Depletion of a Bose-Einstein condensate by laser-induced dipole-dipole interactions

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Abstract

We study a gaseous atomic Bose-Einstein condensate with laser-induced dipole-dipole interactions using the Hartee-Fock-Bogoliubov theory within the Popov approximation. The dipolar interactions introduce long-range atom-atom correlations which manifest themselves as increased depletion at momenta similar to that of the laser wavelength, as well as a ‘roton’ dip in the excitation spectrum. Surprisingly, the roton dip and the corresponding peak in the depletion are enhanced by raising the temperature above absolute zero.

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1 Introduction

One of the novel features of gaseous Bose-Einstein condensates (BECs), when considered from a many-body physics perspective, is the ability to directly manipulate the interatomic interaction using external electromagnetic fields \cite{1}. In the BECs realized thus far the interatomic interactions can be described by a single parameter, the s-wave scattering length, \( a \), and are short-range in comparison to the average interatomic distance. The experiment by Inouye et al \cite{2} demonstrated how the s-wave scattering length can be changed by magnetic fields via a Feshbach resonance. Here we consider enhancing and controlling the interatomic interactions using laser-induced dipole-dipole forces. These interactions are intrinsically long-range and so affect the gas in a way profoundly different from a Feshbach resonance. In particular, dipole-dipole interactions, both laser-induced \cite{3} and static \cite{4}, have been predicted to be capable of introducing a ‘roton’ dip into the Bogoliubov excitation spectrum of the BEC. This is behaviour reminiscent of that of the strongly correlated quantum liquid helium II, and can be understood in terms of Feynman’s relation which provides an upper bound for the spectrum \( E(k) \) of helium II

\[
E(k) \leq \frac{\hbar^2 k^2}{2mS(k)}
\]

where \( S(k) \), known as the static structure factor, is the Fourier transform of the pair distribution function. \( S(k) \) is a measure of 2nd order correlation in the system. Feynman’s formula interprets the roton dip in
helium II as being due to a peak in $S(k)$ and hence as being due to strong correlations, which are at values of $k$ corresponding to phonons having wavelengths on the order of the interatomic spacing (intuitively: the excitations at these wavelengths require less energy to excite in comparison to neighbouring wavelengths which cost more compressional energy).

In a gaseous BEC at zero temperature the Bogoliubov theory of a weakly interacting degenerate Bose gas \cite{5} is valid and shows that 2nd order correlation arises predominantly from pairs of atoms scattering out of the condensate to form the so-called quantum depletion, or ‘above condensate’ fraction. In a homogenous system the atoms in these pairs are prefectly correlated, in the sense of perfect two mode-squeezing \cite{6, 7, 8}. However, in gaseous BECs the quantum depletion, and hence 2nd order correlation, is typically very small on account of their diluteness, as expressed through the gas parameter $na^3 \ll 1$, where $n$ is the atomic density. Indeed, within the Bogoliubov theory the Feynman relation \cite{11} is an exact equality, yet in a measurement of the bulk excitation spectrum of a regular gaseous BEC interacting via s-wave scattering by Steinhauer et al. \cite{9} no roton was seen. Laser-induced dipole-dipole interactions, on the other hand, introduce controllable (via laser intensity etc.) long-range correlations on the scale of the laser wavelength, and one might expect some significant depletion at the corresponding momentum.

In our previous investigations into laser-induced dipole-dipole interactions in a BEC \cite{3, 10} (and references therein) we considered the zero-temperature case, and the calculations were limited to the basic Bogoliubov method \cite{5} which does not treat the depletion self-consistently. Our purpose here is to use the so-called Popov version of the Hartree-Fock-Bogoliubov method \cite{11, 12}, which treats the depletion as a self-consistent mean-field in order to include the effects of the the back-action of the depleted fraction upon the condensate. This method was primarily developed to treat the finite temperature case where depletion is important, but our hope here is to account for significant quantum depletion due to enhanced interactions and also for thermal depletion due to non-zero temperature. The most significant results we find here are that, perhaps contrary to expectation, the depletion can increase the effects of the dipole-dipole interactions, deepening the roton dip in the spectrum, so that the roton is not diminished by finite-temperature effects but can actually be enhanced.

