In search for the vortex charge and the Cooper pair mass

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ABSTRACT
A novel experiment for determination of the charge related to vortices in thin superconducting film is proposed and a number of related experimental set-ups are also theoretically considered. The methods are based on the Torricelli-Bernoulli effect in superconductors and the phenomenology of the effect is briefly discussed. The vortex charge is expressed via the effective mass of the Cooper pairs, thus both parameters, inaccessible by now, could be simultaneously determined. The experiment would require layered metal-insulator-superconductor structures and standard electronics employed in kinetic measurements. The quality of the insulator-superconductor interface should be high enough as to allow for observation of electric field effects similar to those investigated in superconducting field-effect transistors. The development of layer-by-layer growth technology of oxide superconductors provides unique possibility for investigation of new fundamental effects in these materials. In particular, the structures necessary for determination of the vortex charge could be used to study the superconducting surface Hall effect, Bernoulli effect, the superfluid density, etc.. In conclusion, the systematic investigation of new effects in oxide superconductors is envisaged as an important part of the material science underlying the oxide electronics.

Keywords: Vortex charge, effective mass of Cooper pair, Torricelli-Bernoulli effect in superconductors, London electrodynamics, current-induced contact-potential difference, interface Hall effect

1. INTRODUCTION
The sign change of the Hall effect observed in the superconducting state of many high-$T_c$ superconductors is one of the most puzzling problems in the electrodynamics of these materials. One may ask then what is the doping dependence of this Hall anomaly and how the vortex-lattice melting affects the Hall behavior? Alas, due to the complexity of the vortex matter many related problems are still not answered satisfactory if at all. It is quite possible that the sign reversal of the temperature dependence of the Hall effect could be closely related to charging of the vortices. There is no doubt that the experimental solution of this enigma would provide the key towards understanding the various electromagnetic phenomena. On the other hand, the currently existing theoretical models often lead to conflicting results thus making it difficult to discriminate between all those competing explanations. In such a situation we feel it appealing to accelerate the selection by looking for simplicity in experiments with artificial structures where many of the complications typical for the real systems are avoided.

The aim of the present paper is to propose an experiment for determination of the vortex charge employing transport measurement in a layered metal-insulator-superconductor (MIS) system. We shall require that the quality of the insulator-superconductor interface be extremely high and the insulator layer be very thin. Such a layered MIS structure incorporating a high-$T_c$ film can be manufactured by the contemporary technology of atomic-level engineering of superconducting oxide multilayers and superlattices. In fact, structures of the kind are now being in use for purposes of the fundamental research in the physics of high-$T_c$ superconductors, therefore the vortex charge problem can find its solution thanks to the technological progress. The simplest possible idea behind the search for the vortex charge is to study the electrostatic effect related to charged vortices, which is analogous to electrostatic effects originating, in turn, in the Bernoulli effect due to circulating currents in a thin superconducting film in a vortex-free state; the numerical value of the angular momentum $\oint (m^* v + e^* A) \cdot dr$ is irrelevant for the current-induced contact-potential difference. The paper is organized as follow: in Sec. 2 we derive the formula for the vortex charge $q_v$ expressed via the effective mass of Cooper pairs $m^*$ as well as the expression for the interface Hall conductivity $\sigma_{xy}$. In Sec. 3 we will analyze our proposed experimental set-up for determination of the vortex...
charge by measuring the Hall resistance of the vortex charge currents. An overview is made in Sec. 4 of different experimental methods for determination of the Cooper pair mass: the surface Hall current [12] subsection 4.1, the Bernoulli effect [9] subsection 4.2, and the electrostatic charge modulation [11] subsection 4.3. It is finally concluded in Sec. 5 that the vortex charge \( q_v \) and the effective mass \( m^* \) of fluctuation Cooper pairs fall into the class of the last unresolved problems in the physics of superconductivity. These important parameters enter the theories of a number of phenomena related to electrodynamics of superconductors and can be simultaneously determined by standard electronic measurements. Contemporary layer-by-layer growth of layered oxide structures gives the unique chance for finding \( q_v \) and \( m^* \) for high-\( T_c \) materials but some of the proposed experiments can be realized also in MIS structures with conventional superconductors.

2. MODEL

2.1. Type-II superconductors

This section gives an account of the vortex charging due to the Bernoulli effect within the framework of London electrodynamics. For a superconductor in thermodynamic equilibrium the electrochemical potential \( \zeta \) is constant and the space distribution of the electric potential \( \varphi(r) \) is determined by the Bernoulli-Torricelli theorem

\[
\frac{1}{2}m^*v^2(r)n(T) + \rho_{tot}\varphi(r) = \rho_{tot}\zeta.
\]

