Extracting work from the magnetic field coupled Brownian particles

Tian Chen\textsuperscript{1,2,3}, Xiang-Bin Wang\textsuperscript{1,3,4}, and Ting Yu\textsuperscript{2}
\textsuperscript{1}State Key Laboratory of Low Dimensional Quantum Physics, Department of Physics, Tsinghua University, Beijing 100084, People's Republic of China
\textsuperscript{2}Center for Controlled Quantum Systems and the Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030, USA
\textsuperscript{3}Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China
\textsuperscript{4}Jinan Institute of Quantum Technology, Shandong Academy of Information and Communication Technology, Jinan 250101, People's Republic of China

Thermodynamics of the magnetic field coupled Brownian particles is studied. We show that in the presence of the magnetic field, work can be extracted from the reservoir even when the measurement operation and the potential change operation are applied in different spatial directions. In particular, we show that more work can be extracted if the measurements are applied in two different directions simultaneously. In all these cases, we show that the generalized second law involving the measurement information and potential change is satisfied. In addition, we show how the continuous potential change and measurement position affect the work extraction.

PACS numbers: 05.40.Jc, 05.70.Ln

I. INTRODUCTION

Extracting work from a system (reservoir) has attracted a widespread research interest recently\textsuperscript{1,2}. The original Jarzynski equality\textsuperscript{3–10} culminating in the recent developments states that, for a finite time nonequilibrium process, \(e^{-\beta W} = e^{-\beta \Delta F}\), this means that \(W \geq \Delta F\), where \(W\) is the work exerted on the system, and \(\Delta F\) is the equilibrium free energy difference between the initial and final states. More recently, an information-theoretic concept – relative entropy has been introduced\textsuperscript{11–13}, and the original Jarzynski equality is then extended to a new context where the measurement process and the information contribution from the measurement have been incorporated to produce a generalized Jarzynski equality \(e^{-\beta (W-\Delta F)-I} = 1\), where the quantity \(I\) is the obtained information measured by the relative entropy. Then the generalized second law is written as\textsuperscript{14–22},

\[ W \geq \Delta F - I. \]  

From Eq. (1), it becomes clear that, when the parameters are chosen appropriately, the thermal equilibrium states of the system remain the same at the initial and final times, hence the applied work can be negative due to the presence of the information contribution. Physically, it implies that one can extract work from the reservoir. Recently several models have been studied including Szilard machine\textsuperscript{23,24} where the work can be extracted from the thermal reservoir by employing the information of the particle’s position in the box, and the Brownian particle model\textsuperscript{14,17,25} where one can show that the Brownian particle can be driven by using the information of the measurement outcomes of the particle’s position\textsuperscript{14}. Furthermore, a more sophisticated Brownian motion model with variable potential’s stiffness and potential center together with a measurement process has been studied to show how the maximum work can be extracted from the reservoir\textsuperscript{17,25}. Other interesting developments that are devoted to energy and information extraction include the transporting particles devices through writing the information on the bit tape\textsuperscript{26–28}, the electron transport\textsuperscript{19} where one electronic state of a quantum dot is used as a detector, which only affects the entropy production of the probed conventional single electron transistor (SET) etc.\textsuperscript{29–32}. The purpose of this paper is to study the work extracting from the system consisting of the magnetic field coupled Brownian particles by incorporating the obtained information from measurement processes based on a model for the dynamics of the plasma diffusion\textsuperscript{8,33–35}. More specifically, we will consider the work and the measurement relation and explore the work extraction in four interesting setups. First, we consider the measurement operation and the potential change operation in the same direction. Then, we consider a different setup where the two operations are executed in different directions. Next, we consider the work extraction when two independent measurements are considered. Finally, we discuss the extracted work when the potential change operations are made in both two directions, and the measurement takes place only on one direction. We find that, for all four setups, when the measurement effects are taken into account, the generalized second law is valid even when the measurement operation and the changing potential operation are spatially separated. It is interesting to note that more work can be extracted when two independent measurements are performed. In addition, we have studied the magnetic field effect on the thermodynamics of this coupled particles system.

