Large $\theta_{13}^\nu$ and Unified Description of Quark and Lepton Mixing Matrices

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Abstract

We present a revised version of the so-called “yukawaon model”, which was proposed for the purpose of a unified description of the lepton mixing matrix $U_{PMNS}$ and the quark mixing matrix $V_{CKM}$. It is assumed from a phenomenological point of view that the neutrino Dirac mass matrix $M_D$ is given with a somewhat different structure from the charged lepton mass matrix $M_e$, although $M_D = M_e$ was assumed in the previous model. As a result, the revised model predicts a reasonable value $\sin^2 2\theta_{13} \sim 0.07$ with keeping successful results for other parameters in $U_{PMNS}$ as well as $V_{CKM}$ and quark and lepton mass ratios.

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1 Introduction

In a series of papers [1, 2, 3], the authors have investigated a unified description of the lepton mixing matrix $U_{PMNS}$ and the quark mixing matrix $V_{CKM}$. The essential idea is as follows: (i) The Yukawa coupling constants $Y_f$ ($f = u, d, e$, and so on) in the standard model are effectively given by vacuum expectation values (VEVs) of scalars (“yukawaon”) $Y_f$ with $3 \times 3$ components, i.e. by $\langle Y_f \rangle / \Lambda$. Here $\Lambda$ is an energy scale of the effective theory. (The yukawaon model is a kind of the “flavon” model [6].) (ii) The model does not contain any coefficients which are dependent on the family numbers. The hierarchical structures of the effective Yukawa coupling constants originate only in a fundamental VEV matrix $\langle \Phi_0 \rangle$, whose hierarchical structure is ad hoc assumed at present and whose VEV values are fixed by the observed charged lepton masses. (iii) Relations among those VEV matrices are obtained from SUSY vacuum conditions for a given superpotential under family symmetries and $R$ charges assumed. (Since we use the observed charged lepton mass values as the input values, it is a characteristic in the yukawaon model that adjustable parameters are quite few.)

In the previous model [1, 2, 3], the quark and lepton mass matrices (charged lepton mass matrix $M_e$, Dirac neutrino mass matrix $M_D$, down-quark mass matrix $M_d$, neutrino mass matrix $M_\nu$,}
and right-handed Majorana neutrino mass matrix \( M_R \) are given as follows:

\[
\begin{align*}
M_e &= k_e \Phi_0 (1 + a_e X_3) \Phi_0, \\
M_D &= M_e, \\
M_d &= k_d \left[ \Phi_0 (1 + a_d X_3) \Phi_0 + m_\alpha^0 1 \right], \\
M_u &= k_u^\prime M_u \hat{M}_u, \\
\hat{M}_u &= k_u \Phi_0 (1 + a_u X_3) \Phi_0, \\
M_\nu &= M_D M_R^{-1} M_D^T, \\
M_R &= k_R (\hat{M}_u M_e + M_e \hat{M}_u) + \cdots
\end{align*}
\]

where \( M_e, \Phi_0, X_3, \ldots \) are 3 \times 3 numerical matrices which result from VEV matrices of scalar fields. Here the VEV matrices \( \Phi_0, X_3, \) and \( 1 \) have structures given by

\[
\Phi_0 = \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad 1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]

The coefficients \( a_f (f = e, u, d) \) which are important parameters in the model play an essential role in the mass ratios and mixings. On the other hand, the family-number independent coefficients \( k_f \) and \( k_u^\prime \) do not any role in predicting family mixings and mass ratios. The values of \( (x_1, x_2, x_3) \) with \( x_1^2 + x_2^2 + x_3^2 = 1 \) are fixed by the observed charged lepton mass values under the given value of \( a_e \). (In an earlier model [4], the charged lepton mass matrix \( M_e \) was given by \( M_e = k_e^\prime \Phi_e \Phi_e \) and \( M_d \) and \( \hat{M}_u \) are given by those in (1.1) with the replacement \( \Phi_0 \rightarrow \Phi_e \). The structures with \( (1 + a_f X_3) \) were suggested in a phenomenological model by Fusaoka and one of the authors [8].)

The previous models [1][2][3] have given almost successful unified description and predictions of \( U_{PMNS} \) and \( V_{CKM} \). However, these models have failed to give the observed large mixing of \( \theta_{13} \) in \( U_{PMNS} \): the observed value is \( \sin^2 2 \theta_{13} \sim 0.09 \) [10], while the model in Ref.[1] predicts \( \sin^2 2 \theta_{13} \sim 10^{-4} \). Even in a recent revised model [3], the predicted value was, at most, \( \sin^2 2 \theta_{13} \sim 0.03 \). Since the model does not contain enough number of adjustable parameters as it is, it is hard to improve the prediction of \( \sin^2 2 \theta_{13} \) without the cost of other successful predictions. So, an interesting attempt of introducing the structure \( X_2 \) into the model has been done in Ref.[2]. In Ref.[2], the structure \( X_2 \) [see Eq.(1.44)] was introduced in \( M_e \) together with assumption \( M_D = M_e \), but the predicted value of \( \sin^2 2 \theta_{13} \) was still small: \( \sin^2 2 \theta_{13} \sim 10^{-2} \). The \( V_{CKM} \) was not discussed in Ref.[2].

