Optimal Sharing and Fair Cost Allocation of Community Energy Storage

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Abstract—This paper studies an ES sharing model where multiple buildings cooperatively invest and share a community ES (CES) to harness economic benefits from on-site renewable integration and utility price arbitrage. Particularly, we formulate the problem that integrates the optimal ES sizing, operation and cost allocation as a coalition game, which are generally addressed separately in the literature. Particularly, we address the fair ex-post cost allocation which hasn’t been well studied. To overcome the computational challenge of computing the entire information of explicit characteristic functions (takes exponential time), we propose a fair cost allocation based on nucleolus by employing a constraints generation technique. We study the fairness and computational efficiency of the method through a number of case studies. The numeric results imply that the proposed method outperforms the Shapley approach and proportional method either in computational efficiency or fairness. Notably, for the proposed method, only a small fraction of characteristic functions (2.54%) is computed to achieve the cost allocation versus the entire information required by Shapley approach. With the proposed cost allocation, we investigate the enhanced economic benefits of the CES model over individual ES (IES) installation. We see the CES model provides higher cost reduction to each committed buildings. Moreover, the value of storage is obviously improved (about 1.83 times) with the CES model over the IES model.

Index Terms—community energy storage sharing, coalition game, cost allocation, nucleolus, fairness.

I. INTRODUCTION

Energy storage (ES) is a prospective and crucial technology to advance a sustainable, flexible and reliable energy system [1]. In consumer premise, ES can generate stacked economic benefits by enabling on-site renewable integration and electricity market engagement [2]. Specifically, consumers paired with ES can store surplus renewable generation to support future demand. Meanwhile, they can harvest monetary values from time-of-use utility price differentials and grid ancillary service. Though the economic benefit is profound, the commercial deployment of ES is still impeded by the high capital cost and long payback period [3, 4].

In recent years, sharing economy has manifested in transportation and housing [5]. Naturally, such sharing-oriented solution to bring capital-intensive new technologies, such as ES, to the grid is penetrating energy systems [5, 6]. Particularly, multiple users can cooperatively invest and share a central community ES (CES) [7, 8]. The added benefits of CES over individual ES (IES) are comprehensive. First of all, the marginal value of ES can be increased by exploring the complementary energy storage requirements of users. Besides, duplicate installation cost can be avoid, which generally accounts for 20%-50% of the total cost [10]. Moreover, it can gain additional benefits from economy of scale, i.e., purchase ES at scale with a lower price.

A. Related Works

In the literature, a number of works have focused on the two critical issues of ES sharing: i) optimal ES planning (i.e., sizing) and ii) optimal ES operation (i.e., ES resource or payoff allocation among the participants).

For the planning of shared ES, [11] proposed an analytical ES sizing approach based on a Markovian fluid model capturing the stochastic user demand. This paper mainly studied the required minimum ES capacity to support user demands under grid capacity limits. The added economic benefits of the shared ES through coordinating the end-users are not discussed. [12] also studied the optimal sizing of an shared ES. However, the operation of ES is centrally coordinated and the objective is to minimize the total cost of the consumers.

Another main category of related works primarily focuses on the operation or management of the shared ES; whereas most of them do not involve the optimal ES planning or ES capital cost distribution among the users. Typically, [8] proposed a credit-based distributed algorithm for the management of an ES shared by a group of cost-aware households. Specifically, the discharging rate of the shared ES is dynamically allocated among the households by their credits characterizing their accumulated stored energy in the shared ES. [13] proposed a centralized reputation-based management for ES sharing. The optimal operation of the shared ES is coordinated to minimize the reputation-weighted energy cost of the consumers, which are characterized by the proportions of their renewable injection into the shared ES over a historical time period. Similarly, [14] [15] studied the optimal centralized

1 The average revenue gained from per unit of ES resource
2 A Tesla Powerwall costs 7,600$ before installation. However, accounting for the installation cost, a rough estimate of the Tesla Powerwall cost $9,600-$15,600 for a full system installation.
control of a shared ES to optimize the weighted total cost of a group of homes which represents their agreements on cost-saving priorities. Slightly different, [16] proposed a shared ES model working as an energy provider serving consumers with elastic demand. The operation of the ES is managed by an aggregator obligated to minimize the total electricity bill of the consumers. Particularly, a marginal sizing price model was deduced to charge the consumers. [17] studied the optimal operation of a solar-plus-storage system shared across multiple consumers but with the objective to achieve social welfare and profit maximization.

We note that most of the existing works have addressed the optimal ES planning or operation (i.e., resource or payoff allocation) for ES sharing separately. However, for capital-intensive ES resources, they should be jointly considered so as to justify the high upfront cost or maximize the economic benefits. This paper works towards such objective which hasn’t been well studied. One existing close work is [18] which investigated the ES sharing among multiple consumers via a cooperative game formulation. Notably, an explicit fair cost allocation formula based on core was identified. However, this paper only considers price arbitrage under a simplified two-stage utility price setting and can not account for renewable integration. Exceptionally, [19] 20] studied the joint planning and operation of an ES model shared by multiple users with local renewable generation. However, those works focused on maximizing the profit of the ES developer instead of the consumers. Another work that is mostly related to ours is [21] which studied the optimal planning and operation of an ES shared across multiple electricity retailers. The operation of the ES was centrally coordinated to minimize the total cost. Particularly, the ex-post benefit is proportionally distributed among the retailers by their contribution degree (i.e., their electricity cost reduction).

In a nutshell, most existing works on shared ES models, especially CES models, have separately addressed the issues of ES planning (see [11] 12]) and ES operation (see [14] [15]). However, they are required to jointly considered to maximize the economic benefits [18] [21]. Generally, the optimal sharing of the ES capitalizes on dynamical coordination of the users to explore their complementary features. However, this requires a paired cost allocation mechanism to distribute the co-created value among the users. Most existing works have applied heuristic cost allocation mechanisms (see [8] [13] [21]) which is intuitive and easy to implement but do not address the fairness, which is a critical to ensure the stable cooperation and preserve the co-created economic benefits. Some scarce works have discussed the fairness based on the core or Shapley value. However, they either apply for some specific cases (e.g., price arbitrage [18]) or suffer from computational challenges [21].

B. Main contributions

Motivated by the literature, this paper studies an optimal ES sharing model where multiple buildings with on-site renewable generation cooperatively invest and share a community ES (CES). Our main contributions are outlined.

