Determination of correct sign in primary angle calibrations

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Abstract. This paper discusses the critical issue of correct sign determination of angle deviations during primary calibration of angle standards and practical realisation of SI angle unit radian. The cases referring to reading direction of angle measuring devices and standards are presented and correct sign determination process is illustrated for primary method, cross calibration. The information should be of use to the laboratories contemplating the establishment of primary angle measurements or high precision angle metrology applications.

1. Introduction

Precision angle measurement is an important enabling technology with a wide range of scientific and industrial applications [1] and requires traceability to SI angle unit of radian [2]. Traceability in angle measurements is different from length measurements where it is essential to have traceability to a primary standard [2]. In the case of angle measurement, no corresponding primary standard is required, since any angle can be established by appropriate subdivision of the natural and error-free standard of the full circle, 2π rad. In the case of small angles, however, it is usual, for practical reasons to determine the angle as ratio of two lengths referring to SI definition of radian. The ability to calibrate reference and working standards of angle is thus mainly dependent upon the ability to perform accurate subdivision of a full circle. National Metrology Institutes (NMIs) achieve primary angle measurements using high precision angle standards with fundamental method - subdivision of a full circle. One of the most practical application of subdivision of a full circle is cross calibration [3, 4]. During application of cross calibration, determination of correct sign of the measurement results is a critical issue since the sign will be affected by the equipment such as the direction of reading of the autocollimator, the direction of labelling of the polygon faces and the direction of the scale of the rotary/indexing table. The effect of all such factors must be checked in every calibration since reading direction of these devices vary depending on the manufacturer. This process is indeed confusing and results in wrong sign estimation of calibration results. Although there is enough references [3-6] for cross calibration, no detailed information was given for correct determination of sign in this process.

This paper discusses the issue of correct sign determination of angle deviations in cross calibration method during primary calibration of angle standards (e.g. polygons, rotary or indexing tables) and for realisation of SI angle unit radian utilising fundamental method - subdivisional of a full circle.

2. Primary angle measurements and fundamental method - subdivision of a full circle

Angle standards having full closure properties such as polygons, rotary/indexing tables and angle encoders may be calibrated by fundamental subdivisional methods or by direct comparison with...
appropriate reference standards. The method chosen will depend on the equipment available and the accuracy required. Precision rotary or indexing tables, polygons, and high precision electronic autocollimators are mostly used in primary angle measurements.

2.1. Precision indexing / rotary tables

Precision indexing/rotary tables can be used, in conjunction with an autocollimator, for the calibration of precision polygons and some angle gauges and also for calibration of test rotary tables with addition of a single reflector.

2.2. Precision polygons

Precision polygons are hardened steel or glass blocks in the form of regular polygons with flat reflecting faces. They may have up to 72 faces (sides), but 4, 6, 8, or 12 sides are most common. They are used in conjunction with an autocollimator to calibrate angle-measuring devices such as indexing or rotary tables fitted with angle encoders or optical dividing heads.

2.3. Autocollimators

Autocollimators (ACs) are optical devices for measuring small angles of tilt of a reflecting face. They are typically used as reading devices (i.e. optical probes) in the calibration of precision polygons and angle gauges. Currently high resolution electronic autocollimators (resolution of 0.001°) is used in most calibration laboratories particularly for primary angle measurements.

2.4. Fundamental method - Subdivision of a full circle

Suppose we have a circle (circle 1) divided into 4 nominally equal angle intervals, $\alpha_{11}$, $\alpha_{12}$, $\alpha_{13}$, $\alpha_{14}$, each with a small error $T_1$, $T_2$, $T_3$ and $T_4$ and another circle (circle 2) with angle intervals, $\beta_{P1}$, $\beta_{P2}$, $\beta_{P3}$, $\beta_{P4}$, each with a small error $P_1$, $P_2$, $P_3$ and $P_4$. The four unknown errors can be determined by comparing the interval $\beta_{P1}$ of circle 2 with each of the four intervals of circle 1 in turn and observing the differences $AC_{11}$, $AC_{12}$, $AC_{13}$, $AC_{14}$. In this way, four equations are obtained.

\[
P_1-T_1 = AC_{11} \quad (1) \\
P_1-T_2 = AC_{12} \quad (2) \\
P_1-T_3 = AC_{13} \quad (3) \\
P_1-T_4 = AC_{14} \quad (4)
\]

The fact that the sum of the four intervals is complete circle and hence the sum of the four errors must be zero; $T_1+T_2+T_3+T_4 = 0$. Summing equations (1) to (4) gives,

\[
4P_1 - (T_1+T_2+T_3+T_4) = AC_{11}+ AC_{12}+ AC_{13}+ AC_{14} \quad (5)
\]

since $T_1+T_2+T_3+T_4 = 0$, $P_1$ is given by

\[
P_1 = (AC_{11}+ AC_{12}+ AC_{13}+ AC_{14}) / 4 \quad (6)
\]

with this, all deviations are known: $T_1=P_1\cdot AC_{11}$, $T_2=P_1\cdot AC_{12}$, $T_3=P_1\cdot AC_{13}$, and $T_4=P_1\cdot AC_{14}$.

