A Pulsed Synchrotron for Muon Acceleration at a Neutrino Factory

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Abstract. A 4600 Hz pulsed synchrotron is considered as a means of accelerating cool muons with superconducting RF cavities from 4 to 20 GeV/c for a neutrino factory. Eddy current losses are held to less than a megawatt by the low machine duty cycle plus 100 micron thick grain oriented silicon steel laminations and 250 micron diameter copper wires. Combined function magnets with 20 T/m gradients alternating within single magnets form the lattice.Muon survival is 83%.

Historically synchrotrons have provided economical particle acceleration. Here we consider a pulsed muon synchrotron [1] for a neutrino factory [2]. The accelerated muons are stored in a racetrack to produce neutrino beams ($\mu^- \rightarrow e^- \nu_e \nu_\mu$ and $\mu^+ \rightarrow e^+ \nu_e \nu_\mu$). Neutrino oscillations have been observed at experiments [3] such as Homestake, Super–Kamiokande, SNO, and KamLAND. Further exploration using a neutrino factory could reveal CP violation in the lepton sector [4].

This synchrotron must accelerate muons from 4 to 20 GeV/c with moderate decay loss ($\tau_{\mu^\pm} = 2.2 \mu S$), using magnet power supplies with reasonable voltages. To reduce voltage, magnet gaps are minimized to store less magnetic energy. Cool muons [5] with low beam emittance allow this. Acceleration to 4 GeV/c might feature fixed field dogbone arcs [6, 7] to minimize muon decay loss. Fast ramping synchrotrons [6, 8] might also accelerate very cool muons to higher energies for a $\mu^+\mu^-$ collider [9].

We form arcs with sequences of combined function cells within continuous long magnets, whose poles are alternately shaped to give focusing gradients of each sign. A cell has been simulated using SYNCH [10]. Gradients alternate from positive 20 T/m gradient (2.24 m long), to zero gradient (.4 m long) to negative 20 T/m gradient (2.24 m) to zero gradient (0.4 m), etc. See Fig. 1 and Table 1. It is proposed to use 5 such arc cells to form an arc segment. These segments are alternated with straight sections containing RF. The phase advance through one arc segment is $5 \times 72^0 = 360^0$. This being so, dispersion suppression between straights and arcs can be omitted. There are 18 arc segments and 18 straight sections, forming 18 superperiods in the ring. Straight
sections (22 m) without dispersion are used for superconducting RF, and, in two longer straights (44 m), the injection and extraction. To assure sufficiently low magnetic fields at the cavities, relatively long field free regions are desirable. A straight consisting of two half cells would allow a central gap of 10 m between quadrupoles, and two smaller gaps at the ends. Details are given in Table 3. Matching between the arcs and straights is not yet designed. The total circumference of the ring including combined functions magnets and straight sections adds up to 917 m ($18 \times 26.5 + 16 \times 22 + 2 \times 44$).

**Table 1.** Combined function magnet cell parameters. 5 cells/arc. 18 arcs form the ring.

| Parameter                  | Value       |
|----------------------------|-------------|
| Cell length                | 5.28 m      |
| Combined Dipole length     | 2.24 m      |
| Combined Dipole $B_{\text{central}}$ | 0.9 T |
| Combined Dipole Gradient   | 20.2 T/m    |
| Pure Dipole Length         | 0.4 m       |
| Pure Dipole B              | 1.8 T       |
| Momentum                   | 20 GeV/c    |
| Phase advance/cell         | $72^0$      |
| beta max                   | 8.1 m       |
| Dispersion max             | 0.392 m     |
| Norm. Trans. Acceptance    | $4 \pi \text{ mm rad}$ |

