MASS FUNCTION OF GRAVITATIONALLY BOUNDED OBJECTS IN THE INHOMOGENEOUS UNIVERSE

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Abstract

We modify the Press-Schechter formalism to calculate the mass function of gravitationally bounded objects in a local Universe with size $L > 10h^{-1}\text{Mpc}$, i.e. separately in voids ($\Omega_{m,L} < \Omega_m$) and superclusters ($\Omega_{m,L} > \Omega_m$). The dependence of the abundance of gravitationally bounded objects on local matter density is analyzed.

1 Introduction

Although the Universe is spatially flat and homogeneous at large scales ($> 300 \, h^{-1} \text{Mpc}$), at smaller scales it can be parted in over- and under- density regions. We will label these regions as superclusters and voids, respectively. The correctness of such classification is justified by a formal smoothing of the matter density field with a window of large size $L > 10h^{-1}\text{Mpc}$, the obtained smoothed density is then characterized by the local density parameter $\Omega_{m,L}$: by definition, the regions with $\Omega_{m,L} > \Omega_m$ are superclusters and voids, respectively. Obviously, the 3D volume of the Universe is separated on these two regions (depending on the formal parameter $L$) which can be referred as the most extended 'elements' of LSS. Observationally, a typical difference $\Delta \equiv \Omega_{m,L} - \Omega_m$ is about $\sim 0.1 \div 0.2$ in largest voids and superclusters, $L \sim 100h^{-1}\text{Mpc}$. This scale is physically related with the cosmological horizon at the equipartition epoch.

The difference between superclusters and voids appears in many features. In addition to the different sign of $(\delta \rho)_L/\rho$ (averaged with the scale of the consequent element of LSS), superclusters and voids can be distinguished by their population. In supercluster we observe the brightest galaxies, their groups and clusters, galaxies with AGNs, and others, whereas all these bright objects avoid voids (Dalcanton$^1$, Mobasher & Trenthom$^9$) and, may be, $Ly_\alpha$-clouds (Rees$^{13}$, Rauch$^{12}$). The current observational data demonstrate that the main population of voids are dwarf galaxies (Shade$^{14}$, Pustilnik et al$^{11}$, Linder & Einasto$^7$) and, may be, $Ly_\alpha$-clouds (Rees$^{13}$, Rauch$^{12}$). This difference in the populations have to be appear in mass function, but these researches are at the beginning now. We try to solve this problem by modification of Press-Schechter formalism (Press & Schechter$^{10}$) which allows to calculate mass functions in voids and superclusters separately, in other words in the inhomogeneous Universe.
2 Press-Schechter formalism in the inhomogeneous Universe

The Press-Schechter (P&S) formalism is based on two main assumptions: (1) the density contrast field is Gaussian, the probability to find the density contrast \( \delta \) (in a sphere with center in \( x \) and radius \( R \)) larger than a fixed value \( \delta_c \) (White\textsuperscript{15}) is given by the formula:

\[
f(R) = \int_{\delta_c}^{\infty} \frac{1}{\sqrt{2\pi} \sigma(R)} \exp \left( -\frac{\delta^2}{2\sigma(R)^2} \right) d\delta;
\]

(2) the mass distribution of gravitationally bounded objects (“halos”) is the same as the distribution of density peaks. In other words, the collapse of matter is spherical.

To calculate \( \delta_c \) in standard P&S formalism, the linear overdensity of object \( \frac{\delta \rho}{\rho} \) in the uniform background at the collapse moment is considered. In the spatially flat matter-dominated Universe (Gunn & Gott\textsuperscript{3}) \( \delta_c = 1.686 \), cases of spatially flat \( \Lambda \)CDM (\( \Omega_{\text{tot}} \equiv \Omega_m + \Omega_\Lambda = 1 \)) and open cosmological models (\( \Omega_{\text{tot}} < 1 \)) were studied in (Madau et al\textsuperscript{8}, Eke et al\textsuperscript{2}, Lacey & Cole\textsuperscript{4}, Lahav et al\textsuperscript{6}, Lokas & Hoffman\textsuperscript{5}).

In standard form P&S formalism allows to calculate the averaged mass function in the homogeneous Universe. This standard formalism does not take into account the existence of inhomogeneities (such as voids and superclusters), where matter densities (\( \rho_v \) and \( \rho_{\text{sc}} \)) are different from the background value (\( \rho_0 \)). Now let us modify P&S formalism and describe the influence of inhomogeneous background related with existence of voids and superclusters.

To analyze this influence we need: (i) to separate LSS background and density perturbations related with gravitationally bounded halos; (ii) to calculate the growth rates of density perturbations in voids and superclusters separately.

The first step is rather simple one. Let us decompose the density contrast as follows:

\[\delta(R) = \delta(L) + \delta(R, L),\]

where \( \delta(L) \) is density contrast value related with void or supercluster with linear scale \( L \), \( \delta(L, R) \) is density contrast related with gravitationally bounded halos with linear scale \( R \) in void or supercluster.

