Adaptive control for reliable cooperative intersection crossing of connected autonomous vehicles

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Abstract
Rapid advances in vehicle automation and communication technologies enable connected autonomous vehicles (CAVs) to cross intersections cooperatively, which could significantly improve traffic throughput and safety at intersections. Virtual platooning, designed upon car-following behavior, is one of the promising control methods to promote cooperative intersection crossing of CAVs. Nevertheless, demand variation raises safety and stability concerns when CAVs adopt a virtual platooning control approach. Along this line, this study proposes an adaptive vehicle control method to facilitate the formation of a virtual platoon and the cooperative crossing of CAVs, factoring demand variations at an isolated intersection. This study derives the stability conditions of virtual CAV platoons depending on the time-varying traffic demand. Based on the derived stability conditions, an optimization model is proposed to adaptively control CAVs dynamics by balancing approaching traffic mobility and safety to enhance the reliability of cooperative crossing at intersections. The simulation results show that, compared to the nonadaptive control, our proposed method can increase the intersection throughput by 18.2%. Also, time-to-collision results highlight the advantages of the proposed adaptive control in securing traffic safety.

KEYWORDS
connected autonomous vehicles, virtual platooning, cooperative intersection crossing, optimization, stability

1 | INTRODUCTION

Traffic congestion is a severe problem in modern cities that increases travel time, air pollution, fuel consumption, and economic losses. Urban congestion caused about 8.7 billion hours of travel delay and 35 billion gallons of wasted fuel for an estimated congestion cost of $190 billion in 2019. Intersections represent major road bottlenecks where traffic flows from multiple directions cross at the location with a limited capacity. The delay at intersections alone contributes 5%-10% of total traffic delay. Meanwhile, the intense conflicting movements of vehicles result in frequent accidents at intersections. According to the European Union community database on road accidents (CARE), more than 20% of traffic fatalities are intersection-related in the European Union. In the United States, around 30% of fatal crashes occurred in intersection areas between 2015 and 2019. These statistics demonstrate the need to improve traffic mobility and safety at intersections.

Traffic controls at intersections have evolved from traditional fixed-time signal control to recent adaptive signal control to manage...
intersection traffic efficiently. The traditional fixed-time control approaches determine signal phases and green splits based on historical data. Therefore, they cannot adapt to the nonrecurrent traffic pattern. Adaptive traffic signal systems such as the Split Cycle Offset Optimization Technique (SCOOT) and the Real-Time Hierarchical Optimized Distributed Effective System (RHODES) improve the traffic throughput by adjusting signal phases and green splits based on estimations on queue length or vehicle delay.

The recent advent of connected autonomous vehicle (CAV) technology can help control vehicles to improve traffic mobility, safety, and sustainability by removing human errors and timely sharing traffic information among vehicles and infrastructure. Autonomous vehicle technology enables precise control of vehicle movements to enhance safety and mobility. Moreover, the development in wireless communication technology allows vehicles to communicate with each other and infrastructure, which will provide real-time information about the surrounding traffic conditions and upcoming operations. Such information will promote cooperative behaviors among autonomous vehicles to improve traffic efficiency and safety.

The integration of vehicle automation and communication technologies can promote the development of intelligent intersection. Existing studies in the literature have proposed applying the CAV technology to adaptive signal control. For example, Day and Bullock proposed using low-penetration connected-vehicle data to optimize the signal timing plan and offset in real time. Lee et al. used connected vehicles to detect vehicles’ states and estimate road travel times for designing adaptive traffic signal control. However, the inevitable green time loss in traffic signal control causes inefficiency in intersection management. With a focus on this issue, researchers have proposed signal-free intersections by promoting cooperative behaviors among CAVs when these vehicles cross intersections.

Depending on the existence of a central operator for supervising the control process, traffic management strategies for signal-free intersections can be classified as central and distributed control. Centralized cooperative controls can be further categorized as resource reservation approaches or trajectory planning approaches. The resource reservation approaches discretize the intersection space into cells along the temporal and spatial dimensions. The discretized spatiotemporal cells will then be allocated to the approaching vehicles. For example, Dresner and Stone proposed a reservation approach where the vehicle agent reserves a space–time block in the intersection. The intersection controller manages the reservations following the “first come, first serve” policy.

For the other centralized cooperative control strategy, trajectory planning approaches seek to simultaneously determine all vehicles’ paths to ensure that all vehicles enter the signal-free intersection without any conflicts. Trajectory planning-based studies design cooperative intersection control by assigning vehicles the optimized paths. For example, Li and Wang proposed a scheduling algorithm that assigns vehicles to cross an intersection. Their scheduling algorithm finds the minimal travel times for all approaching vehicles based on a spanning tree. Mirheli et al. proposed a consensus-based trajectory planning approach by formulating the problem as nonlinear programming to minimize the travel time and speed variations of each CAV.

