Marching on a rugged landscape: universality in disordered asymmetric exclusion processes

Astik Haldar\textsuperscript{1,*} and Abhik Basu\textsuperscript{1,†}

\textsuperscript{1}Condensed Matter Physics Division, Saha Institute of Nuclear Physics, HBNI, Calcutta 700064, India

(Dated: April 17, 2020)

We develop the hydrodynamic theory for number conserving asymmetric exclusion processes with short-range random quenched disordered hopping rates. We show that when the system is away from half-filling, the spatio-temporal scaling of the density fluctuations is indistinguishable from its pure counterpart, with the associated scaling exponents belonging to the one-dimensional Kardar-Parisi-Zhang universality class. In contrast, close to half-filling, the scaling exponents take different values, indicating the relevance of the quenched disorder and the emergence of a new universality class.

I. INTRODUCTION

Studies on the large-scale, macroscopic effects of quenched disorder in statistical mechanics models and condensed matter systems have a long tradition. For systems with quenched disorders, the impurities are fixed in particular configurations and do not evolve in time, and, as a result, the disorder configuration is not in thermodynamic equilibrium. Studies on the effects of random quenched disorder on pure systems, e.g., the classical $O(N)$ spin model\textsuperscript{1–3} and self-avoiding walks on random lattices\textsuperscript{4–8} clearly reveal the modifications in the critical behaviour brought about by the presence of such impurities. The effects of quenched disorder on driven, nonequilibrium systems are much less understood in comparison with their equilibrium counterparts. This is a question that could be of significance in a number of physical systems, e.g., systems involving flows in random media\textsuperscript{9}. In the absence of any general theories for nonequilibrium systems, it is useful to construct and study simple nonequilibrium models with quench disorders that are easy to analyse and yet can capture some basic features of more complex physical systems. This should help us to characterise and delineate regimes with different macroscopic behaviour.

In this article, we study quenched-disordered asymmetric exclusion processes (TASEP) with particle number conservation. TASEP with open boundaries was originally proposed as a simple model for the motion of molecular motors in eukaryotic cells\textsuperscript{10}. Subsequently, it was reinvented as a paradigmatic one-dimensional (1D) model for nonequilibrium statistical mechanics, that shows boundary induced phase transitions\textsuperscript{11}. In contrast to TASEP with open boundaries, homogeneous periodic TASEPs (e.g., a TASEP on a ring) that necessarily conserves particles, displays uniform steady states. There are, however, local number fluctuations about the steady states that execute slow dynamics due to the overall particle number conservation.

Steady states of periodic TASEPs are known to be affected by quenched but non-random disorders, e.g., by localised “obstacles” or defects. This has been studied either with isolated point defects\textsuperscript{12,13}, or with extended defects\textsuperscript{14,15}. The general outcome from these studies point towards the existence of two generic classes of steady states in which the steady state currents are either independent of or vary explicitly with the particle numbers.

In this article, we revisit the problem of periodic asymmetric exclusion processes with generic short range quenched disorder hopping rates. TASEPs with quenched disorder, both with open boundary conditions and periodic, have been considered. For studies on open TASEPs with quenched disorders, see Refs.\textsuperscript{17–21}. In this work, we are however interested in the large scale fluctuation properties of periodic TASEPs with quenched disorder. Recently, a number of simulations have been performed to study this, which generally point to larger fluctuations and slower dynamics\textsuperscript{18,22,23}. Here, we systematically construct the hydrodynamic theory for it, that to our knowledge is absent till the date. For this, we first define an asymmetric exclusion process with quenched disordered hopping rates on a ring. We then use a continuum hydrodynamic description for this number conserving lattice-gas model to elucidate the universal scaling that characterises the local number fluctuations in the long wavelength limit. We show that away from the half-filling the relevant scaling exponents belong to the Kardar-Parisi-Zhang (KPZ) universality class, rendering disorder irrelevant. On the other hand, close to half-filling, the scaling exponents are modified and new universal scaling behaviour emerges. These form the principal results from this work. Our hydrodynamic theory complements the results of Refs.\textsuperscript{22,23}. Due to the generic nature of the hydrodynamic theory, it is valid for all number conserving asymmetric exclusion processes with short range quenched disorder. The rest of this article is organised in the following manner. We construct the model and set up the continuum hydrodynamic equations of motion in Sec.\textsuperscript{11}. Then in Sec.\textsuperscript{111} we analyse the scaling properties of the density fluctuations in the
nonequilibrium steady states. We first study the symmetric limit of the problem in Sec. IV and move on to the general, asymmetric case analysed in Sec. V. We show that the scaling of the density fluctuations in the half-filled limit is different from when the system is away from half-filling. We finally summarise in Sec. VI. Some of the technical details are discussed in Appendix for interested readers.

II. MODEL

We consider an asymmetric exclusion process (TASEP) on a closed ring with $L$ sites. The unidirectional rate of hopping $m_i$, ($1 \leq i \leq L$) from site $i$ to $i+1$ is a positive definite time-independent random number, i.e., $m_i$ is quenched; see Fig. 1. This model study complements the studies in Ref. 22, 23, where a binary distribution for $m_i$ has been considered. The quenched disordered hopping rates make the system inhomogeneous and break the translational invariance along the ring. The occupation $n_i$ for site $i$, can be either 0 or 1 and follows

$$\frac{\partial n_i(t)}{\partial t} = m_{i-1}n_{i-1}(1-n_i)-m_in_i(1-n_{i+1}),$$

(1)
together with $0 < m_i < 1$. In the thermodynamic limit with $L \to \infty$, it is convenient to introduce a quasi-continuous coordinate $x = i/L$, such that $0 \leq x \leq 1$ with $L \to \infty$ and $\rho(x,t) \equiv n_i(t)$, $m(x) = m_i$. In this continuum limit using a gradient expansion up to the second order in spatial gradients, we obtain

$$\frac{\partial \rho(x,t)}{\partial t} = \nu \frac{\partial^2 \rho(x,t)}{\partial x^2} - \frac{\partial}{\partial x}[am(x)\rho(x,t)(1-\rho(x,t))]
+ \frac{\partial f(x,t)}{\partial x},$$

