Atmospheric and Solar neutrinos in the light of the SuperKamiokande results

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Abstract

The hierarchy $\Delta m^2_{\text{atm}} \gg \Delta m^2_\odot$ and the large $\theta_{23}$ mixing angle, as suggested by neutrino oscillation experiments, can be accounted for by a variety of lepton flavour models. A dichotomy emerges: i) Models were all neutrino masses are bounded by $m_{\text{atm}} \equiv (\Delta m^2_{\text{atm}})^{1/2} \approx 0.03 eV$; ii) Models of quasi-degenerate neutrinos. It is shown how these different patterns of neutrino masses may arise from different lepton flavour symmetries. Physical implications are discussed in the various cases.

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1 Introduction

Measurements of both solar and atmospheric neutrino fluxes provide strong although still indirect evidence for neutrino oscillations \[1\]. The Super-Kamiokande collaboration has measured the magnitude and angular distribution of the $\nu_\mu$ flux originating from cosmic ray induced atmospheric showers \[2\]. Especially, but not only the angular distribution of the $\nu_\mu$ flux calls for an interpretation of the data in terms of large angle ($\theta > 32^\circ$) neutrino oscillations, with $\nu_\mu$ disappearing to $\nu_\tau$ or a singlet neutrino and $\Delta m^2_{atm}$ close to $10^{-3}$eV$^2$. Five independent solar neutrino experiments , using three detection methods \[3\] \[4\] \[5\], have measured solar neutrino fluxes which differ significantly from expectations. The data is consistent with $\nu_e$ disappearance neutrino oscillations, occuring either inside the sun, with $\Delta m^2_\odot$ of order $10^{-5}$eV$^2$, or between the sun and the earth, with $\Delta m^2_\odot$ of order $10^{-10}$eV$^2$. The combination of data on atmospheric and solar neutrino fluxes therefore suggests a hierarchy of neutrino mass splittings: $\Delta m^2_{atm} \gg \Delta m^2_\odot$.

Although this is the physical picture to which we stick in this lecture, two caveats have to be remembered. A problem in one of the solar neutrino experiments or in the Standard Solar Model could still allow comparable mass differences for $\Delta m^2_{atm}$ and $\Delta m^2_\odot$ \[6\]. Furthermore, another experimental result exists \[7\], interpretable as due to neutrino oscillations. The problem with it is that its description together with the atmospheric and solar neutrino anomalies in terms of oscillations of the 3 standard neutrinos is impossible, even if $\Delta m^2_{atm} \approx \Delta m^2_\odot$ is allowed \[6\].

In this lecture I consider theories with three neutrinos. Ignoring the small contribution to the neutrino mass matrix which gives $\Delta m^2_\odot$, there are three possible forms for the neutrino mass eigenvalues:

- **“Hierarchical”**
  \[
  \begin{pmatrix}
  0 \\
  0 \\
  1
  \end{pmatrix}
  \]

- **“Pseudo-Dirac”**
  \[
  \begin{pmatrix}
  1 \\
  1 \\
  \alpha
  \end{pmatrix}
  \]

- **“Degenerate”**
  \[
  \begin{pmatrix}
  0 \\
  0 \\
  1
  \end{pmatrix} + M
  \begin{pmatrix}
  1 \\
  1 \\
  1
  \end{pmatrix}
  \]

where $m_{atm}$ is approximately 0.03 eV, the scale of the atmospheric oscillations. The real parameter $\alpha$ is either of order unity (but not very close to unity) or zero, while the mass scale $M$ is much larger than $m_{atm}$. I have chosen to order the eigenvalues so that $\Delta m^2_{atm} = \Delta m^2_{32}$, while $\Delta m^2_\odot = \Delta m^2_{21}$ vanishes until perturbations much less than $m_{atm}$ are added. An important implication of the Super-Kamiokande atmospheric data is that the mixing $\theta_{\mu\tau}$ is large. It is remarkable that this large mixing occurs between states with a hierarchy of $\Delta m^2$. 

