Scotogenic Dirac neutrino model embedded with leptoquarks: one pathway to addressing all

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If the leptoquarks proposed to account for the intriguing anomalies observed in the B-meson decays, $R_{D^{(*)}}$ and $R_{K^{(*)}}$, as well as in the anomalous magnetic moment of the muon, $(g - 2)_\mu$, can be embedded into the scotogenic Dirac neutrino model, all these flavor anomalies, together with the origin of neutrino masses and the nature of dark matter, would be potentially addressed in a unified picture. Among the minimal seesaw, one-loop, and two-loop realizations of the dimension-4 effective operator $\mathcal{L}_4$ for the Dirac neutrino masses, we show that plenty of diagrams associated with the two-loop realizations of $\mathcal{L}_4$ can support the coexistence of leptoquarks and dark matter candidates. After a simple match of these leptoquarks to those that can accommodate all the flavor anomalies, we establish the scotogenic Dirac neutrino models embedded with leptoquarks, which could address all the problems mentioned above.

I. INTRODUCTION

Despite its glory in elementary particle physics, the Standard Model (SM) fails to explain the origin of tiny neutrino masses and the nature of dark matter (DM). In general, these two problems are considered as two separate topics, and could be solved via completely different mechanisms with unrelated particles and/or interactions. However, addressing them in a unified picture, in which the neutrino mass scale as well as the DM properties and its abundance can be quantitatively connected with each other, would be more intriguing. It is known that the scotogenic models can fulfill the task, because the essence of these models, as demonstrated in the original works [1, 2], is that the neutrino mass generation involves at least one DM propagator. Interestingly enough, the interactions responsible for the neutrino mass generation, though not necessarily, can also successfully account for the DM relic abundance [3]. Thus, in a sense, the scotogenic models can really kill two birds with one stone.

Depending on the nature of neutrinos, being of either Majorana or Dirac type, the scotogenic models are generally classified into two categories. Historically, neutrinos of the Majorana type are relatively more motivated and various mechanisms for their mass generation, based either on the seesaw [4–10] or the loop effects [1, 11–13], have been proposed. However, since no indisputable evidence has been reported for the neutrinoless double beta decay so far [14–18], the nature of neutrinos is still unsettled, and the possibility of Dirac neutrinos should not be discounted. In fact, during the past few years there has been a renewed interest in building the mass generation models for the Dirac neutrinos, in which the tiny masses can be generated either through the seesaw mechanisms [19–36] or the loop effects [3, 37–57]. Among these proposals, if the DM exchanges are also involved, they can be identified as the scotogenic Dirac neutrino models (SDnM) [3], one of the key “stones” of this work. Besides in the mass generation mechanisms, growing interests have also been rekindled in other aspects of the Dirac neutrinos, such as their intimate connections to the baryon asymmetry of the Universe [58, 59], which is another conundrum that the SM of particle physics is now facing.

Besides the problems imposed by neutrinos and DM, several intriguing anomalies in the B-meson decays, particularly in the ratios $R_{D^{(*)}}$ [60–68] and $R_{K^{(*)}}$ [69–74], as well as in the muon $(g - 2)_\mu$ measurements [75, 76] have been reported in recent years. Interestingly, it has been demonstrated that models with only a single leptoquark (LQ) [77–79] or a few of them [80–87] are capable of addressing all the flavor anomalies simultaneously. Inspired by the spirit of SDnM, we will explore if these “stones”, once combined properly, can crack all the flavor anomalies, together with the origin of neutrino masses and the nature of dark matter in a unified picture. Arguably, the simplest approach is to check if whatever accounts for the $B$-meson and $(g - 2)_\mu$ anomalies simultaneously also contributes to the Dirac neutrino mass generation. If these viable LQs could be embedded into the SDnM, not only can all the aforementioned problems be solved in a unified picture, but also their rich phenomenology correlated with the neutrino mass and mixing will make the constructed models more predictable and testable.

Our starting point will be the classification of minimal seesaw (tree-level), one-loop, and two-loop realizations of the Dirac neutrino mass operator $\mathcal{L}_4$ at dimension four [88, 89] (Note that the classification of seesaw and loop realizations of the Dirac neutrino mass operators at dimension five [90–92] and six [93, 94] also exists; our choice here is solely for simplicity). Guided exclusively by the SM gauge symmetry, we will work out a selection scheme for the topologies, in which each topology, with possible diagrams and ultraviolet (UV) completions associated with it, will be examined...
systematically. In contrast to the previous studies [88, 89], the topology we are seeking must involve the exchanges of at least one LQ and one DM candidate, supporting therefore our scenario of SνDM embedded with LQs—only after the dominance of their contributions to the Dirac neutrino masses is established, will the topology-based UV completions be dubbed LQ-SνDM. If the LQ(s) embedded can simultaneously account for the B-meson and (g − 2)μ anomalies as well, the LQ-SνDM will be considered as the mighty “stones” as well. As will be demonstrated in the paper, there exist plenty of such kinds of mighty models. Intriguingly, in some of those models, a close-knit connection between the flavor anomalies and neutrino mass can be established.

The paper is organized as follows. We begin Sec. II by establishing a topology-selection scheme, under which we examine the topologies with their associated diagrams of the two-loop realizations of L4, and identify the ones that can support the coexistence of at least one LQ and one DM candidate. Based on the existing studies aimed at addressing all the flavor anomalies with LQs exclusively, we establish the possible mighty “stones” in Sec. III. Our conclusions are finally made in Sec. IV, and supplementary materials are provided in the appendices.

II. SνDM EMBEDDED WITH LQs

In order to have massive Dirac neutrinos, one needs firstly extend the SM particle contents by adding the right-handed neutrinos νR. Arguably, the simplest way goes with the following Yukawa interaction,

\[ \mathcal{L}_4 = -y \bar{L}_L H \nu_R + \text{H.c.}, \]  

(1)

where \( L_L = (\nu_L, e_L)^T \) is the left-handed lepton doublet, and \( \bar{H} = i \sigma_2 H^* \) with \( H = (\phi^+, \phi^0)^T \) being the SM Higgs doublet. However, being often criticized as unnatural, the Yukawa coupling \( y \) must be tuned to a very small value, \( y \sim O(10^{-15}) \), to account for the sub-eV neutrino masses. To circumvent this problem, more attractive mechanisms have been proposed, such as the Dirac seesaw [19–36] and the radiative mass generation [3, 37–57], all of which are not confined to the Yukawa operator specified by Eq. (1).

In this paper, we will focus on the case in which the Dirac neutrino masses are generated by those attractive mechanisms. In addition, a global lepton-number symmetry U(1)L will be introduced to ensure the absence of Majorana mass terms for \( \nu_R \) at all orders, so that the Dirac nature of the neutrinos is protected. In general, there may be an infinite number of UV completions of \( \mathcal{L}_4 \) that respect the SM and lepton-number symmetries. For simplicity, we will only focus on the fields transforming as singlet, doublet, or triplet under the SU(2)L gauge symmetry. Furthermore, for the sake of minimality, we will introduce limited number of new degrees of freedom carrying colors. Before diving into the classification of minimal seesaw (tree-level), one-loop, and two-loop realizations of \( \mathcal{L}_4 \), and identifying which topology with its associated diagrams can support the coexistence of at least one LQ and one DM candidate, we will firstly give a brief discussion about the LQs and DM candidates, and then establish a topology-selection scheme.

The LQs, due to their ability of turning quarks into leptons and vice versa, have very rich phenomenology in, e.g., the anomalous magnetic moment of the charged leptons, the weak decays of various hadrons, the neutral meson mixings, etc. See Ref. [95] for a recent review. If their interactions with the right-handed Dirac neutrinos are taken into account, there are totally twelve LQs: six scalars and six vectors. Their possible couplings to the SM fermions and νR, as well as their representations under the SM gauge symmetry SU(3)c ⊗ SU(2)L ⊗ U(1)α, are summarized in Table I schematically.

| Scalar LQ | SM Rep. | Vector LQ | SM Rep. |
|-----------|---------|-----------|---------|
| \( S_i \tilde{Q}_L^c \nu_R \) | \((3,1,1/3)\) | \( U_i \nu \tilde{Q}_L^c \nu_R \) | \((3,1,2/3)\) |
| \( S_i \tilde{d}_R^c \nu_R \) | \((3,1,2/3)\) | \( U_i \nu \tilde{d}_R^c \nu_R \) | \((3,1,1/3)\) |
| \( S_i \tilde{d}_L^c \nu_R \) | \((3,1,2/3)\) | \( U_i \nu \tilde{d}_L^c \nu_R \) | \((3,1,1/3)\) |
| \( R_i \tilde{d}_R^c \nu_R \) | \((3,1,2/3)\) | \( U_i \nu \tilde{d}_R^c \nu_R \) | \((3,1,1/3)\) |
| \( R_i \tilde{d}_L^c \nu_R \) | \((3,1,2/3)\) | \( U_i \nu \tilde{d}_L^c \nu_R \) | \((3,1,1/3)\) |
| \( S_i \tilde{d}_L^c \nu_R \) | \((3,1,1/3)\) | \( U_i \nu \tilde{d}_L^c \nu_R \) | \((3,1,1/3)\) |

Table I. Possible couplings of the scalar and vector LQs to the SM fermions and νR, as well as their representations under the SM gauge symmetry SU(3)c ⊗ SU(2)L ⊗ U(1)α. The interactions between LQs and fermions are indicated only schematically and, for simplicity, their coupling constants and Hermitian conjugated terms are not shown explicitly—these could be found, e.g., in Ref. [95]. Our convention for the hypercharge α is given by \( Q_{em} = T_3 + \alpha \), and the charge conjugate of a fermion field \( \psi \) is defined as \( \psi^C \).
thus help the model with such a scalar doublet evade the constraints from direct detection experiments. In general, there can be more than one DM candidate participating in the neutrino mass generation. If so, the lightest one will be deemed the DM.

To prevent the DM from decaying exclusively to the SM particles, an auxiliary symmetry is usually necessary. The most popular choice is the $Z_2$ symmetry, which is commonly used in radiative neutrino mass models to help stabilize the DM candidates. Assuming the symmetry to be exact, we will assign an even ($+$) $Z_2$ parity to all the SM particles, as well as $\nu_R$, whereas an odd ($-$) parity to the DM candidates.

