PHYSICS OBSERVABLES FOR COLOR TRANSPARENCY

Bernard PIRE
Centre de Physique Théorique, Ecole Polytechnique
F91128 Palaiseau, France

Abstract: The physics observables dedicated to the study of color transparency are diverse. After a brief pedagogical introduction, we emphasize the complementarity of the nuclear filtering and color transparency concepts. The importance of quantum interferences leads to suspect pictures based on the preparation of a squeezed state of small transverse size leading to a rapid transverse expansion. The different roles of heavy and light nuclei are emphasized. The possibility of color transparency tests in the vacuum is stressed. Such a test may be quickly addressed in the virtual Compton scattering reactions at CEBAF or HERMES energies.

Color Transparency\[1\] studies are entering the age of maturity. After the first experimental data\[2\] have been analyzed in many details, one understands better that different observables will shed light on this interesting new QCD concept.

1 Mini-hadron scattering in a gauge field theory

Let us first present a somewhat academic but instructive calculation of the high energy limit of a forward hadron-hadron amplitude in perturbative field theory \[3\]. We shall use the optical theorem to relate the total cross section to the imaginary part of the forward amplitude.

Let us first consider the two gluon exchange processes for quark-quark scattering in the region \(-t \ll s \sim -u\). From the 18 diagrams which may be drawn, 14 are vertex or self energy corrections to the one-gluon diagrams, and as such are real. Two of the four remaining diagrams dominate at small \((-t)\).

Denoting as \(q\) the 4-vector of the first gluon emitted by the bottom fermion line, one finds that the two diagrams have identical boson propagators and bottom fermion expression. From the upper fermion line, one gets adding the two diagrams, after some simple algebra:

\[
A \propto \frac{1}{q^- + i\epsilon} - \frac{1}{q^- - i\epsilon}
\]  

leading to a \(\delta(q^-)\) constraint. Moreover the bottom fermion line leads to a \(\delta(q^+)\)

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constraint, so that the $d^4q$ integration boils down to a 2–dimensional transverse space integration. The resulting amplitude is dominantly imaginary:

$$M = iCs \int \frac{d^2q_T}{(2\pi)^2} \frac{1}{(q^2_T + \lambda^2)((q_T - \Delta_T)^2 + \lambda^2)}$$  \hspace{1cm} (2)$$

which may be rewritten as

$$M = iCs \int d^2b e^{i\Delta_T \cdot b} \left( \frac{g^2 K_0(\lambda b)}{2\pi} \right)^2$$  \hspace{1cm} (3)$$

where $\Delta_T$ is the (small) 2–dimensional transfer between the initial and final fermion. The important features of this result is the 2–dimensional nature of the integral and the squared factor reminiscent of the eikonal nature of forward amplitude in the high energy limit.

Let us now go to the almost physical case of meson-meson scattering, that is consider a color singlet formed by a quark–antiquark pair scattering on another pair. A straightforward computation shows that the color factors are the same for all 2–gluon exchange graphs connecting the upper hadron to the lower one, but that an extra minus sign is attached for an antiquark line. The result is then

$$M = iCs \int d^2b e^{i\Delta_T \cdot b} \int d^2r_1 d^2r_2 \psi^* (r_1) \psi^* (r_2) \{ V(x_1 - x_3) - V(x_2 - x_3) - V(x_1 - x_4) + V(x_2 - x_4) \}^2 \psi (r_1) \psi (r_2)$$  \hspace{1cm} (4)$$

with

$$V(x) = \frac{1}{(2\pi)^2} \int \frac{d^2k e^{ik \cdot x}}{k^2 + \lambda^2}$$  \hspace{1cm} (5)$$

Suppose now that $r_1 = x_1 - x_2$ is small (a mini-hadron); then one may approximate $V(x_1 - x_3) - V(x_2 - x_3) \sim r_1 \cdot \nabla (x_1 - x_3)$ and similarly for $V(x_1 - x_4) - V(x_2 - x_4)$, and get:

$$M \propto r_1^2.$$  \hspace{1cm} (6)$$

leading to a total cross section for the mini hadron scattering on a hadron:

$$\sigma \propto \frac{Im M}{s} \propto r_1^2.$$  \hspace{1cm} (7)$$

This is color transparency. Note that if the $q\bar{q}$ was in a color octet state, the resulting amplitude would contain an additionnal term

$$M \propto \ldots + \{ V(x_1 - x_3) - V(x_1 - x_4) \} \{ V(x_2 - x_3) - V(x_2 - x_4) \}$$  \hspace{1cm} (8)$$

which does not vanish in the limit $x_1 \to x_2$. The color singlet nature of the quark-antiquark pair is thus essential for Eq[8] to be valid.
2 How can a hadron behave like a mini-hadron?

The understanding of color transparency experiments would be easy if we had access to hadrons of tunable transverse size. This is almost the case for quarkonia, whose size is governed by the inverse of the heavy quark mass. But life is complicated in this case by the problem of the production process which is still much debated. Hard exclusive scattering offers a promising way since it has been recognized that these processes asymptotically select a compact valence component of the hadron light-cone wave-function, thus forcing a hadron to behave like a mini-hadron. The study of the (reduced) final state interactions of these mini hadrons with spectator hadrons enable us to study the soft scattering of these objects, opening the domain of the strong interactions of small color-singlet objects. This should be of crucial importance for the detailed understanding of Pomeron physics.

