Transport and zero sound in holographic strange metals

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Abstract. It is a challenge to describe the non-Fermi liquid behavior that appears in strongly interacting systems of fermions using traditional field theory techniques. A proposal to model the associated "strange metal" phenomenology is to use a strongly coupled critical point with non-relativistic scale invariance, dual to a Lifshitz geometry, coupled to a finite density of probe charged carriers dual to D-branes. We use this model to study the thermodynamic and transport properties of charge carriers at finite and zero temperature. At zero temperature we show that the large wavelength behavior is dominated by a single mode whose properties are similar to the zero sound of Fermi liquids.

1. Motivation

Gauge-gravity duality \cite{1-3} provides a novel way to compute observables in strongly-coupled, scale-invariant systems at finite density, and thus may be useful for studying condensed matter systems near quantum critical points, such as the "strange metal" phase of some heavy fermion compounds and possibly also of some high-$T_c$ materials. In particular, gauge-gravity duality may be able to reproduce the simple properties of strange metals, such as electrical resistivity scaling linearly with temperature $T$, which cannot be derived in a Fermi liquid description, which assumes the appropriate degrees of freedom are weakly-interacting quasi-particles. Of course the ultimate goal is to understand why some strange metals have a high $T_c$.

In relativistic systems at zero temperature, holographic calculations also reveal the existence of a propagating quasi-particle excitation producing a pole in the charge density and current retarded two point functions \cite{4-6}. In a weakly-coupled Fermi liquid, fluctuations in the shape of the Fermi surface produce a collective excitation at zero temperature, Landau's so-called "zero sound." The dispersion relation of the mode in these holographic systems was identical in form to that of Landau's zero sound, with a dispersion relation linear in momentum and a damping quadratic in momentum, hence the mode was dubbed zero sound by analogy. We will call this mode "holographic zero sound." The physical origin of this mode remains mysterious since it appears in systems with both fermions and scalars—systems that do not fit neatly into either Fermi liquid or quantum Bose liquid theory. Indeed, given the existence of the holographic zero sound, as well as unusual scaling of the heat capacity with temperature, these holographic systems may be new kinds of (strongly-coupled) quantum liquids \cite{4}.

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In ref. [7] we explored the fate of holographic zero sound in the holographic strange metal models. To do so, we computed holographically the low-frequency and low-momentum forms of the retarded correlation functions of the current, for massless charge carriers. We also computed the AC conductivity associated with the charge carriers, generalizing the results of ref. [8], for two spatial field theory directions, to an arbitrary number of spatial directions. Here we give an overview of these results.

This text is organized as follows. We start reviewing the characteristics of the holographic critical point that we will use to model a strange metal in section 2. We then study the thermodynamics properties of these systems in section 3. We compute the current Green’s functions in section 4, and use them to study the holographic zero sound and AC conductivity in section. We conclude with some discussion and suggestions for future research in section 5.

2. Holographic critical point

Gauge-gravity duality is holographic: it equates a weakly-coupled theory of gravity on some spacetime with a strongly-coupled field theory living on the boundary of that spacetime. The spacetime symmetries of the field theory are dual to the isometries of the bulk metric. Here we will focus on field theories invariant under the Lifshitz group, which includes scale transformations of the time coordinate $t$ and spatial coordinates $\vec{x}$ of the form

$$t \rightarrow \lambda^z t, \quad \vec{x} \rightarrow \lambda \vec{x},$$

where $\lambda$ is real and positive and $z$ is called the “dynamical exponent.” The dual gravity theory then lives in Lifshitz spacetime, with metric [9]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{dr^2}{r^2} - \frac{dt^2}{r^2 z^2} + \frac{d\vec{x}^2}{r^2},$$

where $r$ is the holographic radial coordinate: $r \rightarrow \infty$ corresponds to the infrared (IR) of the field theory, while the near-boundary region $r \rightarrow 0$ corresponds to the ultraviolet (UV). Notice that when $z = 1$, the metric becomes that of anti-de Sitter space (AdS), and the dual field theory symmetry is enhanced to the relativistic conformal group.

