A Systematic Design of Stabilizer Controller for Interval Type-2 TSK Fuzzy Logic Systems

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ABSTRACT
This paper proposes a new method for designing a state feedback controller for stabilizing and also making the desired performance of Interval Type-2 Takagi–Sugeno-Kang Fuzzy Logic Systems (IT2 TSK FLS). By using the advantages of WU-Mendel Uncertainty Bounds (WM-UB) method, this paper proposes a new approach for Single-Input and Single-Output (SISO) and Multi-Input and Multi-Output (MIMO) interval type-2 TSK systems, which is based on the Hybrid Compensation Control (HCC) approach. The advantages of this method are no necessity to solving any Linear Matrix Inequalities (LMIs) to find a quadratic Lyapunov function for designing the stabilizer controller and also, the designed controller could compensate the time-varying variations. It should be noted that the inference engine is formulated in closed form and does not require using any type of reduction. Some examples have been conducted in our study to demonstrate the effectiveness of the proposed control design approach and compare this method with the previous approaches. All results illustrate good control performance.

1. Introduction
Fuzzy systems were first introduced by Zadeh [1,2]. By introducing Takagi–Sugeno-Kang (TSK) models, the application of fuzzy theory in system control shows a remarkable growth in popularity [3,4]. The fuzzy models are used in a wide variety of applications such as robotics [5], biomedical engineering [6] and decision making [7].

Type-1 fuzzy models help us to provide a control design in the case where only qualitative or imprecise knowledge is available. When the amount of data and knowledge imprecision increase, the efficiency of these models decrease. Therefore, the type-2 fuzzy sets were introduced by Zadeh to account for uncertainties in fuzzy models [8]. In recent years, the Type-2 Fuzzy Logic Systems (T2 FLSs) have been successfully applied in many areas due to their ability in the modelling of uncertainties [9–14]. A T2 FLS is very similar to a Type-1 Fuzzy Logic System (T1 FLS) and the major structural difference is the type-reduction block, which is added to the structure of T2 FLS. The type reduction reduces the output of T2 FLS into a type-1 fuzzy set. Karnik Mendel (KM) algorithm is one of the most important algorithms which issued for the type reduction [15,16]. This algorithm computes
the left and right endpoints which are needed to define Interval Type-2 Fuzzy Logic Systems (IT2 FLSs). In order to bypass the computational effort of KM algorithms, Wu and Mendel developed uncertainty bounds for IT2 FLSs to approximate type reduction while achieving similar results \cite{17}. By modifying the Wu-Mendel Uncertainty Bounds (WM-UBs), the stable Interval Type-2 TSK Fuzzy Logic Control Systems (IT2 TSK FLCSs) could be designed which is suited for real-time execution \cite{18}.

Common quadratic Lyapunov function is the main and fundamental progress for checking the stability of the system. By using Linear Matrix Inequalities (LMIs) in IT2 TSK FLCSs, stability conditions can be tested. Biglarbegian et al. \cite{18} have proposed an inference mechanism for IT2 TSK FLCSs when antecedents are the type-2 fuzzy sets and consequents are the crisp numbers. The mechanism has a closed mathematical form which makes it more feasible to analyze the stability of FLCSs and the stability conditions have been tested by LMIs.

Several authors have been introduced some methods for analyzing the stability of dynamic fuzzy systems and also designing a suitable controller to stabilize them. Lo and Chen \cite{19} have used Kharitonov theory to derive a sufficient condition for designing the stabilizer fuzzy controller. Johansson and Rantzer \cite{20} have used piecewise-continuous quadratic Lyapunov function and have presented a novel approach for the stability analysis of the fuzzy systems. However, the conditions for the existence of a common Lyapunov function are quite restrictive, and even no common Lyapunov function may exist for many stable fuzzy systems. Hisa et al. \cite{21} have proposed Hybrid Compensation Control (HCC) to design the stable state feedback controllers for the affine TSK fuzzy models. The proposed controller has been designed to compensate for all rules so that the desired control performance could appear in the overall system. Sonbol and Fadali \cite{22} have proposed a new approach for the stability analysis of discrete-Sugeno types 2 and 3 fuzzy systems. In this approach, the arguments similar to those of traditional Lyapunov stability theory with positive and negative definite functions have been introduced. Jafarzadeh and Fadali \cite{23} have introduced three sufficient stability conditions for the discrete-type-2 TSK fuzzy systems with interval uncertainties that do not require a common Lyapunov function and have been applicable to systems with unstable consequents. Jafarzadeh et al. in \cite{24} and \cite{25} have proposed sufficient conditions for the exponential stability of TSK fuzzy systems with type (i) antecedents and type (j) consequents as A(i)-C(j). In \cite{24} authors have introduced the stability conditions for A1-C0, A1-C1, A2-C0 and A2-C1 systems. In \cite{25}, a new controller designing methodology for A1-C0, A1-C1, A2-C0 and A2-C1 systems has been proposed which their method has been based on the stability condition and was introduced by themselves in \cite{24}. On the contrary, the above approaches may become infeasible, when the number of rules is large or when subsystems are distributed in a wide range. In this case, the upper bound of such variations may become large, and the stabilization algorithm may fail. In addition, the control performance of the closed-loop system cannot be anticipated.

In this paper, we introduce a new method to stabilize Single-Input and Single-Output (SISO) and Multi-Input and Multi-Output (MIMO) IT2 TSK FLSSs. The advantages of our approach are as follows:

- In our approach, it is not necessary to solve any LMIs to find a quadratic Lyapunov function. To design the stabilizer controller, we design the controlled system with the desired
performance by choosing the useful nominal system and feedback gain matrix for the nominal system.

- Also designed controller could compensate the time varying variations.
- The inference engine is formulated in closed form and hence does not require using any type reduction algorithm.

This paper is organized as follows: Section 2 describes some fundamental properties of TSK fuzzy systems. Section 3 presents our new approach to design stabilizer controller for SISO and MIMO IT2 FLSs. We demonstrate our method through numerical examples in Section 4. The conclusion is given in Section 5.

