The Resonance Photon-Paraphoton Conversion in Media

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Abstract. We propose the new approach for the direct dark matter particles observation at a laboratory. To do this one should use the violet laser to irradiate the sodium uranyl acetate-filled globular photonic crystal. In this case the laser light quanta will resonantly couple to paraphotons at uranyl unitary polaritons frequencies. Due to high density of states at the uranyl absorption edge this conversion will be extremely efficient. So there is a good opportunity to experimentally observe the paraphotons generation.

1. Introduction
According to the modern astrophysical data the most of the Universe’s matter is “dark” [1]. So it’s very important to explore this terra incognita. We consider paraphotons as candidates for the role of dark matter quanta [2-13]. In this paper we analyze the process of photon-paraphoton conversion to demonstrate the possibilities of dark particles generation at a laboratory.

The paraphoton occurs when the two photons merge into the scalar bound state\[ \gamma + \gamma \rightarrow p\gamma. \] (1)
Because of two photons interaction the Lagrangian must be proportional to the square of the electric field:\[ L \sim g \cdot \psi_i \cdot E^2. \] (2)
The smallness of the conversion cross section $g$ makes the process (1) practically impossible – one has to imply the superstrong electromagnetic field to observe this in the void.

In the paper we offer the new approach to solve the problem – to involve medium to increase the efficiency of the photon-paraphoton conversion due to the resonant interaction of electromagnetic radiation with matter when the laser radiation’s frequency is close to the polar type exciton transition frequencies for single-photon processes of absorption.

2. Theoretical analysis
In the electronic theory of dispersion the motion of an electron in an atom at the external field $E=E_0 \exp(i\omega t)$ is described by the simple model\[ m_0 \ddot{x} = -m_0 \omega_0^2 x + q_0 E_0 e^{i\omega t}. \] (3)
Here $x$ is the electron coordinate; $m_0$ is its mass; $q_0$ is the elementary charge; $\omega_0$ is the natural frequency of an electronic oscillator; $\omega$ is the external field’s photon frequency.
One can see that the solution for (3) is...
\[ x = \frac{q_0}{m_0 \left( \omega_0^2 - \omega^2 \right)} E_0 e^{i \omega t}. \]  

So, the electric polarization for the medium containing \( n_0 \) atoms per cubic meter is

\[ P = n_0 q_0 x = \frac{n_0 q_0^2}{m_0 \left( \omega_0^2 - \omega^2 \right)} E_0 e^{i \omega t}. \]

On the other hand,

\[ P = \varepsilon_0 (\varepsilon - 1) E_0 e^{i \omega t}. \]

Therefore, the material medium dielectric constant depends on the external field frequency the following way:

\[ \varepsilon = 1 + \frac{n_0 q_0^2}{\varepsilon_0 m_0 \left( \omega_0^2 - \omega^2 \right)} \equiv 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} = \frac{\left( \omega_0^2 + \omega_p^2 \right) - \omega^2}{\omega_0^2 - \omega_p^2} \equiv \frac{\omega_0^2 - \omega^2}{\omega_0^2 - \omega_p^2}. \]

In this formula

\[ \omega_p = \sqrt{\frac{n_0 q_0^2}{\varepsilon_0 m_0}} \]  - the plasma frequency; \( \omega_0 = \sqrt{\omega_0^2 + \omega_p^2} \)  - the longitudinal mode frequency.

When many oscillators the formula (7) becomes

\[ \varepsilon(\omega) = \varepsilon_\infty \prod_j \frac{\omega_0^2_j - \omega^2}{\omega_p^2_j - \omega^2}. \]

Here \( \omega_0_j \) and \( \omega_p \) are the characteristic frequencies of the material oscillators; \( \varepsilon_\infty = \lim_{\omega \to \infty} \varepsilon(\omega) \) is the medium high frequency dielectric permittivity.

