Theoretical investigation for the relation (supermassive black hole mass)—(spiral arm pitch angle): a correlation for galaxies with classical bulges

Ismaeel A Al-Baidhany¹, Sami S Chiad¹, Wasmam A Jabbar¹, Rasha A Hussein², Firas F K Hussain³ and Nadir F Habubi¹*

¹Department of Physics, College of Education, Mustansiriya University, Baghdad, Iraq.
²College of Engineering, Al-Muthanna University, Baghdad, Iraq.
³Department of Physics, College of Science, Al-Muthanna University, Baghdad, Iraq.

E-mail: ismaeel_2000@uomustansiriyah.edu.iq, dr.sami@uomustansiriyah.edu.iq, wasmnajabbar@uomustansiriyah.edu.iq, rasha.lasereng@mu.edu.iq, firas.f@mu.edu.iq.

*Corresponding Author. E-mail: nadirfadhil@uomustansiriyah.edu.iq.

Abstract.

In this work, the determination of the masses of supermassive black holes (SMBHs) and the properties of their host spiral galaxies are focused for the purposes of constraining scaling relations and with the aim of understanding the role of SMBHs in the evolution of galaxies.

The measurements of SMBHs mass for a sample of 40 spiral galaxies were studied by applying indirect techniques (the SMBHs mass versus stellar/gas velocity dispersion relation). In addition, spiral arm pitch angle of a sample of nearly face-on spiral galaxies were measured using IRAF (The Image Reduction and Analysis Facility) and two-dimensional Fast Fourier Transform (2DFFT) program.

Finally, we present a new correlation between spiral arm pitch angle (a measure of the tightness of spiral structure) and the mass of supermassive black holes (BHs) in the nuclei of classical galaxies.

Key words: supermassive black holes—spiral galaxies: fundamental parameters—galaxies: pitch angle—galaxies: dispersion velocity.

1. Introduction

Supermassive black holes (SMBHs) are common at the center of all or most of galaxies [1, 2]. As observed at high sensitivities and resolution with the Hubble Space Telescope (HST). In addition, their masses are in the range of hundreds of thousands to billions of solar masses [3-5].
Over the last decade, studies of galaxies have led to the discovery that there are many strong or tight correlations locally between the masses of the SMBHs and the global properties of the spheroid components of their hosts. This suggests an intriguing link between galaxy formation and SMBH growth. As a result, astrophysicists believe that the energy released by growing SMBHs play important role in shaping the properties of the structure of their host galaxies [6].

There is increasing evidence which indicates that relationships between the mass of the SMBH and almost or all the possible parameters of the host galaxy bulges. This suggests that SMBHs play an important role in galaxy formation.

Most galaxy bulges contain a central SBH whose mass strongly correlates with stellar velocity dispersion (σ*) within the effective radius (r_e) (MBH-σ*); [2, 7], with the bulge luminosity or spheroid luminosity of the galaxy (L_{bul}) (M_{bul}-L_{Bulge}) [1,3,8,9,10], with the bulge mass (M_{bulge}) [1,9] and circular velocity [11], with the galaxy light concentration[12], the dark matter halo[11], with the effective radius[13], the Sersic index [14], with the gravitational binding energy and gravitational potential, combination of bulge velocity dispersion, effective radius and/ or intensity [15], with the radio core length [16] and the inner core radius [17].

Seigar et al. 2008 found a strong correlation between M_{BH} and the spiral arm pitch angle using 27 galaxies, as well as Barrier et al. 2013. Davis et al. 2018 found a tight linear correlation between M_{BH} and the spiral arm pitch angle for 44 pseudobulges galaxies [4, 1].

As mentioned above, all scaling laws have led previous authors to the conclusion that SMBH, growth and bulge formation regulate each other [18]. That means that mass of the SMBH is somehow tied to the structural parameters of the rest of the galaxy.

