Whispering gallery mode nanodisk resonator based on layered metal-dielectric waveguide

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Abstract: This paper proposes a layered metal-dielectric waveguide consisting of a stack of alternating metal and dielectric films which enables an ultracompact mode confinement. The properties of whispering gallery modes supported by disk resonators based on such waveguides are investigated for achieving a large Purcell factor. We show that by stacking three layers of 10 nm thick silver with two layers of 50 nm dielectric layers (of refractive index $n$) in sequence, the disk radius can be as small as 61 nm $-\lambda_0/(7n)$ and the mode volume is only $0.0175(\lambda_0/(2n))^3$. When operating at 40 K, the cavity’s Q-factor can be ~670; Purcell factor can be as large as $2.3\times10^4$, which is more than five times larger than that achievable in a metal-dielectric-metal disk cavity in the same condition. When more dielectric layers with smaller thicknesses are used, even more compact confinement can be achieved. For example, the radius of a cavity consisting of seven dielectric-layer waveguide can be shrunk down to $\lambda_0/(13.5n)$, corresponding to a mode volume of $0.005\lambda_0/(2n)^3$, and Purcell factor can be enhanced to $7.3\times10^4$ at 40 K. The influence of parameters like thicknesses of dielectric and metal films, cavity size, and number of dielectric layers is also comprehensively studied. The proposed waveguide and nanodisk cavity provide an alternative for ultracompact light confinement, and can find applications where a strong light-matter interaction is necessary.

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1. Introduction

Optical cavities with high quality factors (Q) as well as small mode volume (V) have attracted great attention for a long time due to their applications such as low-threshold lasers, cavity quantum electrodynamics, sensing, nonlinear enhancement, switching, etc [1]. Purcell factor which is proportional to the ratio of Q/V is a figure of merit of particular interest [1–3]. In conventional dielectric cavities, light is confined by the index contrast between high index cores and low index surroundings. Normally, the mode volume V of a dielectric cavity cannot be shrunk beyond cubic effective wavelength (λ/n)^3 due to the diffraction limit of light [1–4]. However, by stimulating surface plasmon polaritons (SPPs) along a metal-dielectric interface [5, 6], metallic cavities can enable nano-scale confinement. Intensive efforts have been made to create sub-wavelength confinement in recent years [7–18]. Typical SPP waveguides
composing metallic cavities include metal-coated dielectric waveguide [7–9], dielectric- loaded plasmonic waveguide [10, 11], hybrid plasmonic waveguide [12–15], and metal- dielectric-metal (MDM) waveguide [16–18]. Among these, MDM waveguide has no cutoff frequency [5, 19, 20], hence the size of MDM cavity can be very small [16–18]. Simulations in Ref [21] show that the radius of MDM disk can be as shrunk to 88 nm \( \lambda_0 / (5.2n) \) (here \( n \) is the dielectric’s index) for a 2nd order whispering gallery mode (WGM), and ultrahigh Purcell factor can be achieved at 40 K for solid-state quantum electrodynamics applications. However, the thickness of the slot dielectric needs to be very thin, in the referred case 10 nm to obtain such compact confinement, limiting the design freedom for on-chip active devices as well as the coupling and pumping mechanism. It is thus meaningful to look for an alternative approach to obtain a large propagation constant. One example is to engineer the waveguide dispersion, by employing the so-called hyperbolic metamaterial waveguide which consists of infinite metal-dielectric layers [22–26]. Cavity size can be shrunk down to \( \lambda / 12 \) [25], and light-matter interaction enhancement can also be achieved [26].

In this paper, we propose a layered metal-dielectric (LMD) waveguide whose geometry resembles a hyperbolic metamaterial waveguide, while its guiding principle is similar as MDM waveguide and dielectric-metal-dielectric (DMD) waveguide. In contrast to MDM and DMD waveguides, the proposed LMD waveguide achieves much more compact confinement due to the strong mode coupling between multiple metal and dielectric layers. The physical properties of WGM nanodisks based on the LMD waveguide will be systematically studied, including Q-factor, mode volume and Purcell factor, as well as influence of the metal and dielectric thicknesses. We show that when the metal and/or dielectric layer is thin, strong inter-layer coupling occurs and the propagation constant can increase significantly. Simulations show that the radius of the disk cavity can be shrunk down to 61 nm, i.e. \(- \lambda_0 / (7n)\) (here \( n \) is the dielectric’s index) when stacking three layers of 10 nm thick silver alternatively with two layers of 50 nm dielectric layers. The cavity’s mode volume is calculated as small as \(0.0175(\lambda_0 / (2n))^3\), and a Purcell factor around \(2.3 \times 10^4\) can be achieved at 40 K. The calculated Purcell factor is more than five times larger than that achievable in a metal-dielectric-metal disk cavity with identical dielectric thickness. We also show that when using a LMD waveguide with more dielectric layers, the cavity’s properties can be further enhanced. For example, when the number of dielectric layers changes from 1 to 7, the disk radius can be shrunk from 92 nm to 33 nm \(- \lambda_0 / (13.5n)\), and Purcell factor can be dramatically increased from about 1860 to \(7.3 \times 10^4\).

