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Abstract

Why do governments borrow internationally, so much as to risk default? Why do they remain out of financial markets for a while after default? This paper develops a quantitative model of sovereign default with endogenous default costs to propose a novel and unified answer to these questions. In the model, the government has an incentive to borrow internationally due to a difference between the world interest rate and the domestic return on capital, which arises from a friction in the domestic banking sector. Since banks are exposed to sovereign debt, sovereign default causes losses for them, which translate into a financial crisis. When deciding upon repayment, the government trades off these costs against the advantage of not repaying international investors. After default, it only reaccesses international capital markets once banks have recovered, because only then are they able to efficiently allocate the marginal unit of investment again. Exclusion hence arises endogenously. The model is able to generate significant levels of domestic and foreign debt, realistic spreads, quantitatively plausible drops of lending and output in default episodes, and periods of post-default international financial market exclusion of a realistic duration.

Keywords: sovereign default, banking crisis, endogenous cost of default, international capital market exclusion.

JEL classification: F34, E62.
Resumen

¿Por qué los gobiernos se endeudan internacionalmente, tanto como para arriesgarse a quebrar? ¿Por qué permanecen fuera de los mercados financieros durante un tiempo después del impago? Este documento desarrolla un modelo cuantitativo de impago soberano con costes de impago endógenos para proponer una respuesta nueva y unificada a estas preguntas. En el modelo, el gobierno tiene un incentivo para endeudarse en los mercados financieros internacionales debido a una diferencia entre la tasa de interés mundial y el rendimiento sobre el capital nacional, que surge de una fricción en el sector bancario interno. Dada la exposición de los bancos a la deuda soberana, el impago soberano les causa pérdidas, que se traducen en una crisis financiera. Al decidir sobre el reembolso de su deuda, el gobierno compara estos costes con la ventaja de no reembolsar a los inversores internacionales. Después del impago, el país solo vuelve a participar en los mercados financieros internacionales una vez que los bancos se han recuperado, porque solo entonces pueden de nuevo asignar eficientemente cada unidad marginal de inversión. La exclusión surge de manera endógena. El modelo puede generar niveles significativos de deuda interna y externa, spreads realistas, reducciones de crédito interno y PIB cuantitativamente plausibles en episodios de impago, y períodos de exclusión del mercado financiero internacional posterior al impago de una duración realista.

Palabras clave: impago de la deuda soberana, crisis bancaria, costes de impago soberano endógenos, exclusión de los mercados financieros internacionales.

Códigos JEL: F34, E62.
1 Introduction

Sovereign default episodes are relatively frequent, often dramatic and can occur even in developed economies, as the recent European debt crisis has demonstrated. A default episode typically is a drama in three acts: During the first act, the government accumulates such large amounts of debt that repayment becomes uncertain and spreads rise. The second act starts with a tragic twist of fortune: The government defaults, which disrupts in particular the domestic financial sector, leading to reduced economic activity. The government ceases to issue new debt for a number of years. The last act finally sees the economy recovering and the government reaccessing international financial markets once again.

This paper proposes a unified quantitative theory to explain not just the default event itself, but also the beginning and the end of a typical default episode. First, the theory embodies a novel rationale, based on differences in returns, for why governments want to borrow internationally. Second, it provides an explanation for the output costs of default. These costs constitute a commitment device, which explains why and when governments repay and hence why they can borrow in the first place. Third and perhaps most importantly, it explains the temporary breakdown of international borrowing after default and its duration as an endogenous outcome. These three features all result from the same mechanism, relying on interactions between the sovereign and the domestic financial sector. This focus on intermediaries not only reflects the recent experience in the European debt crisis. It is also in line with both existing empirical findings that relate capital flows to financial frictions and sovereign default to financial crises as well as with a new finding documented in this paper that links market reaccess to the recovery of the financial sector.

The model describes a small open economy populated by households, banks and firms, which is governed by a benevolent government maximizing household utility. The government can issue bonds, which are the only domestic asset that foreign investors can also invest in. Firms produce output from labor and capital. Banks issue deposits to the household and invest in loans to firms and sovereign bonds. Their capacity to intermediate private savings within the economy is constrained by a leverage constraint. This friction in the intermediation process drives a wedge between the private and the social return on capital. The inefficiency resulting from this friction implies a motive for the government to borrow from foreign investors. Sovereign debt is commitment free, that is, the government can default on it whenever it finds it opportune to do so. There are no exogenous costs of default. However, since default is nondiscriminatory, default not only hurts foreign investors, but also domestic banks, generating a financial crisis in the domestic economy. This crisis depresses both output and the marginal return on domestic investment. The latter makes it optimal for the government not to access international capital markets for a while. The model is calibrated to Greek data and successfully replicates important empirical moments related to the three novel features of the model.

An encompassing theory of default needs to explain why governments borrow systematically. The literature on sovereign default following Eaton and Gersovitz (1981) and Arellano (2008)
typically embeds the motive to borrow in the utility function by assuming that the borrowing country is impatient relative to the international investors pricing the sovereign bond. My theory provides an alternative endogenous motive to borrow: The explicit modeling of capital and financial frictions implies that the social marginal return on domestic investment is higher than private marginal return. Since in the ergodic distribution the latter is largely determined by the households discount factor, this implies that the social marginal return exceeds the world interest rate (most of the time) even if the domestic household has the same time preference as the international investor. This difference in returns provides a motive for the government to borrow abroad in order to lower taxes and hence stimulate private investment. This return differential motive echoes the literature on international capital flows such as Gourinchas and Jeanne (2013) and Benhima (2013). Unlike the impatience motive, which is usually calibrated without direct evidence to unusually high degrees of impatience, the return differential motive can be disciplined by measured spreads between loan and deposit rates. It is reassuring that my modeling choice delivers reasonable quantitative results. In particular, the sovereign spread is in line with the data and the amount of debt, while smaller than in the data, compares favorably to the literature. Moreover, as will I argue in a few lines, the endogenous borrowing motive is key for explaining the end of default episodes.

An encompassing theory of default furthermore needs to explain why and when sovereign governments repay and default. Early contributions such as Arellano (2008) or Aguiar and Gopinath (2006) answer these questions by assuming that default leads to capital market exclusion and output losses and is hence costly. Subsequently, researchers attempted to endogenously explain the incentives to default. This paper adds to this literature by proposing a model where the costs of default arise from the exposure of the domestic financial sector to domestic government debt: When the government defaults, the balance sheet of intermediaries is hit. Due to financial frictions, this distorts capital allocation and hence leads to inefficiently low output. Quantitatively, when calibrated to Greek data, the mechanism is able to generate drops in output, credit and investment that are of similar magnitude as those observed around the Greek default.

Finally, an encompassing theory of default should explain why default is typically followed by a period of several years of market exclusion and how long this lasts. The conventional view is to explain exclusion as a punishment strategy by lenders and to introduce it into the model as an assumption. However, as argued by Kletzer (1994) this assumption is not straightforward, since it would be profitable for each individual lender to deviate from the punishment strategy. The theory I propose offers a novel interpretation of market exclusion. To the best of my knowledge, my model is the first in which exclusion arises endogenously, as an optimal choice by the government. During the financial crisis following default, the marginal return on domestic investment is depressed. This is because the financial sector, which possesses

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1 Similar recent contributions are compared at the end of this section.
a superior capital allocation technology, is unable to intermediate additional investment and capital is hence allocated less efficiently without the help of banks at the margin. Hence, the difference between the marginal domestic return and the world interest rate drops and may become negative, making borrowing abroad temporarily undesirable for the government. As time passes, however, the financial sector recovers, until it eventually intermediates the marginal unit of investment again. In that moment the domestic marginal return on investment jumps up and the government reenters international capital markets. In the next section I provide novel empirical evidence that is consistent with this link between the recovery of the financial system and the resumption of borrowing. Furthermore, this mechanism allows the model to replicate the empirical distribution of the duration of exclusion well.

Having explored the quantitative implications of the model, it is then used to evaluate a current policy proposal to reduce the exposure of European banks to domestic sovereign debt. The model highlights that this may have unintended consequences: As bank exposure is reduced, banks not only become more resilient to a sovereign debt crisis. Such a policy also reduces the degree to which the government can commit to repay its debt, since the default costs are reduced endogenously. According to the model, a 20% reduction in bank exposure cuts the sustainable amount of foreign debt by one half. Since foreign debt is beneficial for the economy, this comes at welfare costs equivalent to almost 1% of lifetime consumption equivalent. If implemented abruptly, this reform would trigger default with a high probability.

**Related literature.** The paper is related to several strands of the literature on sovereign default. The most innovative feature of my theory is endogenous exclusion. The study by Benjamin and Wright (2013) focuses on a related issue. They model the debt renegotiation process between defaulter and creditor, that – due to uncertainty – can potentially last many years. During this time the country is assumed not to be able to borrow, hence they provide a theory of the duration of renegotiations, but not of exclusion itself. My theory complements theirs, highlighting how the incentives to borrow depend on the state of the domestic financial sector, and can explain exclusion endogenously.\(^2\)

Furthermore, by introducing capital this paper bridges the gap between the sovereign default literature and the canonical RBC model. Guimaraes (2011), Roldán-Peña (2012), Joo (2014), Gornemann (2015), Gordon and Guerrón-Quintana (2017) and Park (2017) also introduce capital into an Eaton and Gersovitz (1981) default model. I add to these paper by also modeling financial frictions and exploring how these can lead to a persistent motive to borrow and endogenous costs of default.

Lastly, this paper contributes to a recent strand of literature that endogenizes the incentives to default. While some contributions explore the distributional incentives of domestic default

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\(^2\)Kletzer and Wright (2000) show how exclusion may arise as a renegotiation proof punishment strategy to make trade across states sustainable in an environment of state-contingent contracts and two-sided limited commitment. However, theirs is not a theory of exclusion since exclusion never arises in equilibrium and its duration off-equilibrium is undetermined.
(D’Erasmo and Mendoza, 2012 and Guembel and Sussman, 2009), many focus on explaining the output losses associated to default, most prominently Mendoza and Yue (2012), who highlight the importance of international trade credit.

Within this strand, several papers also endogenize output costs by introducing a financial sector. Like in my paper, in these models banks are exposed to sovereign debt and hence default affects output. However, in these models banks finance the labor bill in advance through intra-period loans, while in mine banks instead finance the capital stock. The introduction of a domestic inter-period asset not only yields a more realistic bank balance sheet. Modeling financial frictions and capital jointly also gives rise to a new motive to borrow and, moreover, yields post-default exclusion as an endogenous outcome. Sosa-Padilla (2017) develops a stylized model in which banks’ wealth directly serves as an input to production, without explicitly modeling intermediation. In Boz et al. (2014) leverage constrained banks intermediate household funds to firms. In Engler and Grosse Steffen (2016), Mallucci (2014) and Perez (2015) local banks trade on interbank markets, and sovereign default brings these markets to collapse: in the first and second case because sovereign bonds are necessary as collateral to facilitate interbank lending and this collateral disappears in case of default, in the last because interbank borrowing is leverage constrained. The collapse of the interbank markets then leads to efficiency losses. Note that all of these papers simply assume exclusion and – apart from Sosa-Padilla (2017) – they require this assumption to generate the financial crisis and the corresponding output costs of default. The same is true for the trade crisis in Mendoza and Yue (2012). On the contrary, my theory provides an explanation for exclusion and predicts its duration.3

This paper is also related more broadly to the macro literature on the role of frictions in the financial sector, e.g. Gertler and Kiyotaki (2010). Bocola (2016) analyzes sovereign default and its consequences in the context of the former model. While the setup is otherwise similar, he considers the government’s decisions as exogenous.

The rest of the paper is structured as follows. The next section briefly reviews the empirical literature and provides some additional empirical evidence to motivate the quantitative model of sovereign default, which is set up and discussed in section 3. Next, section 4 explains the intuition behind the main mechanisms analytically. Section 5 discusses calibration and solution method and explores the mechanisms further based on numerical illustrations. Section 6 presents the main quantitative results and section 7 uses the model to evaluate the effects of a policy that reduces the exposure of banks to domestic sovereign debt. The last section concludes.

3Furthermore, the idea that sovereign default inflicts costly damage on the financial sector is also developed in a more stylized manner in 3-period models like Acharya et al. (2014), Basu (2010), Bolton and Jeanne (2011), Brutti (2011), Erce (2012), Gennaioli et al. (2014) and Mayer (2011). Due to their 1-shot nature, none of these models can speak about market exclusion or be tested quantitatively. Chari et al. (2016) analyzes the use of financial repression (i.e. forced exposure of banks to the sovereign to gain commitment ) as a policy tool in an analytical setup.
2 Empirical motivation

There is a large body of empirical literature that documents a number of empirical regularities around sovereign default events, which motivate this model. First, external sovereign debt is high for many countries and sovereign default is a relatively frequent event (e.g. Schmitt-Grohe and Uribe, 2017).4 Second, sovereign default typically is followed by significant contractions in economic activity (e.g. Mendoza and Yue, 2012) and periods of exclusion from international capital markets (e.g. Gelos et al., 2011). Third, there exists a tight connection between sovereign defaults and financial crisis. Gennaioli et al. (2014) for example compile an international panel comprising data on the macro economy, public debt and financial institutions’ balance sheets, covering 110 default episodes. They document four regularities that are key for my model: (i) Domestic banks hold significant amounts of domestic sovereign debt. (ii) Defaults often happen in close proximity to financial crises, with the former typically starting earlier or simultaneously. (iii) Credit extended by the financial to the private sector drops significantly in case of default and this drop is stronger, the more exposed banks are to sovereign debt. (iv) Higher exposure of domestic banks to domestic sovereign debt reduces the probability of default.5

By putting the financial system at the center, my model is able to reproduce all of these patterns: First, it provides an explanation for why countries borrow and default. Second, it predicts that default is followed by output losses and exclusion. Third it rationalizes these consequences of default as the result of a domestic financial crisis, which triggers a reduction in credit. Hence it explains how the banking sector’s exposure provides the government with some commitment.

Furthermore, in providing a novel theory of market exclusion, the model yields another testable prediction that has not yet been analyzed empirically. According to the model, market exclusion is a result of the default-induced crisis in the financial sector, and its duration is related to how long it takes for the financial sector to recover from the losses associated to the sovereign default.6 If this mechanism is indeed relevant, we should be able to find a positive relationship between the health of the financial sector and the date of reaccess.

In order to test this implication, I compile a data set covering default episodes since 1980, which builds on existing empirical studies. The post-default exclusion episodes are taken from Gennaioli et al. (2014) (GMR) and Cruces and Trebesch (2013) (CT). Both report default episodes, however using different concepts to time the beginning and end of an episode. The former define the start as the year when a country defaults, and the end as the year in which

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4Schmitt-Grohe and Uribe document 147 defaults in 93 nations between 1975 and 2014.
5Similar evidence is provided by De Paoli et al. (2006), Sturzenegger and Zettelmeyer (2007), Acharya et al. (2014), Bolton and Jeanne (2011), Reinhart and Rogoff (2011b), Sosa-Padilla (2012) and Balteanu and Erce (2017).
6As we shall see, in the model exclusion lasts as long as the financial sector is sufficiently constrained by its leverage constraint as to not be the marginal investor in firm loans.
episodes are combined with data from the IMF’s IFS and the World Bank’s WDI databases, from which I extract annual data on GDP, population and credit. As a proxy for the conditions of the financial sector I use private credit to GDP, that is the volume of loans provided by financial institutions to the private non-financial sector relative to GDP. This choice is both in line with the model, which predicts reaccess only after banks balance sheets and hence credit has recovered, as well as with previous empirical works such as GMR. All variables are in logs.

To validate the model’s prediction that reaccess is associated with recovery of the financial sector, in figure 1 I first report the average evolution of credit to GDP around reaccess events after controlling for country-specific linear trends. Indeed, governments reaccess international capital markets only after credit has recovered from the low levels associated with the previous default period. The figure also shows that real GDP per capita still is below trend when market reaccess happens, which is an indication that credit conditions may matter independently of the overall level of the economic activity.

Figure 1: Credit and GDP around market reaccess. The figure shows the average behavior of credit and GDP around reaccess events. The values shown are the results of an event analysis as in Balteanu and Erce (2017) and Gourinchas and Obstfeld (2012). In particular, the lines report the $\beta_\tau$ estimates of the regression $Y_{it} = \alpha_i + \tilde{\alpha}_t + \sum_{\tau=-4}^{4} \beta_\tau D_{\tau it} + \varepsilon_{it}$ where $D_{\tau it}$ are the $\tau$ years leads and lags of a dummy marking reaccess of country $i$ in year $t$ and where $Y_{it}$ is the log of either credit/GDP or GDP. The estimates are reported in table 5 in appendix E.