Summarizing all the bits and pieces of discussions made above, we can establish the following set of criteria for our later topology (diagram) selection scheme:

- Only fields transforming as singlet, doublet, or triplet under the SU(2)$_L$ gauge symmetry will be considered.
- The DM candidates, expect for being the components of a scalar doublet with $\alpha = \pm 1/2$, must satisfy the condition $\alpha = T_3 = 0$, to avoid the constraints from DM direct detection experiments.
- A minimum of new degrees of freedom carrying colors will be introduced.
- The dark $Z_2$ symmetry introduced to prevent the DM from decaying exclusively to the SM particles must be exact.

Given that the external field contents $L_L$, $\nu_R$, and $H$ in $\mathcal{L}_4$ are all colorless, conservation of SU(3)$_c$ symmetry indicates that colored internal fields must be wrapped into loops. If they all arise in one loop, obviously DM candidates cannot appear in the very same loop, because they are colorless.

Then to have a diagram containing at least one LQ and one DM in two loops. Note that the particle connecting the two loops in structure (I) must be colorless.

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Figure 1. Three possible structures for a diagram containing at least one LQ and one DM in two loops. Note that the particle connecting the two loops in structure (I) must be colorless.

A. Two-loop realizations of the operator $\mathcal{L}_4$

Two-loop realizations of the effective operator $\mathcal{L}_4$ have been studied in Ref. [89]. After removing all the topologies corresponding to tadpoles, self-energy diagrams, and the non-renormalizable diagrams involving, e.g., the three-point vertices with only fermions or the four-point interactions with a fermion insertion, we are left with 5 one-particle-irreducible (1PI) topologies, which are shown in Fig. 2. It can be seen that topologies T1, T2, T3, and T5, share the same structure (III), while T4 has the structure (II).

Based on those topologies, we can build 18 diagrams with the external fermion and scalar fields specified, as depicted in Figs. 3 and 4. It can be seen that all the diagrams except the three ones on the top in Fig. 3 contain a non-local fermion-fermion-scalar vertex, whereas diagrams in Fig. 4 have a non-local scalar-scalar-scalar or scalar-vector-vector vertex.

Consider the diagrams in Fig. 4 first. For a simple demonstration, here we will focus on the diagram T4-i that has the structure (II). If the DM candidates occupy the upper, small loop, to make this diagram our desired one, then at least one LQ must propagate in the lower, big loop, yielding a non-local operator $X_2^2X_5^2H$ with both $X_2$ and $X_5$ carrying colors. On the other hand, if the LQ arises in the upper, small loop, then the DM candidates must propagate in the lower, big one, resulting in another non-local operator $X_2X_5^3H$ with both $X_2$ and $X_5$ carrying colorless, odd $Z_2$ parity. Nevertheless, such an operator in either case, according to Refs. [89, 100] and
Figure 3. Genuine two-loop diagrams (the top three) [89] and the ones with a non-local fermion-fermion-scalar vertex. Note that the dashed line represents either a scalar or a vector field, while the solid line denotes a fermion one.

Figure 4. Diagrams that contain a non-local scalar-scalar-scalar (scalar-vector-vector) vertex. As in Fig. 3, the dashed line represents either a scalar or a vector field, while the solid line denotes a fermion one.

also shown in Fig. 5, cannot prevent the presence of the tree-level vertex \(X_2X_5H\), which renders the two-loop diagram non-genuine [89]. Thus we will not consider the diagram T4-i, as well as the rest diagrams in Fig. 4 (for the same reason), and the structure (II) is completely eliminated in this case.

Move on to the diagrams in Fig. 3. To build their associated, possible UV models, proper quantum numbers of the internal fields in these diagrams have to be pinpointed. Let us firstly assign them possible \(Z_2\) parity and color. Note that assigning concrete SU(3)\(_c\) representations to the internal fields is not necessary at this point; we thus use a simple notation “c” to denote the fields carrying colors. In addition, since these diagrams share the same structure, we can strip away the external fields to make our following procedure as general as possible.

Now it can be seen from Fig. 6 that there are 3 ways to assign \(Z_2\) parity (as depicted in the upper 3 diagrams) and c (as shown in the lower diagrams) to the internal fields separately, yielding 9 combinations in total. Among them, combinations (a)-(B) and (b)-(A) leave no place for DM candidates, and thus shall be eliminated. In addition, (a)-(C), (b)-(C), (c)-(A), and (c)-(B) contain 2 colored, \(Z_2\) odd particles, whereas (c)-(C) predicts 4 colored, \(Z_2\) odd particles. They will also be eliminated according to the selection criteria. Finally, (a)-(A) and (b)-(B) contain 1 colored, \(Z_2\) odd particle, 2 colored, \(Z_2\) even particles (one of them can be a LQ), and 2 colorless, \(Z_2\) odd particles (one of them can be a DM). Thus, these two combinations are what we will assign to the diagrams in Fig. 3. For the convenience of later discussions, we will hereafter use “a” and “b” to denote (a)-(A) and (b)-(B), respectively.

Assigning the LQs in Table I to the internal field contents for both the combinations “a” and “b”, we work out in Tables IV-XVII possible UV models for all the diagrams in Fig. 3 except T2-ii, T3-v, T3-vi, T3-vii, and T3-viii, since their models can be obtained by following the same procedure (note that they all involve at least 1 colored, \(Z_2\) odd fermion). In addition, possible DM candidates are already identified by using the DM candidate condition. As indicated by the quantum numbers of the field contents in Tables XVI and XVII, we have set the LQs and DM candidates occupying the lower and upper loops, respectively, to be the combination “a” for diagrams T3-ix and T3-x. The underlying reason for such an arrangement is to make the LQs’ couplings to the left-handed lepton doublet \(L_L\) directly contribute to the neutrino mass generation, which in turn helps establish the later mighty models, because these interactions are essential to address the flavor anomalies, as will be discussed latter. This indicates that the combination “b” in these two cases, i.e., the LQs and DM candidates occupying the 2 loops in reverse order, must be eliminated, since otherwise no LQ couplings in Table I will contribute to the neutrino mass generation. It is also interest-
the Dirac neutrino masses, all the lower-order contributions to the
Dirac neutrino masses, all the lower-order contributions to the
Dirac neutrino masses by computing the two-loop integral
After the electroweak symmetry breaking, one can obtain the
Yukawa interactions, and
SU
α
After setting the hypercharge
this model consist of 2 SM singlets
X
for an illustration (we refer the readers to Appendix B for
port the coexistence of LQs and DM candidates, and can be
Fig. 6. Possible assignments of the Z
parity (see the upper 3 diagrams) and the color symbol c (see the lower 3 diagrams) to each
piece of the structure diagram. Note that diagrams with all their pieces labeled with c or + have been eliminated, because no place
is left for DM candidates.
These UV models for radiative Dirac neutrino masses support
the coexistence of LQs and DM candidates, and can be
easily read off from Tables IV-XVII. Take model T1-i-a-A-
1 for an illustration (we refer the readers to Appendix B for
the convention of model labeling). The new field contents of
this model consist of 2 SM singlets \( X^2_L \) and \( X^3_S \), 1 colored,
SU(2)_L doublet \( X^5_X \), and 1 LQ \( S_1 \). Note that, for simplicity,
we have chosen \( X_{3,4} \) to be scalar fields—hence the su-
perscript, though they can be vector ones, and \( X_{2,3,5} \) to be
SU(2)_L singlets, even though triplets would work just fine.
After setting the hypercharge \( \alpha = 0 \) for \( X_1 \), both \( X_2 \) and \( X_3 \),
as shown in Table IV, satisfy the DM candidate condition, and
thus the lighter one shall be the DM.

The relevant Lagrangian that generates the diagram T1-i-a
is given by
\[
\mathcal{L} \supset \left[ \lambda_1 Q_L^1 \tau_2 L_L S_1 + \lambda_2 Q_L X^2_L X_4 + \lambda_3 \nu R X^2_L X^3_4 \right] \\
+ \lambda_4 X^1_3 H S^1_1 X_3 - M_{X_3} X^2_L X^2_R + \text{H.c.} \\
- M_{S_1}^2 S^1_1 S_1 - M^2_{X_3} X^3_1 X^1_3 - M^2_{X_4} X^4_1 X^1_4, \quad (2)
\]
where \( X_2 \) is a vector-like fermion, \( \lambda_{1,2,3} \) denote the new
Yukawa interactions, and \( \lambda_4 \) is the coupling constant among
the Higgs \( H \), the LQ \( S_1 \), and two other new scalar fields \( X_{3,4} \).
After the electroweak symmetry breaking, one can obtain the
effective neutrino mass by computing the two-loop integral
of this diagram. For technical details and final expression of
the loop evaluation, we refer to Refs. [101–104]. Following
the same procedure, one can also write out the Lagrangian for
other models and work out the corresponding neutrino masses.

### B. Securing the two-loop dominance

To ensure the dominance of the two-loop contribution to the
Dirac neutrino masses, all the lower-order contributions
must be forbidden. For instance, besides the renormalizable
Yukawa interaction \( \bar{L}_L \nu_R \tilde{H} \), all the tree-level (seesaw) and
one-loop realizations of the effective operators \( \mathcal{L}_4 \) considered,
e.g., in Ref. [88], should be absent. In what follows, we will
consider the soft breaking non-Abelian \( S_4 \) symmetry and apply
it to T1-i-a as a demonstration of how the absence of these
lower-order contributions is guaranteed.
The discrete flavor group \( S_4 \) has been used to predict the
lepton flavor mixing angles and CP violation phases [105–
109], and recently to ensure the dominance of one-loop contrib-
utions to the Dirac neutrino masses [90, 93]. It has five
irreducible representations: two singlets \( 1 \) and \( 1' \), one doublet \( 2 \),
and two triplets \( 3 \) and \( 3' \), with their tensor decomposition
rules given, respectively, by
\[
\begin{align*}
1' \otimes 1' &= 1, \quad 1' \otimes 2 = 2, \quad 1' \otimes 3 = 3', \quad 1' \otimes 3' = 3, \\
2 \otimes 2 &= 1 + 1' + 2, \quad 2 \otimes 3 = 2 \otimes 3' = 3 \oplus 3', \\
3 \otimes 3 &= 3' \otimes 3' = 1 \oplus 2 \oplus 3 \oplus 3', \\
3 \otimes 3' &= 1' \oplus 2 \oplus 3 \oplus 3'.
\end{align*}
\] (3)

