Climate predictability in the Himalayan foothills using fractals

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ABSTRACT. Statistical relationships among meteorological parameters could significantly improve the understanding of climate dynamics and predictability in the Himalayan foothills, where the anthropogenic emissions are increasing and 3-D models have limitations in resolving highly complex topography. Here, we calculate the Predictability Indices (PI) of pressure (P), temperature (T) and rainfall (R) at two low-altitude (Pantnagar, Dehradun) and two high-altitude (Tehri and Mukteshwar) stations in the Himalayan foothills, by considering the observational time series of temperature, pressure and rainfall as a multi-dimensional array and applying the fractal theory. Fractal dimension demonstrated significant variations from station-to-station with the values relatively closer to unity at high-altitude sites demonstrating better climate predictability, as compared to those over the low-altitude stations in the Himalayan foothills.

Long-term computations (2005-2014) for Dehradun observations further revealed strong inter-annual variations in the climate predictability. Pressure and temperature at Dehradun are found to be reasonably predictable (P<0.5 throughout the year, P values of up to 0.93). However, temperature predictability was lower during July (P = 0.43) and the inter-annual variability (1-standard deviation) in this month is also found to be very high (0.29) than those during other months (0.11-0.18) at Dehradun. The predictability index for rainfall (R) is found to be up to 0.5 during summer-Monsoon, however its lower values (0.16-0.34) during other months suggest that the rainfall is less predictable as compared to the temperature and pressure. Hence, fractal based analysis on an array consisting of numerous meteorological variables may be utilized as a tool to understand the predictability of either of the meteorological variables, in order to assure the utility of a statistical model over the dynamical model before employing it for the prediction of meteorological conditions at an observational site.

Key words – Climate predictability, Himalayan foothills, Fractals.
1. Introduction

A comprehensive investigation of the relationships among observed climate variables such as temperature, pressure and rainfall could significantly improve the predictions through statistical modeling, especially in the mountainous regions of the Himalayas, where 3-D climate models show limitations in resolving the highly complex geographical topography (Singh et al., 2016; Pant et al., 2018). Despite of the advantage with statistical modeling to inherit the effects of terrain and correlated variations among various meteorological parameters, comprehensive investigations of such statistical relationships among observed climate variables are lacking over the Himalayan foothills and needs to be studied in detail.

The temporal evolution of climate variables can be formulated by considering the observations of different variables as a multi-dimensional array and combining fractal-based analysis with clustering (Mingkaiet et al., 2016). A variety of methods have been employed for analyzing the nonlinear dynamics in time series (Eckmann and Ruelle, 1985; Abarbanel et al., 1993; Tong, 1993; Abarbanel, 1996; Kantz et al., 1997; Diks, 1999; Galka, 2000; Sprott, 2003). The predictability of climate models depends on the dynamics of climatic parameters, as dynamics and thermodynamic processes are the causes of the non-linear responses in the atmosphere. Therefore for a proper identification, classification and mapping of climate variations, long-term systematic observational datasets are required from a network of stations in conjunction with statistical techniques.

Fractal theory has been widely applied on diverse types of datasets in geophysics as well as in other fields (Mandelbrot and Wallis, 1969; Rehman, 2009) to identify the irregular and complex behaviors of dynamical systems (Men et al., 2004). He et al., 2016, analyzed the spatio-temporal variations in rainfall for flood seasons during 1958-2013 using Hurst exponent in China and concluded that the rainfall trends will persist in the future also having implications for the ecological restoration and farming operations.

The state of Uttarakhand located in the lap of central Himalayan region has been identified as a hotspot of anthropogenic stress and one of the most vulnerable regions for climate mediated risks. The region provides water resources supporting millions of people in South Asia. During the last few decades, the central Himalayan region is observing the cascading effects of the climate change including rise in temperature, receding of glaciers, erratic precipitation patterns etc. (Dimri and Kumar, 2008; Shekhar et al., 2010). In the aforementioned scenario, the Himalayan region is receiving global scientific attention; however, available climate models often have limitation in resolving the highly complex geographical topography in this region. Timely and accurate predictions of climate variables using statistical methods inheriting the relationships among meteorological parameters and terrain effects could therefore complement the understanding of the climate dynamics in the fragile ecosystem of the Uttarakhand. In this study, we compute the Hurst exponent, fractal dimensions and the climate Predictability Index (PI) of pressure (PI_P), temperature (PI_T) and rainfall (PI_R) at two low-altitude (Pantnagar, Dehradun) and two high-altitude (Tehri and Mukteshwar) stations in the Himalayan foothills using the fractal theory.

