Abstract. We present two rather differently based predictions for the quark and lepton spectrum: One provides a rather successful fit to the mass suppressions—the well known fermion mass hierarchy—interpreted as due to most mass terms needing to violate approximately conserved quantum numbers corresponding to the AGUT group $\text{SMG}^3 \times U(1)_f$. This is actually, under certain conditions, the maximal group transforming the known 45 Weyl components of the quark and leptons into each other. From the fit to the fermion spectrum, we get a picture of the series of Higgs fields causing the breakdown (presumably at the Planck scale) of this AGUT to the Standard Model and, thus, providing the small masses of all quarks and leptons except for the top quark. We separately predict the top quark mass to be $173 \pm 5$ GeV and the Higgs mass to be $135 \pm 9$ GeV, from the assumption that there be two degenerate minima in the effective potential for the Weinberg Salam Higgs field with the second one at the Planck field strength.

I INTRODUCTION

All of the charged fermion masses, apart from the top quark, are suppressed relative to the electroweak scale. Indeed the most striking feature of their spectrum is the hierarchy of masses and mixing matrix elements: the masses range over five orders of magnitude, from $1/2$ MeV for the electron to 175 GeV for the top quark. The most promising way of explaining this hierarchy is in terms of mass suppression factors, due to the partial conservation of some chiral flavour quantum numbers beyond the Standard Model (SM). In section III we consider a model [1] based on the Anti-Grand Unified Theory (AGUT) gauge group $\text{SMG}^3 \times U(1)_f$. This means that, near the Planck scale $M_{\text{Planck}} \simeq 10^{19}$ GeV, each quark-lepton generation has its own set of gauge fields quite analogous to those of the Standard Model Group $\text{SMG} = SU(3) \times SU(2) \times U(1)$. In addition there is an extra abelian gauge group called $U(1)_f$. We shall characterize the $\text{SMG}^3 \times U(1)_f$ group as the maximal AGUT group, satisfying some relatively simple assumptions, in section II. This gauge
group is supposed to break down to the Standard Model Group $SMG$, as the
diagonal subgroup of $SMG^3$, about one order of magnitude below $M_{Planck}$.
The vacuum expectation values (VEVs), measured in units of $M_{Planck}$, of the
Higgs fields responsible for the breakdown provide the required fermion mass
suppression factors.

In section IV we present a precise determination of the top quark and Higgs
boson masses within the pure SM, based on the principle of degenerate vacua
and a strongly first order phase transition \cite{2}. The values of the Yukawa
coupling constant $g_t$ and the Higgs self-coupling $\lambda$ then take on fine-tuned values,
rather analogous to a mixture of ice and water taking on a fine-tuned tem-
perature equal to zero degrees Celsius. In winter one often finds a mixture—
slush—of ice and water. This is due to the large latent heat of water giving a
strongly first order phase transition between ice and water. We assume that
the SM is valid up close to the Planck scale; the strongly first order phase
transition condition then implies that the SM effective Higgs potential should
have two degenerate minima, with the corresponding difference in Higgs field
VEVs being of order $M_{Planck}$.

II THE MAXIMAL GROUP

The $SMG^3 \times U(1)_f$ group, with its 37 generators, at first seems a rather ar-
bbitrary choice for a “unified group”. However it can be characterized uniquely
as the gauge group $G$ beyond the SM (i.e. having SMG as a subgroup) satis-
ifying the following 4 postulates:

1. $G \subseteq U(45)$. Here $U(45)$ is the group of all unitary transformations of the
45 species of Weyl fields (3 generations with 15 in each) in the SM.

2. No anomalies. There should be neither gauge anomalies nor mixed
anomalies. We assume that only straightforward anomaly cancellation
takes place and, as in the SM itself, do not allow for a Green-Schwarz
type anomaly cancellation \cite{3}.

