MORE EFFICIENT ADVERSARIAL IMITATION LEARNING ALGORITHMS WITH KNOWN AND UNKNOWN TRANSITIONS*

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ABSTRACT

In this work, we design provably (more) efficient imitation learning algorithms that directly optimize policies from expert demonstrations. Firstly, when the transition function is known, we build on the nearly minimax optimal algorithm MIMIC-MD [Rajaraman et al., 2020] and relax a projection operator in it. Based on this change, we develop an adversarial imitation learning (AIL) algorithm named TAIL with a gradient-based optimization procedure. Accordingly, TAIL has the same sample complexity (i.e., the number of expert trajectories) \( \tilde{O}(H^{3/2}|S|/\varepsilon) \) with MIMIC-MD, where \( H \) is the planning horizon, \( |S| \) is the state space size and \( \varepsilon \) is desired policy value gap. This implies TAIL could be better than conventional AIL methods such as FEM and GTAL in the worst-case since they have a sample complexity \( \tilde{O}(H^2|S|/\varepsilon^2) \). In addition, TAIL is more practical than MIMIC-MD as the former has a space complexity \( O(|S||A|H) \) while the latter’s is about \( O(|S|^2|A|^2H^2) \). Secondly, under the scenario where the transition function is unknown but the interaction is allowed, we present an extension of TAIL named MB-TAIL. The sample complexity of MB-TAIL is still \( \tilde{O}(H^{3/2}|S|/\varepsilon) \) while the interaction complexity (i.e., the number of interaction episodes) is \( \tilde{O}(H^3|S|^2|A|/\varepsilon^2) \). In particular, MB-TAIL is significantly better than the best-known OAL algorithm in [Shani et al., 2021], which has a sample complexity \( \tilde{O}(H^2|S|/\varepsilon^2) \) and interaction complexity \( \tilde{O}(H^4|S|^2|A|/\varepsilon^2) \). The advances in MB-TAIL are based on a new framework that connects reward-free exploration and AIL. To our understanding, MB-TAIL is the first algorithm that shifts the advances in the known transition setting to the unknown transition setting. Finally, we provide numerical results to support our theoretical claims and to explain some empirical observations in practice.

1 Introduction

Reinforcement learning (RL) [Sutton and Barto, 2018] learns the optimal policy from trial and error in unknown environments and suffers from the sample efficiency issue in practice [Mnih et al., 2015, Lillicrap et al., 2016]. On the other hand, imitation learning (IL) [Pomerleau, 1991, Abbeel and Ng, 2004, Ross and Bagnell, 2010, Ho and Ermon, 2016] directly optimizes policies from expert demonstrations, which is more sample-efficient and has been successfully demonstrated in game playing [Ross et al., 2011, Silver et al., 2016], natural language processing [Daumé et al., 2009, Chang et al., 2015], recommendation system [Shi et al., 2019, Chen et al., 2019], and robotics control [Levine et al., 2016, Finn et al., 2016], etc.

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The target of IL is to optimize the policy without the environment reward function. To achieve this, given expert demonstrations, it minimizes the policy value gap between the expert policy and the imitated policy [Ross and Bagnell, 2010, Xu et al., 2020, Rajaraman et al., 2020]. Representative IL methods include behavioral cloning (BC) [Pomerleau, 1991, Ross et al., 2011] and adversarial imitation learning (AIL) [Abbeel and Ng, 2004, Syed and Schapire, 2007]. In particular, BC aims to imitate the expert policy distribution. More specifically, BC builds on supervised learning to minimize the policy distribution discrepancy with the expert policy on the given expert dataset. In contrast, AIL mainly follows the principle of state-action distribution matching [Abbeel and Ng, 2004, Syed and Schapire, 2007, Ho and Ermon, 2016]. In other words, AIL aims to optimize the policy such that the induced state-action distribution could match the counterpart generated by the expert. Algorithmically, it solves a min-max problem: the learner infers an adversarial reward function to maximize the policy value gap and subsequently learns a policy to minimize the policy value gap with the recovered reward function. Based on the above two principles, many practical algorithms [Torabi et al., 2018, Brantley et al., 2020, Fu et al., 2018, Ho and Ermon, 2016, Ke et al., 2019, Kostrikov et al., 2019, 2020] have been developed, and empirical studies suggest that AIL-based algorithms could be better than BC in practice [Ho and Ermon, 2016, Kostrikov et al., 2019, 2020].

**Context.** This paper focuses on the statistical foundations of IL approaches. Concretely, we care about the (expert) sample and interaction complexity, which refer to the number of expert trajectories and interaction episodes with the environment to achieve a small policy value gap, respectively. These two metrics are of interest in practice [Ho and Ermon, 2016, Kostrikov et al., 2019, 2020].

Firstly, when the transition function is known\(^1\), with high probability, the sample complexity of BC [Rajaraman et al., 2020] is \(\tilde{O}(H^2|S|/\varepsilon)\), where \(H\) is the planning horizon, \(|S|\) is the state space size and \(\varepsilon\) is the desired policy value gap. In contrast, the sample complexity of conventional\(^2\) AIL methods such as FEM [Abbeel and Ng, 2004] and GTAL [Syed and Schapire, 2007] is \(\tilde{O}(H^2|S|/\varepsilon^2)^3\). There is an issue coming to mind when interpreting this result.

- **Issue 1 (I1)** By comparison, we know that conventional AIL is inferior to BC when \(\varepsilon < 1\) in the worst-case, contracting with the common empirical observation that AIL usually outperforms BC. Therefore, an explanation is required to clarify whether this mismatch is caused by a coarse theoretical analysis or other reasons.

On the other hand, the lower bound under this scenario is \(\tilde{\Omega}(H^{3/2}/\varepsilon)\) [Rajaraman et al., 2021], suggesting BC and conventional AIL are not minimax optimal. Interestingly, another type of IL algorithm MIMIC-MD [Rajaraman et al., 2020] has a sample complexity \(\tilde{O}(H^{3/2}|S|/\varepsilon)\), which matches the lower bound in terms of \(H\) and \(\varepsilon\). Furthermore, Rajaraman et al. [2021] point out: 1) MIMIC-MD can be viewed as a special AIL method; 2) the main advance in MIMIC-MD is a transition-aware state-action distribution estimator (see (4.1)). Compared with the vanilla estimator (see (4.3)) in FEM and GTAL, this new estimator has a better statistical guarantee. However, the optimization problem in MIMIC-MD cannot be directly solved in polynomial time. Later on, Rajaraman et al. [2021] propose a linear programming (LP) based optimization procedure for MIMIC-MD, which is solvable in polynomial time.

- **Issue 2 (I2)** Unfortunately, the LP formulation for MIMIC-MD leads to a poor space complexity \(\tilde{O}(|S|^2|A|^2H^2)\), which is unbearable in practice when the state space (or the planning horizon) is large (see the evidence in Section 6). A natural question is: can we design a better algorithm that simultaneously matches the statistical lower bound and has a better space complexity?\(^4\)

Secondly, consider a practical setting where the transition function is unknown but the environment interaction is allowed, the main challenge is that the agent can not exactly evaluate a policy any longer. Several theoretical guarantees of the extensions of conventional AIL methods are well-known (shown in Table 1). In particular, seminal works such as FEM and GTAL leverage expert demonstrations to estimate the transition function to perform imitation. Correspondingly, their algorithms are impractical as their sample complexity is unacceptably large. To our best knowledge, only the online apprenticeship learning (OAL) algorithm in [Shani et al., 2021] is promising, which updates the policy and reward function using no-regret algorithms when interacting with the environment. In particular, OAL achieves an expert sample complexity \(\tilde{O}(H^2|S|/\varepsilon^2)\) and interaction complexity \(\tilde{O}(H^4|S|^2|A|/\varepsilon^2)^5\).

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\(^1\)Under this setting, the agent can exactly evaluate a policy (i.e., compute the expected return) for any reward function. This is crucial for the policy optimization step in AIL methods.

\(^2\)Here “conventional” means the state-action distribution estimator is based on the maximum likelihood estimation as in (4.3).

\(^3\)We translate the results in [Abbeel and Ng, 2004, Syed and Schapire, 2007] from the infinite-horizon setting to the episodic setting by 1) replacing the effective planning horizon \(1/(1 - \gamma)\) with the finite planning horizon \(H\); 2) instantiating the linear feature with the one-hot feature under the tabular setting.

\(^4\)It is fundamental to design algorithms that achieve the statistical lower bound and have low sample complexity in the RL community. See e.g. [Jin et al., 2018] (especially Table 1 in [Jin et al., 2018]) for the discussion.

\(^5\)In [Shani et al., 2021], a regret \(\tilde{O}((\sqrt{H}|S|^2|A|K + \sqrt{H^3|S|^2|A|K^2}/m)\) is proved, where \(K\) is the number of interaction episodes and \(m\) is the number of expert trajectories. We convert this regret guarantee into the PAC guarantee (see Appendix C).
Table 1: Sample complexity and interaction complexity to achieve an $\varepsilon$-optimal policy value gap for different IL algorithms. We use $\tilde{O}$ and $\tilde{\Omega}$ to hide logarithmic factors. Here "X" means the algorithm/bound is not applicable under the respective setting.

| Algorithm | Known Transition Setting | Unknown Transition Setting |
|-----------|--------------------------|---------------------------|
| BC [Rajaraman et al., 2020] | $\tilde{O}\left(\frac{H^3}{\varepsilon^2 |S|}\right)$ | $\tilde{O}\left(\frac{H^3 |S|^2}{\varepsilon^2 |A|}\right)$ |
| FEM [Abbeel and Ng, 2004] | $\tilde{O}\left(\frac{H^3}{\varepsilon^2 |S|}\right)$ | $\tilde{O}\left(\frac{H^3 |S|^2}{\varepsilon^2 |A|} + \frac{H^3 |S|^2 |A|}{\varepsilon^2 |A|}\right)$ |
| GTAL [Syed and Schapire, 2007] | $\tilde{O}\left(\frac{H^3}{\varepsilon^2 |S|}\right)$ | $\tilde{O}\left(\frac{H^3 |S|^2 |A|}{\varepsilon^2 |A|}\right)$ |
| OAL [Shani et al., 2021] | $\tilde{O}\left(\frac{H^3}{\varepsilon^2 |S|}\right)$ | $\tilde{O}\left(\frac{H^3 |S|^2}{\varepsilon^2 |A|}\right)$ |
| MIMIC-MD [Rajaraman et al., 2020] | $\tilde{O}\left(\frac{H^3 |S|^2}{\varepsilon^2 |A|}\right)$ | X |
| TAIL (Algorithm 1) | X | X |
| MB-TAIL (Algorithm 3) | X | $\tilde{O}\left(\frac{H^3 |S|^2 |A|}{\varepsilon^2 |A|}\right)$ |
| Lower Bound [Rajaraman et al., 2021] | $\Omega\left(\frac{H^3}{\varepsilon^2 |S|}\right)$ | X |

- **Issue 3 (I3)** Despite the interaction complexity of OAL is large, the sample complexity of OAL is not optimal in the sense that the best sample complexity is $\tilde{O}(H^{3/2} |S|/\varepsilon)$ under the unknown transition setting. To this end, we ask the question: can we design algorithms with better sample complexity and interaction complexity under the unknown transition case?

**Our Contribution.** Firstly, we present numerical evidence and detailed analysis in Section 6 to reconcile the mismatch between theory and practice as mentioned in (I1). Specifically, we consider the Reset Cliff MDP introduced in [Rajaraman et al., 2020], which covers the key characteristics of practical tasks in Gym MuJoCo. For this MDP, we disclose that the worst-case analysis of AIL is actually not tight. Furthermore, our analysis suggests AIL could be significantly better than BC under this MDP, which explains the empirical observation that AIL-based methods are often more sample efficient than BC.

In addition, we provide another MDP instance modified from [Rajaraman et al., 2020]. Under this MDP, we empirically show that the worst-case bound of conventional AIL methods is tight, which further implies the improved estimator in MIMIC-MD [Rajaraman et al., 2020] is indispensable. Combining the result on Reset Cliff, we infer that the mismatch in (II) is due to the worst-case comparison. That is, hard instances for AIL and BC are not the same so that a direct comparison is not meaningful here.

Secondly, based on the improved estimator in [Rajaraman et al., 2020], we present a specific optimization procedure in Section 4 to address the space complexity issue in (I2). In particular, our approach relaxes the projection operator in MIMIC-MD when performing the state-action distribution matching (see Remark 4.2). Based on this change, we leverage the tool of online gradient descent [Shalev-Shwartz, 2012] to solve the min-max problem under the AIL framework. Statistically, the resultant algorithm named TAIL still enjoys the nearly minimax optimal sample complexity of $\tilde{O}(H^{3/2} |S|/\varepsilon)$. Computationally, TAIL is solvable in polynomial time and further has a better space complexity $O(|S| |A| H)$ compared with MIMIC-MD.

Finally, when the transition function is unknown, we propose a better algorithm in Section 5 to answer the question raised in (I3). The design of our algorithm is based on a new framework (see Algorithm 2), which connects AIL and reward-free exploration [Jin et al., 2020, Ménard et al., 2021]. Briefly, reward-free exploration methods (see Definition 5.1) usually establish a transition model by interacting with the environment in the first step; then, they evaluate a policy with the constructed transition model and hope the evaluation is uniformly accurate for any reward function. Since AIL requires to optimize the policy under several recovered reward functions, we can incorporate the idea of reward-free exploration to tackle the challenge from the unknown transition.

Under the developed framework, we combine TAIL and RF-Express [Ménard et al., 2021] (a SOTA reward-free exploration method), and obtain a new algorithm named MB-TAIL. The sample complexity of MB-TAIL is $\tilde{O}(H^{3/2} |S|/\varepsilon)$ while its interaction complexity is $\tilde{O}(H^{3/2} |S|^2 |A|/\varepsilon^2)$. Compared with the mentioned OAL algorithm, our algorithm has significant improvements in both sample complexity and interaction complexity. To our understanding, MIMIC-MD is limited to the known transition setting, and our algorithm is the first one to shift the advances from the known transition setting to the unknown transition setting.
2 Related Work

Researchers have analyzed IL algorithms based on the error bound analysis (i.e., under the “infinite” samples setting). To name a few, Ross and Bagnell [2010] reveal that BC has the compounding error issue, indicating that its error bound is \( \mathcal{O}(H^2) \). DAgger [Ross et al., 2011] improves this error bound to \( \mathcal{O}(H) \) with active queries. By state-action distribution matching, AIL methods can also address the compounding error issue and have the error bound \( \mathcal{O}(H) \) [Wang et al., 2020b, Xu et al., 2020] without additional expert queries.