(2)

where we have neglected terms that are higher order in gradient-expansions; $f$ is a white noise that models the stochasticity of the underlying microscopic dynamics, and $\nu$ is diffusion coefficient: $\nu \sim m_0/L$; see Appendix; see also Ref. 24 for discussions on the principles to obtain the hydrodynamic limit for the Burgers equation. We choose $f(x,t)$ to a zero-mean, Gaussian distributed noise with a variance

$$\langle f(x,t)f(0,0) \rangle = 2D\delta(x)\delta(t),$$

(3)

where $D > 0$ is the nonequilibrium analogue of temperature. We further write $m(x) = m_0 + \delta m(x)$, where $m_0 = m(x)$ is the mean of $m(x)$, and $\delta m(x)$ is the local (in space) fluctuation of $m(x)$ about $m_0$: $\delta m(x) = 0$. We assume $\delta m(x)$ to be Gaussian-distributed with a variance

$$\delta m(x)\delta m(0) = 2D\delta(x).$$

(4)

Equivalently, in the Fourier space,

$$\delta m(k_1,\omega_1)\delta m(k_2,\omega_2) = 2D\delta(k_1+k_2)\delta(\omega_1+\omega_2)\delta(\omega_1).$$

(5)

The last $\delta$-function factor appears due to the fact $\delta m(x)$ is time-independent, and so is the correlator on the rhs of (4). Thus the quenched disorder is short ranged. Here, $\langle \ldots \rangle$ implies averages over the annealed (time-dependent) noise distribution, where as an “overline” refers to averages done over the random quenched disorder distribution.

III. SCALING OF THE DENSITY FLUCTUATIONS

In order to extract the scaling behaviour, we must solve for $\rho(x,t)$ from (2) as a function of $f(x,t)$ and $\delta m(x)$. It is convenient to express $\rho(x,t)$ as $\rho(x,t) = \rho_0 + \phi(x,t)$, where $\int dx \rho(x,t) = \rho_0L$ and $\int dx \phi(x,t) = 0$ at all time $t$. Fluctuation $\phi(x,t)$ then satisfies

$$\frac{\partial \phi(x,t)}{\partial t} = \nu \frac{\partial^2 \phi(x,t)}{\partial x^2} + \lambda_0 \frac{\partial \phi(x,t)}{\partial x} + \lambda_m \frac{\partial \delta m(x)}{\partial x}
- \lambda_0 \frac{\partial^2 \phi(x,t)}{2 \partial x^2} + \lambda_1 \frac{\partial}{\partial x}[\delta m(x)\phi(x,t)]
- \lambda_2 \frac{\partial}{\partial x}[\delta m(x)\phi^2(x,t)] + \frac{\partial f(x,t)}{\partial x},$$

(6)

where $\lambda = 2am_0$, $\lambda_m = a(\rho_0^2 - \rho_0)$, $\lambda_1 = am_0(2\rho_0 - 1)$, $\lambda_2 = a(2\rho_0 - 1)$, $\lambda_3 = a$. Notice that $\phi(x,t)$ is driven by two conserved noises - quenched noise $\partial_x \delta m(x)$ and annealed noise $\partial_x f(x,t)$.
To proceed further, we decompose $\phi(x,t)$ into two parts: $\phi(x,t) = \psi(x) + \delta \rho(x,t)$; $\int dx \psi(x) = 0 = \int dx \delta \rho(x,t)$. Time-independent function $\psi(x)$ satisfies

$$
- \nu \psi_t x \psi - \lambda_1 \psi_t x \psi - \lambda_m \partial_x \delta m + \frac{\partial^2 \psi}{2} \psi^2 \\
- \lambda_{2v} \partial_x [\delta m \psi] + \lambda_{3v} \partial_x [\delta m \psi^2] = 0. \quad (7)
$$

In contrast, $\delta \rho(x,t)$ satisfies the time-dependent equation

$$
\frac{\partial \delta \rho}{\partial t} = \nu_\rho \partial_{xx} \delta \rho + \lambda_{1\rho} \partial_x \delta \rho - \frac{\lambda_{2\rho}}{2} \partial_x \delta \rho^2 + 2 \lambda_{2\rho} \partial_x [\delta m \delta \rho] \\
- \lambda_{3\rho} \partial_x [\delta \rho \psi] - \lambda_{3\rho} \partial_x [\delta m \delta \rho^2] - \lambda_{3\rho} \partial_x [\delta m \delta \rho \psi] + \partial_x f. \quad (8)
$$

Here the model parameters $\nu_\psi, \nu_\rho$ are proportional to $\nu$; $\lambda_\psi, \lambda_\rho, \lambda_{2\rho}, \lambda_{3\rho}$ are proportional to $\lambda_1$; $\lambda_{1\psi}, \lambda_{1\rho}$ are proportional to $\lambda_2$; $\lambda_{2\rho}, \lambda_{3\rho}$ are proportional to $\lambda_3$. We do not use the same set of model parameters in $\psi$ and $\rho$. In order to determine the quenched noise $\delta m(x)$. On the other hand, $\delta \rho(x,t)$ is driven by a time-dependent (annealed) noise $\partial_x f$ additively (i.e., as a “source”) and also by the quenched noise $\delta m(x)$ multiplicatively. In addition, $\psi(x)$ enters into the dynamics of $\delta \rho(x,t)$, where $\psi(x)$ itself is independent of $\delta \rho(x,t)$.

Equations (7) and (8) have two linear terms with first order spatial gradients having coefficients proportional to $\alpha$: For example, in (8) the term $\lambda_{1\rho} \partial_x \delta \rho$ implies the existence of underdamped propagating modes for $\delta \rho$. This linear propagating mode cannot be removed by going to the co-moving frame, i.e., the model does not admit Galilean invariance. This is a consequence of the fact that the variance $\delta m$ is not invariant under a Galilean transformation, Eq. (9) is not invariant under an equivalent tilt of the surface. The presence of the columnar disorders manifestly breaks the tilt invariance of the ordinary KPZ equation.

Away from the half-filled limit, the equal-time auto-correlation functions of $\psi$ and $\delta \rho$ are given by

$$
\langle \psi(k) \rangle^2 = \frac{2D \lambda^2}{\nu_\psi^2 k^2}; \quad (10)
$$

$$
\langle \delta \rho(k) \rangle^2 = \frac{D}{\nu_\rho}; \quad (11)
$$

$$
\langle |h(k,t)|^2 \rangle = \frac{D}{\nu_\rho k^2}; \quad (12)
$$

see also Sec. [122] in Appendix. Clearly, $\psi(k)$ and $\delta \rho(k,t)$, and hence $\psi(x)$ and $\delta \rho(x,t)$ scale the same way. This no longer holds true near the half-filled limit, for which the equal-time correlators are

$$
\langle \psi(k) \rangle^2 = \frac{2D \lambda^2}{\nu_\psi^2 k^2}; \quad (13)
$$

$$
\langle |\delta \rho(k,t)|^2 \rangle = \frac{D}{\nu_\rho}; \quad (14)
$$

$$
\langle |h(k,t)|^2 \rangle = \frac{D}{\nu_\rho k^2}; \quad (15)
$$

Thus, $\psi(k)$ scales differently from $\delta \rho(k,t)$. In particular, $\psi(k)$ is more relevant (in a scaling sense) than $\delta \rho(k)$ in the infra-red (long wavelength) limit. It further implies that $\psi(x)$ is more relevant than $\delta \rho(x)$ (in a scaling sense). Interestingly, in the linear theory, $\langle |\delta \rho(k,t)|^2 \rangle$ and $\langle |h(k,t)|^2 \rangle$ do not depend upon the filling-factor.

