PROPOSED GRAVITATIONAL WAVE BACKGROUND FROM BLACK HOLE–TORUS SYSTEMS

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ABSTRACT

Cosmological gamma-ray bursts may be powered by rotating black holes with contemporaneous emission of gravitational radiation from a surrounding torus. We calculate the resulting stochastic background radiation assuming strong cosmological evolution and a uniform black hole mass distribution of $M = 4-14 M_\odot$. The predicted spectral flux density corresponds to a peak spectral closure density of $(1-2) \times 10^{-7}$ and has comparable contributions at 450 Hz and over 300–450 Hz from nearby and distant sources, respectively, where $\kappa$ refers to an uncertainty factor of order unity in the radius of the torus. For two optimized advanced Laser Interferometer Gravitational-Wave Observatory–type detectors the proposed gravitational wave background could be detectable within a year of integration.

Subject headings: black hole physics — cosmology: miscellaneous — gamma rays: bursts — gravitational waves

1. INTRODUCTION

Cosmological gamma-ray bursts (GRBs) are the most enigmatic transient events in the universe. They show a bimodal distribution in durations of short bursts around 0.3 s and long bursts around 30 s (Kouveliotou et al. 1993). Based on their GRB fluence and time variability, their inner engines should be compact and highly energetic. Leading candidates for GRB progenitors are collapsars and mergers of black holes and neutron stars. In particular, long bursts have been associated with core collapse of massive stars (Woosley 1993; MacFadyen & Woosley 1999), or their hypernova variants, in star-forming regions (Paczynski 1998; Brown et al. 2000). These, and mergers of compact binaries, are believed to result in black hole plus disk or torus systems—see van Putten (2001) for a review.

A torus around a rapidly rotating black hole converts spin energy into various channels, notably gravitational and thermal radiation, winds, and MeV neutrino emissions (van Putten & Levinson 2002). These emissions last for the lifetime of rapid spin of the black hole—the deredshifted duration of tens of seconds for long bursts from black hole–torus systems in suspended accretion (van Putten & Ostriker 2001). The single-source spectrum is here described by a dimensionless evolution factor $e(z)$ and can be parameterized from the $f(f)$ diagram (van Putten & Sarkar 2000) within a frequency range in the vicinity of 1 kHz for a $7 M_\odot$ black hole. GRB energies appear to have a diversity of about 1 order of magnitude (Frail et al. 2001; Piran et al. 2001). Black hole–torus systems are expected to have a distribution in black hole mass $M$ consistent with the recently proposed association with soft X-ray transients in the hypernova proposal of Brown et al. (2000); i.e., $M \approx 4-14 M_\odot$.

In this work, we calculate the gravitational wave (GW) spectra expected from a cosmological distribution of black hole–torus systems, assuming strong cosmological evolution locked to the star formation rate (SFR). We shall estimate the expected contribution to the stochastic background in GWs from the low- and high-redshift populations. Following Schmidt (2001) and Frail et al. (2001), the total event rate is normalized to a local GRB rate of 0.5 yr$^{-1}$ Gpc$^{-3}$ at $z = 0$, assuming a “flat-$\Lambda$” cosmology.

2. THE EVOLVING GRB RATE

The SFR can be parameterized according to the model SF2 of Porciani & Madau (2001), wherein the density rises rapidly by an order of magnitude between $z = 0$ and $z = 1$, peaks between $z = 1$ and 2, and declines gently at higher redshifts:

$$R_{\text{SF2}}(z) = 0.15h_{65}[1 + 22\exp(-3.4z)]^{-1} M_\odot \text{ yr}^{-1} \text{ Mpc}^{-3},$$

with $h_{65}$ denoting the Hubble constant normalized to 65 km s$^{-1}$ Mpc$^{-1}$. This model was derived using the Einstein–de Sitter (EdS) cosmology, so a scaling factor has to be applied for other cosmologies (Porciani & Madau 2001). We define a dimensionless evolution factor $e(z)$ as $R_{\text{SF2}}(z)/R_{\text{SF2}}(0)$ and assume a cutoff for star formation at $z = 5$, because active star formation is believed to have begun at that epoch. The variation of the rate of GRB sources with redshift can be expressed by the event-rate equation

$$dR/dz = 4\pi(c^2r_0/H_0^2)e(z)F(z)/(1 + z),$$

where $r_0$ is their present-epoch rate density and $R(z)$ is the all-sky event rate, as observed in our local frame, for sources out to redshift $z$. The factor $c^2r_0/H_0^2$ has the dimensions of inverse time, as does the event rate $R(z)$. The $(1 + z)$ denominator in equation (2) accounts for the time dilation of the observed rate by cosmic expansion, converting a source-count equation to an event-rate equation. The dimensionless function $F(z)$ is determined by the cosmological model and can be calculated from the Hubble parameter $H(z)$ and the angular size distance (Peebles 1993, p. 332).

