Methods for the Epipolarity Analysis of Pushbroom Satellite Images Based on the Rational Function Model

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ABSTRACT Epipolarity is the foundation of epipolar resampling, which is used to eliminate the vertical disparity between stereo pairs in stereo matching. To analyze the epipolarity of pushbroom satellite images, this paper compares three methods based on the rational function model (RFM): the projection trajectory method (PTM), the piecewise projection trajectory method (PPTM) and our extended projection trajectory method (EPTM). To evaluate the quality of epipolar curves, we defined the deviation coefficient as a metric to evaluate the bending degree of epipolar curves. We also defined the maximum deviation coefficient of an image that can be used to determine the tile size in multiview satellite image 3D reconstruction based on image dividing. Comparison experiments have been carried out with pushbroom satellite images using these three methods. Experimental results show that our EPTM is more convenient and practical. It only needs the forward form of the RFM to analyze the epipolarity and can be used in the epipolarity analysis of a single image. By projecting straight lines in the ground space into the image space, the EPTM can be used to perform comprehensive epipolarity analysis for pushbroom satellite images. In addition, the EPTM can be used to calculate the maximum deviation coefficient of an image that the PTM and the PPTM cannot calculate, which is important in 3D reconstruction using multiview satellite images.

INDEX TERMS Epipolarity, stereo matching, pushbroom satellite image, RFM, projection trajectory method, deviation coefficient.

I. INTRODUCTION

In photogrammetry, epipolarity is the foundation of epipolar resampling [1]–[3], and it is also called stereo rectification in computer vision [4]–[6]. Epipolar resampling is the process of eliminating the vertical disparity between stereo pairs, which is an important step for stereo matching algorithms such as semiglobal stereo matching (SGM) [7]–[9]. Epipolarity can be explained as follows [10], [11]. For a stereo image pair captured by a frame camera, the corresponding point of any point in the left image lies on the corresponding epipolar line in the right image. The corresponding point of any point on the epipolar line in the right image also lies on the corresponding epipolar line in the left image. Both of these two epipolar lines are straight lines, which are called conjugated epipolar lines [12]. This means that the search for the corresponding points can be restricted to epipolar lines. By resampling the stereo pairs along epipolar lines, the search problem in stereo matching can be simplified from two-dimensional to one-dimensional.

The epipolarity of pushbroom satellite images is different from the epipolarity of frame images. In the case of a pushbroom camera, an image is assembled by scanning the lines captured by a moving camera. Each line of the image has its own exterior orientation parameters, so the epipolarity of pushbroom satellite images cannot be strictly modeled as frame images.

To figure out the epipolarity of pushbroom images, researchers have conducted many studies. Kim [13] derived the formula of the epipolar lines in pushbroom images based on the rigorous physical sensor model proposed by Orun and Natarajan [14] and analyzed the epipolar lines.
in pushbroom images. The results show that the epipolar lines in pushbroom images are hyperbolic and the conjugated in pushbroom images. The results show that the epipolar lines in pushbroom images based on the affine projective model [18], [19], and the results also show that the epipolar lines are approximate hyperbolic.

Although the analysis based on the physical sensor models proved that the epipolar lines in pushbroom images are no longer straight, the physical sensor models are often difficult to obtain and different satellites use different physical sensor models. This makes it hard to use the physical sensor models in epipolarity analysis. As an alternative, the rational function model (RFM) provides a more convenient way to analyze the epipolarity of pushbroom images [20], [21].

In fact, the RFM is a fitting model, and the epipolarity analysis based on the RFM was usually achieved by using the projection trajectory method (PTM) in the early days [22]. Using the PTM based on the RFM, Zhao et al. [23] generated and analyzed the approximate epipolar lines of IKONOS stereo images. Idrissa and Beumier [24] proposed a generic epipolar resampling method that can rectify the images captured by both perspective frame cameras and pushbroom cameras. However, the epipolar curves generated by the PTM may contain large errors. To realize the epipolar resampling of pushbroom satellite images more accurately, Oh et al. [25], Oh [26] researched the traditional PTM and proposed the piecewise projection trajectory method (PPTM) based on the RFM. The new proposed method can produce more accurate epipolar curves for pushbroom images. By using a high-order polynomial transformation, they mapped the epipolar curves to straight lines, thus realizing the epipolar resampling of pushbroom image pairs. Based on Oh’s work, Koh and Yang [27] proposed a unified piecewise epipolar resampling method for high resolution pushbroom satellite images.

