Book Review: Supporting mathematics educators through and towards online learning. Karen Hollebrands, Robin Anderson, and Kevin Oliver (Eds). (2021) *Online learning in mathematics education*  
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Globally, online learning in mathematics education has progressed, albeit at varying rates. The COVID-19 crisis has acted as a powerful impetus for countries worldwide to speed up the transition from face-to-face classrooms to virtual classrooms. More than two years have passed since the COVID-19 pandemic struck and, while the future remains uncertain, this emergency may have passed. Ultimately, teachers’ skills are a significant determinant of an educational system’s long-term performance (Hattie, 2003). Hence, now is the time for policymakers, educators, and researchers in the field of mathematics education to reflect and act on the best ways to prepare teachers for online learning in the future.

The book *Online Learning in Mathematics Education*, edited by Karen Hollebrands, Robin Anderson, and Kevin Oliver (Hollebrands et al., 2021), contains chapters that assist mathematics teachers educators (MTEs) and mathematics education researchers in supporting teachers (pre-service teachers, or PSTs, and in-service teachers, or ISTs) through and towards online learning. The book’s sixteen chapters are divided into three main sections:

**Section 1: Chapters 1–6 The design of learning environments for mathematics teacher education**

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Section 2: Chapters 7–10 Formal experiences for prospective and practicing mathematics teachers
Section 3: Chapters 11–16 Self-directed, experiential, and practice-based online learning opportunities for mathematics teachers

In this review, we will briefly summarize the book’s chapters before moving on to a general overview.

1 Overview of chapters

Section 1 has six chapters where readers can learn about the basics of online learning and how MTEs are devising new ways to assist their learners and colleagues to learn online, especially in response to the COVID-19 pandemic. In Chapter 1, “Teaching mathematics education online: Instructional theories, strategies, and technologies,” Angie Hodge-Zickerman, Cindy York, and Patrick Lowenthal present an introduction to instructional theories, along with pedagogical and technological strategies, that can be used to inform educators who are new to providing formal online mathematics courses. The complexity of planning and implementing online mathematics courses is a worry for educators with little expertise or no online teaching training. This first chapter aims to offer new online educators the confidence to begin online learning without being anxious about technology. Even with online learning, educators should perceive that learning theory is the foundation for teaching. The authors begin the discussion by defining the categorization of the online learning environment (synchronous, asynchronous, bisynchronous, and HyFlex—a model where the student is given the option to either attend in campus or online). They also exemplify how some basic pedagogical strategies commonly used in face-to-face classrooms—such as cognitive apprenticeship, individualized/personalized instruction, social learning, inquiry-based mathematics education, and problem-based learning—can be the foundation for building online mathematics course strategies. Then they map the technology ideas and the online teaching techniques (grouping students, case studies, think-pair-share, presentation, manipulative, and adapting traditional activities to online) that are appropriate for these theories.

The mapping of theories, strategies, and technology in Table 1.3 (p. 15) may encourage readers new to online learning to develop an online mathematics course using the pedagogical knowledge they already have from their experience of teaching face-to-face. Readers can explore more about the differences between online learning practice that modifies face-to-face instruction and online learning with technology-enhanced enactment in Chapter 6 (p. 122). Readers who are already experts in online learning may use this chapter as a useful reference when assisting colleagues who are just beginning to learn about online mathematics education.

In Chapter 2, “Using digital technology and blending to change the mathematics classroom and mathematics teacher education,” Marcelo Borba, Johann Engelbrecht, and Salvador Llinares offer an overview of how the use of the Internet and interactive digital technologies has evolved in mathematics education prior to and during the pandemic. They reference Borba’s theory which states that people’s learning experiences, information, knowledge, and understanding are all affected when learning is based on humans-with-Internet interactions. This chapter shows how the Internet can dramatically alter social aspects of traditional mathematics PSTs’ education—for example, online sharing and
discussing teaching practice through narrative texts and videos, working online together to create mind maps, and producing mathematical songs using virtual musical instruments. They also describe two video production activities by students in mathematics classrooms: the Mathematics Performance Festival in Canada and the Festival of Digital Videos and Mathematical Education in Brazil. They propose the idea that "LIVES activities (live presentations conducted by the first author during the pandemic) on YouTube, Facebook, and Instagram have opened a way for mathematics to become more interdisciplinary and accessible to the public. Those examples negate the stereotype that online mathematics learning is technology-focused, anti-social, teacher-centered, and static. Incorporating human-with-Internet relationships into mathematical education has become the foundation of the evolution which enhances human civilization and sharpens the Internet technology itself. As the authors observe: “The Internet is changing communication and knowledge as it shapes what humans are, and conversely and dialectically, humans are transforming the Internet” (p. 36).