In the present paper, we revise the model given in (1.1) by changing the structure only for the neutrino Dirac mass matrix \( M_D \) as follows; the structure \( X_2 \) is introduced in \( M_D \) not in the charged lepton mass matrix \( M_e \) unlikely in Ref.[2], and also it is assumed from a phenomenological point of view that the \( M_D \) is given with a somewhat different coefficient from
Using this form we shall discuss $U_{PMNS}$ as well as $V_{CKM}$ of the model. As to the structure $X_2$, we will discuss in Sec. 2. When once we accept the form (1.3), we predict a reasonable value of $\sin^2 2\theta_{13} \sim 0.07$ together with reasonable other parameters of $U_{PMNS}$, $V_{CKM}$ and quark and lepton mass ratios.

2 Model

We assume that a would-be Yukawa interaction is given as follows:

$$W_Y = \frac{y_e}{\Lambda} e^c_i Y^{ij}_e \ell_j H_d + \frac{y_d}{\Lambda} \nu^c_i Y^{ij}_d \ell_j H_u + \lambda_R \nu^c_i Y^{ij}_R \nu^c_j + \frac{y_d}{\Lambda} d^c_i Y^{ij}_d q_j H_d + \frac{y_u}{\Lambda} u^c_i Y^{ij}_u q_j H_u,$$

where $\ell = (\nu, e_L)$ and $q = (u_L, d_L)$ are SU(2)$_L$ doublets. Under this definition of $Y_f$, the CKM mixing matrix and the PMNS mixing matrix are given by $V_{CKM} = U^T_u U_d$ and $U_{PMNS} = U^T_e U_\nu$, respectively, where $U_f$ are defined by $U^T_f M_f^T M_f U_f = D_f^2$ ($D_f$ are diagonal). (Hereafter, for simplicity, we denote $U_{PMNS}$ and $V_{CKM}$ as $U$ and $V$, respectively.) In order to distinguish each yukawaon from others, we assume that $Y_f$ have different $R$ charges from each other together with $R$ charge conservation (a global U(1) symmetry in $N = 1$ supersymmetry; for example, see Ref.[9]). (Of course, the $R$ charge conservation is broken at an energy scale $\Lambda^\prime$.)

We assume the following superpotential for yukawaons by introducing fields $\Theta^{L}, P, E, \bar{E}, E', \bar{E}', \nu_e$, and $\phi_d$:

$$W_e = \lambda_e \left\{ \phi_e Y^{ij}_e + \frac{1}{\Lambda} (\Phi_0)^{i\alpha} \left( (E^\alpha)^{\alpha\beta} + a_e \frac{1}{\Lambda^2} X_{ak} \bar{E}^{kl} X^T_{l\beta} \right) (\Phi_0^T)^{j\beta} \right\} \Theta^{e}_{ij},$$

$$W_D = \frac{\lambda_D}{\Lambda} \left\{ (E')^{\alpha\beta} Y^{ij}_D (E')^{\beta\delta} + (\Phi_0^T)^{\alpha i} \left( E_{ij} + a_D \frac{1}{\Lambda^2} X^T_{j\gamma} (\bar{E}''')^{\gamma\delta} X_{\delta j} \right) (\Phi_0)^{j\beta} \right\} \Theta^{D}_{\beta\alpha},$$

$$W_u = \frac{\lambda_u}{\Lambda} \left\{ (E')^{\alpha\beta} Y^{ij}_u P_{ij} + (\Phi_0)^{i\alpha} \left( E_{ij} + a_u \frac{1}{\Lambda^2} X_{ak} \bar{E}^{kl} X^T_{l\beta} \right) (\Phi_0^T)^{j\beta} \right\} \Theta^{u}_{ij},$$

$$W_u' = \frac{\lambda_u'}{\Lambda} \left\{ (E')^{\alpha\beta} Y^{ij}_u P_{ij} + (\Phi_0)^{i\alpha} \left( E_{ij} + a_u \frac{1}{\Lambda^2} X_{ak} \bar{E}^{kl} X^T_{l\beta} \right) (\Phi_0^T)^{j\beta} \right\} \Theta^{u'}_{ij}.$$
Here, we have assumed family symmetries $U(3) \times U(3)$ and $R$ charges.

