Within generalized linear response theory, an expression for the dielectric function is derived which is consistent with standard approaches to the electrical dc conductivity. Explicit results are given for the first moment Born approximation. Some exact relations as well as the limiting behaviour at small values of the wave number and frequency are investigated.

I. INTRODUCTION

The dielectric function $\epsilon(\vec{k}, \omega)$ describing the response of a charged particle system to an external, time and space dependent electrical field is related to various phenomena such as electrical conductivity and optical absorption of light. In particular, it is an important quantity for plasma diagnostics, see, e.g., recent applications to determine the parameters of picosecond laser produced high-density plasmas [1]. However, the application of widely used simplified expressions for the dielectric function is questionable in the case of nonideal plasmas.

As well known, the electrical dc conductivity of a charged particle system should be obtained as a limiting case of the dielectric function. However, at present both quantities are treated by different theories. A standard approach to the dc electrical conductivity is given by the Chapman-Enskog approach [2]. In dense plasmas, where many-particle effects are of importance, linear response theory has been worked out to relate the conductivity to equilibrium correlation functions which can be evaluated using the method of thermodynamic Green functions, see [3]. This way it is possible to derive results for the conductivity of partially ionized plasmas not only on the level of ordinary kinetic theory, but to include two-particle nonequilibrium correlations as well [4].

On the other hand, the dielectric function can also be expressed in terms of equilibrium correlation functions, but the systematic perturbative treatment to include collision effects is difficult near the point $\vec{k} = 0$, $\omega = 0$, because an essential singularity arises in zeroth order. Different possibilities are known to go beyond the well-known RPA result. In the static limit, local field corrections have been discussed extensively [5], and the dynamical behavior of the corrections to the RPA in the long-wavelength limit was investigated in the time-dependent mean field theory neglecting damping effects [6], see also [7] for the strong coupling case. At arbitrary $\vec{k}$, $\omega$, approximations are made on the basis of sum rules for the lowest moments [8]. However, these approximations cannot give an unambiguous expression for $\epsilon(\vec{k}, \omega)$ in the entire $\vec{k}$, $\omega$ space.

We will give here a unified approach to the dielectric function as well as the dc conductivity, which is consistent with the Chapman-Enskog approach to the dc conductivity and which allows for a perturbation expansion also in the region of small $\vec{k}$, $\omega$. In the following Section II the method of generalized linear response [9] is presented which allows to find very general relations between a dissipative quantity and correlation functions describing the dynamical behaviour of fluctuations in equilibrium. A special expression for the dielectric function is given in Section III which is related to the use of the force-force correlation function in evaluating the conductivity.

Different methods can be applied to evaluate equilibrium correlation functions for nonideal plasmas. We will use perturbation theory to evaluate thermodynamic Green functions [10]. Results in Born approximation are given in Section IV. Using diagram techniques, partial summations can be performed as shown in Ref. [3]. An alternative to evaluate equilibrium correlation function in strongly coupled plasmas is given by molecular dynamical simulations. It is expected that reliable results for the dielectric function for dense systems by quantum molecular dynamics will be available in the near future. Works in this direction are in progress but will not be discussed in this paper.

To illustrate the general approach, explicit results for the dielectric function in lowest moment Born approximation are given for a Hydrogen plasma in Section V. A sum rule as well as the conductivity are discussed. The simple approximation considered here will be improved in a subsequent paper [11], where a four-moment approach to the two-component plasma is investigated.
II. DIELECTRIC FUNCTION WITHIN GENERALIZED LINEAR RESPONSE THEORY

We consider a charge-neutral plasma consisting of two components with masses \( m_c \) and charges \( e_c \), where the index \( c \) denotes species (electron \( e \), ion \( i \)) and spin, under the influence of an external potential \( U_{\text{ext}}(\vec{r}, t) = e^{i(\vec{k}\vec{r} - \omega t)}U_{\text{ext}}(\vec{k}, \omega) + \text{c.c.} \). The total Hamiltonian \( H_{\text{tot}}(t) = H + H_{\text{ext}}(t) \) contains the system Hamiltonian

\[
H = \sum_{c,p} E^c_p c^+_p c^+_p + \frac{1}{2} \sum_{c,d,p,q} V_{cd}(q) c^+_p d^+_q d^+_q c^+_p
\]  

and the interaction with the external potential

\[
H_{\text{ext}}(t) = U_{\text{ext}}(\vec{k}, \omega) e^{-i\omega t} \sum_{c,p} e_c n^c_{p,-k} + \text{c.c.},
\]  

where \( E^c_p = \hbar^2 p^2/2m_c \) denotes the kinetic energy, \( V_{cd}(q) = e_c e_d/(\epsilon_0 \Omega_0 q^2) \) the Coulomb interaction and \( \Omega_0 \) the normalization volume. Furthermore we introduced the Wigner transform of the single-particle density

\[
n^c_{p,k} = (n^c_{p,-k})^+ = c^+_p c_p + c_{p-k/2} c_{p+k/2}.
\]  

Under the influence of the external potential, a time-dependent charge density

\[
\frac{1}{\Omega_0} \sum_{c,p,k'} e_c \langle \delta n^c_{p,k'} \rangle^t e^{i\vec{k}'\vec{r}} + \text{c.c.} = \frac{1}{\Omega_0} \sum_{c,p} e_c \delta f_c(\vec{p}, \vec{k}, \omega) e^{i(\vec{k}\vec{r} - \omega t)} + \text{c.c.}
\]  

will be induced. Here, \( \delta n^c_{p,k'} = n^c_{p,k'} - \text{Tr} \{ n^c_{p,k'} \rho_0 \} \) denotes the deviation from equilibrium given by

\[
\rho_0 = \exp(-\beta H + \beta \sum_c \mu_c N_c) / \text{Tr} \exp(-\beta H + \beta \sum_c \mu_c N_c).
\]  

The average \( \langle \ldots \rangle^t = \text{Tr} \{ \ldots \rho(t) \} \) has to be performed with the nonequilibrium statistical operator \( \rho(t) \), which is derived in linear response with respect to the external potential in Appendix A. For homogeneous and isotropic systems, we find simple algebraic relations between the different modes \( (k, \omega) \) of the external potential \( U_{\text{ext}}(\vec{k}, \omega) \) and the induced single-particle distribution

\[
\delta f_c(\vec{p}; \vec{k}, \omega) = e^{i\omega t} \langle \delta n^c_{p,k'} \rangle^t
\]  

which allow to introduce the dielectric function \( \epsilon(k, \omega) \), the electrical conductivity \( \sigma(k, \omega) \), and the polarization function \( \Pi(k, \omega) \). From standard electrodynamics we have

\[
\epsilon(k, \omega) = 1 + \frac{i}{\epsilon_0 \omega} \sigma(k, \omega) = 1 - \frac{1}{\epsilon_0 k^2} \Pi(k, \omega),
\]  