In Chapter 3, “Presence in online mathematics methods courses: Design principles across institutions,” Dung Tran and Giang-Nguyen Nguyen address the question of how online course design facilitates the interaction of primary mathematics PSTs to develop their instructional practice. In response, they employ the Community of Inquiry (CoI) theoretical framework as a model for designing online courses for PSTs. The CoI framework represents a process that creates higher-order learning by incorporating three elements: cognitive presence, social presence, and teaching presence.

To exemplify how the model works, the authors selected two online courses offered in Australia and the USA that address two major issues concerning PSTs’ competencies: Course 1 focuses on PSTs’ specialized-content knowledge, and Course 2 prepares PSTs with pedagogical content knowledge. The cognitive presence in these two courses is no different from face-to-face: Course 1 aims to develop understanding, reasoning, and problem-solving skills; Course 2 facilitates PSTs in designing rich tasks and mathematics assessments. The social presence outcome provides participants with the experience of collaborating in the community and discussing synchronously and asynchronously to support the cognitive presence. The teaching presence is about the teacher’s decision to facilitate the online class to foster cognitive presence and the use of social presence. In a synchronous setting, the teacher’s strategy was to highlight the teaching presence to facilitate discussion in zoom breakout rooms, use virtual manipulatives, and engage in small (learner-learner) or cohort (learners-instructor) communities. In an asynchronous setting, the teaching presence was evident in choosing tasks to engage in the teaching practices using Canvas as the Learning Management System (LMS).

Undoubtedly, the authors’ strategy of delving deeper into each component of CoI over the two courses works to answer the question posed. Using a modified version of CoI, which views the cognitive and social presence as the outcome of the teaching presence, they emphasize that the inclusion of human experts (teachers) in online learning cannot be replaced by technology. While they acknowledge that the evidence of students’ engagement in the study is anecdotal, the proposed model for future research into online mathematics courses incorporating student presence as a fourth component of the CoI model (see Fig. 3.9, p. 61), namely learning presence, is intriguing.

In Chapter 4, “Online curriculum modules for preparing teachers to teach Statistics: Design, implementation, and results,” Hollylynne Lee, Rick Hudson, Stephanie Casey, Gemma Mojica, and Taylor Harrison share their design and implementation of the Enhancing Statistics Teacher Education through the E-Modules (ESTEEM) project for providing statistics learning material to be used by MTEs in conducting teachers’ professional
development (TPD). ESTEEM modules use a common cartridge that lets MTEs organize teaching materials based on their needs in different LMS environments, either asynchronously or synchronously. The course comprises three modules (Foundations of Statistics Teaching, Teaching Inferential Reasoning, and Teaching Statistical Association) and two assignments (Screencast and Task Design Assignment). The modules aim to develop teachers’ critical understanding of statistics and awareness of students’ statistics thinking. At the same time, they aim to enhance teachers’ ability to use the Common Online Data Analysis Platform (CODAP, https://codap.concord.org), a free web-based tool for learning and investigating data.

The authors discuss the implementation of ESTEEM during 2018–2020 in the USA, which involved 45 MTEs and over 200 teachers (PSTs and ISTs). The post-implementation survey and interview revealed that teachers and MTEs had a favorable impression of their experience using ESTEEM. Furthermore, they report that teachers’ self-efficacy in teaching statistics increased after using the ESTEEM materials, as did their confidence in teaching statistics. However, the authors also mention several technical and non-technical issues that MTEs encountered while implementing ESTEEM and how the ESTEEM team overcame the issues. For instance, when MTEs indicated that the video was too long and their teachers had difficulty grasping its meaning, the authors revised the video using PlayPosit, an online tool that allows MTEs to provide questions or feedback in video format.

We believe that the authors succeeded in thoroughly explaining how ESTEEM was designed and put into action. The ESTEEM project inspired an iterative process of developing and implementing e-modules for mathematics TPD that are well-structured and focused on a piece of specific pedagogical content knowledge. Moreover, this chapter demonstrates to readers that a strong, long-term collaboration between mathematics education experts makes it possible to elevate the quality of online mathematics teachers’ education. Readers can find more in-depth elaboration on collaboration between teacher educators as an example of the distribution of expertise in designing online learning in Chapter 6.

