Ultralight Scalars and Spiral Galaxies

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Abstract

We study some possible astrophysical implications of a very weakly coupled ultralight dilaton-type scalar field. Such a field may develop an (approximately stable) network of domain walls. The domain wall thickness is assumed to be comparable with the thickness of the luminous part of the spiral galaxies. The walls provide trapping for galactic matter. This is used to motivate the very existence of the spiral galaxies. A zero mode existing on the domain wall is a massless scalar particle confined to (1 + 2) dimensions. At distances much larger than the galaxy/wall thickness, the zero-mode exchange generates a logarithmic potential, acting as an additional term with respect to Newton’s gravity. The logarithmic term naturally leads to constant rotational velocities at the periphery. We estimate the scalar field coupling to the matter energy-momentum tensor needed to fit the observable flat rotational curves of the spiral galaxies. The value of this coupling turns out to be reasonable – we find no contradiction with the existing data.
1 Introduction

Many years ago, Zeldovich, Kobzarev, and Okun considered a network of domain walls in the Universe, which inevitably develops provided the underlying theory has a spontaneously broken discrete symmetry, and came to the conclusion that such a network is ruled out because its existence would lead to an unacceptable cosmology (for a review and more detailed discussion see [3]). In this paper we revisit the issue of the domain wall network in a specific context of an ultralight dilaton field with an ultraweak self-interaction and a universal coupling to the matter fields. We show that such domain walls are not only compatible with experiment, but they may be used to explain salient features of the observed world – the flatness of the spiral galaxies and the constancy of the rotational velocities at the periphery of the spiral galaxies. Our suggestion can be summarized as follows. The mechanism to be discussed below provides a light trapping for the galactic matter. One or several spiral galaxies may be embedded in one wall. The transverse width of the luminous disk of the galaxy (which includes the major part of dark baryon matter) is of the order of the width of the domain wall, which is, in turn, of the order of the inverse mass of the dilaton, \( \Delta \sim m_d^{-1} \sim 10^{20} \text{ cm} \). At distances much larger than the disk thickness, \( L \gg \Delta \), the domain wall can be approximated by a two-dimensional surface. There is a scalar field which is confined to and propagates along this surface. The existence of this 2+1 dimensional massless scalar is an unavoidable consequence of the presence of the wall itself (the zero mode). The zero mode exchange at distances \( L \gg \Delta \) gives rise to a logarithmic modification of the gravitational potential, so that the rotational velocities are constant at distances of a few dozen kiloparsecs, as required by observations.

Our model includes the usual fine-tuning of the cosmological constant. We have nothing to add in this respect, and assume that the cosmological constant is somehow fine-tuned to fit the current estimate,

\[
\varepsilon \lesssim (10^{-3} \text{ eV})^4.
\]

The extremely small dilaton mass and couplings are then automatically protected by Eq. (1) against renormalizations due to virtual matter fields propagating in loops. Quantum gravity loops may pose a problem, see Sec. 2.

2 Ultralight Scalars

Fundamental particle physics theories often deal with some very weakly coupled massless scalar particles, such as dilatons or other moduli. Usually there are dynamical reasons which protect the masslessness of these fields. As long as they stay massless they could mediate gravity-competing forces. This requires the strength of their interactions to be adequately suppressed.
Although the massless limit is a useful theoretical laboratory, in actuality, the dilaton-type fields cannot remain massless – for stabilization a mass term has to be generated. Usually, the mass generation (moduli stabilization) is attributed to some nonperturbative physics responsible for supersymmetry (SUSY) breaking. In view of quantum corrections, it is naturally to expect then the mass in the ballpark $m \sim M_{\text{SUSY}}^2/M_P$, where $M_{\text{SUSY}}$ is the scale of supersymmetry breaking and $M_P$ is the Planck mass. Since, $M_{\text{SUSY}} \gtrsim 1$ TeV, the mass comes out $\gtrsim 10^{-3}$ eV.

For the applications to be discussed below we will need a much lighter scalar field. *A priori* it is not difficult to imagine the existence of a field with the bare mass

$$m_0 \sim M_P \exp\left(-\frac{8\pi^2}{g^2}\right),$$

where $g$ is some coupling constant, $g \lesssim 1$. If, for instance, $g \simeq 0.8$, then we get $m_0 \sim 10^{-33}$ GeV (our favorite number, see below). The problem is that the bare mass that small will not be protected from huge renormalizations in loops. In the full interacting quantum theory, if there is no special reason, the scalar mass will be shifted towards the natural estimate $\sim M_{\text{SUSY}}^2/M_P$ by loop corrections. The question is whether there exists a set-up that would naturally protect a scalar field from renormalizations.

One can pose this question at two different levels. First, one can switch off gravity and consider loops generated by virtual particles of the Standard Model or other fields that may be present in extensions of the Standard Model (these quantum corrections will be referred to as SM loops). We found a mechanism protecting against these SM loops. A model will be presented below in which the mass and coupling constants of a universally coupled scalar field are protected in a natural way.

At the next level, one can start bothering about gravity-generated loops. Generally speaking, the mass of any elementary scalar field is not protected from renormalizations by quantum gravity loops. Thus, a protection against the quantum gravity loops may be needed – perhaps, a fine-tuning of a special kind. Since quantum gravity is not yet a closed theory, we will not ask in the present paper what is the precise nature of this protection, but, rather, focus on astrophysical consequences of an ultralight universally coupled scalar field.

