Abstract

A new evaluation of the constraint on the number of light neutrino species ($N_\nu$) from big bang nucleosynthesis suggests a discrepancy between the predicted light element abundances and those inferred from observations, unless the inferred primordial $^4$He abundance has been underestimated by $0.014 \pm 0.004 \,(1\sigma)$ or less than 10% (95% C.L.) of $^3$He survives stellar processing. With the quoted systematic errors in the observed abundances and a conservative chemical evolution parameterization, the best fit to the combined data is $N_\nu = 2.1 \pm 0.3 \,(1\sigma)$ and the upper limit is $N_\nu < 2.6 \,(95\% \,\text{C.L.})$. The data are inconsistent with the Standard Model ($N_\nu = 3$) at the 98.6% C.L.
Along with the Hubble expansion and the cosmic microwave background radiation, big bang nucleosynthesis (BBN) provides one of the key quantitative tests of the standard big bang cosmology. The predicted primordial abundances of $^4\text{He}$, $\text{D}$, $^3\text{He}$, and $^7\text{Li}$ have been used to constrain the effective number of light neutrino species ($N_\nu$) \[3\]. The neutrino counting includes anything beyond the Standard Model [such as a right-handed (sterile) neutrino] that contributes to the energy density. This constraint is complementary to neutrino counting from the invisible width of $Z$ decays ($N_Z^\nu$), which is sensitive to a much larger mass range ($\lesssim M_Z/2$, where $M_Z$ is the $Z$ mass), but only to neutrinos fully coupled to the $Z$; the current result is $N_Z^\nu = 2.988 \pm 0.023$ \[6\], in agreement with the Standard Model ($N_Z^\nu = 3$).

The primordial $^4\text{He}$ abundance is sensitive to the competition between the early universe expansion rate and the weak interaction rates responsible for the interconversion of neutrons and protons. The expansion rate depends on the overall density and hence on $N_\nu$, while the weak rates are normalized via the neutron lifetime. Recent improvements in neutron lifetime measurements have significantly reduced the uncertainty in the $^4\text{He}$ prediction and, coupled with increasingly accurate astronomical data on extragalactic $^4\text{He}$, have led to tighter constraints on $N_\nu$; at 95% C.L. $N_\nu < 4$ in 1989 \[4\], $< 3.3$ in 1991 \[1\], and $< 3.04$ in 1994 \[5\]. However, a constraint as strong as $N_\nu < 3.04$ hints that the standard theory with $N_\nu = 3$ may not provide a good fit to the observations.

In this Letter we present new BBN limits on $N_\nu$ and the baryon-to-photon ratio ($\eta$) from simultaneous fits to the primordial $^4\text{He}$, $\text{D}$, $^3\text{He}$ and $^7\text{Li}$ abundances [hereafter we use the notation $Y_p$ ($^4\text{He}$ mass fraction), $y_{2p} = \text{D}/\text{H}$, $y_{3p} = ^3\text{He}/\text{H}$, and $y_{7p} = ^7\text{Li}/\text{H}$, fractions by number] inferred from the astrophysical observations. In particular, we incorporate new constraints on $y_{2p}$ \[7\], which are based on a generic chemical evolution parameterization \[8\] and

\[1\] Neglecting the baryon contribution, the total energy density $\rho_{\text{tot}}$ depends on $N_\nu$ as $\rho_{\text{tot}} = \rho_\gamma + \rho_e + N_\nu \rho_{\nu}$, where $\rho_\gamma$, $\rho_e$, and $\rho_{\nu}$ are the energy density of photons, electrons and positrons, and massless neutrinos (one species), respectively.
which significantly improve the prior constraints [9,11]. Our likelihood analysis systematically incorporates the theoretical and observational uncertainties. The theoretical uncertainties and their correlations are estimated by the Monte Carlo method [10,11,12]. Non-Gaussian uncertainties in the observations, such as the adopted systematic error in the value of \( Y_p \), the upper and lower limits for \( D \), and the model-dependent \(^3\)He survival parameter \( (g_3) \), are treated in a statistically well-defined way.