2. Fundamentals

2.1. Affine TSK Type-1 Fuzzy Model

Most of the systems can be described in the set of fuzzy rules and controller part can be designed for them. The considered fuzzy models can be introduced as in [26]. Therefore, consider a discrete-system as follows:

\[
\text{Rule } i: \text{If } x_1(k) \text{ is } X_1^i \text{ and } \ldots \text{ and } x_n(k) \text{ is } X_n^i \\
\text{Then } x_n(k+1) = a_0^i + a_1^i x_1(k) + \ldots + a_n^i x_n(k) + b^i u(k) \tag{1}
\]

where \(x_j(k)\) is input state \(j\) on each rule. Also, from [26] \(x_j(k) = x_{j-1}(k+1)\) for \(j = 2, \ldots, n\), \(u(k)\) is input, \(X_j^i\) are fuzzy sets of input state \(j\) in rule \(i\), and \(a_0^i, a_1^i, \ldots, a_n^i, b^i\) are the parameters for describing the input and output relationships in the \(i\)th fuzzy rule. For a continuous -TSK fuzzy model system, the \(i\)th rule is given as:

\[
\text{Rule } i: \text{If } x_1(t) \text{ is } X_1^i \text{ and } \ldots \text{ and } x_n(t) \text{ is } X_n^i \\
\text{Then } \dot{x}(t) = a_0^i + a_1^i x_1(t) + \ldots + a_n^i x_n(t) + b^i u(t) \tag{2}
\]

where \(x_j(t) = \dot{x}_{j-1}(t)\) for \(j = 2, \ldots, n\) and \(\dot{x}_{j-1}(t)\) is the time derivative of \(x_{j-1}(t)\).

2.2. SISO IT2 TSK FLSs

The general structure of an IT2 TSK A2-C0 model for discrete-systems is given as follows [15]:

\[
\text{Rule } i: \text{If } x(k) \text{ is } \tilde{F}_1^i \text{ and } \ldots \text{ and } x(k-n+1) \text{ is } \tilde{F}_n^i \\
\text{Then } x'(k+1) = a_0^i + a_1^i x(k) + \ldots + a_n^i x(n-k+1) + b^i u(k) \tag{3}
\]

where \(i = 1, \ldots, M, M\) is the number of rules, \(x'(k+1)\) is the output of the \(i\)th rule, \(\tilde{F}_j^i\) represents the IT2 FS of the \(j\)th input in \(i\)th rule, and \(a_j^i\) is the \(j\)th coefficient of the output function for the rule \(i\). In IT2 TSK A2-C0 model, the lower and upper firing strengths of the \(i\)th rule, i.e. \(f_l^i\) and \(f_u^i\), are given as follows:

\[
f_l^i(x) = \mu_{\tilde{F}_1^i}(x(k)) \ast \cdots \ast \mu_{\tilde{F}_n^i}(x(k-n+1)) \tag{4}
\]
\[
\tilde{f}^i(x) = \mu_{\tilde{F}_i}^{\tilde{F}_i}(x(k)) \ast \cdots \ast \mu_{\tilde{F}_i}^{\tilde{F}_i}(x(k - n + 1))
\]

(5)

where \( \mu_{\tilde{F}_i}^{\tilde{F}_i} \) and \( \mu_{\tilde{F}_i}^{\tilde{F}_i} \) represent the lower and upper membership degrees of the \( i \)th rule, respectively. The state vector \( x \) is defined as

\[
x = [x(k), x(k - 1), \ldots, x(k - n + 1)]^T.
\]

(6)

To develop IT2 FLCSs, the control structure must be suited for real-time execution. To satisfy this requirement, the iterative KM inference algorithm \([15,16]\) may not be suitable. Hence, to compute the final output we use WM UBs method as \([18]\):

\[
x(k + 1) = w \frac{\sum_{i=1}^{M} f^i(x)x'(k + 1)}{\sum_{i=1}^{M} f^i(x)} + z \frac{\sum_{i=1}^{M} \tilde{f}^i(x)x'(k + 1)}{\sum_{i=1}^{M} \tilde{f}^i(x)}
\]

(7)

where \( x'(k + 1) \) is given by (3), and \( f^i(x) \) and \( \tilde{f}^i(x) \) are given by (4) and (5). Notice that \( w \) and \( z \) are the design parameters which give the weights to share the lower and upper firing levels of each fired rules. Also, their bounds could be calculated according to the method which has been presented in \([18]\). Figure 1 shows the closed-loop IT2 TSK A2-C0 fuzzy control system. The inputs of the controller are the states of system \( x(k) \) and the output is \( u(k) \). For the controller, \( l \)th rule is considered as follows \([18]\):

Rule \( l \): If \( x(k) \) is \( \tilde{C}_1 \) and \ldots and \( x(k - n + 1) \) is \( \tilde{C}_n \)

Then \( u'(k + 1) = c_1^l x(k) + c_2^l x(k - 1) + \ldots + c_n^l x(n - k + 1) \)

(8)

where \( l = 1, 2, \ldots, Q \) and \( Q \) is the number of rules, \( \tilde{C}_j^l \) represent the T2 FS of the input variable \( j \) in \( l \)th rule, and \( c_j^l \) is the \( j \)th coefficient of the output function for rule \( l \). The controller output, \( u(k) \), can be demonstrated as follows:

\[
u(k) = w' \frac{\sum_{i=1}^{Q} \nu^i(x)u'(k + 1)}{\sum_{i=1}^{Q} \nu^i(x)} + z' \frac{\sum_{i=1}^{Q} \nu'(x)u'(k + 1)}{\sum_{i=1}^{Q} \nu'(x)}
\]

(9)

where \( w' \) and \( z' \) are the tuning parameters of controller, and we have:

\[
\nu^i(x) = \mu_{\tilde{C}_1}^{\tilde{C}_1}(x(k)) \ast \cdots \ast \mu_{\tilde{C}_n}^{\tilde{C}_n}(x(k - n + 1))
\]

