The optimal parameters of the laser triangulation modified thickness meter

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Abstract. In this paper, we search for optimal installation parameters for measuring the thickness of biological tissues. We created the mathematical model of the meter and researched various configurations of the constituent parts of the model. We also proposed the methods for determining the optimal parameters and we found numerical values the most important cases. We considered different options for modifying the laser triangulation method and proposed a scheme of the measurement setup. We conducted experiments and represented the results obtained with computer processing too.

1. Introduction

In the modern world for the solution of problems of research and technical problems in biology and medicine, it is necessary to make a modeling of biological systems [1] and the processes proceeding in them, for example, by means of laser radiation [2,3]. Mathematical modeling for medical research and measurement systems and devices [4] is also necessary. In particular, for the correction of visual impairment in ophthalmic practice, it is necessary to take into account the individual characteristics of aberrations of the focusing system of the human eye [5,6]. If we take into account these aberrations, it is possible to neutralize optical distortions in the formation of the image on the retina (myopia, astigmatism, hyperopia, etc.) by changing the thickness of the corneal layer of the eye.

Therefore, currently in ophthalmic practice [7], it is necessary to measure the thickness of thin biological tissues (for example, cornea, residual corneal layers, sclera) during a surgical intervention in refractive surgery. The thickness meter must meet a number of criteria, the most important of which is the safety requirement (the risk of complication after use should be minimal or absent), measurement by the non-contact method (due to the specific use) and in real time (for express measurement). This new method could also expand the capabilities of the devices [5,6] currently used to diagnose the condition of the eye.

In works [8-10] on the basis of [11-13] the modification of a method of laser triangulation which would allow to develop a device capable of measuring the thickness of biological tissues in compliance with the above restrictions. In our case, fewer parameters are used for the measuring system and the device calibration is simpler and faster. After analysis of works [14-18] devoted to the properties of various biological tissues and their transparency windows for the corresponding radiation, as well as comparing the parameters of existing thickness meters for thin transparent tissues and films [19] and experimental study we created the laser triangulation thickness meter...
for residual corneal layers of the eye [20]. In addition, this setup can be used to determine the depth of the lesion in corneal ulcers in humans and animals eyes.

We investigated the focusing of laser radiation [21] used in this method and its dependence of the spectral characteristics on the temperature of the active laser medium [22,23], and also considered the issue of processing the obtained experimental data [24] using the library for machine vision [25]. In [26,27], we carried out a semi-empirical selection of the optimal values of the parameters of the experimental setup for carrying out measurements. The lower limit of measurement for the thickness of fabrics and films was 0.2 mm.

The purpose of this work is to develop a mathematical model of a laser triangulation meter for corneal thickness and, on the basis of precise analysis of this model, obtain numerically optimal values for the parameters of this setup.

2. Mathematical model of the measuring system

The measurement method is based on the ability of a collimated laser beam to propagate over long distances in media with sufficiently small absorption and scattering coefficients. Figure 1 shows the functional-optical scheme of the measurement setup. Light marks (we will call them triangulagram), obtained by the reflection of a laser beam from the air-measurable biological tissue (cornea) and cornea-moisture of the anterior chamber (substrate), are recorded on the CCD sensor of the photodetector. If we know the distance between the images of the light marks, then we can calculate the thickness of the measured layer.

According to the general principles of modeling [28-30] and the laws of geometric optics in [8], we showed the derivation of a formula that relates the thickness of the measured layer in the case of plane-parallel boundaries (the radius of curvature of the layer \( R \to \infty \)) with the parameters of the meter and the optical properties of the layer itself (refractive index). The biological tissue thickness (film, layer) of \( z \) is related to the viewing angle \( \alpha \), the angle between the receiver of optical radiation (the surface of the CCD matrix) and the optical axis of the system \( \beta \), the linear magnification of the lens \( M = r'/r \), the distance between the image of the light marks on the CCD matrix of the photo-recording device \( x \) and the refractive index \( n \) relate as follows [8]:

\[
z = 2x \frac{r}{r'} \frac{\sin \beta}{\sin 2\alpha} \sqrt{n^2 - \sin^2 \alpha}.
\]

The value of the correction, which would take into account the radius of curvature at the point of measurement, we obtained in [31]:
\[
\begin{cases}
    x_{AB} = AB = d (\cot \xi - \tan \gamma); \\
    d = R \left( 1 - \sqrt{1 - (a/2R)^2} \right); \\
    \tan \gamma = \sin \alpha / \sqrt{n^2 - \sin^2 \alpha}; \\
\end{cases}
\]

where \( \xi = \frac{\pi}{2} - \alpha \), \( a^2 = z(2R - z) \).

