Radiative decays with scalar mesons $a_0(980)$ and $f_0(980)$ in Resonance Chiral Theory

S. Ivashyn* and A. Korchin†

* Institute for Theoretical Physics, NSC “Kharkov Institute of Physics and Technology”, 1, Akademicheskaya str., Kharkov 61108, Ukraine

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The $\phi(1020)$ radiative decays form a good playground for study of the low-lying $a_0(980)$ and $f_0(980)$ scalar mesons. The complicated interplay of kaon loop mechanism, isoscalar meson mixing and momentum dependence of the effective couplings manifests itself in the invariant mass distributions in the $\phi(1020) \rightarrow \pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ spectra. These distributions are fitted in framework of Resonance Chiral Theory.

1. Introduction

The $e^+e^-$ experiments in Novosibirsk [12] and Frascati [34] allow one to study the $\phi(1020) \rightarrow \pi\pi\gamma$ (and $\pi\eta\gamma$) radiative decays. The invariant mass distributions of the pseudoscalar ($P$) pairs in these decays are of considerable interest. The scalar mesons ($S$) $f_0(980)$ and $a_0(980)$ are important intermediate resonances in the $\pi\pi$ and $\pi\eta$ channel correspondingly, and thereby they show up in these spectra.

It is believed that the kaon loop (KL) coupling of $S$ to the vector meson $\phi$ is very important (see, e.g. the data analysis [11,5,6]). Fig. 1 shows schematically the processes with scalar mesons. Although many authors relate the dominance of the KL mechanism to a large $K\bar{K}$ component in the $a_0(980)$ and $f_0(980)$ mesons and to the proximity of the $K\bar{K}$ threshold to the scalar meson mass, in fact the KL mechanism is a feature of the chiral dynamics and reflects the important role of the pseudoscalar mesons in low- and intermediate-energy interactions. It is shown in Refs. [7,8] that the kaon loops in the $\phi \rightarrow S\gamma$ transitions naturally arise in the leading order in Resonance Chiral Theory (R\textchiral{}T) [9], irrespectively of the threshold and mass position, and the internal structure of the scalars. The loop contribution is convergent, gauge invariant and universal: one can use either the analytical expression for $I(a,b)$ [10], or calculate it numerically.

The $\phi$ decays are rather suitable for study of the chiral dynamics, and the framework is outlined in Section 2. We should note that R\textchiral{}T leads to an important feature – momentum dependence of the scalar meson effective couplings. Employing the R\textchiral{}T Lagrangian of [9] we obtained [7] a complete set of $O(p^4)$ contributions to various radiative decays with the scalar mesons. Later on this model was applied [8] to the study of $\pi^0\pi^0$ and $\pi^0\eta$ invariant mass distributions. In Ref. [7,8] we compared the model results with estimates of other related approaches [11,12].

The free parameters of the model are $c_d$, $c_m$, $\tilde{c}_d$ and $\tilde{c}_m$, which are the coupling constants in the Lagrangian [9]. It is possible to fix their val-
ues from the limit of large number of quark colors ($N_c \to \infty$), and by imposing the constraints suggested in Ref. [13]. In the present paper we use the corresponding values in the description of invariant mass distributions in \(\phi(1020) \to \pi^0\pi^0\gamma\) and \(\pi\eta\gamma\) decays.

The application of the model is presented in Section III and future prospects are outlined in Section IV.

2. Scalar meson vertices and propagators in \(R\chi T\)

In the \(R\chi T\) a scalar meson \(S(p)\) decays into the pseudoscalar meson pair \(P_1(p_1) P_2(p_2)\) through the momentum-independent vertex \(ic_{SPP}/f_2^2\) and the momentum-dependent one \(ic_{SPP}(p_1 \cdot p_2)/f_2^2\) (see Table I), where \(f_\pi = 92.4\) MeV. These vertices vanish in the chiral limit, as soon as \(c_{SPP}\) is proportional to the pseudoscalar meson mass squared. The \(f_0 \to \pi\pi\) tree-level decay width is then

\[
\Gamma_{f_0 \to \pi\pi}(p^2) = \tilde{\Gamma}_{f_0 \to \pi\pi}(p^2) \Theta(p^2 - 4m_\pi^2),
\]

