Effect of a finite–time resolution on Schrödinger cat states in complex collisions

L. Benet\textsuperscript{a,b}, S.Yu. Kun\textsuperscript{a,c,d}, Wang Qi\textsuperscript{e}, V. Denisov\textsuperscript{f}

\textsuperscript{a}Centro de Ciencias Físicas, National University of Mexico (UNAM), Cuernavaca, Mexico
\textsuperscript{b}Centro Internacional de Ciencias, Cuernavaca, Mexico
\textsuperscript{c}Center for Nonlinear Physics, RSPhysSE, The Australian National University, Canberra ACT 0200, Australia
\textsuperscript{d}Department of Theoretical Physics, RSPhysSE, The Australian National University, Canberra ACT 0200, Australia
\textsuperscript{e}Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
\textsuperscript{f}Institute for Nuclear Research, 03680 Kiev, Ukraine

Abstract

We study the effect of finite time resolution on coherent superpositions of rotating clockwise and anticlockwise wave packets in the regime of strongly overlapping resonances of the intermediate complex. Such highly excited deformed complexes may be created in binary collisions of heavy-ions, molecules and atomic clusters. It is shown that time averaging reduces the interference fringes acting effectively as dephasing. We propose a simple estimate of the "critical" time averaging interval. For the time averaging intervals bigger than the critical one the interference fringes wash out appreciably. This is confirmed numerically. We evaluate minimal energy intervals for measurements of the excitation functions needed to observe the Schrödinger Cat States. These should be easily observable in heavy-ion scattering. Such an experiment is suggested.

Key words: Schrödinger Cat States; heavy–ion scattering; strongly overlapping resonances; slow spin phase relaxation; atomic clusters.
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Time dependence of the decay of an intermediate complex provides the most detailed information on the collisional molecular dynamics. In order to get such an information experimentally one needs to arrange for a “clock”. The clock should be switched on at the moment when the collision partners start
to interact. It should also fix the moment of the decay of the intermediate complex. For unimolecular reactions this problem was successfully solved using pump and probe laser pulses [1,2]. The pump pulse switches on the clock while the probe pulse determines if the intermediate complex has decayed or not. By changing continuously the time delay between the pump and probe laser pulses one measures the time dependence of the decay of the intermediate complex.

Unfortunately such a real-time monitoring of collisional dynamics cannot be applied for bimolecular collisions unless these are initiated by a laser pulse [1,2]. However it was shown [3,4] that measurements of the cross sections with pure energy resolution enable one to extract the information on the time and scattering angle intensity of the decay of the intermediate complex. This information is equivalent to that obtained in real-time pump/probe laser pulses experiments provided the relative contribution of the direct (fast) processes in the energy averaged cross section is appreciable. Then the energy dependence of the cross section is mainly determined by interference between energy smooth direct reaction amplitude and the fluctuating one corresponding to time delayed processes.

The possibility of formation of coherent superpositions of wave packets rotating clockwise and anticlockwise in the regime of strongly overlapping resonances of the intermediate complex has recently been suggested [5]. In this regime the rotational period is much shorter than the Heisenberg time of the quasibound complex. The wave packets are stable and do not spread appreciably due to the small width of spin off-diagonal phase relaxation as compared to the inverse rotational period and inverse average life-time of the intermediate complex. Such highly excited complexes may be created in binary collisions of heavy–ions, molecules and atomic clusters provided the double–folding potential between the collision partners has a pocket. The calculations support the existence of such pockets for heavy–ion [6] and atomic cluster [7,8] systems. When the relative kinetic energy of the colliding partners dissipates into intrinsic excitation they drop into this pocket, forming a highly excited intermediate complex.

It has been unambiguously demonstrated in double slit like experiments [9] that fullerene beams do indeed represent waves. This supports the conceptual similarity in the quantum mechanical treatment of atomic clusters, i.e. mesoscopic, collisions and heavy–ion collisions [10].

It should be noted that stable wave packets in highly–excited many–body systems have been identified and associated with the existence of invariant manifolds in the classical phase space of the system [11,12].

