Investigation of the effect of rotation speed on the torsional vibration of transmission system

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Abstract
During the operation of vehicles, it is found that dramatic vibration occurs when the engine rotation speed reaches a certain value. In order to study this phenomenon, a theoretical model of automobile transmission system is developed in this paper. This model includes four sub-models of gearbox, drive shafts, main reducer and rear axle, which take into account the inhomogeneous transmission speed of universal joint of drive shafts as well as the effect of time-varying and nonlinear factors of main reducer gears. In this model, the transmission system is an elastic system characterized by mass, stiffness and damping. The torsional vibration responses of transmission system are simulated, and the natural frequencies of transmission system and corresponding mode shapes are calculated using this model. Simulation results indicate that the maximum amplitude of torsional vibration response appears at a certain speed. On the other hand, experimental investigation on the effect of rotation speed on torsional vibration is conducted to verify the theoretical model. Experimental results also show there is the maximum amplitude of torsional vibration response appearing at a certain speed. The results of FEA indicate that the excitation frequencies of drive shaft are quite close to the first order natural frequency of drive shaft, and the resonant vibration of drive shaft would induce the resonant vibration of transmission system, given that the first order natural frequency of drive shaft is quite close to the third order natural frequency of transmission system. In particular, it is discovered that deviations between the rotation speeds corresponding to the maximum amplitude of angular displacement and the rotation speeds corresponding to the maximum amplitude of angular acceleration exist for both theoretical simulation and experimental measurements.

Keywords: Transmission system, Torsional vibration, Rotation speed, Angular displacement, Angular acceleration

1. Introduction

In the process of power transmission, vibration and noise are generated, which have a great effect on vehicle NVH performance. Automobile power transmission system is an important part of automobile, which mainly includes engine, clutch, gearbox, drive shaft, main reducer, half axle and driving wheel. The torque of engine transfers to driving wheel through clutch, gearbox, drive shaft and rear axle. During the operation of vehicles, it is found that dramatic vibration occurs when the engine rotation speed reaches a certain value.

This phenomenon has been investigated experimentally. A torsional vibration test rig was developed to investigate the torsional vibration caused by engine excitation, including the torsional vibration responses of flywheel end of engine and gearbox input shaft (Yang et al., 2017). The torsional vibration signals of transmission system were collected and the results show that increasing the flywheel mass can effectively reduce the effect of engine excitation on the torsional vibration of transmission system. For the investigation of torsional vibration of drive shafts, a field test of torsional vibration of automobile drive shaft shows that the reasonable arrangement of universal joints can reduce the torsional vibration of drive shafts (Wu et al., 2013). The fluctuation of input torque induced by internal-combustion engines is an...
important source of vibration. One experimental research found that the torsional vibration of automobiles can be reduced by inserting a passive torsional vibration isolator made of helical springs between engine and gearbox (Kim et al., 2015). Although experimental investigations have figured out some countermeasures for certain problems, it is still insufficient to illustrate the causes for these phenomena. In the process of automobile driving, there are not only torsional vibration and linear vibration, but also vibrations coupled with each other. Therefore, a theoretical model for transmission system should be developed to investigate these problems.

The torsional vibrations of transmission system generated in shaft parts are influenced by the stiffness, damping and mass of shafting structure. The power output from engine first passes through gearbox, which composes of gears and gear shafts. It is necessary to determine the value of torque as the output torque of engine does not change linearly. A nonlinear torque estimator for accurately estimating non-measurable torque in an automotive vehicle was proposed (Yoshida et al., 2019). The estimator was based on a single degree-of-freedom, nonlinear drivetrain model, which simulated the non-measurable torque arising from measurable engine-output-shaft torque and drive-shaft torque. A nonlinear gearbox model involving the factors such as time-varying mesh stiffness, damping, static transmission error and backlash was developed (Zhao et al., 2015). The results of this research show that large time-varying meshing stiffness can stabilize the torsional vibration responses. In order to reduce the vibration and noise of gearbox, a mathematical model of powertrain was developed (Idehara et al., 2018). Using this model, the vibration response of gearbox was predicted by changing the stiffness and friction moment of clutch. The results show that a reduction of clutch stiffness decreases the natural frequency, and an increase of friction moment reduces the maximum amplitude of gearbox torsional vibration.