One proposal for holographic model building of strange metals is to use probe D-branes in a Lifshitz spacetime [8]. In this approach, an appropriate configuration of fields on the probe D-brane represents a finite density of massive charge carriers [10–12], while the background spacetime represents a strongly-coupled neutral quantum critical theory. Holographic calculations then reveal that such systems can indeed reproduce the simple properties of strange metals, including electrical resistivity linear in temperature [8]. However, ingredients beyond Lifshitz spacetime, for example a nontrivial dilaton, are needed to reproduce all the measured strange metal properties [8].

In gauge-gravity duality, a theory of gravity is dual to some large-$N$, strongly-coupled non-Abelian gauge theory. $N_f$ probe D-branes are dual to $N_f$ fields in the fundamental representation of the gauge group(s) in the probe limit $N_f \ll N$, transforming under a global $U(N_f)$ group [13]. We will call these fields quarks or flavor fields, in analogy with Quantum Chromodynamics, and they will be our charged degrees of freedom. The dynamics of $N_f$ coincident D-branes are determined by the non-Abelian Dirac-Born-Infeld (DBI) action, involving worldvolume scalars and $U(N_f)$ gauge fields. We produce a finite density of charge carriers by introducing a chemical potential for the diagonal $U(1) \subset U(N_f)$. The density operator $J^t$ is the time component of the conserved $U(1)$ current $J^t$, and is holographically dual to $A_t$, the time component of the $U(1)$ gauge field living on the D-branes. We will thus introduce in the bulk D-branes with only $A_t(r)$ present. The action then reduces to the Abelian DBI action.

We will only consider massless flavor fields. A flavor field mass operator would be dual to a scalar field on the D-branes. We omit any such scalar from our calculations. We assume our
D-branes are extended along the \( q \) spatial dimensions of the Lifshitz spacetime. The Abelian DBI action for our D-branes is then

\[
\hat{S} = -N_f T_D V \int dr \, dt \, d^q x \, \sqrt{-\det \left[ g_{ab} + (2\pi \alpha') F_{ab} \right]},
\]

where \( T_D \) is the tension, \( g_{ab} \) the induced metric, and \( F_{ab} \) the \( U(1) \) field strength of the D-branes. The factor \( V \) is the volume of any internal space that the D-brane may be wrapping, and \( 2\pi \alpha' \) is the inverse string tension, which will typically be present in an actual string theory system.

Inserting our ansatz \( A_t(r) \), performing the trivial integrations over \( dt \) and \( d^q x \), and dividing out the subsequent (infinite) volume factors, we obtain the action density \( S \).

3. Thermodynamics

In this section we review the results of refs. [4,7,12,14–16] for the thermodynamics of the charge carriers in these holographic systems. The charge density in the field theory is

\[
\langle \mathcal{J}^t \rangle = \tilde{N} d = \frac{\delta S}{\delta A_t} = N g_{xx}^{q/2} \frac{(2\pi \alpha') A_t'}{\sqrt{|g_{tt}| g_{rr} - A_t'^2}},
\]

where a tilde means there is a \((2\pi \alpha')\) factor. Solving for \( A_t'(r) \) and plugging it back into the action, we find

\[
S = -N \int dr |g_{tt}|^{1/2} g_{rr}^{1/2} \frac{g_{xx}^{q/2}}{\sqrt{g_{xx}^{q/2} + dz^2}}.
\]

We obtain the field theory chemical potential and free energy by performing the integrals for \( A_t'(r) \) and \( S \). The chemical potential is

\[
\mu_0 = \int_0^\infty dr A_t'(r) = \frac{d}{2\pi \alpha'} \int_0^\infty dr \frac{r^{-(z+1)}}{\sqrt{r^{-2q} + d^2}} = \frac{d^{z/q}}{4\pi \alpha' q} \Gamma \left( \frac{z}{2q} \right) \Gamma \left( \frac{1}{2} - \frac{z}{2q} \right) \Gamma \left( \frac{1}{2} \right).
\]

