Damping of long wavelength collective modes in spinor Bose-Fermi mixtures

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Using an effective field theory we describe the low energy bosonic excitations in a three dimensional ultra-cold mixture of spin-1 bosons and spin-1/2 fermions. We establish an interesting fermionic excitation induced generic damping of the usual undamped long wavelength bosonic collective Goldstone modes. Two states with bosons forming either a ferromagnetic or polar superfluid are studied. The linear dispersion of the bosonic Bogoliubov excitations is preserved with a renormalized sound velocity. For the polar superfluid we find both gapless modes (density and spin) are damped, whereas in the ferromagnetic superfluid we find the density (spin) mode is (not) damped. We argue quite generally that this holds for any mixture of bosons and fermions that are coupled through at least a density-density interaction. We discuss the implications of our many-body interaction results for experiments on Bose-Fermi mixtures.

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The interplay of bosons and fermions is ubiquitous throughout physics, ranging from the interaction of light (i.e. photons) and matter (i.e. electrons) to the behavior of a simple metal in the ionic lattice background. In solid state physics the interaction of electrons with slowly moving phonons provides the necessary attractive electron-electron interaction to form Cooper pairs leading to superconductivity [1]. In many itinerant magnetic systems, the interaction between electrons and magnons leads to observable effects. Another well-known example of the solid state manifestation of fermion-boson interaction is the polaron formation in ionic insulators. The itinerant nature of electrons can create an RKKY interaction between localized magnetic impurities [2] in solids. In the description of itinerant quantum phase transitions [3, 4] (relevant for the organic superconductors [5], heavy fermion materials [6], and the high temperature copper [7] and iron [8] based superconductors) the fermions are always coupled to a bosonic collective mode reflecting the ordering of the underlying Fermi gas. A coupled fermion-boson interacting many-body system is thus a fundamental paradigm in condensed matter physics leading to a large number of interesting phenomena.

Coupled fermion-boson systems have also become of interest recently in ultra-cold atomic systems. In the context of cold atoms, a variety of Bose-Fermi mixtures, e.g., $^6\text{Li}$-$^7\text{Li}$ [9, 10], $^{40}\text{K}$-$^{87}\text{Rb}$ [11, 12], $^6\text{Li}$-$^{133}\text{Cs}$ [13, 14], $^6\text{Li}$-$^{174}\text{Yb}$ [15], have been prepared in experiments for different purposes such as implementing sympathetic cooling [11], studying molecule formation [14], engineering dipolar quantum simulators [15], exploring few-body physics [16, 17] or looking for interesting collective excitations [18]. In a recent experiment [19] of particular relevance to our theory to be presented in the current work, laser spectroscopy was used to study the effect of the fermions on the bosonic excitation spectra of $^6\text{Li}$-$^{174}\text{Yb}$ atomic mixtures.

In the absence of fermions, the low energy excitations in Bose-Einstein condensates are well described by Bogoliubov theory and it is well understood that the Bogoliubov quasiparticles (BQs) are damped through higher order interactions in the form of Beliaev [20–23] and Landau [24] damping. Due to destructive quantum interference Beliaev and Landau process are suppressed at low momentum and low energies, thus making the long wavelength collective modes a well defined undamped bosonic excitation; in fact, the long-wavelength damping goes as $q^2$ vanishing rapidly as the wave number $q$ decreases. An important question of fundamental interest, which has also become relevant in view of recent experiments [19, 28], is, however, still open in spite of extensive theoretical activity, namely, how the fermion-boson interaction (specifically the existence of the Fermi surface) affects the bosonic excitation spectrum in a Bose-Fermi cold atom mixture. We address this important question in the current work using field theoretic techniques, finding a generic fermion-induced damping of the long wavelength bosonic collective modes.

We start with a microscopic Hamiltonian for a Bose-Fermi mixture, and derive a low energy effective field theory based on a controlled perturbative expansion, from which the low energy bosonic excitations and damping effects are obtained. A mixture of a spin-1 Bose gas and a spin-1/2 Fermi gas is considered and spin SU(2) symmetry is assumed for both theoretical simplicity and relevance to experimental systems in the absence of magnetic fields. We show quite generally that the linearly dispersive BQs of a bosonic superfluid interacting with fermions become damped due to Fermi surface effects, with a damping rate

$$\gamma_q/\hbar = D|q|,$$

where $D$ is dependent on the microscopic details of the system. The linear momentum dependence in Eq. (1) is drastically different from pure boson systems [20, 21, 24].

