SL(2) sector: weak/strong coupling agreement of three-point correlators

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Abstract

We evaluate three-point correlation functions of single trace operators in $N = 4$ SYM at both weak and strong coupling. We focus on the case where two of the operators belong in a $SL(2)$ sub-sector, and are dual to string solutions in a broad class of solutions with large $S$ and $J$ charges, while the third operator is a BPS state. Perfect agreement between the structure constants at weak and strong coupling is found. Finally, comments on this matching, as well as on the space-time structure of the correlators, are given.
1 Introduction

$\mathcal{N} = 4$ Super Yang-Mills (SYM) theory is an important example of an interacting four dimensional Conformal Field Theory (CFT) which has been thoroughly studied because of the AdS/CFT duality with string theory [1]. Furthermore, it is the first interacting four dimensional theory that we have significant chances to completely "solve". Being a conformal field theory "solving" it means to be, at least, able to identify its primary operators and their conformal dimensions. The second necessary piece of information needed is the structure constants which determine the Operator Product Expansion (OPE) between two primary operators. Recently, huge progress has been made in the computation of the planar contribution to the conformal dimensions of non-protected operators for any value of the coupling constant, using integrability (for a recent review see [2]). On the other hand, very little is known about the structure constants. The aim of this note is to contribute towards this second direction.

Our current knowledge of the OPE coefficients is essentially based on a perturbative expansion around $\lambda = 0$, where standard Feynman diagrams can be used to evaluate the relevant gauge theory correlators, or around $\lambda = \infty$ where the IIB string theory is well approximated by a simpler description. By comparing the 3-point correlators among half-BPS operators in these two different limits, the authors of [3] conjectured that the corresponding structure constants are non-renormalised (i.e. they have a trivial dependence on the 't Hooft coupling). On the contrary the 3-point correlators among non-protected operators receive quantum corrections, as it is shown, for instance, by the correlator between three Konishi operators [4]. On the gauge theory side, the authors of [5–7] studied systematically the structure constants and computed the corrections arising from the planar 1-loop Feynman diagrams. The importance of the operator mixing for the operators participating in the correlators was stressed in [8, 7, 9]. On the string theory side it is more difficult to extract information about non-protected OPE coefficients, since, in the supergravity limit, all non-protected operators acquire large conformal dimension and decouple. The BMN limit [10] represents a different approximation, where it is possible to extract useful information on non-BPS structure constants.

Recently, another approach to the calculation of n-points correlators involving non-BPS states was developed [11–16]. More precisely, the authors of [13] argued that it should be possible to obtain the correlation functions of local operators corresponding to classical spinning string states, at strong coupling, by evaluating the string action on a classical solution with appropriate boundary conditions after convoluting with the relevant to the classical states wavefunctions. In [14, 17, 18], 2-point and 3-point correlators of vertex operators representing classical string states with large spins were calculated. Finally, in a series of papers [15, 16, 19, 21] the 3-point function coefficients of a correlator involving a
massive string state, its conjugate and a third ”light” state state were computed. This was
done by taking advantage of the known classical solutions corresponding to the 2-point
correlators of operators dual to massive string states.

More recently, three-point functions of single trace operators were studied in [22, 23]
from the perspective of integrability. In particular, an intriguing weak/strong coupling
match of correlators involving operators in the $SU(3)$ sector was observed [23]. This
match was found to hold for correlators of two non-protected operators in the Frolov-
Tseytlin limit and one short BPS operator. In this note, we extend this weak/strong
coupling match for correlators involving operators in the $SL(2, R)$ closed subsector of
$N = 4$ SYM theory. One important difference with respect to the $SU(3)$ case is that our
string solutions are not point-like in the $AdS_5$ space which means that their field theory
duals have a large number of covariant derivatives along a light-like direction.

The plan for the rest of this paper is as follows. In Section 2, we give a short review of
classical string solutions in the Frolov-Tseytlin limit having one large spin $S$ in $AdS_5$
and one large spin $J$ in $S^5$. Subsequently, we write down the analytically continued version of
these solutions describing the propagation of a string extending along a light-like direction
on the boundary of the $AdS_5$ into the bulk and back to the boundary. We then proceed
and use the formalism of [15] to evaluate the three-point function coefficient, at strong
coupling, of a correlator involving two of the aforementioned $SL(2, R)$ operators and a
BPS state. In Section 3, we evaluate the same three-point function coefficient at weak
coupling by employing a coherent state description of the $SL(2, R)$ operators valid in the
limit we consider. Perfect agreement between the weak and strong coupling result is
found. It should be noted that the agreement we find is valid for any solution in the
Frolov-Tseytlin limit. Finally, comments on this matching, as well as on the space-time
structure of the aforementioned correlators (see Appendix), are given.

2 Strong coupling regime

In this Section, we give a short review of classical string solutions in the Frolov-Tseytlin
limit having one large spin $S$ in $AdS_5$ and one large spin $J$ in $S^5$. These solutions are
the string counterparts of single trace gauge invariant operators belonging in the $SL(2)$
closed subsector of the full $PSU(2, 2|4)$ algebra of $N = 4$ supersymmetric Yang-Mills
(SYM) theory. The operators we will be focusing on can be written schematically as

$$O_{SJ} = Tr[D^5 S Z^J] + ...$$  \tag{2.1}

\footnote{The anomalous dimension of twist 2 operators has been studied extensively, both at weak coupling [24]
for theories with different amounts of supersymmetry and at strong coupling [25] for the maximally
supersymmetric theory.}
where $Z$ is one of the complex scalars of $N = 4$ SYM and $D_+$ is the covariant derivative along a light-cone direction.