While promising, the centralized cooperative control methods require tremendous computation and communication resources since the centralized controller needs to optimize and control the trajectories of all approaching vehicles. Such a heavy computation burden raises concerns about system efficiency and reliability. Moreover, all the vehicles must communicate with the centralized controller, leading to a heavy communication burden and security issues.

To tackle these issues, distributed intersection control strategies are proposed to promote the development of signal-free intersections. In distributed control strategies, vehicles approaching such intersections apply their own controller to cross the signal-free intersection in a cooperative manner. For example, in some studies, a signal-free intersection is discretized and labeled with tokens. Only one vehicle can hold a token at a time. A cooperative vehicle broadcasts the token it occupied. Other cooperating vehicles cannot receive the token until the vehicle holding the token leaves the designated space and releases the token to ensure vehicle safety in this cooperation mechanism.

Besides using tokens, virtual vehicle platooning is an efficient alternative for distributed cooperative intersection control. Approaching CAVs from different links is mapped into one virtual platoon to ensure the safety of the cooperative crossing. The virtual platoon strategy is implemented for T-intersection and on-ramp merging conditions. Medina et al. applied the virtual platoon-based cooperation control to a four-leg intersection to reduce the risk of accidents and increase intersection throughput. Xu et al. proposed a virtual platoon-based approach and proved its linear string stability.

Existing studies on cooperative intersection crossing at signal-free intersections focus mainly on designing a control strategy given a fixed demand. However, traffic demand varies from time to time, leading to additional delays or safety issues if the control is not designed properly. For example, during peak hours, the increasing demand can surge over the intersection capacity, resulting in oversaturated flows and unpredictable travel times. The overflows can propagate to adjacent intersections, causing urban gridlock. Moreover, the demand variation introduces additional uncertainty to the intensive vehicular interactions at the intersection and affects the system-level travel reliability and mobility of cooperative crossing control. In this context, adaptive control strategies can adjust the control parameters to adapt to demand variation. So far, no studies have focused on designing adaptive control for cooperative intersection crossing to accommodate demand variation.

To fill this gap, this study proposes an adaptive cooperative intersection crossing control to adapt to time-dependent traffic and enhance control reliability considering traffic demand variation. The trade-off between platoon string stability and safety to determine the
optimal cooperative crossing control for CAVs is investigated. Traffic information acquisition systems can collect traffic data from connected-vehicle systems and various traffic sensors, including loop detectors, ultrasonic devices, and image recognition devices. By using the data from upstream traffic, including flow rate, traffic speed, density, and travel time, the traffic demand at the subject intersection can be predicted in real time. In this study, an adaptive virtual platoon control is introduced by adjusting the control parameters adaptively according to the predicted traffic demand of the subject intersection.

The proposed adaptive control aims to achieve three objectives. The first is to ensure the virtual platoon stability to mitigate perturbation effects resulting from demand variation. Second, adaptive control seeks to maintain the traffic throughput at a satisfactory level. Third, the proposed control is intended to ensure a smooth control transition by minimizing the changes of the control parameters. To achieve the three objectives, this study formulates the adaptive virtual platooning as a discrete-time optimization problem assuming that the demand does not deviate from the predicted value in the next time interval. The platoon parameters will be adjusted dynamically based on the predicted demand variation using the proposed optimization to ensure cooperative behavior among CAVs while enhancing CAVs’ travel reliability.

The rest of this paper is organized as follows: the following section presents the proposed discrete-time optimization problem for the adaptive control of virtual platooning. Section 3 carries out simulation experiments and shows the effectiveness of the proposed adaptive control strategy. Section 4 concludes this study and provides future research directions.

2 | METHODOLOGY

2.1 | Virtual platooning

This study considers traffic management at an isolated four-leg intersection, where each entrance or exit has one lane. For simplicity, this study does not consider turning movements, indicating that vehicles in each lane of the intersection can only go straight. The proposed model is developed based on the assumption of 100% penetration of CAVs, implying that all vehicles are automated and equipped with vehicle-to-vehicle (V2V) communication and positioning devices. This study assumes that all vehicles are homogeneous. When these CAVs approach the intersection, they instantly broadcast their states to other vehicles through V2V communications, including position and speed.

As a distributed cooperative intersection control, virtual vehicle platooning has been proposed to improve intersection mobility and safety. It has broad advantages in reducing computational and communication burden to enhance system security and reliability. This study adopts the virtual platoon control strategy to manage CAV traffic at an isolated signal-free intersection. In a virtual platoon control, approaching CAVs on different links are projected into one virtual platoon according to their distance to the center of the intersection to ensure the safety of the cooperative crossing. The projection of the virtual vehicles is shown in Figure 1. The projected virtual vehicle platoon is shown in the dashed line rectangle. The area within the circle is called the cooperation zone. In the cooperation zone, vehicles will form a self-organized car-following maneuver to cross the intersection. Denote R as the radius of the intersection cooperation zone and Q as the center of the intersection.

