Optimization of teeth distribution for promote gearbox used in jack-up offshore platforms using improved genetic algorithm

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Abstract
The mathematical model of teeth distribution optimization for the promote gearbox is presented considering the objective functions such as the volume, efficiency, strength difference for the adjacent stages, and the design variables such as module, sun gear teeth, face width and transmission ratio of all stages. The improved genetic algorithm is used to solve the model and an analysis model of the promote gearbox is established using Masta to verify the proposed model. Results show that, after optimization, the attribute value of the synthesized objective function increases by 82.73%, the volume decreases by 17.61%, and the efficiency increases by 0.02%. The objective function value of equal contact and bending strength decreases by 55.93% and 51.10%, respectively. The difference for the safety coefficients for the adjacent stages decreases obviously. The constraint conditions of single-stage and adjacent stages satisfy the requirements. Through the strength verification calculation by the analysis model, the variation trend of the strength before and after optimization is consistent with the proposed optimization model, which indicates that the multi-objective optimization model established in this paper and the improved genetic algorithm selected are correct and effective.

Keywords: Jack-up offshore platforms, Promote gearbox, Teeth distribution optimization, Improved genetic algorithm, Strength difference between adjacent stages

1. Introduction

The jack-up offshore platforms are widely used in offshore oil and gas exploration and development due to the advantages of flexible operation, low construction cost and good movability. At present, the gear-rack promote platform is widely used and the configuration of the promote gearbox mostly adopts multi-stage NGW planetary transmission series. The transmission ratio distribution and teeth distribution will directly affect the volume, carrying capacity and efficiency of the entire promote gearbox. The optimization of that becomes a critical step in the design process of the promote gearbox. Therefore, the teeth distribution optimization analysis of the promote gearbox has important theoretical significance and engineering application value.

Currently, many scholars have done lots of research on teeth distribution based on the macro parameters optimization. Thompson (Thompson et al., 2000) presented the multi-objective optimal design equation applicable to the two and three-stage spur gear reducer, and evaluated the contact fatigue life after volume reduction by taking contact fatigue life and volume as objective functions. Huang (Huang et al., 2005) solved the multi-objective optimization problem in reference of Thompson (Thompson et al., 2000) based on the interactive physical programming, and applied it in the optimum design procedure for the three-stage spur gear reducer. Gologlu (Gologlu et al., 2009) presented a genetic algorithm-based design procedure and performed the volume optimization for helical gear transmission by introducing static and dynamic penalty functions into the objective function to deal with design constraints. Genetic algorithm was applied by Mendi (Mendi et al., 2010) to optimize the transmission system parameters including the design parameters of shafts, gears and bearings. The optimization results were compared with the results by traditional analysis methods to verify the feasibility and reliability of genetic algorithm in the optimization design of gearbox. Qin (Qin et al., 2020) investigated the transient feature extraction by the improved orthogonal matching pursuit and K-SVD algorithm with adaptive transient dictionary. Savsani (Savsani et al., 2010) proposed two advanced optimization algorithms known as particle swarm optimization and
simulated annealing to find the optimal combination of design parameters for minimum weight of a spur gear train. Bozca (Bozca et al., 2010) took the vibration and noise of the 5-speed gearbox as the objective function, and its bending, contact stress and gear center distance as constraint conditions to optimize the geometric parameters of the gearbox and reduce the vibration and noise of the gearbox. Zhu (Zhu et al., 2010) carried out the fuzzy reliability optimization for transmission system of high-power marine gearbox considering the number of pinion teeth, normal module, helix angle, the gear width and the velocity ratio distribution as design variables. Marjanovic (Marjanovic et al., 2012) described the optimization of material selection, transmission ratio distribution and shaft installation positions of spur gear transmission, and gave the specific definition of the optimization mathematical model. Golabi (Golabi et al., 2014) provided the general form of the volume objective function of gearbox and constraint conditions, and optimized the volume for the one, two and three-stages gear transmission. Miler (Miler et al., 2018) used a genetic algorithm to conduct a multi-objective optimization of gear pair parameters with a goal of reducing the transmission volume and power losses. Fu (Fu et al., 2018) proposed a novel meta-heuristics algorithm for solving the multi-objective optimization problem of minimum volume, maximum surface fatigue life and maximum carrying capacity, taking two-stage gear transmission as the object. Wang (Wang et al., 2019) optimized the geometric shape of the helicopter main reducer by using the objective function with the maximum vibration and noise attenuation based on the dynamic model and the multi-objective multivariable genetic algorithm. A multi-objective optimization model of two-stage cylindrical helical gear transmission was established and solved with the volume and power loss as objective functions and pitting and friction as main constraints (Patil et al., 2019). Based on the reliability and the dynamic models, Qin (Qin et al., 2012) developed a multi-objective optimization model to improve the reliability, minimize the vibration and the overall volume of a multi-stage planetary gearing used in shield machine cutter driver. Luo (Luo et al., 2013) proposed a new transmission ratio distribution method to reduce the gear strength difference between the adjacent stages. Zhou (Zhou et al., 2019) established a the completely new calculation method of envelope surface to optimize the conventional approaches to calculate the face gear tooth surface and develop an efficient approach to automatically generate the 3D models of face gears. Wei (Wei et al., 2018) proposed an equal strength optimal design method to assure the minimum volume and safety coefficient difference in the planetary gear train, and calculated the load sharing coefficient of the system based on the kinetics. Mo (Mo et al., 2020) studied the influence mechanism of flexible support on the load sharing characteristic with floating sun gear and established a dynamic model for HPGT to research natural characteristic. Shao (Shao et al., 2018) developed A data-driven optimization model to collaborative manufacturing system considering geometric and physical performances for hypoid gear product.