\[
\Pi(k, \omega) = \frac{1}{\Omega_0} \sum_{c,p} e_c \delta f_c(\vec{p}, \vec{k}, \omega) \frac{1}{U_{\text{eff}}(k, \omega)}
\]  

with \( U_{\text{eff}}(k, \omega) = U_{\text{ext}}(k, \omega)/\epsilon(k, \omega) \). Using the equation of continuity

\[
\omega \sum_p \delta f_c(\vec{p}; \vec{k}, \omega) = \frac{k}{m_c} \sum_p \hbar p_z \delta f_c(\vec{p}; \vec{k}, \omega),
\]  

where the \( z \) direction is parallel to \( \vec{k}, \vec{k} = k\vec{e}_z \), we can also express

\[
\Pi(k, \omega) = \frac{k}{\omega} \frac{1}{\Omega_0} \sum_{c,p} e_c \hbar p_z \delta f_c(\vec{p}; \vec{k}, \omega) \frac{1}{U_{\text{eff}}(k, \omega)}
\]  

\[
= \frac{k}{\omega} \langle J_k \rangle^t e^{i\omega t} \frac{1}{U_{\text{eff}}(k, \omega)}
\]  

where \( \langle J_k \rangle^t = J_k \) and \( U_{\text{eff}}(k, \omega) \).
with the current density operator

\[ J_k = \frac{1}{\Omega_0} \sum_{c,p} \frac{e_c}{m_c} \hbar p_z n^c_{p,k} . \]  

(10)

The main problem in evaluating the mean value of the current density \( \langle J_k \rangle ^t \), Eq. (10), is the determination of \( \rho(t) \). In linear response theory where the external potential is considered to be weak, the statistical operator \( \rho(t) \) can be found as shown in Appendix A. An important ingredient is that a set of relevant observables \( \{ B_n \} \) can be introduced whose mean values \( \langle B_n \rangle ^t \) characterize the nonequilibrium state of the system. The nonequilibrium statistical operator is constructed using a corresponding set of thermodynamic parameters \( \phi_n(t) \). For weak perturbations, in linear response theory it is assumed that the \( \phi_n(t) \) are linear with respect to the external potential, and a set of generalized response equations is derived which allow to evaluate the response parameters \( \phi_n(t) \). The coefficients of these response equations are given in terms of equilibrium correlation functions which can be evaluated using the methods of quantum statistics.

Solving this set of linear response equations by using Cramers rule, the response parameters can be eliminated. If the current density operator \( J_k \) can be represented as a superposition of the relevant observables \( B_n \), we find

\[ \Pi(k, \omega) = \frac{k^2}{\omega} \beta \Omega_0 \begin{vmatrix} 0 & M_{0n}(k, \omega) \\ M_{mn}(k, \omega) & M_{mn}(k, \omega) \end{vmatrix} / \left| M_{mn}(k, \omega) \right| \]  

(11)

with

\[
M_{0n}(k, \omega) = (J_k; B_n) , \quad M_{mn}(k, \omega) = (B_m; \dot{J}_k) ,
\]

\[
M_{mn}(k, \omega) = (B_m; [\dot{B}_n - i\omega B_n]) + \langle B_m; [\dot{B}_n - i\omega B_n] \rangle_{\omega + i\eta}, \quad \langle B_m; [\dot{B}_n - i\omega B_n] \rangle_{\omega + i\eta} \right) = \frac{\langle \dot{B}_m; J_k \rangle_{\omega + i\eta}}{\langle B_m; J_k \rangle_{\omega + i\eta}} \langle B_m; [\dot{B}_n - i\omega B_n] \rangle_{\omega + i\eta} .
\]

(12)

The equilibrium correlation functions are defined as

\[
(A; B) = (B^+; A^+) = \frac{1}{\beta} \int_0^\beta d\tau ~ \text{Tr} \left[ A(-\hbar \tau) B^+ \rho_0 \right] ,
\]

\[
\langle A; B \rangle_z = \int_0^\infty dt ~ e^{i z t} \langle A(t); B \rangle ,
\]

(13)

with \( A(t) = \exp(iHt/\hbar) A \exp(-iHt/\hbar) \) and \( \dot{A} = \frac{i}{\hbar}[H, A] \), furthermore we used the abbreviation

\[
\dot{J}_k = \epsilon^{-1}(k, \omega) J_k .
\]

(14)

The correlation functions can be evaluated by standard many particle methods such as perturbation theory for thermodynamic Green functions. In this context the correlation functions containing \( J_k \) are obtained from irreducible diagrams to Green functions containing \( J_k \), which do not disintegrate cutting only one interaction line.

The expression (11) for the polarization function is very general. Depending on the set of observables \( \{ B_n \} \), different special cases are possible such as the Kubo formula or the Boltzmann equation to be discussed in the following section. It is also possible to include two-particle nonequilibrium correlations if an appropriate set of \( B_n \) is chosen. We will work out here an approach to the dielectric function which is closely related to the Chapman-Enskog approach to the electrical conductivity.

### III. MOMENT EXPANSION OF THE POLARIZATION FUNCTION

Up to now, \( B_n \) was not specified. It is an advantage of the approach given here that different levels of approximations can be constructed, depending on the use of different sets of \( B_n \). If no finite order perturbation expansion of the correlation functions is performed in evaluating the polarization function (11), all these different approaches are exact and should give identical results. However, evaluating the correlation functions within perturbation theory, different results for the polarization function are expected using different sets of \( B_n \). As has been shown for the electrical conductivity, results from finite order perturbation theory are the better the more relevant observables are considered.

A simple example for a relevant observable \( B_n \) characterizing the nonequilibrium state of the system is the current density (14).
\[ B_n = J_k . \]

During this paper, we will treat this approach in detail. The current density is related to the lowest moment of the distribution function. Possible extensions to more general sets of relevant observables are discussed at the end of this section.

In the approach given by Eq. (15), we have
\[
\Pi(k, \omega) = -\frac{i k^2 \beta \Omega_0}{\omega} \frac{(J_k; J_k) (J_k; \dot{J}_k)}{M_{JJ}},
\]
with
\[
M_{JJ} = -i \omega (J_k; J_k) + \langle \dot{J}_k; J_k \rangle_{\omega + i \eta} - \langle \dot{J}_k; J_k \rangle_{\omega + i \eta} \langle J_k; J_k \rangle_{\omega + i \eta} .
\]

For the derivation we used the property
\[
(\hat{A}; B) = \frac{i}{\hbar \beta} \text{Tr}\{[A, B^\dagger] \rho_0\}
\]
(for proving perform the integral in the definition (13)) so that \((J_k; \dot{J}_k) = \frac{1}{\pi \hbar} \text{Tr}\{[J_k, J_{-k}] \rho_0\} = 0.

