Odd sector of QCD

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Abstract
A systematic study of the odd-intrinsic parity sector of QCD is presented. We briefly describe different applications including \( \pi^0 \rightarrow \gamma\gamma \) decay, muonic \( g - 2 \) factor and test of new holographic conjectures.

Keywords: Chiral Lagrangians, 1/N Expansion, QCD

1. Introduction

The low-energy domain of quantum chromodynamics (QCD) is well understood and described by an effective field theory called chiral perturbation theory (ChPT). The derivative (\( \rightarrow \) momenta \( \rightarrow \) energy) expansion together with the symmetry pattern of the underlying QCD organize the inner structure of the effective Lagrangian and describe the dynamics of the associated Goldstone bosons (GB). The most important ingredient, the so-called chiral symmetry, the approximate global symmetry of QCD which acts independently on left- and right-handed light flavour quarks gives rise to two variants of ChPT. One can distinguish two-flavour ChPT (active degrees of freedom are \( u \) and \( d \) quarks) and three-flavour ChPT (added \( s \) quark as well). Their small but not negligible mass (specially in the case of \( s \) quark) is treated as a perturbation. For further details we refer to the original papers [1] and [2] in the case of the next-to-leading order (NLO) and next-to-next-to-leading order (NNLO), respectively. The number of GB (and in the two-flavour ChPT \( G \)-parity) is strictly conserved in this Lagrangian and we are talking about even-intrinsic parity sector. At this moment, however, the QCD symmetry pattern cannot be complete as it would not be possible to describe e.g. \( KK \rightarrow 3\pi \) or \( \pi \rightarrow 2\gamma \), i.e. well established and existing processes. The missing odd-intrinsic parity sector arises as a consequence of the famous chiral anomaly, i.e. of the well-known fact that the dynamics can be invariant under the chiral symmetry even if operators are not, provided that their variation is a total derivative [3]. The explicit construction at NLO can be found in [4]. More general discussion on chiral anomaly was presented in the talk of O. Teryaev at this conference (cf. also [8]).

The number of monomials in the above constructed Lagrangian either in even or odd intrinsic sector increases with higher orders. Information on low-energy constants (LEC) that stands in front of these monomials thus reflects the predictivity of ChPT. One possibility how to gain an insight into them is to use the phenomenology above the domain of ChPT applicability (\( \sim 1 \) GeV). The first attempt of systematic description of the active degrees of freedom of the low-lying meson resonances with spin \( \leq 1 \) was provided in [5] for NLO and in [6] for NNLO, in both cases for even intrinsic parity sector. This effort was concluded in [7] also for the anomalous sector.

In this work we will focus only on the odd-intrinsic parity sector. We will make a brief overview of the meson-resonance construction of [7] together with so-called resonance saturation and focus on few applications, namely \( \pi^0 \rightarrow \gamma\gamma \) decay and \( VVP \) Green function.
2. Odd-basis construction

In order to obtain the basis one should first construct all possible hermitian operators using the standard chiral building blocks and resonance fields keeping in mind $C$, $P$ and chiral invariance. In order to minimize the set one uses the partial integration, equation of motions, Bianchi identities, Schouten identity and basic relations among the chiral building blocks. The constructed Lagrangian of this meson-resonance odd-parity sector becomes:

$$\mathcal{L}^{(\text{odd})}_{\text{th}} = g^{\mu\nu\rho\sigma} \sum_{X,j} k_i^X O^{\mu\nu\rho\sigma}_j.$$  \hfill (1)

The $X$ stands for the resonances entering the individual terms, linear in resonances: $V$ (18 monomials), $A$ (16), $S$ (2), $P$ (5), quadratic: $VV$ (4), $AA$ (4), $VA$ (6), $VS$ (2), $VP$ (3), $AS$ (2), $AP$ (2), and finally three trilinear monomials: $VVP$, $VAS$, $AAP$. All 67 monomials can be found in [7]. The contribution of $k_i^X$ introduced in [1] to LEC will be discussed in the next section.

3. Resonance saturation

The resonance saturation both for even-intrinsic parity and odd-intrinsic parity is schematically depicted in Fig. 1. LEC are thus expressed by means of resonance

![Figure 1](image_url)

Figure 1: Schematic description of resonance (double line) saturation for both even intrinsic sector (NLO LEC denoted by $L_i$) and anomalous sector (LEC represented by $C_i^W$).

parameters $k_i^X$. One usually uses a term saturation provided we neglect all other contribution. In principle one has exchanged one set of parameters with another. In the latter case we can use, however, richer phenomenological information which includes explicit resonances. In the construction of resonance monomials in [1], the large $N_C$ approximation was tacitly assumed. As so all LEC can be saturated except that they are subleading in large $N_C$. In the anomalous NLO sector we have altogether 23 LEC constants from which 21 can be saturated schematically as

$$C_i^W = a_i N_C^2 + b_i N_C + O(N_C^0).$$ \hfill (2)