- We formulate the problem which integrates the optimal sizing, operation and payoff allocation as a coalition game.
- We address the fair ex-post cost allocation for ES sharing based on nucleolus. Particularly, the proposed method employs a constraint generation technique [22] to overcome the computational challenges of computing the entire information of characteristic functions.
- We demonstrate the fairness and computation efficiency of the proposed method through a number of case studies. We find only a small fraction of the characteristic functions (2.54%) is computed to achieve the cost allocation. Moreover, we compare the method with Shapley approach and proportional method. The numeric results imply that the proposed method outperforms the other two either in fairness or computational efficiency.
- We investigate the enhanced economic benefits of the CES model for individual buildings over the individual ES (IES) installation with the proposed cost allocation. Specifically, the CES model can provide higher cost reduction for each building. Moreover, the value of storage is obviously improved (about 1.83 times) with the CES model over the IES model.

The remainder of this paper is outlined. In Section II, we present the coalition game formulation. In Section III, we propose a fair cost allocation mechanism represented by nucleolus. In Section IV, we conduct the economic analysis of CES model through a number of case studies. In Section V, we briefly conclude this paper.

II. Coalition Game for CES Models

A. The configuration of community energy storage (CES)

The configuration of a CES is displayed in Fig. 1. We consider a grid-connected community microgrid composed of multiple buildings with on-site renewable generation (i.e., wind and solar power). We assume the buildings cooperatively invest and share a central ES to harness economic benefits from local renewable integration and utility price arbitrage. The building can charge the shared ES with local renewable generation or procured electricity from the grid to support their future non-elastic demand. Alternatively, the buildings can sell their surplus renewable generation back to the grid but usually at a relatively lower price. As the CES is shared by multiple buildings, we assume there is a central coordinator obligated to coordinate their charging and discharging behaviors with the objective to maximize the total economic benefits (i.e., minimize the total cost). Supportively, a net metering is attached to each building for monitoring the energy flow.

This paper studies the maximum economic benefits of the shared CES model which integrates the optimal ES sizing, operation as well as the ex-post cost allocation among the buildings. Considering the computational intensity of the long-term planning, we project the problem on a daily basis.

B. Main assumptions

We clarify our main assumptions as below.
we characterize the value of a CES coalition $\nu(S)$ by the total cost which is composed of the buildings’ electricity bills and the ES capital cost. Particularly, the coalition value $\nu(S)$ relies on a two-stage stochastic optimization problem which integrates the optimal ES sizing and the operation. We uses a collection of representative scenarios to capture the volatility of renewable generation. Therefore, we have the following two-stage stochastic optimization problem:

\[
\mathcal{P}: \nu(S) = \min_{E_S, P_S \geq 0} c(x_S) + \sum_{\omega \in \Omega} \rho_\omega g(x_S, \zeta_\omega) \\
\quad g(x_S, \zeta_\omega) = \min_{y^t_{i,t} \in S} g(x_S, y^t_{i,t}, \zeta_\omega) 
\]

subject to

\[
\begin{align*}
y^t_{i,t} &\in Y^t_i, \ i \in S. \\
\sum_{i \in S} c_{b,i} &\leq E_S, t \in T. \\
\sum_{i \in S} p^{c,i}_{t,t} &\leq P_S, t \in T. \\
\sum_{i \in S} p^{d,i}_{t,t} &\leq P_S, t \in T.
\end{align*}
\]

where $i$ and $t$ are building and time indices. $x_S = (E_S, P_S)$ denotes the ES capacity: energy capacity $E_S$ (in kW) and power capacity $P_S$ (in kW). $\omega$, $\zeta_\omega$, $\Omega$ and $p_\omega$ indicate the scenario indices, scenario realizations, scenario collection and scenario probability. $y^t_{i,t}$ denotes the operating strategy for building $i$ under scenarios $\omega$, which includes the charging/discharging of ES and energy trading with the grid. Correspondingly, $\mathcal{Y}^t_\omega$ indicates the admissible operation policies.

- The first-stage objective $\mathcal{P}_U$ captures the total cost which is equal to the ES capital cost $c(x_S)$ plus the weighted operation cost $\sum_{\omega \in \Omega} p_\omega g(x_S, \zeta_\omega)$. For the ES capital cost, we capitalize on an amortized price model $20, 24$: $c(x_S) = k_p P_S + k_i E_S$. $k_p$ ($\$/kWh$) and $k_i$ ($\$/kW$) are the amortized ES price which are obtained according to the projected ES installation cost $100€/kW$ and $300€/kWh$ by 2025 $25$. The calculation of the amortized ES price refers to Appendix A.

- The second-stage objective characterizes the optimal operation cost $g(x_S, y^t_{i,t}, \zeta_\omega)$ for each scenario $\omega$ which is subject to the ES capacity $x_S$ and scenario realization $\zeta_\omega$ and the operating policies of all the buildings in the coalition $Y^t_{\omega} = \{y^t_{i,t}, \forall i \in S\}$. The operation cost is constituted by three blocks: the cost of purchasing electricity from the grid, the revenue of selling energy back to the grid and the demand charge for the peak demand over the billing cycle. We use $c^t_{e,i}$, $c^t_{s,i}$ and $c^t_{\max,i}$ to indicate the purchasing, selling and demand charge price. $p^{c,i}_{t,t}$ and $p^{d,i}_{t,t}$ denote the procured and sold power at time $t$. $p^{c,i}_{\max,t}$ and $p^{d,i}_{\max,t}$ characterize the peak demand over the billing cycle. Then for each scenario $\omega$, we have the operation cost:

\[
g(x, \ y^t_{i,t}, \zeta_\omega) = \sum_{i \in S} \left\{ \sum_{t \in T} \left( c^t_{e,i} p^{c,i}_{t,t} - c^t_{s,i} p^{d,i}_{t,t} \right) + c^t_{\max,i} p^{d,i}_{\max,t} \right\}
\]
For brevity, \( y_t^{\omega} = [p_t^{ch,i,\omega}, p_t^{dis,i,\omega}, e_t^{\omega}, \eta_t^{\omega}, \phi_t^{\omega}, \mu_t^{\omega}, \eta_t^{max,\omega}, \phi_t^{max,\omega}] \) collects the operation of building \( i \) under scenarios \( \omega \), which includes i) the charging and discharging of the CES; \( p_t^{ch,i,\omega}, p_t^{dis,i,\omega} \) ii) the storage energy in the CES; \( e_t^{\omega} \), \( \eta_t^{\omega}, \phi_t^{\omega} \), and the peak demand over the billing cycle; \( \mu_t^{\omega}, \phi_t^{max,\omega} \).

