Cutting pattern of a fabric for architectural hypar

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Abstract. Research represents designing and modeling of summer fabric awning against weather conditions. A membrane is fixed on rigid supporting frame. This frame has four linear sides. There is a presentation of shell construction design, which geometric shape consists of four plane fabric cutting patterns. A number of calculation patterns of geometric parameters of the concerned elements are presented. The method of use of mesh with isolateral tetragonal cells on a curved surface was applied. Contour lines are taken as the longitudinal axis of the cutting elements. This axis follows the geodesic line on the surface and directed orthogonally to the contour lines. The finally cut form of the elements results once it is reorganized to the plane and expansion of the fabric from pretension loads. The manufacture of the shell construction completes with interconnection of four triangular work pieces by its sides. These sides are curvilinear therefore they are straightened by stretching during its installation. The elaborated calculation methods and algorithms are implemented in software and experimentally tested on the designed and manufactured fabric models of hypar.

Key words: membrane, structure, fabric, hypar, cutting, pattern.

1 Introduction
The reversal external loadings are applied to the structure. For example, there is a wind load. An effective resistance against these stresses is maintained by structures which have a shape of negative Gaussian curvature surfaces. The geometry of hyperboloid is most suitable with these terms. There are ruled surfaces, what is shown in figure1. The hyperboloid structures which are composed of system of linear elements are well known in building industry as a framed net-structure.

V.G. Shukhov system towers are good examples. These towers are located in Moscow on Shabolovskaya st. (Russia), in port in Kobe (Japan) and in Guangzhou (China). The hypar geometry in structures was applied by Félix Candela, Kenzō Tange [1], Santiago Calatrava and other famous architects. The use of hyperboloid geometry is extremely effective and natural in structures made of membrane-fabric materials.
Such building contractures are known as a “Fabric Structures or Architectural membranes”. Fabric membranes are not capable of compression stress resistance however they have a high tensile stress resistance. They can take the shape of stable and functional architectural shapes under the biaxial tension. There are three ways to reach this condition: mechanical, pneumatic and hydraulic. Mechanically operated tension is used in described fabric awning. The architectural structure with geometry of hyperbolic paraboloid shape is called Hypar. (Hypar is an acronym Hyperbolic paraboloid). The great example of this architectural decision is an awning at the flower show in Cologne. It is shown in figure 2. The author of the project is a Dutch architect Frei Otto [2, 3], a laureate of Pritzker Architecture Prize 2015. He is a pioneer of membrane architecture.

Figure 2. Dance Pavilion at the Federal Garden Exhibition, 1957, Cologne (Germany).

The authors of this article took part in the design of several tent hypars in Kazan (Russia). The photos of these contractures are shown in figures 3, 4.

Figure 3. Hypar «Sabantuy» (Russia).  
Figure 4. Hypar “AZS” in Kazan (Russia).

The scheme of a hypar with squared parabolas is shown in figure 5. The vertical axial cross sections of the hypar have parabolas shapes with reverse curvature:

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z
\]  
(1)
Examples of equations for the surface of a hypar with the origin at the saddle node are given in an Eq. (1), (2). In this equation the parameters a and b are the semi-axes of the parabolas, q and p are the focal distances. This is a ruled surface. It has two linear guide systems, as can be seen in figure 6.

\[
z = \frac{x^2}{2p} - \frac{y^2}{2q} \tag{2}
\]

**Figure 5.** Hypar with hard line borders.

**Figure 6.** Lines on Hypar.

These systems are described of Eq. (3), (4), (5) and (6):
\[
x = \frac{a}{b} y = 2kz \tag{3}
\]
\[
x = \frac{a}{b} y = \frac{1}{k} \tag{4}
\]
\[
x = \frac{a}{b} y = 2lz \tag{5}
\]
\[
x = \frac{a}{b} y = \frac{1}{l} \tag{6}
\]

**1.1 Research Area**

The study of fabric hypars receives wide attention because of the fact that a large number of high-strength lightweight materials from fabrics and films were developed. The studies described in this article also apply to the specification of tented hypars in the form of film-fabric membranes. Descriptions of studies of such structures can be found in publications [4-9]. Hypars, like shells in the form of hyperbolic paraboloid, have an important structural geometric feature. Parabolic sections of one direction serve a load bearing function. They perceive the gravitational load and the pressure of the transverse parabolas. Transverse parabolas create the required stabilizing stress in the shell and absorb the tensile wind load.

**1.2 Research Object**

Awning hypar designers should solve two specific problems.

The first one is **Form Finding** of the film-fabric shell under the load after its installation and pretensioning. Research on this problem can be found in the articles of the TensiNet-2004 collection [10] and also in articles [11-13].

The second problem is the design of the cutting pattern and the calculation of the parameters of the **Cutting Pattern**. This article is concerned with this problem and it is the object of research.