Here we follow closely [26, 27]. As mentioned above, the string states we are interested in have two non-zero charges, one with respect to one of the isometries of $AdS_5$ and the other with respect to one of the isometries of the five-sphere $S^5$. Thus, it is enough to consider solutions embedded in a $AdS_3 \times S^1$ subspace of the full $AdS_5 \times S^5$ manifold. The metric of this subspace reads

$$ds^2 = - \cosh^2 \tilde{\rho} \, dt^2 + d\tilde{\rho}^2 + \sinh^2 \tilde{\rho} \, d\tilde{\phi}_1^2 + d\gamma_1^2. \quad (2.2)$$

The bosonic part of the Polyakov action can be written as

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d\sigma d\tau g_{\mu\nu}(\partial_\tau X^\mu \partial_\tau X^\nu - \partial_\sigma X^\mu \partial_\sigma X^\nu). \quad (2.3)$$

Besides satisfying the equations of motion that follow from (2.3) the solutions must satisfy the Virasoro constraints

$$g_{\mu\nu} \partial_\tau X^\mu \partial_\sigma X^\nu = 0$$

$$g_{\mu\nu}(\partial_\tau X^\mu \partial_\tau X^\nu + \partial_\sigma X^\mu \partial_\sigma X^\nu). \quad (2.4)$$

Subsequently, one can employ the change of variables

$$\tilde{\phi}_1 = \phi + t \quad \gamma_1 = \phi_3 - t \quad \tilde{\rho} = \frac{\rho}{2} \quad (2.5)$$

and look for solutions satisfying the following ansatz

$$t = k \tau \quad \phi = \phi(\sigma, \tau) \quad \phi_3 = \phi_3(\sigma, \tau) \quad \rho = \rho(\sigma, \tau). \quad (2.6)$$

The Frolov-Tseytlin limit consists in taking

$$k \to \infty \quad \text{while keeping} \quad k \partial_\tau X^\mu = \text{finite}, \quad \partial_\sigma X^\mu = \text{finite} \quad X^\mu = \rho, \phi, \phi_3. \quad (2.7)$$

Then to leading order in $k$ the first Virasoro constraint (2.3) becomes

$$k((\cosh \rho - 1)\partial_\sigma \phi - 2\partial_\sigma \phi_3) = 0. \quad (2.8)$$

This equation can be used to eliminate $\partial_\sigma \phi_3$ in terms of $\partial_\sigma \phi$. Under this substitution and at leading in $k$ order, the action (2.3) takes the form

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d\sigma d\tau \left(k((\cosh \rho - 1)\partial_\tau \phi - 2\partial_\tau \phi_3) - \frac{1}{4}((\partial_\sigma \rho)^2 + \sinh^2 \rho(\partial_\sigma \phi)^2)\right). \quad (2.9)$$
Changing the variable from $\tau$ to $t$ and introducing the effective coupling $\lambda' = J^2 (J \approx \lambda k)$ we get

$$S = \frac{J}{2} \int_0^{2\pi} \frac{d\sigma}{2\pi} \int dt \left( ((\cosh \rho - 1) \partial_t \phi - 2 \partial_t \phi_3) - \left( \frac{\lambda'}{4} ((\partial_\sigma \rho)^2 + \sinh^2 \rho (\partial_\sigma \phi)^2) \right) \right).$$  \hspace{1cm} (2.10)

As shown in [27, 26] the same expression appears also on the field theory side as the effective low energy Lagrangian of the $SL(2)$ spin chain associated to the 1-loop dilatation operator of $N = 4$ SYM restricted to this particular sector. This fact guarantees the agreement, in leading order in $\lambda'$, between the string energies and the anomalous dimensions of the corresponding operators in field theory.

- **Analytic continuation**

  Although the solution (2.5), (2.6) is perfectly fine for calculating the conserved charges of the string, it is not appropriate for calculating the 2-point function of this solution holographically. This is because the string of (2.5), (2.6) lives entirely in the bulk of $AdS$ and it never touches its boundary. What we need is a string solution that tunnels from the boundary of $AdS_5$ to the boundary, in the spirit of [11].

  This solution can be constructed by performing an analytic continuation to both the global and world-sheet time. Namely,

$$t \to -it \quad \tau \to -i\tau. \hspace{1cm} (2.11)$$

After this the metric (2.2) and the action (2.3) become

$$ds^2 = \cosh^2 \tilde{\rho} dt^2 + d\tilde{\rho}^2 + \sinh^2 \tilde{\rho} d\tilde{\phi}_1^2 + d\gamma_1^2. \hspace{1cm} (2.12)$$

$$iS = -S_E = -\frac{\sqrt{\lambda}}{4\pi} \int d\sigma d\tau g_{\mu\nu} (\partial_\tau X^\mu \partial_\tau X^\nu + \partial_\sigma X^\mu \partial_\sigma X^\nu). \hspace{1cm} (2.13)$$

If, as above, we keep only the leading in $k$ terms the expressions for the action and the Virasoro constraints read

$$iS = -S_E = -\frac{\sqrt{\lambda}}{4\pi} \int d\sigma d\tau \left( -ik((\cosh \rho - 1) \partial_\tau \phi - 2 \partial_\tau \phi_3) + \frac{1}{4} ((\partial_\sigma \rho)^2 + \sinh^2 \rho (\partial_\sigma \phi)^2) \right). \hspace{1cm} (2.14)$$

$$k((\cosh \rho - 1) \partial_\sigma \phi - 2 \partial_\sigma \phi_3) = 0 \hspace{1cm} (2.15)$$

$$ik((\cosh \rho - 1) \partial_\tau \phi - 2 \partial_\tau \phi_3) = -\frac{1}{4} ((\partial_\sigma \rho)^2 + \sinh^2 \rho (\partial_\sigma \phi)^2)$$

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3The agreement between string and field theory will not continue to hold at arbitrary order in the $\lambda'$ expansion due to the order of limits problem. On the string theory side both $\lambda$ and $J$ tend to infinity in such a way that $\lambda/J^2$ is small and fixed. On the other hand, on field theory side $\lambda$ (and thus $\lambda/J^2$) is kept small such that perturbation theory can be applied. Obviously these two limits are not the same.

