Sharp Magnetic Field Dependence of the 2D Hall Coefficient Induced by Classical Memory Effects

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We show that a sharp dependence of the Hall coefficient $R$ on the magnetic field $B$ arises in two-dimensional electron systems with randomly located strong scatterers. The phenomenon is due to classical memory effects. We calculate analytically the dependence $R(B)$ for the case of scattering by hard disks of radius $a$, randomly distributed with concentration $n_0 \ll 1/a^2$. We demonstrate that in very weak magnetic fields ($\omega \tau \lesssim n_0 a^2$) memory effects lead to a considerable renormalization of the Boltzmann value of the Hall coefficient: $\delta R/R \sim 1$. With increasing magnetic field, the relative correction to $R$ decreases, then changes sign, and saturates at the value $\delta R/R \sim -n_0 a^2$. We also discuss the effect of the smooth disorder on the dependence of $R$ on $B$.

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The simplest theoretical description of the magnetotransport properties of the two-dimensional (2D) degenerated electron gas is based on the Boltzmann equation which yields the well-known expressions for the components of the resistivity tensor:

$$\rho_{xx} = \frac{m}{e^2 n \tau}, \quad \rho_{xy} = \frac{m \omega_c}{e^2 n} = -RB. \quad (1)$$

Here $\tau$ is the transport scattering time, $\omega_c = |e|B/mc$ is the cyclotron frequency, $R = 1/en < 0$ is the Hall coefficient, and $n$ is the electron concentration. Thus, in the frame of the Boltzmann approach, $\rho_{xx}$ and $R$ do not depend on magnetic field $B$. Experimental measurements of $\rho_{xx}$ and $R$ are widely used to find $\tau$ and $n$.

It is known, that Eqs. (1) may become invalid due to a number of effects of both quantum and classical nature. The most remarkable of them is the Quantum Hall Effect. Another quantum effect, weak localization, leads to negative magnetoresistance (MR) – the decrease of $\rho_{xx}$ with $B$, concentrated in the region of weak magnetic fields [1]. Besides, the dependence of $\rho_{xx}$ on $B$ appears due to quantum effects related to electron-electron interaction [2] (see also [3] for review). At the same time, both weak localization and electron-electron interaction (in frame of standard Altshuler-Aronov theory) do not result in any dependence of $R$ on $B$ (see [4] and [2, 5, 6], respectively).

The dependence of $\rho_{xx}$ on $B$ may also be caused by classical effects. One of the reasons is that in the Boltzmann approach one neglects classical memory effects (ME) arising as a manifestation of non-Markovian nature of electron dynamics in a static random potential. Physically, a diffusive electron returning to a certain region of space “remembers” the random potential landscape in this region, so its motion is not purely chaotic as it is assumed in the Boltzmann picture. For $B = 0$, non-Markovian corrections to kinetic coefficients are usually small. In particular, in a case of hard-core scatterers of radius $a$ (impenetrable disks) randomly distributed with concentration $n_0$, ME-induced relative correction to the resistivity is proportional to the gas parameter $\beta_0 = a/l = 2n_0 a^2 \ll 1$ ($l = 1/2a_0$ is the mean free path). However, for $B \neq 0$ the role of ME is dramatically increased due to a strong dependence of return probability on $B$. Recent studies demonstrated that ME lead to a variety of non-trivial magnetotransport phenomena in 2D disordered systems such as magnetic-field-induced classical localization [7, 8, 9, 10, 11] and positive MR [12], low-field anomalous MR [13, 14, 15], and non-Lorentzian shape of cyclotron resonance [10].

