A Novel Concept for Mass as Complex-Mass towards Wave-particle Duality

R. C. Gupta ¹, Anirudh Pradhan ² and Sushant Gupta ³

¹ Institute of Technology (GLAITM), Mathura-281 406, India
E-mail: rcg.iet@hotmail.com, rcgupta@glaitm.org

² Department of Mathematics, Hindu Post-graduate College, Zamania-232 331, Ghazipur, India
E-mail: acpradhan@yahoo.com, pradhan@iucaa.ernet.in

³ Department of Physics, University of Lucknow, Lucknow-226 007, India
E-mail: sushant1586@gmail.com

Abstract

In the present paper a new concept is introduced that: ‘mass is a complex quantity’. The concept of complex-mass suggests that the total mass M of a moving body is complex sum of: (i) the real-part (grain or rest) mass \( m_g \) establishing its particle behavior and (ii) the imaginary-part mass \( m_p \) governing its wave properties. Mathematically, the complex mass \( M = m_g + im_p \); the magnitude \( |M| = (m_g^2 + m_p^2)^{1/2} \). The theory proposed here explains successfully several effects such as ‘Compton effect’ and ‘refraction of light’ which could not be explained otherwise by a single theory of wave or particle. Also explained are ‘Doppler effect for light’, ‘photo-electric effect’, ‘Uncertainty principle’, ‘Relativity’ and ‘supersymmetry’.

Key words: Complex mass, Wave particle duality, Photon scattering, Compton effect, Relativity.
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1 Introduction

The longest controversial issue in the scientific history is perhaps over the fundamental question of Nature that whether light is wave or particle? Huygens’ wave-theory succeeded over Newton’s corpuscular theory, as the wave theory explained successfully several important phenomena such as refraction and interference whereas particle theory could not. However, in the beginning of 20th century the particle-theory of light (photon) emerged again as it explained clearly
some observations such as photoelectric effect and Compton effect whereas the
wave-theory failed to do so. In order to resolve the controversy, de-Broglie [1]
proposed the hypothesis for wave-particle duality suggesting that light is both,
particle and wave. Moreover he also suggested that a moving electron too can
exhibit wave properties and it was experimentally proved to be so. However,
the wave-particle dilemma especially for light still persists [2 6] in some ways.
It is not clear as to why electromagnetic radiation behaves as particle in one
experiment and as wavy in another experiment. Both the particle and wave
aspects have never been observed simultaneously, as if one aspect comes into
being only if the other aspect is absent and vice-versa. Attempt to identify one
aspect vanishes the other [2 6]. In an attempt to resolve the dilemma, the novel
concept of ‘complex mass’ is presented, and its applicability to several situations
are demonstrated in this paper.

2 Need for the new concept of mass

Many arguments could be given in favour of the need for the new concept of
mass. However, the author would state/quote a few, as follows:

(i) Mass and energy are no more separate quantities, but are interwoven by
Einstein’s mass-energy equation. This may imply that mass loses its indepen-
dent identity and requires a new interpretation.

(ii) It is known, as Ugarov [7] mentions in his book on special relativity that
‘rest mass of a system exceeds the sum of rest masses of constituent particles by
a certain amount estimated in the reference frame in which the total momen-
tum is zero’. He states further that ‘rest mass is not additive quantity’. Such
a property of mass is uncommon in classical mechanics. It is tempting to bring
in some new definition of mass for constituent particle.

(iii) Is mass a scalar-real-quantity, a vector-imaginary-quantity or a complex-
quantity? Rest mass is definitely a real physical quantity. Photon however
has a zero real-physical rest-mass though it has a total mass \((h\nu/c)/c\) due
to its momentum vector \(h\nu/c\), as if it is due to the mass residing therein as
imaginary-part. What about the mass of a moving body having a rest-mass
and momentum-vector? The quaternion was first proposed by Hamilton [8]
as a sum of scalar plus vector [8 - 10]; is mass ‘a quaternion’ for a moving body?
Yes, it seems to be so [11]. Alternatively; the representation of a moving mass
as ‘a complex-quantity’ is more meaningful for its familiarity and ease of writing
& comprehension, and that is what is done in the present paper as a better way
than the earlier paper [11]. Anyway, the mathematics [8 - 10] of ‘quaternion’
and ‘complex number’ are almost equivalent.

(iv) Both the aspects of a moving body i.e., the corpuscular (material) nature
and its wave behavior have never been observed simultaneously. Any attempt
to identify one aspect vanishes the other aspect, as if these two aspects come out of two different entities (say, real scalar-part and imaginary vector-part). Could mass be considered as ‘complex quantity’, the real part of it representing the material-content (rest-mass), and the imaginary part (due to its momentum) governing its wave aspect?

