SUPERCONDUCTING PHASE TRANSITIONS
IN 2+1 DIMENSIONAL QUANTUM FIELD THEORIES MODELING
GENERALIZED POLARONIC INTERACTIONS

I: JAHN-TELLER INSPIRED MODELS

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ABSTRACT

We review the fundamentals of Jahn-Teller interactions and their field theoretical modelings and show that a 2+1 dimensional gauge theory where the gauge field couples to “flavored fermions” arises in a natural way from a two-band model describing the dynamical Jahn-Teller effect. The theory exhibits a second order phase transition to novel finite-temperature superconductivity.

1. Introduction

The discovery of the cuprate high-$T_C$ superconductors \cite{1} together with the fact that there is to date no generally accepted theory for the relevant mechanism \cite{2}, gives physicists reasons to search for novel scenarios responsible for macroscopic quantum coherence phenomena in condensed matter physics.

One of the main problems is the dependence of the nature of superconducting phase transitions on dimensionality. Of course, one has to be careful to make statements like “high-$T_C$ superconductivity is essentially a two-dimensional problem” since it is not a priori clear how the very nature of the superconducting phase transition in the cuprates is related to or affected by some kind of interplane coupling. Nevertheless, there is no doubt that a general (i.e. quantum field theoretical) study of the possibility of idealized two-dimensional superconductivity at finite temperature is a challenging task. There is an old celebrated theorem due to Hohenberg, Mermin and Wagner, and Coleman \cite{3} stating that conventional off-diagonal long range order (ODLRO) is suppressed at any finite temperature in 2+1 dimensional quantum systems. At this point the fundamental question of the very existence of loopholes arises - as often in theoretical physics.

\footnote{1Presented at Workshop at Differential Geometry and Quantum Physics, University of Leipzig, 28.09.92 - 02.10.92, Leipzig, Germany.}

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In the following we present a proposal for a *microscopic* mechanism which may provide such a loophole and possibly explain high-$T_C$ superconductivity or may open the door to even more interesting superconducting materials. It is based on an effective electron-electron (resp. hole-hole) interaction induced by a generalized dynamical Jahn-Teller effect [4]. Akin to fractional statistics this interaction is a *phasing interaction*, i.e. one excitation *modulates* the quantum mechanical phase of the other excitation giving rise to a net attractive interaction for the relevant fermions. Alternatively, a phasing interaction may be viewed as a *renormalization* of the statistical properties of the quanta changing their statistical identity. It is our objective to make plausible that it is the phasing aspect overlooked in more conventional treatments of the Jahn-Teller interaction which may play a crucial role in the mechanism of high-$T_C$ superconductivity.

Our discussion is organized as follows: We start by recalling the early history of the Jahn-Teller theory in molecular physics. We proceed by reviewing the concept of the geometrical phase (now called Berry phase and Aharonov-Anandan phase in the adiabatic and in the non-adiabatic cases, respectively) [5] first analyzed systematically in the context of quantum chemistry in the pioneering work of Mead who dubbed this phenomenon *molecular Aharonov-Bohm effect* [6]. Then we show how to set up a theory of strongly correlated electrons interacting via a generalized dynamical Jahn-Teller interaction.

Motivated by the work of Yu and Anderson [7] we show that our ansatz can be expressed in terms of a double-well tunneling event which we christened *solid state instanton*. It is related to what is known as a *polaron* in solid state physics, but it is not exactly the same thing. The microscopic double-well mechanism allows to break parity and time-reversal *spontaneously* thus giving a *microscopic* reason for the existence of anyons without being restricted to them. This double-well idea is intimately related to T. D. Lee’s early 70’s work on CP violation in elementary particle physics [8]. It is very amusing to observe that already T. D. Lee relates his own concept to geometrical ideas associated with abstract “distortive” deformations. Notice that the actual breaking and its magnitude of these discrete symmetries depend on a fine tuning of the coupling parameters. The effective Lagrangean describing such a polaronic interaction is by no means unique and adding or discarding terms determines whether P and T are broken or not. Moreover, there may exist the possibility of a phase transition between an anyonic and a Berezinskii-Kosterlitz-Thouless-type phase *within* the high-temperature conducting phase.