The organization of our work is as follows, Sec. III introduces the coupled Brownian particles model, the initial system state, and the definitions of the related thermo-
dynamic quantities. In Sec. III we study the effect of the outcomes of different position measurements, measurement accuracies, and frequencies of the potential. We conclude in Sec. IV.

II. MODEL

Equation of motion. The dynamics of the magnetic field coupled Brownian equations can be described by a stochastic Langevin equation [33, 32].

\[ M \frac{d\tilde{u}}{dt} = -\gamma \tilde{u} + \frac{q}{c} \times \vec{B} - k(\vec{x} - \vec{x}_0) + \bar{\mu}(t), \quad (2) \]

with \( \tilde{u} = (v_x, v_y), \vec{x} = (x, y), \vec{x}_0 = (x_0, y_0), \) the noise term \( \bar{\mu}(t) = (\mu_x(t), \mu_y(t)) \) satisfying \( \langle \mu_i(t)\mu_j(t') \rangle = \frac{\gamma k_B T \delta(t-t')}{\gamma} \) (i = x, y) where \( k_B \) is the Boltzmann’s constant and \( T \) is the thermal bath temperature. \( \gamma \) is the friction coefficient. The potential fields along \( x \) and \( y \) directions are \( V_x = \frac{1}{2}k_x(x-x_0)^2, V_y = \frac{1}{2}k_y(y-y_0)^2, \) respectively. The magnetic field \( \vec{B} \) is applied along the \( z \) direction.

The above equation can be explicitly written into a set of coupled differential equations along the \( x \) and \( y \) directions, respectively,

\[ \dot{x}_c = \frac{\gamma}{M} v_x - \frac{k_x}{M} (x - x_0) + \frac{q}{c M} v_y B + \frac{\mu_x(t)}{M}, \quad (3a) \]
\[ \dot{y}_c = v_x, \quad (3b) \]
\[ \dot{v}_c = -\frac{\gamma}{M} v_y - \frac{k_y}{M} (y - y_0) - \frac{q}{c M} v_x B + \frac{\mu_y(t)}{M}, \quad (3c) \]
\[ \dot{\bar{\mu}} = v_y. \quad (3d) \]

Here, for simplicity, we set \( M = 1 \), and employ the Fokker-Planck (FP) equation to describe the system dynamics [30],

\[ \frac{\partial \mathcal{P}(x, v_x, y, v_y, t)}{\partial t} = \left( -\frac{\partial}{\partial x_c} v_x + \frac{\partial}{\partial v_x} (\gamma v_x + k_x (x - x_0) - \frac{q}{c} v_y B) \right. \]
\[ + \frac{\gamma k_B T}{2} \frac{\partial^2}{\partial v_x^2} \frac{\partial}{\partial v_y} v_y \]
\[ + \frac{\partial}{\partial v_y} (\gamma v_y + k_y (y - y_0) + \frac{q}{c} v_x B) \]
\[ + \left. \frac{\gamma k_B T}{2} \frac{\partial^2}{\partial v_y^2} \right) \mathcal{P}(x, v_x, y, v_y, t). \quad (4) \]

where \( \mathcal{P}(x, v_x, y, v_y, t) \) denotes the probability distribution function of the system at time \( t \).

In the overdamped approximation, the system dynamics can be simplified as,

\[ \gamma v_x = -k_x (x - x_0) + \frac{q}{c} v_y B + \mu_x(t), \quad (5a) \]
\[ \gamma v_y = -k_y (y - y_0) - \frac{q}{c} v_x B + \mu_y(t). \quad (5b) \]

