Thermal Fluctuations of Disoriented Chiral Condensate Domains

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Abstract

We argue that disoriented chiral condensate (DCC) domains are not well defined for temperatures above the Ginzburg temperature $T_G \approx 0.7T_c$. Above $T_G$, the dynamics of DCC domains is dominated by thermal fluctuations leading to fluctuating orientation of the chiral field in a given domain. It implies that DCC domains may form even in relatively lower energy collisions where the temperature only reaches $T_G$, and never rises to $T_c$. It also means that detection of DCC can not be taken as a signal for an intermediate chirally symmetric phase of matter. Using these considerations, we estimate the probability distribution for DCC domains as a function of the chiral angle.

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Formation and detection of disoriented chiral condensates (DCC) in laboratory experiments has been a subject of intensive investigations recently. By DCC one essentially means the formation of a chiral condensate in an extended domain, such that the direction of the condensate is misaligned from the true vacuum direction. It has been suggested that regions of DCC may form in high multiplicity hadronic collisions or in relativistic heavy ion collisions [1–4]. It has been argued that, as the chiral field relaxes to the true vacuum in such a domain, it will lead to coherent emission of pions which can be detected as anomalous fluctuations in neutral to charged pion ratio [2].

The most important ingredient in most of the models, which have been proposed for DCC formation, is the assumption of an intermediate stage where chiral symmetry is restored (apart from the effects due to pion mass). Such a stage is naturally expected in the context of relativistic heavy ion collisions. In fact, DCC is thought to provide a relatively clean signal for an intermediate chirally symmetric phase of matter. Even in high multiplicity hadronic collisions such an intermediate stage may arise, though the thermalized region may not be large there.

In this paper, we consider thermal fluctuations of correlation regions and its effects on the formation of DCC domains. We argue that for temperatures below $T_c$ but above certain value $T_G$ (the Ginzburg temperature), regions of correlation length size ($\sim m_\pi^{-1}$, say) can easily fluctuate to chirally symmetric state, making DCC domains ill defined. A well defined DCC domain, inside which the field can evolve towards the true vacuum via field equations, can not exist until temperature has dropped below $T_G$ ($\approx 0.7 T_c$ for $T_c$ in chiral limit, or about $0.8 T_c$ for $T_c$ with non-zero pion mass). One implication of this result is that DCC domains can form even in relatively lower energy heavy-ion collisions where the temperature of the system never rises to $T_c$, but only rises up to $T_G$. Unfortunately, this also implies that detection of DCC, though interesting by itself, will not imply that quark-gluon plasma phase (or, more precisely, chirally symmetric phase of matter) has been detected. A hot hadronic gas, in the spontaneously broken phase of chiral symmetry, can still lead to formation of DCC as long as its temperature reaches a value near $T_G$.

We first recall the basic picture of DCC formation in the framework of linear sigma model. We follow the notations used in ref. [5]. The Lagrangian is expressed in terms of a scalar field $\sigma$ and the pion field $\pi$.

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \pi)^2 - \frac{\lambda}{4} (\sigma^2 + \pi^2 - c^2/\lambda)^2 + \epsilon \sigma. \]  

(1)

Chiral symmetry is explicitly broken due to the last term $\epsilon \sigma$. The three parameters $\lambda$, $c$, and $\epsilon$, are restricted so as to give the proper pion mass, pion decay constant, a reasonable value for the $\sigma$ mass, PCAC (partial conservation of axial vector current), and the condition that the ground state of the theory occur at $\sigma = f_\pi$ and $\pi = 0$. We have,

\[ m^2_\pi = \lambda f_\pi^2 - c^2, \]
\[ m^2_\sigma = 2\lambda f_\pi^2 + m^2_\pi, \]
\[ f_\pi m^2_\pi = \epsilon. \]  

(2) (3) (4)

