ELECTROMAGNETIC FORM FACTORS AT LARGE MOMENTUM TRANSFER

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Abstract

Recent improvements of the hard scattering picture for the large $p_\perp$ behaviour of electromagnetic form factors, namely the inclusion of both Sudakov corrections and intrinsic transverse momentum dependence of the hadronic wave function, are reviewed. On account of these improvements the perturbative contributions to the pion’s and the nucleon’s form factor can be calculated in a theoretically self-consistent way for momentum transfers as low as about 2 and 3 GeV, respectively. This is achieved at the expense of a substantial suppression of the perturbative contribution in the few GeV region. Eventual higher twist contributions are discussed in some detail.

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1. The hard scattering picture

There is general agreement that perturbative QCD in the framework of the hard-scattering picture (HSP) is the correct description of form factors at asymptotically large momentum transfer (see [1] and references therein). In the HSP a form factor is expressed by a convolution of distribution amplitudes (DA) with hard scattering amplitudes calculated in collinear approximation within perturbative QCD. The universal, process independent DAs, which represent hadronic wave functions integrated over transverse momenta, are controlled by long distance physics in contrast to the hard scattering amplitudes which are governed by short distance physics. The DAs cannot be calculated by perturbative means, we have to rely on models. In principle lattice gauge theory offers a possibility to calculate the DAs but with the present-day computers a sufficient accuracy can not be achieved, only a few moments of the pion and the proton DA have been obtained [2]. It is of utmost phenomenological interest whether or not the asymptotic perturbative result can already be applied at experimentally accessible momentum transfers. The major topic of this talk is to answer that question. In order to keep the technical effort simple I am going to discuss the electromagnetic form factor of the pion mainly. The generalization to the phenomenological more important case of the nucleon form factor is straightforward.

Now let us consider the electromagnetic form factor of the pion. To lowest order pertubative QCD the hard scattering amplitude $T_H$ is to be calculated from the two one-gluon exchange diagrams. Working out the diagrams one finds

\[ T_H(x_1, y_1, Q, \vec{k}_\perp, \vec{l}_\perp) = \frac{16\pi \alpha_s(\mu) C_F}{x_1 y_1 Q^2 + (\vec{k}_\perp + \vec{l}_\perp)^2}. \]  

(1.1)

where $Q(\geq 0)$ is the momentum transfer from the initial to the final state pion. $x_1$ ($y_1$) is the longitudinal momentum fraction carried by the quark and $\vec{k}_\perp$ ($\vec{l}_\perp$) its transverse momentum with respect to the initial (final) state pion. The momentum of the antiquark is characterized by $x_2 = 1 - x_1$ ($y_2 = 1 - y_1$) and $-\vec{k}_\perp$ ($-\vec{l}_\perp$). $C_F (= 4/3)$ is the colour factor and $\alpha_s$ is the usual strong coupling constant to be evaluated at a renormalization scale $\mu$. The expression (1.1) is an approximation in so far as only the most important $\vec{k}_\perp$- and $\vec{l}_\perp$-dependences have been kept. Denoting the wave function of the pion’s valence Fock state by $\Psi_0$, the form factor is given by

\[ F^\pi(Q^2) = \int \frac{dx_1 d^2k_\perp}{16\pi^3} \int \frac{dy_1 d^2l_\perp}{16\pi^3} \Psi_0^*(y_1, \vec{l}_\perp) T_H(x_1, y_1, Q, \vec{k}_\perp, \vec{l}_\perp) \Psi_0(x_1, \vec{k}_\perp). \]  

(1.2)

Strictly speaking $\Psi_0$ represents only the soft part of the pion wave function, i.e. the full wave function with the perturbative tail removed from it [1]. Contributions from higher Fock states are neglected in (1.2) since, at large momentum transfer, they are suppressed by powers of $\alpha_s/Q^2$.

