Research Article

A Novel Method for Fault Diagnosis of the Two-Input Two-Output Nonlinear Mass-Spring-Damper System Based on NOFRF and MBPCA

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For fault diagnosis of the two-input two-output mass-spring-damper system, a novel method based on the nonlinear output frequency response function (NOFRF) and multiblock principal component analysis (MBPCA) is proposed. The NOFRF is the extension of the frequency response function of the linear system to the nonlinear system, which can reflect the inherent characteristics of the nonlinear system. Therefore, the NOFRF is used to obtain the original fault feature data. In order to reduce the amount of feature data, a multiblock principal component analysis method is used for fault feature extraction. The least squares support vector machine (LSSVM) is used to construct a multifault classifier. A simplified LSSVM model is adopted to improve the training speed, and the conjugate gradient algorithm is used to reduce the required storage of LSSVM training. A fault diagnosis simulation experiment of a two-input two-output mass-spring-damper system is carried out. The results show that the proposed method has good diagnosis performance, and the training speed of the simplified LSSVM model is significantly higher than the traditional LSSVM.

1. Introduction

At present, fault diagnosis technologies have been widely used in manufacturing equipment, electric machine, wind power system, electronic equipment, and so on. With the increasing requirements of reliability and safety, the studies of fault diagnosis technology become more and more important [1–3]. The mass-spring-damper system is a classical vibration system, which can be used to describe many practical systems [4–7]. During operation, some faults will be occurred due to the influence of aging or external environment. Therefore, it is necessary to study the fault diagnosis of the mass-spring-damper system. In practical engineering, the multivariable nonlinear mass-spring-damper system can be used for modelling [8]. Under normal condition, the nonlinearity of the mass-spring-damper system is weak. When a fault occurred, the parameter or structure will be changed.

Volterra series is an important mathematical model for nonlinear systems. The frequency domain Volterra kernel is called the generalized frequency response function (GFRF), which is a direct generalization of the frequency response function of the linear system in the nonlinear system. The frequency characteristics of the nonlinear system can be described by GFRF [9–12]. The frequency characteristic information is obtained by a generalized frequency response function, which can be used for fault diagnosis of nonlinear systems [13]. The fault diagnosis of a permanent magnet synchronous motor is studied by using the generalized frequency response function and convolutional neural network [14]. For fault diagnosis of nonlinear analog circuit, GFRFs are used to obtain the feature data, and the LSSVM fusion method is used for fault identification [15]. The generalized frequency response function is a multidimensional function, and the computational complexity increases exponentially with the order. In order to reduce the
computational complexity, a nonlinear output frequency response function (NOFRF) is proposed based on GFRF [16]. The NOFRF is a one-dimensional function with less computational complexity. In [17], the stiffness and damper coefficients of the multidegree-of-freedom nonlinear system is obtained according to NOFRF. In [18], the transfer characteristics of NOFRFs of the multidegree-of-freedom system are analyzed. The fault diagnosis of the transmission system of numerical control equipment is studied by using NOFRF [19]. The concept of NOFRF is extended to MIMO nonlinear systems, and the characteristics of NOFRF are analyzed [20].

Principal component analysis (PCA) is a multivariate statistical analysis method, which can be used to extract feature data effectively [21–25]. In [26], the real-time incipient fault diagnosis for the electric drive system in high-speed train is studied based on deep principal component analysis. For degradation of sensor accuracy in the practical system, a hierarchical principal component analysis method based on dynamic fault differential characteristics is used for fault detection [27]. In [28], a distributed fault detection method based on fault-related variable selection and Bayesian reasoning is proposed. Multiblock principal component analysis (MBPCA) methods have been proposed for large-scale data compression and analysis [29–31]. MBPCA divides the data into different blocks according to the characteristics of the data and then conducts principal component analysis. In [32], the measured data of the chemical process are analyzed by MBPCA, and the fault is identified by a subblock contribution graph. In [33], the fault detection of semiconductor devices is studied by using multiblock principal component analysis, and the combination index is constructed by using SPE statistics and Hotelling’s $T^2$ statistics.

Support vector machine (SVM) is a typical machine learning method, which is widely used for fault diagnosis [34–37]. The training of the support vector machine is very complex. In [38], the least squares support vector machine is proposed by changing the risk function of the SVM. The training of LSSVM only needs to solve one linear equation, which is highly efficient. The least square support vector machine model is established to predict the degradation trend of the slewing bearing [39]. In [40], the intelligent location of high-speed train is studied based on LSSVM, and the iterative pruning error minimization and L-0 norm minimization algorithm are used to sparse LSSVM. In [41], S-transform is used for obtaining feature data from induced potential signal and particle swarm optimization LSSVM is used for identifying local demagnetization fault of a permanent magnet linear synchronous motor. An iterative algorithm based on conjugate gradient is used to train LSSVM, and the storage requirement is reduced [42]. In [43], a simplified model of the least squares support vector machine is proposed, which can reduce the computational complexity.

In this study, a fault diagnosis method is proposed for a nonlinear two-input two-output mass-spring-damper system based on nonlinear output frequency response function and multiblock principal component analysis. The nonlinear output frequency response function is used to establish the system model and obtain the original fault feature data. The features are extracted from the amplitudes of NOFRFs by multiblock kernel principal component analysis. A LSSVM multifault classifier is established to identify faults based on a simplified LSSVM model and conjugate gradient algorithm. A simulation experiment for a two-input two-output mass-spring-damper system is used to verify the effectiveness of the proposed fault diagnosis method.

