Experiments of inverted pendulum as a vertical reference of a telescope

H Hanada$^{1,2}$, K Asari$^1$, S Tsuruta$^1$, H Araki$^1$, K Funazaki$^3$, A Satoh$^3$, H Taniguchi$^3$ and M Kikuchi$^3$

1. National Astronomical Observatory, Mizusawa, Oshu, Iwate 023-0861, Japan
2. Oshu Space & Astronomy Museum
3. Iwate University, Morioka, Iwate 020-8551, Japan.

E-mail: h.hanada@uchuyugakukan.com

Abstract. We propose an attitude control system for a telescope utilizing an inverted pendulum. We put a tube with a sharp bottom end on an XY linear translation stage, and surround the top by a ring with 4 pressure gauges inside. We made experiments with a device composed of an inverted pendulum which can move only in one direction. The device is composed of a tube with a conical bottom, a linear translation stage, a monitoring tilt meter, supporting springs, a laser displacement sensor and a frame. The laser displacement sensor which is substitution of pressure gauges measured extension/contraction of the springs between the tube and the frame. We have succeeded in keeping it vertical with the accuracy of better than 1 arc sec.

1. Introduction

Optical telescopes had been used for observation of the rotation of the Earth for many years. However, they are not very important at present for study of the rotation of the Earth since space technologies such as VLBI (Very Long Baseline Interferometer) and SLR (Satellite Laser Ranging) have much higher accuracy. Optical telescopes, on the other hand, still have potential for in-situ observations of rotation of the moon and the planets if they can be small and light weight.

We have developed a telescope for in-situ observations of lunar rotation based on the PZT (Photographic Zenith Tube) used for observations of the rotation of the Earth. It is expected that the accuracy should be better than 1 mas (milli arc seconds) for detecting the weak signal related to the existence of lunar liquid core [1]. Although we have a prospect to attain the expected accuracy from
laboratory experiments and preliminary ground observations [2], the size of the telescope is still large for on boarding an artificial satellite. A factor which hinders the downsizing is the optical system adopted in the telescope. Although the PZT has an advantage in capable of automatic operation which is essential for observation on the Moon, it needs a horizontal reflected surface as a part of optical elements of the telescope. There is no other way than putting liquid surface there. A new system which makes the tube vertical without the mercury pool will widen the possibility of adopting many other optical systems and may lead to a small sized positioning telescope.

It is necessary to have certain connection between the attitude of a telescope and the reference frame of a planet when we observe the rotation of the planet. There are several devices or materials for this purpose such as a liquid surface, bubble tilt meters and plumb bobs. They each have own advantages and disadvantages. For example, liquid surface is easy to evaporate and vibrate. Mercury surface, which is the most popular as the liquid mirror, is easy to react with other metals especially such as copper and aluminum although it almost perfectly follows an equal potential surface and its reflectivity is very high. The plumb bob or a pendulum is affected by friction around a pivot, and it is easy to swing. Tilt meters such as bubble type which are independent to the telescope has a problem of difficulty in relating the attitude of the tilt meter and that of the telescope.

Considering the situation mentioned above, we proposed a new method to control the attitude of the tube by making it be an inverted pendulum [3]. The results of experiments of the inverted pendulum as well as theoretical consideration are shown in this paper.

2. Principle
An inverted pendulum is unstable since the center of gravity is above the pivot point. It is, however, used on many occasions. Keeping the pendulum vertical has been a classical but challenging problem in control engineering, and this technique leads to a bipedal walking robot and two-wheeled vehicle [4]. The inverted pendulum has unique characteristics; the proper period of which can be longer if the force to tilt almost balance the force to prevent it. The Wiechert seismometer is an example of application of the inverted pendulum to being long period [5]. The inverted pendulum with the effective length of 1m prolongs the proper period up to about 10s, which is about five times as long as that of normal pendulum with the same length.

We propose an attitude control system for a telescope utilizing the tube as an inverted pendulum. This is a kind of static and precise control on the contrary to the walking robot which needs rather dynamic control. This method gives little constraint upon optical system of the telescope. There is no need to place the reference of the horizontal plane into the optical system nor a plume bob which may interfere with the incident rays. Therefore, there is some possibility of realizing a small sized telescope using this method.

We put a tube with a sharp bottom end on an XY linear translation stage, and surround the top of it by
a ring with 4 pressure gauges inside of it.

