Sliced Slices: Separating Data and Control Influences

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Abstract—Backward slicing has been used extensively in program understanding, debugging and scaling up of program analysis. For large programs, the size of the conventional backward slice is about 25% of the program size. This may be too large to be useful. Our investigations reveal that in general, the size of a slice is influenced more by computations governing the control flow reaching the slicing criterion than by the computations governing the values relevant to the slicing criterion. We distinguish between the two by defining data slices and control slices both of which are smaller than the conventional slices which can be obtained by combining the two. This is useful because for many applications, the individual data or control slices are sufficient.

Our experiments show that for more than 50% of cases, the data slice is smaller than 10% of the program in size. Besides, the time to compute data or control slice is comparable to that for computing the conventional slice.

I. INTRODUCTION

Program slicing, introduced in 1984 by Mark Weiser [7], strips down a large program to a smaller version based on the requirements of program observation. Many variants of slices such as forward, backward, dynamic, and abstract slice etc. have been devised [2]. They have been used for different purposes like program understanding, debugging, testing, maintenance, software quality assurance and reverse engineering among others. A brief description of various applications of program slices is given by Binkley et. al. [7].

Among static slicing techniques, backward slicing which answers the question “which program statements can influence the given variables at the given statement?” seems more natural and is most common. It identifies the portion of program that one would be interested in while understanding a computation or debugging for an erroneous output. In safety property checking, rather than verifying the property on whole program, one can verify the property on the static backward slice with respect to slicing criterion derived from the specific property of interest. As a result, backward slice helps scaling up of property checking techniques also.

While static slicing is efficient and scalable, the size of the computed slice may remain a matter of concern. Empirical studies [4] have shown that size of a static slice on an average is about 30% of the program size. For large programs, this size could be too large. Our investigations of the factors influencing the size of a slices reveal that most statements are included in a slice due to some conditions governing the reachability of the statement involved in the slicing criteria rather than due to the values of variables in the slicing criteria. These statements are irrelevant for understanding the computations leading to the values of the variables.

As a motivating example, assume that the program in Figure 1(a) computes an erroneous value of \( u \) at line 17. It is obvious that for slicing criterion \( \langle 17, u \rangle \), no static slicing can reduce the program anymore. Therefore using the conventional backward slice is of no help in getting a reduced program for debugging this program. A careful examination reveals that the value of \( u \) does not depend on the values of variables \( i \) or \( t \). These variables are used in the conditions which decide the reachability of line 17 in the execution. Since we know that a wrong value is getting computed at line 17, reachability of line 17 is obvious and need not be established. Thus, computations of \( i \) and \( t \) are irrelevant to our purpose.

Figure 1(b) shows a portion of the program which is sufficient to understand the computation of \( u \) and to debug the reason for its wrong value. Any erroneous statement responsible for an erroneous value has to be contained in this program fragment. The statements that have been removed only alter the reachability of line 17 and not the value computed for \( u \). Note that functions \( fn1 \) and \( fn2 \) have been sliced out as they are not required any longer.

Observe that retaining the program structure requires that a conditionally executed statement in the original program must also be included as a conditionally executed statement in the program slice. However, since the values of variables appearing in the condition are irrelevant, we replace conditions by ‘*’ which stands for a random value chosen from \{true, false\} when it is executed.

It is easy to see that the resulting slice is much smaller in comparison to conventional backward slice (incidentally the whole program in this case) and still sufficient to debug (or understand) the computation of \( u \) at the slicing criterion. We call such a slice as data slice.

Consider a contrasting requirement of debugging the program when line 17 is not getting executed a desired number of times. For this purpose we only need to see how the reachability of line 17 is getting influenced. How the value of \( u \) is computed is irrelevant. Therefore we need to know how values of \( i \) and \( t \) are getting computed as they appear in the conditions that govern whether or not line 17 will be reached. We show the portion of code in Figure 1(c). It is sufficient to understand when line 17 in original program (mapped to
The classical backward slicing tries to find the program fragment that influences a slicing criterion. Forward slicing [7] discovers the statements that are influenced by a given slicing criterion. Chopping [8] discovers statements influenced by a source criterion on paths to a target criterion. A dynamic slice [7] computes a subset of program statements which affect a slicing criterion in a particular run of the program. Assertion slicing [7], [9], [10] is a technique which computes set of statements which are sufficient to ensure a post condition or the statements which will be executed starting from a given pre condition. All these variants use control and data influences in an integrated manner. To the best of our knowledge there is no work which distinguishes between data and control influences.

B. Control flow graph and data dependence

Program model. We present our ideas in context of imperative programs modeled in terms of assignment statements, conditional statements, \texttt{while} loops, and procedure calls. We also allow \texttt{break} and \texttt{continue} statements in loops. Without any loss of generality, we restrict ourselves to goto-less programs with single-entry loops and two-way branching conditional statements at the source level.

Control Flow Graph (CFG). We use the standard notion of control flow graph (CFG) $G = (N,E)$ where $N$ is the set of nodes, $E$ is a set of directed edges in $N \times N$ [7]. \texttt{ENTRY} and \texttt{EXIT} are distinguished nodes representing the entry and exit of program. We use $s \rightarrow l$ and $s \rightarrow t$ to denote unconditional and conditional edges respectively, where $l \in \{true, false\}$ indicates the branch outcome. We assume that two special edges $ENTRY \rightarrow EXIT$ and $EXIT \rightarrow ENTRY$ are added in CFG. There is a one-to-one correspondence between nodes of CFG and statements of the program, hence we will use the terms statement and node interchangeably.

Data dependence. A definition $d$ of a variable $v$ in node $p$ is said to be \textit{reaching definition} [2] for a node point $q$, if there is a control flow path from $p$ to $q$ devoid of any other definition of $v$. A variable $x$ at location $l$ is said to be \textit{data dependent} on a definition $d$ of $x$, if $d$ is reaching definition for $l$. The set of definitions of variables in $X$ reaching $l$ is denoted by $DU(l,X) = \{ d | \exists v \in X.v \text{ at } l \text{ is data dependent on } d \}$. We will use $REF(t)$ to denote set of variables whose value is referred in a statement $t$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Usual backward slice, data slice and control slice}
\end{figure}
C. Program states and traces

Let \( V \) be set of all variables in program \( P \) and \( \mathbb{R} \) be the set of all values which the variables can take in \( P \).

**Definition 1:** (Program state). A program state is a valuation of all variables in the program at a given instant during program execution. It is represented by a map \( \theta : V \rightarrow \mathbb{R} \) such that \( \theta(v) \) denotes the value of \( v \in V \) in the program state \( \theta \).

