Non-Ohmic critical fluctuation Hall conductivity of layered superconductors in strong electric fields

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The excess Hall conductivity, resulting from thermal fluctuations of the superconducting order parameter, is calculated for a layered superconductor for an arbitrarily strong in-plane electric field and a perpendicular magnetic field in the frame of the time-dependent Ginzburg-Landau theory. The fluctuation Hall conductivity is suppressed in high electric fields much stronger than the longitudinal one. For high-temperature superconductors we predict a pronounced non-Ohmic behavior of the excess Hall effect near the critical temperature in moderate magnetic fields and electric fields of the order of 100 V/cm.

The non-Ohmic behavior of the fluctuation conductivity in strong electric fields, studied for the first time for the isotropic case in connection with the low temperature superconductors, can be summarized by saying that reasonably high values of the electric field can accelerate the fluctuating paired carriers to the depairing current, and thus, suppress the lifetime of the fluctuations, which leads to a deviation from Ohm’s law. For a layered superconductor, a situation very much resembling the crystalline structure of the high temperature superconductors (HTSC), the issue has been addressed theoretically starting from a microscopic approach and subsequently in the frame of the time-dependent Ginzburg-Landau theory, in the Gaussian as well as in the self-consistent Hartree approximation. The non-Ohmic fluctuation conductivity was studied recently also in the presence of a perpendicular magnetic field in the Hartree approximation for the TDGL theory, revealing that the simultaneous application of electric and magnetic fields results only in a slight additional suppression of the superconducting fluctuations, compared to the case when the fields are applied individually.

However, a treatment of a possible non-linearity of the off-diagonal (Hall) components of the fluctuation magnetoconductivity tensor in a high electric field has been lacking yet. The non-vanishing Hall current due to fluctuating Cooper pairs, ascribed to a hole-particle asymmetry and a complex relaxation time in the TDGL theory, was first calculated in the Gaussian approximation and subsequently improved by incorporating the fluctuation interaction in the frame of a Hartree approach or based on the single particle-hole renormalization. All the above models for the so-called excess Hall conductivity represent however linear response approximations with respect to the longitudinal electric field and are therefore valid only for small magnitudes of the latter.

Experimentally, several investigations of the Hall effect at high current densities up to 10^6 Acm^-2 have been reported, but with the aim of overcoming the vortex pinning and hence of testing its influence on the Hall conductivity. The intrinsic non-Ohmic effect was neither envisaged nor fortuitously evidenced, since it needs, as we shall see later, even higher current densities in order to be unambiguously discerned.

In this paper we shall treat, in the self-consistent Hartree approach, the thermal fluctuation Hall conductivity for a layered superconductor in a perpendicular magnetic field and for an arbitrarily strong in-plane electric field. To our present knowledge this topic has not been treated yet, not even for the simpler cases of an isotropic superconductor, or in the Gaussian approximation. For our purpose, we shall adopt the Langevin approach to the TDGL equation. Keeping the same notations as in Ref. the gauge-invariant relaxational TDGL equation governing the critical dynamics of the superconducting order parameter in the l-th superconducting plane writes:

$$\Gamma_0^{-1}(1+i\eta)(\partial_t + 2i\frac{eEx}{\hbar})\psi_I + a\psi_I + b|\psi_I|^2\psi_I - \frac{\hbar^2}{2m}\left[\partial_x^2 + \left(\partial_y + \frac{2ie}{\hbar}xB\right)^2\right]\psi_I + \frac{\hbar^2}{2m_s^2}(2\psi_I - \psi_{I-1} - \psi_{I+1}) = \zeta_I(x, t) . \quad (1)$$

Here m and m_s are effective Cooper pair masses in the ab-plane and along the c-axis, respectively, s is the distance between superconducting planes, and e the elementary electric charge. The order parameter has the same physical dimension as in the three-dimensional case and SI units are used. The perpendicular magnetic field B is generated by the vector potential A = (0, xB, 0), with x and y the in-plane coordinates, and the magnetization is neglected. The GL potential a = a_0\varepsilon is parameterized by a_0 = \hbar^2/2m_0^2 = \hbar^2/2m_0\xi_0^2 and \varepsilon = \ln(T/T_0), with T_0 the mean-field transition temperature, while \xi_0 and \xi_{0c} are the in-plane and out-of-plane coherence lengths extrapolated at T = 0, respectively. The real part of the relaxation time in the TDGL equation is given by \Gamma_0^{-1} = \pi\hbar^3/16m_0^2k_BT, while the imaginary part \Gamma_0^{-1}\eta must be introduced in order to break the particle-hole symmetry and allow for a non-vanishing Hall current.