It can be seen from Fig. 1 of Ref. [5] that the angular period of the interference
fringes is about $\pi/I$, where $I$ is the average spin of the intermediate complex. In order to detect the effect experimentally, one must measure the cross section with angular resolution better than $\pi/I$. Panels (f) and (g) in Fig. 1 of Ref. [5] also provide an indication of a strong time dependence of the interference fringes. Therefore an unanswered question is of what time resolution $\Delta t$ is necessary to detect the effect. This question is of a crucial importance for planning the experiment, since this required time resolution determines a minimal energy interval $\Delta E = \hbar/\Delta t$ on which the cross section measurements must be carried out in order to resolve the interference fringes.

First we suggest a simple estimate for the time averaging interval $\Delta t_{cr}$ obtained from the required angular resolution, $\Delta \theta \approx \pi/I$, needed to resolve angular dependence of the interference fringes. The time it takes the intermediate complex to rotate by the angle $\pi/I$ is $\pi/I\omega = T/2I$, where $T = 2\pi/\omega$ is the period of rotation and $\omega$ is the angular velocity of the intermediate system. This suggests $\Delta t_{cr} \approx T/2I$. In what follows this simple estimate will be tested and confirmed numerically.

Following Ref. [5] we consider spinless collision partners in the entrance and exit channels. The time and angle dependent intensity of the decay, $P(t, \theta)$, has been obtained [5] by summing over very large number of strongly overlapping resonance levels, $t \ll \hbar/D$, where $t$ is the average life time and $D$ is the average level spacing of the intermediate complex. As a result $P(t, \theta)$ has the form:

$$P(t, \theta) \propto H(t) \exp\left(-t/\tilde{t}\right) \sum_{J,J'}(2J + 1)(2J' + 1)|W(J)W(J')|^{1/2} \exp\left[i(\Phi - \omega t)(J - J') - \beta|J - J'|t/\hbar\right]P_J(\theta)P_{J'}(\theta).$$

Here $H(t)$ is the Heaviside step function, $\beta$ is the spin relaxation width, $\omega$ is the angular velocity of the coherent rotation, $\Phi$ is the deflection given by the total spin ($J$) derivative of the potential (direct reaction) phase shifts, and the $P_J(\theta)$ are Legendre polynomials. The partial average reaction probability is taken in the $J$–window form, $W(J) =< |\delta S^{J}(E)|^2 > \propto \exp[-(J - I)^2/d^2]$, where $I$ is the average spin and $d \ll I$ is the $J$–window width. This means that we assume a peripheral character of the collision.

In order to study the effect of the time averaging on the interference fringes we average $P(t, \theta)$ in (1), with a constant weight, on the time interval from $t - \Delta t/2$ to $t + \Delta t/2$. This integration is elementary. Then we calculate the time averaged $P(t, \theta)$ numerically.

Like in Ref. [5], we calculate $P(t, \theta)$ with the set of parameters obtained from the description [3] of the experimental cross section energy autocorrelation functions [13] for $^{12}$C+$^{24}$Mg elastic and inelastic scattering at $\theta = \pi$ [14]. For these collisions the analysis [3] indicates a formation of a stable rotational
Fig. 1. (Color online) Time and angular dependence of decay intensity of the highly–excited intermediate complex for different time averaging intervals: (a) $t = T/4$; (b) $t = 7T/16$; (c) $t = T/2$; (d) $t = 3T/4$. $T$ is the period of one complete revolution of the complex. Black lines correspond to ideal time resolution $\Delta t = 0$; blue $\Delta t = \Delta t_{cr}$; magenta $\Delta t = 5\Delta t_{cr}$. The bigger $\Delta t$ the more the spread of the wave packets and the stronger reduction of the interference fringes.

wave packets in spite of strong overlap of resonance levels in the highly–excited intermediate molecule. The set of parameters is [3]: $\Phi = 0$, $d = 3$, $I = 14$, $\beta = 0.01$ MeV, $\hbar \omega = 1.35$ MeV, and $\bar{t} = 2.2 \times 10^{-21}$ sec. It should be noted that analysis of the angular distributions [14] supports a peripheral reaction mechanism, i.e. $d \ll I$, in the $^{12}$C+$^{24}$Mg elastic scattering.

