An algorithm of enemy aircraft and missile threat zone in surface warship air defense

F Long, Z W Ma, H R Ma
Institute of Software and Simulation, Dalian Academy, Dalian, Liaoning, China
doubao.0123@aliyun.com

Abstract. Aiming at the algorithm of the enemy aircraft and missile threat zone when the enemy launches air-to-ship missiles in surface warship air defense, this paper sets up the algorithm coordinate system, and distinguishes the position situation of both parties. Based on these, the paper defines the regional shape of the threat zone, and establishes the calculation model. At last, it develops a demonstration verification program and supplies the application examples in various cases. The enemy aircraft and missile threat zone can be calculated accurately with this algorithm, so it is able to effectively assist the commander to form scientific air defense plans.

1. Background and requirement
In modern naval warfare, surface warship forces may face many hostile threats, including attacks from the air, underwater, surface, and shore. Of all these threats, the air threat from the enemy targets is most urgent and fatal. Launching the air-to-ship missiles is the major combat mode for the enemies to attack our surface warships. Therefore, air defense in naval warfare is one of the main tasks for the commanders of surface warships. There are two types of enemy air targets--enemy aircrafts and air-to-ship missiles. Enemy aircrafts are intercepted by fighters, bombers or surface warships, and enemy air-to-ship missiles are resisted by surface warships [1].

The problem of top priority in the surface warship air defense operation is to acquire the possible flight range of enemy air targets. This paper mainly studies the flight range of enemy air targets when the enemy aircrafts launch air-to-ship missiles, that is, the enemy aircraft and missile threat zone, including the enemy aircraft maneuver areas and missile attack areas. By acquiring the flight trajectory of the enemy air targets, not only the air defense operations of the surface warship can be effective, but the air defense resources can be used to the full. Otherwise, it is blind and inefficient, and is easily broken through. It may even be subjected to fatal air strikes, thus affecting the entire operation [2].

2. Distinction of the conditions of the enemy aircraft and missile threat zone
The enemy aircraft and missile threat zone is of three dimensions with complexity and irregularity. The altitude range is from the sea level to the enemy aircrafts. The horizontal range is the set of the horizontal maneuvering flight path of the enemy aircrafts and missiles. The altitude factor of the enemy air targets has less influence on the air defense operation planning, while the horizontal factor has the main influence.

Mastering the projection of the enemy aircraft and missile threat zone on the sea surface is adequate to formulate air defense plan in practice, so the plane rectangular coordinate system \(xOy\) is established. The initial condition is that the location of our surface warship is Point \(W\), and the enemy airport or aircraft carrier locates in Point \(O\), with a distance of \(d\) in between. The enemy aircraft's
combat radius is $R$, and the maximum range of the enemy air-to-ship missile is $r$. So the enemy's attack distance is $R + r$. An plane rectangular coordinate system can be set up with $O$ as the original point and $OW$ as the Axis $x$ forward. Let's rotate $OW$ 90 degrees counter clockwise and get Axis $y$. The Circle $\Omega_1$ is the enemy combat radius circle, with the center $O$, and the radius $R$; the Circle $\Omega_2$ is the enemy attack circle, with the center $O$, and the radius $R + r$; the Circle $\Omega_3$ is the enemy air-to-ship missile attack circle, with the center $W$, and the radius $r$.

With the change of the distance $|OW|$ between both parties, the shapes of the enemy aircraft and missile threat zones and their algorithms are divided into 3 cases corresponding to $|OW|$‘s range of value. Case 1: our party locates inside the enemy attack circle, and outside the enemy combat radius circle, that is $R < |OW| < R + r$; Case 2: our party locates inside the enemy combat radius circle, and is beyond the maximum range of the enemy air-to-ship missile, that is $r \leq |OW| \leq R$; Case 3: our party locates inside the enemy combat radius circle, and is within the maximum range of the enemy air-to-ship missile, that is $|OW| < r$.

3. Algorithm of enemy aircraft and missile threat zone

3.1. Algorithm of Case 1($R < |OW| < R + r$)

Case 1 is the most common situation, that is, the enemy aircraft launches air-to-ship missiles to attack the surface warships within the range of the enemy air-to-ship missiles. In this way, the enemy can perform rapid strike, thus improving the success rate of the strike.

3.1.1. Establishment of auxiliary geometric figures. The maximum value of the sum of the flight distance of the enemy aircraft and the enemy air-to-ship missile is of fixed value $R + r$, so partial boundary of the enemy aircraft mobile area conforms to the geometric characteristics of an ellipse [3-4]. As shown in Figure 1, the auxiliary Ellipse $P$ is established, the focus is $O, W$, and the length of the long axis is $R + r$. Point $A$ and Point $B$ are the intersections of Ellipse $P$ and Circle $\Omega_1$. Point $C$ and Point $D$ are the intersections of Ellipse $P$ and Axis $y$.