1
What lies behind this pattern of neutrino masses and mixings? The conventional paradigm for models with flavour symmetries is the hierarchical case with hierarchically small mixing angles, typically given by $\theta_{ij} \approx (m_i/m_j)^{1/2}$. If the neutrino mass hierarchy is moderate, and if the charged and neutral contributions to $\theta_{atm}$ add, this kind of approach is probably not excluded by the data [8]. It looks more interesting to think, however, that the neutrino masses and mixings do not follow this conventional pattern, since this places considerable constraints on model building. An attractive possibility is that a broken flavour symmetry leads to the leading order masses of (1), (2) or (3), to a large $\theta_{atm}$, and to acceptable values for $\theta_{\odot}$ and $\Delta m^2_{\odot}$. To this purpose it is essential that the charged lepton mass matrix is discussed at the same time. Although it would be interesting to consider also the quark mass matrices, this problem is only briefly mentioned here.

It turns out that it is simpler to construct flavour symmetries which lead to (1) or (2) with large $\theta_{atm}$, as illustrated in Sect 2. In both hierarchical and pseudo-Dirac cases, the neutrino masses have upper bounds of $(\Delta m^2_{atm})^{1/2}$. In these schemes the sum of the neutrino masses is also bounded, $\Sigma_i m_{\nu i} \leq 0.1 \text{ eV}$, implying that neutrino hot dark matter has too low an abundance to be relevant for any cosmological or astrophysical observation. By contrast, it is more difficult to construct theories with flavour symmetries for the degenerate case [9], were the total sum $\Sigma_i m_{\nu i} = 3 M$ is unconstrained by any oscillation data. While non-Abelian symmetries can clearly obtain the degeneracy of (3) at zeroth order, the difficulty is in obtaining the desired lepton mass hierarchies and mixing angles, which requires flavour symmetry breaking vevs pointing in very different directions in group space. I propose a solution to this vacuum misalignment problem in Sect 4, which can be used to construct a variety of models, some of which predict $\theta_{atm} = 45^\circ$. Along these lines one can also construct a model with bimaximal mixing [10] having $\theta_{atm} = 45^\circ$ and $\theta_{12} = 45^\circ$ [11].

2 "Hierarchical" or "Pseudo-Dirac" neutrino masses

A large $\theta_{\mu\tau}$ mixing angle can simply be attributed to an abelian symmetry which does not distinguish between the muon and the tau left-handed lepton doublets. Since this does not constrain the right handed singlets, some asymmetry between them can be responsible of the $\mu - \tau$ mass difference. This is straightforward, but not enough, however. As emphasized above, the $\Delta m^2_{atm} \gg \Delta m^2_{\odot}$ hierarchy, if real and significant, has to be explained as well.

At least two textures for the neutrino mass matrices have been singled out which can be responsible for the "Pseudo-Dirac" or "Hierarchical" cases respectively and give at the same time a large $\theta_{atm}$ [12] [13]:

$$\lambda^{(0)I}_\nu = \begin{pmatrix} 0 & B & A \\ B & 0 & 0 \\ A & 0 & 0 \end{pmatrix}, \quad \lambda^{(0)II}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & B^2 & B \\ 0 & B & A \end{pmatrix}$$

The important point is that both textures $I$ and $II$ can be obtained by the seesaw mechanism with abelian symmetries and no tuning of parameters. For example, a simple model
for texture II has a single heavy Majorana right-handed neutrino, $N$, with interactions $l_{2,3}NH + MNN$, which could be guaranteed, for example, by a $Z_2$ symmetry with $l_{2,3}, N$ odd and $l_1$ even. A simple model for texture I has two heavy right-handed neutrinos which form the components of a Dirac state and have the interactions $l_1N_1H + l_{2,3}N_2H + MN_1N_2$. These interactions could result, for example, from a $U(1)$ symmetry with $N_1, l_{2,3}$ having charge $+1$, and $N_2, l_1$ having charge $-1$. In both cases, the missing right-handed neutrinos can be heavier, and/or have suitably suppressed couplings.

