How to detect the shortest period binary pulsars in the era of LISA

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ABSTRACT
We discuss a multimessenger strategy to detect radio pulses from Galactic binary neutron stars in a very tight orbit with the period shorter than 10 min. On one hand, all-sky surveys by radio instruments are inefficient for detecting faint pulsars in very tight binaries due partly to the rarity of targets and primarily to the need of correction for severe Doppler smearing. On the other hand, the Laser Interferometer Space Antenna (LISA) will detect these binaries with a very large signal-to-noise ratio and determine the orbital frequency, binary parameters, and sky location to high accuracy. The information provided by LISA will reduce the number of required pointings by two to six orders of magnitude and that of required trials for the corrections by about nine orders of magnitude, increasing the chance of discovering radio pulsars. For making full use of this strategy, it is desirable to operate high-sensitivity radio instruments such as Square Kilometer Array Phase 2 simultaneously with LISA.

Key words: gravitational waves – methods: data analysis – binaries: close – stars: neutron – pulsars: general

1 INTRODUCTION
Radio pulsars are fascinating laboratories of astrophysics (Lorimer & Kramer 2004). In particular, those in short-period binaries serve as useful tools to study binary evolution (Tauris et al. 2017), interacting magnetospheres (e.g. Lyne et al. 2004), and the theory of gravitation (Will 2014). Even after direct detections of gravitational waves from compact binary coalescences, human time-scale monitoring of relativistic binary pulsars could remain undefeatable for constraining some models of modified gravity (Yunes et al. 2016; Abbott et al. 2018). Furthermore, these binaries could be our Schelling point to communicate with extraterrestrial intelligence (Nishino & Seto 2018).

The tightest Galactic binary neutron stars should have much shorter orbital periods than those observed today. On one hand, as of 2018, the shortest period of binary neutron stars hosting a detectable pulsar is 1.88 h of J1946+2052 (Stovall et al. 2018), which will spend 46 Myr before merger. On the other hand, the gravitational-wave event GW170817 suggests that the Galactic merger rate of binary neutron stars is about one in $10^8$ yr (Abbott et al. 2017). Therefore, it is virtually certain that many binary neutron stars with short orbital periods and being close to merger exist in our Galaxy. It is also natural to expect that some of them host detectable radio pulsars.

Pulsars in very tight binaries, however, are difficult to find, because strong Doppler smearing has to be corrected appropriately. While the acceleration search has been adopted to discover binary pulsars (Faulkner et al. 2004), this method assumes a linear drift of the frequency and is applicable only to integration times shorter than $\approx 10\%$ of the orbital period (Johnston & Kulkarni 1991). Thus, pulsars in very tight binaries easily elude this search unless they are extremely luminous. Jerk searches are proposed to improve the situation, but integration times are still restricted to $\lesssim 15\%$ of the orbital period (Johnston & Kulkarni 1991). Thus, pulsars in very tight binaries easily elude this search unless they are extremely luminous. Jerk searches are proposed to improve the situation, but integration times are still restricted to $\lesssim 15\%$ of the orbital period (Bagchi et al. 2013; Andersen & Ransom 2018). The incoherent phase-modulation search is applicable to integration times longer than the orbital period (Joutex et al. 2002; Ransom et al. 2003).

In this paper, we examine an alternative and coherent method to detect pulsars in very tight binaries utilizing information provided by a future space-borne gravitational-wave detector, the Laser Interferometer Space Antenna (LISA; see Armano et al. 2016, 2018, for the results of LISA Pathfinder). LISA is sensitive at mHz bands, and its targets

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including various compact object binaries are summarized in Amaro-Seoane et al. (2017). As we will show later, the merger rate estimated by GW170817 suggests that a number of Galactic binary neutron stars can also be detected with LISA as quasi-monochromatic sources, where O(10) of them are free from the foreground associated with white dwarf binaries. LISA will provide us with accurate information of these binaries such as the orbital frequency (or period), binary parameters, and sky location (Cutler 1998; Seto 2002; Takahashi & Seto 2002). This information enables pulsar surveys by radio instruments to reduce both the searching time and the computational cost of corrections for Doppler smearing significantly. Although we only focus on binary neutron stars for concreteness, our discussion also applies to black hole/white dwarf–neutron star binaries.

