{1 + 4\Phi'/\alpha_0} \right) = \frac{k}{4\pi} \left( \frac{\Phi'_N}{\Xi \alpha_0} \right)^{1/2}. \tag{29}
\]

Here, the constant

\[
\alpha_0 \equiv \left( \frac{3k}{4\pi^2} \right)^{1/2}
\]

can be identified as Milgrom’s constant. We should keep in mind that this form of \( \mu \) is valid under the condition \( \Phi' \ll (4\pi/k)^2 \alpha_0 \). It also offers a criterion for distinguishing the Newtonian and the MOND regimes.

We define a distance,

\[
r_0 = \left( \frac{G_NM_s}{\alpha_0} \right)^{1/2}, \tag{31}
\]

at which the acceleration \( a \) equals Milgrom’s constant \( a_0 \). Then, a particle is in the deep MOND regime if \( \varrho \gg k r_0/4\pi \) and in the Newtonian regime if \( \varrho \ll k r_0/4\pi \).

### 3.2.2. The Deep MOND Regime

In the deep MOND regime, \( \varrho_0 \gg k r_0/4\pi \), \( \mu \) has the form of equation (29). Then, along with equations (19) and (29), the deflection angle for a point-mass model in the deep MOND regime can be arrived at from equation (28) as

\[
\Delta \varphi = \frac{4G_NM_s}{\varrho_0(\Xi + k/4\pi)} \left\{ \Xi + \frac{\pi}{2} \varrho_0 \left[ \frac{\alpha_0(\Xi^2 + k\Xi/4\pi)}{G_NM_s} \right]^{1/2} \right\}, \tag{33}
\]

where \( \Xi \equiv 1 - K/2 - 2\varphi_c \). Since all of \( K, \varphi_c, \) and \( k \) are much less than 1, for a given mass, the deflection angle in the deep MOND regime differs from that in GR (or equivalently in the Newtonian regime; eq. [32]) by an amount almost independent of the distance of the closest approach.

We have to stress that even though we do not know exactly how the deflection angle varies with respect to the closest approach in the intermediate MOND regime, our result based on the deep MOND assumption (i.e., \( \mu \ll 1 \)), which is incorrect in the regime \( \varrho_0 \ll k r_0/4\pi \), approaches the Newtonian prediction while \( \varrho_0 \) decreases (see Fig. 1). Therefore, we believe that even in the intermediate regime, the deflection law based on the deep MOND assumption is approximately correct.

### 3.3. Magnification and Microlensing

From the observational point of view, the measurable data are not deflection angles but positions of the projected sources, i.e., \( \theta \equiv \varrho_0/D_L \). Here, \( D_L \) is the angular distance of the lens (eq. [16]). Therefore, we need a relation to connect \( \theta \) and \( \Delta \varphi \), the deflection angle derived in §3.2. We can obtain their relation from an embedding diagram (Fig. 2) and get the lens equations

\[
\beta = \theta_+ - \alpha(\theta_+), \tag{34}
\]
\[
\beta = \alpha(\theta_-) - \theta_- . \tag{35}
\]

Here the reduced deflection angle is defined as \( \alpha(\theta) = (D_L/S)/D_L \Delta \varphi(\theta) \). In GR it is not important to distinguish the difference between equations (34) and (35) in finding \( \theta \) (Paczynski 1986). This is, however, not the case in TeVeS. In order to keep \( \theta_+ = \theta_- \) when \( \theta \to 0 \), we must use equation (34) for \( \theta_+ \) and equation (35) for \( \theta_- \).

After substituting \( \Delta \varphi \) and \( \theta \) into the lens equation equation (34), we get

\[
\beta = \frac{D_L \Delta S}{D_L D_S} \left( \frac{\Xi}{\Xi + k/4\pi} + \frac{\pi \theta D_L}{2 \varrho_0} \left( \Xi + k/4\pi \right)^{-1/2} \right). \tag{36}
\]

For clarity, we can define \( \theta_0 \equiv \varrho_0/D_L \) and \( \theta_E \) with the form

\[
\theta_E = \left( \frac{4G_NM_s}{D_L D_S} \right)^{1/2}. \tag{37}
\]

