Modeling and Evaluation of Mutual Coupling Effect in Conventional Switched Reluctance Machines Using Space Vector Representation

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ABSTRACT This paper presents a non-linear model for conventional switched reluctance machines (CSRMs) that considers the mutual coupling between phases. Although CSRMs are based on single-phase excitation, there is usually an overlapping between the excited phases. During this overlapping, the CSRMs are at 2-phase excitation and the single-phase representation of motor dynamics is not accurate. Hence, a dynamic model is presented in this paper that that accounts for the mutual coupling between phases in addition to the effects of saturation and spatial harmonics. The proposed method is based on modeling the resultant current vector due to 2-phase excitation in space. The relationship between the resultant current and flux linkage vectors is obtained at different rotor positions using finite element method (FEM). This relationship is saved as a 3D lookup table (LUT). These 3D LUTs are reduced into 2D LUTs independent of rotor position by representing the phase currents as vectors instead of instantaneous values. Similarly, the relationship between the resultant current vector and the electromagnetic torque is obtained and saved as a 3D lookup table. The proposed method is compared with a conventional model where the mutual coupling is not considered. This comparison is conducted using FEM on two different motors; the first motor is 12/8 3-phase 2kW CSRMs and the second motor is 24/16 3-phase 75kW CSRMs. FEM results show that the mutual coupling is significant for high power motors. The proposed dynamic model is compared with experimental results using the 12/8 3-phase 2kW CSRMs.

INDEX TERMS Conventional switched reluctance machine (CSRMs), finite element method (FEM), mutual coupling, vector modeling.

I. INTRODUCTION
Switched reluctance machines (SRMs) are characterized by the simple rotor structure [1], [2]. Conventional SRMs (CSRMs) have a similar operation to stepper motors, as they are both based on the minimum reluctance law with DC current excitation [3], [4]. This explains why some references refer to SRM as variable reluctance stepper motor [5], [6], [7]. CSRMs are based on single-phase excitation where torque production is due to the rate of change of self inductance. Despite this, the overlapping between the excited phases in CSRMs takes place during the commutation [8], [9] and that overlapping is considered as two-phase excitation.

The mutual effect of the excited phases used to be neglected due to the low current levels during overlapping besides the winding configuration of the CSRMs that minimizes the mutual flux paths. However, the advanced control techniques of CSRMs to reduce torque ripple are strongly dependent on sharing the torque between the excited phases [10], [11]. Hence, the current levels during overlapping are not low. Additionally, the current levels at high speeds are usually high due to the large demagnetization time of the excited phase and the advanced turn-on angles. Furthermore, the mutual coupling effect can be significant for CSRMs typologies with high stator and rotor poles. It was proven in [12] that the mutual coupling during overlapping for a 3-phase 24/16 CSRMs is significant even at low current levels, where the modeling error exceeds 25% when a
dynamic model that does not account for mutual coupling is used.

Modeling approaches for SRMs are classified into analytical and lookup table based methods. The first one can be inductance or magnetic circuit modeling. In inductance modeling, the self inductance profile is represented by Fourier expansion and the magnitudes of the Fourier coefficients are functions of the excited phase current magnitude \([13] , [14] , [15] , [16] \). None of the inductance modeling methods have considered the mutual inductance between the two excited phases because of complexity \([17] , [18] , [19] \).

In magnetic circuit modeling, the equivalent magnetic circuit of CSRM is used. This magnetic circuit represents a number of reluctance elements which are usually the stator core, rotor core, stator pole, rotor pole, and the airgap. Although the mutual inductance is considered in the magnetic circuit methods for CSRM in \([20]\), it is dependent on rotor position and independent on the excited phase currents. This means that it cannot describe motor dynamics at saturation. A magnetic circuit model is introduced in \([21]\) that considers mutual coupling and saturation, however, the winding configuration of the CSRM used in this method is different from the standard winding configurations for CSRM and mutually coupled SRM \([22]\). Magnetic circuit methods are also introduced for MCSRMs \([23] , [24]\), these methods are only applicable when the excited phases have equal current levels.