For the investigations of torsional vibration of drive shafts, a coupled torsional vibration model of driveline and rear axle reflecting the relationship between the driveline and the rear axle was developed (Hao et al., 2018), and the torsional vibration responses of drive shafts were acquired using this model. The Multi-Island Genetic Algorithm was applied to optimize this model, and the robustness of driveline torsional vibration was improved after optimization. Structural modification is an effective method to reduce vibration, and a reasonable structure of drive shafts could reduce torsional vibration effectively using numerical method (Liu et al., 2016).

The coupled vibration of transmission system can be generated due to the engaging movement of main reducer gear pair. The time-varying and non-linear factors are the main factors to be considered when developing the model. In order to minimize dynamic transmission errors caused by the periodically time-varying mesh stiffness of gears in transmission system, a non-linear controller was proposed to adjust the gear torque (Dogruer et al., 2017). In this way, the nonlinear meshing of gears can be simplified to linear meshing. A nonlinear dynamic model of spiral bevel gear pair was developed by neglecting the effect of static transmission error (Qiu et al., 2014), and the vibration responses of gear system were acquired using Runge-Kutta method reflecting the effect of time-varying backlash and torque. The simulation results show that large torque is beneficial for stability of system, while the gear backlash has an opposite effect on the stability of system. A FEA model of gear pair including the effect of friction, backlash, and time-varying stiffness was developed (Wang et al., 2011). The vibration responses of an actual gear pair under dynamic loading were simulated using this model. In addition, the maximum resonance frequencies were obtained, which was close to the exciting frequency of rotation speed of input gear.

For these relevant research, only the torsional vibration of shafting structure or the coupled vibration of main reducer is considered, which is insufficient to characterize the actual vibration responses of transmission system. Therefore, these two should be integrated to investigate the vibrations of transmission system. In the previous research, a multi-degree-of-freedom coupled vibration model of transmission system was developed (Xu et al., 2016), including the effect of non-linear factors, and the stiffness coefficient is considered in numerical analysis to analyze the dynamic response of system. The results show that increasing the stiffness of bearing by adding preload can reduce the vibration response at a certain speed. When the stiffness of bearing increases to certain value, the reduction is not evident. Based on the model in (Xu et al., 2016), the effect of angel between drive shafts on the vibration responses of main reducer was investigated (Liu et al., 2018a) and the vibration responses reached the maximum value at a certain rotation speed (Liu et al., 2018b).

Even though the torsional vibrations of components of transmission system have been investigated intensively, the effect of rotation speed on torsional vibration of transmission system which includes gearbox has not been investigated in details. In particular, it is found that a kind of vehicles have a dramatic vibration at a certain speed. No detailed illustration for this phenomenon, however, has ever been provided. In this research, a theoretical model of transmission system is developed, including the gearbox, drive shafts and rear axle. The effects of rotation speed on torsional vibration...
responses of gearbox shafts and drive shafts and the coupled vibration responses of reducer gears are investigated using this model. In addition, finite element analysis is applied to illustrate the reason why a dramatic vibration occurs at a certain rotation speed.

2. Methodologies

The transmission system is a complex elastic system, so it is necessary to simplify the system in the process of analysis. In this part, the transmission system is simplified using lumped mass method. The transmission system is simplified as a system consists of masses, stiffness and damping. The vibration responses of this system are measured by displacement (angular displacement), velocity (angular velocity) and acceleration (angular acceleration).

2.1 Theoretical model

The theoretical sub-models of each part of transmission system are developed, and the model of whole transmission system can be acquired by integrating these sub-models.

