Research on Fault Tolerance Consistency of Multi-Agent Based on Event Triggering Mechanism

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Abstract. For the multi-agent system with directed topology, considering the possible actuator faults of any agent, the robust $H_{\infty}$ fault-tolerant consensus controller design problem is studied when the system is disturbed by external finite energy. Firstly, a distributed event-triggered fault-tolerant consistency protocol is proposed, which is used to transform the fault-tolerant consistency of distributed multi-agent systems into the stability of time-delay systems. Secondly, by using Lyapunov stability theory, a sufficient condition for the fault-tolerant consensus of the multi-agent system is given for the actuator faults that may occur in any agent. A cooperative design method that saves network communication resources and realizes fault-tolerant consensus control is obtained, which makes all states of the fault multi-agent system asymptotically consistent and has a certain disturbance rejection performance. Finally, the feasibility and effectiveness of the proposed method are verified by simulation.

1. Introduction
In the coordinated control of multi-agent, the consistency problem is the most basic control problem of multi-agent system. Consensus problem in multi-agent system has attracted wide attention due to its application in formation control of unmanned aerial vehicles, agent control and scheduling of power grid system [1-2]. Due to the coupling between individuals introduced by distributed information exchange, multi-agent system has more complex structure and dynamic characteristics, which makes multi-agent system more prone to failure than single-agent system and general centralized system. Therefore, more and more researchers are devoted to the fault-tolerant control of multi-agent systems [3-4].

In many practical multi-agent systems, the bandwidth of communication networks and the power supply of agents are inevitably limited. Therefore, a good consensus algorithm for distributed multi-agent systems must be able to save limited communication capacity and energy supply resources while ensuring control performance. The traditional time trigger is the periodic state sampling and information transmission, which will not only lead to the waste of network resources, but also reduce the endurance time of the agent. Compared with the time-triggered mechanism, the event-triggered mechanism algorithm usually requires less information transmission and control task execution when reaching a certain performance level. Inspired by this, event-triggered control has been studied in the network control system [5]. For multi-agent systems, the paper [6] studies the distributed event-triggered transmission strategy of sampling data consistency in multi-agent systems to better achieve the effect of saving network resources.

The above research results are mostly focused on the consensus control problem of multi-agent systems without faults. With the complexity of the structure and the influence of external interference, the probability of fault occurrence increases, and the fault-tolerant control of multi-agent systems is widely
concerned by researchers. For the leader-follower multi-agent system. Reference [7] considered the external environment disturbance and actuator fault simultaneously for the leader-follower multi-agent system, by designing the distributed controller reconfiguration strategy, the performance fault recovery control problem of actuator fault under the time-triggered mechanism was studied.

Although in the existing literature, multi-agent fault-tolerant control has been developed to a certain extent, there is little research on multi-agent fault-tolerant consistency control based on event trigger mechanism. The focus of this paper is on how to design a robust passive fault-tolerant controller based on event trigger mechanism for multi-agent systems with network-induced delay in the case of possible actuator failure and external limited energy interference. The system has good robustness and certain disturbance suppression performance in the case of saving limited network resources.

2. Problem description

2.1 Communication topology

Let $g=(\nu, E, W)$ denote a directed weighted graph of $N$ order, where $\nu = \{v_1, v_2,\ldots, v_N\}$, and $E \subseteq \nu \times \nu$ be the set of nodes and edges. $W = [w_{ij}] \subseteq R^{N \times N}$ represents a weighted adjacency matrix. It is assumed that $w_{ii} = 0$ for any $i$. An edge defined as $e_{ij} = (v_i, v_j)$ implies that node $v_i$ can receive information from node $v_j$. Node $v_j$ is considered as a neighbor of node $v_i$ if $e_{ij} \in E$. The degree matrix of the graph $g$ is denoted by $\Delta = \text{diag}\{\nu_1, \nu_2,\ldots, \nu_N\}$, where the diagonal element is represented as $\nu_i = \sum w_{ii}$ (which is also called as the in-degree of node). Correspondingly, the Laplacian matrix of the directed graph $g$ is defined as $L = D - W$.

