Abstract. Hierarchical Agglomerative Clustering (HAC) algorithms are extensively utilized in modern data science, and seek to partition the dataset into clusters while generating a hierarchical relationship between the data samples. HAC algorithms are employed in many applications, such as biology, natural language processing, and recommender systems. Thus, it is imperative to ensure that these algorithms are fair– even if the dataset contains biases against certain protected groups, the cluster outputs generated should not discriminate against samples from any of these groups. However, recent work in clustering fairness has mostly focused on center-based clustering algorithms, such as $k$-median and $k$-means clustering. In this paper, we propose fair algorithms for performing HAC that enforce fairness constraints 1) irrespective of the distance linkage criteria used, 2) generalize to any natural measures of clustering fairness for HAC, 3) work for multiple protected groups, and 4) have competitive running times to vanilla HAC. Through extensive experiments on multiple real-world UCI datasets, we show that our proposed algorithm finds fairer clusterings compared to vanilla HAC as well as other state-of-the-art fair clustering approaches.

Keywords: Clustering - Hierarchical Agglomerative Clustering - Fairness in Clustering

1 Introduction

Hierarchical Agglomerative Clustering (HAC) refers to a class of greedy unsupervised learning algorithms that seek to build a hierarchy between data points while clustering them in a bottom-up fashion. HAC algorithms are widely utilized in modern data science– such as in genetics [26], genomics [27], and recommendation systems [22]. These algorithms also possess two distinct advantages over non-hierarchical or flat clustering algorithms: 1) they do not require the number of clusters to be explicitly specified initially, and 2) they output a hierarchy between all samples in the dataset as part of the clustering process.

Recently, the machine learning community has realized the importance of designing fair algorithms. Traditional machine learning algorithms do not account
for any biases that may be present (against certain minority groups) in the data, and hence, may end up augmenting them. Machine learning is being increasingly utilized in societal applications such as loan defaulter prediction [33], recidivism rate prediction [17], [21], and many more. Owing to the sensitive nature of these applications and their far-reaching impact on human lives, ensuring that these algorithms are fair becomes of paramount importance. However, work in designing fair clustering algorithms has mostly been focused either on the center-based or graph-based clustering objectives like [8,11,6,10,30,19,5]. Despite all the advantages of HAC algorithms, there has been little work that proposes fair variants to HAC. In this paper, we seek to bridge this gap by making the following contributions:

– The proposed fair HAC algorithm (Section 3) works for multiple protected groups, and we provide results on all the widely used linkage criteria (single-linkage, average-linkage, complete-linkage) for real datasets (Section 4). Our algorithm is especially useful since it can generalize to any natural analytical notions of fairness for HAC (Section 3).

– Our algorithm achieves an asymptotic time complexity of $O(fn^3)$ ($f$ is the number of protected groups), which is comparable to $O(n^3)$ for traditional HAC algorithms as $f$ is usually a small number for most real-world applications (Section 3).

– We provide experimental analysis for different fairness costs for our algorithm, and show that it is more fair than vanilla HAC algorithms (Section 4.2), and the only other fair HAC variant algorithm proposed by [4] (Section 4.3). Also, their algorithm [4] cannot generalize to any arbitrary notion of fairness.

– We undertake an experimental analysis of cluster quality/performance between our proposed algorithm, vanilla HAC, and those of [4] as well, using the Silhouette Coefficient [29] and find that we obtain high quality clusters (Section 4.4).

– We also discuss an alternative to fair hierarchical clustering algorithms—such as making hierarchical clustering trees fair post the vanilla HAC process (Section 5). This approach can have lower asymptotic time complexity of $O(fn^2)$ given an output has already been obtained via vanilla HAC.

Motivating Example: We provide an example similar to the job shortlisting example given in [16], but tailor it to shortlisting households/individuals for bank credit promotions. A dataset, e.g., the creditcard dataset [32], is used by the marketing division of a bank to reach out to prospective customers and offer them loans and available credit opportunities. The dataset contains features such as the potential customer's age, their education level, their weekly work hours, and their capital gains per month. The bank utilizes a hierarchical clustering algorithm to find target audiences for promotional offers and uses the aforementioned attributes as input to the clustering algorithm. On running the algorithm, they obtain hierarchical clusters of people. Since the cluster outputs are hierarchical the bank then chooses an appropriate $k$ number of clusters based on available credit opportunities. Out of these a small number of clusters are
then shortlisted to be targeted for a particular promotion/offer. This shortlisting is undertaken by using some metrics, (e.g., the means of the education and wages-earned features in a cluster) to represent clusters and then select a few of them. It is important to note here that people-of-color (POC) as well as women, tend to earn lower wages than Caucasian males [2], and that POC face more adversities that lead to disparities in their education level [1] as opposed to white demographics. Now, considering these facts on the racial education divide and the wage gap, a clustering algorithm using these attributes will inherently group white households as well as men, as better candidates for better deals and offers (such as mortgages and loans) for each hierarchical level of clusters. As a result, this marketing clustering algorithm has **disparate impact** on POC as well as women, as they are deprived of an opportunity of improvement at each level of the hierarchy/cluster output. Therefore, it is important to study **protected groups** (e.g., ethnicity and gender) and the corresponding fairness in such a hierarchical clustering setting. In this case, race/ethnicity and gender can be considered as **protected groups** and the objective of the fair hierarchical clustering algorithm is to ensure that each group gets a certain minimum representation, for each promotion’s target cluster, for some $k$ number of clusters. Moreover, no group should be overwhelmingly preferred, and thus there should also be a cap on the maximum representation allowed. This example demonstrates how **bounded representation** mitigates disparate impact, specifically in the context of clustering.

