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Does bitcoin provide hedge to Islamic stock markets for pre- and during COVID-19 outbreak? A comparative analysis with gold

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ABSTRACT

This paper applies the DCC-FIGARCH model to investigate the role of Bitcoin as a hedge and safe haven for Islamic stock markets in comparison with gold. We use daily data for the period January 2010–May 2020, which covers the recent COVID-19 pandemic. Empirical results show that the dynamic correlation between Bitcoin and Islamic stock markets is low and usually negative during major economic and political events suggesting that Bitcoin qualifies as a safe haven against Islamic stock markets downturns. Extending our analysis to portfolio management, findings reveal that the diversification benefits of Bitcoin are most times stable and increase significantly during turbulent periods. Thereby, adding Bitcoin in a portfolio of Islamic stocks reduce the risk of portfolio. Finally, as regards the COVID-19 outbreak period, we find that the hedging strategy involving Bitcoin leads to a higher cost during the crisis. These results provide substantial recommendations for Islamic investors and portfolio managers.

1. Introduction

Since the emergence of cryptocurrencies in 2009, investors, professionals and academics have raised the issue of their ability to hedge traditional assets. In addition, the last decade has witnessed the occurrence of several crises including the recent COVID-19 outbreak. This pandemic has raised the uncertainty and thus the investment-risk on international financial markets. In these circumstances, investors seek to reduce the risk of their investment and to accomplish an optimal portfolio diversification by involving new financial assets such as Bitcoin. This is due to the failure of gold to preserve its traditional role as a strong safe haven for post Global financial crisis period. Bekiros et al. (2017) and Shahzad et al. (2019) explain the weakness ability of gold to hedge by the quickening in the financialization process of commodity market and the reaction of gold prices to the different events of the recent years.

Despite the diversity of cryptocurrencies, Bitcoin is arguably the most widely used currency in the world. Compared to other cryptocurrencies, Bitcoin retains for 10 years its first place with a 51% of the capitalization of all the crypto-assets. In addition, the Bitcoin market has experienced a swift growth and substantial development over the last decade. More precisely, the capitalization has significantly increased between 2014 Q3 and 2019 Q3. It goes from around USD 30 million to almost USD 20 billion with a growth rate of almost 1000%.

Fig. 1 illustrates the evolution of the prices of Bitcoin and gold (in logarithmic scale) between 2011 and 2020. Prices are plotted in logarithmic scale in order to visualize periods of extreme increase for the two markets. Some divergences appear between the two assets. The highest value of gold prices is observed during 2012, while the Bitcoin prices reach its maximum value during 2018. This difference may hide dissimilar hedging opportunities for stock markets. Indeed, the hedging opportunity between assets depends on the extent and the sign of correlation between them. However, the divergences in the dynamic prices of gold and Bitcoin lead to differences in their dynamic correlation with Islamic stock indices. This can so provide disparity in their ability to hedge Islamic stocks.

Therefore, several previous studies attempt to compare the role that the two assets can play as a hedge and safe haven for various commodity and financial assets. Klein et al. (2018) compare the volatility,
correlation and portfolio performance of Bitcoin and gold for the period July 2011–December 2017. They conclude that the correlations of Bitcoin with the other markets are most of time opposite to those of gold. They also find that Bitcoin’s volatility dynamics share some aspects with gold but cannot serve as a safe haven during market downturn which represents the principal feature of gold. Finally, they affirm that Bitcoin cannot be considered as new gold.

Shahzad et al. (2019) test whether Bitcoin exhibits the safe haven property for extreme stock market movements. Then, they compare this property to that of gold and general commodity index. They use data for several stock market indices for developed and emerging economies during the period July 2010–February 2018. The main conclusion of the study is that the three assets (Bitcoin/Gold/commodity index) display an overall weak safe haven property for most markets. Further, using the rolling window analysis, their findings show that the safe haven feature is time-varying and differs across the considered stock markets. Even for Islamic stock markets, the role that Bitcoin can play as a hedge and/or safe haven for Islamic investments is not clear. For instance, Mensi et al. (2020) investigate the co-movements between Bitcoin and some world and regional Islamic stock indices as well as Sukuk markets using wavelet-based approach. The noteworthy finding of the paper is that the dynamic correlation and the benefits of portfolio diversification between the considered assets vary across time and frequencies. More precisely, the co-movement between Bitcoin and Islamic stock market is stronger and in same direction at low frequencies indicating that gains of diversification are less important for long-run investments than short-run investments. Withal, the opposite directions of co-movement found at high frequencies suggest that relevant hedging benefits could be achieved in the short-term through diversification between Bitcoin and Islamic equity assets.

More recently, some researches have shifted to the safe haven properties of Bitcoin during the COVID-19 outbreak. Thereby, these studies have attempted to verify the superiority of the Bitcoin compared to gold in terms of portfolio diversification. This is due to the high volatility observed in international stock markets during the pandemic which led to substantial increase of investment-risk. Consequently, investors lean towards alternative assets such as Bitcoin to reduce the risk of their portfolios. Mariana et al. (2021) suggest that Bitcoin exhibits short-term safe haven features before and during the pandemic despite the fact that it is more volatile than gold and S&P 500. Pho et al. (2021) reveal that the choice of the diversifier depends on the investor’s degree of risk aversion. More precisely, they conclude that for China, gold is a preferred portfolio diversifier than Bitcoin for risk-averse investors and vice versa for risk-seeking investors.

This paper aims at studying the role of Bitcoin as a hedge and/or safe haven for Islamic stock markets while comparing it with gold. We thus proceed with a number of research issues. First, we intend to examine the stylized facts of Bitcoin return towards specifying the best fitting model. Second, we seek to explore the dynamic and nonlinear co-movement of Bitcoin with Islamic equity indices while comparing the results with gold price dynamics. Third, we pursue to discover managerial implications regarding portfolio designs and hedging strategies. Note that the economic and theoretical logic of this aim has been developed for an international investor who wishes a successful investment strategy far from discussing why an Islamic investor will prefer a speculative asset for profit and hedging.

The economic and theoretical nexus between cryptocurrencies and Islamic stock markets has been recently documented by Narayan et al. (2019), Mensi et al. (2020) and Rehman et al. (2020). Mensi et al. (2020) suggest that this relationship is more indirect than direct. In fact Bitcoin influences the monetary system and overall stock markets through three channels namely monetary aggregates, foreign exchange rates and inflation. According to the authors, the adoption of Bitcoin can replace conventional money or inherit one or more of the roles of money. This phenomenon leads to the reduction of the circulation of money and consequently the demise of the quantity theory of money.

Narayan et al. (2019) point out that Bitcoin can influence the marginal cost and inflation given that Bitcoin is not only used as an investment asset but it is also perceived as a store of value. Thus, an increase of value and wealth in Bitcoin may stimulate the demand for goods and services and exert upward pressure on prices of inputs of these goods and services. As Bitcoin is not controlled by monetary policy, any action from central bank or policy makers other than regulation to preclude Bitcoin to defuse increasing inflation would fail.