**Definition 2:** (Restricted program state). Given \( X \subseteq V \), a \( X \)-restriction of program state \( \theta \), denoted as \( [\theta]_X \), is a map \( X \rightarrow \mathbb{R} \) such that \( \forall x \in X, [\theta]_X(x) = \theta(x) \).

**Definition 3:** (Execution state). An execution state is a pair \( (n, \theta) \) where \( \theta \) is a program state and \( n \) is a CFG node.

Execution of a program can be seen as a sequence of execution states starting with execution state \( (\text{ENTRY}, \sigma_0) \) where \( \sigma_0 \) is initial program state. The subsequent execution state \( (n, \sigma) \) for a given execution state \( (n, \sigma) \) is decided by semantics of statement corresponding to node \( n \) and program state \( \sigma \). Let function \( \text{TRAN}(n, \sigma) \) provide the subsequent execution state of \( (n, \sigma) \).

**Definition 4:** (Trace). A (possibly infinite) sequence of execution states starting with execution state \( (\text{ENTRY}, \sigma_0) \) is called a trace, provided \( \sigma_0 = \text{ENTRY} \) and \( \sigma_0 \) is initial program state, and \( \forall i \geq 0 : (n_{i+1}, \sigma_{i+1}) = \text{TRAN}(n_i, \sigma_i) \).

When the trace sequence is finite and ends with an execution state \( (\text{EXIT}, \theta) \) then the trace is called a terminating trace. Unless stated otherwise, a trace means a terminating trace in the rest of this paper.

D. Post-dominance and control dependence

Backward slicing algorithms are implemented efficiently using post-dominance and control dependence \([1], [2]\).

**Definition 5:** (Post-dominance). A node \( n_2 \) post-dominates a node \( n_1 \) if every path from \( n_1 \) to \( \text{EXIT} \) contains \( n_2 \). If, in addition \( n_1 \neq n_2 \) then \( n_2 \) is said to strictly post-dominate \( n_1 \).

**Definition 6:** (Control dependence). A node \( n_3 \) is control dependent on an edge \( n_1 \xrightarrow{1} n_2 \) if \( n_3 \) post-dominates \( n_2 \), and \( n_3 \) does not strictly post-dominate \( n_1 \).

Later Podgurski and Clarke \([2]\) introduced concept of strong post-dominance and weak control dependence, to consider execution of a statement being dependent on loop termination. The previous definition of post-dominance and control dependence were termed as weak post-dominance and strong control dependence respectively.

**Definition 7:** (Strong post-dominance). A node \( n_2 \) strongly post-dominates a node \( n_1 \) if every infinite path starting at \( n_1 \) contains \( n_2 \). If, in addition, \( n_1 \neq n_2 \) then \( n_2 \) strictly strongly post-dominate \( n_1 \).

**Definition 8:** (Weak control dependence). A node \( n_3 \) is weakly control dependent on an edge \( n_1 \xrightarrow{1} n_2 \) if \( n_3 \) strongly post-dominates \( n_2 \), and \( n_3 \) does not strictly strongly post-dominate \( n_1 \).

Only \( \text{EXIT} \) node strongly post-dominates a loop exit edge. Therefore, all the nodes that weakly post-dominate a loop condition are weakly control dependent on loop exit edge. Bilardi and Pingali \([2]\) give efficient algorithms for computing strong post-dominance and weak control dependence relationship. For our purpose, we will need to know whether a statement is controlled by a condition through a chain of weak/strong control dependence. For this, we define a transitive closure of weak/strong control dependence.

**Definition 9:** (Transitive control dependence). For given statement \( s \) and condition \( c \), if there is a path \( \pi \) in PDG from \( c \) to \( s \) consisting of only (weakly/strongly) control dependent edges then we say that \( s \) is transitive control dependent on \( c \), written as \( c \xrightarrow{t} s \). If \( \pi \) consists of only strong control dependent edges then we say \( s \) is strongly transitive control dependent on \( c \), written as \( c \xrightarrow{s} s \).

If \( \pi \) starts with a control dependent edge labeled as \( e \) then we qualify the transitive control dependence with edge \( e \) as \( c \xrightarrow{s} s \) or \( c \xrightarrow{t} s \). Obviously, \( c \xrightarrow{t} s \implies c \xrightarrow{s} s \).

Following properties are obvious, for the programs under our discourse.

**SP1** \( (c \xrightarrow{c_1} s \land c_2 \xrightarrow{c_2} s) \implies e_1 = e_2 \)

**SP2** \( c \xrightarrow{c} s \implies \) no path to \( s \) from an edge \( \bar{e} \neq e \) of \( c \) can bypass \( e \).

E. Subprogram and backward slice

An important requirement of a slice is that the behaviour of the slice must be a specified subset of the original program’s behaviour. A subset of original program’s behaviour is specified through a pair \( \Upsilon = (l, V) \), known as slicing criterion, where \( l \) is a statement location and \( V \) is set of variables. It is interpreted as values of variables \( V \) just before executing statement at \( l \). We will use \( \Upsilon \) and \( (l, V) \) interchangeably to denote a slicing criterion. Where context is clear, we will use \( l \) and \( V \) to denote the location and variables set components, respectively. We will use \( LV(t) \) to denote the slicing criterion \( (t, \text{REF}(t)) \).

A subprogram of a program \( P \) is a program carved out of \( P \) by deleting some statements such that program structure remains intact in that for each statement \( n \) that appears in subprogram, if \( n \) is enclosed by a condition in \( P \), then \( n \) must be enclosed by a condition in subprogram too.

An augmented program \((P^A)\) is the result of transforming a given program \( P \) for a slicing criterion \( \Upsilon = (l, V) \) by inserting a SKIP statement at location \( l \). Obviously, an augmented program is equivalent to the original program. To compute a slice of program \( P \) with respect to \( \Upsilon \), we compute the slice \( S \) of the augmented program with respect to \( \Upsilon \) with the restriction that the inserted SKIP statement is retained in \( S \). Now with \( l \) standing for SKIP statement, \( \Upsilon \) can be seen as specification for the desired subset of original program’s behaviour for \( P, P^A \) and \( S \). Henceforth, we assume that SKIP statement, inserted at location \( l \), is part of every slice with respect to \( \Upsilon \).

Henceforth, we will assume that each statement in \( P \) is labeled uniquely and the nodes in CFG are labeled with corresponding statement label. Statements in subprogram will get their label from the ones given in \( P \). As a result, in CFG, \( G^s = (N^s, E^s) \) constructed for a sub program \( S \) of program \( P \), having CFG, \( G : (N, E), N^s \subseteq N \). To be more explicit,
given a node \( n^s \in N^s \) and \( n \in N, n^s = n \) would mean that they represent statement with same label. This also holds for special nodes \textsc{ENTRY} and \textsc{EXIT}.