Finally we show how to model the generalized dynamical Jahn-Teller interaction field theoretically and give an explicit example for a quasi two-dimensional finite temperature ansatz - firstly written down and analyzed by Joe Kapusta and his collaborators (without any reference to the generalized dynamical Jahn-Teller mechanism, however). This model indeed shows up a finite temperature phase transition to novel superconductivity within a finite window of parameters [10]. Its phase structure is very reminiscent of the phase diagram proposed by Chakraverty in 1979 [11] which led Bednorz and Müller [1] to their discovery of novel superconductivity. This model should be seen in a wider context including two-dimensional
scenarios without parity and time-reversal violation. Finally, we give a heuristical argument for macroscopic quantum coherence induced by Jahn-Teller systems.

The field theoretical ideas presented here are based on joint work with Heinz [12].

2. What is the Jahn-Teller effect?

As early as 1929 v. Neumann and Wigner [13] asked a very interesting question: Given a hermitean $n \times n$ matrix whose entries are depending on a sequence of parameters $\kappa_1, \kappa_2, \ldots$ - how many parameters have to be changed in order to get a collision of two eigenvalues of the matrix. The general answer is at least three.

Let us make this explicit for the most simplest case $n = 2$. Any hermitean $2 \times 2$ matrix $A$ may be expanded with respect to the Pauli basis:

$$A = \alpha^\mu \sigma_\mu = \alpha^0 \mathbf{1} + \alpha^1 \sigma_1 + \alpha^2 \sigma_2 + \alpha^3 \sigma_3$$

$$= \begin{pmatrix} \alpha^0 + \alpha^3 & \alpha^1 + i\alpha^2 \\ \alpha^1 - i\alpha^2 & \alpha^0 - \alpha^3 \end{pmatrix}.$$  

Solving the eigenvalue equation $\det(A - \lambda \mathbf{1}) = 0$ gives

$$\lambda = \alpha^0 \pm \sqrt{(\alpha^1)^2 + (\alpha^2)^2 + (\alpha^3)^2}.  \tag{3}$$

Clearly we have to send all three “space-like” $\alpha$’s to zero in order to get a collision!

Evidently, in case of a real hermitean (i.e. symmetric) matrix this reduces to two parameters, in case of a real diagonal matrix to one parameter.

Interpreting the space of independent parameters as a mechanical configuration space and cutting out the point of conicidence, i.e. the level crossing point, we obtain manifolds with non-trivial topological structures. This suggests that in all three cases, once physically realized, we may expect topological quantization effects giving rise to very interesting physics.

To get real physics from this mathematical observations we must find a Hamiltonian having the structure of $A$ and a mechanism which prevents the level crossing. This was done in the pioneering work of Jahn and Teller in 1937 who studied the stability of polyatomic molecules in degenerate electronic states [14].

Essentially the famous Jahn-Teller theorem states the following: A configuration of a polyatomic molecule for an electronic state having orbital degeneracy cannot be stable with respect to all displacements of the nuclei unless the nuclei all lie on a straight line. The original proof of Jahn and Teller [14] is based on a detailed discussion of particular symmetries and their realizations. A more general proof within the framework of induced representations of finite groups was given by Ruch and Schönhofer in 1965 [15]. It is interesting to note that - by the side of the Weiss theory of ferromagnetism - the Jahn-Teller effect is the archetype of what is commonly called spontaneously symmetry breaking (SSB), a fact recently recalled by Nambu [16].
In modern terminology a dynamics determined by a sensible matching of vibration modes and electronic excitations is called a vibronic interaction [4]. There is no doubt that a phonon induced fermion-fermion interaction incorporating Jahn-Teller-type effects may exhibit new peculiarities beyond the marks of the conventional electron-phonon interaction. Due to the non-trivial topological structure of the configuration spaces involved, we may expect highly non-trivial quantization and coherence phenomena, and it is the aim of this discussion to push forward the thesis that the understanding of high-\(T_C\) superconductivity is at least related to the generalized dynamics of vibronic interactions.

3. The geometric phase in Jahn-Teller systems

To sum up, the (static) Jahn-Teller effect is an electronic symmetry breaking phenomenon associated with a spontaneous distortion. To be concrete, let us consider a vibronic interaction of a doubly degenerate electronic state \(E\) with a doubly degenerate vibrational mode \(e\) [17].