The greatest challenge of the solution lies in the fact that the rolls of fabric are not able to cover the surface of the hypar completely without any folds and deformations. Therefore, it is necessary to develop surface cutting patterns from a set of flat elements and draw them in plane. The development
of techniques and methods of imparting a coating of flat coated rolls to a surface with a double curvature is one of the most important tasks in the design of tent membranes. A number of methods are known for cutting surfaces of double curvature. Basically, this is a segmentation method - cutting into segments. Examples are given in figures 7, 8 and 9.

![Figure 7. Examples of Cutting Pattern of Sphere.](image)

![Figure 8. Hiperbolic Paraboloid.](image)

![Figure 9. Cutting Pattern of Hypar.](image)

The main approach currently used in the practice of designing tent hypars also consists in dividing the shell into curved segments [14-18]. Within these segments, the shell is modeled by a linear polyhedron. Its scan is built geometrically taking into account the extensibility of the fabric. This is quite applicable with a small curvature of the shell. With this approach, the traditional pattern of dividing the surface into “cutting” segments has the form shown in Fig. 9. All cutting segments have a curved shape. This is a good method. But there are a number of difficulties. For example, how to use a large number of curved pieces of fabric that remain after cutting. How to connect fabric elements with curved edges.

1.3 The purpose and objectives of the research

The authors set the goal of exploring the possibilities of cutting hypar awning. This requires solving a number of problems:

1. Increasing in the size of flat cutting parts – elements;
2. Minimizing the number of cutting parts;
3. Reducing tissue waste during cutting;
4. Reducing the curvature of the elements edges to optimize the connection technology;
5. Development of algorithms for the automation of calculation and design cutting;
6. Testing on a real model the practicality of the proposed cutting method.
2 Materials and methods

The solution proposed by the authors takes into account three important properties of tent hypars with a rigid linear contour.

2.1 The first property

Fabrics have low shear stiffness at small angular deformations. A number of tent fabrics were tested for diagonal tension. Test illustrations are shown in figures 10 and 11.

![Figure 10. Schem of experience.](image)

![Figure 11. Diagonal Tensile.](image)

It is revealed that the plain weave fabrics have an allowable skew angle until wrinkles on the surface are about 7 degrees. Its can see on figure 12. In this case, the tensile forces do not exceed 1 Kn/m. Such efforts are characteristic of the initial stage of tissue deformations and lead only to a kinematic rearrangement of the tissue structure under tension. They do not significantly affect the stress-strain state of the tissue under biaxial tension. It can see on figure 13. There are the works, written in articles [19-21].

![Figure 12. Angle shears if diagonal stress](image)

![Figure 13. Biaxial fabric stressing](image)
2.2 The second property
It is linearity of surface geometry and bearing contour. So as these edges are linear elements, the scheme, when the fabric material orientation along contour’s lines was selected.

2.3 The third property
The rigid support edge replaced a flexibly cable. So as this edge is a directing line of the hypar, the orientation scheme of the fabric material along the contour lines of the hypar was selected.

The method of applying the Chebyshev mesh is chosen to calculate the boundaries of the cutting elements which have regard to the allowable shear strain of the material structure [22, 23]. To implement computer calculation it was necessary to develop special algorithm and a program for calculating the position on the surface of a point equally spaced from a pair of base points.

2.4 The algorithm of finding of equally spaced node
This is a numerical method for determining the position on a curved surface of a certain point "C", remote at a given distance from two base points of the surface. Schemes explaining the solution of the problem are shown in figures 14, 15 and 16.

![Figure 14. Calculating cross axis.](image1)

![Figure 15. Calculating mesh node.](image2)

![Figure 16. Schema for calculate $d_x$, $d_y$.](image3)

*Description of key operations* of the iterative calculation process.
The following notation is accepted:

- $X_a, Y_a, Z_a, X_b, Y_b, Z_b$ – coordinates of given points $A$ and $B$;
- $X_c, Y_c, Z_c$ – coordinates of the desired point $C$;
- $S$ – the specified distance to the desired point;
- $t$ – the accuracy of distance calculations.
Step 1. The choice of the initial approximate values of the sought coordinates of the point "C" depends on the problem being solved:

The first approximation of the position of point "C" can be taken as the origin of the hypar, the point with coordinates (0,0,0) in the task of constructing the transverse axis of the Chebyshev mesh (figure 14).

It is recommended that the initial values of the coordinates of point "C" should be taken in accordance with Eq. (7) in the task of calculating the coordinates of the nodes of the Chebyshev mesh on the surface. This is illustrated by the circuit in figure 15.

\[ X_c = X_a; \quad Y_c = Y_b. \]  

(7)

Step 2. Calculation of the distance between points “C” and “A” from Eq.(8) and transferring the coordinates of point “C” to the required distance from “A”, which is equal to the value of \( S \), in accordance with Eq. (9), (10) and (2), which essence is definite from the plot in figure 16:

\[ L = \sqrt{dx^2 + dy^2 + dz^2}; \]  

(8)

\[ X_c = X_{a(b)} + dx \frac{S}{L}; \]  

(9)

\[ Y_c = Y_{a(b)} + dy \frac{S}{L}; \]  

(10)

Step 3. The calculation of the coordinate of point “C”, which takes into account the required distance from “B”, is similar to the second step according to Eq. (8), (9), (10) and (2) a new calculation of the distance between points “C” and “A” in accordance with (8);

Step 4. Calculation of the value of \( D_{\text{max}} \), as the maximum absolute distance difference between \( L_a \) and \( L_b \) from the desired point C to points A and B according Eq. (11):

\[ D_{\text{max}} = \max \left| L_a - S, \quad L_b - S \right| \]  

(11)

Step 5. Making a decision by condition of meeting the accuracy Eq. (12):

\[ D_{\text{max}} \leq t, \]  

(12)

If the condition is not met, return to step 2, otherwise stop, end.

