Quantum Hall Effect under Rotation and Mass of the Laughlin Quasiparticles

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We consider the quantum Hall effect induced by magnetic field and rotation, which can drive the Hall samples into the quantum Hall regime and induce fractional excitations. Both the mass and the charge of the Laughlin quasiparticles are predicted to be fractionally quantized. The observable effects induced by rotation are discussed. Based on the usual Hall samples under rotation, we propose an experimental setup for detecting the macroscopic quantization phenomena and the fractional mass of the Laughlin quasiparticles.

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In condensed matter, two-dimensional (2D) electron gases are of particular interest due to the discovery of fractional quantum Hall (QH) effect by Tsui, Stormer, and Gossard [1]. It is one of the spectacular macroscopic quantization phenomena and has been investigated for two decades both theoretically and experimentally [2]. One of the exotic aspects is the Laughlin quasiparticles of fractional charge $e/(l+\frac{1}{2})$ (l odd for fundamental fractional filling fraction) [12] which has been demonstrated recently by several beautiful experiments [17]. Similar ideas have been explored in another strongly correlated system—dilute cold neutral atoms in Bose-Einstein condensates (BECs) in recent years [3, 4, 5, 6]. The rapidly rotating BECs, which can be treated as 2D interacting, atoms, will lie in the lowest landau level (LLL) and the nondegenerate ground state is of the form of the famous Laughlin state [3, 4]. In our previous paper [6], we predicted some macroscopic quantization phenomena which are the “atomic twin of the electronic brother” on the base of the similarity between these two strongly correlated many-body systems. The quantized “mass-conductance” $\sigma^{(m)}$ and the quasiparticles of fractional mass replace, respectively, the electron-conductance $\sigma^{(e)}$ and fractionally charged quasiparticles in the usual QH effect. But the requirements for observing similar QH physics in BECs are demanding due to the difficulty in the current BEC experiments to deposit a large enough angular momentum to reach the QH regime.

Essentially, the phenomena predicted in Ref. [6] in rapidly-rotating strongly-correlated cold atoms are induced by the noninertial force on massive particles in the rotating frame. Thus, these effects should also occur in other strongly correlated many-body systems, even in the electron gases. The detection of effects induced by rotation in magnetizable and metals has a history of more than one hundred year [7]. The famous experiment on this respect was done by Barnett who measured the magnetic field induced by rotation in 1915 [8]. Recent experiments on the rotating effect were to measure the mass of the current carriers (Cooper pairs) in superconductors [9]. Recently, Fischer et al. [10] have studied the rotation in QH samples using a twofold $U(1)$ gauge invariance.

The charge of the current carriers (the Laughlin quasiparticles) in fractional QH effect is fractionized as $e/ve$ for simple Hall droplet, as is now well known. Now a fascinating question arises: Is the mass of the fractionally charged quasiparticles in a fractional QH liquid also fractionized? To the best of our knowledge, this question has not been investigated up to date. Its answer is not as obvious as in a superconductor where the current carriers are the Cooper pairs with mass $m_{CP}$ being duplication of electron’s mass $m_e$, namely, $m_{CP} = 2m_e$. In usual quantum Hall effect an excitation can be induced by piercing a magnetic flux $\Phi_c = \frac{\pi\hbar}{e}$. However, there is no physical principle to determine the mass of the excitations.

Motivated by our previous work [6], in this paper we deal with such kind of problems from a microscopic view. Namely, we are concerned with QH effect under rotation. Of particular interest is the properties (e.g., the mass) of the fractional excitations. By using the usual QH samples under rotation, we propose an experiment to measure the fractional mass of the Laughlin quasiparticles, which could be tested under current experimental conditions.

First, we consider a 2D rotating QH sample (see Fig. 1) under a high magnetic field $-B\hat{z}$, which is perpendicular to the 2D plane. Assume that the sample is confined by a harmonic potential with trapping frequency $\omega_0$ and then rotated, along the $\hat{z}$-direction, at an angular frequency $\omega = \omega_0$. In the rotating frame the 2D electrons are described by

$$H_{tot} = \sum_i H_i + \sum_{i<j} V(r_i - r_j). \quad (1)$$

Here $H_i = \left< \frac{p_i^2}{2m} + e\hat{z} \cdot \hat{B} \right>$ is the single-body Hamiltonian; $V(r_i - r_j)$ describes the two-body interaction, which in this case is the Coulomb interaction $\frac{1}{|r_i - r_j|}$ between electrons as is familiar in QH effect. The Hamiltonian (1) can be naturally realized in a rotating quantum-dot system under the same conditions specified above.

