Model for non-perturbative contributions to the quark distributions

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Abstract. In the present talk we briefly discuss the most general form of QCD Factorization, which we call Basic Factorization, and show how it is related to the conventional forms of factorization: $K_T$-Factorization and Collinear Factorization. When Basic Factorization is applied to hadronic scattering amplitudes, integration over momenta of the partons connecting perturbative and non-perturbative blobs runs over the whole phase space but result of the integration must be finite. This obvious requirement allows us to obtain theoretical restrictions on the fits for the involved non-perturbative contributions. In order to model non-perturbative contributions in Basic Factorization, we suggest the Resonance Model which can universally be used to describe hadronic reactions with unpolarized or polarized hadrons.

1. Introduction

QCD factorization is actually the only available means to apply results of Perturbative QCD to description of hadronic reactions at high energies. It is supposed to compensate, though approximative, lack of knowledge on QCD in the infra-red region. This method proved to be quite efficient for description of all kinds of hadronic processes and such important their ingredients as parton distributions in the hadrons. In essence, factorization works as follows: First, appropriate non-perturbative inputs are introduced and then Evolution Equations (or fixed-order perturbative calculations) evolve them to perturbative domain. However, actual situation is more involved and because of that different forms of factorization were invented. Choice of a particular form depends mostly on perturbative methods used in the calculations. In order to illustrate it, let us consider factorization for the DIS non-singlet structure function $f_{NS}(x, Q^2)$ as an example. We consider it under the approximation of single-parton collisions, when only one parton with momentum $k$ in the initial hadron with momentum $p$ interacts with the virtual photon with momentum $q$ while other partons are spectators. In this case, QCD factorization for each structure function in the hadronic tensor $W_{\mu\nu}$ is conventionally depicted as shown in Fig. 1, where the upper blob corresponds to the parton-photon scattering and the lowest blob corresponds to emission of a single active parton off the hadron¹.

When the DGLAP² or its generalizations to the small-$x$, or to the small-$Q^2$ regions (see e.g. Ref. [3] and Refs therein for detail) are used to describe $f_{NS}(x, Q^2)$, an appropriate non-perturbative input $\phi(x_0, \mu^2)$ is supposed to be on-shell. In this case Collinear Factorization⁴ is usually used, Using the DGLAP equations to evolve the scale $\mu^2$ up to $Q^2$ (with $Q^2 > \mu^2$) and

¹ For an overview of double-parton collisions see recent Ref. [1].
complementing the issue by the $x$-evolution, one arrives at the result which can be written as follows:

$$f_{NS}(x, Q^2) = \int_{\beta_0}^{1} \frac{d\beta}{\beta} \int_{0}^{w(1-x)} \frac{dk_{\perp}^2}{k_{\perp}^2} f^{(pert)}_{NS}(x/\beta, Q^2/k_{\perp}^2) \Phi(\beta, k_{\perp}^2),$$  \hspace{1cm} (1)

where $f^{(pert)}_{NS}$ is the perturbative component. It includes both the coefficient function and anomalous dimension. The factorization scale $\mu^2$, i.e. the starting point of the perturbative $Q^2$-evolution in the literature is usually chosen in the perturbative domain ($x_0 \sim 1$ and $\mu \sim \text{few GeV}$), so the input (integrated quark distribution) $\phi(x_0, \mu^2)$ contains both perturbative and non-perturbative contributions.

