Expectations for Supersymmetric Dark Matter Searches Underground.

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Abstract

We consider the neutralino as a dominant Dark Matter particle in the galactic halo and investigate some general issues of direct DM searches via elastic neutralino-nucleus scattering. On the basis of conventional assumptions about the nuclear and nucleon structure we analyse constraints on SUSY model parameter space accessible by the direct DM searches. This analysis shows that DM detectors fall into the three different categories with respect to their sensitivity to different groups of the SUSY model parameters.

We calculate the event rate for various experimentally interesting isotopes within the Minimal Supersymmetric Standard Model (MSSM) taking into account the known accelerator and cosmological constraints on the MSSM parameter space.

We investigate the role of nuclear spin in elastic neutralino-nucleus scattering. It is found that the contribution of the spin-dependent interaction to this process is subdominant for nuclei with atomic weights $A \geq 50$.

1. Introduction

Analysis of the data on distribution and motion of astronomical objects within our galaxy and far beyond indicates presence of a large amount of non-luminous dark matter (for review see\cite{1}). According to estimations, dark matter (DM) may constitute more than 90\% of the total mass of the universe if a mass density $\rho$ of the universe is assumed to be close to the critical value $\rho_c = 3H^2/8\pi G_N$ ($H$ is the Hubble constant and $G_N$ is the Newtonian gravitational constant). The exact equality $\Omega = \rho/\rho_c = 1$, corresponding to a flat universe, is supported by naturalness arguments and by inflation scenarios. The theory of primordial nucleosynthesis restricts the amount of baryonic matter in the universe to $\sim 10\%$. Thus a dominant component of DM is non-baryonic. The recent
data by the COBE satellite on anisotropy in the cosmic background radiation and the theory of the formation of large scale structures of the universe lead to the conclusion that non-baryonic DM itself consists of a dominant (70%) "cold" DM (CDM) and a smaller (30%) "hot" DM (HDM) component.

A possible HDM candidate is the massive neutrino. The SUSY model neutralino ($\chi$) is currently a favorable candidate for CDM. This is a Majorana ($\chi^c = \chi$) particle with spin $1/2$ predicted by supersymmetric (SUSY) models.

There are four neutralinos in the minimal supersymmetric extension of the standard model (MSSM) (see[5]). They are a mixture of gauginos ($\tilde{W}_3, \tilde{B}$) and Higgsinos ($\tilde{H}_{1,2}$) being SUSY partners of gauge ($W_3, B$) and Higgs ($H_{1,2}$) bosons. The DM neutralino $\chi$ is assumed to be the lightest supersymmetric particle (LSP) and therefore is stable in SUSY models with $R$-parity conservation.

In our galaxy most of the mass is expected to be in form of a spherical dark halo. Microlensing searches have discovered far more MACHOs in the disk than in the halo of the Galaxy. The data obtained from these searches is consistent with the fraction of MACHOs in the halo dark matter less than 30%. Thus, most of the halo of the galaxy must be non baryonic cold dark matter.

In the galactic halo neutralinos are assumed to be Maxwellian distributed in velocities with a mean velocity in the earth frame $v \approx 320$ km/sec. Their mass density in the Solar system is expected to be about $\rho \approx 0.3$ GeV $\cdot$ cm$^{-3}$. Therefore, at the earth surface neutralinos might produce a substantial flux ($\Phi = \rho \cdot v/M$) of $\Phi > 10^7$ cm$^{-2}$ sec$^{-1}$ for a particle mass of $M \sim 1$ GeV. In view of this one may hope to detect DM particles directly, for instance through elastic scattering from nuclei inside terrestrial detector.

The problem of direct detection of the DM neutralino $\chi$ via elastic scattering off nuclei has attracted considerable efforts during the last decade and remains a field of great experimental and theoretical activity[8–16].

In this report we address the questions concerning prospects for the direct detection of the supersymmetric Dark Matter with the current and the near future detectors.

In Sec. 2 we consider general issues of such experiments for exploration of the SUSY model parameter space and classify possible DM detectors with respect to their sensitivity to different domains of this parameter space. We propose and discuss special criterion for assessing an isotope as a target material for a DM detector. In discussing general expectations for DM detection experiments, we avoid the use of specific nuclear and nucleon structure models, but rather base our consideration on the known experimental data about nuclei and nucleon properties.

In Sec. 3 we discuss predictions for the DM detection event rate obtained in the framework of the MSSM. We undertake a systematic exploration of a broad domain of the MSSM parameter space restricted by the well known accelerator constraints and by the cosmological bounds on neutralino relic abundance in the universe. The effect of a non-zero threshold energy of a realistic DM detector is analyzed.

