Spread theory of the special theory of relativity

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Abstract

The transformation of space-time $x_\mu$ and $x'_\mu$ in the two inertial reference frames $\Sigma$ and $\Sigma'$ in which their relative velocity is less than light speed, and the relation of a particle mass $m$ with its movement velocity $v$ and so on are expatiated by Einstein’s special theory of relativity. In this paper, we set forth a new transformation of space-time $x_\mu$ and $x'_\mu$ in two inertial reference frames in which their relative velocity is equal to or more than light speed, the new relation of a particle mass $m$ and energy $E$ with its velocity $v$ ($v \geq c$), the new mass-energy equation and the new dynamics equation of a particle.

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1. Introduction

Einstein’s special relativity modify all the laws of physics, where necessary, so as to make them a metaprinciple: it puts constraints on all the lows of physics. The modifications suggested by the theory, though highly significant in many modern applications, have negligible effect in most classical problems, which is of course why they were not discovered earlier. However, they were not exactly needed empirically in 1905 either. This is a beautiful example of the power of pure thought to leap ahead of the empirical frontier—a feature of all good physical theories, though rarely on such a heroic scale. It has led, among other things, to a new theory of space and time, and in particular to the relativity of simultaneity and the existence of a maximum speed for all particles and signals, to a new mechanics in which mass increases with speed, to the formula $E = mc^2$ \[1 - 5\]. The special theory of relativity is based on two postulates formulated by Einstein [5]:

1. All laws of nature are the same in all inertial reference frames. In other words, we can say that the equations expressing the laws of nature are invariant with respect to transformations of coordinates and time from one inertial reference frame to another.

2. Light always propagates in a vacuum at a definite constant speed $c$ not depending on the state of motion of the emitting body.

Einstein’s two assumption are all right in all inertial reference frames in which their relative velocity $v$ is less than light speed, because all the results are proved by the experiment. However, when two inertial reference frames relative velocity $v$ is $v = c$ or $v > c$, the two assumption in the above don’t hold. For example, when a beam of photons move in the same direction, all photons relative velocity $v$ is zero and isn’t light velocity $c$. So, the assumption about the invariant of the light speed is incorrect in this situation, and the result that light velocity $c$ is a maximum speed is also incorrect, because the relative velocity of two beams of lights moving along opposite direction exceeds light speed $c$. So, when two inertial reference frames relative velocity $v$ is equal to or more than light speed Einstein’s two postulations and some results must be modified.
2. The space-time relation at $v = c$

Consider two inertial reference frames $\Sigma$ and $\Sigma'$ in which their relative velocity is $v$ ($v = c$). We shall select the coordinate axes of these frames so that the axes $x$ and $x'$ are directed along the velocity $v$ of the frame $\Sigma'$, and the axes $y$ and $z$ are parallel to the axes $y'$ and $z'$. Let us assume that at the time $t = 0$ a electromagnetic wave leaves the origin of $\Sigma$ and $\Sigma'$, which propagates with a speed $c$ in all directions in inertial reference frames $\Sigma$, but it propagates with different velocity in different direction in inertial reference frames $\Sigma'$. So, the electromagnetic wave is a spherical wave in $\Sigma$, and it is a ellipsoid wave in $\Sigma'$. That is,

$$c^2 t^2 - x^2 - y^2 - z^2 = 0 \quad (1)$$

$$\frac{(x' + \frac{ac't^2}{2})^2}{(\frac{ac't}{2})^2} + \frac{(y')^2}{(bct')^2} + \frac{(z')^2}{(bct')^2} = 1 \quad (2)$$

where $ac$ is the light velocity along with $x'$ axes’ negative direction, and the light velocity is zero in $x'$ axes positive direction, and $bc$ is the light velocity along with $y'$ and $z'$ axes in the $\Sigma'$ reference frames. Obviously, the events coordinates satisfy

$$x = \gamma'(x' + ct') \quad (3)$$

$$x' = \gamma(x - ct) \quad (4)$$

$$y = y', z = z' \quad (5)$$

From Eqs. (3)-(4), we have

$$t' = \gamma [t - \frac{x}{c} (1 - \frac{1}{\gamma'})] \quad (6)$$

and from Eqs. (1)-(6), we obtain

$$(4b^2 \gamma^2 - 4ab^2 \gamma^2 + 4ab^2 \gamma^2 \frac{1}{\gamma'}) x^2 + (8ab^2 c \gamma^2 - 8b^2 c \gamma^2 - 4ab^2 c \gamma^2 \frac{1}{\gamma'}) xt + (4b^2 c^2 \gamma^2 - 4ab^2 c^2 \gamma^2) t^2$$