Tschorsch and Scheuermann (2016), Mensi et al. (2020) and Rehman et al. (2020) suggest that Bitcoin has its own features and dynamic behavior. More precisely, Bitcoin differs from fiat currencies in terms of operating system, liquidity, maturity, valuation, underlying assets and speculation. Rehman et al. (2020) report that Bitcoin offers an effective diversification opportunity due to its low correlation with conventional and Islamic equity markets. Some existing literature finds that the inclusion of Bitcoin in a diversified portfolio enhances its expected profit despite the volatile behavior of Bitcoin (Mariana et al., 2021). Therefore, others highlight that even if Bitcoin can improve the return of portfolios, its high volatility makes the performance improvement hard to achieve (Smales, 2019; Chemkha et al., 2021).

The paper contributes to the existing literature on asset management in three ways. Firstly, while few recent papers were developed to examine the role of Bitcoin as a hedge and safe haven for conventional stock markets (Chkili, 2016; Mensi et al., 2018), no one has been interested in the dynamic relationship between Bitcoin and Islamic stock markets except Mensi et al. (2020) and Rehman et al. (2020). However, those studies did not clarify the role that this new asset can play as a safe haven for Islamic stock markets since they don’t consider the

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![Fig. 1. Price dynamics of Gold and Bitcoin (in logarithmic scale) from January 2011 to May 2020.](image-url)
time-varying co-movement between the two assets. Such analysis requires the specification of the best approach that incorporates all time series properties such as volatility clustering, asymmetry and long memory. The latter has greater importance given that shocks of volatility decay slowly which can influence the dynamic relationship between the variables. Secondly, we examine the co-movement between Bitcoin and the Islamic equity indices using the FIGARCH model that considers the long memory in the conditional volatility process. This feature is omitted by previous existing literature. Such model allows us on the one hand to analyze the evolution of the correlation between the two markets and, on the other hand, to build the optimal portfolio design that Islamic investor can possess in order to reduce the risk of their investments. Finally, we compare the ability of Bitcoin to hedge Islamic investments with gold that is considered as a sturdy haven in the previous studies. However, a further analysis is devoted to the period of COVID-19 to achieve the optimal portfolio management during stock market downturn. Indeed, the existing literature (Mensi et al. (2020) and Rehman et al., (2020)) is limited to the pre-crisis period and to the best of our knowledge, ours is the first study that examines the role of Bitcoin as a hedge and safe haven for Islamic stock market risks before and during the COVID-19 outbreak. It is worth noting that this task is essential for international investors given the instability of conventional and Islamic markets during the recent pandemic crisis which can lead to great uncertainty on future investments.

The remainder of the paper is organized as follows. Section 2 presents a short literature review related to the subject. Section 3 describes the data and empirical methodology. The empirical results are discussed in section 4. Section 5 concludes the paper.

2. Literature and theoretical background

The development of Bitcoin as a new asset has raised the question about its ability to compete with gold in its traditional role as a haven. The analysis of the ability of gold to hedge stock markets dates to the works of Baur and McDermott (2010) and Baur and Lucey (2010). These papers reveal that gold is a hedge against stock market uncertainties and can be considered as a safe haven for extreme market conditions. However, recent papers provide mixed results according to the empirical methodology and sample countries under study. For example, Chkili (2016) and Mensi et al. (2018) suggest that gold plays an inspiring role for conventional and Islamic market investments providing an alternative opportunity of less risky investments. Raza et al. (2016) reveal that the impact of gold prices varies through markets. This impact is positive on the BRICS stock market and negative on the stock markets of Mexico, Malaysia, Thailand, Chile and Indonesia. In the same vein, Akcoc and Civcir (2019) point out the presence of time-varying co-movement between gold and Turkish stock market when volatility is high, suggesting that gold cannot be considered as a safe haven against volatility risk.

In addition, the emergence of the Covid-19 pandemic has encouraged research on the ability of gold to maintain its traditional role as a safe haven during crises (Akhtaruzzaman et al., 2021; Salsisu et al., 2021; Drake, 2021). Akhtaruzzaman et al. (2021) reveal that the dynamic correlation between gold and some equity returns is not stable during the whole period of the COVID-19 pandemic. Their findings show that the correlation is negative during Phase I of the pandemic suggesting that gold acts as a safe haven asset for stock market indices during this period. However, the safe haven property of gold has weakened during Phase II when governments proceeded with fiscal and monetary stimulus package. Salsisu et al. (2021) point out the ability of gold market to secure investments during the pandemic more than other assets.

The fast development of Islamic stock markets has encouraged researchers to verify the ability of gold as hedge instrument for Islamic financial assets. The results of most studies confirm the role of gold as a safe haven during extreme Islamic stock market conditions (Chkili, 2017; Maghyereh et al., 2019). Furthermore, according to Chkili (2017), the value of gold tends to increase over time even during financial slumps. This allows gold to play a fundamental role in managing Islamic assets. Maghyereh et al. (2019) investigate the dynamic relationship between gold, sukuk and Islamic equity market. They report, on the one hand, that gold hedges sukuk’s risk at short and medium horizons. On the other hand, they reveal that gold serves as a perfect diversifier and hedging instrument across all investment horizons.

The emergence of Bitcoin, the last decade, has attracted national as well as international investors. As a new asset, Bitcoin offers a potential diversification opportunity. In this framework, the question of whether Bitcoin can provide hedge for other assets has been raised by several research studies. Most of these studies verify whether Bitcoin can be a substitute for gold by playing the role of a hedge or safe haven for other financial markets. Using different GARCH models, Guesmi et al. (2018) examine the properties of Bitcoin in some financial markets. They reveal some interaction effects between Bitcoin and financial variables. Moreover, the Bitcoin market allows hedging risk of investment for financial assets. Shahzad et al. (2019) compare the safe haven property of Bitcoin and gold during extreme market conditions. They verify whether such property is similar or different for the two assets. From a sample of developed and emerging economies, they conclude that both Bitcoin and gold can be regarded as a weak safe haven asset in most cases. More precisely, the safe haven roles are not stable but varying over time and differ across markets. Thus, gold is the best safe haven during extreme down movements of stock markets in developed economies given that Bitcoin fails to offer such property. However, Chinese investors can resort to Bitcoin to reduce the risk of their investments. Using the wavelet VaR approach, Bouri et al. (2020) conclude to the superiority of Bitcoin compared to gold and commodities in terms of diversification benefits. Their findings show also that Bitcoin can be classified as the most suitable safe haven followed by gold. Mizerka et al. (2020) suggest that Bitcoin acts differently across developed and emerging countries. More precisely, they find that Bitcoin serves as a strong hedge on emerging stock markets whereas it acts as a weak hedge on developed markets.

In the same vein, Shahzad et al. (2020) find that Bitcoin and gold present substantial difference in their safe haven and hedging properties for the G7 countries. Gold exhibits indisputable safe haven ability for most G7 stock indices. Moreover, gold is regarded as the most effective hedge for the stock indices of France, Germany, Italy, Japan, the United Kingdom, the United States and the MSCI G7 index, whereas Bitcoin is an effective hedge for the Canadian stock index. Będowska-Sójka and Kliber (2021) compare the safe-haven properties of gold, Bitcoin and Ether. They find that only gold serves as a strong safe haven against the stock market indices.

