MSSM with Yukawa Quasi-Unification

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ABSTRACT

We consider the constrained minimal supersymmetric standard model which emerges from one theory with a small deviation from Yukawa unification which is adequate for $\mu > 0$. We show that this model possesses a wide and natural range of parameters which is consistent with the data on $b \to s\gamma$, the muon anomalous magnetic moment, the cold dark matter abundance in the universe, and the Higgs boson masses.

1 Introduction

We study the phenomenological consequences of imposing on the constrained MSSM (CMSSM) an asymptotic relation for the Yukawa couplings at the GUT scale. This assumption (Yukawa unification) naturally restricts [1] the $t$-quark mass to large values compatible with the data. Also, the emerging model is highly predictive [2]. Despite of its appealing, the simple scheme of a single Yukawa for the three third generations at the GUT scale leads to an unacceptable $b$-quark mass. This fact excludes minimal versions of GUT groups with this property, such as Pati–Salam unification ($G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$), $SO(10)$ or $E_6$.

We consider the SUSY GUT model described in Ref.[3] which is based in the $G_{PS}$ as described in Refs.[4] and establish an ‘asymptotic’ relation for the Yukawa couplings that depends on a single complex parameter $c$:

$$h_t : h_b : h_\tau = |1 + c| : |1 - c| : |1 + 3c|,$$

(1)

For simplicity, we will restrict our analysis to real values of $0 < c < 1$. The relative splitting of the Yukawa couplings becomes: $\delta h \equiv -(h_b - h_t)/h_t = (h_\tau - h_t)/h_t = 2c/(1+c)$. This means that the bottom and tau Yukawa couplings split from the top Yukawa coupling by the same amount but in opposite directions, with $h_b$ becoming smaller than $h_t$. 
2 The MSSM with Quasi-Yukawa Unification

This model, below $M_{\text{GUT}}$, reduces to the MSSM supplemented by the ‘asymptotic’ Yukawa coupling quasi-unification condition in Eq.(1). We will assume universal soft SUSY breaking terms at $M_{\text{GUT}}$, i.e., a common mass for all scalar fields $m_0$, a common gaugino mass $M_{1/2}$ and a common trilinear scalar coupling $A_0$. In the present work, we will concentrate on the $\mu > 0$. The case $\mu < 0$ is phenomenologically less interesting, it will be presented in [5]. We follow the notation as well as the RG and radiative electroweak breaking analysis of Ref.[6] for the CMSSM with the improvements of Refs.[7, 3] (recall that the sign of $\mu$ in Refs.[6, 7] is opposite to Ref.[3], which is the one adopted here).

For any given $m_b(M_Z)$ in its 95% c.l. range ($2.684 - 3.092$ GeV for $\alpha_s(M_Z) = 0.1185$), we can determine the parameters $c$ and $\tan \beta$ at $M_{\text{SUSY}} = (m_{\tilde{t}_1} m_{\tilde{t}_2})^{1/2}$ ($\tilde{t}_{1,2}$ are the stop mass eigenstates) so that the ‘asymptotic’ condition in Eq.(1) is satisfied. We use fixed values for the running top quark mass $m_t(m_t) = 166$ GeV and the running tau lepton mass $m_\tau(M_Z) = 1.746$ GeV and incorporate not only the SUSY correction to the bottom quark mass but also the SUSY threshold correction to $m_\tau(M_{\text{SUSY}})$ from the approximate formula of Ref.[8]. After imposing the conditions of gauge coupling unification, successful electroweak breaking and Yukawa quasi-unification in Eq.(1), we are left with three free input parameters $m_0$, $M_{1/2}$ and $A_0$. In order to make the notation physically more transparent, we replace $m_0$ and $M_{1/2}$ equivalently by the mass $m_{\text{LSP}}$ (or $m_{\tilde{\chi}}$) of the lightest supersymmetric particle (LSP), which turns out to be the lightest neutralino ($\tilde{\chi}$), and the relative mass splitting $\Delta_{\tilde{\tau}_2} = (m_{\tilde{\tau}_2} - m_{\tilde{\chi}})/m_{\tilde{\chi}}$ between the lightest stau mass eigenstate ($\tilde{\tau}_2$) and the LSP. In Fig.1 we display the changes on $M_{\text{SUSY}}$ and the mass of the pseudo-scalar Higgs, $m_A$, for several values of $\Delta_{\tilde{\tau}_2}$ and $m_b(M_Z)$. These changes will help us to understand the corresponding predictions for $\Omega_{\text{LSP}} h^2$ in the presence of resonant annihilation channels for values of $m_A \approx 2 \cdot m_{\text{LSP}}$.

