Application of Tikhonov Regularization for 1D Geothermal Heat Flux Ill-Posed Inverse Problem: A Case Study on Chad Sedimentary Basin

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Abstract. Subsurface heat flux information is important in geothermal exploration. With the information, geophysicists can map exactly the thermal potential in a particular area. Based on the surface heat flux, inverse modeling produces the 1D subsurface heat flux distribution. However, inverse problems in the geothermal system are generally ill-posed. Small changes in the data can cause large changes in the solution and the solution may not be unique. To solve the mentioned non-linear and ill-posed equation above, Tikhonov regularization is a choice for stabilizing the inverse calculation. This paper demonstrates how Tikhonov regularization is useful to solve subsurface heat flux distribution both in the synthetic model and real model. Based on surface heat flux distribution from the direct problem, the preconditioned conjugate gradient algorithm calculates the subsurface heat flux. With the correct choice of the regularization parameter, the inverse model fits the initial model. For the testing purposes in real-world conditions, Chad sedimentary basin located in Chad and Nigeria is used as a model. A high geothermal gradient is found in this area. Therefore, geothermal explorations are on the rise recently. Its thermal conductivity, heat production, and stratigraphy data from previous researches provide information about the initial model. The heat flux curve generated from inversion matches the initial noisy model with the error of around $10^{-9}$ mW/m$^2$. Therefore, to answer the increasing energy demand, this method can be highly applicable to future geothermal prospecting.

1. Introduction

In the energy industry, one of the emerging energy sources is geothermal. Geothermal energy is produced from underground heat. As water passes the hot rock beneath the surface, it heats up and generates heat flow that can be produced later by drilling [1]. Geothermal energy can generate steam that runs the steam turbine. This leads to the production of electricity [2]. Geothermal has been proven to succeed in satisfying energy demands in some parts of the world, one of them is Philippines that sees a lot of success in producing geothermal energy. In May 2018, it is reported by the Ministry of Energy in Jakarta, Indonesia, that the country’s geothermal power plants can generate energy for as much as 1,925 MW. It can be concluded that geothermal energy exploration and production see a lot of prospects in the long-term energy demand, because of the uncertainty in fossil fuel production.

Geothermal systems are often discussed using many different terms. One of the main properties to characterize geothermal reservoir properties before it can be produced is heat flux. “A geothermal
reservoir is the section of the geothermal field that is so hot and permeable that it can be economically exploited for the production of fluid or heat" [3]. A geothermal reservoir interacts with its surroundings by heat flow. Heat flow in the geothermal reservoir is mostly by heat conduction between the rock layer and the homogenous bedrocks. When this recharge of heat is absent, heat may also be transferred into the reservoir from the surrounding hot rocks, which are part of the geothermal system, by means of conduction [3]. This subsurface heat flux can be produced by doing the inversion method, given the surface heat flux. A number of researchers have processed geophysical data such as Gravity [4] [5], Magnetic [6], Geoelectric [7], Magnetotelluric [8] and Transient Electromagnetic [9] to estimate subsurface conditions in geothermal areas.

Some research has been done to map the heat flow beneath the surface based on surface temperature and heat flux. The first example is the inversion of geothermal heat flow in the Stokes ice model. But this experiment relies on other data such as surface ice flow velocity. The second example is an inversion of geothermal heat flow if the linear relation between surface heat flux \( Q_0 \) and subsurface (basement) heat flux \( Q_m \) is known to be ill-posed [10]. This can minimize the instability in the inversion process. The third example is a characterization of geothermal reservoirs’ parameters by inverse problem resolution and geostatistical simulations [11]. This characterization requires modeling of the spatial distribution of thermal parameters, water content, and water flow. However, the method was only proposed for the idealized problem, which is a test problem. To date, there have not been enough researches that use real data to analyze new possibilities on whether geothermal exploration can be done on the corresponding area.