2.1.1 Sub-model of gearbox

It is necessary to simplify the front parts of gearbox before simplifying gearbox, which are engine, flywheel and clutch. In this research, the engine is replaced with a motor, and flywheel and clutch are simplified as a mass point. The motor is the first lumped mass point, and the mass point of flywheel and clutch is the second lumped mass point. The clutch is presented as an elastic element in the model. The simplified model of motor, flywheel and clutch is illustrated in Fig. 1.

![Fig. 1 Simplified model of motor, flywheel and clutch.](image1)

In Fig. 1, $\theta_1$ and $\theta_2$ are the angular displacement of motor and clutch; $k_1$ is the stiffness between motor and clutch; $k_2$ is the stiffness between clutch and gearbox; $c_1$ is the damping between motor and clutch; $c_2$ is the damping between clutch and gearbox; $T_D$ is the torque of motor. The torque of engine is the main external excitation of automobile transmission system. The change of gas pressure of internal combustion engine and the periodic force caused by reciprocating motion of crankshaft make the output torque fluctuate periodically. The engine studied in this paper is a four-cylinder four-stroke gasoline engine. The torque fluctuation caused by the second order reciprocating inertia force is the main output excitation of engine. Therefore, only the second order excitation torque is considered, and the input torque is

$$ T_D = T_m + T_{d2} \cos(2\omega, t + \varphi_2) $$

where $T_m$ is the average torque; $T_{d2}$ is the second order fluctuation torque amplitude; $\omega_f$ is the input angular velocity; $\varphi_2$ is the second order initial phase of fluctuation torque.

The gearbox in this research is shown in Fig. 2. Taking the third gear of gearbox as an example, the lumped mass model of gearbox is developed. As a common gear meshing system, the centralized point of inertia of gear is located in the center of gear. The input shaft ((A) shaft in Fig. 2) of gearbox can be regarded as an elastic element, and its inertia is equivalent to the centralized inertia of two ends of shaft. Similarly, the intermediate shaft ((B) shaft in Fig. 2) and output shaft ((C) shaft in Fig. 2) can also be simplified using the same way. Therefore, gearbox is divided into six equivalent centralized inertias, which is shown in Fig. 3.
According to the simplified lumped mass model of gearbox, the dynamic equations are listed as follows,

\[
\begin{align*}
J_i \ddot{\theta}_i &+ c_i (\dot{\theta}_i - \dot{\theta}_j) + k_i (\theta_i - \theta_j) = T_D \\
J_i \ddot{\theta}_i &+ c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) + c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) = 0 \\
J_i \ddot{\theta}_i &+ c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) + c_i (\theta_i - \theta_j) = 0 \\
J_i \ddot{\theta}_i &+ c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) + c_i (\theta_i - \theta_j) = 0 \\
J_i \ddot{\theta}_i &+ c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) + c_i (\theta_i - \theta_j) = 0 \\
J_i \ddot{\theta}_i &+ c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) + c_i (\theta_i - \theta_j) = 0 \\
J_i \ddot{\theta}_i &+ c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) + c_i (\theta_i - \theta_j) = 0 \\
J_i \ddot{\theta}_i &+ c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) + c_i (\theta_i - \theta_j) = 0 \\
J_i \ddot{\theta}_i &+ c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) + c_i (\theta_i - \theta_j) = 0 \\
J_i \ddot{\theta}_i &+ c_i (\theta_i - \theta_j) + k_i (\theta_i - \theta_j) + c_i (\theta_i - \theta_j) = 0 \\
\end{align*}
\]

where \( \theta_i, \dot{\theta}_i \) and \( \ddot{\theta}_i \) \((i=1,2,...,8)\) are the angular displacement, angular velocity and angular acceleration of inertia; \( J_i(i=1,2,...,8) \) is the equivalent inertia of a mass point around axle; \( c_i(i=1,2,...,8) \) is the equivalent torsional damping; \( k_i(i=1,2,...,8) \) is the equivalent torsional stiffness; \( T_D \) and \( T_i \) are the torque of motor input and load torque of transmission output end; \( i \) is the transmission ratio of the third gear. The values above mentioned are lists in Table 1.

