Using the dense linear multihole to control the directional hydraulic fracturing is a significant technical method to realize roof control in mining engineering. By combining the large-scale true triaxial directional hydraulic fracturing experiment with the discrete element numerical simulation experiment, the basic law of dense linear holes controlling directional hydraulic fracturing was studied. The results show the following: (1) Using the dense linear holes to control directional hydraulic fracturing can effectively form directional hydraulic fractures extending along the borehole line. (2) The hydraulic fracturing simulation program is very suitable for studying the basic law of directional hydraulic fracturing. (3) The reason why the hydraulic fracture can be controlled and oriented is that firstly, due to the mutual compression between the dense holes, the maximum effective tangential tensile stress appears on the connecting line of the drilling hole, where the hydraulic fracture is easy to be initiated. Secondly, due to the effect of pore water pressure, the disturbed stress zone appears at the tip of the hydraulic fracture, and the stress concentration zone overlaps with each other to form the stress guiding strip, which controls the propagation and formation of directional hydraulic fractures. (4) The angle between the drilling line and the direction of the maximum principal stress, the in situ stress, and the hole spacing has significant effects on the directional hydraulic fracturing effect. The smaller the angle, the difference of the in situ stress, and the hole spacing, the better the directional hydraulic fracturing effect. (5) The directional effect of synchronous hydraulic fracturing is better than that of sequential hydraulic fracturing. (6) According to the multihole linear codirectional hydraulic fracturing experiments, five typical directional hydraulic fracture propagation modes are summarized.

1. Introduction
Conventional hydraulic fracturing can effectively change the internal structure of coal rock reservoirs to significantly improve the permeability of coal rock. At the same time, the hard roof above the coal seam is weakened to prevent the dynamic disaster caused by the sudden collapse of the roof. At present, this technology has been widely used in mines [1–11]. The geometric morphology of hydraulic fracture is the core of hydraulic fracturing construction design. Many studies showed that the initiation and propagation of hydraulic fractures are controlled by the three-dimensional in situ stress field. When the stress field is fixed, the orientation of the maximum tangential effective tensile stress on the borehole wall is fixed [12, 13], which makes the hydraulic fractures usually initiate and propagate perpendicular to the direction of the minimum principal stress [14–17]. This provides a foundation for the mechanism and application of hydraulic fracturing.

As the initiation and expansion of hydraulic fractures are mainly controlled by far-field in situ stress, some limitations are found while using conventional hydraulic fracturing technology to solve underground related problems, such as roof cutting and pressure release engineering to automatically form the gob-side entry in the working face of an underground coal mine [18], gob management, gob-side entry retaining, and vertical fracture of hardtop coal [19]. The required direction of hydraulic fracture propagation in these projects is usually not consistent with that under the in situ stress field, which means the expansion of hydraulic fractures
in the suspended roof and coal seam needs to be accurately controlled to guarantee the effectiveness of directional fracture treatment [20, 21]. Therefore, directional hydraulic fracturing technology becomes the research topic.

In order to realize the directional expansion of hydraulic fractures, it is necessary to change the stress distribution of surrounding rock by artificial measures to make the maximum effective tensile stress appear first in the direction to be oriented, which can guide the expansion of hydraulic fractures and form the dominant fracture surface. At present, scholars have proposed the following four methods on this technology [19, 22, 23]:

**Directional hydraulic fracturing by water jet preslottting** [24–28]: this method uses the hydraulic slotting device to artificially prefabricate cracks on the borehole wall to change the stress distribution of the borehole wall, making the borehole crack along the preslottting direction. Relevant studies show that preslottting has an obvious directional effect on the initiation of hydraulic fracture. But with the increase of the distance, the propagation of hydraulic fracture gradually turns to the direction of the maximum principal stress, suggesting that a large range of directional fracture cannot be achieved, and its effective range is very limited.

**Directional hydraulic fracturing method controlled by guiding hole** [29, 30]: multiple guiding boreholes are arranged around the injection hole, which can significantly promote the internal stress distribution of the reservoir and effectively avoid local stress concentration. At the same time, it can also play the role of auxiliary free surface in the process of hydraulic fracture expansion. In addition, the mechanical properties of the rock bridge between the injection hole and the guiding hole are significantly reduced, which makes the hydraulic fracture more easily seep and expand along the direction of the guiding hole. The more guiding holes, the better the directional propagation effect and the larger the propagation range. Therefore, to achieve directional hydraulic fracture in coal and rock strata, a large amount of engineering and cost is required to arrange enough number of guiding holes.

**Directional hydraulic fracturing guided by a guiding groove** [31–34]: as the first directional hydraulic fracturing method, it was developed by the Polish General Institute of mining research to weaken the hard roof of the coal mine. The stress concentration of the annular groove leads the borehole to crack along the slotting direction. However, due to the limited stress concentration at the tip of the circular groove, its effect on the initiation and propagation of hydraulic fracture is limited, as well as its application.