The FP equation can be written in a more transparent form if we use the following new notations:

\[ R = x - \frac{q B}{c} y, \quad S = \frac{q B}{c} x + y, \quad H(t) = \frac{k(t)}{\gamma (1 + (qB/c))^2}, \]
\[ J(t) = \frac{k(t)}{\gamma (1 + (qB/c))^2}, \quad N = \frac{k_B T}{\gamma}, \quad L = \frac{q B}{c^2}, \]

then the FP equation of the system distribution \( \mathcal{P}(R, S, t) \) is given by,

\[ \frac{\partial \mathcal{P}(R, S, t)}{\partial t} = \left( H(t) \frac{\partial}{\partial R} (R + L \cdot S) + J(t) \frac{\partial}{\partial S} (S - L \cdot R) \right. \]
\[ - \frac{k_x(t)}{\gamma} \frac{\partial}{\partial R} x_0 - \frac{k_y(t)}{\gamma} \frac{\partial}{\partial S} y_0 + N \frac{\partial^2}{\partial R^2} + N \frac{\partial^2}{\partial S^2} \right) \mathcal{P}(R, S, t). \quad (6) \]

Analytical Solution. The above FP equation can be solved when the Fourier transform is applied:

\[ \mathcal{P}(R, S, t) = \frac{1}{2\pi^2} \int dp dq \exp(iR p + S q) \mathcal{P}(p, q, t). \]

Note that the parameters transformation \( \frac{\partial}{\partial R} \rightarrow ip, \frac{\partial}{\partial S} \rightarrow iq \), \( R \rightarrow i \frac{\partial}{\partial p}, \quad S \rightarrow i \frac{\partial}{\partial q} \). We can decompose the probability function in the frequency domain as, \( \tilde{\mathcal{P}} = \tilde{P}_1 \tilde{P}_2 \tilde{P}_3 \), with \( \tilde{P}_1 = \exp(-ipf(t) - p^2 g(t)), \tilde{P}_2 = \exp(-iqh(t) - q^2 e(t)), \) and \( \tilde{P}_3 = \exp(-pqm(t)) \). Then it is easy to show that the coupled equations for the coefficients \( f(t), h(t), g(t), e(t), \) and \( \kappa(t) \) take the following forms,

\[ -\dot{f}(t) = H(t)f(t) + H(t)L \cdot h(t) - \frac{k_x(t)}{\gamma} x_0, \quad (7a) \]
\[ -\dot{h}(t) = J(t)h(t) - J(t)L \cdot f(t) - \frac{k_y(t)}{\gamma} y_0, \quad (7b) \]
\[ -\dot{g}(t) = 2H(t)g(t) + H(t)L \cdot \kappa(t) - N, \quad (7c) \]
\[ -\dot{c}(t) = 2J(t)c(t) - J(t)L \cdot \kappa(t) - N, \quad (7d) \]
\[ -\dot{\kappa}(t) = H(t)\kappa(t) + J(t)\kappa(t) + 2H(t)L \cdot e(t) - 2J(t)L \cdot g(t). \quad (7e) \]

The inverse Fourier transform is applied, and we obtain the system distribution \( \mathcal{P}(x, y, t) \) as,

\[ \mathcal{P}(x, y, t) = \frac{1}{4\pi} \frac{1}{\sqrt{g(t)e(t) - \kappa(t)^2}} \exp\left(-\frac{1}{|4g(t)e(t) - \kappa(t)^2|} \cdot \left( Q_1(x - A_1)^2 + Q_2(y - A_2)^2 + Q_3(x - A_1)(y - A_2) \right) \right), \quad (8) \]

where \( A_1 = \frac{f(t) + h(t)L}{1 + L^2}, A_2 = \frac{h(t) - f(t)L}{1 + L^2}, Q_1 = e(t) - \kappa(t)L + g(t)L^2, Q_2 = g(t) + \kappa(t)L + e(t)L^2, \) and \( Q_3 = \kappa(t)(L^2 - 1) - 2L(e(t) - g(t)). \)

Initial state. The initial state distributions along the \( x \) and \( y \) directions are given below, respectively,

\[ \chi(x, 0) = \frac{1}{\sqrt{2\pi u_x^2(0)}} \exp\left(-\frac{(x - d_x(0))^2}{2u_x^2(0)}\right), \quad (9a) \]
\[ \chi(y, 0) = \frac{1}{\sqrt{2\pi u_y^2(0)}} \exp\left(-\frac{(y - d_y(0))^2}{2u_y^2(0)}\right), \quad (9b) \]

where \( d_x, d_y(0), u_x^2(0), u_y^2(0) \) denote the initial average and variance in \( x(y) \) prior to a measurement, respectively.