![Figure 1. Enemy aircraft and missile threat zone in Case 1.](image)

3.1.2. Description of the enemy aircraft and missile threat zone

(I) Enemy aircraft mobile area

The shadow part (oblique dotted line) in Figure 1 is the enemy aircraft mobile area $J_1$, which should meet the following conditions at the same time. First, being inside or on the enemy combat radius circle $\Omega_1$. Second, being inside or on the Ellipse $P$. Third, the enemy aircrafts do not reverse the flight during the attack, that is, Area $Q_1$ is excluded. Therefore, $J_1$ is:

$$J_1 = \Omega_1 \cap P - Q_1$$

Of which $Q_1$ is the area formed by the Segment $CD$ and the Arc $CD$. 

2
Therefore, establish the equations of Circle $\Omega_1$ and Ellipse $P$, and calculate the coordinates of Point $A$, Point $B$, Point $C$ and Point $D$, and then, the enemy aircraft mobile area $J_1$ can be obtained.

(II) Air-to-ship missile attack area

The shadow part (transverse dotted line) in Figure 1 is the enemy missile attack area $L_1$, which should meet the following conditions at the same time. First, being inside the air-to-ship missile attack circle $\Omega_3$. Second, the enemy aircraft being inside the combat radius circle $\Omega_1$ and outside the air-to-ship missile attack circle $\Omega_3$. Therefore, $L_1$ is Sector $S_{WAB}$.

Therefore, establish the equations of Circle $\Omega_1$ and circle $\Omega_3$, and calculate the coordinates of Point $A$, Point $B$, so that the missile attack area $L_1$ can be obtained.

### 3.1.3 Calculation of the enemy aircraft and missile threat zone

(I) Enemy aircraft mobile area

(1) Equation of Circle $\Omega_1$

\[ x^2 + y^2 = R^2 \]

(2) Equation of Ellipse $P$

\[ \frac{(x - d/2)^2}{a^2} + \frac{y^2}{b^2} = 1 \]

Of which: semi-major axis $a = (R + r)/2$; semi-minor axis $b = \sqrt{a^2 - (d/2)^2}$

(3) Coordinates of Point $A$ and Point $B$

Point $A$ and Point $B$ are the intersection points of Circle $\Omega_1$, Circle $\Omega_3$ and Ellipse $P$, of which the equation of Circle $\Omega_3$ is:

\[ (x - d)^2 + y^2 = r^2 \]

The coordinates of Point $A$ and Point $B$ can be obtained from simultaneous equations of Circle $\Omega_1$, Circle $\Omega_3$ and Ellipse $P$:

Point $A$ coordinates,
\[
\begin{align*}
    x_A &= \frac{d^2 + R^2 - r^2}{2d} \\
    y_A &= -\sqrt{R^2 - \left(\frac{d^2 + R^2 - r^2}{2d}\right)^2}
\end{align*}
\]

Point $B$ coordinates,
\[
\begin{align*}
    x_B &= \frac{d^2 + R^2 - r^2}{2d} \\
    y_B &= \sqrt{R^2 - \left(\frac{d^2 + R^2 - r^2}{2d}\right)^2}
\end{align*}
\]

(4) Coordinates of Point $C$ and Point $D$

Point $C$ and Point $D$ are on the Ellipse $P$. Place $x_C = 0$, $x_D = 0$ into the equation of Ellipse $P$, and the coordinates of Point $C$ and Point $D$ can be obtained:

Point $C$ coordinates,
\[
\begin{align*}
    x_C &= 0 \\
    y_C &= b\sqrt{1 - \left(\frac{d}{2a}\right)^2}
\end{align*}
\]

Point $D$ coordinates,
\[
\begin{align*}
    x_D &= 0 \\
    y_D &= -b\sqrt{1 - \left(\frac{d}{2a}\right)^2}
\end{align*}
\]

(II) Air-to-ship missile attack area

Refer to the above **Enemy aircraft mobile area** for the equations of Circle $\Omega_1$ and Circle $\Omega_3$, and the coordinates of Point $A$ and Point $B$.

