The Statistical Analysis of Anisotropy Fluctuations of CMB Temperature in Angular Spatial and Temporal Areas

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Abstract. The changes in the nature of the statistical distributions of the anisotropy of the temperature of CMB in the satellite measurements of the “WMAP” probe were analyzed for the presence of laws in them in changes within the adjacent periods of data accumulation. Statistical distributions of changes in anisotropy between adjacent data accumulation periods were analyzed. The probability of the occurrence of such unidirectional changes at different frequencies of satellite measurements was analyzed and the probability of their occurrence under the influence of non-random factors was estimated.

1. Introduction
In [1] describes a method of searching for frequency independent changes in the anisotropy of the temperature of CMB over time in some directions of the celestial sphere. Such changes can be detected using poly-frequency satellite measurement databases such as "WMAP" and "Planck".

These databases are integrated with a cumulative time of one year for the “WMAP” experiments and six months for the “Planck” experiments. The changes in anisotropy of temperature over time are differences in temperature anisotropy values for the same cumulative measurement periods, similar in selected measurement frequencies. At the same time, such changes in temperature anisotropy should be observed at selected measurement frequencies in the same directions of the celestial sphere.

It should also be noted that statistically significant time fluctuations in the temperature of CMB were recorded directly by the “WMAP” probe [2] in 501 directions of the celestial sphere. In the present work the procedure of detection of frequency-independent changes of anisotropy of radiation temperature is considered.

The frequency of such changes in time in different directions of the celestial sphere may exceed their statistically determined frequency due to the randomness of the described phenomenon. In this case, it should be concluded that such changes serve as manifestations of some non-random processes. Since frequency-independent phenomena include gravitational microlensing of electromagnetic radiation, it can be assumed that non-random frequency-independent changes in the anisotropy of the temperature of CMB are caused by its gravitational microlensing.

In [3], the described method of searching for frequency-independent changes in temperature anisotropy is used for their possible detection in the satellite measurement database "Planck" in its low-frequency frequency band. It has been found that the values of such changes can be within one dozen microkelvins, and the number of frequency-independent changes in anisotropy detected between adjacent pairs of accumulation measurement periods can reach several hundred.
The present work is aimed at detecting frequency-independent jumps in anisotropy of CMB using the "WMAP" probe database. This base, in particular, contains the results of satellite measurements made [4] on its five frequency bands K (23 GHz); Ka (33 GHz); Q (41 GHz); V (61 GHz) and W (94 GHz). The results obtained using the two bands K and Ka differ significantly in estimating the measurement error of the anisotropy of the radiation temperature from similar estimates for the three bands Q, V and W. As a result, the totality of temperature anisotropy measurement results cannot be considered close measurements. Therefore for the further analysis the database received with use of eight pairs of radiometers was used; Q1; Q2; V1; V2 as well as W1; W2; W3 and W4, which were grouped according to their alphanumeric designations into the three listed bands.

Estimates of temperature anisotropy are presented in the order range of its absolute values from $10^1 \, K$ to $10^8 \, K$, usually with seven significant digits. At the same time, the error in measuring temperature was seriously influenced by foreground radiation, which was the three main influencing factors. These include synchrotron radiation, free radiation and astrophysical dust radiation. The systematic error due to foreground radiation has a frequency dependence, and it was possible to significantly reduce it by introducing corrections precisely for bands Q, V and W.

It should be noted that the published results [2] of measurements of the anisotropy of the radiation temperature are "cleared" of its dipole component due to the movement of the Earth with respect to the flow of the volumetric spectral density of the radiation power. These results are integral data established over a cumulative time of one year.

The procedure for detecting frequency-independent changes in various directions in the celestial sphere is illustrated using the "WMAP" probe database to resolve $\sigma = 9$ in the HEALPix coordinate system [5]. This resolution corresponds to covering the celestial sphere with numbered pixels, within each of which the anisotropy of temperature is considered constant. The average size of such a pixel in angular terms is 0.1145 angular degrees, or 6.87 angular minutes. Each pixel pulls the body angle $3,9947416 \times 10^{-6}$ sr. Thus, the map of the celestial sphere is composed of 3145728 numbered pixels.