After algebraic transformations using formulas (1) and (2), you can obtain the dependence of the distance between the images of the light marks on the CCD matrix (taking into account the angle of incidence \( \varphi \)):

\[
\begin{cases}
    x = K_1^{-1} \left( z - K_1 K_2 \left( R - \frac{\sqrt{(z-R)^2 + 3R^2}}{2} \right) \right); \\
    K_1 = \frac{2 \sin \beta \sqrt{n_i^2 - \sin^2 \alpha}}{M} \sqrt{\frac{\sin 2\alpha n_i^2 - \sin^2 \varphi}{n_i^2}}; \\
    K_2 = \tan \alpha \sqrt{\frac{n_i^2 - \sin^2 \alpha - \cos \alpha}{n_i^2 - \sin^2 \alpha}}. \\
\end{cases}
\]

Formula (3) is a function of 4 variables \( z = x(\varphi, \alpha, \beta, M) \) with parameters of biological tissue \((z, n, R)\), which is a mathematical description of this meter.

In the limiting case, when \( R \to \infty \) — the boundaries of the tissue being measured are plane-parallel, formula (3) goes into formula (1) up to the angle of incidence \( \varphi \): \( z = xK_1 \).

**Figure 2.** Model of the cornea like a spherical surface: A and B are light marks from the incident rays.

**Figure 3.** Model of the cornea like a plane-parallel layer: A and B are light marks from the incident rays.

### 2.1. Case of a multilayer film

For the case of a multilayer film with different refractive indices for each \( i \) layer, formula (1) will be rewritten as a product of the \([x_i]\) matrix column of \((1 \times m)\) size by \([y_i]\) matrix row \((m \times 1)\):

\[
z = [x_i] \cdot [y_i] = \sum_{i=1}^{m} x_i y_i,
\]

where \( x_i \) — the distance between the image of the light marks for the \( i \)-th layer,

\[
y_i = \left( 2 \sin \beta \sqrt{(n_i^2 - \sin^2 \alpha)(n_i^2 - \sin^2 \varphi_i)} \right) (M n_i \sin 2\alpha)^{-1}, m \) — number of layers.
2.2. Case of a 2-ray’s triangulation
For 2-ray triangulation, using as a model of the corneal layer in a first approximation a segment of a spherical surface (fig. 2), you can get the following formula for the optical thickness $z_{opt} = zn$:

$$z_{opt} = R \left(1 - \frac{R - z_{opt}}{\sqrt{a^2 + (R - z_{opt})^2}}\right),$$

or in the case of plane-parallel boundaries of the layer being measured we used:

$$z = a \cot(\arcsin \frac{\sin \varphi}{n}),$$

where $n$ is a refractive index of layer, that is, the task is reduced to determining the parameter $a$ — the distance between the light marks (fig. 2 and fig. 3).

2.3. Case of a laser scanner
To determine the thickness at the same time at several points, you can use a laser line. Thus, we can make a laser scanner thickness mode (fig. 4). The formula for the layer thickness in this case is as follows:

$$t = d_1 - (d_2/n);$$

$$d_i = \sqrt{R^2 - r_i^2} = \sqrt{R^2 - \left(\frac{L_i}{\Delta \sigma_i}\right)^2};$$

$$\Delta \sigma_i = 2 \arcsin \left(\sqrt{\sin^2(\Delta \varphi/2) + \cos(\varphi_1) \cos(\varphi_2) \sin^2(\Delta \xi/2)}\right);$$

where $(\varphi_i, \xi_i)$ — latitude and longitude of points in radians.

