Nonreciprocal light transmission via optomechanical parametric interactions

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Nonreciprocal transmission of optical or microwave signals is indispensable in various applications involving sensitive measurements. In this paper, we study optomechanically induced directional amplification and isolation in a generic setup including two cavities and two mechanical oscillators by exclusively using blue-sideband drive tones. The input and output ports defined by the two cavity modes are coupled through coherent and dissipative paths mediated by the two mechanical resonators, respectively. By choosing appropriate transfer phases and strengths of the driving fields, either a directional amplifier or an isolator can be implemented at low thermal temperature, and both of them show bi-directional nonreciprocity working at two mirrored frequencies. The nonreciprocal device can potentially be demonstrated by opto- and elecro-mechanical setups in both optical and microwave domains.

Nonreciprocal devices, such as isolators and directional amplifiers, are essential in communication and signal processing, as they protect the signal source from disturbances by the measurements and extraneous noises. A typical way to achieve nonreciprocal transmission of the signal is to divide the signal into two branches and interfere differently for the forward and the backward propagation directions by a controlled phase difference \[1\] \[2\]. Conventional realization of optical and microwave non-reciprocity is built on addressing magnetic-field induced effects, which are hard to integrate on chips \[3\] \[5\]. A more suitable implementation of integrated devices can be realized with superconducting quantum circuits, where the strong Josephson nonlinearity and parametric pumping have been exploited to realize circulators, directional amplifiers \[2\] \[6\] \[8\] and isolators \[9\], but working at microwave frequencies.

Recently, optomechanics \[10\] becomes a new promising platform for realizing nonreciprocity via multi-path interference \[11\] \[17\]. In recent theoretical work, Metelmann and Clerk have proposed that any coherent interaction can be made directionally by reservoir engineering an auxiliary dissipative path \[18\]. The idea has found good implementations in both asymmetric three-mode and symmetric four-mode optomechanical setups, basing on which nonreciprocity in the optical or microwave domain has been demonstrated with two mechanical resonators each mediating both coherent and dissipative couplings \[16\] \[19\] \[20\]. In general, optomechanically induced directional amplification can be realized by including blue-sideband drive tones \[16\] \[21\] \[26\], and isolation \[19\] \[27\] \[29\] can be performed by using drive tones close to the red sidebands. Moreover, optomechanical setups can be used as a nonreciprocal transducer, interfacing microwave and optical elements \[14\] \[28\].

Inspired by the previous schemes, in this paper, we focus on optomechanical interactions inducing nonreciprocal transmission between two cavity modes where two mechanical oscillators (MOs) respectively mediate coherent and dissipative couplings on two transfer paths. Exclusive use of blue-sideband drive tone leads to a linearized Hamiltonian, involving only two-mode squeezing interactions. We find that the optomechanical setup can be configured as both a directional amplifier and an isolator, depending on the driving strengths and the global phase. In contrast to Ref. \[22\], where two optomechanical cavities are driven by blue-detuned lasers with coherent optical (mechanical) couplings, giving rise to a port-to-port net gain of 1 dB, the input and output ports in our scheme are not directly connected and the optomechanically induced net amplification can be larger than 11 dB. Remarkably, the working points for both the directional amplifier and the isolator do not appear at the cavity resonance. Indeed, due to the broken time-reversal symmetry, the system shows a bi-directional nonreciprocal response between two cavity modes at two different frequencies detuned from the probe resonance by sublinewidth. The scheme can potentially be implemented in both the microwave and optical domains, and be used for a microwave-to-optical transducer with the electromechanical systems.

