Sheath-accumulating Propagation of Interplanetary Coronal Mass Ejection

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Abstract
Fast interplanetary coronal mass ejections (ICMEs) are the drivers of strong space weather storms such as solar energetic particle events and geomagnetic storms. The connection between the space-weather-impacting solar wind disturbances associated with fast ICMEs at Earth and the characteristics of causative energetic CMEs observed near the Sun is a key question in the study of space weather storms, as well as in the development of practical space weather prediction. Such shock-driving fast ICMEs usually expand at supersonic speeds during the propagation, resulting in the continuous accumulation of shocked sheath plasma ahead. In this paper, we propose a “sheath-accumulating propagation” (SAP) model that describes the coevolution of the interplanetary sheath and decelerating ICME ejecta by taking into account the process of upstream solar wind plasma accumulation within the sheath region. Based on the SAP model, we discuss (1) ICME deceleration characteristics; (2) the fundamental condition for fast ICMEs at Earth; (3) the thickness of interplanetary sheaths; (4) arrival time prediction; and (5) the super-intense geomagnetic storms associated with huge solar flares. We quantitatively show that not only the speed but also the mass of the CME are crucial for discussing the above five points. The similarities and differences between the SAP model, the drag-based model, and the “snow-plow” model proposed by Tappin are also discussed.

Key words: planets and satellites: magnetic fields – shock waves – solar–terrestrial relations – Sun: coronal mass ejections (CMEs) – Sun: flares – Sun: heliosphere

1. Introduction
Coronal mass ejections (CMEs) are the largest plasma explosions in the solar system, and are characterized by a vast amount (typically, $10^{13}–10^{16}$ g) of the solar coronal plasma being ejected out into the interplanetary space at speeds up to 3000 km s$^{-1}$ (Ilgø & Hundhausen 1986; Aarnio et al. 2012; Porfir’eva et al. 2012; Webb & Howard 2012).

CMEs propagating in interplanetary space are called interplanetary CMEs or ICMEs. When the magnetic fields of ICMEs observed in situ show continuous rotation, they are called magnetic clouds (Klein & Burlaga 1982). Fast ICMEs drive shock waves. The draped plasma compressed and accelerated by the leading shock is called an interplanetary sheath. The sudden jumps in energetic proton flux measured in situ associated with the passage of interplanetary shocks are called energetic storm particle (ESP) events (Bryant et al. 1962). On the other hand, when the magnetic clouds or interplanetary sheaths that arrive at Earth possess southward magnetic fields, they drive intense geomagnetic storms (Dungey 1961; Klein & Burlaga 1982; Tsurutani et al. 1988; Zhang et al. 2004).

One super-intense ESP event was recently recorded on 2012 July 23 by the STEREO-A spacecraft located 0.96 au from the Sun and 121° ahead of Earth at the time. The estimated peak $E > 10$ MeV proton flux measured in situ by STEREO-A was about $6.5 \times 10^{4}$ pfu when the interplanetary shock passed the spacecraft (Russell et al. 2013; Gopalswamy et al. 2014). Liu et al. (2014) carried out a comprehensive study of the ICME characteristics using stereoscopic observations and in situ measurements. The post-shock peak speed measured in situ at STEREO-A is about 2250 km s$^{-1}$. The recorded peak magnetic field strength is 109 nT, one of the largest interplanetary field strengths on record near 1 au. Liu et al. (2014) concluded that we would have had an extreme geomagnetic storm with a minimum DST index of $\sim -1150$ or $-600$ nT had the ICME arrived at Earth, based on two different empirical formulae. The causative CME resulted from the merger of two successive CMEs with a peak speed of about 3050 km s$^{-1}$ near the Sun. Also, a fast CME preceded the July 23 event by 4 days from the same solar active region, and the solar wind trailing the preceding CME had a density as low as 1 cm$^{-3}$. They discussed how both the “preconditioned” low-density upstream solar wind and the CME merger have played a crucial role in producing very fast ICMEs near 1 au, with extremely strong magnetic fields.

Taking into account the fact that ICMEs faster than 1000 km s$^{-1}$ at 1 au are extremely rare (Guo et al. 2010), the ICME of 2012 July 23 event had truly outstanding propagation characteristics. The fundamental condition for such a space-weather-impacting ICME at 1 au remains an issue of crucial importance for space weather research.