We adopt a primordial helium abundance estimated from low metallicity HII regions [13]:

\[
Y_p = 0.232 \pm 0.003 \text{ (stat)} \pm 0.005 \text{ (syst)},
\]

assuming a Gaussian distribution for the 1\(\sigma\) statistical uncertainty and a flat (top hat) distribution with a half width of 0.005 for the systematic uncertainty [12]. The systematic error is similar to that used for previous estimates on \( N_\nu \) [4,1,5] and to that obtained from Pagel’s analysis of the data [14].

New \( D \) constraints were obtained in Refs. [7,8], using pre-solar abundances of \( D \) and \(^3\)He (as inferred from \(^3\)He measurements in the solar wind, meteorites, and lunar soil [15]) and a generic chemical evolution parameterization:

\[
y_{2p} = (1.5 - 10.0) \times 10^{-5} \tag{2}
\]

\[
y_{3p} \leq 2.6 \times 10^{-5} \quad (95\% \text{ C.L.}) \tag{3}
\]

Although these constraints are independent of any specific model for primordial nucleosynthesis, standard BBN or otherwise, they do depend on the adopted \(^3\)He survival fraction \( g_3 \). To be consistent with prior analyses we adopt \( g_3 = 0.25 \) [9,16,11] although the effective \( g_3 \) of most models is significantly larger than this (see later discussion). When the observational bounds in Eqs. 2 and 3 are convolved with the BBN predictions (which are a function of \( \eta \) with \( N_\nu \) fixed at 3), even tighter constraints on \( D \) and \(^3\)He may be inferred [10]:

\[
y_{2p} = (3.5^{+2.7}_{-1.8}) \times 10^{-5} \text{ and } y_{3p} = (1.2 \pm 0.3) \times 10^{-5} \text{ at 95\% C.L.}.
\]

The resulting upper bound to \( y_{2p} \) is roughly 30\% lower than the corresponding bound in Ref. [11] and this has the effect
of raising the lower bound on the allowed range of $\eta$. Our central value for $y_{2p}$ is an order of magnitude smaller than the abundance inferred from a possible D detection in absorption against a high redshift QSO \[17,18\], but consistent with that reported for a different QSO absorption system \[19\].

We estimate the primordial $^7\text{Li}$ abundance from the metal-poor stars in our Galaxy’s halo:

$$y_{7p} = (1.2^{+4.0}_{-0.5}) \times 10^{-10} \quad (95\% \text{ C.L.}).$$

This estimate is consistent with other recent determinations \[20,11\] which take into account possible post big bang production and stellar depletion of $^7\text{Li}$.

For standard ($N_\nu = 3$) BBN, the theoretical predictions with the uncertainties (1$\sigma$) determined by the Monte Carlo technique are displayed as a function of $\eta$ in Fig. 1. Also shown in Fig. 1 are the constraints obtained by our likelihood analysis of the predictions and observations. The result is disturbing: the constraints on $\eta$ from the observed $^4\text{He}$ and D–$^3\text{He}$ abundances appear to be mutually inconsistent.

To explore this more carefully, all four elements are fit simultaneously, yielding the likelihood function for $N_\nu$ shown in Fig. 2 (where the likelihood is maximized with respect to $\eta$ for each $N_\nu$). The BBN predictions for the D, $^3\text{He}$, and $^7\text{Li}$ abundances are sensitive to the baryon-to-photon ratio $\eta$, but only weakly dependent on $N_\nu$. The BBN prediction for $^4\text{He}$ is very weakly dependent on $\eta$ and is approximately proportional to ($N_\nu - 3$). In our likelihood analysis, we have computed the Monte Carlo predictions for all of the element abundances for $1.5 \leq N_\nu \leq 4$ and $10^{-10} \leq \eta \leq 10^{-9}$. The $N_\nu$ and $\eta$ dependences of the uncertainties, the $\eta$ dependence of the correlations among the uncertainties \[21,5,12\], and the correlations between $\eta$ and the $y_{2p}$ and $y_{3p}$ values have all been included in the likelihood function.

Fig. 2 shows that the Standard Model ($N_\nu = 3$) yields an extremely poor fit. The best fit is for $N_\nu = 2.1 \pm 0.3$, and the upper-limit from the joint likelihood (Fig. 2) is

$$N_\nu < 2.6 \quad (95\% \text{ C.L.}).$$

(5)
The ratio of the likelihood of $N_\nu = 3$ to the best fit $N_\nu = 2.1$ is 0.014. This value provides an estimate of the goodness-of-fit of the standard ($N_\nu = 3$) theory.