According to the Jahn-Teller theorem the nuclear motion lifts the electronic degeneracy, i.e. there are nuclear configurations of lower energy than the symmetric state. This \(E \otimes e\) Jahn-Teller effect is the archetype of Berry’s phase: A quantum-mechanical phase shift of purely geometrical origin associated to an adiabatic cycle starting and ending at the same pure state [5]. In our case we start by writing down the quantum-mechanical two-dimensional harmonic oscillator Hamiltonian for a double degenerated vibrational mode

\[
H = \frac{P^2}{2M} + \frac{1}{2}M\Omega^2 Q^2, \tag{4}
\]

with \(P = (P_x, P_y, 0)\) and \(Q = (Q_x, Q_y, 0)\). This Hamiltonian is thought to be acting on two-component (Pauli) wave functions \(\psi_i\) (i=1,2). The vibronic coupling is given by adding a term \(k \cdot \tau Q\) to \(H\), whereby \(\tau_i\) denote the Pauli matrices and \(k\) is a coupling constant. (We use the letter \(\tau\) to avoid confusion with spin degrees of freedom). In analogy to elementary particle physics we call the associated internal quantum number of the electron a Jahn-Teller isospin or flavor.

Explicitly we have

\[
H = \left( \frac{P^2}{2M} + \frac{1}{2}M\Omega^2 Q^2 \right) \cdot 1 + k \cdot \begin{pmatrix} 0 & Q_x - iQ_y \\ Q_x + iQ_y & 0 \end{pmatrix} \tag{5}
\]

\[
= \left( \frac{P^2}{2M} + \frac{1}{2}M\Omega^2 Q^2 \right) \cdot 1 + k \cdot \begin{pmatrix} 0 & Q e^{i\varphi} \\ Q e^{-i\varphi} & 0 \end{pmatrix} \tag{6}
\]

with \(Q_x + iQ_y = Q e^{i\varphi}\) and \(Q_z = 0\). Let us rewrite the Hamiltonian in cylinder coordinates:

\[
H = \left( \frac{P_Q^2}{2M} + \frac{P_{Q\varphi}^2}{2MQ^2} + \frac{1}{2}M\Omega^2 Q^2 \right) \cdot 1 + k \cdot \begin{pmatrix} 0 & Q e^{-i\varphi} \\ Q e^{i\varphi} & 0 \end{pmatrix}. \tag{7}
\]
In the adiabatic or Born-Oppenheimer approximation we neglect the kinetic energy term and diagonalize the remainder. We obtain

\[ H_{BO} = \frac{1}{2} M \Omega^2 Q^2 \cdot 1 + k \cdot \begin{pmatrix} Q & 0 \\ 0 & -Q \end{pmatrix} \]  

(8)

with

\[ \begin{pmatrix} Q & 0 \\ 0 & -Q \end{pmatrix} = U \begin{pmatrix} 0 & Q e^{-i\varphi} \\ Q e^{i\varphi} & 0 \end{pmatrix} U^\dagger. \]  

(9)

Note that the matrix

\[ \begin{pmatrix} e^{i\varphi/2} & e^{-i\varphi/2} \\ \overline{e^{-i\varphi/2}} & \overline{e^{i\varphi/2}} \end{pmatrix} \]  

(10)

is a double-valued function in the polar angle \( \varphi \). The associated energy eigenvalues are

\[ E_{\pm}(Q, \varphi) = \frac{1}{2} M \Omega^2 Q^2 \pm |Q|, \]  

(11)

(where we have written \(|Q|\) for \(Q\)) and correspond to two sheets (the upper cone-like, the lower sombrero-like) coinciding at a point of degeneracy at the origin. This point of zero distortion defines a conical intersection.

Because of the double-valued character of the diagonalizing similarity transformation the eigenstates \( \eta_{\pm} \) corresponding to \( E_{\pm}(Q, \varphi) \) are double-valued in \( \varphi \). The multiple-valuedness may be compensated for by an appropriate local gauge transformation

\[ \eta_{\pm} \rightarrow \exp(i\varphi/2) \eta_{\pm}, \]  

(12)

which in turn induces the change

\[ \nabla \rightarrow \nabla - i A, \]  

(13)

with

\[ A = -e\varphi/2Q \]  

(14)

in the nuclear energy operator. The vector potential \( A \) corresponds to a fictitious flux tube with strength 1/2 confined to the origin. In order to visualize the circuit in \( \varphi \) inducing the quasi-spinorial sign change of the electronic states one takes a look on a typical example for the \( E \otimes e \) Jahn-Teller effect: the trimer. A circuit of distortions - corresponding to the natural motions of the nuclei - avoids the point of symmetry, at which the trimer looks like an equilateral triangle.

