Study of sequential semileptonic decays of $b$ hadrons produced at the Tevatron

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Abstract

We present a study of rates and kinematical properties of lepton pairs contained in central jets with transverse energy $E_T \geq 15$ GeV that are produced at the Fermilab Tevatron collider. We compare the data to a QCD prediction based on the herwig and QQ Monte Carlo generator programs. We find that the data are poorly described by the simulation, in which sequential semileptonic decays of single $b$ quarks ($b \rightarrow l\ c\ X$ with $c \rightarrow l\ s\ X$) are the major source of such lepton pairs.

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I. INTRODUCTION

This study of sequential semileptonic decays of $b$ hadrons completes the review of the heavy flavor properties of jets produced at the Fermilab Tevatron collider presented in Ref. [1]. The data set, collected with the Collider Detector at Fermilab (CDF) in the 1992–1995 collider run, consists of events with two or more jets with transverse energy $E_T \geq 15$ GeV and pseudorapidity $|\eta| \leq 1.5$, and is the same as that used in Ref. [1]. The heavy flavor purity of the sample is enriched by requiring that at least one of the jets contains a lepton ($e$ or $\mu$) with transverse momentum larger than 8 GeV/c. The jet containing the lepton is referred to as lepton-jet, whereas the jets recoiling against the lepton-jet are called away-jets. Since these events have been acquired by triggering on the presence of a lepton with $p_T \geq 8$ GeV/c, we call electron and muon data the samples with an electron- and muon-jet, respectively. Jets containing hadrons with heavy flavor are identified using the CDF silicon micro-vertex detector (SVX) to locate secondary vertices produced by the decay of $b$ and $c$ hadrons inside a jet. These vertices (secvtx tags) are separated from the primary vertex as a result of the long $b$ and $c$ lifetime. The $b$- and $c$-hadron contributions are separated by employing an additional tagging algorithm [2], which uses track impact parameters to select jets with a small probability of originating from the primary vertex of the event (jpb tags). Sequential semileptonic decays are identified by searching lepton-jets for the presence of additional soft leptons ($e$ or $\mu$ with $p_T \geq 2$ GeV/c) that are referred to as slt tags. In Ref. [1], we have used measured rates of secvtx and jpb tags to determine the bottom and charmed content of this data sample; we have then tuned the parton-level cross sections predicted by the simulation, based upon the HERWIG [3] and QQ [4] Monte Carlo generator programs, to match the heavy-flavor content of the data. Reference [1] shows that rates of lepton- and away-jets with secvtx and jpb tags, as well as the relevant kinematical properties of the data, can be modeled by tuning the simulation within the theoretical and experimental uncertainties. However, the number of away-jets with slt tags, which according to the simulation are mostly due to $b\bar{b}$ production, is found to be significantly larger than what predicted by the conventional-QCD simulation. The observed discrepancy is consistent with previously reported anomalies [5, 6, 7], and opens the possibility that approximately 30% of the presumed semileptonic decays of $b$- hadrons produced at the Tevatron is due to unconventional sources.
Therefore, it is of interest to extend the earlier comparison to the yields of SLT tags contained inside lepton-jets. The present analysis is based upon the same samples of data and simulated events used in Ref. [1], and makes use of the same tuning of the simulation. In Sec. II we evaluate rates of lepton-jets containing also one soft lepton tag. In Sec. III we compare the kinematics of these lepton pairs in the data and in the simulation. Section IV contains cross-checks and a discussion of systematic effects. Our conclusions are presented in Sec. V.

II. LEPTON-JETS CONTAINING AN ADDITIONAL SOFT LEPTON

We search lepton-jets for additional soft leptons \( p_T \geq 2 \text{ GeV}/c \) using the SLT algorithm [8, 9, 10, 11]. Pairs of trigger and soft leptons arise from four different sources: sequential semileptonic decays of single \( b \) hadrons, leptonic decays of \( \psi \) mesons, semileptonic decays of two different hadrons with heavy flavor produced by gluons branching into pairs of \( b \) or \( c \) quarks, and hadrons that mimic the experimental signature of a lepton. We compare data and simulation for the following yields of tags:

1. \( \text{Dil} \), the number of lepton-jets containing one and only one additional soft lepton. Since approximately 50% of the \( J/\psi \) mesons produced at the Tevatron do not originate from \( B \) decays [12] and are not modeled by the heavy flavor simulation, dileptons with opposite charge, same flavor, and invariant mass \( 2.6 \leq m_{ee} \leq 3.6 \text{ GeV}/c^2 \) and \( 2.9 \leq m_{\mu\mu} \leq 3.3 \text{ GeV}/c^2 \) are removed from this study.

