Phase Transition of charged Rotational Black Hole and Quintessence

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Abstract

In this paper, we calculate thermodynamical quantity of Kerr-Newman-AdS black hole solution in quintessence matter. Then, we show that how the rotation and cosmological parameters effect to the thermodynamics properties of black hole. Also, we investigate both types of phase transition for different values of $\omega$ parameter in extended phase space. We notice that type one of phase transition occurs for $P < 0.42$ and $a < 0.5$. And also we see that the phase transition point shifts to higher entropy when pressure $P$, rotation parameter $a$ and $\alpha$ increase. Also, we find that by changing parameter $\omega$ from -1 to $-\frac{1}{3}$, the critical point shifts to higher entropy. Then we study type two of phase transition and show critical points increase by increasing parameter $\alpha$. Also, we show that the critical point shifts to higher entropy when $\alpha$, $\omega$ and rotation parameter $a$ decrease. Finally, we find that by decreasing pressure the first critical point shifts to lower entropy and second critical point shifts to higher entropy.

Keywords: Thermodynamics; Phase transition; Quintessence matter; Kerr-Newman-AdS black hole solution.

1 Introduction

As we know black holes play an important role in physics especially quantum gravity. One of the interesting methods to study the quantum gravity is black hole thermodynamics in AdS spacetime [1,2]. First time Hawking and Bekenstein investigated the black hole thermodynamic. They found that all laws of black hole mechanics are similar to ordinary thermodynamics where surface gravity, mass and area of black hole are related to the temperature, energy and entropy respectively [3]. The study of black hole thermodynamic will

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be interesting with presence of a negative cosmological constant. Because such a cosmological constant play important role in holography and AdS/CFT. From AdS/CFT point of view, asymptotically AdS black hole spacetimes admit a gauge duality description with dual thermal field theory. Such theory lead us to interesting phenomenon which is called phase transition. For the first time, the phase transition was studied by Hawking and Page [4]. They found a phase transition between Schwarzschild-AdS black holes and thermal radiation which is known as Hawking-Page phase transition. After that, a lot of studies were done in context of AdS black hole thermodynamics especially in extended phase space [5-9]. In AdS black hole thermodynamics, the cosmological constant of the spacetime treat as pressure and its conjugate quantity as a thermodynamic volume. But in extended phase space, it appear as a thermodynamical variable in the first law of black hole thermodynamics. Here, remarkable matter is that in extended phase space the mass of black hole $M$ is not related to energy but it is associated with enthalpy by $M = H \equiv U + PV$ [3], so the first Law is expressed as,

$$dM = TdS + VdP + \Phi dQ + \Omega dJ.$$  \(1\)

The phase transition plays a key role to study thermodynamical properties of a system near critical point [10-14]. One of the approaches to investigate phase transition is studying the behavior of the heat capacity in different ensembles. In the study of heat capacity, we encounter two types of phase transition as type one and type two. The type one is related to changes of signature in heat capacity where roots of heat capacity are representing phase transitions. The type two is related to divergency of the heat capacity. It means that the singular points of the heat capacity are representing the phase transitions [15,16]. Also, the heat capacity is an interesting thermodynamical quantity to determine stability and instability of the black hole. For general black hole heat capacity is always negative which shows such a black hole is unstable and produces Hawking radiation. But with presence of charge and rotation parameters of black hole the heat capacity could be positive and the phase transition will happen. But the study of phase transition will be interesting when black holes are surrounded by quintessence or other matters. In 2003, Kiselev obtained the Einstein equations solution for the quintessence matter around schwarzschild black hole [17]. These solution is written in terms of $\omega$ and $\alpha$. The state parameter $\omega$ is defined by the equation of state $p = \omega \rho$ where $p$ and $\rho$ are the pressure and energy density of the quintessence respectively. Quintessence is a dark energy model with the state parameter $-1 < \omega < 1$ which the case of $\omega = \frac{1}{3}$ represents radiation and case of $\omega = 0$ represents dust around black hole [18]. To describe the late-time cosmic acceleration, the $\omega$ is restricted to $-1 < \omega < -\frac{4}{3}$ but $\omega$ will not equal $0, -1, -\frac{1}{3}$ [19-23]. The parameter $\omega$ is an important parameter to determine the property of spacetime metric. The spacetime has the asymptotically flat solution for $(-\frac{1}{3} < \omega < 0)$ and it has de Sitter horizon for $(-1 < \omega < -\frac{4}{3})$.