where \(\Theta(x)\) is the Heaviside step function. For the further convenience we employ the analytic function of \(p^2\)

\[
\tilde{\Gamma}_{f_0 \to \pi\pi}(p^2) = \frac{3}{2} \frac{1}{2p^2} \sqrt{p^2/4 - m_\pi^2} G^2_{f_0}(p^2),
\]

which is defined above the threshold and below the threshold with \(\sqrt{p^2/4 - m_\pi^2} = i\sqrt{|p^2/4 - m_\pi^2|}\). The factor 3/2 accounts for both \(\pi^+\pi^-\) and \(\pi^0\pi^0\) final states and \(G_{f_0\pi\pi}\) is a coupling of the \(f_0\) to \(\pi^+\pi^-\). The analogous formulae hold for other decays. We suppose that the scalar iso-singlet \(f_0(980) = (S^{sing}) \cos\theta - S^{oct}\sin\theta\) is a mixture of the singlet \(S^{sing}\) and the octet component of \(S^{oct}\) with the angle \(\theta\).

It is important to note that the scalar meson couplings \(G^2_{SPP}(p^2)/4\pi\) are the essentially momentum-dependent functions, see Table I for explicit expressions. As an example, in Fig. 2 we compare the functions \(G^2_{a_0K^+K^-}(p^2)/4\pi\) and \(G^2_{f_0K^+K^-}(p^2)/4\pi\) to the corresponding constant values of these couplings, which are often discussed.

The total widths \(\Gamma_{S,tot}(p^2)\) of the scalar mesons are

\[
\Gamma_{f_0,tot}(p^2) = \Gamma_{f_0 \to \pi\pi}(p^2) + \Gamma_{f_0 \to K\bar{K}}(p^2), \quad (2)
\]
\[
\Gamma_{a_0,tot}(p^2) = \Gamma_{a_0 \to \pi\pi}(p^2) + \Gamma_{a_0 \to K\bar{K}}(p^2). \quad (3)
\]

In Fig. 4 the mechanism of the scalar-meson production is depicted. The scalar meson propagates and decays to a pair of pseudoscalar mesons. The finite resonance width effects in the invariant mass distributions for \(\pi^0\pi^0\) and \(\pi^0\eta\) in the \(\phi\) radiative decays are known to be important [14].

For the scalar meson propagator one may use

\[
D_S(p^2) = [p^2 - m_S^2 + i\sqrt{p^2\Gamma_{S,tot}(p^2)}]^{-1}, \quad (4)
\]
old via the analytic continuation.

the real part of the self energy below the threshold. It also approximately describes which is supported by the phenomenological analysis in the present work we use the so-called Flatté-like form [16] the scalar-meson propagator which is degenerate and have equal masses.

We use the latter constraint although one may argue whether the large-$N_c$ consideration is applicable to the scalar mesons, especially in view of so-called Inverse Amplitude Method results [17].

The unusual large-$N_c$ behavior of the scalar resonances was recently summarized in [18].

In Ref. [13] based on the short-distance constraints on the flavor-changing $K\pi$, $K\eta$ and $K\eta'$ scalar form-factors it was shown that the values of $c_m$ and $c_d$ couplings in the $\mathcal{O}(p^2)$ resonance chiral Lagrangian satisfy the relation

$$c_m = c_d = \frac{f_\pi}{2} \approx 46.2 \text{ MeV}. \quad (9)$$

This relation, in particular, allows us to reduce the number of adjustable parameters in the fit.

The expression for the invariant mass distribution in the KL model is obtained in Ref. [10], and its physical meaning is explained in terms of the $SPP$ and $\phi PP$ couplings [5]. Taking into account the momentum-dependent vertices we obtain

$$\frac{dB}{dm_{\pi^0\pi^0}} = \frac{1}{2\Gamma_{\phi, \text{tot}}} \frac{\alpha \sqrt{p^2} \sqrt{1 - 4m_{\pi^0}^2/p^2}}{4 \times 48\pi^4m_K^4}$$

$$\times \left| \frac{I(a,b)}{D_{f_0}(p^2)} \right|^2 \left( \frac{M_\phi^2 - p^2}{M_\phi} \right)^3 \times \frac{G_{f_0\pi\pi}(p^2) G_{f_0KK}(p^2)}{4\pi} \left( \frac{\sqrt{2G_V M_\phi}}{f_\pi^2} \right)^2$$

for the $\pi^0\pi^0$ invariant mass ($m_{\pi^0\pi^0} = \sqrt{p^2}$) distribution, here $a = M_\phi^2/m_K^2$, $b = p^2/m_K^2$ and $\alpha \approx 1/137$. The factor $1/2$ takes into account the identity of the neutral pions. The formula for the $\pi^0\eta$ case is analogous to (10).