(10)

\[
\nu'(x) = \mu_{\tilde{C}_1}^{\tilde{C}_1}(x(k)) \ast \cdots \ast \mu_{\tilde{C}_n}^{\tilde{C}_n}(x(k - n + 1))
\]

(11)

**Figure 1.** Closed-loop IT2 TSK A2-C0 fuzzy control system.
By using (9), the output of the closed-loop system, as Figure 1, is:

\[ x(k + 1) = C x(k) \]  

(12)

where

\[
C = \sum_{i = 1}^{M} A^i + \sum_{i = 1}^{M} f(x) A^i + w w' \sum_{i = 1}^{M} \sum_{j = 1}^{Q} f(x) v(x) B^{ij}
\]

\[
+ w z' \sum_{i = 1}^{M} \sum_{j = 1}^{Q} f(x) v(x) B^{ij}
\]

\[
+ z w' \sum_{i = 1}^{M} \sum_{j = 1}^{Q} f(x) v(x) B^{ij}
\]

(13)

in which \( A^i \) and \( B^{ij} \) are \( n \times n \) matrices as follows:

\[
A^i = \begin{bmatrix}
    a_1 & a_2 & \ldots & a_{n-1} & a_n \\
    1 & 0 & \ldots & 0 & 0 \\
    0 & 1 & \ldots & 0 & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    0 & 0 & \ldots & 1 & 0 \\
\end{bmatrix}
\]

(14)

\[
B^{ij} = \begin{bmatrix}
    b_1 c_1 & b_1 c_2 & \ldots & b_1 c_{n-1} & b_1 c_n \\
    1 & 0 & \ldots & 0 & 0 \\
    0 & 1 & \ldots & 0 & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    0 & 0 & \ldots & 1 & 0 \\
\end{bmatrix}
\]

(15)

It should be noted that the output vector is defined as:

\[ x(k + 1) = [x(k + 1), x(k), \ldots, x(k - n + 2)]^T. \]  

(16)

The system is asymptotically stable, if there exists a common positive-definite matrix \( P \) such as the following inequalities are satisfied [18]:

\[
a(A^i)^T P A^i < \frac{1}{36} P(0)
\]

(17)

\[
b(A^i)^T P B^{ij} + b(B^{ij})^T P A^i < \frac{1}{18} P(0)
\]

(18)

\[
c(B^{ij})^T P B^{ij} < \frac{1}{36} P(0)
\]

(19)

where \( i, j = 1, 2, \ldots, M \) and \( l, q = 1, 2, \ldots, Q \), and also:

\[
a = \{w^2, wz, z^2\}
\]

(20)

\[
b = \{w^2 w', z^2 z', wzw', w^2 z', wzz', z^2 w', z^2 z'\}
\]

(21)

\[
c = \{w^2 w^2, z^2 z^2, w^2 w' z', wzw' z', w^2 z^2, wzz'^2, wzw'^2, z^2 w^2, z^2 w' z'\}
\]

(22)
It is clear that, in this case, the number of inequalities is large and it is hard to solve LMIs to find a quadratic Lyapunov function. Further by increasing the number of rules, we have a complicated situation to find a quadratic Lyapunov function.

### 2.3. MIMO IT2 TS FLSs

Consider the general form of rule for the plant in discrete-MIMO system as follows:

Rule 1: If \( x(k) \) is \( \bar{F}_1 \) and \( \ldots \) and \( x(k - n + 1) \) is \( \bar{F}_n \)

Then \( x'(k + 1) = A'x(k) + b'u(k)i = 1, 2, \ldots, M \) \( (23) \)

where \( M \) is the number of rules, \( x'(k + 1) \) is the output in each rule, \( \bar{F}_i \) represents the IT2 FS of input variable \( j \) in rule \( i \), \( x(k) \) is the state vector which is given by \( (6) \), and \( A' \in \mathbb{R}^{n \times n} \), \( b' \in \mathbb{R}^{n \times m} \), and \( u(k) \in \mathbb{R}^m \). The output of the system, i.e. \( x(k + 1) \), is given as follows:

\[
x(k + 1) = w \sum_{i=1}^{M} \bar{f}_i(x) \{ A'x(k) + b'u(k) \} + z \sum_{i=1}^{M} \bar{f}_i(x) \{ A'x(k) + b'u(k) \} \sum_{i=1}^{M} \bar{f}_i(x) \quad (24)
\]

Suppose that the \( l \)th control rule for discrete-MIMO system is as follows \( [27] \)

Rule 1: If \( x(k) \) is \( \bar{C}_1 \) and \( \ldots \) and \( x(k - n + 1) \) is \( \bar{C}_n \) Then \( u_l(k) = F_lx(k) \) \( (25) \)

where \( F_l \) is the \( j \)th feedback gain matrix of the consequent part and \( \bar{C}_i \) represents the T2 FS of input variable \( j \) in rule \( i \). Note that the number of rules for the controller is also \( Q \). The controller output \( u(k) \) is given as \( [18] \):

\[
u(k) = w' \sum_{i=1}^{Q} \nu_i(x)F_ix(k) + z' \sum_{i=1}^{Q} \nu_i(x)F_ix(k) \sum_{i=1}^{Q} \nu_i(x) \quad (26)\]

Substituting \( (26) \) into \( (24) \), the output of the system, i.e. \( x(k + 1) \), can be expressed as

\[
x(k + 1) = w \sum_{i=1}^{M} \bar{f}_i(x)A'x(k) \sum_{i=1}^{M} \bar{f}_i(x) + z \sum_{i=1}^{M} \bar{f}_i(x)A'x(k) \sum_{i=1}^{M} \bar{f}_i(x) \]

\[
+ w' \sum_{i=1}^{Q} \nu_i(x)A'x(k) \sum_{i=1}^{Q} \nu_i(x) + z w' \sum_{i=1}^{Q} \nu_i(x)A'x(k) \sum_{i=1}^{Q} \nu_i(x) \]

\[
+ z \bar{w}' \sum_{i=1}^{Q} \nu_i(x)A'x(k) \sum_{i=1}^{Q} \nu_i(x) + z \bar{w}' \sum_{i=1}^{Q} \nu_i(x)A'x(k) \sum_{i=1}^{Q} \nu_i(x) \quad (27)\]

The purpose of discussing Section 2.2 and 2.3 is using these theorems for designing the appropriate controller as will be discussed in the next sections.

