Dynamics of probabilistic labor markets: statistical physics perspective

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Abstract We introduce a toy probabilistic model to analyze job-matching processes in recent Japanese labor markets for university graduates by means of statistical physics. We show that the aggregation probability of each company is rewritten by means of non-linear map under several conditions. Mathematical treatment of the map enables us to discuss the condition on which the rankings of arbitrary two companies are reversed during the dynamics. The so-called ‘mismatch’ between students and companies is discussed from both empirical and theoretical viewpoints.

1 Introduction

Deterioration of the employment rate is now one of the most serious problems in Japan [1, 2, 3] and various attempts to overcome these difficulties have been done by central or local governments. Especially, in recent Japan, the employment rate in young generation such as university graduates is getting worse. To consider the effective policy and to carry out it for sweeping away the unemployment uncertainty, it seems that we should simulate artificial labor markets in computers to reveal the essential features of the problem. In fact, in macroeconomics (labor science), there exist a lot of effective attempts to discuss the macroscopic properties [4, 5, 6, 7, 8, 9] including so-called search theory [10, 11, 12, 13, 14]. However, apparently, the macroscopic approaches lack of their microscopic viewpoint, namely, in their arguments, the behaviour of microscopic agents such as job seekers or companies are neglected.

Taking this fact in mind, in our preliminary studies [15, 16], we proposed a simple probabilistic model based on the concept of statistical mechanics for stochastic labor markets, in particular, Japanese labor markets for university graduates. In these
papers [15, 16], we showed that a phase transition takes place in macroscopic quantities such as unemployment rate as the degree of high ranking preferential factor increases. These results are obtained at the equilibrium state of the labor market, however, the dynamical aspect seems to be important to reveal the matching process between the students and companies. Hence, in this paper, we shall focus on the dynamical aspect of our probabilistic labor market.

This paper is organized as follows. In section 2 we introduce our probabilistic model according to the references [15, 16]. In the next section 3 we show the aggregation probability of each company is described by a non-linear map. Using the knowledge obtained from the non-linear map, we discuss the condition on which the ranking of arbitrary two companies is reversed in successive two business years in section 4. In section 5 we discuss the global mismatch measurement, namely, ratio of job supply. We compare the result with the empirical evidence in recent Japan. In section 6 we introduce a simple procedure to derive the analytic form of the aggregation probability at the steady state by means of ‘high-temperature expansion’. The last section 7 is summary.

2 Model systems

According to our preliminary studies [15, 16], we shall assume that the following four points (i)-(iv) should be taken into account to construct the our labor markets.

(i) Each company recruits constant numbers of students in each business year.
(ii) If the company takes too much or too less applications which are far beyond or far below the quota, the ability of the company to gather students in the next business year decreases.
(iii) Each company is apparently ranked according to various perspectives. The ranking information is available for all students.
(iv) A diversity of making decision of students should be taken into account by means of maximization of Shannon’s entropy under some constraints.

To construct labor markets by considering the above four essential points, let us define the total number of companies as $K$ and each of them is distinguished by the label: $k = 1, 2, \ldots, K$. Then, the number of the quota of the company $k$ is specified by $v_k^*$. In this paper, we shall fix the value $v_k^*$ and regard the quota as a ‘time-independent’ variable. Hence, the total quota (total job vacancy in society) in each business year $V$ is now given by

$$ V = \sum_{k=1}^{K} v_k^*. $$

(1)

When we define the number of students by $N$ (each of the students is explicitly specified by the index $i$ as $i = 1, 2, \ldots, N$), one assumes that the $V$ is proportional to $N$ as $V = \alpha N$, where $\alpha$ stands for job offer ratio and is independent of $V$ and $N$. 
Apparently, for $\alpha = V/N > 1$, that is $V > N$, the labor market behaves as a ‘seller’s market’, whereas for $\alpha < 1$, the market becomes a ‘buyer’s market’.