Conlon and McGee (2020) look at the safe haven properties of three cryptocurrencies namely Bitcoin, Ethereum and Tether. They find that Bitcoin and Ethereum cannot act as safe haven for international equity markets. However, Tether acts as a safe haven asset for all international indices.

The analysis is also extended to Islamic equity markets. In this context, two papers have been recently developed (Rehman et al., 2020; Mensi et al., 2020). Mensi et al. (2020) conclude that some diversification benefits exist when Islamic investors introduce Bitcoin in their investments. They employ the wavelet technique to investigate the co-movement between the two assets and to test whether Bitcoin can act as a hedge and safe haven for major Islamic market indices. Their findings reveal that the co-movement is stronger and in the same direction at low frequencies, indicating that the benefits of diversification is lower for long-term investors compared to short-term investors. Besides, they state that the co-movement in the opposite direction at high frequencies represents a favorable situation for investors to reduce the risk of their investments in the short run through a diversification strategy involving Bitcoin and Islamic stocks. Using the time varying copulas approach, Rehman et al. (2020) examine the risk dependence between Bitcoin and major Islamic indices. They find that the investment in
Islamic equity indices serves as an effective hedge in a portfolio along with Bitcoin.

Table A1 provides some previous studies on the role of gold and Bitcoin for the hedging of both conventional and Islamic stock markets.

| Study | Stock market | Approach | Main results |
|-------|--------------|----------|--------------|
| Panel A: Gold/Stock markets |
| Baur and McDermott (2010) | Emerging and developing countries | Quantile regression | Gold is both a hedge and safe haven for major European stock markets and the US. |
| Baur and Lacey (2010) | U.S., U.K., Germany and France | Quantile regression | Gold is a hedge on average and safe haven for turbulent period. |
| Coudert and Raymond-Feingold (2011) | U.S., U.K., Germany and France | GARCH | Gold can be regarded as a safe haven against all extreme stock market movements. |
| Hood and Malik (2013) | US stock market | Quantile regression | Gold can serve as a hedge and weak safe haven for US equity market. |
| Gurgun and Unalmis (2014) | Emerging and developing countries | Quantile regression | Gold is both a hedge and a safe haven in most of the sample countries. |
| Choudhry et al. (2015) | US, UK and Japan | Nonlinear causality test | Gold does not perform well as a safe haven however it can be used as a hedge. |
| Chkilı (2016) | BRICS countries | ADCC-GARCH | Gold can act as a safe haven against extreme market movements. |
| Chkilı (2017) | Islamic markets | Markov switching | Gold can act as a weak hedge and a strong safe haven against extreme Islamic equity market fluctuations. |
| Mensi et al. (2018) | BRICS countries | VAR-wavelet approach | Gold can act as a hedge or safe haven asset for the stock market of BRICS countries. |
| Maghyereh et al. (2019) | Dow Jones Islamic stock market | Wavelet DCC-GARCH | Gold plays an important role in the diversification and hedging of Islamic assets. |
| Akhtaruzzaman et al. (2021) | US, Japan, China and EURO STOXX | DCC-GARCH | Gold serves as a safe haven for stock markets during phase I of the COVID-19 pandemic but it loses its safe haven property during Phase II. |
| Salisu et al. (2021) | US stock market | Causality test | Gold acts as an effective safe haven before and during the pandemic. |
| Panel B: Bitcoin/Stock markets |
| Guessmi et al. (2018) | Financial markets | Multivariate GARCH | Bitcoin offers some diversification and hedging benefits for international investors. |

Table A1 (continued)

| Study | Stock market | Approach | Main results |
|-------|--------------|----------|--------------|
| Wang et al. (2019) | China | GARCH-BEKK | Bitcoin can be used as a hedge for equity movements. |
| Stenssas et al. (2019) | Seven developed and six developing markets | DCC-GARCH | Bitcoin acts as a hedge in most of the developing countries and only as a diversifier in developed countries. |
| Shahzad et al. (2020) | G7 stock markets | Quantile regression | Gold and Bitcoin exhibit some dissimilarities in their safe haven, hedging and diversifying characteristics for the stock markets. |
| Mizerka et al. (2020) | Developed and emerging markets | Quantile regression | Bitcoin exhibits superiority over both gold and commodities in terms of diversification benefits. |
| Bouri et al. (2020) | world, developed, and emerging markets, China, and the USA | Wavelet coherence | Bitcoin can provide diversification opportunity for some financial assets. |
| Maghyereh and Abdoh (2020) | S&P 500 index | Quantile cross-spectral approach | Bitcoin does not act as a safe haven during the COVID-19 bear market. |
| Rehman et al. (2020) | Seven Islamic stock markets | Copula approach | Islamic stocks can serve as an effective hedge in a portfolio of Bitcoin. |
| Mensi et al. (2020) | Nine Islamic stock market indices | Wavelet coherence | Some benefits exist from diversification with Bitcoin. |
| Conlon and McGee (2020) | S&P 500 index | Value at Risk model | Bitcoin exhibits short-term safe-haven characteristics for stocks. |
| Mariana et al. (2021) | S&P 500 index | DCC-GARCH | Bitcoin acts as a weak safe haven against the stock market indices. |
| Będowska-Sójka and Kliber (2021) | Four developed indices | DCC-MSV approach | Bitcoin contributes to the diversification of the stock market indices. |

3. Data and empirical methodology

3.1. Data

The dataset consists of the closing daily prices of six Dow Jones Islamic market (DJIM) indices (World, USA, Europe, Asia/Pacific, GCC and developed), Bitcoin and gold for the period January 2011 to May 2020. The considered stock markets allow us to verify how Bitcoin and
gold can serve as a hedge and safe haven for various global and sub-regional Islamic equity indices. As Bouri et al. (2017) and Mensi et al. (2019), we use the CoinDesk1 price index as a measure of the Bitcoin prices. The gold and the Islamic stock market indices are extracted from Datstream. We calculate daily return as the difference between the natural logarithm of two consecutive prices (Mensi et al., 2020: Guesmi et al., 2018; Chkili et al., 2014)

\[
r_t = \log P_t - \log P_{t-1}
\]

Table 1 reports some descriptive statistics and preliminary tests. As shown, Bitcoin exhibits the highest mean return (39.28%) while the DJIM developed market has the lowest mean return (0.26%). The Bitcoin market is also the most volatile market (as measured by the standard deviation) followed by DJI Europe market while the DJI World market appears as the most stable. The Jarque-Berra test statistics are significant in all cases rejecting the null hypothesis of normality for all series. The Ljung-Box and the ARCH test statistics are all significant at 1%. This indicates the presence of serial correlation and ARCH effect, confirming the use of GARCH models.

Panel B presents results of some unit root tests. From the results of the ADF and PP tests, we reject the null hypothesis of unit root given that all calculated statistics are lower than the critical values. This result is confirmed by the two GARCH-based unit root tests namely Perron (2006) and Lee and Strazicich (2003). We can then conclude that all the return series are stationary and can be considered for an empirical analysis.

Panel C reports the pair-wise correlations between Bitcoin, gold and each Islamic stock market index. We can see that all the unconditional correlations are low. This result motivates us, on the one hand, to analyze the role that Bitcoin can play as a hedge and safe haven for Islamic investments. On the other hand, it is to compare their characteristics to gold. In other words, we verify whether Bitcoin can substitute gold in its traditional role as an effective hedge and safe haven.

