The $Q^2$ dependence of the measured asymmetry $A_1$: the test of the Bjorken sum rule

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Abstract

We analyse the proton and deuteron data on spin dependent asymmetry $A_1(x, Q^2)$ supposing the DIS structure functions $g_1(x, Q^2)$ and $F_3(x, Q^2)$ have the similar $Q^2$-dependence. As a result, we have obtained that $\Gamma_p^1 - \Gamma_n^1 = 0.190 \pm 0.038$ at $Q^2 = 10 \text{ GeV}^2$ and $\Gamma_p^1 - \Gamma_n^1 = 0.165 \pm 0.026$ at $Q^2 = 3 \text{ GeV}^2$, what is in the best agreement with the Bjorken sum rule predictions.

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An experimental study of the nucleon spin structure is realized by the measuring of asymmetry $A_1(x, Q^2) = g_1(x, Q^2)/F_1(x, Q^2)$. The best known theoretical predictions on spin dependent structure function $g_1(x, Q^2)$ of the nucleon were made by Bjorken [1] and Ellis and Jaffe [2] for the so called first moment value $\Gamma_1 = \int_0^1 g_1(x) dx$.

The calculation of the $\Gamma_1$ value requires the knowledge of structure function $g_1$ at the same $Q^2$ in the whole $x$ range. Experimentally the asymmetry $A_1$ is measuring at different values of $Q^2$ for different $x$ bins. An accuracy of the past and modern experiments [3, 4] allows to analyze data in the assumption [5] that asymmetry $A_1(x, Q^2)$ is $Q^2$ independent (i.e. the structure functions $g_1$ and $F_1$ have the same $Q^2$ dependence). However, this assumption is not theoretically warranted (see discussions in [6, 7, 8]); the different $Q^2$ dependence of the structure functions $g_1(x, Q^2)$ and $F_1(x, Q^2)$ is expected due to the difference in polarized and unpolarized splitting functions (except for the leading order quark-quark one). Thus, in view of forthcoming more precise data it is important to add the $Q^2$ dependence of the asymmetry.

This article is based on our observation that the $Q^2$ dependence of spin dependent and spin average structure functions $g_1$ and $F_3$ is very similar in a wide $x$ range: $10^{-2} < x < 1$. At the small $x$ region ($x < 10^{-2}$ it could be not true (see [6, 9]), but most of the existed data were measured out off that range.

Let’s consider the nonsinglet (NS) $Q^2$ evolution of structure functions $F_1$, $g_1$ and $F_3$. The DGLAP equation for the NS part of these functions can be presented as:

\[
\begin{align*}
\frac{dg_1^{NS}(x, Q^2)}{dlnQ^2} &= -\frac{1}{2} \gamma_{NS}^-(x, \alpha) \times g_1^{NS}(x, Q^2), \\
\frac{dF_1^{NS}(x, Q^2)}{dlnQ^2} &= -\frac{1}{2} \gamma_{NS}^+(x, \alpha) \times F_1^{NS}(x, Q^2), \\
\frac{dF_3(x, Q^2)}{dlnQ^2} &= -\frac{1}{2} \gamma_{NS}^-(x, \alpha) \times F_3(x, Q^2),
\end{align*}
\]

(1)

where symbol $\times$ means the Mellin convolution. The splitting functions $\gamma_{NS}^\pm$ are the reverse Mellin transforms of the anomalous dimensions $\gamma_{NS}^\pm(n, \alpha) = \alpha \gamma^{(0)}(n)_{NS} + \alpha^2 \gamma^{(1)}_{NS} + O(\alpha^3)$ and the Wilson coefficients $\beta n b^\pm(n) + O(\alpha^2)$:

\[
\begin{align*}
\gamma_{NS}^+(x, \alpha) &= \alpha \gamma^{(0)}(x) + \alpha^2 \left( \gamma^{(1)}_{NS}(x) + 2 \beta_0 b^-(x) \right) + O(\alpha^3),
\end{align*}
\]

(2)

where $\beta(\alpha) = -\alpha^2 \beta_0 - \alpha^3 \beta_1 + O(\alpha^4)$ is QCD $\beta$-function.