We next define a sort of ‘energy function’ for each company which represents the ability (power) of gathering applicants in each business year $t$. The energy function is a nice bridge to link the labor market to physics. We shall first define the local mismatch measurement $h_k(t)$ for each company $k$ ($k = 1, 2, \cdots, K$) as

$$h_k(t) = \frac{1}{V} |v_k^* - v_k(t)| = \frac{1}{\alpha N} |v_k^* - v_k(t)|,$$

where $v_k(t)$ denotes the number of students who seek for the position in the company $k$ at the business year $t$ (they will post their own ‘entry sheet (CV)’ to the company $k$). Hence, the local mismatch measurement $h_k(t)$ is the difference between the number of applicants $v_k(t)$ and the quota $v_k^*$. We should keep in mind that from the fact (i) mentioned before, the $v_k^*$ is a business year $t$-independent constant.

On the other hand, we define the ranking of the company $k$ by $\varepsilon_k(>1)$ which is independent of the business year $t$. Here we assume that the ranking of the company $k$ is higher if the value of $\varepsilon_k$ is larger. In this paper, we simply set the value as

$$\varepsilon_k = 1 + \frac{k}{K}. \quad (3)$$

Namely, the company $k = K$ is the highest ranking company and the company $k = 1$ is the lowest. Then, we define the energy function of our labor market for each company $k$ as

$$E(\varepsilon_k, h_k; t) \equiv -\gamma \log \varepsilon_k + \sum_{l=1}^{\tau} \beta_l h_k(t-l). \quad (4)$$

From the first term appearing in the right hand side of the above energy function (4), students tend to apply their entry sheets to relatively high ranking companies. However, the second term in (4) acts as the ‘negative feedback’ on the first ranking preference to decrease the probability at the next business year $t+1$ for the relatively high ranking company gathering the applicants at the previous business year $t$. Thus, the second term is actually regarded as a negative feedback on the first term. The ratio $\gamma/\beta_l$ ($l = 1, \cdots, \tau$) determines to what extent students take into account the history of the labor market. In this paper, we simply set $\beta_1 = \beta, \beta_2 = \cdots = \beta_\tau = 0$, namely, we assume that each student consider the latest result in the market history.

We next adopt the assumption (iv). In order to quantify the diversity of making decision by students, we introduce the following Shannon’s entropy:

$$H = -\sum_{k=1}^{K} P_k(t) \log P_k(t) \quad (5)$$

Then, let us consider the probability $P_k(t)$ that maximizes the above $H$ under the normalization constraint $\sum_{k=1}^{K} P_k(t) = 1$. To find such $P_k(t)$, we maximize the functional $f$ with Lagrange multiplier $\lambda$, $f = -\sum_{k=1}^{K} P_k(t) \log P_k(t) + \lambda \{\sum_{k=1}^{K} P_k(t) - 1\}$, with respect to $P_k(t)$. A simple algebra gives the solution
This implies that the most diverse making decision by students is realized by a
random selection from among $K$ companies with probability $1/K$. (It should be
noted that we set $Ke^{λ−1} = 1$).

On the analogy of the Boltzmann-Gibbs distribution in statistical mechanics,
we add an extra constraint in such a way that the expectation of energy func-
tion over the probability $P_k(t)$ is constant for each business year $t$, namely,
$$E = \sum_{k=1}^{K} P_k(t)E(ε_k, h_k; t).$$
Taking into account this constraint by means of another La-
grange multiplier $λ'$ and maximizing the functional
$$f = -\sum_{k=1}^{K} P_k(t) \log P_k(t) + λ\left\{\sum_{k=1}^{K} P_k(t) - 1\right\} + λ'\left\{E = \sum_{k=1}^{K} P_k(t)E(ε_k, h_k; t)\right\}$$
with respect to $P_k(t)$, we have the probability $P_k(t)$ that the company $k$ gathers their
applicants at time $t$ as
$$P_k(t) = \frac{\exp\left[-E(ε_k, h_k(t−1))\right]}{Z}, E(ε_k, h_k(t−1)) = −γ\log ε_k + βh_k(t−1)$$
where we defined $Z = \sum_{k=1}^{K} \exp[-E(ε_k, h_k(t−1))]$ stands for the normalization con-
tant for the probability. The parameters $γ$ and $β$ specify the probability from the
macroscopic point of view. Namely, the company $k$ having relatively small $h_k(t)$ can
gather a lot of applicants in the next business year and the ability is controlled by
$β$ (we used the assumption (ii)). On the other hand, the high ranked company can
gather lots of applicants and the degree of the ability is specified by $γ$ (we used the
assumption (iii)).

