Off-Policy Correction for Actor-Critic Algorithms in Deep Reinforcement Learning

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Abstract
Compared to on-policy policy gradient techniques, off-policy model-free deep reinforcement learning (RL) approaches that use previously gathered data can improve sampling efficiency. However, off-policy learning becomes challenging when the discrepancy between the distributions of the policy of interest and the policies that collected the data increases. Although the well-studied importance sampling and off-policy policy gradient techniques were proposed to compensate for this discrepancy, they usually require a collection of long trajectories that increases the computational complexity and induce additional problems such as vanishing or exploding gradients. Moreover, their generalization to continuous action domains is strictly limited as they require action probabilities, which is unsuitable for deterministic policies. To overcome these limitations, we introduce an alternative off-policy correction algorithm for continuous action spaces, Actor-Critic Off-Policy Correction (AC-Off-POC), to mitigate the potential drawbacks introduced by the previously collected data. Through a novel discrepancy measure computed by the agent’s most recent action decisions on the states of the randomly sampled batch of transitions, the approach does not require actual or estimated action probabilities for any policy and offers an adequate one-step importance sampling. Theoretical results show that the introduced approach can achieve a contraction mapping with a fixed unique point, which allows a “safe” off-policy learning. Our empirical results suggest that AC-Off-POC consistently improves the state-of-the-art and attains higher returns in fewer steps than the competing methods by efficiently scheduling the learning rate in Q-learning and policy optimization.

Keywords deep reinforcement learning · actor-critic algorithms · off-policy learning · importance sampling

1 Introduction

The deep reinforcement learning (RL) setting in the control of continuous systems still requires large amounts of data to obtain optimal policies and construct general and scalable reinforcement learning agents [Sutton, 1992]. In order to improve data and sampling efficiency, agents store their experiences in a buffer, experience replay memory [Lin, 1992], and reuse them multiple times to perform gradient steps on their policies and value functions approximated by deep neural networks. Methods that employ experience replay [Lin, 1992] are off-policy as the samples contained in the buffer which are used to learn a target policy are generated by a group of different policies than the one being optimized [Sutton and Barto, 2018]. Although the past experiences of the agents may be beneficial for learning in later stages, deadly triad can occur in off-policy deep RL, a phenomenon that refers to the instability of an RL algorithm when it employs off-policy learning, function approximation, and bootstrapping simultaneously [Sutton and Barto, 2018]. Due to the combination of the mentioned entities, a sudden divergence may arise even if the policy is very close to an optimal one [Sutton and Barto, 2018]. To mitigate the effects of the deadly triad, a common approach in off-policy learning is to disregard or completely discard transitions whose underlying distributions are uncorrelated to the policy distribution to be optimized, also known as off-policy correction [Cicek et al., 2021]. This is done through the relative divergence between the action probabilities generated by the current and other behavioral policies that collected the experiences, which is performed through importance sampling or off-policy policy gradients.
Importance sampling methods weigh the gradients of the approximate value functions according to this relative divergence using off-policy transitions sampled from the experience replay buffer [Lin, 1992]. Importance sampling can consider temporally correlated transitions through the trajectories, e.g., [Watkins and Dayan, 1992, Degris et al., 2012, Munos et al., 2016, Harutyunyan et al., 2016, Espeholt et al., 2018] or through a single transition, e.g., [Schmitt et al., 2020]. Although the widely known importance sampling techniques are shown to be successful in off-policy learning, there are several issues introduced by these approaches [Schmitt et al., 2020]. As the trajectory-based approaches conservatively treat samples, there may be unnecessary trajectory terminations, a large amount of variance or biased action probability estimates [Munos et al., 2016]. For single-transition methods, inaccurate importance weights are one of the broadly observed issues [Schmitt et al., 2020].

Prior to the employment of function approximators in reinforcement learning, many off-policy techniques, specifically Monte-Carlo, did not have an option to evaluate the off-policy policies in terms of the action probabilities [Harutyunyan et al., 2016]. However, with the utilization of value function approximators, temporal difference [Sutton, 1988a] methods provide a continuous link between one-step and Monte-Carlo approaches through *eligibility traces* [Watkins, 1989, Singh and Sutton, 1996]. As the value functions in temporal difference methods assess actions under the cumulative reward objective, they offer a direct correction to the discrepancy between the target and behavioral policies [Harutyunyan et al., 2016]. Off-policy correction in terms of action probabilities via eligibility traces [Watkins, 1989, Singh and Sutton, 1996] is a well-studied artifact for discrete action spaces [Espeholt et al., 2018, Harutyunyan et al., 2016, Munos et al., 2016, Precup et al., 2001, Degris et al., 2012, Schmitt et al., 2020]. Even though actor-critic algorithms are on-policy by construction due to the Bellman optimization [Bellman, 1957], [Schmitt et al., 2020], similar work for the continuous action spaces is very few, e.g., [Humayoo and Cheng, 2019, Zhang et al., 2019, Imani et al., 2018, Gu et al., 2017]. Furthermore, for large action spaces, the gradients can vanish due to the product of importance weights [Han and Sung, 2019]. As an environment’s observation and action spaces start to increase and more challenging tasks are introduced, the vanishing gradient problem due to importance sampling can be detrimental [Dulac-Arnold et al., 2021]. Additionally, if the continuous policy is deterministic, then off-policy correction in terms of the action probability estimates through eligibility traces [Watkins, 1989, Singh and Sutton, 1996] is not possible by construction [Sutton and Barto, 2018].

Motivated by the mentioned problems in the current importance sampling approaches and limited generalization of off-policy correction to continuous action domains, we introduce Actor-Critic Off-Policy Correction (AC-Off-POC), an alternative one-step off-policy correction method for actor-critic algorithms to overcome the effects of deadly triad when learning from off-policy samples. The main contributions of this study can be summarized as follows:

- **We derive a parameter-free importance weighting rule that can operate using randomly sampled transitions on both types of continuous policies, deterministic or stochastic.** Through this rule, we correct the off-policy error to achieve efficient off-policy learning in continuous control and make the standard on-policy policy gradients closer to their on-policy nature.

- **By operating using a randomly sampled batch of transitions and in continuous action spaces, our algorithm prevents the high-variance build-up, vanishing gradients, and unnecessary trajectory terminations observed in traditional importance sampling methods.** Therefore, the presented method can achieve a *safe* off-policy learning by constructing a contraction mapping around the discount factor with a unique fixed point.

- **Our method can readily be adapted to off-policy actor-critic algorithms and experience replay [Lin, 1992] sampling methods that are currently available and will be introduced in the future, which we validate through remarks and empirical results.**

- **We make remarks to indicate the generalizability and behavior of our approach.** An extensive set of empirical studies support our claims in the provided remarks and demonstrate that, in the majority of the tasks, our algorithm improves the performance of the baseline algorithms and outperforms the competing off-policy correction methods in challenging OpenAI Gym [Brockman et al., 2016] continuous control tasks. Moreover, we conduct additional experiments to give insights into the introduced off-policy correction scheme and perform a sensitivity analysis based on the components that may affect off-policy learning.

- **To ensure reproducibility, our source code is made publicly available at the GitHub repository¹.**

## 2 Related Work

Off-policy RL considers learning from experiences generated by the current or past versions of the agent’s policy. Learning from the past experiences of the policy can cause action-values to be erroneously estimated and may degrade

¹[https://github.com/baturaysaglam/AC-Off-POC.git](https://github.com/baturaysaglam/AC-Off-POC.git)
the performance in environments that vastly require learning from on-policy samples, i.e., data collected by the current policy of interest. Importance sampling or off-policy correction can prevent such a discrepancy in off-policy learning. This can be done through a known or estimated probability distribution of the policy [Harutyunyan et al., 2016, Munos et al., 2016, Schmitt et al., 2020, Espeholt et al., 2018]. Initial studies in off-policy correction and importance sampling have been done by Precup et al. [2000], Harutyunyan et al. [2016], Watkins [1989]. Before the deep function approximators, basic approaches in importance sampling through eligibility traces have been studied by Precup et al. [2000], Harutyunyan et al. [2016], Singh and Sutton [1996]. Watkins [1989] proposed an off-policy correction method to learn from delayed rewards. Before employing deep neural networks, Mahmood et al. [2014] introduced a weighted importance sampling with linear function approximators. Degris et al. [2012] introduced a multi-step formulation for the expected return in terms of the importance sampling. Monte-Carlo is employed to compute the value estimates’ importance weights when the action probabilities cannot be computed due to large state and action spaces. The high variance due to the product of importance coefficients motivated further variance reduction techniques such as the work of Precup et al. [2000], which reduces the number of steps before the bootstrapping.

With the introduction of deep function approximators in RL, Munos et al. [2016] utilized full returns as in the on-policy case in their RETRACE algorithm, yet Schmitt et al. [2020] shows that it introduces a zero-mean random variable at each step, yielding an increased variance in both on- and off-policy settings. The variance build-up in importance sampling is overcome by the V-trace algorithm [Espeholt et al., 2018] by trading off the variance for a biased return estimate. Differently from RETRACE [Munos et al., 2016], V-trace [Espeholt et al., 2018] fully recovers the Monte-Carlo return under on-policy learning. Schmitt et al. [2020] presented LASER and show to eliminate the bias in the V-trace [Espeholt et al., 2018] algorithm without introducing an unbounded variance. Finally, Hanna et al. [2021] replaces the behavioral policy’s action probabilities by their maximum likelihood estimate under the observed data. They show that this general technique reduces variance due to sampling error in Monte Carlo style estimators. These methods were mainly introduced for discrete action spaces and have improved performance in classic Atari games and control problems.

In continuous action domains, importance sampling through the probability distributions of the policies is impossible by constructing the deterministic policies [Sutton and Barto, 2018]. However, Metelli et al. [2018] proposed to derive a high-confidence bound for importance weight estimates in on-policy learning. They define a surrogate objective function that optimizes the policy offline when a new batch of trajectories is collected. Detrimental estimation bias due to the clipped importance weights to avoid the large variances is addressed by Han and Sung [2019] and overcome by an adaptive structure that separately clips the importance weight of each action dimension for on-policy algorithms. In addition, Ciccek et al. [2021] previously introduced an experience replay [Lin, 1992] sampling algorithm that prioritizes relatively on-policy samples in training off-policy actor-critic algorithms. Their approach, Batch Prioritizing Experience Replay via KL-divergence (KLPER) [Ciccek et al., 2021], exhibits substantial results in environments that vastly require on-policy experiences to be solved.