The schematic diagram of the device is shown in Fig. 1(a). The setup consists of two cavity modes (with frequencies \(\omega_1\) and \(\omega_2\), and ring down rates \(\kappa_1\) and \(\kappa_2\), respectively) acting as input and output ports, each of which is coupled to two non-degenerate mechanical resonators (with motional frequencies \(\omega_{m,1}\) and \(\omega_{m,2}\), and intrinsic damping rates \(\gamma_1\) and \(\gamma_2\)). The Hamiltonian of the optomechanical system reads (\(\hbar = 1\))

\[
H = \sum_{i=1,2} \omega_i a_i^\dagger a_i + \sum_{j=1,2} \omega_{m,j} b_j^\dagger b_j \\
+ \sum_{i,j} g_{ij} a_i^\dagger a_i (b_j^\dagger + b_j),
\]

where \((a_i^\dagger, a_i)_{i=1,2}\) are the cavity field creation and annihilation operators, and \((b_j^\dagger, b_j)_{j=1,2}\) are the phononic oper-
FIG. 1. Implementation of nonreciprocal transmission. (a) Two cavity modes (denoted by $a_1$, $a_2$) independently couple to two mechanical oscillators (denoted by $b_1$, $b_2$) via optomechanical parametric interactions with the effective pairwise coupling strengths $G_{i,j}$ ($i, j = 1, 2$). $\varphi$ is the global phase introduced by the phase-correlated driving lasers. (b) Fourier driving scheme. The optomechanical cavities are driven at frequencies exactly on the blue motional sides of the mechanical modes.

The optomechanical cavities are respectively driven by two laser beams on the well-resolved blue sidebands ($\omega_{m,j} \gg \kappa_t, \gamma_j$) with the frequencies $\omega_i + \omega_{m,j}$, see Fig. 1(b), which induce two-mode squeezing interactions between the optical modes and the mechanical modes [10], in contrast to previously demonstrated optomechanical isolators that used exclusively drive tones close to the red sidebands [15] [19] [28]. For strong drivings, the cavity field $a_i(t)$ ($i = 1, 2$) can be decomposed into a coherent amplitude $\alpha_i(t)$ oscillating at pump frequencies and a fluctuation part $\delta a_i(t)$. We assume that $\min[|\omega_{m,j}|, |\omega_{m,j} - \omega_{m,2}|] \gg \max[|g_{i,j} \alpha_i(t)|]$, then the coherent parts can be approximately given by $\alpha_i(t) \approx \sum_{j=1,2} \alpha_{i,j} e^{-i \omega_{m,j} t}$, where $\alpha_{i,j} = |\alpha_{i,j}| e^{i \theta_{i,j}}$ are complex number with $\theta_{i,j}$ relying upon the phase of the corresponding laser pump. Using the standard semiclassical approach, we can obtain the linearized Hamiltonian with the fluctuation operators $\delta a_i(t)$ ($\delta b_j(t)$) [renamed as $a_i(t)$ ($b_j(t)$) for concision], which in the frame rotating with $H_0 = \sum_{i=1,2} \omega_i a_i^\dagger a_i + \sum_{j=1,2} \omega_{m,j} b_j^\dagger b_j$ reads

$$H_{\text{Lin}} = G_{11} a_1^\dagger b_1 + G_{21} a_2^\dagger b_1 e^{i \varphi} + G_{22} a_2 b_2 + G_{12} a_1 b_2^\dagger + H.c., \quad (2)$$

where $G_{ij} = |g_{i,j} \alpha_{i,j}|$ are the field-enhanced coupling strengths and the terms oscillating at frequencies close to $\pm (\omega_{m,1} - \omega_{m,2})$ and $2 \omega_{m,j}$ are neglected (i.e. the rotating wave approximation). Moreover, we have made gauge transformations to the operators $a_1 \rightarrow a_1 e^{i \theta_{12}}$, $a_2 \rightarrow a_2 e^{i \theta_{22}}$ and $b_1 \rightarrow b_1 e^{i (\theta_{11} - \theta_{12})}$, leaving only the global phase $\varphi = \theta_{21} - \theta_{22} - (\theta_{11} - \theta_{12})$ here, which can be addressed by using phase-correlated laser lights.