## 2 Laser induced dipole-dipole interactions

The idea of exploiting dipole-dipole forces to modify the properties of atomic BECs first arose in the context of static dipolar interactions. A BEC of atoms with a large permanent magnetic moment (e.g. chromium) ordered by an external magnetic field, or equivalently, polarizable atoms in a static electric field, has been considered by a number of authors \cite{13, 14, 15, 16, 17, 18}. Here, however, we are interested in fully retarded dynamic dipole-dipole interactions, such as those induced by an electromagnetic wave (e.g. laser beam). The dynamic dipole-dipole interaction is distinguished from the static case by a longer range (the retardation gives it an attractive $r^{-1}$ component which can be used in certain geometries to mimic gravity \cite{19}) and a huge enhancement of atomic polarizability close to a resonance.

We consider a cigar-shaped BEC tightly confined in the radial ($x,y$) plane, irradiated by a plane-wave laser as in \cite{3} (see Fig. 1). The laser polarization is along the long z-axis of the condensate to suppress collective (“superradiant”) Rayleigh scattering \cite{20} or coherent atomic recoil lasing \cite{21} that are forbidden along the direction of polarization. The tight confinement along the propagation direction, together with the far off-resonance condition, enables us to treat the electromagnetic field within the Born approximation. The dipole-dipole potential induced by far off-resonance electromagnetic radiation
of intensity $I$, wave-vector $\mathbf{k}_L = k_L \hat{y}$ (along the y-axis), and polarization $\mathbf{e} = \hat{z}$ (along the z-axis) is

$$U_{dd}(r) = \frac{I \alpha^2 (\omega)}{4 \pi c \varepsilon_0} \frac{1}{r^3} \left[ (1 - 3 \cos^2(\theta)) \left( \cos(k_L r) + k_L r \sin(k_L r) \right) - \sin^2(\theta) k_L^2 r^2 \cos(k_L r) \right] \cos(k_L y).$$

(2)

Here $r$ is the interatomic axis, and $\theta$ is the angle between $r$ and the z-axis. The far-zone $(k_L r \gg 1)$ behavior of (2) along the z-axis is proportional to $-\sin(k_L r)/(k_L r)^2$. In terms of the condensate density $n(r) = |\psi(r)|^2$, the mean-field energy functional accounting for atom-atom interactions is taken to be the sum $H_{dd} + H_s$ [13-19], where $H_{dd} = (1/2) \int n(r) U_{dd}(r - r') n(r') d^3r d^3r'$, and $H_s = (1/2)(4\pi \hbar^2/m) \int n^2(r) d^3r$ is due to short-range $(r^{-6})$ van der Waals interactions, which are described, as is usual, by a delta function pseudo-potential

$$U_s(r) = (4\pi \hbar^2/m) \delta(r).$$

In momentum space $H_{dd}$ takes the form, $H_{dd} = (1/2)(2\pi)^{-3} \int \tilde{U}_{dd}(\mathbf{k}) \tilde{n}(\mathbf{k}) \tilde{n}(-\mathbf{k}) d^3k$, where $\tilde{U}_{dd}(\mathbf{k}) = \int \exp[-i \mathbf{k} \cdot \mathbf{r}] U_{dd}(\mathbf{r}) d^3r$ is the Fourier transform of the dipole-dipole potential (2), the explicit form of which, for the laser propagation and polarization as in Fig. 1, we have given in Eq. (4) of [3]. When the radial trapping is sufficiently tight so that the BEC is in its radial ground state we may adopt a cylindrically symmetric gaussian ansatz for the density profile whose width, $w_r$, is

$$n_{3D}(r) = N (\pi w_r^2)^{-1} n(z) \exp \left[ - (x^2 + y^2)/w_r^2 \right],$$

(3)

where $N$ is the total number of atoms and $n(z)$, the 1D axial density, is normalized to 1, but is unspecified for the time being. Denoting by $\tilde{n}_{3D}(\mathbf{k})$ the Fourier transform of $n_{3D}(r)$, we have

$$\tilde{n}_{3D}(\mathbf{k}) = N \tilde{n}(k_z) \exp \left[ -w_r^2(k_x^2 + k_y^2)/4 \right],$$

in which $\tilde{n}(k_z)$ is the Fourier transform of the axial density $n(z)$. This ansatz allows the principal value of the radial integration in $H_{dd}$ to be evaluated analytically so that the dipole-dipole energy reduces to a one dimensional functional along the axial ($\hat{z}$) direction

$$H_{dd} = (N^2/2) \int n(z) n(z') U_{zz}(z - z') dz dz' = (N^2/4\pi) \int \tilde{n}(k_z) \tilde{n}(-k_z) \tilde{U}_{zz}(k_z) dk_z.$$

