Cognitive MAC Protocols for General Primary Network Models

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Abstract—We consider the design of cognitive Medium Access Control (MAC) protocols enabling a secondary (unlicensed) transmitter-receiver pair to communicate over the idle periods of a set of primary (licensed) channels. More specifically, we propose cognitive MAC protocols optimized for both slotted and un-slotted primary networks. For the slotted structure, the objective is to maximize the secondary throughput while maintaining synchronization between the secondary pair and not causing interference to the primary network. Our investigations differentiate between two sensing scenarios. In the first, the secondary transmitter is capable of sensing all the primary channels, whereas it senses only a subset of the primary channels in the second scenario. In both cases, we propose blind MAC protocols that efficiently learn the statistics of the primary traffic on-line and asymptotically achieve the throughput obtained when prior knowledge of primary traffic statistics is available. For the un-slotted structure, the objective is to maximize the secondary throughput while satisfying an interference constraint on the primary network. Sensing-dependent periods are optimized for each primary channel yielding a MAC protocol which outperforms previously proposed techniques that rely on a single sensing period optimization.

Index Terms—Cognitive radios, Spectrum sensing, MAC protocols, Bandit problems, Whittle’s index

1 INTRODUCTION

The radio spectrum resource is of fundamental importance to wireless communication. Recent reports show that most available spectrum has been allocated. However, most of licensed spectrum resources are under-utilized. This observation has encouraged the emergence of dynamic and opportunistic spectrum access concepts, where secondary (unlicensed) users (SU) equipped with cognitive radios are allowed to opportunistically access the spectrum as long as they do not interfere with primary (licensed) users (PU). To achieve this goal, the secondary users must monitor the primary traffic in order to identify spectrum holes or opportunities which can be exploited to transfer data [1].

The main goal of a cognitive MAC protocol is to sense the radio spectrum, detect the occupancy state of different primary channels, and then opportunistically communicate over unused channels (spectrum holes). Specifically, the cognitive MAC protocol should continuously make efficient decisions on which channels to sense and access in order to obtain the most benefit from the available spectrum opportunities. Previous work on the design of cognitive MAC protocols has considered two distinct scenarios. In the first, the primary network is slotted (e.g., [2], [4], [5], [7], [8] and [13]) whereas a continuous structure (un-slotted) of the primary channels is adopted in the second set of works (e.g., [3], [10], [11] and [14]). In this work we propose decentralized cognitive MAC protocols for each of the two models.

For the slotted structure, two cases are considered. The first assumes that the secondary transmitter can sense all the available primary channels before making the decision on which one to access. The secondary receiver, however, does not participate in the sensing process and waits to decode on only one channel. This is the model adopted in [4] and is intended to limit the decoding complexity needed by the secondary receiver. In the sequel, we propose an efficient algorithm that optimizes the on-line learning capabilities of the secondary transmitter and ensures perfect synchronization between the secondary pair. The proposed protocol does not assume a separate control channel, and hence, piggybacks the synchronization information on the same data packet.

In the second scenario, the secondary transmitter can only sense a subset of the available primary channels at the beginning of each time slot. This model was studied in [5] where the optimal algorithm was obtained by formulating the problem as a Partially Observable Markov Decision Process (POMDP). Unfortunately, finding a computationally efficient version of the optimal solution for this problem remains an elusive task. By re-casting the problem as a restless multi-armed bandit problem, a near-optimal index policy was proposed in [7]. The authors of [5] and [7], however, assumed that the primary traffic statistics (i.e., Markov chain transition probabilities) were available a-priori to the secondary users. Here, we develop blind MAC protocols where the protocol must learn the transition probabilities on-line. This can be viewed as the Whittle index strategy of [7] augmented with a similar learning phase to the one proposed in [8] for the multi-armed bandit scenario. Our numerical results show that the performance of this protocol converges to that of the Whittle index strategy.
with known transition probabilities \[7\].

Under the un-slotted primary network set-up, we first assume that the SU radio can be tuned to any combination of the primary channels at the same time. This can be achieved by an Orthogonal Frequency Division Multiplexing (OFDM) technique with adaptive and selective allocation of OFDM subcarriers to utilize any subset of licensed channels at the same time. The SU aims at maximizing its throughput (i.e., maximizing the opportunities discovered and accessed in all primary channels) while imposing minimal interference to the primary network. A similar model was adopted in [3], where the authors developed an optimal sensing period for each of the primary channels by optimizing the tradeoff between the sensing overhead resulting from frequent sensing of the channels and the missed opportunities in the primary channels due to infrequent sensing. However, it was assumed that if a primary transmission is resumed on a channel, the SU will discover this return, via the help of a Genie, and immediately evacuate the channel, thereby causing no interference to the primary transmissions. In this work, we relax this Genie-aided assumption and impose an interference/outage constraint on each primary channel. In [11], an optimal sensing period satisfying a primary network interference constraint was developed. However, the approximations made in [11] deviate considerably from the true values. More importantly, we show that by introducing two different sensing periods, a period if the channel is sensed free and a different period if the channel is sensed busy, the performance can be substantially improved. In particular, this performance improvement becomes more significant when there is a large difference between the expected time a primary channel is busy and the expected time it remains idle. Finally, we consider the scenario when the SU radio can be tuned to only one channel. A SU in this case shall try to access a primary channel as long as it is free. When this channel switches to busy, the SU shall search other primary channels until a free channel is identified. A similar model was adopted in [10], where an optimal sequence of primary channels to be sensed was proposed. This optimal sequence aimed at minimizing the average delay in finding the primary channels at the same time. This can be achieved by an Orthogonal Frequency Division Multiplexing (OFDM) technique with adaptive and selective allocation of OFDM subcarriers to utilize any subset of licensed channels at the same time. The SU aims at maximizing its throughput (i.e., maximizing the opportunities discovered and accessed in all primary channels) while imposing minimal interference to the primary network. A similar model was adopted in [3], where the authors developed an optimal sensing period for each of the primary channels by optimizing the tradeoff between the sensing overhead resulting from frequent sensing of the channels and the missed opportunities in the primary channels due to infrequent sensing. However, it was assumed that if a primary transmission is resumed on a channel, the SU will discover this return, via the help of a Genie, and immediately evacuate the channel, thereby causing no interference to the primary transmissions. In this work, we relax this Genie-aided assumption and impose an interference/outage constraint on each primary channel. In [11], an optimal sensing period satisfying a primary network interference constraint was developed. However, the approximations made in [11] deviate considerably from the true values. More importantly, we show that by introducing two different sensing periods, a period if the channel is sensed free and a different period if the channel is sensed busy, the performance can be substantially improved. In particular, this performance improvement becomes more significant when there is a large difference between the expected time a primary channel is busy and the expected time it remains idle. Finally, we consider the scenario when the SU radio can be tuned to only one channel. A SU in this case shall try to access a primary channel as long as it is free. When this channel switches to busy, the SU shall search other primary channels until a free channel is identified. A similar model was adopted in [10], where an optimal sequence of primary channels to be sensed was proposed. This optimal sequence aimed at minimizing the average delay in finding a free channel. Here, we extend this work by finding the period a free channel shall be accessed in order satisfy an interference/outage constraint on the primary network, which was not considered in [10].