Noting that $\psi(x)$ is independent of $t$, we define the scaling exponents that characterise the auto-correlation functions of $\psi(x)$ and $h(x,t)$:

$$
C_{\psi \psi}(x) \equiv \langle \psi(x) \psi(0) \rangle = |x|^{2 \chi_\psi}; \quad (16)
$$

$$
C_{hh}(x,t) \equiv \langle |h(x,t) - h(0,0)|^2 \rangle = |x|^{2 \chi_h} \Theta(|x|z); \quad (17)
$$

Scaling exponents $\chi_\psi, \chi_h$ are the roughness exponents of $\psi$ and $h$ respectively; $z$ is the dynamic exponent of $\delta \rho$. In the linear theory, these exponents are known $\chi_\psi$ exactly. For instance, for $\rho_0 \neq 1/2$, $\chi_\psi = -1/2$, since $\psi(x)$ is proportional to $\psi(x)$ that indeed scales as $1/|x|$. On the other hand when $\rho_0 \approx 1/2$, Eqs. (12) and (14) give $\chi_\psi = 1/2 = \chi_h$; $z$ continues to remain $2$ at the linear level. It remains to be seen how the various nonlinear terms may affect these scaling exponents.

### IV. SYMMETRIC EXCLUSION PROCESS WITH QUENCHED DISORDER

Before we attempt to solve (7) or (9), it is instructive to first look at the symmetric limit of the
problem: number conserving symmetric exclusion processes (SEP) with random quenched disordered hopping rates. In SEP, particle movement is bidirectional, subject to exclusion; see Fig. 2.

Again writing the local fluctuating density as the sum of $\psi(x)$ and $\delta \rho(x,t)$, the effective long wavelength equations for $\psi(x)$ and $\delta \rho(x,t)$ after discarding the irrelevant terms (in a scaling sense) read

$$D_x m_0 \partial_{xx} \psi(x) = 0, \quad (18)$$

$$\frac{\partial \delta \rho(x,t)}{\partial t} - m_0 \partial_{xx} \delta \rho = \partial_x f, \quad (19)$$

to the leading order in fluctuations; see also Appendix. We thus find that $\psi(x)$ and $\delta \rho(x,t)$ are mutually decoupled to the leading order in fluctuations at all $\rho_0$, and the dynamics of $\delta \rho$ is insensitive to $\rho_0$. With $\delta \rho = \partial_x m$, (19) reduces to the well-known Edward-Wilkinson model for growing surfaces driven by a white noise $f$:

$$\frac{\partial h}{\partial t} - \nu \partial_{xx} h = f. \quad (20)$$

Equation (18) implies $\psi = 0$, where as due to the linearity of (19) or (20), the corresponding scaling exponents are known exactly: $\chi_h = 1/2$, $z = 2$. Thus,

$$C_{hh}(r,t) \equiv \langle [h(x+r,t) - h(x,0)]^2 \rangle = r \Theta(r^2/t). \quad (21)$$

Since $\psi(x) = 0$, the steady state itself is uniform when viewed at sufficiently large scales. Further, with the scaling of $C_{hh}$ being indistinguishable from its pure counterpart, we conclude that any coarse-grained measurements of the density fluctuations cannot detect existence of any short range quenched disorder in a periodic SEP.

V. SCALING IN THE ASYMMETRIC CASE

Having discussed the simpler quenched disordered SEP with particle number conservation, we now go over to the more general (and more complex as we will see below) corresponding asymmetric case.

It is useful to first study the linearised version of the asymmetric case. Equation (17), when away from the half-filled limit, $\psi(k) = -(\lambda_m/\lambda_1\psi) \delta m(k)$ in the Fourier space in the long wavelength limit, meaning $\psi(x)$ is just as rough as $\delta m(x)$. In this linear limit, this implies $\psi(x,t)$ can be obtained by minimising an effective free energy $F_1: \delta F_1/\delta \psi = 0$, where

$$F_1 = \int dx \left[ \lambda_1 \psi(x)^2/2 + \lambda_m \psi(x) \delta m(x) \right]. \quad (22)$$

On the other hand, close to the half-filled limit, $\psi(k) = i\lambda_m/(ik\nu) \delta m(k)$, implying that $\psi(x)$ is distinctly rougher than $\delta m(x)$ sufficiently close to the half-filled limit. Again, in this linear limit, $\psi(x)$ can be obtained by minimising an effective free energy $F_2: \delta F_2/\delta \psi = 0$, where

$$F_2 = \int dx \left[ \nu \psi(x)^2/2 - \lambda_m \psi \partial_x \delta m \right]. \quad (23)$$

Nonlinear terms and the propagating modes in (17) and (23) may change the above simple picture in terms of $F_1$ and $F_2$, that we seek to find now. This cannot be done exactly due to the nonlinear terms. Furthermore, naive perturbative theory can produce diverging corrections to the model parameters in (17), as happens for ordinary Burgers equation [32, 33]. This calls for a systematic dynamic renormalisation group (DRG) analysis.

DRG methods applied on continuum driven hydrodynamic models have a long history of studies in statistical mechanics. These are well-established methods, particularly suitable to delineate the universality classes of the driven systems: see Refs. [24, 30, 33, 35] for applications of DRG methods in related models.