Obviously, Collinear Factorization cannot be applied when BFKL[5] is used to account for perturbative contributions, because the initial partons (gluons) here are essentially off-shell, with virtuality $k^2 \approx -k_{\perp}^2$, and integration over $k_{\perp}$ runs down to zero. In this case another form of QCD factorization, $K_T$-Factorization[6] named also High-Energy Factorization[7], can be applied. $K_T$-Factorization can also be be used beyond the BFKL context. In this case the expression for $f_{NS}(x, Q^2)$ is

$$f_{NS}(x, Q^2) = \int_{\beta_0}^{1} \frac{d\beta}{\beta} \int_{0}^{w(1-x)} \frac{dk_{\perp}^2}{k_{\perp}^2} f^{(pert)}_{NS}(x/\beta, Q^2/k_{\perp}^2) \Phi(\beta, k_{\perp}^2),$$  \hspace{1cm} (2)

where $\Phi(\beta, k_{\perp}^2)$ denotes the non-perturbative input (unintegrated quark distribution) in $K_T$-Factorization, $\beta_0 = x + k_{\perp}^2/w$ and $w = 2pq$. Obviously, $K_T$-Factorization is more general than Collinear Factorization. Momentum $k$ of the active parton in Fig. 1 is parameterized in $K_T$-Factorization as follows:

$$k = \beta p + k_{\perp},$$  \hspace{1cm} (3)

which operates with three parameters: $\beta, |k_{\perp}|$ and the azimuthal angle\(^2\). Nevertheless, momentum $k$ is four-component, so one of its longitudinal components is left unaccounted by the parametrization of Eq. (3).

Recently in Ref. [9] we have derived a new and most general form of QCD factorization: Basic Factorization where the parametrization of $k$ accounts for all four components of $k$ and therefore is most general:

$$k_\lambda = -\alpha \left(q_\lambda - (q^2/w)p_\lambda\right) + \beta \left(p_\lambda - (p^2/w)q_\lambda\right) + k_{\perp}^\lambda,$$  \hspace{1cm} (4)

i.e. it is the standard Sudakov parametrization[8]. In Basic Factorization

$$f_{NS}(x, Q^2) = \int_{\beta_0}^{1} \frac{d\beta}{\beta} \int_{0}^{w(1-x)} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{0}^{w} \frac{d\alpha}{\alpha} f^{(pert)}_{NS}(x/\beta, Q^2/k^2) \frac{B}{k^2} \Psi(\alpha, k^2),$$  \hspace{1cm} (5)

\(^2\) Dependence on the azimuthal angle is often neglected as unessential.
where the factor $B$ is
\[ B = k_\perp^2 + w \left( \alpha^2 + \beta^2 \right)/2. \] (6)

It arises from handling the spinors, while the factors $k^2$ correspond to the propagators of the intermediate (active) quarks. In practical implementations, one can use the approximation $b \approx k_\perp^2$. $\Psi(\alpha, k^2)$ is a new non-perturbative input (totally unintegrated quark distribution). Let us remind that $k^2 = -w\alpha\beta - k_\perp^2$. Optical theorem relates $f_{NS}$ to the non-singlet part of the Compton amplitude $A_{NS}$ in the forward kinematics. In Basic Factorization $A_{NS}$ is given by the following expression:
\[ A_{NS}(x, Q^2) = \int \frac{d\beta}{\beta} dk_\perp^2 d\alpha A_{NS}^{(pert)}(x/\beta, Q^2/k^2) \frac{B}{k^2 k_\perp^2} T(\alpha, k^2), \] (7)

where $A_{NS}$ is the perturbative and $T$ is the non-perturbative input (altogether non-perturbative quark-hadron scattering amplitude). In contrast to Eq. (5), integration over $k$ in Eq. (7) runs over the whole phase space. Basic Factorization approximately can be reduced to $K_T$-Factorization providing the most essential values of $\alpha$ are outlined by the following restriction:
\[ w|\alpha\beta| \ll k_\perp^2, \] (8)

which is the standard restriction for the ladder momenta in all available perturbative Evolution Eqs, including DGLAP and BFKL. On its turn, $K_T$-Factorization can approximately be reduced to Collinear Factorization providing the dependence of $\Phi(\beta, k_\perp^2)$ on $k_\perp^2$ has a specific form: it should be one or several sharp peaks.