The Langevin white-noise forces \zeta_I(x, t) are correlated through \langle \zeta_I(x, t)\zeta_I^*(x', t') \rangle = 2\Gamma_0^{-1}k_BT\delta(x - x')\delta(t - t')\delta_{0v}/s, where \delta(x - x') is the 2-dimensional delta-function concerning the in-plane coordinates. The electric field E is assumed along the x-axis, generated by the
the elements:

$$\langle j_y(t) \rangle = \frac{i e \hbar}{m} \left( \partial_y - \partial_y' \right) \langle \psi_l(x, y, t) \psi_l^*(x', y', t) \rangle \bigg|_{y=y'}$$

$$- \frac{4 e^2}{m \epsilon x B} \left\langle \psi_l(x, y, t) \right| \psi_l(x, y, t) \right\rangle^2 ,$$

so that the fluctuation Hall conductivity is given by

$$\Delta \sigma_{xy} = - \Delta \sigma_{yz} = - \left\langle j_y \right\rangle / E.$$

As mentioned, the quartic term in the thermodynamical potential will be treated in the Hartree approximation, which results in a linear problem with a modified (renormalized) reduced temperature $\tilde{\varepsilon} = \varepsilon + b \left\langle \psi_l^2 \right\rangle / a_0$.

Following the same procedure as in Ref. 4, we introduce the Fourier transform with respect to the in-plane coordinate $y$, the layer index $l$, and time $t$, respectively, and also the Landau level (LL) representation with respect to the $x$-dependence, through the relation:

$$\psi_l(x, y, t) = \int \frac{dk}{2\pi} \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} \sum_{n \geq 0} \sum_{k, \omega} \psi(n, k, \omega) \langle e^{-i k y} e^{-i q y} e^{-i \omega t} u_n \rangle (x - \hbar k / 2 e B),$$

where the functions $u_n(x)$ with $n \in \mathbb{N}$ build the orthonormal eigenfunction system of the harmonic oscillator hamiltonian, so that $(-\hbar^2 \partial_x^2 + 4 \epsilon^2 B^2 x^2) u_n(x) = 2 \hbar e B (2n + 1) u_n(x)$. Equation (1) can be evaluated into the matrix form, after applying the expansion (3):

$$a_0 \sum_n \langle M + \eta P \rangle_{nm} \psi_q(n', k, \omega) = \zeta_q(n, k, \omega)$$

where the new noise terms $\zeta_q(n, k, \omega)$, corresponding to the expansion rule (4), are delta-correlated such as

$$\langle \zeta_q(n_1, k_1, \omega_1) \zeta_{q'}(n_2, k_2, \omega_2) \rangle = 2 \Gamma^{-1} k_B T (2\pi)^3 \delta(k_1 - k_2) \delta(q_1 - q_2) \delta(\omega_1 - \omega_2) \delta_{n_1, n_2},$$

and where the two dimensionless symmetrical triadiagonal matrices $M$ and $P$ have the elements:

$$M_{nn} = -i \omega' + \tilde{\varepsilon}_{nq'}; \quad P_{nn} = \omega'; \quad M_{n,n+1} = M_{n+1,n} = i f \sqrt{n+1}; \quad P_{n,n+1} = P_{n+1,n} = -f \sqrt{n+1}; \quad \tilde{\varepsilon}_{nq'} = \tilde{\varepsilon} + \frac{r}{2} (1 - \cos \theta') + (2n + 1) h; \quad r = \frac{2 \hbar^2}{a_0 m_c s} = \left( \frac{2 \xi_0 c}{s} \right)^2; \quad f = 2 \sqrt{6} \frac{E'}{\sqrt{h}}.$$

Here we have introduced the new variables:

$$\omega' = \frac{\Gamma^{-1}}{a_0} \left( \omega - \frac{E k}{B} \right); \quad q' = qs; \quad k' = \frac{h}{2 e B} k,$$

and the reduced field magnitudes:

$$h = \frac{B}{B_{c2}(0)} \frac{\hbar e B}{m a_0} \quad \text{and} \quad E' = \frac{e E \xi_0 a_0^{-1}}{2 \sqrt{3} a_0 h} = \frac{E}{E_0},$$

with $E_0 = 16 \sqrt{3} k_B T / \pi e \xi_0$ defined as in Refs. 3 and 4.

