An Imaging Method for Spaceborne Cooperative Multistatic SAR Formations With Nonzero Cross-Track Baselines

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Abstract—Spaceborne multistatic synthetic aperture radar (M-SAR) formations can deploy multiple spatially separated receiving phase centers along the along-track (AT) direction to achieve high-resolution wide-swath. However, the cross-track (XT) separation between the spacecraft is inevitable due to the formation design like orbital safety and application potential like the XT interferometry. In addition, the different motion vectors of two or more satellites caused by the Helix formation will lead to the difference of the Doppler parameters at the same range gate. Therefore, nonzero XT baselines and space-variant characteristics pose new challenges to the focusing and phase-preserving of the M-SAR imaging. To this end, an imaging method for spaceborne cooperative M-SAR formations with nonzero XT baselines is proposed in this article. Firstly, a spaceborne cooperative M-SAR formation is demonstrated. Afterward, the imaging method is described in detail. The motion compensation technology and the idea of partitioned equivalent velocity are adopted to solve the problems of nonzero XT baselines and space-variant characteristics, respectively. Finally, the simulations of point targets and distributed targets are carried out to verify the proposed method, and the results show that the precise focusing of spaceborne cooperative M-SAR with nonzero XT baselines can be achieved by the proposed imaging method.

Index Terms—M-SAR formations, M-SAR imaging processing, multistatic synthetic aperture radar (M-SAR), nonzero cross-track (XT) baselines.

I. INTRODUCTION

Spaceborne multistatic synthetic aperture radar (SAR) is an extension of the concept of bistatic SAR (BiSAR), which is characterized by deploying multiple physically separate transceivers to obtain three or more simultaneous echo datasets [1], [2]. Multistatic SAR (M-SAR) has many unique characteristics that meet the application requirements of the next-generation spaceborne SAR system, such as high-resolution wide-swath (HRWS) imaging, along-track (AT) interferometry, cross-track (XT) interferometry, and so on, so it has received extensive attention [3], [4], [5]. Another trend is adopting cheaper systems with the intention to shift the complexity from the space segment to software. Thus, small satellites which received echoes only can be deployed in the formation of M-SAR systems to provide equivalent or superior performance compared to the state-of-the-art SAR systems currently with a minor cost and higher robustness toward failure [6].

The processing methods of spaceborne M-SAR data have been studied by several scholars, and the main purpose is to use the M-SAR data to overcome the contradiction between the swath width and the azimuth resolution of the conventional SAR systems. The reconstruction method proposed in [7] can be used in the case of AT displaced receivers, and Sakar et al. proposed a reconstruction method for M-SAR formation with large AT baselines [8], [9]. In [10], a reconstruction scheme in the 2-D frequency domain is introduced, which is also aimed to the case of AT baselines. However, the XT separation between the spacecraft may be intentional when the spacecrafts are operating in cooperative formations, so that along-track drifts can be tolerated without increasing the collision risk. More importantly, it can form a single-pass XT interferometry system, which has broad application prospects. Unfortunately, a large difference of the slant range history between secondary satellites is produced due to XT baselines, so the existence of XT baselines will bring difficulties to the subsequent imaging processing. The most troublesome problem is that the additional azimuth modulation phase introduced by XT baselines will make the existing reconstruction methods used only for AT direction failure. Some scholars have put forward some ideas to solve this problem. Dogan et al. [11] provided a processing method in the case of nonzero XT baselines, but the derivation of the method is carried out under the assumption of flat earth. In [12], the errors introduced by XT baselines were modeled as channel mismatches, and a compensation method is proposed. However, the model established in [12] is based on the traditional monostatic multichannel system, i.e., receiving apertures are distributed in one platform, so the method proposed in [12] is only practicable for the case where XT baselines are very.
small (less than 10 m). There are other efforts for the problem of nonzero XT baselines, which is mentioned in [10], [13], [14], but no complete solution is given.

For the imaging algorithm, since the cooperative M-SAR system also satisfies the quasi-stationary configuration, so many existing BiSAR imaging algorithms can be considered, such as Loffeld’s bistatic formula (LBF) method [15], [16], extended LBF method [17], [18], series reversion [19], [20], and so on. However, there are still some problems. The different motion vectors of two or more satellites caused by the Helix satellite formation will lead to the difference of the closest slant range and Doppler characteristics at the same range gate. In more general terms, the azimuth modulations are unequal for the primary and secondary satellites due to the different slant ranges and velocities, which is described as the problem of space-variant characteristics in this article. However, the imaging algorithms mentioned above cannot solve the problem of space-variant characteristics well.