(Let us parenthetically note that an obvious possibility is to assume that the scalar field in question is a composite one, with a very low compositness scale.)

The model we suggest is as follows. Consider a field theory which has both dimensionless and dimensionful parameters (the latter include, in particular, the physical masses as well as the ultraviolet regulator masses). In spite of the presence of the dimensionful parameters, one can make the theory scale-invariant if every parameter of mass dimension $\nu$ is multiplied by

$$\exp\left(\frac{\phi}{M_*}\nu\right),$$

where $M_*$ is some ultraviolet regulator mass.
where $\phi$ is a dilaton field. The scale transformation

$$x \to \kappa \cdot x, \quad \Phi \to \Phi \cdot (\kappa)^{-\text{dim}\Phi},$$

where $\Phi$ is a generic field and $\text{dim}\Phi$ is its dimension, must be supplemented by a shift of the dilaton field,

$$\phi \to \phi + M_* \ln \kappa.$$  \hspace{1cm} (5)

The scale invariance is then obvious.

As long as the scale invariance is unbroken, the effective Lagrangian of the dilaton field, all loops included, must be proportional to

$$\varepsilon \exp \left( \frac{4\phi}{M_*} \right).$$  \hspace{1cm} (6)

If the vacuum energy density $\varepsilon$ is fine-tuned as in Eq. (1) and $M_* > M_P$ (as will be the case, see below), the loop renormalizations of the quadratic, cubic, quartic, etc. terms in the dilaton field will be negligibly small, automatically. When we give a mass to the dilaton field, and some self-interactions, we explicitly break the scale invariance in the dilaton sector. If the coupling constant is sufficiently small, however, this breaking will not lead to large renormalizations either. A typical constraint is

$$\lambda M_{\text{SUSY}}^2 < m_d^2,$$  \hspace{1cm} (7)

where $\lambda$ is a quartic coupling constant. Equation (7) implies that $\lambda \ll 10^{-72}$. In fact, in our model $\lambda$ is many orders of magnitude smaller than the estimate above.

The coupling of the dilaton field under consideration to matter is universal, through the trace of the matter energy-momentum tensor $\theta_{\mu\nu}$,

$$\Delta \mathcal{L}_{\text{int}} = \frac{\phi}{M_*} \theta_{\mu}^{\mu}.$$  \hspace{1cm} (8)

We will refer to any such $\phi$'s as the dilatons. In this language, say, the loop correction to the dilaton mass is determined by the zero-momentum two-point function of $\theta_{\mu}^{\mu}$'s

$$(M_*)^{-2} \int d^4x \langle \theta_{\mu}^{\mu}(x) \theta_{\nu}^{\nu}(0) \rangle = (M_*)^{-2}(-4) \langle \theta_{\mu}^{\mu} \rangle.$$  \hspace{1cm} (9)

\footnote{Note that the string theory dilaton does not have such a property and, unless it is heavy, gives rise to the violation of the equivalence principle even in the nonrelativistic approximation. We will not deal with the string dilaton, but, rather, concentrate on a hypothetical dilaton which couples universally to the trace of the matter energy-momentum tensor. This latter can lead to the equivalence-principle violating effects in relativistic experiments (see below).}
The proportionality of this correlator to the vacuum expectation value of $\theta_{\mu}$ is a consequence of the scale Ward identity \[4\). Therefore, the mass renormalization is proportional to the cosmological constant,

$$\Delta (m_d^2) \sim \varepsilon/M_*^2, \quad (10)$$

and can be neglected provided the cosmological constant is fine-tuned to its empiric value.

In this way, we naturally arrive at an ultralight scalar; its Compton wavelength may be of the astronomical (galactic) size $\sim 10^{20}$ cm (corresponding to the mass $m_d \sim 10^{-33}$ GeV). The parameter $M_*$ will be fitted below to generate the observable value of the tails of the rotational curves in the spiral galaxies,

$$M_* \sim \sqrt{M_G m_d C}, \quad (11)$$

where $M_G$ is the mass of the baryon matter in galaxy, $M_G \sim 10^{68}$ GeV, and $C$ is a dimensionless constant of the order of several units times $10^{-7}$ (see Sec. 4). Numerically, Eq. \(11\) implies that $M_*$ is 10 to 100 Planck masses. It is remarkable that a constant of the Planck scale emerges from such macroscopic quantities as the galactic mass and its width $\Delta \sim m_d^{-1} \sim 10^{20}$ cm.

At distances much less than $\Delta$ the dilaton can be considered as massless; it will provide a long-range force competing with the Newton gravity, with a different vertex structure (the scalar rather than tensor exchange). Needless to say that this violates the equivalence principle in relativistic gravitational measurements. Therefore, its coupling must be sufficiently suppressed. We will discuss this issue at length below. Here we only note that if the parameter $M_*$ is larger than

$$M_*^2 \gtrsim (10^3 - 10^4) M_P^2, \quad (12)$$

the effects of the equivalence principle violation are below the currently observable level. Remarkably, the very same value of $M_*$ emerges from the domain wall explanation of the constant tails of the rotational curves.

