Stylized features of single-nucleon momentum distributions

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Background: Nuclear short-range correlations (SRC) typically manifest themselves in the tail parts of the single-nucleon momentum distributions.

Purpose: To develop an approximate flexible method for computing the single-nucleon momentum distributions throughout the whole mass table, thereby including the majority of the effects of SRC. To use this method to study the mass and isospin dependence of SRC.

Method: The framework adopted in this work, corrects mean-field models for central, spin-isospin and tensor correlations by shifting the complexity induced by the SRC from the wave functions to the operators. It is argued that the expansion of these modified operators can be truncated to a low order.

Results: The proposed model can generate the SRC-related high-momentum tail of the single-nucleon momentum distribution. These are dominated by correlations operating on mean-field pairs with vanishing relative radial and angular-momentum quantum numbers. In asymmetric nuclei, the correlations make the average kinetic energy for the minority nucleons larger than for the majority nucleons.

Conclusions: The proposed method explains the dominant role of proton-neutron pairs in generating the SRC and accounts for the magnitude and mass dependence of SRC as probed in inclusive electron scattering. It also provides predictions for the ratio of the amount of correlated proton-proton to proton-neutron pairs which are in line with the observations.

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I. INTRODUCTION

One of the most elusive properties of nuclei is that nucleons forcefully repel each other as they get close. Since the early days of nuclear physics, it has been recognized that this repulsion is an important ingredient of the dynamics of nuclei, and induces pair, triple, \ldots{} correlations in the wave functions for atomic nuclei. This calls for a more sophisticated approach to the quantum mechanical structure of nuclei and the nuclear response to external probes.

Momentum distributions contain all the information about the momentum decomposition of the nuclear motion. The computation of single-nucleon momentum distributions has reached a very high level of sophistication. Ab-initio methods with variational wave functions can be used to compute the momentum distributions for nuclei up to $A = 12$ \cite{1,2}. Also for atomic mass number infinity, or nuclear matter, exact calculations with realistic nucleon-nucleon interactions can be performed \cite{3,4}. Momentum distributions for mid-heavy and heavy nuclei cannot be computed with exact methods to date. Advanced approximate schemes like cluster expansions \cite{5,6,7} and correlated basis function theory \cite{8,9} are able to provide momentum distributions for heavier nuclei.

We wish to develop an approximate practical way of computing the short-range contributions to momentum distributions for stable nuclei over the entire mass range. Thereby, we start from wave functions that can be written as correlation operators acting on a single Slater determinant. The computation of expectation values of one-body and two-body operators for those wave functions involves multi-body effective operators and a truncation scheme is in order. We propose a low-order correlation operator approximation, dubbed LCA, that truncates the modified correlated operator corresponding with an one-body operator to the level of two-body operators. For the computation of the single-nucleon momentum distribution, the LCA model developed in Sec. II preserves some fundamental properties like the normalization conditions.

In Sec. III we illustrate that the LCA method is a practical approximate way of computing the effect of SRC on single-nucleon momentum distributions for nuclei over the entire mass range. It will be shown that after inclusion of central, spin-isospin and tensor correlations, it can capture some stylized features of nuclear momentum distributions. Due to its wide range of applicability, the LCA framework allows one to study the mass and isospin dependence of SRC and to arrive at a comprehensive picture of the impact of SRC throughout the mass table. To assess how realistic the LCA method is, we compare its one-body momentum distributions for $^4$He, $^9$Be and $^{12}$C with those from ab-initio calculations.

Of course, the LCA approximate method is only justified if the resulting physical quantities like radii and kinetic energies are in reasonable agreement with data.

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and results from more realistic approaches. The impact of short-range dynamics on the average nucleon kinetic energies and the rms radii for symmetric and asymmetric nuclei is discussed in Sec. [LV]. As the correlations induce high-momentum components, they increase the average kinetic energies. The isospin dependence of the SRC is at the origin of some interesting features which depend on the asymmetry of nuclei [7, 12]. Also these asymmetry effects will be discussed in Sec. [LV]

II. FORMALISM

A time-honored method to account for correlations in independent particle models (IPM) is to shift the complexity induced by the correlations from the wave functions to the operators [3, 13]. The correlated wave functions |Ψ⟩ are constructed by applying a many-body correlation operator $\hat{G}$ to the uncorrelated wave functions |Φ⟩. The operator $\hat{G}$ corrects the IPM Slater determinant |Φ⟩ for short-range and other correlations:

$$| \Psi ⟩ = \frac{1}{N} \hat{G} | \Phi ⟩,$$  (1)

with the normalization factor $N \equiv ⟨Φ | \hat{G}^† \hat{G} | Φ ⟩$. Determining the operator $\hat{G}$ represents a major challenge [14]. One can be guided, however, by the knowledge of the basic features of the nucleon-nucleon force. As far as the short-range nucleon-nucleon (NN) correlations are concerned, $\hat{G}$ is dominated by the central, spin-isospin and tensor correlations [4, 15, 16].