(v) If introduction of a new concept of mass as ‘complex quantity’ could explain various phenomena and could eliminate/minimize ambiguity; it is worth doing. The new concept may appear to be a bit speculative at first glance, but as it will be seen later that its success and capability to explain several diverse phenomena is striking. All avenues for the ‘Truth’ must be kept open.

3 Complex-mass concept

For the ‘complex mass concept’ introduced here it is proposed that ‘mass is a complex quantity’ and that the total mass \( M \) of a moving body has two components: (i) a real-part (particle at relatively-rest) ‘grain-mass’ \( m_g \) and (ii) an imaginary-part ‘photonic mass’ \( m_p \) due to its momentum. This may tentatively be considered as a postulate for the time being. However, it will be evident later that this novel concept of the ‘complex-mass’ has the potential to explain several phenomena without any real contradiction with the present status of formulation in physical sciences of concern.

The ‘total complex-mass’ \( M \) could be written as the complex sum of \( m_g \) and \( m_p \) as follows:

\[
M = m_g + im_p.
\]  

(1)

When the particle-momentum is zero (particle at rest) its photonic (imaginary-part) mass is zero, whereas if its velocity is \( c \) (as for photon) its rest or grain (real-part) mass is zero. Taking \( x \)-axis corresponding to the rest or gain mass and the \( y \)-axis corresponding to the photonic mass, a diagrammatic representation of complex mass is suggested in Fig. 1.

Total ‘magnitude’ \( |M| \) of the complex-mass \( M \) is ‘Pythagorean-sum’ of the constituents as follows as per property of the complex-number,

\[
|M| = (m_g^2 + m_p^2)^{\frac{1}{2}}.
\]  

(2)

and the ‘grain-photon phase angle’ \( \phi \) (Fig.1) is defined as,

\[
\cos \phi = \frac{m_g}{|M|} \text{ and } \sin \phi = \frac{m_p}{|M|}.
\]  

(3)

Though the ‘grain-photon angle \( \phi \)’ is different from the ‘Schrodinger’s wave-function \( \psi \)’, but seems to have some correlation; greater value of \( \phi \) implies that the particle is more ‘wavy’ than ‘grainy’ i.e., it has more wave-aspects.
Figure 1: Complex-mass representation

\[ M = m_g + i m_p \]

\[ |M| = \left( m_g^2 + m_p^2 \right)^{1/2} \]
Particle’s relative speed is \( v \) and photon’s speed is \( c \). The ‘total’ momentum of the particle is \( |M|/v \). It is considered that the rest (real-part) mass does not take any momentum and the total momentum is taken up by the photonic (imaginary-part) mass \( (m_p) \). Hence, \( |M|/v = m_p c \) or
\[
m_p = |M|/c.
\] (4)
From equations (2) and (4) the following equation is obtained,
\[
|M| = \frac{m_g}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}.
\] (5)
which may also be rewritten as
\[
m_g = |M| \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}.
\] (6)
The cross-relationship between the magnitude-of-mass \( |M| \) and photonic-mass \( m_p \) (as in Eqs. 2 & 4) may appear to be interestingly intriguing showing dependence of one on the other, but is okay since it includes the relativistic effect as reflected in Eqs. (5) and (6).

If \( \phi = 0^\circ \), the particle is a real-particle (grain at rest) as in Fig.1; if \( \phi = 90^\circ \), the particle is in-particle-sense an imaginary-particle (photon in flight). For a particle moving with a velocity \( (v < c) \), \( 0^\circ < \phi < 90^\circ \) and it has dual (complex) nature. Mathematically (from Eqs. 1, 4, 5 & 6),
\[
M = m_g + im_p = m_g + i|M|/c = |M| \left[\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} + i\frac{v}{c}\right].
\] (7)

### 3.1 de-Broglie Wavelength
Photonic-momentum \( p(= m_p c = |M|/v) \) and photonic-energy \( E(= h\nu) \) are as usual related by \( E = pc = |M|/vc \). Rest-mass does not take any momentum, total momentum is taken up by photonic mass \( m_p \) leading to the de-Broglie wavelength \( \lambda \) (Eq. 8) as under: as \( p = |M|/v = m_p c = E/c = h\nu/c \) and that \( c = \nu\lambda \), where \( h \) is the Planck’s constant and \( \nu \) is the frequency, \( \lambda \) is the wavelength and \( c \) is the speed of light,
\[
\lambda = \frac{h}{(|M|/v)}.
\] (8)

