Universal Seesaw Mass Matrix Model
and SO(10)×SO(10) Unification

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Abstract

On the universal seesaw mass matrix model, which is a promising model of the unified description of the quark and lepton mass matrices, the behaviors of the gauge coupling constants and intermediate energy scales in the SO(10)$_L$ × SO(10)$_R$ model are investigated related to the neutrino mass generation scenarios. The unification of the gauge coupling constants in the framework of the non-SUSY model is possible if the SO(10) symmetry is broken via Pati-Salam type symmetries.

Key words: universal seesaw, evolution, SO(10), quark mass matrix, neutrino mass matrix
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1 Introduction

Recently, considerable interest \cite{1,2,3,4,5} in the universal seesaw mass matrix model \cite{6} has been revived as a unified mass matrix model of the quarks and leptons. Suggested by the seesaw mechanism for neutrinos \cite{7}, the model was proposed in order to understand the question why the masses of quarks (except for top quark) and charged leptons are so small compared with the electroweak scale $\Lambda_L \sim 10^2$ GeV). The model has hypothetical fermions $F_i$ in addition to the conventional quarks and leptons $f_i$ (flavors $f = u, d, \nu, e$; family indices $i = 1, 2, 3$), and those are assigned to $f_L = (2,1)$, $f_R = (1,2)$, $F_L = (1,1)$ and $F_R = (1,1)$ of SU(2)$_L \times$ SU(2)$_R$. The $6 \times 6$ mass matrix which is sandwiched between the fields $(f_L, F_L)$ and $(f_R, F_R)$ is given by

$$M_{6 \times 6} = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix},$$

(1.1)

where $m_L$ and $m_R$ are universal for all fermion sectors ($f = u, d, \nu, e$) and only $M_F$ have structures dependent on the flavors $f$. For $\Lambda_L < \Lambda_R \ll \Lambda_S$, where $\Lambda_L = O(m_L), \Lambda_R = O(m_R)$ and $\Lambda_S = O(M_F)$, the $3 \times 3$ mass matrix $M_f$ for the fermions $f$ is given by the well-known seesaw expression

$$M_f \simeq -m_L M_F^{-1} m_R.$$  

(1.2)

However, after the observation \cite{8} of the heavy top quark mass $m_t \sim \Lambda_L$, the model, at one time, became embarrassed, because the observed fact $m_t \sim O(m_L)$ means $O(M_F^{-1} m_R) \sim 1$. This problem was recently solved by Fusaoka and the author \cite{1}, and later by Morozumi et al. \cite{2}. If we can built a model with $\det M_F = 0$ for the up-quark sector ($F = U$), one of the fermion masses $m(U_i)$ is zero [say, $m(U_3) = 0$], so that the seesaw mechanism does not work for the third family, i.e., the fermions $(u_{3L}, U_{3R})$ and $(u_{3R}, U_{3L})$ acquire masses of $O(m_L)$ and $O(m_R)$, respectively. We identify $(u_{3L}, U_{3R})$ as the top quark $(t_L, t_R)$. Thus, we can understand the question why only the top quark has a mass of the order of $\Lambda_L$. Of course, we can successfully describe \cite{3} the quark masses and mixings in terms of the charged lepton masses by assuming simple structures of $m_L, m_R$ and $M_F$. The model also gives an interesting phenomenology for neutrinos \cite{3}.

In spite of such phenomenological successes, there is a reluctance to recognize the model, because the model needs extra fermions $F$. In most unification models, there are no rooms for the fermions $F$. For example, it has been found \cite{4} that when the gauge symmetries $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_Y \times \text{SU}(3)_c$ are embedded
into the Pati-Salam\[9\] type unification $\text{SO}(10) \rightarrow \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_{PS}$, those gauge coupling constants are unified at $\mu = \Lambda_X \simeq 6 \times 10^{17}$ GeV [$\text{SU}(4)$ is broken into $\text{U}(1)_Y \times \text{SU}(3)_c$ at $\mu = \Lambda_R \simeq 5 \times 10^{12}$ GeV]. However, in the $\text{SO}(10)$ model, there is no representation which offers suitable seats to the fermions $F_{L/R} = (1, 1, 4)_{L/R}$ of $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_{PS}$. Whether we can build a unification model in which the fermions $F$ are reasonably embedded will be a touchstone for the great future of the universal seesaw mass matrix model.

For this problem, there is an idea [10]. We can consider that the fermions $F^c_R \equiv C^T F^{T} R$ together with the fermions $f_L$ belong to $16$ of $\text{SO}(10)$, and also $F^c_L$ together with $f_R$ belong to $16$ of another $\text{SO}(10)$, i.e.,

\begin{equation}
(f_L + F^c_R) \sim (16, 1), \quad (f_R + F^c_L) \sim (1, 16),
\end{equation}

of $\text{SO}(10)_L \times \text{SO}(10)_R$. The symmetries are broken into $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_Y \times \text{SU}(3)_c$ at $\mu = \Lambda_S$ and the fermions $F$ have the mass term $\mathcal{M}_{LM} F R$.

In order to examine the idea (1.3), in the present paper, we investigate the evolution of the gauge coupling constants on the basis of the $\text{SO}(10)_L \times \text{SO}(10)_R$ model and estimate the intermediate energy scales $\Lambda_R$ and $\Lambda_S$ together with the unification energy scale $\Lambda_X$. Of the numerical results, especially, we interest in the value of $\kappa \equiv \Lambda_R/\Lambda_L$, because the value is closely related to the neutrino mass generation scenarios as we discuss in the next section. The evolutions of the gauge coupling constants under $\text{SO}(10)_L \times \text{SO}(10)_R$ symmetries have already been done by Davidson, Wali and Cho[10], but their symmetry breaking patterns are somewhat different from that in the present model. We will investigate the possible intermediate energy scales under a constraint $\Lambda_R/\Lambda_S \simeq 0.02$ [11] which were derived from the observed ratio $m_t/m_c$ in the new scenario of the universal seesaw model[1, 2], where the masses $m_t$ and $m_c$ are given by $m_t \sim \Lambda_L$ and $m_c \sim (\Lambda_R/\Lambda_S) \Lambda_L$, respectively.

In Sec. 3, we investigate the case of the symmetry breaking $\text{SO}(10)_L \times \text{SO}(10)_R \rightarrow [\text{SU}(5) \times \text{U}(1)']_L \times [\text{SU}(5) \times \text{U}(1)']_R$. We will ruled out this case, because the results are inconsistent with the observed values of the gauge coupling constants at $\mu = m_Z$. In Sec. 4, we investigate the case $\text{SO}(10)_L \times \text{SO}(10)_R \rightarrow [\text{SU}(2) \times \text{SU}(2)' \times \text{SU}(4)]_L \times [\text{SU}(2) \times \text{SU}(2)' \times \text{SU}(4)]_R$. We will conclude that the case is allowed for the intermediate energy scale $\Lambda_R \sim (10^1-10^6)$ GeV if we accept a model with $\Lambda_X \neq \Lambda_X$, where $\Lambda_X$ and $\Lambda_X$ are the unification scales of $\text{SO}(10)_L$ and $\text{SO}(10)_R$, respectively. Finally, Sec. 5 will be devoted to the conclusions and remarks.
2 Neutrino mass matrix