The virtual platoon differs from the real vehicle platoon due to vehicle crossing relations, which can be categorized into conflict movements and nonconflict movements without considering the turning movements. The conflict movements are perpendicular ones, such as eastbound and southbound movements. Due to safety concerns, vehicles on conflict movements cannot cross the intersection simultaneously. The nonconflict movements are parallel. For example, northbound and southbound movements are nonconflict because vehicles can cross the intersection simultaneously. In Figure 1, vehicles #2 and #3 have nonconflict movements. In this case, vehicle #3 does not necessarily follow vehicle #2 (the nearest preceding vehicle projected on the virtual vehicle platoon) but follows vehicle #1 traveling northbound. Then, vehicles #2 and #3 can cross the intersection simultaneously to increase the intersection throughput.

The complex relationship of different traffic movements at the intersection is crucial for developing an analytical model. To tackle this issue, this paper first defines the car-following schedule set F for cooperating vehicles in the cooperation zone based on their crossing relations. A schedule set F contains the adjacent vehicles in the virtual platoon that can cross the intersection simultaneously. Denote set N containing all vehicles in the cooperation zone. Then, all vehicles in

FIGURE 1 Virtual vehicle platoon projection
the cooperation zone will be clustered into a set of schedule sets $\mathbb{E}_i$, $i = 1, 2, 3, \ldots$, such that, for any vehicle $n \in N$, it only belongs to one $\mathbb{E}_i$. Given the detected vehicles in the intersection cooperation zone, a clustering procedure that is simplified based on research\textsuperscript{22} is conducted as follows: first, this study assumes a pure CAV environment and all vehicles will be willing to cooperate with other vehicles to cross the intersection. Therefore, the virtual platoon vehicles are placed into an order based on their distance to the intersection center. The vehicle closest to the center of the intersection is tagged as vehicle $#1$, and it belongs to $\mathbb{E}_0$. Starting with vehicle $n = 2, 3, 4, \ldots$, in the virtual platoon, the clustering procedure checks whether the vehicle’s crossing movement conflicts with its predecessor’s crossing movement. If not, then this vehicle is clustered into its predecessor’s schedule set, for example, $\mathbb{E}_{i-1}$; otherwise, this vehicle belongs to a new schedule set $\mathbb{E}_i$. If multiple vehicles have the same distance to the intersection center, then they share the same predecessor. For this case, the clustering procedure enumerates all possible clustering solutions such that the number of schedule sets is minimized. The algorithm is shown in Table 1. Then, the virtual platoon vehicles make their following behaviors by the schedule sets $\mathbb{E}_i$, rather than by vehicles. The rationale behind this clustering procedure is to maximize the usage of the intersection capacity by allowing nonconflict crossing movements to share the intersection simultaneously. Note that the car-following schedule set $\mathbb{F}$ can be further extended to accommodate turning movements and multiple lanes in each direction. However, considering the turning movements and possible lane-changing behaviors due to multiple lanes requires another set of derivations of the stability conditions to account for their impacts on traffic dynamics. Such complexities demand further analyses in the future.

Take the virtual platoon in Figure 1 as an example. Vehicles #1 to #4 are clustered into the following schedule set $\mathbb{E}_0 = \{1\}$, $\mathbb{E}_1 = \{2, 3\}$, $\mathbb{E}_2 = \{4\}$, where vehicle 1 is the leading vehicle in the virtual platoon. Because vehicles #2 and #3 have nonconflict movements, they are clustered into the same schedule set $\mathbb{E}_1$ to cross the intersection simultaneously. After obtaining the schedule sets, the topology of the virtual vehicle platoon can be determined. Any vehicle $n \in \mathbb{E}_i (i > 0)$, it will follow a virtual platoon vehicle $m \in \mathbb{E}_{i-1}$ and maintain the desired space gap $d$ while tracking the speed of vehicle $m$. Under an ideal condition, virtual platoon vehicles will converge to the equilibrium state as the following equations:

$$
\lim_{t \to \infty} (v_n(t) - v_m(t)) = 0,
$$
$$
\lim_{t \to \infty} (p_n(t) - p_m(t) - d) = 0,
$$

where $v_n$ and $p_n$ are the speed and position of vehicle $n \in \mathbb{E}_i$, $v_m$ and $p_m$ are the speed and position of vehicle $n \in \mathbb{E}_{i-1}$, and $d$ is the desired space gap. Different underlying car-following models can be used in the virtual vehicle platoon. The above condition only depicts the equilibrium state of the virtual vehicle platoon, which represents the ideal condition. However, demand variations will result in issues including congestion and perturbations on virtual platoon formation.

To solve these issues, this paper proposes an adaptive virtual platoon control by adjusting the underlying car-following dynamics.