### 3. Compensation Control for IT2 TSK FLCSs

In the compensation control approach, for the affine TSK type-1 fuzzy models, by combining two different control design methodologies, the proposed controller is designed to compensate the variations of nominal system, which are shown as rules. Therefore, the desired control performance can appear in the overall system \([21]\).
3.1. The New Compensation Control Approach for SISO IT2 TSK FLCSs

Consider the feedback control system which is shown in Figure 1, where the plant and the controller are IT2 TSK A2-C0 models. Assume that, the general form of rule for the plant is as Equation (3). The output of open-loop system for Rule $i$ can be written as:

$$x'(k + 1) = A^i x(k) + B^i u(k)$$  \hspace{1cm} (28)$$

where

$$A^i = \begin{bmatrix} a_1^i & a_2^i & \cdots & a_{n-1}^i & a_n^i \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \quad B^i = \begin{bmatrix} b_1^i \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$  \hspace{1cm} (29)$$

By using (4) and (5), the output of the plant is as follows:

$$x(k + 1) = w\frac{\sum_{i=1}^{M} f_i(x)A^i x(k) + \sum_{i=1}^{M} f_i(x)B^i u(k)}{\sum_{i=1}^{M} f_i(x)} + z\frac{\sum_{i=1}^{M} f_i(x)A^i x(k) + \sum_{i=1}^{M} f_i(x)B^i u(k)}{\sum_{i=1}^{M} f_i(x)}$$  \hspace{1cm} (30)$$

The nominal system can be defined as:

$$A_0 = \begin{bmatrix} a_1^0 & a_2^0 & \cdots & a_{n-1}^0 & a_n^0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \quad B_0 = \begin{bmatrix} b_1^0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$  \hspace{1cm} (31, 32)$$

The formation of $A_0$ and $B_0$ can be arbitrary as long as they make the closed-loop system stable and the desired control performance appears in the overall system. Therefore, for this study nominal system should be selected in the way that it makes $h_1(x)$ system stable. Usually, a simple arithmetic average is taken to determine the elements of nominal system matrices and that is $a_i^0 = (\text{Max}_i\{a_j^i\} + \text{Min}_i\{a_j^i\})/2$ and $b_i^0 = (\text{Max}_i\{b_i^i\} + \text{Min}_i\{b_i^i\})/2$. The
differences between \((A^i, B^i)\) and \((A_0, B_0)\) are:

\[
A_i = A^i - A_0 = \begin{bmatrix}
a_{1i} & a_{2i} & \cdots & a_{(n-1)i} & a_{ni} \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0
\end{bmatrix}
\]

(33)

\[
B_i = B^i - B_0 = \begin{bmatrix}
b_i \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

(34)

where \(a_{ji} = a^i_j - a^0_j\) and \(b_i = b^i - b^0\). Then relation (28) can be written as

\[
x(k + 1) = w \sum_{i=1}^{M} f^i(x) (A_i + A_0) x(k) + w \sum_{i=1}^{M} f^i(x) (B_i + B_0) u(k) \\
+ z \sum_{i=1}^{M} f^i(x) A_i x(k) + z \sum_{i=1}^{M} f^i(x) B_i u(k)
\]

(35)

Because \((A_0, B_0)\) are related to the nominal system, we can write:

\[
x(k + 1) = (w + z) (A_0 x(k) + B_0 u(k)) + W_1 (x(k), u(k))
\]

(36)

where \(W_1 (x(k), u(k))\) is regarded as the uncertainties of open-loop system and is described as follows:

\[
W_1 (x(k), u(k)) = w \sum_{i=1}^{M} f^i(x) A_i x(k) + w \sum_{i=1}^{M} f^i(x) B_i u(k) \\
+ z \sum_{i=1}^{M} f^i(x) A_i x(k) + z \sum_{i=1}^{M} f^i(x) B_i u(k)
\]

(37)

In our approach, for this system, the general form of rule for the controller is as (25) and the controller output is as (26). The feedback control law is as

\[
u = G_0 x + G_c (x)
\]

(38)

where \(G_0\) is the gain matrix to control the nominal system and \(G_c (x)\) compensate the variations in all rules individually. As \(G_0\) is related to the nominal system we have,

\[
u(k) = (w' + z') G_0 x(k) + G_c (x)
\]

(39)
Substituting (39) into (36), the output of the system, i.e. \( x(k+1) \), can be expressed as follows:

\[
x(k+1) = (w+z)(A_0 x(k) + B_0 (w' + z')G_0 x(k) + B_0 G_c(x)) + w \frac{\sum_{i=1}^{M} f_i' (x) A_i x(k)}{\sum_{i=1}^{M} f_i' (x)} + z \frac{\sum_{i=1}^{M} f_i' (x) B_i x(k)}{\sum_{i=1}^{M} f_i' (x)} + w \frac{\sum_{i=1}^{M} f_i' (x) B_i G_c(x)}{\sum_{i=1}^{M} f_i' (x)} + z \frac{\sum_{i=1}^{M} f_i' (x) B_i G_0 x(k)}{\sum_{i=1}^{M} f_i' (x)} + z \frac{\sum_{i=1}^{M} f_i' (x) B_i G_c(x)}{\sum_{i=1}^{M} f_i' (x)}
\]

(40)

