Study on Model Based Combustion Control of Diesel Engine with Multi Fuel Injection

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Abstract. A controller for model-based control of diesel engine with triple injection were developed with a combustion model. In the combustion model, an engine cycle is discretized into several representative points in order to improve calculation speed, while physical equations are employed to expand the versatility. The combustion model can predict in-cylinder pressure and temperature in these discrete points. Prediction accuracy of the combustion model was evaluated by comparison with experimental result. A controller was designed with the combustion model in order to calculate optimal fuel injection pattern for controlling in-cylinder pressure peak timing. The controller’s performance was evaluated through simulation in which the combustion model was used as a plant model.

1. Introduction

Emission control imposed on diesel car is getting stricter and stricter. At the moment, kinds of exhaust gas after treatment devices are employed in order to meet the control. However, it is required to improve the combustion in engine cylinders itself and reduce the emission of toxic substances like nitrogen oxide (NO\(_x\)) in the future.

The present car engines are controlled by lookup tables for optimal operation of each actuator, called control map. However, it requires huge amount of experimental results to construct control maps, and burden on engineers developing control systems is too severe.

In order to tackle with such problems around development of diesel engine, there are many challenges to introduce model-based control method to diesel engine in this paper. In model-based control method for car engine, a mathematical model of engine dynamics is developed and on-board installed. Then, the optimal operation of each actuator to meet the given performance objectives like thermal efficiency and NO\(_x\) emission is calculated by referring to the model’s predictive result. The method’s advantage is its high versatility when compared to the present method using control map. One control system can be installed to many kinds of engines through minimum effort of parameter tuning by employing physical equations which can be applied to general engines in developing the model.

There are many works in which model-based control method is introduced to car engine. Ravi \(^1\) developed the combustion model with light calculative load aimed at controlling homogeneous-charge compression ignition engine. In the model, a cycle was discretized into some points which provides useful information in order to reduce calculation load. Yasuda \(^2\) developed a discretized combustion model for diesel engine with single fuel injection based on Ravi’s model. Shimizu \(^3\) expanded Yasuda’s model in order to predict the combustion made by double fuel injections. However, more than three times of fuel injections are conducted in...
common diesel engine, so it is necessary to develop a combustion model of diesel engine with more than three
times fuel injections for model-based control of present diesel engines.

A combustion model of diesel engine with triple fuel injections was explained and the model’s prediction accuracy was evaluated through experiments[^4]. Next, as a main outcome of this paper, feed forward controller of diesel engine was developed and the model was embedded into the controller in order to make use of its prediction result. The controller’s performance was evaluated through simulation in which the developed combustion model was used as a plant model.

2. Developing the combustion model for a diesel engine

2.1 Engine specification

The specification of the engine with which the combustion model was developed as a target is shown in Table 1. The engine has four cylinders and its displacement is about 3 L. The diagram of the engine system is shown in Fig. 1. The engine has common-rail fuel injection system, variable geometry turbo charger, and EGR system.

2.2 Discretization of a cycle

In this model, a cycle is discretized into some points which can provide useful information in order to make calculation load so light that the model can be embedded into controller and the controller can be on-board installed. The diagram of discretization and the specification of the discrete points are shown in Fig. 2 and Table. 2. In the model, in-cylinder pressure and temperature at these discrete points is predicted. The model’s prediction target is the situation in which two pilot injection and single main injection of fuel are conducted. In order to predict the combustion in the engine with triple fuel injection, the discrete points for two pilot injection and ignition of pilot combustion were set.

| Engine type              | 4 Cylinder DI Diesel |
|--------------------------|----------------------|
| Bore/Stroke [mm]         | 96 / 103             |
| Displacement [cm³]       | 2982                 |
| Swirl ratio              | 2.3                  |
| Injection system         | Common rail          |
| Intake valve opening / closing [deg BTDC] | 3 / 152 |
| Exhaust valve opening / closing [deg ATDC] | 132 / 0 |