However, the error bound analysis does not fully characterize the sample efficiency. Later on, there emerge lots of finite sample complexity studies [Xu et al., 2020, Rajaraman et al., 2020, 2021, Rashidinejad et al., 2021, Shani et al., 2021]. In addition to results in Table 1, under the unknown transition and no interaction setting, Rajaraman et al. [2020] prove that BC has a sample complexity \( \tilde{\mathcal{O}}(H^2|S|/\varepsilon) \). This upper bound matches the lower bound \( \Omega(H^2|S|/\varepsilon) \) under this setting [Rajaraman et al., 2020]. Interestingly, under the active setting where the agent can query the expert, Rajaraman et al. [2020] show that DAgger does not improve the sample complexity compared with BC; see [Rajaraman et al., 2020] for a detailed explanation.

In addition to the tabular setting considered in this work, Liu et al. [2021] and Cai et al. [2019] consider GAIL with linear function approximation setting while the neural network approximation case is studied in [Wang et al., 2020b, Zhang et al., 2020, Xu et al., 2021].

The reward-free exploration framework is firstly proposed in [Jin et al., 2020] with the goal of 1) isolating the exploration issue and planning issue under a standard RL framework and 2) learning a “robust” environment to cover all possible training scenarios. Following [Jin et al., 2020], there are many advances in this direction [Ménard et al., 2021, Kaufmann et al., 2021, Ménard et al., 2021, Wang et al., 2020a, Ménard et al., 2021, Xu et al., 2020, Rashidinejad et al., 2021, Shani et al., 2021]. For imitation learning, our framework in Section 5 connects the reward-free exploration and adversarial imitation learning.

3 Background

Episodic Markov Decision Process. In this paper, we consider episodic Markov decision process (MDP), which can be described by the tuple \( \mathcal{M} = (S, A, P, r, H, \rho) \). Here \( S \) and \( A \) are the state and action space, respectively. \( H \) is the planning horizon and \( \rho \) is the initial state distribution. \( P = \{P_1, \ldots, P_H\} \) specifies the non-stationary transition function of this MDP; concretely, \( P_h(s_{h+1}|s_h, a_h) \) determines the probability of transiting to state \( s_{h+1} \) conditioned on state \( s_h \) and action \( a_h \) at time step \( h \), for \( h \in [H]\). Similarly, \( r = \{r_1, \ldots, r_H\} \) specifies the reward function of this MDP; without loss of generality, we assume that \( r_h : S \times A \to [0, 1] \), for \( h \in [H]\). A non-stationary policy \( \pi = \{\pi_1, \ldots, \pi_H\} \), where \( \pi_h : S \to \Delta(A) \) and \( \Delta(A) \) is the probability simplex, \( \pi_h(a|s) \) gives the probability of selecting action \( a \) on state \( s \) at time step \( h \), for \( h \in [H]\).

The sequential decision process runs as follows: in the beginning of an episode, the environment is reset to an initial state according to \( \rho \); then the agent observes a state \( s_h \) and takes an action \( a_h \) based on \( \pi_h(a_h|s_h) \); consequently, the environment makes a transition to the next state \( s_{h+1} \) according to \( P_h(s_{h+1}|s_h, a_h) \) and sends a reward \( r_h(s_h, a_h) \) to the agent. This episode ends after \( H \) repeats.

The quality of a policy is measured by its policy value (i.e., the expected return):

\[
V^\pi := \mathbb{E} \left[ \sum_{h=1}^{H} r_h(s_h, a_h)|s_1 \sim \rho; a_1 \sim \pi_1(.|s_1), s_{h+1} \sim P_h(.|s_h, a_h), \forall h \in [H] \right].
\]

To facilitate later analysis, we introduce the state-action distribution induced by a policy \( \pi \):

\[
P_h^\pi(s, a) := \mathbb{P} \left( s_h = s, a_h = a|s_1 \sim \rho; a_1 \sim \pi_1(.|s_1), \forall \ell \in [h] \right).
\]

In other words, \( P_h^\pi(s, a) \) qualifies the visitation probability of state-action pair \((s, a)\) at time step \( h \). In this way, we get an equivalent dual form of the policy value [Puterman, 2014]:

\[
V^\pi = \sum_{h=1}^{H} \sum_{(s, a) \in S \times A} P_h^\pi(s, a)r_h(s, a).
\]

Imitation Learning. The goal of imitation learning is to learn a high quality policy without the environment reward function. To this end, we often assume there is a nearly optimal expert policy \( \pi^E \) that could interact with the environment

\[
[x] \text{ denotes the set of integers from 1 to } x.
\]
to generate a dataset:
\[ D = \{ \text{tr} = (s_1, a_1, s_2, a_2, \ldots, s_H, a_H) : s_1 \sim \rho, a_h \sim \pi^E_h(\cdot|s_h), s_{h+1} \sim P_h(\cdot|s_h, a_h), \forall h \in [H] \} \]

Then, the learner can use the dataset \( D \) to mimic the expert and to obtain a good policy. The quality of imitation is measured by the \textit{policy value gap} [Abbeel and Ng, 2004, Ross and Bagnell, 2010, Rajaraman et al., 2020]: \( V^{\pi^E} - V^\pi \).

Following [Xu et al., 2020, Rajaraman et al., 2020], we assume the expert policy is deterministic.

**Notation.** We denote \( \Pi \) as the set of all deterministic policies for the learner. For a trajectory \( \text{tr} \) in expert demonstrations \( D \), \( \text{tr}(s_h) \) and \( \text{tr}(s_h, a_h) \) denote the specific state and state-action pair at time step \( h \) in the trajectory \( \text{tr} \), respectively. Furthermore, \( |D| \) is the number of trajectories in \( D \). We write \( a \gtrsim b \) if there exists a constant \( C > 0 \) such that \( a \geq Cb \) by ignoring the lower order terms.

## 4 An Improved Transition-aware AIL Algorithm with Known Transitions

In this part, we assume the transition is known, so that the agent can exactly evaluate a policy for any reward function.

### 4.1 The Transition-Aware Estimator

To break the sample barrier issue mentioned in Section 1, we leverage the estimator proposed in [Rajaraman et al., 2020] for the state-action distribution \( P^E_h \). For a better presentation, let us introduce the following notations.

- \( \text{tr}_h \): the truncated trajectory up to time step \( h \), i.e., \( \text{tr}_h = (s_1, a_1, \ldots, s_h, a_h) \).
- \( S_h(D) \): the set of states visited at time step \( h \) in \( D \).
- \( T^D_h = \{ \text{tr}_h : \text{tr}_h(s_h) \in S_\ell(D), \forall \ell \in [h] \} \): the trajectories along which each state has been visited in \( D \) up to time step \( h \).

Now, consider the dataset \( D \) is randomly divided into two equal parts, i.e., \( D = D_1 \cup D_2 \). The estimator in [Rajaraman et al., 2020] is\(^7\):

\[
\hat{P}^E_h(s, a) = \sum_{\text{tr}_h \in T^D_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{\text{tr}_h(s_h, a_h) = (s, a)\} + \frac{\sum_{\text{tr}_h \in D_1} \mathbb{I}\{\text{tr}_h(s_h, a_h) = (s, a), \text{tr}_h \notin T^D_1\}}{|D_1|}, \tag{4.1}
\]

where \( \mathbb{P}^E(\text{tr}_h) \) is the probability of the truncated trajectory \( \text{tr}_h \) induced by the deterministic expert policy \( \pi^E \).

To get a better intuition of this estimator, consider the following key decomposition of \( P^E_h(s, a) \):

\[
P^E_h(s, a) = \sum_{\text{tr}_h \in T^D_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{\text{tr}_h(s_h, a_h) = (s, a)\} + \sum_{\text{tr}_h \notin T^D_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{\text{tr}_h(s_h, a_h) = (s, a)\}. \tag{4.2}
\]

First of all, we see that the first term in the estimator (4.1) is exactly the first part in (4.2). For this part, all state-action pairs up to time step \( h \) are known from \( D_1 \). Therefore, we can compute \( \mathbb{P}^E(\text{tr}_h) \) exactly as the transition function is known\(^8\). Secondly, for the second term in (4.2) (i.e., \( \text{tr}_h \notin T^D_1 \)), we cannot exactly calculate \( \mathbb{P}^E(\text{tr}_h) \) since we do not know some actions in \( \text{tr}_h \), from \( D_1 \). Fortunately, we can leverage the complementary dataset \( D_2 \) to establish an estimator. In fact, the second term in (4.1) is a maximum likelihood estimation for the associated part in (4.2). Finally, since the estimator in (4.1) utilizes the transition function information explicitly, it has a better statistical guarantee.

**Lemma 4.1** ([Rajaraman et al., 2020]). Consider \( D \) is randomly divided into two subsets, i.e., \( D = D_1 \cup D_2 \) with \( |D_1| = |D_2| = m/2 \). Fix \( \varepsilon \in (0, H) \) and \( \delta \in (0, 1) \); suppose \( H \geq 5 \). If the number of trajectories in \( D \) satisfies

\[
m \gtrsim \frac{H^{3/2}|S|}{\varepsilon} \log \left( \frac{H|S|}{\delta} \right),
\]

then with probability at least \( 1 - \delta \), we have \( \sum_{h=1}^{H} \| \hat{P}^E_h - P^E_h \|_1 \leq \varepsilon \).

**Remark 4.1.** This lemma indicates that when \( \varepsilon \leq \sqrt{H} \), the estimator in (4.1) is better than the maximum likelihood estimation in conventional AIL approaches such as FEM [Abbeel and Ng, 2004], GTAL [Syed and Schapire, 2007] and

\[
\mathbb{P}^E(\text{tr}_h) = \rho(\text{tr}(s_1)) \prod_{\ell=1}^{h-1} \mathbb{P}_\ell(\text{tr}(s_{\ell+1})|\text{tr}(s_\ell), \text{tr}(a_\ell)).
\]
The problem we consider is slightly different from the one in MIMIC-MD. More specifically, MIMIC-MD restricts

\[ \pi \]  \quad \text{where} \quad \eta \]

we can use online gradient descent methods [Shalev-Shwartz, 2012] so that the overall objective can finally reach an approximate saddle point. Formally, let us define the objective

\[ f(\pi) = \min_{\pi \in \Pi} \frac{1}{H} \sum_{h=1}^{H} \| P^\pi_h - \tilde{P}^\pi_h \|_1, \quad \text{where} \quad \Pi \]

\[ \hat{P}^\pi_h(s, a) := \sum_{t \in D} \mathbb{1} \{ t(r_{s_h}, a_h) = (s, a) \}, \quad \text{for} \quad \text{Algorithm 1} \]

\[ \text{Remark 4.2. The optimization problem posted in (4.4) is indeed a “projection” problem with the } \ell_1\text{-norm metric. The problem we consider is slightly different from the one in MIMIC-MD. More specifically, MIMIC-MD restricts candidate policies to } \Pi_{BC}(D_1) = \{ \pi \in \Pi : \pi_h(s) = \pi^E_h(s), \forall h \in [H], s \in S_h(D_1) \}, \text{which is the set of BC policies on } D_1. \text{The intuition in MIMIC-MD is that the expert actions are known on } \text{for } MIMIC-MD. \text{It is clear that } \Pi \text{ is more general than } \Pi_{BC}(D_1). \text{We will show that our choice allows an efficient gradient-based optimization procedure without sacrificing any statistical guarantee.} \]

where \( W = \{ w : \| w \|_\infty \leq 1 \} \) is the unit ball. We see that the inner problem in (4.5) is to maximize the policy value of \( \pi \) given the reward function \( w_h(s, a) \) (see (3.1) for the dual form of policy value). For the outer optimization problem, we can use online gradient descent methods [Shalev-Shwartz, 2012] so that the overall objective can finally reach an approximate saddle point. Formally, let us define the objective \( f(t)(w) \):

\[ \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} w_h(s,a) \left( P^\pi_h(s,a) - \tilde{P}^\pi_h(s,a) \right), \quad \text{where} \quad \pi(t) \]

\[ \text{Remark 4.2. The optimization problem posted in (4.4) is indeed a “projection” problem with the } \ell_1\text{-norm metric. The problem we consider is slightly different from the one in MIMIC-MD. More specifically, MIMIC-MD restricts candidate policies to } \Pi_{BC}(D_1) = \{ \pi \in \Pi : \pi_h(s) = \pi^E_h(s), \forall h \in [H], s \in S_h(D_1) \}, \text{which is the set of BC policies on } D_1. \text{The intuition in MIMIC-MD is that the expert actions are known on } D_1 \text{so that direct projection is feasible. Based on the choice of } \Pi_{BC}(D_1), \text{Rajaraman et al. [2021] develop a linear programming based optimization procedure for MIMIC-MD. It is clear that } \Pi \text{ is more general than } \Pi_{BC}(D_1). \text{We will show that our choice allows an efficient gradient-based optimization procedure without sacrificing any statistical guarantee.} \]

We now consider the optimization issue of (4.4). In particular, we utilize the dual representation of \( \ell_1\)-norm and the min-max theorem [Bertsekas, 2016], which help us transform (4.4) into the following min-max optimization problem:

\[ \max_{w \in W} \min_{\pi \in \Pi} \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} w_h(s,a) \left( P^\pi_h(s,a) - \tilde{P}^\pi_h(s,a) \right), \quad \text{where} \quad \mathcal{W} = \{ w : \| w \|_\infty \leq 1 \} \]

\[ \text{Algorithm 1 Transition-aware AIL (TAIL)} \]

**Input:** expert demonstrations \( D \), number of iterations \( T \), step size \( \eta(t) \), and initialization \( w(1) \).

1. Randomly split \( D \) into two equal parts: \( D = D_1 \cup D_2 \) and obtain the estimation \( \hat{P}^\pi_h \) in (4.1).

2. for \( t = 1, 2, \ldots, T \) do

3. \( \pi(t) \) \( \leftarrow \) solve the optimal policy with the reward function \( w(t) \) up to an error of \( \varepsilon_{\text{opt}} \).

4. Compute the state-action distribution \( P^\pi_h(t) \) for \( \pi(t) \).

5. Update \( w(t+1) := \mathcal{P}_W \left( w(t) - \eta(t) \nabla f(t)(w(t)) \right) \), where \( \eta(t) > 0 \) is the stepsize to be chosen later, and \( \mathcal{P}_W \) is the Euclidean projection on the unit ball \( \mathcal{W} \), i.e., \( \mathcal{P}_W(w) := \arg\min_{z \in \mathcal{W}} \| z - w \|_2 \). The procedure for solving (4.5) is outlined in Algorithm 1.

6. end for

7. Compute the mean state-action distribution \( \overline{P}_h(s,a) = \sum_{t=1}^{T} P^\pi_h(t)(s,a)/T \).

8. Derive \( \pi_h(s,a) \leftarrow \mathcal{P}_h(s,a)/ \sum_a \mathcal{P}_h(s,a) \).

**Output:** policy \( \pi \).

**Theorem 4.1.** Fix \( \varepsilon \in (0, H) \) and \( \delta \in (0, 1) \); suppose \( H \geq 5 \). Consider the approach TAIL in Algorithm 1 with \( \pi \) being the output policy. Assume that the optimization error \( \varepsilon_{\text{opt}} \leq \varepsilon/2 \), the number of iterations \( T \geq H^2|S||A|/\varepsilon^2 \),
and the step size \( \eta(t) := \sqrt{|S||A|/(8T)} \). If the number of expert trajectories satisfies

\[
m \geq \frac{H^{3/2}|S|}{\varepsilon} \log \left( \frac{|S|}{\delta} \right),
\]

then with probability at least \( 1 - \delta \), we have \( V_{\pi}^* - V_{\hat{\pi}} \leq \varepsilon \).

