Inflationary Cosmology

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Abstract. The big bang model and the history of the early universe according to the grand unified theories are introduced. The shortcomings of big bang are discussed together with their resolution by inflationary cosmology. Inflation, the subsequent oscillation and decay of the inflaton, and the resulting ‘reheating’ of the universe are studied. The density perturbations produced by inflation and the temperature fluctuations of the cosmic background radiation are discussed. The hybrid inflationary model is described. Two ‘natural’ extensions of this model which avoid the disaster encountered in its standard realization from the overproduction of monopoles are presented. Successful ‘reheating’ satisfying the gravitino constraint takes place after the end of inflation in all three versions of hybrid inflation. Adequate baryogenesis via a primordial leptogenesis occurs consistently with the solar and atmospheric neutrino oscillation data. The primordial lepton asymmetry is turned partly into baryon asymmetry via the sphalerons which are summarized.

1 Introduction

The discovery of the cosmic microwave background radiation (CMBR) in 1964 together with the observed Hubble expansion of the universe had established hot big bang cosmology as a viable model of the universe. The success of the theory of nucleosynthesis in reproducing the observed abundance pattern of light elements together with the proof of the black body character of the CMBR then imposed hot big bang as the standard cosmological model. This model combined with grand unified theories (GUTs) of strong, weak and electromagnetic interactions provides an appropriate framework for discussing the very early stages of the universe evolution.

Despite its great successes, the standard big bang (SBB) cosmological model had a number of long-standing shortcomings. One of them is the so-called horizon problem. The CMBR which we receive now has been emitted from regions of the sky which never communicated causally before sending light to us. The question then arises how come the temperature of the black body radiation from these regions is so finely tuned as the measurements of the cosmic background explorer (COBE) show. Another issue is the flatness problem. The present universe appears almost flat. This requires that, in its early stages, the universe was flat with a great accuracy, which needs some explanation. Also, combined with GUTs which predict the existence of superheavy magnetic monopoles, the SBB model leads to a cosmological
catastrophe due to the overproduction of these monopoles. Finally, the model does not explain the origin of the small density perturbations required for the structure formation in the universe \[1\] and the generation of the observed \[3\] temperature fluctuations in the CMBR.

Inflation \[7,8\] offers an elegant solution to all these problems of the SBB model. The idea behind inflation is that, in the early universe, a real scalar field (the inflaton) was displaced from its vacuum value. If the potential energy density of this field happens to be quite flat, the roll-over of the field towards the vacuum can be very slow for a period of time. During this period, the energy density is dominated by the almost constant potential energy density of the inflaton. As a consequence, the universe undergoes a period of quasi-exponential expansion, which can readily solve the horizon and flatness problems by stretching the distance over which causal contact is established and reducing any pre-existing curvature in the universe. It can also dilute adequately the GUT magnetic monopoles. Moreover, it provides us with the primordial density perturbations which are necessary for explaining the large scale structure formation in the universe \[6\] as well as the temperature fluctuations observed in the CMBR. Inflation can be easily incorporated in GUTs. It occurs during the GUT phase transition at which the GUT gauge symmetry breaks by the vacuum expectation value (vev) of a Higgs field, which also plays the role of the inflaton.

After the end of inflation, the inflaton enters into an oscillatory phase about the vacuum. The oscillations are damped because of the dilution of the field energy density caused by the expansion of the universe and the decay of the inflaton into ‘light’ matter. The radiation energy density generated by the inflaton decay eventually dominates over the field energy density and the universe returns to a normal big bang type evolution. The cosmic temperature at which this occurs is historically called ‘reheat’ temperature although there is actually neither supercooling nor reheating of the universe \[9\].

An important disadvantage of the early realizations of inflation is that they require tiny coupling constants in order to reproduce the COBE measurements on the CMBR. To solve this ‘naturalness’ problem, the hybrid inflationary scenario has been introduced \[10\]. The basic idea was to use two real scalar fields instead of one that was normally used. One field may be a gauge non-singlet and provides the ‘vacuum’ energy density which drives inflation, while the other is the slowly varying field during inflation. This splitting of roles between two fields allows us to reproduce the temperature fluctuations of the CMBR with ‘natural’ (not too small) values of the relevant parameters in contrast to previous realizations of inflation. Hybrid inflation, although initially introduced in the context of non-supersymmetric GUTs, can be ‘naturally’ incorporated \[11,12\] in supersymmetric (SUSY) GUTs.

Unfortunately, the GUT monopole problem reappears in hybrid inflation. The termination of inflation, in this case, is abrupt and is followed by a ‘waterfall’ regime during which the system falls towards the vacuum manifold.
and starts performing damped oscillations about it. If the vacuum manifold is homotopically non-trivial, topological defects will be copiously formed \[13\] by the Kibble mechanism \[14\] since the system can end up at any point of this manifold with equal probability. So a cosmological disaster is encountered in the hybrid inflationary models which are based on a gauge symmetry breaking predicting the existence of magnetic monopoles.

One idea \[13,15,16\] for solving the monopole problem of SUSY hybrid inflation is to include into the standard superpotential for hybrid inflation the leading non-renormalizable term. This term cannot be excluded by any symmetries and, if its dimensionless coefficient is of order unity, can be comparable with the trilinear coupling of the standard superpotential (whose coefficient is \(\sim 10^{-3}\)). Actually, we have two options. We can either keep \[13\] both these terms or remove \[13,16\] the trilinear term by imposing an appropriate discrete symmetry and keep only the leading non-renormalizable term. The pictures which emerge in the two cases are quite different. However, they share an important common feature. The GUT gauge group is already broken during inflation and thus no topological defects can form at the end of inflation. Consequently, the monopole problem is solved.

A complete inflationary scenario should be followed by a successful ‘re-heating’ satisfying the gravitino constraint \[17\] on the ‘reheat’ temperature, \(T_r \sim 10^9\) GeV, and generating the observed baryon asymmetry of the universe (BAU). In hybrid inflationary models, it is \[18\] generally preferable to generate the BAU by first producing a primordial lepton asymmetry \[19\] which is then partly converted into baryon asymmetry by the non-perturbative electroweak sphaleron effects \[20,21\]. Actually, in many specific models, this is the only way to generate the BAU since the inflaton decays into right handed neutrino superfields. The subsequent decay of these superfields into lepton (antilepton) and electroweak Higgs superfields can only produce a lepton asymmetry. Successful ‘reheating’ can be achieved \[15,16\] in hybrid inflationary models in accord with the experimental requirements from solar and atmospheric neutrino oscillations and with ‘natural’ values of parameters.

## 2 The Big Bang Model

We will start with an introduction to the salient features of the SBB model \[1\] and a summary of the history of the early universe in accordance to GUTs.

### 2.1 Hubble Expansion

For cosmic times \(t \simeq t_P = M_P^{-1} \sim 10^{-44}\) sec \((M_P = 1.22 \times 10^{19}\) GeV is the Planck scale) after the big bang, the quantum fluctuations of gravity cease to exist. Gravitation can then be adequately described by classical relativity. Strong, weak and electromagnetic interactions, however, require relativistic quantum field theoretic treatment and are described by gauge theories.
An important principle, on which SBB is based, is that the universe is homogeneous and isotropic. The strongest evidence for this cosmological principle is the observed isotropy of the CMBR. Under this assumption, the four dimensional space-time is described by the Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta \, d\varphi^2) \right], \quad (1)$$

where $r$, $\varphi$ and $\theta$ are ‘comoving’ polar coordinates, which remain fixed for objects that have no other motion than the general expansion of the universe. $k$ is the ‘scalar curvature’ of the 3-space and $k = 0$, $> 0$ or $< 0$ corresponds to flat, closed or open universe. The dimensionless parameter $a(t)$ is the ‘scale factor’ of the universe and describes cosmological expansion. We normalize it by taking $a_0 \equiv a(t_0) = 1$, where $t_0$ is the present cosmic time.

The ‘instantaneous’ radial physical distance is given by

$$R = a(t) \int_0^r \frac{dr}{(1 - kr^2)^{1/2}}. \quad (2)$$

For flat universe ($k = 0$), $\bar{R} = a(t) \bar{r}$ ($\bar{r}$ is a ‘comoving’ and $\bar{R}$ a physical vector in 3-space) and the velocity of an object is

$$\bar{V} = \frac{d\bar{R}}{dt} = \frac{\dot{a}}{a} \bar{R} + a \frac{d\bar{r}}{dt}, \quad (3)$$

where overdots denote derivation with respect to cosmic time. The second term in the right hand side (rhs) of this equation is the so-called ‘peculiar velocity’, $\bar{v} = a(t) \dot{\bar{r}}$, of the object, i.e., its velocity with respect to the ‘comoving’ coordinate system. For $\bar{v} = 0$, (3) becomes

$$\bar{V} = \frac{\dot{a}}{a} \bar{R} \equiv H(t) \bar{R}, \quad (4)$$

where $H(t) \equiv \dot{a}(t)/a(t)$ is the Hubble parameter. This is the well-known Hubble law asserting that all objects run away from each other with velocities proportional to their distances and is the first success of SBB cosmology.

### 2.2 Friedmann Equation

Homogeneity and isotropy of the universe imply that the energy momentum tensor takes the diagonal form $(T^{\mu}_{\nu}) = \text{diag}(-\rho, p, p, p)$, where $\rho$ is the energy density of the universe and $p$ the pressure. Energy momentum conservation $(T^{\mu}_{\nu} ; \nu = 0)$ then takes the form of the continuity equation

$$\frac{dp}{dt} = -3H(t)(\rho + p), \quad (5)$$
where the first term in the rhs describes the dilution of the energy due to the expansion of the universe and the second term corresponds to the work done by pressure. Equation (5) can be given the following more transparent form

\[ d \left( \frac{4\pi}{3} a^3 \rho \right) = -p 4\pi a^2 da , \]  

which indicates that the energy loss of a ‘comoving’ sphere of radius \( a(t) \) equals the work done by pressure on its boundary as it expands.

For a universe described by the metric in (1), Einstein’s equations

\[ R_{\mu}^{\nu} - \frac{1}{2} \delta_{\mu}^{\nu} R = 8\pi G T_{\mu}^{\nu} , \]

where \( R_{\mu}^{\nu} \) and \( R \) are the Ricci tensor and scalar curvature and \( G \equiv M_{P}^{-2} \) is the Newton’s constant, lead to the Friedmann equation

\[ H^2 \equiv \left( \frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} . \]  

Averaging \( p \), we can write \( \rho + p = (1 + w)\rho = \gamma \rho \) and (5) becomes \( \dot{\rho} = -3H\gamma \rho \), which gives \( d\rho/\rho = -3\gamma da/a \) and \( \rho \propto a^{-3\gamma} \). For a universe dominated by pressureless matter, \( p = 0 \) and, thus, \( \gamma = 1 \), which gives \( \rho \propto a^{-3} \). This is interpreted as mere dilution of a fixed number of particles in a ‘comoving’ volume due to the cosmological expansion. For a radiation dominated universe, \( p = \rho/3 \) and, thus, \( \gamma = 4/3 \), which gives \( \rho \propto a^{-4} \). Here, we get an extra factor of \( a(t) \) due to the red-shifting of all wave lengths by the expansion. Substituting \( \rho \propto a^{-3\gamma} \) in (8) with \( k = 0 \), we get \( \dot{a}/a \propto a^{-3\gamma/2} \) and, thus, \( a(t) \propto t^{2/3\gamma} \). Taking into account that \( a(t_0) = 1 \), this gives

\[ a(t) = (t/t_0)^{2/3\gamma} . \]  

For a matter dominated universe, we get the expansion law \( a(t) = (t/t_0)^{2/3} \).

‘Radiation’, however, expands as \( a(t) = (t/t_0)^{1/2} \).

The early universe is radiation dominated and its energy density is

\[ \rho = \frac{\pi^2}{30} \left( N_{b} + \frac{7}{8} N_{f} \right) T^4 \equiv c T^4 , \]  

where \( T \) is the cosmic temperature and \( N_{b(f)} \) the number of massless bosonic (fermionic) degrees of freedom. The quantity \( g_{*} = N_{b} + (7/8) N_{f} \) is called effective number of massless degrees of freedom. The entropy density is

\[ s = \frac{2\pi^2}{45} g_{*} T^3 . \]  

Assuming adiabatic universe evolution, i.e., constant entropy in a ‘comoving’ volume \( (sa^3 \equiv \text{constant}) \), we obtain \( aT = \text{constant} \). The temperature-time relation during radiation dominance is then derived from (8) (with \( k = 0 \)):

\[ T^2 = \frac{M_{P}}{2(8\pi c/3)^{1/2} t} . \]
Classically, the expansion starts at $t = 0$ with $T = \infty$ and $a = 0$. This initial singularity is, however, not physical since general relativity fails for $t \leq t_P$ (the Planck time). The only meaningful statement is that the universe, after a yet unknown initial stage, emerges at $t \sim t_P$ with $T \sim M_P$.

### 2.3 Important Cosmological Parameters

The most important parameters describing the expanding universe are:

i. The present value of the Hubble parameter (known as Hubble constant) $H_0 \equiv H(t_0) = 100 \ h \ \text{km sec}^{-1} \ \text{Mpc}^{-1}$ ($h \approx 0.72 \pm 0.07$ [22]).

ii. The fraction $\Omega = \rho/\rho_c$, where $\rho_c$ is the critical density corresponding to a flat universe. From (8), $\rho_c = 3H^2/8\pi G$ and $\Omega = 1 + k/a^2 H^2$. $\Omega = 1$, $> 1$ or $< 1$ corresponds to flat, closed or open universe. Assuming inflation (see below), the present value of $\Omega$ must be $\Omega_0 = 1$ in accord with the recent DASI observations which yield $\Omega_0 = 1 \pm 0.04$. The low deuterium abundance measurements give $\Omega_B h^2 \approx 0.20 \pm 0.001$, where $\Omega_B$ is the baryonic contribution to $\Omega$. This result implies that $\Omega_B \approx 0.039 \pm 0.077$. The total contribution $\Omega_M$ of matter to $\Omega_0$ can then be determined from the measurements of the baryon-to-matter ratio in clusters. It is found that $\Omega_M \approx 1/3$, which shows that most of the matter in the universe is non-baryonic, i.e., dark matter. Moreover, we see that about 2/3 of the energy density of the universe is not even in the form of matter and we call it dark energy.

iii. The deceleration parameter

$$q = -\frac{(\ddot{a}/\dot{a})}{(\dot{a}/a)} = \frac{\rho + 3p}{2\rho_c}, \quad \text{Eq} \ (13)$$

Measurements of type Ia supernovae [26] indicate that the universe is speeding up ($q_0 < 0$). This requires that, at present, $p < 0$ as can be seen from (13). Negative pressure can only be attributed to the dark energy since matter is pressureless. Equation (13) gives $q_0 = (\Omega_0 + 3w_X \Omega_X)/2$, where $\Omega_X = \rho_X/\rho_c$ and $w_X = p_X/\rho_X$ with $\rho_X$ and $p_X$ being the dark energy density and pressure. Observations prefer $w_X = -1$, with a 95% confidence limit $w_X < -0.6$ [27]. Thus, dark energy can be interpreted as something close to a non-zero cosmological constant (see below).