**Definition 10:** (SC execution state). Given a program \( P \) and a slicing criterion \( \langle l, V \rangle \), execution states of \( P \) having statement location as \( l \) are called SC execution states.

Given a program \( P \) and a slicing criterion \( \Upsilon = \langle l, V \rangle \), let \( \tau : [(n_i, \sigma_i)] \), \( 0 \leq i \leq k \), be the trace for input \( I \) with \( m \) SC-execution states. Let \( \tau^* : [(n^*_i, \sigma^*_i)] \), \( 0 \leq i \leq k^* \), be trace for a slice \( S \) of \( P \), on same input \( I \) with \( m^* \) SC-execution states. Let \( [(n_i, \sigma_i)] \), \( 1 \leq j \leq m \) and \( [(n^*_i, \sigma^*_i)] \), \( 1 \leq j \leq m^* \) be the sequence of SC-execution states in in \( \tau \) and \( \tau^* \) respectively. We say, \( \langle n_i, \sigma_i \rangle \) and \( \langle n^*_i, \sigma^*_i \rangle \) for \( 0 < j < \min(m, m^*) \) are corresponding SC-execution states. Now the window of observation for \( \Upsilon = \langle l, V \rangle \) is the sequence of \( V \)-restricted program states \( [[\sigma_i]_V] \), \( 1 \leq j \leq m \). We will call such a sequence as the trace window of observation \( TW(P, I, \Upsilon) \).

**Definition 11:** (SC-equivalent trace). A trace \( \tau \) of \( P \) on an input \( I \) is called SC-equivalent to trace \( \tau^* \) on slice \( S \) for same input \( I \), if \( TW(P, I, \Upsilon) = TW(S, I, \Upsilon) \).

We express the definition of a backward slice \( P^B_I \) for a program \( P \) and a slicing criterion \( \Upsilon \) as follows.

1) \( P^B_I \) is subprogram of \( P^A \).
2) For every input \( I \) on which original program terminates, \( TW(P, I, \Upsilon) = TW(P^B_I, I, \Upsilon) \).

Of all methods that use some form of data flow to compute the backward slice, the methods of Ferrante et al. \[?\] and Horwitz et al. \[?\] using program dependence graphs (PDG) produce the minimal slice.

### III. Data and control slices

A backward slice is the answer to the question “Given a slicing criterion \( \Upsilon = \langle l, V \rangle \), which program statements can affect the values of the variables \( V \)?” This question can be meaningfully split into two parts:

- **Q1** Which statements decide whether program control will reach \( l \)?
- **Q2** Assuming that control reaches \( l \), which program statements decide the values of the variables \( V \).

As mentioned in the motivating example of Figure\[1\] often we are interested in the separate answers to questions Q1 or Q2, even if we want to use them together. We call the subprogram resulting from the answer to Q1 as a control slice and that to Q2 as a data slice.

In Figure\[2\] P1 is the original program in which the codes for the functions \( f \neq 1 \) and \( f \neq 2 \) are not shown. P2 is the control slice for the slicing criterion \( \{13, \{y\}\} \). The conditions \( c_1 \) and \( x < 10 \) at lines 7 and 12 decide whether program control reaches line 13, and therefore they are part of the control slice. Further, the value of the condition \( x < 10 \) may be decided by the assignment at line 10, which, in turn, is controlled by the condition at line 9. Therefore both line 12 and line 10 are in the control slice. For similar reasons, lines 4, 5, and 6 also have to be included.

To obtain the data slice for P1 for the same slicing criterion, we reason as follows. The value of \( y \) at line 13 is given by assignment at 11 and the condition at line 12 has no impact on this value. Therefore line 11 is included in the data slice but line 12 is not. The value of \( x \) assigned to \( y \) at line 11 is computed by lines 10 and 4. Of these, the value that reaches line 11 during execution is decided by condition at line 9. Therefore lines 4, 9 and 10 are also included in the data slice. Further, line 6 is also included, as it computes the value to be used in the condition at line 9. No other statement affects the value of \( y \) at line 13. The resulting program P3 is the data slice.

We now formalize the notions of control and data slice.

#### A. Control slice

A control slice of a program \( P \) wrt. to the slicing criterion \( \Upsilon = \langle l, V \rangle \), denoted \( P^C_I \), is the backward slice of \( P \) with respect to the slicing criterion \( \langle l, \emptyset \rangle \).

Thus, the sliced program \( P^C \) contains those statements of the original program which merely caused the program control reach the program point \( l \). We now show that the control slice is contained in the backward slice.

**Lemma 1:** \( P^C_I \subseteq P^B_I \)

**Proof:** From the definition of control slice, we have \( P^C_I = P^B_{\langle l, \emptyset \rangle} \). Further, backward slices have the property \[?\] \( V_1 \subset V_2 \) implies \( P^B_{\langle l, V_1 \rangle} \subset P^B_{\langle l, V_2 \rangle} \) and thus \( P^B_{\langle l, \emptyset \rangle} \subseteq P^B_{\langle l, V \rangle} \). Consequently \( P^C_I \subseteq P^B_I \).

Since computing backward slice is a well studied problem, we can compute the control slice by computing the backward slice with respect to the slicing criterion \( \langle l, \emptyset \rangle \) following any of the existing approaches.

#### B. Data slice

While P3 is an answer to the question Q2 posed for the criterion \( \{13, \{y\}\} \) and can therefore be regarded as a data slice, it is not suitable for applications like program understanding and debugging. For example, the information that the statement at line 13 may not be executed at all is missing from P3. Thus apart from the statements that decide on the values of variables at a slicing criterion, we also need to include statements that explicate the paths along which the computation of such values takes place.

Therefore, we also include conditions that impact the reachability of \( l \). This is shown in P4, and the included conditions are shown as \( * \\) indicating a non-deterministic branch. We make such conditions non-deterministic because their values are inconsequential for our purpose. Further, if these conditions were made concrete, then we would also have to include additional statements affecting their values, increasing the size of the slice. We call such a non-deterministic conditional as abstract conditional. During execution, such a conditional can randomly evaluate to true or false.

The form of the slice as shown in P4 is good for program understanding and debugging. However it falls short if used for property verification. The reason is that while the abstract conditional helps in keeping the size of the sliced program

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small, it slides path conditions that are important for verification. As an example, suppose we want to check the property \( y < 20 \) at line 13. This property holds in original program but does not hold in P4. However, if we retain the included conditionals in concrete form and abstract out instead those assignment statements that assign to the variables involved in the conditional, we get P5. The property \( y < 20 \) holds for this program. Note that the assignments to \( x \) cannot be eliminated since they also determine the value of the variables of the slicing criterion.

An abstract assignment of the form \( x = \ast \) assigns to \( x \) a random value from some domain. If \( x \) is an integer, for example, then \( x \) is assigned an integer value between \(-2^{31} + 1\) \([2]\).

