Transitions of two baryons to the H dibaryon in nuclei

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We calculate the suppression in the rate at which two baryons in a nucleus (viz., nucleons or Λ’s) convert to an H dibaryon, using an Isgur-Karl wavefunction for quarks in the baryons and H, and a Bethe-Goldstone wavefunction for the baryons in the nucleus. If $r_H \lesssim 1/3$ $r_N$, we find $\tau_{AAA}$ $\gtrsim \tau_\Lambda$ and the observation of Λ decays from double-Λ hypernuclei does not exclude the existence of the H. If $m_H < 2 m_p$, nuclei are unstable but have very long lifetimes. For reasonable values of $r_H$ and the nuclear wavefunction, the lifetime can be long enough to evade anticipated SuperK limits $\tau_{A_{NN}} \gtrsim \text{few} 10^{29}$ yr, or short enough to be observed. An analysis of SuperK data to look for this possibility should be undertaken.

I. INTRODUCTION

The H dibaryon corresponds to the most symmetric color-spin representation of six quarks (uuddss). It is a flavor singlet state with charge 0, strangeness -2 and spin-isospin-parity $I(J^P) = 0(0^+)$. The existence of the H was predicted by Jaffe in 1977 [1] in the framework of the quark-bag model. Its mass was originally estimated to be around 2150 MeV, making it stable toward strong decay to two Λ particles. Since then, there have been many theoretical efforts to determine its mass and production cross section and, on the experimental side, many inconclusive or unsuccessful attempts to produce and detect it.

Our work is prompted by the possibility that the H is lighter than two nucleons. This is motivated by several lines of reasoning based on hadron phenomenology and non-perturbative QCD modeling[2]. The interpretation of Λ(1405) and Λ(1520) as bound states of gluon plus uds quarks in a flavor-singlet color-octet state, suggests the H has properties similar to a glueball and $m_H \approx 1.3 - 1.8$ GeV[2, 3]. Adding current quark masses to the original Skyrme model calculation with massless quarks[2, 4] gives $m_H \approx 1.8$ GeV. Finally, an instanton-liquid calculation gives $m_H = 1780$ MeV[5].

Being tightly bound, the H is expected to be a spatially compact state. Analogy with the glueball and instanton-liquid results suggest $r_H \approx r_G \approx (3/3 - 1/2)$ $r_\pi \approx (1/6 - 1/4)$ $r_N$ [2, 6]. In the absence of an unquenched, high-resolution lattice QCD calculation capable of a reliable determination of the H mass and size, we will take $r_H/r_N = 1/f$ with $f$ treated as a parameter expected to be in the range 4-6. For a more detailed discussion of motivation and properties of a stable H, and a review of experimental constraints on such an H, see ref. [2].

In this paper we focus on two types of experimental constraints on the transition of two baryons to an H in a nucleus, $A_{BB}$ $\rightarrow$ $A'_H X$. Experiments observing single Λ decays from double Λ hypernuclei $A_{AA}$[7, 8] indicate that $\tau(A_{AA} \rightarrow A'_H X) \lesssim \tau_\Lambda = 3 10^{-10}$ sec. In addition, if the H is lighter than two nucleons, nuclei are unstable toward $\Delta S = -2$ weak decays producing the H particle in the final state. To estimate the rates for these processes requires calculating the overlap of initial and final quark wavefunctions in a nucleus. We will model that overlap using an Isgur-Karl harmonic oscillator model for the baryons and H, and the Bethe-Goldstone wavefunction for a nucleus. Our results will be expressed in terms of the parameter $f = r_N/r_H$ and the nuclear hard core radius. We will find that the stability of nuclei is the more stringent constraint on the properties of a stable H, but is acceptably small if the H is sufficiently compact: $r_H \approx 1/4$ $r_N$ depending on mass and nuclear hard core radius. Adequate suppression of $\Gamma(A_{AA} \rightarrow A'_H X)$ requires $r_H \lesssim 1/3$ $r_N$, whether H is stable or not. Thus a byproduct of this investigation is that the conventional H with mass $2 m_N - m_H < 2 m_\Lambda$ may still be viable in spite of the observation of double-Λ hypernuclei, consistent with the conclusion of ref. [9].

This paper is organized as follows. In section II we describe in greater detail the two types of experimental constraints on the conversion of baryons to an H in a nucleus. In section III we calculate lifetimes for these transitions, using a non-relativistic harmonic oscillator quark model and a phenomenological treatment of the weak interactions. The results are summarized in section IV.