**Remark 4.3.** The optimization problem in line 3 of Algorithm 1 can be solved efficiently to an acceptable error by value iteration [Puterman, 2014] or policy gradient based methods [Agarwal et al., 2020].

**Remark 4.4.** Note that the sample complexity of Algorithm 1 matches the lower bound of \( \Omega\left(H^{3/2}/\varepsilon\right) \) [Rajaraman et al., 2021] in terms of \( H \) and \( \varepsilon \). Under the AIL framework, it improves the existing upper bound \( \tilde{O}(H^2|S|/\varepsilon^2) \) for conventional methods such as FEM and GTAL. The tightness of the sample complexity in Theorem 4.1 is empirically verified in Appendix H.1.

**Remark 4.5.** The space complexity of Algorithm 1 is \( O(|S||A|H) \) for updating \( w^{(t)} \) and \( \pi^{(t)} \). In contrast, the space complexity of MIMIC-MD is \( O\left(|S|^2|A|^2H^2\right) \). Therefore, MIMIC-MD is hard to be applied on large-scale IL problems (see Section 6). As a side note, TAIL can be efficiently solved in polynomial time\(^9\).

## 5 An Improved Transition-aware AIL Algorithm With Unknown Transitions

In this part, we consider a practical setting where the learner does not know the transition function in advance but can interact with the environment. Under this scenario, in addition to the number of expert demonstrations, we also care about the number of environment interactions. Here we refer to the above two measures as **(expert) sample complexity** and **interaction complexity**, respectively. The main challenge under this scenario is that the agent cannot exactly evaluate a policy any longer since the transition function is unknown.

### 5.1 A Framework For AIL Under the Unknown Transition Setting

To address the challenge from the unknown transition, we need to consider the online exploration issue in RL. Without a smart exploration strategy, the agent may take exponentially large interactions to make progress. To overcome this difficulty, we propose a general framework to connect reward-free exploration [Jin et al., 2020, Ménard et al., 2021] and adversarial imitation learning. Under our framework, a proper AIL algorithm that works under the known transition setting could be easily transferred to the unknown transition setting. In the sequel, let us formally introduce the reward-free exploration methods.

**Definition 5.1** ([Ménard et al., 2021]). An algorithm is \((\varepsilon, \delta)\)-PAC for reward-free exploration if

\[
P(\text{for any reward function } r, |V_{\pi^*}^r - V_{\hat{\pi}^*}^r| \leq \varepsilon) \geq 1 - \delta,
\]

where \( \pi^* \) is the optimal policy in the MDP with the reward function \( r \), and \( \hat{\pi}^* \) is the optimal policy in the MDP with the learned transition model \( \hat{P} \) and the reward function \( r \).

This definition suggests the reward-free exploration methods could achieve **uniform policy evaluation** after the exploration. Formally, a reward-free exploration method can ensure that \( \forall r : S \times A \rightarrow [0, 1] \), \( \pi \in \Pi \), we have \( |V_{\pi, \hat{P}, r} - V_{\pi, \hat{P}, r}| \leq \varepsilon \), where \( V_{\pi, \hat{P}, r} \) is the policy value under transition \( \hat{P} \) and reward \( r \) [Jin et al., 2020]. Based on such a learned transition model, an AIL algorithm can perform policy optimization as if this empirical transition is same with the true transition function. We outline such an idea in Algorithm 2.

**Proposition 5.1.** Suppose that

(a) an algorithm \( A \) solves the reward-free exploration problem (see Definition 5.1) up to an error \( \varepsilon_{RFE} \) with probability at least \( 1 - \delta_{RFE} \);

(b) an algorithm \( B \) has a state-action distribution estimator for \( P_h^{\pi^*} \), which satisfies \( \sum_{h=1}^{H} \| P_h^{\pi^*} - \hat{P}_h^{\pi^*} \|_1 \leq \varepsilon_{EST} \), with probability at least \( 1 - \delta_{EST} \);

(c) with the estimator in (b), the algorithm \( B \) solves the projection/optimization problem in (4.4) up to an error \( \varepsilon_{AIL} \).

Then applying algorithm \( A \) and \( B \) under the framework in Algorithm 2 could return a policy \( \pi \), which has a policy value gap (i.e., \( V_{\pi^*}^r - V_{\pi}^r \)) at most \( 2\varepsilon_{EST} + 2\varepsilon_{RFE} + H\varepsilon_{AIL} \) with probability at least \( 1 - \delta_{EST} - \delta_{RFE} \).

\(^9\)The computation complexity (in terms of arithmetic operations) of TAIL is \( \tilde{O}(|S|^3|A|^2H^3/\varepsilon^2) \) while MIMIC-MD by LP is about \( \tilde{O}(d^2\varepsilon^2) \) where \( d = 2|S||A|/H \). We see TAIL is asymptotically more efficient than MIMIC-MD when \( \varepsilon > \tilde{O}((|S|H/|A|)^{0.25}) \).
Remark 5.1. Note the metric (e.g., $\ell_1$-norm) used in the estimation problem (assumption (b)) and the projection problem (assumption (c)) is not unique. For instance, FEM [Abbeel and Ng, 2004] uses the $\ell_2$-norm metric in its algorithm but FEM can be also applied under this framework. As a result, the policy value gap becomes $O(\sqrt{|S||A|} \varepsilon_{\text{EST}} + \varepsilon_{\text{RFE}} + H \varepsilon_{\text{AIL}})$; refer to Appendix F.2 for a formal argument and the explanation of the additional factor $\sqrt{|S||A|}$.

Remark 5.2. Unfortunately, we could not apply the MIMIC-MD algorithm under our framework for two reasons. Firstly, the estimator in MIMIC-MD does not satisfy the assumption (b) under the unknown transition setting. We will discuss this later. Secondly, the projection in MIMIC-MD is restrictive to $\Pi_{\text{BC}}(D_1)$ (see Remark 4.2) and its optimization procedure heavily relies on the true transition function, which are not compatible with the assumption (c).

Algorithm 2 General Algorithmic Framework For AIL Under the Unknown Transition Setting

Input: expert demonstrations $D$.

1: Establish the state-action distribution estimation $\tilde{P}^\pi_h$.
2: $\hat{P} \leftarrow$ invoke a reward-free method to collect $n'$ trajectories and learn a transition model.
3: $\pi \leftarrow$ apply an AIL approach to perform imitation with the estimation $\tilde{P}^\pi_h$ under transition model $\hat{P}$.

Output: policy $\pi$.

5.2 Model-based Transition-aware Adversarial Imitation Learning

Next, we present how to apply TAIL in Algorithm 1 under our framework. As mentioned in Remark 5.2, we need to address the issue that the estimator in (4.1) involves the exact transition function and cannot be directly applied here. Luckily, we could fix this issue by a dataset $D'_\text{env}$ collected by rolling out a BC policy (obtained from $D_1$) with the environment; see Appendix F.3 for more explanation. Based on this trick, the new estimator is formulated as

$$
\tilde{P}^\pi_h(s,a) = \frac{\sum_{\text{tr}_h \in D'_\text{env}} \mathbb{I}\{\text{tr}_h(s_h, a_h) = (s, a), \text{tr}_h \in \text{Tr}_h^D_1\}}{|D'_\text{env}|} + \frac{\sum_{\text{tr}_h \in D'_\text{env}} \mathbb{I}\{\text{tr}_h(s_h, a_h) = (s, a), \text{tr}_h \notin \text{Tr}_h^D_1\}}{|D'_\text{env}|},
$$

(5.1)

With the estimator in (5.1), we develop an extension of TAIL named MB-TAIL presented in Algorithm 3.

Algorithm 3 Model-based Transition-aware AIL (MB-TAIL)

Input: expert demonstrations $D$.

1: Randomly split $D$ into two equal parts: $D = D_1 \cup D'_1$.
2: Learn $\pi \in \Pi_{\text{BC}}(D_1)$ by BC and roll out $\pi$ to obtain dataset $D'_\text{env}$ with $|D'_\text{env}| = n'$.
3: Obtain the estimator $\tilde{P}^\pi_h$ in (5.1) with $D$ and $D'_\text{env}$.
4: Invoke RF-Express to collect $n$ trajectories and learn an empirical transition function $\hat{P}$.
5: $\pi \leftarrow$ apply TAIL to perform imitation with the estimation $\tilde{P}^\pi_h$ under transition model $\hat{P}$.

Output: policy $\pi$.

Theorem 5.1. Fix $\varepsilon \in (0, 1)$ and $\delta \in (0, 1)$; suppose $H \geq 5$. Under the unknown transition setting, consider MB-TAIL displayed in Algorithm 3 and $\pi$ is output policy, assume that the optimization error $\varepsilon_{\text{opt}} \leq \varepsilon/2$, the number of iterations and the step size are the same as in Theorem 4.1, if the expert sample complexity and the total interaction complexity satisfy

$$
m \gtrsim \frac{H^{3/2} |S|}{\varepsilon} \log \left( \frac{H |S|}{\delta} \right), n' \gtrsim \frac{H^2 |S|}{\varepsilon^2} \log \left( \frac{H |S|}{\delta} \right), n \gtrsim \frac{H^3 |S||A|}{\varepsilon^2} \left( |S| + \log \left( \frac{H |S||A|}{\delta \varepsilon} \right) \right)
$$

Then with probability at least $1 - \delta$, we have $V^\pi_h - V^\pi \leq \varepsilon$.

Remark 5.3. Recall that the OAL algorithm in [Shani et al., 2021] has sample complexity $\tilde{O}(H^2 |S|/\varepsilon^2)$ and interaction complexity $\tilde{O}(H^3 |S|^2 |A|/\varepsilon^2)$ under the unknown transition case. Theorem 5.1 implies our approach has substantial improvements over the OAL algorithm in both sample complexity and interaction complexity. This is partially because the reward-free exploration methods are more efficient than the no-regret algorithms used in [Shani et al., 2021] under the scenario where the inferred reward function varies over iterations. To our understanding, our algorithm is the first one to shift the advances from the known transition setting to the unknown transition setting.
6 Case Studies

In this section, we conduct experiments to 1) answer the question raised in (II) in Section 1; 2) validate the sample and interaction efficiency of the proposed algorithms. We focus on two MDPs: Reset Cliff and Standard Imitation shown in Figure 1, which are adapted from [Rajaraman et al., 2020]. Experiment details are given in Appendix G.

Figure 1: Two MDPs modified from [Rajaraman et al., 2020]. Green and blue arrows indicate state transitions under the expert and non-expert actions, respectively. Digits on arrows are reward values.

6.1 Known Transition Setting

We first consider the known transition setting with methods including BC [Pomerleau, 1991], FEM [Abbeel and Ng, 2004], GTAL [Syed and Schapire, 2007], OAL [Shani et al., 2021] and TAIL (see Algorithm 1). We aim to study the dependence on $H$ and $\varepsilon$ appeared in the sample complexity. To achieve this goal, figures have used logarithmic scales so that we can read the order dependence from slopes of curves. Specifically, a worst-case sample complexity $m \gtrsim H^a |S|/\varepsilon^\beta$ implies the policy value gap $V^\pi^k - V^\pi \lesssim H^a/\beta |S|^{1/\beta}/m^{1/\beta}$. Then,

$$\log(V^\pi^k - V^\pi) \lesssim (a/\beta) \log(H) - 1/\beta \log(m) + 1/\beta \log(|S|).$$

For example, the sample complexity $\tilde{O}(H^{3/2} |S|/\varepsilon)$ of TAIL similarly suggests slope $3/2$ w.r.t. $\log(H)$ and slope $-1$ w.r.t. $\log(m)$ for its log policy value gap. It is worth mentioning that these implications are true only on “hard” instances.

Case Study on Standard Imitation. For Standard Imitation (Figure 1a), each state is absorbing and the agent gets +1 reward only by taking the expert action (shown in green). Different from [Rajaraman et al., 2020], the initial state distribution $\rho$ is $(1/|S|, \cdots, 1/|S|)$ to better disclose the sample barrier issue of AIL methods mentioned in Section 1.

Firstly, we focus on the planning horizon dependence issue; see the result in Figure 2a. In particular, the numerical result shows that the policy value gap of all methods grows linearly with respect to the planning horizon. This is reasonable since each state in Standard Imitation is absorbing. Notice that Standard Imitation is not the worst-case MDP for BC due to its absorbing structure. However, Standard Imitation is still challenging for existing AIL approaches (FEM, GTAL, and OAL) and thus can be used to validate the tightness of their sample complexity as we will illustrate later.

Secondly, we display the result regarding the number of expert demonstrations in Figure 2b. Under Standard Imitation, the state distribution of every policy is a uniform distribution at every time step, which raises a statistical estimation challenge for AIL. From Figure 2b, we clearly see that the slopes of FEM, GTAL and OAL w.r.t $\log(m)$ are around $-1/2$. This can be explained by their sample complexity $\tilde{O}(H^{3/2} |S|/\varepsilon^2)$, which implies $\log(V^\pi^k - V^\pi) \lesssim -1/2 \log(m) + \text{constant}$. This result indicates the upper bound of conventional AIL methods is tight and we should improve them from an algorithmic level (rather than the analysis level). As for our method, as shown in Figure 2b, the policy value gap of TAIL diminishes substantially faster than FEM, GTAL and OAL, which verifies the sample efficiency of TAIL. The fast diminishing rate of BC is due to the quick concentration rate of missing mass [McAllester and Ortiz, 2003]; see [Rajaraman et al., 2020] for more explanation.

Case Study on Reset Cliff. For Reset Cliff (Figure 1b), the agent gets +1 reward by taking the expert action (shown in green) on states except the bad state $b$, then the next state is renewed according to the initial state distribution $\rho$. Here, $\rho = (1/(m + 1), \cdots, 1/(m + 1), 1 - ((|S| - 2)/(m + 1), 0)$ [Rajaraman et al., 2020]. Once taking a non-expert action (shown in blue), the agent goes to the absorbing state $b$ and gets 0 reward.

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10 MIMIC-MD is not involved as it runs out of memory on a machine with 128GB RAM when $H \geq 100$. GAIL is not considered as it does not have a formal convergence and sample complexity guarantee. We provide a variant of GAIL and investigate its performance in Appendix G.
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Figure 2: The policy value gap (i.e., $V^{E} - V^{π}$) in Standard Imitation and Reset Cliff. The solid lines are mean of results and the shaded region corresponds to the standard deviation over 20 random seeds (same with the following figure).