2. \( \text{Dil}^{\text{SEC}} \) \( (\text{Dil}^{\text{JPB}}) \), the number of lepton-jets that also contain one soft lepton and a secvtx (JPB) tag due to heavy flavor. Lepton pairs consistent with \( J/\psi \) decays are also removed.

The yields of lepton pairs consistent with \( J/\psi \) decays, which are removed from the present analysis, have been compared to the simulation in Table XIV of Ref. [1]. The comparison has been used to verify the \( b \) purity of the data.

The observed numbers of jets containing a lepton pair are listed in Table II. Rates of dileptons produced by heavy flavor decays in the simulation (as yet unnormalized) are shown in Table III. In the simulation, dileptons are mostly produced by sequential decays of single \( b \) hadrons and have opposite sign charge (OS); approximately 5% of the lepton pairs have
TABLE I: Numbers of lepton-jets containing an additional soft lepton. OS and SS indicate lepton pairs with opposite and same sign charge, respectively. We use the difference OS-SS to remove the contribution of fake leptons. Mistags are the numbers of dileptons in jets with fake secvtx or JPB tags. The yields of lepton pairs (Dil) are also shown for different lepton flavors (ee, μμ, and eμ).

| Flavor | Electron data | | Muon data | | |
|--------|----------------|----------------|----------------|----------------|----------------|
|        | OS  | SS | OS-SS | Mistags | Flavor | OS  | SS | OS-SS | Mistags |
| ee     | 441 | 107 | 334  |         | μμ     | 335 | 107 | 228   |         |
| eμ     | 1009| 232 | 777  |         | μe     | 141 | 33  | 108   |         |
| Dil (ee+eμ) | 1450 | 339 | 1111 |         | Dil (μμ+μμ) | 476 | 140 | 336   |         |
| ee     | 111 | 18  | 93   |         | μμ     | 127 | 25  | 102   |         |
| eμ     | 371 | 61  | 310  |         | μe     | 71  | 14  | 57    |         |
| DilSEC (ee+eμ) | 482 | 79  | 403  | 2.0    | DilSEC (μμ+μμ) | 198 | 39  | 159   | 0.8    |
| ee     | 143 | 20  | 123  |         | μμ     | 143 | 28  | 115   |         |
| eμ     | 414 | 66  | 348  |         | μe     | 72  | 12  | 60    |         |
| DilJPB (ee+eμ) | 557 | 86  | 471  | 7.0    | DilJPB (μμ+μμ) | 215 | 40  | 175   | 3.5    |

same sign charge (SS) and are found in jets produced by gluons branching into pairs of heavy quarks.

In the data the ratio of SS to OS dileptons is appreciably higher ($\approx 20\%$) than in the simulation. The excess of SS dileptons with respect to the simulation is attributed to hadrons that mimic the lepton signature. Therefore we use the number of SS dileptons with a 10% error to estimate and remove the fake-lepton contribution to OS dileptons. This rather intuitive method for estimating this background will be further discussed in Section IV.
TABLE II: Number of lepton-jets containing an additional soft lepton due to heavy ($b$ and $c$) quarks in the simulation not yet tuned according to the fit performed in Ref. [1]; dir, f.exc, and GSP indicate the direct (LO), flavor excitation, and gluon splitting contributions predicted by HERWIG to the numbers of OS/SS lepton pairs with different and same flavor and to the numbers of OS-SS lepton pairs with any flavor. There is no contribution from $c$ direct production. At generator level, we have verified that both the trigger and soft lepton tracks match an electron or muon originating from $b$- or $c$-hadron decays (including those coming from $\tau$ or $\psi$ cascade decays). Lepton pairs consistent with $J/\psi$ decays are removed as in the data.