II. EXPERIMENTAL CONSTRAINTS

A. Stability of nuclei

There are a number or possible reactions by which two nucleons can convert to an H in a nucleus. The initial state is most likely to be $pn$ or $nn$ in a relative s-wave, because in other cases the Coulomb barrier or relative orbital angular momentum suppresses the overlap of the nucleons at short distances which is necessary to produce the H. If $m_H \lesssim 2 m_N - n m_\pi^{-1}$, the final state

\[ ^1 \text{Throughout, we use this shorthand for the more precise inequality } m_H < m_A - m_{A'} - m_X \text{ where } m_X \text{ is the minimum invariant mass of the final decay products.} \]
can be $H \pi^+$ or $H \pi^0$ and $n-1$ pions with total charge 0. For $m_H \gtrsim 1740$ MeV, the most important reactions are $pn \rightarrow H e^+\nu_e$ or the radiative-doubly-weak reaction $nn \rightarrow H \gamma$.

The most sensitive experiments to place a limit on the stability of nuclei are proton decay experiments. Super Kamiokande (SuperK), places the most stringent constraint due to its large mass; it is a water Cerenkov detector with a 22.5 kiloton fiducial mass, corresponding to $8 \times 10^{22}$ oxygen nuclei. SuperK is sensitive to proton decay events in over 40 specific proton decay channels\cite{10}.

Since the signatures for the transition of two nucleons to the H are substantially different from the monitored transitions, a specific analysis by SuperK is needed to place a limit. We will discuss the order-of-magnitude of the limits which can be anticipated.

Detection is easiest if the H is light enough to be produced with a $\pi^+$ or $\pi^0$. The efficiency of SuperK to detect neutral pions, in the energy range of interest (KE = 0 - 300 MeV), is around 70 percent. In the case that a $\pi^+$ is emitted, it can change charge to $\pi^0$ within the detector, or be directly detected as a non-showering muon-like particle with similar efficiency. More difficult is the most interesting mass range $m_H \gtrsim 1740$ MeV, for which the dominant channel $pn \rightarrow H e^+\nu$ gives an electron with $E \sim (2m_N - m_H)/2 \lesssim 70$ MeV. With a rate of order $\alpha$ smaller, the $nn \rightarrow H \gamma$ channel would give a monochromatic photon with energy $(2m_N - m_H) \lesssim 100$ MeV.

We can estimate SuperK’s probable sensitivity as follows. The ultimate background comes primarily from atmospheric neutrino interactions: $\nu N \rightarrow N'(e,\mu)$, $\nu N \rightarrow N'(e,\mu) + n\pi$, $\nu N \rightarrow nN' + n\pi$, which has a rate of about 100 kton$^{-1}$yr$^{-1}$. Without a strikingly distinct signature, it would be difficult to detect a signal rate significantly smaller than this, which would imply SuperK might be able to achieve a sensitivity of order $\tau_{nn \rightarrow H \gamma} \gtrsim \text{few} 10^{29}$ yr. Since the H production signature is not more favorable than the signatures for proton decay, the SuperK limit on $\tau_{AA \rightarrow A'\gamma}$ can at best be $0.1\tau_{\nu}$, where 0.1 is the ratio of Oxygen nuclei to protons in water. Detailed study of the spectrum of the background is needed to make a more precise statement. We can get a lower limit on the SuperK lifetime limit by noting that the SuperK trigger rate is a few Hz\cite{10}, putting an immediate limit $\tau_{O \rightarrow H + X} \gtrsim \text{few} 10^{25}$ yr, assuming the decays trigger SuperK.

While SuperK limits depend on specific decay channels, three other experiments potentially establish limits on the proton lifetime which are independent of the decay channel\cite{11}. They place weaker constraints on the lifetime, due to their smaller size, but are of interest because they measure the stability of nuclei directly. The experiments of Dix et al.\cite{12} and Evans et al.\cite{13} are not sensitive to two nucleon transitions and thus are not applicable to nuclei disintegrating with the emission of an H.

