Prediction of noise transmission through infinite panels using a wave and finite element method

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The transmission of sound through simple infinite isotropic panels can be predicted in a straightforward manner using well-established analytical models. However, such models become difficult to implement for more complicated structures such as laminates etc. In these cases, numerical approaches such as the finite element method are viable alternatives. However, these methods can be computationally intensive as the entire structure must usually be meshed with the surrounding fluid being modelled using finite, infinite or boundary element methods. This paper describes the extension of a Wave and Finite Element (WFE) method for the prediction of sound transmission through infinite, two-dimensional panels. Excitation of the structure by oblique plane waves, a diffuse sound field and a point force are all considered. The WFE method involves a finite element model of a small segment of the panel from which the wave properties and the response to external excitation can be found. Because the WFE method only involves meshing a small segment of the panel, computational times are significantly less than a full finite element simulation. Two example applications of the method are described, namely a thin isotropic panel and a laminated panel.

1. Introduction
The sound transmission characteristics of simple structures (beams, thin isotropic plates etc.) can be investigated analytically [1]. However, for complex structures (laminated, composite, orthotropic or sandwich panels, etc.) analytical methods become difficult and involve various assumptions and approximations [2-4]. Consequently for such structures numerical approaches become valuable. This paper describes one such approach based on the wave and finite element (WFE) method. In essence, this involves a finite element model of just a small segment of the structure, with the resulting mass and stiffness matrices being post-processed to determine the structural and acoustic responses.

The WFE method, reviewed in [5], was developed to determine the characteristics of wave propagation in complex waveguides [6] and 2-dimensional structures [7], including cylinders etc. It can also be used (sometimes using a hybrid finite element/WFE approach) to model wave transmission and reflection through joints, and to predict the forced response of 1-dimensional [8] and 2-dimensional structures [9]. Here the method is extended to predict sound transmission through plane structures. The case of 1-dimensional structures coupling 2-dimensional acoustic spaces has been investigated using this method [10]. In this paper, 2-dimensional structures connecting 3-D acoustic spaces are considered. Sound radiation caused by excitation of the structure by a point force is also considered.

In section 2 the application of the WFE method to calculating sound transmission through infinite structures is presented. In section 3 the application of the WFE method for calculating noise radiation from a point force excited plates is described. In section 4 a series of numerical examples demonstrating the method are presented. These examples include sound transmission through, and radiation from, thin

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isotropic plates and it is shown that the results obtained using the WFE method compare favourably with results obtained using analytical models. The WFE method is also used to calculate sound transmission through a foam-cored panel for which analytical methods are difficult.

2. Noise transmission and WFE models

2.1 Fluid-structure interaction

Consider the situation in figure 1. An incident plane wave excites a structure whose motion in turn excites transmitted and reflected waves in the fluids adjacent to the top and bottom surfaces of the structure. The regions $z_1 > 0$ and $z_2 < 0$ are occupied by acoustic fluids 1 and 2 with wave speeds $c_1$ and $c_2$ respectively. An acoustic wave $p_i \exp[i(\omega t - k_{x,1}x_1 - k_{y,1}y_1 + k_{z,1}z_1)]$ is incident from $z_1 > 0$ at an angle $\theta$ to the normal, giving rise to reflected and transmitted waves

$$p_r e^{-i(k_{x,2}x_2 + k_{y,2}y_2 - k_{z,2}z_2)}, \quad p_t e^{-i(k_{x,2}x_2 + k_{y,2}y_2 - k_{z,2}z_2)},$$

where the time harmonic dependence $\exp(i\omega t)$ has been suppressed and an radiation condition is assumed in the lower half-space. In equation (1)

$$k_{x,1}^2 + k_{y,1}^2 + k_{z,1}^2 = k_1^2, \quad k_{x,2}^2 + k_{y,2}^2 + k_{z,2}^2 = k_2^2, \quad k_{1,2} = \omega/c_{1,2},$$

where

$$k_{x,1,2} = k_1 \sin \theta \cos \phi, \quad k_{y,1,2} = k_1 \sin \theta \sin \phi$$