The result of our simultaneous fit in the $\eta - N_\nu$ plane is shown in Fig. 3. The constraint on the baryon-photon ratio is $\eta = (4.4^{+0.8}_{-0.6}) \times 10^{-10}$ ($1\sigma$). The conflict between the lower and upper bounds on $\eta$ coming from D and $^4$He, respectively, has been noted before [22]. Our results exacerbate this discrepancy to roughly a 3 standard deviation effect, mainly due to our new D constraint.

In setting limits when the likelihood function extends beyond the physical parameter space, it is usually a reasonable (and conservative) prescription to renormalize the probability density distribution within the physical part of parameter space. This implies that one should renormalize the likelihood function for $N_\nu \geq 3$, when constraining any (nonstandard) particle contribution in addition to three massless neutrinos in the Standard Model. Examining the $N_\nu$ limit this way, the 95% C.L. limit for $N_\nu$ extends to 3.25 (for $\eta = 4.6 \times 10^{-10}$). However, we do not advocate this interpretation, since the poorness of the $N_\nu = 3$ fit makes this additional constraint for $N_\nu > 3$ meaningless.

The combined data (D, $^3$He, $^4$He, and $^7$Li) with the adopted uncertainties are inconsistent with standard ($N_\nu = 3$) BBN, for a conservative choice of $^3$He survival factor $g_3 = 0.25$. But what if some of the uncertainties have been underestimated? In particular, the systematic uncertainty in the $^4$He observational data may be 3 or more times larger [23] than the estimate in Ref. [13]. With $\eta$ determined by the combined D–$^3$He and $^7$Li constraints, BBN predicts $Y_p = 0.246 \pm 0.002$ ($1\sigma$), where the error includes the uncertainties from the D–$^3$He and $^7$Li constraints and from the BBN theory calculation. This value for $Y_p$ required for BBN consistency is 0.014 above the adopted observed value [Eq. (1)].

In Fig. 4 we show the $\eta - N_\nu$ constraints when the central value for $Y_p$ is systematically

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2There is no standard procedure to estimate the goodness-of-fit when non-Gaussian uncertainties are involved in a likelihood analysis. In addition to using the ratio of the likelihoods for $N_\nu = 2.1$ and 3, we have also estimated the goodness-of-fit with the standard $\chi^2$ method by approximating the errors with Gaussian distributions: the results from the two methods are consistent [12].
shifted by $\Delta Y$. To be consistent with $N_\nu = 3$, $\Delta Y$ has to be significantly larger than the systematic error adopted in Eq. 1. When $\Delta Y$ is fit as a free parameter with $N_\nu$ fixed to 3, we obtain $\Delta Y = 0.014 \pm 0.004$ at 1$\sigma$. Even allowing $\Delta Y$ to change freely, the $^7$Li and ISM D constraints still bound $\eta$ from above at $6.3 \times 10^{-10}$ (95% C.L.); ISM D alone bounds $\eta$ from above at $9 \times 10^{-10}$. The claim in Ref. [23] that $\eta$ can be as large as $\sim 14 \times 10^{-10}$ is unjustified.

We have also examined (Fig. 5) how the $\eta - N_\nu$ constraint is relaxed when the $^3$He survival factor, which affects the upper limit on $y_{2p}$, differs from that adopted ($g_3 = 0.25$). Relaxing the $y_{2p}$ upper limit so as to be consistent with the Y constraint requires a significantly smaller $g_3$. When $g_3$ is allowed to be a free parameter with $N_\nu = 3$ fixed, we obtain $g_3 \leq 0.10$ at 95% C.L., i.e. stellar destruction of $^3$He would need to be significantly larger than is implied by stellar and chemical evolution models. Although it is difficult to assign statistical probabilities to various values of $g_3$, one can assess the current status of models of Galactic chemical evolution and their associated $^3$He destruction. In this Letter we have adopted an effective $g_3 = 0.25$, a choice based on the fact that $g_3 \geq 0.25$ for any star [16,9,1]. Recent studies [24,25,8] have effective $g_3$'s larger than 0.25, a fact supported by Ostriker and Schramm’s analysis of horizontal branch stars [26] which concludes that $g_3 > 0.3$ and Rood, Bania and Wilson’s observation of $^3$He in planetary nebulae which suggests that low mass stars are net producers of $^3$He [27]. In order for the effective $g_3$ to be lower than 0.25, gas would have to be cycled thru several generations of relatively massive stars (which are the most efficient destroyers of $^3$He) without overproducing metals. Allowing stellar $^3$He production (as evidenced in low mass stars) would effectively increase $g_3$ and therefore exacerbate the present discrepancy between theory and observations. There are models and parameterizations which attempt to address these issues. The models of Olive et al. [28] include stellar $^3$He production in low mass stars and therefore tend towards large $g_3$. 