The values of physical parameters we take are, $m_\pi = 140$ MeV, $f_\pi = 94.5$ MeV, and $m_\sigma = 1$ GeV. To obtain the effective potential, we expand the fields about an arbitrary point as
\[ \sigma(x) = v \cos \theta + \sigma'(x), \quad (5) \]

\[ \pi(x) = v \sin \theta + \pi'(x), \quad (6) \]

where \( v = |v| \). The primes denote fluctuations about the given point. At one loop order, and in the high temperature approximation, the effective potential is

\[ V(v, \theta; T) = \frac{\lambda}{4} v^4 - \frac{1}{2} \left( c^2 - \frac{\lambda T^2}{2} \right) v^2 - \epsilon v \cos \theta. \quad (7) \]

Here \( T \) is the temperature. In the chiral limit one finds, as is well known, a second-order phase transition at the critical temperature \( T_c = \sqrt{2c^2/\lambda} = \sqrt{2f_\pi} \approx 134 \text{ MeV} \) for our choice of parameters). When the non-zero quark masses are taken into account, chiral symmetry is explicitly broken, and is only approximately restored at a temperature \( T \approx 115 \text{ MeV} \) (in the sense that the saddle point at \( \theta = \pi \) disappears above this temperature). In the chiral limit, spontaneous breaking of chiral symmetry for temperatures below \( T_c \) implies that one particular point on the vacuum manifold \( S^3 \) (characterized by \( \pi^2 + \sigma^2 = f_\pi^2 \)) will be chosen as the vacuum state in a given region of space, with all points on \( S^3 \) being equally likely. One may expect this to essentially hold true even in the presence of (small) pion mass. This will lead to a sort of domain structure in the physical space where each domain will have the chiral field aligned in a given direction, but the directions in different domains vary randomly.

This essentially summarizes the physics of the formation of DCC domains. Formation of this type of domain structure in a phase transition has been extensively discussed in the context of topological defects in condensed matter physics and also in particle physics models of the early Universe [1]. For an equilibrium, second order phase transition one expects [4] maximum size of domains to be of the order of \( m_\pi^{-1} \). For such small domains, dramatic signals like fluctuation in neutral to charged pion ratio will not be observable. [For small DCC domains, some other signals may be promising, such as, antibaryon enhancement [7], enhancement in isospin violation [8], etc.] It was proposed by Rajagopal and Wilczek [4] that large DCC domains may arise if the transition happens out of equilibrium. The idea is that the system starts in thermal equilibrium at high temperature where the vacuum expectation value of the field is near zero. After the quench, the field will find itself sitting at the top of the central bump of a low (say, zero) temperature effective potential. The field will then roll down with its dynamics governed by zero temperature field equations. It was shown in ref. [4] that long wavelength modes can grow significantly during such a roll down. However, it has been argued in ref. [4] that due to strong coupling, the roll down of the field is very rapid, leading to small domains, perhaps 1-1.5 fm size.

Large DCC domains may arise in the annealing scenario proposed by Gavin and Müller [10]. Here one uses the fact that it takes some time for the effective potential to change its shape, depending on the cooling rate of the plasma. Initially, at temperatures close to \( T_c \), the effective potential is nearly flat for \( \sigma \approx \pi \approx 0 \). Therefore, the roll down time scale can be very large at first. Only as the potential approaches its zero temperature shape does the roll down become rapid. It was argued in ref. [10] that domains as large as 5 fm may form in this scenario.

We now turn to the consideration of thermal fluctuations of correlated domains. It is well known from the studies of phase transitions in condensed matter systems that, for second
order transitions, thermal fluctuations of the order parameter become completely dominant for temperatures too close to the critical temperature \[ T_c \]. This is the reason that in discussions of the ordered phases in condensed matter systems, as well as in particle theory models of the early Universe (in the context of topological defect formation), the picture of correlation regions with well defined value of the order parameter (i.e. the domains) makes sense only for temperatures below, what is known as, the Ginzburg temperature \( T_G \), see ref. \[ 6 \]. This temperature is defined as the one above which thermal fluctuations in the value of the order parameter, about its mean value, become much larger than the mean value itself \[ 11 \]. Equivalently, one can think of \( T_G \) as the temperature above which a domain of correlation length size can easily fluctuate to the symmetric phase by thermal effects \[ 12,6 \]. Only when such fluctuations become sufficiently suppressed, can one talk about a well defined domain with the order parameter field taking well defined orientation inside the domain (with subsequent evolution via field equations, etc.). This is why, one usually talks about the formation of topological defects via domain structure only for temperatures below \( T_G \), see ref. \[ 6 \]. [Though, there are subtle points here as far as large scale structure of defects is concerned, see ref. \[ 14 \].]

