Coastal navigation by a solar sail

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Abstract. In this paper, we study relative motions of a light spacecraft equipped with a solar sail and tethered somehow to a heavy heliocentric orbital space station. One can say that such spacecraft realizes a kind of a coastal navigation. Formulating a model problem of the spacecraft motion, we state that if the sail can be oriented arbitrary then motions along a rectilinear tether are possible in both directions. Also, we present algorithm of the spacecraft relocation if it is joined with the station by a tether. This algorithm is based on the motion equations solutions that can be interpreted as perturbed motions of a simple pendulum.

1. Introduction

Solar sails are very interesting space propulsions that do not require fuel consumption. Unlike some hypothetical devices, solar sails are elements of spacecrafts in the operating and planned space missions [1,2]. Also, there are hundreds of papers in which the sailing spacecrafts dynamics is studied theoretically. For instance, some proposals for the orbit correction by the solar radiation pressure are offered in [3-12], maneuvers by the solar sail in the Formation Flying is studied in [13], interplanetary spaceflights and flights near asteroids are discussed in [14-17], relative motions of sailing spacecrafts are considered in [18,19], etc. Of course, only a minor part of such papers is cited here.

Note, however, that a solar sail of itself cannot create acceleration directed toward the Sun. Certainly, a solar sail could facilitate to decreasing of the orbit perihelion. But trying 'to move tacks' is pointless for a sailing spaceship. The reason is that the Solar system is not filled with liquid or some other viscous substance. Note, nevertheless, that a sea ship keel factually realizes constraint that restricts the ship motion orthogonally to its course. Tethering the spacecraft to a heavy space station by a cable, one can realize the similar constraint in space.

In this paper we study motions of a light spacecraft equipped with a solar sail with respect to a heavy space station that describes a heliocentric orbit. We assume that the spacecraft motion is restricted by tether(s) which one or both ends fixed on the station. In this case, one might say that the spacecraft makes a kind of a coastal navigation in the orbital frame of reference. Firstly, we study motion of the spacecraft along a rectilinear tether which ends are fixed in two points of the station that is considered sufficiently long. We claim that the spacecraft motion is possible in both directions, excepting the case of the tether that is parallel to the Sun rays, when the motion from the Sun is possible only. We deduce a criterion of the sail and the tether optimal inclinations to the Sun rays to maximize the spacecraft acceleration. Secondly, we study motions of the spacecraft joined with some point of the station by a cable. In this case, if the cable is taut then the spacecraft moves along surface of a sphere with center on the station. We construct an algorithm of the spacecraft displacement between two points of this sphere surface with zero initial and finite velocities. The algorithm is based on the fact that some solutions of
the motion equations might be interpreted as perturbed librations of a simple pendulum, where perturbation arises due to the orbital frame of reference non-inertial.

To achieve these purposes we formulate a model problem of the spacecraft motion. This problem is based on estimations of the forces acting on the spacecraft and on a number of the standard assumptions. We assume that the tether is weightless and inextensible, that the station mass is many times greater than the spacecraft mass, that the spacecraft with the solar sail is a particle equipped with a perfect reflecting flat, that can be placed at any angle to the Sun, and so on. Of course, such assumptions might be revised in future researches as more details might be taken into account.

2. Estimating of acting forces

Using the standard physical units, one seems that the solar radiation pressure is very small. But, there are situations in which the force produced by such kind of pressure is the main force that influence a spacecraft motion. Consider, for instance, a space system consisting of a heavy space station and a light spacecraft \( A \) of mass \( m_A \) near it (\( m_A \leq 1000 \text{ kg} \)). Let the spacecraft be equipped with a perfect reflecting flat of area \( S \), \( S \leq 1000 \text{ m}^2 \). Assume that the station mass center \( B \) describes a heliocentric orbit that is close to circular. Let the orbit radius be close to 1AU. Also, let \( Bxyz \) be right-hand Cartesian coordinate, \( Bx \) be directed as the sunlight, and \( By \) be directed as the station velocity with respect to the Sun. Moreover, let the distance between the spacecraft \( A \) and the station mass center \( B \) be less than 1 km. Theoretically, it might be elongated up to 20-30 km.

The forces acting on the spacecraft in the orbital frame of reference \( Bxyz \) are the gravitational force \( F_G \), the force of the Sun radiation pressure \( F_S \), and, also, the force of moving space \( F^e \) and Coriolis’ force \( F^c \) (figure 1). Note that here, at the difference with [20,21], gravitation between the spacecraft and the station is negligible.