### 3.2 Complex-Mass of a moving Particle and The Two Ways of its Transformation:
#### 3.2.1 Self-inspired self-conversion of rest-mass to photonic-mass without any external agency
Consider a particle at rest (point A in Fig. 2a) having only the material-like grain-mass and no wave-like photonic-mass. So, total mass \( |M| = m_g \).
Now think that this stationary particle suddenly (self-inspired) starts moving at velocity \( v \) (point \( A' \) in Fig. 2b) thus gaining photonic mass \( m_p = |M| \frac{v}{c} \) at the expense of its own grain-mass which is reduced to a new lower value \( m_g' \); total mass \( |M| \) remains same as no external agency is involved in it. Complex mass (for Fig. 2b) is thus expressed as

\[
M = m_g' + im_p.
\]  

(9)

Magnitude wise,

\[
|M|^2 = m_g'^2 + m_p^2 = m_g'^2 + (|M| \frac{v}{c})^2
\]

or

\[
m_g' = |M| \left( 1 - \frac{v^2}{c^2} \right)^{\frac{1}{2}} = m_g \left( 1 - \frac{v^2}{c^2} \right)^{\frac{1}{2}},
\]

(10)

since from Fig. (2a) \( |M| = m_g \). Note that \( m_g \) is the (rest) grain-mass of a stationary-particle whereas \( m_g' \) is the reduced grain-mass of the self-inspired moving particle, also note that in the case of self-inspired self-conversion of (rest) grain-mass, the grain-mass decreases (from \( m_g \) to \( m_g' \)) as the particle moves faster. Equations (10) and (6) are similar. Self-conversion of grain-mass to photonic-mass and vice-versa are manifested fully in ‘annihilation’ and ‘pair-production’ processes for elementary particles and antiparticles.
The total mass $M$ of the particle when $v = 0$ is $|M|$, from Eq. (7), stationary grain-mass is $|M|$ and photonic-mass is zero. If, however, the stationary mass converts itself (self-inspired) into photonic-energy (wave) it may be considered as moving with a photonic-speed ($v = c$) with grain or rest mass as zero and photonic mass as $|M|$ leading to Einstein’s mass energy equation (Eq. 11) as photonic energy $E = |M| \cdot v/c$ as mentioned in section 3.1,

$$E = |M| \cdot c^2.$$  

(11)

3.2.2 Induced-motion due to addition of energy to the particle by external-agency

Consider a particle at rest (point A in Fig. 3a) having only a stationary grain-mass $m_g$ with total mass $|M| = m_g$. Now consider that this mass is pushed or worked-upon or given energy to move forward with a velocity $v$. Thus a photonic-mass is added (Pythagorean way) onto the grain-mass (Fig. 3b) due to the work or energy supplied to it. The total mass thus increases to a higher value $M'$, given by, as complex sum as,

$$M' = m_g + im_p.$$  

(12)

Magnitude wise,

$$|M'|^2 = m_g^2 + m_p^2 = m_g^2 + \left(|M| \cdot \frac{v}{c}\right)^2$$

Figure 3: Pythagorean addition of energy $m_g$ (by external agency) to the particle as photonic mass
or
\[ | M' | = \frac{m_g}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}. \]  

(13)

Thus, there is an overall increase in the total mass from \(| M |\) to \(| M' |\) due to addition of energy. Note that the mass of the stationary-particle is \(| M | = m_g\), whereas when-after addition of work or energy \((W = E = m_p c^2)\) the particle gains photonic mass \(m_p\) it starts moving with greater total mass \(| M' |\).

Equations (13) and (5) are similar. Also note that the Eqs. (13 or 5) is similar and equivalent to the famous Relativity-formula
\[ m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}, \]
where \(m_0\) is the rest-mass and \(m\) is the total moving mass.

4 Collision of particles and conservation laws

Consider two particles 1 and 2, initial masses of which before-collision are \(M_1 = m_{g1} + im_{p1}\) and \(M_2 = m_{g2} + im_{p2}\). During collision, the grain and the photonic masses could be ‘re-distributed’ between them. Consider that after-collision the new masses for the two particles are \(M'_1 = m'_{g1} + im'_{p1}\) and \(M'_2 = m'_{g2} + im'_{p2}\).