Sec. 4 is devoted to the role of nuclear spin in the DM neutralino detection. In
general, the event rate $R$ for elastic $\chi$-nucleus scattering contains contributions from the spin-dependent ($R_{sd}$) and spin-independent ($R_{si}$) neutralino-nucleus interactions: $R = R_{sd} + R_{si}$. We have found that the $R_{si}$ contribution dominates in the total event rate $R$ for nuclei with atomic weight $A > 50$ in the region of the MSSM parameter space where $R = R_{sd} + R_{si} > 0.01 \text{ events kg}^{-1} \text{day}^{-1}$. The lower bound $0.01 \text{ events kg}^{-1} \text{day}^{-1}$ seems to be far below the sensitivity of realistic present and near future DM detectors. Therefore one can ignore the region where $R < 0.01 \text{ events kg}^{-1} \text{day}^{-1}$ as invisible for these detectors.

In view of this result we do not expect crucial dependence of the DM event rate on the nuclear spin for detectors with target nuclei having an atomic weight larger than 50. In other words, we expect essentially equal chances for $J = 0$ and $J \neq 0$ detectors to discover DM events.

In particular, this conclusion supports the idea that presently operating $\beta\beta$-detectors with spinless nuclear target material have good prospects for DM neutralino search.

Sec. 5 gives a conclusion.

2. General Properties of the Neutralino-Nucleus Interactions

A DM event is elastic scattering of a DM neutralino from a target nucleus producing a nuclear recoil which can be detected by a detector. The corresponding event rate depends on the distribution of the DM neutralinos in the solar vicinity and the cross section $\sigma_{el}(\chi A)$ of neutralino-nucleus elastic scattering. In order to calculate $\sigma_{el}(\chi A)$ one should specify neutralino-quark interactions. The relevant low-energy effective Lagrangian can be written in a general form as

$$
L_{\text{eff}} = \sum_q \left( A_q \cdot \bar{\chi} \gamma_\mu \gamma_5 \chi \cdot \bar{q} \gamma^\mu \gamma_5 q + \frac{m_q}{M_W} \cdot C_q \cdot \bar{\chi} \chi \cdot \bar{q} q \right) + O \left( \frac{1}{m_q^4} \right),
$$

where terms with the vector and pseudoscalar quark currents are omitted being negligible in the case of the non-relativistic DM neutralino with typical velocities $v_\chi \approx 10^{-3}c$.

In the Lagrangian (1) we also neglect terms which appear in supersymmetric models at the order of $1/m_q^4$ and higher, where $m_q$ is the mass of the scalar superpartner $\tilde{q}$ of the quark $q$. These terms, as recently pointed out by Drees and Nojiri, are potentially important in the spin-independent neutralino-nucleon scattering, especially in domains of the MSSM parameter space where $m_\tilde{q}$ is close to the neutralino mass $M_\chi$. Below we adopt the approximate treatment of these terms proposed in Ref. which allows "effectively" absorbing them into the coefficients $C_q$ in a wide region of the SUSY model parameter space.

The coefficients $A_q, C_q$ depend on the specific SUSY model and will be considered in the next section.

Here we survey general properties of neutralino-nucleus ($\chi$-$A$) scattering following from the Lagrangian (1).

A general representation of the differential cross section of neutralino-nucleus scattering can be given in terms of three spin-dependent $F_i(q^2)$ and one spin-independent
\[ \frac{d\sigma}{dq^2}(v,q^2) = \frac{8G_F}{v^2} \left( a_0^2 \cdot F_{00}^2(q^2) + a_0a_1 \cdot F_{10}^2(q^2) + a_1^2 \cdot F_{11}^2(q^2) + c_0^2 \cdot A^2 \cdot F_S^2(q^2) \right). \]  

(2)

The last term corresponding to the spin-independent scalar interaction gains coherent enhancement \( A^2 \) (\( A \) is the atomic weight of the nucleus in the reaction). Coefficients \( a_i, c_0 \) do not depend on nuclear structure and relate to the parameters \( A_q, C_q \) of the effective Lagrangian (1) and to the parameters \( \Delta q, f_s, \hat{f} \) characterizing nucleon structure.

One has the relationships

\[
\begin{align*}
a_0 &= (A_u + A_d) (\Delta u + \Delta d) + 2 \Delta s A_s, \\
c_0 &= \hat{f} \frac{m_u C_u + m_d C_d}{m_u + m_d} + f_s C_s + \frac{2}{27} (1 - \hat{f})(C_c + C_b + C_t)
\end{align*}
\]  

(3)

Here \( \Delta q^{p(n)} \) are the fractions of the proton(neutron) spin carried by the quark \( q \). The standard definition is

\[ \langle p(n) | \bar{q}\gamma^\mu\gamma_5 q | p(n) \rangle = 2S^\mu_{p(n)} \Delta q^{p(n)}, \]