However, the epipolarity analysis of pushbroom satellite images, especially multiview pushbroom satellite images, has not been fully studied. To carry out a comprehensive epipolarity analysis of pushbroom satellite images, we summarize the epipolarity analysis methods based on the RFM for pushbroom satellite images in this paper. First, we introduce the PTM and the PPTM. Then, we introduce the epipolarity analysis method proposed in our previous work [28]. Next, we refine the definition of the deviation coefficient for the quality evaluation of epipolar curves, and define the maximum deviation coefficient of a pushbroom satellite image. At last, we compare and summarize these methods by testing them in epipolarity analysis experiments.

The main contributions of our work can be summarized in three points: 1) we proposed a novel method to analyze the epipolarity of pushbroom satellite images; 2) we defined the deviation coefficient as a metric to evaluate the bending degree of epipolar curves; and 3) we defined the maximum deviation coefficient of an image to determine the tile size in multiview satellite image 3D reconstruction based on image dividing.

II. RATIONAL FUNCTION MODEL
Because of its confidentiality and universality, the RFM has become a mainstream sensor model for satellite images. The RFM describes the relative positional relationship between images and ground objects.

The forward form of the RFM is used to describe the transformation from the ground space to the image space [21], [29]:

\[
\begin{align*}
X &= \frac{p_1(X, Y, Z)}{p_2(X, Y, Z)} \\
Y &= \frac{p_3(X, Y, Z)}{p_4(X, Y, Z)}
\end{align*}
\]

where \((r, c)\) and \((X, Y, Z)\) are the normalized image coordinates and ground coordinates, respectively, and the range is from -1 to +1. The normalization of the coordinates is computed using the following equations:

\[
\begin{align*}
X &= \frac{X_u - X_o}{X_s} = \frac{Y_u - Y_o}{Y_s} = \frac{Z_u - Z_o}{Z_s}, \\
r &= \frac{r_u - r_o}{r_s}, \\
c &= \frac{c_u - c_o}{c_s}
\end{align*}
\]

where \((r_u, c_u)\) and \((X_u, Y_u, Z_u)\) are the image pixel coordinates and ground geographic coordinates, respectively, \((X_o, Y_o, Z_o, r_o, c_o)\) are the offset values for the ground geographic coordinates and the image pixel coordinates, and \((X_s, Y_s, Z_s, r_s, c_s)\) are the scale values for the ground geographic coordinates and the image pixel coordinates.

For the polynomials \(p_i(X, Y, Z)\), the maximum power of each coordinate component \(X, Y, Z\) is limited to 3, and the total powers of all coordinate components are also limited to 3. In such a case, each polynomial has 20 cubic terms as follow:

\[
p = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \sum_{k=0}^{m_3} a_{ijk} X^i Y^j Z^k = a_0 + a_1 Z + a_2 Y + a_3 X + a_4 Z X + a_5 X Y + a_7 Z^2 + a_8 Y^2 + a_9 X^2 + a_{10} Z Y X + a_{11} Z^2 Y + a_{12} Z^2 X + a_{13} Z Y^2 + a_{14} Y^2 X + a_{15} Z X^2 + a_{16} Y X^2 + a_{17} Z^3 + a_{18} Y^3 + a_{19} X^3
\]

where \(a_{ijk}\) are the coefficients of the polynomial. They are called rational polynomial coefficients (RPCs) together with the offset values and the scale values.

The inverse form of the RFM is used to describe the transformation from the image space to the ground space:

\[
\begin{align*}
X &= \frac{p_5(r, c, Z)}{p_6(r, c, Z)} \\
Y &= \frac{p_7(r, c, Z)}{p_8(r, c, Z)}
\end{align*}
\]

The detailed expression of the polynomial in the inverse form is similar to the forward form.

Usually, there are two methods for solving the RPCs [30], [31]. 1) The terrain-independent method: With the physical sensor models available, the RPCs can be solved using a virtual ground grid with its grid-point coordinates...
determined using the physical sensor model. Since it does not need to know the specific terrain, it is called the terrain-independent method. The RPCs solved in this way can be regarded as a kind of fitting to the collinear equation described by the physical sensor model. 2) The terrain-dependent method: With no physical sensor model at hand, the 3D virtual ground grid cannot be established. The ground control points (GCPs) collected from maps or digital elevation models (DEMs) are used to solve the RPCs. Since the GCPs are essentially a description of the terrain, this is called the terrain-dependent method. The RPCs solved by this method can be regarded as fitting the ground control points. In this case, the solution is highly dependent on the actual terrain, the number of GCPs, and their distribution across the scene. The terrain-dependent solution may cause large errors when the GCPs are insufficient or distributed unevenly.

III. METHODS FOR EPIPOLARITY ANALYSIS

A. PROJECTION TRAJECTORY METHOD

The PTM is a convenient method to generate epipolar curves. It can be described as in Figure 1. For a stereo pair, a point \( p \) in the left image is projected into the ground space using the inverse form of the RFM. By changing the elevation value \( Z \), we can obtain a projection ray in the ground space. Therefore, we can obtain the projection trajectory \( l \) by projecting the ray into the right image space using the forward form of the RFM. The line \( l \) is the epipolar curve corresponding to the point \( p \). The conjugate point of \( p \) should be located on the line \( l \). This is the so-called PTM.

