Q-stars in scalar-tensor gravitational theories

Athanasios Prikas

Physics Department, National Technical University, Zografou Campus, 157 80 Athens, Greece, tel. +302107722991.\(^1\)

Abstract

We study q-stars in Brans-Dicke gravitational theory. We find that when the Brans-Dicke constant, \(\omega_{BD}\), tends to infinity, the results of General Relativity are reproduced. For other values of \(\omega_{BD}\), the particle number, mass and radius of the star and the absolute value of the matter field are a few percent larger than in the case of General Relativity. We also investigate the general scalar-tensor gravitational theory and find that the star parameters are a few percent larger than in the case of General Relativity.

PACS number(s): 11.27.+d, 04.40.-b

\(^1\)e-mail: aprikas@central.ntua.gr
1 Introduction

Interesting alternative gravitational theories are the scalar-tensor gravitational theories, which appeared at the original paper of Brans and Dicke [1], where the Newtonian constant $G$ was replaced by a scalar field $\phi_{BD}$, and the total action contained kinetic terms for the new field times an $\omega_{BD}$ quantity. $\omega_{BD}$ was regarded as a constant in the original paper. The theory was generalized in a series of papers, [2, 3], mainly in the direction of replacing the constant $\omega_{BD}$ with a function of the Brans-Dicke (BD) scalar field.

Boson stars appeared in the literature as stable field configurations of massive scalar matter with a global $U(1)$ symmetry, coupled to gravity [11, 12], now known as “mini” boson stars due to their small relative magnitude. Other works took into account self interactions [13, 14] or the case of local symmetry [15]. Their common feature is that gravity plays the role of the non-linear interaction that stabilizes the star against decay into free particles.

When the scalar potential is of a special type, admitting stable non-topological soliton solutions in the absence of gravity, the soliton stars appear as relativistic generalizations of the above solitons. The so called “large” soliton stars, with radius of order of lightyears, discussed analytically in a series of papers by Friedberg, Lee and Pang, [16, 17, 18]. Another class of soliton stars appeared as a generalization of q-balls. Q-balls are non-topological solitons in Lagrangians with a global $U(1)$ symmetry, [19], or a local one, [21], or a global $SU(3)$ or $SO(3)$ symmetry, [20]. Q-balls are supposed to appear in the flat directions of the superpotential in supersymmetric extensions of Standard Model, [21, 22], and play a special role in the baryogenesis, [23].

Q-stars are relativistic extensions of q-balls, with one or two scalar fields and a global, [25], or local, [28], $U(1)$ symmetry, non-abelian symmetry, [26], or with fermions and a scalar field, [27] in asymptotically flat or anti de Sitter spacetime, [29]. Any type of the bosonic stars may offer a solution to the problem of Dark Matter, when the q-stars have the additional feature to be of the same order of magnitude as neutron stars and, generally, as other stellar objects.

Within the BD gravitational framework, Gunderson and Jensen investigated the coupling of a scalar field with quartic self-interactions with the metric and the BD scalar field, $\phi_{BD}$, [4]. The properties of boson stars within this framework have been extensively studied in a series of papers [5, 6, 7]. Their results generalized in scalar-tensor gravitational theories, where $\omega_{BD}$ is no more a constant, but a function of the BD field [8, 9]. The case of charged boson-stars in a scalar-tensor gravitational theory has been analyzed in [10].
In the present article we follow the work of [4]-[10]. Our aim is to study the formation of non-topological soliton stars in the context of BD or general scalar-tensor gravitational theory, their stability with respect to fission into free particles and to gravitational collapse, and the influence of $\omega_{BD}$ in the star parameters. We also compare our results with those obtained in the framework of General Relativity.