Due to inclusion of abstract conditions and assignments, there will be multiple execution paths on a given input. In general, the slice produced by abstract assignments will have less number of possible executions paths for same input in comparison to the one produced by abstract conditionals. For example, P4 may have 4 different execution paths on a given input while P5 will have only two such paths on same input. As a result, the slice produced through abstract assignments is more useful in property checking. The flip side is that slice produced by abstract assignments may be larger than the one produced by abstract conditionals. Given a program \( P \), we shall call a subprogram of \( P \) in which some of the assignments and conditionals have been replaced by their abstract versions as an abstract subprogram.

Now we will formally define a data slice and identify the statements which should be part of the data slice.

C. Data slice: Formalization

Given a slicing criterion \((l, V)\), a data slice is required to retain the program’s behavior in computing the value set of the variables in \( V \), but is not required to visit \( l \) as many times as the original program. However, whatever value sets are computed by the slice should match those computed by the original program in a sense that we shall make precise now.

Consider program P7 of Figure [3]. Assume that the number of times the outer loop iterates depends on the input and the inner loop iterates a fixed number of times, say 3. If the outer loop executes twice for an input I, the values of \( x \) at line 8 will be 3,6,9,3,6,9. However, if both the while conditions are replaced by \( \ast \) in a slice, then for the same input, 3,3 is one of the sequence of values generated for \( x \). This sequence does not match the sequence generated by the concrete program. On the other hand, if only the outer loop condition is replaced by \( \ast \), then the output produced will be zero or more occurrences of sequence 3,6,9. These sequences are considered to match the sequence produced by the original program.

Based on these considerations, we identify the necessary properties of a data slice \( P^D \) for a given program \( P \) and slicing criterion \((l, V)\).

1) \( P^D \) is an abstract subprogram of \( P \)
2) For every input \( I \), on which augmented program terminates with trace \( \tau \), there exists a trace \( \tau^d \) of \( P^D \) on same input \( I \), such that \( \tau \) and \( \tau^d \) are SC-equivalent.
3) Let \( \tau^d \) and \( \tau \) be traces of \( P^D \) and \( P \) respectively on an input \( I \). Let \( k \) be the minimum of the numbers of SC-execution states in \( \tau \) and \( \tau^d \). Then for all \( i \leq k \), the \( V \)-restricted program states of \( i^{th} \) SC-execution states of \( \tau \) and \( \tau^d \) are the same.

Clearly, a backward slice \( P^B \) also satisfies the properties of data slice mentioned above. Therefore, \( P^D \subseteq P^B \).

Given a slicing criterion \((l, V)\), we now identify statements which are necessarily in the data slice. We call such statements value-impacting and define the term shortly. Informally speaking, a chain of assignments that determine the value set of \( V \) is value-impacting. Further, a condition that determines which of the several values generated by value-impacting statements reaches \( l \) during execution is also value-impacting. In subsequent discussions, we shall often use “value-impacting statements” to mean both value-impacting assignments and conditionals.

Definition 12: (Value-impacting statement) A statement \( s \) value-impacts \( \Upsilon \), if any of the following conditions hold:

1) \( s \) is the augmented SKIP statement.
2) \( s \) is an assignment, and \( s \in DU(\Upsilon) \).
3) $s$ is an assignment, and there exists a statement $t$ such that $t$ value-impacts $\Upsilon$ and $s \in DU(LV(t))$.

4) $s$ is a condition, and the following holds: From $c$ there exist paths $\pi_1, \pi_2$ to $l$ starting from edges $e_1$ and $e_2$ respectively. Further, there exists a statement $\tau$ such that $\tau$ value-impacts $\Upsilon$ and
   a) $\tau$ is the first value-impacting statement along $\pi_1$.
   b) $\tau$ is not the first value-impacting statement along $\pi_2$.

The triplet $\langle \pi_1, \pi_2, t \rangle$ due to which a condition $c$ satisfies rule (4) will be called a witness for a value-impacting condition. Obviously, there can be more than one witnesses for a condition to be value-impacting. The set of all such witnesses will be referred as $WVI(c, \Upsilon)$.

In Figure 3 we show some examples of value-impacting statements. The CFGs of these programs are shown in Figure 4. In P6, lines 1 and 8 are value-impacting for $(11, \langle x \rangle)$ because the values of $x$ generated at these statements reach 11. In addition, condition $c_2$ is also value-impacting for the reason that of the two paths from $c_2$ to 11, only one has $x = z + 5$ as the first value-impacting statement. As a consequence $c_2$ determines whether the value generated at line 1 or at line 8 reaches 11. However, notice that while line 8 is value-impacting for $(9, \langle x \rangle)$, condition $c_2$ is not, because there is no path to line 9 from the false edge of $c_2$. Similarly, condition $c_1$ value-impacts $(13, \langle x \rangle)$.

In P7 of Figure 3 the definition of $x$ at lines 4 and 7 are value-impacting for $(7, \langle x \rangle)$. Since line 7 is not reachable along the false edge of $c_2$ without passing through line 4, $c_2$ also becomes value-impacting for $(7, \langle x \rangle)$. In P8, we can see that, $c_2$ is value-impacting for $(7, \langle x \rangle)$.

Obviously, if a statement $s$ value-impacts a slicing criterion $\Upsilon = \langle I, V \rangle$, then $s$ can be the cause of an erroneous value of some $x \in V$ at $l$ and should be examined while debugging. Therefore, $s$ must be part of $PD$.

Let $VI(\Upsilon)$ be the set of value-impacting statements of $\Upsilon$. Let $AC(\Upsilon)$ be conditional statements that are not by themselves value-impacting, but on which other value impacting statements are strongly (and transitively) control dependent. Formally:

$$AC(\Upsilon) = (\bigcup_{t \in VI(\Upsilon)} \{c | c \rightarrow t\}) \setminus VI(\Upsilon)$$

We construct an abstract subprogram $P^S = VI(\Upsilon) \cup AC(\Upsilon)$ in which the conditionals in $AC(\Upsilon)$ appear in an abstract form. Obviously, $P^S$ retains the structure of $P^A$ with respect to all statements included in $P^S$. We claim that $P^S$ is a data slice. To show this, we shall first prove that the value sets of $\Upsilon$ produced by execution of $P^S$ match those produced by $P^A$.

**Lemma 2:** Let $\tau$ and $\tau'$ be traces of programs $P^A$ and $P^S$ for an input $I$. Also assume that both the traces go through $l$ at least once. Then the corresponding SC-execution states of $\tau$ and $\tau'$ will be the same when restricted to the variables in $V$.