(4)

Defining the variables

$$\zeta = k_L^2 w_r^2/2, \quad \xi = (k_z^2 - k_L^2) w_r^2/2,$$

(5)

the one-dimensional (1D) axial potential that appears in $H_{dd}$ is

$$\tilde{U}_{zz}(k_z) = \frac{I \alpha^2 k_L^2}{4 \pi c \varepsilon_0} \frac{1}{c} Q[\xi(w_r, k_z), \zeta(w_r)],$$

(6)

where

$$Q[\xi(w_r, k_z), \zeta(w_r)] = \frac{1}{2\xi} \left[ -\frac{\zeta}{\xi} + F(\xi, \zeta) \right],$$

$$F(\xi, \zeta) = 2\xi \exp(\xi - \zeta) \sum_{j=0}^{\infty} \frac{\zeta^j}{j!} \Re\{E_{j+1}(\xi)\},$$

(7)

$\Re\{E_j(\xi)\}$ is the real part of the generalized exponential integral [23]. However, the sum in the expression (6) is rather unwieldy to use in calculations, so instead we will substitute it by the following asymptotic representation for the function $F$:

$$F(\xi, \zeta) \approx \left\{ \begin{array}{ll}
\frac{2\xi}{\zeta + \xi} \left[ 1 - \exp \left( \left( \xi + \frac{\zeta}{\xi} \left( 1 - \sqrt{\frac{\xi}{\zeta}} \right) \right) \right) \right], & \xi < 0 \\
\frac{2\xi}{\zeta + \xi}, & \xi \geq 0
\end{array} \right.$$

(8)
This representation is very good for $\xi > 0$ and gives the correct asymptotics for $\xi \to 0$ ($k_z \to k_L$) and $\xi \to -\xi$ ($k_z \to 0$). For $-\xi < \xi < 0$ the agreement is reasonably good (see Figure 2).

Using the above expression, the Fourier Transform of the total (s-wave plus dipole-dipole) 1D reduced interatomic potential is

$$U_{\text{tot}}^z(k_z) = 4E_R a \left((k_L w_r)^{-2} + IQ(w_r, k_z)\right)$$

where $E_R = h^2 k_L^2 / 2m$ is the photon recoil energy of an atom and $I$ is the dimensionless laser ‘intensity’ parameter

$$I = \frac{I_\alpha^2(\omega) m}{8\pi^2 k_0^2 \hbar^2 a}.$$ (10)

It is emphasized that the radial degree of freedom is contained in $\tilde{u}$ via the radius $w_r$. Note that the Fourier Transform of the total potential is an even function: $U_{\text{tot}}^z(k_z) = U_{\text{tot}}^z(-k_z)$.

3 Hartree-Fock-Bogoliubov-Popov method

The Hartree-Fock-Bogoliubov (HFB) method is concisely described in [12]. A reasonable approach within the HFB method is the Popov approximation [11], which neglects the contribution of the above-condensate fraction to the anomalous correlation function. The advantage of the Popov approximation is that it gives a gapless spectrum of elementary excitations of a degenerate bosonic system in a wide temperature range (up to the critical temperature). In the present paper we apply the Popov approximation to a quasi-1D BEC with the laser-induced dipole-dipole interactions at finite temperature. Of course, for an infinitely long 1D gas there is no condensate, even if this gas is non-ideal. This is because the depletion diverges at infinitely long wavelength. However, taking into account the finite length $\ell$ of the sample, we set an effective cutoff at a wavelength $\sim \ell$ and thus remove the divergence. Hence, a macroscopic population of the ground state becomes possible (in the case of an interacting gas).

