Polyakov loops and monopoles in QCD

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Abstract

Monte-Carlo simulations of abelian projection of $T \neq 0$ pure lattice QCD show that 1) Polyakov loops written in terms of abelian link fields alone play a role of an order parameter of deconfinement transition, 2) the abelian Polyakov loops are decomposed into contributions from Dirac strings of monopoles and from photons, 3) vanishing of the abelian Polyakov loops in the confinement phase is due to the Dirac strings alone and the photons give a finite contribution in both phases. Moreover, these results appear to hold good with any abelian projection as seen from the studies in the maximally abelian gauge and in various unitary gauges.

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I. INTRODUCTION

Color confinement mechanism in QCD is still to be understood. Many works \[1\text{–}13\] have been done to clarify the confinement mechanism on the basis of the idea of abelian projection of QCD \[14\]. The abelian projection of QCD is to extract an abelian theory performing a partial gauge-fixing. After an abelian projection, $SU(3)$ QCD can be regarded as a $U(1) \times U(1)$ abelian gauge theory with magnetic monopoles and electric charges. 'tHooft conjectured that the condensation of the abelian monopoles is the confinement mechanism in QCD \[14\].

An effective $U(1)$ monopole action is derived from vacuum configurations in $SU(2)$ QCD after an abelian projection in a special gauge called maximally abelian (MA) gauge \[2,4,10,15\]. Entropy dominance over energy of the monopole loops, i.e., condensation of the monopole loops seems to occur always (for all values of the coupling constant $\beta$) in the infinite-volume limit when extended monopoles \[16\] are considered \[2,4,10\]. The abelian charge is confined due to the monopole condensation. The confinement of the abelian charge after abelian projection means color confinement as shown in \[14\]. Monopole condensation is known to be the confinement mechanism also in lattice compact QED \[17\text{–}22\].

The string tension is a key quantity of confinement. It vanishes at the deconfinement transition temperature $T_c$ \[23\]. It was shown that the same string tension can be derived from Wilson loops written in terms of abelian link fields alone after the abelian projection in the MA gauge \[1,24\]. The string tension derived from the abelian Wilson loops also vanishes at $T_c$ \[11\]. Moreover, the abelian Wilson loops can be expressed by a product of monopole and photon contributions \[3,9\]. The monopoles alone are responsible for the string tension in $T = 0$ \[3,9\] and $T \neq 0$ \[11\] $SU(2)$ QCD. The same results are obtained also in $SU(3)$ QCD \[25\].

These results strongly support the 'tHooft conjecture \[14\] in $SU(2)$ QCD. However, the above results are restricted to the special MA gauge. What happens in other abelian projections is not known yet. For example, abelian Wilson loops in some unitary gauges are
too small to derive reliably the string tension \[24\], although it does not mean that the string tension can not be derived in the gauges.

A Polyakov loop is another good order parameter, the vanishing of which means color-flux squeezing in the confinement phase. It is the aim of this note to show that 1) a Polyakov loop written in terms of abelian link fields alone is a good order parameter \[26\], 2) it can be written by a product of contributions from Dirac strings of monopoles and from photons, 3) the former alone vanishes in the confinement phase and 4) these results are independent of the gauge choice of abelian projection. The first statement in the MA gauge was already shown in \[26\].

II. DIRAC–STRING AND PHOTON CONTRIBUTIONS TO POLYAKOV LOOPS

We adopt the usual $SU(2)$ Wilson action. To study gauge dependence, we consider here three types of abelian projection, i.e., the MA gauge and two unitary gauges. The MA gauge is given \[27\] by performing a local gauge transformation $V(s)$ such that

$$R = \sum_{s,\mu} \text{Tr} \left( \sigma_3 \tilde{U}(s, \mu) \sigma_3 \tilde{U}^\dagger(s, \mu) \right)$$

is maximized. Then a matrix

$$X_1(s) = \sum_{\mu} \left( \tilde{U}(s, \mu) \sigma_3 \tilde{U}^\dagger(s, \mu) + \tilde{U}^\dagger(s - \hat{\mu}, \mu) \sigma_3 \tilde{U}(s - \hat{\mu}, \mu) \right)$$  \hspace{1cm} (1)

is diagonalized. Here

$$\tilde{U}(s, \mu) = V(s) U(s, \mu) V^{-1}(s + \hat{\mu}).$$  \hspace{1cm} (2)