The rest of the paper is organized as follows. Section 2 presents our modeling assumptions. The proposed cognitive MAC protocols for slotted primary networks are developed in Section 3, whereas the un-slotted scenario is investigated in Section 4. Numerical results for our proposed strategies are reported in Section 5. Finally, Section 6 summarizes our conclusions.

## 2 Network Model

### 2.1 Primary Network

We consider a primary network consisting of \(N\) independent channels. The presence or absence of primary users in each channel can be modeled as alternating time intervals of busy and free states with random durations. For channel

\[ i \in 1, 2, \ldots, N, \]

we model the sojourn time of a busy period as a random variable \(T_i^b\) with the probability density function (p.d.f.) \(f_{T_i^b}(y), y > 0\). Similarly, the p.d.f. of the sojourn time in a free period is given as \(f_{T_i^f}(x), x > 0\). Busy and free periods are assumed to be independent and identically distributed (i.i.d.). We also assume that busy and free periods are independent of each other. The state of channel \(i\) at time \(t\), \(S_i(t)\), is equal to 1 if the channel is free and 0 if busy.

### 2.2 Secondary Pair

The SU can sense any of the primary channels in order to identify the presence of a PU. After sensing the channel, the SU applies the channel access strategies as described in Sections 3 and 4. The performance of the sensing stage is limited by two types of errors. If the secondary transmitter decides to classify a busy channel as busy, a miss detection occurs resulting in interference with primary user. The probability of miss-detection is denoted by \(P_{MD}\). If energy detection is used as a sensing method [12], the minimum required sensing time \(T_s\) that satisfies a certain desired \(P_{FA}\) and \(P_{MD}\) is given by:

\[
T_s = \frac{2}{f_s}[Q^{-1}(P_{FA}) - Q^{-1}(1 - P_{MD})\sqrt{1 + 2\sigma}^2 \sigma^{-2}]
\]

Where, \(f_s\) is the sampling frequency, \(Q(x)\) is the tail probability of a zero-mean unit-variance Gaussian random variable and \(\sigma\) is the PU signal-to-noise ratio [12].

## 3 Slotted Primary Network

In this section, we consider the case of discrete probability distributions for the free and busy periods. We model the duration of these intervals (in terms of the number of time slots they occupy) as geometrically distributed random variables. From the memoryless property of the geometric distribution, we can model the primary users’ traffic in each channel by the two state Markov chain depicted in Figure 1. The channel state transition matrix of the Markov chain is given by

\[
P_i = \begin{bmatrix}
P_{10}^i & P_{11}^i \\
P_{00}^i & P_{01}^i
\end{bmatrix}
\]

We assume that \(P_i\) remains fixed for a block of \(T\) time slots. We use \(j\) to refer to the time-slot index \(j \in \{1, \ldots, T\}\).
We assume that the secondary transmitter can sense $L$ channels ($L \leq N$) and can transmit on only one channel in each slot if the channel it chooses to access is sensed to be free. In this section, $S_i(j)$ denotes the state of channel $i$ at time slot $j$ as sensed by the transmitter, which might not be the actual channel state $S_i(j)$. The secondary receiver does not participate in channel sensing and is assumed to be capable of accessing only one channel [4]. This assumption is intended to limit the decoding complexity needed by the secondary receiver. Another motivation behind restricting channel sensing to the transmitter is the potentially different sensing outcomes at the secondary transmitter and receiver due to the spatial diversity of the primary traffic which can lead to the breakdown of the secondary transmitter-receiver synchronization. Overall, successful communication between the secondary transmitter and receiver occurs only when: 1) they both decide to access the same channel, and 2) the channel is sensed to be free and is actually free from primary transmissions.

Our proposed cognitive MAC protocol can be decomposed into the following stages:

- **Decision stage**: The secondary transmitter decides which $L$ channels to sense. Also, both transmitter and receiver decide which channel to access.
- **Sensing stage**: The transmitter senses the $L$ selected primary channels.
- **Learning stage**: Based on the sensing results from the sensing stage, the transmitter updates the estimated primary channels’ probability transition matrix $P_i$.
- **Access stage**: If the access channel is sensed to be free, a data packet is transmitted to the secondary receiver. This packet contains the information needed to sustain synchronization between secondary terminals, and hence, synchronization does not require a dedicated control channel. The length of the packet is assumed to be large enough such that the loss of throughput resulting from the synchronization overhead is marginal.
- **ACK stage**: The receiver sends an ACK to the transmitter upon successful reception of sent data.

### 3.1 Full Sensing Capability: $L = N$

In this subsection we assume that the secondary transmitter can sense all $N$ primary channels at the beginning of each time slot. Since the receiver doesn’t participate in sensing, and in order to sustain the transmitter-receiver synchronization, the secondary pair must share the same variables which are to be used to decide upon the channel to be accessed. We refer to $\Omega(j) = [\omega_1(j), \ldots, \omega_N(j)]$ as the belief vector at slot $j$, where $\omega_i(j)$ is the probability that $S_i(j) = 1$. Given the sensing outcomes in slot $j$, the belief state in slot $j+1$ can be obtained recursively as follows:

$$\omega_i(j+1) = \begin{cases} P_{i1}^1 & \text{if } S_i(j) = 0 \\ P_{i1}^1 & \text{if } S_i(j) = 1 \\ \omega_i(j)P_{i1}^1 + (1 - \omega_i(j))P_{i0}^1 & \text{if } i \text{ not sensed at } j \end{cases}$$

Due to the different sensing roles between the secondary transmitter and receiver, we introduce the vector $\Omega(j)$ as the common or shared belief vector between the secondary transmitter and receiver. The initial packet sent to the receiver includes estimates for the transition probabilities, and $\Omega(1)$. Once the initial communication is established, the secondary transmitter and receiver implement the same spectrum access strategy described below for $j \geq 1$.

1) **Decision**: At the beginning of time slot $j$, and using belief vector $\Omega(j)$, the secondary transmitter and receiver decide to access channel:

$$i^*(j) = \arg \max_{i=1, \ldots, N} [\hat{\omega}_i(j)B_i]$$

2) **Sensing**: The secondary transmitter senses all channels and captures the sensing vector $\Phi(j) = [\hat{S}_1(j), \ldots, \hat{S}_N(j)]$, where $\hat{S}_i(j) = 1$ if the $i$th channel is sensed to be free, and $\hat{S}_i(j) = 0$ if it is found busy. $\hat{S}_i(j)$ might be different than $S_i(j)$ due to sensing errors. Note that the decision stage precedes the sensing stage in order to maintain the synchronization between the secondary terminals.

3) **Learning**: Based on the sensing results, the transmitter updates the estimates $P_{i1}^0$ and $P_{i1}^1$ for all primary channels as explained below.

4) **Access**: If $\hat{S}_i^*(j) = 1$, the transmitter sends its data packet to the receiver. The packet includes $\Phi(j)$, $P_{i1}^0$ and $P_{i1}^1$. In addition, if the transmission at slot $j - 1$ has failed, the transmitter sends $\Omega(j)$, which is the belief vector computed at the transmitter based on its observations. If the receiver successfully receives the packet, it sends an ACK back to the transmitter. Parameter $K_i^*(j)$ is equal to unity if an ACK is received by the transmitter, and zero otherwise. If the channel is free, the forward transmission and the feedback channel are assumed to be error-free.