Suppose that \( \tilde{A}^i = \left[ \begin{array}{ccc} a_{1i} & a_{2i} & \cdots & a_{ni} \\ \frac{\partial f_i(x)}{\partial x_1} & \frac{\partial f_i(x)}{\partial x_2} & \cdots & \frac{\partial f_i(x)}{\partial x_n} \end{array} \right] \) and \( \tilde{B}^i = \frac{b_i}{\partial x_i} \), then

\[
x(k+1) = (w+z)(A_0 + B_0 (w' + z')G_0) x(k) + B_0 \left[ \left( w \frac{\sum_{i=1}^{M} f_i' (x) \tilde{A}^i}{\sum_{i=1}^{M} f_i' (x)} + z \frac{\sum_{i=1}^{M} f_i' (x) \tilde{A}^i}{\sum_{i=1}^{M} f_i' (x)} \right) \right) x(k) + w((w' + z')) \frac{\sum_{i=1}^{M} f_i' (x) \tilde{B}^i G_0}{\sum_{i=1}^{M} f_i' (x)} + z((w' + z')) \frac{\sum_{i=1}^{M} f_i' (x) \tilde{B}^i G_0}{\sum_{i=1}^{M} f_i' (x)} + (w + z) G_c(x) \]  

(41)

Now, we can write:

\[
x(k+1) = \tilde{A}_0 x(k) + B_0 h_1(x)
\]

(42)

where \( \tilde{A}_0 \) and \( h_1(x) \), respectively, are regarded as the nominal system and the uncertainties of the closed-loop system as follows:

\[
\tilde{A}_0 = (w+z)(A_0 + B_0 (w' + z')G_0)
\]

(43)

\[
h_1(x) = \left( w \frac{\sum_{i=1}^{M} f_i' (x) \tilde{A}^i}{\sum_{i=1}^{M} f_i' (x)} + z \frac{\sum_{i=1}^{M} f_i' (x) \tilde{A}^i}{\sum_{i=1}^{M} f_i' (x)} + w((w' + z')) \frac{\sum_{i=1}^{M} f_i' (x) \tilde{B}^i G_0}{\sum_{i=1}^{M} f_i' (x)} + z((w' + z')) \frac{\sum_{i=1}^{M} f_i' (x) \tilde{B}^i G_0}{\sum_{i=1}^{M} f_i' (x)} + (w + z) G_c(x) \right)
\]

(44)

In order to have a stable system, there must exist symmetry positive definite matrices \( P \) and \( Q \) such that [21]

\[
\tilde{A}_0^T P \tilde{A}_0 - P = -Q
\]

(45)

Define \( v(x(k)) = x^T (k) P x(k) \). Thus, we can write:

\[
\Delta v(x(k)) = -x^T (k) Q x(k) + h_1(x)[\alpha h_1(x) + 2 \beta(x)]
\]

(46)
where $\alpha = \tilde{B}_0^T \rho \tilde{B}_0^T$ and $\beta(x) = \tilde{B}_0^T \rho \tilde{A}_0 x(k)$. It is easy to substantiate that when $h_1(x)[\alpha h_1(x) + 2\beta] \leq 0$, system is stable. Therefore, by making $h_1(x) = 0$, the system can be stable. Compensation part ($G_c(x)$) can be selected in the way that it makes equal to zero. Therefore, $G_c(x)$ can be selected as (47) and the validity of this selection can be verified by replacing it on (44) and finding zero as an outcome.

\[
G_c(x) = - \frac{\left( w \frac{\sum_{i=1}^{M} F_i^i (x) \bar{A}_i}{\sum_{i=1}^{M} F_i (x)} + z \frac{\sum_{i=1}^{M} F_i (x) \bar{A}_i}{\sum_{i=1}^{M} F_i (x)} + w((w' + z')) \right) \sum_{i=1}^{M} F_i^i (x) \bar{B}_i G_0}{w \frac{\sum_{i=1}^{M} F_i^i (x) \bar{B}_i}{\sum_{i=1}^{M} F_i (x)}} + z((w' + z')) \frac{\sum_{i=1}^{M} F_i (x) \bar{B}_i G_0}{\sum_{i=1}^{M} F_i (x)} \right) \bar{A}_0 x(k) \right) (47)
\]

Therefore, by determining a useful nominal system, we can stabilize the overall system. Consequently, $G_c(x)$ could compensate the time-varying variations.

**Lemma 3.1:** Consider the fuzzy model of system with the general form of rule for the plant as (3) and the rule base for the controller as (25). With the compensation part of the control law as (47), the equation $h_1(x) = 0$ holds and the system is stable. If the system has similar membership functions in its antecedent parts and by considering that fact that

\[
G_c(x) = w \sum_{i=1}^{Q} \frac{\nabla^i (x) G_c^i x(k)}{\sum_{i=1}^{Q} \nabla^i (x)} + z \sum_{i=1}^{Q} \frac{\nabla^i (x) G_c^i x(k)}{\sum_{i=1}^{Q} \nabla^i (x)} \right) (48)
\]

Therefore, by supposing that the eigenvalues assigned for each subsystem are the same as that for the nominal system, then the feedback gain matrix for $i$th rule can be approximated by:

\[
G_i = (G_0 - \frac{(w + z)(\tilde{A}_i^i + \tilde{B}_i^i (w' + z'))}{(w' + z')(1 + \tilde{B}_i^i (w + z))}) x(k) \right) (49)
\]

### 3.2. The New Compensation Control Approach for MIMO IT2 TSK FLCSs

In this case, the $i$th rule for the plant is ($i = 1, \ldots, M$)