The total magnitude of the mass \(| M |\) for the colliding particles (which takes into account ‘relativistically’ both, the grain-mass and the photonic-mass) would therefore be ‘conserved’ giving the following equation,
\[ | M_1 | + | M_2 | = | M'_1 | + | M'_2 |. \]  

(14)

In the process of the collision of the particles, the total vector (imaginary-part) photonic mass is also considered to be ‘conserved’ and thus,
\[ | m_{p1} | + | m_{p2} | = | m'_{p1} | + | m'_{p2} |. \]  

(15)

In fact, as it will be more evident later in this paper that the conservation of total mass \(| M |\) (Eq. 14) is the mass-energy conservation and that the conservation of photonic mass \(m_p\) (Eq. 15) is the conservation of momentum.

It may be noted that the ‘complex mass’ introduced in this paper takes into account the relativistic aspects as reflected in Eqs. (5) - (7). Thus, in general, the real-part i.e., the rest (grain) mass is not conserved in collision unless the colliding particles retain their identities. It is also noted that the ‘complex-mass’ is somewhat different from the usual mathematical complex-number; as complex-mass is in fact relativistic.
5 Photon scattering

To study the photon scattering, take a general case of collision (Fig. 4) where a photon strikes (at an angle $\alpha$) on a moving object (say, electron) and scatters away. Consider that before-collision the electron moving with a velocity $v_1$ has a total mass $M_1 = m_g + i \frac{|M_1|}{c}v_1$ where $m_g$ is rest-mass of the electron, and that the incident photon (with a momentum $\frac{h\nu}{c}$) has a total mass $M_2 = 0 + i \frac{h\nu}{c}$.

After collision; the photon (at an angle $\theta$, with momentum $\frac{h\nu'}{c}$) is emerged with a total mass $M'_2 = 0 + i \frac{h\nu'}{c}$, and that the electron comes out (at an angle $\delta$) moving with a velocity $v'_1$ which has a total mass $M'_1 = m_g + i \frac{|M'_1|}{c}v'_1$.

Equation (15) for photonic-mass conservation reduces to the following equations (Eqs. 16 & 17) of conservation corresponding to the momentum conservation in x and y directions,

\[ |M_1|v_1 + \frac{h\nu \cos \alpha}{c} = |M'_1|v'_1 \cos \delta + \frac{h\nu' \cos \theta}{c}, \]  
\[ \frac{h\nu}{c \sin \alpha} = - |M'_1|v'_1 \sin \delta + \frac{h\nu'}{c \sin \theta}. \]  

Equation (14) for total-mass conservation reduces to mass-energy conservation equation (Eq. 18) as,

\[ |M_1|c^2 + h\nu = |M'_1|c^2 + h\nu'. \]  

Figure 4: Photon Scattering
where from Eq. (5) magnitude of total complex masses of the electron before
and after the collision (by photon) are as follows,

\[ |M_1| = \frac{mg}{\left(1 - \frac{v_1^2}{c^2}\right)^\frac{1}{2}} \quad \text{and} \quad |M'_1| = \frac{mg}{\left(1 - \frac{v'_1^2}{c^2}\right)^\frac{1}{2}}. \quad (19) \]

From Eqs. (16), (17), (18) and (19), the following expression is obtained,

\[ (\nu - \nu') = \left(\frac{\hbar \nu \nu' \cos \alpha}{m_g c^2}\right)[1 - \cos (\theta - \alpha)] + \frac{v_1}{c}(\nu \cos \alpha - \nu' \cos \theta). \quad (20) \]

Laugier [12] gave a more accurate derivation of above Eq. (20) as follows,
however, as he himself mentioned that the slight correction has no consequence
for the rest of the paper.

\[ (\nu - \nu') = \left(\frac{\hbar \nu \nu' \cos \alpha}{m_g c^2}\right)[1 - \cos (\theta - \alpha)] + \frac{v_1}{c}(\nu \cos \alpha - \nu' \cos \theta). \quad (21) \]

For cases where the incident photon strikes not the electron but some ‘heavier’
object such as an observer (or an observing instrument), mirror or glass; \(m_g\)
will be replaced with rest-mass of the object in Eq. (20) where the first term
on R.H.S. would become negligible and the equation would thus reduce to,

\[ (\nu - \nu') = (\nu \cos \alpha - \nu' \cos \theta)\frac{v_1}{c}. \quad (22) \]

### 5.1 Compton Effect

For \(v_1 = 0\) and \(\alpha = 0\), the Eq. (20) reduces to the equation for Compton effect
[13] as follows,

\[ \left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{(m_g c^2)}(1 - \cos \theta). \quad (23) \]