are wavenumber components of trace waves propagating in the unbounded panel at an angle $\phi$ to the $x_2$ axis in the $x_2$-$y_2$ plane. Note that the trace wavenumber components are the same for all 3 waves. If the surface has a displacement $w \exp(-ik_{x,2}x_2 - ik_{y,2}y_2)$ then, from continuity of the fluid and structural displacements, it follows that [1]

$$(p_i - p_r) = \frac{i\rho_1 \omega^2}{k_{z,1}} w = D_{f,1}w, \quad p_t = \frac{i\rho_2 \omega^2}{k_{z,2}} w = D_{f,2}w,$$

where $D_{f,1,2}$ are the dynamic stiffnesses of the fluid.
2.2 The WFE model of structures

The WFE method involves modelling a small segment of the structure using the finite element method. From the finite element analysis (FEA) model, mass and stiffness matrices of the segment are determined. Note that the FEA can be done using any FEA package. The mesh might include any number of elements through the thickness of the structure and along the length of the segment (see figure 2). This makes the approach particularly suitable for modelling laminates, sandwich panels etc. At this point, the advantages of the WFE method are clear since users do not need to develop complicated mathematical expressions to describe the core deformation through the thickness [2].

The governing equation of the segment of figure 2 is

\[
[K - \omega^2 M]q = f + e,
\]

where \(q\), \(f\) and \(e\) are \((4n \times 1)\) vectors of nodal DOFs, internal and external nodal forces respectively, \(n\) is the number of degrees of freedom at each hypernode, and \(M\) and \(K\) are the mass and stiffness matrices of the segment. Damping can be included by a viscous damping matrix \(C\) or by \(K\) being complex.

Making use of the fact that the nodal displacements are related by simple periodicity conditions and the fact that the internal nodal forces must satisfy equilibrium equations, the governing equation can be reduced. If there are internal nodes, they can be eliminated via dynamic condensation [5]. The discretization must not be too coarse, i.e. the element size should not be too large compared to the shortest wavelength of structural or acoustic motion. Otherwise the discretized model will not accurately describe the motion of the waveguide.

2.3 Acoustic excitation

The acoustic waves apply pressures \((p_t + p_r)\exp(-ik_{x,1}x_1 - ik_{y,1}y_1)\) and \(-p_t\exp(-ik_{x,2}x_2 - ik_{y,2}y_2)\) on the upper and lower surfaces respectively. In an FE discretization this results in external nodal forces on those DOFs that correspond to displacements in the \(z\)-direction of the lower and upper surfaces, i.e. \(w_1\) and \(w_2\). Vectors \(u_1\) and \(u_2\) are defined, whose elements are all zero apart from that which corresponds to \(w_1\) or \(w_2\) respectively, and which equals 1. Therefore

\[
w_{j,1} = u_1^T q_j, \quad w_{j,2} = u_2^T q_j,
\]

where \(j\) represents the \(j\)th corner of the segment in figure 2(b), \(q_j\) are vectors of degrees of freedoms of the \(j\)th hypernode and is the concatenation of all the nodes at the corner of the segment. The distributed external excitation could merely be lumped at the nodes. Preferably, consistent nodal forces can be calculated as follows. \(e_j\), for example, can be written as

![Fig 2. WFE model. (a) FE mesh of segment of the structure. The mesh can contain any number of elements through the thickness of the structure and along the length of the segment. (b) FE mesh with hypernodes at each corner of the segment. The vectors of nodal DOFs are the concatenation of the individual nodal quantities in figure 2(a).](image-url)
where
\[ \alpha_j = \int e^{-ik_{x,2}x_2 - ik_{y,2}y_2} N_{j,1}(x_2, y_2) \, dx_2 \, dy_2, \]
\[ \beta_j = \int e^{-ik_{x,2}x_2 - ik_{y,2}y_2} N_{j,1}(x_2, y_2) \, dx_2 \, dy_2, \]
\[ \gamma_j = \int e^{-ik_{x,2}x_2 - ik_{y,2}y_2} N_{j,2}(x_2, y_2) \, dx_2 \, dy_2, \]
and where \( N_{j,1,2}(x, y) \) are the shape functions associated with the \( w \)-displacements of the upper and lower surfaces respectively. If there are internal nodes, consistent nodal forces can be calculated for each and combined via dynamic condensation to yield \( \mathbf{e}_j \) at the corners of the segment. Equation (7) explicitly separates the excitation into two components, one on each of the surfaces of the structure.

