Are there nodes in LaFePO, BaFe$_2$(AsP)$_2$, and KFe$_2$As$_2$?

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Abstract

We reexamined the experimental evidence for the possible existence of superconducting (SC) gap nodes in the three most suspected Fe-pnictide SC compounds: LaFePO, BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$, and KFe$_2$As$_2$. We showed that while the $T$-linear temperature dependence of the penetration depth $\lambda(T)$ of these three compounds indicates extremely clean nodal gap superconductors, the thermal conductivity data $\lim_{T \to 0} \kappa_S(H, T)/T$ unambiguously showed that LaFePO and BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ are extremely dirty, while KFe$_2$As$_2$ can possibly be clean. These apparently conflicting experimental data cast a serious doubt on the nodal gap possibility for LaFePO and BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$.

(Some figures may appear in colour only in the online journal)

1. Introduction

Despite the intensive research effort since the discovery of the Fe-based superconductors [1], the pairing symmetry of this new class of superconducting (SC) compounds has not yet been settled. Early theories and experiments appear to best support the sign changing s-wave pairing state (denoted as $s_\pm$ or $\pm s$ state in the literature) [2–6]. However, there exist several Fe-pnictide compounds that are not seemingly compatible with the $\pm s$ state but strongly suggest the presence of nodes in their SC states. Among others, LaFePO [7, 8], BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ [9], and KFe$_2$As$_2$ [10] are the most compelling compounds for the nodal gap, and to a lesser degree Ba(Fe$_{1−x}$Co$_x$)$_2$As$_2$ [11] is also suspected.

Commonly taken evidences for the nodal gap in the above-mentioned compounds are: (1) $T$-linear temperature dependence of penetration depth $\lambda(T)$ down to very low temperatures [7–10], and (2) a strong field dependence in the thermal conductivity slope $\lim_{T \to 0} \kappa(H, T)/T$, which is proportional to $\sqrt{H}$ and $H$, accompanied by a substantial fraction of the residual thermal conductivity $\lim_{T \to 0} \kappa(H, T)/T = \kappa_{00}/T$ [9, 12–14]. These features are the well known signatures of the nodal gap superconductors such as the d-wave superconductivity of the high-$T_c$ cuprates. And although it was recently shown that the strong field dependence of the thermal conductivity $\kappa(H, T \to 0)$ can be equally well explained with the $\pm s$-wave state [15], the extremely close $T$-linear $\lambda(T)$ is hard to reconcile with other than a clean nodal gap superconductor. Furthermore, the finite value of the residual thermal conductivity $\kappa_{00}/T$ measured in all three compounds [9, 12–14]—it is known that the nodal gap SC state produces a universal thermal conductivity slope independent of the impurity concentrations [16–18]—is more evidence for a nodal gap state, so it was widely interpreted as supporting the presence of nodes in these compounds together with the penetration depth data.

In this paper, however, we will show that there is a serious and irreconcilable conflict between the above-mentioned two experimental pieces of evidence for the nodal gap. We notice that (1) the universal value of $\kappa_{00}/T$ delivers no information about the dirtiness of the superconducting sample, however, (2) the normal state value of $\kappa_{N}/T$ tells us the amount of dirt in the sample. Then combining facts (1) and (2), the ratio $\kappa_{S}/\kappa_{N}$, which is the usually plotted data in experiments, is a very good indicator of the dirtiness of the sample. Inspecting the reported data of thermal conductivities of the three compounds, we concluded that the measured samples of LaFePO and BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ should have a large number of impurities and hence cannot be compatible with the $T$-linear $\lambda(T)$ within the nodal gap scenario. In the case of KFe$_2$As$_2$, there exist two very different thermal conductivity data sets by Dong et al [13] and Reid et al [14] with different...
$T_c$ samples, 3 K and 3.8 K, respectively. Our analysis of the thermal conductivity data showed that the sample of $14$ ($T_c \sim 3.8$ K) is cleaner with at least 10 times lower impurity concentration than the sample of $13$ ($T_c \sim 3$ K). Hence, the former sample of KFe$_2$As$_2$ can possibly be compatible with the $T$-linear $\lambda(T)$ data. We conclude that KFe$_2$As$_2$ can remain a possible nodal gap superconductor, but not LaFePO and BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$.