### 5.2 Doppler Effect (For Light)

If the incident light (at \(\alpha = 0\)) reflects back (at \(\theta = 180^0\)) from the moving
object (mirror), the Eq. (22) reduces to the following, after some algebraic
manipulations,

\[ \frac{\nu'}{\nu} = \frac{1 - \frac{v_1}{c}}{1 + \frac{v_1}{c}}. \quad (24) \]

which gives the frequency of the reflected light \(\nu'\) from an object - which is
receding from the source (which emits light of frequency \(\nu\)). It should be noted
that this frequency \(\nu'\) is Doppler-shifted twice, firstly when the moving object
(mirror) receives the light and secondly when it reflects. The result obtained
here is consistent with that predicted by Doppler-effect [14] for light.
5.3 Reflection and Refraction

5.3.1 Reflection

Consider that light of frequency \( \nu \) strikes a stationary mirror \((v_1 = 0)\) and that the frequency of the reflected light is \( \nu' \). From Eq. (22), \( \nu' = \nu \). Moreover, since free surface has zero shear-stress, momentum along free-surface should be same before and after impact. Mathematically,

\[
\frac{h \nu}{c} \sin \alpha = \frac{h \nu'}{c} \sin \beta
\]

gives the reflection-law \[15\] as follows (incident angle \( \alpha \) equals reflected angle \( \beta \)), since \( \nu = \nu' \),

\[
\alpha = \beta.
\]  

(25)

5.3.2 Refraction

As in the case of reflection; for refraction of light into a stationary \((v_1 = 0)\) transparent medium, Eq. (22) gives \( \nu' = \nu \). In case of reflection both the incident and reflected light travel in the same medium, but for refraction it is different. For refraction case, the incident light of frequency \( \nu \) travels in a medium(say, air) wherein speed of light is \( c \) and the refracted light of frequency \( \nu' \) travels in another medium(say, glass) wherein speed of light is \( c' \). Thus the momentum of the incident light is \( \frac{h \nu}{c} \) whereas the momentum of refracted light is \( \frac{h \nu'}{c'} \). Here again, since the interface between the two media can-not take shear stress, momentum conservation along the interface must hold good i.e.,

\[
\frac{h \nu}{c} \sin \alpha = \frac{h \nu'}{c'} \sin \gamma,
\]

which gives the Snell’s law \[15\] for refraction as follows, since \( \nu = \nu' \),

\[
\frac{\sin \alpha}{\sin \gamma} = \frac{c}{c'} = \mu,
\]  

(26)

where \( \mu \) is the refractive index of the second medium-material (say, glass \( \mu > 1 \)) with respect to the first medium (say, air), which indicates that speed of light \( (c') \) in denser medium (glass) would be less than the speed of light \( (c) \) in rarer-medium(air), as expected and is in accordance with the results of wave theory.

It is to recall that the main reason for failure of Newtonian corpuscular-theory was that it predicts wrong results about speed of light in denser medium whereas the wave-theory predicts correctly. The complex-mass concept, which considers light as particle of photonic-mass (which differs in the two media, \( \frac{h \nu}{c} \) in one medium and \( \frac{h \nu}{c'} \) in the other medium), also predicts (Eq. 26) rightly.
5.3.3 Partial refraction and partial reflection

Consider that the incident light (n photons) strikes the stationary \((v_1 = 0)\) glass and that it is partially transmitted \((n_2\) photons) into and partially reflected \((n_1\) photons) back, such that \(n = n_1 + n_2\). As shown earlier, the frequencies: of incident \((\nu)\), transmitted \((\nu'_t)\) and reflected \((\nu'_r)\) light are same. Here also, since the free-surface can-not take shear-stress, momentum conservation along the interface yields,

\[
n \left( \frac{h\nu}{c} \right) \sin \alpha = n_1 \left( \frac{h\nu'_t}{c} \right) \sin \beta + n_2 \left( \frac{h\nu'_r}{c} \right) \sin \gamma,
\]

where \(\alpha\), \(\beta\) and \(\gamma\) are angle(s) of incident, refraction and reflection respectively.

Taking \(n = n_1 + n_2\) and \(\nu = \nu'_t = \nu'_r\) the above equation reduces to

\[
n_1 [\sin \alpha - \sin \beta] + n_2 \left[ \sin \alpha - \left( \frac{c}{\nu'_r} \right) \sin \gamma \right] = 0.
\] (27)

For partial refraction and partial reflection, \(n_2\) and \(n_1\) are the positive integers thus Eq. (27) leads to both, the reflection-law (Eq. 25) and the refraction-law (Eq. 26). Possibly, specific combinations of \(n_1\) and \(n_2\) could lead to the understanding of bunching / anti-bunching of light \([16]\).