![Figure 1. Forces.](image)

One can check easily that for situation under consideration

\[
\frac{|F_S|}{|F_G - F^e|} \sim \frac{S}{m_A} \cdot 2.25 \cdot 10^5, \quad \frac{|F^c|}{|F_S|} \sim \frac{m_A}{S} \sqrt{0.006}, \quad |F_S| = p \frac{S}{m_A} \text{ N},
\]

where \( p \) is the solar radiation pressure. \( p = 9 \cdot 10^{-6} \text{ N/m}^2 \) at 1au from the Sun. In these formulae we assume that \( |F_S| \) has maximum value, i.e. the reflecting flat is orthogonal to the solar rays, and that the spacecraft velocity with respect to the station is less than velocity that is reached after moving 1km with acceleration \( |F_S|/m_A \) that is less than 0.0009 m/s\(^2 \). Let us remark that the spacecraft velocity in \( Bxyz \) will be less in all situations that will be considered below, and, in any case, less than 1 m/s.

Note that the sail material mass is about 7 g/m\(^2 \) presently. So, we can assume \( 1 \text{ m}^2/\text{kg} < S/m_A < 100 \text{ m}^2/\text{kg} \). For instance, \( S/m_A = 6.4 \text{ m}^2/\text{kg} \) for LightSail-2 [1]. Thus, in the case
under consideration, gravitational force is insignificant, and effect of Coriolis’ force is small. Of course, \( F_5 \) is very small objectively, but note that the force of \( 9 \cdot 10^{-3} \text{N} \) can move a mass of \( 1000 \text{ kg} \) by 1km in 4.14 h. (Such situation corresponds to the solar sail of area 1000 m\(^2\)). If such displacement is carried out without any costs, this time does not seem too long.

3. The model problem formulation

Let the spacecraft \( A \) motion be restricted by one or more than one geometrical constraints, for instance, by tether(s) or by some solid guides. (Of course, the last one seems fantastic these days). These constraints can be written as 
\[
\mathcal{F}(x, y, z) \leq 0
\]
(\( x, y, z \) are coordinates of \( A \) with respect to \( x, y, z \)). Given the above, the spacecraft motion equations could be written as
\[
m\ddot{r} = S \mathbf{n} \mathbf{a} + 2m\mathbf{v} + \sum \lambda_i \frac{\partial f_i}{\partial r},
\]
(2)
where \( \mathbf{n} = (n_x, n_y, n_z) \) is a normal to the reflecting flat (\(|\mathbf{n}| = 1\)), \( n_x, n_y, n_z \) are projections of \( \mathbf{n} \) on \( x, y, z \), respectively, \( \mathbf{a} \) is angular velocity of the orbital frame of reference, \( \mathbf{v} = \mathbf{r}' \) is velocity of \( A \) in \( x, y, z \), \( \lambda_i \) are Lagrange’s multipliers. Here we use one of the standard models of a perfect reflecting flat [5,8,15], in which the force of the solar radiation pressure is proportional to \( \mathbf{a} \). Note that if one of the constraints is realized by a tether then this constraint is unilaterial and the correspondent multiplier \( \lambda_i \) is sign-definite. In this case, one can assume without loss of generality that \( \lambda_i \leq 0 \). Note also, that \( \mathbf{a} \neq 0 \) by the problem physics.

Let \( s \) be some fixed distance, for instance, the tether length. Using transformation \( r \rightarrow ar \), one might rewrite (2) as
\[
r'' = n^2 \mathbf{a} - 2 \varepsilon [\mathbf{e}_z, r'] + \sum \mu_i \frac{\partial f_i}{\partial r},
\]
(3)
where \( \mathbf{e}_z \) is a unit vector of \( z \), \( \varepsilon = \sqrt{am/(\mathbf{p}S)} \) is a dimensionless parameter, \( \mu_i \) are Lagrange’s multipliers of the same signs as \( \lambda_i \) respectively, and the prime (\( \cdot \)'\) denotes derivative with respect to dimensionless time \( t = \frac{\sqrt{\mathbf{p}S/(am)}}{m} \).