Using the PTM, we can generate epipolar curves conveniently. However, there is a problem. Since the RFM is a fitting model, the corresponding RPCs are limited in their use range. Generally, the use range is the ground 3D box defined by the offset values and the scale values in the RPCs, as shown in Figure 1. When we generate a projection trajectory by projecting a ray in the ground space into the image space using the RFM, only the projection trajectory of the ray segment inside the range box can describe the epipolar curve accurately. For the ray outside the range box, the projection trajectory will introduce large errors.

B. PIECEWISE PROJECTION TRAJECTORY METHOD

To generate epipolar curves of pushbroom satellite images more accurately, Oh, et al. [25], [26] proposed the PPTM. As shown in Figure 2, a point \( p_1 \) in the left image is projected into the ground space using the inverse form of the RFM. By taking the maximum and minimum of the elevation value \( Z \) in the range box, we obtain two ground points. Then, the two ground points are projected into the right image by the forward form of the RFM, and we obtain the corresponding image points \( q_1 \) and \( q_2 \). The epipolar line corresponding to point \( p_1 \) can be approximated by the straight line between the two image points \( q_1 \) and \( q_2 \). Using the same method, we can get the epipolar line corresponding to \( q_1 \) and \( q_2 \) in the left image. By continuing the projection process, we can get an epipolar curve consisting of the image points in the left image and the corresponding conjugate epipolar curve in the right image. These two epipolar curves constitute an epipolar curve pair. This is the so-called PPTM.

All the projections used in the PPTM are implemented within the range box defined by the RPCs. The epipolar curves generated by the PPTM are more rigorous than the epipolar curves generated by the PTM. As a result, the PPTM has been widely used in the epipolar resampling of pushbroom satellite image pairs.

C. EXTENDED PROJECTION TRAJECTORY METHOD

As described above, the PTM and the PPTM need the RPCs of both the forward form and the inverse form at the same time. However, many satellite image vendors only provide the RPCs of the forward form when providing satellite images. The RPCs of the inverse form are usually not available. This creates a difficulty in the practical epipolarity
analysis of pushbroom satellite images. In addition, since a multiview satellite image dataset with N images can form \( N \times (N-1) \) stereo pairs, the PPTM is too complicated to analyze all the image pairs in the multiview satellite image dataset.

In fact, for an image in space, it can be paired with another image that covers the same object anywhere else in space. Therefore, any straight line in space can be a projection ray. The corresponding projection trajectory of the ray can be an epipolar line. Therefore, we can take special straight lines (such as plumb lines, horizontal lines, diagonal lines, etc.) in the ground range box as virtual projection rays. By projecting these virtual rays into the image space, virtual epipolar curves can be obtained to analyze the epipolarity of the image. The details are shown in Figure 3. We call it the extended projection trajectory method (EPTM).

The advantage of the EPTM is that it can be used to analyze the epipolarity of a single image without forming a stereo pair with another image and does not need the RPCs of the inverse form, which are often not available. By flexibly choosing the virtual rays in the range box defined by the RPCs, a comprehensive epipolarity analysis of the image can be achieved.

**IV. DATASETS AND EVALUATION METRICS**

**A. DATASETS**

To compare these three epipolarity analysis methods, we choose two groups of tristereo images for experiments, including a triplet covering the region of La Dorada acquired by SPOT7 and a triplet covering the region of Melbourne acquired by Pleiades. These two groups of images are respectively shown in Figure 4 and Figure 5.

All the images are accompanied by the RPCs of both the forward form and the inverse form of the RFM. The information of the images is shown in Table 1.

**B. EVALUATION METRICS**

According to the research of Kim [13], although the epipolar lines in pushbroom images are hyperbolic curves, the epipolar lines can be approximately treated as straight lines in small local areas. Using this property, Woo and Pham [32] and Tatar and Arefi [33] achieved the epipolar resampling of pushbroom satellite images by dividing the large images into small tiles. However, how to choose the size of the image tile is a problem that needs to be further studied.

Figure 6 shows the process of searching the corresponding points in a rectified stereo image pair. For a point (the red point) in the left image, the corresponding point lies on the corresponding epipolar curve (the blue curve) in the right image. Usually, stereo matching algorithms search the corresponding point along an image row (the red dotted line in Figure 6) [8]. The vertical parallax of the corresponding points should be less than 1 pixel to ensure the stereo matching of epipolar image pairs. Therefore, the errors introduced by the approximation should be limited to 1 pixel when we approximate the epipolar curves in the divided image tiles as straight lines.