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In the embedding coordinates, the Euclidean continuation of the solution (2.5), (2.6) is

\[ Y^{-1} = \cosh k\tau \cosh \tilde{\rho}, \quad Y^0 = \sinh k\tau \cosh \tilde{\rho}, \]
\[ Y^1 = \cosh (k\tau + i\phi) \sinh \tilde{\rho}, \quad Y^4 = -i \sinh (k\tau + i\phi) \sinh \tilde{\rho}, \]
\[ Y^2 = Y^3 = 0, \quad \gamma_1 = ik\tau + \phi_3(\sigma, \tau), \quad \tilde{\rho} = \frac{\rho}{2} = \tilde{\rho}(\sigma, \tau), \quad \phi = \phi(\sigma, \tau). \tag{2.16} \]

It is instructive to rewrite this solution in Poincare coordinates. It reads:

\[ y^0 = a \tanh k\tau, \quad z = \frac{a}{\cosh k\tau \cosh \tilde{\rho}}, \tag{2.17} \]
\[ y^1 = a \tanh \tilde{\rho} \frac{\cosh (k\tau + i\phi)}{\cosh k\tau}, \quad y^4 = -ia \tanh \tilde{\rho} \frac{\sinh (k\tau + i\phi)}{\cosh k\tau}, \quad \gamma_1 = ik\tau + \phi_3(\sigma, \tau). \]

where \( a \) is an overall scale which we have introduced by rescaling all the Poincare coordinates.

Let us now comment on the solution (2.16), (2.17). Firstly, we would like to note that because of the double Wick rotation of (2.11), the target spacetime is defined by

\[-(Y^{-1})^2 + (Y^0)^2 + (Y^1)^2 + (Y^2)^2 + (Y^3)^2 + (Y^4)^2 = -1 \]

and as a consequence it becomes Euclidean \( AdS_5 \).

Secondly, it is easy to see that (2.17) describes a string which tunnels from the boundary to the boundary of the \( AdS_5 \) space. Indeed, at \( \tau = -\infty \) (2.17) directly gives

\[ z = 0, \quad y^0 = -a, \quad y^1 = a \tanh \tilde{\rho} e^{-i\phi}, \quad y^4 = ia \tanh \tilde{\rho} e^{-i\phi} \tag{2.18} \]

which describes a string sitting on the boundary \((z = 0)\) at \( y^0(-\infty) = -a \) and extending along a light-like direction \((y^1)^2 + (y^4)^2 = 0\). Similarly, at \( \tau = \infty \) we get

\[ z = 0, \quad y^0 = a, \quad y^1 = a \tanh \tilde{\rho} e^{i\phi}, \quad y^4 = -ia \tanh \tilde{\rho} e^{i\phi} \tag{2.19} \]

which also defines a string sitting on the boundary and extending along another light-like direction. This tunnelling behaviour of string solution dual to twist \( J \) operators has already been studied in [17, 20]. It should be stressed that the classical solution of (2.17) does not end at two points on the boundary, as it happens with solutions which are extended only along coordinates of the five-sphere. However, this fact does not signal a problem regarding the positions on the boundary where the string vertex operators and thus the SYM dual operators should be inserted. As shown in [17, 20] these positions are the following points on the boundary \( w_1 = (-a, 0, 0, 0) \) and \( w_3 = (a, 0, 0, 0) \). This implies that the distance between the two operators is given by

\[ |w_{13}| = 2a. \tag{2.20} \]

\[ \text{To pass from the embedding coordinates to the Poincare ones we have used the relations } Y^\mu = x^\mu, \]
\[ \text{where } \mu = 0, 1, 2, 4 \text{ with the direction } 0 \text{ playing the role of time and } Y^{-1} + Y^3 = \frac{1}{2}, \quad Y^{-1} - Y^3 = \frac{e^{2x^0} - e^{-2x^0}}{2}. \]
We are now in position to evaluate the three-point structure constant of two large spin operators dual to the string solution (2.17) and a supergravity state, at strong coupling. For simplicity, we will choose the BPS state to be the BMN vacuum

\[ O_I(x) = \frac{1}{\sqrt{t}} \text{Tr}[Z^t](x), \]  

(2.21)

where \( Z = \frac{\Phi_1 + i\Phi_2}{\sqrt{2}} \) is one of the complex scalar fields of \( N = 4 \) SYM. Following the normalisations of [15] the corresponding spherical harmonic is

\[ Y_I(n) = \left( \frac{n_1 + 2n_2}{\sqrt{2}} \right)^l = \frac{1}{2^l} \sin^l \theta e^{il\gamma_1}, \]  

(2.22)

where \( \mathbf{n} \) is a six-dimensional unit vector defining a point on the 5-sphere,

\[ \mathbf{n} = n_i = (\sin \theta \cos \gamma_1, \sin \theta \sin \gamma_1, \cos \theta \sin a \cos \gamma_2, \cos \theta \sin a \sin \gamma_2, \cos \theta \cos a \cos \gamma_3, \cos \theta \cos a \sin \gamma_3). \]  

(2.23)

In (2.23) \( \gamma_1, \gamma_2, \gamma_3 \) parametrise the three isometries of the sphere. For all the solutions considered in this note \( a = \theta = \pi/2 \) which means that \( n_2 = n_3 = n_5 = n_6 = 0. \)

The correlators on which we will focus are

\[ < \bar{O}_{SI}(w_3) O_{SI}(w_1) O_I(x) > \]  

(2.24)

where \( O_{SI} \) are the operators dual to the string solutions of (2.17). The OPE coefficient we are after is given by [15]

\[ C_{\bar{O}_{SI}O_{SI}O_I} |w_{13}|^\Delta = \frac{2^{\frac{2}{3}}(l + 1) \sqrt{l'\lambda}}{\pi N} \int d\tau d\sigma Y_I(n) z^l \left( \frac{\partial_a X^a X^b X_{\mu} - \partial_a z \partial^a z}{z^2} - \partial_a \mathbf{n} \partial^a \mathbf{n} \right). \]  

(2.25)

In order to proceed we need to evaluate the integrand of (2.25). To this end we rewrite it as

\[ H = \frac{\partial_a X^\mu \partial^a X_\mu - \partial_a z \partial^a z}{z^2} - \partial_a \mathbf{n} \partial^a \mathbf{n} = \frac{\partial_a X^\mu \partial^a X_\mu + \partial_a z \partial^a z}{z^2} + \lambda_a \gamma_1 \partial^a \gamma_1 \]

\[ -2 \frac{\partial_a z \partial^a z}{z^2} - \partial_a \mathbf{n} \partial^a \mathbf{n} - \lambda_a \gamma_1 \partial^a \gamma_1. \]  

