Axial properties of the nucleon with $1/N_c$ corrections

in the solitonic SU(3)-NJL-model

Andree Blotz$^{(1,2)}$ *, Michał Praszałowicz$^{(3)}$ † and Klaus Goeke$^{(2)}$ ‡

(1) Institute for Nuclear Theory (INT), HN-12 University of Washington,
Seattle, WA 98195, USA

(2) Institute for Theoretical Physics II,
P.O. Box 102148, Ruhr-University Bochum,
D-W-44780 Bochum, Germany

(3) Institute of Physics,
Jagellonian University, Reymonta 4,30-059,
Krakow, Poland

(March 26, 2022)

Abstract

Within the semibosonized SU(3)-NJL model the mass splittings of baryons and the axial vector coupling constants of the nucleon are evaluated. The mass splittings of the hyperons are reproduced with great accuracy if second order corrections in $m_s$ are taken into account.

*email:andreeb@elektron.tp2.ruhr-uni-bochum.de

†email:michal@thrisc.if.uj.edu.pl

‡email:goeke@hadron.tp2.ruhr-uni-bochum.de
New corrections to the axial vector currents coming from the subleading terms in the $1/N_c$ expansion are shown to be non-vanishing and substantially improving the phenomenological predictions for axial quantities. These corrections are shown to come from two distinctive sources: 1) anomalous part of the Euclidean effective action related to the Wess-Zumino term of the SU(3) Skyrme model and 2) real, non-anomalous part which in this order of $1/N_c$ has no counterpart within any local effective meson theory. The appearance of the type 2) terms is due to some explicit time-ordering of the collective operators entering the formulae for the axial constants. The question of regularization of these quantities is discussed. The analytic symmetry breaking terms in the strange quark mass play a minor role for $g_A^{(3)}$ and $g_A^{(0)}$. They are however important for $g_A^{(8)}$. Finally the numerical values for the $g_A$’s are $g_A^{(0)} = 0.37$, $g_A^{(3)} = 1.38$ and $g_A^{(8)} = 0.31$ reproducing reasonably well the recent data from lepton scattering.
I. INTRODUCTION

It is a long lasting problem to determine static properties of hadrons from the general theory of the strong interaction, Quantum Chromo Dynamics (QCD) [1]. Therefore there were some attempts in the past to derive an effective theory for the strong interactions in some low energy approximation [2–6]. However none of the derivations could exactly claim the range of validity of the approximations. Although the resulting theory, which coincides with the formerly invented Nambu-Jona-Lasinio (NJL) model [7,8], does not confine it shares the maybe most important features of low-energy QCD relevant for ground states. These are: chiral symmetry and its spontaneous breaking. In this case the lagrangian itself is chirally invariant but the vacuum state breaks the symmetry. The symmetry breaking is driven by a non-vanishing quark condensate, leading to the constituent masses of the formerly massless quarks. As a consequence the Goldstone bosons emerge, namely the pion, kaon and eta. These almost massless excitations of $\bar{q}q$ pairs are expected to play a dominant role in the QCD vacuum not only in the long range limit but also at small distances [9,10].

In the presently investigated NJL model the nucleonic scenario is realized by these Goldstone bosons and explicit quark degrees of freedom. The gluon degrees of freedom are already implicitly contained in the lowest order because the action is obtained, at least formally, after path integration over the gluon fields. The quarks are then bound in a selfconsistent potential based on a non-trivial chiral field configuration in the Hartree approximation [11–13] which is leading in $1/N_c$ expansion. Though this picture resembles much an effective meson theory like the Skyrme model [14], one should note that here the soliton is non-topological, i.e. practically speaking based on the non-perturbative dynamics. On the contrary the Skyrme soliton is topological, i.e. the baryon number totally hinges on the topological winding of the chiral field, while in the NJL model the baryon number is carried by 3 (or $N_c$) valence quarks. On
the basis of the analysis of the proton polarization it was argued in Ref. [15] that the valence quark degrees of freedom, missing in the Skyrme model, do account for some important physics.

There is however another, more profound difference between the present model and the effective meson theories. The effective meson theories are based upon some lagrangian density being local in time and space, whereas the NJL type models are formulated in terms of the path integral over the fermions, which can be regarded as a time-ordered product of the field operators. It has been for a long time overlooked that this time-ordering may bring up some new contributions for various baryonic observables. In the recent papers [16–18] these contributions (hitherto referred to as time-ordered) have been calculated for axial decay constants and for magnetic moments.

The example of the axial constants is perhaps the most persuasive. It is well known that in the nonrelativistic quark model $g_A = (N_c + 2)/3$. This means that there are important $1/N_c$ corrections to $g_A$, which for $N_c = 3$ amount to 60% of the leading result. In the effective meson theories the leading term for $g_A$ scales also as $N_c$, however the next-to-leading correction comes only at the $1/N_c$ level in the SU(2) version of the model. This is due to the fact that effective meson lagrangians are even in field derivatives. In the cranking approximation for the rotating soliton each time derivative counts as $1/N_c$. The only possible source for the contributions linear in the time derivative is the Wess-Zumino term which vanishes identically in the SU(2) case. However in the SU(3) case both in the NJL and in the Skyrme model such corrections appear. In the NJL model in the leading order of LWLA (Long Wave Length Approximation) or gradient expansion they are equal to the Witten’s anomalous current [19] of a local mesonic theory.

It was always believed that the effective quark theories are equivalent to the effective meson theories in a sense that they correspond to some calculable local lagrangian
density. Strictly speaking such a lagrangian density can be derived from the NJL like model only in the limit of the large soliton size. Although this statement remains true it does not mean that the matrix elements of some operators, take as example the axial currents, are the same irrespectively if they are calculated straight away from the fermion path integral or from the equivalent local lagrangian density. This is due to the two facts: 1) upon the semiclassical quantization of the soliton the cranking velocities are promoted to the collective operators which do not commute with the rotation matrix itself and 2) the path integral is time-ordered, i.e. it dictates unambiguously in which order the cranking velocity and the rotation matrix appear in the expressions for the matrix elements of the axial currents. If these non-local properties of the path integral are properly taken into account then one gets the desired $1/N_c$ corrections. These corrections are not small and improve the phenomenological predictions of the NJL model. On the contrary in the local limit of the present effective quark theory they are identically zero.

In the previous paper [17] we have calculated the three SU(3) axial decay constants $g_A^{(3)}$, $g_A^{(8)}$ and $g_A^{(0)}$ in the chiral limit within the semibosonized NJL model. This model reproduces the hyperon spectra [20] and also the isospin splittings within baryon multiplets [21]. Earlier the properties of the axial currents have been investigated in the NJL model only for the case of SU(2) [22,13]. Then the $1/N_c$ corrections for SU(2) have been roughly estimated in [16], neglecting regularization and sea contribution and to full extent in [18]. Beyond this there are only calculations of the axial coupling constants within the pseudoscalar SU(3) Skyrme model [23] and the pseudoscalar vector meson SU(3) Skyrme model [24]. Actually hyperon spectra indicate [24] that the scalar and pseudoscalar NJL model gives a more refined structure of the collective hamilton operator, than the pseudoscalar Skyrme model. A comparable structure can be obtained in the Skyrme model only by introducing explicit vector mesons and modeling
the anomalous and symmetry breaking part of the effective action. This introduces a large number of parameters, whereas in the NJL model the parameters can be fixed completely by requiring proper mesonic masses and decay constants. The symmetry breaking pattern and the anomalous effective action is then uniquely determined.

In contrast to our previous paper now we implement explicit time-ordering within the framework of the effective Euclidean action (EEA). This treatment allows us to make a clear distinction between the terms which emerge from the real or form the imaginary (anomalous) part of the EEA. Actually it turns out that the new explicitly time-ordered terms emerge from the real part of the EEA and therefore have to be regularized. In the present approach the regularization prescription is unique and the regularization function is derived in a well defined manner.

In the present paper we furthermore extend our previous calculations and calculate the $m_s$ corrections to the axial coupling constants. Besides the theoretical interest due to the new explicitly time-ordered corrections, the axial constants are of utmost phenomenological importance as far as the comparison with the recent measurements of the polarized proton and neutron structure functions is concerned. So the main phenomenological concern of this paper will be a comparison of the model predictions with the experimental data for these quantities. Especially we will concentrate on the role of the explicitly time-ordered corrections and furthermore on the so called anomalous quantities, which are dominated by the valence contributions. The latter manifest another conceptual difference between the Skyrme model and the present NJL model.

The organization of the paper is as follows. In Sec. II we review the basic features

---

1The free parameters are: $M$ – constituent quark mass, and cut-off parameters and to some extent $m_s$ – strange quark mass
of the NJL model with special emphasis on the solitonic description. In Sec. III we describe the quantization procedure and summarize the results on the hyperon splittings. We use the mass splittings to fix the parameters of the model. In Sec. IV we derive expressions for axial currents in the chiral limit. Special emphasis is put on the new contributions from the explicit time-ordering and their regularization. Then in Sect. V we discuss mass corrections to the axial currents. Our numerical results are presented in Sect. VI. Section VII contains a brief comparison with the results of the Skyrme model. We present our conclusions in Sect. VIII.

In the Appendices we present useful formulas for the semiclassical quantization (App. A), a derivation of the regularization functions (App. B), a gradient expansion for normal and time-ordered quantities (App. C).

II. THE SU(3) NAMBU-JONA-LASINIO MODEL - SOLITONS

The quark Nambu-Jona-Lasinio model \cite{7,8} can be written in the four-fermion formulation as:

\[ \mathcal{L}_{\text{NJL}} = \bar{q}(x)(i\partial - m)q(x) - 2G[(\bar{q}\lambda^a q)^2 + (\bar{q}\gamma_5\lambda^a q)^2]. \]  

Here the summation over the \( \lambda^a \) matrices is implicit (with \( \lambda_0 = \sqrt{2/3} \)) and \( m \) is the bare quark mass matrix. In the chiral limit the Lagrangian has the desired SU(3)_R \( \otimes \) SU(3)_L symmetry in addition to the U(1)_V \( \otimes \) U(1)_A, where the U(1)_A is the symmetry which is not shared by QCD. In principle one could introduce the 't Hooft term into the Lagrangian, which breaks the U(1)_A explicitly. It could then serve as a source for the \( \eta' \) mass, which otherwise would be a Goldstone boson. This was recently done by Kato et al. \cite{26} and the resulting profile for the SU(2) soliton was very similar to the solutions that are restricted to the chiral circle (i.e. non-linear case) and that are used here. So we conclude that the effects of the 't Hooft determinant on the solitonic...
observables of the present calculations are rather small. However it was proven in Ref. [26] that the ’t Hooft term leads to a stabilization of the linear version of the model, which furthermore supports the use of the non-linear Ansatz for the chiral fields.

Performing the bosonization procedure of Eguchi [27] one arrives immediately at the new classical Lagrangian

\[ L_E = \bar{q}(x) \left( -i\partial + g \left( \sigma^a \lambda^a + i\gamma_5 \pi^a \lambda^a \right) \right) q(x) + \frac{1}{2} \mu^2 \left( \sigma^a \sigma^a + \pi^a \pi^a \right), \tag{2} \]

where the original coupling \( G \) is given by \( G = g^2 / \mu^2 \). For convenience Eq.(2) can be rewritten in the polar decomposition of the chiral fields [28]:

\[ L_E = \bar{q}(x) \left( -i\partial + \frac{P_R}{\sqrt{2}} \mathcal{M} + \frac{P_L}{\sqrt{2}} \mathcal{M}^\dagger + m \right) q(x) + \frac{1}{4} \frac{\mu^2}{g^2} \text{Tr}_\lambda \left( M M^\dagger \right), \tag{3} \]

where \( \mathcal{M} = g \left( \sigma^a \lambda^a + i\gamma_5 \pi^a \lambda^a \right) = \xi_L^\dagger M \xi_R \) is given in terms of the unitary matrices \( \xi_L, \xi_R \) and a hermitian matrix \( M \). The \( P_{R/L} = 1/2(1 \pm \gamma_5) \) are the right/left helicity projection operators. Furthermore the unitary gauge \( \xi_R = \xi_L^\dagger = \xi \) can be chosen to eliminate the redundant degrees of freedom. As an approximation the scalar degrees of freedom in \( M \) are frozen, such that \( M \) is just the constituent quark mass matrix and \( \xi \) is related to the chiral field by \( U = \xi^2 \). The latter can be parametrized as \( \exp \left( i \pi^a \lambda^a / f_\pi \right) \). This parametrization corresponds in SU(2) to the chiral circle condition \( \sigma_{(2)}^2 + \bar{\pi}^2 = \text{const} \), where \( \sigma_{(2)} \) is the SU(2) isoscalar \( \sigma \) field. Then the last term in eq. (3) is just a constant and will be omitted in the following.

Now we will shortly summarize how to calculate quantum corrections to \( L_E \) which are due to the fluctuations of the fermion fields. The fermion determinant obtained by the integration over the quark fluctuations corresponds to the leading contribution in \( 1/N_c \) expansion. The next to leading terms, the so called boson determinant, have been

\[ ^2 \text{We will always denote by E Euclidean and by M Minkowski quantities if it is not obvious from the context.} \]
shown to have minor influence on the mesonic observables \[29\]. Hence the effective Euclidean action (EEA) can be written as:

\[
S_{\text{eff}}[U(x)] = -\text{Sp} \log\left(-i\phi + P_R\mathcal{M}(x) + P_L\mathcal{M}^\dagger(x) + m\right),
\]

(4)

where \(\text{Sp}\) is the functional trace in color, flavor and momentum space.

From eq. (4) the parameters of the model can be fixed by requiring experimental values for the pion decay constant \(f_\pi = 93\) MeV, the pion and the kaon mass, \(m_\pi = 139\) MeV and \(m_K = 496\) MeV, (see Ref. \[20\] for details). As a result the constituent quark mass is the only free parameter of the model, which can e.g. be used to fix baryonic properties \[20,21\].

Solitonic solutions of eq. (4) can be found by making a time-independent hedgehog Ansatz for the chiral field \(U\) and writing the SU(2) action in the chiral limit in terms of a single particle intrinsic hamiltonian:

\[
H = -i\gamma_4(-i\gamma_i\partial_i + MU(x)).
\]

(5)

We will specify the SU(3) extension of the SU(2) Ansatz in the next Section. In the proper time regularization \[30\], the effective action becomes in this case:

\[
S_{\text{eff}} = \text{Sp} \int \frac{du}{u} \phi(u) \exp\left(-u(-\partial_\tau^2 + H^2)\right),
\]

(6)

where \(\phi(u) = c\theta(u-1/\Lambda_1^2) + (1-c)\theta(u-1/\Lambda_2^2)\) is the regularization function of Ref. \[20\], which reproduces common values for the current quark masses and quark condensates in the vacuum. The classical equations of motion can be solved selfconsistently for the chiral field \(U\), resulting in a localized soliton with unit winding number. The energy spectrum of the Hamilton operator \(H\) (eq. (4)) for the baryon number one sector contains a discrete valence level inside a mass gap of the size \(2M\) \[12,11,31\]. Then the classical energy of the soliton can be written as \[31\]:

\[
M_{\text{cl}} = N_c E_{\text{val}} + N_c E_{\text{sea}},
\]

(7)
where $E_{\text{val}}$ is the energy of the valence level and $E_{\text{sea}}$ resembles the polarization the Dirac sea as a sum over the whole spectrum of the Hamilton operator $H$.

