A multiplicity of regimes of transonic flow in channels with branching

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Abstract. The flow in 2D channel with a flat plate located along the plane of symmetry of the channel at a certain distance from the inlet was numerically studied. Straight segments formed the channel walls. At the inlet and the outlet of the channel, a supersonic flow was set. The first and third parts of the channel diverged. The middle part converged. Solutions to the Euler equations were obtained using the Ansys CFX finite volume solver. Four solutions were found that differed in the position of the shock waves. The hysteresis in the shock wave position versus Mach number given at the inlet was found. In a certain range of the inlet Mach number, there were asymmetric solutions of the equations.

1. Introduction
Instabilities of the transonic flow exist in the flow around airfoils and wings with a small curvature of the surface. It is found that the location of the shock waves is sensitive to a small change in the velocity of the free air flow and the angle of attack [1, 2, 3]. The sensitivity is due to the interaction of local supersonic regions near the airfoil. Calculations show the hysteresis of the position of the shock wave. Similar phenomena are also observed in channels of variable cross-section.

Channels with a transition from supersonic to subsonic flow are parts of many devices, such as jet engine air intakes, wind tunnel diffusers [4, 5, 6]. Problems of instability of shock waves were studied in [7, 8] for channels with a curved or broken wall. The calculations revealed the hysteresis of the position of the shock waves in relation to the Mach number of the incoming flow. Transonic flows in a channel with a central body were considered in [9, 10]. The central body divided the stream into two narrow channels. The initial section of the channel to the central body was divergent. Due to the presence of an expanding central body, the following sections in the narrow channels are converging. The side outer walls of the narrow channels had a break that increased the angle of expansion. After the fracture of the narrow channels has expanded.

Numerical simulation of a two-dimensional transonic flow in a channel with a central body revealed the hysteresis of the position of the shock wave in relation to the Mach number set at the channel inlet. This phenomenon exists for both inviscid and viscous gases. The range of Mach numbers at the input includes the domain of existence of asymmetric solutions of the Euler and Reynolds equations.
2. Formulation problem
In this paper, we investigated the flow in a channel in which the central body was a thin plate dividing the channel into two narrow channels. The leading edge of the plate was located at some distance from the entrance of the expanding section of the channel. There were two breaking points on the side walls. After the first break, the channel became converging, after the second break it became diverging (Figure 1). The inviscid gas flow was studied.

![Figure 1. The sketch of the channel.](image)

All dimensions are given in meters. Straight segments constitute lower and upper walls of the channel. The walls include breaking points at $x = 1.3$ and at $x = 2.0$.

- If $0 \leq x \leq 1.3$, then $y = \pm(0.88 + 0.064x)$,
- If $1.3 \leq x \leq 2.0$, then $y = \pm(0.964 - 0.179(x - 1.3))$, and
- If $2.0 \leq x \leq 3.0$, then $y = \pm(0.839 + 0.174(x - 2.0))$.

Inlet (at $x = 0$) and outlet (at $x = 3$) boundaries are vertical straight segments. The thickness of the flat plate is 0.02 m. The leading edge of the plate has the shape of a semicircle and is located at a distance of 0.4 m from the inlet boundary.

3. Numerical method
The unstructured meshes were generated using the Gmsh package. The program written in the Pascal language transformed them into three-dimensional grids, the transverse size of which was equal to one element. The converted meshes are obtained in the TGrid/Fluent format, which is suitable for calculation in the commercial Ansys CFX package. At the walls the absence of heat flow was set. Most calculations are performed with a mesh of about 200,000 elements. Some calculations were repeated with a mesh containing twice as many elements. The finest grid was mainly used for calculations at the inlet Mach numbers, which correspond to the change of flow regimes. At the inlet, a supersonic flow was set, coming from a point source located at a great distance. At the walls the velocity vector was parallel to the walls. Thus, the disturbances created by the walls at the inlet were excluded. At the inlet boundary, the Mach number $M_{in}$, the pressure $p_{in}$, and the temperature $T_{in}$ were set. At the outlet boundary the flow was supersonic. Solutions of Euler equations are obtained with Ansys CFX finite volume solver.
4. Results of calculations

Figures 2 and 3 show two patterns of Euler equations solutions. White line corresponds to Mach number \( M = 1 \). The distance from inlet boundary to shock wave is denoted by \( x_s \) as it is shown in Figure 2. In Figure 2 there is shock wave upstream of the leading edge of the flat plate. The flow pattern has two supersonic regions separated by the wide subsonic one. Increasing of inlet Mach number leads to coalescence of supersonic regions. In Figure 3 the supersonic regions coalesce and form the single supersonic region. There are additional small subsonic region upstream of the edge of the flat plate and subsonic region near flat plate downstream of the edge. At low inlet Mach number \( M_{in} \) the pattern with two supersonic regions is realized. After increasing \( M_{in} \) up to \( M_2 \) the distance \( x_s \) dramatically changes. Supersonic regions coalesce and we receive the flow pattern shown in Figure 3.

**Figure 2.** Two supersonic regions separated by subsonic one. \( M_{in} = 1.67 \).

**Figure 3.** Single supersonic region. \( M_{in} = 1.67 \).

Further reducing of the inlet Mach number leads to increasing of the subsonic region near the flat plate. Supersonic regions split at \( M_{in} = M_1 < M_2 \). So the range of hysteresis of shock wave position \( x_s \) versus Mach number at the inlet \( M_{in} \) is obtained. Figure 4 demonstrates hysteresis.

**Figure 4.** The dependence of the distance \( x_s \) on the inlet Mach number \( M_{in} \): 1 — two supersonic regions, 2 — single supersonic region, 3, 4 — asymmetric solution, 5 — fine mesh.
Asymmetric solutions of Euler equations can be obtained by introducing of asymmetry in the first stage of calculations. For example in the first stage, different inlet Mach numbers for upper and lower parts of inlet boundary are set. At the second stage, asymmetrical solution obtained in the first stage is taken as an initial condition. The boudary conditions are symmetric. The asymmetrical flow pattern is presented in Figure 5. There are two supersonic regions in the upper part of the channel. These supersonic regions are separated by a wide subsonic region. The lower part of the channel includes the single supersonic region. For asymmetric solution the distances $x_s$ in the upper and lower parts of the channel differ from one another. Hysteresis range is greater than the range of existence of asymmetric patterns.

![Figure 5. Asymmetric pattern at $M_{in} = 1.67$.](image)

Qualitatively, the flow regimes and transitions from one regime to another coincided with the modes established for the channel with a wedge-shaped central body [10]. The range of Mach numbers at which the hysteresis exists has increased slightly ($1.52 < M_{in} < 1.7$). As in the case of the wedge, within the hysteresis range there are asymmetric solutions with different structures of shock waves in the upper and lower parts of the channel.

5. Conclusions
Numerical calculations of 2D transonic flow of an inviscid fluid in the channel with flat plate that separate the flow are performed. Channel walls contained breaking points. Calculations revealed the existence of a four different flow regimes in a certain range of Mach numbers set at the inlet of the channel. The realization of one of the four regimes depends on the inlet Mach number and on the initial condition. Hysteresis of the position of shock wave versus inlet Mach number is revealed. Two regimes are asymmetric. Asymmetric solutions are obtained with two stages. In the first stage, different inlet Mach numbers for upper and lower parts of inlet boundary are set. In the second stage boundary conditions are symmetric, and initial condition is the result of the first stage.

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