Four-Wheel-Independent-Driving Electric Vehicles Stability Control Based on Yaw Rate and Side Slip Angle

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Abstract. In this paper, a nonlinear system state observer and a vehicle upper controller suitable for a four-wheel independent drive vehicle have been proposed. The observer uses Unscented Kalman Filter to observe the longitudinal speed, lateral velocity and yaw rate of the vehicle, and then calculates the side slip angle of the vehicle. The upper controller adopts the control method of yaw rate and side slip angle combined with Sliding Mode Control to calculate the target yaw moment. Finally, the CARSIM-MATLAB co-simulation is carried out to verify the proposed control strategy. The simulation results indicate that the proposed control strategy enables the vehicle to effectively track the target yaw rate and side slip angle simultaneously, ensuring vehicle stability and safety.

1. Introduction
In recent years, with the increase in the use of fossil fuels, the demand for alternatives to fossil fuels has become more urgent. The fossil fuel consumption has caused serious global warming problem, such as huge natural disasters and species extinction[1]. In the carbon dioxide emissions from the burning of fossil fuels, automobile exhaust emissions account for more than 20%, and other toxic and harmful gases are also included[2]. In this case, the electric vehicle appears.

As a new type of electric vehicle, Four-Wheel-Drive vehicle involves multiple motor coordination control problems[3]. The control logic is different from conventional vehicles. During normal driving, the vehicle's yaw rate and side slip angle indicate multi-actuator coordination working condition[4]. Moreover, vehicle control strategy achieves better results by using these two parameters as the control

Fig 1.1 Schematic diagram of vehicle chassis controller
targets[5]. The vehicle lateral stability is measured by the yaw rate. There are many factors that affect the yaw rate, such as speed, steering angle, front and rear tire stiffness, etc[6]. Compared with controlling these parameters, controlling the yaw moment can achieve the purpose of controlling the yaw rate more directly and more efficiently[7]. Yaw moment control strategy are divided into an upper controller and a lower controller. The upper controller includes the vehicle state observation and the target yaw moment calculation, and the lower controller is the torque distribution. In this paper, the upper controller is the main research content, and the lower controller adopts the classic quadratic programming[8], which is not mentioned in this paper due to the limited space.

2. System Modeling

2.1. Tire Rotation Model

The tire rotation model is shown in the Fig 2.1. The rotation equation can be described as follows.

\[ \omega_i = \frac{T_i - f_i F_{sl} \omega - q_i \omega_i - F_{si} R}{I_w} \]  

(2.1)

In equation 2.1, \( \omega_i \) is the vehicle rotation speed, \( T_i \) is the driving or brake torque of each tire. \( F_{si} \) is the vertical load at tire road point. \( q_i \) is the viscous damping coefficient of in-wheel motors. \( R \) is the effective radius of each tire. \( f_i \) is the tire rolling resistance coefficient. \( I_w \) is the wheel inertia.

2.2. Simplified 2-DOF Model

To keep the vehicle running as expected, an upper controller based on the sliding mode control is proposed below. The controller is based on a simplified two-degree-of-freedom model, as shown in Fig 2.2.

The differential equation of the model is as follows:

\[
\begin{bmatrix}
    k_1 + k_2 & \frac{1}{V_x} (l_f k_1 - l_r k_2) \\
    l_f k_1 - l_r k_2 & \frac{1}{V_x} (l_f^2 k_1 + l_r^2 k_2)
\end{bmatrix}
\begin{bmatrix}
    \beta \\
    \gamma
\end{bmatrix}
= \begin{bmatrix}
    m (\dot{V}_y + V_x \delta_r) + k_1 \delta \\
    I_z \dot{\gamma} + l_f k_1 \delta
\end{bmatrix}
\]  

(2.2)

Where \( k_1 \) and \( k_2 \) are the lateral stiffness of the front and rear wheels, which is equivalent to the axial stiffness of the vehicle.

