Light sterile neutrinos effects in processes with electron and muon neutrinos

V. V. Khruschov, S. V. Fomichev

NRC Kurchatov Institute, 123182 Moscow, Russia

Abstract

Sterile neutrinos with various masses could be participated in astrophysical and cosmological processes including mixing with active neutrinos, if exist. It is considered the possible effects of mixing of active and sterile neutrinos with masses of the order or less than 1 eV. For oscillation processes involving electron and muon neutrinos, as well as for beta decay and neutrinoless double beta decay processes, the contributions of light sterile neutrinos in the characteristics of these processes are calculated. For this purpose the estimates of the mixing parameters in the model with three active and three sterile neutrinos are made taking into account experimental data. Two cases of sterile neutrinos masses distribution are considered in detail. The results obtained can be used for interpretation of available experimental data, and also for predictions of subsequent experimental results.

Keywords: Neutrino oscillations, Short-baseline anomalies, Sterile neutrinos, Beta decay, Neutrinoless double beta decay.

PACS: 12.10.Kt; 12.90.+b; 13.35.Hb; 14.60.Pq; 14.60.St; 95.35.+d.

1 Introduction

It is well known that in the framework of the Standard Model (SM) of electromagnetic, weak and strong interactions of particles a quantitative agreement between most of experimental and theoretical results was achieved [1]. However, some data are emerging and increasing in number that cannot be well described within the framework of the SM. For instance, it relates to oscillations of active neutrinos, which have been detected experimentally. Nonzero neutrino masses must be invoked for their explanation. In this case SM is called as the modified SM ($\nu$SM). Using the neutrino mass states makes it possible to explain the oscillations of the known neutrino flavor states, i.e. electron, muon and tau neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$) with the help of the Pontecorvo–Maki–Nakagawa–Sakata mixing matrix $U_{\text{PMNS}} \equiv U$. The standard parametrization for matrix $U$ is given in the review [1].

In the present paper some peculiarities of neutrino processes are considered. For their explanation possibly it is necessary to go out beyond not only the SM but also the $\nu$SM. This is especially true in regard to so called neutrino anomalies at
short distances (short baselines, SBL) from the source [2, 3, 4]. The appearance of these anomalies can be explained perhaps with the effects of new particles, namely, light sterile neutrinos (LSN) with the characteristic mass scale about 1 eV. Let us mention that three sterile neutrinos with masses of the order of 1 eV were already introduced in Ref. [5] for the explanation of the LSND anomaly. Generally a LSN number can be arbitrary. The most using model now is the (3+1) model with one LSN, but the (3 + 2) and (3 + 3) models are used as well (see, for example, [6, 7]). Below we consider the LSN effects in processes with electron and muon neutrinos in the framework of the version of the (3+3) model, which was elaborated in Refs. [8, 9, 10].

The content of the paper is as follows. Section 2 provides short reference to SBL neutrino anomalies perceived in a number of experiments [4]. Section 3 contains brief description and some results of the used (3 + 3) model with three LSN [10]. In Section 4 the test values of the model parameters are proposed taking into account experimental data. Thereon calculations of the survival probabilities of electron and muon neutrinos and probabilities of transition of muon neutrinos to electron neutrinos with the help of the obtained parameter values are carried out. The calculation results are represented in the graphic form (Figs. 1, 2 and 3). The effective masses of electron neutrinos, which can be measured in experiments on beta decay and neutrinoless double beta decay, are also estimated. In the final Section 5 the main results of the paper are discussed.

2 Neutrino data anomalies on small distances from neutrino sources

In addition to the known standard data on neutrino oscillations for three active neutrinos, indications are obtained related to anomalous data for SBL neutrino fluxes in a number of processes. These anomalies cannot be explained by using the oscillatory parameters for only known active neutrinos. They include the LSND (or accelerator) anomaly (AA) [11, 12, 13, 14], the gallium (or calibration) anomaly (GA) [15, 16, 17] and the reactor (antineutrino) anomaly (RA or RAA) [18, 19, 20, 21, 22, 23]. AA, GA and RA manifest themselves at small distances, more precisely, at such distances from the source $L$, when the value of the parameter $\Delta m^2 L / E$ is of the order of unity (where $E$ is the neutrino energy and $\Delta m^2$ is the square of the characteristic mass scale of the considered oscillations). $\Delta m^2$ is equal to the difference of the squared masses of the participating neutrinos. Excluding purely experimental problems, the SBL anomalies can be explained by the presence of at least one neutrino with a mass of about 1 eV, which does not interact directly with the $\nu_{\text{SM}}$ gauge bosons, therefore they are called LSN.