2 Q-stars with one scalar field

We consider a static, spherically symmetric metric:

$$ds^2 = -e^{\nu}dt^2 + e^{\lambda}d\rho^2 + \rho^2 d\Omega^2 ,$$

(1)

with $g_{tt} = -e^{\nu}$. In order to realize such a spacetime, we regard both the matter and the BD fields as spherically symmetric and the former with an harmonic time dependence, assuring minimum energy for the matter field, and the latter time independent. If $\phi$ is the matter field and $\phi_{BD}$ the BD field, we write the action for the BD theory:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( \phi_{BD} R - \omega_{BD} g^{\mu\nu} \partial_{\mu} \phi_{BD} \partial_{\nu} \phi_{BD} \right)$$

$$+ \int d^4x \sqrt{-g} \mathcal{L}_{\text{matter}} ,$$

(2)

with:

$$\mathcal{L}_{\text{matter}} = (\partial_{\mu} \phi)^*(\partial_{\nu} \phi) - U ,$$

(3)

and $\omega_{BD}$ a constant in BD gravity and a certain function of the $\phi_{BD}$ field in a generalized scalar-tensor gravitational theory, which we will discuss later.

Varying the action with respect to the metric and scalar fields we obtain the Einstein and Lagrange equations respectively, as follows:

$$G_{\mu\nu} = \frac{8\pi}{\phi_{BD}} T_{\mu\nu} + \frac{1}{\phi_{BD}} (\phi_{BD,\mu;\nu} - g_{\mu\nu} \phi_{BD,;\lambda} ;^\lambda)$$

$$+ \frac{\omega_{BD}}{\phi_{BD}^2} \left( \partial_{\mu} \phi_{BD} \partial_{\nu} \phi_{BD} - \frac{1}{2} g_{\mu\nu} \partial_{\lambda} \phi_{BD} \partial_{\lambda} \phi_{BD} \right) ,$$

(4)

$$\phi_{,\lambda} ;^\lambda - \frac{dU}{d|\phi|^2} \phi = 0 ,$$

(5)

$$\frac{2\omega_{BD}}{\phi_{BD}} \phi_{BD,;\lambda} ;^\lambda - \omega_{BD} \frac{\partial^\lambda \phi_{BD} \partial_{\lambda} \phi_{BD}}{\phi_{BD}^2} + R = 0 .$$

(6)
$G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ the energy momentum tensor for the matter field given by:

$$T_{\mu\nu} = (\partial_{\mu}\phi)^*(\partial_{\nu}\phi) + (\partial_{\mu}\phi)(\partial_{\nu}\phi)^* - g_{\mu\nu}[g^{\alpha\beta}(\partial_{\alpha}\phi)^*(\partial_{\beta}\phi)] - g_{\mu\nu}U \quad (7)$$

and $R$ is the scalar curvature. Tracing eq. 4 we take:

$$-\frac{8\pi}{\phi_{BD}} T - \frac{\omega_{BD}}{\phi_{BD}^2} \partial^\lambda \phi_{BD} \partial_\lambda \phi_{BD} + \frac{3}{\phi_{BD}} \phi_{BD,\lambda} \phi_{BD} = R .$$

with $T$ the trace of the energy-momentum tensor. Substituting in eq. 0 we find:

$$\phi_{BD,\lambda} \phi_{BD} = \frac{8\pi}{2\omega_{BD} + 3} T . \quad (8)$$

The above results hold true for every case of bosonic, spherically symmetric, static field configurations coupled to BD gravity.

We will now insert the q-soliton ansatz writing:

$$\phi(\bar{\rho}, t) = \sigma(\rho)e^{-i\omega t} . \quad (9)$$

with $\omega$ the frequency with which the q-soliton rotates within its internal $U(1)$ space. The Lagrange equation for the $\phi$ field is:

$$\sigma'' + [2/\rho + (1/2)(\nu' - \lambda')]\sigma' + e^\lambda \omega^2 e^{-\nu} \sigma - e^\lambda \frac{dU}{d\sigma^2} \sigma = 0 . \quad (10)$$

We define:

$$A = e^{-\lambda} , \quad B = e^{-\nu} , \quad (11)$$

$$W \equiv e^{-\nu} \left(\frac{\partial \phi}{\partial t}\right)^* \left(\frac{\partial \phi}{\partial t}\right) = e^{-\nu} \omega^2 \sigma^2 , \quad (12)$$

$$V \equiv e^{-\lambda} \left(\frac{\partial \phi}{\partial \rho}\right)^* \left(\frac{\partial \phi}{\partial \rho}\right) = e^{-\lambda} \sigma'^2$$

and rescale:

$$\tilde{\rho} = \rho/m , \quad \tilde{\omega} = \omega/m , \quad \tilde{\phi} = \phi/m , \quad \tilde{U} = U/m^4 , \quad \tilde{W} = W/m^4 , \quad \tilde{V} = V/m^4 . \quad (13)$$

In roughly approximation, gravity becomes important when $R \sim G\mathcal{M}(R)$, where $R$ is defined as the radius, within which the matter field differs from
The radius of a q-star as a function of $\omega_{BD}$ for four different values of $\omega$ in a BD gravitational theory. $\omega$ gives a measure of the gravity strength at the surface, as it is related with the metrics at the surface of the star, through eq. 19.