The lookup table-based modeling methods for CSRM are simpler and more accurate than the analytical methods. Usually, it is based on using two 2-dimensional lookup tables (LUTs); the first LUT describes the non-linear relationship between the phase flux linkage \((\lambda_{ph})\), the phase current \((i_{ph})\), and rotor position \((\theta)\) as \(\lambda_{ph} = f(i_{ph}, \theta)\). The second LUT describes the non-linear relationship between the phase current, the phase torque \((T_c)\), and rotor position as \(T_c = f(i_{ph}, \theta)\). Both LUTs are obtained by experiments or from finite element model using single-phase excitation and mutual coupling between phases is not considered \([25] , [26]\).

In the proposed method, the mutual coupling is considered by representing the currents of the excited phases as a current vector in space. Afterwards, a non-linear relationship between that current vector and the resultant flux linkage vector is obtained. Similarly, the non-linear relationship between that current vector and the resultant torque is obtained. A low- and high-power motors are used in investigation in this paper, finite element analysis (FEA) results show that the mutual coupling is significant for the high-power motor. The paper is organized as; Section II describes the proposed dynamic model, Sections III and IV validate it using the FEA model and experiments, respectively. Finally, section V has the conclusion.

**II. PROPOSED METHOD**

A 24/16 3-phase CSRM is used in the explanation of the proposed method in this section and Table 1 shows the specifications of this motor. The dimensions of this 24/16 CSRM

![Figure 1](image_url)

**TABLE 1. Specifications of the 24/16 CSRM used in investigation.**

| Parameter              | Value       | Parameter              | Value       |
|------------------------|-------------|------------------------|-------------|
| Number of phases       | 3           | Rated torque           | 120N\text{m}|
| Rotor poles            | 16          | peak current           | 100A        |
| Stator poles           | 24          | Rated power            | 75kW        |
| Stator outer radius    | 132 mm      | Stator inner radius    | 100 mm      |
| Rotor outer radius     | 93.5 mm     | Rotor inner radius     | 72 mm       |
| Air-gap length         | 0.5 mm      | Axial length           | 108 mm      |
| Shaft radius           | 26 mm       | Turns per phase        | 13          |

is chosen based on \([27]\), which presents a high power traction motor designed for hybrid electric vehicles \([27]\).

In the conventional modeling methods, the non-linear relationship between the phase current and phase flux linkage is obtained from the finite element model using single-phase excitation. This single-phase excitation results in a current vector in space that has a fixed position corresponds to the excited phase as shown in Fig. 1(a). On the other hand, the proposed method is based on representing a current vector in space, where its position covers the angular displacement between any two excited phases in space as shown in Fig. 1(b), which is 15 degrees for the 24/16 CSRM. Thus, the position of this current vector in space depends on the current magnitudes of the excited phases. In this paper, the two excited phases are referred to as the leading and lagging phases. The leading phase is the phase that was excited first and the lagging phase is the phase that should be excited next to keep the rotation of the rotor. This means that the phase will be defined by a lagging or leading phase based on the rotation direction of the rotor. The current vector in synchronous PM machines results in a rotating magnetic field with a constant speed equal to the synchronous speed and a constant magnitude equal to \(\text{MMF} = \frac{I_m}{2}N_s\), where \(I_m\) is the peak phase current magnitude and \(N_s\) is the number of turns.
In the conventional SRM, the rotating magnetic field has different speeds and magnitudes at each rotor position based on currents magnitude of the excited phases. Additionally, the rotor of synchronous PM motors rotates in the same direction of the rotating magnetic field due to magnetic attraction law between the magnetic fields of stator and rotor. However, the rotor of SRMs rotates in a reverse direction with respect to the stator rotating magnetic field due to the minimum reluctance law.

Since the rotation direction of the rotor of CSRM is opposite to that of the current vector, at the given rotor position in Fig. 1(b), the rotation direction of the rotor will be clockwise, if the rotation direction of the current vector is counter clockwise. Based on that, phase $a$ and phase $b$ are the leading and lagging phases, respectively. Hence, the current vector in space for the 24/16 CSRM can be formulated as:

$$\vec{i} = \vec{i}_{lead} + \vec{i}_{lag}$$  \hspace{1cm} (1a)

$$|i|\phi_i = |i_{lead}|\angle 0^\circ + |i_{lag}|\angle 15^\circ, \ \phi_i = 0^\circ \rightarrow 15^\circ$$  \hspace{1cm} (1b)

where $i$ and $\phi_i$ are the magnitude and angle of the current vector in space, respectively. $i_{lead}$ and $i_{lag}$ represent the currents of the leading and lagging phases, respectively.