Another important result obtained in Ref. [9] is theoretical restrictions on the fits for non-perturbative inputs in all forms of QCD factorization. Integration in Eq. (7) runs over the whole phase space and it should yield a finite result despite that the integrand can have both infra-red (IR) and ultra-violet (UV) divergences. They were regulated in Ref. [9] with imposing restrictions on the non-perturbative inputs. As non-perturbative QCD is known very poor, the only way to specify the non-perturbative inputs is to model them. In Ref. [10] we suggested the Resonance Model for non-perturbative contributions to the quark-hadron scattering amplitudes. It embraces the cases of non-polarized and polarized hadrons with arbitrary orientation of the hadron spin. Applying Optical theorem to the amplitudes makes possible to obtain expressions for the quark distributions in the hadrons. Below we describe an improved version of this model.

2. Resonance Model for non–perturbative inputs

First of all, let us note that the non-perturbative inputs to the non-singlet quark-hadron scattering amplitudes and to the Compton scattering amplitudes off hadrons (the lowest blob in Fig. 1, with no cut) are identical. Fig. 1 explicitly demonstrates that this blob includes both the quark density matrix
\[ \hat{\rho}_q(k, S_q) = \frac{1}{2} \left( \hat{k} + m_q \right) \left( 1 - \gamma_5 \hat{S}_q \right), \] (9)

where $k$, $m_q$ and $S_q$ are the quark momentum, mass and spin respectively. We suggest that the hadron spinor structure can be described similarly:
\[ \hat{T} = (\hat{p} + M_H) T_U - (\hat{p} + M_H) \gamma_5 \hat{S}_H T_S, \] (10)

with $p$, $M_H$ and $S_H$ being the hadron momentum, mass and spin respectively while $T_U$ and $T_S$ are non-perturbative invariant amplitudes, each depends on $k^2$ and the invariant energy $s_1$, with
\[ s_1 = (p-k)^2 \approx k^2 + w\alpha + M_H^2 = w\alpha - w\alpha\beta - k_\perp^2 + M_H^2. \] (11)
Eq. (10) satisfies the conformity: when the hadron is replaced by a quark, \( \hat{T} \) converts into the quark density matrix. Now we have to specify \( T_{U,S} \). As the both of them can be handled identically, we skip the subscripts \( U \) and \( S \) in what follows and will use the generic notation \( T \) for them both, arriving therefore to \( T \) participating in Eq. (7). We also acknowledge that it may be possible that \( T_U = T_S \). Any model for \( T \) should guarantee integrability of the factorization convolutions. For example, the convolution in Eq. (7) can potentially have both infra-red (IR) divergence at \( k^2 = 0 \) and ultra-violet (UV) divergence at large \( |\alpha| \) even with \( A^{(\text{pert})} \) being free of such divergences. As shown in Ref. [10], IR singularities in the convolutions are absent if

\[
T \sim (k^2)^{1+\eta} \quad (12)
\]

at \( k^2 \to 0 \), with \( \eta > 0 \), while UV divergences are killed when

\[
T \sim (\alpha)^{-\chi} = (\alpha)^{1+\eta} (\alpha)^{1-\eta-\chi} \quad (13)
\]

at large \( |\alpha| \), with \( \chi > 0 \). Then, any model for \( T \) should guarantee applicability of the Optical theorem, so \( T \) should have a non-zero imaginary part with respect to the invariant energy \( s_1 \). Finally, after reducing to \( K_T \)-factorization, a dependence of \( T \) on \( k_\perp \) should have a peaked form. We choose the following expression for \( T \) in Basic Factorization:

\[
T = R(k^2) Z(s_1) \quad (14)
\]

with \( R \sim (k^2)^{1+\eta} \) at small \( k^2 \) and \( Z \sim (\alpha)^{-1-\eta-\chi} \) at large \( |\alpha| \). We suggest the following general formula for \( Z \):

\[
Z = Z_n = \prod_{1}^{n} \frac{1}{(s_1 - M^2 + i\Gamma)} \quad (15)
\]

with \( n = 2, 3, \ldots \). The case \( n = 1 \) is excluded by Eq. (13). In what follows we focus on the simplest option \( n = 2 \), where \( Z_2 \) can easily be represented as an interference of two resonances:

\[
Z_2 = \frac{1}{(\Delta M^2 + i\Delta \Gamma)} \left[ \frac{1}{s_1 - M^2 + i\Gamma_1} - \frac{1}{s_1 - M^2 + i\Gamma_2} \right] \quad (16)
\]

where \( \Delta M^2 = M_1^2 - M_2^2, \Delta \Gamma = \Gamma_1 - \Gamma_2 \). Applying the Optical theorem to Eq. (14) allows us to obtain the non-perturbative contribution \( \Psi \) to the quark distributions in the hadrons:

\[
\Psi = \Im s_1 T = \hat{R}(k^2)^T \left[ \frac{\Gamma_2}{(s_1 - M_2^2)^2 + \Gamma_2^2} - \frac{\Gamma_1}{(s_1 - M_1^2)^2 + \Gamma_1^2} \right] \quad (17)
\]

with \( \hat{R}(k^2) = R(k^2) / (\Delta M^2 + i\Delta \Gamma) \). Obviously, the expression \( \Psi \) is of the Breit-Wigner type. Because of that we name our model the Resonance Model. The guiding idea for our suggesting the Resonance Model is based on simple physical grounds: after emitting an active quark off the hadron, the remaining set of partons becomes unstable and therefore it can be described through resonances. Unfortunately, we could not contrive a similar argumentation for specifying \( R \). It is absolutely arbitrary save the only restriction: \( R \sim (k^2)^{1+\eta} \) at small \( k^2 \). For instance, it could be such an option:

\[
R = \left( \frac{k^2}{k^2 + M^2} \right)^{1+\eta} \quad (18)
\]
3. Transition from Basic Factorization to conventional forms of QCD factorization

The expression for the totally unintegrated quark distributions $\Psi(\alpha, \beta, k^2_{\perp})$ in Eq. (17) is obtained for Basic factorization. In order to obtain the unintegrated quark distributions $\Phi(\beta, k^2_{\perp})$ from $\Psi(\alpha, \beta, k^2_{\perp})$, the $\alpha$-dependence of $\Psi(\alpha, \beta, k^2_{\perp})$ should be integrated out. The integration region is given by Eq. (8), so we can use the approximation $k^2 \approx -k^2_{\perp}$ in expressions for $\Psi(\alpha, \beta, k^2_{\perp})$.

Therefore (see Ref. [10] for detail),

$$\Phi(\beta, k^2_{\perp}) = \int_0^\zeta d\alpha \Psi(\alpha, \beta, k^2_{\perp}),$$

with $\zeta = k^2_{\perp}/(w\beta)$. Using Eq. (17), we easily arrive at the following expression for $\Phi$:

$$\Phi = R\left(k^2_{\perp}\right) \left[ \frac{\Gamma_1}{(\zeta - \mu^2_1)^2 + \Gamma^2_1} + \frac{\Gamma_2}{(\zeta - \mu^2_2)^2 + \Gamma^2_2} \right],$$

where we have skipped terms independent of $\zeta$. In Eq. (20) we denoted $\mu^2_1 = M^2_H - M^2_1$, $\mu^2_2 = M^2_H - M^2_2$. Obviously, $\Phi$ in Eq. (20) is also given by expressions of the Breit-Wigner type. Let us note that in Eq. (20) $\zeta > 0$ while signs of $\mu^2_1$ and $\mu^2_2$ cannot be fixed a priori. It means that in the case of positive $\mu^2_1$ and $\mu^2_2$, the both resonances are within their resonant region. On the contrary, if $\mu^2_1 > 0$ and $\mu^2_2 < 0$ (or vice versa), contributions to $\Phi$ come from one sharp resonance and a tail of the other resonance. Such a scenario has a certain resemblance to the Duality conception. Transition from $\Phi(\beta, k^2_{\perp})$ in $K_T$-Factorization to the integrated quark distribution $\phi(\beta)$ in Collinear Factorization (both $\Phi$ and $\phi$ are non-perturbative) can be done with integration of $\Phi(\beta, k^2_{\perp})$ over $k_{\perp}$, leading to the following expressions, depending on the signs of $\mu^2_1$ and $\mu^2_2$: If $\mu^2_1, \mu^2_2 > 0$,