Accordingly, the size of the image tile should depend on the bending degree of the epipolar curves. The more bended the epipolar curves are, the smaller the image tile should be. To evaluate the bending degree of different epipolar curves uniformly, we define a metric.

**Definition 1:** For an epipolar curve, we fit it to a straight line. The deviation coefficient \( \eta \) of the epipolar curve can be calculated as follows.

\[
\eta = \frac{R_{\text{max}} - R_{\text{min}}}{L}
\]
where \( R_{\text{max}} \) is the maximum of the fitting residual (the maximum distance of all the points on the epipolar curve to the fitted line), \( R_{\text{min}} \) is the minimum of the fitting residual (the minimum distance of all the points on the epipolar curve to the fitted line, which is usually a negative value), and \( L \) is the length of the fitted line. The units of these three variables are pixels.

The physical meaning of the deviation coefficient is that when an epipolar curve is shorter than \( 1/\eta \) pixels, the error introduced by approximating the epipolar curve as a straight line is less than 1 pixel.

In stereo matching based on image dividing, the size of the image tile can be determined by the deviation coefficients of the epipolar curves between stereo pairs. However, in multistereo matching, an image may form a stereo pair with any other image. The size of the image tile should be determined by the maximum deviation coefficient of all the epipolar curves in all the possible image pairs. This situation often occurs in multiview satellite image 3D reconstruction [34]–[37]. Therefore, we define another metric.

Definition 2: The maximum deviation coefficient of an image is the maximum deviation coefficient of its all possible epipolar curves in all the possible stereo pairs that it may form.

Depending on the maximum deviation coefficient, we can determine the image dividing size used in multiview satellite image 3D reconstruction.

V. EXPERIMENT AND DISCUSSION
A. COMPARISON EXPERIMENTS
Pleiades-1 and Pleiades-2 were selected to form a stereo pair for comparison experiments. First, five image points were chosen in the left image (Pleiades-1) and the corresponding epipolar curves in the right image (Pleiades-2) were obtained using the PTM. Since the epipolar curves projected from rays outside the range box may introduce large errors, we just projected the rays inside the range box to obtain accurate epipolar curves. Then, five initial points were chosen in the left image, and the corresponding epipolar curves in the left image and right image were generated using the PPTM. Finally, we chose 5 vertical lines, 5 along-track horizontal lines, and 5 cross-track horizontal lines in the range box defined by the RPCs of Pleiades-1, and projected them into the image space of Pleiades-1 using the EPTM. The results are shown in Figure 7. The chosen points are marked with circles. The beginning points of the epipolar curves are marked with diamonds. The ending points of the epipolar curves are marked with stars.

It can be seen from Figure 7 that the epipolar curves generated by the three methods are all approximate straight lines. Due to the limitation of the projection angle, the epipolar curves generated by the PTM are short. As shown in Figure 7 (b), the beginning points and ending points seem to overlap at the same position in the original image. This is because the epipolar curves are only dozens of pixels long, as shown in the enlarged image. The epipolar curves generated by the PTM are too short to cover the entire image. The epipolar curves generated by the EPTM using plumb line projection have similar deficiencies as shown in Figure 7 (e). However, the epipolar curves generated by the PPTM can cover the entire image pair, as shown in Figure 7 (c) and Figure 7 (d). The virtual epipolar curves generated by the EPTM using horizontal line projection can also cover the entire image, as shown in Figure 7 (f).

For further analysis, we fitted (in a least-squares sense) the epipolar curves to straight lines, and calculated the fitting residuals in Figure 8. In terms of the data trend and magnitude, the epipolar curves generated by the PTM are similar to the epipolar curves generated by the EPTM using plumb line projection, and the epipolar curves generated by the PPTM are similar to the epipolar curves generated by the EPTM using along track horizontal line projection.

For quantitative analysis, we calculated the deviation coefficient of each epipolar curve. The results are shown in Tables 2, 3, and 4. Since we only generated the epipolar curves of Pleiades-2 in the PTM experiment, as shown in Figures 7 (a) and (b), we additionally generated the epipolar curves of Pleiades-1 using the PTM and calculated the corresponding deviation coefficients for completeness. The results are listed in Table 2. Since we only generated the epipolar curves of Pleiades-1 in the EPTM experiment, as shown in Figures 7 (e) and (f), we additionally generated the epipolar curves of Pleiades-2 using the EPTM and calculated the corresponding deviation coefficients. The results are listed in the Table 4.

It can be seen in Table 2 that the deviation coefficients of the epipolar curves generated by the PTM are quite different. They have poor consistency and cannot reflect the overall characteristics of the images. The epipolar curves generated by the PTM are so short that they can only be used to analyze the local epipolarity of stereo pairs.