### 2. NOFRF Estimation of the Two-Input Two-Output Nonlinear Mass-Spring-Damper System

A two-input two-output nonlinear mass-spring-damper system is shown in Figure 1.

The motion equation of the system is represented as

\[
\begin{align*}
\dot{y}_1(t) + c_1 \dot{y}_1(t) + c_2 (\dot{y}_1(t) - \dot{y}_2(t)) + k_1 y_1(t) + \\
k_2 (y_1(t) - y_2(t)) + b_1 y_1^2(t) + b_2 (y_1(t) - y_2(t))^2 &= u_1(t), \\
\dot{y}_2(t) + c_3 (\dot{y}_2(t) - \dot{y}_1(t)) + c_4 y_2(t) + k_3 (y_2(t) - y_1(t)) + \\
k_4 y_2(t) - b_2 (y_1(t) - y_2(t))^2 + b_3 y_2^2(t) &= u_2(t),
\end{align*}
\]

where $y_1, y_2$ are the outputs, $u_1, u_2$ are the inputs, and $m_1, m_2, c_1, c_2, c_3, k_1, k_2, k_3, b_1, b_2, b_3$ are the system parameters: mass, damper, linear stiffness, and nonlinear stiffness, respectively.
The mass-spring-damper system can be expressed as Volterra series:

\[ y_i(t) = \sum_{n=1}^{\infty} \sum_{p_1+p_2=n} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h^{(n)}(i,p_1,p_2)(\tau_1,\tau_2, \ldots, \tau_n) \times u_1(t-\tau_1), \ldots, u_i(t-\tau_{p_1}), \ldots, u_N(t-\tau_{p_1+p_2})d\tau_1, \ldots, d\tau_n, \]

(2)

where \( y_i(t) \) is the \( i \)-th output, \( h^{(n)}(i,p_1,p_2)(\tau_1,\tau_2, \ldots, \tau_n) \) is the \( n \)-th Volterra kernel, and \( N \) is the order of the nonlinear system, \( i = 1, 2 \).

The Fourier transform of the \( n \)-th Volterra kernel is expressed as

\[ H^{(n)}(i,p_1,p_2)(j\omega_1, \ldots, j\omega_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h^{(n)}(i,p_1,p_2)(\tau_1,\tau_2, \ldots, \tau_n)e^{-j(\omega_1\tau_1+\cdots+\omega_n\tau_n)}d\tau_1, \ldots, d\tau_n, \]

(3)

where \( H^{(n)}(i,p_1,p_2)(j\omega_1, \ldots, j\omega_n) \) is the \( n \)-th generalized frequency response function of the nonlinear mass-spring-damper system.

The output spectrum of the nonlinear mass-spring-damper system is described as

\[ Y_i(j\omega) = \sum_{n=1}^{\infty} Y_i^{(n)}(j\omega), \]

\[ Y_i^{(n)}(j\omega) = \frac{1}{(2\pi)^{N_0}} \sum_{N_1=1}^{N_n} \sum_{N_2=1}^{N_n} \cdots \sum_{N_{2k-1}=1}^{N_n} \int_{\alpha_1=0}^{2\pi} \cdots \int_{\alpha_{n-1}=0}^{2\pi} \int_{\alpha_{n-2}=0}^{2\pi} \cdots \int_{\alpha_{N_0-1}=0}^{2\pi} \cdots \int_{\alpha_{2k-1}=0}^{2\pi} \cdots d\alpha_{n-1} \cdots d\alpha_{N_0-1} \cdots d\alpha_{2k-1} \cdots Y_{i,j}(j\omega_k) d\sigma_{\alpha_{n-1}} \cdots d\sigma_{\alpha_{N_0-1}} \cdots d\sigma_{\alpha_{2k-1}} \cdots \]

(4)

where \( Y_i(j\omega) \) is the spectrum of the \( i \)-th output, \( Y_i^{(n)}(j\omega) \) is the \( n \)-th order output spectrum, \( U_j(j\omega) \) is the input spectrum, \( N_0 = 0 \), and \( i = 1, 2 \).

The generalized frequency response function is a multidimensional function, which requires a lot of calculation.

In order to reduce the computational complexity, the nonlinear output frequency response function of the multivariable system is proposed [20].

The \( n \)-th NOFRF of the two-input two-output mass-spring-damper system can be expressed as

\[ G^{(n)}(i,p_1=N_1,p_2=N_2)(j\omega) = \frac{Y^{(n)}(i,p_1=N_1,p_2=N_2)(j\omega)}{U^{(n)}(i,p_1=N_1,p_2=N_2)(j\omega)} \]

\[ = \int_{\alpha_1=0}^{2\pi} \cdots \int_{\alpha_{n-1}=0}^{2\pi} \int_{\alpha_{n-2}=0}^{2\pi} \cdots \int_{\alpha_{N_0-1}=0}^{2\pi} \cdots \int_{\alpha_{2k-1}=0}^{2\pi} \cdots \cdots d\alpha_{n-1} \cdots d\alpha_{N_0-1} \cdots d\alpha_{2k-1} \cdots \]

(5)
where \( U_{i,j}(\omega) = \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} \sqrt{n} \\ 2(\pi)^{n^{-1}} \end{pmatrix} \int_{t_{i-1}}^{t_i} a_1(t) a_n(t) e^{ij\omega t} dt \)

\[ \omega \prod_{j=1}^{2} \prod_{k=N_{j}+1}^{N_{j}+N_{j+1}} U_j(j\omega_k) \] is the Fourier transformation of \( u_i(t), \ldots, u_N(t) \) and \( u_j(t), \ldots, u_N(t) \).