5) **ACK**: Finally, the transmitter and receiver update the common belief vector $\Omega(j+1)$ such that:

If $K_i^*(j) = 1$:

$$\hat{\omega}_i(j+1) = \begin{cases} P_{i1}^1 & \text{if } i = i^*(j) \\ \hat{\omega}_i(j)P_{i1}^1 + (1 - \hat{\omega}_i(j))P_{i0}^1 & \text{if } i \neq i^*(j), \hat{S}_i(j) = 1 \\ \mathcal{C}_iP_{i1}^1 + (1 - \mathcal{C}_i)P_{i0}^1 & \text{if } i \neq i^*(j), \hat{S}_i(j) = 0 \end{cases}$$

If $K_i^*(j) = 0$:

$$\hat{\omega}_i(j+1) = \begin{cases} \hat{\omega}_i(j)P_{i1}^1 + (1 - \hat{\omega}_i(j))P_{i0}^1 & \text{if } i = i^*(j) \\ \hat{\omega}_i(j)P_{i1}^1 + (1 - \hat{\omega}_i(j))P_{i0}^1 & \text{if } i \neq i^*(j) \end{cases}$$

where:

$$\mathcal{A}_i = P_T(S_i(j) = 1|\hat{S}_i(j) = 1) = \frac{(1 - P_{FA})\hat{\omega}_i(j)}{(1 - P_{FA}\hat{\omega}_i(j) + P_{MD}(1 - \hat{\omega}_i(j)))}$$

$$\mathcal{C}_i = P_T(S_i(j) = 1|\hat{S}_i(j) = 0) = \frac{P_{FA}\hat{\omega}_i(j)}{P_{FA}\hat{\omega}_i(j) + (1 - P_{MD})(1 - \hat{\omega}_i(j))}$$

$$\mathcal{D}_i = P_T(S_i^*(j) = 1|K_i^*(j) = 0) = \frac{P_{FA}\hat{\omega}_i(j)}{P_{FA}\hat{\omega}_i(j) + (1 - \hat{\omega}_i(j))}$$
$P_{i01}$ and $P_{i11}$ are the most recent shared estimates of $i$th channel transition probabilities. Obviously, in case of perfect sensing, $A_i = 1$, $C_i = 0$ and $D_i = 0$.

In addition, the transmitter computes another belief vector, $\Omega(j+1)$, based on its observations:

If $K_{i*}(j) = 1$, $\omega_i(j+1) = \bar{\omega}_i(j+1)$

If $K_{i*}(j) = 0$:

$$\omega_i(j+1) = \begin{cases} A_i P_{i11} + (1 - A_i) P_{i01} & \text{if } i \neq i^*(j), \bar{S}_i(j) = 1 \\ C_i P_{i11} + (1 - C_i) P_{i01} & \text{if } i \neq i^*(j), \bar{S}_i(j) = 0 \\ D_i P_{i11} + (1 - D_i) P_{i01} & \text{if } i = i^*(j) \end{cases} \quad (9)$$

where $A_i$, $C_i$, and $D_i$ are the same as $\bar{A}_i$, $\bar{C}_i$ and $\bar{D}_i$ with $\bar{\omega}_i(j)$ replaced by $\omega_i(j)$. Note that $\Omega(1) = \Omega(1)$, and $\Omega(j+1)$ differs from $\Omega(j)$ only when $K_{i*}(j) = 0$. If transmission succeeds at the $j$th time slot after one or more failures, the transmitter and receiver set $\bar{\Omega}(j) = \Omega(j)$ before computing $\bar{\Omega}(j + 1)$.

It is noted that although $\Omega_i(j)$ is the updated belief vector which is available to the transmitter, the transmitter and receiver use the degraded $\bar{\Omega}(j)$ in the decision stage instead, in order to maintain the synchronization. So, as an analytical benchmark, we have the following upper-bound on the achievable throughput in this scenario. Assuming that the delayed side information of all the primary channels’ states $S_{i^*}(j-1)$ is known to the secondary receiver as well as the transmitter, the expected throughput per slot is given by:

$$R = \sum_{S_{N=0}}^{1} \cdots \sum_{S_2=0}^{1} \sum_{S_1=0}^{1} \left[ \prod_{i=1}^{N} P_{S_i} \right] \left( \max_i [P_{S_i} B_{i*}] \right) \quad (10)$$

where $P_{S_i}$ denotes the state transition probability for channel $i$ from state $S_i = (0, 1)$ to the free state, and $P_{S_i}$ is the Markov steady state probability of channel $i$ being free or busy. The first term in the summation corresponds to the probability that the $N$ channels are in one of the $2^N$ states, and the second term represents the highest expected throughput given the current joint state for the $N$ channels. The loss in throughput resulting from the use of $\bar{\Omega}(j)$ instead of $\Omega_i(j)$ in the decision is illustrated in Figure [3].

Since we assume that traffic statistics on primary channels ($P_i$) are unknown to the secondary user a-priori, the secondary user needs to estimate these probabilities. When continuous observations of each channel are available, each channel can be modeled as a hidden Markov model (HMM). An optimal learning algorithm for HMM is described in [6] using which the transition probabilities, $P_{AA}$, and $P_{MM}$ can be estimated. However, we here adopt a simple Bayesian learning method. Assume that $P_{01}$ and $P_{11}$ are random variables with distributions $f_{j01}(x)$ and $f_{j11}(y)$ defined on $[0, 1]$; respectively. After sensing all the primary channels at the beginning of each time slot, and depending on the previous state of the channel, the posterior distribution of $P_{01}$ can be updated according to Bayes’ rule; i.e.,

$$f_{j+1}(x)|\{01\}_j = \frac{x f_{j01}(x)}{f_0 f_{j01}(x)} \quad (11)$$

$$f_{j+1}(x)|\{00\}_j = \frac{(1-x) f_{j11}(x)}{f_0 f_{j11}(x)} \quad (12)$$

where the event $\{01\}_j$ represents the state transition from busy at time $j-1$ to free at time $j$. Also, the event $\{00\}_j$ represents the state transition from busy at time $j-1$ to busy at time $j$. The posterior distribution of $P_{01}$ can be updated similarly. In addition, after sensing all the primary channels at the beginning of each time slot, the secondary transmitter shall keep track of the following metrics for each channel:

- Number of state transitions from busy to busy:
  $$N_{i01}(j) = \sum_{l=1}^{j-1} (1 - \bar{S}_i(l))(1 - \bar{S}_i(l + 1)) \quad (13)$$

- Number of state transitions from free to busy:
  $$N_{i10}(j) = \sum_{l=1}^{j-1} \bar{S}_i(l)(1 - \bar{S}_i(l + 1)) \quad (14)$$

- Number of state transitions from free to free:
  $$N_{i11}(j) = \sum_{l=1}^{j-1} \bar{S}_i(l)\bar{S}_i(l + 1) \quad (15)$$