### III. QUANTIZATION OF ZERO MODES AND MASS SPLITTINGS

The purpose of this section is to apply the semiclassical quantization method to the solitons \[12,11,32\] of Sect. \[1\] which result from the classical and time-independent equations of motion. The idea hereby is the following. In order to quantize the system one can perform a time-dependent transformation \[33\], which can be in the direction of the symmetry or orthogonal to it. If the symmetry is at least an approximate symmetry then excitations in this direction should be the dominant contribution to the low lying resonances of the model. In order to check this numerically in the present model, we consider in addition to the usual expansion of the EEA up to the second order in the rotational velocity \textit{consistently} the quadratic corrections from the strange symmetry breaking terms.

Therefore following the treatment of Ref. \[33\] we quantize the soliton by introducing time-dependent SU(3) rotations and impose canonical quantization conditions for the collective coordinates of the rotation matrix. This will allow for the definition of generators of SU(3) and the corresponding baryon states.

First we make use of the trivial embedding of Witten \[34\] of the SU(2) chiral field $U_0(x) = (\sigma(2) + i\gamma_5\vec{\tau}\vec{\pi})/f_\pi$ into the isospin subgroup of SU(3) according to

$$U(x) = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix}.$$  

The soliton solutions of SU(2) are also solutions for SU(3). In the quantization procedure the embedding \[8\] generates, as will be seen later explicitly, the correct quantum numbers for baryons \[34\].

Next one introduces a time-dependent rotation: $U(x,t) = A(t) U(x) A(t)$. This
rotation can be undone by rotating the quark fields: \( \tilde{q} = A(t)\dagger q \) and \( \tilde{\bar{q}} = \bar{q}A(t) \). Then we obtain:

\[
A^\dagger \dot{A} = i\Omega_E = \frac{i}{2}\lambda_a\Omega^a_E
\]

and the relation between Euclidean and Minkowski velocities holds: \( i\Omega_E = \Omega_M \) and \( \Omega^\dagger_E = \Omega_E \).

Expanding \( S_{\text{rot}} = -\text{Sp}\log(\partial_t + H + i\Omega_E - i\gamma_4 A^\dagger mA) \) of Ref. [20] up to the quadratic order in \( \Omega \) (in Minkowski metric and in the chiral limit) one gets:

\[
L_0 = -M_{\text{cl}} + \frac{1}{2}\Omega_a I_{ab} \Omega_b - \frac{N_c}{2\sqrt{3}}\Omega_8
\]

where tensor of inertia \( I_{ab} = \text{diag}(I_1, I_1, I_1, I_2, I_2, I_2, I_2, 0) \) can be found in Ref. [20].

The original path-integral \( \int DU(x,t) \) will be in the following approximated by the integral over the rotation matrices \( A(t) \) only, neglecting translations and other fluctuations [31]. This is known as the quantization of the rotational zero modes [33]. Instead of functionally integrating over all \( A(t) \), one makes usually use of the fundamental relation between the path-integral and the hamiltonian operator formulation [35]:

\[
\int DA(t) \exp \left( - \int_{-T/2}^{T/2} dt L_E^0 \right) = < A(T/2) \mid \exp \left( -TH^{(0)} \right) \mid A(-T/2) >,
\]

where \( H^{(0)} \) is a collective rotational hamiltonian corresponding to the Lagrangian of eq.(10). In this way the path-integral can be evaluated in terms of the eigenstates of the collective hamiltonian. When we consider expectation values of currents similar relations hold. There however one has to pay attention to the fact that the path-integral is time-ordered in a natural way [35,36], whereas in the operator formalism time-ordering has to be introduced explicitly. The latter fact will be of importance when we consider expectation values of currents in the next section and is extensively described in the case of SU(2) in Ref. [18]. Before we calculate axial properties we shall concentrate on mass splittings. This allows to judge the perturbation expansion
in $m_s$ and to fix the remaining parameter of the model - the constituent quark mass $M$.

To calculate the mass splittings one has to expand the effective action in powers of the current quark mass $m = \mu_0 \lambda_0 - \mu_8 \lambda_8 - \mu_3 \lambda_3$ with

$$
\mu_0 = \frac{1}{\sqrt{6}}(m_u + m_d + m_s), \quad \mu_8 = \frac{1}{\sqrt{12}}(2m_s - m_u - m_d), \quad \mu_3 = \frac{1}{2}\Delta m \quad (12)
$$

where $\Delta m = m_d - m_u$.

Here an important remark is in order. There are apparently two small parameters in the present approach: $1/N_c$ and $m_s$. Unfortunately from the explicit calculations one cannot deduce what actually sets the scale for the $m_s$ corrections. In any case $1/N_c$ and $m_s$ can be treated as being of the same order. Therefore to be consistent we expand the effective action up to terms of the order of $m_s, m_s^2, m_s \Omega$ and $\Omega^2$ (expansion in $\Omega$ corresponds to expansion in $1/N_c$):

$$
L_m = -\sigma m_s + \sigma m_s D^{(8)}_{ss}, \quad (13)
$$

$$
L_{m}^{\Omega} = -\frac{2}{\sqrt{3}}m_s D^{(8)}_{ss} K_{ab} \Omega^b, \quad (14)
$$

$$
L_{m^2} = \frac{2}{9} m_s^2 (N_0 (1 - D^{(8)}_{ss})^2 + 3 N_{ab} D^{(8)}_{sa} D^{(8)}_{sb}), \quad (15)
$$

where the constant $\sigma$ is related to the sigma term $\Sigma = 3/2 (m_u + m_d) \sigma$ and $D^{(8)}_{ab} = 1/2 \text{Tr}(A^\dagger \lambda_a A \lambda_b)$. Therefore we can define in this order $L^\text{tot} = L_0 + L_m + L_{m\Omega} + L_{m^2}$. The mass spectrum obtained with the help of $L_0 + L_m + L_{m\Omega}$ was discussed in Refs. [20, 21]; there one can also find explicit formulae for $K_{ab} = \text{diag}(K_1, K_1, K_1, K_2, K_2, K_2, K_2, 0)$. Let us here only remind that the anomalous moments of inertia $K_i$ are nearly entirely given by the valence part, whereas the contribution of the valence level to $I_i$ amounts to approximately 60%. The new feature of the present calculation is the presence of the moments of inertia $N_{ab} = \text{diag}(N_1, N_1, N_1, N_2, N_2, N_2, N_2, N_0/3)$ in $L_{m^2}$ defined as:
\[
N_{ab} = \frac{N_c}{4} \sum_{n,m} < m | \lambda_a \gamma_0 | n > < n | \lambda_b \gamma_0 | m > \mathcal{R}_\beta(E_n, E_m),
\]

where \( \mathcal{R}_\beta(E_n, E_m) \) is given by
\[
\mathcal{R}_\beta(E_n, E_m) = \frac{1}{2\pi} \int \frac{du}{\sqrt{u}} \phi(u) \left[ \frac{E_n e^{-uE_n^2} - E_m e^{-uE_m^2}}{E_n - E_m} \right],
\]

which differs from the regularization function for the usual moment of inertia \( \mathcal{R}_I(E_n, E_m) \) because of the different hermiticity behavior of the mass term and the Coriolis term (\( \Omega \)) in \( S_{\text{eff}}^{\text{rot}} \).

The values of \( N_{0,1,2} \) together with the values of \( I_{1,2} \) and \( K_{1,2} \) for different constituent masses are listed in Tab. I.

The Lagrangian of eq. (10) and eqs. (13-14) reminds the Skyrmion Lagrangian with vector mesons (c.f. Ref. [24]). The quantization proceeds as in the Skyrme model; one defines the quantities (see App. A for details):
\[
J_a = -R_a = I_{ab} \Omega_b - \mu_i D_{ib} K_{ba} - \delta_{a8} \frac{N_c}{2\sqrt{3}}
\]

\((i = 3 \text{ and } 8, \ a, b = 1 \cdots 8)\) which are promoted to the spin operators \( \hat{J}_a = -\hat{R}_a \).

The flavor operators read: \( \hat{T}_a = -D_{ab} \hat{J}_b \). Note that despite the fact that \( \hat{J}_a \) fulfil the SU(3) algebra, only \( \hat{J}_{1,2,3} \) have the meaning of the symmetry generators. That is due to the structure of the SU(3) hedgehog Ansatz and is reflected in the fact that \( \hat{J}_8 = -N_c/\sqrt{12} \) generates a constraint. Therefore the wave function of the baryon state \( B = Y, T, T_3, J, J_3 \) belonging to the SU(3) representation \( \mathcal{R} \) reads (see App. A):
\[
| \mathcal{R}, B > = \sqrt{\dim\mathcal{R}} \left< Y, I, I_3 | D^{(\mathcal{R})}(A) | -Y', J, -J_3 \right>^*,
\]

where the right hypercharge \( Y' \) is in fact constrained to be \( -1 \). The lowest SU(3) representations which contain states with \( Y = 1 \) are: \( \mathcal{R} = \mathbf{8} \) and \( \mathcal{R} = \mathbf{10} \). The quantized collective hamiltonian \( H^{\text{tot}} \) from
\[
H^{\text{tot}} = \sum_a \Omega_a \frac{\partial L^{\text{tot}}}{\partial \Omega_a} - L^{\text{tot}}
\]
reads:

\[ H^{(0)} = M_{\text{cl}} + H_{\text{SU}(2)} + H_{\text{SU}(3)}, \]

\[ H_{\text{SU}(2)} = \frac{1}{2I_1} C_2(\text{SU}(2)), \quad H_{\text{SU}(3)} = \frac{1}{2I_2} \left[ C_2(\text{SU}(3)) - C_2(\text{SU}(2)) - \frac{N_c^2}{12} \right]. \]

Here \( C_2 \) denote the Casimir operators of the spin SU(2) and flavor SU(3). \( M_{\text{cl}} \) is the classical soliton mass. It has been calculated by many authors and its value turns out to be relatively large: \( M_{\text{cl}} \approx 1.2 \text{ GeV} \). This is a common problem for all chiral models. There are however some negative corrections to it, like Casimir energy or rotational band corrections which might bring \( M_{\text{cl}} \) to the right value. In this paper, instead on insisting on the calculation of the absolute masses, we will concentrate on the mass splittings. These are determined in the present model by analytic strange mass contributions in \( \mathcal{O}(m_s) \) and \( \mathcal{O}(m_s^2) \). Whereas linear terms are given by eqs. (13,14), the various contributions from quadratic \( m_s \) corrections can be classified into the following three cases:

- quadratic terms from the expansion of the EEA, corresponding to the Lagrangian in eq. (13). This will be referred to as kinematical correction.

- quadratic terms from replacing the rotational velocities in eq. (18) by the generators \( J_a \) by using eq. (20). This will be referred to as dynamical correction.

- quadratic corrections from sandwiching the linear part \( L_m, L_m \Omega \), which can mix various representations of SU(3), with linear mass corrections from the collective wave-function, as will be discussed in the following. This will be referred to as wave-function correction.

Then the hamiltonian from eq. (20) up to terms quadratic in \( m_s \) reads:
\[ H^{(1)} = \left\{ \sigma - r_2 Y - (\sigma - r_2)D_{88} + \frac{2}{\sqrt{3}}(r_1 - r_2) \sum_{A=1}^{3} D_{8A} J_A \right\} m_s, \]

\[ H^{(2)}_{\text{kin}} = \frac{2}{3} \left\{ r_2 K_2 (1 - D^2_{88}) + (r_1 K_1 - r_2 K_2) \sum_{A=1}^{3} D^2_{8A} \right\} m_s^2, \]

\[ H^{(2)}_{\text{dyn}} = -\frac{2}{9} \left\{ (N_0 + 3N_2) - 2N_0 D_{88} + (N_0 - 3N_2)D^2_{88} + 3(N_1 - N_2) \sum_{A=1}^{3} D^2_{8A} \right\} m_s^2, \]

where \( r_i = K_i/I_i \). According to items above we have split the \( \mathcal{O}(m_s^2) \) hamiltonian into the kinematical part \( H^{(2)}_{\text{kin}} \), and the dynamical part \( H^{(2)}_{\text{dyn}} \).

The Hamiltonian \( H^{(1)} \) mixes states of different SU(3) representations, therefore the wave function is no longer a pure octet or decuplet but rather a mixture:

\[ | B > = | 8, B > + c_{10}^R | 10, B > + c_{27}^R | 27, B >, \]

\[ | B' > = | 10, B' > + c_{27}' | 27, B' > + c_{35}' | 35, B' >, \]

(23)

where \( B = N, \Lambda, \Sigma, \Xi \) and \( B' = \Delta, \Sigma^*, \Xi^*, \Omega \). The coefficients \( c_R^B \) depend linearly on \( m_s \), therefore with this accuracy there is no need to change the normalization of the wave function. In the following we will need their explicit form only for the octet-like states:

\[ c_{10}^R = \frac{\sqrt{5}}{15} (\sigma - r_1) \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} I_2 m_s, \quad c_{27}^R = \frac{1}{75} (3\sigma + r_1 - 4r_2) \begin{bmatrix} \sqrt{6} \\ 3 \\ 2 \\ \sqrt{6} \end{bmatrix} I_2 m_s, \]

(24)

in the basis \([N, \Lambda, \Sigma, \Xi]\). In Fig.1 we plot \( c_R^B \) in dependence on the constituent mass \( M \). The corresponding \( \mathcal{O}(m_s^2) \) contribution to the energy reads:

\[ E_{\text{wf}}^{(2)} = -\left\{ \frac{1}{60} \left( 1 + Y - X + \frac{1}{2} Y^2 \right) (\sigma - r_1)^2 \\
+ \frac{1}{250} \left( \frac{13}{2} + \frac{5}{2} X - \frac{7}{4} Y^2 \right) \frac{1}{9} (3\sigma + r_1 - 4r_2)^2 \right\} I_2 m_s^2 \]

(25)

for the octet and for the decuplet:

\[ E_{\text{wf}}^{(2)} = -\left\{ \frac{1}{16} \left( 1 + \frac{3}{4} Y + \frac{1}{8} Y^2 \right) \frac{1}{9} (3\sigma - 5r_1 + 2r_2)^2 \\
+ \frac{5}{336} \left( 1 - \frac{1}{4} Y - \frac{1}{8} Y^2 \right) (\sigma + r_1 - 2r_2)^2 \right\} I_2 m_s^2. \]

(26)
Here $X = 1 - I(I + 1) + 1/4 Y^2$ is the usual combination entering Gell-Mann–Okubo mass relations.