![Figure 1](image.png)

Figure 1: The mass parameters $m_A$ and $M_{\text{SUSY}}$ as functions of $m_{\text{LSP}}$, with $\alpha_s(M_Z) = 0.1185$ and $A_0 = 0$. On the panel of the left we display various values of $\Delta_{\tilde{\tau}_2}$ (indicated on the curves) for $m_b(M_Z) = 2.888$ GeV. On the right panel we fix $\Delta_{\tilde{\tau}_2} = 1$ and show the curves for $m_b(M_Z) = 2.684$ GeV (dashed lines), 3.092 GeV (dotted lines) or 2.888 GeV (solid lines).
Figure 2: The LSP relic abundance $\Omega_{LSP} h^2$ versus $m_{LSP}$ for various $\Delta_{\tilde{\tau}_2}$'s (indicated on the curves) and with $A_0 = 0$, $m_b(M_Z) = 2.888$ GeV, $\alpha_s(M_Z) = 0.1185$. The solid lines (crosses) are obtained by micrOMEGAs (our alternative method). The upper bound on $\Omega_{LSP} h^2 (=0.22)$ is also depicted.

Here, the LSP ($\tilde{\chi}$) is an almost pure bino. Its relic abundance will be calculated by micrOMEGAs [9], which is the most complete code available. It includes all the coannihilations [10] of neutralinos, charginos, sleptons, squarks and gluinos since it incorporates automatically all possible channels by using COMPHEP [11] (A similar calculation has appeared in Ref.[12].) Also, poles and thresholds are properly handled and one-loop QCD corrected Higgs decay widths [13] are used, which is the main improvement provided by Ref.[9]. The SUSY corrections [14] to these widths are, however, not included. Fortunately, in our case, their effect is much smaller than that of the QCD corrections. From the recent results of DASI [15], one finds that the 95% c.l. range of $\Omega_{CDM} h^2$ is 0.06 − 0.22. Therefore, we require that $\Omega_{LSP} h^2$ does not exceed 0.22.

In order to have an independent check of micrOMEGAs, we also use the following alternative method for calculating $\Omega_{LSP} h^2$ in our model. In most of the parameter space where coannihilations are unimportant, $\Omega_{LSP} h^2$ can be calculated by using DarkSUSY [16] 1. Its neutralino annihilation part is in excellent numerical agreement with the recent exact analytic calculation of Ref.[18], the main defect of its current version is that it uses the tree-level Higgs decay widths. This can be approximately corrected if, in evaluating the Higgs decay widths, we replace $m_b(m_b)$ by $m_b$ at the mass of the appropriate Higgs boson in the couplings of the $b$-quark to the Higgs bosons (see Ref.[9]). In the region of the parameter space where coannihilations come into play, the next-to-lightest supersymmetric particle (NLSP) turns out to be the $\tilde{\tau}_2$ and the only relevant coannihilations are the bino-stau ones [6, 19] (we do not find bino-stop cannihilations [20], potencially important when $A$ is negative). In this region, which is given by $\Delta_{\tilde{\tau}_2} < 0.25$, we calculate $\Omega_{LSP} h^2$ by using an improved version of the analysis of Ref.[6, 7, 21, 23]. The list of bino-stau coannihilation channels appropriate for all $\tan \beta$’s given Ref. [6] has been completed with some additional channels as described in [3](see also Refs.[19, 22]). Their corresponding cross sections are