**A. MODELING OF PHASE CURRENTS**

The electromagnetic characteristics of the motor is described by the relationship between the current vector and flux linkage vector. The generated flux linkage vector depends on the magnitude and space position of the current vector, in addition to the magnetic reluctance defined by the rotor position. Besides, the magnitude and space position of the current vector are functions of the current magnitudes of the leading and lagging phases (see equation (1)). Therefore, the generated flux linkage vector depends on the current magnitudes of the leading and lagging phases, and rotor position.

The finite element model is used to obtain the non-linear relationship between the current vector and flux linkage vector, where the current magnitudes of the lagging and leading phases are swept from zero to the rated value, so that $\phi_i$ covers $15^\circ$ mechanical for different magnitudes of $|i|$. Additionally, the rotor position covers $22.5^\circ$ mechanical which is equivalent to one electrical cycle. The output flux linkage vector from the finite element model can be described by the flux linkage magnitudes of the leading and lagging phases as

$$\vec{\lambda} = \vec{\lambda}_{lead} + \vec{\lambda}_{lag}$$  \hspace{1cm} (2a)

$$|\lambda|\phi_\lambda = |\lambda_{lead}|\angle 0^\circ + |\lambda_{lag}|\angle 15^\circ, \ \phi_\lambda = 0^\circ \rightarrow 15^\circ$$  \hspace{1cm} (2b)

where $\lambda$ and $\phi_\lambda$ are the magnitude and angle of the flux linkage vector in space, respectively. $\lambda_{lead}$ and $\lambda_{lag}$ represent the flux linkages of the leading and lagging phases, respectively. $\lambda_{lead}$ and $\lambda_{lag}$ are functions of leading and lagging current magnitudes, and rotor position ($\theta$) as

$$\lambda_{lag} = f_{\lambda_{lag}}(i_{lead}, i_{lag}, \theta)$$  \hspace{1cm} (3a)

$$\lambda_{lead} = f_{\lambda_{lead}}(i_{lead}, i_{lag}, \theta)$$  \hspace{1cm} (3b)

where $f_{\lambda_{lag}}$ and $f_{\lambda_{lead}}$ are 3-Dimensional (3D) LUTs that describe the current magnitude of the lagging and leading phases, respectively, as functions of the leading and lagging current phases, and rotor position. If these LUTs are used, the model will be referred to as a derivative model since the current will be calculated from the derivative of the phase flux linkage. The derivative model is not preferred as it amplifies the simulation error. Hence, these LUTs are usually inverted, in this case, the flux linkage is calculated from the integration, and the model is considered to be an integral model [1], [28]. Gridfit Matlab function tool is used in [28] and it is used in this paper. The inversion of the LUTs in (3) is done at each rotor position where the 3D LUTs are represented as 2D LUTs ($\lambda_{lag} = f_{\lambda_{lag}}(i_{lag}, i_{lead})$ & $\lambda_{lead} = f_{\lambda_{lead}}(i_{lag}, i_{lead})$). For example, Fig. 2 shows the leading and lagging flux linkages, when the rotor is aligned with the leading phase at $\theta=0$. These 2D LUTs are inverted by using the gridfit tool. After inverting the LUTs at each rotor position, the current LUTs are obtained as

$$i_{lag} = f_{i_{lag}}(\lambda_{lag}, \lambda_{lag}, \theta)$$  \hspace{1cm} (4a)

$$i_{lead} = f_{i_{lead}}(\lambda_{lead}, \lambda_{lag}, \theta)$$  \hspace{1cm} (4b)

where $f_{i_{lag}}$ and $f_{i_{lead}}$ are 3D LUTs that describe the current magnitude of the lagging and leading phases, respectively, as functions of the leading and lagging flux linkages and rotor position. For instance, Fig. 3 shows the inverted LUTs of the leading and lagging phase currents, when the rotor is aligned with the leading phase at $\theta=0$.

