A model-independent analysis of the dependence of the anomalous $J/\psi$ suppression on the number of participant nucleons

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Abstract. A recently published experimental dependence of the $J/\psi$ to Drell-Yan ratio on the measured, by a zero degree calorimeter, forward energy $E_{ZDC}$ in Pb+Pb collisions at the CERN SPS is analyzed. Using a model-independent approach, it is shown that the data are at variance with a earlier published experimental dependence of the same quantity on the transverse energy of neutral hadrons $E_T$. The discrepancy is related to a moderate centrality region: $100 \lesssim N_p \lesssim 200$ ($N_p$ is the number of participant nucleons) and is peculiar only to the data obtained within the ‘minimum bias’ analysis (using the ‘theoretical Drell-Yan’). This could result from systematic experimental errors in the minimum bias sample. A possible source of the errors is discussed.

Recently, the NA50 collaboration published new data on the centrality dependence of the $J/\psi$ suppression pattern in Pb+Pb collisions at CERN SPS. The centrality of the collisions was estimated by measuring the energy of projectile spectator nucleons $E_{ZDC}$ by a zero degree calorimeter. It was, however, mentioned that the new data may be at variance with the ones published by the same collaboration in Refs. where the transverse energy of produced neutral hadrons $E_T$ was used as a centrality estimator. The two sets of data (for brevity, we shall call them respectively ‘$E_{ZDC}$ data’ and ‘$E_T$ data’), having very similar qualitative behavior, seem, nevertheless, to be at variance quantitatively. It was demonstrated in Ref. that using three completely different $J/\psi$ production models if a model is fitted to the $E_T$ data, it does not agree with the $E_{ZDC}$-that and vice versa. Earlier it was mentioned that the co-mover model, which describes very well the $E_T$ data, does not show similar agreement with the $E_{ZDC}$ set.

Although the previous analysis pointed out a possible problem, its results can still be considered as inconclusive. Indeed, one may believe that the problem is related to the used models rather than to the data, and the ‘truly correct model’ should agree with the both data sets simultaneously.

In this letter, we use a model-independent analysis of the data to show that any model would be at variance at least with one of the data sets. This points out presence of systematic errors.

There are two types of data in each set, depending on the analysis procedure used to obtain the ratio $R$ of $J/\psi$ multiplicity to the multiplicity of Drell-Yan pairs. In the standard analysis, the number of both $J/\psi$ and Drell-Yan events are extracted from the measured dimuon spectra, i.e. the physical Drell-Yan is used as a reference. In the ‘minimum bias’ analysis, only the number of $J/\psi$ events and the total number of collisions (minimum bias events) in each centrality bin are found from the experiment.
The ratio $R$ is obtained by dividing the measured ratio of the number of $J/\psi$ events to the number of minimum bias events by the theoretical ratio of Drell-Yan multiplicity to the probability of a minimum bias collision at the corresponding centrality. Then the overall factor is fixed using the standard analysis data. The theoretical Drell-Yan to minimum bias ratio is given by

$$\langle DY \rangle_{E_x} = \frac{\int d^2b P(E_x|b) \langle DY(b) \rangle P_{\text{int}}(b)}{\int d^2b P(E_x|b) P_{\text{int}}(b)},$$

(1)

where $\langle DY(b) \rangle$ is the average number of Drell-Yan dimuon pairs per Pb+Pb collision at impact parameter $b$ and $P_{\text{int}}(b)$ is the probability that an interaction between two nuclei at impact parameter $b$ takes place. Both quantities are calculated in the Glauber approach. The subscript $x$ stands for $T$ or $ZDC$.

The $E_T$-distribution of events at fixed impact parameter $b$, $P(E_T|b)$, is assumed to have a Gaussian form with the central value and dispersion given by

$$\langle E_T(b) \rangle = q N_p(b) \quad \sigma_{E_T}(b) = q \sqrt{\alpha N_p(b)}.$$

(2)

Here $N_p(b)$ is the average number of participant nucleons in a Pb+Pb collision at impact parameter $b$ calculated in the Glauber approach. The parameter values $q = 0.274$ GeV and $\alpha = 1.27$ are fixed from the minimum bias transverse energy distribution [5].

The $E_{ZDC}$-distribution $P(E_{ZDC}|b)$ is assumed to be a Gaussian with the central value and dispersion given by Eqs. (1) and (2) of Ref. [1].