$$\hat{G} ≈ \hat{S} \left( \prod_{i<j=1}^{A} \left[ 1 + \hat{l}(i,j) \right] \right),$$  (2)

with $\hat{S}$ the symmetrization operator and

$$\hat{l}(i,j) = -\hat{g}(i,j) + \hat{s}(i,j) + \hat{t}(i,j)$$

$$= -g_{c}(r_{ij}) + f_{\sigma\tau}(r_{ij}) \hat{\sigma}_{1} \cdot \hat{\sigma}_{2} \hat{\tau}_{1} \cdot \hat{\tau}_{2}$$

$$+ f_{\tau\tau}(r_{ij}) \hat{S}_{ij} \hat{\tau}_{1} \cdot \hat{\tau}_{2}. $$  (3)

Here, $\hat{S}_{ij}$ is the tensor operator and $r_{ij} = |\vec{r}_{i} - \vec{r}_{j}|$. Further, $g_{c}(r_{12})$, $f_{\sigma\tau}(r_{12})$ and $f_{\tau\tau}(r_{12})$ are the central, spin-isospin and tensor correlation functions. This paper uses the central correlation function by Gearhart [17]. For the spin-isospin and tensor correlation functions we use those by Pieper et al. [18]. In Refs. [15, 19] we provided arguments and evidence to support our claim that these correlation functions can be considered realistic.

Evaluating the expectation value of an operator $\hat{Ω}$ between correlated states is far from trivial. The procedure detailed in Ref. [13] for example, amounts to rewriting the matrix element between correlated states

$$⟨ \Psi | \hat{Ω} | \Psi ⟩,$$  (4)

as a matrix element between uncorrelated states

$$\frac{1}{N} ⟨ Φ | \hat{Ω}^{\text{eff}} | Φ ⟩. $$  (5)

Hence, the impact of the correlations is implemented in an effective transition operator $\hat{Ω}^{\text{eff}}$ that corrects the operator $\hat{Ω}$ for the effect of $NN$ correlations

$$\hat{Ω}^{\text{eff}} = \hat{G}^† \hat{Ω} \hat{G}$$

$$= \left( \prod_{i<j=1}^{A} \left[ 1 - \hat{l}(i,j) \right] \right) \hat{Ω} \left( \prod_{k<l=1}^{A} \left[ 1 - \hat{l}(k,l) \right] \right). $$  (6)

For the sake of computing single-nucleon momentum distributions, it suffices to consider one-body operators

$$\hat{Ω} = \sum_{i=1}^{A} \hat{Ω}^{[1]}(i). $$  (7)

The universal character of SRC hints at a local dynamical origin, which naturally truncates a perturbation expansion like the one of Eq. (6). Further, studies of the single-nucleon spectral function in nuclear matter [6] reveal that the correlated part is mainly furnished by three-body breakup processes. For a finite nucleus A this translates into processes with two close-proximity correlated nucleons and a spectator residual $A - 2$ core. This picture has been confirmed in semi-exclusive $A(e,e'p)$ measurements [20, 21]. These observations allow one to treat the SRC as pair correlations to a good approximation. It also justifies a perturbation expansion of the Eq. (6) that truncates the effective operators corresponding with a one-body operator $\hat{Ω}^{[1]}$ to the level of two-body operators.

In the LCA framework used in this work, one adopts a perturbation expansion for the Eq. (6). For an effective operator corresponding with a one-body operator, we truncate the $\hat{Ω}^{\text{eff}}$ to the level of two-body operators and retain the terms that are linear and quadratic in the correlation operator $\hat{l}$. The quadratic terms contain terms with both correlation operators acting on the same particle pair. This results in the following effective operator

$$\hat{Ω}^{\text{eff}} \approx \hat{Ω}^{\text{LCA}} = \sum_{i=1}^{A} \hat{Ω}^{[1]}(i)$$

$$+ \sum_{i<j=1}^{A} \left\{ \hat{Ω}^{[1],l}(i,j) + \left[ \hat{Ω}^{[1],l}(i,j) \right]^{†} + \hat{Ω}^{[1],q}(i,j) \right\}. $$  (8)