Correspondingly, \( y_t^{\omega} \) indicates the admissible operation policies which is constituted by

\[
\begin{align*}
0 \leq p_t^{ch,i} & \leq p_t^{ch,max,\omega}, & (3a) \\
0 \leq p_t^{dis,i} & \leq p_t^{dis,max,\omega}, & (3b) \\
e_t^{\omega} & = e_{t+1}^{\omega} + p_t^{ch,i,\omega} \eta - p_t^{dis,i,\omega} & (3c) \\
0 \leq p_t^{ch,i,\omega} & \leq p_t^{ch,max,\omega}, & (3d) \\
- g_t^{\omega} & \leq p_t^{ch,i,\omega} - p_t^{dis,i,\omega} + p_t^{dis,i,\omega} - p_t^{dis,i,\omega} & (3e) \\
0 \leq p_t^{dis,i,\omega} & \leq p_t^{dis,max,\omega}, & (3f) \\
0 & \leq p_t^{ch,i,\omega} - p_t^{dis,i,\omega} & (3g) \\
0 & \leq p_t^{ch,i,\omega} - p_t^{dis,i,\omega} & (3h)
\end{align*}
\]

where constraints (3a)-(3b) model the physical limits of charging and discharging rate for each building \( i \). \( p_t^{ch,max,\omega} \) and \( p_t^{dis,max,\omega} \) denote the maximum charging and discharging power. Constraint (3c) tracks the stored energy of building \( i \) in the CES with \( e_t^{\omega}, \eta_t^{\omega} \) as ES roundtrip efficiency. Since energy sharing among different buildings is not allowed, each building can not over discharge their stored energy as imposed by constraint (3d). Constraint (3f) models the instantaneous balance of supply and demand for each building \( i \). \( p_t^{ch,i,\omega} \) and \( p_t^{dis,i,\omega} \) denote the non-elastic demand and local renewable generation. Constraint (3g) captures peak demand for each building which determines the demand charge in the billing cycle. Particularly, the complementary constraints (3g)-(3h) enforce each building not to charge (procure) or discharge (sell) simultaneously.

We note that problem (P) is non-linear and non-convex due to the presence of complementary constraints (3g)-(3h), which makes it computationally intractable with the off-the-shelf solvers. However, for the general case with roundtrip efficiency less than unity, i.e., \( \eta_t^{\omega} \eta_t^{dis} < 1 \), and the utility purchasing price higher than the selling price, i.e., \( \phi_t^{+} > \phi_t^{\omega} \), the complementary constraints can be relaxed without affecting the optimal solution of the second-stage optimization problem. An illustrative proof can refer to Appendix B.

We can deduce a convex optimization problem after relaxing the complementary constraints (3g)-(3h). However, the two-stage structure still poses challenges to apply the existing commercial solvers (e.g., CPLEX). To address this issue, we convert the two-stage problem to a single-stage problem:

\[
(P') : \min c(x_S) + \sum_{\omega \in \Omega} \rho_\omega g(x, y^{\omega}, \xi_\omega) 
\]

subject to: (1b) - (1d), (3a) - (3f), \( \forall i \in S \).

var: \( E_S, P_S, y^{\omega}_i, \forall i \in S \).

D. CES Coalition Game Properties

In this part, we study the characteristics of the CES coalition game. We have the following main results.

Theorem 1. The CES coalition game \((N, \nu)\) is subadditive, i.e., \( \nu(S^1 \cup S^2) \leq \nu(S^1) + \nu(S^2) \), \( \forall S^1, S^2 \subseteq N, S^1 \cap S^2 = \emptyset \).

Remark 2. We defer the proof to Appendix C. Theorem 7 implies that it won’t be worse off for two disjoint groups of buildings to merge and invest a single CES. Fundamentally, the economic benefits of CES can be further enhanced through coordination. This implies the buildings within a community microgrid are inclined to form a grand CES coalition \( N \) to maximize the economic benefits. Therefore, this paper focuses on the grand coalition \((N, \nu)\).

III. Cost Allocation based on Nucleolus

In Section II, the enhanced economic benefits of CES have been demonstrated. Specifically, the total cost for a group of buildings can be reduced by cooperatively sharing a central CES over installing individual ES separately. However, the buildings are independent stakeholders, how to allocate the total cost (or payoff through cooperation) among the buildings within a CES coalition is a critical issue and needs to be clarified. From another perspective, the CES coalition can be stabilized only if a fair cost allocation mechanism is paired, otherwise the building participants may deviate from cooperation.

There exist multiple solution concepts conceptualizing a fair cost allocation for a coalition game \([26]\). One prominent solution concept is core which characterizes a stable cost allocation that no groups of players have incentives to deviate from the current coalition \([23, 27]\). There are some standard procedures to compute the cores of a coalition game. Normally, the computation and identification of core correspond to solving a NP-complete linear programming (LP) problem with the characteristic functions explicitly available. For example, for an \( n \)-player coalition game, the computation of the core requires to solve a LP problem with \((2^n - 1)\) linear constraints, corresponding to the characteristic function with \(2^n - 1\) entries. This is a non-trivial task and requires enormous computation as the characteristic functions of a coalition game is generally not explicitly available and characterized by an optimization problem, like the CES coalition game discussed in this paper. Moreover, a cost allocation based on the solution concept of core may suffer from some other drawbacks in practice \([23]\): i) the core of a coalition game can be empty; and ii) the core may not be unique.

Another primary solution concept is the value of game (different from the value of a coalition) with Shapley value as a representative. The underlying idea is to distribute the co-created profit among the players by their marginal contribution. Generally, the marginal contribution of player \( i \) to a coalition \( S \) can be quantified by the increased coalition value of excluding building \( i \) from the coalition, i.e., \( \nu(S \setminus \{i\}) - \nu(S) \). One main advantage of Shapley approach over core is the existence and uniqueness. However, Shapley approach also
suffers computation intensity from computing the characteristic functions while identifying the marginal contribution of each player, which takes exponential time.

Another essential solution concept is nucleus. The nucleus was originally suggested by Schmeidler as a solution of a coalition game which minimizes the maximal dissatisfaction of the players [23]. The motivation behind nucleus is to search for a cost allocation mechanism that satisfy all players. The nucleus for a coalition game generally exists and is unique. However, the standard process for computing the nucleus is even more complex than the Shapley approach.