Formally, this equation can be derived within the framework of the BCS theory using the statistical mechanics methods, but its physical meaning is very simple—it is a consequence of the energy conservation. We shall further stick to the standard notations for the effective mass of Cooper pairs in the \( ab \)-plane \( m^* \), the superfluid velocity \( v \) related to the current density \( j = e^*n(T)v \), the mass density of the superfluid \( m^*n(T) \), and the total charge of the conduction band \( \rho_{tot} = e^*n(T=0) \); at zero temperature all charge carriers are superfluid and according to the BCS theory \( |e^*| = 2|e| \). The temperature dependence of \( n(T) \) can be extracted from that of the London penetration depth for screening currents flowing in the CuO\(_2\) plane,

\[
\frac{1}{\lambda^2(T)} = \frac{\mu_0 n(T)e^*2}{m^*}.
\]

where the use of SI units is implied, \( \mu_0 = 4\pi \times 10^{-7} \). Although the temperature dependence of the superfluid ratio

\[
\frac{n(T)}{n(0)} = \frac{\lambda^2(0)}{\lambda^2(T)}
\]

is related to the gap anisotropy, the hydrodynamic relation Eq. (1) remains invariant.

Consider now a thin cuprate film thread by a perpendicular magnetic field \( B = B_z \hat{z} \). As a first step we determine the distribution of the electric potential as a function of the distance to the vortex line \( r = \sqrt{x^2 + y^2} \).

For \( r \) larger than the Ginzburg-Landau (GL) coherence length in the \( ab \)-plane but smaller than the penetration depth, \( \xi_{ab}(T) \ll r \ll \lambda_{ab}(T) \) one can use the Bohr-Zemmerfeld relation

\[
r m^* v = \hbar.
\]

Substituting \( v(r) = \hbar/m^*r \) from the above equation into the Bernoulli theorem Eq. (1) we derive the current-induced change of the electric potential

\[
\varphi(r) = -\frac{\hbar^2}{2e^*m^* n(0)} \frac{1}{r^2}.
\]
electric potential of the normal plate is set to zero. Far from the vortex core, for \( r > d_{\text{ins}} \), the electric field \( E_z \) of such a plane capacitor can be considered as being homogeneous,

\[
E_z = \frac{\varphi}{d_{\text{ins}}} = \frac{q_{(2D)}}{e_0 d_{\text{ins}}},
\]

which is employed to express the induced on the normal plate surface charge density \( q_{(2D)}(r) \) via the Bernoulli potential \( \varphi(r) \),

\[
q_{(2D)} = \frac{h^2}{2e^* m^*} \frac{e_0 \epsilon_{\text{ins}} n(T)}{d_{\text{ins}}} \frac{1}{r^2},
\]

where \( e_0 = 1/\mu_0 c^2 \), \( c \) being the speed of light, and \( \epsilon_{\text{ins}} \) is the relative dielectric constant of the insulator. We notice that \( q_{(2D)}(r) \) has the same sign as the charge of the Cooper pairs in the superconductor. On the other hand, the Bernoulli potential keeps the Cooper pairs on a circular orbits inside the vortex. The radial electric force is then equal to the centrifugal force

\[
e^* \frac{\partial \varphi}{\partial r} = m^* \frac{v^2}{r}.
\]

The electric potential attracts the Cooper pairs and the charges with the same sign on the normal plate of the plane capacitor. In order to derive the total charge related to the vortex we have to integrate the charge density up to some maximum radius,

\[
r_{\text{max}} = \min \left( \lambda(T), \sqrt{\frac{\Phi_0}{B}} \right),
\]

corresponding to the screening length \( \lambda(T) \) or the typical intervortex distance in case of high area density of vortex lines \( n_v = B/\Phi_0 \), where \( \Phi_0 = 2\pi \hbar / |e^*| = 2.07 \times 10^{15} \text{ m}^2 \) is the flux quantum. Supposing that the insulator layer is thin enough, \( d_{\text{ins}} \ll r_{\text{max}} \), the integration of the surface density gives for the total vortex charge

\[
q_v = \int_{d_{\text{ins}}}^{r_{\text{max}}} q_{(2D)}(r) d(\pi r^2) \approx \frac{\pi h^2}{e^* m^*} \frac{e_0 \epsilon_{\text{ins}} n(T)}{d_{\text{ins}}} \frac{1}{n(0)} \ln r_{\text{max}} d_{\text{ins}} = \frac{\text{sign}(e^*) |e^*| q_0 e_0 \epsilon_{\text{ins}} m_0 \lambda_{ab}(0)}{d_{\text{ins}}} \frac{\lambda_{ab}(T)}{m^*} \ln \kappa_{\text{eff}},
\]

where \( q_0 = 4\pi \epsilon_0 \hbar^2 / e^* m_0 = 53 \text{ pm} \) is the Bohr radius, \( m_0 = 9.11 \times 10^{-31} \text{ kg} \) is the mass of a free electron, and \( \kappa_{\text{eff}} = r_{\text{max}}/d_{\text{ins}} \) is a quantity analogous to the Ginzburg-Landau parameter \( \kappa = \lambda_{ab}(0)/\xi_{ab}(0) \). According to our model the charge related to vortices is localized not in the vortex core but in the adjacent conducting layers: superconducting CuO2 planes in a real high-\( T_c \) crystal or the normal layer in the model MIS system. With this we close the electrostatic consideration of the vortex charge, but the reader is referred to a number of ingenious experiments related to electrostatics of vortices which are suggested in Ref.\textsuperscript{11}. We believe, however, that the standard transport measurement have some advantage even if they are related to observations of pA-range and below.