At large $Q$ one may neglect the $k_\perp$- and $l_\perp$-dependence in the gluon propagator as well; $T_H$ can then be pulled out of the transverse momentum integrals, and these integrations apply only to the wave functions. Defining the DA by

\[ \frac{f_\pi}{2\sqrt{6}} \phi(x_1, \mu_F) = \int \frac{d^2k_\perp}{16\pi^3} \Psi_0(x_1, \vec{k}_\perp), \quad \int_0^1 dx_1 \phi(x_1, \mu_F) = 1, \]  

(1.3)

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one arrives at the celebrated hard-scattering formula for the pion’s form factor

$$F_{\pi}^{HSP}(Q^2) = \frac{f_{\pi}^2}{24} \int dx_1 \, dy_1 \, \phi^*(y_1, \mu_F) \, T_H(x_1, y_1, Q, \mu) \, \phi(x_1, \mu_F), \quad (1.4)$$

which is valid for $Q \to \infty$. $f_{\pi} (= 133 \text{ MeV})$ is the usual $\pi$ decay constant. $\mu_F$ is the scale at which the physics is factorized into its soft and hard components. The DA mildly depends on the factorization scale (QCD evolution).

An appropriate choice of the renormalization scale is $\mu = \sqrt{x_1 y_1 Q}$. This avoids large logs from higher order perturbation theory at the expense, however, of a singular behaviour of $\alpha_s$ at a certain value, which is typically chosen in the range of 0.5 to 0.7. That crude recipe is unsatisfactory although the Sudakov argument itself is correct as will be discussed in the next section. Another often used and very convenient recipe is to pull out of the integral in (1.4) the coupling $\alpha_s$ and to take a suitable fraction of the momentum transfer as its argument. Both the recipes have the unwelcome consequence of introducing an external free parameter in the calculation. The second recipe for the treatment of $\alpha_s$ allows one to cast the HSP prediction for the pion form factor into the simple form

$$Q^2 F_{\pi}^{HSP} = \frac{2\pi}{3} f_{\pi}^2 C_F \alpha_s(\mu) \langle x_1^{-1} \rangle^2 \quad (1.5)$$

which nicely brings to light that the moment $\langle x_1^{-1} \rangle (= \int dx_1 \Phi(x_1)x_1^{-1})$ is the only information on the DA actually required, a fact which approximately remains true if $\alpha_s$ is kept in the integral. One also reads off from (1.5) that the HSP predicts a behaviour of the form factor as $\sim 1/Q^2$ (modulo $\ln Q$) in agreement with dimensional counting.

In the formal limit $Q^2 \to \infty$ the HSP even determines the moment $\langle x_1^{-1} \rangle$ and hence the value of the form factor (except of the uncertainty in the treatment of $\alpha_s$). It can be shown [1] that any DA evolves into the asymptotic form $\Phi_{\pi}^{AS} = 6x_1x_2$ which entails $\langle x_1^{-1} \rangle = 3$ and, using $\alpha_s(\mu) = 0.33$, $Q^2 F_{\pi}^{HSP} = 0.15$. This prediction is too small by about a factor of 2 as compared with the admittedly poor data [3]. In order to obtain better results a larger value of the moment $\langle x_1^{-1} \rangle$ is required. Obviously, this can be achieved with DAs being strongly concentrated in the end-point regions. Such DAs, first proposed by Chernyak and Zhitnitsky (CZ) [4], find a certain justification in QCD sum rules by means of which a few moments of the DAs have been calculated. CZ suggested the DA $\Phi_{\pi}^{CZ} = 30x_1x_2(x_1 - x_2)^2$ yielding $\langle x_1^{-1} \rangle = 5$ and hence a value for the form factor in accord with the data. The CZ moments are subject to considerable controversy: Other QCD sum rule studies provide different values for the moments [5]. Also the results obtained from lattice gauge theory do not well agree with the CZ moments [3]. Finally, the data on the $\pi - \gamma$ transition form factor, which is also determined by the moment $\langle x_1^{-1} \rangle$, favour the asymptotic DA.