Work. If the system Hamiltonian is parametrized by a single quantity \( \lambda \), the work performed during the time
interval $[0, t]$ along one trajectory $z_t$ is given by [3],

$$w(z_t) = \int_0^t d\tau \frac{\partial H(\lambda)}{\partial \lambda}(z_\tau).$$  \hspace{1cm} (10)

Here, $H(\lambda)$ is the system Hamiltonian. Then averaging over all the possible trajectories with the probability $\mathcal{P}(x, y, t)$, one gets the mean work $W$ [3 17]. For the Brownian particle model considered in this paper, the time-dependent terms are $k_x(t)$ and $k_y(t)$, and the total work during the time interval $[0, t]$ can be calculated as,

$$W = \frac{1}{2}(1 + L^2) \int_0^t d\tau \{k_x(\tau)[j^2(\tau) + 2g(\tau) + L^2(h^2(\tau) + 2e(\tau))]
+ 2L(k_x(\tau) + f(\tau)h(\tau)) - 2x_0(1 + L^2)(f(\tau) + Lh(\tau)) + y^2_0(1 + L^2) + k_y(\tau)h^2(\tau) + 2e(\tau) + L^2(f^2(\tau) + 2g(\tau))
- 2L(k_x(\tau) + f(\tau)h(\tau)) - 2y_0(1 + L^2)(h(\tau) - f(\tau)L)
+ y^2_0(1 + L^2)\}.$$  \hspace{1cm} (11)

III. WORK EXTRACTION WITH MEASUREMENT AND POTENTIAL CHANGE

We will consider four different setups where the work extraction and the generalized second law will be investigated. The first setup involves a measurement operation and the potential change operation in the different directions while the second one concerns the special case where the measurement operation and the potential change operation take place in the same direction. In the third setup, two measurements in different directions are considered. For these three setups, only one direction potential is time-dependent ($k_x(t)$ or $k_y(t)$). The fourth setup contains the potential change operations in both directions $x$ and $y$, and the measurement operation is executed along only one direction.

Specifically, for the first setup, we assume that the measurement is along the $y$ direction and the outcomes satisfy the Gaussian distribution $\mathcal{Y}(y_m|y) = \frac{1}{\sqrt{2\pi u^2_{y,m}}} \exp(-\frac{(y - y_m)^2}{2u^2_{y,m}})$ where the coefficients $y_m$ and $u_{y,m}$ stand for the position measurement outcome and accuracy, respectively. The more accurate the measurement, the smaller the $u_{y,m}$ values. The $y$ direction distribution after the measurement is then given by,

$$\mathcal{Y}(y|y_m) = \frac{1}{\sqrt{2\pi u^2_{y,m}}} \exp(-\frac{(y - y_m)^2}{2u^2_{y,m}}).$$  \hspace{1cm} (12)

Note that the distribution along the $x$ direction is

$$\mathcal{X}(x|y_m) = \frac{1}{\sqrt{2\pi u^2_{x,y,m}}} \exp(-\frac{(x - x_m)^2}{2u^2_{x,y,m}}).$$  \hspace{1cm} (13)

In this situation, the distribution along the $y$ direction is

$$\mathcal{Y}(y, 0) = \frac{1}{\sqrt{2\pi u^2_{y}(0)}} \exp(-\frac{(y - y_0)^2}{2u^2_{y}(0)}).$$

Next we introduce the concept of relative entropy. Suppose we measure the particle’s position along the $y$ direction, then the expression of the relative entropy is given by [17],

$$I_m(y_m) = \frac{1}{2} \ln(1 + \frac{u^2_{x,y}(0)}{u^2_{y,m}}) + \frac{u^2_{x,y}(0)}{2u^2_{y}(0) + u^2_{y,m}} \left(\frac{y_m - d_{x,y}(0))^2}{u^2_{y}(0) + u^2_{y,m}} - 1\right).$$  \hspace{1cm} (14)

A similar expression $I_m(x_m)$ can be obtained when the measurement is applied along the $x$ direction.