The dispersion of the halo density contrast is:

\[
\sigma(R, L)^2 = \frac{1}{2\pi^2} \int_0^\infty P(k)|W(kR) - W(kL)|^2 k^2 dk;
\]

(1)

Thus, the mass function of halo in voids and superclusters as follows:

\[
n_L(M) = \sqrt{\frac{2}{\pi}} \frac{\rho_L \delta_{c,L}}{M} \frac{1}{\sigma(R, L)^2} \frac{\partial \sigma(R, L)}{\partial M} \exp \left( -\frac{\delta_{c,L}^2}{2\sigma(R, L)^2} \right),
\]

(2)

Here and further the subscriptions "\( L \)" means a size of local region. It is clear from the Fig.1 that if \( L \to \infty \) we deal with standard P&S curve.

To calculate \( \delta_{c,L} \) in void or supercluster we analyze the collapse of gravitationally bounded object in under- and over- density regions.
3 Collapse in the inhomogeneous Universe

3.1 The case of matter-dominated Universe

Radii of void and supercluster evolve as follows (Landau & Lifshits\(^{16}\)):

void:

\[
\frac{R}{R_v} = \frac{1}{2}(\text{ch} \eta - 1), \quad \frac{t}{t_v} = \frac{1}{\pi} (\text{sh} \eta - \eta),
\]

supercluster:

\[
\frac{R}{R_{scl}} = \frac{1}{2}(1 - \cos \eta), \quad \frac{t}{t_{scl}} = \frac{1}{\pi} (\eta - \sin \eta),
\]

where \(R\) is radius of void or supercluster, \(t\) and \(\eta\) are physical and conformal time (physical time is different in voids and superclusters), \(t_{scl}, t_v, R_{scl}\) and \(R_v\) are constants.

Using the methodology for calculation of density threshold in uniform background (White\(^{15}\)) we applied it for the case of inhomogeneous background. Assuming \(\eta \to 0\) and decomposing (3) and (4) we derive for void/supercluster:

\[
\frac{R}{R_{v,scl}} = \frac{t_v^2}{t_{scl}} \left\{ 1 \pm \frac{1}{20} \left( \frac{6\pi t}{t_{v,scl}} \right)^2 \right\}.
\]

As the density contrast related with halo in void/supercluster is as follows:

\[
\delta_L \equiv \frac{|\rho - \rho_L|}{\rho_L} = \frac{R^3_{L}}{R^3_{h halo}} - 1,
\]

where \(\rho\) is matter density in halo, \(R_{h halo}\) is a radius of halo, \(L\) denotes the void or supercluster. So, the critical density contrast related with halo in discussed regions are as follows:

\[
\delta_{v,scl} = 1.686 \left\{ 1 \pm \left( \frac{t}{t_{v,scl}} \right)^\frac{2}{3} \right\},
\]
where $t_{v,scl}$ is a function of total density $\Omega_{tot,L} = \Omega_m + \Omega_m$ in void or supercluster.

To relate $\delta_{c,L}$ and $\Omega_{tot,L}$ we can use Friedmann equation $3H^2 = 4\pi G\rho$, where all quantities have to be calculated in local region: $H_L = \frac{\dot{R}}{R}$, $\rho = \rho_m + \rho_{curv}$ where the first term is a matter, the second one is curvature.

So, in the superclusters ($\Omega_{tot} > 1$) we can rewrite the Friedman equation as follows:

$$3H^2 = 3\pi^2 \frac{t^2}{t^2_{scl}} \left[ \frac{1}{(1 - \cos \eta)^3} + \frac{\cos \eta}{(1 - \cos \eta)^3} \right],$$

where the first term is a averaged density in the supercluster. Finally, $\Omega_t$ in supercluster as follows:

$$\Omega_{scl} = \frac{\rho_{scl}}{\rho_{cr}} = \frac{9\pi^2}{4} \frac{t^2_{scl}}{t^2} \left( \frac{1}{1 - \cos \eta} \right)^3. \quad (6)$$

For void ($\Omega_{tot} < 1$) similar calculations give rise to:

$$\Omega_{tot}^v = \frac{9\pi^2}{4} \frac{t^2}{t^2_v} \left( \frac{1}{1 - \cosh \eta} \right)^3, \quad (7)$$

where ($t_{v,scl}$) and $\eta$ are related by (6) and (7). Taken together equations (3)-(7) allow to relate $\delta_{c,L}$ and $\Omega_{tot,L}$ in voids and superclusters for cosmological models without $\Lambda$-term (see dotted line on the Fig.2).