The rotation Kiselev and Kerr-Newman kiselev solution was determined in Ref. [24-27] and also Kerr-Newman-AdS solution in the quintessence was obtained in [28]. Then some efforts were made in context of thermodynamic and phase transition for Kiselev and Kerr Kiselev black hole [18,29]. In this paper we want to study thermodynamical relation and phase transition for Kerr-Newman-AdS solution in the quintessence.

The outline of the paper is as follows. In section II, we review Kerr-Newman-AdS black
hole solution in quintessence matter and consider behavior of temperature. In section III, we study the phase transition of this black hole for different value of parameter $\omega$. Finally, in section IV we have some results and conclusion.

2 Kerr-Newman-AdS black hole solution with quintessential energy

The Kerr-Newman AdS metric in quintessence matter is expressed as follows [28],

$$ds^2 = \frac{\Sigma^2}{\Delta_r} dr^2 + \frac{\Sigma^2}{\Delta_\theta} d\theta^2 + \frac{\Delta_\theta \sin^2 \theta (a \frac{dt}{\Xi} - (r^2 - a^2) \frac{d\phi}{\Xi})^2}{\Sigma^2} - \frac{\Delta_r \frac{dt}{\Xi} - a \sin^2 \theta \frac{d\phi}{\Xi})^2}{\Sigma^2}, \quad (2)$$

where

$$\Delta_r = r^2 - 2Mr + a^2 + Q^2 + \frac{r^2}{\ell^2} (r^2 + a^2) - \alpha r^{1-3\omega},$$

$$\Delta_\theta = 1 - \frac{a^2}{\ell^2} \cos^2 \theta \quad \Xi = 1 - \frac{a^2}{\ell^2} \quad \Sigma^2 = r^2 + a^2 \cos^2 \theta. \quad (3)$$

The cosmological constant is $\Lambda = -\frac{3}{\ell^2}$ which interpret as a thermodynamic pressure $P$ by,

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi \ell^2}. \quad (4)$$

The mass of black hole is determined by $\Delta_r(r_+) = 0$, so

$$M = \frac{(r_+^2 + a^2) (\sqrt{r_+^2 + \ell^2}) + Q^2 \ell^2 - \alpha \ell^2 r_+^{1-3\omega}}{2r_+ \ell^2}, \quad (5)$$

the area of event horizon of the black hole is given by,

$$A = 4\pi \frac{(r_+^2 + a^2)}{\Xi}. \quad (6)$$

The Bakenstein-Hawking entropy is given by

$$S = \frac{A}{4} = \pi \frac{(r_+^2 + a^2)}{\Xi}, \quad (7)$$

and Hawking temperature which is related to surface gravity is defined by,

$$T_H = \frac{\kappa(r_+)}{2\pi} = \lim_{\theta \to 0, r \to r_+} \frac{\partial r \sqrt{g_{tt}}}{2\pi \sqrt{g_{rr}}}, \quad (8)$$

Thus, the thermal temperature of Kerr-Newman-AdS solution in quintessential dark energy is obtained as,

$$T = \frac{r_+}{4\pi \Xi (r_+^2 + a^2)} \left( \frac{3r_+^2}{\ell^2} + \frac{a^2}{\ell^2} + 1 - \frac{a^2 + Q^2}{r_+^2} + 3\alpha \omega r_+^{1-3\omega} \right). \quad (9)$$
One can rewrite the temperature in terms of $S$ and $P$ which is given by,

$$T = \frac{\sqrt{S \Xi - a^2}}{4S \Xi^2} \left( \frac{16\pi P}{3} \left( \frac{S \Xi}{\pi} - a^2 \right) + \frac{8SP \Xi}{3} + 1 - \frac{(a^2 + Q^2)}{S \Xi - a^2} + 3\alpha \omega \left( \frac{S \Xi}{\pi} - a^2 \right)^{-\frac{1-3\omega}{2}} \right).$$  \hspace{1cm} (10)

We plot Figure (1) and (2) to study the behavior of the temperature. As we see in figure (1), rotation parameter $a$ and pressure $P$ play an important role in the temperature. The positive range of temperature (physical state) increases by decreasing rotation and pressure. And also, the temperature is always negative for $P > 0.42$ and $a > 0.5$ which Represents a unphysical state. Figure (2) shows that $\omega$ and $\alpha$ parameters have small influence on sign of temperature.