2. Theory

Assuming the quasiparticle excitation $E(k) = \sqrt{(v_1 k_1)^2 + (v_F k_2)^2}$ in the d-wave superconductor ($v_F$ Fermi velocity perpendicular to the Fermi surface (FS), $v_1$ nodal gap velocity parallel to the FS), the universal thermal (electric) conductivity in the nodal gap superconductor has been derived as follows $16–18$.

$$\frac{\kappa_S(T = 0)}{T} \sim \frac{\gamma_F^2}{v_F v_1} \frac{\gamma_n}{\gamma_n^2 + \Delta_0^2},$$

where $\Delta_0$ is the maximum gap value of the d-wave gap $\Delta(\theta)$ and $\gamma_n$ is the impurity induced damping rate at zero energy in the SC state. As is well known, $\gamma_n^{FS}$ indeed becomes universal, independent of the impurity concentrations and scattering strength, but only in the limit of $\Delta_0 \gg \gamma_n$; equation (1) clearly shows that a deviation occurs when $\gamma_n \sim \Delta_0$. The normal state limit of the above is easy to derive as

$$\frac{\kappa_N(T = 0)}{T} \sim \frac{\gamma_n}{v_F v_1^2},$$

where $\gamma_n$ is the impurity induced damping rate in the normal state and in general $\gamma_n \neq \gamma_n^{FS}$ for the same impurity strength and concentration. Knowing that the normal state should have no memory of the superconductivity, the above expression of $\gamma_n^{FS}$ is a disguised form for convenient comparison with equation (1) and $\gamma_n^{FS}$ becomes $\pi/4$ or a material scale of the FS size. Also we do not need to know the material specific parameters like $N(0), v_F, v_1$, etc to estimate the absolute magnitude of the thermal conductivities because for our purpose we only need the ratio

$$\frac{\kappa_S}{\kappa_N} = \frac{\gamma_n}{\gamma_n^{FS} + \Delta_0^2},$$

where $\Gamma = \frac{\gamma_n}{\gamma_n^{FS} + \Delta_0^2}$ is the impurity concentration parameter and in the second line of the above equations we used the results of $\gamma_n \approx \sqrt{\Delta_0^2 \Gamma}$ and $\gamma_n = \Gamma$ assuming the unitary impurity scattering strength. Equation (4) is the key result of this paper.

It was nice to observe the universal value of the thermal conductivity slope $\kappa_S/T$ of equation (1) to confirm a nodal superconductor. On the other hand, it was also a drawback since the universal thermal conductivity slope does not tell us how dirty or clean the sample is. However, as shown in equation (4), the ratio $\kappa_S/\kappa_N$ is an excellent indicator of the dirtiness of the specific SC sample. At this point, we would like to recall the fact that the typical experimental values of $\kappa_S/\kappa_N$ measured in LaFePO and BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ are $\frac{\Gamma}{\Delta_0} \approx 0.2 – 0.4$ which is quite a high level of impurity concentration for a nodal gap superconductor.

3. Numerical calculations and discussions

In this section we will show the full numerical calculations of the field dependence of $\lim_{T \to 0} \kappa_S(H, T)/T$ as well as the specific coefficient $\lim_{T \to 0} C(H, T)/T$ of the canonical d-wave gap ($\Delta(\theta) = \Delta_0 \cos 2\theta$) state with various impurity concentrations. Also the results of the penetration depth $\lambda(T)$ will be shown for the corresponding impurity concentrations.

