About the Interpretation of Gravitationally Induced Neutrino Oscillation Phases

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Abstract

We present some thoughts on how to interpret the gravitationally induced neutrino oscillation phases presented by us in our 1996 Gravity Research Foundation Essay.

In a recent paper we discussed the modification of the neutrino oscillation phases due to the presence of gravity [1]. In a comment by Bhattacharya, Habib and Mottola (BHM) our results were rederived and an interpretation of them were given [2]. Here we would like to present our own interpretation, which stresses less the general relativistic aspect of the results but is more interested in the way gravity and quantum mechanics intermingle with each other [1,3]. In particular our essay studied the detailed interplay of the quantum mechanical principle of the linear superposition and general relativity’s principle of equivalence. From a purely general relativistic point of view, both the considerations of our essay and the physics of neutron interferometry experiments [4] may have nothing exciting [2], but it must be emphasised how the interplay of the principle of superposition and the presence of gravity produces the gravitationally induced neutrino oscillation phases in a manner that ensures compliance with the principle of equivalence.

First, begin with classical and quantum mechanical considerations for a single mass eigenstate. For a single mass eigenstate the classical effects of gravitation may be considered to depend on a force, $\vec{F}$, while the quantum–mechanical
effects are determined by the gravitational interaction energy. The gravitational interaction energy for non-relativistic particle of mass $m$ has the same form as in Newtonian theory and reads $U_{\text{int.}} = m \times \phi$, while for a relativistic particle (as shown below) it is $U_{\text{int.}} = (E/c^2) \times \phi$. The $\phi = -GM/r$ represents the gravitational potential in the weak field limit for an object of mass, $M$, in the standard notation [5]; with $\vec{F} = -\vec{\nabla} \phi$. Along an equi-$\phi$ surface the $\vec{F}$ vanishes and there are no classical effects in this direction. The constant potential along a segment of a equi-potential surface can be removed by going to an appropriately accelerated frame.

These are well known text-book statements [5–7]. However, we now explicitly note what is not always fully appreciated. Quantum mechanically, the mass eigenstate, assumed to have remained stationary at a given spatial position, picks up a global phase factor $\exp(-i m \phi t/\hbar)$. Again, there are no physical consequences. If we now consider a physical state that is in a linear superposition of different mass eigenstates then physically observable relative phases are induced between various mass eigenstates. Specifically, on an equi-$\phi$ surface the gravitational force $\vec{F}$ vanishes, while the relative quantum mechanical phases induced in the evolution of a linear superposition of mass eigenstates do not.

For the Newtonian case considered:

\[
\text{The gravitationally induced (time–)oscillatory phase} = \Phi \times \text{The (time–)oscillatory phase without gravity}, \tag{1}
\]

with $\Phi = \phi/c^2$, the dimensionless gravitational potential. However, as one cannot measure a gravitational potential by a local measurement, one needs to make similar observations on two different equi-$\phi$ surfaces to discern the presence of gravity. The gravitationally induced oscillatory phase, denoted by $\varphi_G^i$ for neutrino oscillations in [1], is an addition to the oscillatory phase without gravity (denoted by $\varphi_0^i$ for neutrino oscillations in [1]). Since neutrino-oscillation experiments are only sensitive to the sum of both phases, one needs to make similar observations on two different equi-$\phi$ surfaces and compare them in order to measure the presence of the gravitationally induced phase. One of these surfaces should be the surface at spatial infinity to extract the full gravitationally induced phase. Alternately, one may wish to compare one’s results with an experiment performed in a freely falling orbiter around the massive object. Specifically, the sense in which this comparison is to be performed is identical to that in which one measures a gravitational red shift of stellar spectra on Earth. The quantum mechanically created clock, via the time-oscillation of the mass eigenstates in the linear superposition, suffers the gravitational

\[^3\] The observability of these phases is not for a local observer, but for an observer making measurements stationed at a different equi-$\phi$ surface.
red shift as demanded by general relativity when the gravitationally induced oscillatory phase is taken into account.