It makes actually sense to speak of neutrino mass textures only in association with corresponding charged lepton mass textures. In turn, these textures have to be obtainable by the same symmetries responsible of $\lambda^{(0)}_I\nu$ or $\lambda^{(0)}_II\nu$. It is easy to see how $\lambda^{(0)}_I\nu$ or $\lambda^{(0)}_II\nu$ can be coupled to

$$\lambda_E^{(0)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & 0 & A \end{pmatrix} \tag{5}$$

I have rotated the right-handed charged leptons so that the entries of the first two columns vanish. As in (4), I take $A$ and $B$ to be non-zero and comparable, since they both occur consistently with the unbroken flavour symmetries. Both in (4) and in (5) the label $^{(0)}$ denotes the fact that suitable perturbations have to be added to obtain fully realistic masses and mixings.

Finally, it is immediate to extend these solutions to quarks, in particular to SU(5) unification, where each field is replaced by its parent SU(5) multiplet ($l_{2,3} \rightarrow \bar{F}_{2,3}, etc$). Some qualitatively successful relations are actually implied by this extension, like, e.g., $V_{cb} = O(m_s/m_b)$.

### 3 Textures for quasi-degenerate neutrinos

What are the possible textures for the degenerate case in the flavour basis? These textures will provide the starting point for constructing theories with flavour symmetries. In passing from flavour basis to mass basis, the relative transformations on $e_L$ and $\nu_L$ gives the leptonic mixing matrix $V$. Defining $V$ by the charged current in the mass basis, $\overline{\nu} V \nu$, I choose to parameterize $V$ in the form

$$V = R(\theta_{23})R(\theta_{13})R(\theta_{12}) \tag{6}$$

where $R(\theta_{ij})$ represents a rotation in the $ij$ plane by angle $\theta_{ij}$, and diagonal phase matrices are left implicit. The angle $\theta_{23}$ is necessarily large as it is $\theta_{\text{atm}}$. In contrast, the Super-Kamiokande data constrains $\theta_{13} \leq 20^\circ \ [3, 14]$, and if $\Delta m^2_{\text{atm}} > 2 \times 10^{-3}$eV$^2$, then the CHOOZ data requires $\theta_{13} \leq 13^\circ \ [15]$. For small angle MSW oscillations in the sun, $\theta_{12} \approx 0.05$, while other descriptions of the solar fluxes require large values for $\theta_{12}$.

Which textures give such a $V$ together with the degenerate mass eigenvalues of eq. (3)? In searching for textures, I require that in the flavour basis any two non-zero entries are
either independent or equal up to a phase, as could follow simply from flavour symmetries. 
This allows just three possible textures for \( m_\nu \) at leading order [11]

\[
\begin{align*}
\text{"A"} \quad m_\nu &= M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + m_{atm} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7} \\
\text{"B"} \quad m_\nu &= M \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + m_{atm} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8} \\
\text{"C"} \quad m_\nu &= M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + m_{atm} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \tag{9}
\end{align*}
\]

Alternatives for the perturbations proportional to \( m_{atm} \) are possible. Each of these textures will have to be coupled to corresponding suitable textures for the charged lepton mass matrix \( m_E \), defined by \( \overline{e}Lm_EeR \). For example, in cases (A) and (B), the big \( \theta_{23} \) rotation angle will have to come from the diagonalization of \( m_E \).

To what degree are the three textures A, B and C the same physics written in different bases, and to what extent can they describe different physics? Any theory with degenerate neutrinos can be written in a texture A form, a texture B form or a texture C form, by using an appropriate choice of basis. However, for certain cases, the physics may be more transparent in one basis than in another, as illustrated later.

4 Degenerate neutrinos from broken non-abelian symmetries

The near degeneracy of the three neutrinos requires a non-abelian flavour symmetry, which I take to be \( SO(3) \), with the three lepton doublets, \( l \), transforming as a triplet. This is for simplicity – many discrete groups, such as a twisted product of two \( Z_2 \)s, would also give zeroth order neutrino degeneracy [11] [16].

