The effect of photometric redshift uncertainties on galaxy clustering and baryonic acoustic oscillations

Jonás Chaves-Montero,1, Raúl E. Angulo2 and Carlos Hernández-Monteagudo2

1High Energy Physics Division, Argonne National Laboratory, Lemont, IL 60439, USA
2Centro de Estudios de Física del Cosmos de Aragón, Plaza San Juan 1, Planta-3, 44001, Teruel, Spain

ABSTRACT
In the upcoming era of high-precision galaxy surveys, it becomes necessary to understand the impact of redshift uncertainties on cosmological observables. In this paper we explore the effect of sub-percent photometric redshift errors (photo-z errors) on galaxy clustering and baryonic acoustic oscillations (BAOs). Using analytic expressions and results from 1000 N-body simulations, we show how photo-z errors modify the amplitude of moments of the 2D power spectrum, their variances, the amplitude of BAOs, and the cosmological information in them. We find that (a) photo-z errors suppress the clustering on small scales, increasing the relative importance of shot noise, and thus reducing the interval of scales available for BAO analyses; (b) photo-z errors decrease the smearing of BAOs due to non-linear redshift-space distortions (RSDs) by giving less weight to line-of-sight modes; and (c) photo-z errors (and small-scale RSD) induce a scale dependence on the information encoded in the BAO scale, and that reduces the constraining power on the Hubble parameter. Using these findings, we propose a template that extracts unbiased cosmological information from samples with photo-z errors with respect to cases without them. Finally, we provide analytic expressions to forecast the precision in measuring the BAO scale, showing that spectro-photometric surveys will measure the expansion history of the Universe with a precision competitive to that of spectroscopic surveys.

Key words: large-scale structure of the Universe—techniques: photometric—surveys—galaxies: distances and redshifts—cosmological parameters—distance scale.

1 INTRODUCTION

A new generation of wide-field cosmological galaxy surveys will soon map the spatial distribution of hundreds of millions of galaxies over a wide range of redshifts. With these, it will be possible to characterize the expansion history of the Universe and the growth of structures with exquisite precision. Moreover, these measurements will set strong constraints on the contributors to the total energy density as a function of redshift, the law of gravity on large scales, and perhaps will offer hints to explain the accelerated expansion of the Universe (see Weinberg et al. 2013, for a review).

Some of these future galaxy surveys will employ spectrographs, which will deliver precise estimates for galaxy redshifts, e.g. Dark Energy Spectroscopic Instrument (DES; DES Collaboration et al. 2016), WEAVE (Dalton et al. 2014), Euclid (Laureijs et al. 2011), and 4-metre Multi-Object Spectroscopic Telescope (de Jong 2011). Other surveys, instead, will rely on either linear variable filters or sets of narrow-band filters, e.g. the Physics of the Accelerating Universe Survey (Martí et al. 2014), the Javalambre Physics of the accelerating universe Astrophysical Survey (J-PAS; Benítez et al. 2014), and the Spectro-Photometer for the History of the Universe, Epoch of Reionization, and Ices Explorer (Doré et al. 2014, 2016). The advantages of the latter class are higher surveying speeds, spectral data for every region of the sky, and a larger number of characterized objects. On the other hand, this approach adds non-negligible uncertainties to the measured redshifts, which propagates to the observed galaxy density field. In order to fully exploit the potential of this class of surveys, the effect of noisy redshift estimators on galaxy clustering has to be carefully studied.

The impact of photometric redshift errors (photo-z errors) on galaxy clustering and baryonic acoustic oscillations (BAOs) has been investigated by several authors (e.g. Seo & Eisenstein 2003; Blake & Bridle 2005; Glazebrook & Blake 2005; Dolney, Jain & Takada 2006; Seo & Eisenstein 2007; Benítez et al. 2009; Cai et al. 2009; Sereno et al. 2015; Ross et al. 2017). These studies have concluded that, in configuration space, adding photo-z errors to galaxy redshifts can be regarded as a smoothing of the galaxy density field along the line of sight (LOS). The analogous effect in
Fourier space is a suppression of LOS k-modes. Despite this, these authors demonstrated that BAO can still be detected and used to measure the expansion history of the Universe through the Hubble parameter $H(z)$ and the angular diameter distance $D_A(z)$ (e.g. Seo & Eisenstein 2003). For instance, for the same number density, the uncertainty on the measured acoustic scale only doubles for photo-z errors of 0.3% per cent with respect to a spectroscopic case (Cai et al. 2009).

In this paper we extend previous studies by developing a complete framework to extract cosmological information from BAO analyses under the presence of photo-z errors. In the first half of the manuscript, we explore the problem analytically. We provide expressions for power spectrum moments (related, but not identical, to Legendre multipoles) and their variances; we study the respective signal-to-noise ratios (SNR); and we quantify the cosmological information encoded in BAOs, all this as a function on the photo-z errors and number density of a given sample. In addition, we provide a formula that forecasts the precision with which $H(z)$ and $D_A(z)$ can be measured, as function number density, linear bias, and typical photo-z uncertainty. In the second half of the paper, we use these results to build an unbiased estimator of the BAO scale from data with photo-z errors. We test the method by applying it to multiple samples drawn from a set of 1000 cosmological N-body simulations.

Our paper is organized as follows. In Section 2 we describe our cosmological simulations and how we compute clustering statistics and mimic photo-z errors. In Section 3 we derive analytic expressions for the impact of photo-z errors on power spectrum moments and their variances, which we compare with the results from our set of simulations. Then, in Section 4, we model how photo-z errors modify the suppression of BAOs and the cosmological information that they encode. In Section 5 we build an unbiased model for the contribution of BAOs to power spectrum moments, and in Section 6 we apply it to simulated samples with different number densities and photo-z errors. In Section 7 we make forecasts for the precision with which cosmological parameters can be measured from galaxy surveys with sub-percent photo-z errors and in Section 8 we summarize our most important results.

## 2 NUMERICAL METHODS

In this section we present the numerical simulations that we analyse, we explain how we measure power spectrum moments and their respective variances, and we describe how we mimic photo-z errors in the simulations.

### 2.1 Numerical simulations

Typically, numerical simulations poorly sample the modes where BAOs are located (but see Angulo & Pontzen 2016). Additionally, owing to periodic boundary conditions, simulations do not consider the coupling with modes larger than the box size. To avoid these complications and obtain accurate results, it is necessary to consider ensembles of N-body simulations of volumes in excess of $1 \ h^{-3} \mathrm{Gpc}^3$ when analysing the BAO feature (Angulo et al. 2008).

In this work we have carried out an ensemble of 1000 N-body simulations, where each of them evolved 1024$^3$ dark matter (DM) particles of mass $1.7 \times 10^{12} \ h^{-1} \mathrm{M}_\odot$ in a cubic box of $3 \ h^{-1} \mathrm{Gpc}$ on a side from different initial conditions. This suite has an aggregated volume of $27 \times 10^{3} \ h^{-3} \mathrm{Gpc}^3$, which will allow us to accurately resolve the BAO feature.

For computational efficiency, we carried out these simulations using the Comoving Lagrangian Acceleration (COLA) method (Tassev, Zaldarriaga & Eisenstein 2013). This algorithm is able to recover the real-space power spectrum of a full N-body simulation to within 2% per cent for $k < 0.3 \ h \mathrm{Mpc}^{-1}$ at a fraction of its computational cost (Howlett, Manera & Percival 2015). Moreover, COLA reproduces the redshift-space power spectrum monopole and quadrupole of HOD galaxies for $k < 0.2 \ h \mathrm{Mpc}^{-1}$ (Koda et al. 2016).

We adopt Gaussian initial conditions created using second-order Lagrangian Perturbation theory. Gravitational forces were computed using a Particle-Mesh algorithm with a Fourier grid of $1024^3$ points, and particles were evolved from $z = 9$ down to $z = 1$ in 10 time steps. The cosmological parameters adopted were $\Omega_m = 0.25$, $\Omega_b = 0.045$, $n_s = 1$, $H_0 = 73 \mathrm{km} \mathrm{s}^{-1} \mathrm{Mpc}^{-1}$, and $\sigma_8 = 0.9$. Each simulation took 3 CPU hours to complete.

The COLA ensemble will allow us to investigate the impact of photo-z errors on power spectrum moments, their variances, and BAOs. We will only explore the $z = 1$ outputs, which is motivated by the target redshift of future surveys, but our results can be readily generalized to any redshift and for a full lightcone. In addition, we will only focus on dark matter statistics, since the typical number density of haloes that can be resolved with at least 100 particles in our simulations, $n = 1.17 \times 10^{-6} \ h^3 \mathrm{Mpc}^{-3}$, is too low for clustering studies (thus BAO scales are dominated by shot noise). Nevertheless, extrapolating our results to biased tracers can be done by considering the adequate number density and fluctuations amplitude.

### 2.2 Power spectrum and covariance measurements

Throughout this paper, we study the matter density field in Fourier space using its power spectrum, $P(k)$, defined by

$$\langle \delta(k)\delta(k') \rangle = (2\pi)^3 \delta_D(k-k') \ P(k),$$

where $\langle \rangle$ indicates an ensemble average, $\delta_D()$ is the Dirac delta function, and $\delta(k)$ is the Fourier transform of the density contrast field, $\delta(x)$. Operationally, we compute $P(k)$ by gridding the DM particles of each simulation on to a $1024^3$ cubic lattice using a cloud-in-cell (CIC) scheme (Hockney & Eastwood 1981). Then, we Fast Fourier Transform this field and correct for the assignment scheme by dividing by the Fourier transform of the CIC window function. We expect this to provide power spectrum estimates accurate to within 0.1 per cent up to $k = 0.4 \ h \mathrm{Mpc}^{-1}$ (Sefusatti et al. 2016).

Within the plane-parallel approximation, the 3D power spectrum has an azimuthal symmetry and it can be decomposed in terms of the following moments:

$$P_{\ell}(k) = \frac{1}{2} \int_{-1}^{1} \mathrm{d} \mu \mu^\ell P(k, \mu),$$

where $k \equiv |k|$ is the modulus of the wave vector $k$, and $\mu = \frac{\hat{k} \cdot \hat{n}}{r}$.