On the one hand, Reset Cliff highlights the compounding error issue and recovers the key characteristics of many practical tasks. Take the Gym MuJoCo task as an example, once the robot takes a wrong action, it would go to the terminate state and obtain 0 reward forever. The numerical result about the planning horizon is given in Figure 2c. In particular, on states uncovered in expert demonstrations, BC randomly selects an action. It is very likely that the agent picks up a non-expert action, goes to the bad state, and gets 0 reward in the remaining time. In contrast, AIL can avoid this issue via state-action distribution matching. To see this, consider a good state (e.g., state 1) at time step $h$ appears in the dataset, then the agent must follow the expert actions at all time steps before $h$; otherwise, the agent will go to the bad state and cannot visit this good state at time step $h$, which violates the state-action distribution matching principle.

On the other hand, we consider the dependence on the number of expert demonstrations; the corresponding numerical result is shown in Figure 2d. From Figure 2d, we see that the slopes of all methods are around $-1$. Combined with the quadratic horizon dependency of BC, we empirically validate that the sample complexity analysis of BC is tight. Note that Reset Cliff is not the worst-case MDP for AIL approaches (FEM, GTAL, and OAL) as we have analyzed, thus AIL approaches indeed have better performance on Reset Cliff compared with the worst-case performance listed in Table 1.

6.2 Unknown Transition Setting

In this part, we study the interaction complexity under the unknown transition setting. We still use the above two MDPs, but they may not be hard instances. Hence, we do not verify the tightness of order dependency. The comparison involves BC [Pomerleau, 1991], OAL [Shani et al., 2021] and MB-TAIL (see Algorithm 3). All algorithms are provided with the same expert demonstrations.

Empirical results are displayed in Figure 3. Note that BC does not need interaction. Similar to the results shown in Figure 2, BC performs worse than MB-TAIL method on Reset Cliff while BC could be better than MB-TAIL on Standard Imitation. Moreover, we see that MB-TAIL outperforms OAL provided with the same number of interactions.
7 Conclusion

To summarize, the contribution of this work is twofold: 1) we reconcile the mismatch between theory and practice when analyzing imitation learning algorithms; 2) we provide more efficient algorithms under the known transition and unknown transition settings. For both practitioners and theorists, it is promising to extend the tabular results in this paper to the general cases with function approximation.

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A Appendix Organization

The Appendix is organized as follows.

- Appendix B lists all notations appeared in the paper.
- Appendix C translates the regret guarantee to PAC guarantee for algorithm OAL Shani et al. [2021].
- Appendix D provides an MDP example to explain the transition-aware state-action distribution estimator.
- Appendix E presents the proof of results in Section 4.
- Appendix F provides the proof of results in Section 5.
- Appendix G offers experimental details such as the information of the MDP and the hyperparameters of algorithm.
- Appendix H provides additional experiment results: the tightness of the sample complexity of TAIL in Theorem 4.1, numerical performance of GAIL, and the study of interaction efficiency under the unknown transition setting.
B  Notation

Table 2: Notations

| Symbol | Meaning |
|--------|---------|
| $S$    | the state space |
| $A$    | the action space |
| $\mathcal{P} = \{P_1, \ldots, P_H\}$ | the transition function |
| $H$    | the planning horizon |
| $\rho$ | the initial state distribution |
| $r = \{r_1, \ldots, r_H\}$ | the reward function |
| $\pi = \{\pi_1, \ldots, \pi_h\}$ | non-stationary policy |
| $\pi^E$ | the expert policy |
| $V_{\pi,\mathcal{P},r}$ | policy value under the transition $\mathcal{P}$ and reward $r$ |
| $\varepsilon$ | the policy value gap |
| $\delta$ | failure probability |
| $P^\pi_h(s)$ | state distribution |
| $P^\pi_h(s,a)$ | state-action distribution |
| $\mathbf{tr} = (s_1, a_1, \ldots, s_H, a_H)$ | the trajectory |
| $\mathbf{tr}_h = (s_1, a_1, \ldots, s_h, a_h)$ | the truncated trajectory |
| $\mathbf{tr}(s_h)$ | the state at time step $h$ in $\mathbf{tr}$ |
| $\mathbf{tr}(a_h)$ | the action at time step $h$ in $\mathbf{tr}$ |
| $\mathbf{tr}(s_h,a_h)$ | the state-action pair at time step $h$ in $\mathbf{tr}$ |
| $\mathcal{D}$ | expert demonstrations |
| $m$ | number of expert demonstrations |
| $\hat{P}^E_h(s,a)$ | maximum likelihood estimator in $\mathcal{D}$ |
| $\tilde{P}^E_h(s,a)$ | transition-aware estimator used in TAIL and MB-TAIL |
| $\mathbb{P}^E(\mathbf{tr})$ | probability of the trajectory $\mathbf{tr}$ under the expert policy $\pi^E$ |
| $\mathbb{P}^E(\mathbf{tr}_h)$ | probability of the truncated trajectory $\mathbf{tr}_h$ under the expert policy $\pi^E$ |
| $S_h(D)$ | the set of states visited in time step $h$ in dataset $\mathcal{D}$ |
| $\mathcal{T}_h(D)$ | the trajectories along which each state has been visited in $\mathcal{D}$ up to time step $h$ |
| $\pi(t)$ | the policy obtained in the iteration $t$ |
| $w(t)$ | the reward function learned in the iteration $t$ |
| $\eta(t)$ | the step size in the iteration $t$ |
| $f(t)(w)$ | the objective function in the iteration $t$ |
| $\bar{\mathcal{P}}^\pi_h(s,a)$ | the mean state-action distribution |
| $\pi$ | the policy derived by the mean state-action distribution |
| $\Pi_{BC}(D_1)$ | the set of policies which take the expert action on states covered in $D_1$ |
| $\hat{\mathcal{P}}$ | the empirical transition function |
| $P^\pi_{h,\mathcal{P}}(s,a)$ | the state-action distribution of $\pi$ under the empirical transition function $\hat{\mathcal{P}}$ |
From Regret Guarantee to PAC Guarantee

In [Shani et al., 2021], authors prove a regret guarantee for their OAL algorithm. In particular, Shani et al. [2021] show that with probability at least \(1 - \delta'\), we have

\[
\sum_{k=1}^{K} V^k_{\pi_k} - V^\pi_k \leq \tilde{O} \left( \sqrt{H^4|S|^2|A|K} + \sqrt{H^3|S||A|K^2/m} \right),
\]

where \(\pi_k\) is the policy obtained at episode \(k\), \(K\) is the number of interaction episodes, and \(m\) is the number of expert trajectories. We would like to comment that the second term in (C.1) involves the statistical estimation error about the expert policy. Furthermore, this term reduces to \(\tilde{O}(\sqrt{H^2|S|K^2/m})\) under the assumption that the expert policy is deterministic.

To further convert this regret guarantee to the PAC guarantee considered in this paper, we can apply Markov’s inequality as suggested by [Jin et al., 2018]. Concretely, let \(\tilde{\pi}\) be the policy that randomly chosen from \(\{\pi_1, \pi_2, \cdots, \pi_K\}\) with equal probability, then we have

\[
P \left( V^\pi_{\tilde{\pi}} - V^\pi \geq \varepsilon \right) \leq \frac{1}{\varepsilon} \mathbb{E} \left[ \frac{1}{K} \sum_{k=1}^{K} V^k_{\pi_k} - V^\pi_k \right] \leq \frac{1}{\varepsilon} \tilde{O} \left( \sqrt{\frac{H^4|S|^2|A|}{K}} + \sqrt{H^2|S|/m} + \delta' H \right).
\]

Therefore, if we set \(\delta' = \varepsilon\delta/(3H)\), and

\[
K = \tilde{O} \left( \frac{H^4|S|^2|A|}{\varepsilon^2 \delta^2} \right), \quad m = \tilde{O} \left( \frac{H^2|S|}{\varepsilon^2} \right),
\]

we obtain that \(P(V^\pi_{\tilde{\pi}} - V^\pi \geq \varepsilon) \leq \delta\). As commented in [Ménard et al., 2021], this transformation leads to a worse dependence on failure probability \(\delta\), but the sample complexity dependence on other terms does not change.
D An Illustrating Example

![Diagram of the Reset Cliff MDP](image)

Figure 4: The Reset Cliff MDP with three states. There are two actions, shown in green and blue. The expert policy always takes the green action. The initial state distribution is \( \rho = (\rho(1), \rho(2), \rho(3)) = (1/100, 99/100, 0) \). At any state except the bad state \( b \), if the agent plays the same action as the expert does, the next state is renewed according to \( \rho \). Otherwise, the agent goes to the bad state \( b \).

Here we provide an example shown in Figure 4 to explain the transition-aware estimator formulated in (4.1). Consider the Reset Cliff MDP with state space \( \mathcal{S} = \{1, 2, 3\} \), where 3 is the bad state, and action space \( \mathcal{A} = \{g, b\} \) where \( g \) and \( b \) denote the green and blue action, respectively. For simplicity, the planning horizon is \( H = 2 \). The number of expert demonstrations \( m = 100 \) and the initial state distribution \( \rho = (\rho(1), \rho(2), \rho(3)) = (1/100, 99/100, 0) \). Consider the dataset \( \mathcal{D} \) includes:

- 97 trajectories of \( \text{tr}^1 = (2, g, 2, g); \)
- 2 trajectories of \( \text{tr}^2 = (1, g, 2, g); \)
- 1 trajectory of \( \text{tr}^3 = (2, g, 1, g). \)

First of all, let us consider the maximum likelihood in (4.3).

\[
\begin{align*}
\text{at time step } h = 1: & \quad \hat{P}^E_1(1, g) = \frac{1}{50}, \quad \hat{P}^E_2(2, g) = \frac{49}{50}, \\
\text{at time step } h = 2: & \quad \hat{P}^E_1(1, g) = \frac{1}{100}, \quad \hat{P}^E_2(2, g) = \frac{99}{100}.
\end{align*}
\]

Hence, \( \ell_1 \)-norm estimation error \( \sum_{h=1}^{2} \| P^E_h - \hat{P}^E_h \|_1 = 0.02. \)

Now we consider the improved estimator shown in (4.1). In particular, we split \( \mathcal{D} \) into two equals parts \( \mathcal{D} = D_1 \cup D_2 \):

- \( D_1 \) contains 49 trajectories of \( \text{tr}^1 = (2, g, 2, g) \) and 1 trajectory of \( \text{tr}^2 = (1, g, 2, g); \)
- \( D_2 \) contains 48 trajectories of \( \text{tr}^1 = (2, g, 2, g) \), 1 trajectory of \( \text{tr}^2 = (1, g, 2, g) \), and 1 trajectory of \( \text{tr}^3 = (2, g, 1, g). \)

Hence, we can enumerate the trajectories to obtain that \( \text{Tr}_1^{D_1} = \{(1, g) (2, g)\}, \text{Tr}_2^{D_1} = \{(1, g, 2, g), (2, g, 2, g)\}). \)

\[
\begin{align*}
\text{at time step } h = 1: & \quad \hat{P}^E_1(1, g) = P^E(1, g) = \frac{1}{100}, \\
\text{at time step } h = 1: & \quad \hat{P}^E_2(2, g) = P^E(2, g) = \frac{99}{100}, \\
\text{at time step } h = 2: & \quad \hat{P}^E_1(1, g) = \frac{1}{50}, \\
\text{at time step } h = 2: & \quad \hat{P}^E_2(2, g) = P^E(2, g) + P^E(2, g) = \frac{99}{100}.
\end{align*}
\]

Therefore, we see \( \ell_1 \)-norm estimation error \( \sum_{h=1}^{2} \| P^E_h - \hat{P}^E_h \|_1 = 0.01. \)

Through this example, we see that the estimator in (4.1) is better than the naive one in (4.3).
E Proof of Results in Section 4

E.1 Proof of Lemma 4.1

For completeness, we provide the proof for Lemma 4.1 [Rajaraman et al., 2020].

Proof. Our target is to upper bound the estimation error of \( \bar{P}_h^{\pi^E} \in \mathbb{R}^{[S] \times [A]} \):

\[
\sum_{h=1}^{H} \left\| \bar{P}_h^{\pi^E} - P_h^{\pi^E} \right\|_1 = \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} \left| \bar{P}_h^{\pi^E}(s,a) - P_h^{\pi^E}(s,a) \right|,
\]

where \( \bar{P}_h^{\pi^E} \) is defined in (4.1):

\[
\bar{P}_h^{\pi^E}(s,a) = \sum_{\text{tr}_h \in \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a) \} + \sum_{\text{tr}_h \notin \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a), \text{tr}_h \notin \mathcal{D}_1 \} / |\mathcal{D}_1|.
\]

Recall that \( \mathcal{D}_1 \) is the set of trajectories along which each state has been visited in \( \mathcal{D}_1 \) up to time step \( h \). Similarly, for \( P_h^{\pi^E} \), we have the following decomposition in (4.2):

\[
P_h^{\pi^E}(s,a) = \sum_{\text{tr}_h \in \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a) \} + \sum_{\text{tr}_h \notin \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a) \}.
\]

Consequently, we obtain for any \((s,a) \in S \times A, h \in [H] \),

\[
\left| \bar{P}_h^{\pi^E}(s,a) - P_h^{\pi^E}(s,a) \right| = \left| \bar{P}_h^{\pi^E}(s,a) - \left( \sum_{\text{tr}_h \in \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a) \} + \sum_{\text{tr}_h \notin \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a) \} \right) \right|
\]

\[
= \left| \sum_{\text{tr}_h \in \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a), \text{tr}_h \notin \mathcal{D}_1 \} / |\mathcal{D}_1| \right| - \sum_{\text{tr}_h \notin \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a) \},
\]

where the last equation is based on the fact that the first term in \( \bar{P}_h^{\pi^E}(s,a) \) and \( P_h^{\pi^E}(s,a) \) is identical. As a result, the estimation error is caused by the unknown expert actions in trajectories that does not fully match with any trajectory in \( \mathcal{D}_1 \).

Since \( \pi^E \) is a deterministic policy, we can write \((s,a) = (s, \pi^E(s)) \). Then, we obtain that

\[
\left| \sum_{\text{tr}_h \in \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a) \}, \text{tr}_h \notin \mathcal{D}_1 \} / |\mathcal{D}_1| \right| - \sum_{\text{tr}_h \notin \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s,a) \}, \text{tr}_h \notin \mathcal{D}_1 \})
\]

\[
= \sum_{\text{tr}_h \in \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s, \pi^E(s)), \text{tr}_h \notin \mathcal{D}_1 \} / |\mathcal{D}_1| \right| - \sum_{\text{tr}_h \notin \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s, \pi^E(s)) \}, \text{tr}_h \notin \mathcal{D}_1 \}.
\]

Fix \( \mathcal{D}_1 \), we see that the above error bound corresponds to the Bernoulli random variables estimation. Specifically, given \( \mathcal{D}_1 \), the first term in (E.1) is an unbiased maximum likelihood estimator for \( \sum_{\text{tr}_h \notin \mathcal{D}_1} \mathbb{P}^E(\text{tr}_h) \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s, \pi^E(s)) \} \). Hence, the absolute value in (E.1) concentrates to 0 as the number of samples \( |\mathcal{D}_1| \) goes to infinity. In the following, we further qualify the concentration rate.