2. Analysis of statistical distributions of temperature anisotropy of CMB

The features of the procedure for detecting frequency-independent changes in the anisotropy of CMB are considered on the example of data [4] obtained for the first two years of the probe operation. Using a pair of radiometers Q1 and V1, which correspond to the wavelengths of the received radiation, equal to 13 mm and 9.1 mm, respectively. It is known that gravitational microlensing phenomena for the visible radiation range are also detected by changes in the luminosity of sources using pairs of wavelengths, in particular about 0.7 μm and 0.4 μm, i.e. using red and blue ends of the spectrum. Similarly, the difference of the analyzed temperatures in the respective directions of the celestial sphere allows an estimate of the size of changes in temperature anisotropy, wherein comparing such changes based on the results of measurements at the two indicated wavelengths allows an estimate of the nature of frequency independence of changes in anisotropy.

Figure 1 shows statistical distributions of the values of anisotropy of the radiation temperature $T$ measured in all directions of the celestial sphere, respectively, for pairs of radiometers Q1 ($a; c$) and V1 ($b; d$). These values, as mentioned above, refer to the second ($a – c$) and first ($b – d$) measurement years, respectively. For purposes of clarity, the axis of the statistical relative frequency $f$ is constructed on a logarithmic scale of units.

The graphs of Fig.1 also contain numbered vertical lines cutting off values on the abscissa axis, respectively: $T_0 (1); T_0 - 5 \cdot \sigma_T (2); T_0 + 5 \cdot \sigma_T (3); T_{min} (4)$ and $T_{max} (5)$.

Table 1 contains the characteristics of these distributions used in the further analysis. The first column of the table contains the name of the radiometer pair. The second column contains the year of operation of the probe, starting with the first one from the beginning of the work. The third and fourth columns contain, respectively, the average value of $T_0$ anisotropy of radiation temperature, as well as its average quadratic deviation $\sigma_T$ from the average value. Columns five and seven contain minimum $T_{min}$ and maximum $T_{max}$ measured temperature anisotropy values. The sixth and eighth columns
contain $T_{\text{min}}$ and $T_{\text{max}}$ values, respectively, expressed in fractions of $\Delta T$ from the fourth column. All values, uniform with temperature, are expressed in millikelvins.

**Figure 1.** Distributions of measured values of anisotropy of radiation temperature $T$ for pairs of radiometers $Q1$ ($a; c$) and $V1$ ($b; d$) relating to the second ($a - b$) and first ($c - d$) measurement years.

**Figure 2.** Distributions of measured values of change $\Delta T$ in anisotropy of radiation temperature $T$ for pairs of radiometers $Q1$ ($a$) and $V1$ ($b$) occurred between the second and first years of measurement.
Figure 3. Probability distributions $P(1)$ for negative (a) and negative (b) values of change $\Delta T$ in anisotropy of radiation temperature $T$ for pairs of radiometers $Q1$ and $V1$ that occurred between the second and first years of measurement.

Table 1. Characteristics of distributions of anisotropy of temperature of $CMB$ measured by pairs of radiometers $Q1$ and $V1$ for the first two years of probe operation.

| Radiometers | Year | $T_0$, mK CMB | $\sigma_T$, mK CMB | $T_{min}$, mK CMB | $T_{max}$, mK CMB | $T_{max}$ / $\sigma_T$, mK CMB |
|-------------|------|----------------|---------------------|--------------------|-------------------|-----------------------------|
|             | 1    |                |                     |                    |                   |                            |
| $Q1$        | first | 0.01172334     | 0.4505729           | -21.77829          | -48.3             | 72.67414                   |
|             | second| 0.01177452     | 0.4494247           | -22.22542          | -49.5             | 73.31463                   |
| $V1$        | first | 0.006021815    | 0.3559442           | -14.50349          | -40.7             | 73.17532                   |
|             | second| 0.006095985    | 0.354233            | -41.16442          | -116              | 205.8138                   |

