Models for neutrino mass with discrete symmetries

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Discrete non-abelian flavor symmetries give in a natural way tri-bimaximal (TBM) mixing as showed in a prototype model. However neutrino mass matrix pattern may be very different from the tri-bimaximal one if small deviations of TBM will be observed. We give the result of a model independent analysis for TBM neutrino mass pattern.

Neutrino data are in well agreement (approximately within 1σ) with the so called tri-bimaximal mixing \([1]\), giving maximal atmospheric angle \(\sin^2 \theta_{23} = 1/2\), zero reactor angle \(\sin^2 \theta_{13} = 0\) and trimaximal solar angle \(\sin^2 \theta_{12} = 1/3\). This is shown in fig.\([1]\) where we give the result of the fits of three different groups \([2,3,4]\), represented by three horizontal bands where the blue band is the 3σ region, the red band is the 1σ region, and the green point is the best fit value.

Non-abelian discrete symmetries has been extensively used in order to explain TBM mixing for neutrino, see for instance \([5]\) and reference therein. We provide a simple example based on the discrete abelian flavor symmetry \(A_4\) \([6,7,8]\), namely the even permutations of four objects. This is the smallest finite group with triplet irreducible representation and seems very adequate to accommodate the three flavor families of the Standard Model.

We consider the Altarelli-Feruglio model defined in table\([1]\). The flavon field \(\varphi_T\) is coupled only to the charged fermions \(L^c h \varphi_T\) and \(\varphi_S\) is coupled only to the dimension five operator \(L L h h \varphi_S\) in order to preserve the additional \(Z_3^{aux}\). After that the flavon fields take vev, \(A_4\) is spontaneously broken as below

\[
\langle \varphi_T \rangle \sim (1, 0, 0) : A_4 \rightarrow Z_3; \\
\langle \varphi_S \rangle \sim (1, 1, 1) : A_4 \rightarrow Z_2. \tag{1}
\]

So \(A_4\) is spontaneously broken into \(Z_3\) in the charged fermion sector while into \(Z_2\) in the neutrino sector where \(Z_2\) and \(Z_3\) are two subgroups of \(A_4\). The misalignment between the two sectors is at the origin of large lepton mixing. In general this is possible thanks to auxiliary abelian symmetries that distinguish the charged and neutral lepton sectors. In the prototype model we are presenting such a auxiliary symmetry is \(Z_3^{aux}\).

It is well know that the alignments \((1,0,0)\) or \((1,1,1)\) in eq.\([1]\) are natural in \(A_4\) since they leave unbroken the \(T\) and the \(S\) generators of \(A_4\) respectively. However assuming complex vevs different solutions can be found from the minimization of the potential, see \([9,10,11]\). In general the solution of the potential containing both the scalar fields \(\varphi_S\) and \(\varphi_T\) is not given by eq.\([1]\) unless terms mixing \(\varphi_S\) and \(\varphi_T\) like \(|\varphi_S|^2|\varphi_T|^2\) are neglected (this is not possible by means of symmetries). It is possible to solve such a problem assuming SUSY \([7]\) or extra-dimension \([8]\).

As a result of the model in table\([1]\) the neutrino mass matrix is \(\mu - \tau\) invariant and trimaxi-

\begin{tabular}{cccccccc}
\hline
 & \(L\) & \(e^c\) & \(\mu^c\) & \(\tau^c\) & \(h_{u,d}\) & \(\varphi_T\) & \(\varphi_S\) & \(\xi\) \\
\hline
\(SU(2)\) & 2 & 1 & 1 & 1 & 2 & 1 & 1 & 1 \\
\(A_4\) & 3 & 1 & 1’ & 1” & 1 & 3 & 3 & 1 \\
\(Z_3^{aux}\) & \(\omega\) & \(\omega^2\) & \(\omega^2\) & \(\omega^2\) & 1 & 1 & \(\omega\) & \(\omega\) \\
\hline
\end{tabular}

Table 1
Matter assignment of a prototype model for TBM mixing.
Figure 1. The blue and red ranges are respectively the 3\(\sigma\) and 1\(\sigma\) values for \(\sin^2\theta_{12}\) (up plot), \(\sin^2\theta_{23}\) (middle plot) and \(\sin^2\theta_{13}\) (bottom plot) from the Ref.[2] (up band), [3] (middle band) and [4] (down band) and the green points are the best fit point. The vertical line are the TBM values and the \(\lambda_C\), \(\lambda_C^2\) deviations.

Shortly the neutrino mass matrix has TBM texture. In general, in almost of the models studied in literature, the three angles receive corrections of the same order from next to leading order contributions, that is \(\sin^2\theta_{23} = 1/2 + \mathcal{O}(\epsilon)\), \(\sin\theta_{13} = \mathcal{O}(\epsilon)\) and \(\sin^2\theta_{12} = 1/3 + \mathcal{O}(\epsilon)\) where \(\epsilon\) is a small perturbation. In order to satisfy solar neutrino data, \(\epsilon\) can be almost of order \(\lambda_C^2\), and the deviation of the reactor angle from TBM must be of the same order. If a larger value for the reactor angle will be measured, currently at 3\(\sigma\) the reactor angle can be as larger as \(\lambda_C\), most of the models will be ruled out.