While the DRG method is well-documented [31] in the literature, we give a brief outline of the method for the convenience of the reader. The momentum shell DRG procedure consists of tracing over the short wavelength Fourier modes of $\psi(x)$ and $\delta \rho(x,t)$, followed by a rescaling of lengths and time. More precisely, we follow the standard approach of initially restricting wavevectors to lie in a 1D Brillouin zone: $|q| < \Lambda$, where $\Lambda$ is an ultraviolet cutoff, presumably of order the inverse of the lattice spacing $a$, although its precise value has no effect on our results. The density fields $\psi(x)$ and $\delta \rho(x,t)$ are separated into high and low wavevector parts $\psi(x) = \psi^>(x) + \psi^<(x)$ and $\delta \rho(x,t) = \delta \rho^>(x,t) + \delta \rho^<(x,t)$, where $\psi^>(x)$ and $\delta \rho^>(x,t)$ have support in the large wave vector (short wavelength) range $\Lambda e^{-\delta t} < |q| < \Lambda$, while $\psi^<(x)$ and $\delta \rho^<(x,t)$ have support in the small wave vector (long wavelength) range $|q| < e^{-\delta t} \Lambda$. We then integrate out $\psi^<(x)$ and $\delta \rho^>(x,t)$. This integration is done...
perturbatively in the anharmonic couplings in \([12]\) as usual, this perturbation theory can be represented by Feynmann graphs, with the order of perturbation theory reflected by the number of loops in the graphs we consider. After this perturbative step, we rescale lengths, with \(x = x'e^\delta\), so as to restore the UV cutoff back to \(\Lambda\). This is then followed by rescaling the long wavelength part of the fields; see Appendix.

It is important to note that there are two classes of Feynman diagrams: (i) the first kind survives in the limit of vanishing disorders and originate from the standard Burgers-nonlinear term \(\lambda_\delta \partial_x \delta \rho^2\) in \([8]\), or the \(\lambda_\delta dxdt\partial_x \delta \rho^2\)-term in the action functional \([12]\); (ii) the second type originates from the nonlinear terms that involve the disorder in \([7]\) and \([8]\), or in the action functional \([12]\); see Appendix.

We distinguish two cases: (i) away from half-filling \((\rho_0 \neq 1/2)\) and (ii) close to half-filling \(\rho_0 \approx 1/2\).

### A. Pure periodic asymmetric exclusion process

In the absence of any quenched disorder, \(\delta m(x) = 0\) and the hopping rate is uniform everywhere. This gives \(\psi(x) = 0\) for all \(x\) identically, and Eq. \([8]\) reduces to

\[
\frac{\partial \delta \rho}{\partial t} = \nu_\rho \partial_x \delta \rho - \frac{\lambda_\rho}{2} \partial_x h \partial_x \delta \rho^2 + \partial_x f,
\]

where the \(\lambda_\delta \partial_x \delta \rho\)-term in \([12]\) has been removed by using the Galilean invariance of \([23]\); \(\nu_\rho\) is the diffusivity. With \(\delta \rho = \partial_x h\), \(h\) satisfies the 1D KPZ equation driven by a white noise:

\[
\frac{\partial h}{\partial t} = \nu_\rho \partial_x x h - \frac{\lambda_\rho}{2} (\partial_x h)^2 + f.
\]

The scaling exponents of \([25]\) are exactly known with \(\nu_\rho = 1/2, z = 3/2 \quad [32, 33]\). This corresponds to \(\nu_\rho = -1/2\).

### B. Away from half-filling

When \(\rho_0 \neq 1/2\), the Feynman diagrams of the second type defined above are all finite. In contrast, the Feynman diagrams of the first type remain infra-red divergent as they are for the pure KPZ/Burgers problem in 1D. Evaluating the relevant one-loop Feynman diagrams and constructing the DRG flow equations give \(\nu_\rho = 1/2\) and \(z = 3/2\) at the DRG fixed point; the detailed calculations are well-document in the literature \([33, 34]\) that we do not reproduce here. Since the disorder-induced nonlinear couplings in \([3]\) are irrelevant in the long wavelength limit for \(\rho_0 \neq 1/2\), the remaining first order in space derivative linear term \([3]\) can be removed by an appropriate Galilean boost, thereby reducing the governing equation for \(\delta \rho\) to the 1D Burgers equation.

In the long wavelength limit, height fluctuations \(h(x, t)\) follows the KPZ equation \([25]\), for which the scaling of the time-dependent correlation function of \(\delta \rho(x, t)\) is known:

\[
\langle [h(x,t) - h(0,0)]^2 \rangle = |x|^{\Theta_h([x]^{3/2}/t)};
\]

coresponding to \(\chi_\rho = -1/2\) and \(z = 3/2 \quad [32, 33]\); \(\Theta_h\) is a scaling function. Therefore, the correlation function \(C_{\rho}(x,t)\) of the density \(\rho(x,t)\) scales as \(x^{-1}\Theta([x]^{3/2}/t)\).

Further, there are no diverging fluctuation corrections to \(\langle \psi(k)^2 \rangle\), and hence it is still given by \([10]\) even in the full anharmonic theory in the long wavelength limit. This gives \(\chi_\psi = -1/2 = \chi_\rho\). Furthermore, the physical picture in the linearised theory that \(\psi(x)\) can be obtained by minimising \(\mathcal{F}_1\) still holds in the long wavelength limit.

We can then conclude that the correlation function of the total density \(\rho(x,t)\)

\[
C_{\rho}(x,t) \equiv \langle [\rho(x,t) - \rho(0,0)]^2 \rangle
\]

also scales as \(x^{-1}\Theta([x]^{3/2}/t)\), same as what it would show in the absence of any quenched disorder. Thus disorder does not affect the universal scaling when the system is away from half-filling. In other words, experimental measurements of the scaling exponents in physical realisations of this model cannot detect if there is any disorder or not.

### C. Scaling near half-filling

The physics near half-filling \((\rho_0 \approx 1/2)\) turns out to be very different from what we discussed above. In fact, the nonlinear terms involving quenched disorder turn out to be relevant, as we argue below. Notice first that the linear first order in space term now vanish. Thus there are no underdamped propagating modes in the dynamics of \(\delta \rho(x,t)\). For \(\rho_0 \approx 1/2\), as explained above, \(\psi(x)\) is more relevant (in a scaling sense) than \(\delta \rho(x,t)\). It is also more relevant than \(\delta m(x)\) at linear level; see Eq. \([13]\) above. This consideration allows us to write the equations for \(\psi(x)\) and \(\delta \rho(x,t)\), retaining only the most leading order nonlinear terms. We find

\[
\nu_\psi \partial_{xx} \psi - \frac{\lambda_\psi}{2} \partial_x \psi^2 + \lambda m \partial_x \delta m = 0,
\]

\[
\partial_t \delta \rho(x,t) = \nu_\rho \partial_x \delta \rho - \lambda_\rho \partial_x (\psi \delta \rho) + \partial_x f.
\]

Notice that Eq. \([29]\) formally resembles the dynamical equation for a passive scalar, advected by a “frozen-in” Burgers-like irrotational velocity field \([30]\). If we define a “height field” \(h(x,t)\) via \(\delta \rho(x,t) = \partial_x h(x,t)\), then in the long wavelength limit \(h(x,t)\) satisfies

\[
\frac{\partial h}{\partial t} = \nu_\rho \partial_x h - \lambda_\rho \psi_0 \partial_x h + f.
\]

As shown in Appendix, the naïve perturbative fluctuation corrections to the model parameters due to nonlinear
terms diverge in the long wavelength limit, necessitating systematic DRG analysis. We confine ourselves to a low-order (one-loop) DRG analysis, following the calculational scheme outlined above.