$\text{Li}$
We begin with the model fermion-boson interacting Hamiltonian, \( \mathcal{H} = \int d^3r (\mathcal{H}_B + \mathcal{H}_F + \mathcal{H}_{BF}) \), with

\[
\mathcal{H}_B = \frac{\hbar^2}{2m_B} \nabla \Phi_a^\dagger \nabla \Phi_a + \frac{U_0}{2} \Phi_a^\dagger \Phi_a \Phi_a^\dagger \Phi_a + \frac{U_2}{2} \Phi_a^\dagger \Phi_a \cdot \Phi_b^\dagger \Phi_b,
\]

\[
\mathcal{H}_F = \frac{\hbar^2}{2m_F} \nabla \sigma^\dagger \nabla \sigma,
\]

\[
\mathcal{H}_{BF} = U_{BF} \Phi_a^\dagger \Phi_a c_\sigma + J \Phi_b^\dagger T_{ab} \Phi_b \cdot c_\sigma + \tilde{\mathcal{H}}_{MF},
\]

where repeated indices are summed over, \( \mathbf{T} \) denotes a vector of spin-1 matrices, and \( \boldsymbol{\sigma} \) denotes a vector of Pauli matrices. The operator \( \Phi_a \) destroys a boson in the \( m_a = 0 \) state and \( c_\sigma \) destroys a fermion with spin \( \sigma \). The interactions \( U_0 \) and \( U_2 \) are related to the scattering lengths of each hyperfine state of the bosons through \( U_0 = (g_B^0 + 2g_B^F)/3 \) and \( U_2 = (g_B^B - g_B^F)/3 \), with \( g_f = 4\pi \hbar^2 a_f/m_B \) for a scattering length \( a_f \) in hyperfine state \( f \).

We focus on a repulsive density-density Bose-Fermi interaction \( U_{BF} > 0 \) and ferromagnetic spin-spin interaction \( J < 0 \). We assume the fermions to be non-interacting, which is valid for bare repulsive interactions because they are strongly irrelevant in the low energy limit.

In the absence of the Fermi gas, the spin-1 Bose gas can become either a FM superfluid for \( U_2 < 0 \) or a P superfluid for \( U_2 > 0 \), and we consider both situations theoretically. In the FM phase, the ground state breaks both the U(1) and SU(2) symmetry of the Hamiltonian and as a result hosts two distinct types of Goldstone modes corresponding to gapless density excitations which are of the BQ form that go as \( q | q \) and of the ferromagnetic spin wave form that go as \( q^2 \). The P superfluid also breaks the U(1) and SU(2) symmetry and also hosts two distinct sets of gapless density and spin wave excitations, however in this case they both take on the linear-in-\( \mathbf{q} \) BQ form.

We are interested in the low energy theory of the system and derive the corresponding effective action for the gas within a path integral framework. The model Hamiltonian corresponds to an action in the grand canonical ensemble \( S = \int dt \left( \mathcal{H}(\tau) + \int d^3r \left( \Phi_a^\dagger \partial_\tau \Phi_a + c_\sigma^\dagger \partial_\tau c_\sigma - \mu_B \Phi_a^\dagger \Phi_a - \mu_F c_\sigma^\dagger c_\sigma \right) \right) = S_B + S_F + S_{BF} \). We have introduced the Bose gas action \( S_B \), the Fermi gas action \( S_F \), and their mutual interaction \( S_{BF} \). To derive the low energy effective action of the bosonic superfluid we first expand the Bose operators about the mean field ground state \( \Phi^T = \Phi^T_M + \phi_1, \phi_0, \phi_{-1} \), which together with the mean field value of \( \mu_B \) gives rise to an effective action for the \( \phi \) degrees of freedom \( S_{B,\text{eff}}[\phi] \) (see the Supplemental Material).