**Rule i:** If $x(k)$ is $\tilde{F}_{i}^i$ and $\ldots$ and $x(k - n + 1)$ is $\tilde{F}_{n}^i$ Then $x^{i}(k + 1) = A_i^i x(k) + B_i^i u(k) \right) (50)$

where

\[
A_i^i = \begin{bmatrix}
  a_{i1}^i & a_{i2}^i & \cdots & a_{in}^i \\
  a_{i1}^i & a_{i2}^i & \cdots & a_{in}^i \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{in}^1 & a_{in}^2 & \cdots & a_{in}^n \\
\end{bmatrix} \right) \right) (51)
\]

\[
b_i^i = \begin{bmatrix}
  b_{i1}^i & b_{i2}^i & \cdots & b_{im}^i \\
  b_{i1}^i & b_{i2}^i & \cdots & b_{im}^i \\
  \vdots & \vdots & \ddots & \vdots \\
  b_{in}^1 & b_{in}^2 & \cdots & b_{in}^m \\
\end{bmatrix} \right) \right) (52)
\]
where \( t \) is the number of outputs and \( s \) is the number of inputs. As mentioned in the previous part, first we introduce the nominal system as

\[
A_0 = \begin{bmatrix}
da_{01} & a_{02} & \cdots & a_{0n} \\
da_{11} & a_{12} & \cdots & a_{1n} \\
da_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
da_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}_{t \times t}
\]

(53)

\[
B_0 = \begin{bmatrix}
b_{01} & b_{02} & \cdots & b_{0m} \\
b_{11} & b_{12} & \cdots & b_{1m} \\
b_{21} & b_{22} & \cdots & b_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
b_{n1} & b_{n2} & \cdots & b_{nm}
\end{bmatrix}_{t \times s}
\]

(54)

The differences between \((A^i, B^i)\) and \((A_0, B_0)\) are

\[
A_i = A^i - A_0 = \begin{bmatrix}
\Delta a_{11}^i & \Delta a_{12}^i & \cdots & \Delta a_{1n}^i \\
\Delta a_{21}^i & \Delta a_{22}^i & \cdots & \Delta a_{2n}^i \\
\vdots & \vdots & \ddots & \vdots \\
\Delta a_{n1}^i & \Delta a_{n2}^i & \cdots & \Delta a_{nn}^i
\end{bmatrix}_{t \times t}
\]

(55)

\[
B_i = B^i - B_0 = \begin{bmatrix}
\Delta b_{11}^i & \Delta b_{12}^i & \cdots & \Delta b_{1m}^i \\
\Delta b_{21}^i & \Delta b_{22}^i & \cdots & \Delta b_{2m}^i \\
\vdots & \vdots & \ddots & \vdots \\
\Delta b_{n1}^i & \Delta b_{n2}^i & \cdots & \Delta b_{nm}^i
\end{bmatrix}_{t \times s}
\]

(56)

where \( \Delta a_{jk}^i = a_{jk}^i - a_{jk}^0 \) and \( \Delta b_{jk}^i = b_{jk}^i - b_{jk}^0 \). By using compensation control approach for the closed-loop system, similar to the previous Section (3.1), the compensation part of the controller could be calculated by

\[
G_c(x) = -\frac{\left( w \sum_{i=1}^{M} f(x) \Delta \tilde{A}^i + z \sum_{i=1}^{M} f(x) \Delta \tilde{B}^i + w((w' + z')) \sum_{i=1}^{M} f(x) \Delta \tilde{G}^i G_0 + z((w' + z')) \sum_{i=1}^{M} f(x) \Delta \tilde{B}^i G_0 \right)}{w \sum_{i=1}^{M} f(x) \Delta \tilde{B}^i + z \sum_{i=1}^{M} f(x) \Delta \tilde{B}^i + (w + z)G_0} x(k)
\]

(57)

For calculating \( \Delta \tilde{A}^i \) and \( \Delta \tilde{B}^i \), we have three cases:

(1) \( s = t \); where the number of outputs is the same as the number of inputs, then

\[
(\Delta \tilde{A}^i)_{s \times t} = (B_0^{-1})_{s \times t} \times (A_i)_{t \times t}
\]

(58)

\[
(\Delta \tilde{B}^i)_{s \times s} = (B_0^{-1})_{s \times t} \times (B_i)_{t \times s}
\]

(59)

Note that, \( B_0 \) must be invertible.
\( s < t \); in this case, the number of outputs is more than the number of inputs, then by using the virtual inverse theorem, we have:

\[
\begin{align*}
\Delta \tilde{A}^i_{s\times t} &= (B_0^T B_0)^{-1}_{s\times s} \times (B_0^T)_{s\times t} \times (A_i)_{t\times t} \\
\Delta \tilde{B}^i_{s\times s} &= (B_0^T B_0)^{-1}_{s\times s} \times (B_0^T)_{s\times t} \times (B_i)_{t\times s}
\end{align*}
\]  

(60)  

(61)

Note that \( \Delta \tilde{A}^i \) and \( \Delta \tilde{B}^i \) must satisfy the following conditions:

\[
\begin{align*}
||A_i - (B_0 \times \Delta \tilde{A}^i)||_2 &\rightarrow 0 \\
||B_i - (B_0 \times \Delta \tilde{B}^i)||_2 &\rightarrow 0
\end{align*}
\]  

(62)  

(63)

\( s > t \); when the number of inputs is more than the number of outputs, by using the virtual inverse theorem, we have:

\[
\begin{align*}
\Delta \tilde{A}^i_{s\times t} &= (B_0^T)_{s\times t} \times (B_0 B_0^T)^{-1}_{t\times t} \times (A_i)_{t\times t} \\
\Delta \tilde{B}^i_{s\times s} &= (B_0^T)_{s\times t} \times (B_0 B_0^T)^{-1}_{t\times t} \times (B_i)_{t\times s}
\end{align*}
\]  

(64)  

(65)

Lemma 3.2: Consider the fuzzy model of the system with the general form of rule for the plant as (49) and rule base for the controller as (25) with similar membership function in their antecedent parts. With the compensation part of the control law as (57), the equation \( h_1(x) = 0 \) holds and the system is stable. By supposing (48) and the same eigenvalues for the nominal system and each subsystem, the feedback gain matrix for \( l \)th rule can be approximated by

\[
G_l = (G_0 - \frac{(w + z)(\Delta \tilde{A}^i + \Delta \tilde{B}^i (w' + z'))}{(w' + z')(1 + \Delta \tilde{B}^i (w + z))})x(k)
\]

(66)

Note: Similarly, we can derive all conditions and lemmas for the continuous-time systems.