Recently, superflares (flares that are 10–1000 times more energetic than the largest ever observed solar flares) on solar-type stars were discovered with Kepler data (Maehara et al. 2012). The possible impacts of superflares on space weather and terrestrial environment have been vigorously discussed (Miyake et al. 2012; Schrijver et al. 2012; Shibata et al. 2013; Tsurutani & Lakhina 2014; Hayakawa et al. 2015; Airapetian et al. 2016; Takahashi et al. 2016).

A landmark for realistic space weather prediction is to connect the coronagraph observation of a CME near the Sun with expected ICME characteristics at 1 au, as well as their arrival time. As for the prediction of the arrival time of ICMEs at 1 au, various methods have been proposed (Dryer & Smart 1984; Smith & Dryer 1990; Cargill et al. 1996; Gopalswamy et al. 2000, 2005; Feng & Zhao 2006). One of the most typical models is the empirical CME arrival (ECA)
model proposed by Gopalswamy et al. (2000). The ECA model assumes “effective” constant deceleration (or acceleration) of CMEs in interplanetary space. Gopalswamy et al. (2001) introduced a deceleration cessation distance of 0.76 au, after which CMEs propagate with a constant speed in order to improve the predictability of the ECA model. The empirical shock arrival model that Gopalswamy et al. (2005) developed, and based on the combination of ECA model prediction and piston-driven shock propagation, predicts a 1 au arrival time for a leading shock front. On the other hand, the drag-based model predicts a 1 au arrival time of CME ejecta, using the deceleration of the CME by aerodynamic (or viscous) drag (Cargill et al. 1996; Vršnak 2001; Cargill 2004; Vršnak et al. 2010, 2013). The “snow-plow” model proposed by Tappin (2006) calculates the deceleration of CME based on the conservation of momentum as the CME sweeps up the slower solar wind ahead of it.

The actual CME propagation in highly structured solar wind plasma involves various dynamical processes, such as the deflection and rotation of CME flux rope, magnetic reconnection between CME and ambient solar wind, deformation of flux rope, etc. They have studied with the use of global MHD simulations in 3D (Manchester et al. 2004; Lugaz & Roussev 2011; Shiota & Kataoka 2016).

Liu et al. (2013) reported the first detailed examination of the Sun-to-Earth propagation characteristics of three fast CMEs and an associated shock front using the combination of wide-angle heliospheric imaging observations by STEREO, interplanetary Type II radio bursts, and in situ observations of solar wind parameters at multiple points. They reported that CME Sun-to-Earth propagation can be approximately formulated into three phases: (1) an impulsive acceleration near the Sun (up to ~0.1 au), (2) rapid deceleration up to the distance of ~0.2–0.4 au, and (3) nearly constant speed propagation or gradual deceleration afterward.

Shock-driving fast CMEs are accompanied by continuously accumulating sheath plasma ahead. We need a model that describes the coevolution of interplanetary sheaths and shock-driving ICME propagation on a single theoretical basis. Such a model would be helpful for understanding the space-weather-impacting, ICME-related disturbances at 1 au, such as the speed, magnetic field strength, and size of both interplanetary sheaths and magnetic clouds, as well as their arrival times. For this purpose, we construct a new model that connects ICME deceleration and interplanetary shock propagation by taking into account the process of upstream solar wind plasma accumulation within the sheath region. We call the model the “sheath-accumulating propagation” (SAP) model of CMEs. We note that the effects of the Lorentz force and gravity are neglected in the SAP model, though they could also be effective, especially in the vicinity of the Sun (Chen & Kunkel 2010).

In Section 2, we introduce the SAP model and investigate the ICME propagation characteristics of the SAP model. In Section 3, we discuss the thickness of the interplanetary sheath ahead of CMEs. In Section 4, we compare the SAP model with the drag-based model. In Section 5, we present an arrival time prediction by the SAP model for 19 Earth-directed CME–ICME pairs and discuss its prediction ability. In Section 6, we discuss the geomagnetic impact of super-massive, super-fast CMEs associated with solar superflares, based on the SAP model.

### 2. The Sheath-accumulating Propagation (SAP) Model

#### 2.1. Basic Assumptions

In the SAP model, we assume the background solar wind as a spherically symmetric flow with a uniform speed \(V_{sw}\) in a heliocentric location \(r > r_0 = 0.1\) au. We express the total mass of the composite of the ICME and interplanetary sheath at time \(t\) as \(M(t)\), and the radial speed and the heliocentric distance of the center of mass as \(V(t)\) and \(R(t)\), respectively. Initially (at \(t = 0\)), the total mass, radial speed, and heliocentric distance of the ICME-sheath composite are \(M(0) = M_0, V(0) = V_0,\) and \(R(0) = r_0\), respectively (Figure 1). For simplicity, we call \(V(t)\) and \(R(t)\) the ICME speed and location, respectively, throughout this paper. We assume that the ICME angular half width \(\theta_0\) is constant during its propagation. In reality, the CME properties near the Sun are basically estimated by coronagraph observation. In the SAP model, we neglect the effect of gravitational and Lorentz force on the propagation of ICME in \(r > r_0\) space.