The remainder of the paper is organized as follows. Section 2 explains the waveguiding principle by considering an analytical model of a 1D straight waveguide composed of three layers of metal and two layers of dielectric films. Section 3 studies the cavity properties including Q-factor, mode volume and Purcell factor of a WGM disk based on the proposed waveguide, and compares with a traditional MDM-based cavity. Then the influence of geometry parameters on the LMD cavity properties is systematically studied in Section 4. Section 5 presents the situation when increasing the number of dielectric layers. Finally, conclusions are given in Section 6.

2. Waveguiding principles

The proposed layered metal-dielectric waveguide, i.e., LMD\(_N\) (\( N = 1, 2 \ldots \)) is composed by interlacing \(N + 1\) metal layers with \(N\) dielectric layers. As schematically shown in Fig. 1(a), a simple 1-dimensional (1D) LMD waveguide with \( N = 2 \) is firstly considered, to explain the waveguiding principle. Here, no spatial variation along \( y \) direction is assumed. The SPP wave propagates along \( x \) direction. The thickness of each metal and dielectric is denoted by \( h_M \) and \( h_2 \), respectively. Then the concerned waveguide’s propagation constant can be analyzed analytically. In the study, metal is assumed as silver; its permittivity is described by Drude
model $\varepsilon_\infty = \varepsilon_\infty - \omega_p^2 / (\omega^2 + i\gamma)$, where $\varepsilon_\infty$ is 3.1, the plasma frequency $\omega_p$ is $1.4 \times 10^{10}$ s$^{-1}$, and the collision frequency $\gamma$ is $3.1 \times 10^{13}$ s$^{-1}$ [21, 27]. The dielectric can be various materials such as III-V, Ge, doped Si, etc. A permittivity of 12.25 is assumed without losing generality.

Figures 1(b) and 1(c) show the field distributions of two highly-confined eigen modes supported by the LMD$_2$ waveguide, namely even TM$_1$ mode and odd TM$_2$ mode. The frequency considered is 193.55 THz for 1550 nm operation, which is much lower than the plasmon frequency of silver. In the simulation, the silver thickness is $h_M = 10$ nm, and the dielectric thickness is $h_d = 50$ nm. From the figures, one can see that the even mode has a symmetric $E_z$ field across the metal separation in the middle, while the $E_z$ field of the odd mode is asymmetric. In addition, the effective wavelength of the odd mode is relatively smaller than that of the even mode, implying that the corresponding propagation constant is larger. Basically, the LMD$_2$ waveguide can be regarded as a $N = 2$ stack consisting of two LMD$_1$ waveguides separated by a metal layer with thickness of $h_M$. When $h_M$ is thin enough for light penetrating through, the isolated LMD$_1$ modes would symmetrically and asymmetrically couple with each other. The mode hybridization can result in mode splitting and create bonding and anti-bonding modes [28, 29], i.e. the even and odd modes as shown in Figs. 1(b) and 1(c). Due to the phase matching condition, the bonding and anti-bonding eigen modes in the LMD$_2$ waveguide would satisfy: $\beta_p^+ - \beta_p^- \pm \Delta$, where $\beta_p^+$ and $\beta_p^-$ are propagation constants of even and odd modes, and $\beta_p$ is the propagation constant of the fundamental mode in LMD$_1$ waveguide, while $\Delta$ represents the momentum shifts due to the mode couplings.