The magnetic form factor of the nucleon can also be analysed within the HSP. The calculations reveal a dramatic dependence of the results on the utilized DA. Asymptotically,
with the DA \( \sim 120x_1x_2x_3 \), one obtains

\[
G_M^p = 0, \quad Q^4 G_M^n = \left( \frac{\alpha_s(Q)}{\alpha_s(\mu_0)} \right)^{4/3\beta_0} f_N^2(\mu_0) \frac{100}{3} (4\pi\alpha_s(Q))^2
\]

where \( \beta_0 = 11 - 2/3n_f = 9 \) and \( \mu_0 \approx 1 \text{ GeV} \). \( f_N(\mu_0) \) \( = (5.0 \pm 0.3) \times 10^{-3} \text{ GeV}^2 \) represents the value of the nucleon wave function at the origin in configuration space. The factor in front of \( f_N \) takes into account the evolution of \( f_N \). These results do not bear any resemblance to the experimental data \( (Q^4G_M^p \sim 1 \text{ GeV}^4 \text{ for } Q^2 \text{ between 10 and 30 GeV}^2, \) see [3]). Again, as in the pion case, strongly end-point concentrated DAs yield good results provided an appropriate value is chosen for \( \alpha_s \). A set of such end-point concentrated DAs which all respect the QCD sum rules constraints, has been determined by Bergmann and Stefanis (BS)[4].

The Pauli form factor \( F_2 \), and hence the electric form factor \( G_E \), cannot be calculated within the HSP since it requires helicity-flip transitions which are not possible for (almost) massless quarks in the collinear approximation. The Pauli form factor is dominated by sizeable higher twist contributions in the few GeV region as we know from experiment [8]. The higher twist nature of the Pauli form factor is clearly visible in Fig. 1: its large \( Q \) behaviour is compatible with \( 1/Q^6 \).

## 2. The Botts-Li-Sterman approach

The applicability of the HSP at experimentally accessible momentum transfers, typically a few GeV, was questioned [5, 9]. It was asserted that in the few GeV region the hard-scattering picture accumulates large contributions from the end-point regions where the parton virtualities are small. This renders the perturbative calculation inconsistent, in particular for the end-point concentrated DAs. The use of the collinear approximation, i.e. the neglect of the transverse momentum dependence of the hard scattering amplitude, see for instance (1.1), is also unjustified in the end-point regions. Obviously, the collinear approximation entails large errors in the final results for the end-point concentrated DAs whereas for the asymptotic DA and similar forms it turns out to be reasonable.

The statements made by the authors of [3, 4] were challenged by Sterman and collaborators [10, 11, 12]. These authors suggest to retain the transverse momentum dependence of the hard scattering amplitude and to take into account Sudakov corrections. In order to include the Sudakov corrections it is advantageous to reexpress (1.2) in terms of the Fourier conjugated variable \( \vec{b} \) in the transverse configuration space

\[
F_{\pi}^\text{pert}(Q^2) = \int \frac{dx_1 dy_1}{(4\pi)^2} \int d^2b \hat{\Psi}_0^*(y_1, \vec{b}) \hat{T}_H(x_1, y_1, b, Q, t) \hat{\Psi}_0(x_1, -\vec{b}) \exp[-S] \tag{2.1}
\]

where the Fourier transform of a function \( f = f(\vec{k}_\perp) \) is denoted by \( \hat{f} = \hat{f}(\vec{b}) \). As the renormalization scale Sterman et al. choose the largest mass scale appearing in \( \hat{T}_H \), the Fourier transform of the lowest order hard scattering amplitude (1.4):

\[
t = \text{Max}(\sqrt{x_1y_1}Q, 1/b). \tag{2.2}
\]
The factor \( \exp[-S] \) in (2.1), termed the Sudakov factor, incorporates the effects of gluonic radiative corrections and therefore represents parts of higher order perturbative corrections to \( T_H \). Botts and Sterman \[10\] have calculated the Sudakov factor using resummation techniques and having recourse to the renormalization group. For the pion case they find a Sudakov exponent of the form

\[
S(x_1, y_1, b, Q, t) = \sum_{i=1}^{2} [s(x_i, b, Q) + s(y_i, b, Q)] - \frac{8}{3\beta_0} \ln \frac{\ln(t/\Lambda_{QCD})}{\ln(1/b\Lambda_{QCD})}
\]  