We should notice that for the probability $P_k(t)$, each student $i$ decides to post their
entry sheet to the company $k$ at time $t$ as
$$a_{ik}(t) = \begin{cases} 1 & \text{(with } P_k(t)) \\ 0 & \text{(with } 1 - P_k(t)) \end{cases}$$
where $a_{ik}(t) = 1$ means that the student $i$ posts his/her entry sheet to the company $k$
and $a_{ik}(t) = 0$ denotes that he/she does not.

We might consider the simplest case in which each student $i$ posts their single
entry sheet to a company with probability $P_k(t)$. Namely, $P_k(t)$ is independent of
$i$. From the definition, the company $k$ gets the entry sheets $NP_k(t)$ on average and
the total number of entry sheets is $N\sum_{k=1}^{K} P_k(t) = N$. This means that each student
applies only once on average to one of the $K$ companies. We can easily extend
this situation by assuming that each student posts his/her entry sheets $a$-times on
average. In this sense, the situation changes in such a way that the company $k$ takes
$aNP_k(t)$-entry sheets on average.

Now it is possible for us to evaluate how many acceptances are obtained by a
student and let us define the number by $s_i(t)$ for each student $i = 1, \cdots, N$. Then,
In this section, we derive a non-linear map for the aggregation probability $P_k(t)$. Under the simple assumption $v_k^*(\equiv \text{const.})$, we have $V = \sum_{k=1}^{K} v_k^* = K v_k^*$. On the other hand, from the definition of job offer ratio $V = \alpha N$, $\beta v_k^*/V = \beta / K$ and $(\beta / V) v_k(t - 1) = (\beta / \alpha) (v_k(t - 1)/N) = (\beta / \alpha) P_k(t - 1)$ should be satisfied. When $NP_k, NP_k(1-P_k)$ are large enough, the binomial distribution of $v_k(t)$, that is, $P(v_k) = \binom{NP_k}{v_k} P_k^{v_k} (1-P_k)^{NP_k-v_k}$ could be approximated by a normal distribution with mean $NP_k$ and variance $NP_k(1-P_k)$. Namely,

$$v_k(t) = NP_k + \sqrt{NP_k(1-P_k)} \mathcal{N}(0,1).$$

This reads $v_k(t)/N = P_k + \sqrt{P_k(1-P_k)/N} \mathcal{N}(0,1)$ and the second term can be dropped in the limit $N \to \infty$ as $\lim_{N \to \infty} v_k(t)/N = P_k(t)$. Thus the probability $P_k(t)$ is now rewritten by

$$P_k(t) = \frac{\exp \left[ \gamma \log \left( 1 + \frac{k}{K} \right) - \frac{\beta}{\alpha} |P_k(t-1) - \frac{a}{K}| \right]}{\sum_{k=1}^{K} \exp \left[ \gamma \log \left( 1 + \frac{k}{K} \right) - \frac{\beta}{\alpha} |P_k(t-1) - \frac{a}{K}| \right]}.$$  

This is nothing but a non-linear map for the probability $P_k(t)$. Hence, one can evaluate the time-evolution of the aggregation probability $P_k(0) \to P_k(1) \to \cdots \to P_k(t) \to \cdots$ by solving the map (12) recursively from an arbitrary initial condition, say, $P_1(0) = P_2(0) = \cdots = P_K(0) = 1/K$. In Fig. 1 we plot the time evolution of the aggregation probability for the lowest ranking company $P_1(t)$ (left) and the highest
Fig. 1 The time evolution of probability for the lowest ranking company $P_1(t)$ (left) and the highest ranking company $P_K(t)$ (right). We set $K = 50$, $\alpha = 1$ and varied the values of parameters $\beta, \gamma$.

ranking company $P_K(t)$ (right). From this figure, we easily find that the $P_K(t)$ oscillates for $\beta \gg \gamma$ due to the second term of the energy function (8), namely, the negative feedback acts on the high-ranking preference term $-\gamma \log \varepsilon_k$.