2.4 Dynamic stiffness and response

The most straightforward solution method, and that which is most useful if fluid loading is significant, is to note that
\[ q_2 = \lambda_x q_1, \quad q_3 = \lambda_y q_1, \quad q_4 = \lambda_x \lambda_y q_1, \]
where
\[ \lambda_x = e^{-ik_{x,2}L_x}, \quad \lambda_y = e^{-ik_{y,2}L_y}. \]

By writing
\[ \mathbf{q} = \mathbf{A}_R \mathbf{q}_1, \quad \mathbf{A}_R = \begin{bmatrix} 1 & \lambda_x \lambda_y & \lambda_y^2 \lambda_x \end{bmatrix}, \]
equilibrium at hypernode 1 gives
\[ \mathbf{A}_L \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \mathbf{0}, \quad \mathbf{A}_L = \begin{bmatrix} 1 & \lambda_x^{-1} & \lambda_y^{-1} & \lambda_y^{-1} \lambda_x^{-1} \end{bmatrix}, \]
and Premultiplying equation (5) by \( \mathbf{A}_L \) results in
\[ \mathbf{D}_s \mathbf{q}_1 = (\mathbf{e}_1 + e^{ik_{x,2}L_x} \mathbf{e}_2 + e^{ik_{y,2}L_y} \mathbf{e}_3 + e^{ik_{x,2}L_x} e^{ik_{y,2}L_y} \mathbf{e}_4), \]
where \( \mathbf{D}_s \) is the reduced dynamic stiffness matrix of the structure. Noting the dynamic stiffnesses of the fluids in equation (4) and the expressions for the excitation in equation (7), equation (15) reduces to
\[ \mathbf{D} \mathbf{q}_1 = p_l (\epsilon_1 + \epsilon_2 \exp(2ik_{z,1}h)) \mathbf{u}_1, \quad \mathbf{D} = \mathbf{D}_s + \epsilon_2 D_{f,1} \exp(i k_{z,1} h) \mathbf{u}_1 \mathbf{u}_1^T + \epsilon_3 D_{f,2} \mathbf{u}_2 \mathbf{u}_2^T, \]
where
\[ \varepsilon_1 = \alpha_1 + e^{ikx_1x_1}e^{ikx_1x_2} + e^{iky_1y_1}e^{iky_2y_2}, \]  
\[ \varepsilon_2 = \beta_1 + e^{ikx_1x_1}e^{ikx_1x_2} + e^{iky_1y_1}e^{iky_2y_2}, \]  
\[ \varepsilon_3 = \gamma_1 + e^{ikx_1x_1}e^{ikx_1x_2} + e^{iky_1y_1}e^{iky_2y_2}. \]  

Note that the dynamic stiffnesses of the fluids contribute to the relevant diagonal elements of the total system dynamic stiffness matrix \( \mathbf{D} \). Equation (16) can be solved for the response vector \( \mathbf{q}_1 \). The reflected and transmitted pressures then follow from equation (4) as

\[ p_r = p_i \exp(2ikz_1h) - D_{f,1} \exp(ikz_1h) \mathbf{u}_1^T \mathbf{q}_1, \quad p_t = D_{f,2} \mathbf{u}_2^T \mathbf{q}_1. \]  

An alternative solution method is to decompose the structural response into free wave components excited by the incident pressure. This approach can be used to find the response to arbitrary structural excitation [8, 9]. If fluid loading effects are negligible (i.e. \( |D_f| \ll |D_s| \)) then peaks in the response or transmission can be related to strong excitation of free waves in the structure, assisting in interpretation of the behavior. Furthermore, in some cases contour integration can then be used to find the total response in a straightforward manner (see [9]).