The grand canonical (Gibbs) free energy density is

\[
\Omega_0 = -S = N \int_0^\infty dr \frac{r^{-(2q+z+1)}}{\sqrt{r^{-2q} + d^2}} = -\tilde{N} \frac{dz}{z + q} \mu_0.
\]

These results are deceptively simple. We have actually removed various power-law divergences at the \( r = 0 \) endpoint of the integrations via analytic continuation. A straightforward calculation reveals that for us

\[
\langle \mathcal{J}^t \rangle = -\frac{\delta \Omega_0}{\delta \mu_0},
\]

which shows that we are working in the grand canonical ensemble. Notice that with massless flavor fields, any finite chemical potential will produce a finite density.

Analytic continuation cannot remove logarithmic divergences, which indeed occur in eqs. (6) and (7) when \( z = (2n - 1)q \). These divergences occur because the operators \((\mathcal{J}^t)^{2n}\) are classically marginal at these \( z \) values [8], and if added to the action, can produce a breaking of scale invariance through quantum corrections, similarly to what happens for multi-trace scalar operators [17].

To study thermodynamics at finite temperature, we need a black hole in the bulk whose Hawking temperature is dual to the field theory temperature [18]. To capture the basic
qualitative physics without committing to a specific system, we will again follow ref. [8] and write a generic Lifshitz black hole metric

$$ds^2 = \frac{dr^2}{f(r)} - \frac{f(r)}{r^{2z}} dt^2 + \frac{d\vec{x}^2}{r^2}.$$  \hspace{1cm} (9)

We do not need an exact form of $f(r)$. We only need to know that $f(r)$ has a simple zero at the horizon $r = r_H$, and that the temperature is fixed by the regularity of the Wick-rotated metric at the horizon,

$$f(r) \sim c_0 \left( 1 - \frac{r}{r_H} \right), \quad T = c_0 \frac{r_H^{-z}}{4\pi}, \hspace{1cm} (10)$$

where $c_0$ is a dimensionless constant that depends on the specific system.

The chemical potential at finite (small) temperature is

$$\mu = \int_0^{r_H} dr A'_t = \mu_0 - \frac{r_H^{-z}}{2\pi a' z} \left( 1 - \frac{z}{2(2q + z)} \frac{r_H^{-2q}}{d^2} + O \left( \frac{r_H^{-3q}}{d^4} \right) \right), \hspace{1cm} (11)$$

The finite-temperature free energy is

$$\Omega = -S = -\tilde{N} \frac{z d}{z + q} \mu - \tilde{N} \frac{4\pi d T}{c_0(z + q)} + O \left( \frac{T^{2q/z+1}}{d} \right), \hspace{1cm} (12)$$

From here we can see that the entropy density at zero temperature is nonzero and proportional to the charge density [4,14,16],

$$s_0 = -\frac{\partial \Omega}{\partial T} \bigg|_{\mu,T=0} = \frac{4\pi N}{c_0(z + q)} d. \hspace{1cm} (13)$$

Indeed, in a string theory system, this entropy should be equivalent to the entropy of a single quark, represented in the bulk by a single string, times the density $d$ of quarks [14]. The specific heat at finite temperature scales with $T$ as

$$c_V = T \frac{\partial s}{\partial T} \sim T^{\frac{2q}{d}}. \hspace{1cm} (14)$$

A linear scaling, as in a Fermi liquid, occurs when $z = 2q$, while a scaling $T^q$, as in a quantum Bose liquid, occurs when $z = 2$ [16]. Notice that both the entropy and the scaling of the specific heat implies that generically the thermodynamic behavior of these models differ from that of real strange metals.

4. Transport

Zero sound should appear at zero temperature as a pole in the Fourier-transformed retarded two-point function of the density operator, $G^t_R(\omega, k)$ [4]. In what follows we will also compute the density-current and current-current retarded two-point functions $G^{tx}_R(\omega, k)$ and $G^{xx}_R(\omega, k)$. The latter will give us the AC conductivity.

