Variation of Single-Particle Energies: Scaling Behavior for the Spectrum of $^{48}$Cr

Praveen C. Srivastava$^1$ and Larry Zamick$^2$

$^1$Department of Physics, Indian Institute of Technology
Roorkee, Roorkee 247667, India and
$^2$Department of Physics and Astronomy, Rutgers University,
Piscataway, New Jersey 08854, USA

June 24, 2020

Abstract

We perform shell model calculations for $^{48}$Cr using GXPF1A and FPD6 interactions by varying single particle energies. We find a scaling behavior which leads to the finding that many different sets of interactions with vastly different single particle energies can give almost identical results for the yrast spectrum of $^{48}$Cr.

1 Introduction

In this work we examine the effects of varying single particle energies in a shell model calculation. We will focus on the yrast spectrum of $^{48}$Cr $J=0,2,4...16$. We will consider 2 interactions GXPF1A [1] and FPD6 [2]. There have been many publications using these interactions in the PF shell with the shell model program ANTOINE [3], notably the works of E. Caurier et al. [4,5] and more recently of V. Kumar et al. [6]. Other works related to $^{48}$Cr include those of K. Hara et al. [7], F.Brandolini et al. [8], E.Caurier et al.[9], Z.C.Gao et al.[10] and R.A. Herrera et al.[11].

The original single particle energies are given in Table 1. The orbits are $0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$ and $1p_{1/2}$. We will then alter the single particle energies as indicated in Tables 2-9. In Table 2 we use the GXPF1A interaction and in Tables 6-9 the FPD6 interaction. In Table 2 we move the orbits $0f_{5/2}$ and $1p_{1/2}$ by a amount $\Delta$ which is taken to be positive; in Table 3 negative. In Tables 4 and 5 we move the orbits $1p_{3/2}$ and $1p_{1/2}$ by an amount $\Delta$; in Table 4 positive and in Table 5 negative. Basically in Tables 2 and 3 we are moving the spin-orbit partners with $j = l - 1/2$ away from the orbits with $j = l + 1/2$. In Tables 4 and 5 we are moving the $1p$ shell orbits away from the $0f$ shell orbits. That is to say we add a constant $\Delta$ to the orbits $1p_{3/2}$ and $1p_{1/2}$. We show corresponding results for the FPD6 interaction in Tables 6-9.
Before embarking on these detailed calculations let us comment on the overall properties of the yrast band in $^{48}$Cr. Clearly there is a lot of collectivity in the spectrum but it cannot be easily classified as rotational or vibrational. In the rotational case the ratio $E(4)/E(2)$ is $20/6=3.3333$ while in the vibrational model it is 2. The calculated value in Table 2 is 2.179. Thus the ratio is closer to vibrational. Concerning electromagnetic properties it was noted by Y.Y. Sharon et al. [12] that in the simplest version of the rotational model the ratio $\frac{Q(2^+)}{\sqrt{B(E2;0^+\rightarrow 2^+)}$ is equal to one. In the vibrational model this quantity is equal to zero. Using the calculated results of Table 2 for GXPF1A this ratio is 0.938, whilst with FPD6 in Table 6 it is 0.975. This is closer to rotational. We could ask if varying single particle energies will take us closer to the vibrational limit or to the rotational limit. This will be discussed near the end.

Table 1: Single particle energies (MeV) of GXPF1A and FPD6.

| Orbit  | GXPF1A       | FPD6        |
|--------|--------------|-------------|
| 0f$_{7/2}$ | -8.6240     | -8.3876     |
| 1p$_{3/2}$ | 2.9447      | 1.8942      |
| 0f$_{5/2}$ | 7.2411      | 6.4910      |
| 1p$_{1/2}$ | 4.4870      | 3.9093      |

2 GXPF1A and FPD6 with Positive Delta

In Table 10 we show the ratio $E(J)_{\Delta}/E(J)$ for $\Delta=1,10$ and 20. We show this for the GXPF1A interaction but results for FPD6 are similar.

Note that although there are some fluctuations the ratios are similar. If the ratios for a given $\Delta$ were all the same we would have perfect scaling. In that idealized situation we would get identical spectra for any finite $\Delta$ with that of $\Delta=0$ by multiplying the entire matrix for that $\Delta$ by that by a constant. Thus, is a phenomenological approach.

If we limited ourselves to fitting the spectra of the 16 yrast states of $^{48}$Cr, we would have an infinite number of choices of combinations of 2 body matrix elements and single particle energies which would yield the same results. In truth as seen in table 10 the ratios are not exactly the same but they are close enough to the idealized situation so that a large range of choices would lead to equally good results for these spectra. Of course if we expanded the data i.e. included other states. the result would be different.

In Table 11 we compare the original spectrum of GXPF1A with that for $\Delta=20$ multiplied by a renormalization factor 1.2. This renormalization factor multiplies the entire matrix including the $\Delta=20$ single particle energies. We see that the spectra are reasonably close- it would be hard to prefer one to the other. However the single particle energies are vastly different. Originally the 0f$_{5/2}$ and 1p$_{1/2}$ are 7.241 and 4.487 MeV above 0f$_{7/2}$. Now they are 27.241 and 24.487 MeV above the 0f$_{7/2}$ orbit.

Note that the GXPF1A and FPD6 single particle energies are also significantly different. This may well be as shown above that one runs into the problem that many different combinations of single particle energies and 2 body matrix
elements can give almost the same results.

In Tables 4 and 5 we move the 1p orbits away from the 0f orbits. The behavior is not as simple as for the case when we move the spin orbit partners. With increasing $\Delta$ the energies of the $2^+$ states go up but the energies of the $16^+$ states go down. There is no scaling.

When one makes truncation in the PF shell by dropping orbits it is more natural to drop the spin orbit partners $0f_5/2$ and $1p_{1/2}$ than it is to drop the $2p$ shell orbits. This was done by Zamick et al. [13] in the context of quadrupole moments and $B(E2)$’s. They studied the effects of dropping spin-orbit partners $0f_5/2$ and $1p_{1/2}$ on these electromagnetic properties. In the present context this is equivalent to setting $\Delta$ to infinity. To a large extent the results of the truncated calculations could be put into line with the full calculations by enlarging the effective charges in the former when the FPD6 interaction is used. The ratio full to truncated for $Q(2^+)^2$, $Q(4^+)^2$, $B(E2, 2^+ \rightarrow 0^+)$ and $B(E2, 4^+ \rightarrow 2^+)$ were all very close to 1.4.

