Instability of Schwarzschild-AdS black hole in Einstein-Weyl gravity

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Abstract

We investigate the classical stability of Schwarzschild-AdS black hole in a massive gravity theory of the Einstein-Weyl gravity. It turns out that the linearized Einstein tensor perturbations exhibit unstable modes featuring the Gregory-Laflamme instability of five-dimensional AdS black string, in contrast to the stable Schwarzschild-AdS black hole in Einstein gravity. We point out that the instability of the black hole in the Einstein-Weyl gravity arises from the massiveness but not a feature of fourth-order derivative theory giving ghost states.

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1 Introduction

Recently, Babichev and Fabbri [1] have shown that the massive linearized equation around the Schwarzschild black hole in both de Rham, Gabadadze, and Tolley (dRGT) theory [2] and its bigravity extension [3] gives rise to an instability of $s(l = 0)$-mode (spherically symmetric mode) with $l$ the spheroidal harmonic index. This was done by comparing it with the four-dimensional linearized equation around the five-dimensional black string where the Gregory-Laflamme (GL) instability was found [4]. It turned out that the bimetric black hole is unstable provided a mass of $m' = m(1 + 1/\kappa)^{1/2}$ satisfies a bound of $0 < m' < \mathcal{O}(1)/r_0$ with $r_0$ the horizon radius in the metric function $f(r) = 1 - r_0/r$. We note that the limit of $\kappa \to \infty$ recovers the black hole in the dRGT theory. The black hole in the dRGT theory is also unstable because $m$ satisfies a bound of $0 < m < \mathcal{O}(1)/r_0$. In addition, the authors [5] have confirmed this result by considering the Schwarzschild-de Sitter black hole and extending the $l = 0$ mode to generic modes of $l \neq 0$. These results may indicate an important fact that the static black holes do not exist in massive gravity theory.

On the other hand, Whitt [6] has insisted thirty years ago that provided both massive spin-0 and spin-2 gravitons are non-tachyonic, the Schwarzschild black hole is classically stable in a massive theory of fourth-order gravity when he uses the linearized-Ricci tensor equation. In this case, one does not worry about the ghost instability arising from the fourth-order gravity theory because the linearized-Ricci tensor satisfies a second-order tensor equation. Recently, the author has revisited this stability issue. As expected, it was shown that the black hole in fourth-order gravity with $\alpha = -3\beta$ is unstable provided the graviton mass of $m_2 = 1/\sqrt{3\beta}$ satisfies a bound of $0 < m_2 < \mathcal{O}(1)/r_0$ [7]. This was performed by comparing the linearized-Ricci tensor equation with the four-dimensional metric perturbation equation around the five-dimensional black string [4].

In this work, we wish to reexamine the classical stability of Schwarzschild-AdS (SAdS) black hole in Einstein-Weyl gravity which was known to be stable against the metric perturbation [8]. By contrast, it is shown that solving both the linearized-Einstein tensor equation and the metric perturbation equation exhibit unstable modes featuring the GL instability of five-dimensional AdS black string [9]. It confirms that the GL instability of the black hole in the Einstein-Weyl gravity is due to the massiveness but not a feature of fourth-order gravity giving ghost states.

Taking into account the number of degrees of freedom (DOF), it is helpful to show
why the SAdS black hole is physically stable in the Einstein gravity\cite{10,11}, whereas the SAdS black hole is unstable in the Einstein-Weyl gravity. The number of DOF of the metric perturbation is 2 in the Einstein gravity, while the number of DOF is 5 in the Einstein-Weyl gravity. The $s$-mode analysis of the massive graviton with 5 DOF shows the GL instability. The $s$-mode analysis is relevant to the massive graviton in the Einstein-Weyl gravity but not to the massless graviton in the Einstein gravity.

