Can inflationary models of cosmic perturbations evade the secondary oscillation test?

Alex Lewin\textsuperscript{1,2} and Andreas Albrecht\textsuperscript{1}
\textsuperscript{1}Department of Physics
One Shields Ave.
University of California
Davis, CA 95616
\textsuperscript{2}The Blackett Laboratory, Imperial College,
Prince Consort Road, London SW7 2BZ, UK

We consider the consequences of an observed Cosmic Microwave Background (CMB) temperature anisotropy spectrum containing no secondary oscillations. While such a spectrum is generally considered to be a robust signature of active structure formation, we show that such a spectrum can be produced by (very unusual) inflationary models or other passive evolution models. However, we show that for all these passive models the characteristic oscillations would show up in other observable spectra. Our work shows that when CMB polarization and matter power spectra are taken into account secondary oscillations are indeed a signature of even these very exotic passive models. We construct a measure of the observability of secondary oscillations in a given experiment, and show that even with foregrounds both the MAP and PLANCK satellites should be able to distinguish between models with and without oscillations. Thus we conclude that inflationary and other passive models cannot evade the secondary oscillation test.

I. INTRODUCTION

Secondary oscillations in the angular power spectrum of CMB anisotropies are broadly understood to be a signature of inflationary models for the origin of the primordial perturbations. The inflationary models are part of a wider class of ‘passive’ models, where the perturbations are set up at very early times, and then evolve according to linear perturbation theory for many orders of magnitude of expansion.

In linear perturbation theory, the physics of gravitational collapse (on scales greater than the Jeans length) results in a ‘squeezing’ effect as the presence of one growing and one decaying solution draws any initial conditions into a preferred region of phase space. On scales below the Jeans length there is oscillatory behavior at early times, but in passive models each mode has already experienced a long period of squeezing by the time the oscillatory behavior kicks in. The region of phase space preferred by the squeezing corresponds to a fixed phase of oscillation in the oscillatory epoch. The fact that different wavenumbers have their oscillations fixed at different phases ultimately results in the characteristic ‘secondary oscillations’ in the CMB angular power spectrum \cite{1–3}.

The active models are fundamentally different \cite{4}. The canonical examples are the cosmic defect models. The key difference is that these models have a matter component (e.g. cosmic defects) which is evolving in a non-linear manner from very early times. One can think of the non-linear component as a source which constantly seeds perturbations in the other matter. This kind of non-linear ‘force term’ disrupts the squeezing effects which are present at the linear level, and those realistic defect models for which calculations have been performed have shown no secondary oscillations whatsoever in the CMB anisotropy power \cite{5–7}.

Thus the detection, or otherwise, of these acoustic oscillations is cited as a strong indication of whether or not the evolution of the perturbations was active or passive. In fact, the absence of these oscillations is regarded as one of the few ways that aspects of inflationary cosmology can be falsified \cite{8}. But what if inflation produced a primordial spectrum with oscillations, such that the CMB temperature power spectrum has no oscillations? Can a $C_l$ spectrum with no oscillations really rule out inflation? In this paper we investigate this possibility.

Suppose that the temperature spectrum measured by PLANCK turns out not to have oscillations. Note that as we see in Section \textsuperscript{15}, the new Boomerang \textsuperscript{11} and Maxima \textsuperscript{10} data can be fit equally well by spectra with and without oscillations, so we have no indication either way as yet\textsuperscript{4}. With future observations we can hope to decide whether

\textsuperscript{1} A different issue is whether the single peak that is clearly evident in the current data is too sharp to be produced by defect models. One of us answers this question in the affirmative in \textsuperscript{11}.
or not the temperature spectrum has oscillations. What could we say about an inflationary model that produced a temperature spectrum without oscillations, from our other observations, those of the polarization and matter power spectra? We look at a model which has oscillations in the primordial power spectrum producing a temperature anisotropy spectrum with no oscillations, to see how this affects the other spectra. We argue that such an inflationary model is forced to have polarization and matter spectra which are easily distinguishable from those of active models, at least when the best data sets are finally in. Thus, even for the special borderline cases we consider here the presence or absence of secondary oscillations remains a robust test of the fundamental nature of the primordial perturbations.

II. OSCILLATIONS IN PRESENT DAY POWER SPECTRA

In this section we review the origin of the oscillations in the angular spectrum for passive models, and discuss what factors can affect them. We develop our discussion based on the approximate scheme of Hu and Sugiyama \[13,14\] which is more intuitive and gives a good enough account of the physics for our immediate purposes. For the full calculations which are reported later in the paper we use the full Boltzmann code CMBFAST \[12\].