When we define the state satisfying $P_1(t) < \varepsilon \equiv 10^{-5}$ as a kind of ‘business failures’, the failure does not take place until $\gamma \simeq 10.3 \equiv \gamma_c$. However, the failure emerges the parameter $\gamma$ reaches the critical value $\gamma = \gamma_c$.

4 Ranking frozen line

Here we discuss the condition on which the order of probabilities $P_k(t) > P_{K-m}(t)$ is reversed at time $t + 1$ as $P_K(t+1) < P_{K-m}(t+1)$. After simple algebra, we find the condition explicitly as

$$\log \left( \frac{2K}{2K-m} \right) > \frac{\beta}{\gamma \alpha}$$

(13)

From this condition, we are confirmed that if the strength of market history $\beta$ is strong, the condition is satisfied easily, whereas if the ranking factor $\gamma$ or job offer ratio $\alpha$ is large, the condition is hard to be satisfied, namely, the ranking of the highest and $m$-th highest is ‘frozen’. In Fig. 2, we draw the example. We set $K = 50$, $\beta / \alpha \gamma = 0.5$. From this figure, we actually find, for example, the highest ranking company ($m = 0$) and 45-th highest ranking company ($m = 45$) cannot be reversed.

5 Global mismatch measurement

We next discuss the global mismatch measurement between students and companies. For this purpose, we should evaluate ratio of job supply defined as
Fig. 2 The boundary of frozen ranking. The solid and the broken lines are $\log(2K/(2K - m))$ and $\beta/\gamma \alpha$, respectively. We set $K = 50, \beta/\gamma \alpha = 0.5$.

$$\Omega \equiv \frac{1}{V} \sum_{k=1}^{K} (v_k^* - m_k) = 1 - \frac{1}{V} \sum_{k=1}^{K} m_k. \quad (14)$$

Here we assumed that the student who gets multiple informal acceptances chooses the highest ranking company to go, namely, $\hat{l}_i = \arg\max_l (\delta_{s_i, l}),$ where the label $s_{il}$ takes 1 when the student $i$ obtains the informal acceptance from the company $l$ and it takes 0 if not. Then, the new employees of company $k$, namely, the number of students whom the company $k$ obtains is given by

$$m_k = \sum_{i=1}^{N} s_{il} \delta_{k, \hat{l}_i}. \quad (15)$$

On the other hand, it should be noticed that the number of all newcomers (all new employees) in the society is given by

$$\sum_{k=1}^{K} m_k = N - N \sum_{i=1}^{N} (\delta_{s_i, 0}) = N(1 - U). \quad (15)$$

From (14) and (15), we have the linear relationship between the unemployment rate $U$ and the ratio of job supply $\Omega$ as

$$U = \alpha \Omega + 1 - \alpha. \quad (16)$$

It should be noted that the location of a single realization $(U, \Omega)$ is dependent on $\beta, \gamma$ and $\alpha$ through the $\Omega$.

Large mismatch is specified by the area in which both $U$ and $\Omega$ are large. We plot the empirical evidence and our result with $\gamma = \beta = 1$ in Fig. 4. We used the empirical data for the job offer ratio $\alpha$ and we simply set $\beta = \gamma = 1$. We find that qualitative behaviours for both empirical and theoretical are similar but quantitatively they are different. This is because we cannot access the information about macroscopic variables $\beta, \gamma$ and the result might depend on the choice. Estimation for these variables by using empirical data should be addressed as our future problem.
Fig. 3 The relationship between $U$ and $\Omega$ for the past Japanese labor market for university graduates. The left panel is empirical evidence provided by ‘Japan-Insight’ by Mizuho Research Institute (2011). The right panel is obtained by our toy model. We used the empirical data for the job offer ratio $\alpha$ and set the ‘unobservable’ parameters as $\beta = \gamma = 1$ (equally weights).

6 Aggregation probability at ‘high temperature’

In the previous sections, we investigate the dynamics of $P_k(t)$ as a non-linear map. We carried out numerical calculations to evaluate the steady state. Here we attempt to derive the analytic form of the aggregation probability at the steady state by means of high-temperature expansion for the case of $\gamma, \beta/\alpha \ll 1$.