The correlations between the mass of supermassive black holes (SMBHs) and properties of their host galaxies helped to understand the mechanism of nuclear energy by the formation and evolution of BHs [19].

In this work, first, we used a correlation to estimate the SMBH mass compared with other correlations (MBH-σ*), where σ* is the bulge velocity dispersion.

Second, we used the correlation with spiral arm pitch angle [1], to study a correlation between SMBHs mass and spiral arm pitch angle for bulges, pseudobulges, barred, nonbarred, non-AGN (Active galactic nuclei), and AGN galaxies.

2. Methods

1. Measuring spiral arm pitch angle

One of the more interesting methods to find SMBH masses in late-type galaxies use the relationship between SMBH mass in the nuclei of disk galaxies and spiral arm pitch angle (P) (a measure of the
tightness of spiral structure), Seigar et al. (2008) found that SMBHs are linked by a strong correlation with P. Additionally, a correlation between P and rotation curve shear (S) was discovered [1].

Previous studies described logarithmic spiral in polar coordinates [20]. This is a special kind of spiral curve that describes the arm in disk galaxies:

$$r = r_0 e^{\theta \tan(\phi)}$$  \hspace{1cm} (1)

where \(r\) is radius, \(\theta\) is central angle, \(r_0\) is initial radius when \(\theta = 0\), and pitch angle is \(-90 \leq \Phi \leq 90\).

Because the spiral arm pitch angle has been shown to be independent of the wavelength at which it is measured, multi-band images were used to determine it for our sample of spiral galaxies [20].

Spiral arm pitch angles were measured using a two-dimensional fast Fourier transform (2DFFT) decomposition with logarithmic spirals of Spitzer/IRAC 3.6 μm images of 63 galaxies, with inclinations of \(30^\circ \leq i \leq 60^\circ\). The 2DFFT program analyzes images of spiral galaxies and categorizes their pitch angles and number of arms. The two-dimensional fast Fourier transform decomposition program is fully described by Seigar et al. (2012) [21].

The amplitude of each Fourier component is given by:

$$A(m, p) = \frac{\sum_{i=1}^l \sum_{j=1}^l I_{ij}(\ln r, \theta) \exp[-i(m\theta + p \ln r)]}{\sum_{i=1}^l \sum_{j=1}^l I_{ij}(\ln r, \theta)}$$  \hspace{1cm} (2)

Where \(r\) and \(\theta\) are polar coordinates, \(I(\ln r, \theta)\) is the intensity at position \((\ln r, \theta)\), \(m\) represents the number of arms or modes, and \(p\) is the variable associated with the pitch angle \(P\) defined by \(P = -(m/p_{max})\).

IRAF (The Image Reduction and Analysis Facility) was used to determine the ellipticity values and major-axis position angle in order to deproject the 3.6 μm galaxy images to fully face-on by assuming circular outer isophotes. ELLIPSE in IRAF was used to derive inclination angle \((\alpha)\); [22], which is defined by:

$$\alpha = \cos^{-1}(b/a)$$  \hspace{1cm} (3)

where \((a)\) is the semi-major axis and \((b)\) is the semi-minor axis. Where the value \(0^\circ\) describes a face-on galaxy and \(90^\circ\) describes an edge-on galaxy.

2. Measurement SMBHs using \((M_{BH} - \sigma_*)\) relation:

The \(M_{BH} - \sigma_*\) relation supports the notion of regulated formation mechanisms and co-evolution for the galaxy’s central black hole mass and the bulge velocity dispersion [23].

The \(M_{BH} - \sigma_*\) relation is one of the most common techniques used to estimate the mass of SMBH at the center of a spiral galaxy [15]. This is done by measuring the velocity dispersion of stars in the galactic bulge. This method was based on the observation that supermassive black hole masses correlate with the dispersion velocity of the surrounding stellar component (bulge) of spiral galaxies.
[24]. Because BH masses found in late-type spirals and spheroidal galaxies have lower mass, we used the $M_{\text{BH}} - \sigma_*$ relation since it has the least scatter [18].