The experiment of Flerov et. al.\cite{14} could in principle be sensitive to such transitions. It searched for decay products from Th$^{232}$, above the Th natural decay mode background of 4.7 MeV $\alpha$ particles, emitted with the rate $\Gamma_{\alpha} = 0.7 \times 10^{-10}$ yr$^{-1}$. The conversion of two nucleons in Th$^{232}$ could result in the following decay chains:

$$\text{Th}_{pp}^{232} \rightarrow \text{Ra}_{H}^{230} X; \text{Ra}_{H}^{230} \rightarrow \text{Ac}_{H}^{230} + \beta(0.99 \text{MeV})$$

$$\text{Th}_{nn}^{232} \rightarrow \text{Ac}_{H}^{230} X; \text{Ac}_{H}^{230} \rightarrow \text{Th}_{H}^{230} + \beta(2.7 \text{MeV})$$

$$\text{Th}_{nn}^{232} \rightarrow \text{Th}_{H}^{230} X; \text{Th}_{H}^{230} \rightarrow \text{Ra}_{H}^{232} + (4.7 \text{MeV}).$$

However in general the transitional nuclear state, denoted e.g., $\text{Ra}_{H}^{230}$, would have additional more complicated decay chains through excited states. Note that the H does not bind to nuclei\cite{15}; it simply recoils with some momentum imparted in its production. The Flerov et al\cite{14} experiment must have cuts to remove the severe background of 4.7 MeV $\alpha$’s. If these cuts do not remove the events with production of an H, it would imply the limit $\tau_{Th^{232} \rightarrow H + X} > 10^{21}$ yr. Unfortunately ref.\cite{14} does not discuss these cuts or the experimental sensitivity in detail. An attempt to correspond with the experimental group, to determine whether their results are applicable to the H, was unsuccessful.

B. Double $\Lambda$ hyper-nuclei detection

There are five experiments which have reported positive results in the search for single $\Lambda$ decays from double $\Lambda$ hypernuclei. We will describe them briefly. The three early emulsion based experiments \cite{16-18} suffer from ambiguities in the particle identification, and therefore are considered less reliable. In the latest emulsion experiment at KEK \cite{8}, a double hypernucleus event has been observed and interpreted with good confidence as the sequential decay of He$^9$$_\Lambda\Lambda$ emitted from a $\Xi^-$ hyperon nuclear capture at rest. The binding energy of the double $\Lambda$ system is obtained in this experiment to be $B_{\Lambda\Lambda} = 1.01 \pm 0.2$ MeV, in significant disagreement with the results of previous emulsion experiments, finding $B_{\Lambda\Lambda} \sim 4.5$ MeV.

The synchrotron based experiment \cite{7} used the $(K^-, K^+)$ reaction on a Be$^9$ target to produce $S=2$ nuclei. That experiment detected pion pairs, coming from the same vertex in the Be target. Each pion in a pair indicates one unit of strangeness change from the (presumably) di-$\Lambda$ system. Peaks in the two pion spectrum have been observed, interpreted as corresponding to two kinds of decay events. The pion kinetic energies in those peaks are (114,133) MeV and (104,114) MeV. The first peak can be understood as two independent single $\Lambda$ decays from $\Lambda\Lambda$ nuclei. The energies of the second peak do not correspond to known single $\Lambda$ decay energies in hypernuclei of interest. The proposed explanation\cite{7} is that they are pions from the decay of the double $\Lambda$ system, through some specific $He$ resonance. The required resonance has not yet been observed experimentally, but its existence is considered plausible. This experiment does
not suffer from low statistics or inherent ambiguities, and one of the measured peaks in the two pion spectrum suggests observation of consecutive weak decays of a double Λ hyper-nucleus. The binding energy of the double Λ system $B_{\Lambda\Lambda}$ could not be determined in this experiment.

The KEK and BNL experiments demonstrate quite conclusively, in two different techniques, the observation of Λ decays from double Λ hypernuclei. Therefore $\tau_{\Lambda\Lambda \rightarrow \Lambda'X}$ cannot be much less than $\approx 10^{-16}$ s. (To give a more precise limit on $\tau_{\Lambda\Lambda \rightarrow \Lambda'X}$ requires a detailed analysis by the experimental teams, taking into account the number of hypernuclei produced, the number of observed Λ decays, the acceptance, and so on.) As will be seen below, this constraint is readily satisfied if the H is compact: $r_H \lesssim 1/3 r_N$.

III. BB TO H TRANSITION RATES INNUCLEI

As discussed in the introduction, the H may be considerably more compact than a nucleon which would suppress its production from a two baryon initial state, due to the small overlap of the initial and final wave functions in position space. We will estimate this suppression using the non-relativistic harmonic oscillator quark model. Additional suppression comes from the hard core repulsion in the nucleon-nucleon wave function in the nucleus. To take that into account, we use the Bethe-Goldstone relative wave function of two baryons in a nucleon.