Of course, while it is true, that we generally assume that the complete system must be described by a single-valued wave function, it is the splitting between the subsystem and its relative environment which introduces the geometrical phase.
factor: Hence the multiple-valuedness of the electronic wavefunction is compensated for by the multiple-valuedness of the nuclear wavefunctions.

4. Beyond the adiabatic approximation

In the last section we observed that in a Jahn-Teller system (hereafter designated by JT) the electronic wave functions are in general multiple-valued functions in the slow nuclear coordinates; in particular they are double-valued in the nuclear polar angle in the case of a trimer. Our claim is that this could give rise to a novel quality of an electron-electron interaction mediated by an oscillating JT ionic configuration. In the following - generalizing this classical dynamical yet adiabatic JT approach somewhat - we will go beyond the approximation of Born and Oppenheimer and write down an ansatz for an effective field theory describing a dynamical non-adiabatic JT interaction.

One reason for this proceeding lies in the fact that e.g. for the case of the octahedron in the La$_2$CuO$_4$ superconductor we encounter neither an appropriate degeneracy nor an appropriate configuration space of distortions justifying the applicability of the “naive” JT theorem and the Mead model described above [18]. Conversely, what we expect is that the point of degeneracy in the relevant JT-like system is smeared-out (just like a smeared-out Aharonov-Bohm flux line or a regularized anyon), such that the adiabatic transformation definitely breaks down and, in addition, the oscillating ionic arrangement does not simply sweep through all configurations classically possible, but tunnels between some of them instead. We propose a scenario in which both delicate aspects appear as natural consequences of the same fundamental mechanism, but nevertheless, the topological quality of the interaction, reminiscent of Meads molecular Aharonov-Bohm effect [3] will survive.

To convert the nuclear Hamiltonian

$$H = \frac{P^2}{2M} + \frac{1}{2}M\Omega^2Q^2 + k \cdot \tau Q$$

(15)

into a field theoretical Hamiltonian describing an electron-electron interaction mediated by an oscillating ionic configuration we simply make the replacement

$$k \cdot \tau Q \rightarrow - \sum_{kk'} (c_{k1}^\dagger, c_{k2}^\dagger) (k \cdot \tau Q) \begin{pmatrix} c_{k1} \\ c_{k2} \end{pmatrix},$$

(16)

and introduce phonon field operators such that we get a two-band field theory with electron-phonon vertices. A more general Hamiltonian may be obtained by introducing weights for the different modes and adding a conventional term. Integrating out phonons in a standard way we obtain a four-fermion BCS-like Hamiltonian representing interband-intraband interactions with some “wrong-sign” couplings.

Englman, Halperin, and Weger proposed a JT theory for the high-$T_C$ superconductivity of the cuprates, in which the coupling between the copper $d_{\theta}$ and $d'$ states leads to a pairing mechanism “of the same form, but opposite sign” to
that of the BCS theory [19]. They argued that their own ansatz is well-supported by band structure calculations and experiments indicating the involvement of both \( d_\theta(z^2) \) and \( d_\epsilon(x^2 - y^2) \) type states in the carrier states of cuprate superconductors. Furthermore they showed that the proposed pairing mechanism is stable against lattice distortion even for strong coupling. It is noteworthy to remark that there are a number of proposals how the JT effect comes into play in superconductors, in particular in high-\( T_C \) superconductors. Some classical papers can be found in Ref. [20] and more current contributions are listed in Refs. [21]. Especially interesting is the recent work of K. H. Johnson et al. [22] who argued that the observed superconductivity at 18 K in potassium-doped fullerene is induced by a cooperative JT coupling leading to a BCS-like mechanism. Topological aspects relating the JT phenomenon and superconductivity have been almost ignored up until now. The only paper, to our knowledge, relating topological quantization effects (especially fractional quantization) to the JT effect and superconductivity was written by Kuratsuji [23].