In Chapter 5, “Rehumanizing mathematics education and building community for online learning,” Naomi Jessup, Jennifer Wolfe, and Crystal Kalinec-Craig raise the issue of pedagogic practices that rehumanize mathematics online learning. This chapter is based on their experience as MTEs facilitating the method and content course for PSTs during the COVID-19 pandemic, where inequity and participation required immediate attention due to the rapid implementation of online-only learning. The authors state that their goal “is to push MTEs to interrogate how the overall design of our courses can foster community building, expand approaches for engaging in formal learning spaces, and disrupt norms that can be experienced as carceral and dehumanizing” (p. 98). For this purpose, they provide several pedagogical practices for rehumanizing online courses to encourage participation in developing a positive identity in PSTs in the learning community. These include using a pre-survey-based online instructional learning design, a social-emotional check in before the online class begins, digital storytelling to promote “listening” so that PSTs have the competence to provide equity in the classroom, and the use of a platform that distributes class authority, such as Google Slides, Zoom breakout room, and Padlet, among others. They also criticize the “mics off, cameras on” rule that is commonly found in Zoom classrooms as carceral participation, which risks undermining learners’ confidence and restricting their freedom to express themselves.

The ideas and critique expressed in this chapter are crucial because online learning practices during a pandemic will leave a trail for optimizing the technology to help rehumanize mathematics education instead of blaming it for regressing human values. We appreciate the authors’ efforts to consolidate their opinions with relevant examples of teaching
practice to help readers imagine rehumanizing pedagogy in online learning. However, the authors’ thoughts are shaped by their own experiences, identities, and positionalities, which also provides space for readers with different identities and positionalities to act differently to achieve a rehumanization in their own online teaching of mathematics.

In Chapter 6, “An interdisciplinary approach to collaborative professional development for teacher educators: Number talks as culturally responsive online teaching,” Jeannette Alarcón, Jennifer Chauvot, Carrie Cutler, and Susie Gronseth bolster the discussion in Chapter 5 regarding the presentation of equitable access in online learning for mathematics PSTs. Due to the COVID-19 pandemic, the authors, three teacher educators from diverse fields of expertise, collaborated to develop online teaching strategies. This collaborative group, called The Collective, established Culturally Responsive Teaching (CRT) as a critical pedagogy in online learning. According to the Collective, number talks (NT) activities represent CRT practices that promote equity and social justice in online mathematics classrooms for K-12 students. Figure 6.1 (p. 119) illustrates how the Technological Pedagogical Content Knowledge (TPACK) model was used as inspiration for synchronous online instruction based on the NT strategy. Table 6.1 (p. 122) shows how the conversion of NT from face-to-face to a synchronous online activity practiced by two members of the Collective with different expertise creates a different atmosphere from the traditional classroom practice while still prioritizing a meaningful, critical, and equitable learning environment. The authors give examples of how Blackboard, Microsoft Teams, Google Jamboard, and Nearpod support CRT practices in NT activities. They also make it clear that collaboration at The Collective gave members a place to reflect on how they could improve their teaching practices to serve as role models for their PSTs.

This conceptual chapter shows how interdisciplinary collegial professional development changes members’ perspectives in that CRT can reveal more embedded systems of inequality in online teacher preparation programs. However, we are not sure whether the NT activity is considered basic knowledge by the global readers who are interested in this book. We thank Carrie Cutler for providing an appendix for the NT resources she uses (see p. 123), and yet we suggest that NT should have been explained more fully at the beginning of the chapter. We recommend that readers without a basic understanding of the concept of NT jump to the appendix link provided or go to the literature review in Chapter 8.2 (p. 148).

While Sect. 1 is more focused on the instructional design of online learning for preparing mathematics PSTs and ISTs, Sect. 2 is focused more on preparing PSTs and ISTs for teaching mathematics in a formal online learning experience. It consists of four chapters. In Chapter 7, “Culturally sustaining pedagogy for emergent bilinguals in a teacher education online course,” Ji-Yeong I, Ricardo Martinez, and Christa Jackson present an empirical study that aims to design a purely asynchronous online mathematics content-specific course focusing on students who are Emergent Bilinguals (EBs). The authors are motivated by the generic nature of TPD programs in the USA in preparing teachers to work effectively with linguistically and culturally diverse students. This is because “the role of language and culture in mathematics is often not visible” (p. 132). This chapter supports the need for a TPD that enables teachers to learn a research-based approach to teaching mathematics to EBs.