We provide the one-loop Feynman diagrams for the model parameters in Appendix. We evaluate the integrals at the fixed dimension $d = 1$, as done for 1D Burgers equation [22].

The differential flow equations for the parameters are

\[ \frac{d \nu_{\psi}}{dl} = \nu_{\psi} \left[ -3 + \frac{g}{2} \right], \]  
\[ \frac{d \lambda_m}{dl} = \lambda_m \left[ \frac{3}{2} - \chi_{\psi} - 2 \bar{g} \right], \]  
\[ \frac{d \Delta}{dl} = \Delta \bar{g}, \]  
\[ \frac{d \lambda_{\psi}}{dl} = \lambda_{\psi} \left[ -3 + \chi_{\psi} - 2g \right], \]  
\[ \frac{d \lambda_{3\psi}}{dl} = \lambda_{3\psi} \left[ -\frac{5}{2} + \chi_{\psi} \right], \]  
\[ \frac{d \nu_{\rho}}{dl} = \nu_{\rho} \left[ z - 2 + g_1 \right], \]  
\[ \frac{d \bar{D}}{dl} = D \left[ -3 + z - 2 \chi_{\rho} + 2g_1 \right], \]  
\[ \frac{d \lambda_{\psi\omega}}{dl} = \lambda_{\psi\omega} \left[ -1 + z + \chi_{\psi} - 2g_1 \right], \]  
\[ \frac{d \lambda_{\rho}}{dl} = \lambda_{\rho} \left[ -z + z + \chi_{\omega} - 2g_1 \right]. \]

Here, the effective coupling constants: $g = \frac{\lambda_{\psi\omega} \lambda_{3\psi} \Delta}{v_{\psi}^2 \lambda_{\psi}}$, $g_1 = \frac{\lambda_{\psi}^2 \lambda_{\omega}^2 \Delta}{v_{\psi}^2 v_{\omega}^2 \lambda_{\omega}}$. The flow equations for the coupling constants are:

\[ \frac{dg}{dl} = g \left[ \frac{11g}{2} - 4\bar{g} + 3 \right], \]  
\[ \frac{d\bar{g}}{dl} = \bar{g} \left[ \frac{9}{2} - 2\bar{g} + 1 \right], \]  
\[ \frac{dg_1}{dl} = g_1 \left[ -6g_1 - \frac{g}{2} - 4\bar{g} + 3 \right]. \]

Coupling constants $g$ and $\bar{g}$ have their origins in [22], where as $g_1$ appears in the dynamics of $\delta \rho$. Unsurprisingly, [39] and [40] do not depend upon $g_1$. At the DRG fixed point, $dg/dl = 0 = d\bar{g}/dl = dg_1/dl$. Flow equations [39] and [40] admit several solutions at the fixed point ($g^*, \bar{g}^*$): (i) $(0, 0)$, (ii) $(6/11, 0)$, (iii) $(0, 1/2)$ and (iv) $(2/9, 4/9)$. Out of these, $(g^*, \bar{g}^*) = (2/9, 4/9)$ is the globally stable fixed point. The RG flow around the fixed points are shown in Fig. 3.

One can find $g^*_1 = 5/27$ at the stable fixed point by setting $dg_1/dl = 0$. We find $\chi_{\psi} = -4/9 \approx -0.444$, $\chi_{\rho} = -1/2 + 5/54 \approx -0.407$ and $z = 2 - 5/27 \approx 1.815$ by using the flow equations of parameters. Now the scaling exponent $\chi_h$ of $\delta \rho(x, t) = \partial_t h(x, t)$ is given by $\chi_h = \chi_{\rho} + 1 = 1/2 + 5/54$. Clearly, the scaling exponents belong to a new universality class that is distinct from the 1D KPZ universality class. Furthermore, these exponents also imply that both $\bar{\psi}(x)^2$ and $(\delta \rho(x, t))^2$ are finite in the thermodynamic limit, as they should be for a number conserving system. This also implies that linear theory-based picture that $\bar{\psi}(x)$ can be obtained by minimizing $F_2$ may still hold in the long wavelength limit if we construct $F_2$ in terms of the renormalised parameters. The trends of results agree with those observed earlier in related numerical studies [23]. It has been found that the relaxation times in presence of disorder is larger than that in the pure system, resulting in a dynamic exponent $z > 3/2$, the dynamics exponent in the pure system, a feature confirmed by our results. Since $\chi_{\rho} > \chi_{\psi}$, $\chi_{\rho}$ and $z$ may be obtained by measurements of the total density correlator

\[ C_{\rho}(r, t) = |r|^{2\chi_{\rho}} \rho^{(x^2 / t)} = |r|^{-0.814} g_{\rho}^{(x^{1.815} / t)}, \]

where $g_{\rho}$ is a scaling function. Nonetheless, due to the numerically close values of $\chi_{\psi}$ and $\chi_{\rho}$, one must go to sufficiently large system size in order to see the scaling in [42]. For appropriately chosen model parameters, it may be possible to access intermediate scales of sufficiently large window where $\chi_{\psi}$ dominates the scaling of $C_{\rho}(r, t)$ and reveals the form of the steady state density $\bar{\psi}(x)$ in that length scales.

VI. SUMMARY

We have thus developed the hydrodynamic theory for the universal scaling in number conserving asymmetric exclusion processes in the presence of short range random quenched disorder. By using a continuum hydrodynamic description and within a one-loop renormalisation group framework, we show that when the system is away from half-filled, the quenched disorder is irrelevant and the local density fluctuations essentially obey the
well-known 1D Burgers equation in the long wavelength limit. Thus experimental measurements of the scaling of the density fluctuations cannot detect any quenched disorder. In contrast, when the system is close to being half-filled, the density fluctuations are strongly affected by the presence of the quenched disorder. Consequently, the resulting density fluctuations display scaling that belongs to a new universality class. We further note that the spatial scaling exponent $\chi_\rho$ is larger when the system is near the half-filled limit than when it is away. Equivalently, in terms of a height field description, the height field is rougher near the half-filled limit than when it is away. Given that the quenched disorder is relevant close to the half-filled limit and irrelevant otherwise, the height field near the half-filled limit has an additional source of roughening in the form of the quenched disorder. Since the quenched disorder is strongly nonuniform, the steady state density too is strongly nonuniform. This nonuniformity in the steady states contributes to the local density fluctuations, enhancing it *vis-a-vis* the pure limit of the problem. Our general conclusion that only near half-filling the quenched disorder is relevant, is in agreement with the conjecture made in Ref. [22]. While this particular result has been obtained in a one-loop approximation, we believe this holds true even at the higher loop orders. This is due to the reason that the presence of propagating modes away from half-filling continues to reduce the infra-red divergence of fluctuation corrections at any loop order. Our results are universal in the sense that these are independent of the disorder strength $\tilde{D}$, annealed noise strength $D$ and the damping coefficients, and hence of the lattice space, strength of the disorder in the hopping rates and inherent stochasticity of the dynamics. This is in the same spirit as the universality of the KPZ equation driven by short range noises [32]. These results can be tested in numerical simulations of appropriate disordered lattice-gas models. Our results should be useful in any asymmetric exclusion processes with number conservation in more complex geometries, e.g., closed networks [37].