Now turning to the model consisting of the magnetic field coupled Brownian particles [33 32]. We assume that at time $t = 0$, the $x$ and $y$ potentials are given by $k_x(0) = k_y(0) = 1, x_0 = y_0 = 0$. The system initial distribution before the measurement is the product of the two thermal states in $x, y$ direction corresponding to the initial potentials ($u_{x,0} = u_{y,0} = 1, d_{x,0} = d_{y,0} = 0$). The system free energy may be calculated in a similar way [17], $F = \ln(\int \exp(-V(x))dx) = -\frac{1}{2} \ln(\frac{\pi q^2}{L^2})$, here $\Delta F$ stands for the equilibrium free energy difference. Further, we assume that the measurement is applied instantaneously, and the magnetic field is applied for a finite period of time. Our purpose is to study the extracted work during this finite time duration. For simplicity, the final condition of the total system is set as the same as the initial condition of the system (Therefore, one has $\Delta F = 0$).

To begin with, for the first setup, we consider the effect of the outcomes of different measurements on the work extraction. We calculate the work $W$ and relative entropy $I_m$ with different measurement results $y_m$. If the applied measurement is very accurate ($u_{y,m} = 0.01$), after the measurement, the $y$ direction distribution can be obtained as,

$$\mathcal{Y}(y|y_m) = \frac{1}{\sqrt{2\pi u^2_{y,m}}} \exp(-\frac{(y - y_m)^2}{2u^2_{y,m}}).$$  \hspace{1cm} (15)

The value of the magnetic field is set as $L(qB/c\gamma) = 0.5$. Fig. 1 shows that, when the outcome of the position measurement is close to the origin point ($y_m = 0$), no work can be extracted from the environment. When the measurement outcomes exceeds a certain threshold value, however, the applied work value becomes negative, which means that the work can be extracted from the environment by employing the continuous potential change. Although $W(y_m)$ is smaller than the free energy difference $\Delta F$, the generalized second law $W(y_m) + I_m(y_m) \geq$
\[ \Delta F = 0 \] is still satisfied \[ 16, 17, 25 \], and this process above is a thermodynamically validated process.

For the second setup, we assume that both measurement and potential change are performed in the x direction. As a comparison, we also discuss the process involving a measurement in the y direction while the potential change is still in x direction. Other parameters are set equally in these two different setups. The work extraction for the both setups is shown in Fig. 2 (a). In both setups, the work extraction is always possible even the measurement and potential change directions are different. However, more work can be extracted when the measurement and potential change directions coincide (all in x-direction). Specifically, the extract work from the two spatially separated operations is realized when the measurement accuracy reaches a certain value. Interestingly, it is shown that in both cases the generalized second law is satisfied.

Moreover, as the third setup which combines the above setups, we discuss the work extraction that the measurement operations are executed along the x and y directions. The potential change is along the x direction, from Fig. 2 (a) above, the extracted work \( W_{x+y} \) is larger than the measurement operation is applied in one direction solely.

Next, we study the extracted work of the fourth setup. The first setup is included here to compare with the work value \( W_{xy} \) from the fourth setup. As shown in Fig. 2 (b), more work is extracted when potential changes take place in both x and y directions.