The matrix element for the transition $A_{NN} \rightarrow A'_{H}X$ is calculated in the ΛΛ pole approximation, as a product of matrix elements for two subprocesses: a transition matrix element for formation of the H from a ΛΛ system, $|M|_{\Lambda\Lambda \rightarrow H X}$, times the amplitude for a weak doubly-strangeness-changing transition, $|M|_{NN \rightarrow \Lambda\Lambda}$. The suppression in the spatial wavefunction overlap enters the ΛΛ → H transition. We calculate this part of the transition amplitude first. The estimate of $|M|_{NN \rightarrow \Lambda\Lambda}$ based on weak interaction phenomenology is given afterwards.

A. Calculation of $|M|_{\Lambda\Lambda \rightarrow H}$

We calculate $|M|_{\Lambda\Lambda \rightarrow H}$ in position space as the overlap of the H and ΛΛ wave functions in the Isgur-Karli (IK) non-relativistic harmonic oscillator quark model[19, 20]. We take the Λ spatial wavefunction to be the same as the nucleon’s. For now we are concerned with the dynamics of the process and we defer discussion of the suppression from the spin-flavor part of the transition amplitude.

The IK model was designed to reproduce the masses of the observed resonances and it has proved to be successful in calculating baryon decay rates [19]. In the IK model, the quarks in a baryon are described by the Hamiltonian

$$H = \frac{1}{2m}(\vec{p}_1^2 + \vec{p}_2^2 + \vec{p}_3^2) + \frac{1}{2}K \sum_{i<j}^3 (\vec{r}_i - \vec{r}_j)^2 \tag{1}$$

where we have neglected constituent quark mass differences. The wave function of baryons can then be written in terms of the relative positions of quarks, while the center of mass motion is factored out. The relative wave function in this model is [19, 20]

$$\Psi_B(\vec{r}_1, \vec{r}_2, \vec{r}_3) = N_B \exp \left[ -\frac{\alpha_B^2}{6} \sum_{i<j}^3 (\vec{r}_i - \vec{r}_j)^2 \right] \tag{2}$$

where $N_B$ is the normalization factor, $\alpha_B = \frac{1}{\sqrt{3} \sigma_B} = \sqrt{3Km}$, and $<\vec{r}_B^2>$ is the baryon mean charge radius squared. Changing variables to

$$\vec{\rho} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}}, \quad \vec{\lambda} = \frac{\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3}{\sqrt{6}} \tag{3}$$

reduces the wave function to two independent harmonic oscillators. In the ground state

$$\Psi_B(\vec{\rho}, \vec{\lambda}) = \left(\frac{\alpha_B}{\sqrt{\pi}}\right)^3 \exp \left[ -\frac{\alpha_B^2}{2} (\rho^2 + \lambda^2) \right]. \tag{4}$$

One of the deficiencies of the IK model is that the value of the $\alpha_B$ parameter needed to reproduce the mass splittings of lowest lying 1/2− and 3/2− baryons corresponds to a mean charge radius squared for the proton of $<r_{ch}^2> = \frac{1}{\alpha_B^2} = 0.49$ fm. This is distinctly smaller than the experimental value of 0.86 fm. Our results depend strongly on the choice of $\alpha_B$ and therefore we should keep in mind this problem. Another concern is the applicability of the non-relativistic IK model in describing quark systems, especially in the case of the tightly bound H. With $r_H/r_N = 1/f$, the quark momenta in the H are $\approx f$ times higher than in the nucleon, which makes the non-relativistic approach more questionable than in the case of nucleons. Nevertheless we adopt the IK model because it offers a tractable way of obtaining a qualitative estimate of the effect of the small size of the H on the transition rate, and there is no other alternative available at this time.

We fix the wave function for the H particle starting from the same Hamiltonian (1), but generalized to a six quark system. For the relative motion part this gives

$$\Psi_H = N_H \exp \left[ -\frac{\alpha_H^2}{6} \sum_{i<j}^6 (\vec{r}_i - \vec{r}_j)^2 \right]. \tag{5}$$

The space part of the matrix element $<A'_{H}|A_{\Lambda\Lambda}>$ is given by the integral

$$\iiint d^3\vec{r}_1 \Psi_\Lambda^6(1, 2, 3) \Psi_H^6(4, 5, 6) \Psi_H(1, 2, 3, 4, 5, 6). \tag{6}$$