**Proof:** Let $\tau_s = [(n_i, \sigma_i)], i \geq 0$ and $\tau_s' = [(n'_j, \sigma'_j)], j \geq 0$ be the sequence of execution states in $\tau$ and $\tau'$ such that $n_0 = n'_0 = ENTRY$ and for $i > 0$ and $j > 0$, $n_i, n'_j \in VI(\Upsilon)$. Let $K$ be minimum of the number of elements in $\tau_s$ and $\tau_s'$. Since $l$ occurs at least once in $\tau$ and in $\tau'$, $K > 0$. We will prove by induction on $i$ that for all $i \leq K$, $n_i = n'_i$ and $\sigma_i = \sigma'_i$ where $Z = REF(n_i)$.

**Base step :** $i = 0$. It holds trivially as $n_0 = n'_0 = ENTRY \land \sigma_0 = \sigma'_0 = I$.

**Induction step:** Let the hypothesis be true for $i \leq K$ and assume that $i + 1 \leq K$ (else the proof holds vacuously). Since $[\sigma_i]_Z = [\sigma'_i]_Z$, the edge followed from $n_i$ and $n'_i$ in $\tau$ and $\tau'$ have to be same. Assume that $n_{i+1} \neq n'_{i+1}$. Let $c$ be a common condition, with edges $e_1$ and $e_2$, occurring between $n_i$ and $n_{i+1}$ in $\tau$ and between $n'_i$ and $n'_{i+1}$ in $\tau'$. Clearly there is such a $c$, otherwise $n_{i+1}$ would have been the same as $n'_{i+1}$. Since both traces have occurrence of $l$ and $n_{i+1}$ and $n'_{i+1}$ are the first value-impacting statements on paths from $e_1$ and $e_2$, the condition $c \in VI(\Upsilon)$ according to the definition. This is contrary to our assumption that $n_{i+1}$ and $n'_{i+1}$ are the first value-impacting statements in $\tau$ and $\tau'$ after $n_i$ and $n'_i$ respectively. Therefore, $n_{i+1} = n'_{i+1}$.

Now suppose that for some variable $x \in Z$, $\sigma_{i+1}(x) \neq \sigma'_{i+1}(x)$. Let $d$ be statement which provides value of $x$ at $n_{i+1}$. But then $d \in VI(\Upsilon)$. If $d$ occurs before or at $n_i$ then it must be there in $\tau'$ also and therefore, $\sigma_{i+1}(x) = \sigma'_{i+1}(x)$. If $d$ occurs after $n_i$ then $(n_{i+1}, \sigma_{i+1})$ cannot be the first element of $\tau_s$ after $(n_i, \sigma_i)$. This is contrary to our assumption and therefore $\sigma_{i+1}(x) = \sigma'_{i+1}(x)$.

Now we prove our claim that $P^S$ is a data slice. We shall show this by constructing a trace $\tau'$ for $P^S$ from a given trace $\tau$ of $P^A$ on an input $I$. This will establish that $P^S$ satisfies property (2) of data slice. Using lemma 2, we shall show that $P^S$ satisfies property (3) as well.

**Theorem 1:** The abstract subprogram $P^S$ satisfies the property for data slice.

**Proof:** Let $\tau$ be a trace for programs $P^A$ on input $I$ that has $K \geq 0$ execution states. Let $\tau' = [(n_i, \sigma_i)], 0 \leq i \leq K$, be the sub-sequence of $\tau$ such that $n_i, i \geq 0$ are nodes in CFG of $P^S$. We show by induction on $i$ that for each $i \leq K$, $[(n_i, \sigma_i)]$ is also the prefix of a trace for $P^S$.

**Base step:** $i = 0$. The lemma holds trivially as $n_0 = ENTRY$.

**Induction step:** Assume that the hypothesis holds for some $i \leq K$. Let $n_i$ be a condition. If $n_i \in AC(\Upsilon)$ then it is abstract and can take either branch. If $n_i \in VI(\Upsilon)$ then by lemma 2 it will have the same value as in trace $\tau$. So for any edge taken out of $n_i$ in $\tau$, there is a trace of $P^S$ which takes the same edge.

Now assume that for none of the traces of $P^S$ is the $(i + 1)^{th}$ node same as $n_{i+1}$. This must be because of some condition $c$ before $n_{i+1}$, but after $n_i$, in $\tau$. But then $c \rightarrow n_{i+1}$ and therefore $c \in P^S$. So $(n_{i+1}, \sigma_{i+1})$ can not be the first execution state after $(n_i, \sigma_i)$ in $\tau'$, a contradiction. Thus, property (3) is satisfied. By lemma 2 it is obvious that property (2) is also satisfied.
D. Computing data slice using data and control flow

We now relate $VI(\Upsilon)$ to control and data dependence. In Figure 3, we show PDG for programs of Fig. 3. Solid, normal and dotted arrows show data dependence, strong control dependence and weak control dependence respectively. In P6 of Fig. 3, c2 is value-impacting for $\{11, \{x\}\}$. In terms of control-dependence, line 11 is not control dependent on c2 while the value-impacting assignment at line 8 is. For exactly similar reasons, condition c1 is value-impacting for $\{13, \{x\}\}$. We generalize the identified condition for a slicing criterion $\Upsilon = \langle l, V \rangle$ as cond1: l is not control dependent on condition c and a value-impacting statement for $\Upsilon$ is transitively control dependent on c.

In P7, c2 is value-impacting for $\{7, \{x\}\}$, a slicing criterion whose program point (line 7) is strongly true-control dependent on c2. The value-impacting assignment for this criterion at line 4 is weakly false-transitively control dependent on c2. Similarly, in the case of P8, condition c2 is value-impacting for $\{7, \{x\}\}$ and 7 is strongly true-control dependent on c2. Moreover, the value-impacting assignment at line 9 is strongly false-control dependent on c2. From these observations, we identify a second condition cond2: l is control dependent on c and l is reachable from one edge and a value-impacting statement is reachable from other edge. We show that the disjunction of cond1 and cond2 is a necessary condition for value-impact. Thus we can use the disjunction to compute an over-approximation of value impacting conditions.

As seen from the examples, to capture value-impact we need to consider both strong and weak control dependence. For this we make use of transitive control dependence.

We now make the following connections between transitive control dependence and weak post dominance.

Claim 1: Given a condition c and a statement s, assume that $c \rightarrow s$. If there is a statement t distinct from c such that there is a path from c to s starting from the edge $\bar{e} \neq e$ and going through t, then $c \rightarrow_{\bar{t}} t$.

Claim 2: Let c be a condition and u be a statement distinct from c such that u is the immediate post-dominator of c. If there is a statement t distinct from c and u such that there is a path from c to u starting with edge e and passing through t, then $c \rightarrow_{e} t$.

We shall now establish that if a condition c is value-impacting, then there must be another value-impacting statement which is transitively-control dependent on c.

