We consider here chiral symmetry breaking through nontrivial vacuum structure with quark antiquark condensates. We then relate the condensate function to the wave function of pion as a Goldstone mode. This simultaneously yields the pion also as a quark antiquark bound state as a localised zero mode in vacuum. We illustrate the above with Nambu Jona-Lasinio model to calculate different pionic properties in terms of the vacuum structure for breaking of exact or approximate chiral symmetry, as well as the condensate fluctuations giving rise to $\sigma$ mesons.
I. INTRODUCTION

Nambu and Jona-Lasinio (NJL) had linked chiral symmetry breaking with properties of hadrons quite sometime back with pion as the Goldstone mode. However, pion is both a quark antiquark bound state and a Goldstone mode. Hence through Goldstone theorem pion state along with its wave function as a quark antiquark pair should also get related to the vacuum structure. It is surprising that this aspect is absent in the very extensive literature on the topic.

We consider phase transition as a vacuum realignment with an explicit structure and use the techniques developed earlier. In this manner for potential models we had obtained the gap equation to be the same as derived through Schwinger Dyson equation. Through Goldstone theorem here we had the extra feature that the pion state as a space localised quark antiquark zero mode of destabilised vacuum also gets determined. We further discussed the effects of approximate symmetry breaking where the gap equation changes giving rise to a change in the pion wave function. We discuss the same here for Nambu Jona-Lasinio model. The reason for doing the same in NJL model is its mathematical simplicity and its present relevance in the context of Salam Weinberg symmetry breaking and top quark mass.

We organise the paper as follows. In section II we consider the vacuum structure with quark antiquark pairs using an ansatz for the same by minimising the energy density. This gives rise to the conventional gap equation and involves a new description of phase transition with an explicit construct for the destabilised vacuum. In section III we identify the pion as Goldstone mode and relate its wave function with functions associated with the vacuum structure. In section IV we consider the vacuum structure for NJL model when chiral symmetry is approximate. We also derive here some familiar results of current algebra in the present framework. In section V we calculate the pion charge radius using the wave function determined from the vacuum structure. In section VI we consider fluctuation of the condensate mode to give a qualitative identification of σ meson. Section VII consists of
The method here consists of using equal time algebra \[1\] along with construction of
the ground state through a variational principle \[5,6,14\].

II. CHIRAL SYMMETRY BREAKING AND VACUUM REALIGNMENT

We shall now proceed in the same manner as earlier \[6\] for the NJL model. Let us start
with the effective Hamiltonian

\[
\mathcal{H}(\vec{x}) = \psi(\vec{x})^\dagger \left(-i\vec{\alpha} \cdot \nabla\right)\psi(\vec{x}) + \int d\vec{y} \psi^\dagger(\vec{x}) \psi^j(\vec{x}) V^{ij,kl}_{\alpha\beta,\gamma\delta}(\vec{x} - \vec{y}) \psi^k(\vec{y}) \psi^l(\vec{y}),
\]

(1)

which has chiral invariance. In the above \(i, j\) stand for color indices and \(\alpha, \beta\) stand for the
spinor indices and \(V^{ij,kl}_{\alpha\beta,\gamma\delta}(\vec{x} - \vec{y})\) is the potential. For effective QCD based vector potential
we may take

\[
V^{ij,kl}_{\alpha\beta,\gamma\delta}(\vec{x} - \vec{y}) = \delta_{\alpha\beta} \delta_{\gamma\delta} \left(\frac{\lambda^a}{2}\right)_{ij} \delta_{kl} V(|\vec{x} - \vec{y}|),
\]

(2a)

where \(\lambda^a\) are the Gellman matrices. We may also have NJL model when we take the contact
potential as

\[
V^{ij,kl}_{\alpha\beta,\gamma\delta}(\vec{x} - \vec{y}) = G \left[(\gamma^0)_{\alpha\beta}(\gamma^0)_{\gamma\delta} - (\gamma^0, \gamma^5)_{\alpha\beta}(\gamma^0, \gamma^5)_{\gamma\delta} \tau^a_{ij} \tau^a_{kl}\right] \delta(|\vec{x} - \vec{y}|).
\]

(2b)

Here \(G\) is the dimensional interaction coupling constant and \(\tau^a\)'s are the isospin matrices.