**Directional hydraulic fracturing controlled by dense linear multihole** [29, 35, 36]: plenty of parallel dense linear boreholes are arranged in the specified direction inside the rock in advance. Then, water is injected into these boreholes simultaneously. Under the action of pore water pressure, the skeleton stress between boreholes is redistributed, and the effective tensile stress is formed in the direction of the drilling line [37, 38]. Thus, the hydraulic fracture can overcome the limitation of the far-field in situ stress, and the hydraulic fractures can be initiated and expanded along the wellbore line to achieve the directional fracturing of rock eventually. This method is superior, and the engineering quantity is small with high efficiency. Compared with the above three methods, it is easier to implement and operate, so this method has gradually become the focus of scholars.

It is an effective method to realize directional hydraulic fracturing by using dense linear multiholes, which has positive significance for the directional treatment of the hard roof, directional transfer of stress, prevention of rockburst, and improvement of gas drainage rate [39–41]. However, research on laws of the initiation and propagation of the directional hydraulic fractures controlled by multihole is limited and needs to be further studied. In addition, the mechanism of dense linear boreholes affecting the distribution of the stress field in the rock mass to realize the directional propagation of hydraulic fractures is not clear. With the continuous progress of computer technology and numerical simulation algorithms, the combination of experimental research and numerical simulations provides a feasible solution to solve these problems mentioned [42–46]. At present, there are three popular numerical simulation methods used to research hydraulic fracturing [47–49]: the finite element method (FEM), extended finite element method (XFEM), and discrete element method (DEM). The basic idea of the FEM is to discretize the elastic body into an equivalent system of small elements [50]. In this method, the crack boundary coincides with the mesh nodes, and a mesh reconstruction method is used to simulate the crack propagation. The hydraulic fracturing model established by this method requires less calculation and has high efficiency. However, the hydraulic fracture can only extend along a preset path, and the FEM cannot simulate the deflection of hydraulic fractures or the formation process of a complex fracture network [51, 52]. The XFEM is based on the FEM and introduces a shape function to represent the discontinuity of the displacement field [53–55], so the description of the discontinuous displacement field is completely independent of the mesh boundary. This method can simulate fracture propagation along any path without grid reconstruction. This is advantageous in the analysis and calculation of fracture problems and greatly improves the calculation efficiency. The disadvantage of the XFEM is that the simulation of microcracks is limited and needs further development. The main idea of the DEM is to use an explicit algorithm to calculate the motion of particles or blocks, that is, update the motion and contact state of particles in each calculation [56, 57]. When the contact force exceeds its bearing limit, the material will demonstrate shear dislocation, compression shear failure, tensile failure, and other rock fracture phenomena [58, 59]. The channel formed between particles can be used to simulate the fluid flow in a pipe. Because the DEM does not have to satisfy the continuity condition, the DEM is very suitable for simulating the initiation and propagation of microcracks in rock. Considering that this paper mainly focuses on the micromechanism of directional hydraulic fracture propagation, the discrete element simulation method is more suitable.

In this paper, the experimental test and a DEM numerical simulation using particle flow code (PFC<sup>2D</sup>) were combined to investigate the laws of the initiation and propagation of
directional hydraulic fracturing controlled by the dense linear multihole. Besides, the sensitivity analysis of the geological condition and hydraulic fracturing condition affecting the directional effect and accuracy was carried out. The research results can provide theoretical guidance for the implementation of directional hydraulic fracturing in coal mines.

2. Particle Flow Method

2.1. Parallel Bond Model. A parallel bonding model has not only standard stiffness and tangential stiffness but also normal tensile strength and shear strength [56]. It consists of two elements: linear model and parallel bond. The parallel bonding model can be compared to a set of springs arranged on rock particles (Figure 1). Under the action of external force, there is force and displacement between particles. If tensile and shear strength of parallel bond element exceeds \( \sigma_{max} \) or \( \tau_{max} \), the parallel bonds break, and the bond material and its associated forces, moments, and stiffness will be removed from the model, and only the linear model will be available. The parameters shown in the figure represent the mesoparameters of the parallel bond model, where \( k_n \) refers to the normal stiffness of particle element, \( k_t \) represents the tangential stiffness of particle element, \( g_s \) refers to the spacing between particles, \( \mu \) represents the friction coefficient between particle elements, \( \sigma_r \) refers to the normal strength of parallel bond, \( \Phi \) represents the friction angle of rock, and \( c \) refers to the cohesion of rock [42, 60-62].

2.2. Fluid-Solid Coupling Model. The fluid-solid coupling model assumes that the geometric space between adjacent particles is the seepage channel of liquid, and the fluid form of liquid is plate flow (Figure 2). The adjacent seepage channels are connected to form a closed “domain.” In the calculation process, the water pressure in the “domain” is continuously updated and acts on the particles.

The liquid flow of the domain is computed as follows [63]:

\[
q = \frac{a^3}{12\mu} \frac{p_2 - p_1}{L},
\]

where \( q \) represents the liquid flow, \( \mu \) represents the dynamic viscosity, \( p_2 - p_1 \) is the pressure difference between two adjacent areas, \( L \) is the length of the pipe, and \( a \) refers to the width of the connecting “pipe.”