3 GXPF1A and FPD6 with Negative Delta

When we make $\Delta$ negative there will be single particle level crossings. Since the $0f_5/2$ single particle energy is 7.2411 MeV when we take $\Delta$ to be -8 MeV or lower the $0f_5/2$ and $1p_{3/2}$ orbits will be below $0f_7/2$ and certainly $1p_{1/2}$. What is surprising is even a $\Delta$=-8 MeV there is no catastrophy in the spectrum. With an eyeball look it appears not to be too different from that of $\Delta$=0 MeV i.e. the original spectrum. The $2^+$ state excitation energy moves from 0.788 to 0.857 MeV whilst the $16^+$ goes from 12.805 to 12.965 MeV.

When $\Delta$ is lowered to -10 MeV the spectrum stays more or less intact except that the J=6 to J=4 splitting becomes very small.

In the original case ($\Delta$ = 0) it is 1.512 MeV, for $\Delta$ = -8 it is 1.520 MeV but for $\Delta$ = -10 it is only 0.154 MeV. This suggests a drastic change is soon to come.

Such a change can best be seen by the evolutions of the quadrupole moments of the $2^+$ states. From $\Delta$ =0 to -10 they are respectively, -0.30, -0.32, -0.32, -0.31, -0.29, -0.27, -0.26, -0.26, -0.26 and -0.18 $eb$. At $\Delta$ = -20 however there is a change of sign $Q(2^+) = +0.24$ $eb$.

Although, the energy levels for $\Delta$ = -20 track fairly well with those at $\Delta$=0 from J=0 to J=10 there is a huge difference from J=12, 14 and J=16. For $\Delta$ = 0 the latter values are 7.722, 9.701 and 12.805 MeV whilst for $\Delta$ = -20 they are 15.588, 26.249 and 28.411 MeV respectively. This can be understood by the fact that the high spin states require $0f_7/2$ contributions and these are now at a higher energy than $0f_5/2$ and $1p_{1/2}$.
Table 2 Energy spectra of $^{48}$Cr using GXPF1A interaction. Here we have keep the single-particle energies of $0f_{7/2}$ and $1p_{3/2}$ as the original one, and changed the single-particle energies of $0f_{5/2}$ and $1p_{1/2}$ moved up by original plus $\Delta$.

| Energy | GXPF1A | $\Delta = 1$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 20 | 40 | 60 | 80 | 100 |
|--------|--------|-------------|---|---|---|---|---|---|---|---|----|-----|-----|-----|----|-----|
| $0^+$  | 0.000  | 0.000       | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $2^+$  | 0.788  | 0.767       | 0.750 | 0.737 | 0.727 | 0.718 | 0.711 | 0.705 | 0.700 | 0.699 | 0.692 | 0.689 | 0.684 | 0.664 | 0.646 | 0.642 |
| $4^+$  | 1.717  | 1.687       | 1.664 | 1.646 | 1.632 | 1.620 | 1.610 | 1.601 | 1.594 | 1.588 | 1.582 | 1.576 | 1.569 | 1.564 | 1.564 | 1.498 |
| $6^+$  | 3.229  | 3.152       | 3.090 | 3.039 | 2.998 | 2.963 | 2.934 | 2.908 | 2.886 | 2.867 | 2.849 | 2.747 | 2.671 | 2.639 | 2.622 | 2.610 |
| $8^+$  | 4.753  | 4.649       | 4.532 | 4.478 | 4.415 | 4.361 | 4.314 | 4.274 | 4.239 | 4.208 | 4.179 | 4.000 | 3.881 | 3.827 | 3.796 | 3.778 |
| $10^+$ | 6.429  | 6.238       | 6.080 | 5.952 | 5.846 | 5.758 | 5.684 | 5.621 | 5.565 | 5.517 | 5.474 | 5.219 | 5.033 | 4.957 | 4.915 | 4.889 |
| $12^+$ | 7.722  | 7.479       | 7.296 | 7.155 | 7.037 | 6.941 | 6.860 | 6.791 | 6.731 | 6.679 | 6.632 | 6.568 | 6.513 | 6.513 | 6.524 | 6.296 |
| $14^+$ | 9.701  | 9.432       | 9.227 | 9.063 | 8.929 | 8.818 | 8.724 | 8.683 | 8.572 | 8.511 | 8.456 | 8.129 | 7.887 | 7.788 | 7.733 | 7.699 |
| $16^+$ | 12.805 | 12.411      | 12.115 | 11.845 | 11.699 | 11.546 | 11.417 | 11.308 | 11.213 | 11.130 | 11.057 | 10.623 | 10.305 | 10.156 | 10.105 | 10.061 |

$B(E2; \ J \rightarrow J-2 \ ) (e^2 fm^4)$

| $4^+$  | 336    | 321        | 310 | 302 | 296 | 291 | 287 | 283 | 280 | 278 | 276 | 262 | 252 | 248 | 245 | 244 |
| $6^+$  | 336    | 317        | 304 | 296 | 290 | 285 | 280 | 277 | 274 | 271 | 270 | 256 | 247 | 244 | 241 | 240 |
| $8^+$  | 306    | 288        | 276 | 268 | 262 | 257 | 253 | 250 | 248 | 245 | 244 | 232 | 223 | 222 | 221 | 217 |
| $10^+$ | 212    | 195        | 192 | 186 | 186 | 185 | 185 | 184 | 184 | 184 | 183 | 181 | 180 | 179 | 179 | 178 |
| $12^+$ | 162    | 163        | 162 | 162 | 162 | 161 | 161 | 160 | 160 | 160 | 159 | 157 | 156 | 155 | 154 | 154 |
| $14^+$ | 126    | 125        | 124 | 123 | 123 | 123 | 123 | 122 | 122 | 121 | 121 | 120 | 118 | 118 | 118 | 117 |
| $16^+$ | 62     | 65         | 66  | 67  | 68  | 68   | 68  | 68  | 68  | 68  | 68  | 68  | 67  | 67  | 67  | 67  |