This paper is organized as follows. We first review the prospect for detecting Galactic binary neutron stars with LISA in Section 2. Next, we discuss how to detect radio pulses from these binaries in Section 3, comparing performance of our multimessenger strategy with that of an all-sky surveys by radio instruments to reduce both the searching time and the computational cost of corrections for Doppler smearing significantly. Although we only focus on binary neutron stars for concreteness, our discussion also applies to black hole/white dwarf–neutron star binaries.

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2 GRavitational-Wave Observation by LISA

First, we estimate statistical errors in extracting parameters of Galactic binary neutron stars with LISA. Gravitational-wave frequency is denoted by \( f \) and the operation period of LISA is denoted by \( t_{\text{LISA}} \). We adopt the quadrupole approximation in this work, and the gravitational-wave frequency is twice the orbital frequency except for higher harmonics associated with the eccentricity.

2.1 Target

The high rate of binary neutron star mergers reported by the LIGO–Virgo collaboration, \( R = 1500 \pm 1220 \) Gpc\(^{-3}\) yr\(^{-1}\) (Abbott et al. 2017), suggests that Galactic binary neutron stars are numerous. The Galactic merger rate may be given by \( R_{\text{MW}} \approx R/n_{\text{MW}} \), where \( n_{\text{MW}} \approx 0.01 \) Mpc\(^{-3}\) is the number density of Milky Way equivalent galaxies. Assuming that the Galactic population of binary neutron stars is stationary on the time-scale of interest, their distribution in frequency is given by (e.g. Kyutoku & Seto 2016)

\[
dN_{\text{MW}} / df = f^{-1} R_{\text{MW}}. \tag{1}
\]

The time derivative of gravitational-wave frequency, or the chirp parameter, is given by (Peters 1964)

\[
f = 96 \pi^{2/3} (GM)^{5/3} f^{11/3} / 5c^5 \tag{2}
\]

\[
= 1.3 \times 10^{-15} \text{ Hz s}^{-1} \left( \frac{f}{4 \text{ mHz}} \right)^{11/3} \left( \frac{M}{1.2 M_\odot} \right)^{5/3}, \tag{3}
\]

where \( M \) is the chirp mass of the binary, in the quadrupole approximation. Hence, the number of Galactic binary neutron stars emitting gravitational waves at frequency higher than \( f \) is estimated to be

\[
N_{\text{MW}}(> f) = \int_f^\infty \frac{dN_{\text{MW}}}{df} df' \tag{4}
\]

\[
= \frac{5c^5 R_{\text{MW}}}{256 \pi^{16/3} (GM)^{8/3} f^{8/3}} \tag{5}
\]

\[
= 5.6 \left( \frac{M}{1.2 M_\odot} \right)^{-5/3} \left( \frac{f}{4 \text{ mHz}} \right)^{-8/3} \times \left( \frac{R_{\text{MW}}}{1.5 \times 10^{-4} \text{ yr}} \right)^{256 \pi^{16/3} (GM)^{8/3} f^{8/3}}. \tag{6}
\]

This estimate indicates that the shortest period class of Galactic binary neutron stars will be observed via gravitational waves in mHz bands by LISA (Amaro-Seoane et al. 2017). The frequency evolution during the operation period of LISA is insignificant for binaries with \( f < 100 \) mHz considered here, and thus we may regard them as quasi-monochromatic. Although numerous binary neutron stars exist also at \( f \lesssim 3 \) mHz, which corresponds to the orbital period longer than 10 min, the sensitivity of LISA will be degraded because of the foreground associated with unresolved white dwarf binaries (Nelemans et al. 2004; Robson et al. 2018).

Galactic binary neutron stars may be classified into two classes according to their eccentricities (see Tauris et al. 2017, for a compilation). If we focus only on binaries that merge within the Hubble time, more than the half will have low eccentricities of \( e < 3 \times 10^{-3} \) at \( f = 4 \) mHz due to gravitational radiation reaction (Peters 1964) and may be regarded as circular for our purpose. However, three binaries, namely PSR B1913+16 (Hulse & Taylor 1975; Weisberg & Huang 2016), B2127+11C (Prince et al. 1991; Jacoby et al. 2006), and J1757–1854 (Cameron et al. 2018), will retain moderate eccentricities of 0.020, 0.028, and 0.035, respectively, at 4 mHz. In this study, we mainly focus on circular binaries, and possible effects of moderate eccentricity \( e = 0.03 \) are briefly discussed when they are relevant.