In gauge-gravity duality the on-shell action is the field theory generating functional [2, 3], hence we need to take two functional derivatives of the on-shell bulk action, which in turn means we need to solve the linearized equations of motion for the fields dual to the density and current operators, namely the $U(1)$ gauge fields $A_t$ and $A_x$ on the probe D-branes. The holographic zero sound pole will appear as a quasi-normal frequency of these fields. We thus
consider fluctuations $a_{\mu}$ of the gauge fields, where because the field theory is invariant under rotations, we only need to consider fluctuations $a_{t}$ and $a_{x}$ with $r$, $t$, and $x$ dependence. We work in a gauge where $a_{t} = 0$. We also expand in a plane wave basis, $a_{\mu} \sim e^{-i\omega t + ikx}$. Finally, we will work with the gauge-invariant electric field $E(r, \omega, k) = \omega a_{x} + ku_{t}$.

Introducing a cutoff at $r = \epsilon$ and integrating by parts, we obtain in the $\epsilon \rightarrow 0$ limit the on-shell quadratic action in terms of $E$

$$S_{a_{2}} = -\frac{N}{2} \int \frac{d\omega dk}{k^{2}} \frac{\epsilon^{z-q+1}}{k^{2}} \tilde{E}(\epsilon) \tilde{E}'(\epsilon).$$  \hspace{1cm} (15)

We now need to solve the equation of motion with an ingoing boundary condition imposed at the $r \rightarrow \infty$ “horizon” [19–21], insert the solution into $S_{a_{2}}$, and then functionally differentiate to obtain the retarded correlators, for example

$$G_{R}^{tt}(\omega, k) = \frac{\delta^{2}}{\delta a_{2}(\epsilon)^{2}} S_{a_{2}} \equiv \left( \frac{\delta E(\epsilon)}{\delta a_{2}(\epsilon)} \right)^{2} \frac{\delta^{2}}{\delta E(\epsilon)^{2}} S_{a_{2}}.$$  \hspace{1cm} (16)

Defining

$$\Pi(\omega, k) \equiv \frac{\delta^{2}}{\delta E(\epsilon)^{2}} S_{a_{2}} ,$$  \hspace{1cm} (17)

we get

$$G_{R}^{tt}(\omega, k) = k^{2} \Pi(\omega, k), \quad G_{R}^{dx}(\omega, k) = \omega k \Pi(\omega, k), \quad G_{R}^{xx}(\omega, k) = \omega^{2} \Pi(\omega, k).$$  \hspace{1cm} (18)

Our goal is to obtain (the low-frequency and low-momentum form of) the variational derivative $\Pi(\omega, k)$, which determines the retarded two-point functions that we want.

We can obtain the low-frequency behavior of $\Pi(\omega, k)$ by solving the equations of motion in two different limits and then matching the two solutions in a regime where the limits overlap, following ref. [4]. Specifically, we first obtain a solution by taking $r$ large and then taking the small frequency and momentum limit, which means $\omega r^{z} \ll 1$ and $kr \ll 1$ with $\omega k^{-z}$ fixed. We then repeat the process, now taking the small frequency and momentum limit first followed by taking $r$ large. The details are explained in ref. [7].

The boundary behavior of the solutions is given by the small $r$ expansion of the solutions: $E \approx C_{1} + k^{2}C_{2} \xi^{q-z}/(q - z)$. When $z < q$, we have that to leading order $E(\epsilon) \simeq C_{1}$, while to leading order $E'(\epsilon) \simeq k^{2}e^{q-z}C_{2}$, so

$$S_{a_{2}} = -\frac{N}{2} \int d\omega dk \frac{\epsilon^{z-q+1}}{k^{2}} \tilde{E}(\epsilon) \tilde{E}'(\epsilon) = -\frac{N}{2} (2\pi \alpha')^{2} \int d\omega dk C_{1} C_{2}.$$  \hspace{1cm} (19)

To find $\Pi(\omega, k)$, we need to find $C_{2}$ in terms of $C_{1}$, which can be done by matching the solutions that we find in the two asymptotic regimes. After that we immediately obtain $\Pi(\omega, k)$.