If the tube deviates from the vertical direction, force acts on the pressure gauges. Then we move the bottom of the tube horizontally until the force becomes zero, and the tube is kept vertical. This attitude control does not restrict the optical system of the telescope because any optical element as the horizontal reference plane like a mercury pool is not necessary, nor nothing comes in the field of view.

3. Experiments

We made experiments with a device composed of an inverted pendulum which can move only in one direction at first. The device is composed of a tube with a conical bottom, XY linear translation stage, a monitoring tilt meter, supporting springs, a laser displacement sensor and a frame as shown in Figure2. The laser displacement sensor measures extension/contraction of the springs between the tube and the frame. We put springs instead of a pressure gauge, which is different from the original idea though, since springs are less expensive and easier to be controlled. A spring is a kind of pressure gauge and we can know the pressure of the tube exerted on the frame from displacement of the spring. It is also possible to make up a pendulum with a long proper period by putting springs between the pendulum and the frame. Equation of motion of the inverted pendulum suspended by two springs as shown in Figure3 is described in Equation (1), and the proper period $T_0$ is given in Equation(2),

\[
I \frac{d^2 \theta}{dt^2} = mgl \sin \theta - ml \cos \theta \frac{d^2 x}{dt^2} - kh^2 \theta
\]  

Figure 1. Basic idea of the attitude control system making use of an inverted pendulum

Figure 2. Device composed of an inverted pendulum with springs

Figure 3. Equation of motion of the inverted pendulum
\[ T_0 = 2\pi \sqrt{\frac{l}{kh^2-mgl}} \] (2)

where \( l=ml^2 \) is the moment of inertia of the pendulum, \( m \) mass of the weight, \( g \) gravitational acceleration, \( l \) arm length of the pendulum, \( \theta \) deviation angle from the vertical, \( x \) position of the bottom end of the pendulum, \( k \) is the equivalent spring constant for combination of the two springs. If we choose the spring constant so as \( kh^2-mgl \) be almost zero, the period becomes very long.

**Figure 2.** The device for experiments of attitude control using an inverted pendulum in one direction. 1: tilt meter, 2: displacement sensor, 3: spring, 4: linear translations stage, 5: base plate

**Figure 3.** A mechanical model of an inverted pendulum suspended by two springs.

3.1. **Measurement of characteristics of the inverted pendulum**

We measured the proper period of the inverted pendulum suspended by two springs in order to know the spring constant.
Figure 4. Free oscillation of the inverted pendulum.

Figure 4 shows free oscillations of the pendulum, and we found the proper period of about 1.3 s. It is not very long but it is not too short to make the experiments difficult. Next, we measured the relation between displacement of the spring and the tilt angle of the pendulum. The pendulum set on a linear translation stage inclines when the stage moves. We measured the tilt angle of the pendulum by the laser displacement sensor as well as the tilt meter. The distance of movement of the stage has linear relations with the tilt angle and the length of the spring as shown in Figure 5. This means that we know the tilt angle of the pendulum from the length of the spring. In this case, the tilt meter reading of 45 arcsec corresponds to displacement of the spring of 22.5 μm from the figure.

Figure 5. Relation between the displacement of the XY linear translation stage and the tile angle of the pendulum measured by the length of the springs (left) and that measured by the tilt meter (right).

3.2. Experiments with an inverted pendulum movable in one direction

Next, we made experiments to control the pendulum being vertical. The sequence of the experiments is as follows: 1) We tilt the frame by controlling the base plate. 3) The pendulum begins to fall. 4) the tilt meter on the top of the pendulum measures the tilt angle. 5) the laser displacement sensor also
measures the tilt angle through the displacement of the spring. We move the linear translation stage so as the tilt angle readings become zero. Two kinds of experiments were made: one used signal from the laser displacement sensor as feedback signals, and another used that from the tilt meter. The tilt meter as well as the laser displacement sensor also monitored the tilt or the displacement, respectively. Figure 6 shows the results: upper left shows the case where the tilt meter was used as feedback signal and the tilt angle was monitored by the same tilt meter, upper right the same experiment as the former one except the monitor by the laser displacement sensor, lower left the case where the displacement sensor was used as the feedback signal and the tilt angle was monitored by the tilt meter, and the lower right the same experiments as the former one except the monitor by the same displacement sensor. The frame swung with the amplitude of about 7 arc sec and the period of about 1 hour, and feedback control started after 5 hours from the beginning. The pendulum was kept almost vertical after the control starts.

