

Possibility of a superfluid in high multiplicity \textit{p-p} and \textit{p-Pb} collisions

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We consider the case where the saturated and unequilibrated initial stage of a high multiplicity \textit{p-p} and \textit{p-Pb} collision, may, due to high occupation number, display superfluid properties. The special case of an unequilibrated plasma of \textit{SU(2)} gluons formed in the collision of two nuclei with deeply saturated gluon wave functions is considered. Color gauge symmetry is broken by the presence of a net non-vanishing, space-time dependent color expectation value. A two component scenario is envisioned, containing momentum states that are densely occupied below the saturation scale and a dilute normal fluid of excitations within this state. The spectrum of these excitations is shown to have an energy gap and thus it is hard to excite quanta out of the condensed state. Consequences for the stress energy tensor are outlined, in particular the possibility of a near invisid flow is discussed.

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tion to demonstrate the presence of superfluidity in such systems. There is also no requirement that full saturation dynamics have set in, only that there is a large enough density of softer gluons that one may use classical fields to describe this state.

Imagine the collision of two protons at very high energy. Both wave functions of the two incoming protons possess gluon distributions that are saturated up to a transverse momentum of $k_\perp \lesssim Q_S \sim \alpha_S N_c x G(x)/(\pi R^2)$, where $\alpha_s$ is the strong coupling constant, $N_c$ is the number of colors, $G(x)$ is the gluon distribution function and $R$ is the transverse radius of the proton. All these gluons represent colored fluctuations from the high momentum (hard) color charges inside the proton. Those that populate the mid-rapidity or $y = \log [p^+/p^−]/2 = 0$ region at LHC collisions with $\sqrt{s} \sim 5$TeV correspond to an $x \sim 10^{-4}$. The prevalent state immediately after the collision, called the glasma, is formed by the fusion of such low-$x$ virtual gluons from the two nuclei, resulting in near on-shell gluons with $k_\perp \lesssim Q_s$. At mid-rapidity, these gluons also have a $k_z \ll k_\perp$, as gluons with larger $k_z$ will appear at larger rapidities. In this Letter we consider the small rapidity region of these collisions at a time $\tau \leq 1/Q_S$. This of course requires $Q_S \gg \Lambda_{QCD}$, which is only true in very high energy collisions.

Without going into further details of the production of the glasma, we consider its properties in very high multiplicity events. As a first simplification, we consider all modes from the lowest momentum scale up to a scale $Q \sim Q_S$ to be considerably over populated, such that one can apply classical field theory to the description of this condensed state. Modes with momenta above $Q$ will be considered as a dilute gas which may interact with and produce excitations on the saturated glasma and eventually equilibrate with it. As such we may write down an effective Lagrangian for the saturated soft modes and excitations of this state by separating the non-abelian vector potential as a classical part (soft modes) and a small fluctuation (hard modes),

$$A_\mu^a = A_\mu^a + \varphi_\mu^a. \quad (1)$$

One can now expand the gauge field Lagrangian as a series in the small fluctuation field $\varphi_\mu^a$, as

$$\mathcal{L}_0 = -\frac{F^{\mu\nu} F_{\mu\nu}}{4} + \mathcal{L}_1 = -\frac{\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + \mathcal{V}}{4}. \quad (2)$$

In the equations above $F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} + g\epsilon^{abc} A^{b\mu} A^{c\nu}$ and $\mathcal{F}_{\mu\nu} = \epsilon^{\alpha\beta\gamma\delta} (\partial_{\mu} \phi_{\beta}^\gamma - \partial_{\nu} \phi_{\gamma}^\delta)$. Interaction terms between the fluctuation field and the condensate are contained within $\mathcal{V}$, which has the form,

$$\mathcal{V} = -\frac{g^2}{2}\epsilon^{abc} A^{b\mu} A^{c\nu} \epsilon^{\alpha\beta} \phi_{\alpha}^\mu \phi_{\beta}^\nu + \epsilon^{abc} \left( A^{ab} \mathcal{A}^{c\nu} + A^{bc} \mathcal{A}^{a\mu} \right). \quad (4)$$

In the equation above, we have naturally dropped all terms that are odd in the fluctuation field, as they will average to zero. We have also dropped all terms that involve derivatives of the condensate. While these are not vanishing, over the range of distances and times inhabited by the fluctuation field, they are small compared to the derivatives of the fluctuation field (as well as the amplitude of the condensate wave-function $A^{\mu}$). In the writing of the above set of equations, we have also simplified the gauge group to SU(2). This is done primarily to simplify the ensuing derivation. With the exception of a one-dimensional weight diagram compared to a two-dimensional weight diagram, no major differences are expected between this and the more physical case of SU(3).