Table 1: Assignments of $SU(2)_L \times SU(3)_c \times U(3) \times U(3)'$ and $R$ charges

|               | $\ell$ | $e^c$ | $\nu^c$ | $q$ | $u^c$ | $d^c$ | $H_u$ | $H_d$ |
|---------------|-------|-------|---------|-----|-------|-------|-------|-------|
| $SU(2)_L$     | 2     | 1     | 1       | 2   | 1     | 1     | 2     | 2     |
| $SU(3)_c$     | 1 1   | 1 1   | 1 1     | 2   | 1     | 1     | 1     | 1     |
| $U(3)$        | 3 3   | 3 3   | 3 3     | 1   | 1     | 1     | 1     | 1     |
| $U(3)'$       | 1 1   | 1 1   | 1 1     | 1   | 1     | 1     | 1     | 1     |
| $R$           | $r_{\ell}$ | $r_{e^c}$ | $r_{\nu^c}$ | $r_q$ | $r_{u^c}$ | $r_{d^c}$ | $r_{H_u}$ | $r_{H_d}$ |

$Y_e$ $Y_D$ $Y_R$ $Y_u$ $Y^u$ $Y_d$ $\Theta^e$ $\Theta^D$ $\Theta^R$ $\Theta_u$ $\Theta^u$ $\Theta^d$

|               | 1     | 1     | 1       | 1   | 1     | 1     | 1     | 1     |
|---------------|-------|-------|---------|-----|-------|-------|-------|-------|
| $6^*$         | 6     | 6     | 6       | 6   | 6     | 6     | 6     | 1     |
| $3^*$         | 3     | 6     | 6       | 3   | 3     | 1     | 1     | 6     |
| $3^*$         | 3     | 6     | 6       | 3   | 3     | 1     | 1     | 6     |

$r_{Y_e}$ $r_{Y_R}$ $r_{Y_u}$ $r_{Y^u}$ $r_{Y_d}$ $r_{\Theta_e}$ $r_{\Theta_D}$ $r_{\Theta_R}$ $r_{\Theta_u}$ $r_{\Theta^u}$ $r_{\Theta^d}$

$r_0$ $\frac{1}{2}(r_E + r''_E - 1)$ $r_E$ $1 - r_E$ $r'_E$ $1 - r'_E$ $r''_E$ $1 - r''_E$ $r_P$ $1 - r_P$ $r_{\phi_e}$ $r_{\phi_d}$

$W_d = \lambda_d \left\{ \phi_d Y_{ij} + \frac{1}{\lambda} \left( (\Phi_0)^{\alpha \beta} + a_d \frac{1}{\lambda^2} X_{\alpha k} \bar{E}^{kl} X_{l \beta}^T \right) (\Phi_0)^{T^i j}_{\alpha \beta} (\bar{E})_{\alpha \beta} (\bar{E})_{\beta}^{i j} \right\} \Theta_{ji}^d, \tag{2.6}$

$W_R = \left\{ \mu_R Y_{ij} + \frac{1}{\lambda} \left[ Y_{\epsilon i} Y_{kl} \bar{E}^{ij} + \bar{E}^{ik} Y_{kl} Y_{\epsilon j} + \xi_{\alpha \beta} \bar{E}^{ik} E_{kl} Y_{\epsilon j} \right] \right\} \Theta_{ji}^R. \tag{2.7}$

Here, we have assumed family symmetries $U(3) \times U(3)'$. The fundamental yukawaon $\Phi_0$ is assigned to $(3, 3)$ of $U(3) \times U(3)'$, although quarks and leptons are still assigned to $(3, 1)$ and yukawaons $Y_j$ are assigned to $(6^*, 1)$ of $U(3) \times U(3)'$. In order to distinguish $R$ charges between $Y_e$ and $Y_d$, we have introduced $U(3) \times U(3)'$ singlet scalar fields $\phi_e$ and $\phi_d$.

