Research Article

Heat and Mass Transfer Enhancement of MHD Hybrid Nanofluid Flow in the Presence of Activation Energy

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In this study, water is apprehended as conventional fluid with the suspension of two types of hybrid nanoparticles, namely, single-walled CNTs (SWCNTs) and multiwalled CNTs (MWCNTs). The influence of a magnetic field, thermal radiation, and activation energy with binary chemical reaction has been added to better examine the fine point of hybrid nanofluid flow. The mathematical structure regarding the physical model for hybrid nanofluid is established and then the similarity variables are induced to transmute the leading PDEs into nonlinear ODEs. These equations were solved using the shooting technique together with RKF 4-5th order for various values of the governing parameters numerically. The results of prominent parameters were manifested through graphs and tables. The results indicate that the hybrid nanofluid SWCNT – (MWCNT/water) is fully adequate in cooling and heating compared to other hybrid nanofluids. In addition, the rise in the value of activation energy (E) upsurges the nanoparticle transfer rate of hybrid nanofluid.

1. Introduction

Hybrid nanofluids are a new kind of working fluid that is made by suspension of two different types of nanoparticles with sizes (under 100 nm) into the conventional fluid (water, oils, ethylene glycol, biological fluids, etc.). This new form of nanofluid is having higher thermal conductivity and thermo-physical properties than conventional fluids. In recent years, these hybrid nanofluids were used in various heat transfer applications such as heat pipe, solar energy, refrigeration as well as heating, heat exchanger, ventilation, air conditioning system, coolant in machining and manufacturing, biomedical, space, ships, defense, etc. A comprehensive review on hybrid nanofluids in heat transfer applications is given in Sarkar et al. [1]; Sidik et al. [2]; Sundar et al. [3]; and Sajid and Ali [4]. Khashi’ie et al. [5] presented analytical and numerical solutions of MHD flow of Cu – Al2O3/water hybrid nanofluid towards a shrinking cylinder. The effect of velocity and thermal slips and chemical and thermal radiation on TiO2/Al2O3-water-based hybrid nanoliquid over a stretching sheet in both steady and unsteady was examined by Santhi et al. [6]. Khan et al. [7] discussed the impact of two different nanoparticles Cu and Al2O3 with two different base fluids H2O and C2H6O2 along a stretched surface under the impact of nonlinear thermal radiation, inclined magnetic field, and mass suction. Their results showed that the (Cu – Al2O3)/(H2O – C2H6O2) hybrid nanofluid is more effective in cooling and heating compared to (Cu – Al2O3)/H2O hybrid nanofluid, and a simple nanofluid (Al2O3)/H2O. Saeed et al. [8] established the analytical solution of Darcy-Forchheimer MHD hybrid nanofluid flow and heat transfer using HAM. They found that increasing values of Brownian motion increases the temperature profile of the hybrid nanofluid. Sheikholeslami et al. [9] investigated the entropy generation and free convection of MHD hybrid nanofluid flow within a porous tank. They observed that the growth of permeability reduces the exergy loss while Nuave augments with rise of Da.
Carbon nanotubes (CNTs) are the cylindrical structures of carbons atoms (graphene). CNTs are classified into three groups depending on the number of graphene layers: single-walled CNTs (SWCNT) consisting of one graphene layer with a diameter of 0.5–1.5 nm, double-walled carbon nanotube (DWCNT), and multiwalled CNTs (MWCNTs) dwelling of many graphene layers interlinked nanotubes, with diameters of ranges between 10 and 100 nm. Due to the unique properties such as high mechanical strength, rigidity, hardness, adhesion, high dimensional ratio, chemical and biochemical sensors, catalytic, conductive plastics, structural composite materials, etc. Prabhavathi et al. [10] analyzed the slip effects of SWCNTs and MWCNTs based Maxwell nanofluid over different geometries. However, a cautious review of the current literature reveals the impact of activation energy with binary chemical reaction on mass and heat transport characteristics of magnetohydrodynamic (MHD) flow of a SWCNT – (MWCNT/water) hybrid nanofluid induced by moving wedge. The resultant equations were solved using shooting technique along with RKF 4-5th order. Results for velocity, temperature, concentration profiles, skin friction coefficient, heat transfer rate, and nanoparticle transfer rate are graphically displayed and debated briefly.

2. Mathematical Formulation

2.1. Physical Description. The steady two-dimensional MHD boundary layer flow of a SWCNT – (MWCNT/water) hybrid nanofluid induced by moving wedge was investigated. It is assumed that the wedge is moving with the velocity \( \vec{u}_w(x) = U_w \hat{x} \) and the free stream velocity \( \vec{u}_f(x,t) = U_f(x) \hat{x} \). Here, \( m = (\beta(2 - \beta)) \) is the Hartree pressure gradient in which \( \beta = (2m/(1 + m)) \) is the wedge parameter which can be symbolized as \( \bar{\Omega} = \pi \beta \) (see Figure 1). Temperature and concentration hybrid nanofluids at the wall are \( \bar{T}_w \) and \( \bar{C}_w \), respectively, while \( \bar{T} \rightarrow \bar{T}_f \) and \( \bar{C} \rightarrow \bar{C}_f \) are ambient temperature and concentration.