In the present paper we will adopt a pragmatic approach and study astrophysical implications of such very weakly interacting and extremely light dilaton field(s) universally coupled to the trace of the energy-momentum tensor, without speculating on its possible origins or trying to explain, from first principles, why its mass might lie in the ballpark of $10^{-33}$ GeV. We will postulate a specific shallow self-interaction potential. It must exhibit a discrete symmetry, which will be spontaneously broken in the vacuum state. Then a network of domain walls can develop. As we will see, this may lead to trapping of the galactic matter within the wall world-volume and, in addition, to a logarithmic long-range interaction potential on the surface of the domain wall. Our goal is to explore whether such set-up can provide an explanation

\footnote{Mind the issue of gravity loops, see above.}
for the existence of the spiral galaxies and for the constancy of the rotational curves of the spiral galaxies. Before passing to astrophysical aspects, we briefly review domain walls in field theory. Our purpose is to formulate what is needed from the dilaton sector to produce the domain walls of the required type.

3 Domain Walls

To begin with, let us assume that we deal with one real field $\phi$, and switch off the gravitational coupling. The above simplifying assumptions allow us to present the main idea without inessential complications. After the mechanism is presented, we will consider a two-field model and will switch on the coupling with gravity.

Let us consider the simplest possible Lagrangian leading to domain walls (for a review of domain walls in the given context, see e.g. Ref. [5]),

$$L = \frac{1}{2} (\partial_{\mu} \phi)^2 - \frac{\lambda}{2} (\phi^2 - \eta^2)^2.$$  \hspace{1cm} (13)

As we will see below, this Lagrangian is not quite realistic; a more involved model is needed to ensure both, the matter trapping and the proper logarithmic potential. However, the simple Lagrangian below demonstrates the main idea in terms of analytic expressions (while in more realistic cases only numerical solutions can be given).

We temporarily drop interactions of $\phi$ with the matter fields. The classical equations of motion of this system possess a solution in the form of a domain wall stretched in the $x, y$ plane,

$$\phi_{cl}(z) = \eta \tanh (\sqrt{\lambda} \eta z).$$  \hspace{1cm} (14)

The $\phi$ mass $m$ and the wall tension $\sigma_{DW}$ are

$$m = 2\sqrt{\lambda} \eta, \quad \sigma_{DW} = 4\sqrt{\lambda} \eta^{3/2}.$$  \hspace{1cm} (15)

Let us consider perturbations around this solution. It is well known that there exists a massless mode $\rho$ which lives on the wall worldvolume. This can be expressed as follows:

$$\phi(t, x, y, z) = \phi_{cl}(z) + \frac{1}{\sqrt{\sigma_{DW}}} \frac{d\phi_{cl}(z)}{dz} \rho(t, x, y).$$  \hspace{1cm} (16)

One can check that $\rho$ satisfies the equation of motion for a free massless (2+1)-dimensional scalar field,

$$\left( \partial_t^2 - \partial_x^2 - \partial_y^2 \right) \rho(t, x, y) = 0.$$  \hspace{1cm} (17)

Therefore, the field $\rho$, when coupled to matter appropriately, mediates logarithmic potential between sources located on the domain wall,

$$\langle \rho(0, x, y) \rho(0, 0, 0) \rangle \sim \ln \sqrt{x^2 + y^2}.$$  \hspace{1cm} (18)
Trapping field

Figure 1: The bell-shaped profile of the field modulus.

The logarithmic potential at large distances takes over the Newtonian $1/r$, with necessity, and generates a constant component in the average rotational velocity (i.e., a component which does not fall off with the distance $r$). From this point of view, the domain wall built from one real scalar field $\phi$ is perfectly sufficient. What makes us consider more complicated dilaton sectors?

There are two reasons. First, we would like the walls to act as a natural trap for matter. Given the profile of the field $\phi$ in the wall of the type (14) we see that the coupling (8) leads to a monotonous variation of the nucleon masses – the nucleons are slightly heavier on one side of the wall and slightly lighter on the other. This is not what we want. We want them to be lighter inside the wall, and heavier outside. To this end the profile of the field coupled to $\theta^\mu_\mu$ must be of the type depicted on Fig. 1.

It is not difficult to achieve such a profile in two-component models, where, in fact, it is quite typical. Assume we have two real fields, $\phi$ and $\chi$, the “primary” domain wall of the field $\phi$ is of the type (14), and it forces a structure in the $\chi$ field of the type

$$\chi_{cl}(z) \propto -\frac{1}{\cosh (mz)}$$

(19)

to develop. This is the case, e.g., in the potentials of the type

$$V(\phi, \chi) = \left( \frac{m^2}{\sqrt{\lambda}} - \sqrt{\lambda}\phi^2 - \alpha\chi^2 \right)^2 + \beta\phi^2\chi^2, \quad \alpha, \beta > 0,$$

(20)

which were recently considered [6] in a different context. The maximal $|\chi|$ is achieved at the point right in the middle of the $\phi$ wall, where $\phi = 0$. Denote this maximal
value by $\eta_\chi$. For our purposes it is necessary that $\eta \ll \eta_\chi$ while $m_\chi \ll m_\phi$ where much less can actually mean a factor of several units.