We would like our fair hierarchical clustering algorithm to output trees that abide by this notion of fairness.

The rest of the paper is organized as follows: Section 2 discusses related work in the field, Section 3 details our proposed algorithm for performing fair HAC, Section 4 describes our results on real data, Section 5 delineates an alternate approach to fair clustering, and Section 6 concludes the paper and discusses the scope of future work.

## 2 Related Works

Recently, many works have focussed on providing clustering algorithms with fairness guarantees, or on proposing fair variants to existing clustering algorithms. As mentioned before, most of this work has looked at center-based clustering (such as k-means, k-center, and k-median clustering) [8,11,6,7,30], and spectral methods [19,5]. This line of work seeks to impose some **fairness** constraints along with the original clustering distance based objective. The first work to do this for k-median and k-center clustering, proposed by Chierichetti et. al. [11] was based on the notion of disparate impact [18], and ensured that in the case of two protected groups (red and blue) each cluster formed had points of both groups (colors) in roughly the same amount, measured using a metric known as **balance**. As a result, a lot of work has followed that improves upon these ideas either in terms of approximation rates ([6] and [8]), allowing for multiple protected groups ([8] and [28]), extending it to other clustering objectives ([19] and [30]), or a combination of these, among others.
Other than our work and Ahmadian et al’s Fair Hierarchical Agglomerative Clustering (AFHAC) [4], no work has investigated fairness in the context of hierarchical clustering (and specifically greedy HAC algorithms) so far. The key difference between our work and AFHAC is that we work with the greedy HAC algorithms and AFHAC works with hierarchical clustering objectives. Ahmadian et al’s work considers objectives for hierarchical clustering that have been proposed recently following Dasgupta’s seminal work [14], such as [24], [12]; however none of these objectives are approximated well-enough by any general distance linkage criteria that are typically used in HAC (except for average-linkage). Moreover, greedy HAC algorithms despite being ad-hoc and heuristic approaches in nature, are very widely utilized in many application areas, especially in the biological sciences. For these reasons, we wanted to ensure fairness for these algorithms specifically, and provide a fair variant to the HAC problem, irrespective of the choice of distance linkage criteria. We also wanted this fair variant to resemble the general HAC algorithms closely, so that it can be readily implemented in applications. The proposed algorithm is also simpler and easier to understand compared to their complex algorithms. Furthermore, our algorithm works for multiple protected groups (but samples can only be assigned to one group at a time). In the results section, we compare the fairness achieved by our algorithms to the AFHAC algorithms (for both the revenue [24] and value [12] hierarchical clustering objectives) on a number of UCI datasets. Through extensive experiments we find that our proposed algorithm achieves better fairness than AFHAC.

3 Performing Fair HAC

First, we need to define the vanilla HAC process formally. Let $X \in \mathbb{R}^{n \times m}$ be our dataset. Then the HAC on $X$ denoted by $HC(X)$ is a hierarchical partitioning of $X$ that is represented by a binary tree $T$ (also called a dendrogram), where each level of $T$ represents a set of disjoint merges between subclusters. Each node of $T$ at any level represents a subcluster of points. A HAC algorithm first considers each of the $n$ samples of $X$ to be singleton subclusters, and then chooses sets of two subclusters to merge together (that is, a point from $X$ can only belong to any one subcluster at a time). The lowest level of $T$ are leaves, and comprise of all the $n$ points of $X$. The root of $T$ is a single node/cluster that contains all of $X$. Let $C_1, C_2, ..., C_s$ be the subclusters at any level of $T$. Then $C_1 \cup C_2 \cup ... \cup C_s = X$. The choice of which two subclusters should be merged is made by finding two subclusters $C_i$ and $C_j$ such that they minimize a linkage criterion denoted by $D(C_i, C_j)$. There are many linkage criteria that can be used. For example, single-linkage is defined as $D(C_i, C_j) = \min_{x_i \in C_i, x_j \in C_j} d(x_i, x_j)$ and complete-linkage is defined as $D(C_i, C_j) = \max_{x_i \in C_i, x_j \in C_j} d(x_i, x_j)$. In the paper $d(x, y)$ is the Euclidean distance between two points $x$ and $y$, but other distance metrics can also be used.