To answer the problems discussed above, we propose a method for finding \( Q_m \) given the \( Q_0 \) using the help of Tikhonov regularization in the inversion process. Cauchy problems for equations as with many other inverse problems are known to be ill-posed [12]. Since the problem is ill-posed, regularization is needed to stabilize computations. We demonstrate that Tikhonov regularization can be implemented efficiently for solving the operator equation. The data used for this research are average surface temperature, thermal conductivity, and heat production at Chad sedimentary basin. In the past, Chad sedimentary basin is known as the petroleum-producing basin. Given its high geothermal gradient, there may be potential in exploring this basin regarding geothermal energy production [13]. Then, a better understanding of geothermal energy prospecting using the inversion method can be obtained. This is because the data realistically demonstrate the geologic setting and the physical properties in the geothermal basin.

The goal of this paper is to solve the non-linear problem and solving a sequence of linear problems using the Tikhonov regularization method. We demonstrate that Tikhonov regularization can be implemented efficiently for the non-linear operator equation.

2. Regional Geology

Chad Basin is located in north-eastern Nigeria with an area of about 2,335,000 km\(^2\) occupies a vast area at an altitude of between 200 m and 500 m above sea level in Central Africa [14]. The Chad basin is situated at the junction of basins that comprises the West African rift. Chad basin becomes active in early Cretaceous when Gondwana started to break up into component plates [15].

Chad Basin made up of four Formations, those are Chad, Fika, Gongila, and Bima Formation. The uppermost formation of Chad consists of mudstone, muddy sandstone, sandstone and claystone which is Pleistocene in age. The Fika Formation is upper lies by Chad Formation which is made up of shale and thin limestone.

The Gongila Formation lies beneath the Fika Formation and it consists of sandstones, clays, shales and limestone layers. The oldest stratigraphic unit of the formations is Bima which is the basal part of Chad Basin. The Bima Formation made up of thin to thick beds of fine to coarse-grained sandstone. (Figure 1) [16].
3. Numerical Solution for Estimating Temperature Distribution

A model for estimating temperature distribution is two-dimensional and confined to the domain \((x, z) \in C^0(\Omega) \cap C^1(\bar{\Omega})\) satisfies, where \(\kappa\) is thermal conductivity and \(A_p\) is heat production. We know that equation is well-posed and we have access to \(Q_m\) (heat flow) and \(T_0\) (surface temperature) and compute \(T(x,z)\) as temperature:

\[
\begin{align*}
(\kappa T_x)_x + (\kappa T_z)_z + A_p = 0, & \quad 0 < x < L_x, \ 0 < z < L_z \\
T_x(0, z) = T_x(L_x, z) = 0, & \quad 0 < z < L_z \\
T(x, 0) = T_0(x), & \quad 0 < x < L_x, \\
\kappa T_z(x, L_z) = Q_m(x), & \quad 0 < x < L_x,
\end{align*}
\]

distribution in the domain \(\Omega\). From the equations, we can define a non-linear operator that \(T_0\) must be fixed. Specifying the heat-flux \(Q_m\) and having a well-posed problem can solve the temperature distribution \(T(x,z)\). [3] For mapping the heat-flow from \(Q_0\) to \(Q_m\), we introduce

\[
Q_0 = \kappa T_z(x, 0) = K(\kappa, A_p, T_0)Q_m, \quad 0 < z < L_z
\]

\[
[T, Q_0] = F(\kappa, A_p, T_0, Q_m), \quad Q_0 = \kappa T_z|z = 0.
\]

In this paper, we produce inverse modeling the 1D subsurface heat flux distribution. We know that generalized heat conduction equation governing heat flow in the subsurface for 3D [18] is given as:

### Table 1. Depth range, lithology, stratigraphic units and thermal conductivity Kasade-1 Well

| Depth range (m) | Lithology           | Lithostratigraphic Units | Measured Thermal conductivity (W/m°C) | Average thermal conductivity (W/m°C) |
|----------------|---------------------|--------------------------|----------------------------------------|------------------------------------|
| 0-840          | Sandstone           | Chad formation           | 2.005                                  | 2.111                              |
| 840-1190       | Sandstone/Shale     | Fika formation           | 1.895                                  |                                    |
| 1190-1420      | Shale               | Gongila formation        | 1.382                                  |                                    |
Table 2. Measured and computed temperatures at some depth