The field operators \(\psi(\vec{x})\) may be expanded as

\[
\psi(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int \left[U_r(\vec{k}) c_Ir(\vec{k}) + V_s(-\vec{k}) \tilde{c}_{Is}(-\vec{k})\right] e^{i\vec{k} \cdot \vec{x}} d\vec{k}
\]

(3)

where \(U\) and \(V\) are given by

\[
U_r(\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \vec{\sigma} \cdot \vec{k} \end{pmatrix} u_{Ir}; \quad V_s(-\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} -\vec{\sigma} \cdot \vec{k} \\ 1 \end{pmatrix} v_{Is}
\]

(4)

for free chiral fields. The perturbative vacuum is defined by this basis when we have
\(c_I | \text{vac} > = 0 = \tilde{c}_I^\dagger | \text{vac} >\). We next consider a trial vacuum state given as \[5,6\]
\[ | vac' > = U | vac > \equiv \exp(B^\dagger - B) | vac > \] (5a)

with

\[
B^\dagger = \int c_{Ir}(\vec{k}) u_{Ir}^\dagger(\vec{\sigma}, \vec{k}) v_{Is} \tilde{c}_{Is}(\vec{k}) f(\vec{k}) d\vec{k}.
\] (5b)

Here \( f(\vec{k}) \) is a trial function associated as above with quark anti-quark condensate. We may recall a similar construction in Bogoliubov Valatin approach [1,2,4]. We shall minimise the energy density of \( | vac' > \) to analyse the possibility of phase transition [5] from \( | vac > \) to \( | vac' > \). For this purpose we first note that with the above transformation the operators which annihilate \( | vac' > \) are given as

\[
b_I(\vec{k}) = Uc_I(\vec{k}) U^{-1},
\] (6)

which with an explicit calculation yields the Bogoliubov transformation

\[
\begin{pmatrix}
  b_{Ir}(\vec{k}) \\
  \tilde{b}_{Is}(\vec{k})
\end{pmatrix} =
\begin{pmatrix}
  \cos f & -\frac{i}{\sqrt{2}} \sin f a_{rs} \\
  \frac{i}{\sqrt{2}} \sin f (a^\dagger)_{sr} & \cos f
\end{pmatrix}
\begin{pmatrix}
  c_{Ir}(\vec{k}) \\
  \tilde{c}_{Is}(\vec{k})
\end{pmatrix}. \tag{7}
\]

Here \( a_{rs} = u_{Ir}^\dagger(\vec{\sigma} \cdot \vec{k}) v_{Is} \). Using the above transformation (6) or (7) the expectation value of the Hamiltonian with respect to \( | vac' > \) is given as

\[
\mathcal{E} = < vac' | H(x) | vac' > \equiv T + V, \tag{8}
\]

where \( T \) and \( V \) are the expectation values corresponding to the kinetic and the potential terms in Eq.(1). With a straightforward evaluation we then obtain that

\[
T = < vac' | \psi^i(\vec{x})^\dagger(-i\alpha \cdot \vec{\sigma}) \psi^i(\vec{x}) | vac' > = -\frac{2N}{(2\pi)^3} \int d\vec{k} | \vec{k} | \cos 2f(k), \tag{9}
\]

where \( N = N_c \times N_f \) is the total number of quarks. Similarly the potential term is given as

\[
V = \frac{1}{(2\pi)^6} \int \tilde{V}_{ij,k\ell}^l(\Lambda_+^+(\vec{k}_1))_{\beta\gamma}(\Lambda_-^-(\vec{k}_2))_{\delta\alpha} d\vec{k}_1 d\vec{k}_2, \tag{10}
\]

where \( \tilde{V}(\vec{k}) \) is the Fourier transform of the potential \( V(\vec{r}) \) given as

\[
\tilde{V}(\vec{k}) = \int V(\vec{r}) e^{i\vec{k} \cdot \vec{r}} d\vec{r}, \tag{11}
\]
\[ \Lambda_{\pm}(\vec{k}) = \frac{1}{2} \left( 1 \pm \gamma^0 \sin2f(k) \pm \vec{\alpha}.\vec{k}\cos2f(k) \right). \] (12)