### 2.2 Adaptive virtual platoon control

Traffic demand at an intersection is stochastic and could vary rapidly. The demand exceeding the capacity of intersections during peak hours causes queues and traffic congestion. Moreover, traffic congestion will increase travel time, air pollution, fuel consumption, and economic losses. Additionally, the demand variation will introduce uncertainty to the intersection, which can lead to perturbations to the upcoming traffic. Such perturbations will result in speed oscillations of vehicles that affect the traffic control reliability, increase collision risk, and raise substantial safety concerns. Therefore, it is crucial to consider the demand fluctuation to foster reliable cooperative intersection crossing to reduce traffic congestion and mitigate the effect of traffic perturbations to enhance traffic reliability and safety.

#### 2.2.1 Optimization model

The adaptive control is built upon the traffic demand of the cooperation zone during an upcoming time interval, which can be predicted using available traffic acquisition technology and traffic prediction algorithms.\textsuperscript{24–26} Based on the predicted traffic demand, this study proposes to optimize the car-following behavior of the cooperative CAVs when they form a virtual platoon responding to the varying traffic density.

If the predicted demand of approaching traffic changes over a specific level in the next time interval as shown in “Predicted traffic demand” in Figure 2, the control system will trigger the implementation of the optimization model to optimize the parameters of the car-following control to enhance traffic mobility and reliability, illustrated by the “Parameter optimization” box in Figure 2. The general form of the optimization model will be shown later in Equation (3) and the formulation of the optimization model will be explicitly presented in Equation (12) in Section 2.2.2. Note that the throughput at the
signal-free intersection and the reliability of cooperative intersection crossing can be theoretically derived through the stability analysis of the virtual platoon. Therefore, analyzing the theoretical properties of the virtual platoon enables the formulation of the demand-driven adaptive control for cooperative intersection crossing. After the parameters are identified, individual vehicle’s velocity and acceleration could be computed spontaneously by applying these suggested values to the car-following model adopted by the vehicle, shown as the “CAV Following behavior adversary” process in Figure 2.

The proposed adaptive control for cooperative intersection crossing seeks to achieve three objectives. The first objective is to ensure the virtual platoon’s string stability. The second objective of the proposed adaptive control aims to guarantee that the traffic throughput is maintained at a satisfactory level while reducing the congestion effect due to demand variation. The third objective is to mitigate the changes in the control parameters to ensure a smooth control mechanism.

String stability considered in the first objective investigates the behavior of the entire platoon in response to external perturbations. If the speed of one CAV is perturbed, its following vehicles in the virtual platoon have to react to the changes due to the distributed car-following control mechanism for the cooperative intersection crossing. The string stability of the virtual platoon ensures that the speed perturbation is not amplified toward the upstream traffic.

The CAVs face continuous perturbations when approaching the intersection. The upstream traffic demands on different legs vary with time, and vehicles’ arrival time at the intersection can be deemed as a random process. Such randomness causes perturbations to the virtual platoon. Additionally, CAVs approaching the cooperation zone need to adjust their speeds to join the virtual platoon, which will introduce perturbations. These perturbations significantly impact the reliability of CAVs’ cooperation behavior. If the effects of perturbations cannot be mitigated, the resulting oscillation of the virtual platoon can raise collision risk and substantial safety concerns, causing the failure of the cooperative intersection crossing. Therefore, platoon string stability must be guaranteed in the perturbed environment to promote reliable cooperation behavior among the CAVs.

This study integrates platoon string stability conditions into the development of adaptive control. The integration considers vehicles’ ability to restabilize back to the equilibrium after perturbations. The equilibrium is defined as the state that every vehicle has the same space gap $d$ with the preceding vehicle at the equilibrium speed. The platoon string stability is defined as follows:

**Definition 1.** If there exist $\epsilon_1, \epsilon_2 > 0$, for any vehicle $n$, it satisfies

$$\lim_{t \to \infty} |v_1(t) - v_0(t)| \leq \epsilon_1$$

then the platoon is at a string stable state.

In Equation (2), $d$ is the space gap between adjacent vehicles at an equilibrium state. Following the derivation in existing research, the string stability condition is obtained for the virtual platoon. The derived platoon string stability condition can be applied to various car-following models. This paper uses function $h(\cdot)$ to represent the string stability condition. In particular, $h < 0$ stands for the case that the platoon is under the string stable condition and $h \geq 0$ stands for the unstable condition. The detailed formation of the string stability condition will be presented later.

The second objective of the proposed adaptive control aims to guarantee that the traffic throughput is maintained at a satisfactory level. This paper uses function $TH(\cdot)$ to denote the estimated intersection throughput given a set of car-following parameters. Notation $D_t$ denotes the predicted traffic demand at time interval $t$ that is the input of the control system. To maintain the intersection throughput at a satisfactory level, the condition $TH - D_t > 0$ denotes the surplus between intersection throughput and traffic demand.