$Q(\epsilon)$

| $2^+$  | -0.30  | -0.29      | -0.29 | -0.28 | -0.27 | -0.27 | -0.27 | -0.26 | -0.26 | -0.25 | -0.24 | -0.23 | -0.23 | -0.23 | -0.25 | -0.25 |
| $4^+$  | -0.40  | -0.39      | -0.38 | -0.38 | -0.36 | -0.36 | -0.36 | -0.35 | -0.35 | -0.34 | -0.34 | -0.33 | -0.31 | -0.31 | -0.30 | -0.29 |
| $6^+$  | -0.40  | -0.38      | -0.37 | -0.36 | -0.35 | -0.34 | -0.33 | -0.33 | -0.32 | -0.32 | -0.31 | -0.30 | -0.29 | -0.28 | -0.26 | -0.23 | -0.22 |
| $8^+$  | -0.41  | -0.38      | -0.36 | -0.34 | -0.33 | -0.32 | -0.31 | -0.30 | -0.29 | -0.28 | -0.28 | -0.26 | -0.22 | -0.21 | -0.21 | -0.22 |
| $10^+$ | -0.21  | -0.17      | -0.16 | -0.15 | -0.13 | -0.13 | -0.12 | -0.12 | -0.12 | -0.11 | -0.11 | -0.10 | -0.09 | -0.09 | -0.09 | -0.09 |
| $12^+$ | -0.03  | -0.02      | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | -0.01 | -0.01 |
| $14^+$ | -0.05  | -0.04      | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | -0.01 | -0.01 |
| $16^+$ | -0.09  | -0.08      | -0.07 | -0.07 | -0.06 | -0.06 | -0.06 | -0.06 | -0.05 | -0.05 | -0.05 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 |
Table 3 Energy spectra of $^{48}$Cr using GXPF1A interaction. Here we have kept the single-particle energies of $0f_{7/2}$ and $1p_{3/2}$ as the original one, and changed the single-particle energies of $0f_{5/2}$ and $1p_{1/2}$ moved up by original minus $\Delta$.

| Energy | $\Delta = -1$ | -2 | -3 | -4 | -5 | -6 | -7 | -8 | -9 | -10 | -20 | -40 | -60 | -80 | -100 |
|--------|---------------|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| $0^+$  | 0.000         | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $2^+$  | 0.816         | 0.845 | 0.865 | 0.872 | 0.878 | 0.882 | 0.876 | 0.857 | 0.842 | 0.910 | 0.490 | 0.385 | 0.359 | 0.349 | 0.340 |
| $4^+$  | 1.751         | 1.808 | 1.861 | 1.915 | 1.968 | 1.995 | 1.973 | 1.890 | 1.760 | 1.710 | 1.424 | 1.134 | 1.061 | 1.025 | 1.007 |
| $6^+$  | 3.316         | 3.376 | 3.352 | 3.359 | 3.433 | 3.496 | 3.494 | 3.410 | 3.234 | 1.864 | 2.650 | 2.153 | 2.036 | 1.9165 | 1.931 |
| $8^+$  | 4.877         | 4.970 | 4.987 | 5.056 | 5.177 | 5.261 | 5.260 | 5.147 | 4.026 | 4.058 | 2.966 | 2.433 | 2.297 | 2.237 | 2.202 |
| $10^+$ | 6.621         | 6.663 | 6.633 | 6.700 | 6.965 | 7.102 | 7.114 | 6.998 | 6.439 | 5.770 | 7.675 | 6.865 | 6.651 | 6.552 | 6.495 |
| $12^+$ | 8.060         | 8.539 | 9.009 | 9.397 | 9.469 | 9.449 | 9.373 | 9.131 | 8.195 | 7.887 | 15.588 | 35.059 | 54.929 | 74.871 | 94.838 |
| $14^+$ | 10.000        | 10.547 | 11.133 | 11.585 | 11.648 | 11.589 | 11.512 | 11.397 | 10.959 | 10.434 | 26.249 | 65.601 | 105.436 | 145.363 | 185.319 |
| $16^+$ | 13.361        | 14.034 | 14.169 | 13.720 | 13.424 | 13.263 | 13.134 | 12.965 | 12.778 | 12.854 | 28.411 | 67.637 | 107.439 | 147.349 | 187.297 |

| $B(E2: J \rightarrow J-2)$ | (e²fm⁴) |
|---------------------------|--------|
| $4^+$                     | 400    | 449    | 484    | 499    | 501    | 492    | 457    | 388    | 242    | 182    | 165    | 161    | 158    | 156    |
| $6^+$                     | 421    | 448    | 466    | 491    | 511    | 516    | 501    | 429    | 172    | 151    | 137    | 114    | 135    | 130    |
| $8^+$                     | 418    | 487    | 523    | 552    | 568    | 571    | 540    | 205    | 199    | 8      | 8      | 21     | 6      | 7      |
| $10^+$                    | 476    | 511    | 517    | 500    | 223    | 191    | 0.12   | 0.06   | 0.05   | 0.05   | 0.05   | 0.05   | 0.05   |
| $12^+$                    | 412    | 440    | 446    | 15     | 198    | 173    | 0.008  | 0.009  | 0.009  | 0.009  | 0.009  | 0.009  | 0.009 |
| $14^+$                    | 390    | 403    | 389    | 0.45   | 6      | 2      | 0.028  | 0.02   | 0.02   | 0.02   | 0.02   | 0.02   | 0.02   |
| $16^+$                    | 209    | 245    | 255    | 248    | 75     | 46     | 0.23   | 0.73   | 0.96   | 0.94   | 1.00   | 1.00   | 1.00   |

| $Q(eb)$                   |
|---------------------------|
| $2^+$                     | -0.32  | -0.32  | -0.31  | -0.29  | -0.27  | -0.26  | -0.26  | -0.26  | -0.18  | +0.24  | +0.22  | +0.22  | +0.23  | +0.22 |
| $4^+$                     | -0.42  | -0.43  | -0.41  | -0.41  | -0.41  | -0.42  | -0.45  | -0.49  | -0.51  | +0.30  | +0.29  | +0.28  | +0.28  | +0.28 |
| $6^+$                     | -0.42  | -0.43  | -0.37  | -0.34  | -0.34  | -0.35  | -0.37  | -0.41  | -0.54  | -0.61  | +0.29  | +0.26  | +0.13  | +0.27  |
| $8^+$                     | -0.43  | -0.43  | -0.40  | -0.39  | -0.38  | -0.38  | -0.38  | -0.46  | -0.70  | -0.08  | -0.40  | -0.38  | -0.38  | -0.38  |
| $10^+$                    | -0.27  | -0.37  | -0.41  | -0.42  | -0.42  | -0.42  | -0.41  | -0.74  | -0.72  | -0.01  | -0.005 | -0.003 | -0.002 | -0.0019 |
| $12^+$                    | -0.05  | -0.10  | -0.30  | -0.42  | -0.42  | -0.41  | -0.40  | -0.75  | -0.74  | -0.72  | -0.48  | -0.46  | -0.45  | -0.45  |
| $14^+$                    | -0.08  | -0.16  | -0.33  | -0.44  | -0.43  | -0.42  | -0.41  | -0.49  | -0.48  | -0.42  | -0.39  | -0.39  | -0.39  | -0.39  |
| $16^+$                    | -0.01  | -0.32  | -0.36  | -0.43  | -0.44  | -0.44  | -0.43  | -0.44  | -0.49  | -0.51  | -0.48  | -0.48  | -0.47  | -0.47  |
Table 4 Energy spectra of $^{48}$Cr using GXPF1A interaction. Here we have kept the single-particle energies of $0f_{7/2}$ and $0f_{5/2}$ as the original one, and changed the single-particle energies of $1p_{3/2}$ and $1p_{1/2}$ moved up by original plus $\Delta$.