When the first \( N \) order NOFRFs are used to describe the nonlinear mass-spring-damper system, the frequency domain output can be expressed as

\[ Y_i(j\omega) = \sum_{n=N_{i-1}+1}^{N_{i}} \sum_{n+N_{i}+1}^{N_{i+N_{i}}} G_{i,n}^{(n)}(j\omega) U_{i,n}^{(n)}(j\omega). \]

The relationship between input and output of the mass-spring-damper system is shown in Figure 2.

Sort \( G_{i,n}^{(n)}(j\omega) \) and \( U_{i,n}^{(n)}(j\omega) \) respectively:

\[ G_{i,n}^{(n)}(j\omega), \quad Y_i(j\omega), \quad U_{i,n}^{(n)}(j\omega), \quad U_{i,n}^{(n)}(j\omega), \]

\[ \omega = 1, 2, \ldots, L_{(m-1)}, \quad (m+1)1/m, m = 2. \]

Let \( G_{i,n}^{(n)}(j\omega) \) represents \( G_{i,n}^{(n)}(j\omega) \), and \( U_{i,n}^{(n)}(j\omega) \) represents \( U_{i,n}^{(n)}(j\omega) \). According to equations (7) and (8), equation (6) can be rewritten as

\[ Y_i(j\omega) = U_A G_i, \]

where \( Y_i(j\omega) = \begin{pmatrix} Y_{i,1}(j\omega) \\ Y_{i,2}(j\omega) \\ \vdots \\ Y_{i,N}(j\omega) \end{pmatrix}, \)

and \( U_A = \begin{pmatrix} a_1 U_1^{(1)}(j\omega) \\ a_1 U_1^{(1)}(j\omega) \\ \vdots \\ a_1 U_1^{(1)}(j\omega) \\ a_N U_1^{(N)}(j\omega) \\ \vdots \\ a_N U_1^{(N)}(j\omega) \\ a_N U_1^{(N)}(j\omega) \end{pmatrix}. \)

According to equation (10), the NOFRFs of the mass-spring-damper system can be obtained based on the least square principle:

\[ G_i = [U_A^T U_A]^{-1} U_A^T Y_i(j\omega). \]

The nonlinear output frequency response function is a one-dimensional function with low computational complexity. When a fault occurred, the nonlinear stiffness coefficient of the mass-spring-damper system will be increased, and the nonlinear output frequency response functions will be changed significantly. Therefore, the original fault feature data obtained by NOFRF can effectively diagnose the nonlinear mass-spring-damper system. In this study, the amplitudes of NOFRFs are selected for fault diagnosis.

### 3. Feature Extraction for NOFRF Based on MBPCA

The data amount of NOFRF amplitudes of the two-input two-output nonlinear mass-spring-damper system is large. In order to reduce the amount of feature data, feature extraction is needed. According to the number of system outputs, the system can be divided into two subsystems. In order to make the extracted NOFRF feature data more fully reflect the system characteristics, a multiblock principal component analysis method is used for feature extraction.

Under the normal state of the mass-spring-damper system, several groups of NOFRF amplitude data are obtained as samples. Divide the sample data into two blocks to obtain \( S = [A_1, A_2] \), where \( A_1 \) and \( A_2 \) are the NOFRF amplitude matrices of the two subsystems, respectively. The MBPCA method proposed by Westerhuis et al. [30] is used to extract feature data. In order to establish the MBPCA model of the mass-spring-damper system, the following optimization problems need to be solved.

\[ \min_{T_1, T_2} \sum_{l=1}^{L} \left\| T_1 W_l - T_2 P_l^T \right\|^2, \]

where \( T_1 = \sum_{l=1}^{L} T_1 W_l \) is the score matrix of the MBPCA model, \( T_2 \) is the score matrix of subblock, \( W_l \) is the weight matrix of subblock, and \( P_1 \) and \( P_2 \) are the load matrices obtained by the two subblocks.

The nonlinear iterative partial least squares method is used to solve the equation (12). Define \( t_{0l} \) as the first principal component vector of \( S \). Initialize \( t_{0l} \), so that \( \| t_{0l} \| = 1 \). Calculate the first load vector of the subblock, respectively:

\[ p_{1l} = (T_1)^T t_{0l}. \]

where \( l = 1, 2 \).

Normalize load vectors \( p_{1l} \) and \( p_{2l} \) to get \( p_{1l}^* \) and \( p_{2l}^* \). Then, calculate the first principal component vector of each block separately:

\[ t_{1l}^* = A_l p_{1l}^*. \]

According to \( t_{1l}^* \) and \( t_{2l}^* \), calculate the weight vector:

\[ w_{1l} = T_1^* t_{1l}^* \]

where \( w_{1l} = [w_{11}, w_{12}] \), and \( w_{11} \) and \( w_{12} \) respectively, represent the first principal component weight of the two subblock data.

Normalize the weight vector \( w_{1l} \), and calculate the principal component vector \( t_{1l}^* \).

\[ t_{1l} = T^* w_{1l}. \]

According to equations (13)–(16), the principal component vector \( t_{1l}^* \) is iteratively calculated until convergence,
and the weights $w_{11}, w_{21}$ and the load vectors $p_{11}, p_{21}$ are obtained.

Calculate the deviation of the estimated value of each subblock matrix from the original matrix separately:

$$E(l) = A_l - \tilde{A}_l,$$

where $\tilde{A}_l = t_{sl} \cdot p_{il}^T, l = 1, 2.$

According to the NOFRF amplitude deviation matrix $E(l)$ of each subblock, the second group weight $w_{2}'$ and the second group load vectors $p_{12}, p_{22}$ can be obtained by equations (13)-(16), and so on, until the $y^{th}$ weight vector and the $y^{th}$ group load vector are obtained, where $y$ represents the number of principal components. According to the weight vectors and the load vector, $P_1, P_2, W_1,$ and $W_2$ can be obtained.