In spite of large number of publications, devoted to the study of the influence of the non-Markovian effects on the MR, the dependence of $R$ on $B$, induced by such effects, was investigated (to the best of our knowledge) only in the context of so-called “circling electrons” [7]. These electrons occupy closed cyclotron orbits which avoid scatterers. As a consequence, they do not participate in diffusion. Though the existence of circling orbits leads to a strong dependence of $\rho_{xx}$ on $B$ in the region of classically strong $B$ ($\omega \tau \gg 1$), the corresponding dependence of $R$ on $B$ was found to be very weak in the whole range of $B$ [7].
In this paper, we propose another mechanism of dependence of $R$ on $B$. It does not rely upon the existence of non-colliding electrons but, in contrast, assumes that transport properties of colliding electrons are modified by classical ME. The mechanism turns out to be especially effective in the region of very weak fields, $\omega_c \tau \lesssim \beta_0$.

We will study dependence of $R$ on $B$ in 2D degenerated electron gas in a system of randomly located hard core scatterers. We restrict ourselves to the study of the case of classically weak fields ($\omega_c \tau \ll 1$) and assume that Fermi wavelength $\hbar/mv_F$ is much smaller than $a$. The latter assumption will allow us to study the electron dynamics on the classical level.

We start with recalling that in the frame of the Boltzmann approach, the collision with a single scatterer is described by differential scattering cross-section $\sigma(\theta)$ (see Fig. 1a) and the collisions with different scatterers are independent. Inverting in time the process shown in Fig. 1a we get a process shown in Fig. 1a, corresponding to scattering by the angle $-\theta$. This implies an important property of a single scattering – the symmetry with respect to replacement of $\theta$ by $-\theta$ (reciprocity theorem): $\sigma(\theta) = \sigma(-\theta)$ [17]. This is the property which provides that the cross-section $\sigma(\theta)$ remains symmetric: $\delta\sigma(\theta) \neq \delta\sigma(-\theta)$, the expression for $\sigma_{xy}$ becomes $\rho_{xy} = m(\omega_c + \Omega)/e^2n = -B(R + \delta R)$, where

$$\Omega = -n_0v_F \int d\theta \delta\sigma(\theta) \sin \theta, \quad \frac{\delta R}{R} = \frac{\Omega}{\omega_c},$$

and $v_F$ is the Fermi velocity. In particular, such an asymmetric correction arises due to ME specific for processes of double scattering on a scatterer after return to it (see Fig. 1b,c). Though such processes are beyond the Boltzmann picture, they can be formally included into the kinetic equation by a slight modification of the Boltzmann collision integral. Specifically, one can introduce a small change of the scattering cross-section $\sigma(\theta) \to \sigma(\theta) + \delta\sigma(\theta)$ on the disk where double scattering takes place (disk 1 in Fig. 1b,c) [18,19]. For $B = 0$, cross-section remains symmetric: $\delta\sigma(\theta) = \delta\sigma(-\theta)$. However, for $B \neq 0$ the time inversion symmetry is broken, so that the cross-section becomes asymmetric: $\delta\sigma(\theta) \neq \delta\sigma(-\theta)$. The point is that the influence of the magnetic field is different for the processes where closed return path is passed counterclockwise (Fig. 1b,c) and clockwise (Fig. 1b,c).

![FIG. 1: Processes of single scattering by angle $\theta$ (a) and $-\theta$ (a') characterized by a scattering cross-section $\sigma(\theta)$ ($\sigma(\theta) = \sigma(-\theta)$) both for $B = 0$ and for $B \neq 0$, and processes of scattering on complexes of scatterers (b,b',c,c') including double scattering on scatterer 1. Correction to the cross-section due to multi-scattering processes remains symmetric for $B = 0$. Magnetic field bends trajectories as shown in b,b',c,c' by dashed lines. As a result, the symmetry with respect to inversion of $\theta$ is broken, so that $\delta\sigma(\theta) \neq \delta\sigma(-\theta)$ for $B \neq 0$.](image)
of the passage $2 \rightarrow 1$. Here $S_0$ is the area of the overlap of the two corridors, surrounding segments $1 \rightarrow 2$ and $2 \rightarrow 1$, respectively. Hence, $W = \exp (-2r/l + n_gS_0)$. The magnetic field pulls out (together) forward and backward trajectories for process shown in Fig. 1c (Fig. 1c') thus decreasing (increasing) $S_0$ and leading to a sharp dependence of $W$ on $B$. The corresponding correction to $\rho_{xx}$ was calculated numerically in [13] and analytically in [14].

