SPONTANEOUS SYMMETRY BREAKING IN PRESENCE OF ELECTRIC AND MAGNETIC CHARGES

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Abstract

Starting with the definition of quaternion gauge theory, we have undertaken the study of $SU(2)_e \times SU(2)_m \times U(1)_e \times U(1)_m$ in terms of the simultaneous existence of electric and magnetic charges along with their Yang-Mills counterparts. As such, we have developed the gauge theory in terms of four coupling constants associated with four-gauge symmetry $SU(2)_e \times SU(2)_m \times U(1)_e \times U(1)_m$. Accordingly, we have made an attempt to obtain the abelian and non-Abelian gauge structures for the particles carrying simultaneously the electric and magnetic charges (namely dyons). Starting from the Lagrangian density of two $SU(2) \times U(1)$ gauge theories responsible for the existence of electric and magnetic charges, we have discussed the consistent theory of spontaneous symmetry breaking and Higgs mechanism in order to generate the masses. From the symmetry breaking, we have generated the two electromagnetic fields, the two massive vector $W^\pm$ and $Z^0$ bosons fields and the Higgs scalar fields.

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1 Introduction

Symmetry plays the central role in determining its dynamical structure. The Lagrangian exhibits invariance under $SU(2) \times U(1)$ gauge transformations for the electroweak interactions. Since the imposition of local symmetry implies the existence of massless vector particles [[1]], Higgs mechanism is used for the spontaneous breaking of gauge symmetry to generate masses for the weak gauge bosons charged as well as neutral particle [[2]]. If these features of the gauge theory are avoided, we obtain massive vector bosons and hence the gauge symmetry must be broken. In the Higgs mechanism a larger symmetry is spontaneously broken into a smaller symmetry through the vacuum expectation value of the Higgs field and accordingly gauge bosons become massive. The simplest way of introducing spontaneous symmetry breakdown is to include scalar Higgs fields by hand into the Lagrangian [[3]]. Recently, we [[4]] have made an attempt to develop the quaternionic formulation of Yang–Mills field equations and octonion reformulation of quantum chromo dynamics (QCD) by taking magnetic monopole into account [[5]]. It has been shown that the three quaternion units explain the structure of Yang-Mill’s field while the seven octonion units provide the consistent structure of $SU(3)_C$ gauge symmetry of quantum chromo dynamics. Our theory differs from the quaternion gauge theory of spontaneously symmetry breaking mechanism already developed by others [[6], [7], [8], [9]] in terms of gauge groups and methodology adopted by them in different manners. In this paper, we have extended our previous results to develop a meaningful gauge theory which may purport a model for massive gauge particles. Starting with the definition of quaternion gauge theory, we have undertaken the study of $SU(2)_e \times SU(2)_m \times U(1)_e \times U(1)_m$ in terms of the simultaneous existence of electric and magnetic charges along with their Yang-Mills counterparts. As such, we have developed the gauge theory in terms of four coupling constants associated with four-gauge symmetry $SU(2)_e \times SU(2)_m \times U(1)_e \times U(1)_m$. Accordingly, we have made an attempt to obtain the abelian and non-Abelian gauge structures for the particles carrying simultaneously the electric and magnetic charges (namely dyons). Starting from the Lagrangian density of two $SU(2) \times U(1)$ gauge theories responsible for the existence of electric and magnetic charges, we have discussed the consistent theory of spontaneous symmetry breaking and Higgs mechanism in order to generate the masses. From the symmetry breaking, we have generated the two electromagnetic fields, the two massive vector $W^\pm$ and $Z^0$ bosons fields and the Higgs scalar fields. Here, we have briefly discussed the Higgs mechanism for the case of general non-abelian local symmetries.
2 Quaternion Gauge Formalism

Let \( \phi(x) \) be a quaternionic field (Q field) and expressed [4, 6, 7, 8, 9] as

\[
\phi = e_0 \phi_0 + e_j \phi_j \quad (\forall j = 1, 2, 3)
\]  