For a group of NOFRF amplitude vectors, the fault feature vectors can be obtained by using the established MBPCA model. The schematic diagram for feature extraction of NOFRF based on multiblock principal component analysis is shown in Figure 3. In Figure 3, $a_i, a_i$ are the NOFRF amplitude vectors of the two subsystems, $P_1, P_2$ are the load matrices of the subblock, $t_1, t_2$ are the principal component vectors of the subsystem, $W_i = \text{diag}(w_{11}, w_{12}, \ldots, w_{14})$ is the weight matrix of the subblock, and $t_0$ is the extracted fault feature vector of the mass-spring-damper system.

4. Fault Identification Based on Simplified LSSVM

After the fault features are extracted by MBPCA, they are used to identify the fault of the mass-spring-damper system. The least square support vector machine is used to construct a multifault classifier.

Define the training sample dataset as $S_i: \{(x_i, y_i)\}_{i=1}^M,$ where $x_i \in R^d$ is the $i^{th}$ input vector, $y_i \in \{-1, 1\}$ is the $i^{th}$ category label, and $M$ is the sample size. The problem of the binary classification of LSSVM can be described as

$$\min \frac{1}{2} \omega^T \omega + \frac{1}{2} C \sum_{i=1}^M \xi_i^2,$$

s.t. $y_i[(\omega^T \varphi(x_i) + b) = 1 - \xi_i, i = 1, 2, \ldots, M,$

where $\omega$ is the weight vector of the classification hyperplane, $C > 0$ is the penalty factor, $\xi_i$ is the slack variable, $\varphi(\cdot)$ is the nonlinear mapping, and $b$ is the classification threshold.

Define the Lagrangian function as

$$L(\omega, b, \xi, \alpha_i) = \frac{1}{2} \omega^T \omega + \frac{1}{2} C \sum_{i=1}^M \xi_i^2 - \sum_{i=1}^M \alpha_i \left[ y_i(\omega^T \varphi(x_i) + b) - 1 + \xi_i \right],$$

where $\alpha_i \geq 0$ is the Lagrange multiplier.

Let the partial derivatives of the Lagrangian function $L(\omega, b, \xi, \alpha_i)$ with respect to $\omega, b, \xi_i$ and $\alpha_i$ be zero:

$$\begin{cases} \frac{\partial L}{\partial \omega} = 0 \Rightarrow \omega = \sum_{i=1}^M \alpha_i y_i \varphi(x_i), \\ \frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{i=1}^M \alpha_i y_i = 0, \\ \frac{\partial L}{\partial \xi_i} = 0 \Rightarrow \alpha_i = C \xi_i, \\ \frac{\partial L}{\partial \alpha_i} = 0 \Rightarrow y_i[(\omega^T \varphi(x_i) + b) - 1 + \xi_i] = 0. \end{cases}$$

(20)

By sorting out equation (20), the constrained optimization problem of LSSVM can be transformed into linear equations:

$$\begin{bmatrix} 0 & Y^T \end{bmatrix} \begin{bmatrix} b \\ Y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where $Y = [y_1, y_2, \ldots, y_M]^T, H = \Omega + C^{-1} \cdot I, \Omega$ is the $M$-dimensional symmetric square matrix, $\Omega_{i,j} = y_i y_j K(x_i, x_j), K(\cdot, \cdot)$ is the kernel function, $I$ is the $M$-dimensional identity matrix, $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_M]^T$ is the Lagrange multiplier vector, and $1 = [1, 1, \ldots, 1]^T$.

The decision function is

$$y(x) = \text{sign} \left( \sum_{i=1}^M \alpha_i y_i K(x, x_i) + b \right),$$

where $x$ is the sample vector to be classified.

It can be seen from equation (21) that the matrix on the left side of the equation is $M+1$ order square matrix. When $M$ is large, the matrix inversion operation needs a large amount of memory. In order to reduce the required storage, an iterative algorithm based on conjugate gradient can be used to train LSSVM [42].

The conjugate gradient algorithm is used to solve the following $M$-variable linear equations:

$$H \eta = Y,$$

$$H \nu = 1.$$

(23)

According to $\eta$ and $\nu$, calculate the classification threshold $b$ and the Lagrangian multiplier vector $\alpha$. 


\[
\begin{align*}
b &= \left( \eta^T I \right) \eta^T Y, \\
\alpha &= v - b \eta.
\end{align*}
\] 

(24)

There are two \( M \)-variable linear equations that need to be solved when using the traditional LSSVM model and conjugate gradient algorithm to train the LSSVM binary classifier. In order to reduce the computational complexity, Li et al. \cite{43} proposed a simplified LSSVM model. In order to improve the training speed and reduce the storage requirement, the LSSVM multifault classifier is trained by the simplified LSSVM model and conjugate gradient algorithm in this study.