In order to quantitatively analyze the mode splitting, effective refractive indices and propagation lengths of the concerned modes as functions of metal and dielectric thicknesses are plotted in Fig. 2. In the analysis, effective mode indices are given by $n_e^\pm = R(\beta_p^\pm) / k_0$, and propagation lengths are evaluated by $L_e^\pm = k_0^\pm / 2I(\beta_p^\pm)$, depicting the 1/e plasmon decay length along x direction. One can see from Fig. 2(a) and 2(c) that for any given geometry, effective indices of bonding and anti-bonding modes satisfy $2n_e = n_e^+ + n_e^-$ and $n_e^+ < n_e^-$. The mode splitting ($n_e^- - n_e^+$) decreases as increasing $h_M$ due to the decreased mode hybridization since less light can penetrate through the metal when metal gets thicker. When the metal is thick enough ($h_M > ~100$nm), the LMD waveguide degenerates into MDM waveguide, and in this case, $n_e^+$ and $n_e^-$ approach asymptotically the effective index of a MDM mode, which has a much lower refractive index as shown in Fig. 2(a). To analyze the influence of substrate,
\( n_{\text{sub}}^+ \) denoting the effective indices of a LMD\(_2\) waveguide sitting on a silica substrate are also calculated, as shown in Fig. 2(a). One can see that \( n_{\text{sub}}^+ \) are almost identical with \( n_{\text{en}}^+ \), suggesting that the substrate hardly affects the waveguide performance. For simplicity, air is used as cladding and substrate material in the following analysis. As for the influence of dielectric thickness \( h_2 \), one can see in Fig. 2(c) that \( n_{\text{en}}^- \) and \( n_{\text{en}}^+ \) increase simultaneously with decreasing \( h_2 \), which is similar as in a MDM waveguide, where better confinement can be achieved with a thinner dielectric. In addition, the mode splitting is relatively less sensitive to the variation of \( h_2 \) than to the silver thickness variation, mainly because the mode coupling is mostly determined by the field penetration through the metal layer. As a comparison to the traditional MDM waveguide, we also plot the effective index \( n_{\text{MDM}} \) of a MDM waveguide with 100 nm thick metal in Fig. 2(c). As one can see for a given dielectric thickness, the antibonding mode’s effective index \( n_{\text{en}}^- \) is significantly larger than \( n_{\text{MDM}} \), suggesting that the mode confinement can be significantly enhanced by operating with TM\(_2\) mode in a LMD\(_2\) waveguide. We then study the propagation lengths of the LMD waveguide as shown in Figs. 2(b) and 2(d). One can find that in the proposed LMD waveguide, there also exists a tradeoff between effective index (confinement) and propagation length. Note that similar tradeoffs exist in all plasmonic waveguides, and one needs to choose suitable design according to specific applications.

![Fig. 2. Influence of metal and dielectric thicknesses on the effective refractive indices and propagation lengths of the concerned modes. Standard parameters are \( h_\perp = 10 \text{ nm} \) and \( h_\parallel = 50 \text{ nm} \). (a) and (b) are variation of effective indices and propagation lengths when changing \( h_\perp \), respectively. (c) and (d) are variation of effective indices and propagation lengths when changing \( h_\parallel \), respectively.](image)

3. Nanodisk cavity based on layered metal-dielectric waveguide with \( N = 2 \) (LMD\(_2\))

In the following paragraphs, the proposed waveguide is exploited to realize superior plasmonic nano cavities. A whispering gallery mode (WGM) cavity based on the LMD\(_2\) waveguide with \( h_2 = 50 \text{ nm} \) and \( h_M = 10 \text{ nm} \) is firstly considered in this section. The 3-dimensional (3D) schematic diagram of the disk is shown in Fig. 3(a), where \( R \) is the disk
radius, \( h_M \) and \( h_2 \) are the respective film thicknesses. Note that such waveguide geometry is serving as an example to demonstrate the concept, the conclusions based on which can be extended for more general cases. 3D finite-difference time-domain (3D-FDTD) method is used to study the properties of the resonant modes numerically. In all the simulations, perfectly matched layers are employed as the boundary conditions. Mesh size along each direction is 2 nm unless otherwise stated. Similar as in the LMD\(_2\) straight waveguide studied in previous section, the disk can also support even (TM\(_1\)) and odd (TM\(_2\)) quasi-TM modes.

Note that although WGM modes with higher radial-orders can be supported in the nanodisk, only the 1st radial-order modes are studied due to their most compact mode confinement. Figures 3(b) and 3(c) show the \( E_z \) field distributions of a TM\(_1\) WGM mode with an azimuthal number \( m = 3 \) along \( \phi \) and \( zr \) cross-sectional planes. Figures 3(d) and 3(e) are for a TM\(_2\) WGM mode with \( m = 3 \), respectively. Note that in order to enable a resonance near 1550 nm with a given azimuthal number \( m \), an appropriate disk radius \( R \) should be chosen.

![Fig. 3.](image)

Fig. 3. (a) 3-D schematic diagram of a whispering gallery mode cavity based on the LMD\(_2\) waveguide. (b) and (c) are the \( E_z \) field distributions of TM\(_1\) resonant mode with an azimuthal number \( m = 3 \) along \( \phi \) and \( zr \) planes, and \( R = 203 \) nm. (d) and (e) are the \( E_z \) field distributions of TM\(_2\) resonant mode with \( m = 3 \) along \( \phi \) and \( zr \) planes, and \( R = 142 \) nm.