To calculate the field dependences of the thermal conductivity and specific heat in the mixed state with applied field, we just need to calculate the position dependent DOS around the vortex core $v_s(r)$, which is given by $15, 19, 20$

$$\hat{G}(k, r, \omega, \theta) = \frac{1}{[\omega + v_s(r) \cdot k]_0 + \epsilon(k) \tau_3 + \Delta_0 \tau_1}$$

where $\tau_1$ are Pauli matrices and the supercurrent around the vortex core $v_s(r)$ is $\sim \frac{1}{\hbar} \frac{\partial}{\partial \theta}$, with $r$ the distance from the vortex core. The position dependent DOS is calculated as $N(\omega, r) = -\frac{1}{\pi} \text{Tr} \text{Im} \sum_k \int d\theta \ G_0(k, r, \omega, \theta)$. Finally, the field dependent quantities are obtained from the areal average DOS per unit volume as $N(\omega, H) = \int_{-\tilde{\omega}}^{r_{H}} \text{d}r \sqrt{\frac{\pi}{2}} \text{cosh} \left( \frac{r}{2} \right)$ with the magnetic length $r_H = \sqrt{\frac{\pi}{2} \hbar}$ (a flux quanta) and the SC coherence length $\xi$.

The impurity scattering is included by the $T$-matrix method $21–23$. The impurity induced self-energies renormalize the frequency and order parameter (OP) as $\omega \rightarrow \tilde{\omega} = \omega + \Sigma(\omega)$ and $\Delta_0 \rightarrow \tilde{\Delta} = \Delta_0 + \Sigma(\omega)$, with $\Sigma(\omega) = \Gamma \cdot T^{0,1}(\omega)$, where $T^{0,1}$ are the Pauli matrices $\tau^{0,1}$ components of the $T$-matrices in the Nambu space. However, $T^1$ is identically zero in the d-wave state. Then all impurity effects and the Volovik effect can be incorporated into the local Green’s function equation (5) by replacing $\omega$ by $\tilde{\omega}$.

After calculating the averaged $\bar{N}(\omega, H)$ for all frequencies, specific heat is calculated as

$$C(T, H) = \int_{-\infty}^{\infty} d\omega \left( \frac{\omega}{T} \right)^2 \frac{\bar{N}(\omega, H)}{\cosh^2 \left( \frac{\omega}{2T} \right)}.$$
Figure 1. Thermal conductivity $\kappa(H)/T$ versus the normalized fields $H/H_{c2}$ of the d-wave SC state, calculated at $T = 0.02\Delta_0$ for various impurity concentrations $\Gamma/\Delta_0 = 0.01, 0.02, 0.05, 0.1, 0.2,$ and 0.4. (unitary impurity). The inset shows the full range of fields up to $H/H_{c2} = 1$.

with

$$K(\omega, H, r, \theta) = \frac{1}{\text{Im} \sqrt{\tilde{z}^2 - \Delta_0^2}} \times \left( 1 + \frac{\tilde{z}^2 - |\Delta_0|^2}{\tilde{z}^2 + |\Delta_0|^2} \right)$$

(8)

where $\tilde{z} = \tilde{\omega} + \mathbf{v}_s(r) \cdot \mathbf{k}_f$, and the longitudinal and transversal thermal conductivities are calculated as $\kappa_1(T, H) = f_{\parallel}^{\text{R}} d^2r \ k(T, H, r)/\pi R_{\parallel}^2$ and $\kappa_1^{-1}(T, H) = f_{\perp}^{\text{R}} d^2r \ k^{-1}(T, H, r)/\pi R_{\perp}^2$, respectively.

3.1. Thermal conductivity

Figure 1 shows the theoretical thermal conductivity $\kappa(T, H)/T$ versus $H$ of the d-wave SC state calculated at the low temperature limit of $T/\Delta_0 = 1/50$ with the varying impurity concentrations of the unitary scatterers, $\Gamma/\Delta_0 = 0.01, 0.02, 0.05, 0.1, 0.2,$ and 0.4. First, the results indeed showed that the universal thermal conductivity $\lim_{T \to 0} \kappa(T, H)/T$ is well reproduced by our numerical calculations for the vast range of impurity concentrations. Second, it showed that the normal state limit of $\kappa(T, H)/T$, which is approached by increasing the field strength $H$ toward $H_{c2}$, is inversely proportional to the impurity concentration as shown in equation (4). The inset shows the results for the full range of $H/H_{c2} = [0 : 1]$ and we can see that $\kappa(H)/T$ sharply increases near $H_{c2}$. This is due to a rapid collapse of the gap $\Delta_0(H)$ toward $H_{c2}$ and our semiclassical approximation faithfully follows the Doppler shifting effect of this rapidly collapsing gap up to $H_{c2}$. While this is the correct calculation result with the semiclassical approximation, it is also known that this semiclassical approximation is not precisely correct near $H_{c2}$ where the quantum effect should become important [25]. So the exact field dependence of $\kappa(T, H)/T$ near $H_{c2}$ in figure 1 should not be taken seriously.

