A general thermodynamical description of the event horizon in the FRW universe

Fei-Quan Tu and Yi-Xin Chen

Zhejiang Institute of Modern Physics, Zhejiang University, Hangzhou, 310027, China

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Abstract. The Friedmann equation in the Friedmann-Robertson-Walker (FRW) universe with any spatial curvature is derived from the first law of thermodynamics on the event horizon. The key idea is to redefine a Hawking temperature on the event horizon. Furthermore, we obtain the evolution equations of the universe including the quantum correction and explore the evolution of the universe in the $f(R)$ gravity. In addition, we also investigate the generalized second law of thermodynamics in Einstein gravity and the $f(R)$ gravity. This perspective also implies that the first law of thermodynamics on the event horizon have a general description in respect of the evolution of the FRW universe.

PACS. 98.80.-k Evolution of the universe – 95.30.Tg Thermodynamics of horizon of the spacetime

1 Introduction

Since the discovery by Bardeen, Bekenstein, Hawking[1,2,3] in the 1970s, the relationship between black hole physics and thermodynamics have been generally accepted by physicists. Decades of research shows that the formula of black hole entropy $S = A/4$ where $A$ is the area of the horizon and the temperature $T = |κ/(2π)|$ where $κ$ is the surface gravity have a certain universality. In 1995, Jacobson[4] argued that Einstein equation could be derived from the relation of thermodynamics (Clausius relation $dE = TdS - PdV$) and pointed out that Einstein equation is an equation of state. This is an important discovery that there exists a deep connection between Einstein gravity theory and thermodynamics. Besides, in a 4-dimensional de Sitter space, analysis of quantum field theory shows that the temperature of the horizon of spacetime is $T = κ/(2π) = 1/(2πR)$ where $κ$ is the radius of the horizon. This implies that there exists a closed relationship between the horizon of spacetime and thermodynamics. Based on above research results, we know that thermodynamics has a certain universality in describing the horizons of spacetime.

For the dynamic black hole, Hayward[8,9,10] introduced the notion of trapping horizon in the 4-dimensional Einstein gravity and showed that Einstein equation is equivalent to the unified first law. Based on these facts, the authors[11,12] generalized these concepts to the FRW universe and investigated the relationship between the unified first law and thermodynamics of the horizon in the FRW universe. Especially, in Ref.[11], they considered the FRW universe as a dynamical spherically symmetric spacetime and defined a trapping horizon. In this way, they showed the equivalency between the unified first law and thermodynamics of the apparent horizon in the FRW universe.

In addition, regarding thermodynamics of the horizon, Padmanabhan[13,14] has shown that the field equations in Einstein gravity and Lanczos-Lovelock gravity for a spherically symmetric spacetime can be expressed as the thermodynamic identity $dE = TdS - PdV$, where the quantities $E$, $T$, $S$, and $V$ are related to the horizon and have the interpretation for energy, temperature, entropy and volume. So Clausius relation $δQ = TdS$ holds on the horizon. On the other hand, in cosmology, there exists an event horizon since the universe is in accelerated expansion according to astronomical observation. Indeed, Li[15] predicted the equation of state of the dark energy and resolved the cosmic coincidence problem by introducing the event horizon in the model of holographic dark energy. Besides, The event horizon of the universe is the largest comoving distance from which light emitted now can ever reach the observer in the future and very similar to the event horizon of the black hole whose thermodynamics have been accepted generally. Therefore, it is natural and interesting to investigate laws of thermodynamics related to the event horizon in the FRW universe.