We assign all the SM fermions to be \( 3 \) while \( \nu_R \) to be \( 3' \). We
note that certain LQs, like \( S_1, R_2, U_1 \), and \( V_2 \), have couplings
with both the SM fermions and right-handed neutrinos, and
thus can generate Dirac neutrino masses through the Ma dia-
grams at one-loop level [88]. To exclude such a scenario, we
divide the LQs into two groups: the first one refers to the LQs
that couple with \( \nu_R \) and transform as \( 1' \), while the second one
to the LQs transforming as \( 3 \). Consequently, the one-loop Ma
diagrams shown in Fig. 9 are excluded. Besides, as indicated
in Table II, the renormalizable Yukawa coupling \( L_L \tilde{H} \nu_R \), as
well as the seesaw realizations of the effective operator \( \mathcal{L}_4 \),
is automatically forbidden by the \( S_4 \) symmetry, while the two-
loop diagram T1-i-a can be generated due to the soft-breaking
coupling \( X^1_3 X^4_3 \). Thus, the \( S_4 \) symmetry can guaran-
tee the two-loop dominance in generating the Dirac neutrino
masses. It should be, however, mentioned that the represen-
tation assignment here is solely for an illustration. It may
not be the best choice, since a compatible Pontecorvo-Maki-
Nakagawa-Sakata mixing matrix must also be induced, once a
specific UV model is concerned under the symmetry. It should
also be pointed out that the choice of the auxiliary symmetry
is highly subjective; other symmetries, either Abelian (e.g.,
\( Z_2, Z_3 \) [93]) or non-Abelian (e.g., \( A_4 \) [90]), have also been
considered to secure the dominance of loop contributions.
In short, once the dominance of two-loop contributions to

### Table II

| Fields | \( L_L \) | \( \nu_R \) | \( H \) | \( X_1 \) | \( X_2 \) | \( X_3 \) | \( X_4 \) | \( X_5 \) |
|--------|---------|---------|---------|--------|--------|--------|--------|--------|
| \( Z_2 \) | + | + | + | + | - | - | + |
| \( S_4 \) | 3 | 3' | 1 | 3 | 3 | 1' | 3 |
clear that, besides the vector LQ options, we will directly match the LQ assertions have been explored to address these flavor anomalies in order to help visualize these possible options, we summarize in Table III the aforementioned LQs that can accommodate the Dirac neutrino masses is established, the UV completions listed in Tables IV-XVII can be identified as LQ-SDνM, and the mighty “stones” we are seeking, if existed, must arise from them.

### III. THE MIGHTY “STONES”

If the LQs that account for the $R_{K^{(*)}}$, $R_{D^{(*)}}$, and $(g-2)_\mu$ anomalies also emerge in the established LQ-SDνM, there is a good chance that these models are the mighty “stones”. Thus, we shall firstly find out the proper LQs.

It is known that the vector LQ $U_1$ can alone address the $R_{D^{(*)}}$ and $R_{K^{(*)}}$ anomalies simultaneously, while the LQ $S_1$, $S_3$, $R_2$, $\bar{R}_2$, and $U_3$ can only account for one of the anomalies [100, 111]. Interestingly, it has been recently shown in Refs. [77, 78] that such a LQ can also explain the $(g-2)_\mu$ anomaly [75, 76, 112], provided that its interactions with the left- and right-handed SM fermions are both present. These findings lead to an exciting observation that all the flavor anomalies can be addressed by such a single vector LQ. Remarkably, $U_1$ is in fact not alone. A very recent study shows that the same achievement of $U_1$ can also be duplicated by another vector LQ $V_2$ [79].

Besides the option with a single vector LQ $U_1$ or $V_2$, one could also resort to a pair of scalar LQs. It is known that at tree-level $S_3$ is the only scalar LQ that can account for the $R_{K^{(*)}}$ anomaly, while either $S_1$ or $R_2$ can address the $R_{D^{(*)}}$ anomaly (see, e.g., Refs. [111, 113, 114]). Meanwhile, both $S_1$ and $R_2$ can address the $(g-2)_\mu$ anomaly, due to the simultaneous presence of their couplings to the SM left- and right-handed chiral fermions [115]. Thus, to solve all the flavor anomalies, the first step is to select the proper LQ pairs. In order to help visualize these possible options, we summarize in Table III the aforementioned LQs that can accommodate the $R_{K^{(*)}}$, $R_{D^{(*)}}$, and/or $(g-2)_\mu$ anomalies. It becomes clear that, besides the vector LQ options $U_1$ and $V_2$, two more are available: (i) $S_1$ and $S_3$, and (ii) $R_2$ and $S_3$. Both options have been explored to address these flavor anomalies in a unified framework [80-87].

We are now ready to establish the mighty “stones”. Given the high freedom in choosing the auxiliary symmetry to secure the two-loop dominance, we will directly match the LQs in Table III with those in the UV models listed in Tables IV-XVII. For the options $U_1$ and $V_2$, clearly plenty of models are available. Among them, the ones associated with the diagrams T1-i-a and T3-iii-a, are probably the optimum for both options, since a minimum of new fields are involved. Here it should be mentioned that the models of T1-i-b, T3-iv-b, T3-ix-a, and T3-x-a can be optimal for the option $U_1$ as well, due to the same reason.

However, for the scalar-LQ options (i) and (ii), none of the models in Tables IV-XVII are available, because at least one LQ has to be introduced additionally. Under such a circumstance, for the scalar-LQ option (i), model A-1 (A-2) associated with the diagrams T1-i-a, T1-i-b, T3-iii-a, T3-iv-b, T3-ix-a, and T3-x-a, after extending to the missing $S_1$ or $S_3$ accordingly, shall be the best option, because it will involve minimal number of new degrees of freedom. Intriguingly, both $S_1$ and $S_3$ will contribute to the neutrino mass generation in the enlarged model A-1 (A-2) of the diagrams T1-i-a and T3-iii-a. The next best option would be the model A-1 (A-2) associated with the diagrams, such as T3-i-a, T3-i-b, T3-ii-a, T3-ii-b, T2-i-a, T2-i-b, T3-iii-b, and T3-iv-a, since one more new field must be added. Similar to the previous option, both $S_1$ and $S_3$ in the modified model A-1 (A-2) of the diagrams T3-i-a, T3-ii-a, T2-i-a, and T3-iv-a will contribute to the neutrino mass generation; interestingly, the updated model T3-ii-a-A-1 (A-2) will involve overall 3 scalar LQs.

Semi-similar conclusions drawn for the scalar-LQ option (i) hold for the scalar-LQ option (ii), too. For instance, the best option shall arise from model B-1 (B-2) enlarged with $S_3$ of the diagrams T1-i-a and T3-iii-a, while the next best one from the updated model B-1 (B-2) of the diagrams: T3-i-a, T3-ii-a, T2-i-a, and T3-iv-a.

It may be interesting to point out that, combining two models—each contains one component of the scalar LQ pair—surely works for both the options (i) and (ii). Often the combined model will involve more new degrees of freedom, rendering them less appealing. Yet interesting ones can arise. For instance, models A-1 (A-2) and B-1 (B-2) of T1-i-a, although each consists of 3 new fields besides the LQs, share 2 of them. Bringing the two models together will thus generate a next best option as well, which consists of 6 new degrees of freedom, just like the updated models such as T3-i-a-A-1 (A-2). Besides, both $S_3$ and $R_2$ in this model will contribute to the neutrino mass generation. Based on this model, one may also build an even larger model by adding the LQ $S_1$. Although the model may be cumbersome, a slight compromise on model minimality might be a good bargain here, especially given that it contains all the scalar LQs in Table III.

We conclude this section by making the following comment. In the mighty models involving the vector LQ $U_1$ or $V_2$, there is a close-knit connection between the flavor anomalies and neutrino mass. Namely, the absence of any particle accounting for the anomalies will fail to generate the neutrino masses (the same idea has been appreciated recently in Refs. [84, 86]). In the mighty models consisting of the scalar LQs in Table III, on the other hand, such a connection is miss-
ing, since removing \( S_1 \) or \( S_3 \) in the enlarged model A-1 (A-2) associated with, e.g., T1-i-a does not render the neutrino massless. To build a close-knit connection for the scalar-LQ options (i) and (ii), one can consider, e.g., the two-loop realizations of effective operator \( L_5 = -\varphi^0 \varphi L_L H X + \text{H.c.} \), where \( X \) is a color-singlet, \( SU(2)_L \)-triplet scalar field with a hypercharge \( \alpha = 0 \). Given that the two-loop diagrams of \( L_5 \) and \( L_4 \) share similar structures (or skeletons) [89, 92], one can obtain the former (at least some of them) by attaching the \( X \) to the propagators of the latter, and in turn build the corresponding UV models. Take the model T3-iii-a-A-1 for an illustration. We firstly set \( X_3 = (1,1,1) \), \( X_4 = (1,1,\alpha) \), \( X_5 = (3,1,\alpha-1/3) \), and \( X_6 = S_1 \) or \( S_3 \). Then as shown in Fig. 7, we can obtain a two-loop realization of \( L_4 \) by attaching \( X \) to \( X_6 \), and build the corresponding UV model, which contains both \( S_1 \) and \( S_3 \) besides the \( X_{3,4,5} \). We further impose an auxiliary symmetry to forbid all the lower-order diagrams, as well as the T3-iii-a, while introduce a soft breaking coupling (marked in red in Fig. 7) to support the two-loop diagram. In this way, both \( S_1 \) and \( S_3 \) become indispensable to the neutrino mass generaton, and the connection between the flavor anomalies and neutrino mass becomes close-knit.

**IV. CONCLUSION**

We have pointed out a potential path to address in a unified picture the problems the SM encounters: the anomalies observed in flavor physics experiments, neutrino mass generation, and the nature of DM. The key ingredient rests on the compatibility of LQs with the SD\( \nu \)M. Based on the minimal seesaw, one-loop, and two-loop realizations of the effective operators \( L_4 \) for Dirac neutrino masses, and guided by the topology-selection criteria outlined in Sec. II, we have found that plenty of diagrams in the two-loop realizations of \( L_4 \) can support the coexistence of LQs and DM candidates, and exhausted the topology-based UV completions in Tables IV-XVII. Matching the LQs of the UV models to those that account for all the flavor anomalies mentioned in this work, we have established the mighty models that can address all the aforementioned problems in a unified picture. On top of that, we have found that a close-knit connection between the flavor anomalies and neutrino mass can be established in the models involving the vector LQ \( V_1 \) or \( V_2 \).