The change of pore fluid pressure is calculated as follows:

\[
\Delta p = \frac{K_f}{V_d} \left( \sum q_{\Delta t} - \Delta V_d \right),
\]

where \( K_f \) refers to the bulk modulus, \( V_d \) is the apparent volume of the domain, \( \sum \) represents the total flow of the domain from the surrounding pipes, and \( \Delta \) refers to the calculation time step:

\[
F_i = \rho n_i s,
\]

where \( n_i \) is the external average unit vector of the connecting line between adjacent particles, \( s \) is the distance from the center of the corresponding particle to the contact point, and \( p \) is the change of fluid pressure in each time step.

In addition, because of the explicit algorithm, in order to ensure the stability of the numerical model, the pressure change caused by water flow must be less than the disturbance pressure:

\[
\Delta t = \frac{2r V_d}{NK_n k_\alpha^3},
\]

where \( N \) is the number of pipes connected to a domain and \( r \) is the average radius of particles around a domain. In addition, in order to ensure the stability of the whole computing domain, the global time step must be the minimum of all local time steps.

3. Dense Linear Multihole Directional Hydraulic Fracturing Experimental Investigation

3.1. Test Block Preparation and Experimental System. In the experiment, a 500 mm × 500 mm × 500 mm cement mortar block is used as the sample to replace the coal rock mass (Figure 3). The physical and mechanical parameters of the test block are shown in Table 1. Three linearly distributed boreholes are prefabricated on the cube test block, named H1, H2, and H3, respectively. The angle between the direction of the linear borehole and the direction of maximum principal stress (\( \sigma_1 \)) is 15°. The distance between two adjacent boreholes is 141 mm. The test block adopts a borehole packer with an outer diameter of 18 mm. There is another naked drill hole at 100 mm in length from the end of the borehole packer. In addition, a large-scale true triaxial hydraulic fracturing experimental system is applied in the test (Figure 4). The experimental system consisted of a bench frame, a loading system, and a monitoring system.

3.2. Morphology of Dense Linear Multihole Directional Hydraulic Fracture. According to the scheme shown in Table 2, directional hydraulic fracturing controlled by the dense linear multihole was carried out on the coal rock mass.
Figure 2: Pipeline and domain in PFC\textsuperscript{2D} [30].

Figure 3: Manufacture of specimens [64].

Table 1: Physical and mechanical parameters of the cement mortar.

| Porosity $\phi$ (%) | Permeability $K$ (mD) | Uniaxial compressive strength $\sigma_c$ (MPa) | Modulus of elasticity $E$ (GPa) | Tensile strength $\sigma_t$ (MPa) | Cohesion $c$ (MPa) | Angle of internal friction $\phi$ (°) | Fracture toughness $K_{IC}$ (N mm$^{3/2}$) |
|---------------------|-----------------------|---------------------------------|-------------------------------|----------------------|-------------------|----------------|-------------------|
| 13.79               | 1.13                  | 15.85                          | 0.92                         | 1.65                 | 2.5443           | 31.29          | 13.23            |
After a large amount of water leakage from the loading frame, the experiment stopped.

After the loading plate is opened, there are visible hydraulic fractures on the upper surface and side of the test block (Figure 5), and the directional hydraulic fractures on the upper surface extend to the edge of the test block along the borehole line direction, and the directional effect is obvious.

Among them, hydraulic fracture on the leftwing of H1 initiated and expanded along the direction of the borehole line, while that of the rightwing of H1 expanded along the direction of the borehole line and intersected with the H2 borehole. The hydraulic fracture on the leftwing of H2 extended along the direction of the borehole line, but it did not intersect with the hydraulic fracture on the rightwing of H1, forming a rock bridge area between them. The hydraulic fractures on the H2 rightwing and H3 leftwing both propagated along the direction of the borehole line and finally intersected smoothly. The hydraulic fracture on the H3 rightwing propagated to the boundary of the test block along the direction of maximum principal stress.

By observing the hydraulic fracture trajectory in each borehole, it was found that in borehole H1-H3, the hydraulic fractures were obviously initiated and expanded along the direction of the borehole line, while they were initiated and expanded basically along the direction of the maximum principal stress outside H1-H3. The fracture trajectory occurred with an obvious deflection at H1 and H3. On the whole, the directional hydraulic fracture surface was approximately a folding failure surface (Figure 6), which also suggested that the initiation and propagation of hydraulic fracture were affected by the far-field stress and the coupling stress between boreholes.