For the researches about thermodynamics on the event horizon, though Wang et al.[16] claimed the event horizon is unphysical from the point of view of the laws of thermodynamics, Chakraborty[6] concluded that the universe bounded by the event horizon may be a Bekenstein system by redefining a Hawking temperature. Based on the temperature defined in Ref.6, we investigated ther-

\[ S = \frac{A}{4} \]
modynamics of the universe bounded by the event horizon and dominated by the tachyon fluid and found that there exists a good thermodynamic description in such universe. However, the definition of the temperature on the event horizon is not general in Ref. [17], and the thermodynamical description is reasonable just in the flat universe and some models. So does there exist a general thermodynamic description of the event horizon in the FRW universe with any spatial curvature? Indeed, we obtain the first law of thermodynamics on the event horizon by redefining a Hawking temperature in Einstein gravity.

Now we can ask the question whether the first law of thermodynamics can hold on the event horizon in other gravity theories such as the $f(R)$ gravity. In fact, in the $f(R)$ gravity, Eling et al. [18] have shown that the correct equation of motion can not be obtained if one uses Hawking temperature, the entropy assumption $S = a f'(R)$ and the first law of thermodynamics. An entropy production term has to be added to the first law of thermodynamics in order to obtain the correct equation. Thus the $f(R)$ gravity is described by the nonequilibrium thermodynamics of spacetime. So the above question turns into the question whether the first law of thermodynamics on the event horizon which is obtained by redefining the Hawking temperature in Einstein gravity can hold in the $f(R)$ gravity. In other words, can thermodynamics of the spacetime in the $f(R)$ theory be described by the equilibrium thermodynamics? Through the investigation, we find that the first law of thermodynamics on the event horizon is also held in the $f(R)$ theory. Therefore, we may conclude that the first law of thermodynamics on the event horizon has a general description in respect of the evolution of the FRW universe.

The present paper is organized as follows. In Section 2, we show that the first law of thermodynamics on the event horizon holds by redefining a Hawking temperature. In Section 3, we derive the evolution equations of the universe based on the first law of thermodynamics on the event horizon where the quantum correction of the entropy is included. These evolution equations of the universe can not be obtained just by Einstein equation, so the method of thermodynamical description is more general. In Section 4, we study the evolution of the universe based on the first law of thermodynamics on the event horizon in the $f(R)$ gravity. In Section 5, we investigate the generalized second law of thermodynamics of the universe bounded by the event horizon in Einstein gravity and the $f(R)$ gravity. We end our paper with the conclusion in Section 6. Throughout the paper, the Greek indices, $\mu, \nu, ...$, etc. run over 0, 1, 2, 3 and the units are chosen with $c = h = k_B = 1$ and the signature of the spacetime is taken as $(-, +, +, +)$.

2 Redefinition of the Hawking temperature on the event horizon

In the homogenous and isotropic universe, the metric can be expressed as

$$ds^2 = h_{ij}dx^i dx^j + R^2 d\Omega_2^2,$$  \hspace{1cm} (1)

where $i, j$ can take value 0 and 1, $R = a(t)r$ in which $a(t)$ is the scale factor and the 2-dimensional metric $h_{ij} = \text{diag}(-1, a^2/(1 - kr^2))$ in which $k$ is the spatial curvature constant. A scalar quantity is defined as

$$\chi = h^{ij}\partial_i R^i R^j. \hspace{1cm} (2)$$

The apparent horizon is defined by the scalar quantity $\chi = 0$, which gives $R_A = \sqrt{b + \frac{1}{4}}$. Then the surface gravity on the apparent horizon is defined as [5,6,16,20]

$$\kappa_A = -\frac{1}{2} \frac{\partial \chi}{\partial R}\big|_{R=R_A} = \frac{1}{R_A} \hspace{1cm} (3)$$

and the corresponding Hawking temperature is

$$T_A = \frac{\kappa_A}{2\pi} = \frac{1}{2\pi R_A}. \hspace{1cm} (4)$$

The study of thermodynamics of the apparent horizon has made great progress in the FRW universe [11,12,16,21,22,23,24]. In Refs. [11,12], it was shown that the function of the surface gravity for any horizon of the FRW universe depends on these variables $R_A$ and $\kappa_A$ and is related to the ratio $R_A/R_\Lambda$ under the frame of the unified first law. On the other hand, Bousso [23] pointed out that a thermodynamic description of the horizon would be approximately valid and it does not matter whether one uses the apparent or the event horizon in the quintessence dominated spacetime (Q-spacetime). Therefore, we assume that the surface gravity on the event horizon (with the radius $R_E$) should have the following form

$$\kappa_E = -\frac{1}{2} \frac{\partial \chi}{\partial R}\big|_{R=R_E} \frac{R_A}{R_E} g(R_E), \hspace{1cm} (5)$$

where $g(R_E)$ is the function which is related to the variable $R_E$.