The expression for \( V \) as in Eq. (10) can be calculated for Eq. (1) or (2a) for an effective potential [2]. We shall however now illustrate the method with Eq. (2b) for NJL model corresponding to the contact potential. The total energy density then becomes

\[ \mathcal{E}\{f\} = \mathcal{E} = -\frac{2N}{(2\pi)^3} \int d\vec{k} \left| \vec{k} \right| \cos2f - 2GN(2N+1)I^2 \] (13)

with

\[ I = \frac{1}{(2\pi)^3} \int \sin2f(k)d\vec{k}. \] (14)

The leading order in \( N \) here corresponds to the Hartree approximation [4]. The energy functional \( \mathcal{E}\{f\} \) here is quadratic in \( f(k) \) and it is to be determined by minimising the energy density. This yields that

\[ \tan2f(k) = \frac{2GI(2N+1)}{k} \equiv M/k; \] (15)

where, \( M \equiv 2GI(2N+1) \) is the dynamically generated mass. Further, substituting the above in Eq. (14) yields the self consistency relation

\[ M = \frac{2G(2N+1)}{(2\pi)^3} \int_{\Lambda} \frac{M}{\sqrt{M^2 + k^2}} d\vec{k}, \] (16)

with \( \Lambda \) above as the ultraviolet cutoff for NJL model. Eq. (16) is usually derived through an approximate solution to Schwinger Dyson equation [3]. We followed here an alternative variational method with phase transition as \textit{vacuum realignment} as in Eq. (5), determined through \textit{minimising energy density functional} [5].

The above equation has a solution with \( M \neq 0 \) (Goldstone phase) provided

\[ GA^2(2N+1) > 2\pi^2. \] (17)

The energy density of \( |\text{vac}'\rangle \) with respect to the perturbative vacuum \( |\text{vac}\rangle \) may be evaluated to be
\[ \Delta \mathcal{E} = \mathcal{E}\{f\} - \mathcal{E}\{f = 0\} = \frac{2N}{(2\pi)^3} \int^{\Lambda} (k - \sqrt{k^2 + M^2})d\vec{k} + \frac{N}{2G(2N + 1)}M^2 \]  

(18)

which is negative when a nontrivial solution to Eq. (16) exists or Eq. (17) is satisfied. The state with condensates \(|\text{vac}'\rangle\) then becomes the physical vacuum. One may also calculate the order parameter \(<\bar{\psi}\psi>|\text{vac}'\rangle\) given as

\[ <\text{vac}'|\bar{\psi}\psi|\text{vac}'\rangle = -\frac{1}{(2\pi)^3} \times 2NM \int^{\Lambda} \frac{d\vec{k}}{\sqrt{k^2 + M^2}} \]  

(19)

III. GOLDSTONE THEOREM AND PION WAVE FUNCTION

We shall now recapitulate \[\text{[6]}\] that the present description of phase transition permits us to define pion also as a quark antiquark pair. From the gap equation we obtained two solutions for the field operators corresponding to \(\sin 2f(k) = 0\) or \(\sin 2f(k) \neq 0\) along with the corresponding ground state as \(|\text{vac}\rangle\) or \(|\text{vac}'\rangle\) respectively.

For the case of chiral symmetry breaking, we have the gap equation

\[ 1 = \frac{2G(2N + 1)}{(2\pi)^3} \int^{\Lambda} \frac{1}{\sqrt{M^2 + k^2}}d\vec{k}, \]  

(20)

which determines the value of the mass parameter \(M\). Once \(M\) is determined, the function \(f(\vec{k})\) becomes known and hence the condensate structure of vacuum becomes known. However, the Hamiltonian of equation (1) had chiral symmetry, which through equation (19) or otherwise is now seen to be broken. Hence we should have a Goldstone mode corresponding to a zero mass particle \[\text{[7]}\]. We shall approach this theorem in a modified manner to obtain the wave function as a quark antiquark pair \[\text{[8]}\]. When chiral symmetry remains good,

\[ Q_5^a|\text{vac}\rangle = 0 \]  