The structure of the LSSVM multifault classifier is “one against one.” For the LSSVM multifault classifier, the training sample set of the \( k \)th subclassifier is defined as \( G_k = \{ g_i, y_i \}, i = 1, 2, \ldots, M_k \), where \( g_i \in \mathbb{R}^p \) is the NOFRF feature vector, \( y_i \in \{-1, 1\} \) is the category label, and \( M_k \) is the sample size.

\[
\begin{bmatrix}
0 \\
Y_k^T \\
H_k
\end{bmatrix}
\begin{bmatrix}
b_k \\
\alpha_k
\end{bmatrix}
= \begin{bmatrix}
0 \\
1
\end{bmatrix},
\]

(25)

where \( Y_k = [y_1, y_2, \ldots, y_{M_k}]^T \), \( H_k = \Omega_k + \eta_k \cdot I_k \), \( \Omega_k \) is the \( M_k \)-dimensional symmetric square matrix, \( \Omega_{ij} = y_i y_j K(x_i, x_j) \), \( K(\cdot, \cdot) \) is the kernel function, \( \eta_k \) is the penalty factor, \( I_k \) is the \( M_k \)-dimensional identity matrix, \( b_k \) is the classification threshold, \( \alpha_k = [\alpha_1, \alpha_2, \ldots, \alpha_{M_k}] \) is the Lagrange multiplier vector, and \( 1 = [1, 1, \ldots, 1]^T \).

The matrix \( H_k \) can be written as

\[
H_k = \begin{bmatrix}
H_k^{M_k-1} & b_k \\
H_k & H_{M_k, M_k}
\end{bmatrix},
\]

(26)

where \( H_k^{M_k-1} \) is the \((M_k - 1) \times (M_k - 1)\) principal square submatrix of \( H_k \), \( b_k \) is the \( M_k - 1 \) dimensional vector formed after the last element is removed from the \( M_k \)-th vector of \( H_k \), \( H_{M_k, M_k} \) is the element in row \( M_k \) and column \( M_k \) of \( H_k \).

Define

\[
\tilde{H}_k = H_k^{M_k-1} - y_{M_k} Y_k^{M_k-1} h_k^T - y_{M_k} h_k (Y_k^{M_k-1})^T + H_{M_k, M_k} Y_k^{M_k-1} (Y_k^{M_k-1})^T,
\]

(27)

\[
Y_k^{M_k-1} = [y_1, y_2, \ldots, y_{M_k}]^T,
\]

(28)

\[
\alpha_k^{M_k-1} = [\alpha_1, \alpha_2, \ldots, \alpha_{M_k}]^T.
\]

(29)

According to equations (25)–(29), the LSSVM simplified model is given by

\[
\begin{cases}
\tilde{H}_k \alpha_k^{M_k-1} = 1 - y_{M_k} Y_k^{M_k-1}, \\
\alpha_k^{(M_k-1)} = -y_{M_k} (Y_k^{M_k-1})^T \alpha_k^{(M_k-1)}, \\
b_k = y_{M_k} - y_{M_k} (H_k \alpha_k^{M_k-1})
\end{cases}
\]

(30)

where \( (H_k \alpha_k^{M_k-1})_{M_k} \) is the \( M_k \)-th element of \( H_k \alpha_k^{M_k-1} \).

The subclassifier of the LSSVM multifault classifier is trained according to equation (30). First, the conjugate gradient algorithm is used to solve the linear equation \( H_k \alpha_k^{M_k-1} = 1 - y_{M_k} Y_k^{M_k-1} \). Then, the Lagrangian multiplier \( \alpha_k^{M_k} \) and classification threshold \( b_k \) can be calculated by \( \alpha_k^{(M_k-1)} = -y_{M_k} (Y_k^{M_k-1})^T \alpha_k^{(M_k-1)} \) and \( b_k = y_{M_k} - y_{M_k} (H_k \alpha_k^{M_k-1}) \).

When the traditional LSSVM is used to train the \( k \)-th LSSVM subclassifier of the multifault classifier, there are two \( M \)-variable linear equations that need to be solved. The main amount of calculation is \( O(3(1 + k_1)M_k^2) \), where \( k_1 \) and \( k_2 \) represent the number of iterations for solving \( H_k \eta_k = Y_k \) and \( H_k \alpha_k = 1 \), respectively. When the LSSVM simplified model is used to train the \( k \)-th LSSVM subclassifier of the multifault classifier, there is one \( M \)-1-variable linear equation that needs to be solved. The main amount of calculation is \( O(3(k_1 + 2)M_k^2) \), where \( k_1 \) represents the number of iteration for solving \( H_k \alpha_k^{(M_k-1)} = 1 - y_{M_k} Y_k^{(M_k-1)} \). Generally, \( k_1 + k_2 \gg k_1 \), so the computational complexity of training the LSSVM multifault classifier is significantly reduced.

The schematic diagram of fault diagnosis for the mass-spring-damper system based on NOFRF and MBPCA is shown in Figure 4. First, the input spectrum and output
spectrum data are obtained by Fourier transform of time domain data, and then, the NOFRFs are estimated by the least square estimation algorithm. Second, the MBPCA is used to extract fault features. Finally, the LSSVM multifault classifier is used for fault identification.

5. Simulation Experiment

The fault diagnosis simulation experiment of a two-input two-output nonlinear mass-spring-damper system is carried out. The nonlinear equations of the system are given by: 

\[
\begin{align*}
\dot{u}_1 &= P_1 u_1 + P_2 u_2 + k_1 b_1 \\
\dot{u}_2 &= P_1 u_2 + P_2 u_1 + k_2 b_2 \\
\dot{k}_1 &= k_3 b_3 + k_1 b_1 \\
\dot{k}_2 &= k_3 b_3 + k_2 b_2 \\
\dot{b}_1 &= k_3 b_3 + b_1 \\
\dot{b}_2 &= k_3 b_3 + b_2 \\
\end{align*}
\]
Figure 5: The outputs of the nonlinear mass-spring-damper system. (a) The first output $y_1(t)$. (b) The second output $y_2(t)$.