(1)

where \( \phi_0 \) and \( \phi_j \) are local Hermitian fields and \( e_0 \) and \( e_j \) are the imaginary basis of \( Q \) which satisfy the following property

\[
e_0^2 = e_0; \quad e_0 e_j = -\delta_{jk} e_0 + \epsilon_{jkl} e_l \quad (\forall j, k, l = 1, 2, 3).
\]  

(2)

In global gauge symmetry, the unitary transformations are independent of space and time. Accordingly, under \( SU(2) \) global gauge symmetry, the quaternion spinor \( \psi \) transforms as

\[
\psi \rightarrow \psi' = U \psi
\]  

(3)

where \( U \) is a \( 2 \times 2 \) unitary matrix and satisfies

\[
U^\dagger U = U U^\dagger = U U^{-1} = U^{-1} U = 1.
\]  

(4)

On the other hand, the quaternion conjugate spinor transforms as

\[
\bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi} U^{-1}
\]  

(5)

and hence the combination \( \bar{\psi} \bar{\psi}' = \bar{\psi} \bar{\psi} = \bar{\psi} \bar{\psi}' = \bar{\psi}' \bar{\psi} \) is an invariant quantity. We may thus write any unitary matrix as

\[
U = \exp \left( i \hat{H} \right) \quad (i = \sqrt{-1})
\]  

(6)

where \( \hat{H} \) is Hermitian \( \hat{H}^\dagger = \hat{H} \). Thus, we express the Hermitian \( 2 \times 2 \) matrix in terms of four real numbers, \( a_1, a_2, a_3 \) and \( \theta \) as

\[
\hat{H} = \theta \hat{1} + \tau_j a_j = \theta \hat{1} + i e_j a_j
\]  

(7)

where \( \hat{1} \) is the \( 2 \times 2 \) unit matrix, \( \tau_j \) are well known \( 2 \times 2 \) Pauli-spin matrices and \( e_1, e_2, e_3 \) are the quaternion units which are connected to Pauli-spin matrices as

\[
e_0 = 1; \quad e_j = -i\tau_j.
\]

Hence, we write the Hermitian matrix \( \hat{H} \) as

\[
\hat{H} = \begin{pmatrix} \theta + a_3 & a_1 - ia_2 \\ a_1 + ia_2 & \theta - a_3 \end{pmatrix}.
\]

Equation (9) is now reduced to be

\[
U = \exp(i\theta) \cdot \exp(-e_ja_j).
\]

For \( SU(2) \) global gauge transformations both \( \theta \) and \( \vec{a} \) are independent of space time. Here \( \exp(i\theta) \) describes the \( U(1) \) gauge transformations while the term \( \exp(-e_ja_j) \) represents the non-Abelian \( SU(2) \) gauge transformations. Thus under global \( SU(2) \) gauge transformations, the Dirac spinor \( \psi \) transforms as

\[
\psi \mapsto \psi' = U\psi = \exp(-e_ja_j)\psi.
\]

The generators of this group \( e_j \) obey the commutation relation;

\[
[e_j, e_k] = 2f_{jkl}e_l
\]

which implies \( e_je_k \neq e_ke_j \) showing that the elements of the group are not commutating giving rise to the non abelian gauge structure whose structure condstant is \( f_{jkl} \). So, the partial derivative of spinor \( \psi \) transforms as

\[
\partial_\mu \psi(x) \mapsto \partial_\mu \psi'(x) = \exp(-e_ja_j)(\partial_\mu \psi).
\]

For \( SU(2) \) local gauge transformation we may replace the unitary gauge transformation as space - time dependent. So on replacing \( U \) by \( S \) in equation (10), we get
\[ \psi \rightarrow \psi' = S \psi \]  

(14)

in which

\[ S = \exp[- \sum_j q e_j \zeta_j(x)] \]  

(15)

where parameter \( \zeta = - \frac{\overrightarrow{a}(x)}{q} \) with \( \overrightarrow{a}(x) \) is infinitesimal quantity depends on space while time and \( q \) is described as the coupling constant. In \( SU(2) \) local gauge symmetry as the partial derivative which contains an extra term is then replaced by a covariant derivative i.e.