- Number of state transitions from free to free:
  $$N_{i00}(j) = \sum_{l=1}^{j-1} (1 - \bar{S}_i(l))\bar{S}_i(l + 1) \quad (16)$$

Thus, if we assume that at $j = 0$, $P_{i01}$ is uniformly distributed in $[0, 1]$ (i.e., $f_{00}(x) = 1$), or in other words no prior information about $P_{i01}$ is available, then using equation (11) it can be shown that $f_{j01}(x)$ satisfies the following Beta distribution:

$$f_{j01}(x) = \frac{N_{i00} + N_{i01} + 1}{N_{i01}^0 + N_{i10}^0} x^{N_{i00}} (1-x)^{N_{i01}} \quad (17)$$

Finally, the expected value of $f_{j01}(x)$, obtained from equation (17), gives the following best estimate for $P_{i01}$ at time $j$:

$$P_{i01}^j = \int_0^1 x f_{j01}(x) dx = \frac{N_{i01}(j) + 1}{N_{i01}^0 + N_{i01}^0} + 2 \quad (18)$$

Using the same approach, the best estimate for $P_{i11}$ at time $j$ is given by:

$$P_{i11}^j = \frac{N_{i11}(j) + 1}{N_{i11}^0 + N_{i11}^0} + 2 \quad (19)$$

This learning strategy can be easily applied to the situation where the primary traffic statistics ($P_i$) changes with time. One simple idea is to consider only a fixed number of previous sensing samples in estimating $P_i$ (i.e., using a sliding window of samples for the estimation).
In order to share the channel transition probabilities between the secondary transmitter and receiver, as dictated by the proposed strategy, any updates in the values of $N_i^{00}(j)$, $N_i^{10}(j)$, $N_i^{11}(j)$ and $N_i^{01}(j)$ must be sent within the transmitted packet. If $K_i^{*}(j) = 1$, the transmitter and receiver update $P_i^{11}(j)$ and $P_i^{01}(j)$. Otherwise, the transmitter only updates $N_i^{00}(j)$, $N_i^{10}(j)$, $N_i^{11}(j)$ and $N_i^{01}(j)$, but uses the old values, available at the last successful transmission, in the decision phase.

In a nutshell, the proposed algorithm uses the full sensing capability of the secondary transmitter to decouple the exploration (i.e., learning) task from the exploitation task. After an ACK is received, both nodes use the common observation-based belief vector to make the optimal access decision. On the other hand, in the absence of the ACK, both nodes cannot use the optimal belief vector in order to maintain synchronization. In this case, the proposed algorithm opts for a greedy strategy in order to minimize the time between the two successive ACKs.

A final remark is now in order. Assuming that $P_i^{11} = P_i^{01}$, the probability of channel $i$ being free, $P_{S_i} = 1$, becomes independent of the previous state, i.e., $P_{S_i} = 1 = P_i^{11} = P_i^{01}$. In this case, the optimal strategy, assuming that the transition probabilities are known, is for the secondary transmitter to access the channel $i^* = \arg \max_{i=1,\ldots,N} [P_{S_i} = B_i]$ and the expected throughput becomes: $R = \max_{i=1,\ldots,N} [P_{S_i} = B_i]$ [8]. Assuming, however, that the transition probabilities are unknown but both nodes know that $P_i^{11} = P_i^{01}$, one can estimate each channel free probability as $P_{S_i} = N_i^{1}(j)/j$, where $N_i^{1}(j)$ is the number of times channel $i$ was sensed to be free until time slot $j$. In Section 3, we quantify the value of this side information by comparing the performance of this strategy with our proposed universal algorithm that does not make any prior assumption about the transition probabilities.

3.2 The Restless Bandit Scenario: $L < N$

Here, we assume that the secondary transmitter can sense only a subset $L < N$ of the primary channels at the beginning of each time slot. Obviously, the problem here is not as simple as the $L = N$ case since the secondary transmitter has to decide on which $L$ channels to sense at each time slot in addition to the channel the transmitter and receiver decide to access. This opportunistic spectrum access network can be modeled as a partially observable Markov decision process (POMDP) where the channel sensing and access of a MAC protocol correspond to a policy for this POMDP [5]. The design objective is to determine, in each slot, which channel to sense so that the expected total reward: $E \left[ \sum_{j=1}^{T} R_i(j) \right]$ obtained in $T$ slots is maximized, where $R_i(j) = S_i(j)B_i$. It has been shown that for any $j$, the belief vector $\Omega(j) = [\omega_1(j), \ldots, \omega_N(j)]$ is sufficient statistic for the design of the optimal action in slot $j$ [16]. A policy for a POMDP is thus given by a sequence of functions, each mapping from the current belief vector to the sensing and access action to be taken in slot $j$. Unfortunately, finding the optimal policy for a general POMDP is computationally prohibitive. In [5], the authors proposed a reduced complexity strategy based on the greedy approach that maximizes the per-slot throughput based on already known information (i.e., at time slot $j$, transmit on channel $i^*(j) = \arg \max_{i=1,\ldots,N} [\omega_i(j)B_i]$). In a more recent work [7], the problem was re-casted as a restless bandit problem.

Restless Multi-armed Bandit Processes (RMBP) are generalizations of the classical Multi-armed Bandit Processes (MBP). In the MBP, a player, with full knowledge of the current state of each arm, chooses one out of $N$ arms to activate at each time and receive a reward determined by the state of the activated arm. Only the activated arm changes its state while the states of passive arms are frozen. The objective is to maximize the long-run reward over the infinite horizon by choosing which arm to activate at each time. The solution to the multi-armed bandit problem should be able to maintain a balance between the exploration and exploitation in order to maximize the total reward. Whittle [9] introduced the RMBP which allow multiple arms to be activated simultaneously and passive arms to also change states. In each slot, the user chooses one of two possible actions to make a particular arm passive or active. Whittle’s index measures how attractive it is to activate an arm based on the concept of subsidy for passivity. In other words, Whittle’s index for a channel is the minimum subsidy that is needed to move a state from the active set to the passive set. In [7], the Whittle index $W_i(j)$ was obtained in closed form and was used to construct a more efficient medium access policy than the greedy approach. The $W_i(j)$ given in [7] can be viewed as a combination of the immediate reward represented by $\omega_i(j)$ and a learning reward obtained from observing the state of the channel. Based on this Whittle index formulation, the maximum reward obtained at each time slot is given by $R(j) = \sum_{i=1}^{N} R_i(j)$, where:

$$R_i(j) = \begin{cases} W_i(j) & \text{if } i(j) = \arg \max_{i=1,\ldots,N} [W_i(j)B_i] \\ W_i(j) - \omega_i(j) & \text{if } i \in U_{L-1}(j) \\ 0 & \text{otherwise} \end{cases}$$

and $U_{L-1}(j)$ represents the set of $L - 1$ channels with the largest $L - 1$ values of $(W_i(j) - \omega_i(j))$ not including channel $i(j) = \arg \max_{i=1,\ldots,N} [W_i(j)B_i]$. Knowing the states of the set of channels $U_{L-1}(j)$ gives the largest observation reward (i.e., exploration) which enhances future access decisions.

