Conformal $\mathcal{N} = 0$, $d = 4$ Gauge Theories from AdS/CFT

Superstring Duality?

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Abstract

Non-supersymmetric $d = 4$ gauge theories which arise from superstring duality on a manifold $\text{AdS}_5 \times S^5/Z_p$ are cataloged for a range $2 \leq p \leq 41$. A number have vanishing two-loop gauge $\beta$–function, a necessary but not sufficient condition to be a conformal field theory.
The relationship of the Type IIB superstring to conformal gauge theory in $d = 4$ gives rise to an interesting class of gauge theories [1–4]. Choosing the simplest compactification [1] on $AdS_5 \times S_5$ gives rise to an $\mathcal{N} = 4$ SU(N) gauge theory which has been known for some time [3] to be conformal due to the extended global supersymmetry and non-renormalization theorems. All of the RGE $\beta$–functions for this $\mathcal{N} = 4$ case are vanishing in perturbation theory.

One of us (PHF) has recently [6] pursued the idea that an $\mathcal{N} = 0$ theory, without spacetime supersymmetry, arising from compactification [1]–16 on the orbifold $AdS_5 \times S_5/\Gamma$ (with $\Gamma \not\subset SU(3)$) could be conformal and, further, could accommodate the standard model. In the present note we systematically catalog the available $\mathcal{N} = 0$ theories for $\Gamma$ an abelian discrete group $\Gamma = \mathbb{Z}_p$. We also find the subset which has $\beta^{(2)}_g = 0$, a vanishing two-loop $\beta$–function for the gauge coupling, according to the criteria of [6]. In a future publication, we hope to find how many if any of the surviving theories satisfy $\beta^{(2)}_Y = 0$ and $\beta^{(2)}_H = 0$ for the Yukawa and Higgs self-coupling two-loop RGE $\beta$–functions respectively. Note that the one-loop $\beta$–functions satisfy $\beta^{(1)}_Y = 0$ and $\beta^{(1)}_H = 0$ because they are leading order in the planar expansion [17–20]. All one-loop $\mathcal{N} = 0$ calculations coincide with those of the conformal $\mathcal{N} = 4$ theory to leading order in $1/N$. However, beyond large $N$ and beyond one-loop this coincidence ceases, in general.

The ideas in Frampton [8] concerning the cosmological constant and model building beyond the standard model provide the motivation as follows. At a scale sufficiently above the weak scale the masses and VEVs of the standard model obviously become negligible. Consider now that the standard model is promoted by additional states to a conformal theory of the $d = 4 \mathcal{N} = 0$ type which will be highly constrained or even unique, as well as scale invariant. Low energy masses and VEVs are introduced softly into this conformal theory such as to preserve the desirable properties of vanishing vacuum energy and hence vanishing cosmological constant. Since no supersymmetry breaking is needed and provided the introduction of scales is sufficiently mild it is expected that a zero cosmological constant can be retained in this approach.
The embedding of $\Gamma = \mathbb{Z}_p$ in the complex three-dimensional space $\mathbb{C}^3$ can be conveniently specified by three integers $a_i = (a_1, a_2, a_3)$. The action of $\mathbb{Z}_p$ on the three complex coordinates $(X_1, X_2, X_3)$ is then:

$$(X_1, X_2, X_3) \xrightarrow{\mathbb{Z}_p} (\alpha^{a_1}X_1, \alpha^{a_2}X_2, \alpha^{a_3}X_3)$$

(1)

where $\alpha = \exp(2\pi i/p)$ and the elements of $\mathbb{Z}_p$ are $\alpha^r$ ($0 \leq r \leq (p-1)$).

The general rule for breaking supersymmetries is that for $\Gamma \subset SU(2)$, there remains $\mathcal{N} = 2$ supersymmetry; $\Gamma \subset SU(3)$ leaves $\mathcal{N} = 1$ supersymmetry; and for $\Gamma \not\subset SU(3)$, no supersymmetry ($\mathcal{N} = 0$) survives.