6 Photo-electric effect

When a photon of energy \((h\nu)\) strikes an electron (in an atom), part of its energy \((W, \text{the work-function})\) is used up for removing the electron from the atom, thus the remaining energy \((h\nu - W)\) is used to give a velocity \(v\) to the electron.

If \(m_e\) is the total mass of an electron in the atom, conservation of total mass (Eq. 14) gives,

\[
m_e + \frac{(h\nu - W)}{c^2} = \frac{m_e}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} + 0,
\]

which gives the following Einstein’s equation (28) for photo-electric effect \([17]\) as follows (neglecting the higher order terms),

\[
h\nu = \frac{1}{2} m_e v^2 + W.
\] (28)

Photon during its flight actually behaves as wave, but when it strikes an objects and it stops (either momentarily as in Compton-effect or completely as in photo-electric effect) the photon exhibits particle-like aspect.

7 Uncertainty Principle

Consider (in Fig.4) that a photon of frequency \(\nu\) strikes (at \(\alpha = 0\)) an electron and reflects back (at \(\theta = \pi\)) with a frequency \(\nu'\) (from Eq. (23) \(\nu\) may be
different from \( \nu' \) but as \( h \) is very small, \( \nu' \approx \nu \).

It can be shown from Eq. (16) that the magnitude of change (uncertainty) in the momentum (for \( \alpha = 0 = \delta, \theta = \pi \)) \( \Delta m v = 2 \frac{h \nu}{c} \).

Position of the electron from the photon-source is \( x \). Distance traveled by the emitted photon to return back is \( 2x \). During this distance it may be considered that the light had \( N \) wavelengths or \( 2x \approx N \lambda \). The uncertainty in the measurement of position (or in \( N \)), would be of the order of an error of one wavelength, thus \( \Delta x \approx \frac{\lambda}{2} \). Therefore,

\[
\Delta m v \Delta x \approx h,
\]

which is of the same order of magnitude as that by the famous Heisenberg’s uncertainty principle [18].

Here not only that it is shown that product of uncertainty in momentum and position is of the order of \( h \) but also shown individually that error in position is half the wavelength and that error in momentum would be twice the photon-momentum. For measurement with high frequency (small \( \lambda \)) photons, \( \Delta x \) would be less but \( \Delta m v \) would be more, product of these \( \approx h \). A more rigorous consideration (scattering in all directions) would result this product as \( \frac{h}{(2\pi)} \).

8 Relativity

From Eqs. (2) and (4) the relativistic energy equation could be written as follows,

\[
| \mathbf{M} | c^4 = m_g^2 c^4 + | \mathbf{M} | v^2 c^2, \quad (30)
\]

\[
E^2 = E_g^2 + p^2 c^2, \quad (31)
\]

where total-energy \( E = | \mathbf{M} | c^2 \), relative-rest-energy \( E_g = m_g c^2 \) and photonic-energy \( E_p = | \mathbf{M} | v c \); total momentum = photonic momentum = \( m_p c = | \mathbf{M} | v = p \).

Considering the momentum vector

\[
| \mathbf{M} | v = | \mathbf{M} | (iv_x + jv_y + kv_z) = (ip_x + jp_y + kp_z) = p,
\]

it could be shown from Eq. (31) that

\[
\left( \frac{E_g}{c} \right)^2 = \left( \frac{E}{c} \right)^2 - (p_x^2 + p_y^2 + p_z^2) = \left( \frac{E'}{c} \right)^2 - (p'_x^2 + p'_y^2 + p'_z^2) \quad (32)
\]

which is nothing but the well-known ‘4-vector invariant’ in unprimed and primed \((t)\) coordinate system [19 - 21]. The 4-vector invariance of momentum is well known in special-relativity.
It may be noted that the relativistic energy equations (30 - 31) are derived from complex-mass concept equations (1 to 7). The concept of ‘complex mass’ finds strength from the fact that reverse is also possible i.e., the complex-mass concept equations (1 to 7) could be derived from the relativistic energy equations (30 - 31).

In fact, the 4-vector invariance (Eq. 32) is equivalent to the invariance of real-part rest (grain) mass, in unprimed and primed coordinate system. This is consistent with the concept of complex mass; because a change of coordinate system would affect velocity to change the vector (imaginary-part) photonic-mass thus changing the total-mass, whereas the scalar (real-part) grain-mass (material) remains unchanged.