The temperature anisotropy $\Delta T \equiv \Delta T^2/T$ is expanded in plane waves and Legendre polynomials \[13,14\],

$$
\Delta T(x, n, \tau) = \int d^3 k e^{i k \cdot x} \Delta T(k, n, \tau) = \int d^3 k e^{i k \cdot x} \sum_l (2l + 1)(-i)^l \Delta T_l(k, \tau) P_l(k \cdot n).
$$

(1)

The Boltzmann equation must be solved for the photon phase space distribution to get $\Delta T_l(k, \tau)$. The observed sky is expanded in spherical harmonics $\Delta T(x = 0, n, \tau_0) = \sum_{lm} a_{lm} Y_{lm}(n)$, so to compare the theoretical predictions to observations it is convenient to calculate the angular power spectrum,

$$
C_{Tl} \equiv \langle |a_{lm}|^2 \rangle = (4\pi)^2 \int k^2 dk P_i(k) |\Delta T_l(k, \tau_0)|^2
$$

(2)

where $\langle \Delta T_l(k, \tau) \Delta T^*_l(k', \tau) \rangle = \delta(k - k') P_i(k) |\Delta T_l(k, \tau)|^2$, $P_i(k)$ is the primordial spectrum, $\Delta T_l(k, \tau)$ is the photon transfer function and $\tau_0$ is the conformal time today. Homogeneity and isotropy mean that these quantities depend only on the modulus of the wave vector $k$. It is important to note that the photon transfer function does not depend on the initial power spectrum.

The angular anisotropy spectra for polarization and the cross-correlation between temperature and polarization are defined similarly,

$$
C_{E l} = (4\pi)^2 \frac{(l - 2)!}{(l + 2)!} \int k^2 dk P_i(k) |\Delta E_l(k, \tau_0)|^2,
$$

$$
C_{Cl} = (4\pi)^2 \frac{(l - 2)!}{(l + 2)!} \int k^2 dk P_i(k) |\Delta T_l(k, \tau_0)\Delta E_l(k, \tau_0)|.
$$

(3)

The $\Delta E_l(k, \tau)$ are rotationally invariant quantities related to the $\Delta P_l(k, \tau)$ that appears in the Boltzmann equation by a spin lowering operator $\Delta E_l(k, \tau) = -\partial^2 \Delta P_l(k, \tau)$ \[13\].

The Boltzmann equations in Fourier space for the gauge-invariant scalar temperature and polarization anisotropies are \[1,16,13\]

$$
\Delta T + (ik\mu + \dot{\kappa}) \Delta T = -\dot{\Psi} - ik\mu \Psi + \dot{\kappa} (\Delta T_0 + i\mu V_b + \frac{1}{2} P_2(\mu)\Pi),
$$

$$
\Delta P + (ik\mu + \dot{\kappa}) \Delta P = \dot{\kappa} \left( \frac{1}{2} (1 - P_2(\mu))\Pi \right).
$$

(4)

where $\mu = k \cdot n$, $\kappa$ is the optical depth, $\Psi$ and $\Phi$ are gravitational potentials, $V_b$ is the baryon velocity and $\Pi = \Delta T_2 + \Delta P_0 + \Delta P_2$. Since there is much less power in the polarization anisotropy than in the temperature, $\Pi \approx \Delta T_2$, and can be ignored in the temperature equation as the temperature monopole and baryon velocity dominate during scattering.
In order to understand the behaviour of the anisotropies, the visibility function $\delta e^{-\kappa}$ can be approximated by a delta-function at the time of recombination $\tau_s$. Solving the Boltzmann equation gives us

$$\Delta_T(k, \tau_0) \approx (\Delta_{T0} + \Psi)(k, \tau_0)(2l + 1)j_l(k(\tau_0 - \tau_*) - (l + 1)j_{l+1}(k(\tau_0 - \tau_*))) + \Delta_{T1}(k, \tau_*)(lj_{l-1}(k(\tau_0 - \tau_*)) - (l - 1)j_{l+1}(k(\tau_0 - \tau_*))) + (2l + 1) \int_{\tau_s}^{\tau_0} d\tau(\dot{\Psi} - \dot{\Phi})j_l(k(\tau_0 - \tau))$$

$$\Delta_{E1}(k, \tau_0) \approx \frac{3}{4}(2l + 1)\Delta_{T2}(k, \tau_*) j_l(k(\tau_0 - \tau_*))(k(\tau_0 - \tau_*))^2$$

