Noise-induced Transition between Dynamic Attractors in the Parametrically Excited Magneto-optical Trap

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We have investigated noise-induced transition of atoms between double or triple phase-space attractors that are produced in the parametrically driven magneto-optical trap. The transition rates between two or three dynamic attractors, measured for various modulation frequencies and amplitudes, are in good agreement with theoretical calculations and Monte-Carlo simulations based on the Langevin equations. Our experiment may be useful to study nonlinear dynamic problems involving transitions between states far from equilibrium.

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There have been much studies on fluctuation-induced transition between states that are in equilibrium or far from equilibrium. For equilibrium systems, after Kramers’ seminal work [1], many theories have been suggested [2] and tested in many different experiments: for example, optically trapped Brownian particles [3, 4], analogue circuits [5], and semiconductor lasers [6]. Recently, there have been several experiments on far from equilibrium systems, such as Penning trap [7], vibration-fluidized granular matter [8], and Josephson junction [9].

Theoretical works on calculation of the transition rates [10] or the transition paths [11], and analysis of the critical exponents [12] have been performed. In particular, interesting phenomena including oscillatory behavior with respect to the noise intensity [13] or the phase [14], and saddle-point avoidance [15] have been also expected. However, except the Penning trap, the only quantitatively investigated experimental system was the analogue electrical circuit [16], which can be considered as analogue simulations. For the Penning trap, on the other hand, experiment was carried out within a very narrow parameter region: only near the bifurcation points.

In this Letter, we report on the experimental study of the noise-induced transition in the parametrically driven magneto-optical trap (MOT) which is a far from equilibrium system. In particular, we have investigated the transition in nearly full parameter regions from supercritical bifurcation points (dynamic double attractors) to sub-critical bifurcation points (dynamic triple attractors). For double attractors, we have measured the transition rates by observing directly the change of populations in each well [10]. We have also investigated the transition during one period of modulation and found no phase dependence of the transition rates [14]. All the experimental results are found in good agreement with theoretical calculations and Monte-Carlo simulations based on the Langevin equations, which describe well the quantum dissipation [17] including spontaneous emission that is the origin of fluctuations of atomic motion in MOT.

As reported in previous papers [18], when the intensity-modulation frequency \( f \) of the cooling laser is around twice the trap frequency between \( f_1 \) and \( f_2 \), atoms are separated into two clouds [Fig. 1(a) Left] and oscillate in out-of-phase motion, which corresponds to the limit-cycle (LC) characterized by dynamic double attractors. Due to the nonlinearity of MOT, as \( f \) is increased above \( f_2 \) but below \( f_1 \), there appears an additional stable attractor at the center of the LC motion, called sub-critical bifurcation [Fig. 1(a) Right]. Given the modulation amplitude \( h \), the corresponding frequencies are, \( f_1 = 2f_0 - (f_0/2)\sqrt{h^2 - h_T^2} \), \( f_2 = 2f_0 + (f_0/2)\sqrt{h^2 - h_T^2} \), and \( f_3 = f_0 + (hf_0/2h_T)\sqrt{4 + h_T^2} \), where \( f_0 \) is the natural trap frequency of MOT, and \( h_T = \beta/(\pi f_0) \) is the threshold value of \( h \) for parametric resonance to occur for a given damping coefficient \( \beta \).

Assuming no fluctuations of atomic motion, initial conditions of atomic position and velocity determine which attractors (represented by red dots) an atom ends up with. Figure 1(b) shows each region of two attractors (Left) and that of three attractors (Right). For example, if the initial condition of an atom lies in the light gray region, the atom approaches the attractor located in the same basin. In reality, however, there exist atomic fluctuations due to spontaneous emission, resulting in broadened distributions of atomic position and velocity near the stable attractors. For large diffusion (or spontaneous emissions), certain atoms may jump far from the original attractor and be transferred to another attractor through the unstable regions near the boundary. In this case, the shape of the atomic phase-space distribution resembles a dumbbell which has a narrow neck at the unstable point. The distribution represents the dynamic potential wells, as discussed in the static potential.
The transition rate $W_n$ can be characterized by the activation law, $W_n \propto \exp(-S_n/D)$, which is the dynamic version of the Kramers’ equation. Here $S_n$ is the activation energy of each state $n$ and $D$ is the amount of diffusion which is proportional to the diffusion constant of MOT. In what follows we set $n = 1$ and 2 for the stable states of two dynamic attractors and $n = 0$ for the stationary state at the MOT center. As observed in the stationary position-space potentials, the transition rate increases as the diffusion increases or the activation energy decreases. Note that since all the nonlinear terms in the equations of motion of MOT have the same sign as the first order term, the activation energy is proportional to $f$ and $h$.

