Numerical simulation of current-driven magnetization dynamics in three-layered magnetic structures

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Abstract. On the basis of qualitative analysis of the dynamical system, which consists of two coupled Landau-Lifshits equations classification of stationary states of three-layered magnetic structure is performed. It was found numerically that at some combinations of the parameters, the phase space of the system is featured with limiting cycles that indicates the precession of the magnetization vector in the structure. Before the system will reach the stable parallel-spin configuration, it passes through a cascade of bifurcations with different types of spin dynamics. For each of them the threshold values of injection current are determinate.

The Slonczewski–Berger theoretical model of current-driven spin dynamics [1–5] in three-layers structure is based on quantum effects of electron scattering on the interface and spin transmitting to ferromagnetic medium when spin-polarized current entering the thin controlled layer. As it was predicted by Slonczewski in [1], such a structure is featured by switching effects and mesoscopic precession of magnetization. The goal of our work is in mathematical modelling and studying of these processes in more detail.

The structure under investigation consists of two thin metal magnetic layers separated by thin metal nonmagnetic one (figure 1). The thicknesses of the layers are $d_1$ and $d_2$ with $d_1 << d_2$. The magnetic materials are of uniaxial anisotropy of easy-axes type. Let the easy axes of the layers be coincident with $x$-axis of the Cartesian coordinates and $z$-axis be orthogonal to the layers. The current $J$ flows along OZ. The external magnetic field $H_0$ is directed along OY. Such a geometry implies that the external magnetic field introduces the second effective anisotropy axis into the structure.

Figure 1. Geometry of the model.
Following Slonczewski [1], we have considered the model based on the Gilbert dissipative version of the Landau–Lifshits equations

\[
\frac{d\mathbf{M}_1}{dt} = -\gamma [\mathbf{M}_1 \times \mathbf{H}_1] + \frac{\gamma \alpha}{M} I_1 [\mathbf{M}_1 \times [s_2 \times \mathbf{M}_1]] + \frac{\alpha}{M} \left\{ \mathbf{M}_1 \times \frac{d\mathbf{M}_1}{dt} \right\},
\]

\[
\frac{d\mathbf{M}_2}{dt} = -\gamma [\mathbf{M}_2 \times \mathbf{H}_2] - \frac{\gamma \alpha}{M} I_2 [\mathbf{M}_2 \times [s_1 \times \mathbf{M}_2]] + \frac{\alpha}{M} \left\{ \mathbf{M}_2 \times \frac{d\mathbf{M}_2}{dt} \right\}.
\]

Here \( \mathbf{M}_1, \mathbf{M}_2 \) are vectors of magnetization of the thin (controlled) layer, and the thick (controlling) layer respectively, \( M \) is the saturation magnetization; \( \gamma \) is the gyromagnetic ratio, \( \alpha \) is a damping parameter, \( I_1 = \frac{h\mathbf{J} \cdot \mathbf{M}}{\gamma \alpha d} \), \( I_2 = \frac{h\mathbf{J} \cdot \mathbf{M}}{\gamma \alpha d^2} \) are normalized parameters of current injection, \( G = [-4 + (1 + P)^2(3 + s_1 \cdot s_2)/4P^2]^{-1} \), \( J \) is the current density, \( P = \frac{n_+ - n_-}{n_+ + n_-} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \) is spin polarization of current in the volume of the thick layer with magnetization \( \mathbf{M}_2; \) \( s_1 = \frac{\mathbf{M}_1}{M} \), and \( s_2 = \frac{\mathbf{M}_2}{M} \). The vectors \( \mathbf{H}_1, \mathbf{H}_2 \) are effective magnetic fields, which in thermodynamic approach are defined as variational derivatives of the total energy density in the magnetic material on magnetization. In our model they include the external magnetic field, the demagnetizing fields, and the fields of anisotropy, which signs depend on the direction of the magnetization projections on the anisotropy axis. Both effective fields must include also fields of magnetostatic and exchange interactions, however, when the sizes of the structure are comparable with the exchange length \( l_0 = \sqrt{A/k} \) they can be omitted.

After excluding the term with time derivative from the right sides of each equation, throwing away the terms proportional to \( \alpha^2 \), and normalizing the equations by the value of saturation magnetization, we obtain the final form of the equations

\[
\frac{d\mathbf{s}_1}{dt} = -\gamma [\mathbf{s}_1 \times \mathbf{H}_1] + \frac{\gamma \alpha}{\beta} [\mathbf{s}_1 \times \left\{ (\mathbf{H}_1 + I_1 \mathbf{d}_1) \times \mathbf{s}_1 \right\}],
\]

\[
\frac{d\mathbf{s}_2}{dt} = -\gamma [\mathbf{s}_2 \times \mathbf{H}_2] + \frac{\gamma \alpha}{\beta} [\mathbf{s}_2 \times \left\{ (\mathbf{H}_2 - I_1 \frac{d_1}{d_2} \mathbf{s}_1) \times \mathbf{s}_2 \right\}] .
\]