Next, we need to define our proposed notion for fairness. It is important to note that in this work we consider data points to only belong to one protected
group, that is, multiple assignments to many protected groups for the same point are not allowed. In most works of fairness in clustering, the notion of balance [11] is used to ensure that clusters contain each protected group in more or less an equal proportion to other protected groups. In this paper, we work with a similar idea, but frame the fairness metric as a cost, unlike balance. Also, instead of balancing all protected groups equally in a cluster (which is what the original balance definition aimed to optimize for), we introduce an input parameter called the ideal proportion for each group \( \phi_g \). This allows us to ensure that the proportion of points for each group in each cluster reach this ideal proportion. Thus, points from protected groups in each cluster do not need to be equal in proportion, and their (ideal) proportion can be set depending on the application context. In the paper, in accordance with the disparate impact doctrine, we let the ideal proportion of each protected group be the proportion of the group in the entire dataset \( X \) [7]. Thus, for a protected group with \( s \) members, the ideal proportion would be \( s/n \). It is also easy to see that this is a general setting for the original balance metric-- if we have \( f \) protected groups, we can replace the ideal proportion for each group with \( 1/f \), which would perfectly balance all protected groups in each cluster. This is trivial to do, and requires no change to our algorithms. Finally, the only other existing work on fair hierarchical clustering [4], employs a definition of fairness where the fairness of all protected groups is upper-bounded by the same input value. We believe our definition is more general, since we can explicitly define different ideal proportions for different groups.

We now define these ideas mathematically.

**Definition 1. (\( \alpha \)-Proportional Fairness)** Let \( F \in \mathbb{R}^{f \times n} \) be the set of all protected groups where each protected group \( g \in F \) is \( \{0, 1\}^n \). Thus, if a data sample from \( X \) belongs to a particular group \( g \) then at that index the vector \( g \) contains a 1, otherwise a 0. Moreover, a cluster \( C = \{x_i| i \in I\} \) where \( C \subset X \), and \( I \) is the index set containing indices of the points in \( X \) which belong to cluster \( C \), that is \( X = \{x_i\}_{i=1}^n \). The proportion of group \( g \) members in \( C \) is denoted by \( \delta_g^C = \frac{1}{|C|} \sum_{x_i \in C} g(i) \) and the ideal proportion \( \phi_g = \frac{1}{n} \sum_{x_i \in X} g(i) \). Then \( \alpha \)-Proportional Fairness for cluster \( C \) and protected group \( g \) is maintained if the following condition holds: \( |\delta_g^C - \phi_g| \leq \alpha \).

**Definition 2. (Max Fairness Cost (MFC))** Let \( HC(X) \) be the output of some hierarchical clustering on \( X \). Then the fairness cost on some level with \( k \) clusters of the \( HC(X) \) tree measures how close each cluster of points at this level (denoted by \( C_i \), where \( i = \{1, 2, ..., k\} \)) is to the ideal proportion \( \phi_g \) for each protected group \( g \) in \( F \). Mathematically, the Max Fairness Cost can then be defined as: \( \max_{i \in [k]} \sum_{g \in F} |\delta_g^C_i - \phi_g| \).

Minimizing the MFC will essentially aim to minimize the cumulative maximum deviation of groups from the ideal proportion, for all \( k \) clusters. As a result of the optimization, all clusters should have proportion of all protected groups as close as possible to the ideal proportions for each group. As mentioned before, we can also aim to balance each protected group equally using the MFC, where the ideal proportion now becomes \( 1/f \), and \( f \) is the number of protected groups.
We will also undertake experimental analysis for the balance metric. For reference, we define the multiple-group version of balance, initially defined by [11] for the 2-group case, and then generalized by [7]:

**Definition 3. (Multi-Group Balance [7])** Following from the notation established above, the balance of a clustering can be defined as:

$$\min_{i \in [k]} \min \{ \frac{\Delta C_i}{\phi_y}, \frac{\phi_x}{\Delta y} \}, \forall g \in F.$$ 

It is easy to see that while MFC needs to be minimized to improve fairness, balance needs to be maximized to do the same. Furthermore, it can be seen that the obtained balance will always lie between $[0, 1]$, where a value of 0 is a completely unbalanced clustering (least fair) and a value of 1 is a perfectly balanced clustering (most fair). In the next subsection, we delineate how fairness metrics such as the MFC or balance can be optimized for HAC using our proposed algorithm.