| Parts          | Mass/kg | Inertia moment /kg m² | Stiffness/N m/rad |
|----------------|---------|-----------------------|-------------------|
| Engine         | \( m_1 = 1 \) | \( J_1 = 0.091 \) | \( k_1 = 10024 \) |
| Clutch         | \( m_2 = 10 \) | \( J_2 = 0.38017 \) | \( k_2 = 1640 \) |
| Gearbox        | \( m_3 = 0.61 \) | \( J_3 = 0.00008 \) | \( k_3 = 27500 \) |
|                | \( m_4 = 1.38 \) | \( J_4 = 0.00100 \) | \( k_4 = 10^{11} \) |
|                | \( m_5 = 2.36 \) | \( J_5 = 0.00132 \) | \( k_5 = 479000 \) |
|                | \( m_6 = 11.4 \) | \( J_6 = 0.00249 \) | \( k_6 = 10^{11} \) |
|                | \( m_7 = 4.53 \) | \( J_7 = 0.00186 \) | \( k_7 = 21400 \) |
|                | \( m_8 = 7.56 \) | \( J_8 = 0.00210 \) | / |
|                | \( m_9 = 1.284 \) | \( J_9 = 0.001695 \) | \( k_9 = 43922 \) |
|                | \( m_{10} = 1.284 \) | \( J_{10} = 0.001695 \) | / |
| Drive shaft     | \( m_{11} = 1.3125 \) | \( J_{11} = 0.00173 \) | \( k_{10} = 43921.799 \) |
|                | \( m_{12} = 1.3125 \) | \( J_{12} = 0.00173 \) | / |
| Driving gear    | \( m_{13} = 0.4254 \) | \( J_{13} = 0.000028 \) | \( k_{13} = 10^{11} \) |
| Driven gear     | \( m_{14} = 1.7016 \) | \( J_{14} = 0.00011 \) | \( k_u = 9.7 \times 10^8 \) |
| Half axle       | \( m_{15} = 13.75 \) | \( J_{15} = 0.0342 \) | / |
|                | \( m_{16} = 8.45 \) | \( J_{16} = 9.9 \) | \( k_n = 1.2 \times 10^4 \) |
|                | \( m_{17} = 8.45 \) | \( J_{17} = 9.9 \) | \( k_{17} = 1.2 \times 10^4 \) |

2.1.2 Sub-model of drive shaft

Drive shaft can be simplified as an elastic element with uniform mass distribution, and its inertia is averagely equivalent to the mass points at the front and back ends. The lumped mass model of drive shafts is illustrated in Fig. 4.
For the transmission system, universal joint is used for the connections between the output shaft of gearbox and the intermediate drive shaft, the intermediate drive shaft and the main drive shaft, as well as the main drive shaft and rear axle. The intermediate drive shaft is equivalent to the inertia points distributed at both ends and connected by stiffness and damping. Similarly, the main shaft is also equivalent to the inertia points. As the drive shaft is a hollow cylinder, its inertia, stiffness and damping can be calculated as

\[
J = \pi \rho (D^4 - d^4)L/32 \\
K = \pi G(D^4 - d^4)/32L \\
c = 2\zeta /\sqrt{K/(1/J_1 + 1/J_2)}
\]

where \( D \) and \( d \) are the outer diameter and inner diameter of drive shaft, \( \rho \) is the material density of drive shaft, \( L \) is the length of drive shaft, \( \zeta \) is damping ratio, \( J_1 \) and \( J_2 \) are the inertia of two ends of drive shaft. Therefore, the parameters of regular parts can be calculated using above formulas, and irregular parts can be calculated using CAD software.

