(NS5, D5, D3) bound state, OD3, OD5 limits and $SL(2, Z)$ duality

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Abstract

We generalize the non-threshold bound state in type IIB supergravity of the form (NS5, D5, D3) constructed by the present authors (in hep-th/0011236) to non-zero asymptotic value of the axion ($\chi_0$). We identify the decoupling limits corresponding to both the open D3-brane theory and open D5-brane theory for this supergravity solution as expected. However, we do not find any non-commutative Yang-Mills theory (NCYM) limit for this solution in the presence of NS5 branes. We then study the $SL(2, Z)$ duality symmetry of type IIB theory for both OD3-limit and OD5-limit. We find that for OD3 theory, a generic $SL(2, Z)$ duality always gives another OD3-theory irrespective of the value of $\chi_0$ being rational or not. This indicates that OD3-theory is self-dual. But, under a special set of $SL(2, Z)$ transformations for which $\chi_0$ is rational OD3-theory goes over to a 5+1 dimensional NCYM theory and these two theories in this case are related to each other by strong-weak duality symmetry. On the other hand, for OD5-theory, a generic $SL(2, Z)$ duality gives another OD5-theory if $\chi_0$ is irrational, but when $\chi_0$ is rational it gives the little string theory limit indicating that OD5-theory is S-dual to the type IIB little string theory.

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I. INTRODUCTION

In a previous paper \[1\], we have constructed various non-threshold bound state solutions of both type IIB and type IIA supergravities of the type (NS5, D\(p\)) (with \(0 \leq p \leq 5\)) and (NS5, D\((p + 2)\), D\(p\)) (with \(0 \leq p \leq 3\)) by applying a series of T- and S-dualities to the known \((q, p)\) 5-brane solution of type IIB supergravity\[1\]. One of the motivations for constructing such solutions is to look at the world-volume theories of NS5-branes in the presence of various D-branes (or various RR electric gauge fields). In \[3\], it was argued that the world-volume theory of NS5-branes in the presence of a near critical RR \((p + 1)\)-form electric gauge field gives a non-gravitational and non-local theory called an open D\(p\)-brane theory in a special low energy limit (decoupling limit) known as the OD\(p\)-limit. The (NS5, D\(p\)) brane supergravity solution in this decoupling limit describes the supergravity dual of OD\(p\)-theories. These theories are analogous to world-volume theories of D\(p\)-branes in the presence of near critical electric fields (NCOS theory) \[4,5\] and world-volume theory of M5-brane in near critical electric 3-form gauge field (OM theory) \[3,6\] and contains fluctuating light open D\(p\)-branes in the world-volume of NS5-branes decoupled from gravity.

Since starting from these (NS5, D\(p\)) bound state solutions it is possible to construct various other bound states containing NS5-branes and several different D-branes by applying a series of T- and S-dualities, it is natural to ask what kind of theories do they correspond to in the decoupling limit. Some of the cases have been studied in \[7,8\]. In this paper we consider a specific case, namely, the (NS5, D5, D3) non-threshold bound state solution of type IIB supergravity. This is an \(SL(2, Z)\) invariant bound state of type IIB theory and the supergravity solution for this state has been constructed in \[1\] for zero asymptotic value of the axion. We rewrite this solution for the non-zero asymptotic value of the axion \((\chi_0)\). This solution can also be regarded as NS5-branes in the presence of a 6-form and a 4-form RR electric gauge fields. There are \(m\) NS5-branes, \(n\) D5-branes and \(p\) D3-branes per

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\(^1\)Some of these solutions are also considered in \[2\] from a different approach.
$(2\pi)^2 \alpha'$ of two codimensional area of NS5 (or D5)-branes in this bound state and preserves half of the space-time supersymmetries of string theory. We then identify both the open D3-brane limit and the open D5-brane limit for this supergravity solution. For the former, the 4-form approaches the critical value, whereas, for the latter the 6-form gauge field approaches the critical value. In this paper we only concentrate on the supergravity solution in the decoupling limit and also instead of writing the RR 6-form gauge field we write its Poincare dual. The existence of OD3- and OD5-limits for this bound state solution may not be surprising, however, to our surprise, we do not find any NCYM-limit in this $SL(2, Z)$ invariant solution. Since, (NS5, D3) state goes over to (D5, D3) state under S-duality, it was argued in [3] that the strong coupling limit of OD3-theory is the (5+1)-dimensional NCYM. So, it is surprising that no NCYM-limit exists for (NS5, D5, D3) state. However, it can be easily checked that when $m = 0$ (i.e. when NS5-branes are absent), the OD3-limit reduces exactly to the NCYM-limit [8].

We then study the $SL(2, Z)$ transformation on both the OD3-limit and the OD5-limit. We find that under a generic $SL(2, Z)$ transformation OD3-limit always gives another OD3-limit irrespective of whether $\chi_0$ is rational or not. Since even for rational $\chi_0$, we get another OD3-limit, we conclude that OD3-theory is self-dual. In other words, strongly coupled OD3-theory is related to the weakly coupled OD3-theory with different set of parameters related by S-duality. This is in accord with recent observation made in [13,14], where it was emphasized that since 5+1 dimensional NCYM is non-renormalizable, so it can not be obtained from OD3-theory by S-duality. Thus OD3-theory must be self-dual and is the UV completion of the 5+1 dimensional NCYM. But, we find that under a special circumstance $SL(2, Z)$

\[\text{SL}(2, Z)\]

The OD3 and OD5 theory described here are different from the usual OD3 and OD5 theory discussed in [3].

\[\text{SL}(2, Z)\]

transformation on the various decoupling limits of (F, D1, D3) bound state has been studied in [1-2].
duality on OD3-limit can give rise to NCYM-limit. In this case $\chi_0$ becomes rational and therefore, the OD3-theory becomes related to the NCYM-theory by the strong-weak duality. Actually, what happens here is that under this special set of $SL(2, \mathbb{Z})$ transformations the transformed charge of NS5-brane vanishes. Therefore, the OD3-limit in the transformed solution reduces to NCYM-limit as we mentioned earlier. However, for OD5-limit, we find that when $\chi_0$ is irrational a generic $SL(2, \mathbb{Z})$ transformation gives another OD5-limit with different parameters. But when $\chi_0$ is rational OD5-limit gives us precisely the little string theory \cite{15–20} limit. Thus we conclude that under the S-duality of type IIB theory OD5-theory goes over to little string theory.