Fig. 1 Engine system
Table 2 Discrete points

|      | Description                  |
|------|------------------------------|
| EVC  | Exhaust valve closing        |
| IVO  | Intake valve opening         |
| P1 INJ | Pilot1 injection            |
| P2 INJ | Pilot2 injection            |
| P IGN | Pilot ignition               |
| M INJ | Main injection              |
| M IGN | Main ignition                |
| PEAK | In-cylinder pressure peak    |
| EVO  | Exhaust valve opening        |
| IVO  | Intake valve opening         |

2.3 Modeling of intake process

In the model, calculation for one cycle start at intake valve opening (IVO), and newly inhaled gas and residual gas at the previous cycle’s exhaust valve closing (EVC) are mixed during the period between IVO and intake valve closing (IVC). It is necessary to know the temperature and composition of the residual gas in order to calculate mixing with the intake gas. Therefore, the information about the residual gas is passed from the result of calculation for the previous cycle. In this way, it is possible to predict the combustion of each cycle continuously.

The residual gas (RG), newly inhaled air (Air), and exhaust gas recirculation (EGR) are supposed to be mixed adiabatically and in cylinder temperature at IVO is derived from equation (1), which means energy conservation between IVO and IVC.

$$C_v \left\{ n_{RG} (T_{RG} - T_{ref}) + \frac{P_{Boost}}{RT_{IVC}} \left( V_{IVC} - V_{EVC} \frac{T_{IVC}}{T_{RG}} \right) (T_{Inmani} - T_{ref}) \right\} = C_v \left\{ n_{RG} + \frac{P_{Boost}}{RT_{IVC}} \left( V_{IVC} - V_{EVC} \frac{T_{IVC}}{T_{RG}} \right) \right\} (T_{IVC} - T_{ref})$$

(1)

$T_{Inmani}$ is used as the temperature of the newly inhaled gas, which is mixture of the air and EGR. In-cylinder pressure between IVO and IVC is supposed to be equal to $P_{Boost}$.

The amount of the mixed gas, newly inhaled air and EGR in cylinder at IVC are derived from equation (2) to (5), using the result of equation (1).

$$n_{Gas,IVC} = \frac{P_{IVC} V_{IVC}}{RT_{IVC}}$$

(2)
\[ n_{Air} = (n_{Gas,IVC} - n_{RG}) \frac{T_{EGR}(1 - x_{EGR})}{T_{EGR}(1 - x_{EGR}) + T_{Air}x_{EGR}} \]  (3)

\[ n_{EGR} = n_{Gas,IVC} - n_{Air} - n_{RG} \]  (4)

\[ n_{O_2,IVC} = n_{O_2,RG} + 0.79n_{Air} + n_{EGR} \frac{n_{O_2,EGR}}{n_{RG}} \]  (5)

2.4 **Modeling of compression process**

In-cylinder gas is compressed by the piston between IVC and pilot 1 injection (P1 INJ), and there are several kinds of heat loss including cooling loss. In order to consider the effect of heat loss, in-cylinder gas’s status is supposed to follow polytropic change, and in-cylinder pressure and temperature at P1 INJ are derived from equation (6) and (7).

\[ P_{P1_INJ} = P_{IVC} \left( \frac{V_{IVC}}{V_{P1_INJ}} \right)^{Y_{Comp}} \]  (6)

\[ T_{P1_INJ} = T_{IVC} \left( \frac{V_{IVC}}{V_{P1_INJ}} \right)^{Y_{Comp}^{-1}} \]  (7)

\( Y_{Comp} \) means the polytropic index, and it is known that the polytropic index has strong correlation with in-cylinder temperature. In order to predict the polytropic index, statistic equation (8) was derived through multiple regression analysis of experimental result.

\[ Y_{Comp} = 1.357 - 2.272 \times 10^{-5} \times T_{IVC} \]  (8)

2.5 **Modeling of pilot combustion process**

2.5.1 **Interference among pilot fuel sprays**

In pilot combustion period, pilot 1 and 2 injection are conducted, and pilot ignition occurs, and pilot combustion continue until main ignition. In this model, pilot ignition timing and reaction rate of in-cylinder gas premixed with fuel are prediction target as pilot combustion characteristics. In order to predict these characteristics, it is necessary to know fuel concentration in cylinder and substitute it into the equations described later. However, it is known that injected fuel is distributed heterogeneously in a cylinder, and the distribution is strongly affected by shape of fuel sprays and interference among fuel sprays. Therefore, the fuel spray’s flow caused by swirl and their overlap is considered in order to take the interference into account.