Figure 6. Results of experiments for keeping the pendulum vertical. The pendulum is controlled using the tilt meter (upper left and right) or the laser displacement sensor (lower left and right). The tilt angle of the pendulum is monitored by the tilt meter (upper and lower left) and by the displacement sensor (upper and lower right). The frame swings with the period of about 1 hour and the amplitude of about 7 arc sec and feedback control begins after 5 hours from the beginning.
Taking a closer look at the results, residuals from the vertical is within 1 arc sec as standard deviation for the tilt record under the control using the tilt meter (upper left), and is within 1 μm (or 2 arc sec) in the displacement record under the control using the displacement sensor (lower right). The other two results, upper right and lower left, show the periodical residual with the amplitude of about 1 arc sec in tilt or about 2 μm (or 4 arc sec) in displacement of the spring. Different kinds of sensors were used for control and monitor, respectively, in these results. The residual in tilt is in phase with the tilt of the frame and the residual in displacement is out of phase with it.

A possible cause for this is that the position of the pendulum is not always at the center of the frame due to the motion of the linear translation stage, and that the springs do not have enough force to adjust the position of the pendulum. In the case of the experiment at the upper right of Figure 6, the spring on the side where the pendulum is falling is pushed and it contracts when the frame inclines. The pendulum becomes vertical when the pivot is just below the center of gravity of the pendulum. However, the length of the spring, or the distance between the frame and the attachment point of the pendulum to the spring, is almost unchanged because the spring cannot extend against the pendulum.

In the case of the experiment at the lower left of Figure 6, the spring contracts as the same as mentioned above. Here, not the tilt but the length of the spring is used as the feedback signal. The length of the spring is almost unchanged even when the pendulum becomes vertical, and the pivot moves further until the spring extends due to tilt of the pendulum to the opposite side. These situations are illustrated in Figure 7. It will be effective to increase the sensitivity of the spring to reduce the amplitude of the periodical residual. Then the feedback signal will be smaller, and the pendulum will hardly tilt.

These results show that the inverted pendulum can be kept vertical within 1 arc sec error when it is controlled by using displacement signal of a spring as a pressure gauge although there remains a problem to be solved.
Figure 7. Variation of the length of the spring in the course of controlling the tilt angle of the pendulum by moving the pivot of the pendulum using tilt signal (upper) or displacement signal (lower) for feedback. They show how the pendulum is controlled when the frame inclines to the left. At the right end of the upper figure, the pendulum becomes vertical, but the spring does not return to the original state even the pendulum is vertical because the spring cannot extend against the inertia of the pendulum.

At the right end of the lower figure, the pendulum becomes vertical when the pivot comes just below the center of gravity of the pendulum, but the pivot moves further in order that the spring extends to the initial state.

4. Discussion

Is it possible to keep an inverted pendulum with an error of 1 mas? We estimate spring constant required for this accuracy supposing a model shown in Figure 3 with the mass of 1 kg and the length of the pendulum of 0.1m. Figure 8 shows how much the spring contracts/extends for spring constant in the rage up to 100 N/m when the pendulum inclines by 1 mas. It is certain that we can measure the length of the spring with the accuracy better than 1 μm as shown in Figure 5 and 6. Then, the spring constant of 40 or less is required for detecting the tilt angle of 1 mas from displacement of spring. For example, if you wind a wire of 0.8mm thickness and 7.8×10⁴ N/mm² rigidity for 100 times around a cylinder of 10mm diameter, it has a spring constant of 40 according to the relation,

\[ k = \frac{Gd^4}{8N_dD^3}, \]  

where \( G \) is rigidity, \( d \) wire diameter, \( N_d \) number of windings and \( D \) outer spring diameter [6]. It is not difficult to make it.

5. Concluding Remarks

We made experiments with a device composed of an inverted pendulum which can move only in one direction, and succeeded in keeping it vertical with the accuracy of better than 1 arc sec. The accuracy is capable of improvement more by using weaker spring and a linear translation stage with higher resolution. Experiments for the pendulum movable to all the direction have not been completed yet. In principle, it is possible to apply the method which succeeded in the experiments in one direction to all the directions. The inverted pendulum supported by two springs can be expected to be effective as a new attitude control.
Figure 8. Relation between displacement of spring and the spring constant for the model shown in Figure 3 with the mass of 1kg and the effective length of the pendulum of 0.1m. The displacement when the pendulum inclines by 1 mas is shown.

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