Here, the linear (l) and quadratic (q) terms read

$$\hat{Ω}^{[1],l}(i,j) = \hat{l}(i,j) [\hat{Ω}^{[1]}(i) + \hat{Ω}^{[1]}(j)] \hat{l}(i,j), $$  (9)

$$\hat{Ω}^{[1],q}(i,j) = \hat{l}(i,j) [\hat{Ω}^{[1]}(i) + \hat{Ω}^{[1]}(j)] \hat{l}(i,j). $$  (10)
The LCA effective operator of Eq. 8 has one- and two-body terms, and can be conveniently rewritten as \( \hat{\Omega}^{\text{LCA}} = \sum_{i<j} A_{i<j}^{\text{LCA}} (i,j) \) with

\[
\hat{\Omega}^{\text{LCA}} (i,j) = \frac{1}{A-1} \left[ \hat{\Omega}^{[1]} (i) + \hat{\Omega}^{[1]} (j) \right] + \hat{\Omega}^{[1],\text{corr}} (i,j),
\]

whereby we have introduced a short-hand notation for that part of the operator associated with the correlations

\[
\hat{\Omega}^{[1],\text{corr}} (i,j) = \hat{\Omega}^{[1],\text{corr}} (i,j) + \left[ \hat{\Omega}^{[1],\text{corr}} (i,j) \right]^\dagger + \hat{\Omega}^{[1],\text{corr}} (i,j). \quad (11)
\]

In the absence of correlations only the first term in the expansion of Eq. 8 does not vanish. At medium internucleon distances \((r_{ij} \geq 3 \text{ fm})\) one has that \(\hat{\Omega}(i,j) \rightarrow 0\) and the effective operator \(\hat{\Omega}^{\text{LCA}}\) equals the uncorrelated operator \(\hat{\Omega}\).

The single-nucleon momentum distribution \(n^{[1]} (p)\) is defined as

\[
n^{[1]} (p) = \int \frac{d^3 \Omega_p}{(2\pi)^3} \int d^3 \vec{r}_1 d^3 \vec{r}'_1 d^3 \Omega_p \{ \vec{r}'_2 - A \} \times \Psi(\vec{r}'_1, \vec{r}'_2 - A) \Psi(\vec{r}_1, \vec{r}_2 - A). \quad (13)
\]

The corresponding single-nucleon operator \(\hat{n}_p\) reads

\[
\hat{n}_p = \frac{1}{A} \sum_{i=1}^{A} \int \frac{d^3 \Omega_p}{(2\pi)^3} e^{-i\vec{p} \cdot (\vec{r}'_i - \vec{r}_i)} = \sum_{i=1}^{A} \hat{n}^{[1]} (i). \quad (14)
\]

This operator and the expansion of Eq. (11) determine an effective two-body operator \(\hat{n}^{\text{LCA}}_p\) from which the correlated single-nucleon momentum distributions at momentum \(p\) can be computed.

In order to preserve the normalization properties \(\int dp \hat{n}^{[1]} (p) = 1\) in the LCA, the normalization factor \(\mathcal{N}\) of Eq. (1) is expanded up to the same order as the operator of Eq. (11),

\[
\mathcal{N} = 1 + \frac{2}{A} \sum_{\alpha<\beta} \sum_{n}\langle \alpha \beta | \hat{l}_1 (1,2) + \hat{l}_1 (1,2) \hat{l}_1 (1,2) + \hat{l}_1 (1,2) | \alpha \beta \rangle \text{nas}.
\]

TABLE I. The norm \(\mathcal{N}\) of Eq. (15) for a wide range of nuclei.

| \(A\) | 1.128 | 1.637 |
|-------|------|------|
| \(^4\text{He}\) | 1.327 | \(^{40}\text{Ca}\) |
| \(^{9}\text{Be}\) | 1.384 | \(^{56}\text{Fe}\) |
| \(^{12}\text{C}\) | 1.435 | \(^{108}\text{Ag}\) |
| \(^{16}\text{O}\) | 1.527 | \(^{197}\text{Ag}\) |
| \(^{27}\text{Al}\) | 1.545 | \(^{208}\text{Pb}\) |