The 1D ansatz for the atomic field operator is $\hat{\Psi}(x, y, z, t) = \hat{\psi}(z, t) \exp[-(x^2 + y^2)/(2w_r^2)]/\sqrt{\pi w_r}$. Then the 1D Hamiltonian reads as

$$\hat{H} = \int dz \hat{\psi}^\dagger(z) \left(\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2}\right) \hat{\psi}(z) + \frac{1}{2} \int dz \int dz' \hat{\psi}^\dagger(z') \hat{\psi}^\dagger(z) U_{\text{tot}}^z(z - z') \hat{\psi}(z) \hat{\psi}(z'),$$

where $U_{\text{tot}}^z(z)$ is the inverse Fourier Transform of $U_{\text{tot}}^z(k_z)$ defined by Eq. (9). Decomposing the atomic field operator in plane waves $\hat{\psi}(z) = \sum_k \ell^{-1/2} \exp(ikz) \hat{a}_k$ we rewrite Eq. (11) in the momentum space:

$$\hat{H} = \sum_k \left(\frac{\hbar^2 k^2}{2m} \hat{a}_k^\dagger \hat{a}_k + \frac{1}{2\ell} \sum_{k', q} \tilde{U}_{\text{tot}}^z(q) \hat{a}_{k+q}^\dagger \hat{a}_{k'} \hat{a}_k \hat{a}_{k'+q} \right).$$

The Hamiltonian of Eq. (12) applies to a system with a fixed number of particles. For simplicity we will work within the grand canonical ensemble and use the chemical potential, $\mu$, to fix the mean particle number $N$. Then the Heisenberg equation of motion for the atomic annihilation operators $\hat{a}_k$, is given by the commutator of $\hat{a}_k$ with $\hat{H} - \mu \sum_k \hat{a}_k^\dagger \hat{a}_k$, and reads

$$i\hbar \frac{\partial}{\partial t} \hat{a}_k = \left(\frac{\hbar^2 k^2}{2m} - \mu\right) \hat{a}_k + \frac{1}{2\ell} \sum_{k', q} \tilde{U}_{\text{tot}}^z(q) \hat{a}_{k+q}^\dagger \hat{a}_{k'} \hat{a}_k \hat{a}_{k'+q}.$$ (13)
Since the state with zero momentum is macroscopically populated, we invoke broken symmetry arguments and replace $\hat{a}_0$ by a c-number $\sqrt{N_c} \exp(i\varphi)$, where $N_c$ is the number of atoms in the condensate. The phase $\varphi$ is not an observable quantity, and its choice is arbitrary, thus we set $\varphi = 0$. The linear (1D) condensate density is defined as $n_c = N_c/\ell$. Atoms with $k \neq 0$ comprise the above-condensate fraction (i.e., depletion) with the linear density

$$n_a = \frac{1}{\ell} \sum_{k \neq 0} \langle \hat{a}^\dagger_k \hat{a}_k \rangle,$$  

which, together with $n_c$, yields the total linear density

$$n = n_c + n_a \equiv N/\ell.$$  

Finally, in the Popov approximation the chemical potential is given by the expression

$$\mu = \tilde{U}_\text{tot}^z(0)(n_c + n_a) + \tilde{W}(0),$$  

where

$$\tilde{W}(k) = \frac{1}{\ell} \sum_{k' \neq -k} \tilde{U}_\text{tot}^z(k') \langle \hat{a}^\dagger_{k'-k} \hat{a}_{k'}, k \rangle.$$  

For the modes with $k \neq 0$ we obtain, from Eq. (13)

$$i\hbar \frac{\partial}{\partial t} \hat{a}_k = \left( \frac{\hbar^2 k^2}{2m} - \mu \right) \hat{a}_k + \left[ \tilde{U}_\text{tot}^z(0) + \tilde{U}_\text{tot}^z(k) \right] n_c \hat{a}_k + \left[ \tilde{U}_\text{tot}^z(0)n_a + \tilde{W}(k) \right] \hat{a}_k + \tilde{U}_\text{tot}^z(k)n_c \hat{a}^\dagger_{-k}. \tag{17}$$

When deriving Eq. (17), we take into account that $\langle \hat{a}^\dagger_k \hat{a}_{k'} \rangle = \langle \hat{a}^\dagger_k \hat{a}_k \rangle \delta_{k, k'}$. Also note that the anomalous correlation functions for the above-condensate operators, such as $\langle \hat{a}_k \hat{a}_{k'} \rangle$, are neglected in Eqs. (15, 17), as required by the Popov approximation.