To ensure that $\Gamma \not\subset SU(3)$ the requirement is that

$$a_1 + a_2 + a_3 \neq 0 \pmod{p}$$

(2)

Each $a_i$ can, without loss of generality, be in the range $0 \leq a_i \leq (p-1)$. Further we may set $a_1 \leq a_2 \leq a_3$ since permutations of the $a_i$ are equivalent. Let us define $\nu_k(p)$ to be the number of possible $\mathcal{N} = 0$ theories with $k$ non-zero $a_i$ ($1 \leq k \leq 3$).

Since $a_i = (0, 0, a_3)$ is clearly equivalent to $a_i = (0, 0, p - a_3)$ the value of $\nu_1(p)$ is

$$\nu_1(p) = \lfloor p/2 \rfloor$$

(3)

where $\lfloor x \rfloor$ is the largest integer not greater than $x$.

For $\nu_2(p)$ we observe that $a_i = (0, a_2, a_3)$ is equivalent to $a_i = (0, p - a_3, p - a_2)$. Then we may derive, taking into account Eq.(2) that, for $p$ even

$$\nu_2(p) = 2 \sum_{r=1}^{\lfloor p/2 \rfloor} r = \frac{1}{4} p(p - 2)$$

(4)

while, for $p$ odd

$$\nu_2(p) = 2 \sum_{r=1}^{\lfloor p/2 \rfloor} r + \lfloor \frac{p}{2} \rfloor = \frac{1}{4} (p - 1)^2$$

(5)

For $\nu_3(p)$, the counting is only slightly more intricate. There is the equivalence of $a_i = (a_1, a_2, a_3)$ with $(p - a_3, p - a_2, p - a_1)$ as well as Eq.(2) to contend with.
In particular the theory \( a_i = (a_1, p/2, p-a_1) \) is a self-equivalent (SE) one; let the number of such theories be \( \nu_{SE}(p) \). Then it can be seen that \( \nu_{SE}(p) = p/2 \) for \( p \) even, and \( \nu_{SE}(p) = 0 \) for \( p \) odd. With regard to Eq. (2), let \( \nu_p(p) \) be the number of theories with \( \sum a_i = p \) and \( \nu_{2p}(p) \) be the number with \( \sum a_i = 2p \). Then because of the equivalence of \((a_1, a_2, a_3)\) with \((p - a_3, p - a_2, p - a_1)\), it follows that \( \nu_p(p) = \nu_{2p}(p) \). The value will be calculated below; in terms of it \( \nu_3(p) \) is given by

\[
\nu_3(p) = \frac{1}{2} \left[ \bar{\nu}(p) - 2\nu_p(p) + \nu_{SE}(p) \right] \quad (6)
\]

where \( \bar{\nu}(p) \) is the number of unrestricted \((a_1, a_2, a_3)\) satisfying \( 1 \leq a_i \leq (p-1) \) and \( a_1 \leq a_2 \leq a_3 \). Its value is given by

\[
\bar{\nu}(p) = \sum_{a_3=1}^{p-1} \sum_{a_2=1}^{p-1} a_2 = \frac{1}{6}p(p^2 - 1) \quad (7)
\]

It remains only to calculate \( \nu_p(p) \) given by

\[
\nu_p(p) = \sum_{a_1=1}^{\lfloor \frac{p}{2} \rfloor} \left( \left\lfloor \frac{p - a_1}{2} \right\rfloor - a_1 + 1 \right) \quad (8)
\]

The value of \( \nu_p(p) \) depends on the remainder when \( p \) is divided by 6. To show one case in detail, consider \( p = 6k \) where \( k \) is an integer. Then

\[
\nu_p(p) = \sum_{a_1=\text{odd}}^{2k-1} \left( 3k + \frac{1}{2} - \frac{3a_1}{2} \right) + \sum_{a_1=\text{even}}^{2k} \left( 3k + \frac{3a_1}{2} + 1 \right) = 3k^2 = \frac{1}{12}p^2 \quad (9)
\]

Hence from Eq. (3)

\[
\nu_3(p) = \frac{1}{2} \left[ \frac{1}{6}p(p^2 - 1) - \frac{1}{6}p^2 + \frac{p}{2} \right] = \frac{p}{12}(p^2 - p + 2) \quad (10)
\]

Taking \( \nu_1(p) \) from Eq. (3) and \( \nu_2(p) \) from Eq. (5) we find for \( p = 6k \)

\[
\nu_{TOTAL}(p) = \nu_1(p) + \nu_2(p) + \nu_3(p) = \frac{p}{12}(p^2 + 2p + 2) \quad (11)
\]