We now introduce approximations regarding the condensate within the effective theory. Rewriting the interaction term between the classical field and the hard fluctuation, we obtain,

$$\mathcal{V} = -\frac{g^2}{2}\epsilon^{abc} S_{(bc)\mu\nu} \phi_{\mu}^a \phi_{\nu}^c, \quad (5)$$

where, all factors of the classical field are contained within $S_{(bc)\mu\nu}$. In the effort to describe the tree level behavior of the fluctuation field in the presence of an average soft background field, an event average over the product of background fields in $S_{(bc)\mu\nu}$ is carried out. As a result, we use the following simple approximation for the product of gauge fields,

$$\langle A_{(\mu)^a} A^{(\mu)^b} \rangle \simeq \frac{\delta_{ab}}{N_c - 1} \frac{1}{4} (A_{(\mu)^a} A^{(\mu)^a}). \quad (6)$$

In this Letter, the interaction of the fields will be described using axial gauge $A_3^a = \varphi_3^a = 0$. In this gauge, the $A_0^a = \phi^a$ field is derived from the equations of motion. The fluctuation $\varphi_0^a = \psi^a$ is introduced as a Lagrange multiplier to integrate out the conjugate momenta using completion of squares.

To determine the effect of the interaction term on the fluctuation field, we solve the classical equations of motion for the background field,

$$\partial_{\mu} F^{\mu\nu} = (\delta^{ac} \partial_{\mu} + g\epsilon^{abc} A^{a\mu}) F^{c\nu} = 0. \quad (7)$$

Over the lifetimes and distances traversed by the higher frequency fluctuation fields, one may ignore
all derivatives of the classical background field in comparison to the amplitude of the fields themselves. As a result, the equations of motion simplify: e.g. for $\nu = 0$, 
\[ e^{abc} A^b_\mu e^{cde} A^\mu_\rho \phi^0 \simeq 0. \]  
(8)

For $\nu = 3$, due to the choice of axial gauge, the leading term has one derivative,  
\[ e^{abc} A^b_\mu \partial_\mu A_\rho \simeq 0. \]  
(9)

The other components $\nu = 1, 2$ will yield equations similar to Eq. (8).

Since the glasma gauge fields have been sourced by the virtual gauge fields in the colliding nucleons, the initial conditions are determined in every event given a model of the transverse gluon fields in the colliding nucleons. The equations above, describe the behavior of the scalar potential $\phi^0$ and the behavior of the transverse gauge fields over short times after the collision. Over longer time periods and larger distances, the higher derivatives will no longer be negligible.

Our goal here is simply to demonstrate the possibility of a mass gap for excitations with momenta above the saturation scale. We now introduce an ansatz for the classical field which will include a condensate. Here we follow that work of Refs. [21, 22], and decompose the classical vector field as 
\[ A^{+j} = \frac{A^{1j} + iA^{2j}}{\sqrt{2}} = \rho^j e^{i\zeta} \quad (j = 1, 2), \]  
(10)
\[ A^{-j} = \frac{A^{1j} - iA^{2j}}{\sqrt{2}} = \rho^j e^{-i\zeta} \& A^{3j} = Z^j, \]  
(11)

where $\rho = \sqrt{\rho_x^2 + \rho_y^2}$ is the amplitude of the condensate and $\zeta$ is the phase (We have tacitly assumed the same phase for both the $x$ and $y$ components: $\rho_x = \rho \cos \theta$ and $\rho_y = \rho \sin \theta$). The projection of $\rho^j e^{\pm \zeta}$ and $Z^j$ along the $(x, y)$-axis are assumed to be factorizable, i.e., $\rho^j = \rho \hat{a}^j$ and $Z^j = Z \hat{b}^j$, where $\hat{a}$ and $\hat{b}$ are 2-dimensional unit vectors. The presence of a condensate naturally breaks both color symmetry and rotational symmetry. In this first attempt we will ignore any effect of the breaking of rotational symmetry. The symmetries are broken explicitly (by the collision of two nucleons) and not spontaneously. As a result there are no massless Goldstone bosons.

One can also decompose the scalar potential in a fashion similar to the transverse fields,  
\[ \phi^+ = V e^{i\eta}, \quad \phi^- = V e^{-i\eta}, \quad \phi^0 = U, \]  
(12)

i.e., the scalar potential can be out of phase with the transverse gauge fields. Substituting these back into Eqs. (8) along with the equation for $\nu = 3$ and obtain two sets of solutions. One of these solutions leads to negative total energy and this will be ignored. In what follows we focus on the positive energy solution. We obtain equations for the gauge fields and their derivatives from Eqs. (8,9), by equating real and imaginary parts; these yield $V = \rho$, and
\[ U = \frac{2\rho^2 Z \cos (\zeta - \eta) \hat{a} \cdot \hat{b}}{2\rho^2 + Z^2(1 - \hat{a} \cdot \hat{b})}. \]  
(13)

The two phases are related to each other as $\eta = \zeta + \Delta\phi$, where $\partial_\mu \Delta\phi = 0$, and $\Delta\phi$ is small enough that $\sin \Delta\phi \sim \Delta\phi$. These approximations, along with the event average taken above, reduce the factor $S^{(bc)}$ in Eq. (5) to its diagonal form $g_\mu^\nu \delta^{bc} S/[4(N_c^2 - 1)]$, with $S$ given as,  
\[ S = -Z^2 \left( 1 - \frac{4\rho^4 \cos^2 (\zeta - \eta) (\hat{a} \cdot \hat{b})^2}{2\rho^2 + Z^2(1 - \hat{a} \cdot \hat{b})} \right). \]  
(14)