With the help of the matrix elements of the $D$ functions and spin operators discussed in the App. A one arrives at the following result for the hyperon splittings:

$$
\Delta M^{(8)} = A - \frac{F}{2} Y - \frac{D}{\sqrt{5}} X - G Y^2,
$$

$$
\Delta M^{(10)} = B - \frac{C}{2\sqrt{2}} Y - H Y^2. \quad (27)
$$

Constants $A$ and $B$ do not contribute to the splittings within the multiplets, however they shift the mass centers and contribute to the 10-8 mass difference. Constants $G$ and $H$ not present in the first order Gell-Mann–Okubo mass formula are of the order of $m_s^2$. Experimentally one gets:

$$
F = \Xi - N = 379 \text{ MeV},
$$

$$
D = \frac{\sqrt{5}}{2} (\Sigma - \Lambda) = 86 \text{ MeV},
$$

$$
G = \frac{1}{4} (3\Lambda + \Sigma) - \frac{1}{2} (N + \Xi) = 6.75 \text{ MeV} \quad (28)
$$

for the octet. For the decuplet the three operators: 1, $Y$ and $Y^2$ do not form a complete basis and therefore there are two independent relations which determine constants $C$ and $H$ with some small uncertainty:

$$
C = \sqrt{2}(\Xi^* - \Delta) = \frac{1}{\sqrt{2}}(\Omega - 2\Delta + \Sigma^*) = 422.5 \pm 3.5 \text{ MeV},
$$

$$
H = \frac{1}{2}(2\Sigma^* - \Xi^* - \Delta) = \frac{1}{6}(3\Sigma^* - 2\Delta - \Omega) = 2.83 \pm 0.33 \text{ MeV}. \quad (29)
$$

In Tab. we list the coefficients $A \ldots H$ for a typical value of $m_s = 180 \text{ MeV}$ as functions of the constituent mass $M$. It can be seen that in order to reproduce the experimental numbers of eqs. (28-29) one has to take the constituent mass of the order of 400 MeV. Then all constants $A \ldots H$ are roughly reproduced. The constant $G$ and $H$ being of the order $O(m_s^2)$ are small. For reasonable strange quark masses $O(m_s^2)$
corrections to $A, B, C$ and $D$ are of the order of 20% of the leading $O(m_s)$ terms with the exception of $F$ for which $O(m_s^2)$ corrections are almost zero.

In order to make phenomenological statements we adopt the following procedure: first for given $M$ we find the optimal $m_s$ which reproduces $10.8$ splitting. To this end we define the mean octet and decuplet values: $\overline{M}_8 = 1/2 (\Lambda + \Sigma) = 1155$ MeV and $\overline{M}_{10} = \Sigma^* = 1385$ MeV. Then $\Delta_{10-8} \equiv \overline{M}_{10} - \overline{M}_8 = 230$ MeV is given by:

$$\Delta_{10-8} = \frac{3}{2I_1} + B - A.$$ (30)

Since $A - B = \text{const.} \times m_s^2$ one can numerically solve eq.(30) for $m_s$. The result is plotted in Fig.2.

In Fig.3 we show the $m_s$ dependence of the deviations $\text{theory} - \text{experiment}$ for each hyperon. On should remember that for each $m_s$ the optimal constituent quark mass $M$ was used, so that $\Delta_{10-8}$ was automatically reproduced for each $m_s$. The smallest deviations $\pm 7$ MeV for all splittings correspond to $m_s \approx 185$ MeV, i.e. $M \approx 425$ MeV.

It is interesting to examine to what extent the new corrections calculated in this paper are important. The Yabu–Ando method of diagonalizing the hamiltonian of $O(m_s)$ exactly is widely spread in the literature [37,24,20,38] and it essentially corresponds, in our language, to taking into account only the wave function corrections of eqs.(25,26). Indeed for $m_s$ of the order of 200 MeV the second order wave function correction almost exactly coincides with the exact result of the Yabu–Ando method. This is illustrated in Fig.4. However consistency requires to include the kinematical and dynamical contributions of eqs.(22) in the same order of $m_s$. Whereas the kinematical corrections are always small the dynamical ones are by no means negligible. This is explicitly shown in Tab. III, where all contributions to constants $A \ldots H$ are displayed for $M = 423$ MeV and $m_s=180$ MeV. Note that $O(m_s^2)$ corrections to $F$ are negligible.

The message here is clear: The wave function contributions (w.f.) are not the whole
story and one has to include consistently all corrections, i.e. in addition kinematical and dynamical contributions, in a given order of $m_s$.

The purpose of this section was twofold: First we have demonstrated the importance of the $O(m_s^2)$ corrections coming from the effective action as compared to the wave function corrections (the Yabu-Ando approach). Second we have used the mass splittings to fix the parameter of the model, namely the constituent mass $M$. Having done this we can proceed to the evaluation of the axial coupling constants.

IV. AXIAL CURRENTS IN CHIRAL LIMIT

In order to calculate observables like the axial vector currents, one has to consider the path integral expectation value of these operators. This can be also done within the formalism of the quark correlation functions, see e.g. Refs. [13,31]. Here will use however the effective action approach and show how the time-ordering within a quark loop together with the collective quantization brings up the corrections linear in the rotational velocity $\Omega$ [16–18]. The approach presented here will be different from the one of Ref. [17], where these new corrections were calculated from the unregularized expressions.

One can express the axial vector current $A^a_\mu$ as a path integral expectation value

$$<A^a_\mu(x)> = N \int D\bar{q}Dq \int DU (\bar{q}\gamma_\mu\gamma_5\lambda^a q) e^{-\int d^4x L_E} = \frac{\delta}{\delta s(x)} \left[ \int D\bar{q}Dq \int DU \exp \left\{ -\int d^4x (L_E - s \bar{q}\gamma_\mu\gamma_5 I_a q) \right\} \right]_{s=0}$$

(31)

with the convention $\gamma_0 = -i\gamma_4$ and the following definitions for $I_a$:

$$g_A^{(0)} : I = 1, \quad g_A^{(3)} : I = \lambda_3, \quad g_A^{(8)} : I = \lambda_8.$$  

(32)

As in Sect. III the quantization is performed by introducing the time-dependent SU(3) rotation matrix $A(t)$. Then we obtain
\[ < A_\mu^a(x) > = \frac{\delta}{\delta s(x)} \left[ \int D\bar{q}Dq \int DU \exp \left\{ - \int d^4x \left( \tilde{L}_E - s \tilde{q} \gamma_\mu \gamma_5 A^\dagger \lambda^a A \tilde{q} \right) \right\} \right]_{s=0} \]  

with \( \tilde{q} = A(t)\dagger q \) and \( \tilde{q} = \bar{q}A(t) \), and the rotated lagrangian \( \tilde{L}_E \)

\[ \tilde{L}_E = \tilde{q} \left( -i\partial + M U(x) + A^\dagger mA - i\gamma_4 A^\dagger \dot{A} \right) \tilde{q}. \]  

Integrating over the quark fields and restricting the \( DU \) integration to the SU(3) rotations [31] gives for the space components of the current

\[ < A_i^a(x) > = \frac{\delta}{\delta s(x)} \left[ \int DA(t) \ Sp_{(to)} \ \log D[s] \right]_{s=0}, \]  

where \( D[s] = \partial_t + H + A^\dagger \dot{A} - i\gamma_4 A^\dagger mA + i \ s \ \gamma_4 \gamma_i \gamma_5 A^\dagger \lambda^a A. \)

Expression (35) needs now a careful explanation. This is due to the fact that we are not going to perform the path-integral over the rotational matrices but, instead, we will use the operator formalism after the quantization of the generalized SU(3) coordinates has been performed. Within the path integral the time-ordering of the operators after the \( DA \) integration is given in a natural way (see e.g. Ref. [36]). In the operator formalism we can make use of the trace properties only if we respect the time-ordering of the operators (intrinsic and collective) within the trace. This is denoted in short by the modified trace \( Sp_{(to)} \), which has the usual properties except for the time component. Only after the explicit time ordering of the operators the rotational frequencies \( \Omega \) can be again considered as time-independent.

One great advantage of retaining the trace in this form is the straightforward applicability of the regularization procedure. Let us split the effective action into real and imaginary part:

\[ < A_i^a(x) > = \int DA(t) \frac{\delta}{\delta s(x)} \left[ \Re Sp_{(to)} \ \log D + i \ \Im Sp_{(to)} \ \log D \right] \bigg|_{s=0} \]

\[ = \int DA(t) \frac{\delta}{\delta s(x)} \frac{1}{2} \left[ Sp_{(to)} \ \log D^\dagger D + Sp_{(to)} \ \log \frac{D}{D^\dagger} \right] \bigg|_{s=0}. \]  

19
It is important to consider \( s(x) \) as explicitly time-dependent (see App. B). Preserving vector gauge invariance \([39]\) by using the proper time regularization \([30]\) we regularize the real part of the effective action:

\[
\text{Sp}(\tau_0) \log D^\dagger D \rightarrow \text{Sp}(\tau_0) \int \frac{du}{u} \phi(u) \exp \left(-uD^\dagger D\right),
\]

where \( \phi(u) \) is given in Sect. II. Note that for symmetric contributions \([17]\), i.e. when the index of a generator \( \hat{J}_a \) is such that it commutes with the \( D_{bc}\)-function, the time-ordering has no influence.

In the following the separation into valence and sea part is done by introducing \( D' = D - \mu \), where \( \mu \) is a chemical potential with \( 0 < \mu < E_{\text{val}} \) \([12]\), such that \( S_{\text{eff}} = S_{\text{val}} + S_{\text{sea}} \) with the definitions \( S_{\text{val}} = S_{\text{eff}}[D'] - S_{\text{eff}}[D] \) and \( S_{\text{sea}} = S_{\text{eff}}[D] \). The subtraction of possible vacuum contributions is implicitly understood.

**A. The lowest order contribution \( \sim \Omega^0 \)**

The axial vector coupling constants \( g_A^a \), defined as the corresponding formfactor in the limit \( q^2 = 0 \), can be calculated from eq. \((36)\). Noting that the functional integral \( \mathcal{D} A \) over the rotation matrices can be replaced by an ordinary integration \( \int d\xi_A \) over the collective coordinates one can define an operator \( \hat{g}_A^a \) such that

\[
g_A^a = \int d\xi_A < B(\xi_A) | \hat{g}_A^a | B(\xi_A) >,
\]

where \( | B(\xi_A) > \) is the baryon wave-function of eq. \((23)\). Therefore \( \hat{g}_A^a \) is obtained by expanding \( < A^a_i(x) > \) in eq. \((36)\) in terms of the rotational frequency and strange mass but without performing the \( \mathcal{D} A\)-integration. The rotational velocity is then replaced by the generators of \((18)\) yielding after integration over 3-dimensional space the collective operator \( \hat{g}_A^a \) in terms of \( J_a \) and Wigner functions \( D_{ab} \).
Therefore the lowest order result in $\Omega$ (i.e. $\Omega^0$) comes from the proper-time regularized real part of the EEA \((37)\). One obtains for $a = 3$ and $8$ (see App. B for details):

\[
\hat{g}_\Lambda^n(\Omega^0, m_s^0) = M_3 D_{a3},
\]

(39)

where $A^\dagger I_a A = D_{ab} \lambda_b$ for $a = 3$ and $8$, and $A^\dagger I_a A = 1$ for $a = 0$. At this level $\hat{g}_\Lambda^0 \equiv 0$.

The quantity $M_3 = M_{3,\text{val}} + M_{3,\text{sea}}$ which comes from the real part of the action is then given by:

\[
M_{3,\text{val}} = N_c < v | \gamma_0 \gamma_3 \gamma_5 \lambda_3 | v >
\]

(40)

and

\[
M_{3,\text{sea}} = -\frac{N_c}{2} \sum_{\text{all } n} < n | \gamma_0 \gamma_3 \gamma_5 \lambda_3 | n > \text{sign}(E_n) R_\Sigma(E_n),
\]

(41)

where the regularization function reads \([25]\):

\[
R_\Sigma(E_n) = \frac{1}{\sqrt{\pi}} \int_0^\infty du u e^{-u} \phi\left(\frac{u}{E_n^2}\right).
\]

(42)

The values of $M_3$ are displayed in Tab. \([V]\).

\section*{B. Anomalous $1/N_c$ corrections from the imaginary part of the EEA}

Taking $1/N_c$ corrections (i.e. terms linear in the rotational velocity $\Omega$) into account, one can make the clear separation into \textit{local} quantities which emerge from the imaginary part of the effective Euclidean action, and \textit{non-local} quantities emerging from the real part due to explicit time-ordering of collective operators. It will turn out that the former quantities are related to Witten’s anomalous axial current \([34]\), whereas the latter ones have no counterpart in a mesonic effective theory. In a certain sense they renormalize the leading contribution of the axial current given by eq.\((39)\) (compare with Refs. \([10,11]\)).
In the chiral limit the anomalous corrections linear in $\Omega$ can be written as:

$$\hat{g}_A^a = -M_{bc}i\{D_{ab}, \Omega^c_E\} = -2M_{bc}D_{ab}\Omega^c_M$$

(43)

with

$$M_{bc} = \frac{N_c}{4} \sum_{n,m} <n|\sigma_3\lambda_b|m><m|\lambda_c|n> \mathcal{R}_\mathcal{M}(E_n,E_m)$$

(44)

and the cutoff independent regularization function $\mathcal{R}_\mathcal{M}$

$$\mathcal{R}_\mathcal{M}(E_n,E_m) = \frac{1}{2} \frac{\text{sign}(E_n - \mu) - \text{sign}(E_m - \mu)}{E_n - E_m}.$$ 

(45)

As noted above the chemical potential $\mu$ is chosen in such a way, that it always lies between the valence level and the positive continuum of states. In this way the quantities $M_{bc}$ correspond to the full baryon number one contribution and therefore contain the sum of the valence and the sea part. Additionally we define for later use $\overline{M}_{8a} = \sqrt{3}M_{8a}$ and $\overline{M}_{a8} = \sqrt{3}M_{a8}$. The only non-vanishing contributions in eq. (43) are: $M_{55}$ and $M_{44} = M_{55} = -M_{66} = -M_{77}$. Using the symmetries of the hedgehog states one can write for $a = 3$ and 8:

$$\hat{g}_A^a = -2\overline{M}_{83}D_{a8}\Omega_3 - 4M_{44}d_{3bb}D_{ab}\Omega_b,$$

(46)

where the sum over $b = 4 \ldots 7$ is understood. The values of the coefficients entering eq.(46) are displayed in Tabs. [IV] and [VI]. It is clear from the form of eq.(46) that the anomalous corrections linear in $\Omega$ vanish in the SU(2) case.

C. Normal 1/$N_c$ corrections from the real part of the EEA

Apart from the anomalous contributions of the preceding subsection, which have their counterparts within the SU(3) Skyrme model [42,24], there exist corrections linear in $\Omega$, which come from the real, non-anomalous part of the EEA and which are due
to some explicit time-ordering of operators. They were recently discussed within the SU(2) \[16,18\] and SU(3) \[17\] version of present model. As advertised in the beginning of this section these terms emerge because the operators \( \hat{J}_a \) and \( D_{ab} \) in the space of the collective coordinates are in general time-dependent and have to be time-ordered, if one makes use of the operator formalism \[21\]. This is described at length in Appendix B and as a result the contribution to \( \hat{g}_A^a \) from these terms can be summarized as:

\[
\hat{g}_A^a = \frac{N_c}{4} i [\Omega^c_E, D_{ab}] \sum_{m,n} < n | \lambda^c | m > < m | \sigma_i \lambda^b | n > R_Q(E_n, E_m) \tag{47}
\]

where the rather complicated regularization function \( R_Q \) is given by:

\[
R_Q(E_n, E_m) = \int_0^1 \frac{d\alpha}{2\pi} \frac{\alpha E_n - (1-\alpha)E_m}{\sqrt{\alpha(1-\alpha)}} c_i \exp \left( -\frac{[\alpha E_n^2 + (1-\alpha)E_m^2]/\Lambda_i^2}{\alpha E_n^2 + (1-\alpha)E_m^2} \right). \tag{48}
\]

Here the proper-time \( u \)-integration for our step-like functions \( \phi(u) \) has been already performed (see App.B for a general expression). In the limit \( \Lambda_i \rightarrow \infty \) eq. (48) immediately reduces to eq. (B13) of App. B and coincides therefore with our former prescription in Ref. \[17\]. However, as we will see later, with the regularization properly taken into account, the physical values will come out much better.