Two unitary gauges considered here are defined by performing a local gauge transformation $V(s)$ such that one of the following two matrices is diagonalized:

$$X_2(s) = \prod_{i=1}^{N_t} \tilde{U}(s + (i - 1)\hat{4}, 4)$$  \hspace{1cm} (3)

$$X_3(s) = \sum_{\mu \neq \nu} \tilde{U}(s, \mu) \tilde{U}(s + \hat{\mu}, \nu) \tilde{U}^\dagger(s + \hat{\nu}, \mu) \tilde{U}^\dagger(s, \nu),$$  \hspace{1cm} (4)
where the sum in (4) is over all plaquette directions. We call the former (the latter) Polyakov (F12) gauge.

After the gauge fixing is over, there still remains a $U(1)$ symmetry. We can extract an abelian link gauge variable from the $SU(2)$ ones as follows;

$$\tilde{U}(s, \mu) = A(s, \mu)u(s, \mu),$$  \hspace{1cm} (5)

where $u(s, \mu)$ is a diagonal abelian gauge field and $A(s, \mu)$ has off-diagonal components corresponding to charged matters. Note that a $U(1)$ invariant quantity written in terms of the abelian link variables $u(s, \mu)$ after an abelian projection is $SU(2)$ invariant [9,26].

Now let us show that an abelian Polyakov loop operator after the abelian projection

$$P = \text{Re}\{\exp\{i\sum_{i=1}^{N_4} J_4(s + (i-1)\hat{4})\theta_4(s + (i-1)\hat{4})\}\},$$  \hspace{1cm} (6)

is given by a product of monopole and photon contributions. Here $J_4(s)$ is an external current taking +1 along the straight line in the fourth direction and $\theta_4(s)$ is an angle variable defined from $u(s, \mu)$ as follows:

$$u(s, \mu) = \begin{pmatrix} e^{i\theta_\mu(s)} & 0 \\ 0 & e^{-i\theta_\mu(s)} \end{pmatrix}.$$  \hspace{1cm} (7)

Using the definition of a plaquette variable $f_{\mu\nu}(s) = \partial_\mu \theta_\nu(s) - \partial_\nu \theta_\mu(s)$ where $\partial_\mu$ is a forward difference, we get

$$\theta_4(s) = -\sum_{s'} D(s - s')[\partial'_\nu f_{\nu4}(s') + \partial_4(\partial'_\nu \theta_\nu(s'))],$$  \hspace{1cm} (8)

where $D(s - s')$ is the lattice Coulomb propagator and $\partial'_\nu$ is a backward difference. We have used $\partial_\nu \partial'_\nu D(s - s') = -\delta_{ss'}$. Since $\partial'_4 J_4(s) = 0$, the second term in the right-hand side of (8) does not contribute to the abelian Polyakov loop (8). Hence we get

$$P = \text{Re}\{\exp\{-i\sum_{i=1}^{N_4} J_4(s + (i-1)\hat{4})\sum_{s'} D(s + (i-1)\hat{4} - s') \partial'_\nu f_{\nu4}(s')\}\}. $$  \hspace{1cm} (9)

The gauge plaquette variable can be decomposed into two terms:

$$f_{\mu\nu}(s) = \bar{f}_{\mu\nu}(s) + 2\pi n_{\mu\nu}(s), \hspace{1cm} (-\pi < \bar{f}_{\mu\nu}(s) \leq \pi).$$
where \( \partial'_\mu \tilde{f}_{\mu\nu}(s) \) includes only a photon field and \( n_{\mu\nu}(s) \) is an integer-valued plaquette variable denoting the number of Dirac strings through the plaquette coming out of monopoles \[19\]. Hence we get

\[
P = \text{Re}[P_1 \cdot P_2],
\]

\[
P_1 = \exp\left\{-i \sum_{i=1}^{N_4} J_4(s + (i-1)\hat{4}) \sum_{s'} D(s + (i-1)\hat{4} - s') \partial'_\nu \tilde{f}_{\nu\mu}(s')\right\},
\]

\[
P_2 = \exp\left\{-2\pi i \sum_{i=1}^{N_4} J_4(s + (i-1)\hat{4}) \sum_{s'} D(s + (i-1)\hat{4} - s') \partial'_\nu n_{\nu\mu}(s')\right\}.
\]