Solving the differential equation gives the expression of the yaw rate and the centroid side yaw angle:
The equations are only valid in linear area. If the road friction coefficient is too low and the tires turn into nonlinear region, the target values of yaw rate and side slip angle must be limited within an upper value, which is given as follows.

\begin{equation}
\gamma_{\text{limit}} = \frac{0.85}{V_x} \mu g, \quad \beta_{\text{limit}} = \tan^{-1}(0.02 \mu g)
\end{equation}

Consequently, the target yaw rate and side slip angle can be described as follows.

\begin{equation}
\gamma = \min (|\gamma|, \gamma_{\text{limit}}) \text{sgn} (\gamma), \quad \beta = \min (|\beta|, \beta_{\text{limit}}) \text{sgn} (\beta)
\end{equation}

### 3. Observer based on Unscented Kalman Filter

Yaw rate and side slip angle are the most evident variable of vehicle stability, many researchers select them as the control variables. According to the modern vehicle dynamics control architecture, the function of the vehicle upper controller is to acquire the desired yaw moment, so that the car can dynamically track the ideal yaw rate. In this paper, the vehicle observer is designed based on the Unscented Kalman Filter, and the upper controller is designed based on the sliding mode control.

Instead of using linearized equations to approximate the nonlinear model as Extended Kalman Filter (EKF), UKF intentionally generates a finite set of points known as sigma points. These sigma points are transformed to a new set of points using nonlinear model. System states and associated error covariance matrices are determined numerically based on the mean and covariance values of the transformed sigma points. Compared with the Extended Kalman Filter, UKF does not need to do a linear fit, avoiding solving the Jacobian matrix. Moreover, the estimation accuracy and convergence speed are significantly improved. The steps of the Unscented Kalman Filter are as follows.

![Fig 3.1 UKF steps](image)

In Fig 3.1, $P_k$ is error covariance, which $X_k$ is the system state, $\Phi_k$ is the system transition matrix, $H_k$ is the measurement matrix, $Q_k$ and $R_k$ indicates the randomness of the noise. The amplitude of the system noise is usually between one thousandth and three thousandths of the sensor.

The process equation consists of nonlinear vehicle dynamics model and wheel rotation model. Reasoning according to Chapter 2, the nonlinear vehicle dynamics process equations can be obtained. To reduce the calculation amount and increase the calculation speed, the wheel rotation equation is simplified by ignoring the resistance torque. The system process equation is obtained as follows.

\begin{equation}
a_x = \dot{v}_x - rv_y, \quad a_y = \dot{v}_y + rv_x, \quad \omega_i = \frac{T_{mi} - F_{sr}r}{J_w}
\end{equation}
The variables of measurement equation include vehicle longitudinal acceleration, lateral acceleration, yaw rate and wheel velocity that can be directly measured by the vehicle's sensors. Meanwhile, the longitudinal acceleration and measured acceleration of the vehicle can be obtained through vehicle dynamics analysis, based on the simplified vehicle model in Chapter 2. The measurement equations are acquired as follows.

\[
\dot{y} = \frac{1}{J_z} [(F_{x2} + F_{x4}) \sin \delta_w + (F_{y1} + F_{y2}) \cos \delta_w] f + \frac{1}{J_z} [(F_{x2} - F_{x1}) \cos \delta_w + (F_{y1} - F_{y2}) \sin \delta_w] \frac{d_f}{2} + \frac{1}{J_z} [(F_{x4} - F_{x2}) \frac{d_f}{2} - (F_{y3} + F_{y4}) l_t]
\] (3.2)

The external inputs of the UKF are: tire longitudinal force, front and rear tire lateral force, front wheel steering angle, motor drive torque.

In summary, the vehicle system dynamic equation variables are:

\[
x = \begin{bmatrix} v_x & v_y & r & \omega_1 & \omega_2 & \omega_3 & \omega_4 \end{bmatrix}^T
\]

\[
z = \begin{bmatrix} a_x & a_y & r & \omega_1 & \omega_2 & \omega_3 & \omega_4 \end{bmatrix}^T
\]

\[
u = \begin{bmatrix} F_{x1} & F_{x2} & F_{x3} & F_{x4} & F_{y1} & F_{y2} & \delta_w & T_{fl} & T_{fr} & T_{dl} & T_{dr} \end{bmatrix}^T
\]

4. Sliding Mode Control

Sliding mode control is essentially a special kind of nonlinear control. It is well known for good robustness, which is effective in adapting to many uncertainties, sensor noises and variation of parameters. Currently, sliding mode control is widely used in many fields. In view of the advantages above, sliding mode control is chosen as the basis of upper controller.

There are two key factors in the sliding mode controller design: (1) Selecting the appropriate approaching law. (2) Designing the switching surface.

Define the sliding film switching surface as following:

\[
s = a(\gamma - \gamma_i) + b(\beta - \beta_i)
\] (4.1)

In equation 4.1, \(\gamma\) and \(\beta\) are the real-time yaw rate and side slip angle, \(\gamma_i\) and \(\beta_i\) the target yaw rate and target side slip angle.