Assembling all the ingredients, we have our main result, the low-frequency and low-momentum form of $\Pi(\omega, k)$:

$$\Pi(\omega, k) \propto \frac{N}{\alpha_{1} k^{2} - \alpha_{2} \omega^{2} - \alpha_{3} G_{0}(\omega)},$$  \hspace{1cm} (20)

where

$$G_{0}(\omega) = \begin{cases} \frac{\omega^{1+2/z}}{\omega^{2} \log(\alpha \omega^{2})} & \text{if } z \neq 2, \\ \frac{\omega^{2}}{2} & \text{if } z = 2, \end{cases}$$  \hspace{1cm} (21)

and $\alpha_{1}$, $\alpha_{2}$ and $\alpha_{3}$ are constants. For $z = 2$ another (dimensionful) constant appears inside the logarithm in $G_{0}(\omega)$, $\alpha = \frac{\omega^{2}}{2} (2d)^{-d/q}$. In the denominator of $\Pi(\omega, k)$, when $z < 2$ the $\omega^{2}$ term is larger than the $G_{0}(\omega)$ term, whereas when $z > 2$ the $G_{0}(\omega)$ term is larger. The dominant term dictates the low-frequency behavior of the retarded Green’s functions, and hence of the holographic zero sound and AC conductivity, as we will show in detail in what follows.
4.1. Zero sound

The denominator of $\Pi(\omega, k)$ vanishes – and hence the retarded Green’s functions have a pole – whenever

$$k(\omega) = \pm \frac{1}{\sqrt{\alpha_1}} \sqrt{\alpha_2 \omega^2 + \alpha_3 G_0(\omega)}. \quad (22)$$

This determines the dispersion relation of the holographic zero sound mode: we need only expand the right-hand side of eq. (22) and then invert to find $\omega(k)$. As mentioned above, the value of $z$ determines whether the $\omega^2$ term or $G_0(\omega)$ term dominates at low frequencies, so we will consider two cases in turn.

4.1.1. $1 \leq z < 2$ For $1 \leq z < 2$, the $G_0(\omega)$ term under the square root has larger norm, so we can expand $k(\omega)$ and then invert to find

$$\omega(k) = \pm k \sqrt{\frac{\alpha_1}{\alpha_2} - \frac{\alpha_3}{2\alpha_2} \left( \frac{\alpha_1}{\alpha_2} \right)^{1/z} k^{2/z} + O\left(k^{-1+4/z}\right)}. \quad (23)$$

The mode behaves as a quasiparticle since the imaginary part goes as $k^{2/z}$, which is smaller than the real part at low momentum. As a check, if we set $z = 1$ we recover the result of ref. [4],

$$\omega(k) = \pm k - i \frac{d^{-1/2} \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2} - \frac{1}{2q}\right) \Gamma\left(\frac{1}{2q}\right)} k^2 + O\left(k^3\right). \quad (24)$$

The speed of the holographic zero sound, $v_0$, is given by

$$v_0^2 = \frac{\alpha_1}{\alpha_2} = \frac{z}{q} d^{2(z-1)/q} \frac{\Gamma\left(\frac{z}{2q}\right) \Gamma\left(\frac{1}{2} - \frac{z}{2q}\right)}{\Gamma\left(\frac{1}{2} + \frac{z-2}{2q}\right) \Gamma\left(\frac{2-z}{2q}\right)}. \quad (25)$$

When $z > 1$ the speed of the holographic zero sound is dimensionful. In the relativistic case, $z = 1$, the speed of holographic zero sound coincides with the speed of normal/first sound.