Note that \( \varepsilon \) is small. Values of \( \varepsilon \) and units of \( t \) for some values of \( S/m_A \) are given in the table 1.

| \( S/m_A \) (\( m^2/\text{kg} \)) | \( \varepsilon \) | \( \tau = 1 \) |
|-------------------------------|--------------------|---------------|
| 1                            | \( 4.21 \cdot 10^{-3} \) | 2.928 h |
| 10                           | \( 1.33 \cdot 10^{-3} \) | 0.926 h |
| 100                          | \( 4.21 \cdot 10^{-4} \) | 0.293 h |

So, to simplify the problem under consideration, one can assume \( \varepsilon = 0 \) and reduce (2) to
\[
r'' = n^2 \mathbf{a},
\]
(4)
4. Motion along a rectilinear tether

Let \( B \) and \( C \) be points of a sufficiently long space station. Moreover, let these points be join by a tether. Suppose that the tether is rectilinear and taut, and that the spacecraft \( A \) can move along this tether (figure 2). Assuming \( \varepsilon = 0 \), one can repute without loss of generality that the tether lays in \( Bx \). In this case (4) is reduced to
\[
\mathbf{s}'' = n^2 (n_x \cos \alpha + n_y \sin \alpha),
\]
(5)
where \( s = BA \) and \( \alpha \) is angle between \( BX \) and \( BC \), \( 0 < \alpha < \pi \). Evidently, right-hand part of (5) can be positive or negative for any \( \alpha \). Hence, the spacecraft can move in both directions. Really, projection of \( \mathbf{F}_S \) on \( BC \) can be directed both from \( B \) to \( C \) and from \( C \) to \( B \). As dimensionless mass of the spacecraft \( A \) can be assumed 1, this projection coincides with the spacecraft acceleration \( \mathbf{a} \). Clearly, \( |\mathbf{a}| = s'' \) if \( s'' \geq 0 \) (this case is depicted in figure 2) and \( |\mathbf{a}| = -s'' \) in the opposite case.

Figure 2. Motions along a rectilinear tether.

Let \( \beta \) be a clockwise angle between \( BX \) and \( n \). One can prove that \( s'' \) reaches its maximum if \( n \) lays in \( Bxy \) and \( \beta \) is the maximum point of \( \Phi(\beta) = \cos^2 \beta \cos(\beta - \alpha) \). This implies that maximum of the spacecraft acceleration directed from \( B \) to \( C \) is reached if

\[
\tan \beta = \frac{\sqrt{9} \cot^2 \alpha + 8 - 3 \cot \alpha}{4}.
\]  

(6)

As it follows from (6), if \( \alpha \to \pi \) then \( s'' \to 0 \), i.e. the ‘sailing close to the wind’ (\( \alpha \) is close to \( \pi \)) is slower than the ‘sailing by the wind’ (\( \alpha > \pi/2 \)) or the ‘sailing wind abeam’ (\( \alpha = \pi/2 \)). In the last case we obtain \( s''_{\text{max}} = 2\sqrt{3}/9 \) for \( \beta = \arctan \sqrt{2}/2 \).

Evidently, if \( \alpha \) is acute then the spacecraft can move from \( B \) to \( C \) even if these points are not tethered, i.e. if the spacecraft moves freely, however, the spacecraft motion along the tether is much faster. The free motion time \( \tau \) as function of \( \alpha \) for \( BC = 1 \) and zero initial velocity is depicted in figure 3 by the dashed curve. This curve is located much higher than the analogous curve for the time of motion along the tether with maximum acceleration (the solid curve in figure 3). Also, the point curve in this figure is the time of motion along the tether, but if the solar sail is placed in accordance with the usual sailing recommendations. We see that the curves have a common point corresponding to the ‘sailing before the wind’ (\( \alpha = 0 \)) in which \( \tau = \sqrt{2} \) and, also, that \( \tau_{\text{min}} = 3^{3/4} = 2.280 \) for \( \alpha = \pi/2 \).

Figure 3. The motion time.
We say that $\alpha$ corresponds to an ‘optimal tack’ if projection of $a$ on the negative ray of $Bx$ is maximum. To find such value of $\alpha$ we need to maximize $G(\alpha, \beta) = -\cos \alpha \cos^2 \beta \cos(\beta - \alpha)$. The maximum point of this function is $\alpha = \pi - \arctan \sqrt{5}$, $\beta = \arctan \sqrt{5}/2$. Substituting this point in $G$, we get 2/27. This is the maximum acceleration with which the sailing spacecraft can move toward the Sun. Also, let us remark that the tether might be non-rectilinear in reality, as in [22].