Here, we relax the assumption of the a-priori available transition probabilities at the secondary transmitter/receiver. This adds another interesting dimension to the problem since the blind cognitive MAC protocol must now learn this statistical information on-line in order to make the appropriate access decisions. Inspired by the previous results of Lai et al. in the multi-armed bandit setup [8], we propose the following simple strategy. The $N$ primary channels are divided into $\lceil \frac{N}{L} \rceil$ channel groups. Then, at the beginning of the $T$ slots, each of the $\Omega$ channel groups are continuously monitored for an initial learning period (LP) to get an estimate for $P_i^{11}$ and $P_i^{01}$. In summary, the proposed strategy works as follows:

1) **Initial learning period**: Each group of channels are continuously sensed for LP time slots. At the end
of the learning period, the transition probabilities are
estimated as
\[ \tilde{P}_i^{11}(j) = \frac{N^{11}(j) + 1}{N^{11}(j) + N^{10}(j) + 2}, \]
\[ \hat{P}_i^{11}(j) = \frac{N^{11}(j) + 1}{N^{11}(j) + N^{10}(j) + 2}. \]

2) Decision: At the beginning of any time slot \( j > \left\lceil \frac{L}{P} \right\rceil \), the secondary transmitter and receiver decide to access channel \( i^*(j) = \arg \max_{i=1,\ldots,N} [\gamma_i(j)B_i] \), where
\[ \gamma_i(j) = \frac{N X_i(j)}{Y_i(j)} + \sqrt{\frac{2 \ln(2)}{Y_i(j)}}, \]
\( X_i(j) \) is the number of time slots where successful communication occurs on channel \( i \), and \( Y_i(j) \) is the number of time slots where channel \( i \) is chosen to sense and access [8].

3) Access: If \( \hat{S}_i(j) = 1 \), the transmitter sends its data packet to the receiver. If the receiver successfully receives a packet, it sends an ACK back to the transmitter.

4) The transmitter and receiver update the following:
\[ Y_i(j+1) = Y_i(j) + 1, \text{ if } i(j) = i^*(j) \]
\[ X_i(j+1) = X_i(j) + 1, \text{ if } K_i(j) = 1, i(j) = i^*(j) \]
\[ \gamma_i(j+1) = \frac{X_i(j+1)}{Y_i(j+1)} + \sqrt{\frac{2 \ln(2)}{Y_i(j+1)}} \]

4 Un-Slotted Primary Network

In this section, we consider the case of continuous probability distributions for the free and busy periods. Whenever the channel enters the busy or free state, the time until the next state transition is governed by the continuous p.d.f. \( f_{T_1}(y) \) or \( f_{T_2}(x) \); respectively. Without loss of generality, we will use the following exponentially distributed busy/free periods for each channel as an illustrative example:
\[ f_{T_1}(x) = \lambda e^{-\lambda x} \]
\[ f_{T_2}(y) = \lambda e^{-\lambda y} \]
The channel utilization \( u_i \) in this case is given by:
\[ u_i = \frac{E[T_2]}{E[T_1] + E[T_2]} \]
\[ = \frac{\lambda T_2}{\lambda T_1 + \lambda T_2} \]

We also assume that the SU is equipped with a single antenna that can be used for either sensing or transmission.

4.1 Multiple Channel Access

Here, the SU can transmit on any combination of the N primary channels simultaneously. However, the SU can sense only one channel at a time. Thus in order to sense any channel, the transmission taking place on any other channels is paused till the end of the sensing event. The goal is to find the optimal access strategy that maximizes the throughput for SU while satisfying the PU interference/outage constraints for each channel.

Since the SU depends only on sensing a channel at specific times to identify the channel’s state, it cannot track the exact state transition of each channel. Hence, the free portion of time between the actual state transition from busy to free until the SU discovers this transition cannot be utilized. In addition, some free periods may remain undiscovered at all if sensing is infrequent. These Unexplored Opportunities are quantified by \( T_i^u \), which is defined as the average fraction of time during which channel \( i \)’s vacancy is not discovered by the SU [3]. On the other hand, the transition of primary activity from free to busy on a channel utilized by the SU causes interference to the primary and secondary receivers until the SU realizes this transition. This Interference Ratio is quantified by \( T_i^f \), which is defined as the average fraction of time at which channel \( i \) is at the busy state but interrupted by SU transmission. Finally, we note that blindly increasing the sensing frequency to reduce interference and discover more opportunities is not desirable because the SU must suspend...
the use of the discovered channel(s) when it senses other channels. This is due to the assumption that data transmission and sensing cannot take place at the same time with one antenna. Thus the Sensing Overhead \( T_i^O \) is defined as the average fraction of time during which channel \( i \) discovered opportunities are interrupted due to the need for sensing any of the \( N \) channels [3]. This trade-off will be captured in the construction of our objective function which is used to find the optimal sensing frequencies/periods. The proposed algorithm relies on the novel idea of using two sensing periods for each channel: free sensing period \( T_i^F \) if \( S_i(t) = 1 \), and busy sensing period \( T_i^B \) if \( S_i(t) = 0 \). Therefore, our optimization task is to identify the optimal sensing periods \( T_i^F \), \( T_i^B \) for each channel, that maximize the total throughput for the SU on the \( N \) channels while satisfying the PU interference constraint on each channel.

The channel as seen by the SU can be modeled by a two state (free/busy) Markov chain, where the transition probabilities from the free or busy state to the free state are: \( P_{i}^{11}(t) = P(S_i(t + t) = 1|S_i(t) = 1) \) and \( P_{i}^{01}(t) = P(S_i(t + t) = 1|S_i(t) = 0) \), \( t \) is the most recent sensing time. For exponentially distributed busy/free periods, \( P_{i}^{11}(t) \) and \( P_{i}^{01}(t) \) are given by [3]:

\[
P_{i}^{11}(t) = (1 - u_i) + u_i e^{-(\lambda_T + \lambda_T) t} \quad (28)
\]

\[
P_{i}^{01}(t) = (1 - u_i) - (1 - u_i)e^{-(\lambda_T + \lambda_T) t} \quad (29)
\]

The ratio of the average number of times the channel is sensed free to the total number of times the channel is sensed can be obtained from the steady state probability that the Markov chain is in the free state:

\[
P_{i}^{SS} = \frac{P_{i}^{01}(T_{i}^{B})}{P_{i}^{01}(T_{i}^{F}) + P_{i}^{01}(T_{i}^{B})} \quad (30)
\]

In case of perfect sensing (i.e., \( P_{FA} = 0 \) and \( P_{MD} = 0 \)), \( P_{i}^{SS} \) represents the probability that the SU senses channel \( i \) with sensing-dependent periods \( T_i^{F} \) and \( T_i^{B} \) and finds it free. In the presence of sensing errors, the probability of finding channel \( i \) free is:

\[
(1 - P_{FA}) \cdot P_{i}^{SS} + P_{MD} \cdot (1 - P_{i}^{SS})
\]

Note that the average time between sensing events on channel \( i \) is given by:

\[
\mu_{i} = P_{i}^{SS} \left[ (1 - P_{FA})T_i^{F} + P_{FA}T_i^{B} \right] + (1 - P_{i}^{SS}) \left[ P_{MD}T_i^{F} + (1 - P_{MD})T_i^{B} \right] \quad (31)
\]