| Energy | $\Delta = 1$ | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 20  | 40  | 60  | 80  | 100 |
|--------|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0$^+$  | 0.000        | 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000|
| 2$^+$  | 0.918        | 1.008| 1.008| 1.007| 1.133| 1.152| 1.165| 1.175| 1.182| 1.187| 1.205| 1.207| 1.206| 1.206| 1.205|
| 4$^+$  | 1.803        | 1.864| 1.907| 1.937| 1.960| 1.976| 1.989| 1.999| 2.007| 2.014| 2.041| 2.051| 2.053| 2.054| 2.054|
| 6$^+$  | 3.293        | 3.312| 3.311| 3.303| 3.293| 3.283| 3.274| 3.265| 3.257| 3.250| 3.205| 3.171| 3.157| 3.150| 3.145|
| 8$^+$  | 4.729        | 4.699| 4.671| 4.647| 4.625| 4.607| 4.591| 4.578| 4.565| 4.555| 4.491| 4.444| 4.425| 4.416| 4.409|
| 10$^+$ | 6.198        | 6.040| 5.940| 5.864| 5.807| 5.762| 5.725| 5.696| 5.670| 5.649| 6.352| 5.455| 5.425| 5.409| 5.399|
| 12$^+$ | 7.296        | 7.108| 6.948| 6.833| 6.747| 6.680| 6.626| 6.582| 6.544| 6.513| 6.346| 6.241| 6.201| 6.179| 6.167|
| 14$^+$ | 9.244        | 8.954| 8.757| 8.614| 8.507| 8.423| 8.355| 8.300| 8.253| 8.213| 8.002| 7.889| 7.819| 7.792| 7.776|
| 16$^+$ | 12.280       | 11.939| 11.702| 11.529| 11.396| 11.292| 11.207| 11.137| 11.078| 11.027| 10.754| 10.578| 10.511| 10.475| 10.453|

B(E2: J → J-2 ) ($e^2$fm$^4$) |

| Energy | 4$^+$ | 127 | 234 | 265 | 186 | 171 | 161 | 153 | 148 | 143 | 139 | 122 | 114 | 110 | 109 | 109 |
|--------|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 6$^+$  | 268   | 224 | 196 | 178 | 166 | 156 | 150 | 145 | 140 | 137 | 122 | 115 | 111 | 111 | 111 | 111 |
| 8$^+$  | 255   | 223 | 203 | 190 | 180 | 173 | 168 | 164 | 160 | 157 | 143 | 135 | 132 | 131 | 131 | 131 |
| 10$^+$ | 184   | 170 | 161 | 155 | 150 | 147 | 143 | 140 | 138 | 135 | 127 | 121 | 119 | 118 | 118 | 117 |
| 12$^+$ | 133   | 145 | 139 | 134 | 131 | 127 | 125 | 123 | 121 | 119 | 112 | 107 | 105 | 103 | 103 | 103 |
| 14$^+$ | 120   | 114 | 110 | 107 | 104 | 102 | 100 | 99  | 98  | 97  | 90  | 87  | 85  | 85  | 84  | 84  |
| 16$^+$ | 60    | 59  | 57  | 56  | 56  | 55  | 54  | 54  | 53  | 53  | 50  | 48  | 48  | 47  | 47  | 47  |

Q(ceb) |

| Energy | 2$^+$ | -0.26 | -0.22 | -0.18 | -0.16 | -0.13 | -0.12 | -0.10 | -0.09 | -0.08 | -0.04 | -0.01 | -0.0091 | -0.0091 | -0.0091 |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 4$^+$  | -0.36 | -0.34 | -0.32 | -0.30 | -0.28 | -0.27 | -0.27 | -0.26 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 |
| 6$^+$  | -0.32 | -0.26 | -0.22 | -0.19 | -0.17 | -0.15 | -0.14 | -0.13 | -0.12 | -0.12 | -0.12 | -0.12 | -0.12 | -0.12 | -0.12 |
| 8$^+$  | -0.33 | -0.27 | -0.23 | -0.20 | -0.18 | -0.17 | -0.15 | -0.15 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 |
| 10$^+$ | -0.15 | -0.12 | -0.10 | -0.09 | -0.08 | -0.08 | -0.07 | -0.07 | -0.06 | -0.06 | -0.06 | -0.06 | -0.06 | -0.06 | -0.06 |
| 12$^+$ | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| 14$^+$ | -0.04 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
| 16$^+$ | -0.06 | -0.05 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
Table 5 Energy spectra of $^{48}$Cr using GXPF1A interaction. Here we have keep the single-particle energies of $0f_{7/2}$ and $0f_{5/2}$ as the original one, and changed the single-particle energies of $0p_{3/2}$ and $1p_{1/2}$ moved up by original minus $\Delta$.

| Energy | $\Delta = -1$ | -2 | -3 | -4 | -5 | -6 | -7 | -8 | -9 | -10 | -20 | -40 | -60 | -80 | -100 |
|--------|---------------|----|----|----|----|----|----|----|----|-----|------|------|------|------|------|-------|
| 0°     | 0.000         | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2°     | 0.638         | 0.532 | 0.540 | 0.795 | 1.232 | 1.294 | 1.317 | 1.333 | 1.344 | 1.352 | 1.367 | 1.361 | 1.358 | 1.355 | 1.353  |
| 4°     | 1.612         | 1.539 | 1.584 | 1.881 | 2.738 | 3.838 | 4.334 | 4.446 | 4.501 | 4.529 | 4.552 | 4.520 | 4.505 | 4.497 | 4.491  |
| 6°     | 3.103         | 2.991 | 3.061 | 3.577 | 5.074 | 7.084 | 8.712 | 9.843 | 10.860 | 21.830 | 41.799 | 61.787 | 81.781 | 101.778 |
| 8°     | 4.753         | 4.745 | 4.863 | 5.389 | 6.785 | 8.699 | 10.701 | 12.712 | 14.683 | 15.949 | 22.968 | 24.139 | 24.139 | 24.139 | 24.139 |
| 10°    | 6.728         | 6.848 | 6.935 | 7.594 | 9.491 | 11.692 | 13.547 | 17.414 | 19.376 | 21.327 | 29.219 | 39.219 | 49.219 | 59.219 | 69.219 |
| 12°    | 8.341         | 8.942 | 9.884 | 9.706 | 11.763 | 14.550 | 17.441 | 21.075 | 22.152 | 24.139 | 32.127 | 32.127 | 32.127 | 32.127 | 32.127 |
| 14°    | 10.459        | 11.392 | 11.312 | 12.011 | 13.547 | 15.467 | 17.414 | 19.376 | 21.327 | 29.219 | 39.219 | 49.219 | 59.219 | 69.219 | 79.219 |
| 16°    | 13.655        | 14.610 | 14.605 | 15.296 | 17.590 | 20.955 | 24.656 | 28.478 | 32.358 | 36.270 | 43.927 | 48.927 | 53.927 | 58.927 | 63.927 |