The next important step is to address the vortex flow regime of the superconducting film when a strong enough dc current density \( j_y \) is applied through the superconducting film. This condition will create small dissipation and give rise to an electric field \( E_y \) parallel to the current density. The electric field, in turn, creates a drift of the vortices with mean drift velocity in \( x \)-direction \( v_y = E_y / B_z \). In a coordinate system moving with the vortex drift velocity \( v_y \) the electric field is zero. We suppose that \( v_y \) is much smaller than the critical depairing velocity \( v_c = \hbar / m^* \xi_{ab}(T) \) and the Bernoulli potential is nearly the same as in the dissipation-free static regime. Along this line let us recall the fact that airplanes fly thanks to the Bernoulli theorem that holds true for a unviscous dissipationless fluid, but the significant part of the ticket price covers the dissipated energy. By the same token, for \( v_y \ll v_c \) the vortex-induced charge has nearly the static value \( q_v \). Since the charge images will follow the vortices as shadows, the vortex flow will create a two dimensional (2D) current density on the surface of the normal metal

\[
j_{xy}^{(2D)} = q_v n_v v_y = \frac{q_v}{\Phi_0} E_y = \sigma_{xy} E_y.
\]

The electric field \( E_y \) resides the superconducting layer, whereas the current \( j_{xy}^{(2D)} \) exists in the normal slab. The 2D Hall conductivity directly gives the vortex charge

\[
\sigma_{xy} = \frac{q_v}{\Phi_0} = \frac{q_v |e^*|}{2\pi \hbar}.
\]
For \( L_x \times L_y \) rectangular shape of the MIS structure the voltage drop in the superconducting layer is \( V_y = E_y L_y \), the total current in the normal layer is \( I_x = L_x j_x^{(2D)} \), and the interface Hall resistivity is size-independent,

\[
R_{xy} \equiv \frac{V_y}{I_x} = \frac{1}{\sigma_{xy}} = \frac{2\pi h}{q_v|e^*|} = \frac{e^2}{q_v|e^*|} R_{QHE} = \frac{1}{2} R_{QHE} \frac{|e|}{q_v},
\]

where \( R_{QHE} = 2\pi h/e^2 = 25.813 \) kΩ is the fundamental resistance determined by the quantum Hall effect (QHE). Since the vortex charge \( q_v \ll |e| \), the experiment would face the problem of measuring huge Hall resistances. This sets the first technological requirement regarding to the quality of the insulating layer—in order to avoid the leakage currents the resistance \( R_{MS} \) of the plane capacitor should satisfy the relation \( R_{MS} = \rho_{ins} d_{ins}/(L_x L_y) \gg R_{xy} \). In the present model we used the hydrodynamic approach applicable for extreme type-II superconductors and completely neglected the influence of the geometrically small vortex core. However the states in vortex core can have some influence in the total charge of vortex core. In order qualitatively to ”interpolate” a real situation with moderate Ginzburg-Landau parameter \( \kappa \) let us analyze the interface Hall current for a type I superconductor. In this case the normal ”cores” are domains of normal metal surrounded by circulating superconducting currents. This problem, certainly, is only of an academic interest and is irrelevant for the oxide superconductors.

### 2.2. Interface Hall current for type-I MIS structure

If the superconducting layer of a MIS structure is of type-I superconductor, in a perpendicular magnetic field \( B_z \) the magnetic field in the normal domains is equal to the thermodynamic one \( B_c(T) \) and is zero in the superconducting domains. The relative area of the normal regions is \( c_N = B_z/B_c(T) \), correspondingly the part of the superconducting area is \( c_S = 1 - B_z/B_c(T) \), thus \( c_N + c_S = 1 \) and the external field is equal to the mean field \( B_z = c_N B_c(T) + c_S \times 0 \). The contact potential difference between the normal and the superconducting phase (see Eq. (15) bellow) is

\[
\varphi_N - \varphi_S = -\frac{1}{e^* n(0)} B^2_c(T). \tag{14}
\]

This contact potential difference creates, in turn, a difference in the charge density at the surface of the normal layer in front of the normal domain

\[
q^{(2D)} = \frac{\epsilon_0 \epsilon_{ins} c_N}{d_{ins}} (\varphi_S - \varphi_N) = \frac{\epsilon_0 \epsilon_{ins} B_c(T)}{2\mu_0 d_{ins} e^* n(0)} B_z, \tag{15}
\]

where a plane capacitor configuration is implied.