The quantum Langevin equations (QLEs) for the fluctuation operators are $\dot{\mu} = -M \mu + \sqrt{\Gamma} \mu_{in}$, where $\mu = (a_1^\dagger, a_2^\dagger, b_1, b_2)^T$, $\Gamma = \text{diag}(\kappa_1, \kappa_2, \gamma_1, \gamma_2)$,

$$M = \begin{bmatrix}
\frac{\kappa_1}{2} & 0 & \frac{\kappa_2}{2} & -i G_{11} \\
0 & \frac{\kappa_1}{2} & \frac{\kappa_2}{2} & -i G_{21} e^{-i \varphi} \\
\frac{G_{11}}{2} & \frac{G_{21}}{2} & 0 & -i G_{22} \\
G_{12} & G_{22} & 0 & \frac{\gamma_1}{2}
\end{bmatrix}, \quad (3)$$

and $\mu_{in} = (a_{1,\text{in}}^\dagger, a_{2,\text{in}}^\dagger, b_{1,\text{in}}, b_{2,\text{in}})^T$, with $a_{1,\text{in}}$ and $b_{j,\text{in}}$ being the zero-mean noise operators for the cavities and mechanical oscillators, satisfying the correlation functions $\langle a_{1,\text{in}}(t) \alpha_{1,\text{in}}(t') \rangle = 0$, $\langle (a_{1,\text{in}}(t) \alpha_{1,\text{in}}(t') \rangle = \delta(t - t')$, $\langle b_{j,\text{in}}(t) \alpha_{j,\text{in}}(t') \rangle = (n_{m,j} + 1) \delta(t - t')$, $\langle b_{j,\text{in}}(t) b_{k,\text{in}}(t') \rangle = (n_{m,j} - 1) \delta(t - t')$ with the thermal phonon number $n_{m,j} = \text{[exp}(h\omega_{m,j}/k_B T) - 1]^{-1}$ and negligible phononic thermal occupation. We then rewrite the QLEs in the frequency domain with the Fourier transformations $o^i(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} o^i(t) e^{i \omega t} dt$, leading to $\mu(\omega) = (M - i \omega I)^{-1} \sqrt{\Gamma} \mu_{in}(\omega)$, where $I$ is the identity matrix. Moreover, by combining the standard input-output relation $\mu_{out}(\omega) = \mu_{in}(\omega) - \sqrt{\Gamma} \mu(\omega)$ [10] [30] [31], we can obtain the spectrum of the output fields $S_{\text{out}}(\omega) = \langle s_{a_1,\text{out}}(\omega), s_{a_2,\text{out}}(\omega), s_{b_1,\text{out}}(\omega), s_{b_2,\text{out}}(\omega) \rangle^T$, which connect to the spectrum of input fields $S_{\text{in}}(\omega) = \langle s_{a_1,\text{in}}(\omega), s_{a_2,\text{in}}(\omega), s_{b_1,\text{in}}(\omega), s_{b_2,\text{in}}(\omega) \rangle^T$ by the transmissivity matrix $T(\omega)$:

$$S_{\text{out}}(\omega) = T(\omega) S_{\text{in}}(\omega), \quad (4)$$

where the matrix elements $T_{ij}(\omega) = |S_{ij}(\omega)|^2$ (with $S(\omega) = I - \sqrt{\Gamma} (M - i \omega I)^{-1} \sqrt{\Gamma}$) describe the scattering probabilities of the signal between the cavity $a_i$ and the cavity $a_j$ via the mechanical oscillators, and the noise correlations in the frequency domain have been included. Thus, the asymmetry of the transmission matrix, i.e. $T_{12}(\omega) \neq T_{21}(\omega)$, implicates nonreciprocity of the device.

To construct a dissipative coupling path for the two ports, we consider the MO 2 is strongly damped, with its damping rate $\gamma_2$ being much larger than the decay rates of the cavity modes (assumed to be equal $\kappa_1 = \kappa_2$...
\(\kappa_2 = \kappa\) and that of the MO 1 (i.e. \(\gamma_2 \gg \kappa, \gamma_1\)). We can then adiabatically eliminate the MO 2, which leads to dissipative coupling between the two cavity modes with the strength \(Q_2 = 2G_{12}G_{22}/\gamma_2\). As a result, the two cavities are coupled through both a coherent path via MO 1 \((a_1 \rightarrow b_1 \rightarrow a_2)\) and a dissipative path intermediated by MO 2 \((a_1 \rightarrow b_2 \rightarrow a_2)\). The coupled equations for \(a_1, a_2,\) and \(b_2\) become