**B. MODELING OF ELECTROMAGNETIC TORQUE**

Similar to the modeling of the flux linkage vector, the electromagnetic torque can be modeled as a function of the
current vector in space in terms of the leading and lagging phases, and rotor position. Therefore, the electromagnetic torque obtained from the FEA can be represented as:

\[ T_e = f_{tor}(i_{\text{lead}}, i_{\text{lag}}, \theta) \]  

(5)

where \( T_e \) is the electromagnetic torque, and \( f_{tor} \) is a 3D LUT that describes the electromagnetic torque. On the other hand, it is not required to invert the torque LUTs. Fig. 4 shows the electromagnetic torque when the rotor is aligned with the leading phase.

C. REDUCING THE SIZE OF LOOKUP TABLES

The dimensions of LUTs in (4) and (5) are 3D which makes them large in size. The size of these LUTs can be reduced significantly by reducing the LUTs’ dimension from 3D to 2D. This can be done by eliminating the third axis that describes the rotor position, \( \theta \). This means the phase current and electromagnetic torque in (4) and (5), respectively, will be described in vector representation instead of the instantaneous value. Hence, the 2D LUTs will describe the magnitude and angle of the current and torque vectors instead of the instantaneous values at a given rotor position.

1) LUTs OF LEADING PHASE CURRENTS

In order to represent the phase current in vector form, first, the harmonic components are found. For example, the harmonic components of the leading phase current when \([\lambda_{\text{lead}}, \lambda_{\text{lag}}]=[0.4, 0.3]\) Wb-t are shown in Fig. 5. It can be noticed from Fig. 5 that the significant harmonic components are the DC offset, the first, the second, and the third order harmonics. Thus, the leading phase current is represented by Fourier series:

\[ i_{\text{lead}}(\theta) = I_o + I_{a1}\sin(\theta) + I_{a2}\sin(2\theta) + I_{a3}\sin(3\theta) + I_{b1}\cos(\theta) + I_{b2}\cos(2\theta) + I_{b3}\cos(3\theta) \]  

(6)

where \( I_0 \) is the DC offset, and \( I_{a1}, I_{a2}, \) and \( I_{a3} \) are the sine Fourier coefficients of the first, second, and third harmonic orders, respectively. \( I_{b1}, I_{b2}, \) and \( I_{b3} \) are the cosine Fourier coefficients of the first, second, and third harmonic orders, respectively. The Fourier coefficients in (6) can be represented as 2D LUTs independent of rotor position in terms of the leading and lagging phase flux linkages as:

\[ I_o = f_{o}(\lambda_{\text{lead}}, \lambda_{\text{lag}}) \]  

(7a)

\[ \sum_{n=1,2,3} I_{an} = f_{an}(\lambda_{\text{lead}}, \lambda_{\text{lag}}) \]  

(7b)

\[ \sum_{n=1,2,3} I_{bn} = f_{bn}(\lambda_{\text{lead}}, \lambda_{\text{lag}}) \]  

(7c)

where \( f_{o}(\lambda_{\text{lead}}, \lambda_{\text{lag}}) \) is a 2D LUT that describes the relationship between the DC offset and leading and lagging phase currents. Similarly, \( f_{an}(\lambda_{\text{lead}}, \lambda_{\text{lag}}) \), and \( f_{bn}(\lambda_{\text{lead}}, \lambda_{\text{lag}}) \) are 2D LUTs that describe the relationship between the sine and cosine Fourier coefficients, respectively, in terms of the leading and lagging phase flux linkages.
lagging flux linkages. For instance, Fig. 6 shows \( I_{a1} \) and \( I_{b1} \) with respect to the leading and lagging phases flux linkage.

For a leading-lagging current grid of \( 15 \times 15 \) and 100 rotor positions as a total size of steps in the FEA model per electrical cycle, the size of the original LUT of the leading phase current in (4)(a) would be \( 15 \times 15 \times 100 \). After representing the leading phase current by 7 Fourier coefficients (see equation (7)), the size of the new LUTs is reduced to \( 15 \times 15 \times 7 \), which is 14 times reduction in the size of the LUTs.