When an electric field \( E_y \) is applied in the superconducting layer the normal domains acquire a drift velocity in \( x \)-direction \( v_x = E_y/B_z \). Again, in the mobile coordinate system the domain structure is static and the mean electric field is zero. The extra charges induced in the normal layer follow the moving normal domains and for the 2D current \( j_x^{(2D)} = q^{(2D)} v_x = \sigma_{xy} E_y \) at the surface of the normal plate we obtain

\[
\sigma_{xy} = \frac{\epsilon_0}{2\mu_0 e^* n(0) d_{ins}} = \frac{1}{R_{xy}}. \tag{16}
\]

This very small interface Hall conductivity vanishes at \( T_c \) and its detection requires fA sensitivity. For comparison with Eq. (10) here we also give the expression for the induced charges per flux quantum

\[
q_v I = q^{(2D)} = \frac{\epsilon_0 \Phi_0}{n_v} B_c(T) \frac{\epsilon_{ins}}{2\mu_0 e^* n(0) d_{ins}}. \tag{17}
\]

Of course, around every normal domain in a type-I superconductor \( |e^* \oint A \cdot d\mathbf{r}/h| \gg 1 \).

Having derived the formulae, Eq. (13) and Eq. (16), concerning the new predicted effect we proceed with more detailed discussion and description of the proposed new experiment in the next section.
3. EXPERIMENTAL SET-UP FOR MEASURING THE VORTEX CHARGE

To begin with, we have sketched a "gedanken" experimental set-up in Fig. 1. The contemporary technology of layer-by-layer growth of oxide superconductors opens the possibility for realization of such a layered structure—a superconducting film protected by an insulating plate. Moreover, we consider that a MIS plane capacitor is one of the simplest possible systems employed in the fundamental research towards further technical applications. Therefore we believe that the suggested experiment could become a standard tool in studying the quality of the insulator-superconductor interface. In order to check whether this idea is another case of a science fiction or, vice versa, is a smoking gun we provide below a numerical example involving an acceptable set of parameters which have been collected from various references:

\[ m^* = 11m_0 \] (Ref. 11),
\[ \xi_{ab}(0) = 1 \text{ nm}, \]
\[ d_{\text{ins}} = 15 \text{ nm}, \]
\[ d_{\text{ins}}/\epsilon_{\text{ins}} = 1 \text{ nm} \] (Ref. 12),
\[ \lambda_{ab}(0) = 150 \text{ nm} \] (Ref. 14).

For an illustration we take as well:
\[ B_z = 100 \text{ mT}, \]
\[ E_y = 1 \text{ V/cm}, \] and
\[ L_x = L_y = 1 \text{ mm}. \]

The value of \( B_z \) we chose imply for the following parameters:
\[ n_v = B_z/\Phi_0 = 4.83 \times 10^{13} \text{ m}^{-2}, \]
\[ L_x L_y n_v = 48 \times 10^6, \]
and
\[ 1/\sqrt{n_v} = 144 \text{ nm} \approx \lambda_{ab}(0). \]

For a model estimate we also take \( r_{\text{min}} \approx 150 \text{ nm}. \) It is now straightforward to work out the vortex charge at liquid-helium temperature, i.e. in the temperature range far below \( T_c. \) In this case the substitution of the above mentioned set of parameters in Eq. (10) gives

\[ q_v |e| = 1 \frac{8}{1} \cdot \frac{53}{1000} \cdot \frac{1}{11} \cdot \ln(10) = 1.386 \times 10^{-3} \approx \frac{1}{1000}. \] (18)

The so estimated \( q_v \approx 10^{-3} |e| \) is in agreement with another model evaluation due to Khomskii and Freimuth. Further, Eq. (13) gives \( R_{xy} = 9.35 \text{ M\Omega} \) and the electric field chosen gives for the voltage \( V_y = 100 \text{ mV}, \) therefore for the Hall current we have \( I_x = R_{xy} V_y = 11 \text{ pA}. \) Lastly, the vortex drift velocity \( v_v = E_y/B_z = 1 \text{ km/s}, \) which is one order of magnitude smaller than the depairing velocity at \( T = 0, v_c = h/m^*\xi_{ab}(0) = 9.6 \text{ km/s}. \) We note that the resistance of the capacitor should be thus at least \( R_{\text{MS}} = 100 \text{ M\Omega}. \)

For conventional superconductors similar evaluations show that effect is less but still observable. One can consider, for example, a thin Nb metal film grown by molecular beam epitaxy, and an Al layer after oxidation in natural
condition could give a good insulator layer. All technologies for planar Josephson junctions provide as a rule metal-insulator interface of sufficient quality. Only the insulator layer should be thick enough to prevent leakage tunneling.

The example analyzed above shows that the proposed experiment is in principle possible to be carried out but we find it difficult to anticipate all problems that could arise in the course of it. For instance, due to a good capacitance cross-talk the noise created by the vortex motion in the superconducting layer will be transmitted to the normal layer thus disturbing the detection of the small Hall current. We believe, however, that similar problems could be surmounted, given the challenge of the novel physics underlying the vortex charge. Furthermore, it is quite possible that the charge, concentrated in the vortex core, is comparable to the charge outside, so only a detailed analysis will analyze similar experiments employing artificial MIS structures.