Assume that both, $\chi$ and $\phi$ are universally coupled to the trace of the energy-momentum tensor,

$$\Delta L_{\text{int}} = \left( \frac{\phi}{M_*} + \frac{\chi}{M_*} \right) \theta_\mu^\mu,$$  \hspace{1cm} (21)

and the constraints above are satisfied.

The main virtue of this set-up is the possibility to trap the galaxy matter inside the domain wall. Since $\eta \ll \eta_\chi$ the main role in the trap is played by the $\chi$ field. The matter interaction (21) is such that nucleons are lighter inside the wall. This can be viewed as a shallow potential \[\text{(4)}\] for particles which constitute the galaxy. The galaxy matter is naturally lightly trapped. The relative nucleon mass variation inside/outside can be easily estimated to be $\delta M/M \sim 10^{-9}$ to $10^{-10}$. The corresponding escape velocity in the direction perpendicular to the wall (galaxy) plane is 3 to 10 km/sec.

At the same time, since $m_\chi \ll m_\phi$ the zero-mode mediating logarithmic potential will be predominantly associated with the field $\phi$, rather than $\chi$. We need this to be the case since it is $d\phi/dz$ that has the proper bell-like shape, rather than $d\chi/dz$. The bell-like shape of the zero mode is needed in order to ensure such (logarithmic) attraction of the distant bodies in the galaxy lying inside the domain wall which would be (approximately) independent of the position of the body in the transverse direction. The profile of $d\chi/dz$ is inappropriate for that purpose.

Another reason for playing with more sophisticated dilaton sectors is the desire to have a network of domain walls with the stable wall junctions. The stability of the wall junctions cannot be attained in the simplest $\mathbb{Z}_2$-based models.

In order to produce the junction solutions one should start from a more complicated Lagrangian for a scalar field, with some $\mathbb{Z}_N$ symmetry. A simplest possibility along these lines would be to start from a $\mathbb{Z}_4$ symmetric potential for a complex scalar field $\varphi$,

$$V(\varphi^+, \varphi) = \frac{\lambda}{M_p^4} \left| \eta^4 - \varphi^4 \right|^2.$$ \hspace{1cm} (22)

There are four minima described by this potential: $\varphi_{\text{vac}} = \eta \exp(i2\pi k/4)$, $k = 0, 1, 2, 3$. The original $\mathbb{Z}_4$ symmetry is spontaneously broken in any of these four vacua. As a result, $\mathbb{Z}_4$ domain walls and junctions could form (see Fig. 2). Each wall in the junction can move along the transverse direction. Hence, there are zero modes on each of them. These latter remain to be zero modes as long as gravitational interactions are neglected. (The corresponding mixing of the zero modes with gravitons is discussed below; it turns out to be unimportant.)

The idea is that the spiral galaxies will be located on the links between the junctions. On the other hand, the junction points can be used to create centers for
Galaxy

Figure 2: A network of domain walls with immersed galaxies.

elliptic galaxies. Of course, we imply that in the realistic situation the domain wall network is more like a soap foam than the regular lattice of Fig. 2.

So far, in discussing the wall structure we ignored the facts that the dilaton fields are coupled to gravity and to matter, i.e. we ignored the back reaction of matter on the walls themselves.

What changes when the interaction of the scalar field with gravity is switched on? Had we an idealized case of a single wall with a single galaxy on it, then the regions of the wall where there is no galaxy matter (i.e., far away from the galaxy disc along the wall) would inflate with the following line element [2, 13]:

$$\begin{align*}
    ds^2 &= \left(1 - H |z|\right)^2 dt^2 - \left(1 - H |z|\right)^2 e^{2Ht} \left(dx^2 + dy^2\right). \\
    \text{(23)}
\end{align*}$$

The inflation would be governed by a Hubble parameter which is defined as $H \sim G_N \sigma_{DW}$. Below we will argue that the wall tension $\sigma_{DW}$ must be smaller than

$$\sigma_{DW} \ll 10^{-5} \text{ GeV}^3,$$

so that $H \ll 10^{-43} \text{ GeV}$. Therefore, the horizon in the $z$ direction, which is due to the Rindler type metric of the domain wall, would be well beyond our true horizon. In reality, however, there are other domain walls and other galaxies surrounding the wall, so that the aforementioned inflationary line element is not applicable.

Another important gravitational effect is the mixing of the $\{zm\}$ components $(m = x, y)$ of the graviton with the zero mode of the $\phi$ field on a $(2 + 1)$-dimensional wall worldvolume, which, in principle, leads to a “mass” of the graviton inside this layer. Let us show that this effect is so small numerically that it can be safely neglected at distances of the size of the galactic disk or larger.
The mixing term emerges due to the interaction
\[ \int dz \sqrt{g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi . \tag{24} \]
When one of the \( \phi \)'s is substituted by the classical solution \( \phi = \phi^{(1)} \), this term gives rise to an effective (2+1)-dimensional mixing of \( \rho \) with the vector component of the metric ("graviphoton"),
\[ A^m(t, x, y) \equiv \sqrt{m_d} \int_{-\Delta/2}^{\Delta/2} dz g^{mz}(x, z) M_P, \quad m = 0, 1, 2 . \tag{25} \]
This (2+1)-dimensional mixing takes the form
\[ m_d \frac{\eta}{M_P} A^m(t, x, y) \partial_m \rho(t, x, y) . \tag{26} \]
The graviton propagation along the wall worldvolume is modified due to this mixing term. This leads to the constraint
\[ \eta \ll M_P \left( \frac{\Delta}{L} \right) , \tag{27} \]
where \( L \) denotes the longitudinal size of the galactic luminous disk, or any appropriate distance along the wall. Since the ratio \( \Delta/L \) is typically of the order \( 10^{-2} - 10^{-3} \), the constraint \( \eta \ll M_P \) is not stringent. We will see below that other considerations restrict the value of \( \eta \) much more severely. In fact, one can go to as large values of \( L \) as \( L \sim 10^{27} \) cm without violating the constraint \( \eta \ll M_P \).