2.2 Control objectives

Consider a distributed multi-agent system consisting of $N$ agents, whose dynamics is described by

$$
\begin{align*}
\dot{x}_i(t) &= Ax_i(t) + BG_i(t) + Dw_i(t) \\
y_i(t) &= Cx_i(t), \quad i = 1, 2, \ldots, N
\end{align*}
$$

(1)

Where, $A,B,C$ are the state of the $i$th agent, input and output, The initial conditions are $x_i(0) = x_i^0$, and $A,B,C,D$ are constant matrices with appropriate dimensions; $w_i(t) \in R^p$ is the external finite energy disturbance, which is $w_i(t) \in L_2[0,\infty)$.

Considering the possible fault of actuators, the form is $G = \text{diag}\{g_1, g_2,\ldots, g_m\}$, $g_i \in [0,1], i = 1, 2,\ldots, m$, which represents the failure degree of the actuator in the system. Its model is described as follows: $g_i = 0$ denotes the complete failure of the $i$th actuator of the system, $g_i = (0,1)$ denotes the partial failure of the $i$th actuator of the system, and $g_i = 1$ means that the $i$th actuator of the system is normal.

The purpose of this paper is to design a distributed fault tolerant controller under the premise of saving network communication resources, when the actuator of the system may fail and is interfered by the external finite energy, so that the system(1)can be asymptotically consistent, that is, $\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0$.

3. Distributed fault-tolerant control under event-triggered communication mechanism

In this paper, an event-triggered communication mechanism based on sampled data is introduced to design a distributed fault-tolerant control scheme. Whether the sampled data of Agent $i$ should propagate at sampling instant $kh$ ($k \in N$) depends on when it reaches its threshold. The sampler samples the state of each agent $i$ with a constant sampling period $h > 0$. The sample data $x_i(kh)$ ($k \in N$) of agent $i$ can be successfully sent to a distributed event processor (DEP), embedding event trigger conditions. When the threshold is violated, the trigger signal of the event trigger is generated.
Denote the $k$th transmitted instant of the sampled-data of agent $i$ by $t_i^k, k \in N$. Hence, the next broadcasting instant $t_{i,k+1}$ of agent $i$ is accordingly determined by

$$t_{i,k+1} = t_i + \min_{l \geq k} \{l \mid [X_i(t_i^l + l) - x_i(t_i^l)] \geq \sigma \eta_i(t_i^l + l) \Xi \eta_i(t_i^l + l)\}$$

(2)

Where, $l_i \in N, \sigma_i > 0$ is the threshold parameter, $\Xi > 0$ is the weighted matrix, sampling data error $\xi_i(t_i^l + l) = x_i(t_i^l + l) - x_i(t_i^l)$, sampling data $\eta_i(t_i^l + l) = \sum_{j=0}^{N} \omega_j [x_j(t_j^l) - x_j(t_i^l)]$, $k_j \triangleq \arg \min_{k} \{t_i^k + l_i - t_i^k > t_i^p, p \in N\}$.

Therefore, the following fault-tolerant consistency control protocol for sampling data is proposed

$$u_i(t) = -K \sum_{j=1}^{N} w_j [x_i(t_i^l) - x_j(t_{j,i}^l)]$$

(3)

Where, $t \in [t_i^l, t_{i,k+1}), k_j \triangleq \arg \min \{t - t_i^p \mid t_i^p > t_i, p \in N\}$ and $K$ is the controller gain.

### 3.1 Model transformation

The measurement error at the $k$th sampling time is defined as

$$e_i(k) \triangleq x_i(k) - x_i(t_i^k), t_i^k \leq k \leq t_{i,k+1}$$

(4)