This paper is organized as follows. In section 2, we give the (NS5, D5, D3) supergravity solution for non-zero asymptotic value of the axion. In section 3, we discuss the OD3- and OD5-limits for this solution. The $SL(2, \mathbb{Z})$ transformation is discussed in section 4. Finally, in section 5, we present our conclusion.

II. (NS5, D5, D3) SOLUTION FOR NON-ZERO $\chi_0$

The non-threshold bound state solution of the type (NS5, D5, D3) of type IIB supergravity was constructed in \cite{1} and has the form:

$$ds^2 = H^{1/2}H''^{1/2} \left[H^{-1}(-dx_0^2 + dx_1^2 + \cdots + dx_3^2) + H'^{-1}(dx_4^2 + dx_5^2) + dr^2 + r^2d\Omega_3^2\right]$$

$$e^{\phi_b} = g_s H'^{-1/2}H''$$

$$\chi = \frac{\tan \psi}{g_s}(H''^{-1} - 1)$$

$$B^{(b)} = 2ma' \sin^2 \theta \cos \phi_1 d\theta \wedge d\phi_2 + \tan \varphi \sin \psi H'^{-1} dx_4 \wedge dx_5$$

$$A^{(2)} = 2na' \sin^2 \theta \cos \phi_1 d\theta \wedge d\phi_2 - \frac{\cos \psi}{g_s} \tan \varphi H'^{-1} dx_4 \wedge dx_5$$

$$A^{(4)} = -po' H'^{-1} \sin^2 \theta \cos \phi_1 dx_4 \wedge dx_5 \wedge d\theta \wedge d\phi_2$$

\begin{equation}
-\frac{\sin \varphi}{g_s} H'^{-1} dx_0^1 \wedge dx_1^1 \wedge dx_2^1 \wedge dx_3^1
\end{equation}

In the above $r = \sqrt{x_0^2 + x_1^2 + x_2^2 + x_3^2}$ and $d\Omega_3^2 = d\theta^2 + \sin^2 \theta d\phi_1^2 + \sin^2 \theta \sin^2 \phi_1 d\phi_2^2$ is the line element of the unit 3-sphere transverse to the 5-branes. $g_s = e^{\phi_0}$ is the string coupling
constant, $B^{(b)}$ and $A^{(2)}$ denote the NSNS and RR two-form potentials. $\chi$ is the RR scalar and $A^{(4)}$ is the RR 4-form gauge field whose field strength is self-dual. The harmonic functions $H$, $H'$ and $H''$ are given as,

\[
H = 1 + \frac{Q_5}{r^2},
\]

\[
H' = 1 + \frac{\cos^2 \varphi Q_5}{r^2},
\]

\[
H'' = 1 + \frac{\cos^2 \varphi \cos^2 \psi Q_5}{r^2}
\]  \(2\)

where the angles $\cos \varphi$, $\cos \psi$ and the charge $Q_5$ are defined as

\[
\cos \varphi = \frac{(m^2 + n^2 g_s^2)^{1/2}}{[m^2 + (p^2 + n^2) g_s^2]^{1/2}},
\]

\[
\cos \psi = \frac{m}{(m^2 + n^2 g_s^2)^{1/2}},
\]

\[
Q_5 = [m^2 + (p^2 + n^2) g_s^2]^{1/2} \alpha'
\]  \(3\)

Here $m$ is the number of NS5-branes $n$ is the number of D5-branes and $p$ is the number of D3-branes per $(2\pi)^2 \alpha'$ of two codimensional area of 5-branes.

Note here that since the harmonic functions in (2) approaches unity asymptotically, so the string metric in (1) becomes Minkowskian in this limit. Also, $e^{\phi_b} \to e^{\phi_{b0}}$ and $\chi \to 0$ asymptotically. In order to obtain this solution for non-zero asymptotic value of the axion ($\chi_0$), we make an $SL(2, R)$ transformation by the matrix

\[
\Lambda = \begin{pmatrix} 1 & \chi_0 \\ 0 & 1 \end{pmatrix}
\]  \(4\)

The solution then takes the form:

\[
ds^2 = H^{1/2} H'^{1/2} \left[ H^{-1} \left( -dx_0^2 + dx_1^2 + \cdots + dx_5^2 \right) + H'^{-1} \left( dx_4^2 + dx_5^2 \right) + dr^2 + r^2 d\Omega_3^2 \right]
\]

\[
e^{\phi_b} = g_s H'^{-1/2} H''
\]

\[
\chi = \frac{\tan \psi}{g_s} (H''^{-1} - 1) + \chi_0
\]

\[
B^{(b)} = 2m \alpha' \sin^2 \theta \cos \phi_1 d\theta \wedge d\phi_2 + \tan \varphi \sin \psi H'^{-1} dx^4 \wedge dx^5
\]

\[
A^{(2)} = 2n \alpha' \sin^2 \theta \cos \phi_1 d\theta \wedge d\phi_2 - \left( \frac{\cos \psi}{g_s} + \chi_0 \sin \psi \right) \tan \varphi H'^{-1} dx^4 \wedge dx^5
\]  \(5\)
and $A^{(4)}$ retains its form as given in (1) since it is $SL(2, R)$ invariant. The harmonic functions retain their form as given in (2), but now the angles and the charge $Q_5$ are given as,

$$
\cos \varphi = \frac{[m^2 + (n + \chi_0 m)^2 g_s^2]^{1/2}}{[m^2 + (p^2 + (n + \chi_0 m)^2 g_s^2)]^{1/2}} \\
\cos \psi = \frac{m}{[m^2 + (n + \chi_0 m)^2 g_s^2]^{1/2}} \\
Q_5 = \left[ m^2 + (p^2 + (n + \chi_0 m)^2 g_s^2) \right]^{1/2} \alpha' 
$$