In addition, we study the effect of the magnetic field and the potential change frequency on the work extraction. Fig. 3 shows the extracted work during a half integer period \( (k_1 t = \pi) \), and \( \Delta F = 0 \). The potential change is taken along the x direction, while the measurement is along the y direction. When the magnetic field is weak, the coupling between the two direction Brownian particles motion is weak, and the motion in these two directions \( (x, y) \) can be approximately seen as two separated motions, so the measurement along the y direction has little effect on the x direction dynamics. It is seen that in this weak-coupling limit the potential change operation cannot extract the work from the reservoir. When the magnetic field coupling strength is relatively large, the x and y direction dynamics are coupled, and one can extract work from the reservoir \( (W < 0) \). However, it is worthwhile to note that, if the magnetic field becomes too strong (exceeding a threshold), the applied work approaches zero. This is because the magnetic field interaction dominates all the interactions of the system rendering the external potentials to be ignorable. When there is no external force applied, the work done on the system is close to zero. As shown in Fig. 3 one can find that in our continuous modulation case, the maximum
work can be obtained for a certain magnetic field parameter. Of course, this value is affected by the frequency of the potential change.

\[ I_{\text{ave}} = \int p(y_m)I_m(y_m)dy_m \]
where \( p(y_m) \) is the probability of the position measurement with outcome \( y_m \), it is interesting to consider the feedback effect on the work extraction. The potential is chosen as \( V_x(t) = \frac{1}{2}(1 - 0.5 \cdot \sin(k_1 \cdot t))x^2 \), the evolvement time is the half integer period \( k_1 \cdot t = \pi \), the constant potential on the \( y \) direction, \( V_y = \frac{1}{2}y^2 \).

The thermal relaxation coefficients are set, \( N = 1 \), the measurement is done on the \( y \) direction, parameters are set, \( u_{y,0} = u_{x,0} = 1 \), \( d_{y,0} = d_{x,0} = 0 \), \( u_{y,m} = 0.01 \), and \( y_m = 4 \).

Now we will consider the average over the outcomes of the position measurement. The mean total work \( W_{\text{ave}} = \int p(y_m)W(y_m)dy_m \), the mutual information

\[ I_{\text{ave}} = \int p(y_m)I_m(y_m)dy_m \]

where \( p(y_m) \) is the probability of the position measurement with outcome \( y_m \), it is interesting to consider the feedback effect on the work extraction. The potential is chosen as \( V_x(t) = \frac{1}{2}(1 - 0.5 \cdot \sin(k_1 \cdot t))x^2 \), the evolvement time is the half integer period \( k_1 \cdot t = \pi \), the constant potential on the \( y \) direction, \( V_y = \frac{1}{2}y^2 \).

The thermal relaxation coefficients are set, \( N = 1 \), the measurement is done on the \( y \) direction, parameters are set, \( u_{y,0} = u_{x,0} = 1 \), \( d_{y,0} = d_{x,0} = 0 \), \( u_{y,m} = 0.01 \), and \( y_m = 4 \).

In summary, we have studied the work extraction in a model consisting of the magnetic field coupled to Brownian particles. We find that, by employing the information contribution, we can extract work from the reservoir even in the case where the measurement operation and the potential change operation are spatially separated. In particular, we show that more work can be extracted when two independent measurements are considered. In all the processes considered in this paper the generalized second law is obeyed. Furthermore, the extracted work for the different magnetic fields and the potential change frequencies as well as the measurement accuracies are discussed. In addition, the feedback effect on the work extraction is discussed, the applied work in the feedback case is shown to less than that in the no-feedback case. Physically, it implies that the feedback process can be used to gain more information from the system. It will be interesting to consider the work extraction in a case where more general measurement processes are considered. This will be the topic of our future investigation.

Note Added: After our manuscript was in print, we became aware of a paper available in arXiv (arXiv:1405.1461v1), which also considers a similar topic as ours.

ACKNOWLEDGEMENT

This work is supported in part by the 10000-Plan of Shandong province, and the National High-Tech Program of China Grants No. 2011AA010800 and 2011AA010803, NSFC Grants No. 11174177 and 60725416 (T.C. and X. B. W.). T.C. is grateful to the financial support from China Scholarship Council.
No. 201206210176. T.Y. acknowledges Grant support from the NSF PHY-0925174 and DOD/AF/AFOSR No. FA9550-12-1-0001.