Here \( \theta_E \) is called the Einstein radius; it is used as a length scale in the standard GL modeling. As \( k \ll 1 \) and \( K \ll 1 \), the lens equation (36) can be reduced to

\[
\beta \approx \frac{\theta_E}{\theta} \left( 1 + \frac{\pi \theta}{2 \theta_0} \right), \tag{38}
\]
with the reduced deflection angle \( \alpha(\theta) \) as

\[
\alpha \simeq \frac{\theta_E^2}{\theta} \left( 1 + \frac{\pi}{2} \frac{\theta}{\theta_0} \right).
\]

(39)

If equation (35) is used instead of equation (34), then the only difference in equation (38) is from \( \beta \rightarrow -\beta \). It is easily seen that the effective contribution of the scalar field can be expressed as \( \pi \theta_E^2/2 \theta_0 \) and is independent of \( \theta_0 \) (or equivalently \( \theta_0 \)).

For a point source, there are always two projected images, which can be obtained from equations (34) and (35). The solutions are

\[
\theta_{\pm} = \pm \frac{1}{2} \left\{ \left( \beta \pm \frac{\pi \theta_E^2}{2 \theta_0} \right) \pm \left[ \left( \beta \pm \frac{\pi \theta_E^2}{2 \theta_0} \right)^2 + 4 \theta_0^2 \right]^{1/2} \right\} > 0.
\]

(40)

Moreover, since gravitational lensing conserves surface brightness, the magnifications of the images to source intensity are determined by their area ratio,

\[
A_{\pm} = \left| \frac{\theta_{\pm} \partial \theta_{\pm}}{\beta \partial \beta} \right|.
\]

(41)

Interestingly enough, the difference in the magnification of the two images is no longer constant in TeVeS, as in GR (see Fig. 3):

\[
|A_+ - A_-| \geq 1.
\]

(42)

In the deep MOND regime, the total magnification is given by

\[
A \equiv A_+ + A_- = \left( \frac{u_1}{2u} \right) \left[ \frac{u_1^3 + 2}{u_1(u_1^3 + 4)^{1/2} + 1} \right] + \left( \frac{u_2}{2u} \right) \left[ \frac{u_2^3 + 2}{u_2(u_2^3 + 4)^{1/2} - 1} \right],
\]

(43)

where

\[
u = \frac{\beta}{\theta_E}, \quad u_1 = u + \frac{\pi \theta_E}{2 \theta_0}, \quad u_2 = u - \frac{\pi \theta_E}{2 \theta_0}.
\]

(45)

The magnification reduces to the standard GR form as \( \theta_0 \rightarrow 1 \). It is then straightforward to study the light curves of microlensing (Paczynski 1986).

Microlensing events result from the relative proper motions of lenses to the sources. Because of proper motions, the alignment between lens and observer and source and observer vary with time,

\[
u(t) = \sqrt{(\beta_{\text{min}}/\theta_E)^2 + (t/t_0)^2},
\]

(46)

and so does the magnification. Here \( \beta_{\text{min}} \) occurs at the greatest alignment, and \( t_0 \) is the time it takes the source to move with respect to the lens by one \( \theta_E \). Putting equation (46) into equations (44) and (45), we can obtain the microlensing light curves expressed in stellar magnitudes \( (\Delta m = 2.5 \log \lambda) \). As is the case in GR, the variation reaches the maximum as \( \beta(t) \rightarrow \beta_{\text{min}} \), and for any given \( \pi \theta_E^2/2 \theta_0 \), the smaller the \( \beta_{\text{min}} \), the greater the amplitude of the light curve becomes. However, the shape of light curves in TeVeS deviates from that in GR, as \( \pi \theta_E^2/2 \theta_0 \) is significant (i.e., in the deep MOND regime). In general, the contribution of \( \pi \theta_E^2/2 \theta_0 \) raises the peak (see Fig. 4).
Even though the differences in microlensing events between TeVeS and GR only occur at source redshift \( z_s \geq 1 \) (Mortlock & Turner 2001), which is beyond the current microlensing projects within the Local Group (see, e.g., Alcock et al. 2000; Paczyński 1996), observations such as several quasar microlensing events (see, e.g., Wambsganss 2001) may help us to decide whether MOND is a viable alternative to the dark matter paradigm.

4. TIME DELAY

Time delay always plays an important role in gravitational lensing. It not only provides a more convenient way to get the lens equation (Schneider 1985; Blandford & Narayan 1986) but also has some important values in its own right (for more complete reviews, see Courbin et al. 2002; Kochanek & Schechter 2004). As it is an alternative to the cold dark matter, it should be very interesting to study the behavior of time delay in TeVeS.