In the universal seesaw mass matrix model, the most general form of the neutrino mass matrix which is sandwiched between \((\nu_L, \nu_R, \bar{N}_L, \bar{N}_R)\) and \((\nu'_L, \nu'_R, \bar{N}'_L, \bar{N}'_R)^T\) is given by

\[
M^{12 \times 12} = \begin{pmatrix}
0 & 0 & m'_L & m_L \\
0 & 0 & m'_R & m'_R \\
m'_L & m_R & M_R & M_D \\
m'_R & M'_R & M'_D & M_L
\end{pmatrix}, \quad (2.1)
\]

under the broken SU(2)\(_L\) × SU(2)\(_R\) × U(1)\(_Y\) symmetries. Here, we have denoted the Majorana mass terms of the fermions \(F'_L\) and \(F'_R\) as \(M_R\) and \(M_L\), respectively, because the fermions \(F_L\) and \(F_R\) are members of \((1, 16^*)\) and \((16^*, 1)\) of SO(10)\(_L\) × SO(10)\(_R\), respectively. The mass terms \(\mathcal{F}_L m_L F_R\) and \(\mathcal{F}_R m_R F'_R\) are generated, for example, by the Higgs scalars \((126, 1)\) and \((1, 126^*)\) of SO(10)\(_L\) × SO(10)\(_R\), respectively, while the mass terms \(\mathcal{F}_L m'_L F'_L\) and \(\mathcal{F}_R m'_R F'_R\) must be generated by Higgs scalars of the type \((16, 16^*)\) of SO(10)\(_L\) × SO(10)\(_R\). Therefore, in the present model, we do not consider the terms \(m'_L\) and \(m'_R\), i.e., we take \(m'_L = m'_R = 0\). (For the special case with \(m'_L \simeq m_L\) and \(m'_R \simeq m_R\), see Ref. [11].) Hereafter, we assume \(m_L \ll m_R \ll M_F\).

Our interest is in a mass matrix for the left-handed neutrino states \(\nu_L\). By using the seesaw approximation for the matrix (2.1), we obtain the \(6 \times 6\) mass matrix for approximate \((\nu'_L, \nu'_R)\) states

\[
M^{6 \times 6} \simeq - \begin{pmatrix}
0 & m_L \\
m_R^T & 0
\end{pmatrix}
\begin{pmatrix}
M_R & M_D \\
M_D^T & M_L
\end{pmatrix}^{-1}
\begin{pmatrix}
0 & m_R \\
m_L^T & 0
\end{pmatrix}
= - \begin{pmatrix}
M_L M_{22}^{-1} m_L^T & m_L M_{21}^{-1} m_R \\
m_R^T M_{12}^{-1} m_L^T & m_R^T M_{11}^{-1} m_R
\end{pmatrix}, \quad (2.2)
\]

where

\[
\begin{pmatrix}
M_R & M_D \\
M_D^T & M_L
\end{pmatrix}^{-1} = \begin{pmatrix}
M_{11}^{-1} & M_{12}^{-1} \\
M_{21}^{-1} & M_{22}^{-1}
\end{pmatrix} \quad (2.3)
\]

\[
M_{11} = M_R - M_D M_L^{-1} M_D^T, \\
M_{22} = M_L - M_D^T M_R^{-1} M_D, \quad (2.4)
\]
\[ M_{12} = M_{21}' = M_D^T - M_L M_D^{-1} M_R . \]

According as the cases (a) \( M_L, M_R \gg M_D \), (b) \( M_L, M_R \sim M_D \), and (c) \( M_L, M_R \ll M_D \), we obtain the following mass matrix for the approximate \( \nu_L \) states.

(a) The case \( M_L, M_R \gg M_D \)

From \( M_{11} \sim M_R, M_{22} \sim M_L \) and \( M_{12} \sim M_L M_D^{-1} M_R \), we obtain

\[ M^{6 \times 6} \simeq \begin{pmatrix} -m_L M_L^{-1} m_L^T & m_L M_L^{-1} M_D^T M_R^{-1} m_R \\ m_R M_R^{-1} M_D M_L^{-1} m_R^T & -m_R M_R^{-1} m_R \end{pmatrix} , \tag{2.5} \]

so that we get the mass matrix for approximate \( \nu_L \) states

\[ M(\nu_L) \simeq -m_L M_L^{-1} m_L^T , \tag{2.6} \]

because of \((M^{6 \times 6})_{11}, (M^{6 \times 6})_{22} \gg (M^{6 \times 6})_{12} \).

(b) The case \( M_L, M_R \sim M_D \)

We consider the case

\[ \det \begin{pmatrix} M_R & M_D \\ M_D^T & M_L \end{pmatrix} \neq 0 . \tag{2.7} \]

(The special case that the determinant is zero has been discussed in Ref. [3].)

Since we consider the case \( m_L \ll m_R \), we can use the seesaw approximation for the expression (2.2), so that we obtain

\[ M(\nu_L) \simeq -m_L M_{22}^{-1} m_L^T + m_L M_{21}^{-1} m_R (m_R M_{11}^{-1} m_R)^{-1} m_R M_{12}^{-1} m_L^T \]

\[ = -m_L (M_{22}^{-1} - M_{21}^{-1} M_{11}^{-1} M_{12}^{-1}) m_L = -m_L M_L^{-1} m_L^T , \tag{2.8} \]

where we have used the relation \( M_L = (M_{22}^{-1} - M_{21}^{-1} M_{11}^{-1} M_{12}^{-1})^{-1} \) in the inverse expression of (2.3). Thus, we obtain the expression (2.6) for the case (b), too. Note that the \( 3 \times 3 \) mass matrix for approximate \( \nu_L \) states is almost independent of the structures of \( M_D \) and \( M_R \) in spite of \( O(M_L) \sim O(M_D) \sim O(M_R) \).

(c) The case \( M_L, M_R \ll M_D \)

From \( M_{11} \sim -M_D M_L^{-1} M_D^T, M_{22} \sim -M_D^T M_R^{-1} M_D \) and \( M_{12} \sim M_D^T \), we obtain the mass matrix

\[ M^{6 \times 6} \simeq \begin{pmatrix} m_L M_D^{-1} M_R M_D^{-1} m_L^T & -m_L M_D^{-1} m_R \\ -m_R M_D^{-1} m_L^T & m_R M_D^{-1} M_L M_D^{-1} m_R \end{pmatrix} . \tag{2.9} \]
The mass matrix gives three light pseudo-Dirac neutrino states \[ \nu_{\pm}^{psD} \simeq (\nu_{iL} \pm \nu_{iR}')/\sqrt{2} \] (\(i = e, \mu, \tau\)), because \((M_{6\times6})_{11}, (M_{6\times6})_{22} \ll (M_{6\times6})_{12}\). This case has been discussed by Bowes and Volkas. The case is very attractive phenomenologically, because the maximal mixing state between \(\nu_{\mu L}\) and \(\nu_{\mu R}\) can give a natural explanation for the recent atmospheric neutrino data. The mass matrix \(M(\nu_{\pm}^{psD})\) in the limit of \(m(\nu_{i+}^{psD}) = m(\nu_{i-}^{psD})\) is approximately given by

\[
M(\nu_{\pm}^{psD}) \simeq -m_L M^{-1}_D m_R .
\] (2.10)