Using the quantization condition for \( \Omega \) and making use of the commutator \([\hat{J}_c, D_{ab}] = i f_{cda}D_{ad} \] \[43\] eq. (47) can be written as:

\[
\hat{g}_A^a = -\frac{i f_{cda}}{I_{cc}} Q_{bc} \]

\[
= - \left( \frac{2iQ_{12}}{I_1} + \frac{2iQ_{45}}{I_2} \right) D_{a3}, \tag{49}
\]

where the quantities \( Q_{bc} \) coming from the real part of the EEA are given by \( Q_{bc} = Q_{bc,\text{val}} + Q_{bc,\text{sea}} \). Explicitly the valence part reads:

\[
Q_{bc,\text{val}} = \frac{N_c}{2} \sum_n < n | \sigma_3 \lambda^b | v > < v | \lambda^c | n > \text{sign}E_n \tag{50}
\]

and the sea part:
\[ Q_{bc,\text{sea}} = \frac{N_c}{2} \sum_{m,n} <n| \gamma_0 \gamma_3 \gamma_5 \lambda_b | m> <m| \lambda_c | n> \mathcal{R}_Q(E_n, E_m). \]

The numerical values for the \( Q_{bc} \) can be found in Tab. V.

One should note however that the appearance of \( Q_{bc} \) is due to the fact that the operators in the EEA are explicitly time-ordered and that in addition the matrix elements \( <n| \gamma_0 \gamma_3 \gamma_5 \lambda_b | m> <m| \lambda_c | n> \) are antisymmetric with respect to the interchange of \( m \) and \( n \).

The valence contribution \( Q_{bc,\text{val}} \) differs from the formula given in Ref. [16], where the existence of such corrections was claimed for the first time. The correct path-integral formula is given by eq. (50) and eq. (51). Numerically however the difference between our expression for \( Q_{bc,\text{val}} \) and the expression of Ref. [16] is quite small. Note also that in Ref. [16] the sea contribution to \( Q_{bc} \) was erroneously claimed to be identically zero. Again numerically \( Q_{bc,\text{sea}} \) is rather small.

Putting all these corrections together one obtains:

\[
\hat{g}^0_a(\Omega^0 + \Omega^1, m_s^0) = \left[ M_3 - \frac{2iQ_{12}}{I_1} - \frac{2iQ_{45}}{I_2} \right] D_{a3} \\
- \frac{2M_{83}}{\sqrt{3}I_1} D_{a8} \hat{j}_3 - 4\frac{M_{44}}{I_2} D_{3b} D_{ab} \hat{j}_b
\]

(b runs over 4 ... 7). Note that all the quantities \( M_3, Q_{bc}, M_{bc} \) and also \( I_{1,2} \) are of the order \( O(N_c) \), such that the \( Q_{bc} \)-terms in the brackets indeed correspond to \( 1/N_c \) corrections to the lowest order result. In other words as far as one neglects the anomalous, purely SU(3) contribution in eq. (52), the ratio of different \( \hat{g}_A^a \)'s has no \( 1/N_c \) correction.

D. The anomalous singlet axial current

The singlet axial vector current was already given in [14] and it gets only anomalous contribution linear in \( \Omega \):

\[
\hat{g}_A^0(\Omega^1, m_s^0) = -\frac{2M_{83}}{I_1} \hat{j}_3.
\]
Note that eq. (53) given here in the context of SU(3) coincides exactly with the SU(2) result of Ref. [13]. This is because only spin eigenvalues \( J_3 \) enter here, whereas the other \( \hat{g}_A \)'s always contain \( D \)-functions, whose matrix elements depend crucially on the SU\((N_{\text{flavor}})\) algebra used.

E. The axial currents in the leading order LWLA

For large size of the soliton one can approximate the different contributions in eq. (52) by the gradient expansion (or long wave-length approximation (LWLA)) [17]. The lowest order result for SU(2) is given in Ref. [22] and it coincides with the expressions from the Skyrme model. Terms linear in \( \Omega \) can be also gradient-expanded. In SU(3) one gets the anomalous contribution coinciding with the Wess-Zumino-Witten term in the Skyrme model. Using the results of App. C one can also calculate the LWLA of the \textit{time-ordered} terms\(^3\). Altogether one obtains:

\[
\hat{g}_A^a = \int drr^2 \left( \theta' + \frac{\sin 2\theta}{r} \right) \left[ \frac{8\pi}{3} f_{\pi}^2 + \frac{M}{4I_1} + \frac{M}{8I_2} \right] D_{a3} \\
-\frac{4N_c}{I_2} \frac{I_1}{6\pi} \int drr' \sin^3 \theta(r) \, d_{3ab} D_{ab} \hat{J}_b. 
\] (54)

It is clear from eq. (54) that the \textit{time-ordered} contributions (i.e. \( Q_{bc} \)) lead to the renormalization of \( g_A \) in the sense that they are also proportional to \( D_{a3} \). In SU(2) where the second line of eq. (54) vanishes the ratio of \( \hat{g}_A^a \)'s for different baryons has no \( 1/N_c \) correction. This was also found by Dashen and Manohar from large \( N_c \) QCD [40,41]. Furthermore the SU(2) result resembles very much the old non-relativistic quark model prediction for the \( 1/N_c \) correction which is given by: \( g_A = N_c/3 + 2/3 \).

Using the value \( I_1 = 1.156 \) fm for \( M = 423 \) MeV from Tab. [4] the SU(2) part of eq. (54) is

\(^3\)The moments \( Q_{bc} \) are evaluated here in the infinite cutoff limit in order to get a simple and cutoff independent result.
(54) gets approximately 50% correction from the $1/N_c$ term. In SU(3) however, there are additional corrections from the third term in the brackets of eq. (54) and from the anomalous terms, such that the simple rescaling factor does not exist any more.

V. MASS CORRECTIONS FOR $G_A$

In this Section we will evaluate the symmetry breaking corrections to the axial currents due to the non-vanishing strange quark mass. These arise from the term $A^\dagger mA = \mu_0 - \mu_8 \lambda^a D_{8a}$. In the linear order in $m_s$ and in the zeroth order in $\Omega$ neither contributions from the imaginary part nor from the explicit time-ordering (because D-functions always commute with each other) exists. Therefore entire symmetry breaking contribution comes from the real part of the EEA. Performing the expansion of the real part of the EEA in $m_s$ one gets:

$$
\hat{g}_A^a(\Omega^0, m_1^s) = -\frac{4m_s}{\sqrt{3}} R_{38} D_{a3} (1 - D_{88}) + \frac{4m_s}{\sqrt{3}} R_{83} D_{a8} D_{83} + 8\frac{m_s}{\sqrt{3}} d_{3b} R_{44} D_{ab} D_{8b}
$$

(55)

with $b = 4 \ldots 7$ The proper time regularized quantities $R_{bc} = R_{bc,\text{val}} + R_{bc,\text{sea}}$ are given by:

$$
R_{bc,\text{val}} = \frac{N_c}{2} \sum_n < n | \sigma_3 \lambda_b | v > < v | \lambda_c \gamma_0 | n > \frac{E_n - E_v}{E_n - E_m} (56)
$$

and

$$
R_{bc,\text{sea}} = \frac{N_c}{4} \sum_{n,m} < n | \sigma_3 \lambda_b | m > < m | \lambda_c \gamma_0 | n > R_\beta(E_n, E_m).
$$

(57)

with

$$
R_\beta(E_n, E_m) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{dt}{\sqrt{t}} \phi(t) \left[ \frac{E_n e^{-tE_n^2} - E_m e^{-tE_m^2}}{E_n - E_m} \right]
$$

(58)

For future use we also define $\overline{R}_{83} = \sqrt{3}R_{88}$ and $\overline{R}_{38} = \sqrt{3}R_{38}$. Note that $R_\beta(E_n, E_m)$ is different from the regularization functions $R_I(E_n, E_m)$ and $R_Q(E_n, E_m)$. The origin
of this difference, which however survives only in the finite cutoff case, is the different
hermiticity behavior of the current and the mass term on the one hand and the Coriolis
term $i\Omega_E$ in eq. (33) on the other hand. The latter one turns out to be antihermitian
in Euclidean space, whereas the former ones are hermitian. Because the proper time
regularization rests on building $D_E^\dagger D_E$ from the very beginning, different signs emerge
and lead to different regularization functions. Their substantial different behavior can
be seen in Fig. 1 of Ref. [44] and Fig.5 of the present paper. We list the new coefficients
$R_{ab}$ in Tab. VII and Tab. VIII.

Apart from these terms from the action we have in addition the $m_s$ terms from
the quantization condition (18). Including these mass corrections and the ones from
the expansion of the effective action we obtain up to the linear order in the symmetry
breaking and the rotational frequency (for $a = 3$ and 8):

$$\hat{g}_A^a = \left[ M_3 - \frac{2iQ_{12}}{I_1} - \frac{2iQ_{45}}{I_2} \right] D_{a3} - \frac{4M_{44}}{I_2} d_{3bb} D_{ab} \hat{J}_b$$

$$- \frac{2M_{83}}{\sqrt{3}I_1} (1 + \frac{4m_s K_1}{\sqrt{3} I_1} D_{83}) D_{a8} \hat{J}_3$$

$$+ \frac{4m_s}{\sqrt{3}} R_{83} D_{a8} D_{83} + \frac{8m_s}{\sqrt{3}} (R_{44} - M_{44} \frac{K_2}{I_2}) d_{3bb} D_{ab} D_{8b}$$

$$- \frac{4m_s}{\sqrt{3}} R_{38} D_{a3} (1 - D_{88}), \tag{59}$$

where, as usually, the index $b$ in $d_{3bb}$ runs over 4...7.

The quantities $R_{83}$ and $M_{83}$ are already known from the expression of the flavor
singlet axial constant [44]. We found there\footnote{Comparison with Ref. [44] can be done by identifying $\overline{M}_{83} = \beta_1$ and $\overline{R}_{83} = \beta_2$. Note also that the sign in Eq.(5) in Ref. [44] is misprinted.} in the same order of the expansion:

$$\hat{g}_A^a = - \frac{2M_{83}}{I_1} \hat{J}_3 - \frac{4m_s}{\sqrt{3}} D_{83} \left( \frac{K_1}{I_1} \overline{M}_{83} - \overline{R}_{83} \right). \tag{60}$$

The main difference between eq. (59) and eq. (60) is that the lowest order term for
$g_A^{(0)}$ is purely anomalous, whereas the corresponding term for the $g_A^a$ is non-anomalous. We come back to this point, when we make the comparison with the Skyrme model in Sect. VI.

VI. NUMERICAL RESULTS FOR AXIAL CURRENTS

The three different measurements of the spin asymmetry in the polarized lepton–nucleon deep inelastic scattering $[46]–[49]$ have been recently reexamined by Ellis and Karliner $[50]$. The message of their work is that whereas the Bjorken sum rule $[51]$ is in agreement with the data, the Ellis–Jaffe sum rule $[52,53]$ is violated and the results read finally:

$$g_A^{(0),exp} = 0.24 \pm 0.09, \quad g_A^{(8),exp} = 0.35 \pm 0.04 \quad \text{and} \quad g_A^{(3),exp} = 1.25. \quad (61)$$

In this section we discuss our numerical results for the three axial decay constants $g_A^{(3)}$, $g_A^{(8)}$ and $g_A^{(0)}$ including the strange mass corrections. They are summarized in Tab. VIII, Tab. IX and Tab. X. Our final values for a constituent quark mass $M = 420\text{MeV}$ are given by

$$g_A^{(0)} = 0.37, \quad g_A^{(8)} = 0.31 \quad \text{and} \quad g_A^{(3)} = 1.38. \quad (62)$$

In Tab. VIII the difference between SU(2) and SU(3) results for $g_A^{(3)}$ can be seen in each order of the $1/N_c$ expansion. Obviously the lowest order contribution ($\Omega^0$) in SU(3) is significantly smaller than in SU(2) due to the fact that the SU(3) expectation value of the corresponding $D$-function $D_{33}$ is only 70% of the SU(2) value. The anomalous contribution of Eq.$(46)$ linear in $\Omega$ which is non-zero only in the SU(3) case acts as a substitute for this group-theoretical reduction. Indeed, it leads to an almost exact readjustment of the SU(3) value to the SU(2) one. For our preferred value of $M = 423 \text{MeV}$ from the hyperon spectra and $m_s = 180 \text{MeV}$ it pushes the leading
order SU(3) result up to \( g_A^{(3)} \approx 0.84 \). These two values of the model parameters \( M \) and \( m_s \) will be used in the following discussion of the numerical results. Due to the presence of the quantities \( Q_{bc} \) from the explicit time-ordering the SU(2), as well as the SU(3) results, have corrections linear in the rotational velocity. These conceptually new terms have no counterparts in the ordinary Skyrme model. Similarly to the old non-relativistic quark model estimates of the \( 1/N_c \) correction, i.e. \( g_A^{(3)} = N_c/3 + 2/3 \), these new terms turn out to be of the order of 50\% of the leading term. For SU(2) they push \( g_A^{(3)} \) from 0.84 to 1.15 and in SU(3) they give the final value of 1.31. Note that the latter value is obtained with regularized \emph{time-ordered} quantities \( Q_{bc} \). In the present work we have derived \( Q_{bc} \) from the proper-time regularized real part of the EEA. In Ref. [17], where the regularization was neglected as a first approximation\footnote{Note that for this purpose the functional trace \( \text{Sp} \) had to be changed to \( \text{Sp}^{(to)} \) which has different properties in time indices.}, the sea part of the quantities \( Q_{bc} \) made a \( \approx 30\% \) contribution to the total value of the \( Q_{bc} \). Although the \( Q_{bc} \) are finite it should be regularized since it is a quantity coming from the real part of the EEA. This is done in the present paper and as a result with the inclusion of a regularization function the sea contribution is less than 3\% (for \( M = 423 \text{MeV} \), see Tab. [V]).

Various contributions from the strange quark mass (kinematical, dynamical and wave function) increase the value of \( g_A^{(3)} \) of about \( \simeq 5\% \) up to \( g_A^{(3)} = 1.38 \), such that the experimental value \( g_A^{(3),\text{exp}} = 1.25 \) is overestimated only by \( \simeq 10\% \).

It has to be stressed that this is in contrast to all calculations within the purely pseudoscalar [23] or pseudoscalar and vector Skyrme model [24,42], in which \( g_A^{(3)} \) is underestimated by \( \simeq 30\% \). That this significant difference is due to the presence of the new terms from the time-ordering of the functional trace is most clearly evident.
from Tab. \( \text{X} \). There the flavour contributions to the axial current are given for the 
Skyrme model and the NJL model with and without the time-ordered (T) corrections. 
Without the new corrections the NJL model resembles very much the numerical results 
of the SU(3) Skyrme model with vector mesons. This was already noted at the level 
of the collective hamiltonian for the mass splittings in Ref. \([20]\) and here can be seen 
numerically for the axial currents with a high accuracy.