4.1. Arrival Time in TeVeS

We apply the static and isotropic metric described by equation (17) and assume that the photons are confined to the equatorial plane without loss of generality. Then, following the standard procedure of light bending (see, e.g., Weinberg 1972), not only can the deflection angle be derived, but also the equation that describes the arrival time of a light ray.

\[
(1 - 4\Phi)\dot{\theta}^2 = \left[ 1 - \frac{1 + 4\Phi}{1 + 4\Phi_0} \right] \left( \frac{\theta_0}{\theta} \right)^2 \dot{\theta}^2. \tag{47}
\]

This equation has an integral as a solution, which states that a photon traveling from \( \theta_0 \) to any distance \( \theta \) (or from \( \theta \) to the closest approach distance \( \theta_0 \)) would require a time

\[
t = \int_{\theta}^{\theta_0} \frac{(1 - 4\Phi)^{1/2}}{1 - \left( 1 + 4\Phi \right) / \left( 1 + 4\Phi_0 \right) \left( \theta_0 / \theta \right)^2}^{1/2} \, d\theta. \tag{48}
\]

Taylor expanding equation (48) to the first order of \( O(\Phi) \), the original integral can be reduced to

\[
t(\theta, \theta_0) = \int_{\theta}^{\theta_0} \left[ 1 - \left( \frac{\theta_0}{\theta} \right)^2 \right]^{-1/2} \, d\theta + \int_{\theta}^{\theta_0} \frac{2(\Phi - \Phi_0)(\theta_0 / \theta)^2}{1 - \left( \theta_0 / \theta \right)^2} \left[ 1/2 \right]^{1/2} \, d\theta
\]

\[
- \int_{\theta}^{\theta_0} \frac{2\Phi}{1 - \left( \theta_0 / \theta \right)^2} \left[ 1/2 \right]^{1/2}. \tag{49}
\]

The second integral of equation (49) differs from equation (27) only by a factor of \( \theta_0 \), whence, from this part, the arrival time contributions due to both gravitational mass and the scalar field are positive. However, this is not the case in the third integral of equation (49). Since \( \Phi_N \) and \( \Phi \) have opposite signs in \( \Phi \), their contributions to the resulting arrival time are also opposite. Moreover, since the third integral is always larger than the second one [it is easy to observe this by multiplying by a factor of \( 1 - (\theta_0 / \theta)^2 \) in both numerator and denominator of the second integral], the time delay due to the scalar field will always be opposite to that due to the gravitational mass, i.e., with an opposite sign. In other words, the arrival time will be shortened rather than enhanced by the positive scalar field; this is a necessary condition in TeVeS for not violating causality (Bekenstein 2004).

4.2. Arrival Time and the MOND Regime

As in the case of the deflection angle, in order to calculate time delay from equation (49), we need some additional information on the free function \( F(\mu) \) to determine the potential \( \Phi \) in TeVeS. However, there are some differences in these two considerations. The first one arises from the fact that time delay is influenced by gravity even far away from the center. This makes a purely Newtonian regime unavailable. Therefore, we only consider the MOND regime here. Moreover, unlike the deflection angle case, we need information about \( \Phi \) in addition to \( \Phi' \) for the time delay. Eliminating \( \mu \) of equation (19) from equation (29) yields

\[
\phi = (\Xi a_0 G m g) \frac{1/2}{k}\ln \frac{4\pi a_0}{k} + \phi_c, \tag{50}
\]

where \( \phi_c \) can be absorbed into the symmetric physical metric \( \tilde{g}_{\mu\nu} \) by rescaling the \( t \) and \( \rho \) coordinates appropriately (Bekenstein 2004). Recalling that \( \Phi = \Xi \Phi_N + \phi \), and putting equation (49) into equation (50), we get the arrival time in the MOND regime as

\[
t(\theta, \theta_0) = \left( \theta^2 - \theta_0^2 \right)^{1/2} + 2\Xi a_0 G m g \left( \frac{\theta - \theta_0}{\theta + \theta_0} \right) \left( \frac{\dot{\theta}}{\dot{\theta}_0} \right) \ln \left( \frac{\theta^2 - \theta_0^2}{\dot{\theta}^2} \right) - 2 \left( \Xi a_0 G m g \right)^{1/2} \left[ \frac{\ln \left( \frac{\theta^2 - \theta_0^2}{\dot{\theta}^2} \right)}{1} \right] \left( \frac{\theta - \theta_0}{\theta + \theta_0} \right) \left( \frac{\dot{\theta}}{\dot{\theta}_0} \right)
\]

\[
- \theta_0^2 \ln \left( \frac{\theta}{\theta_0} \right) \left( \frac{\theta^2 - \theta_0^2}{\dot{\theta}^2} \right)^{-1/2}. \tag{51}
\]

