Consistent relativistic mean-field models: critical parameter values

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(Dated: May 25, 2018)

We revisit the study published in [1], related to the behavior of 34 relativistic mean-field models, previously selected because they satisfy bulk nuclear matter properties, here used to compute the critical parameters of the symmetric nuclear matter. We evaluate their critical temperature, pressure, and density and compare with some values encountered in the literature. We also show that these parameters are correlated with the incompressibility calculated at the zero temperature regime.

I. INTRODUCTION

Nuclear matter is an idealized medium, and all its properties, are derived from experiments indirectly in a model-dependent way. However, the understanding of its properties is of fundamental importance as a guide towards more specific subjects, such as nuclear and hadron spectroscopy, heavy-ion collisions, nuclear multifragmentation, caloric curves, and others. It is well known that theoretical hadronic models predict phase transitions at finite temperature and density. Qualitatively, the isotherms of these hadronic mean-field models typically show a van der Waals-like behavior, where liquid and gaseous phases can coexist [2].

It is important to note that the critical temperature is defined in such a way that it always takes place in symmetric matter. In Ref. [3] the authors have shown that the instability region decreases with the increase of the temperature up to a certain value, which is related to a critical pressure and critical density. The values of these critical parameters are model dependent and there are many nonrelativistic and relativistic models in the literature, which can be used to calculate them. In this work we use the Relativistic Mean-Field (RMF) Approximation.

II. CHOICE OF MODELS

Our study is based on the study presented in Ref. [4] where 263 RMF models were analyzed. These parametrizations had their volumetric and thermodynamical quantities compared with theoretical and experimental data available in the literature. These data were divided into three groups: symmetric nuclear matter (SNM), pure neutron matter (PNM) and a third group named MIX (the mixture of PNM + SNM). This last one encompasses the symmetry energy and its slope at the saturation density as well as reduction of the symmetry energy at half of the saturation density. In Table I we present a summary of these constraints. For more details see Ref. [4].

The analysis has shown that only 35 parametrizations were approved. They are named consistent relativistic mean field (CRMF) parametrizations. We consider in the present study only 34 of them because the point-coupling parametrization does not generate a mass-radius curve, according to Ref. [5], so, it was excluded. The remaining of them are part of two groups out of the seven presented in Ref. [4]. We have shown them in Subsection II A and II B of that reference”.

A. Nonlinear RMF models

The group of the nonlinear RMF parametrizations with $\sigma$ and $\omega$ terms and cross terms involving these fields encompasses thirty parametrizations. The Lagrangian density that describes this model is:

$$\mathcal{L}_{NL} = \overline{\psi} (i \gamma^{\mu} \partial_{\mu} - M) \psi + g_{\sigma} \sigma \overline{\psi} \psi - g_{\omega} \psi \gamma^{\mu} \omega_{\mu} \psi + \frac{g_{\rho}}{2} \psi \gamma^{\mu} \rho_{\mu} \overline{\psi} \psi - \frac{g_{\omega}}{2} \psi \gamma^{\mu} \omega_{\mu} \psi + \frac{B}{4} \sigma^{4} - \frac{1}{4} F_{\mu \nu}^{\mu} F_{\rho \sigma}^{\rho \sigma} + \frac{1}{2} m_{\omega}^{2} \omega_{\mu} \omega_{\mu} + \frac{C}{4} (g_{\omega}^{2} \omega_{\mu} \omega_{\mu})^{2}$$

$$- \frac{1}{4} B_{\mu \nu} B_{\mu \nu} + \frac{1}{2} m_{\rho}^{2} \rho_{\mu} \rho_{\mu} + \frac{1}{2} \alpha' g_{\rho}^{2} g_{\sigma}^{2} \omega_{\mu} \omega_{\mu} \rho_{\mu} \rho_{\mu} + g_{\sigma} g_{\omega}^{2} \sigma \omega_{\mu} \omega_{\mu} \rho_{\mu} \rho_{\mu} + g_{\sigma} g_{\omega}^{2} \rho_{\mu} \rho_{\mu}$$

$$+ \frac{1}{2} \alpha' g_{\rho}^{2} g_{\sigma}^{2} \rho_{\mu} \rho_{\mu} + \frac{1}{2} \alpha' g_{\rho}^{2} g_{\sigma}^{2} \rho_{\mu} \rho_{\mu}$$

with $F_{\mu \nu} = \partial_{\mu} \omega_{\nu} - \partial_{\nu} \omega_{\mu}$ and $B_{\mu \nu} = \partial_{\mu} \rho_{\nu} - \partial_{\nu} \rho_{\mu}$. The nucleon mass is $M$ and the meson masses are $m_{\sigma}, m_{\omega}$, and $m_{\rho}$.