For off-policy learning with stochastic actors, the instability in the policy gradients due to the discrepancy between the target and behavioral policy distributions is addressed by Humayoo and Cheng [2019], Imani et al. [2018] and Gu et al. [2017]. Humayoo and Cheng [2019] alleviate such a discrepancy by employing a smooth variant of the relative importance sampling in their algorithm, Relative Off-Policy Actor-Critic (RIS-Off-PAC), which has a parameter $\beta \in [0,1]$ that controls the smoothness. Additionally, when the behavior policy does not always aim to learn and adopt the optimal policy for the task in off-policy learning, the policy gradient theorem becomes insufficient [Imani et al., 2018]. Imani et al. [2018] introduced a novel off-policy policy gradient through an emphatic weighting scheme in their algorithm, Actor-Critic with Emphatic Weights (ACE). However, the excursion objective used in [Imani et al., 2018] can be misleading about the performance of the target policy. Generalized Off-Policy Actor-Critic (Geoff-PAC) [Zhang et al., 2019] solves this problem by containing a new objective for off-policy learning to compute the policy gradient of the counterfactual objective and use an emphatic approach to get an unbiased sample from this policy gradient. Moreover, Interpolated Policy Gradient (IPG) [Gu et al., 2017] merges on- and off-policy updates for actor-critic methods. They show that off-policy updates with a value function estimator can be interpolated with on-policy policy gradient updates while the performance can still satisfy the bounds. Overall, these methods are well-known off-policy correction techniques in the control of continuous systems and have been shown to exhibit remarkable results. Thus, we employ them as competing methods in our experiments.

3 Background and Notation

In reinforcement learning, an agent interacts with its environment and collect rewards. At each discrete time step $t$, the agent observes its state $s$ and chooses an action $a_t$; it receives a reward $r$ and observes a new state $s_t'$. In fully observable environments, the RL problem is represented by a Markov Decision Process (MDP), a tuple $(S, A, P, \gamma)$ where $S \subseteq \mathbb{R}^m$ and $A \subseteq \mathbb{R}^n$ denotes the state and action spaces, respectively, $P$ is the transition dynamics such that
The agent selects actions depending on its observation of states $s \in S$ and with respect to a policy $\pi : S \rightarrow A$. Policy of an agent $\pi(s)$ is stochastic if it maps states to action probabilities, $a \sim \pi(s)$, or deterministic if it maps states to unique actions, $a = \pi(s)$. The policy of an agent follows a distribution $\pi^\tau(s)$ over the states $s \in S$ visited by the policy. There exists an action-value function $Q^\pi(s, a) = E_{s \sim \pi, a \sim \tau}[R_t | s, a]$, corresponding to each policy $\pi$, which represents the expected return while following the policy after taking action $a$ in state $s$. For a given policy $\pi$, the action-value function is computed through the Bellman operator $T^\pi$ [Bellman, 1957]:

$$T^\pi Q^\pi(s, a) = E_{a' | S'}[r + \gamma Q^\pi(s', a')] ; \quad a' \sim \pi(s').$$ (1)

The Bellman operator [Bellman, 1957] is a contraction for $\gamma \in [0, 1]$ with unique fixed point $Q^\pi(s, a)$, i.e., the resulting error is bounded by $\gamma$ [Bertsekas and Tsitsiklis, 1996]. The optimal action-value function, $Q^*(s, a) = \max_{\pi} Q^\pi(s, a)$, is obtained through the greedy actions of the corresponding policy, i.e., actions that yield the highest action-value.

In deep reinforcement learning, the action-value functions are modeled by differentiable function approximators $Q_\theta(s, a)$, parameterized by $\theta$, also known as Q-networks. The deep action-value function is obtained by temporal difference (TD) learning [Sutton, 1988b], an update rule based on the Bellman equation [Bellman, 1957] which represents a fundamental relationship used to learn the action-value function by bootstrapping from the estimate of the current state-action pair $(s, a)$ to the subsequent state-action pair $(s', a')$, i.e., the Deep Q-Networks (DQN) algorithm [Mnih et al., 2013]:

$$Q_\theta(s, a) \leftarrow r + \gamma Q_\theta(s', a'),$$ (2)

where $\theta'$ is the target network parameters of the Q-network. The usage of target networks is a practical approach in deep reinforcement learning to provide a fixed objective to Q-networks and ensure stability in the updates. The target networks are usually updated by a small proportion $\tau$ at each step, i.e., $\theta' \leftarrow \tau \theta + (1 - \tau) \theta'$, called soft-update, or periodically to exactly match the behavioral networks called hard-update. Q-networks are optimized by minimizing:

$$J(\theta) = \delta^2 ;$$ (3)
$$y := r + \gamma Q_\theta(s', a'),$$ (4)
$$\delta := y - Q_\theta(s, a),$$ (5)

where $y$ is the fixed objective for the Q-network and $\delta$ is the resulting TD-error.

In continuous action spaces, maximum of the action-value function over possible actions, i.e., $\max_a Q^\pi(s, a)$, is intractable. Therefore, a separate network that selects actions on the observed states is employed, called the actor network. The deep reinforcement learning algorithms that utilize actor networks are called actor-critic methods, and they enable to control of continuous systems through the employed actor network. In actor-critic algorithms, actor networks $\pi_\phi$ parameterized by $\phi$, are optimized by one-step gradient ascent over the policy gradient $\nabla_\phi J(\phi)$, where the loss for the actor networks are the Q-values of the selected actions estimated by the Q-network:

$$J(\phi) = Q_\theta(s, \pi_\phi(s)).$$ (6)

For deterministic policies, the policy gradient is found through maximizing the Q-value of the given state-action pair, also known as the Deterministic Policy Gradient algorithm (DPG) [Silver et al., 2014]. DPG [Silver et al., 2014] computes the gradient of the deterministic policy as:

$$\nabla_\phi J_{det}(\phi) = \nabla_\phi Q_\theta(s, a)|_{a = \pi_\phi(s)} \nabla_\phi \pi_\phi(s).$$ (7)

In stochastic policy gradient, the log-derivative trick is used to compute the gradient over the expected cumulative reward:

$$\nabla_\phi J_{sto}(\phi) = \nabla_\phi \log \pi_\phi(a | s) Q_\theta(s, a).$$ (8)

In off-policy deep reinforcement learning, the behavioral policy of agents collects experiences and stores them into the experience replay buffer [Lin, 1992] to use in updating their deep policies and Q-networks. As the agents progressively learn the environment and improve their policies, the distribution of the current policy may not match the distribution under the previously collected transitions, such as experiences collected in the initial time steps. In this case, those experiences are off-policy samples, i.e., experiences gathered by other policies, and the ones collected by the current agent’s policy are on-policy samples. At every update step, the agent samples a batch of transitions through a sampling algorithm that may contain on- and off-policy samples:

$$(S^{[B] \times m}, A^{[B] \times n}, R^{[B] \times 1}, S'^{[B] \times m}) \sim \mathcal{B},$$ (9)
where \(|B|\) is the number of sampled transitions, and \(S^{[B]}_{t} \times m\), \(A^{[B]}_{t} \times n\), \(R^{[B]}_{t} \times 1\), \(S^{[B]}_{t} \times m\) are the state, action, reward and next state matrices or vectors in \(B\), respectively, i.e.:

\[
S^{[B]}_{t} \times m = [s_1^m, s_2^m, \ldots, s_m^m]^T, \\
A^{[B]}_{t} \times n = [a_1^n, a_2^n, \ldots, a_n^n]^T, \\
R^{[B]}_{t} \times 1 = [r_1, r_2, \ldots, r_{|B|}]^T, \\
S^{[B]}_{t} \times m = [s_1^m, s_2^m, \ldots, s_m^m]^T.
\]

Note that we use batch or mini-batch interchangeably throughout the paper. Through batch learning, we consider that the agent optimizes its actor and critic networks using the sampled batch of transitions that do not require to be necessarily temporally correlated. Following Equation (9), policy gradients denoted by Equation (7) and Equation (8), and critic loss denoted by Equation (3) can be adapted to mini-batch setting:

\[
y^{[B]}_{t} = R^{[B]}_{t} \times 1 + \gamma \hat{Q}_\phi(S^{[B]}_{t} \times m, \pi_\phi(S^{[B]}_{t} \times m)), \\
\delta^{[B]}_{t} = y^{[B]}_{t} - \hat{Q}_\phi(S^{[B]}_{t} \times m, A^{[B]}_{t} \times n), \\
J(\theta) = \frac{\|\delta^{[B]}_{t}\|^2_{2}}{|B|}, \\
\nabla_\phi J_{det}(\phi) = \nabla_\phi \frac{1}{|B|} \sum_{B} \hat{Q}_\phi(S^{[B]}_{t} \times m, \pi_\phi(S^{[B]}_{t} \times m)), \\
\nabla_\phi J_{sto}(\phi) = \nabla_\phi \frac{1}{|B|} \sum_{B} \log \pi_\phi(A^{[B]}_{t} \times n | S^{[B]}_{t} \times m) \circ \hat{Q}_\phi(S^{[B]}_{t} \times m, A^{[B]}_{t} \times n),
\]

where sum operators compute the loss over the row vectors of \(B\), \(\circ\) is the Hadamard product, and \(\| \cdot \|_2\) is the L_2 norm.

## 4 Off-Policy Correction for Actor-Critic Algorithms

Our goal is to achieve faster and more efficient learning by disregarding transitions that the distribution under which does not match the distribution of the current policy. To enable safe and efficient off-policy learning, we first aim to mitigate the effects of off-policy error induced by the data collected by divergent behavioral policies. In the following subsections, we introduce AC-Off-POC with two variants that measure the similarity between deterministic or stochastic policies. We show that the effect of each off-policy sample in the optimization can be corrected through the Jensen-Shannon divergence without action probability estimates under a basic multivariate Gaussian assumption on the actions. AC-Off-POC projects the similarity measure within the interval \([0, 1]\), where 1 indicates identical policies and 0 indicates uncorrelated ones, to weigh the gradients of the actor and critic networks. We then introduce a general off-policy actor-critic framework with AC-Off-POC for continuous control and support our claims with theoretical derivations. We show that AC-Off-POC provides a safe off-policy correction through the mentioned bounded \(\gamma\)-contraction mapping. We make remarks throughout the study to emphasize certain points and findings and discuss the empirical results further.