The total mass of the ICME-sheath composite at time \(t\) is expressed as

\[
M(t) = M_0 + M_{sheath}(t),
\]

where \(M_{sheath}(t)\) is the mass of the interplanetary sheath ahead of the ICME (Figure 1). Assuming that a constant fraction \(c_0 \simeq 1\) of the plasma swept by interplanetary shock becomes a part of the interplanetary sheath, we obtain

\[
M_{sheath}(t) = c_0 \Omega_0 \int_0^t \rho_{sw}(R(t'))R(t')^2 \times (V_{shock}(R(t')) - V_{sw}) dt',
\]
where \( \rho_{\text{sw}}(R) \) and \( V_{\text{shock}}(R) \) are the solar wind mass density and interplanetary shock propagation speed at \( r = R \), respectively. The ICME solid angle \( \Omega_0 \) is approximated by the half angular width \( \theta_0 \) as \( \Omega_0 \propto \pi \theta_0^2 \), assuming a circular cross-section of the ICME. In the SAP model, we consider a spherically symmetric wind, i.e., \( \rho_{\text{sw}}(R)R^2V_{\text{sw}} = M_{\text{sw}}/4\pi \), where \( M_{\text{sw}} \) is a solar mass-loss rate by the solar wind, which is a constant. Approximating the shock propagation speed by ICME speed, i.e., \( V_{\text{shock}} \simeq dR/dt \), we get

\[
M_{\text{sheath}}(t) = c_0\rho_{\text{sw}} \frac{\Omega_0 R(t) - r_0 - V_{\text{sw}}t}{4\pi} V_{\text{sw}}. \tag{3}
\]

On the other hand, the conservation of momentum of the ICME-sheath composite is written as

\[
(M_0 + M_{\text{sheath}}(t))V(t) \simeq M_0V_0 + M_{\text{sheath}}(t)V_{\text{sw}}. \tag{4}
\]

### 2.2. The ICME Propagation Characteristics in the SAP Model

By solving Equations (3) and (4), we can express the ICME arrival time \( t \), the sheath mass \( M_{\text{sheath}} \), the ICME speed \( V \), and the deceleration \(-a\) as a function of ICME heliocentric location \( R \), as follows,

\[
t(R) = \frac{R - r_0}{V_{\text{sw}}} \left\{ 1 - \frac{M_0V_0}{M_c(R)V_{\text{sw}}(R)} \epsilon(R) \right\}, \tag{5}
\]

\[
M_{\text{sheath}}(R) = M_0 \frac{V_0}{V_{\text{sw}}} \epsilon(R), \tag{6}
\]

\[
V(R) = (V_0 - V_{\text{sw}}) \left( 1 + \frac{V_0}{V_{\text{sw}}} \epsilon(R) \right)^{-1} + V_{\text{sw}}, \tag{7}
\]

\[
-a(R) = \frac{c_0\Omega_0\rho_{\text{sw}}}{4\pi(M_0 + M_{\text{sheath}}(R))V_{\text{sw}}}(V(R) - V_{\text{sw}})^2, \tag{8}
\]

where \( M_c(R) \) and \( \epsilon(R) \) are,

\[
M_c(R) = c_0\Omega_0 \int_0^R \rho_{\text{sw}}(R')R'^2dR' = c_0\rho_{\text{sw}} \frac{\Omega_0(R - r_0)}{4\pi V_{\text{sw}}}, \tag{9}
\]

\[
\epsilon(R) = \sqrt{1 + 2\frac{M_c(R)V_{\text{sw}}(V_0 - V_{\text{sw}})}{M_0V_0^2} - 1}. \tag{10}
\]

A detailed derivation of Equations (5)–(8) is given in Appendix A.