5. Navigation on the sphere

Now, let the spacecraft and the station be joined by a tether. In this case the spacecraft motion with respect to the station is restricted by some sphere. Denote, for definiteness, by $B$ the center of this sphere. Using, as above, dimensionless variables, one can rewrite (4) as

$$r'' = n_x^2 n + \mu r,$$

where $\mu \leq 0$, $(n, n) = 1$. Let us restrict to a case of the taut tether, i.e. to motions along the sphere surface. Then, using the tether length as dimensionless unit, one can assume $(r, r) = 1$. Combining this equality with (7) we get

$$-\mu = v^2 + n_x^2(n, r) \geq 0,$$

where $v$ is the spacecraft velocity with respect to $Bxyz$. This inequality is called a condition of constrained motion. Moreover, eliminating $\mu$ from (7), we obtain

$$\frac{d}{d \tau} \left( \frac{v^2}{2} \right) = n_x^2(n, r').$$

This implies that if the form $n_x^2(n, dr)$ is integrable then the motion equations (7) have the integral of energy.

5.1. A pendulum motion. Algorithms of relocation between two points of the sphere surface

Let now $n = \text{const}$. In this case the motion equations (7) have two integrals. Let $Bx_1$ be directed as $n$. Then from (9) we get $v^2/2 - n_x^2 n_{x_1} x_1 = h = \text{const}$, where $n_{x_1}$ and $x_1$ are projections of $n$ and $r$ on $Bx_1$ accordingly. Moreover, from (7) we obtain the integral of angular momentum $(r, r', n) = k = \text{const}$. Note that if $k = 0$ then equations (7) coincide in fact with motion equations of the simple pendulum with librations with respect to $Bx_1$.

![Figure 4. The pendulum motion.](image.png)

Let $D(x_D, y_D, z_D)$ and $E(x_E, y_E, z_E)$ be a pair of points on the surface $(r, r) = 1$ (figure 4). Let the spacecraft $A$ be immovable in the point $D$ initially. Moreover, let $Bx_1$ be directed as the angle $\angle DBE$. 
bisector. In this case, if the normal \( \mathbf{n} \) will be directed as \( \mathbf{Bx}_1 \) all the time, then the spacecraft \( \mathbf{A} \) will move from \( D \) to \( E \), and will reach the point \( D \) with zero velocity. Moreover, if one will continue to hold direction of the normal \( \mathbf{n} \) then the spacecraft will return into \( D \). Unfortunately, this simple algorithm of relocation is applicable only if two conditions are fulfilled. Firstly, \( n_x > 0 \) implies \( x_D + x_E > 0 \). Secondly, (8) implies \( \angle DBE \leq \pi \). Particularly, if both points in the pair belong to the semi-sphere \( x \leq 0 \) then the algorithm is inapplicable.

\[ \text{Figure 5. The pendulum motion with an intermediate point.} \]

This problem can be resolved if one use some intermediate point and realize relocation in two steps. Specifically, the pole \( x = 1 \) (\( N \) in figure 5) may be such intermediate point. In other words, if \( x_D + x_E \leq 0 \) then the spacecraft moves initially from \( D \) to the pole \( N \), then from \( N \) to the target point \( D \). Evidently, the both conditions are fulfilled for such process.

Theoretically, the modified algorithm is applicable for any pair of points, excepting the pairs including the pole \( x = -1 \) (\( S \) in figure 5). However, the time of the one stage relocation can be computed by the formulae

\[
T = \frac{2}{n_x} K \left( \sin \frac{\theta_0}{2} \right), \quad \theta_0 = \frac{1}{2} \arccos \left( BD, BE \right),
\]

where \( n_x = (x_D + x_E)/(2 \cos \theta_0) \) and \( K(k) \) is the complete elliptic integral of first kind with modulus \( k \). Evidently, if \( x_D + x_E \to 0 \) then \( T \to \infty \). Hence, even one uses the modified algorithm, some vicinity of the pole \( S \) is not available in fact.

5.2. Refinement of the algorithm
As \( \varepsilon \) is of order of magnitude 0.001 actually, the algorithm from the previous chapter has some inaccuracy since it allows the spacecraft deviation of order of magnitude \( \varepsilon T \) from the goal point. Such deviation can reach up to 50 m if the tether length is about 1km. This might not be acceptable.