The velocity dispersion ($\sigma_*$) of classical bulges and pseudobulges in spiral galaxies were converted to SMBH masses using the following relation [19]:

$$M_{\text{BH}}(\sigma_*) = 10^{8.13\pm0.06}(\sigma_*/200 \text{ km s}^{-1})^{4.02\pm0.32}$$ (4)

Using 40 spiral galaxies observed with Spitzer at 3.6μm along with the $M_{\text{BH}}-\sigma_*$ relation, SMBH masses were determined [19].

3. Results and discussion

In this work, the correlation between SMBH mass and spiral arm pitch angle is re-examined. This was initially determined by Seigar et al. 2008 for 40 spiral galaxies. The $M_{\text{BH}} - P$ correlations are shown in Figures (1), (2), and (3). In Figure (1), we found that Pearson’s linear correlation coefficient for this correlation was 0.833, and the fit is a linear relation:

$$\log_{10} \frac{M_{\text{BH}}}{M_{\odot}} = (8.46 \pm 0.14) - (0.058 \pm 0.02)P$$ (5)

From Figure (1), we found Pearson’s linear correlation coefficient for a correlation between SMBH and $P$ is 0.83 for 40 spiral galaxies. This work confirmed the conclusion by Seigar et al. (2008) and Berrier et al (2013) that a significant correlation exists between SMBH masses and the spiral arm pitch angles [1, 17, 26]. On the other hand, our results are similar to some extent with Seigar’s relation [17].

Figure (2) shows the relations in $M_{\text{BH}}-P$ plots (we assigned markers to these galaxies according to bulges and pseudobulges).

This part is dedicated to the results and discussion of the correlations between $M_{\text{BH}}$ and spiral arm pitch angle, where the study of the correlations is considered a benchmark to research BH vs. host galaxy co-evolution, as well as the study of the location in the scaling relations for the classical bulges and pseudobulges of spiral galaxies.

Spitzer/IRAC provided images of unprecedented quality in the MIR (Fazio et al. 2004) for the 40 galaxies analyzed. IRAC’s high resolution permits the clear identification of morphological features, like identify classical bulges and pseudobulges with our analysis [8]. The spiral galaxies that have a pseudobulge and classical bulge allow us to better investigate at 3.6 μm differences between pseudobulge and classical bulge properties [27][28][29][30].

We found Pearson’s linear correlation coefficient for a correlation between SMBH and $P$ to be 0.92 and 0.78 for classical bulges and pseudobulges, respectively. The best-fitting lines are shown for this diagram:

$$\log_{10} \frac{M_{\text{BH}}}{M_{\odot}} = (8.42 \pm 0.12) - (0.051 \pm 0.015)P$$ (Classical bulges)
These results proved that pseudobulges do not follow the same $M_{\text{BH}} - P$ relation as classical bulge. This means that $M_{\text{BH}} - P$ relation has the same properties of $M_{\text{BH}} - \sigma^*$ relation as pseudobulge and classical bulge galaxies - due to the existence of a strong correlation between bulge central velocity dispersion and spiral arm pitch angle [1, 17].

According to Pearson's linear correlation coefficient for a correlation between SMBH and P, the $M_{\text{BH}} - P$ relation for classical bulge galaxies is the tightest correlation that we measured; and the results is significantly consistent with the result of Seigar et al. (2008). On the contrary, the $M_{\text{BH}} - P$ for pseudobulges galaxies is significantly below the $M_{\text{BH}} - P$ relation.