As a result, on average $S < 0$ and this term behaves in the fluctuation Lagrangian as an $m^2$ term (i.e., a mass term at tree level). As a result, tree level dispersion relations of the fluctuation field $\phi^\mu_{\nu}$, with the inclusion of an interaction with the mean condensate will possess an energy $E > p$, the momentum of the modes. Hence such modes will be difficult to excite and thus the condensate will be unable to equilibrate by populating the higher momentum modes. The incorporation of this mass term in the propagators of the fluctuation field is akin to the Bogoliubov transformation in condensed Bosonic systems [23] and similar to dispersion relations required in the recent condensation of photons [24]. While the analysis in this Letter is carried out in axial gauge, we expect a mass correction to arise in other gauges as well. Such a correction is seen to arise also in the simplified charged scalar theory [24].

As a result of this condition, at small rapidities in $p-p$ and $p-A$ collisions, for a time $\tau > 1/Q_S$, there may exist a superfluid phase in the glasma (at larger rapidities this phase may appear at a later boosted time). The superfluid four-velocity is given by the derivative of the phase of the condensate [21],  
\[ u^\mu = \frac{\partial^\mu \zeta}{\sqrt{\sigma}} = \frac{\partial^\mu \zeta}{\sqrt{\partial^\nu \zeta \partial_\nu \zeta}}. \]  
(15)

As the glasma expands, the magnitudes of the condensates $Z, \rho$ will tend to drop and the magnitude of the energy gap will drop. As a result, with increasing time, it will become progressively easier to excite modes out of the condensate leading to a diminishing of the condensate and population of the “normal fluid” of excitations (hard modes). For small
systems produced in p-p and p-A collisions, it is not clear if there will be a further inviscid fluid phase due to the strong interactions in the normal fluid. It is also interesting to note that with a drop in the amplitude of the gauge fields, the separation scale $Q \sim Q_S$ will also drop with time and thus the fluid will continue to become more strongly interacting with increasing time. If the superfluid were to persist past equilibration, this mechanism would provide another reason for the appearance of a BEC as described in Ref. [20]. To determine the fate of the superfluid phase and the possible appearance of a normal fluid phase would require a numerical simulation which is beyond the scope of this Letter.

In the limit that higher derivatives of the condensate can be ignored, in particular, second derivatives, and if the condensate fraction dominates over the normal fluid, the energy-momentum tensor has a simple form in terms of only the gauge fields. Here we list a few of the components obtained from the expression for the traceless part of the classical energy momentum tensor of a non-abelian theory (we ignore issues related with operator renormalization):

$$
\Pi^{00} = g^2 (\Delta \phi)^2 \left[ (2 \rho^2 + Z^2) p^2 \right],
\Pi^{11} = g^2 [2 \rho^2 + Z^2] (\partial \phi)^2 \left[ \rho^2 (1 - 2 \partial^i \partial^i) \right],
\Pi^{22} = g^2 [2 \rho^2 + Z^2] (\Delta \phi)^2 \rho^2 ,
\Pi^{01} = g^2 \partial_i [2 \rho^2 + Z^2] (\Delta \phi)^2 \rho^2 ,
\Pi^{03} = 0 \text{ etc. (16)}
$$

Such an expression for the stress-energy tensor is only valid in the limit that the amplitude of the gauge fields is very large compared to both the contribution from the normal fluid and from derivative terms in the condensate. With the inclusion of the first set of derivative terms, one will obtain both derivatives of the amplitude of the condensate ($\rho, Z$), as well as derivatives of the phase $\phi$, which will yield factors of velocity $u^\mu$. Even at this stage, the viscosity of the fluid will be effectively vanishing. With the rise in the population of the excited modes (normal fluid), as well as the magnitude of second derivatives of the condensate, viscous terms will appear in the stress energy tensor of this field. As a result, we may hypothesize that very high multiplicity p-p and p-A collisions will behave like an inviscid fluid from a time $\tau > 1/Q_S$, and continue to retain this behavior until there is a sufficient population of the excited states. The determination of the exact time when this will take place requires a numerical simulation of the system, which we leave for a future effort.

In this Letter, we have outlined a possible reason for the appearance of inviscid fluid behavior in high multiplicity p-p and p-A collisions: The appearance of a colored superfluid of gluons. The analysis carried out in this letter is very simplified, the goal was to highlight the means by which an energy gap in the excitations of the normal fluid may arise by interaction with the condensate. The persistence of such large fields leads to vanishingly small viscosity in such systems and may be the underlying reason for the perfect fluid nature in such systems. While such a superfluid would appear a very short time after the collision, the time up to which it will persist is uncertain at this point and will require a more detailed numerical analysis.

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