(6)

Note here that in terms of angles $Q_5$ in (1) can be written as $Q_5 = (m \alpha')/(\cos \varphi \cos \psi)$. Therefore, the harmonic functions in eq.(2) take the forms:

$$
H = 1 + \frac{m \alpha'}{\cos \varphi \cos \psi r^2} \\
H' = 1 + \frac{m \alpha' \cos \varphi}{\cos \psi r^2} \\
H'' = 1 + \frac{m \alpha' \cos \varphi \cos \psi}{r^2} 
$$

(7)

We will use these forms later.

From eq.(6) we deduce the following quantization conditions:

$$
\frac{n}{m} = \tan \psi - \chi_0 \\
\frac{p}{m} = \frac{\tan \varphi}{g_s \cos \psi} 
$$

(8)

Note here that under a general $SL(2, Z)$ transformation by the matrix

$$
\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} 
$$

(9)

where $a, b, c, d$ are integers with $ad - bc = 1$, $Q_5$, $\cos \varphi$, $H$ and $H'$ are invariant. But $\cos \psi$ and $H''$ change as

$$
\cos \hat{\psi} = \frac{\frac{c}{g_s} \tan \psi - (c \chi_0 + d)}{\left( (c \chi_0 + d)^2 + \frac{c^2}{g_s^2} \right)^{1/2}} \cos \psi \\
\hat{H}'' = 1 + \frac{\cos^2 \varphi \cos^2 \hat{\psi} Q_5}{r^2} 
$$

(10)

We will consider the various decoupling limits for this solution in the next section.
III. OD3 AND OD5 LIMITS

(a) OD3 limit: The open D3-brane theory appears as a decoupling limit on the world-volume of NS5-branes in the presence of a near critical RR 4-form electric gauge field and was obtained in [3]. The corresponding supergravity dual [1,21] is given as the decoupling limit of (NS5, D3) bound state solution of type IIB theory. The OD3-limit is given as the following:

\[ \cos \varphi = \epsilon \to 0 \]  (11)

keeping the following quantities fixed,

\[ \alpha'_{\text{eff}} = \frac{\alpha'}{\epsilon} , \quad u = \frac{r}{\epsilon \alpha'_\text{eff}} , \quad G^2_{o(3)} = gs \]  (12)

Note here that if we set \( \cos \psi = 1 \) and \( \chi_0 = 0 \), then the solution given in (5) reduces to (NS5, D3) solution [1]. However, when D5-branes are also present OD3-limit is again the same as in (11) and (12), but in addition we set

\[ \cos \psi = l \text{ (finite)} \quad \text{and} \quad \chi_0 \neq 0 \]  (13)

In the above \( \epsilon \) is a dimensionless parameter, \( (\alpha'_{\text{eff}})^{3/2} \) corresponds to the finite inverse tension of open D3-brane and \( G^2_{o(3)} \) is the coupling constant. In this limit the harmonic functions in (5) reduce to

\[ H = \frac{1}{a^2 \epsilon^2 u^2} \]
\[ H' = \frac{h'}{a^2 u^2} \]
\[ H'' = \frac{h''}{a^2 u^2} \]  (14)

where \( h' = 1 + a^2 u^2 \) and \( h'' = 1 + \tilde{a}^2 u^2 \), with \( a^2 = l \alpha'_{\text{eff}} / m \) and \( \tilde{a}^2 = \alpha'_{\text{eff}} / (lm) \). So, the metric in (3) takes the following form,

\[ ds^2 = \alpha' h'^{1/2} \left[ -d\tilde{x}_0^2 + \sum_{i=1}^3 d\tilde{x}_i^2 + h'^{-1} \sum_{j=4}^5 d\tilde{x}_j^2 + \frac{m}{u^2} \left( du^2 + u^2 d\Omega_3^2 \right) \right] \]  (15)
The finite coordinates in (15) are defined as
\[ \tilde{x}_{0,1,2,3} = \sqrt{\frac{l}{\alpha'_{\text{eff}}}} x_{0,1,2,3} \]
\[ \tilde{x}_{4,5} = \sqrt{\frac{lc'_{\text{eff}}}{\alpha'}} x_{4,5} \] (16)

So, these are precisely the OD3-limit discussed in [3]. In this limit the dilaton and other gauge fields take the form:

\[ e^{\phi_b} = G^2_{o(3)} \frac{h''}{h'^{1/2}} \frac{l}{\tilde{a} u} \]
\[ \chi = -\frac{\sqrt{1 - l^2}}{l} \frac{1}{G^2_{o(3)}} \frac{1}{h''} + \chi_0 \]
\[ B^{(b)}_{\theta \phi_2} = 2m \alpha' \sin^2 \theta \cos \phi_1, \quad B^{(b)}_{45} = \alpha' \frac{\sqrt{1 - l^2} a^2 u^2}{h'} \]
\[ A^{(2)}_{\theta \phi_2} = 2n \alpha' \sin^2 \theta \cos \phi_1, \quad A^{(2)}_{45} = -\alpha' \left( \frac{1}{G^2_{o(3)}} + \frac{\chi_0 \sqrt{1 - l^2}}{l} \right) \frac{a^2 u^2}{h'} \]
\[ A^{(4)}_{45 \theta \phi_2} = -\frac{\alpha'_{\text{eff}}}{\alpha'} \frac{a^2 u^2}{lh'} \sin^2 \theta \cos \phi_1, \quad A^{(4)}_{0123} = -\frac{\alpha'^2}{G^2_{o(3)}} \frac{a^2 u^2}{l^2} \] (17)