By solving Eq. 4, and taking into account the expansion form (3), one obtains the correlation function of the order parameter:

$$\langle \psi_l(x, y, t) \psi^*_l(x', y', t) \rangle = \frac{4 m k_B T}{\hbar^2 s} t_k$$

$$\cdot \left\langle \sum_{n \geq 0} \sum_{n'} \sum_{k, \omega} e^{-i k y} e^{-i q y} e^{-i \omega t} u_n \right\rangle (x - k') \left\langle \sum_{n \geq 0} \sum_{n'} \sum_{k, \omega} e^{i k y} e^{i q y} e^{i \omega t} u_{n'} \right\rangle^{-1},$$

where the notation $[\ldots]^{-1}$ is to be understood as the element of the inverted matrix.

Before proceeding further we point out that the sums over the LL in Eqs. 3, 4 and 5 must be cut-off at some index $N_l$, reflecting the inherent UV divergence of the Ginzburg-Landau theory. The classical procedure is to suppress the short wavelength fluctuating modes through a momentum (or, equivalently, kinetic energy) cut-off condition, which, in terms of the LL representation writes (6) $(heq B/m)(n + \frac{1}{2}) < c_0 = \epsilon h^2 / 2 m \xi_0^2$, with the cut-off parameter $c$ of the order of unity. A total energy cut-off was also recently proposed (7) whose physical meaning was shown to follow from the uncertainty principle. However, in the critical fluctuation region, the two cut-off conditions almost coincide quantitatively, due to the low reduced-temperature $\varepsilon$ with respect to $c$, so that we shall apply for simplicity the momentum cut-off procedure. In terms of the reduced magnetic field $h$, it writes thus $h \left( N_c + \frac{1}{2} \right) = c / 2$. In this way, the matrices $M$ and $P$ are truncated at $N_c + 1$ lines and columns.

The value of $\eta$ can be inferred from the microscopic theory if one considers the energy derivative $N'$ of the density of states $N$ at the Fermi level $\varepsilon_F$, and it writes (8) $\eta = -(k_B T / \varepsilon_F) \alpha$, where the parameter $\alpha$ amounts in the BCS model to $\alpha = 4 \pi F N'/\pi N_B N^2$, with $g_B$ (or $\varepsilon_F$) for HTSC of the order of $10^3$ K (in $k_B$ units) (9) and the hole-particle asymmetry parameter $\alpha$, inferred from fits of excess Hall effect data (10,11) with the models from Refs. 7 and 8 turns out to be of the order of $10^{-2} \div 10^{-1}$, we conclude that $\eta$ is a small parameter, reflecting also the small Hall angle. We shall therefore expand the inverted matrix in Eq. (3) only up to the linear term in $\eta$, such as $[(M + \eta P)^+ \cdot (M + \eta P)]^{-1} = Q - \eta Q \cdot K \cdot Q + O(\eta^2)$, where the Hermitian matrix $Q = (M + \cdot M)^{-1}$, and the symmetrical triangular matrix $K$ has the elements: $K_{nn} = 2 \omega' \tilde{\varepsilon}_{nq'}$; $K_{n,n+1} = K_{n+1,n} = -f \sqrt{n + 1} (\varepsilon_{nq'} + \tilde{\varepsilon}_{n+1,q'})$. By using the correlation function (5) in the current density expression (2), we can eventually write the fluctuation Hall conductivity in the form:
where we have explicitly specified the cut-off in the Landau level sum. We note also that the zeroth order in $\eta$ gives no contribution to $\Delta \sigma_{xy}$, as also established on the general grounds of the Onsager relations. The electric field enters Eq. (9) through the parameter $f$, defined in Eqs. (10). In order to apply the expression for $\Delta \sigma_{xy}$ also in the limit $E \to 0$, we have to expand the $Q$-matrix elements up to the linear term in $f$, namely $Q = Q^{(0)} + f Q^{(1)} + O \left(f^2\right)$. Since $Q^{(0)}$ is diagonal and has the elements $Q^{(0)}_{nn} = \left\{ \omega^2 + \xi^2_n q^2 \right\}^{-1}$, one needs from $Q^{(1)}$ only the elements $Q^{(1)}_{n+1,n} = 2 \sqrt{n+1} \xi_{n+1} Q^{(0)} n_{n+1,n+1} \left( \omega - i \eta \right)$ and finally obtains in the linear response approximation:

$$\Delta \sigma_{xy}^{(0)} = \frac{\eta e^2 h^3}{2 \hbar s} \int_{-\pi}^{\pi} \frac{dq'}{2\pi} \frac{N_{n-1}}{n_{n+1} \xi_{n+1} q'^2 + \frac{1}{2} q'^2}$$