$E \sim \mathcal{M}(R) \equiv \int_0^R \mathcal{E} d^3\rho$, we find that for a q-star:

$$R \sim \epsilon^{-1}, \quad \epsilon \equiv \sqrt{8\pi G \mathcal{M}},$$

so if we redefine:

$$\tilde{r} = \epsilon \tilde{\rho},$$

we expect $\tilde{r} \sim 1$. We also use a suitable rescaled potential, admitting q-ball type solutions in the absence of gravity, namely:

$$\tilde{U} = |\tilde{\phi}|^2 \left( 1 - |\tilde{\phi}|^2 + \frac{1}{3} |\tilde{\phi}|^4 \right) = \tilde{\sigma}^2 \left( 1 - \tilde{\sigma}^2 + \frac{1}{3} \tilde{\sigma}^4 \right).$$

Dropping form now on the tildes, we will use the Lagrange equation to find an analytical solution for the matter field. $\epsilon$ is a very small quantity for $m$ of the order of some (hundreds) $GeV$, so ignoring the $O(\epsilon)$ terms from the Lagrange equation, we find:

$$\sigma^2 = 1 + \omega B^{1/2}, \quad U = \frac{1}{3} (1 + \omega^3 B^{3/2}).$$

The surface is determined by the star radius. The radius of the solitonic configuration is defined as the radius within which the matter Lagrangian
Figure 2: The value of the scalar field $\sigma$ at the center of a q-star as a function of $\omega_{BD}$ for four different values of $\omega$ in a BD gravitational theory.

Figure 3: The mass of a q-star as a function of $\omega_{BD}$ for four different values of $\omega$ in a BD gravitational theory.
Figure 4: The particle number of a q-star as a function of $\omega_{\text{BD}}$ for four different values of $\omega$ in a BD gravitational theory. From figures [1][4] we see that small $\omega$ means a star with larger radius, mass and particle number.

differs from zero. Outside this radius the matter Lagrangian is zero, when the BD Lagrangian may not necessarily be. The surface width is of order of $m^{-1}$. Within this, the matter field varies very rapidly from a $\sigma$ value at the inner edge of the surface, to zero at the outer, but the metric fields vary very slowly. So, dropping from the Lagrange equation the $O(\epsilon)$ terms we take:

$$\frac{\delta(W - U - V)}{\delta\sigma} = 0 .$$

(17)

The above equation can be straightforward integrated and, because all energy quantities are zero at the outer edge of the surface, the result gives the following equation, holding true only within the surface:

$$V + W - U = 0 .$$

(18)

At the inner edge of the surface $\sigma'$ is zero in order to match the interior with the surface solution. So, at the inner edge of the surface the equality $W = U$ together with eq. (16) gives:

$$\omega = \frac{A_{\text{sur}}^{1/2}}{2} = \frac{B_{\text{sur}}^{-1/2}}{2} .$$

(19)

Eq. (19) is the eigenvalue equation for the frequency of the q-star, revealing the relation between a feature of the star and the spacetime curvature.
Redefining:

\[ \Phi_{BD} = \frac{2\omega_{BD} + 3}{2\omega_{BD} + 4} G\phi_{BD} , \]  

(20)

and dropping the \( O(\epsilon) \) terms, the Lagrange equation for the BD field and the Einstein equations take the form respectively:

\[
A \left[ \frac{d^2 \Phi_{BD}}{dr^2} + \left( \frac{2}{r} + \frac{1}{2A} \frac{dA}{dr} - \frac{1}{2B} \frac{dB}{dr} \right) \frac{d\Phi_{BD}}{dr} \right] = \frac{2W - 4U}{2\omega_{BD} + 4} ,
\]