| Well Name | Depth (km) | Tm (°C) | Ta (°C) | Tb (°C) | Tm-T1 | Tm-T2 | Tm-T3 |
|-----------|------------|---------|---------|---------|-------|-------|-------|
| Well-1    | 0          | 27      | 27      | 27      | 0     | 0     | 0     |
|           | 0.5        | 46      | 42.9    | 43.12   | 3.1   | 2.88  | 3.2595|
|           | 4          | 180     | 154.2   | 155.96  | 25.8  | 24.04 | 27.076|

\[ \rho c_p \frac{dT}{dt} = K \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + A \]  

(4)

In 1D sedimentary column where there is no erosion or sediment deposition and constant heat flow, the temperature at any point is steady and therefore equation 4 reduces to:

\[ K \left( \frac{\partial^2 T}{\partial z^2} \right) + A \]  

(5)

We called it as equilibrium geotherm. In the basin case, we assumed that the heat always flows in the vertical direction from the earth interiors and therefore neglecting the x and y coordinates [16] equation 5 reduces to:

\[ \frac{\partial^2 T}{\partial z^2} = -\frac{A}{K} \]  

(6)

where:

- \( A \) = Heat production in the sediment (mWm\(^{-3}\)),
- \( K \) = Thermal conductivity (Wm\(^{-1}\)°C),
- \( T \) = Temperature (°C),
- \( Z \) = depth (m).

For solving equation (4) requires information about the thermal conductivity and heat production within the lithosphere. Thermal conductivity of most rocks at crustal conditions varies inversely with temperature according to the relation [19]:

\[ k = \frac{k_0}{1 + CT} \]  

(7)

where \( k_0 \) is the thermal conductivity at surface conditions and \( C \) is an experimentally determined constant. At relatively low temperatures, thermal conductivity generally decreases with increasing temperature, i.e., \( C > 0 \), while at 300–500°C, when the conductivity of basic rocks varies little with temperature, \( C = 0 \) [19].

Following the pieces of information above, the Chad Basin consists of four lithostratigraphic and the equilibrium geotherm for such models is calculated by considering each layer separately. The radiogenic heat production, \( A \) in each of the four formations is given as:

- \( A = A_1 \) for \( 0 \leq Z < Z_1 \)
- \( A = A_2 \) for \( Z_1 \leq Z < Z_2 \)
- \( A = A_3 \) for \( Z_2 \leq Z < Z_3 \)
- \( A = A_4 \) for \( Z_3 \leq Z < Z_4 \)

Equation (6) can be solved by iterating it twice and the solution of the equation requires two boundary conditions as the temperature \( T_o = 27^\circ \text{C} \) at \( z = 0 \) and surface heat flow \( Q = -k \frac{\partial T}{\partial z} = -Qoatz = 0 \).

This study used the thermal conductivities information from [20] which studied surface temperature distribution of Chad Basin in well Kasade-1 and had written on Table 1 and Table 2. Only one well is used, assuming there is minimal information during the initial exploration.
4. The Linear Inverse Problem and Regularization

We have had equation (1) as a solver to the heat-flow $Q_m$ at the base of the model and the surface temperature $T_0$ and also obtain the temperature distribution $T(x,z)$. However, in practice we cannot measure the heat-flux $Q_m$, therefore we consider equation (2) as a non-linear operator equation,

$$KQ_m = Q_0, \quad K := K(\kappa, A_p, T_0),$$  \hspace{1cm} (8)

That can be solved for the desired heat-flux $Q_m$. The functions $T$, $\kappa$, and $A_p$ are represented by matrices, and $Q_0$, $Q_m$ and $T_0$ are vectors. The matrix-relation of them

$$KQ_m := FDM(\kappa, 0, 0, Q_m).$$  \hspace{1cm} (9)

In the remainder of this section, we are concerned with solving the ill-conditioned linear system of equations,

$$KQ_m = Q_0$$  \hspace{1cm} (10)

The linear system in equation (9) is ill-conditioned and regularization is needed. We use Tikhonov regularization, the regularization is computed by minimizing the Tikhonov functional, where $\lambda > 0$ is a regularization of the parameter.