(4)

where \( S^\mu_{p(n)} = (0, \vec{S}_{p(n)}) \) is the 4-spin of the nucleon. The parameters \( \Delta q^{p(n)} \) can be extracted from data on polarized nucleon structure functions \(^{18,19}\) and hyperon semileptonic decay data \(^{20}\). It has been recently recognized \(^{21}\) that the new preliminary SMC \(^{19}\) measurements of the spin structure function of the proton at \( Q^2 = 10.3 \) GeV\(^2\) may have dramatic implications for calculations of the spin-dependent neutralino-nucleus scattering cross section. The values of \( \Delta q \) extracted from these new data in comparison with previous EMC \(^{18}\) data are much closer to \( SU(3) \) naïve quark model (NQM) predictions \(^{8,22}\). This gives rise to small enhancement of the spin-dependent cross section for nuclei with an unpaired proton and a strong (by a factor of about 30) suppression for nuclei with an unpaired neutron. In view of this we use in the analysis \( \Delta q \) values extracted both from the EMC \(^{18}\) and from SMC \(^{19}\) data.

The other nuclear structure parameters \( f_s \) and \( \hat{f} \) in formula (3) are defined as follows:

\[
\begin{align*}
\langle p(n) | (m_u + m_d)(\bar{u}u + \bar{d}d) | p(n) \rangle &= 2\hat{f} M_{p(n)} \bar{\Psi}\Psi, \\
\langle p(n) | m_s \bar{s}s | p(n) \rangle &= f_s M_{p(n)} \bar{\Psi}\Psi.
\end{align*}
\]  

(5)

The values extracted from the data under certain theoretical assumptions are \(^{23}\):

\[
\hat{f} = 0.05 \quad \text{and} \quad f_s = 0.14.
\]  

(6)

The strange quark contribution \( f_s \) is known to be uncertain to about a factor of 2. Therefore we take its values in the analysis within the interval \( 0.07 < f_s < 0.3 \).\(^{24,25}\)
The nuclear structure comes into play via the form factors $F_{ij}(q^2), F_S(q^2)$ in Eq. (2). The spin-independent form factor $F_S(q^2)$ can be represented as the normalized Fourier transform of a spherical nuclear ground state density distribution $\rho(r)$

$$F_S(q^2) = \int d^3r \rho(r)e^{irq}. \tag{7}$$

In the analysis we use the standard Woods-Saxon inspired distribution. It leads to the forma factor

$$F_S(q^2) = \frac{3j_1(qR_0)}{qR_0}e^{-\frac{1}{2}(qs)^2}, \tag{8}$$

where $R_0 = (R^2 - 5s^2)^{1/2}$ and $s \approx 1 \text{ fm}$ are the radius and the thickness of a spherical nuclear surface respectively, $j_1$ is the spherical Bessel function of index 1.

Spin-dependent form factors $F_{ij}(q^2)$ are much more nuclear model dependent quantities. The last few years have seen a noticeable progress in detailed nuclear model calculations of these form factors. For many nuclei of interest in DM search they have been calculated within the conventional shell model and within an approach based on the theory of finite Fermi systems. We use the simple minded parametrization for $q^2$ dependence of $F_{ij}(q^2)$ in the form of a Gaussian with the rms spin radius of the nucleus calculated in the harmonic well potential. For our purposes this semi-empirical scheme is sufficient.

An experimentally observable quantity is the differential event rate per unit mass of the target material. It reads

$$\frac{dR}{dE_r} = \left[ N \rho_N/m_x \right] \int_{v_{min}}^{v_{max}} dv f(v) v \frac{d\sigma}{dq^2}(v, E_r) \tag{9}$$

Here $f(v)$ is the velocity distribution of neutralinos in the earth frame which is usually assumed to be the Maxwellian distribution in the galactic frame. $v_{max} = v_{esc} \approx 600 \text{ km/s}$ is the escape velocity at the sun position; $v_{min} = (M_A E_r/2M_{red}^2)^{1/2}$ with $M_A$ and $M_{red}$ being the mass of nucleus $A$ and the reduced mass of the neutralino-nucleus system respectively. Note that $q^2 = 2M_A E_r$.

The differential event rate is the most appropriate for comparing with the observable recoil spectrum and allows one to take properly into account spectral characteristics of a specific detector and to separate the background. However, for a more general theoretical discussion the event rate integrated over some domain of recoil energy is more useful and commonly employed for estimating the prospects for dark matter detection, ignoring experimental complications which may occur on the way. Moreover, the integrated event rate is basically less sensitive to details of nuclear structure than the differential one (4). The $q^2$ shape of the form factors $F_{ij}(q^2), F_S(q^2)$ in Eq. (2) may essentially change from one nuclear model to another. Integration over some $q^2$ region reduces these variations drastically.