3. Correct sign determination of angle deviations in cross calibration method

When a subdivisional method is used, it is common for a series of comparisons to be made which is more extensive than that leading to the equations in section 2.4. Each interval to be calibrated is compared in turn with every other interval. A computational procedure is followed which leads to least squares estimates of the individual errors and enables a statistical analysis of the results to be made.

A calibration of this nature, in which the intercomparison of intervals of one device are compared with those of the second device, results in the determination of the errors in both without any prior knowledge of either. In other words, simultaneous calibration of both devices are achieved. This is referred to as a cross calibration method and applied for calibration of polygons using indexing/rotary
tables, calibration of indexing/rotary tables using polygons, or calibration of indexing/rotary tables with another indexing/rotary tables utilising autocollimators. Autocollimators are used for optical probing as reading devices to determine the difference between angle intervals. For calibration of indexing/rotary table with another one, a reflector mounted on the top table is used for optical probing.

3.1. Cases for sign determination

During application of cross calibration, two possible sets of relationships may occur for determination of correct sign; direction of reading of the autocollimator and direction of the scale of the indexing table. The issue for direction of labelling of the test device e.g. polygon faces is removed since the measurement results are placed in the graphical square according to labelling of e.g. polygon as shown in section 3.3. Two cases are set out in Table 1.

| Case | Equation for determination of the difference between angle intervals; where T and P are errors of table and polygon respectively, AC is the autocollimator reading. |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| I    | T-P = AC                                                                                                                             |
| II   | P-T = AC                                                                                                                             |

3.2. Cross calibration and determination of correct sign

Determination of correct sign will be explained through cross calibration process with the example of polygon calibration by indexing table and observations will be shown by symbols indicating the direction of the reading devices.

The polygon is set on the indexing table with the table reading zero and the autocollimator viewing the datum face of the polygon (0° or face 1). The autocollimator reading (AC) is taken and recorded as in Table 2. The table is then rotated through one polygon interval in a direction to give increasing of readings of the indexing table and the autocollimator readings are recorded until the indexing table reaches its original zero position, i.e. intervals of indexing table (0-90-180-270-0)° and polygon (0-90-180-270-0)° are compared. This process is repeated with the direction of rotation reversed. This is illustrated in table 2 as CCW and CW directions. Next, the differences between readings at successive positions and the mean of corresponding differences are calculated in 7th column of table 2. The equations for calculation of these differences are given in the last column of table 2 for two cases.

Later, the equipment is reset so that with the indexing table still reading zero, the autocollimator views the next adjacent face of the polygon (90° or face 2 in this case). This may be done by simply rotating the polygon on top face of the indexing table. Then the second sets of results are taken with indexing table, (0-90-180-270-0)° and polygon (90-180-270-0-90)°.

Resetting the equipment is continued so that with the indexing table reading zero, the autocollimator views the next adjacent face of the polygon. Further sets of results are recorded by repeating the above process until the number of sets of observation is equal to the number of faces of the polygon, i.e. until a set has been recorded with each polygon face in turn corresponding with the zero reading of the indexing table. The differences in the 7th column of table 2 now represent differences between single intervals of the polygon and the chosen intervals of the indexing table taken in all combinations. These differences (e.g. AC(Ti-Pi)) are entered into the appropriate places in a graphical square as (T1-P1) in table 3.

The graphical square shown in table 3 is designed for case I using the equation, T-P=AC, where reading direction of the autocollimator and indexing table is same, i.e. results of the equation is positive if table interval is larger than polygon interval. Table 4 shows the graphical square for case II with the equation, P-T=AC, where reading direction of the autocollimator and indexing table is opposite. The results of the equation will be positive again if table interval is larger than polygon's.