When combining the two data sets, most of the time the settlement periods coincide. Sometimes they do not because GMR bunch default sequences into single episodes. In these cases I follow GMR. Six of CT’s default episodes are not listed in GMR. I these cases I use the list of default episodes in Schmitt-Grohe and Uribe (2017). For robustness I also consider the original GMR set, where I excluded those episodes that are not associated with banking crises according to the definition in GMR. See the cited sources and the associated online appendices for a more detailed description of the definitions they use.
To corroborate this finding, I next estimate the effect of financial sector conditions on the probability of an exclusion period ending. In order to address potential reverse causality issues, I lag my explanatory variable. Hence I estimate the following logit model:

$$Pr(\text{reaccess}|\text{post default exclusion})_{it} = F\left(\alpha + \beta (\text{Credit}_t/\text{GDP})_{it-1} + \gamma X_{it}\right)$$

where $X_{it}$ are controls. Table 1 column (1) reports the results of this regression without controls. Indeed, as the model predicts, credit contributes positively to the probability of reaccess. However this result may be driven by spurious correlation. One may be concerned that credit—even though relative to GDP—is just a reflection of overall economic activity. Therefore, as in CT, I include real GDP growth, real GDP per capita and population, all lagged. Besides one may be concerned that global credit conditions drive domestic credit and reaccess, therefore I include a time dummy. Or one may be concerned that both reaccess and the recovery of credit are simply a matter of time. To account for this possibility I include a third order polynomial of the number years since default. Finally, country- or region-specific characteristics that are correlated to credit to GDP may explain the exclusion duration, hence region or country dummies are considered.$^8$

As the table shows, the coefficients for the measures

|                          | (1)     | (2)     | (3)     | (4)     | (5)     | (6)     | (7)     | (8)     |
|--------------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| Private Credit$_{t-1}$   | 0.0319*** | 0.0266** | 0.0394* | 0.0276** | 0.0336* | 0.0319** | 0.161** | 0.129** |
|                          | (0.000) | (0.037) | (0.052) | (0.033) | (0.086) | (0.044) | (0.021) | (0.035) |
| Real GDP growth$_{t-1}$  | 0.00304*** | 0.00258* | 0.000760 | 0.000157 | 0.00434* | -0.00419 | 0.00153 |
|                          | (0.002) | (0.093) | (0.508) | (0.283) | (0.052) | (0.238) | (0.483) |
| Population$_{t-1}$       | 0.0160*** | 0.0325*** | 0.0255*** | 0.0340*** | 0.00646 | 1.740* | 0.162 |
|                          | (0.009) | (0.001) | (0.000) | (0.000) | (0.515) | (0.087) | (0.927) |
| Real GDP pc$_{t-1}$      | 0.0455*** | 0.0725*** | 0.0457*** | 0.0635*** | 0.0359** | 1.295*** | 1.193*** |
|                          | (0.000) | (0.000) | (0.000) | (0.000) | (0.013) | (0.000) | (0.042) |
| Region dummies           | ✓       | ✓       | ✓       | ✓       | ✓       |          |         |         |
| Year dummies             | ✓       | ✓       | ✓       | ✓       |          | ✓       | ✓       |         |
| Duration dependence      | ✓       | ✓       | ✓       | ✓       |          | ✓       | ✓       |         |
| Country dummies          | ✓       | ✓       | ✓       | ✓       |          | ✓       | ✓       |         |
| default set              | CT      | CT      | CT      | CT      | CT      | GMR     | CT      | GMR     |
| # default episodes       | 60      | 58      | 58      | 58      | 58      | 57      | 44      | 48      |
| # re-accesses            | 52      | 50      | 50      | 50      | 50      | 63      | 48      | 55      |
| # observations           | 686     | 667     | 450     | 450     | 450     | 509     | 502     | 425     |
| Pseudo R$^2$             | 0.021   | 0.088   | 0.201   | 0.229   | 0.270   | 0.240   | 0.727   | 0.597   |

Table 1: Determinants of market reaccess. The reported numbers are average marginal effects and — in parenthesis — p-values, clustered at country level. #observations are adjusted for observations perfectly predicted by dummies.

$^8$Country fixed effects are problematic due to the limited number of observations per country, since exclusion periods are short and most countries do not reappear in the sample. I therefore interpret these specifications as robustness check. I do not report results from the full model with country FE and the CT sample since the number of observations that is not perfectly predicted by dummies drops by 2/3.
of financial sector conditions are positive and statistically significant across all specifications. The estimates are also economically significant: Based on the average marginal effects of the specifications without country fixed effects, a one standard deviation increase in private credit increases the probability of market reaccess, which, depending on the sample, is between 5 and 10% unconditionally, by roughly 3 ppt.9

As common in the empirical literature on sovereign default, these results do not necessarily imply causality.10 However they are certainly consistent with the mechanism that the model highlights. They complement the result in CT, who find that higher haircuts are associated with longer exclusion periods. While haircuts are not modeled in this paper, this finding is consistent with the model’s mechanism too, since, ceteris paribus, higher haircuts imply larger losses for the financial sector and hence longer recovery periods.

3 The model

This section proposes a dynamic small open economy model of sovereign default, where the only costs of default stem from the exposure of the domestic banking sector to the sovereign. The domestic private economy is made up of 3 sectors, namely households, banks and firms and is governed by a benevolent government. The domestic economy is open and can borrow from international lenders (the rest of the world) through government bonds.

3.1 The households

There is a continuum of mass 1 of identical households, which each make a consumption-labor-savings choice. The household can invest into bank deposits \( d_h \) and loans to firms \( k_h \). Bank deposits are promises to pay 1 unit of the final good tomorrow, which can be bought at price \( 1/r \). \( r \) hence is the deposit rate. Loans are risk-free promises to repay \( R' \) units of the final good tomorrow. When lending directly to firms, the household incurs a cost \( \xi \). Both assets are safe and loans cannot be shorted.11 The household’s disposable income is the sum of the value of its assets carried over from the last period \( d_h + Rk_h \), his labor income \( Wl_h \) and the lump sum dividend and profit payments from the banks \( Div \) and the firms \( \pi \), minus a lump sum tax \( T \). The household has rational expectations and chooses labor, consumption and investment to maximize his lifetime utility, taking prices and aggregate states as given. Its problem hence is:

\[ \text{max} \quad U \]
\[ V_h(d, k_h, \Omega) = \max_{c, d'_h, k'_h, l_h} u(c, l_h) + \beta E_{t|\Omega} V_h(d'_h, \Omega') \]

subject to
\[ c + d'_h / r + (1 + \xi)k'_h + T = d_h + Rk_h + Wl_h + Div + \pi \]
\[ k'_h \geq 0 \]

where \( \Omega \) denotes the aggregate state vector and \( V_h \) is its value function. Instantaneous utility is given by a conventional additively separable CRRA function:
\[ u(c, l_h) = \frac{c^{1-\gamma}}{1-\gamma} - \chi \frac{l_h^{1+\nu}}{1+\nu} \]

The corresponding first order conditions are necessary and sufficient:
\[ u_c(c) = \beta r E_{t|\Omega} [u_c(c')] \] (2)
\[ u_c(c)(1 + \xi) = \beta R'E_{t|\Omega} [u_c(c')] + \lambda_h \] (3)
\[ 0 = \min \{ \lambda_h, k'_h \} \] (4)
\[ u_c(c)W = u_l(l) \] (5)

Since, as we shall see, households’ deposits are passed on to firms as loans, the household essentially chooses between intermediated (bank loans) and direct financing of the real economy (think of corporate bonds and entrepreneurial self finance). However, direct investment is less efficient than intermediated investment due to the costs \( \xi \), which is incurred on the first but not on the latter. This assumption is common in the macro models with banks (e.g. Brunnermeier and Sannikov (2014)). The idea behind it is that banks have a superior capital allocation technology, since they are better at screening and monitoring (see Holmstrom and Tirole (1997)).\(^{12}\) It explains why intermediation is desirable and why banks exist in equilibrium. It is hence consistent with the fact that the financial sector intermediates a large part of investment in many economies.\(^{13}\) Furthermore, allowing for a less efficient alternative form of investment is in line with the finding that direct investment partially compensated for the reduction in intermediated investment during banking crisis, documented for example by Fiore and Uhlig (2015) for Europe and by Becker and Ivashina (2014) for the US.

The cost \( \xi \) is allowed to vary with TFP \( \omega \): \( \xi = \hat{\xi} \omega^\xi \). We shall assume that \( \hat{\xi} < 0 \), such that the cost advantage of intermediated financing is particularly strong when the TFP is low. This reflects the idea that the banks’ screening and monitoring technology is most useful, when profitable investment opportunities are scarce.

\(^{12}\) \( \xi \) might also capture liquidity, maturity transformation or diversification services provided by banks.
\(^{13}\) With the notable exception of the US, where market based funding plays an important role.
3.2 The banks

Banks are the key agents in this economy. Banks intermediate the savings from the households to the firms and allocate them more efficiently than if the household invests directly, yet they are constrained by a financial friction that gives rise to a leverage constraint. It is this constraint that will ultimately make foreign debt desirable and default costly for the government.

There is a continuum of mass 1 of banks, each of which is run by a banker. (In this subsection, I drop the index $b$ to simplify notation.) The banker enters each period with a portfolio of 1-period assets and liabilities (his balance sheet) chosen last period. In particular, he holds $k$ units of firm loans, which pay the ex-ante fixed gross return $R$ and $b$ units of government bonds, which return 1 if repaid ($Rep = 1$), otherwise they return nothing ($Rep = 0$); and (3) he owes depositors $d$. The net cash flow of these assets and liabilities defines the bank’s pre-dividend equity $e$.

$$e = Rk + bRep - d$$

Next, the banker chooses how much dividend $div$ to pay to his shareholders. The remaining resources are kept on the balance sheet as post-dividend equity $e - div$. Given the post-dividend equity, the banker next chooses his optimal portfolio of assets and liabilities: he can invest in loans to firms $k'$ at price 1, buy government bonds $b'$ at price $q$, and issue deposits $d'$ at price $1/r$. Note the above mentioned superiority of banks in allocating capital: to provide one unit of loan the bank pays one unit, while the household pays $1 + \xi$. This yields the following end-of-period balance sheet:

| Assets                      | Liabilities               |
|-----------------------------|---------------------------|
| Loans $k'$                  | Deposits $d'/r$           |
| Government bonds $q \cdot b'$ | Post dividend equity $e - div$ |

which can equivalently be expressed by the balance sheet constraint:

$$qb' + k' = e - div + d'/r$$

Besides the balance sheet equation, the banker’s choice is constrained by several other constraints. The first constraint is crucial. Following Kiyotaki and Moore (1997) I assume that bankers face a commitment problem: At the end of a period (after investing) bankers can repudiate their deposit liabilities. In that case the depositors would size a fraction $\theta < 1$ of the banks assets. This gives the banker the opportunity to renegotiate his liabilities. Depositors anticipate this and will not let the value of deposits exceed the value of recoverable assets. This translates into a leverage constraint, requiring the banker to hold a certain minimum of expected pre-dividend equity tomorrow (weighted by the depositors’ stochastic discount factor):\(^{14}\)

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\(^{14}\)The RHS is the household’s expected value of sizing the assets of an individual atomistic bank at the end of period $t$. The LHS is the value of the deposit claim on that bank.
Second, the banker’s dividend choice is constrained downwards, it can not be smaller than a certain \((1 - \eta)\) fraction of equity.

\[
div \geq (1 - \eta)e
\]

This assumption, which is common in the macro literature on financial frictions,\(^{15}\) makes sure that the leverage constraint introduced above bites: One the on hand, the banker cannot simply raise new equity should he require more. On the other hand, the fact that he constantly has to payout a positive dividend ensures that he also cannot overcome the equity constraint over time by accumulating retained earnings. Moreover, this assumption reflects reality. Banks seem to have difficulties raising capital especially in times of crisis and they generally try to smooth their dividends.

Third, the bank has to hold at least a fraction \(\psi\) of government bonds on the balance sheet:

\[
\frac{b'q}{b'q + k'} \geq \psi
\]

This simple constraint is intended to capture, in a reduced form, a number of reasons that induce banks to invest heavily into domestic sovereign bonds. These reasons may for example be related to financial regulation, financial repression or the special role of bonds as liquid or collateral asset.\(^{16}\) Appendix D provides a simple model to rationalize this assumption in a model where banks hold a certain amount of bonds for either liquidity or collateral reasons. Whatever the exact nature of these forces, it is a well documented empirical fact that banks exhibit a strong home bias and are heavily exposed to domestic sovereign debt: Sosa-Padilla (2012) and Gennaioli et al. (2014) report that on average 22% or 12% respectively of the financial sector’s assets in each country in their samples are domestic sovereign debt. Note that this constraint is crucial for the equilibrium of the model, in which bonds will generally pay a lower risk adjusted return than loans and where the banks exposure is necessary to sustain any foreign lending.

Fourth, next period equity must not be negative, even in case of sovereign default.

\[
R'k' \geq d'
\]

Finally, there is no short selling of assets.

\[
b' \geq 0, \quad k' \geq 0
\]

\(^{15}\)E.g. Gertler and Kiyotaki (2010), who motivate an equivalent assumption by retiring bankers.

\(^{16}\)For a model with a liquidity motive for holding government bonds see Engler and Grosse Steffen (2016). Besides, notice that regulation and repression could be considered policy variables as well. I abstract from these considerations. For a model with optimal financial repression see Chari et al. (2016).
The banker is a small member of the household, to whom he pays the dividend, and has rational expectations. He maximizes the discounted value of dividends, taking the household discount factor, the prices and the law of motion of the aggregate state as given. Her objective function reads

$$V_b(d, b, k, \Omega) = \max_{d', b', k'} \left[ (e + d'/r - qb' + k') u_c(c) + \beta E_\Omega [V_b(d', b', k', \Omega')] \right]$$

After multiplying the objective function by $u_c(c)$, using the balance sheet constraint to substitute for $div$, rearranging and abstracting from the short selling constraints, which cannot be binding in any equilibrium with banking, and the non-negativity constraint on equity,\(^{17}\) the banks problem is:

$$V_b(d, b, k, \Omega) = \max_{d', b', k'} \left[ (e + d'/r - qb' + k') u_c(c) + \beta E_\Omega [V_b(d', b', k', \Omega')] \right]$$

subject to:

$$E_\Omega [u_c(c)d'] \leq E_\Omega [u_c(c') \theta(Rep' b' + R' k')]$$

$$(1 - \eta)e \leq e + d'/r - qb' - k'$$

$$\psi k' \leq b'q(1 - \psi)$$

where $e = Rk + bRep - d$

The solution of the bank’s dynamic problem can conveniently be characterized:

**Proposition 1:** The vector of choice variables $[b', d', k'] \in (\mathbb{R}_0^+)^3$ is a solution to the banks problem if and only if together with the vector of multipliers $[\lambda_1, \lambda_2, \lambda_3] \in (\mathbb{R}_0^+)^3$ it solves the system of first order conditions

$$\frac{\partial L}{\partial d'} : 0 = u_c(c)/r - \beta E_\Omega [V_{be}(c', \Omega')] - \lambda_1 E_\Omega [u_c(c')] - \lambda_2/r \quad (6)$$

$$\frac{\partial L}{\partial b'} : 0 = -qu_c(c) + \beta E_\Omega [Rep'V_{be}(c', \Omega')] + \lambda_1 E_\Omega [Rep'u_c(c')] \theta - \lambda_2 q + \lambda_3 q(1 - \psi) \quad (7)$$

$$\frac{\partial L}{\partial k'} : 0 = -u_c(c) + R' E_\Omega [V_{be}(c', \Omega')] + \lambda_1 R' E_\Omega [u_c(c')] \theta - \lambda_2 - \lambda_3 \psi \quad (8)$$

$$\frac{\partial L}{\partial \lambda_1} : 0 = \min \left\{ E_\Omega [u_c(c) \theta(Rep' b' + R' k')] - E_\Omega [u_c(c')d'], \lambda_1 \right\} \quad (9)$$

$$\frac{\partial L}{\partial \lambda_2} : 0 = \min \left\{ \eta e - qb' - k' + d'/r, \lambda_2 \right\} \quad (10)$$

$$\frac{\partial L}{\partial \lambda_3} : 0 = \min \left\{ b'q - (b'q + k) \psi, \lambda_3 \right\} \quad (11)$$

where $V_{be}(c', \Omega')$, the derivative of the value function with respect to equity, is given by $V_{be}(c', \Omega') = u_c(c') + \eta \lambda_2$. The solution is linear in pre-dividend equity $e$.

Proof: appendix C. The last statement implies that the distribution of equity among banks does not matter, which allows us to represent the banking sector by a representative bank.\(^{18}\)

\(^{17}\)The model will be parametrized such that abstracting from the constraint on equity remains irrelevant.

\(^{18}\)The penultimate statement allows us to eliminate all references to the value function from the FOCs and to conveniently characterize the bank’s optimal choice in terms of policy functions alone.
3.3 The firms

There is a continuum of mass 1 of firms. The problem of the firm is a 2-period problem. In the first period the firm borrows $k_f$ units from the banks and households at a fixed interest rate $R'$, which she transforms into capital for period 2. In the second period the firm hires $l_f$ units of labor from the households and produces. The firm uses a standard Cobb-Douglas production function with capital share $\alpha$ and stochastic TFP $\omega$. Capital depreciates at rate $\delta$. TFP $\omega$ follows a log-normal AR(1) process with persistence $\rho$ and innovations $\varepsilon$ with variance $\sigma^2$

$$\log(\omega) = \rho \log(\omega_{t-1}) + \sigma \varepsilon$$

Like the banker, the firm manager is part of the household, to whom any profits or losses $\pi$ are rebated lump sum and whose stochastic discount factor the firm manager takes as given. Her sequential optimization problem is

$$\max_{k_f'} E_{\Omega}\left[\left(\beta \frac{u(c')}{u(c)}\omega' \left(\hat{l}(\omega')\right)^{1-\alpha} + (1 - \delta)k_f' - R'k_f' - W(\omega')\hat{l}(\omega')\right)\right]$$

s.t

$$\hat{l}(\omega') = \arg\max_{l_f'} \omega' \left(k_f'^\alpha \left(l_f'^{1-\alpha} + (1 - \delta)k_f' - Rk_f' - W(\omega')l_f'\right)\right)$$

The FOCs determining the loan rate and the wage are necessary and sufficient:

$$R' = E_\Omega\left[u(c')\left(\omega'\alpha k_f'^{\alpha-1}l_f'^{1-\alpha} + (1 - \delta)\right)\right] / E_\Omega\left[u(c)\right]$$

$$W = \omega(1 - \alpha)k_f'^\alpha l_f'^{-\alpha}$$

I have assumed that firm loans are riskless contracts. Hence it is the firms’ owners (the household) and not the banks who absorb aggregate risk, unlike e.g. in Gertler and Kiyotaki (2010) where banks hold firm equity. My assumption mimics reality: most of banks assets are loans, which have a flat repayment profile as long as they are not defaulted upon. I abstract from firm default both for simplicity and to isolate the effects of sovereign default.\(^{19}\) Besides, note that in the absence of financial frictions, both assumptions are equivalent.