The last issue to be discussed in this section is the back reaction of matter on the wall. Certainly, this interaction distorts the wall profile. The easiest way to picture the distortion is to consider a single infinite domain wall on which a single finite-size galaxy disc is placed. Far away from the galaxy disc (along the wall) the effect of the distortion is negligible. As we come closer to the galaxy the distortion could become noticeable. The wall may swell but it cannot disappear; the trapping due to the \( \chi \) component of the wall is operating.

4 Application to Flat Rotational Curves

One of motivations for the existence of dark matter is the experimental result on flat rotation curves of spiral galaxies \( [8] \). The measured rotational velocity \( v \) can be related to the classical interaction potential \( V(r) \) as follows:
\[ v^2 = r \frac{dV(r)}{dr} . \tag{28} \]

\[ {\text{In addition, there will be induced terms for gravitons} [14] \text{ and gauge fields} [13] \text{ which could give rise to additional interactions} [14, 15]. \text{However, these effects are negligible in the present case due to the low density of the galaxy and domain wall matter.}} \]

\[ {\text{The domain wall profile will also be distorted with respect to that of the galaxy matter, because the domain walls and the matter density follow different laws of the cosmological expansion. However, these effects are negligibly small in our model.}} \]
Experimental data [8] show that the velocity squared (after subtracting from it the part due to the Newtonian attraction) is nearly constant at distances much larger than the radius of the galaxy disk,

\[ v_{\text{exp}}^2 \sim \text{constant}. \quad (29) \]

The conventional Newton potential, \( V(r) \sim M_G/r \), with the constant galaxy mass \( M_G \) does not satisfy (28) provided that (29) is valid. A standard way out is to assume that there is a halo of dark matter which surrounds the galaxy disk, with the density distribution decreasing as \( \sim 1/r^2 \). As a result, the total mass \( M_G \) is raising linearly with \( r \) as long as we are inside the dark matter halo, and (29) is fulfilled.

Dark matter can have a few distinct components. The density of luminous matter is estimated to be \( \Omega_{\text{lum}} \sim (0.003 - 0.006) \). On the other hand, Big Bang Nucleosynthesis (BBN) [9] predicts the baryon density in the ballpark (for discussions, see, e.g. [10]): \( \Omega_B \sim (0.015 - 0.16) \). Since \( \Omega_{\text{lum}} < \Omega_B \), there is a substantial amount of dark baryon matter in the Universe.

It is believed that there is at least one additional very important component of dark matter, cold dark matter (CDM). CDM seems to be necessary for successful structure formation [11]. Although, the CDM could be providing dominant components in the elliptic galaxies, it is not excluded that the mechanism for flat rotational velocity curves of spiral galaxies is different. In particular, we explore below the possibility that flat rotational curves in spiral galaxies are due to the same domain walls which trap the galaxy matter in the disk plane.

Let us now discuss dynamical aspects of our mechanism in more detail. The logarithmic correction to the potential at galactic scales does explain the flat rotational curves,

\[ \delta V(r)|_{\text{few kpc}} \simeq C \ln r, \quad (30) \]

where \( C \simeq v^2 \). The logarithmic potential obviously satisfies Eq. (28) with the asymptotically constant velocity. In our galaxy \( v \simeq 220 \text{ km/s} \), hence

\[ C \simeq 5 \times 10^{-7}. \quad (31) \]

In general, \( v \simeq (60 \text{ to } 300) \text{ km/s} \) and, thus, \( C \sim (10^{-8} \text{ to } 10^{-6}) \).

The authors of Ref. [12] argue that the modified potential can arise due to a change of general relativity (GR) at galactic scales. (A typical galactic size is of the order of several units times \( 10^{22} \text{ cm} \). We take its thickness outside the central domain, the bulge, to be of the order of \( 10^{20} \text{ cm} \).) Eventually this interesting proposal might find some theoretical ground. At present, we are aware of no possibility for constructing a theoretically and phenomenologically viable modification of GR, which would give rise to such a potential. However, the logarithmic interactions can be obtained from domain walls, as we argued above. Let us estimate the force due to this logarithmic potential. The new term generates an interaction of the field \( \rho \)
with \( \tilde{\theta}_\mu \), an effective “three-dimensional” energy-momentum tensor measuring surface energy density in the galactic disk,

\[
\mathcal{L}_{\text{int}}^\rho \simeq \frac{1}{\sqrt{m_d} M_\ast \Delta} \rho \tilde{\theta}_\mu^\mu \equiv g \rho \tilde{\theta}_\mu^\mu ,
\]