Figure 1 plots the resulting GRB rate evolution for three standard cosmologies: $(\Omega_M, \Omega_\Lambda) = (1.0), (0.3, 0.7),$ and (0.3, 0), all with $h_{65} = 1$, using local GRB rate densities of 0.9, 0.5, and 0.6 yr$^{-1}$ Gpc$^{-3}$, respectively; the differing local rate densities reflect the different spatial geometries in the

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3. SINGLE-SOURCE SPECTRA: A GENTLE CHIRP

The single-source spectrum of gravitational waves of a black hole–torus system is modeled by the emission of a fraction of about 10% of the rotational energy of a rapidly spinning black hole (van Putten & Levinson 2002). These emissions correspond to mass inhomogeneities in the torus and have recently been identified as Papaloizou-Pringle buckling modes (van Putten 2002). The lowest frequency emission relevant to the current laser-interferometric gravitational wave experiments is produced by a quadrupole moment of a quadrupole signal at a luminosity distance $d_L(z)$ as expressed (Ferrari, Matarrese, & Schneider 1999a, 1999b; Coward, Burman, & Blair 2001)

$$F_{gw}(f_{obs}, z) = \frac{(c^3/8\pi G)^2 f_{obs}^2 \tilde{h}(f_{obs})^2 / d_L^2(z).}$$

where $\tilde{h}(f_{obs})$, which is in units of m Hz$^{-1}$, is the Fourier transform of the wave amplitude (in meters) at the observed frequency $f_{obs}$, which is related to the source frequency $f$ by the redshift factor: $f_{obs} = f/(1+z)$. The factor $(c^3/8\pi G)$ has dimensions mass/time, equivalent to energy/(area × frequency).

Figure 2 illustrates the spectral fluence for a selection of black hole masses ranging from 14 to $4 M_\odot$ in steps of 1 $M_\odot$, for negative $f$ at a source distance of 100 Mpc ($z \approx 0.02$); via equation (4), the corresponding $f_i$ range over 0.5–1.75 kHz with $-f$ ranging over 0.625–27 Hz s$^{-1}$; the bandwidth $\Delta f$ has been taken to be 10% of $f_i$ in each case, ranging over 175–50 Hz. The figure depicts the decrease of both $f$ and $\Delta f$ and also the steepening of the spectrum, with increasing $M$.

The scaling of the burst duration $T$ follows from equation (4):

$$T \approx \Delta f/f_i \approx 0.1 \frac{f}{f_i} M^2,$$

ranging over 44–3.6 s for $M = (14–4) M_\odot$; we have chosen
matched filtering will increase the signal-to-noise (S/N) ratio by a factor $\sqrt{fT} \approx \sqrt{N}$. Figure 3 shows, for example, that the dimensionless strain amplitude of a single burst event is expressed by $h_{\text{char}} \approx 6 \times 10^{-21}$ for $M = 7 M_\odot$ with $f_{\text{obs}} = 980-880 \text{ Hz}$; for an 11 s emission at a mean frequency of 950 Hz, $h_{\text{char}}/\sqrt{N} \approx 0.06 \times 10^{-21}$ for a single cycle. For comparison with these predictions, a model for the rms dimensionless noise amplitude of an advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) detector (Flanagan & Hughes 1998) is included in the figure.

4. GW BACKGROUND SPECTRA

The stochastic background produced by the proposed radiation from black hole–torus systems may be expressed by their spectral flux density, in units of W m$^{-2}$ Hz$^{-1}$. The contribution from these sources throughout the universe obtains by integrating the product $F_{\text{s}}(f_{\text{obs}}, z)dR/dz$ over the redshift range $z = 0-5$:

$$F_B(f_{\text{obs}}) = \int_0^5 F_{\text{s}}(f_{\text{obs}}, z)(dR/dz)dz ,$$

with $F_{\text{s}}$ and $dR/dz$ given by equations (7) and (2). The background spectral strain, in units of Hz$^{-1/2}$, is calculated directly from this background spectral flux density (Ferrari et al. 1999a, 1999b):

$$\sqrt{S_B(f_{\text{obs}})} = (2G/\pi c^3)^{1/2}f_{\text{obs}}^{-1}F_B(f_{\text{obs}})^{1/2} .$$

The spectral energy density of a GW background is conventionally expressed by the dimensionless spectral “closure density,” defined as the energy density of gravitational waves per logarithmic frequency interval normalized to the cosmological critical density $\rho_c c^2$. It can be obtained from the background spectral flux density (Ferrari et al. 1999a, 1999b) as

$$\Omega_B(f_{\text{obs}}) = f_{\text{obs}}F_B(f_{\text{obs}})/(\rho_c c^2) .$$

The duty cycle (DC) from events out to redshift $z$ is given by

$$\text{DC}(z) = \int_0^z (1+z)\tau(dR/dz)dz ,$$

where the typical duration, $\tau$, of the signal is dilated to $(1+z)\tau$ by the cosmic expansion. A calculated DC of unity or greater implies that the signal is continuous. If DC $\gg 1$, then the central limit theorem tells us that the amplitude distribution can be approximated as a Gaussian. If the calculated DC $\leq 1$, then the amplitude distribution can be simulated (Coward et al. 2002a, 2002b) from a random sampling of the probability distribution based on the event rate equation (2).