(2.3)

The lengthy expression for the Sudakov function \( s = s(\xi_i, b, Q) \), which includes all leading and next-to-leading logarithms, is given explicitly in \[11\]. The most important term in it is the double logarithm

\[
\frac{8}{3\beta_0} \ln \frac{\xi_i Q}{\sqrt{2}\Lambda_{QCD}} \ln \frac{\ln(\xi_i Q/\sqrt{2}\Lambda_{QCD})}{\ln(1/b\Lambda_{QCD})},
\]

(2.4)

where \( \xi_i \) is one of the fractions, \( x_i \) or \( y_i \). The quark-antiquark transverse separation acts as an infrared cut-off. The underlying physical idea is the following: Because of the colour neutrality of a hadron, its quark distribution cannot be resolved by a gluon with a wave length much larger than the \( q - \bar{q} \) separation. Consequently radiation is damped. The infrared cutoff marks the interface between the non-perturbatively soft momenta, which are implicitly accounted for in the hadronic wave function, and the contributions from semi-hard gluons, incorporated in a perturbative way in the Sudakov factor. Whenever \( 1/b \) is large relative to the hard (gluon) scale \( \xi_i Q \), the gluonic corrections are to be considered as hard gluon corrections to \( \hat{T}_H \) and hence are not contained in the Sudakov factor but absorbed in \( \hat{\hat{T}}_H \). For that reason the Sudakov function \( s(\xi_i, b, Q) \) is set equal to zero whenever \( \xi_i \leq \sqrt{2}/bQ \).

The crucial advantage of the modified HSP (2.1) is that the renormalization scale can now be chosen to be \( \mu = \sqrt{x_1 y_1 Q} \) and so large logs from higher order perturbation theory be avoided. The singularity of the “bare” \( \alpha_s \) is suppressed by the Sudakov factor inherently; there is no need for external regulators! This can be observed from Fig. 2 where the exponential of the Sudakov function \( \exp[-s(\xi_i, b, Q)] \) is displayed. For small \( b \) there is no suppression. As \( b \) increases \( \exp[-s] \) decreases and drops to zero faster than any power of \( \ln(1/b\Lambda_{QCD}) \) for \( b\Lambda_{QCD} \rightarrow 1 \) except one is in the dangerous region, \( \xi_i \leq \sqrt{2}/bQ \), where \( s(x_i, b, Q) \) is set to zero. However, in this case \( \exp[-s(1 - \xi_i, b, Q)] \) provides the required suppression. Consequently, the Sudakov factor (2.3) drops to zero faster than any power of \( \ln(1/b\Lambda_{QCD}) \) for \( b\Lambda_{QCD} \rightarrow 1 \) irrespective of the value of \( \xi_i \). This behaviour of the Sudakov factor guarantees the cancellation of the \( \alpha_s \) singularity (owing to the limit \( t \rightarrow \Lambda_{QCD} \)).

For \( b\Lambda_{QCD} \) larger than 1, considered as the true soft region, the Sudakov factor is set to zero. Note that for \( Q \rightarrow \infty \) the Sudakov factor damps any contribution except those from configurations with small quark-antiquark separations. In other words, the hard-scattering contribution (1.4) dominates the form factor asymptotically.
3. The intrinsic $k_{\perp}$-dependence of the wave function

The approach proposed in [10, 11, 12] certainly constitutes an enormous progress in our understanding of exclusive reactions at large momentum transfer. In any practical application of that approach one has however to allow for an intrinsic transverse momentum dependence of the hadronic wave function [13], although, admittedly, this requires a new phenomenological element in the calculation. Fortunately, in the case of the pion the intrinsic transverse momentum of its valence Fock state wave function is rather well constrained. In accordance with (1.3) the wave function can be written as

$$\Psi_0(x_1, \vec{k}_{\perp}) = \frac{f_\pi}{2\sqrt{6}} \phi(x_1) \Sigma(x_1, \vec{k}_{\perp}), \quad (3.1)$$

the function $\Sigma$ being normalized in such a way that

$$\int \frac{d^2k_{\perp}}{16\pi^3} \Sigma(x_1, \vec{k}_{\perp}) = 1. \quad (3.2)$$