These results complement to the recent studies on number conserving TASEP with quenched but spatially smoothly varying (non-random) hopping rates [16], where the focus was on the possible types of steady states, and the existence of a type of universality has been brought out. In contrast, we here concentrate on the universal scaling in the disorder-averaged steady states, a rather different aspect of the problem. Nonetheless, there are some interesting parallels between the results from these two studies. For instance, Ref. [16] has shown that regardless of the specific functional form of the space-dependent hopping rate function, there are only two kinds of steady states: for $\rho_0$ very low (close to zero), or very high (close to unity), the steady states are essentially “smooth” (unless there are discontinuities in the hopping rate function itself). On the other hand, for intermediate filling ($\rho_0$ close to 1/2), the steady state develop shocks, clearly differentiating the intermediate filling system from the low or high filling limits. Similarly, in the present study too, the scaling of the density fluctuations are very sensitive to whether the system is away or close to half-filling. It would be interesting to further explore any deeper connections between the two models.

In this work, we have confined ourselves to study the effects of short ranged quenched disorder. Long range quenched disorder is expected to be relevant and further modify the scaling exponents obtained here. In fact, for sufficiently long range random quenched disorder, the scaling exponents are likely to be affected *even when the system is away from half-filling*. However, even then the present analysis suggest that the scaling exponents near the half-filling will be different from their counterparts away from half-filling. A full quantitative analysis of long range random quench disorder will be presented elsewhere.

**Appendix A: Derivation of the hydrodynamic equations**

1. **Asymmetric case**

Consider a closed 1D lattice with $L$ sites with $a$ as the lattice spacing. Hence, the total length of the lattice is $La$. Let $\bar{N} = L/a$. We are interested in the thermodynamic limit, $\bar{N} \to \infty$, which can be realised, e.g., for $L \to \infty$ for a fixed $a$, or vice versa.

We closely follow Ref. [24] in deriving the hydrodynamic limit. We start by restating the dynamical equations for the occupation number $n_i$ at site $i$ of the lattice:

$$\frac{\partial n_i(t)}{\partial t} = m_{i-1}n_{i-1}(1-n_i) - m_in_i(1-n_{i+1}), \quad (A1)$$

where $m_i$ is the hopping rate from site $i$ to $i+1$. Total particle number $N = \sum_i n_i$ is clearly a constant of motion. We define $x = i/L$, which becomes quasi-continuous in the thermodynamic limit. Further, suitable coarse-graining allows us to obtain a continuous density $\rho(x) = \langle n_i \rangle_c$ and $\rho(x-1/L) = \langle n_{i-1} \rangle_c$, where as $\langle ... \rangle_c$ implies a suitable coarse-graining. Neglecting correlations between neighbouring sites, we obtain

$$\frac{\partial \rho(x)}{\partial t} = m(x-1/L)\rho(x-1/L)[1-\rho(x)] - m(x)\rho(x)[1-\rho(x+1/L)]. \quad (A2)$$

In the hydrodynamic limit, we expand $\rho(x-1/L)$, $\rho(x+1/L)$ and $m(x-1/L)$ up to second order in $1/L$. This together with a ballistic rescaling of time gives [24] in the main text (where a conserved noise has been added). We further identify $\nu = m_0/L$ as the effective diffusion coefficient.
2. Symmetric case

Let us now consider the symmetric case, where particles can move bidirectionally subject to exclusions. The equation motion for occupation \( n_i \) at site \( i \) is given by
\[
\frac{\partial n_i}{\partial t} = m_{i-1} n_{i-1} (1 - n_i) - m_{i-1} n_i (1 - n_{i-1}) - m_i n_i (1 - n_{i+1}) + m_i n_i (1 - n_i). \tag{A3}
\]

As before, coarse-graining and expanding up to \( O(1/L^2) \), we obtain
\[
\frac{\partial \rho}{\partial t} = \frac{1}{L^2} \frac{\partial}{\partial x} \left( m(x) \frac{\partial \rho}{\partial x} \right). \tag{A4}
\]

This gives, upon discarding irrelevant nonlinearities and rescaling \( t \) by \( 1/L^2 \) (diffusive time-scale),
\[
\frac{\partial \rho}{\partial t} = m_0 \frac{\partial^2 \rho}{\partial x^2} + \partial_x f, \tag{A5}
\]

where a conserved noise \( \partial_x f \) has been added; this is identical to the coupled equations [18] and [19] in the main text.

Appendix B: Generating functional

The generating functional [38] corresponding to Eqs. [1] and [5] is
\[
Z = \int D\psi D\hat{\psi} D\rho D\hat{\rho} D\delta m \exp(-S). \tag{B1}
\]

Here, \( \hat{\psi}(x) \) and \( \hat{\rho}(x, t) \) are the dynamic conjugate fields to \( \psi(x) \) and \( \delta\rho(x, t) \), respectively [38]. Further, \( S \) is the action functional given by
\[
S = \int dx \left[ -\nu_1 \partial_x \psi \lambda_1 \psi - \lambda_m \partial_x \delta m + \frac{\lambda_3}{2} \partial_x \psi^2 \right. \\
- \frac{\nu_1}{m_0} \partial_x (\delta m \psi) + \lambda_3 \partial_x (\delta m \psi^2) \left. \right] + \int dx \frac{\delta m^2}{4D} \\
+ \int dx dt D(\partial_x \hat{\rho}) \hat{\rho} + \hat{\rho} \partial_t \delta \rho - \lambda_1 \partial_x \delta \rho - \nu_\rho \partial_x \delta \rho \\
+ \frac{\lambda_4}{2} \partial_x \delta \rho^2 - \lambda_2 \partial_x (\delta m \rho) + \lambda_\rho \partial_x (\delta \rho \psi) \\
+ \lambda_3 \partial_x (\delta m \rho^2) + \lambda_3 \partial_x (\delta m \psi \delta \rho). \tag{B2}
\]