For small deviation of neutrino mass matrix from TBM pattern, see eq. (2), we expect small deviation of TBM mixing. However assuming small deviations (of order \(\lambda_C^2\)) of the lepton mixing matrix from TBM, the deviations of the neutrino mass matrix from the TBM texture [2] can be very large as indicated in [12]. In order to parameterize the deviation of the neutrino mass matrix from the \(\mu - \tau\) exchange symmetry and from the trimaximality, that is \(m_{\nu_{11}} + m_{\nu_{12}} = m_{\nu_{22}} + m_{\nu_{23}}\), in [12] the following parameters has been introduced

\[
\Delta_e = \frac{m_{\nu_e}^\mu - m_{\nu_e}^\tau}{m_{\nu_e}^\mu},
\]

\[
\Delta_{\mu\tau} = \frac{m_{\nu_e}^{\mu\mu} - m_{\nu_e}^{\tau\tau}}{m_{\nu_e}^{\mu\mu}},
\]

\[
\Delta_\Sigma = \Sigma_L - \Sigma_R,\]

where \(\Sigma_L = m_{\nu_e}^{ee} + (m_{\nu_e}^{e\mu} + m_{\nu_e}^{e\tau})/2\) and \(\Sigma_R = m_{\nu_e}^{\mu\mu} + (m_{\nu_e}^{\mu\tau} + m_{\nu_e}^{\tau\tau})/2\). The result of the model independent analysis is shown in fig. (2) where a correlation between the parameter \(\Delta_e\) and \(\sin\theta_{13}\) has been given. Such analysis shows that the deviation of the neutrino mass matrix from TBM texture is small, namely of order 10\(\%\), if \(\sin\theta_{13} < 10^{-3}\) that is very small. When \(\sin\theta_{13} \sim \lambda_C^2\), in the sensitivity of future experiments, the deviation of TBM pattern can be very large \(\Delta_3 > 1\), see fig. (2).

Also if the model independent analysis do not strictly indicate TBM neutrino pattern, non-abelian discrete symmetries remain a simple and economical description of data. For instance, a model with only \(A_4\) symmetry (no auxiliary symmetries) and without flavons has been proposed in [10] where the standard model has been extended assuming extra Higgs doublets transforming as a triplet of \(A_4\). Another example of model based on discrete symmetries with interesting phenomenological consequence is given in [13] with approxi-
mate Fritzsch texture for light neutrino mass matrix

\[
m_\nu \sim \begin{pmatrix} 0 & b & 0 \\ b & a & c \\ 0 & c & d \end{pmatrix}.
\]  

(6)

The model is based on $S_3$ flavor symmetry, the permutation of three objects with singlets and doublet irreducible representations. The charged lepton mass matrix is close to diagonal. The two zeros of the Fritzsch texture, give a strong correlation between the reactor angle $\theta_{13}$ and the Dirac CP phase $\delta$. In fig. 3 we show such a correlation. The central line is obtained assuming all the observables at the best fit, and we have for maximal CP violation $\delta = \pm \pi/2$ that $\sin^2 \theta_{13} \approx 0.01$ close to the indication of ref. [14].

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**REFERENCES**

1. P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B 530, 167 (2002) [arXiv:hep-ph/0202074].
2. T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 10, 113011 (2008) [arXiv:0808.2016 [hep-ph]].
3. M. C. Gonzalez-Garcia, M. Maltoni and J. Salvado, JHEP 1004, 056 (2010) [arXiv:1001.4524 [hep-ph]].
4. G. L. Fogli, E. Lisi, A. Marrone and A. Palazzo, Prog. Part. Nucl. Phys. 57, 742 (2006) [arXiv:hep-ph/0506083].
5. G. Altarelli, F. Feruglio, Rev. Mod. Phys. 82, 2701-2729 (2010). [arXiv:1002.0211 [hep-ph]].
6. K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B 552, 207 (2003) [arXiv:hep-ph/0206292].
7. G. Altarelli and F. Feruglio, Nucl. Phys. B 720, 64 (2005) [arXiv:hep-ph/0504165].
8. G. Altarelli and F. Feruglio, Nucl. Phys. B 741, 215 (2006) [arXiv:hep-ph/0512103].
9. L. Lavoura and H. Kuhbock, Eur. Phys. J. C 55, 303 (2008) [arXiv:0711.0670 [hep-ph]].
10. S. Morisi and E. Peinado, Phys. Rev. D 80, 113011 (2009) [arXiv:0910.4389 [hep-ph]].
11. R. d. A. Toorop, F. Bazzocchi, L. Merlo and A. Paris, [arXiv:1012.1791 [hep-ph]].
12. M. Abbas, A. Y. Smirnov, Phys. Rev. D 82, 013008 (2010). [arXiv:1004.0099 [hep-ph]].
13. D. Meloni, S. Morisi, E. Peinado, [arXiv:1005.3452 [hep-ph]].
14. G. L. Fogli, E. Lisi, A. Marrone et al., Phys. Rev. Lett. 101, 141801 (2008). [arXiv:0806.2649 [hep-ph]].