Figure 6: The amplitudes of the first-order NOFRFs.

Figure 7: The amplitudes of the second-order NOFRFs.
Figure 8: The amplitudes of the third-order NOFRFs.

Figure 9: The amplitudes of the fourth-order NOFRFs.
Table 1: The fault description of the mass-spring-damper system.

| Fault number | Fault mode        | Parameters                                                                 |
|--------------|-------------------|----------------------------------------------------------------------------|
| F0           | Normal            | $c_1, c_2, c_3 \in [19.5, 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4, 1.05 \times 10^4]$ $b_1, b_2, b_3 \in [0.95 \times 10^{07}, 1.05 \times 10^{07}]$ |
| F1           | $b_1$ increased   | $c_1, c_2, c_3 \in [19.5, 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4, 1.05 \times 10^4]$ $b_1 \in [1.45 \times 10^0, 1.55 \times 10^0]$ |
| F2           | $b_2$ decreased   | $c_1, c_2, c_3 \in [19.5, 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4, 1.05 \times 10^4]$ $b_1, b_3 \in [0.95 \times 10^{07}, 1.05 \times 10^{07}]$ |
| F3           | $b_3$ increased   | $c_1, c_2, c_3 \in [19.5, 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4, 1.05 \times 10^4]$ $b_1, b_2 \in [0.95 \times 10^{07}, 1.05 \times 10^{07}]$ |
| F4           | $b_1$ and $b_2$ increased | $c_1, c_2, c_3 \in [19.5, 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4, 1.05 \times 10^4]$ $b_1, b_2 \in [1.45 \times 10^0, 1.55 \times 10^0]$ |

Figure 10: The distributions of the first and second principal components.

Figure 11: The distributions of the third and fourth principal components.
The simulation has been performed using MATLAB R2014a. The CPU clock speed of the computer is 2.3 GHz, and the main memory is 8 GB. Let the input signal be $R_{2014a}$. The CPU clock speed of the computer is 2.3 GHz, and the main memory is 8 GB. Let the input signal be $R_{2014a}$.

The Monte Carlo method is used for fault diagnosis simulation experiment of the mass-spring-damper system. Assume that under normal conditions, the variation ranges of linear stiffness and nonlinear stiffness are within 5%, and the variation range of damper is within 2.5%. When the system fails, the nonlinear characteristics will increase. Five kinds of faults of the mass-spring-damper system are defined, and the fault description is given in Table 1.

Table 2: The recognition rate with different kernel functions.

| Fault number | Fault mode     | Linear kernel (%) | Polynomial kernel (%) | GRB kernel (%) | ERB kernel (%) | MLP kernel (%) |
|--------------|----------------|-------------------|-----------------------|---------------|---------------|---------------|
| F0           | Normal         | 96                | 97                    | 100           | 95            | 97            |
| F1           | $b_1$ increased| 87                | 87                    | 88            | 85            | 82            |
| F2           | $b_2$ decreased| 78                | 94                    | 92            | 95            | 82            |
| F3           | $b_1$ increased| 85                | 97                    | 98            | 93            | 85            |
| F4           | $b_1$ and $b_2$ increased| 81            | 85                    | 91            | 90            | 86            |

Table 3: The training time.

| Kernel function | Traditional LSSVM model (s) | LSSVM simplified model (s) |
|-----------------|-----------------------------|----------------------------|
| Linear kernel   | 0.63                        | 0.56                       |
| Polynomial kernel | 0.67                      | 0.62                       |
| GRB kernel      | 0.78                        | 0.66                       |
| ERB kernel      | 0.90                        | 0.69                       |
| MLP kernel      | 1.28                        | 0.98                       |

\[
\begin{align*}
\dot{y}_1(t) + 20\dot{y}_1(t) + 20(\dot{y}_1(t) - \dot{y}_2(t)) + 10^4 y_1(t) + \\
10^4(y_1(t) - y_2(t)) + 10^7 y_2(t) + 10^7(y_1(t) - y_2(t))^2 = u_1(t), \\
\dot{y}_2(t) + 20(\dot{y}_2(t) - \dot{y}_1(t)) + 20\dot{y}_2(t) + 10^4(y_2(t) - y_1(t)) \\
+ 10^4 y_2(t) - 10^7(y_1(t) - y_2(t))^2 + 10^7 y_2^2(t) = u_2(t). 
\end{align*}
\]

The simulation has been performed using MATLAB R2014a. The CPU clock speed of the computer is 2.3 GHz, and the main memory is 8 GB. Let the input signal be $R_{2014a}$. The CPU clock speed of the computer is 2.3 GHz, and the main memory is 8 GB. Let the input signal be $R_{2014a}$.

The Monte Carlo method is used for fault diagnosis simulation experiment of the mass-spring-damper system. Assume that under normal conditions, the variation ranges of linear stiffness and nonlinear stiffness are within 5%, and the variation range of damper is within 2.5%. When the system fails, the nonlinear characteristics will increase. Five kinds of faults of the mass-spring-damper system are defined, and the fault description is given in Table 1.

200 sets of input and output samples of the mass-spring-damper system are collected for each fault mode. The NOFRFs of the system are obtained by equation (11). After obtaining the amplitudes of NOFRFs, the features are extracted by MBPCA. The principal component distributions of the five fault modes are shown in Figures 10 and 11, where PC1 is the first principal component, PC2 is the second principal component, PC3 is the third principal component, and PC4 is the fourth principal component.