$$= a^2 x^2 - a^2 c^2 t^2 \quad (7)$$

from Eq. (7), we obtain

$$a^2 = 4b^2 \gamma^2 - 4ab^2 \gamma^2 + 4ab^2 \frac{\gamma}{\gamma'} \quad (8)$$
\[8ab^2c\gamma^2 - 8b^2c\gamma^2 - 4ab^2c\frac{\gamma}{\gamma'} = 0\]  \hfill (9)

\[4b^2c^2\gamma^2 - 4ab^2c^2\gamma^2 = -a^2c^2\]  \hfill (10)

from Eqs. (8)-(10), we have

\[\frac{\gamma}{\gamma'} = \frac{a}{2b^2}\]  \hfill (11)

If we let

\[\gamma = \gamma', b = 1\]  \hfill (12)

then

\[a = 2\]  \hfill (13)

By substituting these values into Eqs. (3)-(6), we obtain, finally, the following transformation equations of the events coordinates and velocity

\[x' = x - ct\]  \hfill (14)

\[x = x' + ct\]  \hfill (15)

\[y' = y, z' = z\]  \hfill (16)

\[t' = t\]  \hfill (17)

\[v' x = v_x - c\]  \hfill (18)

\[v' y = v_y, v' z = v_z\]  \hfill (19)

for the general a and b, we have

\[\gamma = \gamma' = \sqrt{\frac{a}{2(a - b)}}\]  \hfill (20)

\[x' = \sqrt{\frac{a}{2(a - b)}}(x - ct)\]  \hfill (21)

\[x = \sqrt{\frac{a}{2(a - b)}}(x' + ct')\]  \hfill (22)

\[t' = \sqrt{\frac{a}{2(a - b)}}(t - \frac{x}{c}(\frac{2b}{a} - 1))\]  \hfill (23)

\[v' x = \frac{v_x - c}{1 - \frac{2b}{a}v_x} - \frac{2b}{a}v_x\]  \hfill (24)
\[ v_y' = \frac{\sqrt{\frac{2(a-b)}{a} \frac{b}{a} - 1}}{v_x} \] (25)

\[ v_z' = \frac{\sqrt{\frac{2(a-b)}{a} \frac{b}{a} - 1}}{v_x} \] (26)

from Eq. (18), when

\[ v_x = -c \] (27)

then

\[ v_x' = -2c \] (28)

from Eq. (28), we find the phenomenon exceeding light velocity can happen in the reference frames of photons movement. This is different from Einstein’s theory of special relativity, which light speed \( c \) is the maximum velocity in the nature.

3. The space-time relation at \( v > c \)

We consider two inertial reference frames \( \Sigma \) and \( \Sigma' \) in which their relative motion is the same as section 2, and the only different is the relative velocity \( v > c \). We can obtain the following equations

\[ x^2 + y^2 + z^2 - c^2t^2 = 0 \] (29)

\[ \left( \frac{x' + \frac{acf'}{2}}{(x+f')^2} \right)^2 + \left( \frac{y'}{(bct')^2} \right)^2 + \left( \frac{z'}{(bct')^2} \right)^2 = 1 \] (30)

where \( ac \) is the amplitude of light velocity along the \( x' \) axes negative direction in the \( \Sigma' \) reference frames which corresponds to the light velocity \( -c \) along the \( x \) axes negative direction in the \( \Sigma \) reference frames, and \( fc \) is the amplitude of light velocity along the \( x' \) axes negative direction in the \( \Sigma' \) reference frames which corresponds to the light velocity \( c \) along the \( x \) axes positive direction in the \( \Sigma \) reference frames, and \( bc \) is the light velocity along with \( y' \) and \( z' \) axes in \( \Sigma' \) reference frames.

The events coordinates in \( \Sigma \) and \( \Sigma' \) reference frames satisfy following equations

\[ x = \gamma'(x' + ct') \] (31)

\[ x' = \gamma(x - ct) \] (32)
\[ y' = y, \ z' = z \] (33)

where \( e = v (v > c) \), from Eqs. (31)-(32), we obtain

\[ t' = \gamma [t - \frac{x}{e} (1 - \frac{1}{\gamma'})] \] (34)

and from Eqs. (29)-(34), we have

\[
4b^2\gamma^2[1 - (a + f)c\frac{1}{e} + (a + f)c\frac{1}{e \gamma'} + \frac{afc^2}{e^2} + \frac{afc^2}{e^2 \gamma^2} - \frac{2afc^2}{e^2 \gamma'}]x^2
\]