For a trajectory \( \text{tr}_h \), let \( E_h^s \) be the event that \( \text{tr}_h \) agrees with expert policy at state \( s \) at time step \( h \) but is not in \( \mathcal{D}_1 \), that is,

\[
E_h^s = \mathbb{I}\{ \text{tr}_h(s_h, a_h) = (s, \pi^E(s)) \} \cap \text{tr}_h \notin \mathcal{D}_1 \}.\]
We consider $E_h^x$ is measured by the stochastic process induced by the expert policy $\pi^E$. Accordingly, its probability is denoted as $\mathbb{P}^E(E_h^x)$. In fact, we see that $\mathbb{P}^E(E_h^x)$ is equal to the second term in (E.1). Moreover, the first term in (E.1) is an empirical estimation for $\mathbb{P}^E(E_h^x)$.

The main idea in [Rajaraman et al., 2020] is to show that $\mathbb{P}^E(E_h^x)$ is well controlled so that applying Chernoff’s bound [Vershynin, 2018] can yield a better concentration rate.

**Lemma E.1** (Chernoff’s bound [Vershynin, 2018]). Let $\bar{X} = 1/n \cdot \sum_{i=1}^n X_i$, where $X_i$ is a Bernoulli random variable with $\mathbb{P}(X_i = 1) = p_i$ and $\mathbb{P}(X_i = 0) = 1 - p_i$ for $i \in [n]$. Furthermore, assume these random variables are independent. Let $\mu = \mathbb{E}[\bar{X}] = 1/n \cdot \sum_{i=1}^n p_i$. Then for $0 < t \leq 1$,

$$
\mathbb{P} \left( |\bar{X} - \mu| \geq t\mu \right) \leq 2 \exp \left( -\frac{\mu t^2}{3} \right).
$$

After applying Lemma E.1 to (E.1) and taking an union bound, with probability at least $1 - \delta/(2|S|H)$ with $\delta \in (0, 1)$ (over the randomness of the dataset $D_1^h$), for each $h \in [H]$, $s \in S$, the following inequality holds,

$$
\left| \sum_{tr_h \in D_1^h} \mathbb{I}\{tr_h(s_h, a_h) = (s, \pi^E(s)), tr_h \notin Tr_{D_1} \} \right| \leq \sum_{tr_h \notin Tr_{D_1}} \mathbb{P}^E(tr_h) \mathbb{I}\{tr_h(s_h, a_h) = (s, \pi^E(s))\}.
$$

Therefore, with probability at least $1 - \delta/2$ (over the randomness of the dataset $D_1^h$), we have

$$
\sum_{h=1}^H \sum_{(s,a) \in S \times A} \left| \hat{P}_h^\pi(s, a) - P_h^\pi(s, a) \right| \leq \sum_{h=1}^H \sum_{s \in S} \mathbb{P}^E(E_h^x) \frac{3|S| \log(4|S|H/\delta)}{m},
$$

where the last step follows the Cauchy–Schwarz inequality. It remains to upper bound $\sum_{s \in S} \mathbb{P}^E(E_h^x)$ for all $h \in [H]$. To this end, we define the event $G_{D_1}^h$: the expert policy $\pi^E$ visits certain states that are uncovered in $D_1$ up to time step $h$. Formally, $G_{D_1}^h = \{ \exists h' \leq h, s_{h'} \notin S_h(D_1) \}$, where $S_h(D_1)$ is the set of states in $D_1$ at time step $h$. Then, for all $h \in [H]$, we have

$$
\sum_{s \in S} \mathbb{P}^E(E_h^x) = \mathbb{P}^E(G_{D_1}^h) \leq \mathbb{P}(G_{D_1}^h),
$$

where the first equality is true because $\bigcup E_h^x$ corresponds to the event that $\pi^E$ does not visit any trajectory in $D_1$, and the last inequality holds since $G_{D_1}^h \subseteq G_{D_1}^h$ for all $h \in [H]$. Conditioned on $D_1$, we further have

$$
\mathbb{P}(G_{D_1}^h) \leq \sum_{h=1}^H \sum_{s \in S} P_h^\pi(s) \mathbb{I}\{s \notin S_h(D_1)\}.
$$

We note that RHS is so-called missing mass [McAllester and Ortiz, 2003, Rajaraman et al., 2020]. In summary, we arrive at, with probability $1 - \delta/2$ (over the randomness of the dataset $D_1^h$),

$$
\sum_{h=1}^H \sum_{(s,a) \in S \times A} \left| \hat{P}_h^\pi(s, a) - P_h^\pi(s, a) \right| \leq \sum_{h=1}^H \sum_{s \in S} \mathbb{P}^E(G_{D_1}^h) \frac{3|S| \log(4|S|H/\delta)}{m}.
$$

We first consider the expectation $\mathbb{E}[\sum_{h=1}^H \sum_{s \in S} P_h^\pi(s) \mathbb{I}\{s \notin S_h(D_1)\}]$, where the expectation is taken over the expert dataset $D_1$.

$$
\mathbb{E}\left[ \sum_{h=1}^H \sum_{s \in S} P_h^\pi(s) \mathbb{I}\{s \notin S_h(D_1)\} \right] \leq \sum_{h=1}^H \sum_{s \in S} P_h^\pi(s) \left(1 - P_h^\pi(s)\right)^{m/2} \leq \frac{8|S|H}{9m},
$$
where the last step uses the numerical inequality\(^{11}\) \(\max_{x \in [0,1]} x(1-x)^m \leq 1/(1+m) \cdot (1 - 1/m)^m \leq 4/(9m)\). To transform the bound in expectation to a high probability form, we can use the following concentration bound from [Rajaraman et al., 2020], which is based on [McAllester and Ortiz, 2003].

**Lemma E.2** (Concentration inequality for missing mass [Rajaraman et al., 2020]). Suppose that

\[
\mathbb{E} \left[ \sum_{h=1}^{H} \sum_{s \in S} P_h^E(s) \mathbb{I} \left\{ s \notin S_h(D_1) \right\} \right] \leq \frac{8|S|H}{9m}.
\]

Fix \(\delta \in (0,1)\) with \(\delta \in (0, \min\{1, H/10\})\), with probability at least \(1 - \delta\) (over the randomness of the expert dataset \(D_1\)), we have

\[
\sum_{h=1}^{H} \sum_{s \in S} P_h^E(s) \mathbb{I} \left\{ s \notin S_h(D_1) \right\} \leq \frac{8|S|H}{9m} + \frac{6 \sqrt{|S|} H \log(H/\delta)}{m}.
\]

With Lemma E.2, with probability at least \(1 - \delta\) with \(\delta \in (0, \min\{1, H/5\})\), we have

\[
\sum_{h=1}^{H} \sum_{(s,a) \in S \times A} \left| \hat{P}_h^E(s,a) - P_h^E(s,a) \right| \\
\leq \sum_{h=1}^{H} \sqrt{\left( \frac{8H^3|S|}{9m} + \frac{6 \sqrt{|S|} H \log(2H/\delta)}{m} \right) 3|S| \log(4|S|H/\delta)} \\
\leq \frac{H^3/2|S|}{m} \log^{1/2} \left( \frac{4H^3|S|}{\delta} \right) \sqrt{\frac{8}{3}} + 18 \log(2H/\delta).
\]

Rearranging yields the desired result. \(\square\)

### E.2 Sample Complexity of the Direct Maximum Likelihood Estimation

Here we consider the sample complexity of the direct maximum likelihood estimation in FEM [Abbeel and Ng, 2004] and GTAL [Syed and Schapire, 2007]. Specifically,

\[
\hat{P}_h^E(s,a) := \sum_{t \in D} \mathbb{I} \{ t(s_h, a_h) = (s,a) \}.
\]

**Lemma E.3.** Fix \(\varepsilon \in (0,H)\) and \(\delta \in (0,1)\), if the number of trajectories in \(D\) satisfies

\[
m \geq \frac{H^2|S|}{\varepsilon^2} \log \left( \frac{H}{\delta} \right),
\]

then with probability at least \(1 - \delta\), we have

\[
\sum_{h=1}^{H} \left\| \hat{P}_h^E - P_h^E \right\|_1 \leq \varepsilon.
\]

**Proof.** For each time step \(h \in [H]\), by the TV distance concentration inequality [Weissman et al., 2003], with probability at least \(1 - \delta\), we have

\[
\left\| \hat{P}_h^E - P_h^E \right\|_1 \leq \sqrt{\frac{2|S|}{m} \log \left( \frac{1}{\delta} \right)}.
\]

By applying union bound, with probability at least \(1 - \delta\),

\[
\sum_{h=1}^{H} \left\| \hat{P}_h^E - P_h^E \right\|_1 \leq H \sqrt{\frac{2|S|}{m} \log \left( \frac{H}{\delta} \right)}.
\]

If the number of trajectories satisfies that

\[
m \geq \frac{H^2|S|}{\varepsilon^2} \log \left( \frac{H}{\delta} \right),
\]

\(\square\)

---

\(^{11}\)The first inequality is based on the basic calculus and the second inequality is based on the fact that \((1 - 1/x)^x \leq 1/e \leq 4/9\) while \(x \geq 1\).
we have $\sum_{h=1}^{H} \| \tilde{P}^\pi_h^R - \bar{P}^\pi_h^R \|_1 \leq \varepsilon$. \hfill \Box

### E.3 Proof of Theorem 4.1

Before we prove Theorem 4.1, we first state two key lemmas: Lemma E.4 and Lemma E.5.

**Lemma E.4.** Consider the adversarial imitation learning approach displayed in Algorithm 1, then we have

$$\sum_{t=1}^{T} f(t) \left( w(t) \right) - \min_{w \in \mathcal{W}} \sum_{t=1}^{T} f(t) \left( w \right) \leq 2H \sqrt{2|\mathcal{S}||\mathcal{A}|T},$$

where $f(t)(w) = \sum_{h=1}^{H} \sum_{(s,a) \in \mathcal{S} \times \mathcal{A}} w_h(s,a)(P^\pi_h(t)(s,a) - \bar{P}^\pi_h(s,a))$.

**Proof.** Lemma E.4 is a direct consequence of the regret bound of online gradient descent [Shalev-Shwartz, 2012]. To apply such a regret bound, we need to verify that 1) the iterate norm $\|w\|_2$ has an upper bound; 2) the gradient norm $\|\nabla w f(t)(w)\|_2$ also has an upper bound. The first point is easy to show, i.e., $\|w\|_2 \leq \sqrt{H|\mathcal{S}||\mathcal{A}|}$ by the condition that $w \in \mathcal{W} = \{w : \|w\|_\infty \leq 1\}$. For the second point, let $\bar{P}^1_h$ and $\bar{P}^2_h$ be the first and the second part in $\bar{P}^\pi_h$ defined in (4.1). Then,

$$\|\nabla w f(t)(w)\|_2 = \sqrt{\sum_{h=1}^{H} \sum_{(s,a) \in \mathcal{S} \times \mathcal{A}} \left( P^\pi_h(t)(s,a) - \bar{P}^\pi_h(s,a) \right)^2}$$

$$= \sqrt{\sum_{h=1}^{H} \sum_{(s,a) \in \mathcal{S} \times \mathcal{A}} \left( P^\pi_h(t)(s,a) - \bar{P}^1_h(s,a) - \bar{P}^2_h(s,a) \right)^2}$$

$$\leq \sqrt{\sum_{h=1}^{H} 3 \sum_{(s,a) \in \mathcal{S} \times \mathcal{A}} \left( P^\pi_h(t)(s,a) \right)^2 + \left( \bar{P}^1_h(s,a) \right)^2 + \left( \bar{P}^2_h(s,a) \right)^2}$$

$$\leq 2\sqrt{H},$$

where the first inequality follows $(a + b + c)^2 \leq 3(a^2 + b^2 + c^2)$ and the second inequality is based on that $x^2 \leq |x|$ if $0 \leq x \leq 1$.

Invoking Corollary 2.7 in [Shalev-Shwartz, 2012] with $B = \sqrt{H|\mathcal{S}||\mathcal{A}|}$ and $L = 2\sqrt{H}$ finishes the proof. \hfill \Box

**Lemma E.5.** Consider the transition-aware adversarial imitation learning approach displayed in Algorithm 1 and $\pi$ is the output policy, then we have

$$\sum_{h=1}^{H} \| P^\pi_h - \bar{P}^\pi_h \|_1 \leq \min_{\pi \in \Pi} \sum_{h=1}^{H} \| P^\pi_h - \bar{P}^\pi_h \|_1 + 2H \sqrt{2|\mathcal{S}||\mathcal{A}| T} + \varepsilon_{opt}.$$

**Proof of Lemma E.5.** With the dual representation of $\ell_1$-norm, we have

$$\min_{\pi \in \mathcal{P}} \sum_{h=1}^{H} \| P^\pi_h - \bar{P}^\pi_h \|_1 = \min_{\pi \in \mathcal{P}} \max_{w \in \mathcal{W}} \sum_{h=1}^{H} \sum_{(s,a) \in \mathcal{S} \times \mathcal{A}} w_h(s,a) \left( \tilde{P}^\pi_h(s,a) - P^\pi_h(s,a) \right).$$

Since the above objective is linear w.r.t both $w$ and $P^\pi_h$, invoking the minimax theorem [Bertsekas, 2016] yields

$$= \max_{w \in \mathcal{W}} \min_{\pi \in \mathcal{P}} \sum_{h=1}^{H} \sum_{(s,a) \in \mathcal{S} \times \mathcal{A}} w_h(s,a) \left( \tilde{P}^\pi_h(s,a) - P^\pi_h(s,a) \right).$$

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where the last step follows the property that for a function $f$, $-\max_x f(x) = \min_x -f(x)$. Therefore, we have

$$\min_{\pi \in \Pi} \sum_{h=1}^H \sum_{(s,a) \in S \times A} w_h(s,a) \left( P^\pi_h(s,a) - \tilde{P}^\pi_h(s,a) \right),$$

where $\varepsilon = \min_{x \in X} f(x)$. Then we consider the term $\min_{\pi \in \Pi} \max_{w \in W} \sum_{h=1}^H \sum_{(s,a) \in S \times A} w_h(s,a) \left( P^\pi_h(s,a) - \tilde{P}^\pi_h(s,a) \right)$.

