Cooling status of three neutron stars

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Abstract. We briefly review a simple method to analyse observations of cooling neutron stars (developed in our previous works) and discuss its applications to XMMU J173203.3–244518, RX J1856.5–3754 and the Vela pulsar.

1. Introduction
The cooling theory of neutron stars (NSs), e.g. [1], is important for exploring superdense NS interiors. An NS cooling rate is regulated by the properties of the superdense matter, such as equation of state and baryon superfluidity (SF), as well as by chemical composition of an outer heat blanketing envelope (HBE). Here, we combine the cooling theory of NSs (including, particularly, the results of [2–5]) with the X-ray spectral analysis. Spectra of several NSs have been studied for a wide range of $M$ and $R$ that allows one to constrain $M$ and $R$. We consider three NSs, XMMU J173203.3–244518 (hereafter XMMU J1732), RX J1856.5–3754 (RX J1856), and PSR B0833–45 (Vela).

2. Analytic description of neutron star cooling
In $\lesssim 100$ yr after an NS birth its interior becomes isothermal except for a thin outer HBE [1]. Then the main cooling equation is $C \ddot{T}/dt = -L_\nu^\infty - L_\gamma^\infty$, where $\dot{T} = T\sqrt{g_{00}}$ is the redshifted internal temperature that is constant over the thermally relaxed stellar interior at any moment $t$; $T$ is a local temperature, $g_{00}$ is the metric tensor component; $C$, $L_\nu^\infty$, and $L_\gamma^\infty$ are the total NS heat capacity, neutrino and photon luminosities, respectively. The superscript ‘$\infty$’ denotes quantities in the restframe of a distant observer.

In a thermally relaxed NS, the main contribution into $C$ and $L_\nu^\infty$ comes from the NS core, with $C(\dot{T}) \propto \dot{T}$ and $L_\nu^\infty(\dot{T}) \propto \dot{T}^n$; typically, $n$ is either 6 or 8. We have approximated [4] these quantities as $L_\nu^\infty = \Lambda(M,R)\dot{T}^n$ and $C = \Sigma(M,R)\dot{T}$, where $\Lambda$ and $\Sigma$ are analytic fit functions of $M$ and $R$. They appear to be the same for a wide class of nucleonic equations of state in NS cores [4]. $L_\nu^\infty$ is the sum of contributions from several neutrino mechanisms in the core, and $C$ is the sum of heat capacities of different particle species ($n$, $p$, $e$ or $\mu$). All these partial contributions have been calculated (neglecting SF) and fitted by the expressions which have similar structure [4]. Below we will mainly consider the case in which the main contribution to $L_\nu^\infty$ comes either from the modified Urca process ($\Lambda = \Lambda_{MU}$) or from neutron-neutron neutrino bremsstrahlung ($\Lambda = \Lambda_{nn}$). The former case is for non-superfluid NSs (the standard cooling), and the latter is for strong proton superfluidity but normal neutrons (the slowest neutrino cooling [6]). In both cases $n = 8$. In what follows, we need $C$ for the same two cases.
of a non-superfluid NS contains the contributions of all particles in the core \((\Sigma = \Sigma_{\text{npe}})\); strong proton SF fully suppresses the contribution of protons \((\Sigma = \Sigma_{\text{n}})\).

The photon luminosity \(L_{\gamma}^\infty = 4\pi\sigma R^2(T^\infty_s)^4 \) (\(\sigma\) being the Stefan-Boltzmann constant and \(R^\infty = R/\sqrt{1 - 2GM/(Rc^2)}\)) can be determined using a relation between the surface temperature \(T_s^\infty\) and \(T\) from the theory of HBEs [7, 8]. If HBE consists of one ion species and the effects of magnetic fields are neglected, \((T^\infty_s)^4 \propto \bar{T}\), where \(\alpha \approx 2\) slightly depends on the HBE composition. For onion-like accreted HBEs and/or magnetized HBEs, this scaling is modified [7].

Assuming \(L_{\gamma}^\infty \propto \bar{T}\), the cooling equation is approximately solved analytically [5],

\[
\tilde{T}(t) \approx (s/q)^{(n-\alpha)} \left[ \left( (\alpha - 2) s^k q^{-\gamma} t + 1 \right)^{k/\gamma} - 1 \right]^{-1/(n-2)},
\]

where \(k = (n-2)/(n-\alpha), \gamma = (\alpha - 2)/(n-\alpha); q = A/\Sigma\) and \(s = L_{\gamma}^\infty / (\Sigma \bar{T})\) are the functions of \(M\) and \(R\). For the iron HBE, \(\alpha \approx 2.2\), and the maximum relative fit error of (1) is \(\delta_{\text{max}} \lesssim 7\%\). For the carbon HBE, \(\alpha \approx 2.3\) and \(\delta_{\text{max}} \lesssim 11\%\). At the neutrino cooling stage (1) reduces to a well-known formula \(\tilde{T} \approx [(n - 2)q]^{-1/(n-2)}\).