The wave function (3.1) is subject to the following constraints: it is normalized to a number $P_{qq} \leq 1$, the probability of the valence quark Fock state; the value of the configuration space wave function at the origin is determined by the $\pi$ decay constant; the process $\pi^0 \rightarrow \gamma\gamma$ provides a third relation. Finally, the charge radius of the pion provides a lower limit on the root mean square (r.m.s.) transverse momentum; actually it should be larger than 300 MeV. The $k_{\perp}$-dependence of the wave function is parameterized as a simple Gaussian

$$\Sigma(x_1, \vec{k}_{\perp}) = 16\pi^2\beta^2 g(x_1) \exp\left(-g(x_1)\beta^2 k_{\perp}^2\right), \quad (3.3)$$

g($x_1$) being either 1 or $1/x_1 x_2$. The latter case goes along with a factor $\exp(-\beta^2 m_q^2/x_1 x_2)$ in the DA where $m_q$ is a constituent quark mass (330 MeV). The Gaussian (3.3) is consistent with the required large-$k_{\perp}$ behaviour of a soft wave function. Several wave functions have been employed in [13]. Here, in this talk, only the results for the two extreme cases utilized in [13] are quoted. That is, on the one hand, the CZ wave function $\sim \Phi_{CZ}$, $g = 1$ [4] which is the example most concentrated in the end-point region and, on the other hand, the modified asymptotic (MAS) wave function $\sim \Phi_{AS}$, $g = 1/x_1 x_2$. The MAS wave function is the example least concentrated in the end-point regions. The parameter $\beta$ in the wave function is fixed by requiring specific values for the r.m.s. transverse momentum. For a value of 350 MeV all the constraints on the pion wave functions are well respected [13].

Li and Sterman [11] assume that the dominant $b$-dependence of the integrand in (2.1) arises from the Sudakov factor and that the Gaussian in the Fourier transformed $\Sigma$ can consequently be replaced by 1. Numerical evaluations of the pion’s form factor through (2.1), using the various wave functions mentioned above, reveal that this assumption leads to an overestimate of the perturbative contribution (see Fig. 3). For momentum transfers of the order of a few GeV the wave function damps the integrand in (2.1) more than the Sudakov factor which takes over only for very large values of $Q$. The suppression caused by
the intrinsic transverse momentum dependence of the wave function, is particular strong for the end-point concentrated wave functions. These observations confirm the statements made at the beginning of Sect. 2: the so-called success of the end-point concentrated DAs is only fictitious; for finite values of $Q$ the HSP formula (1.4) does not represent a reasonable approximation to (1.2) for such DAs.

The numerical studies reveal that the modified hard scattering approach is self-consistent for $Q \geq 2$GeV in the sense that less than, say, 50% of the result is generated by soft gluon exchange ($\alpha_s > 0.7$). The exact value of $Q$ at which self-consistency sets in depends on the wave function. It is larger for the end-point concentrated wave functions than for the MAS DA. However, the perturbative contribution (2.1), although self-consistent, is presumably too small as compared with the (poor) data.

The Gaussian $k_\perp$ dependence of the wave function is a plausible but special case and one may suspect that more complicated functions lead to a better agreement with the data. Recently, in QCD sum rule analyses similar to those determining the moments of the DAs $\langle x^n \rangle$, the lowest $k_\perp$-moments, $\langle k^n_\perp \rangle$ have been estimated [14, 15]. The ratio $\langle k^4_\perp \rangle / \langle k^2_\perp \rangle^2$ is found to have a value in the range $5 - 8$. For comparison the Gaussian (3.3) provides a value of about 2. The large value of the ratio implies a $k_\perp$ distribution being not strongly concentrated around the point $k_\perp = 0$. Modelling a wave function which respects that new constraint, e. g. a Gaussian plus a $\delta$-function, does however not lead to a substantially larger perturbative contribution to the pion form factor.