The special-relativity in the concept of complex-mass is also reflected in Eqs. (5 - 6) and Eqs. (13 & 10) in fact, the special-relativity is inherent in the concept of complex mass.

9 Supersymmetry and Links between Complex-mass and Supersymmetry

Supersymmetry \[\text{[22]}\] means that there is transformation which relates the particles of integral spin such as photon (boson) to the particles of half-integral spin such as electron (fermion). Bosons are the ‘mediators’ of the fundamental forces while fermions make up the ‘matter’. The supersymmetry solves the ‘hierarchy problem’ for grand-unification. Also, for unification of forces, with supersymmetry the promising string-theory becomes the better and famous superstring theory \[\text{[23]}\]. But as yet, no supersymmetric -partner particles have been found.

An ambitious attempt is made, as follows, to answer: ‘why superpartners exist only in \textit{principle} but not in \textit{reality’}.

Referring to the ‘complex mass concept’, it may be noted that the grain-mass \(m_g\) signifies the material content due to atoms/molecules (group of fermions) whereas the photonic-mass \(m_p\) is due to associated photon (boson). So, in a way, the complex mass \(M = m_g + im_p\) is ‘marriage’ of fermion plus boson. Thus the supersymmetry seems to be inherently embedded in the complex mass concept. It is as if, fermion & boson are ‘coupled to each other’ and that fermion-part behaves as particle & boson-part shows the wave nature. Wave-particle duality seems, thus in a way, due to supersymmetry embedded in the complex mass.

It seems that supersymmetric partners (\& thus the supersymmetry) can exists only in ‘married’ state as complex sum as \(M = m_g + im_p\), of fermionic-part (grain-mass) and bosonic-part (photonic-mass).
10 Discussions

It is well known that, as per Relativity-theory, total moving mass is more than its rest mass, which is also evident from Eq. (13) of section-3.2.2. But this is only a part of the story; the rest (grain) mass itself can decrease if the motion is self-inspired on its own as given in Eq. (10) of section-3.2.1. Note that the equations 5 & 6 (similar to equations (13) & (10) respectively) are basically the two facets of the same coin but have different meanings as explained in sections-3.2.2 & 3.2.1.

To avoid ambiguity of wave-particle, some new terminology is suggested as follows. A particle at rest (in laboratory) is let called as ‘grainon’, a particle moving with a speed \( v (v < c) \) is let called as ‘complexion’ and the particle moving with speed of light is called as ‘photon’. All; grainon, complexion and photon, let be ‘said’ as particles! However, the grainon (at relative-rest \( v = 0 \)) has corpuscular (material) nature, the photon (in flight \( v = c \)) has the wave nature and the complexion (in motion \( v < c \)) has both corpuscular and wave natures.

However, as per Eq. (11) inter-conversion of photon and grainon is possible. Moreover, when a photon is in motion \( (v = c) \) it behaves as wave (during its flight), but when it strikes an object it stops \( (v = 0, \) momentarily in ‘Compton-effect’ and completely in ‘photo-electric effect’ experiments) and thus the photon thereafter behaves as grainon. In experiments such as of ‘interference’ the two photon-streams interfere in the flight itself before coming to rest on screen, thereby showing the wave-nature, the essential-characteristic necessary for such experiments. That’s why photon shows sometimes (in flight) wavy-aspect and sometimes (when it strikes another particle) corpuscular-nature.

Furthermore, an atom at rest may be apparently considered as grainon but it is composed of complexion (moving electron) and grainon (stationary nucleus), the nucleus in turn is composed of several particles of varied nature. In a way, one can say that an atom is a ‘compound’ particle.

The concept of complex-mass is new, interesting, promising and is of fundamental importance. Its compatibility with special-relativity suggests that it is okay but at this stage there is no point in weighing this (present theory) with full grown 4-vector special-relativity theory. However, a comparison could be made between the two for clarity as follows in Table-1.
Table 1: Comparison between ‘Complex-mass approach’ and ‘Special-Relativity theory’ for mass.