Appel pointed out to me that if high-\( T_C \) superconductivity is due to a JT-like scenario then the description of the relevant mechanism surely has to go far beyond the adiabatic approximation [24]. In particular he was inspired by the work of Cohen and collaborators [25] who emphasized that - due to the fundamental instability of the oxygen ion which causes its motions influencing the charge density between the copper and oxygen - anharmonic double-well potentials for normal modes may give larger coupling then expected from harmonic phonons and are less sensitive to the mass. Appel argued that the local phonon ansatz by Yu and Anderson [7] - considered as the non-adiabatic extension of the dynamical JT effect - provides a suitable framework to describe the fundamental interaction. Note that we are not interested in the exact details of the interaction (e.g. apex in-plane charge interaction, apex positional splitting, out-of-plane motions etc.); we only assume that the essential dynamics is governed by an inharmonic potential.

In a Yu-Anderson-type model we restrict ourselves to a one mode description replacing the “double sheeted sombrero” by a “double sheeted double well”. Explicitly we have

\[
H_{el-ph} = - \sum_{k,k+q} \left( c_{k+q,s}^\dagger, c_{k+q,p}^\dagger \right) \begin{pmatrix} 0 & \lambda Q \\ \lambda Q & 0 \end{pmatrix} \begin{pmatrix} c_{ks} \\ c_{kp} \end{pmatrix},
\]

To diagonalize the phonon matrix we introduce the chiral - i.e. left-handed and right-handed - linear combinations

\[
c_{kL} = \frac{1}{\sqrt{2}}(c_{ks} - c_{kp}), \quad (18)
\]

\[
c_{kR} = \frac{1}{\sqrt{2}}(c_{ks} + c_{kp}), \quad (19)
\]
and get

\[ H_{el-ph} = - \sum_{k,k+q} (c_{k+qL}^\dagger c_{k+qR}^\dagger) \begin{pmatrix} \lambda Q & 0 \\ 0 & -\lambda Q \end{pmatrix} \begin{pmatrix} c_{kL} \\ c_{kR} \end{pmatrix}, \]  

(20)

i.e. an interaction term proportional to \( \tau_3 \). Yu and Anderson proceed by “integrating out” the electron degrees of freedom and calculate the dynamical modification of the harmonic oscillator potential giving a “dynamical double well” \( \propto (Q^2 - h)^2 \) replacing the more singular \( Q^2 - |Q| \) term. The calculation is very involved and relies heavily of path integral techniques reminiscent of instanton calculations in quantum field theory.

An effective non-relativistic Lagrangean for this model may have the form

\[ \mathcal{L} = \left( \frac{1}{c^2} \partial_{\mu}\varphi \partial^{\mu}\varphi - \partial_k \varphi \partial_k \varphi \right) - V(\varphi) + \psi^\dagger i \partial_{\mu}\psi - \frac{1}{2m} |\partial_k \psi|^2 - \frac{1}{2m} \lambda \varphi (\psi^\dagger \tau_3 \psi), \]  

(21)

where \( V(\varphi) \) is a quartic term and \( \psi \) is a two-component Schrödinger field. That anharmonicity modifies the mass-frequency relation of a quantum mechanical oscillator and hence the isotope effect is due to the non-analytic character of the solution of the double-well tunneling problem - a rather general feature. In field theory tunneling events are called instantons and it is, to our opinion, appropriate to name the Yu-Anderson local phonon a \textit{solid state instanton}. Summarizing, complementary to the BCS-like four fermion interaction à la Engelman, Halperin, and Weger [19] which is obtained by integrating out the phonons, we get an effective anharmonic phonon potential by integrating out the fermions.

A very interesting point lies in the fact that \textit{double-well system} is intimately related to a \textit{two-level system} in that the lowest states of the former are to be identified with the only states spanning the latter. Now the dynamics of the two-level system considered as an abstract spin-1/2 system is driven by an abstract external magnetic field - self-consistently generated through the local phonon tunneling dynamics. In case of a \textit{real} spin-1/2 system the driving external magnetic field introduces definitely an oddness under time reversal. We do not really know under which conditions this oddness under time reversal carries over to the abstract case, but at least as a possibility it remains. The oddness under \( T \) is also suggested by the fact that a hidden parity violation is already present in the model due to the interference of odd and even modes and due to the fact that PT should be a good symmetry in solid state physics.