**Table 2.** Superconducting RF.

| Parameter                  | Value       |
|----------------------------|-------------|
| Frequency                  | 201 MHz     |
| Gap                        | .75 m       |
| Gradient                   | 15 MV/m     |
| Stored Energy              | 900 J       |
| Muons per train            | $5 \times 10^{12}$ |
| Orbits (4 to 20 GeV/c)     | 12          |
| No. of RF Cavities         | 160         |
| RF Total                   | 1800 MV     |
| $\Delta U_{\text{beam}}$  | 110 J       |
| Energy Loading             | .082        |
| Voltage Drop               | .041        |
| Acceleration Time          | 37 $\mu$S  |
| Muon Survival              | .83         |

The superconducting RF (see Fig. 1 and Table 2 and note that 11 MV/m has been achieved so far [11]) must be distributed around the ring to avoid large differences between the beam momentum (which rises in steps at each RF section) and the magnetic field (which rises continuously). The amount of RF is a tradeoff between cost and muon survival. Time dilation permits extra orbits with little muon decay if the RF sags.

**Table 3.** Straight section lattice parameters.

| Parameter      | Value   |
|----------------|---------|
| $\phi$         | $77^0$  |
| $L_{\text{cell/2}}$ | 11 m   |
| $L_{\text{quad}}$ | 1 m    |
| $dB/dx$        | 7.54 T/m |
| a              | 5.8 cm  |
| $\beta_{\text{max}}$ | 36.6 m |
| $\sigma_{\text{max}}$ | 1.95 cm |
| $B_{\text{pole}}$ | 0.44 T |
| $U_{\text{mag/quad}}$ | $\approx 3000 J$ |

**Table 4.** Permeability ($B/\mu_0 H$). Grain oriented silicon (3% Si) steel has a far higher permeability parallel ($||$) to than perpendicular ($\perp$) to its rolling direction [12]. Grain oriented silicon steel permits high fields with little energy ($B^2/2\mu$) stored in the yoke.

| Material        | 1.0 T | 1.5 T | 1.8 T |
|-----------------|-------|-------|-------|
| 1008 Steel      | 3000  | 2000  | 200   |
| Grain Oriented ($||$) | 40000 | 30000 | 3000   |
| Grain Oriented ($\perp$) | 4000  | 1000  |       |

The muons accelerate from 4 to 20 GeV. If they are extracted at 95% of full field they will be injected at 19% of full field. For acceleration with a plain sine wave, injection occurs at $11^0$ and extraction occurs at $72^0$. So the phase must change by $61^0$ in $37 \mu$S. Thus the sine wave goes through $360^0$ in $218 \mu$sec, giving 4600 Hz.

Estimate the energy stored in each 26.5 m long combined function magnet. The gap is about .14 m wide and has an average height of $h = .06$ m. Assume an average field of...
1.1 Tesla. The permeability constant, \(\mu_0\), is \(4\pi \times 10^{-7}\). \(W = B^2/2\mu_0[\text{Volume}] = 110 000\) Joules. Next given one turn (N = 1), an LC circuit capacitor, and a 4600 Hz frequency; estimate current, inductance, capacitance, and voltage.

\[
B = \frac{\mu_0 NI}{h} \rightarrow I = \frac{Bh}{\mu_0 N} = 52\text{kA}; \quad W = \frac{1}{2}LI^2 \rightarrow L = \frac{2W}{I^2} = 80\text{\mu H} \quad (1)
\]

\[
f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \rightarrow C = \frac{1}{L(2\pi f)^2} = 15\text{\mu F}; \quad W = \frac{1}{2}CV^2 \rightarrow V = \sqrt{\frac{2W}{C}} = 120\text{kV} \quad (2)
\]

The stack of SCRs driving each coil might be center tapped to halve the 120 kV. Nine equally spaced 6 cm coil slots could be created in the top and bottom of each yoke using 6 cm of taller laminations to cut the voltage by ten, while leaving the pole faces continuous. 6 kV is easier to insulate than 120 kV. It will be useful to shield [1] and/or chamfer [13] magnet ends to avoid large eddy currents where the field lines typically do not follow laminations. Neutrino horn power supplies are of interest.