For each fault mode, select 100 sets of feature data of NOFRF as training samples and the remaining 100 sets as test samples. The fault diagnosis simulation of the mass-spring-damper system is carried out by the LSSVM simplified model and the traditional LSSVM model based on the conjugate gradient algorithm, respectively. The linear kernel function, polynomial kernel function, Gaussian radial basis (GRB) kernel function, exponential radial basis (ERB) kernel function [44], and multilayered perceptron (MLP) kernel function [45] are chosen as kernel functions of the LSSVM multifault classifier, respectively.

The fault recognition rates with different kernel functions are given in Table 2, and the training time is given in Table 3. As can be seen from Table 2, the LSSVM based on the GRB kernel function has the best result for fault identification. As can be seen from Table 3, the training time of the multifault classifier based on the simplified LSSVM model, respectively, is 0.56 s, 0.62 s, 0.66 s, 0.69 s, and 0.98 s, while that based on the traditional LSSVM model, respectively, is 0.63 s, 0.67 s, 0.78 s, 0.90 s, and 1.28 s. The training time of the multifault classifier based on the simplified model is significantly reduced. Therefore, the proposed diagnosis
method for the two-input two-output mass-spring-damper system has good diagnostic performance and fast training speed.

6. Conclusions

In this work, we studied the fault diagnosis of the nonlinear two-input two-output mass-spring-damper system combining NOFRF and MBPCA. In order to obtain the original feature data which can fully reflect the system information, the NOFRFs of the mass-spring-damper system are used to obtain original fault feature data. To reduce the number of feature variables, the multiblock kernel principal component analysis method is used for feature extraction. Based on a simplified LSSVM model and conjugate gradient algorithm, a multifault classifier is constructed for fault identification, which improves the training speed and reduces the storage requirement. A fault diagnosis simulation experiment of a nonlinear two-input two-output mass-spring-damper system is used to verify the effectiveness of the proposed method. The results demonstrate that the performance of the proposed method is good, and the training speed of the multifault classifier is fast.

Due to the serious disturbance in practical engineering, the fault diagnosis accuracy of will be affected. Therefore, the identification of NOFRF and the design of the LSSVM multifault classifier will be deeply studied further to improve the estimation accuracy and fault diagnosis rate.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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References