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TABLE I: Set of updated constraints (SET2a) used in Ref. [4]. For more details concerning each constraint see the reference.

| Constraint | Quantity | Density Region | Range of constraint |
|------------|----------|----------------|--------------------|
| SM1        | $K_0$    | at $\rho_0$    | 190 - 270 MeV      |
| SM3a       | $P(\rho)$| $2 < \frac{\rho}{\rho_0} < 5$ | Band Region        |
| SM4        | $P(\rho)$| $1.2 < \frac{\rho}{\rho_0} < 2.2$ | Band Region        |
| PNM1       | $\xi_{\text{PNM}}/\rho$| 0.017 $< \frac{\rho}{\rho_0} < 0.108$ | Band Region        |
| MIX1a      | $I$      | at $\rho_0$    | 25 - 35 MeV        |
| MIX2a      | $L_0$    | at $\rho_0$    | 25 - 115 MeV       |
| MIX4       | $\tilde{\delta}(\rho/2)$| at $\rho_0$ and $\rho_0/2$| 0.57 – 0.86 |

We can derive from Eq. (1) the equation of state for symmetric nuclear matter ($\gamma = 4$). The pressure is given by

$$P_{NL} = -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{A}{3}\sigma^3 - \frac{B}{4}\sigma^4 + \frac{C}{2}m_\omega^2\omega_0^2 + \frac{C}{4}(g_\sigma^2\sigma^2)^2 + g_\sigma g_\sigma^2\sigma^2\omega_0^2\left(\alpha_1 + \frac{\alpha_1}{2}\right) + \frac{\gamma}{6\pi^2}\int_0^\infty \frac{dk}{k^2 + M^2} \left[n(k,T,\mu^*) + \bar{n}(k,T,\mu^*)\right]$$

where

$$n(k,T,\mu^*) = \frac{1}{e^{(E^* - \mu^*)/T} + 1}, \quad \bar{n}(k,T,\mu^*) = \frac{1}{e^{(E^* + \mu^*)/T} + 1}$$

are the Fermi-Dirac distributions for particles and antiparticles, respectively. The effective energy, nucleon mass, and chemical potential are $E^* = (k^2 + M^2)^{1/2}$, $M^* = M - g_\sigma\sigma$, and $\mu^* = \mu - g_\omega\omega_0$, respectively. Furthermore, the (classical) mean-field values of $\sigma$ and $\omega_0$ are found by solving the following system of equations,

$$m_\sigma^2\sigma = g_\sigma\rho_s - A\sigma^2 - B\sigma^3 + g_\sigma G_\omega\omega_0^2(\alpha_1 + \alpha_1'\partial\sigma)$$

$$m_\omega^2\omega_0 = g_\omega - Cg_\omega(\omega_0)^3 - g_\sigma G_\omega\omega_0^2(2\alpha_1 + \alpha_1'\partial\sigma),$$

with

$$\rho = \frac{\gamma}{2\pi^2}\int_0^\infty dk k^2 \left[n(k,T,\mu^*) - \bar{n}(k,T,\mu^*)\right],$$

$$\rho_s = \frac{\gamma}{2\pi^2}\int_0^\infty \frac{dkM^*k^2}{(k^2 + M^2)^{3/2}} \left[n(k,T,\mu^*) + \bar{n}(k,T,\mu^*)\right].$$