Analysis of Figure 1, as well as Table 1, shows that the temperature anisotropy distributions measured on each of the two frequency channels during the two adjacent accumulation periods have sufficiently close sample mean $T_0$ as well as sample mean square deviations $\sigma_T$ for each of the accumulation periods to be considered pairs of samples of one of the same general population. In particular, the sample averages $T_0$ for the two adjacent accumulation periods for the pairs of radiometers $Q1$, as well as $V1$ taken separately, differ by less than one percent, and the corresponding pairs of estimates of average quadratic deviations $\sigma_T$ – by no more than several percent.

At the same time, the width of the tails of the respective distributions can vary by two or more times or more from one accumulation period to another accumulation period, which indicates the existence of noticeable changes in the anisotropy of the radiation temperature in some directions of the celestial sphere.

Some of these changes should be considered statistically random. For the remainder of these changes, a possible list of non-accidental processes leading to such changes should be identified. In particular, if such changes are found to be frequency-independent, they may be due to the effect of the gravitational microlensing phenomenon on $CMB$.

It should be noted that the frequency independence in the case of microlensing the radiation observed in the optical and radio wave band is subjected to a fairly strict check for proximity to the unit of luminosity ratio in two different frequency bands of observation. The deviation of this deviation from the unit usually does not exceed several percent. However, in the present case of possible microlensing of $CMB$, this ratio can be substantially large. This is evidenced by the fact that when comparing the statistical characteristics of radiation obtained using two different pairs of radiometers, it turns out that their estimates of the average temperature $T_0$ can differ up to two times, and estimates of the average quadratic deviation $\sigma_T$ by more than 20%. Such differences may be related, in particular, to features of behavior in the frequency domain of systematic temperature measurement errors caused by foreground radiation.
3. Analysis of statistical distributions of change in temperature anisotropy of CMB

Next, we will go directly to the statistical analysis of changes in the measured temperature values within adjacent pairs of accumulation periods for each of the frequency-differing pairs of radiometers $Q1$ and $V1$ during two adjacent accumulation periods of data starting from the first year of operation of the probe.

Figure 2 (a) shows the distribution of changes in the anisotropy of the radiation temperature between the second and first accumulation periods for a pair of radiometers $Q1$, and Figure 2 (b) shows the distribution of changes in the anisotropy of the radiation temperature for a pair of radiometers $V1$. For purposes of clarity, the relative frequency axis $f$ of these graphs is constructed on a logarithmic scale of units, as in the graphs of Fig.1.

The graphs of Fig.1 also contain numbered vertical lines cutting off values on the abscissa axis, respectively: $\Delta T_0$ (1); $\Delta T_0 - 5 \cdot \sigma_{\Delta T}$ (2); $\Delta T_0 + 5 \cdot \sigma_{\Delta T}$ (3); $\Delta T_{\min}$ (4) and $\Delta T_{\max}$ (5).

Table 2 contains the characteristics of these distributions, similar to Table 1. The first column of this table contains the name of the radiometer pair. The second column indicates the years of operation of the probe, starting with the first one from the beginning of operation, for which the temperature difference is calculated. The third and fourth columns contain, respectively, the average value of $\Delta T_0$ change of anisotropy of radiation temperature, as well as its average quadratic deviation $\sigma_{\Delta T}$ from the average value. Minimum $\Delta T_{\min}$ and maximum $\Delta T_{\max}$ measured temperature anisotropy change values are entered in columns 5 and 7, respectively. The sixth and eighth columns contain the values of $\Delta T_{\min}$ and $\Delta T_{\max}$, respectively, expressed in fractions of $\sigma_{\Delta T}$ from the fourth column. All values contained in Table 2 and uniform with temperature are expressed, unlike Table 1, in microkelvins.

