Fragmentation of thinking structure and its impact to students’ algebraic concept construction and problem solving

B Usodo1, I I Aulia1*, A N Wulandari1, Sutopo1, R Setiawan1, I Kurniawati1, Y Kuswardi1

1Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Sebelas Maret, Indonesia

*Corresponding author: isyfania@student.uns.ac.id

Abstract. The term “fragmentation” is adopted from the term of data storage which is used inefficiently in a computer. The analogy of fragmentation in thinking process is inefficient phenomenon of information storage within the brain which obstructs the process of reconstructing and solving mathematical problems. This descriptive qualitative research aims to describe the fragmentation of thinking structure and its impact on concept construction and algebraic problem solving in Junior High School Students. The subjects consisted of 8th grade students of junior high school in Surakarta, Indonesia. Data of this research were collected using a task-based in-depth interview method. This research shows that the students have the following fragmentation of thinking structure: (1) pseudo construction, which occurs when the students’ answers seem right even though the reasoning are wrong, (2) hole construction, which occurs when students do not thoroughly understand the concepts of addition, multiplication, algebraic power forms, and mathematical problems-modeling, (3) mis-analogical thinking, which occurs when students consider an algebraic form to be the same as another algebraic form.

1. Introduction
Mathematics consists of some concepts that related to each other [1]. Students often have to understand certain concepts in order to understand another related concept [1]. It is interesting to discuss about how students construct their mathematics concept. In the process of mathematical concept construction, students often make mistake [2]. The mistake is not just some mistakenly writing answers or miscalculated results, but more like mismatched ideas, way of thinking, incorrect analogy as one problem is mistakenly compared to the other, etc. [3]. Subanji states that these kinds of mistake are called fragmentation of thinking structure [4,5].

The term “fragmentation” is adopted from the term of data storage which is used inefficiently in a computer. It is a condition in which a file that is placed in a storage media does not occupy sector sequentially [3]. The analogy of the term fragmentation in thinking process is inefficient phenomenon of information storage within the brain which obstructs the process of reconstructing and solving mathematical problems [4]. Fragmentation in thinking process often happens as the result of meaningless learning, especially when it only stresses in memorizing formulas and how a procedure is done [3].

Subanji (2016) formulates a theory about fragmentation in students’ mathematical thinking process and it is classified as follows: (1) pseudo construction occurs when students’ answer seems right but in further interrogation, students do not thoroughly understand their answer, or they do not know why
certain rules or formula apply in the answer, (2) hole construction happens when there is a hole in students’ thinking structure as they answer question or conclude mathematical concept, (3) mis-analogical construction arises when students try to understand a concept analogizing it with another concept, assuming they are the same while in fact they are not, and (4) mis-logical construction which happens when students fail to evaluate whether a mathematical statement is right or whether a rule can apply [4,6].

Fragmentation of students’ thinking structure has been studied by many researchers in different contexts and terms. For instance, Vinner (1970) used the term pseudo-analytic in routine mathematical problem solving. Leron and Hazzam (2009) applied Dual Process Theory of Kahneman in context of solving algebra problems, Brodie (2010) addressed it as concept error, Subanji and Nusantara (2013) introduced the term faulty-thinking in constructing mathematical concept, and Netti (2016) once studied about failure construct proof based on Piaget’s assimilation and accommodation [7].

One of the most popular and important branches of mathematics among junior high school students is algebra [4]. Starting to know algebra, students encounter new terms such as variable and coefficient. Algebra introduces symbols and new concepts that they never encounter before. It is not easy at first for student to learn algebra, there are times when they experience difficulties [8]. These difficulties can cause students experience error in their thinking structure [9]. These kinds of error have an impact to student’s concept construction and problem solving [10,11].

Knowing a description of how student encounters fragmentation helps teachers to evaluate learning process in class [12]. It also helps them to guide students in constructing their mathematical knowledge without having fragmentation in thinking structure. This research intends to describe all of the above.

2. Methodology
This qualitative research describes students’ fragmentation of thinking structure and its impacts to their algebra concept constructions and problem solving. The subjects consisted of 8th grade students of junior high school in Surakarta, Indonesia. Data of the research were collected using a task-based in-depth interview method. There are two instruments that help collecting the data, i.e. written test and interview guidelines.