4. HOW TO MEASURE THE COOPER PAIR MASS

Before addressing the problem of measuring the Cooper pair effective mass \( m^* \) let us analyze a parallel between the latter issue and the civil engineering, where in a static approximation only the weight \( W = mg \) is essential for a construction. In this approximation the masses could reach colossal values if we renormalize the earth acceleration \( g \rightarrow 0 \). The uncertainty, however, immediately disappears during the first earthquake when a dynamical problem should be solved. Just the same is the situation with the superconducting order parameter \( \Psi \) in the static GL theory the superfluid density \( n = |\Psi|^2 \) and the effective mass \( m^* \) are inaccessible separately. They are contained in the experimental parameters, such as the penetration depth Eq. (2), only via the ratio \( n/m^* \). In order to determine the effective mass one has to investigate some dynamic phenomenon, which is time-dependent. Due to phase invariance, however, the time \( t \) could participate only in the gauge invariant derivative \( (i\hbar \partial/\partial t - e^*\varphi) \Psi \), that is why electric field effects in superconductors are to be studied. The subtle point is that the latter are already dynamic effects even if the electric fields are static. One therefore needs to perturb the thermodynamic equilibrium of the superconductor as slightly as possible and all methods for determination of the effective mass \( m^* \) of Cooper pairs thus become effectively ac methods, based on the electrostatic effects in superconductors. The set-up proposed to determine the vortex charge, Fig. 1, is a MIS device having four terminals. Probably the most simple method to accomplish the task would be to use the same MIS structure without making any contacts on the superconducting layer and to investigate the surface Hall current \( J \) as described in the next subsection.

4.1. Surface Hall current

This physical effect refers to the 2D surface currents \( j^{(2D)} \) at the surfaces of a thin \( (d_{\text{film}} \ll \lambda_{ab}(0)) \) superconducting film induced by a normal-to-the-layer electric induction \( D_n \) and parallel-to-the-layer magnetic field \( B_t \),

\[
\mathbf{j}^{(2D)} = \frac{e^*}{m^*} d_{\text{film}} \frac{\lambda_{ab}^2(0)}{\lambda_{ab}^2(T)} \mathbf{D}_n \times \mathbf{B}_t,
\]

where the Cooper pair mass \( m^* \) is the material constant of the effect. This is an electrostatic effect and the superconducting film is in vortex-free state. The dissipation is zero and the superconductor is in thermodynamic equilibrium. A symmetric layered structure is grown by capping of the superconducting film with an insulator layer. Two normal metal layers are evaporated on the protecting insulator layer and on the back side of the substrate thus achieving a plane capacitor configuration. The normal-metal electrodes are circles with radius \( R \) and a cartoon of the experimental set-up in Corbino geometry is shown in Fig. 2. Exploiting the axial symmetry of the geometry Eq. (19) reads as \( j^{(2D)} \propto D_z B_r \) and for the total magnetic moment of the circulating currents we have

\[
M(t) = \int_0^R (\pi r^2) j^{(2D)}(r) dr = \frac{e^*}{m^*} d_{\text{film}} \frac{\lambda_{ab}^2(0)}{\lambda_{ab}^2(T)} \int_0^R (\pi r^2) B_r(r) dr
\]

This small magnetic moment could be difficult to detect against the large background due to the dc magnets creating \( B_r \). We derive a static magnetic moment and the next natural step is to consider in a quasistatic approximation the electric induction \( D_z \) as being time-dependent, \( D_z = D_z(t) \). The ac magnetic moment can be detected by the electromotive voltage

\[
\mathcal{E}(t) = -\mu_0 v \frac{dM(t)}{dt},
\]
Figure 2. Set-up for observation of surface Hall current induced by a normal to the superconducting film electric induction $D_z$ and nearly homogeneous parallel-to-the-film magnetic field $B_r$. The core ingredient is a layered MIS structure (see text) in the field of a plane capacitor (Corbino geometry; schematically, not to be scaled). The ac voltage generator creates current $I$ through the plane capacitor, and the dc current source generates opposite oriented magnetic poles in the drive coils and a radial magnetic field $B_r$ in the plane of the superconducting film. A many-turn solenoid is used to detect the ac magnetic moment $M_z$ of the circulating surface Hall currents $j^{(2D)}_\varphi$.

induced in the solenoid having $\nu$ turns per unite length. The total charge of the capacitor is $(\pi R^2)D_z$ and the time derivative of the electric induction,

$$\frac{dD_z(t)}{dt} = \frac{I(t)}{\pi R^2}$$

(22)

can be expressed by the current $I(t)$ charging the capacitor. For the electromotive voltage we finally obtain the equation

$$E(t) = R_{\text{eff}} I(t) - M_{12} \frac{dI(t)}{dt},$$

(23)

where

$$R_{\text{eff}} = -\frac{e^*}{m^*} \frac{\nu d_{\text{film}}}{\pi R^2} \lambda_{ab}^2(0) \int_0^{R} (\pi r^2) B_r(r) dr$$

(24)

is the effective resistance describing this new electrodynamic effect created by the effective mass $m^*$. The experimental difficulties might be related with the careful compensation of the mutual inductance $M_{12}$ between the solenoid and the ac generator charging the MIS plane capacitor. The rigorous analysis of the experiment requires the knowledge of the break-through voltages of the MIS structure and the noise induced in the detecting coil, but in any case this auxiliary experiment would be easier to perform than the detection of vortex charge currents.