For the promote gearbox, Lin (Lin et al., 2018) established an analytical coupled dynamic model solved by harmonic balance method for four-stage planetary gearbox considering time-varying mesh characteristics. Xu (Xu et al., 2016) studied how to make the structure more compact and the cost lower by taking the transmission ratio as the optimization objective for the promote gearbox used in the offshore platform. Zhang (Zhang et al., 2015) established the multi-objective mathematical model by taking reliability and lightweight as objective and proposed a method to coordinate reliability and economy after the optimization of the promote gearbox used in jack-up offshore platforms.

For the work mentioned above, the research is mainly focused on the parallel stage gear transmission and two-stage planetary gear transmission. Many researchers regarded the volume, weight, efficiency, reliability and cost of the gearbox as one or more objective functions, and teeth number, module, face width, transmission ratio, pressure angle, meshing angle, etc. as design variables. The constraint conditions were determined according to the working situations. Most optimization models were solved by genetic algorithms. However, few studies have been carried out on the teeth distribution optimization of planetary gear transmissions with three or more stages used for promote gearbox.

On the basis of analyzing the structure and transmission principle of the promote gearbox, a multi-objective optimization mathematical model of teeth distribution for promote gearbox is established with 16 design variables, 4 objective functions and more than 50 design constraint conditions. The variation of objective functions, key constraints and power density before and after optimization are compared and analyzed using improved genetic algorithm. Also, a systematic analysis model is established using Masta to verify the analytical results by the multi-objective optimization mathematical model.

2. Structure and transmission principle analysis of promote gearbox

The jack-up offshore platform investigated here has 4 pile legs and each pile leg has 18 promote gearboxes. The platform has a total of 72 promote gearboxes. The output gears of all promote gearboxes mesh with the racks on the pile
legs to drive the four pile legs downward to the seabed after the platform reaches the working place. At the same time, the platform is driven away from the sea level and up to the pre-load height, then the platform will be locked for pre-load work. The pile legs are released by the locking devices and the platform is driven by promote gearboxes to the working height after pre-load. Finally, the platform carries out the general operation after locking the racks. The promote gearbox adopts four-stage NGW planetary transmission series. The transmission schematic diagram is shown in Fig. 1(a), and the 3D model established by Solidworks is shown in Fig. 1(b). All the gears in the figure are involute spur gears, and the annular gears $A$ are fixed to the housing. The torque $T$ is applied on the sun gear $S_1$ of the 1st stage. Then the planet carriers $H$ are designed as the output. Finally, the output gear shaft driven by the 4th stage planet carrier $H_4$ meshes with the rack on the pile leg to drive the platform promoting or support platform.

![Fig. 1 Transmission system of the promote gearbox](image)

3. The teeth distribution optimization model of promote gearbox

3.1 The teeth distribution optimization mathematical model

In the design process of the promote gearbox, if we consider all the design variables including sun gear tooth number, planet gear tooth number, annular gear tooth number, modulus, face width, meshing angle, transmission ratio, pressure angle, etc., the optimization model will be much too complicated and the computational efficiency will be much lower. So, independent variables such as module $m_i$, face width $b_i$, transmission ratio $u_i$ and sun gear number $z_i$ are taken as design variables in the process of transmission ratio distribution and teeth distribution optimization for the promote gearbox. Considering the concentricity condition for planetary gearing, the number of planet gear teeth $z_{pi}$ and annular gear teeth $z_{ai}$ is

$$
\begin{align*}
    z_{pi} &= \frac{(u_i-2)}{2}z_i, \quad i = 1, 2, 3, 4 \\
    z_{ai} &= (u_i - 1)z_i
\end{align*}
$$

(1)

The design variables are expressed by

$$
X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16})^T
$$

$$
= (m_1, m_2, m_3, m_4, z_1, z_2, z_3, z_4, b_1, b_2, b_3, b_4, u_1, u_2, u_3, u_4)^T
$$

(2)
of planetary gear system of the 1\textsuperscript{st} to 4\textsuperscript{th} stage respectively. $u_1$, $u_2$, $u_3$, $u_4$ are the transmission ratio of planetary gear system of the 1\textsuperscript{st} to 4\textsuperscript{th} stage respectively.

Considering the design requirements of high-power density, high reliability, high efficiency and coordinated sizes design for the adjacent stages of the promote gearbox, the multi-objective function of teeth distribution optimization is established from three aspects of volume, efficiency and strength difference for the adjacent stages.

In terms of the volume objective function, the volume of sun gear, planet gear and annular gear at all stages is considered and the overall volume of the promote gearbox can be calculated by

$$F_1 = \sum_{i=1}^{4}(V_{si} + N_{pi}V_{pi} + V_{ai})$$

Where, $V_{si}$ denotes the volume of the $i$-stage sun gear, $V_{pi}$ denotes the volume of the $i$-stage planet gear, $V_{ai}$ means the volume of the $i$-stage annular gear, $N_{pi}$ denotes the number of $i$-stage planet gear.