### 3.1 Fair HAC: The FHAC Algorithm

The goal for our fair algorithm will then be to minimize the fairness cost and ensure that at a level with some clusters, we have at least maintained some $\alpha'$-Proportional Fairness where $\alpha'$ is some constant value. We also run the algorithm till some $k$ clusters are remaining, and return after that. To enforce these constraints, we keep tightening the bound on proportional fairness and simultaneously keep loosening the bound on minimizing distance, as we start getting closer to $k$ clusters. The greedy Fair Hierarchical Agglomerative Clustering (FHAC) algorithm is described as Algorithm 1 (which works irrespective of the choice of linkage criteria). Algorithm 1 resembles the working of vanilla HAC except for some key distinctions that allow it to be fairer than traditional HAC. The key difference is in incorporating fairness constraints by foregoing the minimum distance linkage criterion constraint to allow for the selection of other clusters that can be merged, and lead to a fairer output tree. It is important to note that throughout, we maintain a distance matrix $D \in \mathbb{R}^{n \times n}$ between clusters to improve the runtime of the algorithm, and this can be done by computing the distances using the linkage criterion provided as input. Also, $d_{\text{min}}$ signifies the initialization of the distance (computed using the linkage criterion) between the clusters chosen to be merged for this level. Next, the proposed Algorithm 1 essentially tightens the fairness constraint (and loosens the distance constraint) as we keep constructing the clustering tree from bottom to top. The fairness constraint tightening is achieved using the function $Z_\alpha : \mathbb{R} \rightarrow \mathbb{R}$ and the distance constraint loosening is achieved using the $Z_\beta : \mathbb{R} \rightarrow \mathbb{R}$. $Z_\alpha$ and $Z_\beta$ are both monotonically increasing functions and are parameterized appropriately for the dataset $X$ (their parameterization is discussed later). We compute the actual constraint bounds $\alpha$ and $\beta$ using $Z_\alpha(|C| - k)$ (line 6) and $Z_\beta(n - |C|)$ (line 7), respectively, where $C$ in each iteration of the while loop (line 3) denotes the current state of clusters at some level of the tree. This can be understood intuitively— if both functions are monotonically increasing, then $\alpha$ reduces as we
get closer to \( k \) clusters, tightening the fairness constraint, whereas \( \beta \) increases as we get closer to \( k \) clusters and thus, loosens the distance bound. We use \( Z_\alpha(x) = e^{\theta_1 x + \alpha_0} \) and \( Z_\beta(x) = e^{\theta_2 x + \beta_0} \) for our experiments and results, but any monotonic function can be used.

Algorithm 1: Proposed FHAC Algorithm

**Input:** \( X, F, k, D(.,.), Z_\alpha : \mathbb{R} \to \mathbb{R}, Z_\beta : \mathbb{R} \to \mathbb{R} \)

**Output:** Fair HAC tree \( T_{\text{fair}} \)

1. set \( C \leftarrow X \)
2. compute \( D_{n_1,n_2} = D(n_1,n_2), \forall (n_1,n_2) \in X \times X \)
3. while \( |C| \geq k \) do
4. \( P_g = 0, \forall g \in F \)
5. \( d_{\text{min}} \leftarrow \infty \)
6. \( \alpha \leftarrow Z_\alpha(|C| - k) \)
7. \( \beta \leftarrow Z_\beta(n - |C|) \)
8. for each \((c_i,c_j) \in C \times C, \text{s.t. } c_i \neq c_j\) do
9. \( \text{for each } g \in F \text{ do} \)
10. \( \delta_{g}^{c_i+c_j} \leftarrow \frac{\delta_{g}^{c_i}|c_i|+\delta_{g}^{c_j}|c_j|}{|c_i|+|c_j|} \)
11. \( \text{if } |\delta_{g}^{c_i+c_j} - \phi_g| \leq \alpha \text{ then } P_g' = 1, \text{ else } P_g' = 0 \)
12. \( \text{end for} \)
13. \( \text{if } \sum_{g \in F} P_g' \geq \sum_{g \in F} P_g \text{ then} \)
14. \( \text{if } d_{\text{min}} + \beta > D_{c_i,c_j} \text{ then} \)
15. \( d_{\text{min}} \leftarrow D_{c_i,c_j} \)
16. \( P_g \leftarrow P_g', \forall g \in F \)
17. \( (c_m^{n_1},c_m^{n_2}) \leftarrow (c_i,c_j) \)
18. \( \text{end if} \)
19. \( \text{end if} \)
20. \( \text{end for} \)
21. \( \text{merge } c_m^{n_1} \leftarrow c_i^{n_1} + c_j^{n_2} \)
22. \( \text{update } C \text{ with newly merged clusters} \)
23. \( \text{recompute } D_{c_1,c_2} = D(c_1,c_2), \forall (c_1,c_2) \in C \times C \)
24. \( \text{update } T_{\text{fair}} \text{ with merge} \)
25. \( \text{end while} \)
26. \( \text{return } T_{\text{fair}} \)