It can be seen in Table 3 that the deviation coefficients of the epipolar curves generated by the PPTM vary slightly. They have good consistency and can reflect the overall characteristics of the images. The PPTM can generate complete epipolar curves, which can be used to analyze the overall epipolarity of stereo pairs.

As shown in Table 4, the deviation coefficients of the virtual epipolar curves projected from plumb lines are quite
different. The virtual epipolar curves projected from the plumb lines are similar to the epipolar curves generated by the PTM. They have the same problem in that they are too short to analyze the overall epipolarity of an image.

The deviation coefficients of the virtual epipolar curves projected from horizontal lines vary slightly. They have good consistency and can reflect the overall characteristics of the images. In fact, the virtual epipolar curves projected from along-track horizontal lines are similar to the epipolar curves generated by the PPTM. Both of them can cover the whole image, and their deviation coefficients are very close. This is because the epipolar curves projected from along-track
horizontal lines are consistent with the epipolar curves generated by the PPTM in direction. Since the directions of the virtual epipolar curves projected from cross-track horizontal lines and the epipolar curves generated by the PPTM are significantly different, their corresponding deviation coefficients may be quite different, as in the case of Pleiades-2, as shown in Table 4.

The experiments in this section show that our EPTM can be used to replace the PTM and the PPTM for the epipolarity analysis of pushbroom satellite images. Since the EPTM does not require the RPCs of the inverse form and can be used to analyze the epipolarity of a single image, it is more convenient to use the EPTM to analyze the epipolarity of satellite images.

**B. MAXIMUM DEVIATION COEFFICIENT**

From the analysis in the previous section, it can be seen that only the PPTM and the EPTM can generate complete epipolar curves for pushbroom satellite images. To calculate the maximum deviation coefficient of an image, we tested the PPTM and the EPTM using the Pleiades triplet and the SPOT7 triplet. First, two groups of tristereo images were combined into stereo pairs, respectively. We generated epipolar curves for each stereo pair using the PPTM and calculated the corresponding deviation coefficients. Then, for each image, four diagonal lines were selected in the range box defined by the RPCs. We generated virtual epipolar curves using the EPTM by projecting the diagonal lines into the image space. The corresponding deviation coefficients were calculated. The results are shown in Tables 5, 6, 7, and 8.

As shown in Tables V and VI, for the PPTM tests, the deviation coefficients of an image in different stereo pairs are different and they vary slightly. This is because all the images in the same triplet are acquired in a single track, and the epipolar curves of the same image generated in two different stereo pairs have a consistent direction; therefore, the
TABLE 4. The deviation coefficients of the epipolar curves generated by the EPTM.

| Image  | Plumb 1 | Plumb 2 | Plumb 3 | Plumb 4 | Plumb 5 | Mean  | Std   |
|--------|---------|---------|---------|---------|---------|-------|-------|
| Pleiades-1 | 0.008   | 0.073   | 0.034   | 0.022   | 0.063   | 0.040 | 0.025 |
| Pleiades-2 | 0.024   | 0.108   | 0.132   | 0.496   | 0.314   | 0.215 | 0.169 |
| Image  | Cross 1 | Cross 2 | Cross 3 | Cross 4 | Cross 5 | Mean  | Std   |
| Pleiades-1 | 1.873   | 1.872   | 1.871   | 1.871   | 1.873   | 1.872 | 0.001 |
| Pleiades-2 | 2.96    | 2.958   | 2.957   | 2.956   | 2.956   | 2.957 | 0.001 |
| Image  | Along 1 | Along 2 | Along 3 | Along 4 | Along 5 | Mean  | Std   |
| Pleiades-1 | 1.822   | 1.859   | 1.896   | 1.933   | 1.97    | 1.896 | 0.052 |
| Pleiades-2 | 1.879   | 1.864   | 1.847   | 1.828   | 1.806   | 1.845 | 0.026 |

*Plumb* n represents the epipolar curve projected from the plumb line. *Cross* n represents the epipolar curve projected from the cross-track horizontal line. *Along* n represents the epipolar curve projected from the along-track horizontal line. The deviation coefficients in the table have been multiplied by 10^4.