Without a doubt, rich phenomenology can arise from each mighty model. However, since detailed analysis has to be conducted on a case-by-case basis, we leave it for future works.

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**Appendix A: Systematic classification of tree-level and one-loop realizations of the operator \( L_4 \)**

In this appendix, following Refs. [88, 90], we will present a systematic classification of tree-level and one-loop realizations of the effective operator \( L_4 \).

The tree-level and one-loop realizations of the Dirac neutrino mass operator \( L_4 \) have been studied in Ref. [88]. It is shown that there are only four ways to achieve the seesaw (tree-level) realizations, i.e., the Dirac neutrino masses can be generated with the insertion of a Dirac singlet \( N \), a Dirac doublet \( E = (E^0, E^-)^T \), and \( \Sigma = (\Sigma^+, \Sigma^0, \Sigma^-)^T \) denote a Dirac singlet, a Dirac doublet, and a Dirac triplet fermion, respectively, whereas \( \eta = (\eta^+, \eta^0, \eta^-)^T \) is a scalar triplet [88].

If the seesaw realizations discussed above are not available, the Dirac neutrino masses may still arise at one-loop. As demonstrated in Ref. [88], there are only 2 Feynman diagrams for the one-loop generation of the Dirac neutrino masses, which are shown in Fig. 9. It can be seen that both diagrams contain three propagators, two bosonic and one fermionic in
responding UV models can then be identified as the SD from them or their neutral components [88, 93]. If so, the cor-
X to the fields
With the appropriate assignments of the SM gauge charges

Figure 9. One-loop realizations of the Dirac neutrino mass operator \( L_{\nu} \). The dashed line represents either a scalar or a vector field, while the solid line a fermion one.

Fig. 9 (a), whereas 1 bosonic and 2 fermionic in Fig. 9 (b). With the appropriate assignments of the SM gauge charges to the fields \( X_{1,2,3} \), the DM candidates can potentially arise from them or their neutral components [88, 93]. If so, the corresponding UV models can then be identified as the SD/\( \nu \)M.

Appendix B: UV completions and dark matter candidates

In Sec. II, we have shown that the diagrams depicted in Fig. 3 can support the coexistence of the LQs and DM candi-

dates. To pinpoint the possible quantum numbers of the messenger fields, we have then explored possible assignments of the \( Z_2 \) parity and color \( c \), and found that the combinations "a" and "b" can support our selection criteria better. In this appendix, guided by the two combinations and selection criteria, we shall present in Tables IV-XVII the possible quantum numbers of all the internal fields in each diagram, and simultaneously the UV models of the diagram. Although we have uniformly labeled the models in these tables by \( x-y \) with \( x=A, B, C, \ldots \) and \( y=1, 2, 3, \ldots \), they are properly referred by, e.g., T1-i-a-x-y in the main text, where T1-i-a denotes the diagram T1-i with the combination "a". To help identify possible DM, we have specified the fermionic messenger field and its transformation property under the \( Z_2 \) symmetry. By setting down their hypercharges according to the DM candidate condition, we have selected the possible DM candidates in each model.

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Table IV. Possible assignments of quantum numbers under the SU(3)_c ⊗ SU(2)_L ⊗ U(1)_Y ⊗ Z_2 symmetry to the mediators in diagram T1-i-a, where m ⊕ n denotes that either an SU(2)_L m- or an n-plet works. The possible DM candidates are identified under proper conditions for the hypercharge \( \alpha \).

| Model | \( X_1^F \) | \( X_2^F \) | \( X_3 \) | \( X_4 \) | \( X_5 \) | Dark matter |
|-------|-------------|-------------|-------------|-------------|-------------|---------------|
| A-1   | \( Q_L \)   | \((1, 1 ⊕ 3, \alpha)^-\) | \((1, 1 ⊕ 3, \alpha)^-\) | \((3, 2, \alpha + \frac{1}{6} \alpha)^-\) | \( S_{1,3}^I \) | \([X_2, X_3]|_{\alpha=0} \) |
| A-2   | \( \bar{Q}_L \) | \((1, 2, \alpha)^-\) | \((1, 2, \alpha)^-\) | \((3, 1 ⊕ 3, \alpha + \frac{1}{6} \alpha)^-\) | \( S_{1,3}^I \) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| B-1   | \( u_R \)   | \((1, 1 ⊕ 3, \alpha)^-\) | \((1, 1 ⊕ 3, \alpha)^-\) | \((3, 1, \alpha - \frac{1}{3} \alpha)^-\) | \( R_2^L \) | \([X_2, X_3]|_{\alpha=0} \) |
| B-2   | \( u_R \)   | \((1, 2, \alpha)^-\) | \((1, 2, \alpha)^-\) | \((3, 2, \alpha - \frac{1}{3} \alpha)^-\) | \( R_2^L \) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| C-1   | \( d_R \)   | \((1, 1 ⊕ 3, \alpha)^-\) | \((1, 1 ⊕ 3, \alpha)^-\) | \((3, 1 ⊕ 3, \alpha + \frac{1}{3} \alpha)^-\) | \( \bar{R}_2^L \) | \([X_2, X_3]|_{\alpha=0} \) |
| C-2   | \( d_R \)   | \((1, 2, \alpha)^-\) | \((1, 2, \alpha)^-\) | \((3, 2, \alpha + \frac{1}{3} \alpha)^-\) | \( \bar{R}_2^L \) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| D-1   | \( Q_L \)   | \((1, 1 ⊕ 3, \alpha)^-\) | \((1, 1 ⊕ 3, \alpha)^-\) | \((3, 2, \alpha - \frac{1}{3} \alpha)^-\) | \( U_{1,3}^I \) | \([X_2, X_3]|_{\alpha=0} \) |
| D-2   | \( Q_L \)   | \((1, 2, \alpha)^-\) | \((1, 2, \alpha)^-\) | \((3, 1 ⊕ 3, \alpha - \frac{1}{6} \alpha)^-\) | \( U_{1,3}^I \) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| E-1   | \( \bar{u}_R \) | \((1, 1 ⊕ 3, \alpha)^-\) | \((1, 1 ⊕ 3, \alpha)^-\) | \((3, 1 ⊕ 3, \alpha + \frac{1}{3} \alpha)^-\) | \( \bar{V}_2^I \) | \([X_2, X_3]|_{\alpha=0} \) |
| E-2   | \( \bar{u}_R \) | \((1, 2, \alpha)^-\) | \((1, 2, \alpha)^-\) | \((3, 2, \alpha + \frac{1}{3} \alpha)^-\) | \( \bar{V}_2^I \) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| F-1   | \( d_R \)   | \((1, 1 ⊕ 3, \alpha)^-\) | \((1, 1 ⊕ 3, \alpha)^-\) | \((3, 1 ⊕ 3, \alpha - \frac{1}{3} \alpha)^-\) | \( V_2^I \) | \([X_2, X_3]|_{\alpha=0} \) |
| F-2   | \( d_R \)   | \((1, 2, \alpha)^-\) | \((1, 2, \alpha)^-\) | \((3, 2, \alpha - \frac{1}{3} \alpha)^-\) | \( V_2^I \) | \([X_3]|_{\alpha=\frac{1}{2}} \) |

Table V. Possible assignments of quantum numbers under the SU(3)_c ⊗ SU(2)_L ⊗ U(1)_Y ⊗ Z_2 symmetry to the mediators in diagram T1-i-b, where m ⊕ n denotes that either an SU(2)_L m- or an n-plet works.