An intuitive way of estimating the value of \( T_G \) is as follows. \( T_G \) can be obtained by equating the temperature to the energy \( E_d \) needed to fluctuate a correlation domain to the chirally symmetric phase. \( E_d \) is equal to the energy density difference \( \Delta V(T) \) between the bottom of the effective potential and the top of the central bump, times the volume of a region of size correlation length. Here, \( m_\sigma^{-1} \) should determine the appropriate correlation length \[ 6,11,13 \]. Though, for a conservative estimate of \( T_G \), we will take \( m_\sigma^{-1} \) (as this is the largest length scale). Thus we get,

\[
E_d \equiv m_\sigma^{-3}(T) \Delta V(T) \simeq T .
\] (8)

Here \( m_\sigma(T) \) is the temperature dependent pion mass \[ 4 \], and \( \Delta V(T) \) is calculated from Eq.(7). We mention here that we are neglecting any gradient energy terms in this estimate of \( E_d \), as we are only interested in rough estimate of \( T_G \). Also, if we take the correlation length to be given by \( m_\sigma^{-1} \) (which is more appropriate) in the above equation, then \( T_G \) comes out to be extremely small \( \simeq 2 - 4 \) MeV. Clearly such a small value of \( T_G \) will have drastic implications. We should point out that this \( T_G \) will be larger (though still much less than the value obtained from Eq.(8)) if gradient terms are also incorporated in Eq.(8). [In fact, the gradient term can make the orientational fluctuations of the chiral field in a small domain to be energetically unfavorable compared to the magnitude fluctuations. Though such estimates of orientational fluctuations may not be entirely justifiable, pion being the (approximate) Goldstone boson \[ 13 \].] At this preliminary stage, we will not worry about these complications as they do not seem to have too significant effect on the estimates of \( T_G \) obtained from Eq.(8). The main aim of this paper is to explore the physical consequences of the considerations of thermal fluctuations with the estimate of \( T_G \) which is significantly lower than \( T_c \). We hope to present a more detailed calculation of \( T_G \) in a future work.

An interesting aspect for the case of chiral transition is that the bottom of the potential is tilted. Thus the value of \( T_G \), determined by solving Eq.(8) for \( T \), will depend on the chiral angle \( \theta \). Figure 1 shows the plot of \( T_G \) as a function of the chiral angle \( \theta \). [Though, we mention that the true value of \( T_G \), in the sense of defining the regime where fluctuations
are dominant, corresponds to the value of \( T_G \) at \( \theta = \pi \) in Fig.1. The plot of \( T_G \) vs. \( \theta \) is in the context of DCC formation, and shows the values of temperature at which domains with various values of \( \theta \) become unstable due to thermal fluctuations.]

There are two important points to be noted here. First, note that these values of \( T_G \) are significantly lower than the value of \( T_c \), especially for large \( \theta \). For \( \theta = \pi \), \( T_G \approx 0.6T_c \). For heavy-ion collisions, in the longitudinal expansion model of Bjorken [15], the temperature of the expanding fluid is proportional to \( \tau^{-1/3} \), \( \tau \) being the proper time. If we take that \( T = T_c \) is achieved at \( \tau \approx 3 - 10 \) fm, then the temperature will drop to \( 0.6T_c \) at \( \tau \approx 14 - 46 \) fm. These values of \( \tau \) are large from the point of view of the expected life time of the hadronic gas before freeze out. Also, note that in the context of the annealing scenario of DCC formation [10], for temperatures as low as \( 0.6T_c \), the bottom of the effective potential is not that flat any more (compared to the situation close to \( T_c \)), and the roll down of the chiral field will be much faster. The net result will be that the resulting DCC domain will not be as large. For example, \( m_\sigma \) may grow [4] by about a factor four as temperature drops from \( T_c \) to \( 0.6T_c \), leading to DCC domains which are smaller by a factor of two or so, say of size 2 - 2.5 fm.