Fig. 2 displays the evolution of prices for Bitcoin, gold and the six considered Islamic equity indices. The Bitcoin prices present a stable period until 2016. From 2017, it experiences a bullish phase to reach its highest value in 2018. Henceforth, the prices seem unstable until the end of the reporting period. Gold shows a first period of expansion between 2012 and 2014. Then, the prices decrease to reach a minimum value in 2014 and afterwards they increase gradually starting from 2015. The Islamic markets experience a continuously increasing trend with some little swings within the study period. Fig. 3 plots the return series. We observe that periods of high (low) volatility tend to be followed by periods of high (low) volatility. This characteristic corresponds to the volatility clustering and justifies the use of the GARCH models to an appropriate description of the return volatility dynamics.

3.2. Empirical methodology

The main objective is to verify whether Bitcoin can serve as a hedge and/or safe haven for Islamic stock markets while comparing its dynamics to gold. More precisely, we inspect if Bitcoin can play the role of safe haven for extreme stock market conditions. However, such analysis requires an investigation of the time-varying relationship between Bitcoin, gold and Islamic markets using the suitable model. For this purpose, we employ the multivariate DCC-FIARCH model that incorporates the long memory feature in the volatility dynamic process. This feature is omitted in the related literature. This framework is based on the Dynamic Conditional Correlation (DCC) process of Tse and Tsui (2002) augmented by the FIARCH volatility model developed by Baillie et al. (1996).

Let \( y_t \) be a return series, the conditional mean equation is defined as follows:

\[
\begin{align*}
\mathbf{y}_t &= \mathbf{c} + \mathbf{D}
\end{align*}
\]

\[
\begin{align*}
\mathbf{e}_t &= \mathbf{H}_t^{1/2} \mathbf{\eta}_t
\end{align*}
\]

where \( \mathbf{y}_t \) is the vector of return on gold, Bitcoin and Islamic stock markets. \( \mathbf{D} \) is the vector of estimated coefficients. \( \mathbf{e}_t \) is the vector of error terms and \( \mathbf{H}_t \) is the conditional variance-covariance matrix. Following Tse and Tsui (2002) this matrix is defined as follows:

\[
\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t
\]

where \( \mathbf{R}_t \) is the \((N \times N)\) symmetric matrix of conditional correlations. \( \mathbf{D}_t \) denotes the \((N \times N)\) diagonal matrix of conditional standard deviations that is defined as:

\[
\mathbf{D}_t = \text{diag}\left(\sqrt{h_{1t}}, \ldots, \sqrt{h_{Nt}}\right)
\]

\[
h_t \text{ is assumed to follow univariate FIGARCH process:}
\]

\[
h_t = \omega + \beta(L)h_t + \{1 - \beta(L)\} \epsilon_t^2 - (\varphi(L)(1 - L)^d \epsilon_t^2.
\]

Where \( d \) is the fractional differencing parameter that should satisfy the appropriate stationary condition of \( 0 \leq d \leq 1 \), to guarantee the existence of the variance. \( (1 - L)^d \) is the fractional differencing operator. \( \beta(L) \) and \( \varphi(L) \) are polynomials in the lag operator of orders \( p \) and \( q \), respectively.

The matrix \( \mathbf{R}_t \) is specified as:

\[
\mathbf{R}_t = \mathbf{Q}_{t}^{-1} \mathbf{Q}_{t}^{-1}
\]

where \( \mathbf{Q} \) is a symmetric positive definitive conditional variances-covariances matrix:

\[
\mathbf{Q}_t = \mathbf{Q} + a \mathbf{\epsilon}_{t-1} \mathbf{\epsilon}_{t-1}' + b \mathbf{Q}_{t-1}
\]

\( \mathbf{Q}_t^* \) is a diagonal matrix containing the elements of \( \mathbf{Q}_t \).

\[
\begin{align*}
\mathbf{Q}_t^* &= \begin{bmatrix}
\sqrt{q_{11}} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \sqrt{q_{NN}}
\end{bmatrix}
\end{align*}
\]

Note that \( \mathbf{Q} \) is unconditional covariance of the standardized errors of univariate FIGARCH mode determined as:

\[
\mathbf{Q} = \frac{1}{T} \sum_{t=1}^{T} \mathbf{e}_t \mathbf{e}_t'^{-1}
\]

Finally, the dynamic conditional correlation is expressed as follows:

\[
\rho_{\gamma} = \frac{q_{ij}}{\sqrt{q_{ii}q_{jj}}}
\]

4. Empirical results

4.1. Long memory results

The choice of the appropriate model is based on its ability to capture the stylized facts of time series such as clustering volatility, persistence and long memory. The latter represents, in the recent years, an important feature in the conditional variance process of both commodity and financial time series. Given that, we start our analysis by testing the presence of long memory property in the considered markets. We use two tests namely the GPH and Robinson tests. As suggested by several previous studies (Mabrouk and Saadi, 2012; Chkili et al., 2012; Mensi et al., 2014; Mabrouk, 2016), these tests should be applied to squared returns and absolute returns. Table 2 reports the calculated values of the two tests for various bandwidths. From the reported results, we observe that the statistics of the two tests are significant at 1% level. This finding is detected for all considered markets and for all selected bandwidths. We thus reject the null hypothesis of no long-range memory in favor of

1 https://www.coindesk.com.
the alternative hypothesis of the presence of long memory property in the variance process of the commodity and financial markets. Similar results are highlighted by Chkili et al. (2014) and Aloui and Mabrouk (2010) for commodity market and El Mehdi and Mghaieth (2017) for Islamic stock markets. However, Mensi et al. (2019) point out the existence of long memory in both mean and volatility of cryptocurrency market. So, we choose the multivariate DCC-FIGARCH model to incorporate this attribute in our analysis and describe the time-varying relationship between Bitcoin, gold and Islamic stock indices.

4.2. FIGARCH estimation results

Table 3 presents the estimation results for the relationship between Bitcoin, gold and Islamic stock markets. Referring to the Akaike information criterion and Hanann-Quin information criterion, the AR(1) is the appropriate model for the conditional mean equation. As shown, the AR(1) coefficients are positive and statistically significant for most Islamic equity indices suggesting that returns are affected by their own past values.