Applying integration by part (57), the expression (16) can be rewritten as
\[
\Pi(k, \omega) = -\frac{i k^2 \beta \Omega_0}{\omega} \frac{(J_k; \dot{J}_k) (J_k; J_k)_{\omega + i \eta}}{(J_k; J_k)_{\omega - \eta} \langle J_k; J_k \rangle_{\omega + i \eta}}
\]
Performing the limit \( \eta \to 0 \), for finite values of the correlation function \( \langle J_k; J_k \rangle_{\omega + i \eta} \) we obtain the simple result
\[
\Pi(k, \omega) = -\frac{i k^2 \beta \Omega_0}{\omega} \langle J_k; \dot{J}_k \rangle_{\omega + i \eta}
\]
which is also denoted as the Kubo formula for the polarization function. Similarly, the Kubo formula can also be obtained from more general sets of observables \( \{B_n\} \). A direct derivation of the Kubo formula is obtained from Appendix A, Eq. (22), if the set of relevant observables \( B_n \) is empty. Different approaches based on different sets of relevant observables \( B_n \) are formally equivalent as long as no approximations in evaluating the correlation functions are performed.

However, expressions (16) and (20) are differently suited to perform perturbation expansions. For this we consider the static conductivity \( \sigma = \sigma(0,0) \) which follows from
\[
\sigma(k, \omega) = i \omega k^2 \Pi(k, \omega)
\]
in the limit \( k \to 0, \omega \to 0 \).

Comparing the Kubo formula
\[
\sigma = \beta \Omega_0 \langle J_0; \dot{J}_0 \rangle_{i \eta}
\]
with the result according to (16),
\[
\sigma = \beta \Omega_0 \frac{(J_0; J_0) (J_0; \dot{J}_0)}{\langle J_0; J_0 \rangle_{i \eta} - \langle J_0; J_0 \rangle_{i \eta}^{-1} \langle J_0; J_0 \rangle_{i \eta}^{-1} \langle J_0; J_0 \rangle_{i \eta}^{-1}},
\]
it is evident that perturbation theory cannot be applied to (22) because in zeroth order this expression is already diverging. In contrast, (23) allows for a perturbative expansion. For instance, in Born approximation the Faber–Ziman result for the electrical conductivity is obtained. The expression \( \sigma^{-1} \sim \langle J_0; \dot{J}_0 \rangle_{i \eta} \) is also known as the force–force correlation function expression for the resistivity. More precisely, the resistivity should be given in terms of stochastic forces which are related to the second term in the denominator of Eq. (23), see also Eq. (58) in App. A. The applicability of correlation functions for the inverse transport coefficients has been widely discussed.

The approach to the dielectric function given in the present paper is based on the choice (13) for the set of relevant observables and may be considered as the generalization of the force–force correlation function method for the electrical resistivity to the dielectric function. Possible extensions of the set of relevant observables have been investigated in evaluating the dc conductivity in Ref. [1] and will be considered in evaluating the dielectric function in a forthcoming paper [11].
IV. EVALUATION OF CORRELATION FUNCTIONS

Within the generalized linear response approach, the polarization function is given in terms of correlation functions which, in general, are elements of matrices. Within a quantum statistical approach, the correlation functions are related to Green functions which can be evaluated by diagram techniques. This has been discussed in detail in the case of the static electrical conductivity \[3\] and will not be detailed here. Instead, we will consider only the lowest orders of perturbation theory (Born approximation).

In the case considered here, the relevant observable \(J_k\) \([10]\) is given by a single particle observable. The correlation functions occurring in \([10]\) will contain the operators \(n^i_{p, k} = c^+_p c_{p+k/2}\) and \(n^i_{p, k} = -i(\hbar p^c_k/m_c)\) \(n^i_{p, k} + v^c_{p, k}\), with

\[
u^c_{p, k} = \frac{i}{\hbar} \sum_{q, q'} V_{cd}(q) \left[ c^+_{p-k/2-q} d^+_{p'+q} c_{p+k/2} - c^+_{p-k/2} d^+_{p'+q} c_{p+k/2+q} \right].
\]

(24)

To evaluate the correlation functions, we perform a perturbation expansion with respect to the interaction \(V\), see App. B. In addition to the zeroth order terms, which reproduce the RPA result, we consider the Born approximation. Up to second order with respect to \(V\) we have

\[
\Pi(k, \omega) = -\frac{i \beta \Omega_0 k^2 \langle J_k J_k^{(0)} \rangle_{\omega+iq}}{1 + \sum_{cd,pp} \frac{\hbar^2}{m_c m_d} \rho_{c,p} \rho_{d,p'} \sum_{n} \frac{(e_n^{(0)} p^c_n)^2}{(e_n^{(0)} p^c_n)^2} \frac{1}{\eta - i\omega + i m_c p_k} + \frac{1}{\eta - i\omega + i m_c p_k} \frac{(J_n^{(0)} J_n^{(0)} p^c_n)^2}{(J_n^{(0)} J_n^{(0)} p^c_n)^2}}.
\]

(25)

The evaluation of the correlation functions for the non-degenerate case is shown in App. B. We obtain the following expression

\[
\Pi(k, \omega) = -\frac{\beta \sum c^2 \frac{e^2}{m_c} [1 + z_c D(z_c)]}{1 - i \frac{\bar{\epsilon}^2 e^2}{k^2 (4\pi \epsilon_0)^2 n_c/m_c} \sum_c e^2 n_c / m_c} \int_0^\infty dp \ e^{-p^2} \left( \ln \frac{\lambda - 1}{\lambda + 1} + \frac{2}{\lambda + 1} \right) W(p)
\]

with

\[
W(p) = \frac{2}{3} p \left( \frac{e_c}{m_c} - \frac{e_i}{m_i} \right)^2 \sum_c e^2 n_c [1 + z_c D(z_c)]
\]

\[
- \frac{M_{el}^{1/2}}{\mu_{el}^{1/2}} \left( \frac{e_c}{m_c} - \frac{e_i}{m_i} \right) \int_{-1}^1 dc \ c \left[ e_c D(z_c) - \sqrt{\frac{m_c}{m_{el} cp}} + e_i D(z_c) + \sqrt{\frac{m_i}{m_{el} cp}} \right].
\]

(27)

Here, \(z_{ci} = \# \sqrt{\frac{M_{el}}{2k_B T}}\), \(z_c = \# \sqrt{\frac{m_c}{2k_B T}}, \lambda(p) = (\hbar^2 k^2) / (4\mu_{el} k_B T p^2) + 1\), and

\[
D(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z^2 - x^2} dx = i \sqrt{\pi} e^{-z^2} [1 + \text{Erf}(iz)]
\]

(28)

denotes the Dawson integral. Note that a statically screened potential was used in \([24]\) to obtain a convergent collision integral, the screening parameter is given by \(\kappa^2 = \sum c^2 n_c / (\epsilon_0 k_B T)\). From \([26]\) it is immediately seen that the RPA result is obtained in the limit of vanishing interactions, \(W(p) = 0\).