This paper studies a fair cost allocation for the CES game based on nucleus. To overcome the computational challenges to search for the nucleus, this paper develops a cost allocation mechanism based on constraints generation technique proposed in [22].

A. Definitions

We first give the some important definitions for cost allocation and core.

Definition 1. (Cost allocation) A cost allocation of a coalition game $(S, \nu)$ is usually indicated by a vector $x \in \mathbb{R}^{|S|}$ (| | indicates the carnality of a set), where the entry $x_i$ denotes the cost allocated to player $i$.

Definition 2. (Imputation) The imputation corresponds to the cost allocations $x \in \mathbb{R}^{|N|}$ of a grand coalition $N$, which are both efficient, i.e., $\nu(N) = x(N)$ and individually rational (i.e., $\nu(i) \geq x_i, \forall i \in N$), where we have $x(S) = \sum_{i \in S} x_i, \forall S \subseteq N$. Generally, the imputation of a grand coalition $(N, \nu)$ is given by

$$I = \left\{ x \in \mathbb{R}^N : x(N) = \nu(N) \text{ and } \nu\{\{i\}\} \geq x_i, \forall i \in N \right\}$$

Definition 3. (Core) The core refers to the set of imputations that no subsets of players has any incentives to deviate from the grand coalition $(N, \nu)$. We have the specific definition of core given by

$$C = \left\{ x \in I : x(S) \geq \nu(S), \forall S \subseteq N \right\}$$

Beside, we give two important concepts that relates to the solution concept of nucleus.

Definition 4. (Lexicographical order) Suppose we have two $n$-dimensional vectors $y, z \in \mathbb{R}^n$ with their elements arranged in non-increasing order, i.e., $y_i \geq y_j$ and $z_i \geq z_j$ if $i < j$. We claim vector $y$ is lexicographically smaller than vector $z$, i.e., $y \prec_{lex} z$, if and only if there exists an integer $k < n$ such that $y_i = z_i, \forall i < k$ and $y_k < z_k$ (if $k = 1$, we have $y_1 < z_1$).

Definition 5. (Excess of coalition) For a coalition $S$ with the given cost allocation $x \in \mathbb{R}^{|S|}$, the excess of coalition $S$ under the cost allocation $x$ is defined as

$$e(x, S) = x(S) - \nu(S)$$

For a coalition game characterized by the minimal total cost, the excess of coalition $e(x, S)$ can be interpreted as the dissatisfaction of cost allocation $x$ in coalition $S$.

The solution concept of nucleus is deduced from lexicographical order. Generally, nucleus is the imputation with the lexicographically smallest excess of coalitions. We display the specific definition as below.

Definition 6. (Nucleus) The nucleus of a coalition game is the lexicographically minimal imputation $x \in I$. Specifically, for a coalition game $(N, \nu)$ (the total number of coalitions is $2^n - 1$), let vector $O(x) \in \mathbb{R}^{2^n - 1}$ be the excess of the coalitions for cost allocation $x \in \mathbb{R}^n$ arranged in a non-increasing order, i.e., $O_i(x) \geq O_j(x), \forall i < j$, then a cost allocation $x \in \mathbb{R}^n$ is the nucleus if we have

$$O(x) \prec_{lex} O(x'), \forall x' \in I \setminus \{x\}.$$
1) Start with the least core: The least core of a coalition game \((\mathcal{N}, \nu)\) is a set of optimal solutions to the following LP problem \([26]\):

\[
\begin{align*}
z^* &= \min z \\
\text{subject to:} & \quad x(\mathcal{N}) = \nu(\mathcal{N}) \\
& \quad z \leq \nu(S) - x(S), \quad \forall S \subseteq \mathcal{N} \setminus \{\emptyset, \mathcal{N}\}.
\end{align*}
\] (5)

We note that problem (5) is equivalent to \(z^* = \min_{x \in \mathcal{X}} \max_{S \subseteq \mathcal{N} \setminus \{\emptyset, \mathcal{N}\}} (x(S), S)\), therefore the intuitive interpretation of least core is the set of imputations with the minimum maximum excess of coalition, i.e., \(O_1(x) \leq O_2(x') \iff \forall x' \in \mathcal{I}\). Note that the least core is always non-empty and the nucleolus must locate in the least core but the converse is not true (Definition 3). All illustrative example: assume a 3-player coalition game with the imputations \(x = [5, 3, 3], y = [5, 4, 2]\), and \(z = [6, 4, 2]\). We have the least core \(x\) and \(y\) which includes the unique nucleolus \(x\).

Though the least core is well-defined and generally can narrow the search scope for nucleolus, computing least core by solving problem (5) is computationally intensive as it requires the entire characteristic functions (i.e., \(\nu(S), \forall S \subseteq \mathcal{N} \setminus \{\emptyset, \mathcal{N}\}\)). To overcome the computational challenge, we capitalize on a constraint generation technique to obtain the least core which mainly contains three steps: i) Start with the relaxed problem (6) corresponding to a small subset of coalitions \(\mathcal{F}_1 \subseteq \mathcal{N}\) (e.g., \(\mathcal{F}_1 = \{\{1\}, \{2\}, \ldots, \{N\}\}\)); ii) identify the most “violated” coalitions (i.e., maximum excess of coalition) with the obtained solution (cost allocation); iii) add the new constraint corresponding to the most “violated” coalition to problem (6). This process is repeated until no “violated” coalitions exist with the obtained cost allocation. This implies the optimal solution of problem (5) (i.e., least core) is approached. We defer the implementation of the constraint generation technique later.

\[
\begin{align*}
z^{*,1} &= \min z \\
\text{subject to:} & \quad x(\mathcal{N}) = \nu(\mathcal{N}) \\
& \quad z \leq \nu(S) - x(S), \quad \forall S \subseteq \mathcal{F}_1 \setminus \{\emptyset, \mathcal{N}\}.
\end{align*}
\] (6)

2) Lexicographically optimization: Intuitively, if the least core is unique, the nucleolus is approached. However, that is not the usual case and the algorithm has to be carried on to identify the lexicographically minimal solution \(O_2(x) \in \{x \mid O_1(x) = z^{*,1}\}\). Consider a general situation, we introduce the lexicographical minimization of identifying the cost allocations that minimize \(O_k(x)\) within \(\{x \mid O_k(x) = z^{*,k}, \forall i = 1, 2, \ldots, k-1\}\) (at step \(k\)).