$$\phi = R(\mu^2_1, \beta) + R(\mu^2_2, \beta)$$

and if $\mu^2_1 > 0, \mu^2_2 < 0$, then

$$\phi = R(\mu^2_1, \beta).$$

We remind that, being altogether non-perturbative, $\phi$ does not coincide with the integrated distributions $\phi$ conventionally used in Collinear Factorization (see e.g. Eq. (1)), where both perturbative and non-perturbative contributions are present. Transition from $\phi$ to $\phi$ is done with applying perturbative evolution means to $\phi$ (see Ref. [10] for detail).

4. Conclusion

In the present talk we have given a brief description of Basic Factorization which is the most general form of QCD factorization and demonstrated its relations to the conventional forms of QCD factorization: $K_T$-Factorization and Collinear Factorization. We presented the Resonance Model which is supposed to universally describe non-perturbative contributions to the quark-hadron scattering amplitudes and quark distributions in hadrons, where the hadrons can be either polarized or unpolarized.

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References

[1] Szczurek A 2015 Acta Phys. Polon. Supp. 8 483
[2] Altarelli G and Parisi G 1977 *Nucl. Phys.* B **126** 297;
    Gribov V N and Lipatov L N 1972 *Sov. J. Nucl. Phys.* **15** 438;
    Lipatov L N 1972 *Sov. J. Nucl. Phys.* **20** 95;
    Dokshitzer Yu L 1977 *Sov. Phys. JETP* **46** 641

[3] Ermolaev B I, Greco M and Troyan S I 2010 *Riv. Nuovo Cim.* **33** 57

[4] Amati D, Petronzio R and Veneziano G 1978 *Nucl. Phys.* B **140** 54;
    Efremov A V and Radyushkin A V 1980 *Teor. Mat. Fiz.* **42** 147; 1980 *Theor. Math. Phys.* **44** 573; 1980 *Teor. Mat. Fiz.** bf 44 17; 1976 *Phys. Lett.* B **63** 449; 1977 *Lett. Nuovo Cim.* **19** 83;
    Libby S and Sterman G. 1978 *Phys. Rev.* D **18** 3252;
    Brodsky S J and Lepage G P 1979 *Phys. Lett.* B **87** 359; 1980 *Phys. Rev.* D **22** 2157;
    Collins J C and Soper D E 1981 *Nucl. Phys.* B **193** 381;
    Collins J C and Soper D E 1982 *Int Nucl. Phys.* B **194** 445;
    Collins J C, Soper D E and Sterman G 1985 *Nucl. Phys.* B **250** 199;
    Efremov A V and Radyushkin A V *Report JINR E2-80-521; 2009 Mod. Phys. Lett. A* **24** 2803

[5] Kuraev E A, Lipatov L N and Fadin V S 1976 *Sov. Phys. JETP* **44** 443; 1977 *Sov. Phys. JETP* **45** 199;
    Balitsky I I and Lipatov L N 1978 *Sov. J. Nucl. Phys.* **28** 822

[6] Catani S, Ciafaloni M and Hautmann F 1990 *Phys. Lett.* B **242** 97; 1991 *Nucl. Phys.* B **366** 135

[7] Collins J C and Ellis R K 1991 *Nucl. Phys.* B **360** 3

[8] Sudakov V V 1956 *Sov. Phys. JETP* **3** 65

[9] Ermolaev B I , Greco M and Troyan S I 2011 *Eur. Phys. J. C* **71** 1750; 2012 *Eur. Phys. J. C* **72** 1953

[10] Ermolaev B I, Greco M and Troyan S I 2015 *Eur. Phys. J. C* **75** 306.