Following ref [11], I work in a supersymmetric theory and introduce a set of “flavon” chiral superfields which spontaneously break \( SO(3) \). For now I just assign the desired vevs to these fields; later I show how to construct potentials which force these orientations. Also, for simplicity I assume one set of flavon fields, \( \chi \), couple to operators which give neutrino masses, and another set, \( \phi \), to operators for charged lepton masses. Fields are labelled according to the direction of the vev, e.g. \( \phi_3 = (0, 0, v) \). For example, texture A, with

\[
m_E = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta_2 & D_2 \\ 0 & \delta_3 & D_3 \end{pmatrix} \equiv m_{II}, \tag{10}
\]
results from the superpotential

$$W = (l \cdot l)hh + (l \cdot \chi_3)^2hh + (l \cdot \phi_3)\tau h + (l \cdot \phi_2)\tau h + (l \cdot \phi_3)\xi_{\mu}h + (l \cdot \phi_2)\xi_{\mu}h$$  \hspace{1cm} (11)

where the coefficient of each operator is understood to be an order unity coupling multiplied by the appropriate inverse power of the large flavour mass scale $M_f$. The lepton doublet $l$ and the $\phi, \chi$ flavons are all SO(3) triplets, while the right-handed charged leptons ($\epsilon, \mu, \tau$) and the Higgs doublets, $h$, are SO(3) singlets. The form of eqn. (11) may be guaranteed by additional Abelian flavour symmetries; in the limit where these symmetries are exact, the only charged lepton to acquire a mass is the $\tau$. These symmetries are broken by vevs of flavons $\xi_{\epsilon,\mu}$, which are SO(3) and standard model singlet fields. The hierarchy of charged fermion masses is then generated by\n
$$\langle \xi_{\epsilon,\mu} \rangle / M_f$$\n
The ratios $\langle \phi_{2,3} \rangle / M_f$ and $\langle \chi \rangle / M_f$ generate small dimensionless SO(3) symmetry breaking parameters. The first term of (11) generates an SO(3) invariant mass for the neutrinos corresponding to the first term in (7). The second term gives the second term of (7) with $m_{atm} / M_f = \langle \chi_3 \rangle^2 / M_f^2$. The remaining terms generate the charged lepton mass matrices. Note that the charged fermion masses vanish in the SO(3) symmetric limit — this is the way I reconcile the near degeneracy of the neutrino spectrum with the hierarchical charged lepton sector.

In this example we see that the origin of large $\theta_{atm}$ is due to the misalignment of the $\phi$ vev directions relative to that of the $\chi$ vev. This is generic. In theories with flavour symmetries, large $\theta_{atm}$ will always arise because of a misalignment of flavons in charged and neutral sectors. To obtain $\theta_{atm} = 45^\circ$, as preferred by the atmospheric data, requires however a very precise misalignment, which can occur as follows. In a basis where the $\chi$ vev is in the direction $(0, 0, 1)$, there should be a single $\phi$ field coupling to $\tau$ which has a vev in the direction $(0, 1, 1)$, where an independent phase for each entry is understood. As we shall now discuss, in theories based on SO(3), such an alignment occurs very easily, and hence should be viewed as a typical expectation, and certainly not as a fine tuning.

Consider any 2 dimensional subspace within the $l$ triplet, and label the resulting 2-component vector of SO(2) as $\ell = (\ell_1, \ell_2)$. At zeroth order in SO(2) breaking only the neutrinos of $\ell$ acquire a mass, and they are degenerate from $\ell \cdot \ell hh$. Introduce a flavon doublet $\chi = (\chi_1, \chi_2)$ which acquires a vev to break SO(2). If this field were real, then one could do an SO(2) rotation to set $\langle \chi_2 \rangle = 0$. However, in supersymmetric theories $\chi$ is complex and a general vev has the form $\langle \chi_i \rangle = a_i + ib_i$. Only one of these four real parameters can be set to zero using SO(2) rotations. Hence the scalar potential can determine a variety of interesting alignments. There are two alignments which are easily produced and are very useful in constructing theories:

**“SO(2)” Alignment:** \hspace{1cm} $W = X(\chi^2 - M^2)$; \hspace{1cm} $m_{\chi^2}^2 > 0$; \hspace{1cm} $\langle \chi \rangle = M(0, 1)$.  \hspace{1cm} (12)

The parameter $M$, which could result from the vev of some SO(2) singlet, can be taken real and positive by a phase choice for the fields. The parameter $m_{\chi^2}$ is a soft mass squared for the doublet $\chi$. 