Here again, \( G \) differs by a factor of \( \Xi + k/4\pi \) from the usual Newtonian constant, which is measured in the solar system experiments. We would like to stress that as a particular case of our consideration in § 4.1, the sum of last two terms in equation (51) is always negative.

Intrinsically, we may think that time delay (photon deviation of the path length) should increase with deflection angle as read off from the embedding diagram (Fig. 2). However, this is only true if we do not consider the deflection potential. Actually, a positive scalar field will offer a hyperbolic-like spacetime, in which the distance is shorter than that of a flat universe (see Fig. 5).

4.3. Measurable Time Delay

Since it is the positions of sources and lenses and not the distances from sources (or observers) to lenses that are observed, time delay is conventionally expressed in terms of these dimensionless angles. It is appropriate to rewrite equation (49) as

\[
t = \int_{\theta_0}^{\theta} \frac{1}{2} \, d\theta, \tag{52}
\]

where \( dl = \left[ 1 - (1 + 4\Phi)/(1 + 4\Phi_0) \right] \left( \theta_0 / \theta \right)^2 \left( \theta_0 / \theta \right)^2 \left( \theta_0 / \theta \right)^2 \) is a line element like that of Euclidean geometry, i.e., it obeys \( dl^2 = d\theta^2 + d\varphi^2 \). Traditionally, we call the first integral in equation (52) the geometric arrival time, because it can be read off in a geometric way (see Fig. 2) as

\[
t_{\text{geom}}(\theta_1, \theta_2) = \sqrt{(\theta_0 - \theta_1)^2 + D_{1X}^2} + \sqrt{D_{1X}^2 + D_{1Z}^2}. \tag{53}
\]
Here $\eta = D_S \beta$. Indeed, if we let a photon move from $q_2$ to $q_0$ and from $q_0$ to $q_1$, it can be proved that the contributions to the arrival time calculated from the first two integrals of equation (49) are identical to the geometric arrival time (cf. eq. [53]). On the other hand, the third integral in equation (49), which equals the second integral of equation (52), contributes to what is called the “potential time delay.” Adding the geometric and potential contributions to the arrival time and subtracting the arrival time for an unlensed ray from $S$ to $O$, and considering the expansion of the cosmological background, we obtain the time delay of a possible ray compared to an undeflected ray,

$$\Delta t = (1 + z) \left[ \frac{D_d D_S}{2D_S} \left( \frac{\theta_0}{D_S} - \frac{\eta}{D_S} \right) - \psi(\theta_0) \right] + B, \quad (54)$$

where $\psi(\theta_0)$, called the “deflection potential,” can be calculated from the potential time delay. Once again we only consider what happens in the MOND regime. For any given sources and lenses, the distances from the lenses to the observer (or sources) are decided, whence the potential time delay, which is given from the second and the fifth terms in equation (51), can be reduced to some constant plus the deflection potential contributions

$$\psi_{\text{GR}} = 4G_N m_\gamma \ln \theta_0, \quad (55)$$

$$\psi = 4 \left( \frac{a_0 G_N m_\gamma}{1 + k/4\pi} \right)^{1/2} \left( \frac{\pi}{2} \theta_0 \right)^{1/2}, \quad (56)$$

$$\psi_{\text{corr}} = -\frac{4G_N m_\gamma}{1 + 4\pi k/k} \ln \theta_0. \quad (57)$$

We have shown in § 4.2 that the time delay (relative to the undeflected path) is reduced rather than enhanced in the TeVeS framework. Unfortunately, this phenomenon cannot be measured in any GL system. For any given source at position $\beta$, we are only able to measure the time difference for two images $\theta_{01}$ and $\theta_{02}$ that share the same lens potential and lens-observer and lens-source distances. In other words, the constant $B$ in equation (54) is the same for all light rays from the source plane to the observer. Therefore, in regard to the potential time delay, we can only measure the contributions from the deflection potential, which possesses the same sign in GR and TeVeS (Fig. 6). Moreover, since the deflection potential does not depend on the length scale $k r_0/4\pi$, the free choice of $\phi_0$ has no influence on the measurable time delay in GL systems.