| Model | \( X_1^F \) | \( X_2^F \) | \( X_3 \) | \( X_4 \) | \( X_5 \) | Dark matter |
|-------|-------------|-------------|-------------|-------------|-------------|---------------|
| A-1   | \((1, 1 ⊕ 3, \alpha)^-\) | \( \bar{d}_R \) | \( S_1 \) | \((3, 1 ⊕ 3, \alpha - 3 \alpha)^-\) | \((1, 2, \alpha - \frac{1}{2} \alpha)^-\) | \([X_1, X_3]|_{\alpha=0} \) |
| A-2   | \((1, 2, \alpha)^-\) | \( \bar{d}_R \) | \( S_1 \) | \((3, 2, \alpha - 3 \alpha)^-\) | \((1, 1 ⊕ 3, \alpha - \frac{1}{2} \alpha)^-\) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| B-1   | \((1, 1 ⊕ 3, \alpha)^-\) | \( Q_L \) | \( \bar{R}_2 \) | \((3, 2, \alpha - 3 \alpha)^-\) | \((1, 2, \alpha - \frac{1}{2} \alpha)^-\) | \([X_1, X_3]|_{\alpha=0} \) |
| B-2   | \((1, 2, \alpha)^-\) | \( Q_L \) | \( \bar{R}_2 \) | \((3, 1 ⊕ 3, \alpha - \frac{1}{6} \alpha)^-\) | \((1, 1 ⊕ 3, \alpha - \frac{1}{2} \alpha)^-\) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| C-1   | \((1, 1 ⊕ 3, \alpha)^-\) | \( \bar{u}_R \) | \( \bar{S}_1 \) | \((3, 1 ⊕ 3, \alpha - \frac{3}{2} \alpha)^-\) | \((1, 2, \alpha - \frac{1}{2} \alpha)^-\) | \([X_1, X_3]|_{\alpha=0} \) |
| C-2   | \((1, 2, \alpha)^-\) | \( \bar{u}_R \) | \( \bar{S}_1 \) | \((3, 2, \alpha - \frac{3}{2} \alpha)^-\) | \((1, 1 ⊕ 3, \alpha - \frac{3}{2} \alpha)^-\) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| D-1   | \((1, 1 ⊕ 3, \alpha)^-\) | \( u_R \) | \( U_1 \) | \((3, 1 ⊕ 3, \alpha - \frac{3}{2} \alpha)^-\) | \((1, 2, \alpha - \frac{1}{2} \alpha)^-\) | \([X_1, X_3]|_{\alpha=0} \) |
| D-2   | \((1, 2, \alpha)^-\) | \( u_R \) | \( U_1 \) | \((3, 2, \alpha - \frac{3}{2} \alpha)^-\) | \((1, 1 ⊕ 3, \alpha - \frac{3}{2} \alpha)^-\) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| E-1   | \((1, 1 ⊕ 3, \alpha)^-\) | \( \bar{Q}_L \) | \( \bar{V}_2 \) | \((3, 2, \alpha - \frac{3}{2} \alpha)^-\) | \((1, 2, \alpha - \frac{1}{2} \alpha)^-\) | \([X_1, X_3]|_{\alpha=0} \) |
| E-2   | \((1, 2, \alpha)^-\) | \( \bar{Q}_L \) | \( \bar{V}_2 \) | \((3, 1 ⊕ 3, \alpha - \frac{3}{2} \alpha)^-\) | \((1, 1 ⊕ 3, \alpha - \frac{3}{2} \alpha)^-\) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| F-1   | \((1, 1 ⊕ 3, \alpha)^-\) | \( d_R \) | \( \bar{U}_1 \) | \((3, 1 ⊕ 3, \alpha - \frac{3}{2} \alpha)^-\) | \((1, 2, \alpha - \frac{1}{2} \alpha)^-\) | \([X_1, X_3]|_{\alpha=0} \) |
| F-2   | \((1, 2, \alpha)^-\) | \( d_R \) | \( \bar{U}_1 \) | \((3, 2, \alpha - \frac{3}{2} \alpha)^-\) | \((1, 1 ⊕ 3, \alpha - \frac{3}{2} \alpha)^-\) | \([X_3]|_{\alpha=\frac{1}{2}} \) |
| Model | $X_1^P$ | $X_2^P$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | Dark matter |
|-------|---------|---------|-------|-------|-------|-------|-------------|
| A-1   | $Q_L$   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha + \frac{1}{2})^-$ | $S_{1,3}^L$ | $[X_2, X_3, X_4]_{\alpha=0}$ |
| A-2   | $\bar{Q}_L$ | $(1, 2, \alpha)^-$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $S_{1,3}^L$ | $[X_3, X_4]_{\alpha=\frac{1}{2}}$ |
| B-1   | $u_R$   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha - \frac{1}{2})^- | $R_{2}^L$ | $[X_2, X_3, X_4]_{\alpha=0}$ |
| B-2   | $u_R$   | $(1, 2, \alpha)^-$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha - \frac{1}{2})^- | $R_{2}^L$ | $[X_3, X_4]_{\alpha=\frac{1}{2}}$ |
| C-1   | $d_R$   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha + \frac{1}{2})^- | $\bar{R}_{2}^L$ | $[X_2, X_3, X_4]_{\alpha=0}$ |
| C-2   | $d_R$   | $(1, 2, \alpha)^-$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha + \frac{1}{2})^- | $\bar{R}_{2}^L$ | $[X_3, X_4]_{\alpha=\frac{1}{2}}$ |
| D-1   | $Q_L$   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha - \frac{1}{2})^- | $U_{1,3}^L$ | $[X_2, X_3, X_4]_{\alpha=0}$ |
| D-2   | $Q_L$   | $(1, 2, \alpha)^-$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha - \frac{1}{2})^- | $U_{1,3}^L$ | $[X_3, X_4]_{\alpha=\frac{1}{2}}$ |
| E-1   | $\bar{u}_R$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha + \frac{1}{2})^- | $\bar{V}_{2}^L$ | $[X_2, X_3, X_4]_{\alpha=0}$ |
| E-2   | $\bar{u}_R$ | $(1, 2, \alpha)^-$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha + \frac{1}{2})^- | $\bar{V}_{2}^L$ | $[X_3, X_4]_{\alpha=\frac{1}{2}}$ |
| F-1   | $d_R$   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha - \frac{1}{2})^- | $V_{2}^L$ | $[X_2, X_3, X_4]_{\alpha=0}$ |
| F-2   | $d_R$   | $(1, 2, \alpha)^-$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha - \frac{1}{2})^- | $V_{2}^L$ | $[X_3, X_4]_{\alpha=\frac{1}{2}}$ |

Table VI. Possible assignments of quantum numbers under the $SU(3)_c \otimes SU(2)_L \otimes U(1)_\alpha \otimes Z_2$ symmetry to the mediators in diagram T3-i-a, where $m \oplus n$ denotes that either $SU(2)_L$ $m$- or an $n$-plet works.

| Model | $X_1^P$ | $X_2^P$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | Dark matter |
|-------|---------|---------|-------|-------|-------|-------|-------------|
| A-1   | $(1, 1 \oplus 3, \alpha)^-$ | $\bar{Q}_L$ | $S_1$ | $(3, 2, \frac{1}{2})^-$ | $(3, 1 \oplus 3, \frac{1}{2} - \alpha)^-$ | $(1, 2, - \frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| A-2   | $(1, 2, \alpha)^-$ | $\bar{Q}_L$ | $S_1$ | $(3, 2, \frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, - \frac{1}{2} - \alpha)^-$ | $X_6|_{\alpha=\frac{1}{2}}$ |
| B-1   | $(1, 1 \oplus 3, \alpha)^-$ | $u_R$ | $R_2$ | $S_1^L$ | $(3, 2, \frac{1}{2} - \alpha)^-$ | $(1, 2, - \frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| B-2   | $(1, 2, \alpha)^-$ | $u_R$ | $R_2$ | $S_1^L$ | $(3, 1 \oplus 3, \frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, - \frac{1}{2} - \alpha)^-$ | $X_6|_{\alpha=\frac{1}{2}}$ |
| C-1   | $(1, 1 \oplus 3, \alpha)^-$ | $\bar{u}_R$ | $S_1$ | $\bar{R}_2$ | $(3, 1 \oplus 3, - \frac{1}{2} - \alpha)^-$ | $(1, 2, - \frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| C-2   | $(1, 2, \alpha)^-$ | $\bar{u}_R$ | $S_1$ | $\bar{R}_2$ | $(3, 2, - \frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, - \frac{1}{2} - \alpha)^-$ | $X_6|_{\alpha=\frac{1}{2}}$ |
| D-1   | $(1, 1 \oplus 3, \alpha)^-$ | $u_R$ | $U_1$ | $(3, 2, \frac{1}{2})^-$ | $(3, 1 \oplus 3, \frac{1}{2} - \alpha)^-$ | $(1, 2, - \frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| D-2   | $(1, 2, \alpha)^-$ | $u_R$ | $U_1$ | $(3, 2, \frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, - \frac{1}{2} - \alpha)^-$ | $X_6|_{\alpha=\frac{1}{2}}$ |
| E-1   | $(1, 1 \oplus 3, \alpha)^-$ | $Q_L$ | $\bar{V}_2$ | $\bar{U}_1$ | $(3, 2, - \frac{1}{2} - \alpha)^-$ | $(1, 2, - \frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| E-2   | $(1, 2, \alpha)^-$ | $Q_L$ | $\bar{V}_2$ | $\bar{U}_1$ | $(3, 1 \oplus 3, - \frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, - \frac{1}{2} - \alpha)^-$ | $X_6|_{\alpha=\frac{1}{2}}$ |
| F-1   | $(1, 1 \oplus 3, \alpha)^-$ | $d_R$ | $\bar{U}_1$ | $\bar{V}_2$ | $(3, 1 \oplus 3, - \frac{1}{2} - \alpha)^-$ | $(1, 2, - \frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| F-2   | $(1, 2, \alpha)^-$ | $d_R$ | $\bar{U}_1$ | $\bar{V}_2$ | $(3, 2, - \frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, - \frac{1}{2} - \alpha)^-$ | $X_6|_{\alpha=\frac{1}{2}}$ |

Table VII. Possible assignments of quantum numbers under the $SU(3)_c \otimes SU(2)_L \otimes U(1)_\alpha \otimes Z_2$ symmetry to the mediators in diagram T3-i-b, where $m \oplus n$ denotes that either $SU(2)_L$ $m$- or an $n$-plet works.
Table VIII. Possible assignments of quantum numbers under the SU(3)_c ⊗ SU(2)_L ⊗ U(1)_α ⊗ Z_2 symmetry to the mediators in diagram T3-ii-a, where m ⊕ n denotes that either an SU(2)_L m- or an n-plet works.

| Model | X_1^F | X_2^F | X_3 | X_4 | X_5 | X_6 | Dark matter |
|-------|-------|-------|-----|-----|-----|-----|-------------|
| A-1   | Q_L   | (1, 1 ⊕ 3, α)⁻ | (1, 1 ⊕ 3, α)⁻ | (3, 2, α⁻ + 2)⁻ | R_2 | S^l_{1,3} | [X_2, X_3]| α=0 |
| A-2   | Q_L   | (1, 2, α)⁻   | (1, 2, α)⁻   | (3, 1 ⊕ 3, α⁻ + 1)⁻ | R_2 | S^l_{1,3} | X_3| α=0 |
| B-1   | u_R   | (1, 1 ⊕ 3, α)⁻ | (1, 1 ⊕ 3, α)⁻ | (3, 1, α⁻ + 2)⁻ | S_1 | R^l_2 | [X_2, X_3]| α=0 |
| B-2   | d_R   | (1, 2, α)⁻   | (1, 2, α)⁻   | (3, 2, α⁻ + 2)⁻ | S_1 | R^l_2 | X_3| α=0 |
| C-1   | d_R   | (1, 1 ⊕ 3, α)⁻ | (1, 1 ⊕ 3, α)⁻ | (3, 1 ⊕ 3, α⁺ + 2)⁻ | S^l_{1,3} | R^l_2 | [X_2, X_3]| α=0 |
| C-2   | d_R   | (1, 2, α)⁻   | (1, 2, α)⁻   | (3, 2, α⁺ + 2)⁻ | S^l_{1,3} | R^l_2 | X_3| α=0 |
| D-1   | Q_L   | (1, 1 ⊕ 3, α)⁻ | (1, 1 ⊕ 3, α)⁻ | (3, 2, α⁻ + 2)⁻ | V_2 | U^l_{1,3} | [X_2, X_3]| α=0 |
| D-2   | Q_L   | (1, 2, α)⁻   | (1, 2, α)⁻   | (3, 1 ⊕ 3, α⁻ + 1)⁻ | V_2 | U^l_{1,3} | X_3| α=0 |
| E-1   | u_R   | (1, 1 ⊕ 3, α)⁻ | (1, 1 ⊕ 3, α)⁻ | (3, 1 ⊕ 3, α⁺ + 2)⁻ | U_1 | V^l_2 | [X_2, X_3]| α=0 |
| E-2   | d_R   | (1, 2, α)⁻   | (1, 2, α)⁻   | (3, 2, α⁺ + 2)⁻ | U_1 | V^l_2 | X_3| α=0 |
| F-1   | d_R   | (1, 1 ⊕ 3, α)⁻ | (1, 1 ⊕ 3, α)⁻ | (3, 1 ⊕ 3, α⁻ + 1)⁻ | U_1 | V^l_2 | [X_2, X_3]| α=0 |
| F-2   | d_R   | (1, 2, α)⁻   | (1, 2, α)⁻   | (3, 2, α⁻ + 1)⁻ | U_1 | V^l_2 | X_3| α=0 |