Apart from \( g_A^{(3)} \) we list also the values for \( g_A^{(0)} \), partially given already in Ref. \([14]\), 
and \( g_A^{(8)} \). The spin of the proton, that is carried by the quarks, and is equal to the 
matrix element of the flavour singlet axial vector current, has no contribution in the 
order \( \Omega^0 \), but gets a first non-vanishing contribution in the linear order of \( \Omega \). This, 
as can be seen from Tab. \( \text{IX} \), is also the dominating contribution which gets only 
a very small strange mass correction from the kinematical or dynamical parts. The 
corrections from the wave-function vanish in the linear order of \( m_s \), because the spin 
operator is diagonal in space of the higher SU(3) representations. For \( M = 423 \) MeV, 
the theoretical value of \( g_A^{(0)} \approx 0.37 \) is a little bit above the experimental error bars, 
evertheless one has to keep in mind that the analysis of the experimental data is still 
under debate \([50]\).

Experimental extraction of \( g_A^{(8)} \) from the hyperon semi-leptonic decays depends on 
how the strange quark mass corrections to the SU(3) symmetric result are taken into 
account. Therefore the experimental error bars on this quantity may be at present 
too small. In the present calculation we obtain \( g_A^{(8)} = 0.31 \) to be compared with the 
'experimental' number of \([50]\) \( g_A^{(8), \text{exp}} = 0.35 \pm 0.04 \).

So from our calculations one can conclude that for the 'fixed' mass of \( M = 423 \) MeV 
and \( m_s = 180 \) MeV all three axial vector coupling constants are quite close to the 
experimental values of \([50]\). From Fig. 6 it can be seen that for larger mass of \( M \approx 550 \) MeV, \( g_A^{(3)} \) and \( g_A^{(0)} \) almost coincide with the experimental values, whereas \( g_A^{(8)} \),
having relative large negative strange quark mass correction, deviates from the central value 0.35. However, ignoring the large $m_s$ corrections (as it is usually assumed in the analysis of the hyperon semileptonic decays [54]), even the value of $g_A^{(8)}$ coincides with the experimental value of Ref. [50].

In our view there is no reason to talk of a spin crisis of the nucleon. In the present approach the proton is treated as a many-body system of valence and sea quarks and the data of the EMC-experiments are basically reproduced.

**VII. COMPARISON WITH THE SKYRME MODEL**

Now we want to compare our results with the Skyrme model, which can be regarded as a large constituent quark mass limit of the NJL model [12], or – equivalently – the large soliton size limit. It was already mentioned in Ref. [25] that the formula for the mass splitting of the present SU(3) NJL model is much more subtle than the corresponding expression for the pseudoscalar Skyrme model. This is because certain important anomalous terms dominated by the valence quarks are missing in the effective action of the Skyrme model. This has been cured in the Skyrme model only by introducing vector mesons [12,24]. Therefore we will focus here on the Skyrme model, in which vector mesons and in addition kaon fluctuations and the gauged Wess-Zumino term are added. Then the collective operator has the following structure [24]:

$$\hat{g}_A^3 = a_1 D_{33} + a_2 d_{3aa} D_{38} + a_3 D_{38} + a_4 d_{3aa} D_{3a} D_{8a} + a_5 D_{38} (1 - D_{88}) + a_6 D_{38} D_{83}$$

(63)

which corresponds effectively to the expression for the NJL model. Although the origin of the various coefficients are quite different in the NJL and Skyrme model, both approaches give effectively the same operator structure for $g_A$.

However one should stress here two important differences: first of all the new
corrections linear in $\Omega$ which arise due to the time-ordering within the fermion loop vanish in the Skyrme model identically. The Skyrme model is based upon a local lagrangian density which, apart from the Wess-Zumino term, is even in time derivatives and therefore the spatial components of the axial currents are also even not allowing for terms linear terms in $\Omega$. Second, even if one restricts oneself to the terms not including the corrections due to the time-ordering (local limit) the contribution of the valence quarks in the present model makes our results qualitatively very different from the ones of the Skyrme model. The coefficient $a_2$ e.g. is in the Skyrme model with purely pseudoscalar mesons dominated by the induced kaon fluctuations [23], which are neglected in the present NJL model. If the vector mesons are included in the Skyrme approach the situation does not change qualitatively [24]. In the Skyrme model $a_2$ gives only a 10% contribution to $g_A$ [23], whereas the dominating valence contribution to $M_{44}$ (compare Tab. [1]) in the NJL-model gives almost a 30% contribution to $g_A$ if the terms due to the time-ordering are neglected. The fact that the total values for $g_A$ in the local limit of the present approach and in the vector meson pseudoscalar Skyrme model in Ref. [23] roughly coincide hinges also on the rescaling procedure for the parameter $e$ in Ref. [23], which tends to increase $g_A$. The Wess-Zumino contributions to $g_A$ in the Skyrme approach play a minor role, in the NJL model the anomalous part of the action, containing the WZ term in lowest order of the gradient expansion, gives $\simeq 1/4$ of the total amount.

VIII. SUMMARY AND DISCUSSION

In this paper we have investigated mass corrections to the hyperon masses and to the axial currents of the semibosonized SU(3) Nambu-Jona-Lasinio model. Moreover we have extended our recent analysis of the corrections to the axial currents which appear due to the time-ordering of the quark loop and semiclassical quantization [17].
to the case of the regularized effective action. In the semibosonized NJL model baryons are understood as solitonic solutions of the classical equations of motion. However the solitons do not carry proper quantum numbers and the semiclassical quantization procedure has to be applied in order to describe the mass splittings within the strange baryon multiplets. This treatment is based on introducing time-dependent rotations in the direction of the zero modes followed by a canonical quantization of the collective coordinates of the rotation matrix. Since these zero modes contribute significantly to the mass splittings \[20\], it was a challenging task to look at the axial currents which can be related to the recent measurements of the spin structure functions \[50,47,48,46\].

First the model parameters were fixed by looking at the hyperon mass splittings up to the terms quadratic in \(m_s\). These are reproduced with unexpectedly high accuracy and point towards a constituent quark mass of \(M \simeq 420\) MeV. We have also explicitly shown that the wave function corrections and the corrections due to the expansion of the effective action are comparable and therefore it is somehow inconsistent to perform only the Yabu-Ando diagonalization of the first order hamiltonian.

Second we considered the axial vector currents with the inclusion of the linear corrections in the rotational velocity. The new contributions which appear due to the time-ordering of the quark loop and semiclassical quantization have been shown explicitly to come from the real part of the effective action. If the proper-time regularization is implemented they are dominated by the contribution of the valence quarks. In the SU(3) model there are also other contributions linear in \(\Omega\) which come from the imaginary part of the effective action and as such do not require regularization. They are also dominated by the valence contribution. This concerns the leading term of \(g_A^{(0)}\), which vanishes in the pure pseudoscalar Skyrme model, whereas it is non-vanishing (however small) in the present model in rough agreement with experiment.

The expression for \(g_A^{(3)}\) has a \(\simeq 25\%\) rotational contribution from the imaginary
part of the effective Euclidean action, which vanishes in the SU(2) case and which can be related to Witten’s formula for the axial vector current from the Wess-Zumino effective action. Moreover it has a $\simeq 30\%$ contribution due to the explicit time-ordering $(Q_{bc})$ of the collective operators. These terms are not present in local theories like the Skyrme model. In the present model they are entirely due to the non-locality of the fermion determinant. Performing the gradient expansion of these quantities, it can be shown that these terms have the same mesonic structure as the lowest order term, such that one can define a profile independent renormalization of the lowest order $g_A$. This is similar to recent findings of Dashen and Manohar [41] within large-$N_c$ QCD and to the old non-relativistic quark model result of $g_A^{(3)} = (N_c + 2)/3$.

Quantitatively despite the fact that the lowest order SU(3) result is reduced by a group theoretical factor of 0.7 with respect to the SU(2) case the new time-ordering and anomalous contributions push the total value of $g_A^{(3)}$ upwards.

Next we have considered corrections linear in the strange quark mass. They are consistently derived from the expansion of the effective action, the quantization condition as well as from the higher representations of the wave function in the spirit of the Yabu-Ando diagonalization. However the effect on $g_A^{(3)}$ is not large and finally one ends up with $g_A^{(3)} = 1.38$ for $M = 423$ MeV, which is only $\simeq 10\%$ above the experimental value of $g_A^{(3),exp} = 1.25$. Here it should be stressed, that such nice agreement was never obtained within the pseudoscalar or pseudoscalar vector Skyrme model [23,24]. This qualitative and quantitative difference comes from the new $1/N_c$ corrections of the time-ordering from the non-local structure of the present NJL model.

The $g_A^{(0)}$ exists already in SU(2) and the only effect in SU(3) is a small shift due to the finite symmetry breaking $m_s$. This is in contrast to $g_A^{(8)}$, which vanishes in the SU(2) case, and which in SU(3) gets entire contribution from the rotation and from the strange quark mass. In the chiral limit the values for $g_A^{(8)}$ and $g_A^{(0)}$ are quite
close to each other, as suggested in [55], however the strange mass corrections reduce
the value of $g_A^{(8)}$ by $\simeq 25\%$, whereas the explicit symmetry breaking has almost no
influence on $g_A^{(0)}$. This holds at least if we take all linear $m_s$ corrections into account
and even the $m_s^2$-corrections, which can be calculated in this framework from the
non-symmetric wave-functions [21]. The final values $g_A^{(0)} \simeq 0.37$ and $g_A^{(8)} \simeq 0.31$ for
$M = 423$ MeV and $m_s = 180$ MeV are to be compared with the experimental data
extracted from the recent Ellis and Karliner analysis [50], i.e. $g_A^{(0),exp} \simeq 0.24 \pm 0.09$
and $g_A^{(8),exp} \simeq 0.35 \pm 0.04$. Apparently the theoretical values are only slightly outside.

Qualitatively the following can be said: $g_A^{(0)}$, which represents the part of the proton
spin carried by the quarks, gets a non-vanishing expectation value entirely due to the
anomalous part of the EEA. In a constituent quark model, when the total spin of the
proton is entirely carried by three quarks, this $g_A^{(0)}$ equals one. The present model gives
values close to experiment since the proton is treated entirely as many body system
rotating in spin-isospin space. Hence one cannot attribute the spin of the proton
to single elementary particles but only to the whole system. Hence the fact that in
agreement with the experiments only a fraction of about $20 - 30\%$ of the nucleon spin
is carried by the quark-spins is not surprising at all in the present model. One should
note that in the Skyrme models a non-vanishing value of $g_A^{(0)}$ can be obtained only by
adding vector mesons for the anomalous part [56,54,24].

Altogether the picture which emerges is quite satisfactory. Mass splittings are
accurately reproduced and axial currents are in good agreement with the experimental
data if rotational $1/N_c$ corrections are taken into account. In particular the spin of the
proton originates in this model to about $35\%$ from the spin of the quarks, a number
being in reasonable agreement to the world data reported by SMC [48]. Together with
the numerical results for the Gottfried sum [57] the model provides a good reproduction
of experimental data so far.
On purely theoretical side the presence of the new terms linear in $\Omega$ calculated in this paper poses a serious problem to effective meson theories like the Skyrme model, where such terms vanish identically. Another theoretical question which deserves a comment is the convergence of the expansion in $\Omega$. The large size of the corrections calculated in this paper, although expected from the quark model calculations, poses a problem of reliability of the numerical results. One way to tackle this problem would be to calculate the $\Omega^2$ corrections to the axial currents. This is the highest power of $\Omega$ one can think of, since the collective hamiltonian itself is of that order. Despite the technical difficulties in calculating these terms the preliminary estimates indicate that they are not negligible\textsuperscript{6}. Therefore one has to conclude that the expansion in $\Omega$ is slowly convergent. Moreover the formalism of the collective quantization has to be revised if one wants to include terms higher than $\Omega^2$. In addition despite the fact that mass splittings are well reproduced the absolute energies provide still some problems which are associated with zero-point corrections \[58,20\] and boson fluctuations. These questions are certainly beyond the scope of this paper, where we had to content ourselves with the linear corrections alone.

ACKNOWLEDGMENTS

The work was partially supported by Alexander von Humboldt Stiftung and Polish Research Grant KBN 2.0091.91.01 (M.P.), Graduiertenstipendium des Landes NRW (A.B.), and the Bundesministerium fuer Forschung und Technologie (BMFT, Bonn). One of us (A.B.) would also like to thank the Ruth und Gerd Massenberg-Stiftung and The Institute of Physics (IOP) for financial support. Discussions with Ch. Christov

\textsuperscript{6}M. Wkamatsu, private communication
(Bochum), D. Diakonov, V. Petrov, M. Polyakov (Petersburg), M. Wakamatsu (Osaka) and T. Watabe (Tokyo) are acknowledged.