Next, unlike traditional HAC algorithms, we now also maintain the proportion of protected group \( g \) members for each possible cluster pair to be merged (line 10). Another distinction is the \( P_g \in \{0,1\}^f \) and \( P_g' \in \{0,1\}^f \) vectors, that help us keep track of how many protected groups for a potential cluster merge we have met the proportional fairness constraints for so far. If \( P_g' = 1 \) we have met the proportionality condition for group \( g \) in this iteration, otherwise \( P_g' = 0 \). \( P_g \) is the same but maintains global state— that is, it keeps track of the same condition but for the best cluster merge pair found so far. Thus, in line 11, we check to see if we have met \( \alpha \)-Proportional Fairness if clusters \( c_i \) and \( c_j \) were to be merged, and then appropriately set the value for \( P_g' \). Next, once we have done this for all
the groups (lines 9-12), we compare the current cluster pair with the best
cluster merge pair found so far (line 13). If the current cluster pair is a better choice, we
proceed to checking for whether we improve on the minimum distance constraint
(relaxed using \( \beta \)) on line 14, and then update variables accordingly. Gradually
we build up the HAC tree and return it as \( T_{\text{fair}} \).

**Note.** As is evident, Algorithm 1 achieves an asymptotic time complexity of
\( O(fn^3) \) which is comparable to vanilla HAC (\( O(n^3) \)) since \( f \) is usually small for
real-world applications. Also, in implementing Algorithm 1, we can further speed-
up the clustering process by making locally optimal cluster merges as long as
they meet the fairness (and distance) criteria, as opposed to searching the entire
space of possible cluster pairs to merge. Furthermore, the benefit of employing
Algorithm 1 for fair HAC, is that it can be utilized to minimize any cost function
that is required for ensuring fairness of the hierarchical clustering process. In the
previous section, we defined the MFC and balance for the level with \( k \) clusters,
which can be minimized by Algorithm 1 through appropriate parameterization of
\( Z_\alpha \) and \( Z_\beta \). In the results section, we will present experimental results for both
these fairness metrics, demonstrating the generality of our approach.

**Calculation of hyperparameters.** For our empirical results we let
\( Z_\alpha(x) = e^{\theta_1 x + \alpha_0} \) and \( Z_\beta(x) = e^{\theta_2 x + \beta_0} \). Thus, we have to estimate the parameters \( \alpha_0, \beta_0, \theta_1, \) and \( \theta_2 \) that minimize MFC or balance. For the results in the paper, we utilize
a simple grid-search and then choose parameters based on the values obtained
for the choice of fairness cost. Alternatively, a black-box hyperparameter search
algorithm can also be utilized for this purpose, such as [31].

We can now show how Algorithm 1 fares in terms of fairness compared with
traditional HAC when optimized for MFC:

**Proposition 1.** For appropriately parameterized monotonically increasing func-
tions \( Z_\alpha \) and \( Z_\beta \), Algorithm 1 computes a \( T_{\text{fair}} \) that is more (or equivalently) fair
to the vanilla HAC tree \( T \), according to the chosen fairness cost metric (MFC or
balance).

*Proof.** Provided in the supplementary file.

### 3.2 Results on Toy Data

We generate two-dimensional data from a uniform distribution ([0, 250]) for our
toy example and there are 25 points in total (\( n \)) and \( k = 4 \). Clusters are denoted
in Figure 1 using 4 colors (red, green, blue, and yellow). Moreover, there are
two protected groups, denoted by \( \circ \) and \( \times \), with ideal proportions denoted by
\( \phi_\circ = 0.56 \) and \( \phi_\times = 0.44 \), respectively. Therefore, a fair distribution of protected
groups across clusters should be \( \phi_\circ \approx \phi_\times \approx 0.5 \). Also, \( Z_\alpha(x) = \theta_1 x + \alpha_0 \) and \( Z_\beta(x) = \theta_2 x + \beta_0 \). As mentioned above, we run
FHAC (Algorithm 1) iteratively and estimate hyperparameters, and then compare
how fair the final clusters are for vanilla single-linkage HAC and single-linkage
FHAC. The values of the parameters are as follows: \( \alpha_0 = 1.0, \theta_1 = 5.0, \beta_0 =
11.031, \theta_2 = 0.1826 \). Moreover, MFC for vanilla HAC with single-linkage is 1.12,
and for FHAC with single-linkage is 0.38. Thus, we find that the fairness achieved by our algorithm is much better, and we obtain proportionally fair clusters as a result. The clusters for traditional HAC are shown in Figure 1(a), and for our proposed FHAC are shown in Figure 1(b). Visually, it is easy to see that clusters in Figure 1(b) are more well-balanced than in Figure 1(a), where the red cluster has a large number of points, and the yellow cluster has only 1 point. Figure 1(b) instead has all points from each of the two protected groups distributed according to their ideal proportions.