In the following we will also provide an elementary derivation of the formula for the surface Hall current Eq. (19) using the London electrodynamics. Let us trace the trajectory of a London superconducting electron (i.e. a Cooper pair) crossing the circular superconducting film during the charging of the MIS plane capacitor, Fig. 2. The superconducting electron leaves the initial surface of the film with zero velocity $v_z(t_i) = 0$, experiences the Lorentz force while traveling across the film

$$m^* \frac{dv_z(t)}{dt} = e^* B_r$$

(25)

and arrives at the opposite surface of the film at the $t_f$, i.e.

$$\int_{t_i}^{t_f} v_z(t) dt = d_{\text{film}}$$

(26)
with an additional azimuthal velocity component

\[ v_\phi = \frac{e^*}{m^*} \frac{d}{d \text{film}} B_r. \]  

(27)

For \( T = 0 \) all charges are superfluid and the electric induction determines the surface (or 2D) excess charge density \( D_z = e^* n^{(2D)} \). For the surface current density of these polarization charges we therefore have

\[ j^{(2D)}_\phi = e^* n^{(2D)} v_\phi = D_z v_\phi = \frac{e^*}{m^*} \frac{d}{d \text{film}} D_z B_r. \]  

(28)

For non-zero temperatures one has to take into account the thermal dissociation of the superconducting electrons, \( e^* \to e + e \), and the appearance of a normal fluid. Thus, taking into account the superfluid part,

\[ j^{(2D)}_\phi(T > 0) = \frac{n(T)}{n(T = 0)} j^{(2D)}_\phi(T = 0) \]  

(29)

we recover the basic equation Eq. (19). The BCS treatment certainly gives the same result because the London electrodynamics is not a mere, naive phenomenological alternative to the microscopic BCS theory, instead it should be viewed as an efficient tool to apply the BCS theory to low frequencies \( \omega \ll \Delta/\hbar \) and small wave-vectors \( k \xi_{ab}(0) \).

Analogous experiment could be performed with a bulk crystal or thick film \( d_{\text{film}} \gg \lambda_{ab}(0) \). In this case in the initial Eq. (19) and the final result, Eq. (24), the thickness of the film \( d_{\text{film}} \) should be replaced with the penetration depth \( \lambda_{ab}(T) \) and the formula for the surface current then reads as

\[ j^{(2D)}_\phi = \frac{e^*}{m^*} \frac{\lambda^2_{ab}(0)}{\lambda^2_{ab}(T)} D_n \times B. \]  

(30)

The investigation of the temperature dependence of this effect can give a new method for determination of the temperature dependence of the penetration depth \( \lambda_{ab}(T) \). A SrTiO\(_3\) layer should be grown on the fresh cleaved surface of Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_8\) crystal and a circular Au electrode needs to be overgrown on the protecting layer. One plate of the capacitor is the bulk high-\( T_c \) crystal and the other one is the Au layer. In order to avoid frozen vortices the constant magnetic field of the dc drive coil must be applied after cooling down to low temperatures. An ac voltage should be applied to the plane capacitor, a lock-in ammeter will measure the polarization current, and the induced due to the effect ac magnetic moment can be detected by a lock-in voltmeter connected to the detector coil. For derivation of the above formula Eq. (30) we have to use: (i) the distribution of the vector-potential at depth \( |z| \) in the superconductor and some fixed radius \( r \),

\[ A_\phi(z) = B_r \lambda_{ab}(T) \exp \left( -\frac{|z|}{\lambda_{ab}(T)} \right), \]  

(31)

where \( B_r(r) \) is the tangential magnetic field at the superconducting surface; (ii) the London-BCS formula for the current response of the superconductor (the polarization operator),

\[ j_\phi = -\frac{A_\phi}{\mu_0 \lambda^2_{ab}(T)}, \]  

(32)

and (iii) the formula for the bulk (3D) density of the superfluid polarization charges

\[ e^* n(z) = D_z \frac{\lambda^2_{ab}(0)}{\lambda^2_{ab}(T)} \delta(z), \]  