There are three terms that come from the surface of last scattering. The first two terms contributing to the temperature perturbation are the first two multipoles $\Delta_{T0} + \Psi$ and $\Delta_{T1}$, which come from the intrinsic density fluctuations and velocity fluctuations on the last scattering surface respectively. These are created during the tight coupling epoch before last scattering, where the photons and baryons are coupled via Compton scattering. The Bessel functions act as projectors from fluctuations in $k$ on the last scattering surface to fluctuations in $l$ that we see today. The quadrupole $\Delta_{T2}$ which produces photon polarization is created during last scattering, as the electrons and protons recombine.

The third term contributing to the temperature fluctuation is the integrated Sachs-Wolfe effect (ISW), where photons travelling towards us are perturbed by fluctuations in the gravitational field. Large angles are dominated by the ISW effect, small angles by the contributions made during scattering. Oscillations in the power spectrum are created on small scales (inside the horizon at last scattering) as shown below, so the ISW effect will not be relevant here.

With adiabatic initial conditions, on small scales during tight coupling the monopole and dipole terms are

$$\Delta_{T0}(k, \tau) \approx (1 + R)^{-1/4}\Delta_{T0}(k, 0) \cos(kr_s(\tau)),$$
$$\Delta_{T1}(k, \tau) \approx -\sqrt{3}(1 + R)^{-3/4}\Delta_{T0}(k, 0) \sin(kr_s(\tau)),$$

where $R$ is the ratio of baryons to photons normalised to 3/4 at equality and $r_s$ is the sound horizon. We can set $\Delta_{T0}(k, 0)$ to 1 for all $k$ as the initial conditions are specified in the initial power spectrum. The dipole (velocity contribution) oscillates out of phase with the monopole (density contribution), and with reduced amplitude. So the temperature anisotropy today, $\Delta_{T1}(k, \tau_0)$, is dominated by the density fluctuation at last scattering.

From the solution to the Boltzmann equation we can see that at times shortly after $\tau_*$ the quadrupole behaves like $\Delta_{T2} \sim \Delta_{T1}$, so the behaviour of the polarization anisotropy today is dominated by the velocity fluctuation during last scattering.

So for adiabatic initial conditions the temperature perturbations on the surface of last scattering are dominated by a cosine oscillation, and the polarization perturbations are dominated by a sine oscillation. The Bessel functions act as projectors from Fourier mode $k$ to multipole $l$, so for large $l$ we only need to consider large $k$ contributions to the angular power spectra to see their oscillatory structure,

$$C_{T1} \sim P_l(l/(\tau_0 - \tau_*)) \cos^2(lr_s(\tau_*)/(\tau_0 - \tau_*)),$$
$$C_{E1} \sim P_l(l/(\tau_0 - \tau_*)) \sin^2(lr_s(\tau_*)/(\tau_0 - \tau_*)),$$
$$C_{TE1} \sim P_l(l/(\tau_0 - \tau_*)) \sin(2lr_s(\tau_*)/(\tau_0 - \tau_*)),$$

for adiabatic initial conditions.

The oscillatory contributions to the $C_{0S}$ come only from the physics of the photon-baryon fluid, which is the same regardless of initial power spectrum. Therefore we may try to produce a non-oscillatory $C_{T1}$ spectrum with an oscillating primordial spectrum. We will show this in the next section.

We see that $C_{T1}, C_{E1}$ and $C_{TE1}$ all oscillate with the same frequency, but $C_{T1}, C_{E1}$ are half a period out of phase and $C_{TE1}$ is a quarter of a period out of phase with both $C_{T1}$ and $C_{E1}$.

Therefore we cannot erase oscillations in more than one of these spectra at a time simply by changing the initial power spectrum of perturbations. If we produce a non-oscillatory spectrum of temperature perturbations, we must necessarily produce enhanced oscillations in the polarization and temperature polarization cross-correlation spectra. The question arises, however, of whether these enhanced oscillations would be observable. We look at this in section VI.