TABLE 5. Deviation coefficients of the epipolar curves generated in the Pleiades triplet by the PPTM.

| Image  | Stereo | Curve 1 | Curve 2 | Curve 3 | Curve 4 | Curve 5 | Max   |
|--------|--------|---------|---------|---------|---------|---------|-------|
| Pleiades-1 | 1-2    | 2.023   | 2.022   | 2.021   | 2.025   | 2.036   | 2.036 |
|         | 1-3    | 2.037   | 2.024   | 2.024   | 2.024   | 2.018   | 2.037 |
| Pleiades-2 | 1-2    | 1.899   | 1.967   | 1.985   | 2       | 2.05    | 2.05  |
|         | 2-3    | 1.815   | 1.799   | 1.819   | 1.832   | 1.796   | 1.832 |
| Pleiades-3 | 1-3    | 3.709   | 3.696   | 3.699   | 3.696   | 3.689   | 3.709 |
|         | 2-3    | 3.531   | 3.548   | 3.548   | 3.532   | 3.564   | 3.564 |

*Stereo* represents the stereo pair. The deviation coefficients in the table have been multiplied by 10^4.

TABLE 6. Deviation coefficients of the epipolar curves generated in the SPOT7 triplet by the PPTM.

| Image  | Stereo | Curve 1 | Curve 2 | Curve 3 | Curve 4 | Curve 5 | Max   |
|--------|--------|---------|---------|---------|---------|---------|-------|
| SPOT7-1 | 1-2    | 0.245   | 0.307   | 0.3     | 0.322   | 0.398   | 0.398 |
|         | 1-3    | 0.278   | 0.331   | 0.332   | 0.367   | 0.453   | 0.453 |
| SPOT7-2 | 1-2    | 0.098   | 0.164   | 0.162   | 0.198   | 0.26    | 0.26  |
|         | 2-3    | 0.138   | 0.172   | 0.164   | 0.215   | 0.308   | 0.308 |
| SPOT7-3 | 1-3    | 0.698   | 0.655   | 0.636   | 0.624   | 0.552   | 0.698 |
|         | 2-3    | 0.671   | 0.65    | 0.669   | 0.624   | 0.538   | 0.671 |

*Stereo* represents the stereo pair. The deviation coefficients in the table have been multiplied by 10^4.

The maximum deviation coefficients of Pleiades-1, Pleiades-2, and Pleiades-3 are 0.00029, 0.00043, and 0.00062, respectively. This means that when the lengths of the epipolar curves are shorter than 3448, 2325, and 1612 pixels, respectively, the errors introduced by the approximation are less than 1 pixel. That is, no matter which image they are paired with, the errors introduced by the approximation are less than 1 pixel when the diagonal lengths of the tile images are less than 3448, 2325, and 1612 pixels, respectively.

The maximum deviation coefficients of SPOT7-1, SPOT7-2, and SPOT7-3 are 0.00094, 0.00039, and 0.00129, respectively. This means that when the lengths of the epipolar curves are shorter than 1063, 2564, and 775 pixels, respectively, the errors introduced by the approximation are less than 1 pixel. That is, no matter which image it is paired with, the corresponding deviation coefficients are close. The deviation coefficients of different images may vary widely. The reason may be that different images differ in the incidence angle and the accuracy of RPCs.

As shown in Tables 7 and 8, for the EPTM tests, the deviation coefficients of the epipolar curves projected from diagonal lines are quite different. This mainly resulted from their different directions.

For a certain image, the deviation coefficients of the epipolar curves projected from the diagonal lines are larger than the deviation coefficients of the epipolar curves projected from other ground lines, including the vertical and horizontal lines in the previous section. The maximum deviation coefficients obtained by the EPTM are also larger than the maximum deviation coefficients obtained by the PPTM.
the errors introduced by the approximation are less than 1 pixel when the diagonal lengths of the tile images are less than 1063, 2564, and 775 pixels, respectively.

In addition, by observing the above experiment results, it can be found that for a certain image, the bending degree of an epipolar curve is mainly related to its position and direction in the image. When the positions and directions of the epipolar curves are close, their deviation coefficients are also close. When the positions or directions of the epipolar curves differ widely, their deviation coefficients also differ widely. For different images acquired by the same satellite, the bending degree of their epipolar curves may vary widely. This may be mainly due to their different acquisition angles and the different accuracy of their RPCs. For different images acquired by different satellites, the bending degree of their epipolar curves may vary more widely since it may be affected by many factors, such as the resolution, orientation accuracy, and so on.

VI. CONCLUSIONS

In this paper, we studied three methods based on the RFM for the epipolarity analysis of pushbroom satellite images, including the PTM, the PPTM, and our EPTM. We defined the deviation coefficient as a metric to evaluate the bending degree of epipolar curves. We also defined the maximum deviation coefficient of an image, which can be used to determine the image dividing size in the epipolar resampling of stereo pairs.

The EPTM is convenient and practical. It only needs the RPCs of the forward form and can generate complete epipolar curves for a single image. By flexibly choosing virtual rays in the range box, it can be used to comprehensively analyze the epipolarity of pushbroom images. By calculating the maximum deviation coefficient of the image, it can be used to determine the image dividing size in the epipolar resampling of multiview 3D reconstruction.