For \( p = 6k + 1 \) or \( p = 6k + 5 \) one finds similarly

\[
\nu_3(p) = \frac{1}{12}(p - 1)^2(p + 1) \quad (p = 6k + 1 \ or \ 6k + 5) \quad (12)
\]
\[ \nu_{TOTAL} = \frac{1}{12} (p - 1)(p + 1)(p + 2) \quad (p = 6k + 1 \; or \; 6k + 5) \] (13)

For \( p = 6k + 2 \) or \( p = 6k + 4 \)

\[ \nu_3(p) = \frac{1}{12} (p + 1)(p^2 - 2p + 4) \quad (p = 6k + 2 \; or \; 6k + 4) \] (14)

\[ \nu_{TOTAL} = \frac{1}{12} (p^3 + 2p^2 + 2p + 4) \quad (p = 6k + 2 \; or \; 6k + 4) \] (15)

and finally for \( p = 6k + 3 \)

\[ \nu_3(p) = \frac{1}{12} (p^3 - p^2 - p - 3) \quad (p = 6k + 3) \] (16)

\[ \nu_{TOTAL} = \frac{1}{12} (p^3 + 2p^2 - p - 6) \quad (p = 6k + 3) \] (17)

The values of \( \nu_1(p), \nu_2(p), \nu_3(p), \nu_{TOTAL}(p) \) and \( \sum_{p'=2}^{p} \nu_{TOTAL}(p') \) for \( 2 \leq p \leq 41 \) are listed in Table 1.

The next question is: of all these candidates for conformal \( \mathcal{N} = 0 \) theories, how many if any are conformal? As a first sifting we can apply the criterion found in [8] from vanishing of the two-loop RGE \( \beta \)–function \( \beta^{(2)}_g = 0 \), for the gauge coupling. The criterion is that \( a_1 + a_2 = a_3 \). Let us denote the number of theories fulfilling this by \( \nu_{alive}(p) \).

If \( p \) is odd there is no contamination by self-equivalent possibilities and the result is

\[ \nu_{alive} = \sum_{r=1}^{\frac{p-1}{2}} (p - 2r) = \frac{1}{4}(p - 1)^2 \quad (p = odd) \] (18)

For \( p \) even some self equivalent cases must be subtracted. The sum in Eq. (18) is \( \frac{1}{4}p(p - 2) \) and the number of self-equivalent cases to remove is \( \lfloor p/4 \rfloor \) with the results

\[ \nu_{alive} = \frac{1}{4}p(p - 3) \quad (p = 4k) \] (19)

\[ \nu_{alive} = \frac{1}{4}(p - 1)(p - 2) \quad (p = 4k + 2) \] (20)

In the last two columns of Table 1 are the values of \( \nu_{alive}(p) \) and \( \sum_{p'=2}^{p} \nu_{alive}(p') \).
Asymptotically for large $p$ the ratio $\nu_{\text{alive}}(p)/\nu_{\text{TOTAL}}(p) \sim 3/p$ and hence vanishes although $\nu_{\text{alive}}(p)$ diverges; the value of the ratio is e.g. 0.28 at $p = 5$ and at $p = 41$ is 0.066. It is being studied how the two-loop requirements $\beta_Y^{(2)} = 0$ and $\beta_H^{(2)} = 0$ select from such theories. That result will further indicate whether any $\nu_{\text{alive}}(p)$ can survive to all orders.

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Table 1. Values of $\nu_1(p)$, $\nu_2(p)$, $\nu_3(p)$, $\nu_{\text{TOTAL}}(p)$, $\sum_{p'=1}^p \nu_{\text{TOTAL}}(p')$, $\nu_{\text{alive}}(p)$ and $\sum_{p'=2}^p \nu_{\text{alive}}(p')$ for $2 \leq p \leq 41$.