\[ \partial_\mu \psi \rightarrow S \partial_\mu \psi + (\partial_\mu S) \psi \rightarrow D_\mu \psi \]  

(16)

where the covariant derivative \( D_\mu \) is defined in terms of two \( Q \)-gauge fields \[ \] i.e

\[ D_\mu \psi = \partial_\mu \psi + A_\mu \psi + B_\mu \psi \]  

(17)

where \( A_\mu = -i A^j_\mu \tau_j = A^0_\mu e^0_j = \overrightarrow{A}_\mu \cdot \overrightarrow{e} \) and \( B_\mu = -i B^j_\mu \tau_j = B^0_\mu e^0_j = \overrightarrow{B}_\mu \cdot \overrightarrow{e} \). Two gauge fields \( A_\mu \) and \( B_\mu \) are respectively associated with electric and magnetic charges of dyons (i.e. particles carrying the simultaneous existence of electric and magnetic charges). Thus the gauge field \( \{A_\mu\} \) is coupled with the electric charge while the gauge field \( \{B_\mu\} \) is coupled with the magnetic charge (i.e. magnetic monopole). These two gauge fields are subjected by the following gauge transformations

\[ A'_\mu \rightarrow S A_\mu S^{-1} + (\partial_\mu S) S^{-1}; \quad B'_\mu \rightarrow S B_\mu S^{-1} + (\partial_\mu S) S^{-1}. \]  

(18)

For the limiting case of infinitesimal transformations of \( \zeta \), we may expand \( S \) by keeping only first order terms in the following manner as

\[ S \approx 1 + \overrightarrow{e} \cdot \overrightarrow{a}(x); \quad S^{-1} \approx 1 - \overrightarrow{e} \cdot \overrightarrow{a}(x); \quad \partial_\mu (S) \approx \overrightarrow{e} \cdot \partial_\mu \{ \overrightarrow{a}(x) \}. \]  

(19)

Thus, the corresponding gauge fields associated with electric and magnetic charges are expressed as,

\[ A_\mu = g_e e_0 A^0_\mu + g^i e_i A^i_\mu; \quad B_\mu = g_m e_0 B^0_\mu + g^i e_i B^i_\mu. \]  

(20)
In the above equation (20) \( g_e, g_m, g'_e, g'_m \) are the four coupling constants corresponding to the symmetry \( U(1)_e, U(1)_m, SU(2)_e, SU(2)_m \) so that the gauge fields \( A_\mu \) and \( B_\mu \) are transformed \([4, 7]\) as

\[
A'_\mu = UA_\mu U + U \partial_\mu U \quad B'_\mu = VB_\mu V + V \partial_\mu V. \tag{21}
\]

where \( U \) and \( V \) are two tuples of unitary gauge groups.

3 Quaternion Spontaneous Symmetry Breaking

Let us consider the local gauge invariance of the Lagrangian, we get

\[
L = (D_\mu \phi)(D^\mu \phi)^\ast - V(\phi^2) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} F_a^{\mu\nu} F_a^{\mu\nu} - \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} \tag{22}
\]

where \( F_{\mu\nu} \) is the electromagnetic field tensor for the description of electric charge, \((\ast)\) is used for complex conjugation when quaternions are compactified to complex numbers, \( G_{\mu\nu} \) is identical to the dual of \( F_{\mu\nu} \) and is responsible for the existence of magnetic charge while the potential term \( V(\phi^2) \) contains the usual mass and quadratic self-interaction terms described as

\[
V(\phi^\dagger \phi) = m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2. \tag{23}
\]

where \( m^2 \) and \( \lambda \) are real constant parameters while the symbol \((\dagger)\) denotes Hermitian conjugation. \( \lambda \) should be positive to ensure the stable vacuum. Furthermore, higher power terms of \((\phi^\dagger \phi)\) are not allowed in order to look the theory to be renormalized. Equation (22) is invariant under the local gauge transformations

\[
\phi' = \exp \left[ ie_0 (g_e + g_m) + e_j \left( g'_e + g'_m \right) \right] \phi. \tag{24}
\]