Unlike the single electron in the Penning trap, there are more than $10^7$ atoms in the initial MOT and each cloud of the LC motion under parametric excitation is almost equally populated. In order to monitor the transition between two dynamic attractors, it is needed to blow away one of the clouds because the atomic number transferred from attractor 1 to 2 is the same as that from 2 to 1. When atoms, say in attractor 1, are removed, one can observe some atoms in attractor 2 are transferred to 1 with time. Due to the two-way transitions the population difference decreases exponentially. For many particle systems the transitions can be described by the following simple rate equations,

\[
\begin{align*}
\frac{dN_1}{dt} &= R - \gamma N_1 - W_1 N_1 + W_2 N_2, \\
\frac{dN_2}{dt} &= R - \gamma N_2 - W_2 N_2 + W_1 N_1,
\end{align*}
\]

where $N_1$ ($N_2$) is the population of the attractor 1 (2), $R$ is the trap loading rate, $\gamma$ is the loss rate due to collisions by the background atoms, and $W_1$ ($W_2$) is the transition rate from the attractor 1 to 2 (2 to 1). Since $R$ and $\gamma$ are small in our experiment (as discussed later), we neglect these terms and also assume $W = W_1 = W_2$ due to the symmetry. Then the steady-state solution of the population difference $\Delta N = N_2 - N_1$ is given by

\[
\Delta N = \Delta N_0 e^{-2W_1},
\]

where $\Delta N_0$ is the difference with one attractor (say, $n = 1$ state) empty, or $N_1 = 0$.

Our experimental setup is similar to that described in previous work [13], but with new features: we have used a photodiode array [19] to measure the transition rates and used a resonant laser to blow away selectively one atomic cloud (say, $n = 1$). The blowing laser was cylindrically focused at 5 mm left from the center of the LC motion. The intensity of the laser was over 5 times saturation intensity and it was turned on for 3 ms in order to remove only one atomic cloud.

Experimental results for double (triple) attractors are obtained at the intensity of 0.034 $I_s$ (0.06 $I_s$) and the detuning of -2.7 $\Gamma$ for the cooling laser along the longitudinal $z$-axis, and the magnetic field gradient of 10 G/cm, and -3.0 $\Gamma$, respectively. For these parameters, the measured trap frequency is 33.4 Hz (43.9 Hz, for triple states), and the detected damping coefficient is 45.4 s$^{-1}$ (for triple states, 85.9 s$^{-1}$), which are in good agreement with the simple Doppler theory. Note that the sub-Doppler nature of MOT is dramatically suppressed and thus neglected when the transverse laser detuning is slightly different from that of the $z$-axis laser, as discussed in depth in our previous work [13].

The typical data about the transition between dynamic...
double wells are presented in Fig. 2. Figure 2(a) shows
countour plot generated by the atomic absorption signals
recorded on the 16 channel photo-diode array. The
vertical axis represents the atomic position and the lon-
gitudinal axis the time evolution. In Fig. 2(a), large ab-
sorption (or large atomic number) is indicated by bright
color. From the plot one can trace the oscillating LC
attractors and measure the atomic number. It can be
observed that one attractor is made empty at 7 half cy-
cles from the left, which is then repopulated with time,
as plotted in Fig. 2(b). Figure 2(c) presents the tem-
poral variation of the population difference ($\Delta N$), which
clearly shows the exponential decay as expected in Eq. 2
where the decay rate is twice the transition rate. Note
that the total number of atoms is nearly conserved dur-
ing the transitions [asterisks in Fig. 2(b)], which justifies
neglect of the loading rate $R$ and loss rate $\gamma$ in Eq. 1.
In reality, the loss rate of our MOT is 0.15 s$^{-1}$ which is
much smaller than the typical transition rates.

The experimental results in Fig. 3 were obtained by
exponential fitting of Fig. 2(c) for (a) various modula-
tion frequencies at $h = 0.72$ and (b) modulation amplitu-
es at $f/f_0 = 1.89$. The curves in Fig. 3 represent
the Monte-Carlo simulation results obtained from simple
Doppler equations based on two-level atom and random
spontaneous emission. We have first calculated the tra-
jectories of atomic motion of $10^4$ atoms initially in $n = 2$
state and monitored the subsequent time evolution of the
population of the two states. We have then obtained the
transition rates by fitting Eq. 2. As can be observed in
Fig. 3, the experimental results are in good agreement
with the simulations when diffusion constant is 0.7 times
DD, where DD represents the Doppler diffusion coeffi-
cient (Eq. (22) in Ref. [21]).