Release time interval $[t_i^l, t_{i,k+1}) = \bigcup_{l=k}^{\infty} (kh, (k+1)h)$, by dividing $[t_i^l, t_{i,k+1})$ into $[t_i^l, t_{i,k+1})$ sampling intervals, and from (1), (3) and (4), the closed-loop equations of the multi-agent system are obtained as

$$\dot{x}(t) = Ax(t) - BGK \sum_{j=1}^{N} \omega_j [x_j(k) - e_i(k)] + Dw_i(t)$$

(5)

let $x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T$, $e(k) = [e_1^T(k), e_2^T(k), \ldots, e_N^T(k)]^T$, $z(t) = [z_1^T(t), z_2^T(t), \ldots, z_N^T(t)]^T$, $\zeta(k) = [\zeta_1^T(k), \zeta_2^T(k), \ldots, \zeta_N^T(k)]^T$ with $z_i(t) = x_i(t) - x_i(t_i^k), w(t) = [w_1^T(t), w_2^T(t), \ldots, w_N^T(t)]^T$, $\zeta_i(k) = e_i(k) - e_i(k)$, $z(t) = (E_i \otimes I_i) x(t)$, $x(t) = (E_2 \otimes I_2) z(t) + (I \otimes I_2) x(t)$, $\zeta(k) = (E_i \otimes I_i) e(k)$ and $e(k) = (E_2 \otimes I_2) \zeta(k) + (I \otimes I_2) e(k)$, respectively, where $E_i = [1, -I_{N_i-1}]$ and $E_2 = [0, -I_{N-1}]$. Then

$$\dot{\zeta}(t) = (I_{N_i-1} \otimes A) z(t) - (E \otimes BGK) \zeta(k)h + (E \otimes BGK) \zeta(k)h + (I_{N_i-1} \otimes D) w(t)$$

$$y(t) = C \zeta(t), \quad kh \leq t \leq (k+1)h$$

(6)

where, $E = E_2 \otimes I_2 \in R^{(N-1) \times (N-1)}$, Define a delay $\tau(t) = t - kh$, $kh \leq t < (k+1)h$. Clearly, $\tau(t)$ is piecewise-linear with the derivative $\dot{\tau}(t) = 1$ at $t = kh$ and is discontinuous at $t = kh$. It is easily known that $0 \leq \tau(t) < h$. Hence, the system (6) can be written as

$$\dot{\zeta}(t) = (I_{N_i-1} \otimes A) z(t) - (E \otimes BGK) \zeta(t) - (E \otimes BGK) \zeta(t) - (I_{N_i-1} \otimes D) w(t)$$

$$y(t) = C \zeta(t), \quad kh \leq t \leq (k+1)h$$

(7)

### 3.2 Stability analysis and controller design

Suppose that the directed graph has a spanning tree. Then, under the event-triggered transmission strategy (2), fault-tolerant consensus of the multi-agent system (1) with (3) can be asymptotically achieved if and only if the system (7) is asymptotically stable.

**Theorem 1.** Given $h > 0$ and $A = \text{diag} \{\sigma_1, \sigma_2, \ldots, \sigma_N\}$, the system (7) under the event-triggered transmission strategy (2) is globally asymptotically stable if the graph $g$ has a directed spanning tree and there exist real matrices $P > 0, Q > 0, R > 0, X > 0, Y > 0$, and some matrices $M_k (k = 1, 2)$ of appropriate dimensions, satisfy $H_\infty$ performance index: $J = \int_0^\infty (y^T(t)y(t) - \gamma^2 w^T(t)w(t))dt$ and the following linear matrix inequality (8), (9).
Where
\[ \Pi_{11} = \xi_1 \xi^T \mathcal{P} + \xi_1 \xi^T \mathcal{Q} \xi_1 - \xi_2 \xi_2^T M_2 + \xi_2 \xi_2^T \mathcal{M}_1 - \xi_3 \xi_3^T M_1 - \xi_3 \xi_3^T \mathcal{M}_2 + \lambda \xi_3 \xi_3^T \mathcal{E}^T \mathcal{L} \mathcal{E} \xi_3 + (I_{N-1} \otimes \mathcal{E}) \xi_3 - \gamma^2 \xi_3 \xi_3^T \mathcal{Q} \xi_3 + \xi_3 \xi_3^T \mathcal{C} \mathcal{C}^T \xi_3, \] \[ \Phi_{11} = \xi_1 \xi^T \mathcal{X} + \xi_1 \xi^T \mathcal{Q} \xi_1 + \xi_2 \xi_2^T \mathcal{M}_1 + \xi_2 \xi_2^T \mathcal{M}_2 + \lambda \xi_3 \xi_3^T \mathcal{E}^T \mathcal{L} \mathcal{E} \xi_3 + (I_{N-1} \otimes \mathcal{E}) \xi_3 - \gamma^2 \xi_3 \xi_3^T \mathcal{Q} \xi_3 + \xi_3 \xi_3^T \mathcal{C} \mathcal{C}^T \xi_3, \] 