In this article, an imaging method for spaceborne cooperative M-SAR formations with nonzero XT baselines is proposed under the assumption that the error of elevation accuracy is zero, and the unambiguous recovery of the M-SAR data and the problem of space-variant characteristics are aimed to be solved. Firstly, remove the extra phase error in the echo data caused by nonzero XT baselines. Since nonzero XT baselines are similar to the motion error, so it is possible to consider applying Motion Compensation (MoCo) technology to compensate for the extra phase error introduced by XT baselines [21], [22]. After that, the case of nonzero XT baselines is converted to the case of zero XT baselines, and many mature reconstruction methods for the data of AT displacements can be used to obtain the unambiguous echo signal. Secondly, an imaging algorithm based on the idea of partitioned equivalent velocity instead of the hyperbolic range history framework is proposed, which refers to the quasi-monostatic method proposed by Bamler et al. [23], and the range-variant and azimuth-variant equivalent velocity is used to solve the problem of space-variant characteristics in the Bi/M-SAR imaging.

This article is arranged as follows. In Section II, a kind of spaceborne cooperative M-SAR formations with nonzero XT baselines is demonstrated, and the problem brought by nonzero XT baselines is discussed. An imaging method for spaceborne cooperative M-SAR formations with nonzero XT baselines is proposed in Section III. The simulations of point targets and distributed targets are carried out in Section IV to verify the proposed method. Finally, Section V concludes the article.

II. FORMATION AND PROBLEM

Although spaceborne cooperative M-SAR formation with nonzero XT baselines can provide many application potentials that cannot be ignored, it brings some difficulties of signal processing. Therefore, a kind of spaceborne cooperative M-SAR formation is demonstrated firstly, and the coverage capability of this formation is given. Then, the problem caused by nonzero XT baselines is also discussed.

A. Formation Configuration

A conceptual diagram of an M-SAR formation applied to multibaselines interferometry is given in Fig. 1. The formation consists of a primary satellite, which transmits radar signals and three secondary satellites which receive echo signals only. In the proposed M-SAR formation, the three secondary satellites follow the primary satellite in a cooperative formation, and the flexible baselines can be selected between them. In addition, the inherent system limitation of HRWS imaging is overcome by placing the receiving phase centers on separate small satellites. For the convenience of expression, $Sat_0$ is used to represent the primary satellite, and $Sat_n$ ($n = 1, 2, 3$) is used to denote the $n$th secondary satellite.

The Helix formation can be adopted in the proposed formation, which uses the double-Helix formation of TanDEM-X for [24]. The coverage capability of the proposed formation is analyzed in Fig. 2. The specific configurations of the low-latitude formation are taken as an example, and the six orbital elements of the proposed M-SAR formation are shown in Table I. Three sets of configurations are formed by the coordination of baselines between the $Sat_0$ and $Sat_n$, and they are represented by the lines of three colors in Fig. 2.

B. Problem

Usually, the control in XT baselines has to take into account the topographic variations within the footprint, since topography
SIX ORBITAL ELEMENTS OF THE PROPOSED M-SAR FORMATION. a, e, I, ω, and Ω REPRESENT THE SEMIMAJOR AXIS, ECCENTRICITY, ARGUMENT OF PERIGEE, RIGHT ASCENSION OF ASCENDING NODE, AND MEAN ANOMALY, RESPECTIVELY.

| Parameters | Sat0 | Sat1 | Sat2 | Sat3 |
|------------|------|------|------|------|
| α (km)     | 7354.4884 | 7354.4884 | 7354.4884 | 7354.4884 |
| ε          | 0.001087 | 0.001051 | 0.001043 | 0.001034 |
| I (°)      | 99.3938  | 99.3938  | 99.3938  | 99.3938  |
| ω (°)      | 10.0000  | 12.385133 | 13.006672 | 13.635838 |
| Ω (°)      | 11.0921  | 11.097391 | 11.098713 | 11.100036 |
| M (°)      | 90.0000  | 87.615731 | 86.994408 | 86.362758 |

"Satn" refers to the primary satellite, and Satn denotes the nth secondary satellite, n ∈ 1, 2, 3.

![Fig. 3](image-url) impacts of the XT baseline bxt in terms of the phase error [25]. (a) The phase error εφ changes with the topographic variation δh and the XT baseline bxt. (b) The phase error εφ changes with and the AASR and the number of satellites Nrx.

The conclusion is that a threshold between 10λ and 20λ covering topographic variations up to 1–2 km is an acceptable compromise when the compensation of nonzero XT baselines is not considered, where λ is carrier wavelength. Otherwise, the azimuth ambiguity level of the reconstructed signal will raise dramatically. However, the applications potential of too short XT baselines (5 m in L-band and 0.6 m in X-band) is limited in the M-SAR system used for the multibaselines interferometry. Thus, the problem to be considered is how to compensate for the extra phase error introduced by XT baselines when they are further increased. For the proposed formation in this article, phase errors caused by the difference of the slant range history due to nonzero XT baselines are shown in Fig. 4. Obviously, larger XT baselines will produce nonnegligible phase errors, which causes the reconstruction process to fail. Therefore, the method proposed in this article can eliminate these phase errors to achieve fine focusing of spaceborne cooperative M-SAR formations with nonzero XT baselines.