2.2 LISA Observation

The signal-to-noise ratio \( \rho \) of gravitational waves can be evaluated by a quasi-monochromatic approximation (e.g. Kyutoku & Seto 2016). Taking the average with respect to the sky location and binary orientation, we have

\[
\rho = \frac{4 \sqrt{3} \pi^{2/3} (GM)^{5/3} f^{2/3} S_{\text{LISA}}(f)^{1/2}}{5c^5 D [\{3/20 S_n(f)\}^{1/2}]} \tag{7}
\]

\[
= 200 \left( \frac{M}{1.2 M_\odot} \right)^{5/3} \left( \frac{D}{10 \text{ kpc}} \right)^{1/2} \left( \frac{S_{\text{LISA}}}{2 \text{ yr}} \right)^{1/2} \times \left( \frac{f}{4 \text{ mHz}} \right)^{2/3} \left( \frac{S_n(f)}{2 \times 10^{-40} \text{ Hz}^{-2}} \right)^{-1/2}, \tag{8}
\]

where \( D \) is the distance to the binary and \( S_n(f) \) is the effective noise spectral density. We normalize \( S_n(f) \) according to Robson et al. (2018). Hereafter, we use the values in this equation as our fiducial parameters.

We estimate 1σ statistical errors of various quantities with LISA observations using the results derived with the Fisher analysis in Seto (2002) and Takahashi & Seto (2002), which are valid for \( t_{\text{LISA}} \gtrsim 2 \) yr. The extrinsic parameters are
estimated with typical errors
\[ \frac{\Delta A}{A} = 0.015 \left( \frac{\rho}{200} \right)^{-1}, \]
(9)
\[ \Delta \phi = 0.015 \left( \frac{\rho}{200} \right)^{-1}, \]
(10)
\[ \Delta \Omega = 0.036 \text{deg}^2 \left( \frac{\rho}{200} \right)^{-2} \left( \frac{f}{4 \text{mHz}} \right)^{-2}, \]
(11)

where A, \( \phi \), and \( \Omega \) are the intrinsic gravitational-wave amplitude, gravitational-wave phase, and sky location of the binary, respectively. Because the error of the amplitude is known to be correlated strongly with the inclination angle \( i \), we use \( \Delta A / A \) as a proxy for the statistical error of \( \Delta (\sin i) \).

Although we find by repeating the Fisher analysis that this approximation sometimes underestimates \( \Delta (\sin i) \) by a factor of 2–3, it turns out later that the precise value of \( \Delta (\sin i) \) is not important for radio observations. Because the wave amplitude is inversely proportional to the distance, the relative error in the distance is also given by \( \Delta A / A \) as far as the chirp mass is determined precisely (Takahashi & Seto 2002). Although the Galactic electron distribution is not fully understood (Yao et al. 2017), the distance error may be useful to constrain loosely the characteristics of the target pulsar (see Section 3).

The errors in determining the gravitational-wave frequency and its time derivative are given by (Takahashi & Seto 2002)
\[ \Delta f = 1.7 \times 10^{-10} \text{Hz} \left( \frac{\rho}{200} \right)^{-1} \left( \frac{t_{\text{LISA}}}{2 \text{yr}} \right)^{-1}, \]
(12)
\[ \dot{\Delta} f = 5.4 \times 10^{-18} \text{Hzs}^{-1} \left( \frac{\rho}{200} \right)^{-1} \left( \frac{t_{\text{LISA}}}{2 \text{yr}} \right)^{-2}. \]
(13)

Note that the matched-filtering analysis, where the detector output is cross-correlated with theoretical waveform models, allows us to estimate these parameters more accurately than the naïvely estimated frequency resolution. The gravitational-wave frequency is determined virtually perfectly, as is the orbital frequency. Because \( \Delta M / M \approx (3/5) \Delta f / f \), where \( f \) is given by equation (3), the chirp mass will be determined to sub-per cent accuracy. Because the chirp mass of white dwarf binaries should be small (Farmer & Phinney 2003), binary neutron stars will be securely selected.

Although the total mass \( M \) is required to derive the orbital separation \( a = |GM/(\pi f)^2|^{1/3} \) from the gravitational-wave frequency, it is not determined unless a large eccentricity enables us to detect the periastron advance (Seto 2001). For circular binaries, we must adopt a plausible value of the mass ratio \( q \), which we define as the mass of the lighter member divided by that of the heavier, to infer the total mass from the chirp mass. If we assume \( 0.7 \leq q \leq 1 \) motivated by current observations (Tauris et al. 2017; Abbott et al. 2017), the uncertainty in the total mass is \( \approx 2 \text{ percent} \). We caution that \( a \) is different from the orbital radius of either component, \( a/(1 + q) \) or \( qa/(1 + q) \).