Let us take the variation in \( \phi \) as,

\[
\delta \phi = \left[ ie_0 (g_e + g_m) + e_j \left( g'_e + g'_m \right) \right]. \tag{25}
\]
After taking the variations, the Lagrangian (22) yields the following expression for current density $j_{\mu}$

\[ J_{\mu} = (j_{\mu})_{U(1)e} + (j_{\mu})_{U(1)m} + (j_{\mu})_{SU(2)e} + (j_{\mu})_{SU(2)m} \]  

(26)

where

\[ (j_{\mu})_{U(1)e} = i\epsilon_0 g_e \left[ \phi^\dagger D_\mu \phi - \phi D_\mu \phi^\dagger \right] ; \]  

(27)

\[ (j_{\mu})_{U(1)m} = i\epsilon_0 g_m \left[ \phi^\dagger D_\mu \phi - \phi D_\mu \phi^\dagger \right] ; \]  

(28)

\[ (j_{\mu})_{SU(2)e} = i\epsilon_j g'_e \left[ \phi^\dagger D_\mu \phi - \phi D_\mu \phi^\dagger \right] ; \]  

(29)

\[ (j_{\mu})_{SU(2)m} = i\epsilon_j g'_m \left[ \phi^\dagger D_\mu \phi - \phi D_\mu \phi^\dagger \right] ; \]  

(30)

are the currents respectively associated with the gauge groups $U(1)e$, $U(1)m$, $SU(2)e$ and $SU(2)m$. The condition for the minimum potential leads to

\[ \frac{\partial V}{\partial \phi} = 0 \Rightarrow \phi \left( m^2 + 2\lambda \phi^2 \right) . \]  

(31)

Now may discuss the different cases of $m^2$. For $m^2 > 0$, we have the situation for a massive scalar field particle, and $\phi = \phi_{\text{min}}$. Here $\phi = 0$, as the vacuum having $V = 0$. For the case $m^2 < 0$, we see that $\phi = \phi_{\text{min}}$ and $\phi = \pm v = \pm \left( \frac{-m^2}{2\lambda} \right)$. So, in this case the local gauge symmetry is spontaneously broken and we may obtain the vacuum expectation value for the scalar field $\phi$ as,

\[ \phi = \begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix} \]  

(32)

with $v = \sqrt{-\frac{m^2}{2\lambda}}$. For particle spectrum, the vacuum expectation value is modified as

\[ \phi = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} (v + \eta(x)) \end{bmatrix} \]  

(33)

where $\eta$ is the arbitrary parameter for excited state. Applying the equation (33), Lagrangian (22) is modified as
L' = \frac{1}{2} \partial_\mu \eta \partial_\mu \eta + \frac{1}{2} 2m^2 \eta^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} F_{\mu}^a F_{\mu}^a - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a - \lambda \nu \eta^3 - \frac{\lambda}{4} \eta^4 \\
+ \frac{1}{2} [v + \eta(x)]^2 [g_2^2 (A_{\mu}^0)^2 + g_2^2 (B_{\mu}^0)^2 + g_3^2 (A_{\mu}^2)^2 + g_3^2 (B_{\mu}^2)^2 + 2g_e g_m A_{\mu}^0 B_{\mu}^0 + 2g_e g_m A_{\mu}^0 B_{\mu}^0 + 2g_e g_m A_{\mu}^0 B_{\mu}^0 + 2g_e g_m A_{\mu}^0 B_{\mu}^0]

This Lagrangian describes the gauge group $SU(2)_c \times SU(2)_m \times U(1)_c \times U(1)_m$. We see that the Lagrangian density (34) is symmetric under $\eta \rightarrow -\eta$ so that the term $\lambda \nu \eta^3$ breaks the symmetry. It is to be noted that the Lagrangian density (22) is invariant under the changes taken to ground expectation value $\langle \phi \rangle_0 = v$ and $\phi = v + \eta$, but the new Lagrangian (34) breaks the symmetry and is no more invariant under these changes. It is only due to the mechanism of spontaneous symmetry breaking. Lagrangian (33) is also expressed as

$$L' = L_0 + L_I$$

where $L_0$ contains the kinetic energy and mass terms i.e.