III. DATA PROCESSING

In this section, an imaging method for spaceborne cooperative M-SAR formations with nonzero XT baselines is proposed, and the diagram of the data processing is shown in Fig. 5.

A. Preprocessing

Two tasks, i.e., synchronization phase error compensation and platform mismatch calibration, are needed to be completed in the preprocessing step.

1) Synchronization Phase Error Compensation: The independent oscillators are used in the transmitter and receivers, and any deviation between the oscillators will cause a residual modulation of the recorded SAR raw data [27], [28]. An advanced noninterrupted synchronization scheme is adopted in the LuTan-1 (LT-1) mission [29], and the LT-1 is a spaceborne BiSAR mission. For the spaceborne cooperative M-SAR system, the synchronization scheme used in the current BiSAR system can be used for reference. Pulse compression is firstly performed to the synchronization signals and the peak phases are extracted in the compressed synchronization signals. Then, orbit parameters are used to correct the Doppler effects and the relativistic effect [30]. Afterward, the coarse-compensation phase can be
Firstly, the phase center \( \phi_{n,m} \) and \( \tau_{n,m} \) denote averaging operation along the azimuth and range time, respectively; \( m \) and the range \( \eta \) are the constant phase error and the range \( \tau \) when there is no XT baseline, as shown \( \eta, \tau \) is the internal calibration phase. Further, the high-accuracy compensation phase \( \phi_{\text{syn},n} \) can be obtained through the Kalman filter and interpolation \[31\]. Finally, compensate \( \phi_{\text{syn},n} \) into the echo signals of the \( n \)th secondary satellite.

**B. Platform Mismatch Calibration**

The receiving links of different secondary satellites are also independent, and the platform mismatch error caused by the hardware link and atmosphere must be considered \[32\], \[33\]. A method called azimuth cross correlation can be used in calibration \[34\] and the excellent effect can be achieved. Assume that the echo signals of every secondary satellite after the synchronization phase error compensation is \( \phi_{n,m} \) and the range sampling time delay \( \Delta \tau_{m} \) can be obtained as follows \[34\]:

\[
\phi_{m} = \arg \left( E_{a} \left[ S_{n}^{*} (\eta, f_{\tau}) \cdot S_{m} (\eta, f_{\tau}) \right] \right) - f_{\tau} \partial \left( \arg \left( E_{a} \left[ S_{n}^{*} (\eta, f_{\tau}) \cdot S_{m} (\eta, f_{\tau}) \right] \right) \right) / \partial f_{\tau}
\]

\[
\Delta \tau_{m} = 1 / 2 \pi \partial \left( \arg \left( E_{a} \left[ S_{n}^{*} (\eta, f_{\tau}) \cdot S_{m} (\eta, f_{\tau}) \right] \right) \right) / \partial f_{\tau}
\]

where \( m \) refers to subscripts of other secondary satellites other than the reference secondary satellite, and \( m = 2, 3 \) in this article; \( \eta \) and \( \tau \) are the range and azimuth time, respectively; \( f_{\tau} \) represents the range frequency; \( \Delta b_{\text{XT},m} \) is the phase center distance between the \( m \)th secondary satellite and the reference secondary satellite; \( v_{x,m} \) is the velocity of the \( m \)th secondary satellite; \( \phi_{m} \) and \( \tau_{m} \) are the constant phase error and the range sampling time delay, respectively, which is the error that needs to be estimated and compensated.

Perform the operation of azimuth cross correlation to the \( m \) echo signals, and the constant phase error \( \phi_{m} \) and the range sampling time delay \( \Delta \tau_{m} \) can be obtained as follows \[34\]:

\[
\phi_{m} = \frac{1}{2} (\phi_{0,n} - \phi_{n,0}) + \phi_{\text{cal}}
\]

where \( \phi_{0,n} \) and \( \phi_{n,0} \) are the peak phases of the synchronization signals from the primary satellite and the \( n \)th secondary satellite, respectively; \( \phi_{\text{cal}} \) is the internal calibration phase. Further, the high-accuracy compensation phase \( \phi_{\text{syn},n} \) can be obtained through the Kalman filter and interpolation \[31\]. Finally, compensate \( \phi_{\text{syn},n} \) into the echo signals of the \( n \)th secondary satellite.