[1] Z. Gao, C. Cecati, and S. X. Ding, “A survey of fault diagnosis and fault-tolerant techniques-part I: fault diagnosis with model-based and signal-based approaches,” IEEE Transactions on Industrial Electronics, vol. 62, no. 6, pp. 3757–3767, 2015.
[2] Z. W. Gao, C. Cecati, and S. X. Ding, “A survey of fault diagnosis and fault-tolerant techniques-part II: fault diagnosis with knowledge-based and hybrid/active approaches,” IEEE Transactions on Industrial Electronics, vol. 62, no. 6, pp. 3768–3774, 2015.
[3] Z. W. Gao and X. X. Liu, “An overview on fault diagnosis, prognosis and resilient control for wind turbine systems,” Processes, vol. 9, no. 2, p. 300, 2021.
[4] D. Wang and C. Mu, “Adaptive-critic-based robust trajectory tracking of uncertain dynamics and its application to a spring-mass-damper system,” IEEE Transactions on Industrial Electronics, vol. 65, no. 1, pp. 654–663, 2018.
[5] Z. Chen, L. Zhang, and R. W. Yeung, “Analysis and optimization of a dual mass-spring-damper (DMSD) wave-energy converter with variable resonance capability,” Renewable Energy, vol. 131, pp. 1060–1072, 2019.
[6] Y. Yu, B. Ackmese, and M. Mesbah, “Mass-spring-damper networks for distributed optimization in non-Euclidean spaces,” Automatica, vol. 112, Article ID 108703, 2020.
[7] B. Janecek, V. Kracik, J. Skiba et al., “Simple friction model of the guiding device of a mechanical system: mass, spring and damper,” Journal of Vibroengineering, vol. 13, no. 4, pp. 891–899, 2011.
[8] C. Zeng, S. Liang, Y. S. Sun et al., “Discrete dynamics analysis for nonlinear collocated multivariable mass-spring-damper intelligent mechanical vibration systems,” Journal of Vibroengineering, vol. 16, no. 2, pp. 633–644, 2014.
[9] M. Alizadeh, S. Amin, and D. Rönnow, “Measurement and analysis of frequency-domain Volterra kernels of nonlinear dynamic 3×3 MIMO systems,” IEEE Transactions on Instrumentation and Measurement, vol. 66, no. 7, pp. 1893–1905, 2017.
[10] K. Worden, W. E. Becker, T. J. Rogers, and E. J. Cross, “On the confidence bounds of Gaussian process NARX models and their higher-order frequency response functions,” Mechanical Systems and Signal Processing, vol. 104, pp. 188–223, 2018.
[11] K. Worden, G. Manson, and G. R. Tomlinson, “A harmonic probing algorithm for the multi-input Volterra series,” Journal of Sound and Vibration, vol. 201, no. 1, pp. 67–84, 1997.
[12] R. M. Lin and T. Y. Ng, “A new method for the accurate measurement of higher-order frequency response functions of nonlinear structural systems,” ISA Transactions, vol. 81, pp. 270–285, 2018.
[13] H. Tang, Y. H. Liao, J. Y. Cao, and H. Xie, “Fault diagnosis approach based on Volterra models,” Mechanical Systems and Signal Processing, vol. 24, no. 4, pp. 1099–1113, 2010.
[14] L. R. Chen, Z. R. Zhang, and J. F. Cao, “A novel method of combining generalized frequency response function and convolutional neural network for complex system fault diagnosis,” PLoS One, vol. 15, no. 2, Article ID e0228324, 2020.
[15] J. L. Zhang, “Fault diagnosis of nonlinear analog circuit based on generalized frequency response function and LSSVM classifier fusion,” Mathematical Problems in Engineering, vol. 2020, Article ID 8274570, 11 pages, 2020.
[16] Z. Q. Lang and S. A. Billings, “Energy transfer properties of non-linear systems in the frequency domain,” International Journal of Control, vol. 78, no. 5, pp. 345–362, 2005.
[17] Z. K. Peng, Z. Q. Lang, and S. A. Billings, “Linear parameter estimation for multi-degree-of-freedom nonlinear systems using nonlinear output frequency-response functions,” Mechanical Systems and Signal Processing, vol. 21, no. 8, pp. 3108–3122, 2007.
[18] Z. Q. Lang, G. Park, C. R. Farra et al., “Transmissibility of nonlinear output frequency response functions with application in detection and location of damage in MDOF structural systems,” International Journal of Non-linear Mechanics, vol. 46, no. 6, pp. 841–853, 2011.
[19] J. Cao, L. Chen, J. Zhang, and W. Cao, “Fault diagnosis of complex system based on nonlinear frequency spectrum fusion,” Measurement, vol. 46, no. 1, pp. 125–131, 2013.
[20] Z. K. Peng, Z. Q. Lang, and S. A. Billings, “Non-linear output frequency response functions for multi-input non-linear
volterra systems,” *International Journal of Control*, vol. 80, no. 6, pp. 843–855, 2007.
[21] T. Z. Wang, H. Xu, J. G. Han, E. Elbouchikhi, and M. E. H. Benbouzid, “Cascaded H-bridge multilevel inverter system fault diagnosis using a PCA and multiclass relevance vector machine approach,” *IEEE Transactions on Power Electronics*, vol. 30, no. 12, pp. 7086–7018, 2015.
[22] G. Georgoulas, M. O. Mustafa, I. P. Tsoumas et al., “Principal component analysis of the start-up transient and hidden Markov modeling for broken rotor bar fault diagnosis in asynchronous machines,” *Expert Systems with Applications*, vol. 40, no. 17, pp. 7024–7033, 2013.
[23] O. Solomon, R. Cohen, Y. Zhang et al., “Deep unfolded robust PCA with application to clutter suppression in ultrasound,” *IEEE Transactions on Medical Imaging*, vol. 39, no. 4, pp. 1051–1063, 2020.
[24] T. Loutas, N. Eleftheroglou, G. Georgoulas, P. Loukopoulos, D. Mba, and I. Bennett, “Valve failure prognostics in reciprocating compressors utilizing temperature measurements, PCA-based data fusion, and probabilistic algorithms,” *IEEE Transactions on Industrial Electronics*, vol. 67, no. 6, pp. 5022–5029, 2020.
[25] X. L. Li, X. Zhang, Z. Zhang et al., “Fault data detection of traffic detector based on wavelet packet in the residual sub-space associated with PCA,” *Applied Sciences-Basel*, vol. 9, no. 17, p. 3494, 2019.
[26] H. T. Chen, B. Jiang, N. Y. Lu, and Z. Mao, “Deep PCA based real-time incipient fault detection and diagnosis methodology for electrical drive in high-speed trains,” *IEEE Transactions on Vehicular Technology*, vol. 67, no. 6, pp. 4819–4830, 2018.
[27] F. N. Zhou, J. H. Park, and Y. J. Liu, “Differential feature based hierarchical PCA fault detection method for dynamic fault,” *Neurocomputing*, vol. 202, pp. 27–35, 2016.
[28] Q. C. Jiang, X. F. Yan, and B. Huang, “Performance-driven distributed PCA process monitoring based on fault-relevant variable selection and Bayesian inference,” *IEEE Transactions on Industrial Electronics*, vol. 63, no. 1, pp. 377–386, 2016.
[29] Z. Q. Ge, “Improved two-level monitoring system for plant-wide processes,” *Chemometrics and Intelligent Laboratory Systems*, vol. 132, pp. 141–151, 2014.
[30] J. A. Westerhuis, T. Kourt, and J. F. Macgregor, “Analysis of multiblock and hierarchical PCA and PLS models,” *Journal of Chemometrics*, vol. 12, no. 5, pp. 301–321, 1998.
[31] A. K. Smilde, J. A. Westerhuis, and S. de Jong, “A framework for sequential multiblock component methods,” *Journal of Chemometrics*, vol. 17, no. 6, pp. 323–337, 2003.
[32] L. B. Bie and X. D. Wang, “Fault detection and diagnosis of continuous process based on multiblock principal component analysis,” in *Proceedings of the International Conference on Computer Engineering and Technology*, pp. 200–204, Singapore, January 2009.
[33] G. A. Cherry and S. J. Qin, “Multiblock principal component analysis based on a combined index for semiconductor fault detection and diagnosis,” *IEEE Transactions on Semiconductor Manufacturing*, vol. 19, no. 2, pp. 159–172, 2006.
[34] S. Ben Salem, K. Bacha, and A. Chaari, “Support vector machine based decision for mechanical fault condition monitoring in induction motor using an advanced Hilbert-Park transform,” *ISA Transactions*, vol. 51, no. 5, pp. 566–572, 2012.
[35] J. Huang, X. Hu, and F. Yang, “Support vector machine with genetic algorithm for machinery fault diagnosis of high voltage circuit breaker,” *Measurement*, vol. 44, no. 6, pp. 1018–1027, 2011.