Table IX. Possible assignments of quantum numbers under the SU(3)_c ⊗ SU(2)_L ⊗ U(1)_α ⊗ Z_2 symmetry to the mediators in diagram T3-ii-b, where m ⊕ n denotes that either an SU(2)_L m- or an n-plet works.

| Model | X_1^F | X_2^F | X_3 | X_4 | X_5 | X_6 | Dark matter |
|-------|-------|-------|-----|-----|-----|-----|-------------|
| A-1   | (1, 1 ⊕ 3, α)⁻ | d_R   | S_1 | (3, 1 ⊕ 3, ¼⁻ α)⁻ | (1, 1 ⊕ 3, −α)⁻ | (1, 2, −2⁻ α)⁻ | [X_1, X_3, X_6]| α=0 |
| A-2   | (1, 2, α)⁻   | d_R   | S_1 | (3, 2, ½⁻ α)⁻   | (1, 2, −α)⁻   | (1, 1 ⊕ 3, −2⁻ α)⁻ | [X_5, X_6]| α=0 |
| B-1   | (1, 1 ⊕ 3, α)⁻ | Q_L   | R_2 | (3, 2, ½⁻ α)⁻   | (1, 1 ⊕ 3, −α)⁻ | (1, 2, −2⁻ α)⁻ | [X_1, X_3, X_6]| α=0 |
| B-2   | (1, 2, α)⁻   | Q_L   | R_2 | (3, 1 ⊕ 3, 2⁻ α)⁻ | (1, 2, −α)⁻ | (1, 1 ⊕ 3, −2⁻ α)⁻ | [X_5, X_6]| α=0 |
| C-1   | (1, 1 ⊕ 3, α)⁻ | u_R   | S_1 | (3, 1 ⊕ 3, −2⁻ α)⁻ | (1, 1 ⊕ 3, −α)⁻ | (1, 2, −2⁻ α)⁻ | [X_1, X_3, X_6]| α=0 |
| C-2   | (1, 2, α)⁻   | u_R   | S_1 | (3, 2, −2⁻ α)⁻   | (1, 2, −α)⁻ | (1, 1 ⊕ 3, −2⁻ α)⁻ | [X_5, X_6]| α=0 |
| D-1   | (1, 1 ⊕ 3, α)⁻ | u_R   | U_1 | (3, 1 ⊕ 3, 2⁻ α)⁻ | (1, 1 ⊕ 3, −α)⁻ | (1, 2, −2⁻ α)⁻ | [X_1, X_3, X_6]| α=0 |
| D-2   | (1, 2, α)⁻   | u_R   | U_1 | (3, 2, 2⁻ α)⁻   | (1, 2, −α)⁻ | (1, 1 ⊕ 3, −2⁻ α)⁻ | [X_5, X_6]| α=0 |
| E-1   | (1, 1 ⊕ 3, α)⁻ | Q_L   | V_2 | (3, 2, −2⁻ α)⁻ | (1, 1 ⊕ 3, −α)⁻ | (1, 2, −2⁻ α)⁻ | [X_1, X_3, X_6]| α=0 |
| E-2   | (1, 2, α)⁻   | Q_L   | V_2 | (3, 1 ⊕ 3, −2⁻ α)⁻ | (1, 2, −α)⁻ | (1, 1 ⊕ 3, −2⁻ α)⁻ | [X_5, X_6]| α=0 |
| F-1   | (1, 1 ⊕ 3, α)⁻ | d_R   | U_1 | (3, 1 ⊕ 3, −2⁻ α)⁻ | (1, 1 ⊕ 3, −α)⁻ | (1, 2, −2⁻ α)⁻ | [X_1, X_3, X_6]| α=0 |
| F-2   | (1, 2, α)⁻   | d_R   | U_1 | (3, 2, −2⁻ α)⁻ | (1, 2, −α)⁻ | (1, 1 ⊕ 3, −2⁻ α)⁻ | [X_5, X_6]| α=0 |
Table X. Possible assignments of quantum numbers under the SU(3)_c ⊗ SU(2)_L ⊗ U(1)_Y ⊗ Z_2 symmetry to the mediators in diagram T2-i-a, where m ⊕ n denotes that either an SU(2)_L m- or an n-plet works.

| Model | X_{1}^\pm | X_{2}^\pm | X_{3} | X_{4} | X_{5} | X_{6} | Dark matter |
|-------|-----------|-----------|-------|-------|-------|-------|-------------|
| A-1   | Q_L       | (1, 1 ⊕ 3, α)^- | (1, 1 ⊕ 3, α^-) | (3, 1 ⊕ 3, α^- 1) | (3, 2, α 1/2^-) | S_1^\pm | [X_2, X_3]_{α=0} |
| A-2   | Q_L       | (1, 2, α^-) | (1, 2, α^-) | (3, 2, α^- 1/2^-) | (3, 1 ⊕ 3, α 1/2^-) | S_1^\pm | X_{3|α=1/2} |
| B-1   | u_R       | (1, 1 ⊕ 3, α^-) | (1, 1 ⊕ 3, α^-) | (3, 2, α^- α^- 1/2^-) | (3, 1 ⊕ 3, α^- 1/2^-) | R_2^- | [X_2, X_3]_{α=0} |
| B-2   | u_R       | (1, 2, α^-) | (1, 2, α^-) | (3, 1 ⊕ 3, α^- 1/2^-) | (3, 2, α^- α^- 1/2^-) | R_2^- | X_{3|α=1/2} |
| C-1   | d_R       | (1, 1 ⊕ 3, α^-) | (1, 1 ⊕ 3, α^-) | (3, 2, α^- α^- 1/2^-) | (3, 1 ⊕ 3, α^- α^- 1/2^-) | R_2^- | [X_2, X_3]_{α=0} |
| C-2   | d_R       | (1, 2, α^-) | (1, 2, α^-) | (3, 1 ⊕ 3, α^- 1/2^-) | (3, 2, α^- α^- 1/2^-) | R_2^- | X_{3|α=1/2} |
| D-1   | Q_L       | (1, 1 ⊕ 3, α^-) | (1, 1 ⊕ 3, α^-) | (3, 1 ⊕ 3, α^- 1/2^-) | (3, 2, α^- 1/2^-) | U_{1,3}^- | [X_2, X_3]_{α=0} |
| D-2   | Q_L       | (1, 2, α^-) | (1, 2, α^-) | (3, 2, α^- 1/2^-) | (3, 1 ⊕ 3, α^- 1/2^-) | U_{1,3}^- | X_{3|α=1/2} |
| E-1   | u_R       | (1, 1 ⊕ 3, α^-) | (1, 1 ⊕ 3, α^-) | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (3, 1 ⊕ 3, α^- α^- 1/2^-) | V_{1,2}^- | [X_2, X_3]_{α=0} |
| E-2   | u_R       | (1, 2, α^-) | (1, 2, α^-) | (3, 2, α^- α^- 1/2^-) | (3, 1 ⊕ 3, α^- α^- 1/2^-) | V_{1,2}^- | X_{3|α=1/2} |
| F-1   | d_R       | (1, 1 ⊕ 3, α^-) | (1, 1 ⊕ 3, α^-) | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (3, 1 ⊕ 3, α^- 1/2^-) | V_{1,2}^- | [X_2, X_3]_{α=0} |
| F-2   | d_R       | (1, 2, α^-) | (1, 2, α^-) | (3, 2, α^- α^- 1/2^-) | (3, 2, α^- α^- 1/2^-) | V_{1,2}^- | X_{3|α=1/2} |

Table XI. Possible assignments of quantum numbers under the SU(3)_c ⊗ SU(2)_L ⊗ U(1)_Y ⊗ Z_2 symmetry to the mediators in diagram T2-i-b, where m ⊕ n denotes that either an SU(2)_L m- or an n-plet works.