![Fig.1: Toy Example](image)

4 Results

We utilize three real-world datasets to demonstrate the working of FHAC (Algorithm 1) and show that it computes *fairer* (or equivalently fair) hierarchical clustering solutions compared to vanilla/traditional HAC (for average, complete, and single linkage criteria) in Section 4.2, as well as the algorithms of Ahmadian et al for fair hierarchical clustering [4] (for both value and revenue clustering objectives) in Section 4.3. Furthermore we also analyze the quality of the fair clustering outputs obtained (using the Silhouette Coefficient [29] clustering performance metric) between these algorithms in Section 4.4, and find that our algorithms compute clusters of comparable quality. For Algorithm 1, we have $Z_\alpha(x) = e^{\theta_1 x}$ and $Z_\beta(x) = e^{\theta_2 x}$ for all experiments (that is we also set $\alpha_0 = \beta_0 = 0$). We have also open-sourced our implementations and all code used on Github.

4.1 Datasets Used

We have run our experiments for comparative analysis of FHAC on the following three datasets obtained from the UCI ML repository. We subsample datasets ($n = 1000$) much like previous work [11] [3]. We let merges occur up to $k = 4$ (that is, we start 1000 singleton clusters and merge them till we have 4 clusters and we have obtained a hierarchical cluster tree) for all experiments and all
datasets. These datasets are utilized in most research to evaluate fair clustering algorithms [11], [7]:

1. **Default of Credit Card Clients Dataset** [32]: This is the dataset of customers default payments in Taiwan, denoted as `creditcard`. The features used are `age`, `bill-amt 1 — 6`, `limit-bal`, `pay-amt 1 — 6`. The sensitive attribute used is `education` and the four protected groups are `graduate school` ($\phi_0 = 0.25$), `university` ($\phi_1 = 0.25$), `high school` ($\phi_2 = 0.378$), `others` ($\phi_3 = 0.122$).

2. **Bank Marketing Dataset** [23]: This is a dataset related to the telephonic marketing campaigns of a Portuguese bank, denoted as `bank`. The features used are `age`, `balance` and `duration`. The sensitive attribute is `marital status`, and the three protected groups are `married` ($\phi_0 = 0.334$), `single` ($\phi_1 = 0.334$), `divorced` ($\phi_2 = 0.332$), respectively.

3. **Census Income Dataset** [20]: This is a set of 14 attributes of adults obtained via a census survey in 1996, denoted as `census`. The features used are `age`, `education-num`, `final-weight`, `capital-gain`, `hours-per-week`. The sensitive attribute is `sex` and the two protected groups are `male` ($\phi_0 = 0.376$), `female` ($\phi_1 = 0.624$).

### 4.2 Comparison between FHAC (Algorithm 1) and Vanilla HAC

The fairness results (including parameter values) for running Algorithm 1 and vanilla HAC over all the aforementioned datasets for different linkage criteria for MFC as the fairness metric are shown in Table 1. The lower the MFC, the more fair the obtained clustering is. The results when balance is selected as the fairness metric are shown in Table 2. Here, the higher the balance, the fairer the clustering obtained. Clearly, Algorithm 1 achieves fairer solutions to vanilla HAC on real data.

| Dataset   | Linkage | $\theta_1$ | $\theta_2$ | MFC (Vanilla) | MFC (FHAC) |
|-----------|---------|------------|------------|---------------|-------------|
| census    | Average | 0.001      | 0.001      | 0.0853        | 0.0853      |
| census    | Complete| 0.001      | 0.001      | 0.1806        | 0.0853      |
| census    | Single  | 0.001      | 0.65       | 1.248         | 0.752       |
| creditcard| Average | 0.00005    | 0.005      | 1.2439        | 1.0         |
| creditcard| Complete| 0.00005    | 0.005      | 0.744         | 0.60        |
| creditcard| Single  | 0.0075     | 0.5        | 1.5           | 0.778       |
| bank      | Average | 0.0001     | 0.05       | 1.332         | 0.6679      |
| bank      | Complete| 0.5        | 0.05       | 1.332         | 0.4457      |
| bank      | Single  | 0.001      | 0.65       | 1.332         | 0.6653      |
Table 2: Comparison of Balance between Vanilla HAC and FHAC (Ours)