Hence a relativistic ansatz for a \( T, P, \text{and } C \) invariant effective Lagrangean based on pseudoscalar anharmonic phonons may be written

\[ \mathcal{L} = \frac{1}{2} ((\partial_{\mu}\varphi)^2 - V(\varphi) + \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - m)\psi - \bar{\psi}\gamma_5\psi \cdot \varphi, \]  

(22)

with

\[ V(\varphi) = -\frac{1}{2} \lambda \varphi^2 + \frac{1}{4} \kappa \varphi^4, \]  

(23)
where we demand \( \lambda > 0, \kappa > 0 \), and the relativistic spin degrees of freedom are identified with the two bands of the generalized JT interaction in the non-relativistic limit.

5. A phase transition towards two-dimensional superconductivity

Our relativistic Lagrangean is identical to the one studied by T. D. Lee in the early 70’s as a simple example for spontaneous T violation \([8, 12]\), a phenomenon discussed in the framework of anyon physics \([20]\). Unfortunately, the experimental situation is compatible with the absence of anyons in high-\( T_C \) materials rather than with their presence \([24]\). Nevertheless, if high-\( T_C \) superconductivity is still a really two-dimensional phenomenon, then we should continue to study anyon superconductivity because it is a nice ‘toy model’ possibly exhibiting features of the true theory. Therefore our strategy is to develop a theory in which anyons appear as a consequence of a microscopic mechanism and induce finite temperature superconductivity. At the end we will try to find out how to modify the model in order to preserve P and T.

Assuming they exist, anyons are never elementary particles like electrons such that the question remains: How can we get anyons and hence spontaneously T violation from fundamental electronic interactions? We think it is near at hand and much more natural to reverse the standard argumentation and consider the violation of time reversal and not necessarily the validity of the two-dimensional description as the main problem. This view is also supported by Wilczek’s axion electrodynamics \([28]\) and the chiron model by Chaplin and Yagamishi \([29]\). Both models go beyond two dimensions while preserving the P and T violating character. Indeed, we should find the reason for anyons - departing from a microscopic picture. Our picture is that it is a background of tunneling impurities which does generate an effective four fermion interaction. In particular, we expect that it should be possible to derive anyon physics as a consequence of this ansatz casted in the form of T. D. Lee’s Lagrangean.

Let us briefly sketch the original Lee mechanism. Though the Lagrangean is invariant under time reversal and parity, the vacuum expectation value

\[
\langle \varphi \rangle_{\text{vac}} = \rho \neq 0
\]

of the \( \varphi \) field is not: It changes its sign under P and T.

In the tree graph approximation \( V'(\varphi) = 0 \) we have \( \rho^2 = \lambda / \kappa \). Quantum fluctuations

\[
\varphi = \rho + \delta \varphi
\]

yield an effective potential

\[
V = V_0 + \frac{1}{2} \mu^2 (\delta \varphi)^2 + \kappa \rho (\delta \varphi)^3 + \frac{1}{4} (\delta \varphi)^4
\]

with \( V_0 = -\kappa^{-1} \lambda^2 / 4 \) and \( \mu^2 = 2 \lambda \). The mass of the \( \delta \varphi \) quantum does not vanish: Since T is a discrete symmetry, we have no Goldstone modes here.
To conclude, we just have described an effective field theory of a generalized dynamical JT effect incorporating spontaneous T violation. The main input are the two flavors and the double-well, i.e. the breaking and restoring of a symmetry associated to a microscopic degeneracy. How can we derive an effective two-dimensional theory from this picture?

Evidently, there must exist a description of this scenario in terms of a four-fermion interaction. In the spirit of Bjorken’s work who - motivated by the BCS theory - investigated the general possibility of constructing a gauge field from fundamental fermionic interactions [30], we are looking for a relativistic ansatz incorporating a non-propagating gauge coupling similar to our local phonon. It was found by Ogievetskí and Polubarinov (OP), who have shown that it works in field theories with antisymmetric tensor gauge bosons [31]. It is amusing that the latter have been called phonon modes by Balachandran et al. in an entirely different context [32].