Lemma 1[6]. Under the event-triggered transmission strategy (2), the following inequality holds for any \( k \in \mathbb{N} \)

\[ \epsilon_i^T (kh) \Xi e_i(kh) \leq \sigma \bar{h}_i (kh) \Xi \bar{h}_i (kh) \] \[ (10) \]

where \( \bar{h}_i (kh) = \sum_{j=1}^{N} w_j [x_i(t_{m+j}(k)) - x_j(t_{m+j}(k))] \), \( m(k) \equiv \arg \min_p \{ k - t'_p \mid k \geq t'_p, p \in \mathbb{N} \} \). 

Lemma 2[8]. Let there exist positive numbers \( v_i (i = 1, 2, 3) \) and a Lyapunov functional \( V(t, z, \dot{z}) : R \times R^n \times R^n \rightarrow R \) such that \( V(t, z, \dot{z}) \) is continuous from the right for \( t \neq kh \) and satisfies

\[ v_1 \| z(t) \|^2 \leq V(t, z, \dot{z}) \leq v_2 \| z(t) \|^2 \] \[ (11) \]

\[ \lim_{t \rightarrow kh} V(t, z, \dot{z}) \geq V(t_{kh} , z_{kh}) \] \[ (12) \]

\[ \dot{V}(t, z, \dot{z}) \leq v_3 \| z(t) \|^2, t \neq kh \] \[ (13) \]

Lemma 3[9]. For any constant matrix \( R \in \mathbb{R}^{n \times n} \), \( R = R^T > 0 \), \( M \in \mathbb{R}^{n \times k} \), time-varying function \( \tau(t) \) satisfying \( 0 < \tau(t) < h \), and vector function \( \dot{z} : [-h, 0] \rightarrow \mathbb{R}^n \) such that the following integration is well defined, let

\[ \int_{t_{r(t)}}^{t_{r(t)}} \dot{z}(s)ds = E \phi(t) \] where \( E \in \mathbb{R}^{n \times n} \) and \( \phi(t) \in \mathbb{R}^n \). Then the following inequality holds

\[ -\int_{t_{r(t)}}^{t_{r(t)}} \dot{z}(s)R \dot{z}(s)ds \leq \phi^T(t)(\tau(t)M^T R^{-1}M - E^T M - M^T E) \phi(t) \] 

Proof of theorem. Choose a Lyapunov–Krasovskii functional as

\[ \dot{V}(t, z, \dot{z}) = \sum_{i=1}^{N} V_i(t, z, \dot{z}), \forall t \in [kh, (k+1)h] \] \[ (14) \]

where

\[ V_i(t, z, \dot{z}) = \dot{z}(t)Pz(t) + \int_{t-h}^{t} \dot{z}(s)Qz(s)ds + \int_{t-h}^{t} \int_{t-h}^{t} \dot{z}(s)R \dot{z}(s)dsd\theta, \] \[ V_i(t, z, \dot{z}) = (h - \tau(t)) \] 

[\( \tau(t) \rightarrow 0 \), \( \theta \in [-h, 0], P > 0, Q > 0, R > 0, X > 0, Y > 0 \)] For the simplicity, let \( \psi(t) = [\tau(t)z(t), \dot{z}(t), z(t-h), \zeta(t-h), w(t)] \) and \( \xi_i \) be a block entry matrix with \( \xi_1 = \xi_j - \xi_i \). For example, \( \xi_1 = [I_{N-1}0, 0, 0, 0] \) and \( \xi_j = [I_{N-1}0, 0, 0, 0] \). Taking the derivative of (14) along the trajectory (7) with respect to \( t \rightarrow (kh, (k+1)h) \) and using the fact that \( \hat{e}(t) = 1, t \neq kh \), we have