In Fig. 1 we demonstrate the dependence of the quantity $AP(t, \theta)/<\sigma(E, \theta)>$ on the time averaging interval for the four moments of time. Here $<\sigma(E, \theta)> \propto \int_0^\infty dt P(t, \theta)$ is the energy average differential cross section for the time-delayed collision. In Fig. 1, $P(t, \theta)$ is scaled with $<\sigma(E, \theta)>$ for the reason discussed in Ref. [5]. The constant $A$ is derived from the condition $AP(t = 0, \theta = 0)/<\sigma(E, \theta = 0)> = 1$, where $P(t = 0, \theta = 0)$ is not averaged over the time.

From Fig. 1 one can see that an increase of the time averaging interval results in a reduction of the amplitude of the interference fringes.

In Fig. 2 we plot a ratio, $r = P^{\Delta t}(t = T/2, \theta)/P^{\Delta t=0}(t = T/2, \theta)$, of the time
Fig. 2. (Color online) Ratio $r = P^{\Delta t}(t = T/2, \theta)/P^{\Delta t=0}(t = T/2, \theta)$ of the time averaged $P^{\Delta t}(t = T/2, \theta)$ and the $P^{\Delta t=0}(t = T/2, \theta)$ corresponding to ideal time resolution. The time averaging has been performed around $t = T/2$ with $T$ being the period of one complete revolution of the complex. Black color is for ideal time resolution $\Delta t = 0$; red $\Delta t = t_{cr}$; green $\Delta t = 3t_{cr}/2$; blue $\Delta t = 2t_{cr}$. The minima of the curves reflect suppression of the peaks of the interference fringes (see Fig. 1) due to the time averaging. The maxima signify filling in of the interference deeps (see Fig. 1) due to the time averaging. The bigger the time averaging interval $\Delta t$ the greater deviation from the constant unity.
Thus, for the critical time averaging interval, the reduction of the interference maximum/minimum ratio is given by a factor of about 1.5. Since, for the ideal time resolution, the interference maximum/minimum ratio exceeds one order of magnitude (see Fig. 1), this ratio is still of about a factor 6 after the averaging over $\Delta t_{cr}$. In order to have the time resolution $\Delta t_{cr}$ one has to measure the cross section on the energy interval $\Delta E \simeq \hbar/\Delta t_{cr} = (I/\pi)\hbar\omega$. For the above considered $^{12}\text{C} + ^{24}\text{Mg}$ elastic and inelastic scattering [13,14] we have $I = 14$ and $\hbar\omega = 1.35 \text{ MeV}$ [3]. This yields $\Delta E \simeq 6 \text{ MeV}$. The measurements of the $^{12}\text{C} + ^{24}\text{Mg}$ elastic and inelastic scattering at $\theta = \pi$ [14] and the consequent extraction of the cross section energy autocorrelation functions [13] were performed on the energy intervals bigger than 10 MeV and 9 MeV, respectively. Clearly, the measurements of the excitation functions on such energy intervals for many backward angles $130^\circ \leq \theta \leq 180^\circ$ with a fine angular resolution $\Delta \theta \leq \pi/3I \simeq 4 - 5$ degrees and a fine angular step would allow to search for Schrödinger Cat States in $^{12}\text{C} + ^{24}\text{Mg}$ elastic scattering. Such an experiment would be desirable since the Schrödinger Cat States predicted [5] for $^{12}\text{C} + ^{24}\text{Mg}$ scattering involve $\sim 10^4$ many-body configurations of the highly excited intermediate complex. To the best of our knowledge, the internal interactive complexity of these quantum macroscopic superpositions dramatically exceeds [5] all of those previously experimentally realized.

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