The effect on the matter power spectrum is much simpler. As we look at this in Fourier space and we are only considering linear behaviour, the Fourier modes do not couple and we can relate the spectrum today directly to the primordial spectrum with a transfer function.
\[ P_f(k) = T^2(k)P_t(k), \]  

(9)

where \( P_f(k) = |\delta_f(k)|^2 \) is the power spectrum today.

Transfer functions for all species fall off for \( k > k_{eq} \) as perturbations inside the horizon decay during the radiation dominated era. The baryon transfer function has oscillations inside the horizon before and during recombination, due to the oscillations in the photon-baryon fluid as described above, while the CDM transfer function is smooth since CDM interacts only gravitationally. After recombination however, the baryons stop interacting with the photons, effectively have no pressure, and only interact gravitationally, falling into the gravitational wells created by the CDM. The CDM and baryons therefore end up with the same transfer function today. In models with low baryon density the transfer function is smooth today, for models with high baryon density the oscillations may not have disappeared by the present day. In general, there could also be a hot or warm component to the dark matter, and these would also tend to damp the oscillations. For models in which the present day transfer function has no oscillations, oscillations in the primordial spectrum would still be present today. If the transfer function still has oscillations, eg due to a large baryon fraction, we would expect them to be at least partially cancelled by the primordial oscillations. In Section VI we calculate the present day spectrum and discuss whether these oscillations are observable.

We note that while we focus on the standard adiabatic picture here, one could construct a very similar discussion for the isocurvature case. Although specific aspects of the oscillations are different for isocurvature perturbations, the tendency for the fluid to oscillate and the resulting impact on the different observations is similar. In fact, a more general analysis would include a combination of all possible perturbation modes, as was done recently in [19]. It is clear from the results in [19] that the differentiating power of the polarization is present even when general mixtures of perturbation modes are considered. Introducing a wider fitting parameter space will naturally require more precise data to pin down the parameters, but we expect the main thrust of our analysis to persist under a more general multi-mode treatment.

### III. INITIAL POWER SPECTRUM

There has been much interest recently in models of inflation which produce a broken-scale-invariant spectrum of perturbations. There is a model which produces oscillations in the primordial spectrum from an oscillating inflaton field [17,18], and there has been work on models with two fields in a potential with a step, producing oscillations in the spectrum below some scale [20,21]. Also multiple periods of inflation could produce oscillations in the spectrum [22].

We have chosen not to use a specific model to produce the initial spectrum, but to use a spectrum which completely cancels oscillations in the temperature spectrum created during the tight coupling era, as a worst case scenario, as this would be most confused with active models. The two models in this paper are examples for illustration of the ideas and analysis of how well they fit the data. They are intended to demonstrate some general features of this type of model. While Barrow and Liddle [23] argue that it might not be possible to construct a realistic inflaton that can produce the sort of primordial spectrum we discuss here, they only consider single-dimensional inflation space. If the data really do not show oscillations it is quite likely that some inflation enthusiast will find a model that tries to evade the secondary oscillation test. In any case our discussion is valid for the more general question passive vs. active perturbations.

We have looked at two models: one with a cosmological constant and a low value of \( H_0 \) (Model 1) and one with a higher Hubble constant and a tilted initial spectrum (Model 2). Model 1 has \( \Omega_b = 0.061, \Omega_m = 0.32, \Omega_\Lambda = 0.68 \) and \( h = 0.47 \). Model 2 has \( \Omega_b = 0.05, \Omega_m = 0.3, \Omega_\Lambda = 0.7 \) and \( h = 0.7 \). Figure 1 shows the initial spectra with oscillations for the two models. Also shown are non-oscillatory spectra for comparison.
IV. TEMPERATURE ANISOTROPY SPECTRUM

First we will compare the models to existing data on the microwave background. We have chosen to look only at high $l$ since the spectra agree at low $l$. Figure 2 shows the usual and modified $C_T l$ spectra for the two models, with all the data points currently available with an effective $l$ of over 180. This includes Saskatoon [24], MSAM [25], Python [26,27], MAT [28], CAT [29], OVRO [30], SUZIE [31], Boomerang [1] and MAXIMA [10] data.

In order to compare the predictions from the models to experimental data we use the package Radpack [32]. This calculates the $\chi^2$ between data and model in band powers, using a transformation of the $C_l$s into approximately Gaussian distributed quantities [33].