### 3.2 The case of non-zero $\Lambda$-term

To calculate $\delta_{c,L}$ in the case we use the approach proposed by Lahav et al. (Lahav et al\cite{Lahav1991}) and developed by others (Lacey & Cole\cite{Lacey1993}, Eke et al\cite{Eke1996}, Lokas & Hoffman\cite{Lokas2000}). Our calculations are mainly based on the last article (we refer it as LH00). To apply LH00 formalism we need to modify its basement. The first change is related to Hubble constant calculation. In LH00 $H$ does not depend on $\Omega_{tot}$ and is equal to observable value (65 km sec$^{-1}$ Mpc$^{-1}$). In our case $H \equiv H_L$ is a function of local regions. The second modification of LH00 is the introduction of the above-mentioned $L$, which is analogous to the scale factor in voids or superclusters.
To calculate $H_L$ we solve Friedman equation:

$$\left(\frac{H_L}{H_0}\right)^2 = \frac{\Omega_{m,L}}{L^3} + \Omega_\Lambda \pm \frac{\Omega_{m,L}}{L^2},$$

(8)

$H_0$ is the current value of Hubble constant in the homogeneous Universe, $\Omega_{m,L}$ is matter density in void or supercluster.

To relate $L$ with time we need a similar formula for the evolution of Hubble parameter in the Universe:

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_m}{a^3} + \Omega_\Lambda.$$  

(9)

Assuming that physical time interval is the same in the different regions of the Universe it can be written as follows: $dt = ad\eta = Ld\tau$, where $a$ is scale factor in the total Universe. Thus, we can rewrite last equation in more convenient form:

$$\int_{\beta_0}^\beta \frac{d\beta}{\sqrt{(\frac{1}{\beta} + \beta^2 \pm c)}} = \int_{\alpha_0}^\alpha \frac{d\alpha}{\sqrt{(\frac{1}{\alpha} + \alpha^2)}},$$

(10)

where $\alpha \equiv \left(\frac{\Omega_{m,L}}{\Omega_\Lambda}\right)^{-\frac{1}{2}} a$, $\beta \equiv \left(\frac{\Omega_{m,L}}{\Omega_\Lambda}\right)^{-\frac{1}{2}} L$, $c = \frac{(\Omega_{m,L}/\Omega_\Lambda)^{\frac{1}{2}}}{a}$. The subscription "0" means that the consequent quantity is taken now ($a_0 = 1$). Found $H_L$ and $L$ have to be substitute into eq.(19) of LH00. The result of numerical calculations is presented on Fig.3 (dashed and long dashed lines).

### 4 Mass function in voids and superclusters

Assuming results obtained in Section 2 and 3 we can calculate the new mass functions in void and supercluster separately. They are designed on Fig.3 for cosmological model with $\Omega^{\text{eol}}_m = 0.7$, $\Omega^v_m = 0.1$, $\Omega_\Lambda = 0.6$. All curves normalized by $\sigma_8$ (Eke et al. 1998).

It is clear from Fig.3 that the difference between mass functions calculated for slightly varying $\Omega_t$ is significant for all considered masses. In this case the ratio between mass functions

![Figure 3: The galaxy cluster mass functions for the Universe regions with different $\Omega_{t,L}$. The dashed line corresponds to $\Omega_{t,L} = 1.3$ (supercluster), the dotted line – to $\Omega_{t,L} = 1.0$ (flat homogeneous background), the solid line corresponds $\Omega_{t,L} = 0.7$ (void). $\Omega_\Lambda = 0.6$.](image)
in superclusters and voids can be approximated by exponential law:

\[ \frac{N_{\text{scl}}(> M)}{N_{v}(> M)} = 20.1 \exp \left( \frac{M}{9.4 \times 10^{12} h^{-1} M_{\odot}} \right). \]

This effect increases in models with larger \( \Omega_{\Lambda} \).

5 Discussion and Conclusions

To provide the correct calculation \( N_{L}(> M) \) in local region we take into account the dependence of the threshold density contrast (\( \delta_{c,L} \)) on the mean matter density in considered regions (\( \Omega_{m,L} \)). We find with 10% accuracy the following approximation for \( \delta_{c,L} \):

\[ \delta_{c,L} = 1.686 - 0.05\Omega_{\Lambda} - 2.42\Delta(1 + 1.78\Omega_{\Lambda}) + 1.7\Delta^2(1 + 3.1\Omega_{\Lambda}), \]

where \( \Delta = \Omega_{m,L} - \Omega_{m} \).

The significant difference between mass function in void and supercluster is found. We suggest an analytical approximation for the differential mass function in voids and superclusters for the spatially flat Universe with \( \Omega_{\Lambda} = 0.6 \) (the accuracy is less than 10%:

\[ n(M) = 3 \cdot 10^{-5} \cdot \left( \frac{M}{M_{0}} \right)^{-1.8} \exp \left( -\frac{M}{M_{0}} \right), \]

where the parameter \( M_{0} \) depends on \( \Omega_{m,L} \):

\[ M_{0} = [(1 + 21.8\Omega_{m,L})^2 + 0.4] \cdot 2.5 \cdot 10^{12}. \]

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