Mathematical critical thinking ability of students at vocational highschool (adolescence)

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Abstract. Mathematical critical thinking ability is very important for vocational students and at that level critical thinking ability is currently developing. The research has a purpose to describe mathematical critical thinking ability based on the age range of vocational high school students. With purposive sampling, six students from the Vocational High School (SMKN 4 Bandung) were chosen as research subjects. This qualitative-descriptive research uses test instruments and interview guidelines. The data interpretation of this research is produced based on these indicators: 1) Giving an answer with relevant reasons. 2) Giving a solution with explanation to check a correct result. 3) Analyzing and clarifying questions, answers, and arguments. 4) Evaluating and considering source of truth. The result of this research is the students with 15 years and 16 years old were able to complete the first, second and fourth indicators, otherwise 17 years old could complete all indicators. The conclusion of the research is math critical thinking ability between 15, 16, 17 years old are relative same.

1. Introduction
Facing life with development in all aspects is certainly not easy. Everyone is required to be tough and have high self-quality, so they can compete and survive. Thus someone needs to have a high ability to solve various problems faced, one of which is to be able to think critically [1,2]. Critical thinking ability are also needed to succeed in the future [3]. In improving self quality, of course not only cognitive aspects that need to be developed but also aspects of attitude. For example, getting a needed and an unneeded information are very easy from social media. Whereas every information that comes always has two possibilities, namely the possibility of right and wrong. So that, critical thinking ability is always needed for making every decision carefully in a particular problem [4].

Critical thinking can be regarded as the ability and "habit of thinking" where individuals interact with the world by questioning it, rather than just accepting it [5]. Critical thinking can also be defined as reflective and reasonable practical thinking[6]. One of the subjects that facilitates the formation of a critical thinking process is mathematics. This is because critical thinking processes are one of the goals of mathematics learning [7] and mathematics learning can develop students' critical thinking ability [3]. So that, mathematical critical thinking ability is one of the important abilities possessed by students. Learning in principle provides opportunities for students to think and manage abilities so that learning is meaningful [8], as well as in mathematics learning. In this case the mathematics learning in question
is to provide a problem that is easy to understand but challenging for students [9]. The importance of other mathematical critical thinking is that it can shape personal and good character for students. Where students can account for their opinions accompanied by logical reasons [7].

Although critical thinking ability are important for students, it turns out that their achievements have not been optimal, including the achievement of vocational high school students. From the research conducted by Kharisma in 2018 on vocational high school students it was found that both students with high, medium and low initial mathematical abilities had an average low critical thinking ability, so it needed to be improved [10].

Keating argues that adolescence is a transition period that is important in the development of critical thinking [11]. Santrock suggests that there are cognitive changes that allow an increase in critical thinking during the teenage period [11]. Some developmental experts distinguish adolescence into three parts [12, 13]. Likewise, Blum distinguish adolescence consists of three parts, namely early adolescence (10-14 years), middle adolescence (15-17 years) and late adolescence (18-19 years) [14].

Based on some developmental experts distinguishing, can be said that critical thinking is very important for students while adolescence is a period in which critical thinking ability develop. So it is necessary to do further investigation regarding the mathematical critical thinking ability of students at middle adolescence, especially vocational high school students.

2. Methodology
This qualitative research uses a descriptive method to describe the mathematical critical thinking ability of students in middle adolescence, especially vocational students based on age range. There are three of age ranges in this research 15 years old, 16 years old and 17 years old. Data was collected from 16 high school students who entered middle adolescence with high mathematical abilities. The characteristics of the 16 students are shown in Table 1.

| Age       | Number of men | Number of women | Total Student Number |
|-----------|---------------|-----------------|----------------------|
| 15 years  | 2             | 2               | 4                    |
| 16 years  | 4             | 1               | 5                    |
| 17 years  | 7             | 0               | 7                    |

After collecting the data, the researcher selected two students with purposive sampling technique from each criteria to be investigated more. The selection is created by these criterias. The research subjects must be in the same age, male, and also have a high math ability.

| Student name       | Initial | Age     |
|--------------------|---------|---------|
| M. Kevin Safaat    | MKS     | 15 years|
| Hanif Abyan A      | HAA     | 15 years|
| Alvaro Dwi O.      | ADO     | 16 years|
| Nugraha A. S       | NAS     | 16 years|
| Gemilang A.        | GA      | 17 years|
| Daud Ibrahim H.    | DIH     | 17 years|

The main instrument in this research is the researchers themselves with supporting instruments in the form of instruments of critical thinking ability and interview guidelines. The instrument for testing critical thinking ability used in this research is an instrument made by Joko about the concept of the two-variable equation system in 2016 [7]. This instrument is used because the material tested must be
material that has been studied by each subject of research. Indicators of mathematical critical thinking ability in this research refer to the Ennis indicators in Baron and Sternberg in 1987 [7], among others: 1) Answering questions along with relevant reasons, 2) Checking the truth of the results or solutions accompanied by explanations, 3) Analyzing and clarifying questions, answers and arguments; and 4) Evaluating and considering reliable sources or arguments.