Generally, massive CMEs experience only weak deceleration and although the deceleration \(-a(R)\) decays slower with \( R \), massive CMEs do not lose their speeds as fast as light CMEs, where \(-a(R)\) is initially very strong (see Figure 1 of Vršnak et al. 2013) as an example in the drag-based model. Figures 2(a)–(d) show \( V(R) \), \(-a(R)\), \( t(R) \), and \( M_{\text{sheath}}(R)/M_0 \) with six different CME parameters in the SAP model. When the CME mass is \( M_0 = 3 \times 10^{15} \) g (thin lines), the ICME experiences rapid deceleration before \( R \sim 0.3 \) au, followed by gradual deceleration afterward. In that case, the ICME arrives at 1 au with almost the same speed as the background solar wind. This behavior is consistent with the two-phased deceleration characteristics of ICMEs reported by Liu et al. (2013). When the CME is as heavy as \( M_0 = 3 \times 10^{16} \) g (thick lines), the ICME arrives at 1 au, with a larger speed compared to the \( M_0 = 3 \times 10^{15} \) g cases. We also note that the SAP model, as well as the drag-based model, predicts that faster CMEs will experience stronger deceleration. This tendency is consistent with the observed deceleration of the shock front from the Sun to 1 au, which was reported by Woo et al. (1985). In Section 2.3, we discuss the fundamental condition for fast ICMEs at Earth (e.g., \( V(1 \text{ au}) \gtrsim 1000 \text{ km s}^{-1} \)), as in the case of the 2012 July 23 super-intense ESP event.

### 2.3. The Fundamental Condition for Extremely Fast ICMEs at 1 au

First, we consider the heliocentric distance \( R_c \) at which the relative speed of ICMEs with respect to the solar wind is halved, i.e., \( V(R_c) - V_{\text{sw}} = (V_0 - V_{\text{sw}})/2 \). From Equation (7), \( R_c \) satisfies the following relation,

\[
\epsilon(R_c) = \frac{V_0}{V_{\text{sw}}}. \tag{11}
\]

From Equations (6) and (11) this leads to

\[
M_{\text{sheath}}(R_c) = M_0. \tag{12}
\]

From Equations (8), (12), and \( V(R_c) - V_{\text{sw}} = (V_0 - V_{\text{sw}})/2 \), we get \( a(R_c) = a(r_0)/8 \), which means the rapid ICME deceleration almost ceases at \( r = R_c \). We call \( R_c \) a “deceleration cessation distance,” which was originally discussed in the empirical models (Gopalswamy et al. 2001). Solving Equation (11), \( R_c \) can be written in terms of \( M_0 \) and \( V_0 \) as follows,

\[
R_c = r_0 + (1 \text{ au} - r_0) \frac{M_0}{M_c(1 \text{ au})} \left( 1 + \frac{3}{2} \frac{V_0}{V_0 - V_{\text{sw}}} \right). \tag{13}
\]

When the CME mass \( M_0 \) is larger than \( M_c(1 \text{ au}) \), \( R_c \) is always larger than 1 au. This means that CMEs heavier than \( M_c(1 \text{ au}) \) with any initial speed will arrive at 1 au without significant deceleration. In this sense, we call \( M_c(1 \text{ au}) \) the “critical CME mass” for 1 au travel.

The thin and thick lines in Figure 3(a) show \( R_c \) against \( V_0 \), with CME masses of \( M_0 = 3 \times 10^{15} \) g and \( M_0 = 3 \times 10^{16} \) g, respectively, in a slow background solar wind. Figure 3(b) shows \( R_c \) with the same CME parameters as in panel (a) but in a fast background solar wind. The \( M_c(1 \text{ au}) \) in cases of slow and fast winds are \( M_c = 9.1 \times 10^{16} \) g and \( M_c = 3.0 \times 10^{16} \) g, respectively. When the CME is heavier than or comparable to the critical mass (as in the case of the thick line in Figure 3(b)), ICME will remain fast when arriving at Earth.

We compare the extremely fast ICME in the 2012 July 23 event and the SAP model prediction. Before the arrival of the ICME, the solar wind speed and density measured in situ by STEREO-A at the distance of 0.96 au from the Sun were roughly \( V_{\text{sw}}(0.96 \text{ au}) \approx 500 \text{ km s}^{-1} \) and \( n_{\text{sw}}(0.96 \text{ au}) \approx 1-3 \text{ cm}^{-3} \), respectively, with little variation. The corresponding critical CME mass is \( M_c(0.96 \text{ au}) = 1 \times 10^{15} - 3 \times 10^{15} \) g. The peak CME speed near the Sun is \( V_0 \approx 3050 \text{ km s}^{-1} \) (Liu et al. 2014) and the CME mass estimated with SOHO/LASCO was about \( M_0 = 3.2 \times 10^{16} \) g. With these values, Equation (7) predicts the ICME speed at 0.96 au to be between \( 2.0 \times 10^3 \text{ km s}^{-1} \) and \( 2.6 \times 10^3 \text{ km s}^{-1} \), which is consistent with the post-shock peak speed of 2250 km s\(^{-1}\) measured at STEREO-A. We assumed \( c_0 = 1 \) and \( \theta_0 = \pi/4 \) in the above estimation.
### 3. The Thickness of the Interplanetary Sheath

Although the thickness of the sheath is an important parameter for understanding space weather storms such as geomagnetic storms, they are rarely considered in modeling or observations. In this section, we discuss the thickness of the interplanetary sheath $D(R)$ expected by the SAP model.