After cutting the test block along the trajectory of directional hydraulic fracture, it can be found that the directional fracture surface was formed in the H1-H3 section and a two-layered fracture surface appeared. The fracture surface I was formed by the expansion of hydraulic fractures on both wings of H1, and fracture surface II was formed by the expansion of hydraulic fractures on both wings of H2. The area between the two fracture surfaces is the rock bridge area mentioned above. The expansion range of fracture surface I was relatively small. The intensive fluctuation of the fracture surface I indicates that the hydraulic fracture expanded with obvious deflection, and the orientation effectiveness was poor. However, the second fracture zone covered the whole

---

**Table 2: Experimental scheme of directional hydraulic fracturing controlled by dense linear multihole.**

| Test name                  | Principal stress (MPa) | Borehole spacing (mm) | Arrangement angle of boreholes (°) | The injection rate (ml/min) | Pumping mode |
|---------------------------|------------------------|-----------------------|------------------------------------|----------------------------|--------------|
| 1-1 (laboratory test)     | 7                      | 5                     | 3                                 | 141                        | 15           | 50           | Synchronous |
| 1-2 (simulation test)     | 7                      | 3                     | 141                               | 15                         | 50           | Synchronous |

---

**Figure 4: True triaxial hydraulic fracturing test system.**
block, and the fracture surface II showed a large expansion range and good orientation, indicating that the directional effect of the hydraulic fracture in the middle borehole was stronger than that in the surrounding borehole. This is because the stress coupling degree between the middle boreholes is the highest.

### 4. Numerical Investigation of Dense Linear Multihole Directional Hydraulic Fracturing

#### 4.1. Calibration of Numerical Model Parameters

To guarantee the mechanical properties and failure characteristics of DEM simulation samples consistent with the laboratory test results, a numerical model with a length of 100 mm and a width of 50 mm was established to calibrate the mechanical parameters (Figure 7). The microparameters were adjusted repeatedly until the stress-strain curve and the final fracture mode of the simulated specimen were consistent with the laboratory test. Table 3 shows the microscopic parameters of the simulated sample. The calibration results show that the selection of mesoparameters is reasonable.

The rock parameters obtained above were used for numerical modeling. As Figure 8 shows, the size of the square model is 500 mm in length and 500 mm in width, which contains approximately 14683 particles. The fluid injection point is located in the center of the borehole. The angle between the drilling line and the direction of the maximum principal stress is also 15°. The confining pressure of the test block was loaded by applying a stress constraint on the boundary (wall) of the model.

#### 4.2. Morphology of Directional Hydraulic Fracture

According to the scheme shown in Table 2, the simulation results are shown in Figure 9. The hydraulic fractures in the three boreholes are initiated and propagated along the direction of the drilling line. The hydraulic fracture of the H1 leftwing extended to the edge of the test block along the maximum main stress direction, while the hydraulic fracture of the H1 rightwing extended a short distance along the maximum main stress direction and then turned to the borehole line with obvious turning points in the fracture trajectory. Finally, they intersected with that of the H2. The hydraulic fracture of the H2 leftwing extended to H1 along the borehole line, but it did not intersect with the hydraulic fracture of the H1 rightwing, forming a rock bridge. The hydraulic fractures of H2 rightwing and H3 leftwing both extend in the direction of the drilling line and finally merged smoothly. Hydraulic fracture of H3 rightwing extended to the edge of test block along

![Figure 5: Propagation morphology of directional hydraulic fracturing [64].](image)

![Figure 6: The integrated morphology of the failure plane [64].](image)

![Figure 7: Laboratory test and numerical simulation of the stress-strain curve and final fracture mode of the intact specimen [30].](image)
the direction of maximum principal stress. The shape of directional hydraulic fracture in the numerical simulation was very consistent with that of the laboratory test, which fully verifies the rationality of the particle flow numerical simulation test.

5. The Mechanism of Directional Hydraulic Fracturing Controlled by Dense Linear Multihole

The simulation results were furtherly analyzed to explore the directional fracture mechanism. In this section, the mechanism of dense linear multihole controlling the directional hydraulic fracturing from the fracture initiation stage and fracture propagation stage was analyzed.

5.1. Initiation Stage of Hydraulic Fracture. According to the theory of hydraulic fracturing, the resistance that hydraulic fracture needs to overcome is the least when it propagates along the direction perpendicular to the minimum principal stress, which means the direction perpendicular to the minimum principal stress in the far-field stress field is the dominant propagation direction. When the water injection hole is excavated, due to the constraint of the far-field stress field, the borehole wall will occur compression deformation, resulting in the stress concentration effect around the wellbore. If multiple boreholes are arranged on the same straight line, due to the mutual compression between multiple boreholes, the stress concentration between boreholes affected by the superposition of boreholes will be significantly higher than that of single boreholes, and the tangential tensile stress field perpendicular to the centerline of boreholes is generated. With the injection of high-pressure water, the pore water pressure gradient is formed on the hole wall, which creates a new additional stress field around the borehole, and the additional stress field will be superimposed on the surrounding rock stress formed after excavation.

It is assumed that the stress in the original reservoir rock mass is uniformly distributed. In this study, the complex three-dimensional stress problem is simplified as a two-dimensional stress model. The additional stress can be calculated by the thick-walled cylinder formula. The stress of surrounding rock mass can be calculated as follows [65]:

\[
\begin{align*}
\sigma_r &= \frac{a^2 (b^2 - r^2)}{r^2 (b^2 - a^2)} p = \frac{a^2}{r^2} p, \\
\sigma_\theta &= -\frac{a^2 (b^2 + r^2)}{r^2 (b^2 - a^2)} p = -\frac{a^2}{r^2} p,
\end{align*}
\]

where \( p \) represents the water pressure, \( a \) is the radius of the borehole, and \( b \) refers to the outer diameter.