Now let us first consider the single-body Hamiltonian. As is done in our previous paper [6], one can deal with the rotation and the magnetic field in a uniform way,
one inputs a particle current in the sample, the current
E where \(A= -\frac{i}{2} B \vec{z} \times \mathbf{r} \) (with \(B = \nabla \times A \) and \(B^* = \nabla \times A^* \)) and \(q^* \) is the effective charge. Then single-body Hamiltonian is simplified to be
\[
H = \frac{(\mathbf{p} - q^* A^*)^2}{2m} = \frac{[\mathbf{p} - (m\omega + \frac{eB}{\pi}) \vec{z} \times \mathbf{r}]^2}{2m},
\]
which is completely analogous to the Hamiltonian with only effective magnetic field \(B^* \). So when the magnetic field \(B \) or/and the rotating frequency \(\omega \) is large enough, the system can be studied only in the lowest landau level (LLL). In LLL the single-body states read \(\psi_l(\eta) = N_l \eta^l \exp[-|\eta|^2/(4a_0)^2] \) \((l = 0, 1, 2, ...)\) in terms of the 2D complex coordinate \(\eta = x + iy \equiv a_0 \bar{\eta} \). Here \(N_l = \sqrt{\pi l!(2a_0)^{l+1}} \) is the normalization constant and \(a_0 = \sqrt{\hbar/(2m\omega + eB)} \) is the effective magnetic length.

We can also define the filling fraction as usual
\[
\nu = \frac{2\pi n \hbar}{q^* B^*} = \frac{2\pi n \hbar}{2m\omega + eB}.
\]

In this case the many-body system will show some macroscopic quantum phenomena due to the repulsive two-body interaction as in the electron or atom QH effect. Its nondegenerate ground state is the Laughlin state
\[
\Psi_l = N_l \prod_{i<j} (\bar{\eta}_i - \eta_j) \exp[-\frac{i}{\hbar} \sum_i |\bar{\eta}_i|^2],
\]
which was first put forward by Laughlin in a classic paper in 1983 [12] to explain the fundamental filling fraction in the fractional QH effect. Here \(N_l \) is an unimportant normalization constant, and \(l > 0 \) is an even number for bosons or an odd number for fermions. Then in this state the filling fraction will be quantized as usual \(\nu = \frac{1}{l} \). The “atomic Hall-conductance” in atomic QH effect [6] and the usual quantized Hall-conductance in electron QH effect [2] is naturally unified as
\[
\frac{e^2}{\sigma^{(e)} 2\pi \hbar} + \frac{ma^2}{\sigma^{(m)} 2\pi \hbar} = l.
\]
This means that neither of the two Hall conductances is quantized, but their combination is quantized.

The equivalent electron-number current (the real current carriers are not the electron but the quasiparticles, as will be discussed below) is
\[
j_e = \frac{\nu}{2\pi \hbar} (eE_y + mg_y),
\]
where \(E_y \) and \(g_y \) are induced Hall electric field and gravitational-like acceleration field, respectively. Equation [6] is identical to the twofold \(U(1) \) gauge invariant current obtained in Ref. [10]. It means that when one inputs a particle current in the sample, the current will induce not only the Hall electrostatic field but also the gravitational-like acceleration field. The two induced fields are caused by the two-body interaction. Then the equivalent particle current induced by the change of magnetic field \(B \) (piercing a magnetic flux \(\Phi_e = \frac{2\pi \hbar}{e} \) or rotating velocity \(\omega \) (piercing a vortex \(\Phi_m = \frac{2\pi \hbar}{m} \)) can be described in a uniform way
\[
j = \frac{d\nu}{dt} = \frac{\nu}{2\pi \hbar} \frac{d\Phi}{dt}.
\]

where \(\Phi = m \oint \frac{\mathbf{r} \times d\mathbf{r} \cdot e}{\mathbf{A}} \) or piercing a vortex \(\Phi_m = \frac{2\pi \hbar}{m} \) in a disk or cylinder sample will induce a particle current as long as the sample is in Hall regime.