However, the important points for our purpose are: (1) at both limits, the universal limit value of $\lim_{T \to 0} \kappa(T, H)/T$ and the normal state limit value $\kappa(H = H_{c2})/T$ are exact and (2) the overall field dependence of the initially slow rise and then a rapid rise of $\kappa(H)/T$ near $H_{c2}$ is the genuinely correct behavior regardless of different theoretical treatments [26]. The main conclusions of this paper rely only on these two points. The main panel in figure 1 shows the results for the limited region of $H \leq 0.9H_{c2}$ for a better resolution of the low field behavior of $\kappa(H)/T$.

In figure 2, we replotted the theoretical results of figure 1 with two different normalizations and the experimental data of $[\kappa_S(H)/T]/[\kappa_N(T)]$ are overlaid. Figure 2(a) normalized $\kappa_S(H)/T$ of figure 1 with its $\kappa_S(H = H_{c2})/T$ values for each impurity concentration, and figure 2(b) used the $\kappa_S(H = 0.9H_{c2})/T$ values for normalization. The second normalization plot by the $H = 0.9H_{c2}$ values was chosen because it is the typical point of saturation before the sharp rise as seen in the inset of figure 1. and this concave-down saturation behavior approaching $H_{c2}$ is the typical observation in experiments [9, 12, 13]. Different normalizations yield different line shapes of the normalized $\kappa_S(H)/T$ which is supposed to be compared to the experimental $[\kappa_S(H)/T]/[\kappa_N(T)]$. The true behavior should be somewhere in between figures 2(a) and (b), but we
emphasize that this fine detail is irrelevant to our main conclusions and analysis. The overlaid experimental data are BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ [9], LaFePO [12], and KFe$_2$As$_2$ [13, 14].

Regardless of the choice of the normalizations, the experimental values of the residual thermal conductivity of BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ [9], LaFePO [12], and KFe$_2$As$_2$ [13] unambiguously indicate that these compounds should have the impurity concentration $\Gamma/\Delta_0 > 0.4$, which is an extremely dirty superconductor. Our theoretical calculations are with a single d-wave gap band. In reality, if there exists a nodal gap in these multiband Fe-pnictide compounds, the total gap function should consist of a nodal gap + one or two s-wave gaps, for example, a nodal ±s-wave gap [12]. If that is the case, the total $\kappa_N/T$ should increase due to the additional contributions from other bands. However, these additional s-wave gap bands have negligible contributions to the residual thermal conductivity $\lim_{T \to 0} \kappa_S(H)/T$ because they are fully gapped at low fields and low temperatures. Therefore, we need to have even higher impurity concentration than $\Gamma/\Delta_0 > 0.4$ in order to match the experimental data [9, 12, 13], of the normalized residual thermal conductivity $\lim_{T \to 0} \kappa_S(H)/T)/[\kappa_N(T)$.

On the other hand, the data of KFe$_2$As$_2$ by Reid et al [14] are very different from the data of KFe$_2$As$_2$ by Dong et al [13]. We can see that the data of Reid et al [14] reasonably fit the theoretical result in figure 2(a) with the impurity concentration, $\Gamma/\Delta_0 < 0.02$, which is a relatively clean limit. As discussed in [14], the discrepancy between the data of the two groups is understood by the sample purity. Judging from the $T_c$ of two samples (3.80 K and 3 K, respectively) and our theoretical calculations of $\kappa_S/\kappa_N$ in figure 2, the sample of Reid et al must be much cleaner, by about 10–20 times, than the one of Dong et al and it appears to be consistent with the result of the clean d-wave calculation with $\Gamma/\Delta_0 < 0.02$ in figure 2(a).

Summarizing the cases of BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ [9] and LaFePO [12], if we interpret the thermal conductivity data of these two compounds with a nodal gap scenario, we are led to conclude that both compounds are very dirty nodal gap superconductors. However, if this is true, the linear-$T$ penetration depth measurements [7–9] of these two compounds are in serious conflict with the dirty nodal gap scenario.