5. NUMERICAL RESULTS

The stochastic background produced by systems with a uniform black hole mass distribution $M = 4-14 M_\odot$ for three standard cosmologies, expressed in spectral flux density, is shown in Figure 4. For the flat-$\Lambda$ cosmology, it exhibits a broad plateau of $5 \times 10^{-11} \text{ W m}^{-2} \text{ Hz}^{-1}$ at about 300–450 Hz. At 300 Hz, the main contributions are from
cosmologically distant sources at about $z = 0.6-1$. The edge at 450 Hz represents contributions from more local sources and is strongly dependent on the single-source fluence. For the EdS cosmology, the plateau is similar in bandwidth but scaled up to $10^{-10}$ W m$^{-2}$ Hz$^{-1}$. The open cosmology shows a noticeable peak corresponding to contributions from local higher mass black holes.

The variation in spectral flux density among the flat-$\Lambda$, EdS, and open cosmologies can be attributed to the cosmology dependence of the luminosity distance. Figure 5 plots the ratio of inverse luminosity distance squared for the open- and EdS-to-flat-$\Lambda$ cosmologies. It shows a maximum of 1.25 at $z \approx 1$ for the open-to-flat-$\Lambda$ case, consistent with the ratio of the spectral flux density in Figure 4 for these cosmologies. The EdS-to-flat-$\Lambda$ case, the ratio in Figure 5 increases steadily to 2.35 at $z = 5$, which is consistent with the ratio of the plateau heights in Figure 4 for these cosmologies.

The spectral strain, shown in Figure 6 for the same cosmologies, exhibits a maximum of about $3.5 \times 10^{-26}$ Hz$^{-1/2}$ at about 20–250 Hz. The tails at frequencies above 500 Hz in Figures 4 and 6 represent the contributions from low-mass black holes; evidently, the spectra are dominated by high-mass black holes.

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al. (2001) translates to a net GRB event rate throughout the universe of a few per minute, as seen in our frame. This signal is comprised of discrete pulses, originating largely from sources at redshifts of 1–3. Since these pulses last typically about 40–80 s (in our frame), they overlap and produce a near-continuous background, to within the approximations made in our treatment.

6. DETECTABILITY

A stochastic background will manifest itself in a single detector as excess noise. The signal from the proposed GRB GW background is expected to be far below the noise level in any single planned ground-based detector. The S/N in terms of amplitude for either an interferometer or resonant-mass detector is (Maggiore 2000)

\[
S/N = [FS_n(f)/S_n(f)]^{1/2};
\]

(13)

here \( F \) is a pattern function, which is a measure of the angular efficiency of the detector, \( S_n(f) \) is the signal power spectral density (density in frequency space), and \( S_n(f) \) is the noise power spectral density, both in units of Hz\(^{-1}\). Figure 9 is a plot of the S/N of the proposed background as a function of frequency, using \( S_n(f) \) calculated from the flat-\( f \) spectral strain shown in Figure 6 and from a model for the noise power spectrum from an “advanced LIGO-type” interferometer (Flanagan & Hughes 1998). The S/N is small, with a sharp maximum of \( 1 \times 10^{-2} \) near 100 Hz; the rapid fall to the left of this is due mainly to the absence of signal below about 100 Hz.

The most promising detection strategy is that of cross correlation of the output of two neighboring detectors, as recently reviewed by Allen & Romano (1999) and Maggiore (2000). For the signals in them to be correlated, the detectors must be separated by less than one reduced wavelength, which is about 100 km for frequencies around 500 Hz, where we expect the GRB-generated background to peak. The detectors also need to be sufficiently well separated that their noise sources are largely uncorrelated.

Under these conditions, assuming Gaussian noise in each detector and optimal filtering, a filter function chosen to maximize S/N for two such detectors yields the formula

\[
\left( \frac{S}{N} \right)^2 \approx \frac{9H_0^4}{20\pi^4} T \int_0^\infty \frac{\gamma(f) \Omega_{B}(f)}{f^6 S_{n1}(f) S_{n2}(f)} \, df.
\]

(14)

Here \( \gamma(f) \) is an “overlap reduction function,” which accounts for the separation and relative orientation of the detectors, and \( \Omega_{B}(f) \) are the noise power spectral densities of the detectors. As the optimal filter depends on \( \Omega_{B}(f) \), a range of filter functions based on theoretical expectations of this function will need to be used.