With respect to the conditional variance equation, we observe that the ARCH parameters are significant for Bitcoin, gold, Asia and GCC Islamic market indices. This indicates that the volatility of these markets is affected by its past own shocks. However, the GARCH parameters are significant for all markets indicating that conditional volatility depends significantly on its past values. In addition, the fractional differencing parameters \(d\) obtained from the FIGARCH model are highly significant in all cases. This finding confirms the results of LM tests and proves the presence of long memory property in the conditional volatility dynamics for all examined markets. Our findings are thus in line with several papers in the previous literature. For instance, El Mehdi and Mghaieth (2017) apply GARCH-family models for some conventional and Islamic stock markets. They conclude to the presence of long memory process in

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**Table 1** Summary statistics of daily return series.

|              | Bitcoin | Gold | World | USA | Europe | Asia | GCC | Develop. |
|--------------|---------|------|-------|-----|--------|------|-----|----------|
| **Panel A: Descriptive statistics** |          |      |       |     |        |      |     |          |
| Mean         | 0.3928  | 0.0046 | 0.0233 | 0.0364 | 0.0124 | 0.0108 | 0.0037 | 0.0037   |
| S.D          | 6.5522  | 0.9473 | 0.7937 | 0.9342 | 1.0435 | 0.8393 | 0.8380 | 0.8714    |
| Skewness     | -0.795  | -0.0991 | -0.6390 | -0.5367 | -0.4329 | -0.3983 | -0.7785 | -0.6187  |
| Kurtosis     | 27.838  | 6.4346 | 4.6754 | 4.6099 | 4.1041 | 2.6702 | 13.914 | 4.7330    |
| JB           | 74558** | 4121.5** | 2260.1** | 2154** | 1692.6* | 746.99** | 18858** | 2302.5** |
| ARCH(5)      | 129.26** | 23.918** | 69.911** | 95.920** | 61.411** | 38.516** | 109.51** | 80.413** |
| ARCH(10)     | 68.162** | 13.667** | 39.883** | 52.222** | 35.755** | 25.094** | 59.291** | 44.938** |
| Q(10)        | 842.72** | 199.30** | 759.36** | 1017.4** | 760.21** | 493.71** | 849.48** | 869.45** |
| **Panel B: Stationarity tests** |          |      |       |     |        |      |     |          |
| ADF          | -41.83** | -48.07** | -40.88** | -48.55** | -47.23** | -44.66** | -43.40** | -42.38** |
| PR (2006)    | -43.93** | -49.81** | -41.52** | -49.25** | -47.88** | -45.09** | -44.54** | -43.01** |
| LS (2003)    | -42.07** | -48.22** | -40.97** | -48.59** | -47.42** | -44.89** | -43.74** | -42.45** |
| **Panel C: unconditional correlation** |          |      |       |     |        |      |     |          |
| Bitcoin      | 1.000   | 0.0378 | 0.0174 | 0.0183 | 0.0293 | -0.0155 | 0.0008 | 0.0210    |
| Gold         | -1.000  | 0.0601 | 0.0091 | 0.1006 | 0.0618 | 0.0073 | 0.0537 |

Note: The table presents summary statistics for Bitcoin, gold and six Dow Jones Islamic equity markets namely World, USA, Europe, Asia/Pacific (Asia), Gulf Cooperation Council (GCC) and Developed (Develop.). SD is the standard deviation. JB is the Jarque-Berra test of normality. ADF and PP are the statistics of the augmented Dickey–Fuller and Philip-Perron unit root tests, respectively. PR (2006) is the Perron (2006) unit root test with break. LS (2003) is the Lee and Strazicich (2003) minimum LM unit root test with two breaks.

![Fig. 2. Prices dynamics of Bitcoin, Gold and Islamic stock indices.](image-url)
all the stock market volatility dynamics. Chkili (2021) tries specifying the appropriate model for the volatility dynamics of Bitcoin. The author employs two types of frameworks namely long memory and Markov switching models. His empirical results show strong evidence of long memory process in the conditional variance.

The average conditional correlations ($\rho_{31}$) between Bitcoin and Islamic markets have negative but non-significant values indicating that Bitcoin can offer a new investment opportunity when combined with Islamic stocks. Such analysis requires a thorough investigation for the evolution of the dynamic conditional correlation between markets.

Fig. 4 traces the time-varying conditional volatility for all considered markets. Some high volatility common phases can be detected for the Islamic stock markets. More precisely, all Islamic stock markets, except the GCC, exhibit a first period of high volatility between 2011 and 2012 which coincides with the Euro-Zone debt crisis (EZDC). This crisis has appeared in Greece and has affected most conventional and Islamic markets worldwide. The plots also show that conditional variance of Islamic markets increase significantly between 2015 and 2016, as a response to the oil price shock. Finally, the drop of financial markets during the last year followed by a turbulent period can be explained by the effects of the Covid-19 pandemic. Several studies conclude that during the current pandemic, the indices of developed and emerging markets have plummeted, followed by a spike in volatility (Izzeldin et al., 2021; Uddin et al., 2021; Liu et al., 2021; Hasan et al., 2021).

Hasan et al. (2021) evaluate the effect of COVID-19 outbreak on both conventional and Islamic equity market indices. Specifically, they seek each difference in the volatility dynamics of Islamic and conventional markets during the COVID-19 crisis. Their findings point out that both markets switch to volatile regime since the COVID-19 was affirmed a global pandemic in the beginning of March 2020.

### Table 2
Long memory test results.

|                      | GPH test       | Robinson test  |
|----------------------|----------------|---------------|
|                      | $m = T^{0.5}$  | $m = T^{0.6}$  | $m = T^{0.7}$  | $m = T^{1/4}$ | $m = T^{1/8}$ | $m = T^{1/16}$ |
| Panel A: Squared returns |
| Bitcoin              | 0.4345***      | 0.1508*       | 0.1435***      | 0.4015***    | 0.2034***    | 0.1591***     |
| Gold                 | 0.5862***      | 0.2968***     | 0.2225***      | 0.3221***    | 0.3806***    | 0.4114***     |
| DJIM World           | 0.6784***      | 0.5326***     | 0.3913***      | 0.3480***    | 0.3723***    | 0.3565***     |
| DJIM USA             | 0.5813***      | 0.3982***     | 0.3874***      | 0.3332***    | 0.3929***    | 0.4567***     |
| DJIM Europe          | 0.6840***      | 0.6065***     | 0.4254***      | 0.3048***    | 0.4047***    | 0.4588***     |
| DJIM Asia            | 0.6087***      | 0.6174***     | 0.4728***      | 0.2935***    | 0.2294***    | 0.2343***     |
| DJIM GCC             | 0.2278*        | 0.2284***     | 0.2660***      | 0.3315***    | 0.3817***    | 0.4024***     |
| DJIM Dev.            | 0.6872***      | 0.5109***     | 0.3850***      | 0.4106***    | 0.4108***    | 0.3891***     |
| Panel B: Absolute returns |
| Bitcoin              | 0.5995***      | 0.4121***     | 0.3970***      | 0.4106***    | 0.4108***    | 0.3891***     |
| Gold                 | 0.8344***      | 0.6257***     | 0.4403***      | 0.3472***    | 0.4561***    | 0.5629***     |
| DJIM World           | 0.7962***      | 0.6184***     | 0.4904***      | 0.4039***    | 0.4652***    | 0.5554***     |
| DJIM USA             | 0.7182***      | 0.5610***     | 0.5358***      | 0.4009***    | 0.4601***    | 0.5142***     |
| DJIM Europe          | 0.8077***      | 0.6364***     | 0.4969***      | 0.3846***    | 0.4703***    | 0.5785***     |
| DJIM Asia            | 0.8095***      | 0.7281***     | 0.5876***      | 0.3794***    | 0.4847***    | 0.5866***     |
| DJIM GCC             | 0.3816***      | 0.3476***     | 0.3075***      | 0.3579***    | 0.3557***    | 0.4284***     |
| DJIM Dev.            | 0.7994***      | 0.6154***     | 0.5024***      | 0.4048***    | 0.4664***    | 0.5486***     |

Note: ** indicates significance at the 1% level, $T$ is the sample size. $m$ is the bandwidth.
4.3. Time-varying dynamic correlation

Fig. 5 shows the evolution of the time-varying correlation between Bitcoin and each Islamic stock market. We also report the dynamic relationship between gold and the different Islamic markets for comparative purposes. The dynamic correlation for the couple Bitcoin/DJIM indices is low for all plots. More interestingly, the correlations switch between positive and negative values during the period under investigation. Therefore, we can see a significant drop in the conditional correlation during the year 2011 and for the period 2013–2014. During these two episodes, the correlation is often negative, attesting that the two markets evolve in opposite directions. Not surprisingly, most conventional and Islamic stock markets have experienced a fall in their prices in response to the Eurozone crisis (EZDC) of 2011–2012, and the falling of crude oil prices in 2014. This last event is transmitted to the financial markets given the intense financialization of commodity markets, in particular oil market. Some previous studies point out similar results (Creti et al., 2013; Zhang et al., 2017; Ding et al., 2021).