\[
\begin{align*}
\text{B(E2: } J \rightarrow J-2 \text{ ) (e}^2\text{fm}^4) \\
\text{Q(eb)}
\end{align*}
\]

| Energy | 2°     | 4°     | 6°     | 8°     | 10°    | 12°    | 14°    | 16°    |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0°     | -0.35  | -0.39  | -0.40  | -0.35  | +0.13  | +0.25  | +0.26  | +0.25  |
| 2°     | -0.45  | -0.50  | -0.53  | -0.55  | -0.53  | -0.50  | +0.29  | +0.32  |
| 4°     | -0.49  | -0.56  | -0.61  | -0.63  | -0.61  | -0.49  | -0.14  | +0.01  |
| 6°     | -0.51  | -0.60  | -0.67  | -0.69  | -0.67  | -0.63  | -0.34  | -0.32  |
| 8°     | -0.38  | -0.51  | -0.60  | -0.69  | -0.66  | -0.35  | -0.32  | -0.29  |
| 10°    | -0.08  | -0.29  | -0.60  | -0.69  | -0.67  | -0.35  | -0.32  | -0.29  |
| 12°    | -0.08  | -0.57  | -0.70  | -0.68  | -0.66  | -0.58  | -0.48  | -0.46  |
| 14°    | -0.14  | -0.59  | -0.71  | -0.70  | -0.68  | -0.66  | -0.54  | -0.42  |
| 16°    | -0.14  | -0.59  | -0.71  | -0.70  | -0.68  | -0.66  | -0.63  | -0.59  |
Table 6 Energy spectra of $^{48}$Cr using FPD6 interaction. Here we have kept the single-particle energies of $0f_{7/2}$ and $1p_{3/2}$ as the original one, and changed the single-particle energies of $0f_{5/2}$ and $1p_{1/2}$ moved up by original plus $\Delta$.

| Energy | FPD6 | $\Delta = +1$ | +2 | +3 | +4 | +5 | +6 | +7 | +8 | +9 | +10 | +20 | +40 | +60 | +80 | +100 |
|--------|------|---------------|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| 0$^+$  | 0.000| 0.000         | 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000|
| 2$^+$  | 0.788| 0.768         | 0.747| 0.728| 0.711| 0.696| 0.682| 0.674| 0.661| 0.662| 0.644| 0.659| 0.671| 0.662| 0.659| 0.662|
| 4$^+$  | 1.940| 1.889         | 1.841| 1.796| 1.757| 1.724| 1.694| 1.669| 1.647| 1.628| 1.611| 1.510| 1.440| 1.397| 1.384| 1.372|
| 6$^+$  | 3.657| 3.550         | 3.443| 3.346| 3.262| 3.191| 3.129| 3.078| 3.032| 2.992| 2.958| 2.757| 2.619| 2.565| 2.536| 2.519|
| 8$^+$  | 5.568| 5.393         | 5.218| 5.064| 4.931| 4.818| 4.722| 4.640| 4.569| 4.506| 4.452| 4.138| 3.919| 3.834| 3.788| 3.761|
| 10$^+$ | 7.664| 7.362         | 7.076| 6.833| 6.631| 6.462| 6.320| 6.194| 6.095| 6.005| 5.927| 5.476| 5.167| 5.046| 4.983| 4.942|
| 12$^+$ | 9.218| 8.340         | 8.063| 8.063| 7.825| 7.629| 7.465| 7.328| 7.210| 7.109| 7.020| 6.163| 6.025| 5.952| 5.906| 5.870|
| 14$^+$ | 11.360| 10.809       | 10.372| 10.024| 9.741| 9.508| 9.314| 9.149| 9.008| 8.885| 8.778| 8.161| 7.732| 7.563| 7.473| 7.417|
| 16$^+$ | 14.620| 13.868       | 13.286| 12.826| 12.457| 12.155| 11.904| 11.691| 11.509| 11.352| 11.214| 10.423| 9.873| 9.656| 9.540| 9.468|

$B(E2; J \rightarrow J-2)$ ($e^2fm^4$)

| Energy | $B(E2)$ | 4$^+$ | 6$^+$ | 8$^+$ | 10$^+$ | 12$^+$ | 14$^+$ | 16$^+$ |
|--------|----------|------|------|------|-------|-------|-------|-------|
| 4$^+$  | 437      | 412  | 395  | 380  | 370   | 361   | 354   | 349   |
| 6$^+$  | 453      | 420  | 397  | 382  | 369   | 360   | 353   | 347   |
| 8$^+$  | 428      | 388  | 363  | 346  | 335   | 326   | 319   | 313   |
| 10$^+$ | 342      | 294  | 267  | 251  | 244   | 236   | 232   | 227   |
| 12$^+$ | 131      | 149  | 150  | 150  | 150   | 150   | 150   | 150   |
| 14$^+$ | 138      | 32   | 130  | 128  | 127   | 127   | 126   | 126   |
| 16$^+$ | 69       | 70   | 72   | 72   | 73    | 73    | 73    | 73    |

$Q(\sigma b)$

| Energy | $Q(\sigma b)$ |
|--------|---------------|
| 2$^+$  | -0.35         |
| 4$^+$  | -0.45         |
| 6$^+$  | -0.47         |
| 8$^+$  | -0.49         |
| 10$^+$ | -0.42         |
| 12$^+$ | -0.08         |
| 14$^+$ | -0.10         |
| 16$^+$ | -0.09         |
Table 7 Energy spectra of $^{48}$Cr using FPD6 interaction. Here we have kept the single-particle energies of $0f_{7/2}$ and $0p_{3/2}$ as the original one, and changed the single-particle energies of $0f_{5/2}$ and $1p_{1/2}$ moved up by original plus $\Delta$.