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\[
S_{\text{eff}}[\phi] = S_{B,\text{eff}}[\phi] + \text{Tr}(\hat{V}(\phi)\hat{G}_0) + \frac{1}{2} \text{Tr} \left( (\hat{V}(\phi)\hat{G}_0)^2 \right).
\]

The first term after \( S_{B,\text{eff}} \) corresponds to the tadpole diagram in Fig. 1(a) and the second to the spinful particle-hole bubble Fig. 1(b). We find that the mean field value of \( \mu_B \) exactly cancels the contribution from the tadpole diagram, and only the fermionic bubble renormalizes \( S_{B,\text{eff}}[\phi] \).

**Ferromagnetic Ground State:** Focusing on the ferromagnetic case \( U_2 < 0 \) \((\Phi_{MF}^T \Phi_{MF}^T) = \rho_0 \hat{\mathbf{z}} \), where \( \rho_0 \) is the boson density), the mean field expectation value of the Bose operators can be written as a three component spinor, \( \Phi_{MF}^T = (\sqrt{\rho_0}, 0, 0) \). The chemical potential of the bosons at the mean field level is

\[
\mu_B^{FM} = g_B^B \rho_0 + J M_z + U_{BF} n_F,
\]

where \( M_z = (\langle n_\uparrow \rangle - \langle n_\downarrow \rangle)/2 \) and \( n_F = \sum_\sigma \langle n_\sigma \rangle \), with \( n_\sigma = c_\sigma^\dagger c_\sigma \). Expanding the Bose operators about their mean field ground state results in the fermions seeing an effective magnetic field \( \rho_0 \hat{\mathbf{z}} \) in the \( \hat{\mathbf{z}} \) direction and it breaks the up-down spin symmetry of the Fermi gas. This gives rise to the non-interacting Green function, \( \hat{G}_0^{-1} = -\delta_{\sigma,\sigma'}(\partial_\tau - \hbar^2/(2m_F))\nabla^2 - \mu_F + J \rho_0 \sigma / 2 \), where \( \sigma = \pm 1 \).
for $\uparrow$ and $\downarrow$, and we have shifted the fermionic chemical potential $\mu_F = \mu_B - \rho_0 U_{BF}$. The matrix $V_F$ is given by

$$V_{FM}(\phi) = \left( J(\sqrt{\rho_0/2}\phi_n^0 + \frac{1}{2} S^z_0)\sigma_+ + \text{h.c.} \right)$$

$$+ \left( U_{BF}(\sqrt{\rho_0}\phi_0^1 + \phi_0) + J(\sqrt{\rho_0}\phi_1^1 + \phi_1) + S^z_0 \right)\sigma_z,$$

where $I$ denotes the identity matrix, $\sigma_i$ are the Pauli matrices, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$, and we have introduced $n_\sigma = \sum \phi_i\phi_{i\sigma}$ and $S^z_\sigma = \phi_i^0 T^z_{\sigma\sigma} \phi_i$.

Evaluating the trace for the particle-hole bubble leads to $\chi^\alpha_{\sigma\beta}(q) = \int dk G_{0\alpha}(k + q) G_{0\beta}(k)$, where we are using the shorthand notation $q = (\nu_F q, \phi)$ for bosonic Matsubara frequency, $k = (\omega_n, k)$ for fermionic Matsubara frequency, and the integral $\int dq \equiv \int \frac{d^2q}{(2\pi)^2} \sum \sigma$. The Green function in momentum space is $G_{0\sigma}(k, i\omega_n)^{-1} = \omega_n - \epsilon_\sigma + \mu_F$, with a spin dependent dispersion $\epsilon_\sigma = \hbar^2 k^2/(2m_F) + \rho_0 \sigma\delta/2$. For the spin diagonal case $\alpha = \beta$ this reduces to the well known Lindhard function \[15\], which at sufficiently low temperature in the low energy limit, $|q|/k_F \ll 1$ and $\nu_F/(\hbar v_F |q|) \ll 1$ (where $v_F$ is the Fermi velocity of the spin $\sigma$ electrons), becomes

$$\chi^0_{\sigma\sigma}(q, i\nu_n) = -\eta^2 \left( 1 - \frac{\pi}{2} \frac{\nu_n}{\hbar v_F |q|} - \frac{1}{3} \left( \frac{|q|}{2k_F} \right)^2 \right)$$