The material of this work was presented at the *International Workshop on the Quark Structure of Baryons* in Trento, Italy, at the new *European Centre for Theoretical Studies* (ECT*), October 4-15, 1993.
TABLE I. Moments of inertia for different constituent masses

| $M$ [MeV] | $\Sigma$ [SU(2)] [MeV] | $I_1$ [fm] | $I_2$ [fm] | $K_1$ [fm] | $K_2$ [fm] | $N_0$ [fm] | $N_1$ [fm] | $N_2$ [fm] |
|---|---|---|---|---|---|---|---|---|
| 363. | 60.32 | 1.512 | 0.720 | 0.606 | 0.372 | 0.765 | 0.647 | 0.496 |
| 395. | 58.14 | 1.285 | 0.618 | 0.438 | 0.290 | 0.704 | 0.500 | 0.408 |
| 419. | 56.14 | 1.178 | 0.569 | 0.369 | 0.255 | 0.668 | 0.438 | 0.370 |
| 423. | 55.52 | 1.156 | 0.560 | 0.357 | 0.250 | 0.658 | 0.426 | 0.362 |
| 465. | 51.86 | 1.015 | 0.496 | 0.276 | 0.210 | 0.599 | 0.349 | 0.311 |

TABLE II. Coefficients of Eqs.(22) as functions of constituent mass $M$ for $m_s = 180$ MeV

| $M$ [MeV] | $A$ | $F$ | $D$ | $G$ | $B$ [fm] | $C$ [fm] | $H$ |
|---|---|---|---|---|---|---|---|
| 363. | 489.04 | 427.02 | 94.67 | 1.96 | 450.89 | 510.24 | 6.70 |
| 395. | 481.32 | 399.60 | 97.41 | 1.70 | 449.48 | 457.80 | 5.56 |
| 419. | 469.60 | 381.66 | 97.17 | 1.51 | 441.70 | 427.02 | 4.86 |
| 423. | 465.57 | 377.47 | 96.79 | 1.48 | 438.60 | 420.32 | 4.70 |
| 465. | 441.39 | 349.75 | 94.91 | 1.25 | 419.82 | 376.11 | 3.72 |
TABLE III. Different contributions to coefficients of Eqs.(22) for a constituent mass $M = 423$ MeV and for $m_s = 180$ MeV

|       | $O(m_s)$ | $O(m_s^2)$ | total | exp. |
|-------|----------|------------|-------|------|
|       |          |            |       |      |
|       | kin.     | dyn.       | w.f.  |      |
| A     | 546.10   | 10.94      | -64.64| -0.15|
|       |          |            |       |      |
| B     | 546.10   | 10.85      | -64.55| -53.81|
|       |          |            |       |      |
| F     | 381.20   | 1.18       | -27.67| 22.76|
|       |          |            |       |      |
| D     | 120.76   | -0.02      | -11.78| -0.07|
|       |          |            |       |      |
| C     | 348.16   | 1.20       | -19.97| 90.92|
|       |          |            |       |      |
| G     | 0.00     | 0.61       | -0.66 | 1.53 |
|       |          |            |       |      |
| H     | 0.00     | -0.29      | 0.41  | 4.57 |

TABLE IV. Quantities $M_3$ and $M_{44}$ for the SU(3) Nambu–Jona-Lasinio model in dependence on the constituent quark mass $M$

|       | $M_3$     | $M_{44}$ [fm] |
|-------|----------|--------------|
| $M$ [MeV] | val | sea | tot | val | sea | tot |
| 363    | -2.293  | -0.468 | -2.761 | -0.414  | -0.011 | -0.425 |
| 395    | -2.235  | -0.384 | -2.619 | -0.326  | -0.012 | -0.338 |
| 419    | -2.209  | -0.316 | -2.525 | -0.288  | -0.012 | -0.301 |
| 423    | -2.205  | -0.307 | -2.511 | -0.283  | -0.013 | -0.295 |
| 465    | -2.173  | -0.203 | -2.377 | -0.238  | -0.014 | -0.251 |
TABLE V. Quantities $Q_{12} = -2iQ_{-+}$ and $Q_{45} = i\tilde{Q}_{45}$ for the SU(3) Nambu–Jona-Lasinio model in dependence on the constituent quark mass $M$

| $M$ [MeV] | $Q_{-+}$ [fm] | $\tilde{Q}_{45}$ [fm] |
|-----------|---------------|------------------------|
|           | val | sea | tot | val | sea | tot |
| 363       | .408 | .025 | .433 | -.410 | -.023 | -.433 |
| 395       | .317 | .021 | .339 | -.318 | -.020 | -.338 |
| 419       | .279 | .019 | .298 | -.279 | -.018 | -.297 |
| 423       | .272 | .019 | .291 | -.273 | -.017 | -.290 |
| 465       | .226 | .016 | .242 | -.226 | -.015 | -.241 |

TABLE VI. Quantities $\overline{M}_{83}$ and $\overline{R}_{83}$ for the SU(3) Nambu–Jona-Lasinio model in dependence on the constituent quark mass $M$

| $M$ [MeV] | $\overline{M}_{83}$ [fm] | $\overline{R}_{83}$ [fm] |
|-----------|------------------------|------------------------|
|           | val | sea | tot | val | sea | tot |
| 363       | -0.683 | -0.015 | -0.699 | -0.293 | -0.094 | -0.387 |
| 395       | -0.500 | -0.016 | -0.516 | -0.150 | -0.090 | -0.240 |
| 419       | -0.422 | -0.016 | -0.438 | -0.095 | -0.091 | -0.186 |
| 423       | -0.409 | -0.016 | -0.425 | -0.086 | -0.091 | -0.177 |
| 465       | -0.316 | -0.017 | -0.333 | -0.025 | -0.090 | -0.115 |
TABLE VII. Quantities $R_{44}$ and $\overline{R}_{38}$ for the SU(3) Nambu–Jona-Lasinio model in dependence on the constituent quark mass $M$

| $M$ [MeV] | $R_{44}$ [fm] val | $R_{44}$ [fm] sea | $R_{44}$ [fm] tot | $\overline{R}_{38}$ [fm] val | $\overline{R}_{38}$ [fm] sea | $\overline{R}_{38}$ [fm] tot |
|-----------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 363       | -0.253            | -0.023            | -0.277            | 0.074             | -0.049            | 0.024             |
| 395       | -0.178            | -0.029            | -0.207            | 0.083             | -0.066            | 0.017             |
| 419       | -0.148            | -0.030            | -0.179            | 0.086             | -0.073            | 0.012             |
| 423       | -0.144            | -0.031            | -0.175            | 0.087             | -0.075            | 0.012             |
| 465       | -0.111            | -0.033            | -0.143            | 0.090             | -0.083            | 0.006             |

TABLE VIII. The axial vector coupling constant $g_A^{(3)}$ for the SU(3) Nambu–Jona-Lasinio model in dependence on the constituent quark mass $M$. The strange current quark mass is chosen as $m_s = 180$ MeV. The final model predictions are given by $g_A^{(3)}(\Omega^0)$ in SU(2) and $g_A^{(3)}(\Omega^1, m_s)$ in SU(3). The experimental value is given by $g_A^{(3), \text{exp}} = 1.25$.

| $M$ [MeV] | SU(2) |SU(3) |
|-----------|-------|-------|
|           | $g_A^{(3)}(\Omega^0)$ | $g_A^{(3)}(\Omega^1)$ | $g_A^{(3)}(\Omega^0, m_s)$ | $g_A^{(3)}(\Omega^1, m_s)$ | $g_A^{(3)}(\Omega^1, m_s)$ |
| 363       | 0.920 | 1.302 | 0.644 | 1.482 | 1.603 |
| 395       | 0.873 | 1.224 | 0.611 | 1.381 | 1.473 |
| 419       | 0.841 | 1.179 | 0.589 | 1.328 | 1.407 |
| 423       | 0.837 | 1.173 | 0.585 | 1.314 | 1.380 |
| 465       | 0.792 | 1.109 | 0.554 | 1.250 | 1.308 |
TABLE IX. The axial vector coupling constant $g_A^{(0)}$ and $g_A^{(8)}$ for the SU(3) Nambu–Jona-Lasinio model in dependence of the constituent quark mass $M$. The strange current quark mass is chosen as $m_s = 180$ MeV. The final model predictions are given by $g_A^{(0)}(\Omega^1, m_s)$ and $g_A^{(8)}(\Omega^1, m_s)$. The experimental values are given by $g_A^{(8),\text{exp}} = 0.35 \pm 0.04$ and $g_A^{(0),\text{exp}} = 0.24 \pm 0.09$ (Ellis and Karliner [50]).

| $M$ [MeV] | $g_A^{(8)}(\Omega^0, m_s)$ | $g_A^{(8)}(\Omega^1, m_s^0)$ | $g_A^{(8)}(\Omega^1, m_s^1)$ | $g_A^{(0)}(\Omega^1, m_s^0)$ | $g_A^{(0)}(\Omega^1, m_s^1)$ |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 363      | 0.159           | 0.443           | 0.328           | 0.462           | 0.475           |
| 395      | 0.151           | 0.408           | 0.316           | 0.401           | 0.409           |
| 419      | 0.145           | 0.389           | 0.309           | 0.371           | 0.377           |
| 423      | 0.144           | 0.385           | 0.308           | 0.364           | 0.370           |
| 465      | 0.137           | 0.363           | 0.301           | 0.328           | 0.331           |

TABLE X. Various contributions to the axial vector current of the proton in terms of u,d and s quarks. A comparison is made between the Skyrme model with vector mesons [24] (Skyrme,vector), the NJL model without (NJL,scalar) and with the time-ordered corrections of this paper (NJL,scalar,T). In the last column experimental values from Ellis and Karliner [50] are given.

|         | Skyrme (vector) | NJL (scalar) | NJL (scalar,T) | 'experiment' |
|---------|-----------------|--------------|----------------|--------------|
| $\Delta u$ | 0.63           | 0.64         | 0.902          | 0.806        |
| $\Delta d$ | -0.31          | -0.24        | -0.478         | -0.444       |
| $\Delta s$ | -0.03          | -0.02        | -0.054         | -0.120       |
TABLE XI. Matrix elements of the operators for $g_A^a$ in the proton state with spin up, where the index $i$ is always running from 1 to 3 and $b$ from 4 to 7.

|       | $D_{33}$ | $D_{38}$ | $D_{88}$ | $d_{33b}D_{3b}\hat{J}_b$ | $d_{33b}D_{8b}\hat{J}_b$ | $d_{33b}D_{3b}D_{8b}$ | $d_{33b}D_{8b}D_{8b}$ |
|-------|----------|----------|----------|--------------------------|--------------------------|------------------------|------------------------|
| $D_{33}$ | $-7/30$  | $\sqrt{3}/30$ | $3/10$  | $7/60$  | $1/(20\sqrt{3})$ | $-11/(90\sqrt{3})$ | $1/30$ |
| $D_{38}D_{83}$ | $-1/45$  | $-4/45$  | $-\sqrt{3}/30$ | $0$ | $7/20$ | $\sqrt{3}/45$ | $-\sqrt{3}/45$ |
APPENDIX A: SEMICLASSICAL QUANTIZATION

In this Appendix we collect all essential ingredients of the semiclassical quantization scheme. For the purpose of simplicity we will consider only $L_0$ without mass corrections. The generalization to include $m_s$ terms is straightforward. To this end one defines curvilinear angular velocities:

$$A^i \dot{A}_i = \frac{i}{2} \Omega_i \lambda_i = \frac{i}{2} \dot{\theta}_k e_{ki}(\theta) \lambda_i,$$

where the vielbeins fulfill: $e_{ki} \sigma_{kj} = \delta_{ij}$ and $e_{ik} \sigma_{jk} = \delta_{ij}$ (the bar denotes the inverse vielbein). The generalized momenta are defined as:

$$\pi_i = \frac{\partial L_0}{\partial \dot{\theta}_i} = \dot{\theta}_j e_{jm} I_{mn} e_{in} - c e_{i8},$$

where $c = N_c/\sqrt{12}$. Here one postulates canonical commutation rules:

$$[\hat{\pi}_i, \hat{\theta}_j] = -i \delta_{ij}$$

and then the differential representation for the operator $\hat{\pi}$ is, as usual: $\hat{\pi}_i = -i \partial / \partial \theta_i \equiv -i \partial_i$.

It is convenient to define the quantities $J_i$:

$$J_j = e_{ij} \pi_i = \Omega_m I_{mj} - c \delta_{8j}.$$

The normal quantization procedure is here slightly subtle, since the tensor $I$ is not invertible and hence $J_8 = -c$ is a constraint rather than the dynamical variable. To circumvent this difficulty let us introduce a small moment of inertia $I_{88} = \varepsilon$ and then take $\varepsilon \to 0$. Then one can proceed in a normal way, inverting Eq. (A2) for velocities and calculating the hamiltonian:

\[\text{We are greatful to P. Pobylitsa for discussion and for making clear to us many subtleties of the group theory.}\]
\[ H^{(0)} = M_{\text{cl}} + \frac{1}{2I_1} \sum_{i=1}^{3} J_i^2 + \frac{1}{2I_2} \sum_{i=4}^{7} J_i^2 + \frac{1}{2\varepsilon} (J_8 + c)^2. \]  
\quad \text{(A5)}

In the limit \( \varepsilon \to 0 \) one recovers the constraint.

Upon quantization \( J_i \) are promoted to the operators: \( \hat{J}_i = -i \varepsilon_{ij} \partial_j \). It is straightforward, but tedious to calculate the commutation relations for \( \hat{J}_i \)'s. To this end one has to use the following identities:

\[ \partial_j e_{ik} - \partial_i e_{jk} = e_{im} e_{jn} f_{mnk}, \]
\[ \varepsilon_{jm} \partial_j e_{in} - \varepsilon_{jn} \partial_j e_{im} = -\varepsilon_{ik} f_{kmn}. \]  
\quad \text{(A6)}

Finally one arrives at:

\[ [\hat{J}_i, \hat{J}_j] = i f_{ijk} \hat{J}_k. \]  
\quad \text{(A7)}

The next question is how to interpret operators \( \hat{J}_i \). There are two global symmetries of the rotation matrix \( A \) which do not change the Lagrangian \( L_0 \):

\[ \text{left : } A \to e^{i \frac{1}{2} \varepsilon \vec{\lambda}} A \quad \text{and} \quad \text{right : } A \to A e^{-i \frac{1}{2} \varepsilon \vec{\lambda}}. \]  
\quad \text{(A8)}

In the case of the right multiplication only \( \lambda_{1,2,3} \) enter, since due to the form of the hedgehog Ansatz the right multiplication has to commute with the right multiplication by \( \lambda_8 \). Then it becomes clear that the right symmetry corresponds to rotations, since it can be undone by the rotation of vector \( \vec{n} \) entering the hedgehog Ansatz. On the contrary the left multiplication is just a global symmetry of the Lagrangian and it can be interpreted as a flavor symmetry. To calculate the generators of these symmetries one uses the Noether construction promoting \( \varphi \) to a time dependent quantity. Then, for infinitesimal transformations, one gets that:

\[ \text{left : } \Omega_i \to \Omega_i + \dot{\varphi}_j D_{ji} \quad \text{and} \quad \text{right : } \Omega_i \to \Omega_i - \dot{\varphi}_i \]

and subsequently:
\[ -\frac{\partial}{\partial \phi_i} L_0 = -D_{ij} J_j \equiv T_i \quad \text{and} \quad -\frac{\partial}{\partial \phi_i} L_0 = J_i. \quad (A9) \]

Here \( D_{ij} \equiv D_{ij}^{(8)} = 1/2 \Tr(A_i^\dagger \lambda_i A \lambda_j) \) are the Wigner matrices in the adjoint representation of the SU(3) group. We have already shown that upon the quantization \( J_i \) are promoted to the SU(3) generators, however only \( \hat{J}_{1,2,3} \) correspond to the symmetry generators, namely to spin. To evaluate the commutation relations for \( \hat{T}_i \) one has to convince oneself that:

\[ [\hat{J}_i, D_{aj}] = i f_{ijk} D_{ak}. \]

Then it follows that:

\[ [\hat{T}_i, \hat{T}_j] = i f_{ijk} \hat{T}_k. \quad (A10) \]

Making use of the first of Eqs. (A9) and of the orthogonality relation for the \( D \) functions: \( D_{ac} D_{bc} = D_{ca} D_{cb} = \delta_{ab} \) one can rewrite \( H^{(0)} \) as in eq. (21).