The volume of the sun gear, planet gear and annular gear can be obtained by

$$V_{si} = \frac{\pi}{6}m_i^2z_i^2b_i$$
$$V_{pi} = \frac{\pi}{6}m_i^2z_i^2b_i$$
$$V_{ai} = \frac{\pi}{6}m_i^2z_i^2b_i$$

By combining Eqs. (1), (3) and (4), the volume objective function of the promote gearbox is

$$F_1 = \sum_{i=1}^{4}\left(\frac{\pi}{6}m_i^2z_i^2b_i \left[1 + N_{pi}(u_i - 1)^2\right]\right)$$

In terms of equal contact and bending strength, the carrying capacity of multi-stage planetary reducer depends on the strength of the weakest gear, and the weak spot of each stage is the external meshing pair. The waste of carrying capacity is often caused by large gaps in the strength of weak spots at all stages. Therefore, equal strength design is usually regarded as the priority principle in the transmission ratio distribution process of planetary reducer. The minimum differences of the safety coefficients of contact strength and bending strength for adjacent stages are selected as the objective respectively, then

$$S_{Himin} = \text{min}(S_{Hs1}, S_{Hpi})$$
$$S_{Fimin} = \text{min}(S_{Fsi}, S_{FPi})$$
$$f_1 = |S_{H2min} - S_{H1min}|$$
$$f_2 = |S_{H3min} - S_{H2min}|$$
$$f_3 = |S_{H4min} - S_{H3min}|$$
$$f_4 = |S_{FP2min} - S_{FP1min}|$$
$$f_5 = |S_{FP3min} - S_{FP2min}|$$
$$f_6 = |S_{FP4min} - S_{FP3min}|$$

Where $S_{Hsi}$ is contact strength safety coefficient of the $i$-stage sun gear, $S_{Hpi}$ is contact strength safety coefficient of the $i$-stage planet gear, $S_{Fsi}$ is bending strength safety coefficient of the $i$-stage sun gear, $S_{FPi}$ is bending strength safety coefficient of the $i$-stage planet gear, $S_{Himin}$ is the smaller safety coefficient of contact strength for the $i$-stage sun gear and planet gear, $S_{Fimin}$ is the smaller safety coefficient of bending strength for the $i$-stage sun gear and planet gear.

From Eq. (6), the objective function of equal contact strength and equal bending strength of the promote gearbox can be written as

$$F2 = w_1 \times f1 + w_2 \times f2 + w_3 \times f3$$
$$F3 = w_4 \times f4 + w_5 \times f5 + w_6 \times f6$$

The power losses of promote gearbox including the friction loss of gear pairs, friction loss of bearings and hydraulic loss are considered when the efficiency objective function is established. The efficiencies corresponding to the three losses are $\eta_m$, $\eta_n$ and $\eta_s$, respectively. The friction loss of bearings is small and can be neglected. The efficiency of the promote gearbox $\eta$ and the efficiency of the $i$-stage gear pair $\eta_{mi}$ can be calculated by

$$\eta = \prod_{i=1}^{4}\eta_i\eta_{mi}$$
$$\eta_{mi} = 1 - \frac{u_{i-1}}{u_i}\psi_{i}^x$$

Where $u_i$ is the transmission ratio of the $i$-stage, $\psi_{i}^x$ is the power loss coefficient of the $i$-stage transmission mechanism, namely

$$\psi_{i}^x = \psi_{m}^x + \psi_{mai}^x$$

Where $\psi_{m}^x$ denotes the meshing loss between sun gear $S$ and planet gear $P$, $\psi_{mai}^x$ denotes the meshing loss between annular gear $A$ and planet gear $P$. The meshing loss of all external meshing pairs $\psi_{msi}^x$ and all internal meshing pairs $\psi_{mai}^x$ can be calculated by the following equations.
The efficiency objective function of promote gearbox is obtained as

\[ F4 = \prod_{i=1}^{7} \eta_{si} \eta_{mi} = \prod_{i=1}^{7} \eta_{si} \left[ 1 - \frac{u_{i-1}}{u_{i}} (\psi_{msi}^{x} + \psi_{mai}^{x}) \right] \]  \hspace{1cm} (14)

Since the four objective functions expressed in Eq. (5), Eq. (7) and Eq. (14) have different physical meanings and magnitudes, the volume function should be treated accordingly. Then, the four objective functions are matched with the weight as the final synthesized objective function of the teeth distribution optimization for the promote gearbox considering different weighting factors as shown in the following.

\[ f = w_{1} \times F1 + w_{5} \times F2 + w_{7} \times F3 + w_{10} \times F4 \]  \hspace{1cm} (15)

Where \( F1 \) denotes the volume of original design for promote gearbox.

Considering the importance of the volume, efficiency, equal contact strength and equal bending strength, the weighting factors for the multi-objective function are shown in Table 2.

| \( w_{1} \) | \( w_{2} \) | \( w_{3} \) | \( w_{4} \) | \( w_{5} \) | \( w_{6} \) | \( w_{7} \) | \( w_{8} \) | \( w_{9} \) | \( w_{10} \) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| \( \frac{2}{3} \) | \( \frac{2}{3} \) | \( \frac{2}{3} \) | \( \frac{2}{3} \) | \( \frac{2}{3} \) | \( \frac{2}{3} \) | \( \frac{2}{3} \) | \( \frac{6}{6} \) | \( \frac{6}{6} \) | \( \frac{6}{3} \) |

The design constraint conditions of teeth distribution optimization of the promote gearbox are divided into single-stage constraint condition and constraint condition for the adjacent stages. The single-stage constraint condition includes boundary constraints, tooth constraints, strength constraints and contact ratio constraints, and the constraint condition for the adjacent stages is mainly the coordinated sizes design constraints for the adjacent stages.