To study quantitatively the influence of the cavity size on the cavity properties, cavity Q-factors, mode volume \( V \) and Purcell factor \( F \) are investigated for different azimuthal numbers. In the investigation, the deterioration of the metal quality due to the quantum size effect when the film is thin is also considered. Namely, the collision frequency in the Ag’s Drude model is modified according to the experimental data as \( \gamma_{\text{thin}} = 6.462 \times 10^{13} \text{s}^{-1} \) for the 10 nm Ag film [30] in our simulations. Similar as in strong coupling like laser or single photon system [7, 8, 21, 31], low temperature operation is also assumed in our simulations at 40 K. The low temperature permittivity of silver is modelled as described in [7, 8, 21]. Figure 4 shows the simulated properties as functions of azimuthal number \( m \) for the TM\(_1\) and TM\(_2\) WGM modes in LMD\(_2\) nanodisk and the WGM mode in a traditional MDM nanodisk with a metal thickness as \( h_M = 100 \) nm for comparison. Note that the disk radii for azimuthal number \( m \) from 1 to 7 need to be carefully selected to enable a resonance within 1550 ± 20 nm. The selected radii are shown in Table 1. In Fig. 4(a), the \( E_z \) field distributions of the concerned WGM modes with \( m = 2 \) along \( zr \) cross-sectional half plane (\( r > 0 \)) are shown. One can see that in each case, light is tightly localized in the 100 nm thick dielectric layers. However, the respective radius of MDM, TM\(_1\) and TM\(_2\) LMD\(_2\) nanodisk gradually decreases from 197 nm to 146 nm, and then to 103 nm. One can see from Table 1 that the selected radii of MDM cavities with a 100 nm thick dielectric layer fit well with the simulation results in Ref [21]. Moreover, for a fixed \( m \), TM\(_2\) LMD\(_2\) nanodisk has the smallest radius due to its strongest light confinement; while MDM nanodisk has the largest cavity size due to the weakest confinement capability.
Table 1. Selected Radii for \( m \)th Azimuthal Order Resonance within 1550 ± 20 nm (Unit: nm)

| \( m \) | 1     | 2     | 3     | 4     | 5     | 6     | 7     |
|-------|-------|-------|-------|-------|-------|-------|-------|
| MDM   | NA    | 197   | 273   | 343   | 415   | 482   | 547   |
| TM1 LMD \(_2\) | 86    | 146   | 203   | 256   | 309   | 362   | NA    |
| TM2 LMD \(_2\) | 61    | 103   | 142   | 180   | 218   | 254   | NA    |

Next, by studying the temporal decay rate of the resonant modes, cavity Q-factors \( Q \) representing the cavity’s photon life time are extracted for each cavity. Moreover, based on the cavity mode distributions, effective mode volume \( V_{\text{eff}} \) can be evaluated by

\[
V_{\text{eff}} = \frac{\int \varepsilon(x,y,z) |E(x,y,z)|^2 \, dx \, dy \, dz}{\max(\varepsilon(x,y,z) |E(x,y,z)|^2)} \quad [2],
\]

where \( \varepsilon(x,y,z) \) and \( E(x,y,z) \) are the cavity permittivity profile and the mode electric field distribution, respectively. By normalizing \( V_{\text{eff}} \) with effective wavelength, a more commonly used unit-less normalized mode volume can be calculated as \( V = V_{\text{eff}} / (\lambda_0 / (2n))^3 \). Then Purcell factor as a figure of merit for a strong coupling cavity is evaluated by

\[
F = \frac{3}{4 \pi^2} \frac{Q}{V_{\text{eff}}} \left(\frac{\lambda_0}{n}\right)^3 \quad [2],
\]

describing the cavity-resulted enhancement of the spontaneous emission rate. It can further be expressed as

\[ F = 6Q / (V \cdot \pi^2). \]