2.2. Flow Analysis. The modeled equations based on the above assumptions are stated as follows [6]:

\begin{equation}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\end{equation}

\begin{equation}
\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} - \bar{u}_e \frac{\partial \bar{u}}{\partial x} = \frac{\mu_{hnd}}{\rho_{hnd}} \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\sigma B^2}{\rho_{hnd}} (\bar{u}_e - \bar{u}),
\end{equation}

\begin{equation}
\frac{\partial \bar{T}}{\partial x} + \frac{\partial \bar{T}}{\partial y} \left( \frac{\rho C_P}{\rho C_h}_{hnd} \right) \frac{\partial \bar{q}_r}{\partial y} = \frac{k_{hnd}}{(\rho C_P)_{hnd}} \frac{\partial^2 \bar{T}}{\partial y^2},
\end{equation}

\begin{equation}
\left[ \bar{T}^4 \equiv 4 \bar{\Omega} \bar{T}^3 - 3 \bar{\Omega} \bar{T}^4, q_r = \frac{4 \sigma^*}{3k^*} \frac{\partial \bar{T}^4}{\partial y} = \frac{16 \sigma^* \bar{\Omega}^2 \bar{T}^3}{3k^*} \frac{\partial \bar{T}}{\partial y} \right],
\end{equation}

\begin{equation}
\bar{u} \frac{\partial \bar{C}}{\partial x} + \bar{v} \frac{\partial \bar{C}}{\partial y} + k_e^* (\bar{C} - \bar{C}_f) \left( \frac{\bar{T}}{T_f} \right)^\eta_s e^{\left(-E_e/\kappa T_w\right)} = D_e \left( \frac{\partial^2 \bar{C}}{\partial y^2} \right) + D_r \left( \frac{\partial^2 \bar{T}}{\partial y^2} \right),
\end{equation}

\section*{2.3. Results and Discussion}

The research regarding the boundary layer flow with heat and mass transfer, involving hybrid nanofluid, has revealed to be realistically significant in engineering processes. However, a cautious review of the current literature reveals that the flow over a moving wedge embedded in the hybrid nanofluid was not taken into account. Due to broad fascinating in energy sector, the present work investigate the impact of activation energy with binary chemical reaction on mass and heat transport characteristics of magnetohydrodynamic (MHD) flow of a SWCNT – (MWCNT/water) hybrid nanofluid induced by moving wedge. The resultant equations were solved using shooting technique along with RKF 4-5th order. Results for velocity, temperature, concentration profiles, skin friction coefficient, heat transfer rate, and nanoparticle transfer rate are graphically displayed and debated briefly.
with

\[ \bar{u} \big|_{y=0} = \bar{u}_w(x) = \bar{U}_w x^m, \]

\[ \bar{v} \big|_{y=0} = 0, \]

\[ \bar{T} \big|_{y=0} = \bar{T}_w, \]

\[ \bar{C} \big|_{y=\infty} = \bar{C}_w, \]

\[ \bar{u} \big|_{y=\infty} = \bar{u}_e(x) = \bar{U}_e x^m, \]

\[ \bar{T} \big|_{y=\infty} = \bar{T}_e, \]

\[ \bar{C} \big|_{y=\infty} = \bar{C}_e. \]

In (4), \( k^2/(\bar{T}/\bar{T}_e)^{\varphi} \) exemplify the modified Arrhenius equation in which \( k_r \) is the rate of chemical reaction rate, \( n_k \) is fitted rate constant, and \( E_a \) is the modified Arrhenius function.

2.3. Transformation. In order to transform the modeled equations, the following transformations are established [22]:

\[ \psi = \left[ \frac{2\theta_j \bar{U}_fs}{(m + 1)} \right]^{0.5} x^{0.5(m+1)} f'(\zeta), \]

\[ \zeta = \left[ \frac{(m + 1)\bar{U}_fs}{2\theta_j} \right]^{0.5} x^{0.5(m-1)} \phi, \]

\[ \theta(\zeta) = \frac{\bar{T} - \bar{T}_fs}{\bar{T}_w - \bar{T}_fs}, \]

\[ \varphi(\zeta) = \frac{\bar{C} - \bar{C}_fs}{\bar{C}_w - \bar{C}_fs}, \]

with the components of velocity \((\bar{u}, \bar{v})\) and stream function \((\psi)\) as

\[ \bar{u} = \frac{\partial \psi}{\partial y} = \bar{U}_fs x^m f'(\zeta), \]

\[ \bar{v} = -\frac{\partial \psi}{\partial x} = \left[ \frac{2(m+1)\theta_j \bar{U}_fs}{2} \right]^{0.5} x^{0.5(m-1)} \cdot \left[ f'(\zeta) + \left[ (m - 1) x^{m-1} f'(\zeta) \right] \right]. \]