\[
-(2e - 2(a + f)c + (a + f)c\frac{1}{e \gamma'} + 2afc\frac{1}{e} - 2afc\frac{1}{e \gamma'})xt
\]

\[
+(e^2 - (a + f)ce +afc^2)t^2]
\]

\[ = (a - f)^2x^2 - (a - f)^2e^2t^2 \] (35)

from the Eq. (35), we can obtain

\[
4b^2\gamma^2[1 - (a + f)c\frac{1}{e} + (a + f)c\frac{1}{e \gamma'} + \frac{afc^2}{e^2} + \frac{afc^2}{e^2 \gamma^2} - \frac{2afc^2}{e^2 \gamma'}] = (a - f)^2 \] (36)

\[
2e - 2(a + f)c + (a + f)c\frac{1}{e \gamma'} + 2afc\frac{1}{e} - 2afc\frac{1}{e \gamma'} = 0 \] (37)

\[
4b^2\gamma^2[e^2 - (a + f)ce +afc^2] = -(a - f)^2e^2 \] (38)

from the Eqs. (36)-(38), we find

\[
\gamma^2 = \frac{(a - f)^2e^2}{4b^2[(a + f)ce - e^2 - afc^2]} \] (39)

\[
\gamma'^2 = \frac{4b^2afc^2(e^2 - (a + f)ce +afc^2)}{(a - f)^2(e^4 - ace^3 - fce^3 + af^2e^2 - e^2c^2 + fce^3 + ace^3 - af^2e^4)} \] (40)

from Eqs. (31)-(34), we obtain

\[
v_x' = \frac{v_x - e}{1 - \frac{e}{v_x} \gamma' v_x} \] (41)

when

\[ v_x = -c \] (42)

then

\[
v_x' = \frac{-c - e}{1 + \frac{e}{c} - \frac{e}{e \gamma'}} = -ae \] (43)
and when
\[ \nu_x = c \]  \hspace{1cm} (44)
then
\[ \nu_x' = \frac{c - e}{1 - \frac{2}{\nu} e} = -fc \]  \hspace{1cm} (45)
from Eqs. 39-45, we have
\[ e^2\gamma^2 = b^2\gamma^2e^2\gamma'^2 - b^2\gamma^2\gamma'^2e^2 + 2c^2\gamma' - c^2 \]  \hspace{1cm} (46)
for \( e > c \), the Eq. (46) is identical equation. So, we have
\[ b^2\gamma^2 = 1 \]  \hspace{1cm} (47)
\[ -b^2\gamma^2\gamma'^2e^2 - e^2 + 2c^2\gamma' = 0 \]  \hspace{1cm} (48)
from the Eqs. (47)-(48), we find
\[ \gamma = \gamma' = 1, b = 1 \]  \hspace{1cm} (49)
By substituting these values into Eqs. (31)-(34), we can obtain the transformation equations of the events coordinates and velocity in the \( \Sigma \) and \( \Sigma' \) reference frames.
\[ x = (x' + \nu t') \]  \hspace{1cm} (50)
\[ x' = (x - \nu t) \]  \hspace{1cm} (51)
\[ y' = y, z' = z \]  \hspace{1cm} (52)
\[ t = t' \]  \hspace{1cm} (53)
\[ \nu_x' = \nu_x - e \]  \hspace{1cm} (54)
\[ \nu_y' = \nu_y, \nu_z' = \nu_z \]  \hspace{1cm} (55)
Now, we can obtain the following result: when the relative velocity of two inertial reference frames \( \Sigma \) and \( \Sigma' \) is \( \nu = c \) and \( \nu > c \) they have the same transformation relation about space-time, and they are different from the Lorentz transformation in the special theory of relativity.
4. Mass and energy of a particle

In special theory of relativity, Einstein found the relation of a particle mass and energy with its motion velocity \( v \), which is less than light speed \( c \). In this section, when a particle motion velocity \( v \) is large than light speed \( c \) or equal to \( c \), the relation of its mass and energy with its velocity \( v \) will be established. We consider the collision of two particles which they are identical particle. The \( \sum \) is laboratory frame, and the \( \sum' \) is the mass center frame, and two inertial reference frame relative velocity is \( c \). Before colliding, two particle’ velocity is \( d \ (d > c) \) and \( v_2 \) in the \( \sum \) frame, and it is \( -v' \) and \( v' \) in the \( \sum' \) frame. After colliding, all particle’ velocity is \( c \), and their velocity are zero in \( \sum' \) frame. 