Model 1 with a scale invariant initial spectrum has a $\chi^2$ per degree of freedom of 2.9 and the same model with an oscillating spectrum has $\chi^2$ per degree of freedom of 3.8. Model 2 has $\chi^2$ per degree of freedom of 0.94 with a non-oscillatory initial spectrum and 0.95 with an oscillating primordial spectrum, so fits the data very well. Introducing oscillations into the primordial power spectrum does not alter the level of fit appreciably. Clearly we cannot say anything yet about the presence or absence of oscillations.

Future observations should be able to tell if there are oscillations in the CMB temperature spectrum. Figure 3 shows errorbars (including the beam and uniform, uncorrelated noise, assuming foregrounds have been removed) predicted for the PLANCK satellite for $C_T l$ spectra with non-oscillatory spectra. Also shown are the spectra produced by an oscillatory primordial spectrum.
The errorbars are calculated in the standard way, assuming that the $a_{lm}$ are Gaussian distributed and that there is spatially-uniform, Gaussian noise \[ \sigma_{lm} \approx \frac{1}{\sqrt{8 \ln 2}} \] \[ \omega_{T} B_{T}^{2} = \sum_{c} \omega_{T,c} B_{T,c}^{2}, \quad \omega_{T,c} = \text{variance of the noise in the temperature measurement in channel } c, \quad B_{T,c} = \text{variance of the error in channel } c. \]

The angular resolution of the beam is given by $\sigma_{c} = \theta_{fwhm,c}/\sqrt{8 \ln 2}$ where $\theta_{fwhm}$ is the full width at half max of the beam in radians. Incomplete sky coverage is partially taken into account by the factor $f_{sky}$ which is the fraction of sky used to estimate the $C_{ll}$. For PLANCK we have used $\omega_{T}^{-1} = (0.011 \mu K)^2$, combining the 143 GHz and 217 GHz channels at full width half max of 8.0' and 5.5' respectively, and $f_{sky} = 0.33 \quad [38]$. For the MAP satellite \[ \omega_{T}^{-1} = (0.106 \mu K)^2, \] with the highest frequency channels (40 GHz, 60 GHz and 90 GHz) at full width half max of 0.47°, 0.35° and 0.21° respectively and $f_{sky} = 0.66$. The estimates of the errors are averaged in arbitrary bins to make the plots clearer.

In order to determine whether or not future experiments will be able to distinguish between two models we need a different measure from the $\chi^2$ statistic used to compare theory with observation. Since now we have two models which we wish to compare on an equal footing, we need a statistic that is symmetric in the two theories. We consider simulating $\hat{C}_{ll}$ from each theory with mean $C_{ll}$ and standard deviation $\Delta C_{ll}$. We use a Normal distribution to approximate the distribution of the $\hat{C}_{ll}$s since we are only interested in the higher Is.

There is a standard way to test if two samples have the same underlying mean: if $C_{ll}^{-1}$ and $\hat{C}_{ll}^{-1}$ are both Gaussian-distributed with the same mean but possibly different variances then their difference will be Gaussian-distributed with mean zero and variance $\left(\Delta C_{ll}^{-1}\right)^2 + \left(\Delta \hat{C}_{ll}^{-1}\right)^2$. So for each $l$ we should use the statistic

$$\chi^2(l) = \frac{(\hat{C}_{ll}^{-1} - C_{ll}^{-1})^2}{\left(\Delta C_{ll}^{-1}\right)^2 + \left(\Delta \hat{C}_{ll}^{-1}\right)^2}$$

and sum them over $l$.

In fact we can use this idea to give a measure of how likely it is to confuse two models without performing simulations. The expectation of $\chi^2(l)$ is

$$d_l = 1 + \frac{(\hat{C}_{ll}^{-1} - C_{ll}^{-1})^2}{\left(\Delta C_{ll}^{-1}\right)^2 + \left(\Delta \hat{C}_{ll}^{-1}\right)^2}. \quad (11)$$

We will interpret the sum over the $d_l$ terms in a $\chi^2$ distribution, in order to find a probability with which the two theories could be confused. We define $d$ to be the average of the $d_l$. Note that if the two theories are exactly the same, $d = 1$ as expected. We can use this measure to see how similar two theories are ideally, that is in the limit of no instrumental effects. To look at how well a particular experiment will distinguish the theories we put the noise, beam and fractional sky coverage effects into the spread in the $C_{ll}$s, the $\Delta C_{ll}$ as described above.

