Screening masses in the equilibrium thermodynamics of gauge theories were measured. The spectroscopy of these screening masses gives strong evidence for dimensional reduction at $T \geq 2T_c$. A perturbative explanation of these masses is ruled out. The mass ratios in the high temperature phase are consistent with those in the pure gauge theory in three dimensions.

I. INTRODUCTION

Upto the present the only argument for dimensional reduction of finite temperature field theories has been in terms of a mode expansion of fields. Then it is argued that this mode expansion may be truncated to the lowest Matsubara mode when considering long distance physics \[1\]. The consistency of this perturbative argument has been checked by many recent works \[2\].

Here I present a fully non-perturbative argument for dimensional reduction \[3\], involving a measurement of the spectrum of screening masses in a equilibrium 3+1 dimensional gauge theory. Degeneracies of the spectrum yield information on the symmetry group of the spatial transfer matrix. This turns out to be the rotation group in one lower dimension.

I also present one out of several arguments that this spectrum has no perturbative explanation. The rest may be found in \[3\].

Thermodynamics of field theories can be formulated in terms of a spatial transfer matrix, $T$, which propagates field information from one point to another. On the lattice this is a regulated object whose continuum limit has to be taken in the usual way. The eigenvalues of $T$ can be ordered ($\Lambda_0 \leq \Lambda_1 \leq \cdots$). The free energy, and hence all thermodynamics is entirely given by $\Lambda_r (T)$, where $T$ is the temperature.

The screening masses, $\mu_r$, are defined by the long distance exponential fall of static current-current correlation functions

$$C_r(x) \equiv \langle J_r(0) J_r(x) \rangle \sim \exp(-\mu_r x), \quad (1)$$

where $r$ is a complete set of quantum numbers and $x$ a space-like separation. In terms of the eigenvalues of $T$, $\mu_r = \log(\Lambda_0/\Lambda_r)$, where the relevant eigenvalue depends on the quantum number $r$. Thus, the screening masses contain more information on the transfer matrix than bulk thermodynamics. In particular, the degeneracies of $\mu_r$ tell us of the degeneracies of $\Lambda_r$, and hence the effective symmetries of the transfer matrix.

It helps intuition to note that the temporal transfer matrix of a zero-temperature field theory is the exponential of the time integral of the Hamiltonian of the theory. Hence its eigenstates are the physical states of the theory and the eigenvalues are their masses.

II. GROUP THEORY

In $d + 1$ dimensions, a transfer matrix propagates information from one $d$-dimensional slice of space-time to another. The transfer matrix is invariant under any symmetry operation of this slice.

At zero temperature, an Euclidean field theory has $O(4)$ rotational symmetry. Hence the three dimensional slices relevant for temporal and spatial transfer matrices are isomorphic. Each such slice has $O(3)$ rotational symmetry. On the lattice this breaks to the symmetry of a cube, $O_h$. This group includes the parity $P$ with action $(x, y, z) \rightarrow (x, -y, -z)$.

At finite temperature, $T$, the Euclidean time direction has extent $1/T$, and hence is special. One consequence of this is well-known to practitioners of perturbative finite-temperature field theory. It is that 4-tensors have to be decomposed into spatial and temporal (and various types of mixed) parts which transform differently. For the classification of eigenstates of the spatial transfer matrix the consequences are simple. The $O(3)$ symmetry of the $T = 0$ theory breaks down to the cylindrical symmetry, $C$, consisting of all operations in $O(3)$ which do not mix the time coordinate with $x$ and $y$. On the lattice, $O_h$ breaks to the dihedral group $D_h^4$. The groups $C$ and $D_h^4$ contain a parity $P$ with action $(t, x, y) \rightarrow (-t, -x, -y)$, and the reflection $\sigma_2$ with action $(t, x, y) \rightarrow (-t, x, y)$. The operators $P$ and $\sigma_2$ commute.

If dimensional reduction happens to be a good approximation to the long distance physics, then the lowest eigenstates of the transfer matrix can be classified into approximate multiplets of the two dimensional rotation group, $O(2)$. On the lattice this would be the group $C_{\Lambda}^4$.

Both these contain the two dimensional parity $\Pi$ with action $(x, y) \rightarrow (x, -y)$. The three $Z_2$ generators are related by $\Pi = P \sigma_z$.

The real irreps of $O(2)$ can be obtained from the $O(3)$ angular momentum irreps. There are two one dimensional irreps $0^+ \mu$ and $0^- \mu$ (the $J_z = 0$ part of even and odd $J$ irreps) and a two dimensional irreps for each distinct $\pm J_z$ projections of any $O(3)$ irrep $J$. 

