Deep Learning-based Metamodeling Technique for Nonlinear Seismic Response Quantification

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Abstract. Classical physics-based numerical techniques such as finite element method (FEM) usually takes a huge computational resource and time for simulation-based uncertainties in structural analysis. Especially, it is expensive when nonlinear time history analysis is involved. The metamodeling technique becomes an alternative with an ability to predict the time history response of both elastic and inelastic structural system in a data-driven fashion. Various machine learning and deep learning-based techniques have been attempted to have their limitations in handling huge and sequential data while it comes to predict the whole response time history given stochastic ground motion acceleration. In this regard, Long Short Term Memory (LSTM) approach is found to be useful. The present study used LSTM deep learning network based metamodeling approach for nonlinear seismic response history approximation. The novelty of the approach lies in its capability of capturing record to record variability even for an inelastic structure. This metamodel can work with the desired level of accuracy with very limited data. The proposed approach has shown satisfactory results to approximate seismic response of a nonlinear single degree of freedom system.

1. Introduction

The finite element method is more efficient for structural dynamic analysis but demands huge computational effort. It is a well known fact that the finite element method based vibration response analysis of nonlinear systems demands a small time step and considerably small mesh size to provide an accurate and numerically stable solution. Monte Carlo simulation is the best way to evaluate the reliability when uncertainty in external load as well as internal structural parameters is involved. This method requires a huge number of simulations. Also, incremental dynamic analysis while doing the structural fragility assessment demands numerous simulations. In these cases, the finite element method fails in terms of time and computational demand. This limitation demands an alternative approach that is metamodel i.e. a fast running surrogate model, which is mathematically trained and learns the response prediction laws from very few available inputs and output data and reproduces it for unknown input data. Various emerging adaptive machine learning-based approaches e.g. artificial neural network (ANN) [1,2], Kriging [3], polynomial chaos expansion (PCE) [4,5], and support vector machines (SVM) [6] are notable in this regard. But such metamodeling approaches have some limitation such as selection and extraction of feature requires human intervention. These are not suitable for very large and structured data such as sequential data. Recent developments in the deep neural networks (DNNs) allow learning sequential models for forecasting and predicting in the field of nonlinear structural analysis. DNN includes Multi-layer perceptron (MLP), convolutional neural network (CNN), recurrent neural network (RNN), auto-encoder decoder, etc. Multi-layer perceptron (MLP), a classical ANN-based network has shown state-of-the-art performance in the field of structural damage diagnosis [7, 8]. Though, MLP is limited to the dynamic analysis of the linear system and fails to model a very complex nonlinear system. CNN is capable of handling spatial data with grid-like topology though a modified version i.e. 1-
dimensional CNN has been used in the field of seismic response prediction but while large plastic deformation is involved, the accuracy of 1D CNN drastically falls in predicting the response [9, 10]. Zhang et al. [11] have developed a data-driven physics-guided CNN model for seismic response prediction. It is trained with limited data gained from simulation; and the network output is constrained by available laws of physics and dynamics to alleviate the overfitting issue. The recent development of RNN in handling sequential data and huge success in the field of speech and handwriting recognition [12, 13] motivate to apply this in the field of nonlinear response history prediction of structure. As discussed, this study focuses on quantifying uncertainty in the record to record variability of ground motion acceleration by establishing a surrogate model that can predict the whole response time history. The RNN is found most suitable for this. However, the major two drawbacks of the RNN are vanishing gradient and exploding gradient problems which prevents the model to update the weights at the beginning of the sequence resulting in a zero-learning situation [14]. For this RNN is not capable of predicting response involving long term dependency and complex nonlinearity in time-varying data.

The most robust solution is to use a gated recurrent unit that enables us to learn and forget information selectively through the gates and keep a long track of memory even in case complex nonlinearity is involved. Such a gated cell called long short term memory (LSTM) is explored little in nonlinear stochastic dynamic response approximation. Using the LSTM network Yang et al. [15] have developed a deformation pattern of a landslide along with time, which is complex, nonlinear, and periodically dependent on external factors such as rainfall, fluctuation of reservoir-water-level, etc. To make a well-informed decision-making system for an emergency responder to attend the most critically affected area after severe earthquake, Mangalathu et al. [16] have developed an LSTM based classification model based on textual description gathered from the social media platform. Using semantic analysis i.e. converting language to information, an indexed library is made that classifies description of the severity of building damage. Yin et al. [17] have developed a model to detect anomaly of earthquake precursor data using LSTM. Zhang et al. [18] have developed a seismic response prediction model with a stacked LSTM model. The present study used LSTM deep learning network based metamodeling approach for nonlinear seismic response history approximation. The novelty of the approach lies in its capability of capturing record to record variability even for an inelastic structure. The capability of the proposed metamodel to approximate nonlinear seismic response with the desired level of accuracy with very limited data is numerically demonstrated.

2. Long short-term memory (LSTM) network

The network architecture of LSTM is made of LSTM and other hidden layers having input and output layers at the two surfaces. The LSTM layers are made of several LSTM cells which are the constitutive component of the LSTM architecture.