Table 2. Characteristics of distributions of change of anisotropy of temperature of CMB measured by pairs of radiometers $Q1$ and $V1$ for the first two years of probe operation.

| Radiometers | Year | $\Delta T_0$, $\mu K$ | $\sigma_{\Delta T}$, $\mu K$ | $\Delta T_{\min}$, $\mu K$ | $\Delta T_{\max}$, $\mu K$ | $\Delta T_{\max} / \sigma_{\Delta T}$, $\mu K$ CMB |
|-------------|------|----------------------|------------------|------------------|------------------|----------------------------------|
| ---- |  |   |   |   |   |   |
| $Q1$  | second | 0.05117946 | 0.2525774 | 3.61435 | 14.3 | 2.67885 |
| $V1$  | and first | 0.07416961 | 0.3132958 | -3.09434 | -0.89 | 2.37407 |

Analysis of Figure 2 as well as Table 2 shows that the distributions of temperature anisotropy changes measured on each of the two frequency channels during the two adjacent accumulation periods have a sufficiently high centering. Estimates of mean values of $\Delta T_0$ change differ from zero value within several hundredths of microlevin. Estimates of the mean quadratic deviations $\sigma_{\Delta T}$ reach several tenths of the microlevin, which is three orders of magnitude less than the estimates of the mean quadratic deviation $\sigma_T$ of the temperature anisotropy itself contained in Table 1. At the same time, at both frequencies of measurement, jumps in temperature anisotropy are observed, which are an order of magnitude higher than their average square deviation and reach microlevin units.

It should be determined whether such significant changes in temperature anisotropy are frequency independent, i.e. whether they are realized in the tails of the statistical distributions of temperature anisotropy in the same directions of the celestial sphere at two frequencies. The presence of such implementations may serve as a basis for believing the corresponding changes in temperature anisotropy to be statistically non-random, that is, to consider them to be manifestations of non-random processes to be established.

Then we analyze probabilities of variation of anisotropy of radiation temperature measured between the second and the first accumulation periods into tails of statistical distributions of these changes, which were implemented between the second and the first accumulation periods using pairs of radiometers $Q1$ and $V1$ differing in frequency.

4. Analysis of statistical distributions tails of change in temperature anisotropy of CMB

Note that the analyzed tails of temperature anisotropy change distributions correspond to both positive and negative temperature anisotropy change values. Tail bearing negative values of change...
corresponds to decrease of temperature anisotropy during transition from previous of two adjacent pairs of accumulation periods to subsequent accumulation period. The tail bearing the positive values of the change in anisotropy corresponds to an increase in temperature.

We establish for subsequent reasoning that both tails of the distribution of changes in the anisotropy of temperature, both positive and negative, have the same power, equal to half of the $0.5\cdot N_0$ general distribution of these $N_0$. Place in ascending order the absolute temperature anisotropy changes measured for each of the two frequencies. We fix at any frequency, in either of the two tails of the distribution of the change in anisotropy, any direction on the celestial sphere, which is characterized by some number $j_1$ in the general ascending order of the modulus of change in anisotropy. We will consider this value a boundary for the corresponding tail. Then the probability of accidentally hitting the previously fixed value in the tail in question is equal to the ratio of the width of this tail, i.e. $j_1$, to the half-volume of $0.5\cdot N_0$ of the general distribution. In the same way, for the same direction in the celestial sphere, the probability of accidentally hitting the corresponding value of the change in anisotropy in the same tail of the distribution obtained by measuring at another frequency is equal to the ratio of the width $j_2$ of this tail to the half-volume $0.5\cdot N_0$ of the general population.

Then the probability $P$ of claiming that changes in anisotropy measured at two frequencies hit both corresponding tails is randomly described by multiplication:

$$P = \frac{4j_1 j_2}{N_0^2}. \quad (1)$$

The lower the probability value (1) of accidental ingress of temperature anisotropy values into the tails of distributions at both frequencies, the greater the probability of claiming that such an ingress occurred in a non-random manner.