In a HO basis, a transformation from \((\vec{r}_1, \vec{r}_2)\) to \((\vec{r}_{12} = \vec{r}_1 - \vec{r}_2, \vec{R}_{12} = \vec{r}_{12} + \vec{r}_{12})\) for the nas two-nucleon state can be readily performed 22

\[
| \alpha \beta \rangle_{\text{nas}} = \sum_A C^A_{\alpha \beta} | A\rangle,
\]

with \(A = \{ n \ell \ell' m \ell', \text{NL} M_{\ell'}, \text{TM} M' \}\) and \(C^A_{\alpha \beta} \equiv \langle A | \alpha \beta \rangle_{\text{nas}}\). Here, \(n\) and \(l\) are the radial and orbital angular-momentum quantum numbers corresponding with the relative motion of the pair. The \(j m_j\) are the quantum numbers of the total angular momentum of the pair. The \(T M_F\) (S) determine the isospin (spin) quantum numbers of the pair. The c.m. wave function is described by the quantum numbers \(NLM_L\).

Table I lists the computed values of the normalization factors of Eq. (15) for a range of nuclei from \(^2\text{H}\) to \(^{208}\text{Pb}\). The model dependence of the computed \(\mathcal{N}\) is related to the choices made with regard to the IPM basis and the correlation functions. The deviation of \(\mathcal{N}\) from 1 can be interpreted as a quantitative measure for the total effect of the SRC operators on the IPM ground-state wave function. For the deuteron, the tensor correlation operator acting on the relative S-wave of the IPM nucleon pair wave function is responsible for the D-wave component. The LCA is a crude approximation for the proton-neutron deuteron system. Nevertheless, the tail part of the LCA deuteron momen-
FIG. 2. The measured magnitude of the EMC effect, \( -\frac{dR_{EMC}}{dx} \), is plotted as a function of the computed \( R_2(A/2H) \) ratios defined in Eq. (18). The values of the EMC magnitude are from the analysis presented in Ref. [24]. The fitted dashed line obeys the equation \( -\frac{dR_{EMC}}{dx} = (0.033 \pm 0.035) + (0.071 \pm 0.009) \cdot R_2(A/2H) \).

### III. SINGLE-NUCLEON MOMENTUM DISTRIBUTION

In order to test the realistic character of the LCA method, in Fig. 3 we compare the LCA results for the \( n^{[1]}(p) \) with those obtained with quantum Monte-Carlo (QMC) methods using realistic two-nucleon and three-nucleon Hamiltonians [1]. To facilitate the comparison over the various nuclei we adopt the normalization \( 1 = \int dp \ p^2 n^{[1]}(p) \). Clearly, the predicted momentum dependence of the QMC and LCA methods is qualitatively comparable. Up to the characteristic nuclear Fermi momentum \( p_F = 1.25 \text{ fm}^{-1} \), the shape of \( n^{[1]}(p) \) is very Gaussian in both approaches. For \( p > p_F \) the distribution is heavy-tailed. The QMC and the LCA method predict a comparable exponential-like fat tail, which is very remarkable given the very different frameworks in which the results are obtained. It is not surprising that for \(^4\text{He}\) and \(^9\text{Be}\) the LCA and QMC display some differences at low \( p \), given that LCA does not account for the complicated long-range cluster structures of those nuclei. For \(^{12}\text{C}\), the LCA and QMC results are in reasonable agreement. In this context, it is worth mentioning that the nuclear-matter studies of Ref. [7] have clearly illustrated that the fat tails of the single-nucleon distributions are sensitive to the adopted realistic nuclear-nucleon interaction. This is related to the fact that the short-range part of the NN force is not well constrained by a fit to scattering data.

The computed momentum dependence of the \( n^{[1]}(p) \) are displayed in Fig. 4 for a range of nuclei from \(^4\text{He}\) to Ag. Some stylized features which apply to all studied nuclei are emerging from the LCA calculations. For \( p \lesssim 1.5 \text{ fm}^{-1} \) the distribution is dominated by the IPM contribution. The fat tails are induced by the correlations whereby one distinguishes two regions. For \( 1.5 \lesssim p \lesssim 3 \text{ fm}^{-1} \) the tensor correlations dominate. The effect of the central correlations extends over a large momentum range and for \( p > 3.5 \text{ fm}^{-1} \), it represents the dominant contribution to \( n^{[1]}(p) \) (with the tensor part gradually losing in importance). For all nuclei the crossover between the tensor and the central correlated part of the tail of \( n^{[1]}(p) \) occurs at a momentum slightly larger than \( 3 \text{ fm}^{-1} \). At momenta approaching \( 4 \text{ fm}^{-1} \) the central correlations provide about half of the the \( n^{[1]}(p) \). A major source of strength to the other half is due to the interference between the central and spin-isospin correlations (not shown separately in Fig. 4). This qualitative behavior is in line with the ab-initio \(^4\text{He}\) results of Ref. [8] (see Figure 3 of that reference). The abovementioned conclusions which apply to the correlated part of the one-body momentum distributions of all nuclei studied here, are qualitatively in line with the nuclear-matter results of Ref. [7]. This illustrates that the effect of SRC on single-nucleon momentum distributions can be summarized in some universally applicable principles.