(32)

where

\[
\Delta \equiv \frac{1}{m_d} \sim \frac{1}{\sqrt{\lambda \eta}} \sim 10^{20} \text{ cm}, \quad g = \frac{\sqrt{m_d}}{M_\ast}, \quad \tilde{\theta}_\nu^\mu \sim \theta_\nu^\mu \Delta .
\]

(33)

Here, \( \Delta \) is the wall thickness, of the order the transverse width of the luminous disk. Let us make a comment here. When we go to the distances which are larger than the disk diameter the interactions along the disk plane are mediated by the exchange of a zero-mode field \( \rho \). All other \((2 + 1)\)-dimensional massive modes of the \( \phi \) field are decoupled at those distances. However, at distances which are smaller than the disk diameter, at the solar system distances for instance, all those massive states give rise to sizable contributions. The net result of these states is equivalent to the exchange of a single \((3 + 1)\) dimensional scalar mode \( \phi \). This exchange is suppressed by \( 1/M_\ast \) and its effect is negligible for the solar system data.

We come to the following scenario. At the scale of distances less than the galactic thickness (i.e. \( \ll 10^{20} \) cm), in particular, in the Solar system, the impact of the field \( \phi \) reduces to a small \( 10^{-3} \) to \( 10^{-4} \), see below) correction to the Newtonian law. This correction shows up in the form of discrepancy between the measurements for the static and relativistic matter (e.g. light deflection). At such distances the mass of the \( \phi \) quantum can be neglected, and it acts as a massless \( 3+1 \)-dimensional field. This field couples to the galactic matter with yet-to-be-determined coupling \( 1/M_\ast \).

However, at distances \( \gg \Delta \), in particular at distances of the order the galactic disk size or larger, one can neglect the thickness of the galactic disk/domain wall. Then, \( \rho \) behaves as a \( 2+1 \)-dimensional excitation that mediates the logarithmic potential,

\[
\delta V(r)_{\text{few \ kpc}} \simeq M_G g^2 \ln r .
\]

(34)

Using the known value of the rotational velocity we find

\[
M_G g^2 \sim C \sim 5 \times 10^{-7} .
\]

(35)

This can be used to determine \( M_\ast \). Using the mass of the luminous galaxy for \( M_G \) one would underestimate \( M_\ast \). Instead, we take into account dark baryon matter in the galaxy disk. This gives (see Eq. (11))

\[
M_\ast^2 \sim \left( 10^{41} - 10^{42} \right) \text{ GeV} \simeq \left( 10^3 - 10^4 \right) M_P^2 .
\]

(36)

Therefore, we conclude that the interaction of the four-dimensional scalar with matter is sufficiently suppressed. It does not contradict the Solar system data such as
bending of light rays and precession of the Mercury perihelion which now are measured to one percent accuracy.

The modification of the Newton law discussed above will lead to a peculiar “two-component” interaction between distant galaxies located inside distinct wall cells (Fig. 1). First, there is a conventional Newtonian attraction through the bulk (due to conventional four-dimensional gravitons). This is not the end of the story, however. In addition, our dilaton gravity will propagate through the domain wall network\footnote{We are grateful to Tonnis ter Veldhuis who posed this question.}. Consider, for instance, two galaxies on adjacent domain walls connected by a common junction. Assume the distance of each galaxy to the junction is \( R \). Then each of the galaxies will experience an additional (non-Newtonian) attraction force towards the junction of order of \( (M_G/M_\ast)^2 (R\Delta)^{-1} \). This force falls off with the distance as the first rather than the second power of \( R \). It becomes comparable with the Newtonian force at distances \( R \sim 10^4 \Delta \sim 10^{24} \text{cm} \). If the number of the walls \( N \) joined in the given junction is large, say, \( N \sim 10 \), the extra force will be further suppressed by \( 1/N \) because the force line flux approaching the junction from one wall, after the junction will be shared by \( N-1 \) walls. This would shift the critical \( R \) even to larger distances, \( R \sim 10^{25} \text{cm} \).

Typically distant galaxies will be separated by several (perhaps, many) junctions. If the number of junctions is \( k \), the additional suppression factor one gets is \( (1/N)^k \).

The “simulated” attraction of the galaxies to the junction will tend to distort the form of the junctions themselves. This effect can be easily estimated by comparing the energy (per unit area) added to the domain wall due to the dilaton gravity induced by the given galaxy,

\[
\sigma_{DG} \sim \left( \frac{M_G}{M_\ast} \frac{1}{\Delta R} \right)^2 \Delta \lesssim 10^{-11} \text{ GeV}^3 ,
\]

if \( R \gtrsim 10^3 \Delta \sim 10^{23} \text{ cm} \). Since in our model the wall tension \( \sigma_{DW} \lesssim 10^{-11} \text{ GeV}^2 \), the distortion is not drastic and can be neglected at the level of accuracy we maintain here.