### 2.4 Particle Horizon

Light travels only a finite distance from the time of big bang ($t = 0$) until some cosmic time $t$. From (14), we find that the propagation of light along the radial direction is described by $a(t)dr = dt$. The particle horizon, which is the ‘instantaneous’ distance at $t$ travelled by light since $t = 0$, is then

$$d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')} \quad \text{Eq} \ (14)$$
The particle horizon is an important notion since it coincides with the size of the universe already seen at time \( t \) or, equivalently, with the distance at which causal contact has been established at \( t \). Equations (9) and (14) give

\[
d_H(t) = \frac{3\gamma}{3\gamma - 2} t, \quad \gamma \neq 2/3 .
\]  

(15)

Also,

\[
H(t) = \frac{2}{3\gamma} t^{-1}, \quad d_H(t) = \frac{2}{3\gamma - 2} H^{-1}(t) .
\]  

(16)

For 'matter' ('radiation'), these formulae become

\[
d_H(t) = 2H^{-1}(t) = 3t \quad (d_H(t) = H^{-1}(t) = 2t).\]

Assuming matter dominance, the present particle horizon (cosmic time) is

\[
d_H(t_0) = 2H_0^{-1} \approx 6,000 \, h^{-1} \text{ Mpc} \quad \text{(} t_0 = 2H_0^{-1}/3 \approx 6.5 \times 10^9 \, h^{-1} \text{ years). \ The present } \rho_c = 3H_0^2/8\pi G \approx 1.9 \times 10^{-29} \, h^2 \, \text{gm/cm}^3.\]

2.5 Brief History of the Early Universe

We will now briefly describe the early stages of the universe evolution according to GUTs [2]. We take a GUT based on the gauge group \( G (= SU(5), SO(10), SU(3)^3, ...) \) with or without SUSY. At a superheavy scale \( M_X \sim 10^{16} \text{ GeV} \) (the GUT mass scale), \( G \) breaks to the standard model gauge group \( G_S = SU(3)_c \times SU(2)_L \times U(1)_Y \) by the vev of an appropriate Higgs field \( \phi \). (For simplicity, we consider that this breaking occurs in one step.) \( G_S \) is, subsequently, broken to \( SU(3)_c \times U(1)_{em} \) at the electroweak scale \( M_W \).

GUTs together with the SBB model provide a suitable framework for discussing the early history of the universe for cosmic times \( t > \sim 10^{-44} \text{ sec.} \) They predict that the universe, as it expands and cools after the big bang, undergoes a series of phase transitions during which the gauge symmetry is gradually reduced and several important phenomena take place.

After the big bang, \( G \) was unbroken and the universe was filled with a hot 'soup' of massless particles which included not only photons, quarks, leptons and gluons but also the weak gauge boson \( W^\pm, Z^0 \), the GUT gauge bosons \( X, Y, \ldots \) and several Higgs bosons. (In the SUSY case, all the SUSY partners of these particles were also present.) At cosmic time \( t \sim 10^{-37} \text{ sec} \) corresponding to temperature \( T \sim 10^{16} \text{ GeV} \), \( G \) broke down to \( G_S \) and the \( X, Y, \ldots \) gauge bosons together with some Higgs bosons acquired superheavy masses of order \( M_X \). Their out-of-equilibrium decay could, in principle, produce the observed BAU (with the reservation at the end of Sect.14.2). Important ingredients are the violation of baryon number, which is inherent in GUTs, and C and CP violation. This is the second (potential) success of SBB.

During the GUT phase transition, topologically stable extended objects such as monopoles [1], cosmic strings [3], or domain walls [4] can also be produced. Monopoles, which exist in most GUTs, can lead into cosmological problems which are, however, avoided by inflation [5] (see Sects.3.3 and 4.3). This is a period of an exponentially fast expansion of the universe which
can occur during some GUT phase transition. Cosmic strings can contribute to the primordial density perturbations necessary for structure formation in the universe whereas domain walls are absolutely catastrophic and GUTs should be constructed so that they avoid them (see e.g., [33]) or inflation should be used to remove them from the scene.

At $t \approx 10^{-10}$ sec or $T \approx 100$ GeV, the electroweak transition takes place and $G_S$ breaks to $SU(3)_c \times U(1)_{em}$. $W^\pm$, $Z^0$ and the electroweak Higgs fields acquire masses $\sim M_W$. Subsequently, at $t \approx 10^{-4}$ sec or $T \sim 1$ GeV, color confinement sets in and the quarks get bounded forming hadrons.

The direct involvement of particle physics essentially ends here since most of the subsequent phenomena fall into the realm of other branches. We will, however, sketch some of them since they are crucial for understanding the earlier stages of the universe evolution where their origin lies.

At $t \approx 180$ sec ($T \approx 1$ MeV), nucleosynthesis takes place, i.e., protons and neutrons form nuclei. The abundance of light elements ($D$, $^3He$, $^4He$ and $^7Li$) depends crucially on the number of light particles (with mass $\lesssim 1$ MeV), i.e., the number of light neutrinos, $N_\nu$, and $\Omega_B h^2$. Agreement with observations is achieved for $N_\nu = 3$ and $\Omega_B h^2 \approx 0.020$. This is the third success of SBB cosmology. Much later, at the so-called ‘equidensity’ point, $t_{eq} \approx 5 \times 10^4$ years, matter dominates over radiation.

At cosmic time $t \approx 200,000$ $h^{-1}$years ($T \approx 3,000$ K), we have the ‘decoupling’ of matter and radiation and the ‘recombination’ of atoms. After this, radiation evolves as an independent (not interacting) component of the universe and is detected today as CMBR with temperature $T_0 \approx 2.73$ K. The existence of this radiation is the fourth success of the SBB model. Finally, structure formation in the universe starts at $t \approx 2 \times 10^9$ years.

### 3 Shortcomings of Big Bang

The SBB cosmological model has been very successful in explaining, among other things, the Hubble expansion of the universe, the existence of the CMBR and the abundances of the light elements which were formed during primordial nucleosynthesis. Despite its great successes, this model had a number of long-standing shortcomings which we will now summarize:

#### 3.1 Horizon Problem

The CMBR, which we receive now, was emitted at the time of ‘decoupling’ of matter and radiation when the cosmic temperature was $T_d \approx 3,000$ K. The decoupling time, $t_d$, can be calculated from

$$\frac{T_0}{T_d} = \frac{2.73 \text{ K}}{3,000 \text{ K}} = \frac{a(t_d)}{a(t_0)} = \left(\frac{t_d}{t_0}\right)^{2/3}.$$  \hspace{1cm} (17)

It turns out that $t_d \approx 200,000$ $h^{-1}$ years.
The distance over which the CMBR has travelled since its emission is

$$a(t_0) \int_{t_d}^{t_0} \frac{dt'}{a(t')} = 3t_0 \left[ 1 - \left( \frac{t_d}{t_0} \right)^{2/3} \right] \approx 3t_0 \approx 6,000 \text{ } h^{-1} \text{Mpc}, \quad (18)$$

which essentially coincides with the present particle horizon size. A sphere around us with radius equal to this distance is called the ‘last scattering surface’ since the CMBR observed now has been emitted from it. The particle horizon size at $$t_d$$ was $2H^{-1}(t_d) = 3t_d \approx 0.168 \text{ } h^{-1} \text{Mpc}$ and expanded until now to become equal to $0.168 \text{ } h^{-1}(a(t_0)/a(t_d)) \text{ Mpc} \approx 184 \text{ } h^{-1} \text{Mpc}$. The angle subtended by this ‘decoupling’ horizon at present is $\theta_d \approx \frac{184}{6,000} \approx 0.03 \text{ rads} \approx 2^\circ$. Thus, the sky splits into $4\pi/(0.03)^2 \approx 14,000$ patches which never communicated causally before sending light to us. The question then arises how come the temperature of the black body radiation from all these patches is so accurately tuned as the results of COBE require.

### 3.2 Flatness Problem

The present energy density of the universe has been observed to be very close to its critical energy density corresponding to a flat universe ($\Omega_0 = 1 \pm 0.04$). Equation (18) implies that $(\rho - \rho_c)/\rho_c = 3(8\pi G \rho_c)^{-1}(k/a^2)$ is proportional to $a$, for matter dominated universe. Thus, in the early universe, we have $|\rho - \rho_c|/\rho_c \ll 1$ and the question arises why the initial energy density of the universe was so finely tuned to be equal to its critical value.

### 3.3 Magnetic Monopole Problem

This problem arises only if we combine the SBB model with GUTs which predict the existence of magnetic monopoles. As already indicated, according to GUTs, the universe underwent a phase transition during which the GUT gauge symmetry group, $G$, broke to $G_S$. This breaking was due to the fact that, at a critical temperature $T_c$, an appropriate Higgs field, $\phi$, developed a non-zero vev. Assuming that this phase transition was a second order one, we have $\langle \phi \rangle(T) \approx \langle \phi \rangle(T = 0)(1 - T^2/T_c^2)^{1/2}$, $m_H(T) \approx \lambda \langle \phi \rangle(T)$, for the temperature dependent vev and mass of the Higgs field respectively at $T \leq T_c \approx 184/6,000\approx 0.03$ rads $\approx 2^\circ$. Thus, the sky splits into $4\pi/(0.03)^2 \approx 14,000$ patches which never communicated causally before sending light to us. The question then arises how come the temperature of the black body radiation from all these patches is so accurately tuned as the results of COBE require.

The GUT phase transition produces monopoles which are localized deviations from the vacuum with radius $\sim M_X^{-1}$, mass $m_M \sim M_X/\alpha_G$ and $\phi = 0$ at their center ($\alpha_G = g_G^2/4\pi$ with $g_G$ being the GUT gauge coupling constant). The vev of the Higgs field on a sphere, $S^2$, with radius $\gg M_X^{-1}$ around the monopole lies on the vacuum manifold $G/G_S$ and we, thus, obtain a mapping: $S^2 \rightarrow G/G_S$. If this mapping is homotopically non-trivial the topological stability of the monopole is guaranteed.

Monopoles can be produced when the fluctuations of $\phi$ over $\phi = 0$ between the vacua at $\pm \langle \phi \rangle(T)$ cease to be frequent. This occurs when the free energy
needed for $\phi$ to fluctuate from $\langle \phi \rangle(T) = m_H^2(T)$ to zero in a region of radius equal to the Higgs correlation length $\xi(T) = m_\xi^2(T)$ exceeds $T$. This condition reads

$$
\frac{4\pi}{3}\xi^3 \Delta V \approx T,
$$

where $\Delta V \sim \lambda^2 \langle \phi \rangle^4$ is the difference in free energy density between $\phi = 0$ and $\phi = \langle \phi \rangle(T)$. The Ginzburg temperature $T_G$ corresponds to the saturation of this inequality. So, at $T \approx T_G$, the fluctuations over $\phi = 0$ stop and $\langle \phi \rangle$ settles on $G/G_S$. At $T_G$, the universe splits into regions of size $\xi_G \sim (\lambda^2 T_c)^{-1}$, the Higgs correlation length at $T_G$, with $\phi$ being more or less aligned in each region. Monopoles are produced at the corners where such regions meet (Kibble mechanism [4]) and their number density is estimated to be $n_M \sim p\xi_G^{-3} \sim p\lambda^2 T_c^3$, where $p \sim 1/10$ is a geometric factor. The ‘relative’ monopole number density then turns out to be $r_M = n_M/T^3 \sim 10^{-6}$. We can derive a lower bound on $r_M$ by employing causality. The Higgs field $\phi$ cannot be correlated at distances bigger than the particle horizon size, $2t_G$, at $T_G$. This gives the causality bound

$$
n_M \gtrsim \frac{p}{4\pi (2t_G)^3},
$$

which implies that $r_M \gtrsim 10^{-10}$.

The subsequent evolution of monopoles, after $T_G$, is governed by [3]

$$
\frac{dn_M}{dt} = -Dn_M^2 - 3\dot{a} n_M,
$$

where the first term in the rhs (with $D$ being an appropriate constant) describes the dilution of monopoles by their annihilation with antimonopoles, while the second term corresponds to their dilution by Hubble expansion. The monopole-antimonopole annihilation proceeds as follows. Monopoles diffuse towards antimonopoles in the plasma of charged particles, capture each other in Bohr orbits and eventually annihilate. The annihilation is effective provided that the mean free path of monopoles in the plasma does not exceed their capture distance. This holds for $T \approx 10^{12}$ GeV. The overall result is that, if the initial relative monopole density $r_{M,\text{in}} \approx 10^{-9} \sim 10^{-9}$, the final one $r_{M,\text{fin}} \approx 10^{-9} \sim 10^{-9}$). This combined with the causality bound yields $r_{M,\text{fin}} \approx 10^{-10}$. However, the requirement that monopoles do not dominate the energy density of the universe at nucleosynthesis gives

$$
r_M(T \approx 1 \text{ MeV}) \approx 10^{-19},
$$

and we obtain a clear discrepancy of about ten orders of magnitude.

### 3.4 Density Perturbations

For structure formation [3] in the universe, we need a primordial density perturbation, $\delta \rho/\rho$, at all length scales with a nearly flat spectrum [36]. We also need an explanation of the temperature fluctuations of the CMBR observed by COBE [3] at angles $\theta \approx \theta_d \approx 2^\circ$ which violate causality (see Sect. 3.1).
Let us expand $\delta \rho/\rho$ in plane waves

$$\frac{\delta \rho}{\rho}(\vec{r}, t) = \int d^3k \delta_k(t)e^{i\vec{k}\cdot\vec{r}}, \quad (22)$$

where $\vec{r}$ is a ‘comoving’ vector in 3-space and $\vec{k}$ is the ‘comoving’ wave vector with $k = |\vec{k}|$ being the ‘comoving’ wave number ($\lambda = 2\pi/k$ is the ‘comoving’ wave length and the physical wave length is $\lambda_{\text{phys}} = a(t)\lambda$). For $\lambda_{\text{phys}} \leq H^{-1}$, the time evolution of $\delta_k$ is described by the Newtonian equation

$$\ddot{\delta}_k + 2H \dot{\delta}_k + \frac{v_s^2 k^2}{a^2} \delta_k = 4\pi G \rho \delta_k, \quad (23)$$

where the second term in the left hand side (lhs) comes from Hubble expansion and the third is the ‘pressure term’ ($v_s$ is the velocity of sound given by $v_s^2 = \frac{dp}{d\rho}$). The rhs corresponds to the gravitational attraction.