2. Methodology

2.1 Fractal analysis and Hurst exponent

Fractal are characterized by self-similarity property having similar characteristics when analyzed over a large range of scales and that the parts of any size of a fractal have characteristics similar to that of the whole fractal (Schroeder, 1991; Mandelbrot and Wallis, 1968, 1969; Mandelbrot and Van, 1968; Hurst et al., 1965; Fluegeman et al., 1989; Hsui et al., 1993; Turcotte, 1997; Rangarajan and Sant, 1997; Rangarajan and Sant, 2004). The fractal dimension of a time series recognizes a process between the deterministic and random (Peters, 1996). If the fractal-dimension (FD) for a time series is 1.5, a usual Brownian motion develops resulting into no correlations between amplitude changes corresponding to successive time intervals. The Hurst exponent (H) (Peitgen et al., 1992; Rehman and Siddiqi, 2007) is a measure of the long-term memory spread of a data set. According to its value, a time series is classified as persistent (0.5 < H ≤ 1) or anti-persistent (0 ≤ H < 0.5). If H = 0.5, then the subsequent data are not inter-correlated, meaning that the future values of the time series are not influenced by the present or past values (Sakalauskiene, 2003) indicating that the series is unpredictable.

In terms of Asymptotic scaling relation, the Hurst exponent of real valued time series is defined as:

\[ \left( \frac{R(n)}{S(n)} \right) = Cn^{H} \]  

where, C is a constant, angular brackets \( \langle \cdots \rangle \) denote expected value, S(n) is the Standard Deviation of the first \( n \) data of the series \( \{X_1, X_2, \cdots, X_n\} \), \( R(n) \) is their range:
\[ R(n) = \max\{X_1, X_2, \ldots, X_n\} - \min\{X_1, X_2, \ldots, X_n\} \]

The Hurst exponent \( H \) is calculated from rescaled range technique and can also be computed from wavelets method for the time series \( \{X_1, X_2, \ldots, X_n\} \).

2.1.1. Estimate of the Hurst Exponent (H\(_w\)): Wavelets method

The wavelets method is valid for self-affine traces, where the variance is not constant as the window size increases. If \( f(t) \) is a self-affine random process, \( t \) a position parameter (time or distance), \( a \geq 0 \) is a scale (dilatation) parameter, \( w(t) \) is a mother wavelet,

\[ w_{t,a}(t) = \frac{1}{\sqrt{a}} w\left(\frac{t-t}{a}\right) \]

is its shifted, dilatated and scaled version, then the continuous wavelet transform of, \( f(t) \) is defined as:

\[ W(t,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} w_{t,a}(t') f(t') dt' \]

(2)

If the time series \( f(t) \) is self-affine, the variance of \( W(t,a) \) will scale with the dilatation parameter asymptotically as:

\[ V(a) = \left\{ \frac{\langle W^2 \rangle}{\langle W \rangle^2} \right\} \propto a^\delta \]

(3)

The exponent \( \delta \) is between minus 1 and plus 3, \(-1 \leq \delta \leq 3\). The Hurst exponent is defined as:

\[ H_w = \begin{cases} \frac{\delta + 1}{2} & \text{if } -1 \leq \delta < 1 \quad (\text{FGN}) \\ \frac{\delta - 1}{2} & \text{if } 1 \leq \delta \leq 3 \quad (\text{FBM}) \end{cases} \]

(4)

where, FGN is Fractal Gaussian Noise, FBM is Fractional Brownian Motion, The Hurst exponent is linked with fractal dimension (D) and define as:

\[ H = 2 - D \]

(5)

This wavelet-based method is used to find \( H \) for rainfall data (Rangarajan and Sant, 2004) and the climate predictability index is given as:

\[ PI = 2[D - 1.5] = 2(0.5 - H) \]

(6)

If \( PI \) is close to zero, climate is unpredictable. Closer the \( PI \) is to 1, the more predictable climate is.

### Table 1

| Site     | Latitude (°) | Longitude (°) | Altitude (m) |
|----------|--------------|---------------|--------------|
| Pantnagar| 28.97° N     | 79.41° E      | 344          |
| Dehradun | 30.34° N     | 78.04° E      | 682          |
| Tehri    | 30.38° N     | 78.48° E      | 1750         |
| Mukteshwar| 29.47° N   | 79.64° E      | 2271         |

#### 2.2. Study region and observational dataset

The study stations include two low-altitude (Pantnagar, Dehradun) and two high altitude (Tehri and Mukteshwar) stations in the Himalayan foothills as mentioned in Table 1. Dehradun and Tehri districts are in the Garhwal region, whereas Pantnagar and Mukteshwar are in the Kumaun region, Pantnagar and Dehradun may be considered as the boundaries of Gangetic plain and to the south most of central Himalaya. Dehradun is the capital of the Uttarakhand state and experiencing rapid increase in the traffic and population. Despite of relatively less population in Pantnagar, previous observations of chemical tracers and model results showed potential mixing of pollution from this station with a Himalayan station especially during pre-monsoon (Ojha et al., 2012; Sarangi et al., 2014). The observations of meteorological variables used for the analysis are conducted by the India Meteorological Department (IMD).