| Flavor | $b$ dir | $b$ f.exc | $c$ f.exc | $b$ GSP | $c$ GSP |
|--------|---------|-----------|-----------|---------|---------|
| $ee$   | 63/0    | 105/1     | 2/0       | 57/12   | 12/0    |
| $e\mu$ | 148/0   | 266/4     | 0/0       | 157/28  | 20/0    |
| $Dil$ ($ee+e\mu$) | 211     | 367       | 2         | 174     | 32      |
| $ee$   | 24/0    | 39/1      | 0/0       | 25/7    | 0/0     |
| $e\mu$ | 73/0    | 118/3     | 0/0       | 61/8    | 2/0     |
| $Dil^{SEC}$ ($ee+e\mu$) | 97      | 153       | 0         | 71      | 2       |
| $ee$   | 31/0    | 61/1      | 0/0       | 31/8    | 4/0     |
| $e\mu$ | 81/0    | 146/2     | 0/0       | 85/13   | 2/0     |
| $Dil^{JPB}$ ($ee+e\mu$) | 112     | 204       | 0         | 95      | 6       |

**Electron simulation**

| Flavor | $b$ dir | $b$ f.exc | $c$ f.exc | $b$ GSP | $c$ GSP |
|--------|---------|-----------|-----------|---------|---------|
| $\mu\mu$ | 28/0   | 65/2      | 3/1       | 52/12   | 12/0    |
| $\mu e$ | 26/0   | 27/0      | 0/0       | 31/3    | 5/0     |
| $Dil$ ($\mu\mu+\mu e$) | 54      | 90        | 2         | 68      | 17      |
| $\mu\mu$ | 14/0   | 35/2      | 1/0       | 25/7    | 0/0     |
| $\mu e$ | 21/0   | 15/0      | 0/0       | 16/1    | 0/0     |
| $Dil^{SEC}$ ($\mu\mu+\mu e$) | 35      | 48        | 1         | 33      | 0       |
| $\mu\mu$ | 18/0   | 38/2      | 1/0       | 33/8    | 5/0     |
| $\mu e$ | 19/0   | 21/0      | 0/0       | 18/2    | 1/0     |
| $Dil^{JPB}$ ($\mu\mu+\mu e$) | 37      | 57        | 1         | 41      | 6       |

**Muon simulation**
A. Rates of soft leptons due to heavy flavor in the data and in the normalized simulation

Table III lists numbers of OS-SS lepton pairs due to heavy flavor in the data and in the simulation normalized according to the fit described in Sec. VIII of Ref. [1]. Table IV lists the contribution of the various production mechanisms to the numbers of OS-SS lepton pairs in the normalized simulation.

In the data there are $1447 \pm 65$ lepton pairs in the same jet (the statistical error is $\pm 43.8$ and the systematic uncertainty of the fake-lepton removal is $\pm 48.0$). The simulation predicts $1180.7 \pm 128.7$ dileptons (the systematic uncertainty due to the SLT tagging efficiency is $\pm 118$ and the uncertainty due to the fit used to tune the simulation in Ref. [1] and to the simulation statistical error is $\pm 51.5$).

In lepton-jets with JPB tags, we find $635.5 \pm 30.5$ lepton pairs (the statistical error is $\pm 27.8$ and the systematic uncertainty due to the fake-lepton removal is $\pm 12.6$). The simulation predicts $530.9 \pm 39.7$ lepton pairs (the systematic uncertainty due to the SLT tagging efficiency is $\pm 26.5$ and the uncertainty due to the tuning of the simulation and to the simulation statistical error is $\pm 29.5$). The small excess of the data with respect to the simulation is approximately a $2 \sigma$ effect. In the next section, we study some kinematical properties of these lepton pairs.
TABLE III: Number of OS-SS lepton pairs due to heavy flavors in the data and in the normalized simulation.

| Tag type | Electrons | | | Muons | | |
|----------|-----------|-----------------|---------|-----------------|---------|
|          | Data      | Simulation      | Data    | Simulation      |
| Dil      | 1111.0 ± 54.2 | 893.0 ± 101.3 | 336.0 ± 28.5 | 287.7 ± 38.6 |
| Dil<sup>SEC</sup> | 401.0 ± 25.0 | 366.6 ± 32.1 | 158.2 ± 15.9 | 143.0 ± 17.6 |
| Dil<sup>JPB</sup> | 464.0 ± 26.8 | 387.3 ± 32.0 | 171.5 ± 16.5 | 143.6 ± 16.7 |