The dominant role of the tensor correlations for intermediate nucleon momenta \( 1.5 \lesssim p \lesssim 3 \text{ fm}^{-1} \), has
some important implications for the isospin dependence of the effect of short-range correlations. With the aid of Eq. (11) one can write

\[ n^{[1]}(p) = n^{[1]}_{pp}(p) + n^{[1]}_{nn}(p) + n^{[1]}_{pn}(p), \]  

with

\[ n^{[1]}_{N_1N_2}(p) = \frac{1}{N} \sum_{\alpha < \beta} \delta_{\alpha,N_1} \delta_{\beta,N_2} \times \langle \alpha\beta | \hat{L}\text{^{LCA}}(1,2) | \alpha\beta \rangle_{\text{nas}}. \]  

The LCA results for \( n^{[1]}_{N_1N_2}(p) \) are shown in Fig. 5. The ratio \( r_{N_1N_2}(p) \equiv n^{[1]}_{N_1N_2}(p)/n^{[1]}(p) \) quantifies the relative contribution of \( N_1N_2 \) pairs to \( n^{[1]}(p) \) at given momentum \( p \). In a naive IPM one expects momentum-independent values of \( r_{pp} = Z(Z-1)/A(A-1), \ r_{nn} = (N(N-1))/A(A-1) \) and \( r_{pn} = 2NZ/A(A-1) \). For \( p < p_F \) the plotted ratios in the bottom panel of Fig. 5 very much follow these naive expectations. The tensor dominated momentum range is characterized by an increase of the \( pn \) contribution to \( n^{[1]}(p) \).

The above discussion provides a natural explanation for the observation that SRC-sensitive reactions like two-
nucleon knockout ($A(e,e'pN)$ and $A(p,ppN)$ reactions for example) are very much dominated by the $pn$ channel in the tensor-dominated region which roughly corresponds with $1.5 \lesssim p \lesssim 3$ fm$^{-1}$. The bottom panels of Fig. 5 suggest that under those conditions the $pn$ channel can represent 90% of the correlated strength, leaving a mere 5% for the $pp$ channel. This prediction seems to be in line with the experimental observations.

Indeed, the small ratio of $pp$-to-$np$ pairs above the Fermi momentum was experimentally verified in $^{12}$C($e,e'p$) measured at Jefferson Lab [30]. The quoted $pp$ to $pn$ ratio of $1.2^{+0.35}_{-0.35}$, displayed in Fig. 5, is compatible with the LCA predictions thereby assuming that the $pp$ and $nn$ contributions are equal for $N=Z$ nuclei. From an analysis of the ratio $^{12}$C($p,pp$)/$^{12}$C($p,pn$), it could be inferred that the removal of a proton from the nucleus with initial momentum 275–550 MeV/c is 92$^{+8}_{-18}$ % of the time accompanied by a neutron [31]. Also this result is in line with the LCA predictions for $^{12}$C contained in Fig. 5. Our results indicate that similar anomalously large $r_{pn}/r_{pp}$ ratios may be found for heavier nuclei when probing the tensor-dominated tail of the single-nucleon momentum distribution.

Another interesting feature of the results of Fig. 5 is that the $r_{pp}(p) [r_{pn}(p)]$ reaches its minimum (maximum) at $p \approx 2$ fm$^{-1}$. For $p > 2$ fm$^{-1}$ the $r_{pp}(p)$ grows and the $r_{pn}(p)$ decreases. Experimental evidence supporting this prediction has been recently obtained in the simultaneous measurement of exclusive $^4$He($e,e'pp$) and $^4$He($e,e'pn$) at ($e,e'p$) missing momenta from 2 to 4.3 fm$^{-1}$ [29]. In those measurements, the kinematics is tuned to probe a nucleon at a given momentum $p > p_F$ in conjunction with its correlated partner. These are precisely the SRC induced two-nucleon processes which systematically dominate the LCA $n^{[1]}(p)$ above the Fermi momentum. One may be tempted to connect $A(e,e'pN)$ cross sections to two-nucleon momentum distributions (TNMD). First, even after cross-section factorization no direct connection between the cross sections and TNMD can be established [32]. Second, as has been pointed out in Ref. [3], a nice pictorial description is given in Figure 12 of that reference, the correlated part of the TNMD receives large SRC contributions from three-nucleon configurations. Thereby the correlation is mediated through a third nucleon. The exclusive $A(e,e'pN)$ measurements are not kinematically optimized to probe those three-nucleon configurations. The $A(e,e'pN)$ kinematical settings are optimized to probe SRC-related two-nucleon configurations, and it is precisely those configurations which are the source of strength of the tails of the single-nucleon momentum distributions.