We can rewrite this in a more convenient form, changing variables to

$$\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, \vec{r}_5, \vec{r}_6 \rightarrow \vec{\rho}, \vec{\lambda}, \vec{\rho}, \vec{\lambda}, \vec{a}, \vec{R}_{CM} \tag{7}$$
where $\overline{R}_{CM}^{(b)}$ and $\overline{\lambda}_{\alpha}^{(b)}$ are defined as in eq (3), with $a(b)$ referring to coordinates 1, 2, 3 (4, 5, 6). (Since we are ignoring the flavor-spin part of the wavefunction, we can consider the six quarks as distinguishable and not worry about fermi statistics at this stage.) We also define the center-of-mass position and the separation, $\bar{a}$, between initial baryons $a$ and $b$:

$$R_{CM} = \frac{R_{CM}^{a} + R_{CM}^{b}}{2}, \quad \bar{a} = R_{CM}^{a} - R_{CM}^{b}. \quad (8)$$

Using these variables, the H ground state wave function becomes

$$\Psi_{H} = \left( \frac{3}{2 \pi} \right)^{1/4} \left( \frac{\alpha_{H}}{\sqrt{\pi}} \right)^{15/2} \times \exp\left[ -\frac{\alpha_{H}^{2}}{2} (a^{2} + \lambda_{a}^{2} + \rho_{a}^{2} + \lambda_{b}^{2} + \rho_{b}^{2} + \frac{3}{2} \bar{a}^{2}) \right] \quad (9)$$

and the overlap of the space wave functions is given by

$$|M|_{\Lambda\Lambda \rightarrow H} = \int \prod_{i=a,b} d^{3}p_{i} d^{3}\lambda_{i} d^{3}a \psi_{H}^{a} \psi_{H}^{b} \psi_{\Lambda}^{a} \psi_{\Lambda}^{b} \psi_{nuc} \quad (10)$$

where the center of mass dependence has been factored out, $\psi_{\Lambda}^{a,b} = \psi_{\Lambda}^{a,b} (\overline{R}_{CM}^{a,b}, \overline{\lambda}_{\alpha}^{a,b})$, and $\psi_{nuc} = \psi_{nuc}(\bar{a})$ is the relative wavefunction function of the two $\Lambda$’s in the nucleus. The integration over the center of mass position of the system gives a 3 dimensional momentum delta function. In the case of pion or lepton emission, plane waves of the emitted particles should be included in the integrand. For brevity we use here the zero momentum transfer, $\vec{k} = 0$, approximation, which we have checked holds with good accuracy; this is not surprising since typical momenta are $\lesssim 0.3$ GeV.

To describe two $\Lambda$’s or nucleons in a nucleus we will use solutions of the Bruecker-Bethe-Goldston equation describing the interaction of a pair of fermions in an independent pair approximation; see, e.g., [21]. The two particle potential in a nucleus is poorly known at short distances. Measurements (the observed deuteron form at short distance is chosen for technical convenience constrained for distances below 0.7 fm. Rather, their form at short distance is chosen for technical convenience or aesthetics.

For the s-wave, the B-G wavefunction is

$$\Psi_{BG}(\bar{a}) = \begin{cases} N_{BG} \frac{u(k_{F}a)}{k_{F}a} \text{ for } a > \frac{\bar{a}}{k_{F}} \\ 0 \text{ for } a < \frac{\bar{a}}{k_{F}} \end{cases} \quad (11)$$

where $R_{H}$ is the hard core radius. The function $u$ vanishes at the hard core surface by construction. It then rapidly approaches the unperturbed value 1, crossing over at that value at the so called “healing distance”. Expressions for $u$ and $N_{BG}$ can be found in [21].

After performing the Gaussian integrals analytically, the overlap of the space wave functions becomes

$$|M|_{\Lambda\Lambda \rightarrow H} = \frac{1}{4} \left( \frac{2f}{1 + f^{2}} \right)^{6} \left( \frac{3}{23} \right)^{1/4} \left( \frac{\alpha_{H}}{\sqrt{\pi}} \right)^{3/2} \times N_{BG} \int_{k_{F}}^{\infty} d^{3}a \frac{u(k_{F}a)}{k_{F}a} e^{-\frac{3}{2}a^{2}} \quad (12)$$

where the factor 1/4 comes from the probability that two nucleons are in a relative s-wave, and $f$ is the previously-introduced ratio of nucleon to H radius; $\alpha_{H} = f \cdot \alpha_{H}$. Since $N_{BG}$ has dimensions $V^{-1/2}$ the spatial overlap $|M|_{\Lambda\Lambda \rightarrow H}$ is a dimensionless quantity, characterized by the ratio $f$, the Isgur-Karl oscillator parameter $\alpha_{H}$, and the value of the hard core radius. It is shown in Fig. 1 for a range values of hard-core radius and $f$, using the standard value of $\alpha_{H}$ for the IK model[20].