Applying Lemma E.4 yields that

$$\min_{\pi \in \Pi} \sum_{h=1}^H \sum_{(s,a) \in S \times A} w_h(s,a) \left( P^\pi_h(s,a) - \tilde{P}^\pi_h(s,a) \right) \leq \sum_{t=1}^T \sum_{h=1}^H \sum_{(s,a) \in S \times A} w_h(s,a) \left( P^\pi_h(s,a) - \tilde{P}^\pi_h(s,a) \right) + \varepsilon_{opt}. $$

Note that $\pi$ is induced by the mean state-action distribution, i.e., $\pi_h(a|s) = \sum_a \pi_h(s,a) / \sum_a \pi_h(s,a)$. Based on Proposition 3.1 in [Ho and Ermon, 2016], we have that $P^\pi_h(s,a) = \bar{P}_h(s,a)$, and hence the last equation holds. Combined with (E.2), we have that

$$\min_{\pi \in \Pi} \sum_{h=1}^H \left| P^\pi_h - \tilde{P}^\pi_h \right|_1$$

$$\geq - \min_{w \in W} \sum_{h=1}^H \sum_{(s,a) \in S \times A} w_h(s,a) \left( P^\pi_h(s,a) - \tilde{P}^\pi_h(s,a) \right) - 2H \frac{2|S||A|}{T} - \varepsilon_{opt}$$

$$= \max_{w \in W} \sum_{h=1}^H \sum_{(s,a) \in S \times A} w_h(s,a) \left( \tilde{P}^\pi_h(s,a) - P^\pi_h(s,a) \right) - 2H \frac{2|S||A|}{T} - \varepsilon_{opt}$$
= \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1 - 2H \sqrt{\frac{2|S||A|}{T}} - \varepsilon_{\text{opt}},

where the last step again utilizes the dual representation of \( \ell_1 \)-norm. We complete the proof.

**Proof of Theorem 4.1.** Let \( \pi \) be the policy output by Algorithm 1. With Lemma E.5, we establish the upper bound on the \( \ell_1 \) deviation between \( P_h^\pi (s, a) \) and \( \tilde{P}_h^\pi (s, a) \).

\[
\sum_{h=1}^{H} \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1 \leq \min_{\pi \in \Pi} \sum_{h=1}^{H} \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1 + 2H \sqrt{\frac{2|S||A|}{T}} + \varepsilon_{\text{opt}}.
\]

Since \( \pi^E \in \Pi \), we further obtain that

\[
\sum_{h=1}^{H} \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1 \leq \sum_{h=1}^{H} \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1 + 2H \sqrt{\frac{2|S||A|}{T}} + \varepsilon_{\text{opt}}.
\]

Fix \( \varepsilon \in (0, H) \) and \( \delta \in (0, 1) \), when the number of trajectories in \( D \) satisfies that \( m \gtrsim \frac{|S|H^{3/2}}{\varepsilon} \log \left( \frac{|S|H}{\delta} \right) \), with probability at least \( 1 - \delta \), we have

\[
\sum_{h=1}^{H} \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1 \leq \varepsilon \frac{8}{S}.
\]

Moreover, with \( T \gtrsim \frac{|S||A|H^2}{\varepsilon^2} \) and \( \varepsilon_{\text{opt}} \leq \varepsilon \frac{4}{S} \), we can obtain that

\[
\sum_{h=1}^{H} \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1 \leq \varepsilon + \varepsilon_{\text{opt}} \leq \varepsilon + \varepsilon_{\text{opt}} \leq 7\varepsilon \frac{8}{S}.
\]

Finally, with the dual representation of policy value, we can upper bound the policy value gap by the state-action distribution error.

\[
\left| V^\pi - \hat{V}^\pi \right| = \left| \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} \left( P_h^\pi (s, a) - P_h^\pi (s, a) \right) r_h (s, a) \right|
\]

\[
\leq \sum_{h=1}^{H} \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1
\]

\[
\leq \sum_{h=1}^{H} \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1 + \sum_{h=1}^{H} \left\| P_h^\pi - \tilde{P}_h^\pi \right\|_1
\]

\[
\leq \varepsilon \frac{8}{S} + 7\varepsilon \frac{8}{S} = \varepsilon.
\]

\( \square \)
F Proof of Results in Section 5

F.1 Proof of Proposition 5.1

Proof. Let \( \widehat{P}_h^\pi(s, a) \) be an expert state-action distribution estimator and \( \hat{P} \) be a transition model learned by a reward-free method. We define the following events.

\[
E_{\text{EST}} := \left\{ \sum_{h=1}^{H} \left\| \widehat{P}_h^\pi - P_h^\pi \right\|_1 \leq \varepsilon_{\text{EST}} \right\},
\]

\[
E_{\text{RFE}} := \left\{ \forall r : S \times A \rightarrow [0, 1], \pi \in \Pi, \left| V^\pi, P, - V^\pi, \hat{P} \right| \leq \varepsilon_{\text{RFE}} \right\}.
\]

According to assumption (a) and (b), we have that \( \mathbb{P}(E_{\text{EST}}) \geq 1 - \delta_{\text{EST}} \) and \( \mathbb{P}(E_{\text{RFE}}) \geq 1 - \delta_{\text{RFE}} \). Applying union bound yields

\[\mathbb{P}(E_{\text{EST}} \cap E_{\text{RFE}}) \geq 1 - \delta_{\text{EST}} - \delta_{\text{RFE}}.\]

The following analysis is established on the event \( E_{\text{EST}} \cap E_{\text{RFE}} \). Let \( \pi \) be the output of Algorithm 2.

\[
\left| V^\pi, P - V^\pi, \hat{P} \right| \leq \left| V^\pi, P - V^\pi, \hat{P} \right| + \left| V^\pi, \hat{P} - V^\pi, \hat{P} \right| \leq \left| V^\pi, P - V^\pi, \hat{P} \right| + \varepsilon_{\text{RFE}}.
\]

The last inequality follows the event \( E_{\text{RFE}} \). Then we consider the error \( \left| V^\pi, P - V^\pi, \hat{P} \right| \). From the dual form of the policy value in (3.1), we have that

\[
\left| V^\pi, P - V^\pi, \hat{P} \right| = \sum_{h=1}^{H} \sum_{(s, a) \in S \times A} \left( P_h^\pi(s, a) - \hat{P}_h^\pi(s, a) \right) r_h(s, a) \leq \sum_{h=1}^{H} \left\| P_h^\pi - \hat{P}_h^\pi \right\|_1,
\]

where \( \hat{P}_h^\pi(s, a) \) is the state-action distribution of the policy \( \pi \) under the transition model \( \hat{P} \). Then we get that

\[
\sum_{h=1}^{H} \left\| P_h^\pi, P - \hat{P}_h^\pi \right\|_1 \leq \sum_{h=1}^{H} \left\| P_h^\pi, P - P_h^\pi \right\|_1 + \sum_{h=1}^{H} \left\| \hat{P}_h^\pi, P - \hat{P}_h^\pi \right\|_1
\]

\[
\leq \varepsilon_{\text{EST}} + \sum_{h=1}^{H} \left\| \hat{P}_h^\pi - P_h^\pi \right\|_1.
\]

The last inequality follows the event \( E_{\text{EST}} \). Combining the above three inequalities yields

\[
\left| V^\pi, P - V^\pi, \hat{P} \right| \leq \sum_{h=1}^{H} \left\| \hat{P}_h^\pi - P_h^\pi \right\|_1 + \varepsilon_{\text{EST}} + \varepsilon_{\text{RFE}}.
\]

According to assumption (c), with the estimator \( \hat{P}_h^\pi(s, a) \) and transition model \( \hat{P} \), algorithm B solves the projection problem in (4.4) up to an error \( \varepsilon_{\text{AIL}} \) and \( \pi \) is the output of the algorithm B. Formally,

\[
\frac{1}{H} \sum_{h=1}^{H} \left\| \hat{P}_h^\pi - P_h^\pi \right\|_1 \leq \min_{\pi \in \Pi} \frac{1}{H} \sum_{h=1}^{H} \left\| \hat{P}_h^\pi - P_h^\pi \right\|_1 + \varepsilon_{\text{AIL}}.
\]

Then we get that

\[
\left| V^\pi, P - V^\pi, \hat{P} \right| \leq \sum_{h=1}^{H} \left\| \hat{P}_h^\pi - P_h^\pi \right\|_1 + \varepsilon_{\text{EST}} + \varepsilon_{\text{RFE}}
\]

\[
\leq \min_{\pi \in \Pi} \sum_{h=1}^{H} \left\| \hat{P}_h^\pi - P_h^\pi \right\|_1 + H \varepsilon_{\text{AIL}} + \varepsilon_{\text{EST}} + \varepsilon_{\text{RFE}}
\]

\[
\leq \sum_{h=1}^{H} \left\| \hat{P}_h^\pi - P_h^\pi \right\|_1 + H \varepsilon_{\text{AIL}} + \varepsilon_{\text{EST}} + \varepsilon_{\text{RFE}}
\]

\[
\leq \sum_{h=1}^{H} \left\| \hat{P}_h^\pi - P_h^\pi \right\|_1 + H \varepsilon_{\text{AIL}} + \varepsilon_{\text{EST}} + \varepsilon_{\text{RFE}}
\]
\[
\sum_{h=1}^{H} \left\| P_h^{\pi_E, P} - P_h^{\pi_E, \hat{P}} \right\|_1 + H\varepsilon_{AIL} + 2\varepsilon_{EST} + \varepsilon_{RFE},
\]

where inequality (1) holds since \( \pi_E \in \Pi \) and inequality (2) follows the event \( E_{EST} \). With the dual representation of \( \ell_1 \)-norm, we have that

\[
\sum_{h=1}^{H} \left\| P_h^{\pi_E, \hat{P}} - P_h^{\pi_E, \hat{P}} \right\|_1 \leq \max_{w \in \mathcal{W}} \sum_{h=1}^{H} w_h(s, a) \left( P_h^{\pi_E, P}(s, a) - P_h^{\pi_E, \hat{P}}(s, a) \right)
\]

where \( \mathcal{W} = \{ w : \|w\|_{\infty} \leq 1 \} \), \( V_{\pi_E, \hat{P}, w} \) is the value of policy \( \pi_E \) with the transition model \( \hat{P} \) and reward function \( w \). The last inequality follows the event \( E_{RFE} \). Then we prove that

\[
\left| V_{\pi_E, P} - V_{\pi, \hat{P}} \right| \leq 2\varepsilon_{EST} + 2\varepsilon_{RFE} + H\varepsilon_{AIL}.
\]

\[\square\]

### F.2 The Application of FEM on Proposition 5.1

Here we present the policy value gap if we apply FEM [Abbeel and Ng, 2004], which \( \ell_2 \)-norm based estimation and projection. Under our framework in Algorithm 2, the assumption (b) becomes: with probability at least \( 1 - \delta_{EST} \),

\[
\sum_{h=1}^{H} \left\| \hat{P}_h^{\pi_E} - P_h^{\pi_E} \right\|_2 \leq \varepsilon_{EST}.
\]

Besides, the assumption (c) becomes: with estimation \( \hat{P}_h^{\pi_E}(s, a) \) and transition model \( \hat{P} \), the policy \( \pi \) output by FEM satisfies

\[
\frac{1}{H} \sum_{h=1}^{H} \left\| \hat{P}_h^{\pi_E} - P_h^{\pi_E} \right\|_2 \leq \min_{\pi \in \Pi} \frac{1}{H} \sum_{h=1}^{H} \left\| \hat{P}_h^{\pi_E} - P_h^{\pi_E} \right\|_2 + \varepsilon_{AIL}.
\]

Following the same idea in the proof of Proposition 5.1, we can get that

\[
\left| V_{\pi_E, P} - V_{\pi, \hat{P}} \right| \leq \left| V_{\pi_E, P} - V_{\pi, \hat{P}} \right| + \varepsilon_{RFE}
\]

\[
\leq \sum_{h=1}^{H} \left\| P_h^{\pi_E, P} - P_h^{\pi_E, \hat{P}} \right\|_1 + \varepsilon_{RFE}
\]

\[
\leq \sum_{h=1}^{H} \left\| P_h^{\pi_E, P} - \hat{P}_h^{\pi_E} \right\|_1 + \sum_{h=1}^{H} \left\| \hat{P}_h^{\pi_E} - P_h^{\pi_E} \right\|_1 + \varepsilon_{RFE}.
\]

For an arbitrary vector \( x \in \mathbb{R}^n \), we have that \( \|x\|_2 \leq \|x\|_1 \leq \sqrt{n} \|x\|_2 \). Then we show that

\[
\left\| P_h^{\pi_E, P} - \hat{P}_h^{\pi_E} \right\|_1 \leq \sqrt{|S||A|} \left\| P_h^{\pi_E, P} - \hat{P}_h^{\pi_E} \right\|_2 \leq \sqrt{|S||A|} \varepsilon_{EST}.
\]

Then we continue to consider the policy value gap.

\[
\left| V_{\pi_E, P} - V_{\pi, \hat{P}} \right| \leq \sum_{h=1}^{H} \left\| P_h^{\pi_E, P} - P_h^{\pi_E, \hat{P}} \right\|_1 + \sqrt{|S||A|} \varepsilon_{EST} + \varepsilon_{RFE}
\]

\[
\leq \sqrt{|S||A|} \sum_{h=1}^{H} \left\| \hat{P}_h^{\pi_E} - P_h^{\pi_E} \right\|_2 + \sqrt{|S||A|} \varepsilon_{EST} + \varepsilon_{RFE}
\]

\[
\leq \sqrt{|S||A|} \left( \min_{\pi \in \Pi} \sum_{h=1}^{H} \left\| P_h^{\pi_E} - P_h^{\pi_E} \right\|_2 \right) + H\varepsilon_{AIL} + \sqrt{|S||A|} \varepsilon_{EST} + \varepsilon_{RFE}.
\]
The last inequality holds since that FEM performs $\ell_2$-norm projection with $\hat{P}_h^{\pi E}(s,a)$ and $\hat{P}$ up to an error of $\varepsilon_{AIL}$. Then we have that
\[
\left| V_{\pi E}^{r,P} - V^{\pi,P} \right| \leq \sqrt{|S||A|} \sum_{h=1}^{H} \left| \hat{P}_h^{\pi E} - P_h^{\pi E,\hat{P}} \right|_2 + \sqrt{|S||A|} (H\varepsilon_{AIL} + \varepsilon_{EST}) + \varepsilon_{RFE}
\]
\[
\leq \sqrt{|S||A|} \left( \sum_{h=1}^{H} \left| \hat{P}_h^{\pi E} - P_h^{\pi E,\hat{P}} \right|_2 + \sum_{h=1}^{H} \left| P_h^{\pi E,\hat{P}} - P_h^{\pi E,\hat{P}} \right|_2 \right) + \sqrt{|S||A|} (H\varepsilon_{AIL} + \varepsilon_{EST}) + \varepsilon_{RFE}
\]
\[
\leq \sqrt{|S||A|} \left( \varepsilon_{EST} + \sum_{h=1}^{H} \left| P_h^{\pi E,\hat{P}} - P_h^{\pi E,\hat{P}} \right|_2 \right) + \sqrt{|S||A|} (H\varepsilon_{AIL} + \varepsilon_{EST}) + \varepsilon_{RFE}
\]
\[
\leq \sqrt{|S||A|} \sum_{h=1}^{H} \left| P_h^{\pi E,\hat{P}} - P_h^{\pi E,\hat{P}} \right|_1 + \sqrt{|S||A|} (H\varepsilon_{AIL} + 2\varepsilon_{EST}) + \varepsilon_{RFE}
\]
\[
\leq \sqrt{|S||A|} (H\varepsilon_{AIL} + 2\varepsilon_{EST} + \varepsilon_{RFE}) + \varepsilon_{RFE}.
\]
In the last inequality, we use the dual representation of $\ell_1$-norm and policy value. Furthermore, $\hat{P}$ satisfies that for any policy $\pi \in \Pi$ and reward $r \in S \times A \rightarrow [0, 1]$, $|V_{\pi E}^{r,P} - V^{\pi,\hat{P},r}| \leq \varepsilon_{RFE}$.