2 Linearized Einstein-Weyl gravity

We start with the fourth-order gravity in AdS\(_{4}\) spacetimes

\[
S_{\text{FO}} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - 2\Lambda + \alpha R_{\mu\nu} R^{\mu\nu} + \beta R^2 \right]
\]

with two arbitrary parameters $\alpha$ and $\beta$. Although this theory is renormalizable in Minkowski spacetimes\cite{12}, the massive spin-2 graviton suffers from having ghosts. A massive spin-0 graviton is decoupled for the choice of $\alpha = -3\beta$, which leads to a critical gravity with $\beta = -1/2\Lambda$\cite{13,14}. For a higher dimensional critical gravity, see a reference of\cite{15}.

The Einstein-Weyl gravity is defined under the condition of $\alpha = -3\beta$ as

\[
S_{\text{EW}} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - 2\Lambda - \frac{\beta}{6} C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} \right]
\]

with

\[
C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} = 2 \left( R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right) + (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2).
\]

Here the last of Gauss-Bonnet term could be neglected because it does not contribute to equation of motion.

From (1), the Einstein equation is derived to be

\[
G_{\mu\nu} + E_{\mu\nu} = 0,
\]

where the Einstein tensor is given by

\[
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu}
\]

and $E_{\mu\nu}$ takes the form

\[
E_{\mu\nu} = 2\alpha \left( R_{\mu\rho\sigma} R^{\rho\sigma} - \frac{1}{4} R^{\rho\sigma} R_{\rho\sigma} g_{\mu\nu} \right) + 2\beta \left( R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} \right)
\]

\[+ \alpha \left( \nabla^2 R_{\mu\nu} + \frac{1}{2} \nabla^2 R g_{\mu\nu} - \nabla_{\mu} \nabla_{\nu} R \right) + 2\beta \left( g_{\mu\nu} \nabla^2 R - \nabla_{\mu} \nabla_{\nu} R \right).
\]
It is well-known that Eq. (4) provides the SAdS black hole solution [13, 8]

\[ ds_{SAdS}^2 = \overline{g}_{\mu\nu} dx^\mu dx^\nu = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2 \]  
with the metric function
\[ f(r) = 1 - \frac{r_0}{r} - \frac{\Lambda}{3} r^2, \quad \Lambda = -\frac{3}{\ell^2}. \]  

Here \( \ell \) denotes the curvature radius of AdS_4 spacetimes. We note that a mass parameter of \( r_0 = r_+ (1 + r_+^2/\ell^2) \) is not the horizon radius \( r_+ \) which is obtained as a solution to \( f(r_+) = 0 \). Hereafter we denote the background quantities with the “overbar”. In this case, the background Ricci tensor is given by
\[ \overline{R}_{\mu\nu} = \Lambda \overline{g}_{\mu\nu}. \]

It is easy to show that the SAdS black hole solution (7) to the Einstein equation of \( G_{\mu\nu} = 0 \) is also the solution to the Einstein-Weyl gravity when one substitutes (9) together with \( \overline{R} = 4\Lambda \) into (3). To perform the stability analysis, we usually introduce the metric perturbation around the SAdS black hole
\[ g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \]

Then, the linearized Einstein equation takes the form
\[
\left[ 1 + 2\Lambda (\alpha + 4\beta) \right] \delta G_{\mu\nu} + \alpha \left[ \nabla^2 \delta G_{\mu\nu} + 2\overline{R}_{\rho\sigma\mu\nu} \delta G^{\rho\sigma} - \frac{2\Lambda}{3} \delta R \bar{g}_{\mu\nu} \right] \\
+ (\alpha + 2\beta) \left[ -\nabla_\mu \nabla_\nu \bar{g}_{\mu\nu} + \bar{g}_{\mu\nu} \nabla^2 \bar{g} + \Lambda \bar{g}_{\mu\nu} \right] \delta R = 0,
\]  

where the linearized Einstein tensor, Ricci tensor, and Ricci scalar are given by
\[
\delta G_{\mu\nu} = \delta R_{\mu\nu} - \frac{1}{2} \delta R \bar{g}_{\mu\nu} - \Lambda h_{\mu\nu}, \\
\delta R_{\mu\nu} = \frac{1}{2} \left( \nabla^\rho \nabla_\mu h_{\nu\rho} + \nabla^\rho \nabla_\nu h_{\mu\rho} - \nabla^2 h_{\mu\nu} - \nabla_\mu \nabla_\nu h \right), \\
\delta R = \bar{g}^{\mu\nu} \delta R_{\mu\nu} - h^{\mu\nu} \overline{R}_{\mu\nu} = \nabla^\mu \nabla_\nu h_{\mu\nu} - \nabla^2 h - \Lambda h.
\]  