4. Examples

This section provides numerical examples that demonstrate our control design. There are three case studies, which are discrete-SISO, discrete-MIMO and finally continuous-MIMO IT2 TSK FLSs. Moreover, in the first and second examples, we compare our approach with prior methods.

Example 4.1: Consider the following SISO interval type-2 TSK fuzzy logic control system [18]:

Rule 1: If \( x(k) \) is \( \tilde{F}^1 \) and \( x(k - 1) \) is \( \tilde{F}^2 \) Then \( x^1(k + 1) = 2.3x(k) - 2x(k - 1) + 0.7u(k) \)

Rule 2: If \( x(k) \) is \( \tilde{F}^2 \) and \( x(k - 1) \) is \( \tilde{F}^1 \) Then \( x^2(k + 1) = 1.5x(k) - x(k - 1) + 0.01u(k) \)

This system has two control rules as follows:

Rule 1: If \( x(k) \) is \( \tilde{C}^1 \) and \( x(k - 1) \) is \( \tilde{C}^2 \) Then \( u^1(k + 1) = -0.9x(k) - 1.08x(k - 1) \)

Rule 2: If \( x(k) \) is \( \tilde{C}^2 \) and \( x(k - 1) \) is \( \tilde{C}^1 \) Then \( u^2(k + 1) = 1.4x(k) - 2.1x(k - 1) \)
The fuzzy sets for the plant, $\tilde{F}^1$ and $\tilde{F}^2$, and for the controller, $\tilde{C}^1$ and $\tilde{C}^2$, are shown in Figure 2. The $A^i$ and $B^{ij}$ matrices, according to (14) and (15), are obtained as follows:

\begin{align*}
A^1 &= \begin{bmatrix} 2.3 & -2 \\ 1 & 0 \end{bmatrix}, \\
A^2 &= \begin{bmatrix} 1.5 & -1 \\ 1 & 0 \end{bmatrix}, \\
B^{1,1} &= \begin{bmatrix} -0.63 & -0.756 \\ 1 & 0 \end{bmatrix}, \\
B^{1,2} &= \begin{bmatrix} -0.009 & -0.011 \\ 1 & 0 \end{bmatrix}, \\
B^{2,1} &= \begin{bmatrix} 0.98 & -1.47 \\ 1 & 0 \end{bmatrix}, \\
B^{2,2} &= \begin{bmatrix} 0.014 & -0.021 \\ 1 & 0 \end{bmatrix}.
\end{align*}

By using the method of [18], for analyzing the stability of the system and designing the proper controller, we need to check 204 LMIs, which make a difficult situation for analyzing and designing. Note that in this example we consider $w$ and $z$, which are the plant parameters, are given as $w = 0.1$ and $z = 0.1$ and the controller tuning parameters $w'$ and $z'$, both of them are 0.2. The bounds of the tuning parameters have been calculated based on the method introduced in [18]. To use our approach for SISO IT2 TSK FLCSs, first we introduce the nominal system as follows:

\begin{align*}
A_0 &= \begin{bmatrix} 1.9 & -1.5 \\ 1 & 0 \end{bmatrix}, \\
B_0 &= \begin{bmatrix} 0.355 \\ 0 \end{bmatrix}.
\end{align*}
Suppose that the initial conditions are \( x(1) = 0.1 \) and \( x(2) = 0.01 \), by using the nominal gain matrix as \( G_0 = \begin{bmatrix} 0.6 & 2.8 \end{bmatrix} \), we can design a new stabilizer controller for this system with a good performance. By using this gain matrix, the controller gain matrices for each rule are designed by (49) and the fuzzy rules are as follows:

Rule 1 : If \( x(k) \) is \( \tilde{C}^1 \) and \( x(k - 1) \) is \( \tilde{C}^2 \) Then \( u_1^1(k + 1) = 1.7x(k) - 0.4x(k - 1) \)

Rule 2 : If \( x(k) \) is \( \tilde{C}^2 \) and \( x(k - 1) \) is \( \tilde{C}^1 \) Then \( u_2^2(k + 1) = -12x(k) + 2.8x(k - 1) \)

The response of the system is shown in Figure 3. It is clear that the proposed controller has better performance than the controller of [18]. It is clear that in the new method the settling time and the value of undershooting is very low. It must be noted that in our new method, it is not necessary to analyze any LMIs to find a quadratic Lyapunov function, while in the method of [18], we should solve 204 LMIs.

**Example 4.2:** In this case study, we apply our approach to design the stabilizer controller to track a predefined trajectory by the car. In Figure 4, \( x_0 \) is the angle that the car makes with the horizontal axis, which is given in radians and \( x_1 \) is the vertical position of the rear end of the car. The control objective is to track the car model from the given initial location to the position, where \( x_0 = x_1 = 0 \) with no backward movement [18].

Consider the following rules for the plant [18]:

Rule 1 : If \( x_0(k) \) is 'about 0,' then \( x(k + 1) = A_1x(k) + b_1u(k) \)

Rule 2 : If \( x_0(k) \) is 'about \( \pi \) or -\( \pi \),' then \( x(k + 1) = A_2x(k) + b_2u(k) \)

**Figure 3.** Closed-loop system response using different controllers.
Figure 4. Coordinate system used to describe the car position and orientation.

