Polarization contributions to the spin-dependence of the effective interaction in neutron matter

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We calculate the modification of the effective interaction of particles on the Fermi surface due to polarization contributions, with particular attention to spin-dependent forces. In addition to the standard spin-spin, tensor and spin-orbit forces, spin non-conserving effective interactions are induced by screening in the particle-hole channels. Furthermore, a novel long-wavelength tensor force is generated. We compute the polarization contributions to second order in the low-momentum interaction $V_{\text{low }k}$ and find that the medium-induced spin-orbit interaction leads to a reduction of the $^3P_2$ pairing gap for neutrons in the interior of neutron stars.

Introduction. — Landau-Fermi liquid theory is a powerful effective theory for strongly interacting Fermi systems at low temperatures. It has been successfully applied to liquid $^3$He, nuclear matter and nuclei. While the free interaction between $^3$He atoms is almost state-independent, the nuclear interaction is complicated due to large non-central spin-orbit and tensor forces, which are crucial for understanding nuclear phenomena. For investigations of matter under extreme conditions, such as nuclei with large proton or neutron excess and asymmetric nuclear matter in neutron stars, the role of non-central forces in the effective interaction must be understood.

As part of a program to determine effective nuclear interactions using renormalization group methods, we analyze the spin-dependence of the quasiparticle interaction and the low-energy scattering amplitude in the presence of non-central forces. We focus on pure neutron matter, and as an application, we estimate the modification of the $^3P_2$ pairing gap in neutron star interiors due to the screening of the nucleon-nucleon interaction. This is a long-standing problem in neutron star structure, and since polarization effects suppress the S-wave gaps by a factor four, large effects may be expected.

Symmetry considerations. — In general, the two-body interaction is hermitian and constrained by symmetries, specifically, time-reversal and parity invariance, as well as invariance under exchange of particle labels. In addition, in a non-relativistic theory in vacuum, the potential is Galilean invariant, i.e., independent of the particle-pair momentum $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$ or the difference $\mathbf{P} = \mathbf{p}_1 - \mathbf{p}_2$.

The possible operators are well-known: scalar: 1, spin-spin: $\sigma_1 \cdot \sigma_2$, spin-orbit: $i(\sigma_1 + \sigma_2) \cdot \mathbf{q} \times \mathbf{q}'$, tensor: $S_{12}(\mathbf{q}) \equiv \sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q} - 1/3 q^2 \sigma_1 \cdot \sigma_2$ (and the exchange thereof $S_{12}(\mathbf{q}')$) as well as the quadratic spin-orbit force: $\mathbf{q} \cdot \mathbf{q}' (\sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q}' + \sigma_1 \cdot \sigma_2 \frac{1}{3} q^2 \mathbf{q} \cdot \mathbf{q}' - 2/3 q_1 \cdot q'_1 \sigma_1 \cdot \sigma_2)$. The momentum transfer is $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_3$ and in the exchange term $\mathbf{q}' = \mathbf{p}_1 - \mathbf{p}_3$.

In the many-body medium the presence of the Fermi sea defines a preferred frame. Therefore, the effective two-body interaction depends on the two-body center of mass (cm) momentum. This is physically clear, since the magnitude of the cm momentum, for given momentum transfers, determines where the interacting particles are relative to the Fermi sea. For particles on the Fermi surface $q^2 + q'^2 + P^2 = 4k_F^2$ and the momentum dependence of possible invariants is constrained geometrically, since $\mathbf{q}$, $\mathbf{q}'$ and $\mathbf{P}$ are orthogonal. As a consequence, the quadratic spin-orbit force vanishes in this case.

Effective interactions on the Fermi surface. — In the presence of a Fermi sea, additional operators are possible. For particles on the Fermi surface, these are

\begin{alignat}{2}
S_{12}(\mathbf{P}) &= \text{cm tensor} \quad (1) \\
D_{12}(\mathbf{q}, \mathbf{P}) &= i(\sigma_1 - \sigma_2) \cdot \mathbf{q} \times \mathbf{P} \quad \text{diff vector} \quad (2) \\
A_{12}(\mathbf{q}', \mathbf{P}) &= (\sigma_1 \times \sigma_2) \cdot (\mathbf{q}' \times \mathbf{P}) \quad \text{cross vector} \quad (3)
\end{alignat}

Operators $D_{12}$ and $A_{12}$ are related by exchange. These are antisymmetric in spin and thus do not conserve the spin of the interacting particle pair. In this Letter, we explore the microscopic origin of these forces and compute the contributions to the quasiparticle interaction in neutron matter. Our results can be used as input for calculations of neutron star properties. For particles not on the Fermi surface, further invariants are possible.