$L_0 = \frac{1}{2} \partial_\mu \eta \partial_\mu \eta + \frac{1}{2} 2m^2 \eta^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} F_{\mu}^a F_{\mu}^a - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \frac{1}{2} v^2 [g_2^2 (A_{\mu}^0)^2 \\
+ g_2^2 (B_{\mu}^0)^2 + g_3^2 (A_{\mu}^2)^2 + g_3^2 (B_{\mu}^2)^2 + 2g_e g_m A_{\mu}^0 B_{\mu}^0 + 2g_e g_m A_{\mu}^0 B_{\mu}^0 + 2g_e g_m A_{\mu}^0 B_{\mu}^0 + 2g_e g_m A_{\mu}^0 B_{\mu}^0]

and the interaction term becomes,

$L_I = - \lambda \nu \eta^3 - \frac{\lambda}{4} \eta^4 + \left[ v \eta + \eta^2 \right] [g_2^2 (A_{\mu}^0)^2 + g_2^2 (B_{\mu}^0)^2 + g_3^2 (A_{\mu}^2)^2 + g_3^2 (B_{\mu}^2)^2 + 2g_e g_m A_{\mu}^0 B_{\mu}^0 + 2g_e g_m A_{\mu}^0 B_{\mu}^0 + 2g_e g_m A_{\mu}^0 B_{\mu}^0 + 2g_e g_m A_{\mu}^0 B_{\mu}^0]

We may now write the masses for gauge bosons and their scalar interaction terms in the form of $4 \times 4$ matrix as,
\[
\beta = \begin{bmatrix}
A_0^\mu & B_0^\mu & A_i^\mu & B_i^\mu
\end{bmatrix}
\begin{bmatrix}
g_e^2 & g_e g_m & g_e g_e' & g_e g_m \\
g_e g_m & g_m^2 & g_m g_e' & g_m g_m' \\
g_e g_e' & g_m g_e' & g_e'^2 & g_e' g_m' \\
g_e g_m' & g_m g_m' & g_e' g_m' & g_m'^2
\end{bmatrix}
\begin{bmatrix}
A_0^\mu \\
B_0^\mu \\
A_i^\mu \\
B_i^\mu
\end{bmatrix}
\] (38)

which can be further reduced to

\[
\beta = g_e^2 (A_0^\mu)^2 + g_m^2 (B_0^\mu)^2 + g_e'^2 (A_i^\mu)^2 + g_m'^2 (B_i^\mu)^2 + 2 g_e g_m A_0^\mu B_0^\mu + 2 g_e g_e' A_i^\mu A_i^\mu \\
+ 2 g_e g_m A_0^\mu B_i^\mu + 2 g_e g_m A_i^\mu B_0^\mu + 2 g_e g_m A_i^\mu B_i^\mu. 
\] (39)

4 Mass Generation due to Symmetry Breaking

Now we may discuss the different cases for \( SU(2) \times U(1) \) gauge groups of electro-weak unification as under:

**Case-I** For usual electroweak unification i.e. \( SU(2)_e \times U(1)_e \), we assume \( g_m' = g_m = 0 \) so that the mass contribution term is given by

\[
L_{mass} = \left( \frac{v g_e'}{2} \right)^2 W_\mu^+ W_\mu^- + \frac{v^2}{8} \left( A_0^\mu \ A_1^\mu \right) \begin{bmatrix}
g_e^2 & g_e g_e' \\
g_e g_e' & g_e'^2
\end{bmatrix} \begin{bmatrix}
A_0^\mu \\
A_1^\mu
\end{bmatrix}. 
\] (40)