Comparing the Model 1 $C_{ll}$ spectra with and without oscillations, using the MAP noise and beam parameters, gives a value for $d$ of 2.6, which if interpreted as a $\chi^2$ per degree of freedom would mean that confusing the two models is ruled out at a $\sigma$ level of 43. For Model 2 $d$ is 1.2, which corresponds to a $\sigma$ level of 3.9. The Model 2 spectra are harder to distinguish, since having a tilted initial spectrum suppresses the oscillations in the $C_{ll}$s. The PLANCK satellite will be more discriminating: with PLANCK characteristics $d = 4.8$ for Model 1 and 5.0 for Model 2.

FIG. 3. Temperature anisotropy spectra for models with and without oscillations shown with predicted errorbars as measured by PLANCK. Left: Model 1, right: Model 2
Attempts have been made [39–41] to estimate the effect of foregrounds on our ability to measure the angular power spectrum. We can use the degradation factors for the temperature error bars in [41] to calculate $d$ for the satellites taking into account their foreground models. The degradation factors $D_l$ increase the errorbars, $\Delta C^F_G = D_l \Delta C_l$. We hope that the estimates for $D_l$ given in [41] would not vary too much with different underlying cosmological models. We use an upper limit $D$ for the $D_l$ to give some room for variation.

This gives us

$$d^{FG} = 1 + (d - 1)/D^2$$

For the main foreground scenario in [41] $D_l \leq 1.01$ for all $l$ values and both satellites which means that $d$ is essentially unchanged. For their more pessimistic model $D_l \leq 2$ which leads to $d^{FG} \geq 2.0$ for PLANCK for both models, and $d^{FG} \geq 1.4$ and $d^{FG} \geq 1.0$ for MAP for Model 1 and Model 2 respectively. MAP will not be able to tell whether or not a tilted model has oscillations, but PLANCK can distinguish in both cases we consider here.

To get the above values for $d$, we have not used $l$ values below 50, as $C_l$ are not Gaussian distributed for low $l$. The models we are interested in do not differ at those $l$, so $d$ would decrease if we included them, but not enough to make a difference to the conclusions we draw. We have calculated each $d_l$ separately, without binning, but this would make errorbars smaller and therefore $d$ even bigger.

V. MATTER POWER SPECTRUM

In this section we look at how the altered primordial spectrum affects the present day matter power spectrum and whether these changes are observable. We would like to compare the model predictions for the matter power spectrum with current data, to see what we can already say about models with oscillations in the primordial spectrum. In the currently favoured models the transfer functions are smooth, and any features in the initial spectrum would show up in the present day matter power spectrum. For such models the power spectrum of the matter density field is the best probe of possible oscillations in the initial spectrum.

![Figure 4](image-url)  
**FIG. 4.** Present day matter power spectrum from the models with oscillations in the primordial spectrum, shown with data from [42]. Left: Model 1, right: Model 2.

Figure 4 shows the matter power spectrum from the models with and without oscillations and data from [42]. We use the decorrelated linear spectrum which has effectively uncorrelated points. It is important to have independent points since oscillations would be smoothed out in correlated data. We have calculated the $\chi^2$ for our models binned in the same way as the data. For Model 1 with a scale invariant initial spectrum the $\chi^2$ per degree of freedom is 3.2 and with oscillations in the initial power spectrum it is 3.4. For Model 2 with oscillations the $\chi^2$ per degree of freedom is 1.2 with and without oscillations. This model fits the data well but we cannot say whether or not the spectrum has oscillations.

We can look at averaged values, such as $\sigma_8$. For Model 1 this is 0.53 and 0.56 without and with oscillations in the primordial spectrum, normalised to COBE. For the Model 2 the values are 0.68 and 0.74 without and with oscillations. We see that oscillations in the spectrum have very little effect on the value of $\sigma_8$. We would expect this, as we have not changed the overall tilt of the spectrum.

It does not seem that the current data on the matter power spectrum can tell us much about the presence or absence of oscillations as we have discussed here. In order to draw conclusions about this we need data points with
a smaller coherence length. We hope that new data from redshift surveys will constrain the power spectrum much more than has been possible so far \cite{43}.

One possible complication would be if the preferred model turns out to have an oscillatory matter transfer function (e.g., because the matter has a high proportion of baryons). Even in that case, with good enough data it might be possible to tease apart the different effects.

VI. POLARIZATION IN THE MICROWAVE BACKGROUND

As we saw in Section \ref{sec:pol}, the power in the polarization anisotropy oscillates out of phase with that of the temperature anisotropy, so the extra power on certain scales in the initial spectrum leads to enhanced polarization oscillations. Here we show examples for our models. Note that active models have completely different polarization spectra so there is no opportunity for confusion \cite{44,45}.