Akin to the search for least core, we employ the constraint generation technique to overcome the computational challenges. Similarly, the solution of problem (7) mainly contains three steps: i) we start with a relaxed problem (7) corresponding to a subset of coalitions \(\mathcal{F}_k\) with explicit characteristic functions (the characteristic function for any specified coalition \(S\) can be obtained by solving problem (4)). Particularly, we use \(z^{*,i}\) to denote the optimal value at step \(i\) and \(\Gamma_i\) collects the binding constraints at step \(i\). This enforces the search within the scope of cost allocations that satisfying \(O_i(x) = z^{*,i}, \forall i = 1, 2, \ldots, k-1\). ii) we apply the constraint generation technique to identify the most “violated” coalitions with the obtained solution (cost allocation proposal). iii) we add a new constraints corresponding to the identified coalition to problem (7) (i.e., \(\mathcal{F}_k\)). This process is repeated until no “violated” coalition exists. Similarly, if the solution to problem (7) is unique, the nucleolus is approached, otherwise we carry on the lexicographically minimization with \(k + 1\).

\[
\begin{align*}
z^{k,*} &= \begin{cases}
\max z \\
\text{subject to:}
\end{cases} \\
& \quad x(\mathcal{N}) = \nu(\mathcal{N}) \\
& \quad z \leq \nu(S) - x(S), \quad \forall S \in \mathcal{F}_k \setminus \bigcup_{i \leq k-1} \Gamma_i \\
& \quad z^{i,*} = \nu(S) - x(S), \quad \forall S \in \Gamma_i, i = 1, 2, \ldots, k-1.
\end{align*}
\] (7)

3) Constraint Generation: This part introduces the implementation of constraints generation technique to identify the most “violated” coalition for the least core and lexicographically optimization in the CES coalition game.

Suppose we have a cost allocation proposal \(x^{k,*}\) for lexicographically optimization at step \(k\) (the solution of problem (6) or (7) with a specific \(\mathcal{F}_k\)). The constraint generation technique requires to identify the most “dissatisfied” or “violated” coalition (the subset of players) out of \(\mathcal{F}_k\). This can be achieved by maximizing the excess of coalitions over the coalitions \(\mathcal{N} \setminus \mathcal{F}_k\).

To address such issue, we define some binary variables \(s_i \in \{0, 1\}, \forall i \in \mathcal{N}\) to indicate whether player \(i\) is in the identified coalition or not. Therefore, we can interchangeably indicate a coalition by \(S^j \subseteq \mathcal{N}\) or a binary vector \(s^j = \{s_1^j, s_2^j, \ldots, s_N^j\}\). We have \(s_i^j = 1\) if player \(i\) is in coalition \(S^j\), otherwise \(s_i^j = 0\). Therefore, the problem to identify the most “dissatisfied” coalition out of \(\mathcal{F}_k\) can be formulated as

\[
c^* = \max_{S \subseteq \mathcal{N}} \sum_{j \in S} x^{k,*}_j + z^{k,*} - \nu(S)
\]

subject to:

\[
\begin{align*}
1 & \leq \sum_{i \in \mathcal{N}} s_i \leq N - 1. & (8a) \\
\sum_{i \in \mathcal{N}} s_i + \sum_{j \in \mathcal{N}} (1 - s_i) & \geq 1, \quad \forall j \in \mathcal{S}^{i} \in \mathcal{F}_k. & (8b) \\
s_i & \in \{0, 1\}, \forall i \in \mathcal{N}. & (8c)
\end{align*}
\]

where constraint (8a) is imposed to exclude the empty coalition \(\emptyset\) and grand coalition \(\mathcal{N}\). Constraint (8b) enforces to exclude all the coalitions in \(\mathcal{F}_k\).

We note that problem (8) requires the explicit characteristic function \(\nu(S)\). However, for the CES coalition game, the characteristic function \(\nu(S)\) is characterized by a stochastic optimization problem. To address this issue, we reformulate problem (8) for the CES coalition game as the following
mixed-integer linear programming (MILP):
\[
c^* = \max \sum_{j \in S} x_j^{k,*} + z_j^{k,*} - (c(x_S) + \sum_{\omega \in \Omega} \rho_{\omega} g(x_S, \zeta_\omega))
\]
subject to:\n\[
(1b) - (1d), \quad \forall i \in N.
\]
\[
p_{b,t}^i - p_{r,t}^i \geq s_i p_{b,t}^i + p_{d,t}^i - p_{r,t}^i, \quad p_{b,t}^i - p_{d,t}^i \leq s_i p_{g,\text{max}}^i
\]
1 ≤ \sum_{i \in N} s_i \leq N - 1.
\[
\sum_{i \in N} (1 - s_i) \geq 1, \quad \forall j \mid S^j \in \mathcal{F}^k.
\]
where \( P_{g,\text{max}} \) indicates the maximum trading power (purchase or sell) power with the grid for each building. Correspondingly, the objective of problem (9) indicates the excess of coalition for the CES coalition game. Particularly, we impose the relaxed constraints \( p_{b,t}^i - p_{r,t}^i \geq s_i p_{b,t}^i + p_{d,t}^i - p_{r,t}^i \) to capture the balance of supply and demand for all the buildings in or out of the identified coalition. More specifically, for the buildings in the identified coalition, we would have \( s_i = 1 \) and the procured electricity from the grid at least should satisfy the demand (increase the objective); otherwise we have \( s_i = 0 \) and the demand of such buildings is released while minimizing the objective function.

Note that by solving problem (9), the most “dissatisfied” coalition \( S^* \) can be identified, which can be added to \( \mathcal{F}_k \) (i.e., \( \mathcal{F}_k = \mathcal{F}_k \cup \{S^*\} \) ) of problem (7) (or (9)) to adjust the cost allocation.

The procedures of identifying the most “violated” coalition and adjusting the cost allocation are alternated until no “violated” coalition is found, i.e., the excess of coalition is non-positive (\( c^* \leq 0 \)). This implies the optimal solution of problem (7) or (6) is approached. In other word, the least core or the set of cost allocations with the smallest \( O_i(x) \) is identified.