The next comment concerns the Tully-Fisher (TF) relation \footnote{We are grateful to Tonnis ter Veldhuis who posed this question.}. This is a connection between the flat rotational velocity and luminosity \( L \) of a galaxy: \( L \propto v^\alpha \). The coefficient \( \alpha \) is fit by the data and ranges for various galaxies in the interval \( \alpha = (2.5 - 4) \). If the luminosity were a linear function of the galaxy mass \( M_G \), the TF relation would give rise to the dependence \( v^\alpha \propto M_G \). This would certainly exclude our scenario, since the latter predicts \( v^2 \propto M_G \). However, the relation between the luminosity and the mass does not need to be linear. The total mass of the galaxy (without the CDM contribution) is composed of the luminous disk matter mass \( M_d \), the mass of the gaseous matter \( M_g \), the mass of the central bulge region of the galaxy \( M_b \) and, finally, of the dark baryon mass. Although the luminous disk mass could be linearly dependent on the luminosity, the other three components do not have to. Moreover, these components constitute a very important part of the galaxy mass. Just for the demonstrational purposes we present
the mass contents of a few spiral galaxies (for detailed discussions, see, e.g. \[18\]): (UGC2885, \(M_g = 5.0\), \(M_d = 25.1\), \(M_b = 5.7\)), (NGC5533, \(M_g = 3.0\), \(M_d = 2.0\), \(M_b = 17.0\)), (NGC6674, \(M_g = 3.9\), \(M_d = 2.5\), \(M_b = 15.5\)), (NGC5907, \(M_g = 1.1\), \(M_d = 7.2\), \(M_b = 2.5\)). All masses here are given in the units of \(10^{10} M_{\odot}\).

Therefore, it is conceivable to assume that the relation \(v^2 \propto M_G\), predicted by the present scenario, does not contradict the known data.

So far we assumed that only a single galaxy disk can be embedded in a given domain wall. However, this condition is not necessary. For instance, one could imagine a pair of nearby galaxies which are rotating with respect to each other inside one and the same domain wall. If so, there is an additional logarithmic force between them due to the exchange of \(\rho\). Due to this logarithmic force, the nearby binary galaxies would seem to be “confined” to each other. This can be used to explain the old puzzling data of Ref. \[19\] where it was observed that binary galaxies attract each other with forces stronger than those exerted by their disks. On the other hand, distant galaxies can and should be located inside distinct domain walls. Then, the logarithmic potential between them will not be present and they will be interacting by means of Newtonian force.

5 Other data

Let us now check whether this type of domain wall scenarios are allowed by cosmological and astrophysical data.

- Domain Wall Domination
  
  First we have to make sure that the tension of the domain wall is small enough so that its contribution to the density of the Universe is not large and the evolution of the Universe is not domain wall dominated. The standard estimate \[1\] leads:

  \[
  \Omega_{DW} \sim \frac{\sigma_{DW}}{10^{-5} \text{ GeV}^3} .
  \]

  Given that \(m_d \sim (10^{-33} - 10^{-32}) \text{ GeV}\) and assuming that \(\Omega_{DW} \ll 1\), we get \(\eta \ll 10^{13} \text{ GeV}\).

- The CMBR anisotropy

  The presence of the domain wall network in the Universe would lead to an anisotropy in Cosmic Microwave Background Radiation. The CMBR temperature anisotropy \(\delta T/T\) is measured with very high accuracy. Therefore, the anisotropy introduced by the domain walls should be constrained as follows:

  \[
  \frac{\delta T}{T} \sim \frac{\sigma_{DW}}{H_0 M_p^2} < 10^{-4} .
  \]
This gives rise to a more stringent bound on $\eta$,

$$\eta < 10^{11} \text{ GeV}.$$  

Note that if $\eta$ is close to its upper bound, then the coupling constant $\lambda$ is extremely small, $\lambda \sim 10^{-88}$. Therefore, self-interactions of the $\phi$ field are harmless.

There are additional constraints which should be imposed on the mass and decay constant of $\phi$. These come from:

- Star cooling
- Big Bang Nucleosynthesis

In the former case one must make sure that the stars do not cool too fast due to the emission of energy in the form of the light $\phi$ quanta. Moreover, for the purpose of successful BBN the decay $\phi \rightarrow \gamma\gamma$ should be suppressed enough to avoid overproduction of entropy which would spoil the standard BBN scenario. Using the analysis of [20] one can see that the scalar field with the mass $m_d \sim 10^{-32}$ to $10^{-33}$ GeV and the constant $M_* \sim 10^2 M_P$ satisfies these constraints.

6 Cosmological evolution

A few words are in order regarding the cosmology of the “dilatonic” wall formation. The domain walls that can survive till the present epoch must have been formed after the inflation. This implies that the discrete symmetry must be restored either during the inflation or, at least, during reheating. For the “dilatonic” type walls under consideration this issue is somewhat subtle, as we will now discuss.

First, during the inflation the VEV of $\phi$ cannot vanish, due to the fact that expectation value of $\theta^\mu$ is nonzero and it generates a tadpole for the $\phi$ field. As a result, the pre-existing domain walls would be inflated away. If the only coupling to matter is through the term $e^{\phi/M_*} \theta_\mu$, the symmetry cannot be restored after reheating either, since the thermal average of the $\theta_\mu$ vanishes and $\phi$ is never in the thermal equilibrium. Thus, we are left with no walls.