The $\phi(1020) \to \pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ spectra fitted by varying only the masses of scalar mesons and the singlet-octet mixing angle $\theta$ are shown in

| Table 1 |
|----------------|
| R\chi T effective couplings (see Table 9 in Ref. [7]). |

\[ c_{f\pi} = -m_\pi^2(4\tilde{c}_m \cos \theta - 2\sqrt{2}/3 c_m \sin \theta), \]

\[ c_{fKK} = -m_K^2(4\tilde{c}_m \cos \theta + 2\sqrt{2}/3 c_m \sin \theta). \]

\[ \tilde{c}_{f\pi} = 4\tilde{c}_d \cos \theta - 2\sqrt{2}/3 c_d \sin \theta, \]

\[ \tilde{c}_{fKK} = 4\tilde{c}_d \cos \theta + 2\sqrt{2}/3 c_d \sin \theta. \]

\[ c_{aKK} = -\sqrt{2} c_m m_K^2, \]

\[ c_{a\pi\eta} = -2Z\sqrt{2}/3 c_m m_\pi^2. \]

\[ \tilde{c}_{aKK} = \sqrt{2} c_d, \]

\[ \tilde{c}_{a\pi\eta} = 2Z\sqrt{2}/3 c_d. \]

\[ G_{f_0KK} \equiv \frac{1}{f_\pi^2} \left( \tilde{c}_{f_0KK}(m_K^2 - p^2/2) + c_{f_0KK} \right), \]

\[ G_{f_0\pi\pi} \equiv \frac{1}{f_\pi^2} \left( \tilde{c}_{f_0\pi\pi}(m_\pi^2 - p^2/2) + c_{f_0\pi\pi} \right), \]

\[ G_{aKK} \equiv \frac{1}{f_\pi^2} \left( \tilde{c}_{aKK}(m_K^2 - p^2/2) + c_{aKK} \right), \]

\[ G_{a\pi\eta} \equiv \frac{1}{f_\pi^2} \left( \tilde{c}_{a\pi\eta}(m_\pi^2 - p^2/2 + c_{a\pi\eta}) \right). \]

The factor $Z = \frac{\cos \theta_0 - \sqrt{2} \sin \theta_0}{\cos(\theta_0 - \theta_0)} \approx 1.53$, where $\theta_0$ and $\theta_0$ are the $\eta - \eta'$ mixing angles.

where the scalar meson mass is denoted by $m_S$. A more advanced form of the propagator including both real and imaginary parts of the scalar-meson self energy $\Pi_S(p^2)$ was proposed recently [15]. As an option, in the present work we use the so-called Flatté-like form [16] the scalar-meson propagator

$$D_S(p^2) = \left[ p^2 - m_S^2 + i\sqrt{p^2 \Gamma_S, \text{tot}(p^2)} \right]^{-1}, \quad (5)$$

where

$$\tilde{\Gamma}_{f_0, \text{tot}}(p^2) = \tilde{\Gamma}_{f_0 \to \pi\pi}(p^2) + \tilde{\Gamma}_{f_0 \to KK}(p^2), \quad (6)$$

$$\tilde{\Gamma}_{a, \text{tot}}(p^2) = \tilde{\Gamma}_{a \to \pi\eta}(p^2) + \tilde{\Gamma}_{a \to \pi\pi}(p^2) \quad (7)$$

which is supported by the phenomenological analysis, see [3]. The propagator in the form of eq.(5) accounts for a contribution of the $KK$ channel to the imaginary part of the self energy above the $KK$ threshold. It also approximately describes the real part of the self energy below the threshold via the analytic continuation.

3. Application to the radiative decays

A priori the scalar octet and singlet have independent couplings $c_d, c_m$ and $\tilde{c}_d, \tilde{c}_m$ respectively.