With \( h = h^{\rho\rho} \). It is very difficult to solve the linearized equation (11) directly because it is a coupled second-order equation for \( \delta G_{\mu\nu} \) and \( \delta R \). Thus, we attempt to decouple \( \delta R \) from (11).
For this purpose, we take the trace of (11) which leads to

\[
4(\alpha + 3\beta)\bar{\nabla}^2 - 2\delta R = 0.
\] (15)

It implies that the D’Alembertian operator could be removed if one chooses

\[
\alpha = -3\beta.
\] (16)

In this case, the linearized Ricci scalar is constrained to vanish

\[
\delta R = 0.
\] (17)

Plugging \(\delta R = 0\) into Eq. (11) leads to the equation for the linearized Einstein tensor solely

\[
\left(\bar{\nabla}^2 - 2\frac{\Lambda}{3} - \frac{1}{3\beta}\right)\delta G_{\mu\nu} + 2\bar{R}_{\mu\rho\sigma\nu}\delta G^{\rho\sigma} = 0.
\] (18)

This shows clearly why we consider the Einstein-Weyl gravity (2) with \(\alpha = -3\beta\) instead of the fourth-order gravity action (1) with arbitrary \(\alpha\) and \(\beta\).

Before we proceed, we wish to mention that the metric perturbation is not suitable for analyzing the SAdS black hole stability in the Einstein-Weyl gravity. For simplicity, we consider the AdS\(_4\) spacetimes background whose curvature tensor takes a simple form

\[
\bar{R}_{\mu\nu\rho\sigma} = \frac{\Lambda}{3} (\bar{g}_{\mu\rho}\bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma}\bar{g}_{\nu\rho}).
\] (19)

After choosing the transverse-traceless gauge (TTG)

\[
\bar{\nabla}^\mu h_{\mu\nu} = 0 \text{ and } h = 0,
\] (20)

Eq. (18) leads to a fourth-order differential equation [13]

\[
\left(\bar{\nabla}^2 - 2\frac{\Lambda}{3}\right)\left(\bar{\nabla}^2 - \frac{4\Lambda}{3} - \frac{1}{3\beta}\right)h_{\mu\nu} = 0
\] (21)

which may imply a massless spin-2 graviton equation

\[
\left(\bar{\nabla}^2 - 2\frac{\Lambda}{3}\right)h_{\mu\nu}^m = 0
\] (22)

and a massive spin-2 graviton equation

\[
\left(\bar{\nabla}^2 - 2\frac{\Lambda}{3} - M^2\right)h_{\mu\nu}^M = 0.
\] (23)
Here the mass squared is given by

\[ M^2 = \frac{2\Lambda}{3} + \frac{1}{3\beta} = \frac{1}{3\beta} (1 + 2\beta\Lambda). \] (24)

In AdS₄ spacetimes, the stability condition is given by the absence of tachyonic instability \((M^2 \geq 0)\) [8], which implies that \(\beta\) must satisfy

\[ 0 < \beta \leq -\frac{1}{2\Lambda} = \frac{\ell^2}{6}. \] (25)

In the massless case of \(\beta = -1/2\Lambda = \ell^2/6\), Eq. (21) leads to that for a critical gravity

\[ \left( \nabla^2 - \frac{2\Lambda}{3} \right)^4 h^{\log}_{\mu\nu} = 0, \quad \left( \nabla^2 - \frac{2\Lambda}{3} \right)^2 h^m_{\mu\nu} = -h^m_{\mu\nu}. \] (26)

However, it was shown that a general mode of \(h_{\mu\nu} = c_1 h^{\log}_{\mu\nu} + c_2 h^m_{\mu\nu}\) suffers from negative norm states unless one truncates out the log-mode by imposing appropriate AdS₄ boundary conditions. Up to now, there is no consistent truncation mechanism to eliminate the log-mode. We recall that this problem arises because we work with the fourth-order derivative equation (21) for the metric perturbation. In this work, we do not consider a new unitary gravity for \(-\frac{9}{4\ell^2} \leq M^2 < 0\) [14, 16] because it has still a non-unitarity problem like the critical gravity.