The calculation of $R$ is quite analogous to the calculation of $\rho_{xx}$ presented in [14]. The easiest way to find $R$ is to use the expression for $\delta \sigma(\theta)$ derived in [14]:

$$
\delta \sigma(\theta) = \frac{1}{4l} \int_{\phi_0}^{\phi_f} \frac{d \phi}{2\pi} e^{-2r/l} \int_0^{2\pi} d \psi \int_0^{2\pi} d \psi_f \left( \sigma(\phi_0) \sigma(\phi_f) e^{n_0 S_0 [\delta(\theta - \phi_{\phi_0}) + \delta(\theta - \pi) - \delta(\theta - \phi_{\phi_f}) - \delta(\theta - \phi_{\phi_0})] \right)
$$

Here $\phi_{\phi_0,\phi_f} = (\pi + \phi_0 + \phi_f)(\text{mod} 2\pi)$, $\sigma(\phi) = (a/2)^2 \sin(\phi/2)$ is the single scattering cross-section, $S_0 = \int_0^\infty d \alpha d \theta d \phi (2 \alpha - |\phi - r^2/R_c|) \theta [2 \alpha - |\phi - r^2/R_c|]$ is the Heaviside step function, $\phi = \Phi + r/R_c$, $R_c$ is the cyclotron radius and $\Phi \approx (a/r) [\cos(\phi_0/2) + \cos(\phi_f/2)]$ (see Fig. 1c). Introducing dimensionless variables $T = r/l$, $z = \omega_c \tau / \beta_0$ and using Eq. (2), we get

$$
\frac{\delta R}{R} = g(z) = - \int_0^\infty \frac{d T}{T} e^{-2T} \int_0^\pi d \alpha \int_0^{2\pi} d \gamma \times \sin(\alpha + \gamma) \sin^2 \alpha \sin^2 \gamma \frac{e^{x_0} - e^{x_0}}{2z}
$$

Here $x_0 = \int_0^\infty d t \left( 1 - \left| \zeta - \frac{x_0^2}{2} \right| \right) \theta \left( 1 - \left| \zeta - \frac{x_0^2}{2} \right| \right)$, $\zeta = (\cos \alpha + \cos \gamma) / 2 T + z T / 2$, $s_0 = s_{\tau = 0}$. Function $g(z)$ calculated numerically with the use of Eq. (4) is plotted in Fig. 2. For $z \ll 1$, $g(z) \approx 0.064 - 4z^2$. For $z \gg 1$, $g(z)$ decreases as $0.35/\sqrt{z}$. It worth emphasizing that $\delta R/R \sim 1$ for $z \ll 1$. This means that the correction is not parametrically small in a gas parameter $\beta_0$ which is usually considered as expansion parameter for ME-induced corrections.

Next we calculate $\delta R$ for stronger fields, $\beta_0 \ll \omega_c \tau \ll 1$. At such fields empty corridor effect is suppressed and returns after one scattering (Fig. 1c,c') and after a number of scatterings (Fig. 1b,b') equally contribute to $\delta R$. In this case, one can also introduce the effective scattering cross-section [19] which turns out to be frequency-dependent and for $\omega = 0$ reads [18]

$$
\delta \sigma(\theta - \theta') = v_F \int [\sigma(\theta - \varphi) - \sigma_0 \delta(\theta - \varphi)]
$$

FIG. 2: Magnetic field dependence of the relative correction to the Hall coefficient caused by empty corridor effect.

$$
\times \tilde{G}(0, \varphi - \varphi') \sigma(\varphi' - \theta') - \sigma_0 \delta(\varphi' - \theta') d \varphi d \varphi'.
$$