In the Kaiser approximation, the full 2D power spectrum can be written using combinations of $\ell = 0, 2,$ and 4 moments only. Even when considering non-linearities, almost all cosmological information is encoded in these three moments, which is why we will only consider these in our subsequent analysis.

Note that we chose to adopt moments of the power spectrum, rather than the more common Legendre multipoles, for simplicity in the analytic expressions and in the numerical analysis we will present later. In practice, this choice is unimportant as any Legendre multipole of order $\ell$ can be written as a linear combination of moments of order $\ell \leq \ell'$.

To minimize the impact of a discrete sampling of wavenodes, specially on large scales, we compute $P_{\ell}(k)$ using an $\alpha$-point Gauss-
3 EFFECT OF PHOTOMETRIC REDSHIFT ERRORS ON POWER SPECTRUM MOMENTS

In this section we derive analytic expressions for the impact of photo-z errors on power spectrum moments and their variances. In all cases, we compare these predictions with numerical results obtained from the COLA ensemble.

3.1 Power spectrum moments

3.1.1 General expressions

Let us consider a set of galaxies with a real-space overdensity \( \delta_r(k) \) discretely sampling a field of covariance \( P(k) \), and whose redshifts are measured through a noisy but unbiased estimator. The observed redshifts are thus \( z + \delta z \), where \( \delta z \) is the photo-z error. Assuming that the PDF of the photo-z errors, \( P[\delta_r] \), is identical for every galaxy, the redshift-space overdensity field within the Gaussian dispersion model (Kaiser 1987; Peacock & Dodds 1994) is

\[
\delta_r(k, \mu) = \delta_r(k) F(k, \mu),
\]

where \( \beta \equiv b^{-1} \ln D(a) / \ln a \), \( b \) is the large-scale bias of the sample, \( D(a) \) is the linear growth factor for dark matter, \( a = (1 + z)^{-1} \) is the cosmological scale factor, \( \sigma_v \) is a velocity dispersion induced by non-linear dynamics in units of the speed of light, and \( F(k, \mu) \) is the Fourier transform of \( P[\delta_r] \). The first and second terms of the right-hand side of equation (9) encode large- and small-scale RSDs generated by the peculiar velocity of the galaxies, respectively, and the third the effect of photo-z errors. Hereafter, for brevity we will not write explicitly the dependence of \( F(k, \mu) \) on \( \mu \) and \( k \).

We have chosen to adopt a Gaussian form for distribution of small-scale velocities. However, other forms better fit the distributions measured in cosmological simulations (Scoccimarro 2004; Orsi & Angulo 2017). Our results can be easily generalized to those distributions by replacing the exponential term in equation (9) by the desired velocity distribution function.

From equations (2), (8), and (9), we can derive the relation between power spectrum moments in redshift space, \( P_\ell \), and the \( l = 0 \) moment of the real-space power spectrum, \( P_0^r \):

\[
P_\ell = \langle \mu^\ell \mathcal{F}^2 \rangle |_{0} \equiv P_0^r \langle \mu^\ell \mathcal{F}^2 \rangle |_{0},
\]

where, here, and in the remainder of this paper, the \( \langle \ldots \rangle_k \) brackets denote an angular average.

3.1.2 The Gaussian case

For a Gaussian \( P[\delta_r] \), \( F(k, \mu) = \exp \left[-0.5(\mu a_s(1 + z) c H(z))^2 \right] \), and the \( \ell = 0, 2, \) and 4 moments can be derived analytically:

\[
\langle \mathcal{F}^2 \rangle^\ell_k = \frac{\sqrt{\pi}}{2} \frac{\text{Erf}(x)}{x} \left( 1 + \frac{\beta}{x^2} + \frac{3\beta^2}{4x^4} \right)
- \frac{\beta e^{-x^2}}{2x^2} \left( 1 + \frac{3\beta}{4x^2} H_1(x) \right),
\]

where \( x^2 = \frac{2\mu^\ell \langle \delta_r \rangle}{3\sigma^2} \).
\[
\langle \mu^2 \mathcal{F}^2 \rangle_k = \frac{\sqrt{\pi} \operatorname{Erf}(x)}{4 x^3} \left( 1 + \frac{3 \beta}{x^2} + \frac{15 \beta^2}{4 x^4} \right) - \frac{e^{-x^2}}{2 x^2} \left( 1 + \frac{3 \beta}{x^2} \mathcal{H}_1(x) + \frac{15 \beta^2}{4 x^4} \mathcal{H}_2(x) \right), \tag{13}
\]

\[
\langle \mu^4 \mathcal{F}^4 \rangle_k = \frac{3 \sqrt{\pi} \operatorname{Erf}(x)}{8 x^5} \left( 1 + \frac{5 \beta}{x^2} + \frac{35 \beta^2}{4 x^4} \right) - \frac{3 e^{-x^2}}{4 x^4} \left( \mathcal{H}_1(x) + \frac{5 \beta}{x^2} \mathcal{H}_2(x) + \frac{35 \beta^2}{4 x^4} \mathcal{H}_3(x) \right), \tag{14}
\]

where \( \mathcal{H}_n(x) = \sum_{i=0}^{\infty} \frac{x^{2i}}{(2i+1)!} x^2i \) denotes the double factorial, \( x = k \sigma_{\text{eff}} \), and \( \sigma_{\text{eff}} = \sqrt{\sigma^2 + \sigma_\nu^2 (1+z) c / H(z)} \), i.e. the redshift uncertainties and small-scale peculiar velocities are added in quadrature (Peacock & Dodds 1994). Note that these expressions diverge as \( x \to 0 \), and in general \( \langle \mu^n \mathcal{F}^n \rangle_k \) expressions are valid only when \( x > 3 \). To obtain valid expressions when \( x < 3 \), we expand \( F(k \mu) \) into a power series.

The expressions in real space can be trivially obtained by setting \( \beta = 0 \) and \( \sigma_{\text{eff}} = \sigma \), recovering the expression for equation (12) provided in Peacock & Dodds (1994). In that case, we can see that photo-z errors always generate an apparent anisotropic clustering, even if the underlying galaxy field is isotropic. In redshift space the effect is more complex as photo-z errors couple with RSD parameters. We will explore this next.

### 3.1.3 Comparison with numerical simulations

In the top panels of Fig. 1 we display the average power spectrum moments from our analytic expression and from the COLA ensemble. Symbols and colours indicate the results for samples with different number densities and photo-z errors, as stated in the legend. To compute our model, we employ the average \( P(k) \) from the COLA ensemble with \( n = 0.03 \) \( h^3 \) Mpc\(^{-3} \) and \( \sigma_v = 3 \times 10^{-4} \), where \( \sigma_v \) is obtained by fitting our analytic model to power spectrum moments of samples without photo-z errors.

We can see that our numerical and analytical results show good agreement. The discrepancies, of the order of 10 per cent, arise from the inaccuracy of our RSD model and shot noise subtraction. In the bottom panels of Fig. 1, we display the ratio of shot noise corrected moments and their shot noise level. The long dashed line indicates when the shot noise level is 10 times greater than the amplitude of shot noise corrected moments. As we can see, below this line the shot noise subtraction is no longer precise.

Overall, we can see that photo-z errors suppress the amplitude of \( P_0, P_2, \) and \( P_4 \) for all wavenumbers, specially on small scales. The Poisson noise, however, is unaltered. Therefore, for a given number density of objects, photo-z reduce the number of modes and scales useful for cosmological analyses. This is the main effect for galaxy clustering.

### 3.2 Variance of power spectrum moments

#### 3.2.1 General expressions

The effect of photo-z errors is not only to modify the amplitude of the power spectrum \( P \) but also its Gaussian covariance. Let us first consider the diagonal elements of the power spectrum covariance matrix:

\[
\sigma^2[P](k) = \frac{2}{N_k} \sum_{k_i} \langle |\delta(k_i)|^2 \rangle - \langle P(k_i) \rangle^2, \tag{15}
\]

Figure 1. Top panels: impact of photo-z errors on \( P_0, P_2, \) and \( P_4 \). Symbols show the average results from an ensemble of 1 000 N-body simulations and lines our analytic predictions (equations 10–14), which employ as input the average \( P(k) \) from simulations with \( n = 0.03 \) \( h^3 \) Mpc\(^{-3} \) and \( \sigma_v = 3 \times 10^{-4} \). The colour and shape of lines and symbols indicate the size of Gaussian photo-z errors and the number density for each sample, respectively, as stated in the legend. Results for different number densities are vertically displaced by \( 0.03 \) \( n_0 \). The long dashed line indicates the redshift unaccuracy of our RSD model and shot noise subtraction. In the bottom panels of Fig. 1, we display the ratio of shot noise corrected power spectrum moments from simulations and shot noise. On the scales where \( (2 \ell + 1) | n P_\ell | < 0.1 \) (indicated by long-dashed lines) the shot noise correction is no longer accurate.
where \( \langle \ldots \rangle \) denotes the ensemble average over multiple realizations/universes. Factor 2 appears because only half of the modes of the power spectrum are independent due to the reality of \( \delta(x) \).

Assuming that the real and imaginary parts of \( \hat{\delta}(k) \) are Gaussian random variables with zero mean and standard deviation \( P/2 \), and combining equations (10) and (15), we obtain the following expression for the variance of power spectrum moments in redshift space under the presence of shot noise, phot-z errors, and small-scale velocities:

\[
\sigma^2[P_\ell] = \frac{2}{N_k} \left[ (\mu^{2\ell} F^2)_k + \frac{2(\mu^{2\ell} F^2)_k}{nP_0^2} \right] + \frac{1}{(2\ell + 1)(nP_0^2)} (P_0^2)^2,
\]

where this expression reduces to that provided by Colombi et al. (2009) for \( \ell = 0 \) in real space without phot-z errors (\( F = 1 \)). Note that to compute the diagonal terms of the covariance between two power spectrum moments \( P_\ell \) and \( P_\ell' \), we only have to substitute 2\( \ell \) by \( \ell + \ell' \) in the previous equation.