In fact, the detection of periastron advance improves the situation only marginally. By following Seto (2001), the error in the eccentricity is found to be \( \Delta e > 3 \times 10^{-3} \) irrespective of the value of \( e \), with assuming that \( S_m(3f/2) = S_m(f) \).

The error in the total mass is determined by the accuracy in determining the frequency of third orbital harmonics and is at best \( \approx 1 \text{ percent} \) for \( t_{\text{LISA}} = 2 \text{ yr} \). This amounts to constraining the mass ratio to \( 0.78 \lesssim q \leq 1 \). We neglect this possible improvement for simplicity, commenting that a long operation will reduce the error in the total mass as \( t_{\text{LISA}}^{-3/2} \).

Before concluding this section, we comment on the benefit from a long operation of \( LISA \). Taking \( \rho \approx t_{\text{LISA}}^2 \) shown in equation (8) into account, the errors in the phase (equation 10) and the sky location (equation 11) decrease as \( t_{\text{LISA}}^{-1/2} \) and \( t_{\text{LISA}}^{-1} \), respectively, when \( t_{\text{LISA}} \) increases. As we will describe in Section 3.2, these improvements directly reduce the computational cost required for detecting radio pulses. Although we adopted the fiducial value of \( t_{\text{LISA}} = 2 \text{ yr} \), the current nominal plan is \( 4 \text{ yr} \) with a possible extension to \( 10 \text{ yr} \) (Amaro-Seoane et al. 2017). Thus, the statistical errors will be improved compared to those stated in this paper, and the computational cost may be reduced by a factor of 3–10.

### 3 PULSAR SURVEY

Next, we describe a strategy to detect radio pulses from the shortest period class of Galactic binary neutron stars utilizing information obtained by \( LISA \). Because \( LISA \) is planned to be launched around 2030 (Amaro-Seoane et al. 2017), we aim at detecting the radio pulses with the Square Kilometer Array (SKA) Phase 2\(^1\), which may complete construction around that time. The radio frequency is denoted by \( v \) and the integration time of SKA is denoted by \( t_{\text{SKA}} \). We mainly focus on observations at popular 1.4 GHz with the bandwidth \( B = 500 \text{ MHz} \). Our results can be applied to other facilities by scaling relevant parameters.

#### 3.1 Target

The promising target is a mildly recycled pulsar with the spin period of \( P \approx 30 \text{ ms} \). Indeed, most of the known pulsars in binary neutron stars are mildly recycled (Tauris et al. 2017). Theoretically, the evolution scenario generally predicts that one of the neutron stars in these binaries is mildly recycled and the other is young. The difference in detectability may come from the beaming fraction, where that of recycled pulsars is usually considered to be larger than that of young pulsars. For example, Levin et al. (2013) argue that the former is 0.4–1 compared to 0.2 of the latter. Our strategy can also find the young pulsar, whereas it may have a small beaming fraction. In addition, young pulsars tend to spin down rapidly and could pass the death line before entering the \( LISA \) band (Tauris et al. 2017).

In this study, we take the fiducial value of the sampling time \( t_{\text{amp}} \) to be 100 \text{ ms} following plans of normal pulsar surveys (Smits et al. 2009). However, we speculate that \( t_{\text{amp}} \approx 1 \text{ ms} \) may be acceptable for detecting mildly recycled pulsars. The reason is that the typical intrinsic pulse width of mildly recycled pulsars is likely to be a few milliseconds (Kramer et al. 1998; Lorimer et al. 2006) and that the effective width \( W \) must be broader due to dispersion mismatch and scattering (Bhat et al. 2004). If we accept this coarse sampling, the computational cost of data analysis will be reduced substantially, in particular for a long integration time.

\(^1\) [https://astronomers.skatelescope.org/ska/](https://astronomers.skatelescope.org/ska/)
Detailed timing observations can be performed to resolve the pulse profile after the discovery. An obvious drawback is that the coarse sampling will miss very narrow pulses. On another front, if the scattering is so severe that the pulses become wider than their separations (i.e. spin period), observations at high frequency will be required.