First, we suppose the following symmetry breaking pattern (hereafter, we will refer to it as the case A):

\[
\begin{align*}
\text{SO}(10)_L \times \text{SO}(10)_R & \quad \downarrow \quad \mu = \Lambda_{X_{10}} \\
& \quad [\text{SU}(5) \times \text{U}(1)']_L \times [\text{SU}(5) \times \text{U}(1)']_R \\
& \quad \downarrow \quad \mu = \Lambda_N \\
& \quad \text{SU}(5)_L \times \text{SU}(5)_R \\
& \quad \downarrow \quad \mu = \Lambda_{X_5} \\
& \quad [\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)]_L \times [\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)]_R \\
& \quad \downarrow \quad \mu = \Lambda_S \\
& \quad \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_Y \\
& \quad \downarrow \quad \mu = \Lambda_R \\
& \quad \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \\
& \quad \downarrow \quad \mu = \Lambda_L \\
& \quad \text{SU}(3)_c \times \text{U}(1)_{em} .
\end{align*}
\] (2.11)

At the energy scale \(\mu = \Lambda_N\), the gauge symmetries \(U(1)'_L \times U(1)'_R\) are completely broken, so that the neutral leptons \(N_L\) and \(N_R\) acquire Dirac and Majorana masses of the order of \(\Lambda_N\). At \(\mu = \Lambda_S\), the remaining fermions \(F_L\) and \(F_R\) (except for \(U_{3L}\) and \(U_{3R}\)) acquire masses of the order of \(\Lambda_S\) by Higgs bosons \(\Phi\) (as we discuss in the next section), and \(SU(3)_L \times SU(3)_R\) and \(U(1)_L \times U(1)_R\) are broken into \(SU(3)_{L+R} \equiv SU(3)_c\) and \(U(1)_{L+R} \equiv U(1)_Y\), respectively. If this scenario A is true, the mass matrices \(M_L, M_R\) and \(M_D\) are of the order of \(\Lambda_N\), so that we suppose that the order of the neutrino masses \(m(\nu_i)\) are given by

\[
m(\nu_i) \sim \Lambda_L^2/\Lambda_L \sim (\Lambda_L \Lambda_S / \Lambda_R \Lambda_N) m(e_i) ,
\] (2.12)
from the result (2.8) in the case (b), the neutrino masses are suppressed by a factor $\frac{\Lambda_L}{\Lambda_R} (\Lambda_S/\Lambda_N)$ compared with the charged lepton masses $m(e_i)$.

Next, we can suppose another symmetry breaking (case B):

$$\text{SO}(10)_L \times \text{SO}(10)_R$$

$$\text{SU}(2)_L \times \text{SU}(2)'/\times\text{SU}(4)_L \times [\text{SU}(2) \times \text{SU}(2)' \times \text{SU}(4)]_R$$

$\mu = \Lambda_S$

$\text{SU}(2)_L \times \text{SU}(2)_R \times U(1)_Y \times \text{SU}(3)_c$

$\mu = \Lambda_R$

$\text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y$,

$\mu = \Lambda_L$

$\text{SU}(3)_c \times U(1)_{em}$.

(2.13)

If this scenario B is true, since $M_L \sim M_R \sim M_D \sim M_S$, we suppose

$$m(\nu_i) \sim \frac{\Lambda^2_L}{\Lambda_S} \sim (\frac{\Lambda_L}{\Lambda_R}) m(e_i),$$

(2.14)

so that the neutrino masses $m(\nu_i)$ are suppressed by a factor $\frac{\Lambda_L}{\Lambda_R}$ compared with the charged lepton masses $m(e_i)$.

What is of the great interest is to estimate the possible values of such intermediate energy scales $\Lambda_R, \Lambda_S$, and so on.

Although the Bowes-Volkas model\cite{13} is very interesting, the model cannot apply to the universal seesaw model based on the $\text{SO}(10)_L \times \text{SO}(10)_R$ unification, because the case $M_L, M_R \ll M_D$ is not likely in the $\text{SO}(10)_L \times \text{SO}(10)_R$ model, and, if it is possible, the relation (2.10) leads to the wrong prediction $m(\nu_i) \sim m(e_i)$ for $M_D \equiv M_N \sim M_F$ ($F \neq N$).

3 Case of $\text{SO}(10) \rightarrow \text{SU}(5) \times U(1)$

In the present section, we investigate the case A with the symmetry breaking pattern (2.11). At the energy scale $\mu = \Lambda_S$, the symmetries $[\text{SU}(3) \times \text{SU}(2) \times U(1)]_L \times [\text{SU}(3) \times \text{SU}(2) \times U(1)]_R$ are broken into $\text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y$ by the following Higgs scalars $\Phi_Y$:

$$\Phi_{2/3} \sim (3^*; 1; 3, 1)_{Y=2/3},$$
\[\Phi_{4/3} \sim (3, 1; 3^*, 1)_{Y=4/3},\]  
\[\Phi_2 \sim (1, 1; 1, 1)_{Y=-2},\]  
of \([\text{SU}(3) \times \text{SU}(2)]_L \times [\text{SU}(3) \times \text{SU}(2)]_R\), where \(\text{SU}(3)_c \equiv \text{SU}(3)_{L+R}\), \(U(1)_Y \equiv U(1)_{L+R}\), and \(Y = Y_L = Y_R\). Our interest is in the region \(\Lambda_L < \mu \leq \Lambda_{X5}\). Hereafter, we call region \(\Lambda_L < \mu \leq \Lambda_R\), \(\Lambda_R < \mu \leq \Lambda_S\), and \(\Lambda_S < \mu \leq \Lambda_{X5}\) regions I, II, and III, respectively.

The electric charge operator \(Q\) is given by

\[Q = I_3^L + \frac{1}{2}Y'\]  \hspace{1cm} (Region I),  
\[\frac{1}{2}Y' = I_3^R + \frac{1}{2}Y\]  \hspace{1cm} (Region II),  
\[\frac{1}{2}Y = \frac{1}{2}Y_L + \frac{1}{2}Y_R\]  \hspace{1cm} (Region III).

We denote the gauge coupling constants corresponding to the operators \(Q, Y', Y, Y_L, Y_R, I^L,\) and \(I^R\) as \(g_{em} \equiv e, g'_1, g_1, g_{1L}, g_{1R}, g_{2L},\) and \(g_{2R}\), respectively. The boundary conditions for these gauge coupling constants at \(\mu = \Lambda_L, \mu = \Lambda_R,\) and \(\mu = \Lambda_S\) are as follows:

\[\alpha_{em}^{-1}(\Lambda_L) = \alpha_{2L}^{-1}(\Lambda_L) + \frac{5}{3}\alpha_{1}^{-1}(\Lambda_L),\]  
\[\frac{5}{3}\alpha_{1}^{-1}(\Lambda_R) = \alpha_{2R}^{-1}(\Lambda_R) + \frac{2}{3}\alpha_{1}^{-1}(\Lambda_R),\]  
and

\[\frac{2}{3}\alpha_{1}^{-1}(\Lambda_S) = \frac{5}{3}\alpha_{1L}^{-1}(\Lambda_S) + \frac{5}{3}\alpha_{1R}^{-1}(\Lambda_S),\]  
respectively, correspondingly to Eqs. (3.2), (3.3) and (3.4), where \(\alpha_i \equiv g_i^2/4\pi\) and the normalizations of the \(U(1)_Y, U(1)_Y, U(1)_{Y_L}\) and \(U(1)_{Y_R}\) gauge coupling constants have been taken as they satisfy \(\alpha'_1 = \alpha_{2L} = \alpha_3, \alpha_{1L} = \alpha_{2L} = \alpha_{3L},\) and \(\alpha_{1R} = \alpha_{2R} = \alpha_{3R}\) in the SU(5) grand-unification limit and \(\alpha_1 = \alpha_3 \equiv \alpha_4\) in the SU(4) unification limit \([\alpha_4 = \alpha_{2L} = \alpha_{2R}\) in the SO(10) unification limit\], respectively. We also have the following boundary conditions at \(\mu = \Lambda_S\) and \(\mu = \Lambda_{X5}\):

\[\alpha_{3}^{-1}(\Lambda_S) = \alpha_{3L}^{-1}(\Lambda_S) + \alpha_{3R}^{-1}(\Lambda_S),\]  
\[\alpha_{1L}^{-1}(\Lambda_{X5L}) = \alpha_{2L}^{-1}(\Lambda_{X5L}) = \alpha_{3L}^{-1}(\Lambda_{X5L}).\]
Therefore, from the relations (3.7), (3.8) and \( b \), we have also shown the values of \( R \) that of SU(5).