We observe the photon \((P_p)\) and the Dirac-string \((P_m)\) contributions separately:

\[
P_p = \text{Re}[P_1] \quad \text{and} \quad P_m = \text{Re}[P_2].
\]

\[\text{III. THE VILLAIN FORM OF QED}\]

The above separation can be done also in the case of compact QED. We first measure \(P_p, P_m\) and \(P\) adopting the Villain form \[28\] of the partition function on a \(8^4\) lattice. Since there are natural monopoles and DeGrand-Toussaint monopoles in the Villain case of QED, we observe \(P_m\) in terms of the two types of the monopoles. Since the auto-correlation time is long for \(\beta > \beta_c\), we have to perform Monte-Carlo simulations carefully. We follow the same method as done in \[13\]. The results are shown in Fig. 1. We find the following.

1. The monopole Dirac-string data vanish in the confinement phase, whereas the photon data remain finite and change gradually for all \(\beta\). The characteristic features of the Polyakov loops are then due to the behaviors of the Dirac-string contributions alone.

2. Monopole Polyakov loops show more enhancement than the total ones for \(\beta > \beta_c\).

3. Both types of monopoles give almost the same results.
IV. THE MA GAUGE IN $SU(2)$ AND $SU(3)$ QCD

The Monte-Carlo simulations were done in $SU(2)$ on $16^3 \times 4$ lattice from $\beta = 2.1$ to $\beta = 2.5$ in the MA gauge and in the unitary gauges. In $SU(3)$ QCD, we adopted $10^3 \times 2$ lattice from $\beta = 5.07$ to $\beta = 5.12$. All measurements were done every 50 sweeps (40 sweeps in the $SU(3)$ case) after a thermalization of 2000 sweeps. We took 50 configurations totally for measurements. The gauge-fixing criterion in the MA gauge is the same as done in Ref. [29].

The results in the MA gauge are shown in the following:

1. We plot the $SU(2)$ data in the MA gauge in Fig. 2. The abelian Polyakov loops remain zero in the confinement phase, whereas they begin to rise from the critical temperature $\beta_c = 2.298$ [30]. This was observed already in [26]. It is interesting that the Dirac-string contribution shows similar behaviors more drastically. It is zero for $\beta < \beta_c$, whereas it begins to rise rapidly and it reaches $\sim 1.0$ for large $\beta$. On the other hand, the photon part has a finite contribution for both phases and it changes only slightly. Characteristic behaviors of the abelian Polyakov loops as an order parameter of deconfinement transition are then explained by the Dirac-string part of monopoles alone. This is consistent with the results in [3,4] stating monopoles alone are responsible for the value of the string tension.

2. The same results are obtained also in pure $SU(3)$ QCD in the MA gauge as shown in Fig. 3. The monopole Dirac string alone is seen to be responsible for the flux squeezing. There is a clear hysteresis behavior showing the first order transition.

V. THE UNITARY GAUGES

We next study the case in the unitary gauges.

1. The data in the unitary gauges are plotted in Fig. 4 and Fig. 5. It is very interesting to see that qualitative features are similar to those in the MA gauge. Namely, the
abelian and the Dirac-string Polyakov loops are zero in the confinement phase, which suggests occurrence of flux squeezing in the unitary gauges, too. They show finite contribution above the critical temperature $\beta_c$. Photon contributions are finite and change gradually in both phases. The Dirac-string part of monopoles is responsible for the essential features of an order parameter. These are the first phenomena suggesting gauge independence of the 'tHooft conjecture.

2. Comparing the figures in the unitary gauges, one can see both are almost equal, although the gauge fixing conditions are quite different. The finite values in the deconfinement phase in Fig. 4 and Fig. 5 are much smaller than those in Fig. 2 in the MA gauge. It is similar to the behaviors of Wilson loops in these gauges studied in [24]. Namely, abelian Wilson loops in the MA gauge enhance drastically and then we could determine the string tension. On the other hand, the Wilson loops in the unitary gauges are too small to fix the string tension reliably.