and we have defined the density states for spin $\sigma$, $\eta_\sigma = m_F k_F \sigma/(2\pi^2 \hbar^2)$. We stress the approximation $\nu_n/(\hbar v_F |q|) \ll 1$ is consistent while considering Bose-Fermi mixtures with $v_n, 0 \ll v_F$, where $v_n, 0$ is the sound velocity of the BO excitations in the absence of the Fermi gas. For the case $\sigma \neq \sigma'$, in the low frequency, low momentum limit we can always treat $\hbar^2 |q|^2/(2m_F) \ll |J| \rho_0$, $\hbar v_F |q| \ll |J| \rho_0$, and $|\nu_n| \ll |J| \rho_0$. Within this approximation we find

$$\chi^0_{\sigma\sigma}(q, i\nu_n) = \frac{1}{\rho_0 J} (|\nu_n| - (\nu_n)) \left( \tilde{\sigma} - \frac{i\nu_n}{\rho_0 J} \right)$$

$$+ \left( \hbar^2 n_\sigma/2m_F + \frac{\sigma}{5\rho_0 J} (\hbar v_F |q|^2 (\nu_n) - (\hbar v_F |q|^2 (\nu_n)) \right) |q|^2 / \rho_0 J.$$

With all of these results in hand we can now determine the effective action to quadratic order. Writing the action in separate parts we have $S^F_{\text{eff}}[\phi] = \int dq \left( L_\phi + L_{\phi_0} + L_{\phi_{-1}} \right)$, where

$$L_\phi = \phi^\dagger_0 q - i\nu_n + \frac{\hbar^2 |q|^2}{2m_B} \phi_0(1)$$

$$+ \left( \Delta^{FM}_1 + B^{FM}_1 \phi_0^2 + B^{FM}_0 \phi_1^2 \right) \frac{1}{2} \phi^\dagger_0(q) + \phi_1(-q)^2,$$

$$L_{\phi_0} = (-A^{FM}_0 i\nu_n + B^{FM}_0 |q|^2) \phi_0^0(q) \phi_0(0),$$

$$L_{\phi_{-1}} = \left( M - i\nu_n + \frac{\hbar^2 |q|^2}{2m_B} \right) \phi^\dagger_{-1}(q) \phi_{-1}(q).$$

In order to simplify the presentation we have introduced the constants $\Delta^{FM}_1$, $A^{FM}_1$, $B^{FM}_1$, $A^{FM}_0$, and $B^{FM}_0$ which are explicitly given in the Supplemental Material. Now that we have the Green function for each spin state from the effective action, we can determine the excitation of each mode from the poles \[34\]. A few remarks are in order: The $\phi_{-1}$ mode is gapped with a value $M = 2(\rho_0 |U_2| + M_s/|J|)$. In contrast, the spin wave excitations that correspond to $\phi_0$, are given by $\omega_{\phi_0}^{FM} = \hbar^2 q^2/(2m_B^*)$ where the coefficient is altered from $\hbar^2/(2m_B)$, and the spin excitations acquire a renormalized effective mass $\hbar^2/(2m_B^*) = B_0^{FM}/A_0^{FM}$, which is not a function of $U_{BF}$. Therefore, the renormalization of $M$ and $m^*$ is due entirely to the paramagnon excitations of the Fermi gas through the coupling $J$. The density modes ($\phi_0^1 + \phi_0^\dagger$) acquire damping through the additional contribution of $|\nu_n|/|q|$ which arises due to the particle-hole excitations of the Fermi gas (see Eq. \[20\] below). It is useful to note that the constants $A_1^{FM}$ and $B_1^{FM}$ are functions of both $U_{BF}$ and $J$. The explicit form of the damping rate $\gamma_0$ and dispersion $E(q)$ are given below after we discuss the P superfluids low energy theory.

**Polar Ground State:** We now discuss the polar ground state of the bosons when $U_2 > 0$, ($\langle \Phi^{FM}_T T \Phi^{FM}_T \rangle = 0$). In this case we can take the mean field expectation value to be $\Phi^{FM}_T = (0, \sqrt{\rho_0} \phi_0 + n_0)$. The matrix $V_F$ is defined as

$$V_F(\phi) = U_{BF}(\sqrt{\rho_0}\phi_0^1 + \phi_0) + J S^z \sigma_+ / 2$$

$$+ \left( J(\sqrt{\rho_0/2}\phi_0^1 + \phi_1) + \frac{1}{2} S^z \sigma_+ + \text{h.c.} \right).$$

At this point it is useful to compare this with the FM case. From the effective bosonic action $S_{B,\text{eff}}[\phi]$ in the P case we know that the density mode is related to $\delta n(q) = (\phi^\dagger_0(q)) \phi_0(-q)$, while the spin excitations are given by $\delta S(q) = (\phi^\dagger_0(q), \phi_0(-q)).$ As a result, it is quite natural that the density excitations only couple through $U_{BF}$ and the spin excitations through $J$. This is quite different from the case of the FM superfluid where the BOs are a combination of density and spin along the $z$ direction.