3. The various irreducible representations of Weyl fields for the SMG remain
irreducible under $G$. This postulate is motivated by the observation that
combining SM irreducible representations into larger unified representa-
tions introduces symmetry relations between Yukawa coupling constants,
whereas the particle spectrum exhibits a hierarchy between essentially all
the fermion masses rather than exact degeneracies.

4. $G$ is the maximal group satisfying the other 3 postulates.

A rather complicated calculation shows that, modulo permutations of the
various SM fermion irreducible representations, we are led to the result $G =
SMG^3 \times U(1)_f$ with the usual SMG embedded as the diagonal subgroup of
SMG^3. Apart from the various permutations of the particle names, the U(1)_f group is unique. The U(1)_f charges Q_f can then be chosen so that the only non-zero values are carried by the right-handed fermions of the second and third proto-generations:

\[ Q_f(\tau_R) = Q_f(b_R) = Q_f(c_R) = 1 \quad Q_f(\mu_R) = Q_f(d_R) = Q_f(t_R) = -1 \]  

(1)

III THE FERMION MASS SPECTRUM

The Yukawa couplings of the SM fermions to the Weinberg-Salam Higgs field \( \phi_{WS} \) are mainly forbidden by the gauge quantum numbers of the AGUT group. In the AGUT theory, these transitions between the left- and right-handed Weyl fields also involve the Higgs fields responsible for the breakdown of the SMG^3 × U(1)_f group. We assume that all the fundamental couplings of the AGUT theory are of order unity and that there exists a rich spectrum of vector-like fermion states at the Planck scale, which can mediate all the required transitions. Then the gauge quantum numbers of the quark-lepton and Higgs fields determine the combinations of Higgs fields needed to provide non-zero values for the various elements in the effective SM Yukawa coupling matrices \( Y_U \), \( Y_D \) and \( Y_E \) for the up quarks, down quarks and charged leptons respectively. We have the freedom to choose the quantum numbers of the Higgs fields and, guided by phenomenology, we select four Higgs fields S, W, T and \( \xi \), in addition to \( \phi_{WS} \). In fact we specify their U(1) charges and use a natural generalisation of the SM charge quantisation rule to determine their non-abelian representations. The Higgs field S is supposed to have a VEV of order unity in Planck units and, therefore, does not contribute to the fermion mass suppression. In this way we find the following order of magnitude SM Yukawa coupling matrices:

\[
Y_U \simeq \begin{pmatrix}
S^\dagger W^\dagger T^2 (\xi^\dagger)^2 & W^\dagger T^2 \xi & (W^\dagger)^2 T \xi \\
S^\dagger W^\dagger T^2 (\xi^\dagger)^3 & W^\dagger T^2 & (W^\dagger)^2 T \\
S^\dagger (\xi^\dagger)^3 & 1 & W^\dagger T^\dagger
\end{pmatrix}
\]  

(2)

\[
Y_D \simeq \begin{pmatrix}
SW(T^\dagger)^2 \xi^2 & W(T^\dagger)^2 \xi & T^3 \xi \\
SW(T^\dagger)^2 \xi & W(T^\dagger)^2 & T^3 \\
SW^2(T^\dagger)^4 \xi & W^2(T^\dagger)^4 & WT
\end{pmatrix}
\]  

(3)

\[
Y_E \simeq \begin{pmatrix}
SW(T^\dagger)^2 \xi^2 & W(T^\dagger)^2 (\xi^\dagger)^3 & (S^\dagger)^2 WT^4 \xi^\dagger \\
SW(T^\dagger)^2 \xi^5 & W(T^\dagger)^2 & (S^\dagger)^2 WT^4 \xi^2 \\
S^3(W(T^\dagger)^5 \xi^3 & (W^\dagger)^2 T^4 & WT
\end{pmatrix}
\]  

(4)

where the Higgs fields are replaced by their VEVs in Planck units.