(21)

where \(Q_5^a\) is the chiral charge operator given as

\[ Q_5^a = \int \bar{\psi}(\vec{x})^\dagger \frac{1}{2} \gamma^5 \psi(\vec{x})d\vec{x}. \]  

(22)

For symmetry broken case however
\[ Q_5^a | \text{vac}' > \neq 0. \]  \tag{23}

We expect that this will describe a pion of zero total momentum. Since it will be massless it will also have zero energy, corresponding to the pion state. To show this we first note that
\[ [Q_5^a, H] = 0 \]  \tag{24}

irrespective of whether \( Q_5^a \) and \( H \) are written in terms of field operators corresponding to \( \sin 2f = 0 \) or \( \sin 2f \neq 0 \) since the anticommutation relation between the operators remain unchanged by the Bogoliubov transformation. Clearly, for the Goldstone phase, \( |\text{vac}' > \) is an approximate eigenstate of \( H \) with \( \mathcal{E}V \) as the approximate eigenvalue (\( V \) being the total volume). With \( H_{\text{eff}} = H - \mathcal{E}V \), we then obtain from Eq.(23) that
\[ H_{\text{eff}} Q_5^a |\text{vac}' > = 0 \]  \tag{25}
i.e. the state \( Q_5^a | \text{vac}' > \) with zero momentum has also zero energy, thus corresponding to the massless pion. Explicitly, using Eq.(3) and Eq.(7), we then obtain with \( q_I \) now as two component isospin doublet corresponding to \((u,d)\) quarks above,
\[ | \pi^a(\vec{0}) > = N_\pi \int \tilde{q}_I(k) \frac{\tau^a}{2} \tilde{q}_I(-k) \sin 2f(k) d\vec{k} | \text{vac}' >, \]  \tag{26}
where, \( N_\pi \) is a normalisation constant. The wave function for pion thus is given as proportional to \( \tilde{u}(k) \equiv \sin 2f(k) \). The isospin and spin indices of \( q^i \) and \( \tilde{q} \) for quarks have been suppressed. Further, with
\[ < \pi^a(\vec{0}) | \pi^b(\vec{p}) > = \delta^{ab} \delta(\vec{p}), \]  \tag{27}
the normalisation constant \( N_\pi \) is given by
\[ N_\pi^2 \times \frac{N_c N_f}{2} \int \sin^2 2f(k) d\vec{k} = 1. \]  \tag{28}

Clearly the state as in Eq.(26) as the Goldstone mode will be accurate to the extent we determine the vacuum structure sufficiently accurately through variational or any other
method so that $|\text{vac}'\rangle$ is an eigenstate of the Hamiltonian. The above results relate pion wave function with vacuum structure for any example of chiral symmetry breaking.

We may note that we could relate pion wave function to the vacuum structure since vacuum had an explicit structure as in equations (5). In fact, the two body condensate as in equations (5) for the destabilised vacuum is strictly related to the presence of the zero mode as seen here.

IV. APPROXIMATE CHIRAL SYMMETRY

While considering chiral symmetry breaking, we often use results from current algebra so that we may obtain numbers for approximate chiral symmetry breaking. For the sake of completeness, with the present mechanism, we elaborate [4] these results so as to use the same for NJL model. For this purpose we may add a small mass term to the Hamiltonian that breaks the chiral symmetry explicitly. Then $Q_5^a | \text{vac}' \rangle$ will not be a zero mode and will have finite mass. In fact the mass of the pion in the lowest order will now be $m_\pi$ formally given as

$$< \pi^a(\vec{0}) | H_{sb} | \pi^a(\vec{0}) > = m_\pi \delta(\vec{0}), \quad (29)$$

where $H_{sb}$ is the symmetry breaking part of the Hamiltonian corresponding to the Hamiltonian density $H_{sb} = m \bar{\psi} \psi$, $m$ being the current quark mass. The above may be related to $N_\pi$ and pion decay constant as follows. Firstly we note that the identity for pion decay constant is [13]

$$< 0 | J_5^{0a} | \pi(\vec{p}) > = \frac{i}{(2\pi)^{3/2}} \times \frac{c_\pi \times p_0}{\sqrt{2p_0}} \times e^{ip\cdot x}, \quad (30)$$