Note that for \( \tilde{a} u \ll 1 \) (which implies that both \( au \ll 1 \) and \( \tilde{a} u \ll 1 \)) i.e. in the IR region the supergravity description is valid (in this case the curvature \( \alpha' R \sim 1/m \) remains small for large enough \( m \), the number of NS5-branes) if \( \tilde{a} u \gg G^2_{o(3)} \). In this case \( G^2_{o(3)} \ll 1 \). However, this condition is not satisfied in the extreme IR region, where \( e^{\phi_b} \) becomes large. In that case we have to go to the S-dual frame and we will describe this in the next section. In any case, in the IR region the OD3-theory flows to (5+1) dimensional SYM theory.

For \( au \gg 1 \), i.e. in the UV region, the string coupling \( e^{\phi_b} = G^2_{o(3)} = \text{fixed} \). So, when \( G^2_{o(3)} \ll 1 \), we have valid supergravity description and the metric in (15) reduces to that of ordinary D3-branes smeared in 4, 5 directions. However, for \( G^2_{o(3)} \gg 1 \), we have to go to the S-dual frame and will be discussed in the next section.

We would like to emphasize that the OD3 theory we are discussing here whose supergravity dual is given in (15) and (17) are different from the usual OD3 theory in that the field configurations in this case are different. This is also manifested in the extra parameter defined in (13). Note that when the parameter \( l = 1 \), it implies \( n = 0 \) and \( \chi_0 = 0 \) and
therefore the configuration (3) reduces to (NS5, D3) solution. In the decoupling limit (11) and (12), it will give the supergravity dual of ordinary OD3 theory [3,1,21]. In this case we note that \( B^{(b)}_{45} \) in (17) vanishes. On the other hand if the parameter \( l = 0 \), it implies that \( m = 0 \) i.e. there are no NS5 branes in the supergravity configuration. In this case as we will explain later, the OD3 limit reduces to NCYM limit. When \( l \) is in between 0 and 1, we get a new OD3 theory which is noncommutative as \( B^{(b)}_{45} \neq 0 \) and therefore the coordinates \( x^4 \) and \( x^5 \) are noncommutative. It is clear from the expression of \( B^{(b)}_{45} \) in (17) that noncommutativity is related with the parameter \( l \) as \( \sqrt{1-l^2}/l \). This gives a physical interpretation of the parameter \( l \) in this new OD3 theory.

(b) OD5 limit: As in the previous case open D5-brane theory arises as a decoupling limit on NS5-branes when the RR 6-form electric gauge field approaches a critical value. The dual supergravity solution is obtained from (NS5, D5) bound state \( \text{I} \) in the decoupling limit. In the present case of (NS5, D5, D3) solution the OD5-limit is given in the following:

\[
\cos \psi = \epsilon \to 0 \quad (18)
\]

keeping the following quantities fixed,

\[
\alpha_{\text{eff}}' = \frac{\alpha'}{\epsilon}, \quad u = \frac{r}{\epsilon \alpha_{\text{eff}}'}, \quad G_{\alpha(5)}^2 = \epsilon g_s \quad (19)
\]

and also we have

\[
\cos \varphi = \tilde{l} = \text{finite}, \quad \text{and} \quad \chi_0 \neq 0 \quad (20)
\]

Under this decoupling limit the harmonic functions take the forms:

\[
H = \frac{1}{a^2 \epsilon^2 u^2} \\
H' = \frac{1}{\tilde{a}^2 \epsilon^2 u^2} \\
H'' = \frac{h''}{\tilde{a}^2 u^2} \quad (21)
\]

where \( h'' = 1 + \tilde{a}^2 u^2 \), with \( \tilde{a}^2 = \alpha_{\text{eff}}'/(m \tilde{l}) \), \( a^2 = \tilde{l} \alpha_{\text{eff}}'/m \).

The metric in (3) is now given by
\[ ds^2 = \alpha' h'^{1/2} \left[ -d\tilde{x}_0^2 + \sum_{i=1}^{5} d\tilde{x}_i^2 + \frac{m}{u^2} \left( du^2 + u^2 d\Omega_3^2 \right) \right] \] (22)

where the finite coordinates are defined as
\[ \tilde{x}_{0,1,2,3} = \sqrt{\frac{l}{\alpha'_\text{eff}}} x_{0,1,2,3}, \quad \tilde{x}_{4,5} = \frac{1}{\sqrt{l\alpha'_\text{eff}}} x_{4,5} \] (23)

The dilaton, axion and other gauge fields in the decoupling limit are given as,
\[ e^{\phi_b} = G_{o(5)}^2 \frac{h''}{\tilde{a}u} \]
\[ \chi = -\frac{1}{G_{o(5)}^2} \frac{1}{h''} + \chi_0 \]
\[ B_{\theta\phi_2}^{(b)} = 2m\alpha' \sin^2 \theta \cos \phi_1, \quad B_{45}^{(b)} = \frac{\alpha'^2}{m} \frac{\sqrt{1-\tilde{l}^2}}{l} u^2 \]
\[ A_{\theta\phi_2}^{(2)} = 2n\alpha' \sin^2 \theta \cos \phi_1, \quad A_{45}^{(2)} = \frac{\alpha'^2}{m} \frac{\chi_0}{l} \frac{\sqrt{1-\tilde{l}^2}}{l} u^2 \]
\[ A_{45\theta\phi_2}^{(4)} = -\frac{p}{m} \alpha'^3 u^2 \sin^2 \theta \cos \phi_1, \quad A_{0123}^{(4)} = -\alpha'^3 \frac{1}{m} \frac{\sqrt{1-\tilde{l}^2}}{l} u^2 \] (24)

This is precisely the OD5-limit discussed in [3]. However, instead of the RR 6-form electric gauge field, we have given here its Poincare dual. One can indeed check that the corresponding 6-form approaches the critical value given there in this limit.