Culturally Relevant Pedagogy (CRP) and Culturally Sustaining Pedagogy (CSP) are two theories used to guide the online course design. The CRP theory focuses not only on students’ academic achievement but also on students’ cultural competence and how students can develop an understanding and critique of societal norms, which requires critical reflection in guiding action. Meanwhile, CSP (theoretically grounded in Funds of Knowledge and Third Space) explicitly emphasizes pluralistic societies and views students’ cultural and linguistic assets as the center of the classroom environment. The integration of “Funds of Knowledge into instruction and classroom environments is crucial because they [the students] experience
significant linguistic and cultural differences between home and school” (p. 133). The Third Space principle of CSP supports a hybrid space created for EBs’ learning by bringing together elements of the school culture and the home culture to create something new. Translanguaging (“a natural way bi/multilingual use language in which two or more languages interact to perform the best meaning-making within their language system” (p. 133)) and Community Cultural Wealth (CCW) are two other theories considered catalysts for EBs teachers. Table 7.1 (p. 135) offers a useful overview of the theoretical principles adopted in this chapter, along with reference to their core literatures and how they informed the course design.

This chapter has provided a broad insight into the four theoretical frameworks (CRP, CSP, translanguaging, and CCW) of online course design for EBs’ mathematical learning and their interrelationship. It also briefly describes the course development journey, the revision, and how prior research and theoretical frameworks informed that process. However, the practical explanation of how this perspective is related to mathematics learning has not been clearly reflected and is difficult for readers to find. Although mathematical modeling is supposed to be presented as a strategy that provides many opportunities for learning mathematics for EBs, examples of mathematical modeling tasks that directly show the relationship between students’ cultural and language assets with mathematics have not been well described.

In Chapter 8, “Number talks in asynchronous online classrooms for more equitable participation and as formative assessment of student thinking,” Simon Byeonguk Han and Eva Thanheiser describe how they apply number talks (NT) in the context of asynchronous mathematics content courses for elementary school PSTs, which differs from the NT context as discussed in Chapter 6. This study was motivated by the desire to discover a method for all students to share their thinking and strategies and to interact with each other’s thinking and strategies and also for the teacher to access all of the students’ thinking in learning mathematics when mathematics teaching moves online. Since several previous studies revealed that asynchronous online learning could be collaborative, it was possible to find a suitable strategy to enhance students’ number sense by conducting NT in the online environment. For this reason, the Post Before You See Other Responses function was utilized to enable all PSTs to participate in each NT by sharing their solutions before seeing their classmates’ solutions. Their three main research questions were as follows:

1. Can NT be implemented successfully in an online asynchronous classroom?
2. What strategies emerge when PSTs participate in asynchronous online NT?
3. What do PSTs claim to learn from online asynchronous NT? (p. 147).

In their literature review, the authors offer an overview of NT, with examples of strategies for addition. Responding to the research questions, first, the authors concluded that NT can be executed successfully in an online asynchronous setting. Second, they found that the online environment allowed the emergence of the same strategies as in the face-to-face classroom. Finally, they also found that PSTs noticed that NT helped them learn different ways to solve problems and compare strategies with others. Han and Thanheiser also conclude that NT is likely to play a role in the formative assessment of students’ thinking. Overall, the discussion on implementing NT in the context of asynchronous mathematics content courses for elementary school PSTs in this chapter is very well explained, systematic, and easy to understand. For example, Table 8.2 (p. 152) offers a summary of the types of examples, their goals/rationales, and the researchers’ anticipated strategies. Examples of contributions made by the participant teachers in supporting the answers to each research question, and their reflections, are presented clearly and directly in text and in numerous tables.
In Chapter 9, “A three-part synchronous online model for middle-grade mathematics teachers’ professional development,” Jeffrey Choppin, Julie Amador, Cynthia Callard, Cynthia Carson, Ryan Gillespie, Jennifer Kruger, Stephanie Martin, and Genie Foster explore ways to encourage rural middle school mathematics teachers to engage in ambitious mathematics instruction. Promoting students in solving and reasoning complex mathematics problems, listening and responding to their contributions, expanding participation in mathematical discourse, and positioning students as mathematically competent are all instances of ambitious mathematics instruction. Since this chapter targets rural mathematics teachers, this effort was accomplished by creating a fully online professional development model with three parts in the implementation process.

Grounded in three theories of action concerning teacher change, the authors offer three online activities as interventions: orchestrating mathematical discussions, teaching labs, and coaching cycles designed with targeted goals carried out sequentially and systematically. They discuss the conceptualization and analysis of the model, as well as the appropriate and cutting-edge technological applications that were utilized in this study. For example, the various online platforms, such as Zoom, Google Docs, and Google Drawings, allowed teachers to jointly develop and share artifacts, including approaches to solving mathematics problems, in real time. In addition, the Swivl platform was utilized to create attractive videos that allow teachers to record video lessons without someone else operating the camera. Teacher interactions in discussing pedagogical principles and exploring the impact of online instruction and assignments on student learning can also be assessed using a combination of Zoom and Panopto. This project demonstrates that the online environment is advantageous for professional development of rural teachers by providing opportunities to share, reflect, and discuss their ambitious mathematics teaching.