3.4 The foreign investors

The model describes a small open economy. The rest of the world is represented by perfectly competitive risk neutral deep pocket foreign investors. I assume that lending to domestic private agents requires local know-how and is hence not possible or profitable for foreign investors.

\(^{19}\)Assuming that banks carry some of the risk associated with firm investment would change the quantitative properties of the model, since it would introduce a negative correlation between the innovation in TFP and bank equity and would hence reduce the degree to which default incentives increase in TFP.
but they can invest in the 1-period government bond. They demand any amount of government bonds if their expected return equals the world interest rate $1/\bar{q}$. Foreign demand for government bonds, denoted by $B'_X$, is hence given by

$$B'_X \in \begin{cases} 0 & \text{if } E_{\Omega} \text{Rep}'/\bar{q} < 1/\bar{q} \\ [0, M] & \text{if } E_{\Omega} \text{Rep}'/\bar{q} = 1/\bar{q} \end{cases}$$

where $M$ is an arbitrarily large number. This upper bound is assumed to never bind in equilibrium, however it rules out Ponzi schemes.

3.5 Market clearing

Having discussed all private agents, we are now ready to close the private economy by the corresponding market clearing conditions. From now I will denote aggregate choice variables by capital letters, i.e. $\int_0^1 x_i di = X_I$ where $i$ is the respective agent’s index (which was suppressed for banks before) and $I$ its type. Labor market clearing requires

$$L_F = L_H$$

Loan and deposit market clearing implies

$$K'_F = K'_H + K'_B$$

$$D'_B = D'_H$$

Finally, bond markets clear when the total government debt issuance $B'_T$ equals the total demand by foreigners $B'_X$ and local banks $B'_B$

$$B'_B + B'_X = B'_T$$

By Walras’ law the goods market clears as well.

This concludes the set up of the private economy. Before we move to the problem of the government, which will choose its tax, borrowing and repayment policy, it is convenient to define the notion of a private equilibrium, given a arbitrary set of government policies $T(\Omega), B_T(\Omega), \text{Rep}(\Omega)$.

**Definition 1: Private equilibrium** Given set of state dependent government policy functions $T(\Omega), B_T(\Omega), \text{Rep}(\Omega)$ a stationary private equilibrium consists of a set of state dependent policy functions $C(\Omega), L_H(\Omega), D'_H(\Omega), K'_H(\Omega), D'_B(\Omega), B'_B(\Omega), K'_B(\Omega), K'_F(\Omega), L_F(\Omega), B'_X(\Omega)$ price functions $r(\Omega), q(\Omega), R'(\Omega), W(\Omega)$ and shadow price functions $\lambda_1(\Omega), \lambda_2(\Omega), \lambda_3(\Omega), \lambda_H(\Omega)$ such that the maximization problems of the household, the bank and the firm are solved, the foreign lenders’ demand is satisfied, markets clear and such that the policy functions imply the state transition function that underlies the agents’ expectations.
Call the vector of policy, price and shadow price functions \( X(\Omega) \). Denote the system of functional equations over the domain \( \Omega \), which is defined by equations (2)-(18) after replacing all individual variables with the corresponding aggregate variables, by \( F(X, T, B_T, Rep|\Omega) = 0 \). Then, since all first order conditions are necessary and sufficient, an equivalent way of defining a stationar y private equilibrium is:

\[
\text{such that } F(X, T, B_T, Rep|\Omega) = 0 \quad \forall \Omega \text{ given some } T(\Omega), B_T(\Omega), Rep(\Omega).
\]

What are the state variables in this problem? First, we need to keep track of TFP \( \omega \), which is the only exogenous state. Second, we need to account for the balance sheets of households and banks. This encompasses the loans extended by the household \( K_H \); the deposits issued by the bank and bought by the household, \( D \); the loans extended by the bank \( K_B \); the bonds bought by the bank \( B_B \) and the loan rate \( R \). Third, \( \Omega \) encompasses the amount of outstanding sovereign debt held abroad \( B_X \), because this variable influences the government policy choices, that are discussed next. The state vector is hence \( \Omega = [\omega, K_H, D, B_B, K_B, R, B_X] \).

### 3.6 The government

The model economy is governed by a benevolent government, which chooses its actions such as to maximize the households’ utility. It needs to finance some fixed government expenditures \( G \) and it can do so by taxing the household through a lump sum tax \( T \) and issuing 1-period government bonds \( B_T' \), which are a promise to repay 1 unit tomorrow and are traded at price \( q \). These bonds are sold on anonymous markets, where both local banks and international investors can buy them. Note that unlike the government, private agents can not trade assets directly with the rest of the world. This is a simplifying assumption, which could generally be relaxed as long as foreign lenders have some disadvantage in direct investment. At the same time it is a plausible approximation for many less financially integrated economies, where private borrowing makes up for only a small part of the total net foreign asset position, such as Greece for example.

While it is known who buys the bonds (banks buy \( B_B' \), foreign lenders invest \( B_X' \)), the government cannot discriminate buyers, neither at the time of issuance nor at the time of repayment. This assumption is key and discussed below.

Following standard practice in the literature I assume that sovereign debt is non-enforceable and the government cannot commit to future actions; i.e. the government is free to choose to repay its debt \( (Rep = 1) \) or to fully default on its obligations \( (Rep = 0) \) at the beginning of each period after observing the realization of the TFP shock.

In case of default there is no direct punishment: in particular foreign lenders do not categorically refrain from lending, and no direct output costs arise. Yet I assume that following

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20Following the literature, I rule out explicitly state contingent contracts and I assume lump sum taxation. The latter simplifies the model by avoiding a motive for distortion smoothing.
default the government effectively can not save abroad, because its foreign assets would else be seized by the creditors.\footnote{An equivalent alternative to this assumption would be a sufficiently high spread between the borrowing and savings rate. In the optimization problem of the government below, I enforce this constraint also before default. This simplifies matters and is innocuous since in equilibrium the constraint only binds after default for the part of the parameter space that we are interested in.} This gives rise to the following government budget constraint:

$$\begin{align} q \cdot B_T' + T & \geq B_T \cdot \text{Rep} + G \\
B_X' & \geq 0 \end{align}$$

It is important to note that the government has rational expectations and — unlike the private agents — is non-atomistic. This means it understands how its own decisions influence the choices and expectations of households, banks, firms and foreign lenders. While no direct default costs are assumed, the government understands that defaulting erodes the banks’ equity, reduces their capacity to intermediate and hence leads to lower capital and therefore less production and income for the household. This is how default becomes costly to the government — despite the absence of any direct punishment — and hence why it may choose to repay ex post. Furthermore, the government understands that its decision how much new debt to issue influences the private sector’s actions, in particular the households’ savings choice, and expectations, in particular on the probability of default and hence the current bond price. Furthermore, the government understands its commitment problem and correctly anticipates how the behavior of the private sector and the government in the future depend on the future state vector.

We can summarize the government’s problem as follows

$$V_G(\Omega) = \max_{\text{Rep}, T, B_T', X} V_H(\Omega)$$

$$\begin{align} q \cdot B_T' + T & \geq B_T \cdot \text{Rep} + G \\
B_X' & \geq 0 \end{align}$$

$$0 = F(X, T, B_T', \text{Rep} | \Omega)$$

Note that the condition (22) constrains the government’s choices to be consistent with a private equilibrium. Also note that in (19) we do not need to distinguish between the value of being in default or not, as is common in the default literature. This is because we make no particular assumptions about post default periods, such as exogenous costs of default.

**Definition 2: Full equilibrium** A stationary equilibrium in this economy is defined by a set of value functions $V_H(\Omega)$, $V_G(\Omega)$, government policy functions $T(\Omega)$, $B_T'(\Omega)$, $\text{Rep}(\Omega)$, private policy functions $C'(\Omega)$, $L_H'(\Omega)$, $D_H'(\Omega)$, $K_H'(\Omega)$, $D_B'(\Omega)$, $B_B'(\Omega)$, $K_B'(\Omega)$, $K_F'(\Omega)$, $L_F'(\Omega)$, $B_X'(\Omega)$, price functions $r(\Omega)$, $q(\Omega)$, $R'(\Omega)$, $W(\Omega)$ and shadow price functions $\lambda_1(\Omega)$, $\lambda_2(\Omega)$. 
λ_3(Ω), λ_H(Ω) such that the governments problem (19) is solved subject to its budget constraint (20), the non-negativity constraint (21) and the equilibrium conditions of the private economy (22). Appendix A presents the government’s problem in full detail.

Intuitively, one can think about the equilibrium as an infinitely repeated game. Each period the government moves first choosing the new debt and tax levels and whether to default, anticipating correctly the reactions of the private agents, which in turn have correct expectations about the future. Yet another way to think about the full equilibrium is a private equilibrium that is associated with a set of time-consistent (i.e. commitment free) optimal government policy functions. Notice that this equilibrium notion coincides both with that of time consistent policy in Klein et al. (2008) and the equilibrium applied in the sovereign default literature in the tradition of Eaton and Gersovitz (1981).\textsuperscript{22}

Before moving on to the next section, a few words are in order on the assumption that the government cannot default selectively and cannot bail out domestic banks after default. These assumptions are key to generate the endogenous costs of default that sustain an equilibrium with external debt. If the government could choose to default only on foreign lenders, or equivalently to inject equity into domestic banks after defaulting, external default would be costless and hence always ex-post optimal. Since foreigners would anticipate that, no foreign lending could be sustained.

Theoretically, both assumptions can be justified by secondary markets, as Broner et al. (2010) show. In anonymous markets, if the government planned to default selectively (or to bail some lenders out) the defaulted upon would simply sell to the exempted (bailed out). The non-selectivity of default is plausible also empirically. While there have been a few cases of selective default, they were typically not with respect to the holder, but to the currency (Moody’s (2008)) or the legislation (Reinhart and Rogoff (2011a)) of the bond. It seems rather hard to target the holders of the bond. The empirical case for no bailouts seems a bit harder to make, given that we do see that government defaults are often accompanied by bank bailouts. I nevertheless follow the literature (e.g. Gertler and Kiyotaki (2010), Sosa-Padilla (2012), Engler and Grosse Steffen (2016)) and rule them out. This must be understood as an approximation for the fact that these rescue packages (1) typically do not fully compensate for the full default losses, (2) are politically costly,\textsuperscript{23} (3) need to be financed by distortionary taxation and (4) are subject to complicated legal constraints and does justice to the fact that financial crises, despite all policy efforts, are very damaging for the real economy.

\textsuperscript{22}Nicolini et al. (2015) compares this equilibrium notion to alternative ones, where the government lacks the first mover advantage, which can give rise to self-fulfilling equilibria.

\textsuperscript{23}Political costs can arise because the population perceives that banks were “accomplices” in bringing upon the dire situation the country finds itself in or because they are provided by an external agent like the IMF who enforces some conditionality.
4 Dissecting the mechanism analytically: Financial frictions, the motive to borrow and endogenous exclusion

This section analyzes the two novel features of the model analytically: The borrowing motive and self-exclusion. I start by showing how the financial frictions affect investment in capital. Then I present a simplified version of the model. This model is then used in the subsequent two subsections to explain how financial frictions gives rise to a motive to borrow and why this motive may vanish, leading to post default exclusion from international financial markets.

4.1 Underinvestment due to the financial friction

In the model economy the household has two ways to invest into physical capital: either by directly lending to the firm, or indirectly by depositing savings at the bank, which in turn lends to the firm. Bank intermediated investment is assumed to be more efficient, but the capacity of banks to intermediate is constrained by a leverage constraint.

**Intermediated loans.** As is well known from the literature on financial frictions, the leverage constraint — when it binds — drives a wedge between the return on bank investment and the cost of deposits \( r \), even though banks behave competitively. To see this combine equations (6)-(8) and abstract for convenience from the covariance terms to get

\[
  r = \left[ \psi E[\Omega | Rep] \right] q + (1 - \psi) R' + \frac{\beta E[\Omega V_{be}(e', \Omega')] + \lambda_1 \theta E[\Omega [u_e(c')]}{\beta E[\Omega V_{be}(e', \Omega')] + \lambda_1 E[\Omega [u_c(c')]} \right]
\]

This tells us that in equilibrium the deposit rate \( r \) equals the expected return on the bank’s investment portfolio if the leverage constraint is slack, i.e. if the associated multiplier \( \lambda_1 = 0 \). On the contrary, if the constraint binds and hence \( \lambda_1 > 0 \), then the deposit rate has to be lower than the bank’s return (since \( \theta < 1 \)). I shall refer to this difference as the bank’s profit margin.

If banks hold only loans, i.e. for \( \psi \to 0 \), this margin converges to the deposit-loan spread.

Notice that we can expect the bank’s profit margin and the deposit loan spread to weakly decrease in equity. When the constraint binds, an increase in equity \( E \) ceteris paribus leads to higher demand for assets and deposits by the bank. Since ceteris paribus the households’ supply of deposits increases in the deposit rate, and the firms’ demand for loans decreases in the loan rate, in general equilibrium we can expect the deposit rate \( r \) (loan rate \( R' \)) to increase (decrease) in \( E \). Hence the profit margin and deposit-loan spread decrease in \( E \). When \( E \) becomes so large that the leverage constraint turns slack, the profit margin reaches zero.

We shall assume that the leverage constraint indeed binds most of the time in the ergodic distribution, which is assured by assuming a minimum dividend payout ratio \( 1 - \eta \) sufficiently larger than \( 1 - \beta \). The profit margin implies that the return on intermediated investment, which households privately perceive, is lower than its social return. Households therefore under-invest in deposits and hence in intermediated loans. At the heart of this underinvestment in banks
is a positive externality: When making their deposit choice, households only take the deposit
rate into account, but not the profit margin of the bank. In doing so the household ignores
that these profits on the one hand relax the leverage constraint in the future and on the other
hand flow back to the household as lump sum dividend payments.

**Direct loans.** The households’ direct investment technology is not affected by the leverage
constraint, it is however assumed to be less efficient. This technology will hence only be used,
if the deposit-loan spread on intermediated investment is large enough. To see this combine
the household’s FOCs (2) and (3) to get

\[ \xi r - (R' - r) = \lambda_H / \left( \beta E_{\Omega} \left[ u_c(c') \right] \right) \]

If the deposit-loan spread is small enough, i.e. if \( R' - r < r\xi \), then the multiplier on the
non-negativity constraint on direct loans \( \lambda_H \) is positive and hence the household undertakes no
direct investment \( K'_H = 0 \). If the spread reaches its maximum \( R' - r = r\xi \), then \( \lambda_H = 0 \) and
the household may extend direct loans \( K'_H \geq 0 \).

Depending on how scarce capital and hence how high the deposit-loan spread is, the non-
negativity constraint on direct investment \( K'_H \) may bind or not. I assume that parameters are
such that in the ergodic distribution the constraint binds most of the time, i.e. banks usually
will intermediate all investment. I will refer to such times as “normal times”.

**Marginal investors.** Notice that as long as \( \lambda_H > 0 \) and \( K'_H = 0 \) the bank is the marginal
investor in loans, even if the leverage constraint binds: A larger supply of savings by the
household increases the bank’s profit margin, the deposit-loan spread and \( \lambda_1 \). The widening
of the profit margin relaxes the leverage constraint, accommodating additional intermediated
investment. However, once the spread reaches its maximum such that \( R' - r = r\xi \), the household
may become the predominant marginal investor. To see this, combine the leverage constraint
(9), the dividend payout rule (10), the bond holding requirement (11) (assume they all bind),
and the foreign bond demand (14) and abstract for convenience from covariance terms to get

\[ K'_B = \eta E \left[ \frac{\psi}{1 - \psi} \left( 1 - \theta \frac{1}{r\bar{q}} \right) + \left( 1 - \theta \frac{R'}{r} \right) \right]^{-1} \]

Now if \( R' - r = r\xi \) and banks hold mainly loans \( \psi \to 0 \), then this expression simplifies to
\( K'_B = \eta E / (1 - \theta(1 + \xi)) \). Hence the amount of loans the bank can intermediate \( K'_B \) is fixed
and any additional savings must flow into direct investment \( K'_H \). The household now is the
marginal investor in loans. The situation is a little more complicated if \( R' - r = r\xi \) and
\( \psi > 0 \). In this case an increase in the supply of savings drives down the deposit and loan
rates proportionally: \( R' = r(1 + \xi) \). This potentially increases the bank’s profit on its bond
holdings and thus loosens the above constraint on bank loans \( K'_B \) through the \( r \) inside the left
parenthesis. Banks and households are both marginal investors. However, for small \( \psi \) such
as in my calibration this effect – and hence the banks contribution to the marginal loan – is
negligible. The household therefore is the predominant marginal investor.
4.2 A simplified version of the model

In the context of an open economy model with a benevolent government, underinvestment in intermediated loans implies a motive for the government to borrow abroad. However, this motive may disappear when banks cease to be the (only) marginal investors and the households instead (predominantly) undertake marginal investment, hence giving rise to self-exclusion from international financial markets.

To explain this time varying borrowing motive analytically, let me simplify the model. In particular, for the rest of this section I shall abstract from uncertainty ($\omega = 1$), government spending ($G = 0$), labor ($\nu \to \infty$) and domestic holdings of bonds ($\psi = 0$), assume full depreciation ($\delta = 1$) and that the government can borrow internationally in non-contingent bonds under full commitment up to a limit $B_X^{\text{max}} > 0$.