3 Phase transition of Kerr-Newman-AdS black hole with quintessence

In this section, we are going to study the phase transition of Kerr-Newman-AdS black hole in Quintessential dark energy. The parameter $\omega$ is an important parameter to determine the property of spacetime metric. Here, we investigate two type of phase transition for different values of $\omega$ and compare the obtained result with each other. As mentioned in the introduction, the type one of phase transition occurs when the temperature vanishes. We plotted behavior of temperature with respect to entropy in figures (1) and (2). As we see, type one of phase transition occurs for $P < 0.42$ and $a < 0.5$ and the phase transition point
shifts to higher entropy when pressure $P$, rotation parameter $a$ and $\alpha$ increase. Also we see that by changing parameter $\omega$ from -1 to $-\frac{1}{3}$, the critical point shifts to higher entropy. We need to calculate the heat capacity of the black hole to study type two of phase transition. The heat capacity is an interesting thermodynamical quantity to determine stability and instability of the black hole. As we know the heat capacity is negative for general black hole then it is unstable and one can say a such black hole produce Hawking radiation. But for charged and rotating black hole the heat capacity could be positive and the black hole can have phase transition. Now, we calculate heat capacity for different $\omega$ and consider the phase transition.

**Case one:** $\omega = -\frac{1}{3}$

The heat capacity is given by,

$$C = T \left( \frac{\partial S}{\partial T} \right) = T \left( \frac{\partial S}{\partial r_+} \frac{\partial r_+}{\partial T} \right)$$

(11)

Form equation (7), (9) and (11), we obtain heat capacity for $\omega = -\frac{1}{3}$.

$$C = \frac{2\pi r_+^2 (r_+^2 + a^2) \left( \frac{3r_+^2}{r_+^2} + \frac{a^2}{r_+^2} + 1 - \frac{a^2 + Q^2}{r_+^2} - \alpha \right)}{\Xi \left( \frac{3r_+^2}{r_+^2} (r_+^2 + 3a^2) + \frac{a^2 + Q^2}{r_+} (3r_+^2 + a^2) + (a^2 - r_+^2)(\frac{a^2}{r_+^2} + 1 - \alpha) \right)}.$$ 

(12)

One can rewrite equation (12) in terms of entropy $S$ and pressure $P$ by substituting $r_+ = \sqrt{\frac{8\Xi}{\pi} - a^2}$ and $b^2 = \frac{3}{8\pi P}$. In figures (3) and (4), we have plotted heat capacity with respect
Figure 3: Left plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $a = 0.1$, $\alpha = 0.5$, $\omega = -\frac{1}{3}$ and different values of $P$; Right plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $P = 0.1$, $\alpha = 0.05$, $\omega = -\frac{1}{3}$ and different values of $\omega$.

to entropy. The left plot of Fig. (3) show that heat capacity will be divergent for $P < 0.35$ which is representing type two of phase transition. As we see, there are two critical points. For first case the system translate to an unstable phase afterward it will have a transition to a completely stability phase. In right plot of fig. (3) we see that phase transition will happen for $a < 0.4$ and critical points increases when rotation parameter decreases. By observing fig. (4) we notice that the system will be divergent for $\alpha < 0.2$ and $Q < 0.1$. Also these two figures show that the critical point shifts to higher entropy by decreasing pressure, rotation and $\alpha$ parameters.