2.4. Thermophysical Properties of SWCNT – (MWCNT/water)-Based Hybrid Nanofluid. The dynamic viscosity \( \mu_{\text{hf}} \), the effective dynamic density \( \rho_{\text{hf}} \), the specific heat or heat capacitance \( \rho C_p \), and the thermal conductivity \( k_{\text{hf}} \) of the hybrid nanofluid are specified as

\[ \mu_{\text{hf}} = \frac{\mu_1}{\left(1 - \varphi_1 \right)^{2.5} (1 - \varphi_2)^{2.5}}, \]

\[ \rho_{\text{hf}} = \varphi_2 \rho_{\text{SWCNT}} + (1 - \varphi_2) \left( (1 - \varphi_1) \rho_f + \varphi_1 \rho_{\text{MWCNT}} \right), \]

\[ (\rho C_p)_{\text{hf}} = \varphi_2 (\rho C_p)_{\text{SWCNT}} + (1 - \varphi_2) \cdot \left( (1 - \varphi_1) \rho C_p \right)_{\text{MWCNT}}. \]

\[ k_{\text{hf}} = k_{nf} \cdot \left( k_{MWCNT} + 2k_{nf} - 2\varphi_1 (k_{nf} - k_{SWCNT}) / (k_{SWCNT} + 2k_f + 2\varphi_1 (k_f - k_{SWCNT})) \right), \]

where \( k_{nf} = k_f \cdot (k_{SWCNT} + 2k_f - 2\varphi_1 (k_f - k_{SWCNT})) / (k_{SWCNT} + 2k_f + 2\varphi_1 (k_f - k_{SWCNT})) \).

\( \varphi_1 \) and \( \varphi_2 \) are used for the solid volume fraction of MWCNT and SWCNT respectively, \( \rho_f \), \( \rho C_p \), and \( k_f \) are the respective densities, specific heat, and thermal conductivity of the conventional fluid. The thermophysical properties of the present flow modeled are listed in Table 1.

2.5. Transformed Problems. Plugging the above-mentioned transformation and thermophysical properties, SWCNT – (MWCNT/water)-based hybrid nanofluid, the momentum, and thermal and concentration boundary layers become
\[ f'' - \frac{A_1}{A_4} \left( \frac{2m}{m+1} \right) (f'^2 - 1) - f' f'' = 0, \]
\[ + A_1 M \left( \frac{2}{m+1} \right) (1 - f') = 0, \]
\[ \left[ 1 + \frac{R}{A_4} \right] \theta'' - \frac{Pr A_1}{A_4} \left( \frac{4m}{m+1} \right) f' \theta - f \theta' = 0, \]
\[ \phi'' + \left( \frac{Nt}{Nb} \right) \theta'' - Le \left[ \frac{4m}{m+1} \right] f' \varphi - f \varphi' \]
\[ - Le \sigma (1 + \delta \varphi) \exp \left( \frac{-E}{1 + \delta \varphi} \right) \varphi = 0, \]
subject to
\[ f|_{z=0} = 0, \]
\[ f'|_{z=0} = \gamma, \]
\[ \theta|_{z=0} = 1, \]
\[ \phi|_{z=0} = 1, \]
\[ f'|_{z=\infty} = 1, \]
\[ \theta|_{z=\infty} = 0, \]
\[ \phi|_{z=\infty} = 0, \]
where
\[ A_1 = (1 - \varphi_1)^{-2.5} (1 - \varphi_2)^{-2.5}, \]
\[ A_2 = (1 - \varphi_2) \left[ (1 - \varphi_1) + \varphi_1 \left( \frac{\rho_{SWCNT}}{\rho_f} \right) + \varphi_2 \left( \frac{\rho_{MWCNT}}{\rho_f} \right) \right], \]
\[ A_3 = (1 - \varphi_2) \left[ (1 - \varphi_1) + \varphi_1 \left( \frac{\rho_{MWCNT}}{\rho_{SWCNT}} \right) \right], \]
\[ + \varphi_2 \left( \frac{\rho_{MWCNT}}{\rho_{SWCNT}} \right), \]
\[ A_4 = \frac{k_{\text{SWCNT}}}{k_{\text{MWCNT}}}. \]