The $^4$He data points shown in Fig. 5 are extracted from the $^4$He($e,e'pp$)/$^4$He($e,e'pn$) cross-section ratios
of Ref. [29], whereby we have assumed that \( r_{nn} = r_{pp} \). The \( r_{np} \) and \( r_{pp} \) cannot be directly connected to the \( ^4\text{He}(e,e'p)/^4\text{He}(e,e') \) and \( ^4\text{He}(e,e'p)/^4\text{He}(e,e') \) cross-section ratios also shown in Fig. 2 of Ref. [29]. Indeed, for \( p > p_p \) the \( r_{N_1N_2}(p) \) encodes information about correlated pairs, whereas the \( ^4\text{He}(e,e') \) cross sections also contain contributions from other sources like final-state interactions and triple correlations.

As the central correlations, which are blind for the isospin of the interacting pairs, gain in importance with increasing \( p \) one observes that the \( r_{N_1N_2}(p) \) ratios gradually approach a limiting value which is different from the IPM values, in particular for heavier nuclei.

The above discussions indicate that the LCA framework in combination with central and tensor correlations, captures the stylized features of the SRC including its mass and isospin dependence. We now wish to shed light on the underlying physics mechanics of the correlated part of the momentum distribution. More in particular we address the question: “What are the quantum numbers of the IPM pairs which are most affected by the correlations?” This discussion will lead to an understanding of the high \( p \) limits in the bottom panels of Fig. 5.

One can determine the contributions from the relative quantum numbers \( nl \) of the IPM pairs to the correlated part of \( n^{[1]}(p) \) (denoted by \( n^{[1],\text{corr}}(p) \)) by means of the expansion of Eq. [17]. One finds,

\[
\hat{\delta}^{[1],\text{corr}}_{nl,n'\ell'}(p) = \sum_{\alpha<\beta} \sum_{A,B} (C^{\alpha}_{\alpha'})^\dagger \left[ C_{\beta} \delta_{nn,A} \delta_{l\ell,d\ell'n\ell'} \right] \langle A | \hat{n}^{[1],\text{corr}}(1,2) | B \rangle ,
\]

(21)

where the operator \( \hat{n}^{[1],\text{corr}}(1,2) \) has been defined as in Eq. [12]. Obviously, one has

\[
\sum_{nl} \sum_{n'\ell'} \hat{\delta}^{[1],\text{corr}}_{nl,n'\ell'}(p) = n^{[1],\text{corr}}(p) .
\]

(22)

The \( \hat{\delta}^{[1],\text{corr}}_{nl,n'\ell'}(p) \) that provide the largest contribution to \( n^{[1]}(p) \) are shown in Fig. 6. It is clear that correlation operators acting on \( nl = 00 \) IPM pairs are responsible for the major fraction of the \( n^{[1]}(p) \) for \( p \gtrsim 2 \text{ fm}^{-1} \). For heavier nuclei, the contributions from pairs with \( n > 0 \) increase in importance. Non-diagonal \( \hat{\delta}^{[1],\text{corr}}_{nl,n'\ell'}(p) \) represent a small fraction of the high-momentum tail.

We wish to stress that correlation operators acting on IPM pairs can change the quantum numbers. For example, the tensor operator acting on the deuteron’s \( l = 0 \) IPM pair generates the correlated \( l = 2 \) state. The dominant role of \( nl = 00 \) IPM pairs in the creation of high-momentum components, provides support for our proposed method to quantify the SRC by counting the number of \( nl = 00 \) IPM pairs [13] [22] [32]. Consequently, for high \( p \) the central correlations dominate and the \( r_{N_1N_2}(p) \) ratios of Fig. 4 are connected with the amount of \( N_1N_2 \) IPM pairs with \( nl = 00 \). Using the computed number of \( nl = 00 \) pairs in \( ^{12}\text{C} \) we find \( r_{pp} = r_{nn} = 0.16 \) and \( r_{pn} = 0.68 \). For \(^{108}\text{Ag} \), a similar calculation leads to \( r_{pp} = 0.14, r_{nn} = 0.20 \) and \( r_{pn} = 0.66 \). For high \( p \) these numbers are fair predictions for the computed ratios \( r_{N_1N_2}(p) \) in Fig. 5.