For the moment, put $H=0$ (static universe). There exists then a characteristic wave number $k_J$, the Jeans wave number, given by $k_J^2 = 4\pi G a^2 \rho/v_s^2$ and having the following property. For $k > k_J$, pressure dominates over gravitational attraction and the density perturbations just oscillate, whereas, for $k < k_J$, attraction dominates and the perturbations grow exponentially. In particular, for ‘matter’, $v_s = 0$ and all scales are Jeans unstable with

$$\delta_k \propto \exp\left(t/\tau\right), \quad \tau = (4\pi G \rho)^{-1/2}. \quad (24)$$

Now let us take $H \neq 0$. Since the cosmological expansion pulls the particles apart, we get a smaller growth:

$$\delta_k \propto a(t) \propto t^{2/3}, \quad (25)$$

in the matter dominated case. For ‘radiation’ ($\rho \neq 0$), we get essentially no growth of the density perturbations. This means that, in order to have structure formation in the universe, which requires $\delta\rho/\rho \sim 1$, we must have

$$\left(\frac{\delta\rho}{\rho}\right)_{\text{eq}} \sim 4 \times 10^{-5}(\Omega_M h^2)^{-1}, \quad (26)$$

at the ‘equidensity’ point, since the available growth factor for perturbations is given by $a_0/a_{\text{eq}} \sim 2.5 \times 10^4 \Omega_M h^2$. The question then is where these primordial density perturbations originate from.

## 4 Inflation

Inflation [7,8] is an idea which solves simultaneously all four cosmological puzzles and can be summarized as follows. Suppose there is a real scalar field $\phi$ (the inflaton) with (symmetric) potential energy density $V(\phi)$ which is quite flat near $\phi = 0$ and has minima at $\phi = \pm \langle\phi\rangle$ with $V(\pm \langle\phi\rangle) = 0$. At
high enough $T$’s, $\phi = 0$ in the universe due to the temperature corrections to $V(\phi)$. As $T$ drops, the effective potential approaches the $T=0$ potential but a little potential barrier separating the local minimum at $\phi = 0$ and the vacua at $\phi = \pm \langle \phi \rangle$ still remains. At some point, $\phi$ tunnels out to $\phi_1 \ll \langle \phi \rangle$ and a bubble with $\phi = \phi_1$ is created in the universe. The field then rolls over to the minimum of $V(\phi)$ very slowly (due to the flatness of the potential). During this slow roll-over, the energy density $\rho \approx V(\phi = 0) \equiv V_0$ remains essentially constant for quite some time. The Lagrangian density

$$L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$  \hspace{1cm} (27)$$

gives the energy momentum tensor

$$T_\mu^\nu = -\partial_\mu \phi \partial'^\nu \phi + \delta_\mu^\nu \left( \frac{1}{2} \partial_\lambda \phi \partial'^\lambda \phi - V(\phi) \right),$$  \hspace{1cm} (28)$$

which during the slow roll-over takes the form $T_\mu^\nu \approx -V_0 \delta_\mu^\nu$. This means that $\rho \approx -p \approx V_0$, i.e., the pressure is negative and equal in magnitude with the energy density, which is consistent with (5). As we will see, $a(t)$ grows fast and the ‘curvature term’, $k/a^2$, in (8) diminishes. We thus get

$$H^2 \equiv \frac{\dot{a}^2}{a} = \frac{8\pi G}{3} V_0,$$  \hspace{1cm} (29)$$

which gives $a(t) \propto e^{Ht}$, $H^2 = (8\pi G/3)V_0 = \text{constant}$. So the bubble expands exponentially for some time and $a(t)$ grows by a factor

$$\frac{a(t_f)}{a(t_i)} = \exp H(t_f - t_i) \equiv \exp H\tau,$$  \hspace{1cm} (30)$$

between an initial ($t_i$) and a final ($t_f$) time.

The inflationary scenario just described, known as ‘new’ inflation (with the inflaton starting from zero), is not the only realization of the idea of inflation. Another possibility is to consider the universe as it emerges at $t_P$. We can imagine a region of size $\ell_P \sim M_P^{-1}$ (the Planck length) where the inflaton acquires a large and almost uniform value and carries negligible kinetic energy. Under certain circumstances, this region can inflate (exponentially expand) as $\phi$ rolls down towards the vacuum. This type of inflation with the inflaton starting from large values is known as ‘chaotic’ inflation.

We will now show that, with an adequate number of e-foldings, $N = H\tau$, the first three cosmological puzzles are easily resolved (we leave the question of density perturbations for later).

### 4.1 Resolution of the Horizon Problem

The particle horizon during inflation

$$d(t) = e^{Ht} \int_{t_i}^{t} \frac{dt'}{e^{Ht'}} \approx H^{-1} \exp H(t - t_i),$$  \hspace{1cm} (31)$$
for \( t-t_i \gg H^{-1} \), grows as fast as \( a(t) \). At the end of inflation \( t = t_f \), \( d(t_f) \approx H^{-1} \exp H \tau \) and \( \phi \) starts oscillating about the minimum of the potential at \( \phi = \langle \phi \rangle \). It finally decays and ‘reheats’ the universe at a temperature \( T_r \sim 10^9 \) GeV. The universe then returns to normal big bang cosmology. The horizon \( d(t_f) \) is stretched during the \( \phi \)-oscillations by a factor \( \sim 10^9 \) depending on details and between \( T_r \) and the present by a factor \( T_r/T_0 \). So it finally becomes equal to \( H^{-1} \exp H \tau \left( \frac{10^9 \text{GeV}}{10^9 \text{GeV}} \right)^2 \), which should exceed \( 2H_0^{-1} \) in order to solve the horizon problem. Taking \( V_0 \approx M_X^4 \), \( M_X \sim 10^{16} \) GeV, we see that, with \( N = H \tau \approx 55 \), the horizon problem is evaded.

4.2 Resolution of the Flatness Problem

The ‘curvature term’ of the Friedmann equation, at present, is given by

\[
\frac{k}{a^2} \approx \left( \frac{k}{a^2} \right)_{b_i} e^{-2H \tau} \left( \frac{10^{-13} \text{GeV}}{10^9 \text{GeV}} \right)^2 ,
\]

where the terms in the rhs correspond to the ‘curvature term’ before inflation, and its growth factors during inflation, during \( \phi \)-oscillations and after ‘reheating’ respectively. Assuming \( (k/a^2)_{b_i} \sim (8\pi G/3)\rho \sim H^2 (\rho \approx V_0) \), we obtain \( \Omega_0 - 1 = k/a_0^2 H_0^2 \sim 10^{48} e^{-2H \tau} \) which is \(< 1\), for \( H \tau \gg 55 \). Strong inflation implies that the present universe is flat with a great accuracy.

4.3 Resolution of the Monopole Problem

For \( N \approx 55 \), the monopoles are diluted by at least 70 orders of magnitude and become irrelevant. Also, since \( T_r \ll m_M \), there is no monopole production after ‘reheating’. Extinction of monopoles may also be achieved by non-inflationary mechanisms such as magnetic confinement. For models leading to a possibly measurable monopole density see e.g., [40,41].

5 Detailed Analysis of Inflation

The Hubble parameter is not exactly constant during inflation as we, naively, assumed so far. It actually depends on the value of \( \phi \):

\[
H^2(\phi) = \frac{8\pi G}{3} V(\phi) .
\]

To find the evolution equation for \( \phi \) during inflation, we vary the action

\[
\int \sqrt{-\det(g)} \, d^4 x \left( \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) + M(\phi) \right) ,
\]

where \( g \) is the metric tensor and \( M(\phi) \) represents the coupling of \( \phi \) to ‘light’ matter causing its decay. We find

\[
\ddot{\phi} + 3H \dot{\phi} + \Gamma_\phi \dot{\phi} + V'(\phi) = 0 ,
\]
where the prime denotes derivation with respect to $\phi$ and $\Gamma_\phi$ is the decay width \cite{42} of the inflaton. Assume, for the moment, that the decay time of $\phi$, $t_d = \Gamma_\phi^{-1}$, is much greater than $H^{-1}$, the expansion time for inflation. Then the term $\Gamma_\phi \dot{\phi}$ can be ignored and (35) becomes

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0.$$ \hspace{1cm} (36)

Inflation is by definition the situation where $\ddot{\phi}$ is subdominant to the ‘friction term’ $3H\dot{\phi}$ (and the kinetic energy density is subdominant to the potential one). Equation (36) then reduces to the inflationary equation \cite{43}

$$3H\dot{\phi} = -V'(\phi),$$ \hspace{1cm} (37)

which gives

$$\ddot{\phi} = -\frac{V''(\phi)\dot{\phi}}{3H(\phi)} + \frac{V'(\phi)}{3H^2(\phi)}H'(\phi)\dot{\phi}.$$ \hspace{1cm} (38)

Comparing the two terms in the rhs of this equation with the ‘friction term’ in (36), we get the conditions for inflation (slow roll conditions):

$$|\eta| \equiv \frac{M_P^2}{8\pi} \left| \frac{V''(\phi)}{V(\phi)} \right| \leq 1,$$ \hspace{0.5cm} $\epsilon \equiv \frac{M_P^2}{16\pi} \left( \frac{V'(\phi)}{V(\phi)} \right)^2 \leq 1.$ \hspace{1cm} (39)

The end of the slow roll-over occurs when either of these inequalities is saturated. If $\phi_f$ is the value of $\phi$ at the end of inflation, then $t_f \sim H^{-1}(\phi_f)$.

The number of e-foldings during inflation can be calculated as follows:

$$N(\phi_i \rightarrow \phi_f) \equiv \ln \left( \frac{a(t_f)}{a(t_i)} \right) = \int_{t_i}^{t_f} Hdt = \int_{\phi_i}^{\phi_f} \frac{H(\phi)}{\phi} d\phi = -\int_{\phi_i}^{\phi_f} \frac{3H^2(\phi)d\phi}{V'(\phi)}.$$ \hspace{1cm} (40)

where (30), (37) and the definition of $H = \dot{a}/a$ were used. For simplicity, we can shift the field $\phi$ so that the global minimum of the potential is displaced at $\phi = 0$. Then, if $V(\phi) = \lambda \phi^\nu$ during inflation, we have

$$N(\phi_i \rightarrow \phi_f) = -\int_{\phi_i}^{\phi_f} \frac{3H^2(\phi)d\phi}{V'(\phi)} = -8\pi G \int_{\phi_i}^{\phi_f} \frac{V(\phi)d\phi}{V'(\phi)} = \frac{4\pi G}{\nu} (\phi_f^2 - \phi_i^2).$$ \hspace{1cm} (41)

Assuming that $\phi_i \gg \phi_f$, this reduces to $N(\phi) \approx (4\pi G/\nu)\phi_i^2$.

6 Coherent Oscillations of the Inflaton

After the end of inflation at $t_f$, the term $\ddot{\phi}$ takes over in (36) and $\phi$ starts performing coherent damped oscillations about the global minimum of the potential. The rate of energy density loss, due to ‘friction’, is given by

$$\dot{\rho} = \frac{d}{dt} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) = -3H\dot{\phi}^2 = -3H(\rho + p),$$ \hspace{1cm} (42)
where $\rho = \dot{\phi}^2/2 + V(\phi)$ and $p = \dot{\phi}^2/2 - V(\phi)$. Averaging $p$ over one oscillation of $\phi$ and writing $\rho + p = \gamma \rho$, we get $\rho \propto a^{-3\gamma}$ and $a(t) \propto t^{2/3\gamma}$ (see Sect. 2.2).

The number $\gamma$ can be written as (assuming a symmetric potential)

$$\gamma = \int_0^T \dot{\phi}^2 dt / \int_0^T \rho dt = \int_0^{\phi_{\text{max}}} \dot{\phi} d\phi / \int_0^{\phi_{\text{max}}} (\rho / \dot{\phi}) d\phi ,$$

(43)

where $T$ and $\phi_{\text{max}}$ are the period and the amplitude of the oscillation. From $\rho = \dot{\phi}^2/2 + V(\phi) = V_{\text{max}}$, where $V_{\text{max}}$ is the maximal potential energy density, we obtain $\dot{\phi} = \sqrt{2(V_{\text{max}} - V(\phi))}$. Substituting this in (43) we get [44]

$$\gamma = \frac{2 \int_0^{\phi_{\text{max}}} (1 - V/V_{\text{max}})^{1/2} d\phi}{\int_0^{\phi_{\text{max}}} (1 - V/V_{\text{max}})^{-1/2} d\phi} ,$$

(44)

For $V(\phi) = \lambda \phi^\nu$, we find $\gamma = 2\nu/(\nu + 2)$ and, thus, $\rho \propto a^{-6\nu/(\nu + 2)}$ and $a(t) \propto t^{(\nu + 2)/3\nu}$. For $\nu = 2$, in particular, $\gamma = 1$, $\rho \propto a^{-3}$, $a(t) \propto t^{2/3}$ and $\phi$ behaves like pressureless matter. This is not unexpected since a coherent oscillating massive free field corresponds to a distribution of static massive particles. For $\nu = 4$, we obtain $\gamma = 4/3$, $\rho \propto a^{-4}$, $a(t) \propto t^{1/2}$ and the system resembles radiation. For $\nu = 6$, one has $\gamma = 3/2$, $\rho \propto a^{-9/2}$, $a(t) \propto t^{4/9}$ and the expansion is slower (the pressure is higher) than in radiation.

### 7 Decay of the Inflaton

Reintroducing the ‘decay term’ $\Gamma_\phi \dot{\phi}$, (35) can be written as

$$\dot{\rho} + (3H + \Gamma_\phi) \dot{\rho} = -4H \rho + \gamma \Gamma_\phi \rho ,$$

where $\rho_f$ is the energy density at $t_f$. The second and third factors in the rhs of this equation represent the dilution of the field energy due to the expansion of the universe and the decay of $\phi$ to ‘light’ particles respectively.

All pre-existing radiation (known as ‘old radiation’) was diluted by inflation, so the only radiation present is the one produced by the decay of $\phi$ and is known as ‘new radiation’. Its energy density satisfies [44] the equation

$$\dot{\rho}_r = -4H \rho_r + \gamma \Gamma_\phi \rho ,$$

where the first term in the rhs represents the dilution of radiation due to the cosmological expansion while the second one is the energy density transfer...
from \( \phi \) to radiation. Taking \( \rho_r(t_f) = 0 \), this equation gives

\[
\rho_r(t) = \rho_f \left( \frac{a(t)}{a(t_f)} \right)^{-4} \int_{t_f}^{t} \left( \frac{a(t')}{a(t_f)} \right)^{4-3\gamma} e^{-\gamma \Gamma_\phi (t' - t_f)} \gamma \Gamma_\phi dt'.
\] (48)

For \( t_f \ll t_d \) and \( \nu = 2 \), this expression is approximated by

\[
\rho_r(t) = \rho_f \left( \frac{t}{t_f} \right)^{-8/3} \int_0^{t} \left( \frac{t'}{t_f} \right)^{2/3} e^{-\Gamma_\phi t'} dt',
\] (49)

which, using the formula

\[
\int_0^u x^{p-1} e^{-x} dx = e^{-u} \sum_{k=0}^{\infty} \frac{u^{p+k}}{p(p+1) \cdots (p+k)},
\] (50)

can be written as

\[
\rho_r = \frac{3}{5} \rho \Gamma_\phi t \left[ 1 + \frac{3}{8} \Gamma_\phi t + \frac{9}{88} (\Gamma_\phi t)^2 + \cdots \right],
\] (51)

with \( \rho = \rho_f (t/t_f)^{-2} \exp(-\Gamma_\phi t) \) being the energy density of the field \( \phi \) which performs damped oscillations and decays into ‘light’ particles.