Figure 5. Type-2 fuzzy sets for Example 2.

where

\[
A_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 \\ 0.003183 & 1 \end{bmatrix}
\]

\[
b_1 = \begin{bmatrix} 0.357143 \\ 1 \end{bmatrix} \quad b_2 = \begin{bmatrix} 0.357143 \\ 1 \end{bmatrix}
\]

The fuzzy sets are shown in Figure 5. The controller is designed as follows:

Rule 1 : If \( x_0(k) \) is ‘about 0,’ then \( u(k) = f_1x(k) \)

Rule 2 : If \( x_0(k) \) is ‘about \( \pi \) or \( -\pi \),’ then \( u(k) = f_2x(k) \)
wherein [27] the gain matrices were obtained by pole-placement method as

\[
\begin{align*}
  f_1 &= \begin{bmatrix} -0.4212 & -0.02933 \end{bmatrix} \\
  f_2 &= \begin{bmatrix} -0.0991 & -0.00967 \end{bmatrix}
\end{align*}
\]

Note that to analyze the stability of the system and to design the proper controller, we need to check 5 LMIs [18]. Suppose that the initial conditions are \( x_0(0) = \pi \) radian and \( x_1(0) = 20 \) meter and the tuning parameters are \( w = -0.8 \), \( z = 0.7 \), \( w' = 0.5 \) and \( z' = 1.9 \). In order to use our approach for MIMO IT2 TSK FLCSs, first we introduce the nominal system and the nominal gain matrix as follows:

\[
\begin{align*}
  A_0 &= \begin{bmatrix} 1 & 0 \\ 0.5016 & 1 \end{bmatrix} \\
  B_0 &= \begin{bmatrix} 0.3571 \\ 0 \end{bmatrix} \\
  G_0 &= \begin{bmatrix} 0.71 & -0.38 \end{bmatrix}
\end{align*}
\]

By using this nominal system and our approach to design the stabilizer controller for MIMO IT2 TSK FLCSs, we can design a new controller for this system with gain matrices as follows:

\[
\begin{align*}
  f_1 &= \begin{bmatrix} 0.5258 & -0.38 \end{bmatrix} \\
  f_2 &= \begin{bmatrix} 0.8942 & -0.38 \end{bmatrix}
\end{align*}
\]

which has better performance and lower settling time and lower undershoot. Figures 6 and 7, show the performance of closed-loop system with the controller of [18] and our designed controller.

**Example 4.3:** In this example, we apply our approach to design the controller to stabilize an inverted pendulum by applying a horizontal force to the system which is an example of a benchmark problem often used in the design of controllers. Figure 8 shows an inverted pendulum located on the cart. The inverted pendulum system has nonlinear dynamics, and

![Figure 6. Angular position of the car model using two controllers.](image)
Figure 7. Trajectories of the car model using two controllers.

Figure 8. Inverted pendulum.

the equations of motion are given as follows:

\[
\dot{x}_1(t) = x_2(t)
\]

\[
\dot{x}_2(t) = \frac{gsin(x_1(t)) - amlx_2^2(t)sin(2x_1(t))/2 - acos(x_1(t))u(t)}{4l/3 - amlcos^2(x_1(t))}
\]

where \(x_1(t)\) and \(x_2(t)\) are the angular position and the velocity of the pendulum mass respectively, \(M\) is the cart mass, \(2l\) is the length of the pendulum, and \(a \equiv 1/(m + M)\). To avoid these nonlinear equations, the system is modelled as interval type-2 TSK fuzzy system with rules for the plant, which are as follows [18]

\(R_1\) : If \(x_1(t)\) is ‘about 0,’ then \(\dot{x}(t) = A_1x(t) + b_1u(t)\)

\(R_2\) : If \(x_1(t)\) is ‘about \(\pi/2\) or \(-\pi/2\),’ then \(\dot{x}(t) = A_2x(t) + b_2u(t)\)
Table 1. Some selected controller tuning parameters and their gain matrices.

| w'  | z'  | f₁       | f₂       |
|-----|-----|----------|----------|
| 1   | 0.9 | [97.8, 11.83] | [74.11, 11.83] |
| 2.45 | −0.38 | [96.82, 11.83] | [75.09, 11.83] |
| 1.72 | 1.32 | [123.75, 16.02] | [108.94, 16.02] |

where

\[
A_1 = \begin{bmatrix} 0 & 1 \\ 17.3118 & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 1 \\ 9.3696 & 0 \end{bmatrix} \\
\]

\[
b_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix} \quad b_2 = \begin{bmatrix} 0 \\ -0.0052 \end{bmatrix}
\]

The structure of the controller is given as

\[
R_1 : \text{If } x_1(t) \text{ is ‘about 0,’ then } u(t) = f_1 x(t)
\]

\[
R_2 : \text{If } x_1(t) \text{ is ‘about } \pi/2 \text{ or } -\pi/2, \text{ then } u(t) = f_2 x(t)
\]

Define \( x(t) = [x_1(t), x_2(t)]^T \), where \( x_1(t) \) and \( x_2(t) \) are the state variables, i.e. the angular position and the velocity of pendulum. Figure 9 shows the membership functions of antecedent parts.

By using our method for MIMO IT2TSK FLCSs, we try to design the controller in order to stabilize this system. To begin with, the nominal system and the nominal gain matrix are introduced

\[
A_0 = \begin{bmatrix} 0 & 1 \\ 13.3406 & 0 \end{bmatrix} \quad B_0 = \begin{bmatrix} 0 \\ -0.0909 \end{bmatrix} \\
G_0 = \begin{bmatrix} 167 & 23 \end{bmatrix}
\]

\(^1\) \( x_1 \) is given in radians.
Then by using (66) we can design the controller for this system. Table 1 shows the different controllers for different $w'$ and $z'$. Suppose that the initial conditions are $x_1(0) = 0.105$ and $x_2(0) = 1$, the performance of the closed-loop system is shown in Figures 10 and 11. It is clear that for $w' = 2.45$ and $z' = 1.38$, the system has the best performance and lowest settling time. Therefore by selecting proper $w'$ and $z'$ system can have a faster convergence rate.

5. Conclusion

In this paper, we proposed a new method for designing a suitable controller for SISO and MIMO interval type-2 TSK fuzzy logic systems. The main novelty of this approach is that it does not need to solve any LMI to find a quadratic Lyapunov for designing the stabilizer.
controller. Also, the designed controller could compensate for the time-varying variations of the nominal system. Also, the designer can select variables and volumes in the way that overall systems have a faster convergence rate, as applied to several different systems. The inference engine is formulated in closed form and hence does not require using any type reduction algorithms. Various examples as SISO and MIMO systems are conducted in this paper. All results showed good control performance with our proposed controller.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

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