Firstly, a fuel spray’s shape is supposed to be conic growing from a injector hole, and the spray’s length and vertical angle are predicted by the experimental equations (9) and (10) found by Reitz [5]. The spray’s diagram is shown in Fig. 3.

\[ L_{Spray,PK} = 2.95 \left( \frac{\Delta P}{\rho_{Fuel}} \right)^{0.25} \sqrt{d_{Hole}t} \]  (9)

\[ \tan(\psi_{Spray,PK}) = \left\{ 3.0 + 0.28 \left( \frac{L_{Nozzle}}{d_{Hole}} \right) \right\}^{-1} \frac{4\pi \sqrt{\rho_{Gas} \sqrt{3}}}{\rho_{Fuel} \sqrt{6}} \]  (10)

The spray’s expansion is supposed to stop when its front reaches the combustion chamber wall, and the distance between the injector hole and the combustion chamber wall is supposed to be equal to the combustion chamber radius.
Secondly, flow and overlap of the fuel sprays injected at P1 INJ and P2 INJ (P1 spray and P2 spray) is considered in the model. The diagram of their flow and overlap is shown in Fig. 4. There are three zones called zone 1, 2, and 3 in the sprays shown in Fig. 4. Zone 1 is included in P1 spray, but not included in P2 spray, and zone 2’s definition is the reverse. On the other hand, zone 3 is shared by P1 and P2 spray. Zone 1, 2, and 3 are defined in order to calculate fuel concentration in cylinders as quotient between the amount of fuel injected and the volume of zone 1, 2, and 3. In this way, influence of flow and overlap of the sprays are considered in the prediction of pilot combustion characteristics.

Fig. 3 Fuel spray

Fuel sprays are supposed to turn with their conic shape kept after their front reach the combustion chamber wall. Their turning is caused by the swirl, and the angular velocity of the swirl is supposed to be directly proportional to that of the engine shaft. The angular velocity of the swirl is derived from the equation (11). \( \omega_{\text{Swirl}} \) is a parameter which needs configuration.

\[
\omega_{\text{Swirl}} = c_{\text{Swirl}} \omega_{\text{Engine}} \tag{11}
\]

### 2.5.2 Pilot ignition timing

Pilot ignition timing is predicted by using the fuel concentration derived from the methods previously noted. Firstly, the in-cylinder gas’s state is supposed to follow polytropic change between P1 INJ, P2 INJ, and pilot ignition timing (P IGN), and in-cylinder pressure and temperature at each discrete points are derived from the equations from (12) to (15).

\[
P_{P2 \text{INJ}} = P_{P1 \text{INJ}} \left( \frac{V_{P1 \text{INJ}}}{V_{P2 \text{INJ}}} \right)^{\gamma} \tag{12}
\]

\[
T_{P2 \text{INJ}} = T_{P1 \text{INJ}} \left( \frac{V_{P1 \text{INJ}}}{V_{P2 \text{INJ}}} \right)^{\gamma^{-1}} \tag{13}
\]

\[
P_{\text{P IGN}} = P_{P2 \text{INJ}} \left( \frac{V_{P2 \text{INJ}}}{V_{\text{P IGN}}} \right)^{\gamma_{\text{Comp}}} \tag{14}
\]

\[
T_{\text{P IGN}} = T_{P2 \text{INJ}} \left( \frac{V_{P2 \text{INJ}}}{V_{\text{P IGN}}} \right)^{\gamma_{\text{Comp}}^{-1}} \tag{15}
\]

Pilot ignition timing is predicted by Livengood-Wu integration\(^6\), which is experimental equation about ignition of air-fuel premixed gas. Livengood-Wu integration is described like the equation (16) and (17), and the fuel-air premixed gas ignites when the time integration reaches the threshold \( K \).