The diagonal elements in all 3 Yukawa matrices have the same form, up to complex conjugation, giving the order of magnitude SU(5)-like results \( m_b \approx m_\tau \) and \( m_s \approx m_\mu \), but the off-diagonal elements dominate \( Y_U \) making \( m_t \) and
TABLE 1. Best fit to experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

|               | $M_t$  | $m_h$ | $m_{\tau}$ | $m_c$ | $m_s$ | $m_{\mu}$ |
|---------------|--------|-------|------------|-------|-------|-----------|
| **Fit**       | 192 GeV| 8.3 GeV| 1.27 GeV   | 1.02 GeV| 400 MeV| 88 MeV    |
| **Data**      | 180 GeV| 6.3 GeV| 1.78 GeV   | 1.4 GeV| 200 MeV| 105 MeV   |

|               | $m_u$  | $m_d$  | $m_e$      | $V_{us}$| $V_{cb}$| $V_{ub}$  |
|---------------|--------|--------|------------|--------|--------|----------|
| **Fit**       | 3.6 MeV| 7.0 MeV| 0.87 MeV   | 0.18   | 0.018  | 0.0039   |
| **Data**      | 4 MeV  | 9 MeV  | 0.5 MeV    | 0.22   | 0.041  | 0.0035   |

$m_c$ respectively larger. The diagonal and off-diagonal contributions to the lowest eigenvalue of $Y_D$ are approximately equal, giving $m_d \gtrsim m_u \approx m_e$. A three parameter order of magnitude fit [1], including random complex factors of order unity, with $S = 1$ fixed, $\langle W \rangle = 0.179$, $\langle T \rangle = 0.071$ and $\langle \xi \rangle = 0.099$ successfully reproduces the 9 masses and 3 mixing angles—see Table 1.

IV TOP QUARK AND HIGGS MASSES

We now apply our principle of degenerate vacua and the strongly first order phase transition requirement to the pure SM, with a desert up to the Planck scale [2]. It is well-known that, with loop corrections, the SM effective Higgs potential can have two minima. So we are led to our two crucial assumptions:

a) The two minima in the Standard Model effective Higgs potential are degenerate: $V_{\text{eff}}(\phi_{\text{min}1}) = V_{\text{eff}}(\phi_{\text{min}2})$.

b) The second minimum has a Higgs field squared of the order of unity in Planck units: $\langle |\phi_{\text{min}2}|^2 \rangle = \mathcal{O}(M_{\text{Planck}}^2) \sim (10^{-19} \text{GeV})^2$.

We use the renormalisation group improved tree level effective potential, identifying the renormalisation point with the field strength $\phi$:

$$V_{\text{eff}}(\phi) = \frac{1}{2} m_h^2 (\mu = |\phi|) |\phi|^2 + \frac{1}{8} \lambda (\mu = |\phi|) |\phi|^4$$ (5)

The condition $V_{\text{eff}}(\phi_{\text{min}1}) = V_{\text{eff}}(\phi_{\text{min}2})$, where one of the minima corresponds to our vacuum with $\phi_{\text{min}1} = 246$ GeV, defines (part of) the well-known vacuum stability curve in the $M_t - M_H$ plane, for $\phi_{\text{min}2} < M_{\text{Planck}}$. For values of $\phi$ of order $M_{\text{Planck}}$, the $|\phi|^4$ term dominates the $|\phi|^2$ term and the vacuum degeneracy condition requires $\lambda(\phi_{\text{min}2})$ to vanish. Also the derivative of $V_{\text{eff}}(\phi) \approx \frac{1}{8} \lambda(\phi) |\phi|^4$ should be zero:

$$\frac{dV_{\text{eff}}(\phi)}{d\phi}|_{\phi_{\text{min}2}} = \frac{1}{2} \lambda(\phi) \phi^3 + \frac{1}{8} \frac{d\lambda(\phi)}{d\phi} \phi^4 = \frac{1}{8} \beta_\lambda \phi^3 = 0$$ (6)

and thus the beta function $\beta_\lambda(\phi_{\text{min}2})$ vanishes as well. So we impose the conditions $\beta_\lambda = \lambda = 0$ near the Planck scale and, using the renormalisation
group equations, determine a single point on the SM vacuum stability curve. In this way our two assumptions, illustrated in Figure 1, lead to our predictions for the top quark and Higgs boson pole masses:

\[ M_t = 173 \pm 4 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV}. \]  