Similarly, we obtain

\[
\alpha^{-1}_L(R) = \alpha^{-1}_R(R) = \alpha^{-1}_3(R),
\]

(3.10)

where, for convenience, we distinguish the unification scale of SU(5)_L, \( \Lambda_{X5L} \), from that of SU(5)_R, \( \Lambda_{X5R} \).

The evolutions of the gauge coupling constants \( g_i \) at one-loop are given by the equations

\[
\frac{d}{dt} \alpha_i(\mu) = -\frac{1}{2\pi} b_i \alpha_i^2(\mu),
\]

(3.11)

where \( t = \ln \mu \). Since the quantum numbers of the fermions \( f \) and \( F \) are assigned as those in Table 4, the coefficients \( b_i \) are given in Table 2. In the model with \( \det M_U = 0 \), the heavy fermions \( F_L \) and \( F_R \) except for \( U_{3L} \) and \( U_{3R} \) are decoupled for \( \mu \leq \Lambda_S \) and the fermions \( u_{3R} \) and \( U_{3L} \) are decoupled for \( \mu \leq \Lambda_R \). In Table 2 we have also shown the values of \( b_i \) for the conventional case without such the constraint \( \det M_U = 0 \) in parentheses.

By substituting \( \alpha^{-1}_2(L) = \alpha^{-1}_3(L) \) with the relations at one-loop

\[
\alpha^{-1}_2(L) = \alpha^{-1}_2(\Lambda_S) + b^{III}_{2L} \frac{1}{2\pi} \ln \frac{\Lambda_{X5L}}{\Lambda_S},
\]

(3.12)

\[
\alpha^{-1}_3(L) = \alpha^{-1}_3(\Lambda_S) + b^{III}_{3L} \frac{1}{2\pi} \ln \frac{\Lambda_{X5L}}{\Lambda_S},
\]

(3.13)

we obtain

\[
\alpha^{-1}_3(\Lambda_S) - \alpha^{-1}_2(\Lambda_S) + (b^{III}_{3L} - b^{III}_{2L}) \frac{1}{2\pi} \ln \frac{\Lambda_{X5L}}{\Lambda_S} = 0.
\]

(3.14)

Similarly, from the condition \( \alpha^{-1}_1(L) = \alpha^{-1}_2(L) \), we obtain

\[
\alpha^{-1}_2(\Lambda_S) - \alpha^{-1}_1(\Lambda_S) + (b^{III}_{2L} - b^{III}_{1L}) \frac{1}{2\pi} \ln \frac{\Lambda_{X5L}}{\Lambda_S} = 0.
\]

(3.15)

By eliminating \( \ln(\Lambda_{X5L}/\Lambda_S) \) from Eqs. (3.14) and (3.15), we obtain

\[
(b^{III}_{2L} - b^{III}_{1L}) \alpha^{-1}_3(\Lambda_S) + (b^{III}_{3L} - b^{III}_{2L}) \alpha^{-1}_1(\Lambda_S) - (b^{III}_{3L} - b^{III}_{1L}) \alpha^{-1}_2(\Lambda_S) = 0.
\]

(3.16)

Similarly, we obtain

\[
(b^{III}_{2R} - b^{III}_{1R}) \alpha^{-1}_3(\Lambda_S) + (b^{III}_{3R} - b^{III}_{2R}) \alpha^{-1}_1(\Lambda_S) - (b^{III}_{3R} - b^{III}_{1R}) \alpha^{-1}_2(\Lambda_S) = 0.
\]

(3.17)

Therefore, from the relations (3.7), (3.8) and \( b^{III}_{1L} = b^{III}_{1R} \equiv b^{III}_1 \), we obtain

\[
(b^{III}_{2} - b^{III}_{1}) \alpha_3^{-1}(\Lambda_S) + (b^{III}_{3} - b^{III}_{2}) \alpha_1^{-1}(\Lambda_S).
\]
The right-hand side of (3.20) gives the value $-\frac{3}{5}(b_3^{III} - b_4^{III}) + (b_3^{III} - b_4^{III}) \alpha_{2R}^{-1}(\Lambda_R) \\ \ (3.18)$

which leads to

$$\left[\frac{3}{5}(b_3^{III} - b_4^{III}) + (b_3^{III} - b_4^{III}) \right] \alpha_{2R}^{-1}(\Lambda_R)$$

$$= (b_3^{III} - b_4^{III}) \alpha_{3}^{-1}(\Lambda_L) + (b_3^{III} - b_4^{III}) \alpha_{3}^{-1}(\Lambda_L) - (b_3^{III} - b_4^{III}) \alpha_{2L}^{-1}(\Lambda_L). \ \ (3.19)$$

For the model with $\det M_U = 0$, the relation (3.19) becomes

$$13\alpha_{2R}^{-1}(\Lambda_R) + \frac{391}{15} \frac{1}{2\pi} \frac{\Lambda_S}{\Lambda_R} - \frac{178}{15} \frac{1}{2\pi} \frac{\Lambda_R}{\Lambda_L} = \frac{127}{15} \alpha_{3}^{-1}(\Lambda_L) + \frac{17}{6} \alpha_{3}^{-1}(\Lambda_L) - \frac{113}{30} \alpha_{2L}^{-1}(\Lambda_L). \ \ (3.20)$$

The right-hand side of (3.20) gives the value $-97.82$ for the input values $\alpha_1(m_Z) = 0.01683$, $\alpha_L(m_Z) = 0.03349$ and $\alpha_3(m_Z) = 0.1189$ [4], where for convenience, we have used the initial values at $\mu = m_Z$ instead of those at $\mu = \Lambda_L$. The relation (3.20) puts a lower bound on the ratio $\Lambda_R/\Lambda_L$: For $\alpha_{2R}^{-1}(\Lambda_R) \geq 1$, we obtain $\Lambda_R/\Lambda_L \geq 2 \times 10^{35}$ (for $\Lambda_S/\Lambda_R = 50$ [4]) and $\Lambda_R/\Lambda_L \geq 3 \times 10^{22}$ (for $\Lambda_S/\Lambda_R \geq 1$). Such a large value of $\Lambda_R/\Lambda_L$ is physically unlikely, so that the case A is ruled out.

By similar discussion to the relation (3.19), it turn out that the conclusion that the case A is ruled out is still unchanged for the model without the condition $\det M_U = 0$ and also for the minimal SUSY version of the present model.