\[
\Delta t = c_{\text{Swirl}} \omega_{\text{Engine}} \tag{16}
\]

\[
\Delta t = c_{\text{Swirl}} \omega_{\text{Engine}} - K \tag{17}
\]
\[
\frac{1}{\tau} = A[Fuel]^B [O_2]^C \exp\left(-\frac{E}{RT}\right)
\]  

(16)

\[
K = \int_{INJ}^{IGN} \frac{1}{\tau} \, dt
\]

(17)

A, B, C, E in the equation are parameters which need configuration. However, it is impossible to calculate the time integration in the model because a cycle is discretized into the discrete points. Therefore, the equation (16)’s value is presented by that at P2 INJ and the time integration is simply calculated. Pilot ignition timing is derived from the equation (18).

\[
\theta_{P\,IGN} = \theta_{P2\,INJ} + \omega K \frac{\omega K}{A} \left( \frac{n_{p1} + n_{p2}}{V_{zone1} + V_{zone2} + V_{zone3}} \right)^B \left( \frac{n_{O2,IVC}}{V_{P\,IGN}} \right)^C \exp\left(-\frac{E}{RT_{P\,IGN}}\right)
\]

(18)

The fuel concentration is calculated as the quotient between the amount of injected fuel and the volume of the volume of zone 1, 2 and 3, and O\(_2\) is supposed to be distributed homogeneously in the cylinder.

### 2.5.3 The reaction rate in pilot combustion

The reaction rate of the combustion is predicted by Arrhenius’s equation described like the equation (19).

\[
-\frac{d}{dt}[Fuel] = \alpha[Fuel]^B [O_2]^Y \exp\left(-\frac{\varepsilon}{RT}\right)
\]

(19)

\(\alpha, \beta, \gamma, \varepsilon\) are parameters which need configurations. However, it is impossible to calculate the equation (19) continuously, because a cycle is discretized into the discrete points. Therefore, the equation (19)’s value is represented by the value of that at P IGN and predict the amount of fuel which burns between P IGN and M IGN. The rate of reaction of fuel-air premixed gas between P IGN and M IGN is the equation (20).

\[
-\frac{d}{dt}[Fuel] = \alpha \left( \frac{n_{p1} + n_{p2}}{V_{zone1} + V_{zone2} + V_{zone3}} \right)^B \left( \frac{n_{O2,IVC}}{V_{P\,IGN}} \right)^C \exp\left(-\frac{\varepsilon}{RT_{P\,IGN}}\right)
\]

(20)

In-cylinder pressure and temperature at P IGN, M INJ, and M IGN are predicted by considering energy conservation.

### 2.6 Modeling of main combustion process

#### 2.6.1 The ignition timing and reaction rate of main combustion

Main ignition timing and the reaction rate of fuel-air premixed gas during main combustion are predicted by using Livengood-wu integration and Arrhenius equation like pilot combustion. In order to predict these characteristics, it is necessary to calculate the concentration of fuel in cylinder during main combustion. It is supposed that there exist fuel injected at M INJ and unburned fuel injected at P INJ, and they form premixed gas with air and burn in the main spray with conic shape. Unburned fuel injected at P INJ is supposed to form premixed gas homogeneously in a cylinder. Fuel spray injected at M INJ is supposed to be conic and grow from injector hole. Some amount of premixed gas of unburned fuel injected at P INJ is taken into main spray, and the concentration of fuel in cylinder is calculated as quotient of the amount of fuel exist in main spray zone and the volume of main spray.

Main ignition timing is predicted by the equation (21) which is derived from Livengood-Wu integration.

\[
\theta_{M\,IGN} = \theta_{M\,INJ} + \frac{\omega K'}{A'} \exp\left(\frac{E'}{RT_{M\,INJ}}\right)
\]

\[
A'\left\{ \frac{n_{M}}{V_{Spray,M}} + \frac{1 - x_P (n_{p1} + n_{p2})}{V_M\,INJ} \right\}^{B'} \left[ \frac{n_{O2,IVC} - 18.5x_P(n_{p1} + n_{p2})}{V_M\,INJ} \right]^{C'}
\]

(21)
However, the value of the equation (16) is represented by that at M INJ in order to calculate Livengood-Wu integration for prediction of main ignition timing.