Equations (9)–(12) include the nondimensional magnetic parameter \( M \), radiation parameter \( R \), Prandtl number \( Pr \), Brownian movement parameter \( Nt \), thermophoresis parameter \( Nb \), Lewis number \( Le \), moving wedge parameter \( \gamma \), temperature relative parameter \( \delta \), chemical reaction parameter \( \sigma \), and activation energy parameter \( E \) which are defined as follows:

\[ M = \frac{\sigma B^2_0}{\rho_f U_{\infty}}, \]
\[ R = \frac{16 \sigma^4 T^3_{\infty}}{3k^4 f}, \]
\[ Pr = \frac{\theta_f}{\alpha_f}, \]
\[ Nt = \frac{\tau D_T (T_w - T_{fs})}{T_{fs} \delta_f}, \]
\[ Nb = \frac{\tau D_p (C_w - C_{fs})}{\delta_f}, \]
\[ Le = \frac{\theta_f}{D_{f} H}, \]
\[ \gamma = \frac{\bar{U}_w}{U_{fs}}, \]
\[ \delta = \frac{(T_w - T_{fs})}{T_{fs}}, \]
\[ \sigma = k_c \epsilon_0 x^{2m}, \]
\[ E = \frac{E_w}{\kappa T_{fs}}. \]

Table 1: Physical properties of base fluids (H₂O) and nanoparticles CNTs [19].

| Property                     | Conventional fluid (H₂O) | MWCNT | Nanoparticles |
|------------------------------|--------------------------|-------|--------------|
| \( C_p \) (kJ kg⁻¹ K⁻¹)     | 4179                     | 796   | 425          |
| \( k \) (W m⁻¹ K⁻¹)         | 0.613                    | 3000  | 6600         |
| \( \rho \) (kg m⁻³)         | 997.1                    | 1600  | 2600         |

2.6. Physical Quantities. The dimensionless forms of skin friction coefficient \( \bar{S}_{f x} = (\rho u_{\text{SWCNT}}/\rho_f u_{\text{MWCNT}}^2) (\partial u/\partial y)|_{z=0} \), heat transfer rate \( \bar{H}_{fs} = \bar{x}(\rho u_{\text{SWCNT}}^2/\rho_f u_{\text{MWCNT}}^2) (\partial T/\partial y)|_{z=0} \), and nanoparticle transfer rate \( \bar{C}_{fs} = -(\bar{x}(\partial C/\partial y)|_{z=0})/\bar{C}_{\text{SWCNT}} - \bar{C}_{\text{MWCNT}} \) are defined as...
where \( \text{Re}_x = \frac{u_e(x)\bar{x}}{\partial} \) is the Reynolds number.

### 3. Computational Scheme

A shooting technique together with RKF 4-5\(^{th}\) order integration scheme is adopted to establish the computational results of nonlinear differential equations (9)–(11) with the boundary restrictions represented in equation (12). Following the basic methodology of shooting technique, we reduce this dimensionless boundary value flow system into first-order system by using the pursuing procedure:

\[
\begin{align*}
    f &= u_1, \\
    f' &= u_2, \\
    f'' &= u_3,
\end{align*}
\]

\[
\begin{align*}
    f^{(n)} &= u_4\left[\frac{2m}{m+1}\left(u_2^2 - 1\right) - u_1u_5\right] - A_1M\left(\frac{2}{m+1}\right) \\
    &\quad\cdot (1 - u_2), \\
    \theta &= v_1, \\
    \theta' &= v_2,
\end{align*}
\]

\[
\begin{align*}
    v_2' &= \frac{\Pr}{1 + \left(R/A_4\right)} A_2 \left[\frac{4m}{m+1}\right] u_3v_1 - u_1v_2, \\
    \varphi &= u_1, \\
    \varphi' &= w_2, \\
    w_2' &= -\frac{N\bar{T}}{N\bar{b}} v_2' + \left(\frac{4m}{m+1}\right) u_3w_1 - u_1w_2 \\
    &+ \text{Le}\sigma(1 + \delta v_1)^\delta \exp\left(\frac{-E}{1 + \delta v_1}\right) w_1,
\end{align*}
\]

with

\[
\begin{align*}
    u_1\mid_{z=0} &= 0, \\
    u_2\mid_{z=0} &= \gamma, \\
    v_1\mid_{z=0} &= 1, \\
    w_1\mid_{z=0} &= 1, \\
    u_3\mid_{z=\infty} &= a_1, \\
    v_2\mid_{z=\infty} &= a_2, \\
    w_2\mid_{z=\infty} &= a_3.
\end{align*}
\]
4. Outcomes and Discussions