![Figure 1: Log10 of $|M|_{\Lambda\Lambda \rightarrow H}^{2}$ versus hard core radius in fm, for ratio $f = R_{N}/R_{H} = 4, 5, 6, 7$.](image)

**B. Lifetime of doubly-strange nuclei**

We can now estimate the decay rate of a doubly-strange nucleus:

$$\Gamma_{\Lambda\Lambda \rightarrow \Lambda\Lambda \pi} \approx K^{2} (2\pi)^{4} \frac{m_{q}^{2}}{2(2m_{\Lambda\Lambda})} \times \Phi_{2}[|M|_{\Lambda\Lambda \rightarrow H}^{2}] \quad (13)$$

where $\Phi_{2}$ is the two body phase final space factor, defined as in [11], and $m_{\Lambda\Lambda}$ is the invariant mass of the $\Lambda$’s, $\approx 2m_{\Lambda}$. The factor $K$ contains the transition element in spin flavor space. It can be estimated by counting the total number of flavor-spin states a $uuddss$ system can
occupy, and taking $K^2$ to be the fraction of those states which has the correct quantum numbers to form the H. That gives $K^2 \sim 1/1440$. Thus we obtain the lifetime estimate

$$\tau_{\Lambda\Lambda\rightarrow A'_{H}} \approx \frac{1.4}{K^2|\mathcal{M}|_{\Lambda\Lambda\rightarrow H}^2} \times 10^{-21} \text{ s}, \quad (14)$$

where the phase space factor was calculated for $m_H = 1.5$ GeV.

Fig. 2 shows $|\mathcal{M}|_{\Lambda\Lambda\rightarrow H}^2$ in the range of $f$ and hard-core radius where its value is in the neighborhood of the experimental limits. Evidently, $|\mathcal{M}|_{\Lambda\Lambda\rightarrow H}^2 \lesssim 10^{-8}$ is satisfied even for relatively large H, e.g., $r_H \lesssim 1/3$ for the canonical choice $0.4$ fm for hard-core radius. This suppresses $\Gamma(\Lambda\Lambda \rightarrow A'_{H}X)$ sufficiently that some $\Lambda$'s in a double-$\Lambda$ hypernucleus will decay prior to formation of an H. Thus the observation of single $\Lambda$ decay products from double-$\Lambda$ hypernuclei cannot be taken to exclude the existence of an H with mass below $2m_{\Lambda}$ unless it can be demonstrated that $r_H \geq 1/3 r_N$.

![Figure 2: Log$_{10}$ of $|\mathcal{M}|_{\Lambda\Lambda\rightarrow H}^2$ versus hard core radius in fm, for $f=2, 3, 4$.](image)

C. Calculation of the $|\mathcal{M}|_{BB\rightarrow \Lambda\Lambda\Lambda\Lambda}$ matrix element

Transition of a two nucleon system to $\Lambda\Lambda$ requires two strangeness changing weak reactions. Possible $\Delta S = 1$ sub-processes to consider are a weak transition with emission of a pion or lepton pair and an internal weak transition. These are illustrated in Fig. 3 for a three quark system. We estimate the amplitude for each of the sub-processes and calculate the overall matrix element for transition to the $\Lambda\Lambda$ system as a product of the sub-process amplitudes.

The matrix element for weak pion emission is estimated from the $\Lambda \rightarrow N\pi$ rate:

$$|\mathcal{M}|_{\Lambda\rightarrow N\pi}^2 = \frac{1}{(2\pi)^4} \frac{2m_{\Lambda}}{m_{\Sigma}} \frac{1}{\tau_{\Lambda\rightarrow N\pi}} \approx 0.8 \times 10^{-12} \text{ GeV}^2. \quad (15)$$

By crossing symmetry this is equal to the desired $|\mathcal{M}|_{N \rightarrow \Lambda\pi}$, in the approximation of momentum-independence which should be valid for the small momenta in this application. Analogously, for lepton pair emission we have

$$|\mathcal{M}|_{\Lambda\rightarrow N\nu\bar{\nu}}^2 = \frac{1}{(2\pi)^4} \frac{2m_{\Lambda}}{m_{\Sigma}} \frac{1}{\tau_{\Lambda\rightarrow N\nu\bar{\nu}}} \approx 3.0 \times 10^{-12}. \quad (16)$$