The two-point autocorrelation functions of the fields in the harmonic theory, i.e., neglecting all the nonlinear terms are
\[
\langle |\psi_k| \rangle^2 = \frac{2D\lambda_0^2 \delta(\omega)}{\lambda_0^2 + \nu^2 k^2} \approx \frac{2D\lambda_0^2 \delta(\omega)}{\lambda_0^2 + \nu^2 k^2}, \tag{B3}
\]
\[
\langle |\delta \rho_k| \rangle^2 = \frac{2Dk^2}{(\omega - \lambda_1 k)^2 + \nu^2 k^4}, \tag{B4}
\]
\[
\langle |h_k| \rangle^2 = \frac{2Dk^2}{(\omega - \lambda_1 k)^2 + \nu^2 k^4}. \tag{B5}
\]

Appendix C: Perturbation theory for \( \rho_0 \neq 1/2 \)

We set up the perturbation theory by expanding in the coefficients of the nonlinear terms. The nonlinear vertices are represented diagrammatically as given in Fig. 4.

\[\text{FIG. 4: Diagrammatic representations of the nonlinear vertices in action [32].}\]

\[\text{FIG. 5: Diagrammatic representations of the lines representing the propagators and correlators of } \rho, \psi \text{ and } \delta m.\]

The bare or unrenormalised propagators and the correlators are diagrammatically represented as given in Fig. 5.

\[\text{FIG. 5: \textbf{(a)} } \hat{\psi} \rightarrow \psi, \text{ \textbf{(b)} } \hat{\rho} \rightarrow \rho, \text{ \textbf{(c)} } \delta m \rightarrow \delta m.\]
FIG. 6: One-loop corrections to \( \nu_\psi \), \( \tilde{D} \) and \( \lambda_m \), originated from nonlinear vertices of action (B2).

One loop corrections for parameters in (7):

\[
\begin{align*}
\text{Fig. (a)} & \sim \frac{\lambda_1^2 \lambda_m^2 \tilde{D}}{i \lambda_\psi^3} \int \frac{dq}{2\pi}, \\
\text{Fig. (b)} & \sim \frac{\lambda_1^2 \tilde{D}}{i \lambda_\psi^3} \int \frac{dq}{2\pi}, \\
\text{Fig. (c)} & \sim \left[ \frac{2 \tilde{D} \lambda_\psi^2 \lambda_m^2}{\lambda_1^2} \right]^2 \int \frac{dq}{2\pi}, \\
\text{Fig. (d)} & \sim \frac{\tilde{D} \lambda_m^2 \lambda_\psi^2}{\lambda_1^2} \int \frac{dq}{2\pi}. \\
\end{align*}
\]

The one-loop corrections for the propagator and autocorrelator for \( \rho \) that in turn contribute to the fluctuation corrections of \( \nu_\rho \) and \( D \) are shown below:

FIG. 7: One-loop corrections to \( \nu_\rho \) and \( D \), originated from nonlinear vertices of action (B2).

One loop corrections for parameters in (8) are

\[
\begin{align*}
\text{Fig. (a)} & \sim \frac{\lambda_1^2 \rho_0^2 \tilde{D} \lambda_m^2 \omega^2}{2 \lambda_1^2 \lambda_\rho^2} \int \frac{dq}{2\pi}, \\
\text{Fig. (b)} & \sim \frac{\lambda_1^2 \tilde{D} \lambda_m^2 \omega^2}{2 \lambda_1^2 \lambda_\rho^2} \int \frac{dq}{2\pi}, \\
\text{Fig. (c)} & \sim \frac{i \lambda_1^2 \tilde{D} \lambda_m^2 \omega^2}{2 \lambda_1^2 \lambda_\rho^2} \int \frac{dq}{2\pi}, \\
\text{Fig. (d)} & \sim \frac{i \lambda_1^2 \tilde{D} \lambda_m^2 \omega^2}{2 \lambda_1^2 \lambda_\rho^2} \int \frac{dq}{2\pi}. \\
\end{align*}
\]

Appendix D: Perturbation theory for \( \rho_0 \approx 1/2 \): DRG analysis

The action functional now reads

\[
S = \int dx \frac{\partial}{\partial \psi} \left[ -\nu_\psi \partial_x \psi - \lambda_m \partial_x \delta m + \frac{\lambda_\psi}{2} \partial_x \psi^2 \right. \\
+ \left. \lambda_3 \partial_x \omega \delta m \right] + \frac{\delta m^2}{4D} + \int dx dt \left( \partial_x \delta \rho \right) \hat{\rho} + \hat{\rho} \left[ \partial_t \delta \rho - \nu_\rho \partial_x \delta \rho + \lambda_\rho \partial_x \delta \rho \right].
\]

The autocorrelation functions of the fields in harmonic theory for the action (D1) are given by

\[
\begin{align*}
\langle |\psi_k, \omega|^2 \rangle & = \frac{2 \tilde{D} \lambda_k^2 / \nu_\rho^2 k^2}{\omega^2}, \\
\langle |\delta \rho_k, \omega|^2 \rangle & = \frac{2 D k^2}{\omega^2 + \nu_\rho^2 k^4}, \\
\langle |h_k, \omega|^2 \rangle & = \frac{2 D}{\omega^2 + \nu_\rho^2 k^4}.
\end{align*}
\]

We highlight a technical point here. With (13), at half-filling

\[
\langle |\psi(x)|^2 \rangle = \frac{2 \tilde{D} \lambda_m^2}{\nu_\rho^2} \int^{1/2} \frac{dk}{2\pi} \frac{1}{k^2}
\]

grows linearly with \( L \) without any bound. On the other hand, \( \langle \delta \rho(x, t) \rangle \) remains finite in the thermodynamic limit for any filling-factor and \( \langle |\psi(x)|^2 \rangle \) remains finite in the thermodynamic limit when the system is away from half-filling. That \( \langle |\psi(x)|^2 \rangle \) diverges in the thermodynamic limit is not consistent with the particle number conservation in the system. We shall see below that nonlinear effects make \( \langle |\psi(x)|^2 \rangle \) finite in the thermodynamic limit.

For \( \rho_0 \approx 1/2 \), some of the nonlinear coefficients in the action functional (B2) vanish, giving rise to the action (D1), hence some of the one-loop diagrams that exist for \( \rho_0 \neq 1/2 \) now vanish.