\[ \dot{V}(t, z, \dot{z}) = \psi^T(t)(2\xi_1^T \mathcal{P} + \xi_1 \xi^T \mathcal{Q} \xi_1 + \xi_2 \xi_2^T \mathcal{M}_1 + \xi_2 \xi_2^T \mathcal{M}_2 + \lambda \xi_3 \xi_3^T \mathcal{E}^T \mathcal{L} \mathcal{E} \xi_3 + (I_{N-1} \otimes \mathcal{E}) \xi_3 - \gamma^2 \xi_3 \xi_3^T \mathcal{Q} \xi_3 + \xi_3 \xi_3^T \mathcal{C} \mathcal{C}^T \xi_3, \] 

Applying Lemma 3, obtain
\[
\phi_1 + \phi_2 \leq \psi^T(t) \left\{ (h - r(t))M_1 R^{-1}M_1 - \xi_{23}^T M_1 M_1 \xi_{23} + r(t)M_2^T (R + Y)^{-1} M_2 - \xi_{21}^T M_1 - M_1 \xi_{21} \right\} \psi(t)
\]

Due to the fact that \( \zeta^T(kh) (I_{N-1} \otimes \Phi) \zeta(kh) = e^T(kh) (E_1^T E_1 \otimes \Xi) e(kh) \leq \gamma e^T(kh) (I_{N-1} \otimes \Phi) e(kh) \) and applying Lemma 1, obtain

\[
\zeta^T(kh) (I_{N-1} \otimes \Phi) \zeta(kh) \leq \lambda (z(kh) - \zeta(kh))^T \left( E_1^T L^T A \Xi - \Xi^T A^T L \right) (z(kh) - \zeta(kh))
\]

Form(15)-(16), have

\[
2T(t, z) = \left( \int_{t_0}^{t} \gamma \psi \psi + \Sigma \right) \begin{cases} \in \begin{pmatrix} & \xi \xi \\ & \Sigma \end{pmatrix} \end{cases} \\
1 = T H F R F \Omega, 2 = T M R M \Omega - \Omega = +, 3 = T M R Y M \Omega - \Omega = +, \text{ when } \Sigma < 0, \text{ then } T(t, z) \leq \gamma \tau w(t) + \psi^T(t) \Sigma \psi(t), \text{ obtain } V(t, z) - V(t_0, z_0) \leq \int_{t_0}^{t} \gamma \tau w(t) + \psi^T(t) \Sigma \psi(t). \text{ Under zero initial conditions, when } t \to \infty, \text{ there is } T(t) \leq \gamma \tau w(t) dt, \text{ which is } T(t) \leq \gamma \tau w(t) dt.
\]

Notice that \( \Sigma \) is a convex combination of \( \Phi \Omega + \Omega \) on \( (t, c, a) \), therefore, \( \Sigma < 0 \) if

\[
\Pi_{i+1} + \Omega \gamma \tau \Phi + \Omega \gamma \tau \Omega < 0
\]

Applying the Schur complement to (17) and (18), we arrive at (8) and (9). The proof is completed.

**Theorem 2.** Given \( h > 0, \mu > 0, S > 0 \) and \( \Lambda = \text{diag} \{ \sigma_1, \sigma_2, \ldots, \sigma_N \} \), the system (7) under the event-triggered transmission strategy (2) is globally asymptotically stable if the graph \( g \) has a directed spanning tree and there exist real matrices \( P > 0, Q > 0, R > 0, \tilde{R} > 0, \gamma \) and some matrices \( \tilde{M}_k (k = 1, 2) \) of appropriate dimensions, satisfy \( H_\infty \) performance index:

\[
J = \int_{t_0}^{t} \gamma \tau \psi^T(t) \psi(t) - \gamma \tau \psi^T(t) \psi(t)
\]

and the following linear matrix inequality (19), (20).