$$\|KQ_m^\lambda - Q_0\|_2^2 + \lambda^2\|Q_m^\lambda\|_2^2$$  \hspace{1cm} (11)

That techniques for the case of temperature-independent coefficients the inverse problem, Tikhonov regularization can solve it. Also, we obtain a functional relation,

$$Q_m^\lambda := \langle \kappa, A_p, T_0, Q_0, \lambda \rangle.$$  \hspace{1cm} (12)

For maintaining small residues and stability, also preventing noise too much being enlarged, we must put the right choice for the $\lambda$ parameter. We have some standard methods to choose the parameter, those are the Discrepancy principle and the L-curve.

The L-curve technique is used to identify a suitable value for $\lambda$ [21]. The L-Curve is a plot of the residual norm against different values of $\lambda$ parameter and a graphical tool for analysis of ill-posed problems. For finding a good value of the regularization parameter the L-curve is used and it can minimize the Tikhonov functional for a range of parameter values $\lambda$. Applying the Tikhonov regularization method with the right choice of a regularization parameter $\lambda$ gives good results. These methods are compatible even though the data contains significant amounts of noise the reconstruction of the heat flux.

5. Non-linear Problem

The core problem in this study to solve the non-linear case, where both the thermal conductivity and the heat production $A_p$ depend on the temperature. We have solved the linearized problem using Tikhonov regularization, and now for the new approximate heat-flux at the base of the model, is then used to compute the next temperature distribution.

$$T^{(k+1)} := FDM(\kappa(T^{(k)}), A_p(T^{(k)}), T_0, Q_m^{(k+1), \lambda}).$$  \hspace{1cm} (13)

The process is repeated until the update $\|T^{(k+1)} - T^{(k)}\|_p$ is sufficiently small. When we start guess $k = -1$ where $T^{(0)} = 0$ was used and the stopping criteria for the iterations was a relative residual of magnitude less than $10^{-6}$ Note that even $Q_m^{(k), \lambda}$ has converged the temperature $T^{(k+1), \lambda}$ can still change due to the fact that the problem is non-linear and $\kappa(T^{(k), \lambda})$ is recomputed in each step [10].

6. Methodology

The research was successfully conducted using MATLAB. This research used source code which was modified from the previous research that found $Q_m$ given the $Q_0$ [10]. In illustrating geothermal energy, a numerical result with heat-flux $Q_m$ is used. The same model is also used for thermal conductivity ($k$) and heat production ($A_p$). The model used numerically generated test problems.

Heat flux surface $Q_0$ is output from the process. With heat flux surface $Q_0$ can use as input for the inverse problem. When low temperatures the thermal conductivity decreases with increasing temperatures. The physical size of our model was $L_x = 400$ km with $L_z = 4$ km. So, some graphs explain
the Frobenius norm $\| T(k) - T(k-1) \|_F$, thermal conductivity ($k$), and heat production ($A_p$). That we know that, coefficients $k$ and $A_p$ are independent of the temperature.

$$KQ_m = Q_0 \quad K := K(k, A_p, T_0)$$ (14)

With this, we can solve and find heat-flux $Q_m$. To get a solution that resolves non-linear conduction problems, repeated iterations are performed to excellent accuracy. The result is a convergence graph that will appear. We start our model with large error, needed 15 iterations to achieve such a small error. This is shown in Figure 2. With thermal conductivity and heat production, we can get the temperature distribution of the direct program, as seen in Figure 3. Next, we will have lambda that will be used for Tikhonov to solve the problem. We used lambda = $5\times10^{-7}$. After that, the singular value of the matrix $K$ will be displayed to show the situation of the operator, which is mildly ill-posed. Because we want to get an appropriate value of lambda and the regularization parameter, we use the L-curve and minimize the Tikhonov functional, so we can get the range of parameter values $\lambda$. L-curve works is by plotting solution norm against the residual norm.