\[
\begin{align*}
\aleph_{min i} & \leq \aleph_{i} \leq \aleph_{max i} \\
\zeta_{min i} & \leq \zeta_{i} \leq \zeta_{max i} \\
b_{min i} & \leq b_{i} \leq b_{max i} \\
u_{min i} & \leq u_{i} \leq u_{max i}
\end{align*}
\]  \hspace{1cm} (16)

Where \( \aleph_{min i} \) and \( \aleph_{max i} \) are the lower and upper limits of the module \( \aleph_{i} \), respectively. \( \zeta_{min i} \) and \( \zeta_{max i} \) are the lower and upper limits of the tooth number \( \zeta_{i} \) of the sun gear, respectively. \( b_{min i} \) and \( b_{max i} \) are the lower and upper limits of the face width \( b_{i} \), respectively. \( u_{min i} \) and \( u_{max i} \) are the lower and upper limits of the transmission ratio \( u_{i} \), respectively.

In addition to meeting the transmission ratio conditions, the assembly conditions should be satisfied in determining the gear tooth number of planetary gear train, namely concentricity, adjacent and installation conditions. The transmission ratio condition and concentricity condition have been guaranteed when the number of planet gear and annular gear is determined. So, the adjacent and installation conditions, as well as transmission ratio deviation, should also be considered. The range of transmission ratio is ±2% according to the operating requirement of the promote gearbox. The designed transmission ratio \( U_{0} \) is taken as the input condition, and the determined transmission ratio needs to satisfy Eq. (17). In order to ensure that the addendum of adjacent planet gears at all stages do not collide with each other, there is a certain gap between the addendum of planet gears on their connecting lines. The addendum circle diameter of adjacent planet gears should be less than the center distance of adjacent planet gears, that is, the adjacency condition of Eq. (18) should be satisfied. Eq. (19) is the installation condition to ensure that all planet gears are evenly distributed between the two center wheels and meshing well with the center wheels without dislocation.
\[
\frac{|u_2 u_3 u_4 - u_0|}{u_0} \leq 2\%
\] (17)
\[
d_{API} < 2a_{spi} \sin \frac{n}{N_{pi}}
\] (18)
\[
z_i + z_{api} = u_i z_i = C
\] (19)

Where \(d_{API}\) denotes the addendum circle diameter for planet gears, \(a_{spi}\) denotes the center distance between sun and planet gears, \(N_{pi}\) denotes the number of planet gears, \(C\) is an integer.

Generally, the strength of the internal meshing pairs in the planetary gear train is greater than that of the external meshing pairs, so it is enough to guarantee the strength of external meshing pairs. Eq. (21) is the contact ratio condition. In order to ensure the gear transmission stability, the contact ratio of external meshing pairs \(e_{spi}\) and the contact ratio of internal meshing pairs \(e_{pai}\) must be greater than 1, generally greater than or equal to 1.2.

\[
\begin{align*}
S_{HSI} & \geq [S_H] \\
S_{HPI} & \geq [S_H] \\
S_{FSI} & \geq [S_F] \\
S_{FPi} & \geq [S_F] \\
\epsilon_{spi} & \geq 1.2 \\
\epsilon_{pai} & \geq 1.2
\end{align*}
\] (20)

Where \(S_{HSI}\) and \(S_{FSI}\) are the safety coefficients of contact and bending strength of the \(i\)-stage sun gear, \(S_{HPI}\) and \(S_{FPi}\) are the safety coefficients of contact and bending strength of the \(i\)-stage planet gear, \([S_H]\) and \([S_F]\) are the allowable safety coefficients of contact and bending strength. The safety coefficients of contact and bending strength are calculated according to ISO 6336, as shown in Eq. (25) and Eq. (28). For normal jacking condition, the required \([S_H]\) and \([S_F]\) are 1.2 and 1.5, respectively. For pre-load and storm holding condition, the required \([S_H]\) and \([S_F]\) are 1.25 and 1.1, respectively.

The safety coefficients of contact strength are calculated as shown in the following.

\[
\begin{align*}
\sigma_{HO} &= z_H z_E z_\beta \sqrt{\frac{K_h}{b u}} \\
\sigma_{H1} &= z_B \sigma_{HO} \sqrt{K_A K_V K_{H_B} K_{Ha}} \\
\sigma_{H2} &= z_D \sigma_{HO} \sqrt{K_A K_V K_{H_B} K_{Ha}} \\
S_H &= \frac{S_{lim} z_{NT} z_{R} z_{W} z_{X}}{\sigma_H}
\end{align*}
\] (25)

Where \(\sigma_{HO}\) is the nominal contact stress at the point, \(z_H\) is the zero factor, \(z_E\) is the elasticity factor, \(z_\beta\) is the contact ratio factor, \(z_B\) is the helix angle factor, \(F_t\) is the nominal tangential load, \(d\) is the reference diameter of pinion, \(b\) is the face width, \(u\) is the gear ratio, \(z_B\) is the pinion single pair tooth contact factor of the pinion, \(z_D\) is the single pair tooth contact factor of the wheel, \(K_A\) is the application factor, \(K_V\) is the dynamic factor, \(K_{H_B}\) is the face load factor for contact stress, \(K_{Ha}\) is the transverse load factor for contact stress, \(\sigma_{lim}\) is the allowable stress number (contact), \(\sigma_H\) is the calculated contact stress and the smaller values of pinion root stress \(\sigma_{H1}\) and wheel root stress \(\sigma_{H2}\) shown in Eq.(23) and Eq.(24), \(z_{NT}\) is the life factor for test gears for contact stress, \(z_L\) is the lubricant factor, \(z_V\) is the velocity factor, \(z_R\) is the roughness factor, \(z_W\) is the work hardening factor, \(z_X\) is the size factor for contact stress.