3. Results and Discussion

The following is the research data in the form of a description of mathematical critical thinking ability of the research subjects seen from indicators that refer to Ennis indicators in Baron and Sternberg in 1987 [7], among others:

3.1. Answering questions with relevant reasons

This indicator can be seen when students are able to answer questions correctly about the presence or absence of a solution to a system of equations two variables and explain the relevant reasons related to the answers given. The questions used to obtain data related to this indicator are as follows:

“The equation \( x + 2y = 8 \) and \( 3x + 6y = 30 \) is known. Does the SPLDV have a settlement set? If yes, how do you solve it? If not, what is the reason?”

From the research subject’s interview result, are known that they all understood with the question purpose above. They also gave a conclusion regarding to the question above, it means that SPLDV doesn’t have any collection of solution. Here will be explained more detail about the result

The fifteen years old students, MKS and HAA gave a same reason which is when they did an elimination \( x \) or \( y \), both were directly eliminated. MKS gave a calculation result about what he did, but HAA didn’t. After interview process with HAA, HAA said he didn’t do a calculation on the paper that already provided. How MKS and HAA answered the question could be seen on Figure 1 and 2.

![Figure 1. HAA’s Answer on Indicator 3.1](image)

Meanwhile the sixteen years old students, ADO and NAS gave a different result. ADO gave the reason that the coefficient of variable in the question was created same but gave a different value. On the other hand, NAS gave a reason that the various methods don’t give a value \( x \) and \( y \). For making their reason arguable, they gave a calculation reason that could be seen on Figure 3 and 4.

![Figure 2. MKS’s Answer on Indicator 3.1](image)
Figure 3. ADO’s Answer on Indicator 3.1

The seventeen years old students, GA and DIH gave a same reason that they couldn’t find a value x and y from SPLDV in the question. They made their reasons stronger, they used an elimination result for their calculation that could be seen in Figure 5 and 6.

Figure 4. NAS’s Answer on Indicator 3.1

Based on all given reasons, where known that several research subject did the same way to make a conclusion. The way is to find x and y value. The different points are when NAS tried to suppose x and y values to fulfill the first equation then enter an x and y value to a second equation, so he could know that the x and y value didn’t fulfill the second equation. The NAS’s answer could be seen in Figure 7.

Figure 5. GA’s Answer on Indicator 3.1

Figure 6. DIH’s Answer on Indicator 3.1

Figure 7. Answers to other NAS in Indicator 3.1

Based on the facts and analysis of the data above, researchers can interpret that both research subjects aged 15 years, 16 years and 17 years, they have been able to answer questions along with relevant reasons.

3.2. Check the correctness of results or solutions accompanied by explanations

This indicator could be seen when student could decide the right solution from all alternatives of the equation of two variables system. The student also gave a reason regarding the choice of their solution. The question that uses to get a related indicator data could be seen below.
“Given the equation \( x + 2y = 11 \) and \( 2x - y = 7 \). According to Uma, the settlement of the equation is \( x = 5 \) and \( y = 3 \). Whereas according to Meydia the set of resolutions of the equation is \( x = 3 \) and \( y = 5 \). Whose answer correct? Give an explanation!”

The interview results from all research subjects were known they understood with the purpose of question. They also gave a conclusion regarding why they choose Uma’s answer. Here are the reasons from all the research subjects.

Fifteen years old students, MKS and HAA gave a based explanation from their calculation result. HAA did once right calculation, but MKS did twice calculation, one of those is wrong calculation. How they answer the question could be seen in Figure 8 and 9.

![Figure 8. HAA’s Answer on Indicator 3.2](image1)

![Figure 9. MKS’s Answer on Indicator 3.2](image2)

Sixteen years old students, ADO and NAS gave a calculation result as their conclusion explanation. Both of them did a right calculation. ADO did once calculation, but NAS did twice calculation. How ADO and NAS answered the question could be seen in Figure 10 and 11.

![Figure 10. ADO’s Answer on Indicator 3.2](image3)
Seventeen years old students, GA and DIH gave a calculation result as their conclusion explanation. They did a right calculation with the same methods which are elimination and substitution. How GA and DIH answer could be seen in Figure 12 and 13.