Dimensional Reduction in High Temperature Gauge Theories

Saumen Datta and Sourendu Gupta

Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India

(March 27, 2022)
The various groups are related by
\[ O(3) = SO(3) \times Z_2(P) \supset O_h = O \times Z_2(P) \]
\[ C = O(2) \times Z_2(\sigma_z) \supset D_h^4 = D^4 \times Z_2(P) \]  
(2)
\[ O(2) \supset C_v^4 \]

The irreps of the continuum groups can now be easily written. The representation theory for \( O_h \) in terms of loop operators was first presented in [4]. The identification of \( C \) and \( D_h^4 \) as the appropriate symmetries at \( T \neq 0 \) was made in [5] where the representation theory of \( D_h^4 \) in terms of gluon operators was given. Similar work was also done in [6].

Full details of the reduction formulæ can be found in [7]. Here we only quote the results necessary to make sense of the numerical data to be presented later. If dimensional reduction occurs, then some representations of \( D_h^4 \) collapse pairwise into representations of \( C_v^4 \). We write the results for the one-dimensional irreps of the two groups—
\[ A^{P,\bar{P}}_{i,\bar{j},C} \rightarrow A^{\bar{P},P}_{i,\bar{j},C}, \]
\[ B^{P,\bar{P}}_{i,\bar{j},C} \rightarrow B^{\bar{P},P}_{i,\bar{j},C}. \]  
(3)
Here \( C \) denotes the charge conjugation quantum number. If lattice artifacts are small, then the splitting between the \( B^{++} \) and \( B^{-+} \) irreps of \( C_v^4 \) vanishes.

III. DIMENSIONAL REDUCTION

In this section we present results from simulations of the \( SU(3) \) gauge theory. The simulations were performed using a heat-bath algorithm [8]. Particulars of the runs are summarised in Table I. Correlation functions between loop operators were measured in the long direction of the lattice, after averaging over the transverse slice. Along this long direction we measured correlation functions at separations of more than \( 1/T \). Previous measurements at these temperatures show that the gauge coupling \( g > 1 \) [9], so that \( 1/g^2T < 1/gT < 1/T \).

### Table I

| \( T \) | \( \beta \) | Size | Statistics |
|---|---|---|---|
| \( T_c \) | 5.7 | \( 4.8^{+1.6} \) | \( 1000 + 5000 \times 50 \) |
| \( T_c/4 \) | 5.9 | \( 4.8^{+2.16} \) | \( 400 + 5000 \times 10 \) |
| \( 2T_c \) | 6.0 | \( 4.8^{+2.12} \) | \( 400 + 5000 \times 10 \) |
|          |     | \( 4.8^{+2.16} \) | \( 400 + 10000 \times 10 \) |
|          |     | \( 4.12^{+2.16} \) | \( 400 + 10000 \times 10 \) |

### Table II

| \( O(2) \) | \( C_v^4 \) | \( D_h^4 \) | \( T_c \) | \( \mu/T \) |
|---|---|---|---|---|
| \( O^+ \) | \( A^{++} \) | \( A^{++}_1 \) | \( 3.4 \pm 0.4 \) | \( 2.56 \pm 0.04 \) | \( 2.60 \pm 0.04 \) |
| \( O^+ \) | \( A^{++} \) | \( A^{++}_2 \) | \( 3.2 \pm 0.1 \) | \( 2.8 \pm 0.2 \) | \( 2.8 \pm 0.2 \) |
| \( O^+ \) | \( A^{-+} \) | \( A^{-+}_1 \) | \( 6.3 \pm 0.1 \) | \( 6.3 \pm 0.2 \) | \( 6.3 \pm 0.2 \) |
| \( 2^+ \) | \( B^{++} \) | \( B^{++}_1 \) | \( 4.9 \pm 0.3 \) | \( 4.9 \pm 0.3 \) | \( 4.8 \pm 0.1 \) |
| \( 2^+ \) | \( B^{--} \) | \( B^{++}_2 \) | \( 5.1 \pm 0.3 \) | \( 5.5 \pm 0.3 \) | \( 5.6 \pm 0.4 \) |

We measured 95 operators for \( A^{++}_1 \), 80 for \( B^{++}_1 \), 55 for \( A^{-+}_1 \) and 40 each for \( A^{++}_2 \), \( B^{++}_2 \) and \( B^{++}_2 \). For noise reduction we used a variant of the usual techniques [9]. The lowest screening mass in each channel was found by an algorithm [10] which implements the following idea—

a linear combination of operators is taken in the representation \( r \),
\[ \mathcal{O}' = \sum O_i \alpha_i \mathcal{O}_i, \]  
(4)

(the index \( i \) runs over all operators in the measured set) and the coefficients \( \alpha_i \) are varied to minimise the measured screening mass.