Compared with the PTM and the PPTM, the EPTM has three advantages: 1) the EPTM only needs the forward form of the RFM, while the PTM and the PPTM need both the forward form and the inverse form; 2) the EPTM can generate virtual epipolar curves for a single image, while the PTM and the PPTM can only generate epipolar curves for a stereo pair; and 3) the EPTM can be used to calculate the maximum deviation coefficient of an image that the PTM and the PPTM cannot calculate.

Considering the EPTM from another view, for a pushbroom satellite image, the projection trajectory projected from any straight line in the ground space is no longer straight, but rather a curve. Since the bending degree of the curves are related to the imaging quality of the satellite, the maximum deviation coefficient obtained by the EPTM can also reflect the imaging quality of the pushbroom satellite image to some extent.

In future research, we will study the epipolar resampling of pushbroom satellite images by dividing large images into small tiles based on the methods studied in this paper.

REFERENCES

[1] J. Hu, G.-S. Xia, and H. Sun, “An SRTM-aided epipolar resampling method for multi-source high-resolution satellite stereo observation,” Remote Sens., vol. 11, no. 6, p. 678, Mar. 2019.

[2] M. Jannati, M. Valadan Zoej, and M. Mokhtarzade, “Epipolar resampling of cross-track pushbroom satellite imagery using the rigorous sensor model,” Sensors, vol. 17, no. 12, p. 129, Jan. 2017.
[3] M. Jannati, M. J. Valadan Zoeg, and M. Mokhtarzade, “A novel approach for epipolar resampling of cross-track linear pushbroom imagery using orbital parameters model,” ISPRS J. Photogramm. Remote Sens., vol. 137, pp. 1–14, Mar. 2018.

[4] J. Mallon and P. F. Whelan, “Projective rectification from the fundamental matrix,” Image Vis. Comput., vol. 23, no. 7, pp. 643–650, Jul. 2005.

[5] A. Fusiello and L. Irsara, “Quasi-Euclidean uncalibrated epipolar rectification,” in Proc. 19th Int. Conf. Pattern Recognit., Dec. 2008, pp. 1–4.

[6] A. Fusiello and L. Irsara, “Quasi-Euclidean epipolar rectification of uncalibrated images,” Mach. Vis. Appl., vol. 22, no. 4, pp. 663–670, Jul. 2011.

[7] H. Hirschmuller, “Accurate and efficient stereo processing by semi-global matching and mutual information,” in Proc. Comput. Vis. Pattern Recognit. (CVPR), 2005, pp. 807–814.

[8] H. Hirschmuller, “Stereo processing by semiglobal matching and mutual information,” IEEE Trans. Pattern Anal. Mach. Intell., vol. 30, no. 2, pp. 328–341, Feb. 2008.

[9] N. Tatar, M. Saadatseresht, H. Arefi, and A. Hadavand, “Quasi-epipolar resampling of high resolution satellite imagery for semi global matching,” Int. Arch. Photogramm., Remote Sens. Spatial Inf. Sci., vol. 40, no. 1, p. 707, 2015.

[10] R. I. Hartley and A. Zisserman, Multiple View Geometry in Computer Vision, 2nd ed. New York, NY, USA: Cambridge Univ. Press, 2003, pp. 239–247.

[11] J.-I. Kim and T. Kim, “Comparison of computer vision and photogrammetry approaches for epipolar resampling of image sequence,” Sensors, vol. 16, no. 3, p. 412, Mar. 2016.

[12] D. Fritschi, “Some stuttgart highlights of photogrammetry and remote sensing,” Photogrammetric Week, 15th ed. D. F. Wichmann. Berlin, Germany: Offenbach, 2015, pp. 3–20.

[13] T. Kim, “A study on the epipolarity of linear pushbroom images,” Photogramm. Eng. Remote Sens., vol. 66, no. 8, pp. 961–966, 2000.

[14] A. B. Orun and K. Natarajan, “A modified bundle adjustment software for SPOT imagery and photography—Tradeoff,” Photogramm. Eng. Remote Sens., vol. 60, no. 12, pp. 1431–1438, 1994.

[15] A. F. Habib, M. Morgan, S. Jeong, and K.-O. Kim, “Analysis of epipolar geometry in linear array scanner scenes,” Photogramm. Rec., vol. 20, no. 109, pp. 27–47, Mar. 2005.

[16] R. Gupta and R. I. Hartley, “Linear pushbroom cameras,” IEEE Trans. Pattern Anal. Mach. Intell., vol. 19, no. 9, pp. 963–975, May 1997.

[17] A. F. Habib, M. F. Morgan, S. Jeong, and K.-O. Kim, “Epipolar geometry of line cameras moving with constant velocity and attitude,” ETRI J., vol. 27, no. 2, pp. 172–180, Apr. 2005.