We graphically analyze the behavior of various sundry parameters on fluid velocity, temperature, and concentration dissemination \( f'(\zeta), \theta(\zeta) \) and \( \phi(\zeta) \) in Figures 2–21. The ranges of constraints in this research are considered as 0.1 \( \leq m \leq 0.4, 0.1 \leq M \leq 3, 0.5 \leq R \leq 2, 0.1 \leq Pr \leq 2, 0.2 \leq Nt \leq 0.8, 0.5 \leq Nb \leq 2, 1 \leq Le \leq 2.5, 0.1 \leq \sigma \leq 0.7, 0 \leq \delta \leq 3, 1 \leq E \leq 4, 0.5 \leq \gamma \leq 5, 0.01 \leq \varphi_1, \varphi_2 \leq 0.1 \). Figures 2–7, given the impact of \( \varphi_1 \) and \( \varphi_2 \) on \( f'(\zeta), \theta(\zeta) \) and \( \phi(\zeta) \). The intensification of \( \varphi_1 \) and \( \varphi_2 \) reduces the thickness of the boundary layer flow, which arises in an increase of fluid velocity. We boost the quantity of both CNTs in H\(_2\)O, the heat absorbing capacity of the fluid intensifies. As a result, the temperature profile is enhanced. Additionally, the higher values of \( \varphi_1 \) and \( \varphi_2 \) decay the mass transfer rate. Figures 8 and 9 demonstrate the effects of \( m \) (Hartree pressure gradient parameter) on \( f'(\zeta) \) and \( \theta(\zeta) \). The augmented values of maugmented the
surface drag force and \( f'(\zeta) \) increases. A reverse configuration is grasped for growing values of \( \min \theta(\zeta) \).

The variation of \( f'(\zeta) \) on \( M \) (magnetic parameter) is shown in Figures 10 and 11. An increment in magnetic strength strengthens the velocity and weakens the temperature. This is due to the fact that with growing values of \( M \), the Lorentz forces enhances, which raises the resistive force to the hybrid nanofluid motion. The response of \( f'(\zeta) \) and \( \theta(\zeta) \) to the variation of \( \gamma \) (moving wedge parameter) are illustrated in Figures 12 and 13. Rising of \( \gamma \) increases the velocity distribution and diminishes the temperature distribution. Furthermore, the thickness of the hydrodynamic boundary layer depreciation and thermal boundary layer are upgraded.

Figure 14 demonstrates the effect of \( R \) (Radiation parameter) on \( \theta(\zeta) \). It explains that, due to the increment of \( R \), depreciate \( k^* \) (the mean absorption coefficient), consequently increases \( (\partial q_r/\partial y) \) (heat flux radiation) and the radiative heat transfer rate into the hybrid nanofluid that caused an increase in \( \theta(\zeta) \). Figures 15 and 16
show the disparities of $\theta(\zeta)$ and $\varphi(\zeta)$ on up surging values of $Pr$ is the proportion of the molecular diffusivity over thermal diffusivity, increase in $Pr$ in the hybrid nanofluid has reduce the thermal diffusivity. Therefore, the temperature of SWCNT– (MWCNT/water)-based hybrid nanofluid worsens as $Pr$ intensifies, while the reverse phenomena occurs in $\varphi(\zeta)$. The variation of $\varphi(\zeta)$ via $Nt$ (thermophoresis parameter) and $Nb$ (Brownian motion) is displayed in Figures 17 and 18. With increase in the value of $Nt$, thickening the concentration boundary layer, $\varphi(\zeta)$ increases. $Nb$ portrays a declining performance of $\varphi(\zeta)$. Figure 19 reveals that an elevation in $Le$ (Lewis number) diminishes the mass diffusion and $\varphi(\zeta)$; this is because that $Le$ inversely related to the mass diffusion.

Figure 20 illustrates the $\varphi(\zeta)$ profiles decline due to the intensification of $\sigma$ (dimensionless reaction rate) in the entire flow region. Physically, growing values of $\sigma$ causes an increment in the term $\sigma (1 + \delta\theta)^{\alpha}\exp\left(-E/\left(1 + \delta\theta\right)\right)$, it helps to calamitous $k_c$ (chemical reaction rate) that decreases $\varphi(\zeta)$. Figure 21 shows the abating phenomena of hybrid nanoparticle concentration $\varphi(\zeta)$ due to the fluctuation of $\delta$. The portrayal of $E$ (activation energy) in distribution of $\varphi(\zeta)$ is shown in Figure 22. It is found that $\varphi(\zeta)$ attained a maximum level when $E$ is assigned the maximum value. Physically, due to involvement of $E$ afford some extra energy to system, which enhances the chemical reaction rate and hence $\varphi(\zeta)$ increases with higher values of $E$.

The current results of the present problem are compared with the available literature and divulged in Table 2. Tables 3–5 show the computational results of skin friction, heat transfer rate, and nanoparticle transfer rate against certain physical parameters. These tables show that $\tilde{S}_fRe_x^{0.5}$ enhances for larger values of ($m$), ($M$), ($\varphi_f$) and ($\varphi_s$), while it decays for ($\gamma$). $H_fRe_x^{0.5}$ upsurgs for increasing values of ($R$), ($\varphi_f$), and ($\varphi_s$), while decreases for ($m$), ($M$), ($Pr$) and
Figure 22: $E$ variation on $\varphi(\zeta)$.