We list the assignments of $SU(2)_L \times SU(3)_c \times U(3) \times U(3)'$ and $R$ charges for the fields in the present model in Table 1. The assignments of $R$ charges are done so that the total $R$ charge of the superpotential term is $R(W) = 2$. The $r$ parameters in Table 1 must satisfy the following relations: $r_{H_u} = 2 - r_{\ell} - r_D - r_{\nu c} - r_{Y_e} = 2 - r_q - r_{u c} - r_{Y_u}$, $r_{H_d} = 2 - r_{\ell} - r_{\nu c} - r_{Y_d} = 2 - r_q - r_{u c} - r_{Y_d}$, $r_{\Theta_e} = 2 - r_{Y_e} - r_{\phi_e}$, $r_{\Theta_d} = 2 - r_{Y_D} - 2 r'_E$, $r_{\Theta_R} = 2 - r_{Y_R}$, $r_{\Theta_u} = 2 - r_{Y_u} - 2 r_P$, $r_{\Theta^d} = 2 - r_{Y^d} - 2 r''_E$, $r_{\Theta^R} = 2 - r_{Y^R}$.
\( \hat{r} \Theta_u = 1 + r_E - \hat{r}_Y u \), and \( r \Theta_d = 2 - r_Y d - r_\phi d \). Here, the \( R \) charges of these fields must satisfy the following relations: 
\[ 2r_0 + r''_E = r_Y e + r_\phi e = r_Y d + r_\phi d = \hat{r}_Y u + 1 - r_E, \]
\[ 2r_0 + r_E = r_Y D + 2r'_D, \]
and
\[ r_Y R = r_Y e + \hat{r}_Y u = 2r_Y D + r_E. \]
Since we consider that family symmetries \( U(3) \) and \( U(3)' \) are gauge symmetries, the model must be anomaly free. However, as seen in Table 1, the present model has anomaly coefficients \( A(SU(3)) = 9 \) and \( A(U(3)') = 7 \), so that we need further fields \( (6^* + 3^* + 3^*, 1) \) and \( (1, 6^*) \) of \( U(3) \times U(3)' \). However, since roles of such additional fields in the present model are, at present, not clear, we do not discuss such fields.

From Eqs.(2.2) and (2.3) [and also (2.5) and (2.6)], we obtain
\[ R(E) + R(\bar{E}) = R(E'') + R(\bar{E}''). \quad (2.8) \]
The VEVs of the introduced fields \( E, \bar{E}, P, \) and \( \bar{P} \) are described by the following superpotential by assuming \( R(E\bar{E}) = R(P\bar{P}) = 1 \):
\[ W_{E,P} = \frac{\lambda_1}{\Lambda} \text{Tr}[\bar{E}EPP] + \frac{\lambda_2}{\Lambda} \text{Tr}[\bar{E}E \text{Tr}[\bar{P}P]], \quad (2.9) \]
which leads to
\[ \langle E \rangle \langle \bar{E} \rangle \propto 1, \quad \langle P \rangle \langle \bar{P} \rangle \propto 1. \quad (2.10) \]

We assume specific solutions of Eq.(2.10):
\[ \frac{1}{v_E} \langle E \rangle = \frac{1}{\bar{v}_E} \langle \bar{E} \rangle = 1, \quad (2.11) \]
\[ \frac{1}{v_P} \langle P \rangle = \frac{1}{\bar{v}_P} \langle \bar{P} \rangle^\dagger = \text{diag}(e^{-i \phi_1}, e^{-i \phi_2}, 1), \quad (2.12) \]
as the explicit forms of \( \langle E \rangle, \langle \bar{E} \rangle, \) and \( \langle \bar{P} \rangle \). We assume similar superpotential forms for \( E'' \) and \( \bar{E}'' \), and for \( E' \) and \( \bar{E}' \).

From SUSY vacuum conditions \( \partial W/\partial \Theta = 0 \), we obtain the following relations:
\[ \langle Y_e \rangle = k_e \langle \Phi_0 \rangle (1 + a_e XX^T) \langle \Phi_0^T \rangle, \quad (2.13) \]
\[ \langle Y_D \rangle = k_D \langle \Phi_0^T \rangle (1 + a_D XX^T) \langle \Phi_0 \rangle, \quad (2.14) \]
\[ \langle P \rangle \langle Y_u \rangle \langle P \rangle = k'_u \langle \hat{Y} u \rangle \langle \hat{Y} u \rangle, \quad (2.15) \]
\[ \langle \hat{Y} u \rangle = k_u \langle \Phi_0 \rangle (1 + a_u XX^T) \langle \Phi_0^T \rangle, \quad (2.16) \]
\[ \langle Y_d \rangle = k_d \left[ \langle \Phi_0 \rangle (1 + a_d XX^T) \langle \Phi_0^T \rangle + n^0_d \mathbf{1} \right], \quad (2.17) \]
\[ \langle Y_R \rangle = k_R \left( \langle Y_e \rangle \langle \hat{Y} u \rangle + \langle \hat{Y} u \rangle \langle Y_e \rangle + \xi^0_e \langle Y_D \rangle \langle Y_D \rangle \right), \quad (2.18) \]
where, for convenience, we have already put \( \langle E \rangle = 1 \), and so on. Here, since we have assumed that all \( \Theta \) fields take \( \langle \Theta \rangle = 0 \), we do not need to consider vacuum conditions for other fields \( \partial W / \partial Y_e = 0 \), because those always contain \( \langle \Theta \rangle \). Thus, mass matrices are given by \( M_e = \langle Y_e \rangle \), \( M_D = \langle Y_D \rangle \), \( M_u = \tilde{k}_u \tilde{M}_u \tilde{M}_u \), \( \tilde{M}_u = \langle \tilde{Y}^u \rangle \), \( M_d = \langle Y_d \rangle \), \( M_\nu = M_D M_R^{-1} M_D^T \), and \( M_R = \langle Y_R \rangle \).