2.1. The LSTM cell

LSTM cells are equivalent to the unrolling of the recurrent operation by a standard RNN network but with some additional gated operation. These gates keep the relevant information memorised throughout a long sequence. Each LSTM cell has an independent set of weights and biases that is shared across the entire temporal space within that specific layer and get updated through training. This works through three elementary operations (i) selective writing in the memory (ii) selective reading from the memory and (iii) selectively forgetting the irrelevant part from the memory. The selectively written hidden state from the previous cell and new input to the current cell, collectively generate a temporary cell state. The flow of input activation is controlled by the input gate. Selective forget operation is conducted by the forget gate which produces the cell state for the next time step combining with previous cell state. Then the output gate helps in producing the hidden state which is served as the output from that particular time step. This way the LSTM cell forget irrelevant information adaptively and store the relevant information for long term dependencies [19].

2.2. The architecture

An LSTM network is composed of one or more LSTM layers followed by fully connected (FC) or dense layers. Each LSTM layer is composed of multiple LSTM cells or nodes. Each node in the layer is analogous to a mini-network which is exposed to the input sequence and produces an output. The number
of cells in a layer represents the number of times the recurrent operation is repeated. The input sequence is processed one time step at a time. Each step of input for a node results in output and an internal state. Both are used in the processing of the subsequent time step. Next, the FC layer which is provided to obtain the necessary number of output features available at the output layer. FC layers have full connections to all activation nodes in the previous layer. Adding dropout layers [20] is a good way to prevent overfitting which is a very common problem in deep learning. Typically, it is applied before the FC layer due to having a large number of learnable parameters which make this layer prone to overfitting.

2.3. Sequence to sequence modelling
In this study an LSTM network is trained where a stochastically varied ground motion signature is taken as input and the response time history obtained from nonlinear time history analysis (NLTHA) is taken as the target. Input ground motion records are denoted as \( X = \{ x_1, x_2, x_3, \ldots, x_n \}^{T} \in \mathbb{R}^{n \times p} \) and the output structural response \( Y = \{ y_1, y_2, y_3, \ldots, y_n \}^{T} \in \mathbb{R}^{n \times q} \). Here \( n \) is the number of time steps, \( p \) and \( q \) are the numbers of input and output features respectively. In the current study \( p \) and \( q \), both are considered to be one i.e. one set of ground motion acceleration corresponds to a set of displacement time history which made both \( X \) and \( Y \) of dimension \( n \times 1 \). LSTM architecture with multiple datasets prescribes three-dimensional structuring of the input and output layer which is discussed in detail in the numerical study section.

3. Numerical study: A non-linear single degree of freedom (SDOF) system
The nonlinear spring-mass SDOF system is considered to elucidate the effectiveness of the present LSTM network. A nonlinear spring is connected to the ground having a lumped mass on its free end. The nonlinear spring behaviour is described in figure 2 (b) [21]. Rayleigh type damping is assumed where it is proportional to the mass and stiffness.

![Figure 1](image.png)

**Figure 1.** (a) The SDOF system and (b) the nonlinear spring behaviour.

The nonlinear seismic response of the SDOF system is obtained by numerical integration in the OpenSees platform which is taken from the study by Ghosh et al.[22]. All the structural parameters are assumed to be deterministic. The various parameters characterising the nonlinear SDOF system is described in table 1.

In the current study, a suite of synthetically generated accelerograms with the most vulnerable surface wave magnitude (\( M \)) and epicentral distance (\( R \)) combination are used whose properties are determined by probabilistic seismic hazard analysis (PSHA). More details on this may be seen in Ghosh et al. [22]. For the current study, 100 numbers of synthetic accelerograms are generated with different combinations of \( M \) ranging from 6.0 to 8.0 and \( R \) ranging from 100 to 300 km for rock site. The stochastic method as proposed by Boore [23] is considered for the generation of synthetic accelerograms. The formulation, methodology, and MATLAB codes are adapted from the study of Ghosh et al. [22]. The ground motion generated represents the earthquake nature of the Guwahati region of north-east India.
Table 1. Values of the SDOF system parameters.

| System parameters                               | Values  |
|-------------------------------------------------|---------|
| Natural frequency \( (\text{rad/sec}) \omega \) | 6.28    |
| Damping ratio, \( \zeta \)                      | 0.02    |
| Yield force \( (N) \), \( F_y \)               | 2.5     |
| The ratio of post-yield to elastic stiffness, \( \alpha \) | 0.1     |