Note that for a Gaussian \( P[\delta_{rz}] \), all terms in equation (16) have an analytic expression, which we provided in Section 3.1.2. We can thus analytically compute the variance of any power spectrum moment.

3.2.2 Comparison with simulations: diagonal terms

In Fig. 2 we display the variance of \( P_0 \), \( P_2 \), and \( P_4 \) for the same samples shown in Fig. 1. In the bottom panels we compare our analytic model and the results from our simulations. Overall, our expressions correctly capture the effect of phot-zs – the agreement is to within 5 per cent, 10 per cent, and 20 per cent for \( P_0 \), \( P_2 \), and \( P_4 \), respectively. The differences originate from assuming that the matter density field is Gaussian, and thus from neglecting in our calculations the contribution of a non-zero trispectrum.

We see that phot-z errors reduce the variance, especially on intermediate scales where BAO are located, which is analogous to their effect on the amplitude of power spectrum moments. On the other hand, at a fixed scale, the contribution of shot noise progressively dominates as the order of the moment increases. We can understand this from equation 16: since the last term in brackets of equation does not depend on phot-z errors, it will be more important as the redshift uncertainty increases.

3.2.3 Comparison with simulations: off-diagonal terms

In Fig. 3 we display the diagonal and off-diagonal elements of the auto-correlation matrices of \( P_0 \), \( P_2 \), and \( P_4 \), and their cross-correlations for \( \sigma_z = 0 \) and 0.5 per cent. The results shown are computed using 1 000 samples from the COLA ensemble with \( n = 0.01 h^3 \text{Mpc}^{-3} \). The top panels display the cross-correlation between different moments, which is scale-independent for samples with no errors. However, for samples with phot-z errors, they decrease by increasing the value of \( k \). This is the consequence of phot-z errors modifying the information content of power spectrum moments, we further analyse this in the following section.

On the bottom panels we display the off-diagonal terms of the correlation matrices at \( k = 0.2 h \text{Mpc}^{-1} \). They are negligible at this scale for samples with and without phot-z errors. This is because phot-z errors do not modify the structure of the covariance matrix. In particular, in the plane-parallel approximation if the real-space power spectrum covariance matrix is diagonal for samples with no...
For an ensemble average over a given k-bin we have that the SNR per radial k-interval reads:

\[
\text{SNR} = \frac{1 + \eta_{21} \exp(-k^2 \sigma_{\text{eff}}^2 \Delta \mu^2)}{\sqrt{1 + \eta_{21}^2}} \frac{\Delta \mu^2 - \mu_2^2}{\eta_{21}^2} \eta_{21} \exp(-2k^2 \sigma_{\text{eff}}^2 \Delta \mu^2),
\]

(18)

with \( \Delta \mu^2 = \mu_2^2 - \mu_1^2 \) and \( \eta_{21} = \eta(\mu_2)/\eta(\mu_1) \). From this expression, we shall consider three different cases:

(i) No photo-z errors nor small-scale RDSs, \( k \sigma_{\text{eff}} = 0 \). In this case

\[
\text{SNR} = \frac{1 + \eta_{21}}{\sqrt{1 + \eta_{21}^2}},
\]

(19)

where the SNR is always below \( \sqrt{2} \), which is the value corresponding to real space.

(ii) Very large photo-z errors, \( k \sigma_{\text{eff}} \to \infty \). In this limit, the value of the SNR is 1. It is smaller than that in the first case because all information along k-modes parallel to the LOS is lost.

(iii) Small photo-z errors, \( k \sigma_{\text{eff}} \to 0 \). In this case, to first order in \( (k \sigma_{\text{eff}})^2 \), we obtain

\[
\text{SNR} = \frac{1 + \eta_{21} \exp(-k^2 \sigma_{\text{eff}}^2 \Delta \mu^2)}{\sqrt{1 + \eta_{21}^2}} \frac{\Delta \mu^2 - \mu_2^2}{\eta_{21}^2} \eta_{21} \exp(-2k^2 \sigma_{\text{eff}}^2 \Delta \mu^2) \times (k \sigma_{\text{eff}})^2 + O[(k \sigma_{\text{eff}})^4],
\]

(20)

where in this limit the SNR increases with respect to the case without photo-z errors nor small-scale RDSs as \( \eta_{21} > 1 \). This behaviour must thus yield a local maximum in the SNR, since for larger \( k \sigma_{\text{eff}} \) values we must recover the second case. This reflects that in this limit photo-z errors affect more the standard deviation of \( P_0 \) than its amplitude, and thus they slightly increase the SNR.

From equation (18) we find that the scale corresponding to the local maximum of the SNR, \( \partial(\text{SNR})/\partial(k \sigma_{\text{eff}})^2 = 0 \), is \( (k \sigma_{\text{eff}})^2 = 1/\sigma_{\text{eff}}^2 \Delta \mu^2 \). That is \( (k \sigma_{\text{eff}})^2 \simeq 1 \), as shown in the top panel of Fig. 4.

3.3.2 Comparison with simulations

We now compare our analytic expressions for the SNR of power spectrum moments (i.e. those derived in the previous two subsections) with the results from the COLA ensemble. In Fig. 4 we show the SNR of \( P_0, P_2, \) and \( P_3 \) relative to that of the real-space power spectrum with \( n = 0.03 \ h^3 \text{Mpc}^{-3} \) and no photo-z errors. We present the redshift-space results for two number densities, as indicated by the legend. In all cases we can see that our model, indicated by lines, approximately reproduces the numerical data, displayed by symbols. The differences for \( n = 10^{-2} \ h^3 \text{Mpc}^{-3} \) on large scales are driven by our model for the shot noise subtraction. Independently of the size of photo-z errors, on large scales the SNR of each moment is lower than that of the real-space power spectrum. This implies that, in the regime where shot noise is dominant and despite the clustering enhancement due to large-scale RSD, in redshift space the SNR of power spectrum moments is lower than that in real space. This confirms the predictions of the toy model introduced in the previous section.

For samples with photo-z errors, we appreciate an increase in the SNR relative to the case with no photo-z errors on scales where \( k \sigma_{\text{eff}} \simeq 1 \), and a decrease on scales where the contribution of shot noise is the dominant in equation (16). Moreover, for \( \sigma_z \lesssim 0.5 \) per cent, the enhancement occurs on the scales where BAO are located. As BAOs are suppressed by the non-linear evolution of the matter density field

\[
\text{Figure 3.} \quad \text{Diagonal and off-diagonal elements (top and bottom panels, respectively) of the auto-correlation matrices of } P_0, P_2, \text{ and } P_3, \text{ and their cross-correlations. The left-hand and right-hand panels show the results for } \sigma_z = 0 \text{ and } 0.5 \text{ per cent, respectively, which are computed using 1 000 samples from the COLA ensemble with } n = 0.01 \ h^3 \text{Mpc}^{-3}. \text{ For samples with photo-z errors the diagonal terms of the cross-correlations decrease on small scales (we do not display those of the auto-correlations because their value is one by definition). Off-diagonal terms are computed at } k = 0.2 \ h \text{Mpc}^{-1}, \text{ and their amplitudes are small independently of the size of photo-z errors.}
\]
and RSD, this enhancement could imply that stronger cosmological constraints are derived from samples with sub-percent photo-z errors. We will return to this in the next section. Note that these apparent advantages may disappear after applying reconstruction procedures to the density field.

In Table 1 we show the ratio between the total SNR of $P_0$, $P_2$, and $P_4$ (computed taking into account the covariances among them; see Fig. 3) and that of the real-space power spectrum with no photo-z errors. This ratio is computed from 0.05 to 0.3 $h^{-1}$ Mpc, the range of scales we use for BAO analyses in Section 6. We find that $\frac{\text{SNR}_{\text{reduc}}}{\text{SNR}_{\text{prev}}} = 1$ for samples with no photo-z errors and $n = 10^{-2} h^{-1}$ Mpc, and thus there is the same amount of information in redshift space as in real space. Nevertheless, this information is not equally distributed in both spaces. In real space and for samples with no photo-z errors all the information is stored in the $\ell = 0$ moment, whereas in redshift space it is distributed between the $\ell = 0$, 2, and 4 moments. In this case, we find that the $\ell = 0$ moment accounts for $\approx 97 \%$ of the signal, the $\ell = 2$ moment the remaining $\approx 3 \%$, and the $\ell = 4$ moment does not contain any new information. Examining samples with a lower number density we find that the $\ell = 0$ moment encodes even more information, whereas $P_4$ starts to provide as much information as $P_2$. This motivates the analysis of $P_0$, $P_2$, and $P_4$ in Section 6.

The results of Table 1 indicate that the value of $\frac{\text{SNR}_{\text{reduc}}}{\text{SNR}_{\text{prev}}}$ decreases by increasing the size of photo-z errors and $n = 0.03 h^{-1}$ Mpc. This ratio, $\frac{\text{SNR}_{\text{reduc}}}{\text{SNR}_{\text{prev}}}$, is computed from 0.05 to 0.3 $h^{-1}$ Mpc, the range of scales that we use for BAO analyses in Section 6.

\begin{table}[h]
\centering
\caption{Ratio of the total SNR of $P_0$, $P_2$, and $P_4$ and the SNR of the real-space power spectrum with no photo-z errors and $n = 0.03 h^{-1}$ Mpc. This ratio is computed from 0.05 to 0.3 $h^{-1}$ Mpc, the range of scales used for BAO analyses in Section 6.}
\begin{tabular}{cccc}
\hline
$n [h^{-3} \text{Mpc}^3]$ & $\sigma_z$ [per cent] & $\frac{\text{SNR}_{\text{reduc}}}{\text{SNR}_{\text{prev}}}$ \\
\hline
$10^{-2}$ & 0 & 1.00 \\
$10^{-2}$ & 0.3 & 0.61 \\
$10^{-2}$ & 0.5 & 0.52 \\
$10^{-2}$ & 1.0 & 0.39 \\
$10^{-3}$ & 0 & 0.70 \\
$10^{-3}$ & 0.3 & 0.40 \\
$10^{-3}$ & 0.5 & 0.31 \\
$10^{-3}$ & 1.0 & 0.22 \\
$10^{-4}$ & 0 & 0.21 \\
$10^{-4}$ & 0.3 & 0.11 \\
$10^{-4}$ & 0.5 & 0.08 \\
$10^{-4}$ & 1.0 & 0.05 \\
\hline
\end{tabular}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Ratio of the SNR of power spectrum moments, $P_0/P_0$, to that of a case with no photo-z errors and $n = 0.03 h^{-1}$ Mpc in real space, $P'/P'$. On large scales, the SNR of $P_0$ is greater for samples with a large number of densities and photo-z errors. However, on small scales the SNR of power spectrum moments decreases by increasing $\sigma_z$.}
\end{figure}

4 EFFECT OF PHOTO METRIC REDSHIFT ERRORS ON BAO

In this section we investigate the effect of photo-z errors on the BAO feature imprinted in power spectrum moments. In Section 4.1 we study the suppression of BAOs due to the combined effect of photo-z errors, the non-linear evolution of the matter density field, and RSD. In Section 4.2 we analyse the cosmological information encoded in BAOs, in Section 4.3 we introduce a model to estimate the uncertainty in measuring the BAO scale, and in Section 4.4 we address how to extract cosmological information from the joint analysis of power spectrum moments.