The most curious assumption is to assume the VEV matrix form of the scalar \( X \) as

\[
\frac{1}{v_X} \langle X \rangle_{\alpha i} = \frac{1}{2} \begin{pmatrix}
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0 
\end{pmatrix}_{\alpha i}.
\]

(2.19)

The form (2.19) leads to

\[
(\langle X \rangle \langle X^T \rangle)_{\alpha \beta} = \frac{3}{2} (X_3)_{\alpha \beta}, \quad (\langle X^T \rangle \langle X \rangle)_{ij} = \frac{3}{2} (X_2)_{ij},
\]

(2.20)

together with \( \langle X \rangle \langle X \rangle = \langle X \rangle \), where \( X_3 \) and \( X_2 \) is defined by Eqs. (1.2) and (1.4), respectively, and, for simplicity, we have put \( v_X = 1 \) because we are interested only in the relative ratios among the family components.

At present, there is no idea for the origin of this form (2.19). We may speculate that this form is related to a breaking pattern of \( U(3) \times U(3)' \) (for example, discrete symmetries \( S_2 \times S_3 \)). In the present paper, the form (2.12) is only ad hoc assumption. However, as seen later, we can obtain a good fitting for the neutrino mixing angle \( \sin^2 2\theta_{13} \) due to this assumption.

3 Parameter fitting

We summarize our mass matrices in the present model as follows:

\[
M_e = k_e \Phi_0 (1 + a_e X_3) \Phi_0^T, 
\]

(3.1)

\[
M_D = k_D \Phi_0^T (1 + a_D X_2) \Phi_0, 
\]

(3.2)

\[
P \tilde{M}_u P = \tilde{k}_u \tilde{M}_u \tilde{M}_u, 
\]

(3.3)

\[
\tilde{M}_u = k_u \Phi_0 (1 + a_u e^{i\alpha_u} X_2) \Phi_0^T, 
\]

(3.4)

\[
M_d = k_d \left[ \Phi_0 (1 + a_d X_3) \Phi_0^T + m_d^0 1 \right], 
\]

(3.5)

\[
M_\nu = M_D M_R^{-1} M_D^T, 
\]

(3.6)

\[
M_R = k_R \left( M_e \tilde{M}_u u + \tilde{M}_u u M_e + \xi_\nu^0 M_D M_D \right), 
\]

(3.7)

where, for convenience, we have dropped the notations “(” and “)”. Since we are interested only in the mass ratios and mixings, hereafter, we will use dimensionless expressions \( \Phi_0 = \text{diag}(x_1, x_2, x_3) \), \( P = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1) \), and \( E = \text{diag}(1, 1, 1) \). For simplicity, we have regarded
Table 2: Process for fitting parameters. Of course, since these parameters listed in each step can slightly affect predicted values listed in the other steps, we need fine tuning after the step 5th.

| Step | Inputs $N_{inp}$ | Parameters $N_{par}$ | Predictions |
|------|------------------|----------------------|-------------|
| 1st  | $m_e, m_{\mu}$, $m_{\tau}$, $\sin^2 \theta_{23}$ | $a_e, a_u$ | $\sin^2 2\theta_{13}, \delta_{CP}^{\ell}$, 2 Majorana phases |
| 2nd  | $\sin^2 \theta_{12}, R_\nu$ | $a_D, \xi^0_\nu$ | $|V_{ub}|, |V_{td}|, \delta_{CP}^{\ell}$ |
| 3rd  | $m_c/m_t$ | $a_d$ | not affect to other predictions |
| 4th  | $|V_{us}|, |V_{cb}|$ | $(\phi_1, \phi_2)$ | $(m_{\nu1}, m_{\nu2}, m_{\nu3}), \langle m \rangle$ |
| 5th  | $m_d/m_s$ | $m_{\nu3}^0$ | $\Delta m_{\text{atm}}^2$ |
| option | $\Delta m_{\text{atm}}^2$ | $m_{\nu3}$ | 11 |

The parameter $a_d$ as real correspondingly to the parameter $a_e$. The parameters are re-refined by Eqs.(3.1)-(3.5).