IV. SINGLE-NUCLEON KINETIC ENERGIES AND RMS RADII

In a non-relativistic framework, the diagonal single-nucleon kinetic energy operator \( \hat{T}^{[1]} \) can be written as

\[
\hat{T}^{[1]} = \sum_{i=1}^{A} \hat{T}^{[1]}(i) = \sum_{i=1}^{A} \frac{-h^2}{2M_i} \nabla_i^2 ,
\]

(23)

where \( M_i \) is the nucleon mass. In the IPM, the average kinetic energy \( \langle T_p \rangle \) per proton is given by

\[
\langle T_p^{\text{IPM}} \rangle = \frac{1}{Z} \sum_{\alpha} \delta_{\alpha,p} \langle \alpha | \hat{T}^{[1]}(1) | \alpha \rangle .
\]

(24)

A similar definition is adopted for the average kinetic energy per neutron \( \langle T_n \rangle \). In the LCA framework developed in Sec. II one has

\[
\langle T_p^{\text{LCA}} \rangle = \frac{1}{N} \sum_{\alpha<\beta} \langle n^{[1],\text{corr}}(1,2) | \alpha \rangle \langle \alpha | \hat{T}^{[1]}(2) | \beta \rangle ,
\]

(25)

where the operator \( \hat{T}^{[1]} \) can be obtained from Eq. [11]. Since we work in a non-relativistic framework, we have adopted a hard cutoff of \( 4.5 \text{ fm}^{-1} \) for the maximum nucleon momentum in the calculations of the kinetic energy.

Table I compares the IPM and LCA predictions for the kinetic energies per proton and neutron. Obviously, as the kinetic energies can be associated with the fourth moments of the \( n^{[1]}(p) \), they are highly sensitive to the fat tails. Indeed, inclusion of the correlations increases the \( \langle T_p \rangle \) and \( \langle T_n \rangle \) by a factor of about two. For the sake of reference, the average kinetic energy of a one-component nuclear Fermi gas is 21 MeV. For the heaviest nuclei studied in this work we find values which are about 50% larger. The LCA results for the average kinetic energies for \(^{9}\text{Be} \) are comparable those of realistic calculations quoted in Table 1 of Ref. [33] — \( \langle T_p \rangle = 29.82 \text{ MeV} \) and \( \langle T_n \rangle = 27.09 \text{ MeV} \). As can be appreciated from Table I the LCA predictions for the correlated kinetic energies \( \langle T_N \rangle \) are comparable with those of the realistic model of Ref. [33]. The predictions for \( \langle T_N \rangle \) from the variational calculations of Ref. [36] are systematically smaller.

The parameter \( x_p = \frac{x}{4} \) is the proton fraction and is a measure for the asymmetry of nuclei. As expected for a non-interacting two-component Fermi system, is \( \langle T_p \rangle < \langle T_n \rangle \) for asymmetric nuclei (\( x_p < 0.5 \)) in the IPM. As can be appreciated from Fig. 7 after inclusion of the correlations, the situation is reversed with the minority component having a larger average kinetic energy. This can be attributed to the tensor correlations, which are
We now discuss the effect of the correlations on the computed rms radii. The IPM predictions which are obtained with the operator \( \hat{R} = \sum_i \langle \vec{r}_i \rangle \) that dominate the high momentum tail. The purple dashed line is the summed contribution of the the \( n_{nl,n'}^{[1]}(p) \) which are not shown separately.

stronger between \( pm \) than between \( pp \) and \( nn \) pairs. The difference between \( \langle T_p \rangle \) and \( \langle T_n \rangle \) increases roughly linearly with decreasing proton fraction \( x_p \). For the most asymmetric nucleus considered here, \(^{48}\text{Ca}\), \( \langle T_p \rangle \) is about 10% larger than \( \langle T_n \rangle \).