V. RESULTS FOR HYDROGEN PLASMAS

The expression \([26]\) for the polarization function is simplified for a system consisting of protons and electrons, where \(e_i = -e_e, n_i = n_e, \text{ and } m_i / m_c = 1836:\)

\[
e(k, \omega) = 1 + \frac{e^2 n}{\epsilon_0 k_B T k^2} \left[ 2 + z_c D(z_c) + z_i D(z_i) \right] \left[ 1 - i \frac{\omega}{k^2 (4\pi \epsilon_0)^2 n_c} \mu_{el}^{1/2} (k_B T)^{5/2} / (2\pi)^{1/2} \int_0^\infty dp \ e^{-p^2} \left( \ln \frac{\lambda - 1}{\lambda + 1} + \frac{2}{\lambda + 1} \right) \right]
\]

\[
\times \left\{ \frac{2}{3} p \left[ 2 + z_c D(z_c) + z_i D(z_i) \right] - \left( \frac{M_{el}}{\mu_{el}} \right)^{1/2} \left[ \int_{-1}^1 dc \left( D(z_c) - \sqrt{\frac{m_c}{m_{el} cp}} - D(z_i) + \sqrt{\frac{m_i}{m_{el} cp}} \right) \right]^{-1} \right\}
\]

(29)
We first discuss the limiting case of small $k$. For $k \ll \omega \sqrt{m_e/(2k_B T)}$ we use the expansion
\[
D(z) = i\sqrt{\pi} e^{-z^2} - \frac{1}{z} - \frac{1}{2z^3} \pm \ldots
\] (30)
so that after expanding also with respect to $cp/z_{ei}$ we have
\[
\epsilon(0, \omega) = 1 - \frac{\omega^2}{\omega^2 + i\omega/\tau}
\] (31)
with $\omega_{pl}^2 = e^2 n/(\epsilon_0 \mu_{ei})$ and
\[
\tau = \frac{(4\pi\epsilon_0)^2}{e^4} \frac{(k_B T)^{3/2}}{n} \frac{\mu_{ei}^{1/2}}{4(2\pi)^{1/2}} \left[ \int_0^\infty \frac{dp}{p} e^{-p^2} \left( \ln \frac{\lambda - 1}{\lambda + 1} + \frac{2}{\lambda + 1} \right) \right]^{-1}
\] (32)
According to (8), the dc conductivity
\[
\sigma(0, \omega \to 0) = \omega_{pl}^2 \epsilon_0 \tau
\] (33)
is obtained, what coincides with the Faber-Ziman formula at finite temperatures [1].

On the other hand, in the limiting case of small $\omega$ we use for $\omega \ll \sqrt{2k_B T/m_e k}$ the expansion
\[
D(z) = i\sqrt{\pi} e^{-z^2} - 2z + \frac{4}{3} z^3 \pm \ldots
\] (34)
and obtain
\[
\lim_{k \to 0} \lim_{\omega \to 0} \epsilon(k, \omega) = 1 + \frac{\kappa^2}{-i\omega + dk^2} \left( 1 + \frac{i\omega}{2k_B T} \right)
\] (35)
with
\[
d^{-1} = -\frac{e^4}{(4\pi\epsilon_0)^2} \frac{4(2\pi)^{1/2}}{(k_B T)^{3/2}} \frac{\mu_{ei}^{1/2}}{n} \left[ \int_0^\infty \frac{dp}{p} e^{-p^2} \left( \ln \frac{\lambda - 1}{\lambda + 1} + \frac{2}{\lambda + 1} \right) \right]
\] (36)
Here, in evaluating the last expression of [23], also $z_{ei} + \sqrt{m_e/m_c cp}$ is considered as a small quantity, whereas $z_{ei} - \sqrt{m_e/m_c cp}$ is large in the region of relevant $p$. For small values $k < \sqrt{2k_B T/(\pi m_e)}$ the second term in the numerator of (35) can be neglected, and the diffusion type form of $\epsilon(k, \omega)$ is obtained, see [12].

As an example, a dense plasma is considered with parameter values $T = 50$ eV and $n_e = 3.2 \times 10^{23}$ cm$^{-3}$. Such parameter values have been reported recently in laser produced high-density plasmas by Sauerbrey et al., see [1]. We will use Rydberg units so that $T = 3.68$ in Ryd and $n_e = 0.0474$ in $a_B^{-3}$. At these parameter values, the plasma frequency is obtained as $\omega_{pl} = 1.54$, and the screening parameter as $\kappa = 0.805$.

First we discuss the dependence of the dielectric function on frequency for different values of $k$, see Figs. 1-4. For large values of $k$ our result for the dielectric function coincides with the RPA result. At decreasing $k$ strong deviations are observed. Both the RPA expression as well as the expression (29) for the dielectric function fulfill important relations such as the Kramers-Kronig relation and the condition of total screening. The validity of the sum rule
\[
\int_0^\infty \omega \text{Im} \epsilon(k, \omega) d\omega = \frac{\pi}{2} \omega_{pl}^2
\] (37)
is checked by numerical integration. The RPA result coincides with the exact value $\omega_{pl}^2 \pi/2 = 3.74$ to be compared with expression (29) which gives 3.74 at $k = 1$, 3.75 at $k = 0.1$, 3.71 at $k = 0.01$ and 3.74 at $k = 0.001$. The small deviations are possibly due to numerical accuracy.

To investigate the behavior at small $k$, we give a log-log plot of $\text{Im} \epsilon(k, \omega)$ as function of $\omega$ for different values $k$ in Fig. 5. For $\omega > \sqrt{2k_B T/m_e k}$ is 3.84$k$ the Drude-like behaviour (31) is clearly seen, with $\tau = 8.36$.

Considering the limit of small $\omega$, a log-log plot of $\text{Im} \epsilon(k, \omega)$ as function of $k$ for different values $\omega$ is shown in Fig. 6. The diffusion behavior (32) occurs for $k < \sqrt{2k_B T/(\pi m_e)} = 0.00732$ at $k > \sqrt{m_e/(2k_B T)} = 11.17$ with $d = 13.8$. Altogether the numerical evaluation of the general expression (29) for the dielectric function confirms the validity of the simple limiting formulae (13) and (17).

In this paper we have focussed the discussion only to the properties of $\epsilon(k, \omega)$. Related quantities such as $\epsilon^{-1}(k, \omega)$ will be investigated in a forthcoming paper [11]. The parameter values for density and temperature can be extended to other nondegenerate plasmas like ordinary laboratory plasmas or the solar plasma. This has been done with results showing the same qualitative behavior of the expression (29) in comparison with the RPA expression, but at shifted values of $k$ and $\omega$. 

6
VI. CONCLUSIONS

An expression for the dielectric function of Coulomb systems is derived which is consistent with the Chapman-Enskog approach to the dc conductivity. For a two-component plasma, explicit calculations have been performed in the lowest moment approach. In Born approximation, expressions are given which allow the determination of \( \epsilon(k, \omega) \) in an analytical way. It is shown that general relations such as sum rules are fulfilled as well as the dc conductivity is obtained in the form of the Ziman-Faber result.