2.5 Transmission loss and diffuse incident field

The transmission loss for a wave incident at a pair of angles \((\theta, \phi)\) is defined as

\[ TL = -10 \log[\tau(\theta, \phi)], \]  

where the power transmission coefficient \( \tau \) is the ratio of the transmitted and incident powers, i.e.

\[ \tau(\theta, \phi) = \frac{\text{Real}[\{(kz_2/\rho_2)p^2_2\}]}{(kz_1/\rho_1)|p^2_1|}. \]  

For a diffuse incident field the transmission loss, found by integrating over all possible incident angles \( \theta \) and heading angles \( \phi \), is

\[ \tau_d = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/2} \tau(\theta, \phi) \sin \theta \cos \theta d\theta d\phi. \]  

This integral would typically need to be evaluated numerically.

3. Noise radiation from point excited plates

3.1 External loading

Assume the half space \( z > 0 \) is occupied by fluid with density \( \rho_1 \) and sound velocity \( c_1 \), bounded by a thin elastic plate of infinite extent and thickness \( h \) lying in the \( x-y \) plane (as shown in figure(3)). The region below the plate is assumed to be vacuum. The plate is driven by a concentrated time-harmonic force \( F \) located at the origin as well as the unknown acoustic surface pressure \( p_{rad}(x, y) \). Define a pair of Fourier transforms,
Fig 3. Geometry of the infinite plate and coordinates defining the observation point.

\[ p(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{p}(k_x, k_y) e^{-ik_xx} e^{-ik_yy} dk_x dk_y, \]  

(24)

where

\[ \tilde{p}(k_x, k_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F\delta(x)\delta(y) e^{ik_xx} e^{ik_yy} dx dy = F \]  

(25)

is the Fourier transform of the force per unit area \( F\delta(x)\delta(y) \). Now the point force has been converted into an integral of exciting plane waves in the wavenumber domain. Each plane wave \( F e^{-ik_xx} e^{-ik_yy} \) is described by a pair of corresponding wavenumbers \( (k_x, k_y) \).

3.2 Farfield sound pressure

The vibrating plate excites the motion of the surrounding fluid in the \( z > 0 \) space. In return the fluid creates radiation loading \( p_{rad}(x, y) \) on the top surface of the plate. The acoustic radiation in the fluid is governed by the wave equation

\[ \frac{\partial^2 p_{rad}}{\partial x^2} + \frac{\partial^2 p_{rad}}{\partial y^2} + \frac{\partial^2 p_{rad}}{\partial z^2} = \frac{1}{c_f^2} \frac{\partial^2 p_{rad}}{\partial t^2}. \]  

(26)

Applying the two dimensional Fourier transform to equation (26) gives

\[ \left( k_1^2 - k_x^2 - k_y^2 + \frac{\partial^2}{\partial z^2} \right) p_{rad}(k_x, k_y, z) = 0. \]  

(27)

The solution to equation (27) is given by

\[ p_{rad}(k_x, k_y, z) = C \exp\left( ik_{z_1}^{1/z} z \right), \]  

(28)

where \( C \) is determined by requiring that the fluid particle displacement in the \( z \)-direction at \( z = 0 \) is equal to the surface displacement. This requirement gives

\[ C = -i\omega^2 \rho w(k_x, k_y) \]  

(29)

The radiated sound pressure is thus given by
\[
p_{\text{rad}} = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} i\omega^2 \rho \tilde{w}(k_x, k_y) \frac{e^{-ik_x x} e^{-ik_y y} e^{-ik_x^2} d k_x d k_y}{\sqrt{k_1^2 - k_x^2 - k_y^2}}.
\]

where \( \tilde{w} = i\omega \bar{v} \). For observation points in the acoustic far-field, the double integral can be evaluated asymptotically using the method of stationary phase \[1\] which gives the approximate expression of the radiated sound pressure as

\[
p_{\text{rad}}(R, \theta, \phi) \approx -\omega^2 \rho \tilde{w}(\bar{k}_x, \bar{k}_y) \frac{e^{ikR}}{2\pi R}.
\]

where \((\bar{k}_x, \bar{k}_y) = (k \sin \theta \cos \phi, k \sin \theta \sin \phi)\) represents a pair of stationary phase points which corresponds to wavenumber components in the \( x \) and \( y \) directions of a plane-wave propagating towards the observer from the origin of the coordinate system, \( 0 < \theta < \pi/2, 0 < \phi < 2\pi \).