**C. Nonzero XT Baselines Compensation**

Nonzero XT baselines compensation based on MoCo technology is the core step of the proposed method. After the phase error caused by nonzero XT baselines in the echo signal is eliminated, the relevant azimuth reconstruction algorithm can be used to obtain the unambiguous echo signal.

1) **Calculate Virtual-Zero-Cross (VZC) Positions**: Firstly, calculate the VZC positions of all secondary satellites. The so-called VZC positions refer to the positions of the \( Sat_{n} \) relative to the \( Sat_{0} \) when there is no XT baseline, as shown in Fig. 6. Suppose coordinates of the \( Sat_{0} \) and every \( Sat_{n} \) are \( \vec{p}_{Sat_{0}}(\eta) \) and \( \vec{p}_{Sat_{n}}(\eta) \), respectively. Calculate the length of AT baselines from the every \( Sat_{n} \) to the \( Sat_{0} \), and denote them as \( b_{\text{XT},n}(\eta) \). It should be noted that the calculation of baselines may be deviated, and we will estimate this deviation and correct it in the subsequent step.

Then, the Range-Doppler (RD) geolocation algorithm \[35\] is used to obtain ground aiming point (GAP) coordinates and
zero-Doppler (ZD) vectors of the \( Sat_0 \). Assuming that GAP coordinates are represented as \( \hat{p}_{n,k} \), where \( \eta \) indicates the \( \eta \)-th azimuth moment and \( k \) represents the \( k \)-th ZD vector in this period. Thus, the ZD vector at the \( \eta \)-th azimuth moment corresponding to \( \hat{p}_{n,k} \) can be calculated as \( \hat{r}_{ZD,n,k} (\eta) = \hat{p}_{Sat_0}(\eta) - \hat{p}_{n,k} \). At any azimuth moment, the cross product of any two ZD vectors is performed to obtain the forward vector \( \hat{r}_{f,0}(\eta) \) of the \( Sat_0 \) at this moment. Thus, the VZC positions of the every \( Sat_n \) relative to the \( Sat_0 \) can be calculated as

\[
\hat{p}_{VZC,n}(\hat{\eta}) = \hat{p}_{Sat_0}(\eta) + \hat{r}_{f,0}(\eta) \cdot \hat{B}_{XT,n}(\eta)
\]

where \( \hat{\eta} \) indicates that there may be deviation in azimuth direction because the calculated VZC positions may deviate from the ideal case.

2) First-Order Error Compensation: The slant range error specific to the scene center between the every \( Sat_n \) and the corresponding VZC position is compensated in the first-order error compensation. Assuming that the coordinate of the scene center is \( \hat{p}_{0} \), and the slant range error can be calculated as

\[
\Delta R_{0,n}(\hat{\eta}) = |\hat{p}_{VZC,n}(\hat{\eta}) - \hat{p}_{0}| - |\hat{p}_{Sat_n}(\eta) - \hat{p}_{0}|
\]

Thus, first-order phase error compensation for the received echoes is performed as

\[
s_n(\tau, \hat{\eta}) = s_n(\tau, \eta) \cdot \exp \left[ j4\pi \frac{\Delta R_{0,n}(\hat{\eta})}{\lambda} \right]
\]

where \( s_n(\tau, \eta) \) is the echo signals after preprocessing. Then, the range resampling is performed as

\[
s_n(\tau, \hat{\eta}) = \text{IFFT}_r \left[ \text{FFT}_r \left[ s_n(\tau, \hat{\eta}) \right] \cdot \exp \left( j4\pi \frac{\Delta R_{0,n}(\hat{\eta})}{c} f_r \right) \right]
\]

where \( c \) is speed of light; \( f_r \) represents the range frequency; \( \text{FFT}_r[\cdot] \) and \( \text{IFFT}_r[\cdot] \) refer to Fast Fourier Transform and Inverse Transform in range direction.

3) Second-Order Error Compensation: The residual slant range error specific to all sampling points in the scene is compensated in the second-order error compensation. Firstly, pulse compression in range direction is performed using the matched filter \( H_r(f_r) \) as follows:

\[
H_r(f_r) = \exp \left( j\pi \frac{f_r^2}{K_r} \right)
\]

\[
s_n(\tau, \hat{\eta}) = \text{IFFT}_r \left[ \text{FFT}_r \left[ s_n(\tau, \hat{\eta}) \right] \cdot H_r(f_r) \right]
\]

where \( K_r \) is range frequency-modulated rate. Secondly, divide the range directions into \( K \) segments. Use the RD geolocation algorithm to calculate the GAP coordinates and ZD vectors in \( k \)-th range segment of the \( Sat_0 \) in every azimuth moment in combination with Digital Elevation Model, where \( k \) is \( 1,2,\ldots,K \). The slant range error between every \( Sat_n \) and the corresponding VZC position relative to GAP coordinates \( \hat{p}_{n,k} \) can be calculated as

\[
\Delta R_{k,n}(\hat{\eta}) = |\hat{p}_{VZC,n}(\hat{\eta}) - \hat{p}_{n,k}| - |\hat{p}_{Sat_n}(\eta) - \hat{p}_{n,k}|
\]