To correct the algorithm, one might do the following. Let \( By_1 \) be directed as \( DE \), \( Bx_1y_1z_1 \) be a right-hand Cartesian coordinate system and \( x_1, y_1, z_1 \) are coordinates of the spacecraft \( A \) with respect to this system. Also, denote by \( q_1, q_2, q_3 \) coordinates of the normal \( \mathbf{n} \) with respect to \( Bx_1y_1z_1 \). Let \( z_1 \equiv 0 \) at all time of motion. Then, from (3) implies that

\[
(q_1y_1 + q_2y_2 + q_3y_3)^2 - q_3 = 2\varepsilon (\omega_y y_1' - \omega_y x_1'),
\]

where \( y_2, y_3, y_2 \) are direction cosines of \( Bx \) with respect to \( Bx_1y_1z_1 \); \( \omega_x, \omega_y, \omega_z \) are coordinates of \( \mathbf{e}_z \) with respect to \( Bx_1y_1z_1 \). Let \( x_1 = \cos \theta, y_1 = \sin \theta \). Then one could reduce (3) to
\[ \theta'' = (q_1 y_1 + q_2 y_2 + q_3 y_3)^2 (q_2 \cos \theta - q_1 \sin \theta). \]  
(12)

Assume that \( q_2, q_3 \) are of order of magnitude \( \varepsilon \). Then, combining (11) and the result of integrating (12), and taking into account the terms of order of magnitude \( \varepsilon \) only, we get

\[ q_3 = \frac{2\varepsilon}{y_1} \sqrt{2(\cos \theta - \cos \theta_0)} (\omega_x \cos \theta + \omega_y \sin \theta). \]  
(13)

One might rewrite it as

\[ q_3 = \frac{2\varepsilon z}{y_1} \sqrt{2(x_1 - x_{D1})}, \]  
(14)

where \( x_{D1} \) is coordinate of \( D \) with respect to \( Bx_1 \).

Further, one must take into account that the spacecraft velocity in the goal point \( E \) must be zero. Basing on this requirement and taking into account only terms of order of magnitude \( \varepsilon \), one can express \( q_2 \) from the result of integration of (12) as

\[ q_2 = \frac{2\varepsilon y_3}{y_1^2 \sin \theta_0} \int_{-\theta_0}^{\theta_0} \sin^2 \theta \sqrt{2(\cos \theta - \cos \theta_0)} d\theta, \]  
(15)

or in explicit form

\[ q_2 = \frac{8\varepsilon y_3}{15y_1^2 \sin \theta_0} \left( (2 \cos 2\theta_0 + 7)E \left( \sin \frac{\theta_0}{2} \right) - (2 \cos 2\theta_0 - 8 \cos \theta_0 + 7)K \left( \sin \frac{\theta_0}{2} \right) \right), \]  
(16)

where \( E(k) \) is the complete elliptic integral of second kind with modulus \( k \). Note that unlike \( q_3, q_2 \) is constant. Analyzing (14-16), we see that if the spacecraft moves in the plane of the station orbit \( Bxy \) then \( q_2 = q_3 = 0 \) and correction of the initial algorithm is not required. Also, if \( B\overrightarrow{D}, E\overrightarrow{D} \) and \( Bx \) are coplanar then \( q_2 = 0 \). Evidently, (14-16) are applicable only if \( n_x \) is not small.

One can easily check that the improved algorithm allows the spacecraft deviation of order of magnitude \( \varepsilon^2 T^2 \) from the goal point. For the tether of length 1km this deviation can be estimated as less than 1 meter. It can be considered acceptable for a real situation.

6. Conclusion

In this paper the model problem of constrained motion of a spacecraft with a solar sail near a heavy heliocentric space station is formulated. Factually, a kind of ‘the coastal shipping’ in space is studied. The motion equations of the spacecraft with respect to station are deduced basing on estimations of the acting forces. Some criterion for the optimal placing of the sail is formulated if the spacecraft moves along a rectilinear tether. Also, motions of the spacecraft tethered to the station are considered. In this case some sphere restricts the spacecraft motion. Algorithm of the spacecraft relocation between two points of such sphere surface is offered. This algorithm provides such relocation as a kind of perturbed motions of a simple pendulum.

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