In the present model, we have 9 adjustable parameters except for $x_i$ [$a_e, a_D, (a_u, \alpha_u), a_d, (\phi_1, \phi_2), m_d^0$, and $\xi^0_\nu$] for the 16 observable quantities (6 mass ratios in the up-quark-, down-quark-, and neutrino-sectors, 4 CKM mixing parameters, and 4+2 PMNS mixing parameters). In order to fix these parameters, we use, as input values, the observed values for $m_c/m_t, m_u/m_c, \sin^2 \theta_{23}$ as shown later. The relative ratios of parameters $x_i$ in $\Phi_0$ are fixed by the ratios of the charged lepton masses $m_e/m_\mu$ and $m_\mu/m_\tau$. The process of fixing parameters are summarized in Table 2.

Now let us present the details of parameter fitting. Since we do not change the mass matrix structures for $M_e$, $M_u$, and $M_d$ from the previous paper [3], we use the following parameter values of $a_e$ and $(a_u, \alpha_u)$

$$(a_e, a_u, \alpha_u) \sim (8, -1.35, \pm 8^\circ), \quad (3.8)$$

which are fixed from the observed values of $m_c/m_t$, $m_u/m_c$, and $\sin^2 \theta_{23}$:

$$r_{12}^0 = \sqrt{\frac{m_u}{m_c}} = 0.045^{+0.013}_{-0.010}, \quad r_{23}^0 = \sqrt{\frac{m_c}{m_t}} = 0.060 \pm 0.005, \quad (3.9)$$

at $\mu = m_Z$ [11], and $\sin^2 2\theta_{23} > 0.95$ [12]. (These values will be fine-tuned in whole parameter fitting of $U$ and $V$ later.) Note that the neutrino sector of the model is different from the previous model, however the predicted values of $\sin^2 2\theta_{23}$ are almost the same before.
First, let us investigate lepton sector. In the revised model, a new parameter $a_D$ is added. We illustrate the behaviors of Lepton mixing parameters $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$, and $\sin^2 \theta_{13}$, and the neutrino mass squared ratio $R_{\nu}$ versus the parameter $a_D$. ("solar", "atm", "t13", and "10 R" denote curves of $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$, and $R_{\nu} \times 10$, respectively. Other parameter values are taken as $a_e = 7.5$, $a_u = -1.35$, and $\alpha_u = 7.6^\circ$.

The non-zero parameter $\xi_{\nu}$ has phenomenologically been brought in order to adjust the predicted value of $R_{\nu}$. The predicted values of $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$, and $\sin^2 2\theta_{13}$ are almost unchanged against the parameter $\xi_{\nu}$. In order to fit the neutrino mass ratio $R_{\nu}$, we take $\xi_{\nu} = -0.78$.

Next, we discuss quark sector. Since we have fixed the five parameters $a_e$, $a_u$, $\alpha_u$, $a_D$, and $\xi_{\nu}$, we have remaining four parameters for six observables (2 down-quark mass ratios and 4 CKM mixing parameters). The parameters $a_d$ and $m_d^0$ are used to fit the observed down-quark mass ratios \[ r_{23}^d \equiv \frac{m_s}{m_b} = 0.019^{+0.006}_{-0.006}, \quad r_{12}^d \equiv \frac{m_d}{m_s} = 0.053^{+0.005}_{-0.003}. \]
Figure 2: Contour plots in the ($\phi_1$, $\phi_2$) parameter plane, which are shown by using experimental constraints on CKM mixing matrix elements: $|V_{us}| = 0.2252 \pm 0.0009$, $|V_{cb}| = 0.0406 \pm 0.0013$, $|V_{ub}| = 0.00389 \pm 0.00044$, and $|V_{td}| = 0.0084 \pm 0.0006$. The CKM elements depends on only the parameter set of $[a_e, (a_u, \alpha_u), a_d, m_d^0, \phi_1, \phi_2]$. Here we present contour plots of the CKM elements in the ($\phi_1$, $\phi_2$) parameter plane by taking the values of other parameters as $a_e = 7.5$, $a_u = -1.35$, $\alpha_u = 7.6^\circ$, $a_d = 25$, and $m_d^0 = 0.0115$. We find that ($\phi_1$, $\phi_2$) = (177.0$^\circ$, 197.4$^\circ$) is consistent with all the CKM constraints.

respectively. Therefore, the four CKM mixing parameters are described only by two parameters ($\phi_1$, $\phi_2$). As shown in Fig. 2, all the experimental constraints on CKM mixing matrix elements are satisfied by fine tuning with use of only two parameters ($\phi_1$, $\phi_2$).