In the case of KFe$_2$As$_2$, we have two seemingly contradicting thermal conductivity measurements [13, 14] as seen in figures 2(a) and (b). However, despite the large difference in $T_c$ and the line shapes of $\kappa_S(H)/\kappa_N$, both samples reported a similar value of the residual thermal conductivity: $\kappa_{S0}/T = 3.7 \pm 0.4$ mW K$^{-2}$ cm$^{-1}$ [14] and $\kappa_{S0}/T = 2.27 \pm 0.02$ mW K$^{-2}$ cm$^{-1}$ ([13], here we ignored the correction by geometric factor discussed in [14]), respectively. This fact itself is strong supporting evidence for the nodal gap in the KFe$_2$As$_2$ compound. The different zero field intercepts of the data of the two samples in figure 2 are due to the normalization by the normal state thermal conductivities: $\kappa_S/T = 7.36 \pm 0.04$ mW K$^{-2}$ cm$^{-1}$ for Dong et al [13] and $\kappa_N/T \approx 109$ mW K$^{-2}$ cm$^{-1}$ for Reid et al [14].

Figure 3 shows the normalized specific heat coefficient $\gamma(H)$ versus fields $H/H_{c2}$ calculated at $T = 0.02\Delta_0$ of the d-wave SC state for various impurity concentrations $\Gamma/\Delta_0 = 0, 0.02, 0.05, 0.1, 0.2, 0.4$ (unitary impurity).

The data of Reid et al [14] can fit reasonably well with the clean nodal gap calculation ($\Gamma/\Delta_0 < 0.02$) for most of the low field region as seen in figure 2(a). The data of Dong et al [13] however, do not fit with any calculational results in figure 2. While the residual thermal conductivity value of it can be fitted with a dirty nodal gap with $\Gamma/\Delta_0 \approx 0.4$, the data for $H > 0$ increase much more rapidly at low fields and saturate to become flat for $H > 0.4H_{c2}$, not even close to any theoretical results in figure 2. But, if we assume the additional bands with s-wave gaps in addition to a nodal gap, as in a nodal ±s-wave state, it would be possible to fit the data of both clean and dirty limits of [13, 14]. The detailed SC properties of the nodal ±s-wave state will be reported in a future publication.

### 3.2. Specific heat coefficient and superfluid density

To foster the above discussions, we calculated the field dependence of the specific heat coefficient $\gamma(H)$ = $\lim_{T \to 0} C(H, T)/T$ and the temperature dependence of the superfluid density $\rho(T)$ = $1/\lambda^2(T)$ of the d-wave state with various concentrations of the unitary impurities as in figures 1 and 2. Figure 3 is the normalized $\gamma(H)/\rho_{fl}$ for $\Gamma/\Delta_0 = 0.4, 0.2, 0.1, 0.05, 0.02$, and 0 (no impurity limit). It shows that the expected behavior as the $\sqrt{H}$ behavior of $\gamma(H)$ in the clean limit becomes flattened with increasing impurity concentration. The only point that we want to emphasize for our purpose is that even a small amount of impurity, for example, $\Gamma/\Delta_0 = 0.02$, immediately creates a substantial fraction of the specific heat coefficient $\gamma(H = 0) \approx 0.2\gamma\rho_{fl}$. This demonstrates that a nodal gap such as the d-wave state is extremely vulnerable to the unitary impurity scattering to create the low energy excitations.