Let us note that the formal large $N_C$ enhancement ($a_i \neq 0 \text{ for } i = 6, 8, 10$) is due to $\eta'$ exchange. The explicit form for all $a_i$ and $b_i$ can be found in [7]. A systematic elimination of $k_i$ in [3] leads to one relation which includes only LEC and the resonance parameters from the even sector

$$\frac{F V}{2 G_V} C_{12}^W = F_V (C_{14}^W - C_{15}^W) + G_V C_{22}^W. \hfill (3)$$

4. Decay $\pi^0 \to \gamma\gamma$

Studying the odd-intrinsic parity sector of QCD one cannot avoid the discussion on the most important anomalous process $\pi^0 \to \gamma\gamma$. This decay was crucial in establishing the role of the anomaly for the gauge theory [9]. Without the anomaly it was a long standing puzzle how to overcome the implication of the Sutherland theorem which stated that this process should be suppressed by power of $m_\pi^2/(1 \text{ GeV}^2)$ to its actual measured value. A new experiment PrimEx [10] with a total uncertainty of 2.8% (with planned improvement) motivates even to better understand these Sutherland contributions. Presumably accidental non-existence of the leading logarithm (i.e. terms $\sim m_\pi^2 \log m_\pi^2$) [11] leads to a necessity to calculate even higher orders in $m_\pi$. In the language sketched in Introduction one needs to calculate this process up to NNLO in ChPT. This was performed for two-flavour ChPT in [12] with remarkable simple analytic result. In order to use a phenomenological information the transition to three-flavour ChPT must be employed [12] [13] [11]. Apart from LO and logarithms one ends up with two LEC $C_7^W$ and $C_8^W$ on which $\pi^0 \to \gamma\gamma$ depends. The first one can be set using the resonance phenomenology as explained in the previous section. The second one can be obtained from the decay $\eta \to \gamma\gamma$: here we have similar dependence on $C_7^y$ and $C_8^y$ as for $\pi^0$ decay. However, this process is known only at the NLO and again it contains no leading logarithms. First attempt to calculate higher-order logarithms can be found in [14]. Assuming these corrections would be small we may obtain [12]

$$\Gamma_{\pi^0 \to \gamma\gamma} = (8.1 \pm 0.1) \text{ eV}. \hfill (4)$$

5. Renormalization group equation and leading logs

In the previous section we were discussing the most significant process of the odd sector $\pi \to \gamma\gamma$ and mentioned an important concept of the leading logarithms. These logarithms were obtained calculating two-loop diagrams without any simplifications. It is however well
known that the leading logs (up to all orders) can be obtained from one-loop diagrams only \[15\]. This enables to open program of calculating the leading logs to a sufficiently high order estimating general prescription and eventually resum the series. It should in any case help to understand the connection of Taylor expansion in ChPT and asymptotic series of QCD \[17\] in some intermediate region. This program was started with massive \(O(N)\) non-linear sigma model in \[16\]. Among other things the authors calculated the physical mass and decay constant \(F_\pi\) up to five-loop order. As \(O(N+1)/O(N)\) for \(N = 3\) is isomorphic to \(SU(2) \times SU(2)/SU(2)\) i.e. to two-flavour ChPT (without isospin breaking) we can use the same program to make further prediction also in the anomalous sector for two flavours. This program is in progress \[18\].

6. VVP Green function

The previous example \(\pi^0 \to \gamma\gamma\) can be obtained from a three-point Green function involving two vector currents and one pseudoscalar density, in short VVP. As this function can be further related to other interesting physical quantities we will first discuss some of its general properties. Let us start with the definition

\[
\Pi_{\mu\nu}^{\text{VVP}}(p, q) = \int d^4x \, d^4y \, e^{\imath (p + q) \cdot x} \langle 0 | T [V_\mu^a(x)V_\nu^b(y)] P^0(0)|0\rangle, \tag{5}
\]

with

\[
V_\mu^a(x) = \bar{q}(x)\gamma_\mu q(x), \quad P^0(x) = \bar{q}(x)\gamma_5 q(x). \tag{6}
\]

This object can be calculated using the Lagrangian \(\mathcal{L}_V\). Nine of odd-intrinsic parity resonance parameters \(k_i\) contribute: \(k_2^V, k_3^V, k_4^V, k_5^PV, k_6^PV\) and \(k_7^VVP\). We can assume that the result should correctly describe also high-energy region which is accessible directly by perturbative QCD (procedure known as operator product expansion (OPE)). Using this link we arrive to the result which depends only on two unknown parameters \(k_3^PV\) and \(k_4^PV\). Our theoretical construction can be connected with an existing phenomenological model known as LMD+P \[19\] (for LMD see below). Here we will write the result only in the limiting on-shell case (working in the chiral limit it means \(r^2 = 0\)) when one parameter drops out and we have a simple dependence only on one of them:

\[
\frac{\langle 1 \rangle}{B_0} \Pi^{\text{OPE}}(p^2, q^2; r^2 \to 0) = \frac{1}{(p^2 - M_\pi^2)(q^2 - M_\pi^2)} \times \left\{ \frac{1}{2}(p^2 + q^2)(32 \sqrt{2} d_{\mu} F_{\mu\nu} k_3^{PV} + F^2) - \frac{M_\pi^2 N_c}{8\pi^2} \right\}. \tag{6}
\]