This algorithm was implemented on “HYPAR” software.

2.5 Design cutting computer program

The program interface is shown in figure 17. This program was used to build the cutting pattern of the experimental model of hypar on a rigid diamond-shaped contour. The response sequence is given in the calculation. To develop a software application for the calculation and design of cutting, a step by step algorithm was compiled.

The lateral boundaries of the plot are transferred to the workpiece from parallel fabric canvases (figure 21).

The general drawing of the hypar is shown in figure 5. The sequence of calculation of cutting includes several important stages.

Stage 1. Dividing of the shell into cutting parts. It is rational to divide the shell into 4 parts, enclosed between the contour and the axes OX and OY. A diagram of this division of the hypar is shown in figure 18.

Stage 2. Selection of the optimal size of the quadrangular cell of the mesh model of the shell as a Chebyshev mesh. Chebyshev mesh is a linear-pinned mesh with equilateral quadrangular cells. In this case, two opposite aspects must be taken into account. It is important to note, that the smaller cells provide with the more accurate polygonal model of the curved surface, since the difference in the
lengths of the chord and arc between two points on the surface decreases. At the same time, the smaller cells require more unknowns for numerical calculation. Calculations show that for building membranes structures the optimal cell sizes can be taken in the range from 1/100 to 1/50 of the magnitude of the span of the hypar.

Stage 3. Calculate coordinates of the axes and mesh of Chebyshev on the hypar surface. It is shown in figure 19. As noted above, it is rational to accept the boundary guide line of the hypar as the longitudinal axis. Considering the fact that the structure of the direction of the fabric threads is orthogonal, it is evident to take the direction of the transverse axis perpendicular to the longitudinal. In the case of complete symmetry of the hypar, the center of the local coordinate system of the mesh on the surface is the midpoint on the contour side. A numerical algorithm was used to find a point on the surface of the shell equidistant from the two given, to construct the transverse axis, which is the shortest line on the surface of the shell to the center of the hypar. Its description is given above.
Stage 4. Design of the development of the desired part of the shell. First, a Chebyshev mesh is developed in the area of the shell. Then the limiting points of its contour are calculated at the points of intersection with the axes of the hypar. This allows calculating the length of the mesh threads and transmit the mesh on a plane (figure 20). An example of the form of a flat cutting pattern of a section of the hypar surface obtained in this way is shown in figure 21.

![Figure 20](image.png)  ![Figure 21](image.png)

Figure 20. Mesh transmit on plan.  Figure 21. Cutting Pattern.

Stage 5. Assembling the hypar shell. All parts of the shell are connected by the sides with tension until they straighten. This is shown in figure 22.

![Figure 22](image.png)

Figure 22. Assembly parts to shell.

Experimental verification. This is the method of the cutting pattern on model of hypar. At the beginning the fabric hypar model cutting pattern was calculated. This is geometrical parameters:
- 1000 mm – spans of a parabolas along axes;
- 300 mm – deflection of the supporting parabola;
- 300 mm – lifting the upper tension parabola;

First model had free edges without an edge cable. This model is shown in figure 23. To increase the rigidity of the contour of the tissue membrane its edges were reinforced with metal tubes. The appearance of the reinforced model is shown in figure 24.

Metal elements perform two functions:
1. Create hardness edge;
2. Fix the position of the all nodes of hypar.

Using a laser scanner, geometric model parameters were measured along the hypar axes. The results are plotted along the 0-X axis in figure 25. And results are plotted along the 0-Y axis are show in figure 26.
3 Results and discussions

The proposed method of cutting allows solving the following problems:

1. Increase the dimensions of the flat cutting parts and reduce their number.
2. Reduce the curvature of the edges of the cutting elements and reduce the consumption of fabric.
3. Create and test computer cut calculation algorithms and software.
4. Experimentally confirm the possibility of tissue orientation along the hypar edge direction.

4 Conclusion

1. This method can be used for shells of large curvature.
2. The proposed cutting option has a number of useful advantages: Fabric economy, Simple cutting and fabrication technology. Possibility to create blocked architectural compositions using linearity of contour.
3. Application of proposed type of cutting is justified for summer protective shelters against rain and sun.
4. It is necessary addition to the research of the bearing capacity of this type of hypar
5. This is also important because the main stresses in the shell are directed diagonally to the fabric structure and along the joints.

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