Fig. 5 also depicts a low correlation phase that spans between 2015 and 2017. This period is characterized by the decrease in the Islamic stock indices. The more important decrease is observed for the GCC Islamic market which achieves its minimum value for the period under analysis. Indeed, the fall of the SSE Composite Index about 43% in mid-2015 in just over 2 months is followed by a turbulence phase of Chinese stock market, a devaluation of the Yuan, a slowing of the Chinese economic growth and a fall of oil prices. As a result, investors sold shares globally, which in turn culminated in the decline of the value of conventional and Islamic stock prices globally.

The low or negative correlations between the two markets, observed during the period of falling DJIM indices, suggest that Bitcoin can serve as a safe haven during extreme Islamic stock market conditions. Comparing this finding to the gold results, we notice that usually the dynamic correlation for the pair gold/Islamic markets is higher than the correlation for the pair Bitcoin/Islamic markets. This suggests that gold

| Table 3 Estimation results for DCC-FIAPARCH model. |
|---------------------------------------------------|
| **Panel A: estimation results**                     |
| **Bitcoin** | **Gold** | **World** | **USA** | **Europe** | **Asia** | **GCC** | **Develop** |
| C(m)        | 0.208** (0.088) | 0.010 (0.017) | 0.065*** (0.014) | 0.079*** (0.014) | 0.035** (0.017) | 0.042** (0.016) | 0.036** (0.016) | 0.067*** (0.014) |
| AR(1)       | 0.056* (0.034) | −0.001 (0.023) | 0.134*** (0.023) | −0.035 (0.022) | −0.012 (0.022) | 0.068*** (0.021) | 0.068*** (0.023) | 0.097*** (0.023) |
| GARCH       | 1.292** (0.530) | 0.048* (0.029) | 0.031*** (0.010) | 0.051*** (0.016) | 0.051** (0.022) | 0.022* (0.014) | 0.070 (0.047) | 0.033** (0.028) |
| ARCH        | 0.090* (0.089) | 0.395** (0.176) | −0.007 (0.098) | 0.025 (0.091) | −0.007 (0.091) | 0.162 (0.110) | 0.020* (0.130) | −0.012 (0.095) |
| GARCH       | 0.056*** (0.107) | 0.589*** (0.176) | 0.594*** (0.126) | 0.416** (0.178) | 0.576*** (0.178) | 0.391*** (0.064) | 0.461*** (0.128) | 0.242** (0.111) |
| d           | 0.793*** (0.101) | 0.275*** (0.048) | 0.515*** (0.094) | 0.576*** (0.166) | 0.391*** (0.064) | 0.461*** (0.128) | 0.242** (0.111) | 0.532** (0.104) |

**Panel B: Estimates of DCC model**

| **ρ_{21}** | −0.007 (0.454) | −0.009 (0.323) | −0.002 (0.034) | 0.006 (0.023) | −0.007 (0.017) | −0.008 (0.045) |
| **ρ_{31}** | −0.031 (0.464) | −0.023 (0.034) | 0.021 (0.036) | −0.028 (0.024) | −0.024 (0.021) | −0.028 (0.048) |
| **ρ_{32}** | 0.085* (0.051) | 0.057* (0.035) | 0.074* (0.039) | 0.047* (0.023) | 0.022 (0.018) | 0.082* (0.052) |
| **a**      | 0.008*** (0.003) | 0.012 (0.011) | 0.004*** (0.002) | 0.010*** (0.005) | 0.008*** (0.003) | 0.008*** (0.003) |
| **b**      | 0.986*** (0.007) | 0.972*** (0.045) | 0.990*** (0.005) | 0.931*** (0.026) | 0.986*** (0.008) | 0.986*** (0.006) |

**Student-t**

| 5.258*** (0.252) | 5.067*** (0.237) | 5.274*** (0.258) | 5.529*** (0.286) | 5.110*** (0.234) | 5.237*** (0.250) |

**Log L**

| −11886.6 | −12266.1 | −12598.8 | −12656.6 | −12382.8 | −11953.8 |

**Panel C: Diagnostic tests**

| **AIC** | 10.317 | 10.645 | 10.934 | 10.595 | 10.417 | 10.375 |
| **SIC** | 10.376 | 10.705 | 10.993 | 10.655 | 10.475 | 10.435 |
| **Q2(10)** | 5.151 [0.881] | 7.234 [0.703] | 8.752 [0.556] | 12.750 [0.238] | 5.538 [0.852] | 4.195 [0.938] |
| **Q2(20)** | 24.189 [0.234] | 11.862 [0.921] | 15.015 [0.775] | 18.216 [0.573] | 12.504 [0.998] | 19.066 [0.517] |

Note: C(m) and C(v) are the constants of the mean and variance equations, respectively. Standard deviations are reported in parentheses. p-values of statistical tests are presented in brackets. Asterisks ***, ** and * denote the significance at 1%, 5% and 10% levels, respectively.
has ceded its role as a haven to new assets such as Bitcoin. However, the latter appears to be a strong refuge during this past decade. We also notice that unlike Bitcoin, the price of gold decreased during the period 2015–2016, which explains the positive correlation between gold and Islamic market indices. This result is in line with Bouri et al. (2020) who reveal that the overall connectedness between gold/ commodities/Bitcoin and the stock markets is not very strong over time. In addition, the wavelet coherency approach results show that Bitcoin displays the weakest dependence with equity markets. Whereas, Shahnaz et al. (2019) find that Bitcoin, gold, and commodities exhibit a weak safe haven propriety for most stock market indices.

The last period of low correlation has emerged at the end-2019 and coincided with the period of the COVID-19 downturn which led to a severe global economic recession phase. The impacts of the COVID-19 pandemic became more visible in the beginning of 2020 with the stock market crashes. This finding confirms the role that Bitcoin can play during the severe economic, political and social events. More precisely, Bitcoin can serve as a safe haven during COVID-19 stock market shocks. Huang et al. (2021) assert that Bitcoin can act as a safe haven in Europe, the UK, and the US during the virus pandemic, suggesting that investors in these countries should hedge their portfolios using Bitcoin. Mariana et al. (2021) find that the daily returns of Bitcoin and Ethereum are negatively correlated with S&P 500 return during the COVID-19 pandemic suggesting their safe haven features during extreme stock market downturn. Therefore, our results are different from Shehzad et al. (2021) who state that most of the time during the COVID-19, gold investments were evinced to be more beneficial than Bitcoin.

