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Strengthening container shipping network connectivity during COVID-19: A graph theory approach

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1. Introduction

Shipping has a positive impact on facilitating international trading activities, since more than 80% of the total world trade by volume is carried by sea (Zheng et al., 2021). In 2020, the total global container port throughput was 815.6 million Twenty-Foot Equivalent Units (TEUs). Containerized trade accounts for about 70% of total world trade in volume terms, making container shipping the most important mode of maritime transport (UNCTAD 2021). Liner shipping companies provide regular container services to connect ports all over the world, configuring a global container shipping network consisted of ports as nodes and shipping services as links. Connectivity level of the container shipping network indicates how well (to what extent) the ports in the network connect to each other. Good connectivity plays a significant role in ensuring the smooth delivery of goods between trading countries and regions.

However, the COVID-19 pandemic throughout the world had a significantly negative impact on the container shipping businesses (Xu et al., 2021a; Menhat et al., 2021). To slow down the spreading speed of the virus, containment measures such as temporary border closings, manufacturing plant shutdowns and mandated social distancing are implemented by countries (Dirzka and Acciaro 2022). These anti-epidemic measures restrict economic activities, hinder international trade and disrupt commodity imports and exports (Xu et al., 2021b). As a response to the uncertain cargo demand, liner companies reschedule or cancel services, resulting in decreased connectivity level of the container shipping network (Chen et al., 2021), because those cancellation of services in the shipping market is equivalent to the interruption of links in the container shipping network, which may lead to local disconnection of the shipping network. The cancellation of shipping market services is equivalent to the interruption of links in the container shipping network, which may lead to local disconnection of
the shipping network.

Port failures induced by COVID-19 have also affected shipping network connectivity. Reductions in working hours and shortages of labor and equipment at ports lead to longer port call times for ships. Such delays caused ports failures and ports’ ability to connect to others in the container shipping network decreased. For example, in 2021, the Port of Los Angeles and Port of Long Beach suffered from serious traffic jams. According to the Marine Exchange of Southern California, at its peak, more than 100 container ships floated off the U.S. West Coast, which were stuck at anchor or in drift areas (Xinhua 2021). To avoid high transport costs caused by container vessels waiting for available berths, liner shipping companies have to jump congestion ports so that blank sailings weaken the container shipping network connectivity (Guerrero et al. 2022).

The negative trend has been observed in liner shipping connectivity levels on a global scale (Notteboom et al. 2021). During public health emergencies such as COVID-19, it is extremely important to maintain shipping network connected and ensure the availability of shipping services. Therefore, it is critical that to optimize the container shipping network to be better connected, which has the ability to deliver goods between any two ports via several shipping links in the event of unexpected ports failures and service cancellations. Although a number of studies have focused on the optimization of container liner shipping networks of individual liner shipping companies with the aim of maximizing their profits (Gao et al., 2022; Chen et al., 2022), far fewer studies have investigated container route design while considering the role of a container shipping network as a whole in providing global connectivity. Consequently, the main questions addressed in this paper are as follows:

- How can we strengthen the connectivity of the container shipping network to improve network performance and mitigate negative impact of the COVID-19 pandemic on container shipping?
- Which shipping links can be added to strengthen a container shipping network’s connectivity the most when the number of shipping links that can be added is limited?

We propose a model for strengthening a container shipping network’s connectivity. The major contributions of this paper are as follows. First, we propose a model for strengthening a container shipping network’s connectivity using a graph theory approach. This model considers both the network topology and the possibility of opening new shipping links between ports. Network connectivity is measured by algebraic connectivity, and the possibility of opening new shipping links is estimated by an extended gravity model. We show the proposed model strengthens network connectivity the most when the number of shipping links that can be added is limited. Second, to efficiently solve the model, a heuristic algorithm based on Fiedler vector is used to find optimal solutions that indicate the optimal shipping links. Third, simulation experiments have been conducted on a real-world container shipping network. These simulations show how to strengthen the connectivity of a container shipping network by adding a set of shipping links and how to use the findings to obtain policy insights in the context of the COVID-19 pandemic.

The reminder of this paper is organized as follows. Section 2 provides a brief review of the related literature. Section 3 formulates the model to strengthen the container shipping network’s connectivity. In Section 4, a heuristic algorithm is developed to solve the model. Section 5 reports on simulation experiments conducted under different scenarios to explore container routing strategies. Section 6 concludes the paper and provides policy implications for decision makers.