TABLE IV: Predicted numbers of OS-SS lepton pairs, listed by production mechanisms.

| Electron simulation | Muon simulation |
|---------------------|----------------|
| Tag type            | b dir | b f.exc | c f.exc | b GSP | c GSP | b dir | b f.exc | c f.exc | b GSP | c GSP |
| Dil                 | 215.1 ± 28.6 | 381.9 ± 56.4 | 2.2 ± 1.7 | 248.1 ± 49.0 | 45.7 ± 14.5 |
| Dil<sup>SEC</sup>   | 100.1 ± 12.4 | 161.7 ± 22.9 | 0 | 102.5 ± 21.1 | 2.2 ± 1.9 |
| Dil<sup>JPB</sup>   | 93.9 ± 11.0 | 175.0 ± 23.6 | 0 | 111.4 ± 21.6 | 7.0 ± 3.4 |
| Dil                 | 58.1 ± 10.3 | 99.2 ± 17.8 | 2.4 ± 2.4 | 102.4 ± 23.0 | 25.6 ± 9.3 |
| Dil<sup>SEC</sup>   | 38.2 ± 7.0 | 53.6 ± 10.4 | 0.9 ± 1.0 | 50.3 ± 12.9 | 0 |
| Dil<sup>JPB</sup>   | 32.8 ± 5.8 | 51.6 ± 9.4 | 1.0 ± 1.0 | 50.8 ± 12.2 | 7.4 ± 3.6 |
III. DILEPTON KINEMATICS

Figures 1 to 3 compare distributions of invariant mass and opening angle of dileptons contained in the same jet in the data and in the simulation \(^1\). The small excess of lepton pairs with respect to the simulation prediction (see Table III) appears to be concentrated at invariant masses smaller than 2 GeV/\(c^2\) and opening angles smaller than 0.2 rad. For dilepton invariant masses larger than 2 GeV/\(c^2\) data and simulation are in reasonable agreement. The shapes of the transverse momentum distributions of the trigger and soft leptons, shown in Fig. 4 are compatible with that of the expectation.

\(^1\) The fake-lepton background is removed by subtracting the distribution of SS dileptons from that of OS dilepton, both in the data and in the simulation. In the data, errors include the \(\pm 10\%\) systematic uncertainty of this removal.
FIG. 1: Dilepton invariant mass distributions in the inclusive electron (a) and muon (b) samples are compared to the simulation prediction. Distributions of the dilepton opening angle, $\theta$, in the two samples are shown in (c) and (d).
FIG. 2: Distributions of the dilepton (a) invariant mass, $M$, and (b) opening angle, $\theta$, for the inclusive lepton (electron+muon) sample and for its simulation.
FIG. 3: Distributions of (a) invariant mass and (b) opening angle of lepton pairs contained in jets tagged by the secvtx or (c and d) JPB algorithms for the inclusive lepton sample and its simulation.
FIG. 4: Transverse momentum distributions of the trigger lepton (lep) and the accompanying soft lepton (slt).
IV. SYSTEMATICS

The excess of lepton pairs with respect to the simulation is all concentrated at dilepton opening angles smaller than 11°. Since this is approximately the angle covered by a central calorimeter tower, we have investigated at length the possibility that the efficiency of the lepton selection criteria, described in Sec. IV of Ref. [1], is not simulated properly when two lepton-candidate tracks hit the same calorimeter tower. However, for opening angles smaller than 11°, the excess with respect to the simulation when the leptons are contained in the same tower is smaller, but consistent with that observed when the leptons hit two neighboring towers. We have also inspected all distributions of the tracking and calorimeter informations that are used to select leptons. We have compared these distributions for lepton pairs with opening angle smaller and larger than 11° without discovering any sensible difference. Therefore, we have investigated other possible causes; since they are of general interest, we present these studies in the following. In subsection A we verify the method used to estimate and remove the fake-lepton contribution. In subsection B we check the simulation of sequential b-decays that represents the largest contribution to lepton pairs. Finally, in subsection C we study a handful of events containing three leptons.