4 Case of $SO(10) \rightarrow SU(2) \times SU(2) \times SU(4)$

Next, we investigate the case B, $SO(10)_L \times SO(10)_R \rightarrow [SU(2) \times SU(2)']_L \times [SU(2) \times SU(2)']_R$. At the energy scale $\mu = \Lambda_S$, the symmetries $[SU(2)']_L \times [SU(2)']_R$ are broken into $U(1)_Y \times SU(3)_c$ by Higgs scalars

$$\Phi_Y \sim (1, 2, 4; 1, 2, 4),$$
$$\Phi_L \sim (1, 1, 10; 1, 1, 1),\ \ (4.1)$$
$$\Phi_R \sim (1, 1, 1; 1, 1, 10),$$
of \([SU(2) \times SU(2)'] \times SU(4)]_L \times [SU(2) \times SU(2)'] \times SU(4)]_R\), where Higgs scalars \(\Phi_V, \Phi_L\) and \(\Phi_R\) generate the masses \(M_F, M_L\) and \(M_R\), respectively. In the present section, we call the regions \(\Lambda_L < \mu \leq \Lambda_R\), \(\Lambda_R < \mu \leq \Lambda_S\), and \(\Lambda_S < \mu \leq \Lambda_X\) regions I, II, and III, respectively.

The electric charge operator \(Q\) is given by Eqs. (3.2) and (3.3) in the regions I and II, respectively, but the relation (3.4) is replaced by

\[
\frac{1}{2} Y = I_3^L + \frac{1}{2} Y_L + I_3^R + \frac{1}{2} Y_R ,
\]

so that the boundary condition (3.7) is replaced by

\[
\frac{2}{3} \alpha_{1}^{-1}(\Lambda_S) = \alpha_{2L}^{-1}(\Lambda_S) + \frac{2}{3} \alpha_{1L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S) + \frac{2}{3} \alpha_{1R}^{-1}(\Lambda_S) .
\]

The boundary conditions at \(\mu = \Lambda_S\) and \(\mu = \Lambda_X\) are as follows:

\[
\alpha_{3}^{-1}(\Lambda_S) = \alpha_{3L}^{-1}(\Lambda_S) + \alpha_{3R}^{-1}(\Lambda_S) ,
\]

\[
\alpha_{1L}^{-1}(\Lambda_S) = \alpha_{3L}^{-1}(\Lambda_S) = \alpha_{4L}^{-1}(\Lambda_S) ,
\]

\[
\alpha_{1R}^{-1}(\Lambda_S) = \alpha_{3R}^{-1}(\Lambda_S) = \alpha_{4R}^{-1}(\Lambda_S) ,
\]

\[
\alpha_{2L}^{-1}(\Lambda_{XL}) = \alpha_{2L}^{-1}(\Lambda_{XL}) = \alpha_{4L}^{-1}(\Lambda_{XL}) ,
\]

\[
\alpha_{2R}^{-1}(\Lambda_{XR}) = \alpha_{2R}^{-1}(\Lambda_{XR}) = \alpha_{4R}^{-1}(\Lambda_{XR}) ,
\]

where, for convenience, we have again distinguished the unification scale of \(SO(10)_L, \Lambda_{XL}\) from that of \(SO(10)_R, \Lambda_{XR}\).

Since \(b_{2L}^{III} = b_{2R}^{III} \equiv b_{2}^{III} \neq b_{2L}^{III} = b_{2R}^{III} \equiv b_{2}^{III}\), we obtain

\[
\alpha_{2L}^{-1}(\Lambda_S) - \alpha_{2L}^{-1}(\Lambda_S) = (b_{2L}^{III} - b_{2L}^{III}) \frac{1}{2\pi} \ln \frac{\Lambda_S}{\Lambda_{XL}} ,
\]

\[
\alpha_{2R}^{-1}(\Lambda_S) - \alpha_{2R}^{-1}(\Lambda_S) = (b_{2R}^{III} - b_{2R}^{III}) \frac{1}{2\pi} \ln \frac{\Lambda_S}{\Lambda_{XR}} ,
\]

i.e.,

\[
\alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S)
\]

\[
= \alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S) + 2(b_{2}^{III} - b_{2}^{III}) \frac{1}{2\pi} \ln \frac{\Lambda_X}{\Lambda_S} ,
\]
where \( \Lambda_X = (\Lambda_{XL} \Lambda_{XR})^{1/2} \). On the other hand, from Eqs.(4.3)-(4.6), we obtain

\[
\alpha_3^{-1}(\Lambda_S) + \frac{3}{2} \left[ \alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S) \right] - \alpha_1^{-1}(\Lambda_S) = 0, \tag{4.12}
\]

so that

\[
\alpha_3^{-1}(\Lambda_S) + \frac{3}{2} \left[ \alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S) \right] - \alpha_1^{-1}(\Lambda_S) + 3(b_2^{II} - b_2^{III}) \frac{1}{2\pi} \ln \frac{\Lambda_X}{\Lambda_S} = 0. \tag{4.13}
\]

Similarly, from Eq.(4.7), we obtain

\[
\alpha_{3L}^{-1}(\Lambda_S) - \alpha_{2L}^{-1}(\Lambda_S) + (b_4^{II} - b_2^{III}) \frac{1}{2\pi} \ln \frac{\Lambda_{XL}}{\Lambda_S} = 0, \tag{4.14}
\]

so that, together with the equation with (L \(\rightarrow\) R) in (4.14), we obtain

\[
\alpha_3^{-1}(\Lambda_S) - \left[ \alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S) \right] + 2(b_4^{II} - b_2^{III}) \frac{1}{2\pi} \ln \frac{\Lambda_X}{\Lambda_S} = 0. \tag{4.15}
\]

By eliminating \( \Lambda_X/\Lambda_R \) from (4.13) and (4.15), we obtain

\[
c_3 \alpha_3^{-1}(\Lambda_S) + c_2 \left[ \alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S) \right] - c_1 \alpha_1^{-1}(\Lambda_S) = 0, \tag{4.16}
\]

where

\[
c_1 = b_4^{II} - b_2^{III}, \tag{4.17}
\]

\[
c_2 = \frac{3}{2} (b_4^{II} - b_2^{III}), \tag{4.18}
\]

\[
c_3 = b_4^{II} - b_2^{III} - \frac{3}{2} (b_2^{II} - b_2^{III}). \tag{4.19}
\]