| Energy | $\Delta = 1$ | -2 | -3 | -4 | -5 | -6 | -7 | -8 | -9 | -10 | -20 | -60 | -80 | -100 |
|--------|--------------|----|----|----|----|----|----|----|----|-----|------|-----|-----|------|
| 0°     | 0.000        | 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000|
| 2°     | 0.807        | 0.817| 0.820| 0.819| 0.816| 0.815| 0.823| 0.866| 1.011| 1.278| 0.984| 0.884| 0.856| 0.843|
| 4°     | 1.982        | 2.007| 2.013| 2.008| 1.999| 1.984| 1.965| 1.967| 2.084| 2.540| 2.544| 2.283| 2.150| 2.111|
| 6°     | 3.731        | 3.733| 3.681| 3.632| 3.602| 3.583| 3.570| 3.587| 3.673| 3.768| 3.451| 3.750| 3.597| 3.527|
| 8°     | 5.685        | 5.686| 5.623| 5.571| 5.558| 5.514| 5.497| 5.514| 5.555| 5.710| 4.779| 4.089| 3.914| 3.835|
| 10°    | 7.880        | 7.913| 7.866| 7.819| 7.781| 7.746| 7.717| 7.726| 7.875| 7.962| 10.625| 9.726| 9.489| 9.316|
| 12°    | 9.810        | 10.420| 10.704| 10.692| 10.607| 10.535| 10.508| 10.573| 10.198| 10.150| 18.194| 37.492| 57.318| 97.193|
| 14°    | 12.041       | 12.808| 13.371| 13.447| 13.328| 13.211| 13.185| 13.277| 13.583| 13.724| 29.644| 68.812| 108.603| 188.453|
| 16°    | 15.590       | 16.358| 16.591| 16.387| 16.142| 15.960| 15.856| 15.862| 15.114| 15.126| 32.136| 71.170| 110.926| 190.750|

$\text{B(E2: } J \rightarrow J-2 \text{)} (\text{e2 fm}^4)$

| Energy | 4°    | 6°    | 8°    | 10°   | 12°   | 14°   | 16°   |
|--------|-------|-------|-------|-------|-------|-------|-------|
| 4°     | 465   | 495   | 518   | 533   | 540   | 541   | 532   |
| 6°     | 492   | 525   | 541   | 552   | 559   | 559   | 549   |
| 8°     | 484   | 482   | 518   | 543   | 558   | 561   | 549   |
| 10°    | 419   | 482   | 518   | 543   | 558   | 561   | 549   |
| 12°    | 102   | 272   | 435   | 483   | 498   | 499   | 461   |
| 14°    | 152   | 210   | 352   | 433   | 447   | 447   | 407   |
| 16°    | 68    | 90    | 247   | 288   | 298   | 297   | 283   |

$\text{Q(eb)}$

| Energy | 2°    | 4°    | 6°    | 8°    | 10°   | 12°   | 14°   | 16°   |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| 2°     | -0.35 | -0.36 | -0.36 | -0.36 | -0.36 | -0.36 | -0.34 | -0.30 |
| 4°     | -0.46 | -0.45 | -0.43 | -0.42 | -0.41 | -0.42 | -0.43 | -0.45 |
| 6°     | -0.47 | -0.44 | -0.40 | -0.38 | -0.37 | -0.37 | -0.38 | -0.41 |
| 8°     | -0.48 | -0.43 | -0.42 | -0.40 | -0.40 | -0.40 | -0.41 | -0.43 |
| 10°    | -0.45 | -0.44 | -0.43 | -0.43 | -0.42 | -0.42 | -0.42 | -0.42 |
| 12°    | -0.13 | -0.32 | -0.45 | -0.44 | -0.43 | -0.43 | -0.42 | -0.41 |
| 14°    | -0.14 | -0.27 | -0.44 | -0.44 | -0.44 | -0.43 | -0.42 | -0.42 |
| 16°    | -0.11 | -0.41 | -0.46 | -0.44 | -0.44 | -0.43 | -0.42 | -0.42 |
Table 8 Energy spectra of $^{48}$Cr using FPD6 interaction. Here we have keep the single-particle energies of $0f_{7/2}$ and $0f_{5/2}$ as the original one, and changed the single-particle energies of $1p_{3/2}$ and $1p_{1/2}$ moved up by original plus $\Delta$.

| Energy | $\Delta = 1$ | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 20  | 40  | 60  | 80  | 100 |
|--------|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $0^+$  | 0.000         | 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000| 0.000|
| $2^+$  | 0.942         | 1.088| 1.208| 1.299| 1.367| 1.417| 1.453| 1.480| 1.501| 1.516| 1.569| 1.577| 1.576| 1.574| 1.573|
| $4^+$  | 2.071         | 2.189| 2.283| 2.354| 2.405| 2.444| 2.473| 2.496| 2.513| 2.527| 2.581| 2.598| 2.601| 2.602| 2.602|
| $6^+$  | 3.795         | 3.885| 3.923| 3.927| 3.914| 3.895| 3.875| 3.854| 3.835| 3.818| 3.713| 3.639| 3.611| 3.597| 3.587|
| $8^+$  | 5.508         | 5.597| 5.557| 5.515| 5.473| 5.433| 5.400| 5.370| 5.344| 5.320| 5.185| 5.093| 5.058| 5.039| 5.028|
| $10^+$ | 7.450         | 7.235| 7.057| 6.918| 6.809| 6.722| 6.653| 6.595| 6.547| 6.506| 6.292| 6.160| 6.111| 6.085| 6.069|
| $12^+$ | 8.500         | 8.138| 7.815| 7.579| 7.401| 7.263| 7.153| 7.064| 6.990| 6.928| 6.610| 6.424| 6.356| 6.322| 6.300|
| $14^+$ | 10.596        | 10.055| 9.669| 9.384| 9.116| 9.000| 8.866| 8.756| 8.665| 8.588| 8.191| 7.954| 7.808| 7.824| 7.796|
| $16^+$ | 13.733        | 13.105| 12.653| 12.318| 12.062| 11.860| 11.698| 11.565| 11.453| 11.359| 10.864| 10.560| 10.448| 10.390| 10.354|