The calculation results of the considered three types of resonant modes are shown in Figs. 4(b)-4(d). \( Q \) and \( V \) of the MDM disk as functions of \( m \) are shown in Fig. 4(b), while Fig. 4(c) shows the \( Q \) and \( V \) of the TM\(_1\) and TM\(_2\) LMD\(_2\) disks. Generally, there are two loss origins in a plasmonic cavity, i.e. absorption loss due to the metal dissipations, radiation loss due to the sharp bendings. Potential scattering losses due to fabrication imperfections are not considered in this simulation work. Hence, the numerically achieved Q-factors of the studied cavities satisfy \( 1/Q = 1/Q_{\text{rad}} + 1/Q_{\text{abs}} \) \[21\], where \( Q_{\text{rad}} \) and \( Q_{\text{abs}} \) are radiation loss and absorption loss related Q-factors. When increasing the cavity radius, \( Q_{\text{rad}} \) grows exponentially owing to decreasing radiation loss, while \( Q_{\text{abs}} \) is insensitive to radius variations since the energy loss in a unit period is mainly determined by the mode distributions, which only changes slightly with radius. If one looks at the \( Q \) of MDM and TM\(_1\) LMD\(_2\) nanodisk as shown in Fig. 4(b) and 4(c), when \( m \) changes from smallest to largest values, \( Q \) of both cavities increases exponentially with \( m \) at first, then the slopes gradually decrease, and at last \( Q \) keeps constant. The changing processes correspond to the conditions when \( Q_{\text{rad}} \) is much smaller than \( Q_{\text{abs}} \), \( Q_{\text{rad}} \) is comparable with \( Q_{\text{abs}} \) and \( Q_{\text{rad}} \) is much larger than \( Q_{\text{abs}} \). The simulated Q-factors of MDM cavities as shown in Fig. 4(a) are consistent with the study in Ref \[21\]. For the TM\(_2\) LMD\(_2\) disk as shown in Fig. 4(c), since the mode confinement is very strong as discussed in the previous section, even for \( m = 1 \), \( Q_{\text{rad}} \) is already much larger than \( Q_{\text{abs}} \), hence \( Q \) almost keeps constant when \( m \) increases from 1 to 6. When the azimuthal number \( m = 2 \), the \( Q \) for MDM disk, TM\(_1\) and TM\(_2\) LMD\(_2\) disk is \( Q_{\text{MDM}} \sim 320 \), \( Q_{\text{TM1}}^{\text{N2}} \sim 715 \) and \( Q_{\text{TM2}}^{\text{N2}} \sim 700 \) 700, respectively. It is worth to mention that by optimizing the thin film metal quality to decrease the influence of quantum size effect, \( Q_{\text{TM1}}^{\text{N2}} \) and \( Q_{\text{TM2}}^{\text{N2}} \) can further be enhanced. In Figs. 4(b)-4(c), normalized mode volumes \( V \) as functions of \( m \) are also shown. One can see that when increasing the cavity size \( \pi R^2 \), \( V \) also increases continuously. For a fixed azimuthal number \( m = 2 \), \( V_{\text{TM1}}^{\text{N2}} = \frac{1}{3} V_{\text{MDM}} \) and \( V_{\text{TM2}}^{\text{N2}} = \frac{1}{8} V_{\text{MDM}} \), suggesting that a much more compact mode volume can be achieved in LMD\(_2\) nanodisks. Figure 4(d) plots the Purcell factors of the studied WGM nanodisks. One can see that the
maximal Purcell factor $F$ achievable by MDM disk is ~3500 when the azimuthal number is $m = 4$, i.e. the disk radius is $R = 343$ nm, while the maximal $F$ achievable by TM$_1$ LMD$_2$ disk is ~3870 when $m = 2$, i.e. $R = 146$ nm. In the TM$_2$ LMD$_2$ disk however, the highest Purcell factor $F$ can be as large as ~$2.3 \times 10^4$, when $m = 1$ with a radius $R = 61$ nm. Compared with the traditional MDM nanodisk, LMD$_2$ cavity can enhance the achievable Purcell factor by more than five times.

![Fig. 4. (a) $E_z$ field distributions of the whispering gallery modes of MDM, TM$_1$ and TM$_2$ LMD$_2$ nanodisks along $z\tau$ cross-sectional half plane ($r+$). Here, the azimuthal number is $m = 2$. (b) $Q$ and $V$ of MDM disk as functions of $m$. (c) $Q$ and $V$ of TM$_1$ and TM$_2$ LMD$_2$ disks as functions of $m$. (d) Purcell factors $F$ as functions of $m$.](image)

### 4. Influence of metal and dielectric thicknesses on TM$_2$ LMD$_2$ nanodisk cavity

In this section, the influence of the metal and dielectric thicknesses on the cavity properties is systematically analyzed, aiming to provide useful insights on design guidelines. As discussed in previous sections, the WGM nanodisk cavity based on TM$_2$ LMD$_2$ waveguide operated with an azimuthal number $m = 1$ possesses the highest achievable Purcell factor, due to the low radiation loss and compact mode confinement. Hence, the following analysis focuses on the properties of 1st order WGM mode of a TM$_2$ LMD$_2$ cavity.