The indirect MBH measurements for spiral galaxies would be reliable to study the important and analyse the behaviour of $(M_{\text{BH}} - P)$ for pseudobulges and classical bulge galaxies.

$$\log_{10}\frac{M_{\text{BH}}}{M_\odot} = (8.08 \pm 0.19) - (0.049 \pm 0.013)P \quad \text{(Pseudobulges)}$$

**Figure 1.** Supermassive black hole masses from $(M_{\text{BH}} - \sigma)$ relation as a function of spiral arm pitch angle $(P)$. The solid line is the fit to all spiral galaxies.
Figure 2. SMBH masses as a function of spiral arm pitch angle. The linear regression are shown as dash dot dot dot, dash dot and dashed, respectively, for classical bulge, pseudobulge galaxies.

Figure (3) shows a plot of SMBHs masses from (MBH-P) relation, for non-barred, non-AGN, and AGN galaxies respectively. Pearson's linear correlation coefficients were found for a correlation between SMBH and P to be 0.83, 0.87, and 0.78 respectively. Pearson's linear correlation coefficients values were noted for all types of galaxies. The significance level at which the null hypothesis of zero correlation is disproved is 3σ.

The best-fitting lines are shown for this diagram:

\[ \log_{10} \frac{M_{BH}}{M_\odot} = (8.403 \pm 0.15) - (0.05 \pm 0.011)P \] (Non – Barred)

\[ \log_{10} \frac{M_{BH}}{M_\odot} = (8.39 \pm 0.13) - (0.059 \pm 0.013)P \] (Non – AGN)

\[ \log_{10} \frac{M_{BH}}{M_\odot} = (8.36 \pm 0.17) - (0.051 \pm 0.017)P \] (AGN)
Figure 3. Supermassive black hole masses from \((M_{BH}-\sigma)\) relation as a function of spiral arm pitch angle \((P)\). The linear regression are shown as long dash, dash dot dot dot, dash dot and dashed, respectively, for non-barred, Non-AGN, and AGN galaxies.

Table 1. Linear correlation coefficient and linear regression coefficients of SMBHs as a function of the spiral arm pitch angle \([\log(M_{BH})]=\alpha + \beta(P)\):
Comparison with previous studies

The previous works by Seigar et al. (2008), Berrier et al. 2013, and Davis et al. 2018, attempted to use a new techniques of measuring the spiral arm pitch angle of the host galaxy such as the two-dimensional fast Fourier transform decomposition program by several bands (K, 3.6µm), and they found \( M_{\text{BH}}-P \) relation [1,18,24].

The most prominent results of previous studies are as follows:
1-They found the differences of the best-fit parameters.
2- All of them reached different values for the slope of the spiral galaxies \( M_{\text{BH}}-P \) correlation.
3- Seigar et al. 2008 found a strong correlation between \( M_{\text{BH}} \) and the spiral arm pitch angle using 27 galaxies, as well as Barrier et al. 2013 have confirmed Seigar’s result using 67 galaxies.
4- Davis et al. 2018 found a tight linear correlation between \( M_{\text{BH}} \) and the spiral arm pitch angle for 44 pseudobulges galaxies.

of these, only a few were done on K-band and 3.6 µm which were available to us for comparison with our results. In Table (2), the results were compared with three other results: Seigar et al. (2008), Berrier et al. 2013, Davis et al. 2018) [19,20], and our work.

In all cases, the previous studies found different values for the slope of the galaxies in the \( M_{\text{BH}}-P \) correlation.