B(E2: J = J-2) ($e^2fm^6$)

| $4^+$  | 376 | 321 | 277 | 242 | 215 | 196 | 181 | 169 | 160 | 152 | 133 | 97  | 91  | 89  | 87  |
| $6^+$  | 380 | 312 | 256 | 215 | 186 | 166 | 153 | 142 | 134 | 128 | 98  | 85  | 81  | 80  | 78  |
| $8^+$  | 357 | 298 | 256 | 227 | 206 | 191 | 180 | 172 | 166 | 161 | 137 | 125 | 122 | 119 | 116 |
| $10^+$ | 264 | 215 | 188 | 170 | 159 | 149 | 143 | 137 | 133 | 130 | 113 | 105 | 101 | 100 | 99  |
| $12^+$ | 145 | 142 | 139 | 135 | 131 | 128 | 125 | 123 | 121 | 119 | 110 | 104 | 102 | 100 | 98  |
| $14^+$ | 127 | 120 | 115 | 111 | 108 | 105 | 103 | 102 | 100 | 99  | 91  | 86  | 85  | 84  | 83  | 81  |
| $16^+$ | 65  | 63  | 61  | 60  | 59  | 58  | 57  | 56  | 56  | 55  | 52  | 50  | 49  | 48  | 48  | 47  |

Q(ibs)

| $2^+$  | -0.32 | -0.28 | -0.25 | -0.22 | -0.19 | -0.17 | -0.15 | -0.13 | -0.12 | -0.10 | -0.04 | -0.0331 | +0.008 | +0.01 | +0.01 |
| $4^+$  | -0.42 | -0.39 | -0.37 | -0.35 | -0.33 | -0.32 | -0.30 | -0.29 | -0.28 | -0.28 | -0.25 | -0.23 | -0.22 | -0.22 | -0.22 |
| $6^+$  | -0.42 | -0.35 | -0.29 | -0.24 | -0.20 | -0.18 | -0.16 | -0.14 | -0.13 | -0.12 | -0.08 | -0.05 | -0.05 | -0.05 | -0.04 |
| $8^+$  | -0.42 | -0.36 | -0.31 | -0.27 | -0.24 | -0.21 | -0.19 | -0.18 | -0.16 | -0.16 | -0.10 | -0.07 | -0.07 | -0.06 | -0.06 |
| $10^+$ | -0.30 | -0.22 | -0.18 | -0.15 | -0.14 | -0.12 | -0.11 | -0.10 | -0.09 | -0.07 | -0.05 | -0.05 | -0.05 | -0.04 | -0.04 |
| $12^+$ | -0.03 | -0.03 | -0.02 | -0.02 | -0.02 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 |
| $14^+$ | -0.07 | -0.06 | -0.05 | -0.04 | -0.04 | -0.04 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.02 |
| $16^+$ | -0.06 | -0.04 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.009 | -0.008 | -0.008 | -0.008 |
Table 9 Energy spectra of $^{48}$Cr using FPD6 interaction. Here we have keep the single-particle energies of $0f_{7/2}$ and $0f_{5/2}$ as the original one, and changed the single-particle energies of $1p_{3/2}$ and $1p_{1/2}$ moved up by original plus $\Delta$.

| Energy | $\Delta = 1$ | -2 | -3 | -4 | -5 | -6 | -7 | -8 | -9 | -10 | -20 | -40 | -60 | -80 | -100 |
|--------|--------------|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| 0°     | 0.000        | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2°     | 0.612        | 0.593 | 0.594 | 0.702 | 1.048 | 1.328 | 1.358 | 1.362 | 1.364 | 1.364 | 1.346 | 1.339 | 1.336 | 1.334 |     |
| 4°     | 1.827        | 1.779 | 1.810 | 1.978 | 2.415 | 3.216 | 4.176 | 4.372 | 4.376 | 4.362 | 4.237 | 4.160 | 4.133 | 4.120 | 4.112 |
| 6°     | 3.525        | 3.473 | 3.561 | 3.872 | 4.651 | 6.101 | 7.802 | 9.109 | 10.165 | 11.170 | 21.085 | 41.027 | 61.007 | 80.998 | 100.992 |
| 8°     | 5.531        | 5.537 | 5.714 | 6.057 | 6.757 | 7.990 | 9.675 | 11.541 | 13.477 | 15.319 | 25.025 | 44.861 | 64.808 | 84.783 | 104.767 |
| 10°    | 7.816        | 7.945 | 8.161 | 8.608 | 9.635 | 11.769 | 13.239 | 14.974 | 16.816 | 18.710 | 38.370 | 78.247 | 118.211 | 158.193 | 198.183 |
| 12°    | 9.980        | 10.403 | 10.616 | 10.975 | 12.157 | 14.089 | 16.581 | 19.253 | 21.346 | 23.240 | 42.748 | 82.532 | 122.464 | 162.431 | 202.411 |
| 14°    | 12.578       | 13.108 | 13.257 | 13.697 | 14.992 | 17.412 | 20.497 | 23.529 | 26.885 | 29.244 | 58.683 | 118.458 | 178.389 | 238.546 | 298.836 |
| 16°    | 15.826       | 16.660 | 16.762 | 17.121 | 18.344 | 20.729 | 23.898 | 27.429 | 31.129 | 34.921 | 74.209 | 153.942 | 233.862 | 313.824 | 393.801 |

$B(E2; J \rightarrow J-2 ) \ (e^2 fm^4)$

| Energy | 4°     | 6°     | 8°     | 10°    | 12°    | 14°    | 16°    |
|--------|--------|--------|--------|--------|--------|--------|--------|
|        | 497    | 517    | 491    | 423    | 235    | 172    | 83     |
|        | 547    | 567    | 532    | 463    | 368    | 275    | 165    |
|        | 570    | 582    | 529    | 454    | 375    | 225    | 170    |
|        | 534    | 542    | 466    | 393    | 345    | 275    | 164    |
|        | 377    | 434    | 317    | 275    | 246    | 225    | 164    |
|        | 76     | 233    | 191    | 191    | 191    | 191    | 160    |
|        | 92     | 20     | 76     | 10     | 2      | 1      | 2      |
|        | 202    | 20     | 10     | 1      | 0.06   | 1      | 0.5    |
|        | 171    | 4      | 2      | 0.5    | 0.02   | 0.5    | 0.4    |
|        | 154    | 1      | 1      | 0.5    | 0.01   | 1      | 0.4    |
|        | 111    | 0.5    | 1      | 0.5    | 0.01   | 1      | 0.4    |
|        | 99     | 5      | 1      | 0.5    | 0.01   | 1      | 0.4    |
|        | 95     | 2      | 0.5    | 0.5    | 0.01   | 1      | 0.4    |
|        | 94     | 1      | 0.5    | 0.5    | 0.01   | 1      | 0.4    |
|        | 93     | 0.5    | 0.5    | 0.5    | 0.01   | 1      | 0.4    |