Numerical values of these couplings, in principle, can be determined from the underlying QCD. From the assumption $N_c \to \infty$ it was demonstrated [19] that the octet and singlet (with “tilde”) chiral couplings obey the constraints

$$\tilde{c}_m = \frac{c_m}{\sqrt{3}}, \quad \tilde{c}_d = \frac{c_d}{\sqrt{3}}. \quad (8)$$

In this limit the octet and singlet mesons become degenerate and have equal masses.
Fig. 3. The invariant mass distributions in $\phi \rightarrow \pi^0\pi^0\gamma$ (left panel) and $\phi \rightarrow \pi^0\eta\gamma$ (right panel). Theoretical calculation is performed with the values of parameters $c_m = c_d = f_\pi/2 \approx 46.2$ MeV [13]. The scalar resonance masses and the mixing angle $\theta$ obtained from the fit are shown in the legend. The two variants of the scalar meson propagator are discussed in the text. Data: [13] (left) and [24] (right).

Usually the experimental results are interpreted in terms of the constant values of the $SPP$ couplings $g_{SPP}^2/4\pi$ instead of $G_{SPP}(p^2)$ functions. Constant $g_{SPP}$ couplings can be justified in the phenomenological approaches, however this is not supported by chiral models. An importance of the derivative-coupling interactions was emphasized in Ref. [11] some time ago, and recently discussed in [19]. The replacement of $G_{SPP}(p^2)$ functions by the constant values may have a dramatic influence on the invariant mass distributions. In fact, in these distributions $p^2$ varies within the wide region – from the $PP$ threshold up to the $\phi$ mass squared – and the scalar resonances may “feel” the variation of $G_{SPP}(p^2)$ along the spectrum. This becomes more manifest in the case of a broad $\sigma$ meson, which is sometimes included in the fit.

4. Prospects and conclusion

We fit the $\phi(1020) \rightarrow \pi^0\pi^0\gamma$ and $\pi\pi\gamma$ spectra by varying only the masses of scalar mesons...
and the singlet-octet mixing angle $\theta$. The $\sigma = f_0(600)$ meson is not included yet. It is interesting that the strict constraint \(c_m = c_d = f_\pi / 2 \approx 46.2 \text{ MeV}\) still leaves a possibility to fit the data with a reasonably small $\chi^2 / \text{d.o.f}$. At the same time one realizes that the subtraction of a rather nontrivial non-scalar resonance contributions may be very important for a study of scalar resonance features.

Although our fit gives somewhat overestimated values for the mass of $f_0(980)$ scalar resonance, there may be a way out. An inclusion of the self-energy in the scalar meson propagator in an advanced form may result in the mass lowering.

The dominant scalar resonance contribution of Fig.1 exhibits just a part of the $e^+e^- \to \pi\pi\gamma$ and $e^+e^- \to \pi\eta\gamma$ mechanisms. The other contributions, which interfere with the scalar meson contribution, are in general complicated. For example, in the case of the neutral particles in the final state there are $\phi \to \rho^0\pi^0 \to \pi^0\pi^0\gamma$, $\phi \to \omega\eta \to \eta\pi^0\gamma$, $\phi \to \rho^0\pi^0 \to \eta\pi^0\gamma$ and other processes. Such non-scalar resonance channels and background are important for the precise analysis of the data (see, for example, Ref. [6]). These contributions can also be included in framework of $R\chi T$ with a relatively few number of free parameters. The approach can be extended to a wide interval of the $e^+e^-$ center-of-mass energy, covering not only the $\phi$ resonance [20].

The model allows for an extension to the charged particles production in $e^+e^-$ annihilation. In addition to the $\phi$ decays, the framework is useful for a detailed study of other radiative decays involving the light scalar mesons. Among the most interesting ones are the radiative decays into vector mesons $f_0/a_0 \to \gamma V$, $V = \rho, \omega$, currently under study in Jülich [21].

To summarize, the $R\chi T$ approach for the decays with scalar mesons at order $p^2$ is outlined. Though it does not specify the internal structure of scalar mesons, the important features of these resonances are reproduced. The model allows to investigate a complicated interplay of kaon loop mechanism, scalar-meson mixing and momentum dependence of the effective couplings in the invariant mass distributions.

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