The effects of proton or neutron SFs in the core can be described by phenomenological cooling factors (e.g., [5]) \(f_t = q/q_{\text{SC}}\) and \(f_C = C/C_{\text{npe}} = s_{\text{SC}}/s\). Here, \(q_{\text{SC}}\) and \(s_{\text{SC}}\) refer to the case of standard cooling. Strictly speaking, the cooling factors are time-dependent. The factor \(f_C\) is mainly affected by triplet-state SF of neutrons and can vary from \(\lesssim 2\) (if \(\bar{T}\) is just below the maximum critical temperature for neutron SF) to \(\Sigma_{\mu}/\Sigma_{\text{npe}} \approx 0.1\) (if neutron and proton SFs are strong). In the absence of neutron SF, proton SF affects \(f_C\) much weaker, by a factor from \(\sim 0.75\) to \(\sim 1.25\). The factor \(f_t\) can vary from \(\Lambda_{\text{nn}}/\Lambda_{\text{MU}} \approx 0.01\) for strong proton SF and normal neutrons to \(\sim 100\) if triplet-state SF of neutrons strongly enhances \(L_{\gamma}^\infty\). In what follows we will treat \(f_t\) and \(f_C\) as constants.

If strong proton SF greatly suppresses the neutrino cooling, it is convenient to introduce another cooling factor \(f_{\ell p}\) [3] as \(q = f_{\ell p} q_{\text{SC}} + \Lambda_{\text{nn}}/\Sigma_{\text{n}}\). It is better to use \(f_{\ell p}\) instead of \(f_t\) at \(f_{\ell p} \ll 1\) but it is simple to calculate \(f_t\) if \(f_{\ell p}\) is known.

Then we can take \(\bar{T}\) from (1) or its simplification at the neutrino cooling stage, use the appropriate \(q, s\) and \(T_s - \bar{T}\) relation to obtain a star’s cooling curve \(T^\infty_s(t)\).

3. Cooling status of RX J1856, XMMU J1732, and Vela
Detailed analysis of XMMU J1732 is given in [3]; the spectral and cooling analyses of RX J1856 are described in [9] and [5], respectively. The previous spectral analysis of Vela was performed [10, 11] at \(M = 1.4 M_\odot\) and \(R = 10\) km; we have reanalyzed (Zyuzin & Ofengeim, in prep.) these results taking the Vela’s X-ray spectrum from archival Chandra observations (ObsID 127, 131 and 1852). We have employed a magnetic hydrogen (\(n_{\text{max}}\)) atmosphere model from the Sherpa package for a wide range of \(M\) and \(R\) at the distance \(D = 290\) kpc [12]. However there are indications [11] that the Vela’ surface has a small hot region. It cannot be properly described by the atmosphere model we use and the problem deserves further studies.

The derived confidence levels on the \(M - R\) plane for Vela (figure 1a) show that the preferable values of Vela’s \(M\) and \(R\) lie far from the values assumed in [10]. By comparing these confidence contours with the constraints from [13], we conclude that Vela is likely to be a massive NS.

Figure 1b shows \(T^\infty_s(t)\) for all three NSs compared to the standard cooling scenario (black line). XMMU J1732 is much hotter than for the standard cooling, while Vela and RX J1856 are much colder. Other colored curves give examples of possible explanations of the data. Most suitable atmosphere models allow XMMU J1732 to have a carbon HBE and allow RX J1856 to have an iron HBE (but hydrogen atmosphere). Thus the red cooling curve cannot explain
RX J1856 cooling though it hits its error box. Vela is too cold for its age for having a large fraction of accreted matter in its HBE; in addition, it should have enhanced neutrino emission.

Each cooling scenario with constant cooling factors is equivalent to a point on the $f_t - f_C$ diagram (figure 1c). Particularly, each curve on the middle panel is represented by a point of corresponding color. Note that the magenta point that refers to XMMU J1732 lies in the $(f_t, f_C)$-domain which is determined by proton SF without neutron SF. Grey-shaded regions with smooth boundaries are forbidden on some theoretical grounds [5].

We have no access to the results of the RX J1856 spectral analysis. Thus we have analysed its cooling in a simple way. We have determined all those $(f_t, f_C)$ values which allow us to hit the error box on the $T_s^\infty - t$ plane, adopting the best-fit $M$ and $R$ (figure 1c, a double-hatched strip) [5]. It indicates that RX J1856 has neutron SF in the core.