In Chapter 10, “The impact of an online teacher education program on the development of prospective secondary mathematics teachers’ noticing,” Ceneida Fernández, Salvador Llinares, and Yoilyn Rojas present their research in Costa Rica to reveal the potential of online environments to enhance PSTs’ noticing skills in teaching–learning situations. This study extends the field of noticing development by designing an online environment to support PSTs’ noticing skills while interning at schools. The design brought together two theoretical perspectives: the enactive stance (knowing is doing and doing is knowing) and the process of becoming aware. It is characterized by a cycle in which a narrative is written and shared on an online forum, followed by the writing of a new narrative that incorporates the forum’s feedback. Hence, this study addresses two research questions:

- How do prospective teachers develop their noticing skills in a formal online environment where they have to share narratives about their practices and share them over online forums?
- How do the design elements of the formal online environment influence the prospective teachers’ learning? (p. 125)

For the purpose of writing the narratives, PSTs are given guided questions as shown in Appendix 1 (p. 204) so that their narratives focus on noticing students’ mathematical thinking competence, thereby supporting the development of awareness. The authors use the Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) analytical framework to answer the first research question. It enables them to analyze the narratives written by the PSTs to show how their skills at noticing students’ mathematical thinking improved (or not) and whether the PSTs took advantage of what they noticed about the students’ thinking to improve their teaching actions.
findings indicated that all five PSTs who took part in this study improved their noticing skills, which resulted in changes in classroom management. At the same time, only one participant failed to demonstrate the changes in lesson planning. For the second research question, the authors elaborate on four specific design elements that support PSTs’ learning: (a) the narrative-forum-feedback-new narrative cycle which provides an online collaborative group between PSTs and MTE, (b) the online forum which allows PSTs to improve the way they articulate reasons for their specific teaching action, (c) the cycle which also links the development of noticing and change of actions, and (d) the guiding questions that “helped PTs to write their narratives [and] focused their attention on students’ thinking and on what the teacher’s role should be to promote students’ learning” (p. 203). The authors also express concern about the possibility of other factors (e.g., mathematical content) also influencing the development of PSTs’ noticing skills. Aside from that, the designs they provide may potentially improve PSTs’ noticing skills in formal online environments.

Self-directed, experiential, and practice-based online learning opportunities for mathematics teachers are the focus of Sect. 3. It contains six chapters, beginning with Chapter 11. In this chapter, “Theory-based intervention framework to improve mathematics teachers’ motivation to engage in online professional development,” Nathan Hawk, Margaret Bowman, and Kui Xie show readers how teachers’ motivation to use online learning affects their professional development. They refer to research that shows the importance of teachers’ motivation and its impact on teacher engagement in TPD, collecting and synthesizing relevant literature on mathematics TPD as well as increasing the teachers’ positive values and expectancy for success.

The authors believe that a more solid and multi-dimensional theoretical framework based on Expectancy-Value Theory is needed in looking at teacher motivation. They offer five principles (promote intrinsic value, highlight utility value, foster attainment value, reduce perceived cost, increase expectancy for success) that can be used as a framework to support and increase the value of positive perceptions of mathematics teachers in online learning. Table 11.1 (p. 202) offers a useful summary of the design principles with descriptions and examples. Readers of this chapter will discover that each proposed principle has significant implications for how online mathematics TPD should be carried out. The authors recommend that future online mathematics TPD designers consider these five principles to provide more positive teacher involvement and influence.

In Chapter 12, “Mathematics for the citizen, m@t.abel, and MOOCs: From paper to online environments for mathematics teachers’ professional development,” Ferdinando Arzarello, Ornella Robutti, and Eugenia Taranto discuss how the Italian TPD program has changed over time. The first program, Mathematics for the Citizen, ran from 2000 to 2005 and was a website-accessible book-based TPD program. This program was later changed to m@t.abel which ran from 2006 to 2012 using a blended model with online platform support by teacher trainers. The final program discussed in this chapter is Math MOOC UniTo, which uses the massive open online course (MOOC) platform and was implemented between 2015 and 2020. The MOOC resources deal with the didactics of a MOOC module designed by a team of researchers and teacher-researchers following the Italian National Curriculum.