The energy density of the ‘new radiation’ grows relative to the energy density of the oscillating field and becomes essentially equal to it at a cosmic time \( t_d = \Gamma_\phi^{-1} \) as one can deduce from (51). After this time, the universe enters into the radiation dominated era and the normal big bang cosmology is recovered. The temperature at \( t_d \), \( T_r(t_d) \), is historically called the ‘reheat’ temperature although no supercooling and subsequent reheating of the universe actually takes place. Using (12), we find that

\[
T_r = \left( \frac{45}{16\pi^3 g_*} \right)^{1/4} (\Gamma_\phi M_P)^{1/2},
\] (52)

where \( g_* \) is the effective number of degrees of freedom. For \( V(\phi) = \lambda \phi^2 \), the total expansion of the universe during the damped field oscillations is

\[
\frac{a(t_d)}{a(t_f)} = \left( \frac{t_d}{t_f} \right)^{\frac{\nu+2}{\nu}}.
\] (53)

8 Density Perturbations from Inflation

We will now sketch how inflation solves the density perturbation problem described in Sect. 7.4. As a matter of fact, inflation not only homogenizes the universe but also provides us with the primordial density perturbations needed for structure formation. To understand the origin of these fluctuations,
we will introduce the notion of event horizon. Our event horizon, at a cosmic
time $t$, includes all points with which we will eventually communicate sending
signals at $t$. The ‘instantaneous’ (at $t$) radius of the event horizon is

$$d_e(t) = a(t) \int_t^\infty \frac{dt'}{a(t')}.$$  \hfill (54)

It is obvious, from this formula, that the event horizon is infinite for ‘mat-
ter’ or ‘radiation’. For inflation, however, we obtain a slowly varying e vent
horizon with $d_e(t) = H^{-1} < \infty$. Points, in our event horizon at $t$, with which
we can communicate sending signals at $t$, are eventually pulled away by the
exponential expansion and we cease to be able to communicate with them
emitting signals at later times. We say that these points (and the correspon-
ding scales) crossed outside the event horizon. The situation is similar to that
of a black hole. Indeed, the exponentially expanding (de Sitter) spa ce is like
a black hole turned inside out. We are inside and the black hole surroun ds
us from all sides. Then, exactly as in a black hole, there are quantum fluc-
tuations of the ‘thermal type’ governed by the Hawking temperat ure $T_H = H/2\pi$. It turns out that the quantum fluctuations of all mass-
less fields (the inflaton is nearly massless due to the flatness of the potential)
are $\delta \phi = T_H$. These fluctuations of $\phi$ lead to energy density perturbations
$\delta \rho = V'(\phi)\delta \phi$. As the scale of these perturbations crosses outside the event
horizon, they become classical metric perturbations.

The evolution of these fluctuations outside the event horizon is quit e sub-
tle due to the gauge freedom in general relativity. However, there is a simple
gauge invariant quantity $\zeta \approx \delta \rho/(\rho + p)$, which remains constant outside
the horizon. Thus, the density perturbation at any present physical (‘co-
moving’) scale $\ell$, $(\delta \rho/\rho)_\ell$, when this scale crosses inside the post-inflationary
particle horizon ($p=0$ at this instance) can be related to the value of $\zeta$ when
the same scale crossed outside the inflationary event horizon (at $\ell / H^{-1}$).
This latter value of $\zeta$ is found, using (55), to be

$$\zeta |_{\ell / H^{-1}} = \left. \left( \frac{\delta \rho}{\rho} \right) \right|_{\ell / H^{-1}} = \left. \left( \frac{V'(\phi)H(\phi)}{2\pi \phi^2} \right) \right|_{\ell / H^{-1}} = -\left. \left( \frac{9H^3(\phi)}{2\pi V'(\phi)} \right) \right|_{\ell / H^{-1}}.$$  \hfill (55)

Taking into account an extra $2/5$ factor from the fact that the universe is
matter dominated when the scale $\ell$ re-enters the horizon, we obtain

$$\left( \frac{\delta \rho}{\rho} \right)_{\ell} = \frac{16\sqrt{6\pi}}{5} \frac{V^{3/2}(\phi_{\ell})}{M_p^2V'(\phi_{\ell})}.$$  \hfill (56)

The calculation of $\phi_{\ell}$, the value of the inflaton field when the ‘comov-
ing’ scale $\ell$ crossed outside the event horizon, goes as follows. A ‘comoving’
(present physical) scale $\ell$, at $T_r$, was equal to $\ell(a(t_f)/a(t_0)) = \ell(T_0/T_r)$. Its
magnitude at the end of inflation ($t = t_f$) was equal to $\ell(T_0/T_r)(a(t_f)/a(t_0)) = \ell(T_0/T_r)(t_f/t_0)^{(\nu+2)/3\nu} \equiv \ell_{\phys}(t_f)$, where the potential $V(\phi) = \lambda\phi^\nu$ was
assumed. The scale $\ell$, when it crossed outside the inflationary horizon, was equal to $H^{-1}(\phi_\ell)$. We, thus, obtain

$$H^{-1}(\phi_\ell)e^{N(\phi_\ell)} = \ell_{\text{phys}}(t_f).$$

(57)

Solving this equation, one can calculate $\phi_\ell$ and, thus, $N(\phi_\ell) \equiv N_\ell$, the number of e-foldings the scale $\ell$ suffered during inflation. In particular, for our present horizon scale $\ell \approx 2H_0^{-1} \sim 10^4$ Mpc, it turns out that $N_{H_0} \approx 50 - 60$.

Taking the potential $V(\phi) = \lambda \phi^4$, (41), (56) and (57) give

$$\left(\frac{\delta \rho}{\rho}\right)_\ell = \frac{4\sqrt{6\pi}}{5} \lambda^{1/2} \left(\frac{\phi_\ell}{M_P}\right)^3 = \frac{4\sqrt{6\pi}}{5} \lambda^{1/2} \left(\frac{N_{H_0}}{\pi}\right)^{3/2}. $$

(58)

From the result of COBE [3], $(\delta \rho/\rho)_{H_0} \approx 6 \times 10^{-5}$, one can then deduce that $\lambda \approx 6 \times 10^{-14}$ for $N_{H_0} \approx 55$. We thus see that the inflaton must be a very weakly coupled field. In non-SUSY GUTs, the inflaton is necessarily gauge singlet since otherwise radiative corrections will make it strongly coupled. This is not so satisfactory since it forces us to introduce an otherwise unmotivated very weakly coupled gauge singlet. In SUSY GUTs, however, the inflaton could be identified [51] with a conjugate pair of gauge non-singlet fields $\phi, \bar{\phi}$ already present in the theory and causing the gauge symmetry breaking. Absence of strong radiative corrections from gauge interactions is guaranteed by the mutual cancellation of the D-terms of these fields.

The spectrum of density perturbations which emerge from inflation can also be analyzed. We will again take the potential $V(\phi) = \lambda \phi^\nu$. One then finds that $(\delta \rho/\rho)_{H_0}$ is proportional to $\phi_{\ell}^{(\nu+2)/2}$ which, combined with the fact that $N(\phi_\ell)$ is proportional to $\phi_\ell^2$ (see (41)), gives

$$\left(\frac{\delta \rho}{\rho}\right)_\ell = \left(\frac{\delta \rho}{\rho}\right)_{H_0} \left(\frac{N_{H_0}}{N_{H_0}}\right)^{\alpha_s}.$$

(59)

The scale $\ell$ divided by the size of our present horizon ($\approx 10^4$ Mpc) should equal exp$(N_\ell - N_{H_0})$. This gives $N_\ell/N_{H_0} = 1 + \ln(\ell/10^4)^{1/N_{H_0}}$ which expanded around $\ell \approx 10^4$ Mpc and substituted in (59) yields

$$\left(\frac{\delta \rho}{\rho}\right)_\ell \approx \left(\frac{\delta \rho}{\rho}\right)_{H_0} \left(\frac{\ell}{10^4 \text{ Mpc}}\right)^{\alpha_s},$$

(60)

with $\alpha_s = (\nu + 2)/4N_{H_0}$. For $\nu = 4$, $\alpha_s \approx 0.03$ and, thus, the density perturbations are essentially scale independent.

9 Density Perturbations in ‘Matter’

We will now discuss the evolution of the primordial density perturbations after their scale enters the post-inflationary horizon. To this end, we introduce
the ‘conformal time’, η, so that the Robertson-Walker metric takes the form of a conformally expanding Minkowski space:

\[ ds^2 = -dt^2 + a^2(t) \, d\bar{r}^2 = a^2(\eta) \, (-d\eta^2 + d\bar{r}^2), \]  

where \( \bar{r} \) is a ‘comoving’ 3-vector. The Hubble parameter now takes the form

\[ H \equiv \frac{\dot{a}(t)}{a(t)} = \frac{a'(\eta)}{a^2(\eta)} \]  

and the Friedmann equation (8) is rewritten as

\[ \frac{1}{a^2} \left( \frac{a'}{a} \right)^2 = \frac{8\pi G}{3}\rho, \]  

where primes denote derivation with respect to \( \eta \). The continuity equation (5) takes the form

\[ \frac{\rho'}{\rho} = -3 \tilde{H}(\rho + p) \]  

with \( \tilde{H} = \frac{a'}{a} \). For ‘matter’, \( \rho \propto a^{-3} \) which gives \( a = (\eta/\eta_0)^2 \) and \( a'/a = 2/\eta \) (\( \eta_0 \) is the present value of \( \eta \)).

The Newtonian equation (23) can now be written in the form

\[ \delta''_k(\eta) + a \frac{a'}{a} \delta'_k(\eta) - 4\pi G a^2 \delta_k(\eta) = 0, \]  

and the growing (Jeans unstable) mode \( \delta_k(\eta) \propto \eta^2 \) and is expressed \[53\] as

\[ \delta_k(\eta) = \epsilon_H \left( \frac{k_H}{2} \right)^2 \hat{s}(k), \]  

where \( \hat{s}(k) \) is a Gaussian random variable satisfying

\[ < \hat{s}(k) >= 0, \quad < \hat{s}(k)\hat{s}(k') > = \frac{1}{k^3} \delta(k - k'), \]  

and \( \epsilon_H \) is the amplitude of the perturbation when its scale crosses inside the post-inflationary horizon. The latter can be seen as follows. A ‘comoving’ (present physical) length \( \ell \) crosses inside the post-inflationary horizon when

\[ \frac{a\ell}{2\pi} = \frac{H^{-1}}{\ell} = \frac{a^2/\ell}{a'} \]  

which gives \( \ell/2\pi = k^{-1} = a/a' = \eta_H/2 \) or \( k\eta_H/2 = 1 \), where \( \eta_H \) is the ‘conformal time’ at horizon crossing. This means that, at horizon crossing, \( \delta_k(\eta_H) = \epsilon_H \hat{s}(k) \). For scale invariant perturbations, the amplitude \( \epsilon_H \) is constant. The gauge invariant perturbations of the scalar gravitational potential are given \[52\] by the Poisson’s equation

\[ \Phi = -4\pi G \frac{a^2}{k^2} \rho \delta_k(\eta). \]  

From the Friedmann equation (8), we then obtain

\[ \Phi = \frac{3}{2} \epsilon_H \hat{s}(k). \]  

The spectrum of the density perturbations can be characterized by the correlation function (\( \hat{x} \) is a ‘comoving’ 3-vector)

\[ \xi(\bar{r}) \equiv < \hat{\delta}^* (\bar{x}, \eta) \hat{\delta} (\bar{x} + \bar{r}, \eta) >, \]  

where \( \hat{\delta} \) is the perturbation field.
where

\[ \tilde{\delta}(\vec{x}, \eta) = \int d^3k \delta_k(\eta) e^{i\vec{k}\cdot\vec{x}}. \]  

(69)

Substituting (64) in (68) and then using (65), we obtain

\[ \xi(\vec{r}) = \int d^3k e^{-i\vec{k}\cdot\vec{r}} \epsilon_H^2 \left( \frac{k\eta}{2} \right)^4 \frac{1}{k^3}, \]  

(70)

and the spectral function \( P(k, \eta) = \epsilon_H^2(\eta^4/16)k \) is proportional to \( k \) for \( \epsilon_H \) constant. We say that, in this case, the spectral index \( n = 1 \) and we have a Harrison-Zeldovich [36] flat spectrum. In the general case, \( P \propto k^n \) with \( n = 1 - 2\alpha_s \) (see (60)). For \( V(\phi) = \lambda\phi^4 \), we get \( n \approx 0.94 \).

10 Temperature Fluctuations

The density inhomogeneities produce temperature fluctuations in the CMBR. For angles \( \theta > 2^\circ \), the dominant effect is the scalar Sachs-Wolfe [54] effect. Density perturbations on the ‘last scattering surface’ cause scalar gravitational potential fluctuations, \( \Phi \), which then produce temperature fluctuations in the CMBR. The reason is that regions with a deep gravitational potential will cause the photons to lose energy as they climb up the well and, thus, appear cooler. For \( \theta < 2^\circ \), the dominant effects are: i) Motion of the ‘last scattering surface’ causing Doppler shifts, and ii) Intrinsic fluctuations of the photon temperature which are more difficult to calculate since they depend on microphysics, the ionization history, photon streaming and other effects.

The temperature fluctuations at an angle \( \theta \) due to the scalar Sachs-Wolfe effect turn out [54] to be \( (\delta T/T)_\theta = -\Phi_\ell/3 \), with \( \ell \) being the ‘co-moving’ scale on the ‘last scattering surface’ which subtends the angle \( \theta \) [\( \ell \approx 100 \ h^{-1}\text{(degrees) Mpc} \)] and \( \Phi_\ell \) the corresponding scalar gravitational potential fluctuations. From (71), we then obtain \( (\delta T/T)_\theta = (\epsilon_H/2)\hat{s}(\bar{k}) \), which using (64) gives the relation

\[ \left( \frac{\delta T}{T} \right)_\theta = \frac{1}{2} \left( \frac{\delta\rho}{\rho} \right)_{\ell \approx 2\pi k^{-1}}. \]  

(71)

The COBE scale (present horizon) corresponds to \( \theta \approx 60^\circ \). Equations (41), (56) and (71) give

\[ \left( \frac{\delta T}{T} \right)_\ell \propto \left( \frac{\delta\rho}{\rho} \right)_\ell \propto \frac{V^{3/2}(\phi_\ell)}{M_p^3 V'(\phi_\ell)} \propto N^{1/2}_\ell. \]  

(72)

Analyzing the temperature fluctuations in spherical harmonics, one can obtain the quadrupole anisotropy due to the scalar Sachs-Wolfe effect:

\[ \left( \frac{\delta T}{T} \right)_{Q-S} = \left( \frac{32\pi}{45} \right)^{1/2} \frac{V^{3/2}(\phi_\ell)}{M_p^3 V'(\phi_\ell)}. \]  

(73)
For \( V(\phi) = \lambda \phi^\nu \), this becomes

\[
\left( \frac{\delta T}{T} \right)_{Q-S} = \left( \frac{32\pi}{45} \right)^{1/2} \frac{\lambda^{1/2} \phi^{\nu+2}_t}{\nu M_P^3} = \left( \frac{32\pi}{45} \right)^{1/2} \frac{\lambda^{1/2}}{\nu M_P^3} \left( \nu M_P^2 \right)^{\nu+2} N_{\ell}^{\nu+2}.
\]

Comparing this with the COBE \([3]\) result, \((\delta T/T)_Q \approx 6.6 \times 10^{-6}\), we obtain \(\lambda \approx 6 \times 10^{-14}\) for \(\nu = 4\) and number of e-foldings suffered by our present horizon scale during the inflationary phase \(N_{\ell-H_z^{-1}} \equiv N_Q \approx 55\).