We performed exploratory calculations to illustrate how the generalized linear response approach works. Obviously an improvement of the results can be obtained if i) the Born approximation is improved including higher order of perturbation theory, ii) higher moments of the single-particle distribution are taken into account. Both points have been discussed for the limiting case of the dc conductivity [3], where a virial expansion of the inverse conductivity was given.

A four moment approach will be presented in a subsequent paper [11] where also the comparison with the Kubo approach and computer simulations are discussed. Within the approach given here it is also possible to treat the degenerate case. Work in this direction is in progress.

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APPENDIX A: GENERALIZED LINEAR RESPONSE THEORY

To construct the nonequilibrium statistical operator \( \rho(t) \) we use the density matrix approach [9,13]. Characterizing the nonequilibrium state of the system by the mean values \( \langle A_n \rangle_t \) of a set of relevant observables \( \{ A_n \} \), the generalized Gibbs state

\[
\rho_{\text{rel}}(t) = e^{-S(t)/k_B},
\]  

where

\[
\frac{1}{k_B} S(t) = \Phi(t) + \sum_n \alpha_n(t) A_n
\]

is the entropy operator and

\[
\Phi(t) = \ln \text{Tr} \exp \left\{ - \sum_n \alpha_n(t) A_n \right\}
\]

is the Massieu-Planck function, follows from the maximum of the entropy

\[
\langle S(t) \rangle_t = -k_B \text{Tr} \{ \rho_{\text{rel}}(t) \ln \rho_{\text{rel}}(t) \}
\]

at given mean values

\[
\text{Tr} \{ A_n \rho_{\text{rel}}(t) \} = \langle A_n \rangle_t.
\]

The thermodynamic parameters (Lagrange multipliers) \( \alpha_n(t) \) are determined by the self-consistency conditions [42] and will be evaluated within linear response theory below.

The relevant statistical operator [35] does not solve the von Neumann equation, but it can serve to formulate the correct boundary conditions to obtain the retarded solution of the von Neumann equation. Using Abel’s theorem, the nonequilibrium statistical operator [3] is found with the help of the time evolution operator \( U(t, t') \),

\[
\frac{i\hbar}{\partial t} U(t, t') = H_{\text{tot}}(t) \ U(t, t'); \quad U(t', t') = 1,
\]

as
\[ \rho(t) = \eta \int_{-\infty}^{t} dt' \, e^{-\eta(t-t')} \, U(t, t') \, \rho_{\text{rel}}(t') \, U(t', t), \]  

(44)

where the limit \( \eta \to 0 \) has to be taken after the thermodynamic limit. Partial integration of (44) gives

\[ \rho(t) = \rho_{\text{rel}}(t) + \rho_{\text{irrel}}(t) \]  

(45)

with

\[ \rho_{\text{irrel}}(t) = -\int_{-\infty}^{t} dt' \, e^{-\eta(t-t')} U(t, t') \left\{ \frac{i}{\hbar} \left[ H_{\text{tot}}(t'), \rho_{\text{rel}}(t') \right] + \frac{\partial}{\partial t'} \rho_{\text{rel}}(t') \right\} U(t', t). \]  

(46)

The self-consistency conditions (42) which determine the Lagrange multipliers take the form

\[ \text{Tr}\{A_n \rho_{\text{irrel}}(t)\} = 0. \]  

(47)

For a weak external field \( U_{\text{ext}} \), the system remains near thermal equilibrium described by \( \rho_0 \) \(^\dagger\), so that \( \rho(t) \) (45) can be expanded up to the first order with respect to \( U_{\text{ext}} \). For this we specify the set of relevant observables \( \{A_n\} \) as \( \{H, N_c, B_n(\vec{r})\} \) (note that summation over \( n \) in (39) also means integration over \( \vec{r} \)) and the corresponding Lagrange parameters \( \{\alpha_n\} \) as \( \{\beta, -\beta_{tc}, -\beta \phi_n(\vec{r}, t)\} \),

\[ \frac{1}{k_B} S(t) = \Phi(t) + \beta H - \beta \sum_c \mu_c N_c - \beta \sum_n \int d^3r \, \phi_n(\vec{r}, t) \, B_n(\vec{r}) . \]  

(48)

Expanding the nonequilibrium statistical operator up to first order with respect to \( U_{\text{ext}} \) and \( \phi_n(\vec{r}, t) \) it is convenient to use the Fourier representation\(^\dagger\) so that

\[ \int d^3r \, \phi_n(\vec{r}, t) \, B_n(\vec{r}) = \phi_n(\vec{k}, \omega) \, e^{-i\omega t} B_n^+ + \text{c.c.} \]  

(49)

with

\[ \phi_n(\vec{r}, t) = e^{i(\vec{k}\vec{r} - \omega t)} \, \phi_n(\vec{k}, \omega), \quad B_n = \int d^3r \, B_n(\vec{r}) \, e^{-i\vec{k}\vec{r}}. \]  

(50)

The contributions to (43) are

\[ \rho_{\text{rel}}(t) = \rho_0 + e^{-i\omega t} \int_0^\beta d\tau \sum_n B_n^+(i\hbar\tau) \, \phi_n(\vec{k}, \omega) \, \rho_0 + \text{c.c.} \]  

(51)

and, applying the Kubo identity

\[ [A, \rho_0] = \int_0^\beta d\tau \, e^{-\tau H} \, [H, A] \, e^{\tau H} \, \rho_0, \]  

(52)

we find

\[ \rho_{\text{irrel}}(t) = -\int_{-\infty}^{t} dt' \, e^{-\eta(t-t')} \, e^{-i\omega t'} \int_0^\beta d\tau \left\{ \sum_{c,p} e_c \hat{n}_{p, -k}(t' - t + i\hbar\tau) \, U_{\text{ext}}(\vec{k}, \omega) \right. \]

\[ + \sum_n \left[ \hat{B}_n^+(t' - t + i\hbar\tau) - i\omega B_n^+(t' - t + i\hbar\tau) \right] \phi_n(\vec{k}, \omega) \right\} \rho_0 + \text{c.c.} \]  

(53)

\(^\dagger\)In general we have \( \phi_n(\vec{r}, t) = \sum_{k'} \int d^d\vec{r'} \, e^{i(\vec{k}' \cdot \vec{r} - \omega t)} \phi_n(\vec{k}', \omega) \) and \( B_{n, k'} = \int d^d\vec{r} \, B_n(\vec{r}) \, e^{-i\vec{k}' \cdot \vec{r}} \). The selfconsistency equations (47) must be fulfilled for any time \( t \) so that \( \omega' = \omega \) follows. Furthermore, the equilibrium correlation functions \( \text{Tr}(A_k B_n^+ \rho_0) \) do not vanish only if \( k' = k \) so that \( \langle A_k (\eta - i\omega) ; B_k \rangle \sim \delta_{kk'} \). The well-known property of linear response that only such fluctuations are induced where the wave vector and frequency coincide with the external potential is a consequence of homogeneity in space and time.
Inserting this result in the self-consistency conditions (17) we get the response equations

\[- \langle B_m; A \rangle_{\omega + i\eta} \ U_{\text{eff}}(k, \omega) = \langle B_m; C \rangle_{\omega + i\eta} \]

with the correlation functions defined by (33),

\[A = \sum_{c,p} e^c \hat{n}_{p,k}^c = i k \Omega_0 \hat{J}_k, \]

and

\[C = \sum_n \left[ \hat{B}_n - i \omega B_n \right] \phi_n(k, \omega). \]