Baryon wave functions should fulfill the following identities:

\[ \left[ \exp(i \omega^a \hat{T}_a) \psi \right](A) = \psi \left( e^{-i \omega^a \lambda_a / 2} A \right), \left[ \exp(i \omega^a \hat{J}_a) \psi \right](A) = \psi \left( A e^{i \omega^a \lambda_a / 2} \right). \quad (A11) \]

The phase convention in Eq. (A11) is chosen in such a way that the \( \psi \)'s are faithful, \( i.e. \):

\[ \left[ \exp(i \omega^a_{(2)} \hat{T}_a) \exp(i \omega^a_{(1)} \hat{T}_a) \psi \right](A) = \psi \left( \left\{ e^{i \omega^a_{(2)} \lambda_a / 2} e^{i \omega^a_{(1)} \lambda_a / 2} \right\}^{-1} A \right), \]

\[ \left[ \exp(i \omega^a_{(2)} \hat{J}_a) \exp(i \omega^a_{(1)} \hat{J}_a) \psi \right](A) = \psi \left( A e^{i \omega^a_{(2)} \lambda_a / 2} e^{i \omega^a_{(1)} \lambda_a / 2} \right). \]

The problem of constructing the baryon wave functions reduces to the construction of the functions of \( A \) which transform under left and right rotations as the tensors of the irreducible representations \( \mathcal{T} \) and \( \mathcal{J} \) respectively. In the matrix representation that means:
\[
\left[ \exp(-i\omega_a \hat{T}_a) \psi^{(T,J)} \right]_{tj} (A) = \psi^{(T,J)}_{t'j'} (A) \ D^{(T)}_{t't} \left( e^{-i\omega_a \lambda_a} \right),
\]
\[
\left[ \exp(i\omega_a \hat{J}_a) \psi^{(T,J)} \right]_{tj} (A) = \psi^{(T,J)}_{t'j'} (A) \ D^{(J)}_{j'j} \left( e^{i\omega_a \lambda_a} \right). \tag{A12}
\]

Replacing LHS in Eqs.\ref{eq:A12} by RHS of Eqs.\ref{eq:A11} and making substitution: \(A \rightarrow 1\) and \(\exp(i\omega_a \lambda_a) \rightarrow A\) one gets:
\[
\psi^{(T,J)}_{t'j'} (A) = c_{t'j'} D^{(T)}_{t't} (A^\dagger), \quad \psi^{(T,J)}_{t'j'} (A) = c_{t'j'} D^{(J)}_{j'j} (A) \tag{A13}
\]

where \(c_{tj} = \psi^{(T,J)}_{tj} (1)\).

In order to calculate the constants \(c_{tj}\) let us observe that:
\[
\psi^{(T,J)}_{t'j'} (L^\dagger AR) = \psi^{(T,J)}_{t'j'} (A) \ D^{(T)}_{t't} (L) \ D^{(J)}_{j'j} (R). \tag{A14}
\]

We will now make use of the following identity connecting elements of the Wigner matrices in representation \(T\) and its complex conjugate \(\overline{T}\):
\[
D^{(T)}_{ij} (L) = (-)^{Q(i)-Q(j)} D^{(\overline{T})}_{-j-i} (L^\dagger), \tag{A15}
\]
where \(Q\) denotes the charge of the state: \(Y, I, I_3\) and \(minus\) before the index \(i\) or \(j\) denotes the complex conjugate state: \(-Y, I, -I_3\). Putting \(A = 1\) in Eq.\ref{eq:A14} we we find that \(\psi(L^\dagger R)\) is proportional to the product of \(D^{(T)}(L^\dagger) \ D^{(J)}(R)\). On the other hand, following Eq.\ref{eq:A13}, \(\psi(L^\dagger R)\) should be proportional either to \(D^{(T)}(L^\dagger R)\) or \(D^{(J)}(L^\dagger R)\). That means immediately that
\[
\overline{T} = J.
\]

One can also easily convince oneself that:
\[
c_{tj} = (-)^{-Q(t)} \delta_{-tj}.
\]

So we get two equivalent expressions for the baryon wave function:
\[
\psi^{(T)}_{t'j'} (A) = (-)^{Q(j)} D^{(T)}_{j'j} (A^\dagger) = (-)^{Q(t)} D^{(\overline{T})}_{-t'j} (A). \tag{A16}
\]
Throughout this derivation we have assumed that charges are integer. Finally making use of the fact that $D_{ij}^{(T)}(A) = D_{ji}^{(T)}(A)^*$ we get our final expression for the baryon wave function:

$$\psi_{(Y,T,Y',J,J_3)}^{(T)}(A) = \sqrt{\dim T} \frac{(-)^{Y'/2+J_3}}{2+J_3} [\langle Y,T,J_3 | D^{(T)}(A) | -Y', J, -J_3 \rangle]^*.$$

(A17)

where the normalization factor has been included. Remember that the right hypercharge is constrained: $Y' = -N_c/3$

The action of the collective operators on these wave functions is straightforward: flavor operators $\hat{T}_a$ and spin operators $\hat{J}_b$ (also for $b = 4 \ldots 8$) act on $\psi$ as the SU(3) generators in representation $T$ act on the state $(Y,T,T_3)$ and $(Y', J, J_3)$ respectively. The action of the $D$ functions entering the collective operators can be calculated with the help of the SU(3) Clebsch-Gordan coefficients:

$$\dim T_3 \int dA D_{\gamma_{\gamma_{\gamma}}(T_3)}^{(T)}(A)^* D_{\gamma}(T_2) D_{\gamma_{\gamma_{\gamma}}(T_1)}^{(T)}(A) = \sum \gamma \left( T_1 \ T_2 \ T_3 \right) \left( \begin{array}{ccc} t_1 & t_2 & t_3 \\ j_1 & j_2 & j_3 \end{array} \right), \quad (A18)$$

where $\gamma$ is the degeneracy index. In Tab. [X] we list some of the proton spin up matrix elements of the collective operators which enter the expressions for $g^a_\Lambda$.

**APPENDIX B: DERIVATION OF THE REGULARIZATION FUNCTIONS FOR THE AXIAL CURRENT**

Here we want to give an explicit derivation of the $\Omega^0$ and $\Omega^1$ contributions to the axial current. We emphasize the method of regularization of non-anomalous quantities from the explicit *time-ordering* of the collective operators within the proper-time regularization scheme [30]. Then the real part can be written as:

$$\text{Re } A_i^a(x) = -\frac{1}{2} \frac{\delta}{\delta s_i^a(x)} \text{Sp(to)} \int \frac{du}{u} \phi(u) \exp \left( -u D^1 D \right)$$

(B1)
where

$$D = \partial_t + H + i\Omega_E - is_i^a \gamma_4 \gamma_i \gamma_5 A^\dagger \lambda^a A$$

(B2)

and

$$D^\dagger = -\partial_t + H - i\Omega_E - is_i^a \gamma_4 \gamma_i \gamma_5 A^\dagger \lambda^a A,$$

(B3)

such that

$$D^\dagger D = -\partial^2 + H^2 + \Omega^2_E - i[\Omega_E, H] - i\{\Omega_E, \partial_t\} - is_i^a \{\gamma_4 \gamma_i \gamma_5 A^\dagger \lambda^a A, H\}$$

$$+ s_i^a \gamma_4 \gamma_i \gamma_5 [A^\dagger \lambda^a A, \Omega_E] - i\gamma_4 \gamma_i \gamma_5 [s_i^a A^\dagger \lambda^a A, \partial_t].$$

(B4)

Then one has to expand $D^\dagger D$ around $D^\dagger_0 D_0$ with $D_0 = \partial_t + H$, i.e. one expands in terms of $s_i^a$ and $\Omega_E$. This is done by using the Schwinger Dyson formula:

$$\exp \left( -uD^\dagger D \right) = \exp \left( -uD^\dagger_0 D_0 \right)$$

$$-u \int_0^1 d\alpha \exp \left( -u\alpha D^\dagger_0 D_0 \right) \left[ D^\dagger D - D^\dagger_0 D_0 \right] \exp \left( -u(1 - \alpha) D^\dagger_0 D_0 \right)$$

$$+ u^2 \int_0^1 d\beta \int_0^{1-\beta} d\alpha e^{-u\alpha D^\dagger_0 D_0} \left[ D^\dagger D - D^\dagger_0 D_0 \right] e^{-u\beta D^\dagger_0 D_0} \left[ D^\dagger D - D^\dagger_0 D_0 \right] e^{-u(1-\alpha-\beta) D^\dagger_0 D_0}$$

$$+ \ldots.$$  

(B5)

In the lowest order $\Omega^0$, one obtains

$$\text{Re} A_i^a(x) = -\frac{1}{2} \frac{\delta}{s_i^a(x)} \text{Sp}(\omega) \int \frac{du}{u} \phi(u) \int d\alpha \exp \left( -u\alpha D^\dagger_0 D_0 \right)$$

$$\left( uis_i^a \{\gamma_4 \gamma_i \gamma_5 A^\dagger \lambda^a A, H\} \right) \exp \left( -u(1 - \alpha) D^\dagger_0 D_0 \right),$$

(B6)

which after some simple manipulations gives eq. (39) (see Ref. [22]).

Now we want to consider the $\Omega^1_E$ corrections to the current. Let us define

$V_1 = -i[\Omega_E, H] - i\{\Omega_E, \partial_t\}$, $V_2 = -is_i^a \{\gamma_4 \gamma_i \gamma_5 A^\dagger \lambda^a A, H\}$, $V_3 = s_i^a \gamma_4 \gamma_i \gamma_5 [A^\dagger \lambda^a A, \Omega_E]$ and $V_4 = -i\gamma_4 \gamma_i \gamma_5 [s_i^a A^\dagger \lambda^a A, \partial_t]$. Consistently in the order $\Omega^1_E$ one has to consider combinations of $V_1$ and $V_2$ as well as the single sum $V_3 + V_4$. Note that it is important
to retain $s^a_t$ as time-dependent in eq. (B4), because otherwise the two terms $V_3 + V_4$ cancel. This can be seen using $[A^\dagger \lambda_a A, \partial_t] = i[\Omega_E, A^\dagger \lambda_a A]$. After some lengthy algebra the operator $\hat{g}_A^a$ defined in Eq.(38) can be written as:

$$\hat{g}_A^a = - \frac{N_c}{4} \int dt \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} \frac{2\omega E_n + 2\omega' E_m}{(\omega^2 + E_n^2)(\omega'^2 + E_m^2)} \exp(i(\omega - \omega')(t - t_0)) \int_0^\infty du \ u \phi(u) \exp\left(-u[\alpha(\omega^2 + E_n^2) + (1 - \alpha)(\omega'^2 + E_m^2)]\right)$$

Performing now the $dt$ integration with special care to the time-ordered product $\mathcal{T}(\Omega_E^c(t)D_{ab}(t_0))$ one gets the relation:

$$\int dt \exp(i(\omega - \omega')(t - t_0)) \mathcal{T}[\Omega_E^c(t)D_{ab}(t_0)] = \frac{1}{i} \left[\text{PP} \frac{1}{\omega - \omega'} + i\pi\delta(\omega - \omega')\right] D_{ab}(t_0)\Omega_E^c - \frac{1}{i} \left[\text{PP} \frac{1}{\omega - \omega'} - i\pi\delta(\omega - \omega')\right] \Omega_E^c D_{ab}(t_0)$$

Note that after the time-ordering the angular velocities are again assumed to be time-independent in order to perform the $\int dt$ integration. Last term in eq. (B8) vanishes because the $\delta$-function makes the integral in eq. (B7) odd in $\omega$. Therefore if the indices of $\Omega_E^c$ and $D_{ab}$ are such that $[D_{ab}, \Omega_E^c] = 0$ eq. (B8) gives identically zero. Evaluating the $\omega, \omega'$ integration finally gives:

$$\hat{g}_A^a = - \frac{N_c}{4} \frac{i f_{cab} D_{ab}}{I_{cc}} \sum_{m,n} <n | \lambda^c | m > < m | \sigma_3 \lambda^b | n > \mathcal{R}_Q(E_n, E_m),$$

where the regularization function is given by

$$\mathcal{R}_Q(E_n, E_m) = \frac{1}{2\pi} \int du \phi(u) \int_0^1 d\alpha \frac{\alpha E_n - (1 - \alpha)E_m}{\sqrt{\alpha(1 - \alpha)}} \exp\left(-u[\alpha E_n^2 + (1 - \alpha)E_m^2]\right),$$

which in contrast to the regularization function for the usual moment of inertia $\mathcal{R}_{I}(E_n, E_m)$ or $\mathcal{R}_{\beta}(E_n, E_m)$ is antisymmetric with respect to $E_m$ and $E_n$. The $du$
integration can be performed analytically in the case of step like regularization functions \( \phi(u) = c_i \theta(u - 1/\Lambda_i^2) \) and gives

\[
\mathcal{R}_Q(E_n, E_m) = c_i \int_0^1 \frac{d\alpha}{2\pi} \frac{\alpha E_n - (1 - \alpha)E_m}{\sqrt{\alpha(1 - \alpha)}} \frac{\exp\left(-[\alpha E_n^2 + (1 - \alpha)E_m^2]/\Lambda_i^2\right)}{\alpha E_n^2 + (1 - \alpha)E_m^2}.
\] (B11)

Using the formula

\[
\int_0^1 \frac{d\alpha}{\sqrt{\alpha(1 - \alpha)}} \frac{1}{q - \alpha p} = \frac{\pi}{\sqrt{q(p - q)}}, \quad 0 < p < q
\] (B12)

the infinite cutoff limit of eq. (48) is given by (p. 219 of Ref. [59]):

\[
\mathcal{R}_Q(E_n, E_m) = \frac{1}{|E_n - E_m|} \frac{\text{sign}E_n - \text{sign}E_m}{2}
\] (B13)

and was used in Ref. [17] to calculate the \(1/N_c\) corrections. Defining

\[
Q_{bc} = \frac{N_c}{4} \sum_{m,n} <n | \lambda^c | m > <m | \sigma_3 \lambda^b | n > \mathcal{R}_Q(E_n, E_m)
\] (B14)

the operator \(\hat{g}_A^a\) can be rewritten as:

\[
\hat{g}_A^a = -\frac{2iQ_{12}}{I_1} D_{a3} - \frac{2iQ_{45}}{I_2} D_{a3}.
\] (B15)

Equation (B15) follows directly from the real part of the Euclidean effective action given by eq. (B1). Therefore the corrections described above have no counterpart in the Wess-Zumino term which follows from the imaginary part of the Euclidean action. As such they vanish identically in any local mesonic theory like the Skyrme model for instance.

**APPENDIX C: COMPARISON WITH THE GRADIENT EXPANSION**

In order to check the results of the numerical diagonalization one should always consult the long wave length expansion of the coefficients appearing in the expressions for the observables. This technique is described at length in Ref. [13]. It also clarifies the question, whether the exact numerical value can be approximated by the gradient expanded quantities, or in other words, whether the local mesonic theory like the Skyrme model for instance, is a good approximation to the NJL-model.
1. The lowest order result from the real part of the EEA

For the lowest order ($\Omega^0$) only the quantity $M_3$, which already exists in SU(2), contributes to $g^{(3)}_A$. Its gradient expansion can be found by expanding $D^\dagger_0 D_0 = -\partial^2 + M^2 + iM\gamma_i\partial_i U(x)$ in terms of the gradients $\partial_i U(x)$. The result is:

$$M_3^{\text{grd}} = \frac{2}{3} \int d^3x (\sigma \partial_i \pi_i - \pi_i \partial_i \sigma). \quad (C1)$$

Eq. (C1) can be rewritten in terms of the chiral angle $\theta$ and for $\pi$ and $\sigma$ on the chiral circle $\sigma(r) = \cos \theta(r)$ and $\pi(r) = \sin \theta(r)$:

$$M_3^{\text{grd}} = \frac{8\pi}{3} f_\pi^2 \int dr \, r^2 \left( \theta' + \frac{2\sin \theta \cos \theta}{r} \right). \quad (C2)$$

For the simplest case of a linear profile $\theta(r) = \pi(1 - r/2R)$, it reduces to:

$$M_3^{\text{grd}} = -\frac{32\pi^2}{9} f_\pi^2 R^2 \left( 1 - \frac{3}{2\pi^2} \right). \quad (C3)$$

This quadratic behaviour of $M_3^{\text{grd}}$ is explicitly checked by using a large $R$ profile function $\theta$ as an input for the quark wave functions of the exact formula for $M_3$.