It is clear from above that in the IR (\( \tilde{a}u \ll 1 \)) the supergravity solution is valid if \( \tilde{a}u \ll G_{o(5)}^2 \). In that case, \( G_{o(5)}^2 \ll 1 \). But in the extreme IR, this relation is not satisfied and we need to go to the S-dual frame. On the other hand, in the UV, the solution is valid if \( \tilde{a}u \ll G_{o(5)}^{-2} \). This again is not satisfied in the extreme UV region and we need to go to the S-dual frame which we will discuss in the next section.

As in the case of OD3 theory, the OD5 theory whose supergravity dual is given by (22) and (24) is also different from the usual OD5 theory [3,1,21]. The field contents for this new OD5 theory is different. In this case there is an additional parameter \( \tilde{l} \) defined in (20). When \( \tilde{l} = 1 \), this new OD5 theory reduces to ordinary OD5 theory. However, when \( \tilde{l} = 0 \), the configuration (3) reduces to that of D3 brane with two additional isometries in \( x^4 \) and \( x^5 \) directions. Under the decoupling limit (18) and (19) the supergravity description becomes
invalid as the dilaton blows up. So, $\tilde{l} \neq 0$ and when it lies in between 0 and 1, we get a noncommutative OD5 theory since $B_{45}^{(b)}$ does not vanish as can be seen from (24). Again the coordinates $x^4$ and $x^5$ are noncommutative. The noncommutativity parameter in this case is proportional to $\alpha' \sqrt{1 - \tilde{l}^2/\tilde{l}}$ i.e. it is not fixed but scales as $\alpha'$. This should be contrasted with OD3 theory where the noncommutativity parameter is fixed as mentioned earlier.

Thus we have obtained both the OD3-limit and OD5-limit for the (NS5, D5, D3) supergravity solution. This as we mentioned in the introduction is quite expected. However, contrary to our expectation, we do not find any NCYM limit for this solution. We just like to point out that the OD3-limit discussed in this section takes the form of NCYM limit when we set ‘$m$’ the number of NS5-branes exactly to zero. Since $m = 0$, $\chi_0 = 0$, implies $\cos \psi = 0$, so the harmonic function $H'' = 1$. Also, note that since $m/\cos \psi = ng_s$, so, the other two harmonic functions in the limit (11) and (12) reduce to

$$H = 1 + \frac{ng_s \alpha'}{\cos \varphi r^2} \rightarrow \frac{\tilde{b}^2}{a^2 u^2 \alpha'^2},$$

$$H' = 1 + \frac{ng_s \alpha' \cos \varphi}{r^2} = \frac{h}{a^2 u^2}$$

where we have defined $h = 1 + a^2 u^2$ with $a^2 = \tilde{b}/(ng_s)$. Note that here we are interpreting $\alpha'_{\text{eff}} = \tilde{b}$ as the non-commutativity parameter and the Yang-Mills coupling $g_{\text{NCYM}}^2 = (2\pi)^3 g_s = \text{fixed}$. With these forms of the harmonic functions the metric and the dilaton take precisely the same form as the dual of NCYM theory [8]:

$$ds^2 = \alpha' \left[ \frac{u}{R} \left( -d\tilde{x}_0^2 + \sum_{i=1}^{3} d\tilde{x}_i^2 \right) + h^{-1} \sum_{j=4}^{5} d\tilde{x}_j^2 \right] + \frac{R}{u} \left( du^2 + u^2 d\Omega_3^2 \right)$$

where $R = \tilde{b}/a$ and the fixed coordinates are

$$\tilde{x}_{0,1,2,3} = x_{0,1,2,3}; \quad \tilde{x}_{4,5} = \frac{\tilde{b}}{\alpha'} x_{4,5}$$

and

$$e^{\phi_b} = g_s \left( \frac{u}{R} \right) \frac{\tilde{b}}{h^{3/2}}$$

(28)
Since NCYM limit is nothing but OD3-limit of (NS5, D5, D3) solution with $m = 0$, so it is not clear whether there exists an NCYM limit for this solution (for $m \neq 0$) independent of OD3-limit.

**IV. SL(2, Z) TRANSFORMATION**

Type IIB string theory as well as its low energy limit, the corresponding supergravity theory, are well-known to possess an $SL(2, Z)$ duality symmetry. Since the (NS5, D5, D3) solution is $SL(2, Z)$ invariant, we would like to know what happens to the OD3-limit and OD5-limit under an $SL(2, Z)$ transformation. Under a general $SL(2, Z)$ transformation by the matrix given in (9) the various fields of type IIB supergravity transform as:

$$g^E_{\mu\nu} \rightarrow g^E_{\mu\nu}, \quad \lambda \rightarrow \frac{a\lambda + b}{c\lambda + d}, \quad \begin{pmatrix} B^{(b)} \\ A^{(2)} \end{pmatrix} \rightarrow \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \begin{pmatrix} B^{(b)} \\ A^{(2)} \end{pmatrix}$$

$$A^{(4)} \rightarrow A^{(4)}$$

(29)

where $g^E_{\mu\nu}$ denotes the Einstein metric and $\lambda = \chi + ie^{-\phi_b}$. Under (29) the axion and the dilaton transform as

$$\hat{\chi} = \frac{(a\chi + b)(c\chi + d) + ace^{-2\phi_b}}{|c\lambda + d|^2}$$

$$e^{\hat{\phi}_b} = |c\lambda + d|^2 e^{\phi_b}$$

(30)

Since the Einstein metric $g^E_{\mu\nu} = e^{-\phi_b/2}g_{\mu\nu}$ (where $g_{\mu\nu}$ is the string metric) remains invariant under $SL(2, Z)$ transformation so, the string metric would transform as

$$d\hat{s}^2 = |c\lambda + d|ds^2$$

(31)

If we now insist that the transformed metric should asymptotically be Minkowskian then the transformed metric would be given as

$$d\hat{s}^2 = \frac{|c\lambda + d|}{|(c\chi_0 + d)^2 + \frac{a^2}{g_4^2}|^{1/2}}ds^2$$

(32)
We will show in the following how the metric and the dilaton would transform under the \( SL(2, Z) \) for both the OD3-limit and OD5-limit. The transformation of the other gauge fields can be obtained in a straightforward manner.