To meet the requirements of the National Curriculum for teacher education, the authors explain how they adapted the previous program’s activities into online learning modules. They also explain how mathematics teachers were able to collaborate online to learn and use new teaching practices, resources, and technologies. They employed the Meta-didactic Transposition (MDT) framework. In this, the researchers designed an educational program
and coached the teachers who participated in implementing it. The teachers actively participated, drawing on their professional expertise and experience. This implies a deep intertwining between practical and theoretical issues for both groups, and the research draws heavily on Chevallard’s Anthropological Theory of Didactics.

The authors also describe the profiles of participants and the MOOC team, teaching activities, and interactions between teachers in the program “to show how they learned and made use of new teaching practices, new resources, and new technologies for teaching mathematics” (p. 235). We believe this chapter has developed an inspiring TPD in mathematics, which continues to adapt to technological developments, evolves, and provides space for shared praxeologies between researchers and teachers. MDT, along with other related theoretical frameworks, has the potential to lead to new approaches to classroom instruction.

In Chapter 13, “Tweeting or listening to learn: Professional networks of mathematics teachers on Twitter,” Anne Garrison Wilhelm and Jaymie Ruddock examine Twitter’s role in connecting mathematics educators. They believe a network is required to connect teachers with different competencies so they can share their teaching experiences. This chapter is a descriptive case study of Twitter as a site for teacher professional development, focusing on mathematics teachers who utilize MathTwitterBlogSphere. The authors employ a digital method to collect, observe, and characterize educator participation by analyzing their digital footprint on Twitter. In addition, they demonstrate that the network gives teachers greater access to highly relevant skills based on its scope. This chapter’s discussion gives readers insight into Twitter usage and how it inspires teachers to fulfill their potential. The authors observe the potential for Twitter to extend teachers’ advice networks and suggest opportunities for future research on the professional network of mathematics teachers who are listening (e.g., how do teachers listen?) or retweeting, as well as how their participation with Twitter changes over time.

In Chapter 14, “A distributed leadership model for informal, online faculty professional development,” Erica Miller and Emily Braley discuss how distributed leadership (DL) and communities of practice (CoP) theories can be used to structure informal, online faculty professional development. DL provides a supportive context and actions by which CoP can be created and maintained. On the other hand, CoP contributes one way of enabling the DL approach. For this reason, the authors conduct a study to create and analyze the connection between these two theories through online book study groups for college mathematics faculty members. This online book study group allows mathematics faculty members to practice facilitating conversations about teaching mathematics equitably and in an online setting. Three macro-activity tasks of the online book study group are carried out systematically to explain how the two theories are interrelated, namely: (a) launching the book study group, (b) supporting the participants, and (c) supporting the facilitators. This study’s results indicate that the online setting’s structure allowed participants to broaden their professional networks outside of their home institutions, providing more flexibility for participation and allocating adequate practice time. For those involved in mathematics teacher education and professional development, the distributed leadership model for book study groups can be used to engage teachers in informal conversations about mathematics education literature.

This chapter shows the significant contribution of the online book study group in facilitating mathematics faculty in teaching mathematics equitably. Various theoretical arguments, as well as macro and micro stages, are well explained to the readers. However, contributions related to the development of the mathematics community concerning mathematics content and learning do not seem to have a clear emphasis.
In Chapter 15, “Confronting teachers with contingencies to support their learning about situation-specific pedagogical decisions in an online context,” Amanda Brown, Irma Stevens, Patricio Herbst, and Craig Huhn illustrate how they adapted the StoryCircles process with a contingency card design which teachers found useful for facilitating whole-class discussion around open and novel tasks. StoryCircles is a form of professional education that gathers teachers to collectively represent a lesson through iterative phases of scripting, visualizing, and arguing about alternatives—with teachers’ visualization of lesson details supported through the production of storyboards. Each activity is based on the idea that teachers can only improve their professionalism by engaging in, learning from, and reflecting on their practice. Specifically, the authors focus on how they derived teachers’ practical rationale about the subject-specific moves teachers make and how they used that information. Contingency cards are a way for TPD facilitators to provoke conversations in order to show teachers how to prepare for the real-life situations (or contingencies) they might face in practice. The authors show how contingency cards can enable teachers and researchers alike to explore and become more aware of how the givens of a pedagogical situation can play a role in shaping teachers’ decisions within that situation. Uniquely, this process was designed in an online environment, thus opening up opportunities for teachers across geographically distant districts to work together, practice, and receive feedback on subject-specific pedagogical decisions.