We now implement the standard Bogoliubov transformation $\hat{a}_k = u_k \exp(-i\omega_k t) \hat{b}_k - v_k \exp(i\omega_k t) \hat{b}^\dagger_{-k},$ where the new (quasiparticle) operators $\hat{b}_k$, $\hat{b}^\dagger_{-k}$ obey the usual bosonic commutation relations. The solution of the generalized eigenvalue problem gives the dispersion relation

$$\hbar \omega_k = \sqrt{T(k) \left[ T(k) + 2\tilde{U}_\text{tot}^z(k)n_c \right]}, \tag{18}$$

where

$$T(k) = \frac{\hbar^2 k^2}{2m} + \tilde{W}(k) - \tilde{W}(0). \tag{19}$$

The transformation coefficients are

$$u_k = \sqrt{\frac{T(k) + \tilde{U}_\text{tot}^z(k) n_a}{2\hbar \omega_k}} + \frac{1}{2}, \quad v_k = \sqrt{\frac{T(k) + \tilde{U}_\text{tot}^z(k) n_a}{2\hbar \omega_k}} - \frac{1}{2}. \tag{20}$$
Note that if $\hat{U}_{\text{tot}}(k)$ were $k$-independent (as it is in the case of absence of the external laser radiation, $I = 0$) then $T(k)$ would coincide with the free atom kinetic energy, and the well-known dispersion relation for a BEC with short-range interactions in the Popov approximation [12, 11] would hold. At a finite temperature $\Theta$ (measured in energy units) we have [12]

$$\langle \hat{a}_k \hat{a}_k \rangle = u_k^2 \exp(\hbar \omega_k / \Theta) - 1 + v_k^2. \tag{21}$$

The Hartree-Fock-Bogoliubov-Popov equations should be solved self-consistently to obtain the condensate and above-condensate occupation numbers, under the constraint that the total density is fixed, $n = n_c + n_a$. This involves an iterative procedure [12]:

1) As a starting point we choose the occupation numbers given by the $\{u_k, v_k\}$ coefficients found from the basic Bogoliubov scheme [5] (which does not treat the depletion self-consistently).
2) The occupation numbers are used to calculate $\hat{W}(k)$ [Eq. (16)] and hence the dispersion relation $\hbar \omega_k$ [Eq. (18)].
3) The occupation numbers are summed to give the new total density $n'$, which is used to renormalize the occupation numbers of the condensate and above condensate modes, in order to keep the total density fixed.
4) The dispersion relation, incorporating the new occupation numbers, is then used to recalculate the $\{u_k, v_k\}$ coefficients [Eq. (20)].
5) Repeat above steps 2–4 until convergence is achieved.

4 Results and Discussion

We have solved the HFB equations with the Popov approximation, as described above for the basic setup shown in Figure 1. In Figures 3–6 we present the results for two sets of parameters:

Case 1: $k_L w_r = 2.4$, $na = 0.66$, $\ell / w_r = 74$, $\Theta / E_R = 0.68$, $I = 1.12$;
Case 2: $k_L w_r = 8.0$, $na = 4.7$, $\ell / w_r = 28$, $\Theta / E_R = 2.3$, $I = 0.28$.

For example, for $^{87}\text{Rb}$ these sets of parameters mean explicitly:

Case 1: The condensate radius $w_r$ is set at 0.3 $\mu$m, the 3D density at the trap centre is equal to $4 \cdot 10^{14}$ cm$^{-3}$, $\ell = 22$ $\mu$m, $\Theta = 120$ nK. The laser intensity can be found from the relation $I / \Delta^2 = 0.26$ W/(cm $\cdot$ GHz)$^2$, where $\Delta$ is the laser detuning from resonance.

Case 2: $w_r = 1.0$ $\mu$m, the peak 3D density is the same as in case 1, $\ell = 28$ $\mu$m, $\Theta = 410$ nK, $I / \Delta^2 = 0.064$ W/(cm $\cdot$ GHz)$^2$.

Case 1 is representative of a BEC at the crossover between the ideal-gas and Thomas-Fermi limits with respect to the radial motion, while in case 2 the radial motion satisfies the conditions of the Thomas-Fermi limit. The parameters in cases 1 and 2 are chosen so that a significant ‘roton’ dip appears only in the more sophisticated Hartree-Fock-Bogoliubov-Popov method, but not in the simple Bogoliubov approach.

