Thermodynamics of Expansive Nondecelerative Universe

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Abstract

The present contribution deals with thermodynamic aspects of the model of Expansive Nondecelerative Universe. In this model, in the matter era a dependence \( T_{CBR} \approx E_{CBR} \approx a^{-3/4} \) holds for the energy of cosmic background radiation, \( E_{CBR} \) and its temperature, \( T_{CBR} \), while the proportionality of the energy density \( \varepsilon_{CBR} \) to the gauge factor \( a \) can be expressed as \( \varepsilon_{CBR} \approx a^{-3} \). The given relationships comply with experimental observations of the cosmic background radiation as well as with a surprising finding that the Universe expansion is not decelerated by gravitational forces. It is rationalized that the specific entropy is proportional to \( a^{-1/4} \), i.e. it is gradually decreasing in time.

1 Introduction

In the model of Expansive Nondecelerative Universe (hereinafter ENU) it holds

\[
a = c.t_c = \frac{2G.m_U}{c^2}
\]  

(1)

where \( a \) is the gauge factor, \( t_c \) is the cosmological time, and \( m_U \) is the mass of the Universe. The present-time values provided by calculations based on ENU do not differ substantially from the generally accepted values and are as follows: \( a_{pt} \approx 1.3 \times 10^{26} \) m, \( t_{c(pt)} \approx 1.4 \times 10^{10} \) yr, \( m_{U(pt)} \approx 8.6 \times 10^{52} \) kg. It stems from (1) that the Universe mass is time-increasing. Since the mass-energy of the Universe must be time-independent (and equal to zero), simultaneously with the matter creation also an equivalent amount of gravitational field energy is formed which is, however, negative. This is why
the Universe can expand with a constant velocity equal to the velocity of light $c$ in ENU. In ENU, Schwarzschild metric is replaced by Vaidya metric $^3$ originally elaborated to solve the problems of radiating stars, latter shown by Virbhadra $^4, 5$ and by us $^6$ that its application is more general.

It is postulated in ENU that the energy density of the Universe is just critical and it is expressed as

$$\varepsilon_{\text{crit}} = \frac{3c^4}{8\pi G a^2}$$ \hspace{1cm} (2)

Till the end of the radiation era, there was a thermodynamic equilibrium of matter and radiation, and energy, temperature and gauge factor were related as follows

$$E_{\text{CBR}} \approx T_{\text{CBR}} \approx a^{-1/2}$$ \hspace{1cm} (3)

The fact that the energy density in (2) is proportional to $a^{-2}$ and not to $a^{-3}$ can be rationalized by matter creation. Thus, ENU describes the Universe in which eternal inflation occurs. In classical inflationary models of universe, after completing its inflation stage the Universe should decelerate due to effects of gravitational forces. As a consequence, in the models of inflationary universe a new matter incessantly emerges from behind the event horizon and in this way the proportionality $\varepsilon \approx a^{-2}$ stated by (2) is explained.

In the ENU model relation (2) holds permanently, i.e. also in the matter era and the energy density matches well with the accepted gauge factor value. Detailed and precise observations performed in the last few years have led to a conclusion that the predicted decrease in the Universe expansion rate has not occurred. Contrary, a nonzero value of the cosmological constant $\Lambda$ or a newly elaborated quintessential model $^7$ gave rise to a presumption stated that the Universe expansion accelerates and that such an acceleration started at the beginning of the matter era. The postulate on the Universe acceleration leads, however, directly to a very important conclusion concerning impossibility of relation (2) validity in the matter era and also to a conclusion on impossibility of critical energy density preserving.

2 Thermodynamics of ENU

It is generally accepted that the radiation era ended approximately in the time

$$t_r \approx 7 \times 10^5 \text{ yr}$$ \hspace{1cm} (4)

when the temperature of radiation approached to

$$T_r \approx 5 \times 10^3 \text{ K}$$ \hspace{1cm} (5)
(In the contribution, the subscripts \(p_t\), \(r\) and \(m\) refer to the present-time, the end of radiation era, and matter era, respectively). The present-time temperature is

\[
T_{p_t} \approx 2.735 \text{ K}
\]  

(6)

Taking into account that the Universe expansion did not decelerate in the matter era, a presumption emerges stating that not only event horizon but also original part of the Universe extended in about four orders

\[
\frac{t_{p_t}}{t_r} = \frac{a_{p_t}}{a_r} \approx 10^4
\]

(7)

Based on the fact that from the end of the radiation era the of cosmic background radiation temperature has decreased by three orders but the gauge factor increased by four orders, a relation between the energy of cosmic background radiation \(E_{CBR}\) and its temperature \(T_{CBR}\) follows

\[
T_{CBR} \approx E_{CBR} \approx a^{3/4}
\]

(8)

Introducing the value of \(a_{p_t}\) into calculation of the present-time critical energy density of the Universe it follows that

\[
\varepsilon_{crit(p_t)} \approx 8.577 \times 10^{-10} \text{ J/m}^3
\]