The third objective seeks to mitigate the perturbation on the control parameters to ensure a smooth control mechanism. Adjustment of the CAV car-following parameters will affect the platooning vehicle’s dynamics. Abrupt adjustments introduce perturbations to CAVs dynamics that raise safety concerns. Therefore, a smaller change in car-following parameter adjustment implies a smoother control. Our model denotes the car-following parameters as a vector $\theta$. Without loss of generality, this study applies the Euclidean distance to measure the extent of adjustment of platoon control parameters. Other functions, such as $L_2$-norm or vehicle’s jerk could be integrated into the measurement of control smoothness.

This paper proposes an optimization model to achieve the three control objectives. If the predicted demand of approaching vehicles changes beyond a threshold in the next time interval, the proposed optimization will optimize the parameters in CAVs’ car-following control to help them form a virtual vehicle platoon adapting to the new traffic demand while enhancing the platoon reliability. The optimization problem is formulated as follows:

$$\text{minimize } \alpha \times h + \beta \times (TH - D_t) + \gamma \|\theta - \hat{\theta}\|_2$$

s.t. $h(\theta) < 0$

$\theta_{\min} \leq \theta \leq \theta_{\max}$

(3)

where the CAV car-following control parameters are in $\theta$. The coefficients $\alpha$, $\beta$, and $\gamma$ in the objective function denote the weights assigned to the three control objectives. Note that $\alpha > 0, \beta < 0, \gamma > 0$. 

**Figure 2** Adaptive control diagram

- **CAV Following behavior adversary**
- **Parameter optimization**
- **Virtual platoon**
- **Actuating**
- **Predicted traffic demand**
In the objective function, the first component $\alpha \times h$ seeks to warrant the string stability condition. The optimized parameters are expected to be further away from the stability transition surface to gain a better stability condition. Therefore, the value of the first component $\alpha \times h$ should be the smaller, the better. The second component $\beta \times (TH - D_l)$ denotes the surplus between intersection throughput and predicted traffic demand level. A negative weight is imposed on this component to enlarge the surplus to mitigate traffic congestion. The third component $\gamma \| \theta - \theta^\ast \|_2$ is the weighted Euler distance between the current parameters ($\theta$) and the optimal ones for the varied demand ($\theta^\ast$). This component ensures that platoon control is smooth and reduces the perturbations caused by parameter adjustments. The weights $\alpha$, $\beta$, and $\gamma$ can be dynamically adjusted to balance the traffic throughput and the platoon stability condition. The optimization problem contains two constraints. The first constraint ensures the stability of the virtual vehicle platoon. The second constraint represents the adjustment ranges for the platoon control parameters.

2.2.2 | Underlying car-following model: Intelligent driver model

The proposed optimization model can be adapted to different car-following models for describing underlying CAVs’ following behavior when they form the virtual platoon. Various car-following models have been developed in the literature, such as Newell’s model, Pipes Model, the Optimal Velocity Model, and the Intelligent Driver Model (IDM). This study adopts the IDM due to the following advantages. First, the IDM is a multiregime model, which captures the dynamics of different traffic congestion levels more realistically than other models. Second, it provides collision-free behavior and smooth traffic flow. Third, it is well accepted to model connected autonomous vehicles’ longitudinal dynamics. In the IDM, a vehicle’s acceleration is formulated as the following differential equation:

$$\dot{\psi}_n(t) = \left[ 1 - \frac{\psi_n}{\psi_0} \right] \delta - \left( \frac{s^*(\psi_n, \Delta \psi_n)}{s_0} \right)^2,$$

$$s^*(\psi_n, \Delta \psi_n) = s_0 + \psi_n T + \frac{\psi_n \cdot \Delta \psi_n}{2 \sqrt{ab}},$$

where $\dot{\psi}_n(t)$ and $\psi_n$ denote the acceleration and speed of vehicle $n$ at time $t$; $a$ and $b$ denote the maximum acceleration and deceleration of the vehicle, respectively; $\psi_0$ denotes the free-flow speed; $\delta$ is the acceleration exponent parameter; $\Delta \psi_n$ denotes the speed difference between vehicle $n$ and its preceding vehicle $n - 1$; $s_0$ represents the minimum bumper-to-bumper gap in traffic jam states; $T$ is the desired time gap to follow the preceding vehicle $n - 1$; and $s_n$ denotes the net distance $s_n = p_m - p_n - l$, where $n \in \mathbb{N}$, $m \in \mathbb{N} + 1$, and $l$ is the vehicle length. In the model, the vehicle speed at equilibrium state, desired time gap $T$, and maximum acceleration $a$ are the adaptive control parameters.

With the IDM model adopted, the three components in the objective function in Equation (3) can be specified. For the first objective to guarantee the platoon string stability, the stability condition $h$ is obtained for the IDM model as follows:

$$\left( \psi_n^2 \right) + \frac{1}{2} \alpha^2 + \gamma \psi_n < 0,$$

where

$$\left( \psi_n^2 \right) = \frac{\psi_n^2}{\psi_0},$$

$$g_n^+ = \frac{2\alpha}{\psi_0} \left( \frac{s_0 + \psi_n T}{s_0} \right)^2,$$

$$g_n^- = -2\alpha \left( \frac{s_0 + \psi_n T}{s_0} \right)^2,$$

$$g_n^{\Delta \psi} = \frac{\psi_n}{s_0} \sqrt{\frac{\alpha}{\beta}} \frac{s_0 + T \psi_n}{s_0},$$

where $\psi_n$ and $s_0$ denote the vehicle speed and the net distance between vehicles at equilibrium state, respectively.