It is possible to select such a boundary value $j_0$ of the number limiting the tail of the temperature anisotropy distribution that when the conditions $j_1 < j_0$ and $j_2 < j_0$ are met, the corresponding value $P_0$ of the probability $P$

$$P_0 = \frac{4j_0^2}{N_0^2}. \quad (2)$$

cannot be satisfied with any temperature anisotropy value that would accidentally fall into the tail of the distribution at both frequencies. It should then be considered fair to claim that such a hit is the result of a non-random frequency-independent phenomenon, such as gravitational microlensing of CMB. It should then be considered fair to claim that such a hit is a consequence of a non-random frequency-independent phenomenon, such as gravitational microlensing of CMB.

Figure 3 shows statistical distributions of probability values $P$ (1) calculated in all directions of the celestial sphere for two adjacent annual accumulation periods, starting from the first year of operation of the satellite, for pairs of radiometers $Q1$ and $V1$. Figure 3 (a) graph corresponds to negative changes of anisotropy of temperature in tail, and Figure 3 (b) graph corresponds to its positive changes. For purposes of clarity, the relative frequency axis $f$ is constructed on a logarithmic scale of units.

Table 3 contains the characteristics of these distributions used in the further analysis. The first column of the table indicates whether the temperature anisotropy change in the temperature distribution is positive or negative. In the second column, the number of $N_0/2$ characterizing the volume of the corresponding distribution is placed. The third column contains the value of the value inverse of $N_0/2$, i.e. the probability $P_1$ of falling from the first time into the tail of the distribution of the change in temperature anisotropy of any of its values, which is measured at any one of the frequencies. The fifth column contains the value of the value inverse to the square of $N_0/2$, i.e. an estimate of the probability $P_2$ of falling out from the first time into the tails of the distributions of the change in anisotropy of the temperature of any of its values, which is measured at both frequencies. In the fourth column, that is, between the values $P_1$ and $P_2$, the probability $P$ (1) detected by statistical analysis of the database is placed.
Table 3. - Characteristics of probability distributions $P(1)$ measured by pairs of radiometers $Q1$ and $V1$ for the first two years of probe operation.

| Tail sign | $N_0/2$ | $1/(N_0/2)$ | $P(1)$ | $1/(N_0/2)^2$ |
|-----------|---------|-------------|--------|---------------|
| positive  | 1573005 | 6.357x10^{-7} | 3.641x10^{-12} | 4.041x10^{-13} |
| negative  | 1572723 | 6.358x10^{-7} | 1.003x10^{-9}  | 4.043x10^{-13} |

The analysis of Figure 3 and Table 3 shows that the distributions of positive and negative changes in temperature anisotropy differ slightly in their volumes within a tenth of the percentage, that is, almost half the volume of the total $N_0$. The values $P(1)$, both for the distribution with positive changes in temperature anisotropy and for the distribution with negative changes, are located inside the segment of the probability scale $f$ limited by the values of $P_1$ and $P_2$. At the same time, the obtained result indicates that in the tails of the distribution of both positive and negative values of temperature anisotropy changes measured at both frequencies, in some directions of the celestial sphere, such changes may be contained, in respect of which it is true that their presence in the tails of both distributions is caused by some non-random factor.

Next, we will analyze the estimated number of frequency-independent changes observed in satellite experiments in the temperature anisotropy of $CMB$, which can be observed between adjacent periods of data accumulation and which exceeds their number corresponding to the influence of statistically random factors.