Both the cm tensor and the cross vector operator survive in the long-wavelength limit, $q \to 0$, and thus contribute to the quasiparticle interaction in nuclear matter. The exchange tensor in the quasiparticle interaction was considered in \cite{4}, and Landau parameters were computed in \cite{2} and \cite{3}. We introduce the Fermi liquid parameters, $H_k$, $K_1$, and $L_1$ for the non-central interactions,

\begin{equation}
\mathcal{F}_{\sigma_1 \sigma_2}^{\nu \epsilon} (\mathbf{q}', \mathbf{P}) = H(\cos \theta_{q'}) S_{12}(\mathbf{q}') + K(\cos \theta_{q'}) S_{12}(\mathbf{P}) \\
+ L(\cos \theta_{q'}) A_{12}(\mathbf{q}', \mathbf{P}).
\end{equation}

The tensor operators are defined with unit vectors $\mathbf{q}$ and the dependence on Landau angle $\theta_{q'}$ is expanded in Legendre polynomials, $H(x) = \sum H_k P_k(x)$ etc. The novel Fermi-liquid interactions $K$ and $L$ have not been considered in previous work. Analyticity of the quasiparticle interaction implies $H \sim q'^2$ as $q' \to 0$, $K \sim P^2$ as $q' \to 2k_F$ ($P \to 0$) and $L \sim q'$ as $q' \to 0$ (as well as $L \sim P$ as $q' \to 2k_F$). Many-body effects may give rise
to singularities in the effective interaction, which modify these limits. This is illustrated by $K(\cos \theta_{q'})$ in Fig. 11 where the pairing singularity cancels the zero of the cm tensor in the limit $q' \rightarrow 2k_F$ ($\cos \theta_{q'} \rightarrow 1$).

**Effects of the many-body medium.** At second-order, there are contributions to the effective four-point vertex from scattering in the BCS channel with intermediate particle-particle and hole-hole excitations of cm momentum $\mathbf{P}$, as well as scattering in the direct particle-hole or zero sound (ZS) channel and in the exchange particle-hole (ZS') channel. The ZS and ZS' channels include intermediate states with particle-hole excitations of momentum $\mathbf{q}$ and $\mathbf{q}'$ respectively. For the quasiparticle interaction only the BCS and ZS' channel contribute. General recoupling arguments imply that the interference of the spin-spin with the tensor force in the particle-hole channels leads to a large renormalization of the tensor interaction. Moreover, the presence of a third particle in intermediate states induces novel contributions to the effective interaction, of the form of Eqs. (11 - 13).

For an antisymmetrized interaction $f_{\sigma_1,\sigma_2}^{d}(\mathbf{q}, \mathbf{q'}) = V_{\sigma_1,\sigma_2}(\mathbf{q}, \mathbf{q'}) - P_\sigma V_{\sigma_1,\sigma_2}(\mathbf{q}', \mathbf{q})$ (where $P_\sigma$ is the spin exchange operator and the superscript $d$ labels the driving interaction), the particle-hole contributions in the ZS' channel are given by

$$a_{\sigma_1,\sigma_2}^{ZS'}(\mathbf{q}, \mathbf{q'}, \mathbf{P}) = -P_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_p + \mathbf{q'}/2 - n_p - \mathbf{q}/2}{\varepsilon_p + \mathbf{q'}/2 - \varepsilon_p - \mathbf{q}/2}$$

$$\times \text{Tr} \, f_{\sigma_1,\sigma_2}^{d}(\mathbf{q}', \frac{\mathbf{P} + \mathbf{q}}{2} - \mathbf{p}) f_{\sigma_1,\sigma_2}^{d}(\mathbf{q'}, \mathbf{p} - \frac{\mathbf{P} - \mathbf{q}}{2}),$$

where $n_p$ denotes the Fermi-Dirac distribution function and $\varepsilon_p$ is the quasiparticle energy for which a single-particle spectrum with effective mass is used. The exchange tensor, iterated in the particle-hole ladder, yields an unusual ordering of the spin operators $\sigma_1 \cdot t' \sigma_1 \cdot t \sigma_2 \cdot t'$, where $t' = \pm (\mathbf{P}/2 \pm \mathbf{q}/2 - \mathbf{p})$. This ordering gives rise to a particular coupling between the spin and the angular motion, which leads to antisymmetric spin operators. Furthermore, particle-hole polarization contributions involving spin-orbit forces always result in spin non-conserving interactions.