(2.26)

On the right hand side of (2.26) and in the first line one can easily recognise the Lagrangian density of the bosonic part of the Polyakov action. It can be most easily evaluated using (2.14) and the second of the Virasoro constraints (2.15). Using (2.17) one can evaluate the different term appearing in (2.26) to get

\[ \frac{\partial_a X^\mu \partial^a X_\mu + \partial_a z \partial^a z}{z^2} = \left( (\partial_a \rho)^2 + \sinh^2 \rho (\partial_a \phi)^2 \right) \]

\[ \frac{\partial_a z \partial^a z}{z^2} = \frac{k^2 \sinh^2 k\tau}{\cosh^2 k\tau} + \ldots \]

\[ \partial_a \mathbf{n} \partial^a \mathbf{n} = -k^2 + \ldots \]

\[ \partial_a \gamma_1 \partial^a \gamma_1 = -k^2 + \ldots \]  

(2.27)
where the dots in (2.27) denote terms with subleading powers of \( k \) which can be ignored.

For the same reason the first line of (2.27) is subleading with respect to the contributions coming from the other terms in (2.26). Overall we obtain

\[
H = -2k^2 \frac{\sinh^2 k\tau}{\cosh^2 k\tau} + 2k^2 + \ldots = \frac{2k^2}{\cosh^2 k\tau} + \ldots \tag{2.28}
\]

As a result, the structure coefficient becomes [15]

\[
C_{\mathcal{O}_{S_J} \mathcal{O}_{S_J} \mathcal{O}_I}^{(\text{strong})}(2\alpha) = \sqrt{\frac{\lambda}{2\pi l+3(l+1)}} \frac{k}{\pi N} \int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \frac{1}{2^{l/2}} \frac{e^{ik\tau + \phi_3}}{\cosh^l k\tau \cosh^l \hat{\rho} \cosh^2 k\tau}. \tag{2.29}
\]

Although both \( \hat{\rho} \) and \( \phi \) depend on \( \tau \) the denominator \( 1/\cosh^{(l+2)} k\tau \) localises the integrand of (2.29) around \( \tau = 0 \) [23]. The \( \tau \) integration can then be easily performed to give

\[
\int_{-\infty}^{\infty} d\tau \frac{e^{-lk\tau}}{\cosh^{l+2} k\tau} = \frac{2^{l+1}}{k(l + 1)} \tag{2.30}
\]

Using this result we finally obtain

\[
C_{\mathcal{O}_{S_J} \mathcal{O}_{S_J} \mathcal{O}_I}^{(\text{strong})} = \sqrt{\frac{\lambda}{2\pi}} \frac{k}{N} \int_0^{2\pi} d\sigma \frac{J \sqrt{l}}{2\pi} \int_0^{2\pi} d\sigma \frac{1}{2\pi} \frac{e^{il\phi_3(0,\sigma)}}{\cosh^l \hat{\rho}(0, \sigma)}. \tag{2.31}
\]

This is the final and main result of this Section. In the next Section we will compute the same quantity at weak coupling to find agreement with (2.31).

3 Weak coupling regime

In this Section we present the weak coupling computation of the structure constants where two of the operators are non-protected operators with large spins while the third one is the BMN vacuum. Since the operators we are considering belong to an \( SL(2) \) subsector of the full \( PSU(2,2|4) \) superconformal algebra of \( N = 4 \) SYM the orthodox way to proceed would be to find the Bethe eigenstates of this sector’s one-loop dilatation operator, which has been shown to be equivalent to the Hamiltonian of the \( XXX_{-1/2} \) spin chain [28], and plug them in the correlator to extract the structure constant. However, the exact Bethe eigenstates for operators having large values of \( S \) and \( J \) (see (2.1)) are very complicated entangled quantum states. Fortunately, when the length of the spin chain is large, i.e. in the large \( J \) limit a huge simplification occurs. Namely, the exact eigenstates can be approximated by coherent states [29]. Although these coherent states are not exact eigenstates of the 1-loop dilatation operator they have the following important property. If one computes with them the average of any classical quantity, such as the energy or the
spin, the result one gets agrees with the exact result obtained from the exact eigenstates up to finite size corrections [23]. Let us mention that the effective low-energy dynamics of the coherent spin chain states are governed by (2.10) (after removing the total derivative term) [27,26].

The coherent state approach in the $\text{SL}(2)$ sector has been studied in [27,26]. At each site of the spin chain one has a coherent state parametrised by a point on the upper sheet of the two-dimensional hyperboloid

$$l = (\cosh \rho, \sinh \rho \sin \phi, \sinh \rho \cos \phi), \quad l^2 = l_0^2 - l_1^2 - l_2^2 = 1, \quad l_0 > 0.$$  (3.32)

The $\text{SL}(2)$ coherent state is defined by applying the following ‘rotation’ operator on the lowest weight state $|0\rangle = |\frac{1}{2}, \frac{1}{2}\rangle$ of an infinite dimensional irreducible representation of the $\text{SL}(2)$ algebra.

$$|l\rangle = e^{\xi J_+ - \bar{\xi} J_-}|0\rangle, \quad \xi = \frac{\rho}{2} e^{i\phi}. $$  (3.33)

In (333) $J_+$ and $J_-$ denote the creation and annihilation operators which together with $J_0$ define the $\text{SL}(2)$ algebra through the commutation relations

$$[J_-, J_+] = 2J_0, \quad [J_0, J_{\pm}] = \pm J_{\pm}, \quad J_{\pm} = J_2 \mp iJ_1.$$  (3.34)

One can show that that the the coherent state can also be expressed in the form [26]

$$|l\rangle = \frac{1}{\cosh \frac{\rho}{2} \cosh \frac{\rho}{2}} \sum_{m=0}^{\infty} e^{im\phi} \tanh^m \frac{\rho}{2} |\frac{1}{2}, \frac{1}{2} + m\rangle. $$  (3.35)