This case the medium electromagnetic field wavenumber is

\[ k(\omega) = \frac{\omega}{c} n = \frac{\omega}{c} \sqrt{\varepsilon(\omega) \mu} = \frac{\omega}{c} \sqrt{\varepsilon_\infty \prod_j \frac{\omega_0^2_j - \omega^2}{\omega_p^2_j - \omega^2} \mu}. \]

And the group velocity of the electromagnetic wave is equal to

\[ v(\omega) = \frac{d\omega}{dk} = \left( \frac{d k(\omega)}{d\omega} \right)^{-1}. \]

Also the effective mass of the relevant quanta (photons and polaritons) is

\[ m = \left( \frac{d^2 W}{dp^2} \right)^{-1} = \hbar \left( \frac{d^2 \omega}{dk^2} \right)^{-1} = \hbar \left( v(\omega) \frac{dv(\omega)}{d\omega} \right)^{-1}. \]

The formulas (8) - (10) show that the group velocity tends to zero when \( \omega \to \omega_p \). At known conditions this leads to the formation of polaritons Bose-Einstein condensation with the critical temperature

\[ T_c = \left( \frac{n}{\zeta(3/2)} \right)^{\frac{2}{3}} \frac{\hbar^2}{2 \pi m k_n}. \]

Here \( n \) is the concentration of polaritons; \( m \) is its effective mass; \( \hbar \) is the Planck constant; \( k_n \) is the Boltzmann constant; \( \zeta \) is the Riemann Zeta-function, \( \zeta \left( \frac{3}{2} \right) = 2.6124... \)
Thus, there is a great opportunity to create the ultra-high density polariton states at material medium oscillators frequencies \( \omega_j \). Because of (2) this leads to the resonant amplification for the photon-paraphoton conversion. Besides, to realize (1) one has to meet the energy and the momentum conservation. This case the medium refractive index \( n \) must be equal to one. It is possible at the medium unitary polaritons [4] frequencies: the equation (8) shows that in media, there are the frequencies \( \omega_u \) at which the absolute value of dielectric constant is equal to one:

\[
|\varepsilon(\omega_u)| = \left| \varepsilon_u \prod_j \frac{\omega_j^2 - \omega_u^2}{\alpha_j^2 - \omega_u^2} \right| = 1. \tag{13}
\]

Thus, at these frequencies the medium is absolute transparent and the process (1) is welcomed. Therefore, the photon to paraphoton conversion (1) in medium is best observable for the photons of \( \omega_u \approx \omega_j \) when \( \omega_u \) is close to one of \( \omega_j \). As seen from (13), the more factors involved in the product, the higher probability that there is such \( j \) that \( \omega_j \approx \omega_u \). So, it is necessary to use a material medium with the greatest possible number of electronic oscillators. These elements are located at the bottom of the Periodic table and known as actinides or f-elements. Unfortunately these elements are unstable, and with great difficulty we managed to find an affordable non-radioactive uranium-containing sample—sodium uranyl acetate \( \text{NaU}_{238}\text{O}_{238}(\text{CH}_3\text{COO})_3 \).

It is well-known [14-16] that the uranyl acetate absorption edge is located near 480 nm. It means that at the violet spectral region, there are a lot of oscillators, so there is the one with \( \omega_j \approx \omega_u \). In the paper [17] the uranyl resonant absorption wavelengths \( \lambda_{0j} \) was found (see table 1). So, to use the data we managed to rewrite (8) following way:

\[
\varepsilon(\lambda) = 1 + \sum_j \frac{f_j}{\lambda^2 - \lambda_{0j}^2}. \tag{14}
\]

Since the oscillator strengths \( f_j \) are unknown, we assume that all of \( f_j \) are equal. The value of \( f_j \) was found by equating the calculated refractive index at sodium D-line (\( \lambda_D = 589.3 \text{ nm} \)) to the known value of \( n_D = 1.504 \) [18]:

\[
f_j = \frac{\varepsilon(\lambda_D) - 1}{\sum_j \frac{1}{\lambda_D^2 - \lambda_{0j}^2}} = \frac{n_D^2 - 1}{\sum_j \frac{1}{\lambda_D^2 - \lambda_{0j}^2}}. \tag{15}
\]

This way \( f_j = 1.3 \times 10^{-14} \text{ m}^2 \).