The fast mode Mach number of the leading shock is approximated as

$$\mathcal{M}_f(R) = \left( \frac{V(R) - V_{sw}}{C_f} \right),$$

with $C_f$ being the phase speed of the fast mode MHD wave in the background solar wind plasma. On the other hand, the sheath mass is approximately written as

$$M_{\text{sheath}}(R) = \frac{\rho_{sw}(R) \Omega_0 R^2 D(R)}{4\pi \frac{M_0 \chi(R) D(R)}{V_{sw}}},$$

where $\chi(R)$ is the compression ratio of the interplanetary shock, which depends not only on $\mathcal{M}_f(R)$ but also on upstream plasma beta and the angle between the shock normal and upstream magnetic field. From Equations (6) and (14), $D(R)$ can be approximated as

$$D(R) \approx \frac{4\pi M_0 V_0 \epsilon(R)}{\Omega_0 M_{sw} \chi(R)}.$$

If we assume the values of $M_0$, $V_0$, $V_{sw}(1 \text{ au})$, and $n_{sw}(0.96 \text{ au})$ as those assumed in the previous section for the 2012 July 23 event, and also assume $\chi \approx 3$, the sheath thickness estimated with Equation (15) becomes $D(0.96 \text{ au}) \approx 0.25 \text{ au}$. The estimated sheath thickness of $\sim 0.25 \text{ au}$ is substantially larger than the typical thickness of $\sim 0.05 \text{ au}$ reported by Russell & Mulligan (2002), because of exceptionally large $V_0$ and $M_0$ in the 2012 July 23 event. Assuming the sheath speed during its passage at the spacecraft to be $\sim 2 \times 10^3 \text{ km s}^{-1}$, the transit time of a sheath with a thickness of 0.25 au would be $\approx 5 \text{ hr}$. The actual solar wind disturbance detected by STEREO-A is known to be the merger of two successively launched CMEs (Liu et al. 2014). The leading edges of the two CMEs passed the spacecraft after the

\[\text{Figure 2. ICME propagation properties in the case of slow background solar wind as a function of ICME heliocentric distance } R. \text{ ICME speed } (V(R)), \text{ deceleration } (-a(R)), \text{ arrival time } (t(R)), \text{ and sheath mass in units of initial CME mass } M_{\text{sheath}}(R)/M_0 \text{ are plotted in panels (a)-(d), respectively, with six different pairs of } M_0 \text{ and } V_0 \text{ values. The CME mass is chosen to be } M_0 = 3 \times 10^{15} \text{ g or } 3 \times 10^{16} \text{ g, while the CME speed takes three values, which are } V_0 = 500, 1000, \text{ or } 3000 \text{ km s}^{-1}. \]
arrival of the leading shock front by 2 and 6 hr, respectively. The predicted sheath transit time of 5 hr is in between the two but more consistent with the latter.

4. Comparison of the SAP Model with the Drag-based Model and the “Snow-plow” Model of Tappin (2006)

From the relation \( M_{sw} = 4\pi \rho_{sw}(R)R^2V_{sw} \), the ICME acceleration in the SAP model (Equation (8)) can be expressed as

\[
a = -\frac{c_0A_{sw}}{M_0 + M_{sheath}}(V - V_{sw})^2,
\]

with \( A = \Omega_0R^2 \) being the cross-sectional area of the ICME.

On the other hand, the CME acceleration in the drag-based model is

\[
a = -\frac{c_dA_{sw}}{M_0 + M_V}(V - V_{sw})|V - V_{sw}|,
\]

where \( c_d \) is a drag coefficient of order unity and \( M_V = \rho_{sw}V/2 \) is a “virtual mass,” with \( V \) being instantaneous CME volume (Vršnak et al. 2013; see also Cargill 2004 and references therein).

The “virtual mass” formulation incorporated in the drag-based model is based on the assumption that a potential flow passed a solid sphere (Landau & Lifshitz 1959). However, if \( V - V_{sw} \) is transonic or supersonic, the flow around the CME would be substantially different from the potential flow, due to strong compressibility (e.g., a sheath forms ahead of the CME). In such a case, the SAP model would give a straightforward estimation of the virtual mass \( M_V = M_{sheath} \).