\( (a) \) \( SL(2, Z) \) transformation and OD3-limit: We would like to point out that the numerator appearing in the transformation of the angle \( \cos \psi \) in (10) may vanish for a particular choice of a set of \( SL(2, Z) \) transformations. It can be seen that when \( c \tan \psi / g_s - (c \chi_0 + d) = 0, \cos \hat{\psi} = 0 \) and \( \hat{H}'' = 1 \). As it is clear from the discussion in section 3, the OD3-limit in this case would reduce to the NCYM-limit. So, in the following discussion we would consider the two cases \( (c \tan \psi / g_s - (c \chi_0 + d) \neq 0 \) and \( = 0 \) separately.

\( (i) \) \( c \chi_0 + d \neq c \tan \psi / g_s \):

From the form of \( \chi \) and \( e^{\phi_b} \) of OD3-limit given in (17) we find that

\[
|c \lambda + d| = \frac{\tilde{h}''^{n1/2}}{\tilde{h}''^{n1/2}}
\]

where

\[
\tilde{h}'' = \left( \frac{c}{G_\omega^{(3)}} \frac{\sqrt{1 - l^2}}{l} - (c \chi_0 + d) \right)^2 + \tilde{a}^2 u^2 \left( \frac{c^2}{G_\omega^{(3)}} + (c \chi_0 + d)^2 \right)
\]

We thus find from (32) and (15) that the transformed metric has the form

\[
d\hat{s}^2 = \alpha' \frac{\tilde{h}''^{n1/2}}{(c \chi_0 + d)^2 + \tilde{a}^2 u^2} \left[ -d\hat{x}_0^2 + \sum_{i=1}^{3} d\hat{x}_i^2 + h'^{-1} \sum_{j=4}^{5} d\hat{x}_j^2 + \frac{m}{u^2} (du^2 + u^2 d\Omega_3^2) \right]
\]

By writing \( \tilde{h}'' \) as

\[
\tilde{h}'' = \left( \frac{c}{G_\omega^{(3)}} \frac{\sqrt{1 - l^2}}{l} - (c \chi_0 + d) \right)^2 (1 + \tilde{a}^2 u^2)
\]

\[
\tilde{h}'' = \left( \frac{c}{G_\omega^{(3)}} \frac{\sqrt{1 - l^2}}{l} - (c \chi_0 + d) \right)^2 \tilde{h}''
\]

where

\[
\tilde{a}^2 = \left( \frac{c}{G_\omega^{(3)}} \frac{\sqrt{1 - l^2}}{l} - (c \chi_0 + d) \right)^2 \tilde{a}^2
\]
we can write the metric in (35) precisely in the same form as that of the OD3-limit in (15) i.e.
\[
d\hat{s}^2 = \alpha' \hat{h}^{\mu/2} \left[ -d\hat{s}_0^2 + \sum_{i=1}^{3} d\hat{x}_i^2 + h'^{-1} \sum_{j=4}^{5} d\hat{x}_j^2 + \frac{\hat{m}}{u^2} \left( du^2 + u^2 d\Omega_3^2 \right) \right]
\]
(38)

where
\[
\hat{m} = \left( \frac{c}{G_{o(3)}^2} \frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d) \right) \left[ (c\chi_0 + d)^2 + \frac{c^2}{G_{o(3)}^2} \right]^{1/2} m
\]
(39)
and also we have rescaled the coordinates as
\[
\hat{x}_{0,1,...,5} = \left( \frac{c}{G_{o(3)}^2} \frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d) \right)^{1/2} \left[ (c\chi_0 + d)^2 + \frac{c^2}{G_{o(3)}^2} \right]^{1/4} \tilde{x}_{0,1,...,5}
\]
(40)

Note that (37) gives the transformation of \( \tilde{a}^2 = \alpha'_{\text{eff}}/(ml) \) under \( SL(2, Z) \), however, \( a^2 = l\alpha'_{\text{eff}}/m \) remains invariant.

The form of the dilaton can be obtained from (30) and (17) as,
\[
e^{\hat{\phi}_b} = \hat{G}_{o(3)}^2 \frac{\hat{h}''}{\hat{h}'^{1/2}} \frac{\hat{l}}{\hat{a}u}
\]
(41)

Where the coupling constant of the \( SL(2, Z) \) transformed OD3-theory is given by
\[
\hat{G}_{o(3)}^2 = \frac{1}{G_{o(3)}^2} \left[ (c\chi_0 + d)^2 G_{o(3)}^4 + c^2 \right]
\]
(42)
and
\[
\hat{l} = \cos \hat{\psi} = \left( \frac{c}{G_{o(3)}^2} \frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d) \right) \left[ (c\chi_0 + d)^2 + \frac{c^2}{G_{o(3)}^2} \right]^{1/2} \cos \psi
\]
(43)