Firstly, changing of the silver thickness $h_M$ from 10 nm to 100 nm at a step of 10 nm is considered. In order to rule out the influence of the quantum size effect and concentrate on $h_M$ itself, silver with different thicknesses is described by same Drude model as used for bulk material. Figure 5(a) shows a 2D colormap of the resonant wavelength $\lambda_{Res}$ (in nm) as a function of the disk radius $R$ and the silver thickness $h_M$. One can see that $\lambda_{Res}$ increases dramatically when $R$ increases and/or $h_M$ decreases. This is because for the nanodisk operates at very small azimuthal number (1 in this case), slight change of the disk’s equivalent optical path results in significant shift of the resonant wavelength. As shown by the markers in Fig. 5(a), suitable combinations of $R$ and $h_M$ should be chosen, to render a resonant wavelength within $1550 \pm 20$ nm. The effective refractive index of the whispering gallery mode can then be evaluated by $n_{WGM}^e = m \lambda_{Res}^e / (2\pi R)$, where $m = 1$ and $\lambda_{Res}^e \sim 1550$ nm. One can see from Fig. 5(b) that as $h_M$ increases from 10 nm to 100 nm, $n_{WGM}^e$ decreases from around 4.1 to 2.5, and the decline slope also decreases gradually. This is similar as in a straight LMD$_2$ waveguide discussed in Section 2, i.e. the mode hybridization is highly dependent on the
metal thickness, and it decreases as metal gets thicker and eventually the LMD$_2$ mode degenerates to MDM mode. As one may find the effective index of the whispering gallery mode $n^w$ calculated here is generally smaller than 1D waveguide index $n_e$ as shown in Fig. 2(a). Beside the reason that the bending mode has a slightly different mode distribution as compared to a straight waveguide, the main reason is that in the equation evaluating the WGM index, the physical radius of the disk is used for simplicity. However, the actual bending radius of the gallery mode is smaller than the physical radius, considering the WGM has a spatial distribution ranging from the perimeter of the nanodisk to several tens of nanometers towards the disk center. Hence, such simplification would underestimate the WGM index while it doesn’t affect the information it can provide.

Then Q-factors and normalized mode volumes as functions of the metal thickness are studied as shown in Fig. 5(c). Here, the cavity and radiation Q-factor, i.e. $Q$ and $Q_{rad}$ are numerically calculated by the FDTD simulation by considering a lossy and lossless silver film, respectively. The metal absorption-related Q-factor of the cavity $Q_{abs}$ is deduced correspondingly. As one can see from Fig. 5(c), when $h_M$ increases from 10 nm to 100 nm, $Q_{rad}$ decreases several orders of magnitude from around $2.86 \times 10^4$ to 370, due to the decreased WGM effective index $n^w$ as shown in Fig. 5(b). This is because with a smaller $n^w$, the cavity suffers larger radiation originating from the sharp bendings, hence has a lower Q-factor. On the other hand, the absorption-related Q-factor $Q_{abs}$ increases but much more slowly when $h_M$ increases as shown in Fig. 5(c). This is because the waveguide absorption would decrease when $h_M$ increases as shown in Fig. 2(b). However, the influence of $h_M$ on $Q_{abs}$ is much less dramatic, because $Q_{abs}$ is fundamentally determined by the metal’s intrinsic property other than the waveguide geometry. As a result, the cavity Q-factor $Q$, as a function of $Q_{rad}$ and $Q_{abs}$ changes slightly when $h_M$ is small, i.e. $Q_{rad} \gg Q_{abs}$, while it drops significantly when $h_M$ becomes larger, i.e. $Q_{rad} \sim < Q_{abs}$ as shown in Fig. 5(c). The normalized mode volume and Purcell factor are then analyzed, as shown in Figs. 5(c) and 5(d). One can find that when $h_M$ increases from 10 nm to 100 nm, $V$ increases from about 0.0175 to 0.039, while the Purcell factor $F$ drops from about $4.8 \times 10^4$ to 5250.
Next, we perform a similar analysis on the influence of the dielectric thickness by altering $h_2$ from 10 nm to 100 nm at a step of 10 nm, while the silver thickness is fixed at 10 nm. The 2D colormap of resonant wavelength as a function of $h_2$ and the disk radius are shown in Fig. 6(a). Here, the markers represent the suitable parameters to enable a 1st order WGM resonance around 1550 nm. When $h_2$ gets thicker from 10 nm to 50 nm then to 100 nm, the corresponding disk radius increases rapidly from 31 nm to 61 nm then gradually to 69 nm.