**Case two: $\omega = -\frac{2}{3}$**

By substituting $\omega = -\frac{2}{3}$ in equation (9) and by differentiating equation (7) and (9), we determine heat capacity as follows,

$$C = \frac{2\pi r_+^2 (r_+^2 + a^2) \left(\frac{3r_+^2}{r_+^2} + \frac{a^2}{r_+^2} + 1 - \frac{a^2 + Q^2}{r_+^2} - 2\alpha r_+\right)}{\Xi \left(\frac{3r_+^2}{r_+^2} (3r_+^2 + 3a^2) + \frac{a^2 + Q^2}{r_+^2} (3r_+^2 + a^2) + (a^2 - r_+^2)(\frac{a^2}{r_+^2} + 1) - 4\alpha a^2 r_+\right)}.$$  (13)

In figures (5) and (6) we draw differentiation of heat capacity with respect to entropy. Figure (5) show that heat capacity have two divergent points for $P < 0.15$ and $a < 0.125$. As pervious case ($\omega = -\frac{1}{3}$), the first divergent point imply an unstable transition and a stable transition occurs for second critical point. Also we see that critical point shifts to lower entropy by increasing pressure and rotation parameter. In left plot of fig. (6) we see that the phase transition happen for $Q < 0.1$ also right plot show that $\alpha$ parameter does not affect on phase transition for $\omega = -\frac{2}{3}$. 
Figure 4: Left plot: Heat capacity $C$ with respect to entropy $S$ for $P = 0.05$, $a = 0.05$, $\alpha = 0.05$, $\omega = -\frac{1}{3}$ and different values of $Q$; Right plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $P = 0.05$, $a = 0.05$, $\omega = -\frac{1}{3}$ and different values of $\alpha$.

Figure 5: Left plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $a = 0.1$, $\alpha = 0.5$, $\omega = -\frac{2}{3}$ and different values of $P$; Right plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $P = 0.05$, $\alpha = 0.05$, $\omega = -\frac{2}{3}$ and different values of $a$. 
Case three: $\omega = -\frac{2}{3}$

We can calculate heat capacity as previous steps. For this case, the heat capacity is obtained as,

$$C = \frac{2\pi r_+^2(r_+^2 + a^2)\left(\frac{3r_+^2}{r_+^2} + \frac{a^2}{r_+^2} + 1 - \frac{a^2+Q^2}{r_+^2} - 3\alpha r_+^2\right)}{\Xi\left(\frac{3r_+^2}{r_+^2} - 3\alpha r_+^2\right)(r_+^2 + 3a^2) + \frac{a^2+Q^2}{r_+^2}(3r_+^2 + a^2) + (a^2 - r_+^2)(\frac{a^2}{r_+^2} + 1)}. \quad (14)$$

We plot the behavior of the heat capacity with respect to entropy $S$ in figures (7) and (8). By looking at Fig (7), we find that phase transition will occur for $P < 0.45$ and $a \leq 0.5$. And Fig (4) show that the heat capacity will be divergent for $Q < 0.1$ and $\alpha \leq 0.5$. Here, as pervious cases, the critical point move to higher entropy by decreasing four parameters $p, a, \alpha$ and $Q$. The only difference than two pervious cases is that the $C - Q$ plot has two divergent points.

Case four: $\omega = -\frac{1}{2}$

The heat capacity is determined as follows,

$$C = \frac{2\pi r_+^2(r_+^2 + a^2)\left(\frac{3r_+^2}{r_+^2} + \frac{a^2}{r_+^2} + 1 - \frac{a^2+Q^2}{r_+^2} - \frac{3}{2}\alpha r_+^2\right)}{\Xi\left(\frac{3r_+^2}{r_+^2} + 3a^2\right) + \frac{a^2+Q^2}{r_+^2}(3r_+^2 + a^2) + (a^2 - r_+^2)(\frac{a^2}{r_+^2} + 1) - \frac{3}{4}\alpha r_+^2(3a^2 - r_+^2)}. \quad (15)$$

We plot the heat capacity behavior with respect to entropy in figures (9) and (10). Fig. (9)
Figure 7: Left plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $a = 0.1$, $\alpha = 0.5$, $\omega = -1$ and different values of $P$; Right plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $P = 0.05$, $a = 0.05$, $\omega = -1$ and different values of $\alpha$.