For this preliminary study of the detectability of the GRB-generated background, we assume an optimized value of close to unity for \( \gamma(f) \) of two detectors situated within several kilometers. We take the flat-\( A \) \( \Omega_{B}(f) \) as shown in Figure 7 and use a piecewise parameterized model for \( S_{n1}(f) \) and \( S_{n2}(f) \) for proposed advanced LIGO detectors (Flanagan & Hughes 1998), assuming a pair of similar detectors. Figure 10 plots the resulting S/N as a function of integration time, yielding a value of about 8 for 1 yr of integration. This preliminary result suggests that the proposed GRB GW background is potentially detectable given
optimized cross correlation between two advanced LIGO detectors.

We also note that given the uncertainty in the sign of $f(f)$ in the single-source emission model, the peak flux may occur at higher frequencies. If so, then the GRB GW background may be detectable using cross correlation with a resonant-mass interferometer detector pair. The LIGO-WA interferometer and ALLEGRO resonant-mass detector pair, when co-aligned, yield a value for $\gamma$ close to unity across the bandwidth 1–1000 Hz (Whelan et al. 2002). Unfortunately, this bandwidth is not fully utilized in the ratio of GW power–noise power appearing in the integral in equation (14), because ALLEGRO, as a resonant-mass detector, has a narrow bandwidth, centered on a resonant frequency of about 900 Hz in this case.

7. DISCUSSION

We draw several conclusions:

1. Black hole–torus systems associated with GRBs are expected to give a substantial contribution to the stochastic GW background in a frequency window of $300 - 450 \text{ Hz} \times \kappa$, where $\kappa$ denotes an uncertainty factor of order unity in the radius of the torus. If black hole–torus systems exist also as transient sources independent of GRBs, their event rate will be higher with a commensurably more pronounced stochastic GW background.

2. It is instructive to compare the presented results with radiation from rapidly rotating neutron stars. Black hole–torus systems produce spectral flux densities that are similar to the contribution from $r$-mode instabilities as described in Ferrari et al. (1999b). Here the large output provided by the spin energy of the black hole compensates for an event rate that is less than the formation rate of neutron stars by some 3 orders of magnitude. The frequencies and DC of the former are markedly higher, respectively, lower than those predicted for radiation from neutron star modes, however. The DC of the signals from neutron star $r$-modes may be as high as $10^9$ (Ferrari et al. 1999b) in contrast to our DC estimate of order unity for the signals from GRB sources. This comparison becomes more favorable with recent understanding of $r$-modes in more detailed and realistic scenarios. The $r$-modes are driven by a generally weak gravitational radiation–reaction force and, in the perturbative limit, effectively decoupled from the sources (Schenk, Arras, & Flanagan 2002). A recent appreciation of various channels for creating viscosity (Lindblom & Owen 2002; Rezolla et al. 2001a, 2001b; Wu, Matzner, & Arras 2001) renders their saturation energies small, and less so than previously thought (Arras et al. 2002).

3. The high output in gravitational radiation powered by the spin energy of the black hole gives rise to a major contribution to the predicted GW background by a relatively nearby population of sources. This is apparent in the edge at $450 \text{ Hz} \times \kappa$ in Figure 4 to the $300 - 450 \text{ Hz} \times \kappa$ flat-A spectral flux density plateau due to sources at higher redshifts. The comparable contributions by the nearby and distant sources is a robust result, depending only on the assumption that the proposed black hole–torus systems are tightly locked to the SFR.

4. As the local GRB rate (assumed in this work) is about $1 \text{ yr}^{-1}$ within a radius of 100 Mpc, the average background spectral flux density should be representative of sources from $z = 0.02 - 5$ for an observation time of 1 yr. One can view the background spectral flux density shown in Figure 4 as the result of averaging the flux from a discrete number distribution over an infinite observation time. We plan to investigate these issues further in connection with the detectability of this predicted background using advanced LIGO-type detectors.

5. The estimated frequency range of about $200 - 2000 \text{ Hz}$ of the proposed background and foreground radiation in gravitational waves from black hole–torus systems in association with GRBs defines a new source with a well-defined event rate in the high-frequency bandwidth of LIGO/VIRGO detectors as well as for some of the current bar detectors. Cross-correlating the output between pairs of detectors promises to be the optimum detection strategy. For an idealized advanced LIGO pair of detectors, the S/N could be high enough for detection of the proposed background in a year of integration. Using present detector technology and locations, the optimum detector pair combination may be a resonant-mass and interferometric detector, such as the LIGO-WA and ALLEGRO pair.

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