### 4.1 Deterministic Policies

To measure the mismatch between the distributions under the off-policy experiences and current policy, we assume without loss of generality that at every time step \(t\), the multi-dimensional continuous actions are samples of a multivariate Gaussian distribution which is not known during the training. As the agent is progressively learning, each action selected by a different behavioral policy may be a sample of a different multivariate Gaussian distribution. However, each dimension in an action vector may be correlated to the rest of the dimensions. This is realistic since each dimension in action vectors often has effects on other dimensions [Todorov et al., 2012, Parberry, 2013]. For instance, consider a continuous control task where a robot lies on the ground and tries to stand up, and the \(n\)-dimensional action space represents the angles of the robot’s joints. Intuitively, an angle of a joint may restrict or affect the motion of other joints. This implies the correlation across the action dimensions. A multivariate probability distribution can represent such an action vector, specifically, multivariate Gaussian distribution, for which dimensions of actions are correlated. In particular, the mean vector can represent the deterministic action chosen by the policy, and the covariance matrix can represent the noise introduced by the exploration and approximation through deep neural networks.

However, an agent’s policy may have multiple modes in some environments, e.g., jumping over an obstacle or passing it by crouching in a 2D view. In such cases, the distributions under the actions selected by each policy mode will be
We now have the batch of current policy’s action decisions on the states from the off-policy transitions. We now show how to derive importance weights for the off-policy experiences. The agent samples a batch of off-policy transitions from behavioral policies of the agent that generated the sampled off-policy transitions. To construct a multivariate Gaussian distribution from the action difference batch, each action in the experience replay buffer [Lin, 1992] corresponds to a multivariate Gaussian distribution from the action difference batch of past policies’ decisions, which corresponds to the temporal difference learning [Sutton, 1988b] becomes:

\[
\mathcal{L}(\theta) = \sum_{t} (r_t + \gamma \mathcal{V}(s_{t+1}) - \mathcal{V}(s_t))^2.
\]

To measure the similarity between the current policy and the policies that executed the off-policy transitions from the sampled batch \( \mathcal{B} \), we first forward pass the states from \( \mathcal{B} \) through the actor-network corresponding to the current policy:

\[
\hat{A}^{[|\mathcal{B}| \times n]} = \pi_\phi(S^{[|\mathcal{B}| \times m]}).
\]

We now have the batch of current policy’s action decisions on the states from the off-policy transitions \( \hat{A}^{[|\mathcal{B}| \times n]} \), and the batch of past policies’ decisions \( A^{[|\mathcal{B}| \times n]} \) from \( \mathcal{B} \). Let \( \hat{A}^{[|\mathcal{B}| \times n]} \) be the batch of numerical differences in action decisions, that is:

\[
\hat{A}^{[|\mathcal{B}| \times n]} := A^{[|\mathcal{B}| \times n]} - \hat{A}^{[|\mathcal{B}| \times n]}.
\]

Observe that \( \hat{A}^{[|\mathcal{B}| \times n]} \) indicates the numerical deviation between the action decisions of the current policy and previous behavioral policies of the agent that generated the sampled off-policy transitions. To construct a multivariate Gaussian distribution from the action difference batch \( \mathcal{N}(\hat{\mu}^{[|\mathcal{B}| \times n]}, \Sigma_{n \times n}) \), let:

\[
\hat{\mu}^{[|\mathcal{B}| \times n]} = \frac{1}{|\mathcal{B}|} \sum_{i=1}^{[|\mathcal{B}|]} A_i^{[|\mathcal{B}| \times n]},
\]

\[
\Sigma_{n \times n} = \frac{1}{|\mathcal{B}| - 1} \sum_{i=1}^{[|\mathcal{B}|]} (A_i^{[|\mathcal{B}| \times n]} - \hat{\mu}^{[|\mathcal{B}| \times n]})^T (A_i^{[|\mathcal{B}| \times n]} - \hat{\mu}^{[|\mathcal{B}| \times n]}),
\]

where \( A_i^{[|\mathcal{B}| \times n]} \) represents the \( i \)th row vector in \( A^{[|\mathcal{B}| \times n]} \) which corresponds to the \( i \)th transition. Then, define the dissimilarity measure as:

\[
\rho = \text{JSD}(\mathcal{N}(\hat{\mu}^{[|\mathcal{B}| \times n]}, \Sigma_{n \times n}), \mathcal{N}(\hat{\mu}^{[|\mathcal{B}| \times 1]}, \Sigma_{1 \times 1})),
\]

where \( \sigma \) is the standard deviation of the exploration noise, and \( I^{n \times n} \) is the identity matrix. In addition, \( \text{JSD} \) denotes the Jensen-Shannon divergence, the symmetrized and smoothed version of the Kullback–Leibler (KL) divergence defined by:

\[
\text{JSD}(P || Q) = \frac{1}{2} \text{KL}(P || M) + \frac{1}{2} \text{KL}(Q || M);
\]

\[
M = \frac{1}{2}(P + Q),
\]

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where KL denotes the KL-divergence, $P$ and $Q$ can be any probability distribution. We choose JSD for symmetric similarity measurement as the similarity of two policy distributions should not be assumed to be directed. Although KL-divergence is well-known for penalizing a distribution that is completely different from the distribution in interest, two policies in the same environment cannot be completely distinct [Sutton and Barto, 2018]. Hence, we do not need to penalize two distinct distributions heavily.

**Remark 1.** Jensen-Shannon divergence provides a symmetric similarity measurement between two probability distributions by utilizing Kullback-Leibler divergence. Moreover, KL-divergence heavily penalizes distributions that are entirely different from each other. Therefore, the smoothed version of KL-divergence, JS-divergence, should be used to measure the similarity between two behavioral policies throughout the training, as two policies in the same environment cannot be completely distinct.

Rather than directly comparing the action difference with a zero multivariate Gaussian in Equation (25), we compute JSD using a reference matrix of zero-mean Gaussian with a diagonal covariance matrix having the standard deviation of the exploration noise in the diagonal entries. Otherwise, policies closer to the current policy that executed a transition with exploration noise may be rejected as the action decisions numerically deviate from the actual policy due to the additive exploration noise such as additive Gaussian noise [Williams, 1992] or Ornstein-Uhlenbeck noise process [Uhlenbeck and Ornstein, 1930] utilized by the Deep Deterministic Policy Gradient algorithm [Lillicrap et al., 2016]. Thus, we couple the standard deviation of the exploration noise with the reference matrix to form a parameter-free algorithm.

Naturally, if all transitions in the sampled batch match the distribution under the current policy, we have $\rho = 0$ and $\rho \in (0, \infty)$. To project the similarity measure into the interval $[0, 1]$ for bounded weights, a non-linear transformation can be applied:

$$\lambda = e^{-\rho}.$$  \hspace{1cm} (28)

Notice that for two identical policies we have $\lambda = 1$, and for completely distinct policies we have $\lambda = 0$, making Equation (28) a similarity measure between two policies.

Intuitively, the deterministic variant of AC-Off-POC first computes the numerical difference between the actions chosen by the target and behavioral policies, then compares the difference with zero. This approach is equivalent to an implicit comparison of the distributions under the current policy and the policies that executed the off-policy transitions. However, one concern with the deterministic variant of AC-Off-POC may be that the minority of the transitions within the sampled batch may be executed by policies very similar to the current agent’s policy. Since we take the average of the action batch in computing the importance weight, i.e., Equation (23), those transitions may be weighted by a fixed weight close to 0, which results in loss of information. We address this issue and conclude our off-policy correction scheme for deterministic policies in Remark 2, and provide an intuitive analysis on the impact of mini-batch size in Observation 1.

**Remark 2.** The deterministic variant of AC-Off-POC measures the overall discrepancy between the sampled batch of transitions and the underlying distribution of the current policy. If the distribution under the majority of the transitions in a sampled batch diverges from the current agent’s policy, the corresponding importance weights will be small, or vice versa. Nevertheless, it should be expected that the policies corresponding to the majority of the transitions in a sampled batch must be close to the current policy since the networks in actor-critic methods are already optimized through mini-batch learning.

**Observation 1.** Let $|B|$ and $|R|$ are the fixed mini-batch and experience replay buffer [Lin, 1992] sizes used in the training. If $|B| \rightarrow |R|$, then the mean indices of the sampled transitions will be 0.5 in the expectation, and the deterministic AC-Off-POC weights become slightly less than 0.5 due to the non-linear transformation. If $|B| \rightarrow 0$, then the AC-Off-POC weights cannot be accurately estimated. Therefore, very large or small mini-batch sizes may result in poor performance.

### 4.2 Stochastic Policies

The extension of AC-Off-POC to stochastic policies is trivial as probability distributions represent the policies. Although the known probability distributions can be directly used in the traditional importance sampling methods through eligibility traces [Watkins, 1989, Singh and Sutton, 1996], they require sufficiently long trajectories executed by each policy, such as in [Espeholt et al., 2018, Schmitt et al., 2020, Munos et al., 2016, Harutyunyan et al., 2016]. Instead, stochastic AC-Off-POC can apply an off-policy correction on randomly selected off-policy transitions to control continuous systems.

In this variant of AC-Off-POC, we do not need assumptions on the type of distribution the actions are sampled. Therefore, we can store the parameters of the policy distributions in the replay buffer [Lin, 1992]. For example, if the stochastic policy of an agent is represented by a Beta distribution, $B(\alpha, \beta)$, we store the parameters $\alpha$ and $\beta$ in...
the experience replay buffer [Lin, 1992] and sample these parameters along with states, actions, and rewards at every update step.