3. Result and Discussion
The data shows that students perform 3 kinds of fragmentations, namely pseudo construction, hole construction, and mis-analogical construction.

3.1. Pseudo construction
Students’ answer can seem right, but when it is reviewed deeper, there might be some mistaken ways of thinking.

Given an incomplete expression \((x + y)^2 = \ldots\), the expected answer is \(x^2 + 2xy + y^2\) as the students have to understand power forms, two-term algebraic multiplication, and how to simplify the answer. At first it seems like most students can answer correctly as what shown in Figure 1. When they are asked the way to expand power form expression, they tend to recall the formula they memorized.

![Figure 1](image)

**Figure 1** Student’s written test answer \((x + y)^2 = x^2 + 2xy + y^2\)

Though students have answered correctly, they cannot explain how they get the answer \(x^2 + 2xy + y^2\). They do not have any idea that \(x^2 + 2xy + y^2\) can be obtained from \((x + y)^2\) as \(x + y\) is multiplied by itself like so \((x + y)(x + y)\). From the multiplication we get \(x^2 + 2xy + y^2\). Here is the related interview.

| R | Did you just know that the answer is \(x^2 + 2xy + y^2\)? Or is there any methods that you did? |
|---|-------------------------------|
| S | I just knew, I memorized it |
The following illustration shows the students’ fragmentation of thinking structure.

![Diagram showing the students' fragmentation of thinking structure](image)

**Figure 2** Students’ fragmentation of thinking structure (pseudo construction)

Common mistake students do in solving mathematical problem is that they lack of understanding in algebraic operation, especially power form concept. Students’ attitude toward the expression \((x + y)^2 = x^2 + 2xy + y^2\) is that they accept it as what it is, not as something that is gained from doing some particular things such as multiplied one term by itself or practicing distributive properties. Though this method can generate the right answer, but sure it has impact to students’ concept construction. Students might get used to memorize every single formula that comes across them. It is not a bad thing, because memorizing is also concept construction process, but it is a very week one [13]. Yet memorizing is not what teachers want students do when they teach them. Also, this will impact badly to students’ concept construction attitude, as they memorize formula without having curiosity why some certain formulas can apply. This method can imply more pseudo constructions to happen to students [14]. It will take a great deal for them to also memorize more complicated higher power forms.

3.2. **Hole construction**

There might be some holes in students’ thinking structure where they don’t seem to understand prerequisite concepts. The following are descriptions of hole construction that students have experienced.

3.2.1. **Students perform incorrect concept of algebraic operation**

Algebraic addition only applies to similar terms by performing distributive properties of multiplication and addition. This differs from algebraic multiplication that it can apply to both similar and non-similar terms. Students are confused at practicing these concepts. They are not totally sure what to do when they encounter algebraic multiplication. This shows when students state that \((x + y)^2 = xy^2\) and \(-10x + 10x = -x\). Figure 3 and Figure 4 show this condition:

**Figure 3** Student’s written test answer

\[(x + y)^2 = \ldots\]

\[xy^2\]

**Figure 4** Student’s written test answer

\[-10x + 10x = -x\]

Instead of performing algebraic addition to similar terms, students perform it to terms which have similar coefficients. For instance students work out that \(x + y = xy\) giving excuse that both terms \(x\)
and \( y \) have coefficient 1 so we obtain: \( x + y = 1x + 1y = 1xy = xy \). In another case, students work out that \(-10x + 10x = -x; -xy + (-xy) = xy^2; 2x^2 + 3x = 5x^3\). The following illustrations show the students’ fragmentation of thinking structure.

**Figure 5** Students’ fragmentation of thinking structure (hole construction)

Figure 5 indicates students experiencing unstructured concept understanding. They do not understand simple rule in adding algebraic form, which is adding only the similar terms. This will impact their skill in performing other algebraic operations since addition is the simplest one there is.