(33)

where \( \delta \) stands for the Dirac \( \delta \)-function. The effective mass \( m^* \) can be determined not only by the surface Hall effect but also from the Bernoulli effect for which the BCS theory was developed by Omel’yanchuk and Beloborod’ko as well as from all other predictions of the London theory. The existence of Bernoulli effect for conventional superconductors is experimentally confirmed; some references can be found, for example, in Ref. \footnote{13} In the next subsection we give a brief account of the suggested here Cooper pair mass spectroscopy.
Figure 3. Cooper pair mass spectroscopy based on the Bernoulli potential (after Ref. 9). (a) top view (b) cross section, (c) equivalent electric scheme. Two electrodes, circle- (1) and ring-shaped electrode (2), should be produced on the insulating layer capping the superconducting film. (3) and (4) denote the contacts of the drive coil with inductance $L_d$ and resistance $R_d$; (5)—insulator layer with thickness $d_{ins}$; (6)—superconducting film with thickness $d_{film} < \lambda_{ab}(0)$; (7)—substrate; $M_{12}$—mutual inductance; $L_1$, $L_2$—variable inductances; $R$—load resistor; $V$—voltmeter; $A$—ammeter; $SW$—switch; $C_d$—capacitor of the drive resonance contour with resonance frequency $\omega$; $G$—Bernoulli voltage generator with doubled frequency $2\omega$; $C_1$, $C_2$—capacitances between the superconducting film and metal electrodes (1) and (2). This figure and the underlying author’s idea have been used in the discussions in Refs. 3, 4 on the vortex charge problem; for distribution of the electric force lines of the circulating currents see Fig. 1 of Ref. 4.

4.2. Bernoulli effect in thin superconducting film

The experimental set-up for a current-induced Cooper pair mass spectroscopy is presented in Fig. 3. The Bernoulli effect is related to a current-induced contact-potential difference that can be measured by the electrostatic polarization of a normal metal electrode which covers the surface of the superconductor, forming a plane capacitor. For the averaged change of the electric potential beneath the electrode the Bernoulli theorem Eq. (1) gives

$$\langle \phi \rangle = -e^* \langle n(T) \rangle \frac{1}{2} m^* v^2,$$

where $e^*$ is the kinetic inductance which can be measured directly by means of the mutual inductance method, $n$ is the area density of Cooper pairs and $\langle (j^{(2D)})^2 \rangle$ is the averaged square of the 2D supercurrent beneath the electrode whose distribution has to be found by solving a magnetostatic problem. If two electrodes were grown on the superconductor surface, the Bernoulli voltage

$$V_{Bernoulli} = \langle \phi \rangle_2 - \langle \phi \rangle_1$$
can be considered as a voltage generator sequentially connected to two capacitors $C_1$ and $C_2$ as depicted in Fig. 3 (c). The currents induced in the superconductor film are proportional to the current through the drive coil $I_{\text{drive}}$, Fig. 3 (b,c). The coefficient $A_n$ of this proportion ($\langle j^{(2D)} \rangle = R_{\text{drive}}^2/A_n$) has dimension of area. According to Eq. (35) an ac current $I_\omega \propto \cos(\omega t)$ will create an ac Bernoulli voltage of doubled frequency $V_{\text{Bernoulli}} \propto \cos(2\omega t)$. Initially, in the switched-off regime, when the detector contour resonates at frequency $\omega = 1/(L_2 C)^{1/2}$, where $C = C_1 C_2/(C_1 + C_2)$ the parasite mutual inductance between the drive coil contour and the detector contour must be carefully annulled by a small tunable mutual inductance $M_{12}$. After that taking $L_2 \approx L_1/3$ in switched-on regime the detecting contour will resonate at doubled frequency $2\omega = 1/(L C)^{1/2}$, $L = L_1 L_2/(L_1 + L_2)$. In resonance conditions the Bernoulli voltage can be directly detected by a lock-in voltmeter with a low noise preamplifier. If we know the penetration depth $\lambda_{ab}(T)$ the measured Bernoulli voltage, according to the Eq. (35), gives the effective mass of Cooper pairs $m^*$.

If thick films, $d_{\text{film}} \gg \lambda_{ab}(0)$, or bulk single crystals are to be used for such experiment we have to substitute in Eq. (34) the London formula for the velocity $v = \mu_0 A$, which is a trivial consequence of the Newton equation $m^* \dot{v} = e^* E$ for a nearly homogeneous electric field $E(t) = -\partial A/\partial t$. Combining with Eq. (33) we obtain

$$
\langle \phi \rangle = -R_{\text{LH}} (p_B), \quad p_B = \frac{B^2}{2\mu_0}, \quad R_{\text{LH}} = \frac{1}{e^* n(T = 0)},
$$

(38)

where $p_B$ is the pressure of the tangential to the superconducting surface magnetic field, and the $R_{\text{LH}}$ is the temperature independent London-Hall constant expressed via the total volume density of conduction band $\rho_{\text{tot}} = e^* n(0) = 1/R_{\text{LH}}$. We consider the Greiter, Wilczek and Witten \cite{Greiter} prediction for a temperature dependence of the London-Hall constant as being erroneous and the problem still waits for its experimental solution. For type-I superconductors the Eq. (38) can be applied up to $B_c(T)$ obtaining in this way the contact potential difference Eq. (14). It is still questionable whether the thermal-induced contact-potential difference

$$
\varphi(T_2) - \varphi(T_1) = -\frac{1}{e^* n(0)} \frac{B^2(T_2) - B^2(T_1)}{2\mu_0}
$$

(39)

may be measured, but if the answer is positive this effect can be used to determine the thermodynamic critical field $B_c(T)$ even for type-II superconductors. In any case the fluctuation of the temperature should be taken into account in the experiments aiming to observe the Bernoulli effect.