Figure 4 shows the normalized superfluid density $\rho_\delta(T) \sim 1/\lambda^2(T)$ of the d-wave state with corresponding impurity concentrations of figure 3. It also shows the well known behavior of $\rho_\delta(T)$ of the d-wave state with impurities. The typical $T$-linear $\rho_\delta(T)$ behavior in the clean
limit changes to the $T^2$ behavior at low temperatures with impurities. Similarly to the evolution of the specific heat coefficients in figure 3, even a small number of impurities changes quite a wide temperature region into the $T^2$ behavior. For example, the impurity concentration of $\Gamma/\Delta_0 = 0.02$ makes $\rho_S(T) \sim T^2$ for $0 < T < 0.2 T_c$. In view of the fact that all three pnictide compounds studied in this paper reported the $T$-linear behaviors of $\lambda(T)$ down to extremely low temperatures, LaFePO (0.02 < $T/T_c$ [7], and 0.08 < $T/T_c$ [8]), BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ (0.025 < $T/T_c$ [9]), and KFe$_2$As$_2$ (0.05 < $T/T_c$ [10]), the required purity of these samples for the nodal gap scenario is $\Gamma/\Delta_0 < 0.01$ or even cleaner. However, this clean nodal gap scenario is totally in contradiction to the thermal conductivity measurements $\kappa_S(H)/T$ in the cases of BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ [9] and LaFePO [12], but possibly not with the case of KFe$_2$As$_2$ [13, 14].

4. Remark on the quantum oscillation (QO) experiments

It is well known that the observation of QO such as de Haas–van Alphen (dHvA) oscillation is possible only with very clean samples and, therefore, is often quoted as an indication of the extreme purity of the probed sample. All three compounds studied in this paper have reported the QO experiments [27–29], hence we should worry about the consistency between the QO experiments, which warrant that the probed samples are clean, and our analysis of the thermal conductivity, which indicate that most of these compounds are not so clean. To begin with, we note that the criteria of the cleanliness for the QO signal and the SC properties are different; the former is $\Gamma < \hbar \omega_c (= \hbar e B/m^*; m^* =$ renormalized mass) and the latter is $\Gamma < \Delta_0$. Also, we need some interpretation for the typical dirtiness deduced from our thermal conductivity analysis, i.e. $\Gamma/\Delta_0$, which was based on the single d-wave model. We concluded in previous sections that only KFe$_2$As$_2$ is possibly consistent with a nodal gap, but LaFePO and BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ are not consistent with a nodal gap state but would be more consistent with a $\pm s$-wave gap model with a small isotropic gap on the major band and a larger isotropic gap on the minor band, namely, $\Delta_S \ll \Delta_L$ and $N(0)_S \gg N(0)_L$. Therefore, in the cases of LaFePO and BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$, for example, the deduced damping rate $\Gamma/\Delta_0 \sim 0.4$ should be understood as $\Gamma/\Delta_S \sim 0.4$ while $T_c$ of the compound is mostly governed by $\Delta_L$ [30]. Bearing these in mind, let us examine the cases of each compound below.

4.1. LaFePO ($T_c \approx 6 K$)

The QO measurement on LaFePO has observed signals with magnetic fields above ~9 T but the practically useful signals were obtained above ~20 and up to 45 T [27]. The estimated cyclotron frequency at $B = 20$ T is $\hbar \omega_c \sim 1.2–2.4$ meV with the renormalized mass $m^*/m_0$ (free electron mass) ~1–2 [27] and the estimated damping rate from our analysis is $\Gamma = 0.4\Delta_0 \sim 0.35$ meV assuming the BCS relation $\Delta_0/T_c = 1.75$. Therefore, the condition for the QO observation $\hbar \omega_c > \Gamma$ sufficiently holds for all fields $B > 20$ T, and, therefore, without invoking further argument of the multiple gaps, $\Delta_S$ and $\Delta_L$, the observation of the QO in LaFePO has no contradiction with the damping rate estimated in our analysis.

4.2. KFe$_2$As$_2$ ($T_c \approx 3 K$)

The QO signals on KFe$_2$As$_2$ [28] were obtained in the field range of 10–17.5 T. The estimated cyclotron frequency is $\hbar \omega_c \sim 0.2$ meV at $B = 10$ T with the heavily renormalized mass $m^*/m_0 \sim 6$ of this compound [28]. As discussed in section 4.1, there exist two very different thermal conductivity experiments [13, 14]; the sample by Reid et al [14] seems to be clean ($\Gamma < 0.02\Delta_0$) and the other one by Dong et al [13] seems to be dirtier ($\Gamma \approx 0.4\Delta_0$). The estimated damping rates are $\Gamma \sim 0.0087$ meV for the clean one and $\Gamma \sim 0.175$ meV for the dirty one, respectively, assuming the BCS relation $\Delta_0/T_c = 1.75$ with $T_c \approx 3$ K. Therefore, if the sample used for the QO experiment [28] is close to the cleaner one, there is absolutely no problem to observe the QO signals ($\hbar \omega_c > \Gamma; \hbar \omega_c \sim 0.2$ meV, $\Gamma \sim 0.0087$ meV). On the other hand, if the sample were on the side of the dirtier one, the observation of the QO signals should be very weak at best ($\Gamma \sim 0.175$ meV, $\hbar \omega_c \sim 0.2$ meV). Putting this together, there exists a wide range of sample purity between 0.0087 meV < $\Gamma < 0.175$ meV with which the QO experiment was possible.