The rate of reaction of fuel in main spray zone is predicted by the equation (22) which is derived from Arrhenius equation.

\[ \frac{d}{dt} [Fuel] = \alpha' \left[ \frac{n_M}{V_{Spray,M}} \right] + \left( 1 - x_P \right) \frac{n_{D,IVC}}{V_{MIGN}} \frac{n_{P1} + n_{P2}}{V_{MIGN}} \exp \left( \frac{-e'}{RT_{MIGN}} \right) \]  

(22)

However, the value of the equation (19) is representative at M IGN.

### 2.6.2 In-cylinder pressure peak

In order to predict in-cylinder pressure peak (PEAK) timing, it is necessary to predict the amount of fuel burning from M IGN to PEAK. There are two types of combustion in a diesel engine. One is premixed combustion, in which the premixed gas of fuel self-ignite and the rate of reaction is mainly determined by chemical reaction. The other is diffusive combustion, in which the drop of fuel evaporate, get mixed with hot gas, and ignite. Diffusive combustion’s rate is mainly determined by the rate of evaporation and diffusion of fuel, so it is not suitable to apply Arrhenius equation to diffusive combustion. As for main combustion in diesel engine with multi fuel injection, premixed combustion is dominant just after main ignition, and diffusive combustion gets dominant just before in-cylinder pressure peak timing.

In order to predict the amount of fuel burning from M IGN to PEAK, it is supposed that in-cylinder pressure gets its peak when premixed combustion is finished, and the reaction rate of premixed gas of fuel from M IGN to PEAK is predicted by using Arrhenius equation. The ratio of the fuel which is consumed by premixed combustion \( x_{premixed} \) is predicted by the equation (23), which is derived through multiple regression analysis of experimental result.

\[ x_{premixed} = 0.8036 - 401.1 \frac{\theta_{MIGN} - \theta_{MINJ}}{\omega_{Engine}} + 1.32 \times 10^{-9} P_{Commonrail} \]  

(23)

### 2.7 Modeling of expansion process

In-cylinder gas expands from PEAK to exhaust valve opening (EVO), and the state of in-cylinder gas is supposed to follow polytropic change. In-cylinder pressure and temperature at EVO are derived from the equation (24) and (25).

\[ P_{EVO} = P_{PEAK} \left( \frac{V_{PEAK}}{V_{EVO}} \right)^{Y_{Exp}} \]  

(24)

\[ T_{EVO} = T_{PEAK} \left( \frac{V_{PEAK}}{V_{EVO}} \right)^{Y_{Exp} - 1} \]  

(25)

The polytropic index \( Y_{Exp} \) is derived from the equation (26).

\[ Y_{Exp} = 1.357 - 2.272 \times 10^{-5} \times T_{PEAK} \]  

(26)

### 2.8 Modeling of exhaust process

In-cylinder gas expands through exhaust valve and reaches variable geometry turbo and EGR cooler from EVO to exhaust valve closing (EVC). In-cylinder gas’s state is supposed to follow polytropic change, and in-cylinder pressure from EVO to EVC is supposed to be equal to boost pressure. In-cylinder temperature at EVC are derived from the equation (27).
\[ T_{EVC} = T_{EVO} \left( \frac{P_{EVC}}{P_{EVO}} \right)^{\frac{\gamma_{ex}}{\gamma_{ex} - 1}} \]  

In-cylinder gas at EVC is the residual gas succeeded to the next cycle, and the model’s calculation for a cycle is finished when temperature and composition of in-cylinder gas at EVC is handed to the calculation for the next cycle as the characteristics of the residual gas.

2.9 Evaluation of the prediction accuracy and the calculation speed

The model’s performance is evaluated from the viewpoints of the prediction accuracy and the calculation speed. The model can predict in-cylinder pressure and temperature at the discrete points, and the timing of pilot ignition, main ignition, and in-cylinder pressure peak. The prediction accuracy of the model evaluated through the comparison between experimental and predictive result about these prediction target.