The matrix element for internal conversion, $(uds) \rightarrow (udd)$, is proportional to the spatial nucleon wave function when two quarks are at the same point:

$$|\mathcal{M}|_{\Lambda \rightarrow N\nu\bar{\nu}} \approx <\psi_{\Lambda}|\delta^3(\vec{r}_1 - \vec{r}_2)|\psi_N> \frac{G_F \sin \theta_e \cos \theta_e}{m_q} \approx 0.4 \times 10^{-2} \text{ GeV}^3. \quad (17)$$

The delta function term can be also inferred phenomenologically in the following way, as suggested in [23]. The Fermi spin-spin interaction has a contact character depending on $\sigma_1 \sigma_2 / m_q^2 \delta(\vec{r}_1 - \vec{r}_2)$, and therefore the delta function matrix element can be determined in terms of electromagnetic or strong hyperfine splitting:

$$<\psi_{\Lambda}|\delta^3(\vec{r}_1 - \vec{r}_2)|\psi_N> \approx \left( \frac{G_F}{\sqrt{2} \pi} \right)^3 \approx 0.4 \times 10^{-2} \text{ GeV}^3. \quad (18)$$

The method used in eqn 18, namely:

$$m_{\Sigma} - m_{\Sigma^+} - (m_n - m_p) = \frac{2\pi}{3m_q} <\delta^3(\vec{r}_1 - \vec{r}_2) > (19)$$

$$m_{\Delta} - m_{N} = \frac{8\pi}{3m_q} <\delta^3(\vec{r}_1 - \vec{r}_2) > (20)$$

where $m_q$ is taken to be $m_q/3$ is the quark mass. Using the first form to avoid the issue of scale dependence of $\alpha_S$ leads to a value three times larger than predicted by the method used in eqn 18, namely:

$$<\psi_{\Lambda}|\delta^3(\vec{r}_1 - \vec{r}_2)|\psi_N> = 1.2 \times 10^{-2} \text{ GeV}^3. \quad (21)$$

We average of the expectation values (18) and (21) and find

$$|\mathcal{M}|_{\Lambda \rightarrow N\nu\bar{\nu}}^2 = 4.4 \times 10^{-15}. \quad (22)$$
In this way we have roughly estimated all the matrix elements for the relevant sub-processes based on weak-interaction phenomenology.

D. Nuclear decay rates

NN → HX requires two weak reactions. For the process $A_{NN} \to A_H^* \pi \pi$, the rate is thus approximately

$$\Gamma_{A_{NN} \to A_H^* \pi \pi} \approx K^2 \frac{(2\pi)^4}{2(2m_N)} \Phi_3$$

$$\times \left( \frac{|M|^2_{N\to\Lambda\pi} |M|^2_{\Lambda\Lambda\to H}}{(2m_\Lambda - m_H)^2} \right)^2$$

where the denominator is introduced in the spirit of the $\Lambda \Lambda$ pole approximation, to make the 4 point vertex amplitude dimensionless. The lifetime for this decay is

$$\tau_{A_{NN} \to A_H^* \pi \pi} \approx \frac{0.03}{K^2 |M|^2_{\Lambda\Lambda\to H}} \text{ yr}$$

(24)

taking $m_H = 1.5$ GeV in the phase space factor. For the process with one pion emission and an internal conversion, the rate estimate is

$$\Gamma_{A_{NN} \to A_H^* \pi} \approx K^2 \frac{(2\pi)^4}{2(2m_N)} \Phi_2$$

$$\times \left( \frac{|M|^2_{N\to\Lambda\pi} |M|^2_{N\to\Lambda} |M|^2_{\Lambda\Lambda\to H}}{(2m_\Lambda - m_H)^2} \right)^2$$

leading to the lifetime for $m_H = 1.5$ GeV of

$$\tau_{A_{NN} \to A_H^* \pi} \approx \frac{2 \times 10^{-3}}{K^2 |M|^2_{\Lambda\Lambda\to H}} \text{ yr}$$

(26)

If $m_H \geq 1740$ MeV, pion emission in a nucleus is kinematically suppressed and the relevant final states are $e^+\nu$ or $\gamma$; we now calculate these rates, taking $m_H = 1.8$ GeV. For the transition $A_{NN} \to A_H^* e \nu$, the rate is

$$\Gamma_{A_{NN} \to A_H^* e \nu} \approx K^2 \frac{(2\pi)^4}{2(2m_N)} \Phi_3$$

$$\times \left( \frac{|M|^2_{N\to\Lambda\pi} |M|^2_{N\to\Lambda} |M|^2_{\Lambda\Lambda\to H}}{(2m_\Lambda - m_H)^2} \right)^2$$

leading to the lifetime

$$\tau_{A_{NN} \to A_H^* e \nu} \approx \frac{70}{K^2 |M|^2_{\Lambda\Lambda\to H}} \text{ yr}$$