First, synthetically generated ground motions are normalised with their peak ground acceleration (PGA) values. The duration of each ground motion is 40 seconds having 2000 data points. A set of randomly generated PGA values lying between 0.1 and 1.0 are multiplied with all the normalised ground motions. The set is divided into two parts each having 50 ground motion accelerations. The first one is used in training while the other is kept for the prediction purpose. Both training and prediction data set is formatted and prepared in three-dimensional tensor as suggested earlier. The training input (ground motion acceleration) format is [50, 2000, 1] whereas the training output (response time history) format is [50, 2000, 1]. Here, the first dimension depicts the number of events or samples, the second dimension denotes the total number of time steps, and the third dimension denotes the number of input or output features which is one for both the cases in the present study. As the equal number of samples is preserved for the prediction; the data format will be the same as of training data. Two LSTM layers each consist of 100 nodes are provided. An FC layer with 100 nodes followed by a dropout layer with 20 percent is provided. Dense layer with node number equal to the number of features i.e. one (displacement time history of the SDOF system) in the current study connects the previous FC layer. The data are normalised between 0 to 1 using `MinMaxScaler`. From the training set, the ground motion (as input) and the displacement response (as output) obtained from NLTHA (as target) are fed into the model for training. During the training process, at each epoch, the training set is shuffled to increase the speed of convergence and 15 samples are kept for validation while the rest of 35 samples are used in training. Unlike the feed-forward neural network, the biases and weights are updated after each epoch using the backpropagation through time (BPTT) algorithm that results in reducing both the training and validation loss as shown in figure 2. In this study, it is observed that approximately after 10,000 epochs of training, validation loss decreased to a much lower value which indicates satisfactory performance. The network has a validation error of \( 2.8532 \times 10^{-5} \) is saved as the best model. Total training time is 3.7 hours i.e. 1 sec/epoch is required. A batch size of five is used. Adam (Adaptive Momentum Estimation) optimiser is used with a learning rate of 0.002 and a decay rate of 0.00001 which is decided by the grid search method. The loss function is taken as a mean squared error (MSE) for this regression problem.

![Figure 2. Decrement of ‘train’ and ‘test’ loss with progression of training.](image)

The best model is saved, fed with the prediction-ground motion set and response time history is predicted. Along with this, the response is calculated with NLTHA from the prediction set of ground motion and compared with the response predicted from LSTM based metamodel. Such compared
response for the two typical ground motions is shown in figure 3. It is seen that the predicted response is quite the same as the true response obtained from NLTHA.

Figure 3. The response of the nonlinear SDOF system at unknown ground motion.

To further study the accuracy of the prediction, regression analysis is conducted. Let, $\bar{y}_{i,t}^{(j)}$ is denoted as the drift of the $j^{th}$ sample from the test set at the $t^{th}$ time step of the response time history predicted by the LSTM model. Similarly, $y_{i,t}$ denotes the response time history generated from NLTHA. The regression analysis is performed on the prediction data set (50 samples) by plotting the histogram normalised by the probability density of the Pearson’s coefficient of correlation obtained for each time step, with the help of 50 pairs of data points. Pearson’s correlation coefficient ($r$) is calculated as $r_{XY} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$ where, $X = \{y_{i}^{(1)}, y_{i}^{(2)}, \ldots, y_{i}^{(50)}\}, Y = \{\bar{y}_{i}^{(1)}, \bar{y}_{i}^{(2)}, \ldots, \bar{y}_{i}^{(50)}\}$ and $t \in \{1, 2, \ldots, 2000\}$ all the $r_{XY}$ is prepared for all $t \in \{1, 2, \ldots, 2000\}$, and the probability density function is plotted with 2000 generated Pearson’s coefficient values. As the correlation coefficient is calculated over the sample points at each time step, it truly reflects the temporal distribution of the error. A near to 1 correlation coefficient depicts a superior performance while a close to zero value indicates poor performance. Figure 4 represents the probability distribution of the Pearson’s correlation coefficient. It can be seen that the probability of correlation coefficient being near 1 is very high. Notably, the majority of the correlation coefficients are over 0.9 and very few are below 0.7 or less than that which indicates a high prediction accuracy of the model.

Figure 4. Regression analysis: Probability distribution of the Pearson’s correlation coefficient calculated with 50 pairs of true and predicted response values at each time step showing the temporal distribution of error.

To illustrate the range of error for each sample from the prediction set, a box-whisker plot of normalised error percentage is presented in figure 5. The normalised error is defined as $\frac{|y_{i,true} - y_{i,pred}|}{\max(y_{i,true})} \times 100$. It is a series of errors at each time step for a particular sample. 1st and 3rd quartile values are shown by the blue box where the red mark in middle shows the median in figure 5. The majority of the sample shows a median of error below 5 and very few shows above 10. Red ‘+’ marks in the figure indicate the outliers. The figure also indicates the maximum and minimum error for each sample.
4. Summary and Conclusions
An LSTM based metamodeling approach is applied in seismic response approximation which is flexible in handling long term dependency in sequential data. The present algorithm is demonstrated for seismic response approximation of a nonlinear SDOF system subjected to ground motion generated synthetically. A satisfactory result is found through regression analysis and a box-whisker plot of normalised error. Outliers can be treated as a good sign as it is found below the lowest error which implies better accuracy of prediction. However, the accuracy could not be achieved without optimising the hyperparameters involved in the training phase. It is important to choose these juristically and optimise using the grid search method. From the result obtained, it is shown that the LSTM algorithm proposed is potentially having an ample scope to explore further in the seismic safety assessment of structures. It is reliable and robust. The proposed algorithm is generic and can be readily extended to a multi-degree of freedom system considering system parameter uncertainty which is under consideration at this stage.

5. References
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