The above mentioned Mellin transforms mean that

\[
f(n, Q^2) = \int_0^1 dx x^{n-1} f(x, Q^2),
\]

(3)

3We use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

4Because we consider here the structure functions themselves but not the parton distributions. Note that $b_{1,NS}(n)$ and $b_{2,NS}(n)$ have more standard definition as $b_{1,NS}(n) = b_{2,NS}(n) - b_{L,NS}(n)$ and $b_{3,NS}(n)$.
where \( f = \{ \gamma_{\text{NS}}^{(0)}, \gamma_{\text{NS}}^{(1)}, b_{\text{NS}}^{+}, \gamma_{ij}^{(k)}, \gamma_{ij}^{(k)*}, b_i, b_i^* \} \) with \( k = 1,2 \) and \( \{i,j\} = \{S,G\} \).

Eqs. (1) show the \( Q^2 \) dependence of NS parts of \( g_1 \) and \( F_3 \) is the same (at least in first two orders of the perturbative QCD \([10]\)) and differs from \( F_1 \) already in the first subleading order \( \left( \gamma_{\text{NS}}^{+} \neq \gamma_{\text{NS}}^{(1)} \right) \) and \( b_{\text{NS}}^+ - b_{\text{NS}}^- = (8/3)x(1-x) \).

For the singlet parts of \( g_1 \) and \( F_1 \) evolution equations are:

\[
\frac{dg_1^S(x, Q^2)}{d\ln Q^2} = -\frac{1}{2}\left[ \gamma_{SS}^0(x, \alpha) \times g_1^S(x, Q^2) + \gamma_{SG}^0(x, \alpha) \times \Delta G(x, Q^2) \right],
\]

\[
\frac{dF_1^S(x, Q^2)}{d\ln Q^2} = -\frac{1}{2}\left[ \gamma_{SS}^0(x, \alpha) \times F_1^S(x, Q^2) + \gamma_{SG}^0(x, \alpha) \times G(x, Q^2) \right],
\]

where

\[
\gamma_{SS}(x, \alpha) = \alpha \gamma_{SS}^0(x) + \alpha^2 \left( \gamma_{SS}^1(x) + b_G(x) \times \gamma_{GS}^0(x) + 2\beta_0 b_S(x) \right) + O(\alpha^3),
\]

\[
\gamma_{SG}(x, \alpha) = \frac{e}{f} \left[ \alpha \gamma_{SG}^0(x) + \alpha^2 \left( \gamma_{SG}^1(x) + b_G(x) \times \left( \gamma_{GG}^0(x) - \gamma_{SS}^0(x) \right) + 2\beta_0 b_G(x) \right) \right] + O(\alpha^3)
\]

where \( e = \sum_i e_i^2 \) is the sum of charge squares of \( f \) active quarks. The equations for polarized anomalous dimensions \( \gamma_{SS}(x, \alpha) \) and \( \gamma_{SG}(x, \alpha) \) are similar. They can be obtained by replacing \( \gamma_{SG}^0(x) \rightarrow \gamma_{SG}^0(x) \), \( \gamma_{SS}^1(x) \rightarrow \gamma_{SS}^1(x) \) and \( b_i(x) \rightarrow b_i^*(x) \) \( (i = \{S, G\}) \).

Note here the gluon term is not negligible for \( F_1 \) at \( x < 0.3 \) but for \( g_1 \) we can neglect them for \( x > 0.01 \). The value \( b_i^*(x) \) \( (b_i(x)) \) coincides with \( b_i^-(x) \) \( (b_i^+(x)) \). The difference between \( \gamma_{NS}^{-(1)} \) and \( \gamma_{SS}^1 + b_G(x) \times \gamma_{GS}^0(x) \) is negligible because it does not contain a power singularity at \( x \rightarrow 0 \) (i.e. a singularity at \( n \rightarrow 1 \) in momentum space). Moreover, it decreases as \( O(1-x) \) at \( x \rightarrow 1 \). Contrary to this, the difference between \( \gamma_{SS} + b_G(x) \times \gamma_{GS}^0(x) \) and \( \gamma_{SS}^1 + b_G^*(x) \times \gamma_{GS}^0(x) \) contains the power singularity at \( x \rightarrow 0 \) (see for example \([11]\)).