According to equation (5), it can be found that in the process of hydraulic fracturing, the tangential tensile stress perpendicular to the borehole line will concentrate in the rock mass, and its stress intensity fluctuates periodically with the change of distance (Figure 10). The closer the borehole hole is, the higher the stress intensity is, and the tensile stress is the largest at the intersection of the borehole center line and the borehole wall. Therefore, with the continuous accumulation of water pressure in the hole, the water pressure in the direction of the borehole line will first overcome the tensile strength of the rock and cause a fracture; thus, the hydraulic fracture begins to germinate. That explains why the hydraulic fracture is most likely to initiate along the borehole line, when the directional hydraulic fracturing controlled by dense linear multihole is used.

5.2. Propagation Stage of Hydraulic Fracture. By extracting the maximum principal stress distribution data, the dynamic evolution of the stress field in the test block can be obtained and shown in Figure 11.

After the borehole wall cracked, the high-pressure water entered into the hydraulic fracture and penetrated the boundary area of the fracture tip, forming the pore pressure gradient. At this time, the stress was disturbed by the hydraulic fracturing, and the stress disturbance field appeared around the hydraulic fracture, where the tensile disturbance stress concentrated in the tip of the hydraulic fracture, and the compressive stress was distributed on both sides of the hydraulic fracture. The stress disturbance field superimposed the original stress field of surrounding rock, forming a new stress distribution field in the boundary area of the fracture tip and leading to the deflection of the direction of the maximum principal stress along the propagation path in front of the hydraulic fracture. However, because the size and scope of the tensile disturbance stress generated by the fracture tip were limited, the stress disturbance fields around different
hydraulic fracture tips did not superimpose with each other, making the far-field in situ stress still dominate the key factor of hydraulic fracture propagation direction.

With the continuous injection of high-pressure water, the length of hydraulic fracture increased, and the size and range of disturbed tensile stress at the tip of hydraulic fracture gradually increased. At this time, the influence of the far-field stress on hydraulic fracture propagation was weakened. The superposition and fusion of different tensile stress disturbance fields resulted in the formation of a new maximum principal stress concentration strip between adjacent boreholes, which is named “guided fracturing control zone between boreholes.” The maximum principal stress strip showed an “\(\int\)” shape and connected the tips of adjacent hydraulic fractures, which promoted the hydraulic fractures in the guided fracturing control area to expand along the borehole line (Figure 12). In addition, multiple maximum principal stress concentration strips developed along the borehole line and paralleled to each other.

With the further increase of hydraulic fracture length, the range and internal stress of the maximum principal stress concentration zone between boreholes also increased. It can be seen that the shape of the stress strip also changed from “\(\int\)” to “\(\int\)” to “\(\int\).” In the process of morphological change, the propagation direction of hydraulic fracture also changed continuously. In the expansion process, the hydraulic fracture in each borehole not only deflected as a whole and tended to expand along the borehole line but also locally deflected around the borehole axis, which made these hydraulic fractures intersected and fused to form a directional fracture surface along the borehole axis and realize the directional fracture of rock reservoir. After the adjacent hydraulic fractures merge with each other, the maximum principal stress strip gradually decreased and disappeared.

6. Analysis of Influencing Factors of the Multihole Linear Directional Hydraulic Fracture Propagation

When directional hydraulic fracturing controlled by dense linear multihole is carried out in the coal mine [66–72], the in situ stress conditions, borehole arrangement angle, borehole spacing, and borehole pumping mode will significantly affect the shape and orientation accuracy of the directional hydraulic fracture, so this section will carry out sensitivity analysis for the above factors.

6.1. Discussion: Angle between the Line of the Borehole and the Direction of Maximum Principal Stress. According to the scheme shown in Table 4, four groups of arrangement angles of boreholes \(\theta\) are set, and the angles are 15, 45, 60, and 75, respectively. The final test results are shown in Figure 13. When the value of \(\theta\) was 15°, except for hydraulic fracture of the H3 rightwing extended along the direction of maximum main stress, the hydraulic fractures of other holes all extended along the direction of the borehole line. The hydraulic fractures of the H2 leftwing and the H1 rightwing expanded and gradually fused, and the hydraulic fracture of the H2 rightwing was connected with the H3. In addition, the hydraulic fracture of the H3 leftwing turned obviously, forming a rock bridge with the hydraulic
fracture of H2 rightwing and resulting in a directional hydraulic fracture crossing each borehole without obvious bifurcation phenomenon.

When the value of \( \theta \) was 45°, the hydraulic fractures of each borehole initiate along the direction of the maximum principal stress \( \sigma_1 \) and extend for a short distance. With different hydraulic fractures approaching each other, the propagation direction changed, and the hydraulic fractures expanded along the borehole line. Finally, the hydraulic fractures intersected with adjacent injection holes. In addition, it should be noted that the hydraulic fractures on both sides of different boreholes did not fuse. Thus, multiple rock bridge areas were formed, and the fracture morphology was more tortuous and rough, forming a "ladder"-shaped directional hydraulic fracture connecting each borehole with obvious stratification.