Following the same steps as before, we compute the trace over the square term in Eq. \[15\], only now the equal spin particle-hole bubble comes into play which is given by Eq. \[16\]. Due to the spin symmetry the Fermi gas parameters are now $\sigma$ independent. The effective action to quadratic order for the polar case is given by, $S^F_{\text{eff}}[\phi] = \int dq \left( \frac{1}{2} L_{\delta n} + L_{\delta S} \right)$, where $L_{\delta n} = \delta n(q)\delta n(q)^\dagger$ and $L_{\delta S} = \delta S(q)\delta S(q)^\dagger$. The bosonic Green functions can be written in terms of the free bosonic
FIG. 2: Damping rate in the FM case (a) and the density mode of the P case (b) as a function of $q/k_F$, where $k_F = (k_F + k_{F_1})/2$. We consider the Fermi gas to be $^6$Li with a density of $n_F = 10^{13}\text{cm}^{-3}$, for the Bose gas in the FM case we have considered $^{87}\text{Rb}$ and the P case $^{23}\text{Na}$, with a density $\rho_0 = 10^{15}\text{cm}^{-3}$. We have used scattering lengths of each taken from Ref. \[39\]. The value of $U_{BF}$ is given in the legend, while keeping the ratio $U_{BF}/J = U_0/U_2$ fixed. A complete numerical solution of Eq. (19) using the full energy and momentum dependent Lindhard function \[35\] gives results indistinguishable from the analytical results up to rather large wave number of $O(k_F)$. At large $q > k_F$ (not shown), the fermion-induced bosonic damping vanishes.

Green function and the self energy using Dyson’s equation

$$G_{a}^{\prime}(i\nu_n, \mathbf{q})^{-1} = G_{a}^{\prime}(i\nu_n, \mathbf{q})^{-1} - \Sigma_{a}(\nu_n, \mathbf{q}),$$

which are two by two matrices

$$G_{a}^{\prime}(i\nu_n, \mathbf{q})^{-1} = \begin{pmatrix} -i\nu_n + \frac{\hbar^2}{2m_B} \mathbf{q}^2 & 0 \\ 0 & -i\nu_n + \frac{\hbar^2}{2m_B} \mathbf{q}^2 \end{pmatrix},$$

and

$$\Sigma_{a}(i\nu_n, \mathbf{q}) = -\left( \Delta_{a}^{P} + \mathcal{A}_{a}^{P} \frac{\nu_n}{|\mathbf{q}|} + \mathcal{B}_{a}^{P} |\mathbf{q}|^2 \right) \left( \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right).$$

We give the explicit forms of $\Delta_{a}^{P}$, $\mathcal{A}_{a}^{P}$, and $\mathcal{B}_{a}^{P}$ in the Supplemental Material. In this case, the $\delta n$ constants obviously do not depend on $J$ and the $\delta S$ constants are independent of $U_{BF}$. Interestingly, in the polar case we find both BQ modes to be damped through the appearance of $\nu_n/|\mathbf{q}|$. We remark that the density mode for the FM case in Eq. (11) can also be converted to the above form.

BQ Excitation Spectrum and Damping: We have derived the effective field theory for both the ferromagnetic and polar bosonic ground states interacting with a spin-1/2 Fermi gas. In each case we find the Bogoliubov excitations acquire a self energy $\Sigma_{a}$. To diagonalize the action using the Bogoliubov transformation we find $u_k$ and $\nu_k$ to be functions of $\nu_n$. The generic form of the Green function for the BQs is given by Eqs. (16)-(18). To determine the dispersion and damping rate we find the poles of the bosonic Green function $G_{a}$ (with $a = \phi_1, \delta n, \delta S$) \[34\], after analytically continuing the Matsubara frequency to real frequency $\nu_n \to \omega + i0^+$, we solve the algebraic equation