Table 2: Comparison of $\tilde{S}_{f_2}Re_0^{0.5}$ at selected values $m$ when $M = \varphi_1 = \varphi_2 = 0$.

| $m$  | Ullah et al. [23] | Gopikrishna and Shanmugapiya [22] | Present results |
|------|-------------------|-----------------------------------|-----------------|
| 0    | 0.4696            | 0.4696                            | 0.4696          |
| 1/11 | 0.6550            | 0.6550                            | 0.6550          |
| 1/5  | 0.8021            | 0.8021                            | 0.8021          |
| 1/3  | 0.9277            | 0.9277                            | 0.9277          |
| 1/2  | 1.0389            | 1.0389                            | 1.0389          |
| 1    | 1.2326            | 1.2326                            | 1.2326          |

Table 3: Numerical values of $\tilde{S}_{f_2}Re_0^{0.5}$.

| $m$  | $M$  | $\gamma$ | $\varphi_1$ | $\varphi_2$ | $(1/(1 - \varphi_1)^{2.5} (1 - \varphi_2)^{2.5}) \sqrt{(m + 1)/2} f''(0)$ |
|------|------|----------|-------------|-------------|--------------------------------------------------------------------------------|
| 0.1  | 0.1  | 0.5      | 0.1         | 0.1         | 0.4450                                                                           |
| 0.2  | 0.1  | 0.5      | 0.1         | 0.1         | 0.4846                                                                           |
| 0.3  | 2.0  | 0.5      | 0.1         | 0.1         | 0.5160                                                                           |
| 0.4  | 3.0  | 0.5      | 0.1         | 0.1         | 0.5415                                                                           |
|      | 0.4  | 0.1      | 0.1         | 0.1         | 0.6516                                                                           |
|      | 1.0  | 0.1      | 0.1         | 0.1         | 0.9402                                                                           |
|      | 2.0  | 0.1      | 0.1         | 0.1         | 1.2855                                                                           |
|      | 3.0  | 0.1      | 0.1         | 0.1         | 1.5563                                                                           |
|      | 0.5  | 0.1      | 0.1         | 0.1         | 0.4450                                                                           |
|      | 1.5  | 0.1      | 0.1         | 0.1         | -0.5354                                                                          |
|      | 3.0  | 0.1      | 0.1         | 0.1         | -2.5889                                                                          |
|      | 5.0  | 0.1      | 0.1         | 0.1         | -6.1693                                                                          |
|      | 0.01 | 0.1      | 0.1         | 0.1         | 0.4455                                                                           |
|      | 0.03 | 0.1      | 0.1         | 0.1         | 0.4451                                                                           |
|      | 0.05 | 0.1      | 0.1         | 0.1         | 0.4449                                                                           |
|      | 0.10 | 0.1      | 0.1         | 0.1         | 0.4450                                                                           |
|      | 0.01 | 0.1      | 0.1         | 0.1         | 0.4610                                                                           |
|      | 0.03 | 0.1      | 0.1         | 0.1         | 0.4570                                                                           |
|      | 0.05 | 0.1      | 0.1         | 0.1         | 0.4532                                                                           |
|      | 0.10 | 0.1      | 0.1         | 0.1         | 0.4450                                                                           |

Table 4: Numerical values of $\tilde{H}_{s_2}Re_0^{0.5}$.

| $m$  | $M$  | $R$  | $Pr$ | $\gamma$ | $\varphi_1$ | $\varphi_2$ | $-(k_{hsl}/k_{nl})\sqrt{(m + 1)/2} f''(0)$ |
|------|------|------|------|----------|-------------|-------------|---------------------------------------------|
| 0.1  | 0.1  | 1.0  | 6.2  | 0.5      | 0.1         | 0.1         | -0.8698                                     |
| 0.2  | 0.2  | 1.0  | 6.2  | 0.5      | 0.1         | 0.1         | -0.9865                                     |
| 0.3  | 0.3  | 1.0  | 6.2  | 0.5      | 0.1         | 0.1         | -1.0765                                     |
| 0.4  | 0.4  | 1.0  | 6.2  | 0.5      | 0.1         | 0.1         | -1.1486                                     |
|      | 0.4  | 1.0  | 6.2  | 0.5      | 0.1         | 0.1         | -0.8935                                     |
|      | 1.0  | 1.0  | 6.2  | 0.5      | 0.1         | 0.1         | -0.9183                                     |
|      | 2.0  | 1.0  | 6.2  | 0.5      | 0.1         | 0.1         | -0.9400                                     |
Table 4: Continued.

| m  | M  | R  | Pr | γ  | φ₁ | φ₂  | \(-(k_{\text{inh}}/k_{\text{ad}})\sqrt{(m + 1)/2}[1 + R]\theta'(0)\) |
|----|----|----|----|----|----|----|---------------------------------------------------------------|
| 3.0 | 0.5 | -0.9529 |
|     | 1.0 | -0.9173 |
|     | 1.5 | -0.8698 |
|     | 2.0 | -0.8292 |
|     | 0.1 | -0.7945 |
|     | 0.72| -0.1741 |
|     | 1.0 | -0.3265 |
|     | 2.0 | -0.3782 |
|     | 0.5 | -0.8698 |
|     | 1.5 | -0.3888 |
|     | 3.0 | -1.2311 |
|     | 5.0 | -1.6444 |
|     | 0.01| -2.0780 |
|     | 0.03| -1.0176 |
|     | 0.05| -0.9842 |
|     | 0.10| -0.9512 |
|     | 0.01| -0.8698 |
|     | 0.03| -0.9865 |
|     | 0.05| -0.9529 |
|     | 0.10| -0.8698 |