4. The magnetic form factor of the nucleon

The nucleon’s magnetic form factor $G_M$ can be analysed within the modified HSP along the same lines as the pion’s form factor. The basic formula for the perturbative contribution is an expression similar to (2.1). Since the nucleon’s valence Fock state consists of three quarks, one has two independent $x$ (and $y$) variables and two independent transverse interquark separations. The wave function is parameterized analogously to (3.1) with the DA being a linear combination of Appell polynomials $\tilde{\Phi}^n$ which are the eigenfunctions of the evolution equation [1]

$$\Phi(x_1, x_2, x_3) = 120 x_1 x_2 x_3 \sum_n B_n \left( \frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)} \right)^{\gamma_n/\beta_0} \tilde{\Phi}^n(x_1, x_2, x_3).$$  (4.1)

The exponents $\gamma_n$, driving the evolution behaviour of the DA, are positive fractional numbers increasing with $n$. For the BS set of DAs [7] the sum in (4.1) is truncated at $n = 5$ (corresponding to the polynomial order 2) and the coefficients $B_n$ are determined in the context of QCD sum rules.

Now the Sudakov exponent $S$ consists of a sum of six Sudakov functions $s(\xi_l, \tilde{\beta}_l, Q)$ ($\xi_l = x_l$ or $y_l$, $l = 1, 2, 3$) and of the integral over the anomalous dimensions (see (2.3)) which depends on the mass scales associated with the hard gluons. The choice of the infrared cut-off parameters $\tilde{\beta}_l$ is not as simple as in the pion case. These parameters are naturally related to but not uniquely determined by the mutual separations of the three quarks. Li, in his pioneering analysis of the proton form factor within the modified HSP [12], chooses
\[ \tilde{b}_l = b_l \text{ where } b_1 (b_2) \text{ is the transverse separation of quark 1 (2) and 3 and } b_3 \text{ that of quark 1 and 2. However, Li’s analysis is seriously flawed: the cancellation of the } \alpha_s \text{ singularities for } \tilde{b}_l \Lambda_{QCD} \to 1 (x_l \text{ fixed}) \text{ is incomplete for his choice of the infrared cut-offs. The uncompensated singularities are of the form} \]

\[ \sim \ln^{-\kappa}(1/\tilde{b}_l \Lambda_{QCD}) \]

The maximum degree of divergence \( \kappa \) is 7/9 if evolution is ignored but larger than 123/81 if it is taken into account (note, the factorization scale \( \mu_F \) is min\{1/\( \tilde{b}_l \)\}). The Sudakov factor does not necessarily vanish fast enough to guarantee the cancellation of these singularities. Consider for example the string like configurations: \( x_1 \leq \sqrt{2}/(\tilde{b}_1 Q) \) and \( \tilde{b}_1 \Lambda_{QCD} \to 1 \), the region where \( s(x_1, \tilde{b}_1, Q) \) is set to zero, and \( \tilde{b}_2 \Lambda_{QCD} \simeq b_3 \Lambda_{QCD} \simeq 1/2 \). Then, none of the other two Sudakov functions, \( s(x_2, \tilde{b}_2, Q) \) and \( s(x_3, \tilde{b}_3, Q) \), tends to infinity and hence the Sudakov factor does not provide any suppression.

In a recent paper another choice for the infrared cut-offs has been suggested \[16\], namely a common cut-off \( \tilde{b} = \tilde{b}_1 = \tilde{b}_2 = \tilde{b}_3 = \max\{b_l\} \). Obviously, for this choice, termed the “MAX” prescription, at least one of the Sudakov functions tends to infinity if \( b_l \Lambda_{QCD} \to 1 \) (at least one of the \( x_l \) is larger than \( \sqrt{2}/bQ \)). The “MAX” prescription does not only lead to a regular integral but also to a non-singular integrand which is a prerequisite for the self-consistency of the modified HSP.