4.1 The shape of the BAO signal

Let us begin by considering the following quantity:

$$B_\ell(k) \equiv \frac{P\_\ell(k)}{P\_\ell^{\text{no wiggle}}(k)} - 1,$$

where $P\_\ell^{\text{no wiggle}}$ is a no-wiggle version of $P\_\ell$. This is a quantity with the same broad-band shape as $P\_\ell$ but no BAO. Therefore, $B_\ell$ is
insensitive to the overall shape of the observed moments and isolates the BAO wiggles.

Motivated by Renormalized Perturbation Theory (Crocce & Scoccimarro 2008), we write the non-linear redshift-space power spectrum as

\[ P(k, \mu) = \left[ P_{\text{lin}}(k) G(k, \mu) + P_{\text{nc}}(k, \mu) \right] b^2 \mathcal{F}^2, \tag{22} \]

where \( P_{\text{lin}} \) is the linear theory power spectrum in real space, \( P_{\text{nc}} \) denotes contribution of the coupling between different k-modes, and \( G \) is a propagator that controls the suppression of BAOs due to non-linearities and RSD. This propagator is well approximated by a 2D exponential function:

\[ G(k, \mu) = \exp \left\{ - \left[ (1 - \mu^2) k^2 \sigma_\parallel^2 + \mu^2 k^2 \sigma_\perp^2 \right] \right\}, \tag{23} \]

where \( \sigma_\parallel \) and \( \sigma_\perp \) are parameters that control the suppression of k-modes along and perpendicular to the LOS, respectively. Note that, in redshift space \( \sigma_\parallel > \sigma_\perp \). This is a consequence of peculiar velocities and the non-linear mapping between real and redshift-space positions, and it implies that the larger the value of \( \mu \), the greater the suppression of BAO for a given wavemode (e.g. Seo & Eisenstein 2007; Sánchez, Baugh & Angulo 2008).

Let us now write a theoretical model for \( B \):\n
\[ B_\ell(k) \approx B_{\text{lin}}(k) G_{\text{eff}, \ell}(k), \tag{24} \]

where \( B_{\text{lin}} = P_{\text{lin}}/P_{\text{lin}}^{\text{sm}} \), and

\[ G_{\text{eff}, \ell}(k) = \left( \frac{\langle G(k, \mu) \mu^\ell \mathcal{F}^2 \rangle_k}{\langle \mu^\ell \mathcal{F}^2 \rangle_k} \right)^{2/\epsilon \mu}, \tag{25} \]

In deriving these expressions, we have assumed that \( P \approx [P_{\text{lin}} + (P_{\text{lin}} - P_{\text{nc}})] G b^2 \mathcal{F}^2 \) and \( P_{\text{nc}} \approx P_{\text{lin}} b^2 \mathcal{F}^2 \). These assumptions are not strictly correct because they do not take into account the \( P_{\text{nc}} \) term in equation (22), and thus equation (25) does not predict a shift in the BAO scale due to non-linearities as the full model does (Crocce & Scoccimarro 2008).

The term \( G_{\text{eff}, \ell} \) gives the effective suppression of BAOs in \( P_\ell \), as a function of the scale. As we can see, this quantity is a weighted average of \( G(k, \mu) \), where the weights are given by photo-z errors and RSD. Therefore, a balance between these two aspects will determine the final appearance of BAOs, as we will see next.

4.1.1 The Gaussian case and comparison with simulations

In the particular case of Gaussian photo-z errors, \( G_{\text{eff}, \ell} \) has an analytic expression. In the top, middle, and bottom panels of Fig. 5, we show \( G_{\text{eff}, 0}, G_{\text{eff}, 2}, \) and \( G_{\text{eff}, 4} \), respectively. To build this figure, we used \( \sigma_\parallel = 6.8 \, h^{-1} \) Mpc and \( \sigma_\perp = 4.3 \, h^{-1} \) Mpc (these values were set by our fits to the numerical simulations presented in Section 6). The vertical dotted and dashed lines indicate the scale at which the suppression of BAO is 50 per cent and 90 per cent, respectively.

As we can see, the BAO suppression is weaker for samples with larger photo-z errors. This has an interesting consequence, photo-z errors make BAO wiggles to appear sharper. Furthermore, for a given photo-z error, the suppression is stronger for higher order moments. This counter-intuitive result can be understood by recalling that LOS modes – where BAO smearing is more significant – contribute less to a given moment, thus, when photo-z are included, BAOs appear more alike to the less-damped real-space case.

In order to check the accuracy of the predictions of equation (25), we measure \( B \) from the COLA ensemble. To obtain the no-wiggle power spectrum \( P_{\text{sm}} \), we fit \( P_\ell \) using the following model:

\[ P_{\text{sm}, \ell}(k) = A_{0, \ell} (\mu^\ell \mathcal{F}^2)_k P_{\text{lin}}(k) + A_{3, \ell} k^2 + A_{2, \ell} k + A_{3, \ell} k^{-2} + A_{2, \ell} k^{-3}, \tag{26} \]

where \( P_{\text{sm}} \) is the non-wiggle power spectrum from (Eisenstein & Hu 1998), and the factors \( A_{0, \ell} \) give us enough freedom to fit the broad-band shape of \( P_\ell \) without fitting the oscillations. We explicitly show this in Fig. 6, where we display the average value of \( B \) measured from samples of the COLA ensemble with \( n = 10^{-2} \, h^3 \) Mpc^{-3}. In descending order, the panels show the results for \( P_0, P_2, \) and \( P_4 \). Note that for samples with sub-percent photo-z errors, \( B_3 \) and \( B_4 \) are not displayed up to \( k \sim 0.3 \, h^{-1} \) Mpc^{-1}. This is because on small scales the measurements start to be progressively dominated by shot noise.

We find that in real space the BAO feature is the identical in the cases with and without photo-z errors. This is expected from equation (25), as in real space \( \sigma_\parallel \gg \sigma_\perp \), and thus \( G_{\text{eff}, 0} \approx e^{-k^2 \sigma_\parallel^2} \). However, in redshift space, BAOs are less suppressed for samples with greater photo-z errors, confirming the predictions of Fig. 5 and our analytic model.

4.2 Cosmological information encoded in BAO

We now explore the cosmological information encoded in the BAO feature. Let us consider a given scale \( k = \sqrt{k_\parallel^2 + k_\perp^2} \) and angle \( \mu = k_\parallel/k \) in the two-dimensional power spectrum. Assuming a fiducial cosmology, they are observed as

\[ k^{\text{fid}} = k/\alpha = k \sqrt{\sigma_\parallel^2 + (1 - \mu^2)\sigma_\perp^2}, \tag{27} \]

\[ \mu^{\text{fid}} = \mu \equiv \frac{\mu}{\sqrt{\mu^2 + (1 - \mu^2)\sigma_\parallel/\sigma_\perp}}. \tag{28} \]
Figure 6. Average \( B_\ell \) computed using 1000 samples from the COLA ensemble with \( n = 0.01 \ h^3 \text{Mpc}^{-3} \) for \( P_0 \), \( P_2 \), and \( P_4 \). The grey and purple areas denote the 1σ confidence region for samples with \( \sigma_z = 0 \) and 1 per cent, respectively. For samples with photo-z errors we do not show the results up to \( k \sim 0.3 \ h \text{Mpc}^{-1} \) because our procedure for extracting \( B_\ell \) does not work well on scales where power spectrum moments are dominated by shot noise. In real space the amplitude of BAO is not modified by photo-z errors. In redshift space the amplitude of BAO is not modified by photo-z errors. In real space the amplitude of BAO is not modified by photo-z errors. In redshift space the amplitude of BAO is not modified by photo-z errors.

where \( \alpha_{\parallel} = \frac{H_{\text{fid}}}{H(z)} \) and \( \alpha_{\perp} = \frac{D_{\text{ac}}(z)}{D_{\text{ac}}(z)} \). In the above expressions, \( r_s \) is the sound horizon scale, \( D_{\Lambda} \) is the angular diameter distance, \( H \) is the Hubble parameter, and fid denotes these quantities in the fiducial cosmology. The observed redshift-space power spectrum moments are thus

\[
P_\ell(k/a_\ell) = a_\ell^{-1} a_\parallel^{-2} (\epsilon^{\ell+1} \mu^\ell) \left[ 1 + \epsilon^2 \mu^2 (\alpha_\parallel^2 - 1) \right] \times F^2(k/\alpha, \epsilon \mu) P_0'(k/\alpha),
\]

where assuming an incorrect cosmology would cause isotropic and anisotropic deformations. For the full expression of deformations in observed multipoles, see Padmanabhan & White (2008) in Fourier space and Xu et al. (2013) in configuration space.