Information about how emerging digital technologies (e.g., LessonSketch—a platform to represent and create teaching practices) can be leveraged to support the delivery of online, practice-based professional learning experiences for teachers was presented very well and systematically. A practical example demonstrates the use of StoryCircles in supporting secondary in-service geometry teachers’ professional growth online related to the Tangent Circle Problem, giving the reader a brief explanation of how the practice-based professional learning experiences were developed. The development of a particularly salient illustration of teacher and student moves in the mathematical learning process offers a useful guide for readers and researchers. The authors note that teachers’ practical rationality enabled decisions such as suggesting pedagogical alternatives not traditionally valued. In addition, online administration ensured that “teachers have regular and routine opportunities to practice and receive feedback on subject-specific pedagogical decisions” (p. 312). Finally, this chapter also highlights how the virtual environment afforded “opportunities for teachers to learn in, from, and for practice” (p. 312).

In Chapter 16, “Virtual field experiences as an opportunity to develop pre-service teachers’ efficacy and equitable teaching practice,” Liza Bondurant and Joel Amidon discuss their work on using the Mursion™ as a Virtual Field Experience (VFE) platform to help PSTs practice equitable teaching. The VFE mediates and supports the practice of teaching through interactions with avatars. Based on Situated-Learning Theory, the authors suggest that PSTs should gain real-world experience as teachers in schools, particularly in the area of equitable teaching practice, which was increasingly hard to achieve during the COVID-19 pandemic. The VFE in this study offers a simulated classroom with five student avatars designed to be racially ambiguous. The PSTs attempt to instruct those avatars in the way a teacher should in a real classroom.

The primary goal of this study was to determine how VFE promotes or hinders the efficacy and equitable teaching practice of PSTs. To answer this question, the authors used a variety of instruments that had been validated based on previous research findings. The authors found that PSTs with more teaching practice hours using VFE demonstrated more equitable teaching practices as evidenced by a higher percentage of student participation, a more equitable distribution of participation, and a higher percentage of questions
requiring student justification. The authors argue that VFE gives PSTs a realistic level of teaching complexity, which (not surprisingly) resulted in a diverse efficacy response among participants.

The authors indicate that this study had the major limitation of having only four PSTs and make it clear that the results should not be generalized. However, this final chapter helps readers to appreciate another way that online teacher preparation programs have evolved: using virtual students to simulate classroom settings. Even after the pandemic, we agree that VFE should be considered to prepare PSTs before they are willing to deal with students in real classrooms. Furthermore, VFE enables MTE to present challenges to PSTs regarding specific cases of students’ behavior for which it may not always be easy to find actual samples. However, we should consider that VFE technology is, or could be, still too complex to be realized globally for the time being.

2 General overview of the book

The COVID-19 pandemic has prompted a re-examination of how face-to-face mathematics education instruction has been transformed into online learning and how the dimensions of teacher professionalism have developed through online learning, making this book highly relevant to the current state of mathematics education. The readership of this book will primarily include MTEs and mathematics education researchers, although it could also be helpful to others in the field.

It is not easy to find books that help mathematics teacher educators do their work; this is one of the few. Even though teacher educators play a central role in teacher education and are potentially present throughout a teacher’s career, there is still some concern for the profession as a whole (Murray, 2016). However, as Alarcón et al. note at the beginning of Chapter 6, MTEs are communities that need to be ready to switch to online learning and have to deal with two problems. First, MTEs may know how their pedagogy works in person, but they may not know how to use it online. Second, online practice means getting their teachers ready to handle a virtual learning environment where they may still be learners themselves. This book offers insights into the multi-dimensional mathematics classroom that has emerged from various proven online learning practices for MTEs. Such insights may be helpful for those who are just starting their careers in online learning as well as those who are already familiar with online learning.

Schoenfeld (2020) outlines five dimensions for achieving an effective mathematics classroom: mathematical content; cognitive demands; equitable access; agency, ownership, and identity; and formative assessment. Understanding how these five dimensions transform from face-to-face to online instruction is a crucial issue for the MTE profession. The reader will find that these dimensions are thoroughly discussed in the book. The mathematics content specifically addressed in online learning is statistics (Chapter 4), number (Chapters 6 and 8), and geometry (Chapters 9 and 15). Dimensions of cognitive demand for students, such as mathematical problem solving, reasoning, and modeling, are discussed in Chapters 3, 7, 8, 9, 10, and 16. Chapters 5, 6, 7, 8, 14, and 16 discuss equitable access and mathematical identity in an online environment. Meanwhile, online learning assessment is discussed in Chapters 3, 8, and 12.

In addition to the online mathematics classroom dimensions, this book offers suggestions on how educators and researchers can collaborate on the professional development of MTEs (Chapters 4 and 6) and professional development of teachers (Chapters 7, 9,
10, and 12), even in informal settings (Chapters 13 and 14). Researchers in mathematics education as well as MTEs will benefit from this book, as it presents a wide range of research studies on online mathematics education and opens the door to future research with much unexplored territory. This book also provides policymakers with a perspective on how online TPD programs’ collaborative long-term agendas can enhance mathematics teachers’ competencies.