The stability transition surface can be obtained from the neutral stability criterion in Equation (5). Figure 3 shows the stability transition surface in the space of vehicle equilibrium speed, desired time gap, and maximum acceleration. The virtual platoon is stable when the control parameters are above the stability transition surface; otherwise, the platoon is unstable. The figure shows that the stability transition surface moves downward with the increase of the maximum acceleration $a$ and the desired time gap $T$. This trend indicates that the stability condition of the vehicle platoon will be improved if the maximum acceleration or the desired time gap is increased.

For the second objective, the form of throughput $TH$ can be specified based on the underlying car-following model, that is, the IDM model in the present study. The intersection throughput can be approximated using the virtual vehicle platoon traffic flow $f$ at equilibrium state by the following equation:

$$TH = \lambda \times f,$$

where $\lambda$ is the scaling parameter.

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**FIGURE 3** Stability transition surface
As described previously, two vehicles in a virtual platoon with nonconflict movements can cross the intersection simultaneously. Therefore, the intersection throughput is scaled up by $\lambda(1 \leq \lambda \leq 2)$ times of the platoon traffic flow at equilibrium. Here, $\lambda = 1$ represents the extreme scenario that there is only one vehicle in each schedule set, indicating that only one vehicle crosses the intersection at each time, while $\lambda = 2$ represents the other extreme scenario with maximum throughput, in which two vehicles in every schedule set form a vehicle pair to cross the intersection simultaneously without conflicts. The $\lambda$ can be estimated when the schedule sets are determined. Note that the schedule sets depend on the predicted vehicle arrivals that can be given by available sensing technology.

The following analysis focuses on estimating traffic flow at equilibrium, given CAV car-following parameters. Based on Equation (4), vehicle acceleration $v_a(t)$ equals to 0 when achieving the equilibrium state. By using this condition, the traffic flow at equilibrium state is represented by

$$
\begin{align*}
    f &= \frac{v_e}{s_e + t}, \\
    s_e &= \frac{s_0 + v_e T}{\sqrt{1 - \left(\frac{v_e}{v_0}\right)^2}}. \\
\end{align*}
$$

Equation (11) shows that the maximum acceleration does not affect the traffic flow at equilibrium. The relationship between the traffic equilibrium flow, equilibrium speed, and desired time gap can be shown by the surface in Figure 4. As the figure shows, the increase in equilibrium speed $v_e$ and decrease of desired time gap $T$ will increase the traffic equilibrium flow of the vehicle platoon.

Based on the above analysis, the formulation of the optimization problem with the specified IDM model is as follows:

$$
\begin{align*}
    \text{minimize} & \quad \mu \times \left( v_e^i \right)^\gamma + \frac{1}{2} g_n^\lambda + g_n^\lambda \right) \\
    & \quad + \beta \times \left( \lambda \times \frac{v_e}{s^* + l} - D_1 \right) + \gamma \| \theta - \hat{\theta} \|_2 \\
    \text{s.t.} & \quad \left( v_e^i \right)^\gamma + \frac{1}{2} g_n^\lambda + g_n^\lambda < 0 \\
    & \quad \gamma_{\text{min}} \leq v_e \leq \gamma_{\text{max}} \\
    & \quad T_{\text{min}} \leq T \leq T_{\text{max}} \\
    & \quad a_{\text{min}} \leq a \leq a_{\text{max}},
\end{align*}
$$

where $\theta = [v_e, T, a]$ denotes the car-following control parameters of CAVs. In this paper, these parameter values are from the study that provided calibrated IDM parameters to represent the common behaviors in the real world. In the model, the last three constraints represent the bounds for the car-following control parameters. This study applies the grid search method to solve the optimization problem.

### 3 | NUMERICAL STUDY

We conducted a simulation study to verify the proposed adaptive cooperative intersection crossing control. This study adopts the IDM model to analyze the string stability, enabling us to include the closed-form formula in the objective function of the proposed optimization problem. However, the simulation platforms apply different car-following models to describe vehicle dynamics. For verification purposes, our simulation conducted in this study in Python excludes the impact of car-following models, allowing us to concentrate our analyses on the effects of random arrival, arriving speeds, and demand variations.

This simulation considers an isolated intersection scenario without turning movements shown in Figure 1. When CAVs enter the cooperation zone, they communicate and share information with each other. These CAVs coordinate their movements and cross the intersection cooperatively. The simulation time is set to be 1 h, which is divided into four quarters. The warming-up time is not considered in the analysis. We assume that the average demand can be estimated in real time and vehicles’ arrivals from the four legs follow a negative exponential distribution with the mean headway $\mu$ equal to 7.2, 6, 5.1, and 4.9 s during the first, second, third, and fourth quarter, respectively. The resultant approaching traffic flow follows the Poisson distribution, with the corresponding mean demand being 2000, 2400, 2800, and 2900 vehicles per hour (vph), respectively.