According to the lumped mass model of drive shaft, the equations of drive shaft are listed as follows,

\[
\begin{align*}
J_8 \ddot{\theta}_8 + c_8 (\dot{\theta}_8 - \dot{\theta}_9) + k_8 (\theta_8 - \theta_9) &= T_2 \\
J_9 \ddot{\theta}_9 + c_9 (\dot{\theta}_9 - \dot{\theta}_10) + k_9 (\theta_9 - \theta_{10}) &= T_3 \\
J_{10} \ddot{\theta}_{10} + c_{10} (\dot{\theta}_{10} - \dot{\theta}_9) + k_{10} (\theta_{10} - \theta_9) &= T_4 \\
J_{11} \ddot{\theta}_{11} + c_{11} (\dot{\theta}_{11} - \dot{\theta}_{12}) + k_{11} (\theta_{11} - \theta_{12}) &= T_5 
\end{align*}
\]

where \( T_2 \) and \( T_3 \) are the input torque and load torque of intermediate drive shaft; \( T_4 \) and \( T_5 \) are the input torque and load torque of main drive shaft; The parameters are listed in Table 1.

As there is an angle between the shafts, the relationship between the output angular displacement of intermediate drive shaft and the input angular displacement of main drive shaft can be expressed as

\[
\tan \theta_{10} = \tan \theta_{11} \cos \alpha_2
\]

where \( \alpha_2 \) is the angle between intermediate drive shaft and main drive shaft, thus

\[
\begin{align*}
\theta_{11} &= \arctan \frac{\tan \theta_{10}}{\cos \alpha_2} \\
\dot{\theta}_{10} &= \frac{\dot{\theta}_{11}}{A - B \cos 2\theta_{10}} \\
\ddot{\theta}_{10} &= \frac{\dot{\theta}_{11}}{A - B \cos 2\theta_{10}} - \frac{2B \sin 2\theta_{10}}{(A - B \cos 2\theta_{10})^2} (\dot{\theta}_{10})^2
\end{align*}
\]

where \( A = 1 + \cos^2 \alpha_2 \), \( B = 1 - \cos^2 \alpha_2 \).

The energy loss is negligible, so the relationship between input torque and output torque can be expressed as

\[
T_4 = (A - B \cos 2\theta_{10})T_5
\]
2.1.3 Sub-model of main reducer gear pair

As one important part of rear axle assembly, the main function of main reducer is to reduce rotation speed and increase torque. Vibrations are generated due to the meshing movement of gear pair and the elasticity of bearing. For the convenience of analysis, the working load of gear uniformly distributes over the contact line, which is substituted by a concentrated force acting on the middle point of gear tooth. The contact forces of pinion are illustrated in Fig. 5.

![Fig. 5 Force analysis of pinion](image)

![Fig. 6 The coupled vibration model of hypoid gears](image)

$F_n$ is the normal dynamic meshing force of pinion; $F_x$, $F_y$ and $F_z$ are the meshing force of pinion in $x$, $y$ and $z$ direction. These contact forces can be expressed as,

$$
\begin{align*}
F_n &= k_m(t) f(x_n) + c_m \dot{x}_n \\
F_x &= -F_n \cos \alpha_n \sin \beta_1 \cos \delta_1 + F_x \sin \alpha_n \sin \delta_1 \\
F_y &= -F_n \cos \alpha_n \cos \beta_1 \\
F_z &= -F_n \cos \alpha_n \sin \beta_1 \cos \delta_1 - F_z \sin \alpha_n \sin \delta_1
\end{align*}
$$

(8)

where $k_m(t)$ is the gear pair meshing stiffness function; $f(x_n)$ is the gear backlash function; $c_m$ is the average damping of hypoid gear; $\dot{x}_n$ is the relative velocity between two gears at meshing point in normal direction; $\delta_1$ is the pitch cone angle of pinion; $\beta_1$ is the average spiral angle; $\alpha_n$ is the average pressure angle, whose values are listed in Table 2.

| Table 2. The parameters of pinion and gear |
|------------------------------------------|
| **Pinion**                                | **Gear**       |
| The number of teeth                       | 10             | 41             |
| Pitch cone angle                          | 12.75°         | 76.50°         |
| Average pressure angle                    | 21.25°         | 21.25°         |
| Average spiral angle                      | 47.97°         | 28.27°         |
| Radius of nodal circle                    | 18.8mm         | 71.7mm         |