On the other hand, in the BCS channel the dependence on the cm momentum enters only through the phase space and the ordering of the spin operators is the same as in vacuum. Consequently, the BCS channel does not yield antisymmetric spin operators. The second order contribution is given by

$$a_{\sigma_1,\sigma_2}^{BCS}(\mathbf{q}, \mathbf{q'}, \mathbf{P}) = \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \frac{1 - n_p + \mathbf{p}/2 + n_p - \mathbf{p}/2}{2\mu - \varepsilon_p + \mathbf{p}/2 - \varepsilon_p - \mathbf{p}/2}$$

$$\times f_{\sigma_1,\sigma_2}^{d}(\mathbf{p} - \mathbf{k}_f, \mathbf{p} + \mathbf{k}_f) f_{\sigma_1,\sigma_2}^{d}(\mathbf{k}_i - \mathbf{p}, \mathbf{k}_i + \mathbf{p}),$$

where $\mathbf{k}_i, \mathbf{k}_f = (\mathbf{q'} \pm \mathbf{q})/2$ denotes initial/final relative momenta and $\mu$ is the chemical potential.

**Boost corrections.** In addition to the dynamical effects, there are kinematical contributions from boosting the two-body interaction to the rest frame of the Fermi sea. To leading order in $k_F^2/m^2$, the boost to the frame where the nucleon pair carries momentum $\mathbf{P}$ is given in terms of the vacuum interaction in the cm frame 10 [11]

$$\delta F_{\sigma_1,\sigma_2}(\mathbf{q}, \mathbf{q'}, \mathbf{P}) = -\frac{P^2}{4m^2} F_{\sigma_1,\sigma_2}(\mathbf{q}, \mathbf{q'})$$

$$+ \frac{i}{16m^2} \left(\sigma_1 - \sigma_2\right) \cdot \mathbf{P} \cdot (\mathbf{q} - \mathbf{q'}) F_{\sigma_1,\sigma_2}(\mathbf{q}, \mathbf{q'})$$

$$- \frac{i}{16m^2} F_{\sigma_1,\sigma_2}(\mathbf{q}, \mathbf{q'}) \left(\sigma_1 - \sigma_2\right) \times \mathbf{P} \cdot (\mathbf{q'} + \mathbf{q}),$$

where $F_{\sigma_1,\sigma_2}(\mathbf{q}, \mathbf{q'}) = m^* k_F f_{\sigma_1,\sigma_2}^{a}(\mathbf{q}, \mathbf{q'})$. In Eq. 11 the particles are restricted to the Fermi surface and direct and exchange terms are included. In the long-wavelength limit, we find

$$\delta F_{\sigma_1,\sigma_2}(\mathbf{q'}, \mathbf{P}) = -\frac{P^2}{4m^2} \left\{ F_{a}(\cos \theta_{q'}) + G_{a}(\cos \theta_{q'}) \sigma_1 \cdot \sigma_2 + H_{a}(\cos \theta_{q'}) S_{12}(\mathbf{q'}) \right\}$$

$$- \frac{1}{4m^2} A_{12}(\mathbf{q'}, \mathbf{P}) \left\{ G_{a}(\cos \theta_{q'}) + \frac{1}{6} H_{a}(\cos \theta_{q'}) \right\},$$

with standard notation for the scalar and spin-spin parts, $F_{a}$ and $G_{a}$. We note that the boost corrections contribute to the spin non-conserving part, but not to the cm tensor Eq. 11. These kinematical effects are of order $k_F^2/m^2$, while the many-body effects are of order...
interaction average over the interaction in the loop integral. As shown in P-waves in free space, and similarly for a realistic assessment because it is crucial for an accurate reproduction of the phenomena, one needs the scattering amplitude at finite momentum. For instance, the exchange tensor at this order is comparable to the exchange tensor at this order.

Results. We start from the free-space low-momentum interaction $V_{\text{low}}$ with a density-dependent cutoff $\Lambda = \sqrt{2} k_F$. Detailed results for both neutron and nuclear matter will be presented elsewhere. Here, we focus on the novel spin-dependent interactions. The contributions to the Fermi liquid parameters are given in Table I and the dependence on $\cos \theta_q$ is shown in Fig. 1. We find a substantial renormalization of the exchange tensor, which necessitates a self-consistent treatment within, e.g., the RG approach, and significant contributions to the new interaction terms. In particular, the cm tensor is comparable to the exchange tensor at this order.