(9)

The energy density of radiation can be extracted from Stefan-Boltzmann law and for cosmic background radiation is generally given as

\[
\varepsilon_{CBR} = \frac{4 \sigma T^4}{c}
\]

(10)

Based on (6) and (10) the present-time energy density value of cosmic background radiation reaches

\[
\varepsilon_{CBR(p_t)} \approx 4.229 \times 10^{-14} \text{ J/m}^3
\]

(11)

It follows from (8) and (10) that during the matter era

\[
\varepsilon_{CBR(m)} \approx a^{-3}
\]

(12)

and at the same time, stemming from (2), (8), (10), (11) and (12) it follows

\[
\frac{a_{p_t}}{a_r} = \frac{\varepsilon_{crit(p_t)}}{\varepsilon_{CBR(p_t)}}
\]

(13)

Based on (13) the gauge factor value at the end of the radiation era can be calculated

\[
a_r \approx 6.41 \times 10^{21} \text{ m}
\]

(14)
together with the cosmological time value at the same time
\[ t_r \cong 6.6 \times 10^5 \text{ yr} \] (15)

Treatment of relations (8), (9), and (14) leads to
\[ \frac{T_{pt}}{T_r} = \left( \frac{a_r}{a_{pt}} \right)^{3/4} \] (16)

Stemming from (16) the temperature at the end of the radiation era is directly calculable and it reaches the value of
\[ T_e \cong 4650 \text{ K} \] (17)
being in excellent agreement with the generally accepted value obtained using other independent modes of calculations.

3 Specific Entropy

Total average number of relict photons \( n(h\nu)_m \) in a cubic meter during the matter era relates to the gauge factor according to (5) and (12) as follows
\[ n(h\nu)_m \approx a^{-9/4} \] (18)
and that of protons \( n(p)_m \) (representing the matter particles) based on (2) as
\[ n(p)_m \approx a^{-2} \] (19)

Dependence of the specific entropy \( S \), defined as a number of relict photons per one proton, on the gauge factor is in the matter era expressed as
\[ S \approx a^{-1/4} \] (20)

At the time being, the temperature of cosmic background radiation (2.735 K) leads to the following number of relict photons in a volume unit
\[ n(h\nu)_{pt} = \frac{\varepsilon_{CBR(pt)}}{E_{CBR(pt)}} \cong 4 \times 10^8 \] (21)
where \( E_{CBR(pt)} \) is the mean energy of actual relict photons.

Given the present time energy density (4) and gauge factor values, a number of protons in a volume unit reaches
\[ n(p)_{pt} \cong 5 \] (22)
The present-time specific entropy calculated as a ratio of values provided in (21) and (22) is of the order

$$S_{pt} \approx 10^8$$  \hspace{1cm} (23)

At the end of the radiation era, the specific entropy value approached

$$S_r \approx 10^9$$  \hspace{1cm} (24)

Comparison of (23) and (24) verifies the correctness of (20), i.e. a slow decrease in specific entropy with time.

Within a discussion on a time-dependence of specific entropy some contradictions emerge. If the specific entropy is constant, i.e. if relation (24) is valid at the present-time number of relict photons and gauge factor, the Universe density would have to have subcritical value. The assumption of permanent critical (nearly critical) density, however, excludes a constant value of specific entropy. The majority of current cosmological models take, however, critical mass-energy density a priori into account and tries to solve this discrepancy introducing some “exotic” nonbaryonic forms of matter.

## 4 Thermodynamics and Gravitation

In the period of time starting with the Universe expansion and finishing with the end of the radiation era, energy densities given by (2) and (10) are identical which corresponds to (3). It must therefore hold for the mean energy value of the photons of cosmic background radiation

$$E_{CBR} = E_{Pc} \left( \frac{l_{Pc}}{a} \right)^{1/2} = \left( \frac{\hbar^3 c^7}{G a^2} \right)^{1/4}$$  \hspace{1cm} (25)

where $l_{Pc}$ and $E_{Pc}$ are the Planck length and Planck energy [8], respectively

$$l_{Pc} = \left( \frac{G \hbar}{c^3} \right)^{1/2} = 1.606151 \times 10^{-35} \text{ m}$$  \hspace{1cm} (26)

$$E_{Pc} = \left( \frac{\hbar c^5}{G} \right)^{1/2} = 1.2211 \times 10^{19}$$  \hspace{1cm} (27)

Since proportionality (8) holds in the matter era, the mean energy value of the photons of relict radiation is expressed as

$$E_{CBR} = \left( \frac{\hbar^3 c^7 a_r}{G a^3} \right)^{1/4}$$  \hspace{1cm} (28)
The ENU model allows to localize gravitational field energy. The wave function of gravitational field is described \[9\] as

\[ \Psi_g = \exp \left[ i \cdot t \left( \frac{m \cdot c^5}{\hbar \cdot a \cdot r^2} \right)^{1/4} \right] \]  