$$\det G_{a}(\omega, \mathbf{q})^{-1} = 0,$$

to find the poles at $\hbar \omega = E_{\mathbf{q}} - i\gamma_{\mathbf{q}}$, where $\gamma_{\mathbf{q}}/\hbar \geq 0$ is the damping rate. We obtain

$$\gamma_{\mathbf{q},a} = \frac{\hbar^2}{2m_B} A_{a} |\mathbf{q}|,$$

(see Fig. 2) and find the Bogoliubov form remains $E_{\mathbf{q}}(\mathbf{q}) = \sqrt{v_{s,a} |\mathbf{q}|^2 + b_{a} \mathbf{q}^4}$, with the renormalized parameters

$$v_{s,a} = \sqrt{\frac{\hbar^2}{2m_B} \left( 2\Delta_{a} - \frac{\hbar^2}{2m_B} A_{a}^2 \right)},$$

$$b_{a} = \frac{\hbar^2}{2m_B} \left( \frac{\hbar^2}{2m_B} + 2\mathcal{B}_{a} \right).$$

It is useful to note each correction to the superfluid parameters in $E_{\mathbf{q}}(\mathbf{q})$ is of order $\rho_0$ and should therefore produce significant (and observable) quantitative effects. We have also done a complete numerical solution of Eq. (19) using the exact Lindhard function \[37\], which gives results indistinguishable from the analytical results shown in Fig. 2 up to a rather large wave number of $O(k_F)$. Thus, all BQ modes present in a spin-1 Bose gases coupled to a Fermi gas become damped at low energies and are thus no longer true collective modes even at long wavelengths!

A few comments about our main results shown in Eqs. (20)-(22) are in order: (i) Our results are valid to all orders in the Bose-Fermi coupling provided $v_{s,0} \ll v_F$ since all higher-order vertex corrections in the self-energy are negligible by virtue of Migdal’s theorem \[37\]; (ii) the fermion-induced bosonic damping being linear in $q$ strongly dominates any intrinsic bosonic Beliaev damping at long wavelength \[23\]; (iii) Eqs. (20) and (21) imply that the bosonic collective mode frequency vanishes without becoming overdamped if the damping becomes comparable or larger than the mode energy itself since both the frequency and the damping go linear in wave number; and finally, (iv) at large wave numbers, $q > k_F$, eventually the fermion-induced bosonic damping vanishes because the imaginary part of the Lindhard function \[35\] itself vanishes by energy-momentum conservation, but the intrinsic bosonic damping (e.g. Beliaev damping) becomes dominant at large values of $q$.

We are now in a position to provide a general argument for any spin-$S$ Bose gas coupled to a Fermi gas (either spinful or spinless) through at least a density-density interaction $U_{BF}n_Bn_F$. Under the reasonable assumption that the spin-$S$ Bose gas will condense in a particular channel $\alpha$ and host BQs (this can be a linear combination of the modes in each $m_\alpha$ state), the Bose-Fermi interaction becomes $\sqrt{\rho_0} (\phi_\alpha + \phi_\alpha^\dagger) c^\dagger$. Following the previous discussions, integrating out the fermions will produce damped BQ excitations in the $\alpha$ channel. Therefore, our results are valid for all Bose-Fermi mixtures in general, independent of the spin of the Bose gas.
Our theoretical predictions can be tested in ultra cold Bose-Fermi mixtures with weakly interacting fermions, through two-photon Bragg spectroscopy [27] that couples to the bosonic species. We expect the damping will give rise to a broadened Bragg line, i.e. an intrinsic line width, in the presence of fermions that will depend explicitly on energy-momentum as shown in Eqs. (20)-(22).

We have studied a spin-1 Bose gas coupled to a spin-1/2 Fermi gas in three dimensions. We have shown BQ excitations are damped at long wavelength as a result of particle-hole excitations of the Fermi gas while the functional form of the BQ dispersion is unchanged. We have argued this phenomenon should apply to Bose-Fermi mixtures in general independent of the spin structure of either species.