where, $c_\pi = 94 MeV$. The normalisation constant $N_\pi$ in Eq. (28) is then given by using

$$N_\pi^{-2} \times \delta(\vec{0}) = < \text{vac}' | Q_5^a Q_5^a | \text{vac}' > = \int < \text{vac}' | Q_5^a | \pi_b(\vec{p}) > d\vec{p} < \pi_b(\vec{p}) | Q_5^a | \text{vac}' >, \quad (31)$$
where we have saturated the intermediate states with pions. The index $b$ is summed and there is no summation over the index $a$. With Eq.(30) and Eq.(31) we then have

$$N_{\pi}^{-2} = \frac{1}{2} \cdot (2\pi)^3 \cdot m_{\pi} c_{\pi}^2,$$

which links $N_{\pi}$ of vacuum structure with pion mass and pion decay constant. We shall now substitute explicitly the pion state as in equation (26) in equation (29). We shall also substitute the value of normalisation constant in equation (26) by equation (32). On using straightforward commutation relations, we then obtain that

$$<\pi^a(\vec{0}) | H_{sb} | \pi^a(\vec{0})> = \frac{2}{m_{\pi} c_{\pi}^2} \times \frac{1}{(2\pi)^3} \times <vac' | Q^a_{5} H_{sb} Q^a_{5} | vac'>$$

$$= \frac{2}{m_{\pi} c_{\pi}^2} \times \frac{1}{(2\pi)^3} \times \frac{1}{2} <vac' | [[Q^a_{5}, H_{sb}], Q^a_{5}] | vac'>$$

$$= \frac{2}{m_{\pi} c_{\pi}^2} \times \frac{m}{2} <vac'|\bar{\psi}\psi|vac' > \times \delta(\vec{0}).$$

(33)

(34)

From equation (34) and equation (29) we then obtain that

$$m_{\pi}^2 = -\frac{m}{c_{\pi}^2} <\bar{\psi}\psi >.$$

(35)

which is the familiar result for current algebra.

V. CHARGE RADIUS OF PION

With the wave function of the pion as above, we may next estimate the size of the Goldstone pion as related to the vacuum structure. The pion state with momentum $\vec{p}$ using translational invariance from Eq.(26) becomes

$$|\pi^+(\vec{p})> = N_{\pi} \int d\vec{k} q_i^i (\vec{k} + \vec{p}/2)^i (\tau^+)_{ij} \bar{q}^j_i (\vec{k}) \bar{u}(\vec{k}) |vac'>.$$

(36)

In Breit frame the electric form factor is given by [11]

$$G_E(t) = (2\pi)^3 <\pi^+(-\vec{p}) | J_0 | \pi^+(\vec{p}) >$$

(37)

where $t = -4p^2$ and $J_0 = \bar{\psi}^\dagger \psi$. This may be evaluated directly as
\[ G_E(t) = eN_e^2 \times \int d\vec{k} \bar{u}(\vec{k} - \frac{\vec{p}}{2})^* \bar{u}(\vec{k} + \frac{\vec{p}}{2}) \]
\[ \times \left\{ u_1(\vec{k} - \vec{p})u_1(\vec{k} + \vec{p}) + (k^2 - p^2)u_2(\vec{k} - \vec{p})u_2(\vec{k} + \vec{p}) \right\}. \] (38)

In the above
\[ u_1(\vec{k}) = \sqrt{\frac{1 + \sin^2 f(|\vec{k}|)}{2}}; \quad u_2(\vec{k}) = \frac{1}{|\vec{k}|} \sqrt{\frac{1 - \sin^2 f(|\vec{k}|)}{2}}. \] (39)

To calculate the charge radius we expand the above in powers of $\vec{p}$ and the coefficient of $p^2$ will be related to the charge radius through
\[ G_E(t) = e\left(1 + \frac{1}{6} R^2_{ch} + \cdots \right). \] (40)

With $G_E(t)$ as in Eq.(38) we then obtain that
\[ < R^2_{ch} > = \frac{1}{2} \int d\vec{k} \left[ \frac{1}{4} (u_0'(k)^2 - \frac{2}{k} u_0' u_0 - u_0'' u_0) + u_0^2 \left\{ (u_1'' - \frac{2}{k} u_1' - u_1'' u_1) + 3 u_2^2 + k^2 (u_2'' - \frac{2}{k} u_2' - u_2'' u_2) \right\} \right], \] (41)

where we have substituted
\[ u_0(k) = \frac{1}{\sqrt{\left( f \sin^2 f(k) dk \right)}} \times \bar{u}(k), \] (42)

and, primes denote differentiation with respect to $k$.