At each gradient step, the agent samples a batch of off-policy transitions along with the parameters that define the policy distribution:

\[ (S^{[B]}_1 \times m, A^{[B]}_1 \times n, R^{[B]}_1 \times 1, S^{[B]}_1 \times m, \alpha_1, \ldots, \alpha_{|B|}) \sim B, \]

where \( \alpha_1, \ldots, \alpha_{|B|} \) represent the distribution parameters for each behavioral policy that executed the transitions indexed by \( 1, \ldots, |B| \), respectively. Each behavioral policy \( \eta_i \) that executed the \( i \)-th transition can be represented with the corresponding distribution parameters, \( \eta_i(\cdot; \alpha_i) \). Instead of comparing the difference batch with a reference Gaussian, we can directly measure the similarity between the distribution under the current policy \( \eta^\pi(\cdot) \) and \( \eta_i(\cdot; \alpha_i) \):

\[ \rho_i = \text{JS}(\eta^\pi(\cdot) || \eta_i(\cdot; \alpha_i)), \]
\[ \lambda_i = e^{-\rho_i}. \]

Then, we construct an importance weight vector containing the weights for the sampled batch of transitions:

\[ \lambda^{[B]}_1 := [\lambda_1 \ldots \lambda_{|B|}]^T. \]

Similar to Equation (19) and Equation (20), we can derive the weighted policy gradient and critic loss for stochastic actor-critic as:

\[ \nabla_{\phi} J_{\text{sto}}(\theta) = \frac{1}{||\lambda^{[B]}_1||_1} \sum_{i=1}^{|B|} \lambda^{[B]}_i \log \pi_\theta(A^{[B]}_i | S^{[B]}_i) \circ Q_\theta(S^{[B]}_i, A^{[B]}_i), \]
\[ J_{\text{sto}}(\theta) = \frac{||\lambda^{[B]}_1 \circ \delta^{[B]}_1||_2^2}{||\lambda^{[B]}_1||_1^2}, \]

where \( || \cdot ||_1 \) represents the L_1 norm.

In the stochastic variant, each off-policy transition has a unique weight as it can be computed using a known mean vector and covariance matrix for each policy corresponding to that transition. Thus, we compute the loss for actor and critic parameters using a weighted sum of the off-policy transitions. However, for deterministic policies, all off-policy transitions in \( B \) are weighted by the same importance weight \( \lambda \). Therefore, the deterministic variant weighs the sampled batch through the overall discrepancy instead of a weighted sum. As a result, the concern with the loss of information for the deterministic variant is overcome in the stochastic policy case. Nonetheless, the loss of information with deterministic policies can still be neglected as the sampled batch should be similar to the current policy on average [Fujimoto et al., 2019]. We adapt Remark 2 to the stochastic variant of AC-Off-POC in the following.

**Remark 3.** Due to the transition-wise similarity measurement in the stochastic variant of AC-Off-POC, the off-policy correction is not affected by the mini-batch size. However, the impact of the batch size can still be observable on the underlying off-policy actor-critic algorithm, which may affect the performance of stochastic AC-Off-POC application.

We refer to the resulting parameter-free algorithm as **Actor-Critic Off-Policy Correction (AC-Off-POC)** and provide pseudocode in Algorithm 1. In the next section, we describe a generic off-policy actor-critic framework with AC-Off-POC, explain the generalizability of our method, and make a theoretical analysis of the contraction mapping produced by the off-policy correction coefficients.

**4.3 Off-Policy Actor-Critic with AC-Off-POC**

As discussed, trajectory-based importance sampling methods can introduce high-variance build-up or vanishing gradients. Specifically, when each importance weight in a trajectory is larger than 1, the multiplication of the weights causes a detrimental accumulated variance that can diminish the accuracy of the trajectory weight. In contrast, when the weights are closer to 0, the trajectory weight becomes nearly 0, which yields the gradients to vanish; therefore, no updates are to be performed. However, these issues cannot be observed in AC-Off-POC since our method does not consider temporally correlated samples. Instead, single transitions are weighted by coefficients constrained in the interval of \([0, 1]\), i.e., weights of the off-policy transitions in a single update do not affect the weights in the next update. Thus, continuous control with AC-Off-POC offers safe and efficient off-policy learning. We summarize through the following remark how AC-Off-POC can overcome the mentioned issues observed in the importance sampling methods and why the introduced approach is preferable to providing off-policy corrections compared to standard importance sampling.
We introduce a general off-policy actor-critic framework with AC-Off-POC in Algorithm 2 and perform an intuitive similarity measure to correct the extrapolation error. Another concern with the off-policy correction is that some environments may broadly require off-policy samples to solve, e.g., transitions of a random policy can also be used to improve a suboptimal policy. In such cases, an off-policy correction that filters out the on-policy samples may degrade the performance. This is addressed by Equation (28) and Remark 1. Therefore, the AC-Off-POC weights should be near zero only when the policies are quite different. In this particular example, the variance in the fitted normal distribution would mitigate the unnecessary trajectory termination issue since AC-Off-POC only considers transition-wise or temporally uncorrelated batch-wise similarities.

One concern with the introduced approach is that the mean difference could be zero even when the behavior policies are quite different from the current policy. Suppose the action range is $[-10, 10]$, and the current policy outputs action 0, half of past actions are -10, and the other half are 10. The fitted normal distribution will have a mean zero even though the policies are quite different. In this particular example, the variance in the fitted normal distribution would mitigate this issue, i.e., the empirical variance would be greater than the exploration noise added to the current policy, and hence, the similarity weights will be small.

Another concern with the off-policy correction is that some environments may broadly require off-policy samples to be solved, e.g., transitions of a random policy can also be used to improve a suboptimal policy. In such cases, an off-policy correction that filters out the on-policy samples may degrade the performance. This is addressed by Equation (28) and Remark 1. Therefore, the AC-Off-POC weights should be near zero only when the distributions under two policies are nearly distinct, allowing beneficial off-policy samples to be included in the training. How AC-Off-POC performs this is explained in Remark 5.

Remark 4. As a single coefficient weighs each transition, vanishing or exploding gradient problem due to the product of weights in the trajectory-based importance sampling methods does not occur in AC-Off-POC. This is also valid for the unnecessary trajectory termination issue since AC-Off-POC only considers transition-wise or temporally uncorrelated batch-wise similarities.

Algorithm 1 Actor-Critic Off-Policy Correction (AC-Off-POC)

1: Input: $\pi_\phi, B$
2: Output: $\lambda \lor \lambda^{[|B|\times 1]}$
3: if $\pi_\phi$ is deterministic then
4: Compute the current policy’s action decisions on the sampled batch of off-policy states:
   $\hat{A}^{[|B|\times n]} = \pi_\phi(S^{[|B|\times m]}).$
5: Compute the numerical differences between the current agent’s actions and off-policy actions:
   $\hat{A}^{[|B|\times n]} := A^{[|B|\times n]} - \hat{A}^{[|B|\times n]}.$
6: Construct the multivariate Gaussian distribution, $\mathcal{N}(\mu^{1\times n}, \Sigma^{n\times n})$ where:
   $\mu^{1\times n} = \frac{1}{|B|} \sum_{i=1}^{|B|} \hat{A}_i^{[|B|\times n]}$, 
   $\Sigma^{n\times n} = \frac{1}{|B|-1} \sum_{i=1}^{|B|} (\hat{A}_i^{[|B|\times n]} - \mu^{1\times n})^T(\hat{A}_i^{[|B|\times n]} - \mu^{1\times n}).$
7: Compute the similarity $\lambda$: $\rho = JSD(\mathcal{N}(\hat{\mu}^{1\times n}, \hat{\Sigma}^{n\times n})||\mathcal{N}(0^{1\times n}, \sigma I^{n\times n})) \Rightarrow \lambda = e^{-\rho}$.
8: else
9: Obtain the batch containing the policy distribution parameters:
   $(S^{[|B|\times m]}, A^{[|B|\times n]}, R^{[|B|\times 1]}, S'^{[|B|\times m]}, \alpha_1, \ldots, \alpha_{|B|}) \sim B.$
10: Compute the similarities $\lambda^{[|B|\times 1]} = [\lambda_1 \ldots \lambda_{|B|}]^T$; $\rho_i = JSD(\eta^\pi(\cdot)||\eta(\cdot; \alpha_i)) \Rightarrow \lambda_i = e^{-\rho_i}$.
11: end if
12: return $\lambda \lor \lambda^{[|B|\times 1]}$

Remark 5. In environments that vastly require off-policy transitions to be solved, AC-Off-POC can still include transitions that diverge from the current agent’s policy to an extent due to the non-linear transformation $e^{-\rho}$ and Jensen-Shannon divergence used in the similarity measurements, which does not heavily penalize two different distributions unless they are nearly distinct.

Furthermore, as AC-Off-POC operates as a learning rate scheduler to adjust the contribution of each gradient with respect to off-policy samples, it can be employed in learning from a fixed dataset such as batch constrained RL algorithms and imitation learning. By allowing off-policy transitions correlated to the distribution under the current policy, AC-Off-POC can inhibit the extrapolation error, a phenomenon caused by the mismatch between the distributions corresponding to the off-policy data collected by a different agent and the latest agent’s policy [Fujimoto et al., 2019]. Thus, it can be applied to various off-policy reinforcement learning applications. However, batch constraint RL and imitation learning are not in the scope of this work as they require additional data generation modules.

We introduce a general off-policy actor-critic framework with AC-Off-POC in Algorithm 2 and perform an intuitive complexity analysis in Remark 6. Furthermore, we emphasize the “safeness” of our off-policy correction method in Corollary 1, where we show that the presented method defines a contraction mapping around the produced weights, i.e.,
Algorithm 2 A General Framework for Off-Policy Actor-Critic with AC-Off-POC

1: Initialize the agent with actor $\pi_\phi$ and critic $Q_\theta$ networks with parameters $\phi, \theta$, respectively.
2: Initialize target and additional behavioral networks if required.
3: Initialize the experience replay buffer $\mathcal{R}$.
4: for each exploration time step do
5: Obtain transition tuple $\tau$.
6: Store transition tuple $\tau$ in $\mathcal{R}$.
7: end for
8: for each training iteration do
9: Sample a batch of transitions $B$ from $\mathcal{R}$ by a sampling algorithm.
10: Obtain the importance weight(s): $\lambda \lor \lambda^{[B]\times 1} = \text{AC-Off-POC}(\pi_\phi, B)$.
11: Update actor and critic networks with an off-policy continuous actor-critic algorithm using policy gradients and losses weighted by $\lambda \lor \lambda^{[B]\times 1}$, e.g., Equation (19), Equation (20), Equation (33) and Equation (34).
12: Update target networks if required.
13: end for

14: Update actor and critic networks with an off-policy continuous actor-critic algorithm using policy gradients and losses weighted by $\lambda \lor \lambda^{[B]\times 1}$, e.g., Equation (19), Equation (20), Equation (33) and Equation (34).

15: end for

16: end for

17: end for

the Q-value error interval shrinks as there exists a large similarity between behavioral policies corresponding to the sampled off-policy transitions and the policy of interest.

Remark 6. In the worst case, i.e., AC-Off-POC consists of a single forward pass through the actor network and several arithmetic matrix operations for deterministic policies. It is known that a single pass through a feedforward neural network has a time complexity of $O(n^2)$. This value dominates the rest of the operations, which have at most quadratic runtime of $O(n^2)$.