To make the relation between the response equations (54) and the Boltzmann equation more closely, see [3], we introduce the 'stochastic' part of forces applying partial integrations

\[-ik \Omega_0 \left( B_m; \hat{J}_k \right) U_{\text{eff}}(k, \omega) = \frac{(B_m; J_k) + \langle \hat{B}_m; J_k \rangle_{\omega + i\eta} - \langle \hat{B}_m; J_k \rangle_{\omega + i\eta}}{(B_m; J_k)_{\omega + i\eta}} \langle B_m; C \rangle_{\omega + i\eta} \]

\[= (B_m; C) + \left\{ \hat{B}_m - \langle \hat{B}_m; J_k \rangle_{\omega + i\eta} B_m ; \right\} \left\{ C - \frac{(B_m; C)_{\omega + i\eta}}{(B_m; J_k)_{\omega + i\eta}} J_k \right\}_{\omega + i\eta} \]

Then, we find the following form for the response equations

\[-ik \Omega_0 \ M_{m0} U_{\text{eff}}(k, \omega) = \sum_n M_{mn} \phi_n(k, \omega) \]

with

\[M_{m0} = \left( B_m; \hat{J}_k \right) \]

and

\[M_{mn} = \left( B_m; [\hat{B}_n - i \omega B_n] \right) + \left\{ \hat{B}_m - \frac{(B_m; J_k)_{\omega + i\eta}}{(B_m; J_k)_{\omega + i\eta}} B_m ; \right\} \left\{ [\hat{B}_n - i \omega B_n] \right\}_{\omega + i\eta}. \]

The system of equations (59) can be solved applying Cramers rule. Then, the response parameters are represented as a ratio of two determinants.

With the solutions \( \phi_n \) the explicit form of \( \rho(t) \) is known, and we can evaluate mean values of arbitrary observables. In particular, we are interested in the evaluation of \( \langle J_k \rangle^t \exp(i\omega t) \) to calculate the polarization function (4) using (53), (23),

\[\langle J_k \rangle^t e^{i\omega t} = \beta \sum_n \left\{ \langle J_k; B_n \rangle - \langle J_k; [\hat{B}_n - i \omega B_n] \rangle_{\omega + i\eta} \right\} \phi_n(k, \omega) \]

\[-ik \Omega_0 \beta \langle J_k; \hat{J}_k \rangle_{\omega + i\eta} U_{\text{eff}}(k, \omega). \]

If \( J_k \) can be represented by a linear combination of the relevant observables \( \{ B_n \} \), we can directly use the selfconsistency conditions (12) and have

\[\langle J_k \rangle^t e^{i\omega t} = \text{Tr} \left[ J_k \ \rho_{\text{rel}}(t) \right] e^{i\omega t}. \]

Comparing with (22) we see that the remaining terms on the rhs of (22) compensate due to the response equations (29). After expanding \( \rho_{\text{rel}}(t) \) up to first order in \( \phi_n(k, \omega) \), Eq. (71), we have

\[\langle J_k \rangle^t e^{i\omega t} = \beta \sum_n \langle J_k; B_n \rangle \phi_n(k, \omega). \]

Inserting the solutions for \( \phi_n \) in the form of determinants, we get the same result as obtained if we expand the numerator determinant (11) with respect to its first row.
APPENDIX B: EVALUATION OF THE COLLISION TERM IN BORN APPROXIMATION

Let us first consider the lowest order of perturbation theory where we have for the correlation functions

\[
\langle n^d_{p,k}; n^c_{p',k} \rangle = \hat{f}_{p,k}^c \delta_{pp'} \delta_{cd}
\]

\[
\langle n^d_{p,k}; n^c_{p',k} \rangle_{\omega+i\eta} = (\eta - i\omega + i\hbar p_z k/m_c)^{-1} \hat{f}_{p,k}^c \delta_{pp'} \delta_{cd},
\]

(65)

where

\[
\hat{f}_{p,k}^c = (\beta \hbar^2 p_z k/m_c)^{-1} (f_{p-k/2}^c - f_{p+k/2}^c)
\]

(66)

Notice that \( \lim_{k \to 0} f_{p,k}^c = f_p^c = \{ \exp[\beta(E^c_p - \mu_c)] + 1 \}^{-1} \). In the classical limit where the Fermi function can be replaced by the Maxwell distribution, we have in lowest order with respect to the Coulomb interaction

\[
(J_k; J_k)^{(0)} = \frac{k_B T}{\Omega_0} \sum_c \frac{e^2}{m_c} n_c,
\]

(67)

\[
\langle J_k; J_k \rangle_{\omega+i\eta}^{(0)} = -i \frac{\omega}{k^2} \frac{1}{\Omega_0} \sum_c e_c^2 n_c \left[ 1 + z_c D(z_c) \right]
\]

(68)

with \( z_c = \frac{\omega}{k} \sqrt{\frac{m_c}{2\pi k_B T}} \) and

\[
D(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} \frac{dx}{x - z - i\eta}.
\]

(69)

Furthermore we have

\[
\langle \dot{J}_k; J_k \rangle_{\omega+i\eta}^{(0)} = -k_B T \frac{1}{\Omega_0} \sum_c \frac{e^2}{m_c} n_c - \frac{\omega^2}{k^2} \frac{1}{\Omega_0} \sum_c e_c^2 n_c \left[ 1 + z_c D(z_c) \right]
\]

\[
\quad = -\langle J_k; \dot{J}_k \rangle_{\omega+i\eta},
\]

(70)

\[
\langle \dot{J}_k; J_k \rangle_{\omega+i\eta}^{(0)} = -i\omega \frac{k_B T}{\Omega_0} \sum_c \frac{e^2}{m_c} n_c - i \frac{\omega^3}{k^2} \frac{1}{\Omega_0} \sum_c e_c^2 n_c \left[ 1 + z_c D(z_c) \right],
\]

(71)

so that from Eq. (14) the random phase approximation (RPA)

\[
\Pi^{(0)}(k, \omega) = -\beta \sum_c e_c^2 n_c \left[ 1 + z_c D(z_c) \right]
\]

(72)

is obtained.