Thus, the residual slant range error can be obtained as

\[
\delta R_{k,n}(\hat{\eta}) = \Delta R_{k,n}(\hat{\eta}) - \Delta R_{0,n}(\hat{\eta})
\]

Then, the interpolation along the range direction is carried out to obtain the residual slant range error \( \delta R_{n}(\hat{\eta}) \) of all sampling points in the scene, and phase error compensation is performed as follows:

\[
s_n(\tau, \hat{\eta}) = s_n(\tau, \hat{\eta}) \cdot \exp \left( j4\pi \frac{\delta R_{n}(\hat{\eta})}{\lambda} \right)
\]

4) Azimuth Resampling: As shown in Fig. 6, there may be deviations between calculated VZC positions and ideal VZC positions in azimuth direction, which is caused by the inaccurate calculation of AT baselines \( \hat{B}_{XT,n}(\eta) \). These deviations can be estimated by an optimization problem. The coordinate of \( Sat_0 \) (precision), AT baselines (deviation), and XT baselines (deviation) are used to estimate the coordinates of \( Sat_n \), and the distance between the estimated and real coordinates of \( Sat_n \) can be calculated. When the distance is small enough, it can be considered that the estimated coordinates of \( Sat_n \) are precise enough, i.e., the estimated baselines are precise enough. Thus, we can let \( \hat{B}_n(\eta) = \{ \hat{B}_{XT,n}(\eta) \} \) be the estimated baselines, and the distance \( D[\hat{B}_n(\eta)] \) between the estimated and real coordinates of \( Sat_n \) is chosen as the criterion for optimization.

The forward vector \( \hat{r}_{f,0}(\eta) \) of the \( Sat_0 \) and the forward vectors \( \hat{r}_{f,n}(\eta) \) of the \( Sat_n \) are used to perform the cross product, and the vectors \( \hat{r}_{f,n}(\eta) \) along the XT baselines direction can be obtained. Thus, the estimated coordinates of \( Sat_n \) can be calculated as follows:

\[
\hat{p}_{Sat_n}(\hat{\eta}) = \hat{p}_{VZC,n}(\hat{\eta}) + \hat{r}_{f,n}(\eta) \cdot \hat{B}_{XT,n}(\eta)
\]

where \( \hat{p}_{VZC,n}(\hat{\eta}) \) can be obtained according to (6). Thus, the distance between the estimated and real coordinates of \( Sat_n \) is given as follows:

\[
D[\hat{B}_n(\eta)] = ||\hat{p}_{Sat_n}(\hat{\eta}) - \hat{p}_{Sat_n}(\eta)||
\]

where \( || \cdot || \) denotes the module of a vector. Therefore, the optimal estimation of baselines is

\[
\hat{B}_n(\eta) = \arg \min_{B_n(\eta)} D[\hat{B}_n(\eta)].
\]

Some optimal methods, such as steepest descent method and Newton method, have been studied to solve the unconstrained optimization problem in (17). The gradient descent method (GDM) is used here as an example. The optimal estimated baselines \( \hat{B}_{n,opt}(\eta) \) are obtained when the last iteration is done. Based on GDM, the steps for the optimization problem are given as follows [36]:

Step 1 (Initialization): Given initial values of baselines \( \hat{B}_{n,0}(\eta) \)

Step 2: Calculate the search direction. \( \Delta \hat{B}_{n,j}(\eta) = \nabla D[\hat{B}_{n,j}(\eta)] \).

Step 3 (Linear search): Choose step size \( t \) via exact or backtracking line search.

Step 4 (Update): Calculate \( \hat{B}_{n,j+1}(\eta) = \hat{B}_{n,j}(\eta) + t \cdot \Delta \hat{B}_{n,j}(\eta) \).
Step 5: If the stopping criterion is satisfied, stop; otherwise, return to step 2.

The $\nabla D(\hat{B}_n(\eta))$ can be expressed as

$$\nabla D \left[ \hat{B}_n(\eta) \right] = \begin{bmatrix} \frac{\partial D}{\partial B_{AT,n}(\eta)} + v_{s,n} \frac{\partial D}{\partial B_{XT,n}(\eta)} \end{bmatrix}. \quad (18)$$

Thus, the deviations between the calculated and ideal VZC positions can be calculated as follows:

$$\Delta \hat{B}_{AT,n}(\eta) = \hat{B}_{AT,n,opt}(\eta) - \hat{B}_{AT,n}(\eta) \quad (19)$$

where $\hat{B}_{AT,n,opt}(\eta)$ is obtained from $\hat{B}_{n,opt}(\eta)$. Thus, the sinc interpolation is used in azimuth resampling to compensate deviations of VZC positions in azimuth direction.