There are also ‘tensor’ fluctuations \([55]\) in the temperature of the CMBR. The ‘tensor’ quadrupole anisotropy is

\[
\left( \frac{\delta T}{T} \right)_{Q-T} \approx 0.77 \frac{V^{1/2}(\phi_t)}{M_P^2}.
\]

The total quadrupole anisotropy is given by

\[
\left( \frac{\delta T}{T} \right)_Q = \left[ \left( \frac{\delta T}{T} \right)^2_{Q-S} + \left( \frac{\delta T}{T} \right)^2_{Q-T} \right]^{1/2},
\]

and the ratio

\[
r = \frac{(\delta T/T)^2_{Q-T}}{(\delta T/T)^2_{Q-S}} \approx 0.27 \left( \frac{\nu M_P V'(\phi_t)}{V(\phi_t)} \right)^2.
\]

For \( V(\phi) = \lambda \phi^\nu \), we obtain \(r \approx 3.4 \nu/N_H \ll 1\), and the ‘tensor’ contribution to the temperature fluctuations of the CMBR is negligible.

11 Hybrid Inflation

11.1 The non-Supersymmetric Version

The basic disadvantage of inflationary scenarios such as the ‘new’ \([37]\) or ‘chaotic’ \([38]\) ones is that they require tiny coupling constants in order to reproduce the results of COBE \([3]\). This has led Linde \([10]\) to propose, in the context of non-SUSY GUTs, the hybrid inflationary scenario. The idea was to use two real scalar fields \(\chi\) and \(\sigma\) instead of one that was normally used. \(\chi\) provides the ‘vacuum’ energy density which drives inflation, while \(\sigma\) is the slowly varying field during inflation. This splitting of roles between two fields allows us to reproduce the COBE results with ‘natural’ (not too small) values of the relevant parameters in contrast to previous realizations of inflation.

The scalar potential utilized by Linde is

\[
V(\chi, \sigma) = \kappa^2 \left( M^2 - \frac{\chi^2}{4} \right)^2 + \frac{\lambda^2 \chi^2 \sigma^2}{4} + \frac{m^2 \sigma^2}{2},
\]

(78)
where $\kappa, \lambda$ are dimensionless positive coupling constants and $M, m$ are mass parameters. The vacua lie at $\langle \chi \rangle = \pm 2M, \langle \sigma \rangle = 0$. Putting $m=0$, we see that $V$ possesses a flat direction at $\chi = 0$ with $V(\chi = 0, \sigma) = \kappa^2 M^4$. The mass$^2$ of $\chi$ along this direction is $m_{\chi}^2 = -\kappa^2 M^2 + \lambda^2 \sigma^2/2$. So, for $\chi = 0$ and $|\sigma| > \sigma_c = \sqrt{2\kappa M/\lambda}$, we obtain a flat valley of minima. Reintroducing $m \neq 0$, this valley acquires a non-zero slope and the system can inflate as it rolls down this valley. This scenario is called hybrid since the ‘vacuum’ energy density ($\approx \kappa^2 M^4$) is provided by $\chi$, while the slowly rolling field is $\sigma$.

The $\epsilon$ and $\eta$ criteria (see (39)) imply that, for the relevant values of parameters (see below), inflation continues until $\sigma$ reaches $\sigma_c$, where it terminates abruptly. It is followed by a ‘waterfall’, i.e., a sudden entrance into an oscillatory phase about a global minimum. Since the system can fall into either of the two minima with equal probability, topological defects (monopoles, cosmic strings or domain walls) are copiously produced [13] if they are predicted by the particular particle physics model employed. So, if the underlying GUT gauge symmetry breaking (by $\langle \chi \rangle$) leads to the existence of monopoles or domain walls, we encounter a cosmological catastrophe.

The onset of hybrid inflation requires [56] that, at $t \sim H^{-1}$, $H$ being the inflationary Hubble parameter, a region exists with size $\sim H^{-1}$, where $\chi$ and $\sigma$ are almost uniform with negligible kinetic energies and values close to the bottom of the valley of minima. Such a region, at $t_P$, would have been much larger than the Planck length $\ell_P$ and it is, thus, difficult to imagine how it could be so homogeneous. Moreover, as it has been argued [57], the initial values (at $t_P$) of the fields in this region must be strongly restricted in order to obtain adequate inflation. Several possible solutions to this problem of initial conditions for hybrid inflation have been proposed (see e.g., [58,59,60]).

The quadrupole anisotropy of the CMBR produced during hybrid inflation can be estimated, using (73), to be

$$\left( \frac{\delta T}{T} \right)_Q \approx \left( \frac{16\pi}{45} \right)^{1/2} \frac{\lambda \kappa^2 M^5}{M_P m^2} \cdot$$

The COBE result, $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$, can then be reproduced with $M \approx 2.86 \times 10^{16}$ GeV, the SUSY GUT vev, and $m \approx 1.3 \kappa \sqrt{A} \times 10^{15}$ GeV. Note that $m \sim 10^{12}$ GeV for $\kappa, \lambda \sim 10^{-2}$.

11.2 The Supersymmetric Version

Hybrid inflation is [11] ‘tailor made’ for globally SUSY GUTs except that an intermediate scale mass for $\sigma$ cannot be obtained. Actually, all scalars acquire masses $\sim m_{3/2} \sim 1$ TeV (the gravitino mass) from soft SUSY breaking.

Let us consider the renormalizable superpotential

$$W = \kappa S(-M^2 + \delta \phi) \ ,$$

(80)
where \( \tilde{\phi}, \phi \) is a pair of \( G_S \) singlet left handed superfields belonging to non-trivial conjugate representations of the GUT gauge group \( G \) and reducing its rank by their vevs, and \( S \) is a gauge singlet left handed superfield. The parameters \( \kappa \) and \( M (\sim 10^{16} \text{ GeV}) \) are made positive by field redefinitions. The vanishing of the F-term \( F_S \) gives \( \langle \tilde{\phi} \rangle \langle \phi \rangle = M^2 \), and the D-terms vanish for \( \langle |\tilde{\phi}| \rangle = \langle |\phi| \rangle \). So, the SUSY vacua lie at \( \langle \tilde{\phi} \rangle^* = \langle \phi \rangle = \pm M \) and \( \langle S \rangle = 0 \) (from \( F_{\tilde{\phi}} = F_\phi = 0 \)). We see that \( W \) leads to the spontaneous breaking of \( G \).

\( W \) also gives rise to hybrid inflation. The potential derived from it is

\[
V(\tilde{\phi}, \phi, S) = \kappa^2 |M^2 - \tilde{\phi} \phi|^2 + \kappa^2 |S|^2 (|\tilde{\phi}|^2 + |\phi|^2) + D \text{ - terms} .
\]

D-flatness implies \( \tilde{\phi}^* = e^{i \theta} \phi \). We take \( \theta = 0 \), so that the SUSY vacua are contained. \( W \) has a \( U(1)_R \) R-symmetry: \( \tilde{\phi} \phi \rightarrow \tilde{\phi} \phi, S \rightarrow e^{i \alpha} S, W \rightarrow e^{i \alpha} W \). Actually, \( W \) is the most general renormalizable superpotential allowed by \( G \) and \( U(1)_R \). Bringing \( \tilde{\phi}, \phi, S \) on the real axis by \( G \) and \( U(1)_R \) transformations, we write \( \tilde{\phi} = \phi \equiv \chi/2, S = \sigma/\sqrt{2} \) where \( \chi, \sigma \) are normalized real scalar fields. \( V \) then takes the form in \( [12] \) with \( \kappa = \lambda \) and \( m = 0 \). So, Linde’s potential is almost obtainable from SUSY GUTs but without the mass term of \( \sigma \) which is, however, crucial for driving the inflaton towards the vacua.

One way to generate a slope along the inflationary valley \( \langle \tilde{\phi} = \phi = 0, |S| > S_c \equiv M \rangle \) is \([12]\) to include the one-loop radiative corrections. In fact, SUSY breaking by the ‘vacuum’ energy density \( \kappa^2 M^4 \) along this valley causes a mass splitting in the supermultiplets \( \tilde{\phi}, \phi \). We obtain a Dirac fermion with mass \( M^2 = \kappa^2 |S|^2 \) and two complex scalars with mass \( \tilde{\phi}^2 = \kappa^2 |S|^2 \pm \kappa^2 M^2 \). This leads to the existence of one-loop radiative corrections to \( V \) on the inflationary valley which are found from the Coleman–Weinberg formula \([12],[22]\):

\[
\Delta V = \frac{1}{64\pi^2} \sum_{i} (-1)^{F_i} \frac{M^4}{M^2} \ln \left( \frac{M^2}{\Lambda^2} \right) ,
\]

where the sum extends over all helicity states \( i \), with fermion number \( F_i \) and mass \( \kappa M^2 \), and \( \Lambda \) is a renormalization scale. We find that \( \Delta V(|S|) \) is

\[
\kappa^2 M^4 \frac{\kappa^2 N}{32\pi^2} \left( 2 \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1}) \right) ,
\]

where \( z = x^2 = |S|^2/M^2 \) and \( N \) is the dimensionality of the representations to which \( \tilde{\phi}, \phi \) belong. For \( z \gg 1 \) \( (|S| \gg S_c) \), the effective potential on the inflationary valley can be expanded as \([12],[22]\):

\[
V_{\text{eff}}(|S|) = \kappa^2 M^4 \left[ 1 + \frac{\kappa^2 N}{16\pi^2} \left( \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + \frac{3}{2} - \frac{1}{12 z^2} + \cdots \right) \right] .
\]

The slope on this valley from these radiative corrections is \( \Lambda \)-independent.

From \([41],[72]\) and \([83]\), we find the quadrupole anisotropy of the CMBR:

\[
\left( \frac{\delta T}{T} \right)_Q \approx \frac{8\pi}{\sqrt{N}} \left( \frac{N_Q}{45} \right)^{1/2} \left( \frac{M}{M_P} \right)^2 x_Q^{-1} y_Q^{-1} \Lambda (x_Q^{-1})^{-1} ,
\]

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with
\[ A(z) = (z + 1) \ln(1 + z^{-1}) + (z - 1) \ln(1 - z^{-1}), \] (86)
\[ y_Q^2 = \int_1^{x_Q^2} \frac{dz}{z} A(z)^{-1}, \quad y_Q \geq 0. \] (87)

Here, \( N_Q \) is the number of e-foldings suffered by our present horizon scale during inflation, and \( x_Q = |S_Q|/M \), with \( S_Q \) being the value of \( S \) when our present horizon scale crossed outside the inflationary horizon. For \( |S_Q| \gg S_c \), \( y_Q = x_Q(1 - 7/12 x_Q^2 + \cdots) \). Finally, from (83), one finds
\[ \kappa \approx \frac{8\pi^2}{\sqrt{NN_Q}} y_Q \frac{M}{M_P}. \] (88)

The slow roll conditions (see (39)) for SUSY hybrid inflation are \( \epsilon, |\eta| \leq 1 \), where
\[ \epsilon = \left( \frac{\kappa^2 M_P}{16\pi^2 M} \right)^2 \frac{N^2 x^2}{8\pi} A(x^2)^2, \] (89)
\[ \eta = \left( \frac{\kappa M_P}{4\pi M} \right)^2 \frac{N}{8\pi} \left( (3z + 1) \ln(1 + z^{-1}) + (3z - 1) \ln(1 - z^{-1}) \right). \] (90)

Note that \( \eta \to -\infty \) as \( x \to 1^+ \). However, for most relevant values of the parameters (\( \kappa \ll 1 \)), the slow roll conditions are violated only ‘infinitesimally’ close to the critical point at \( x = 1 \) (\(|S| = S_c\)). So, inflation continues practically until this point is reached, where the ‘waterfall’ occurs.

From the COBE [3] result, \( (\delta T/T)_Q \approx 6.6 \times 10^{-6} \), and eliminating \( x_Q \) between (85) and (88), we obtain \( M \) as a function of \( \kappa \). For \( x_Q \to \infty, y_Q \to x_Q \) and \( x_Q y_Q A(x_Q^2) \to 1^- \). Thus, the maximal \( M \) is achieved in this limit and equals about \( 10^{16} \) GeV (for \( N = 8, N_Q \approx 55 \)). This value of \( M \), although somewhat smaller than the SUSY GUT scale, is quite close to it. As a numerical example, take \( \kappa = 4 \times 10^{-3} \) which gives \( M \approx 9.57 \times 10^{15} \) GeV, \( x_Q \approx 2.633, y_Q \approx 2.42 \). The slow roll conditions are violated at \( x - 1 \approx 7.23 \times 10^{-5}, \) where \( \eta = -1 \) (\( \epsilon \approx 8.17 \times 10^{-8} \) at \( x = 1 \)). The spectral index of density perturbations \( n = 1 - 6 + 2\eta \approx 0.985 \).