| Model | X_{1}^\pm | X_{2}^\pm | X_{3} | X_{4} | X_{5} | X_{6} | Dark matter |
|-------|-----------|-----------|-------|-------|-------|-------|-------------|
| A-1   | (1, 1 ⊕ 3, α^-) | d_R | S_1 | (3, 2, α^- 1/2^-) | (3, 1 ⊕ 3, α^- 1/2^-) | (1, 2, α^- 1/2^-) | [X_1, X_6]_{α=0} |
| A-2   | (1, 2, α^-) | d_R | S_1 | (3, 1 ⊕ 3, α^- 1/2^-) | (3, 2, α^- 1/2^-) | (1, 1 ⊕ 3, α^- 1/2^-) | X_{6|α=1/2} |
| B-1   | (1, 1 ⊕ 3, α^-) | Q_L | R_2^- | (3, 1 ⊕ 3, α^- 1/2^-) | (3, 2, α^- 1/2^-) | (1, 2, α^- 1/2^-) | [X_1, X_6]_{α=0} |
| B-2   | (1, 2, α^-) | Q_L | R_2^- | (3, 2, α^- 1/2^-) | (3, 1 ⊕ 3, α^- 1/2^-) | (1, 1 ⊕ 3, α^- 1/2^-) | X_{6|α=1/2} |
| C-1   | (1, 1 ⊕ 3, α^-) | u_R | S_1^- | (3, 2, α^- α^- 1/2^-) | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (1, 2, α^- α^- 1/2^-) | [X_1, X_6]_{α=0} |
| C-2   | (1, 2, α^-) | u_R | S_1^- | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (3, 2, α^- α^- 1/2^-) | (1, 1 ⊕ 3, α^- α^- 1/2^-) | X_{6|α=1/2} |
| D-1   | (1, 1 ⊕ 3, α^-) | u_R | U_1^- | (3, 2, α^- α^- 1/2^-) | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (1, 2, α^- α^- 1/2^-) | [X_1, X_6]_{α=0} |
| D-2   | (1, 2, α^-) | u_R | U_1^- | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (3, 2, α^- α^- 1/2^-) | (1, 1 ⊕ 3, α^- α^- 1/2^-) | X_{6|α=1/2} |
| E-1   | (1, 1 ⊕ 3, α^-) | Q_L | V_2^- | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (3, 2, α^- α^- 1/2^-) | (1, 2, α^- α^- 1/2^-) | [X_1, X_6]_{α=0} |
| E-2   | (1, 2, α^-) | Q_L | V_2^- | (3, 2, α^- α^- 1/2^-) | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (1, 1 ⊕ 3, α^- α^- 1/2^-) | X_{6|α=1/2} |
| F-1   | (1, 1 ⊕ 3, α^-) | d_R | U_1^- | (3, 2, α^- α^- 1/2^-) | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (1, 2, α^- α^- 1/2^-) | [X_1, X_6]_{α=0} |
| F-2   | (1, 2, α^-) | d_R | U_1^- | (3, 1 ⊕ 3, α^- α^- 1/2^-) | (3, 2, α^- α^- 1/2^-) | (1, 1 ⊕ 3, α^- α^- 1/2^-) | X_{6|α=1/2} |
| Model | $X_1^F$  | $X_2^F$  | $X_3^F$  | $X_4$  | $X_5$  | $X_6$  | Dark matter |
|-------|----------|----------|----------|--------|--------|--------|-------------|
| A-1   | $Q_L$    | $d_R$    | $(1, 1 \oplus 3, \alpha)^-\ (1, 1 \oplus 3, \alpha)^-\ (3, 1 \oplus 3, \alpha - \frac{\alpha}{2})^-\ S_{1,3}^1$ | $1$ | $[X_3, X_4]|_{\alpha=0}$ |
| A-2   | $\bar{Q}_L$ | $\bar{d}_R$ | $(1, 2, \alpha)^-\ (1, 2, \alpha)^-\ (3, 2, \alpha - \frac{\alpha}{2})^-\ S_{1,3}^1$ | $1$ | $X_4|_{\alpha=\frac{1}{2}}$ |
| B-1   | $u_R$    | $(3, 2, \frac{\alpha}{3})$ | $(1, 1 \oplus 3, \alpha)^-\ (1, 1 \oplus 3, \alpha)^-\ (3, 2, \alpha - \frac{\alpha}{2})^-\ R_{1,3}^L$ | $[X_3, X_4]|_{\alpha=0}$ |
| B-2   | $u_R$    | $(3, 2, \frac{\alpha}{3})$ | $(1, 2, \alpha)^-\ (1, 2, \alpha)^-\ (3, 1 \oplus 3, \alpha - \frac{\alpha}{2})^-\ R_{1,3}^L$ | $X_4|_{\alpha=\frac{1}{2}}$ |
| C-1   | $\bar{d}_R$ | $Q_L$    | $(1, 1 \oplus 3, \alpha)^-\ (1, 1 \oplus 3, \alpha)^-\ (3, 2, \alpha - \frac{\alpha}{2})^-\ R_{1,3}^L$ | $[X_3, X_4]|_{\alpha=0}$ |
| C-2   | $d_R$    | $Q_L$    | $(1, 2, \alpha)^-\ (1, 2, \alpha)^-\ (3, 1 \oplus 3, \alpha - \frac{\alpha}{2})^-\ R_{1,3}^L$ | $X_4|_{\alpha=\frac{1}{2}}$ |
| D-1   | $u_R$    | $Q_L$    | $(1, 1 \oplus 3, \alpha)^-\ (1, 1 \oplus 3, \alpha)^-\ (3, 1 \oplus 3, \alpha - \frac{\alpha}{2})^-\ U_{1,3}^L$ | $[X_3, X_4]|_{\alpha=0}$ |
| D-2   | $u_R$    | $Q_L$    | $(1, 2, \alpha)^-\ (1, 2, \alpha)^-\ (3, 2, \alpha - \frac{\alpha}{2})^-\ U_{1,3}^L$ | $X_4|_{\alpha=\frac{1}{2}}$ |
| E-1   | $\bar{u}_R$ | $\bar{Q}_L$ | $(1, 1 \oplus 3, \alpha)^-\ (1, 1 \oplus 3, \alpha)^-\ (3, 2, \alpha + \frac{\alpha}{2})^-\ \bar{V}_2^L$ | $[X_3, X_4]|_{\alpha=0}$ |
| E-2   | $\bar{u}_R$ | $\bar{Q}_L$ | $(1, 2, \alpha)^-\ (1, 2, \alpha)^-\ (3, 1 \oplus 3, \alpha + \frac{\alpha}{2})^-\ \bar{V}_2^L$ | $X_4|_{\alpha=\frac{1}{2}}$ |
| F-1   | $\bar{d}_R$ | $(3, 2, \frac{\alpha}{3})$ | $(1, 1 \oplus 3, \alpha)^-\ (1, 1 \oplus 3, \alpha)^-\ (3, 2, \alpha - \frac{\alpha}{2})^-\ V_2^L$ | $[X_3, X_4]|_{\alpha=0}$ |
| F-2   | $\bar{d}_R$ | $(3, 2, \frac{\alpha}{3})$ | $(1, 2, \alpha)^-\ (1, 2, \alpha)^-\ (3, 1 \oplus 3, \alpha - \frac{\alpha}{2})^-\ V_2^L$ | $X_4|_{\alpha=\frac{1}{2}}$ |

Table XII. Possible assignments of quantum numbers under the SU(3)$_c$ $\otimes$ SU(2)$_L$ $\otimes$ U(1)$_\alpha$ $\otimes$ Z$_2$ symmetry to the mediators in diagram T3-iii-a, where $m \oplus n$ denotes that either an SU(2)$_L$ $m$- or an $n$-plet works.

| Model | $X_1^F$  | $X_2^F$  | $X_3^F$  | $X_4$  | $X_5$  | $X_6$  | Dark matter |
|-------|----------|----------|----------|--------|--------|--------|-------------|
| A-1   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $d_R$ | $S_1$ | $(3, 2, -\frac{\alpha}{2} - \alpha)^-$ | $(1, 2, -\frac{1}{2} - \alpha)^-$ | $[X_1, X_6]|_{\alpha=0}$ |
| A-2   | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $\bar{d}_R$ | $S_1$ | $(3, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $[X_2, X_6]|_{\alpha=\frac{1}{2}}$ |
| B-1   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $Q_L$ | $\bar{R}_2$ | $(3, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $(1, 2, -\frac{1}{2} - \alpha)^-$ | $[X_1, X_6]|_{\alpha=0}$ |
| B-2   | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $Q_L$ | $\bar{R}_2$ | $(3, 2, -\frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $[X_2, X_6]|_{\alpha=\frac{1}{2}}$ |
| C-1   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $\bar{u}_R$ | $\bar{S}_1$ | $(3, 2, -\frac{1}{2} - \alpha)^-$ | $(1, 2, -\frac{1}{2} - \alpha)^-$ | $[X_1, X_6]|_{\alpha=0}$ |
| C-2   | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $\bar{u}_R$ | $\bar{S}_1$ | $(3, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $[X_2, X_6]|_{\alpha=\frac{1}{2}}$ |
| D-1   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $u_R$ | $U_1$ | $(3, 2, -\frac{1}{2} - \alpha)^-$ | $(1, 2, -\frac{1}{2} - \alpha)^-$ | $[X_1, X_6]|_{\alpha=0}$ |
| D-2   | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $u_R$ | $U_1$ | $(3, 1 \oplus 3, \frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $[X_2, X_6]|_{\alpha=\frac{1}{2}}$ |
| E-1   | $Q_L$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $\bar{Q}_L$ | $\bar{V}_2$ | $(3, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $(1, 2, -\frac{1}{2} - \alpha)^-$ | $[X_1, X_6]|_{\alpha=0}$ |
| E-2   | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $Q_L$ | $\bar{V}_2$ | $(3, 2, -\frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $[X_2, X_6]|_{\alpha=\frac{1}{2}}$ |
| F-1   | $d_R$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $d_R$ | $U_1$ | $(3, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $(1, 1 \oplus 3, -\frac{1}{2} - \alpha)^-$ | $[X_2, X_6]|_{\alpha=\frac{1}{2}}$ |

Table XIII. Possible assignments of quantum numbers under the SU(3)$_c$ $\otimes$ SU(2)$_L$ $\otimes$ U(1)$_\alpha$ $\otimes$ Z$_2$ symmetry to the mediators in diagram T3-iii-b, where $m \oplus n$ denotes that either an SU(2)$_L$ $m$- or an $n$-plet works.
Table XIV. Possible assignments of quantum numbers under the SU(3) ⊗ SU(2)_L ⊗ U(1)_{\alpha} ⊗ Z_2 symmetry to the mediators in diagram T3-iv-a, where m ⊕ n denotes that either an SU(2)_L m- or an n-plet works.