After we have considered the collisionless plasma, we will now treat the general case of an interacting system where the correlation functions have to be evaluated with the full Hamiltonian \( H \). The evaluation of equilibrium correlation functions for an interacting many-fermion system can be performed within perturbation theory such as a Green function approach, and many-particle effects can be treated in a systematic way. We will give here the lowest order contribution with respect to the screened Coulomb interaction (Born approximation), a systematic treatment of higher orders can be done as indicated in [3] for the case of static conductivity.

In the numerator of (16), the higher order expansion for \( \langle J_k; J_k \rangle \) lead to the replacement of the occupation numbers \( f_p^c \) for the free fermion gas by the occupation numbers in an interacting fermion gas. This corrections in Born approximation can be given as shift of the single-particle energies and can be replaced by a shift of the chemical potential.

We will investigate here the collision terms where the Born approximation leads to essential contributions. For this we use the relations (proof by partial integration (57))

\[
\langle n^d_{p,k}; n^d_{p',k} \rangle_{\omega+i\eta} = (\eta - i\omega + i\hbar p_z k/m_c)^{-1} \left[ \langle n^d_{p,k}; n^d_{p',k} \rangle + \langle n^d_{p,k}; n^d_{p',k} \rangle_{\omega+i\eta} \right],
\]

(73)
\begin{align*}
\langle v_{p,k}^c; n_{p',k}^d \rangle_{\omega + i\eta} &= (\eta - i\omega + i\hbar p_z k/m_d)^{-1} \left[ (v_{p,k}^c; n_{p',k}^d) - (v_{p,k}^c; v_{p',k}^d)_{\omega + i\eta} \right], \\
&= \frac{(J_k; J_k)^2}{\langle J_k; J_k \rangle_{\omega + i\eta}} + \sum_{cd,pq} \frac{\hbar^2}{\Omega_0} e_c e_d \frac{p_z p_z'}{m_c m_d} \left\{ -1 + \frac{(J_k; J_k)}{\langle J_k; J_k \rangle_{\omega + i\eta}} \left[ \frac{1}{\eta - i\omega + i\hbar p_z k/m_d} + \frac{1}{\eta - i\omega + i\hbar p_z k/m_c} \right] \right\}. 
\end{align*}

In the Born approximation for the frequency and wave vector dependent collision term we take the evolution operator due to the noninteracting part $H^0$ of the Hamiltonian \[ H \] so that the correlation functions are immediately evaluated using Wick's theorem. Dropping single-particle exchange terms what can be justified for the Coulomb interaction in the low-density limit, we find

\begin{align*}
\langle v_{p,k}^c(\eta - i\omega); v_{p',k}^d \rangle &= -\frac{\pi}{\hbar} \sum_{c'p'q} \exp(\beta \hbar \omega) - \frac{1}{\beta \hbar \omega} V_{c'c}(q) f_{c'p'q}(1 - f_{c''p'}) \\
&\times \left\{ f_{p+k/2-2q}(1 - f_{p'-k/2}) \delta(E_{p+k/2-2q} + E_{p'-k/2} - E_{p+k/2} - \hbar \omega) \\
&\times V_{c'c}(-q) \delta_{c,c'}(\delta_{p',p} - \delta_{p',p}) + V_{c'c}(-k + q) \delta_{c,c'}(\delta_{p',p'} - \delta_{p',p'} + \delta_{p',p'}/2) - f_{p+k/2}(1 - f_{p'-k/2}) \delta(E_{p+k/2} + E_{p'-k/2} - E_{p+k/2} - \hbar \omega) \\
&\times [V_{c'c}(-q) \delta_{c,c'}(\delta_{p',p} - \delta_{p',p'}) + V_{c'c}(-k + q) \delta_{c,c'}(\delta_{p',p'} - \delta_{p',p'} + \delta_{p',p'}/2)] \right\}. 
\end{align*}

We evaluate the matrix element $M_{I,J}$, Eq. (75) in Born approximation to obtain the polarization function $\Pi(k,\omega)$, Eq. (16). Using \[ \Pi, \] \( \Pi \), \[ \Pi \], \[ \Pi \], \[ \Pi \], \[ \Pi \], \[ \Pi \] we introduce

\begin{align*}
R &= \frac{(J_k; J_k)^{00}}{\langle J_k; J_k \rangle_{\omega + i\eta}} = ik_B T \frac{k^2}{\omega} \sum_{c} e_c^2 n_c/m_c \\
&\times \left\{ e_c^2 n_c \left[ 1 + z_c D(z_c) \right] \right\} 
\end{align*}

and find the perturbation expansion $M_{I,J} = M_{I,J}^{(0)} + M_{I,J}^{(1)}$, where

\begin{align*}
M_{I,J}^{(0)} &= R \langle J_k; J_k \rangle^{00}, \\
M_{I,J}^{(1)} &= \frac{\hbar^2}{\Omega_0} \sum_{c'd'p} \frac{e_c e_d}{m_c m_d} p_z l_z \langle v_{p,k}^c; v_{p',k}^d \rangle_{\omega + i\eta} \left\{ -1 + R \left[ \frac{1}{\eta - i\omega + i\hbar p_z k/m_c} + \frac{1}{\eta - i\omega + i\hbar l_z k/m_d} \right] \right\} 
\end{align*}

Evaluating the correlation functions $\langle v_{p,k}^c; v_{p',k}^d \rangle_{\omega + i\eta}$ in Born approximation \[ (74) \], we have for small $k, \omega$

\begin{align*}
M_{I,J}^{(1)} &= 2 \frac{\pi \hbar}{\Omega_0} \sum_{lq} \frac{V_{c'c}(q)f_{c'f} \delta(E_{p+q} + E_{l-q} - E_p - E_l)}{q z} \left( \frac{e_c}{m_c} - \frac{e_l}{m_l} \right) \left( \frac{e_p}{m_p} + \frac{e_l}{m_l} \right) - 2R \left( \frac{p_z}{\hbar k_p z_m - \omega + \eta} + \frac{e_p}{m_p} + \frac{l_z}{\hbar k l_z/m_l - \omega + \eta} \right) 
\end{align*}