Using the lumped mass method, the gear coupled vibration model is developed as shown in Fig. 6. In the model, pinion and gear are equivalent to lumped mass or rotation inertia, and the shaft of pinion is considered as massless rigid body and the bearing is considered as a massless spring and a damper. $R_5$ and $R_6$ are the radius of nodal circle of pinion and gear at meshing point; $\theta_p$ is the torsional degree of freedom of pinion, and $\theta_g$ is the torsional degree of freedom of gear. These two freedoms result in torsional vibration in the process of vehicle running. The linear displacement degree of freedoms of pinion and gear result in linear vibrations in $x$, $y$ and $z$ direction. Therefore, the sub-model of main reducer gears can be expressed as,
where $x_p, y_p$ and $z_p$ are the displacement of pinion in $x$, $y$ and $z$ direction; $x_g, y_g$ and $z_g$ are the displacement of gear in $x$, $y$ and $z$ direction; $m_{px}, m_{py}$ and $m_{pz}$ are the equivalent mass of pinion in three directions are equal in value; $m_{gx}, m_{gy}$ and $m_{gz}$ are the equivalent mass of gear in three directions are equal in value; $c_{px}, c_{py}$ and $c_{pz}$ are the equivalent damping of pinion in $x$, $y$ and $z$ direction; $c_{gx}, c_{gy}$ and $c_{gz}$ are the equivalent damping of gear in $x$, $y$ and $z$ direction; $K_{px}, K_{py}$ and $K_{pz}$ are the equivalent stiffness of pinion in $x$, $y$ and $z$ direction, and their values are listed in Table 3.

Table 3. The stiffness, damping and equivalent mass of pinion and gear

| Stiffness (N/m) | Damping (N·s/m) | Equivalent mass (kg) |
|----------------|----------------|----------------------|
| $K_{px}$       | $8.6\times10^8$ | $c_{px}$ 1050        | $m_{px}$ 1.037       |
| $K_{py}$       | $1.6\times10^9$ | $c_{py}$ 1050        | $m_{py}$ 1.037       |
| $K_{pz}$       | $1.6\times10^9$ | $c_{pz}$ 1050        | $m_{pz}$ 1.037       |
| $K_{gx}$       | $2.8\times10^9$ | $c_{gx}$ 1000        | $m_{gx}$ 6.162       |
| $K_{gy}$       | $6.9\times10^8$ | $c_{gy}$ 1000        | $m_{gy}$ 6.162       |
| $K_{gz}$       | $2.8\times10^9$ | $c_{gz}$ 1000        | $m_{gz}$ 6.162       |

2.1.4 Sub-model of rear axle

The drive axle composes of main reducer, half axle and axle housing, etc. In torsional vibration model, the pinion shaft is simplified as two inertias. Main reducer gears and differential assembly are simplified as the mass point, and the half axle is simplified as elastic element. The simplified lumped mass model of rear axle is illustrated in Fig. 7.

![Fig. 7. Lumped mass model of rear axle](image)

According to the lumped mass model, the dynamic equations of the rear axle are listed as follows,
where $T_{RL}$ and $T_{RR}$ are the load torque of left and right wheel; $i_1$ and $i_2$ are the transmission ratio of gearbox and main reducer, whose value are 1.35 and 4.1. The parameters are calculated in Table 1.

The sub-model of gearbox, drive shafts, main reducer and rear axle have been developed, which can be expressed by Eq. (2), (4), (9) and (10).

### 2.2 Experimental method

The test rig and the position of sensors are displayed in Fig. 8.

![Fig. 8. Test rig and sensors distribution](image)

As illustrated in Fig. 8, there are 4 torsional vibration signal measuring points. These 4 torsional vibration signal measuring points are photoelectric sensors (i and ii) which are used to measure the angular displacement and angular acceleration of output shaft of gearbox and input shaft of rear axle respectively, and electromagnetic sensors (iii and iv) which are used to measure the angular displacement and angular acceleration of left and right hubs. The excitation torque of engine is replaced by a motor. The speed of motor is from 1000 RPM to 2500 RPM. The torsional vibration and linear vibration responses of these measuring points can be measured at continuous speed.