The sensitivity of a search can be estimated by the radiometer equation (Lorimer & Kramer 2004). Following and extending the notation of Nishino & Seto (2018), we write the signal-to-noise ratio (S/N) of a radio source with the flux density \( S \) as

\[
S/N = \frac{A_{\text{eff}} S \sqrt{N_p B_{\text{SKA}}}}{k_B T} \sqrt{P - W} \tag{14}
\]

where \( y \) is a factor of \( O(1) \) determined by the detector configuration, \( A_{\text{eff}} \) is the effective area, \( N_p \) is the number of polarization modes, and \( T \) is the system temperature. Accordingly, the minimum detectable flux density for a given threshold of the signal-to-noise ratio, \( S/N_{\text{min}} \), is given by

\[
S_{\text{min}} = 10^{-4} \, \text{mJy} \left( \frac{S/N_{\text{min}}}{5} \right) \left( \frac{A_{\text{eff}}/T}{10^4 \, \text{m}^2 \, \text{K}^{-1}} \right)^{-1} \times \left( \frac{N_p}{2} \right)^{-1/2} \left( \frac{B}{500 \, \text{MHz}} \right)^{-1/2} \left( \frac{\text{SKA}}{12 \, \text{h}} \right)^{-1} \left( \frac{X}{1} \right), \tag{15}
\]

where \( X = \sqrt{W/(P - W)} \) becomes unity when \( W = 0.5P \). The value of \( A_{\text{eff}}/T \) is normalized aiming at the planned SKA Phase 2 (Dewdney et al. 2009).^2

This indicates that 12-h integration of SKA Phase 2 will enable us to detect pulsars down to the pseudo-luminosity of 0.01 mJy kpc^2 within 10 kpc as far as the Doppler smearing is corrected for appropriately. This sensitivity should be sufficient for finding very faint pulsars at 1.4 GHz (Kramer et al. 1998; Burgay et al. 2013), and non-detection would indicate the absence of pulses due to the beaming or cessation. The flux density typically behaves as \( \nu^{-1.6} \) (Kramer et al. 1998; Jankowski et al. 2018), and thus the search at high frequency will be a little more challenging. Actual observations will benefit from the distance estimated to 1 percent accuracy with LISA to determine the required integration time for a given pseudo-luminosity.

### 3.2 LISA-informed radio observation

The sky localization of LISA ensures covering of the target pulsar by a single pointing with 15 m dishes of SKA. Quantitatively, the size of a pencil beam with the dish diameter \( D_{\text{dish}} \) is approximately given using the wavelength \( \lambda := c/\nu \) by (Smits et al. 2009)

\[
\Omega_{\text{dish}} \approx \frac{\lambda^2}{D_{\text{dish}}^2} \tag{16}
\]

\[
= 0.67 \, \text{deg}^2 \left( \frac{\nu}{1.4 \, \text{GHz}} \right)^{-2} \left( \frac{D_{\text{dish}}}{15 \, \text{m}} \right)^{-2} \tag{17}
\]

and is much larger than the localization error of LISA, equation (11). If the entire area of a single-dish beam, equation (17), is observed by tied-array beams with high sensitivity in a coherent manner, this information saves the observation time significantly as we later discuss in Section 3.3. Accordingly, long integration times could be spent for observing a single target to achieve high sensitivity as shown in equation (15). Exceptionally, if we have to look at the region in the vicinity of the Galactic Center, high frequency of \( \sim 10 \, \text{GHz} \) may be required to overcome severe scattering, and the single pointing may not be sufficient.

Even if it would be computationally prohibitive to resolve the single-dish beam completely with an array of sparsely distributed dishes (Smits et al. 2009; Keane et al. 2015), the localization accuracy of LISA is still powerful. The size of a tied-array beam formed by dishes distributed within \( D_{\text{array}} \sim 1 \, \text{km} \) is given by

\[
\Omega_{\text{array}} \approx \frac{\lambda^2}{D_{\text{array}}^2}, \tag{18}
\]

where the exact value of \( D_{\text{array}} \) depends on the configuration and the observing strategy (Smits et al. 2009). The number of tied-array beams required to span the localization area of LISA is given by

\[
N_{\text{array}} \approx \frac{\Delta \Omega}{\Omega_{\text{array}}} \tag{19}
\]

\[
= 240 \left( \frac{P}{200} \right)^{-2} \left( \frac{f}{4 \, \text{mHz}} \right)^2 \left( \frac{\nu}{1.4 \, \text{GHz}} \right)^2 \left( \frac{D_{\text{array}}}{1 \, \text{km}} \right)^2, \tag{20}
\]

and this is smaller by a factor of \( \Omega_{\text{dish}}/\Omega_{\text{array}} \approx 20 \) than that required to resolve fully a single-dish beam. We expect that this number of beam forming will be affordable with computational resources of SKA (Keane et al. 2015).