Lemma 3: Assume that for a slicing criterion $\Upsilon = \langle l, V \rangle$, $c \in VI(\Upsilon)$. Then one of the following holds:

$\exists (\pi, \pi, t) \in WV1(c, \Upsilon) : (\neg (c \rightarrow l) \land c \rightarrow t)$, or

$\exists (\pi, \pi, t) \in WV1(c, \Upsilon) : (e \rightarrow_{e} l \land \neg (c \rightarrow_{e} t) \land c \rightarrow_{e} t)$

where $e_1$ and $e_2$ are starting edges of paths $\pi_1$ and $\pi_2$ respectively.

Proof: Let $c \in VI(\Upsilon)$. The proof is by case analysis:

Case 1: Assume $\neg (c \rightarrow l)$. Then there must exist $u \neq c$ such that u is immediate post dominator of c. Since $c \in VI(\Upsilon)$, by definition there are paths $\pi_1$ and $\pi_2$ starting from $e_1$ and $e_2$ and a value-impacting statement t such that at least one of the $\pi_1$ and $\pi_2$ should have t before u. Without loss of generality, assume that t is on $\pi_1$. Then by claim 2 $c \rightarrow_{e} t$. Thus we have proved that for the witness $\langle \pi_1, \pi_2, t \rangle \in WV1(c, \Upsilon), \neg (c \rightarrow l)$ and $c \rightarrow_{e} t$. 

Fig. 3: Programs to explain value-impact

Fig. 4: Control flow graphs

Fig. 5: Program dependence graphs
Case 2: Assume \(c \rightarrow l\), and call the starting edge in the chain of control dependence from \(c\) to \(l\) as \(c,\) i.e. \(c \rightarrow e_2 \rightarrow l\). From the definition of value-impacting condition, there is a path from \(c\) to \(l\) that has a statement, say \(t\), as the first value-impacting statement and another that does not have \(t\) as the first value-impacting statement. Now there are two cases:

1. Let the path that has \(t\) as the first value-impacting statement leave \(c\) through the edge \(e_1\). For the kind of programs under consideration, no path from \(c\) to \(l\) can bypass \(e_2\). Therefore \(t\) on \(\pi_1\) must be between \(e_1\) and \(e_2\). Thus we have \(\neg(c \rightarrow e_2 \rightarrow w)\), and by claim \(\text{I} \ c \Rightarrow w\). The witness in this case being \((\pi_1, \pi_2, t)\).

2. Now let the path that has \(t\) as the first value-impacting statement leave \(c\) through the edge \(e_2\). Once again the path that goes from \(c\) to \(l\) through \(e_1\) must go through \(e_2\) and there must be a first value-impacting statement \(w\) between \(e_1\) and \(e_2\). Now we have a witness \((\pi_1, \pi_2, w)\) for which \(\neg(c \rightarrow e_2 \rightarrow w)\), and by claim \(\text{I} \ c \Rightarrow w\).

Based on lemma \(\text{I}\) we shall give criteria for computing a set of statements \(CVI(T)\), \(\text{computed value-impacting statements}\). \(CVI\) uses data and control dependence and computes an over-approximation of \(VI\).

A statement \(s\) is in \(CVI(T)\) if:

1) \(s\) is the augmented \(\text{SKIP}\) statement at \(l\).
2) \(s\) is an assignment such that \(s \in DU(T)\).
3) \(s\) is an assignment, and there is a statement \(t\) such that \(t \in CVI(T)\) and \(s \in DU(LV(t))\).
4) \(s\) is a condition \(c\) with two outgoing edges, labeled as \(e_1\) and \(e_2\), and \(\exists t \in CVI(T)\) satisfying one of the following:
   a) \(c \rightarrow t \land \neg(c \rightarrow l)\)
   b) \(c \rightarrow e_1 \land \neg(e \rightarrow e_1) \land c \rightarrow t\)

Condition 4) of \(CVI\) is motivated by lemma \(\text{I}\).

We shall now show that \(CVI(T)\) computes an over-approximation of value-impacting statements \(VI(T)\). In subsequent proofs, we use \(MLP(s_1, s_2)\) to denote maximum length of a loop free path in a CFG from \(s_1\) to \(s_2\). In particular, \(MLP(s, s) = 0\).

Lemma 4: \(VI(T) \subseteq CVI(T)\)

Proof: Let \(s \in VI(T)\). We will prove the result by induction on \(MLP(s_1, s_2)\).

Base step: \(MLP(s, l) = 0\). \(s\) must be the augmented \(\text{SKIP}\) statement and therefore in \(CVI(T)\).

Induction step: Let the hypothesis be true for all \(s\) such that \(MLP(s, l) \leq i\) and consider a \(s\) for which \(MLP(s, l) = i + 1\) and \(s \in VI(T)\). If \(s\) is an assignment, then \(s \in CVI(T)\) from definition. Let \(s\) be a condition \(c\). Let \((\pi_1, \pi_2, t) \in WVI(c, T)\) satisfy the criteria of lemma \(\text{I}\). Obviously \(MLP(t, l) \leq i\), and \(t \in CVI(T)\) by the induction hypothesis. By definition, \(c \in CVI(T)\).

We will now show that though \(CVI\) is an over-approximation of \(VI\), it is contained within the backward slice \(P_B^l\).

Lemma 5: \(CVI(T) \subset P_B^l\).

Proof: Once again the proof is by induction on \(MLP(s, l)\).

Base case: \(i = 0\). \(s\) must be the augmented \(\text{SKIP}\) statement and therefore \(s \in P_B^l\).

Induction step: Let the hypothesis be true for all \(s\) such that \(MLP(s, l) \leq i\) and consider a \(s\) for which \(MLP(s, l) = i + 1\) and \(s \in CVI(T)\). By definition of \(CVI(T)\), we have the following cases:

1) \(s\) is an assignment and \(s \in DU(T)\). Clearly \(s \in P_B^l\).
2) \(s\) is assignment and \(\exists t \in CVI(T) : s \in DU(LV(t))\). Clearly \(MLP(t, l) \leq i\) and therefore \(t \in P_B^l\). By construction of backward slice, \(s \in P_B^l\).
3) \(s\) is a condition \(c\), say with two edges \(e_1\) and \(e_2\). By definition of \(CVI(T)\), \(\exists t \in CVI(T)\), satisfying one of the following:
   Case a: \(c \rightarrow t \land \neg(c \rightarrow l)\). Obviously, \(MLP(t, l) \leq i\) and \(c \rightarrow t\). By the induction hypothesis, \(t \in P_B^l\). Therefore \(c \in P_B^l\).
   Case b: \(c \rightarrow l \land \neg(c \rightarrow e_1) \land c \rightarrow t\). In this case \(c \rightarrow l\) and therefore, by construction of backward slice, \(c \in P_B^l\).