1 An updated version of this code is now available [17].
Figure 3: Restrictions on the $m_{\text{LSP}} - \Delta \tau_{\tilde{\tau}}$ plane for $A_0 = 0$, $\alpha_s(M_Z) = 0.1185$. From left to right, the dashed (dotted) lines depict the lower bounds on $m_{\text{LSP}}$ from $m_A > 110$ GeV, BR($b \to s\gamma$) > $1.9 \times 10^{-4}$ and $\delta a_\mu < 58 \times 10^{-10}$, and the upper bound on $m_{\text{LSP}}$ from $\Omega_{\text{LSP}} h^2 < 0.22$ for $m_b(M_Z) = 2.684$ GeV (3.092 GeV). The left (right) solid line depicts the lower (upper) bound on $m_{\text{LSP}}$ from $m_h > 114.1$ GeV ($\Omega_{\text{LSP}} h^2 < 0.22$) for $m_b(M_Z) = 2.888$ GeV. The allowed area for $m_b(M_Z) = 2.888$ GeV is shaded.

combined with the results of DarkSUSY as described in [3]. The results presented in Fig. 2 show an impressive agreement of the two methods.

We calculate BR($b \to s\gamma$) using the formalism of Ref.[24], where the SM contribution is factorized out. This contribution includes the next-to-leading order (NLO) QCD and the leading order (LO) QED corrections. The charged Higgs boson contribution to BR($b \to s\gamma$) is evaluated by including the NLO QCD corrections from Ref.[25]. The dominant SUSY contribution includes the NLO QCD corrections from Ref.[26], which hold for large $\tan \beta$. With the considerations of [3] the 95% c.l. range of this branching ratio then turns out to be about $(1.9 - 4.6) \times 10^{-4}$.

According with the latest measurement [27] of the anomalous magnetic moment of the muon $a_\mu \equiv (g_\mu - 2)/2$, the deviation of from its predicted value in the SM [28], $\delta a_\mu$, is found to lie, at 95% c.l., in the range from $-4.7 \times 10^{-10}$ to $56 \times 10^{-10}$ when the SM calculations based in $e^+e^-$ data and in $\tau$ data are both taken into account. The calculation of $\delta a_\mu$ in the CMSSM is performed here by using the analysis of Ref.[29], the updating of the experimental bounds does not introduce significant differences respect the results presented in [3].

We will also impose the 95% c.l. LEP bound on the lightest CP-even neutral Higgs boson mass $m_h > 114.1$ GeV. In the CMSSM, this bound holds almost always for all $\tan \beta$’s, at least as long as CP is conserved. The CP-even neutral Higgs boson mass matrix by using FeynHiggsFast [30]. Finally, for the values of $\tan \beta$ which appear here (about 60), the CDF results yield the 95% c.l. bound $m_A > 110$ GeV [31].
3 The Allowed Parameter Space

The restrictions on the $m_{LSP} - \Delta_{\tilde{t}_2}$ plane, for $A_0 = 0$ and with the central value of $\alpha_s(M_Z) = 0.1185$, are shown in Fig. 3. The dashed (dotted) lines correspond to the 95\% c.l. lower (upper) experimental bound on $m_b(M_Z)$ which is 2.684 GeV (3.092 GeV), while the solid lines correspond to the central experimental value of $m_b(M_Z) = 2.888$ GeV. From left to right, the dashed (dotted) lines depict the lower bounds on $m_{LSP}$ from the constraints $m_A > 110$ GeV, $\text{BR}(b \rightarrow s\gamma) > 1.9 \times 10^{-4}$ and $\delta a_\mu < 58 \times 10^{-10}$, and the 95\% c.l. upper bound on $m_{LSP}$ from $\Omega_{LSP} h^2 < 0.22$. The constraints $\text{BR}(b \rightarrow s\gamma) < 4.6 \times 10^{-4}$ and $\delta a_\mu > -6 \times 10^{-10}$ do not restrict the parameters since they are always satisfied for $\mu > 0$. The left solid line depicts the lower bound on $m_{LSP}$ from $m_h > 114.1$ GeV which does not depend much on $m_b(M_Z)$, while the right solid line corresponds to $\Omega_{LSP} h^2 = 0.22$ for the central value of $m_b(M_Z)$. We see that $m_A$ is always smaller than $2m_{LSP}$ but close to it. Thus, generally, the neutralino annihilation via the $s$-channel exchange of an $A$-boson is by far the dominant (co)annihilation process. We also observe that, as $m_{LSP}$ or $\Delta_{\tilde{t}_2}$ increase, we move away from the $A$-pole, which thus becomes less efficient. As a consequence, $\Omega_{LSP} h^2$ increases with $m_{LSP}$ or $\Delta_{\tilde{t}_2}$ (see Fig. 2).