The safety coefficients of bending strength are calculated as shown in the following.

\[
\begin{align*}
\sigma_{F0} &= \frac{F_p}{b m_n} Y_p Y_S Y_B \\
\sigma_F &= \sigma_{F0} K_A K_V K_{FB} K_{FA} \\
S_p &= \frac{\sigma_{F0}}{\sigma_{F0}} Y_{ST} Y_{NT} Y_{F} Y_{Rel} Y_{Rel} Y_{X} \\
&= K_A K_V K_{FB} K_{FA}
\end{align*}
\] (26)

Where \(\sigma_{F0}\) is the nominal tooth stress shown in Eq.(26), \(m_n\) is the normal module, \(Y_p\) is farm factor, \(Y_S\) is the stress correction factor, \(Y_B\) is the helix angle factor, \(K_{FB}\) is the face load factor for tooth root stress, \(K_{FA}\) is the transverse load factor for tooth root stress, \(\sigma_{lim}\) is the nominal stress number (bending), \(Y_{NT}\) is the stress correction factor, \(Y_{F}\) is the life factor for tooth root stress, \(Y_{Rel}\) is the relative notch sensitivity factor, \(Y_{Rel}\) is the relative surface factor, \(Y_X\) is the size factor relevant to tooth root strength.

Since a multi-stage planetary gear train is selected for the promote gearbox used in jack-up offshore platforms, the difference between the sizes of the adjacent stages should be considered and coordinated. And the diameter ratio of the adjacent annular gears should be limited to between 1 and 1.2.

\[
1.0 \leq \frac{d_{i+1}}{d_i} \leq 1.2
\] (29)

Where \(d_i\) and \(d_{i+1}\) are the pitch circle diameter of annular gears for the \(i\)-stage and \(i+1\)-stage planetary gear train, respectively.

Finally, the multi-objective optimal design model can be expressed as
\[ (X = (m_1, m_2, m_3, m_4, z_1, z_2, z_3, z_4, b_1, b_2, b_3, b_4, u_1, u_2, u_3, u_4))^T \]

\[
\begin{align*}
\text{max } f &= -w_1 \times \frac{F_1}{F_1^*} - w_2 \times F_2 - w_3 \times F_3 + w_4 \times F_4 \\
\text{s.t. } m_{\text{min}} \leq m_i \leq m_{\text{max}} \quad i \\
z_{\text{min}} \leq z_i \leq z_{\text{max}} \quad i \\
b_{\text{min}} \leq b_i \leq b_{\text{max}} \quad i \\
u_{\text{min}} \leq u_i \leq u_{\text{max}} \quad i \\
\frac{\lvert u_1, u_2, u_3 - u_4 \rvert}{u_0} \leq 2\% \\
\theta_{\text{api}} < 2\alpha_{\text{spi}} \sin \frac{\pi}{N_{\text{spi}}} \\
\frac{z_{\text{pi}}}{N_{\text{pi}}} = \frac{u_{\text{pi}}}{N_{\text{pi}}} = C \\
S_{\text{HSi}} \geq [S_H] \\
S_{\text{Hpi}} \geq [S_H] \\
S_{\text{FSi}} \geq [S_F] \\
S_{\text{Fpi}} \geq [S_F] \\
\varepsilon_{\text{spi}} \geq 1.2 \\
\varepsilon_{\text{pali}} \geq 1.2 \\
1.0 \leq \frac{d_{\text{r}(i+1)}}{d_i} \leq 1.2
\end{align*}
\]

(30)

3.2 The improved genetic algorithm

An improved genetic algorithm (Zhou et al., 2018) is applied to solve the proposed teeth distribution optimization model. The algorithm focuses on the improvement of crossover and mutation, and optimizes crossover rate and mutation rate adaptively with nonlinear curve mode, so as to increase the diversity of population, improve the search ability and convergence speed. Fig. 2 shows the adjustment curve of crossover rate and mutation rate. In practical application, the value of \( a \) in Sigmoid function is 0.9903. The technique flowchart for the work in this paper is shown in Fig. 3. The adjustment formulas of the adaptive crossover rate and mutation rate can be represented by

\[
P_c = \begin{cases} 
\frac{P_c_{\text{max}} - P_c_{\text{min}}}{1 + \exp(k_1 (f' - f_a))} + P_c_{\text{min}}, & f' \geq f_a \\
\frac{P_c_{\text{max}} - P_c_{\text{min}}}{1 + \exp(k_1 (f - f_a))} + P_c_{\text{min}}, & f < f_a 
\end{cases}
\]

(31)

\[
P_m = \frac{h_2 (P_m_{\text{max}} - P_m_{\text{min}})}{\sqrt{2\pi}} \exp \left( -\frac{(f - f_a)^2}{2} \right) + P_m_{\text{min}}
\]

(32)

Where \( f_a, f_{\text{max}} \) and \( f' \) are the average individual fitness of the population, the maximum individual fitness and the larger fitness of the two individuals to cross, respectively. \( k_1 \) and \( k_2 \) are curve smoothing and curve height parameters. \( P_c_{\text{max}} \) and \( P_c_{\text{min}} \) are the maximum and minimum values of the crossover rate. \( P_m_{\text{max}} \) and \( P_m_{\text{min}} \) are the maximum and minimum values of mutation rate.