We note that the metric and the dilaton have the same forms as those of the OD3-limit obtained in eqs.(17) and (17) in the previous section. From (34), it is clear that no matter whether \( c\chi_0 + d \neq 0 \) (\( \chi_0 \) is irrational) or \( c\chi_0 + d = 0 \) (\( \chi_0 \) is rational) the forms of the metric and the dilaton in (35) and (11) are always the same as those of the original OD3-limit. The gauge fields of the \( SL(2, Z) \) transformed OD3-theory can also be obtained from eqs.(23).
We thus conclude that OD3-theory is self-dual. For rational $\chi_0$, the metric and the dilaton are given by the same expressions as in (38) and (41) with a simpler form of $\tilde{h}''$, $\tilde{a}^2$, $\tilde{m}$, $\tilde{x}_{0,1,...,5}$ and $\tilde{G}^2_{\alpha(3)}$. They are given in the following

$$\tilde{h}'' = \frac{c^2}{G_{\alpha(3)}^4} \frac{1-l^2}{l^2} (1 + \tilde{a}^2 u^2)$$
$$\tilde{a}^2 = \frac{l^2}{1-l^2} \tilde{a}^2, \quad \tilde{m} = \frac{\sqrt{1-l^2}}{l} m$$
$$\tilde{x}_{0,1,...,5} = \frac{(1-l^2)^{1/4}}{\sqrt{l}} \tilde{x}_{0,1,...,5}, \quad \tilde{G}^2_{\alpha(3)} = \frac{c^2}{G_{\alpha(3)}^2}$$

(44)

These are the various variables and parameters for the supergravity dual of S-dual OD3-theory. We note that since $a^2 = l\alpha'_{\text{eff}}/m$ remains invariant and $l$, $m$ transform exactly in the same way (see (39) and (43)), so, $\alpha'_{\text{eff}}$ is invariant.

(ii) $c\chi_0 + d = c \tan \psi/g_s$:

We have already mentioned that in this case $\cos \hat{\psi} = 0$ or in other words, $\tilde{m}$, the transformed number of NS5-branes vanishes. So, in the final theory the axion $\hat{\chi}$ and its asymptotic value $\hat{\chi}_0$ must vanish also. This indicates that $\chi_0$ in the original theory must be rational i.e. $c\chi_0 + d = 0$. This also implies that $\sin \psi$ in the original theory vanishes i.e. there are no D5-branes in the original theory. From (34) we notice that when the above condition is satisfied we get

$$\tilde{h}'' = \frac{\tilde{a}^2 u^2 c^2}{G_{\alpha(3)}^4}$$

(45)

Then the metric in (31) has exactly the same form as in (26). Also, from (30), the dilaton is given by,

$$e^{\tilde{\phi}_b} = \hat{g}_s \left( \frac{u}{R} \right) \frac{\tilde{b}}{h'^{1/2}}$$

(46)

where $\hat{g}_s = c^2/g_s$ and $h' = 1 + a^2 u^2$, with $a^2 = \alpha'_{\text{eff}}/(\hat{n}\hat{g}_s) = \tilde{b}/(\tilde{n}\hat{g}_s)$, $R = \tilde{b}/a$. Note that in this case $a^2 = \tilde{a}^2$ since $l = 1$. This is precisely the NCYM limit, we discussed in section 3, with $\tilde{b}$ as the non-commutativity parameter and $g_{\text{NCYM}}^2 = (2\pi)^3 \hat{g}_s$, the Yang-Mills coupling. Thus we find NCYM-limit from a special set of $SL(2, Z)$ transformations of OD3-limit for
which $\chi_0$ is rational. Therefore, OD3-theory and NCYM theory in 5+1 dimensions can be related as strong-weak duality symmetry as discussed in [3].

(b) $SL(2, \mathbb{Z})$ transformation and OD5-limit: Unlike in the previous case, we note here that the expression $c \tan \psi / g_s - (c\chi_0 + d)$ can not become equal to zero while considering OD5-limit. The reason is, as we have already mentioned, when the above expression vanishes, it implies that $\sin \psi = 0$ in the original theory i.e. there are no D5-branes. In other words, since $\cos \psi = 1$ in this case we can not take OD5-limit in the original theory. So, we take $c \tan \psi / g_s - (c\chi_0 + d) \neq 0$. Now proceeding exactly in the same way as in subsection (a) we first find the form of $|c\lambda + d|$ from the expressions of $\chi$ and $e^{\phi_b}$ in (24) as,

$$|c\lambda + d| = \frac{\tilde{h}^{n/2}}{\tilde{h}^{''n/2}}$$

where

$$\tilde{h}'' = \left[ \frac{c}{G_5^{(5)}} - (c\chi_0 + d) \right]^2 + \tilde{a}^2 u^2 (c\chi_0 + d)^2$$

Note that since the form of $\tilde{h}''$ changes completely for irrational $\chi_0$ and rational $\chi_0$, we study these two cases separately.

Irrational $\chi_0$

When $c\chi_0 + d$ is not equal to zero, we find from (12) and (22) that the transformed metric takes the form:

$$ds^2 = \alpha' \frac{\tilde{h}^{n/2}}{(c\chi_0 + d)} \left[ -d\tilde{x}_0^{'2} + \sum_{i=1}^{5} d\tilde{x}_i^{'2} + \frac{m}{u^2} \left( du^2 + u^2 d\Omega_3^{(5)} \right) \right]$$

Redefining

$$\tilde{h}'' = \left[ \frac{c}{G_5^{(5)}} - (c\chi_0 + d) \right]^2 (1 + \tilde{a}^2 u^2)$$

$$= \left[ \frac{c}{G_5^{(5)}} - (c\chi_0 + d) \right]^2 \hat{h}''$$

where $\hat{h}'' = 1 + \hat{a}^2 u^2$ and
\[ \hat{a}^2 = \frac{(c\chi_0 + d)^2}{\left[ \frac{c}{G_{o(5)}^2} - (c\chi_0 + d) \right]^2} \]  

we can rewrite the metric in (49) exactly in the same form as that of OD5-limit, namely,  

\[ d\hat{s}^2 = \alpha^{'h}^{m/2} \left[ -d\hat{x}_0^2 + \sum_{i=1}^{5} d\hat{x}_i^2 + \hat{m} u^2 \left( du^2 + u^2 d\Omega_3^2 \right) \right] \]  

where  

\[ \hat{m} = \frac{\frac{c}{G_{o(5)}^2} - (c\chi_0 + d)}{(c\chi_0 + d)} m \]  

we have also redefined the coordinates as,  

\[ \hat{x}_{0,1,\ldots,5} = \frac{\frac{c}{G_{o(5)}^2} - (c\chi_0 + d)}{(c\chi_0 + d)^{1/2}} \tilde{x}_{0,1,\ldots,5} \]  