4. Numerical Examples

4.1 Thin isotropic plate

An infinite, elastic, homogeneous and isotropic plate \((E = 2 \times 10^{11}\text{Nm}^{-2}; \rho = 7800\text{kgm}^{-3}; \nu = 0.2)\) lies in the \( x-y \) plane. The thickness of the plate is 3 mm. A small segment of the plate is meshed using one ANSYS Solid 45 element of side \( L_x = L_y = 3\text{mm} \).

The wavenumbers from analytical solutions are

\[
k_s = \omega \sqrt{\frac{2\rho(1 + \nu)}{E}}, k_e = \omega \sqrt{\frac{\rho(1 - \nu^2)}{E}}, \sqrt{k_x^2 + k_y^2} = \pm k_b, \pm ik_b,
\]

where

\[
k_b = \sqrt{\frac{4\rho \omega^2}{D}}.
\]
is the flexural wavenumber and where $k_s$, $k_e$ and $D$ are the shear wavenumber, extensional wavenumber and bending stiffness respectively. To illustrate the frequency range in which this WFE model can give accurate acoustic predictions, the dispersion curves are shown in figure (4) for free wave propagation in the direction $\theta = 0^\circ$ where the structural damping is neglected. There are three propagating wave branches, $a$, $b$ and $c$, representing bending, shear and extensional waves. The numerical wavenumbers show good agreements with those from analytical solutions below around 80kHz. As frequency is increased above approximately 80kHz, differences can be seen for $k_xL_x > \pi/3$ due to discretisation errors. For example, obvious disagreement in shear wavenumbers can be seen at higher frequencies. Therefore, smaller elements are needed to mesh the segment for predictions at higher frequencies. As a rule of thumb, there should be at least 6 elements per acoustic trace wavelength. The details concerning numerical issues of applying the WFE method can be found in the reference [11].

4.1.1 Transmission loss

Assume the plate is submerged in fluids both of which are air. Figure 5(a) shows the transmission loss, predicted from the analytical expression [1] and from the WFE solution, for an angle of incidence $\theta = 60^\circ$. The agreement can be seen to be very good. There is a notch in the transmission loss at around 5500 Hz due to coincidence: at this frequency the trace wavenumber of the acoustic wave equals the wavenumber of free bending wave propagation in the plate $k_b$. For this choice of structure and fluid the fluid loading effects are generally weak, although it should be noted that they are significant at and around the coincidence frequency. Figure 5(b) shows the diffuse field transmission loss, calculated by numerical integration. Again the agreement is very good.

4.1.2 Sound radiation

Consider first the case where the fluid in the half space $z > 0$ is air. Structural damping is considered by introducing a complex Young’s modulus using a loss factor $\eta = 0.01$. The analytical solution of radiated sound pressure from thin isotropic plates can be found in [1].

Figure 6(a) shows the far-field sound pressure predicted by the WFE solution and the analytical expression, for an excitation frequency $f = 8$kHz. The agreement can be seen to be very good. There is a peak in the radiated pressure at an angle $\theta = 45.6^\circ$.

![Figure 5](image)

**Fig 5.** Transmission loss for the isotropic plate, air to air: (a) angle of incidence $\theta = 60^\circ$; (b) diffuse incident field; ……… analytical solution; --------- WFE results; ______ mass law.

(a)
Fig 6. Radiated sound pressure in the far field: (a) air occupies the $z > 0$ space; (b) water occupies the $z > 0$ space; .......... analytical solution; ------ WFE results.