5. Analysis of assumed numbers of occurrence of frequency-independent changes of temperature anisotropy of $CMB$

Figure 4 shows time diagrams of the amounts of $M$ satellite observations of frequency-independent changes in the temperature anisotropy of $CMB$ over nine annual cycles of accumulation of measurement results. Figure 4 $(a-b)$ corresponds to the two observation frequency ranges discussed above with pairs of radiometers $Q1$ and $V1$. These two diagrams contain ten graphs of the change of the number $M$ from time, which for the sake of clarity are depicted with different scales of the magnitude $M$. To construct these graphs, five values of the number $J$ of frequency-independent changes in temperature anisotropy are selected, which can fall into the tails of the distribution at two measurement frequencies statistically randomly.

![Figure 4](image-url)
Figure 5. Diagrams of number \( k \) dependencies on observation time at different values of factor \( J \) for two pairs of radiometers \( Q1 \) and \( V1 \) (a - b), as well \( V1 \) and \( W1 \) (c - d)

Schedule lines are assigned conditional numbers 1; 2; 3; 4; 5, which correspond to the calculated values of the number \( J \), respectively equal to 100; 10; 1; 0.1; 0.01.

If, for example, at any point in Figure 3 to which \( J = 1 \) corresponds, the number \( M \) of observed changes in anisotropy exceeds one, this means that in the corresponding tails of the distribution of anisotropy of the radiation temperature obtained at each of the two frequencies, there is only one change in anisotropy of temperature, which can be explained from the points of chance. The remaining \( M-1 \) changes arose due to the effect on radiation of some non-random factors.

In the designation of all graphs of Figures 4, except for the numerical designation, there is a sign "\(+\)" or "\(-\)" for example, "\(3+\)" or "\(3-\)" The sign "\(+\)" corresponds to positive increments of temperature anisotropy in the corresponding tail of the distribution during the transition from the previous period of accumulation of measurement results to the next, and the sign "\(-\)" – negative.

To improve the reliability of the calculations, they also included another combination of pairs of satellite radiometers - \( V1 \) (with a frequency of 9.1 mm) and \( W1 \) (with a frequency of 3.2 mm). The diagram of Figure 4 (c – d) for the specified pair of radiometers completely repeats the diagram of Figure 4 (a – b) for the pair of radiometers \( Q1 \) and \( V1 \).

In order to compare the numerical values of \( M \) and \( J \), Figure 5 shows time diagrams of the number \( k \) equal to the ratio \( M/J \). Numeric symbols of graphs of Fig.5 fully correspond to symbols of graphs of Fig.4. At the same time, the greater \( k \), the greater the probability of anisotropy temperature changes in the tailings due to the presence of non-random physical processes in relation to the number of temperature anisotropy changes due to the random combination of temperature anisotropy values.
Table 4. Statistical characteristics of the number $M$ at different values of factor $J$ for two pairs of radiometers $Q1$ and $V1$, as well as $V1$ and $W1$.

| Radiometers pairs | Tail sign | $J$ | $M_0$, units | $\sigma_M$, units | $M_{\text{max}}$, units | $M_{\text{min}}$, units |
|-------------------|-----------|-----|--------------|----------------|--------------------------|--------------------------|
|                   |           | 1   | 2            | 3             | 4                        | 6                        | 7                        | 8                        |
| $Q1$ $V1$         | positive  | 100 | 111          | 21,3          | 118                      | 78                       |
|                   |           | 10  | 23,4         | 10,9          | 30                       | 8                        |
|                   |           | 1   | 10,1         | 7,74          | 23                       | 2                        |
|                   |           | 0,1 | 6,71         | 5,48          | 15                       | 2                        |
|                   |           | 0,01| 4,43         | 4,42          | 12                       | 0                        |
|                   | negative  | 100 | 114          | 21,0          | 116                      | 78                       |
|                   |           | 10  | 16,7         | 6,18          | 24                       | 7                        |
|                   |           | 1   | 5            | 4,45          | 13                       | 1                        |
|                   |           | 0,1 | 2,71         | 2,79          | 8                        | 0                        |
|                   |           | 0,01| 2            | 2,58          | 7                        | 0                        |
| $V1$ $W1$         | positive  | 100 | 102          | 20,7          | 117                      | 70                       |
|                   |           | 10  | 12,9         | 4,21          | 16                       | 7                        |
|                   |           | 1   | 2,57         | 3,27          | 8                        | 0                        |
|                   |           | 0,1 | 1,14         | 2,32          | 6                        | 0                        |
|                   |           | 0,01| 0,86         | 1,61          | 4                        | 0                        |
|                   | negative  | 100 | 102          | 19,3          | 110                      | 69                       |
|                   |           | 10  | 4,12         | 14            | 18                       | 4                        |
|                   |           | 1   | 2,29         | 1,76          | 5                        | 0                        |
|                   |           | 0,1 | 0,42         | 0,81          | 2                        | 0                        |
|                   |           | 0,01| 0,143        | 0,382         | 1                        | 0                        |