Finally, we do fine-tuning of whole parameter values in order to give more improved fitting with the whole data. Our final result is as follows: under the parameter values

$$a_e = 7.5, \quad a_D = 9.01, \quad (a_u, \alpha_u) = (-1.35, -7.6^\circ), \quad a_d = 25,$$

$$m_d^0 = 0.0115, \quad \xi_\nu^0 = -0.78, \quad (\phi_1, \phi_2) = (177.0^\circ, 197.4^\circ),$$

we obtain

$$r_{12}^u = 0.0465, \quad r_{23}^u = 0.0614, \quad r_{12}^d = 0.0569, \quad r_{23}^d = 0.0240,$$

$$\sin^2 2\theta_{23} = 0.969, \quad \sin^2 2\theta_{12} = 0.860, \quad \sin^2 2\theta_{13} = 0.0711, \quad R_\nu = 0.0324,$$

$$\delta_{CP}^\ell = -131^\circ \quad (J_\ell^\ell = -2.3 \times 10^{-2}),$$

$$|V_{us}| = 0.2271, \quad |V_{cb}| = 0.0394, \quad |V_{ub}| = 0.00347, \quad |V_{td}| = 0.00780,$$

$$\delta_{CP}^q = 59.6^\circ \quad (J_q = 2.6 \times 10^{-5}).$$

Here, $\delta_{CP}^\ell$ and $\delta_{CP}^q$ are Dirac CP violating phases in the standard conventions of $U$ and $V$, respectively.
Even if we choose a value of $\xi_0^\nu$ which gives a value of $R_\nu$ within $1\sigma$ given in Eq.(3.10), our predicted value of $\sin^2 2\theta_{13}$ is $\sin^2 2\theta_{13} = 0.0711^{+0.003}_{-0.004}$, which is still somewhat small compared with the observed value $\sin^2 2\theta_{13} = 0.098 \pm 0.013$ [12]. So far, we have assumed that the parameter $\xi_0^\nu$ is real. If we consider that the parameter $\xi_0^\nu$ is complex, $\xi_0^\nu \to \xi_0^\nu e^{i\alpha}$, we can adjust the value of $\sin^2 2\theta_{13}$ without changing other predicted values as seen in Fig.3. However, such a modification by the parameter $\alpha$ is not essential in the present model, so that we will regard that the parameter $\xi_0^\nu$ is real.

Figure 3: Lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared ratio $R_\nu$ versus the phase parameter $\alpha$. (“solar”, “atm”, “10 t13”, and “10 R” denote curves of $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13} \times 10$, and $R_\nu \times 10$, respectively. Here the $\alpha$ dependence is presented under the parameter values given by (3.12).

We can also predict neutrino masses, for the parameters given (3.12) with real $\xi_0^\nu$,

$$m_{\nu 1} \simeq 0.0048 \text{ eV}, \quad m_{\nu 2} \simeq 0.0101 \text{ eV}, \quad m_{\nu 3} \simeq 0.0503 \text{ eV},$$

by using the input value [13] $\Delta m_{32}^2 \simeq 0.00243 \text{ eV}^2$. We also predict the effective Majorana neutrino mass [14] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = |m_{\nu 1}(U_{e1})^2 + m_{\nu 2}(U_{e2})^2 + m_{\nu 3}(U_{e3})^2| \simeq 7.3 \times 10^{-4} \text{ eV}.$$

Finally, let us comment on sensitivity of the predicted values Eq.(3.14) to the input parameter values Eq.(3.12). For simplicity, we show the sensitivity of only the lepton mixing and up-quark mass ratios to the input parameters $a_D$, $a_u$ and $\alpha_u$ in Table 3. (We do not show sensitivity for the predicted CKM parameters, because it can be easily seen in Fig. 3.) In Table 3, values $\Delta x$ ($x = a_D$, $a_u$ and $\alpha_u$) for the parameter values $x$ are taken such as $(\Delta x)/x = 0.05,$
Table 3: Sensitivity of the predicted values to the input parameter values. In the table, values $\Delta x$ ($x = a_D$, $a_u$ and $\alpha_u$) for the parameter values $x$ are taken such as $\Delta x/x = 0.05$, where the values $x$ are given in Eq.(3.12). Note that $r_{12}^u$ and $r_{23}^u$ are independent of the parameter $a_D$.