D. Reconstruction Processing

After eliminating the extra phase error introduced by the nonzero XT baseline, the physical meaning of signal acquisition is transformed into the case that acquires echoes along only the AT direction, and the spacing of the receiving phase centers is the length of AT baselines between the $Sat_n$ and the $Sat_0$. Select the $Sat_1$ as the reference secondary satellite in this article, which is similar to the reference receive channel in the multichannel SAR system, and $Sat_m$ refers to other secondary satellites other than the reference secondary satellite. Thus, the traditional reconstruction method based on the filter bank [7] is adopted in this article.

The reconstruction filter $P[f_\eta; \Delta x_n(\eta)]$ is adopted as follows:

$$P[f_\eta; \Delta x_n(\eta)] = H^{-1}[f_\eta; \Delta x_n(\eta)] \quad (20)$$

where $\Delta x_n(\eta)$ represents the spacing between the $Sat_n$ and the $Sat_1$. Besides, the elements in the prefiler $H[f_\eta; \Delta x_n(\eta)]$ can be expressed as follows [7]:

$$H_n[f_\eta; \Delta x_n(\eta)] = \exp \left[ -j\pi \frac{\Delta x_n(\eta)}{v_{s,n}} f_\eta \right] \quad (21)$$

where $v_{s,n}$ is the velocity of the $n$th secondary satellite; $f_\eta$ refers to the azimuth frequency.

E. Imaging Processing

After reconstruction processing, the unambiguous echo signal is obtained. Considering the approximation of the quasi-monostatic configuration is still satisfied in the cooperative M-SAR formation; the imaging method proposed by Bamler et al. for BiSAR imaging can be used for [23]. Thus, the imaging geometry of the cooperative M-SAR formation can be considered as the geometry between the $Sat_0$ and the reference secondary satellite $Sat_1$.

The equivalent velocity $v_e$ is the most important parameter in the linear track case. For moderate bistatic configurations, the slant range history can be approximated when some assumptions are satisfied [23]

$$R(\eta) \approx \hat{R}(\eta) = \sqrt{R_0^2 + \frac{v_e^2}{2}} \quad (22)$$

where $R_0$ is the closest slant range. For the sake of accommodating orbit curvature, a solution of the equivalent velocity is given as follows [37]:

$$v_e = \sqrt{R_0 \cdot \hat{R}(\eta)}. \quad (23)$$

This solution is optimal for the center of the aperture but degenerates toward its boundaries. Thus, the best approximation is to use the least mean square error fit over the entire aperture time [23], [37]. In the imaging algorithm proposed in this article, the range-variant and azimuth-variant equivalent velocity, i.e., partitioned equivalent velocity, is adopted, so as to achieve fine focusing of the entire scene including edge points.

Thus, the slant range history of the reference secondary satellite $Sat_1$ corresponding to the reference point of the scene can be given by

$$R_{lb}(\eta) = |\vec{p}_{Sat_0}(\eta) - \vec{p}_0| + |\vec{p}_{Sat_1}(\eta) - \vec{p}_0|. \quad (24)$$

Using the approximation of quasi-monostatic, the history given in (24) can be expressed as

$$R_{mono}(\eta) = \frac{R_{lb}(\eta)}{2} \simeq \sqrt{R_{0,ref}^2 + v_{e,ref}^2 (\eta - \eta_0)^2} \quad (25)$$

where the subscript ref refers to “reference.” Using the method of minimum mean square error to fit $[R_{lb}(\eta)/2]^2$, the closest slant range $R_{0,ref}$ and the equivalent velocity $v_{e,ref}$ of the reference point of the scene can be obtained.

The first step of the imaging processing is bulk range processing applied in 2-D frequency domain, which includes range cell migration correction (RCMC), secondary range compression (SRC), and the compensation of all higher-order phase terms for all points located at the reference range [23]. The transfer function is given as follows [23]:

$$H_R(f_\eta, f_r; R_{0,ref})$$

$$= \exp \left( -j \frac{4\pi}{K_r} R_{0,ref} \cdot \left[ \frac{(f_0 + f_r)^2 - \frac{c^2 f_n^2}{4v_{e,ref}^2}}{2} \right] + \sqrt{1 - \frac{c^2 f_n^2}{4v_{e,ref}^2}} - j \pi \frac{f_n^2}{K_r} \right). \quad (26)$$

where $f_0$ is the carrier frequency.