| Model | $X_1^F$ | $X_2^F$ | $X_3^F$ | $X_4$ | $X_5$ | $X_6$ | Dark matter |
|-------|---------|---------|---------|-------|-------|-------|-------------|
| A-1   | $Q_L$   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 2, \alpha + \frac{1}{6})^-$ | $S_1^L, 3$ | $[X_2, X_4]_{\alpha=0}$ |
| A-2   | $\bar{Q}_L$ | $(1, 2, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 1, \alpha + \frac{1}{6})^-$ | $S_1^L, 3$ | $[X_2, X_4]_{\alpha=-\frac{1}{2}}$ |
| B-1   | $u_R$   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 2, \alpha - \frac{1}{6})^-$ | $R_2^L$ | $[X_2, X_4]_{\alpha=0}$ |
| B-2   | $\bar{u}_R$ | $(1, 2, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 2, \alpha - \frac{1}{6})^-$ | $R_2^L$ | $[X_2, X_4]_{\alpha=-\frac{1}{2}}$ |
| C-1   | $d_R$   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 2, \alpha + \frac{1}{6})^-$ | $R_2^L$ | $[X_2, X_4]_{\alpha=0}$ |
| C-2   | $\bar{d}_R$ | $(1, 2, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 2, \alpha + \frac{1}{6})^-$ | $R_2^L$ | $[X_2, X_4]_{\alpha=-\frac{1}{2}}$ |
| D-1   | $Q_L$   | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 2, \alpha - \frac{1}{6})^-$ | $U_1^L, 3$ | $[X_2, X_4]_{\alpha=0}$ |
| D-2   | $\bar{Q}_L$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 2, \alpha - \frac{1}{6})^-$ | $U_1^L, 3$ | $[X_2, X_4]_{\alpha=-\frac{1}{2}}$ |
| E-1   | $\bar{u}_R$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 2, \alpha - \frac{1}{6})^-$ | $\bar{V}_2^L$ | $[X_2, X_4]_{\alpha=0}$ |
| E-2   | $\bar{d}_R$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{6})^-$ | $(3, 2, \alpha - \frac{1}{6})^-$ | $\bar{V}_2^L$ | $[X_2, X_4]_{\alpha=-\frac{1}{2}}$ |
| F-1   | $\bar{d}_R$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{6})^-$ | $(3, 1, \alpha + \frac{1}{6})^-$ | $\bar{V}_2^L$ | $[X_2, X_4]_{\alpha=0}$ |
| F-2   | $\bar{d}_R$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{6})^-$ | $(3, 1, \alpha + \frac{1}{6})^-$ | $\bar{V}_2^L$ | $[X_2, X_4]_{\alpha=-\frac{1}{2}}$ |

Table XV. Possible assignments of quantum numbers under the SU(3)_c ⊗ SU(2)_L ⊗ U(1)_{\alpha} ⊗ Z_2 symmetry to the mediators in diagram T3-iv-b, where m ⊕ n denotes that either an SU(2)_L m- or an n-plet works.

| Model | $X_1^F$ | $X_2^F$ | $X_3^F$ | $X_4$ | $X_5$ | $X_6$ | Dark matter |
|-------|---------|---------|---------|-------|-------|-------|-------------|
| A-1   | $(1, 1 \oplus 3, \alpha)^-$ | $Q_L$ | $\bar{d}_R$ | $S_1$ | $(3, 2, \alpha - \frac{1}{6} - \alpha)^-$ | $(1, 2, \alpha - \frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| A-2   | $(1, 2, \alpha)^-$ | $\bar{Q}_L$ | $\bar{d}_R$ | $S_1$ | $(3, 1 \oplus 3, \alpha - \frac{1}{6} - \alpha)^-$ | $(1, 1 \oplus 3, \alpha - \frac{1}{2} - \alpha)^-$ | $X_{\alpha=-\frac{1}{2}}$ |
| B-1   | $(1, 1 \oplus 3, \alpha)^-$ | $d_R$ | $Q_L$ | $\bar{R}_2$ | $(3, 1 \oplus 3, \alpha - \frac{1}{6} - \alpha)^-$ | $(1, 1 \oplus 3, \alpha - \frac{1}{2} - \alpha)^-$ | $X_{\alpha=-\frac{1}{2}}$ |
| B-2   | $(1, 2, \alpha)^-$ | $d_R$ | $Q_L$ | $\bar{R}_2$ | $(3, 2, \alpha - \frac{1}{6} - \alpha)^-$ | $(1, 1 \oplus 3, \alpha - \frac{1}{2} - \alpha)^-$ | $X_{\alpha=-\frac{1}{2}}$ |
| C-1   | $(1, 1 \oplus 3, \alpha)^-$ | $(3, 2, -\frac{1}{6})$ | $\bar{u}_R$ | $S_1$ | $(3, 2, -\frac{1}{6} - \alpha)^-$ | $(1, 1 \oplus 3, \alpha - \frac{1}{2} - \alpha)^-$ | $X_{\alpha=-\frac{1}{2}}$ |
| C-2   | $(1, 2, \alpha)^-$ | $(3, 2, -\frac{1}{6})$ | $\bar{u}_R$ | $S_1$ | $(3, 2, -\frac{1}{6} - \alpha)^-$ | $(1, 1 \oplus 3, \alpha - \frac{1}{2} - \alpha)^-$ | $X_{\alpha=-\frac{1}{2}}$ |
| D-1   | $(1, 1 \oplus 3, \alpha)^-$ | $Q_L$ | $u_R$ | $U_1$ | $(3, 2, \alpha - \frac{1}{6} - \alpha)^-$ | $(1, 2, -\frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| D-2   | $(1, 2, \alpha)^-$ | $Q_L$ | $u_R$ | $U_1$ | $(3, 1 \oplus 3, \alpha - \frac{1}{6} - \alpha)^-$ | $(1, 1 \oplus 3, \alpha - \frac{1}{2} - \alpha)^-$ | $X_{\alpha=-\frac{1}{2}}$ |
| E-1   | $(1, 1 \oplus 3, \alpha)^-$ | $\bar{u}_R$ | $\bar{Q}_L$ | $\bar{V}_2$ | $(3, 2, -\frac{1}{6} - \alpha)^-$ | $(1, 2, -\frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| E-2   | $(1, 2, \alpha)^-$ | $\bar{u}_R$ | $\bar{Q}_L$ | $\bar{V}_2$ | $(3, 2, -\frac{1}{6} - \alpha)^-$ | $(1, 1 \oplus 3, \alpha - \frac{1}{2} - \alpha)^-$ | $X_{\alpha=-\frac{1}{2}}$ |
| F-1   | $(1, 1 \oplus 3, \alpha)^-$ | $(3, 2, -\frac{1}{6})$ | $d_R$ | $\bar{U}_1$ | $(3, 2, -\frac{1}{6} - \alpha)^-$ | $(1, 2, -\frac{1}{2} - \alpha)^-$ | $[X_1, X_6]_{\alpha=0}$ |
| F-2   | $(1, 2, \alpha)^-$ | $(3, 2, -\frac{1}{6})$ | $d_R$ | $\bar{U}_1$ | $(3, 1 \oplus 3, \alpha - \frac{1}{6} - \alpha)^-$ | $(1, 1 \oplus 3, \alpha - \frac{1}{2} - \alpha)^-$ | $X_{\alpha=-\frac{1}{2}}$ |
Table XVI. Possible assignments of quantum numbers under the SU(3)$_c$ symmetry to the mediators in diagram T3-ix-a, where $m \oplus n$ denotes that either an SU(2)$_L$ $m$- or an $n$-plet works.

| Model | $X_1$ | $X_2$ | $X_3^F$ | $X_4^F$ | $X_5$ | $X_6^F$ | Dark matter |
|-------|-------|-------|---------|---------|-------|---------|-------------|
| A-1   | $S_1^T$ | $d_R$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha + \frac{1}{2})^-$ | $Q_L$ | $X_3 | \alpha = -\frac{1}{2}$ |
| A-2   | $S_1^T$ | $d_R$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $\bar{Q}_L$ | $X_3 | \alpha = 0$ |
| B-1   | $\bar{R}_2^T$ | $\bar{Q}_L$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $d_R$ | $X_4 | \alpha = -\frac{1}{2}$ |
| B-2   | $\bar{R}_2^T$ | $\bar{Q}_L$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha + \frac{1}{2})^-$ | $d_R$ | $X_3 | \alpha = 0$ |
| C-1   | $U_1^T$ | $\bar{u}_R$ | $(1, 1, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha + \frac{1}{2})^-$ | $\bar{Q}_L$ | $X_4 | \alpha = -\frac{1}{2}$ |
| C-2   | $U_1^T$ | $\bar{u}_R$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha + \frac{2}{3})^-$ | $Q_L$ | $X_3 | \alpha = 0$ |
| D-1   | $\bar{V}_2^T$ | $Q_L$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha - \frac{1}{2})^-$ | $\bar{u}_R$ | $X_4 | \alpha = -\frac{1}{2}$ |
| D-2   | $\bar{V}_2^T$ | $Q_L$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha - \frac{1}{2})^-$ | $\bar{u}_R$ | $X_3 | \alpha = 0$ |

Table XVII. Possible assignments of quantum numbers under the SU(3)$_c$ $\otimes$ SU(2)$_L$ $\otimes$ U(1)$_a$ $\otimes$ Z$_2$ symmetry to the mediators in diagram T3-x-a, where $m \oplus n$ denotes that either an SU(2)$_L$ $m$- or an $n$-plet works.

| Model | $X_1$ | $X_2^F$ | $X_3$ | $X_4$ | $X_5^F$ | $X_6^F$ | Dark matter |
|-------|-------|-------|-------|-------|---------|---------|-------------|
| A-1   | $S_1^T$ | $d_R$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha + \frac{1}{2})^-$ | $Q_L$ | $X_3 | X_4 | \alpha = -\frac{1}{2}$ |
| A-2   | $S_1^T$ | $d_R$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $\bar{Q}_L$ | $X_3 | X_4 | \alpha = 0$ |
| B-1   | $\bar{R}_2^T$ | $\bar{Q}_L$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $d_R$ | $X_3 | X_4 | \alpha = -\frac{1}{2}$ |
| B-2   | $\bar{R}_2^T$ | $\bar{Q}_L$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha + \frac{1}{2})^-$ | $d_R$ | $X_3 | X_4 | \alpha = 0$ |
| C-1   | $U_1^T$ | $\bar{u}_R$ | $(1, 1, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha + \frac{1}{2})^-$ | $\bar{Q}_L$ | $X_3 | X_4 | \alpha = -\frac{1}{2}$ |
| C-2   | $U_1^T$ | $\bar{u}_R$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha + \frac{2}{3})^-$ | $Q_L$ | $X_3 | X_4 | \alpha = 0$ |
| D-1   | $\bar{V}_2^T$ | $Q_L$ | $(1, 2, \alpha)^-$ | $(1, 1 \oplus 3, \alpha + \frac{1}{2})^-$ | $(3, 1 \oplus 3, \alpha - \frac{1}{2})^-$ | $\bar{u}_R$ | $X_3 | X_4 | \alpha = -\frac{1}{2}$ |
| D-2   | $\bar{V}_2^T$ | $Q_L$ | $(1, 1 \oplus 3, \alpha)^-$ | $(1, 2, \alpha + \frac{1}{2})^-$ | $(3, 2, \alpha - \frac{1}{2})^-$ | $\bar{u}_R$ | $X_3 | X_4 | \alpha = 0$ |