Second point is that, due to the dependence of \( T_G \) on \( \theta \), domains of smaller \( \theta \) will become stable first (against thermal fluctuations) and domains with larger values of \( \theta \) will stabilize later. [For \( \theta \) between 0 and \( \pi \). Things being symmetric about \( \theta = \pi \).] During this time interval, a region with large value of \( \theta \) will still keep fluctuating to chirally symmetric phase (as the temperature is still above \( T_G \) relevant for that value of \( \theta \)). During these fluctuations, the magnitude of the chiral field is fluctuating between zero and the value corresponding to the broken phase. This then leads to fluctuating orientation of the chiral field in that region, with decreasing probability for the region to retain its large value of \( \theta \), as \( \theta \) in that region can take any value after each fluctuation. [For large \( \theta \) one may have to worry about the gradient energy terms. However, as for Eq.(8), we will not worry about it for the present rough estimates.] This leads to a probability distribution for different \( \theta \) values, with smaller values of \( \theta \) being most probable and large values of \( \theta \) being suppressed. We now calculate the time evolution of this probability distribution. For simplicity we discretize \( \theta \) between 0 to \( \pi \) in \( m \) segments (we will use \( m = 6 \)). Rate of change of the number of DCC domains \( N_i(\tau) \) with value of the chiral angle in the ith segment, at proper time \( \tau \), can be written as,

\[
\frac{dN_i(\tau)}{d\tau} = -A \exp[-E_d(i, T)/T] \left( \frac{m-1}{m} \right) N_i + \sum_{j, j \neq i} A \exp[-E_d(j, T)/T] \frac{1}{m} N_j . \tag{9}
\]

Here, the factors \( (m-1)/m \), and \( 1/m \), give probabilities for fluctuation of a domain out of, and into, the ith segment respectively. We will only be interested in rough estimates of the relative values of \( N_i(\tau) \), so we drop the pre-exponential factor \( A \) for the thermal fluctuation rate in the above equation (assuming that \( A \) does not depend on \( \theta \)). \( E_d(i, T) \) denotes the value of \( E_d \) (Eq.(8)) for the domain with \( \theta \) in the ith segment. For the time evolution of the temperature we assume longitudinal expansion model of Bjorken [15] for the plasma,
\[
T(\tau) = T(\tau_0)\left(\frac{\tau_0}{\tau}\right)^{1/3}.
\]

We take \(\tau_0 = 3\) fm as an example, corresponding to the chiral symmetry breaking transition, with \(T_0 = T_c \simeq 134\) MeV. Initial values of \(N_i\) are prescribed at a temperature \(T \simeq 110\) MeV (at \(\tau \simeq 5\) fm), when the central bump has already appeared in the effective potential in Eq.(7) (see, ref. [3]). Figure 2 shows the plots of \(N(\tau)\). [We mention again that the plots in Fig.2 are only supposed to represent relative values of \(N_i\)s, as we dropped the pre-exponential factor \(A\) in Eq.(9).]