The results are given in Table I. The measured values of \( \mu/T \) in the \( A^{++}_1 \) and \( A^{++}_2 \) irreps of \( D_h^4 \) should become degenerate when dimensional reduction works. This happens at \( T = 2T_c \) but not for \( T \leq 3T_c/2 \). Also at \( 2T_c \), the screening masses in the irreps \( B^{++}_1 \) and \( B^{++}_2 \) of \( D_h^4 \) become degenerate, giving additional evidence for dimensional reduction on the lattice.

Lattice artifacts are under reasonable control and the continuum physics is not very far away. Evidence for this comes from the splitting between the screening masses of the \( B^{++} \) and \( B^{-+} \) irreps of \( C_v^4 \), which correspond to the same irrep of the continuum group. This splitting is less than 14% of the average of these two screening masses.

The screening mass ratios compare well with the glueball mass ratios in \( SU(3) \) pure gauge theory in three dimensions. We find
\[ \frac{\mu(A^{++})}{\mu(B^{++})} = 0.54 \pm 0.02, \]
\[ \frac{\mu(B^{++})}{\mu(A^{-+})} = 0.76 \pm 0.02. \]  
(5)

In the 3-d \( SU(3) \) pure gauge theory these ratios are 0.60 and 0.78 respectively [11].

We have also made measurements in the \( SU(2) \) gauge theory [3]. We find dimensional reduction at \( 2T_c \) in that the screening masses are organised by \( C_v^4 \). In \( SU(2) \) the lattice artifacts are larger, and a comparison with pure gauge theory is still not possible. Finite-size effects are currently under investigation.

TABLE I. Details of runs on \( N_t = 4 \) lattices for \( SU(3) \) pure gauge theory. The correlation functions are measured in the long direction which is always kept greater than \( 4/T \). The statistics is quoted as discarded \( + \) configurations \( \times \) seperation.

TABLE II. Screening masses in various irreps of \( D_h^4 \). The pattern of degeneracies follows closely the organisation into irreps of \( C_v^4 \). The organisation into irreps of the continuum dimensionally reduced group \( O(2) \) is still not very good.
IV. MAGNETIC MASSES

One interesting question is whether the data on screening masses can be interpreted in terms of two basic masses— the Debye screening mass, $M_D$, and the magnetic mass, $M_m$. Complete details are given in [3].

In order to do this we construct a gluon field operator associated with each link of the lattice. It must be an element of the Lie algebra of the gauge group, and when exponentiated, must give the group element associated with the link. The representations turn out to be gauge independent, although the field values are not. With this construction in hand, we can go on to construct multigluon operators. By matching the representation content of these with the measured correlation functions we will be in a position to test perturbation theory.

The gluons have $C = -1$. Hence all $C = 1$ irreps are obtained from exchange of an even number of gluons and all $C = -1$ from an odd number. Gauge-invariant zero-momentum two gluon operators

$$ O_{\mu\nu} = \sum_k Tr[G_{\mu}(k)G_{\nu}(-k)] $$

have $P = 1$ after imposing symmetry under exchange of the two gluons. $P = -1$ and $C = 1$ irreps can only be obtained by exchange of at least four gluons.

If we associate with each gluon some mass, and assume a perturbative dispersion relation for the gluons, then the observed systematic degeneracy of $P = 1$ and $P = -1$ states obviously implies that one of the following conditions is false—

1. The dispersion relations are perturbative.
2. The gluons can be assigned a non-vanishing mass.

Since the degeneracy of opposite parity states is required for dimensional reduction, it is not consistent with a perturbative spectrum of screening masses.

V. SUMMARY

We have studied gauge invariant screening masses in pure gauge theories at high temperatures. We have found evidence of dimensional reduction in the spectrum at $T = 2T_c$, but not at lower temperature. In the $SU(3)$ theory, the screening mass spectrum agrees with the glueball mass spectrum of 3-d pure gauge $SU(3)$ theory. Group theoretical considerations show that dimensional reduction is not consistent with a perturbative spectrum of screening masses.