2) LUTs OF LAGGING PHASE CURRENTS
The same analysis done to reduce the size of the leading phase current LUT, can also be done to reduce the size of the lagging phase current LUT. Furthermore, the same LUTs that describe the leading phase current in (7) can also be used to describe the lagging phase current. Fig. 7 shows the leading phase current when \( [\lambda_{\text{lead}}, \lambda_{\text{lag}}] = [0.4, -0.15] \text{ Wb-t} \), it also shows the lagging phase current when the leading and lagging flux linkages are reversed \( [\lambda_{\text{lead}}, \lambda_{\text{lag}}] = [-0.15, 0.4] \text{ Wb-t} \). It can be noticed from Fig. 7 that the leading and lagging phase currents are symmetrical around the Y-axis in addition to 120 degrees electrical phase shift between the two waveforms.

In order to represent the symmetry between the two waveforms around the Y-axis, the sine coefficients of the lagging phase currents have the same magnitude of those of the leading phase currents but with opposite sign, while the cosine Fourier coefficients and the DC offset of the lagging and leading phase currents are the same. This means the same LUTs in (7) can be used to obtain the magnitude of the Fourier coefficients of the lagging phase currents but with reversing the axes of the LUTs. Finally, the lagging phase current can be formulated as:

\[
i_{\text{lag}}(\theta) = I_o - I_{a1}\sin(\theta - 120^\circ) - I_{a2}\sin(2\theta - 120^\circ)
- I_{a3}\sin(3\theta - 120^\circ) + I_{b1}\cos(\theta - 120^\circ)
+ I_{b2}\cos(2\theta - 120^\circ) + I_{b3}\cos(3\theta - 120^\circ)
\] (8)

where the magnitude of Fourier coefficients in (8) can be obtained from (7) after reversing the axes of the LUTs.

3) LUTs OF ELECTROMAGNETIC TORQUE
The harmonic components of the electromagnetic torque when \( [i_{\text{lead}}, i_{\text{lag}}] = [100, 100] \text{ A} \) are shown in Fig. 8. It can be noticed that the considerable harmonic components are the first to the tenth order with exception to the eighth order. Therefore the torque waveform can be described as:
be 15

This size is reduced to 15 × 15 × 100. This size is reduced to 15 × 15 × 16 due to considering 16 Fourier coefficients (see equation (10)) which is 6 times less than the size of the original LUT.

$$T_e(\theta) = \sum_{n=1,2,3}^{4,5,6,7,9} T_{an} \sin(n\theta) + T_{bn} \cos(n\theta)$$

where $T_{an}$ and $T_{bn}$ correspond to the sine and cosine Fourier coefficients of the harmonic torque component of order $n$. 

$$\sum_{n=1,2,3}^{4,5,6,7,9} T_{an} = f_{an}(i_{lead}, i_{lag})$$

$$\sum_{n=1,2,3}^{4,5,6,7,9} T_{bn} = f_{bn}(i_{lead}, i_{lag})$$

where $f_{an}$ and $f_{bn}$ are 2D LUTs describe the non-linear relationship between the sine and cosine Fourier coefficients, respectively, of the torque waveform and the leading and the lagging phase currents. For instance, the first order Fourier coefficients $T_{a1}$ and $T_{b1}$ with respect to the leading and lagging phase currents are shown in Fig. 9.

For the same leading-lagging current grid of 15 × 15 and 100 rotor positions per cycle in the FEA model, the size of the original LUT of the electromagnetic torque in (5) would be 15 × 15 × 100. This size is reduced to 15 × 15 × 16 due to considering 16 Fourier coefficients (see equation (10)) which is 6 times less than the size of the original LUT.

$$\theta_{on,u} \leq \theta < \theta_{on,v} : (i, \lambda, \theta)_{lead} = (i, \lambda, \theta)_w$$

$$\theta_{on,v} \leq \theta < \theta_{on,w} : (i, \lambda, \theta)_{lag} = (i, \lambda, \theta)_u$$

$$\theta_{on,w} \leq \theta < \theta_{on,u} : (i, \lambda, \theta)_{lead} = (i, \lambda, \theta)_v$$

$$\theta_{on,u} \leq \theta < \theta_{on,v} : (i, \lambda, \theta)_{lag} = (i, \lambda, \theta)_w$$

### E. FINAL MODEL

The proposed method diagram is shown in Fig. 11. The flux linkages are calculated where the simultaneously excited phases are categorized into leading and lagging phases on (11). Those flux linkages are input to the LUTs in (7) to output the cosine and sine Fourier coefficients of the phase currents. The instantaneous current values of the leading and lagging phases are calculated at a given rotor position based on (6) and (8), respectively. The instantaneous current values of the leading and lagging phases are used to estimate the Fourier coefficients of the electromagnetic torque in (10). The instantaneous value of electromagnetic torque is calculated at a given rotor position using (9).