Besides, I simplify the modeling of the banking sector. I assume that banks charge an exogenously given relative deposit-loan spread $s$ and they are subject to an exogenous constraint on the size of their asset holdings. This means that the banks’ FOCs can be replaced by the deposit pricing equation $s r = R'$ and an upper bound on bank loans $K_B' \leq K_B^{\text{max}}$. Furthermore assume that $s \in [1, 1 + \xi]$ and that $K_B^{\text{max}}$ is such that it binds only when $s$ takes its maximal value. The latter assumption determines the identity of the marginal investor, in the same way that it endogenously arises in the full model: Banks are the marginal investor in “normal times” when the spread is moderate, the household is the marginal investor when the spread is at its maximum. This simplified model of the banking sector can be understood as a partial equilibrium representation of the full model, where the government does not take into account the effect of its choices on the deposit-loan spread and the banks’ balance sheet capacity.

Furthermore, for convenience define the world interest rate $\bar{R} \equiv 1/\bar{q}$. This results in the following model:

$$ V_G(\Omega) = \max_{C,T,\lambda_H,r,r',B_X',K_H',K_B'} \frac{C_{-\gamma}^{1-\gamma}}{1-\gamma} + \beta E[V'_G] $$

s.t.

**Household FOCs**

- $C^{-\gamma} = \beta r E[C'^{-\gamma}]$
- $C^{-\gamma}(1 + \xi) = \beta R'E[C'^{-\gamma}] + \lambda_H$
- $0 = \min\{\lambda_H, K_H\}$
- $(K_B + K_H)^{\alpha} = C + K_B' + K_H' + T$

**Firm & Bank FOCs**

- $R' = \alpha(K_B + K_H)^{\alpha-1}$
- $s r = R$
- $K_B' \leq K_B^{\text{max}}$

**Govt. Budget Cons.**

- $T = B_X - B_X'/\bar{R}$
- $0 \leq B_X' \leq B_X^{\text{max}}$

This stripped down model is simple enough to be analytically tractable, yet it incorporates the same financial frictions that make international borrowing desirable and at times can lead to market exclusion as the full model: The household’s savings choice (and hence its Euler equation) is distorted by a spread, its investment technology is inferior to that of banks, and firms cannot borrow from abroad. Let us solve the model, distinguishing between the two cases where either the bank or the household is the marginal investor.
Case 1: Banks as marginal investors. First assume that the spread is below its maximal value and that the bank is the marginal investor \((K_H = 0, \lambda_H > 0, K_B \leq K_B^{\text{max}} \text{ not binding})\). After substituting out \(C, T, r, R', B'_X\) and \(K'_H\) and ignoring the inequality constraints, we can derive a tractable FOC for \(K'\). After substituting \(R'\) and \(C\) back in it reads:

\[
0 = \frac{\partial V_G}{\partial K_B} = \left(\frac{1}{2} \frac{1}{\beta s} \frac{1}{R_B} K_B^{\alpha} + \bar{R} K_B^{\alpha} \right)^{\gamma - 2} \left( C \left( \frac{\alpha s^{-1} K_B^{\alpha} - 1}{s} + \bar{R} \right) \right)^{-\gamma} \frac{1}{\gamma K_B} \times \frac{(1 - \alpha) C}{K_B} \left( \frac{\beta R'/s}{R'} \right)^{\frac{1}{2}} + \bar{R} \left( \frac{R' - s \bar{R}}{R'} \right)^{\gamma} \left( \frac{s \bar{R}}{S'} + \frac{1}{s} \beta^{\frac{1}{2}} + \frac{\bar{R}}{(\beta R'/s)^{\frac{1}{2}}} + 1 \right) \left( \frac{R' - \bar{R}}{R} \right)
\]

This condition implicitly defines an optimal marginal product of capital (MPK) \(\hat{R}'\), which the government would like to achieve – if the inequality constraints allow. Inspecting the second line of this condition provides two insights. First, the optimal MPK \(\hat{R}'\) is time-varying and slightly higher than the world interest rate \(\bar{R}\): \(\bar{R} < \hat{R}' < \bar{R}s\). Second, if the marginal return on capital \(R'\) exceeds this threshold \((R' > \hat{R}'\)), then the derivative \(\frac{\partial V_G}{\partial K_B}\) is positive. That is, the government wants to borrow internationally as long as the marginal return on capital \(R'\) exceeds a certain threshold \(\hat{R}'\).

Note that the higher the preference for consumption smoothing \(\gamma\), the higher the relative weight of the expression in the last bracket. I.e., as \(\gamma \to \infty\) the FOC implies that the threshold value equals the world interest rate \(\hat{R}' = \bar{R}\). Conversely, as \(\gamma \to 0\) the FOC becomes \(\hat{R}' = s\bar{R}\.\) For convenience, let us make the approximation that \(\hat{R}' \approx \bar{R} – \) either because \(\gamma\) is large or \(s\) close to 1. Under this approximation the government’s optimal policy is

\[
B'_X = \begin{cases} 
B'_X^{\text{max}} & \text{if } R'(B'_X^{\text{max}}) > \bar{R} \\
\hat{B}'_{X1} & \text{if } \hat{B}'_{X1} \in [0, B'_X^{\text{max}}] \\
0 & \text{if } R'(0) < \bar{R}
\end{cases}
\]

I.e. the government would like to borrow so much as to implement the allocation where the MPK equals the world interest rate. If this objective violates the bounds on \(B'_X\) it chooses the closest corner solution. If the government borrows, it does so because the household, anticipating future taxes, invests most of its additional disposable income due to lower current taxes in domestic capital, which has a marginal social return \(R'\) that exceeds the cost of international funds \(\bar{R}\).

\[\footnote{When agents do not value consumption smoothing, the deposit rate is constant and so is the capital stock. Borrowing by the government hence does not lead to additional savings. In this situation the desirability of borrowing depends on whether the deposit rate (i.e. \(1/\beta\)) is above or below the world interest rate.} \]
Case 2: Households as marginal investors. Let us next assume that the household is the marginal investor in the current period. I.e. assume that \( s = 1 + \xi \) and that the upper-bound on the banks balance sheet binds \( K'_B = K'^{\max}_B \) for all possible debt levels \( B'_X \in [0, B'^{\max}_X] \).\(^{25}\) The FOC of this version of the government’s problem, ignoring the inequality constraints on \( B'_X \) and \( K'_H \), is: \(^{26}\)

\[
\frac{\partial V_G}{\partial K'_H} = C^{1-\gamma} \left( \frac{R'}{(1 + \xi)} - \bar{R} \right) = 0
\]

Again, this condition defines an optimal MPK \( \hat{R}'_2 \equiv \bar{R}(1 + \xi) \). The inequality constraints permitting, the government hence would like to borrow so much from abroad \( B'_X \) that the total capital stock \( K'_H + K'^{\max}_B \) is such that the MPK is \( \hat{R}'_2 \). Analogously to case 1, taking the bounds on \( B'_X \) into account yields the following optimal foreign debt level

\[
B'_X = \begin{cases} 
B'^{\max}_X & \text{if } R'(B'^{\max}_X) > \bar{R}(1 + \xi) \\
\hat{B}'_{X2} & \text{if } \hat{B}'_{X2} \in [0, B'^{\max}_X] \\
0 & \text{if } R'(0) < \bar{R}(1 + \xi)
\end{cases}
\]

where \( \hat{B}'_{X2} \equiv \left( B'_X \in \mathbb{R} \mid R'(B'_X) = \bar{R}(1 + \xi) \right) \)

Summary of cases 1 & 2. The optimal borrowing choice for both cases can conveniently be summarized as follows. Define the marginal social return on investment \( R'_m \) as

\[
R'_m \equiv \begin{cases} 
R' & \text{if } K_H = 0 \\
R'/(1 + \xi) & \text{if } K_H > 0
\end{cases}
\]

Lemma 1: Under the approximation \( \hat{R}'_1 \approx \bar{R} \), it is optimal for the government to borrow

\[
B'_X = \begin{cases} 
B'^{\max}_X & \text{if } R'_m(B'^{\max}_X) > \bar{R} \\
\hat{B}'_X & \text{if } \hat{B}'_X \in [0, B'^{\max}_X] \\
0 & \text{if } R'_m(0) < \bar{R}
\end{cases}
\]

This implies the following optimality condition: The government borrows if the social marginal return \( R'_m \) exceeds the world interest rate \( \bar{R} \). The government does not borrow if the world interest rate exceeds the social marginal return.

4.3 A new motive to borrow internationally

The simple model delivered an intuitive and simple optimality condition. Recall, that in section 4.1 I assumed that in the full model the bank is the marginal investor in “normal times”. That

\(^{25}\)The assumption that the constraint on \( K'_B \) binds regardless of \( B'_X \) is wlg. It can be relaxed to allow that the constraint binds only for \( B'_X \) exceeding a certain threshold at the expense of complicating expressions.

\(^{26}\)Once \( K_H > 0 \) the household’s relevant Euler equation is undistorted and the private equilibrium conditions besides \( K'_B = K'^{\max}_B \) are not strictly binding.
is case 1 is the case that best resembles the full model for most of the ergodic distribution. Therefore let us consider the optimal debt level in the steady state in case 1 (assuming a SS exists).

Can we expect the condition $R'_m > \bar{R}$ to hold in steady state? In steady state the deposit rate $r$ is equal to the discount factor $1/\beta$. Since the marginal social return in case 1 is the loan rate ($R'_m = R'$), which is given by the deposit rate times the spread ($R' = rs$), the condition is satisfied if $\frac{r}{\beta} > \bar{R}$. That is, even if – as I shall assume – the household is as patient as the foreign investors pricing the bond ($\beta = \bar{q} \equiv 1/\bar{R}$), foreign borrowing is desirable in the long run as long as the deposit-loan spread $s$ is positive.

In sum, the existence of capital, together with the financial frictions that prevent the marginal social return on capital $R'$ to converge to the world interest rate $\bar{R}$ over time, constitute a non-transitory motive for government borrowing: When choosing whether to finance current government expenditures with taxes or international borrowing, the government prefers the latter, because that allows the household to invest in domestic capital through banks at a social return $R'$ that exceeds the world interest rate $\bar{R}$.

This return differential motive hence provides an explanation for why many governments systematically and heavily rely on international debt, which can not be explained by the consumption-smoothing motive alone. It arguably constitutes an improvement in realism over the previous literature on sovereign default, which for convenience usually abstracts from capital and instead hardwires the borrowing motive into the government’s utility function by assuming a low discount factor $\beta$ relative to the inverse world interest rate $\bar{q}$. In section 6 we will see that the return differential motive delivers quantitatively reasonable predictions. More importantly, as explained in the next subsection, allowing for an endogenous, time-varying borrowing motive can explain why we observe post-default market exclusion.

The motive for the government to borrow internationally relies on the model’s feature that the social return on investment is higher than the cost of international funds. In the model, this is the result of a financial friction, that raises the social return of investment above the private. This feature is in line with the literature on capital flows such as Gourinchas and Jeanne (2013) and Benhima (2013), who argue that country specific “capital wedges” between the private and social returns of investment can explain empirical cross-country differences in

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27 Namely, the domestic banks’ leverage constraint and partial integration into international capital markets.
28 If consumption smoothing were the only motive for borrowing, it would (on average) call for precautionary savings due to the endogenous borrowing limit resulting from the governments lack of commitment.
29 With the time period being one quarter, Arellano (2008) uses a $\beta$ of 0.953 and a net world interest rate $\bar{R} - 1$ of 1.7%, which implies a $\bar{q} = 0.983$. Aguiar and Gopinath (2006) use $\beta = 0.8$ and $\bar{R} - 1 = 1\%$. Mendoza and Yue (2012) use $\beta = 0.88$ and $\bar{R} - 1 = 1\%$. Guimaraes (2011), Roklán-Peña (2012), Joo (2014), Gornemann (2015), Gordon and Guerrón-Quintana (2017) and Park (2017) also consider a sovereign default models with capital. All but one of them rely on similar parameter values for $\beta$ and $\bar{R}$ in order to motivate borrowing in the long run. Guimaraes also considers an economy where the MPK is the only reason to borrow. However, he restricts his attention to analytical results with numerical illustrations. Moreover, in his model this motive to borrow vanishes as the domestic capital stock converges to its steady state level where $MPK = \bar{R}$. 

Electronic copy available at: https://ssrn.com/abstract=3216992
long run investment rates, while yielding similar private returns on investment across countries. The wedges estimated in this literature tend to be higher for poorer countries, which also tend to be riskier borrowers. This squares with the models predictions: The bigger the incentive to borrow, the more default risk countries are willing to take. Micro evidence for financial frictions that can explain these wedges has also been documented in the development literature. The literature review in Banerjee and Duflo (2005) shows how especially in less developed countries access to finance for private agents is often difficult and very expensive, much more than what can reasonably be explained by default and intermediation costs.

Finally, consider the case of Greece, which I will calibrate the model to. Domestic bank loans are the primary source of finance for Greek firms and loan rates paid by Greek firms certainly exceeded those paid by the Greek sovereign substantially, the latter — prior to the crisis — being virtually equal to the return on the German bonds, and close to deposit rates. Further evidence for access to finance being particularly difficult for Greek firms can be found in the ECB’s survey on the access to finance of enterprises. The fraction of small and medium enterprises that name “access to finance” as their principal concern is roughly twice as large in Greece as it is in the Euro zone as a whole since the start of the biannual survey in 2009; and this difference has somewhat increased since the Greek default.

4.4 Endogenous exclusion from international capital markets

Having explained why the government usually borrows in this framework, let us next use the simplified model to understand why after default the government may temporarily choose to self-exclude itself from international capital markets. As mentioned above, the assumption that the household does not invest directly \( K'_{H} = 0 \) — case 1 — is valid in the full model for the calibration chosen below for most of the ergodic distribution. However, in the full model large losses of bank equity due to sovereign default may increase the endogenous spread so much that direct investment becomes attractive to the household and he becomes the predominant marginal investor. In that case the full model resembles case 2.

In case 2, when the household is the marginal investor, the marginal social return is lower due to the household’s competitive disadvantage at allocating capital \( R'_{m} = R'/(1 + \xi) \). If the household’s disadvantage \( \xi \) is large enough, the marginal return of investment may be lower than the cost of international funds \( R'_{m} = R'/(1 + \xi) < \bar{R} \), which makes international borrowing undesirable by lemma 1 — even if \( R' > \bar{R} \). In this situation the government hence chooses not to issue any new debt. It self-excludes itself from international capital markets.

To sum up, the marginal return on domestic investment drops discretely once the households technology is used for the marginal unit of investment. If it falls below the cost of international funds, borrowing from abroad becomes undesirable. As we shall see in the next section, this situation typically arises as a consequence of sovereign default in the full model for the chosen calibration.
Through financial frictions, the model is hence able not only to explain why governments borrow, and why default is costly (see next section), but it also provides an endogenous explanation for exclusion from international capital markets. While other authors have provided models that endogenize the output costs of default, they still rely on the assumption of exclusion. In most cases such as Mendoza and Yue (2012), exclusion is actually necessary to generate the output costs. This is the first model that can also account for exclusion endogenously.

Furthermore, this explanation for why we observe exclusion is based on a novel mechanism. The argument behind the assumption of exclusion typically refers to some coordinated punishment of the borrower by the lenders. Yet this argument has one weakness: as Kletzer (1994) argues, it would be optimal for individual (small) lenders to deviate from this collective punishment strategy.30 The explanation for exclusion proposed here is immune to such criticism, because it is optimal for both sides not to trade. There is simply no price at which the borrowing country and the lender would want to trade. At the same time both this and the conventional theory are observationally equivalent.

5 Dissecting the mechanism numerically: The effects and incentives of default

In this section I illustrate numerically how the borrowing motive and exclusion play out in the full model in conjunction with the endogenous default choice of the government and the associated default costs. For this purpose I first discuss the calibration and computation. Next I relate the simple to the full model. Then I discuss the consequences and incentives to default in the full model.

5.1 Calibration and Computation

**Calibration.** I calibrate the model to Greek data. As we have seen, the model describes a small open economy whose government has access to international capital markets, with an important domestic financial sector, no foreign investment into the private sector and without the option to inflate away its debt. Greece satisfies this description to a large extent. First, Greece is a small open economy with respect to both the EU and the world. Second, the Greek government has borrowed extensively: the mean debt to GDP ratio for the post-Euro pre-default period 2000-2010 is 111%.31 This debt was held both domestically (48%) as well as abroad according to Bank of Greece data. Third, credit to the local economy is largely supplied by the domestic banking sector and non-intermediated investment plays a minor role (see e.g. Demirguc-Kunt and Levine (1999)). Similarly, domestic financial institutions account

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30 In the Greek crisis for example, it did not take long for rumors about Greece talking with Russia about potential loans.
31 Greece fixed its exchange rate to the Euro in 2000 and introduced currency in 2001.
for most of the domestically held sovereign debt (80% according to Bank of Greece data). Fourth, private investment of foreigners in Greece play a minor role. As figure 7 shows in appendix E, the net foreign asset position of Greece is dominated by government debt and what little net private investment there is largely flows out of Greece rather than in. Fifth, since the accession to the Euro zone, Greece can not inflate its debt away. The main difference between the model and the Greek scenario probably is the policy response: Both Greek banks as well as the government were to some extent bailed out. While the model abstracts from bailouts altogether, the bailouts that were implemented certainly did not go anywhere as far as to compensate entirely for the consequences of default.32

The data on Greece used for the calibration and the model evaluation comes from several sources discussed in appendix E, table 6. For GDP and its components and TFP I use annual data covering the pre-default period 1980 to 2010, while for the financial variables the observation period is restricted to 2000 to 2010. This is both due to data availability and to account for the structural change of accessing the Euro, which should have a stronger impact on these variables. GDP, TFP and credit are expressed as deviation from a linear trend.33

Table 2: Parameter values

| Parameter                  | Value | Target/Explanation                           |
|----------------------------|-------|---------------------------------------------|
| α  capital share           | 0.36  | standard value                              |
| δ  depreciation            | 0.14  | 21% investment/GDP                          |
| β  discount factor         | 0.96  | same as $\bar{q}$                           |
| γ  risk aversion           | 2     | standard value                              |
| χ  labor weight in utility | 1.55  | mean of labor = 1                           |
| ν  inverse Frisch elasticity| 0.5   | standard value                              |
| G  government expenditures | 0.28  | 21% govt. consumption/GDP                   |
| θ  financial constraint   | 0.7   | 28% equity ratio                            |
| η  share of retained equity| 0.803 | 6% deposit-loan spread                      |
| ψ  share of bonds on balance sheet | 0.14 | 14% exposure                               |
| $\bar{\xi}$ average cost of direct investment | 0.071 | 1.5 ppt increase of deposit-loan           |
| $\xi$ dependence of cost of direct investment on GDP | -0.45 | spread in case of default, costs vary across the cycle |
| $\bar{q}$ inverse world interest rate | 0.96 | real rate Germany                          |
| $\rho$ persistence of TFP shock | 0.76 | estimate                                   |
| σ  standard deviation of TFP shock | 0.057 | 7.5% standard deviation of GDP              |

32 At the same time the model also fits other default episodes, such as the Argentinian 2001 crisis where concerns regarding the damage to the financial sector also played a dominant role in the policy debate before and after default. See Sosa-Padilla (2012), Perry and Serven (2003) and Kumhof (2004).