Afterward, the differential RCMC (DRCMC) is performed in the RD domain to correct the residual range cell migration (RCM) of targets at ranges of $R_{0,i} \neq R_{0,ref}$ by sinc interpolation, as well as the space-variant correction in range direction [23]

$$\delta R(f_\eta; R_{0,i}, v_{e,i}) = \Delta R(f_\eta; R_{0,i}, v_{e,i})$$

$$- \Delta R(f_\eta; R_{0,ref}, v_{e,ref}) \quad (27)$$

where $\Delta R(f_\eta; R_{0,i}, v_{e,i})$ is the RCM at the $i$th range gate, which $i = 1, 2, \ldots, N_r$, and $N_r$ is the sampling numbers in range;
\( \Delta R(f_n; R_{0,ref}, v_{e,ref}) \) is the RCM at the reference range gate; and the specific form is expressed as follows:

\[
\Delta R(f_n; R_{0,i}, v_e) = \frac{R_{0,i}}{D(f_n; v_e)} - R_{0,i}
\]  

(28)

where \( D(f_n; v_e) \) is the migration factor in the RD domain, and it is given by

\[
D(f_n; v_e) = \sqrt{1 - \frac{\omega^2 f_n^2}{4v_e^2}}
\]

(29)

where \( v_{e,i} \) represents the equivalent velocity corresponding to the \( i \)th range gate.

In the case of wide swath, the residual high-order phase errors caused by bulk range processing cannot be ignored. Further, the residual space-variant characteristics in range direction are considered in order to achieve the high phase-preserve imaging. Therefore, the signal of the RD domain after DRCMC is divided into \( N \) blocks along the range direction and transformed into 2-D frequency domain. Then, a range-variant transfer function is given by

\[
H_{src,var}(f_n, f_r; R_{0,i}, v_{e,i}) = \exp \left( -j \frac{4\pi}{c} \frac{(R_{0,i} - R_{0,ref})}{f_n} \right) \left[ 1 - D(f_n; v_{e,i})^2 \frac{f_n^2}{2!f_0D(f_n; v_{e,i})^2} \right] \left[ 1 - D(f_n; v_{e,i})^2 \frac{f_n^2}{3!f_0^2D(f_n; v_{e,i})^2} \right] + \Phi_{high}
\]

(30)

where \( \Phi_{high} \) refers to high-order residual phase; \( n = 1, 2, \ldots, N; R_{0,i} \) and \( v_{e,i} \) are the closest slant range and the equivalent velocity corresponding to each block of the signal. Use (30) to complete the compensation of each block of the signal in the 2-D frequency domain. Then, transform each block of the signal into the RD domain for splicing. It should be noted that the purpose of segmenting the signal in range direction is to compensate the residual phase error after the bulk range processing, and will not cause the difference in imaging quality of each block. After the above operation, the space-variant characteristic in range direction can be corrected through the range-variant equivalent velocity.

Finally, the imaging processing is azimuth compression, which the space-variant characteristic correction in azimuth direction is also performed in this step. The signal in the RD domain is divided into \( M \) blocks along the azimuth direction, and the azimuth matched filter of each block is constructed according to the equivalent velocity \( v_{e,m} \) and closest slant range \( R_{0,m} \) of the reference range gate corresponding to the each block

\[
H_{AC,var}(f_n; R_{0,m}, v_{e,m}) = \exp \left[ -j \frac{4\pi R_{0,m}(1 - D(f_n; v_{e,m}))}{\lambda} \right]
\]

(31)

where \( m = 1, 2, \ldots, M \). Use the azimuth matched filters of (31) to perform \( M \) times matched filtering on the overall signal in azimuth, i.e., multiple the signal and the matched filter in the RD domain and then transform them into the azimuth time domain to obtain all imaging results after matched filtering. In the azimuth time domain, the images focused by each filters are extracted and spliced according to the segmentation strategy to obtain a complete focusing image. So far, the 2-D space-variant characteristics are compensated, and all pixels in the scene (including edge points) can achieve high phase-preserve fine focusing.

IV. SIMULATION

In order to demonstrate the validity of the proposed method, the simulations of point targets and distributed targets are carried out in this section. An array composed of nine point targets is adopted in the simulation, which are laid on a 100 km \( \times \) 100 km grid in ground range/azimuth directions, and the geometry of the designed scene is presented in Fig. 7. P5 is the center of the scene (which is also the reference point target). In order to evaluate the focusing effect of the imaging algorithm on the edge of aperture, point targets of P1, P5, and P9 are chosen to evaluate the focusing effect of the imaging algorithm on the scene.