The need to correct for dispersion delay \( t_{\Delta \nu} \) is not affected by LISA observations unless the Galactic electron distribution is understood to high accuracy. For completeness, we estimate the required number of trials for the dispersion measure (i.e. the column density of free electrons) DM. The step in trial values is determined by requiring that the delay across the bandwidth (e.g. section 6.1.1.2 of Lorimer & Kramer 2004), which is approximately given by

\[
\Delta \nu = \frac{\epsilon^2 \nu^2 B}{\pi m_e e^3} \tag{21}
\]

changes by the sampling time. This condition derives the required number of trials to be

\[
N_{DM} \approx \frac{\epsilon^2 D_{\text{max}} B}{\pi m_e e^3 \nu^3} \tag{22}
\]

\[
= 15000 \left( \frac{D_{\text{max}}}{1000 \, \text{pc cm}^{-3}} \right) \left( \frac{t_{\Delta \nu}}{100 \, \text{ps}} \right)^{-1} \times \left( \frac{\nu}{1.4 \, \text{GHz}} \right)^{-3} \left( \frac{B}{500 \, \text{MHz}} \right) \tag{23}
\]

This large number of trials is the absolute maximum and would have to be optimized in realistic data analysis. It is possible that the distance estimated with LISA will weakly constrain the range of DM.

LISA information will be invaluable for correcting pulse arrival times for the orbital modulation (see Fig. 1 for a schematic representation). Because we aim at detecting possibly faint pulsars in very tight binaries devoting long integration times that exceed the orbital period, the acceleration search is not applicable (Johnston & Kulkarni 1991). While the incoherent phase-modulation search could be an option.

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^2 See also URL in the Footnote 1.
analysis similar to that performed in Seto (2002), the accuracy in estimating the gravitational-wave phase outside the operation period of LISA deteriorates so rapidly that it becomes useless. This means that simultaneous operations of LISA and radio instruments such as SKA are beneficial for making full use of our multimessenger strategy.

Following consideration of the dispersion measure, we determine the steps of trial values for the amplitude and phase of the modulation by requiring that the pulse arrival time changes by the sampling time. The number of required trials for the modulation amplitude is given by

\[
N_{\text{amp}} \approx \frac{\Delta t_{\text{mod}}}{t_{\text{amp}}} \leq 350 \left( \frac{\rho_{\text{amp}}}{0.2} \right) \left( \frac{m_c/M}{1/2} \right) \left( \frac{M}{2.8M_\odot} \right)^{1/3} \times \left( \frac{f}{4 \text{ mHz}} \right)^{-2/3} \left( \frac{\sin \iota}{\pi/4} \right)^{-1} t_{\text{amp}}^{100 \mu s}^{-1},
\]

and that for the modulation phase is given by

\[
N_{\text{phase}} = \frac{t_{\text{mod}} \Delta \phi}{t_{\text{amp}}} \leq 26 \left( \frac{\rho}{200} \right)^{-1} \left( \frac{m_c/M}{1/2} \right) \left( \frac{M}{2.8M_\odot} \right)^{1/3} \times \left( \frac{f}{4 \text{ mHz}} \right)^{-2/3} \left( \frac{\sin \iota}{\pi/4} \right)^{-1} t_{\text{amp}}^{100 \mu s}^{-1}.
\]

Additionally, the uncertainty in the eccentricity of \(\Delta e \approx 0.003\) could change the arrival time by \(t_{\text{mod}} \Delta e\), and this will require trials of \(N_{\text{ecc}} \approx 5t_{\text{amp}}/100 \mu s\) for our fiducial parameters. We expect that extension to a slightly eccentric orbit will be straightforward.

We should remark on the spin precession. Its period will be \(> 1 \text{ month} \gg t_{\text{amp}}\) for our fiducial parameters (Barker & O’Connell 1975), and we expect its effect to be marginal. We leave the careful assessment as future work, and information from LISA will prove useful even when the precession must be corrected for.