6.1 The high temperature expansion

Let us first consider the zero-th approximation. Namely, we shall rewrite the aggregation probability in terms of $e^x = 1$. This immediately reads $Z = \sum_{k=1}^{K} 1 = K$ and we have

$$P_{k}^{(0)} = \frac{1}{K}. \quad (17)$$

This is nothing but ‘random selection’ by students in the high temperature limit $\gamma, \beta/\alpha \to 0$. This is a rather trivial result and this type of procedure should be proceeded until we obtain non-trivial results.

In order to proceed to carry out our approximation up to the next order $P_{k}^{(1)}$, we first consider the case of $P_{k}^{(1)} > \alpha/K$. By making use of $e^x = 1 + x$, we obtain $Z = K + \gamma \sum_{k=1}^{K} \log(1 + k/K) - (\beta/\alpha) + \beta$. Hence, if one notices that

$$\sum_{k=1}^{K} \log(1 + k/K) = K \cdot \frac{1}{K} \sum_{k=1}^{K} \log(1 + k/K) \simeq K \int_{0}^{1} \log(1 + x) dx = K(2 \log 2 - 1)$$

holds for for $K \gg 1$, the normalization constant $Z$ leads to $Z = K(1 + 2\gamma \log 2 - \gamma) - (\beta/\alpha) + \beta$. By setting $P_{k}(t) = P_{k}(t-1) = P_{k}^{(1)}$ (steady state), we have
Dynamics of probabilistic labor markets

\[ P_k^{(1)} = \frac{1 + \gamma \log(1 + k/K) - \frac{\beta}{K} P_k^{(1)} + \frac{\beta}{K}}{Z}. \]

By solving the above equation with respect to \( P_k^{(1)} \), one finally has

\[ P_k^{(1)} = \frac{1 + \frac{\beta}{K} + \gamma \log(1 + k/K)}{K(1 + 2\gamma \log 2 - \gamma) + \beta}. \]

For the case of \( P_k^{(1)} < \alpha/K \), the same argument as the above gives

\[ P_k^{(1)} = 1 - \frac{\beta}{K} + \frac{\gamma \log(1 + k/K)}{K(1 + 2\gamma \log 2 - \gamma) - \beta}. \]

It should be noted that \( P_k^{(1)} \) is independent of the job offer ratio \( \alpha \). We also should notice that \( P_k^{(1)} = P_k^{(0)} = 1/K = P_k^{(0)} \) is recovered by setting \( \beta = 0 \) in the \( P_k^{(1)} \).

\[ P_k^{(2)} = \frac{1}{K} \sum_{k=1}^{K} \left( \frac{\log(1 + k/K)}{K} \right)^2 \simeq \int_0^1 \left( \frac{\log(1 + x)}{x} \right)^2 \, dx = 2 \left( \frac{\log 2}{2} - 2\log 2 + 1 \right) \] (22)

we obtain \( P_k^{(2)} \) as
\[ P^{(2)}_{k(\pm)} = \frac{\psi_{\pm} + \frac{v_k^2}{2}(\log(1 + \frac{\beta}{K}))^2 + \frac{\beta^2}{2K^2}(1 + (\frac{K + \beta}{\alpha})^2)}{\frac{P(\psi_{\pm} = 1)}{\alpha} + B\gamma^2 K\chi_1 - \frac{B\gamma}{\alpha}(2K\gamma\chi_1 + (K + \beta)\chi_2)} \]  

where we defined
\[ \chi_1 \equiv (\log 2)^2 - 2\log 2 + 1, \chi_2 \equiv 2\log 2 - 1 \]
\[ \psi_{\pm} \equiv \gamma \log(1 + k/K) \pm (\beta/K) + 1, B \equiv K(2\gamma \log 2 - \gamma + 1) + \beta. \]  

We also defined \( P^{(2)}_{k+} \) as a solution for \( P_k > \alpha/K \), whereas \( P^{(2)}_{k-} \) is a solution for \( P_k < \alpha/K \).

As we saw in the above argument of high-temperature expansion, one can obtain the higher-order approximation for the aggregation probability \( P_k \) systematically. However, we should bear in mind that the above approximation should be broken down for \( \gamma, \beta/\alpha \geq 1 \).