One loop corrections for parameters in (28), (29) and hence in near the half-filled limit action (D1) are
Fig. (8) = \frac{\tilde{D} \lambda_m^2 \lambda^2_{\psi}}{2 \nu^2_{\psi}} \int \frac{dq}{2\pi} \frac{1}{q^4} \tag{D6}

Fig. (9) = \frac{\tilde{D}^2 \lambda_m^2 \lambda^2_{\psi}}{\nu^2_{\psi}} \int \frac{dq}{2\pi} \frac{1}{q^4} \tag{D7}

Fig. (10) = 2 \lambda^2_{\psi} \Delta \lambda^2_{\psi} \int \frac{dq}{2\pi} \frac{1}{q^4} \tag{D8}

Fig. (11) = 2 \lambda^2_{\psi} \Delta \lambda^2_{\psi} \int \frac{dq}{2\pi} \frac{1}{q^4} \tag{D9}

\begin{align*}
\lambda^\psi_m &= \lambda_m \left[ 1 - \frac{2 \lambda^\psi_m \lambda^\psi_\lambda \nu^4}{\nu^4} \int \frac{dq}{2\pi} \frac{1}{q^4} \right], \\
\hat{D}^\psi &= \hat{D} \left[ 1 + \frac{\hat{D} \lambda^\psi_m \lambda^\psi_\lambda \nu^4}{\nu^4} \int \frac{dq}{2\pi} \frac{1}{q^4} \right], \\
\nu^{\psi}_m &= \nu^\psi \left[ 1 + \frac{\nu^\psi \nu^\rho}{\nu^\psi} \int \frac{dq}{2\pi} \frac{1}{q^4} \right], \\
D^\psi &= D \left[ 1 + \frac{2 \nu^\psi \nu^\rho}{\nu^\psi} \int \frac{dq}{2\pi} \frac{1}{q^4} \right], \\
\lambda^{\psi\rho}_m &= \lambda^{\psi\rho} \left[ 1 - \frac{2 \nu^\psi \nu^\rho}{\nu^\psi} \int \frac{dq}{2\pi} \frac{1}{q^4} \right].
\end{align*}

(D10)

FIG. 9: One-loop vertex correction diagram for \( \lambda_{2\psi} \) when \( \rho_0 \approx 1/2 \).

One-loop contributions for parameters of nonlinearity in \( D_{11} \) are

\begin{align*}
\text{Fig. (5b)} &= \frac{2 \tilde{D} \lambda^2_{m\psi}}{\nu^2_{\psi}} \int \frac{dq}{2\pi} \frac{1}{q^4}, \\
\text{Fig. (5d)} &= \frac{2 \tilde{D} \lambda^2_{m\psi}}{\nu^2_{\psi}} \int \frac{dq}{2\pi} \frac{1}{q^4},
\end{align*}

Once the fields having support in the wavevector range \( \Lambda/b \) to 0 are integrated out, we obtain “new” model parameters corresponding to a modified action \( S^\psi \) having \( \Lambda/b \) as the wavevector upper cutoff. We obtain

\begin{align*}
\nu^{\psi}_m &= \nu^\psi \left[ 1 + \frac{\hat{D} \lambda^\psi_m \lambda^\psi_\lambda \nu^4}{\nu^4} \int \frac{dq}{2\pi} \frac{1}{q^4} \right], \\
\lambda^{\psi}_m &= \lambda_m \left[ 1 - \frac{2 \lambda^\psi_m \lambda^\psi_\lambda \nu^4}{\nu^4} \int \frac{dq}{2\pi} \frac{1}{q^4} \right], \\
\hat{D}^\psi &= \hat{D} \left[ 1 + \frac{\hat{D} \lambda^\psi_m \lambda^\psi_\lambda \nu^4}{\nu^4} \int \frac{dq}{2\pi} \frac{1}{q^4} \right], \\
\nu^{\psi}_m &= \nu^\psi \left[ 1 + \frac{\nu^\psi \nu^\rho}{\nu^\psi} \int \frac{dq}{2\pi} \frac{1}{q^4} \right], \\
D^\psi &= D \left[ 1 + \frac{2 \nu^\psi \nu^\rho}{\nu^\psi} \int \frac{dq}{2\pi} \frac{1}{q^4} \right], \\
\lambda^{\psi\rho}_m &= \lambda^{\psi\rho} \left[ 1 - \frac{2 \nu^\psi \nu^\rho}{\nu^\psi} \int \frac{dq}{2\pi} \frac{1}{q^4} \right].
\end{align*}

(D10)

Scaling of momentum and frequency: \( q \rightarrow bq, \omega \rightarrow b^2 \omega \). Together with the rescaling of space(or momentum) and time(or frequency), long wavelength parts of the fields are rescaled:

\begin{align*}
\hat{\psi}(q) &= b^{-1} \hat{\psi}(bq), \\
\delta m(q) &= b^{1/2} \delta m(bq), \\
\hat{\rho}(q, \omega) &= b^{-1} \hat{\rho}(bq, b^2 \omega), \\
\delta \rho(q, \omega) &= b^{1+\chi^\psi} \delta \rho(bq, b^2 \omega).
\end{align*}

(D15)

Appendix E: Generation of higher order nonlinear terms

For \( \rho_0 \approx 1/2, \chi^\psi > 0 \) in the linear theory. This implies naive perturbation expansion should generate higher order nonlinear terms in \( \psi \) that will remain relevant since \( \chi^\psi > 0 \). In our above DRG analysis, we have neglected such perturbatively generated nonlinearities. In the renormalised theory, \( \chi^\psi < 0 \) rendering all such perturbatively generated higher order nonlinear terms irrelevant near the DRG fixed point. Technically speaking, if \( g_h \) is an effective coupling constant that has its origin from one such higher order nonlinear term, then in the renormalised theory near the DRG fixed point, writing schematically,

\[ \frac{dg_h}{dl} = \Delta g_h + O(g_h^2), \tag{E1} \]

where \( \Delta g_h < 0 \) is the scaling dimension of \( g_h \) near the stable DRG fixed point obtained above, evaluated using renormalised \( \chi^\psi < 0 \). Hence, for sufficiently small (bare) \( g_h \), it flows to zero in the long wavelength limit. For larger values of bare \( g_h \), it is possible that \( g_h \) grows in
the long wavelength limit near the DRG fixed point, depending upon the signs of the possible higher order terms in $E_1$. Since in the present theory, such perturbatively generated higher nonlinear terms should essentially scale with $\tilde{D}$, large bare values for such nonlinear terms should imply a large $\tilde{D}$, for which positivity of $m(x)$ is not guaranteed and we do not expect our theory to be valid in that regime.

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