| $p$ | $\nu_1(p)$ | $\nu_2(p)$ | $\nu_3(p)$ | $\nu_{\text{TOTAL}}(p)$ | $\sum \nu_{\text{TOTAL}}$ | $\nu_{\text{alive}}(p)$ | $\sum \nu_{\text{alive}}(p)$ |
|-----|------------|------------|------------|--------------------------|-----------------------------|--------------------------|-----------------------------|
| 2   | 1          | 0          | 1          | 2                        | 2                           | 0                        | 0                           |
| 3   | 1          | 1          | 1          | 3                        | 5                           | 1                        | 1                           |
| 4   | 2          | 2          | 5          | 9                        | 14                          | 1                        | 2                           |
| 5   | 2          | 4          | 8          | 14                       | 28                          | 4                        | 6                           |
| 6   | 3          | 6          | 16         | 25                       | 53                          | 5                        | 11                          |
| 7   | 3          | 9          | 24         | 36                       | 89                          | 9                        | 20                          |
| 8   | 4          | 12         | 39         | 55                       | 144                         | 10                       | 30                          |
| 9   | 4          | 16         | 53         | 73                       | 217                         | 16                       | 46                          |
| 10  | 5          | 20         | 77         | 102                      | 319                         | 18                       | 64                          |
| 11  | 5          | 25         | 100        | 130                      | 449                         | 25                       | 89                          |
| 12  | 6          | 30         | 134        | 170                      | 619                         | 27                       | 116                         |
| 13  | 6          | 36         | 168        | 210                      | 829                         | 36                       | 152                         |
| 14  | 7          | 42         | 215        | 264                      | 1093                        | 39                       | 191                         |
| 15  | 7          | 49         | 261        | 317                      | 1410                        | 49                       | 240                         |
| 16  | 8          | 56         | 323        | 387                      | 1797                        | 52                       | 292                         |
| 17  | 8          | 64         | 384        | 456                      | 2253                        | 64                       | 356                         |
| 18  | 9          | 72         | 462        | 543                      | 2796                        | 68                       | 424                         |
| 19  | 9          | 81         | 540        | 630                      | 3426                        | 81                       | 505                         |
| 20  | 10         | 90         | 637        | 737                      | 4163                        | 85                       | 590                         |
Table 1 (continued)

|   | $\nu_1(p)$ | $\nu_2(p)$ | $\nu_3(p)$ | $\nu_{TOTAL}(p)$ | $\sum \nu_{TOTAL}$ | $\nu_{alive}(p)$ | $\sum \nu_{alive}(p)$ |
|---|-------------|-------------|-------------|-------------------|---------------------|-----------------|---------------------|
| 21| 10          | 100         | 733         | 843               | 5006                | 100             | 690                 |
| 22| 11          | 110         | 851         | 972               | 5978                | 105             | 795                 |
| 23| 11          | 121         | 968         | 1100              | 7078                | 121             | 916                 |
| 24| 12          | 132         | 1108        | 1252              | 8330                | 126             | 1042                |
| 25| 12          | 144         | 1248        | 1404              | 9734                | 144             | 1186                |
| 26| 13          | 156         | 1413        | 1582              | 11316               | 150             | 1336                |
| 27| 13          | 169         | 1577        | 1759              | 13075               | 169             | 1505                |
| 28| 14          | 182         | 1769        | 1965              | 15040               | 175             | 1680                |
| 29| 14          | 196         | 1960        | 2170              | 17210               | 196             | 1876                |
| 30| 15          | 210         | 2180        | 2405              | 19615               | 203             | 2079                |
| 31| 15          | 225         | 2400        | 2640              | 22255               | 225             | 2304                |
| 32| 16          | 240         | 2651        | 2907              | 25162               | 232             | 2536                |
| 33| 16          | 256         | 2901        | 3173              | 28335               | 256             | 2792                |
| 34| 17          | 272         | 3185        | 3474              | 31809               | 264             | 3056                |
| 35| 17          | 289         | 3468        | 3774              | 35583               | 289             | 3345                |
| 36| 18          | 306         | 3796        | 4110              | 39693               | 297             | 3642                |
| 37| 18          | 324         | 4104        | 4446              | 44139               | 324             | 3966                |
| 38| 19          | 342         | 4459        | 4820              | 48959               | 333             | 4299                |
| 39| 19          | 361         | 4813        | 5193              | 54152               | 361             | 4660                |
| 40| 20          | 380         | 5207        | 5607              | 59759               | 370             | 5030                |
| 41| 20          | 400         | 5600        | 6020              | 65779               | 400             | 5430                |