![Crossover rate adjustment curve](attachment:image1.png)

(a) Crossover rate adjustment curve

![Mutation rate adjustment curve](attachment:image2.png)

(b) Mutation rate adjustment curve

**Fig. 2 Crossover and mutation rate adjustment curve**
4. Results comparison before and after optimization

The structure of the promote gearbox used in jack-up offshore platforms is shown in Fig. 1. The input power of the system is 23 kW with input speed 990 r/min, and the working life is 2200 h, which is the normal jacking condition. The total transmission ratio is 2216.375, and the allowable transmission ratio deviation is ±2%. The material of sun gear and planet gear for the promote gearbox is 17CrNiMo6 and the heat treatment is carburizing.

The design variables before and after optimization is shown in Table 3. After optimization, the module of the 3rd stage remains unchanged, the module of the 2nd stage increases, and the module of the 1st and 4th stage decreases. And, the number of sun gear teeth of the 1st stage increases, the number of sun gear teeth of the 2nd and 3rd stage decreases, and the number of sun gear teeth of the 4th stage remains unchanged. The face width of the 1st and 2nd stage decreases, and the face width of the 3rd and 4th stage is unchanged, which can satisfy the operating requirement of large output load. And the transmission ratio of all stages is redistributed. The transmission ratio of the 1st and 2nd stages increases, while that of the 3rd and 4th stage decreases.

Table 3 Design variables before and after optimization

| Design variables | \( m_1 \) (mm) | \( m_2 \) (mm) | \( m_3 \) (mm) | \( m_4 \) (mm) | \( z_1 \) | \( z_2 \) | \( z_3 \) | \( z_4 \) |
|------------------|----------------|----------------|----------------|----------------|--------|--------|--------|--------|
| Before optimization | 4              | 4              | 8              | 12             | 15     | 21     | 24     | 24     |
| After optimization | 3              | 5              | 8              | 10             | 17     | 18     | 22     | 24     |

| Design variables | \( b_1 \) (mm) | \( b_2 \) (mm) | \( b_3 \) (mm) | \( b_4 \) (mm) | \( u_1 \) | \( u_2 \) | \( u_3 \) | \( u_4 \) |
|------------------|----------------|----------------|----------------|----------------|--------|--------|--------|--------|
| Before optimization | 36             | 46             | 81             | 122            | 12.1123| 8.0585 | 5.3411 | 4.2514 |
| After optimization | 33             | 44             | 81             | 122            | 12.4441| 8.4412 | 5.1428 | 4.1187 |
The objective function values before and after optimization are shown in Table 4. The equal strength objective function values before and after optimization are shown in Table 5. $f_1$, $f_2$ and $f_3$ are divisional objective function of equal contact strength. And $f_4$, $f_5$ and $f_6$ are divisional objective function of equal bending strength. The columnar section of equal contact strength and bending strength objective function values before and after optimization is shown in Fig 4.

#### Table 4 The comparison of objective function values before and after optimization

| Objective function values | $f$ | $F_1$ ($10^6$ mm$^3$) | $F_2$ | $F_3$ | $F_4$ |
|---------------------------|-----|-----------------------|-------|-------|-------|
| Before optimization      | 0.2976 | 0.21782 | 0.2993 | 1.7265 | 0.9056 |
| After optimization       | 0.5438 | 0.17946 | 0.1319 | 0.8443 | 0.9058 |
| Decrease amplitude       | -82.73% | 17.61% | 55.93% | 51.10% | -0.02% |

#### Table 5 Divisional objective functions of equal strength before and after optimization

| Divisional objective values of equal strength | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ |
|---------------------------------------------|-------|-------|-------|-------|-------|-------|
| Before optimization                         | 0.5555 | 0.1664 | 0.1761 | 4.7746 | 0.0106 | 0.3943 |
| After optimization                          | 0.2966 | 0.0473 | 0.0518 | 1.7864 | 0.6191 | 0.1274 |
| Decrease amplitude                          | 46.6% | 71.5% | 70.5% | 62.5% | -5740.57% | 67.69% |

The total objective function $f$ attribute value increases by 82.73%, the volume $F_1$ decreases by 17.61%, and the efficiency $F_4$ increases by 0.02%. After optimization, the divisional objective function values of equal contact strength decrease by 46.6%, 71.5% and 70.5%, respectively, and the synthesized objective function $F_2$ value decreases by 55.93%. The divisional objective function values of equal bending strength decrease by 62.5%, -5740.57% and 67.69%, respectively, and the synthesized objective function $F_3$ value decreases by 51.10%. After optimization, the volume decreases, the efficiency remains almost invariant, and the minimum difference of the contact and bending strength for the adjacent stages decreases greatly.