![Figure 2](image.png)

**Figure 2.** The convergence history $\| T^{(k+1)} - T^{(k)} \|_F$, measured in the Frobenius norm
Figure 3. The temperature distribution $T(x,z)$ in the lithosphere

Figure 4. The Tikhonov solution with $\lambda = 6.3096 \times 10^{-4}$

From the L-curve in Figure 4, we have the value of lambda for $6.3096 \times 10^{-4}$, besides the physical lambda is not exactly L-shaped. With this lambda value, the inverse problem was computed. After getting 15 times iteration, norm size $\| T(k) - T(k-1) \|_F$ is smaller and noise is almost smooth.

However, we need to know the output if we increase Lambda for more regularization and decrease for less. Moreover, we plot the result of Tikhonov to get $Q_{th}$ and exact solution $Q_m$. The methodology for running is described in Figure 5.

7. Results
The research was successfully conducted using the methodology described above. The model was produced to match the real geological model: a basin.
The model is demonstrated in Figure 6 and 7, with the different k and A values at every layer. With the direct problem, the initial or predicted $Q_0$ was produced, with its surface temperature in Figure 8. This helps the inversion process to understand what the model should look like.

![Diagram of methodology for running process](image)

**Figure 5.** The diagram of methodology for running process

In the research described above, the result is the comparison between predicted $T_0$ and $Q_0$ obtained from the inversion method. Figure 9 shows the initial direct problem by obtaining $Q_m$ from $Q_0$. The Tikhonov solution $Q_m$ $\lambda$ is depicted in the black curve and the exact data is in the blue curve. The result demonstrates the fitness of the inversion model to the initial model. The error between the actual temperature and the result is shown in Table 3 and Figure 10. Figure 11 shows the convergence history after 15 iterations. To support the experiment by providing the “ill-posedness” degree of the problem, singular values decay was plotted in Figure 12.

**Table 3.** The computational information obtained during the running process

| Iteration number | Elapsed time (s) | Error   |
|------------------|------------------|---------|
| 15               | 1397             | $6.2 \times 10^{-8}$ |
Figure 6. The thermal conductivity (k)

Figure 7. The heat production (Ap)

Figure 8. The data vector $T_0$
Figure 9. The heat-flux $Q_m$ at the base model

Figure 10. The data vector of $Q_m$ and $Q_{tk}$ with Cauchy data is simulated

Figure 11. The convergence history $\|T^{(k+1)}-T^{(k)}\|_F$, measured in the Frobenius norm
8. Discussion

The computational work was successfully demonstrated in the methodology and results. In those sections, it is shown that Tikhonov regularization can help as a “damping factor” for the inversion process. With the correct choice of λ, the error can be minimized. Also, to make sure that the result makes sense in the real-world geological setting, it is needed to provide a realistic model during the computation.

Firstly, a basin model that refers to the real-world data was provided. The methodology, as described in [10], assumes a near-homogeneity of the parameters within the boundary. The heterogeneity only happens when the lithostratigraphy changes. The horizontal range of 400 km is used to cover a vast area of the basin that may be in the region of interest. The surface heat flux produced from the direct problem matches the real-world investigation. An anomalous zone was intentionally created in the simulation, where the heat flow value is the greatest. This helps the geoscientists to analyze the place where the geothermal potential can be capitalized.

For the linear system, it is optional to display the linear operator K explicitly. In this paper, we choose to show K as a large sparse matrix using the finite difference to create a linear system. The size is computationally expensive, with the matrix dimension of 75000x75000. To create K, a sparse LU decomposition is needed. If applied in the iteration process, it can slow down the iteration since the LU decomposition is done repeatedly. Because of this, one can choose whether to store this linear operator or not. However, the K matrix helps us to analyze the degree of ill-posedness of the problem. Figure 12

**Figure 12.** The convergence history \( \|T^{(k+1)} - T^{(k)}\|_F \), measured in the Frobenius norm

**Figure 13.** The singular value for computed matrix K.
shows the relation between the singular value of $K$ and its decay. With the graphs of the parameter above, it is known that the linear system is mildly ill-posed and requires regularization by using Tikhonov regularization, due to the case of the temperature-independent coefficients. This is also illustrating that the operator is mildly ill-posed.