We list now two important properties of the coherent states. The first one is that they have unit norm. The second one is that they are not orthogonal.

$$\langle l|l\rangle = 1$$

$$\langle l_1|l_2\rangle = \frac{1}{\cosh \frac{\rho_1}{2} \cosh \frac{\rho_2}{2} - \sinh \frac{\rho_1}{2} \sinh \frac{\rho_2}{2} e^{i(\phi_2 - \phi_1)}}$$  (3.36)

We are now ready to write down the coherent state representation of the operators dual to the string solutions considered in the previous Section. The physical picture is that of a varying spin wave pointing in the direction $l$ which is slowly changing from site to site. The operator $O_1$ will be represented by the following coherent state

$$\langle O_1 | = \prod_{i=1}^{J_1} \otimes | l(l_i)\rangle.$$  (3.37)
Similarly, the second non-protected operator \( O_1 \) will be given by
\[
|O_2⟩ = \prod_{i=1}^{J_2} \otimes |\lambda(J_2)⟩. \tag{3.38}
\]

We should mention that the function \( \lambda(\sigma) = \lambda(2\pi \tau) \) appearing in (3.37) and (3.38) is the same. This is the incarnation that \( O_1 \) is almost the complex conjugate of \( O_2 \). Finally the BMN vacuum \( \frac{1}{\sqrt{l}} \text{Tr} [Z^l] \) can be written as
\[
|O_1⟩ = \frac{1}{\sqrt{l}} \prod_{i=1}^{l} \otimes |\lambda_0(i⟩), \tag{3.39}
\]

where \( \lambda_0(\sigma) \) is the constant vector \( \lambda_0(\sigma) = (1, 0, 0) \). This is fully consistent with the fact that the BMN vacuum \( \text{Tr} [Z^l] \) lives in the centre \( \rho = 0 \) of the AdS_5 space.

A first observation is that the norm of (3.37) and (3.38) is 1 due to the first equation in (3.36). A second one is that the R-symmetry imposes the condition
\[
J_1 = J_2 + l. \tag{3.40}
\]

Notice also that the vectors \( \lambda(i \cdot J) \) depend also on time since the spin chain coherent state is dynamical. However, following [23] and taking into account that in the strong coupling result the main contribution comes from the region around \( \tau = 0 \) we have set \( t = 0 \) to all coherent state vectors appearing in (3.37) and (3.38). To simplify notation we have suppressed the \( t = 0 \) argument in \( \lambda(i \cdot J) \), i.e. \( \lambda(i \cdot J) = \lambda(i \cdot J, t = 0) \).

The three-point structure constant is obtained by Wick contracting the three operators of (3.37), (3.38) and (3.39). The operator \( O_2 \) can be Wick contracted with the 'long' operator \( O_1 \) at sites \( k, k+1, \ldots, J_2 + k - 1 \). As a result, the BPS state \( O_I \) should then be contracted with \( O_1 \) at sites \( J_2 + k, J_2 + k + 1, \ldots, J_1, 1, \ldots, k - 1 \) (see Figure 1). Finally, one should sum over the insertion starting point \( k \). Let us start by evaluating the Wick contractions between \( O_1 \) and \( O_2 \). These are given by
\[
\mathcal{I}_k = \prod_{i=k}^{J_2+k-1} (\lambda(i \cdot J_1)|\lambda(i \cdot J_2)⟩) = \exp \sum_{i=k}^{J_2+k-1} \log (\lambda(i \cdot J_1)|\lambda(i \cdot J_2)⟩) =
\]
\[
\exp \int_{\frac{2\pi (J_2+k-1)}{J_1}}^{2\pi (J_2+k-1)} (-1)J_1 \frac{d\sigma}{2\pi} \log \left( \cosh \frac{\rho(i \cdot J_1)}{2} \cosh \frac{\rho(i \cdot J_2)}{2} - \sinh \frac{\rho(i \cdot J_1)}{2} \sinh \frac{\rho(i \cdot J_2)}{2} e^{i(\phi(i \cdot J_1) - \phi(i \cdot J_2))} \right) =
\]
\[
\exp \int_{\frac{2\pi k}{J_1}}^{2\pi k} (-1)J_1 \frac{d\sigma}{2\pi} \log \left( \cosh \frac{\rho(\sigma)}{2} \cosh \frac{\rho(\sigma \cdot J_2)}{2} - \sinh \frac{\rho(\sigma)}{2} \sinh \frac{\rho(\sigma \cdot J_2)}{2} e^{i(\phi(\sigma \cdot J_1) - \phi(\sigma \cdot J_2))} \right)
\]

(3.41)
Figure 1: Tree level contractions contributing to the 3-point structure constant at weak coupling. \( O_1 \) and \( O_2 \) are large \( (J_1 \approx J_2 >> 1) \) operators in the \( SL(2) \) sector which are almost conjugate to each other and are well approximated by coherent states. \( O_1 \) is the small \( (l << J_1) \) BPS operator. \( k \) is the starting point for the contractions between \( O_1 \) and \( O_2 \). R-charge conservation implies that \( J_1 = J_2 + l \).

In passing from the first to the second line of (3.41) we have use the second equation in (3.36), as well as the fact that in the large \( J_1 \) limit \( \sum_i \rightarrow \int J_1 \frac{d\sigma}{2\pi} \). The next step is to evaluate the logarithm appearing in (3.41). To this end we use the expansions

\[
\cosh \left( \frac{\rho(\frac{J_1}{J_2})}{2} \right) = \cosh \left( \frac{\rho(\sigma)}{2} \right) + \frac{1}{2} \sinh \left( \frac{\rho(\sigma)}{2} \right) \partial_\sigma \rho(\sigma) \frac{l\sigma}{J_2} + O\left( \frac{1}{J_2^2} \right)
\]

\[
\sinh \left( \frac{\rho(\frac{J_1}{J_2})}{2} \right) = \sinh \left( \frac{\rho(\sigma)}{2} \right) + \frac{1}{2} \cosh \left( \frac{\rho(\sigma)}{2} \right) \partial_\sigma \rho(\sigma) \frac{l\sigma}{J_2} + O\left( \frac{1}{J_2^2} \right)
\]

\[\phi(\frac{J_1}{J_2}) = \phi(\sigma) + \partial_\sigma \phi(\sigma) \frac{l\sigma}{J_2}. \quad (3.42)\]