### III. FINITE ELEMENT VALIDATION AND EFFECT OF MUTUAL INDUCTANCES

In this section, the proposed method is validated by FEA for two different motors. The first motor is a high-power 3-phase 24/16 CSRM that was used in the investigation in Section II. The second motor is a low power 3-phase 12/8 CSRM and the specs of this motor is shown in Table 2. The proposed
A. HIGH POWER MOTOR

The effect of mutual coupling can be observed by simulating two distinct models: one model which considers mutual coupling using the proposed method, while the other model is the conventional one which ignores mutual coupling. Each model will generate distinct phase voltage waveforms according to the operating conditions shown in Table 3. The phase voltage waveforms are then passed into the FEA model where the phase currents, flux linkages, and electromagnetic torque from the proposed model and the FEA model are compared. It is worth mentioning that the initial rotor position is assumed to be the alignment position, this means the non-alignment position is at 180° electrical, which is used as the turn-on angle of the excited phases. The results of this test are shown in Fig. 12 and Fig. 13, when the mutual coupling is and is not considered, respectively. It can be noticed from Fig. 12 that motor variables from the dynamic model and the FEA model are in a good agreement when the mutual coupling is considered using the proposed dynamic model. The maximum percentage error in phase currents, flux linkages, and the electromagnetic torque are 2.6%, 0.4% and 4.1%, respectively. On the other hand, these errors are extremely higher when the conventional modeling method is used, where the mutual coupling is ignored as it can be noticed from Fig. 13. The percentage errors in phase currents, phase flux linkages, and the electromagnetic torque are 20.2%, 13.5% and 21.2%, respectively. It is worth mentioning that the voltage profiles in Fig. 13, obtained from the dynamic model when the mutual coupling effect is ignored, results in higher magnitudes of phase currents when they are imported in the FEA model. These higher phase currents in return generate higher electromagnetic torque. Table 4 compares the maximum percentage errors when the effect of the mutual coupling is considered by using the proposed dynamic model with the conventional model when the mutual coupling is ignored.

B. LOW POWER MOTOR

Similar to the previous section, the phase voltages output from the proposed dynamic model and the conventional model, where the mutual coupling is considered and ignored, respectively, are imported into the FEA model of the 12/8 CSRM. Figs. 14 and 15 show the FEA results and Table 4 summarizes the maximum error between both dynamic models.
models and the FEA model. It can be noticed from Figs. 14 and 15, and Table 4 that the modeling error in the phase flux linkage when the mutual coupling is ignored is the same for both the high power 24/16 and the low power 12/8 CSRM. However, that flux linkage modeling error did not contribute to a large error in phase current modeling for the low power 12/8 CSRM, unlike the case of the high power 24/16 CSRM. Therefore, it can be noticed that the mutual coupling is more significant for the high power CSRM.

IV. EXPERIMENTAL VALIDATION

The proposed method is validated by experiments using the low power 12/8 CSRM (Table 2). The experimental drive is shown in Fig. 16 and its parameters are shown in Table 5. The CSRM is connected to an interior permanent magnet synchronous machine (IPMSM) based dynamometer.
The torque control of the CSRM is performed using a Torque Sharing Function (TSF), which optimizes the current references and the conduction angles as a function of the reference torque and speed respectively. A PI+Δ modulation based current control algorithm is used for current control to minimize the current ripple, where Δ modulation control is a fixed-switching frequency equivalent of the conventional hysteresis controller [30]. When the SRM drive is operated at a low speed of 500 RPM with multi-phase excitation, the overlap between the subsequent phases is significant which consequently induces the mutual coupling effect.

The phase currents of the CSRM are recorded and compared with the phase current from the proposed dynamic model at the same operating conditions. The comparison is shown in Fig. 17 and it reveals that the proposed dynamic model is accurate enough to model mutual coupling during the commutation of the excited phases. Additionally, due to the limited band width of the torque transducer to capture...
torque ripples, the phase currents from the experimental setup are imported to the FEA model, where the output electromagnetic torque is compared with the one from the proposed model. Fig. 17 shows that they are in a good correspondence.