2. Literature review

2.1. Connectivity analysis of container shipping network

Containers are transported through a container shipping network. Well-designed topological structures are of great importance for a network to reduce the operational cost and facilitate international trade. Consequently, there are numerous studies that have assessed a network’s structure by a variety of measures. Most of the researchers’ efforts have been dedicated to the study of the following measures: connectivity, vulnerability, robustness, resilience and reliability (Jiang et al., 2021; Asadabadi and Miller-Hooks 2020). Among these measures, connectivity, or the extent to which a network is connected, is one of the fundamental structural properties of the network and can be considered as a core element characterizing transport network (Reggiani et al., 2015). Connectivity analysis of container shipping network has gained a lot of attention. In container shipping network, once a set of ports or shipping services breakdown, the connected shipping network would become disconnected. Thus, the higher the connectivity, the less vulnerable, the more reliable and the more robust the shipping network.

Graph theory, the study of mathematical objects known as graphs which consist of nodes connected by edges, is usually used to model a container shipping network and analyze its structural properties. Graphs are so named because they can be represented graphically. In graph theory, vertex connectivity or edge connectivity is defined to measure the extent to which a graph is connected, measured simply by the smallest number of vertices or edges removed to make a connected graph be disconnected (Bondy and Murty 1976). In maritime transport, the concept of connectivity has been extended, specifically in relation to the transport service level that can be provided by a shipping network. In 2004, United Nations Conference on Trade and Development (UNCTAD) launched the Liner Shipping Connectivity Index (LSCI) to reveal the positions of coastal countries in terms of maritime connectivity. The LSCI measures deployment of services and ships by liners to a country’s ports of call, in other words, the number of companies that provide services, the number of services, the number of ships that call per month, the total annualized deployed container carrying capacity, and ship sizes (Hoffmann 2005). The LSCI was updated in 2019 to be on the port level, incorporating the number of countries that can be reached without the need for transshipment, also known as port LSCI (PLSCI). In recent years, LSCI has become a key performance indicator for ports, and researchers have paid more attention to the issue of port connectivity, from defining concepts and selecting indicators to developing evaluation methods. Table 1 briefly summarizes the connectivity analysis approach in maritime transport, illustrating that network graph theory is one of the most applicable tools for modeling maritime connectivity.

To date, there is no commonly accepted definition of connectivity in maritime transport. Most research on connectivity investigates the connectivity of individual ports to better understand the ports’ competitiveness and support port managers and policy-makers in their decision making on port planning and infrastructure investment (Jia et al., 2017; Martinez-Moya and Feo-Valero 2020; Tovar and Wall 2022). Some research focuses on the analysis of the influence of individual ports on the global or specific region’s international trade or maritime transport (Jiang et al. 2015; Pan et al., 2019; Cheung et al., 2020).

2.2. Shipping network connectivity strengthening

There is less research on strengthening the shipping network’s connectivity. Strengthening the connectivity of a network, in general, belongs to the area of network evolution. The Barabási-Albert (BA) model may be the best known model related to shipping network evolution, using growth and the preferential attachment mechanism to add new nodes and new links to existing networks (Barabási and Albert 1999). Jiang et al. (2019) applied the BA model to simulate the performance
ports that can serve large container vessels have become hub ports, 
vessels, which appeared in 1957 with only 226 TEU (the size of a regular 
container shipping network.

To achieve economies of scale in container transport, the size of a 
container vessel increases gradually. The first generation of container 
were designed to analyze the improvement of the network’s capacity 
performance. The study concerned the evolution and dynamics of 
shipping networks using the complex network theory, but it ignored the 
spatial elements in the analysis that have a great impact on the config-
uration of a transport network.

To summarize, there is still very little work available on strengthen-
ing connectivity. Moreover, most of the conventional models to 
strengthen connectivity only consider topology, while the possibility of 
opening new shipping links between ports in reality is ignored. There-
fore, this study proposes an approach to consider network connectivity 
and the possibility of opening new shipping links between ports at the 
same time.

3. Research method

3.1. Problem description

A container shipping network consists of container ports and vessel 
routes. To achieve economies of scale in container transport, the size of a 
container vessel increases gradually. The first generation of container 
vessels, which appeared in 1957 with only 226 TEU (the size of a regular 
container), has grown to 23,964 TEU (UNCTAD 2021). Meanwhile, 
ports that can serve large container vessels have become hub ports, 
while others are feeder ports. Fig. 1 illustrates the topology of the 
container shipping network.