Even though we cannot measure the opposite contribution to the arrival time in TeVeS, time delay between two images due to the deflection potential offers another constraint on the mass ratio, which is not a priori identical to that due to deflection angle.

In order to illustrate this, we consider a general case: the value of $\theta_{01}$ is $n$ times $\theta_{02}$, whence the time delay between the two images is

$$\Delta \psi = 4G_N m_\gamma \left[ \ln n - \frac{\pi}{2} \frac{n - 1}{n} \frac{\theta_{02}}{\theta_{01}} \right]. \quad (58)$$

For the extreme case, the difference between $\theta_{01}^1$ and $\theta_{02}^2$ is negligible compared to either distance. Hence, $\ln (\theta_{01}^{1}/\theta_{02}^{2}) \simeq \delta \theta_{01}/\theta_{02}$, and equation (58) can be reduced to

$$\Delta \psi = 4G_N m_\gamma \frac{\delta \theta_{01}}{\theta_{01}} \left( 1 + \frac{\pi}{2} \frac{\theta_{01}}{\theta_{02}} \right). \quad (59)$$

If we compare equation (59) with equation (39), we find that the similarity of the two equations makes the mass ratio $m_{l,\gamma}/m_{l,\gamma}$ exactly the same under the requirement of either identical deflection angle or time delay. However, if the difference between $\theta_{01}^{1}$ and $\theta_{02}^{2}$ is not small, then the mass ratio given by the time delay will be less than that obtained from the deflection angle (Fig. 7).

### 4.4. Time Delay and Lensing Equation

As shown by Schneider (1985) and Blandford & Narayan (1986), time delay with Fermat’s principle offers an alternative for deriving the lens mapping. Moreover, since Fermat’s principle has proved valid under a very general metric (Kovner 1990; Perlick 1990), we should be able to rederive the lens equation from our
time delay result. Fermat’s principle asserts that a light path is true if and only if its arrival time or, what amounts to the same, its time delay is stationary with respect to variation of the turning point. In other words, \( \partial(\Delta \nu) / \partial \theta_0 = 0 \). With this sufficient and necessary condition for Fermat’s principle, we can obtain the lens equation in the MOND regime,

\[
\eta = \frac{D_S}{D_L} \theta_0 - D_{LS} \nabla \psi.
\]  

(60)

Here \( \nabla \psi \) is composed of three parts, and when \( \theta \) is very large

\[
\nabla \psi_{GR} = 4 \frac{G \kappa m_g}{\theta_0},
\]

\[
\nabla \psi_{\phi} = 2\pi \left( \frac{a_0 G \kappa m_g}{1 + k/4\pi G} \right)^{1/2},
\]

\[
\nabla \psi_{\text{corr}} = -4 \left( \frac{G \kappa m_g}{1 + 4\pi \kappa} \right).
\]

(61) \hspace{1cm} (62) \hspace{1cm} (63)

Obviously, the sum of the last three equations equals \( \Delta \varphi \), given by equation (33), and hence equation (60) is identical to the lens equations given by equation (34).

5. DISCUSSION

Since the appearance of Bekenstein’s relativistic MOND gravitation, the long-lasting incompleteness of Milgrom’s modified Newtonian dynamics seems to be filled up. Now we can investigate the GL phenomenon, such as the deflection angle or time delay, in the MOND paradigm, which could not be done before.

In this paper, we have investigated the GL phenomenon under the following approximations and presuppositions:

1. The GL lens is assumed to be static, spherically symmetric, and following the thin lens formalism.
2. The motions of light rays are described in the framework of a Schwarzschild lens (i.e., a point-mass model).
3. The revised physical metric is obtained by adding a positive scalar field into the potential of the standard Schwarzschild metric in symmetric coordinates.