![Figure 4. To the formula for calculating the thickness using a laser line.](image)

3. Search for optimal values for meter parameters
To get a value for the thickness $z$ of the layer being measured, we need to know the distance between the image of the light labels $x$, that is, allow 2 separate light labels on the photodetector. Mathematically, this corresponds to the condition $x \rightarrow x_{max}$. Thus, the problem is reduced to the unconditional optimization of the function $x = x(\varphi, \alpha, \beta, M)$ — the search for its global or at least local (absolute) maximum. The value of the function $x$ is limited only by the physical dimensions of the photodetector matrix.

We used mathematical software packages [32,33] or their open source counterparts [34,35], in which absolute extremum search algorithms are implemented using mathematical analysis [36] to find the optimal values of system parameters $(\varphi, \alpha, \beta, M)$ for the thickness of the biological tissue (for example, cornea of the human eye) $z = 0.1$ mm with the refractive index $n = 1.36$ and the radius of curvature at the measuring point $R = 7.8$ mm; when in the experiment it was
not possible to resolve 2 separate light marks on the image of a triangular diagram: $\varphi \approx 6.9^\circ$, $\alpha \approx 31^\circ$, $\beta \to 0^\circ$, $M \approx 4.4$. However, such a value of the parameter $\beta$ gives an infinitely small energy value that falls into the photodetector, and, strictly speaking, is not technically feasible. In experiment [37], we used a 1080P Full HD SONY IMX322 Low illumination 0.01 Lux CMOS H.264 AEC AEB AGC camera (production: China) as a photodetector, for which the minimum angle was $\beta_{\text{min}} \approx 7^\circ - 8^\circ$. If $\alpha \approx 50, 4\circ$, and $\beta \approx 9.7^\circ$, as we obtained by semi-empirical selection in [26,27], then $M \approx 1.1 \cdot 10^6$, which is difficult technically feasible, since in this case the distance from the object (biological tissue) to the lens should be approximately equal to the focal length $d = F + \delta(F)$, where $\delta(F) \to 0$. This is difficult to achieve at such values of $M$ due to the finite value of the physical dimensions of the focusing system (lens thickness) of the objective.

![Figure 5. Dependence of the distance between the light marks on the photo image on the radius of curvature $R$ and the refractive index $n$ of the layer at optimal parameters for $z = 0.1$ mm.](image)

It should be noted that the radius of curvature of the $R$ surfaces has little effect on the distance between the light marks on the $x$ image. This shows the numerical simulation (fig. 5) and the experiment for different thicknesses $z$. We can convert the values for $x$ from pixels to millimeters using the scaling coefficient $k$, the values of which we determine using instrument calibration: $x_{\text{mm}} = k x_{\text{pxl}}$.

4. Experiment and processing results
The scheme of the measuring unit developed by us is presented in fig. 6. Structurally, it is a portable device consisting of three blocks: measuring, calibration, and a unit for adjusting the angle of incidence of laser radiation. The measuring unit consists of a laser 1 with a block forming a flat-front beam 3, a focusing system 8 and a CCD camera 9. The calibration unit consists of a laser module 2 and a diffraction grating 6. The unit for adjusting the angle of incidence of the laser radiation on the surface of the object under study consists of a dividing mirror 4 and a CMOS camera 5. The linear dimensions of the installation depend on the dimensions of the composite parts and assemblies but do not exceed 8-10 cm in each direction.

![Figure 6. The scheme of the measuring unit: 1 and 2 — laser modules, 3 — beams forming unit with a flat front, 4 — dividing mirror, 5 — CMOS camera, 6 —diffraction grating, 7 — eyes, 8 — focusing system, 9 — CCD camera, 10 — PC.](image)
As a diagnostic 1 and calibrating 2 lasers, we use a semiconductor laser module with
a wavelength in the range of 650-660 nm (model HML1845 Viper Series, Dragon Lasers,
manufacturer Changchun New Industries Optoelectronics Technology Co., Ltd., country of origin
is China) and diode-pumped solid-state laser with a wavelength of 532 nm (model LR-021 Viper
Series, Dragon Lasers, manufacturer Changchun New Industries Optoelectronics Technology
Co., Ltd., the country of origin is China). In addition, lasers that generate radiation at the
same wavelength can be used as a calibrator and diagnostics. The lasers are powered from a
constant current source with parameters of 3-3.5 V and 0.1-0.3 A.