The analysis discussed above allows us to conclude the function \( A_1^* \):

\[
A_1^*(x) = \frac{g_1(x, Q^2)}{F_3(x, Q^2)}
\]

should be practically \( Q^2 \) independent at \( x > 0.01 \).

The r.h.s. of Eqs.\([11]\) and \([13]\) contain integrals of structure functions and, hence, the approximate validity of \([11]\) should be observed only for the similar \( x \)-dependence of \( g_1(x, Q^2) \) and \( F_3(x, Q^2) \) at fixed \( Q^2 \). But it is the case (see \([13]\) at \( Q^2 = 3GeV^2 \), for example).

The asymmetry \( A_1 \) at \( Q^2 =< Q^2 > \) can be defined than as:

\[
A_1(x_i, < Q^2 >) = \frac{F_3(x_i, < Q^2 >)}{F_3(x_i, Q_i^2)} \cdot \frac{F_1(x_i, Q_i^2)}{F_1(x_i, < Q^2 >)} \cdot A_1(x_i, Q_i^2),
\]
where \( x_i(Q^2) \) means an experimentally measured value of \( x(Q^2) \).

We use SMC and E143 proton and deuteron data on asymmetry \( A_1(x, Q^2) \) \[3, 4\]. To get \( F_1(x, Q^2) \) we take NMC parametrization of \( F_2(x, Q^2) \) \[14\] and SLAC parametrization of \( R(x, Q^2) \) \[15\] (\( F_1 \equiv F_2/2x[1 + R] \)). To get the values of \( F_3(x, Q^2) \) we parametrize the CCFR data \[16\] (see the parametrization in Appendix).

First, using Eq.(5), we recalculate the asymmetry measured by SMC\[3\] and E143\[4\] on the proton and deuteron targets at \( Q^2 = 10 \text{ GeV}^2 \) (SMC) and 3 GeV\(^2\) (E143), which are average \( Q^2 \) of these experiments respectively. Obtained values of \( \int g_1(x)dx \) through the measured \( x \) ranges are shown in the Table 1.

To get the values of the first moments \( \Gamma_1^{p(d)} \) we estimate unmeasured regions of SMC and E143 using their original machinery. Our estimations coincide with original ones except to the results in small \( x \) region unmeasured by SMC. We obtain the following results for central values of \( \Delta \Gamma_1^{p(d)} = \int_0^{0.003} g_1(x)dx \) at \( Q^2 = 10 \text{ GeV}^2 \): \( \Delta \Gamma_1^p = 0.003 \) and \( \Delta \Gamma_1^d = 0.0022 \), which are smaller then the corresponding SMC estimations: \( \Delta \Gamma_1^p = 0.004 \) and \( \Delta \Gamma_1^d = 0.0028 \). The errors coincide with ones cited in \[3\]. The E143 estimations for \( \int_0^{0.029} g_1(x)dx \) are not changed because \( Q^2 \)-evolution of the asymmetry is negligible at \( x \sim 0.03 \). Results on the \( \Gamma_1 \) values are shown also in the Table 1.

We would like to note that the E143 and SMC machinery may lead to underestimation of \( g_1^{p(d)}(x, Q^2) \) at small \( x \) and, hence, to underestimation of \( \Delta \Gamma_1^{p(d)}(Q^2) \) (see the careful analysis in first paper in ref. \[8\]). Unfortunately, our procedure is not at work at \( x \leq 0.01 \) and we cannot check the SMC and E143 estimations of unmeasured regions here. To clear up this situation it is necessary to add a careful small \( x \) analysis to this consideration that is a subject of our future large article \[17\].