### B. Density-dependent models

The four remaining parametrizations belong to the density-dependent group. Two of them include the $\delta$ meson. Their Lagrangian density is given by

$$\mathcal{L}_{\text{DD}} = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \Gamma_\sigma(\rho)\bar{\psi}\gamma^\mu\sigma\psi - \Gamma_\omega(\rho)\bar{\psi}\gamma^\mu\omega_\mu\psi - \frac{\Gamma_\rho(\rho)}{2}\bar{\psi}\gamma^\mu\tilde{\rho}_\mu\tilde{\psi} + \Gamma_\delta(\rho)\bar{\psi}\gamma^\mu\delta\psi - \frac{1}{4}F^\mu\nu F_{\mu\nu}$$

$$+ \frac{1}{2}(\partial_\nu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2) + \frac{1}{2}m_\omega^2\omega_\mu\omega_\mu - \frac{1}{4}B^\mu\nu B_{\mu\nu}$$

$$+ \frac{1}{2}m_\rho^2\tilde{\rho}_\mu\tilde{\rho}^\mu + \frac{1}{2}(\partial_\nu\delta\partial_\mu\delta - m_\delta^2\delta^2),$$

where

$$\Gamma_i(\rho) = \Gamma_i(\rho_0)f_i(x); \quad f_i(x) = \frac{a_i + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}$$

for $i = \sigma, \omega, \delta$ and $x = \rho/\rho_0$.

The expression for the pressure for these models can be obtained from Eq. (8) and reads:

$$P_{\text{DD}} = \rho\Sigma_R(\rho) - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2$$

$$+ \gamma\frac{\gamma}{6\pi^2}\int_0^\infty \frac{dkk^4}{(k^2 + M^2)^{1/2}} \left[n(k,T,\mu^*) + \bar{n}(k,T,\mu^*)\right],$$

with the rearrangement term defined as

$$\Sigma_R(\rho) = \frac{\partial\Gamma_\omega}{\partial\rho}\omega_0\rho - \frac{\partial\Gamma_\sigma}{\partial\rho}\sigma\rho_s.$$
TABLE II: Critical values for Consistent RMF models

| Model     | Ref. | $T_c$ (MeV) | $\rho_c$ (fm$^{-3}$) | $P_c$ (MeV/fm$^3$) |
|-----------|------|-------------|----------------------|---------------------|
| BKA20     | 17   | 14.92       | 0.0458               | 0.209               |
| BKA22     | 7    | 13.91       | 0.0442               | 0.178               |
| BKA24     | 7    | 13.83       | 0.0450               | 0.177               |
| BSR8      | 8    | 14.17       | 0.0440               | 0.185               |
| BSR9      | 8    | 14.11       | 0.0450               | 0.196               |
| BSR10     | 8    | 13.90       | 0.0439               | 0.176               |
| BSR11     | 8    | 14.00       | 0.0442               | 0.179               |
| BSR12     | 8    | 14.15       | 0.0448               | 0.185               |
| BSR15     | 8    | 14.53       | 0.0456               | 0.199               |
| BSR16     | 8    | 14.44       | 0.0454               | 0.196               |
| BSR17     | 8    | 13.32       | 0.0451               | 0.191               |
| BSR18     | 8    | 13.25       | 0.0451               | 0.189               |
| BSR19     | 8    | 14.28       | 0.0451               | 0.190               |
| BSR20     | 8    | 14.41       | 0.0464               | 0.197               |
| FSU-III   | 9    | 14.75       | 0.0461               | 0.205               |
| FSU-IV    | 9    | 14.75       | 0.0461               | 0.205               |
| FSUGold   | 10   | 14.75       | 0.0461               | 0.205               |
| FSUGold4  | 11   | 14.80       | 0.0456               | 0.204               |
| FSUGZ03   | 12   | 14.11       | 0.0450               | 0.185               |
| FSUGZ06   | 12   | 14.44       | 0.0454               | 0.196               |
| IU-PSU    | 14   | 14.49       | 0.0457               | 0.196               |
| G2*       | 13   | 13.48       | 0.0468               | 0.192               |
| Z271s2    | 14   | 17.97       | 0.0509               | 0.303               |
| Z271s3    | 14   | 17.97       | 0.0509               | 0.303               |
| Z271s4    | 14   | 17.97       | 0.0509               | 0.303               |
| Z271s5    | 14   | 17.97       | 0.0509               | 0.303               |
| Z271s6    | 14   | 17.97       | 0.0509               | 0.303               |
| Z271v4    | 14   | 17.97       | 0.0509               | 0.303               |
| Z271v5    | 14   | 17.97       | 0.0509               | 0.303               |
| Z271v6    | 14   | 17.97       | 0.0509               | 0.303               |
| DD-F      | 15   | 15.24       | 0.0505               | 0.245               |
| TW39      | 6    | 15.17       | 0.0509               | 0.241               |
| BSM      | 16    | 15.17      | 0.0509               | 0.241               |
| DD-ME4    | 17   | 15.32       | 0.0491               | 0.235               |