The dynamics of sheath accumulation discussed in the SAP model is basically close to the “piston-driven” shock formation process (Vršnak & Cliver 2008). On the basis of the piston-driven mechanism, the shock-driving CME is not necessarily supersonic. Sheeley et al. (1985) reported that the shocks tend to be associated with faster CMEs (with their speeds larger than 500 km s\(^{-1}\)), while they are sometimes associated with slower CMEs with speeds between 200 and 400 km s\(^{-1}\). If the sheath thickness is comparable to or larger than the lateral extent of the CME, i.e., \( D(R) \geq R\theta_0 \), a large part of the shocked plasma would escape from the sides of the CME, deviating from the piston-driven mechanism. In that case, the drag-based model, rather than the SAP model, would give a more appropriate description of the CME deceleration. The typical thickness of the sheath at 1 au reported by Russell & Mulligan (2002) is \( \sim 0.05 \) au, which is likely much smaller than the typical widths of CMEs at 1 au of \( \sim 1 \) au, assuming \( \theta_0 \sim 1 \). From this, we expect the SAP model can be widely applied to the decelerating propagation of CMEs.

Figure 3. \( R_c \) against \( V_0 \) in slow (panel (a)) and fast (panel (b)) background solar winds. The thin and solid lines show \( R_c \) in the cases of heavy \((M_0 = 3 \times 10^{15} \text{ g})\) and super-heavy \((M_0 = 3 \times 10^{16} \text{ g})\) CMEs, respectively. The solar wind density and speed at 1 au are chosen to be \( n_{sw}(1 \text{ au}) = 9 \text{ cm}^{-3} \) and \( V_{sw}(1 \text{ au}) = 350 \text{ km s}^{-1} \) for the slow wind case, and \( n_{sw}(1 \text{ au}) = 3 \text{ cm}^{-3} \) and \( V_{sw}(1 \text{ au}) = 600 \text{ km s}^{-1} \) for the fast wind case, respectively. \( M_c(1 \text{ au}) \) for the slow and fast wind cases are \( M_c = 9.1 \times 10^{15} \text{ g} \) and \( 3.0 \times 10^{16} \text{ g} \), respectively. \( \theta_0 = \pi/4 \) and \( c_0 = 1 \) are assumed in all cases.

Figure 4. Predicted vs. observed 1 au arrival time. The relation between the observed 1 au arrival times \( (t_{\text{obs}}(1 \text{ au})) \) and those predicted by the SAP model \( (t_{\text{SAP}}(1 \text{ au})) \) for the 19 CME–ICME pairs are plotted as a correlation plot.
The “snow-plow” model proposed by Tappin (2006) is mathematically very similar to the SAP model. A detailed comparison of the formulas of the two models is given in Appendix B. The SAP model is basically differs from the “snow-plow” model in that it tracks the evolution of the sheath, and it is an analytical model with the assumption of the uniform solar wind speed.

5. 1 au Arrival Time Prediction Based on the SAP Model

Mäkelä et al. (2016) studied the relation between the radial speed and expansion speed of 19 Earth-directed CMEs that occurred from 2010 January to 2012 September, when STEREO and SOHO were viewing the Sun in near quadrature. We apply the SAP model to predict the 1 au arrival time of the same set of CME–ICME pairs as studied in Mäkelä et al. (2016). The average angular half width of the 19 CMEs measured by STEREO/COR2 is $\theta_0 = 0.23\pi$. The mass is estimated for 16 out of 19 CMEs and listed in the online SOHO/LASCO CME catalog (Yashiro et al. 2004). The average CME mass of the 16 CMEs is $M_0 = 9.6 \times 10^{15}$ g. We note that the mass estimation of the Earth-directed CMEs, based on SOHO/LASCO data, is based on many assumptions that possibly introduce significant uncertainty into the mass estimation. We refer to the observed 1 au arrival time of interplanetary shock as $t_{\text{obs}}$. We refer to the predicted 1 au arrival time based on the SAP model as $t_{\text{SAP}}$.