Plugging these expansions in the the logarithm appearing in (3.41) one gets

\[
\log \left(1 - i \sinh \frac{\rho(\sigma)}{2} \partial_\sigma \phi(\sigma) \frac{l\sigma}{J_2} \right) = -i \sinh \frac{\rho(\sigma)}{2} \partial_\sigma \phi(\sigma) \frac{l\sigma}{J_2} + O\left( \frac{1}{J_2^2} \right)
\]

\[= -i \partial_\sigma \phi_3(\sigma) \frac{l\sigma}{J_2} + O\left( \frac{1}{J_2^2} \right), \quad (3.43)\]

where in order to get the last line of (3.43) we have used the first of the Virasoro constraints (2.15) to express the derivative of the \( AdS_5 \) angle \( \phi \) in terms of the derivative of the \( S^5 \).
angle $\phi_3$. Plugging this result in (3.41) one gets

$$I_k = \exp \left[ \int_{2\pi J_1}^{2\pi J_1 \left( J_2 + k - J_1 - l \right)} J_1 \frac{d\sigma}{2\pi} i\partial_\sigma \phi_3 \left| \frac{J_1}{2} \right. \right] =$$

$$\exp \left[ il \frac{J_1}{J_2 2\pi} \left( \left( \sigma \phi_3 \right) \frac{2\pi (J_2 + k - J_1)}{2\pi J_1} - \int_{2\pi k J_1}^{2\pi k J_1} d\sigma \phi_3 \right) \right] =$$

$$e^{il\phi_3 \left( \frac{2\pi k}{J_1} \right)} e^{-il\Phi} + O \left( \frac{1}{J_1} \right), \quad \Phi = \int_{2\pi k J_1}^{2\pi k J_1} \frac{d\sigma}{2\pi} \phi_3 (\sigma) = \int_0^{2\pi} \frac{d\sigma}{2\pi} \phi_3 (\sigma) + O \left( \frac{1}{J_1} \right). \quad (3.44)$$

Let us now evaluate the result of the contractions between the BPS operator and $O_1$. The sites of $O_1$ that are contracted are $(J_2 + k = J_1 - l + k, ..., J_1, 1, ..., k - 1)$ (see Figure 1). These contractions give

$$J_k = \frac{l}{\sqrt{l}} \prod_{i=-J_1 - l + k}^{k-1} \langle l \left| \frac{j}{J_1} \right| 0 \rangle \quad (3.45)$$

The factor if $1/\sqrt{l}$ appearing in (3.45) is coming from the normalisation of the BPS operator while the factor of $l$ from the fact that there are $l$ different ways of contracting the BPS operator with the operator $O_1$. This is so because one can contract any of the Z fields with the first site $J_2 + k = J_1 - l + k$ from which the contractions of the BPS state and $O_1$ start. Furthermore, since the BPS operator is small $l << J_1$ and the spin wave describing $O_1$ varies slowly from site to site one can approximate $\langle l \left| \frac{j}{J_1} \right| 0 \rangle$ (up to $1/J_1$ corrections) by its value at the last site $k - 1$ or better by its value at site $k$

$$\langle l \left| \frac{j}{J_1} \right| 0 \rangle = \frac{1}{\cosh \frac{\rho \left( \frac{j}{J_1} \right)}{2}}. \quad (3.46)$$

So one gets

$$J_k = \sqrt{l} \frac{1}{\cosh \frac{\rho \left( \frac{j}{J_1} \right)}{2}}. \quad (3.47)$$

Putting together (3.44) and (3.47) we obtain the final result for the three point structure constant at weak coupling.

$$C_{O_{S,j} O_{S,j} O_I}^{(weak)} = \frac{1}{N} \sum_{k=1}^{J_1} I_k J_k = e^{-il\Phi} \frac{J_1 \sqrt{l}}{N} \int_0^{2\pi} \frac{d\sigma}{2\pi} e^{il\phi_3 (\sigma)} \quad (3.48)$$

By taking into account the $\tilde{\rho} = \frac{\rho}{2}$ we see that (3.48) is in agreement with the strong coupling result (2.31) up to an overall phase factor $e^{-i\Phi}$. This phase factor can also
appear in the strong coupling result. When we were writing the ansatz for the isometry of $S^5$ which is conjugate to the angular momentum $J$ we were having $\gamma_1 = ik\tau + \phi_3(\sigma, \tau)$. But we could very well add any constant angle to $\gamma_1$ and the equations of motion, as well as the Virasoro constraints would still be satisfied. Consequently, if we write the solution for $\gamma_1$ as $\gamma_1 = ik\tau + \phi_3(\sigma, \tau) - \Phi$ then complete agreement between the weak and strong coupling structure constants is obtained since the spherical harmonic will give $e^{il\gamma_1} = e^{-ik\tau + il\phi_3(\sigma, \tau)}e^{-il\Phi}$.

In any case, as pointed out in [23], the three-point coefficients are not completely unambiguous even if we canonically normalise the operators participating in the correlator. This is so because one can multiply any of these correlators by a constant phase. This phase will not alter the two-point functions of each of the operators but generically it will change the structure constant by a phase.

Another important comment is that the weak/strong coupling match we have found holds for any solution in the Frolov-Tseytlin limit.

Some additional comments are in order. The exponential appearing in the weak coupling structure constant (3.48) should be interpreted through the Virasoro constraint since the $S^5$ angle $\phi_3$ is meaningless from the field theory point of view. The $\phi_3$ appearing in (3.48) should be understood as the solution of the Virasoro constraint $2\partial_\sigma \phi_3 = (\cosh \rho - 1)\partial_\sigma \phi$ with periodic boundary conditions.