\[
\dot{\mu}'(t) = -M'\mu'(t) + \sqrt{\Gamma'}\mu_{in}(t) - i\sqrt{\Lambda}B_{2,in}, \tag{5}
\]

with \(\mu'(t) = (a_1^+, a_2^+, b_1^+, b_2^+, \Gamma') \mu_{in}(t) = (a_1^+, a_2^+, b_1^+, b_2^+, \Gamma')^T\), \(\Gamma' = \text{diag}(\kappa, \kappa, \gamma_1, \gamma_1, \Lambda) = \text{diag}(\gamma_{1,2}, \gamma_{2,2}, 0\), \(B_{2,in} = (b_{2,in}, b_{2,in}, b_{2,in})^T\) and the coefficient matrix

\[
M' = \begin{bmatrix}
\kappa_{-1,2} & -Q_2 & -iG_{11} & 0 & 0 \\
-Q_2 & \kappa_{-2,2} & -iG_{21}e^{-i\varphi} & iG_{21} & 0 \\
iG_{11} & iG_{21}e^{i\varphi} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}, \tag{6}
\]

where \(\gamma_{1,2} = 4G_{12}^2/\gamma_2\) and \(\gamma_{2,2} = 4G_{22}^2/\gamma_2\) are corrections of the cavity decay rates induced by the strongly dissipative MO 2. Using the Fourier transform and the standard input-output relation again, we get the output vector in the frequency domain

\[
\mu_{out}'(\omega) = S'(\omega)\mu_{in}(\omega) - iL'(\omega)B_{2,in}, \tag{7}
\]

with \(S'(\omega) = \sqrt{\Gamma'}(M' - i\omega I)^{-1}\sqrt{\Gamma'} - I\) and \(L'(\omega) = \sqrt{\Gamma'}(M' - i\omega I)^{-1}\sqrt{\Lambda} \). It follows that the transmission coefficients between the two cavity modes read

\[
S_{12}'(\omega) = \frac{\kappa(Q_{1} + Q_{2})}{D(\omega)}, \quad S_{21}'(\omega) = \frac{\kappa(Q_{1} + Q_{2})}{D(\omega)}, \tag{8}
\]

with

\[
D(\omega) = \left[\frac{\kappa_{1,2}}{2} - i(\omega + \omega_{1})\right] \left[\frac{\kappa_{2,2}}{2} - i(\omega + \omega_{2})\right] - (Q_{1} + Q_{2}) (Q_{1} + Q_{2}). \tag{9}
\]

where we have introduced the frequency \(\omega\) dependent corrections induced by the coherent path for the cavity decay rates \(\kappa_{i,2} = \kappa - \gamma_{i,1} - \gamma_{i,2}\), the coupling strengths \(Q_{1,2} = 2G_{12}G_{22}\Sigma(\omega)e^{\pm i\varphi}\) with \(\Sigma(\omega) = (\gamma_1 - i\omega)^{-1}\), the damping rates \(\gamma_{i,1} = 4G_{12}^2/\Sigma(\omega)^2\gamma_2\) and the “optical spring” frequency shifts \(\omega_{i,1} = 4G_{12}^2/\Sigma(\omega)^2\omega_{2}\). One should note that the dissipative path by the MO 2 can not induce such frequency selective properties for the cavity transmission. Consequently, a non-reciprocal device (with \(|Q_{1} + Q_{2}| \neq |Q_{1} - Q_{2}|\) ) can not be realized at \(\omega = 0\) by engineering the global phase \(\varphi\), but intriguingly, the signal light may be directionally amplified or fully isolated for \(\omega \neq 0\), see the details later.

As the first example in Fig. 2(a), we show the scattering probabilities \(|S_{12}'(\omega)|^2\) and \(|S_{21}'(\omega)|^2\) in units of dB between the cavities as a function of \(\omega\) for \(\varphi = -1.25\pi\). We look for the regime where the device is configured as a directional amplifier with lossless nonreciprocal transmission from the opposite direction, i.e., \(|S_{12}'|^2 > 0\) dB and \(|S_{12}'|^2 = 0\) dB. Such a regime appears bijective around \(\omega/\kappa = \pm 0.081\), where one can achieve 11 dB of amplification and frequency-insensitive lossless transmission with respect to the flat local minimum around \(|S_{12}'|^2 = 0\) dB.