Ports are connected by shipping links. For shipping companies, it is 
possible to operate direct shipping links between all ports due to 
operating cost considerations. The feeder ports in different trade regions 
are usually connected with each other via hub ports. For example, hub 
ports A and E serve as bridging nodes to transport goods between feeder 
ports B and G, or D and F. In this case, the shipping link connecting hub 
port A and hub port E plays a crucial role, determining whether regions I 
and II are connected by waterway. This shipping link can be considered 
as the most vulnerable part of the shipping network because if it is 
disrupted, network performance will degrade the most.

In addition, hub ports are usually located in developed countries, 
while maritime connectivity of the least developed countries, land-
locked developing countries and small island developing states without 
hub ports is poor. For example, the ports in trade region III cannot even 
connect to hub port A directly. The addition of a shipping link con-
necting ports A and H can significantly enhance the maritime connec-
tivity between regions I and III. Meanwhile, the maritime connectivity 
between regions I and II will also be strengthened, as container trans-
shipment at port H becomes possible and more container shipping links 
are available between ports in regions I and II.

Based on these considerations, we strengthen the container shipping 
network’s connectivity by adding a set of shipping links to the most 
vulnerable part of the network. The connectivity strengthening model is 
formulated as follows.

3.2. Modeling

Let graph \( G(V, E, W) \) be the representation of a container shipping 
network. In this graph, \( V \) is the set of nodes representing ports, \( E \) is the 
set of edges representing shipping services between port pairs, and \( W \) is 
the vector of edge weights. For the convenience of computer processing, 
graph \( G \) is represented as a weighted adjacency matrix \( A \), whose ele-
ments are \( a_{ij} \). If link \( (i, j) \in E \), \( i, j \in V \), \( a_{ij} = w_{ij} \), where \( w_{ij} \) indicates 
the number of shipping services connecting port \( i \) and port \( j \); otherwise, \( a_{ij} = 0 \). Let \( D \) be the diagonal matrix of \( A \), then the graph Laplacian 
can be defined as \( L = D - A \). To eliminate biases towards high-frequency nodes, 
the normalized graph Laplacian matrix is obtained by \( N = D^{-0.5}L D^{-0.5} \). 
The second smallest eigenvalue of the normalized graph Laplacian \( N \) for 
network \( G \), namely algebraic connectivity, is presented to locate the cut 
with the least total link weight reflecting the most vulnerability of the 
network, which can be associated with the connectivity of a network 
(Fiedler, 1973). Hence, algebraic connectivity is applied here to measure 
network connectivity, and the objective of the connectivity strength-
ening model is set as the maximal increase in algebraic connectivity. Let 
the decision variables be the links to be added, which are denoted as 
\( \Delta e = \{(i,j) \in E \} \forall i, j \in V \), then the connectivity strengthening model 
is formulated as:

\[
\begin{align*}
\text{Max} \cdot C &= \lambda_2(G + \Delta e) \\
\text{s.t.} \cdot |\Delta e| &= K \\
\Delta e &\leq W_{\Delta e}
\end{align*}
\]

The objective function (1) maximizes the connectivity of a container 
shipping network, where \( \lambda_2 \) is the algebraic connectivity of the network. 
Constraint (2) ensures that at most \( K \) shipping links can be added to the 
existing network since the opening of new shipping links needs extra 
operation and management cost, where \( K \) is the maximum number of
The links to be added correspond to new shipping links to be opened by real container liner shipping companies. Therefore, it is reasonable to consider the possibility of opening new shipping links in the connectivity strengthening model. The more container traffic that flows between two ports, the higher the possibility that shipping companies will open new shipping links. Consequently, this paper extends the classical gravity model to determine the possibility of opening new shipping links in the connectivity strengthening model. The classical gravity model describes the attraction force between two entities by considering their respective masses and distance between these entities, which is based on Newton’s Law (Nijkamp 1975). The model has been widely applied to estimate container traffic flows between two entities by considering their respective masses and distance between these entities, which is based on Newton’s Law (Nijkamp 1975). The model has been widely applied to estimate container traffic flows between ports (Tu et al., 2018; Cheng and Wang 2021). Therefore, the possibility of opening new shipping links between port \( i \) and port \( j \) is calculated as:

\[
p_i = k_Q Q_i f(d_{ij}) \forall i,j \in V
\]

where \( Q_i \) is the total container volume that departs from port \( i \); \( f(d_{ij}) \) is the distance between port \( i \) and port \( j \) calculated based on latitude and longitude, and \( k_Q \) is an adjustment coefficient associated with the container traffic attraction between port \( i \) and port \( j \). As the total possibility of opening \( K \) new links is \( p = \sum_{i,j} p_{ij} \), the objective function of the connectivity strengthening model is modified as:

\[
\max \quad C = \alpha \lambda_2 (G + \Delta e) + (1 - \alpha) p
\]

where \( \alpha \) is the coefficient for balancing the two objectives of the model. To coordinate the two objectives in the objective function, we standardized the value of \( p_i \) to \([0, 1]\) since the value of algebraic connectivity ranges from 0 to 1.

4. Algorithm

To add a new link to an undirected network with \( n \) nodes and \( m \) links, the total number of candidate links is \( \frac{1}{2}n(n-1) - m \). It is challenging to efficiently solve the connectivity strengthening model since maximizing the algebraic connectivity, which is an eigenvalue of a weighted adjacency matrix, by adding links to the associated network is NP hard. To represent the network \( G \) as an incidence matrix \( B \in \mathbb{R}^{n \times m} \), such that \( B_{ij} = -1 \) if the link \( e_i \) leaves node \( i \), 1 if it enters node \( i \), and 0 otherwise. \( B = [b_1, b_2, \ldots, b_m] \), such that the Laplacian matrix \( N \) is equal to \( \sum_{e=1}^{m} b_e b_e^T \) (Xu et al., 2021). After adding a new link, the Laplacian matrix is \( N' = N + \sum_{e=1}^{m} b_e b_e^T \), where \( \Delta w_e \geq 0 \) denotes the weight of the new link. According to the Spectral Theorem (Chen et al., 2021), the second smallest eigenvalue of the Laplacian matrix \( \lambda_2 = \min_{\lambda \in \mathbb{R}} \lambda f(Nf) \), where \( f \) is its corresponding eigenvector. Then the partial derivative of \( \lambda_2(Nf) \) with respect to \( \Delta w_e \) is \( \frac{\partial \lambda_2(Nf)}{\partial \Delta w_e} = f^T \frac{\partial Nf}{\partial \Delta w_e} f = f^T \frac{\partial Nf}{\partial \Delta w_e} f \) (Bell et al., 2017). As \( \frac{\partial Nf}{\partial \Delta w_e} = \sum_{e=1}^{m} b_e b_e^T \), then \( \frac{\partial \lambda_2(Nf)}{\partial \Delta w_e} = f^T \sum_{e=1}^{m} b_e b_e^T f = f^T N f = (f(v_i) - f(v_j))^2 \). Thus, link \( e=(i,j) \) with the maximal \( (f(v_i) - f(v_j))^2 \) is chosen from all candidate links, that is, the algebraic connectivity increases the most if adding a link to a node pair with the largest absolute difference between the corresponding elements in the Fiedler vector. For this reason, before solving the model, the absolute difference in Fiedler vector \( f \) of any port pairs, or candidate links, in the container shipping network is calculated and arranged in descending order. To effectively search the optimal solution, only the first \( R \) links out of all candidate links are selected as the feasible solutions of the model. Then, the connectivity strengthening simulation experiment is carried out according to the following Algorithm 1:

**Algorithm 1. Fiedler vector based algorithm.**

1. **Input**: Weighted adjacency matrix \( A \), feasible solutions \( I \), coefficient \( \alpha \). \( K \) number of links to be added
2. **Output**: the second smallest eigenvalue \( \lambda_2 \), possibility of opening \( K \) new links \( p \)
3. Initialize the set of \( OPT \leftarrow \emptyset \), calculate \( C, \lambda_2, \Phi_2, p \)
4. While \( K > 0 \)
5. \( G = G_i \)
6. For \( e = (i,j) \in I \)
7. \( C = \alpha \lambda_2 (G + \Delta e) + (1 - \alpha) p \)
8. End
9. add the solution that increases \( C \) the most into \( OPT \)
10. \( K = K - 1 \)
11. End

Max \( \quad C = \alpha \lambda_2 (G + \Delta e) + (1 - \alpha) p \) (5)

5. Case study

5.1. Data collection and parameters setting

To construct a real container shipping network, we used the liner services data from Alphaliner’s online database (Alphaliner company, 2021). Liner services in the database were provided by more than 200 liner shipping companies, involving almost all container transport routes in the world. We downloaded 1766 liner services from the Alphaliner’s online database and extracted related details about these services, including port name, port call order, frequency of vessels...
deployed, and vessel capacity. The longitude and latitude, country, and region of ports were collected from the Sea-Web database and the website of China Ports. Our dataset contained 1082 ports and 5915 undirected links, covering 176 countries.