The realistic experiment proposed in Ref. \cite{Lamb} can be substantially simplified (cf. Ref. \cite{Lamb}): the ring electrode capacitor can be substituted by a short circuit, and the central one could cover the whole facet. We stress that at least one capacitive connection is indispensable. The voltmeters do not measure any voltage difference but just the difference in the electrochemical potential (even nowadays almost 90% of the experimentalists are unaware of what a voltmeter really measures)! An error of the kind has prevented Lewis \cite{Lewis} during his pioneer investigations in the period 1953–1955 from observing the Bernoulli effect in superconductors soon after it has been predicted by London \cite{Lamb}. Lewis did not use the capacitive connection but he introduced all other necessary ingredients: lock-in voltmeter with nV sensitivity, ac magnetic field and doubling of the frequency. Now it is worthwhile measuring both the Bernoulli effect and the surface Hall effect in the same sample. At known total charge density $\rho_{\text{tot}}$, Eq. (38), and penetration dept $\lambda_{ab}(0)$, Eq. (39), the Cooper pair mass can be determined as $m^* = \mu_0 e^* \rho_{\text{tot}} \lambda_{ab}^2(0)$. Despite the 40 × 10^3 papers published on high-$T_c$ superconductivity, without the Cooper pair mass the physics of superconductivity remains Hamlet without the Prince, with only the role of Ophelia performed by onnagata \cite{Ohn}. In the next subsection we briefly describe the only, to the best of our knowledge, reliable experiment for determination of effective mass $m^*$.

4.3. Electric charge modulation of the kinetic inductance

When an electric voltage is applied to a MIS plane capacitor the charging of the superconducting surface will create a change of the 2D superfluid charge density

$$
e^* n^{(2D)} = e^* d_{\text{film}} n(T) + D_z \frac{n(T)}{n(0)} = (d_{\text{film}} \rho_{\text{tot}} + D_z) \frac{\lambda_{ab}^2(0)}{\lambda_{ab}^2(T)}
$$

(40)

*female impersonator in kabuki theater
It is then easily worked out from Eq. (36) that this creates a modulation of the kinetic inductance and the derivative determines the effective mass

\[ m^* = -e^* L_\varnothing(0) L_\varnothing(T) \frac{\delta D_z}{\delta L_\varnothing(T)}. \]  

(41)

This simple picture gets complicated due to \( T_c \)-changing upon electrostatic doping of the material, but below the critical region this experiment confirms a temperature independent effective mass \( m^* \). When \( m^* \) and all other parameters of the superconductor are already determined we can turn to the vortex charge problem.

5. DISCUSSION AND CONCLUSIONS

The preceding analysis demonstrates that the proposed electronic measurements are feasible and the suggested experimental programme could be soon realized. The appearance of the first good samples would immediately lead to the solution of the problem concerning the vortex charge and Cooper pair mass. These two parameters, \( q_v \) and \( m^* \), might fall in the line-light of the physics of superconductivity in the nearest future. As a by-product the Cooper pair mass spectroscopy could become a standard tool for testing the quality of the superconducting films for future superconductor electronics. Even in the present paper we suggested two or three new effects thus there is no doubt that new physics will emerge from the development of the layer-by-layer oxide technology. Let us also list some of the main results of this study: the formulae for vortex charge Eqs. (10) and (18), vortex conductivity Eqs. (12) and (13), surface Hall current for bulk crystals Eq. (30), interface Hall conductivity for type-I superconductors Eq. (16), thermal-induced contact-potential difference Eq. (39), etc..

Finding a solution to the vortex charge problem by employing a model system, where the superconducting and the polarized layers are separated, will immediately trigger the answer to the question about what is the charge induced in the adjacent \( \text{CuO}_2 \) layers by a pancake vortex. One may further ask about the fate of the charge cloud when the pancake vortices "polymerize" in a vortex line, and what is the influence of the vortex charge in the vortex-vortex interaction and correlation. According to our analysis of the Bernoulli effect the charge will concentrate at the end of vortex lines, at kinks and sharp turns of stacks of pancake vortices. Needless to say, the clear solution of some model problems is always useful in the search for solution to the complex problems in material science.

The problem of determining the vortex charge by a transport measurement brings us back to one of the first ideas of the electron physics. Only two months after the discovery of the electron, Francis Mott made the first attempt to observe the influence of the electric fields and surface charges on the conductivity of Pt. Likewise, the vortex charge current has led us to another immortal idea of the XIX century—the Kelvin vortex model of the "atom". Starting from a hydrodynamic approach, we were able to realize that the hydrodynamic excitations can propagate as particles and that the charge related to vortex "atoms" gives a measurable electric current.

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