A. Fake-lepton estimate

The technique of removing the fake-lepton background by subtracting SS dileptons from OS dileptons has been used by CDF in several measurements of the Drell-Yan cross section [13]. We prefer this technique to the use of the standard parametrized probability of finding a fake lepton in a jet, derived using large samples of generic-jet data [1, 10, 11], because the latter method might not be applicable to jets that already contain a lepton (generic-jet data do not contain enough lepton pairs to construct a reliable parametrization of this fake probability).

The simulated inclusive electron sample contains 955 ± 108 OS and 63 ± 9 SS dileptons produced by heavy-quark decays. In the data, there are 1450 OS and 339 SS dileptons. After removing the dilepton rates predicted by the simulation, we would like to explain in terms of fake-lepton background the remaining 495 ± 114 OS and 276 ± 20 SS dileptons.

\[2\] The standard simulation ignores b-hadron mixing and underestimates the rate of SS dileptons due to the
In principle, rates of OS and SS dileptons due to misidentification background could be different. On average, jets contain the same number of positive and negative lepton-candidate tracks. Therefore, when searching for an additional soft lepton jets in which one track has been already identified as the trigger lepton, the number of OS candidates is larger than the number of SS candidates (these numbers will be approximately equal only for jets with a very large number of candidate tracks).

We have investigated this scenario by using samples of generic jets (JET 20, JET 50, and JET 70) and their simulation described in Refs. [1, 10, 11]; the simulation has been also tuned to reproduce the rates of secvtx and jpb tags observed in the data. We select jets containing an slt tag, and for these jets we count the number of additional slt candidate tracks, $N_C$, with opposite or same charge. We also count the number of OS and SS additional soft lepton tags, $Dil$, found in these jets. These rates are listed in Table V. As expected, the table shows a large difference between the number $N_C$ of OS and SS candidates. However, in generic jets, which are not rich in heavy flavor, the rate of OS and SS soft lepton pairs inside the same jet is approximately equal (to within 13%). After removing the heavy flavor contribution predicted by the simulation, we derive $P_{fk}$, the probability of converting slt candidate tracks into a fake slt tag in jets that already contain an slt tag (see Table V). We note that in generic-jet data the probability $P_{fk}$ for OS candidates is 65% of that for SS candidates.

Our standard estimate of the number of OS dileptons due to misidentification background assumes that it is equal to the number of observed SS dileptons minus the number of predicted SS dileptons due to heavy flavor \([339 - (63 \pm 9) = 276 \pm 20]\). We use two additional methods to verify this estimate.

In the inclusive electron sample, e-jets contain $N_c(OS) = 54938$ and $N_c(SS) = 34744$ candidate tracks, respectively. In the first method, we multiply the numbers of candidates by the corresponding probabilities $P_{fk}$ derived in generic-jet data. We predict a slightly smaller OS background \((236 \pm 38)\) and an amount of SS background \((229 \pm 14)\) in agreement with the estimate that includes $b$-hadron mixing.

\begin{footnote}
\small

\textit{decays of two different $b$ hadrons. This is a small effect; when using the time-integrated mixing parameter $\bar{\chi} = 0.118$, the simulation predicts $915 \pm 105$ OS and $103 \pm 15$ SS dileptons produced by heavy-quark decays. Therefore, the difference between data and simulation, which should be attributed to the fake-lepton background, becomes $535 \pm 111$ OS and $236 \pm 24$ SS dileptons.}
\end{footnote}
An additional estimate of the OS background can be derived by applying the standard parametrization [1, 10, 11] of the fake SLT probability to all OS candidate tracks. This method yields a slightly higher background estimate of 302 ± 30 OS fake dileptons.