### 6.2 Analytic solution for unemployment rate

We next show that one can obtain the analytic solution for the employment rate in terms of the aggregation probability at high temperature. Here we show the solution for the first order approximation \( P^{(1)}_k \) of the aggregation probability, however, the extension to the higher-order is straightforward.

As we already saw, for \( P_k = P^{(1)}_k \) and \( P_k > \alpha/K, N P_k, N P_k(1 - P_k) \gg 1 \), we have \( \lim_{N \to \infty} v_k/N = P_k \). Namely, the probability that the company \( k \) obtains \( v_k \)-entry sheets is written as \( P(v_k) = \delta(v_k - N P_k)\Theta(P_k - \alpha/K) + \delta(v_k - N \hat{P}_k)\Theta(\alpha/K - P_k) \).

We should notice that there exists a point \( k \) at which \( \hat{P}_k = \hat{P}_k = \alpha/K \) holds.

When we define \( P(s_{ik} = 1|v_k) \) as the probability that student \( i \) receives an informal acceptance from the company \( k \) which gathered \( v_k \)-entry sheets from the students including the student \( i \), the probability \( P(s_{ik} = 1|v_k) \) is given by

\[ P(s_{ik} = 1|v_k) = P(s_{ik} = 1|a_{ik} = 1, v_k > v_k^*)P(a_{ik} = 1)\Theta(v_k - v_k^*) + P(s_{ik} = 1|a_{ik} = 1, v_k < v_k^*)P(a_{ik} = 1)\Theta(v_k^* - v_k). \]  

Then, taking into account the fact \( P(s_{ik} = 1|a_{ik} = 1, v_k > v_k^*) = v_k^*/v_k, P(s_{ik} = 1|a_{ik} = 1, v_k < v_k^*) = 1 \), and \( P(a_{ik} = 1) = P_k \), we immediately obtain

\[ P(s_{ik} = 1|v_k) = \left( \frac{v_k^*}{v_k} \right) P_k\Theta(v_k - v_k^*) + \hat{P}_k\Theta(v_k^* - v_k). \]  

Obviously, the above \( P(s_{ik} = 1|v_k) \) depends on \( v_k \). Hence, we calculate the average of \( P(s_{ik} = 1|v_k) \) over the probability \( P(v_k) \), namely \( P(s_{ik} = 1) = \sum_{v_k} P(v_k)P(s_{ik} = 1|v_k) \). Then, we have
Dynamics of probabilistic labor markets

\[ \phi_k \equiv \frac{\alpha}{K} \Theta(P_k - \alpha/K) + \hat{P}_k \Theta(\alpha/K - \hat{P}_k) \]  

(28)

where we canceled the index \( i \) because it is no longer dependent on specific student

\[ \gamma = \beta = 0.01, \alpha = 1 \]

Fig. 5 \( P(s_k = 1) \equiv \phi_k \) evaluated by means of \( P_k^{(1)} \).

\( i \) and we define \( \phi_k \) as the probability that an arbitrary student gets the informal acceptance from the company \( k \). In Fig. 5, we plot the typical behaviour of the \( \phi_k \).

Therefore, the probability that an arbitrary student who sent three entry sheets to three companies \( k, l, m \) could not get any informal acceptance is given by \( (1 - \phi_k)(1 - \phi_l)(1 - \phi_m)/3! \). From this observation, we easily conclude that analytic form of the unemployment rate is obtained as

\[ U = \sum_{k_1 < k_2 < \cdots < k_a} \frac{(1 - \phi_{k_1}) \cdots (1 - \phi_{k_a})}{K(K - 1) \cdots (K - a + 1)} \]  

(29)

7 Summary

In this paper, we introduced a toy probabilistic model to analyse job-matching processes in recent Japanese labor markets for university graduates by means of statistical physics. We succeeded in deriving a non-linear map with respect to the aggregation probability. By analyzing the map and evaluating the steady states by means of high-temperature expansion, we could discuss several analytic results and their mathematical properties. Global mismatch between students and companies was discussed from both empirical and theoretical viewpoints. However, as we saw in Fig. 4, there still exists a gap between our result and the empirical evidence. To bridge the gap, we should estimate the non-observable parameters such as \( \gamma, \beta \) using the data collected by appropriate questionnaires. It should be addressed as one of our future studies.
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