We define the Secondary Utilization \( T_{i}^{SU}(T_{i}^{F}, T_{i}^{B}) \) as the expected fraction of time during which channel \( i \) is sensed or utilized by the SU,

\[
T_{i}^{SU}(T_{i}^{F}, T_{i}^{B}) = \left[ (1 - P_{FA})P_{i}^{SS} + P_{MD}(1 - P_{i}^{SS}) \right] \frac{T_i^{F}}{\mu_{i}} \quad (32)
\]

The total SU un-interrupted transmission time is equivalent to the expected throughput that can be achieved by the SU on all \( N \) channels, and is given by:

\[
R = \sum_{i=1}^{N} \left[ T_{i}^{SU}(T_{i}^{F}, T_{i}^{B}) - T_{i}^{1}(T_{i}^{F}, T_{i}^{B}) - T_{i}^{0}(T_{i}^{F}, T_{i}^{B}) \right] \quad (33)
\]

Now, in order to find expressions for the expected unexplored opportunities and interference ratio we need first to find expressions for \( \delta_i^{1}(t) \) and \( \delta_i^{0}(t) \), which are defined as the expected time in which a channel is free during the time between \( t_{s} \) and \( t_{s} + t \) provided that \( S_i(t_{s}) = 1 \) or \( S_i(t_{s}) = 0 \); respectively. Based on the theory of alternating renewal processes, the remaining time \( \tilde{t} \) for a channel to be in the same state from any sampling time \( t_{s} \) can be shown to have the p.d.f. \( e^{-T_{i}^{1}/r}(x) \) [15], where \( F_{T_{i}^{1}/r}(x) \) is the c.d.f. of the free or busy period. Therefore, it can be easily shown that:

\[
\delta_i^{1}(t) = t \int_{0}^{t} F_{T_{i}^{1}}(x) dx + \int_{0}^{t} \frac{1 - F_{T_{i}^{1}}(x)}{E[T_{i}^{1}]} \tilde{t}^{1}(t - x) dx \quad (34)
\]

\[
\delta_i^{0}(t) = \int_{0}^{t} \frac{1 - F_{T_{i}^{0}}(y)}{E[T_{i}^{0}]} \tilde{y}^{1}(t - y) dy \quad (35)
\]

where \( \tilde{t}^{1}(t) \) and \( \tilde{y}^{1}(t) \) are the same as \( \delta_i^{1}(t) \) and \( \delta_i^{0}(t) \) if the change in state happens exactly at \( t_{s} \). That is,

\[
\delta_i^{1}(t) = t \int_{0}^{t} f_{T_{i}^{1}}(x) dx + \int_{0}^{t} f_{T_{i}^{1}}(x)(x + \tilde{y}^{0}(t - x)) dx \quad (36)
\]

\[
\delta_i^{0}(t) = \int_{0}^{t} f_{T_{i}^{0}}(y) \tilde{t}^{1}(t - y) dy \quad (37)
\]

Using Laplace transform, \( \delta_i^{1}(t) \) and \( \delta_i^{0}(t) \) for exponentially distributed busy/free periods can be obtained as: (see Appendix A for a complete derivation)

\[
\delta_i^{0}(t) = (1 - u_i) \cdot \left( t + \frac{e^{-(\lambda_T + \lambda_T) t} - 1}{(\lambda_T + \lambda_T)} \right) \quad (38)
\]

\[
\delta_i^{1}(t) = t - u_i \cdot \left( t + \frac{e^{-(\lambda_T + \lambda_T) t} - 1}{(\lambda_T + \lambda_T)} \right) \quad (39)
\]

The unexplored opportunities \( T_{i}^{U} \) can now be obtained as:

\[
T_{i}^{U}(T_{i}^{F}, T_{i}^{B}) = (1 - P_{MD}) \cdot (1 - P_{i}^{SS}) \cdot \left( \frac{\delta_i^{0}(T_{i}^{B})}{\mu_{i}} \right) + P_{FA} \cdot P_{i}^{SS} \cdot \left( \frac{\delta_i^{1}(T_{i}^{B})}{\mu_{i}} \right) \quad (40)
\]
Fig. 2. For a 2 channel primary network, this figure illustrates the periodic sensing using 2 periods per channel, the interference ratio, the unexplored opportunities, the sensing overhead, the secondary utilization, and the secondary achieved throughput on both channels.

Similarly, $T^i_I$ is given by:

$$T^i_I(T^F_i, T^B_i) = (1 - P_{FA}) \cdot P^{SS}_i \cdot \left( \frac{T^F_i - \delta^1_i(T^F_i)}{\mu_i} \right) + P_{MD} \cdot (1 - P^{SS}_i) \cdot \left( \frac{T^F_i - \delta^0_i(T^F_i)}{\mu_i} \right)$$  (41)

Finally, the sensing overhead $T^i_O$ is given by:

$$T^i_O(T^F_i, T^B_i) = (T^{SU}_i - T^i_I) \sum_{j=1}^{N} \left( \frac{T_s}{\mu_i} \right)$$  (42)

It is worth noting that the first term $(T^{SU}_i - T^i_I)$ represents the average fraction of time channel $i$ is utilized by the SU without interference from the PU. This is the useful time that is interrupted by the need for sensing. The second term $\sum_{j=1}^{N} \left( \frac{T_s}{\mu_i} \right)$ represents the aggregate sensing overhead given by the ratio of the sensing time to the average sensing period.

In summary, given a maximum interference constraint per primary channel, $T^i_{I_{max}}$, our optimization problem can be expressed as follows:

Find: $T^i_{F^*}, T^i_{B^*}$

that maximize: $\sum_{i=1}^{N} T^i_{SU}(T^F_i, T^B_i) - T^i_I(T^F_i, T^B_i)$

subject to: $T^i_I(T^F_i, T^B_i) \leq T^i_{I_{max}}, i = 1, \ldots, N$

This is equivalent to minimizing:

$$\sum_{i=1}^{N} \left[ T^i_{U}(T^F_i, T^B_i) + T^i_{O}(T^F_i, T^B_i) \right]$$

subject to the same interference constraint. $T^i_{F^*}$ and $T^i_{B^*}$ can be obtained numerically (as demonstrated by our numerical results in Section 5).

4.2 Single Channel Access

Here, we assume that the SU can transmit on only one single channel. Our goals are 1) to find the optimal transmission period upon accessing any channel in order to satisfy an interference/outage constraint on the primary network and 2) to find the optimal sequence of primary channels to be sensed to minimize the average delay in finding a free channel.