The sample container shipping network is represented by a graph. In this graph, nodes are ports, edges are shipping services between consecutive ports, and weight of each edge is the number of services connecting the port pairs of this edge. The container volume of a port is calculated as the total capacity of all container vessels called at the port. For quantifying the container traffic attraction, the policy factor is the most important factor that cannot be ignored. Governments usually issue related policies to encourage shipping companies to open new services for boosting volumes and profitability of ports and promoting bilateral trade between countries. China’s 21st Century Maritime Silk Road initiative is the most representative policy focusing on maritime connectivity. The container traffic attraction of links corresponding to the initiative is set to be twice that of other links.

### 6. Results

In the experiments, the values of objective coefficient $\alpha$ were 0, 0.5 and 1. All tests were implemented using MATLAB R2016a on an Intel(R) Core(TM) i5-6600 CPU 3.30 GHz personal computer with 8 GB of RAM. Suppose that there are at most $K$ new links to be added at each time. To demonstrate how the proposed model works, the global optimal solutions are obtained by iterating through all candidate links when $K = 1$. There were 578,906 candidate port pairs, so the number of feasible solutions $R$ was set to be 578,906. The results are shown in Table 2.

As shown in Table 2, three optimal links were found when $\alpha = 0, 0.5, 1$. When $\alpha = 0$ in the objective function, the optimal link (Los Angeles–Long Beach) indicates that the new service is most likely to open on this link. These two ports are not only the gateway ports of the West Coast of America, but also the important international hub ports in the Americas. When both the possibility of opening new shipping links and the topology of the shipping network’s connectivity are considered, that is, $\alpha$ is set to 0.5 in the objective function, the optimal link is (Shanghai–Istanbul). The container throughput of the Shanghai port is ranked first in the world, while that of the Istanbul port is not outstanding. The optimal solution is obtained when balancing two objectives of the connectivity strengthening model. Results of the optimal link (Shoreham–Saumlak) when $\alpha = 1$, corresponds to the result that the model only considers the maximization of the container shipping network’s connectivity. Regardless of the container traffic attraction between ports, the optimal link increases algebraic connectivity of the shipping network the most.

The average computation time to exhaust all these node pairs when $K = 1$ is about 176 h. If more links are added to the existing shipping network, the solution time of the model increases exponentially. Therefore, the following tests show the application of the proposed algorithm to find the optimal links when $K$ is set as 100. In the first group of experiments, the global optimal solutions are obtained by iterating through all candidate links. At the same time, our program records the value of algebraic connectivity and its corresponding eigenvector $f$ of each optimal solution. We found that the absolute difference in $f$ of any port pairs, or candidate links, has much larger value in the first fifth of feasible solutions. According to the idea of our proposed algorithm, it means that the optimal solution will be obtained most likely in the first fifth of feasible solutions. Thus, we change the value of $R$ to one fifth. The computation time is correspondingly reduced by about four-fifths. The optimal solutions visualized by ArcGIS software when $\alpha = 1, 0.5$ and 0 are shown in Fig. 2, Fig. 3 and Fig. 4, respectively. In the three figures, red dots represent origin ports, blue dots represent destination ports, and the size of a dot represents the container volume of a port.

The optimal solutions as shown in Fig. 2, when $\alpha = 1$, consists of 100 shipping links involving 76 origin ports represented by red dots and 5 destination ports represented by blue dots.