The above formula applies for any known vacuum realignment with condensates. Let us now estimate the charge radius in Nambu Jona-Lasinio model. We shall also use Eq.(34) for pion decay constant so that chiral symmetry was approximately true. With $H_{sb} = m\bar{\psi}\psi$, the extra contribution to the energy density is $-m \times 2NI$. On extremisation the modified gap function is given by
\[ \tan^2 f(k) = \frac{2G(2N + 1)I + m}{|k|^2} = \frac{M'}{k}, \] (43)

where $M' = 2G(2N + 1)I + m$ satisfies the equation parallel to Eq.(16) given as
\[ 1 = \frac{2G(2N + 1)}{(2\pi)^3} \times \int^\Lambda \frac{d\vec{k}}{\sqrt{k^2 + M'^2}} + m/M'. \] (44)
Thus here we may also have a vacuum realignment.

We shall now choose an optimal set of parameters for $G\Lambda^2$, $\Lambda$ and $m$. Then, for example, with $\Lambda = 420$ MeV, $G\Lambda^2=2.24$ and $m=16$ MeV, we get $M=305$ MeV, $< -\bar{\psi}\psi >^{1/3} = 220$ MeV, $R^2_\pi=0.25$ fermi$^2$ and $c_\pi=94$ MeV. Here we have taken $m_\pi = 138$ MeV. As a further illustration to see how the the corresponding quantities change with parameters, for $\Lambda=500$ MeV, $G\Lambda^2=2.15$, $m=10$ MeV, we have, $M=320$ MeV, $< -\bar{\psi}\psi >^{1/3} = 255$ MeV, $R^2_\pi=0.20$ fermi$^2$ and $c_\pi=93$ MeV. We note that the pion structure as arising from vacuum realignment appears to give a smaller value of charge radius than would be expected. In fact, with $G\Lambda^2 = 2.0$, $\Lambda = 700$ MeV and $m=5$ MeV $[^3]$, we obtain that $M=360$ MeV, $< -\bar{\psi}\psi >^{1/3} = 342$ MeV, $R^2_\pi=0.13$ fermi$^2$ and $c_\pi=103$ MeV. Thus the above set of parameters do not appear to be acceptable $[^6]$. We may also note that the four component Dirac field operators for the quarks will change the above numbers as examined elsewhere $[^2]$, which however does not change the above remarks. The above illustrates the nature of constraints derived for symmetry breaking through determination of pion wave function. A parallel approach with Bogoliubov transformations and Schwinger Dyson equation has been used to obtain Salpeter wave function for the pion, which, however does not permit the definition of pion as a state since the wave function is not normalisable and therefore did not give rise to such constraints.

VI. NEW MODES IN VACUUM

When vacuum has a structure, there can be excitations present due to such a structure. For chiral symmetry breaking, let us substitute

$$\bar{\psi}(\vec{x})\psi(\vec{x}) = < \bar{\psi}(\vec{x})\psi(\vec{x}) > + M^2_{sc}\sigma(\vec{x}) $$

$$\equiv \mu^3 + M^2_{sc}\sigma(\vec{x}),$$

(45)

where $M_{sc}$ is a mass parameter and $\sigma(\vec{x})$ may represent the scalar field of vacuum fluctuations. Then $\sigma(\vec{x})$ can represent quantum fluctuations of the condensate. In fact, we may evaluate
\[<\text{vac}'|(\bar{\psi}(\vec{x})\psi(\vec{x}) - \mu^3)(\bar{\psi}(\vec{y})\psi(\vec{y}) - \mu^3)|\text{vac}'> = M_{sc}^4 <\text{vac}'|\sigma(\vec{x})\sigma(\vec{y})|\text{vac}'> \]

\[\simeq \frac{M_{sc}^4}{(2\pi)^2} \int \frac{e^{i\vec{k}.(\vec{x} - \vec{y})}}{2(\vec{k}^2 + m_\sigma^2)^{1/2}} d\vec{k}, \tag{46}\]

where we approximate \(\sigma(\vec{x})\) by a free field of mass \(m_\sigma\). Let us define

\[I(\vec{k}) = \int d\vec{x} \exp(-i\vec{k}.\vec{x}) <\text{vac}'|(\bar{\psi}(\vec{x})\psi(\vec{x}) - \mu^3)(\bar{\psi}(\vec{0})\psi(\vec{0}) - \mu^3)|\text{vac}'> \tag{47}\]