Now let’s determine the form of the function $g(R_E)$. In the model of the flat Q-spacetime (the scale factor $a(t)$ is $t^{\alpha}$ ($\alpha > 1$) and the spatial curvature constant $k$ is 0), the radius of the apparent horizon is $R_A = \frac{a}{\alpha}$ and the radius of the event horizon is $R_E = \frac{a}{\alpha}$, and the surface gravity on the event horizon can be reduced to the following form [6]

$$\kappa_E = -\frac{1}{2} \frac{\partial \chi}{\partial R}\big|_{R=R_E} \frac{R_A}{R_E}. \hspace{1cm} (6)$$

So the simplest form of the function $g(R_E)$ is

$$g(R_E) = \frac{R_E}{R_E} \hspace{1cm} (7)$$

Up to now, we obtain the surface gravity on the event horizon

$$\kappa_E = -\frac{1}{2} \frac{\partial \chi}{\partial R}\big|_{R=R_E} \frac{R_A}{R_E}. \hspace{1cm} (8)$$

2 The trapping horizon coincides with the apparent horizon $R_A$ in the context of the FRW universe, so we use $R_A$ to denote the radius of the trapping horizon.
According to the relation between Hawking temperature and the surface gravity on spacetime horizons, we get the temperature on the event horizon

\[ T_E = \frac{|\kappa_E|}{2\pi} = \frac{H}{2\pi} \left( \frac{k}{a^2} - \hat{H} \right) \frac{R_E^2}{R_E}. \quad (9) \]

Now we would like to show the universality of this temperature on the event horizon. The energy flux across the event horizon during an infinitesimal time interval \( dt \) can be calculated as \[ \delta Q = AT_{\mu\nu}k^\mu k^\nu dt \mid_{r=R_E} \]

By redefining Hawking temperature (Eq.(9)), we confirm the validity of the first of thermodynamics on the event horizon in the above section. In the following sections, we take the first law of thermodynamics \( \delta Q = TdS \) on the event horizon as the fundamental starting point to derive the dynamic evolution equations of the universe.

In this section, we will consider the quantum correction of the entropy of the event horizon and derive these evolution equations of the universe including quantum correction effects.

As we have pointed out in Introduction, the property of the event horizon of spacetime is similar to that of black hole. Due to the similarity, we take the form of the quantum corrected entropy of black hole as the entropy of the event horizon. (we will discuss it later.)

\begin{align}
S &= \frac{A}{4L_p^2} + \alpha \ln \left( \frac{A}{4L_p^2} \right), \quad (14)
\end{align}

where \( \alpha \) is a constant and \( L_p = \sqrt{\hbar G/c^3} \) is the Planck length. According to Ref. [20], \( \alpha \sim O(1) \). Thus we obtain

\begin{align}
TdS &= \frac{HR_E^2}{G} \left( 1 + \frac{\alpha L_p^2}{\pi R_E^2} \left( \frac{k}{a^2} - \hat{H} \right) \right) dt, \quad (15)
\end{align}

and the energy flux is

\begin{align}
\delta Q &= 4\pi R_E^2 H (\rho + p) dt. \quad (16)
\end{align}

Based on the first law of thermodynamics \( \delta Q = TdS \), we get

\begin{align}
\left( \frac{k}{a^2} - \hat{H} \right) (1 + \frac{\beta}{R_E}) = 4\pi L_p^2 \rho (\rho + p), \quad (17)
\end{align}

where \( \beta = \frac{\alpha L_p^2}{c} \) is a constant. This is the Friedmann equation with quantum correction describing the evolution of the universe. (we will discuss it later.)