AA was noticed firstly by the LSND collaboration in the reaction with the transition of a muon antineutrino into an electron antineutrino [11]. Then this result was confirmed and extended by adding the results of the reaction with the transition of a muon neutrino into an electron neutrino in the MiniBooNE experiment [12, 13] and with less significance in the MicroBooNE experiment [14, 24]. GA was discovered during the calibration of detectors for the Ga-Ge experiment at the SAGE and GALLEX facilities [15, 17] and has been confirmed
in the BEST experiment \[25, 26\]. Now the confidence level of the AA and GA in the mentioned experiments lies in the \(4 - 5\sigma\) CL interval \[24, 25\].

After recalculating the values of the antineutrino flux from the reactor, the theoretical values turned out to be 3% higher than those used before \[18, 19\] that led to RA about the 3\(\sigma\) level \[20\]. However, it should be noted that the \(\beta\)-spectra of the decay products of uranium and plutonium isotopes introduce large systematic uncertainties into the reactor spectra \[27\].

3 Some propositions and results of the \((3+3)\) model

In spite of that the SBL anomalies can be explained, as mentioned above, with the presence only one LSN with the characteristic mass scale about 1 eV, the number of additional sterile neutrinos with different masses, in principle, can be arbitrary \[2, 28, 29\]. Phenomenological models with \(N\) SN are usually denoted as \((3 + N)\) models.

\((3 + N)\) models are often used to describe SBL anomalies as well as some astrophysical data \[30\]. It is desirable that the \(N\) number would be minimal that is why \((3+1)\) and \((3+2)\) models are mostly used \[6\]. However, taking into account the possible left-right symmetry of weak interactions, \((3 + 3)\) models attract considerable attention (see, e.g. \[7, 31\]). In this paper, to take into account the LSN effects, the \((3 + 3)\) model is also used \[10\], which includes three known active neutrinos \(\nu_a\) \((a = e, \mu, \tau)\) and three new (in this case light sterile) neutrinos: sterile neutrino \(\nu_s\), hidden neutrino \(\nu_h\) and dark neutrino \(\nu_d\). Thus, the model contains six neutrino flavor states and six neutrino mass states, therefore a 6×6 mixing matrix is used. This matrix is dubbed as the generalized mixing matrix or the generalized Pontecorvo–Maki–Nakagawa–Sakata matrix \(U_{\text{GPMNS}} \equiv U_{\text{mix}} \[8\].

\(U_{\text{mix}}\) can be represented as the matrix product \(VP\), where \(P\) is a diagonal matrix containing the Majorana CP-phases \(\phi_i, i = 1, \ldots, 5\), that is \(P = \text{diag}\{e^{i\phi_1}, \ldots, e^{i\phi_5}, 1\}\). Below we will use only some particular forms of matrix \(U_{\text{mix}}\). In this case, we will denote the Dirac CP-phases as \(\delta_i\) and \(\kappa_j\), and the mixing angles as \(\theta_i\) and \(\eta_j\). In doing so, \(\delta_1 \equiv \delta_{\text{CP}}, \theta_1 \equiv \theta_{12}, \theta_2 \equiv \theta_{23}\) and \(\theta_3 \equiv \theta_{13}\). Only the normal order (NO) of the active neutrino mass states and the value \(\delta_{\text{CP}} = 1.2\pi\) will be considered.

For compactness of formulas, we introduce symbols \(\nu_b\) and \(\nu_i\)′ for sterile left flavor fields and sterile left mass fields, respectively. So fields \(\nu_b\) with index \(b\) contain fields \(\nu_s, \nu_h\) and \(\nu_d\), while \(i\)′ denotes a set of indices 4, 5 and 6. A total 6×6 mixing matrix \(U_{\text{mix}}\) can be represented in the form of 3×3 matrices \(R, T, V\) and \(W\):

\[
\begin{pmatrix}
\nu_a \\
\nu_b
\end{pmatrix} = U_{\text{mix}} \begin{pmatrix}
\nu_i \\
\nu_i\prime
\end{pmatrix} \equiv \begin{pmatrix}
R & T \\
V & W
\end{pmatrix} \begin{pmatrix}
\nu_i \\
\nu_i\prime
\end{pmatrix}.
\] (1)

Let us represent the matrix \(R\) in the form of \(R = \varkappa U_{\text{PMNS}}\), where \(\varkappa = 1 - \epsilon\), and \(\epsilon\) is a small quantity. The matrix \(T\) in the equation (1) must also be a small matrix as compared with the Pontecorvo–Maki–Nakagawa–Sakata 3×3 matrix for active neutrinos \(U_{\text{PMNS}} \equiv U (UU^+ = I)\). So, active neutrinos mix by means of the \(U\) matrix, as it should be in the \(\nu\text{SM}\), when choosing the appropriate normalization.
In the present state of the art, it is enough to restrict ourselves only to a minimal number of parameters of matrix $U_{\text{mix}}$, that allows one to interpret available (still rather heterogeneous) experimental data. The transition to the full matrix with all parameters should be done later on, when additional data related to the SBL anomalies will be obtained.