When the value of \( \theta \) was 60°, the hydraulic fractures of the three boreholes initiated and expanded along the direction of \( \sigma_1 \). Under the influence of coupling stress between holes, the propagation direction of hydraulic fractures in each borehole gradually approached the borehole line from the direction of the maximum principal stress with a small deflection angle. Although the hydraulic fractures tended to connect with each other, the directional hydraulic fractures were not formed in the end.

When the value of \( \theta \) was 75°, the hydraulic fractures of the three boreholes initiated and propagated along the direction of the maximum principal stress \( \sigma_1 \) without deflection.

The three main hydraulic fractures are parallel to each other, and the propagation behavior of hydraulic fractures is basically determined by the in situ stress field. In addition, there is an obvious stress shadow effect. Multiple fractures compete with each other, and the fracture propagation trajectories repel each other. The lateral hydraulic fracture will deflect to the outside due to the action of the medial hydraulic fracture. The medial hydraulic fracture is squeezed under the influence of the induced stress field of the outer hydraulic fracture, and the width and length of the fracture become narrower, which seriously affects the effect of hydraulic fracturing.

By comparing the directional hydraulic fracture morphology under different arrangement angles of boreholes \( \theta \), it can be found that with the increase of the angle, the directional effect and accuracy gradually weaken. When the angle increased to a certain extent, the directional effect disappeared, and the directional hydraulic fracture was not formed. When the directional hydraulic fractures were 30° and 45°, it was found that the orientation accuracy of directional hydraulic reduced within a certain range. However, the angle between the formed directional fractures and the direction of maximum principal stress significantly increased, and the effect range of the directional hydraulic fractures in the reservoir was also significantly improved. In addition, if the arrangement angle of boreholes \( \theta \) is too large, it is difficult to form directional hydraulic fractures, and there will be an obvious stress shadow effect, which seriously affects the effect of conventional hydraulic fracturing.
Table 4: Experimental scheme of directional hydraulic fracturing.

| Test name | Principal stress (MPa) | Borehole spacing (mm) | Arrangement angle of boreholes (°) | The injection rate (ml/min) | Pumping mode |
|-----------|------------------------|-----------------------|------------------------------------|----------------------------|--------------|
| 2-1       | 7                      | 3                     | 120                                | 15                         | 50           | Synchronous |
| 2-2       | 7                      | 3                     | 120                                | 45                         | 50           | Synchronous |
| 2-3       | 7                      | 3                     | 120                                | 60                         | 50           | Synchronous |
| 2-4       | 7                      | 3                     | 120                                | 75                         | 50           | Synchronous |

Figure 13: Initiation and propagation process of directional hydraulic fractures at different angles between the direction of borehole line and maximum principle stress $\sigma_1$. 
### Table 5: Experimental scheme of directional hydraulic fracturing.

| Test name | Principal stress (MPa) | Borehole spacing (mm) | Arrangement angle of boreholes (°) | The injection rate (ml/min) | Pumping mode |
|-----------|------------------------|-----------------------|-----------------------------------|-----------------------------|--------------|
| 3-1       | 7                      | 1                     | 141                               | 15                          | 50           | Synchronous |
| 3-2       | 7                      | 3                     | 141                               | 15                          | 50           | Synchronous |
| 3-3       | 7                      | 5                     | 141                               | 15                          | 50           | Synchronous |

**Figure 14**: Initiation and propagation process of directional hydraulic fractures at different principle stress differences.
6.2. Discussion: The Principal Stress Difference. According to the scheme shown in Table 5, three different horizontal in situ stress differences 2, 4, and 6 MPa were set, and the test results are shown in Figure 14.

When the value of $\Delta \sigma$ was 2 MPa, the hydraulic fractures of the three boreholes initiated and expanded along the direction of the borehole line, and the hydraulic fractures of the adjacent boreholes finally fused each other to form a linear directional hydraulic fracture coplanar with all the injection holes. The directional hydraulic fracture surface was smooth and flat with obvious directivity and high orientation accuracy.

When the value of $\Delta \sigma$ was 4 MPa, the hydraulic fractures on both sides of H3 extended to the edge of the test block along the direction of the maximum principal stress, and the hydraulic fractures on both sides of H1 and H2 extended along the direction of the borehole line and finally approached the adjacent boreholes to form directional hydraulic fractures. The directional hydraulic fractures of each hole did not fuse, and the formed rock bridge area showed an obvious stratification phenomenon.

When the value of $\Delta \sigma$ was 6 MPa, the hydraulic fractures of each borehole basically initiated and expanded along the direction of the maximum principal stress. In the expansion process, the hydraulic fractures slightly deflected and paralleled to each other, and no directional hydraulic fractures were formed, indicating the poor directional effect.