Going back to the SAdS black hole [7], we rewrite Eq. (18) as a second-order equation for the linearized Einstein tensor

\[ \nabla^2 \delta G_{\mu\nu} + 2\tilde{R}_{\rho\mu\sigma\nu} \delta G^{\rho\sigma} = M^2 \delta G_{\mu\nu}. \] (27)

If one introduces the Lichnerowicz operator

\[ \Delta_L \delta G_{\mu\nu} = -\nabla^2 \delta G_{\mu\nu} - 2\tilde{R}_{\rho\mu\sigma\nu} \delta G^{\rho\sigma} + 2\Lambda \delta G_{\mu\nu}, \] (28)

the corresponding equation could be rewritten as

\[ (\Delta_L - 2\Lambda + M^2) \delta G_{\mu\nu} = 0. \] (29)

Taking into account the TTG [20], the linearized Einstein tensor reduces to

\[ \delta G_{\mu\nu} = -\frac{1}{2}(\Delta_L - 2\Lambda) h_{\mu\nu}. \] (30)

Then, one can rewrite (29) as a fourth-order equation for \(h_{\mu\nu}\)

\[ (\Delta_L - 2\Lambda + M^2)(\Delta_L - 2\Lambda) h_{\mu\nu} = 0 \] (31)
which is similar to (21) in AdS\(_4\) spacetimes. Eq. (31) may imply a linearized massless equation around the SAdS black hole [8]

\[ \nabla^2 h^m_{\mu\nu} + 2 \bar{R}_{\rho\mu\sigma\nu} h^{m\rho\sigma} = 0. \] (32)

and a linearized massive equation for \( h^M_{\mu\nu} \)

\[ \nabla^2 h^M_{\mu\nu} + 2 \bar{R}_{\rho\mu\sigma\nu} h^{M\rho\sigma} = M^2 h^M_{\mu\nu}. \] (33)

At this stage, we wish to point out the difference between (27) and (33). The former equation is a second-order equation for the linearized Einstein tensor, whereas the latter is a suggesting second-order equation from the fourth-order equation (31) for the metric perturbation. It is known that the introduction of fourth-order derivative terms gives rise to ghost-like massive graviton [12], which may imply an instability of a black hole even if a black hole solution exists. Hence, even though (22) [(23)] were frequently used as a linearized massless [massive] equation around the AdS\(_4\) spacetimes [8, 13, 14, 15, 16], their validity is not yet proved because they are free from ghost states. In order to check whether (23) [(33)] are reliable or not, we note that our action (2) reveals ghosts when we perform the metric perturbation \( h_{\mu\nu} \) around the Minkowski spacetimes with \( \Lambda = 0 \) [12]. Eq. (21) [(31)] take the form in the Minkowski background [17]

\[ \Box \left( \Box - m_2^2 \right) h_{\mu\nu} = -T_{\mu\nu}, \quad m_2^2 = \frac{1}{3\beta} \] (34)

with an external source \( T_{\mu\nu} \). Replacing \( \Box \) by \( -p^2 \), the metric perturbation is given by

\[ h_{\mu\nu} \sim \frac{T_{\mu\nu}}{p^2} - \frac{T_{\mu\nu}}{p^2 + m_2^2} \] (35)

which the last term spoils the unitarity. Hence, splitting (21) [(31)] into two second-order equations (22) [(32)] and (23) [(33)] is dangerous because the ‘−’ sign in the front of (23) [(33)] is missed. As is shown in (35), the ghost arises from this sign when one performs the partial fraction. To this end, the authors in [8] have found the two on-shell energies on the AdS\(_4\) spacetime background