The following comparison analysis will be conducted from the aspects of transmission ratio, contact ratio, bending strength, contact strength and power density. The total transmission ratio deviation can be calculated based on Eq. (33).

$$\frac{\vert u_0 - \frac{u_1 u_2 u_3}{u_4 u_5 u_6} \vert_{\text{deviation}}}{{(u_0)}_{\text{average}}} \times 100\% = 0.389\% \leq 2\%$$

The strength of the promote gearbox is checked under pre-load jacking, storm holding, pre-load and normal jacking conditions to prove the effectiveness of the optimization process. The safety coefficients of contact and bending strength of the external meshing pairs under normal jacking and pre-load jacking conditions are shown in Table 6 and Table7, respectively. The safety coefficient of bending strength of the external meshing pairs under storm holding, normal and pre-load holding conditions is shown in Table 8. Figs. 5 and 6 are scatter diagrams of safety coefficient of contact and bending strength under normal jacking and pre-load jacking conditions, and Fig. 7 is the scatter diagram of safety coefficient of bending strength under normal and pre-load holding and storm holding conditions.
Table 6 The strength of external meshing pairs under normal jacking condition

| Stages   | Gears | $S_H$ Before optimization | $S_H$ After optimization | $S_F$ Before optimization | $S_F$ After optimization |
|----------|-------|---------------------------|---------------------------|---------------------------|---------------------------|
| 1st stage| S1    | 1.7027                    | 1.4873                    | 6.6836                    | 4.1847                    |
|          | P1    | 2.0574                    | 1.7972                    | 7.0710                    | 4.4272                    |
| 2nd stage| S2    | 1.1465                    | 1.2134                    | 1.9069                    | 2.3983                    |
|          | P2    | 1.4773                    | 1.5342                    | 1.9844                    | 2.4957                    |
| 3rd stage| S3    | 1.3133                    | 1.2380                    | 1.8979                    | 1.7792                    |
|          | P3    | 1.5354                    | 1.4473                    | 2.3666                    | 2.2185                    |
| 4th stage| S4    | 1.4897                    | 1.2898                    | 2.2930                    | 1.6518                    |
|          | P4    | 1.7174                    | 1.4870                    | 2.8869                    | 2.0796                    |
| Allowable safety coefficient | 1.2 | 1.5 |

![Scatter diagrams of safety coefficients under normal jacking condition](image)

Fig. 5 Scatter diagrams of safety coefficients under normal jacking condition

Under normal jacking condition, the contact strength safety coefficient of the 2nd stage sun gear before optimization is 1.1465, which does not satisfy the design requirement. And there is a large difference for the contact and bending strength safety coefficients for the adjacent stages. After optimization, the contact and bending strength safety coefficients of all sun and planet gears satisfy the requirements, and the distance between scattered points of the optimized safety coefficient and allowable safety coefficient line decreases. The difference for the safety coefficients for the adjacent stages decreases. The minimum contact strength safety coefficient is the 2nd stage sun gear, and the minimum bending strength safety factor is the 3rd stage sun gear before and after optimization.

Table 7 The strength of external meshing pairs under pre-load jacking condition

| Stage   | Gears | $S_H$ Before optimization | $S_H$ After optimization | $S_F$ Before optimization | $S_F$ After optimization |
|---------|-------|---------------------------|---------------------------|---------------------------|---------------------------|
| 1st stage| S1    | 1.4666                    | 1.2811                    | 4.9588                    | 3.1048                    |
|          | P1    | 1.7722                    | 1.5480                    | 5.2462                    | 3.2847                    |
| 2nd stage| S2    | 0.9881                    | 1.0556                    | 1.4164                    | 1.7794                    |
|          | P2    | 1.2732                    | 1.3215                    | 1.4740                    | 1.8517                    |
| 3rd stage| S3    | 1.3314                    | 1.0664                    | 1.4086                    | 1.3201                    |
|          | P3    | 1.3227                    | 1.2467                    | 1.7564                    | 1.6460                    |
| 4th stage| S4    | 1.2830                    | 1.1110                    | 1.7011                    | 1.5255                    |
|          | P4    | 1.4792                    | 1.2808                    | 2.1416                    | 1.5429                    |
| Allowable safety coefficient | 1 | 1.25 |
Combining with Table 7, it can be seen from Fig. 7 that, under pre-load jacking condition, the safety coefficient of the contact strength of the 2nd stage sun gear before optimization is 0.9881, which does not satisfy the design requirement. After optimization, the safety coefficients of contact and bending strength of all sun and planet gears satisfy the requirements. The difference for the safety coefficients for the adjacent stages decreases. The minimum contact strength safety coefficient is the 2nd stage sun gear, and the minimum bending strength safety coefficient is the 3rd stage sun gear before and after optimization.

**Table 8** Bending strength of external meshing pairs

| Stage   | Gears | Normal and pre-load holding | Storm holding |
|---------|-------|-----------------------------|---------------|
|         |       | Before optimization | After optimization | Before optimization | After optimization |
| 1st stage | S1    | 5.1241                     | 3.2083         | 4.2701             | 2.6736             |
|         | P1    | 5.4211                     | 3.3942         | 4.5176             | 2.8285             |
| 2nd stage | S2    | 1.4636                     | 1.8387         | 1.2197             | 1.5322             |
|         | P2    | 1.5231                     | 1.9134         | 1.2692             | 1.5945             |
| 3rd stage | S3    | 1.4555                     | 1.3641         | 1.2129             | 1.2067             |
|         | P3    | 1.8149                     | 1.7009         | 1.5124             | 1.4174             |
| 4th stage | S4    | 1.7578                     | 1.2664         | 1.4648             | 1.1553             |
|         | P4    | 2.2130                     | 1.5943         | 1.8442             | 1.3286             |
| Allowable safety coefficient | 1.25 | 1.1 |