The main procedures to search for the nuleolus of the CES coalition game are displayed in Algorithm 1. Particularly, we clarify two main points in the algorithm. First, the algorithm corresponds to two-loops: outer-loop and inner-loop. The outer-loop correspond to the sequence of lexicographically optimization indicated by the iteration \( k \). Correspondingly, the inner-loop solves the lexicographically optimization problem of each step \( k \) by iteratively performing constraints generation technique. The inner-loop will terminate until no “violated” coalition is found (i.e., \( c^* \leq 0 \)), which implies the set of cost allocation with the smallest \( O_i(x), \forall i = 1, 2, \ldots, k \) has been identified. Second, there is one crucial step before switching to the outer-loop from the inner-loop (line 12). At the end of each inner-loop, the active or binding constraints (i.e., coalitions in \( \mathcal{F}_k \)) are required to be identified. This can be achieved by checking the group of inequality constraints in problem (7). More specifically, at the end of each inner-loop with the obtained cost allocation \( x^{k,*}, z^{k,*}, \forall S \in \mathcal{F}_k \setminus \bigcup_{l \leq k-1} \Gamma_l \) with \( z^{k,*} = \nu(S) - x(S) \), we have \( S \in \Gamma_k \). Besides, we note the out-loop will terminate until an unique solution for a lexicographically optimization is approached.

Algorithm 1: Search for nuleolus of the CES coalition game based on constraints generation

\begin{itemize}
  \item \textbf{Initialize:} \( N := \{1, 2, \ldots, N\} \); building participants.
  \item \textbf{Output :} \( x \in \mathbb{R}^N \); cost allocation for the buildings.
  \item \textbf{Initialize:} \( k \rightarrow 1, \mathcal{F}_1 = \{\{1\}, \ldots, \{N\}\} \). \textbf{STOP} := false;
  \item \textbf{while} \textbf{STOP} \textbf{do}
    \item \textbf{End} \textbf{if} \textbf{STOP} \textbf{do}
      \item \textbf{Stop} := true;
      \item \textbf{break};
  \item \textbf{end}
  \item \textbf{Solve} problem (7) \textbf{or} problem (6) \textbf{if} \( k = 1 \) \textbf{and obtain} the solution \( x^{k,*} \) \textbf{and} \( z^{k,*} \);
  \item \textbf{if} the solution is unique \textbf{then}
    \item \textbf{STOP} := true;
    \item \textbf{break};
  \item \textbf{else}
    \item \textbf{Identify} the active \textbf{and} binding constraints \( \Gamma_k \);
    \item \( k := k + 1; \)
  \item \textbf{end}
\end{itemize}

IV. Case Study

This section reports the numeric results of the proposed CES model. Particularly, i) we study the fairness and computational efficiency of the cost allocation based on nuleolus by compared with Shapley approach and proportional allocation; ii) we investigate the enhanced economic benefits of the CES model for individual building over IES model.

A. Simulation setup

The case studies of this section depend on the real data for building demand [29] and renewable generation (i.e., wind and solar power) [30]. To account for the complementary feature of demand, multiple types of buildings (e.g., office, hotel, school, hospital and restaurant) are considered. We display one typical demand profile for each type of building in Fig. 2 and one representative wind and solar output in Fig. 3.

We set the ES roundtrip efficiency as \( \eta_{\text{ES}} = 0.9 \). To capture the amortized ES capital price, we set the annual interest rate as \( r = 0.06 \) and the ES lifetime as \( L = 10 \) years. We refer to the time-of-use utility price in Singapore: \( \hat{c}_{k}^U = 0.1271 \text{S$/kW}$ (off-peak 23:00−7:00) and \( \hat{c}_{k}^F = 0.2085 \text{S$/kW}$ (peak 8:00−22:00) and demand charge \( c_{\text{ES},\text{max}} = 0.1335 \text{S$/kW}$ (we set selling price as \( c_{k}^F = 0 \)). The maximum trading power with the grid is set as \( P_{g,\text{max}} = 1000 \text{kW} \) for each building.

B. Fairness and computational efficiency

This part investigates the fairness and computational efficiency of the cost allocation mechanism for the CES coalition
game (Algorithm 1). We consider four case studies with different number of buildings \(N \in \{3, 5, 8, 10\}\) sharing a CES. We compare the proposed cost allocation with the proportional method [21] and Shapley approach [23]. The proportional allocation is an empirical cost allocation that is easy to compute; whereas Shapley value method can provide a more sophisticated cost allocation but is computationally intensive as it requires the entire information of the characteristic functions.

- **Proportional method**: This method proportionally distributes the ES capital cost among the buildings based on their operation cost (electricity bill) reduction relative to no ES case (benchmark). Specifically, we denote the operation cost of the buildings with no ES as \(x_{\text{Opex}} \in \mathbb{R}^N\) (this can be easily calculated as the buildings have to purchase the supplement from the grid to balance their non-elastic demand at each operating point with no ES). Besides, we can obtain the operation cost \(x_{\text{Opex}} \in \mathbb{R}^N\) and the ES capital cost \(y_{\text{Cap}} \in \mathbb{R}\) for the CES model by solving problem (4). As the operation cost for each building is readily available through the net-metering in the CES model, the proportional method only distributes the ES capital according to

\[
x_{i, \text{Cap}} = (x_{i, \text{Opex}} - x_{\text{Opex}}) \frac{y_{\text{Cap}}}{\mathbf{1}^T(x_{\text{Opex}} - x_{\text{Opex}})}, \quad \forall i \in N.
\]

where \(\mathbf{1} \in \mathbb{R}^N\) denotes a \(N\)-dimensional unit vector. Thus, the total cost for each building amounts to \(x_{i, \text{Opex}} + x_{i, \text{Cap}}\) (\(\forall i \in N\)).

- **Shapley approach**: The underlying idea of Shapley approach is to distribute payoff among the players based on their marginal “contribution” to the value of the coalition. Specifically, for a coalition game \((\mathcal{N}, \nu)\), the payoff assigned to each player is calculated as [23]

\[
x_i(\mathcal{N}) = \sum_{S \subseteq \mathcal{N}, i \in S} \frac{|S|!(|\mathcal{N}| - |S| - 1)!}{|\mathcal{N}|!} [\nu(S \cup \{i\}) - \nu(S)],
\]

where \(S\) denotes all subsets of players of \(\mathcal{N}\) that exclude player \(i\). It’s clearly seen that the assigned payoff for player \(i\) is built by its marginal “contribution” to each coalition \(S\), which is quantified by \(\nu(S \cup \{i\}) - \nu(S)\).

In this paper, we use the minimum satisfaction (SAT) of players as an indicator of fairness, which can be quantified by the minimum excess of coalition with a cost allocation proposal \(x\):

\[
\text{STA} = \min_{S \subseteq \mathcal{N}, S \neq \emptyset, \mathcal{N}} x(S) - \nu(S) \quad \text{(10)}
\]

Specifically, we refer to a cost allocation as fair if all the payers are satisfied with the cost allocation, i.e., the excess of coalition is non-positive (STA \(\leq 0\)). Intuitively, we prefer a cost allocation with a more negative STA.

