On interaction of P-waves with one-dimensional photonic crystal consisting of weak conducting matter and transparent dielectric layers

A A Yushkanov, N V Zverev
Moscow Region State University, 10a Radio st., 105005 Moscow, Russia
E-mail: zverev_nv@mail.ru

Abstract. An influence of quantum and spatial dispersion properties of the non-degenerate electron plasma on the interaction of electromagnetic P-waves with one-dimensional photonic crystal consisting of conductor with low carrier electron density and transparent dielectric matter, is studied numerically. It is shown that at the frequencies of order of the plasma frequency and at small widths of the conducting and dielectric layers of the photonic crystal, optical coefficients in the quantum non-degenerate plasma approach differ from the coefficients in the classical electron gas approach. And also, at these frequencies one observes a temperature dependence of the optical coefficients.

1. Introduction
At present, a study of dependence the characteristics of the conducting matter on the temperature and other external parameters represents a large interest [1]. It is motivated first of all by a development of the fine bio- and nanotechnologies. The conducting substances having visible temperature behavior of a number of parameters, are substances with low carrier electron number density having order $10^{-5} - 10^{-2}$ of atomic one and hence, they possess a weak conductivity. These substances are semimetals, semiconductors and their compounds. The conductance electron gas of the matters at such densities is almost non-degenerate. But at the same time, in case of little widths of these low conducting substances, the quantum properties of the conductivity electrons should be also taken into account.

In the paper, one studies numerically an interaction of the electromagnetic P-waves with one-dimensional photonic crystal. This crystal consists of alternating thin layers of weak conducting matter with low carrier electron density, and a transparent dielectric substance. One compares the optical power coefficients for the photonic crystal evaluated in the quantum non-degenerate electron plasma approach, with those in the classical electron gas theory. And also, the temperature dependence of the optical power coefficients is studied in the case of quantum approach.

2. The model
The photonic crystal has $N$ flat layers of the low conducting matter with the same width $d_1$, and the $N-1$ flat layers of the transparent dielectric matter with the permittivity $\varepsilon_2$ having the similar width $d_2$ and separating the weak conducting layers. Such a photonic crystal is localized between two transparent dielectric media with permittivities $\varepsilon_1$ and $\varepsilon_3$.

Let the electromagnetic P-wave i.e. wave when the $E$ vector lies in the incidence plane, is incident on the photonic crystal under the incidence angle $\theta$ from the medium with permittivity $\varepsilon_1$. Then the
optical power coefficients, namely, the reflectance \( R \), transmittance \( T \), and absorptance \( A \), for the one-dimensional photonic crystal look as \([2 - 4]\): 

\[
R = \left| \frac{m_{21}}{m_{22}} \right|^2, \quad T = \left| m_{21} - \frac{m_{11} m_{22}}{m_{21}} \right|^2 \text{Re} \left( \frac{\bar{\varepsilon}_3 \cos \theta'}{\varepsilon_1 \cos \theta} \right), \quad A = 1 - R - T_r. \tag{1}
\]

Here \( m_{\alpha} \) are the matrix elements of the transfer matrix for the photonic crystal:

\[
\begin{pmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{pmatrix} = \mathbf{M}^{-1}_3 \mathbf{M}_2 \mathbf{M}^{-1}_1 \mathbf{M}_1, \tag{2}
\]

where the matrices \( \mathbf{M}_{\alpha \alpha} \) and \( \mathbf{M}_{2\alpha} \) in case of the P-wave look as \((\alpha = 1, 2, 3)\):

\[
\mathbf{M}_{\alpha \alpha} = \begin{pmatrix}
\cos \theta - Z_\alpha^{(i)} \sqrt{\varepsilon_a} & - \cos \theta + Z_\alpha^{(i)} \sqrt{\varepsilon_a} \\
- \cos \theta + Z_\alpha^{(2)} \sqrt{\varepsilon_a} & \cos \theta + Z_\alpha^{(2)} \sqrt{\varepsilon_a}
\end{pmatrix}, \tag{3}
\]

\[
\mathbf{M}_{2\alpha} = \begin{pmatrix}
\exp(i k_z d_z) & 0 \\
0 & \exp(-i k_z d_z)
\end{pmatrix}. \tag{5}
\]