from the law of momentum conservation, we have 

\[
m_1d + m_2v_2 = (m_1 + m_2)c \tag{56}
\]

from Eq. (18), we obtain 

\[
d = v' + c \tag{57}
\]

\[
v_2 = -v' + c \tag{58}
\]

By substituting Eqs. (57)-(58) into Eq. (56), we have 

\[
m_1(d) = m_2(v_2) \tag{59}
\]

\[
d = 2c - v_2 \tag{60}
\]

from Eqs. (59)-(60), we obtain 

\[
m_1(d) = m_2(v_2) = \frac{m_0}{\sqrt{1 - \frac{v_2^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{(2c-d)^2}{c^2}}} \tag{61}
\]

So, when a particle has movement velocity \( d \ (d > c) \), its mass is:

\[
m = \frac{m_0}{\sqrt{1 - \frac{(2c-d)^2}{c^2}}} \tag{62}
\]

where \( m_0 \) is its rest mass. In the following, we calculate a particle mass when it moves at light speed \( c \). It is similar to calculating \( m(d) \ (d > c) \) the above. Before colliding, two particle’s velocity are \( d \ (d > c) \) and \(-c \) in the \( \sum \) frame, and they are \(-f' \) and \( f' \) in the \( \sum' \) frame respectively. After colliding, all particle’
velocity are \( c \), and their velocity are zero in \( \Sigma' \) frame.

In according to the law of momentum conversation, we have

\[
m_1 d - m_2 c = (m_1 + m_2)c \tag{63}
\]

from Eq. (18), we obtain

\[
d = c + f \tag{64}
\]

\[
-c = c - f \tag{65}
\]

from Eqs. (63)-(65), we have

\[
m_1(d) = m_2(c) \tag{66}
\]

\[
d = 3c \tag{67}
\]

So, we have

\[
m_2(c) = m_1(d) = \frac{m_0}{\sqrt{1 - (2c-d)^2/c^2}} = \frac{m_0}{\sqrt{1 - \frac{c^2}{c^2}}} = \infty \tag{68}
\]

from the special theory of relativity, the relation of a particle mass with its velocity \( v \) \( (v < c) \) is:

\[
m(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{69}
\]

from the Eqs. (62), (68), and (69), we get the following results: when a particle’s velocity \( v \) is less then light speed \( c \), its mass \( m \) is increased as its speed \( v \) increases. when a particle’s velocity is \( c \), its mass is infinity. when a particle’s velocity \( v \) is larger then light velocity \( c \), its mass \( m \) is decreased as its speed \( v \) increases. when a particle’s velocity \( v \) is \( v > 3c \), its mass is imaginary number.

In the following, Let’s consider the relation of a particle energy with the amplitude of its velocity \( V \) \( (V > c) \), and mass-energy equation, and its dynamics equation.

a particle’s momentum \( \vec{P} \) and exerted force \( \vec{F} \) are defined as:

\[
\vec{P} = \frac{m_0 \vec{V}}{\sqrt{1 - \frac{(2c-V)^2}{c^2}}} \tag{70}
\]

\[
\vec{F} = \frac{d}{dt} \vec{P} \tag{71}
\]
and
\[
\frac{d}{dt} w = \overrightarrow{F} \cdot \overrightarrow{V}
\] (72)

where \( w \) is a particle energy. From the Eq. (70)-(72), we have
\[
w = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{2c - V}{c^2}\right)^2}} \frac{(3c - 2V) V}{(2c - V)c}
\] (73)

and mass-energy equation
\[
P^2 \left(\frac{(3c - 2V)c}{2c - V}\right)^2 - w^2 = 0
\] (74)

and a particle’s dynamics equation
\[
\overrightarrow{F} = \frac{m_0}{\sqrt{1 - \left(\frac{2c - V}{c^2}\right)^2}} \frac{d}{dt} \overrightarrow{V} - \frac{m_0 \overrightarrow{V}}{(\sqrt{1 - \left(\frac{2c - V}{c^2}\right)^2})^3} \frac{2c - V}{c^2 V} \overrightarrow{V} \cdot \frac{d}{dt} \overrightarrow{V}
\] (75)

5. conclusion

To summarize, we have discussed the time-space transformation in two inertial reference frames which their relative velocity is equal to or more than light speed and researched the relation of a particle’s mass and energy with its motion velocity and its dynamics equation. It is different from the special theory of relativity, which based on Einstein’s principle of relativity and the principle of constancy of the speed of light. When two inertia reference frames’ relative velocity and a particle’s velocity is less than light velocity they are described by the special theory of relativity. However, when two inertial reference frames’ relative velocity and a particle’s velocity is equal to or larger than light velocity it must be described by the spread theory of the special theory of relativity.

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