|     | $\Delta a_D = \pm 0.451$ | $\Delta a_u = \pm 0.068$ | $\Delta \alpha_u = \pm 0.38$ |
|-----|-------------------------|-------------------------|-------------------------|
| $r_{12}^\nu$ | 0.0465 | 0.0465$^{+0.0239}_{-0.0179}$ | 0.0465$^{+0.0023}_{-0.0022}$ |
| $r_{23}^\nu$ | 0.0614 | 0.0614$^{+0.0054}_{-0.0075}$ | 0.0614$^{+0.0017}_{-0.0016}$ |
| $\sin^2 2\theta_{12}$ | 0.860$^{+0.092}_{-0.149}$ | 0.860$^{+0.036}_{-0.045}$ | 0.860$^{+0.004}_{-0.004}$ |
| $\sin^2 2\theta_{23}$ | 0.969$^{+0.021}_{-0.040}$ | 0.969$^{+0.034}_{-0.023}$ | 0.969$^{+0.002}_{-0.002}$ |
| $\sin^2 2\theta_{13}$ | 0.0711$^{+0.0012}_{-0.0016}$ | 0.0711$^{+0.0094}_{-0.0091}$ | 0.0711$^{+0.0001}_{-0.0001}$ |

where the values $x$ are given in Eq.(3.12). Here, we consider no change of values for the parameters $a_e$ and $x_i$ ($i = 1, 2, 3$) because those have been fixed by the observed charged lepton masses with high accuracy. We also do not discuss the parameter dependence of $R_\nu$ and $r_{12}^d = m_d/m_s$, because those are freely adjustable by the parameters $\xi_\nu^0$ and $m_d^0$, respectively, without almost affecting other observables. As seen in Table 3, the predicted value $\sin^2 2\theta_{13}$ is sensitive to the parameter value $a_u$, so that we can take a value of $a_u$ which gives $\sin^2 2\theta_{13} \simeq 0.08$ at the cost of other fitting. Also, we find that those predicted values are practically insensitive to the parameter value $\alpha_u$.

6 Concluding remarks

In conclusion, by assuming VEV matrix forms of the yukawaons Eqs.(3.1) - (3.7), we have obtained reasonable CKM and PMNS mixing parameters together with quark and neutrino mass ratios. The major change from the previous yukawaon models is in the form of $M_D$. Although we give the form by assuming the VEV matrix $X_{\alpha i}$ which is given by Eq.(2.19), and by considering the mechanism $(XX^T)_{\alpha\beta} = (X_3)_{\alpha\beta}$ versus $(X^T X)_{ij} = (X_2)_{ij}$, it is still phenomenological and somewhat factitious. However, when once we accept the form of $M_D$, we can obtain $\sin^2 2\theta_{13} \sim 0.07$ whose value is not sensitive to the other parameters.

Note that the present model does not have any family-dependent parameters except for $(x_1, x_2, x_3)$ in $\langle \Phi_0 \rangle$ and $(\phi_1, \phi_2)$ in $\langle P \rangle$. The parameter values $(x_1, x_2, x_3)$ have been fixed by the observed charged lepton masses. Therefore, the model have only 9 adjustable parameters for 16 observables. The 5 parameter values of 9 parameters, $(a_e, a_D, (a_u, \alpha_u)$, and $\xi_\nu^0$, have been fixed by the observed values $m_u/m_c, m_c/m_t, \sin^2 2\theta_{12}, \sin^2 2\theta_{23}$, and $R_\nu$. Especially, the parameter $a_D$ has been fixed the observed value $\sin^2 2\theta_{12}$. The parameter $\xi_\nu^0$ has been introduced only in order to adjust the ratio $R_\nu$. (In other words, the value of $\xi_\nu^0$ has been fixed by $R_\nu^{obs}$, the value (3.10).) Logically speaking, we need four observed values in order to fix the four parameters $a_e, a_D$, and $(a_u, \alpha_u)$. However, as seen in Fig.1, the values $\sin^2 2\theta_{23} \sim 0.9$ and $\sin^2 2\theta_{13} \sim 0.07$
are almost determined independently of the parameter $a_D$ when we fix $a_e$ and $(a_u, \alpha_u)$ from the observed up-quark mass ratios. Therefore, $\sin^2 2\theta_{23} \sim 0.9$ and $\sin^2 2\theta_{13} \sim 0.07$ can substantially be regarded as predictions in the present model. Of course, after we fix the 5 parameters, predictions are 6 quantities: $\sin^2 2\theta_{13}$, 2 neutrino mass ratios, $CP$-violating phase parameter, and 2 Majorana phases.

Of the remaining 4 parameters $a_d, m_d^0$, and $(\phi_1, \phi_2)$, the parameters $a_d$ and $m_d^0$ are determined by the down-quark mass ratios $m_s/m_b$ and $m_d/m_s$, respectively. Therefore, the 4 CKM mixing parameters are predicted only by adjusting the two parameters $(\phi_1, \phi_2)$. We can obtain reasonable predictions of the CKM mixing parameters.

The present model is still in a level of a phenomenological model. Nevertheless, it seems that the yukawaon model offers us a promising hint for a unified mass matrix model for quarks and leptons, i.e. it seems to suggest an idea that the observed family mixings and mass ratios of quarks and leptons are caused by a common origin.

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