The exotic fractional excitations, which are the current carriers, are of particular interest in electron QH effect [12] and in atom QH effect [6]. What is the relation between these two kinds of excitations? The answer to this question can be approached in the present case. As usual, the fractional charge should read \(Q^* = \frac{2\pi}{\nu} \). In order to see clearly its meaning we add the external field \(E^* \), which is induced by piercing flux \(\Phi \) [3] and reads
\[
Q^* E^* = \frac{eE + mg}{l} = \frac{e}{l} E + \frac{m}{l} g.
\]
This relation means that the quasiparticles will respond to the electric field as \(\frac{e}{l} \) and acceleration field as \(\frac{m}{l} \). This fact implies that the quasiparticles are of both the fractional charge and the fractional mass. Thus, piercing a magnetic flux \(\Phi_e = \frac{2\pi \hbar}{e} \) and/or piercing a vortex \(\Phi_m = \frac{2\pi \hbar}{m} \) in the rotating Hall sample will induce excitations from the unique vacuum state (i.e., the Laughlin state) and lead to the same Laughlin quasiparticle state [3, 12]. Consequently, the excitations possess simultaneously fractional charge and fractional mass. This result is necessarily the property of the nondegenerate ground state and does not depend on the origin of the ground state.
state. So what we got above should be correct even in usual non-rotating Hall sample under high magnetic field $\mathbf{B}$, namely, the fractional excitations with charge $\frac{2}{m}$ in electron QH effect possess the fractional mass $\frac{2}{m}$.

It seems that the electron-conductance will not be quantized as usual due to the relation $I_x = \frac{e}{\hbar}V(eV + m\Phi)$ ($\Phi$ is the induced gravitational-like potential $[9]$, $I_x$ is the equivalent electron current) if we add source and drain on the rotating QH sample. But this is not the case. In fact, the quantum Hall voltage or the gravitational-like potential are all caused by the two-body interaction, which is now the Columb interaction. The gravitational-like force is also induced by the two-body interaction and behaves as another electrostatic field $eV' \equiv m\Phi$. What is actually measured is the total Hall voltage $V_{tot} = V + V'$. Then the quantization relation in usual QH effect is formally preserved

$$\sigma^{(em)} = \frac{eI_x}{V_{tot}} = \nu \frac{e^2}{2\pi\hbar}. \quad (10)$$

Now let us consider how to measure the effects induced by rotation, in particular, the fractional mass of the quasiparticles. This issue is currently difficult for BECs for which the current experiments are far from the Hall regime. We turn to other samples, such as the electron Hall sample, to find a way to measure these effects for there have been some experiments to measure the rotating effects and the mass of Cooper pairs in superconductors $[8]$. In the following we will consider a special case and propose a method to measure the physical effects induced by rotation and the fractional mass of the Laughlin quasiparticles.

First we consider a usual Hall Corbino disk sample $[2]$ without rotation. The Hamiltonian describing the system is

$$H_{tot} = \sum_i H_i' + \sum_{i<j} V(r_i - r_j), \quad (11)$$

where $H_i' = \frac{(p_i + eA_i)^2}{2m}$. Laughlin’s gedanken experiment is to pierce a magnetic flux $\Phi = \frac{2\pi e}{\hbar}$, which will induce a fractional charge transfer in the Hall sample, as has already been demonstrated by experiments $[14, 15, 16]$. Based on the similar sample one can also measure the mass of the fractionally charged quasiparticles which are the current carriers in QH sample.