(28)

For $A_{NN} \to A_H^* \gamma$, the rate is approximated as

$$\Gamma_{A_{NN} \to A_H^* \gamma} \approx K^2 \frac{(2\pi)^4}{2(2m_N)} \Phi_2 |M|^2_{N\to\Lambda} |M|^2_{\Lambda\Lambda\to H}$$

$$\times \left( \frac{|M|^2_{N\to\Lambda\pi} |M|^2_{\Lambda\Lambda\to H}}{(2m_\Lambda - m_H)^2} \right)^2$$

leading to the lifetime

$$\tau_{A_{NN} \to A_H^* \gamma} \approx \frac{1.8 \times 10^3}{K^2 |M|^2_{\Lambda\Lambda\to H}} \text{ yr}$$

(29)

One sees from Fig. 1 that a lifetime bound of $\gtrsim$ few $10^{29}$ yr is not a very stringent constraint on this scenario if $m_H$ is large enough that pion final states are not allowed. E.g., with $K^2 = 1/1440$ the rhs of eqn (28) is $\gtrsim$ few $10^{29}$ yr, for the a hard core radius of 0.45 fm and $r_H \approx 1/5 r_N$ – in the middle of the range expected based on the glueball analogy. If $m_H$ is light enough to permit pion production, experimental constraints are much more powerful. $m_H \lesssim 1740$ MeV is disfavored but not excluded; the allowed region in the $f$-hard core radius plane may be reasonable, depending on how strong limits SuperK can give.

| mass $m_H$ [GeV] | final state | final momenta | partial lifetime $\times 10^3 |M|^2_{\Lambda\Lambda\to H}$ [yr] |
|------------------|-------------|---------------|---------------------|
| 1.5              | $\pi \pi$   | $170^*$       | 0.03                |
| 1.5              | $e \nu$     | $48^*$        | 70                  |
| 1.8              | $\gamma$    | 96            | 2.54                |

TABLE I: The final particles and momenta for nucleon-nucleon transitions to H in nuclei. For the 3-body final states marked with *, the momentum given is for the configuration with H produced at rest.

IV. CONCLUSIONS

We have considered the stability of nuclei and hypernuclei with respect to conversion to an H dibaryon. If the binding of the H dibaryon is strong, possibly resulting in $m_H < 2m_N$ as conjectured in refs. [2, 3], then the size of the H is expected to be much smaller than the size of a nucleon and comparable to the size of a glueball: $r_H \approx r_G \approx (1/6 – 1/4) r_N$. We used the Isgur-Karl wavefunctions for quarks in baryons and the H, and the Bethe-Goldstone wavefunction for nucleons in a nucleus, to obtain a rough estimate of the wavefunction overlap for the process $A_{BB} \to A_H^* X$. We find that observation of A decays in double-A hypernuclei does not exclude an H – stable or not – as long as $r_H \lesssim 1/3 r_N$. Combing our wavefunction overlap estimates with phenomenological weak interaction matrix elements, permits the lifetime for conversion of nuclei to H to be estimated. These estimates have uncertainties of greater than an order of magnitude: the weak interaction matrix elements are uncertain to a factor of a few, factors of order 1 were ignored, a crude statistical estimate for the flavor-spin overlap was used, mass scales were set to $m_N/3$, and most importantly, the calculation of the wavefunction overlap used models which surely oversimplify the physics. While the overlap is highly uncertain because it depends on nuclear wavefunctions and hadronic dynamics which are not adequately understood at present, the enormous suppression of H production
which we found in this calculation forces us to conclude that an absolutely stable H is not excluded by these considerations.

SuperK can place important constraints on the conjecture of an absolutely stable H, or conceivably discover evidence of its existence, through observation of the pion(s), positron, or photon produced when two nucleons in an oxygen nucleus convert to an H. We estimated that SuperK could achieve a lifetime limit \( \tau > \text{few } 10^{29} \text{ yr} \). Until the properties of the H and the dynamics of production of the H in nuclei are better understood, this limit would be insufficient to rule out a stable H. However such a sensitivity would access the estimated lifetime range for \( m_H \gtrsim 1740 \text{ MeV} \) and \( r_H \approx 1/5 r_N \), and an experimental search is warranted.

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