Since here we are mostly interested in the information encoded in the BAO scale, we follow Ross, Percival & Manera (2015, R15 hereafter) and focus only on the stretch parameters \( \alpha_\ell \) as a function of \( \alpha_{\parallel} \) and \( \alpha_{\perp} \). As in R15, we will also assume that the information on \( \alpha_\ell \) is separable from the overall shape of power spectrum moments and that the information in different \( \mu \) bins is independent. Under these assumptions,

\[
\alpha_\ell(k) = \frac{\langle \mu^\ell F^2 \alpha \rangle_k}{\langle \mu^\ell F^2 \rangle_k},
\]

where the scale dependence of \( \alpha_\ell \) emerges from the scale dependence of \( F^2 \). The previous expression reduces to equation (6) of R15 for samples with no photo-z errors nor small-scale RSD. Therefore, small-scale RSD and/or photo-z errors induce a scale dependence in the stretch parameter.

As in R15, we find that the first-order expansion of \( \alpha_\ell \) around the fiducial solution can be expressed as \( \alpha_\ell(k) = \alpha_{\parallel}^{m(k)} \alpha_{\perp}^{n(k)} \), where \( m_\ell \) and \( n_\ell \) are given by

\[
m_\ell(k) = \frac{\partial \langle \alpha_\ell \rangle}{\partial \alpha_{\parallel}} \bigg|_{\alpha_{\perp}=1} = \frac{\langle \mu^{\ell+2} F^2 \rangle_k}{\langle \mu^\ell F^2 \rangle_k},
\]

\[
n_\ell(k) = \frac{\partial \langle \alpha_\ell \rangle}{\partial \alpha_{\perp}} \bigg|_{\alpha_{\parallel}=1} = 1 - m_\ell(k),
\]

where the higher the value of \( m_\ell \), the more sensitive \( \alpha_\ell \) is to the Hubble parameter. For the case of Gaussian photo-z errors, \( m_\ell \) and \( n_\ell \) have analytic expressions.

Going back to the stretch parameter, the known case with \( m_0 = 1/3 \) and \( n_0 = 2/3 \) (Eisenstein et al. 2005) is only recovered in real space without photo-z errors. In redshift space, there is a dependence of \( m_\ell \) and \( n_\ell \) on \( \beta \) even if \( \sigma_z = 0 \). In general, the effect of photo-z errors and small-scale RSD is to decrease the sensitivity of \( \alpha_\ell \) on \( H \), whereas large-scale RSD have the opposite effect. Nonetheless, the exact degeneracy between \( \alpha_\ell \) and \( \alpha_{\perp} \) also depends on the properties of analysed sample, such as its large-scale bias.

4.3 Toy model for the uncertainty in the stretch parameter

In Section 3.3 we studied the SNR of power spectrum moments and in the previous section we showed that the suppression of the BAO feature depends on \( G_{\text{eff}} \). In this section we use this to derive an analytic estimation of the uncertainty in \( \alpha_\ell \) as a function of large-scale bias, number density, photo-z error, and cosmology.

To build an estimator for the precision measuring \( \alpha_\ell \), we will assume that its uncertainty is given by the convolution of the one

\[1\text{In what follows we will assume that } \alpha_{\parallel}^2/\alpha_{\perp}^2 = 1 \text{ and thus } \epsilon = 1.\]
measuring $P_\ell$ and the amplitude of BAO wiggles in $P_\ell$. The latter obviously depends on the suppression of BAO due to non-linearities and RSD, which as we explained before is captured by $G_{\text{eff, }\ell}$. For conventional approaches using Fisher matrix forecast, see for instance Seo & Eisenstein (2003).

We estimate the amplitude of unsuppressed BAOs in linear theory by measuring the average value of the local extrema of $1 + B_{\text{lin}}$. Then, we do a linear fit of these values, $B_{\text{amp}}$. The product of this quantity and equation (25), $G_{\text{eff, }\ell} B_{\text{amp}}$, gives thus the maximum amplitude of BAO wiggles in $P_\ell$ under the presence of non-linearities, RSD, and photo-z errors.

In BAO analyses, the stretch parameter $\alpha_\ell$ is extracted from an interval of scales. As this parameter is scale-dependent due to small-scale velocities and photo-z errors, it is useful to define an effective stretch parameter $\alpha_{\text{eff, }\ell}$. We estimate the uncertainty in this parameter as the uncertainty in a moment time the amplitude of BAO wiggles

$$\hat{\sigma}[\alpha_{\text{eff, }\ell}] = \frac{A_\ell}{(k_{\text{max}} - k_{\text{min}})^{\frac{1}{2}}} \times \left( \int_{k_{\text{min}}}^{k_{\text{max}}} dk \frac{P_\ell^2 G_{\text{eff, }\ell}^2 B_{\text{amp}}}{\sigma^2[P_\ell]} \right)^{-\frac{1}{2}},$$

(33)

where $A_\ell$ and $c$ are free parameters. Note that we assumed that the off-diagonal terms of the covariance matrices of power spectrum moments are negligible on the range of scales where BAOs are located (see Section 3.2.3).

### 4.4 The scale dependence of cosmological information

In Section 4.2 we showed that there is a scale dependence for the cosmological information encoded in BAOs, which introduces an additional complication while extracting information from BAO analyses. We present in the top, middle, and bottom panels of Fig. 7 the value of $m_0$, $m_2$, and $m_4$ as a function of the scale, respectively.

Solid lines indicate the results for different photo-z errors, $b = 1$, and our adopted cosmology (cf. Section 2.1). Dashed lines denote when the sensitivity of BAOs to the Hubble parameter is the same as in real-space, $m = 1/3$. As we can see, the higher is the order of the moment, the greater is the sensitivity on the Hubble parameter. For example, if we look at samples with $\sigma_z = 0.3$ per cent, $m_0$, $m_2$, and $m_4$ are greater than $1/3$ up to $k \lesssim 0.1$, 0.2, and $0.3\, h\, \text{Mpc}^{-1}$, respectively. Therefore, the analysis of BAOs in the $\ell = 4$ moment of samples with $\sigma_z = 0.3$ per cent is more sensitive to the Hubble parameter than in real space. This highlights the importance of analysing BAOs in the $l = 0$ and $l = 2$ moments. In addition, we can see that there is a scale dependence of $m_\ell$ for samples with no photo-z errors, which is caused by small-scale velocities.

We can derive the relation between the precision in measuring $\alpha_\ell$ and the radial and perpendicular components of $\alpha - $ Hubble parameter and angular diameter distance, respectively – using equations (31) and (32):

$$\sigma^2[\alpha_{\ell, c}] = m_\ell^2 \sigma^2[\alpha_{\parallel}] + n_\ell^2 \sigma^2[\alpha_{\perp}],$$

(34)

where the uncertainties in the radial and perpendicular components are $\sigma[\alpha_{\parallel}]$ and $\sigma[\alpha_{\perp}]$, respectively. Consequently, to extract the precision in measuring both components, we need to perform a joint fit of at least two power spectrum moments.

In the same way that we computed an effective value for the stretch parameter in the previous section, we can now estimate an overall degeneracy between the parallel and perpendicular components of $\alpha_\ell$ when they are estimated from an interval of scales, $m_{\text{eff, }\ell}$. To do this, we compute the variance-weighted average of $m_\ell$ and $n_\ell$ over the desired $k$-interval, where the variances are given by equation (33). Explicitly,

$$m_{\text{eff, }\ell} = \frac{\int_{k_{\text{min}}}^{k_{\text{max}}} dk P_\ell^2 G_{\text{eff, }\ell}^2 B_{\text{amp}}}{\int_{k_{\text{min}}}^{k_{\text{max}}} dk P_\ell^4 G_{\text{eff, }\ell}^4},$$

(35)

and thus, the degeneracy between the overall precision in the parallel and perpendicular components of $\alpha_{\text{eff, }\ell}$ is given by

$$\sigma^2[\alpha_{\text{eff, }\ell}] = m_{\text{eff, }\ell}^2 \sigma^2[\alpha_{\parallel}] + n_{\text{eff, }\ell}^2 \sigma^2[\alpha_{\perp}].$$

(37)

Nevertheless, we are interested in the precision measuring $H$ and $D_A$. In Section 6 we conduct a joint analysis of BAO in $P_0$, $P_2$, and $P_4$ to compute their uncertainties, as it is required the analysis of at least two moments to break the degeneracy in cosmological information along and perpendicular to the LOS. From now on, we drop the subindex eff for simplicity. In this way,

$$\Sigma = M^+ E (M^+)^T,$$

(38)

$$\Sigma = \begin{pmatrix} \sigma^2[\alpha_{\parallel}] & \sigma[\alpha_{\parallel \perp}] \\ \sigma[\alpha_{\parallel \perp}] & \sigma^2[\alpha_{\perp}] \end{pmatrix},$$

(39)

$$M = \begin{pmatrix} m_0 & n_0 \\ m_2 & n_2 \\ m_4 & n_4 \end{pmatrix},$$

(40)

$$E = \begin{pmatrix} \sigma^2[\alpha_{\parallel}] & \sigma[\alpha_{\parallel \perp}] & \sigma[\alpha_{\parallel \perp}] & \sigma^2[\alpha_{\perp}] \\ \sigma[\alpha_{\parallel \perp}] & \sigma^2[\alpha_{\parallel}] & \sigma[\alpha_{\parallel \perp}] & \sigma^2[\alpha_{\perp}] \\ \sigma[\alpha_{\parallel \perp}] & \sigma[\alpha_{\parallel \perp}] & \sigma^2[\alpha_{\parallel}] & \sigma^2[\alpha_{\perp}] \\ \sigma[\alpha_{\parallel \perp}] & \sigma[\alpha_{\parallel \perp}] & \sigma[\alpha_{\parallel \perp}] & \sigma^2[\alpha_{\perp}] \end{pmatrix},$$

(41)

where $M^+$ is the pseudoinverse of $M$ (Moore 1920; Bjerkholm 1951; Penrose 1955), $\Sigma$ is the covariance matrix of $H$ and $D_A$, and $E$ is computed from BAO analysis of power spectrum moments.