In each of Figures 3–6 we have plotted three curves. The solid line gives the results of the Hartree-Fock-Bogoliubov-Popov calculation described in section 3. At convergence we found that in case 1 55% of atoms remain in the condensate and in case 2 the condensate fraction is 82%. The two other curves in the figures give the results obtained within the simple Bogoliubov approach and differ in the density that appears in the Bogoliubov dispersion relation. The variant (a) uses as the density in the Bogoliubov dispersion relation the condensate density $n_c$ obtained in the numerical calculations. The variant (b) uses in the same context as the density the total density of the system, $n = n_c + n_a$. Similarly, we calculate
the depletion within the simple Bogoliubov approach for the variants (a) and (b). The position of the dip centre remains unchanged with temperature.

Figures 3 and 5 show that when the depletion is taken into account self-consistently, the ‘roton’ dip in the excitation spectrum can actually become deeper. This means that the ‘roton’ dip persists at finite temperatures (of course, below the condensation temperature). Thus the sample heating due to spontaneous scattering of laser photons need not prevent observation of the ‘roton’ dip in the spectrum of the elementary excitations. Contrary to what one might expect, finite temperature effects not only do not wash out or diminish these intermode correlations, but rather serve to enhance them. This is surprising, considering the noisy character of finite-temperature density fluctuations. This might in part be because the dipole-dipole potential has a momentum dependence such that it is more effective at finite interatomic momenta (see Fig. 2).

One may ask: what happens if the laser intensity is increased so that the roton dip passes through the zero-energy axis, so that we obtain imaginary excitation frequencies? Two scenarios are then possible. The first one is that this would create a dynamical instability similar to that found in a BEC of atoms with negative scattering length [24]. The instability would grow exponentially, finally leaving us with a totally depleted BEC. Another scenario is the following: when the laser intensity reaches the value at which the roton minimum touches the zero-energy axis we may access a new ground state that is periodically modulated along the $z$-axis: a supersolid. An analogous behaviour had been predicted for superfluid liquid helium passing through a pipe [25]. The formation of a supersolid state in a BEC with laser-induced dipole-dipole interactions has recently been studied [26] by numerically integrating the time-dependent generalized Gross-Pitaevskii equation describing a cigar-shaped BEC irradiated by a circularly polarized laser (in contrast to the linear case considered in the present paper), whose wave vector is along the major axis, $z$, of the cigar. This new supersolid state has to be stable against small perturbations. One can choose between these two scenarios by numerical investigation of the BEC dynamics in the case of gradually-increasing laser intensity, based on a time-dependent generalization of the Popov approximation. Such an investigation is beyond the scope of the present paper.

Finally, we would like to briefly mention experimentally detectable signatures of the roton minimum. Whilst the most convincing method would certainly be to map out the dispersion formula using Bragg spectroscopy, as in the measurement by Steinhauer et al [9], Bragg spectroscopy is technically difficult and time consuming. A simpler method would be to perform a time-of-flight measurement after releasing the BEC from the trap in order to detect the enhanced number of atoms having momenta around the roton minimum.

To summarize, we have presented a method allowing self-consistent calculation of the effects of laser-induced dipole-dipole interactions upon a BEC in the presence of significant condensate depletion. This is especially relevant to the experimental situation where the temperature cannot be significantly lower than the chemical potential. This may help us cope with the adverse effects of the Rayleigh scattering of laser light [3], which tend to heat up the sample.

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Figure 1: The laser beam and condensate geometry.

Figure 2: Comparison of the exact (dashed line) and approximate (solid line) expressions for $F(\xi, \zeta)$ for
(1) $\zeta = 2.9$, the solid and dashed lines are practically indistinguishable, (2) $\zeta = 32$, the difference between
the two lines is small but visible.
Figure 3: The spectrum of the elementary excitations of a BEC with laser-induced dipole-dipole interactions, for the set of parameters of case (1), see text. The solid line corresponds to the exact numerical solution, the dashed and dot-dashed lines correspond to the Bogoliubov spectrum for the variants (a) and (b), respectively. The frequency unit is the recoil frequency $\omega_R = E_R/\hbar$.

Figure 4: The depletion (above condensate fraction) of a BEC with laser-induced dipole-dipole interactions, for the set of parameters of case (1), see text. The solid line corresponds to the exact numerical solution, the dashed and dot-dashed lines correspond to the depletion calculated for the variants (a) and (b), respectively. Note the correspondence between the position of the peak in the mean occupation number and that of the ‘roton’ dip in Fig. 3 (see Ref. 3).
Figure 5: The same as in Fig. 3 for the set of parameters of case (2), see text.

Figure 6: The same as in Fig. 4 for the set of parameters of case (2), see text.