The beam-forming unit with a flat front consists of a system of successively located two
diaphragms with a diameter of 1-1.5 mm, scattering and collecting lenses, cutting out the central
part of the laser beam, and assembled as a bulk structure with an arrangement 1.5-2 cm apart.
A more detailed description and scheme you can find in [20]. We also experimentally determined
that the attenuation coefficient of such a system for laser radiation was 20 dB.

![Figure 7. Light marks (triangula-gram) on the cornea of a pig’s eye.](image1)

![Figure 8. The results of computer processing photo from figure 7 (distance between the light marks).](image2)

If we use a meter on the front tissues of the eye (cornea), the output power of the continuous
laser radiation should not exceed 0.1-1 mW [27-31], and the waist radius or neck of the caustic
surface of the laser beam on the surface of the biological object under study should be at least
0.1-0.5 mm and the power density of the laser radiation, respectively, no more than 10 mW/mm².

![Figure 9. Laser lines on the cornea of porcine eyes: 1 — cornea, 2 —laser line on the
anterior surface of the cornea, 3 — laser line on the posterior surface of the cornea, 4 —
sclera, 5 — clamp.](image3)

Using the developed setup, we carried out ex vivo measurements of the thickness of the cornea
of the eyeball (fig. 7) of a conditionally healthy domestic pig donor (lat. Sus scrofa domesticus)
of the breed: large white. The cornea thickness in the center is \( z = 0.52-0.65 \) mm, along the
edges \( z = 1.0-1.3 \) mm. The refractive index of the cornea is \( n = 1.336-1.339 \). The computer
program [24], written in Python 3, allows determining the distance in pixels on a photo image
between the global and first local maximum in intensity, which correspond to the laser triangle
tags, and stores the processing results (fig. 8) with the image of the calculated intensities of the entire photo in the format *.png, *.jpg or *.bmp.

We also conducted experiments using a laser line (fig. 9) and on the phantoms of the corneal layers of the eye [8-10], which were films with a refractive index from 1.35 to 1.55 made of polyethylene, polyvinyl chloride, polystyrene and gelatin in the visible wavelength range on a water substrate.

4.1. Measurement error

The magnitude of the absolute error, that we were looking for using the method of processing indirect single measurements, was 0.05-0.10 mm using arbitrary specified installation parameters. With the optimal parameters set, the thickness measurement error is 0.01-0.02 mm in the measurement range from 0.1 mm to 1.5 mm.

5. Conclusions

In this study, we developed a mathematical model of a laser triangulation measuring the thickness of the cornea and conducted its numerical analysis. The optimal values for the parameters of the plant operating according to the proposed method of measurement were obtained.

For thicknesses of biological tissue (like a layers of the cornea of the human’s eye) $z = 0.1$ mm with refractive index $n = 1.36$ and the radius of curvature at the measuring point $R = 7.8$ mm:

1) the angle of incidence of laser radiation on the surface of the measured layer $\varphi \approx 6.9^\circ$;
2) viewing angle $\alpha \approx 31^\circ$;
3) the angle between the optical radiation receiver and the optical axis of the system $\beta \approx 7^\circ - 8^\circ$;
4) linear magnification of the lens $M \approx 4.4$.

The obtained result will reduce the lower limit for the thickness of biological tissues and films during experimental measurements.

The numerical analysis of the calculation formula with optimal parameters is carried out and experiments on transparent films and the pig’s cornea with computer processing of the obtained photo images are carried out. We also got the formula to calculate the layer thickness using laser line using haversine formula. The magnitude of the absolute error was 0.01-0.02 mm using optimal set installation parameters in the measurement range from 0.1 mm to 1.5 mm.

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