\begin{align*}
E_m &= -\frac{3\beta M^2}{2T} \int d^4x \sqrt{-\bar{g}} (\nabla^0 h^m_{\mu\nu}) \dot{h}^m_{\mu\nu} > 0, \quad (36) \\
E_M &= \frac{3\beta M^2}{2T} \int d^4x \sqrt{-\bar{g}} (\nabla^0 h^M_{\mu\nu}) \dot{h}^M_{\mu\nu} < 0 \quad (37)
\end{align*}
when they compute each Hamiltonian which satisfies (22) and (23), respectively. Thus, for $M^2 \neq 0$, ghost-like massive excitation is not avoidable. In order for the theory to be free from ghosts, one needs to choose $M^2 = 0 (\beta = -1/2\Lambda)$ which corresponds to the critical gravity where a massive graviton becomes a massless graviton. Because of a missing of ‘−’ sign, we may insist that Eq.(23)[(33)] by itself do not represent a correct linearized equation for studying the stability of the SAdS black hole in the Einstein-Weyl gravity. However, the overall ‘−’ sign in (23)[(33)] does not make any difference unless an external source is introduced in the right-hand side as Eq.(34) does indicate. Therefore, the fourth-order gravity does not automatically imply the instability of the black hole even if one uses (33).

Hopefully, if one uses (27) instead of (33), one is free from the ghost issue because (27) is a genuine second-order equation.

### 3 SAdS black hole stability in Einstein-Weyl gravity

In Einstein gravity, the linearized equation around the Schwarzschild black hole is given by $\delta R_{\mu\nu}(h) = 0$ with $\delta R_{\mu\nu}(h)$ (13). Then, the metric perturbation $h_{\mu\nu}$ is classified depending on the transformation properties under parity, namely odd and even. Using the Regge-Wheeler [18] and Zerilli gauge [19], one obtains two distinct perturbations: odd with 2 DOF and even with 4 DOF. This implies that one starts with 6 DOF after choosing the Regge-Wheeler gauge, leading to 2 DOF (1 for odd and 1 for even) for a massless spin-2 graviton propagation. The Schwarzschild black hole is stable against the metric perturbation [20, 21]. Performing the stability analysis of the SAdS black hole in Einstein gravity, one has to use the linearized equation

$$\delta G_{\mu\nu}(h) = \delta R_{\mu\nu}(h) - \frac{\bar{g}_{\mu\nu}}{2} \delta R(h) - \Lambda h_{\mu\nu} = 0,$$

which was tuned out to be stable by following the Regge-Wheeler prescription [10, 11, 22]. In these cases, the $s(l = 0)$-mode analysis is not necessary to show the stability of the Schwarzschild and SAdS black holes because the massless spin-2 graviton requires modes with $l \geq 2$.

However, the $s$-mode analysis is responsible for detecting an instability of a massive graviton propagating on the SAdS black hole in Einstein-Weyl gravity. The even-parity metric perturbation is designed for a single $s$-mode analysis in the massive gravity and
whose form is given by $H_{tt}$, $H_{tr}$, $H_{rr}$, and $K$ as

$$
h^e_{\mu\nu} = e^{\Omega u} \begin{pmatrix} H_{tt}(r) & H_{tr}(r) & 0 & 0 \\
H_{tr}(r) & H_{rr}(r) & 0 & 0 \\
0 & 0 & K(r) & 0 \\
0 & 0 & 0 & \sin^2 \theta K(r) \end{pmatrix}.
$$

(39)

Even though one starts with 4 DOF, they are related to each other when one uses the TTG [20]. Hence, we expect to have one decoupled equation for $H_{tr}$.

For a massive gravity theory in the Minkowski background, there is correspondence between linearized Ricci tensor $\delta R_{\mu\nu}$ and Ricci spinor $\Phi_{ABCD}$ when using the Newman-Penrose formalism [23]. Here the null real tetrad is necessary to specify polarization modes of a massive graviton, as the massive gravity requires null complex tetrad to specify six polarization modes [24, 25]. This implies that in fourth-order gravity theory, one may take the linearized Ricci tensor $\delta R_{\mu\nu}$ [13] with 6 DOF as physical observables [7]. Requiring $\delta R = 0$ further, the DOF of $\delta R_{\mu\nu}$ is five which is the same DOF for the metric perturbation $h_{\mu\nu}$ in massive gravity theory.