We take the ±10% discrepancy between the different background estimates as a measure of its uncertainty. We note that the simulation of the SLT algorithm relies on parametrizations based on the data and does not provide a good understanding of why, in generic-jet data, the probability $P_{fk}$ is smaller for OS candidates than for SS candidates. However, imposing the conditions that the excess of 535 ± 111 OS pairs with respect to the simulation prediction is due to fake-lepton background and that the probability $P_{fk}$ does not depend on the pair sign produces a paradoxical result. In the inclusive electron sample, all 339 SS pairs are attributed to fake background whereas the simulation predicts 103 ± 15 SS pairs due to heavy flavor; in contrast, generic jets require the presence of 73 ± 25 SS pairs due to heavy flavor, not predicted by the simulation and of the same size of the predicted number of OS pairs due to heavy flavor (63 ± 24 in Table V).

We use the generic-jet data to also verify that OS and SS dileptons due to misidentification background have similar invariant mass distributions. For this purpose, we use two data sets, the jets of which have an average transverse energy comparable to that of jets in the inclusive lepton samples. The first data set is selected requiring the presence of at least one jet with transverse energy larger than 20 GeV (JET 20). The second data set is selected requiring the presence of at least four jets with transverse energy larger than 15 GeV and total transverse energy larger than 125 GeV ($\sum E_T 125 4CL$). In order to emulate the inclusive lepton sample requirement of one lepton with transverse momentum larger than 8 GeV/c, we select jets with at least one track with $p_T \geq 8$ GeV/c inside a cone of radius 0.4 around their axis. We then search these jets for soft lepton tags. Figure 5 shows that the invariant mass distributions of the high $p_T$ track and the soft lepton track for OS and SS combinations are indeed quite similar.

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3 This method for estimating the fake-lepton contribution has been previously used by CDF in a measurement of the $b$-quark fragmentation fractions of strange and light $B$ mesons by using dimuons with invariant mass $m_{\mu\mu} \leq 2.8$ GeV/$c^2$. 

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TABLE V: Number of tracks candidate to become soft lepton tags, $N_C$, in jets which already contain a soft lepton tag. $Dil$ is the number of jets containing two soft lepton tags. $Dil(h.f.)$ is the number of jets containing dileptons produced by heavy quark decays predicted by the simulation. The difference between data and simulation prediction, $Dil(fk)$, is attributed to fake dileptons. $P_{fk} = (Dil - Dil(h.f.))/N_C$ is the probability that a track produces a fake soft lepton tag.

|       | $N_C$  | $Dil$ | $Dil(h.f.)$ | $Dil(fk)$ | $P_{fk}$       |
|-------|--------|-------|-------------|-----------|----------------|
| JET 20 |
| OS    | 7684   | 44    | 17.7 ± 7.9  | 26.3 ± 10.3| 0.0034 ± 0.0013|
| SS    | 5606   | 40    | 0           | 40        | 0.0071 ± 0.0011|
| JET 50 |
| OS    | 14467  | 85    | 21.0 ± 10.3 | 64.0 ± 13.8| 0.0044 ± 0.0009|
| SS    | 11460  | 71    | 0           | 71        | 0.0062 ± 0.0007|
| JET 70 |
| OS    | 18329  | 107   | 23.8 ± 11.7 | 83.2 ± 15.6| 0.0045 ± 0.0008|
| SS    | 14666  | 97    | 0           | 97        | 0.0066 ± 0.0006|
| Sum   |        |       |             |           | 0.0043 ± 0.0007|
| OS    | 40480  | 236   | 62.6 ± 23.8 | 173.4 ± 28.3| 0.0066 ± 0.0004|
| SS    | 31732  | 208   | 0           | 208       | 0.0066 ± 0.0004|
FIG. 5: Invariant mass distributions between a track with $p_T \geq 8$ GeV/c and a soft lepton track contained in the same jet; OS and SS distributions are normalized to the same area.

B. Simulation of sequential $b$-decays

As shown in Table IV, most lepton pairs contained in the same jet are produced by sequential decays of single $b$-hadrons. Sequential decays of $b$ hadrons are modeled with the CLEO Monte Carlo generator (QQ) [4]. The discrepancy in the shape of the invariant mass
and opening angle distributions in the data and in the simulation could be due to an inaccurate modeling of these decays. We cannot verify this by using any of our data, and we do it through a comparison of the process $e^+e^- \rightarrow b\bar{b}$ at the $Z$-pole using the JETSET 7.3 Monte Carlo generator [16] as implemented at LEP by the DELPHI collaboration [17] and our simulation which uses the HERWIG and QQ generators. At our request, the DELPHI collaboration has compared dilepton invariant mass distributions in hadronic $Z$-decays to their simulation using selection criteria that could be easily reproduced in the CDF detector [18].