4.1.2. $z \geq 2$ We consider first $z > 2$. Now the first term under the square root in eq. (22) has larger norm, so after inverting we find

$$\omega(k) = \left( \frac{\alpha_1}{i\alpha_3} \right) \frac{z}{\alpha_2} k^{\frac{2z}{z+2}} + \frac{\alpha_2}{\alpha_3} \frac{z}{z+2} \left( \frac{\alpha_1}{\alpha_3} \right)^{2(z-1)/z+2} k^{\frac{4(z-1)}{z+2}} + O\left(k^{\frac{2(3z-4)}{z+2}}\right). \quad (26)$$

Note that the leading term has a coefficient that is complex. In particular, the real and imaginary parts are of the same order, hence the excitation is not a quasi-particle.

Finally, when $z = 2$ we have

$$k = \pm \frac{\omega}{\sqrt{\alpha_1}} \sqrt{\alpha_2 + \alpha_3 \log (\alpha \omega^2)}. \quad (27)$$

Although this mode is gapless, the dispersion relation differs from the holographic zero sound mode by logarithmic factors.
4.2. AC Conductivity

In this section we provide some physical intuition for the change in behavior of the holographic zero sound mode as a function of \( z \) by studying the low-frequency AC conductivity \( \sigma(\omega) \), defined as

\[
\sigma(\omega) \equiv -\frac{i}{\omega} G_{xx}^{R}(\omega, k = 0) . \tag{28}
\]

Recalling that the imaginary part of the retarded Green’s function is proportional to the spectral function of \( J^x \), we see that the real part of the conductivity provides a measure of the density of states that couple to \( J^x \).

To obtain \( \sigma(\omega) \) for our system, we first set \( k = 0 \) in our result for \( \Pi(\omega, k) \) in eq. (20) and then plug into eq. (18). After this we obtain the AC conductivity at small frequency from eq. (28)

\[
\sigma(\omega) = -\frac{i}{\omega} G_{xx}^{R}(\omega, k = 0) \begin{cases} 
\omega^{-1} \alpha_{2}^{-1} & z < 2 \\
\omega^{-2} / z \alpha_{3}^{-1} (\omega \log(\alpha \omega^2))^{-1} & z = 2 \\
\omega^{-2} / z & z > 2 
\end{cases} . \tag{29}
\]

Our result for \( \sigma(\omega) \) not only generalizes that of ref. [8] to any number of spatial dimensions \( q \), but is an independent confirmation of the result, since we derived it via a slightly different route from that of ref. [8].

When \( z \neq 2 \), \( \alpha_{2} \) is a real number while \( \alpha_{3} \) is complex. When \( z < 2 \), the conductivity is purely imaginary and has a simple pole at zero frequency. A Kramers-Kronig relation then implies that the real part of the conductivity, and hence the spectral function, consists only of a delta function at zero frequency. The system has no charged states at small frequencies. Notice that the \( i\omega^{-1} \) behavior is expected in a system with free charge carriers (or with translation invariance), and indeed is formally identical to the high-frequency or “collisionless” limit of the standard Drude result. When \( z > 2 \) the conductivity, and hence the spectral function, has a power-law dependence, and no clean holographic zero sound quasi-particle exists.

Something analogous happens in interacting systems with a Fermi surface. Here the charge density spectral function includes a continuum of particle-hole pairs at frequencies \( \omega < v_F k \), where \( v_F \) is the Fermi velocity. The zero sound gives a contribution at \( \omega = v_0 k \). If \( v_0 > v_F \), then the zero sound is outside the continuum, and will produce a sharp quasi-particle peak in the spectral function. However, if \( v_0 < v_F \), the zero sound mode suffers Landau damping through scattering with particle-hole pairs, and hence will be reduced to a broad resonance with no good quasi-particle interpretation. A similar mechanism appears to be at work in the holographic models we have studied: whenever the spectral function is non-vanishing at small frequencies, the holographic zero sound mode becomes a broad resonance.

5. Conclusions

We determined the low-frequency and low-momentum form of the density and current retarded Green’s functions for massless charge carriers in the holographic model of strange metals proposed in ref. [8]. Our main physical results are that for \( z < 2 \) the system’s spectrum includes a quasi-particle – the holographic zero sound – whose dispersion relation we computed, whereas when \( z > 2 \) the quasi-particle is “washed out” by a continuum of low-frequency states. In addition, we recovered the results of ref. [8] for the AC conductivity from this single contribution to the spectral function.