SUSY hybrid inflation is considered ‘natural’ for the following reasons:

i. There is no need of tiny coupling constants (\( \kappa \sim 10^{-3} \)).

ii. \( W \) in (80) has the most general renormalizable form allowed by \( G \) and \( U(1)_R \). The coexistence of the \( S \) and \( S \partial \phi \) terms in \( W \) implies that the combination \( \partial \phi \) is ‘neutral’ under all symmetries of \( W \) and, thus, all the non-renormalizable terms of the form \( S(\partial \phi)^n, n \geq 2 \), are also allowed. The leading term of this type \( S(\partial \phi)^2 \), if its dimensionless coefficient is of order unity, can be comparable to \( S \partial \phi \) (recall that \( \kappa \sim 10^{-3} \)) and, thus, play a role in inflation (see Sect.12). All higher order terms of this type with \( n \geq 3 \) give negligible contributions to the inflationary potential.
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Note that $U(1)_R$ guarantees the linearity of $W$ in $S$ to all orders excluding terms such as $S^2$ which could generate an inflaton mass $\gtrsim H$ and ruin inflation by violating the slow roll conditions.

iii. SUSY guarantees that the radiative corrections do not invalidate inflation, but rather provide a slope along the inflationary trajectory, needed for driving the inflaton towards the SUSY vacua.

iv. Supergravity (SUGRA) corrections can be brought under control leaving inflation intact. The scalar potential in SUGRA is given by

$$V = \exp \left( \frac{K}{m_P^2} \right) \left( (K^{-1})^i_j F_i F_j - 3 \frac{|W|^2}{m_P^2} \right),$$

where $K$ is the Kähler potential, $m_P = M_P/\sqrt{8\pi} \approx 2.44 \times 10^{18}$ GeV is the 'reduced' Planck scale, $F^i = W^i + K^i W/m_P^2$, and upper (lower) indices denote derivation with respect to the scalar field $\phi_i (\phi^* j)$. $K$ is expanded as $K = |S|^2 + |\bar{\phi}|^2 + |\phi|^2 + \alpha |S|^4/m_P^2 + \cdots$, where the quadratic terms constitute the 'minimal' Kähler potential. The term $|S|^2$, whose coefficient is normalized to unity, could generate a mass $\sim \kappa^2 M^4/m_P^2 \sim H^2$ for $S$ on the inflationary path from the expansion of the exponential prefactor in (91). This would ruin inflation. Fortunately, with this form of $W$ (including all the higher order terms), this mass is cancelled in $V$ (11, 65). The linearity of $W$ in $S$, guaranteed to all orders by $U(1)_R$, is crucial for this cancellation. The $|S|^4$ term in $K$ also generates a mass $\sim \kappa^2 M^4/m_P^2 \sim H^2$ for $S$ via the factor $(\partial^2 K/\partial S \partial S^*)^{-1} = 1 - 4\alpha |S|^2/m_P^2 + \cdots$ in (91), which is however not cancelled (see e.g., 66). In order to avoid ruining inflation, one has then to assume that $|\alpha| \lesssim 10^{-3}$. All other higher order terms in $K$ give suppressed contributions on the inflationary path (since $|S| \ll m_P$). So, we see that a mild tuning of just one parameter is adequate for controlling SUGRA corrections. (In other models, tuning of infinitely many parameters is required.) Moreover, note that with special forms of $K$ one can solve this problem even without a mild tuning. An example is given in 61, where the dangerous mass could be cancelled in the presence of fields without superpotential but with large vevs generated via D-terms. These properties practically persist even in the extensions of the model we will consider in Sect.12.

In summary, for all these reasons, we consider SUSY hybrid inflation (with its extensions) as an extremely 'natural' inflationary scenario.

12 Extensions of Supersymmetric Hybrid Inflation

In trying to apply (SUSY) hybrid inflation to higher GUT gauge groups which predict the existence of monopoles, we encounter the following problem. Inflation is terminated abruptly as the system reaches the critical point...
on the inflationary path and is followed by the ‘waterfall’ regime during which the scalar fields \( \phi, \phi \) develop their vevs starting from zero and the spontaneous breaking of the GUT gauge symmetry takes place. The fields \( \phi, \phi \) can end up at any point of the vacuum manifold with equal probability and, thus, monopoles are copiously produced \[13\] via the Kibble mechanism \[14\] leading to a cosmological disaster (see e.g., \[67\]).

One of the simplest GUTs predicting monopoles is the Pati-Salam (PS) model \[68\] with gauge group \( G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R \). These monopoles carry two units of ‘Dirac’ magnetic charge \[69\]. We will present solutions \[13,15\] of the monopole problem of hybrid inflation within the SUSY PS model, although our mechanisms can be extended to other semi-simple gauge groups such as the ‘trinification’ group \( SU(3)_c \times SU(3)_L \times SU(3)_R \), which emerges from string theory and predicts \[41,70\] monopoles with triple ‘Dirac’ charge, and possibly to simple gauge groups such as \( SO(10) \).

### 12.1 Shifted Hybrid Inflation

One idea \[17\] for solving the magnetic monopole problem is to include into the standard superpotential for hybrid inflation (shown in (80)) the leading non-renormalizable term, which, as explained in Sect.11.2, cannot be excluded by any symmetries. If its dimensionless coefficient is of order unity, this term can compete with the trilinear coupling of the standard superpotential (whose coefficient is \( \sim 10^{-3} \)). The coexistence of these terms reveals a completely new picture. In particular, there appears a non-trivial (classically) flat direction along which \( G_{PS} \) is spontaneously broken with the appropriate Higgs fields acquiring constant values. This ‘shifted’ flat direction can be used as inflationary trajectory with the necessary slope obtained again from one-loop radiative corrections \[12\]. The termination of inflation is again abrupt followed by a ‘waterfall’ but no monopoles are formed in this transition since \( G_{PS} \) is already spontaneously broken during inflation.

The spontaneous breaking of the gauge group \( G_{PS} \) to \( G_S \) is achieved by the vevs of a conjugate pair of Higgs superfields

\[
\begin{align*}
\hat{H}^c &= (4,1,2) = \begin{pmatrix} \bar{H}_c^\nu & \bar{H}_c^\nu & \bar{H}_c^\nu & \bar{H}_c^\nu \\
d_H^\nu & d_H^\nu & d_H^\nu & e_H^\nu \end{pmatrix}, \\
H^c &= (\bar{4},1,2) = \begin{pmatrix} u_H^c & u_H^c & u_H^c & v_H^c \\
d_H^c & d_H^c & e_H^c & e_H^c \end{pmatrix},
\end{align*}
\]

in the \( \bar{H}_c^\nu, \nu_H^c \) directions. The relevant part of the superpotential, which includes the leading non-renormalizable term, is

\[
\delta W = \kappa S(-M^2 + \hat{H}^c \hat{H}^c) - \beta S(\bar{H}^c H^c)^2 M_S^2,
\]

where \( M_S \approx 5 \times 10^{17} \) GeV is the string scale and \( \beta \) is taken positive for simplicity. D-flatness implies that \( \hat{H}^c * = e^{i\theta} \hat{H}^c \). We restrict ourselves to the
direction with $\theta = 0$ ($\dot{H}^c = H^c$) containing the ‘shifted’ inflationary path (see below). The scalar potential derived from $\delta W$ then takes the form

$$V = \left[ \kappa (|H|^2 - M^2) - \beta \frac{|H|^4}{M^4_S} \right]^2 + 2\kappa^2 |S|^2 |H|^2 \left[ 1 - \frac{2\beta}{\kappa M^4_S} |H|^2 \right]^2. \quad (94)$$

Defining the dimensionless variables $w = |S|/M$, $y = |H^c|/M$, we obtain

$$\dot{V} = \frac{V}{\kappa^2 M^4} = (y^2 - 1 - 4\xi y^4)^2 + 2w^2 y^2(1 - 2\xi y^2)^2, \quad (95)$$

where $\xi = \beta M^2 / \kappa M^4_S$. This potential is a simple extension of the standard hybrid inflation for SUSY (which corresponds to $\xi = 0$) and appears in a wide class of models incorporating the leading non-renormalizable correction to the standard hybrid inflation superpotential.

For constant $w$ (or $|S|$), $\dot{V}$ in (95) has extrema at

$$y_1 = 0, \quad y_2 = \frac{1}{\sqrt{2\xi}}, \quad y_{3\pm} = \frac{1}{\sqrt{2\xi}} \sqrt{(1 - 6\xi w^2) \pm \sqrt{(1 - 6\xi w^2)^2 - 4\xi(1 - w^2)}}. \quad (96)$$

Note that the first two extrema (at $y_1, y_2$) are $|S|$-independent and, thus, correspond to classically flat directions, the trivial one at $y_1 = 0$ with $\dot{V}_1 = 1$, and the ‘shifted’ one at $y_2 = 1/\sqrt{2\xi}$ = constant with $\dot{V}_2 = (1/4\xi - 1)^2$, which we will use as inflationary path. The trivial trajectory is a valley of minima for $w > 1$, while the ‘shifted’ one for $w > w_0 = (1/8\xi - 1/2)^{1/2}$, which is its critical point. We take $\xi < 1/4$, so that $w_0 > 0$ and the ‘shifted’ path is destabilized (in the chosen direction $\dot{H}^c = H^c$) before $w$ reaches zero. The extrema at $y_{3\pm}$, which are $|S|$-dependent and non-flat, do not exist for all values of $w$ and $\xi$, since the expressions under the square roots in (96) are not always non-negative. These two extrema, at $w = 0$, become SUSY vacua. The relevant SUSY vacuum (see below) corresponds to $y_{3-(w = 0)}$ and, thus, the common vev $v_0$ of $\dot{V}^c$, $H^c$ is given by

$$\left( \frac{v_0}{M} \right)^2 = \frac{1}{2\xi} \left( 1 - \sqrt{1 - 4\xi} \right). \quad (97)$$

We will now discuss the structure of $\dot{V}$ and the inflationary history for $1/6 < \xi < 1/4$. For fixed $w > 1$, there exist two local minima at $y_1 = 0$ and $y_2 = 1/\sqrt{2\xi}$, which has lower potential energy density, and a local maximum at $y_{3+}$ between the minima. As $w$ becomes smaller than unity, the extremum at $y_1$ turns into a local maximum, while the extremum at $y_{3+}$ disappears. The system then falls into the ‘shifted’ path in case it had started at $y_1 = 0$. As we further decrease $w$ below $(2 - \sqrt{36\xi - 5})^{1/2} / 3\sqrt{2\xi}$, a pair of new extrema, a local minimum at $y_{3-}$ and a local maximum at $y_{3+}$, are created between $y_1$ and $y_2$. As $w$ crosses $(1/8\xi - 1/2)^{1/2}$, the local maximum at $y_{3+}$ crosses $y_2$ becoming a local minimum. At the same time, the local minimum at $y_2$
turns into a local maximum and inflation ends with the system falling into the local minimum at \( y_{3-} \) which, at \( w = 0 \), becomes the SUSY vacuum.

We see that, no matter where the system starts from, it passes from the ‘shifted’ path, where the relevant part of inflation takes place. So, \( G_{PS} \) is broken during inflation and no monopoles are produced at the ‘waterfall’.

After inflation, the system could fall into the minimum at \( y_{3+} \) instead of the one at \( y_{3-} \). This, however, does not happen since in the last e-folding or so the barrier between the minima at \( y_{3-} \) and \( y_{2} \) is considerably reduced and the decay of the ‘false vacuum’ at \( y_{2} \) to the minimum at \( y_{3-} \) is completed within a fraction of an e-folding before the \( y_{3+} \) minimum even appears. This transition is further accelerated by the inflationary density perturbations.

The mass spectrum on the ‘shifted’ path can be evaluated \( \ddagger \). We find that the only mass splitting in supermultiplets occurs in the \( \bar{\nu}_H, \nu_H \) sector. Namely, we obtain one Majorana fermion with mass\( ^2 \) equal to \( 4\kappa^2|S|^2 \), which corresponds to the direction \( (\delta \bar{\nu}_H + \delta \nu_H)/\sqrt{2} \), and two normalized real scalars \( \text{Re}(\delta \bar{\nu}_H + \delta \nu_H) \) and \( \text{Im}(\delta \bar{\nu}_H + \delta \nu_H) \) with \( m^2 = 4\kappa^2|S|^2 + 2\kappa^2 m^2 \). Here, \( m = M(1/4\xi - 1)^{1/2} \) and \( \delta \bar{\nu}_H = \bar{\nu}_H - v, \delta \nu_H = \nu_H - v \) with \( v = (\kappa M^2 S^2/2\beta)^{1/2} \), being the common value of \( \bar{H}^c, H^c \) on the trajectory.

The radiative corrections on the ‘shifted’ path can then be constructed using \( (52) \) and \( (53) \) with \( \kappa \) can be evaluated. We find the same expressions as in \( (55) \) and \( (58) \) with \( N = 2 \) \( (N = 4) \) in the formula for \( (\delta T/T)_Q (\kappa) \) and \( M \) generally replaced by \( m \). The COBE \( \ddagger \) result can be reproduced, for instance, with \( \kappa \approx 4 \times 10^{-3} \), which corresponds to \( \xi = 1/5, v_0 \approx 1.7 \times 10^{16} \) GeV (we put \( N_Q \approx 55, \beta = 1 \)). The scales \( M \approx 1.45 \times 10^{16} \) GeV, \( m \approx 7.23 \times 10^{15} \) GeV, the mass of the inflaton \( m_{\text{infl}} \approx 4.1 \times 10^{13} \) GeV and the ‘inflationary scale’, which characterizes the inflationary ‘vacuum’ energy density, \( v_{\text{infl}} = \kappa^{1/2} m \approx 4.57 \times 10^{14} \) GeV. The spectral index \( n = 0.954 \).

### 12.2 Smooth Hybrid Inflation

An alternative solution to the monopole problem of hybrid inflation has been proposed \( \ddagger \) some years ago. We will present it here within the SUSY PS model of Sect. 12.1, although it can be applied to other semi-simple (and possibly some simple) gauge groups too. The idea is to impose an extra \( Z_2 \) symmetry under which \( H^c \rightarrow -H^c \). The whole structure of the model remains unchanged except that now only even powers of the combination \( \bar{H}^c H^c \) are allowed in the superpotential terms.

The inflationary superpotential in \( (53) \) becomes

\[
\delta W = S \left( -\mu^2 + \frac{(\bar{H}^c H^c)^2}{M^2_S} \right), \tag{98}
\]

where we absorbed the dimensionless parameters \( \kappa, \beta \) in \( \mu, M_S \). The resulting scalar potential \( V \) is then given by

\[
\hat{V} = \frac{V}{\mu^4} = (1 - \bar{\chi}^4)^2 + 16\bar{\delta}^2 \chi^6, \tag{99}
\]
where we used the dimensionless fields $\tilde{\chi} = \chi / 2(\mu M_S)^{1/2}$, $\tilde{\sigma} = \sigma / 2(\mu M_S)^{1/2}$ with $\chi$, $\sigma$ being normalized real scalar fields defined by $\bar{\nu}_H = \nu_H = \chi / 2$, $S = \sigma / \sqrt{2}$ after rotating $\bar{\nu}_H$, $\nu_H$, $S$ to the real axis.

The emerging picture is completely different. The flat direction at $\tilde{\chi} = 0$ is now a local maximum with respect to $\tilde{\chi}$ for all values of $\tilde{\sigma}$, and two new symmetric valleys of minima appear [13,16] at
\[
\tilde{\chi} = \pm \sqrt{6} \tilde{\sigma} \left[ \left( 1 + \frac{1}{36 \tilde{\sigma}^4} \right)^{1/2} - 1 \right]^{1/2}.
\] (100)

They contain the SUSY vacua which lie at $\tilde{\chi} = \pm 1$, $\tilde{\sigma} = 0$. These valleys are not classically flat. In fact, they possess a slope already at the classical level, which can drive the inflaton towards the vacua. Thus, there is no need of radiative corrections in this case. The potential on these paths is [13,16]
\[
\tilde{V} = 48 \tilde{\sigma}^4 \left[ 72 \tilde{\sigma}^4 \left( 1 + \frac{1}{36 \tilde{\sigma}^4} \right) \left( \left( 1 + \frac{1}{36 \tilde{\sigma}^4} \right)^{1/2} - 1 \right) - 1 \right]
\]
\[
= 1 - \frac{1}{216 \tilde{\sigma}^4} + \cdots, \quad \text{for } \tilde{\sigma} \gg 1.
\] (101)

The system follows, from the beginning, a particular inflationary path and, thus, ends up at a particular point of the vacuum manifold leading to no production of disastrous monopoles.