(21)

\[
A \left( \frac{A - 1}{r^2} + \frac{1}{r} \frac{dA}{dr} \right) = \frac{2\omega_{BD} + 3}{(2\omega_{BD} + 4)\Phi_{BD}} \left( -W - U - \frac{2W - 4U}{2\omega_{BD} + 3} \right) - \frac{\omega_{BD} A}{2\Phi_{BD}^2} \left( \frac{d\Phi_{BD}}{dr} \right)^2 - \frac{A}{2\Phi_{BD} B} \frac{dB}{dr} \frac{d\Phi_{BD}}{dr} ,
\]

(22)

\[
A \left( \frac{A - 1}{r^2} - \frac{A}{B} \frac{dB}{dr} \right) = \frac{2\omega_{BD} + 3}{(2\omega_{BD} + 4)\Phi_{BD}} \left( W - U - \frac{2W - 4U}{2\omega_{BD} + 3} \right) + \frac{\omega_{BD} A}{2\Phi_{BD}^2} \left( \frac{d\Phi_{BD}}{dr} \right)^2 + \frac{A}{\Phi_{BD}} \left( \frac{d^2 \Phi_{BD}}{dr^2} + \frac{1}{2A} \frac{dA}{dr} \frac{d\Phi_{BD}}{dr} \right) ,
\]

(23)

with boundary conditions:

\[
A(0) = 1 , \quad A(\infty) = 1/B(\infty) = 1 , \quad \Phi_{BD}' = 0 , \quad \Phi_{BD}(\infty) = 1 ,
\]

(24)

where the first condition reflects the freedom to define the \( g_{\rho\rho} \) metric at least locally when the second arises from the flatness of the spacetime. One may alternatively, instead of eq. (22), use the relation resulting from the Schwarzschild formula: \( A(\rho) = 1 - \frac{2G M(\rho)}{\rho} \), which can be written with our rescalings as:

\[
A = 1 - \frac{M(r)}{4\pi r} .
\]

(25)

With the new variable eq. (22) takes the form:

\[
\frac{1}{4\pi r^2} \frac{dM}{dr} = \frac{2\omega_{BD} + 3}{(2\omega_{BD} + 4)\Phi_{BD}} \left( -W - U - \frac{2W - 4U}{2\omega_{BD} + 3} \right) - \frac{\omega_{BD} (1 - \frac{M}{4\pi r})}{2\Phi_{BD}^2} \left( \frac{d\Phi_{BD}}{dr} \right)^2 - \frac{1 - \frac{M}{4\pi r}}{2\Phi_{BD} B} \frac{dB}{dr} \frac{d\Phi_{BD}}{dr} ,
\]

(26)

with \( M(0) = 0 \) which reflects the absence of anomalies at the center of the star.
The stability of the star results from a conserved Noether charge. There is a Noether current due to the global $U(1)$ symmetry defined as:

$$j^\mu = \sqrt{-g} g^{\mu\nu} \{ \phi^* \partial_\nu \phi - \phi \partial_\nu \phi^* \}$$  \hspace{1cm} (27)

and a conserved Noether charge defined as:

$$Q = \int d^3x j^0 = 8\pi \int dr r^2 \omega \sigma^2 \sqrt{B/A} .$$  \hspace{1cm} (28)

In our figures $R$ is in $(8\pi G m^4)^{-1/2}$ units, the total mass in $(8\pi G)^{-3} m^{-2}$ units and the charge in $(8\pi G m)^{-3}$ units. The total charge equals to the particle number if every single particle is assigned with a unity “baryon” number. The particle number also equals to the total energy of the free particles as their mass is taken to be unity. So, when the particle number exceeds the total mass, the star decays into free particles as the energetically favorable case. All the field configurations depicted in our figures are stable. An experimental lower limit for $\omega_{BD}$ is 500, [4]. The results obtained in the BD context coincide with general relativity when $\omega_{BD} \rightarrow \infty$. We investigate the phase space of the star with $\omega_{BD}$ varying between 5 and 1000, following the works of [4]-[10], so as to explore thoroughly the influence of $\omega_{BD}$ in the features of the star. As a general result we find that the star parameters, mass, particle number, radius and absolute value of the scalar field at the center of the star, increase when $\omega_{BD}$ decreases.