We now discuss the effect of the correlations on the root-mean-square (rms) radii of the nuclear matter distribution. The rms radii can be computed with an operator of the form

\[
\hat{R}^2 = \frac{1}{A} \sum_i \left( \vec{r}_i - \vec{R}_{cm} \right)^2,
\]

with \( \vec{R}_{cm} = \frac{1}{A} \sum_i \vec{r}_i \). Using a procedure which is completely similar to the one used for the kinetic energy, in the LCA the operator \( \hat{R}^2 \) becomes a correlated operator with a one-body and a two-body part. Table III compares the IPM and the LCA predictions for the rms radii. The IPM predictions which are obtained with the global parameterization of Eq. (16) tend to overestimate the measured radii for light and heavy nuclei, and underestimate them for mid-heavy nucleus. All in all, the effect of the correlations on the computed rms radii is rather modest. The LCA predictions are in acceptable agreement with the experimental values and the predic-
V. SUMMARY

We have introduced an approximate flexible method, dubbed LCA, for the computation of the SRC contributions to the single-nucleon momentum distributions \( n^{[1]}(p) \) throughout the whole mass table. A basis of single-particle wave functions and a set of correlation functions serves as an input to LCA. For the numerical calculations presented here, we have included the central, spin-isospin and tensor correlations and mass-independent correlation functions. The approximate LCA method predicts the characteristic high-momentum part of the single-nucleon momentum distribution for a wide range of nuclei. For the light nuclei \( ^4\text{He} \), \( ^9\text{Be} \) and \( ^{12}\text{C} \), the LCA predictions for the tails of the single-nucleon momentum distributions are in line with those of sophisticated QMC calculations with realistic Hamiltonians. The predicted aggregated effect of SRC and its mass dependence closely matches the observations from inclusive electron scattering (\( a_2 \) coefficients and the magnitude of the EMC effect).

In the LCA, one can separate contributions of the central, spin-isospin and tensor correlations and study how these affect the relative strength of \( nn \), \( pp \) and \( pn \) pairs in the high-momentum tail of \( n^{[1]}(p) \). For \( 1.5 \lesssim p \lesssim 3 \text{ fm}^{-1} \) the \( n^{[1]}(p) \) is dominated by tensor-induced \( pn \) correlations. Our prediction for the relative strength of \( pp \) and \( pn \) pairs in the tail part of \( n^{[1]}(p) \) is in line with observations in exclusive two-nucleon knockout studies. We have shown that the high-momentum tail of \( n^{[1]}(p) \) is dominated by the correlation operators acting on mean-field pairs with vanishing relative radial quantum number and vanishing orbital angular momentum, i.e. IPM pairs in a close-proximity configuration. Another prediction of the LCA is that in asymmetric nuclei, the correlations are responsible for the fact that the average kinetic energy of the minority nucleons is larger than for the majority nucleons. The LCA method provides results for the correlated average kinetic energies and nuclear radii which are in line with those of alternate many-body approaches.

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TABLE III. Results from the IPM and LCA framework for the rms radii for a variety of nuclei. The results are compared with those from the Unitary Correlation Operator Method (UCOM) [35] and experimental values (Expt) [34]. All radii are in fm.

| \( A \)  | IPM | LCA | UCOM [35] | Expt [34] |
|---|---|---|---|---|
| \(^4\text{He}\) | 1.84 | 1.70 | 1.35 | 1.6755 ± 0.0028 |
| \(^9\text{Be}\) | 2.32 | 2.13 | 2.51 | 2.4702 ± 0.0022 |
| \(^{12}\text{C}\) | 2.46 | 2.23 | 2.36 | 2.6991 ± 0.0052 |
| \(^{16}\text{O}\) | 2.59 | 2.32 | 2.28 | 3.0610 ± 0.0031 |
| \(^{27}\text{Al}\) | 3.06 | 2.72 | 2.82 | 3.4771 ± 0.0020 |
| \(^{40}\text{Ca}\) | 3.21 | 2.84 | 2.93 | 3.7377 ± 0.0016 |
| \(^{48}\text{Ca}\) | 3.47 | 3.05 | 3.20 | 3.7377 ± 0.0016 |
| \(^{56}\text{Fe}\) | 3.63 | 3.20 | 3.20 | 3.7377 ± 0.0016 |
| \(^{108}\text{Ag}\) | 4.50 | 3.94 | 4.6538 ± 0.0025 |
| \(^{197}\text{Au}\) | 5.73 | 5.21 | 5.4371 ± 0.0038 |
| \(^{208}\text{Pb}\) | 5.83 | 5.28 | 5.5012 ± 0.0013 |

Weations from the UCOM framework of Ref. [35]. We stress that our IPM results are obtained with a single Slater determinant with HO wave functions from the global parameterization of Eq. [16]. It is likely that one can find a slightly modified parametrization that brings the LCA rms radii closer to the data.

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