IV. DATA SLICE COMPUTATION

As stated earlier, \(CVI(T)\) provides the core set of statements of the data slice. To make the slice executable, conditions are added using one of the methods of creating abstract sub programs described before. We present an algorithm to compute \(CVI\) only; adding the abstract conditions and assignments is straightforward. As an example, the conditions \(c\) to be abstracted are given by \(\{c \mid c \rightarrow l\} \setminus CVI(T)\).

Algorithm 1 Identifying \(CVI\) conditions

1: procedure getCVIConds(t, lconds)
2: begin
3: \(R = \emptyset\)
4: \(tconds = tcntrls(t)\)
5: for all conditions \(c\) appearing in \(tconds\) do
6: \(\text{lsttab}[T] = ((c, T, true) \in lconds)\)
7: \(\text{lsttab}[F] = ((c, F, true) \in lconds)\)
8: \(\text{tsttab}[T] = ((c, T, true) \in tconds)\)
9: \(\text{tsttab}[F] = ((c, F, true) \in tconds)\)
10: \(\text{tswtab}[T] = ((c, T, false) \in lconds)\)
11: \(\text{tswtab}[F] = ((c, F, false) \in lconds)\)
12: if \(\neg\text{lsttab}[T] \land \neg\text{lsttab}[F] \land (\text{tsttab}[T] \lor \text{tsttab}[F])\) then
13: add \(c\) to \(R\)
14: else
15: if \((\text{lsttab}[T] \land \neg\text{lsttab}[T] \land \text{tswtab}[F])\) \lor 
\((\text{tsttab}[F] \land \neg\text{tsttab}[F] \land \text{tswtab}[T])\) then
16: add \(c\) to \(R\)
17: end if
18: end if
19: end for
20: return \(R\)
21: end

A. Computing CVI(T)

In computing CVI(T), the critical part is to identify conditional expressions which satisfy criteria for being in CVI. We assume that the PDG already exists with weak and strong control dependences and data dependences. Such a PDG can be computed efficiently by an algorithm of Bilardi and Pingali [7]. From the PDG, for a given statement s, we can find the set \(\text{conds}(s)\) of pairs \((c, e, b)\), such that \(c \rightarrow^* s\) when \(b = \text{false}\) and \(c \rightarrow^* s\) when \(b = \text{true}\). Using \(\text{conds}(s)\), we can compute the set of conditions on which \(s\) is transitively control dependent. We call this set as \(\text{tcntrls}(s)\). We do so by traversing the PDG and taking a transitive closure of \(\text{conds}(s)\). By the definition of CVI(T), we need to examine only the conditions \(c\) which appear in \(\text{tcntrls}(t)\) for a given statement \(t\).

Algorithm 1 computes the set of conditions which satisfy the control depdendent conditions of PDG. We call this set as \(\text{tcntrls}(s)\). We do so by traversing the PDG and taking a transitive closure of \(\text{conds}(s)\). By the definition of CVI(T), we need to examine only the conditions \(c\) which appear in \(\text{tcntrls}(t)\) for a given statement \(t\).

Algorithm 2 computes the complete set CVI(T) for a given statement \(t\) which is already in CVI(T). Lines 6 to 11 identify the kinds of transit control dependence that \(t\) has on the outgoing edges of \(c\). While \(\text{tstab}[e] = \text{true}\) means \(c \rightarrow^* t\), \(\text{tsstab}[e] = \text{true}\) means \(c \rightarrow^* t\). Algorithm 2 computes the complete set CVI(T) using a worklist based approach. A node comes on the worklist only once. The final result is denoted by a boolean array \(\text{inwsl}\) having value true for every statement included in CVI(T).

Algorithm 2: Computation of CVI(T)

```latex
\begin{algorithm}
  \textbf{begin}
  \textbf{procedure} computeCVISet(l, V)
  \begin{algorithmic}
  \State initialize \text{inslice}, \text{inwl} with \text{false}
  \State \text{inslice}[i] = true; \text{lconds} \equiv \text{tcntrls}(l)
  \State \text{inwl} = \{\}
  \State \text{duset} \equiv \text{DU}(l, V)
  \For {\text{s} \in \text{duset}}
  \State add \text{s} to \text{wl} ; \text{inwl}[s] = \text{true}
  \EndFor
  \While {\text{wl} \text{is not empty}}
  \State remove next element \text{w} from \text{wl}
  \State \text{inslice}[w] = \text{true}
  \State \text{duset} \equiv \text{DU}(\text{LV}(w))
  \State \text{cset} \equiv \text{getCVIConds}(w, \text{lconds})
  \For {\text{s} \in \text{\text{duset} \cup \text{cset}}} \text{do}
  \If {\text{inwl}[s] = \text{false}}
  \State add \text{s} to \text{wl} ; \text{inwl}[s] = \text{true}
  \EndIf
  \EndFor
  \EndWhile
  \State \text{end}
  \end{algorithmic}
\end{algorithm}
```

B. Algorithm complexity

Assume there are \(N\) nodes, \(E^d\) data dependent edges and \(E^c\) control dependent edges, giving a total of \(E = E^d + E^c\) edges in the PDG. In getCVIConds, computing tcntrls for the given node, will take \(O(E^c)\) time. The checks in lines 6 to 11 can be done in \(O(1)\) time with a space complexity of \(O(N)\). Since the checks have to be made for all conditions occurring in tcntrls, the worst case complexity of getCVIConds would be \(O(N + E^c)\). In algorithm, computeCVISet, a node goes in the worklist only once, therefore there would be maximum \(N\) invocations of getCVIConds. The worst case complexity of entire algorithm is \(O(E^c \times N + N^2 + E^c)\). However in practice, the loop at line 5 in getCVIConds will be executed much fewer times than \(N\) and nodes going in worklist will also be of the size of the data slice. As a result, the algorithm’s average complexity will be \(O(N + E^c + E^d)\). In contrast, the backward slice is computed in \(O(E)\) time, in worst case. Note that in both cases the time complexity is arrived at by assuming that PDG have already been built. In practice, our results have also shown that there is only a marginal increase in time in computing data slice from that taken in backward slice when compared to time taken in building the PDG itself.

V. IMPLEMENTATION AND MEASUREMENTS

We implemented the algorithm to compute data slices using our in-house data flow analysis framework called PRISM which is based on the JAVA platform. It can construct PDGs and can compute the conventional backward slices. It has been used for developing static analysis tools [7], [8]. We have used a context and flow sensitive points-to analysis. The backward slicing algorithm is also context sensitive and field sensitive. Thus it represents the state of the art in backward slicing. We computed data slice using condition abstraction approach which is suitable for debugging and program understanding.