[1] U. Seifert, Rep. Prog. Phys. 75, 126001 (2012) and references therein.
[2] K. Maruyama, F. Nori, and V. Vedral, Rev. Mod. Phys. 81, 1 (2009).
[3] C. Jarzynski, Phys. Rev. Lett. 78, 2690 (1997).
[4] C. Jarzynski, Phys. Rev. E 56, 5018 (1997).
[5] M. Esposito, U. Harbola, and S. Mukamel, Rev. Mod. Phys. 81, 1665 (2009).
[6] M. Campisi, P. Hänggi, and P. Talkner, Rev. Mod. Phys. 83, 771 (2011).
[7] M. Esposito, and S. Mukamel, Phys. Rev. E 73, 046129 (2006).
[8] J. I. Jiménez-Aquino, F. J. Uribe, and R. M. Velasco, J. Phys. A 43, 255001 (2010).
[9] Y. Subaşı, and B. L. Hu, Phys. Rev. E 85, 011112 (2012).
[10] B. Leggio, A. Napoli, H.-P. Breuer, and A. Messina, Phys. Rev. E 87, 032113 (2013).
[11] T. Sagawa and M. Ueda, Phys. Rev. Lett. 100, 080403 (2008).
[12] A. Gomez-Marin, J. M. R. Parrondo, and C. Van den Broeck, Phys. Rev. E 78, 011107 (2008).
[13] K. Jacobs, Phys. Rev. A 80, 012322 (2009).
[14] T. Sagawa and M. Ueda, Phys. Rev. Lett. 104, 090602 (2010).
[15] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki, and M. Sano, Nat. Phys. 6, 988 (2010).
[16] L. Granger and H. Kantz, Phys. Rev. E 84, 061110 (2011).
[17] D. Abreu and U. Seifert, Euro. Phys. Lett. 94, 10001 (2011).
[18] T. Sagawa and M. Ueda, Phys. Rev. Lett. 109, 180602 (2012).
[19] P. Strasberg, G. Schaller, T. Brandes, and M. Esposito, Phys. Rev. Lett. 110, 040601 (2013).
[20] T. Sagawa, and M. Ueda, New J. Phys. 15, 125012 (2013).
[21] T. Munakata, and M. L. Rosinberg, J. Stat. Mech. (2013) P06014.
[22] K. Funo, Y. Watanabe, and M. Ueda, Phys. Rev. E 88, 052121 (2013).
[23] L. Szilard, Z. Phys. 53, 840 (1929).
[24] S. W. Kim, T. Sagawa, S. De Liberato, and M. Ueda, Phys. Rev. Lett. 106, 070401 (2011).
[25] M. Bauer, D. Abreu and U. Seifert, J. Phys. A: Math. Theor. 45, 162001 (2012).
[26] D. Mandal and C. Jarzynski, Proc. Natl. Acad. Sci. U.S.A. 109, 11641 (2012).
[27] A. C. Barato, and U. Seifert, Euro. Phys. Lett. 101, 60001 (2013).
[28] J. M. Horowitz, T. Sagawa, and J. M. R. Parrondo, Phys. Rev. Lett. 111, 010602 (2013).
[29] J. M. Horowitz and J. M. R. Parrondo, New J. Phys. 13, 123019 (2011).
[30] J. M. Horowitz and J. M. R. Parrondo, Euro. Phys. Lett. 95, 10005 (2011).
[31] J. M. Horowitz and J. M. R. Parrondo, Euro. Phys. Lett. 95, 40004 (2011).
[32] M. Esposito and C. Van den Broeck, Euro. Phys. Lett. 95, 40004 (2011).
[33] J. M. Horowitz, and J. M. R. Parrondo, arXiv:1305.6281v2.
[34] J. B. Taylor, Phys. Rev. Lett. 6, 262 (1962).
[35] J. I. Jiménez-Aquino and M. Romero-Bastida, Phys. Rev. E 86, 061115 (2012).
[36] J. I. Jiménez-Aquino and R. M. Velasco, Phys. Rev. E 87, 022112 (2013).
[37] H. Risken, The Fokker-Planck Equation: Methods of Solution and Applications (Springer-Verlag Berlin Heidelberg 1989).