$$t_{\text{SAP}} = t_{\text{in}} + t(1\text{ au}) - \frac{1}{2} t_s,$$  (18)

where $t_{\text{in}} \simeq r_0/V_0$ is the time for a CME to travel from the Sun to the solar wind shock radius, and $t_s = D(1\text{ au})/V(1\text{ au})$ is the sheath transit time at Earth. We assumed $M_{\text{sheath}}(1\text{ au}) \gg M_0$, so the ICME center of mass when $R(t) = 1$ au is almost at the midpoint of the sheath region. The average of the mass ratio $M_{\text{sheath}}/M_0$ expected from the SAP model using Equation (6) is 5.5. We assumed a typical slow background solar wind with $V_{sw} = 350$ km s$^{-1}$ and $n_{sw}(1\text{ au}) = 9$ cm$^{-3}$ (Schwenn 2006). $M_0 = M_0$ and $\theta_0 = \theta_0$ are assumed in the calculation of $t(1\text{ au})$, so $t_{\text{SAP}}$ only depends on initial CME speed $V_0$ in this study. In the evaluation of $t_s$, we assumed the shock compression ratio to be 3 in all cases, for simplicity. We tried three different values of $c_0$: 0.8, 0.9, and 1.0. We found that $c_0 = 0.9$ minimized the root-mean-square (rms) of the observed-minus-calculated transit time differences. We note that, solely using this finding, we cannot draw the conclusion that 90% of shocked solar wind plasma has been accumulated in the sheath on average, partly because the assumed CME parameters, especially the mass, contain large uncertainty. The rms and the maximum value of the observed-minus-calculated transit time differences in the $c_0 = 0.9$ case were 7.5 hr and 15.8 hr, respectively. Figure 4 shows the correlation plot between the predicted ($t_{\text{SAP}}$) and observed ($t_{\text{obs}}$) arrival times in the $c_0 = 0.9$ case.

6. Super-intense Geomagnetic Storms Associated with Solar Superflares

The ICME-driven westward electric field at Earth ($E_y = V B_z$, with $V$ and $B_z$ being the speed and the southward magnetic field of the ICME) is the crucial space plasma quantity that drives intense geomagnetic storms (Burton et al. 1975; Yermolaev et al. 2007). The magnetic cloud core field is known to be correlated with the ICME speed (Gonzalez et al. 1998) and the upper limit can be estimated by the equipartition field strength (Takahashi et al. 2016) as

$$B_z,\text{upperlimit}(1\text{ au}) \simeq \sqrt{4\pi \rho_{sw}(1\text{ au})} (V(1\text{ au}) - V_{sw}).$$  (19)

The upper limit of $E_y$ at 1 au can be expressed by

$$E_y,\text{upperlimit}(1\text{ au}) \simeq V(1\text{ au})B_z,\text{upperlimit}(1\text{ au}).$$  (20)

From Equations (19) and (20), $E_y,\text{upperlimit}(1\text{ au})$ is determined solely by the ICME speed (and the background solar wind density) at 1 au, which depends both on the mass and speed of causative CMEs, due to Equation (7).

Based on the scaling relations among CME properties and flare soft-X-ray peak flux ($F_{\text{SXR}}$) at 0.1-0.8 nm, measured by the X-ray detector on board GOES (discussed in Takahashi et al. 2016), the upper limit of the mass and speed of CMEs are expressed as

$$M_{0,\text{upperlimit}} \simeq 3 \times 10^{16} \times \left(\frac{F_{\text{SXR}}}{F_{\text{SXR},10}}\right)^{2/3} \text{g},$$  (21)

$$V_{0,\text{upperlimit}} \simeq 4.2 \times 10^3 \times \left(\frac{F_{\text{SXR}}}{F_{\text{SXR},10}}\right)^{1/6} \text{km s}^{-1},$$  (22)

with $F_{\text{SXR},10} = 0.001$ W m$^{-2}$ being the $F_{\text{SXR}}$ of an X10 class flare. In Equation (21), we assumed the maximum CME mass associated with the X10 flare to be $\sim 3 \times 10^{16}$ g (Aarnio et al. 2012). Applying Equations (21) and (22) for the evaluation of $E_y,\text{upperlimit}(1\text{ au})$ in Equation (20), we find $E_y,\text{upperlimit}(1\text{ au})$ as a function of $F_{\text{SXR}}$. $E_y,\text{upperlimit}(1\text{ au})$ against $F_{\text{SXR}}$ (or flare class), with the fast and slow background solar winds plotted in Figure 5. When $M_{0,\text{upperlimit}} \gtrsim M_1(1\text{ au})$, $V(1\text{ au}) - V_{sw} \simeq V_0$ holds, and $E_y,\text{upperlimit}(1\text{ au})$ approximately scales as $E_y,\text{upperlimit}(1\text{ au}) \propto V_0^2 \propto F_{\text{SXR}}^{1/3}$. Based on the discussion above, the $E_y,\text{upperlimit}$ associated with the X10 flare, for example, will be $\sim 2 \times 10^5$ nT m$^{-1}$. If such $E_y$ continues for $\sim 2$ hr, $Dst$ would be $Dst \sim -2 \times 10^3$ nT, following the formula by Burton et al. (1975). On the other hand, the upper limit of $Dst$ inherent in geomagnetism is evaluated to be $\sim 2500$ nT in Vasyltov (2011), which is comparable to the upper limit of $Dst$ associated with the X10 flare above.