The numerical results for the proton’s magnetic form factor are compared to the data \[6\] in Fig. 4. The hatched band indicates the range of predictions derived for the BS set of DAs. Each DA is multiplied by a Gaussian of the type \( (3.3) \) and the full wave function is normalized to unity fixing the parameter \( \beta \). The values for the r. m. s. transverse momentum range from 270 to 320 MeV for the various wave functions. These values are to be considered as minimum values, they may be larger (providing stronger suppression of the perturbative contribution). The band of predictions does not overlap with the data, the predictions are too low by about a factor of 2 to 3. Even if the intrinsic transverse momentum is ignored there is no overlap. The perturbative contribution becomes self-consistent for \( Q \geq 3 \text{ GeV} \): 50% of the result is accumulated in regions where \( \alpha_s^2 \leq 0.5 \).

The onset of self-consistency mildly depends on the wave function used.

An analysis of the neutron’s magnetic form factor \[16\] yields results similar in trend but in apparently better agreement with the data. There is overlap between the band of predictions and the data at the largest measured momentum transfers where, incidentally, the theoretical calculation becomes self-consistent.

Finally, it should be noted that Hyer \[17\] has investigated the magnetic form factor of the proton in the time-like region within the modified HSP. Comparing with the data of the Fermilab E760 collaboration \[26\] (see Fig. 5), also Hyer’s result is too small.

5. Soft contributions to the form factors

As discussed in sections 3 and 4 the perturbative contributions to the pion’s and the nucleon’s form factor are too small. Hence other contributions must play an important role in the few GeV region. Obviously, for a perturbative calculation one may suspect higher
order contributions to be responsible for the discrepancy between theory and experiment. In analogy to the Drell-Yan process such contributions may be condensed in a K-factor multiplying the lowest order result for the form factor

\[ K = 1 + \frac{\alpha_s(\mu)}{\pi} B(Q, \mu) + \mathcal{O}(\alpha_s^2). \]  

Calculateds of the one-loop corrections \[18, 19\] to the pion form factor reveal that the magnitude of the K-factor strongly depends on the renormalization scale. It is in general large except the renormalization scale is chosen like \( \mu = \sqrt{x_1 y_1 Q} \) (see Sect. 1). For this choice and the use of the asymptotic DA, K is about 1.3 in the few GeV region. For DAs broader than the asymptotic one, i.e. for such with a stronger weight of the end-point regions, B seems to be negative. Note that at least part of the K-factor is included in the Sudakov factor. With regard to the new developments discussed above it is perhaps advisable to reanalyse the one-loop corrections.

Disregarded soft contributions offer another explanation of the eventual discrepancy between theory and experiment. As the \( k_\perp \)-effects discussed above such contributions are of higher twist type and do not respect the quark counting rules. Dominance of such contributions in the case of the pion’s form factor and perhaps in other exclusive quantities would leave unexplained the apparent success of the counting rules.

There are several possible sources for such soft contributions:

i) Genuine soft contributions like VMD contributions or contributions from the overlap of the soft parts of the hadronic wave functions. The overlap contribution can be estimated with the aid of the famous Drell-Yan formula \[20\] (note that the HSP represents the contribution from the overlap of the perturbative tails of the hadronic wave functions). In the pion case it reads

\[ F_{\pi \text{soft}}(Q^2) = \int \frac{dx_1 d^2k}{16\pi^3} \Psi^*(x_1, \vec{k}_\perp + x_2 \vec{q}) \Psi(x_1, \vec{k}_\perp) (5.2) \]

\((Q^2 = \vec{q}^2)\). The integral is dominated by the region near \( x_1 = 1 \), other regions are strongly damped by the wave function. Hence \( F_{\pi \text{soft}} \) sensitively reacts to the behaviour of the wave function. Hence \( F_{\pi \text{soft}} \) is found to hold even up to 6 GeV \[14\].