The effective index of the whispering gallery mode $n_{WGM}$ is also calculated, and plotted in Fig. 6(b). One can see that the mode index, i.e. mode confinement can be significantly enhanced, by decreasing the dielectric thickness. Similar tendency can also be observed in the 1D straight waveguide as shown in Fig. 2(c). Using the aforementioned simulation methods, $Q_{rad}$, $Q_{abs}$, and $Q$ are evaluated as shown in Figs. 6(b) and 6(c). As one can seem from Fig. 6(b), when increasing the dielectric thickness, $Q_{rad}$ drops exponentially ($Q_{rad} \sim 1.6 \times 10^6$ when $h_2 = 20$ nm, and $Q_{rad} \sim 2000$ when $h_2 = 100$ nm), due to the significantly decreased mode confinement. On the other hand, Fig. 6(c) shows that $Q_{abs}$ increases gradually with increased $h_2$, since the waveguide propagation loss is decreased, which is similar for the 1D straight waveguide as shown in Fig. 2(d). As a result, one can see from Fig. 6(c) that when $h_2$ increases from 10 nm to 40 nm, the intrinsic $Q$ follows the trend of $Q_{abs}$ because in those cases, $Q_{rad}$ as shown in Fig. 6(b) is much larger than $Q_{abs}$ as shown in Fig. 6(c). However, when $h_2$ gets even larger, the rapid decrease of $Q_{rad}$ dominates the contribution to cavity $Q$, thus $Q$ decreases gradually. The normalized mode volume $V$ and Purcell factor $F$ are then plotted in Fig. 6(c) and 6(d). Due to the strong field localization and shrinkage of the cavity radius, $V$ can be very small in a disk cavity with $h_2 = 10$ nm. The volume is approximately only one-ninth of that of a cavity with $h_2 = 100$ nm, while the Purcell factor is enhanced tenfold.

Fig. 5. (a) 2D colormap of the resonant wavelength as a function of silver height $h_M$ and the disk radius. The unit of colorbar is nm. Black markers represent resonant conditions around 1550 nm. (b) Effective index of the whispering gallery mode as a function of $h_M$. (c) $Q$-factors and normalized mode volume as functions of $h_M$; here, $Q$, $Q_{rad}$ and $Q_{abs}$ are the cavity’s intrinsic $Q$, radiation-related and absorption related $Q$-factors, respectively. (d) Purcell factor as a function of $h_M$. 

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5. Nanodisk cavity based on LMD$_N$ waveguide with $N > 2$

In this section, the situation when increasing the number of dielectric layers is considered. The thickness of each dielectric layer in the composed LMD$_N$ ($N = 3, 4, ...$) waveguide is $h_N = 100/N$ nm, and the metal thickness $h_M$ is 10 nm. The total dielectric thickness $h_D$ is fixed at 100 nm in these cases, to be consistent with previous sections.

Firstly, WGM nanodisks based on LMD$_3$ waveguides are analyzed. Similarly as the mode splitting in a LMD$_2$ waveguide, the multi-layer coupling in a LMD$_3$ waveguide also results in three hybridized TM modes, namely TM$_1$, TM$_2$ and TM$_3$. The corresponding $E_z$ field distributions of the WGM cavities operating under these modes are shown in Fig. 7(a), and the respective cavity radii are 86 nm, 60 nm and 48 nm for an azimuthal number $m = 1$. One may find that similar as the TM$_2$ mode in a LMD$_2$ waveguide, the TM$_3$ mode in a LMD$_3$ waveguide with asymmetric $E_z$ fields across each metal separation also possesses the largest propagation constant. In fact, such reversed mode ordering applies not only for LMD$_2$ waveguide with $N = 2$ and 3, but also for $N > 3$ LMD waveguide. Generally, a LMD$_N$ waveguide can be regarded as $N$th cascading of LMD$_1$ waveguides separated by a thin metal section. As discussed in Section 2, the asymmetric coupling (anti-bonding) between adjacent LMD$_1$ modes would result in the enhancement of the propagation constant. Hence, when each LMD$_1$ mode asymmetrically couples with adjacent ones, the overall enhancement of the propagation constant is the largest. In Fig. 7(a), the WGM distributions of the nanodisks based on LMD$_N$ ($N = 4, 5, 6$ and 7) waveguides are also shown. One can see that each waveguide has an asymmetric field which has the most compact confinement. Figure 7(b) shows the selected disk radius for $m = 1$ and the WGM effective index as functions of the number of dielectric layers. When $N$ increases from 1 to 7, the corresponding disk radius drops from 92 nm to 33 nm $\sim \lambda_0 / (13.5n)$, and $n_{\text{eff}}^\text{WGM}$ increases from about 2.7 to 7.5. Then, the Q-factors and mode volume of the LMD$_N$ cavities are simulated, as shown in Fig. 7(c). On one hand, when $N$ increases from 2 to 7, $Q$ decreases gradually from 680 to 615 due to the enhanced metal absorption loss. On the other hand, $Q$ drops dramatically to $\sim 190$ for $N = 1$. 
The sudden drop can be understood by comparing the radiation Q-factors for \( N = 1 \) and \( N = 2 \) cavities. As shown in Fig. 7(c), \( Q_{\text{rad}} \) for LMD\(_1\) cavity is only 215, while \( Q_{\text{rad}} \) is as large as \( 2.86 \times 10^4 \) for a LMD\(_2\) cavity, as discussed in Section 4. Hence, for \( N > 1 \), since \( Q_{\text{rad}} \gg Q_{\text{abs}} \) for \( N > 1 \), \( Q \) is determined by \( Q_{\text{abs}} \). On the contrary, the cavity’s \( Q \) is largely limited by \( Q_{\text{rad}} \) for \( N = 1 \), resulting in the discontinuity as shown in Fig. 7(c). At last, the normalized mode volume and Purcell factor of each cavity are plotted in Fig. 7(c) and 7(d). One can see that by increasing the number of dielectric layer from 1 to 7, \( V \) can be decreased from about 0.06 to 0.005, while Purcell factor can be enhanced from around 1860 to \( 7.3 \times 10^4 \) which is about two times larger than that of a \( N = 2 \) cavity.