In conclusion, the principal stress difference has a significant effect on directional hydraulic fracturing. With the increase of principal stress difference, the effect and accuracy of orientation become worse. The hydraulic fracture propagation direction of each borehole gradually deviates from the borehole line, and the angle between the maximum principal stress and the hydraulic fracture propagation direction gradually decreases. Hydraulic fractures are more difficult to fuse and easy to form rock bridge area. When the stress difference is greater than a certain extent, the directional effect is completely invalid. The hydraulic fractures are parallel to each other, which cannot achieve the goal of directional fracturing.

6.3. Discussion: Borehole Spacing. According to the scheme shown in Table 6, three groups of different borehole spacing $D$ were set, and the borehole spacing is 105, 141, and 176 mm. The final test results are shown in Figure 15.

When the value of $D$ was 105 mm, the hydraulic fractures in the directional fracturing section (H1-H3) obviously initiated and expanded along the borehole line direction and finally intersected with each other and approached adjacent injection holes to form a directional hydraulic fracture. For the range outside the directional fracturing section (H1-H3), the hydraulic fractures extended to the edge of the test block along the direction of the maximum principal stress. The directional fracture surface was coplanar with all injection holes. It was smooth and flat with obvious directivity and without other bifurcation and stratification.

When the value of $D$ was 141 mm, the hydraulic fractures of the H1 leftwing and the H3 rightwing extended to the edge of the test block along the direction of maximum principal stress $\sigma_1$ and then extended along the direction of the borehole line with an obvious turning point in the fracture trajectory. The hydraulic fractures of H2 finally extended to two adjacent holes along the borehole line and did not intersect with the adjacent hydraulic fractures, forming a rock bridge in the oblique direction of $\sigma_1$. Besides, there is delamination in directional hydraulic fracture.

When the value of $D$ was 176 mm, the hydraulic fractures of the three boreholes all initiated and expanded along the direction of the maximum principal stress. After expanding for a certain distance, the direction of expansion deflected. However, due to the long distance between the boreholes, the coupling stress between the boreholes was weak and the degree of deflection was small, and finally, there was no directional fracture formed.

With the increase of hole spacing, the directional fracture zone became more difficult to be formed by overlapping the stress disturbance zones at the tip of hydraulic fractures. The coupling stress between holes was smaller, making it more difficult for hydraulic fractures of different boreholes to intersect. The stratification phenomenon and the number of rock bridges formed in the process of hydraulic fracture intersection significantly increased, and the directional effect and accuracy were reduced.

6.4. Discussion: Borehole Pumping Mode. During the hydraulic fracturing construction in the coal mine, due to the pipeline loss and the difference in the strength of rock around the borehole, the initiation of hole wall cracking was not synchronous. In order to reveal the influence of borehole pumping mode on directional hydraulic fracturing, the test scheme is shown in Table 7, and three different pumping modes were constructed, including the priority of middle hole, the priority of outer hole, and the simultaneous initiation of three holes, respectively. The test results are shown in Figure 16.

When the middle borehole cracked first, the hydraulic fractures initiated along the borehole line and expanded along the direction of the maximum principal stress. After
4000 time steps, the water began to be injected into the outer borehole. The hydraulic fractures of H1 leftwing and H3 rightwing extended to the edge of the test block along the direction of the maximum principal stress. The hydraulic fractures of the H1 rightwing and H3 leftwing initiated and expanded along the borehole line. Under the effect of the...
Table 7: Experimental scheme of directional hydraulic fracturing.

| Test name | Principal stress (MPa) | Borehole spacing (mm) | Arrangement angle of boreholes (°) | The injection rate (ml/min) | Pumping mode |
|-----------|------------------------|-----------------------|-----------------------------------|----------------------------|--------------|
| 5-1       | σ₁ 7                   | σ₂ 5                  | σ₃ 141                            | 15                         | 50           | Sequence (1) |
| 5-2       | σ₁ 7                   | σ₂ 5                  | σ₃ 141                            | 15                         | 50           | Sequence (2) |
| 5-3       | σ₁ 7                   | σ₂ 5                  | σ₃ 141                            | 15                         | 50           | Synchronous  |

Figure 16: Initiation and propagation process of directional hydraulic fractures at borehole pumping mode.
hydraulic fractures of adjacent boreholes, the hydraulic fractures of two H2 wings deflected and expanded along the borehole line. Among them, hydraulic fractures of H2 rightwing and H3 leftwing were fused, and hydraulic fracture of H2 leftwing connected with that of the H1, forming a rock bridge between H2 leftwing and H1 rightwing and a stratified directional hydraulic fracture.

When the outer boreholes cracked first, the hydraulic fractures of the two holes initiated and expanded along the direction of the maximum principal stress. After 4000 time steps, the water began to be injected into the middle hole, and the hydraulic fractures of two H2 wings initiated and expanded along the borehole line, finally connecting the adjacent boreholes. During the expansion process, the hydraulic fractures of the three holes did not intersect, forming rock bridges on the directional hydraulic fracture surface.

When the three boreholes started to crack simultaneously, the hydraulic fractures initiated and expanded along the direction of the borehole line, and the directional hydraulic fractures were formed by the fusion of the hydraulic fractures in the adjacent drilling holes. These fractures were smooth and flat in shape, with obvious directivity and good directional effect but without an obvious turn in the expansion.