$Q(eb)$

| Energy | 2°     | 4°     | 6°     | 8°     | 10°    | 12°    | 14°    | 16°    |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|        | -0.38  | -0.48  | -0.52  | -0.34  | -0.53  | -0.28  | -0.20  | -0.20  |
|        | -0.40  | -0.51  | -0.56  | -0.39  | -0.61  | -0.62  | -0.63  | -0.63  |
|        | -0.41  | -0.53  | -0.59  | -0.63  | -0.71  | -0.72  | -0.72  | -0.72  |
|        | -0.40  | -0.54  | -0.62  | -0.70  | -0.71  | -0.72  | -0.72  | -0.72  |
|        | -0.29  | -0.53  | -0.39  | -0.69  | -0.71  | -0.72  | -0.72  | -0.72  |
|        | -0.24  | -0.36  | -0.39  | -0.67  | -0.67  | -0.67  | -0.67  | -0.67  |
|        | +0.25  | +0.30  | +0.30  | +0.32  | +0.32  | +0.29  | +0.28  | +0.28  |
|        | +0.24  | +0.32  | +0.32  | +0.27  | +0.27  | +0.26  | +0.26  | +0.26  |
|        | +0.21  | +0.29  | +0.29  | +0.25  | +0.25  | +0.24  | +0.24  | +0.24  |
|        | +0.21  | +0.27  | +0.27  | +0.24  | +0.24  | +0.23  | +0.23  | +0.23  |
|        | +0.21  | +0.26  | +0.26  | +0.24  | +0.24  | +0.23  | +0.23  | +0.23  |
|        | +0.21  | +0.25  | +0.25  | +0.24  | +0.24  | +0.24  | +0.24  | +0.24  |
|        | +0.21  | +0.24  | +0.24  | +0.23  | +0.23  | +0.23  | +0.23  | +0.23  |


Table 10 Ratio $E(J)_{\Delta}/E(J)$ with GXPF1A for $\Delta=1,10$ and 20.

| $J/\Delta$ | 1   | 10  | 20  |
|------------|-----|-----|-----|
| 2          | .973| .878| .849|
| 4          | .983| .921| .901|
| 6          | .976| .882| .851|
| 8          | .978| .879| .842|
| 10         | .977| .851| .812|
| 12         | .968| .859| .823|
| 14         | .972| .872| .838|
| 16         | .969| .863| .829|
| $Q(2^+)$   | .967| .867| .867|
| $\sqrt{B(E2)}$ | .975| .896| .876|

Table 11 Comparison of original energies with normalized ones for $\Delta=20$ using GXPF1A. Renormalization factor =1.2.

| $J$ | Original Spectrum | $\Delta=20$ Renormalized |
|-----|-------------------|---------------------------|
| 0   | 0.000             | 0.000                     |
| 2   | 0.788             | 0.802                     |
| 4   | 1.717             | 1.854                     |
| 6   | 3.279             | 3.294?                    |
| 8   | 4.752             | 4.801                     |
| 10  | 6.420             | 6.268?                    |
| 12  | 7.722             | 7.627                     |
| 14  | 9.701             | 9.755                     |
| 16  | 12.805            | 12.748                    |

4 Closing Remarks

In the introduction we noted that one tries to associate configuration mixing with collective behavior. For $^{48}$Cr there is a mixed bag. The spectrum is not rotational but the value of $1.103Q(2^+)/\sqrt{B(E2,0^+ \rightarrow 2^+}$ is very close to the rotational limit of one, and far away from the vibrational limit of zero. One of our motivations was to see what light would be shed by varying the single particle energies. Would one choice of $\Delta$ take us closer to the rotational limit and another to the vibrational limit? But then we found that there was very little change in the spectra for positive $\Delta$. For example the ratio $E(4)/E(2)$ in Table 2 is 2.179 for $\Delta=0$ and 2.365 for $\Delta=20$. Our first reaction was “the results are dull because not much is happening” but then it changed to “the results are exciting because not much is happening”. We found this scaling behavior leading the fact that we can find whole sets of interactions which will give almost the same results for the yrast spectrum of $^{48}$Cr, and some have drastically different single particle energies than others. We are of course looking at limited data but it is data that is mostly focused on when phenomenological interactions are being constructed. And for negative $\Delta$ we have the intriguing results that even
when spin orbit partners are inverted one can or a while at least get results very similar to those for the normally ordered spectrum. Clearly these observations deserve further study.

ACKNOWLEDGEMENTS

P C Srivastava acknowledges a research grant from SERB (India), CRG/2019/000556 and Kalam cluster at Physics Department, IIT-Roorkee.

References

[1] M. Honma, T. Otsuka, B. A. Brown, and T. Mizusaki, Phys. Rev. C 65, 061301 (R), (2002); Phys. Rev. C 69, 034335 (2004).

[2] W.A. Richter, M.G. Van Der Merwe, R.E. Julius and B.A. Brown, Nucl. Phys. A 523, 325 (1991).

[3] E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves, and A. P. Zuker, Rev. Mod. Phys. 77, 427 (2005).

[4] E. Caurier, A.P. Zuker, A. Poves and C. Martinez-Pinero, Phys. Rev. C50, 225 (1994).

[5] E. Caurier, F. Nowacki, A. P. Zuker, G. Martinez-Pinedo, A. Poves, and J. Retamosa, Nuclear Physics A 654, 747 (1999).

[6] V. Kumar, P. C. Srivastava and A. Kumar, Acta Physica Polonica B 51, 961 (2020).

[7] Kenji Hara, Yang Sun, and Takahiro Mizusaki, Phys. Rev. Lett. 83, 1922 (1999).

[8] F. Brandolini and C. A. Ur, Phys. Rev. C 71, 054316 (2005).

[9] E. Caurier, J. L. Egido, G. Martinez-Pinedo, A. Poves, J. Retamosa, L. M. Robledo, and A. P. Zuker Phys. Rev. Lett. 75, 2466 (1995).

[10] Zao-Chun Gao, Mihai Horoi, Y. S. Chen, Y. J. Chen, and Tuya, Phys. Rev. C 83, 057303 (2011).

[11] R. A. Herrera and C. W. Johnson, Phys. Rev. C 95, 024303 (2017).

[12] Y.Y. Sharon, N. Benczer-Koller, G. J. Kumbartzki, L. Zamick, R. F. Casten, Nuclear Physics A 980, 131 (2018).

[13] L. Zamick, Y.Y.Sharon, S.J.Q. Robinson and M. Harper, Phys. Rev. C 91, 064321 (2015).