Under these presuppositions, we find that when \( \theta \) is larger than \( \theta_0 \) (\( \equiv r_0 / D_L \); the MOND length scale), the reduced deflection angle \( \alpha \) will approach a constant for a given mass. This special prediction is a feature of GL in TeVeS, which is similar to that obtained by Mortlock & Turner (2001) with an intuitional approach. This is no surprise, because just as in GR, the deflection of photons is simply twice the deflection of a massive particle with the speed of light in TeVeS, and thus Mortlock & Turner (2001) started from the correct premise. However, there is still something that was unknown before the appearance of TeVeS. For a static spherically symmetric spacetime, the case \( \mu \ll 1 \), which yields a relation of equation (29), is valid only when \( |\nabla \psi| \ll (4\pi / k) \phi \). This condition is valid in the MOND regime when \( |\nabla \psi| \) goes up to a couple of orders of magnitude above \( \phi_0 \), or equivalently when \( \theta_0 \) is around one order of magnitude below \( r_0 \) (Bekenstein 2004).

We should address that the criteria of mass discrepancy for GL effects in TeVeS consists with that of stellar dynamics. In other words, only when \( \theta_0 \gg kr_0 / 4\pi \) does the “missing mass” appear. This corresponds to the demarcation of the high surface brightness (HSB) and low surface brightness (LSB) galaxies from the dynamical analysis (Sanders & McGaugh 2002).

When we apply the deflection angle law to magnification, we find that in TeVeS the difference in the magnifications of the two images in the point-mass model depends on the lens mass and source positions and is always larger than one. This differs from traditional gravitational lensing, which says that the difference must always be one. Tens of thousands of the multiple-image lensings found by the Sloan Digital Sky Survey (SDSS; Stoughton et al. 2002) might be used to check this prediction. For micro-lensing, light curves in TeVeS in the deep MOND regime also differ from that in GR. To observe the discrepancy, the sources have to be located at about \( z \geq 1 \) (Mortlock & Turner 2001).

Concerning time delay, the result is even more exotic. Adding an arbitrary positive scalar field into the primary Schwarzschild metric (Bekenstein 2004), the resulting contributions of the scalar field will reduce rather than enhance the potential time delay. Unfortunately, this phenomenon is unmeasurable in GL systems. What we can determine is only the time delay between two images produced by the same source. Therefore, the opposite feature of the time delay in TeVeS, which is the same for all images, will be canceled out, and only those parts contributed from the deflection potential can be observed.

Even though the opposite effect on the potential time delay due to the scalar field cannot be measured in a GL system, the time delay between two images offers another constraint on the needed mass in GR and TeVeS, which usually differs from that given by the deflection angle. Actually, the mass ratio obtained from these two approaches will be the same; otherwise, the mass ratio given by time delay would always be smaller than that given by the deflection angle. In other words, the MOND and CDM paradigms are not mutual alternatives in a GL system, when we consider deflection angle and time delay at the same time.

The first time delay for the gravitational lens Q0957+561 was measured in 1984 (Florentin-Nielsen 1984), and since then more than 11 time delay lenses have been found, including seven systems with a good quality of astrometric data and two systems with serious problems (Kochanek & Schechter 2004). It would be very interesting if those debatable systems could be explained in the framework of TeVeS. However, we emphasize that the
measurement of time delay is controversial. Actually, there were almost 20 years of debate between a short delay and a long delay for the first observed time delay source (Petters et al. 2001).

It also worth noting that our conclusion is based on Bekenstein’s approach to investigating gravitational lenses. He assumes a priori that the potential dominating in the massive particle dynamics agrees with the potential influences on the GL system. In other words, the temporal and spatial components of the metric, respectively, have a form $(1 + 2\Phi)$ and $(1 - 2\Phi)$, whence $\tilde{g}_{\mu\nu} \tilde{g}^{\mu\nu} \approx 1$. However, there is still a chance that these two potentials are not the same and even that $\tilde{g}_{\mu\nu} \tilde{g}^{\mu\nu} \neq 1$ under the framework of TeVeS. If so, the conclusion of this paper may be different (Edery 1999; Bekenstein et al. 2000).

Since we can observe the influence of gravity on gravitational lens phenomena at distances farther away than we can for the dynamics of massive particles, GL systems offer a good method to distinguish between GR and TeVeS. Although we need more than the point-mass model to fit astrometric data, this simplified model has in principle told us the exotic predictions for the deflection angle and time delay. It would be very interesting if we could develop a more realistic model to fit the known data in GL systems.

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