In our 1996 Gravity Research Foundation Essay [1] the above noted non-relativistic observations were appropriately modified and applied to neutrino weak flavor eigenstates which are empirically indicated to be linear superposition of mass eigenstates. We confirmed the demands of general relativity in a quantum context.

BHM have shown in their communication [2] that by a local measurement one cannot measure a local gravitational potential. We agree.

In reference to BHM’s comment [2] one may wish to note:

(1) Exactly how the relativistic expression for $U_{int}$, mentioned above, is obtained. In the weak field limit the force on a mass eigenstate, of mass $m$, in the Schwarzschild gravitational environment mass of M reads [7]

$$\vec{F} = -\frac{GMm\gamma}{r^{3}} \left[ \left( 1 + \beta^2 \right) \vec{r} - \left( \vec{r} \cdot \vec{\beta} \right) \vec{\beta} \right].$$

In this equation, $\beta = \vec{v}/|\vec{v}|$, with $\vec{v}$ the velocity of mass eigenstate, and $\gamma = (1 - \beta^2)^{-1/2}$. Assuming the mass eigenstate to be relativistic and setting $m\gamma = E/c^2$, we have

$$U_{int} = \int_{\infty}^{r} \vec{F} \cdot d\vec{r} = -\frac{GME}{r c^2} = -(E/c^2) \times \phi .$$

(2) Theoretically, the prediction of general relativity as regards any clock (classically driven, or quantum mechanically, with non-relativistic mechanism or relativistic) and the prediction arising from quantum evolution that incorporates gravity via an interaction energy term (in a manner similar to the classic neutron interferometer experiments and their analysis [4,8]) are in mutual agreement. However, recently it has been suggested that the atmospheric and solar neutrino data could be explained by a violation of the equivalence principle [9] and it is, therefore, an important matter to understand in detail how the quantum mechanics and gravity work together for neutrino oscillations. Apart from this motivation, the problem is of interest in its own right to understand explicitly, and in detail, how the principle of equivalence and the principle of linear superposition of quantum mechanics intermingle [3].

(3) In reference to the discussion following Eq. (10) of the comment [2] by BHM one needs to note that it is not necessary (even in a semi-classical framework), or even correct, to note (as the authors of the comment [2] under consideration do) "Since the energy is fixed but the masses are different, if interference is to be observed at the same final spacetime point
$(r_B, t_B)$, the relevant components of the wave function could not both have started from the same initial spacetime point $(r_A, t_A)$. It needs to be appreciated that a wave function, after having evolved over a certain distance, may develop more than one spacial peak (perhaps corresponding to each of the mass eigenstate) and yet “collapse” to a single space–time point (or, appropriately defined spacial region governed by the uncertainty principle) in a weak flavor measurement. This, we believe, is the orthodox text–book wisdom [10–12] and we see no need to violate [2,13] it in neutrino oscillation phenomenology.

We intend to take up the subject matter in more detail elsewhere and show how to extend the conceptual framework to terrestrial experiments (with atomic systems and superconducting devices) where the weaker gravity of the Earth can be compensated by integrating the gravitationally induced effects over time.

In summary, and in reference to the BHM comment [2], we stand by our conclusions presented in [1].

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[13] In order to prevent confusion arising when comparing the results of BHM’s communication [2] and our essay [1], let us stress that we defined the four-momentum $p^\mu$ as the one from special relativity and treated the difference between $\tilde{g}^{\mu\nu}$ and the flat metric $\eta^{\mu\nu}$ as perturbation on top of the flat metric $\eta^{\mu\nu}$. Thus the symbols $p^\mu$, $E$ and $\vec{p}$ in [1] and the corresponding symbols in [2] do not describe the same objects. Within our framework $[E - (G M E/c^2 r)]$ is conserved and our results are obtained to first order in $\Delta m^2$ and $\Phi$. The derivation of the kinematic and the gravitationally induced oscillatory phases starting from Eq. (4) of [1] is a straightforward exercise (with the algebraic results also rederived by BHM in [2]) to any one familiar with the standard neutrino oscillation phenomenology and was therefore skipped in [1].