Since

\[
\alpha_1^{-1}(\Lambda_S) = \frac{5}{2} \alpha_1^{-1}(\Lambda_L) - \frac{3}{2} \alpha_2^{-1}(\Lambda_R) + b_1^{II} \frac{1}{2\pi} \ln \frac{\Lambda_S}{\Lambda_R} + \frac{5}{2} b_1^{II} \frac{1}{2\pi} \ln \frac{\Lambda_R}{\Lambda_L}, \tag{4.20}
\]

\[
\alpha_{2L}^{-1}(\Lambda_S) = \alpha_{2L}^{-1}(\Lambda_L) + b_2^{II} \frac{1}{2\pi} \ln \frac{\Lambda_S}{\Lambda_R} + b_2^{II} \frac{1}{2\pi} \ln \frac{\Lambda_R}{\Lambda_L}, \tag{4.21}
\]

\[
\alpha_{2R}^{-1}(\Lambda_S) = \alpha_{2R}^{-1}(\Lambda_R) + b_2^{II} \frac{1}{2\pi} \ln \frac{\Lambda_S}{\Lambda_R}, \tag{4.22}
\]
\[ \alpha_3^{-1}(\Lambda_S) = \alpha_3^{-1}(\Lambda_L) + b_3^I \frac{1}{2\pi} \ln \frac{\Lambda_S}{\Lambda_R} + b_3^I \frac{1}{2\pi} \ln \frac{\Lambda_R}{\Lambda_L}, \quad (4.23) \]

the relation (4.16) leads to the constraint for \( \Lambda_R/\Lambda_L \):

\[ 0 = \left( c_2 + \frac{3}{2} c_1 \right) \alpha_2^{-1}(\Lambda_R) + (c_3 b_3^I + 2 c_2 b_2^I - c_1 b_1^I) \frac{1}{2\pi} \ln \frac{\Lambda_S}{\Lambda_R} \]

\[ + \left( c_3 b_3^I + c_2 b_2^I - \frac{5}{2} b_1^I \right) \frac{1}{2\pi} \ln \frac{\Lambda_R}{\Lambda_L} + c_3 \alpha_3^{-1}(\Lambda_L) + c_2 \alpha_2^{-1}(\Lambda_L) - \frac{5}{2} \alpha_1^{-1}(\Lambda_L) \]

\[ = 19.5 \alpha_2^{-1}(\Lambda_R) + 19.67 \log \frac{\Lambda_R}{\Lambda_L} + 32.31 \log \frac{\Lambda_S}{\Lambda_R} - 193.96, \quad (4.24) \]

where we have used the values of \( b_i \) given in Table 2 and the same input values of \( \alpha_1^{-1}(\Lambda_L) \), \( \alpha_2^{-1}(\Lambda_L) \) and \( \alpha_3^{-1}(\Lambda_L) \) as those used in (3.20). For \( \Lambda_S/\Lambda_R = 50 \), the relation (4.24) leads to

\[ \log \frac{\Lambda_R}{\Lambda_L} = 7.071 - 0.9915 \alpha_2^{-1}(\Lambda_R), \quad (4.25) \]

so that, for \( \alpha_2^{-1}(\Lambda_R) \geq 1 \), we obtain the constraint

\[ \kappa \equiv \Lambda_R/\Lambda_L \leq 1.20 \times 10^6. \quad (4.26) \]

Similarly, we can obtain the constraint for \( \Lambda_X/\Lambda_S \):

\[ \log \frac{\Lambda_X}{\Lambda_S} = 4.098 + 0.8517 \alpha_2^{-1}(\Lambda_R). \quad (4.27) \]

We show the values of \( \Lambda_R, \Lambda_S \) and \( \Lambda_X \) for the typical values of \( \alpha_2^{-1}(\Lambda_R) \) in Table 3. The values of \( \Lambda_{XL} \) and \( \Lambda_{XR} \) depend not only on the input value of \( \alpha_2^{-1}(\Lambda_R) \) but also on that of \( \alpha_4^{-1}(\Lambda_S) \), because

\[ \alpha_4^{-1}(\Lambda_S) = \alpha_2^{-1}(\Lambda_S) + (b_2^{III} - b_4^{III}) \frac{1}{2\pi} \ln \frac{\Lambda_X}{\Lambda_S} \]

\[ = \frac{1}{2} \left[ \alpha_2^{-1}(\Lambda_S) - \alpha_2^{-1}(\Lambda_S) + \alpha_3^{-1}(\Lambda_S) \right] + (b_4^{III} - b_4^{III}) \frac{1}{2\pi} \ln \frac{\Lambda_X}{\Lambda_X R} \]

\[ = -3.785 - 0.1964 \alpha_2^{-1}(\Lambda_R) + 1.405 \log \frac{\Lambda_X}{\Lambda_X R}, \quad (4.28) \]
i.e.,
\begin{align}
\log \frac{\Lambda_X}{\Lambda_{XR}} &= 2.694 + 0.1398 \alpha^{-1}_{2R}(\Lambda_R) + 0.7118 \alpha^{-1}_{4R}(\Lambda_S), \quad (4.29)
\end{align}
where we have used \(\Lambda_S/\Lambda_R = 50\). For \(\alpha^{-1}_{2R}(\Lambda_R) \geq 1\) and \(\alpha^{-1}_{4R}(\Lambda_S) \geq 1\), the relation (4.29) gives the constraint
\begin{align}
\Lambda_{XL}/\Lambda_{XR} \geq 1.26 \times 10^7. \quad (4.30)
\end{align}
The relation (4.29) concludes that a model with \(\Lambda_{XL} = \Lambda_{XR}\) is ruled out. Values of \(\Lambda_{XR}\) and \(\Lambda_{XL}\) for typical values of \(\alpha^{-1}_{2R}(\Lambda_R)\) and \(\alpha^{-1}_{4R}(\Lambda_S)\) are also listed in Table 3.

Considering the present results \([14]\) of the experimental searches for the right-handed weak bosons, we take \(\kappa \equiv \Lambda_R/\Lambda_L \geq 10\), so that we conclude that the allowed regions of \(\kappa\), the intermediate energy scale \(\Lambda_S\) and the unification scale \(\Lambda_X \equiv (\Lambda_{XL}\Lambda_{XL})^{1/2}\) are
\begin{align}
\kappa &= 1.3 \times 10^1 - 1.2 \times 10^6, \\
\Lambda_S &= (6.0 \times 10^4 - 5.5 \times 10^9) \text{ GeV}, \quad (4.31) \\
\Lambda_X &= (9.8 \times 10^{13} - 4.9 \times 10^{14}) \text{ GeV},
\end{align}
corresponding to the values \(\alpha^{-1}_{2R}(\Lambda_R) = 6 - 1\). Behaviors of the gauge coupling constants for a typical case are illustrated in Fig. 1.

5 Conclusions

In conclusion, in order to examine the idea that the extra fermions \(F_R\) and \(F_L\) in the universal seesaw mass matrix model, together with the conventional three family quarks and leptons \(f_L\) and \(f_R\), are assigned to \((f_L + F^c_L) \sim (16,1)\) and \((f_R + F^c_R) \sim (1,16)\) of \(\text{SO}(10)_L \times \text{SO}(10)_R\), we have investigated the evolution of the gauge coupling constants and intermediate mass scales. The case A, \(\text{SO}(10)_L \times \text{SO}(10)_R \rightarrow [\text{SU}(5) \times \text{U}(1)']_L \times [\text{SU}(5) \times \text{U}(1)']_R\), is ruled out because the results are inconsistent with the observed values of the gauge coupling constants at \(\mu = m_Z\). The case B, \(\text{SO}(10)_L \times \text{SO}(10)_R \rightarrow [\text{SU}(2) \times \text{SU}(2)']_L \times [\text{SU}(2) \times \text{SU}(2)']_R\), is allowed for the intermediate energy scale \(\Lambda_R \sim (10^1 - 10^6) \text{ GeV}\) if we accept a model with \(\Lambda_{XL} \neq \Lambda_{XR}\), where \(\Lambda_{XL}\) and \(\Lambda_{XR}\) are the unification scales of \(\text{SO}(10)_L\) and \(\text{SO}(10)_R\), respectively. We have obtained the allowed regions \(\kappa \simeq
$10^1 - 10^6$, $\Lambda_S \simeq (6 \times 10^4 - 6 \times 10^9)$ GeV, and $\Lambda_X = (\Lambda_{XL}\Lambda_{XR})^{1/2} = (5 \times 10^{14} - 10^{14})$ GeV correspondingly to $\alpha_{2R}^{-1}(\Lambda_R) \simeq 6 - 1$.