The selection of the compact (sometimes optimistically called point-like) component of the hadronic wave function should not be misidentified as the preparation (in the sense of quantum mechanics) of a squeezed wave-packet which would have a dramatic tendency to transversally expand at a rate governed by Heisenberg uncertainty relations such as $\Delta p_T \Delta b_T > \hbar$. On the contrary, one should realize that the selection process is due to a subtle destructive interference pattern which annihilates all the contributions from the large states. The perturbative treatment of the simplest occurrence of this phenomenon, namely the meson form factor, is much instructive. The pattern of gluon radiation indeed leads to a severe Sudakov suppression of the amplitude when the transverse separation of quarks exceeds some value of order $(1/Q)$.

As often in quantum field theories, one should not be surprised that a reasoning based on the trajectories of the hadronic components, or equivalently to the expanding transverse hadronic size, leads one to erroneous statements. The observable to be calculated, although hopefully factorizable in short and long distance quantities, is not adequately described by a semi-classical picture where the process is chronologically cut into pieces: firstly, preparation of a state through some hard scattering process, secondly, evolution of this state, and finally soft scattering and absorption phenomena.

3 Nuclear Filtering in heavy nuclei

The first piece of evidence for something like color transparency came from the Brookhaven experiment on pp elastic scattering at 90° CM in a nuclear medium. These data lead to a lively debate. The special feature of hadron hadron elastic scattering at fixed angle is that in addition to a clear cut short distance amplitude, there is an infrared sensitive process (the so-called independent scattering mechanism) which allows not so small configurations to scatter.
elastically. The phenomenon of colour transparency is thus replaced by a nuclear filtering process: elastic scattering in a nucleus filters away the big component of the nucleon wave function and thus its contribution to the cross-section. Since the presence of these two competing processes had been analysed as responsible for the oscillating pattern seen in the scaled cross-section $s^{10} d\sigma/dt$, an oscillating color transparency ratio emerges. One way (“attenuation method”) to understand data is to define a survival probability related in a standard way to an effective attenuation cross section $\sigma_{\text{eff}}(Q^2)$ and to plot this attenuation cross section as a function of the transfer of the reaction. One indeed obtains values of $\sigma_{\text{eff}}(Q^2)$ decreasing with $Q^2$ and quite smaller than the free space inelastic proton cross section. The survival probability is even found to obey a simple scaling law in $Q^2/A^{1/3}$.

The SLAC NE18 experiment measured the color transparency ratio up to $Q^2 = 7\text{GeV}^2$, without any observable increase. This conclusion follows only if the hard scattering part of the process is assumed to be the same as in free space, canceling out in forming the transparency ratio. While this assumption is not needed in the attenuation method, the precision of the data were not sufficient to conclude much using the less model-dependent test. While the majority view is that these data cast doubt on the most optimistic onset of color transparency, emphasizing the importance of a sufficient boost to get the small state quickly out of the nucleus, this conclusion remains tentative and something to be tested.

The diffractive electroproduction of vector mesons at Fermilab and Cern exhibit an interesting increase of the transparency ratio for data at $Q^2 \simeq 7\text{GeV}^2$. In this case the boost is high since the lepton energy is around $E \simeq 200\text{GeV}$ but the problem is to disentangle diffractive from inelastic events.

### 4 Color Transparency signals with Small Nuclei

Although color transparency was first mostly considered as an effect to be studied on rather large nuclei, it became recently clear that small nuclei had much to teach us about this physics item. Deuteron electrodesintegration reactions $d(e, e'p)n$ for instance, both polarized and unpolarized, is much interesting. The idea is simple: let us examine the case where the virtual photon mostly hit the proton in the deuteron. The neutron momentum distribution is then due to the combination of both Fermi momentum effects in the initial state and final state interactions. These two components are well known at small $Q^2$. At large $Q^2$, the hard process selects small-sized proton, and the interaction minimiprot-on-neutron is much weaker. This is where Pomeron physics enters, at least if energy is high enough. High $Q^2$ electroproduction data appear then as an unexpected testing bench of soft physics, with the important bonus of a controllable variable sized hadron scattering on a normal one. Whether the small size justifies completely a perturbative treatment of this small transfer
amplitude is still an open question.

The cross section for \(d(e,e'p)n\) may be written as

\[
\frac{d\sigma}{dE_e/d\Omega_e/d^3p_p} = \sigma_{ep} \ D_d(q,p_p,p_n) \ \delta(q_0 - M_d - E_p - E_n) \tag{9}
\]

where \(D_d(q,p_p,p_n)\) is the joint probability of the initial proton having a Fermi momentum \(p_p - q\) and the final proton (neutron) a momentum \(p_p (p_n)\). A poor energy resolution would restrict the physics to a qualitative observation of color transparency, whereas a very good one would allow a quantitative determination of the miniproton-neutron scattering cross-section as a function of the miniproton size (i.e. \(Q^2\)), provided one controls and deconvolutes Fermi momentum effects in the deuteron.