Table 1. The input data for the calculations

| \( j \) | \( \lambda_{0j}, \text{ nm} \) | \( f_j, 10^{-14} \text{ m}^2 \) |
|---|---|---|
| 1 | 480.5 | |
| 2 | 462.2 | |
| 3 | 446.7 | |
| 4 | 432.4 | |
| 5 | 419.0 | |
| 6 | 414.5 | |
| 7 | 409.5 | |
We use the table 1 data to calculate the dependence of the uranyl acetate dielectric permittivity on the external electromagnetic field wavelength (8) – Figure 1. One can see the poles (\( \lambda_{0j} \)) and the zeros (\( \lambda_{lj} \)). There are at least dozen resonances located at the absorption edge. Note the wavelengths at which \( \varepsilon = 1 \) – it’s the unitary polaritons ones.

| \( j \) | \( \lambda_{0j} \), nm | \( f_{j} \), \( 10^{14} \text{m}^2 \) |
|-----|------------------|------------------|
| 8   | 402.5            |                  |
| 9   | 398.0            |                  |
| 10  | 391.2            |                  |
| 11  | 388.3            |                  |
| 12  | 380.5            |                  |
| 13  | 364.5            |                  |
| 14  | 361.8            |                  |
| 15  | 351.5            |                  |
| 16  | 346.0            |                  |
| 17  | 343.0            |                  |

Then we calculated the dispersion law (9) – Figure 2. At the picture the thick lines represent the medium polaritons dispersion (9) and the thin one illustrates the dispersion law in the vacuum: \( \omega = ck \). The intersections mark the unitary polaritons positions (\( \omega_{u} \)) and the zeros – the LO-modes locations (\( \omega_{l} \)).

**Figure 1.** The spectral behavior (8) for \( \varepsilon (\lambda) \).
Figure 2. The dispersion of polaritons in the sodium uranyl acetate (thick lines). The thin line illustrates the dispersion law in the vacuum: \( \omega = c \times k \). The intersections correspond to the unitary polaritons.

Next we found the electromagnetic waves group velocity (10) in the crystal – Figure 3 (Figure 3a is plotted at the linear scale, Figure 3b at the logarithmic one). One can see the anomalous speed of light slowing near the absorption edge. Due to the conservation of the energy flux density \( \mathcal{S} = \mathcal{E}_0 \mathbf{E}^2 = w \nu \); so, for the slow light \( \mathcal{E}^2 = \mathcal{E}_0^2 \frac{c}{\nu} \) and the Lagrangian (2) is proportional to the deceleration: \( L \sim \left( \frac{c}{\nu} \right) \). It means that the light deceleration to 1 m/s increases the conversion (1) probability by 8 orders of magnitude.

Figure 3. The light group velocity dispersion in the sodium uranyl acetate calculated by (10)

At last we established the effective masses of light quanta in the sample – Figure 4. At unitary polaritons frequencies the light quanta have mass the order of \( 10^{-34} \) kg.
Figure 4. The polaritons effective mass dispersion (11) in the sodium uranyl acetate.

In the paper [19] we shown that it is possible to adjust medium natural resonant frequencies by using the photonic crystal – according the Bragg’s law one should to rotate crystal to shift the lines:

\[ 2\alpha n \sin \vartheta = \lambda_n . \]  

(16)

So it is very intriguing to analyze the conversion process (1) in the photonic crystal filled by the sodium uranyl acetate.