This paper designs two simulation scenarios. The first scenario applies the proposed adaptive control. The parameters $v_e, T, a$ (i.e., the equilibrium speed, desired time gap, and maximum acceleration) of the CAVs are optimized by the model Equation (12) at the 15th, 30th, and 45th min to adapt to the upcoming demand fluctuation. This study applies the grid search method to solve the optimization problem. The proposed optimization problem is not convex due to the string stability condition formula in the objective function. Therefore, the solution solved by the grid search method may not be the global optimum. The second simulation scenario is labeled as the nonadaptive control scenario, which simply maintains the car-following parameters as constants during the 1-h simulation. The simulation parameters are listed in Tables 2 and 3, summarizing the platoon parameters in four different time intervals optimized by Equation (12) for the case of adaptive control.

Since the vehicles’ arrival follows the negative exponential distribution, which introduces stochasticity in the simulation, this numerical study executed each 1-h simulation scenario 10 times. For the scenario of adaptive control, 2523 vehicles on average crossed...
traffic throughput under the nonadaptive control stays at a low level compared to the increased traffic demand after 15-min simulation. The induced queue propagated outside the cooperation zone. Such queue propagation can lead to oversaturation or even gridlock in the real world. The results demonstrate that the performance of nonadaptive control is not acceptable when facing significant traffic demand fluctuations.

Furthermore, Figure 6 shows the vehicle speed profiles when the car-following parameters are adjusted around the 15th, 30th, and 45th min in the adaptive control scenario. Vehicles from four directions are represented using four different colors. From Figure 6, one can find that vehicles from different directions reach the equilibrium state fast when crossing the intersection. Meanwhile, the perturbations induced by the car-following control parameters do not propagate toward the upstream. The platoon stability is guaranteed when the parameters are changed. When the demand varies, the CAVs entering the cooperation zone can adaptively adjust their car-following behavior when they enter the cooperation zone and reach the stable state with consensus in a short time. These results demonstrate the effectiveness of the proposed adaptive control strategy in terms of maintaining the traffic throughput at a satisfactory level as well as maintaining the platoon stability when facing the varying demand. As for the nonadaptive control scenario, the vehicle speed profile maintains the same pattern during the 1-h simulation since the parameters do not change.
We further analyze the optimized parameters. Figure 7 illustrates the changes of the optimized parameters in four time intervals, shown as points with different colors in the space composed of the equilibrium speed, desired time gap, and maximum acceleration. The dashed lines with arrows represent the changes of the optimized parameters. The stability transition surface to warrant string stability is shown in gray. The platoon is string stable above the surface; otherwise, the platoon is unstable when the state is below the surface. One can find that four points are all above the stability transition surface, indicating that the platoon stability is guaranteed in all time intervals. The desired time gap $T$ in the second, third, and fourth quarters is decreased by 0.438, 0.637, and 0.950, respectively, compared to the initial value. The desired time gap $T$ is the crucial parameter to be adjusted because a small decrease of the desired time gap can significantly improve the traffic throughput. As for the equilibrium speed, one can find that the equilibrium speed remains the same in the first three quarters. This is because the third component in the objective function of Equation (12) aims to smooth the control, which suppresses the adjustment of the equilibrium speed. In the last quarter, due to the further increased demand, the desired time $T$ decreases significantly, which will decrease the platoon’s string stability. Therefore, the equilibrium speed decreases in the last quarter to ensure platoon string stability. The maximum acceleration $a$ in the last three quarters reaches its upper bound of 1.3 m/s². This is because the maximum acceleration $a$ will not affect the intersection throughput. However, the increase of maximum acceleration will improve the platoon’s stability, which is further from the stability transition surface.

This study then analyzes how parameters can be adjusted to balance intersection throughput and platoon stability in the 1-h
TABLE 4 Time exposed to-collision (TTC) indicator (TET)

| Time interval (min) | 0–15 | 15–30 | 30–45 | 45–60 |
|---------------------|------|-------|-------|-------|
| Poisson arrival average headway (s) | 7.2  | 6     | 5.1   | 4.9   |
| Average demand level (vph) | 2000 | 2400  | 2800  | 2900  |

| Adaptive control | TTC threshold | TET | TET | TET | TET |
|------------------|---------------|-----|-----|-----|-----|
|                  | 2s            | 0s  | 0s  | 0s  | 0s  |
|                  | 3s            | 0s  | 0s  | 0s  | 0s  |
|                  | 4s            | 0s  | 0s  | 0.8s| 5.3s|