The experimental condition is shown in Table 3. In experiment, each parameter was changed in the range shown in Table 3. Firstly, experimental result of in-cylinder pressure and predictive result of the discretized points’s in-cylinder pressure when pilot 1 injection quantity is changed are shown in Fig. 5. In Fig 5, the discrete points lie upon the lines, and it is shown that the model has good prediction accuracy of in-cylinder pressure. In other conditions, the model had good prediction accuracy.

Next, the model’s calculation speed is evaluated. It is assumed that the model is on-board installed through being embedded into the controller, and its prediction result is used in order to calculate optimal input to the actuators every cycle. Therefore, it is necessary that the time required for the model to finish calculation for a cycle is much shorter than the time required for ECU to get the information from sensors and conduct Pilot 1 injection. The power of commercially available ECU is lower than that of personal computer, but it is assumed to improve greatly. Therefore, the model’s calculation speed is evaluated through comparison between the time required for the model’s calculation for a cycle on PC and the time allowed for calculation on ECU.

The time required for the model’s calculation for a cycle on PC was about 0.0001s. On the other hand, the time required for ECU to get the information from sensors and conduct pilot 1 injection is about 0.015s when engine speed is 1,500 rpm. Therefore, the time required for the model’s calculation for a cycle is much shorter than the time allowed, and it can be said that the power of the model is enough.

| Table 3 Experimental condition |
|--------------------------------|
| Pilot 1 injection timing [deg ATDC] | -29~21 |
| Pilot 1 injection quantity [mm³] | 1~5 |
| Pilot 2 injection timing [deg ATDC] | -19~11 |
| Pilot 2 injection quantity [mm³] | 1~5 |
| Main injection timing [deg ATDC] | -6~2 |
| Main injection quantity [mm³] | 16~24 |
| Boost pressure [kPa] | 100~120 |
| External EGR ratio [%] | 0.2~0.4 |
| Injection pressure [MPa] | 60~100 |
| Engine speed [rpm] | 1500 |
3. Model-based control

3.1 Development of controller

A feed-forward and feed-back controller were developed with the combustion model. The controller’s inputs and outputs are shown in Table 4. The controller is intended to control in-cylinder pressure peak timing by changing main injection timing optimally as a first step to evaluate availability of the model. The diagram of the calculation in the controller is shown in Fig. 6. The controller is composed of a feed-forward controller and a feed-back controller, and the combustion model is embedded in the feed-forward controller and can predict in-cylinder pressure peak timing when given a series of condition.

The process of calculating optimal main injection timing is as follows. Firstly, the combustion model in the feed-forward controller is given the conditions obtained from ECU, main injection timing default, and predicts in-cylinder pressure peak timing. Secondly, the difference between the prediction result and the target of in-cylinder pressure timing is calculated and integrated. The integration is multiplied with the constant and the difference between main injection timing default and the integration is calculated. Finally, the difference is passed to the combustion model as renewed main injection timing, and the model predicts in-cylinder pressure peak timing again. These process is repeated several times and the error between the prediction result and the target of in-cylinder pressure peak timing is decreased. After these process, the controller outputs main injection timing to ECU as control input, and in-cylinder pressure peak timing is measured.

Secondly, main injection timing is corrected according to the error between the target and the measured result of in-cylinder pressure peak timing. Difference between the target and measured result of in-cylinder pressure peak timing is calculated and integrated, and difference between present main-injection timing and the integration multiplied with the constant is passed to ECU as the corrected main-injection timing input.