4.4. Portfolio designs

It is worth noting that the ambition of international investors is to achieve the optimal portfolio allocation. In this respect, financial analysts and portfolio managers attempt to specify the suitable model that describes the volatility dynamics of financial and commodity markets. They seek then to propose the optimal portfolio design. Using the estimation results of our long memory model, we determine the optimal portfolio weights composed by Bitcoin and Islamic stocks. The objective is to build the portfolio that allows to minimize the risk without lowering expected returns. Following Kroner and Ng (1998), Chkili et al. (2014) and Chkili (2016), the optimal holding weight of Bitcoin in a one dollar invest in Bitcoin/Islamic stocks portfolio is calculated as follows:

$$w_{SB}^t = \frac{h_S^t - h_B^t}{h_B^t - 2h_{SB}^t + h_S^t}$$

Following Kroner and Ng (1998), the optimal weight of Bitcoin is between 0 and 1, so that:

$$w_{SB}^t = \begin{cases} 0 & \text{if } w_{SB}^t < 0 \\ w_{SB}^t & \text{if } 0 < w_{SB}^t < 1 \\ 1 & \text{if } w_{SB}^t > 1 \end{cases}$$

where $h_S^t$ and $h_B^t$ designate, respectively, the conditional variances of the Islamic stock market and Bitcoin derived from the estimation of the FIGARCH model. $h_{SB}^t$ measures the covariance between the Islamic stock market index and Bitcoin at time $t$. We can also compute the weight of Islamic asset in the one-dollar Bitcoin/Islamic stocks portfolio as the
difference between one and the weight of Bitcoin \((1 - w_{SB})\). Finally, we note that this methodology is applied likewise to the portfolio composed by gold and Islamic stocks.

The summary statistics of the optimal weights for both Bitcoin and gold are reported in Table 4. Starting with the Bitcoin, the average value of the portfolio weight ranges between 0.0357 for the Islamic world index and 0.0528 for the Islamic Europe index. This indicates that for one-dollar Bitcoin/Islamic stock’s portfolio, 3.57 cents should be allocated to Bitcoin while the remaining 96.43 cents should be invested in the world Islamic market index. Regarding the GCC Islamic market, the optimal investment weights are 4.46 and 95.54 respectively. More precisely, to reduce the risk of their portfolio without lowering the expected returns, investors in the GCC’s countries should invest 4.46% of their wealth on the Bitcoin market while 95.54% should be devoted to hold Islamic equities. Quite similar results are uncovered by Rehan et al. (2020). The authors suggest that investors accomplish diversification benefits by investing in Bitcoin along with Islamic stocks.

Panel B exhibits statistics of the optimal weight values between gold and Islamic stock index. We see that for all considered Islamic markets the mean values are high compared to Bitcoin. The lowest value is observed for the DJIM World/Gold pair which equal to 0.3803. However, the highest weight is detected for the Islamic Europe index which reaches the value of 0.5262. This suggests that in order to minimize the risk without diminishing the profitability of their investments, Islamic investors should invest between 38.03% and 52.62% of their budget in gold market and the rest in Islamic equity markets.

Fig. 6 plots the evolution of the optimal weights of Bitcoin and gold over time. We see that the conditional weight is not stable but varies through periods and across markets. More interestingly, the optimal weight for the Bitcoin/Islamic equity pairs is weaker and more stable than that for the gold/Islamic equity pairs. This suggests that the opportunities of diversification are entirely different and distinguishable between the two assets.

For most of the Islamic stock markets, we witness a common evolution of Bitcoin weight during the period under investigation. The diversification benefits for Bitcoin are stable for most of time and in contrast than that for the gold/Islamic equity pairs. This suggests that the opportunities of diversification are entirely different and distinguishable between the two assets.

|                      | Mean    | Median  | S.D     | Min     | Max     |
|----------------------|---------|---------|---------|---------|---------|
| DJIM world/Bitcoin   | 0.0357  | 0.0245  | 0.0338  | 0.0002  | 0.2170  |
| DJIM USA/Bitcoin     | 0.0496  | 0.0348  | 0.0488  | 0        | 0.3478  |
| DJIM Europe/Bitcoin  | 0.0528  | 0.0353  | 0.0493  | 0        | 0.3273  |
| DJIM Asia/Bitcoin    | 0.0443  | 0.0316  | 0.0415  | 0.0008  | 0.2591  |
| DJIM GCC/Bitcoin     | 0.0446  | 0.0283  | 0.0478  | 0        | 0.3706  |
| DJIM Dev./Bitcoin    | 0.0374  | 0.0256  | 0.0357  | 0        | 0.2327  |

4.5. Hedge and risk reduction

Some previous studies suggest that Bitcoin can be used to hedge financial portfolio and to reduce risk. More precisely, it comes to determine the appropriate position in different markets. For this purpose, we calculate the optimal hedge ratio \(\beta_{SB}\) using the FIGARCH estimation results. This means that a short position (selling) in the Islamic stock market index should be hedged by a long position (buying) of \(\beta_{SB}\) dollar in the Bitcoin market. The optimal hedge ratio is developed by Kroner and Sultan (1993) as follows:

\[
\beta_{SB} = \frac{h_{SB}}{\sqrt{h_{SB}^2 + h_{He}^2}}
\]

This methodology is also applied by previous literature to check the hedge ability among several commodities and financial assets (Mensi et al., 2018; Chkili et al., 2014; Hamoudeh et al., 2010) or among Bitcoin and different asset types (Guesmi et al., 2018). Given our objective of comparison, we calculate in the same way the optimal hedge ratio for gold.

To refine our analysis, we also calculate the hedging effectiveness (HE) index. It represents an indicator of the performance of the applied hedging strategy. Practically, this index measures the gain or loss in the variance of hedged portfolio compared to the unhedged portfolio. The hedging effectiveness index is calculated as follows:

\[
HE = \frac{\text{variance}_{unhedged} - \text{variance}_{hedged}}{\text{variance}_{unhedged}}
\]

Table 4

Summary statistics for portfolio optimal weights.

|                      | Mean    | Median  | S.D     | Min     | Max     |
|----------------------|---------|---------|---------|---------|---------|
| DJIM world/Gold      | 0.3803  | 0.3522  | 0.1698  | 0.0514  | 0.8929  |
| DJIM USA/Gold        | 0.4630  | 0.4361  | 0.1795  | 0.0885  | 0.9477  |
| DJIM Europe/Gold     | 0.5262  | 0.5236  | 0.1454  | 0.0572  | 0.9065  |
| DJIM Asia/Gold       | 0.4384  | 0.4221  | 0.1400  | 0.0600  | 0.8296  |
| DJIM GCC/Gold        | 0.4242  | 0.4370  | 0.1552  | 0.0066  | 0.8908  |
| DJIM Dev./Gold       | 0.3951  | 0.3651  | 0.1719  | 0.0555  | 0.8988  |

Looking at the gold results, we notice that all the hedge ratios are higher than those of the Bitcoin. This suggests that Islamic investors need much smaller budget in Bitcoin to hedge Islamic equity investments. More precisely, the average values of the hedge ratio switch between 0.0036 for the GCC Islamic market and 0.0792 for the Europe Islamic market. The positive average value of 0.0792 for the couple DJI Europe/gold indicates that a USD 1000 short position in Europe Islamic stocks can be hedged by implementing a long position in the gold market of USD 79.2. This amount is reduced to USD 3.6 for the GCC Islamic market. The evolution over time of the hedge ratio between Bitcoin (gold) and each Islamic market is displayed in Fig. 7 (Fig. 8). The most important finding that we can notice is that the hedge ratio is not stable.
and varies between positive and negative values during the period under study.