### 3. Results

Using the theoretical model developed above, the torsional vibration responses and natural frequencies of transmission system can be calculated. In addition, the results of measurement have been illustrated in this part.

#### 3.1 Simulation results

It is found that the torsional vibration of transmission system changes with the engine speed as presented in Fig. 9.

![Fig. 9 Simulation results of vibration responses of transmission system (a) angular displacement, (b) angular acceleration](image)
As can be seen in Fig. 9 (a), the angular displacement amplitude of output of gearbox and the input of rear axle are larger. As the left hub is symmetrical to the right hub, only the angular displacement amplitude of one side of hub is plotted, and the amplitude of torsional vibration of hub is smaller. The angular displacement amplitudes of these three are the largest at the rotation speed of 1700 RPM, and the angular displacement amplitudes decrease diminishingly when the rotation speed exceeds 2000 RPM. It can be seen in Fig. 9 (b) that the angular acceleration of amplitudes reach the maximum at the rotation speed of 1750 RPM. Comparing Fig. 9 (a) with (b), the rotation speed which corresponding to the maximum amplitude of angular displacement is different from the rotation speed which corresponding to the maximum amplitude of angular acceleration, and the deviation value of rotation speed is 50 RPM.

The differential equation of transmission system in free vibration is

$$\{J\} \{\ddot{\theta}\} + \{K\} \{\dot{\theta}\} = 0$$

(11)

where $\{J\}$ is the inertia matrix of transmission system; $\{K\}$ is the stiffness matrix transmission system; $\{\ddot{\theta}\}$ is the angular acceleration vector; $\{\dot{\theta}\}$ is the angular displacement vector.

The solution of Eq. (11) is expressed as,

$$\theta_i = A_i \sin(\omega t + \phi)$$

(12)

when Eq. (12) and its second derivative are substituted into Eq. (11), Eq. (11) can be converted to,

$$\left([K] - \omega^2 [J]\right) \{A\} = 0$$

(13)

where $\omega$ is the natural frequency, $\{A\}$ is the amplitude vector.

Solving Eq. (13), the natural frequencies can be acquired, and the first 8 order natural frequencies results are listed in Table 4.

| Order | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|
| Frequency/Hz | 8.85 | 34.07 | 44.49 | 674.64 | 1156.69 | 1272.34 | 2083.78 | 2282.45 |

In particular, the first 4 order mode shapes are presented in Fig. 10.

Fig. 10 Mode shapes of transmission system (a) first order mode shape, (b) second order mode shape, (c) third order mode shape, (d) fourth order mode shape.

In Fig. 10, each point on the x-coordinate represents corresponding inertia element of transmission system, i.e., point 1 and 2 are the inertial elements of motor and clutch; point 3 to 7 are the inertial points of gearbox shaft; point 8 and 9 are the inertial elements of intermediate drive shaft and main drive shaft; point 10 is the inertial elements of pinion shaft; point 11 and 12 are the inertial elements of pinion and gear; point 13 is the inertial element of hub. The first mode shape and the second mode shape in Fig. 10 (a) and (b) indicate that the inertia points of gearbox (point 3-7) vibrate dramatically; the third mode shape in Fig. 10 (c) shows that the inertia point of gearbox (point 3) and inertia elements of drive shafts...
(point 8, 9) vibrate dramatically; the fourth mode shape in Fig. 10 (d) shows that the inertia elements of pinion shaft (point 10), pinion (point 11) and gear (point 12) vibrate dramatically.

### 3.2 Measurement results

The measured angular displacement and acceleration of test rig are presented in Fig. 11.