(7)

**FIGURE 1.** This symbolic graph of the effective potential \( V_{\text{eff}}(\phi) \) for the Standard Model Higgs field illustrates the two assumptions which lead to our prediction of the top quark and Higgs boson masses: 1) Two equally deep minima, 2) achieved for \(|\phi|\) values differing, order of magnitudewise, by unity in Planck units.

**V CONCLUSION**

We have found a rather good fit, that in principle should only work to order of magnitude accuracy, for the nine charged quark and lepton masses and the three mixing angles; it even fits the CP-violation reasonably well. Three suppression factors were used, indentified in the model with expectation values of three Higgs fields, \( T, W, \) and \( \xi \). There was also a Higgs field \( S \) causing no suppression. Even the overall mass scale could be thought of as being correctly predicted, in as far as it is part of our model that unforbidden Yukawa couplings are of order of magnitude unity and therefore the top quark mass corresponds to the electroweak scale. Since our fit is a priori based on the assumption of the AGUT gauge group, \( SMG^3 \times U(1)_f \), one may at first think that we could take the goodness of our fit as evidence for this gauge group really being realized at some very high energy scale—say the Planck scale, to which we actually extrapolated in our detailed fit. However, first the field \( S \) caused a breakdown of this group to the subgroup \( SMG^2 \times U(1) = SMG_{12} \times SMG_3 \times U(1) \). Thus our model for the fermion masses really only
used the quantum numbers of this subgroup and checked for its presence in
the gauge group. Secondly we did not have to use the non-abelian parts of
the group, but obtained our results using just the U(1) factors. So concluding
back to the relevance of the proposed AGUT group is somewhat doubtfull.

However, we also pointed out that our $SMG^3 \times U(1)_f$ gauge group could
be specified rather simply by means of four suggestive postulates. Thus when
this relatively easy to characterize group, $SMG^3 \times U(1)_f$, turns out to provide
a consistent mass matrix fit, it is rather suggestive after all that it is indeed
the correct gauge group.

An at first rather unrelated calculation gave us the top quark mass with
good accuracy $M_t = 173 \pm 5$ GeV, and not only its order of magnitude as
in the just mentioned AGUT fit. We required the Weinberg-Salam Higgs
potential of the pure Standard Model to have two degenerate minima, one
being at the Planck scale, as in Figure 1. From the same requirement, the
Higgs mass is predicted to have that value which barely allows the stability of
the SM vacuum: $M_H = 135 \pm 9$ GeV.

The two calculations are actually connected, in as far as our fine structure
constant predictions, described in Don Bennett’s talk [4], are based on the
assumptions underlying both calculations: the AGUT gauge group (ignoring
though the $U(1)_f$) and the principle that there shall be degenerate vacua.

It should be stressed that the more precise top quark mass prediction was
based on the assumption that there should be a desert almost up to the Planck
scale. So it would be a falsification of the simplest—and presumably the only
sensible—version of our model, if new physics particles such as, for example,
SUSY particles are found, which would give so strong contributions to the
renormalisation group running for the top and Higgs couplings that it would
disturb our top quark mass prediction.

However, our order of magnitude fit to the quark and lepton spectrum is not
very sensitive to the scale at which our proposed AGUT gauge group should
be found. It is true that we extrapolate to the Planck scale and thereby use
the simple suppression factors to fit the Planck scale Yukawa couplings. This
extrapolation essentially just provides each lepton mass with an extra factor
3 to 4 relative to the quark masses, because the lepton Yukawa couplings are
running less than the quark Yukawas. But a factor 3 to 4 is not sensitively
tested when we only work with orders of magnitude.

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