We choose $T$ in the form of $T = \sqrt{1 - \kappa^2} a$, where $a$ is an arbitrary unitary $3 \times 3$ matrix ($aa^+ = I$), then $U_{\text{mix}}$ can be written in the following form:

$$U_{\text{mix}} = \left( \begin{array}{cc} R & T \\ V & W \end{array} \right) \equiv \left( \begin{array}{cc} \kappa U & \sqrt{1 - \kappa^2} bU \\ \sqrt{1 - \kappa^2} a & \kappa \end{array} \right),$$

(2)

where $b$ is also an arbitrary unitary $3 \times 3$ matrix ($bb^+ = I$), moreover $c = -ba$.

Under these conditions the $U_{\text{mix}}$ matrix will be unitary, too ($U_{\text{mix}}^+ U_{\text{mix}} = I$). In particular, we will use the following $a$ and $b$ matrices:

$$a = \left( \begin{array}{ccc} \cos \eta_2 & \sin \eta_2 & 0 \\ -\sin \eta_2 & \cos \eta_2 & 0 \\ 0 & 0 & e^{-i\kappa_2} \end{array} \right),$$

(3a)

$$b = - \left( \begin{array}{ccc} \cos \eta_1 & \sin \eta_1 & 0 \\ -\sin \eta_1 & \cos \eta_1 & 0 \\ 0 & 0 & e^{-i\kappa_1} \end{array} \right),$$

(3b)

where $\kappa_1$ and $\kappa_2$ are mixing phases between active and sterile neutrinos, while $\eta_1$ and $\eta_2$ are mixing angles between them. The remaining elements of the matrix $U_{\text{mix}}$ are obtained in a standard way.

To make calculations more specific, we will use the following test values of the new mixing parameters:

$$\kappa_1 = \kappa_2 = -\pi/2, \quad \eta_1 = 5^\circ, \quad \eta_2 = \pm 15^\circ, \pm 30^\circ,$$

(4)

and restrict the values of the small parameter $\epsilon$ as $\epsilon \lesssim 0.1$.

Let us specify the neutrino masses by the set of values $\{m\} = \{m_i, m_{\nu}\}$. For active neutrino masses, we take the estimates presented in the works [31, 32] for the NO case (in units of eV), which do not contradict recent experimental data: $m_1 \approx 0.0016, m_2 \approx 0.0088, m_3 \approx 0.0497$. The values of mixing angles $\theta_{ij}$ for three active neutrinos, which define the Pontecorvo–Maki–Nakagawa–Sakata matrix, are calculated from the relations $\sin^2 \theta_{12} \approx 0.318, \sin^2 \theta_{23} \approx 0.566$ and $\sin^2 \theta_{13} \approx 0.0222$. These relations are obtained on the basis of processing of experimental data for the NO-case and are given in the paper [33].

In order to choose the values of the masses $m_4$ and $m_5$, we use the results of the experiments BEST, DANSS, NEUTRINO-4, MiniBooNE and MacroBooNE [23, 24, 25, 26]. The BEST, DANSS and NEUTRINO-4 experiments are devoted to testing the existence of GA and RA associated with a deficit of electron neutrinos and antineutrinos at short distances from the source, respectively. It would be expected that the values of the sterile neutrino mass determined in these two experiments would be practically the same. However, if the value of $m_4 = 1.1$ in eV agrees with the data of the BEST experiment, but for the NEUTRINO-4 experiment the analogous value is equal to $m_4 = 2.5$ eV. Since further we will
use one more result of the BEST experiment for the value of $R$ (see below), we choose $m_4 = 1.1$ eV. For the value of $m_5$, we take either the value 0.6 eV close to the results of the MiniBooNE and MacroBooNE experiments, or 0.002 eV [34].

The value $m_4 = 1.1$ eV coincides with the value of this parameter, which was previously used in the $(3 + 3)$ model [10]. As for the value of $m_6$, its justified choice can be made according to the results of future special experiments. Description of the results of the experiments under consideration, due to the use of a specific mixing matrix form (3a), do not depend in this context on a value of $m_6$. Nonetheless we choose $m_6$ equal to 0.001 eV following Ref. [34].