Considering a simple deep actor-critic algorithm that involves a single actor and critic networks, the backpropagation operations will run in $O(n^2)$. Therefore, even under the most straightforward deep actor-critic algorithm, the complexity of AC-Off-POC will be dominated by the backpropagation through the actor and critic networks.

Definition 1. The general expectation operator for one-step importance sampling in return-based off-policy algorithms is defined in the bootstrapped form by:

$$H Q(s, a) := Q(s, a) + \mathbb{E}_{\eta} \sum_{t \geq 0} \gamma^t \lambda_t \left( r_t + \gamma \mathbb{E}_\pi Q(s_{t+1}, \cdot) - Q(s_t, a_t) \right) ,$$

(35)

for transition tuple $(s_t, a_t, r_t, s_{t+1})$ and some non-negative one-step importance sampling coefficient $\lambda_t$ at time step $t$, and any behavioral policy $\eta$, where we write $\mathbb{E}_\pi Q(s, \cdot) := \sum_{a} \pi(a|s)Q(s, a)$.

Lemma 1. The difference between $H Q$ and its fixed point $Q^\pi$ is expressed by:

$$H Q(s, a) - Q^\pi(s, a) = \mathbb{E}_{\eta} \sum_{t \geq 1} \gamma^t \left( \mathbb{E}_\pi \left( \left( Q - Q^\pi \right)(s_t, \cdot) \right) - \lambda_t (Q - Q^\pi)(s_t, a_t) \right)$$

(36)

Proof. The proof reduces the proof of Lemma 1 by Munos et al. [2016] to one-step importance sampling. First, rewrite Equation (35) in terms of the next state-action pair:

$$H Q(s, a) = \sum_{t \geq 0} \gamma^t \mathbb{E}_\eta \left[ \lambda_t (r_t + \gamma \mathbb{E}_\pi Q(s_{t+1}, \cdot) - \lambda_{t+1} Q(s_{t+1}, a_{t+1})) \right] ,$$

(37)

Since $Q^\pi$ is the fixed point of $H$, we have:

$$Q^\pi(s, a) = H Q^\pi(s, a) = \sum_{t \geq 0} \gamma^t \mathbb{E}_\eta \left[ \lambda_t (r_t + \gamma \mathbb{E}_\pi Q^\pi(s_{t+1}, \cdot) - \lambda_{t+1} Q^\pi(s_{t+1}, a_{t+1})) \right] .$$

(38)

Letting $\Delta Q := Q - Q^\pi$, we infer that:

$$H Q(s, a) - Q^\pi(s, a) = \sum_{t \geq 0} \gamma^t \mathbb{E}_\eta \left[ \lambda_t \left( \mathbb{E}_\pi \Delta Q(s_{t+1}, \cdot) - \lambda_{t+1} \Delta Q(s_{t+1}, a_{t+1}) \right) \right] .$$

(39)

Decrementing a single step, the latter equation is equal to the following due to the bootstrapping in the temporal difference learning [Sutton, 1988b]:

$$H Q(s, a) - Q^\pi(s, a) = \sum_{t \geq 1} \gamma^t \mathbb{E}_\eta \left[ \mathbb{E}_\pi \left( \Delta Q(s_t, \cdot) \right) - \lambda_t \Delta Q(s_t, a_t) \right] .$$

(40)
Taking the \( \sum_{t \geq 1} \gamma^t \) term into the expectation as \( \gamma \) is fixed and expanding \( \Delta Q \), we have:

\[
\mathcal{H}Q(s, a) - Q^\pi(s, a) = \mathbb{E}_t \left[ \sum_{t \geq 1} \gamma^t (\mathbb{E}_\pi [(Q - Q^\pi)(s_t, \cdot)] - \lambda_t (Q - Q^\pi)(s_t, a_t)) \right],
\]

(41)

**Theorem 1.** The operator \( \mathcal{H} \) defined by Definition 1 has a unique fixed point \( Q^\pi \). Moreover, if for each action selected by the policy \( a_t \in A \) and sampled batch of transitions \( B_t \) at time step \( t \), we have \( \lambda_t = \lambda(a_t, B_t) \in [0, e^{-\rho}] \). Then for any \( Q \)-function, we have \( \mathcal{H} \), which is a \( \gamma \)-contraction mapping around \( Q^\pi \):

\[
\| \mathcal{H}Q - Q^\pi \| \leq \gamma \| Q - Q^\pi \|,
\]

(42)

under the current policy \( \pi \).

**Proof.** Follows from the adaptation of proof of Theorem 1 by Munos et al. [2016] to one-step importance sampling. It is trivial to observe from Definition 1 that \( Q^\pi \) is the fixed point of the operator \( \mathcal{H} \) since:

\[
\mathbb{E}_{s_{t+1} \sim P(\cdot|s_t, a_t)} [r_t + \gamma \mathbb{E}_a Q^\pi(s_{t+1}, \cdot) - Q^\pi(s_t, a_t)] = (\mathcal{H}Q^\pi - Q^\pi)(s_t, a_t) = 0,
\]

(43)
as \( Q^\pi \) is the fixed point of \( \mathcal{H}^\pi \). Let \( \Delta Q := Q - Q^\pi \) and from Lemma 1, we have:

\[
\mathcal{H}Q(s, a) - Q^\pi(s, a) = \sum_{t \geq 1} \gamma^t \mathbb{E}_{a_t, s_t} \mathbb{E}_\pi [\Delta Q(s_{t+1}, \cdot) - \lambda_t \Delta Q(s_t, a_t)],
\]

(44)

\[
= \sum_{t \geq 1} \gamma^t \mathbb{E}_{a_t, s_t} [\mathbb{E}_\pi [\Delta Q(s_t, \cdot)] - \mathbb{E}_a [\lambda_t (a_t, B_t) \Delta Q(s_t, a_t)] | B_t],
\]

(45)

\[
= \sum_{t \geq 1} \gamma^t \mathbb{E}_{a_t, s_t} \left[ \sum_b (\pi(b|s_t) - \eta(b|s_t) \lambda_t(b, B_t)) \Delta Q(s_t, b) \right].
\]

(46)

Since \( \lambda \in [0, 1] \), we have \( \pi(a|s_t) - \eta(a|s_t) \lambda_t(a, B_t) \geq 0 \) [Munos et al., 2016]. This yields:

\[
\mathcal{H}Q(s, a) - Q^\pi(s, a) = \sum_{y, b} w_{y, b} \Delta Q(y, b),
\]

(47)

which is a linear combination of \( \Delta Q(y, b) \) weighted by non-negative coefficients \( \lambda \):

\[
w_{y, b} := \sum_{t \geq 1} \gamma^t \mathbb{E}_{a_t, s_t} (\pi(b|s_t) - \eta(b|s_t) \lambda_t(b, B_t) \mathbb{I}(s_t = y)),
\]

(48)

where \( \mathbb{I}(\cdot) \) is the indicator function. The sum of those coefficients over \( y \) and \( b \) is:

\[
\sum_{y, b} w_{y, b} = \sum_{t \geq 1} \gamma^t \mathbb{E}_{a_t, s_t} \mathbb{E}_b [\pi(b|s_t) - \eta(b|s_t) \lambda_t(b, B_t)],
\]

(49)

\[
= \sum_{t \geq 1} \gamma^t \mathbb{E}_{a_t, s_t} [1 - \lambda_t(a_t, B_t)] = \sum_{t \geq 1} \gamma^t \mathbb{E}_{a_t, s_t} [1 - \lambda_t],
\]

(50)

\[
= \mathbb{E}_\eta [\sum_{t \geq 1} \gamma^t - \sum_{t \geq 1} \gamma^t \lambda_t]
\]

(51)

Clearly, we have \( \sum_{y, b} w_{y, b} \leq \gamma \) for any \( t \geq 1 \) and \( \lambda_t \in [0, 1] \). Therefore, \( \mathcal{H}Q(s, a) - Q^\pi(s, a) \) is a sub-convex combination of \( \Delta Q(y, b) \) weighted by non-negative coefficients \( \omega_{y, b} \) which sum to at most \( \gamma \). Hence, \( \mathcal{H} \) is a \( \gamma \)-contraction mapping around \( Q^\pi \). 

**Corollary 1.** In the proof of Theorem 1, notice that the term \( \mathbb{E}_\eta [\sum_{t \geq 1} \gamma^t \lambda_t] \) depends on \( (s, a) \) since \( \lambda_t \) already depends on the sampled batch \( B \). Let:

\[
\xi(s, a) := \sum_{y, b} w_{y, b}.
\]

(52)

Then, we can show that:

\[
|\mathcal{H}Q(s, a) - Q^\pi(s, a)| \leq \xi(s, a)\|Q - Q^\pi\|,
\]

(53)

Thus, \( \xi(s, a) \in [0, \gamma] \) is a contraction coefficient based on \( (s, a) \) where \( \xi(s, a) = \gamma \) if \( \lambda = 0 \), i.e., when there is no similarity, and close to zero when the behavioral policies corresponding to the sampled batch \( B \) match the current policy.
5 Experiments

We conduct experiments to evaluate the effectiveness of our off-policy correction approach. In Section 5.2, we present the simulation results for the comparative evaluation with different importance sampling methods and off-policy policy gradient techniques. We conduct experiments under different batch and replay memory sizes, experience replay sampling methods, and divergence measures in Section 5.3 for the sensitivity analysis of the introduced off-policy correction scheme. In addition, weights produced by AC-Off-POC are visualized and discussed in Section 5.4. We use the MuJoCo [Todorov et al., 2012] and Box2D [Parberry, 2013] benchmarks interfaced by OpenAI Gym [Brockman et al., 2016]. For reproducibility, we do not modify the environment dynamics and reward functions. The computing infrastructure for reproducibility is summarized in our repository.

5.1 Experimental Setup and Implementation

We apply our method to three baseline off-policy actor-critic methods: Deep Deterministic Policy Gradient (DDPG) [Lillicrap et al., 2016], Soft Actor-Critic [Haarnoja et al., 2018] and Twin Delayed DDPG (TD3) [Fujimoto et al., 2018]. For comparative evaluation, we consider each baseline with and without AC-Off-POC, the continuous importance sampling method, RIS-Off-PAC [Humayoo and Cheng, 2019], and off-policy policy gradient methods, ACE [Imani et al., 2018], Geoff-PAC [Zhang et al., 2019] and IPG [Gu et al., 2017]. In the sensitivity analysis, we use the experience replay [Lin, 1992] sampling methods of Combined Experience Replay (CER) [Zhang and Sutton, 2017], Experience Replay Optimization (ERO) [Zha et al., 2019] and Prioritized Experience Replay (PER) [Schaul et al., 2016].