| Dataset | Linkage    | $\theta_1$ | $\theta_2$ | Balance(Vanilla) | Balance(FHAC) |
|---------|------------|------------|------------|------------------|---------------|
| census  | Average    | 0.001      | 0.001      | 0.8865           | 0.8865        |
| census  | Complete   | 0.001      | 0.001      | 0.7599           | 0.8865        |
| census  | Single     | 0.0005     | 0.0         | 0.9385           |               |
| creditcard | Average | 0.05       | 0.1         | 0.427            |               |
| creditcard | Complete | 0.0005     | 1.0         | 0.7105           |               |
| creditcard | Single   | 0.0005     | 1.0         | 0.7959           |               |
| bank    | Average    | 0.045      | 0.09       | 0.5567           |               |
| bank    | Complete   | 0.5        | 0.05       | 0.3327           |               |
| bank    | Single     | 0.0005     | 1.0         | 0.8099           |               |

4.3 Comparison between FHAC (Algorithm 1) and AFHAC

In this subsection, we compare the performance of our proposed algorithm FHAC with AFHAC with regards to the fairness metric MFC and balance defined in Section 3. AFHAC works with the following hierarchical clustering objectives: Cohen-Addad et al’s value objective [25] and Moseley et al’s revenue objective [13]. Experiments were run to calculate MFC and balance of the clusters formed by AFHAC optimizing revenue (denoted as AFHAC-R) as well as value objectives (denoted as AFHAC-V).

The AFHAC algorithm proposed by Ahmadian et al utilizes average-linkage HAC as part of their clustering process [4]. Thus, for comparisons to be justifiable, we compare their algorithm with our proposed FHAC algorithm with average-linkage as the linkage criterion. Another important consideration is the difference in fairness enforcement. The AFHAC algorithms aim to ensure that all clusters (and their merged sub-clusters) should have each protected group’s proportion upper-bounded by some value $\alpha$ provided at runtime. Extrapolating for the level with $k$ clusters, it is then evident that their algorithms cannot accommodate different ideal proportions for each group as they attempt to use only the same $\alpha$ for each group. Mathematically, they aim to ensure that $\delta_{Ci}^g \leq \alpha, \forall g \in F, i \in [k]$. Therefore, to make comparisons fair, we set $\alpha = \max_{g \in F} \{\phi_g\}$ for our experiments using their algorithms. We can then compare their approach with ours.

Table 3: Comparison of MFC for FHAC (Ours), AFHAC-R, and AFHAC-V

| Dataset | MFC (FHAC) | MFC (AFHAC-V) | MFC (AFHAC-R) |
|---------|------------|---------------|---------------|
| creditcard | 1.0        | 2.8505        | 1.5825        |
| census   | 0.0853     | 4.0           | 4.0           |
| bank     | 0.6679     | 0.03648       | 2.0027        |
We calculated MFC values for AFHAC-R and AFHAC-V on all the datasets mentioned in Section 4.1 and compare with Algorithm 1. These results are shown in Table 3. We do similar experiments for balance and show the results in Table 4. Observing both Table 3 and Table 4, we can see that we outperform both AFHAC-V and AFHAC-R for both the MFC and balance metrics for the creditcard and census datasets. For the bank dataset, we outperform AFHAC-R for both MFC and balance, but AFHAC-V has better results than ours. Despite this, we believe in the merits of our approach due to the lack of generalization capability of the AFHAC algorithms to any fairness notions, and their overall inconsistent performance on the other datasets.

Table 4: Comparison of Balance for FHAC (Ours), AFHAC-R, and AFHAC-V

| Dataset | Balance(FHAC) | Balance(AFHAC-V) | Balance(AFHAC-R) |
|---------|---------------|------------------|------------------|
| creditcard | 0.427         | 0.0              | 0.416            |
| census   | 0.8865        | 0.2              | 0.0              |
| bank     | 0.5567        | 0.9474           | 0.0              |

4.4 Comparing Clustering Performance/Quality of Fair Clusters

As a result of enforcing fairness constraints on a clustering, we are reducing the clustering quality compared to the original optimal clustering. This happens because we opt for more fair cluster merges over ones that minimize distance between clusters in Algorithm 1. That is, as we are improving fairness (either in terms of balance or MFC) for our proposed FHAC algorithm, we are reducing clustering performance compared to vanilla HAC. In our approach, this directly relates to increasing the $\beta$ relaxation of the distance criterion while finding pairs of clusters to merge together. Similarly, even for the AFHAC algorithms (and other fair algorithmic variants), clustering performance decreases at the cost of finding fairer solutions.