Here $\sigma_0 = \int d \varphi \sigma(\varphi)$ is the total cross-section for single scattering, $G(0, \varphi - \varphi') = \tilde{G}(r, \varphi, \varphi')|_{r = 0}$, $G(0, \varphi, \varphi') = G(r, \varphi, \varphi') - G^{\text{ball}}(r, \varphi, \varphi')$, $G(r, \varphi, \varphi')$ is the Green function of the stationary Boltzmann equation, $G^{\text{ball}}(r, \varphi, \varphi') = \exp(-1/\beta r) \delta(\varphi - \varphi + \theta_r r/2) \delta(\varphi' - \varphi - \theta_r r/2)$ is the Green function of the Boltzmann equation without in-scattering term, $\varphi_r$ is the angle of vector $r$, and $\theta_r = 2 \arcsin(3r/2l)$. Substituting Eq. (5) into Eq. (2) and using the property $\int d \varphi d \varphi' G(0, \varphi, \varphi') \sin(\varphi - \varphi') = 0$ [20], we get after some algebra

$$
\frac{\delta R}{R} = - n_0 r^2 / (2\pi) \ll 1,
$$

where $\sigma_{\tau r} = \int d \theta \sigma(\theta)(1 - \cos \theta) = 8a/3$. Hence, with increasing $B$ relative correction decreases according to Eq. (4), then changes sign and saturates at small negative value. It is noteworthy that, as follows from the above derivation, Eq. (6) is valid not only for the case of impenetrable disks but also for any type of well-separated scatterers.

Above we discussed an idealized system where only strong scatterers are present. Let us now assume that in addition to strong scatterers there is a weak smooth random potential $U(r)$ with the rms amplitude $U$ and the correlation length $d$ ($a \ll d \ll l$). The presence of such a potential does not influence the empty corridor effect provided that $\lambda \gg l$, where $\lambda \sim d(E_F/U)^{2/3}$ is the Lyapunov length, characterizing the di-
vergence of the electron trajectories in the potential $U(r)$. In the opposite limit, $\lambda \ll l$, one should restrict integration over $r$ in Eq. (4) by $\lambda$. In this case, relative correction to $R$ decreases: $\delta R/R \sim \lambda/l$. On the other hand, the field needed for suppression of the empty corridor effect increases and can be found from the following estimate $\omega_c \tau \sim (l/\lambda)^2$. At such a field two corridors, corresponding to passage $1 \to 2$ and $2 \to 1$ (see Fig. 1c,c˙) between disks 1 and 2 separated by a distance $r \approx \lambda$, cease to overlap.

To study the effect of the smooth disorder at stronger fields, one should add a term $(F/m)\partial/\partial \nu$ in the l.h.s. of the Boltzmann equation, where $F = - \partial U/\partial r$. Treating this term as a small perturbation, we find correction to the Boltzmann collision integral $\delta T = (F(\partial/\partial \nu) G F(\partial/\partial \nu)/m^2)$, where $\langle \cdots \rangle$ stands for averaging over realizations of $U(r)$ and $G$ is the operator with the kernel $G(r, \varphi, \varphi')$. This kernel can not be found explicitly for the above-discussed case of scattering on impenetrable disks. However, the exact solution is possible for short range scatterers, where $\sigma(\theta) = \sigma_0/2\pi = \text{const}$. In this case, after cumbersome but straightforward calculations we get (in addition to Eq. (4))

$$\frac{\delta R}{R} \approx \frac{17(\omega_c \tau)^2}{192 E_F^2 l^2} \int_0^\infty dr r K(r) \sim -(\omega_c \tau) \frac{2^2 U^2}{l^2 E_F^2}.$$ 

We see that smooth disorder leads to appearing of a very weak parabolic dependence of $R$ on $B$.

To conclude, we have shown that in a 2D system with rare hard-core scatterers classical ME lead to a very sharp dependence of $R$ on $B$ concentrated in the region of very weak fields ($\omega_c \tau \ll a/l$). The total variation of $R$ in this region of fields is on the order of the Boltzmann value of $R$. At larger fields, where $a/l \ll \omega_c \tau \ll 1$, the ME lead to a small field-independent correction to $R$ and (in presence of smooth disorder) to a very weak parabolic dependence.

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