Let us consider the case of piercing a vortex $\Phi_m = \frac{2\pi e}{m}$ in the sample by rotating the fractional Hall Corbino disk. Due to Faraday’s law $[6, 10]$, this process will induce an acceleration field and excite the electron gas. As described above it will induce a radial Hall mass current $[6]

$$m_{jr} = \nu \frac{e^2}{2\pi\hbar}g_\phi. \quad (12)$$

Note that the current carriers are quasiparticles. Assuming $e^*$ and $m^*$ are the charge and mass of the quasiparticles, respectively, one can get the electric current $e_{jr} = \nu \frac{e^*}{m^*} \frac{e^2}{2\pi\hbar}g_\phi$ with $g_\phi = \frac{\Phi}{2\pi R}$. The charge transferred through this process is

$$Q = 2\pi R \int e_{jr}^* dt = \nu \frac{e^*}{m^*} \frac{m^2}{2\pi\hbar} \int dt \frac{e^*}{m^*} \frac{e^2}{2\pi\hbar}g_\phi$$

$$= \nu \frac{e^*}{m^*} \frac{m^2}{2\pi\hbar} \int dt \Phi = \nu \frac{e^*}{m^*} \frac{m^2}{2\pi\hbar} \frac{2\pi e}{\hbar} \frac{m^2}{2\pi\hbar}$$

$$= \nu e^* \frac{m^2}{m^*}. \quad (13)$$

According to the above results that $e^* = \frac{e}{m}$ and $m^* = \frac{m}{m}$ we will get $Q = \nu e$. This transferred charge is equal to piercing a flux $\Phi = \frac{2\pi e}{m}$. While the fractional charge of the Laughlin quasiparticles has been demonstrated by several experiments through measuring the shot noise in tunneling effect $[11]$, the fractional mass is unnoticed and thus, not determined up to date. So this process opens a door to determine the fractional mass of the Laughlin quasiparticles.

For this purpose we propose a way to measure the proportion $\frac{e^*}{m^*}$ between the charge and the mass of the Laughlin quasiparticles to demonstrate the fractional mass. The experimental device is depicted in Fig. 2. This device has been used in QH effect to test Laughlin’s gedanken experiment $[13, 14]$. The Corbino disk has reached the fractional quantum Hall regime under high magnetic field $\mathbf{B}$ perpendicular to the 2D plane. Then rotating the sample to excite the quasiparticles will induce charge to transfer from one edge to the other. The charge transferred in this process is

$$Q = \frac{e^*}{m^*} \frac{e^2}{m^*} \frac{e^2}{m^*} 2\pi\hbar \frac{2\pi e}{\hbar} \frac{2\pi e}{\hbar}$$

$$= \frac{e^*}{m^*} \frac{m^2}{2\pi\hbar} \frac{2\pi e}{\hbar} \frac{2\pi e}{\hbar}$$

$$= \frac{e^*}{m^*} \frac{m^2}{2\pi\hbar} \frac{2\pi e}{\hbar} \frac{2\pi e}{\hbar}.$$

FIG. 2: Schematic view of the Corbino sample to verify the fractional mass of the Laughlin quasiparticles. Rotating the sample to induce $g$ will excite the quasiparticles and induce a voltage in the capacitance. The voltage plateau will demonstrate the fractional mass.
where $R$ is the radius of the Corbino disk, $\triangle \omega$ is the rotation angular velocity. This transferred charge can be detected by measuring the voltage in capacitance, $V = \frac{Q}{C}$. Now the rotation only plays the role of exciting the QH sample. Since the accuracy of the typical measurements in QH effect is high, the rotation frequency does not need to be too large, say, about $10^3 \sim 10^4 \text{s}^{-1}$, is enough for current experimental conditions [8, 9, 10] to observe the predicted effects. The methods to observe similar phenomenon in QH effect by changing magnetic field in this sample are familiar in the usual electron QH effect, where the quantized conductance is measured. This result is also available to integer QH effect where $\nu = \text{integer}$ and the current carriers are electrons.

In summary, we have exploited the quantum Hall effect induced by magnetic field and/or rotation. In this case both the magnetic field and the rotation are available to reach the quantum Hall regime and can induce the fractional excitations. Both the mass and charge of the quasiparticles, which are the current carriers in the Hall sample, are fractionally quantized. The observable effects induced by rotation have been discussed. An experimental proof of the macroscopic quantization phenomena and the fractional mass of the Laughlin quasiparticles has been proposed. We hope that our prediction can be verified in the near future by using the usual quantum Hall samples under rotation.

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