Figure 5 shows the angular power spectra for polarization for the two models using a scale invariant primordial spectrum with 1σ errors \cite{15} expected from the PLANCK satellite. The parameters used are the same as for the temperature spectra earlier.

The figures show clearly the enhancement of the polarization spectrum. To see if these differences are observable we have used the same measure $d$ as in Section \ref{sec:pol} to compare two models. Using errorbars predicted for PLANCK $d = 6.5$ for Model 1 and 2.8 for Model 2. For MAP the values are 1.01 and 1.00 respectively. So we would not consider these two curves different under MAP observations, but PLANCK should be able to distinguish them.

In the main (supposedly most realistic) model of polarization foregrounds from \cite{41} the degradation factor $D_I$ for errorbars on the angular polarization spectrum is less than 2 for PLANCK at all $l$, which gives $d_{FG}^G \geq 2.4$ for Model 1 and $d_{FG}^G \geq 1.5$ for Model 2. So PLANCK will be able to tell these models apart even in the presence of this level of foregrounds. The more pessimistic model gives $D_I < 10$ for PLANCK at $l > 50$, giving $d_{FG}^G > 1.06$ for Model 1 and $d_{FG}^G > 1.02$ for Model 2, so if the foregrounds are this bad even PLANCK will not separate these spectra.

Figure 6 shows the temperature-polarization cross-correlation for the various models. Here again we can see the shift in oscillations produced by altering the primordial spectrum. Looking at the cross-correlation power should be more valuable than the polarization on its own, for two reasons. The first is that the noise begins to dominate at higher $l$ for temperature than for polarization, so we will get a clearer signal for the cross-correlation than for the polarization alone. Using errorbars predicted for PLANCK, $d = 5.0$ for Model 1 and 3.2 for Model 2. For MAP the values are 1.09 and 1.01 respectively. These are too low to allow MAP to distinguish between the models, but PLANCK would not be confused. We see that for these models the cross-correlation is in fact not such a good discriminator between models with and without oscillations in the temperature spectrum. This is because although the errorbars are smaller, the models are closer together. The effect of oscillations in the primordial spectrum is clearer in the present day polarization spectrum than in the temperature-polarization correlation since in the polarization the oscillations are completely out of phase, but in the cross-correlation they are only half a period out of phase.

There are no estimates of the degradation factor for the cross-correlation in \cite{41}. We make a rough estimate using the fractional errors on $C_{l}^{TT}$ with foregrounds obtained by \cite{39}. These give degradation factors slightly larger than the main foreground scenarios in \cite{41} for the temperature and polarization spectra as found be PLANCK. We find

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig5a}
\includegraphics[width=0.45\textwidth]{fig5b}
\caption{Angular polarization power spectra for models with and without oscillations, with errorbars forecast for the PLANCK satellite. The errorbars are shown for the models with scale-invariant initial spectra. Left: Model 1, right: Model 2}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig6a}
\includegraphics[width=0.45\textwidth]{fig6b}
\caption{Temperature-polarization cross-correlation for various models. Left: Model 1, right: Model 2}
\end{figure}
\( d^{FG} \gtrsim 1.6 \) for Model 1 and \( d^{FG} \gtrsim 1.4 \) for Model 2, hence we estimate that with this level of foreground contamination the PLANCK data should discriminate between these models with and without oscillations.

The second reason for looking at the cross-correlation power is that a strong prediction for active models is that due to their incoherence the cross-correlation in models where active sources play the major role in structure formation will be close to zero. We have calculated \( d \) for the cross-correlation using the models with oscillations in the primordial spectrum and a 'model' where all the \( C_{TE} \) are zero, to see if the cross-correlation could be consistent with zero. The value of \( d \) for Model 1 is 15.4, and for Model 2 the value is 13.5. We would not consider either of these cross-correlation spectra to be consistent with a vanishing spectrum. Therefore an observation of a smooth temperature anisotropy spectrum and vanishing cross-correlation would be strong evidence against passive fluctuations.