\[
\begin{bmatrix}
\tilde{\Pi}_{11} + h \Phi_{11} & h \tilde{F}^T \\
* & h (\tilde{\Pi}_{22} + \mu^2 \tilde{Y}) & 0 & 0 \\
* & * & \tilde{h} & 0 \\
* & * & * & - \tilde{j}
\end{bmatrix} < 0
\]

\[
\begin{bmatrix}
\tilde{\Pi}_{11} & h \tilde{F}^T \\
* & h \tilde{M}_{2}^T & \tilde{H} \\
* & * & \tilde{h} \\
* & * & - h (\tilde{R} + \tilde{Y}) \\
* & * & * & - I
\end{bmatrix} < 0
\]

where

\[
\tilde{\Pi}_{11} = \xi_{T} \tilde{F} + \tilde{F}^T \xi_{1} - \delta_{21} \xi_{1} (I_{N-1} \otimes \tilde{P}) \xi_{21}^T + \xi_{T} \tilde{F} - \xi_{1} \tilde{Q}_{T} \xi_{1} - \xi_{23} \xi_{1} \tilde{M}_{1} - \xi_{23} \xi_{1} \tilde{M}_{1} - \xi_{23} \xi_{1} \tilde{M}_{2} - \xi_{23} \xi_{1} \tilde{M}_{2} + \lambda \xi_{T} \xi_{1} \left( E_{1}^T L^T A \Xi - \Xi^T A^T L \right) \Lambda \Xi E_{2} \otimes \Xi \xi_{24} - \xi_{T} \xi_{24} \left( I_{N-1} \otimes \Xi \right) \xi_{24} - \xi_{T} \xi_{24} \text{ \Phi}_{11} = \text{diag} \left( \xi_{T} \tilde{F} + \tilde{F}^T \xi_{1} \right), \tilde{H} = \xi_{T} (I_{N-1} \otimes \tilde{C} \tilde{P}), \tilde{F} = (I_{N-1} \otimes A \tilde{P}) \xi_{1}, (I_{N-1} \otimes D \tilde{P}) \tilde{S}, \tilde{F}_{22} = \mu^{2} \tilde{R} - \mu^{2} (I_{N-1} \otimes \tilde{P}) \text{ with } \lambda \text{ is given in Theorem 1.}
\]

Moreover, the consensus controller gain is given by \( K = \tilde{K} \tilde{P} \).

**Proof.** Let \( P = I_{N-1} \otimes U \) for the conditions (8) and (9) in Theorem 1, define

\[
X = \delta P \tilde{P} = U^{-1}, \tilde{Q} = P^{-1} Q P^{-1}, \tilde{R} = P^{-1} R P^{-1}, \tilde{Y} = P^{-1} Y P^{-1}, \tilde{I} = P^{-1} I P^{-1}, J = \text{diag} \{ P^{-1}, P^{-1}, P^{-1}, P^{-1}, \}, \tilde{S} = U^{-1} \Xi U^{-1}, \tilde{M}_{k} = J M_{k} (k = 1, 2),
\]

5
and $\tilde{K} = KU^{-1}$, Pre- and post-multiplying both the sides of (8) by $\text{diag}\{J_1,(R+Y)^{-1},P^{-1},P^{-1}\}$ and both the sides of (9) by $\text{diag}\{J_1,R^{-1},P^{-1},P^{-1}\}$, respectively, one can obtain (19) and (20), where the term $P^{-1}(\tilde{R}+\tilde{Y})^{-1}P^{-1}$ is resolved by the inequality $-PR^{-1}P \leq \mu^2R - 2\mu P$, The proof is completed.

4. Simulation example
Consider a multi-agent system consisting of four linear agents. The communication topology of the multi-agent system is shown in figure 1.

![Figure 1](image)

**Figure 1.** A directed graph with spanning tree $g$

Suppose the weight coefficient $ω_{ij} = 1$. According to the knowledge of graph theory, the Laplacian matrix of graph $g$ can be written as

$$L = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 2 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix}$$

The parameters of the state equation (1) of the system are described as follows:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.4 \end{bmatrix}, \quad B = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