We use the stochastic variant of the SAC algorithm [Haarnoja et al., 2018] to show the performance improvement when stochastic AC-Off-POC is applied. Environment-specific and shared parameters of SAC [Haarnoja et al., 2018] follows the tuned hyper-parameters from OpenAI Baselines3 Zoo [Raffin, 2020]. As the actions are sampled from the environment’s action space in the initial exploration steps, policy distribution parameters are not accessible for stochastic AC-Off-POC. Rather, we start storing these parameters after a multiple of the number of exploration time steps and remove the exploration transitions from the experience replay buffer [Lin, 1992]. Our implementation of DDPG [Lillicrap et al., 2016] also closely matches the hyper-parameter setting outlined in the original paper. Different from the proposed setting, we add 1000 exploration time steps with frozen parameters at the beginning of each training and increase the batch size from 64 to 256 for the agents to encounter more off-policy samples in each update. Moreover, we replace Ornstein–Uhlenbeck exploration noise with a zero-mean Gaussian with a standard deviation of 0.1, shown when stochastic AC-Off-POC is applied. Environment-specific and shared parameters of SAC [Haarnoja et al., 2018] are applied to the baselines by replacing the policy gradient computation. The importance sampling method, RIS-Off-PAC [Humayoo and Cheng, 2019] is directly applied on top of the baseline algorithms. We follow the original papers to implement ACE [Imani et al., 2018], IPG [Gu et al., 2017], and RIS-Off-PAC [Humayoo and Cheng, 2019].

Our implementation of Geoff-PAC [Zhang et al., 2019] is based on the code from authors’ GitHub repository. We follow the exact structure given in the original papers to implement CER [Zhang and Sutton, 2017] and ERO [Zha et al., 2019]. Additionally, the repository is used to implement PER [Schaul et al., 2016].

The considered competing algorithms introduce hyper-parameters that either control the bias-variance trade-off or on-policyness of the updates. We use the tuned hyper-parameters provided in the original articles. In particular, we use the bias-variance trade-off value of $v = 1$ for IPG [Gu et al., 2017] as it is defined for actor-critic methods with corrected off-policy samples. As stated by Zhang et al. [2019], ACE [Imani et al., 2018] is not sensitive for the bias parameter $\lambda_1$ on OpenAI Gym [Brockman et al., 2016] benchmarks and $\lambda_1 = 0$ can produce sufficiently good results. Hence, we set $\lambda_1 = 0$ for ACE [Imani et al., 2018]. For Geoff-PAC [Zhang et al., 2019], however, we employ the tuned hyper-parameters of $\lambda_1 = 0.7$, $\lambda_2 = 0.6$, and $\hat{\gamma} = 0.2$ since they are reported to result in good empirical performance. Finally for RIS-Off-PAC [Humayoo and Cheng, 2019], we use 0.2 for the smoothing control parameter $\beta$. For all simulations, each algorithm is run for 1 million time steps with evaluations occurring every 1000 time steps, where an evaluation of an agent records the average reward over ten episodes in a distinct evaluation environment without exploration noise and updates. We report the average evaluation return of ten random seeds for initializing networks.

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1. https://github.com/DLR-RM/rl-baselines3-zoo
2. https://github.com/sfujim/LAP-PAL
3. https://github.com/sfujim/TD3
4. https://github.com/ShangtongZhang/DeepRL
5.2 Comparative Evaluation

Evaluation results for the DDPG [Lillicrap et al., 2016], SAC [Haarnoja et al., 2018] and TD3 [Fujimoto et al., 2018] algorithms are reported in Figure 1, 2, and 3, respectively. As learning curves are intertwined and hard to follow for some of the environments, we also report the average of the last ten evaluation returns, i.e., where the algorithms converge, in Table 1. Note that for some of the tasks, the baseline actor-critic algorithms have worse performance than reported in the original articles. This is due to the stochasticity of the simulators and used random seeds. Nevertheless, the performance difference between competing methods would be consistent if we used different sets of random seeds, regardless of where the baselines converge. Therefore, our comparative evaluations are fair per the standard deep RL benchmarking [Henderson et al., 2018].

We observe that both variants of AC-Off-POC significantly improve the performance of the baseline methods and either match or outperform the competing off-policy correction techniques in all of the domains and baselines tested. This shows that our off-policy correction method is scalable to many off-policy actor-critic algorithms, as previously discussed. The performance improvement obtained by AC-Off-POC is mainly more observable when the baselines are stuck at the local optima in challenging tasks, e.g., Ant, BipedalWalker, Humanoid, and Swimmer [Henderson et al., 2018]. The adversity of these environments usually comes from the dimensionality, e.g., large state-action spaces or state-action intervals, or internal simulation dynamics [Henderson et al., 2018]. Thus, the fatal effects of off-policy samples are further amplified in these tasks, and the resulting improvement offered by an effective off-policy correction technique becomes more prominent.

Furthermore, we observe more substantial improvement by AC-Off-POC in the BipedalWalker, Hopper, Swimmer, and Walker2d environments. As shown by Henderson et al. [2018], on-policy algorithms, i.e., which learn from the data that the policy of interest collected, outperform the off-policy methods by a considerable margin in these tasks. Hence, we deduce that these environments may vastly require on-policy samples to be solved for a stable and better learning process. Hence, corrected off-policy transitions maximize the improvement compared to other environments where off-policy algorithms are often superior to the on-policy algorithms, e.g., Ant, HalfCheetah, and Humanoid [Henderson et al., 2018]. Nonetheless, we still observe improved performance in these domains. We believe that this improvement is caused by only eliminating very divergent off-policy samples collected throughout the learning process since the
| Method                  | Ant         | BipedalWalker | HalfCheetah | Hopper      | Humanoid | LunarLanderContinuous | Swimmer | Walker2d |
|------------------------|-------------|---------------|-------------|-------------|----------|-----------------------|---------|----------|
| DDPG                   | 816.99 ± 165.29 | -139.73 ± 36.10 | 3798.16 ± 1586.74 | 2137.20 ± 537.64 | 206.80 ± 105.63 | 26.69 ± 73.32 | 19.71 ± 5.90 | 1270.86 ± 332.23 |
| DDPG + AC-Off-POC      | 1052.28 ± 239.11 | -104.79 ± 38.39 | 5012.32 ± 2155.64 | 2335.00 ± 765.72 | 1115.81 ± 641.76 | 173.02 ± 85.59 | 28.95 ± 7.88 | 2054.85 ± 555.45 |
| DDPG + AC-CE           | 829.96 ± 133.42 | -122.38 ± 33.63 | 4043.05 ± 1672.13 | 1742.74 ± 488.32 | 204.14 ± 100.09 | 161.61 ± 58.27 | 20.82 ± 12.27 | 1742.84 ± 414.89 |
| DDPG + Geof-PAC        | 784.67 ± 133.77 | -125.71 ± 36.54 | 4660.93 ± 1951.33 | 1767.06 ± 676.08 | 235.75 ± 115.61 | 210.53 ± 70.60 | 27.80 ± 5.45 | 2030.25 ± 625.53 |
| DDPG + IPG             | 833.51 ± 88.32 | -121.91 ± 39.11 | 4248.09 ± 1796.15 | 1905.45 ± 731.71 | 208.46 ± 84.78 | 207.11 ± 69.60 | 24.55 ± 5.47 | 1601.75 ± 516.69 |
| DDPG + RIS-Off-PAC     | 837.10 ± 107.24 | -107.97 ± 31.83 | 4171.61 ± 1753.42 | 1819.95 ± 450.00 | 198.50 ± 81.86 | 196.59 ± 66.94 | 21.93 ± 6.86 | 1644.48 ± 462.71 |
| SAC                    | 4864.24 ± 296.94 | 285.37 ± 50.74 | 9871.42 ± 273.98 | 3351.43 ± 317.07 | 2689.22 ± 1499.29 | 271.37 ± 9.82 | 53.11 ± 4.68 | 4103.54 ± 805.03 |
| SAC + AC-Off-POC       | 5983.03 ± 341.04 | 343.96 ± 6.93 | 11320.82 ± 262.23 | 3846.05 ± 777.89 | 5098.33 ± 529.73 | 284.83 ± 7.09 | 130.29 ± 19.46 | 5366.81 ± 819.00 |
| SAC + AC-CE            | 5019.17 ± 549.69 | 307.18 ± 17.98 | 9375.53 ± 410.10 | 3367.48 ± 313.79 | 4545.63 ± 415.48 | 277.18 ± 7.91 | 60.89 ± 5.46 | 3239.37 ± 1193.46 |
| SAC + Geof-PAC         | 5281.92 ± 471.66 | 297.24 ± 29.89 | 10604.40 ± 173.26 | 2636.44 ± 791.47 | 4897.81 ± 508.99 | 279.75 ± 9.46 | 96.24 ± 14.80 | 4367.23 ± 1217.94 |
| SAC + IPG              | 5173.16 ± 274.65 | 318.80 ± 9.41 | 9993.52 ± 399.77 | 2906.38 ± 609.73 | 4768.96 ± 457.70 | 276.29 ± 11.56 | 84.20 ± 16.11 | 3456.34 ± 1486.13 |
| SAC + RIS-Off-PAC      | 4613.55 ± 430.51 | 313.67 ± 12.80 | 10267.73 ± 325.33 | 3036.12 ± 534.42 | 4646.68 ± 459.60 | 276.06 ± 11.19 | 61.90 ± 9.43 | 4035.21 ± 976.43 |
| TD3                    | 4846.66 ± 418.23 | 271.77 ± 79.60 | 10050.95 ± 168.03 | 3174.56 ± 401.05 | 4929.35 ± 331.30 | 277.36 ± 5.42 | 52.18 ± 4.45 | 4107.08 ± 349.81 |
| TD3 + AC-Off-POC       | 5472.19 ± 276.49 | 322.38 ± 6.92 | 11735.35 ± 196.97 | 3658.85 ± 354.52 | 5297.80 ± 308.16 | 308.05 ± 15.24 | 114.02 ± 21.51 | 5219.89 ± 583.36 |
| TD3 + AC-CE            | 5064.34 ± 376.07 | 306.96 ± 15.10 | 10543.87 ± 148.98 | 3215.57 ± 379.31 | 5115.29 ± 219.01 | 279.49 ± 5.27 | 57.18 ± 5.30 | 4179.44 ± 267.09 |
| TD3 + Geof-PAC         | 5266.21 ± 416.70 | 319.52 ± 15.19 | 11467.56 ± 148.70 | 3041.78 ± 516.34 | 4805.35 ± 397.55 | 268.35 ± 6.36 | 80.36 ± 16.91 | 4795.22 ± 232.31 |
| TD3 + IPG              | 5322.47 ± 367.49 | 295.09 ± 44.95 | 11064.32 ± 143.90 | 3524.52 ± 247.26 | 4908.21 ± 482.69 | 280.46 ± 5.32 | 59.94 ± 8.01 | 4751.56 ± 153.17 |
| TD3 + RIS-Off-PAC      | 5045.43 ± 488.06 | 307.22 ± 16.90 | 10811.51 ± 166.86 | 3552.53 ± 128.64 | 5015.17 ± 340.12 | 278.38 ± 6.68 | 56.18 ± 5.13 | 4421.31 ± 194.74 |

Table 1: Average return of last 10 evaluations over 10 trials of 1 million time steps. ± captures a 95% confidence interval over the trials. Bold values represent the maximum under each baseline algorithm and environment.
employed JSD measure assigns very low scores only to transitions that correspond to very distinct behavioral policies. Therefore, although transitions corresponding to different behavioral policies are not similar, they are still included in the Q-learning and policy optimization. Consequently, the latter two empirical findings validate Remarks 1 and 5.