Soft contributions of the type \((5.2)\) are also discussed in \[9, 21\]. Strong soft contributions to form factors are also obtained from QCD sum rules \[5, 22\].

ii) There may be orbital angular momentum components in the hadronic wave function other than zero. New phenomenological functions appear in general which is certainly a disadvantage but may lead to a better quantitative description of the form factor data. \( L \neq 0 \) components have the appealing consequence of violating the helicity sum rule for finite values of \( Q \) \[23\]. This may offer a possibility to calculate the Pauli form factor of the proton.
iii) Contributions from higher Fock states are another source of higher twist contributions. Also in this case new phenomenological functions have to be introduced.

iv) For baryons one may also think of quark-quark correlations in the wave functions which constitute higher twist effects. In a series of papers (see [24, 25] and references therein) the idea has been put forward that such correlations can effectively be described by quasi-elementary diquarks. A systematic study of all exclusive photon-proton reactions has been carried out in the diquark model, which is a variant of the unmodified HSP: form factors in the space-like and in the time-like regions, virtual and real Compton scattering, two-photon-annihilations into proton-antiproton as well as photoproduction of mesons. A fair description of all the data has been achieved utilizing in all cases the same proton DA (as well as the same values for the parameters specifying the diquarks). The diquark model allows to calculate helicity flip amplitudes and consequently to predict for instance the Pauli form factor of the proton. The results for it, shown in Fig. 1, are in agreement with the data. Results for the magnetic form factor in both the time-like and the space-like regions are shown in Fig. 5.

6. Summary

The modified HSP which includes both the Sudakov corrections and the intrinsic $k_{\perp}$-dependence of the hadronic wave function constitutes an enormous progress in our understanding of exclusive reactions although there are still some theoretical problems left. It provides an explicit scheme for the infrared protection of the “bare” coupling constant. The modified HSP allows to calculate the perturbative contribution to form factors in a theoretically self-consistent way for momentum transfers as low as few GeV (about 2(3) GeV in the pion (nucleon) case). This is, however, achieved at the expense of strong suppressions of the perturbative contributions as compared to those obtained with the original unmodified HSP. Now the perturbative contributions are too small as compared to data. It thus seems that other contributions (higher order corrections to the hard scattering amplitudes and/or higher twists) also play an important role in the few GeV region as already indicated by the recently measured Pauli form factor of the proton. An interesting task for the future is to find out the size of such higher twist contributions and to elucidate their physical nature. Finally, I would like to emphasize that the approximate validity of the quark counting rules, for which the HSP offers an explanation, would remain a mystery if all large momentum transfer data for exclusive processes are dominated by soft contributions.

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Figure 1: The Pauli form factor of the proton scaled by $Q^6$. Data are taken from [8]. The solid line represents the results obtained from the diquark model [25].

Figure 2: The exponential of the Sudakov function $s(\xi_l, b, Q)$ vs. $\xi_l$ and $\tilde{b} \Lambda_{QCD}$ for $Q = 30 \Lambda_{QCD}$. In the pion case $\tilde{b}$ equals $b$. In the hatched area the Sudakov function is set equal to zero.

Figure 3: (Left) The pion's form factor as a function of $Q^2$ evaluated with the CZ wave function and $\Lambda_{QCD} = 200$ MeV. The dash-dotted line is obtained from (1.4) with $\alpha_s$ frozen at 0.5 and the dashed line from (2.1) ignoring the intrinsic $k_\perp$-dependence [11]. The solid line represents the complete result obtained from (2.1) ($\langle k_\perp^2 \rangle^{1/2} = 350$ MeV). Data are taken from [3] (◦ 1976, ♦ 1978). (Right) As left figure but using the MAS wave function. Note the modified scale of the abscissa.

Figure 4: The proton’s magnetic form factor vs. $Q^2$. Data are taken from [6] (filled (open) circles $G_M (F_1)$). Theoretical results are obtained with the “MAX” prescription using the DAs given in [7] ($\Lambda_{QCD} = 180$ MeV). The wave functions are normalized to unity.

Figure 5: The magnetic form factor of the proton in the time-like and space-like (at $Q^2 = -s$) regions. The time-like data are taken from [26], the space-like data from [4]. The solid lines represent the predictions of the diquark model [24], the dashed line is Hyer’s prediction [17].
Fig. 1
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exp[-s(\xi, \bar{b}, Q)]
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Fig. 3a
Fig. 3b
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