In this way, the controller calculates the optimal main injection timing every cycle, and it is necessary that the time required by the calculation is much shorter than the time of a cycle. Therefore, the number of the repeated calculation around the combustion model is required to meet the regulation.
Table 4 Controller’s inputs and outputs

| Target input                        | Description                                      |
|-------------------------------------|--------------------------------------------------|
| $\theta_{\text{PEAK}}$              | Peak pressure timing [deg ATDC]                  |
| Input from ECU                      |                                                  |
| $Q_{P1}$                            | Pilot1 injection quantity [mm$^3$/st]            |
| $\theta_{P1,\text{INJ}}$            | Pilot1 injection timing [deg ATDC]               |
| $Q_{P2}$                            | Pilot2 injection quantity [mm$^3$/st]            |
| $\theta_{P2,\text{INJ}}$            | Pilot2 injection timing [deg ATDC]               |
| $Q_M$                               | Main injection quantity [mm$^3$/st]              |
| $P_{\text{Boost}}$                  | Boost pressure [kPa]                             |
| $X_{\text{EGR}}$                    | External EGR ratio [-]                           |
| $P_{\text{Commonrail}}$             | Fuel injection pressure [MPa]                    |
| $N_e$                               | Engine speed [rpm]                               |
| $T_{\text{Inman}}$                  | Intake manifold temperature[K]                   |
| Input to ECU                        |                                                  |
| $\theta_{M,\text{INJ}}$             | Main injection timing [deg ATDC]                 |

3.2 Simulation of the controller

Performance of the controller was evaluated through the simulation in which the developed combustion model was used as a plant model. The condition of the simulation is shown in Table 5.

The simulation was conducted on the conditions in which in-cylinder pressure peak timing target was changed like a step function in the range shown in Table 5. The controller calculated the optimal main injection timing in order to keep in-cylinder pressure peak target and passed it to the plant model as a control input. However, difference about external EGR ratio between the value passed to the controller and that passed to the plant model as a disturbance. The plant model was passed external EGR ratio with the error like sin wave, whose range is 0.2. In this way, both the feed-forward and the feed-back controller’s performance was evaluated.
Table 5: Simulation condition

| Parameter                          | Value                  |
|-----------------------------------|------------------------|
| In-cylinder pressure peak timing target [deg ATDC] | 10, 11, 12, 13, 14  |
| Pilot1 injection timing [deg ATDC]      | -25                    |
| Pilot1 injection quantity [mm$^3$]    | 3                      |
| Pilot2 injection timing [deg ATDC]      | -15                    |
| Pilot 2 injection quantity [mm$^3$]    | 3                      |
| Main injection timing [deg ATDC]       | Control input          |
| Main injection quantity [mm$^3$]       | 20                     |
| Boost pressure [kPa]                 | 110                    |
| External EGR ratio [-]               | 0.3 ± 0.2              |
| Injection pressure [MPa]             | 80                     |
| Engine speed [rpm]                  | 1500                   |

The result of simulations are shown in Fig. 7. Fig. 7 show the target and the prediction result of in-cylinder pressure peak timing. In Fig. 7, red plots express in-cylinder pressure peak timing of every cycle predicted by the plant model, and blue line shows transition of given in-cylinder pressure peak timing target. It is shown that the error between the target and the prediction result of in-cylinder pressure peak timing is less than 1 deg. Therefore, it can be said that the controller can control in-cylinder pressure peak timing according to the transition of the target, and the disturbance of external EGR ratio.

![Fig. 7 In-cylinder pressure peak timing](image)

**4. Conclusion**

The combustion model and the controller for model-based control of diesel engine with triple injection were developed and their performance was evaluated in this work.

In the model, a cycle was discretized into some representative points including pilot ignition, main ignition, in-cylinder pressure peak. The model aims to predict in-cylinder pressure peak timing, and in-cylinder pressure and temperature at the discretized points. The model employed physical equation with less parameters which need configuration in order to improve the versatility. Finally, the accuracy of the prediction model on in-cylinder pressure and temperature of the discretized points was evaluated by comparing with the experimental result, and it was shown that the model can predict the combustion with good accuracy. Moreover, the time required for the prediction calculation was enough short to be on-board installed and used for calculating the optimal actuation in every cycle.

The feed-forward and feed-back controller was also developed with the combustion model. The controller aims to control in-cylinder pressure peak timing by calculating the optimal main injection timing. Performance of the controller was evaluated through the simulation, in which in-cylinder pressure peak timing target were changed...
with a disturbance of external EGR ratio, and it was shown that the controller can control in-cylinder pressure peak timing in good accuracy.

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