#### 3. Results and discussion

Fig. 1 shows a comparison of Fractal Dimension (FD), Hurst exponent (H) and temperature predictability index (PI) among the four stations: Pantnagar, Dehradun, Mukteshwar and Tehri. FD values are found to show strong station-to-station and seasonal variations in the Himalayan foothills. During the winter, both low-altitude sites in the Indo-Gangetic Plain (Pantnagar and Dehradun) show high FD values (1.38-1.52 at Pantnagar and 1.35-1.47 at Dehradun) resulting in the lower temperature predictability (0.03-0.22 at Pantnagar and 0.05-0.3 at Dehradun). In contrast, FD values at high-altitude stations are found to be closer to unity leading to better temperature predictability. Over Tehri, FD values remain lower and closer to unity throughout the year (1.15±0.07), except during summer-monsoon (1.44 during Aug-Sep) leading to average temperature predictability value of 0.68±0.13. Over Mukteshwar FD values are close to unity (1.12±0.1) except during January (1.4), leading to average temperature predictability value of 0.76±0.2 during Feb-Dec. Better climate predictability in the high-altitude stations indicates the potential to complement the
Fig. 1. Seasonal variations of the Fractal Dimension (FD), Hurst index (H) and the temperature predictability index (Plt) at four stations in the Himalayan foothills during 2005.

Numerical simulations of 3-D models which have limitation in resolving the complex terrains in this region. In general, on daily or monthly time scales the predictability will depend upon the correlations among the various meteorological parameters, which needs to be tested at various scales with a long-term data set. However, here the same is tested for the year 2005 and indicates towards the good predictability over the period of study.

To study the inter-annual variations in the climate predictability, a long-term computation of predictability indices for pressure, temperature and rainfall at Dehradun were performed for 2005-2014 period, as shown in the form of box plots in the Fig. 2. The range in box represent the difference between 25th percentile to 75th percentile i.e., the variability in data after reducing the effects of outliers by excluding data below (above) the 25th (75th) percentile. Whiskers provide the additional information of minimum and maximum values in the data. The practicability indices show significant variability which is somewhat expected in the Himalayan foothills due to modulations of strong local dynamics and large-scale impacts (Singh et al., 2016; Pant et al., 2018). Nevertheless, mean as well as median values indicate a reasonable predictability most of the year for temperature as well as pressure. Average pressure predictability index is found to be 0.8 or more at Dehradun, except during the summer-monsoon. Accurate predictions of pressure variations could help in identifying sudden buildups of low-high pressure systems resulting in disastrous events in this region, especially those occurring outside the monsoon season when we find good predictability. Average temperature predictability index for the study period is observed to be more than 0.5 in all the months (0.56-0.93) at Dehradun, except during July (0.43).
Moreover the year-to-year variability (1-standard deviation values) is found to be varying from 0.11 to 0.18, while during July the variability is also very high (0.29). It can be concluded that except during rainy season, pressure as well as temperature show reasonable predictability at Dehradun. The predictability index for rainfall ($P_l$) is found to be up to 0.5 during summer-monsoon, however, it is generally lower (0.16-0.34) during other months. Lower $P_l$ values indicate that rainfall is less predictable as compared to temperature and pressure. Lack of sufficient number of rainfall events inhibited to derive the correlated variability and $P_l$ values during winter. Fractal analysis approach may further be tested for more stations and long term data e.g. over a century, in order to understand the utility of statistical models particularly over the regions where dynamical models have limitations to resolve the topography.

4. Conclusions

In this short communication, Fractal analysis approach which is basically a mathematical method used to analyze a pattern repeated at every scale (and can’t be represented by classical geometry), is successfully utilized here to study and understand the predictability of various meteorological parameters at intra-seasonal scale. Significant variability is observed from station to station in the predictability of temperature, pressure and rainfall. Maximum Fractal dimension considered is 1.5 and the predictability index for a meteorological variable closer to unity indicates that it is quite predictable, whereas near to zero means non-predictable. Such an approach may be employed to test the predictability over a station or region by acquiring long term data series and multiple meteorological variables in an array arrangement. This may corroborate and complement the limitations of dynamical models where it becomes very uncertain to predict a meteorological variable. In the future course of study this approach is to be tested for meteorologically different zones and over complex topographical regions.

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