for $T_c = 17.9 \pm 0.4$ MeV, but also for $P_c = 0.31 \pm 0.07$ MeV/fm$^3$, and $\rho_c = 0.06 \pm 0.01$ fm$^{-3}$, all of them related to symmetric nuclear matter.

By first analyzing the critical temperature, we can see that only the family Z271 (that encompasses all 8 related parametrizations), presents $T_c$ compatible with five of the eight experimental points, including the more recent one [24]. The density-dependent models present the critical temperature inside the range of $15 \leq T_c \leq 19$ MeV proposed by [21]. The other critical parameters of Ref. [24], namely, pressure and density, are also compatible with the ones computed for the Z271 family.

The density dependent family also agrees with this experiment.

TABLE III: Summary of experimental data of Refs. [18–24]

| Reference | $T_c$ (MeV) | $\rho_c$ (fm$^{-3}$) | $P_c$ (MeV/fm$^3$) |
|-----------|-------------|----------------------|---------------------|
| 18        | 19.1 ± 3    | -                    | -                   |
| 19        | 16.60 ± 0.89| -                    | -                   |
| 20        | 20 ± 3      | -                    | -                   |
| 21        | 17 ± 2      | -                    | -                   |
| 22        | 19.5 ± 1.2/16.5 ± 1.0 | - | - |
| 23        | 17.9 ± 0.4  | 0.06 ± 0.01          | 0.31 ± 0.07         |

FIG. 1: Critical (a) temperature, (b) pressure, and (c) density of CRMF parametrizations versus symmetry energy at saturation density. Circles: nonlinear model. Squares: density dependent model.

FIG. 2: Critical (a) temperature, (b) pressure, and (c) density of CRMF parametrizations versus the slope of symmetry energy at saturation density. Circles: nonlinear model. Squares: density dependent model.

If we look at the structure of Eq. (2), we can understand this agreement with the experimental data based on the only term that distinguish such model from the $\sigma^3 - \sigma^4$ one, which is those containing the $C$ constant.

In this case $C \neq 0$.

If we look at the critical temperature, we can think of a similar structure, since the nonlinear behavior of the $\sigma$ field can be represented somehow in the thermodynamical quantities, by the density-dependent constant $\Gamma_\sigma(\rho)$. The same occurs with the $\omega_\sigma$ field, i.e., the strength of the repulsive interaction is also a density-dependent quantity, $\Gamma_\omega(\rho)$.

We have also tried to verify if there are correlations between the critical parameters and the observables of nuclear matter at zero temperature and at the saturation density. We investigate possible correlations between $T_c$, $P_c$ and $\rho_c$ with the symmetry energy, its slope and incompressibility. The results are shown in Figs. 1, 2, and 3, respectively.
dependent models are not seen. However, the picture changes when we look at the incompressibility ($K_0$). From Fig. 3, one can observe an increasing behavior of $T_c$, $P_c$ and $\rho_c$ as $K_0$ increases.

IV. SUMMARY

In this work, we present the results obtained in the calculation of the critical parameters: temperature, pressure, and density in symmetric nuclear matter. In our analysis, we verified that the nonlinear models, whose parameterizations were grouped in the family Z271, show a good agreement with the experimental data [24] for all critical parameters analyzed. The density-dependent family also shows an agreement with the data given in [24] for the pressure and density. Concerning $T_c$, the agreement is only found with data presented in Ref. [21].

In the search for possible correlations, we can see that the incompressibility at zero temperature and at saturation density show a clear increasing behavior with the critical parameters analyzed. The same does not occur with the symmetry energy and its slope.

Acknowledgments

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Brazil under grants 301155/2017-8 and 310242/2017-7. This work is also a part of the project CNPq-INCT-FNA Proc. No. 464898/2014-5.

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