A way out would be to add some additional, $Z_N$-preserving interactions which would couple $\phi$ to matter fields that are in thermal equilibrium during reheating. Let $X$ be such scalar matter field. Then the relevant interaction is

$$\frac{\phi^2}{M_P^2} X^4. \quad (39)$$

We choose the sign of this coupling to be positive. Although $\phi$ is not in the equilibrium, the above coupling generates an effective potential for it, which can restore the
symmetry. The relevant diagrams are “butterfly” diagrams with two $X$ loops and external $\phi$ legs \[21\]. Each $X$ loop contributes a $T^2$ factor ($T$ being the temperature), so that the resulting effective operator in the free energy has the form

$$\phi^2 \frac{T^4}{M_P^2},$$

(40)

with the positive coefficient. Since in the radiation dominated epoch $\frac{T^4}{M_P^2} \sim H^2$, the field $\phi$ gets a positive mass term of the order of the Hubble parameter. At the same time, there is no tadpole since $\theta_{\mu}^{\mu}$ vanishes. Thus, the discrete symmetry may be restored until the zero temperature potential starts dominating again, and the phase transition takes place with the subsequent wall formation.

Note that the coupling (39), if included in a non-universal way, can induce an unacceptably large zero-temperature mass renormalization of $\phi$, which should be readjusted to zero along with the possible quantum gravity corrections. This does not add additional fine-tuning to the model.

7 Comments on the literature

It should be noted that models with ultralight (pseudo)Goldstone bosons developing domain walls and characterized by phase transitions at late time, after the decoupling of the microwave background, had been suggested long ago \[22\]. The Compton wave length of the ultralight boson considered, was in the ballpark of 1 to 100 Mpc, its mass being protected by a continuous $U(1)$ symmetry, of which the ultralight boson in question is the Goldstone boson. It was found that this set-up may be helpful from the phenomenological standpoint, for the large-scale structure formation, a welcome feature.

While this model shares certain common elements with ours, differences are crucial. The main distinction is that the coupling of the ultralight boson \[22\] with fermions is nonuniversal and pseudoscalar (which, naturally, does not allow for coherent effects inherent to the dilaton universally coupled to the trace of the energy-momentum tensor)\[^{\text{6}}\]. It remains to be seen whether our dilaton-based model enjoys the same favorable environment for the large-scale structure formation as that of Ref. \[22\].

An ad hoc logarithmic long-range potential coupled to baryon number, as a possible explanation of the constancy of the rotational curves, was also discussed in \[24\]. Since this potential was assumed to exist in the three-dimensional bulk, this led to irreconcilable contradictions with the microlensing data. The authors themselves noted that their construction was ruled out. None of the drawbacks of \[24\] exist in our scheme.

\[^{\text{6}}\]CP violation might turn the pseudoscalar coupling into scalar \[23\]. The amount of CP violation is severely limited, however, by the neutron dipole moment data.
8 Discussion

We presented a scenario in which spiral galaxies have their natural habitat inside very fat domain walls. The existence of these walls gives rise to trapping of matter within the wall which may provide a natural explanation for the very formation of flat spiral galaxies. In addition, a scalar particle living on the walls (zero modes), generates a logarithmic potential which takes over the Newtonian $1/r$ at distances $\gtrsim 10^{22}$ cm. This explains the constancy of the tails of the rotational curves in the spiral galaxies.

Although our mechanism ensuring a logarithmic component in the attraction potential of matter is very simple, surprisingly it seemed to escape attention of the previous investigators. As far as we can see, it contradicts no existing data. However, there are several issues which should be studied further. Among those are the questions: Are there subtler effects which might limit the applicability of this mechanism? Do “exceptional” galaxies which are perpendicular to the walls exist? Is there a place in this picture for elliptic galaxies and galaxy clusters? (Near the junctions?) Could the velocity dispersion in the elliptic galaxies be explained by the presence of dark baryons alone? All these questions should be analyzed further. However, since these issues cannot be studied analytically, but rather require involved numerical simulations, we did not address them here.

To our mind, a hint that there might be some truth in the suggested picture is the remarkable numerical proximity of the parameter $M_\star$ obtained in Sec. 4 from the galactic parameters, to the Planck scale. It is unlikely that it is accidental. Another positive feature that makes us optimistic is the fact that we found a set-up ensuring “almost no renormalization” environment for the ultralight scalar (at least, as far as the SM loops are concerned). The universal coupling to the trace of the energy-momentum tensor, in conjunction with the scale Ward identities, provides a natural protection. The parameters of the scale symmetry breaking in the dilaton sector we need for the success of the model are such that this protection is operative.

We would like to stress that the mechanism outlined in the present paper can be confronted with experiment in the near future. Indeed, the coupling of the $\phi$ field to matter is suppressed with respect to the gravitational coupling only by a three to four orders of magnitude. This is a borderline where the equivalence principle, as well as relativistic gravitational effects, are tested experimentally. Further improvements in the measurements by one or two orders of magnitude could be decisive for this proposal.

There is another tantalizing experimental possibility. Assume, in a given spiral galaxy two independent measurements are performed: gravitational lensing and the rotational curve. One could reconstruct the dark matter distribution that would fit the result of the gravitational lensing measurement. Assume the reconstruction has sufficient precision that would enable one to say that the amount of dark matter obtained from gravitational lensing is insufficient to describe the rotational curve. This would be a very strong argument in favor of our conjecture.
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