While the application of four blue-sideband drive tones enables amplification, the system stability should be examined carefully and can be checked by using the Routh-Hurwitz criterion \[31\], which imposes the conditions given by

\[
\sum_{j=1,2} 4(G_{1j}^2 + G_{2j}^2) - \gamma_1 \gamma_2 - 2\gamma_2 \kappa - 4\kappa^2 > 0, \tag{10}
\]

\[
\sum_{j=1,2} 4(G_{1j}^2 + G_{2j}^2)(\gamma_{2/j} + \kappa) - \gamma_1 \gamma_2 (2\gamma_1 + \kappa) > 0, \tag{10}
\]

\[
\frac{G_{1j}^2G_{21} + G_{11}^2G_{22}}{\gamma_1 \gamma_2 \kappa^2} - \sum_{j=1,2} \frac{G_{1j}^2 + G_{2j}^2}{4\gamma_j \kappa} + \frac{1}{16} > 0 . \tag{10}
\]

We have confirmed that the nonreciprocity shown in all figures can be realized under the stable parameters.

Second, the setup with exclusive use of blue-sideband drive tones can be alternatively configured as an isolator with lossless nonreciprocal transmission, i.e. \(|S_{12}'|^2 \rightarrow -\infty\) and \(|S_{21}'|^2 = 0\) dB in Fig. 2(b). Indeed, the absolute isolation from the cavity 2 to the cavity 1 can be obtained as \(Q_{1} - Q_{2} = 0\), giving rise to

\[
\frac{\gamma_1}{\gamma_2} = -\frac{G_{1j}G_{21}}{G_{1j}G_{22}}\cos \varphi, \tag{11}
\]

with \(\varphi = \arctan(2|\omega|/\gamma_1)\). Taking the stability of system into account, we then choose \(\gamma_1 = \gamma_2/16 = \kappa, G_{11} = G_{21} = 0.323\kappa, G_{12} = G_{22} = 1.198\kappa\). As such, the absolute isolation appears at the frequencies \(\omega = \gamma_1 \tan \varphi/2 \approx \pm 0.3\kappa, \) see Fig. 2(b), namely the isolation can be implemented bidirectionally at two working points with the probe detuning being \(\pm 0.3\kappa\).

Finally, the nonreciprocal device can be nicely configured as both an directional amplifier as well as an isolator for the transmission from the other direction, e.g.
In this regime, $G \vert \phi$ with the system approaching the boundary between the phase $\phi$ achievable gain under stable conditions. Moreover, we first show the forward gain $S_{\text{out}, a_2} (\omega)$ via the coherent and dissipative paths, respectively [21, 33, 34]. The added noise is then simply given by $N'(\omega) = S_{\text{out}, a_2} (\omega) / G - 1 / 2$, with $G = |S_{21} (\omega)|^2$. For the directional amplifier implemented as in Fig. 3(d), we find that the added noise is $N'(-0.1\kappa) = 4.35$ noise quanta in the absence of thermal mechanical noise, while for the directional amplifier as in Fig. 2(a), the added noise is $N'(\pm 0.081\kappa) = 8.46$ even for $n_{m, 1} = n_{m, 2} = 3$.

In conclusion, we have shown that directional amplifier and isolator can be realized in a single optomechanical setup involving two cavities and two mechanics exclusively drive tones on the blue sidebands. The directional amplification with absolute isolation in the reversal direction appears at the working points detuned from the resonance, which gives rise to bi-directional nonreciprocity at two mirrored frequencies for signal propagating from opposite directions. The amplifier gain can in principle be enhanced by increasing the strengths of the driving fields, and is however limited by the stability condition of the system. Such devices carry great promise for integrated nonreciprocal optical and microwave devices, as well as the interface between the two frequency domains.

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