The DELPHI data consist of 573474 hadronic $Z$-decays and the simulation of 992988 events which are normalized to the same luminosity of the data. The DELPHI samples consist of events with $|\cos(\theta_T)| \leq 0.95$, where $T$ is the thrust axis [19]. The two leptons ($e$ or $\mu$) are required to belong to the same hemisphere, as defined by the thrust axis. Leptons are selected with pseudorapidity $|\eta| \leq 1$ and momenta larger than 3 GeV/c to mimic closely the selection criteria of the SLT algorithm used by CDF. Dileptons are divided into OS and SS pairs. The comparison of the invariant mass distribution of OS-SS dileptons in the DELPHI data and simulation is shown in Fig. 6. The JETSET generator models correctly the data at the $Z$-pole, where $b$ quarks are produced with an energy comparable to that in our inclusive lepton sample; the DELPHI simulation correctly models all fake-lepton backgrounds including photon conversions, which are clearly visible in Fig. 6(a). CDF identifies and removes conversions in the data and in the simulation, and then uses SS pairs to estimate the remaining fake-lepton background. Our technique for removing misidentification background is supported by the plot in Fig. 7. For $Z \rightarrow b\bar{b}$ events generated with the DELPHI simulation, the invariant mass distribution of all OS-SS lepton pairs is almost identical to that for OS-SS dilepton pairs identified at generator level as coming from $b$ decays ($J/\psi$ mesons are removed as in our analysis).
FIG. 6: Invariant mass distributions of all OS-SS lepton pairs identified in events due to hadronic \( Z \)-decays (●) by the DELPHI experiment at LEP are compared to the DELPHI simulation based on the \textsc{jetset 7.3} generator (open circle) for (a) all dileptons, (b) \( ee \) pairs, (c) \( \mu\mu \) pairs, and (d) \( e\mu \) pairs.
FIG. 7: The invariant mass distribution of all OS-SS lepton pairs identified in the DELPHI simulated sample of $Z \rightarrow b\bar{b}$ events is compared to the distribution of OS-SS dileptons identified at generator level as coming from $b$-quark decays. As in our study, $J/\psi$ mesons have been removed.

Figure 8 compares invariant mass distributions of OS-SS dileptons in a simulation of the process $e^+e^- \rightarrow b\bar{b}$ at the $Z$-pole using the Jetset 7.3 generator and the DELPHI detector simulation and in a simulation of the same process that uses the generator package Herwig 5.6+QG 9.1 and the CDF detector simulation. Dileptons are identified at generator level as coming from $b$ decays ($J/\psi$ mesons are excluded). Given the fair agreement between the two event generators, it is unlikely that a simulation deficiency is responsible for the large discrepancy between the observed and predicted shapes of the invariant mass and opening angle distributions of lepton pairs arising from sequential decays of $b$ hadrons produced at the Tevatron (see Fig. 2).
FIG. 8: Invariant mass distributions of OS-SS dileptons in a simulation of the process $e^+e^- \rightarrow b\bar{b}$ at the $Z$-pole using the JETSET 7.3 generator and the generator package HERWIG 5.6 + QQ 9.1.

C. Events with three leptons

Events in which the lepton-jet contains an additional SLT tag and the away-jet also contains a soft lepton tag are the most interesting because the rate of a-jets with SLT tags is also larger than the prediction [1]. Unfortunately, there is only a handful of these events. As shown in Fig. 2, the dilepton opening angle distribution is correctly modeled by the simulation for $\cos \theta \leq 0.975$; in contrast, only 60% of the events is accounted for by the simulation for $\cos \theta \geq 0.975$. We compare the fraction of SLT tags in away-jets recoiling a jet containing a lepton pair with opening angle $\cos \theta \geq 0.975$ and $\cos \theta \leq 0.975$ to verify if there is a correlation between the excess of dilepton pairs and the excess of SLT tags in the away-jets. Rates of events with three leptons are listed in Table VII. The fraction of away-jets with SLT tags is $(2.34 \pm 0.76)\%$ for $\cos \theta \geq 0.975$ and $(1.07 \pm 1.07)\%$ for $\cos \theta \leq 0.975$. The two fractions are statistically compatible, however a large correlation is also not excluded.
TABLE VI: Numbers of away-jets recoiling against a lepton-jet that also contains a soft lepton, \( N_{a-jet} \). \( N_{a-jet}^{SLT} \) is the number of away-jets with an \( SLT \) tag due to heavy flavor; the listed number of fake \( SLT \) tags (mistags) has been removed. The numbers of a-jets are split according to \( \theta \), the opening angle of the lepton pair (\( Dil \)). The SS dilepton contributions are used to remove the fake-lepton background to OS lepton pairs.