(29)

where \( \Psi_g \) is the wave function of gravitational field quanta created by a body with the mass \( m \) at the distance \( r \). The mentioned thermodynamic equilibrium in the radiation era means that the total mass of the relict radiation is equal to the total mass of matter particles. This is why this mass can be expressed as

\[ m \approx \frac{a \cdot c^2}{G} \]  

(30)

When taking the Universe as one system into account, the following general equation must always hold

\[ r = a \]  

(31)

Introducing (30) and (31) into (29), we obtain the Universe wave function in the form \[9\]

\[ \Psi_g = \exp \left[ i \cdot t \left( t_{PC} \cdot t_c \right)^{-1/2} \right] \]  

(32)

Here we can formulate a hypothesis stating that the mean energy of a photon of relict radiation is modulated by the energy of gravitational quanta \( E_g \) and we will intent to justify this hypothesis and prove its correctness. In such a case the absolute values of the corresponding mean energies must be identical, i.e.

\[ |E_g| = E_{CBR} \]  

(33)

Writing a Schrödinger-like equation for the energy of gravitational quanta \( E_g \)

\[ E_g \cdot \Psi_g = i \cdot \hbar \frac{d\Psi_g}{dt} \]  

(34)

it comes from (32) and (34) that

\[ |E_g| = \frac{\hbar}{(t_{PC} \cdot t_c)^{1/2}} = \left( \frac{\hbar^3 \cdot c^7}{G \cdot a^2} \right)^{1/4} \]  

(35)

which is a relation identical to that of (25).

Situation is quite different in the matter era. Stemming from the validity of (12) for the total mass of the photons of relict radiation, \( m_{CBR(total)} \) it must hold

\[ m_{CBR(total)} \approx V \cdot a^{-3} = \text{const.} \]  

(36)
At the same time
\[ m_{CBR(\text{total})} \approx \frac{a_r c^2}{G} \approx 10^{49} \text{ kg} \tag{37} \]

This value is really constant. At the present-time volume of the Universe being
\[ V_U \approx 10^{79} \text{ m}^3 \tag{38} \]

and the energy density of relict radiation \([11]\) one can easily come, in an independent way, to the mass given by \((37)\). This mass generates the gravitational field that modulates the mean value of the energy of relict photons in the matter era. This allows to introduce \((31)\) and \((37)\) into \((29)\) and obtain
\[
\Psi_g = \exp \left[ i.t \left( \frac{a_r c^7}{\hbar G a^3} \right)^{1/4} \right] \tag{39}
\]

The mean energy of photons in the matter era, stemming from \((34)\) and \((39)\) is
\[
|E_g| = E_{CBR} = \left( \frac{\hbar^3 c^7 a_r}{G a^3} \right)^{1/4} \tag{40}
\]

Relation \((34)\) is identical to postulated relation \((28)\). In this way, using the ENU model we are able to harmonize theory and observation. It holds that the mean energy of the photons of relict radiation is equal to the absolute value of the energy of gravitational quanta that are generated by the total mass of relict radiation \(m_{CBR(\text{total})}\).

5 Conclusions

All values presented in this contribution comply with experimentally observed or generally accepted values.

Stemming from a supposed acceleration of the Universe expansion hypothesized in the models including nonzero value of cosmological constant \(\Lambda\) in those introducing quintessence (or other “exotic” forms of matter and energy), values of the Universe-related quantities might differ in a substantial extent in the future. Some of the open questions can be answered by exact measurements of the parameter \(\omega\) (defined as the pressure-to-energy density ratio)
\[
\omega = \frac{p}{\varepsilon} \tag{41}
\]

that in the models accepting the nonzero \(\Lambda\) should reach the value
\[
\omega = -1 \tag{42}
\]
in case of quintessential models it should be of the range

\[ \omega = (-1; -1/3) \]  

(43)

and in our ENU model \[10\]

\[ \omega = -1/3 \]  

(44)

It is worth mentioning, however, that to obtain exact value of \( \omega \), the exact value of Hubble constant must be known.

Summarizing the conclusions offered in the present contribution it should be pointed out that in the majority of conventional models it is postulated that

\[ E_{CBR} \approx T_{CBR} \approx a^{-1}, \ \varepsilon_{CBR} \approx a^{-4}, S = \text{const.} \]

In the ENU, \( E_{CBR} \approx T_{CBR} \approx a^{-3/4}, \ \varepsilon_{CBR} \approx a^{-3}, S \approx a^{-1/4} \). Observations are in accord with the values derived by ENU model. Other models explain the above mentioned observed relations as a consequence of the matter emerging from behind event horizon due to the Universe expansion deceleration. The latest measurements, however, suggest that the Universe expansion might accelerate and exclude its deceleration. In the ENU the expansion neither decelerates nor accelerates, it is constant and equal to the velocity of light.

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