Table 5: Numerical values of \(\bar{C}_m\)Reₐ⁰.⁵

| m  | M  | R  | Pr | Nt | Nb | Le  | σ  | δ  | E  | γ  | φ₁ | φ₂  | \(\sqrt{(m + 1)/2}\theta'(0)\) |
|----|----|----|----|----|----|-----|----|----|----|----|----|----|--------------------------------|
| 0.1 | 0.1 | 1.0 | 6.2 | 0.1 | 0.1 | 1.0 | 0.5 | 0.5 | 0.5 | 0.5 | 0.1 | 0.1 | -0.4039 |
| 0.2 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4549 |
| 0.3 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4944 |
| 0.4 |    |    |    |    |    |     |    |    |    |    |    |    | -0.5261 |
| 0.4 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4142 |
| 1.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4235 |
| 2.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4302 |
| 2.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4336 |
| 0.5 |    |    |    |    |    |     |    |    |    |    |    |    | -0.3977 |
| 1.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4039 |
| 1.5 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4341 |
| 2.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4598 |
| 0.1 |    |    |    |    |    |     |    |    |    |    |    |    | -0.8084 |
| 0.72|    |    |    |    |    |     |    |    |    |    |    |    | -0.7443 |
| 1.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.7198 |
| 2.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.6435 |
| 0.2 |    |    |    |    |    |     |    |    |    |    |    |    | 0.0095 |
| 0.4 |    |    |    |    |    |     |    |    |    |    |    |    | 0.8364 |
| 0.6 |    |    |    |    |    |     |    |    |    |    |    |    | 1.6634 |
| 0.8 |    |    |    |    |    |     |    |    |    |    |    |    | 2.1903 |
| 0.5 |    |    |    |    |    |     |    |    |    |    |    |    | -0.7347 |
| 1.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.7760 |
| 1.5 |    |    |    |    |    |     |    |    |    |    |    |    | -0.7898 |
| 2.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.7967 |
| 1.0 | 0.2 |    |    |    |    |     |    |    |    |    |    |    | -0.4039 |
| 1.5 |    |    |    |    |    |     |    |    |    |    |    |    | -0.6148 |
| 2.0 |    |    |    |    |    |     |    |    |    |    |    |    | -0.7849 |
| 2.5 |    |    |    |    |    |     |    |    |    |    |    |    | -0.9302 |
| 0.1 |    |    |    |    |    |     |    |    |    |    |    |    | -0.3967 |
| 0.3 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4003 |
| 0.5 |    |    |    |    |    |     |    |    |    |    |    |    | -0.4039 |
\( \varphi \) also the larger values of \( \text{Pr} \), \( \text{Nt} \) and \( E \) intensifies \( \tilde{C}_{tx} \text{Re}^{-0.5} \). Thus, hybrid nanofluid evinced an eminent performance in \( \tilde{S}_{tx} \text{Re}^{0.5} \), \( \tilde{H}_{tx} \text{Re}^{-0.5} \), and \( \tilde{C}_{tx} \text{Re}^{-0.5} \) compared to other nanofluid.

5. Conclusions

Effect of thermal radiation and activation energy with binary chemical reaction of SWCNT + (MWCNT/H\(_2\)O) hybrid nanofluid is studied. The key findings were as follows:

- Widening of \( (M) \) caused skin friction \( (\tilde{S}_{tx} \text{Re}^{0.5}) \) increases and heat transfer \( (\tilde{H}_{tx} \text{Re}^{-0.5}) \) and mass transfer \( (\tilde{C}_{tx} \text{Re}^{-0.5}) \) rates to be decreased.
- Heat transfer \( (\tilde{H}_{tx} \text{Re}^{-0.5}) \) is intensified subject to large radiation \( (R) \).
- Upsurge of \( (\gamma) \) leads to enhance \( f'(\zeta) \) and decline \( \theta(\zeta) \) and \( \varphi(\zeta) \).
- \( \varphi(\zeta) \) upgrades via \( \text{Pr} \), \( \text{Nt} \) and it degrades \( \text{Nb} \).
- \( \varphi(\zeta) \) enhances by larger activation energy \( (E) \) and it decays through \( (\sigma), (\delta) \).
- An increase in \( (\varphi_1) \) and \( (\varphi_2) \) diminish \( f'(\zeta) \) and \( \varphi(\zeta) \) and enhances \( \theta(\zeta) \).
- From the above analysis, we can conclude that the impact of heat transfer in SWCNT – (MWCNT/water)-based hybrid nanofluid is more powerful compared to the common fluid.