| A       | \( \beta \) | N   | Ref.                     |
|---------|-------------|-----|-------------------------|
| 8.44±0.10 | 0.076±0.005 | 27  | Seigar et al. 2008 (Direct and indirect method) |
| 8.36±0.15 | 0.076±0.008 | 67  | Berrier et al. 2013 (Direct and indirect method) |
| 8.12±0.16 | 0.062±0.009 | 34  | Berrier et al. 2013 (Direct method) |
| 8.47±0.24 | 0.089±0.013 | 23  | Berrier et al. 2013 (Indirect) |
| 7.01±0.07 | 0.171±0.017 | 44  | Davis et al. 2018 (pseudobulges) (Direct method) |
| 8.46 ± 0.14 | 0.058 ± 0.02 | 40  | This work (Indirect) |
| 8.42 ± 0.012 | 0.051 ± 0.015 | 25  | This work (classical bulges) |
| 8.08 ± 0.19 | 0.049 ± 0.013 | 15  | This work (pseudobulges) |
| 8.40 ± 0.15 | 0.05 ± 0.011 | 14  | This work (Non-Barred) |
| 8.39 ± 0.13 | 0.059 ± 0.013 | 18  | This work (AGN) |
| 8.36 ± 0.17 | 0.051 ± 0.017 | 23  | This work (Non-AGN) |
Table 3. Estimated Galaxies Parameters. Columns: (1) galaxy name. Columns: (2) Hubble type taken from the Hyper-Leda catalogue and the NASA/IPAC Extragalactic database (NED). Columns: (3) Dispersion velocity in km s\(^{-1}\) taken from HyperLeda catalogue; (4) Spiral arm pitch angle (P). The spiral arm pitch angle (P) taken from Berrier et al. (2013)(1), and Davis et al. (2012)(2) and this work (3). Columns: (5) \(\log (M_{\text{BH}}/M_{\odot})\) calculated by using \(M_{\text{BH}}-\sigma\) relation.

| Name     | Leda Type | P (deg.) | \(\sigma_{*}\) (km/sec) | \(\log (M_{\text{BH}}/M_{\odot})\) |
|----------|-----------|----------|--------------------------|------------------------------------|
| Circinus | Sb        | 26.7\(^{(3)}\) | 75                       | 6.418±0.1                          |
| IC 2560  | SBb       | 16.3\(^{(3)}\) | 137                      | 7.469±0.2                          |
| NGC 224  | Sb        | 8.5±1.3\(^{(3)}\) | 160±8                    | 7.794±0.23                         |
| NGC 613  | Sbc       | 23.68±1.77\(^{(1)}\) | 125.3±18.9              | 7.309±0.2                          |
| NGC 1022 | SBa       | 19.83±3.6\(^{(1)}\) | 99                      | 6.902±0.3                          |
| NGC 1068 | Sb        | 17.3±2.2\(^{(2)}\) | 151±7                    | 7.63±0.05                          |
| NGC 1097 | SBb       | 16.7±2.62\(^{(1)}\) | 150                      | 7.627±0.18                         |
| NGC 1300 | Sbc       | 12.7±1.8\(^{(3)}\) | 145±22                   | 7.568±0.17                         |
| NGC 1350 | Sab       | 20.57±5.38\(^{(1)}\) | 120.9±2.08              | 7.251±0.04                         |
| NGC 1353 | Sb        | 36.6±5.4\(^{(1)}\) | 83                      | 6.594±0.13                         |
| NGC 1357 | Sab       | 16.16±3.48\(^{(1)}\) | 121±14                  | 7.252±0.03                         |
| NGC 1365 | Sb        | 15.4±2.4\(^{(3)}\) | 151±20                   | 7.639±0.07                         |
| NGC 1398 | SBab      | 6.2±2\(^{(3)}\) | 216±20                   | 8.264±0.08                         |
| NGC 1433 | SBab      | 25.82±3.79\(^{(1)}\) | 84±9                    | 6.615±0.05                         |
| NGC 1566 | SABb      | 21.31±4.78\(^{(3)}\) | 100±10                  | 6.919±0.07                         |
| NGC 1672 | Sb        | 18.22±14.07\(^{(3)}\) | 130.8±2.09             | 7.388±0.14                         |
| NGC 1808 | Sa        | 23.65±7.77\(^{(1)}\) | 148                     | 7.601±0.11                         |
| NGC 2442 | Sbc       | 14.95±4.2\(^{(1)}\) | 140.7±2.18              | 7.516±0.12                         |
| NGC 3031 | Sab       | 15.4±8.6\(^{(3)}\) | 143±7                   | 7.544±0.04                         |
| NGC 3227 | SABA      | 12.9±9\(^{(3)}\) | 128±13                  | 7.35±0.16                          |
| NGC 3368 | SABA      | 14±1.4\(^{(3)}\) | 122±(28.24)             | 7.267±0.06                         |
| NGC 3511 | SABc      | 28.21±2.27\(^{(1)}\) | 93.56±2.04            | 6.803±0.07                         |
| NGC 3521 | SABb      | 21.86±6.34\(^{(2)}\) | 130.5±7.1              | 7.384±0.05                         |
| NGC 3673 | Sb        | 19.34±4.38\(^{(1)}\) | 117.45±2.07           | 7.2±0.011                          |
| NGC 3783 | SBab      | 22.73±2.58\(^{(1)}\) | 95±10                  | 6.83±0.021                         |
| NGC 3887 | Sbc       | 24.4±2.6\(^{(2)}\) | 102.01±2.05           | 6.954±0.04                         |
| NGC 4030 | Sbc       | 19.8±3.2\(^{(2)}\) | 122.43±2.1            | 7.544±0.06                         |
NGC 4151  SABa  11.8±1.8$^{(3)}$  156±8  7.696±0.07
NGC 4258  SABb  7.7±4.2$^{(3)}$  146±15  7.58±0.012
NGC 4462  SBab  17.2±5.42$^{(1)}$  146±8  7.579±0.02
NGC 4594  Sa  6.1$^{(2)}$  240±12  8.448±0.01
NGC 4699  SABb  6.2±2.2$^{(1)}$  215±10  8.256±0.05
NGC 5054  Sbc  25.57±3.73$^{(1)}$  104.48±2.05  6.996±0.06
NGC 5055  Sbc  14.9±6.9  101±5  6.937±0.08
NGC 6300  SBb  24.3±3.8$^{(1)}$  94±5  6.811±0.05
NGC 6902  SBab  13.71±2.3$^{(3)}$  145.86±2.1  7.117±0.07
NGC 7213  Sa  7.05±0.28$^{(1)}$  185±20  7.578±0.04
NGC 7531  SABb  18.31±9.09$^{(1)}$  108.7±5.6  7.993±0.03
NGC 7582  SBab  14.7±7.44$^{(3)}$  137±20  7.065±0.09
NGC 7727  SABa  15.94±6.39$^{(1)}$  181±10  7.469±0.09