Before performing OPE we can assume the construction of VVP using the vector resonances only (i.e. dropping pseudoscalar resonances) and connecting the currents only via lowest lying vector resonance. This procedure is known as vector meson dominance (VMD). It can be achieved in \(\Pi\) by setting \(k_3^{PV}\) to

\[
k_3^{PV} = \frac{F^2}{32 \sqrt{2} d_{\mu} F_{\mu\nu}}, \tag{7}
\]

so the first term on the second line of \(\Pi\) cancels out. Such a fine-tuning is, however, not natural in our formalism and we prefer to fix its value from phenomenology. Defining the on-shell pion-\(\gamma^*\) formfactor

\[
\mathcal{F}_{\pi\gamma^*}(p^2, q^2) = \frac{2}{3BF} \lim_{r^2 \to 0} r^2 \Pi(p^2, q^2; r^2) \tag{8}
\]

we can connect the measurements of this formfactor with the value of \(k_3^{PV}\) in \(\Pi\). Using the combined fit on CLEO and BABAR data \[20\] we get

\[
k_3^{PV} = -0.047 \pm 0.018. \tag{9}
\]

From information on \(\pi(1300)\) decay modes we can set \(\Pi\) also the value for the last parameter entering VVP

\[
k_4^{PV} = (-0.57 \pm 0.13) \text{ GeV}. \tag{10}
\]

6.1. \(g\)-2

Having study VVP correlator it is impossible to avoid discussion on its role in the famous muon \(g - 2\) factor. One of the major systematic error in determining this factor lies in the so-called hadronic light-by-light (LbL) scattering. This has several contributions, one of which can be related to VVP correlator defined above. Using the values in \(\Pi\) and \(\Pi\) we can obtain a prediction for the \(e^+e^-\)-exchange in hadronic LbL scattering:

\[
d_{\mu}^{\text{LbL}; e^+e^-} = (65.8 \pm 1.2) \times 10^{-11}, \tag{11}
\]

which is the most precise up-to-date determination of this quantity, very close to the recent AdS/QCD studies in \[21\].

7. AdS/QCD and anomaly

A direct connection between low energy QCD and its high energy version in the odd-intrinsic parity sector using new holographic methods \[22\] was suggested recently in \[23\]. Within a wide range of holographic

\footnote{Note that after this work was presented a new study on the anomalous VVA in the soft-wall holographic model has appeared \[24\].}
models Son and Yamamoto have established a relation which connects the longitudinal and transverse part of the VVA (it can be directly connected with VVP of the previous section) and two-point correlator (LR)

\[ w_L(Q^2) - 2w_T(Q^2) = -\frac{2N_c}{F^2}\Pi_{LR}(Q^2), \tag{12} \]

valid for all \( Q^2 \). This relation was tested in pQCD domain with not very satisfactory result [25]. We will address here its validity for a small \( Q^2 \) within a non-perturbative regime of QCD. Using the language of LEC we can rewrite (12) into

\[ C_{22}^W = -\frac{N_c}{32\pi^2 F^2} L_{10}, \tag{13} \]

which connects the odd-intrinsic parity LEC (\( C_{22}^W \)) with even parity LEC (\( L_i \)) at NLO. Using the resonance saturation discussed in Section 3 (for \( L_{10} \) see [5]) we start with

\[ C_{22}^W = -\frac{F_W \kappa_{17}^V}{\sqrt{2M_V^2}} - \frac{F_W \kappa_{17}^{PV}}{2M_V^2}, \tag{14} \]

\[ L_{10} = \frac{F_A^2}{4M_A^2} - \frac{F_V^2}{4M_V^2}. \tag{15} \]

Employing OPE of the previous section together with further asymptotic behaviour (namely Weinberg sum rule and OPE on the axial pion form factor) we end up with a condition

\[ k_3^{PV} \approx \frac{M_V^2 N_c - 16\pi^2 F^2}{1024\pi^2 d_{\text{ms}} F} \approx 0.019 \pm 0.01, \tag{16} \]

which is numerically close to zero \(^2\) and thus in a disagreement with the phenomenological study (cf. [4]). On the other hand \( k_3^{PV} = 0 \) is just a result of the so called LMD (lowest meson dominance) model [26] (i.e. where one drops systematically \( P \)). It is thus interesting to notice that the studied class of holographic models produces a relation [13] which seems to be valid in the LMD model.

8. Conclusion

A systematic study of the odd-intrinsic parity sector was presented. It concludes similar study in the even sector. Variety of applications was briefly described using different methods. We have shown a general concept of the resonance saturation for LEC in the odd sector; showed its implication for the important decay of this sector: \( \pi^0 \to \gamma\gamma \) decay and sketched further program for it using renormalization group equations. The general VVP Green function and its phenomenological applications were shortly discussed. The last section was devoted to the modern holographic study focused on this sector with a particular check.

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