### Table 1. The first moment values of \( g_1 \) of the proton and deuteron.

| \( x_{min} - x_{max} \) | \( < Q^2 > \) | target type | \( f_{x_{max}}^{x_{min}} g_1 dx \) | \( \Gamma_1 \) | experiment |
|--------------------------|-------------|-------------|----------------|---------|-----------|
| \( .003 - 0.7 \)         | 10 GeV\(^2\)| proton      | 0.130         | 0.134 ± 0.011| SMC      |
| \( .003 - 0.7 \)         | 10 GeV\(^2\)| deuteron    | 0.038         | 0.036 ± 0.009| SMC      |
| \( .029 - 0.8 \)         | 3 GeV\(^2\) | proton      | 0.123         | 0.130 ± 0.004| E143     |
| \( .029 - 0.8 \)         | 3 GeV\(^2\) | deuteron    | 0.043         | 0.044 ± 0.003| E143     |

As the last step we calculate the difference which is predicted by the Bjorken sum rule \( \Gamma_1^p - \Gamma_1^n \):

\[
\Gamma_1^p - \Gamma_1^n = 2\Gamma_1^p - 2\Gamma_1^d/(1 - 1.5 \cdot \omega_D),
\]

where \( \omega_D = 0.05 \) \[3, 4\] is the probability of the deuteron to be in a D-state.
At $Q^2 = 10 \text{ GeV}^2$ we get the following results:
\[ \Gamma_1^p - \Gamma_1^n = 0.190 \pm 0.038 \]  
(7) 

to be compared with the SMC published value
\[ \Gamma_1^p - \Gamma_1^n = 0.199 \pm 0.038 \]  
(SMC [3])

and the theoretical prediction
\[ \Gamma_1^p - \Gamma_1^n = 0.187 \pm 0.003 \]  
(Theory)

At $Q^2 = 3 \text{ GeV}^2$ we get for E143 data:
\[ \Gamma_1^p - \Gamma_1^n = 0.165 \pm 0.026 \]  
(8)

to be compared with
\[ \Gamma_1^p - \Gamma_1^n = 0.163 \pm 0.026 \]  
(E143 [4])
\[ \Gamma_1^p - \Gamma_1^n = 0.171 \pm 0.008 \]  
(Theory)

Note that only the statistical errors are quoted here. To the considered accuracy they coincide with the errors cited in ([3, 4]). The above cited theoretical predictions for the Bjorken sum rule have been computed in \[ [18] \] to the third order in the QCD $\alpha_s$.

As a conclusion, we would like to note

- The value of $\Gamma_1^p - \Gamma_1^n$ obtained in our analysis is in the best agreement with the Bjorken sum rule prediction.
- The values of $\Gamma_1^p$ and $\Gamma_1^n$ themselves obtained here do not change essentially. The improvement for the Bjorken sum rule is the result of the opposite changes of the $\Gamma_1^p$ and $\Gamma_1^n$ values, when Eq. (5) is used.
- our observation that function $A_1^1(x)$ is $Q^2$ independent at large and intermediate $x$ is supported by good agreement of present analysis with other estimations \[ [19, 7, 8] \] of the $Q^2$ dependence of the $A_1$. A detail analysis will be present later in the separate large article \[ [17] \].

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Appendix

The parametrization is used for CCFR data [16]:

$$x F_3(x, Q^2) = F_3^a \cdot \left( \frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)} \right) F_3^b,$$

where

$$F_3^a = x^{C_1} \cdot (1 - x)^{C_2} \cdot \left( C_3 + C_4 \cdot (1 - x) + C_5 \cdot (1 - x)^2 + C_6 \cdot (1 - x)^3 + C_7 \cdot (1 - x)^4 \right) \cdot \left[ C_8 + C_9 \cdot x + C_{10} \cdot x^2 + C_{11} \cdot x^3 \right]$$

$$F_3^b = C_{12} + C_{13} \cdot x + \frac{C_{14}}{x + C_{15}}$$

and $Q_0^2 = 10$ GeV$^2$, $\Lambda = 200$ MeV.

| $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------|-------|-------|-------|-------|
| 0.8064 | 1.6113 | 0.70921 | -2.2852 | 1.8927 |
| $C_6$ | $C_7$ | $C_8$ | $C_9$ | $C_{10}$ |
| 6.0810 | 4.5578 | 0.7464 | -0.3006 | 3.9181 |
| $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{14}$ | $C_{15}$ |
| -0.1166 | 10.516 | -5.7336 | -37.114 | 3.7452 |

Table 2. The values of the coefficients of CCFR data parametrization.

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