Finally, the sum and mean of each row and column are calculated by following the equations 1-6. The estimated errors in the intervals of the polygon and of the indexing table can be calculated using...
the mean values given in 7th row and 7th column of the graphical squares (Table 3 and 4). But the correct sign for these values has to be evaluated. Column 8 and row 8 may be designed according to the equations explained for case I or II. This is also illustrated in the 8th column and 8th row of table 3 and table 4 as 'adjusted'. Depending on the selected equations of case I or II, sign of the resulted mean values is reversed or not in the adjusted section. For example in table 3, calculated ‘-P1, -P2, -P3 and -P4’ values (using eq. 6) in 7th row have negative sign (calculated as inverted value) and they must be reversed as ‘P1, P2, P3 and P4’ in the adjusted row. However, ‘T1, T2, T3 and T4’ values in 7th column of table 3 have positive sign (non-inverted) and reverse is not required in adjusted column. By designing the graphical squares with these considerations, correct sign of deviations may be determined.

Table 2. Observations for a cross calibration - indexing table and polygon

| Index | Polygon or Index Table (°) | CCW Rotation | Difference (Case I) | Clockwise Rotation | Difference (Case I) | Mean difference (Case I) | Equations for calculation of difference values to be used in the graphical square for Case I and Case II |
|-------|----------------------------|--------------|---------------------|--------------------|---------------------|--------------------------|----------------------------------------------------------------------------------|
| 0     | 0 (1)                      | AC0_CCW      | -                   | AC0_CCW            | -                   | AC0_CCW                  | AC0 - AC1 = AC(T1;P1) or AC(P4;T1)                                               |
| 90    | 90 (2)                     | AC90_CCW     | AC90_CCW            | AC90_CCW           | -                   | AC90_CCW                 | AC90 - AC10 = AC(T2;P1) or AC(P4;T2)                                           |
| 180   | 180 (3)                    | AC180_CCW    | AC180_CCW           | AC180_CCW          | -                   | AC180_CCW                | AC180 - AC10 = AC(T2;P2) or AC(P4;T2)                                          |
| 270   | 270 (4)                    | AC270_CCW    | AC270_CCW           | AC270_CCW          | -                   | AC270_CCW                | AC270 - AC10 = AC(T2;P3) or AC(P4;T3)                                          |
| 0     | 0 (1)                      | AC0_CCW      | -                   | AC0_CCW            | -                   | AC0_CCW                  | AC0 - AC10 = AC(T1;P1) or AC(P4;T1)                                           |
| 90    | 90 (2)                     | AC90_CCW     | AC90_CCW            | AC90_CCW           | -                   | AC90_CCW                 | AC90 - AC10 = AC(T2;P1) or AC(P4;T2)                                           |
| 180   | 180 (3)                    | AC180_CCW    | AC180_CCW           | AC180_CCW          | -                   | AC180_CCW                | AC180 - AC10 = AC(T2;P2) or AC(P4;T2)                                          |
| 270   | 270 (4)                    | AC270_CCW    | AC270_CCW           | AC270_CCW          | -                   | AC270_CCW                | AC270 - AC10 = AC(T2;P3) or AC(P4;T3)                                          |

Table 3. Graphical square for Case I.

| Polygon Table | 0-90 (1) | 90-180 (2) | 180-270 (3) | 270-0 (4) | Sum | Mean | Adjusted |
|---------------|----------|------------|-------------|-----------|-----|------|----------|
| 0-90          | T1-P1    | T2-P2      | T3-P3      | T4-P4    | 4T1| T1   | T1       |
| 90-180        | T1-P1    | T2-P2      | T3-P3      | T4-P4    | 2T1| T2   | T2       |
| 180-270       | T1-P1    | T2-P2      | T3-P3      | T4-P4    | 2T1| T3   | T3       |
| 270-0         | T1-P1    | T2-P2      | T3-P3      | T4-P4    | 4T4| T4   | T3       |

Table 4. Graphical square for Case II.

| Polygon Table | 0-90 (1) | 90-180 (2) | 180-270 (3) | 270-0 (4) | Sum | Mean | Adjusted |
|---------------|----------|------------|-------------|-----------|-----|------|----------|
| 0-90          | P1-P1    | P2-P2      | P3-P3      | P4-P4    | 4P1| P1   | P1       |
| 90-180        | P1-P1    | P2-P2      | P3-P3      | P4-P4    | 2P1| P2   | P2       |
| 180-270       | P1-P1    | P2-P2      | P3-P3      | P4-P4    | 2P1| P3   | P3       |
| 270-0         | P1-P1    | P2-P2      | P3-P3      | P4-P4    | 4P4| P4   | P4       |

4. Conclusions

The issue for determination of correct sign of angle measurements during primary calibration of angle measurement standards was discussed and different cases related reading direction of autocollimator and indexing/rotary tables were presented with proposed solutions. It is hoped that this information will be off assistance to a laboratory in performing primary calibrations of angle standards.

5. References

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