When the SU finds a channel busy, it switches between channels and senses them until a free channel is found. Upon finding a vacant channel, the SU transmits for a period of $T^F_i$. In order to obtain the interference imposed on the primary transmission when the SU utilizes the channel for $T^F_i$, we note that this case is equivalent to setting $T^B_i = 0$ in the expressions derived in the previous section. Thus, the interference $T^i_I(T^F_i)$ is given by:

$$T^i_I(T^F_i) = (1 - P_{FA}) \cdot \left( \frac{T^F_i - \delta^1_i(T^F_i)}{T^F_i} \right) + P_{MD} \cdot \left( \frac{T^F_i - \delta^0_i(T^F_i)}{T^F_i} \right)$$  (43)
Assuming error-free sensing, the interference $T_i^i(T_i^F)$ for exponentially distributed busy/free periods becomes:

$$T_i^i(T_i^F) = u_i \left( 1 + \frac{e^{-(\lambda_{p1} + \lambda_{p2})T_i^F}}{T_i^F \cdot (\lambda_{p2} + \lambda_{p1})} - 1 \right)$$ (44)

Let $T_i^{I_{max}}$ be the maximum fraction of outage/interference that the primary users can tolerate on all primary channels. Since only one channel is accessed at a given time, satisfying the interference constraint on each single channel is sufficient for ensuring that the total interference constraint is satisfied. Hence, assuming that the SU sensed channel $i$ to be free at time $t$ (i.e., $S_i(t) = 1$), the sensing period $T_i^F$ for each channel must satisfy the constraint: $T_i^i(T_i^F) \leq T_i^{I_{max}}$. In order to maximize the SU throughput, the optimal sensing period for each channel $T_i^F$ is $T_i^F$ which satisfies: $(T_i^i(T_i^F) = T_i^{I_{max}})$.

Now, we focus on the case when a channel is sensed to be busy. We need to find the optimal sequence of channels to sense in order to find a free channel as soon as possible, thereby minimizing the average delay in finding free channels. It is shown in [10] that in order to minimize the average delay in finding a free channel, assuming all channels have the same capacity, the SU should attempt to access the channels in descending order of the channel index $\gamma_i$, where:

$$\gamma_i = \begin{cases} \frac{P_{i1}(t_1 - t_s)}{t_s} & \text{if } S_i(t_s) = 1 \\ \frac{P_{i1}(t_2 - t_s)}{t_s} & \text{if } S_i(t_s) = 0 \end{cases}$$

In brief, the proposed strategy works as follows:
1) Sense the $N$ channels in descending order of $\gamma_i$ until a free channel is found.
2) Access the free channel $i$ for the calculated $T_i^F$.
3) When $T_i^F$ ends, recalculate $\gamma_i$ for all channels, then repeat the previous steps.

In order to synchronize the SU transmitter and receiver to the same channel while hopping, in the presence of sensing errors or in the case of spatially varying spectrum opportunities, we propose using Request To Send (RTS) and Clear To Send (CTS) packets. We assume that the packet duration of RTS and CTS is negligible compared to $T_i^F$ for all channels. If the transmitter senses a channel free, it will send RTS. If the receiver gets the RTS, it replies with CTS, then transmission proceeds for $T_i^F$. If the receiver does not receive the RTS, it waits for the duration of CTS then switch to the next channel and when the transmitter does not receive CTS it also switches to the next channel. If the transmitter senses a channel busy, it waits for the duration of RTS, then switches to the next channel to sense. When the receiver does not receive the RTS, it shall switch to next channel, Thus synchronization between the secondary transmitter and receiver is always sustained. Note that in the case of multiple channel access, the secondary receiver is assumed to decode all primary channels, thus, no synchronization is needed.

5 NUMERICAL RESULTS

5.1 Slotted Primary Networks

In this subsection we present simulation results for the two scenarios discussed earlier in Section 3. Throughout this subsection, we assume that the number of primary channels $N = 5$, each with bandwidth $B_i = 1$. The spectrum usage statistics of the primary network were assumed to remain unchanged for a block of $T = 10^4$ time slots for Figures 3, 4 and 5 and for a block of $T = 10^5$ time slots for Figure 6. The transition probabilities for each channel, i.e., $P_{i1}^{11}$ and $P_{i1}^{01}$, were generated uniformly between 0.1 and 0.9. The plotted results are the average over 1000 simulation runs. The discount factor used to obtain Whittle’s index is 0.9999. In all our simulations, perfect sensing is assumed and the average throughput per time slot is plotted.

Figure 3 reports the throughput comparison between the different cognitive MAC strategies, all with prior knowledge about the channels transition probabilities. First, it is noted that if the secondary transmitter can sense all $N$ channels neglecting the need for synchronization with the receiver, the probability of always finding a free channel $\approx 1$ for large $N$. However, since the strategies we consider sustain the secondary pair synchronization, a loss of throughput occurs as a price for synchronization. The topmost curve in Figure 5 gives the throughput obtained using equation (10) assuming the delayed side information of all the primary channels’ states is known to the secondary pair. The proposed strategy for the $L = N$ case is shown to achieve a throughput very close to the upper bound. Unsurprisingly, there is slight throughput gain offered by the full sensing capability as compared to the
$L = 1$ scenario. The difference, however, is not significant due to the constraint of maintaining synchronization which dictates choosing a channel to access followed by sensing. It is also seen from Figure 4 that the proposed strategies achieve a higher throughput than the protocol proposed in [4] where the primary transmitter and receiver are assumed to access the channels in a predetermined sequence, agreed upon a-priori, in which the channel with highest steady state probability of being free is always chosen for access.

Figure 4 illustrates the convergence of the throughput of the proposed blind strategy for $L = N$ to the informed case with prior knowledge of the transition probabilities as $T$ grows. In Figure 5 we assume that $P_i^{11} = P_i^{01}$ for all channels. It is shown that even if the secondary users are unaware of this fact, and apply the proposed universal strategy, the achievable throughput converges asymptotically to the achievable performance when the fact that $P_i^{11} = P_i^{01}$ is known a-priori, albeit at the expense of a longer learning phase. Interestingly, both strategies are shown to converge asymptotically to the genie-aided upper bound (when the transition probabilities are known). It is noted in Figures 4 and 5 that the proposed blind strategies converge to the strategy with known transition probabilities in the range between $10^2$ and $10^3$ time slots.

Finally, Figure 6 demonstrates the tradeoff between the learning time overhead in the blind strategy of Section 3.2 and the final achievable throughput at the end of the $T$ slots assuming $L = 1$. Clearly, this figure supports the intuitive conclusion that for large $T$ blocks, one should use a longer learning phase in order to maximize the steady state achievable throughput.

5.2 Un-slotted primary channels structure

In this subsection we present simulation results for the multiple channel access scenario discussed in Section 4. In Figures 7 and 8 we compare the performance of the proposed strategy which adopts the sensing-dependent periods $T_i^F$ and $T_i^B$, and the strategy proposed in [3] which uses a single sensing period for each channel. In the simulations, we assume $N = 5$ primary channels with exponentially distributed busy/free periods, where $\lambda_{T_1} = [0.2; 0.17; 0.15; 0.13; 0.11]$ and $\lambda_{T_0} = [1; 0.9; 0.8; 0.7; 0.6]$. Perfect sensing is assumed and the channel sensing duration is assumed to be $T_s = 0.01$. The plotted results are the average over 100 simulation runs and the average throughput per time unit is plotted. The total available opportunities in the primary spectrum (upper bound on SU throughput) for the given values of $\lambda_{T_1}$ and $\lambda_{T_0}$ are: $(\sum_{i=1}^{5} (1 - u_i)) = 4.205$. Instead of the assumption that the SU will immediately detect returning PUs and evacuate the channel, which was used in [3], we impose an interference constraint for each primary channel.