|         | \( \cos \theta \geq 0.975 \) |         | \( \cos \theta \leq 0.975 \) |
|---------|-------------------------------|---------|-------------------------------|
|         | \( N_{a-jet} \)            | \( N_{a-jet}^{SLT} \) (mistags) | \( N_{a-jet} \)            | \( N_{a-jet}^{SLT} \) (mistags) |
| **Electron data**                        |                         |                         |                              |
| **Dil** |                               |                         |                              |
| OS      | 1010                          | 22.8 (15.2)             | 560                          | 8.7 (8.3)                      |
| SS      | 198                           | 6.4 (2.6)               | 170                          | 5.5 (2.5)                      |
| OS-SS   | 812 ± 34.7                    | 16.4 ± 6.6              | 390 ± 27.0                   | 3.2 ± 5.0                      |
| **Muon data**                            |                         |                         |                              |
| **Dil** |                               |                         |                              |
| OS      | 330                           | 10.6 (5.4)              | 191                          | 2.7 (3.3)                      |
| SS      | 94                            | 2.5 (1.5)               | 61                           | 0.3 (0.7)                      |
| OS-SS   | 236 ± 20.6                    | 8.1 ± 4.4               | 130 ± 15.1                   | 2.4 ± 2.6                      |
V. CONCLUSIONS

We have studied rates and kinematical properties of sequential semileptonic decays of single $b$ hadrons produced at the Fermilab Tevatron collider. This study completes the review of the heavy flavor properties of jets produced at the Tevatron reported in Ref. [1]. As in the previous analysis, we use events with two or more central jets with $E_T \geq 15$ GeV, one of which (lepton-jet) is consistent with a semileptonic bottom or charmed decay to a lepton with $p_T \geq 8$ GeV$/c$. In the previous study, we have used measured rates of lepton- and away-jets containing displaced vertices (secvtx tags) or tracks with large impact parameter (jpb tags) to determine the bottom and charmed content of the data; we have then tuned the parton-level cross section predicted by the HERWIG-based simulation to match the observed heavy-flavor content. The simulation, tuned within the experimental and theoretical uncertainties, models correctly rates of lepton- and away-jets with secvtx and jpb tags, as well as the relevant kinematical properties of the data; however, it underestimates by 50% the number of away-jets containing a soft ($p_T \geq 2$ GeV$/c$) lepton.

The present study uses the same tuning of the simulation, and extends the comparison to the rates of soft leptons in the lepton-jet. We compare rates of jets containing a lepton pair (the trigger lepton and a soft lepton) with opposite (OS) and same (SS) sign charge in the data and in the conventional-QCD simulation. The data have a 20% excess of OS-SS dileptons with respect to the simulation ($2\sigma$ systematic effect). The distributions of the dilepton invariant mass and opening angle in the data are strikingly different from the simulation prediction for the observed excess is all concentrated at dilepton invariant masses smaller than $2$ GeV$/c^2$ and opening angles smaller than $11^\circ$.

Since all QCD-based simulations predict that rates of OS-SS dileptons with small opening angle are dominated by sequential semileptonic decays of single $b$-hadrons, observed rates of lepton pairs with invariant masses smaller than that of a $b$ quark are frequently used by collider experiments to determine or verify the $b$ purity of data samples used to measure $b$-hadron properties or to calibrate efficiencies for detecting $b$ hadrons. The present study shows that there is at least a difficulty in modeling rates and kinematical properties of such lepton pairs.
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