| Items                      | Complex-Mass Approach | Special-Relativity Theory | Remarks                                      |
|----------------------------|-----------------------|---------------------------|---------------------------------------------|
| **Masses**                 |                       |                           |                                             |
| Grain (rest) mass (real-part) | $m_g$                | $m_0$                     | $m_g = m_0$                                 |
| Photonic mass (imaginary-part) | $m_p = |M| \frac{v}{c}$ | -                         | $m_p = (m^2 - m_0^2)^{1/2}$                    |
| Total magnitude of mass    | $|M|$                 | $m$                       | $|M| = m$                                   |
| **Momentum**               |                       |                           |                                             |
| Total momentum             | $|M| v$               | $mv$                      |                                             |
| Photonic momentum          | $m_pc = |M| v$          | -                         |                                             |
| **Energy**                 |                       |                           |                                             |
| Total energy               | $|M| c^2$             | $E = mc^2$                |                                             |
| Rest-mass energy           | $m_g c^2$             | $E_0 = m_0 c^2$           |                                             |
| Photonic energy            | $|M| vc$              | -                         |                                             |
| Kinetic energy             | -                     | $m c^2 - m_0 c^2$         | $\approx \frac{1}{2}mv^2$                  |
| **Conservation laws of**   |                       |                           |                                             |
| Photonic mass              | $m_p$                 | momentum                  | $m_pc = mv$                                 |
| Total mass & energy        | $|M|$                 | mass-energy               | $|M| c^2 = mc^2$                             |
| **Invariance of**          |                       |                           |                                             |
| real-part of mass (energy) | Grain (rest) mass     | 4-vector momentum         | both are equivalent                          |
| **Representation**         |                       |                           |                                             |
| Diagram                    | Complex-mass         | Minkowskian space         |                                             |
|                           | representation       |                            |                                             |
| Dimensions                 | Real-part scalar(1) + | space(3) + time(1)        | both, 4-dimensions                          |
|                           | imaginary-part vector(3) |                            |                                             |
| **Expression**             |                       |                           |                                             |
| Equation                   | $M = m_g + im_p$      | $(E/c)^2 = (\frac{E_0}{c})^2 - \frac{(p_x^2 + p_y^2 + p_z^2)}{c^2}$ | Inter-derivable                             |
The authors, however, like to stress that the complex-mass components are not merely a re-arrangement of relativistic masses but the important point is the introduction of the concept that mass of a moving body is a ‘complex quantity’ i.e., it has two components: (i) a scalar (real-part) component as grain-mass (exhibiting the corpuscular material nature) and (ii) a vector (imaginary-part) component as photonic-mass (governing the wave aspects). It may also, however, be noted that the ‘complex mass’ is not simply a conventional type of mathematical complex-number but is a new type of complex-quantity encompassing the relativistic aspects too, as reflected in Eq. (7).

Although several applications of complex-mass have already been considered earlier in some depth, the author would cite the following example where the importance of photonic (vector imaginary-part) mass may possibly be further emphasized and usefulness of the complex-mass concept may be appreciated.

In quantum mechanical explanation of interference pattern of electrons in the famous double-slit experiment, we are forced to accept that ‘an’ electron passes through ‘both’ the slits simultaneously! Whereas in view of the ‘complexion’ concept of complex-mass; it may be considered that the electron (rest scalar mass) just-pass through one of slits only and the associated wave (vector imaginary-part photonic-mass) split-passes through both the slits, or in other words the ‘complexion’ divided into two parts, or the ‘complexion’ momentarily divides into two different complexions.

As discussed in section-9, it seems that the supersymmetry is embedded in coupled-state into the complex-mass.

In the complex-mass concept, in fact, both the corpuscular and the wave aspects are merged together. The concept may provide a bridge-link between quantum-mechanics and special-relativity as well as between micro world and macro world. Special theory of relativity has close links to Electro-magnetism [24]. General theory of relativity [25] is the theory of Gravity, which tells that gravity is there because the 4-dimensional space-time is curved and that this curvature is because of presence of ‘mass’ there. Even the alternative theory [26] of gravity suggests that gravity is due the material mass. ‘Mass’ is in the central theme everywhere from atom to galaxy. What if the mass is a ‘complex’ quantity: the complex-mass is in accordance with special-relativity but how is it with general-relativity? Although at present stage it may appear that the concept of ‘complex’ mass raises more questions than it provides answers, but the concept is of fundamental importance and its full potentials will be realized in time to come.
11 Conclusions

The concept of ‘complex-mass’ is new, simple and useful. It has the mathematical ingredients and the special-relativity is naturally embedded in it. It is able to explain phenomena which could not otherwise be explained by a single theory of wave or of particle. The wave-particle duality is mathematically incorporated in the complex mass. The novel concept that ‘mass is a complex quantity’ is of fundamental importance and has the potential to explain several diverse phenomena. It easily explains (derives) de-Broglie hypothesis, thus it has ability to minimize the quantum-mystery.

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