Since we cannot solve the problem analytically, the finite difference is effective at approximating the direct problem. Using the initial guess in the direct problem with $T_0=0$, it is not surprising that the initial error is substantial. However, this iteration reaches convergence fast because the $K$ matrix is not yet needed to be produced. “Direct backslash” method for the linear equation can solve the problem much faster in this case.

In our inverse problem, we solve the same problem again using the iterative implementation, but this time finding $Q_m$ given the $Q_0$ from the direct problem. Because the noise was introduced to the Cauchy data [$T_0$, $Q_0$] and the problem becomes underdetermined, infinitely many solutions can produce the same $Q_m$ graph. Since the noise can be magnified and destroy the characteristics of the solution, this problem is indeed ill-posed. This leads to inverse problems, in which it is required to determine the equation coefficients from the information about the solution of the direct problem.

Because of the reason explained above, the Tikhonov regularization parameter is needed. If we pick a larger $\lambda$ value than it is supposed to, the solution will cut off the noise entirely, but the characteristics of the curve produced do not reflect the desired curve. If $\lambda$ is too small, the damping is not significant and the noise can create a highly fluctuating curve since it also satisfies the minimum $\text{Frobenius}$ norm between temperature found and the real temperature. Thus, not only using Tikhonov regularization but choosing the right value is also a crucial part of this research.

However, the trial-and-error method of determining the optimal value of $\lambda$ costs time and is not effective for a much larger case. L-curve works well at determining the optimal value of $\lambda$. How L-curve works are by plotting solution norm against the residual norm. Then, what we do is finding the corresponding $\lambda$ value at the point where the curve starts from being vertical to horizontal, hence its name L-curve. In Figure 4, it is shown that $\lambda = 6.3096 \times 10^4$ is the most appropriate value for the regularization parameter. Even though the L-curve is not exactly L-shaped, it can be clearly shown that the corresponding $\lambda$ value is located between the sharpest curvature.

After finding the right regularization parameter, the inverse problem was computed using both finite difference and $\lambda$ applied to the related least-square problem using conjugate gradient iterative method (PCG). Again, the initial guess for the computation was $T_0=0$ and $Q_0=0$. This resulted in the large value of initial errors. However, after 15 iterations, the $\text{Frobenius}$ norm of $T(k+1)$ and $T(k)$ became significantly smaller to the order of 10^-8, shown in Figure 11. The iteration also achieved convergence after eight iterations. The reason that the computation was done in 15 iterations was to look better at the point of which the convergence started to take place. Even though the $\text{Frobenius}$ norm is $10^-8$, the residual only converges to $3\times10^-5$. In the beginning, we expected the PCG residual to be very small, just like the $\text{Frobenius}$ norm. However, because of the noise and the more-fluctuate initial data, it is harder to achieve a very smooth result.

The final result is a plot between $Q_m$ from the inversion and $Q_m$ from the initial data. At the start, we defined $Q_m$ in terms of the Fourier series. The results show that both of the curves match each other. However, at the left and right end, there are some discrepancies between computed $Q_m$ and initial $Q_m$. The values seem to be spiking and reach zero eventually. This is caused by our assumption that the value of kappa and Ap at both ends are nearly zero. The assumption affects the calculation heavily. Besides that, the other parts of the curve match very closely.

9. Conclusion
In this paper, we have demonstrated how to produce 1D subsurface heat flux given the surface heat flux, thermal conductivity, heat production, and 2D geological model. This application is important since a lot of geothermal exploration depends on the information about subsurface temperature. Also, this paper helps the audience to get better understanding of the inversion process using Tikhonov regularization. Because many inverse problems arise in these fields as noisy Cauchy data. After redefining the problem as a linear equation, by doing the regularization, the computation is numerically stable and reaches convergence relatively fast. And also have the right value of lambda is really important, the L-curve provides the best way to choose a regularization parameter. Particularly values around the corner provides the best compromise between minimal residual and keeping penalty term...
small. In the future, the inversion method using other ways of regularization such as truncated singular value decomposition (TSVD) can be done and compared to this method. 3D modeling using this method is one of our tasks to do in the future for improving this research.

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