Table 4 contains numerical characteristics of Fig. 4 diagrams. The first column of the table contains the designation of a pair of radiometers. The second column of the table indicates whether the temperature anisotropy change in its distribution is positive or negative. In the third column, the value of factor $J$. In the fourth column, the mathematical expectation of the $M_0$ number $M$ is indicated, in the fifth - its average quadratic deviation $\sigma_M$, in the sixth and seventh columns - respectively the maximum $M_{\text{max}}$ and the minimum $M_{\text{min}}$ values of the number $M$.

Table 5 contains numerical characteristics of Fig. 5 diagrams. Construction of Table 5 is similar to construction of Table 4. Columns of Table 5 from the first to the third coincide with the corresponding columns of Table 4. In the fourth column, the mathematical expectation $k_0$ of the number $k$ is indicated, in the fifth - its average quadratic deviation $\sigma_k$, in the sixth and seventh columns - respectively the maximum $k_{\text{max}}$ and the minimum $k_{\text{min}}$ of the number $k$. 
Table 5. Statistical characteristics of the factor $k$ at different values of factor $J$ for two pairs of radiometers $Q1$ and $V1$, as well as $V1$ and $W1$.

| Radiometers pairs | Tail sign | $J$  | $k_0$ | $\sigma_k$ | $k_{max}$ | $k_{min}$ |
|-------------------|-----------|------|-------|------------|-----------|-----------|
| $Q1$ V1           | positive  | 100  | 1.11  | 0.213      | 1.78      | 0.78      |
|                   |           | 10   | 2.34  | 1.09       | 3.9       | 0.8       |
|                   |           | 1    | 10.1  | 7.74       | 23        | 2         |
|                   |           | 0.1  | 67    | 54         | 150       | 20        |
|                   |           | 0.01 | 443   | 442        | 1200      | 0         |
|                   | negative  | 100  | 1.15  | 0.210      | 1.16      | 0.78      |
|                   |           | 10   | 1.67  | 0.617      | 2.4       | 0.7       |
|                   |           | 1    | 5     | 4.45       | 13        | 1         |
|                   |           | 0.1  | 22.1  | 27.9       | 80        | 0         |
|                   |           | 0.01 | 200   | 258        | 700       | 0         |
| $V1$ W1           | positive  | 100  | 1.02  | 0.206      | 1.17      | 0.7       |
|                   |           | 10   | 1.29  | 0.421      | 1.6       | 0.7       |
|                   |           | 1    | 2.57  | 3.27       | 8         | 0         |
|                   |           | 0.1  | 11.6  | 23.15      | 60        | 0         |
|                   |           | 0.01 | 85.7  | 161        | 400       | 0         |
|                   | negative  | 100  | 1.02  | 0.196      | 1.1       | 0.69      |
|                   |           | 10   | 1.4   | 0.412      | 1.8       | 0.8       |
|                   |           | 1    | 2.29  | 1.28       | 5         | 0         |
|                   |           | 0.1  | 4.39  | 0.0601     | 20        | 0         |
|                   |           | 0.01 | 14.3  | 38.2       | 100       | 0         |

The analysis of the diagrams of Fig.4 and Fig.5, as well of the data of Table 4 and Table 5 shows that with the growth of factor $J$, the number $M$ increases, but the factor $k$ decreases. In other words, the number of points in the tails of temperature anisotropy changes distributions, which are candidates for frequency-independent changes, increases. However, among them, the number of changes increases disproportionately rapidly, the presence of which can be explained by a random combination of values of changes in temperature anisotropy at two different frequencies. Therefore, at $J = 100$ as well as $J = 10$, it is likely that in the case of the assumption of changes in temperature anisotropy as frequency-independent, these changes are due only to a random combination of their numerical values in the tails of distributions at different measurement frequencies.