[18] S. Hattori, T. Ono, C. Fraser, and H. Hasegawa, “Orientation of high-resolution satellite images based on affine projection,” Int. Arch. Photogramm. Remote Sens., vol. 33, nos. 3–1, pp. 359–366, 2000.

[19] T. Ono, S. Hattori, H. Hasegawa, and S. Akamatsu, “Digital mapping using high resolution satellite imagery based on 2D affine projection model,” Int. Arch. Photogramm. Remote Sens., vol. 33, no. 3, pp. 672–677, 2000.

[20] C. Tao and Y. Hu, “The rational function model: A tool for processing high resolution imagery,” Earth Observat. Mag., vol. 10, pp. 13–16, 2001.

[21] Y. Zhang, J. Liu, and D. Gong, High Resolution Remote Sensing Satellite Application. 2nd ed. Beijing, China: Science Press, 2014.

[22] G. Danchao, Z. Yongsheng, and D. Xueqing, “A study on the epipolarity of linear CCD push-broom images,” J. Remote Sens., vol. 2, pp. 97–101, Dec. 2004.

[23] D. Zhao, X. Yuan, and X. Liu, “Epipolar line generation from IKONOS imagery based on rational function model,” Int. Arch. Photogramm., Remote Sens. Spatial Inf. Sci., vol. 37, no. B4, pp. 1293–1297, 2008.

[24] M. Idrissa and C. Beumier, “Generic epipolar resampling method for perspective frame camera and linear push-broom sensor,” Int. J. Remote Sens., vol. 37, no. 15, pp. 3494–3504, Aug. 2016.

[25] J. Oh, W. H. Lee, C. K. Toth, D. A. Grejner-Brzezinska, and C. Lee, “A piecewise approach to epipolar resampling of pushbroom satellite images based on RPC,” Photogramm. Eng. Remote Sens., vol. 76, no. 12, pp. 1353–1363, Dec. 2010.

[26] J. Oh, “Novel approach to epipolar resampling of HRSI and satellite stereo imagery-based georeferencing of aerial images.” Ph.D. dissertation, Dept. Philosophy, Ohio State Univ., Columbus, OH, USA, 2011.

[27] J. Koh and H. Yang, “Unified piecewise epipolar resampling method for pushbroom satellite images,” Eurasip J. Image Vide., vol. 2016, no. 1, p. 11, 2016.

[28] H. Yi, X. Chen, D. Wang, and S. Liu, “A novel analysis method for epipolar geometry in pushbroom satellite image based on extended projection trajectory,” in Proc. 6th China High Resolution Earth Observ. Conf., 2019, pp. 192–206.

[29] C. V. Tao, “3D reconstruction methods based on the rational function model,” Photogramm. Eng. Remote Sens., vol. 68, no. 7, pp. 705–714, 2002.

[30] C. V. Tao and Y. Hu, “A comprehensive study of the rational function model for photogrammetric processing,” Photogramm. Eng. Remote Sens., vol. 67, no. 12, pp. 1347–1357, Dec. 2001.

[31] Q. Sheng and H. Xiao, Satellite Remote Sensing and Photogrammetry. Beijing, China: Science Press, 2015.

[32] D. Woo and T. Pham, “Epipolar resampling of pushbroom satellite images using piecewise linear implementation of pseudo epipolar line,” Int. J. Distrib. Comput. Eng., vol. 11, no. 5, pp. 45–55, 2018.

[33] N. Tatar and H. Arefi, “Stereo rectification of pushbroom satellite images by robustly estimating the fundamental matrix,” Int. J. Remote Sens., vol. 40, no. 23, pp. 8879–8898, Jan. 2019.

[34] C. De Franchis, E. Meinhardt-Llopis, J. Michel, J. Morel, and G. Facciolo, “On stereo-rectification of pushbroom images,” in Proc. Int. Conf. Image Process. (ICIP), Oct. 2014, pp. 5447–5451.

[35] M. Bosch, Z. Kurtz, S. Hagstrom, and M. Brown, “A multiple view stereo benchmark for satellite imagery,” in Proc. Appl. Imagery Pattern Recognit. Workshop, 2017, pp. 1–9.

[36] G. Facciolo, C. De Franchis, and E. Meinhardt-Llopis, “Automatic 3D reconstruction from multi-date satellite images,” in Proc. IEEE Conf. Comput. Vis. Pattern Recognit. Workshops, May 2017, pp. 1542–1551.

[37] K. Gong and D. Fritschi, “DSM generation from high resolution multi-view satellite stereo imagery,” Photogramm. Eng. Remote Sens., vol. 85, no. 5, pp. 379–387, May 2019.
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