3. There is a problem in extracting monopoles in the unitary gauges in comparison with the case in the MA gauge, as seen from the histogram of $f_{\mu\nu}$ in some configurations. The data in both gauges are shown in Fig. 6 and Fig. 7. The data in the MA gauge was first studied in [31]. In the MA gauge, quantum fluctuation is small and it is reliable to separate Dirac strings out of $f_{\mu\nu}$. However, in the unitary gauges, the separation between Dirac strings and quantum fluctuation is more ambiguous as seen from the bump around $\pm\pi$ in the histogram. To be noted, the bump becomes smaller as $\beta$ becomes larger. This is because quantum fluctuation becomes smaller for larger $\beta$. Hence, we may expect that the correct string tension can be derived even in unitary gauges when we consider the $T = 0$ case for large $\beta$ on larger lattices.
VI. CONCLUSION AND REMARKS

In conclusion, our analyses done here strongly suggest that abelian monopoles are responsible for confinement in $SU(2)$ QCD and condensation of the monopoles is the confinement mechanism. We have found first the data suggesting gauge independence of the 'tHooft conjecture.

Our data show the following picture of color confinement due to monopole condensation. Choose any $U(1)$ out of $SU(2)$ through an abelian projection. Then quarks and gluons behave like charged matters with respect to the $U(1)$ symmetry after the abelian projection. There are always monopoles with magnetic charges with respect to the magnetic $U(1)$ symmetry dual to the $U(1)$ chosen. The electric $U(1)$ charge is confined due to the dual Meissner effect caused by the condensation of the corresponding monopoles. The electric charge confinement after abelian projection is equivalent to color confinement as proved in [14]. Gauge independence of the confinement mechanism appears in this way.

In [24], abelian Wilson loops have been measured in various gauges. The abelian Wilson loops in the unitary gauges do not show abelian dominance and take a similar value to that without gauge-fixing. Suggested from the study, we have tried to measure abelian Polyakov loops without gauge-fixing, although such quantities are gauge variant. Abelian link fields are defined by choosing any one of isospin directions. Surprisingly enough, we have obtained a similar behavior as shown in Fig. 8. The abelian Polyakov loop is zero in the confinement phase and shows rising at the critical $\beta_c$. The responsibility of the Dirac string is also seen. The abelian quantity without gauge-fixing is variant under local $SU(2)$ transformation. Hence to see whether the above behavior includes some physical meaning or not, we have also measured abelian Polyakov loops composed of abelian links defined randomly at each site. Namely abelian links in the time direction at each site are defined to take a random isospin direction. The data are shown in Fig. 8. Qualitatively similar behaviors are obtained, although the finite values of the total and the Dirac-string contributions in the deconfinement region are smaller. These data may support the above
picture of gauge independence of the 'tHooft idea. To prove gauge independence definitely is to be studied in future.

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FIG. 1. Monopole Dirac string and photon contributions to Polyakov loops in the Villain model of compact QED.
FIG. 2. Monopole Dirac string and photon contributions to Polyakov loops in the MA gauge in $SU(2)$ QCD.
FIG. 3. Monopole Dirac string and photon contributions to Polyakov loops in the MA gauge in $SU(3)$ QCD.
FIG. 4. Monopole Dirac string and photon contributions to Polyakov loops in the Polyakov gauge.
FIG. 5. Monopole Dirac string and photon contributions to Polyakov loops in the $F_{12}$ gauge.
FIG. 6. Histogram of $f_{\mu\nu}$ in the MA gauge for some $\beta$. 
FIG. 7. Histogram of $f_{\mu\nu}$ in the Polyakov gauge for some $\beta$. 

SU2: POLYAKOV GAUGE

- beta=2.1
- beta=2.5
FIG. 8. Monopole Dirac string and photon contributions to abelian Polyakov loops without gauge-fixing.
FIG. 9. Monopole Dirac string and photon contributions to abelian Polyakov loops without gauge-fixing. Abelian links are defined randomly at each site.