Let the initial states of the multi-agent system be: $x_1(0) = [0; 1], x_2(0) = [0.5; 1.5], x_3(0) = [-1; 0], x_4(0) = [0; 1]$. And suppose the sampling period is $h = 0.06$, and the event trigger parameters are $σ_1 = 0.010, σ_2 = 0.007, σ_3 = 0.016, σ_4 = 0.013$, The interference of external bounded energy by each independent agent is

$$w_i(t) = \cos(2\pi t) \times \exp(-0.5t), \quad i = 2, 3, 4$$

Consider the normal and various failure cases of the actuators of any agent as $G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, G_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix}, G_3 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Where, $G_1$ means that both actuators are normal, $G_2$ means that both actuators are partially failed, and $G_3$ means that one of the actuators is completely failed. Given $δ = 1, μ = 1$, according to theorem 2, the controller gain matrix and event triggering weight matrix can be obtained as follows

$$K = \begin{bmatrix} 2.5243 & 5.0990 \\ 0.1827 & 0.6580 \end{bmatrix}, \quad Ξ = \begin{bmatrix} 0.4492 & 1.0472 \\ 1.0472 & 2.9108 \end{bmatrix}$$

When agent 1 is under $G_1$, all other agents are normal, that is, all four agents have no failure and are interfered by external finite energy, the position / velocity state trajectory of the multi-agent system (1) under information topology diagram 2 is shown in (a) and (b) in figure 2. It can be clearly seen from the figure that the position and velocity of the system gradually converge respectively, which demonstrates the effectiveness of the proposed design method.

When agent 1 is under $G_2$, and other agents are all normal and are disturbed by the external finite energy, the position/velocity state trajectory of the multi-agent system (1) under the information topology
diagram 2 is shown in (a) and (b) of figure 3. It can be clearly seen from the figure that the position and velocity of the system gradually converge respectively, which demonstrates the effectiveness of the proposed design method.

When agent 1 is under fault $G_1$, all other agents are normal and interfered by external finite energy, the position/velocity state trajectory of the multi-agent system (1) under the information topology diagram 2 is shown in (a) and (b) of figure 4. It can be clearly seen from the figure that the position and velocity of the system gradually converge respectively, which demonstrates the effectiveness of the proposed design method.

Figure 2. Position/velocity state response curve of multi-agent system without fault. (a) Position state response curve. (b) Velocity state response curve.

Figure 3. System position/velocity state response curve when failure $G_2$ occurs to agent 1. (a) Position state response curve. (b) Velocity state response curve.

Figure 4. System position/velocity state response curve when failure $G_3$ occurs to agent 1. (a) Position state response curve. (b) Velocity state response curve.

Figure 5. System position/velocity state response curve when agent 1, agent 2 and agent 3 all fail, (a) Position state response curve. (b) Velocity state response curve.

When agent 1 and agent 2 have a fault $G_2$, agent 3 has a fault $G_3$, and agent 4 has no fault and interfered by external finite energy, the multi-agent system (1) is shown in the information topology diagram 2 is shown in (a) and (b) of figure 5. It can be clearly seen from the figure that the position and velocity of the system gradually converge respectively, which demonstrates the effectiveness of the proposed design method.
In order to highlight the advantages of the proposed event-triggered communication mechanism, figures 6 and 7 compare the periodicity and event-triggered communication of the agent 1 in the position and velocity states, respectively. (a) is the traditional periodic communication mechanism. (b) is the event-triggered communication mechanism. The horizontal axis represents the broadcasting instants and the vertical axis represents the release intervals. The larger the value on the vertical axis, the longer the time interval between the two data broadcasting instants, the less data will be transmitted in a certain period of time, the better the system performance will be, and the more conducive to saving network resources. Therefore, it can be seen that (b) is more conducive to saving network resources, and the system performance is better.

![Figure 6. Event instants and intervals of position state of agent 1 under (a) periodic communication and (b) event-triggered communication.](image)

![Figure 7. Event instants and intervals of velocity state of agent 1 under (a) periodic communication and (b) event-triggered communication.](image)

5. Conclusion
In this paper, for the purpose of saving network communication resources and realizing fault tolerant consistent control, a new fault-tolerant consistent control protocol for sampling data is proposed for leaderless multi-agent system with network induced delay, taking into account various possible failures and external finite energy interference. As a result, the fault-tolerant consistency of the distributed multi-agent system can be transformed into the stability of the time-delay system. And by using Lyapunov-Krasovskii method, a new sufficient condition is obtained, which can make all states of distributed multi-agent system asymptotically converge.

6. References
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