In the low damping regime, one can also find that the
transition rates vary approximately around the bifurca-
tion point as

$$W = c_1 \exp \left[ -\frac{c_2}{\hbar D} (f - f_1)^2 \right], \quad (3)$$

where $f_1$ is the modulation frequency at the bifurcation
point, and $c_1$ and $c_2$ are constants [10]. The constant
$c_2 = (1/f_0^2)\sqrt{2/\hbar T}$ was obtained by direct calculation

[10]. The constant $c_1$ and the diffusion $D$ were used as fit-
ting parameters. The consequent diffusion $D$ for our
experimental conditions is 0.071(8), which are in the same
order to the calculated value with simple Doppler theory
$$D = 3A_0/ (2m^2\omega_0^2) \ DD \ [10].$$

Here the coefficient $A_0$ is nonlinear coefficient of a term $z^3$ in the equation of motion of a parametrically-driven atom, which is presented
as Eq. (7) in Ref. [20]. The triangles in Fig. 3(a) and 3(b) represent the fitted results. Even though the calcula-
tions are mainly well applied in the limit $f \gtrsim f_1$ and $\beta \ll \omega_0$, we can see these fittings very well describe the
experimental results.

Since $W$ and $f$ are of the same order in our experi-
ment, we also have studied the time dependence of the transition during one period of parametric oscillation.
By averaging the absorption signals over 20 times, we have obtained the population evolution during each pe-
riod. The experiments were performed at $f = 1.76f_0$ and $h = 0.72$, with the same laser conditions and magnetic
field gradient as above. The transition rate $W$ was 15.4
s$^{-1}$, which was measured during three oscillation periods
until the population difference became negligible. In par-
icular, we have not found any phase dependence of the transition rates, which indicates that transitions occur
constantly without any dependence of location of atomic
clouds in phase space. Note that these results are very
different from those observed in periodic driven systems
[14], which were studied only theoretically. Experimental
results are also in very good agreement with simulations,
which do not show any clear variation of the transition
rates during one period.

For the case of dynamic triple attractors, direct obser-
vation of the transition is difficult in our setup. When
$f_2 < f < f_3$, it is numerically observed that transition
occurs only between one of the two dynamic states (1
or 2) and the stable state (0), with no direct transitions
from 1 (2) to 2 (1). Moreover, unlike the double wells,
the transition rates $W_{i0}$ from state $i$ to 0 are not necessarily the same as $W_{0i}$ from state 0 to $i$ ($i = 1, 2$). Thus the method of just removing the state 1 and monitoring the time evolution of the population difference does not provide the necessary information of the relevant transition rates. Therefore we have studied the transition in the sub-critical region by using an indirect method of measuring the populations in each state. The ratios of the population $N_1$ (or $N_2$) to $N_0$ can be used to obtain the transition rates, that is, $N_1/N_0 = N_2/N_0 = W_0/2W$, where we have employed $W_{i0} = W_{0i} = W$ and $W_{01} = W_{02} = W/2$. Note that for a larger modulation frequency, $N_0$ becomes larger than $N_1$ (or $N_2$) 10, which is confirmed in our experiment.

The experimental and simulation results for dynamic triple attractors are presented in Fig. 4. Figure 4(a) shows the motions of atomic clouds during single period of oscillation, which provides the population in each state. States 1 and 2 show out-of-phase oscillating motions, while state 0 stays at the center. Our simulations [Fig. 4(b)] have very similar results to the absorption image [Fig. 4(a)], with different spatial resolutions (1 mm for Fig. 4(a) and 0.1 mm for Fig. 4(b)).

The data of $(N_1 + N_2)/N_T$ (filled boxes) and $N_0/N_T$ (filled circles) in Fig. 4(c) are obtained by fitting the profiles of Fig. 4(a) at 0.58 $\pi$ phase. The dashed curves ($(N_1 + N_2)/N_T$) and dotted curves ($N_0/N_T$) are derived from Fig. 4(b) at 0.5 $\pi$. Here $N_n$ ($n = 0, 1, 2$) is the population of the state $n$ and $N_T = N_0 + N_1 + N_2$. The profile shows three peaks, representing three states, are fitted by triple Gaussian functions. Filled triangles (experiment) and solid curves (simulation) are just the ratios of $N_1 + N_2$ to $N_0$, which also provides the ratio of the transitions rates ($W_0/W$). One can observe $N_0$ is increased as $f$ increases, whereas $N_1$ and $N_2$ are decreased. It shows that the transition from state 1 (2) to state 0 becomes easier than that from 0 to 1 (2) as $f$ increases. These results are in good agreement with the theoretical calculation $W_0/W = \exp[(S_1 - S_0)/D]$ where $S_n$ is the activation energy of state $n$ 10.

The parametrically driven MOT is an ideal experimental realization of dynamic double and triple attractors under the conditions far from equilibrium. We have investigated the noise-induced transition at various $f$ and $h$ from the super-critical to sub-critical bifurcation region.

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