In order to put a place for results of future experiments let us to consider three possibilities for LSN mass values. Masses of rather heavy LSN (HLSN) fall within the range between 0.2 and 3 eV, masses of intermediate LSN (ILSN) between 0.003 and 0.2 eV, masses of very light SN (VLSN) between 0.0002 and 0.003 eV (see, for instance, [5] [31] [32] [33] [34]). Then one can consider four cases, namely, cases A and B, when SN mass states belong to HLSN and VLSN ($\Lambda_1$ and $\lambda_i$ states), case C, when SN mass states belong only to HLSN ($P_1$ states), case D, when SN mass states belong only to VLSN ($\rho_i$ states). In this paper let us consider the A case ($\Lambda_i$ states, $i = 1, 2$ and $\lambda$ state, $\eta_2 = \pi / 6$) and the B case ($\Lambda$ state and $\lambda_i$ states, $i = 1, 2$, $\eta_2 = \pi / 12$). We will use in eV $\Lambda = 1.1$, $\lambda_2 = 0.002$, $\lambda_1 = 0.001$, $\Lambda_1 = 1.1$, $\Lambda_2 = 0.6$, $\lambda = 0.001$. The A case uses the $m_4$ and $m_5$ mass values which are close to results based on experimental data from Refs. [37] [38]. The B case uses the $m_4$, $m_5$ and $m_6$ mass values which are close to ones from Ref. [31].

One can generalize analytical expressions for the probabilities of transitions and conservation of various neutrino flavors [29] to the case of decaying neutrinos. Using equations for propagation of various neutrino flavors (see, for example, [8]), it is possible to obtain analytical expressions for the transition probabilities of various flavors of stable neutrinos/antineutrinos in a vacuum as a function of distance from the source. If $\bar{U} = U_{\text{mix}}$ is a generalized $6 \times 6$ mixing matrix in the form of expression (2), and if one uses notation $\Delta \equiv \Delta m_{\nu}^2 L / (4E)$, then, following [29], it is possible to calculate the transition probabilities from $\nu_\alpha$ to $\nu_{\alpha'}$, or from $\bar{\nu}_\alpha$ to $\bar{\nu}_{\alpha'}$ by the formula

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \delta_{\alpha'\alpha} - 4 \sum_{i \neq k} \text{Re}(\bar{U}_{\alpha i} U_{\alpha' k}) \sin^2 \Delta_{\alpha i} \pm 2 \sum_{i \neq k} \text{Im}(\bar{U}_{\alpha i} U_{\alpha' k}) \sin 2\Delta_{\alpha i},$$

(5)

where the upper sign (+) corresponds to neutrino transitions $\nu_\alpha \rightarrow \nu_{\alpha'}$ while the undersign (−) corresponds to antineutrino transitions $\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}$. Note that the flavor indices $\alpha$ and $\alpha'$ (as well as the summation indices $i$ and $k$ over mass states) are applied to all neutrinos, that is, to active and sterile neutrinos. Moreover, as follows from the equation (5), the relation $P(\nu_\alpha \rightarrow \nu_\alpha) \equiv P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha)$ is fulfilled exactly due to the CPT-invariance condition [29].

The expressions (5) given above are directly generalized to the case of decaying neutrino mass states $k$ with including of their decay widths $\Gamma_k$ [10]. To do this, it is necessary to substitute $E_k - i \Gamma_k / 2$ instead of the neutrino energy $E_k$ into the original equations for the propagation of neutrino flavors [8], where $\Gamma_k \approx m_k \Gamma_k^{(0)} / E_k$ is the decay width in the laboratory frame, and $\Gamma_k^{(0)}$ is the same in the
rest frame. Then, the probabilities of transitions from $\nu_\alpha$ to $\nu_{\alpha'}$, or from $\bar{\nu}_\alpha$ to $\bar{\nu}_{\alpha'}$, will be calculated as follows:

$$P(\nu_\alpha(\bar{\nu}_\alpha) \rightarrow \nu_{\alpha'}(\bar{\nu}_{\alpha'})) = \delta_{\alpha\alpha'} - 2 \sum_{i>k} \text{Re}(\bar{U}_{\alpha'i} U^*_{\alpha'i} \bar{U}_{\alpha'k} U_{\alpha'k})(T_{ki} \cos 2\Delta_{ki} - 1)$$

$$+ 2 \sum_{i>k} \text{Im}(\bar{U}_{\alpha'i} U^*_{\alpha'i} \bar{U}_{\alpha'k} U_{\alpha'k}) T_{ki} \sin 2\Delta_{ki},$$