In these equations, \( \theta_i = \theta \) is the angle of incidence, but \( \theta_2 \) and \( \theta_3 = \theta' \) are the refraction angles into the dielectric layers with \( \varepsilon_2 \) and into the medium with \( \varepsilon_3 \), respectively. The latter angles are evaluated according to relation

\[
\sqrt{\varepsilon_i} \sin \theta = \sqrt{\varepsilon_2} \sin \theta_2 = \sqrt{\varepsilon_3} \sin \theta'. \tag{6}
\]

Further \( k_{z_2} = \frac{\omega}{c} \sqrt{\varepsilon_2} \cos \theta_2 \) is the projection of wave vector \( \mathbf{k}_2 \) in a dielectric layer with \( \varepsilon_2 \), onto the Z axis which is orthogonal to the layers surfaces of the photonic crystal and is directed towards the medium with \( \varepsilon_3 \). Here \( \omega \) is the wave frequency and \( c \) is the vacuum speed of light.

And at the end, \( Z_\alpha^{(j)} \) \((j = 1, 2)\) are the dimensionless surface impedances for the P-wave incident on the layer of the conducting matter. The impedances were evaluated in the paper \([5]\) in the case of mirror reflection of the carrier electrons from the layer surfaces of the matter and look as follows:

\[
Z_\alpha^{(j)} = \frac{2 i c \omega}{d_1} \sum_n \frac{1}{k_n^2} \left( \frac{k_n^2}{\omega^2 \varepsilon_1 (\omega, k_n)} + \frac{(m_1/d_1)^2}{\omega^2 \varepsilon_1 (\omega, k_n) - (c k_n)^2} \right). \tag{7}
\]

Here \( k_n = \sqrt{\frac{m_1}{d_1}} + k_s^2 \) and \( k_s = \frac{\omega}{c} \sqrt{\varepsilon_1} \sin \theta \) is the projection of the wave vector \( \mathbf{k} \) of incident wave onto the X axis which lies in the incidence plane and is parallel to the layer surfaces of the photonic crystal. Further, summation in \((7)\) is performed over all odd numbers \( n = \pm 1, \pm 3, \pm 5, \ldots \) when \( j = 1 \), and over all even \( n = 0, \pm 2, \pm 4, \ldots \) when \( j = 2 \). And finally, \( \varepsilon_1 \) and \( \varepsilon_2 \) are the longitudinal and transverse dielectric functions of the electron plasma.

One has taken for investigating the dielectric functions of the quantum non-degenerate electron plasma with invariable collision frequency evaluated in \([6, 7]\):

\[
\varepsilon_1^{(q)} (\omega, k) = 1 - \frac{2}{\Omega^2} \left[ \frac{G(\Omega + i \gamma, \Omega + i \gamma, Q) G(0, Q)}{\Omega G(0, Q) + i \gamma G(\Omega + i \gamma, Q)} \right], \tag{8}
\]

\[
\varepsilon_2^{(q)} (\omega, k) = 1 - \frac{1}{\Omega^2} \left[ \frac{G(\Omega + i \gamma, \Omega + i \gamma, Q) + i \gamma G(0, Q)}{\Omega + i \gamma} \right], \tag{9}
\]

where

\[
G(\Omega + i \gamma, Q) = \frac{Q^2}{\sqrt{\pi}} \int_0^{\infty} \frac{[\Omega_+ + i \gamma](\Omega_+ + i \gamma) + (Q \xi)^2 \exp(-\xi^2)}{[\Omega_+ + i \gamma]^2 - (Q \xi)^2} \, d\xi, \tag{10}
\]

\[
[\Omega_+ + i \gamma]^2 - (Q \xi)^2 = 0. \tag{11}
\]
\[ \Omega = \frac{\omega}{\omega_p}, \quad \Omega_\epsilon = \Omega \pm \frac{1}{2 m_\epsilon \omega_p} k^2, \quad \gamma = \frac{1}{\omega_p \tau}, \quad Q = k \frac{2k_B T}{m_e \omega_p^2}. \] (11)

Here \( \omega_p \) is the frequency of the conductance electron plasma, \( \tau \) is the electron relaxation time owing to collisions of the plasma electrons, \( m_\epsilon \) is the effective carrier electron mass, \( h \) is the Planck constant, \( k_B \) is the Boltzmann constant, and \( T \) is the temperature of electron plasma.

The dielectric functions (8) and (9) take into account both the spatial dispersion i.e. dependence of the functions on the wave number \( k \), and the quantum wave properties of the electron plasma reflected in the terms containing the Planck constant.