Define the integral event rate as:

$$R(E_1, E_2) = \int_{E_1}^{E_2} \frac{dR}{dE_r} \theta(E_{max} - E_r)dE_r \tag{10}$$
Here $E_{\text{max}} = 2M_{\text{red}}^2v_{\text{esc}}/M_A$ It is a common practice in theoretical papers to analyze the total event rate $R = R(0, \infty)$. For the realistic DM detector one should take into account a non-zero threshold energy $E_1 \geq E_{\text{thr}}$. In what follows we use the total event rate $R$ in a general discussion. The effect of a DM detector threshold $E_{\text{thr}}$ is analyzed later on in Sec. 2.

Now let us address the question of possible constraints on SUSY models reachable in the direct DM search experiments. As seen from Eq. (2) there are just three parameters $a_{0,1}, c_0$ accumulating all SUSY model dependence via parameters $A_q, C_q$. Therefore, in the experiments discussed here the only three constraints on the combined SUSY model parameters $a_{0,1}, c_0$ are accessible.

For getting a more transparent physical meaning of these constrains it is useful to adopt the approximation of the odd group model (OGM). It assumes for odd-even nuclei that a dominant contribution to the nuclear spin comes from the odd nucleon group. For the most of nuclei of interest for DM searches this approximation is fairly good. If so, one can write the following formula for the total event rate

\[ R_p(n) = \left[ \phi_p(n) \cdot a_{p(n)}^2 + \phi_0 \cdot c_0^2 \right] \frac{\text{events}}{\text{kg} \cdot \text{day}}, \]

(11)

for the case of proton(neutron) odd group nucleus. Here, $a_{p(n)} = a_0 \pm a_1$. The parameters $\phi_i$ depend on properties of the target nucleus as well as on the mass density and the average velocity of DM particles in the solar vicinity.

The quantities $a_{p,n}$ and $c_0$ contain all dependence on the parameters of the effective Lagrangian (1) and do not depend on nuclear properties. This factorization leads us to the following conclusions.

From Eq. (11) we see again that measuring the event rate $R$ we can study just three special combinations of fundamental parameters $a_{p,n}$ and $c_0$. This is the only information about fundamental parameters accessible in DM search experiments. R is a linear combination of the quantities $a_{p,n}$ and $c_0$. To extract experimental limitations for each of them one should search for DM with different target nuclei. We can distinguish three categories of DM detectors with respect to their sensitivity to $a_{p,n}$ and $c_0$. These are detectors built of spin-non-zero target nuclei with an odd proton(neutron) group probing a linear combination $a_{p(n)}$ and $c_0$ and spin-zero target nuclei sensitive only to the scalar part $c_0$ of the neutralino-nucleus interaction.

We would like to stress again the following. To extract all possible information about SUSY-model parameters from direct DM search one should have three above mentioned types of DM detectors. No other information can be obtained from the direct DM search experiments. Different detectors can only improve the data on the three above-defined groups of SUSY-model parameters.

Accessible constraints can be represented as experimental constraints to the effective parameters $a_{p,n}$ and $c_0$ accumulating all SUSY model dependence of the event rate. According to formula (11) the corresponding exclusion plots in the $a_p - c_0$ and $a_n - c_0$ planes have a form of ellipses as presented in Fig.1 for nuclei with neutron...
FIG. 1. Exclusion plots for the effective SUSY parameters $a_{p(n)}$ (Axial), $c_0$ (Coherent) for nuclei with odd neutron (left panel) and proton (right panel) groups respectively.

and proton odd groups respectively. The plots correspond to a DM detector sensitivity $R > 1 \text{ events kg}^{-1} \text{ day}$. It is easy to see that the same picture holds for any sensitivity. The only effect is the rescaling of the $a_{p(n)} - c_0$ axes. These plots allow one to assess which isotope gives more restrictive constraints at the same detector sensitivity $R \geq R_{\text{det}}$. For instance, it is seen that among p-odd nuclei $^{205}$Tl has potentially better prospects as a target material for DM detectors than $^{23}$Na, $^{35}$Cl, $^{69}$Ga, $^{127}$I. Of course, one should remember that such a criterion is very superficial because it does not take into account limitations imposed by specific conditions under which an isotope can be used in a realistic DM detector.

3. SUSY-model Predictions

In order to obtain quantitative predictions for the DM detection event rate one should calculate the parameters $A_q$ and $C_q$ of the effective Lagrangian in the specific SUSY model. We follow the MSSM with the GUT unification conditions for gauge coupling constants and for soft SUSY breaking parameters. This model is specified by the standard $SU(3) \times SU(2) \times U(1)$ gauge couplings as well as by the low-energy superpotential and soft SUSY breaking terms.

The effective low-energy superpotential is:

$$\tilde{W} = \sum_{\text{generations}} \left( h_L \hat{H}_1 \hat{L} \hat{E} + h_D \hat{H}_1 \hat{Q} \hat{D} - h_U \hat{H}_2 \hat{Q} \hat{U} \right) - \mu \hat{H}_1 \hat{H}_2. \quad (12)$$

Here $\hat{L}$, $\hat{E}$ are lepton doublets and singlets; $\hat{Q}$ are quark doublets, $\hat{U}$, $\hat{D}$ are up and
down quark singlets; $\hat{H}_1$ and $\hat{H}_2$ are the Higgs doublets with a weak hypercharge $Y = -1, +1$, respectively.