![Fig. 11 Measurement results of vibration responses of transmission system (a) angular displacement, (b) angular acceleration](image)

As can be seen in Fig. 11 (a), the angular displacement amplitudes are the largest at the rotation speed is 1701 RPM, and the vibration responses decrease when the rotation speed exceeds 1701 RPM. It can be seen in Fig. 11 (b) that the angular acceleration amplitudes reach the maximum at the rotation speed of 1739 RPM. Comparing Fig. 11 (a) with (b), the rotation speed which corresponding to the maximum amplitude of angular displacement is different from the rotation speed which corresponding to the maximum amplitude of angular acceleration, and the deviation value of rotation speed is 38 RPM.

### 4. Discussion

For the convenience of comparison, simulated and measured responses of angular displacement are presented in Fig. 12 (a) and (b), respectively.

![Fig. 12. Angular displacement amplitude of transmission system (a) simulation results, (b) measurement results](image)

As can be seen in Fig. 12 (a) that the transmission system vibrate dramatically at the rotation speed of 1700 RPM, while the measurement results in Fig. 12 (b) show that the transmission system vibrate dramatically at the speed of 1701 RPM.

The simulated and measured responses of angular acceleration are presented in Fig. 13 (a) and (b), respectively.
As can be seen in Fig. 13 (a), the angular acceleration amplitudes reach the maximum at the speed of 1750 RPM; while measurement results in Fig. 13 (b) show that the angular acceleration amplitudes reach the maximum at the speed of 1739 RPM.

A FEA model is developed to investigate the reason why the transmission system vibrates dramatically at a certain rotation speed, and the mode frequencies of drive shaft are listed in Table 5.

### Table 5. Mode frequencies of drive shaft

| Order | Frequency/Hz |
|-------|--------------|
| 1     | 43.45        |
| 2     | 47.79        |
| 3     | 177.08       |
| 4     | 189.35       |

In particular, the first order vibration mode of drive shaft is presented in Fig. 14.

The excitation frequencies of drive shaft at the rotation speed of 1700 RPM and 1750 RPM, which can be calculated with Eq. (14). These excitation frequencies are quite close to the first order natural frequency of drive shaft (43.45 Hz), acquired with FEM as illustrated in Table 5. Therefore, it seems that when the excitation frequencies are close to the first order natural frequency of drive shaft, the resonant vibration of drive shaft is triggered. Furthermore, the resonant vibration of the shaft would induce the resonant vibration of transmission system, given that the first order natural frequency of drive shaft is quite close to the third order natural frequency of transmission system, which is 44.49 Hz as listed in Table 4. In addition, the inertia elements of drive shafts vibrate dramatically at the third order natural frequency of transmission system as illustrated in Fig. 10 (c), which can support the conclusion of this research.

### 5. Conclusions

A theoretical model of transmission system is developed in this research, and torsional vibration responses of transmission system is simulated using this model. The simulation results show that the angular displacement amplitudes
of transmission system reach the maximum value at the rotation speed of 1700 RPM and the angular acceleration amplitudes of transmission system reach the maximum value at the rotation speed of 1750 RPM. Furthermore, the results of natural frequencies of transmission system and corresponding mode shapes have been calculated. The test results show that the angular displacement amplitudes reach the maximum at the rotation speed of 1701 RPM and the angular acceleration amplitudes of transmission system reach the maximum value at the rotation speed of 1739 RPM. Therefore, these measurements could verify the theoretical model. In addition, FEA is applied to investigate the reason why the system vibrates dramatically at certain rotation speed. The results of FEA indicate that the excitation frequencies of drive shaft are quite close to the first order natural frequency of drive shaft, and the resonant vibration of drive shaft is triggered. Furthermore, the resonant vibration of drive shaft would induce the resonant vibration of transmission system, given that the first order natural frequency of drive shaft is quite close to the third order natural frequency of transmission system. In particular, both theoretical and experimental results indicate that there are deviations between the rotation speeds that corresponding to the maximum amplitude of angular displacement and the rotation speeds that corresponding to the maximum amplitude of angular acceleration.

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