Calculate the resistive energy loss in the copper coils. There are two 5 cm square copper conductors each 5300 cm long. \(R = 5300 (1.8 \mu\Omega\text{-cm})/(2) (5^2) = 190 \mu\Omega\). So, \(P = I^2R \int_0^{2\pi} \cos^2(\theta) d\theta = 260 000\) w/magnet. Eighteen magnets give a total loss of 4680 kW. But the neutrino factory runs at 30 Hz. Thirty half cycles of 109 \(\mu\text{sec}\) per second gives a duty factor of 300 and a total \(I^2R\) loss of 16 kW. Muons are orbited in opposite directions on alternate cycles. If this proves too cumbersome, the duty cycle factor could be lowered to 150. See if .25 mm (30 gauge) wire is usable. The skin depth [14], \(\delta\), of copper at 4600 Hz is \((\rho / \pi f \mu_0)^{1/2} = (1.8 \times 10^{-8} / \pi 4600 \mu_0)^{1/2} = 0.97\) mm.

Now calculate the dissipation due to eddy currents [15] in a w = .25 mm wide conductor, which consists of transposed strands to reduce this loss [13, 15]. To get an idea, take the maximum B-field during a cycle to be that generated by a 0.025m radius conductor carrying 26 kA. The eddy current loss in a conductor made of square wires .25 mm wide (Litz wire [16]) with a perpendicular magnetic field is as follows. \(B = \mu_0 I/2\pi r = 0.2\) Tesla.

\[
P = [\text{Volume}] \frac{(2\pi f Bw)^2}{24\rho} = [2.05^2 \times 53] \frac{(2\pi 4600 .2 .00025)^2}{(24) 1.8 \times 10^{-8}} = 1400\text{ kW} \quad (3)
\]

Multiply by 18 magnets and divide by a duty factor of 300 to get an eddy current loss in the copper of 85 kW. Stainless steel water cooling tubes will dissipate a similar amount of power [6]. Alloy titanium cooling tubes would dissipate half as much.

Grain oriented silicon steel is chosen for the yoke due to its high permeability at high field at noted in Table 4. The skin depth [14], \(\delta\), of a lamination is \((\rho / \pi f \mu)^{1/2} = (47 \times 10^{-8} / \pi 4600 1000 \mu_0)^{1/2} = 160 \mu\text{m}. \rho\) is resistivity. Take \(\mu = 1000\mu_0\) as a limit on magnetic saturation and hence energy storage in the yoke. Next estimate the fraction of the yoke inductance that remains after eddy currents shield the laminations [17]. The lamination thickness, \(t\), is 100 \(\mu\text{m}\) [18]. \(L/L_0 = (\delta/t) (\sin(t/\delta) + \sinh(t/\delta)) / (\cosh(t/\delta) + \cos(t/\delta)) = 0.995\). So it appears that magnetic fields can penetrate 100 \(\mu\text{m}\) thick laminations at 4600 Hz. Thicker 175 \(\mu\text{m}\) laminations [12] would be half as costly and can achieve a bit higher packing fraction. \(L/L_0(t = 175\ \mu\text{m}) = 0.956\).

Do the eddy current losses [15] in the 100 \(\mu\text{m}\) thick iron laminations. Use equation 3 with a quarter meter square area, a 26.5 m length, and an average field of 1.1 Tesla. \(P = [(26.5) (5^2)] (2\pi 2600 1.1 .0001)^2 / [(24) 47 \times 10^{-8}] = 5900\) kW. Multiply by 18
magnets and divide by a duty factor of 300 to get an eddy current loss in the iron laminations of 350 kW or 700 watts/m of magnet. So the iron will need some cooling. The ring only ramps 30 times per second, so the $\int H \cdot dB$ hysteresis losses will be low, even more so because of the low coercive force ($H_c = 0.1$ Oersteds) of grain oriented silicon steel. This value of $H_c$ is eight times less than 1008 low carbon steel.

The low duty cycle of the neutrino factory leads to eddy current losses of less than a megawatt in a 4600 Hz, 917 m circumference ring. Gradients are switched within dipoles to minimize eddy current losses in ends. Muon survival is 83%.

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