Finally, we calculate the hedging effectiveness from the variances of the hedged (incorporating Bitcoin or gold and Islamic stocks) and unhedged portfolio (incorporating only Islamic stocks). A great HE value means a perfect hedging strategy. Comparing the HE for the two assets, the results show that Bitcoin exhibits higher HE values than gold for all Islamic equity markets except for the Europe Islamic market. This suggests that Bitcoin possess hedging benefits more substantial than the traditional hedge asset. The harvest result confirms previous research arguing that Bitcoin is isolated from financial assets and hence offers risk diversification possibilities for portfolio investment (Corbet et al., 2018; Bouri et al., 2020).

4.6. The COVID-19 outbreak effects

The COVID-19 pandemic has increased the uncertainty and volatility of stock markets worldwide. Consequently, the risk associated with conventional and Islamic stock investments has significantly risen. In these circumstances, international investors seek to reach the optimal portfolio management and to determine the effective hedging strategies. Thereby, this period requires further analysis to investigate the potential diversification opportunity involving Islamic stocks and Bitcoin or gold. We also verify the ability of Bitcoin and gold to hedge Islamic investment during this pandemic turmoil. To achieve this issue, we compute the optimal portfolio weights, the hedging ratio and the hedging effectiveness index. Finally, to check the successfulness of hedging instru-
ment, we calculate the difference between the HE values during COVID-19 pandemic and the HE values in the full sample period as follows:

$$\Delta HE = HE_{\text{during COVID-19}} - HE_{\text{full period}}$$

Note that positive values of $\Delta HE$ suggest beneficial hedging strategy during COVID-19 crisis. The period of crisis spans from January 1, 2020 to May 20, 2020 which covers the first wave of the pandemic. The results are reported in Table 6. As shown, the optimal weights for the Bitcoin are lower than the gold in all cases. This indicates that Islamic investors required less of Bitcoin than gold to reduce the risk of their portfolio during the COVID-19 outbreak. In addition, the optimal weight values are slightly more important during the pandemic suggesting that investors should invest more in the Bitcoin or gold market during the recent crisis. For example regarding the Bitcoin findings, the optimal portfolio weight skips from 0.0357 (DJIM world/Bitcoin) and 0.0496 (DJIM USA/Bitcoin) for the whole period to 0.0457 and 0.0709 during the COVID-19 outbreak, respectively. This finding confirms the role that Bitcoin can play for Islamic investments. As regards the hedge ratio, the table displays an increase in their absolute values compared to full sample. Thus, the hedging strategy leads to higher costs during the crisis. However, the hedging strategy shows greater performance during the COVID-19 outbreak as the $\Delta HE$ values are positive in all cases except for DJIM Asia Pacific. This result confirms that Bitcoin provide beneficial reduction of investment-risk during the turmoil period. Wang et al. (2021), Rubbaniy et al. (2021) and Kumar (2020) find evidence of benefits from diversification with Bitcoin during the COVID-19 pandemic.

5. Discussion and conclusion

The instability of financial markets owing to different financial crises has amplified uncertainty of international investments. This uncertainty
persisted over the last two years with the rising trade conflict between the USA and China and the economic turmoil associated with the COVID-19 pandemic. Given these circumstances, it becomes challenging for investors to accomplish diversification benefits. Even the diversification opportunity offered by commodity markets has weakened with the process of financialization of these markets. All these events encouraged international investors to look for new financial markets such as Bitcoin that would provide a potential hedging opportunity.

In this study, we have verified whether Bitcoin can serve as a hedge for Islamic equity markets. Notably, we have investigated the benefits of hedging through the diversification between Bitcoin and Islamic equity indices. Such analysis allows us to compare its hedging ability to that of gold. We have used the DCC-FIGARCH model that considers some stylized facts of time return series such as volatility clustering, conditional heavy tails and long memory property. The findings show that all the time series exhibit the presence of long memory property in the conditional volatility dynamics of the considered equity and commodity markets. More interestingly, shocks of volatility persist and don’t decease swiftly. The analysis of the dynamic correlation shows some divergences between the couple Bitcoin/Islamic index and the couple gold/Islamic index. Interestingly, the correlation is time-varying, and its evolution varies across markets. In addition, this correlation is lower for the pair Bitcoin/Islamic index than for the pair gold/Islamic index. The connection of Bitcoin to Islamic market shows also variation across periods and it is most times close to zero or negative. This suggests that some diversification benefits exist between the two assets.

The results provide important implications for Islamic investors and portfolio managers. In fact, the risk management requires better understanding of the type of links between assets. Thus, the composition of the optimal portfolio depends on the market conditions. On the whole, Islamic investors should invest more in Bitcoin market during bearish market conditions than during normal periods. This allows investors to achieve the benefits of diversification through computing optimal weights of each asset which lead to lowering portfolio risk without decreasing the expected returns. Urom et al. (2020) point out that international investors should consider Bitcoin as part of their portfolio diversification. Bouri et al. (2017) also underline that Bitcoin can serve as an effective diversifier for most financial and commodity assets.

Our results for the COVID-19 period show that Bitcoin offers a better diversification opportunity to reduce the risks of major Islamic equity markets. More interestingly, investors should place a lower amount on Bitcoin market during the COVID-19 compared to the pre-COVID-19 period in order to minimize their portfolio risk. The fund managers and financial analysts can rely on these findings in order to help investors reach a well-diversified portfolio during the COVID-19 crisis and achieve an optimal risk reduction.

Finally, our results highlight the ability of Bitcoin to hedge Islamic market risk. Hence, its hedging property differs according to market conditions. Mensi et al. (2020) and Sensoy (2016) argue that the systematic risk of Islamic equity markets varies over time which make hedge ratio not constant. However, in average, a low amount of Bitcoin provides a high hedging effectiveness of Islamic investments. This amount has substantially increased during the recent COVID-19 crisis suggesting that hedging strategies require higher costs during crisis period.

CRediT attribution statement

Walid Chkili: Conceptualization, Methodology, Data curation, Software, Visualization, Investigation, Writing-original draft, Validation, Project administration, Supervision, Writing-review & editing, Formal analysis. Aymen Ben Rejeb: Conceptualization, Investigation, Validation, Writing-original draft, Visualization, Writing-review & editing, Supervision. Mongi Arfaoui: Software, Data curation, Writing-original draft, Investigation, Validation, Writing-review & editing, Formal analysis.

Declaration of Competing Interest

We declare that there are no conflicts of interest.

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