A second comment is related to the particular form of the operators used. It is well known that in the case where $J_1 = J_2 + l$, i.e. that is in the case where there are no contraction between operators $O_2$ and $O_I$, the mixing of the single trace operators $O_1$ with double trace operators is important in the evaluation of the structure constant [30, 5]. However, this kind of correlators have been recently studied in the literature at strong coupling without taking into account the mixing with multi-string states. For the same kind of correlators, involving operators in the $SU(3)$ sector, agreement between the weak and the strong coupling structure constants was found in [23]. There it was argued that either the same effect (meaning the effect of mixing with double traces or double string states) is being forgotten both at weak and strong coupling leaving a remainder quantity that can still be matched or that the effect of the mixing is suppressed in the large $J$ limit. The agreement we have found can be viewed from the same perspective. It is clearly desirable to find a more refined approach where these effects are taken into account.

Another issue is the fate of this agreement when one includes genuine one-loop contributions to the three-point functions. If both the strong and weak coupling results accept expansions in terms of the quantity $\frac{1}{J}$, it would be interesting to see if some of the corresponding coefficients agree, as it happens with the anomalous dimensions/string energies. This agreement, even if it is there, should fail at some point due to the well-known by now order of limits problem. As commented in [23], this agreement should be, more de-
cently, viewed as a guide to the all-loop result. Finally, it should be nice to generalise the weak/strong coupling agreement found in the $SU(3)$ sector \cite{23} and the one found in this note to the full $PSU(2,2|4)$ algebra.

4 Appendix

In this Appendix we are having a closer look at the spacetime structure of the correlators we have considered in the main text. The careful reader should have noticed that we have treated these correlation function as if they were scalar. However, this is not true for correlators involving the twist $J$ operators of (2.1).

First of all let us specify the light cone directions along which the derivatives of the operators (2.1) are taken. These directions are determined from the orientation of the string when it touches the boundary. By inspecting (2.18) we conclude that the derivatives of the operator $O_{SJ}(w_1)$ are taken along the light-like direction $e^\mu_+ = (y^0, y^1, y^2, y^4) = \frac{1}{\sqrt{2}}(0, 1, 0, i)$. In a similar way, (2.19) implies that in its conjugate $\bar{O}_{SJ}(w_3)$ the derivatives are taken along $e^\mu_- = (y^0, y^1, y^2, y^4) = \frac{1}{\sqrt{2}}(0, 1, 0, -i)$. By defining the light-like coordinates $y^+ = \frac{y^1+iy^4}{\sqrt{2}}$ and $y^- = \frac{y^1-iy^4}{\sqrt{2}}$ and bearing in mind that the boundary of $AdS_5$ has Euclidean signature we obtain $\eta_{+-} = \frac{\partial y^\mu}{\partial y^+} \frac{\partial y^\nu}{\partial y^-} \delta_{\mu\nu} = 1$. Let us first consider the two-point function of $O_{SJ}(w_1)$ and its conjugate $\bar{O}_{SJ}(w_3)$. Conformal invariance of the theory dictates the form of the two-point functions of operators which are symmetric and traceless in the vector indices to be \cite{31,32}

$$\langle \bar{O}_{\mu_1...\mu_S}(w_3) O_{\nu_1...\nu_S}(w_1) \rangle = \frac{\text{sym}[J_{\mu_1\nu_1}(w_{13}) J_{\mu_2\nu_2}(w_{31}) ... J_{\mu_S\nu_S}(w_{31})]}{w_{13}^{2\Delta}}$$ \hspace{1cm} (4.49)

where "sym" denotes the symmetrisation and subtraction of traces performed in each group of indices $\mu_1...\mu_S$, $\nu_1...\nu_S$ and $J_{\mu\nu}$ is the inversion tensor

$$J_{\mu\nu}(y) = \delta_{\mu\nu} - 2 \frac{y^\mu y^\nu}{y^2}.$$ \hspace{1cm} (4.50)

These symmetric and traceless operators belong in irreducible representations of the conformal group in Euclidean space.

The twist $J$ operators that we have consider in this note are exactly of this form. For the case in hand all $\mu_i = -$, $i = 1,...S$ while all $\nu_j = +$, $j = 1,...S$. Furthermore since $w_{31}^\mu = (2a, 0, 0, 0)$ we see that $w_{31}$ has no component along either of the light-cone directions + or −, i.e. $w_+ = w^- = 0$. As a result

$$J_{\mu,\nu}(w_{31}) = \eta_{+-} = 1$$ \hspace{1cm} (4.51)
and our two-point function becomes
\[ \langle \bar{O}_{SJ}(w_3) O_{SJ}(w_1) \rangle = \left( \frac{S!}{w_3^{2\Delta}} \right)^2. \] (4.52)

We now turn to the conformal structure of the three-point functions of the form (2.24).

To simplify notation, let us consider the case where one of the $SL(2)$ operators has only two indices, i.e. it is a symmetric tensor of rank two. In this case conformal invariance imposes that the three-point correlator should have the form \[ 31 \]
\[ \langle \bar{O}^{(\Delta_3)}_{\mu_1...\mu_S}(w_3) O^{(\Delta_1)}_{\mu\nu}(w_1) O^{(\Delta_2)}(w_2) \rangle = \left[ A_1(\lambda) Y^1_{\mu\nu}(w_2w_3) Y^3_{\mu_1...\mu_S}(w_2w_1) + A_2(\lambda) \left( \sum_{k=1}^{S} J_{\mu\nu k} (w_31) Y^3_{\mu_1...\mu_k...\mu_S}(w_2w_1) \right) \right] D_2(w_1, w_2, w_3), \] (4.53)
where the hat denotes the absence of the corresponding index and $A(\lambda)$, $B(\lambda)$ and $C(\lambda)$ are coupling dependent constants that can not be determined from just the conformal nature of the theory. Let us now define the quantities appearing in (4.53). Besides the inversion tensor $J_{\mu\nu}$, a second conformal vector is of importance \[ 33 \].

\[ Y^1_{\mu}(w_2w_3) = \frac{w^\mu_2}{w^\mu_1} - \frac{w^\mu_3}{w^\mu_1}. \] (4.54)

This structure is conformally covariant at point $w_1$ and invariant at points $w_2$ and $w_3$.