On the other hand, when $J = 0.01$, $k$ reaches several hundred. However, in this case, the number $M$ does not exceed several units. As a result, the probability of erroneously considering frequency-independent changes in temperature anisotropy as a result of random combinations of numerical values of temperature anisotropy in the tails of distributions at different measurement frequencies is high.

Summing up this, it can be assumed that the number of candidates for frequency-independent changes in the temperature anisotropy of $CMB$ can be several tens of pieces during the annual accumulation period of satellite measurements.

6. Conclusion
The analysis of the anisotropy of the temperature of $CMB$ according to the satellite measurements of the “WMAP” probe along various directions in the celestial sphere, as well as for various time periods of accumulation of measurement results and for various frequencies of satellite measurements, made it possible to establish a number of features of the change in anisotropy of the temperature of $CMB$ in angular spatial as well as time areas.
Tails of statistical distributions of temperature anisotropy can experience changes in length at different measurement frequencies, which exceed several tens of average quadratic deviations of temperature anisotropy from its average value. This makes it possible to set the task of searching for the manifestation of physical laws in changes in the anisotropy of temperature in the angular directions of the celestial sphere, as well as in time.

Analysis of the tails of distributions of changes in the anisotropy of the temperature of \(CMB\) makes it possible to establish that within the adjacent periods of accumulation of measurement results at various frequencies of satellite measurements, changes in the temperature of anisotropy are observed, reaching three to five microkelvins. This makes it possible to set the task of estimating the probability of such unidirectional changes in the tails of their distributions within the same adjacent periods of accumulation of measurement results in the same directions of the celestial sphere simultaneously at different frequencies.

Analysis of distributions of probabilities of appearance of unidirectional changes of anisotropy of \(CMB\) results in the same directions of celestial sphere simultaneously at different frequencies makes it possible to conclude that probability of occurrence of such changes calculated on the basis of satellite measurements can exceed by ten times statistical probability of their random appearance. This also makes it possible to search for the effect of the physical laws of occurrence of such changes in temperature anisotropy, as well as to estimate the number of such events within different periods of accumulation of measurement results.

The number of candidates for frequency-independent changes of anisotropy of temperature of \(CMB\) is estimated at several tens during the annual accumulation period of satellite measurements taking into account resolution of these measurements in 6.87 angular minutes and corresponding number of separately set directions in the celestial sphere equal to 3145728 [2] in the coordinate system \(HEALPix\) [5].

References

[1] Vargashkin V Ya 2018 The analysis of frequency-independent jumps of CMB according to the «Planck» data Journal of Physics: Proceeding of XX International Meeting «Physical Interpretations of Relativity Theory» (London: IOP published house) 1051 012027

[2] Bennett C L et al 2013 Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results \textit{ApJS} 208 20B

[3] Vargashkin V Ya 2020 The analysis of CMB anisotropy to temporary domain according to WMAP and Planck probes databases Journal of Physics: Proceeding of XXI International Meeting «Physical Interpretations of Relativity Theory» (London: IOP published house) 1557 012037

[4] Jarosik N et al 2003 Implementation And Testing u/f ehe Map Radiometers \textit{ApJS} 145 413

[5] Górski K M. \textit{et al} 2005 HEALPix - a Framework for High Resolution Discretization, and Fast Analysis of Data Distributed on the Sphere \textit{ApJ} 622 759