Opposed to the massive Maxwell-Dirac Lagrangean
\[
L = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{m^2}{2} A^2_\mu - \bar{\psi} (\gamma_\mu \partial^\mu + M) \psi + ie \bar{\psi} \gamma_\mu \partial^\mu \psi A_\mu
\] (27)
the most simple massive OP phonon quantum electrodynamics is described by
\[
L = -\frac{1}{12} (\partial_\mu A_{\nu\lambda} + \partial_\nu A_{\mu\lambda} + \partial_\lambda A_{\mu\nu})^2 + \frac{m^2}{4} A^2_{\mu\nu} - \bar{\psi} (\gamma_\mu \partial^\mu + M) \psi - \frac{1}{2} g \varepsilon_{\mu\nu\lambda\delta} \bar{\psi} \gamma^\nu \psi \partial^\mu A^{\lambda\delta}.
\] (28)

Here the massless limit \( m \rightarrow 0 \) is taken at the end of the computation. Since in this gauge interaction picture the double well is no longer present, we have to input the two flavors by hand and obtain as the final effective Lagrangean
\[
L = -\frac{1}{12} (\partial_\mu A_{\nu\lambda} + \partial_\nu A_{\mu\lambda} + \partial_\lambda A_{\mu\nu})^2 + \frac{m^2}{4} A^2_{\mu\nu} - \bar{\psi} (\gamma_\mu \partial^\mu + M) \psi - \frac{1}{2} g \varepsilon_{\mu\nu\lambda\delta} \bar{\psi} \gamma^\nu \tau_3 \psi \partial^\mu A^{\lambda\delta}.
\] (29)

And now comes the point of the story: Reducing this Lagrangean down to a non-relativistic 2+1 dimensional situation what is done by cancelling the third row and column of the antisymmetric tensor field \( A_{\mu\nu} \) and identifying the rest with the dual of a 2+1 dimensional vector potential \( a_\mu \) such that the interaction term must have the form “field strength times a current diagonal in flavor” we get a Lagrangean of the form
\[
L = \frac{1}{2} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \psi^\dagger i D_\psi - \frac{1}{2m} |D_\mu \psi|^2 - \frac{1}{2m} f_{12} (\psi^\dagger \tau_3 \psi),
\] (30)

where \( f_{12} = \partial_1 a_2 - \partial_2 a_1 \) and \( D_\mu = \partial_\mu - i a_\mu \).

Note that the coefficient of the Chern-Simons term is \textit{a priori} undetermined, it is \textit{a posteriori} fixed by the correspondence of the number of flavors and the

\[\text{In general, care must be exercised in taking } m \rightarrow 0 \text{ since the constraint structure of the underlying operator algebra changes discontinuously in the limit.}\]
Table 1: Possible realizations of symmetry (after Ref. 35)

| symmetry of Hamiltonian | neutral charged | Wigner-Weyl | Nambu-Goldstone | Berezinski-Kosterlitz-Thouless |
|------------------------|----------------|-------------|-----------------|-------------------------------|
| symmetry of ground state | $[H,Q] = 0$ | $[H,Q] = 0$ | $[H,Q] = 0$ | $[H,Q] = 0$ |
| energy gap to charged excitations | $Q|0\rangle = 0$ | $Q|0\rangle = 0$ | $Q|0\rangle \neq 0$ | $Q|0\rangle \neq 0$ |
| long wave excitations | none | none | NG bosons | BKT bosons |
| local order parameter | 0 | 0 | $\neq 0$ | 0 |
| correlator of order field as $r \to \infty$ | 0 | $e^{-rm}$ | $const$ | $r^{-\alpha}$ |

statistics parameter according to Mavromatou et al. and others [33]. The final 2+1 dimensional Lagrangean coincides with the Lagrangean of Kapusta et al. who suggested that the internal degree of freedom may be identified with the spin, though they did not forbid other interpretations [10]. From this we get a statistical magnetic field

$$b = -g \left( \langle \psi_R^\dagger \psi_R \rangle - \langle \psi_L^\dagger \psi_L \rangle \right),$$

(31)

such that a breaking of the “chiral” invariance in the generalized JT model at low temperatures gives $b$ a finite value. In a study of the finite temperature Meißner-Ochsenfeld effect Kapusta et al. arrived at a set of four coupled integro-differential equations, indicating that superconductivity terminates at $T = T_C$. With a mean field approximation and certain values for the coupling constants and effective mass Kapusta et al. arrived at reasonable $T_C$’s. Thus Kapusta and collaborators have shown that there exist 2+1 dimensional gauge theories exhibiting a true second order superconducting phase transition at finite temperature.