The transformed dilaton can be obtained from (30) and (24) as,  

\[ e^{\hat{\phi}_b} = \hat{G}_{o(5)}^2 \hat{h}''_{a'b'} \]  

where  

\[ \hat{G}_{o(5)}^2 = (c\chi_0 + d) \left[ \frac{c}{G_{o(5)}^2} - (c\chi_0 + d) \right] G_{o(5)}^2 \]  

Here we note that since \( \cos \varphi = \tilde{l} = a/\hat{a} = \) invariant under SL(2, Z), so both \( \hat{a} \) and \( a \) must transform in the same way. The transformation of \( \hat{a} \) is given in (51). However, \( \cos \psi = \epsilon = m/\left( \sqrt{m^2 + (n + \chi_0 m)^2 g_s^2} \right) \) is not SL(2, Z) invariant, but it must transform as \( m \) i.e.  

\[ \cos \hat{\psi} = \hat{\epsilon} = \frac{\frac{c}{G_{o(5)}^2} - (c\chi_0 + d)}{(c\chi_0 + d)} \cos \psi \]  

Thus we observe from (52) and (53) that the transformed metric and the dilaton have exactly the same form as those of OD5-limit obtained in (22) and (24). Also since \( \tilde{a}^2 = \alpha'_e/(m\tilde{l}) \), so the effective tension of the SL(2, Z) transformed OD5-theory would be given as,  

\[ \hat{\alpha}'_{e} = \frac{(c\chi_0 + d)}{\left[ \frac{c}{G_{o(5)}^2} - (c\chi_0 + d) \right]} \alpha'_e \]
The forms of the other gauge fields of the $SL(2, Z)$ dual OD5-theory can be obtained from (29).

**Rational $\chi_0$**

Now if $\chi_0$ is rational i.e. $c\chi_0 + d = 0$, then from (47) we have

$$|c\lambda + d| = \frac{\tilde{h}^{n/2}}{\tilde{h}^{n/2}}$$

where $\tilde{h}'' = c^2/G^4_{\tilde{a}(5)}$. So, the metric and the dilaton would now take the forms,

$$d\hat{s}^2 = \left[ -d\tilde{x}_0^2 + \sum_{i=1}^{5} d\tilde{x}_i^2 + \alpha'_{\text{eff}} \frac{m}{u^2}(du^2 + u^2d\omega_3^2) \right]$$

$$e^{\hat{\phi}_b} = \frac{c^2}{G^2_{\tilde{a}(5)}} \frac{1}{\tilde{a}u}$$

Here $G^2_{\tilde{a}(5)}$ is just a finite quantity which can be absorbed in $\tilde{a}$. We have also redefined the coordinates $\tilde{x}_{0,1,...,5}$ by $\sqrt{\alpha'_{\text{eff}}} \tilde{x}_{0,1,...,5}$ and $\alpha'_{\text{eff}} = \alpha'/\epsilon = \text{finite}$. Note also that although $e^{\hat{\phi}_b}$ is finite, the coupling constant for the dual theory $\hat{G}^2_{\tilde{a}(5)} = \hat{g}_s = (c^2/G^2_{\tilde{a}(5)})\epsilon \to 0$. Eq.(60) represents precisely the supergravity dual of little string theory $^{20,19}$. We thus conclude that the S-dual of OD5-theory is the little string theory.

**V. CONCLUSION**

To summarize, we have studied in this paper the various decoupling limits of an $SL(2, Z)$ invariant bound state of the type (NS5, D5, D3) in type IIB supergravity. This solution can also be regarded as NS5-branes in the presence of both a 4-form and a 6-form RR electric gauge fields. In particular, we have identified an OD3-limit and an OD5-limit for this solution. In these decoupling limits (NS5, D5, D3) solution represent the supergravity dual of OD3-theory and OD5-theory respectively. In both the cases we obtained noncommutative theories and are different from the usual OD3 and OD5 theories. We have mentioned that when NS5-branes are absent, the OD3-limit reduces to an NCYM limit. But we do not find an independent NCYM limit in the presence of NS5-branes. We then studied the $SL(2, Z)$ transformation of both OD3-limit and OD5-limit. The generic $SL(2, Z)$ transformation of
OD3-limit always gives another OD3-limit with different set of parameters irrespective of whether the asymptotic value of axion is irrational or not. When the asymptotic value of the axion is rational the two OD3-theories are related to each other by strong-weak duality symmetry. We thus conclude that OD3-theory is self-dual. However, under a special set of $SL(2, Z)$ transformations we find that the OD3-limit reduces to NCYM-limit. But for these set of $SL(2, Z)$ transformations the transformed charge of NS5-brane vanishes. Thus it is not surprising that we get an NCYM-limit for these transformations as we have already mentioned in section 3. In this case $\chi_0$ is rational and so, the OD3-theory and NCYM-theory are related by strong-weak duality symmetry and this case has already been studied in [3]. On the other hand, for OD5-limit, we find that when the asymptotic value of axion is irrational a generic $SL(2, Z)$ transformation gives another OD5-limit with different set of parameters characterizing the $SL(2, Z)$ transformed OD5-theory. But when $\chi_0$ is rational OD5-limit reduces to little string theory limit. So, we conclude that OD5-theory and little string theory are related to each other by type IIB S-duality symmetry.

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