It directly gives the masses of charged \( W \) bosons as,

\[
m_W = \frac{v g_e'}{2 g_e}; 
\] (41)

\[
W_\mu^\pm = \left( A_0^\mu \pm e_3 A_3^\mu \right); 
\] (42)

\[
Z^0 = \frac{e_0 g_e A_0^\mu + e_1 g_e' A_1^\mu}{\sqrt{g_e^2 + g_e'^2}}. 
\] (43)

**Case - II** For magnetoweak unification i.e.\( SU(2)_m \times U(1)_m \), we assume \( g_e' = g_e = 0 \) so that the mass contribution term is given by

\[
L_{mass} = \left( \frac{v g_m'}{2} \right)^2 W_\mu^+ W_\mu^- + \frac{v^2}{8} \left( B_0^\mu \ B_1^\mu \right) \begin{bmatrix}
g_m^2 & g_m g_m' \\
g_m g_m' & g_m'^2
\end{bmatrix} \begin{bmatrix}
B_0^\mu \\
B_1^\mu
\end{bmatrix}. 
\] (44)

Here we have another type of electroweak interaction due to the presence of magnetic
monopole for which the masses for the charged W bosons are obtained as

\[ m_W = \frac{v}{2} g_m; \]

\[ W^\pm = \left[ B^2_\mu \pm e_3 B^3_\mu \right]; \]  

\[ Z^0 = \frac{e_0 g_mB^0_\mu + e_1 g_mB^1_\mu}{\sqrt{g^2_m + g^2_e}}. \]  

**Case - III** For magneto-electroweak unification i.e. \( SU(2)_c \times U(1)_m \), we assume \( g'_m = g_e = 0 \) and the mass contribution to Lagrangian density is thus given by

\[ L_{mass} = \left( \frac{v g'_m}{2} \right)^2 W^\pm W^\mp + \frac{v^2}{8} \left( B^0_\mu \ A^1_\mu \right) \left( \begin{array}{cc} g^2_m & g_m g'_e \\ g_m g'_e & g^2_e \end{array} \right) \left( A^0_\mu \ B^1_\mu \right) \]  

which leads to the charged W boson masses as,

\[ m_W = \frac{v}{2} g'_m; \]

\[ W^\pm = A^2_\mu \pm e_3 A^3_\mu; \]

\[ Z^0 = \frac{e_1 g_e A^1_\mu + e_0 g_mB^0_\mu}{\sqrt{g^2_m + g^2_e}}. \]

**Case - IV** For electro-magnetoweak unification i.e. \( SU(2)_m \times U(1)_e \), we assume \( g'_e = g_m = 0 \) and the mass contribution to Lagrangian density is given by

\[ L_{mass} = \left( \frac{v g'_e}{2} \right)^2 W^\pm W^\mp + \frac{v^2}{8} \left( A^0_\mu \ B^1_\mu \right) \left( \begin{array}{cc} g^2_e & g_e g'_m \\ g_e g'_m & g^2_m \end{array} \right) \left( B^0_\mu \ A^1_\mu \right) \]  

which leads to the charged W boson masses as,

\[ m_W = \frac{v}{2} g'_e; \]

\[ W^\pm = B^2_\mu \pm e_3 B^3_\mu; \]

\[ Z^0 = \frac{e_1 g_m B^1_\mu + e_0 g_e A^0_\mu}{\sqrt{g^2_e + g^2_m}}. \]

From the foregoing analysis we find that the discovery of the spontaneous symmetry breaking (SSB) and the Higgs mechanism in the non-Abelian gauge theories describe a
great break through towards the unification of electromagnetic and weak interactions for different coupling parameters of quaternion gauge theories. We may also conclude that the zeroth model of quaternion leads the usual theory of electroweak interactions of standard model. On the other hand the imaginary units quaternion enlarges the gauge group leading to various gauge bosons which play an important role in extra dimensions of string theory. As such, the real understanding of the mechanism of the spontaneous breakdown and the Higgs mechanism is still extremely challenging problem to be solved in field theories.

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