At a given dependence $\langle J/\psi(b) \rangle$ of the average number of $J/\psi$’s per Pb+Pb collision on the impact parameter $b$, the ratio $R$ in a finite-size $E_x$-bin, $E_{x\text{min}} \leq E_x \leq E_{x\text{max}}$ can be calculated as:

$$R(E_{x\text{min}} \leq E_x \leq E_{x\text{max}}) = \frac{\int_{E_{x\text{min}}}^{E_{x\text{max}}} dE_x \int d^2b P(E_x|b) \langle J/\psi(b) \rangle P_{\text{int}}(b)}{\int_{E_{x\text{min}}}^{E_{x\text{max}}} dE_x \int d^2b P(E_x|b) \langle DY(b) \rangle P_{\text{int}}(b)}.$$

(3)

In absence of essential systematic errors, the $E_T$- and $E_{ZDC}$ data should be consistent with each other: the formula (3) should fit the both data sets simultaneously: the $\chi^2$ per degree of freedom, $\chi^2/\text{ndf}$, that includes $E_T$ as well as $E_{ZDC}$ data points should be in the vicinity of 1. In the following we estimate in a model-independent way the lower bound for $\chi^2/\text{ndf}$ and show that it is in fact essentially larger than 1.

Instead of $\chi^2$ per degree of freedom, we shall calculate the $\chi^2$ per experimental point, $\chi^2/\text{nep}$, which provides a lower estimate for $\chi^2/\text{ndf}$:

$$\chi^2/\text{ndf} \geq \chi^2/\text{nep},$$

(4)

because $\text{ndf} \leq \text{nep}$.

We approximate the unknown function $\langle J/\psi(b) \rangle$ in the following way:

- We fix $J$ nodes $b_j$ on the $b$-axes: $b_0 = 0$, while $b_J$ is taken sufficiently large ($b_J = 22$ fm) so that the probability of interaction of two Pb-nuclei and nuclear modification of $J/\psi$ production at $b = b_J$ are negligibly small. In our calculations, we fix the rest of nodes in two different ways:
  - (a) equidistant
    $$b_{j+1} - b_j = \Delta b = \text{const}, \quad j = 0, \ldots, J - 1$$
    (5)
  - and
    - (b) so that the difference of the average number of participant nucleons between every two neighboring nodes is the same:
      $$N_p(b_j) - N_p(b_{j+1}) = \Delta N_p = \text{const}, \quad j = 0, \ldots, J - 1.$$
      (6)
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- The values of $\langle J/\psi(b) \rangle$ in all nodes except the last one:
  $$\langle J/\psi(b_j) \rangle = x_j, \quad j = 0, \ldots, J - 1,$$
  \hspace{1cm} (7)
  are treated as free parameters. The value in the node $b_j$ is fixed assuming the same $J/\psi$ to Drell-Yan ratio in extremely peripheral Pb+Pb collisions as in p+p:
  $$\langle J/\psi(b_j) \rangle = 53.5 \langle DY(b_j) \rangle$$
  \hspace{1cm} (8)

- The value of $\langle J/\psi(b) \rangle$ between the nodes is given by a linear interpolation:
  $$\langle J/\psi(b) \rangle = \frac{(b_{j+1} - b)x_j + (b - b_j)x_{j+1}}{b_{j+1} - b_j}, \quad b_j < b < b_{j+1}.$$  \hspace{1cm} (9)

- There is no reason to expect oscillating dependence of the $J/\psi$ multiplicity on the centrality. Therefore, $x_j$ are subjected to the constraint
  $$\frac{x_{j-1}}{\langle DY(b_{j-1}) \rangle} \leq \frac{x_j}{\langle DY(b_j) \rangle},$$
  \hspace{1cm} (10)
  which ensures monotonicity of the $J/\psi$ to Drell-Yan ratio $R$ as function of $b$.

- One more constraint is needed to avoid negative values of $J/\psi$ multiplicity:
  $$x_0 \geq 0.$$

Any non-negative function $\langle J/\psi(b) \rangle$ related to a monotonic $R(b)$, including even a disconnected one, can be arbitrarily well approximated in the above way, provided that $J$ is sufficiently large. Therefore, minimizing of $\chi^2$ with respect to all $x_j$, $0 \leq j < J$, at sufficiently large $J$ one gets the value, which is, at least, not larger than the minimum $\chi^2$ in any reasonable model.