In the allowed (shaded) area of Fig. 3 which corresponds to the central value of $m_b(M_Z)$, the parameter $c$ (tan $\beta$) varies between about 0.15 and 0.20 (58 and 59). For $m_b(M_Z)$ fixed to its lower or upper bound, we find that, in the corresponding allowed area, the parameter $c$ (tan $\beta$) ranges between about 0.17 and 0.23 (59 and 61) or 0.13 and 0.17 (56 and 58). We observe that, as we increase $m_b(M_Z)$, the parameter $c$ decreases and we get closer to exact Yukawa unification. This behavior is certainly consistent with the fact that the value of $m_b(M_Z)$ which corresponds to exact Yukawa unification lies well above its 95\% c.l. range. The LSP mass is restricted to be higher than about 138 GeV for $A_0 = 0$ and $\alpha_s(M_Z) = 0.1185$, with the minimum being practically $\Delta_{\tilde{t}_2}$-independent. At this minimum, $c \approx 0.16 - 0.20$ (c \approx 0.13 - 0.23) and $\tan \beta \approx 59$ (tan $\beta \approx 58 - 61$) for $m_b(M_Z) = 2.888$ GeV ($m_b(M_Z) = 2.684 - 3.092$ GeV).

In Fig. 3, we present the restrictions on the $m_{LSP} - A_0/M_{1/2}$ plane for $m_b(M_Z) = 2.888$ GeV, $\alpha_s(M_Z) = 0.1185$ and fixed values of $\Delta_{\tilde{t}_2}$. The most significant restriction on the allowed area are due to the displacement of the $\Omega_{LSP} h^2 = 0.22$ line as $\Delta_{\tilde{t}_2}$ increases, showing clearly the effect of the bino-stau coannihilations on the left panel. On the right panel we can observe that the allowed area becomes narrower as $|A_0/M_{1/2}| \neq 0$.

4 Conclusions

We showed that, in the particular model with Yukawa quasi-unification considered, there exists a wide and natural range of CMSSM parameters which is consistent with all the above constraints. We found that, within the investigated part of the overall allowed parameter space, the parameter $\tan \beta$ ranges between about 58 and 61 and the ‘asymptotic’ splitting between the bottom (or tau) and the top Yukawa couplings varies in the range $26 - 35\%$ for central values of $m_b(M_Z)$ and $\alpha_s(M_Z)$.

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Figure 4: Restrictions on the $m_{\text{LSP}} - A_0/M_{1/2}$ plane for $\Delta \tilde{\tau}_2 = 0$ (left) and 1 (right), $m_b(M_Z) = 2.888$ GeV, $\alpha_s(M_Z) = 0.1185$. The solid, dashed, dot-dashed and dotted lines correspond to the lower bounds on $m_{\text{LSP}}$ from $m_A > 110$ GeV, $\text{BR}(b \to s\gamma) > 1.9 \times 10^{-4}$, $\delta a_\mu < 58 \times 10^{-10}$ and $m_h > 114.1$ GeV respectively. The upper bound on $m_{\text{LSP}}$ from $\Omega_{\text{LSP}} h^2 < 0.22$ does not appear in the left panel since it lies at $m_{\text{LSP}} > 500$ GeV. The allowed area is shaded. For comparison, we also display on the left panel the bounds from $\Omega_{\text{LSP}} h^2 < 0.22$ (double dot-dashed lines) for $\Delta \tilde{\tau}_2 = 0.1$ and 0.03, as indicated. The upper bound on $m_{\text{LSP}}$ from the cosmological constraint $\Omega_{\text{LSP}} h^2 < 0.22$ corresponds to the double dot-dashed line on the left panel.

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