In calculations of transport processes and pairing phenomena, one needs the scattering amplitude at finite momentum, $q \neq 0$. In particular, the in-medium modification of the spin-orbit interaction is of special interest, because it is crucial for an accurate reproduction of the P-waves in free space, and similarly for a realistic assessment of P-wave pairing in neutron stars. As shown in

\[ \Delta_{3P_2} = \frac{k_F^2}{m} \exp \left( \frac{\pi}{2 k_F m V_{\text{low}} k_F^3 P_2 \left( k_F, k_F \right)} \right), \]

where the arguments of $V_{\text{low}}$ are the magnitude of the relative momenta $|k_{i,f}| = k_F$. When particle-hole screening effects are included, the pairing interaction $V_{\text{low}} k_F^3 P_2 (k_F, k_F)$ is replaced by the $3P_2$ projection of the effective interaction in the particle-particle channel. The high-lying states in the BCS channel are included in $V_{\text{low}}$ while the low-lying part is accounted for through the (approximate) solution of the gap equation.

We find that spin-dependent polarization effects reduce the $3P_2$ pairing gap (C compared to A and B in

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**FIG. 1:** Angular dependence ($\cos \theta_q = 1 - q'^2/2k_F^2$) of the non-central quasiparticle interactions in neutron matter for $k_F = 1.7$ fm$^{-1}$. The $V_{\text{low}}$ contribution as well as the polarization effects from the exchange particle-hole (ZS') and the particle-particle (BCS) channel are shown for $m^*/m = 0.83$.

**FIG. 2:** Comparison of the particle-hole induced spin-orbit amplitude to the free-space interaction. Results are presented for back-to-back scattering, $P = 0$, $k_F = 1.7$ fm$^{-1}$ (thick lines) and $k_F = 1.0$ fm$^{-1}$ (thin lines). Here, the (antisymmetrized) spin-orbit force is shown in units of the density of states with the operator $V_{\text{SO}}(\cos \theta_q) i(\sigma_1 + \sigma_2) \cdot q/k_F \times q'/k_F$.

Fig. 2 we find a repulsive induced spin-orbit interaction due to particle-hole screening. In order to qualitatively understand the resulting interaction, we assume a contact spin-spin and an averaged spin-orbit matrix element. Then, the ZS and ZS' channel for $P = 0$ are repulsive

\[ V_{\text{SO}}^{\text{ind}}(\cos \theta_q) = -G_0 \frac{m^*}{2m} \langle V_{\text{SO}}^{\text{d}} \rangle \left( U(q/k_F) + U(q'/k_F) \right), \]

for an attractive spin-orbit force. Here $U(q/k_F)$ denotes the (positive) static Lindhard function and $q^{(i)} = k_F \sqrt{2 \pm 2 \cos \theta_q}$. As in Fig. 2, $V_{\text{SO}}^{\text{ind}}$ is only weakly dependent on $\cos \theta_q$. The contributions due to the mixing of spin-orbit and tensor forces are also repulsive, with a similar but more complicated momentum dependence.

**Triplet pairing.** To illustrate the importance of non-central induced interactions, we estimate the angle-averaged gap using weak coupling BCS theory (for details see [13, 14]). The coupling to the $3F_2$ partial wave is neglected in this exploratory calculation. In the weak coupling approximation, the $3F_2$ gap is then given by

\[ V_{\text{SO}}^{\text{ind}}(\cos \theta_q) = -G_0 \frac{m^*}{2m} \langle V_{\text{SO}}^{\text{d}} \rangle \left( U(q/k_F) + U(q'/k_F) \right), \]

where $V_{\text{SO}}^{\text{ind}}$ is the magnitude of the relative momenta $|k_{i,f}| = k_F$. When particle-hole screening effects are included, the pairing interaction $V_{\text{low}} k_F^3 P_2 (k_F, k_F)$ is replaced by the $3P_2$ projection of the effective interaction in the particle-particle channel. The high-lying states in the BCS channel are included in $V_{\text{low}}$ while the low-lying part is accounted for through the (approximate) solution of the gap equation.

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Fig. 3. The angle-averaged pairing gap $\Delta_{3P_2}$ in the $3P_2$ channel versus the Fermi momentum in neutron matter. The direct ($V_{\text{lowk}}$) pairing gap, computed with the free and with an effective neutron mass, as well as the gap including particle-hole polarization effects on the pairing interaction are shown. We also give the gap, obtained when only the central and only the spin-orbit polarization contributions are taken into account. For reference, we show the results of Baldo et al. [15], obtained by solving the BCS gap equation in the coupled $^{3}P_2-^{3}F_2$ channel for different free-space interactions. $V_{\text{lowk}}$ is obtained from the CD Bonn potential.

