How many quantum gates do gauge theories require?
Lattice Conference – Fermilab 2023

Edison M. Murairi, Michael J. Cervia, Hersh Kumar, Paulo F. Bedaque, Andrei Alexandru

The George Washington University and University of Maryland College Park

July 31, 2023
Table of Contents

1 Introduction

2 Approach

3 Applications
Introduction

Objective
Assess the complexity of simulating gauge theories on near-term quantum computers

Methodology
- Measure the complexity in terms of the CNOT count
- Improve on existing methods to reduce the CNOT count
- Test the method on gauge theories
Edison M. Murairi, Michael J. Cervia, Hersh Kumar, Paulo F. Bedaque, and Andrei Alexandru

“How many quantum gates do gauge theories require?”
PRD 106, 094504 (2022). arXiv:2208.11789.
Codes https://github.com/emm71201/QC-Hamiltonian-Compilation

Edison Murairi and Michael J. Cervia
“Reducing Circuit Depth with Qubitwise Diagonalization.”
arXiv:2306.00170
Codes https://github.com/emm71201/Reducing-Circuit-Depth-with-Qubitwise-Diagonalization
Table of Contents

1 Introduction

2 Approach

3 Applications
Step 0: Trotterization

Consider a Hamiltonian: $H = \sum_j h_j$

The unitary time evolution operator can be approximated as:

$$U(\delta t) = e^{-iH \delta t}$$
$$= e^{-i \sum_j h_j \delta t}$$
$$\approx \prod_j e^{-i h_j \delta t}$$  (1)
Step 1: Expand the Hamiltonian in the Pauli basis

Given a Hamiltonian $H$, expand $H$ as:

$$H = \sum_i c_i P_i$$

where

$$P_i \in \{1, X, Y, Z\} \otimes_n$$

$$c_i = \frac{\text{Tr} [P_i H]}{2^n}$$

**Notation:** Drop the $\otimes$ symbol: $X \otimes Z \otimes Y \mapsto X Z Y$
Step 2: Group commuting Pauli terms

- Two Paulis commute if the number of qubits where the Pauli matrices anti-commute is even.

![Diagram showing commuting Paulis](image)

**Figure:** Example of two commuting Paulis

Checking commutations of $n$-qubit Paulis: Complexity: $O(n)$ operations.
Step 2: Group commuting Pauli terms – Continued

Graph coloring Problem:

Figure: Grouping commuting Paulis with Graph Coloring.
See Murairi et al., “How many quantum gates do gauge theories require?”
Step 3: Diagonalize each set of commuting Paulis

- Let $C$ be a set of commuting Paulis
- Construct a unitary $V$ simultaneously diagonalizing $C$
- We guarantee circuit depth of $\mathcal{O}(n \log r)$ where ($r \leq n$)

Edison M. Murairi and Michael J. Cervia. “Reducing Circuit Depth with Qubitwise Diagonalization”. In: (May 2023). arXiv: 2306.00170 [quant-ph]

Code source: https://github.com/emm71201/Reducing-Circuit-Depth-with-Qubitwise-Diagonalization
Step 4: Construct the time evolution circuit of a diagonal Hamiltonian

- A $n$-qubits diagonal Hamiltonian $D$ has the form:

$$D = \sum_{c_i} P_i$$

(5)

where $P_i \in \{I, Z\}^\otimes n$

- The quantum circuit of $e^{-iDt}$ can be constructed with a tree traversal algorithm.

Edison M. Murairi et al. “How many quantum gates do gauge theories require?” In: Phys. Rev. D 106 (9 Nov. 2022), p. 094504. DOI: 10.1103/PhysRevD.106.094504. URL: https://link.aps.org/doi/10.1103/PhysRevD.106.094504
Step 5: Putting everything together

\[ H = \sum_i c_i P_i = \sum_j h_j \] (6)

where \( h_j \) consists only of mutually commuting Paulis. Let \( V_j \) be a unitary diagonalizing \( h_j \): \( h_j = V_j D_j V_j^\dagger \).
Randomly generated Hamiltonians

- Randomly generate diagonal Hamiltonians with \( N \) Paulis.

![Graph](image.png)

Figur: The average CNOT gate count required to simulate a diagonal Hamiltonian as function of the number of Pauli strings for \( n = 8 \) qubits. The blue data points are obtained using the compilation method described in step 4.
Lattice Gauge Theories

| SU(2) Lattice Gauge Theory | $n_{\text{link}}$ | CNOT   | $R_z$  |
|----------------------------|-------------------|---------|--------|
| Orland et. al.\(^1\)     | 2                 | 180     | 64     |
| D. Horn\(^2\)            | 3                 | 17,168  | 16,384 |

\(^1\)Peter Orland and Daniel Rohrlich. “Lattice gauge magnets: Local isospin from spin”. In: *Nuclear Physics B* 338.3 (1990), pp. 647–672. [DOI](https://doi.org/10.1016/0550-3213(90)90646-U).

\(^2\)D. Horn. “Finite matrix models with continuous local gauge invariance”. In: *Physics Letters B* 100.2 (1981), pp. 149–151. [ISSN](http://dx.doi.org/10.1016/0370-2693) 0370-2693. [DOI](https://doi.org/10.1016/0370-2693(81)90763-2).
Thank You

THE END