33 Note that in doing so I follow Arellano (2008), who also uses a linear long-run trend, whereas many other papers using emerging economy data use more flexible trends to focus on the business cycle frequency fluctuations (e.g. Mendoza and Yue (2012), Aguiar and Gopinath (2006)). While these differences are relatively unimportant for the much studied 2001 default of Argentina, they are significant for Greece, which experienced a much more prolonged and deep downturn, which a medium-frequency filter would attribute to the trend. Furthermore, using data only till 2010 to construct the trend, prevents the estimate of the trend to be affected by the crisis.
The calibration is summarized in table 2. As in the data the time period is one year. The utility function features standard parameter values: The discount factor $\beta$ is set to 0.96 while the risk aversion coefficient $\gamma$ is chosen to be 2. The labor weight $\chi$ is set to normalize the mean of labor to 1, while $\nu$ is chosen to imply a Frisch elasticity of labor of 2, a common value and within the range of estimates reported by Greenwood et al. (1988). The capital share $\alpha$ is set to 36%. Depreciation $\delta$ is set such as to match the 21% average investment to GDP ratio for Greece. $G$ is chosen to match the average government consumption to GDP ratio of 21%.

The inverse world interest rate $\bar{q}$ is set to 0.96, in line with the average real interest rate for Germany for the period 1980 to 2010. While this choice might not seem very surprising, setting $\beta = \bar{q}$ is highly unusual in the context of quantitative models of sovereign default, which usually assume $\beta \gg \bar{q}$ in order to generate a motive for the government to borrow.

The choice of the financial sector parameters is the least straightforward. I choose these values in order to match moments of financial variables for Greece. The dividend payout parameter $\eta$ is chosen to match an average spread between the loan and the deposit rate of 6%. This is in line with the average spread between 1 year deposits and firm loans for Greece. The share of domestic bonds on banks balance sheets $\psi$ is set to 14%, which is the mean of the Greek credit institutions’ exposure to sovereign bonds in the Bank of Greece data. $\theta$, which determines the tightness of the banks’ leverage constraint, is set such as to yield a ratio between the exposure to the government and equity of 54%, which is the reduction that Greece banks book equity suffered during the period that the haircut was imposed, from 2011 to April 2012. This value implies an equity ratio of 26% percent, which is roughly in line with Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) and guarantees that sovereign default does not drive the banks into default. Hence the higher-than-realistic equity ratio compensates for the simplifying assumption that default is complete. Finally, the parameters that govern the relative cost advantage of intermediated over direct investment $\xi$ and $\hat{\xi}$ are simultaneously chosen such that $\xi$ so as to match the 1.5ppt increase in the deposit-loan spread after default observed in the Greek data and so as to guarantee that intermediate investment is marginally more competitive than direct investment even for the highest values of TFP considered (given the average deposit-loan spread). This implies that $\xi$ varies only mildly.\(^{34}\)

Finally, the persistence parameter of the TFP process $\rho$ is estimated using detrended TFP data, while the standard deviation $\sigma$ is set such as to such as to match the standard deviation of detrended GDP in the data.\(^{35}\)

**Computation.** Given the high complexity of the model, only an approximate numerical solution of the government’s problem (19) is feasible. I briefly summarize the computational

\(^{34}\)\(\xi\) moves from 6% to 8% as log TFP moves from 3.5 standard deviations above to 3.5 standard deviations below the mean. I analyze the sensitivity to these two parameters below.

\(^{35}\)As commonly found in the RBC literature (e.g. Cooley and Prescott (1995)), the estimated volatility of TFP is too small for the model to explain observed volatilities. Hence, to capture the observed volatility of output in a simple model with just one shock I scale up the estimate of $\sigma$. 


The general idea is to replace the full vector of endogenous pre-decision state variables (agents portfolios) by a smaller vector of post-decision state variables (agents cash-at-hand). To get to the post-decision state variable, one has to anticipate certain future decisions that depend on the pre-decision state space. Here, the decision to be anticipated is tomorrow’s default decision, which hence becomes a choice variable today. This is why it is convenient that evaluating expectations exactly yields a function that is continuous in the exogenous state. This method, which is the second methodological innovation, is explained in the appendix and in its general applicability in Thaler (2016).

Solving the model poses some significant computational difficulties due to, first, the high dimensionality of the state space and, second, the complexity of the government maximization problem (19), which is a mathematical program with equilibrium constraints (MPEC). To address the first problem, I use a novel variant of post-decision state variables that reduces the computationally necessary state space. Furthermore, the use of smooth interpolation and analytical integration allows the use of a low number of grid points. To solve the MPEC, I rely on a modern, derivative-based maximization algorithm (KNITRO).

5.2 International borrowing in the full model

The simplified model in the previous section was useful to deliver the intuition behind why and when governments borrow. But it left out some important elements of the full model, which may alter the optimal borrowing choice. Let us sequentially add some of the missing elements to the simple model to see how they affect the results.

First add uncertainty: Stochastic TFP now moves the domestic rates, in particular the marginal social return on capital $R'_m$. For the model to be able to rationalize substantial debt in the long run, the condition $R'_m > \bar{R}$ must therefore hold in the ergodic distribution, not in steady state. For small enough shocks the ergodic distribution will be concentrated around the steady state. Therefore, if $1 + \xi \gg s \gg 0$ then the condition $R'_m > \bar{R}$ will hold for most of the ergodic distribution.

Second, consider additionally limited commitment and endogenous default costs. Now the government faces a soft borrowing constraint instead of a hard. However the logic of the simple model summarized in lemma 1 still applies locally: As long as foreign debt is so low that the approach here. In doing so, I highlight two computational innovations that were integrated in the algorithm. The appendix provides more details.

Solving a DSGE model with continuous choice variables numerically typically requires the uses of 3 tools: function approximation, numerical integration and numerical maximization. I approximate the policy and value functions $Z(\Omega)$ by twice continuously differentiable functions $\tilde{Z}(\Omega)$ over the whole state space (including exogenous states). Given these approximations, I *exactly* evaluate the integrals contained in the expectations of future values of these functions, such as $E_{\Omega} [Z'] \approx \int_0^{\infty} pdf(\omega'|\Omega) \tilde{Z}(\omega'|\Omega) d\omega$. This approach is novel and different from the quadrature approaches, that are usually used to evaluate expectations. Finally, to find the stationary solution to the optimization problem of the government, I solve problem (19) recursively using a time iteration algorithm, that jointly iterates on the policy and value functions.
default probability is 0, the government will want to borrow more if \( R'_m > \bar{R} \). As soon as a default premium arises, the government will have to trade off the benefits of borrowing not just against the cost of funds, but also against the expected cost of default. It will optimally choose a debt level such that the default probability is positive but not too high.

Finally, the simplified model misses some other features that affect the optimal choice of the government to borrow. Therefore the optimality condition from the lemma 1, “borrow if \( R'_m > \bar{R} \)”, which for low \( s \) was an approximation even in the simplified model, holds only approximately in the full model. Nevertheless, as figure 2 shows this approximation does a good job for my calibration. It hence is useful to deliver the intuition for the borrowing motive.

The two conditional histograms in figure 2 show how \( R'_m \) compares to \( \bar{R} \) in the ergodic distribution of the calibrated model. The left histogram shows the density function of \( R'_m \) conditional on the government not borrowing from abroad, while the right histogram is conditional on the government borrowing (\( B'_X > 0 \)). Note that borrowing is optimal most of the time, the unconditional probability is \( p(B'_X > 0) = 96\% \). Furthermore, as the right panel shows, the government borrows almost only when \( R'_m > \bar{R} \) holds, in line with lemma 1. Conversely, as can be seen in the left panel, the government typically does not borrow when \( R'_m < \bar{R} \). However there are a few periods, where the government does not borrow even though the marginal return on investment is above the world interest rate.

This can be due to two reasons. First, recall that in the previous section for \( K_H = 0 \) we approximated \( \hat{R}'_1 \approx \bar{R} \) even though \( \hat{R}'_1 > \bar{R} \). Second, in the simple model we ignored that the spread \( s \) is endogenous. Taking this into account gives rise to an additional distributional motive. Even if international borrowing is “profitable” on a aggregate level, it redistributes wealth away from the banks by reducing the loan rate \( R' \). This makes the borrowing constraint bind tighter in the future, which may be enough to make foreign borrowing undesirable even if \( R'_m - \bar{R} \) is positive but small.
5.3 The costs of default

Having understood the intuition behind the motive to borrow internationally and how it potentially can turn around and become a motive not to borrow, we are now ready to analyze the consequences of default in the full model. As I stressed before, the model assumes no direct costs of default, neither the typical output costs, nor exclusion. But this does not mean that default is costless. On the contrary, the model predicts seizable and long lasting costs of default.

Figure 3 illustrates the consequences of default. Before $t = 0$ the economy rests at its risk adjusted steady state, in which the government has substantial foreign liabilities. Default occurs unexpectedly at $t = 0$. The graph shows how the economy evolves back to the steady state in the absence of further shocks. Note that this default event does not reflect optimal behavior by the government, but merely serves to illustrate the pure costs of default, absent changes of TFP. Typical (i.e. optimal) default episodes are analyzed in the section 6.

Since banks are exposed to sovereign debt, upon default in $t = 0$ they lose a significant fraction of their equity (panel a). This implies that the leverage constraint now binds more tightly (see panel b for the corresponding multiplier $\lambda_1$). As a consequence, banks have to reduce the size of their balance sheets, and hence the amount of loans they extend (pan. c) and the amount of deposits they raise. At the same time the tightening of the leverage constraint is reflected in the deposit-loan spread, which jumps up to the maximum given by $\xi r$ (pan. d). At this high level of the spread, direct investment, which was unprofitable before $t = 0$ due to the lower capital allocation efficiency of the household, becomes profitable for the household. Due to these costs however, the shift to direct financing can only partially compensate the reduction in bank financing and total loans, i.e. total capital drops (pan. c). At the same time, the household decreases his consumption and, after a small initial increase, his labor supply (pan. e and f).\(^{37}\)

Since capital drops and labor does not fully compensate, output falls (pan. h). Notice that the drop in output is persistent and v-shaped. The output costs of default increase for the first years. This is so because underinvestment decreases the capital stock gradually for a number of periods (pan. c). Output follows a similar pattern. This feature can help to explain the sluggishness of the recovery of GDP after default observed for many countries.\(^{38}\)

In the periods following default, banks slowly recapitalize by accumulating the profits generated by the higher spread on their lending activity (pan. b and d). As they do so, they expand their balance sheet once again, and more and more of the direct investment is replaced by intermediated investment (pan. c). The economy slowly converges back.

\(^{37}\)Note that labor moves little relative to capital. The path of labor largely reflects the path of capital (and hence the wage) which drops until year 9 and then recovers quickly until year 11 because banks are marginal investors again.

\(^{38}\)In models with exogenous output costs of default such as Arellano (2008) the costs usually are a increasing function of output. Since default is typically triggered by a negative shock, the output costs of default also increase over time after a typical default event. Note however that in my model the costs increase even independently of TFP.
Figure 3: **Post default dynamics**: Equity, total loans, labor, output and consumption are normalized by their respective steady state values. Direct and bank loans are fractions of total loans. The black line in panel 9 marks the world interest rate $\bar{R}$. The non-smoothness of the responses is the result of the *occasionally binding constraints* switching on and off in this *non-stochastic* simulation: The first jump at $t = 0$ is associated to the default event, the second at $t = 8$ to $B_X \geq 0$ stopping and $K_H \geq 0$ starting to bind, the third at $t = 10$ to the endogenous borrowing limit starting to bind (i.e. the default probability becoming positive).

Panel h) shows the government’s optimal foreign debt choice. After default has wiped out all foreign debt, the government does not borrow any new funds abroad. This is so, because the banks’ losses drive up the deposit-loan spread (pan. d) such that the household becomes the marginal investor and the marginal return of investment $R'_m$ hence drops (pan. i). Since $R'_m$ drops below the world interest rate, by lemma 1 borrowing is no longer desirable. $R'_m$ remains depressed for several years, until it jumps back up once banks have re-accumulated enough equity to become marginal investors again ($K_H = 0$). That is when the government reaccesses international capital markets, in line with lemma 1.39

The model hence predicts that the government “self-excludes” itself from the market for a period of several years. The duration of this period is endogenous and depends on the time it takes for the marginal return on investment to recover, which in turn depends on the how long it takes for banks to recover a sufficient level of equity.

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39 As figure 2 showed exclusion may last a little longer or shorter, since lemma 1 holds only approximately.
5.4 Default incentives

In equilibrium all agents, including the government, correctly anticipate the consequences of default just discussed: They understand that defaulting on foreigners per se is a free lunch. But they also understand that defaulting on local banks causes a financial crisis. Therefore the government ex post optimally repays all outstanding debt whenever the costs of default outweigh the benefits. This makes external debt sustainable ex ante. The financial vulnerability of the economy essentially serves as a commitment device for a government that cannot commit otherwise.\(^{40}\)

To understand what the default decision depends on, it is useful to define the default set as those states where the government defaults in equilibrium: \(\mathcal{D} = \{\Omega \in \Omega : \text{Rep}(\Omega) = 0\}\). This default set has a number of features that resonate similar findings in Eaton and Gersovitz (1981) and Arellano (2008). We start with two analytical results proven in appendix C:

**Proposition 2** Assume that \(\Omega_1 = [\omega, K, D, B_B, K_B, R, B_X] \in \mathcal{D}\). Then the following holds:

\[
\Omega_2 = [\omega, K_H, D, B_B, K_B, R, \hat{B}_X] \in \mathcal{D} \text{ if } \hat{B}_X \geq B_X.
\]

Proposition 2 says that if default is optimal for a given level of foreign debt \(B_X\), it must be optimal for higher levels of foreign debt, ceteris paribus. I.e. the default set is bounded below with respect to \(B_X\). This is so because the benefit of defaulting increases in foreign debt, but the costs remain unchanged.

**Proposition 3** Assume that \(\Omega_1 = [\omega, K, D, B_B, K_B, R, B_X] \in \mathcal{D}\). Assume additionally that the government can transfer \(Z \geq 0\) of the bank’s equity to the household in a lump sum fashion, but not the other way around. Then the following holds:

\[
\Omega_3 = [\omega, K, \hat{D}, \hat{B}_B, K_B, R, B_X] \in \mathcal{D} \text{ if } \hat{B}_B \leq B_B \text{ and } B_B - D = \hat{B}_B - \hat{D}.
\]

Proposition 3 states that lower levels of sovereign exposure of the bank, keeping constant pre-dividend equity in case of repayment, make default more attractive. This is intuitive, given that the post default equity of the bank decreases in the bank’s deposit financed exposure.\(^{41}\)

Figure 4 illustrates the default set for the calibrated model. Panel a) illustrates proposition 2: Higher levels of foreign debt \(B_X\) increase the incentives to default.

Furthermore, panel a) illustrates an additional quantitative feature of the default set, which was found to be robust in numerical experiments: lower levels of TFP \(\omega\) make default more likely.\(^{42}\) This is so because TFP affects the aggregate resources of the economy and the expected

\(^{40}\)Notice that this entails another externality: Banks do not get remunerated for the value of the commitment that their investment in bonds entails.

\(^{41}\)The additional assumption necessary to proof proposition 3 does not change the equilibrium of the economy, if the value function of the government is decreasing in the share of total resources held by the bank, because then \(Z^* = 0\). I find that, for the calibrated model, in equilibrium the value function indeed satisfies this condition across the whole state space.

\(^{42}\)As the appendix shows, the numerical solution algorithm is based on the guess that part 1 holds. This guess is verified ex post at each grid point.
Figure 4: **Default Set:** The blue area is the default region, the yellow area is the repayment region. The white area is outside the ergodic distribution and hence off the grid. In each panel, the remaining states are set to their non-stochastic risk-adjusted steady state values, which are indicated by the black dot. The levels of foreign and domestic debt are normalized by the output at the non-stochastic risk adjusted steady state. The equity ratio on the x-axis in panel b) is function of all state variables: It refers to the banks’ pre-dividend equity ratio conditional on repayment: \((RK_B + BB - D)/(RK_B + BB)\).

return on capital positively. When TFP is high optimal savings are therefore higher than when TFP is low. Thus, bank equity, which is not affected by TFP and which is needed to intermediate savings, is less scarce at lower levels of TFP. Hence, the loss of bank equity due to sovereign default is less costly at lower TFP levels. At the same time, marginal utility and therefore the benefit of not repaying foreign debt \(B_X\) is higher when TFP is low.\(^{43}\) Figure 8 in the appendix illustrates how the costs and benefits depend on default in more detail. Notice, that together with the borrowing motive and the commitment value of default, this feature guarantees that default happens along the equilibrium path.