where $T_{ki} = \exp\left\{-\frac{i}{\hbar} (\Gamma_k + \Gamma_i) \right\} \equiv \exp\left\{-\frac{i}{\hbar} (m_k \Gamma_k^{(0)} + m_i \Gamma_i^{(0)}) \right\}$. The upper sign (+) corresponds to neutrino transitions $\nu_\alpha \rightarrow \nu_{\alpha'}$, and the lower sign (−) corresponds to antineutrino transitions $\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}$. For this, $(\Delta m_{ik}^2/L)/(4E\hbar c)$ is equivalent to $1.27(\Delta m_{ik}^2) E_i$, if $\Delta m_{ik}^2$ is given in eV$^2$, $E_i$ is given in MeV, and $L$ is given in meters. Respectively, $L(\Gamma_k + \Gamma_i)/2\hbar c$ is equivalent to $0.253 \cdot 10^7 (\Gamma_k + \Gamma_i)$, if $L$ is given in meters, and $\Gamma_k$ and $\Gamma_i$ are given in eV. However, $\Gamma_k$ and $\Gamma_i$ typical evaluations admit to neglect their values below.

### 4 Determination of the mixing among active and sterile neutrinos with the experiment BEST results and neutrino characteristics for some processes with LSN contributions

In the model under consideration, the value of the mixing parameter $\epsilon$ between active and sterile neutrinos is of great importance. Let’s estimate its value according to the results of experiment BEST [25, 26]. In this experiment, the detector filled with liquid gallium is divided into two cavities. The internal cavity is a sphere with a radius of about one meter, into which a cylinder with a radioactive source is inserted. Inside the sphere, it can be neglected by the influence of oscillations of active neutrinos themselves, but the effect of electron neutrino oscillations due to mixing with LSN must be taken into account. To do this, we find the survival probability of electron neutrinos themselves, but the effect of electron neutrino oscillations due to mixing with LSN must be taken into account. To do this, we find the survival probability of electron neutrinos for an internal target, it is necessary to integrate over the interior of the sphere according to the formula [39]

$$R \approx \frac{\int V L^{-2} \sum_i P_{ee}(E_i, L) B_i \sigma_i}{\int V L^{-2} \sum_i B_i \sigma_i}. \tag{7}$$

In the formula [7], $L$ is the distance from the source to a point inside the detector, $B_i$ are the partial ratios, $\sigma_i$ are the cross sections of capture by $^{71}$Ga of electron neutrinos emitted from discrete levels of $^{54}$Cr, $P_{ee}(E_i, L)$ is the survival probability of electron neutrinos in the considered $(3+3)$ model, which is given by expression [6]. Thus, an estimate of the average value of $R_{in}$ can be obtained, which, according to [25, 26], should be equal to experimental value of $R_{in} = 0.791 \pm 0.05$. At the same time, it is necessary take into account that in 81.63% of cases electron neutrinos are emitted with an energy of 747 keV, in 8.95% of cases they are emitted with an energy of 427 keV, in 8.49% of cases they are emitted with an
energy of 752 keV, and in 0.93% of cases they are emitted with an energy of 432 keV. Calculations were carried out for different values of the sphere radius from 50 to 70 cm. To estimate the value of $\epsilon$, the value $L_{in} = 67$ cm was chosen as the boundary value of the radius, which is the geometric radius of the sphere. We perform calculations with various values of $\epsilon$ from 0.01 to 0.1. The most suitable for concordance with the experimental value of $R_{in}$ are $\epsilon = 0.08$ for the A case and $\epsilon = 0.07$ for the B case. Fig. 1 shows the behaviour of $P_{ee}(L) = \sum_i P_{ee}(E_i, L)B_i\sigma_i / \sum_i B_i\sigma_i$ inside the sphere depending on the distance $L$ to the neutrino source.

After determining the value of $\epsilon$, using the values of the other parameters given above, one can calculate and plot graphs for the survival probability of electron neutrinos and the probability of transition of muon neutrinos to electron neutrinos depending on the ratio $L/E$ in m/MeV, that is, on the ratio of the distance $L$ from the source to the neutrino energy $E$. These dependencies are important for interpreting the results of the experiments BEST, DANSS, NEUTRINO-4, MiniBooNE and MacroBooNE (see Fig. 1, Fig. 2 and Fig. 3). The advantage of the considered model (the A variant) is the capacity to describe processes of survival and transition for different neutrinos types in the single model. Really in the $(3 + 3)$-model in the general case three additional neutrino masses are available instead of only one mass in the $(3 + 1)$-model.