For the case where the fluid is water, figure 6(b) shows the radiated pressure for an excitation frequency $f = 50\text{kHz}$. The WFE solution shows good agreement with the analytical solution again. The excitation frequency is far below the smallest coincidence frequency, about 80kHz. The coincidence frequencies vary with coincidence angle and can be calculated by

$$ksin\theta = k_B,$$

which reduces to

$$f_c = \sqrt{\Omega[2\pi(sin\theta)^2]^{-1}},$$

where $\Omega = \rho he^4/D$ and $f_c$ represent the coincidence frequency.

4.2 Sandwich panel

A more complex example is an infinite laminated panel consisting of three layers. One ANSYS Solid 45 element is used to mesh each skin (0.6mm thickness) of a segment from the structure while 15 Solid 45 elements are used to for the 15mm-thickness core. The skins of the sandwich panel are made of aluminum ($E = 7.1 \times 10^{10}\text{Nm}^{-2}, \nu = 0.3296, \rho = 2700\text{kgm}^{-3}$) and the core material is foam ($E = 3 \times 10^7\text{Nm}^{-2}, \nu = 0.2, \rho = 48\text{kgm}^{-3}$). The structural damping is neglected here but could be included, for example, by using a complex Young’s modulus.

4.2.1 Transmission loss

The dispersion curves are shown in figure 7(a). There are three wave types below approximately 5kHz, which are the flexural wave (branch a), shear wave (branch c) and extensional wave (branch b) respectively. With the increase of frequency, two additional waves (branch d and branch f) cut on at around 4.9kHz involving the out-of-phase motion of the skins of the panel. At around 8kHz, two wave
Fig 7. Dispersion curves: (a) different wave types of the sandwich panel; (b) the detailed behaviour of higher order waves at around 8kHz.

branches (branch b and branch e) cut on again, and are shown in figure 7(b) for the detailed behaviour. The complexity of high order wave types are such that the veering part of the extensional wave (branch b) spans a comparatively broader frequency range while the veering part of branch e exists over a fairly narrow frequency region.

In contrast to isotropic plates, the high order wave types of laminated panels are significant in affecting acoustic performance. Specifically, it is the veering part representing symmetric bending wave mode of branch b that influences mainly the characteristics of sound transmission. For a certain incident angle $\theta = 45^\circ$, for instance, the sound transmission loss is shown in figure 8(a). Here the trace wavenumber equals the free flexural wavenumber (in the veering part of branch b), which contributes to the notch at 7.871kHz (symmetric coincidence frequency). The diffuse field transmission loss is shown in figure 8(b).

4.2.2 Sound radiation
Consider the case where air occupies the space $z > 0$. Figure 9(a) shows the radiated sound pressure normalized to that of $\theta = 0^\circ$ for a given excitation frequency $f = 1.5$kHz. Figure 9(b) shows the radiated pressure when $z > 0$ is occupied by water.

5. Conclusions
This paper has described an approach for the numerical calculation of sound transmission through 2-dimensional structures coupling 3-dimensional acoustic spaces. It involves a finite element analysis of a small segment of the structure to determine the mass and stiffness matrices, which are then post-processed in a straightforward manner to yield the structural response. The prediction of sound radiation from a structure excited by a time-harmonic point force using this method is also considered.

The numerical examples of sound transmission and sound radiation of isotropic structures are first conducted using the WFE method and analytical approaches respectively, which validates the WFE method. Subsequently, the WFE method is applied to a linear viscoelastic laminate whose material properties vary throughout the thickness of the section. In spite of the complexity of the structure, it can be readily modelled using this method due to being homogeneous in the direction of wave propagation.
Fig 8. Transmission loss for the sandwich panel, air to air: (a) angle of incidence $\theta = 45^\circ$; (b) diffuse incident field; -------WFE results; red mass law.

Fig 9. Radiated sound pressure from the sandwich panel: (a) air occupies the $z > 0$ space; (b) water occupies the $z > 0$ space.

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