As indicated in Fig. 2, the optimal solution to strengthen the connectivity of the container shipping network is to add undirected links between Asian ports and ports in Northwest Europe when only the network topology is considered. The five European ports, represented by blue dots, are Montrose, Shoreham, Kristinehamn, Karlstad and Vanersborg, of which the first two are British and the last three Swedish.
In our collected dataset, only one feeder service connecting these ports, and five small ships with a capacity of less than 300, are deployed on the service, leading to the poor connectivity status of these ports. The main reason for this result is the maximization of algebraic connectivity is the optimization objective of the proposed model, which locates the cut with the least total link weight reflecting the most vulnerability of the network, so adding a set of links to the most vulnerable parts of the shipping network will increase the value of algebraic connectivity the most, meaning maximum improvement in the connectivity of the whole shipping network. Moreover, this optimal solution also involves 76 ports, represented by red dots, including 74 east and Southeast Asia ports and two east Russia ports, most of which have relatively good connectivity. Table 3 lists the 10 best-connected ports in the fourth quarter of 2021 in descending order of PLSCI (United Nations Conference on Trade and Development, 2021).

In Table 3, the ten ports are all located in East Asia; six are in China, led by Shanghai; the remaining four are in Korea and Japan. According to the UNCTAD, less than 6% of ports have a PLSCI greater than 50, so these ten ports are no doubt the best connected ports in the world. The result implies that enhancing maritime cooperation between ports with strong connectivity and ports with relatively weak connectivity will greatly improve the connectivity of the whole shipping network. Moreover, the last two columns in Table 3 represent the container volume of a port calculated as the total capacity of container vessels called at this port, that is $Q_i$, and the actual port container throughput from the Sea-Web database, respectively. The change trend of the $Q_i$ value is quite similar to the real container throughput, which reflects that the estimation of $Q_i$ in Equation (4) is reasonable and verifies the reasonable estimation of the possibility of opening new shipping links since the estimation of this possibility is based on the calculation of container volume of ports. In addition, the 100 links corresponding to locally searched optimal solutions contain the link (Shoreham, Saumlaki), which is also a global optimal solution, as illustrated in Table 2. This finding verifies the effectiveness of the proposed algorithm.

For considering both the network topology and the possibility of opening new routes to strengthen the connectivity of a container shipping network, the value of $\alpha$ is set as 0.5. The optimal solution consists of 100 shipping links involving 23 origin ports represented by red dots and 8 destination ports represented by blue dots, as shown in Fig. 3. It can be observed from Fig. 3 that most of the optimal links connect
ports in Asia and Europe with ports in Europe. In addition, most of the origin ports represented by red dots are well-connected, while ports represented by blue dots are under-connected. Compared with the result that only considering network topology to strengthen the connectivity of a container shipping network in Fig. 2, this result involves three more Northern Europe ports represented by blue dots with relatively weak connectivity, namely Kaskinen in Finland and Skelleftehamn and Iggesund in Sweden. It also can be observed that well-connected European ports are geographically close to the under-connected destination ports. This result is mainly due to the consideration of maximization of route opening possibility, which is determined by not only total container volume that departs from ports but also shipping distance between ports. In addition to European ports, the remaining red dots illustrate five Asian ports: Busan, Shanghai, Ningbo, Shenzhen and Singapore, all of which are hub ports in Asia. It further shows that if ports with relatively weak connectivity are connected with ports with strong connectivity, the connectivity of the whole shipping network significantly improves. It also indicates that the role of ports with good connectivity is extremely important in improving a container shipping network’s connectivity.

When only the possibility of opening new routes is considered, that is, $\alpha = 0$, the optimal undirected links are shown in Fig. 4.

Fig. 4 shows 64 ports illustrated by blue and red dots corresponding to the newly added shipping links. Specifically, there are 19 ports in Europe, five ports in Africa, 32 ports in Asia, and nine ports in America, which mainly serve the East-West trade routes. This finding is in line with the reality that East–West containerized trade increased to 78.5 million TEU in 2021, accounting for more than half of the global volume (United Nations Conference on Trade and Development, 2021). The first five links in the optimal solution are shown in Table 4.

Different from the optimal solutions shown in Figs. 2 and 3, when $\alpha = 0$, the optimal solution no longer contains ports with poor connectivity. It can be inferred that when only the possibility of opening new routes is considered, shipping companies tend to give priority to open shipping links between hub ports. Cheung et al. (2020) also found that shipping activities are highly concentrated in hub ports. This is not only because large ships with economics of scale can only call at hub ports, but also because hub ports are relatively easy to connect to the world’s container shipping networks.