Panel b) illustrates another numerically robust feature: Default is more attractive when the banks are better capitalized, keeping constant the total capital stock \(K_B + K_H\), domestic bond-holdings \(BB\) and foreign sovereign bond-holdings \(B_X\).\(^{44}\) This is because, on the one hand, default is less distortionary and hence less costly when banks are better capitalized. To see this consider a situation where banks are so well capitalized that the leverage constraint is slack, even after default. The current deposit-loan spread would hence be zero and would not be affected by default. Neither would the current size of the banks balance sheet be affected. Compare this to a situation where banks are less well capitalized such that the leverage constraint binds even under repayment. In this situation default would increase the current deposit-loan spread

\(^{43}\)This feature is common: In Arellano (2008) countries also default in bad times, when resources are scarce and the costs of default are assumed to be low.

\(^{44}\)Given these restrictions, higher bank equity may result either from a higher loan rate \(R\), lower deposit liabilities \(D\) or a higher share of bank loans \(K_B\).
and/or force banks to reduce their balance sheets. On the other hand, the benefits of default increase as banks are better capitalized, since well capitalized banks allow a less distorted allocation of the resources not paid to the foreigners $B_X$.

Panel c) illustrates that the default incentives increase in the amount of foreign debt (again proposition 1) but decrease in the amount of domestic debt. This illustrates the disciplining role of domestic debt: the more exposed the domestic sector is, the more foreign debt will be repaid. Note that in panel c) deposits $D$ are kept constant. That means the domestic debt along the y-axis is equity financed. So the graph illustrates that for the calibrated model default incentives increase also in equity financed debt (proposition 3 referred to deposit financed debt).

6 Quantitative performance

This section assesses the model’s quantitative performance in terms of business cycle moments and the patterns observed around default episodes.

6.1 Cyclical moments

Table 3 reports the most important business cycle moments of the model and compares them to their empirical counterparts in Greek data.

The data used to generate the empirical moments has been discussed before, with the exception of the default frequency. This frequency is difficult to measure since default is a rare event. If one considers the longest possible observation period, that is since Greece’s independence in 1829, there have been 5 default events: The 4 reported by Reinhart and Rogoff (2008) in 1843, 1860, 1893, 1932 plus the recent crisis. This yields an annual default frequency of 2.7%. Yet, this estimate is based on the assumption that the underlying economy has not changed over time. This is hardly the case. According to my model for example, the development of a financial sector arguably has made default more costly. Therefore I consider this estimate an upper bound for the default frequency at best.

The model’s moments are obtained from 100,000 periods of simulated data. The full sample is used to calculate the default frequency. Mimicking the treatment of the real data, I then take pre-default subsamples of 31 periods, normalize them by their mean and calculate the respective moments for each pre-default episode. Table 3 reports the means of these moments. Notice that, with the exception of the output volatility, none of these moments were targeted.

Focus first on the default-related moments. The model predicts a total debt to GDP ratio of 44% and a foreign debt to GDP ratio of 18%. While these values are less than half of their empirical counterparts, the model makes reasonable predictions about the share of

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45The initial observations are discarded. The trade balance and the spread are not normalized. Episodes where another default happened in the 31 years prior to the default are excluded.

46Low debt levels are common in models with 1-period debt. As Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) show, more realistic ratios can be obtained with long term debt, an extension beyond the scope of this paper for computational reasons.
domestically held debt: It predicts 59% of domestic debt, while this number is 47% in the data. Since the model feature risk neutral lenders, the default frequency and the mean of the sovereign spread, by construction, almost coincide. In fact the model matches the spread very well, which implies that the predicted default frequency is much lower than the “historical upper bound estimate”. Furthermore, the model yields a negative correlation between GDP and the spread. This not only is in line with the Greek data, but is robust feature generally found in studies of sovereign debt (see e.g. Mendoza and Yue (2012)). At the same time the model underestimates the volatility of the spreads. The magnitude of the volatility of the debt to GDP ratio lies in the ballpark of what is measured in the data.

These results are interesting for three reasons. First, the debt to GDP ratios are higher much than the numbers found usually in the literature. For example, Arellano (2008) and Mendoza and Yue (2012) obtain 1.5% and 5.75% foreign debt to annual GDP. Second because the model allows to distinguish between foreign and domestically held debt. Third, because they result from the novel borrowing motive: In the two aforementioned papers the reason for the country to borrow so much as to risk default is the exogenous difference in its time preference relative to the rest of the world $1/\beta - 1/\bar{q}$ of 14% and 64% annually. Contrarily, the calibration I chose features $\beta = \bar{q}$. The country instead borrows because of an endogenous (time-varying) difference between the domestic return on capital and the world rate $R' - 1/\bar{q}$ of 6% (on average). The magnitude of this spread is much lower than the difference in time preferences commonly used and, unlike the latter, is disciplined by the measured deposit-loan spread.

Besides, the model preserves the typical features of the real business cycle model: As in the data, consumption is slightly less volatile and investment significantly more volatile than output. Moreover, the model matches the data well in predicting a very high correlation between consumption and GDP, and a somewhat lower correlation between investment and output. Furthermore the model predicts a trade balance that is about as volatile and uncorrelated to GDP as in the data.

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47 The following relationship holds $\text{spread} = \frac{P(\text{Rep}=0)}{1-P(\text{Rep}=0)}(1-\bar{q})$, which for $\bar{q}$ and $(1-P(\text{Rep}=0))$ close to 1 can be approximated as $\text{spread} \approx P(\text{Rep} = 0)$. Furthermore, the default frequency refers to the whole sample and the spread to pre-default periods.

48 Or 6% and 23% debt to quarterly GDP. However, they are also targeting higher default frequencies.
Table 3: Cyclical moments

| Moment       | Data  | Model |
|--------------|-------|-------|
| \(E(\bar{B}/Y)\) Total debt to GDP ratio | 121%  | 44%   |
| \(E(B_X/Y)\) Foreign debt to GDP ratio | 51%   | 18%   |
| \(E(B_X/\bar{B})\) Share of domestic debt | 47%   | 59%   |
| \(E(\text{spread})\) Sovereign spread | 0.48% | 0.50% |
| \(E(\text{Rep})\) Default frequency | < 2.7% | 0.44% |

| Volatilities  |                   |       |
|---------------|-------------------|-------|
| \(\sigma(Y)\) GDP | 7.5% | 7.4% |
| \(\sigma(\bar{B}/Y)\) Total debt to GDP ratio | 12.3% | 9.8% |
| \(\sigma(TB)\) Trade balance | 3.7% | 2.9% |
| \(\sigma(\text{spread})\) Sovereign spread | 0.55% | 0.26% |
| \(\sigma(C)/\sigma(Y)\) Relative consumption volatility | 71% | 83% |
| \(\sigma(I)/\sigma(Y)\) Relative investment volatility | 226% | 310% |

| Correlations of GDP with |       |
|--------------------------|-------|
| \(\text{corr}(Y,C)\) Consumption | 96% | 93% |
| \(\text{corr}(Y,I)\) Investment | 91% | 80% |
| \(\text{corr}(Y,\text{spread})\) Sovereign spread | -28% | -70% |
| \(\text{corr}(Y,TB)\) Trade balance | 12% | -1% |

6.2 Default episodes

Next I study the model’s dynamics around defaults using event study techniques. Similarly as before, I extract the period starting 5 years before default and ending 25 years after from the simulated data and normalize these values by their respective means. Figure 5 plots the median of the evolution of each variable. To highlight the stochastic properties of these paths, their 10th to 90th percentiles are also plotted. The figure furthermore shows the evolution of the respective variables in the data. The default period 0 corresponds to 2011 for the data.\(^{49}\) GDP values beyond 2016 are IMF forecasts.

Several features are worth noticing. First, in line with two stylized facts documented by Mendoza and Yue (2012) the model predicts that default happens in bad times: On the one hand, prior to default output is below trend 2/3 of the time and default itself is associated with a strong drop in GDP. On the other hand, prior to default the external debt ratio is above average and peaks in the default period. Second, the magnitude of the output drop associated with default predicted by the model coincides with the what we observed in Greece, where

\(^{49}\)The default of Greece was a somewhat gradual process. While Greece effectively had lost access to capital markets by the end of 2010, Greek banks registered their first losses on sovereign bonds in August 2011 when they participated in the securities exchange program. The process of debt restructuring was finished in April 2012.
GDP dropped by a quarter after default. Third, the model predicts a credit crunch: Bank loans drop, the spread between the deposit and loan rate (not shown) spikes and direct lending as an imperfect substitute can only partially compensate for the loss of intermediated funding, such that total capital drops for a number of periods. Again, the magnitudes predicted by the model are similar to what we observe in the data. Fourth, while the model understates the magnitude of the drop in consumption, it does a fairly good job with respect to investment. Fifth, the responses predicted by the model are less protracted than what we observe in Greece, despite the fact, that the output costs increase over time (recall figure 3). This divergence is explained by the relatively quick reversal of TFP and the absence of adjustment costs in the model and the linear detrending applied to the data.

Most importantly, the model predicts that default is followed by a period of several years, during which the government does not issue any new debt. Unlike in existing models of sovereign default, this is an equilibrium outcome. The average duration of this period is 5.8 years, which is close to the values usually assumed in the default literature. Greece made its return to international capital markets in 2017, 6 years after the start of the episode in 2011. Furthermore, the prediction of the model is well within the range of 1 to 8 years estimated in Gelos et al. (2011), Richmond and Dias (2009) and Cruces and Trebesch (2013). Furthermore, note that the endogenous duration of exclusion is stochastic. As panel a) of figure 6 shows, its distribution...
a) Length of default episodes

b) Credit to GDP around reaccess

Figure 6: Length of default episodes and credit to GDP around reaccess: Panel a): The bar histogram refers to the model. The distribution function, which uses bigger bins, is copied from Schmitt-Grohe and Uribe (2017), figure 13.1. Panel b): The red line reproduces the estimates of credit to GDP from figure 1, with the corresponding 90 and 95% confidence intervals in gray. The green line shows the same estimates for simulated data.

is strongly skewed to the right, with a mean of 5.8 and a median of 4 years. This property of the model is in accordance with empirical observations: Using a sample of 147 defaults since 1975 Schmitt-Grohe and Uribe (2017) report a mean duration of default of 8 and a median of 5 years. However, their sample contains a lot of developing countries. Constraining the sample to the 50% more financially developed countries according to the financial institutions development index proposed by Svirydzenka (2016) reduces these values to 3 and 6.

Furthermore, the model predicts a pattern of credit to GDP around exit that is similar to that documented for the data in section 1.1. This can be seen in panel b) of figure 6, which compares the path of credit to GDP in the model and in the data using the event analysis approach from section 1.1. Exclusion periods end after the financial sector’s conditions measured by credit over GDP have substantially recovered.

50I consider a country to have re-accessed capital markets if it borrows for two consecutive periods. This way, the exclusion periods include a few episodes where the government interrupts its no-borrowing period for one period due to one or several unusually large shocks.

51A caveat: Existing studies differ in their definitions of the beginning and the end of a default episode. Schmitt-Grohe and Uribe (2017) report the duration until settlement, not until market reaccess. Combining their data to date entry into default and Cruces and Trebesch (2013) to date reaccess to the markets and restricting the set to the more developed countries leaves us with only 38 episodes and yields a median of 9 and a mean of 7.5. Gelos et al. (2011) measure the time from default to reaccess and report a mean of 4.7 and a median of 3.5 period across 45 default episodes. While average durations differ across studies, the skewness of their distribution is a robust feature.

52To generate a comparable sample of simulated data, I treated each default episode in my long simulation as a different country. For each such “country” I kept a sample that is 4 times as long as the average default episode, randomly selecting a time window around the default episode.

53However, unlike in the data, in the model the recovery of credit to GDP is not yet complete at time of reaccess. This is because reaccess happens around the time that banks become the marginal investors. At this time bank equity has largely but not yet fully recovered. This implies that the deposit-loan spread is still elevated relative to normal times, causing an elevated loan rate and hence an elevated MPK in turn. A higher MPK means a lower capita/output and therefore loan/output ratio.
6.3 Robustness

The results are robust to a number of modeling changes. The exact shape of the financial friction for example does not influence the mechanism of the model significantly.\footnote{In my model, banks leverage is constraint by the collateral value of their assets as in Kiyotaki and Moore (1997). Alternatively, I could have assumed a simple leverage constraint \( d'/r \leq \theta(k' + qb) \) or a state dependent leverage constraint \( E_\Omega [V'_B] \geq \theta(k' + qb) \) as in Gertler and Kiyotaki (2010). These three different constraints have slightly different implications: For a simple leverage constraint maximum leverage is constant. For the Kiyotaki and Moore constraint, the higher today’s deposit-loan spread, the higher the collateral value, the higher maximal leverage. The Gertler and Kiyotaki constraint extends this to the infinite future: the higher the discounted value of all future profits, the higher maximal leverage. Said differently, the first constrains the book-value leverage, the second the market-value leverage, and the second lies in between.} Furthermore, the main results are qualitatively robust to using GHH preferences or abstracting from labor all together. Using government spending shocks yields similar results as well.

Likewise, most of the conventional parameters have relatively little quantitative effect on the default-related moments of the model. This is in particular true for the parameters of the instantaneous utility function, the production function and government spending. However, making the household impatient relative to the world interest rate, as the literature routinely does, yields significantly higher default rates.

The financial sector parameters play a more important role, since they determine the costs of default. The intensity of the financial friction \( \theta \) determines the leverage of banks. The higher the leverage, the higher the costs of default and hence the amount of sustainable debt.

The dividend payout parameter \( \eta \) pins down the spread between the bank lending and the deposit rate. The higher the spread, the higher the difference between the steady state return on domestic capital and the world rate, hence the stronger the incentive for the government to borrow – more and at higher risk.

The cost advantage of intermediated investment \( \xi \) has an ambiguous effect on the default incentives. For simplicity consider first the case that \( \xi \) is constant, i.e. \( \check{\xi} = 0 \). A lower cost advantage of intermediation \( \bar{\xi} \) has two effects: First, it lowers the social cost of scarcity of equity, second it lowers the maximum spread that banks can charge. The first effect makes default less costly, the second however makes default more costly because it slows down the re-accumulation of equity after default. For the chosen calibration the second effect is found to dominate the first (locally): lowering \( \bar{\xi} \) marginally lengthens the exclusion period and increases the sustainable amount of debt. In the extreme case of \( \bar{\xi} = 0 \) however, banks become obsolete and no debt is sustainable. The other extreme, \( \check{\xi} = \infty \), means shutting down direct investment. In that case upon default the economy experiences a very strong and very short financial crisis, during which the deposit-loan spread shoots up to empirically implausible levels, which allows the bank to recapitalize almost fully within one period. The cost of this strong but short crisis is much smaller than the cumulative cost of the protracted crisis resulting from the chosen intermediate value of \( \check{\xi} \). Hence, the amount of debt sustainable is lower. Allowing \( \xi \) to move inversely with TFP \( (\check{\xi} < 0) \) makes sovereign default more attractive in bad times relative to good times, and hence slightly increases the default probability (by 0.1 ppt) and the debt to...
good times, and hence slightly increases the default probability (by 0.1 ppt) and the debt to GDP ratio (by 1pt) and the exclusion time (1 period). The parameter governing the banks exposure $\psi$, is discussed next.

7 Policy application: The EU plan to reduce the exposure of banks to domestic sovereign debt

In the aftermath of the sovereign debt crisis, policy makers in the EU are discussing new rules to weaken the sovereign-bank nexus. One proposal being discussed is to limit the exposure of banks to domestic sovereign debt through various measures like ceilings or higher capital requirements on domestic sovereign exposure. The intention of this proposal is to reduce the risk of a “diabolic loop”, in which problems of sovereign debt sustainability lead to losses for banks, which in turn affect government revenues negatively or generate additional expenditures due to bailouts, hence making the sovereign debt burden even more unsustainable.

Yet, looking at this proposal through the lens of this model, such a reform may have unintended consequences: Reducing banks' to domestic sovereign debt, ceteris paribus, reduces the cost of default for the domestic economy. This has two implications. First, default will be more attractive ex post. Changing the rules from one day to the other may hence trigger sovereign defaults that would otherwise not have occurred. Second, the anticipation of lower default costs by international lenders ex ante means that governments will not be able to borrow as much as before. Hence, the benefits derived from international lending will shrink. While the second implication resembles the reasoning in the theoretical analysis of Chari et al. (2016), the first is absent in their deterministic model.

To illustrate these consequences quantitatively, I analyze the effects of a reduction of the banks exposure to sovereign debt $\psi$ by 20%. Table 4 shows the long run effects of this change: As as the banks exposure is reduced by 20%, the total debt to GDP ratio drops by one third and the foreign debt ratio by half. The effect of $\psi$ on the debt capacity is highly nonlinear and has a significant utility cost of about 0.9% in consumption equivalent, since foreign lending is desirable. Note the importance of the borrowing motive for this welfare result: If the country were to borrow because of impatience instead, a reduction in its debt capacity would imply an increase in long run welfare.

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55 See for example Brunnermeier et al. (2016) or “Sovereign debt rule changes threaten EU bank finances”, Financial Times, 8.6.2016

56 In the policy discussion it is sometimes argued that the reduced demand for government bonds by domestic banks may increase the interest rates of sovereign bonds as an unintended side effect. Yet this argument is simply based on lower demand, while the argument made here is based on the lower commitment value. In fact the model abstracts from the demand argument altogether, since foreign demand is perfectly elastic.