The obtained results are compared with the data for the dielectric functions of the classical electron gas in the Drude – Lorentz approach [3, 5]:

\[ \varepsilon^{(DL)}_{\varepsilon} (\omega) = \varepsilon^{(DL)}_{e} (\omega) = 1 - \frac{1}{\Omega (\Omega + i\gamma)}. \] (12)

These dielectric functions of the classical electron gas are obtained from the functions (8) and (9) in the long-wave limit \( k \to 0 \). One sees that the functions reflect neither spatial dispersion nor quantum wave properties of the electron plasma.

3. Numerical results

To perform numerical calculation of the optical power coefficients for the photonic crystal with low conducting matter, one has to evaluate the plasma frequency and the electron relaxation time. These values are obtained according to the equations:

\[ \omega_p = \sqrt{\frac{e^2 n_\epsilon}{\varepsilon_0 m_\epsilon}}, \quad \tau = \frac{m_\epsilon}{e^2 n_\epsilon \rho_0}, \]

where \( e \) is the elementary charge, \( \varepsilon_0 \) is the electric constant in the SI units, \( n_\epsilon \) is the carrier electron number density, \( \rho_0 \) is the static resistivity of the electron plasma. The latter two of them were taken from [8] for graphite when assuming the density \( n_\epsilon \) is \( 10^{-5} \) times the atomic one and the effective electron mass \( m_\epsilon \) is equal to the free electron one. Thus one gets \( \omega_p = 5.67 \times 10^{13} \text{s}^{-1} \) and \( \tau = 9 \times 10^{-12} \text{s} \).

**Figure 1.** The reflectance \( R \) (left plot) and absorbance \( A \) (right plot) as functions of the frequency \( \omega \) at \( N = 6 \), \( \varepsilon_1 = 1, \varepsilon_2 = \varepsilon_1 = 2, \theta = 75^\circ, d_1 = 100 \text{ nm}, d_2 = 5 \mu \text{m}, T = 295 \text{ K}, \omega_p = 5.67 \times 10^{13} \text{s}^{-1} \): 1 – quantum non-degenerate electron plasma (solid line), 2 – classical electron gas (dotted line).

Using the values, one performs the numerical evaluations of the optical power coefficients according to the equations (1) – (12). Results of the calculations show first that for the frequencies \( \omega \ll \omega_p \) when the conductor width is much less than the skin depth i.e. \( d_1 \ll c/\omega_p \), and the dielectric layer width \( d_2 \) is of order of \( c/\omega_p \), the optical power coefficients in the quantum non-degenerate electron plasma
approach with the dielectric functions (8) and (9) differ from the ones for the classical electron gas with functions (12) (see fig.1). So, an influence of both the spatial dispersion and quantum wave properties of the electron plasma on the optical power coefficients of the one-dimensional photonic crystal has been demonstrated. In comparison with the classical electron gas, additional resonant peaks of the power coefficients in the quantum non-degenerate plasma approach at the frequencies $\omega > \omega_p$ (fig.1) are caused by oscillations of the longitudinal plasmons [5] within the conducting layers of the photonic crystal that occurs just for the $P$-waves.

One observes also a temperature dependence of the optical coefficients at the frequencies $\omega < \omega_p$ for the quantum non-degenerate plasma (see fig.2). Such dependence has also resonant peaks due to the plasmon oscillations. One sees that the growth of the temperature leads to increase of the peak frequencies.

![Figure 2](image)

**Figure 2.** The reflectance $R$ (left plot) and absorbance $A$ (right plot) as functions of the temperature $T$ for quantum non-degenerate electron plasma at $N = 6$, $\varepsilon_1 = 1$, $\varepsilon_2 = \varepsilon_3 = 2$, $\theta = 75^\circ$, $d_1 = 200$ nm, $d_2 = 4$ $\mu$m, $T_0 = 295$ K, $\omega_p = 5.67 \times 10^{13}$ s$^{-1}$: $1 - \omega = 1.07 \omega_p$ (solid line), $2 - \omega = 1.08 \omega_p$ (dashed line), $3 - \omega = 1.09 \omega_p$ (dotted line).

The above effects of the quantum non-degenerate plasma on the power coefficients of the photonic crystal containing low conducting layers should be accounted for at the constructing and exploiting of fine optical devices having such photonic crystals.

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