Now, in order to see the evolution properties of the universe clearly, we take the scale factor \( a(t) = t^c (c > 1) \) and employ \( G \) to denote \( L_p^2 \). Thus, the radius of the event horizon turns into \( R_E = \frac{k}{\beta} \hat{H}^{-1} \) and Eq.(17) turns into

\begin{align}
\left( \frac{k}{a^2} - \hat{H} \right) (1 + \lambda \hat{H}^2) = 4\pi G (\rho + p), \quad (18)
\end{align}
where \( \lambda = \beta \left( \frac{-1}{2} \right)^2 \) is a constant. Compared with the standard Friedmann equation, we see that this equation has an extra term \( \lambda H^2 \) which is caused by the quantum correction. At present, this term is very small, that’s \( \lambda H^2 \ll 1 \), so we can obtain

\[
\frac{k}{a^2} - \dot{H} = 4\pi G (\bar{\rho} + \bar{\rho}), \tag{19}
\]

where we redefine the effective energy density \( \bar{\rho} \) and the effective pressure \( \bar{p} \),

\[
\bar{\rho} = (1 - \lambda H^2)\rho \tag{20}
\]

and

\[
\bar{p} = (1 - \lambda H^2)p \tag{21}
\]

respectively. On the other hand, the continuity equation integrating the resulting equation, we finally obtain

\[
\dot{\bar{\rho}} + 3H (\bar{\rho} + \bar{p}) = 0. \tag{22}
\]

Substituting Eq.(20), Eq(21), Eq(22) into Eq.(19) and integrating the resulting equation, we finally obtain

\[
H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (1 - \lambda H^2)\rho. \tag{23}
\]

This is another Friedmann equation under the quantum correction. In order to see the properties of the accelerated expansion of the universe clearly, we combine Eq.(19) and Eq.(23), and get the result

\[
\frac{\ddot{a}}{a} = \frac{4}{3} \pi G (\rho + 3p)(1 - \lambda H^2) \cr = \frac{4}{3} \pi G (\rho + 3p) \lambda H^2. \tag{24}
\]

Comparing with the equation \( \frac{\ddot{a}}{a} = -\frac{4}{3} \pi G (\rho + 3p) \) which can be obtained by Einstein equation, we find Eq.(24) has an extra term \( \frac{1}{3} \pi G (\rho + 3p) \lambda H^2 \). From the above derivation, we know \( \lambda \sim O(L_p^2) \), so the extra term contains the factor \( L_p^2H^2 \) which represents quantum correction effects.

It should be noticed that the equation describing the evolution of the universe in the whole history is Eq.(17). From this equation, we know that the evolution of the universe depends on the event horizon \( R_E \) and the term \( \beta/R_E^2 \) cannot be ignored at the early time. So this equation does not only show physical consistency with classical limit but also describes quantum effects which are described by the event horizon. Hence we can conclude that the thermodynamical description based on the event horizon under the redefinition of Hawking temperature is more general than Einstein equation in describing the dynamic evolution of the universe.

### 4 Evolution of the universe based on the first law of thermodynamics on the event horizon in the \( f(R) \) theory

In this section, we will investigate the evolution property of the universe in the theory of \( f(R) \) gravity. According to Eq.(10), the energy flux is

\[
\delta Q = 4\pi R_E^2 \rho (\bar{\rho} + \bar{p}) dt, \tag{25}
\]

where \( \bar{\rho} = \rho + \rho_g \) is the total energy density of the matter energy density \( \rho \) and the effective gravity energy density \( \rho_g \), and \( \bar{p} = p + p_g \) is the total pressure of the matter pressure \( p \) and the effective gravity pressure \( p_g \). In this gravity theory the relation of entropy-area is

\[
S = \frac{Af(R)}{4G}. \tag{26}
\]