V. CONCLUSION
A dynamic modeling method for CSRMs is presented in this paper that accounts for the mutual coupling effect between phases. Although the mutual coupling effect is usually neglected in CSRMs during commutation, FEA results show that the mutual coupling is significant in high power CSRMs such as those used in electric and hybrid electric vehicles. The proposed method is validated by experiments using the low power CSRMs. FEA results show that the mutual coupling is significant for high power motors like those used in electric and hybrid electric vehicles and less significant for low power CSRMs. Thus, the modeling error in current and torque exceeds 20% when the mutual coupling is ignored using the conventional modeling methods for a 70kW 24/16 CSRM. The proposed method is validated by experiments using the low power CSRM.

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REFERENCES
[1] P. Azer, B. Bilgin, and A. Emadi, “Mutually coupled switched reluctance motor: Fundamentals, control, modeling, state of the art review and future trends,” IEEE Access, vol. 7, pp. 100099–100112, 2019.
[2] P. Azer, B. Bilgin, and A. Emadi, “Comprehensive analysis and optimized control of torque ripple and power factor in a three-phase mutually coupled switched reluctance motor with sinusoidal current excitation,” IEEE Trans. Power Electron., vol. 36, no. 6, pp. 7150–7164, Jun. 2021.
[3] D. Sudhoff and P. C. Krause, “Analysis of steady-state operation of a multistack variable-reluctance stepper motor using qd0 variables,” IEEE Trans. Energy Convers., vol. 6, no. 4, pp. 693–699, Dec. 1991.
[4] P. J. Clarkson and P. P. Acarnley, “Simplified approach to the dynamic modelling of variable-reluctance stepping motors,” IEE Proc. B-Electr. Power Appl., vol. 136, no. 1, pp. 1–10, Jan. 1989.
[5] J. Bernat, J. Kolota, and S. Stepień, “Proportional-integral-derivative position control of variable reluctance stepper motor,” in Proc. VRD Int. Symp. Theor. Electr. Eng., Lubbecke, Germany, Jun. 2009, pp. 1–4.
[6] J. W. Finch, H. M. B. Metwally, and J. A. Agber, “Performance prediction in saturated variable reluctance and hybrid motors,” in Proc. 4th Int. Conf. Power Electron. Variable-Speed Drives, London, U.K., Jul. 1990, pp. 231–236.
[7] B. C. Mecrow, “Fully pitched-winding switched-reluctance and stepping-motor arrangements,” IEE Proc. B, Electr. Power Appl., vol. 140, no. 1, pp. 61–70, Jan. 1993.
[8] P. Azer and J. Bauman, “An asymmetric three-level T-Type converter for switched reluctance motor drives in hybrid electric vehicles,” in Proc. IEEE Trans. Electr. Comput. Eng. Expo (ITEC), Jun. 2019, pp. 1–6.
[9] P. Azer, J. Ye, and A. Emadi, “Advanced fault-tolerant control strategy for switched reluctance motor drives,” in Proc. IEEE Trans. Electr. Comput. Eng. Expo (ITEC), Jun. 2018, pp. 20–25.
[10] V. P. Vujčić, “Minimization of torque ripple and copper losses in switched reluctance drive,” IEEE Trans. Power Electron., vol. 27, no. 1, pp. 388–399, Jan. 2012.
[11] P. L. Chapman and S. D. Sudhoff, “Design and precise realization of optimized current waveforms for an 8/6 switched reluctance drive,” IEEE Trans. Power Electron., vol. 17, no. 1, pp. 76–83, Jan. 2002.
[12] M. Kordić, “Dynamometer testing and characterization of switched reluctance motors (SRMs) for electrified powertrains,” Ph.D. dissertation, Dept. Elect. Comput. Eng., McMaster Univ., Hamilton, ON, Canada, Sep. 2019.
[13] X. Ding, M. Rashed, C. I. Hill, and S. Bozhko, “Analytical modelling approach for switched reluctance machines with deep saturation,” in Proc. Int. Conf. Elect. Syst. Aircr., Railway, Ship Props. Road Vehicles Int. Transp. Electrict. Conf., Toulouse, France, Nov. 2016, pp. 1–6.
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