**Remark F.1.** Note that the additional factor $\sqrt{|S||A|}$ is partially caused by the $\ell_2$-norm. In particular, the original assumption in FEM [Abbeel and Ng, 2004] is that there exists some $w_h$ such that $r_h(s,a) = w_h^\top \phi_h(s,a)$. When $\phi_h(s,a)$ is the one-hot feature used in the tabular MDP in this paper, $w_h(s,a) = r_h(s,a)$. According to our assumption that $r_h(s,a) \in [0, 1]$, such an $w_h$ satisfies $\|w_h\|_2 \leq \sqrt{|S||A|}$, which is different from the assumption $\|w_h\|_2 \leq 1$ in [Abbeel and Ng, 2004]. However, this mismatch may not be a big issue since the concentration rate for $\ell_2$-norm metric is faster than $\ell_1$-norm when the estimation error is small.

### F.3 Explanation of The Estimator in (5.1)

Here, we prove that the estimator shown in (5.1) is an unbiased estimation. We consider the decomposition of $P_h^{\pi E}(s,a)$.

\[
P_h^{\pi E}(s,a) = \sum_{\text{tr}_h \in T_h^{\Pi}} P^\pi(\text{tr}_h) I \{ \text{tr}_h(s_h, a_h) = (s, a) \} + \sum_{\text{tr}_h \notin T_h^{\Pi}} P^\pi(\text{tr}_h) I \{ \text{tr}_h(s_h, a_h) = (s, a) \}.
\]

where $\pi \in \Pi_{BC}(D_1)$ and the last equality follows Lemma F.1. Recall the definition of the new estimator.

\[
\hat{P}_h^{\pi E}(s,a) = \frac{\sum_{\text{tr}_h \in D_{env}^\pi} P^\pi(\text{tr}_h) I \{ \text{tr}_h(s_h, a_h) = (s, a) \} + \sum_{\text{tr}_h \notin D_{env}^\pi} P^\pi(\text{tr}_h) I \{ \text{tr}_h(s_h, a_h) = (s, a) \}}{|D_{env}^\pi|}.
\]

where $D_{env}$ is the dataset collected by the policy $\pi \in \Pi_{BC}(D_1)$. Notice that the two terms in RHS are Monte Carlo estimations of $\sum_{\text{tr}_h \in T_h^{\Pi}} P^\pi(\text{tr}_h) I \{ \text{tr}_h(s_h, a_h) = (s, a) \}$ and $\sum_{\text{tr}_h \notin T_h^{\Pi}} P^\pi(\text{tr}_h) I \{ \text{tr}_h(s_h, a_h) = (s, a) \}$ based on the dataset $D_{env}^\pi$ and $D_{env}^\pi$, respectively. Therefore, $\hat{P}_h^{\pi E}(s,a)$ is an unbiased estimation of $P_h^{\pi E}(s,a)$.

**Lemma F.1.** We define $\Pi_{BC}(D_1)$ as the set of policies, each of which takes expert action on states contained in $D_1$. For each $\pi \in \Pi_{BC}(D_1)$, $\forall h \in [H]$ and $(s,a) \in S \times A$, we have

\[
\sum_{\text{tr}_h \in T_h^{\Pi}} P^\pi(\text{tr}_h) I \{ \text{tr}_h(s_h, a_h) = (s, a) \}
\]

\[
= \sum_{\text{tr}_h \in T_h^{\Pi}} P^\pi(\text{tr}_h) I \{ \text{tr}_h(s_h, a_h) = (s, a) \}.
\]

**Proof.** Let $\Pi_{BC}(D_1)$ denote the set of policies, each of which exact takes expert action on states contained in $D_1$. Fix $\pi \in \Pi_{BC}(D_1)$, $h \in [H]$ and $(s,a) \in S \times A$, we consider the probability $P^\pi(\text{tr}_h)$ of a truncated trajectory $\text{tr}_h \in T_h^{\Pi}$. Since $\pi$ exactly takes expert action on states contained in $D_1$, we have $P^\pi(\text{tr}_h)$
which completes the proof.

Therefore, we obtain that

\[ \sum_{\mathbf{tr}_h \in \mathcal{D}_h} \mathbb{P}^E(\mathbf{tr}_h) \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a) \} = \sum_{\mathbf{tr}_h \in \mathcal{D}_h} \mathbb{P}^E(\mathbf{tr}_h) \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a) \}, \]

which completes the proof. \qed

The sample complexity and interaction complexity of the estimator (5.1) are given in the following.

**Lemma F.2.** Given expert dataset \( \mathcal{D} \) and \( \mathcal{D} \) is divided into two equal subsets, i.e., \( \mathcal{D} = \mathcal{D}_1 \cup \mathcal{D}_1^c \) with \( |\mathcal{D}_1| = |\mathcal{D}_1^c| = m/2 \). Fix \( \pi \in \Pi_E(\mathcal{D}_1^c) \), let \( \mathcal{D}_E \) be the dataset collected by \( \pi \) and \( |\mathcal{D}_E| = n' \). Fix \( \varepsilon \in (0, 1) \) and \( \delta \in (0, 1) \): suppose \( H \geq 5 \). Consider the estimator \( \tilde{P}_h^E \) shown in (5.1), if the expert sample complexity (i.e., \( |\mathcal{D}| \)) and the interaction complexity (i.e., \( |\mathcal{D}_E| \)) satisfies

\[ m \gtrsim \frac{H^{1/2} |S|}{\varepsilon} \log \left( \frac{|S| H}{\delta} \right), \quad n' \gtrsim \frac{H^2 |S|}{\varepsilon^2} \log \left( \frac{|S| H}{\delta} \right), \]

then with probability at least \( 1 - \delta \), we have

\[ \sum_{h=1}^H \left\| \tilde{P}_h^E - P_h^E \right\|_1 \leq \varepsilon. \]

**Proof.** We aim to upper bound the estimation error.

\[ \sum_{h=1}^H \left\| \tilde{P}_h^E - P_h^E \right\|_1. \]

Recall the definition of the estimator \( \tilde{P}_h^E(s, a) \).

\[ \tilde{P}_h^E(s, a) := \frac{\sum_{\mathbf{tr}_h \in \mathcal{D}_E} \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a), \mathbf{tr}_h \in \mathcal{D}_h \} |\mathcal{D}_h|}{|\mathcal{D}_E|} + \frac{\sum_{\mathbf{tr}_h \in \mathcal{D}_E} \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a), \mathbf{tr}_h \notin \mathcal{D}_h \} |\mathcal{D}_E|}{|\mathcal{D}_E|}. \]

Similarly, we utilize the decomposition of \( P_h^E(s, a) \) as we have done in the proof of Lemma 4.1.

\[ P_h^E(s, a) = \sum_{\mathbf{tr}_h \in \mathcal{D}_h} \mathbb{P}^E(\mathbf{tr}_h) \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a) \} + \sum_{\mathbf{tr}_h \notin \mathcal{D}_h} \mathbb{P}^E(\mathbf{tr}_h) \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a) \}. \]

Then, for any \( h \in [H] \) and \( (s, a) \in S \times A \), we have

\[ \left| \tilde{P}_h^E(s, a) - P_h^E(s, a) \right| \leq \left| \sum_{\mathbf{tr}_h \in \mathcal{D}_E} \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a), \mathbf{tr}_h \in \mathcal{D}_h \} \frac{|\mathcal{D}_E|}{|\mathcal{D}_h|} \right| - \left| \sum_{\mathbf{tr}_h \in \mathcal{D}_h} \mathbb{P}^E(\mathbf{tr}_h) \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a) \} \right| \]

\[ + \left| \sum_{\mathbf{tr}_h \notin \mathcal{D}_h} \mathbb{P}^E(\mathbf{tr}_h) \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a) \} \right| - \left| \sum_{\mathbf{tr}_h \notin \mathcal{D}_h} \mathbb{P}^E(\mathbf{tr}_h) \mathbb{I} \{ \mathbf{tr}_h(s_h, a_h) = (s, a) \} \right|. \]

Thus, we can upper bound the estimation error.

\[ \sum_{h=1}^H \left\| \tilde{P}_h^E - P_h^E \right\|_1. \]
When the interaction complexity satisfies that

\[ E^\pi \leq \sum_{h=1}^{H} \sum_{s,a} \mathbb{P}^{\pi \in \mathcal{E}}(s, a) \mathbb{I}\{ \mathcal{T}_b(s, a) = (s, a), \mathcal{T}_b \notin T^D_h \} - \sum_{\mathcal{T}_b \in T^D_h} \mathbb{P}^{\pi \in \mathcal{E}}(\mathcal{T}_b) \mathbb{I}\{ \mathcal{T}_b(s, a) = (s, a) \} \]

\[ + \sum_{h=1}^{H} \sum_{s,a} \mathbb{P}^{\pi \in \mathcal{E}}(s, a) \mathbb{I}\{ \mathcal{T}_b(s, a) = (s, a), \mathcal{T}_b \notin T^D_h \} - \sum_{\mathcal{T}_b \in T^D_h} \mathbb{P}^{\pi \in \mathcal{E}}(\mathcal{T}_b) \mathbb{I}\{ \mathcal{T}_b(s, a) = (s, a) \} . \]

We first analyze the term Error A. Recall that dataset \( \mathcal{D}'_{\text{env}} \) is collected by the policy \( \pi \in \Pi_{\text{BC}}(\mathcal{D}_1) \) with \( |\mathcal{D}'_{\text{env}}| = n' \), and \( \sum_{\mathcal{T}_b \in \mathcal{D}'_{\text{env}}} \mathbb{I}\{ \mathcal{T}_b(s, a) = (s, a), \mathcal{T}_b \notin T^D_h \}/|\mathcal{D}'_{\text{env}}| \) is a maximum likelihood estimator for \( \mathbb{P}^{\pi \in \mathcal{E}}(\mathcal{T}_b) \mathbb{I}\{ \mathcal{T}_b(s, a) = (s, a) \} \). Let \( E^\pi_h \) be the event that \( \mathcal{T}_b \) agrees with expert policy at state \( s \) at time step \( h \) and appears in \( T^D_h \). Formally,

\[ E^\pi_h = \mathbb{I}\{ \mathcal{T}_b(s, a) = (s, \pi \in \mathcal{E}(s)) \cap \mathcal{T}_b \in T^D_h \}. \]

By Lemma E.1, for each \( s \in S \) and \( h \in [H] \), with probability at least \( 1 - \frac{\delta}{2^{|S|H}} \) over the randomness of \( \mathcal{D}' \), we have

\[ \mathbb{P}^{\pi \in \mathcal{E}}(\mathcal{T}_b(s, a) = (s, \pi \in \mathcal{E}(s))) \mathbb{I}\{ \mathcal{T}_b(s, a) \notin T^D_h \} \leq \mathbb{P}^{\pi \in \mathcal{E}}(\mathcal{T}_b(s, a) = (s, \pi \in \mathcal{E}(s))) \frac{3 \log (4|S|H/\delta)}{n'}. \]

By union bound, with probability at least \( 1 - \frac{\delta}{2} \) over the randomness of \( \mathcal{D}'_{\text{env}} \), we have

\[ \text{Error A} \leq \sum_{h=1}^{H} \sum_{s,a} \mathbb{P}^{\pi \in \mathcal{E}}(E^\pi_h) \frac{3 \log (4|S|H/\delta)}{n'} \]

\[ \leq \sum_{h=1}^{H} \frac{|S|}{\sqrt{|S|}} \sum_{s,a} \mathbb{P}^{\pi \in \mathcal{E}}(E^\pi_h) \frac{3 \log (4|S|H/\delta)}{n'}. \]

The last inequality follows the Cauchy-Schwartz inequality. It remains to upper bound \( \sum_{s,a} \mathbb{P}^{\pi \in \mathcal{E}}(E^\pi_h) \) for all \( h \in [H] \).

To this end, we define the event \( G^D_h \) that expert policy \( \pi \in \mathcal{E}^D \) visits states covered in \( \mathcal{D}_1 \) up to time step \( h \). Formally,

\[ G^D_h = \mathbb{I}\{ \forall h' \leq h, s_{h} \in S_h(\mathcal{D}_1) \}, \]

where \( S_h(\mathcal{D}_1) \) is the set of states in \( \mathcal{D}_1 \) at time step \( h \). Then, for all \( h \in [H] \), we have

\[ \sum_{s,a} \mathbb{P}^{\pi \in \mathcal{E}}(E^\pi_h) = \mathbb{P}^{\pi \in \mathcal{E}}(G^D_h) \leq \mathbb{P}(G^D_{h+1}). \]

The last inequality holds since \( G^D_h \subseteq G^D_{h+1} \) for all \( h \in [H] \). Then we have that

\[ \text{Error A} \leq H \sqrt{3|S| \log (4|S|H/\delta) \frac{3 \log (4|S|H/\delta)}{n'}}. \]

When the interaction complexity satisfies that \( n' \geq \frac{|S|H^2}{\epsilon} \log \left( \frac{|S|H}{\delta} \right) \), with probability at least \( 1 - \frac{\delta}{2} \) over the randomness of \( \mathcal{D}' \), we have Error A \( \leq \frac{\delta}{2} \). For the term Error B, we have analyzed it in the proof of Lemma 4.1. When the expert sample complexity satisfies that \( m \geq \frac{|S|H^3/2}{\epsilon} \log \left( \frac{|S|H}{\delta} \right) \), with probability at least \( 1 - \frac{\delta}{2} \) over the randomness of \( \mathcal{D} \), we have Error B \( \leq \frac{\delta}{2} \). Applying union bound finishes the proof.

\[ \square \]

F.4 Proof of Theorem 5.1

Proof of Theorem 5.1. Firstly, we state the theoretical guarantee of RF-Express algorithm [Ménard et al., 2021], which corresponds to assumption (a) in Proposition 5.1.
Then with probability at least \(1 - \delta\), for any policy \(\pi\) and any bounded reward function \(w\) between \([-1, 1]\), we have\(^{12}\) \(|V^\pi, \mathcal{P}, w - V^\pi, \tilde{\mathcal{P}}, w| \leq \varepsilon/2\); furthermore, for any bounded reward function \(w\) between \([-1, 1]\), we have \(\max_{\pi} V^{\pi, w} \leq V^\pi, w + \varepsilon\), where \(\tilde{\pi}^*_w\) is the optimal policy under empirical transition function \(\tilde{\mathcal{P}}\) and reward function \(w\).