We apply the three method to achieve ex-post cost allocation for the CES coalition game. The allocated cost for each buildings with different number of buildings \(N \in \{3, 5, 8, 10\}\) sharing a CES are displayed in TABLE I. We observe that for the cases with \(N = 3, 5\) buildings, all three methods have achieved fair cost allocation among the players as indicated by the negative STA; whereas for the cases with \(N = 8, 10\), we observe obviously positive STA for the proportional method, which indicates the dissatisfaction of some players with the cost allocation. This implies that both the proposed method and the Shapley approach achieve the fairness of cost coalition for the CES coalition game whereas the proportional method fails in some cases.

We also study the computational efficiency of the cost allocations. Particularly, we quantify the computation cost by the number of characteristic functions computed (the number of times solving the stochastic optimization problem (4)) to achieve the cost allocation. The computation cost of the case studies \((N \in \{3, 5, 8, 10\})\) are displayed in TABLE II. First of all, we observe the highest computation cost for Shapley approach, which requires the entire information of characteristic functions \((2^{N} - 1)\). On the contrary, proportional method is most computationally efficient and only requires \(N + 1\) characteristic functions to achieve the cost allocation \((N\) corresponds to computing the cost for each building with no storage and 1 corresponds to computing the total cost for grand coalition). Notably, the proposed method based on nucleolus also takes approximately linear computation w.r.t. the scale \(N\), but with a minor increase compared with the proportional method. Particularly, the computation of the proposed method is determined by the scale \(N\) and the number of constraints \(K\) generated to approach the nucleolus. The numeric results imply that the fraction of the generated constraints is approximately linear with the scale \(N\). It’s noteworthy that for the CES coalition game with 10 buildings, only 2.54\% \((26/1023)\) of the entire characteristic functions is actually computed to achieve cost allocation with the proposed method. This demonstrates the computational efficiency of the proposed
method against the Shapley approach.

Therefore, we imply that the proposed method outperforms the Shapley approach and the proportional method either in computation or fairness.

### TABLE I: The fairness of cost allocations

| Building No. | Proportional \( \times 10^2 \) (\$) | Shapley Value \( \times 10^2 \) (\$) | Nucleolus \( \times 10^2 \) (\$) |
|--------------|-----------------------------------|-----------------------------------|-----------------------------------|
| N = 3        |                                   |                                   |                                   |
| B1           | 2.63                              | 2.64                              | 2.69                              |
| B2           | 4.46                              | 4.49                              | 4.44                              |
| B3           | 2.40                              | 2.37                              | 2.38                              |
| STA          | -5.58(Y)                          | -6.37(Y)                          | -5.80(Y)                          |
| N = 5        |                                   |                                   |                                   |
| B1           | 2.57                              | 2.57                              | 2.59                              |
| B2           | 4.43                              | 4.43                              | 4.48                              |
| B3           | 2.30                              | 2.27                              | 2.26                              |
| B4           | 6.66                              | 6.72                              | 6.75                              |
| B5           | 4.72                              | 4.68                              | 4.60                              |
| STA          | -2.00(Y)                          | -6.73(Y)                          | -10.18(Y)                         |
| N = 8        |                                   |                                   |                                   |
| B1           | 2.61                              | 2.59                              | 2.64                              |
| B2           | 4.44                              | 4.40                              | 4.43                              |
| B3           | 2.32                              | 2.28                              | 2.27                              |
| B4           | 6.59                              | 6.62                              | 6.60                              |
| B5           | 4.66                              | 4.64                              | 4.61                              |
| B6           | 5.82                              | 5.79                              | 5.77                              |
| B7           | 7.54                              | 7.56                              | 7.54                              |
| B8           | 6.69                              | 6.79                              | 6.82                              |
| STA          | 8.34(N)                           | -0.64(Y)                          | -4.98(Y)                          |
| N = 10       |                                   |                                   |                                   |
| B1           | 2.62                              | 2.58                              | 2.61                              |
| B2           | 4.44                              | 4.44                              | 4.43                              |
| B3           | 2.22                              | 2.30                              | 2.22                              |
| B4           | 6.68                              | 6.59                              | 6.64                              |
| B5           | 4.60                              | 4.68                              | 4.59                              |
| B6           | 5.74                              | 5.79                              | 5.75                              |
| B7           | 7.51                              | 7.48                              | 7.50                              |
| B8           | 6.77                              | 6.71                              | 6.82                              |
| B9           | 6.44                              | 6.45                              | 6.46                              |
| B10          | 7.97                              | 7.96                              | 7.97                              |
| STA          | 10.49(N)                          | -1.48(Y)                          | -4.85(Y)                          |

Note: Y: satisfied N: not satisfied

### TABLE II: Computational efficiency

| #Buildings | Proportional Computation | Shapley Computation | Nucleolus Computation |
|------------|--------------------------|--------------------|-----------------------|
| 3          | 4                        | 7                  | 8 (4)                 |
| 5          | 6                        | 31                 | 13 (7)                |
| 8          | 9                        | 255                | 24 (15)               |
| 10         | 11                       | 1023               | 37 (26)               |
| N          | \( N + 1 \)              | \( 2^N - 1 \)      | \( N + 1 + K(\bar{K}) \) |

C. Economic benefits of CES model

In this part, we study the economic benefits of the CES model. We consider a community microgrid with \( N = 10 \) buildings sharing a central CES. We compare the CES model with the IES models (buildings invest their ES separately). For the IES model, the optimal ES size and operation can be obtained by solving problem (4) with \( N = 1 \) for each building. Whereas for the CES model, the allocated cost for each building is computed using the proposed method. Using no ES as benchmark, we display the cost for each building (B1-B10) with the IES and CES model in Fig. 4. Specifically, the cost of each building is stacked by the ES capital cost (CAPex), the operation cost (Opex) as well as cost reduction (Reduction) compared with no ES case. Obviously, we see the CES model can provide a higher percentage of cost reduction for each building over the IES model. Further, we investigate the total cost of the community microgrid (electricity bill of all buildings plus the ES capital cost) with the two ES models as shown in Fig. 5. We find the total cost is reduced by 7.2% with the CES model versus 2.1% with the IES model. This implies the CES model can provide enhanced economic benefits to the community microgrid.

Fig. 4: The cost of each building with the IES and CES model (no ES as benchmark).