The two other NSs are at the neutrino cooling stage ($L_\gamma^\infty \gg L_\nu^\infty$). Their cooling is independent of $f_C$ that simplifies our study. XMMU J1732 is a slowly cooling NS with strong proton SF and normal neutrons. It is conveniently described by the factor $f_{fp}$. For each pair of $M$ and $R$, we derive $T_s^\infty$ from spectral analysis. Then by employing a carbon HBE model [8] with fixed amount of accreted carbon, one can derive $\tilde{T}$ and $f_{fp}$. Such a plot is given in figure 2. The mass $\Delta M$ of accreted carbon can be parameterized by a maximum density $\rho_C$ near the bottom of the carbon layer. For instance, figure 2 employs $\rho_C = 3 \times 10^9$ g cm$^{-3}$. Since proton SF vanishes at high densities, we take $f_{fp} \geq 1/60$ [3]. Thus, the outer region of the curve $f_{fp} = 1/60$ on figure 2 becomes forbidden. In addition, one has $\rho_C \lesssim 3 \times 10^9$ g cm$^{-3}$ (e.g. [3]). Therefore, the inner area of the $f_{fp} = 1/60$ curve on figure 2 is the widest and possible $M - R$ values are located at the intersection of this domain and the confidence levels. Thus the cooling theory sets additional constraints on $M$ and $R$ of XMMU J1732.

Similar analysis has been done for Vela to constrain $f_t$ using the ‘onion’ model of the HBE [7] with the accreted mass $\Delta M$. Figure 3 shows our results for $\Delta M = 10^{-14}$ $M_\odot$. Since Cooper pairing neutrino emission can provide $f_t < 100$ only, the inner region of the thick black curve on

**Figure 1.** (a) $M - R$ confidence levels for the Vela (blue), XMMU J1732 (red), and RX J1856 (1σ, green). Solid, dashed and dotted lines are for the 90%, 68%, and 50% confidence levels, respectively. Crosses mark the best-fits. The greyed shaded area is the area of $M$ and $R$ that was claimed to be the 90% confidence area in [13]. (b) Cooling curves $T_s^\infty(t)$; the black line shows the standard candle cooling with the iron HBE; the red curve shows how to hit the XMMU J1732 error box with fully carbon HBE and strong proton SF in the core; the blue and green curves illustrate different possibilities to hit the error box for RX J1856 assuming the iron HBE (the green curve is for the slow neutrino emission and the blue one is for the enhanced one). The blue curve also hits the Vela’s error box. Corresponding values of $f_t$ or $f_{fp}$ and $f_C$ are presented next to the curves. (c) $f_t - f_C$ diagram. The colored points correspond to the cooling curves plotted in the middle panel. The singly hatched area is a schematic ($f_t - f_C$) domain for NSs with normal neutrons and partially SF protons. The double-hatched strip is the restriction on RX J1856 cooling though it hits its error box.
Figure 2. The map of $f_{\ell\rho}$ over the $M - R$ plane for XMMU J1732 with $\rho_C = 3 \times 10^9$ g cm$^{-3}$. Thin black lines are the same confidence contours as the red ones on figure 1a. Thick black line shows $f_{\ell\rho} = 1/60$ that we treat as the minimum value. The intersection of the 68% confidence region and the $f_{\ell\rho} > 1/60$ domain is grey-shaded. White lines show levels of constant $f_{\ell\rho} = 0.005, 0.010, \ldots, 0.025$.

Figure 3. The map $f_{\ell}$ over the $M - R$ plane for the Vela pulsar with $\Delta M = 10^{-14} M_\odot$. Thin black lines are the same confidence contours as the blue ones on figure 1a. Thick black lines show $f_{\ell} = 100$ which we treat as the maximum value. The grey-shaded area in the white-dashed contour is the $M - R$ area which was considered as the 90% confidence level in [13]. White lines show levels of constant $\log f_{\ell} = 1.4, 1.6, \ldots, 2.2$.

If $\Delta M$ increases, the forbidden area increases too; $\Delta M = 3 \times 10^{-13} M_\odot$ can prohibit all intersections of the Vela confidence contours and the constraints from [13]. Thus, $\Delta M$ in Vela’s HBE does not exceed this value.

4. Conclusions
We have outlined a method to analyse cooling NSs. The method is based on simple analytic approximations valid for a wide class of nucleonic equations of state. Introducing the phenomenological cooling factors, we have made this analysis free of any specific SF model. The method has been applied to the three cooling NSs, XMMU J1732, RX J1856, and the Vela pulsar. The Vela’s spectrum has been reanalysed.

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