In the case B, since $M_L \sim M_R \sim M_N \sim M_F$ ($F \neq N$), the case gives effective neutrino mass matrix $M(\nu) \simeq -m_L M_F^{-1} m_T^T$, so that the conventional neutrino masses $m(\nu_i)$ are of the order of $m(\epsilon_i)/\kappa$. However, for the condition $\alpha_{2R}^{-1}(\Lambda_R) \geq 1$, which is a condition that the model is perturbative, the value of $\kappa$ has been constrained by (4.26), i.e., $\kappa \leq 1.20 \times 10^6$. This suggests that $m(\nu_\tau) \sim m(\tau)/\kappa \geq 10^3$ eV. Such a large value of $m(\nu_\tau)$ is unlikely. Therefore, the straightforward application of the case B to the neutrino mass generation scenario is ruled out.

However, the numerical results in Sec. 4 should not be taken rigidly, because the calculation was done at one-loop. Moreover, the results are dependent on the input value $\Lambda_R/\Lambda_S$. The value $\Lambda_R/\Lambda_S = 0.02$ have been quoted from Ref. [1], where the value was determined from the observed value of $m_\epsilon/m_t$ on the basis of a specific model for $M_L$, $M_R$ and $M_F$. Exactly speaking, the value 0.02 means $y_L v L H R v_R/y_S v_S = 0.02$, where $y$'s and $v$'s are the Yukawa coupling constants and vacuum expectation values, respectively. Because of the numerical uncertainty of $y_L$, $y_R$, and $y_S$, the numerical results may be changed by one or two order. The case B cannot still be ruled out.

In the present paper, the cases for SUSY version of the model have not been investigated systematically, because many versions for the energy scale of the SUSY partners of the super heavy fermions $F$ can be considered. Nevertheless, the case A can easily be ruled out by simple consideration. On the other hand, for the case B, it is a future task whether the SUSY version is allowed or not.

When we take the numerical result of the constraint (4.26), we can consider a minimum modification of the case B. In the case B, the Dirac mass matrix $M_D$ is generated by the Higgs scalar $\Phi_\nu \sim (1, 2, 4; 1, 2, 4)$ of $[SU(2) \times SU(2)' \times SU(4)]_L \times [SU(2) \times SU(2)' \times SU(4)]_R$, while the Majorana mass matrices $M_L$ and $M_R$ are generated by the Higgs scalars $\Phi_L \sim (1, 1, 10; 1, 1, 1)$ and $\Phi_R \sim (1, 1, 1; 1, 1, 10)$, respectively. We assume that the symmetries $SU(4)_L$ and $SU(4)_R$ are broken into $[SU(3) \times U(1)]_L$ and $[SU(3) \times U(1)]_R$ at $\mu = \Lambda_{NL} \equiv O(M_L)$ and $\mu = \Lambda_{NR} \equiv O(M_R)$, respectively, and the energy scales $\Lambda_{NL}$ and $\Lambda_{NR}$ sufficiently larger than $\Lambda_S \equiv O(M_D)$, at which all the fermions $F$ (not $f$) have Dirac masses $M_F$ and the symmetries $SU(3)_L \times SU(3)_R$ and $U(1)_L \times U(1)_R$ are broken into $SU(3)_L \times SU(3)_R$ and $U(1)_L \times U(1)_R \equiv SU(3)_c$ and $U(1)_{L+R} \equiv U(1)_Y$, respectively. Then, the neutrino mass generation scenario is changed from the scenario (b) to the scenario (a). Although the expression of $M_L$ is still given by $M_L \simeq -m_L M_F^{-1} m_T^T$, the suppression factor for neutrino masses is changed from $1/\kappa$ to $(1/\kappa)(\Lambda_S/\Lambda_{NL})$. By taking $\Lambda_S/\Lambda_{NL} \sim 10^{-3}$, we can obtain
reasonable values of the neutrino masses for the case $\alpha_{2R}^{-1}(\Lambda_R) \simeq 1$. Of course, in the modified version with $\Lambda_{XL} \gg \Lambda_{NL} \gg \Lambda_S$, the unification scales of $\Lambda_{XL}$ and $\Lambda_{XR}$ are changed by an order of one or two. However, $\Lambda_R$ and $\Lambda_S$ are insensitive to the present modification.

In the present paper, we have not discussed the evolution of the Yukawa coupling constants. The phenomenological success in Ref.[1] has been obtained by taking $b_e = 0$, $b_u = -1/3$ and $b_d = -e^{i\beta_d} (\beta_d = 18^\circ)$, where $M_F = m_0 \lambda_f \text{diag}(1, 1, 1 + 3b_f)$ in the basis on which $M_F$ is diagonal. The shapes (not the magnitudes) of $M_E = m_0 \lambda_e \text{diag}(1, 1, 1)$ and $M_U = m_0 \lambda_u \text{diag}(1, 1, 0)$ are almost invariant under the evolution, while the shape of $M_D \simeq m_0 \lambda_d \text{diag}(1, 1, -2)$ is not invariant. The following problems remain as our future tasks: (i) what value of $b_d$ is favorable at the unification scale $\mu = \Lambda_X$; (ii) whether we can still assert $\lambda_u \simeq \lambda_d$ or not; (iii) whether the mass matrix $m_R$ can still be approximately diagonal on the basis on which $m_L$ is diagonal; and so on. The numerical results in Ref.[1] will be somewhat changed under the present SO(10)$_L \times$ SO(10)$_R$ model.

In any case, for the universal seesaw mass matrix model based on the SO(10)$_L \times$ SO(10)$_R$ unification, if we consider the symmetry breaking SO(10)$_L \times$ SO(10)$_R \rightarrow [\text{SU}(2) \times \text{SU}(2)' \times \text{SU}(4)]_L \times [\text{SU}(2) \times \text{SU}(2)' \times \text{SU}(4)]_R$ and we accept the case $\Lambda_{XL} \neq \Lambda_R$, where $\Lambda_{XL}$ and $\Lambda_R$ are the unification scales of SO(10)$_L$ and SO(10)$_R$, respectively, we can find a solution of the intermediate energy scales $\Lambda_L$ and $\Lambda_S$ for the unified description of the quark and lepton mass matrices, where only the top quark mass $m_t$ is given by $m_t \sim \Lambda_L$ in contrast with $n_q \ll \Lambda_L$ ($q \neq t$) and the neutrino masses $m(\nu_i)$ are reasonably suppressed compared with the charged lepton masses $m(e_i)$. The model is worth while being taken seriously as a promising unified model of the quarks and leptons.