Finally, the higher order tensors are defined by
\[ Y^1_{\mu_1...\mu_n}(w_2w_3) = Y^1_{\mu_1}(w_2w_3) ... Y^1_{\mu_n}(w_2w_3) - \text{traces}, \] (4.55)
while
\[ D_2(w_1, w_2, w_3) = \frac{1}{(w_2^2)^{\frac{\Delta_1+\Delta_2-\Delta_1-S+2}{2} w_1^{\frac{\Delta_1-\Delta_2+\Delta_1-S-2}{2}} w_3^{\frac{\Delta_1+\Delta_2+\Delta_1+S-2}{2}}}}. \] (4.56)

From (4.53) it is clear how to generalise this expression to the case where the operator at point $w_1$ has $S' < S$ indices. This three-point correlator should have $S' + 1$ different terms which means that to completely determine it one needs $S' + 1$ different constants $A_i$, $i = 1, 2, ..., S' + 1$. The first term should be similar to the first term in the right hand side of (4.53) (the term multiplying $A_1$) except that it should have a tensor structure like $Y^1_{\nu_1...\nu_S'}(w_2w_3) Y^3_{\mu_1...\mu_S}(w_2w_1)$. The second term (the one involving $A_2$) should involve
$Y_{\nu_1...\nu_k...\nu_{s'}}$ and so on. The only other difference is the definition of $D_2(w_1, w_2, w_3)$ which becomes

$$D_2^{S'S'}(w_1, w_2, w_3) = \frac{1}{(w_{23}^2)^{\Delta_4/2+\Delta_2/2+\Delta_1/2-\Delta-S-s'}} (w_{13}^2)^{\Delta_4/2+\Delta_2/2+\Delta_1/2-\Delta-S-s'} (w_{12}^3)^{\Delta_4/2+\Delta_2/2+\Delta_1/2-\Delta-S-s'}. \quad (4.57)$$

Let us now write down the final form of the three-point function as this becomes for operators with the characteristics of the string solution of Section 2. Bearing in mind that $w_{13}^2 = w_{13}^2 = 0$ we obtain

$$\langle \bar{O}^{(\Delta_3)}_{\mu_1=...\mu_s=-(w_3)} O^{(\Delta_1)}_{\nu_1=+...\nu_{s'}=+}(w_1) O_I(w_2) (\Delta_2) \rangle = \sum_{l=0}^{S'} A_{S'+1-l}(\lambda) \frac{S!S'!}{(S-S'+l)!(S'-l)!} \frac{1}{(w_{13}^2)^{S'-l}} \left( \frac{w_{32}^{-}}{w_{32}^{+}} \right)^{s-S'+1} \left( \frac{w_{12}^{+}}{w_{12}^{-}} \right)^{l} \bar{D}_2^{S'S'}(w_1, w_2, w_3). \quad (4.58)$$

To derive the right hand side of (4.58) we have used (4.51) to set all $J$’s to one as well as the fact that $Y^{3}_{\mu=-(w_2^2)} = w_{32}^{-} \frac{w_{12}^{-}}{w_{32}^{+}} w_{13}^{-} = w_{13}^{-} \frac{w_{12}^{-}}{w_{32}^{+}}$ since $w_{13}^{-} = 0$. Furthermore, all traces are zero due to the vanishing of traces when both indices are + or −.

We conclude that the correlators we are interested in, (2.24), are determined by conformal symmetry up to a huge number of structure constants $A_i$, $i = 1, ..., S' + 1$. A natural question arises. Which one of these constants have we calculated in the main text and for which have we found agreement between weak and strong coupling?

At weak coupling the answer to this question is obvious. It is $A_{S'+1}$ the one we have computed in Section 3 (the corresponding term in (4.53) is the term involving $A_3$). This structure constant $A_{S'+1}$ is the $l = 0$ term of (4.58). Notice that all other terms of (4.58) include fractions like $w_{13}^{-}/w_{12}^{-}$ because for all other terms $l \neq 0$. The argument is that these fractions can never appear at tree level because there are no propagators connecting operators at points $w_1$ and $w_2$ (see also Figure 1). Or in other words, $A_{S'+1} = O(\lambda^0)$, whereas $A_{S'} = O(\lambda)$.

Next we argue that it is the same structure constant $A_{S'+1}$ that the method of [15] isolates. One way to see this is the following. In order to find the structure constant the author of [15] multiplies the quantity $< O_I(w_2) >_W$ by $w_2^{2\Delta_I} (\Delta_I = \Delta_2)$ and then sends $w_2 \to \infty$ keeping $w_1$ and $w_3$ fixed (see equation (2.5) of [15]). From (4.58) it is obvious that in this limit it is the $l = 0$ term that dominates. Another way that leads to the same result is the following. One can keep $w_2$ fixed and let both $w_1$ and $w_3$ approach zero. This is so because the theory is conformal so one can rescale all three points in such a way
way that brings the large $w_2$ to a finite value. But then $w_1 \approx w_3 \approx 0$ which also means that $w_{13} \to 0$. By inspecting (4.58) one can easily see that in the limit $w_{13} \to 0$ it is the $l = 0$ term (the one which has as coefficient $A_{S'+1}$) that dominates because all other terms behave as the $A_{S'+1}$ term times $|w_{13}|$ to some positive power. Consequently both at weak and at strong coupling it is the coefficient $A_{S'+1}(\lambda)$ that we have calculated. It is for this one agreement is found.

Another important comment is that the dominating $l = 0$ term of (4.58) is a scalar quantity in the limit we are considering. This is so because for the string solution we use, and as a result for the dual operators, it holds that $S = S'$, i.e. the operators at point 1 and 3 have the same $AdS_5$ spin although they have slightly different $S^0$ spins. Then from (4.58) it is obvious that

$$\left(\frac{w_{23}}{w_{13}}\right)^{S-S'+1} \left(\frac{w_{13}}{w_{23}}\right)^l = 1 \text{ for } l = 0.$$ 

This fact justifies our manipulations in Sections 2 and 3 where we have treated the three-point function as a scalar quantity.

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