Frankfurt et al [14] estimate sizable effects at CEBAF energies, which amount to \(Q^2\) values in the range \(\sim 4\text{GeV}^2 \leq Q^2 \leq 10 (\text{GeV}/c)^2\). Prospects are brighter within ELFE conditions.

5 Helicity (non-)conservation and Color Transparency signals

The hadronic helicity conservation rule in hard exclusive reactions [15] follows from two assumptions:

- quark masses can be neglected;
- valence states (with only fermions) dominate.

These assumptions which asymptotically are quite solid in reactions such as \((e,e',p)\) are less justified in the hadronic case [16]. They are exactly what leads to the result of color transparency. It thus follows that the helicity non-conserving contributions must be filtered away in a nuclear medium. It is thus most interesting, at a given value of \(Q^2\) to compare the nuclear absorption of amplitudes violating the helicity conservation rule. For this measurement to be possible, we must consider cases where such amplitudes are quite large at reasonable \(Q^2\) values. This is not the case for the proton form factor \(F_2\), but maybe for the \(p - \Delta\) transition form factor [17], measured at \(Q^2 = 3.2\text{GeV}^2\).

In the case of hadronic reactions, it has been predicted [16] that the amount of helicity non conservation seen for instance in the helicity matrix elements of the \(\rho\) meson produced in \(\pi p \to \rho p\) at \(90^\circ\) would be filtered out in a nucleus. Experimental data in free space [15] yield \(\rho_{1-1} = 0.32 \pm 0.10\), at \(s = 20.8\text{GeV}^2\), \(\theta_{CM} = 90^\circ\), for the non-diagonal helicity violating matrix element. If the persistence of helicity non-conservation is correctly understood as due to independent scattering processes which do not select mini-hadrons and thus are not subject to color transparency, helicity conservation should be restored at the same \(Q^2\).
in processes filtered by nuclei. One should thus observe a monotonic decrease of $\rho_{1-1}$ with $A$.

6 Color Transparency in the Vacuum, the Virtues of Virtual Compton Scattering

Since the vacuum is highly non-trivial in QCD, one may dream of observable color transparency effects without any nuclear reinteraction. Let us illustrate this statement via the study of synchrotron radiation due to the bending of quarks moving in the vacuum.

Synchrotron radiation of soft photons has been proposed [19] as a powerful way to test the vacuum structure of QCD. The physical picture [20] is the following: consider the QCD vacuum as described by domains of size $a$ (phenomenologically determined around 0.35 fm) where colour fields are highly correlated. A fast quark propagating in this domain will be subject to a Lorentz chromomagnetic force which will bend its trajectory and thus force it to emit synchrotron radiation. Thus, for a hadron of normal transverse size of order 1 fm, one is allowed to add the contributions to synchrotron radiation from the partons in a hadron incoherently. The story will be completely different for a mini-hadron of tunable size as expected in hard exclusive reactions. Asymptotically, the part of the wave function which is able to contribute to the scattering amplitude is of zero transverse size and the color singlet nature forbids it to interact with the chromo-magnetic background field. A vanishing Lorentz force will then lead to a vanishingly low synchrotron radiation rate.

Consider now the exclusive electroproduction of a photon on a proton:

$$e(k) + p(p) \rightarrow e(k') + p(p') + \gamma(q')$$  \hspace{1cm} (10)

where the Bethe Heitler process and the virtual Compton scattering amplitudes interfere. This reaction has been much discussed recently in a particular kinematical range as a tool to extract off-diagonal partonic distributions [21].

The other kinematical range we may focus is the very soft photon case. The transverse size of the incoming and out-going hadrons is a crucial parameter in the description of a QCD-synchrotron radiation process since the gauge nature of QCD leads to a destructive interference pattern when one coherently adds contributions of the different partons constituting a colorless object, leading eventually to a vanishing radiation amplitude at zero transverse separation.

To quantify the expected effect, we need a model which interpolates between incoherent synchrotron radiation for a low $Q^2$ process to a coherent sum of the three valence quark radiation amplitude at intermediate $Q^2$ and a vanishing rate at large $Q^2$. Work is in progress along these lines [22].

Let us finally note that the neutron case $e + n \rightarrow e + n + \gamma$ may be particularly interesting since it clearly suppresses much contamination from usual
7 Conclusion

The 15 – 30GeV continuous electron beam ELFE project is presently discussed at the European level [23]. Besides the determination of hadronic valence wave functions through the careful study of many exclusive hard reactions in free space, the use of nuclear targets to test and use color transparency is one of its major goals. The (e,e',p) reaction should in particular be studied in a wide range of $Q^2$ up to 21GeV$^2$, thus allowing to connect to SLAC data (and better resolution but similar low $Q^2$ data from CEBAF) and hopefully clearly establish this phenomenon in the simplest occurrence. The measurement of the transparency ratio for photo- and electroproduction of heavy vector mesons, in particular of $\psi$ and $\psi'$ will open a new regime where the mass of the quark enters as an other scale controlling the formation length of the produced meson.

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