TABLE II

| Parameters                   | Value       |
|------------------------------|-------------|
| Satellite height             | 984 km      |
| Satellite velocity           | 7.45 km/s   |
| Carrier frequency            | 9.6 GHz     |
| Pulsetwidth                  | 50 \( \mu \)s |
| Bandwidth                    | 300 MHz     |
| Sampling rate                | 360 MHz     |
| Pulse repetition frequency   | 3200 Hz     |
| Numbers of received satellites| 3          |
| \( B_{X,T,1} \)              | 300.25~300.36 m |
| \( B_{X,T,2} \)              | -300.30~300.21 m |
| \( B_{X,T,3} \)              | 300.22~300.32 m |

Fig. 7. Nine simulated point targets which are processed by the proposed method in this article.
Fig. 8. Azimuth spectrum of one point target after performing the azimuth reconstruction algorithm. (a) and (c) The error introduced by nonzero XT baselines is not compensated. (b) and (d) The error introduced by nonzero XT baselines is compensated by the proposed method. The first line is in the 2-D frequency domain (the range and azimuth axis are scaled several times for the convenience of presentation), and the second line is in the 1-D frequency domain.

Fig. 9. A complete imaging result of the azimuth profile of the reference point (P5), and the nonbandlimited real antenna patterns in azimuth direction is considered. (a) The result of simulated single satellite case. (b) The result processed by the proposed scheme after merging simulated three satellites’ echoes.

Fig. 10. Imaging result of distributed targets processed by the proposed method. The selected position is the coast along the northeast of China (42.16° N 130.22° E).

The suppression effect of the proposed method on the azimuth ambiguity is shown in Fig. 9, and the nonbandlimited real antenna patterns in azimuth direction is considered here. Obvious ghosts appear in the simulated single satellite case due to the undersampling in azimuth direction, as shown in Fig. 9(a). The result processed by the proposed method after merging simulated three satellites’ echoes is shown in Fig. 9(b). The performance of the azimuth ambiguity is greatly improved, and the intensity of ghosts is weakened by about −30 dB. In fact, the azimuth ambiguity is worse when the traditional filter bank method is used to deal with the case of large AT baselines. This problem is not the focus of this article, and the methods proposed in [8], [25] can be used to solve the reconstruction problem of large AT baselines.

Finally, the simulation of distributed targets is carried out to verify the performance of the proposed method, and a real SAR image that is extracted by GaoFen-3 is used as the target radar cross-sectional information for simulation purpose. The selected position is the coast along the northeast of China (42.16°N 130.22°E), and the imaging result of distributed targets
Fig. 11. Imaging results of point targets processed by the proposed method. Contour plots (the first line), azimuth profiles (the second line) and range profiles (the third line) of P1 (the first column), P5 (the second column) and P9 (the third column).

TABLE III

| Targets | IRW(m) | PSLR(dB) | ISLR(dB) | IRW(m) | PSLR(dB) | ISLR(dB) | Residual Phase Error |
|---------|--------|----------|----------|--------|----------|----------|----------------------|
| Theoretical | 0.992  | -13.26   | -9.80    | 0.443  | -13.26   | -9.80    | 0°                   |
| P1      | 0.995  | -13.28   | -9.95    | 0.444  | -13.27   | -9.91    | 0.0545°             |
| P5      | 0.994  | -13.27   | -9.94    | 0.444  | -13.27   | -9.88    | 0.0423°             |
| P9      | 0.995  | -13.28   | -9.95    | 0.445  | -13.26   | -9.90    | 0.0625°             |

processed by the proposed method is shown in Fig. 10. The fine imaging of the M-SAR formation with nonzero XT baselines can be achieved by the proposed method, and the good suppression of the ambiguities is also achieved.

V. CONCLUSION

In the M-SAR formation, AT baselines are necessary to achieve the HRWS, and the XT separation may be existed due to orbital safety and application potentials. Thus, how to deal with the problem of nonzero XT baselines for the signal processing of M-SAR needs to be considered urgently. On the other hand, different motion vectors of multiple satellites caused by the Helix formation will lead to the difference of the closest slant range and Doppler characteristics at the same range gate. Thus, space-variant characteristics pose challenge to the focusing and phase-preserving of the Bi/M-SAR imaging, which needs to be solved in the imaging algorithm. Firstly, an M-SAR formation which consists of one transmitting primary satellite and three receiving only secondary satellites are demonstrated in this article. Afterward, an imaging method for spaceborne cooperative M-SAR formations with nonzero XT baselines is proposed, which is aimed to solve the extra error introduced by nonzero XT baselines and the problem of space-variant characteristics.
Finally, the simulations of point targets and distributed targets are carried out to verify the proposed method, and the results demonstrate the effectiveness of the proposed method.

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