### Table 3
The 10 best-connected ports in descending order of PLSCI.

| Rank | Port     | PLSCI | Country | Region | $Q_i$/ million TEU | Throughput/ million TEU |
|------|----------|-------|---------|--------|--------------------|------------------------|
| 1    | Shanghai | 147.6 | China   | East   | 2894               | 4703                   |
| 2    | Ningbo-Zhoushan | 128.3 | China   | East   | 2152               | 3108                   |
| 3    | Qingdao  | 99.8  | China   | East   | 1094               | 2371                   |
| 4    | Kaohsiung | 87.1  | China   | East   | 775                | 986                    |
| 5    | Dalian   | 62.5  | China   | East   | 238                | 367                    |
| 6    | Kwangyang | 61.8  | Korea   | East   | 192                | 213                    |
| 7    | Yokohama | 60.5  | Japan   | East   | 396                | 266                    |
| 8    | Taipei   | 55.2  | China   | East   | 102                | 160                    |
| 9    | Tokyo    | 51.6  | Japan   | East   | 243                | 426                    |
| 10   | Kobe     | 50.1  | Japan   | East   | 190                | 265                    |

### Table 4
The first five links in the optimal solution when $\alpha = 0$.

| ID | Port i Country | Region | Port j Country | Region |
|----|----------------|--------|----------------|--------|
| 1  | Los Angeles    | America| North America  | LongBeach | America |
| 2  | Cristobal      | Panama | Central America| Colon   | Panama |
| 3  | Yantai         | China  | East Asia      | Ningbo-Zhoushan | China |
| 4  | London         | UK     | Western Europe | Europe   |
| 5  | Yantai         | China  | East Asia      | Shenzhen | China |

### 7. Conclusion
This paper has investigated the problem of strengthening container shipping network connectivity, which is formulated to maximize both structural connectivity and the possibility of opening new shipping links between ports by adding a set of shipping links. In the proposed model, algebraic connectivity has been employed to measure the structural connectivity of the shipping network by locating the cut with the least total link weight, reflecting the most vulnerability of the network, and an extended gravity model has been applied to estimate the possibility of opening new shipping links. An effective algorithm based on Fiedler vector has been designed to quickly find optimal solutions of the proposed model, since maximizing the algebraic connectivity is NP hard. We tested the proposed model and algorithm by conducting two groups of experiments on a real container shipping network which contains 1082 ports and 5915 undirected links between 176 countries. In the first group of experiment, the global optimal solutions are obtained by iterating through all feasible links; In the second group of experiment, the proposed heuristic algorithm is applied to find the optimal links from a set of feasible links.

Simulation experiments show that linking ports with relatively weak connectivity with ports that have strong connectivity (most of them are hub ports) would greatly improve the connectivity of the whole shipping network. The maximization of algebraic connectivity is the optimization objective of the proposed model, which locates the cut with the least total link weight reflecting the most vulnerability of the network, so adding a set of links to the most vulnerable parts of the shipping network will increase the value of algebraic connectivity the most, meaning maximum improvement in the connectivity of the whole shipping network. It would be possible to apply our method to provide shipping route opening strategies for port authorities and shipping stakeholders to enhance maritime cooperation between specific economy regions in the post COVID-19 era. In particular, for the small island developing states and the least developed countries whose consumption and production depend more on international trade, and they are facing greater challenges than the countries that possess hub ports during the pandemic. Maritime transport is a lifeline for these countries to boost foreign trade, but anti-epidemic and control measures restrict maritime activities commodity imports and exports, potentially causing supply chain disruptions, inflation and social unrest. To strengthen their shipping connectivity would mitigate negative impact of the pandemic on their economies. Therefore, it is important to understand which new shipping links to open and where encouraging shipping activity would best improve their port connectivity.

Simulation experiments also show that new shipping links tend to be opened between hub ports, in particular to serve the East-West trade routes, when only the possibility of opening new routes is considered. However, the negative trend has been observed in liner shipping connectivity levels on a global scale during COVID-19. Therefore, the government cannot completely obey the rules of shipping market operation. It is necessary to intervene moderately in the domestic shipping industry, such as providing some policy support to help improve the shipping connectivity in specific regions, so as to mitigate negative...
impact of the pandemic on container shipping. The paper proposes a graph theory based approach for strengthening connectivity of container shipping network. The proposed model is implemented with a heuristic algorithm based on Fiedler vector by using the properties of the Laplace matrix of the network. The heuristic algorithm is computationally less costly. After this work, the attention will be on the analytical solution for the model.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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