From all research subject calculations, we know that they tried to find a solution based on the data from the question. After they got the solution, they tried to pair the alternative solution from question. But in Fifteen years old student they still did a wrong thing in making a calculation.

Based on the facts and analysis of the data above, the researcher can interpret that the two research subjects were 15 years, 16 years and 17 years, they have been able to check the truth of the results or solutions accompanied by explanations.

3.3. Analyze and clarify questions, answers and arguments
This indicator can be seen when the students are able to analyze and clarify questions, answers and arguments related to the problem of the system of two variable equations. The questions used to obtain data related to this indicator are as follows:

“In a shop, Febi bought 1 snack and 3 bottles of drinks for IDR 30,200. While Resti at the same shop bought 2 snacks and 1 bottle of drinks for IDR 25,900. Suppose Fathoni at the same shop
wants to buy 1 snack and 2 bottles of drinks and only carries IDR 25.000. Is enough Fathoni money? Give your opinion!

From interviews result of all research subjects could directly understood the purpose of the questions, except NAS. NAS explained that he couldn't understood because the basketball game noise disturb his focus. Here are the conclusions and the reason from each research subjects could be seen below.

The fifteen years old students, MKS and HAA gave a right conclusion regarding the problem. The Fathoni money is enough. This conclusion because the money that Fathoni need is less than IDR 25.000. But both have different calculation regarding how much money that Fathoni need. The result from HAA is IDR 23.340, but the result from MKS is IDR 21.300. Both did a wrong calculation. How HAA and MKS answers could be seen on Figure 14 and 15.

![Figure 14. HAA’s Answer on Indicator 3.3](image1)

![Figure 15. MKS’s Answer on Indicator 3.3](image2)

Sixteen years old students, ADO and NAS have a different conclusion. ADO did a right calculation but wrong conclusion which is Fathoni doesn’t have enough money. Other than that, NAS did a wrong calculation but gave a right conclusion, which is Fathoni has enough money. After interview process, ADO gave a wrong assumption that the money that Fathoni needed is IDR 25.000 and the money that Fathoni had is IDR 23.300. But NAS gave a conclusion based on his assumption. How ADO and NAS could be seen in Figure 16 and 17.
GA and DIH which are seventeen years old, both of them gave a right conclusion and also gave a right calculation to make their opinion stronger. How GA and DIH answers could be seen in Figure 18 and 19.
Figure 19. DIH’s Answer on Indicator 3.3

Based on the facts and analysis of the data above, researchers can interpret that the subject of research aged 15 years has not been able to analyze and clarify questions, answers and arguments, this is because both are still making mistakes in calculations. For research subjects aged 16 years, they have not been able to analyze and clarify questions, answers and arguments, this is because 16-year-old research subjects who still make mistakes in calculations. As for the 17-year-old research subject, they were able to analyze and clarify questions, answers and arguments.

3.4. Evaluate and consider trusted sources or arguments
This indicator can be seen when the student is able to evaluate the adequacy of the data provided by the system questions of two variable equations and consider the arguments related to the results of the evaluation given. The questions used to obtain data related to this indicator are as follows:

“In a park, Andi saw that there were some people who were walking around carrying their own pet cats. Then he counted the number of legs of people and cats there were 60 feet. To find out how many people were walking in the park, were the problems above enough data? Describe your opinion!”

From the results of interviews, all research subjects except GA were known to not understand the intentions of the question. While looking at the answers to the test results of all the research subjects both MKS and HAA who were 15 years old, ADO and NAS who were 16 years old and GA and DIH who were 17 years old, they were all able to evaluate the adequacy of the data and provide arguments regarding the evaluation results. The way to answer all research subjects is relatively the same where they assume there is only one equation that can be made from the problems in the problem in this indicator, namely $2x + 4y = 60$ where $x$ states the number of people and $y$ states the number of cats. For reasons related to the evaluation results that all research subjects provide, the point is "to complete a minimum two-variable equation system, two equations are needed.”

Based on the facts and analysis of the data above, the researcher can interpret that the research subjects, both aged 15 years, 16 years and 17 years, have been able to evaluate and consider reliable sources or arguments.

4. Conclusion
All fifteen and sixteen years old students could give the right answer with a reasonable reason, re-check their answer and solution, give an evaluation and consideration based on source of truth. In other side seventeen years old students could achieve all indicators, including the ability to make analysis and clarification of concern. From all facts that collected, we could conclude that the math critical thinking ability in 15, 16, 17 years old are relative same and on the high level. The critical
thinking ability is relative because all the research subjects in the same age, is middle adolescence. But the high critical thinking ability could be happened if the math ability of each subject is high. The suggestion of the next research is with the students that have middle ability in math, so we could get more variety result from all the age, early, middle and late adolescence.

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