The approaching law is classified into 4 types: constant velocity approach law, exponential approach law, power approach law and general approach law. The fundamental cause of system chattering is that when the system trajectory reaches the switching surface, the inertia causes the moving point to traverse the switching surface, thereby resulting in the chattering superimposed on the ideal sliding model. The exponential approach control law track approaches the switching surface in a smaller speed, which effectively eliminate chattering. In addition, due to the \(-k_s \text{sgn}(s)\) in equation 4.2, even if \(s\) approaches zero, the approaching speed of the trajectory is not zero, which ensures that the trajectory must reach the switching surface. Therefore, this paper chooses the exponential approach law, and the approach law is defined as following:

\[
\dot{s} = -k_s \text{sgn}(s) - k_s s
\] (4.2)

Derivation of formula can be obtained:
\[
\dot{s} = a(\dot{r} - \dot{r}_i) + b(\dot{\beta} - \dot{\beta}_i) \quad (4.3)
\]

Since:
\[
M = I_x \ddot{\gamma} \quad (4.4)
\]

Substituting equations 4.1, 4.2, 4.3 and 4.4 into equations 2.2, the desired yaw moment is obtained as follows.

\[
M_{\text{desire}} = f(F_{x1}) = I_x \left\{ \dot{\gamma} + \frac{1}{a} \left[ -k_1 \text{sgn}(s) - k_2 s - b(\dot{\beta} - \dot{\beta}_i) \right] \right\} - f(F_{y1})
\]

In which:

\[
f(F_{x1}) = l_f (F_{x1} + F_{x2}) \sin \delta + \frac{d}{2} (F_{x2} - F_{x1}) \cos \delta + (F_{x4} - F_{x3}) \frac{d}{2}
\]

\[
f(F_{y1}) = l_f (F_{y1} + F_{y2}) \cos \delta + \frac{d}{2} (F_{y2} - F_{y1}) \sin \delta - (F_{y3} + F_{y4}) l_x
\]

The switching surface involves four parameters: \(k_1, k_2, a, b\). These four parameters not only affect the chattering of sliding mode control, but also determine the effect of sliding mode tracking.

After several adjustments, it is finally determined that the values of the parameters are \(k_1 = 0.01, k_2 = 50, a = 1,\) and \(b = -1\).

5. Simulation

5.1. Observer Effect Verification

Fig 5.1 0.3 Road adhesion coefficients condition test results
To verify the correctness of the side slip angle observations, the longitudinal and lateral velocities from the observer are compared with the carsim output value.

The initial vehicle speed is set to 80km/h, the vehicle performs open-loop sinusoidal hysteretic steering. The wheel steering angle changes according to the sine law, amplitude 100° cycle 2s, while the drive torque, amplitude 300 cycle 6s, is applied to each wheel. Moment. After several simulations, the road adhesion coefficient chooses 0.3 and 0.8 to simulate unstable and stable driving condition.

According to fig 5.1, when the vehicle is in an unstable state on a low-adhesion road surface, the vehicle's side slip angle, longitudinal speed, and lateral velocity still maintain excellent real-time and observation accuracy. According to fig 5.2, due to the influence of the driving torque, although longitudinal speed of the vehicle is fluctuating, UKF observer still follow the real longitudinal speed, lateral speeds and side slip angle of the vehicle very well. The data has high accuracy and good real-time performance. The wheel speed figure shows that the wheel slipped only in the first 2 seconds.

![Fig 5.2 0.8 Road adhesion coefficients condition test results](image)

5.2. Yaw Motion Control Strategy Verification
This paper chose the Closed-Loop Steering Double Lane Change test to verify the vehicle control strategy. In the simulation, the road adhesion coefficient is 0.8, the initial speed is 80km/h, and the sampling period is 0.01s. Set the wheel weights to 1.0 for the front wheel and 1.5 for the rear wheel.

According to the fig 5.3, the vehicle is seriously destabilized without control, the yaw rate and the side slip angle fluctuate irregularly. With control, the yaw rate and the centroid side angle track the calculated ideal value. The error is small and has excellent real-time performance.

6. conclusion
This paper has proposed a new vehicle state observation approach and an upper controller strategy. With a detailed introduction of the designed observer and control strategy, this paper presents a series of comparative simulation results, which consist of state with and without control under the Double Lane Change driving test. The simulation results and analysis demonstrate that the proposed observer and controller have higher control performance and significant effectiveness.

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