5
The second example is:

“U(1)” Alignment: \[ W = X \phi^2; \quad m_\phi^2 < 0; \quad \langle \phi \rangle = V(1, i) \text{ or } V(1, -i). \] (13)

It is now the negative soft mass squared which forces a magnitude \( \sqrt{2} |V| \) for the vev.

The vev of the \( SO(2) \) alignment, (12), picks out the original \( SO(2) \) basis; however, the vev of the \( U(1) \) alignment, (13), picks out a new basis \( (\phi_+ , \phi_-) \), where \( \phi_\pm = (\phi_1 \pm i\phi_2)/\sqrt{2} \). If \( \langle (\phi_1 , \phi_2) \rangle \propto (1, i) \), then \( \langle (\phi_-, \phi_+) \rangle \propto (1, 0) \). An important feature of the \( U(1) \) basis is that the \( SO(2) \) invariant \( \phi_1^2 + \phi_2^2 \) has the form \( 2\phi_+\phi_- \). In the \( SO(3) \) theory, we usually think of \( (l \cdot l)hh \) as giving the unit matrix for neutrino masses as in texture A. However, if we use the \( U(1) \) basis for the 12 subspace, this operator actually gives the leading term in texture B, whereas if we use the \( U(1) \) basis in the 23 subspace we get the leading term in texture C.

Using this trick, it is simple to write down a variety of models for quasi degenerate neutrinos and charged leptons which naturally give rise to \( \theta_{23} = \theta_{\text{atm}} = 45^0 \) and possibly also to \( \theta_{12} = 45^0 \), up to corrections vanishing as \( m_\mu/m_\tau \) or \( m_e/m_\mu \) go respectively to zero [11]. In particular, quasi-degenerate neutrinos with \( \theta_{23} = \theta_{12} = 45^0 \) and \( \theta_{13} = 0 \) are known [17] to meet the condition that makes them evading not only the upper bound on the absolute mass from oscillation experiments but also the constraint from neutrinoless Double Beta decay.

5 Conclusions

The hierarchy \( \Delta m_{\text{atm}}^2 \gg \Delta m_{\odot}^2 \) and the large \( \theta_{23} \) mixing angle, as suggested by neutrino oscillation experiments, can be accounted for by a variety of lepton flavour models. A dichotomy emerges.

Models were all neutrino masses are bounded by \( m_{\text{atm}} \equiv (\Delta m_{\text{atm}}^2)^{\frac{1}{2}} \approx 0.03 \text{eV} \) are easily constructed, based on abelian flavour symmetries, even though special attention has to be payed to the origin of \( \Delta m_{\text{atm}}^2 \gg \Delta m_{\odot}^2 \). These models can be straightforwardly extended to quarks and, in particular, to \( SU(5) \) unification.

On the contrary, models of quasi-degenerate neutrinos are more likely based on non-abelian flavour symmetries. In the limit of exact flavour symmetry, the three neutrinos are massive and degenerate, while the three charged leptons are massless. Such zeroth-order masses result when the three lepton doublets form a real irreducible representation of some non-Abelian flavour group — for example, a triplet of \( SO(3) \). A sequential breaking of the flavour group then produces both a hierarchy of charged lepton masses and a hierarchy of neutrino \( \Delta m^2 \). The problem of extending these non-abelian symmetries to the quark sector is an open one.

An independent indication in favour of a non-abelian flavour symmetry may come from one or maybe two of the mixing angles being close to \( 45^0 \). A feature of the \( SO(3) \) symmetry breaking is that it may follow a different path in the charged and neutral sectors, leading to a vacuum misalignment with interesting consequences. Mixing angles
of 45° do arise from the simplest misalignment potentials. Such mixing can explain the atmospheric neutrino data, and can result in rotating away the $\beta\beta_0$ process, allowing significant amounts of neutrino hot dark matter.

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