4.3. BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ ($T_c \approx 30$ K)

Shishido et al [29] have performed the QO experiments with BaFe$_2$(As$_{1-x}$P$_x$)$_2$. However, the QO signals were obtained only with $1 \geq x \geq 0.41$ for the field range from 17 to 55 T, and the $x = 0.33$ sample never produced meaningful signals up to 55 T. On top of that, even in the samples of $1 \geq x \geq 0.41$ only the electron band FSSs ($\alpha$ and $\beta$ bands in their notations) produced signals but the holeband FSSs never produced measurable signals. With these, we can estimate that the overall damping rate of the $x = 0.33$ sample should be
higher than $\hbar \omega_c \sim 0.55$ meV using the renormalized mass $m^*/m_0 \approx 3$ and the maximum field strength $B = 55$ T used in experiments [29]. On the other hand, our estimated damping rate is $\Gamma = 0.4 \Delta_0 \sim 1.4$ meV using the BCS relation and $T_c = 30$ K. So it is consistent with the failure of the QO experiment for the $x = 0.33$ sample. In reality, since we have argued that the $\pm$-$s$-wave state is more consistent with BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$, if we understood $\Gamma = 0.4 \Delta_0$ as $\Gamma = 0.4 \Delta_s$, the real damping rate $\Gamma$ should be $< 1.4$ meV but still $> 0.55$ meV.

5. Conclusions

In conclusion, we have carefully reexamined the experimental evidence for the possible existence of the SC gap nodes in the three most suspected Fe-pnictide compounds, LaFePO, BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$, and KFe$_2$As$_2$. We have derived an exact relation for the ratio between the universal residual thermal conductivity $\kappa_S/T$ and its normal state value $\kappa_N/T$ in the d-wave state. Using this ratio $\kappa_S/\kappa_N \approx \Gamma/\Delta_0$ as an indicator to determine the dirtiness of the SC sample, we have shown that the reported experimental data of the thermal conductivity in BaFe$_2$(As$_{0.67}$P$_{0.33}$)$_2$ [9] and LaFePO [12] indicate that the measured samples are the dirty limit superconductors, hence contradicting the clean limit nodal gap scenario deduced from the penetration depth measurements [7–9]. To this end, if the nodal gap scenario fails for these two compounds, we propose a dirty $\pm$-$s$-wave state as a possible scenario to reconcile the apparently contradictory experiments of thermal conductivity and penetration depth measurements—the large residual thermal conductivity slope $\kappa_S/T$ and the $T$-linear $\lambda(T)$. In this scenario one isotropic s-wave gap is much smaller than the other one and the small gap is almost filled with the impurity band caused by a sufficient amount of impurity scattering.

In the case of KFe$_2$As$_2$, there exist two qualitatively different data of the thermal conductivity measurements [13, 14]. It appears that the one of Reid et al [14] is a clean sample but the one by Dong et al [13] contains at least 10 times more impurities. We concluded that the clean sample result can be consistent with a nodal gap scenario both for the thermal conductivity and penetration depth measurements. On the other hand, the thermal conductivity data of the dirty sample [13] can be understood with a dirty nodal gap plus additional isotropic gaps as in the nodal $\pm$-$s$-wave gap, but the $T$-linear $\lambda(T)$ cannot be compatible with this sample in any scenario. Therefore, the gap symmetry of KFe$_2$As$_2$ should be further investigated by the cross-examination of the penetration depth and thermal conductivity measurements with samples with various purities.

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