Undoubtedly, this book contains numerous online technology platforms developed by the authors (Chapters 4 and 16) and those utilizing existing technologies (Zoom, Canvas, Moodle, Nearpod, PlayPosit, LessonSketch, CODAP, etc.). Clearly, this book is not about how to use online technology per se. However, we believe that educators, in general, will benefit from the numerous online alternatives presented throughout the book. Despite this, we believe that teachers will still prefer both free and adaptable technology to meet the needs of their specific classroom environments (McCulloch et al., 2018).

This book’s strength is that it is multidimensional, rich in theory, practice, research, references, and technology associated with supporting mathematics educators through and towards online learning. This book is a great source of knowledge, and we appreciate the placement of the chapters into each section. The chapter organization within each section provides the reader with a logical progression of conceptual understanding, experience, examples, and empirical research. In Sect. 1, for instance, the editors included the central idea of regarding the similarity and dissimilarity between traditional and online mathematics classrooms in the first two chapters. This means that both novices and experts in online learning should have the same starting point for exploring the remainder of the book’s content.

However, we believe that in addition to identifying the many positive impacts of online learning that are discussed in this book, several authors have neglected to mention that, just like face-to-face classroom instruction, online learning will never be flawless. Therefore, it is vitally important to have an explicit reflection by authors on what should be revised in terms of future innovation, with warnings for readers about what preventative actions could be needed. For example, we appreciate that Lee et al. (Chapter 4) describe how the ESTEEM project they created continues to evolve through a process of revisions based on feedback from MTEs as users. In addition, Miller and Braley make it clear in Chapter 14 that the informal online faculty professional development program that they provided would have been difficult to manage if the participants had not demonstrated the required level of commitment.

A further example of diversity in this book is that different words are used to discuss things that may have similar meanings. For instance, “instructors” (Chapter 1), “university tutor” (Chapter 10), and “MTEs” (Chapters 4, 5, and 6); “prospective teachers” (Chapter 10), “teacher candidates” (Chapter 5), and “PSTs” (Chapters 3, 4, 6, and 16); and also “teachers” in Chapters 9 and 15 mean the same thing as “ISTs.” Therefore, we strongly advise readers whose first language is not English to familiarize themselves with the context of each chapter so that differences in terminology do not obscure the interrelationships between chapters.

This book is a combination of theoretical-conceptual review chapters and chapters with a focus on research on the development of online learning. Throughout the theoretical-conceptual chapters, the authors have succeeded in presenting their perspectives based on in-depth theoretical studies as well as their implications and application examples, as in Chapters 1, 2, 6, and 11. Some noticeable observations in the online learning development research chapters are essential to mention. To begin, we discovered that only Wilhelm and Ruddock (Chapter 13) and Bondurant and Amidon (Chapter 16) mentioned their research methods explicitly. Other chapter authors appear to prefer to leave their research methods
obscured in their narratives. In fact, disclosure of the methodology helps readers evaluate its strength and weakness. Second, in some chapters, we note the need to triangulate research findings by using multiple instruments (questionnaires, surveys, interviews, observations, etc.) or different groups of participants. Those chapters tended to use anecdotes as a single instrument, as Tran and Nguyen acknowledge in Chapter 3. Triangulation will further convince readers that online learning innovations have internal and external validity (Creswell & Creswell, 2018). However, in relation to the book’s purpose of inspiring and providing examples of innovative practices in online education for MTEs, the editors’ exclusion of these two methodological approaches is understandable.

Many of the chapters in this book discuss how to ensure that students of all genders, ethnicities, and socio-cultural backgrounds have equal access to online mathematics education, and we believe that this book supports that goal. While the majority of authors as well as the research locations are situated in the USA, this book also examines online learning mathematics innovation in various other countries, including Australia, Brazil, Canada, Italy, and Costa Rica. Regarding infrastructure, human resources, policy, and curriculum, clearly not all countries are at the same level of readiness to implement online mathematics education. Therefore, it would be beneficial if the authors or editors reminded the reader to consider the relevance of the research to their respective socio-cultural and other contexts.

Overall, this review suggests that the strength of this book would have benefitted readers even more if the editors had included a concluding chapter to summarize and highlight the book’s multidimensionality. This concluding chapter, in our opinion, would also be advantageous for the editors in communicating their perspectives on the aforementioned review points. With this cautionary note in mind, we recommend that anyone interested in online mathematics education should read this book.

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