Figure 7. Sensitivity of BAOs to the Hubble parameter as a function of the scale, $m_\ell$. The top, medium, and bottom panels show the results for BAOs in $P_0$, $P_2$, and $P_4$, respectively. The vertical lines indicate the scale at which the dependence of BAO on the Hubble parameter is smaller than for samples with no photo-z errors in real space ($m = 1/3$). As expected, the larger the photo-z error, the smaller is the dependence of BAOs on the Hubble parameter. We can see that samples with no photo-z errors also present a dependence of $m_\ell$ on the scale, which occurs due to small-scale velocities.
5 EXTRACTING INFORMATION FROM BAO

In the previous sections we showed how photo-z errors modify the amplitude of power spectrum moments, their variances, the suppression of BAOs, and the cosmological information encoded in them. In this section we employ all this information to create a model to unbiasedly extract the BAO scale, $\alpha$, from observational and/or simulated data, even under the presence of photo-z errors. We will employ this model on simulated catalogues in Section 6.

5.1 Modelling power spectrum moments

Based on the expressions provided in Section 4.1, we can write the following four parameter model for power spectrum moments:

$$P_{f,\ell} = P_{\ell}^{\text{sm}}(k)[B_{\text{proj}}(k/\alpha, f)G_{\ell}(k/\alpha, \sigma_{\text{eff}}, \sigma_\perp, f) + 1]$$

(42)

where $P_{\ell}^{\text{sm}}$ is computed using the same approach as in Section 4.1, the parameter $\alpha_f$ allows for stretching of the BAO, $\sigma_\perp$ and $f = \sigma_f/\sigma_\perp$ control BAO suppression. Photo-z errors enter only through $G_{\ell}$. Note that all these parameters only appear in the expression for $B_{\text{proj}}$ and $G_{\ell}$, and thus they are only constrained by BAO information, i.e. our model extracts cosmological information regardless of the overall shape of the analysed moment.

In the following sections we will not vary $\sigma_{\text{eff}}$, we will use the value that corresponds to the photo-z error introduced in each simulation. This is done to avoid degeneracies between $\sigma_\perp$ and $\sigma_{\text{eff}}$, as both control the suppression of BAOs. In any case, we checked that the value of $\alpha_f$ and its uncertainty are largely independent of fixing $\sigma_{\text{eff}}$ or not.

5.2 Parameter likelihood calculation

To extract the information encoded in BAOs, we jointly fit $P_0$, $P_2$, and $P_4$. For this, we assume that the probability of observing $d = [P_0, P_2, P_4]$ is given by a multivariate Gaussian distribution:

$$P(d|\pi) \propto \exp \left[ -\frac{1}{2} (d-t)^T C^{-1} (d-t) \right] ,$$

(43)

where $t(k, \pi) = [P_{f,0}, P_{f,2}, P_{f,4}]$, and $\pi = \{\alpha_0, \alpha_2, \alpha_4, \sigma_\perp, f\}$ denote the five free parameters of our model. The priors on these parameters are assumed to be flat over the range: $\alpha_\ell \in [0.75, 1.25]$ and $\sigma_\perp \in [0.5, 10]$, and $f \in [1, 10]$. We set the minimum value of $f$ to be 1 because in redshift space the combination of RSD and non-linearities make $k$-modes along the LOS to be the most suppressed (see Section 4.1). We note that the results do not change if we set wider priors. $C^{-1}$ is the data precision matrix of $d$, which we compute from the COLA ensemble as described in Section 2.2. The interval of scales considered is $k = [0.05-0.30]$ h Mpc$^{-1}$.

We sample the posterior probability distribution function of $\pi$ employing the publicly available code EMCEE (Foreman-Mackey et al. 2013). This code is an affine invariant MCMC ensemble sampler that has been widely tested and used in multiple scientific studies. We configure the code to analyse $d$ using a chain of 100 random walkers with 5,000 steps each, and a burn-in phase of 500 steps. We checked that this burn-in phase is sufficient to obtain well-behaved chains.

Additionally, we checked that the standard deviations of the best-fit values from the COLA ensemble are compatible with the uncertainties estimated from the likelihood of each simulated catalogue.

6 RESULTS FROM SIMULATED CATALOGUES

In this section we start by studying the quality of the BAO fitting model introduced in the previous section. Then, we apply this model to simulated catalogues with different photo-z errors and number densities. In addition, we study whether introducing photo-z errors following PDFs different from Gaussian impacts our results. Finally, we present a simple procedure to undo the effect of peculiar velocities on power spectrum moments.

We note that in this section we analyse power spectrum moments computed from DM particles. In principle, this could introduce a shift in the stretch parameter with respect to computing power spectrum moments from haloes/galaxies. However, Angulo et al. (2014) addressed this by measuring the stretch parameter from the real-and redshift-space monopole of galaxies selected by star formation and stellar mass in the Millennium XXL simulation (Angulo et al. 2012), finding no additional shifts with respect to the stretch parameter measured from DM particles. Therefore, we will restrict our analysis to DM particles.

6.1 Quality of our model

In Fig. 8 we show the result of fitting the model introduced in equation (42) to samples from the COLA ensemble with $\sigma_\ell = 0$ and $n = 10^{-2} h^3$ Mpc$^{-3}$. Symbols indicate the average value of $B_\ell$ computed from the COLA ensemble, and lines the average of the best-fit to each mock. We display the results for $\ell = 0, 2, 4$, and $\ell = 4$ in black, red, and blue, respectively, where they are offset for clarity. Shaded areas enclose the 1σ region from mock to mock. In the bottom panel we plot the relative difference between the model and data, where we can see that in all cases the typical deviations are statistically insignificant. In addition, our model reproduces the simulated data to within 1 per cent on the scales shown, which is more than enough for the next generation of galaxy surveys. We also checked that we obtain similar results for samples with sub-percent photo-z errors.
We start by studying the shift in the stretch parameter with respect to its fiducial value, $\alpha_\ell - 1$, from the BAO analysis of real- and redshift-space power spectrum moments of 1 000 samples from the COLA ensemble. In the left-hand and right-hand panels of Fig. 9 we present the distribution of these parameters and their uncertainties, respectively, from the analysis of samples with different photo-z errors. Arrows point to the mean of each distribution. For samples with sub-percent photo-z errors, the value of $\alpha_\ell$ is compatible with the fiducial cosmology at the 1σ level ($\alpha_\ell - 1 = 0$). Therefore, sub-percent photo-z errors do not introduce an additional shift in the position of BAO. As we can see in the right-hand panels, in real space the precision measuring $\alpha_\ell$ decreases with the size of photo-z errors. Nevertheless, in redshift space samples with small photo-z errors measure $\alpha_\ell$ with more precision than samples with no errors. This is because the suppression of BAO in power spectrum moments decreases with the size of photo-z errors (see Section 4.1).

### 6.2 Effect of redshift errors

In this section we apply our BAO analysis procedure to power spectrum moments of samples with $n = 10^{-3} h^3$ Mpc$^{-3}$ and Gaussian photo-z errors of different sizes.

We start by studying the shift in the stretch parameter with respect to its fiducial value, $\alpha_\ell - 1$, from the BAO analysis of real- and redshift-space power spectrum moments of 1 000 samples from the COLA ensemble. In the left-hand and right-hand panels of Fig. 9 we present the distribution of these parameters and their uncertainties, respectively, from the analysis of samples with different photo-z errors. Arrows point to the mean of each distribution, and we gather these values in Table 2. As we can see in the left-hand panels, the average value of $\alpha_\ell - 1$ is compatible with zero at the 1σ level for all samples. This implies that for samples with sub-percent photo-z errors our estimator is unbiased relative to the case with no errors. However, for larger photo-z errors this is no longer correct, as we can see for $\alpha_\ell$ and $\sigma_\ell = 1.0$ per cent. This is because our procedure to calculate a smooth version of power spectrum moments does not work well if the amplitude of the moment is of the order of the shot noise level.

In real and redshift space the stretch parameter presents a small and positive shift, which is caused by the non-linear evolution of the matter density field (e.g. Angulo et al. 2008; Crocce & Scoccimarro 2008; Smith, Scoccimarro & Sheth 2008; Padmanabhan & White 2009). We also find that this shift slightly decreases with the size of photo-z errors. This is because photo-z errors strongly suppress $k$-modes along the LOS on small scales, which are the $k$-modes that present a greater coupling due to non-linearities and RSD, and thus reduce the effective shift in the volume-averaged stretch parameters. Whereas the shifts that we find are statistically significant for the volume of the COLA ensemble, 27 000 $h^{-3}$ Gpc$^3$, for a single simulation they are compatible with zero to within 1σ. Furthermore, these shifts can in principle be approximately corrected for via re-
constructions algorithms (e.g. Eisenstein et al. 2007; Schmittfull et al. 2015) or by recalibrating the estimator employed.

The top-right panel of Fig. 9 displays the uncertainty in the stretch parameter measured from samples in real space. As we expect, the precision measuring $\alpha_z$ decreases with the size of photo-z errors. This is the consequence of photo-z errors suppressing the clustering on small scales, which increases the relative contribution of shot noise to the variance, and thus reduces the SNR.

In redshift space the precision measuring $\alpha_z$ is a combination of the effect explained in the previous paragraph and the fact that photo-z errors reduce the overall suppression of BAOs (see Fig. 5). As explained in Section 4.1, they reduce the weight of $k$-modes parallel to the LOS when doing the angular average of the 3D power spectrum, where these modes are noisier than the perpendicular ones owing to RSD. In the bottom-right panel we can see that the balance between both effects causes $\sigma(\alpha_z)$ not to be a monotonic function of the size of photo-z errors. We find that $\alpha_z$ can be measured with more precision from samples with small photo-z errors (0.5 per cent) than from samples with no errors. Nevertheless, if we increase the size of photo-z errors this is no longer true, for instance $\sigma(\alpha_z)$ for samples with $\sigma_z = 1.0$ per cent is greater than for samples with no photo-z errors. For higher order moments the results are different because the range of scales for which the variance is not dominated by shot noise is smaller. However, for samples with $\sigma_z = 0.3$ per cent we still find that $\alpha_z$ can be measured with more precision or the same precision, respectively, as from samples with no photo-z errors.