VII. DISCUSSION

We have investigated the secondary oscillation test, which is recognized as the most fundamental way in which scenarios with perturbations from inflation can be falsified. While current data are not good enough for the test to be applied, we look toward the future when much better data will be available. We have studied particular situations where the inflationary model conspires to evade the test by building oscillations into the inflaton potential to effectively ‘cancel out’ the natural oscillations that would appear in the CMB anisotropies. We have shown that such conspiratorial models will produce the characteristic oscillations in other observable spectra, and thus will be unable to evade the test.

One might think that reionization could smooth the peaks of the angular spectra and confuse us, but reionization also produces a different signal in the polarization. Reionization damps the temperature perturbations on all scales smaller than the sound horizon at the start of the reionization. Homogeneous reionization causes approximately exponential damping in \( l \) and so cannot alter the oscillations in the spectrum. In order to do this one would have to have damping starting on smaller scales than the sound horizon at recombination; clearly the reionization scale is larger than that of recombination. Reionization damps the polarization spectrum as well but also produces more polarization on large scales due to rescattering at the second ‘last scattering surface’, resulting in a boost at low \( l \) in the polarization angular spectrum. The oscillations in the spectra might be washed out enough to be unobservable if the optical depth to reionization is large enough, but increasing optical depth increases the extra power in the polarization at low \( l \) \([10]\), providing a strong indication of the presence of reionization. So inflationary perturbations with homogeneous reionization could not evade the secondary oscillation test.

The situation might be different in non-standard reionization where the visibility function is broader, but this causes a large amount of damping and is probably ruled out already by the temperature observations. In inhomogeneous reionization models the first order temperature spectrum comes entirely from the homogeneous reionization background. The second order effect due to the inhomogeneities is too small compared to the first order to make any difference to the peaks \([15,18]\).

Work has been done recently on models with both active and passive fluctuations \([19]\). The application of the fine tuning we have done here to those models represents an even more exotic corner of parameter space which we have not examined.
The matter power spectrum is affected by a number of issues (such as the baryon fraction) which make the interpretation of the presence or absence of oscillations less than clear-cut. However, we have shown that the polarization power spectrum and the temperature-polarization cross-correlation will offer a much more straightforward test, assuming the cosmic signal can be separated from the foregrounds. It is these spectra we are relying on to draw our optimistic conclusions.

There is no question that the models we do consider are viewed by most cosmologists (including ourselves) as extremely unnatural and fine-tuned. A very reasonable stand might be that good taste alone would require us to abandon an inflationary origin to the perturbations rather than embrace the conspiratorial models we study here. The main point of this work is that modern datasets will leave nothing to good taste. By the time the new data is in, even the exotic part of model space we explore here will be exposed to confrontation with observations.

ACKNOWLEDGEMENTS

We thank Ben Wandelt, Jochen Weller, Lloyd Knox and Hume Feldman for helpful discussions and Lloyd Knox, Andrew Hamilton and Max Tegmark for providing us with data points. This work was funded in part by PPARC, DOE grant DE-FG03-91ER40674 and UC Davis.
[35] L. Knox, Phys. Rev. D. **52** 4307 (1995).
[36] J. R. Bond, G. Efstathiou and M. Tegmark, M.N.R.A.S. **291L**, 33 (1997).
[37] http://astro.estec.esa.nl/Planck/
[38] http://map.gsfc.nasa.gov/
[39] F. R. Bouchet, S. Prunet and S. K. Sethi, M.N.R.A.S. **302**, 663 (1999).
[40] L. Knox, M.N.R.A.S. **307** 977 (1999).
[41] M. Tegmark, D. Eisenstein, W. Hu and A de Oliveira-Costa, Ap. J. **530** 133 (2000).
[42] A. J. S. Hamilton and M. Tegmark, astro-ph/0008392
[43] M. S. Vogeley, to appear in ‘Ringberg Workshop on Large Scale Structure’, ed. D. Hamilton
[44] U. Seljak, U.-L. Pen and N. Turok, Phys.Rev.Lett. **79** 1615 (1997).
[45] D. Spergel and M. Zaldarriaga, Phys.Rev.Lett. **79** 2180 (1997).
[46] M. Zaldarriaga, Phys. Rev. D **55**, 1822 (1997).
[47] A. Gruzinov and W. Hu, Ap. J. **508**, 435 (1998).
[48] L. Knox, R. Scoccimarro and S. Dodelson, Phys. Rev. Lett. **81**, 2004 (1998).
[49] R. Battye, and J. Weller, Phys. Rev. D **61** 043501 (2000); R. Battye, J. Magueijo and J. Weller astro-ph/9906093