The bending strength is the main consideration as the promote gearbox mainly plays the role of supporting platform under normal and pre-load holding and storm holding conditions. The safety coefficients of contact and bending strength for all sun and planet gears satisfy the requirements before and after optimization. In addition, the weakest spot of the
promote gearbox under the two working conditions is the 3rd stage sun gear before optimization, while the weakest spot is the 4th stage sun gear after optimization. The change of contact ratio before and after optimization is shown in Table 9. After optimization, the contact ratio of internal and external meshing pairs of the 1st stage and 4th stage increases slightly, and that of the 2nd stage and 3rd stage decreases slightly. The sizes for adjacent stages before and after optimization is shown in Fig. 8.

| Stage | Gear pairs | Contact ratio | Before optimization | After optimization |
|-------|------------|---------------|---------------------|-------------------|
| 1st   | S1-P1      | 1.4573        | 1.4906              |
|       | P1-A1      | 1.6160        | 1.6259              |
| 2nd   | S2-P2      | 1.4915        | 1.4636              |
|       | P2-A2      | 1.6137        | 1.6043              |
| 3rd   | S3-P3      | 1.4676        | 1.4526              |
|       | P3-A3      | 1.5828        | 1.5756              |
| 4th   | S4-P4      | 1.4384        | 1.4447              |
|       | P4-A4      | 1.5545        | 1.5572              |
| Requirement |       | ≥1.2         |                     |

Before optimization, the value of \( \frac{d_2}{d_1} \) is 0.9827, less than 1, while the value of \( \frac{d_3}{d_2} \) is 1.2436, greater than 1.2, which does not satisfy the design requirement. After optimization, the pitch circle diameter ratio for the adjacent stages is between 1 and 1.2, which satisfies the design requirement and makes the size of all stages more coordinating. The promote gearbox shows an obvious ladder-like after optimization, rather than spoon-shaped before optimization. After optimization, the annular gear diameter of the 1st, 3rd and 4th stage decreases significantly, while the annular gear diameter of the 2nd stage increases slightly, which is an intuitive reflection of the volume reduction after optimization.

The power density is calculated according to Eq. (34), which shows that the reduction of gearbox volume will increase the power density. The power density before and after optimization is shown Table 10. Power density of all stages under normal jacking condition can be calculated by

\[
q_{spi} = \frac{P_i}{a_i b_i} = \frac{2 \times P_i}{m_i b_i (z_i + z_{pi})} = \frac{2 \times P_i}{m_i b_i (z_{ai} - z_{pi})} \tag{34}
\]

Where \( P_i \) is the power of the i-stage, \( a_i \) is the center distance of the i-stage, \( b_i \) and \( m_i \) are the face width and modulus of the i-stage, \( z_i \), \( z_{pi} \) and \( z_{ai} \) the sun gear number, planet gear number and annulus gear number respectively.
Table 10 The power density calculation before and after optimization

| Stage   | Power density ($\times10^{-4}$ kW/mm$^2$) | Increase amplitude |
|---------|------------------------------------------|--------------------|
|         | Before optimization | After optimization |                        |
| 1$^{st}$ stage | 12.000 | 14.000 | 16.67%                  |
| 2$^{nd}$ stage | 8.9607 | 8.7330 | -2.54%                  |
| 3$^{rd}$ stage | 3.6623 | 3.8847 | 6.07%                   |
| 4$^{th}$ stage | 1.1980 | 1.3820 | 15.36%                  |

The power density of the 1$^{st}$ to 4$^{th}$ stage increase 16.67%, -2.54%, 6.07% and 15.36%, which is consistent with the change trend of the diameter of all annular gears. It also reflects that the volume reduction is mainly from the original 1$^{st}$ and 4$^{th}$ stage.

5. Strength verification by Masta

In order to further verify the correctness and effectiveness of the calculated results of teeth distribution optimization, the analysis model of the promote gearbox is established using Masta before and after optimization, as shown in Fig. 9. The difference between (a) and (b) lies in the outline size and its change among stages.

The contact strength safety coefficients of the external meshing pairs under different working conditions based on the simulation model are shown in Fig. 10. The bending strength safety coefficients of the external meshing pairs under different working conditions based on the simulation model are shown in Fig. 11. Before optimization, the contact strength of the 2$^{nd}$ stage sun gear is 1.1065 under normal jacking condition, which can not satisfy the requirement. After optimization, the contact and bending strength of the promote gearbox’s external meshing pairs satisfies the requirements under all working conditions. The difference for the safety coefficients for the adjacent stages decreases, which is consistent with the change trend of the strength computed by the proposed model before and after optimization.
6. Conclusion

(1) The structure and transmission principle of the promote gearbox used in jack-up offshore platforms is analyzed. The mathematical model of teeth distribution optimization for the promote gearbox is presented considering the objective functions such as the volume, efficiency, strength difference for the adjacent stages, and the design variables such as module, sun gear teeth, face width and transmission ratio of all stages. The model is solved based on the improved genetic algorithm.

(2) After optimization, the attribute value of the synthesized objective function increases by 82.73%, the volume decreases by 17.61%, and the efficiency remains almost invariant. The objective function value of equal contact and bending strength decreases by 55.93% and 51.10%, respectively. The difference for the safety coefficients for the adjacent stages decreases obviously. The constraint conditions of single-stage and adjacent stages satisfy the requirements.

(3) An analysis model of the promote gearbox is established using Masta to verify the proposed model. The safety
coefficients of contact and bending strength satisfy the requirements, and the difference for the safety coefficients for the adjacent stages decreases after optimization, which is consistent with the results by the teeth distribution optimization model.

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