The effect of "soft" supersymmetry breaking can be parametrized at the Fermi scale as a part of the scalar potential:

$$V_{\text{soft}} = \sum_{i=\text{scalars}} m_i^2 |\phi_i|^2 + h_L A_L H_1 \tilde{L} \tilde{E} + h_D A_D H_1 \tilde{Q} \tilde{D} - h_U A_U H_2 \tilde{Q} \tilde{U} -$$

$$- (\mu B H_1 H_2 + \text{h.c.})$$

and a "soft" gaugino mass term

$$\mathcal{L}_{FM} = -\frac{1}{2} \left[ M_1 \tilde{B} \tilde{B} + M_2 \tilde{W}^k \tilde{W}^k + M_3 \tilde{g}^a \tilde{g}^a \right] - \text{h.c.}$$

As usual, $M_{1,2,3}$ are the masses of the $SU(3) \times SU(2) \times U(1)$ gauginos $\tilde{g}, \tilde{W}, \tilde{B}$. $m_i = \{ m_L, m_E, m_Q, m_U, m_D, m_{H_1}, m_{H_2} \}$ are the mass parameters of scalar fields.

To reduce the number of free parameters we use the following unification conditions at the GUT scale $M_X$:

$$A_U = A_D = A_L = A_0,$$

$$m_L = m_E = m_Q = m_U = m_D = m_0,$$

$$M_1 = M_2 = M_3 = m_{1/2},$$

$$g_1(M_X) = g_2(M_X) = g_3(M_X) = g_{\text{GUT}},$$

where $g_3, g_2, g_1$ are the $SU(3) \times SU(2) \times U(1)$ gauge coupling constants equal to $g_{\text{GUT}}$ at the unification scale $M_X$.

At the Fermi scale $Q \sim M_W$ these parameters can be evaluated on the basis of the renormalization group equations (RGE). Equation (17) implies at $Q \sim M_W$

$$M_1 = \frac{5}{3} \tan^2 \theta_W \cdot M_2, \quad M_2 = 0.3 m_{\tilde{g}}.$$ 

Here $m_{\tilde{g}} = M_3$ is the gluino mass. One can see from (15)-(18) that we do not exploit the complete set of GUT unification conditions for the soft supersymmetry breaking parameters, which leads to the supergravity scenario with radiative electroweak gauge symmetry breaking. Specifically, we do not unify Higgs soft masses $m_{H_1}, m_{H_2}$ with the others in Eq. (14). Otherwise strong correlations in the supersymmetric particle spectrum would emerge, essentially attaching the analysis to a particular supersymmetric scenario.

We analyze the Higgs sector of the MSSM at the 1-loop level with taking into account $\tilde{t}_L - \tilde{t}_R$, $\tilde{b}_L - \tilde{b}_R$ mixing between the third-generation squarks. Diagonalization of the Higgs mass matrix leads to three neutral mass-eigenstates: two $CP$-even states, $h, H$, with the masses $m_h, m_H$ and the relevant mixing angle $\alpha_H$ as well as one $CP$-odd state $A$ with the mass $m_A$. We take the mass $m_A$ as an independent free parameter.
A complete list of the essential free parameters we use in the analysis is

\[ \tan \beta, A_0, \mu, M_2, m_A, m_0, m_t. \]  

(20)

The angle \( \beta \) is defined by the vacuum expectation values of the neutral components of the Higgs fields: \( \tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle \).

We fix further for definiteness \( m_t = 174 \text{ GeV} \) which corresponds to the CDF value of the top quark mass\(^3\).

There are four neutralino mass eigenstates \( \chi_i \) in the MSSM

\[ \chi_i = N_{i1} \tilde{W}^3 + N_{i2} \tilde{B} + N_{i3} \tilde{H}_2^0 + N_{i4} \tilde{H}_1^0. \]  

(21)

which are linear combinations of zero charged gauginos (\( \tilde{W}^3, \tilde{B} \) and Higgsinos \( \tilde{H}_2^0, \tilde{H}_1^0 \)). The unitary matrix \( N \) rotates the neutralino 4 \( \times \) 4 mass matrix to the diagonal form. As usual, we denote the lightest neutralino \( \chi_1 \) as \( \chi \).

Having specified the model one can derive the effective Lagrangian \( L_{\text{eff}} \) of low-energy neutralino-quark interactions. In the MSSM the first term of \( L_{\text{eff}} \) in Eq. (1) is induced by the \( Z \)-boson and \( \tilde{q} \) exchange whereas the second one is due to the Higgs boson and \( \tilde{q} \) exchange.