One can also estimate the effective masses of the electron neutrino $m_\beta$ and $m_{\beta\beta}$, which are used in calculating the probabilities of beta decay and neutrinoless double-beta decay (the corresponding set of Majorana phases should be used for the minimal value of $m_{\beta\beta}$), that are for the A and B variants:

$$m_\beta = (\Sigma_i |U_{\text{mix},ei}|^2 m_i^2)^{1/2}, \quad m_{\beta\beta} = |\Sigma_i U_{\text{mix},ei}^2 m_i|.$$  \hspace{1cm} (8)

The obtained values in eV for $m_\beta \approx 0.39$ for both cases and $m_{\beta\beta} \approx 0.1$ for case A and $m_{\beta\beta} \approx 0.13$ for case B do not contradict with the currently available experimental results \cite{40, 41} for large values of the confidence probability.

5 Discussion and conclusions

In this work, we have used the results of the experiments BEST, MiniBooNE and MacroBooNE to obtain estimates of a number of parameters of the phenomenological $(3 + 3)$ neutrino model with active and sterile neutrinos. Note that these values, including LSN masses, do not have to match the parameters of the $(3 + 1)$ model, and to determine them from experimental data it needs to apply an appropriate scheme that are independent of the $(3 + 1)$ model. The values of parameters, for which there are currently no experimental data, were selected as test values. If in the future the existence of light sterile neutrinos will be reliably confirmed, this will lead to a significant change of $\nu$SM and explanation of some phenomena in neutrino physics. Moreover sterile neutrinos with various masses can participate in astrophysical and cosmological processes \cite{30}.

The main results of this work, which are important both for interpretation of the data obtained in ongoing experiments and for predicting the results of planned neutrino experiments are briefly outlined below. Based on the results
Figure 1: (a) The survival probability for $\nu_e$ depending on the distance $L$ from the source for the conditions of the BEST experiment with the values of the parameters of the (3+3) model $m_4 = 1.1$ eV, $m_5 = 0.6$ eV, $\eta_2 = 30^\circ$, $\epsilon = 0.08$ (A), or $m_5 = 0.002$ eV, $\eta_2 = 15^\circ$, $\epsilon = 0.07$ (B). $L_{in} = 67$ cm, $R = 0.8$. (b) The survival probability for $\nu_e$ ($\bar{\nu}_e$) depending on the ratio of the distance $L$ from the source to the neutrino energy $E$ in the beams $\nu_e$ ($\bar{\nu}_e$) for sterile neutrinos in the (3+3) model with parameter values $m_4 = 1.1$ eV, $m_5 = 0.6$ eV, $\eta_2 = 30^\circ$, $\epsilon = 0.08$ (A), or $m_5 = 0.002$ eV, $\eta_2 = 15^\circ$, $\epsilon = 0.07$ (B).
Figure 2: The survival probability of $\nu_\mu$ depending on the ratio of the distance $L$ from the source to the neutrino energy $E$ in the $\nu_\mu$ beams for sterile neutrinos in the (3+3) model with parameter values $m_4 = 1.1$ eV, $m_5 = 0.6$ eV, $\eta_2 = 30^\circ$, $\epsilon = 0.08$ – top panel (A) and $m_5 = 0.002$ eV, $\eta_2 = 15^\circ$, $\epsilon = 0.07$ – bottom panel (B).
Figure 3: The appearance probability of $\nu_e$ depending on the ratio of the distance $L$ from the source to the neutrino energy $E$ in the $\nu_{\mu}$ beams for sterile neutrinos in the (3+3) model with parameter values $m_4 = 1.1$ eV, $m_5 = 0.6$ eV, $\eta_2 = 30^\circ$, $\epsilon = 0.08$ – top panel (A) and $m_5 = 0.002$ eV, $\eta_2 = 15^\circ$, $\epsilon = 0.07$ – bottom panel (B).
of the BEST, MiniBooNE and MacroBooNE experiments, the possible values of masses $m_4$ and $m_5$ of sterile mass states $\nu_4$ and $\nu_5$ have been given. According to the value of $R$, which characterizes the deficit of electron neutrinos and measured in the BEST experiment, the main mixing parameter $\epsilon$ between active and sterile neutrinos in the $(3+3)$ neutrino model has been estimated. The values of angles and phases of mixing $\kappa_1$, $\kappa_2$, $\eta_1$ and $\eta_2$ are fixed as trial values.

The probability of conservation of electron and muon neutrinos and the probability of transition of muon neutrinos to electron neutrinos depending on the ratio of the distance $L$ from the source to the neutrino energy $E$ have been calculated in the considered model and presented in the graphical form in Figs 1-3. Estimates of the electron neutrino effective masses, which can be measured in the experiments on beta decay and neutrinoless double-beta decay, have also been evaluated in the framework of considered model. The important characteristic feature of the used model as compared with the usual $(3+1)$-model is the possibility to describe processes with distinct scales as $\nu_e \nu_e$, $\nu_\mu \nu_e$, and $\nu_\mu \nu_\mu$ processes within the unified approach.