Using Eqs. (3) and (9) one gets a linear dependence of $R$ on $x_j$:

$$R^{(n)} = \sum_{j=0}^{J} A_j^{(n)} x_j.$$  \hspace{1cm} (12)

Here

$$A_0^{(n)} = \frac{\int_{E_x^{(n), \text{min}}}^{E_x^{(n), \text{max}}} dE_x a_0^+ (E_x)}{\int_{E_x^{(n), \text{min}}}^{E_x^{(n), \text{max}}} dE_x \int_0^\infty dbbP(E_x | b) \langle DY(b) \rangle P_{\text{int}}(b)};$$  \hspace{1cm} (13)

$$A_j^{(n)} = \frac{\int_{E_x^{(n), \text{min}}}^{E_x^{(n), \text{max}}} dE_x \left[ a_j^-(E_x) + a_j^+(E_x) \right]}{\int_{E_x^{(n), \text{min}}}^{E_x^{(n), \text{max}}} dE_x \int_0^\infty dbbP(E_x | b) \langle DY(b) \rangle P_{\text{int}}(b)}, \quad j = 1, \ldots, J - 1;$$  \hspace{1cm} (14)

$$A_J^{(n)} = \frac{\int_{E_x^{(n), \text{min}}}^{E_x^{(n), \text{max}}} dE_x a_J^-(E_x)}{\int_{E_x^{(n), \text{min}}}^{E_x^{(n), \text{max}}} dE_x \int_0^\infty dbbP(E_x | b) \langle DY(b) \rangle P_{\text{int}}(b)};$$  \hspace{1cm} (15)

where

$$a_j^+(E_x) = \int_{b_j}^{b_{j+1}} dbbP(E_x | b) \frac{b_{j+1} - b}{b_{j+1} - b_j} P_{\text{int}}(b), \quad j = 0, \ldots, J - 1;$$  \hspace{1cm} (16)

$$a_j^-(E_x) = \int_{b_{j-1}}^{b_j} dbbP(E_x | b) \frac{b - b_{j-1}}{b_j - b_{j-1}} P_{\text{int}}(b), \quad j = 1, \ldots, J.$$  \hspace{1cm} (17)

In our fitting procedure, the initial values of $x_j$ are chosen randomly (using a random number generator). Then the $\chi^2$ is minimized in such a way that the constraint (10) is fulfilled at each iteration step.
The result of the fit of the NA50 minimum bias data is presented in Tab. and in Figs. 1 and 2. As is seen from the table at $J \geq 100$ the value of $\chi^2/\text{nep}$ saturates and ceases to depend on the way, how the nodes are fixed. Therefore $J = 200$ can be accepted as sufficiently large to provide a reliable lower estimate for $\chi^2/\text{nep}$. First, we fitted the $E_T$- and $E_{ZDC}$ data separately (‘$E_T$ fit’ and the ‘$E_{ZDC}$ fit’, respectively). In both cases the lower estimate of $\chi^2/\text{nep}$ is smaller than 1, which indicates that the difference between the data from 1996 and 1998 runs within each minimum bias dataset can be well explained by statistical fluctuations only. But if one compares the $E_T$ data and $E_T$ dependence of the $J/\psi$ to Drell-Yan ratio, calculated with $x_j$’s found from the $E_{ZDC}$ fit (see Fig. 1), one can observe clear discrepancy at moderate centrality ($N_p = 100–200$). Similar discrepancy is observed, when the $E_{ZDC}$ dependence resulting from the $E_T$ fit is compared with the $E_{ZDC}$ data.

Finally, we tried to fit the both sets simultaneously (the ‘combined fit’). The calculations show that $\chi^2/\text{nep} = 3.28$. This means that $\chi^2/\text{ndf} \geq 3.28$ in any model. Again, it is clearly seen from Figs. 1 and 2 that the problem is related to the centrality region $N_p = 100–200$. The discrepancy cannot be explained by statistical fluctuations (this hypothesis is excluded at the level of about $10^{-19}$!). Therefore, at least one of the data sets contains essential systematic errors.

There are reasons to suspect that these are the $E_{ZDC}$ data that contain the systematic errors and the source of these errors is a distortion of the minimum bias sample. Indeed, a crucial point of the minimum bias analysis is the assumption that both the measured Drell-Yan multiplicity as well as the experimental minimum bias centrality distribution are exactly proportional to the corresponding theoretical values, i.e. to the numerator and denominator of Eq. 1, respectively. The experimental and theoretical minimum bias probabilities should cancel (up to a constant factor), when one divides the experimental $J/\psi$ to minimum bias ratio by the theoretical Drell-Yan to minimum bias ratio (1), leaving finally only $J/\psi$ to Drell-Yan ratio.