Note that above studied LMD\(_N\) cavities of \( h_D = 100 \) nm and \( h_M = 10 \) nm are only serving as examples to elaborate the principle of scaling the number of metal-dielectric layers. Since LMD\(_N\) has similar waveguiding principle as LMD\(_2\), varying the metal and dielectric thicknesses has similar influence on LMD\(_N\) (\( N > 2 \)) cavities as on LMD\(_2\) cavity as discussed in section 4. Namely, the bending radii and normalized mode volume decrease when reducing the silver and dielectric thicknesses, while Purcell factors increase. To fabricate the proposed LMD\(_N\) cavities, methods used to realize hyperbolic metamaterial devices can be applied. For examples, one can use either liftoff processes to deposit multiple metal-dielectric layers as explained in Ref [25], or focused ion beam to define and etch the as-deposited multi-layer structures as in Ref [32]. In practical implementations, one can balance the desired performance and needed fabrication requirements, and choose proper design of layer numbers as well as metal-dielectric thicknesses.

6. Conclusions

We propose a layered metal-dielectric (LMD) waveguide consisting of a periodic stack of \((N + 1)\) metal and \( N \) dielectric layers to achieve an ultra-compact mode confinement. The waveguiding principle, as well as the Q-factor, mode volume and Purcell factor of the nanodisk cavity based on the proposed waveguide are systematically analyzed. By utilizing a LMD\(_2\) waveguide with thicknesses of metal and dielectric layers of 10 nm and 50 nm
respectively, the radius of the disk cavity can be shrunk down to 61 nm \( \sim \frac{\lambda_0}{(7n)} \), where \( n \) is the dielectric index, and the mode volume is as small as 0.0175\( (\frac{\lambda_0}{(2n)})^3 \). Compared with a traditional metal-dielectric-metal cavity, the proposed cavity has a five times larger achievable Purcell factor around \( 2.3 \times 10^3 \) at 40 K. To provide useful design guidelines, the influence of metal and dielectric thicknesses on the cavity properties are also presented. Slight change of the disk parameter can result in significant resonance shift, and the exhibiting high sensitivity promise potential device applications in sensing. At last, we show that when maintaining the dielectric thickness at 100 nm, the disk radius can be further decreased to 33 nm \( \sim \frac{\lambda_0}{(13.5n)} \) by increasing the number of dielectric layers to 7, and the corresponding mode volume is around 0.005\( (\frac{\lambda_0}{(2n)})^3 \). The resulted Purcell factor can be further enhanced about three-fold. The proposed waveguide and whispering gallery mode cavity are an alternative approach to achieve ultra-high confinement and strong coupling cavity. Compared with traditional metal-dielectric-metal cavity, the proposed scheme has advantages including: (1) significant enhancement of confinement as well as Purcell factor at a fixed dielectric thickness, (2) convenient vertically optical pumping due to thin metal thickness, (3) design freedoms including metal thickness and the stacking number and (4) cascading capability. Hence, we believe the presented structures may find potential applications in strong light-matter interaction systems.

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