Although we have described our algorithm at an intra-procedural level, our implementation performs interprocedural analysis by summarising procedure calls by a sequence of assignments simulating the use-def summary of called procedure. We trigger additional slicing criteria at call points based upon the values needed at procedure entry point in a context sensitive manner. We computed data slice for these additional slicing criteria and at the end took a union of all of them. This may introduce some imprecision but is sound.

Our experiments have been carried out on 3.0 GHz Intel Core2Duo processor with 2 GB RAM and 32 bit OS. Measurements were performed on 42 modules of varying sizes of a proprietary code base of a large navigational system of an automobile. Due to space constraints, we have presented summary data of only 20 modules in Figure 6. Column (1) gives anonymized program names. Column (2) lists the number of CFG nodes. Column (3) gives number of slices computed for the program. For this purpose we randomly selected the return statements of functions returning some value as the conditioned expression which is suitable for debugging and program understanding.

Columns (4), (5) and (6) provide the average sizes of slices in terms of the number of nodes for backward slice (BS), data slice (DS) and control slice (CS) respectively. Columns (7), (8) and (9) provide the average sizes of slices as a percentage of the program size (as given in column (3)) for BS, DS and CS respectively. The average time taken (in seconds) to computing these slices is shown in columns (10), (11) and (12) in same order. This time includes the time taken for constructing PDGs.

Figure 7 shows the distribution of sizes of slices in terms of percentage of program sizes through a graph. The X axis shows the size of the slice and Y axis shows the percentage of slicing criteria exhibiting that size. In the figure, BS, DS and CS stand for backward slice, data slice and control slice respectively. It is clear that in more than 60% of the cases, the size of a data slice is smaller than 10% of the code size. Besides, the time...
| Program | Size (nodes) | number of slices | Avg. Size (in nodes) |  |  |  | Avg Size as % of program | time in seconds |
|---------|-------------|------------------|----------------------|---|---|---|--------------------------|-----------------|
|         |             |                  | BS | DS | CS | BS | DS | CS | BS | DS | CS |
| (1)     |             |                  | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| 1       | 1063        | 10               | 229 | 60 | 227 | 21.63 | 5.65 | 21.37 | 26.33 | 26.34 | 26.32 |
| 2       | 1183        | 10               | 266 | 123 | 240 | 22.49 | 10.43 | 20.36 | 32.05 | 32.07 | 32.04 |
| 3       | 2944        | 10               | 360 | 257 | 236 | 12.26 | 8.55 | 12.65 | 39.46 | 39.51 | 39.45 |
| 4       | 881         | 10               | 148 | 62 | 148 | 16.89 | 7.12 | 16.81 | 60.43 | 60.47 | 60.42 |
| 5       | 1607        | 10               | 226 | 137 | 203 | 14.08 | 8.55 | 12.65 | 39.46 | 39.51 | 39.45 |
| 6       | 2246        | 10               | 447 | 348 | 444 | 19.93 | 15.49 | 19.79 | 56.47 | 57.24 | 56.45 |
| 7       | 2493        | 5                | 167 | 81 | 163 | 6.70 | 3.26 | 6.54 | 51.45 | 51.48 | 51.42 |
| 8       | 2635        | 10               | 842 | 257 | 842 | 31.98 | 9.79 | 31.97 | 44.20 | 44.31 | 44.19 |
| 9       | 2992        | 9                | 437 | 149 | 429 | 14.63 | 5.01 | 14.34 | 52.67 | 52.76 | 52.65 |
| 10      | 1625        | 10               | 190 | 94 | 178 | 11.70 | 5.84 | 10.98 | 63.80 | 63.83 | 63.79 |
| 11      | 3413        | 10               | 733 | 341 | 728 | 21.50 | 10.01 | 21.35 | 92.74 | 92.97 | 92.70 |
| 12      | 3105        | 5                | 571 | 412 | 563 | 18.41 | 13.29 | 18.13 | 71.41 | 89.70 | 71.35 |
| 13      | 4452        | 1                | 369 | 70 | 317 | 8.29 | 1.57 | 7.12 | 102.29 | 102.27 | 102.26 |
| 14      | 5236        | 1                | 982 | 25 | 982 | 18.75 | 0.48 | 18.75 | 116.92 | 116.47 | 116.53 |
| 15      | 2616        | 10               | 948 | 761 | 945 | 36.27 | 29.12 | 36.15 | 208.23 | 230.40 | 208.24 |
| 16      | 3883        | 10               | 1202 | 1202 | 30.97 | 4.64 | 30.96 | 215.99 | 216.14 | 215.97 |
| 17      | 802         | 10               | 92 | 56 | 90 | 11.48 | 7.01 | 11.28 | 53.85 | 53.86 | 53.84 |
| 18      | 8116        | 10               | 3489 | 1637 | 3143 | 42.99 | 20.18 | 38.73 | 447.69 | 512.58 | 447.30 |
| 19      | 6746        | 10               | 1928 | 1558 | 1923 | 28.58 | 23.10 | 28.51 | 272.10 | 293.40 | 271.74 |
| 20      | 11104       | 10               | 4096 | 1214 | 4053 | 36.89 | 10.94 | 36.50 | 301.26 | 322.31 | 301.13 |

**Fig. 6:** Average slice sizes and computation time

**Fig. 7:** Slice size distribution. X-axis shows slice size as percentage of program size and Y-axis shows percentage of slicing criteria for which this size was observed.

Note that the average size of backward slices is 25% of the code size which matches the observation by Binkley et al. [?]. suggesting our slices are comparable in precision. Average size of data slice is found to be 10% of the code size. Given such a reduction, data slices may be very helpful in debugging, property checking and program understanding. This data corroborates our intuition that most statement are included in a backward slice because they influence the reachability of the slicing criterion rather than the value computed. It is not surprising then that the size of a control slice is comparable to that of the corresponding backward slice in majority of the cases. The average size of control slice turns out to be of 24.4% of program size which is comparable to the size of backward slice (24.89%).

**VI. CONCLUSION**

Different applications of program understanding require different combinations of influences governing data computations and control flow. For example, in the case of debugging for wrong output values, the influences governing the reachability of the statement of interest are irrelevant.

It follows that separating the influences of data and control in a backward slice by constructing separate data and control slices is an effective way of producing smaller programs for debugging, program understanding and property checking. In the case of debugging for wrong output values, a data slice provides a much smaller piece of code to investigate than that provided by a backward slice for the same slicing criterion.

We have provided formal definitions of data and control slices, defined algorithms to compute them, have shown the soundness of the algorithms, and have presented the results of our empirical experiments. Our measurements show that a data slice is indeed much smaller than the corresponding backward slice and is computable in comparable time.

In future, we would like to investigate the minimality of data slices and efficient algorithms to compute them. We would also like to explore the effectiveness of data slices for much larger programs.