The end of inflation is not abrupt in this case since the inflationary path is stable with respect to $\tilde{\chi}$ for all $\tilde{\sigma}$’s. The value $\tilde{\sigma}_0$ of $\tilde{\sigma}$ at which inflation is terminated smoothly is found from the $\epsilon$ and $\eta$ criteria, and the derivatives [16] of the potential on the inflationary path:
\[
\frac{d\tilde{V}}{d\tilde{\sigma}} = 192 \tilde{\sigma}^3 \left[ (1 + 144 \tilde{\sigma}^4) \left( \left( 1 + \frac{1}{36 \tilde{\sigma}^4} \right)^{1/2} - 1 \right) - 2 \right],
\] (102)
\[
\frac{d^2\tilde{V}}{d\tilde{\sigma}^2} = 16 \tilde{\sigma} \left[ (1 + 504 \tilde{\sigma}^4) \left[ 72 \tilde{\sigma}^4 \left( \left( 1 + \frac{1}{36 \tilde{\sigma}^4} \right)^{1/2} - 1 \right) - 1 \right] \right.
\]
\[
- (1 + 252 \tilde{\sigma}^4) \left( \left( 1 + \frac{1}{36 \tilde{\sigma}^4} \right)^{-1/2} - 1 \right) \right].
\] (103)

The quantities $(\delta T/T)_Q$ and $N_Q$ can be found using (102). One important advantage of this scenario is that the common vev of $\bar{H}^c$, $H^c$, which is equal to $v_0 = (\mu M_S)^{1/2}$, is not so rigidly constrained and, thus, can be chosen equal to the SUSY GUT scale ($v_0 \approx 2.86 \times 10^{16}$ GeV). From COBE [3] and for $N_Q \approx 57$, we then obtain $M_S \approx 4.39 \times 10^{17}$ GeV and $\mu \approx 1.86 \times 10^{15}$ GeV, which are quite ‘natural’. The value of $\sigma$ at which inflation ends corresponds to $\eta = -1$ and is $\sigma_0 \approx 1.34 \times 10^{17}$ GeV. The value of $\sigma$ at which our present horizon crosses outside the inflationary horizon is $\sigma_Q \approx 2.71 \times 10^{17}$ GeV. The inflaton mass is $m_{\text{infl}} = 2\sqrt{2}\mu^2/v_0 \approx 3.42 \times 10^{14}$ GeV.
13 ‘Reheating’ and the Gravitino Constraint

A complete inflationary scenario should be followed by a successful ‘reheating’ which satisfies the gravitino constraint [7] and generates the observed BAU. We will discuss ‘reheating’ within a SUSY GUT leading to standard hybrid inflation. We consider a moderate extension of the minimal supersymmetric standard model (MSSM) based on the left-right symmetric gauge group

\[ G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \]  

(104)

(see [62,71,72]). The breaking of \( G_{LR} \) to \( G_S \) is achieved via a conjugate pair of \( SU(2)_R \) doublets \( \tilde{c}^i \), \( \tilde{c}^i \) with \( B - L \) (baryon minus lepton number) equal to -1, 1, which acquire vevs along their right handed neutrino directions \( \nu_H^c \), \( \nu_H^c \) corresponding to \( \phi, \phi \) in Sect.11.2. The relevant superpotential is

\[ W = \kappa S(-M^2 + \tilde{F} \ell^c), \]

(105)

where \( \kappa, M \) are made positive by field redefinitions. This superpotential leads to hybrid inflation exactly as (80). (\( \delta T/T \)) and \( \kappa \) are given by (83) and (88) with \( N = 2 \) since \( \ell^c, \ell^c \) have two components each.

\( G_{LR} \) implies the presence of right handed neutrino superfields \( \nu^c \) (with \( i = 1, 2, 3 \)), which form \( SU(2)_R \) doublets \( \nu^c = (\nu^c_1, \nu^c_2, \nu^c_3) \) with the \( SU(2)_L \) singlet charged antileptons \( c^c_i \). Intermediate scale \( \nu^c \) masses are generated via the superpotential terms \( \gamma_i \tilde{c}^i \nu^c L_i^c L_i^c / m_p \) (in a basis with diagonal and positive \( \gamma \)'s). These masses are \( M_i = 2\gamma_i M^2 / m_p \) (\( \langle \ell^c \rangle, \langle \ell^c \rangle > 0 \) by a \( B - L \) rotation).

Light neutrinos acquire hierarchical seesaw masses and, thus, cannot play the role of hot dark matter (HDM) in the universe. (This requires degenerate neutrino masses which can be obtained [27,71] via \( SU(2)_L \) triplets [74].) They are suitable for a universe with non-zero cosmological constant favored by recent observations [24]. In this case, HDM is not necessary [73,76]. The terms generating the \( \nu^c \) masses also cause the decay of the inflaton (see Sect.13).

After the end of inflation, the system falls towards the SUSY vacuum and performs damped oscillations about it. The inflaton (oscillating system) consists of the two complex scalar fields \( \theta = (\delta \bar{\nu}_H^c + \delta \nu_H^c) / \sqrt{2} (\delta \bar{\nu}_H^c = \bar{\nu}_H^c - M, \delta \nu_H^c = \nu_H^c - M) \) and \( S \), with equal mass \( m_{\text{infl}} = \sqrt{2}\kappa M \).

The oscillating fields \( \theta \) and \( S \) decay into a pair of right handed neutrinos (\( \psi_{\nu^c} \)) and sneutrinos (\( \nu^c_\nu \)) respectively via the superpotential couplings \( \ell^c \ell^c L_i^c L_i^c \) and \( \tilde{S} \ell^c \ell^c \). The relevant Lagrangian terms are:

\[ L^0_{\text{decay}} = -\sqrt{2}\gamma_i \frac{M}{m_p} \theta \psi_{\nu^c_i} \psi_{\nu^c_i} + h.c. , \]

(106)

\[ L^S_{\text{decay}} = -\sqrt{2}\gamma_i \frac{M}{m_p} S^* \nu_{\nu^c_i} \nu_{\nu^c_i} m_{\text{infl}} + h.c. , \]

(107)

and the common, as it turns out, decay width is given by

\[ \Gamma = \Gamma_{\theta \rightarrow \psi_{\nu^c_i} \tilde{\psi}_{\nu^c_i}} = \Gamma_{S \rightarrow \nu_{\nu^c_i} \nu_{\nu^c_i}} = \frac{1}{8\pi} \left( \frac{M}{M} \right)^2 m_{\text{infl}} \]

(108)
provided that the relevant $\nu^c$ mass $M_i < m_{\text{infl}}/2$.

To minimize the number of small coupling constants, we assume that

$$M_2 < \frac{1}{2} m_{\text{infl}} \leq M_3 = \frac{2 M^2}{m_P} \quad (\text{with } \gamma_3 = 1),$$

so that the inflaton decays into the second heaviest right handed neutrino superfield $\nu^c_2$ with mass $M_2$. The second inequality in (109) implies that $y_Q \leq \sqrt{2 N_Q/\pi} \approx 3.34$ for $N_Q \approx 55$. This gives $x_Q \approx 3.5$. As an example, choose $x_Q \approx 1.05$ (bigger values cannot give adequate BAU) which yields $y_Q \approx 0.28$. From the COBE [3] result, we then obtain $M \approx 4.06 \times 10^{15}$ GeV, $\kappa \approx 4 \times 10^{-4}$, $m_{\text{infl}} \approx 2.3 \times 10^{12}$ GeV and $M_3 \approx 1.35 \times 10^{13}$ GeV.

The ‘reheat’ temperature $T_r$, for the MSSM spectrum, is given by [62]

$$T_r \approx \frac{\Gamma M_P}{17},$$

and must satisfy the gravitino constraint [57], $T_r \lesssim 10^9$ GeV, for gravity-mediated SUSY breaking with universal boundary conditions. To maximize the ‘naturalness’ of the model, we take the maximal $M_2$ (and, thus, $\gamma_2$) allowed by this constraint. This is $M_2 \approx 2.7 \times 10^{10}$ GeV ($\gamma_2 \approx 2 \times 10^{-3}$). Note that, with this $M_2$, the first inequality in (109) is well satisfied.

14 Baryogenesis via Leptogenesis

14.1 Primordial Leptogenesis

In hybrid inflationary models, it is [18] generally not so convenient to generate the observed BAU in the usual way, i.e., through the decay of superheavy color (anti)triplets. Some of the reasons are:

i. $B$ is practically conserved in most models of this type. In some cases [77], this is due to a discrete ‘baryon parity’ symmetry. In the left-right model under consideration, $B$ is exactly conserved because of a $U(1)_R$.

ii. The gravitino constraint would require that the mass of the (anti)triplets does not exceed $10^{10}$ GeV. This suggests strong deviations from the MSSM gauge coupling unification and possibly proton instability.

It is generally preferable to produce an initial lepton asymmetry [43] which is then partly turned into baryon asymmetry by sphalerons [20, 21]. In the left-right model we consider and in many other models, this is the only way for obtaining the BAU since the inflaton decays into right handed neutrino superfields. Their subsequent decay to lepton (antilepton) $L$ ($\bar{L}$) and electroweak Higgs superfields can only produce a lepton asymmetry. It is important to ensure that this asymmetry is not erased [73] by lepton number violating $2 \to 2$ scattering processes such as $LL \to h^{(1)^*} h^{(1)*}$ or $L h^{(1)} \to L h^{(1)*}$ at all $T$'s
between $T_r$ and 100 GeV ($h^{(1)}$ is the Higgs $SU(2)_L$ doublet which couples to up type quarks). This is satisfied since the lepton asymmetry is protected by SUSY at $T$'s between $T_r$ and $T \sim 10^7$ GeV and, for $T \lesssim 10^7$ GeV, these scattering processes are well out of equilibrium provided $m_{\nu_\tau} \lesssim 10$ eV.

For MSSM spectrum, the observed BAU $n_B/s$ is related to the primordial lepton asymmetry $n_L/s$ by $n_B/s = (-28/79)n_L/s$ (see Sect. 14.2).

The lepton asymmetry is generated via the decay of the superfield $\nu_2^c$, produced by the inflaton decay, to electroweak Higgs and (anti)lepton superfields. The relevant one-loop diagrams are both of the vertex and self-energy type with an exchange of $\nu_3^c$. The resulting lepton asymmetry is

$$n_L/s \approx 1.33 \frac{9T_r}{16\pi m_{\text{infl}}} \frac{M_2}{M_3} \frac{c^2s^2 \sin 2\delta (m^D_3 - m^D_2)^2}{|\langle h^{(1)} \rangle|^2 (m^D_3 s^2 + m^D_2 c^2) c^2},$$

(111)

where $|\langle h^{(1)} \rangle| \approx 174$ GeV (for large $\tan \beta$), $m^D_{2,3}$ are the ‘Dirac’ masses of the neutrinos (in a basis where they are diagonal and positive), and $c = \cos \theta$, $s = \sin \theta$, with $\theta$ and $\delta$ being the rotation angle and phase which diagonalize the Majorana mass matrix of the $\nu^c$'s. Equation (111) holds provided that $M_2 \ll M_3$ and the decay width of $\nu_3^c$ is $\ll (M_3^2 - M_2^2)/M_2$, which are satisfied in our model. Here, we considered only the two heaviest families ($i = 2, 3$) ignoring the first one since the analysis of the CHOOZ experiment has shown that the solar and atmospheric neutrino oscillations decouple.

The light neutrino mass matrix is given by the seesaw formula:

$$m_\nu \approx - m^D \frac{1}{M} m^D,$$

(112)

where $m^D$ is the ‘Dirac’ neutrino mass matrix and $M$ the Majorana $\nu^c$ mass matrix. The determinant and the trace invariance of the light neutrino mass matrix imply two constraints on the (asymptotic) parameters:

$$m_2 m_3 = \frac{(m^D_2 m^D_3)^2}{M_2 M_3},$$

(113)

$$m_2^2 + m_3^2 = \frac{(m^D_2 c^2 + m^D_3 s^2)^2}{M_2^2} + \frac{(m^D_3 c^2 + m^D_2 s^2)^2}{M_3^2} + 2(m^D_3 m^D_2 c^2 s^2 \cos 2\delta)M_2 M_3,$$

(114)

where $m_2 = m_{\nu_\mu}$ and $m_3 = m_{\nu_\tau}$ are the (positive) eigenvalues of $m_\nu$.

The $\mu - \tau$ mixing angle $\theta_{23} = \theta_{\mu\tau}$ lies in the range

$$|\varphi - \theta^D| \leq \theta_{\mu\tau} \leq \varphi + \theta^D, \text{ for } \varphi + \theta^D \leq \pi/2,$$

(115)

where $\varphi$ is the rotation angle which diagonalizes the light neutrino mass matrix in the basis where the ‘Dirac’ mass matrix is diagonal and $\theta^D$ is the
‘Dirac’ mixing angle, i.e., the ‘unphysical’ mixing angle with zero Majorana masses of the right handed neutrinos.

We take $m_{\nu_{\mu}} \approx 2.6 \times 10^{-3}$ eV and $m_{\nu_{\tau}} \approx 7 \times 10^{-2}$ eV which are the central values from the small angle MSW resolution of the solar neutrino problem [5] and SuperKamiokande [6]. We choose $\delta \approx -\pi/4$ to maximize $-n_L/s$. Finally, we assume that $\theta^D \approx 0$, so that maximal $\nu_{\mu} - \nu_{\tau}$ mixing, which is favored by SuperKamiokande [6], corresponds to $\varphi \approx \pi/4$.

From (113) and (114) and the diagonalization of $m_\nu$, we determine the value of $m_3^D$ corresponding to $\varphi \approx \pi/4$ for any given $\kappa$. A solution for $m_3^D$ exists provided that $M_2 \lesssim 0.037M_\Lambda$. For the numerical example in Sect. 13, we find $m_3^D \approx 8.3$ GeV, $m_2^D \approx 0.98$ GeV and $n_L/s \approx -2.23 \times 10^{-10}$, which satisfies the baryogenesis constraint. Thus, with ‘natural’ values of $\kappa (\approx 4 \times 10^{-1})$ and the other relevant parameters ($\gamma_2 \approx 2 \times 10^{-3}$, $\gamma_3 \approx 1$), we were able not only to reproduce COBE [3] but also to have a successful ‘reheating’ satisfying the gravitino and baryogenesis constraints together with the requirements from solar and atmospheric neutrino oscillations. Similar results hold [13,14] for the shifted and smooth hybrid inflationary models of Sect. 12.