The simplest 1D-photonic crystal is the layered medium consists of two alternating layers. Let’s call them layer №1 and layer №2. Each layer has its own thickness (\( a_i \)) and refractive index (\( n_i \)). And for the 1D-photonic crystal the dispersion law is [20]

\[ \left( k_\omega(\mathbf{a}_1,\mathbf{a}_2) \right)^2 = k_\omega^2(\mathbf{a}_1) + k_\omega^2(\mathbf{a}_2) \]

(17)

In case of the globular photonic crystal based on the artificial opal [21] at the practically mostly used direction at the normal to natural growth lattice plane (111):

\[ k(\omega) = \frac{1}{D} \sqrt{\lambda \bigg( \frac{\omega}{c} n_1(\omega) \bigg)^2 (1-\eta) D \sqrt{\frac{2}{3}}} \cdot \cos\bigg( \frac{\omega}{c} n_1(\omega) \bigg) \cdot \sin\bigg( \frac{\omega}{c} n_2(\omega) \cdot \eta D \sqrt{\frac{2}{3}} \bigg) \]

(18)

\[ -\frac{1}{2} \left( n_1(\omega) \frac{n_2(\omega)}{n_1(\omega)} \right) \left[ \sin\bigg( \frac{\omega}{c} n_1(\omega) \bigg) \cdot (1-\eta) \cdot \frac{2}{3} \bigg] \sin\bigg( \frac{\omega}{c} n_2(\omega) \cdot \eta D \sqrt{\frac{2}{3}} \bigg) \]

Here \( D \) is the quartz globules diameter (typically \( D \approx 250 \) nm); \( \eta = 0.26 \) is the porosity of the crystal gcc-lattice; \( n_1 \) and \( n_2 \) are layers refractive indices (the index 1 corresponds to the crystal matrix, 2 – for the substance in pores); \( \omega \) is the wave frequency and \( c = 3 \cdot 10^8 \) m/s is the speed of light in the vacuum.

For the \( n_1 \) there is the Sellmeier equation [22] (\( \lambda \) is in \( \mu m \)):

\[ n_1^2(\lambda) = 1 + \frac{0.6961663\lambda^2}{\lambda^2 - 0.0684043^2} + \frac{0.4079426\lambda^2}{\lambda^2 - 0.1162414^2} + \frac{0.8974794\lambda^2}{\lambda^2 - 9.896161^2}. \]

(19)

And \( n_2 \) is calculated via (14):

\[ n_2^2(\lambda) = \epsilon(\lambda) = 1 + \sum J f \frac{f_i}{\lambda^2 - \lambda_{0j}^2}. \]

(20)

The simulation (18) - (20) results are shown by Figure 5. One can see the increased density of modes at the uranyl absorption edge due to the photonic crystal lattice.
Figure 5. The dispersion law $\omega = \omega(k)$ for the globular photonic crystal on the artificial opal with $D = 250$ nm filled by the sodium uranyl acetate.

Let's analyze the group speed of light (10) in the photonic crystal with uranyl. The spectral dependences for the electromagnetic waves quanta propagation speed are presented at Figure 6. Note that in photonic crystal light deceleration is greater than without it. Moreover there are additional zero light quanta speed regions due to the photonic crystal band structure.

Figure 6. The light quanta speed in the globular photonic crystal filled by the sodium uranyl acetate. The crystal's globules diameter is $D = 250$ nm.

3. Discussion

The simulation shows that in medium it is possible to obtain deceleration of light velocity over to 8 orders of magnitude. Moreover, in the photonic crystal there are the zero light quanta speed spectral bands. Because of the Lagrangian proportionality to $(c/v)$, it means that this case the conversion (1) becomes resonant and the process probability tends to 1. The theory predicts the best external field frequencies for this: it is the unitary polaritons ones at the uranyl absorption edge at the violet edge of the visible spectral region. Moreover, in discussed case we wait to observe not only spontaneous but the stimulated conversion which probability increases resonant way. We propose the sodium uranyl acetate-filled globular photonic crystal with globules of 250 nm as an active media for the potential dark matter generator under powerful laser emission excitation.
4. Conclusion
The results show that there is an opportunity to observe photon-paraphoton conversion at a laboratory by using specific media. The best of them is the non-radioactive commercially available uranyl compounds. We predict the best conditions for the photon-to-paraphoton transformation (1) at the uranyl absorption edge. The advantage of process (1) over photon to axion conversion is the unnecessary of the magnetic field. So it is the opportunity to make the proper conditions for dark matter quanta generation at a laboratory.

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