Further discussion is needed to evaluate the actual upper limit of $Dst$ associated with the huge solar flares of $\geq X10$ class.

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Appendix A

The Derivation of Equations (5)–(8)

In this appendix, we derive Equations (5)–(8) from Equations (3) and (4).

We first define two new variables, $R(t) = R(t) - r_0 - V_{sw} t$ and $\tilde{V}(t) = d\tilde{R}(t)/dt = V(t) - V_{sw}$, so that Equations (3) and
If we substitute Equation (29) into Equation (23), we get the sheath mass \( M_{\text{sheath}}(R) \) as Equation (6).

Substituting Equation (29) into Equation (26), we get \( V = \dot{V} + V_{\text{sw}} \), as in Equation (7).

Lastly, we derive Equation (8). The acceleration \( a \) can be deformed as

\[
a = \frac{dV}{dt} = \frac{dV}{dR} \frac{dR}{dt} = \frac{dV}{dR}.
\]

(30)

On the other hand, making the derivative of Equation (26) with respect to \( \tilde{R} \), we get

\[
\frac{dV}{dR} = -\frac{c_0 \Omega_0 M_{\text{sw}} V_0}{4 \pi M_0 V_{\text{sw}}} \left( 1 + \frac{c_0 \Omega_0 M_{\text{sw}} \tilde{R}(t)}{4 \pi M_0 V_{\text{sw}}} \right)^{-2}.
\]

(31)

Substituting Equations (23) and (26) into Equation (31), we get,

\[
\frac{dV}{dR} = -\frac{c_0 \Omega_0 M_{\text{sw}}}{4 \pi (M_0 + M_{\text{sheath}}) V_{\text{sw}}} \dot{V}.
\]

(32)

Substituting Equation (32) into Equation (30), we get the ICME deceleration \(-a\) as Equation (8).

**Appendix B**

**Comparison between the “Snow-plow” Model of Tappin (2006) and the SAP Model**

The “snow-plow” model proposed in Tappin (2006) is a set of two coupled differential equations:

\[
\frac{dV_i}{dt} = -\frac{dM_i}{dt} \frac{V_i - V_{\text{sw}}}{M_i},
\]

(33)

\[
\frac{dM_i}{dt} = \Omega_0 \rho_{\text{sw}} R^2 (V_i - V_{\text{sw}}),
\]

(34)

where \( R, M_i, \) and \( V_i \) are the heliocentric distance, the mass, and the speed of a transient. The transient gets heavier by sweeping up solar wind plasma ahead of it.

On the other hand, the SAP model is based on the combination of Equations (3) and (4). Making the time derivative of Equation (3) and using the relations \( \rho_{\text{sw}} R^2 V_{\text{sw}} = M_0 / 4 \pi \), and \( M = M_0 + M_{\text{sheath}} \), we get the following differential equation:

\[
\frac{dM}{dt} = c_0 \Omega_0 \rho_{\text{sw}} R^2 (V - V_{\text{sw}}).
\]

(35)

Then, if we make the time derivative of Equation (4), we get

\[
\frac{dV}{dt} = -\frac{dM}{dt} \frac{V - V_{\text{sw}}}{M_i}.
\]

(36)

If we assume \( M = M_i \) and \( c_0 = 1 \) in Equation (35) of the SAP model, we get Equation (34) of the “snow-plow” model. If we further assume \( V = V_i \), Equation (36) becomes equivalent to Equation (33).

**References**

Aarnio, A. N., Matt, S. P., & Stassun, K. G. 2012, ApJ, 760, 9

Airapetian, V. S., Gloer, A., Gronoff, G., et al. 2016, NatGe, 9, 452

Bryant, D. A., Cline, T. L., Desai, U. D., & McDonald, F. B. 1962, JGR, 67, 4983

Burton, R., McPherron, R., & Russell, C. 1975, JGRA, 80, 4204

Cargill, P. J. 2004, SoPh, 221, 135

Cargill, P. J., Chen, J., Spicer, D. S., & Zalesak, S. T. 1996, JGR, 101, 4855