References

[1] R. L. Workman et al. (Particle Data Group), The Review of Particle Physics, \textit{Prog. Theor. Exp. Phys.} \textbf{2022}, 083C01 (2022).

[2] K. N. Abazajian et al., Light Sterile Neutrinos: A White Paper, \texttt{arXiv:1204.5379 [hep-ph]} (2012).

[3] S. Böser, C. Buck, C. Giunti, J. Lesgourgues, L. Ludhova, S. Mertens, A. Schukraft and M. Wurm, Status of Light Sterile Neutrino Searches, \textit{Prog. Part. Nucl. Phys.} \textbf{111}, 103736 (2020).

[4] M. A. Acero et al., White Paper on Light Sterile Neutrino Searches and Related Phenomenology, \texttt{arXiv:2203.07323 [hep-ex]} (2022).

[5] André de Gouvêa, Seesaw Energy Scale and the LSND Anomaly, \textit{Phys. Rev. D} \textbf{72}, No. 3, 033005 (2005).

[6] J. Kopp, P. A. N. Machado, M. Maltoni and T. Schwetz, Sterile Neutrino Oscillations: The Global Picture, \textit{JHEP} \textbf{1305}, 050 (2013).

[7] J. M. Conrad, C. M. Ignarra, G. Karagiorgi, M. H. Shaevitz and J. Spitz, Sterile Neutrino Fits to Short-Baseline Neutrino Oscillation Measurements, \textit{Adv. High Energy Phys.} \textbf{2013}, 163897 (2013).

[8] V. V. Khruschov and S. V. Fomichev, Sterile Neutrinos Influence on Oscillation Characteristics of Active Neutrinos at Short Distances in the Generalized Model of Neutrino Mixing, \textit{Int. J. Mod. Phys. A} \textbf{34}, No. 29, 1950175 (2019).

[9] V. V. Khruschov, S. V. Fomichev and S. V. Semenov, Properties of Active-Neutrino Oscillations and Double-Beta Decay in the Presence of Sterile-Neutrino Contributions, \textit{Phys. At. Nucl.} \textbf{84}, No. 3, 328-338 (2021).
[10] V. Khruschov and S. Fomichev, Oscillations of Active Neutrinos at Short Baseline in the Model with Three Decaying Sterile Neutrinos, *Universe* 8, No. 2, 97 (2022).

[11] C. Athanassopoulos et al. (LSND Collab.), Evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$ Oscillations from the LSND Experiment at the Los Alamos Meson Physics Facility, *Phys. Rev. Lett.* 77, No. 15, 3082-3085 (1996).

[12] A. A. Aguilar-Arevalo et al. (MiniBooNE Collab.), Improved Search for $\bar{\nu}_\mu \to \bar{\nu}_e$ Oscillations in the MiniBooNE Experiment, *Phys. Rev. Lett.* 110, 161801 (2013).

[13] A. A. Aguilar-Arevalo et al. (MiniBooNE Collab.), Significant Excess of Electronlike Events in the MiniBooNE Short-Baseline Neutrino Experiment, *Phys. Rev. Lett.* 121, 221801 (2018).

[14] P. B. Denton, Sterile Neutrino Search with MicroBooNE’s Electron Neutrino Disappearance Data, *Phys. Rev. Lett.* 129, 061801 (2022).

[15] J. N. Abdurashitov et al. (SAGE Collab.), Measurement of the Solar Neutrino Capture Rate with Gallium Metal. III. Results for the 2002–2007 Data-Taking Period, *Phys. Rev. C* 80, 015807 (2009).

[16] F. Kaether, W. Hampel, G. Heusser, J. Kiko and T. Kirsten, Reanalysis of the Gallex Solar Neutrino Flux and Source Experiments, *Phys. Lett. B* 685, No. 1, 47-54 (2010).

[17] C. Giunti, M. Laveder, Y. F. Li and H. W. Long, Pragmatic View of Short-Baseline Neutrino Oscillations, *Phys. Rev. D* 88, 073008 (2013).

[18] Th. A. Mueller, D. Lhuillier, M. Fallot, A. Letourneau, S. Cormon, M. Fechner, L. Giot, T. Lasserre, J. Martino, G. Mention, A. Porta and F. Yermia, Improved Predictions of Reactor Antineutrino Spectra, *Phys. Rev. C* 83, 054615 (2011).