A by-product of the MCMC analysis of power spectrum moments is the value of the parameters $(\sigma_\perp, f)$, which encode the suppression of the moments perpendicular and parallel to the LOS $(\sigma_\parallel = \sigma_z)$, respectively. We find that their average value and uncertainty are approximately the same independently of the size of photo-z errors. For samples with no photo-z errors they are $\sigma_\perp = 4.02 \pm 0.13$ and $f = 1.06 \pm 0.05$ in real space, and $\sigma_\perp = 4.29 \pm 0.38$ and $f = 1.68 \pm 0.25$ in redshift space. As expected, RSD increase the suppression of $k$-modes parallel to the LOS: for the COLA ensemble at $z = 1$ this suppression is 68 per cent greater than for $k$-modes perpendicular to the LOS. This is the main reason behind getting more precise results in $\alpha_z$ from samples in redshift space with sub-percent photo-z errors for which shot noise is not relevant on BAO scales.

### 6.3 Effect of different PDFs for photometric redshift errors

In general, photo-z errors do not follow a Gaussian PDF. For instance, the comparison between photometric and spectroscopic redshifts in the COSMOS survey shows that the PDF of photo-z errors is well described by a Lorentzian variate (Ilbert et al. 2009). Additionally, for low redshift galaxies the PDF of photo-z errors usually shows a tail towards higher redshifts, which is a natural consequence of imposing $z > 0$ in an otherwise symmetric PDF. In this section we investigate whether photo-z drawn from non-Gaussian PDFs may bias the results from BAO analyses when assuming that they are drawn from a Gaussian PDF.

We will consider two families of functional forms for photo-z errors:

\[ \Pr[\delta_z] dz = \frac{1}{2 \Delta \Gamma \left( 1 + \frac{1}{p_1} \right)} \exp \left( -\frac{|z|^p_1}{{\Delta}^2} \right) dz, \] (44)

\[ \Pr[\delta_z] dz = -\frac{1}{p_2 z \sqrt{2\pi}} \exp \left[ -\frac{1}{2 p_2^2} \ln^2 \left( -\frac{p_2 z}{\Delta} \right) \right] dz, \] (45)

where $\Gamma$ is the Gamma function, $\Delta$ controls the width of the distributions, $p_1$ the excess kurtosis for the family PDF1, and $p_2$ the skewness and excess kurtosis for the family PDF2. The distributions from PDF1 are symmetric (zero skewness) and show different levels of excess kurtosis. We find that for $p_2 = 2$ the distributions of this family are Gaussians, for $p_2 < 2$ show extended wings like a Lorentzian, and for $p_2 > 2$ are boxier than a Gaussian. The distributions from PDF2 are asymmetrical and not centred in zero. We find one famous distribution of this family for $\Delta = p_2$ and $p_2 < 0$, the log-normal distribution.

In Fig. 10 we display the PDF of distributions that we use to introduce photo-z errors in the COLA ensemble. We note that the ones with $p_2 \neq 0$ are in general more extreme than the PDF of photo-z errors from real data. Therefore, if we were to find that the results from BAO analyses assuming a Gaussian distribution are the same for all these distributions, we could conclude that BAO analyses are insensitive to the shape of the PDF of photo-z errors.

In Fig. 11 we present the value of $\alpha_0$, $\alpha_z$, and $\alpha_\parallel$ extracted from the BAO analysis of the average moments of 100 samples from the COLA ensemble after assuming a Gaussian PDF in the analysis. The number density of these samples is $n = 10^{-2} h^3 \text{Mpc}^{-3}$, their photo-z errors are drawn from the distributions displayed in Fig. 10, and the difference between the 84th and 16th percentiles of those distributions is set to be $\sigma_z = 0.3$ per cent. The grey coloured regions indicate the 1 $\sigma$ confidence region for a Gaussian PDF, and the error bars for the other distributions. For extreme PDFs, this

---

2We note that this procedure has never been applied to reconstruct the 3D density field of galaxy samples with photo-z errors.

---

Figure 10. Distributions from which photo-z errors are drawn in Section 6.3. The parameter $p_1$ controls the excess kurtosis of distributions from PDF1 and $p_2$ the skewness and excess kurtosis of that from PDF2. All distributions from PDF1 are symmetric (zero skewness), where the ones with $p_1 = 2$, $p_1 > 2$, and $p_1 < 2$ are Gaussians, boxier than Gaussians, and Gaussians with extended wings like Lorentzians, respectively. The distributions from PDF2 are positively skewed, and their skewness and excess kurtosis grow with $p_2$.

(i) PDF1:

\[ \frac{1}{2 \Delta \Gamma \left( 1 + \frac{1}{p_1} \right)} \exp \left( -\frac{|z|^p_1}{{\Delta}^2} \right) dz, \] (44)

(ii) PDF2:

\[ -\frac{1}{p_2 z \sqrt{2\pi}} \exp \left[ -\frac{1}{2 p_2^2} \ln^2 \left( -\frac{p_2 z}{\Delta} \right) \right] dz, \] (45)
could in principle introduce systematic errors in the estimation of \( \alpha_i \). In practice we can see that even considering extreme PDFs and assuming a Gaussian PDF in the BAO analysis, the results are compatible to within 1\( \sigma \). In addition, the shift in the stretch parameter is largely insensitive to the actual shape of the PDF.

In this section we have disregarded the possibility of interlopers – objects systematically assigned to incorrect redshifts. This may happen to unobscured quasars and star-forming galaxies in medium- and narrow-band surveys, as pairs of emission lines at different redshifts may fall in the same filters, which is translated into different redshifts may fall in the same filters, which is translated into near-IR spectra from the same quasar.

We note that all expressions in this work are given for arbitrary PDFs. Therefore, if it is possible to know the PDF of photo-z errors, its actual shape could be used in BAO analyses.

### 6.4 Extracting cosmological information from BAO and impact of number density

As explained in Section 6.2, the constraining power of BAO depends on the scale at which the shot noise level starts dominating the amplitude of power spectrum moments. In this section we address how the precision in measuring cosmological parameters from BAO analyses (\( H \) and \( D_A \)) depends on the number density of the analysed sample. For this, we re-analyse our COLA samples randomly diluted to have \( n = [10^{-2}, 3 \times 10^{-3}, 10^{-3}, 3 \times 10^{-3}, 10^{-4}] h^3 \text{Mpc}^{-3} \).

In the top and bottom panels of Fig. 12 we present the average precision in measuring \( \sigma_\parallel \) and \( \sigma_\perp \) from 1 000 COLA samples with \( n = 10^{-2} \) and \( 10^{-3} h^3 \text{Mpc}^{-3} \), respectively. Contours enclose the 1\( \sigma \) confidence region for samples with different Gaussian photo-z errors, as stated in the legend. The uncertainty in \( \sigma_\parallel \) always increases with the size of photo-z errors, which is because the error ellipses rotate anticlockwise as photo-z errors grow. Nevertheless, the uncertainty in \( \sigma_\perp \) (after marginalizing over \( \sigma_\parallel \)) always increases with the size of photo-z errors, which is because the error ellipses rotate anticlockwise as photo-z errors grow. For the number densities studied here, we find that for \( \sigma_\parallel = 0.3 \) per cent the precision in measuring \( D_A \) is greater than for samples with \( \sigma_\parallel = 0 \), which highlights that the analysis of power spectrum moments might not optimally extract the cosmological information encoded in BAO perpendicular to the LOS. We further explore this in the next section.
To continue exploring the constraints on cosmological parameters as a function of number density and photo-z errors, in the left-hand, middle, and right-hand panels of Fig. 13 we display the precision in $H$, $D_A$, and the FoM of both parameters, respectively. The uncertainty in each parameter is computed after marginalizing over the other. Symbols and lines indicate the results from simulations and the analytic model introduced in equation (33), respectively, where the free parameters of the model are fitted to reproduce the results from simulations. Their value is $A_0 = 0.35$, $A_2 = 0.50$, $A_4 = 0.55$, and $c = 0.3$. As expected, the uncertainty in $H$ grows with the size of photo-z errors and by decreasing the number density. However, it is important to notice that the precision in $H$ is the same for samples with $\sigma_z = 0.5$ per cent and $n = 10^{-3} h^3$ Mpc$^{-3}$ and as for samples no photo-z errors and $10^{-4} h^3$ Mpc$^{-3}$. As spectro-photometric surveys detect in general fainter objects than spectroscopic surveys, future wide-field surveys with dozens of photometric bands such as J-PAS will be competitive with spectroscopic surveys measuring cosmological parameters from BAO analyses.

The precision measuring $D_A$ and the FoM of $H$ and $D_A$ shows a non-monotonic behaviour with $\sigma_z$ for samples with a large number of densities, whereas at smaller number densities they are proportional to the size of photo-z errors. As we have commented above, we leave the discussion of this to the following section.

Our analytic model, which only employs the real-space $l = 0$ moment as input, reasonably fits the results from simulations. Therefore, it can be used to make forecasts for the precision measuring cosmological parameters from samples with different number densities, linear biases, photo-z errors, and cosmologies. Furthermore, we can use this model to look for the best sample to constrain cosmology. All the above considerations should be taken into account for the optimal design of future galaxy surveys. For instance, the photo-z errors for a given galaxy sample might depend not only on the hardware employed, but also on the intrinsic properties of galaxies (e.g. brighter objects having more accurate redshift estimates). In such a case, the sample that delivers the strongest constraints on cosmological parameters is not necessarily the one with the smallest photo-z errors.

6.5 Loss of transverse information from the analysis of power spectrum moments

As we showed in the previous section, for a large number of densities $D_A$ is measured with more precision from samples with sub-percent photo-z errors than from samples with no errors. We obtain the same results from simulations and from the analytic model introduced in equation (33). As the introduction of photo-z errors cannot increase the amount of cosmological information, this highlights that the analysis of power spectrum moments does not extract all the cosmological information encoded in BAO. Therefore, it is worth to follow other approaches such as the analysis of the full anisotropic power spectrum (e.g. Ballinger, Peacock & Heavens 1996).