4. Conclusion
Despite the enormous interest in studying the scaling relations between SMBHs and the structural parameters of the host galaxies, based on direct and indirect methods of the different bands to measure SMBH mass, we found relatively few published studies by other authors who use MBH-P relation in their studies (e.g., Seigar et al. (2008); Berrier et al. 2013; Davis et al. 2018).

In this work, the following conclusions are made:

1- Scaling relations were studied between SMBHs mass in the center of spiral galaxies, and spiral arm pitch angles.

2- Our findings are in agreement with Seigar et al. (2008) who found a strong correlation between $M_\bullet$ and P using both direct and indirect methods.

3- Finally, we found the relation between SMBHs and spiral arm pitch angle (P), and the best-fitting lines of regression were:

$$\log_{10} \frac{M_{BH}}{M_\odot} = (8.37 \pm 0.13) - (0.056 \pm 0.01)P$$  (Classical bulges)

$$\log_{10} \frac{M_{BH}}{M_\odot} = (8.41 \pm 0.11) - (0.049 \pm 0.012)P$$

$$\log_{10} \frac{M_{BH}}{M_\odot} = (8.05 \pm 0.17) - (0.045 \pm 0.011)P$$  (Pseudobulges)

$$\log_{10} \frac{M_{BH}}{M_\odot} = (8.307 \pm 0.13) - (0.052 \pm 0.01)P$$  (Non – Barred)

$$\log_{10} \frac{M_{BH}}{M_\odot} = (8.38 \pm 0.12) - (0.058 \pm 0.013)P$$  (Non – AGN)
Our analysis reveals that the strong M•−P relation found by Segar et al. (2008) are different relations when galaxies with the different types (i.e., at bulges, pseudobulges, barred, nonbarred, non-AGN, and AGN galaxies.) are taken into account.