3 General scalar-tensor theory

In the original BD gravitational theory $\omega_{BD}$ is a constant. In a more general theory it may be regarded as a function, usually of the BD field. We will use one of the forms that Barrow and Parsons [30] investigated in a cosmological framework namely:

$$2\omega_{BD} + 3 = \omega_0 \phi_{BD}^n ,$$  \hspace{1cm} (29)

with $\omega_0$ and $n$ constants. This form for $\omega_{BD}$ gives an analytical solution, [31], for the metrics within the above mentioned cosmological framework. The Lagrange equation for the BD field is:

$$\phi_{BD,\lambda}^{\Lambda} = \frac{1}{2\omega_{BD} + 3} \left( 8\pi T - \frac{d\omega_{BD}}{d\phi_{BD}} \phi_{BD,\rho}^\rho \phi_{BD,\rho}^\rho \right) ,$$  \hspace{1cm} (30)

If we rescale:

$$\tilde{\omega}_0 = \left( \frac{2\omega_{BD} + 3}{2\omega_{BD} + 4} \right)^n G^n \omega_0 ,$$  \hspace{1cm} (31)
and the other quantities as in eqs. and drop the tildes and the $O(\epsilon)$ quantities we take for the Einstein and the Lagrange equation for the BD field:

\[
G^t_t = \frac{\omega_0}{\omega_0 \Phi_{BD} + 1} \left[ -W - U - \frac{1}{\omega_0 \Phi_{BD}} \times \left( 2W - 4U - \frac{A\Phi_{BD}^2 \omega_0 \Phi_{BD} + 1}{2 \Phi_{BD}} \right) \right] \]

\[
= \frac{\omega_0 \Phi_{BD} - 3 A\Phi_{BD}^2}{2 2\Phi_{BD}^2} - \frac{A\Phi_{BD}^2}{2\Phi_{BD} B} - \frac{A\Phi}{2\Phi_{BD} B} , \quad (32)
\]

\[
G^r_r = \frac{\omega_0}{\omega_0 \Phi_{BD} + 1} \left[ W - U - \frac{1}{\omega_0 \Phi_{BD}} \times \left( 2W - 4U - \frac{A\Phi_{BD}^2 \omega_0 \Phi_{BD} + 1}{2 \Phi_{BD}} \right) \right] \]

\[
+ \frac{\omega_0 \Phi_{BD} - 3 A\Phi_{BD}^2}{2 2\Phi_{BD}^2} + \frac{A\Phi_{BD}^2}{2\Phi_{BD}} + \frac{A\Phi}{2\Phi_{BD}} , \quad (33)
\]

\[
A \left[ \Phi_{BD}^r + \left( \frac{2}{r} + \frac{A'}{2A} - \frac{B'}{2B} \right) \Phi_{BD}^r \right] = \frac{1}{\omega_0 \Phi_{BD} + 1} \left[ 2W - 4U - \frac{A\Phi_{BD}^2 \omega_0 \Phi_{BD} + 1}{2 \Phi_{BD}} \right] . \quad (34)
\]

We solved the coupled Einstein and and Lagrange equations for several integral or half-integral values of $n$ and found that the star parameters are rather constant. This owes to the $\Phi_{BD} \sim 1$ relation, because $\omega_0$ and $\omega_{BD}$ are not small enough so as to deviate considerably from the results of General Relativity. When $\omega_0$ decreases the star parameters are larger than in the case of General Relativity, and when $\omega_0 \to \infty$ its results are reproduced.

### 4 Concluding remarks

We investigated q-stars in a BD gravitational theory. We also studied the case of q-stars in the framework of generalized scalar tensor theories, with $\omega_{BD}$ a simple polynomial function of the BD scalar. All the field configurations discussed here are stable with respect to fission into free particles as the ratio of their energy to the energy of the free particles, equal to the mass
Figure 5: The radius of a q-star as a function of $\omega_0$ for four different values of $\omega$ in a scalar-tensor gravitational theory with $n = 1$.

Figure 6: The value of the scalar field $\sigma$ at the center of a q-star as a function of $\omega_0$ for four different values of $\omega$ in a scalar-tensor gravitational theory with $n = 1$. 
Figure 7: The mass of a q-star as a function of $\omega_0$ for four different values of $\omega$ in a scalar-tensor gravitational theory with $n = 1$.