We see from Fig.2 that the numbers of DCC domains with different values of \(\theta\) rapidly diverge from each other, approaching somewhat constant values asymptotically. In fact, the memory of initial condition seems to be lost rapidly and curves of a given \(\theta\) approach approximately same asymptotic value for both of the sets corresponding to two different initial conditions. This shows the robustness of the distribution of DCC domains with different \(\theta\) values. Of course, at later times, when thermal fluctuations are suppressed, classical evolution of the field towards the true vacuum will take over, enhancing the number of domains with \(\theta = 0\) and depleting others. We neglect any such evolution as our aim is only to provide a reasonable estimate of relative populations of DCC domains with different \(\theta\), which can then be taken as the initial condition to study the roll down of the field and domain growth etc. Around at \(\tau \simeq 16\) fm (which corresponds to \(T \simeq 0.6T_c\)) we find that ratio between number of DCC domains with \(\theta \simeq \pi\) and \(\theta \simeq 0\) is about 0.3. A nontrivial feature of these plots is that for all values of \(\theta\), the asymptotic value of \(N(\tau)\) remains non-zero, with \(N(\tau)\) for \(\theta \simeq \pi/2\) approaching a value appropriate for a uniform \(\theta\) distribution.

This brings us to another very important aspect of these considerations of thermal fluctuations of DCC domains. As domains keep fluctuating, and keep re-distributing \(\theta\), for any temperature not too low compared to \(T_G\), it gives a way to disorient the chiral vacuum without ever achieving an intermediate stage where the chiral symmetry is restored. Conventionally, in most approaches, such a stage is believed to be essential for disorienting the chiral vacuum. However, our considerations show that DCC domains can form even if the temperature of the hadronic matter, in heavy ion collisions or in hadronic collisions, only reaches a value near \(T_G\) (\(\simeq 0.7T_c\) for our parameters) and never rises above \(T_c\). This means that DCC formation may be accessible even in experimental situations with relatively lower center of mass energy.

However, at the same time, one is forced to conclude that detection of DCC (via any of the signals which have been proposed in the literature) will not imply that a new phase of the hadronic matter has been detected in which chiral symmetry is restored. DCC domains can form in the hadronic matter in the chiral symmetry broken phase itself, as long as its temperature reaches a value near \(0.7T_c\) (corresponding to \(T_G\) for \(\theta = 0\)). In some sense the chirally symmetric state would have been probed by the thermal fluctuations leading to DCC formation at such low temperatures. Still, reaching a different phase of the matter in the thermodynamic sense is not required. Certainly, detection of DCC will be interesting in itself as it probes the non-perturbative aspects of the chiral model, and may provide explanation for the Centauro events [2].

We summarize by emphasizing the main points of our results. We have argued that thermal fluctuations of correlation regions make DCC domains ill defined until the temperature of the hadronic system drops to the Ginzburg temperature \(T_G\) (\(\simeq 0.7T_c\) for our choice of
parameters). This means that studies of domain growth and the roll down of the chiral field towards the true vacuum etc. via field equations, can be performed only at temperatures lower than $T_G$. By considering thermal fluctuations and resulting re-distribution of $\theta$ in different DCC domains, we estimate how relative proportions of DCC domains with different values of $\theta$ change in time. Our results imply that DCC formation may be accessible even in relatively lower energy collisions. At the same time it implies that DCC can not be thought of as a signal for detecting an intermediate, chirally symmetric, phase of matter.

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FIGURE CAPTIONS

Fig.1: Ginzburg temperature $T_G$ (signifying the onset of fluctuations for a domain with given value of $\theta$) as a function of the chiral angle $\theta$ (in degrees) for $T_c = 134$ MeV.

Fig.2: Evolution of the relative proportion of DCC domains with different values of the chiral angle $\theta$. The set of curves with solid lines denotes a uniform initial distribution of $\theta$ with $N_i = 10, i = 1, \ldots, 6$, while the set of dashed curves corresponds to the initial condition with $N_1$ (corresponding to $\theta = 0^\circ - 30^\circ$ segment) equal to 60 while all other $N_i$s are zero initially. For both the sets, curves from the top to the bottom correspond to values of $\theta = 15^\circ, 45^\circ, 75^\circ, 105^\circ, 135^\circ$, and $165^\circ$. 
Fig. 2

$N(\tau)$ vs $\tau$ (fm)