In a word, the fracturing mode significantly affects directional hydraulic fracturing. If the sequence fracturing mode is

Figure 17: Dense linear multi-hole directional hydraulic fracture propagation modes.
adopted, the coupling stress effect between the holes will be relatively poor, making it difficult to form the directional fracturing strip inside the test block. The formed directional fracture surface will be prone to stratification, the directional hydraulic fracture trajectory will become more tortuous, and the directional effect and accuracy are not as good as synchronous fracturing.

7. Dense Linear Multihole Directional Hydraulic Fracture Propagation Modes

According to the strength of directional effect and directional accuracy, the following five typical hydraulic fracture propagation behavior modes are finally extracted:

Mode 1: hydraulic fractures of adjacent boreholes intersect smoothly along the drilling line to form a linear directional hydraulic fracture, the fracture morphology is similar to "/". In this mode, the effect and accuracy of hydraulic fracture orientation are the best (Figure 17(a)).

Mode 2: the hydraulic fractures extend along the borehole line and connect with adjacent boreholes, but the hydraulic fractures of each borehole do not intersect, and a rock bridge is formed between different hydraulic fractures obliquely in direction (a). The shape of directional hydraulic fractures is similar to "\" (Figure 17(b)).

Mode 3: the hydraulic fracture first extends along the direction of the maximum principal stress $\sigma_1$, for a certain distance, then turns to expand along the drilling line direction and intersect with each other, forming a ladder-shaped directional hydraulic fracture, similar to "\" (Figure 17(c)).

Mode 4: the hydraulic fracture of the outer borehole propagates along the direction of the maximum principal stress, and the hydraulic fracture of the middle borehole propagates along the direction of the borehole line and communicates with the adjacent outer borehole; the shape of directional hydraulic fractures is similar to "\" (Figure 17(d)).

Mode 5: the hydraulic fractures of each hole expand along the direction of the maximum principal stress $\sigma_1$. In the process of expansion, the hydraulic fractures do not deflect and are parallel to each other, forming "\"-shaped hydraulic fractures, and the orientation effect is the worst (Figure 17(e)). All the above hydraulic fracture propagation modes are reflected in the previous sections.

8. Conclusion

The experimental results demonstrate the following:

(1) When directional hydraulic fracturing controlled by dense linear multihole drilling was carried out, due to the coupled effect between boreholes, the hydraulic fracture propagation tends to intersect and run through and finally forms the directional failure plane along the direction of the connecting line of boreholes, which effectively realizes the directional failure of rock strata

(2) The results show that the directional effect is mainly concentrated in the internal dense boreholes. The hydraulic fractures of the external dense boreholes basically extend along the direction of the maximum principal stress, so the directional effect is poor. In addition, the numerical simulation results are in good agreement with the laboratory test results, which fully verifies the effectiveness and rationality of the directional hydraulic fracturing numerical simulation model based on the two-dimensional discrete element numerical software (PFC$^{2D}$)

(3) After arranging the dense boreholes in the rock, the maximum tangential tensile stress is generated in the direction perpendicular to the drilling line due to the mutual extrusion of boreholes, so the borehole wall along the drilling line direction is most prone to crack initiation. With the increase of the length of the hydraulic fracture, the disturbed stress concentration areas will overlap each other and form the maximum principal stress band, which makes the hydraulic fracture deflect further towards the borehole line

(4) The angle ($\beta$) between the borehole line and the direction of the maximum principal stress, the principal stress difference ($\Delta \sigma$), and the hole spacing ($D$) have significant effects on the directional hydraulic fracturing effect. The smaller the angle, the difference value of the in situ stress, and the hole spacing, the better the directional hydraulic fracturing effect. In addition, the directional effect of synchronous hydraulic fracturing is better than that of sequential hydraulic fracturing

(5) According to the multihole linear codirectional hydraulic fracturing experiments, five typical directional hydraulic fracture propagation modes are summarized: (1) directional hydraulic fracture completely along the drilling line, (2) directional hydraulic fractures that connect boreholes but do not intersect, (3) propagation forming a ladder pattern of directional hydraulic fracture, (4) directional hydraulic fracture propagation both along the borehole line and parallel to the direction of maximum principal stress, and (5) parallel hydraulic fractures extending completely along the direction of maximum principal stress

Nomenclature

- $E$: Elastic modulus (GPa)
- $\nu$: Poisson’s ratio
- $k_n$: Normal stiffness of particle element (N/m)
- $k_t$: Tangential stiffness of particle element (N/m)
- $\mu$: Friction coefficient between particle elements
- $p$: Particle unit density (kg/m$^3$)
- $\sigma_1$: Maximum horizontal geostress (MPa)
- $\sigma_3$: Minimum horizontal geostress (MPa)
- $p$: Water pressure (MPa)
- $c_0$: Cohesion of rock (MPa)
- $\phi$: Friction angle of rock (°)
helped to complete this paper.

The authors would like to thank Yuqi Zhang for his support and permission to publish this paper. We would also like to acknowledge Tao Zhang for various contributions that helped to complete this paper.

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