The further evaluation is done with introducing total and relative momenta \[ \vec{P} = \vec{p} + \vec{l}, \vec{p}' = (m_p \vec{p} - m_l \vec{l})/M_{ei}, \vec{P}' = \vec{p}' + \vec{l}, M_{ei} = m_e + m_i, \mu_{ei} = m_e^{-1} + m_i^{-1} \] so that

\begin{align*}
M_{I,J}^{(1)} &= 2 \frac{\pi \hbar}{\Omega_0} \frac{e_c e_l}{e_o} \frac{2 \pi h^2}{m_k B T} \left( \frac{2 \pi h^2}{m_k B T} \right)^{3/2} \left( \frac{2 \pi h^2}{m_k B T} \right)^{3/2} \int d^3 P \int d^3 P' \int d^3 P'' \\
&\times e^{-\frac{e_o^2}{2 \lambda^2}} e^{-\frac{e_c^2}{2 \lambda^2}} e^{-\frac{e_l^2}{2 \lambda^2}} (\omega - \omega')^2 + (\vec{p}' - \vec{p})^2 (\vec{p}' - \vec{p}) \left( \frac{e_c}{m_c} - \frac{e_l}{m_l} \right) \\
&\times \left\{ \vec{p}' \left( \frac{e_c}{m_c} - \frac{e_l}{m_l} \right) - 2R \frac{M_{ei} \omega}{\hbar \omega} \left( \frac{e_c}{P_z + M_{ei} \omega p_z'} - \frac{M_{ei} \omega}{M_{ei} \omega - i\eta} \right) + \frac{e_c}{p_z - M_{ei} \omega p_z' - \frac{M_{ei} \omega}{M_{ei} \omega - i\eta}} \right\}. 
\end{align*}
Furthermore we introduce dimensionless variables $\lambda P(2M_e k_B T)^{1/2}$, $\lambda p'$ $(2M_e k_B T)^{1/2}$, $\lambda = (h^2 \kappa^2) / (4M_e k_B T p'^2) + 1$ and spherical coordinates $p' = \{p'(1 - c^2)^{1/2}, 0, p' c\}$, $p'' = \{p'' (1 - z^2)^{1/2} \cos \phi, p'' (1 - z^2)^{1/2} \sin \phi, p'' z\}$ and perform the integral over $\phi$ according to

$$\int_0^{2\pi} \frac{d\phi}{|\lambda - c z - \sqrt{1 - c^2} \sqrt{1 - z^2 \cos \phi}|^2} = \frac{2\pi}{\lambda - c z} \sqrt{\lambda^2 - 1 + c^2 - 2\lambda c z + z^2}^{3/2}$$

so that

$$M_{JJ}^{(1)} = \frac{1}{8\lambda^2 n_e n_i} \frac{e^2 \bar{c}_e^2}{e_0} \left( \frac{\mu_{ei}}{2k_B T} \right)^{1/2} \frac{1}{2} \int_0^\infty \frac{dp'}{p'} e^{-p'^2} \int_{-1}^1 \frac{dz}{\pi^{3/2}} \frac{1}{\lambda - c z} \sqrt{\lambda^2 - 1 + c^2 - 2\lambda c z + z^2}^{3/2} (z - c) \left\{ p'^2 c \left( \frac{e_e}{m_e} - \frac{e_i}{m_i} \right)^2 \right\}$$

$$+ i R p' \left( \frac{e_e}{m_e} - \frac{e_i}{m_i} \right) \omega \lambda \left[ \frac{M_e}{\mu_{ei}} \sqrt{\frac{M_e}{2k_B T}} \right] + i R \left( \frac{e_e}{m_e} - \frac{e_i}{m_i} \right) \omega \lambda \left[ \frac{M_e}{\mu_{ei}} \sqrt{\frac{M_e}{2k_B T}} \right]$$

Now, the integrals over $z$ and $p'$ can be performed. Using

$$\int_{-1}^1 \frac{dz}{\lambda - c z} \frac{\lambda - c z}{(\lambda^2 - 1 + c^2 - 2\lambda c z + z^2)^{3/2}} (z - c) = c \left( \ln \frac{\lambda - 1}{\lambda + 1} + \frac{2}{\lambda + 1} \right)$$

we finally find

$$M_{JJ}^{(1)} = \frac{1}{8\lambda^2 n_e n_i} \frac{e^2 \bar{c}_e^2}{e_0} \left( \frac{\mu_{ei}}{2k_B T} \right)^{1/2} \frac{1}{4\pi^{3/2}} \int_0^\infty \frac{dp}{p} e^{-p^2} \left( \ln \frac{\lambda - 1}{\lambda + 1} + \frac{2}{\lambda + 1} \right)$$

$$\times \left\{ \frac{2}{3} p \left( \frac{e_e}{m_e} - \frac{e_i}{m_i} \right)^2 + i R \left( \frac{e_e}{m_e} - \frac{e_i}{m_i} \right) \omega \lambda \left[ \frac{M_e}{\mu_{ei}} \sqrt{\frac{M_e}{2k_B T}} \right] \right\}$$

with $z_{ei} = \frac{\sqrt{M_e}}{M_e}$. Together with (78), (67), this result can be inserted in expression (14) to evaluate $\Pi(k, \omega)$.

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Figure captions:
Fig.1: $\epsilon(k, \omega)$ as function of $\omega$ (in Ryd/$\hbar$) at $k = 1/a_B$ for a hydrogen plasma, $n_e = 3.2 \times 10^{23}$ cm$^{-3}$, $T = 50$ eV.
  a: Re $\epsilon$, b: Im $\epsilon$.
broken line: RPA, full line: first moment Born approximation.
Fig.2: The same as Fig.1 for $k = 0.1/a_B$.
Fig.3: The same as Fig.1 for $k = 0.01/a_B$.
Fig.4: The same as Fig.1 for $k = 0.001/a_B$.
Fig.5: Im $\epsilon(k, \omega)$ as function of $\omega$ for different $k$.
Fig.6: Im $\epsilon(k, \omega)$ as function of $k$ for $\omega = 0.000001$ Ryd/$\hbar$. 

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\[ \text{Re} \epsilon(k=1 \text{a}_B^{-1}, \omega) \]

frequency \( \omega \) [Ryd]
$\text{Im } \varepsilon(k=1 \text{ a}_B^{-1}, \omega)$

frequency $\omega$ [Ryd]
The diagram shows the real part of the energy $\epsilon(k=0.1 \, a_B^{-1}, \omega)$ as a function of frequency $\omega$ [Ryd]. The scale for the real part $\text{Re}(\epsilon)$ ranges from -20.0 to 80.0, while the frequency $\omega$ is logarithmically scaled from $10^{-6}$ to $10^{2}$ [Ryd]. The graph features two curves, one solid and one dashed, indicating different states or conditions of the system.
\text{Im} \varepsilon(k=0.1 \text{a}_B^{-1}, \omega)
$\text{Im} \varepsilon(k=0.01 \, a_B^{-1}, \omega)$

against frequency $\omega$ [Ryd]
$\Re \varepsilon(k=0.001 \text{ a}_B^{-1}, \omega)$

frequency $\omega$ [Ryd]
\[ \text{Im } \varepsilon(k=0.001 \text{ a}_B^{-1}, \omega) \]

\text{frequency } \omega \text{ [Ryd]}
$\text{Im } \varepsilon(k, \omega)$

- $k = 10^{-5} a_B^{-1}$
- $k = 10^{-4} a_B^{-1}$
- $k = 10^{-3} a_B^{-1}$
- $k = 10^{-2} a_B^{-1}$

frequency $\omega$ [Ryd]
\[ \omega = 10^{-6} \text{ Ryd} \]

\[ \text{Im} \varepsilon(k, \omega) \]

\( k \) in units of \( a_B^{-1} \)