Our formulas for the coefficients \( A_q \) and \( C_q \) of the effective Lagrangian take into account squark mixing \( \tilde{q}_L - \tilde{q}_R \) and the contribution of both CP-even Higgs bosons \( h, H \). As pointed out in Ref.\(^3\) the contribution of the heavier Higgs boson \( H \) can be important in certain cases. At some values of the angles \( \alpha_H, \beta \) and the neutralino composition coefficients \( N_{13}, N_{14} \) the contribution of the heavier Higgs boson \( H \) to the coefficients \( C_q \) can be larger than the contribution of the lightest Higgs boson \( h \). The above formulas coincide with the relevant formulas in Ref.\(^3\) neglecting the terms \( \sim 1/m_A^4 \) and higher. As stated in Sec. 2, we adopt the approximate treatment proposed in Ref.\(^3\). It allows one to take into account these terms "effectively" by introducing an "effective" stop quark \( \tilde{t} \) propagator. In the limit \( \theta_q \to 0 \), where \( \theta_q \) is the \( \tilde{q}_L - \tilde{q}_R \) mixing angle, our formulas agree with Ref.\(^3\) except for the relative sign between the \( Z \) and \( \tilde{q} \) exchange terms in the coefficients \( A_q \) and up to the overall sign in the coefficients \( C_q \). These errors in Ref.\(^3\) were also observed in Ref.\(^3\). Now we are ready to calculate the event rate of neutralino nucleus scattering.

In our numerical analysis we scan the MSSM parameter space within a broad domain

\[ 20 \text{ GeV} < M_2 < 1 \text{ TeV}, \quad |\mu| < 1 \text{ TeV}, \]  

\[ 1 < \tan \beta < 50, \quad |A_0| < 1 \text{ TeV}, \]  

\[ 0 < m_0 < 1 \text{ TeV}, \quad 50 \text{ GeV} < m_A < 1 \text{ TeV}. \]  

(22)

In the region where \( \tan \beta \gtrsim 35 \) the top Yukawa dominance approximation is not applicable in the RGE. Therefore, we use procedure developed in Ref.\(^3\) which takes into account the bottom and tau Yukawa couplings as well.
Further limitations on the parameters space are imposed by the known experimental lower bounds on supersymmetric particle and Higgs boson masses from LEP and Tevatron measurements.

The neutralino relic density $\Omega_\chi$ is also under control in our analysis. We calculate it following the standard procedure on the basis of the approximate formula:

$$\Omega_\chi h_0^2 = 2.13 \times 10^{-11} \left(\frac{T_\chi}{T_\gamma}\right)^3 \left(\frac{T_\gamma}{2.7K}\right)^3 \frac{N_F^{1/2}}{a x_F + b x_F^2/2}.$$  \hspace{1cm} (23)

Here $T_\gamma$ is the present day photon temperature, $T_\chi/T_\gamma$ is the reheating factor, $x_F = T_F/M_\chi \approx 1/20$, $T_F$ is the neutralino freeze-out temperature and $N_F$ is the total number of relativistic degrees of freedom at $T_F$. The coefficients $a, b$ are determined from the expansion

$$< \sigma_{\text{ann}} v > \approx a + b x$$  \hspace{1cm} (24)

of the thermally-averaged cross section $< \sigma_{\text{ann}} v >$ of neutralino annihilation. We use an approximate treatment ignoring complications which occur when expansion \hspace{1cm} (24) fails. We take into account all possible channels of the $\chi - \chi$ annihilation.

It is well known that cosmologically acceptable neutralinos should produce a relic density in the interval

$$0.025 < \Omega_\chi h_0^2 < 1.$$  \hspace{1cm} (25)

In this case neutralinos do not overclose the universe and account for a significant fraction of the halo DM.
As an example of our event rate calculations, we show in Fig. 2(left) the total event rate $R$ for $^{73}$Ge. The scatter plot was obtained by random point generation in the MSSM parameter space with the constraints discussed above. More detailed presentation of these results including other isotopes is given in Ref. 15.

A non-zero threshold energy $E_{\text{thr}}$ for the case of a realistic DM detector may essentially modify pictures displayed in Fig. 2(left). To quantify the effect of a non-zero detector threshold we introduce the ratio

$$\nu(E_{\text{thr}}) = \frac{R(E_{\text{thr}}, \infty)}{R(0, \infty)}$$

In Fig. 2(right) this threshold factor is plotted for the isotope $^{73}$Ge. It is seen that for the lighter neutralino $\nu(E_{\text{thr}})$ falls faster than for the heavier one. One can easily understand this dependence noticing that a sizable $R$ can be obtained for $E_{\text{thr}} < 10^{-6} M \chi$. The latter corresponds to the mean kinetic energy of the DM neutralino.