Figure 8: Left plot: Heat capacity $C$ with respect to entropy $S$ for $P = 0.05$, $a = 0.05$, $\alpha = 0.05$, $\omega = -1$ and different values of $Q$; Right plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $P = 0.05$, $a = 0.05$, $\omega = -1$ and different values of $\alpha$. 
Figure 9: Left plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $a = 0.1$, $\alpha = 0.5$, $\omega = -\frac{1}{2}$ and different values of $P$; Right plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $P = 0.05$, $\alpha = 0.05$, $\omega = -\frac{1}{2}$ and different values of $a$.

show that the system will have a phase transition for $P < 0.08$ and $a < 0.125$. In left plot of fig. (10), we see that there are two critical points for $Q \leq 0.2$. This plot is similar to left plot of fig. (8) for $\omega = -1$ but here critical points occur in lower entropy. As we see in right plot of fig. (10), the heat capacity will be diverge for $\alpha \leq 0.5$ and also we see that critical points decrease by decreasing $\alpha$ parameter.

**Case five: $\omega = -\frac{2}{5}$**

The heat capacity is given by,

$$C = \frac{2\pi r^2_+ (r^2_+ + a^2) \left(\frac{3r^2_+}{\ell^2} + \frac{a^2}{r^2_+} + 1 - \frac{a^2 + Q^2}{r^2_+} - \frac{6}{5} \alpha r^\frac{1}{2}_+\right)}{\Xi\left(\frac{3r^2_+}{\ell^2}(r^2_+ + 3a^2) + \frac{a^2 + Q^2}{r^2_+}(3r^2_+ + a^2) + (a^2 - r^2_+)(\frac{a^2}{r^2_+} + 1) - \frac{12}{25} \alpha r^\frac{1}{2}_+(3a^2 - 2r^2_+)\right)}. \quad (16)$$

We draw $C - S$ diagram in figures (11) and (12). From Fig.(11), we notice that phase transition will happen for $P < 0.055$ and $a < 0.1$ and also right plot of figure is similar to right plot of fig. (9). And left plot of Fig.(11) show that phase transition exist for $Q < 0.2$ and also this plot is alike to left plot of fig. (4). The difference is that critical point occurs in lower entropy here. By observing the right plot of figure (12), we find that the heat capacity will be diverge for $\alpha \leq 0.5$. And also divergent points decrease by decreasing $\alpha$ parameter which is similar to right plot of figure (10).
Figure 10: Left plot: Heat capacity $C$ with respect to entropy $S$ for $P = 0.05$, $a = 0.05$, $\alpha = 0.05$, $\omega = -\frac{1}{2}$ and different values of $Q$; Right plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $P = 0.05$, $a = 0.05$, $\omega = -\frac{1}{2}$ and different values of $\alpha$.

Figure 11: Left plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $a = 0.1$, $\alpha = 0.5$, $\omega = -\frac{2}{5}$ and different values of $P$; Right plot: Heat capacity $C$ with respect to entropy $S$ for $Q = 0.05$, $P = 0.05$, $\alpha = 0.05$, $\omega = -\frac{2}{5}$ and different values of $a$. 
4 conclusion

In this paper, we calculated thermodynamical quantity of Kerr-Newman-AdS black hole solution in quintessence matter. Then we considered qualitative behavior of temperature and showed that the rotation parameter and cosmological constant have key role on behavior of temperature where the temperature will be always negative for \( P > 0.42 \) and \( a > 0.5 \) but \( \omega \) and \( \alpha \) parameters have small influence. Also we investigated both types of phase transition for different values of \( \omega \) parameter. By studying behavior of heat capacity we noticed that type one of phase transition occurs for \( P < 0.42 \) and \( a < 0.5 \). We saw that the critical point shifts to higher entropy when pressure \( P \), rotation parameter \( a \) and \( \alpha \) increase. Also we found that by changing parameter \( \omega \) from -1 to \(-\frac{1}{3}\), the critical point shifts to higher entropy. Then we studied type two of phase transition and observed critical points increase by increasing parameter \( \alpha \). Also we noticed that the critical point shifts to higher entropy when \( \alpha \), \( \omega \) and rotation parameter \( a \) decrease. Finally, we found that by decreasing pressure the first critical point shifts to lower entropy and second critical point shifts to higher entropy.

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