### 3.3 Comparison with an all-sky survey

To elucidate usefulness of the multimessenger observation with LISA, we compare its performance with an all-sky survey of radio instruments. To keep the sensitivity fixed, we assume that the integration time per pointing is the same for both observation strategies. We also fix the sampling time. A comparison with an all-sky survey based on the phase-modulation search, with which the sensitivity is not maximal, is briefly made in the end of this section.

First, the all-sky survey trivially requires a large number of pointings, and thus the integration time must actually be shortened. If we do not restrict the search to a limited portion of the sky, e.g., the Galactic plane, equation (17) suggests that \(O(10^5)\) pointings in terms of a single-dish beam are required to find a limited number of targets. Even if instruments with a large field-of-view are available (Smits et al. 2009), the number of pointings may not be reduced below \(O(10^5)\). If we evaluate the number of pointings in terms of tied-array beams, the all-sky survey requires \(O(10^6)\) pointings, which is larger by \(O(10^6)\) compared to the multimessenger observation with LISA, equation (20). However, results...
of the all-sky survey can be used for various applications including standard pulsar searches, and thus this comparison is valid only when we focus on detections of pulsars in the shortest period class of binary neutron stars.

Next, the coherent integration becomes significantly costly because of brute-force corrections for the orbital modulation. We have to search over a large parameter space of not only the modulation amplitude and phase but also the orbital frequency. By requiring that the phase error caused by the frequency mismatch accumulated during the integration time does not induce a timing error larger than the sampling time, particularly for the harmonic summing by the frequency mismatch accumulated during the orbital frequency. By requiring that the phase error caused by the frequency mismatch accumulated during the integration time does not induce a timing error larger than the sampling time, particularly for the harmonic summing by the frequency mismatch accumulated during the integration time.

In reality, the all-sky survey for short-period binaries will likely be conducted with incoherent but efficient phase-modulation searches, and thus it would be informative to compare performance of our strategy with theirs. The number of pointings is reduced significantly by a factor of \( \approx 10^6 \) and \( \approx 10^9 \) for the all-sky survey of the shortest period class of binary neutron stars for given sensitivity. It is highly likely that realistic situations limit the integration time per pointing for an all-sky survey significantly and that we will miss faint and distant pulsars. The massive computational cost of the survey without \( LISA \) information can be mitigated by incoherent search techniques but only at the expense of the sensitivity. Finally, if observations of \( LISA \) and \( SKA \) do not occur simultaneously, we will lose information of the phase so that the efficiency of the multimessenger strategy will be reduced by two orders of magnitude. Although the improvement is still significant, simultaneous observations are preferable.

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### REFERENCES

- Abbott B. P., et al., 2017, Phys. Rev. Lett., 119, 161101
- Abbott B. P., et al., 2018, arXiv:1811.00364
- Amaro-Seoane P., et al., 2017, arXiv:1702.00786
- Andersen B. C., Ransom S. M., 2018, ApJ, 863, L13
- Armano M., et al., 2016, Phys. Rev. Lett., 116, 231101
- Armano M., et al., 2018, Phys. Rev. Lett., 120, 061101
- Bagchi M., Lorimer D. R., Wolfe S., 2013, MNRAS, 432, 1303
- Barber B. M., O’Connell R. F., 1975, Phys. Rev. D, 12, 329
- Bhat N. D. R., Cordes J. M., Camilo F., Nice D. J., Lorimer D. R., 2004, ApJ, 605, 759
- Burgay M., et al., 2013, MNRAS, 433, 259
- Cameron A. D., et al., 2018, MNRAS, 475, L57
- Cutler C., 1998, Phys. Rev. D, 57, 7089
- Dewdney P. E., Peter H. J., Schlüter R. T., Lazio T. J. L. W., 2009, IEEE Proceedings, 97, 1482
- Farmer A., Phinney E. S., 2003, MNRAS, 346, 1197
- Faulkner A. J., et al., 2004, MNRAS, 355, 147
- Hulse R. A., Taylor J. H., 1975, ApJ, 195, L51
- Jacoby B. A., Cameron P. B., Jenet F. A., Anderson S. B., Murty R. N., Kulkarni S. R., 2006, ApJ, 644, L113
- Jankowski F., van Straten W., van der Klis M., 2002, A&A, 384, 532

- Abboud M., Samarakoon R., Stappers B. W., Jonker P., van der Klis M., 2002, A&A, 384, 532