Combining plots Fig. 2(left) and Fig. 2(right) we can estimate the maximal value of the event rate $R(H-M)$ integrated from $E_{\text{thr}} = 48$ KeV which corresponds to the threshold recoil energy of the Heidelberg-Moscow germanium detector. As seen for $M \chi \approx 200$ GeV, it approaches $R(H-M) = 0.2 \text{events kg}^{-1} \text{day}^{-1}$ comparable with the sensitivity quoted by this collaboration. Therefore, one may expect this experiment to provide in the near future the DM constraints on the MSSM parameter space. Detailed analysis leads us to the conclusion that in the similar position is another germanium experiment by the Caltech-PSI-Neuchatel collaboration. Other DM experiments are more distant from the reach of the allowed part of the MSSM parameter space as yet.

Note that in the analysis we ignore possible rescaling of the local neutralino density $\rho$ which may occur in the region of the MSSM parameter space where $\Omega h^2 < 0.05$. This effect, if it took place, could modify the predictions for the event rate $R$. We assume that neutralinos constitute a dominant component of the DM halo of our galaxy with a density $\rho = 0.3 \text{ GeV} \cdot \text{cm}^{-3}$ in the solar vicinity.

4. The role of target nucleus spin

To study the role of nuclear spin in elastic $\chi$-nucleus scattering we consider the ratio

$$\eta = \frac{R_{\text{sd}}}{R_{\text{si}}}$$

characterizing the relative contribution of spin-dependent and spin-independent interactions. Here $R_{\text{sd}}$ and $R_{\text{si}}$ are the spin-dependent and spin-independent parts of the total event rate $R$ respectively. The quantity $\eta + 1$ determines the expected relative sensitivity of DM detectors with spin-non-zero ($J \neq 0$) to those with spin-zero ($J = 0$) nuclei as target material, if their atomic masses are close in value. If $\eta < 1$, then detectors with spin-non-zero and spin-zero target materials have approximately equal sensitivities to the DM signal, otherwise if $\eta > 1$, the spin-non-zero detectors are more sensitive than the spin-zero ones.
Let us consider separately the dependence of $\eta$ on the nuclear structure parameters and on the parameters of neutralino-quark interactions determined in a specific SUSY model. Within our approximations one can write:

$$\eta = \eta_A \eta_{\text{susy}}^{p(n)}$$

(28)

The factorization (28) of the nuclear structure $\eta_A$ from the supersymmetric part of the neutralino-nucleus interaction $\eta_{\text{susy}}$ is essentially based on the assumption of the odd group model$^{17}$ about a negligible contribution of the even nucleon group to the total nuclear spin. $\eta_A$ is a factor depending on the properties of the nucleus $A$, while $\eta_{\text{susy}}^{p(n)}$ is defined by the SUSY-model which specifies the neutralino composition and the interactions with matter. The SUSY-factor also depends on the nucleon matrix element parameters (4), (5) and on the shell-model class to which nucleus $A$ belongs, being $\eta_{\text{susy}}^n$ for the shell-model ”neutron” ($^{3}\text{He}$, $^{29}\text{Si}$, $^{73}\text{Ge}$, $^{129,131}\text{Xe}$,...) and $\eta_{\text{susy}}^p$ for the shell-model ”proton” ($^{19}\text{F}$, $^{23}\text{Na}$, $^{35}\text{Cl}$, $^{127}\text{I}$, $^{205}\text{Tl}$,...).

Fig.3 shows the nuclear factor $\eta_A$ versus the atomic weight $A$. The height of the symbols in the picture represents the variation of the ratio $\eta_A$ within the explored interval of the neutralino mass of $20 \text{ GeV} \leq M_\chi \leq 500 \text{ GeV}$.

It follows from Fig.3 that $\eta_A < 1$ for $A > 50$. Thus at $A > 50$ there is no nuclear structure enhancement of the spin-dependent event rate as compared to the spin-independent one.
The next step is calculation of the SUSY-factor $\eta_{\text{susy}}^{p(n)}$ within the MSSM. We have performed numerical analysis of the MSSM parameter space as described in the previous section. The following absolute upper bound for the SUSY-factor in Eq. (28) was found:

$$\eta_{\text{susy}} < 2,$$

in the subdomain of the parameter space where the total event rate is $R \geq 0.01 \text{ events/kg.day}$. Fig.4 shows the distribution of the points in the $R - \eta_{\text{susy}}$ plane. Plots are given for two representative nuclei with an unpaired proton (p-like), $^{71}\text{Ga}$, and with an unpaired neutron (n-like), $^{73}\text{Ge}$. The nuclei are taken for convenience near the point $A = 50$ (see Fig.3). For heavier nuclei we have obtained basically the same picture and our further conclusions correspond to all nuclei with $A > 50$. One can see the above quoted (29) upper bound $\eta_{\text{susy}} \lesssim 2$ for both cases presented in Fig.4.