Another quantity from the real part of the EEA emerges due to the presence of a finite $m_\pi$. One obtains in leading order:

$$R_{83}^{\text{grd}} = \frac{1}{6g} \int d^3x \partial_i \pi^i(x), \quad (C4)$$

where we have used the formula for the normalization of the kinetic term for mesons, which fixes the cutoff for the quadratically divergent integral $I_2(M)$. As a total divergence the behaviour of this term is determined just by the asymptotics of the pion field.

Therefore it vanishes for the linear profile discussed above or in the case of finite $m_\pi$, when the profile decreases exponentially. In order to check our numerical calculations we have used $\theta = 2\arctan(-R^2/r^2)$. Then $R_{83}^{\text{grd}} = -(4\pi/3g)f_\pi R^2$. As stated in the
text, such term is not found in the Skyrme model and it would actually give a vanishing contribution due to the asymptotics of the profile. In the present non-local quark model \( \tilde{R}_{83} \) does have a non-vanishing contribution (compare with Tab. VI).

2. The anomalous terms from the imaginary part of the EEA

The axial vector current gets a contribution from the imaginary part of the EEA, which is non-vanishing only in the SU(3) case. In a local mesonic theory it can be derived from the Wess-Zumino term [34]. Here want to show shortly how to derive this contribution from the non-local EEA of the present NJL model. Consider a quantity:

\[
\text{Im} \ A_i^a(x) = \frac{1}{2} \int \mathcal{D}A(t) \frac{\delta}{\delta s^a(x)} \text{Sp}(t_0) \left[ \frac{1}{D} - \frac{1}{D^\dagger} \right] i\gamma_4 \gamma_i \gamma_5 \lambda^b D_{ab} s^a(x). \tag{C5}
\]

with \( D = \partial_t + H + i\Omega_E \). Going to the operator form and using \( D^\dagger D \) one can write

\[
\text{Im} \hat{A}_i^a(x) = \frac{1}{2} \frac{\delta}{\delta s^a(x)} \text{Sp}(t_0) \frac{1}{D^\dagger D} \left[ D^\dagger i\gamma_4 \gamma_i \gamma_5 \lambda^b D_{ab} - i\gamma_4 \gamma_i \gamma_5 \lambda^b D_{ab} D \right] s^a(x) \tag{C6}
\]

Expanding \( D^\dagger D \) again in terms of the gradients leads after some laborious algebra:

\[
M_{44}^{\text{grad}} = \frac{N_c}{16\pi^2} \frac{1}{f_\pi^3} \epsilon_{0\mu\nu} \epsilon_{3ab} \int d^3x \partial_\mu \pi^a(x) \partial_\nu \pi^b(x) \sigma(x). \tag{C7}
\]

In the case of the hedgehog Ansatz i.e. \( \sigma(r) = \cos \theta(r) \) and \( \pi(r) = \sin \theta(r) \) Eq.(C7) reduces to:

\[
M_{44}^{\text{grad}} = -\frac{N_c}{6\pi} \frac{1}{f_\pi^3} \int drr \partial_r \sigma(r) \pi(r)^2 = \frac{N_c}{6\pi} \int drr \theta'(r) \sin^3 \theta(r). \tag{C8}
\]

For the linear profile \( \theta(r) = \pi(1 - r/2R) \) we obtain a compact expression:

\[
M_{44}^{\text{grad}} = -\frac{2}{3\pi} R. \tag{C9}
\]

This linear behaviour for large size chiral fields can be seen in Fig. 7.
3. The $\Omega^1$-terms from the real part of the EEA

In this Appendix we derive the gradient expansion for the non-local terms ($Q_{ab}$). The formulae below are given only for this part of the axial vector current operator $\hat{g}_A^a$:

$$\hat{g}_A^a = -\frac{1}{2} \text{Tr} \gamma_{\tau, c} \int d^3 x \ dt < \vec{x}, t_0 | t > < t | \frac{1}{\partial_t + H} \gamma_0 \gamma_i \gamma_5 \lambda_b | \vec{x}, t_0 > \mathcal{T} [\Omega_c(t) D_{ab}(t_0)] ,$$

(C10)

where the regularization is neglected here for simplicity. Inserting eigenstates of $\partial_t$ and $H$ and using eq. (B8) we can define

$$\hat{g}_A^a = \left[ \frac{X_{12}^3}{I_1} + \frac{X_{45}^3}{I_2} \right] D_{a3},$$

(C11)

where the X-quantities can be calculated from:

$$X_{bc}^i = \text{Tr} \int \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} PP \frac{1}{\omega - \omega'} < \vec{x} | \frac{1}{-i\omega + H} \lambda_c \frac{1}{-i\omega' + H} \gamma_0 \gamma_i \gamma_5 \lambda_b | \vec{x} > .$$

(C12)

Then the recipe is to multiply denominators and numerators by the hermitian conjugate of the denominators and recover $H^2 = -\partial_i^2 + M^2 + iM \gamma_i \partial_i U(x)$ in denominators, which can be expanded in terms of the gradients. Then these expressions can be straightforwardly simplified to the pure SU(2) quantity:

$$X_{12}^3 = -\frac{N_c M}{48\pi} \int d^3 x \frac{1}{f_\pi^2} \left( \pi^i \partial_i \sigma + \sigma \partial_i \pi^i \right)$$

$$+ \frac{N_c M}{16\pi} \int d^3 x \frac{1}{f_\pi^2} \left( \sigma \partial_i \pi^i - \pi^i \partial_i \sigma \right)$$

$$= -\frac{N_c M}{4} \int dr r^2 \left( \theta' + \frac{\sin 2\theta}{r} \right)$$

$$- \frac{N_c M}{12} \int dr r^2 \left( \theta' \cos 2\theta + \frac{\sin 2\theta}{r} \right) ,$$

(C13)

and the pure SU(3) quantity.
\[ X_{45}^3 = \frac{N_c M}{32\pi} \int d^3x \frac{1}{f_{\pi}^2} (\sigma \partial_i \pi^i - \pi^i \partial_i \sigma) \]
\[ = -\frac{N_c M}{8} \int drr^2 \left( \theta' + \frac{\sin 2\theta}{r} \right), \quad (C14) \]

where the first line for \( X_{12}^3 \) is a total divergence and vanishes for chiral fields, which vanish at least as \( 1/r^2 \) for \( r \to \infty \). Assuming physical profiles, which vanish exponentially with the pion mass, the axial vector current operator can be written as

\[ \hat{g}_A^a = \int d^3r \left( \theta' + \frac{\sin 2\theta}{r} \right) \left[ \frac{8\pi}{3} f_{\pi}^2 + \frac{M}{4I_1} + \frac{M}{8I_2} \right] D_{a3}. \quad (C15) \]

Note that \( I_1, I_2 \sim N_c \), such that the last two terms in eq. (C13) represent a \( 1/N_c \) correction. Therefore eq. (C13) resembles very much the result of Dashen and Manohar \[ \[1, 40\], which states that the \( 1/N_c \) corrections to the axial current lead only to a renormalization of \( g_A \). Or in other words, the ratio of different coupling constants has no \( 1/N_c \) correction.
FIGURES

FIG. 1. Constituent quark mass $M$ as a function of of $m_s$ such that the octet-decuplet splitting is reproduced.

FIG. 2. The model deviations (i.e. theory-experiment) for octet and decuplet baryons as functions of $m_s$ for the optimal constituent quark mass $M$ of Fig.1

FIG. 3. The theoretical deviations from the experimental values of the $\Sigma$ and $\Lambda$ particle for $M = 419$ MeV. Compared are a perturbative treatment (pert1) of the wave functions in zeroth order, a linear corrections of order $m_s$ (pert2) and an exact diagonalization (YA), proposed by Yabu and Ando [37].

FIG. 4. The coefficients $c_{10}$ and $c_{27}$ of the higher representations $\bar{10}$ and $27$ of the proton as a function of the constituent quark mass $M$. The strange current quark mass is chosen as $m_s = 180$ MeV

FIG. 5. The anomalous quantity $M_{44}$ compared with the leading term of a gradient expansion, which comes exactly from the Wess-Zumino action for $SU(3)$ pseudoscalar fields. This is done for a fixed linear profile in dependence of the radius $R$ for $M = 372$ MeV.

FIG. 6. The $g_0^A$, $g_3^A$ and $g_8^A$ are shown for selfconsistent chiral fields in dependence of the constituent quark mass. The strange current quark mass is chosen as $m_s = 180$ MeV, according to a best fit to the hyperon spectra.

FIG. 7. The regularization function $\mathcal{R}_Q(E_n, E_m)$ for the time-ordered expressions (reg) for fixed $E_n$ and $M = 400$ MeV in dependence of $E_m$. This is compared to the function (noreg), which is obtained in the infinite cutoff limit.
REFERENCES

[1] D. Gross, Talk delivered at the Third International Symposium on the History of Particle Physics, June 26, 1992, Princeton preprint PUPT 1329, 1992.

[2] R. Cahill, Aust. J. Phys. 44, 105 (1991).

[3] R. Cahill, Nucl. Phys. A543, 63c (1992).

[4] D. Diakonov and V. Petrov, Nucl. Phys. B272, 457 (1986).

[5] D. Diakonov and V. Petrov, in Skyrmions and Anomalies, edited by M. Jezabak and M. Praszalowicz (World Scientific, Singapore, 1987).

[6] R. D. Ball, in Workshop on Skyrmions and Anomalies (World Scientific, Singapore, 1987).

[7] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

[8] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 124, 246 (1961).

[9] E. Shuryak, The QCD vacuum, Hadrons and the Superdense Matter (World Scientific, Singapore, 1988).

[10] E. Shuryak, Rev. Mod. Phys. 65, 1 (1993).

[11] H. Reinhardt and R. Wuensch, Phys. Lett. B215, 577 (1988).

[12] Th. Meissner, F. Gruemmer, and K. Goeke, Phys. Lett. B227, 296 (1989).

[13] M. Wakamatsu and H. Yoshiki, Nucl. Phys. A524, 561 (1991).

[14] T. Skyrme, Proc. R. Soc. A260, 127 (1961).

[15] S. Forte, Phys. Rev. D47, 1842 (1993).

[16] M. Wakamatsu and T. Watabe, Phys. Lett. B312, 184 (1993).
[17] A. Blotz, M. Praszałowics, and K. Goeke, Phys. Lett. B317, 195 (1993).

[18] C. Christov et al., RUB-TH-53/93, Phys. Lett. B in press.

[19] E. Witten, Nucl. Phys. B223, 422 (1983).

[20] A. Blotz et al., Nucl. Phys. A555, 765 (1993).

[21] M. Praszałowics, A. Blotz, and K. Goeke, Phys. Rev. D47, 1127 (1993).

[22] Th. Meissner and K. Goeke, Z. Phys. A339, 513 (1991).

[23] N. Park, J. Schechter, and H. Weigel, Phys. Rev. D43, 869 (1991).

[24] N. Park and H. Weigel, Nucl. Phys. A541, 453 (1992).

[25] A. Blotz et al., Phys. Lett. B287, 29 (1992).

[26] M. Kato, W. Bentz, K. Yazaki, and K. Tanaka, Nucl. Phys. A551, 541 (1993).

[27] T. Eguchi, Phys. Rev. D14, 2755 (1976).

[28] M. Wakamatsu and W. Weise, Z. Phys. A133, 173 (1988).

[29] A. Blotz and K. Goeke, Int. J. Mod. Phys. A, in press (1993).

[30] J. Schwinger, Phys. Rev. 93, 664 (1951).

[31] D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B306, 809 (1988).

[32] M. Wakamatsu, Phys. Lett. B234, 223 (1990).

[33] G. Adkins, C. Nappi, and E. Witten, Nucl. Phys. B228, 552 (1983).

[34] E. Witten, Nucl. Phys. B223, 433 (1983).

[35] T. Lee, Particle Physics and Introduction to Field Theory (Harwood Academic Publishers, New York, 1981).
[36] P. Ramond, *Field Theory: A modern Primer* (Addison-Wesley, Redwood City, California, 1990).

[37] H. Yabu and K. Ando, Nucl. Phys. **B301**, 601 (1988).

[38] H. Weigel, R. Alkofer, and H. Reinhardt, Nucl. Phys. **B387**, 638 (1992).

[39] R. Ball, Phys. Rep. **182**, 1 (1989).

[40] R. Dashen and A. Manohar, Baryon-Pion Couplings from Large-\(N_c\) QCD, University of California at San Diego, UCSD/PTH 93-16, 1993.

[41] R. Dashen and A. Manohar, \(1/N_c\) Corrections to the Baryon Axial Currents in QCD, University of California at San Diego, UCSD/PTH 93-18, 1993.

[42] N. Park and H. Weigel, Phys. Lett. **B268**, 155 (1991).

[43] N. Toyota, Prog. Theor. Phys. **77**, 688 (1987).

[44] A. Blotz, M. Polyakov, and K. Goeke, Phys. Lett. **B302**, 151 (1993).

[45] J. R. Aitchison and C. Frazer, Phys. Rev. **D31**, 2608 (1985).

[46] EMC Collaboration, J. Ashman et al., *Phys. Lett. B206* (1988) 364, *Nucl. Phys. B328* (1989) 1.

[47] SMC Collaboration, B. Adeva et al., Phys. Lett. **B302**, 533 (1993).

[48] SMC Collaboration, B. Adeva et al., Phys. Lett. **B320**, 400 (1994).

[49] E142 Collaboration, E. Hughes et al., Determination of the Neutron Structure Function, SLAC-PUB-6217, 1993, to be published in: *Proceedings, Rencontres des Morionds: QCD and High Energy Interactions*.

[50] J. Ellis and M. Karliner, Analysis of data on polarized lepton-nucleon scattering, CERN-TH-6898/93, 1993.
[51] J. Bjorken, *Phys. Rev.* **148** (1966) 1467; **D1** (1970) 1376.

[52] J. Ellis and R. Jaffe, Phys. Rev. **D9**, 1444 (1974).

[53] J. Ellis and R. Jaffe, Phys. Rev. **D10**, 1669 (1974).

[54] J. Schechter, V. Soni, A. Subbaraman, and W. Weigel, The effective Lagrangian approach to the proton spin puzzle and the issue of two components, Syracuse University report No. SU-4228-493, 1991.

[55] V. Bernhard, N. Kaiser, and U. Meissner, Phys. Lett. **B237**, 545 (1990).

[56] P. Jain *et al.*, Phys. Rev. **D37**, 3252 (1988).

[57] A. Blotz, M. Praszalowics, and K. Goeke, Talk delivered at the *International Workshop on the Quark Structure of Baryons*, October 4-15, (to be published), 1993.

[58] P. Pobylitsa *et al.*, J. Phys. **G18**, 1455 (1992).

[59] I. Gradshteyn and I. Ryzhik, *Tables of Integrals, Series and Products* (Academic Press, San Diego, 1980).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403314v1
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403314v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403314v1
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403314v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403314v1
This figure "fig2-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403314v1
This figure "fig2-4.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403314v1