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3The $g_3$ used in previous BBN analyses is an effective $g_3$ in that it represents the $g_3$ per star integrated over all stars and cycled thru some number of stellar generations.
They conclude that “the only way to reduce $g_3$ below that of the massive stars (around 0.3) would be to argue that the gas in the region has been cycled through stars several times. Such an assumption however would invariably predict $^4\text{He}$ abundance factors of 2-4 higher than those observed.” Vangioni-Flam and Casse [29] find that the effective $g_3$ can be small, but the associated metals overproduction requires the revision of classical models of chemical evolution (e.g., including metal depletion by outflow). The interplay between the lower bound to $g_3$ and metal overproduction is reflected in Copi, Schramm and Turner’s [30] ‘stochastic history’ parameterization of chemical evolution. Their 95% C.L. lower bounds to $\eta$ are greater than or equal to ours provided they satisfy the metallicity constraint. It is our conclusion that our D constraint is robust and probably overly conservative - most models of chemical evolution yield D constraints which make the fit between theory and observation for $N_\nu = 3$ worse than we report here. For example, if we assume that $g_3$ is equally likely to be between 0.25 and 0.5, standard BBN would be ruled out at the 99.1% C.L..

The standard ($N_\nu = 3$) BBN predictions for the primordial $^4\text{He}$ and D abundances appear to be inconsistent with those inferred from observations, unless the inferred primordial $^4\text{He}$ mass fraction has been underestimated by $\Delta Y = 0.014 \pm 0.004$ or the $^3\text{He}$ survival fraction, $g_3$, is smaller than 0.10. While it may be that the crisis lies in the observational data and/or its extrapolation to primordial abundances, it is possible to alter standard BBN in order to reduce the $^4\text{He}$ prediction to the level consistent with the D constraint. The effective $N_\nu$ can be reduced to the range $2.1 \pm 0.3$ in several ways: massive tau neutrinos, neutrino degeneracy, or new decaying particles to name but a few.

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REFERENCES

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[1] T. P. Walker, G. Steigman, D. N. Schramm, K. A. Olive, and H. Kang, Astrophys. J. 376, 51 (1991).

[2] M. S. Smith, L. H. Kawano, and R. A. Malaney, Astrophys. J. Suppl. 85, 219 (1993).

[3] G. Steigman, D. N. Schramm, J. E. Gunn, Phys. Lett. B 66, 202 (1977).

[4] L. Kawano and D. N. Schramm, Nucl. Instr. Meth. A 284, 84 (1989); G. Steigman, Ann. NY Acad. Sci. 578, 138 (1989).

[5] P. Kernan and L. Krauss, Phys. Rev. Lett. 72, 3309 (1994).

[6] Particle Data Group, Phys. Rev. D 50, 1171 (1994).

[7] N. Hata, R. J. Scherrer, G. Steigman, D. Thomas, and T. P. Walker, Ohio State University Report No. OSU-TA-26/94, 1994 (to be published, Astrophys. J. 1995).

[8] G. Steigman and M. Tosi, Ohio State University Report No. OSU-TA-12/94, 1994 (to be published in Astrophys. J. Nov. 1, 1995).

[9] J. Yang, M. S. Turner, G. Steigman, D. N. Schramm, and K. Olive, Astrophys. J. 281, 493 (1984).

[10] L. M. Krauss and P. Romanelli, Astrophys. J. 358, 47 (1990).

[11] C. Copi, D. N. Schramm, and M. S. Turner, Science 267, 192 (1995).