Now we may combine the bound (29) with the values of the nuclear factor $\eta_A$ represented in Fig.3. Then we obtain the conservative estimate:

$$\eta = R_{sd}/R_{si} = \eta_A\eta_{\text{susy}}^{p(n)} \lesssim 1.6 \quad \text{for nuclei with } A > 50$$

at a detector sensitivity up to $R > 0.01 \text{ events/kg.day}$. However, as is seen in Fig.4, the majority of points generated in the domain (22) of the MSSM parameter space are concentrated at $\eta \leq 1$. The tendency is that at higher sensitivities (lower $R$ accessible) we get $\eta \leq 1$ for heavier nuclei and vice versa.

In Fig.5 we also present plots of the ratio $r(A) = R_{sd}(A)/R_{sd}(^{73}\text{Ge})$ of the spin-dependent part $R_{sd}$ of the event rate for some target materials ($A$) to that for $^{73}\text{Ge}$. 

FIG. 4. Scatter plots of the total event rate $R$ vs the ratio $\eta_{\text{susy}}$. Two representative nuclei with an unpaired proton ($^{71}\text{Ga}$) and an unpaired neutron ($^{73}\text{Ge}$) are presented.
FIG. 5. The ratio \( r(A) = R_{sd}(A)/R_{sd}(^{73}\text{Ge}) \) of the spin-dependent event rate \( R_{sd} \) for nuclei \(^{19}\text{F} \) and \(^{129}\text{Xe} \) to the spin-dependent event rate \( R_{sd}(^{73}\text{Ge}) \) for \(^{73}\text{Ge} \).

This ratio for \(^{129}\text{Xe} \) is \( r(^{129}\text{Xe}) \approx 1.2 \) being fairly independent of the neutralino mass \( M_X \). As explained in Ref. \[15\] this is the case because both \(^{73}\text{Ge} \) and \(^{129}\text{Xe} \) are nuclei with an unpaired neutron.

It follows from our detailed analysis and is illustrated by Fig. 5 that the maximal values of the total event rate for \(^{129}\text{Xe} \), \(^{73}\text{Ge} \) and \(^{129}\text{Xe} \) are typically the same while for \(^{129}\text{Xe} \) they are lower by about a factor of 5. On the other hand, the sensitivity of \(^{129}\text{Xe} \) to the spin-dependent part of the neutralino-nucleus interaction is by about a factor of 10 larger than that of \(^{129}\text{Xe} \), \(^{73}\text{Ge} \) and \(^{129}\text{Xe} \). The last three materials have approximately an equal spin sensitivity.

5. Conclusion

The central point of this report is that the operating Dark Matter detectors are in the position to probe the MSSM parameter space in the near future. We argued that there are the only three sorts of constraints attainable in direct DM searches. In this respect DM detectors fall into three different categories probing different combinations of SUSY model parameters. They are detectors built of target nuclei with zero spin and with non-zero spin having odd proton, odd neutron groups.

It was pointed out that for sufficiently heavy nuclei with atomic weights \( A > 50 \) the spin-independent event rate \( R_{si} \) is typically larger than the spin-dependent one \( R_{sd} \) if low rate DM signals with total event rates \( R = R_{sd} + R_{si} < 0.01 \frac{\text{events}}{\text{kg} \cdot \text{day}} \) are ignored. This cut-off condition reflects the realistic sensitivities of the present and the near-future DM detectors.

The main practical issue is that two different DM detectors with \((J = 0,A_1)\) and with \((J \neq 0,A_2)\) nuclei as a target material have equal chances to discover DM events if \(A_1 \sim A_2 > 50\).

Another aspect of the DM search is the investigation of the SUSY-model param-
eter space from nonobservation of DM events. For this purpose experiments both with
$J = 0$ and $J \neq 0$ nuclei are important.

We have compared several examples of popular (see for instance$^4$ and references
therein) materials with non-zero spin nuclei as a target in a DM detector. We have
not found an essential difference between NaI, $^{73}$Ge and $^{129}$Xe as a target material
for DM detectors from the point of view of their total and spin sensitivity. We expect
these materials to have better prospects as compared with CaF$_2$ for discovering DM
events. The former materials have a total event rate by about a factor of 5 larger than
the latter one. On the other hand, CaF$_2$ can give a more stringent constraint on the
spin-dependent part of the event rate, having a spin sensitivity by about a factor of 10
larger than NaI, $^{73}$Ge and $^{129}$Xe.

Estimating prospects for various isotopes as target materials of DM detectors
one may use the criterion given in Sec. 1 in the form of exclusion plots (Fig.1) for the
effective SUSY model parameters. These plots characterize sensitivity of an isotope to
the DM event ignoring possible complications of their employment in a detector.

In conclusion we would like to stress that the efforts to improve existing and
to construct new DM detectors are justified by the expectation that these detectors
will be able to tackle supersymmetry from the side unreachable for the accelerator
experiments. Therefore, these two types of experiments are complementary and should
be considered as equally important in searching for supersymmetry.

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