While the experimental minimum bias $E_T$ distribution perfectly agrees with the Glauber model (see Fig. 1 of Ref. [5]), the agreement of the $E_{ZDC}$ minimum bias distribution (see Fig. 1 of Ref. [1]) seems to be not so perfect. Unfortunately, the data plotted in Ref. [1] have never been published in numerical form and the value of $\chi^2/\text{dof}$ characterizing the quality of the fit of the minimum bias $E_{ZDC}$ distribution has not been quoted. Nevertheless, it is seen from the plot that the experimental points lie slightly above the theoretical curves in the region $25 \lesssim E_{ZDC} \lesssim 18$ TeV (which corresponds to $100 \lesssim N_p \lesssim 200$). Although the difference does not exceed even the size of the point symbols, it, nevertheless, indicates a sizable discrepancy, due to logarithmic scale of the vertical axis of the plot. Because of this discrepancy, the measured and calculated minimum bias distribution do not cancel completely, which may lead to the observed systematic errors.

The question about the origin of the distortion of the minimum bias distribution is not simple. The final answer can hopefully be given after a detailed analysis of the raw experimental data supplemented by Monte-Carlo simulations. This is out of the scope of our present analysis. We restrict ourselves only to formulating a ‘working hypothesis’, which is to be tested in course of more detailed investigations. The observed discrepancy may come from interactions of reaction fragments with nuclei inside the zero degree calorimeter. If such an interaction takes place, a fraction of the

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Footnote: Disagreement is observed only in the low $E_T$ region, because the efficiency of the target identification algorithm is less than unity there.
produced particles (those having large transverse momenta) may leave the calorimeter without depositing their energy in it. This may cause losses of the zero-degree energy, which can result in distortion of the shape of the minimum bias distribution. If our hypothesis is correct, the data analysis procedure should be modified to take into account the losses of the zero degree energy. Otherwise the mentioned problem will persist even in a minimum bias analysis of the new, year 2000 data.

From Fig. 3, one can easily see that the minimum bias sample is much more sensitive to this effect than the dimuon one. Indeed, if spectators from a peripheral collision loose an essential part of their energy, this event is registered by the apparatus as a moderate centrality collision. The number of peripheral minimum bias events is about an order of magnitude larger than the number of events at moderate centrality. Therefore, even if the fraction of peripheral events that suffer from losses of the zero degree energy is small, the contamination at moderate centrality can be sizable. In contrast, the number of Drell-Yan events at moderate centrality is larger than the number of peripheral ones. Therefore the Drell-Yan sample is much less influenced.

This is supported by our comparison of the standard analysis data. Even the newest data \[9\], which have smaller statistical errors than older ones, do not show any contradiction between \(E_T\) and the \(E_{ZDC}\) data sets: according to our analysis, the lower bound of \(\chi^2/ndf\) for the combined fit of the new standard analysis data is 0.62.

The region of low \(E_{ZDC}\) (\(E_{ZDC} < 9\) TeV), i.e. the region of the ‘second drop’ of the \(E_{ZDC}\) data, deserves special attention. All models that have been successful describing the \(E_T\) data at \(E_T \gtrsim 100\) GeV \[9\] \[10\] \[11\] \[12\] explain the ‘second drop’ of the \(J/\psi\) to Drell-Yan ratio by \(E_T\)-losses in the dimuon event sample with respect to the minimum bias one and/or by the influence of \(E_T\)-fluctuations on the \(J/\psi\) production or suppression. Both effects do not influence the \(E_{ZDC}\) data, this means that all the models \[9\] \[10\] \[11\] \[12\] fail to describe the \(E_{ZDC}\) data at \(E_{ZDC} < 9\) TeV. If the drop at low \(E_{ZDC}\) indeed exists, the correct model should explain the high-centrality behavior of both \(E_T\) and \(E_{ZDC}\) data by a sharp decrease of the \(J/\psi\) multiplicity at small impact parameter.

It is, however, not unlikely that the ‘second drop’ in the \(E_{ZDC}\) data is an artifact of the data distortion. Indeed, Fig.1 of Ref.\[1\] shows clear disagreement of the minimum bias distribution with the Glauber model at low \(E_{ZDC}\). This, as was explained above, invalidates the minimum bias analysis procedure. Therefore, the apparent agreement between the \(E_T\)- and \(E_{ZDC}\)-data in high centrality region obtained within our analysis (see Figs.1 and 2) may be purely accidental. In this case, there would be no contradiction with the models \[9\] \[10\] \[11\] \[12\]. Hence, refining of the \(E_{ZDC}\) data is very important. It would be hopefully able to narrow the set of successful models in this field, which recently became too broad.