simulation. We calculate the value of the first component of the objective function based on Equation (12) for the four quarters, which represents the stability margin\textsuperscript{41} from the stability transition surface. The values of stability margin are \(-9.2, -11.8, -9.24,\) and \(-1.57\) in each quarter, respectively. A larger absolute value of stability margin indicates that the traffic state is farther away from the stability transition state. One can find that the margin during the second quarter is the largest, representing the best string stability condition against external perturbation. This is because though the time gap \(T\) is decreased to enhance the throughput in the second quarter, the increase of maximum acceleration significantly improves the stability, which counteracts the effect of decreased time gap \(T\). The stability margin in the third and fourth quarters decreases compared with the second quarter because the desired time gap decreases to enhance throughput. The virtual platoon sacrifices the platoon string stability condition for a higher intersection throughput in these two quarters.

This study further analyzes the time to collision as a measurement to investigate the safety issue of the proposed adaptive control strategy using the safety indicator TET (time exposed to-collision [TTC] indicator).\textsuperscript{42} The TET is calculated for adaptive and nonadaptive control scenarios in four quarters using three TTC thresholds: 2, 3, and 4 s. The results are summarized in Table 4. For the scenario under adaptive control, the TETs in four quarters are all 0 s for TTC thresholds 2 and 3 s, indicating that there is no vehicle exposed to potential risk with TTC threshold values below 2 and 3 s. With the TTC threshold increased to 4 s, the TET value in the third quarter is increased to 0.8 s. In particular, for the 708 vehicles arriving at the intersection during the third quarter, their average collision exposure time per vehicle is only 1.1 ms in the 15 min. The TET value in the last quarter is further increased to 5.3 s. For the 714 vehicles arriving at the intersection during the last quarter, the average exposure time per vehicle is only 7.4 ms. The increase of the TET value in the last two quarters resulted from the decreased stability condition and decreased desired time headway. The virtual platoon sacrifices part of the safety for a higher intersection throughput in these two quarters. Note that the TTC threshold of 3 and 4 s is highly conservative for CAVs.\textsuperscript{43} The average exposure time per vehicle meets the vehicle-to-everything communication safety latency requirements, ranging from 3 to 100 ms based on the latest 5G and C-V2X communication protocols, guaranteeing performance reliability.\textsuperscript{44,45} Therefore, the TTC results highlight the advantages of the proposed adaptive intersection crossing control in securing performance reliability.

4 | CONCLUSIONS

In this investigation, an adaptive cooperative intersection crossing control is developed to account for the time-dependent traffic arrivals and enhance cooperative travel reliability impacted by demand variation. An adaptive virtual platoon control has been introduced by adjusting the CAV car-following parameters according to the changing demand. The proposed adaptive control aims to achieve three objectives. The first objective is to ensure the virtual platoon stability to mitigate perturbation effects resulting from demand variations. Second, the control seeks to maintain the traffic throughput at a satisfactory level. The third is to ensure smooth control. To achieve the three objectives, this study formulates the adaptive virtual platooning control as an optimization problem, assuming that the demand does not change significantly in a given time interval. The platoon parameters are adjusted dynamically based on the detected demand to ensure cooperative behavior among CAVs while enhancing their travel reliability.

The simulation results have demonstrated the effectiveness of the proposed adaptive control under demand variations. The traffic throughput results show that, with proper adaptive controls, the traffic throughput can be maintained at a satisfactory level against the varying demand. Moreover, speed profiles show that vehicles can adaptively adjust their speeds to reach the steady state with consensus when they enter the cooperation zone perturbed by demand variations. The research further analyzes the trade-off between traffic throughput and platoon string stability. The analysis demonstrates that the increase of maximum acceleration can improve the stability condition. The decrease of desired time gap can enhance the traffic throughput to reduce congestion; however, it sacrifices platoon stability as a trade-off. Finally, this study conducts a TTC analysis to investigate CAV travel reliability when participating in the cooperative intersection crossing. The result demonstrates the advantages of the proposed adaptive control in protecting vehicle safety when seeking a higher intersection throughput.

In the future, the proposed adaptive control for cooperative intersection crossing can be further extended in several directions. First, the presented research does not consider the turning movements at the intersection. The proposed adaptive control is flexible to incorporate turning movements and multiple lanes by adjusting the car-following schedule sets, while it requires derivations of the stability conditions to account for their impacts on traffic dynamics. Second, future research can focus on investigating the impact of communication-side effects—such as communication topology and communication latency—on the reliability of cooperative intersection control to facilitate a better design of control.
strategies for connected vehicles. Third, this study does not consider the mixed traffic consisting of human-driven vehicles in transition to the pure CAV travel environment. To address this issue, the proposed approach can possibly be extended to accommodate human-driven vehicles by revising the optimization problem based on assumptions of heterogeneous driving behaviors. Fourth, extending the research to a corridor with multiple signal-free intersections will be an interesting research direction, where traffic coordination strategies can further enhance the reliability and throughput of cooperative intersection crossing.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author.

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