Thus, in this subsection, we wish to analyze the clustering performance of our FHAC algorithm to the AFHAC algorithms. Considering the vanilla HAC clustering performance as the optimal, we obtain results to observe the extent to which clustering performance worsens and how it compares to the AFHAC algorithms. To measure clustering quality, we utilize the Silhouette Coefficient/Score [29] which is a widely used clustering performance metric. It aims to capture intra-cluster similarity and inter-cluster dissimilarity and gives an output between $[-1, 1]$ for a given clustering as input. A score of $-1$ indicates an incorrectly assigned clustering and a score of 1 indicates a dense and well-separated clustering. A score of 0 indicates overlapping clusters.
Since we are comparing Silhouette Coefficients for fair clusters obtained via Algorithm 1 and the AFHAC algorithms, it does not make sense to compare clustering solutions that are explicitly unfair according to the fairness metric. In our work, this corresponds to solutions that have balance equal to 0. Therefore, for any solutions such as these, we denote their Silhouette Score as \(-1\) (incorrect clustering). The results comparing the Silhouette Scores for Algorithm 1 with average-linkage for MFC (denoted as FHAC(MFC)), Algorithm 1 with average-linkage for balance (denoted as FHAC(Balance)), AFHAC-V, and AFHAC-R, on the *creditcard*, *bank*, and *census* datasets are shown in Figure 2(a), Figure 2(b), and Figure 2(c), respectively.

For Figure 2(a), we see that FHAC for both MFC and for balance outperforms the AFHAC algorithms in terms of performance. Moreover, the Silhouette Score for FHAC(MFC) and the vanilla HAC average-linkage algorithm (optimal) is very close, with values of 0.621 and 0.624, respectively. For Figure 2(b) we see that AFHAC-V obtains a better Silhouette Score, that is very close to the optimal vanilla average-linkage HAC score, with values of 0.78 and 0.8, respectively. However, FHAC(MFC) and FHAC(Balance) obtain good scores of 0.68 and 0.36 as well, indicating satisfactory clustering performance. Finally for Figure 2(c), we see that FHAC(MFC), FHAC(Balance), and AFHAC-V obtain very good clustering performance, close to the optimal vanilla average-linkage HAC Silhouette Score of 0.52. Thus, we can see that the clustering performance of the FHAC algorithm compares well to vanilla HAC performance, as well as the
AFHAC algorithms. Furthermore, the AFHAC-V algorithm performs well in a few cases, but the AFHAC-R generally outputs unfit clusters in terms of performance.

5 Discussion

5.1 Alternate Approach to FHAC

We also consider an alternate way of improving fairness for HAC given an output tree from vanilla HAC, \( T \). The goal is to come up with a tree \( T_{fair} \) that is fairer than the tree \( T \) with respect to the fairness cost (for the level of \( T \) with \( k \) clusters, as before). We provide a recursive approach that achieves this with a time complexity of \( O(fn^3) \). The general approach is to temporarily swap clusters between one of the \( k \) clusters one at a time, and make a swap permanent only if it improves fairness. By greedily making swaps that reduce the MFC (or balance), the result is a fairer tree. However, the algorithm is complex, and its implementation is not trivial. Due to space limitations, we provide this algorithm and present its asymptotic time complexity analysis in the supplementary file (Algorithm A.1). We defer the study of such post-clustering fairness algorithms to future work.

5.2 Clustering Performance Metrics for Comparing Fair Algorithms

In Section 4.4 we compared our proposed algorithm to the AFHAC algorithms and vanilla HAC in terms of clustering performance. For this, we used the Silhouette Score \[29\] as the performance metric, which is a general-purpose and commonly used clustering quality metric. While metrics such as the Silhouette Score do give higher scores to convex clusters, they are reliable to get a sense of clustering performance. They are also convenient for comparing fair clusters outputted by different fair clustering algorithms. Other performance metrics such as the Calinski-Harabasz index \[9\] and the Davies-Bouldin index \[15\] can also be used, but are harder to interpret since they are unbounded. Compared to these, the Silhouette Score always lies between -1 and 1. To the best of our knowledge, other than our paper, only one other related work also considers using the Silhouette Score to compare cluster quality \[3\], and we espouse more papers to undertake similar analysis.

6 Conclusion

In this paper, we have proposed the FHAC algorithm (Algorithm 1) for performing HAC (Section 3). Our algorithm works for multiple protected groups, generalizes to natural fairness notions for HAC, and for any linkage criterion used. We provide results on UCI datasets such as bank, creditcard, and census comparing our approach to vanilla HAC as well as the only other fair hierarchical clustering approach of \[4\], for both the MFC fairness cost and the balance fairness metric of \[11\] (Section 5). The results demonstrate that our proposed algorithm achieves better fairness and is more robust than the other clustering approaches. Furthermore, Algorithm 1 has an asymptotic time complexity of \( O(fn^3) \) (Section 3). We further
compare the clustering performance of our approach and show that it outputs clusters of quality comparable to the optimal as well as other related approaches. For future work, we aim to improve the proposed post-clustering fair algorithm (Algorithm A.1 in the appendix) and study alternate approaches to making HAC fair.

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