Figure 8: The particle number of a q-star as a function of $\omega_0$ for four different values of $\omega$ in a scalar-tensor gravitational theory with $n = 1$. 
of the free particles times the particle number, is smaller than unity. We investigated their properties, particle number, mass, radius of the matter field configuration and the value of the matter scalar field at the center of the star. The free parameters of their phase space are mainly the eigenfrequency, straightforwardly connected with the surface gravity, and the value of the \( \omega_{\text{BD}} \) or the \( \omega_0 \) for the generalized scalar-tensor theory.

We found that the star parameters, mass, particle number, radius and absolute value at the center, are in generally larger when \( \omega_{\text{BD}} \) or \( \omega_0 \) is small and coincide with the results of general relativity when \( \omega_{\text{BD}}, \omega_0 \to \infty \).

**ACKNOWLEDGMENTS**

I wish to thank N. D. Tracas and E. Papantonopoulos for helpful discussions.

**References**

[1] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

[2] P. J. Bergmann, Int. J. Theor. Phys. **1**, 25 (1968).

[3] R. V. Wagoner, Phys. Rev. **D 1**, 3209 (1970).

[4] M. A. Gunderson and L. G. Jensen, Phys. Rev. **D 48**, 5628 (1993).

[5] J. Balakrishna and Hisa-aki Shinkai, Phys. Rev. **D 58**, 044016 (1998).

[6] A. W. Whinnett, [gr-qc/9711080](http://arxiv.org/abs/gr-qc/9711080)

[7] D. F. Torres, F. E. Schunck and A. R. Liddle, Class. Quant. Grav. **15**, 3701 (1998).

[8] D. F. Torres, Phys. Rev. **D 56**, 3478 (1997).

[9] G. L. Comer and Hisa-aki Shinkai, Class. Quant. Grav. **15**, 669 (1998).

[10] A. W. Whinnett and D. F. Torres, Phys. Rev. **D 60**, 104050 (1999).

[11] D. J. Kaup, Phys. Rev. **172**, 1331 (1968).

[12] R. Ruffini and S. Bonazzola, Phys. Rev. **187**, 1767 (1969).

[13] E. W. Mielke and R. Scherzer, Phys. Rev. **D 24**, 2111 (1981).
[14] M. Colpi, S. L. Shapiro, and I. Wasserman, Phys. Rev. Lett. **57**, 2485 (1986).
[15] Ph. Jetzer and J. J. van der Bij, Phys. Lett. **B 227**, 341 (1989).
[16] R. Friedberg, T. D. Lee, and Y. Pang, Phys. Rev. **D 35**, 3640 (1987).
[17] R. Friedberg, T. D. Lee, and Y. Pang, Phys. Rev. **D 35**, 3658 (1987).
[18] R. Friedberg, T. D. Lee, and Y. Pang, Phys. Rev. **D 35**, 3678 (1987).
[19] S. Coleman, Nucl. Phys. **B 262**, 263 (1985).
[20] A. M. Safian, S. Coleman, and M. Axenides, Nucl. Phys. **B 297**, 498 (1988).
[21] M. Axenides, E. G. Floratos, G. K. Leontaris, and N. D. Tracas, Phys. Lett. **B 447**, 67 (1999).
[22] A. Kusenko, Phys. Lett. **B 405**, 108 (1997).
[23] I. Affleck and M. Dine, Nucl. Phys. **B 249**, 361 (1985).
[24] K. Lee, J. A. Stein-Schabes, R. Watkins, and L. M. Widrow, Phys. Rev. **D 39**, 1665 (1989).
[25] B. W. Lynn, Nucl. Phys. **B 321**, 465 (1989).
[26] S. Bahcall, B. W. Lynn, and S. B. Selipsky, Nucl. Phys. **B 321**, 430 (1989).
[27] S. Bahcall, B. W. Lynn, and S. B. Selipsky, Nucl. Phys. **B 325**, 606 (1989).
[28] A. Prikas, Phys. Rev. **D 66**, 025023 (2002).
[29] A. Prikas, Gen. Rel. Grav. **36**, 1841 (2004).
[30] J. D. Barrow and P. Parsons, Phys. Rev. **D 55**, 1906 (1997).
[31] J. D. Barrow and J. P. Mimoso, Phys. Rev. **D 50**, 3746 (1994).