At this stage, we stress again that [27] is considered as the second-order equation with respect to $\delta G_{\mu\nu}$, but not the fourth-order equation [21] for $h_{\mu\nu}$. Hence, we propose $\delta G_{\mu\nu}$ as physical observables propagating on the SAdS black hole background instead of $\delta R_{\mu\nu}$ on the Schwarzschild black hole background. Also, we have the tracelessness of $\delta G_{\mu}^{\mu} = -\delta R = 0$ and the transversality of $\nabla^\mu \delta G_{\mu\nu} = 0$ from the contracted Bianchi identity. Then, $\delta G_{\mu\nu}$ describe exactly five DOF propagating on the SAdS black hole background without ghosts.

Since Eq. (33) is the same linearized equation for four-dimensional metric perturbation around five-dimensional black string, we follow the GL instability analysis in AdS$_4$ space-times [9]. Eliminating all but $H_{tr}$, Eq. (33) reduces to a second-order equation for $H_{tr}$

$$
A(r; r_0, \ell, \Omega^2, M^2) \frac{d^2}{dr^2} H_{tr} + B \frac{d}{dr} H_{tr} + C H_{tr} = 0,
$$

(40)

where $A, B$ and $C$ were given by (20) in [9, 26]. We stress again that the s-mode perturbation is described by single DOF but not 5 DOF. The authors in [9] have solved (40) numerically and found unstable modes for $0 < m < \frac{O(1)}{r_0}$. See Fig. 1 that is generated from the numerical analysis. We note that $r_+ = 1, 2, 3$ correspond to $r_0 = 1.01, 2.08, 4.64$, respectively. From the observation of Fig. 1 with $O(1) \simeq 0.85$, we find unstable modes for

$$
0 < M < \frac{O(1)}{r_0}
$$

(41)

9
Figure 1: Plots of unstable modes on three curves with $r_+ = 1, 2, 4$ and $l = 10$. The $y(x)$-axis denote $\Omega(M)$. The smallest curve represents $r_+ = 4$, the medium denotes $r_+ = 2$, and the largest one shows $r_+ = 1$.

with the mass

$$M = \sqrt{\frac{1}{3\beta} - \frac{2}{l^2}}.$$  

(42)

As the horizon size $r_+$ increases, the instability becomes weak as in the Schwarzschild black hole [26].

Similarly, we find Eq. (33) when we replace $\delta G_{\mu\nu}$ by $h_{\mu\nu}$ in (27). Hence, a relevant equation for $\delta G_{tr}$ takes the same form

$$A(r; r_0, \ell, \Omega^2, M^2) \frac{d^2}{dr^2} \delta G_{tr} + B \frac{d}{dr} \delta G_{tr} + C \delta G_{tr} = 0$$  

(43)

which shows the same unstable modes appeared in Fig. 1. This implies that even if one uses (33) as a linearized massive equation [8], our conclusion remains unchanged because (33) and (27) are the same equation for different tensors.

Consequently, the instability arises from the massiveness ($M > 0$) but not from a feature of the fourth-order equation which gives the ‘−’ sign (ghost= negative norm state) when one splits it into two second-order equations. This implies that static black holes in massive gravity theory do not exist and/or they do not form in the gravitational collapse. If a black hole was formed in the massive gravity theory, one may ask what is the end-state of such instability. For unstable black strings of SAdS×R with translational symmetry, there are
some evidences that break-up occurs [27]. However, we consider a spherically symmetric black hole in four dimensions. A possible end-state may be a spherically symmetric black hole endowed with a graviton cloud [5, 28].

Finally, in the case of $M = 0 (\beta = -1/2\Lambda)$, the theory becomes massless and is stable against the Einstein tensor perturbation. However, this corresponds precisely to the critical gravity when one uses the metric perturbation. Here we have a non-unitarity issue due to the log-mode like (26). Also, one finds that $\mathcal{M}$ (ADT mass) = 0 and $\mathcal{S}$ (Wald’s entropy) = 0 at the critical point, leading to a vacuum but not a black hole [13].

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