Although the performance enhancements achieved under the SAC [Haarnoja et al., 2018] and TD3 [Fujimoto et al., 2018] algorithms remain practically the same, the overall performance improvement for stochastic policies is slightly more notable than the deterministic policies when DDPG [Lillicrap et al., 2016] is also considered. Nevertheless, the performance improvement on deterministic policies becomes prominent when mini-batch learning is applied as AC-Off-POC completely disregards transitions in the sampled batch. These simulation results verify Remark 2 and 3, that is, more improvement for stochastic policies is expected, yet if the distribution of the sampled transitions is similar to the distribution under the current policy on average, off-policy deterministic actor-critic methods can be further improved. These empirical results suggest that our method effectively boosts the learning of off-policy methods and makes the agents converge to higher evaluation returns in a shorter time by disregarding highly divergent off-policy samples.

Competing methods exhibit notable stability in unstable environments such as Ant, Hopper, and Walker2d. However, the performance improvement is still not promising as they directly disregard off-policy samples through policy gradients or importance sampling. In detail, we observe that Geoff-PAC [Zhang et al., 2019] and IPG [Gu et al., 2017] perform best with similar performances among the competing algorithms, while RIS-Off-PAC [Humayoo and Cheng, 2019] obtains the worst performance and ACE [Imani et al., 2018] remains in the middle. As discussed, RIS-Off-PAC [Humayoo and Cheng, 2019] uses the action-value generated from the behavioral policy in the reward function to train the algorithm rather than from the target policy. However, the behavioral policy that generates the action-value is usually suboptimal throughout the learning process, which yields underperformed baselines. Moreover, the empirical performance of ACE [Imani et al., 2018] is limited to simple domains such as simple Markov chains or cart-pole balancing, as it was previously proposed for linear function approximation [Zhang et al., 2019]. Its generalization to deep neural network function approximation, Geoff-PAC [Zhang et al., 2019], obtains an improved baseline performance, as expected. Although the emphatic approach in Geoff-PAC [Zhang et al., 2019] can get an unbiased sample from the policy gradient, we believe its estimation of the policy gradient is still erroneous and insufficient for optimal performance. Finally, as stated by Gu et al. [2017], IPG yields an on-policy deterministic actor-critic method when $v = 1$ is used. Our previous discussion suggests that a complete on-policy policy gradient can completely disregard the off-policy samples. However, off-policy samples may sometimes be beneficial depending on the task. Therefore, we do not observe the maximal performance by IPG [Gu et al., 2017] but AC-Off-POC.
In summary, off-policy samples usually degrade the performance due to the underlying distribution that substantially diverges from the current agent’s policy. Nonetheless, off-policy methods may still require off-policy samples to learn the environment [Henderson et al., 2018]. AC-Off-POC solves this issue by employing the Jensen-Shannon divergence, which obtains a smooth similarity measurement prior to the non-linear transformation. Its symmetric similarity measurement prevents the off-policy transitions from being heavily penalized and allows them to contribute to the learning progress even with a small proportion. Considering the increased computational complexity of the introduced hyper-parameters in the competing approaches and the superior performance of parameter-free AC-Off-PAC, we believe that our method provides significant gains over the prior approaches.

5.3 Sensitivity Analysis

As our technique is an off-policy method, batch and replay memory sizes and different experience replay sampling methods can considerably impact the learning. Therefore, we perform experiments on both variants of AC-Off-POC under different replay buffer and batch sizes and experience replay [Lin, 1992] sampling algorithms. We evaluate the baseline and AC-Off-POC with mini-batch sizes of 32, 256, 1024, and 2048, and replay memory [Lin, 1992] of sizes 1 million (1M) and 100,000 (100K). We also investigate the performance improvement by AC-Off-POC under the sampling algorithms of CER [Zhang and Sutton, 2017], ERO [Zha et al., 2019], and PER [Schaul et al., 2016]. The same experimental setting given in Section 5.1 is used. Unless otherwise stated, a mini-batch size of 256, replay memory of size 1 million transitions, and uniform sampling are used in all experiments.

5.3.1 Impact of the Batch Size

Table 2 presents the batch size sensitivity analysis over ten random seeds. First, the batch size of 256 gives the best performance for all methods in terms of both mean and confidence as they are already tuned, as outlined in the original papers of the baselines [Haarnoja et al., 2018, Fujimoto et al., 2018]. For stochastic AC-Off-POC used in the SAC algorithm [Haarnoja et al., 2018], we observe that the relative performance improvement by AC-Off-POC practically remains constant over different batch sizes. This is due to the transition-wise similarity measurement offered by the stochastic variant. Hence, when applied to stochastic policies, AC-Off-POC is not affected by the mini-batch learning and mini-batch size. Moreover, the mini-batch size considerably affects the batch-wise similarity measurement in the deterministic variant for TD3 [Fujimoto et al., 2018]. For all environments, a larger batch size degrades the performance of AC-Off-POC and underperforms the baseline. Similarly, a performance drop exists for smaller batch sizes caused...
1992, baseline SAC [Haarnoja et al., 2018] produces better results in HalfCheetah as AC-Off-POC further filters out changes, we observe a significant performance variation in terms of both mean rewards and confidences. In Table 3: Average return over the last 10 evaluations over 10 trials of 1 million time steps, comparing the impact of maximum under each baseline algorithm and environment.

Table 2: Average return over the last 10 evaluations over 10 trials of 1 million time steps, comparing the impact of replay buffer sizes \{32, 256, 1024, 2048\}. \(\pm\) captures a 95% confidence interval over the trials. Bold values represent the maximum under each baseline algorithm and environment.

by the inaccurate estimation of JSD of two Gaussians with an insufficient number of samples. Overall, our sensitivity analysis on the batch size confirms Observation 1 and Remark 3.

5.3.2 Impact of the Replay Memory Size

Impact of the replay memory sizes is reported in Table 3. When the size of the experience replay memory [Lin, 1992] changes, we observe a significant performance variation in terms of both mean rewards and confidences. In the HalfCheetah environment, a smaller replay memory [Lin, 1992] significantly degrades the performance under all methods. As discussed, this is caused by the environment dynamics of HalfCheetah, which usually requires off-policy samples to be efficiently solved. In all experiments, we consider First-In, First-Out (FIFO) buffer. Thus, the smaller replay memory [Lin, 1992] contains more on-policy samples than the buffer of size 1 million transitions. For Hopper and Walker2d, smaller memory yields better performance which supports the evaluation results found in Section 5.2, i.e., off-policy correction in these environments yields a further performance improvement compared to HalfCheetah as the policy is trained in a more on-policy fashion. When we decrease the size of the experience replay buffer [Lin, 1992], baseline SAC [Haarnoja et al., 2018] produces better results in HalfCheetah as AC-Off-POC further filters out the off-policy transitions that may be contained in the limited replay buffer and thus spoil the off-policy learning, which is outlined in Remark 5. Nonetheless, stochastic AC-Off-POC can still correct for off-policy samples and improve the performance of SAC [Haarnoja et al., 2018] in the rest of the environments. Furthermore, similar results can be found

Table 3: Average return over the last 10 evaluations over 10 trials of 1 million time steps, comparing the impact of replay buffer sizes \{100000, 1000000\}. \(\pm\) captures a 95% confidence interval over the trials. Bold values represent the maximum under each baseline algorithm and environment.

| Setting          | HalfCheetah       | Hopper           | Walker2d         |
|------------------|-------------------|------------------|------------------|
| SAC (32)         | 9092.21 ± 312.47  | 3304.77 ± 348.16 | 3707.69 ± 844.41|
| SAC (256)        | 9871.42 ± 273.98  | 3351.43 ± 317.07 | 4103.54 ± 805.03|
| SAC (1024)       | 8766.95 ± 290.90  | 2700.17 ± 342.32 | 3848.68 ± 927.21|
| SAC (2048)       | 8477.68 ± 322.43  | 2930.11 ± 324.41 | 3222.36 ± 930.57|
| SAC + AC-Off-POC (32) | 11022.58 ± 276.83 | 3470.82 ± 777.71 | 5135.93 ± 860.30|
| SAC + AC-Off-POC (256) | 11320.82 ± 262.23 | 3846.05 ± 777.89 | 5366.81 ± 819.00|
| SAC + AC-Off-POC (1024) | 10459.58 ± 284.13 | 3269.79 ± 833.98 | 5242.90 ± 823.80|
| SAC + AC-Off-POC (2048) | 9215.60 ± 303.78  | 3558.28 ± 893.85 | 4300.63 ± 918.97|