Hence

\[
T dS = f'(R) H \left( \frac{k}{a^2} - \dot{H} \right) \frac{R_E^2}{G} dt. \tag{27}
\]

Based on the first law of thermodynamics, we get the following equation

\[
\left( \frac{k}{a^2} - \dot{H} \right) f'(R) = 4\pi G (\bar{\rho} + \bar{p}). \tag{28}
\]

However, \( \bar{\rho} \) and \( \bar{p} \) can’t be determined just by the first law of thermodynamics. So this evolution equation of the universe can’t be also determined just by thermodynamics alone.

In order to determine the total energy density \( \bar{\rho} \) and the total pressure density \( \bar{p} \), we employ the variational principle. In the \( f(R) \) theory, the Einstein-Hilbert action can be written as

\[
S = \int d^4 x \sqrt{-g} \left( f(R) + 2\kappa^2 L_m \right), \tag{29}
\]

where \( \kappa^2 = 8\pi G \). We employ \( f \) to denote the function \( f(R) \) in the following content. Using the variational principle \( \delta S = 0 \), we obtain

\[
G_{\mu\nu} = \kappa^2 \left( \frac{1}{f'} T^{(m)}_{\mu\nu} + \frac{1}{8\pi G} T^{(g)}_{\mu\nu} \right) \equiv \kappa^2 T_{\mu\nu}. \tag{30}
\]

where \( G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \) is the Einstein tensor, \( T^{(m)}_{\mu\nu} = (\rho + p) u_\mu u_\nu + pg_{\mu\nu} \) is the energy-momentum tensor of the matter, and

\[
T^{(g)}_{\mu\nu} = \frac{1}{f'} \left[ \frac{f - Rf'}{2} g_{\mu\nu} + \nabla_\mu \nabla_\nu f' - g_{\mu\nu} \nabla^2 f' \right] \tag{31}
\]

is the energy-momentum tensor of the gravity. Then we get the effective gravity energy density \( \rho_g \) and the effective gravity pressure \( p_g \)

\[
\rho_g = \frac{1}{8\pi G} \left( R f' - f - 3H f'' \dot{R} \right) \tag{32}
\]

and

\[
p_g = \frac{1}{8\pi G} \left( \frac{f - Rf'}{2} + f'' \dot{R}^2 + 2H f'' \dot{R} \right), \tag{33}
\]

respectively.
Thus, substituting Eq.(32) and Eq.(33) into Eq.(28), we finally get the Friedmann equation in the FRW universe
\[
\frac{k}{a^2} \ddot{H} f' + \frac{1}{2} (H f'' - f'') \ddot{R} = 4\pi G (\rho + p). \tag{34}
\]
On the other hand, the continuity equation for the effective perfect fluid in the \(f(R)\) gravity is
\[
\dot{\rho} + 3H(\rho + p) = 0. \tag{35}
\]
Combining Eq.(34) and Eq.(35), we obtain another Friedmann equation
\[
H^2 + \frac{k}{a^2} = \frac{8\pi G}{3f'} \left[ \rho + \frac{1}{8\pi G} \left( \frac{Rf' - f}{2} - 3H f'' \dot{R} \right) \right]. \tag{36}
\]
These Friedmann equations Eq.(34) and Eq.(36) are the same as those of Refs.[33,32] which describe the evolution of the universe in other ways. Therefore, the equivalency between the first law of thermodynamics on the event horizon and Friedmann equations of the FRW universe with any spatial curvature holds not only in Einstein gravity but also in the \(f(R)\) theory. This implies that the thermodynamical description is general in describing the evolution of the universe. Besides, it is also indicated that the FRW universe can be described by the equilibrium thermodynamics on the event horizon in the \(f(R)\) gravity.