### Nomenclature

\[ f: \text{Dimensionless velocity} \]
\[ \theta: \text{Dimensionless temperature} \]
\[ \varphi: \text{Dimensionless concentration} \]
\[ \tilde{u}, \tilde{v}: \text{Velocity components of } x \text{ and } y \text{ direction} \]

| \( m \) | \( M \) | \( R \) | \( \text{Pr} \) | \( \text{Nt} \) | \( \text{Nb} \) | \( \text{Le} \) | \( \sigma \) | \( \delta \) | \( E \) | \( \gamma \) | \( \varphi_1 \) | \( \varphi_2 \) |
|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0.7  |      |      |      |      |      |      |      |      |      |      |      |      |
| 0.0  |      |      |      |      |      |      |      |      |      |      | -0.4075 |      |
| 1.0  |      |      |      |      |      |      |      |      |      |      | -0.3973 |      |
| 2.0  |      |      |      |      |      |      |      |      |      |      | -0.4160 |      |
| 3.0  |      |      |      |      |      |      |      |      |      |      | -0.4520 |      |
| 0.5  |      |      |      |      |      |      |      |      |      |      | -0.5704 |      |
| 0.1  |      |      |      |      |      |      |      |      |      |      | -0.4963 |      |
| 0.01 |      |      |      |      |      |      |      |      |      |      | -0.4039 |      |
| 0.03 |      |      |      |      |      |      |      |      |      |      | -0.4725 |      |
| 0.05 |      |      |      |      |      |      |      |      |      |      | -0.5648 |      |
| 0.10 |      |      |      |      |      |      |      |      |      |      | -0.6712 |      |
| 0.01 |      |      |      |      |      |      |      |      |      |      | -0.3424 |      |
| 0.03 |      |      |      |      |      |      |      |      |      |      | -0.3169 |      |
| 0.05 |      |      |      |      |      |      |      |      |      |      | -0.4039 |      |
| 0.01 |      |      |      |      |      |      |      |      |      |      | -0.2908 |      |
| 0.03 |      |      |      |      |      |      |      |      |      |      | -0.2925 |      |
| 0.05 |      |      |      |      |      |      |      |      |      |      | -0.3180 |      |
| 0.10 |      |      |      |      |      |      |      |      |      |      | -0.3431 |      |
| 0.10 |      |      |      |      |      |      |      |      |      |      | -0.4039 |      |

- \( T_w, \tilde{C}_{\text{avg}} \): Temperature and concentration of the hybrid nanofluids at the wall
- \( T_w, \tilde{C}_{\text{avg}} \): Temperature and concentration far away from the surface
- \( U_w, \tilde{U}_{\text{avg}} \): Velocities near and far away from the surface
- \( \text{Re}_x \): Reynolds number
- \( \tilde{S}_x \): Skin friction coefficient
- \( \text{Ht}_x \): Heat transfer rate
- \( \tilde{C}_{tx} \): Nanoparticle transfer rate
- \( m \): Hartree pressure gradient
- \( \text{Pr} \): Prandtl number
- \( \text{Le} \): Lewis number
- \( M \): Magnetic field
- \( \beta \): Wedge angle parameter
- \( R \): Radiation parameter
- \( \text{Nt} \): Thermophoresis parameter
- \( \text{Nb} \): Brownian motion parameter
- \( \gamma \): Moving wedge parameter
- \( \text{Nb} \): Dimensionless reaction rate
- \( E_a \): Activation energy
- \( \varphi_{\text{f}}, \varphi_{\text{h}} \): Solid volume fraction of MWCNT and SWCNT
- \( D_{\text{hl}}, D_{\text{rl}} \): Diffusion coefficient
- \( \gamma_{\text{f}}, \gamma_{\text{h}} \): Kinematic viscosity of the fluid
- \( \alpha_{\text{f}}, \alpha_{\text{h}} \): Diffusion coefficient
- \( \rho_{\text{hnf}} \): Effective dynamic density of the hybrid nanofluid
- \( \zeta \): Similarity variable
- \( \sigma \): Stefan-Boltzmann constant
- \( k_f, k_h \): Mean absorption coefficient
- \( B_0 \): Magnetic induction parameter
- \( \rho_p \): Specific heat of hybrid nanofluid
- \( \mu_{\text{hnf}} \): Dynamic viscosity of the hybrid nanofluid
- \( k_{\text{hnf}} \): Thermal conductivity of the hybrid nanofluid
- \( \rho_{\text{p}} \): Effective heat capacity of the nanoparticle
$\rho_f$: Density of the fluid

$(\rho c_p)_f$: Specific heat of the fluid

$k_f$: Thermal conductivity of the fluid

Subscripts

$f$: Fluid

$nf$: Nanofluid

$hnf$: Hybrid nanofluid

$w$: Quantities at wall

$fs$: Quantities at free stream.

Data Availability

No data were associated with this submission.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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