Fig. 5: The total cost of the community microgrid with the IES and CES model (no ES as benchmark).

Further, we investigate the value of storage (VoS) for different ES models, which is defined as the average operation cost.
Fig. 6: The value of storage (VoS) for each building with the IES and CES model.

reduction harnessed from per unit of ES capital investment:

\[ \text{VoS} = \left[ x^{\text{Opex}} - x^{\text{Cap}} \right] / x^{\text{Cap}} \]

where \( x^{\text{Opex}} \) and \( x^{\text{Cap}} \) denote the incurred operation cost and ES capital cost for an ES model while minimizing the total cost. \( x^{\text{Opex}} \) represents the operation cost with no ES. Intuitively, it will be economically beneficial to invest ES if and only if VoS > 1. Moreover, we would prefer an ES model with a higher VoS. In this part, we study the VoS for each individual building with the CES and IES model. As shown in Fig. 6, we observe that the VoS for each building is apparently improved with the CES model versus IES model. Specifically, the VoS for each building with the CES model is about 1.83 times over the IES model. This demonstrates the improved ES efficiency with the CES model by allowing the buildings in a community microgrid to share ES.

V. Conclusion

This paper studied a community energy storage (CES) model where multiple buildings cooperatively invest and share a central ES. We formulate the problem to maximize the economic benefits as a CES coalition game which integrates the optimal sizing, operation and fair payoff allocation. Particularly, to address the computational issue of fair ex-post cost allocations across the building participants, we proposed a cost allocation mechanism based on *nucleolus* by using constraints generation technique. We demonstrated the fairness and computational efficiency of the method by comparing with *Shapley approach* and *proportional method* through case studies. The numeric results imply that the proposed method outperforms the other two either in fairness and computational efficiency. Notably, for the proposed method, only a small fraction of characteristic functions (2.54%) is actually computed to achieve fair cost allocation versus the entire information (exponential with the scale) required by the Shapley approach.

We studied the enhanced economic benefits for each building participant with the CES model over individual energy storage (IES) model. We found that the CES model can provide high cost reduction for each building (7.2%) versus the IES model (2.1%). Moreover, the CES model achieves 1.83 times of value of storage (VoS) compared with the IES model.

**Appendix A**

*AMORTIZED ES CAPITAL PRICE*

The amortized ES capital price is calculated as [20] [24]

\[ k_p = K_p \frac{(1 + r)^L}{L \cdot D} \]

and

\[ k_c = K_c \frac{(1 + r)^L}{L \cdot D} \]

where \( r \) is the annual interest rate, \( L \) is the nominal lifetime of the ES in years (typically 10-20 years), \( D \) = 365 represents the number of days per year. \( K_p \) (in $/kWh) and \( K_c \) (in $/kW) denotes the ES price for energy and power capacity.

**Appendix B**

*Proof of Proposition 1*

Proof. i) We first prove constraints \( p_{i,t}^{\text{ch}, \omega} > p_{i,t}^{\text{dis}, \omega} > 0 \) is redundant. This is intuitive as it won’t happen that \( p_{i,t}^{\text{ch}, \omega} > 0 \) and \( p_{i,t}^{\text{dis}, \omega} > 0 \) when the purchase price is higher than the selling price and the objective is to minimize the total cost. If \( p_{i,t}^{\text{ch}, \omega} > 0 \) and \( p_{i,t}^{\text{dis}, \omega} > 0 \), the objective function becomes:

\[ \min \left( \sum_{t \in T} \left( \sum_{\omega \in \Omega} \left( p_{i,t}^{\text{ch}, \omega} \cdot \text{CWh} + p_{i,t}^{\text{dis}, \omega} \cdot \text{CWh} \right) \right) \right) \]

which can be discarded without affecting the optimal solution by investigating two possible cases.

*Case I:* We assume an optimal operation with \( p_{i,t}^{\text{ch}, \omega} > p_{i,t}^{\text{dis}, \omega} > 0 \). Since \( \eta_{\text{ch}, \omega} > \eta_{\text{dis}, \omega} < 1 \), it’s easy to construct another feasible operation for the ES: \( \tilde{p}_{i,t}^{\text{ch}, \omega} = p_{i,t}^{\text{ch}, \omega} - p_{i,t}^{\text{dis}, \omega} / (\eta_{\text{ch}, \omega} - \eta_{\text{dis}, \omega}) \) and \( \tilde{p}_{i,t}^{\text{dis}, \omega} = 0 \). It’s easy to verify that \( \tilde{p}_{i,t}^{\text{ch}, \omega} > 0 \) and \( \tilde{p}_{i,t}^{\text{dis}, \omega} = 0 \). Therefore, the constructed operation has a lower objective function.

*Case II:* We assume an optimal operation with \( p_{i,t}^{\text{ch}, \omega} > p_{i,t}^{\text{dis}, \omega} > 0 \). Similarly, we can construct another feasible ES operation \( \tilde{p}_{i,t}^{\text{ch}, \omega} = 0 \) and \( \tilde{p}_{i,t}^{\text{dis}, \omega} = p_{i,t}^{\text{dis}, \omega} - p_{i,t}^{\text{ch}, \omega} / (\eta_{\text{ch}, \omega} - \eta_{\text{dis}, \omega}) \). It’s easy to verify that \( \tilde{p}_{i,t}^{\text{ch}, \omega} = 0 \) and \( \tilde{p}_{i,t}^{\text{dis}, \omega} > 0 \). Therefore, the constructed operation has a lower objective function.

Clearly, the constructed solution will yield lower cost which contradicts the assumption.

**Appendix C**

*Proof of Theorem 1*

Proof. We prove the results by definition. Suppose there are two disjoint coalitions: \( S_1, S_2 \subseteq S \) and \( S_1 \cap S_2 = \emptyset \). We assume \( x_{S_1}^*, [y_{S_1}]_{\omega \in \Omega} \) and \( x_{S_2}^*, [y_{S_2}^*]_{\omega \in \Omega} \) are the optimal solution of the coalition \( S_1 \) and \( S_2 \), respectively. Since both the objective function and the constraints of problem 3 are linear, it’s easy to verify that \( [x_{S_1}^*, x_{S_2}^*, [y_{S_1}^*]_{\omega \in \Omega}, [y_{S_2}^*]_{\omega \in \Omega}] \) are viable solution for the coalition \( S_1 \cup S_2 \). Therefore, we have

\[ \nu(S_1) + \nu(S_2) \leq \nu(S_1 \cup S_2) \]

\[ \square \]
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