5 Generalized second law of thermodynamics of the universe bounded by the event horizon

The generalized second law of thermodynamics of the universe bounded by the event horizon in Einstein gravity has been investigated in Ref.[33,34], in which the authors assume that the universe can be described by the equilibrium thermodynamics. But in this paper, we have shown the validity of the first law of thermodynamics on the event horizon in Section 2, namely the universe bounded by the event horizon can be described by the equilibrium thermodynamics. This conclusion is particularly important for the \(f(R)\) gravity, because it has been pointed out that the spacetime in the \(f(R)\) gravity is described by the nonequilibrium thermodynamics if one uses the usual Hawking temperature[18]. Next, using the method of Ref.[33], we will present the generalized second law of equilibrium thermodynamics of the universe bounded by the event horizon in Einstein gravity and the \(f(R)\) gravity.

For the holographic dark energy (DE) model[15], the density of holographic DE of the universe bounded by the event horizon is
\[
\rho_D = \frac{3c^2}{8\pi G} R_E^{-2}, \tag{37}
\]
where \(c\) is a numerical constant. And the equation of state of holographic DE can be written as
\[
\rho_D = \omega_D p_D, \tag{38}
\]
and
\[
\rho_D = 3H\rho_d = 0 \tag{39}
\]
separately, where \(\rho_d\) is the energy density of dust matter (for the dust matter, its pressure \(p_d\) is 0).

According to Eq.(10), the energy flux is
\[
\delta Q = 4\pi R_E^3 H (\rho_d + p_D + p_D) dt. \tag{41}
\]
In Section 2, we have shown the validity of the first law of thermodynamics on the event horizon, so it is the equilibrium thermodynamics and the effective temperature of the matter (dust matter and DE) distribution can be considered to be the same as that of the event horizon[33,34,35]. Thus we can use the following Gibbs’s relation[34,35]
\[
T_E dS_m = dE_m + p_D dV, \tag{42}
\]
where \(S_m\) and \(E_m\) are the entropy and energy of the matter distribution. We obtain the following equation
\[
dS_m = \frac{4\pi R_E^2}{T_E} (\rho_d + p_D + p_D) dR_E + \frac{H R_E^3}{T_E} \left( H - \frac{k}{a^2} \right) dt, \tag{43}
\]
where these relations \(E_m = \frac{4}{3}\pi R_E^3 \rho_d\) and \(V = \frac{4}{3}\pi R_E^3\) are used. Substituting Eq.(37) and Eq.(38) into Eq.(40), we get
\[
dR_E = \frac{3}{2} R_E H (1 + \omega_D) dt. \tag{44}
\]
Hence the change of the total entropy \(S_{tot} = S_m + S_E\) where \(S_E\) is the entropy of the event horizon which is determined by Eq.(13) is
\[
\frac{dS_{tot}}{dt} = \frac{6\pi R_E^2}{T_E} (\rho_d + p_D + p_D)(1 + \omega_D). \tag{45}
\]
We see that the result is the same as that of Ref.[33]. When the holographic DE satisfies the weak energy condition
\[
\rho_D + p_D = (1 + \omega_D) \rho_D \geq 0, \tag{46}
\]
the generalized second law of thermodynamics will be valid for the universe bounded by the event horizon.

For the \(f(R)\) gravity, as we have shown in Section 4, the first law of thermodynamics holds on the event horizon, so Gibbs’s relation (42) can be used. Thus we obtain
\[
dS_m = \frac{4\pi R_E^2}{T_E} (\rho_d + p_g + p_g) dR_E + \frac{H R_E^3}{T_E} \left( H - \frac{k}{a^2} \right) dt, \tag{47}
\]
where we employ the dust as the matter. According to the definition of the event horizon \(R_E = a(t) \int_t^\infty \frac{dt'}{a(t')}\), we get
\[
dR_E = (H R_E - 1) dt, \tag{48}
\]
so the change of the total entropy is
\begin{equation}
\frac{dS_{\text{tot}}}{dt} = \frac{4\pi R_E^2}{T_E} (\rho_d + \rho_g + P_g) (H R_E - 1) + (1 - f') \frac{H R_E^3}{G T_E} \left( \dot{H} - \frac{k}{a^2} \right). \tag{49}
\end{equation}