(10)

which matches the formula found in Ref. 8 and is also coincident, after performing the $q'$-integral and replacing $\eta = - (k_B T / \varepsilon_F) \alpha$, with the expression given by Ref. 8. If one neglects the cut-off procedure (i.e. makes $N_c \to \infty$), and also the renormalization (i.e. replaces $\bar{\varepsilon}$ by $\varepsilon$), the result (10) agrees with the Hall conductivity expression in weak electric field and arbitrary magnetic field found from microscopical calculations in Ref. 20. Taking further also the limit of weak magnetic field, one can find the expression given by Ref. 21 in the framework of the Boltzmann kinetic equation.

Starting from Eq. (8) we are able to compute also the fluctuation Cooper pair density $\left\langle |\psi|^2 \right\rangle$, keeping only the dominant zero-th order term in $\eta$, and write the self-consistent equation for the renormalized reduced temperature parameter $\bar{\varepsilon}$, as already found in Ref. 8:

$$\bar{\varepsilon} = \ln \frac{T_0}{T_c} + g T_0 4 \hbar \int_{-\infty}^{\infty} \frac{dq'}{2\pi} \int_{-\pi}^{\pi} \frac{dq'}{2\pi} \sum_n Q_{nn} (q', \omega') .$$

(11)

Here, we have introduced the parameter $g = 2 \mu_0 \kappa^2 e^2 \xi_0^2 k_B / (\pi \hbar^2 s)$ and we have taken into account the expression of the quartic term coefficient $b = 2 \mu_0 \kappa^2 e^2 \hbar^2 / m^2$, with $\kappa$ the Ginzburg-Landau parameter $\kappa = \lambda_0 / \xi_0$. In analogy with the Gaussian fluctuation case, we shall adopt as definition for the critical temperature $T_c(E, B)$ the vanishing of the reduced temperature, $\bar{\varepsilon} = 0$. The relationship between $T_0$ and $T_c(0, 0) = T_{c0}$, corresponding to Eq. (11) taken in the zero-fields limit at $\bar{\varepsilon} = 0$, has been already found in Ref. 8 and writes

$$T_0 = T_{c0} \left\{ \sqrt{c/r} + \sqrt{1 + (c/r)} \right\}^{2 g T_{c0}} .$$

We shall take as example the optimally doped YBa$_2$Cu$_3$O$_{7-x}$, for which typical characteristic parameters are: $s = 1.17$ nm, $\xi_0 = 1.2$ nm, $\xi_{0c} = 0.14$ nm, $\kappa = 70$ and $T_{c0} = 92$ K. We assume a positive (hole-like) normal-state Hall conductivity $\sigma_{xy}^N$, obeying the Anderson’s formula $\sigma_{xy}^N / \sigma_{xy}^N = A T^2$ with a generic $A = 0.07$ K$^{-2}$ at $B = 1$ T, and a linear extrapolation for the normal state resistivity vanishing at $T = 0$, with a typical value $\rho_{xx}^N = 84 \mu \Omega$cm at $T = 200$ K. The cut-off parameter $c = 1$. For the Fermi energy we take $\varepsilon_F = k_B T / 10$ K$^2$ while the parameter $\alpha$ will be given the positive value $\alpha = 0.01 \frac{\Delta \sigma_{xy}}{\Delta \sigma_{xy}}$ in order to have $\eta < 0$ and $\Delta \sigma_{xy} < 0$, and thus to account for the Hall effect’s sign change occurring in the transition region.

It should be mentioned that the relevance of the $N^\sigma$-sign to the sign change of the Hall effect is still open to debate. The conventional $s$-wave weak coupling BCS theory predicts a positive excess Hall effect in the under-doped cuprates for a hole-like Fermi surface, in contrast to the experimental reports on which recent theoretical approaches based on the presence of preformed pairs$^{24}$ on an additional contribution to the particle-hole asymmetry coming from the quadratic electron spectrum$^{25}$ or on the proximity of an electronic topological transition$^{26}$ point out the possibility of an electron-like Hall sign in the hole-like doping range. Our purpose is however to illustrate the high electric field effect on the fluctuation Hall conductivity, for which $\alpha$ (or, equivalently, $\eta$) represents merely a prefactor that could be inferred from fits to the measurements. The experimental study of the non-Ohmic Hall effect could provide thus a supplementary and better tool for assessing the particle-hole asymmetry parameter $\alpha$, without the uncertainty introduced by the previous$^{18,22}$ need to estimate the background normal state contribution $\sigma_{xy}^N$, because the difference $\sigma_{xy}(E) - \sigma_{xy}(0) = \Delta \sigma_{xy}(E) - \Delta \sigma_{xy}(0)$ would be independent of $\sigma_{xy}^N$. An eventual experimental evidence of the non-Ohmic Hall effect behavior would also bring a strong argument in favour of the superconducting fluctuations in the long-lasting debate on the causes of the Hall effect sign change in HTSC.