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Table 1: Quantum numbers of the fermions $f$ and $F$ and Higgs scalars $\phi_L$, $\phi_R$ and $\Phi$ for $SU(2)_L \times SU(2)_R \times U(1)_Y$.

|                | $I_3^L$ | $I_3^R$ | $Y$ | $I_3^L$ | $I_3^R$ | $Y$          |
|----------------|---------|---------|-----|---------|---------|--------------|
| $u_L$          | $\pm \frac{1}{2}$ | 0       | $\frac{1}{3}$ | $u_R$  | $0$       | $\pm \frac{1}{2}$ | $\frac{1}{3}$ |
| $d_L$          | $\pm \frac{1}{2}$ | 0       | $\frac{1}{3}$ | $d_R$  | $0$       | $-\frac{1}{2}$  | $\frac{1}{3}$ |
| $\nu_L$        | $\pm \frac{1}{2}$ | 0       | $-1$ | $\nu_R$ | $0$       | $\pm \frac{1}{2}$ | $-1$         |
| $e_L$          | $\pm \frac{1}{2}$ | 0       | $-1$ | $e_R$   | $0$       | $-\frac{1}{2}$  | $-1$         |
| $U_L$          | 0       | 0       | $\frac{4}{3}$ | $U_R$  | 0         | $\frac{4}{3}$   |             |
| $D_L$          | 0       | 0       | $-\frac{2}{3}$ | $D_R$  | 0         | $-\frac{2}{3}$  |             |
| $N_L$          | 0       | 0       | 0    | $N_R$   | 0         | 0             |             |
| $E_L$          | 0       | 0       | $-2$ | $E_R$   | 0         | $-2$          |             |
| $\phi_L^0$     | $\pm \frac{1}{2}$ | 0       | 1   | $\phi_R^0$ | 0         | $\pm \frac{1}{2}$ | 1            |
| $\phi_L^+$     | $\pm \frac{1}{2}$ | 0       | 1   | $\phi_R^+$ | 0         | $\pm \frac{1}{2}$ | 1            |

Table 2: Coefficients in the evolution equations of gauge coupling constants. The cases A and B are cases with the symmetry breaking patterns $SO(10) \rightarrow SU(5) \times U(1)$ and $SO(10) \rightarrow SU(2) \times SU(2) \times SU(4)$, which are discussed in Secs. 3 and 4, respectively.

|                | $\Lambda_L < \mu \leq \Lambda_R$ | $\Lambda_R < \mu \leq \Lambda_S$ | $\Lambda_S < \mu \leq \Lambda_X$ |
|----------------|----------------------------------|----------------------------------|----------------------------------|
|                | Case A                           | Case B                           |                                  |
| SU(3)$_c$      | $b_3^I = 7$                      | $b_3^{II} = 19/3$ (7)            |                                  |
|                | $\{ b_3^{III}_{3L} = 6 \}$       | $b_3^{III}_{3R} = 6$             |                                  |
|                |                                  | $b_3^{III}_{4L} = 7$             |                                  |
| SU(2)$_L$      | $b_2^{II} = 19/6$               | $b_2^{II} = 19/6$ (19/6)         | $b_2^{III}_{2L} = 19/6$          |
|                |                                  |                                  |                                  |
| SU(2)$_R$      | $b_2^{II} = 19/6$ (19/6)        | $b_2^{II} = 19/6$ (19/6)         | $b_2^{II} = 19/6$ (19/6)        |
|                |                                  |                                  |                                  |
| U(1)$_Y$       | $b_1^I = -41/10$                | $b_1^{II} = -43/6$ (−9/2)        |                                  |
|                | $\{ b_1^{III}_{1L} = -53/10 \}$ | $b_1^{III}_{1R} = -53/10$        |                                  |
|                |                                  | $b_2^{II}_{2L} = -13/6$          |                                  |
|                |                                  | $b_2^{II}_{2R} = -13/6$          |                                  |
Table 3: Intermediate mass scales $\Lambda_R$ and $\Lambda_S$ versus $\alpha^{-1}_{2R}(\Lambda_R)$ in the case of $\text{SO}(10) \to \text{SU}(2) \times \text{SU}(2) \times \text{SU}(4)$. Input values $\Lambda_R/\Lambda_S = 0.02$ and $\Lambda_L = m_Z = 91.2$ GeV are used. The upper and lower rows of $\Lambda_{XR}$ and $\Lambda_{XL}$ correspond to the values for $\alpha^{-1}_{4R}(\Lambda_S) = 1$ and $\alpha^{-1}_{4R}(\Lambda_S) = 2$, respectively.

| $\alpha^{-1}_{2R}(\Lambda_R)$ | 1     | 2     | 4     | 6     |
|-------------------------------|-------|-------|-------|-------|
| $\Lambda_R/\Lambda_L$        | $1.20 \times 10^6$ | $1.23 \times 10^5$ | $1.27 \times 10^3$ | $1.32 \times 10^1$ |
| $\Lambda_R$ [GeV]             | $1.10 \times 10^8$ | $1.12 \times 10^7$ | $1.16 \times 10^5$ | $1.21 \times 10^3$ |
| $\Lambda_S$ [GeV]             | $5.48 \times 10^9$ | $5.59 \times 10^8$ | $5.81 \times 10^6$ | $6.04 \times 10^4$ |
| $\Lambda_X$ [GeV]             | $4.88 \times 10^{14}$ | $3.53 \times 10^{13}$ | $1.86 \times 10^{14}$ | $9.75 \times 10^{13}$ |
| $\Lambda_{XR}$ [GeV]          | $1.39 \times 10^{11}$ | $7.29 \times 10^{10}$ | $2.01 \times 10^{10}$ | $5.54 \times 10^9$ |
|                               | $2.69 \times 10^{10}$ | $1.41 \times 10^{10}$ | $3.90 \times 10^9$ | $1.08 \times 10^9$ |
| $\Lambda_{XL}$ [GeV]          | $1.71 \times 10^{18}$ | $1.71 \times 10^{18}$ | $1.71 \times 10^{18}$ | $1.71 \times 10^{18}$ |
|                               | $8.83 \times 10^{18}$ | $8.83 \times 10^{18}$ | $8.83 \times 10^{18}$ | $8.83 \times 10^{18}$ |
Figure 1: Behaviors of $\alpha_i^{-1}(\mu)$ (dotted line) in $\Lambda_L < \mu \leq \Lambda_R$, $\alpha_1^{-1}(\mu)$ (dotted line) in $\Lambda_R < \mu \leq \Lambda_S$, $\alpha_2^{-1}(\mu)$ (solid line) in $\Lambda_L < \mu \leq \Lambda_{XL}$, $\alpha_2^{-1}(\mu)$ (solid line) in $\Lambda_R < \mu \leq \Lambda_{XR}$, $\alpha_3^{-1}(\mu)$ (dashed line) in $\Lambda_L < \mu \leq \Lambda_S$, $\alpha_2^{-1}(\mu)$ (dotted line) in $\Lambda_S < \mu \leq \Lambda_{XL}$, and $\alpha_1^{-1}(\mu)$ (dotted chain line) in $\Lambda_S < \mu \leq \Lambda_{XR}$, where $\Lambda_L = 91.2$ GeV, $\Lambda_R = 1.10 \times 10^8$ GeV, $\Lambda_S = 5.48 \times 10^9$ GeV, $\Lambda_{XR} = 1.39 \times 10^{11}$ GeV and $\Lambda_{XL} = 1.71 \times 10^{18}$ GeV. The values $\alpha_1^{-1}(\Lambda_L) = 59.42$, $\alpha_2^{-1}(\Lambda_L) = 29.86$, $\alpha_3^{-1}(\Lambda_L) = 8.410$, $\alpha_2^{-1}(\Lambda_R) = 1$ and $\alpha_4^{-1}(\Lambda_S) = 1$ are used as the input values.