The effective field of the thin layer is

\[
\mathbf{H}_1 = \mathbf{H}_0 + \mathbf{H}_{d1} + \mathbf{H}_{a1},
\]

where \( \mathbf{H}_{d1} = M(\mathbf{d}_{11} \cdot \mathbf{s}_1) + (\mathbf{d}_{12} \cdot \mathbf{s}_2) \) is the field if demagnetization in the thin layer, \( \mathbf{d}_{11}, \mathbf{d}_{12} \) are matrices (formfactors), which depend on the form and size of the element, \( \mathbf{H}_{a1} = k_1 \mathbf{s}_1 \cdot \mathbf{n} \) is the field of anisotropy, \( \mathbf{n} \) is the unit vector of \( x \)-direction, \( k_1 \) is a constant of anisotropy for the thin layer. Similarly, for the thick layer

\[
\mathbf{H}_2 = \mathbf{H}_0 + \mathbf{H}_{d2} + \mathbf{H}_{a2},
\]

where \( \mathbf{H}_{d2} = M(\mathbf{d}_{21} \cdot \mathbf{s}_1) + (\mathbf{d}_{22} \cdot \mathbf{s}_2) \), \( \mathbf{H}_{a2} = k_2 \mathbf{s}_2 \cdot \mathbf{n} \), \( k_2 \) is a constant of anisotropy for this layer. In our calculations, based on the work [6], the values \( \mathbf{d}_{11}, \mathbf{d}_{12}, \mathbf{d}_{21}, \mathbf{d}_{22} \) (formfactors) were found to be

\[
\mathbf{d}_{11} = \frac{1}{4\pi} \begin{pmatrix} -0.4513 & 0 & 0 \\ 0 & -0.4513 & 0 \\ 0 & 0 & -11.6637 \end{pmatrix}, \quad \mathbf{d}_{12} = \frac{1}{4\pi} \begin{pmatrix} -0.0448 & 0 & 0 \\ 0 & -0.0448 & 0 \\ 0 & 0 & 0.0896 \end{pmatrix},
\]

\[
\mathbf{d}_{21} = \frac{1}{4\pi} \begin{pmatrix} -0.4471 & 0 & 0 \\ 0 & -0.4471 & 0 \\ 0 & 0 & 0.8941 \end{pmatrix}, \quad \mathbf{d}_{22} = \frac{1}{4\pi} \begin{pmatrix} -0.0453 & 0 & 0 \\ 0 & -0.0453 & 0 \\ 0 & 0 & -12.4759 \end{pmatrix},
\]
the constant of current polarization \( P = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \) was taken to be 0.5.

The other parameters for calculations were as follows: \( \gamma = 2 \cdot 10^{11} \text{s}^{-1} T^{-1}, \alpha = 0.1, k_1 = 2 \cdot 10^6 J/m^3, k_2 = 2 \cdot 10^4 J/m^3, M = 10^6 A/m, d_1 = 0.01 \mu m, d_2 = 0.1 \mu m, J = 0-10^2 A/m^2, H_0 = 0 - 10^5 A/m. \) The choice of the parameters does not correspond to any real physical structure, but looks more or less realistic, since the goal of simulation is to find main general features of the process that cannot be found in other ways. For example, in cobalt and permalloy, typical materials for spin-valve structures, \( \alpha \cong 0.01 \div 0.2. \) We took \( \alpha = 0.1, \) for which the condition \( \alpha < 1 \) is fulfilled.

Taking into account that \( |s_1|=1 \quad |s_2|=1, \) let’s note that \( s_{1z} = \sqrt{1-s_{1x}^2-s_{1y}^2}, \quad s_{2x} = \sqrt{1-s_{2x}^2-s_{2y}^2}. \) Thus, the full phase space \( \Omega \) of the dynamical system (DS) (3)–(4) is four-dimensional and represents the direct production of two spherical surfaces \( \Omega_S \) and \( \Omega_S': \) \( \Omega = \Omega_S \times \Omega_S'. \) If the coordinates \( s_{1x}, s_{1y}\) of the vector \( s_1 \) and \( s_{2x}, s_{2y} \) of the vector \( s_2 \) are chosen as independent variables, then the arbitrary point in \( \Omega \) has the coordinates \((s_{1x}, s_{1y}, s_{2x}, s_{2y})\).

\[
P(s_{1x}, s_{1y}, s_{2x}, s_{2y}) = T(s_{1x}, s_{1y}) \times T'(s_{2x}, s_{2y}).
\]

The important characteristics of any DS, which define the features of its dynamics, are the number and the type of the DS singular points (SP). The number of SP and their location can be found from the condition of right sides of the DS equal to zero. We looked for singular points numerically using Newton method. The results were as follows.