[19] P. Huber, Determination of Antineutrino Spectra from Nuclear Reactors, *Phys. Rev. C* 84, 024617 (2011); *Phys. Rev. C* 85, 029901(Erratum) (2012).

[20] G. Mention, M. Fechner, Th. Lasserre, Th. A. Mueller, D. Lhuillier, M. Cribier and A. Letourneau, Reactor Antineutrino Anomaly, *Phys. Rev. D* 83, 073006 (2011).

[21] Y. J. Ko et al. (NEOS Collab.), Sterile Neutrino Search at the NEOS Experiment, *Phys. Rev. Lett.* 118, 121802 (2017).

[22] I. Alekseev et al., Search for Sterile Neutrinos at the DANSS Experiment, *Phys. Lett. B* 787, 56-63 (2018).

[23] A. P. Serebrov et al., Search for Sterile Neutrinos with the Neutrino-4 Experiment and Measurement Results, *Phys. Rev. D* 104, 032003 (2021).
[24] A. A. Aguilar-Arevalo et al. (MiniBooNE Collab.), MiniBooNE and MicroBooNE Joint Fit to a 3 + 1 Sterile Neutrino Scenario, [arXiv:2201.01724 [hep-ex]] (2022).

[25] V. V. Barinov et al., Search for Electron-Neutrino Transitions to Sterile States in the BEST Experiment, [Phys. Rev. C 105, 065502 (2022)].

[26] V. V. Barinov et al., Results from the Baksan Experiment on Sterile Transitions (BEST), [Phys. Rev. Lett. 128, 232501 (2022)].

[27] V. Kopeikin, M. Skorokhvatov and O. Titov, Reevaluating Reactor Antineutrino Spectra with New Measurements of the Ratio between $^{235}\text{U}$ and $^{239}\text{Pu}$ Spectra, [Phys. Rev. D 104, L071301 (2021)].

[28] S. M. Bilen’kii and B. M. Pontekorvo, Lepton mixing and neutrino oscillations, [Sov. Phys. Usp. 20, No. 10, 776-795 (1977)].

[29] S. M. Bilenky, Some Comments on High-Precision Study of Neutrino Oscillations, [Phys. Part. Nucl. Lett. 12, No. 4, 453-461 (2015)].

[30] K. N. Abazajian, Neutrinos in Astrophysics and Cosmology, [arXiv:2102.10183 [hep-ph]] (2021).

[31] N. Yu. Zysina, S. V. Fomichev and V. V. Khruschov, Mass Properties of Active and Sterile Neutrinos in a Phenomenological (3 + 1 + 2) Model, [Phys. Atom. Nucl. 77, No. 7, 890-900 (2014)].

[32] A. V. Yudin, D. K. Nadyozhin, V. V. Khruschov and S. V. Fomichev, Neutrino Fluxes from a Core-Collapse Supernova in a Model with Three Sterile Neutrinos, [Astron. Lett. 42, No. 12, 800-814 (2016)].

[33] P. F. de Salas, D. V. Forero, S. Gariazzo, P. Martínez-Miravé, O. Mena, C. A. Ternes, M. Tórtola and J. W. F. Valle, 2020 Global Reassessment of the Neutrino Oscillation Picture, [JHEP 2102, 071 (2021)].

[34] V. V. Khruschov and S. V. Fomichev, Active and sterile neutrino oscillations inside the Sun in a phenomenological (3+1+2)-model, [arXiv:1310.5817 [hep-ph]] (2013).

[35] P. C. de Holanda and A. Yu. Smirnov, Homestake result, Sterile neutrinos and Low energy solar neutrino experiments, [arXiv:hep-ph/0307266v4] (2004).

[36] A. de Gouvêa, G. J. Sanchez, K. J. Kelly, Very Light Sterile Neutrinos at NOvA and T2K, [Phys. Rev. D 106, 055025 (2022)].

[37] V. V. Sinev, Joint Analysis of Spectral Reactor Neutrino Experiments, [arXiv:1103.2452v3 [hep-ex]] (2015).

[38] I. Alekseev, Recent results from DANSS Experiment, [arXiv:2305.07417 [hep-ex]] (2023).
[39] C. Giunti and M. Laveder, Statistical Significance of the Gallium Anomaly, 
*Phys. Rev. C* **83**, 065504 (2011).

[40] M. Aker *et al.* (KATRIN Collab.), Direct Neutrino-Mass Measurement with 
Sub-Electronvolt Sensitivity, *Nature Phys.* **18**, No. 2, 160-166 (2020).

[41] S. D. Biller, Combined Constraints on Majorana Masses from Neutrinoless 
Double Beta Decay Experiments, *Phys. Rev. D* **104**, 012002 (2021).