### 3.2 Algorithm of Case 2 ($r \leq |OW| \leq R$)

In Case2, our surface warship is more threatened than is in Case1. In Case2, our surface warship is within the combat radius of the enemy aircrafts, so the enemy aircraft can launch air-to-ship missiles from any direction. Therefore, the enemy will greatly increase the penetration probability, and our surface warship may be subjected to a full 360 degree attack.
3.2.1. Establishment of auxiliary geometric figures. The geometric properties of Case 2 and Case 1 are basically the same, with only an area added. With the change of $|OW|$, which is the distance between the enemy and our party, the area is divided into two cases: Case 2.1: When $R - r \leq |OW| \leq R$, schematic diagram of the calculation of enemy aircraft and missile threat zone is as shown in Figure 2. Case 2.2, when $r \leq |OW| < R - r$, schematic diagram of the calculation of enemy aircraft and missile threat zone is as shown in Figure 3. An auxiliary Sector $S_{OA_1B_1}$ is established. Straight line $OA_1$ and $OB_1$ are tangent to Circle $\Omega_3$, intersect Circle $\Omega_1$ respectively at Point $A_1$ and Point $B_1$, and intersect Ellipse $P$ respectively at Point $C_1$ and Point $D_1$. Through geometric characteristics analysis, our surface warship is inside Circle $\Omega_1$, and the enemy aircraft is mobile inside Sector $S_{OA_1B_1}$. Both sides can launch air-to-ship missiles to attack.

![Figure 2. Enemy aircraft and missile threat zone in Case 2.1.](image)

![Figure 3. Enemy aircraft and missile threat zone in Case 2.2.](image)

3.2.2. Description of the enemy aircraft and missile threat zone

(I) Enemy aircraft mobile area

The shadow parts (oblique dotted line) in Figure 2 and 3 are the enemy aircraft mobile area $J_2$, that is, on the basis of $J_1$, Sector $S_{OA_1B_1}$ is superimposed. Therefore, $J_2$ is:

$$J_2 = (\Omega_1 \cap P - Q_3) \cup S_{OA_1B_1}$$

In fact: in Case 2.1, $J_2$ has an additional area of $Q_2 \cup Q_3$ relative to $J_1$, wherein $Q_2$ is the area enclosed by Segment $A_1D_1$, Arc $AD_1$ and Arc $AA_1$, and $Q_3$ is the area enclosed by Segment $B_1C_1$, Arc $BC_1$ and Arc $BB_1$. In Case 2.2, $J_2$ has an additional area of $Q_4$ relative to $J_1$, wherein $Q_4$ is the area enclosed by Segment $A_1D_1$, Segment $B_1C_1$, Arc $A_1B_1$ and Arc $C_1D_1$.

Therefore, on the basis of the algorithm of Case 1, calculate the coordinates of Point $A_1$, Point $B_1$, and then, the enemy aircraft mobile area $J_2$ can be obtained.

(II) Air-to-ship missile attack area

The shadow part (transverse dotted line) in Figure 3 and 4 is the enemy missile attack area $L_2$, which should meet the same conditions simultaneously as in Case 1. Therefore, $L_2$ is:

$$L_2 = \Omega_1 \cap \Omega_3$$

In fact: in Case 2.2, $L_2 = \Omega_3$.

Therefore, establish the equations of Circle $\Omega_1$ and Circle $\Omega_3$, and calculate the coordinates of Point $A$, Point $B$, so that the missile attack area $L_2$ can be obtained.
3.2.3. Calculation of the enemy aircraft and missile threat zone

(I) Enemy aircraft mobile area

(1) Equation of Straight line \( OA_1 \) and Straight line \( OB_1 \)

\[
\pm kx - y = 0
\]

The distance between Point \( F (d, 0) \) and Straight line \( OA_1 \) or Straight line \( OB_1 \) is \( r \). According to this, \( k = \frac{\pm \sqrt{r^2/(d^2 - r^2)}}{d} \).

The coordinates of Point \( A_1 \) and \( B_1 \) can be obtained from simultaneous equations of Circle \( \Omega_2 \):

Point \( A_1 \) coordinates, \( \begin{aligned} x_{A_1} &= R \sqrt{1 - r^2/d^2} \\ y_{A_1} &= -Rr/d \end{aligned} \); Point \( B_1 \) coordinates, \( \begin{aligned} x_{B_1} &= R \sqrt{1 - r^2/d^2} \\ y_{B_1} &= Rr/d \end{aligned} \)

(II) Air-to-ship missile attack area

Refer to Case 1 for the equations of Circle \( \Omega_1 \) and Circle \( \Omega_3 \), and the coordinates of Point \( A \) and Point \( B \).

3.3. Algorithm of Case 3 (|\( OW \)\( | < r \))

Case 3 is the most rare situation, but once this situation happens, the surface warship will be facing the greatest threat. In this case, the enemy aircraft can launch air-to-ship missiles at any time, that is, the surface warship may be subjected to the rapid and sudden attack of the enemy air-to-ship missiles at any time. So the surface warship should try to avoid this situation.