[12] D. Thomas, N. Hata, R. J. Scherrer, G. Steigman, and T. P. Walker, work in progress.

[13] K. A. Olive and G. Steigman, Astrophys. J. Suppl. 97, 49 (1995).

[14] B. E. J. Pagel, Proc. Nat. Acad. Sci. 90, 4789 (1993).

[15] J. Geiss, in Origin and Evolution of the Elements, edited by N. Prantzos, E. Vangioni-
[16] D. S. P. Dearborn, D. N. Schramm, and G. Steigman, Astrophys. J. 302, 35 (1986).

[17] A. Songaila, L. L. Cowie, C. Hogan, and M. Rugers, Nature 368, 599 (1994).

[18] R. F. Carswell, R. J. Weymann, A. J. Cooke, and J. K. Webb, Mon. Not. R. Astron. Soc. 268, L1 (1994).

[19] D. Tytler, talk given at Munich Absorption Line Meeting, Munich, Germany, December 1994.

[20] S. Vauclair and C. Charbonnel, Astron. Astrophys. 295, 715 (1995); P. Molaro, F. Primas, and P. Bonifacio, Astron. Astrophys. 295, 47 (1995).

[21] T. P. Walker, in Relativistic Astrophysics and Particle Cosmology, edited by C. Akerlof and M. Srednicki, (The New York Academy of Sciences, New York, 1993) p. 745.

[22] A. Dar, J. Goldberg, and M. Rudzsky, Technion Report No. PHR-92-12, 1992 (unpublished); Copi, Schramm, and Turner, in Ref. [11]; L. M. Krauss and P. J. Kernan, Phys. Lett. B 347, 347 (1995).

[23] D. Sasselov and D. Goldwirth, Astrophys. J. Lett. 444, 5 (1995).

[24] G. Steigman and M. Tosi, Astrophys. J. 401, 15 (1992).

[25] E. Vangioni-Flam, K. A. Olive, and N. Prantzos, Astrophys. J. 427, 618 (1994).

[26] J. Ostriker and D. N. Schramm, in preparation (1994).

[27] R. T. Rood, T. M. Bania, and T. L. Wilson, Nature 355, 618 (1992).

[28] K. A. Olive, R. T. Rood, D. N. Schramm, J. Truran, and E. Vangioni-Flam, Astrophys. J. 444, 680 (1995).

[29] E. Vangioni-Flam and M. Casse, submitted to Astrophys. J. (1994).
[30] C. Copi, D. N. Schramm, and M. S. Turner, astro-ph/9506094: FERMILAB-PUB-95/140-A 1995.
FIGURES

FIG. 1. BBN predictions (solid lines) for $Y_p$, $y_{2p}$, and $y_{7p}$ with the theoretical uncertainties (1σ) estimated by the Monte Carlo method (dashed lines). Also shown are the regions constrained by the observations at 68% and 95% C.L. (shaded regions and dotted lines, respectively).

FIG. 2. The likelihood function for $N_\nu$ when the observations for $Y_p$, $y_{2p}$, $y_{3p}$, and $y_{7p}$ are fit simultaneously. For each $N_\nu$, the likelihood function is maximized for $\eta$. The upper limit is $N_\nu < 2.6$ (95% C.L.) The fit for the Standard Model ($N_\nu = 3$) is excluded at 98.6% C.L.

FIG. 3. The combined fit of the observations to $N_\nu$ and $\eta_{10} \equiv \eta \times 10^{10}$.

FIG. 4. The combined fit of the observations when the systematic uncertainty in the $^4$He observation ($\Delta Y_{sys}$) is fixed to 0, 0.005, 0.010, and 0.015.

FIG. 5. The combined fit of the observations when the $^3$He survival factor ($g_3$) is fixed to 0.10, 0.25, and 0.50.
Figure 2
Figure 3
BBN constraints (95% C.L.)

\( N_\nu = 3 \)

\( \Delta Y_{sys} \)

- - - - 0.0
- - - - 0.005
- - - - - 0.010
- - - - -- 0.015

Figure 4
BBN constraints (95% C.L.)

- $g_3 = 0.10$
- $g_3 = 0.25$
- $g_3 = 0.50$

$N_\nu = 3$

Figure 5