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Supplemental Material

In the supplemental material we give the explicit expressions for $S_{B,\text{eff}}$ and the equations defining the constants that we have introduced in the main text in terms of the parameters of the model. For the ferromagnetic case, the effective bosonic action including the bosonic chemical potential is

\[
S_{B,\text{eff}}^{\text{FM}}[\phi] = \int d\tau d^3r \left( \phi_a^\dagger \partial_\tau \phi_a + \frac{\hbar^2}{2m_B} \sum_a \nabla \phi_a^\dagger \nabla \phi_a + g_2 \rho_0 \frac{1}{2} (\phi_1^\dagger + \phi_1)(\phi_1^\dagger + \phi_1) + 2 \rho_0 |U_2| \phi_1^\dagger \phi_{-1} \right) \tag{S1}
\]

\[- \int d\tau d^3r \left( (J M_z + U_{BF} n_F) \left[ \sqrt{\rho_0} (\phi_1^\dagger + \phi_1) + \sum_a \phi_a^\dagger \phi_a \right] \right), \tag{S2}
\]

whereas for the polar case, the effective bosonic action takes the form

\[
S_{B,\text{eff}}^P[\phi] = \int d\tau d^3r \left( \phi_a^\dagger \partial_\tau \phi_a + \frac{\hbar^2}{2m_B} \sum_a \nabla \phi_a^\dagger \nabla \phi_a + \frac{U_0 \rho_0}{2} (\phi_0^\dagger + \phi_0)(\phi_0^\dagger + \phi_0) + U_2 \rho_0 (\phi_1^\dagger + \phi_{-1})(\phi_1^\dagger + \phi_1) \right) \tag{S3}
\]

\[- \int d\tau d^3r U_{BF} n_F \left( \sqrt{\rho_0} (\phi_0^\dagger + \phi_0) + \sum_a \phi_a^\dagger \phi_a \right). \tag{S4}
\]

To simplify the presentation of the main text we have defined the following constants for the ferromagnetic case

\[
\Delta_1^{\text{FM}} = \rho_0 g_2^P - \rho_0 \left( U_{BF}^2 + \frac{1}{4} J^2 \right) \sum_{\sigma} \eta_{\sigma} - \rho_0 U_{BF} J \sum_{\sigma} \sigma \eta_{\sigma}, \tag{S5}
\]

\[
A_1^{\text{FM}} = \rho_0 \left( U_{BF}^2 + \frac{1}{4} J^2 \right) \frac{\pi}{2} \sum_{\sigma} \eta_{\sigma} \frac{\hbar v_{F,\sigma}}{k^0_{F,\sigma}} + \rho_0 U_{BF} J \frac{\pi}{2} \sum_{\sigma} \frac{\eta_{\sigma}}{\hbar v_{F,\sigma}}, \tag{S6}
\]

\[
B_1^{\text{FM}} = \rho_0 \left( U_{BF}^2 + \frac{1}{4} J^2 \right) \frac{1}{12} \sum_{\sigma} \eta_{\sigma} \frac{\hbar v_{F,\sigma}}{k^0_{F,\sigma}} + \rho_0 U_{BF} J \frac{1}{12} \sum_{\sigma} \frac{\eta_{\sigma}}{k^2_{F,\sigma}}, \tag{S7}
\]

\[
A_0^{\text{FM}} = 1 + \frac{M_z}{\rho_0}, \tag{S8}
\]

\[
B_0^{\text{FM}} = \frac{\hbar^2}{2m_B} + \frac{\eta_F}{\rho_0} \frac{\hbar^2}{2m_F} + \frac{1}{10 \rho_0^2 J} \sum_{\sigma} (\hbar v_{F,\sigma})^2 n_{\sigma}. \tag{S9}
\]

For the polar case we have introduced the constants

\[
\Delta_{SS}^P = \rho_0 U_2 - \frac{1}{2} \rho_0 \eta J^2, \tag{S10}
\]

\[
A_{SS}^P = \frac{\pi \rho_0 \eta J^2}{4 \hbar v_F}, \tag{S11}
\]

\[
B_{SS}^P = \frac{\rho_0 \eta J^2}{2 \hbar v_F}, \tag{S12}
\]

\[
\Delta_{sn}^P = \rho_0 U_0 - 2 \rho_0 \eta U_{BF}^2, \tag{S13}
\]

\[
A_{sn}^P = \frac{\pi \rho_0 \eta U_{BF}^2}{\hbar v_F}, \tag{S14}
\]

\[
B_{sn}^P = \frac{\rho_0 \eta U_{BF}^2}{6 k_F}. \tag{S15}
\]