In conclusion, we performed a model independent comparison of two data sets published by the NA50 collaboration: the \(E_T\)- and \(E_{ZDC}\)-dependence of the \(J/\psi\) to Drell-Yan ratio \(R\) in Pb+Pb collisions at SPS. We have shown that the two sets are at variance. This problem seems to be related only to the data obtained within the minimum bias procedure. A possible reason for the problem may be a distortion of the minimum bias data sample. We argue that refining of the data is important to clarify the qualitative and quantitative behavior of the ratio \(R\) at \(E_{ZDC} < 9\) TeV.
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References

[1] M. C. Abreu et al. [NA50 Collaboration], Phys. Lett. B 521 (2001) 195.
[2] A. P. Kostyuk, H. Stocker and W. Greiner, arXiv:nucl-ex/0207018.
[3] M. C. Abreu et al. [NA50 Collaboration], Phys. Lett. B 410 (1997) 337.
[4] M. C. Abreu et al. [NA50 Collaboration], Phys. Lett. B 450 (1999) 456.
[5] M. C. Abreu et al. [NA50 Collaboration], Phys. Lett. B 477 (2000) 28.
[6] A. Capella and D. Sousa, arXiv:nucl-th/0110072.
[7] B. Chaurand, quoted in Ref. [10].
[8] L. Ramello [NA50 Collaboration], Talk given to the International Conference “Quark Matter 2002”, Nantes, France, July 18-24, 2002.
[9] A. Capella, E. G. Ferreiro and A. B. Kaidalov, Phys. Rev. Lett. 85 (2000) 2080 [arXiv:hep-ph/0002300];
A. Capella, A. B. Kaidalov and D. Sousa, Phys. Rev. C 65 (2002) 054908 [arXiv:nucl-th/0105021].
[10] J. Blaizot, M. Dinh and J. Ollitrault, Phys. Rev. Lett. 85 (2000) 4012 [nucl-th/0007020].
[11] A. P. Kostyuk, M. I. Gorenstein, H. Stocker and W. Greiner, J. Phys. G 28 (2002) 2297 [arXiv:hep-ph/0204180];
A. P. Kostyuk, arXiv:hep-ph/0209139.
[12] L. Grandchamp and R. Rapp, Nucl. Phys. A 709 (2002) 415 [arXiv:hep-ph/0205305];
arXiv:hep-ph/0209141.
Figure 1. The dependence of the $J/\psi$ to Drell-Yan ratio $R$ on the transverse energy $E_T$. The corresponding average number of participant nucleons $N_p$ are shown on the upper axis. The points with error bars are the NA50 ‘minimum bias’ analysis data (the $E_T$ data). The dotted and dot-dashed line are fitted to the $E_T$ data and the $E_{ZDC}$ data (see Fig. 2), respectively. The solid line represents the best possible fit to the both data sets simultaneously.
Figure 2. The dependence of the $J/\psi$ to Drell-Yan ratio $R$ on the energy of zero degree calorimeter $E_{ZDC}$. The corresponding average number of participant nucleons $N_p$ are shown on the upper axis. The points with error bars are the NA50 ‘minimum bias’ analysis data (the $E_{ZDC}$ data). The meaning of lines is the same as in Fig. 1.
Figure 3. The probability distribution of minimum bias and Drell-Yan events as a function of the number of spectator nucleons $N_{sp}$ in arbitrary units (a.u.), obtained within the Glauber approach.
Table 1. The value of $\chi^2$ per experimental point at different values of the number of nodes $J$ and for two different ways of choosing the nodes (a) and (b) (see text for details).

| J  | $E_T$ fit (a) | $E_T$ fit (b) | $E_{ZDC}$ fit (a) | $E_{ZDC}$ fit (b) | Combined fit (a) | Combined fit (b) |
|----|--------------|--------------|-------------------|-------------------|------------------|------------------|
| 3  | 6.62         | 3.93         | 21.6              | 69.9              | 25.5             | 51.8             |
| 6  | 2.07         | 1.60         | 1.31              | 6.19              | 3.96             | 6.72             |
| 12 | 0.84         | 0.83         | 0.86              | 1.14              | 3.44             | 3.46             |
| 25 | 0.69         | 0.68         | 0.82              | 0.83              | 3.32             | 3.34             |
| 50 | 0.67         | 0.67         | 0.81              | 0.81              | 3.29             | 3.29             |
| 100| 0.67         | 0.67         | 0.80              | 0.80              | 3.28             | 3.29             |
| 200| 0.67         | 0.66         | 0.80              | 0.80              | 3.28             | 3.28             |