| Setting          | HalfCheetah       | Hopper           | Walker2d         |
|------------------|-------------------|------------------|------------------|
| TD3 (32)         | 8955.77 ± 181.85  | 2961.18 ± 434.44 | 3966.54 ± 377.57|
| TD3 (256)        | 10050.95 ± 168.03 | 3174.56 ± 401.05 | 4107.08 ± 349.81|
| TD3 (1024)       | 9963.48 ± 185.11  | 2847.66 ± 408.16 | 3881.20 ± 351.81|
| TD3 (2048)       | 8712.29 ± 194.69  | 2630.64 ± 455.97 | 4033.33 ± 351.51|
| TD3 + AC-Off-POC (32) | 9104.90 ± 249.96  | 3112.20 ± 411.33 | 4951.56 ± 698.94|
| TD3 + AC-Off-POC (256) | 11735.35 ± 196.97 | 3658.85 ± 354.52 | 5219.89 ± 583.36|
| TD3 + AC-Off-POC (1024) | 8791.17 ± 252.00  | 3126.42 ± 426.60 | 5040.96 ± 607.94|
| TD3 + AC-Off-POC (2048) | 6982.50 ± 248.80  | 2176.53 ± 469.73 | 4713.68 ± 730.17|

| Setting          | HalfCheetah       | Hopper           | Walker2d         |
|------------------|-------------------|------------------|------------------|
| SAC (100K)       | 8596.65 ± 326.68  | 3446.68 ± 368.41 | 4117.61 ± 937.39|
| SAC (1M)         | 9871.42 ± 273.98  | 3351.43 ± 317.07 | 4103.54 ± 805.03|
| SAC + AC-Off-POC (100K) | 8290.28 ± 309.43  | 3959.37 ± 744.74 | 5471.21 ± 788.24|
| SAC + AC-Off-POC (1M) | 11320.82 ± 262.23 | 3846.05 ± 777.89 | 5366.81 ± 819.00|
| TD3 (100K)       | 9343.67 ± 188.52  | 3200.30 ± 403.49 | 4183.48 ± 350.12|
| TD3 (1M)         | 10050.95 ± 168.03 | 3174.56 ± 401.05 | 4107.08 ± 349.81|
| TD3 + AC-Off-POC (100K) | 9068.78 ± 255.80  | 3790.57 ± 494.83 | 5302.83 ± 670.45|
| TD3 + AC-Off-POC (1M) | 11735.35 ± 196.97 | 3658.85 ± 354.52 | 5219.89 ± 583.36|
in the deterministic variant, that is, AC-Off-POC with a smaller buffer underperforms the baseline in HalfCheetah while substantially improving the deterministic baseline algorithm in Hopper and Walker2d.

### 5.3.3 Impact of Different Sampling Algorithms

Table 4 shows the resulting performances under the considered sampling algorithms. We first observe that CER [Zhang and Sutton, 2017] is superior among all the sampling methods. This is due to the most recent collected transition, which is included in each update. Hence, all updates always occur with at least a single on-policy sample. A performance improvement can still be obtained when AC-Off-POC is applied to the baselines while sampling is performed through CER [Zhang and Sutton, 2017]. However, AC-Off-POC slightly underperforms the baselines in the HalfCheetah environment under CER [Zhang and Sutton, 2017] because of the discussed off-policy sample requirement. ERO [Zha et al., 2019] performs poorly when applied to the baseline algorithms due to the internal structure of the algorithm, which is not in the scope of this work. Nevertheless, AC-Off-POC still improves ERO [Zha et al., 2019] when applied to the baseline algorithms. In addition, PER [Schaul et al., 2016] usually underperforms when applied to an actor-critic method. This is an expected result since it was initially proposed for discrete action spaces where a single network, i.e., Q-network, is used to select and evaluate action. Transitions with high prediction errors for Q-network are prioritized. However, in actor-critic methods, the actor network cannot be effectively trained through Bellman optimization [Bellman, 1957] where the prediction error for the critic is large [Sutton and Barto, 2018]. Although ERO [Zha et al., 2019] and PER [Schaul et al., 2016] exhibit a poor performance due to the discussed drawbacks, AC-Off-POC can still attain higher rewards. Therefore, we infer that AC-Off-POC can be readily adapted to any experience replay [Lin, 1992] sampling algorithm.

| Setting                     | HalfCheetah | Hopper    | Walker2d  |
|-----------------------------|-------------|-----------|-----------|
| SAC (uniform)               | 9871.42 ± 273.98 | 3351.43 ± 317.07 | 4103.54 ± 805.03 |
| SAC (CER)                   | 10539.72 ± 285.12 | 3442.93 ± 311.60 | 4105.39 ± 712.35 |
| SAC (ERO)                   | 9497.97 ± 284.17 | 3393.43 ± 315.16 | 4033.84 ± 843.72 |
| SAC (PER)                   | 8953.30 ± 296.70 | 2891.34 ± 317.16 | 3844.24 ± 832.49 |
| SAC + AC-Off-POC (uniform) | 11320.82 ± 262.23 | 3846.05 ± 777.89 | 5366.81 ± 819.00 |
| SAC + AC-Off-POC (CER)      | 9642.60 ± 209.10 | **3905.79 ± 760.39** | **5449.23 ± 802.84** |
| SAC + AC-Off-POC (ERO)      | 11006.71 ± 272.90 | 3865.37 ± 765.77 | 5409.00 ± 827.40 |
| SAC + AC-Off-POC (PER)      | 10388.39 ± 297.94 | 3493.84 ± 840.43 | 4402.69 ± 963.22 |
| TD3 (uniform)               | 10050.95 ± 168.03 | 3174.56 ± 401.05 | 4107.08 ± 349.81 |
| TD3 (CER)                   | 10628.20 ± 173.55 | 3190.81 ± 400.69 | 4151.20 ± 342.95 |
| TD3 (ERO)                   | 9591.56 ± 168.77 | 3187.00 ± 403.67 | 4013.53 ± 361.32 |
| TD3 (PER)                   | 9578.80 ± 172.61 | 2637.58 ± 407.95 | 3447.57 ± 350.60 |
| TD3 + AC-Off-POC (uniform) | **11735.35 ± 196.97** | 3658.85 ± 354.52 | 5219.89 ± 583.36 |
| TD3 + AC-Off-POC (CER)      | 9851.44 ± 177.24 | **3692.22 ± 335.98** | **5367.62 ± 486.86** |
| TD3 + AC-Off-POC (ERO)      | 11345.45 ± 205.72 | 3664.95 ± 352.06 | 5039.16 ± 589.69 |
| TD3 + AC-Off-POC (PER)      | 10648.33 ± 231.66 | 3467.80 ± 415.79 | 4357.27 ± 622.60 |

Table 4: Average return over the last 10 evaluations over 10 trials of 1 million time steps, comparing the impact of uniform, CER, ERO and PER sampling methods. ± captures a 95% confidence interval over the trials. Bold values represent the maximum under each baseline algorithm and environment.
transitions, and since the agent does not effectively use them, the performance drops. Overall, these empirical studies also verify Remark 1.

5.4 Weight Analysis

We highlight that the deterministic variant of AC-Off-POC does not employ temporally correlated trajectories to compute the importance weights as in the state-of-the-art importance sampling methods [Degris et al., 2012, Munos et al., 2016, Harutyunyan et al., 2016, Watkins and Dayan, 1992, Espeholt et al., 2018], and instead, compute the weights through only randomly sampled off-policy transitions. We perform experiments to show the accuracy of the weights produced by deterministic AC-Off-POC through the experiences of an actor trained under the TD3 algorithm [Fujimoto et al., 2018] on the OpenAI Gym tasks HalfCheetah and Hopper [Brockman et al., 2016], over 1 million time steps. Throughout the training, we store the transitions executed by the agent in the temporal order. Then, we start by sampling the transitions with a window size of 256 centered at each transition. We compute the importance weight for each sampled batch with the deterministic variant of AC-Off-POC. Figure 4 depicts the normalized values for time steps and importance weights produced by AC-Off-POC. Note that these weights are derived according to the expert agent’s policy. From the first transition executed by a random policy to the last transition executed by the expert agent, the sampled batches become on-policy with respect to the trained agent’s policy. We observe that as the center of the sliding window approaches the last transition, the AC-Off-POC weights approach the value 1. Hence, the weights increases as the on-policyness of the sampled batch increases, which we observe from the linear path taken by the AC-Off-POC weights. Overall, we infer that the deterministic variant of the AC-Off-POC produces accurate estimates, although it does not employ temporally correlated trajectories or any action probability estimate. Furthermore, some of the AC-Off-POC weights are above the time step line; that is, off-policy transitions are valued more than they should be, which shows that AC-Off-POC can mix on- and off-policy data.
We conduct the same set of experiments to verify the accuracy of the stochastic variant of AC-Off-POC. As the stochastic variant assigns unique weights to each off-policy sample, we directly depict the weights assigned to each transition contained in the expert agent’s experience replay buffer [Lin, 1992]. In Figure 5, we observe that the stochastic variant also produces accurate importance weights to schedule the contribution of each gradient with respect to off-policy samples. Moreover, the confidence of the weights is stronger, and weights are more accurate due to the direct use of the policy distributions in computing the dissimilarity between the target and behavioral policies. In conclusion, as discussed in Remark 4, AC-Off-POC can produce accurate importance weights by preventing vanishing or exploding gradients and the employment of trajectories due to the transition-wise or temporally uncorrelated batch-wise similarities.

6 Conclusion

In this paper, we discuss the theme of off-policy correction, which mitigates the potential effects of off-policy samples whose underlying distribution is divergent from the current policy of interest. As the action probabilities cannot be known and estimated, off-policy correction through eligibility traces for deterministic policies is not available in continuous control. To make the actor-critic methods closer to their on-policy nature, we introduce a novel off-policy correction approach with two variants for stochastic and deterministic policies, AC-Off-POC, which achieves an efficient one-step off-policy correction, enables a safe off-policy learning and improves the data-efficiency by reweighting the contribution of off-policy samples to the gradients of the value functions. We support our claims with solid theoretical analysis that the introduced off-policy correction schemes obtain a bounded contraction mapping which enables a safe one-step importance sampling.

An extensive set of empirical studies shows that AC-Off-POC improves the baseline state-of-the-art and outperforms the competing off-policy correction methods by a considerable margin. By overcoming the perturbation induced by the off-policy samples contained in the experience replay buffer [Lin, 1992], our method can attain faster convergence and optimal policies by disregarding the transitions executed by behavioral policies that highly deviate from the current policy in terms of the numerical action decisions. Our method also resolves the issues regarding the computational cost by not introducing any hyper-parameters or networks. Furthermore, we support our remarks with experimental results throughout the paper and show that AC-Off-POC can readily be applied to any off-policy actor-critic method through a generic approach.

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