Many avenues remain open for future research. As emphasized in ref. [8], a realistic analysis requires massive charge carriers, with a mass gap much larger than the temperature. In the bulk that means introducing a scalar field dual to the flavor mass operator on the probe D-brane worldvolume.
In relativistic string theory settings, the holographic zero sound quasi-particle is absent at any finite temperature, apparently as a consequence of the probe limit [6]. In these systems, a finite temperature produces an energy density of order $N^2$, which is enormous compared to the probe flavor’s order $N_f N$ energy density. The low-frequency dynamics is thus dominated by adjoint-sector physics, which induces charge diffusion. The spectral function grows linearly with frequency in this case, so the finite-temperature width of the holographic zero sound is big and the pole is smeared into a broad resonance. Presumably the same is true for any $z < 2$. A natural question is what happens if we leave the probe limit. Does the holographic zero sound remain a sharp quasi-particle when $N_f$ is on the order of $N$?

Lastly, the model building proposed in ref. [8] is worth pursuing. As noted there, some studies of the high-$T_c$ cuprate strange metals find that the low-frequency AC conductivity scales as $\omega^{-2/3}$, which suggests that any holographic model reproducing such scaling may not exhibit a zero sound pole. A holographic system that reproduces all measured strange metal properties will be more complicated than just probe D-branes in a Lifshitz geometry however, so the question of whether a holographic zero sound quasi-particle exists in these more complicated models should be studied on a case-by-case basis.

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References
[1] Maldacena J M 1998 Adv. Theor. Math. Phys. 2 231–252 (Preprint hep-th/9711200)
[2] Gubser S S, Klebanov I R and Polyakov A M 1998 Phys. Lett. B428 105–114 (Preprint hep-th/9802109)
[3] Witten E 1998 Adv. Theor. Math. Phys. 2 253–291 (Preprint hep-th/9802150)
[4] Karch A, Son D T and Starinets A O 2008 (Preprint 0806.3796)
[5] Kulaxizi M and Parnachev A 2008 Phys. Rev. D78 086004 (Preprint 0808.3953)
[6] Kim K Y and Zahed I 2008 JHEP 12 075 (Preprint 0811.0184)
[7] Hoyos-Badajoz C, O’Bannon A and Wu J M S 2010 JHEP 09 086 (Preprint 1007.0590)
[8] Hartnoll S A, Polchinski J, Silverstein E and Tong D 2009 (Preprint 0912.1061)
[9] Kachru S, Liu X and Mulligan M 2008 Phys. Rev. D78 106005 (Preprint 0808.1725)
[10] Kobayashi S, Mateos D, Matsuura S, Myers R C and Thomson R M 2007 JHEP 02 016 (Preprint hep-th/0611099)
[11] Karch A and O’Bannon A 2007 JHEP 09 024 (Preprint 0705.3870)
[12] Karch A and O’Bannon A 2007 JHEP 11 074 (Preprint 0709.0570)
[13] Karch A and Katz E 2002 JHEP 06 043 (Preprint hep-th/0205236)
[14] Karch A, Kulaxizi M and Parnachev A 2009 JHEP 11 017 (Preprint 0908.3493)
[15] Benincasa P 2009 (Preprint 0911.0075)
[16] Lee B H and Pang D W 2010 (Preprint 1006.4915)
[17] Witten E 2001 (Preprint hep-th/0112258)
[18] Witten E 1998 Adv. Theor. Math. Phys. 2 505–532 (Preprint hep-th/9803131)
[19] Son D T and Starinets A O 2002 JHEP 09 042 (Preprint hep-th/0205051)
[20] Herzog C P and Son D T 2003 JHEP 03 046 (Preprint hep-th/0212072)
[21] van Rees B C 2009 Nucl. Phys. Proc. Suppl. 192-193 193–196 (Preprint 0902.4010)