**Figure 6** Students’ fragmentation of thinking structure (hole construction)

Figure 6 also indicates students experiencing unstructured concept understanding. They do not understand simple rule in adding algebraic form when using distributive properties. Distributive here is multiplication distributive, so \((0)x\) is multiplication form and is not standing alone each other which left \( x \), but equals 0. This indicates there is a hole in students’ thinking structure indicates they do not seem to understand how distributive properties is used. This will impact their skill in performing other algebraic operations.

**Figure 7.** Students’ fragmentation of thinking structure (hole construction)
Figure 7 shows us that students do understand step by step to answer the question. Students could perform distributive properties in multiplying 2 terms algebraic form. Then again, Figure 7 also indicates students experiencing unstructured concept understanding. They do not understand how to add algebraic form as they confused with multiplying them. Not only they mistaken in doing the operation, they also miswritten the symbol as writing what should be $(xy)^2$ to be $xy^2$ claiming that it has the same meaning. It looks simple, but if it keeps happening, sure it will impact their skill in performing other algebraic operations. Students may not be able to differ which operation is addition and which is multiplication [13]. Students could also have the right answer in their thinking, but will end up with the wrong answer on paper.

### 3.2.2. Students cannot develop a mathematical model based on given case

Mathematical modeling is an important skill for students [15]. Though students often get the model wrong because they are affected by language features or problem given. Interview goes as follows.

| S | Yes, it can be subtracted like so $20a - 3b - 2c$, that is the final score |
| R | Why is it subtracted? |
| S | Because Ali’s score is $20a$ then 3 questions is answered wrong, so it has to be subtracted by $3b$, then 2 unanswered question, again it has to be subtracted by $2c$ |

The effect of the word “wrong answered” and “unanswered” tend to make students think that it lessen the score. So they subtract the score with it. This reason causes students to develop wrong mathematical model of given mathematical problem. Students think that variable in algebra can only be substituted by positive numbers, so they put minus sign at the terms. This indicates there is a hole in students’ thinking structure, that they do not understand the concept of variable. Students learn algebra in order to solve mathematical problem, but if this hole construction keeps happening, it will impact their thinking skill [4]. Students may feel like they model the problem well, but if it is not then they will get the wrong answer and fail to solve given problem. Students’ thinking plot will stop by the modeling, which is not sure it is right or not, then the solution of given problem is still be questioned.

### 3.3. Mis-analogical construction

Students assume that $(x + y)^2 = x^2 + y^2$ is one of the algebraic properties that applies as how $(xy)^2 = x^2y^2$ applies. Students believe that these two algebraic expression can apply because they appear the same way, as they have brackets and power forms. Students also have an idea that the brackets of $(x + y)^2 = x^2 + y^2$ and $(xy)^2 = x^2y^2$ can be “opened” to be $x^2 - y^2$ and $x^2y^2$. The following illustrations show the students’ fragmentation of thinking structure.

![Figure 8: Students’ fragmentation of thinking structure (mis-analogical construction)](image)

Figure 8 shows students’ attitude toward the expression $(x + y)^2 = x^2 + y^2$ is that they accept it as they accept $(xy)^2 = x^2y^2$, not as something that is gained from doing some particular things such as...
multiplied one term by itself or practicing distributive properties. Students seem like they get used to memorize every single formula that comes across them. This circumstance will impact badly to students’ concept construction attitude, as they memorize formula without having curiosity why some certain formulas can apply [13]. This also can make students get used to generate one expression can apply to other expressions.

4. Conclusion
Students evidently experience fragmentation of thinking structure and it is also impacting their concept constructions and algebraic problem solving. The following are the conclusion of students’ fragmentation: (1) pseudo construction, which occurs when students know correctly one algebraic expression, yet turns out they just memorize it without knowing why it applies. This can impact students’ concept construction as they will get used to memorize every formula they face. (2) Hole construction, which occurs when students do not thoroughly understand the concepts of addition, multiplication, algebraic power forms, and mathematical problems-modeling. This can impact students’ skill in performing more advanced algebraic operations and mathematical modeling. (3) Mis-analogical thinking, which occurs when students consider an algebraic expression to be the same as another algebraic expression. This will impact students’ attitude in facing mathematical expression as they will get used to generate one expression easily can apply to other expressions.

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