This reduction of the cosmological information available from BAO analyses only appears in redshift space. RSDs suppress more strongly parallel $k$-modes, and when we take the angular average of the power spectrum to compute its moments, we treat in the same way all $k$-modes, even when the ones parallel to the LOS are noisier. Consequently, the resulting power spectrum obtained after averaging over all $k$-modes in a $k$-bin is noisier than if only perpendicular $k$-modes are considered. For samples with photo-z errors this is not the case, as they reduce the weight of parallel $k$-modes during the angular average, and thus the uncertainty after
angular averaging over all $k$-modes is approximately the same as for perpendicular $k$ modes (cf. Section 4.1).

To probe that the angular average of $k$-modes with different uncertainties causes $D_A$ to be measured with smaller precision, we deconvolve the effect of RSD from power spectrum moments:

$$P(k) = \left( \frac{n\tilde{P}(k, \mu) - 1}{n\tilde{F}^2(k, \mu)} \right)_{k}. \quad (46)$$

In Fig. 14 we show the average uncertainty in cosmological parameters computed from 1000 samples of the COLA ensemble with $n = 10^{-2} h^3$ Mpc$^{-3}$ and no photo-$z$ errors. The purple, grey, and blue ellipses indicate the results in real space, redshift space, and redshift space after deconvolving the effect of RSD. Our deconvolution procedure increases the precision measuring cosmological parameters from BAO analyses, and in the previous section we showed that this model approximately reproduces the results from numerical simulations. In this section we use this expression to forecast the precision in $H$ from future spectro-photometric surveys at $z = 1$. We note that the results of this section are illustrative.

In what follows, we will assume that the number density of galaxies linearly scale with the size of photo-$z$ errors (see table 8 of Benitez et al. 2014) and that the analysed volume is $v = 78.7$ Gpc$^3$, i.e. the same volume as each COLA simulation. In particular, we take the relation between number density and photo-$z$ errors to be $n = 5(1 + 10^3\sigma_\alpha)10^{-4} h^3$ Mpc$^{-3}$. In Fig. 15 we display the precision measuring $H$ after marginalizing over $D_A$ from galaxy samples with $b = 2$ and different photo-$z$ errors. For samples with $\sigma_\alpha \leq 0.3$ per cent, the precision measuring $H$ is approximately the same. On the other hand, for $\sigma_\alpha > 0.3$ per cent the uncertainty in $H$ is rapidly increased. This encourages the employment of spectroscopic and spectro-photometric surveys with $\sigma_\alpha < 0.5$ per cent to study the expansion history of the Universe.

In summary, to design and fully exploit galaxy surveys that employ noisy estimators to compute redshifts, it is necessary to carefully select the properties of the target galaxy sample.

8 CONCLUSIONS

The next generation of galaxy surveys will dramatically increase the precision of measurements for the expansion and growth history of the Universe. Some of these surveys will observe large areas of the sky with linear variable filters or dozens of narrow-bands, providing a low-resolution spectra for every region of the sky. In addition, they will measure the redshift of millions of galaxies with sub-percent accuracy, offering a promising way of constraining cosmological parameters. Nevertheless, to fully exploit this new kind of data, it is necessary to fully characterize the effect of photo-$z$ errors on cosmological observables.

In this work we presented a detailed study of the impact of sub-percent photo-$z$ errors on the clustering of galaxies in Fourier space, with an emphasis on the BAO signal. We derived analytic expressions for how photo-$z$ errors modify power spectrum moments, their
variances, and the smearing of BAO, which we compared with the results from 1 000 N-body simulations.

Our main findings can be summarized as follows:

(i) In real space photo-z errors suppress power spectrum moments on intermediate and small scales. This increases the interval of scales dominated by shot noise, which reduces the range of scales available for BAO analyses. There is an additional effect in redshift space: the suppression of angular-averaged BAO wiggles gets weaker with increasing photo-z errors. This is because photo-z errors reduce the weight of LOS k-modes in computing power spectrum moments which have more dilated BAO signal due to non-linear RSD.

(ii) We derived how the cosmological information encoded in BAO depends on the properties of the galaxy sample studied. We showed that small-scale RSD and/or photo-z errors induce a scale dependence on this information, where the dependence on the Hubble parameter (angular diameter distance) decreases (increases) with the size of photo-z errors.

(iii) Based on these findings, we built a model for extracting cosmological information from the analysis of power spectrum moments. Then, we applied it to simulated galaxy catalogues with different number densities and photo-z errors. We found that photo-z errors do not introduce an additional shift in the position of the BAO scale with respect to the no photo-z error case. Therefore, they do not bias the cosmological information encoded in BAOs. In addition, we found that assuming that photo-z errors are Gaussian in BAO analyses, even when they are drawn from PDFs with large excess kurtosis and skewness, does not bias the results.

(iv) In Section 6.4 we analysed the precision measuring the Hubble parameter and the angular diameter distance from samples with different number densities and photo-z errors. We found that for the same number density, the uncertainty in measuring $H$ decreases with the size of photo-z errors. Nevertheless, it is still possible to measure $H$ with the same (or more) precision from samples with sub-percent photo-z errors as from samples with no errors if the number density of the first is increased. Finally, we also found that the analysis of power spectrum moments artificially decreases the precision in measuring $D_A$. We suggest to analyse the 2D power spectrum in future studies.

Our results encourage the measurement of cosmological parameters from spectro-photometric surveys, as in general they are deeper than spectroscopic surveys for the same integration time. In Section 7 we forecast the precision in measuring the Hubble parameter from BAO analyses assuming that the number density of galaxies linearly scales with the size photo-z errors. Roughly, we found the same results for galaxy samples with redshift uncertainties smaller than $\sigma_z = 0.4$ percent. This means that galaxy surveys with sub-percent photo-z errors could set constraints on the dark energy equation of state as precise as spectroscopic surveys.

Along this paper we put a focus on extracting cosmological information from galaxy samples with sub-percent photo-z errors. Recently, Ross et al. (2017) conducted a similar investigation in configuration space for samples with photo-z errors of a few per cent, finding that for those the BAO feature mostly constrains $D_A(z)$ and that the projected correlation function is enough for extracting all cosmological information. Their findings agree with ours for samples with $\sigma_z = 1$ per cent, as we can see in Figs 12 and 13. Nevertheless, as we show along this work, for samples with smaller photo-z errors $H(z)$ can be constrained from the 3D galaxy clustering.

Finally, our paper highlights that photo-z errors substantially increase the complexity of the extraction of cosmological information from BAO analyses. Therefore, it is crucial a thorough understanding and modelling of photo-z errors in galaxy clustering. We hope our work to have clarified some the most important aspects of this issue, and that it will help in the cosmological analysis of future spectro-photometric surveys.

ACKNOWLEDGEMENTS

We thank the anonymous referee for the thorough review, insightful comments, and positive suggestions. We acknowledge discussions with Raul Abramo, Andreu Font-Ribera, Licia Verde, and Carlos López-Sanjuan. Argonne National Laboratory’s work was supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under contract DE-AC02-06CH11357. The authors acknowledge support from the Spanish Ministry of Economy and Competitiveness (MINECO) through the project AYA2015-66211-C2-2. JCM acknowledges support from the Fundación Bancaria Ibercaja for developing this research. REA acknowledges support from the European Research Council through grant number ERC-StG/716151. CHM acknowledges support from the Ramón y Cajal Fellow Program of the Spanish MINECO. This project has received funding from the European Union’s Horizon 2020 Research and Innovation Programme under the Marie Skłodowska-Curie grant agreement No. 734374.

REFERENCES

Abramowitz M., Stegun I. A., 1972, Handbook of Mathematical Functions, United States Department of Commerce, National Institute of Standards and Technology, New York: Dover

Alcock C., Paczynski B., 1979, Nature, 281, 358

Angulo R. E., Pontzen A., 2016, MNRAS, 462, L1

Angulo R. E., Baugh C. M., Frenk C. S., Lacey C. G., 2008, MNRAS, 383, 755

Angulo R. E., Springel V., White S. D. M., Jenkins A., Frenk C. S., 2012, MNRAS, 426, 2046

Angulo R. E., Springel V., Henriques B., 2014, MNRAS, 442, 2131

Ballinger W. E., Peacock J. A., Heavens A. F., 1996, MNRAS, 282, 877

Benitez N. et al., 2009, ApJ, 691, 241

Benitez N. et al., 2014, arXiv:prints

Bjerhammar A., 1951, Trans. Roy. Inst. Tech. Stockholm, 1951, 86 pp. (2 plates)

Blake C., Bridle S., 2005, MNRAS, 363, 1329

Cai Y.-C., Angulo R. E., Baugh C. M., Cole S., Frenk C. S., Jenkins A., 2009, MNRAS, 395, 1185

Chaves-Montero J. et al., 2017, MNRAS, 472, 2085

Colombi S., Jaffe A., Novikov D., Pichon C., 2009, MNRAS, 393, 511

Crocco M., Scoccimarro R., 2008, Phys. Rev. D, 77, 023533

Dalton G. et al., 2014, in Proc. SPIE Conf. Ser., Vol. 9147, Ground-Based and Airborne Instrumentation for Astronomy V, SPIE, Bellingham, p. 91470L

de Jong R., 2011, The Messenger, 145, 14

DESI Collaboration et al., 2016, arXiv:prints

Dolney D., Jain B., Takada M., 2006, MNRAS, 366, 884

Doré O. et al., 2014, arXiv:prints

Doré O. et al., 2016, arXiv:prints

Eisenstein D. J., Hu W., 1998, ApJ, 496, 605

Eisenstein D. J. et al., 2005, ApJ, 633, 560

Eisenstein D. J., Seo H.-J., Spergel D. N., 2007, ApJ, 664, 675

Foreman-Mackey D., Hogg D. W., Lang D., Goodman J., 2013, PASP, 125, 306

Glazebrook K., Blake C., 2005, ApJ, 631, 1

Hartlap J., Simon P., Schneider P., 2007, A&A, 464, 399

MNRAS 477, 3892–3909 (2018)