(I) Enemy aircraft mobile area

According to the rule that the enemy aircrafts do not reverse the flight during the attack, and the analysis of Case 2, in Case 3 the sector angle of Sector \( \sigma_1 \) is defined as \( \pi \), and \( S_{OA_1B_1} \) properly include \( \Omega_1 \cap P - Q_1 \). Therefore, \( J_3 \) is:

\[
J_3 = S_{OA_1B_1}
\]

On the basis of the equation of Circle \( \Omega_1 \), the enemy aircraft mobile area \( J_3 \) can be obtained by calculating the coordinates of Point \( A_1 \) and Point \( B_1 \):

Point \( A_1 \) coordinates, \( \begin{aligned} x_{A_1} &= 0 \\ y_{A_1} &= -R \end{aligned} \); Point \( B_1 \) coordinates, \( \begin{aligned} x_{B_1} &= 0 \\ y_{B_1} &= R \end{aligned} \)

(II) Air-to-ship missile attack area

The enemy missile attack area \( L_3 \) should meet the following conditions at the same time. First, being inside the air-to-ship missile attack circle \( \Omega_3 \). Second, the enemy aircraft being inside the enemy aircraft mobile area \( J_3 \) and the air-to-ship missile attack circle circle \( \Omega_3 \). Therefore, \( L_3 \) is:

\[
L_3 = J_3 \cap \Omega_3
\]

On the basis of the equation of Circle \( \Omega_3 \), the enemy missile attack area \( L_3 \) can be obtained by calculating the coordinates of Point \( E \) and Point \( F \):

Point \( E \) coordinates, \( \begin{aligned} x_e &= 0 \\ y_e &= -\sqrt{r^2 - d^2} \end{aligned} \); Point \( F \) coordinates, \( \begin{aligned} x_f &= 0 \\ y_f &= \sqrt{r^2 - d^2} \end{aligned} \)

4. Application example of the enemy aircraft and missile threat zone

The content above establishes the algorithm model of the enemy aircraft and missile threat zone. This paper is to verify the validity of the algorithm model with the computer demonstration and verification method [5-6], and conduct the calculation and display of enemy aircraft and missile threat zone of various air defense combat factors combination.

Here is a set of assumed numeric values for calculating the enemy aircraft and missile threat zones in various cases. It is known that the position of our surface warship is Point \( W \); the position of the enemy airport or aircraft carrier is Point \( O \); the operational radius of the enemy aircraft \( R \) is 950km; the maximum range of the enemy air-to-ship missile \( r \) is 420km, corresponding to Case 1 and case 2 (2.1 and 2.2); assume that the distances between the enemy and our party \( d \) are 1100km, 680km and 475km, then the enemy aircraft and missile threat zones \( J_1 \) and \( L_1, J_{2,1}, L_{2,1}, J_{2,2} \) and \( L_{2,2} \) are
calculated. The gray shadow areas in Figure 4-6 are the enemy aircraft mobile areas, and the Shadow areas of transverse solid lines are missile attack areas.

![Figure 4](image1.png)  ![Figure 5](image2.png)  ![Figure 6](image3.png)

Figure 4. Calculation example of enemy aircraft and missile threat zone in Case 1 ($d=1100$km).  
Figure 5. Calculation example of enemy aircraft and missile threat zone in Case 2.1 ($d=680$km).  
Figure 6. Calculation example of enemy aircraft and missile threat zone in Case 2.2 ($d=475$km).

5. Conclusion
This paper has solved a key requirement in air defense operations of the surface warships. The mathematical model of the enemy aircraft and the missile threat zone is set up with the plane analytic geometry method. It has provided the two-dimensional plane shape of the enemy aircraft and missile flight range. It has assumed all the typical cases in the air defense operations of the surface warships, and verified the validity of the mathematical model with the computer demonstration and verification method.

The auxiliary calculation of the enemy aircraft and the missile threat zone is one of the main contents of the assistant decision-making function of the command information system. On the basis of this function, the commanders can use and deploy all the forces and weapons of the air defense resources pertinently and efficiently to deal with the air threat of enemy aircrafts and missiles.

References
[1] SONG Yuan, WANG Yongchun. Theory and Application of Naval Air Defense Operation Situation Estimation [M]. Beijing: National Defense Industry Press, 2014.
[2] MIAO Xudong. Theory and Application of Collaborative Air Defense Mission Planning for Warship Formation [M]. Beijing: National Defense Industry Press, 2013.

[3] YIN Chengyi, TAN Ansheng, GUO Jianglong. Sector Angle Prediction Model for Air Threat of Surface Warship Fleet [J]. Command Control and Simulation, 2013, 35(3): 67-70.

[4] TAN Ansheng. Planning and Management Analysis for Shipboard Early-Warning Helicopter Operations [M]. Beijing: National Defense Industry Press, 2017.

[5] HUO Yafei. Qt Creator Quick Start [M]. Beijing: Beijing University of Aeronautics and Astronautics Press, 2017.

[6] ZHANG Hongyan. Qt5 Android Training [M]. Beijing: Posts and Telecommunications Press, 2015.