57 In their setup default is discriminatory and borrowing from banks is directly desirable for tax smoothing motives. In contrast, in my model default is non-discriminatory, and borrowing from banks is desirable indirectly because it generates commitment to borrow from international lenders at the (favorable) world interest rate. Furthermore their setup differs from the experiment here, in that they consider the exposure of banks a policy variable that can be adjusted each period.
The short run effect is equally stark: Assume the country finds itself at the non-stochastic risk adjusted steady state of the baseline economy at the end of period $t$. Assume furthermore that between periods $t$ and $t + 1$ $\psi$ is changed unexpectedly, effective as of next period. Then there is a 10% probability that TFP turns out so bad that the government will default at the beginning of $t + 1$ because it can only partially roll over its outstanding foreign debt and prefers default to the tax hike otherwise needed. If we assume instead that the regulatory change is effective at the end of $t$, the effect is even more striking: After the sudden change, the government not only is incapable of rolling over its debt, but also finds its banks less exposed. It will default with 86% probability. While these sudden change scenarios are arguably too stylized, they exemplify the point that a gradual implementation of the reform is to be recommended, if the default probability along the transition to the lower $\psi$ is to be contained.

8 Conclusion

Motivated by the recent European sovereign debt crisis, this paper proposes a unified theory which explains why countries want and are able to borrow so much as to risk default, and why default episodes are associated with financial crisis, output drops and exclusion from international markets, and when countries return to markets after default.

To this end I develop a model of sovereign debt under limited commitment, with international lenders and an explicitly modeled domestic economy, which consists of households, banks and firms. Savings are intermediated from households to firms by banks. Since there is a friction in the intermediation process, which makes bank equity both necessary and scarce, the economy never reaches its optimal level of investment. This fact generates a strong incentive for the government to borrow on international markets. While the government can always default, the fact that domestic banks are also exposed to sovereign debt generates commitment: If the government chooses to default, banks suffer losses, which leads to a period of credit shortage.

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Table 4: Effect of reduction of exposure

| Moment   | Baseline | $\psi$-20% |
|----------|----------|------------|
| $E(\tilde{B}/Y)$ | Total debt to GDP ratio | 43% | 29% |
| $E(B_X/Y)$ | Foreign debt to GDP ratio | 18% | 9% |
| $E(B_X/B)$ | Share of domestic debt | 60% | 69% |
| $E(\text{spread})$ | Sovereign spread | 0.47% | 0.65% |
| $E(\text{Rep})$ | Default frequency | 0.44% | 0.62% |
| $E(V)$ | Welfare in lifetime. cons. equ. | 100% | 99.1% |

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58Assume that 20% of the government debt on the domestic bank’s balance sheet is forced to be sold to the household.
This translates into a reduced capital stock and output losses. Furthermore, while the financial crisis lasts, the domestic return on investment drops, which discourages the government to borrow. Hence default is followed not only by a period of depressed output, but also by a period during which the government issues no new bonds. These two consequences of default arise endogenously and make a limited amount of debt sustainable.

Conceptually, this paper therefore contributes to the existing literature in three ways. First, it introduces the return differential motive for borrowing into the sovereign debt literature. Second, it proposes a new mechanism to explain the output costs of default, reflecting recent experiences with the sovereign-bank nexus. Third, it proposes a novel explanation for why we observe temporary breakdowns of international borrowing in the aftermath of default and for how long this period lasts, two questions which the theoretical literature has not addressed before.

Quantitatively I show that the model calibrated to other moments predicts empirically plausible magnitudes of the consequences of default: Both the drop in output, investment and credit as well as the duration of market shutdown are roughly in line with the data. Moreover the model offers insights that are relevant for a current policy debate on reducing banks exposure to domestic sovereign debt. I show that any such policy not only makes banks more resilient against sovereign debt crisis, which would be desirable per se. It also reduces the sustainability of sovereign debt, which is undesirable, at least in the context of the model.

Furthermore this model also contributes an explanation to why some countries like Japan or Italy seem to be able to sustain debt to GDP ratios that are higher than what other countries can sustain. It suggests that countries which have a higher share of domestically held debt and which have more leveraged financial sectors would suffer more in case of default and are hence less likely to renege on their obligations. The same argument may apply over time: The development of leveraged financial sectors across the developed world may explain the increase of debt to GDP ratios in the post war period.
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Online-Appendix

Appendix A: The optimization problem of the government

Be $X$ the vector of private sector variables $X = [C, L, D', K'_H, K'_B, B'_B, B'_X, K'_F, r, q, R', W, \lambda_1, \lambda_2, \lambda_3, \lambda_H]$. Then the government’s problem, after substituting out some variables (HH value function, to-morrow’s labor supply and dividend), expressing the foreign bond demand as a complementarity constraint and dropping a few redundant indices, is:

$$V_G(\Omega) = \max_{Rep,T,B',X} \frac{C^{1-\gamma}}{1-\gamma} + \frac{L^{1+\nu}}{1+\nu} + \beta E[V'_G]$$

s.t.

Government budget constraint

$$qB'_T + T = B_T Rep + G$$

$$B'_X \geq 0$$

Bank FOCs

$$0 = C^{-\gamma}/r - \beta E[V'_B] - \lambda_1 E[C'^{-\gamma}] + \lambda_2/r$$

$$0 = -qC^{-\gamma} + \beta E[Rep'V'_B] + \lambda_1 E[Rep'C'^{-\gamma}] \theta - \lambda_2 q + \lambda_3 q(1 - \psi)$$

$$0 = -C^{-\gamma} + \beta R'E[V'_B] + \lambda_1 R'E[C'^{-\gamma}] \theta - \lambda_2 - \lambda_3 \psi$$

$$0 = \min \{ E[C'^{-\gamma}(Rep' B'_B + R' K'_B)] - E[C'^{-\gamma}D'] , \lambda_1 \}$$

$$0 = \min \{ q(RK_B + RepB_B - D) - qB'_B - K'_B + D'/r , \lambda_2 \}$$

$$0 = \min \{ qB'_B - (qB'_B + K'_B) \psi , \lambda_3 \}$$

Household FOCs

$$C^{-\gamma} = \beta r E[C'^{-\gamma}]$$

$$C^{-\gamma} = \beta R/(1 + \xi) E[C'^{-\gamma}] + \lambda_h$$

$$0 = \min \{ \lambda_h, K'_H \}$$

$$C^{-\gamma}W = \chi L^\nu$$

Firm FOC

$$R' = E \left[ \left( C'^{-\gamma} \omega \right)^{\frac{1-\alpha}{1+\alpha}} \alpha K_F^{\frac{1-\alpha}{\alpha}} \left[ (1 - \alpha) K_F^{\frac{1-\alpha}{\alpha}} / \chi \right]^{\frac{1-\alpha}{1+\alpha}} / E[C'^{\gamma}] + (1 - \delta) \right]$$
\[ W = \left[\omega(1 - \alpha)K_F^\alpha L^{-\alpha}\right] \]

Foreign lenders’ bond demand

\[ 0 = \min \{q - E[Rep']\bar{q} + B'_X\} \]

Market clearing

\[ B'_T = B'_B + B'_X \]

\[ K'_F = K'_B + K'_H \]

Resource constraint

\[ C + K'_B + K'_H(1 + \xi) + B'_Bq = B'_Tq + \omega K_F^\alpha L^{1-\alpha} + (1 - \delta)K_F - B_TRep + B_B - G \]

Notice that in this problem, the government not only needs to form expectations over tomorrows value function \( V'_G \), but also over the marginal utility of consumption \( u_c(C') = C'^{-\gamma} \), the marginal value of bank equity \( V'_{Be} = C'^{-\gamma} + \eta\lambda'_2 \), the repayment choice \( Rep' \) and products thereof.

**Appendix B: Computation**

**State space reduction:**

The (pre-decision) state space of the model described above is 7 dimensional \( \Omega = [\omega, \bar{B}, K_H, D, B_B, K_B, R] \).

To reduce the computational burden it us helpful to reduce its dimensionality. This can be done using post-decision state variables, i.e. by anticipating the default decision. In the following I will explain this trick, which is an application of the method explained in Thaler (2016), in two steps.

First assume that for a given state \( \Omega \) we know the default optimal decision. In that case, we can compute cash-at-hand (or pre-dividend equity) of the bank after debt repayment directly from the information contained in \( \Omega \):

\[ E \equiv RK_B + RepB_B - D \]

Cash-at-hand for the household, after debt repayment (but before receiving the proceedings of new debt issuance) is given by the difference of the total resources of the economy minus bank equity:

\[ W = \omega(K_B + K_H)^\alpha L^{1-\alpha} + (1 - \delta)(K_B + K_H) - Rep(B_T - B_B) - G - E \]
This variable contains the non-predetermined choice $L$ though.\footnote{If $L$ is constant it is enough to know $E$ and $W$. Furthermore, given separable utility, one could apply the trick that I apply to the repayment decision also to $L$. Then, even with variable labor, knowing $E$, $W$ and $\omega$ is sufficient.} Therefore, we need to keep track separately of

$$K_F = K_B + K_H$$

and

$$\bar{B} = Rep (B_T - B_B)$$

Once we know the three variables $[E, \bar{B}, K_F]$, plus the exogenous state $\omega$, we have sufficient information to solve the governments problem for all variables other than $Rep$, that is we can find $(T, \bar{B}', X)$.\footnote{To see this note that the state variables only appear in the above problem in these combinations.} Call this alternative “state vector” $[\omega, \tilde{\Omega}] = [\omega, [E, K_F, \bar{B}]]$. Assume we solve the governments problem across this alternate state vector.

But what about the repayment decision, which after all depends on all the 7 state variables in $\Omega$? This bring us to step two. Whenever we solve for $(T, \bar{B}', X)$ given the state $[\omega, \tilde{\Omega}]$, we determine the values of all the endogenous elements in $\Omega$. That means, we can determine the value of tomorrow’s endogenous alternative state conditional on repayment: $\tilde{\Omega}'|Rep'$. Furthermore assume that for each $(B_T, K_H, D, B_B, K_B, R)$ there exists a threshold level of the exogenous shock $\bar{\omega}$, below which default is optimal and above which repayment is optimal. (This assumption is verified ex post.) Once we know this threshold value $\bar{\omega}$, we can determine tomorrow’s endogenous state as a function of the exogenous state: $\tilde{\Omega}'(\omega') = [\bar{E}'(\omega'), \bar{B}(\omega'), K_F']$. This is all we need to know in order to compute the expectations over future variables, given we have approximated them across the state $[\omega, \tilde{\Omega}]$. Finally, we need to ensure that we picked the right $\bar{\omega}$. We know that at $\bar{\omega}$ the government must be indifferent between default and repayment, i.e. $V (\bar{\omega}, \tilde{\Omega}|Rep' = 1) = V (\bar{\omega}, \tilde{\Omega}|Rep' = 0)$. To find this threshold level, we therefore augment the above optimization problem by the variable $\bar{\omega}$ and the condition $V (\bar{\omega}, \tilde{\Omega}|Rep' = 1) = V (\bar{\omega}, \tilde{\Omega}|Rep' = 0)$. I call this trick “anticipation of future choices” because what we essentially do here is to explicitly anticipate one of the choices that the government has to make tomorrow and incorporate it into todays problem — with the aim of being able to reduce the state space.

This means, that by complicating the optimization problem only slightly (by adding one equation and one variable), we are able to reduce the computationally necessary state space from 7D to 4D\footnote{A reduction to 5D is feasible without this trick, and a further reduction to 3D would be feasible by applying the same trick to the labor decision.}. This brings about a massive reduction in computation time.
Analytical evaluation of integrals:

The state space reduction method just described yields a transformed problem, in which the default threshold tomorrow $\bar{\omega}$ is a choice variable. This means that the approximation of the integral in the expectations over tomorrows policy function becomes more important: If we used discrete approximation of the exogenous state variable, the integral would be a step function in $\bar{\omega}$. If we used the trapezoidal rule, the integral would exhibit kinks in $\bar{\omega}$. This would hamper the efficiency of the use of derivative based solvers for smooth problems. Instead I approximate the expectations over tomorrows policy function by the exact evaluation of the integral given the smooth cubic-spline approximation of the policy function. That is, if $Z(\Omega)$ is a policy or value function and $\tilde{Z}(\Omega)$ a smooth approximation thereof, then

$$
E_{\Omega}(Z') = \int_{0}^{\infty} pdf(\omega' | \Omega) Z(\omega' | \Omega) d\omega \simeq \int_{0}^{\infty} pdf(\omega' | \Omega) \tilde{Z}(\omega' | \Omega) d\omega.
$$

This method differs from the quadrature approaches usually employed in the dynamic macro literature. It extends or 'inverts' the insight of Gaussian quadrature methods, that integrals over polynomials can be evaluated exactly given a sufficient number of function values, to the piecewise-polynomial cubic-spline function.

To illustrate the method, consider for example the continuous version of the canonical Arellano (2008) model. Be $B$ the debt level and $\bar{\omega}$ the default threshold (the level of exogenous GDP below which it is optimal to default). Then the expectations of the value function are given by

$$
E_{\Omega}(V(\omega' | B')) = \int_{\bar{\omega}}^{\infty} pdf(\omega' | \omega) V^{def}(\omega' | B') d\omega + \int_{0}^{\infty} pdf(\omega' | \omega) V^{rep}(\omega') d\omega.
$$

Therefore $E_{\Omega}(V(\omega' | B'))$ is a function not only of the endogenous state $B'$, but also of the default threshold $\bar{\omega}$. Continuous interpolation/approximation of $V$ implies smoothness of the expectation in the endogenous state. If combined with exact integration it also implies smoothness of the expectation with respect to the default threshold.

It is easy to use for 1 dimensional integrals and has two advantages: First, it avoids an additional layer of approximation. Second, and more importantly for the current application, it generates expectations that are twice continuously differentiable not only in the endogenous states, but also in the exogenous states. This facilitates the application of continuous solution methods to optimization problems where the exogenous state appears as a variable. This is the case once we represent the government’s problem with post decision state variables.

This approach could be applied given any approximating function. Given a piecewise linear approximation of $Z(\Omega)$, it is equivalent to trapezoidal rule, but then it does not deliver the second advantage. For chebychev polynomials, the other commonly used twice continuous approximating function, it is equivalent to piecewise Gaussian quadrature with enough quadrature points.

Algorithm:

Apart from the way to deal with the state space and to evaluate integrals (expectations), the time iteration algorithm I use is a standard one loop algorithm, commonly used in the sovereign default literature.
1. Define a Cartesian grid over the 4D alternative state vector. Instead of using $[\omega, E, K_F, \tilde{B}]$ I rotate the grid so as to reduce the inclusion of regions of the state space into the grid, which are never visited along the equilibrium path. In particular I use $[\omega, \tilde{W}, E/\tilde{W}, K_F - (\varsigma_0 + \varsigma_1 W)]$ where $\tilde{W} = (K_B + K_H)\alpha + (1 - \delta) (K_B + K_H) - Rep(B_T - B_B)$ and $\varsigma_0 + \varsigma_1 \tilde{W}$ is the result of a regression of $K_F$ on $\tilde{W}$. I use (8,10,8,5) points.

2. Make an initial guesses for the functions $u_c(C)$, $\lambda_2$ and $V_G$ across the points of this grid.

3. From these guesses and this grid, construct a interpolant for each of the 3 functions. In particular I use using cubic splines with not-a-knot end conditions as recommended by Hatchondo et al. (2007). I extrapolate points outside the grid using these splines, but choose the grid such as to ensure that the maximum distance from the grid remains small and the probability of leaving the grid marginal.

4. For each of the points on the grid, solve the optimization problem from appendix A transformed as described above, i.e. augmented by the additional variable $\tilde{\omega}$ and the additional equation $V (\tilde{\omega}, \tilde{\Omega}|Rep' = 1) = V (\tilde{\omega}, \tilde{\Omega}|Rep' = 0)$ and using the previous iterations approximations $\tilde{Z}(\Omega)$ to evaluate expectations.

5. Check the difference between the previous guess and the solutions at the grid points obtained. If they are very similar stop. Else update the initial guess and return to point 3.

The whole code is written in MATLAB. To solve the continuous optimization problem at each of the grid points at step 4 of the algorithm, I use the solver KNITRO, which is able to solve smooth complementarity problems fast and reliably. To improve the performance of the solver, I supply analytical first derivatives, which are largely computed and coded automatically using MATLAB’s symbolic toolbox. Furthermore the code is executed in parallel.

**Precision:** Despite the fact that the policy functions exhibit minor kinks but are approximated by smooth functions, the precision of the result is satisfactory: The algorithm converges successfully up to an average (across the grid) absolute change of the forward looking variables of 0.0001%. Across a very long simulation the mean of the absolute value of the difference between (1) the approximated solution $\tilde{Z}(\Omega)$ and (2) the solution as in step 4 of the algorithm for the (also known as Euler Error, see Judd (1998)) is around 0.05% for the variables $C$ and $\lambda_2$ and 0.005% for the value function $V_G$.