This is in contrast to the increase of the $^{3}P_2$ gap, which one obtains when polarization effects due only to central forces are included (dotted vs. dashed line in Fig. 3). The reduction of the gap is predominantly due to the repulsion from the medium-induced spin-orbit force, discussed above. This effect was not taken into account in earlier work. Note that for the densities given in Fig. 3 the ratio of the induced pairing matrix element to the free-space $V_{\text{lowk}}$ contribution ranges from 0.15 – 0.5. The significant reduction of the gap for only moderate changes of the pairing interaction is due to the singular dependence on the matrix element in Eq. (16) for small gaps. We emphasize that our results are qualitative; a quantitative calculation should include the full polarization contributions for non-central interactions and the solution of the full coupled-channel BCS gap equation. We present results only for $k_F \lesssim 2$ fm$^{-1}$, where the NN interaction is strongly constrained by data.

Conclusions. – In summary, we have found novel non-central effective nuclear interactions, with spin non-conserving forces induced by particle-hole polarization effects. In microscopic shell model calculations, these are implicitly included in the polarization force of Kuo and Brown [17]. Furthermore, the renormalization of the spin-orbit interaction in the medium has important consequences for P-wave pairing. The resulting suppression of the superfluid gap has direct impact on the properties of neutron stars and on their cooling by neutrino emission [18, 19]. The implications of the new interactions for nuclear spectra, for the spin-isospin response and neutrino transport in stellar collapse, for magnetic susceptibilities (see [4]), for deformations of the Fermi surface in spin-polarized systems, and for spin relaxation and mixing of spin and density waves remain to be investigated. Since the new interactions contribute only to spin non-conserving transitions, it may be possible to observe these effects in scattering with polarized beams.

We are grateful to Gerry Brown, Dick Furnstahl, Emma Olsson, Chris Pethick and Dan-Olof Riska for useful discussions. The work of AS is supported by the NSF under Grant No. PHY-0098645 and PHY-0139973.

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[1] A. Schwenk, B. Friman, G.E. Brown, Nucl. Phys. A713 (2003) 191, ibid. A703 (2002) 745.
[2] S. Okubo, R.E. Marshak, Ann. Phys. 4 (1958) 166.
[3] The given tensor operators are linearly dependent, i.e., $S_{12}(\hat{q}) + S_{12}(\hat{q}') + S_{12}(\hat{P}) = 0$. In this work, we have kept all tensor operators explicitly and eliminate $S_{12}(\hat{q})$ for long-wavelength and $S_{12}(\hat{P})$ for pairing interactions.
[4] These are a spin-orbit like $q\cdot P \text{D}_{12}(q, q')$ and $q\cdot q' \text{D}_{12}(q', P)$, and finally $q\cdot P S_{12}(q, P)$ as well as the corresponding operators obtained by exchange. The last operator has the same spin-momentum structure as the quadratic spin-orbit with $q'$ replaced by $P$.
[5] J. Dąbrowski, P. Haensel, Ann. Phys. 97 (1976) 452.
[6] P. Haensel, J. Dąbrowski, Nucl. Phys. A254 (1975) 211.
[7] S.-O. Bäckman, O. Sjöberg, A.D. Jackson, Nucl. Phys. A321 (1979) 10.
[8] W.H. Dickhoff et al., Nucl. Phys. A405 (1983) 534.
[9] E. Olsson, C.J. Pethick, Phys. Rev. C66 (2002) 065803, and private communication.
[10] R.A. Krajcik, L.L. Foldy, Phys. Rev. D10 (1974) 1777, J.L. Friar, Phys. Rev. C12 (1975) 695.
[11] J.L. Forest, V.R. Pandharipande, J.L. Friar, Phys. Rev. C52 (1995) 568.
[12] S.K. Bogner et al., Phys. Lett. B576 (2003) 265, S.K. Bogner, T.T.S. Kuo, A. Schwenk, Phys. Rep. 386 (2003) 1.
[13] B. Patton, A. Zaringhalam, Phys. Lett. A55 (1975) 95.
[14] M. Baldo et al., Nucl. Phys. A536 (1992) 349.
[15] M. Baldo et al., Phys. Rev. C58 (1998) 1921.
[16] C.J. Pethick, D.G. Ravenhall, Ann. N.Y. Acad. Sci. 647 (1991) 503.
[17] T.T.S. Kuo, G.E. Brown, Nucl. Phys. 85 (1966) 40.
[18] A.D. Kaminker, D.G. Yakovlev, O.Y. Gnedin, Astron. Astrophys. 383 (2002) 1076.
[19] P.F. Bedaque, G. Rupak, M. Savage, nucl-th/0305032