Choosing dy or dx? Students' Preference for constructing integral form on Area-related problem

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Abstract. Students are often found dilemmatic when constructing an integral form of a calculus problem. This is a descriptive-explorative study aiming at investigating college students’ preference for constructing integral form of area-related problem. Data were collected from 63 responses of college students who were taking an Integral Calculus course at a university in Surabaya on an integral problem related to area between curves containing two regions. Data were analysed by categorizing students’ responses regarding the successesfulness of responses and preference regarding symbol notations : upper and lower limit, integrand, and variable of integration (dy or dx). The results showed that for the first region, 71% of students could successful to construct integral form, while 24% of them were unsuccessful, and 5% students did not give any responses. Meanwhile, for the second region, 10% of students could successful to construct integral form, while 78% of students unsuccessful, and 13% students did not give any responses. The preference of students in choosing dy or dx as part of integral notation was explained and the students’ misconceptions that arise in constructing the integral notation were also discussed.

1. Introduction
Calculus is one of the fundamental subjects for mathematics students [1]. More over, the students believed that the calculus support a strong foundation in analytical and computational abilities which is needed in the higher level mathematics[2]. There are two main ideas of first-year calculus that students should understand and need special attention, first of all is derivative and the other is integral [3]. There are two main approaches for introducing integral, the first one by the area under the curve and the second one by approximation [4]. Several authors reported that students often do some misconceptions and shortcomings regarding the definite integral as related area and they do not seem to understand that the concept of limit is the key idea to connects the notions of approximating rectangles and area under the curve[5].

The majority of students studying integral calculus can apply basic procedures to find antiderivatives, but their understanding of the underlying concept is limited[6]. Kiat [7] found that students often suggesting a procedural understanding without any including of the connection between definite integrals and area. They could evaluate the area under the curve, But they do not know why the area
represented by definite integral [8-9]. Therefore students should improve their conceptual understanding to make a connection between definite integral and area.

The purpose of this study is to investigate of students’ preference for constructing integral form on area-related problem. The result and discussion of this study can be a reference for teachers or educators to understand better the students misconception while constructing the integral form of area and preference to choose x-axis or y-axis integration.

2. Research Method
This is a descriptive-explorative research which aims to investigate college students’ preference for constructing integral form of area-related problem. Participants were 63 college students who were taking an Integral Calculus course at a university in Surabaya. Data were collected through an integral-area task. The task used to probe students’ mathematical performance for constructing integral form of the area given (figure 1). In this task, there are two areas, namely the first area or area A and the second area or area B. In this article students are expected to be able to construct an integral form of the two areas to further find solutions to the problem.

Data were analyzed descriptively by categorizing students’ responses regarding the successfulness of responses and preference regarding symbol notations : upper and lower limit, integrand, and variable of integration (dy or dx). The phenomena or misconception of students’ work would be explained based on ground theory.

3. Results and Discussion
Based on the results of the analysis, overall, It was not difficult for students to make integral forms for area A. In detail, it could be seen that 71% of students successesfull to represent the area as an integral form, 24% students unsuccesfull, and 5% did not give any responses (figure 2). In contrast to Area B, There were difficulties experienced by students in constructing the integral form for area B. It was seen that more than half of the students were wrong in constructing the integral form of the area (figure 2). In detail, only 10% of students successesfull to represent the area as integral form, meanwhile 78% students unsuccesfull, and 13% did not give any responses.
Furthermore, in constructing the integral form, in area A all students chose to find the area with respect to the x-axis and none chose the y-axis. In detail, there are 45 students answered correctly, 15 students answered incorrectly and 3 students did not give any responses. On the other hand, in area B, most students still chose the x-axis integral, but the answers were all incorrect and of the 26 students who chose to find the area with respect to the x-axis, only 6 students answered correctly and 20 students answered wrong (Table 1). This was certainly illustrated that students had difficulty attempting integral forms in area B.

**Table 1. Preference of students’ to find the area with respect to x-axis and y-axis**

| Area             | Finding the area with respect to the x-axis | Finding the area with respect to the y-axis |
|------------------|--------------------------------------------|--------------------------------------------|
|                  | Correct | Incorrect | Correct | Incorrect | No response |
| First Region (A) | 45      | 15        | 0       | 0         | 3           |
| Second region (B)| 0       | 29        | 6       | 20        | 8           |

Based on the observations, there were three things that have become mistakes in constructing the integral form. First of all, the selection of dy or dx to find the area, the second was integrant and third was determination of the lower limit and the upper limit. The most common mistake was constructing the integral form for area B using the integral with respect to the x-axis but not appropriate.

Student error when constructing the integral, namely choosing dx in the integration area B (Figure 3). Students assumed that the area of area B was like area A which is constructed by two curves with 0 to p as lower and upper limit respectively. This is certainly not true, because area A has to be divided into two areas if you want to use the integral with respect to the x-axis.
Another student mistake was specifying not appropriate integrand. When students could find that the area of area B is easier to find using y-axis integration because only one integral is involved, students found that it was difficult to determine integrand. A student used C as the boundary curve and p as the upper limit, but no longer understands what C is and what p is and what to do afterward (figure 4).

Students can determine the integral shape of areas A and B correctly when they are able to understand what partitions and curves constrain them to further determine the integral shape on the y-axis by taking the C curve as $y = cx^2$. However, note that the x in the upper limit should be $x_p$, which is the abscissa of point P (figure 5). The integral form could be determined if they had developed an understanding of relationship between area-related problem on integral and riemann sum. In addtition, they should have a good proficiency in algebraic manipulation. For instance, they should understand that for using y-axis integration, they have to change curve $y = cx^2$ to $x = \sqrt{y/c}$ and understand the curve was on the first quadrant.
The result in this paper showed that most students did not have deep understanding of the integral area-problem. They have not been able to properly construct an integral form of an area which was marked by several misconception. This students’ misconception is inline with the previous studies which explains that several misconceptions about ideas of integration in integral calculus arise because of lacking strong understanding of definition of integral[9]. Moreover, students' procedural knowledge was better developed than their conceptual knowledge [6] and their knowledge about why such a procedure used is limited [1]. Besides that, Sealey [10] reveals that the difficulties experienced by students are more related to making layers of the Riemann integral concept as a bridge to the definite integral. Therefore, it is better if in teaching the riemann sums and integral concepts, students should be given meaningful experiences to understand them well. As Grundmeier [9] argues that the instructor's calculus should prioritize conceptual understanding more than procedural understanding so that students are not trapped in routine calculations.

4. Conclusion
The number of students who are successful in constructing integral forms in problem areas A and B is presented in figure 1. Furthermore, students’ preferences in choosing x-axis integration or y-axis integration can be seen in table 1.

We conclude that majority of students in our studies do not have satisfactory conceptual understanding of the concept integral-area problem. They are still a lot skilled at procedural knowledge than conceptual understanding. In fact, conceptual understanding should be strengthened because conceptual understanding will improve procedural abilities and efficiency.
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