In that case, clearly free field approximation for \(\sigma(\vec{x})\) corresponds to

\[I(\vec{k}) = \frac{M_{sc}^4}{2\sqrt{m_\sigma^2 + \vec{k}^2}}. \tag{48}\]

Explicit evaluation of the left hand side of equation (47) in the limit of small \(|\vec{k}|\) yields

\[I(\vec{k}) \simeq \frac{1}{\pi^2} \left[ \left( \frac{\Lambda^3}{3} - M^2 \Lambda + M^3 \tan^{-1}\left( \frac{\Lambda}{M} \right) \right) \right. \]

\[\left. - \vec{k}^2 \left\{ \frac{\Lambda^5}{6(\Lambda^2 + M^2)^2} + \frac{1}{8} \frac{\Lambda^3 M^2}{(\Lambda^2 + M^2)^2} + \frac{3}{16} \frac{\Lambda M^2}{(\Lambda^2 + M^2)} - \frac{3}{16} M \tan^{-1}\left( \frac{\Lambda}{M} \right) \right\} \right] \]

\[= \frac{1}{8\pi^3} \left[ \frac{M_{sc}^4}{2m_\sigma} - \vec{k}^2 \frac{M_{sc}^4}{4m_\sigma^2} \right]. \tag{49}\]

where, in equation (48) we have kept terms up to \(\vec{k}^2\). Equating equal powers of \(\vec{k}\) in equation (49) and eliminating \(M_{sc}\) in favor of \(m_\sigma\) yields, with \(x = M/\Lambda\),

\[m_\sigma^2 = \frac{\Lambda^2}{2} \times \left[ \frac{x^3 - x^2 + x^3 \tan^{-1}\left( \frac{1}{x} \right)}{6(1+x^2)^2} - \frac{3}{16} x \tan^{-1}\left( \frac{1}{x} \right) + \frac{3}{16} \frac{x^2}{(1+x^2)^2} + \frac{1}{8} \frac{x^2}{(1+x^2)^2} \right]. \tag{50}\]

We may next estimate the mass of such a mode for different values of \(\Lambda\) and \(M\) obtained in the previous section. For example, for \(\Lambda = 420\) MeV and \(M = 305\) MeV, \(m_\sigma \simeq 2.07M\); for \(\Lambda = 500\) MeV and \(M = 320\) MeV, \(m_\sigma \simeq 2.27M\) and for \(\Lambda = 700\) MeV and \(M = 360\) MeV, \(m_\sigma \simeq 2.65M\). These may be compared with the mass of \(\sigma\) field obtained through an approximate determination of the pole of the propagator with polarisation insertion, which is given as \(m_\sigma = (4M^2 + m_\pi^2)^{1/2}\) [4].

VII. DISCUSSIONS

We thus consider here chiral symmetry breaking as a vacuum realignment with an explicit construct for destabilised vacuum. The new feature of this approach [3] is that it enables
us to relate the function that describes the vacuum structure determined variationally to the wave function of the pion as the localised Goldstone mode in a straightforward manner. This language is not only physically appealing reproducing the conventional results but also puts severe constraints on the parameters for symmetry breaking as illustrated here for NJL model. Some other aspects of low energy hadronic properties as related to the vacuum structure for chiral symmetry breaking have been discussed elsewhere [12].

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[16] There is a disagreement over the value of $\Lambda$. Further, Eq.(17) and the corresponding equation by Bernard in Ref.3 have a difference of a factor 2. We presume that this is a printing error since $G\Lambda^2$ they have taken does not satisfy their bound where as it is consistent with Eq.(17).

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