Substituting Eq. (32) and Eq. (33) into Eq. (49), we get
\begin{equation}
\frac{dS_{\text{tot}}}{dt} = \frac{R_E^2}{2G T_E} (\rho_d + f'' \dot{R}^2 + f'' \dot{R} - H f'' \dot{R})(H R_E - 1) + (1 - f') \frac{H R_E^3}{G T_E} \left( \dot{H} - \frac{k}{a^2} \right). \tag{50}
\end{equation}

So the generalized second law of thermodynamics can be satisfied as long as the above expression is not less than 0.

Now we would like to make some remarks regarding the generalized second law of thermodynamics. (i) From the above derivation, we know that the Gibbs’s relation (42) is important in order to obtain the change of the total entropy. Indeed, we have established the Gibbs’s relation on the event horizon in Section 2, i.e. the first law of thermodynamics on the event horizon. By contrast, the authors in Ref. [33] just assumed the validity of the first law of thermodynamics on the event horizon and a temperature of the event horizon whose expression is unknown. (ii) For the $f(R)$ gravity, if one does not redefine the Hawking temperature, then the horizon is described by the nonequilibrium thermodynamics [11, 18]. As we have known, the Gibbs’s relation (42) can not be used for the nonequilibrium thermodynamics, so the method in Ref. [33] is invalid. However, the first law of thermodynamics on the event horizon holds and the Gibbs’s relation (42) can be used in this paper. (iii) For the $f(R)$ gravity, the form of the change of the total entropy is analytical, so it is convenient to discuss the generalized second law of thermodynamics if some physical quantities are given.

6 Conclusion

So far, the study of thermodynamics of the event horizon is rare while the research of thermodynamics of the apparent horizon has made great progress in the FRW universe. However, there exists an event horizon since the universe is in accelerated expansion. What is more, the concept of the event horizon of the universe is very similar to the horizon of the black hole whose thermodynamics have been accepted generally. Hence it is natural and important to study thermodynamics of the event horizon in the FRW universe. As far as we know, the difficulty of studying thermodynamics on the event horizon is the definition of the temperature. For example, in Ref. [16] the authors employed the temperature on the event horizon $T_E = 1/(2\pi R_E)$ whose form is similar to that of the apparent horizon and showed that the first law of thermodynamics on the event horizon is out of work. In Ref. [6], the author redefined the Hawking temperature but the Hawking temperature is not general, and his conclusions are only suitable for the flat spacetime and some models.

In order to solve these difficulties, we redefine the surface gravity and the corresponding Hawking temperature on the event horizon. Subsequently, we show the equivalency between the first law of thermodynamics on the event horizon and Friedmann equations of FRW universe with any spatial curvature in Einstein gravity. That is to say, the first law of thermodynamics on the event horizon holds in the FRW universe with any spatial curvature in Einstein gravity. This is a very important property which indicates the event horizon can be described by equilibrium thermodynamics.

Then, starting with the first law of thermodynamics on the event horizon, we obtain Friedmann equation including the quantum correction and show that the evolution of the universe is related to the event horizon. As an example, we present the evolution of the universe at present and get corresponding quantum corrected Friedmann equations which are consistent with the standard Friedmann equations under classical limit. Furthermore, we obtain Friedmann equations of the FRW universe with any spatial curvature in the $f(R)$ gravity based on the first law of thermodynamics. Subsequently, we explore the generalized second law of thermodynamics of the universe bounded by the event horizon and get these conditions which satisfy the generalized second law of thermodynamics in Einstein gravity and the $f(R)$ gravity. In summary, we conclude that the first law of thermodynamics on the event horizon has a general description in respect of the evolution of the FRW universe.

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