Figure 1 shows the Hall conductivity $\sigma_{xy} = \sigma_{xy}^N + \Delta \sigma_{xy},$ normalized to the magnetic field value, for $B = 1$ T (dotted lines) and $B = 2$ T (solid lines) at different magnitudes of the in-plane electric field. One can notice a strong suppression of the fluctuation contribution
FIG. 1: Hall conductivity normalized to the magnetic field $B$ as a function of temperature for two values of the magnetic field, at several magnitudes of the (a) in-plane electric field; or (b) current density. A detail is shown in the inset.

to the Hall conductivity $\Delta \sigma_{xy}$ due to the high electric field. A higher magnetic field leads to a stronger reduction of the fluctuation Hall conductivity per se and thus, the fluctuation suppression as a function of the electric field becomes relatively smaller. For $E > 400$ V/cm the fluctuation part $\Delta \sigma_{xy}$ becomes negligible with respect to the normal-state component $\sigma_{xx}^N$, so that the Hall conductivity turns out to be much more sensitive to high electric fields than the longitudinal one $\sigma_{xx}$. The latter preserves a significant superconducting fluctuation component even for fields $E \approx 1500$ V/cm and $B = 11$ T, as shown in Ref. [28]. This might be not too surprising, since $\sigma_{xy}$ is altered much more than $\sigma_{xx}$ also by a magnetic field alone (Ref. [22]).

Throughout an experiment, constant current density $j$ can be achieved much easier than constant $E$. Thus, Fig. (b) presents the same non-Ohmic effect on $\sigma_{xy}$ with $j$ as the parameter. For this purpose, the equation $j = (\sigma_{xx}^N (T) + \Delta \sigma_{xx} (T, E, B)) E$ was firstly solved with respect to $E$ at fixed $T$, $B$ and $j$, where for the longitudinal fluctuation conductivity $\Delta \sigma_{xx}$ the result of Ref. [28] has been used:

$$\Delta \sigma_{xx} = \frac{e^2}{\hbar s} \int_{-\infty}^{\infty} \frac{dw'}{2\pi} \int_{-\pi}^{\pi} \frac{dq'}{2\pi} \sum_{n=0}^{N_s-1} \sqrt{n+1} \text{Im} (Q_{n,n+1}).$$

(12)

One can see in Fig. (b) that, due to the very low resistivity at lower temperatures, high electric field values are difficult to attain, so that the non-Ohmic effect can be discerned only at the beginning of the transition, and only for current densities higher than 2 MAcm$^{-2}$. This explains why the non-Ohmic effect on the Hall conductivity has not been detected experimentally so far. Nevertheless, attaining current densities of a few MAcm$^{-2}$, with minimal self-heating, on cuprate thin films of a typical $d = 100$ nm thickness, might not be such a difficult task, if one uses very short current pulses of the order of tens of nanoseconds, so that only the phonon mismatch at the film-substrate interface practically contributes to the sample temperature rise. According to literature data, a thermal boundary resistance of about $R_{bd} = 0.5$ mK-cm$^2$/W between YBa$_2$Cu$_3$O$_{7-x}$ and the substrate MgO would imply at about 100 K (where $\rho_{xx}^N \simeq 40$ $\mu$Omega) a temperature rise $\Delta T = dR_{bd} j^2 \rho_{xx}^N \simeq 0.8$ K for a current density $j = 2$ MAcm$^{-2}$. This accuracy could be sufficient to discern the non-Ohmic behavior.

In summary, we have calculated the critical fluctuation Hall conductivity for a layered superconductor in an arbitrary in-plane electric field and perpendicular magnetic field in the frame of the TDGL theory using the self-consistent Hartree approximation. The main result is the formula (10) that was found to reduce to previous results in the linear response limit Eq. (10). Qualitatively, high electric fields result in a strong suppression of the fluctuation contribution to the Hall conductivity, in particular in moderate magnetic fields where order-parameter fluctuations are still strong.

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