6. Conclusions

Let us take a look at the familiar conventional 3+1 dimensional superconducting phase transition from a quantum field theoretical point of view: The electric local gauge symmetry is “spontaneously broken” in the superconducting phase giving rise to a would-be Goldstone boson absorbed into the Meißner-Ochsenfeld effect. (We use the quotation marks indicating that from a rigorous point of view...
local gauge symmetries are never spontaneously broken according to the Elitzur-Lüscher theorem \[34\]. Complementarily, we may view the transition from the superconducting state to the normal state as a spontaneously symmetry breaking of a magnetic gauge symmetry whose generator is a magnetic charge quantum number ("vorticity") carried by fictitious infinite long fluxlines of infinite energy \[35\]. In this framework the Goldstone bosons are the photons or, physically speaking, they manifest themselves as the absence of the Meißner-Ochsenfeld effect \[35\].

Conversely, in two space dimensions conventional superconductivity does not exist at any finite temperature because fluctuations overcome energy in destroying off-diagonal long range order. This is the essential conclusion of the Hohenberg-Mermin-Wagner-Coleman theorem \[3\], and one reason to consider anyon superconductivity was the possibility of an evasion of this theorem. There might exist other ways to evade the assumptions underlying this theorem and according our table borrowed from Ref. \[35\] the Berezinskii-Kosterlitz-Thouless transition is an obvious choice \[9\]. Kovner and Rosenstein motivated this choice by the “growing body of experimental data that points to the KT nature of the superconducting phase transition in CuO materials” \[36\]. They propose a Lagrangean in which a doublet of two-component complex Dirac spinors couples to a vector field. In this theory the electric gauge symmetry is implemented in a BKT mode and the vector field reminiscent of our OP phonon degree of freedom represents the corresponding BKT boson.

Contrary to the case of thin superconducting metal films where we observe an vortex-antivortex unbinding transition \[37\] the Kovner-Rosenstein vortices are charged. This is very reminiscent of anyonic superconductivity where vortices and quasi-particles are one and the same entities. From a conservative point of view, one may argue that BKT excitations tend to suppress two-dimensional superconductivity and this suppression will be probably damped by interlayer interactions enhancing three-dimensional superconductivity. In the case of the Kovner-Rosenstein vortices the interplane coupling shifts the values obtained in a two-dimensional theory only by a certain amount \[36\] thus preserving the overall two-dimensional character. This seems to be compatible with our JT anyon ‘toy model’ \[12\].

We think there is still a lot of experimental work to be done to find out what is really happening microscopically. But there is one point which cannot be overemphasized: It is by no means sufficient to confine ourselves to a discussion of the critical behavior of the systems we are interested in. Critical phenomena encode a wide category of specifications into the same universality class (e.g. ‘3D XY’). Critical behavior is distinguished by the equal importance of all length and time scales at a certain point in the phase diagram. Thus there is no logic which allows us to deduce statements about the microscopic mechanism from critical behavior \[38\]. Even the dimensionality and the internal symmetry of a system cannot be read off from the critical behavior of a system (e.g. asymptotic freedom of two dimensional sigma models vs. asymptotic freedom four dimensional Yang-Mills theories). Expressed in different words, the question whether the superconducting phase transition in the cuprates is of a novel type cannot be answered from the
study of critical indices alone.

In conclusion, we have formulated a theoretical model, in which the fermions are interacting via a generalized JT interaction leading to a superconducting phase transition violating parity and time reversal. It is an example for a quantum mechanical distinction between left- and right-handedness different from other interesting proposals doing this [39]. Our model may be regarded as a variant of a more general BKT mechanism for two-dimensional superconductivity including scenarios without P and T violation. Moreover, a phase transition between a T preserving and T violating phase within the superconducting state seems to be possible.

7. Acknowledgements

I would like to thank J. Appel, C. Bandte, M.E. Carrington, A. Heinz, J. Kapusta, C.A. Mead, K.A. Müller, Y. Nambu, R.L. Stuller, and M. Weger for useful discussions. Thanks also to Professor K.A. Müller for drawing my attention onto Ref. [20, 38] and to Professor R. Haag for calling my attention to Ref. [15]. Financial support by IBM is also greatly appreciated.

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