We found a new correlation between SMBHs mass and spiral arm pitch angle for classical bulges galaxies.

Acknowledgments:

The authors are grateful to the anonymous referee for the constructive comments and suggestions. This work has been supported by Al-Mustasiryah University (www.mustansiryah.edu.iq) Baghdad-Iraq. We acknowledge the usage of the HyperLeda database (http://leda.univ-lyon1.fr). This research has made use of the NASA/IPAC extragalactic database (NED).

References

1- Magorrian J Tremaine S and Richstone D. 1998 AJ, 115 2285.
2- Ferrarese L and Merritt D 2000 ApJ 539 L9–L12.
3- Kormendy J and Richstone D O 1995ARA&A 33 581.
4- Richstone D1998Nature 395 A14.
5- Kormendy J and Gebhardt K 2001 in AIP Conf. Proc. 586. 20th Texas.
6- Benson A J and Bower R. 2010 MNRAS 405 1573.
7- Gebhardt K 2000, ApJ, 539 L13.
8- Sani E Marconi A Hunt L K and Risaliti G 2011, MNRAS413 1479-94.
9- Häring N and Rix H.-W 2004, ApJ 604 L89.
10-Gültekin K 2009 ApJ. 695 1577.
11- Ferrarese L 2002 ApJ 578 90–97.
12- Graham A W Onken C. A Athanassoula E and Combes F 2011 412 (4) 2211-2228
13- Marconi A and Hunt L K 2003 ApJ 589 L21–L24.
14- Graham Aand Driver S P 2007 ApJ 655 77.
15- Aller M Cand Richstone D O 2007 ApJ 665 120.
16- Lauer T R Faber S M and Richstone D 2007 ApJ. 662 808.
17- Seigar, M. S., Kennefick, D., Kennefick, J., & Lacy, C.H.S. 2008, ApJ, 678, L93.
18- Carollo C M 2002 Astron. J. 123 159.
19- Tremaine S Gebhardt K and Bender R 2002 ApJ. 574 740.
20- Seigar M S Ho L Cand Barth A J 2006 Bulletin of the AAS 38 1190.
21- Seigar M S Lacy C H Sand Puerari I 2012 ApJS 199 33.

\[
\log_{10} \frac{M_{\text{BH}}}{M_\odot} = (8.31 \pm 0.15) - (0.051 \pm 0.011)P \quad (\text{AGN})
\]
22- Hubble E P 1926 ApJ 64 321.
23- Fisher D B and Drory N 2008 AJ 136 773.
24- Ferrarese L and Merritt D 2000 ApJ 539 L9–L12.
25- Berrier JOEL C 2013 ApJ 769 132.
26- Sani E Marconi A Hunt L. Kand Risaliti G 2011, MNRAS. 413 1479-94.
27- Davis, L. B., Graham A. W., & Seigar, M. S. 2018, Updating the (supermassive black hole mass) (spiral arm pitch angle) relation: a strong correlation for galaxies with pseudobulges, MNRAS (1-18)
28- Al-Baidhany I Rashid H. G Chiad S S Habubi N F Jandow N N Jabbar W A and Abass k H 2018 IOP Conf. Series Journal of Physics Conf. Series 1003 012107.
29- Al-Baidhany I A Habubi N F Jandow N N Chiad S S Abass K H Jabbar W A Majeed H C 2017 The Fourth Scientific - The First International Conference, special volume 653-680.
30- Zain Albedeen F Sh Jandow N N Al-Baidhany I A Chiad S S Habubi N F and Jabbar W A 2016 Third Women scientific conference For the period (7-8) December 2016, Baghdad Science Journal, special volume 178-196.