At the current \( J = 10^2 A/m^2 \) and external field \( H_0 = 10 A/m, \) we found that the system (3)–(4) has six singular points \( P_1 \left(T_1, T_1'\right), \quad P_2 \left(T_2, T_2'\right), \quad P_3 \left(T_3, T_3'\right), \quad P_4 \left(T_4, T_4'\right), \quad P_5 \left(T_5, T_5'\right), \quad P_6 \left(T_6, T_6'\right). \) Among them, there are two attractors (stable focuses) \( T_1 \) and \( T_2, \) two repellers (unstable focuses) \( T_3 \) and \( T_6, \) and two points of the saddle type \( T_5 \) and \( T_4. \) At low values of the magnetic field, they are located closely to the points of intersection of the spheres and coordinate axes. So, at low values of the current and field, the stable position of the vectors \( s_1 \) and \( s_2 \) is such one that they are directed antiparallel.

With current growth the focuses displace slightly by the equator from axis OX to the axis OY, while the saddles displace in the direction of OX.

At the current \( J = 10^3 A/m^2, \) the attracting focuses \( T_1 \) and \( T_2 \) take the positions on the axis OY, in such a manner that the stable equilibriums \( P_1 \left(T_1, T_1'\right) \) and \( P_2 \left(T_2, T_2'\right) \) correspond to the co-directed location of magnetization in the phase space \( \Omega \) — switching does happen. Therewith, the unstable points \( T_5 \) and \( T_6 \) do not change their locations, and saddle points \( P_3 \) and \( P_4 \) take the positions on the axis OX.

Investigation of switching in more detail allowed us to recognize several intervals of the current values, where the character of the magnetization dynamics is qualitatively the same.

(i) \( 0 < J < 3.2 \cdot 10^8 A/m^2 \)

In this interval the dynamics of magnetization is described above.

It must be noted that the term “dynamics” is related here only to the process of returning of magnetization vector to the equilibrium, if it was deflected from it for some reason. For example, if the current jumped from zero (or some another value) to the constant value of this interval. At both constant external field and current, the whole system is in equilibrium either of statical or dynamical type (precession).

(ii) \( 3.2 \cdot 10^8 < J < 5 \cdot 10^8 A/m^2 \)

In this range the Andronov–Hopf bifurcation occurs in the thin layer. A pair of stable limiting cycles around the stable focuses \( T_1 \) and \( T_2 \) comes into being with simultaneous change of the type of focuses that become unstable. The projection of the phase space onto second sphere remains the same as above. The cycles are increased in size with growth of the current value.
This takes place until one of them reaches the vertical axis. The threshold current value is $J = 5 \cdot 10^8 \text{A/m}^2$.

(iii) $5 \cdot 10^8 < J < 5 \cdot 10^9 \text{A/m}^2$

In this range, the dynamics of the magnetization is featured with the oscillations from one stable point to another in both layers that means the existence of the limiting cycle in the phase space of the higher dimension (namely, four). We observed such oscillations up to the current value $J = 5 \cdot 10^9 \text{A/m}^2$.

(iv) $5 \cdot 10^9 < J < 10^{10} \text{A/m}^2$

From the current $J = 5 \cdot 10^9 \text{A/m}^2$, the precession of magnetization takes place in both layers around OZ axis (figure 2). There is one unstable limiting cycle (does not shown in the figure) between two stable ones $O_1 \left(C_1, C'_1\right)$ and $O_2 \left(C_2, C'_2\right)$, and only two singular points in each layer.

(v) $10^{10} < J < 2 \cdot 10^{10} \text{A/m}^2$

Further, from the current value $J = 2 \cdot 10^{10} \text{A/m}^2$, when the cycles reach y-axis, the precession around y-axis takes place in the both layers. This kind of dynamics is a forerunner of switching. At this, two pairs of new singular points come into being: a pair of unstable points: $P_7 \left(T_7, T'_7\right)$, $P_8 \left(T_8, T'_8\right)$ (repelling focuses), and a pair of unstable ones $P_3 \left(T_3, T'_3\right)$, $P_4 \left(T_4, T'_4\right)$ (of saddle type) (figure 3). With current growth, the size of cycles is reduced, until they reach the OY axis and vanish — the inverse Andronov–Hopf bifurcation occurs. Therewith, the unstable focuses come to be stable.

(vi) $J > 10^{11} \text{A/m}^2$

Finally, at current $J = 10^{11} \text{A/m}^2$, we observed strict switching without precession that gives another example of switching effect predicted in [1].

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**Figure 2.** Simultaneous switching to z-axes with precession. The parameters are $H_0 = 10 \text{A/m}$, $\alpha = 0.1$, $J = 10^{10} \text{A/m}^2$.

**Figure 3.** Switching to y-axes with precession. The parameters are $H_0 = 10 \text{A/m}$, $\alpha = 0.1$, $J = 2 \cdot 10^{10} \text{A/m}^2$.

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