### 14.2 Sphaleron Effects

To see how the lepton asymmetry partly turns into baryon asymmetry, we must first discuss the non-perturbative baryon and lepton number violation [57] in the standard model. Consider the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ in the limit where the Weinberg angle $\theta_W = 0$ and concentrate on $SU(2)_L$ ($\theta_W \neq 0$ does not alter the conclusions). Also, for the moment, ignore the fermions and Higgs fields so as to have a pure $SU(2)_L$ gauge theory. This theory has [68] infinitely many classical vacua which are topologically distinct and are characterized by a ‘winding number’ $n \in \mathbb{Z}$. In the ‘temporal gauge’ ($A_0 = 0$), the remaining gauge freedom consists of time independent transformations and the vacuum is a pure gauge

$$A_i = \frac{i}{g} \partial_\mu g(\vec{x})g^{-1}(\vec{x}) \, , \, i = 1, 2, 3 \, . \tag{116}$$

Here $g$ is the $SU(2)_L$ gauge coupling constant, $\vec{x} \in$3-space, $g(\vec{x}) \in SU(2)_L$, and $g(\vec{x}) \rightarrow 1$ as $|\vec{x}| \rightarrow \infty$. Thus, the 3-space compactifies to a sphere $S^3$ and $g(\vec{x})$ gives a map: $S^3 \rightarrow SU(2)_L$ ($SU(2)_L$ is topologically equivalent to $S^3$). These maps are classified into homotopy classes constituting the third homotopy group of $S^3$, $\pi_3(S^3)$, and are characterized by a ‘winding number’

$$n = \int d^3x \, \epsilon^{ijk} \text{tr} \left( \partial_\mu g(\vec{x})g^{-1}(\vec{x}) \partial_\mu g(\vec{x})g^{-1}(\vec{x}) \partial_\nu g(\vec{x})g^{-1}(\vec{x}) \right) \, . \tag{117}$$

The corresponding vacua are denoted as $|n\rangle$, $n \in \mathbb{Z}$.

The tunneling amplitude from the vacuum $|n_-\rangle$ at $t = -\infty$ to the vacuum $|n_+\rangle$ at $t = +\infty$ is given by the functional integral

$$\langle n_+ | n_- \rangle = \int (dA) \, e^{-S(A)} \tag{118}$$
over all gauge field configurations satisfying the appropriate boundary conditions at \( t = \pm \infty \). Performing a Wick rotation, \( x_0 \equiv t \to -ix_4 \), we go to Euclidean space-time. Any Euclidean field configuration with finite action is characterized by an integer known as the Pontryagin number

\[
q = \frac{g^2}{16\pi^2} \int d^4x \, \text{tr} \left( F^{\mu\nu} \tilde{F}_{\mu\nu} \right),
\]

where \( \mu, \nu = 1, 2, 3, 4 \) and \( \tilde{F}_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu\lambda\rho} F^{\lambda\rho} \) is the dual field strength. It is known that \( \text{tr}(F^{\mu\nu} \tilde{F}_{\mu\nu}) = \partial^\mu J_\mu \), where \( J_\mu \) is the ‘Chern-Simons current’ given by

\[
J_\mu = \epsilon_{\mu\nu\alpha\beta} \text{tr} \left( A_\nu F_{\alpha\beta} - \frac{2}{3} g A_\nu A^\alpha A^\beta \right).
\]

In the ‘temporal gauge’ (\( A_0 = 0 \)),

\[
q = \frac{g^2}{16\pi^2} \int d^4x \, \partial^\mu J_\mu = \frac{g^2}{16\pi^2} \Delta x_{4=\pm\infty} \int d^3x \, J_0 = \frac{1}{24\pi^2} \Delta x_{4=\pm\infty} \int d^3x \, \epsilon^{ijk} \text{tr} \left( \partial_i g g^{-1} \partial_j g g^{-1} \partial_k g g^{-1} \right) = n_+ - n_-.
\]

Thus, the Euclidean field configurations which interpolate between the vacua \( | n_+ \rangle \), \( | n_- \rangle \) at \( x_4 = \pm \infty \) have Pontryagin number \( q = n_+ - n_- \) and the path integral in (118) should be performed over all these configurations.

For a given \( q \), there is a lower bound on \( S(A) \),

\[
S(A) \geq \frac{8\pi^2}{g^2} |q|,
\]

which is saturated if and only if \( F_{\mu\nu} = \pm \tilde{F}_{\mu\nu} \), i.e., if the configuration is self-dual or self-antidual. For \( q=1 \), the self-dual classical solution is called instanton [89] and is given by (in the ‘singular’ gauge)

\[
A_{a\mu}(x) = \frac{2\rho^2}{g(x-z)^2} \frac{\eta_{a\mu\nu}(x-z)^\nu}{(x-z)^2 + \rho^2},
\]

where \( \eta_{a\mu\nu} = (a, 1, 2, 3) \) are the t’ Hooft symbols with \( \eta_{aij} = \epsilon_{aij} \) (\( i, j = 1, 2, 3 \)), \( \eta_{a4i} = -\delta_{ai} \), \( \eta_{ai4} = \delta_{ai} \) and \( \eta_{a44} = 0 \). The instanton depends on four Euclidean coordinates \( z_\mu \) (its position) and its scale (or size) \( \rho \). Two successive vacua \( | n \rangle \), \( | n + 1 \rangle \) are separated by a potential barrier of height \( \propto \rho^{-1} \). The Euclidean action of the interpolating instanton is always equal to \( 8\pi^2/g^2 \), but the height of the barrier can be made arbitrarily small since the size \( \rho \) of the instanton can be taken arbitrarily large.

We now reintroduce the fermions into the theory and observe [87] that the baryon and lepton number currents carry anomalies, i.e.,

\[
\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = -n_g \frac{g^2}{16\pi^2} \text{tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}),
\]
where \( n_g \) is the number of generations. Consequently, the tunneling from \( |n_-\rangle \) to \( |n_+\rangle \) is accompanied by a change of the baryon and lepton numbers \( \Delta B = \Delta L = -n_g q = -n_g (n_+ - n_-) \). We should note that i) \( \Delta (B - L) = 0 \), and ii) for \( q=1 \), \( \Delta B = \Delta L = -3 \) which means that one lepton per family and one quark per family and color are annihilated (12-point function).

We, finally, reintroduce the electroweak Higgs doublet \( h \) whose vev is

\[
<h> = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad v \approx 246 \text{ GeV}. \tag{125}
\]

The instanton then ceases to exist as an exact solution. It is replaced by the so-called ‘restricted instanton’ \([90]\) which is an approximate solution for \( \rho \ll v^{-1} \). For \( |x - z| \ll \rho \), the gauge field of the ‘restricted instanton’ essentially coincides with that of the instanton and the Higgs field is

\[
h(x) \approx \frac{v}{\sqrt{2}} \begin{pmatrix} (x - z)^2 \\ (x - z)^2 + \rho^2 \end{pmatrix}^{1/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{126}
\]

For \( |x - z| \gg \rho \), the gauge and Higgs fields decay to a pure gauge and the vev in (125) respectively. The action of the ‘restricted instanton’ is

\[
S_{ri} = \frac{8\pi^2}{g^2} + \pi^2 v^2 \rho^2 + \cdots,
\]

and thus the contribution of big size ‘restricted instantons’ to the path integral in (118) is suppressed.

The height of the potential barrier between the vacua \( |n\rangle, |n + 1\rangle \) cannot be now arbitrarily small. Indeed, the static energy of the ‘restricted instanton’ at \( x_4 = z_4 \) (\( \lambda \) is the Higgs self-coupling),

\[
E_b(\rho) \approx \frac{3\pi^2}{g^2} \frac{1}{\rho} + \frac{3}{8} \pi^2 v^2 \rho^2 + \frac{\lambda}{4\pi^2} v^4 \rho^3,
\]

is minimized for

\[
\rho_{\text{min}} = \frac{\sqrt{2}}{g v} \left( \frac{\lambda}{g^2} \right)^{1/2} \left( \frac{1}{64} + \frac{\lambda}{g^2} \right)^{-1/2} \sim M_W^{-1},
\]

and, thus, the minimal height of the potential barrier is \( E_{\text{min}} \sim M_W/\alpha_W \) (\( \alpha_W = g^2/4\pi \)). The static solution which corresponds to the top (saddle point) of this potential barrier is called sphaleron \([91]\) and is given by

\[
\bar{A} = v \frac{f(\xi)}{\xi} \hat{r} \times \bar{\tau}, \quad h = \frac{v}{\sqrt{2}} t(\xi) \hat{r} \cdot \bar{\tau} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{129}
\]

where \( \xi = 2M_W r \), \( \hat{r} \) is the radial unit vector in 3-space and the 3-vector \( \bar{\tau} \) consists of the Pauli matrices. The functions \( f(\xi), t(\xi) \), which can be determined numerically, tend to zero as \( \xi \to 0 \) and to 1 as \( \xi \to \infty \). The mass (static energy) of the sphaleron solution is estimated to be

\[
E_{\text{sph}} = \frac{2M_W}{\alpha_W} k, \quad 1.5 \leq k \leq 2.7, \quad \text{for } 0 \leq \lambda \leq \infty, \tag{130}
\]
and lies between 10 and 15 TeV.

At $T = 0$ the tunneling from $|n\rangle$ to $|n+1\rangle$ is utterly suppressed \cite{6} by the factor $\exp(-8\pi^2/g^2)$. At high $T$'s, however, thermal fluctuations over the potential barrier are frequent and the tunneling rate is \cite{9} enhanced. For $M_W \lesssim T \lesssim T_c$ ($T_c$ is the critical temperature of the electroweak transition), this rate is calculated \cite{21} by expanding around the sphaleron. We find

$$\Gamma \approx 10^3 n_g \frac{v(T)^9}{T^8} \exp(-E_{\text{sph}}(T)/T).$$

For a second order electroweak transition, $v(T)$, $E_{\text{sph}}(T) \propto (1 - T^2/T_c^2)^{1/2}$. Furthermore, for $T \geq T_c$, where the sphaleron ceases to exist, it was argued \cite{20,21} that we still have $\Gamma \gg H.$ The overall conclusion is that non-perturbative baryon and lepton number violating processes are in equilibrium in the universe for $T \gtrsim 200$ GeV. Note that $B - L$ is conserved by these processes.

Given a primordial lepton asymmetry, one can calculate \cite{78,79} the resulting $n_B/s$. In MSSM, the $SU(2)_L$ instantons produce the effective operator

$$O_2 = (qqql)^{n_g}(\tilde{h}^{(1)}\tilde{h}^{(2)})\tilde{W}^4,$$

and the $SU(3)_c$ instantons the operator

$$O_3 = (qq\ell\ell^c)^{n_g}\tilde{g}^6,$$

where $q$ and $l$ are the quark and lepton $SU(2)_L$ doublets, $\ell^c$ and $d^c$ the up and down type antiquark $SU(2)_L$ singlets, $\tilde{h}^{(2)}$ the Higgs $SU(2)_L$ doublet which couples to down type quarks, $g$ and $W$ the gluons and $W$ bosons, and tilde denotes the superpartner. These interactions as well as the usual MSSM interactions are in equilibrium at high $T$'s. The equilibrium number density of an ultrarelativistic particle species $\Delta n \equiv n_{\text{part}} - n_{\text{antipart}}$ is given by

$$\frac{\Delta n}{s} = \frac{15g}{4\pi^2 g_s} \left( \frac{\mu}{T} \right) \epsilon,$$

where $g$ is the number of internal degrees of freedom of the particle, $\mu$ its chemical potential and $\epsilon = 2$ or 1 for bosons or fermions. For each interaction in equilibrium, the algebraic sum of the $\mu$'s of the particles involved is zero. These constraints leave only two independent chemical potentials, $\mu_q$ and $\mu_{\tilde{g}}$. The baryon and lepton asymmetries are then expressed \cite{79} as

$$\frac{n_B}{s} = \frac{30}{4\pi^2 g_s T} \left( 6n_g\mu_q - (4n_g - 9)\mu_{\tilde{g}} \right),$$

$$\frac{n_L}{s} = -\frac{45}{4\pi^2 g_s T} \left( n_g(14n_g + 9) + \mu_q + \Omega(n_g)\mu_{\tilde{g}} \right),$$

where $\Omega(n_g)$ is a known \cite{79} function. Soft SUSY breaking couplings come in equilibrium at $T \gtrsim 10^7$ GeV since their rate $\Gamma_S \approx m_{3/2}^2/T \gtrsim H \approx 30 T^2/M_P$. 


In particular, the non-vanishing gaugino mass implies \( \mu_{\tilde{g}} = 0 \). Equation (135) then gives

\[
\frac{n_B}{s} = 4\left(1 + 2n_g\right) \frac{n_{B-L}}{22n_g + 13}.
\]

Equating \( n_{B-L}/s \) with the primordial \( n_L/s \), we get

\[
\frac{n_B}{s} = \left(-\frac{28}{79}\right)n_L/s,
\]

for \( n_g = 3 \). Note that it is crucial to generate a primordial \( n_{B-L}/s \) and not only a \( n_B/s \) (and \( n_L/s \)) since otherwise the final \( n_B/s \) will vanish. This is another reason which disfavors the creation of the BAU via the decay of superheavy color (anti)triplets since their interactions usually conserve \( B-L \).

15 Conclusions

We have summarized the shortcomings of the SBB model. We have then shown how they are resolved by inflationary cosmology which suggests that the universe, in its early stages, underwent a period of exponential expansion driven by an almost constant ‘vacuum’ energy density. This may have happened during the GUT phase transition at which the Higgs field which breaks the GUT gauge symmetry was displaced from the vacuum. This field (inflaton) could then, for some time, roll slowly towards the vacuum providing the ‘vacuum’ energy density. Inflation generates the primordial density perturbations which are necessary for the large scale structure formation in the universe and the observed temperature fluctuations of the CMBR. After the end of inflation, the inflaton performs damped oscillations about the vacuum and eventually decays into light particles ‘reheating’ the universe.

The early realizations of inflation required ‘unnaturally’ small coupling constants. This problem was solved by the so-called hybrid inflationary scenario which uses two real scalar fields instead of one that was customarily used. One of them provides the ‘vacuum’ energy density for inflation while the other one is the slowly rolling field. Hybrid inflation arises ‘naturally’ in many SUSY GUTs. However, the cosmological disaster from the overproduction of GUT monopoles, which was avoided in earlier inflationary models, reappears in hybrid inflation. We have constructed two ‘natural’ extensions of SUSY hybrid inflation which do not suffer from the monopole problem.

We have shown that successful ‘reheating’ satisfying the gravitino constraint on the ‘reheat’ temperature takes place after the end of inflation in all three versions of hybrid inflation we have considered here. Adequate baryogenesis via a primordial leptogenesis occurs consistently with the solar and atmospheric neutrino oscillation data. The primordial lepton asymmetry is turned partly into baryon asymmetry via the electroweak sphaleron effects.

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