- Abbott B. P., et al., 2017, Phys. Rev. Lett., 119, 161101
- Abbott B. P., et al., 2018, arXiv:1811.00364
- Amaro-Seoane P., et al., 2017, arXiv:1702.00786
- Andersen B. C., Ransom S. M., 2018, ApJ, 863, L13
- Armano M., et al., 2016, Phys. Rev. Lett., 116, 231101
- Armano M., et al., 2018, Phys. Rev. Lett., 120, 061101
- Bagchi M., Lorimer D. R., Wolfe S., 2013, MNRAS, 432, 1303
- Barber B. M., O’Connell R. F., 1975, Phys. Rev. D, 12, 329
- Bhat N. D. R., Cordes J. M., Camilo F., Nice D. J., Lorimer D. R., 2004, ApJ, 605, 759
- Burgay M., et al., 2013, MNRAS, 433, 259
- Cameron A. D., et al., 2018, MNRAS, 475, L57
- Cutler C., 1998, Phys. Rev. D, 57, 7089
- Dewdney P. E., Peter H. J., Schlüter R. T., Lazio T. J. L. W., 2009, IEEE Proceedings, 97, 1482
- Farmer A., Phinney E. S., 2003, MNRAS, 346, 1197
- Faulkner A. J., et al., 2004, MNRAS, 355, 147
- Hulse R. A., Taylor J. H., 1975, ApJ, 195, L51
- Jacoby B. A., Cameron P. B., Jenet F. A., Anderson S. B., Murty R. N., Kulkarni S. R., 2006, ApJ, 644, L113
- Jankowski F., van Straten W., van der Klis M., 2002, A&A, 384, 532

4 SUMMARY

We have studied a multimessenger strategy to detect pulsars in the shortest period class of Galactic binary neutron stars with \( LISA \) and radio instruments such as \( SKA \). These binaries will be observed by \( LISA \) as quasi-monochromatic gravitational-wave sources with a large signal-to-noise ratio of \( > 100 \). A single pointing in terms of a single dish of \( SKA \) will be sufficient to entirely cover the localization area of \( LISA \), and accordingly long integration times may become available to improve the sensitivity. While this long integration time prohibits the acceleration search, the Doppler smearing caused by the rapid orbital motion can be corrected efficiently by using orbital frequency and binary parameters derived with \( LISA \).
How to detect the shortest period binary pulsars

Keane E. F., et al., 2015, Advancing Astrophysics with the Square Kilometre Array (AASKA14), p. 40
Kramer M., Xilouris K. M., Lorimer D. R., Doroshenko O., Jessner A., Wielebinski R., Wolszczan A., Camilo F., 1998, ApJ, 501, 270
Kyutoku K., Seto N., 2016, MNRAS, 462, 2177
Levin L., et al., 2013, MNRAS, 434, 1387
Lorimer D. R., Kramer M., 2004, Handbook of pulsar astronomy. Cambridge University Press
Lorimer D. R., et al., 2006, MNRAS, 372, 777
Lyne A. G., et al., 2004, Science, 303, 1153
Nelemans G., Yungelson L. R., Portegies Zwart S. F., 2004, MNRAS, 349, 181
Nishino Y., Seto N., 2018, ApJ, 862, L21
Peters P. C., 1964, Physical Review, 136, B1224
Prince T. A., Anderson S. B., Kulkarni S. R., Wolszczan A., 1991, ApJ, 374, L41
Ransom S. M., Greenhill L. J., Herrnstein J. R., Manchester R. N., Camilo F., Eikenberry S. S., Lyne A. G., 2001, ApJ, 546, L25
Ransom S. M., Cordes J. M., Eikenberry S. S., 2003, ApJ, 589, 911
Robson T., Cornish N., Liu C., 2018, arXiv:1803.01944
Seto N., 2001, Phys. Rev. Lett., 87, 251101
Seto N., 2002, MNRAS, 333, 469
Smits R., Kramer M., Stappers B., Lorimer D. R., Cordes J., Faulkner A., 2009, A&A, 493, 1161
Stovall K., et al., 2018, ApJ, 854, L22
Takahashi R., Seto N., 2002, ApJ, 575, 1030
Tauris T. M., et al., 2017, ApJ, 846, 170
Weisberg J. M., Huang Y., 2016, ApJ, 829, 55
Will C. M., 2014, Living Reviews in Relativity, 17, 4
Yao J. M., Manchester R. N., Wang N., 2017, ApJ, 835, 29
Yunes N., Yagi K., Pretorius F., 2016, Phys. Rev. D, 94, 084002

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