In Figure 7 the interference constraint for each channel $T_i^{max} = 0.25 u_i$. Under this assumption, our optimization method results in: $T_i^{F*} = [0.6133; 0.6800; 0.7637; 0.8714; 1.0148]$ and $T_i^{B*} = [0.3001; 0.3155; 0.3338; 0.3561; 0.3839]$. Using these values, the expected rate for the SU is given by $R = 3.8068$. The optimization for the strategy proposed in [3] results in the single sensing period per channel $T_{p_i}$, where: $T_{p_i}^{*} = [0.6345; 0.7032; 0.7908; 0.9034; 1.0533]$ and an expected rate of $R = 3.7531$. In Figure 8 we set a more relaxed interference constraint for each channel $T_i^{max} = 0.75 u_i$. Our optimization method results in: $T_i^{F*} = [3.8847; 4.3127; 4.8462; 5.5318; 6.4457]$, $T_i^{B*} = [0.2793; 0.2950; 0.3135; 0.3359; 0.3637]$, and $R = 4.1085$. The optimization for the strategy proposed in [3] results in: $T_{p_i}^{*} = [1.0444; 1.1035; 1.1403; 1.1886; 1.2532]$, and $R = 3.7731$. Overall, we can see from the two figures that the throughput of the proposed strategy outperforms that of the strategy proposed in [3] at different interference constraints. However, one can observe the following trend: As the interference constraint becomes more strict (i.e., $T_i^{max} \ll u_i$), more frequent sensing is required, resulting in a decreased throughput and a reduced advantage of the proposed strategy.

6 Conclusions

Assuming a slotted structure for the primary network, we proposed blind cognitive MAC protocols that do not require any prior knowledge about the statistics of the primary traffic. Our work differentiated between two distinct scenarios, based on the complexity of the cognitive transmitter. In the first, the full sensing capability of the secondary transmitter was
fully utilized to learn the statistics of the primary traffic while ensuring perfect synchronization between the secondary transmitter and receiver in the absence of a dedicated control channel. The second scenario focuses on a low-complexity cognitive transmitter capable of sensing only a subset of the primary channels at the beginning of each time slot. For this case, we proposed an augmented Whittle index MAC protocol that allows for an initial learning phase to estimate the transition probabilities of the primary traffic. Our numerical results demonstrate the convergence of the blind protocols performance to that of the genie-aided scenario where the primary traffic statistic are known a-priori by the secondary transmitter and receiver. For un-slotted primary networks, in order to maximize the secondary throughput achieved when being capable of transmitting on all primary channels simultaneously, we proposed a novel cognitive MAC protocol that utilize two sensing periods for each channel depending on the sensing outcome. Our numerical results show the superiority of the proposed protocol as compared to the protocols that rely on optimizing a single sensing period for each channel. On the other hand, when the SU can transmit on only one single channel, we show the optimal transmission period upon accessing any channel in order to satisfy an interference/outage constraint on the primary network.

Fig. 7. Performance comparison between the proposed multi-channel access strategy and the strategy proposed in [3] for an interference constraint $T_{i}^{max} = 0.25 u_{i}$

Fig. 8. Performance comparison between the proposed multi-channel access strategy and the strategy proposed in [3] for an interference constraint $T_{i}^{max} = 0.75 u_{i}$

APPENDIX A

DERIVATION OF $\delta_{i}^{0}(t)$ AND $\delta_{i}^{1}(t)$

By taking the Laplace transforms for equations (32), (35), (36) and (37):

$$\delta_{i}^{0}(s) = \frac{F_{T_{i}^{0}}(s)\tilde{\delta}_{i}^{1}(s)}{E[T_{i}^{0}]}$$

$$\delta_{i}^{1}(s) = \frac{1 - F_{T_{i}^{1}}(s)}{s^{2}E[T_{i}^{1}]} + \frac{F_{T_{i}^{1}}(s)\tilde{\delta}_{i}^{0}(s)}{E[T_{i}^{1}]}$$

$$\tilde{\delta}_{i}^{0}(s) = f_{T_{i}^{0}}(s)\tilde{\delta}_{i}^{1}(s)$$

$$\tilde{\delta}_{i}^{1}(s) = \frac{1 - f_{T_{i}^{1}}(s)}{s^{2}} + \frac{f_{T_{i}^{1}}(s)\tilde{\delta}_{i}^{0}(s)}{E[T_{i}^{1}]}$$

where $F_{T_{i}}(t) = 1 - F_{T_{i}}(t)$.

Hence,

$$\delta_{i}^{0}(s) = \frac{F_{T_{i}^{0}}(s)}{s^{2}E[T_{i}^{0}]} \left[ 1 - f_{T_{i}^{1}}(s) \right]$$

$$\delta_{i}^{1}(s) = \frac{1}{s^{2}E[T_{i}^{1}]} \left[ 1 - F_{T_{i}^{1}}(s) \left( 1 - \frac{1 - f_{T_{i}^{1}}(s)}{1 - f_{T_{i}^{0}}(s)} \right) \right]$$

And since for exponential distributions:

$$f_{T_{i}}(s) = \frac{\lambda_{T_{i}}}{s + \lambda_{T_{i}}}$$

$$F_{T_{i}}(t) = e^{-\lambda_{T_{i}}t}$$

$$F_{T_{i}}(s) = \frac{1}{s + \lambda_{T_{i}}}$$

We reach that:

$$\delta_{i}^{0}(s) = \frac{1}{s^{2} + \lambda_{T_{i}}^{0}} \cdot \frac{1 - \frac{\lambda_{T_{i}}^{1}}{s + \lambda_{T_{i}}^{1}}}{\frac{\lambda_{T_{i}}^{0}}{s + \lambda_{T_{i}}^{0}} - \frac{\lambda_{T_{i}}^{1}}{s + \lambda_{T_{i}}^{1}}}$$

$$\delta_{i}^{0}(t) = \int_{y=0}^{t} \left[ \int_{x=0}^{y} \lambda_{T_{i}^{0}} e^{-(\lambda_{T_{i}^{0}} + \lambda_{T_{i}^{1}})x} dx \right] dy$$

$$\delta_{i}^{0}(t) = \frac{\lambda_{T_{i}^{0}}}{(\lambda_{T_{i}^{0}} + \lambda_{T_{i}^{1}})} \int_{y=0}^{t} \left[ 1 - e^{-(\lambda_{T_{i}^{0}} + \lambda_{T_{i}^{1}})y} \right] dy$$

$$\delta_{i}^{0}(t) = (1 - u_{i}) \cdot \left( t + \frac{e^{-(\lambda_{T_{i}^{0}} + \lambda_{T_{i}^{1}})t} - 1}{(\lambda_{T_{i}^{0}} + \lambda_{T_{i}^{1}})} \right)$$

Similarly,

$$\delta_{i}^{1}(t) = t - u_{i} \cdot \left( t + \frac{e^{-(\lambda_{T_{i}^{0}} + \lambda_{T_{i}^{1}})t} - 1}{(\lambda_{T_{i}^{0}} + \lambda_{T_{i}^{1}})} \right)$$

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