ON THE SUPER-EARTHS LOCKED IN THE 3:2 MEAN-MOTION RESONANCE

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Abstract. The first study of migration-induced resonances in a pair of Earth-like planets has been performed by Papaloizou and Szuszkiewicz [2005]. They concluded that in the case of disparate masses embedded in a disc with the surface density expected for a minimum mass solar nebula at 5.2 au, the most likely resonances are ratios of large integers, such as 8:7. For equal masses, planets tend to enter into the 2:1 or 3:2 resonance. In Papaloizou and Szuszkiewicz (2005) the two low-mass planets have masses equal to 4 Earth masses, chosen to mimic the very well known example of two pulsar planets which are close to the 3:2 resonance. That study has stimulated quite a few interesting questions. One of them is considered here, namely how the behaviour of the planets close to the mean-motion resonance depends on the actual values of the masses of the planets. We have chosen a 3:2 commensurability and investigated the outcome of an orbital migration in the vicinity of this resonance in the case of a pair of equal mass super-Earths, whose mass is either 5 or 8 Earth masses.

1 Introduction and numerical set-up

In the first study of migration-induced resonances in a pair of low-mass planets Papaloizou and Szuszkiewicz [2005] performed simulations of two interacting planets embedded in a disc with which they also interact (see also Szuszkiewicz, this volume). The simulations were carried out for a variety of planet masses, initial orbital separations and initial surface densities in order to explore the possible outcomes. In the case of planets with equal masses when the relative migration is slow, Papaloizou and Szuszkiewicz [2005] have found that the planets become trapped in the nearest available resonance. In their study, the two low-mass planets each have a mass of 4 Earth masses, chosen to mimic the very well known example of two pulsar planets which are close to the 3:2 resonance. In this work

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we have modelled the evolution of a pair of planets of equal mass in a gaseous protoplanetary disc, again in the vicinity of a 3:2 mean-motion resonance, but for two different, larger, values of the mass, namely, 5 and 8 Earth masses.

We have used the hydrodynamical code NIRVANA, originally written by Ziegler (1997) and then adapted to disc-planet simulations. More details about the code can be found in the work by Nelson et al. (2000). The kinematic viscosity of the disk is zero and no magnetic field is present. The accretion of matter onto planets is not included. The units used in the code are the following. The mass of the central object is taken to be the mass unit. The initial distance $r_{p2}$ of the inner planet from the central body is the length unit. The time is measured in $(GM_\odot/r_{p2}^3)^{-1/2}$. The profile of the initial surface density in the disc is essentially flat and it is described in detail in Papaloizou and Szuszkiewicz (2003). The initial value of the surface density $\Sigma_0$ at the locations of the planets is $\Sigma_0 = 6.0 \times 10^{-4}$ in our units. The computational domain in polar coordinates extends from 0.33 to 4 length units in the radial direction and covers all azimuthal angles from 0 to $2\pi$, and was divided into 384 radial slices and 512 angular sectors. The disc model is locally isothermal with aspect ratio $H/r = 0.05$

2 Results

In order to investigate how orbital evolution in the vicinity of a 3:2 resonance depends on the mass of the planets, we first considered two planets with 5 Earth masses each and then we repeated the calculation for two planets with 8 Earth masses each. We assume that previous evolution brought the two planets to a configuration close to the 3:2 commensurability. For the low-mass planets embedded in a disc with the surface density distribution given in the form $\Sigma(r) \sim r^{-\alpha}$ one can estimate the characteristic time of migration $\tau$ on a circular orbit from the following equation (see Tanaka, Takeuchi and Ward (2002))

$$\tau = (2.7 + 1.1\alpha)^{-1} \frac{M_c}{M_p} \frac{M_p}{\Sigma_p r_p^2} \left( \frac{c}{r_p \Omega_p} \right)^2 \Omega_p^{-1}, \quad (2.1)$$

where $M_c$ is the mass of the central body, $M_p$ is the mass of the planet, $r_p$ is the semi-major axis of the orbit of the planet, $\Omega_p$ is the angular velocity of the planet and $c$ is the sound speed in the disc at the distance $r_p$ from the central body. We expect that each pair of planets will enter the 3:2 mean motion resonance (MMR) because of their convergent migration, clearly following from Equation (2.1). Among two planets with the same mass, the planet located further out from the central body will migrate faster.

To analyse the system dynamics close to the 3:2 commensurability in the presence of the disc we monitor the evolution of the orbital elements, mainly the semi-major axes, eccentricities and resonant angles defined as follows

$$\phi_i = 3\lambda_1 - 2\lambda_2 - \omega_i, \quad (2.2)$$
where $i = 1, 2$, $\lambda_i$ is the mean longitude of the $i^{th}$ planet and $\omega_i$ is the longitude of the pericentre of the $i^{th}$ planet. To identify the occurrence of the commensurability we have used the fact that near the 3:2 resonance, the resonant angles $2(\lambda_i - \lambda_j)$ and $\omega_1 - \omega_2$ should librate about equilibrium values. In both cases the planets of $5M_\oplus$ and $8M_\oplus$ entered the 3:2 MMR and stayed there throughout the whole experiment from the beginning till the end, for 25000 dimensionless time units as is seen in Figure 1.
3 Conclusions

The planets with 5 and 8M_⊕ entered the resonance and continued evolution in this state till the end of our experiment. The mean values of the orbital eccentricities are excited to the value of \( \sim 0.008 \) in both cases and keep oscillating around this value. The eccentricities of the internal and external planets are roughly the same. The ratio of the orbital semi-major axes slowly approaches the exact resonance. A full discussion of these results will appear in a forthcoming paper.

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