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62For computational convenience the governments problem is solved assuming the bond holding constraint (11) holds with equality. It is then verified by simulation that $\lambda_3 \gg 0$ in the ergodic distribution.
Appendix C: Proofs

Proposition 1: Five steps are necessary to arrive to this result. First, note that the value function of the bank $V_b(d, b, k, \Omega)$ can be summarized as a function $V_b(e, \Omega)$ of only one endogenous state variable $e = Rk + bRep - d$ since $(b, d, k)$ only enter the problem as this linear combination. Second, the solution of the problem of the bank $[b’, d’, k’]$ and its value function $V_b(e, \Omega)$ is linear in $e$. To proof this second claim, assume that $V_b(e’, \Omega)$ is linear in $e’$. Be $x$ a solution to the bank’s problem given $e$ and $\Omega$. Denote the associated value function by $\hat{V}_b(e, \Omega)$. Then we can conclude that $\alpha x$ is optimal given $\alpha e$ for any $\alpha \in \mathbb{R}^+$ since both the objective and the constraints are linear in both $e$ and $x$. (If there existed $\alpha x’$ that is feasible given $\alpha e$ such that $\hat{V}_b(\alpha x’, \Omega) > \hat{V}_b(\alpha x, \Omega)$ then by linearity $x’$ would also be feasible given $e$ and by homogeneity it would dominate $x$.) Therefore the solution of the banks problem – given a linear $V_b(e’, \Omega’)$ – is linear in $e$. Furthermore, by linearity the value of this solution $\hat{V}_b(e, \Omega)$ also needs to be linear in $e$. Since the same reasoning applies to the value function of all the following periods, the initial assumption that $V_b(e’, \Omega)$ is linear in $e’$ must hold. Third, the first and second result together imply $V_b(e’, \Omega’)$ is linear in $[b’, d’, k’]$. Fourth, given the third result it is obvious that both the constraints and the objective of the optimization problem are affine functions. Hence, the first order conditions are necessary and sufficient. Fifth, to determine the derivative of the value function w.r.t equity $V_{be}(e’, \Omega’)$ we can apply the envelope theorem for maximization problems with inequality constraints: $V_{be}(e’, \Omega’) = u_e(e’) + \eta \lambda’_2$. 

Proposition 2: First, note that the value of defaulting $V_{GR}(\Omega)$ is independent of the level of $B_X$, hence $V_{GD}(\Omega_1) = V_{GD}(\Omega_2)$. Second, if $[T_2^*, B_{T2}^*, X_2^*]$ denotes the optimal level of all choice variables under repayment given state $\Omega_2$, then $[T_2^*, B_{T2}^*, X_2^*]$ is also feasible under repayment given state $\Omega_1$ by the government budget constraint. Hence it must be that $V_{GR}(\Omega_1) \geq V_{GR}(\Omega_2)$. Summarizing, we have $V_{GD}(\Omega_1) = V_{GD}(\Omega_2) > V_{GR}(\Omega_1) \geq V_{GR}(\Omega_2)$, and hence that $\Omega_2 \in \mathcal{D}$. 

Proposition 3: First, note that conditional on default or repayment, the vector $[\omega, E, K_F, B_X]$ sufficiently summarizes the 7 dimensional state $\Omega$ (Here $E$ denotes the pre-dividend pre-transfer bank equity $E = RK_B + RepB_B - D$). Second, note that the values of repayment are equal $V_{GR}(\Omega_1) = V_{GR}(\Omega_3)$ since the elements of the vector $[\omega, E, K_F, B_X]$ conditional on repayment are equal. Third, note that conditional on default, $[\omega, K_F, B_X]$ are equal but the banks pre-dividend pre-transfer equity is lower at $\Omega_1$. Denote the optimal level of all choice variables under default given state $\Omega_1$ by $[T_1^*, B_{T1}^*, X_1^*, Z_1^*]$ where $Z$ denotes the transfer from the bank to the household. Then, by choosing an appropriate transfer $Z_3 > Z_1$ the allocation $[T_3^*, B_{T2}^*, X_1^*]$ must be feasible given state $\Omega_3$. By optimality it must hence be that $V_{GD}(\Omega_3) \geq V_{GD}(\Omega_1)$. Hence $V_{GD}(\Omega_3) \geq V_{GD}(\Omega_1) \geq V_{GR}(\Omega_1) = V_{GR}(\Omega_3)$ and therefore $\Omega_3 \in \mathcal{D}$. 

Electronic copy available at: https://ssrn.com/abstract=3216992
Appendix D: Microfounding banks exposure to the sovereign

In the main text banks were simply assumed to hold a fixed share of their assets in bonds. This appendix presents a simple model to endogenize this assumption, where banks hold government bonds because they are more liquid.

Assume that each period has two sub-period. In the first sub-period the continuum of banks make the same choices as in the main text, subject to the same constraints but the bond holding constraint. In the second sub-period banks are hit by idiosyncratic shocks: The firms that borrowed from the bank in the first sub-period either require more funds (tap credit lines) or return excess fund (early repayment). The bank is contractually obliged to accept these changes in its loan portfolio and it can not default on this obligation. These flows are assumed to be of size $(\varepsilon - 1)k'$ where $\varepsilon \sim f(\varepsilon)$ is an iid idiosyncratic random variable with mean 1. Since there is a continuum of banks these flows cancel out on aggregate, but they require the individual banks to adjust their balance sheets. Deposits are slow moving, they can not be adjusted in the second sub-period. A bank that experiences a positive credit demand shock $\varepsilon > 1$ therefore can do 2 things: It needs to either sell bonds or sell firm loans. Conversely, a bank that experiences a negative credit demand shock $\varepsilon < 1$ can invest either in bonds or in loans. Loans are assumed to be partially illiquid: When a loan is sold, a fraction $1 - \kappa$ of the capital financed by the loan is lost. Let $\hat{b}'$ denote the amounts of bonds bought (or sold if negative) in the secondary market and $\hat{k}' + \geq 0$ the amount of loans bought and $\hat{k}' - \leq 0$ the amount of bonds sold. Let $\hat{p}$ and $\hat{q}$ denote the prices at which bonds are traded in the secondary market. Note that these prices as well as the aggregate balance sheet of the financial sector at the end of the sub-period are certain at the end of the first sub-period, since there is no aggregate uncertainty about the second sub-period. The banks sequential problem can be expressed a single optimization problem. In the first sub-period, it chooses both its beginning of period portfolio $b', k', d'$ and its end of period portfolio conditional in the realization of the idiosyncratic shock $\hat{b}'(\varepsilon), \hat{k}'_-(\varepsilon), \hat{k}'_+(\varepsilon)$:

$$V_b(d, b, k, \Omega) = \max_{b', k', d'} (e + d'/r - qb' + k') u_c(c') + \beta E_{\Omega} [V_b(e', \Omega')]
\text{st.}
E_{\Omega} [u_c(c')d'] \leq E_{\Omega} [u_c(c')\theta(Rep' b' + R' k')]
(1 - \eta) e \leq e + d'/r - qb' - k'
\hat{b}' \geq 0
\hat{k}' \geq 0 \forall \varepsilon$$

where $e' = R' \left( \hat{k}'_+ + \hat{k}' - \kappa \hat{p} + k'(\varepsilon - 1) \right) = 0 \forall \varepsilon$

$b + \hat{b}' \geq 0 \forall \varepsilon$

$\hat{k}'_+ \geq 0 \forall \varepsilon$

$-\hat{k}'_- \geq 0 \forall \varepsilon$

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Assume that each period has two sub-period. In the first sub-period the continuum of banks make the same choices as in the main text, subject to the same constraints but the bond holding constraint. In the second sub-period banks are hit by idiosyncratic shocks: The firms that borrowed from the bank in the first sub-period either require more funds (tap credit lines) or return excess fund (early repayment). The bank is contractually obliged to accept these changes in its loan portfolio and it can not default on this obligation. These flows are assumed to be of size $(\varepsilon - 1)k'$ where $\varepsilon \sim f(\varepsilon)$ is an iid idiosyncratic random variable with mean 1. Since there is a continuum of banks these flows cancel out on aggregate, but they require the individual banks to adjust their balance sheets. Deposits are slow moving, they can not be adjusted in the second sub-period. A bank that experiences a positive credit demand shock $\varepsilon > 1$ therefore can do 2 things: It needs to either sell bonds or sell firm loans. Conversely, a bank that experiences a negative credit demand shock $\varepsilon < 1$ can invest either in bonds or in loans. Loans are assumed to be partially illiquid: When a loan is sold, a fraction $1 - \kappa$ of the capital financed by the loan is lost. Let $\hat{b}'$ denote the amounts of bonds bought (or sold if negative) in the secondary market and $\hat{k}'_+ \geq 0$ the amount of loans bought and $\hat{k}'_- \leq 0$ the amount of bonds sold. Let $\hat{p}$ and $\hat{q}$ denote the prices at which bonds are traded in the secondary market. Note that these prices as well as the aggregate balance sheet of the financial sector at the end of the sub-period are certain at the end of the first sub-period, since there is no aggregate uncertainty about the second sub-period. The banks sequential problem can be expressed a single optimization problem. In the first sub-period, it chooses both its beginning of period portfolio $b', k', d'$ and its end of period portfolio conditional in the realization of the idiosyncratic shock $\hat{b}'(\varepsilon), \hat{k}'_-(\varepsilon), \hat{k}'_+(\varepsilon)$:

$$V_b(d, b, k, \Omega) = \max_{b', k', d'} (e + d'/r - qb' + k') u_c(c') + \beta E_{\Omega} [V_b(e', \Omega')]
\text{st.}
E_{\Omega} [u_c(c')d'] \leq E_{\Omega} [u_c(c')\theta(Rep' b' + R' k')]
(1 - \eta) e \leq e + d'/r - qb' - k'
\hat{b}' \geq 0
\hat{k}' \geq 0 \forall \varepsilon$$

where $e' = R' \left( \hat{k}'_+ + \hat{k}' - \kappa \hat{p} + k'(\varepsilon - 1) \right) = 0 \forall \varepsilon$

$b + \hat{b}' \geq 0 \forall \varepsilon$

$\hat{k}'_+ \geq 0 \forall \varepsilon$

$-\hat{k}'_- \geq 0 \forall \varepsilon$
For markets to clear in the second sub-period it must hold that

\[
\int_{\bar{\varepsilon}}^{\varepsilon} f(\varepsilon) \hat{b}'(\varepsilon) d\varepsilon + \hat{B}'_x = 0
\]

\[
\int_{\bar{\varepsilon}}^{\varepsilon} f(\varepsilon) \hat{k}'_+ (\varepsilon) d\varepsilon + \int_{\bar{\varepsilon}}^{\varepsilon} f(\varepsilon) \hat{k}'_-(\varepsilon) d\varepsilon \kappa = 0
\]

Here, the foreign demand for government debt in the second sub-period is determined by the same condition as in the first sub-period. Since the government, by assumption, does not issue any new bonds in this sub-period and since the aggregate demand for funds in the second sub-period is 0, foreigners will not be buy bonds in the second sub-period, and all the assets sold by the banks with a need for funds will be purchased by those banks with excess funds. However, the foreigners demand pins down the price \( \hat{q} = q \).

Under further simplifying assumptions, the solution to the bank’s problem is exactly the same as that in the text. In particular, assume that \( \varepsilon \) is discretely distributed, with a maximum value of \( \bar{\varepsilon} \). Assume that \( \text{prob}(\bar{\varepsilon}) \) is high enough and that \( \kappa \) is small enough (e.g. 0). It is hence optimal for the bank to hold \( b'q = \bar{\varepsilon}k \) just as in the main text the bank holds \( b'q = \psi(1 - \psi)k \).

More generally, the behavior for the bank in this model will look very much like in the model in the main text, if the \( \kappa \) is small enough and \( f(\bar{\varepsilon}) \gg 0 \). Simulations of the model, replacing the banks problem with the one outlined here with a uniformly distributed \( f(\varepsilon) \) have confirmed that.

Furthermore, one could reinterpret this model as a model of collateralized lending. Under this view, \( \hat{q} \) is the inverse interbank rate and \( \hat{b}' \) the interbank claim. \( b \geq -\hat{b}' \) then is the collateral constraint (one would only have to adjust \( e' \), but that has no implications since the distribution of bank equity is irrelevant and banks are risk neutral with respect to idiosyncratic risk)
Appendix E: Additional tables and figures

| Time Horizon         | Credit/GDP   | Real GDP p.c. |
|----------------------|--------------|---------------|
| 4 periods ahead      | -0.169***    | -0.0791***    |
|                      | (0.009)      | (0.000)       |
| 3 periods ahead      | -0.145**     | -0.0837***    |
|                      | (0.022)      | (0.000)       |
| 2 periods ahead      | -0.109*      | -0.0676***    |
|                      | (0.083)      | (0.001)       |
| 1 period ahead       | -0.0279      | -0.0667***    |
|                      | (0.644)      | (0.001)       |
| Contemporaneous      | -0.01000     | -0.0489**     |
|                      | (0.864)      | (0.012)       |
| 1 period after       | 0.0165       | -0.0407**     |
|                      | (0.780)      | (0.036)       |
| 2 periods after      | 0.00523      | -0.0359*      |
|                      | (0.930)      | (0.065)       |
| 3 periods after      | -0.0129      | -0.0324*      |
|                      | (0.828)      | (0.097)       |
| 4 periods after      | -0.00865     | -0.0221       |
|                      | (0.883)      | (0.260)       |
| Country specific linear trend | ✓ | ✓ |
| # countries | 187 | 194 |
| # re-accesses | 52 | 53 |
| # observations | 6022 | 6318 |
| Pseudo R² | 0.869 | 0.992 |

Table 5: Credit to GDP and GDP around market reaccess. This table reports the results of regression from figure 1: The point estimates for $\beta_r$ and $-\beta_t$ in parenthesis — p-values. Credit to GDP and GDP per capita are both on logs.
| Variable                          | Source                                      | Start   | Remark                                                                 |
|----------------------------------|---------------------------------------------|---------|------------------------------------------------------------------------|
| Real GDP per capita              | IMF IFS and WEO (for the forecast)          | 1980 A  | in constant prices                                                      |
| Private Consumption/GDP          | OECD                                        | 1980 A  | since 1995: private nondurable consumption ($P311A+P311B$) / GDP ($B1\_GE$); before 1995 due to missing data: private consumption ($P31S14\_S15$) * the average of durable to total consumption 1995-2010 |
| Investment/GDP                   | OECD and WEO (for the forecast)             | 1980 A  | OECD: Gross capital formation ($P5$) / GDP ($B1\_GE$)                   |
| Government Consumption/GDP       | OECD                                        | 1980 A  | Final consumption expenditure of general government ($P3S13$) / GDP ($B1\_GE$) |
| Trade Balance/GDP                | OECD                                        | 1980 A  | External balance of goods and services ($B11$) / GDP ($B1\_GE$)         |
| TFP                              | AMECO                                       | 1980 A  |                                                                        |
| Balance sheet composition of Greek banks | Bank of Greece                           | 2001 Q  | Aggregated balance sheet CI                                            |
| Deposit-loan spread              | Bank of Greece                              | 2001 Q  | Spread between short term household deposits and loans according to Bank interest rates on new euro-denominated deposits and loans vis-à-vis euro area residents |
| International investment position| Bank of Greece                              | 2001 Q  | International investment position (BPM6)                                 |
| Holders of sovereign debt        | Bank of Greece                              | 2001 Q  | Financial Liabilities broken down by holding sectors ($S13$ Q)          |
| Sovereign spread                 | IFS for Greece and Bundesbank for Germany   | 2000 A  | difference between the returns Greek and German on one year government bonds |
| Private credit/GDP               | IMF IFS                                     | 1998 A  | Financial sector credit to the real economy (line 22d) / GDP (line99)   |
| Sovereign debt/GDP               | IMF Historical Public Debt Database         | 2000 A  |                                                                        |

Table 6: Data definitions and sources for the Greek case study. A stands for annual, Q for quarterly data. Data from the Bank of Greece data is available only from 2001. Credit is used from 1998 onwards since it displays a kink before that. Consumption, investment and credit per capita are constructed multiplying the ratios with GDP.
Figure 7: **International investment position:** Nominal values reported by Bank of Greece. The figure distinguishes between public and private liabilities, and between debt liabilities and total liabilities, including equity. For the government the latter two are almost identical.

Figure 8: **Costs and benefits of default:** This figure illustrates the costs and benefits of default, and how they depend on the TFP.

The blue line plots the difference in the path of key variables for 2 scenarios, starting from the same initial conditions (the risk adjusted steady state): In scenario (1) a negative TFP shock hits the economy at \( t = 0 \) and the government repays. In scenario (2) the same TFP shock hits the economy but the government defaults. No further shocks occur in either scenario. The red line repeats the exercise for a good TFP shock. All plots are normalized by their steady state values. Panel i) shows the evolution of TFP in the two cases.

The blue line in panel e) for example means: Given a negative TFP shock of -14% in year 0, consumption is higher in years 0 and 1 under default than under repayment. The difference equals 2.4% and 0.2% of steady state consumption in years 0 and 1.
The positive shock is a 1 standard deviation shock. In this case repayment is optimal $V_{GD}(t = 0) < V_{GR}(t = 0)$. The negative shock is chosen large enough to make default optimal $V_{GD}(t = 0) > V_{GR}(t = 0)$ (see panel h). Kinks are the result of occasionally binding constraints in each scenario, as in the figure 3.

This figure exemplifies how the costs and benefits of default depend on the level of TFP. Panels e) and f) show that default leads to relatively more favorable paths for consumption and labor if TFP is low: The difference in consumption is higher; the difference in labor is lower (on average). Therefore, under default welfare is marginally higher in period 0 for the negative TFP shock, but much lower for the positive TFP shock (pan. h).

But why does default lead to more desirable paths of consumption and labor only if TFP is low? In states of high TFP the households would like to invest more (both because expected returns are high and to smooth consumption), relative to the case of low TFP. At the same time, banks equity is independent of TFP. Hence, the destruction of bank equity due to default has stronger distortionary effects when TFP is high. This can be seen in panel d): The total amount of loans (i.e. capital) drops much more (relative to the repayment case) under the positive shock than under the negative shock. These distortions constitute the cost of default. Given that the resource gain from defaulting on the rest of the world is independent of the state, but its utility value is higher in bad times, the net costs of default are bigger in states of high TFP.
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