When the number of trajectories collected by RF-Express satisfies
\[
n \gtrsim \frac{H^3|S||A|}{\varepsilon^2} \left( |S| + \log \left( \frac{|S|H}{\delta} \right) \right),
\]
for any policy \(\pi \in \Pi\) and reward function \(w : S \times A \to [0, 1]\), with probability at least \(1 - \delta/2\), \(|V^\pi, \mathcal{P}, w - V^\pi, \tilde{\mathcal{P}}, w| \leq \varepsilon/16 = \varepsilon_{\text{RF}}\). In a word, the assumption (a) in Proposition 5.1 holds with \(\delta_{\text{RF}} = \delta/2\) and \(\varepsilon_{\text{RF}} = \varepsilon/16\).

Secondly, we note that the assumption (b) in Proposition 5.1 holds by Lemma F.2. More concretely, if the expert sample \(\varepsilon/\Pi\) for any policy \(\pi \in \Pi\) and reward function \(\Pi, V\), we have \(\min_{\pi} V^\pi, \mathcal{P}, w \leq V^\pi, \tilde{\mathcal{P}}, w + \varepsilon\), where \(\tilde{\pi}^*_w\) is the optimal policy under empirical transition function \(\tilde{\mathcal{P}}\) and reward function \(w\).

Thirdly, we aim to verify that the assumption (c) in Proposition 5.1 holds with \(\tilde{\pi}^*_h, \mathcal{P}^*_h, w\) and \(\tilde{\mathcal{P}}\). With the dual representation of \(\varepsilon_{\text{RF}}\) and the minimax theorem, we get that
\[
\min_{\pi \in \Pi} \sum_{h=1}^{H} \left| \tilde{\pi}^*_h - \mathcal{P}^*_h \right| \leq \min_{w \in \Pi} \sum_{\pi \in \Pi} \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} w_h(s,a) \left( \tilde{\pi}^*_h - \mathcal{P}^*_h \right).
\]
Recall that \(w^{(t)}\) is the reward function inferred by MB-TAIL in the iteration \(t\). Then we have
\[
\min_{w \in \Pi} \sum_{\pi \in \Pi} \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} w_h(s,a) \left( \tilde{\pi}^*_h - \mathcal{P}^*_h \right) \leq \max_{\pi \in \Pi} \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} \left( \frac{1}{T} \sum_{t=1}^{T} w_h^{(t)}(s,a) \right) \left( \tilde{\pi}^*_h - \mathcal{P}^*_h \right)
\]
\[
= \max_{\pi \in \Pi} \frac{1}{T} \sum_{t=1}^{T} \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} w_h^{(t)}(s,a) \left( \tilde{\pi}^*_h - \mathcal{P}^*_h \right)
\]
\[
\leq \frac{1}{T} \sum_{t=1}^{T} \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} w_h^{(t)}(s,a) \left( \tilde{\pi}^*_h - \mathcal{P}^*_h \right) + \varepsilon_{\text{opt}}.
\]

In the last inequality, the policy \(\pi^{(t)}\) is the nearly optimal policy w.r.t \(w^{(t)}\) and \(\tilde{\mathcal{P}}\) up to an error of \(\varepsilon_{\text{opt}}\). Then we have that
\[
\min_{\pi \in \Pi} \sum_{h=1}^{H} \left| \tilde{\pi}^*_h - \mathcal{P}^*_h \right| \geq -\frac{1}{T} \sum_{t=1}^{T} \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} w_h^{(t)}(s,a) \left( \tilde{\pi}^{(t)} - \mathcal{P}^*_h \right) - \varepsilon_{\text{opt}}.
\]

\(^{12}\)This is implied by the stopping rule in RF-Express algorithm and Lemma 1 in [Ménard et al., 2021].
This is because that IL without a known transition is intrinsically harder than online RL problem and the minimax
ε / T.
Applying Proposition 5.1 finishes the proof. With probability at least
1 − δ,
Therefore, the assumption (c) in Proposition 5.1 holds with
ε_{opt} = H \sqrt{\frac{2|S||A|}{T}}.

Note that the reward function \( w^{(t)} \) is updated by online projected gradient descent with objective function
\[ f^{(t)}(w) = \sum_{h=1}^{H} \sum_{(s,a) \in S \times A} \alpha_{h,s,a}(s,a) \left( P_{h}^\pi(s,a) - P_{h}^\pi(s,a) \right) - \varepsilon_{\text{opt}} - 2H \sqrt{\frac{2|S||A|}{T}}. \]

When \( \varepsilon_{\text{opt}} \leq \varepsilon/2 \) and \( T \geq |S||A|H^2/\varepsilon^2 \) such that
\[ 2H \sqrt{\frac{2|S||A|}{T}} \leq \varepsilon/4, \]
we have that
\[ \sum_{h=1}^{H} \left\| P_{h}^\pi(s,a) - P_{h}^\pi(s,a) \right\| \leq \frac{3\varepsilon}{4} = H \varepsilon_{\text{AIL}}. \]

Therefore, the assumption (c) in Proposition 5.1 holds with \( \varepsilon_{\text{AIL}} = 3\varepsilon/4H \).

Now, we summarize the conditions when we have obtained:

- The assumption (a) in Proposition 5.1 holds with \( \delta_{\text{RFE}} = \delta/2 \) and \( \varepsilon_{\text{RFE}} = \varepsilon/16 \).
- The assumption (b) in Proposition 5.1 holds with \( \delta_{\text{EST}} = \delta/2 \) and \( \varepsilon_{\text{EST}} = \varepsilon/16 \).
- The assumption (c) in Proposition 5.1 holds with \( \varepsilon_{\text{AIL}} = 3\varepsilon/4H \).

Applying Proposition 5.1 finishes the proof. With probability at least \( 1 - \delta \),
\[ V^\pi - V^\pi \leq 2\varepsilon_{\text{RFE}} + 2\varepsilon_{\text{EST}} + H \varepsilon_{\text{AIL}} = \varepsilon. \]

\( \square \)

Remark F.2. We conjecture that the dependence of the interaction complexity on \( |S| \) and \( H \) is tight in the worst case. This is because that IL without a known transition is intrinsically harder than online RL problem and the minimax optimal complexity of online RL is already \( \tilde{O}(H^3|S||A|/\varepsilon^2) \) [Menard et al., 2021]. As for an instance dependent complexity, one observation is that the reward functions in AIL are not arbitrary as in reward-free exploration. If we can leverage this property, we may derive sharper results.
G Experiment Details

G.1 Known Transition Setting

We first consider the known transition setting. We use the optimal policy to collect expert demonstrations. The methods for comparison include BC [Pomerleau, 1991], GTAL [Syed and Schapire, 2007], FEM [Abbeel and Ng, 2004], OAL [Shani et al., 2021] and TAIL (see Algorithm 1). All methods are provided the same number of expert demonstrations. All experiments run with 20 random seeds. The detailed information on tasks is listed in Table 3. All experiments run on the machine with 32 CPU cores, 128 GB RAM and NVIDIA GeForce RTX 2080 Ti.

BC directly estimates the expert policy from expert demonstrations. The information on the number of iterations \( T \) of FEM, GTAL, OAL and TAIL is summarized in Table 4. In each iteration, with the recovered reward function, GTAL, FEM and our algorithm TAIL utilize value iteration to solve the optimal policy and OAL performs a policy mirror descent. As discussed in [Zahavy et al., 2020], the optimization problem of FEM can be solved by Frank Wolfe (FW) algorithm [Frank et al., 1956]. In particular, the step size of FW is determined by line search. GTAL uses multiplicative weights to solve the outer problem in (4.5). OAL and our algorithm TAIL utilize online gradient descent to update the reward function. To utilize the optimization structure, an adaptive step size [Orabona, 2019] is implemented for GTAL, OAL and our algorithm TAIL:

\[
\eta_t = \frac{D}{\sqrt{\sum_{i=1}^{T} \| \nabla_w f^{(i)} (w^{(i)}) \|_2^2}},
\]

where \( D = \sqrt{2H|S||A|} \) is the diameter of the set \( W \). After the training process, we evaluate the policy value via exact Bellman update. The running time of different algorithms on Reset Cliff is presented in Table 5. However, due to its computation issue, MIMIC-MD runs out of memory on the machine with 128 GB RAM when solving Reset Cliff with horizon exceeds 100.

Table 3: Information about tasks under known transition setting.

| Tasks                        | Number of states | Number of actions | Horizon   | Number of expert demonstrations |
|------------------------------|------------------|-------------------|-----------|---------------------------------|
| Standard Imitation (Figure 2a) | 500              | 5                 | \(10^1 \to 10^3\) | 300                             |
| Standard Imitation (Figure 2b) | 500              | 5                 | 10        | \(10^2 \to 10^4\)              |
| Reset Cliff (Figure 2c)       | 20               | 5                 | \(10^1 \to 10^3\) | 5000                           |
| Reset Cliff (Figure 2d)       | 5                | 5                 | 5         | \(10^2 \to 10^4\)              |

Table 4: The number of iterations \( T \) of different algorithms on Standard Imitation and Reset Cliff.

| Tasks                        | FEM  | GTAL | OAL  | TAIL |
|------------------------------|------|------|------|------|
| Standard Imitation (Figure 2a) | 500  | 500  | 1400 | 500  |
| Standard Imitation (Figure 2b) | 8000 | 8000 | 8000 | 8000 |
| Reset Cliff (Figure 2c)       | 300  | 300  | 300  | 300  |
| Reset Cliff (Figure 2d)       | 20000 | 20000 | 20000 | 20000 |

Table 5: Running time of different algorithms on Reset Cliff. The unit is second.

| Task             | BC        | FEM       | GTAL      | OAL       | TAIL       |
|------------------|-----------|-----------|-----------|-----------|------------|
| Reset Cliff      | 1960.94 ± 369.05 | 2120.42 ± 451.09 | 2194.20 ± 452.88 | 1916.20 ± 107.99 | 2353.71 ± 488.59 |

G.2 Unknown Transition Setting

Under the unknown transition setting, the methods for comparison include BC, OAL and MB-TAIL. All experiments run with 20 random seeds. Table 6 summaries the detailed information on tasks under the unknown transition setting.

Conclusions about the sample complexity and computational complexity do not change by this adaptive step size.
In particular, OAL is a model-based method and uses mirror descent (MD) [Beck and Teboulle, 2003] to optimize policy and reward. The step sizes of MD are set by the results in the theoretical analysis of [Shani et al., 2021]. During the interaction, OAL maintains an empirical transition model to estimate Q-function for policy optimization. To encourage exploration, OAL adds a bonus function to the Q-function. The bonus function used in the theoretical analysis of [Shani et al., 2021] is too big in experiments and hence, OAL requires too many interactions to reach a good and stable performance. Therefore, we simplify their bonus function from $b_k^h(s,a) = \sqrt{\frac{4H^2|S|\log(3H^2|A|\frac{n}{\delta})}{n_k^h(s,a)\vee 1}}$ to $b_k^h(s,a) = \sqrt{\frac{\log(H|S||A|\frac{n}{\delta})}{n_k^h(s,a)\vee 1}}$, where $n$ is the total number of interactions, $\delta$ is the failure probability and $n_k^h(s,a)$ is the number of times visiting $(s,a)$ at time step $h$ until episode $k$.

MB-TAIL first establishes the estimator in (5.1) with half of the environment interactions and learns an empirical transition model by invoking RF-Express [Ménard et al., 2021] to collect the other half of trajectories. Subsequently MB-TAIL performs policy and reward optimization with the recovered transition model. In MB-TAIL, the policy and reward optimization steps are the same as in TAIL.

| Tasks          | Number of states | Number of actions | Horizon | Number of expert demonstrations |
|----------------|------------------|-------------------|---------|---------------------------------|
| Reset Cliff    | 20               | 5                 | 20      | 100                             |
| Standard Imitation | 100             | 5                 | 10      | 400                             |

Table 6: Information about tasks under unknown transition setting.
H Additional Results

H.1 Tightness of Theorem 4.1

Here we empirically validate that the sample complexity of TAIL presented in Theorem 4.1 is tight by empirical evaluations. In particular, we verify that the horizon dependency and sample complexity dependency of the policy value gap are $\tilde{O}(H^{3/2})$ and $\tilde{O}(1/m)$, respectively. We consider Three State MDP shown in Figure 6. Three State MDP is utilized to establish the lower bound of IL algorithms under the known transition setting in [Rajaraman et al., 2021].

The numerical result regarding the planning horizon is shown in Figure 5a. We see that the slope of TAIL w.r.t $\log(H)$ is around $3/2$, suggesting that the horizon dependency is $\tilde{O}(H^{3/2})$. Besides, as shown in Figure 5b, we see that the slope w.r.t $\log(m)$ is about $-1$, which indicates that the dependency on the number of expert demonstrations is $\tilde{O}(1/m)$.

In summary, we empirically verify that the sample complexity of TAIL is tight. Due to the transition design, GTAL performs similarly to TAIL on the Three State MDP. This is because that the worst case sample complexity of GTAL is not sharp on the Three State MDP.

Figure 5: The policy value gap (i.e., $V^π_e - V^π$) in Three State with different planning horizons and the number of expert demonstrations.

Figure 6: Three State MDP [Rajaraman et al., 2021]. There are two actions, shown in green and blue. There is only a single action on state 1 and 3. The expert policy always takes the green action. The initial state distribution is $\rho = (\rho(1), \rho(2), \rho(3)) = (1, 0, 0)$. Digits on arrows are corresponding rewards. When taking the only action on state 1 and 3, the agent goes to state 2 with a probability of $1/m$ and stays still with a probability of $1 - 1/m$. When taking the green action on state 2, the agent deterministically goes to state 1. Otherwise, the agent goes to state 3.

H.2 GAIL

Under the known transition setting, we also test a famous practical AIL method named GAIL [Ho and Ermon, 2016]. Let $D = (D_1, \cdots, D_H)$ with $D_h : S \times A \to [0, 1]$ for $h \in [H]$. The min-max objective of GAIL is shown as follows.

$$\min_{\pi \in \Pi} \max_D \sum_{h=1}^H \mathbb{E}_{(s,a) \sim P^\pi_h} [\log (1 - D_h(s, a))] + \sum_{h=1}^H \mathbb{E}_{(s,a) \sim P^\pi_h} [\log (D_h(s, a))].$$

(H.1)

Ho and Ermon [2016] provide a practical implementation of GAIL under the unknown transition setting. Specifically, GAIL uses stochastic gradient descent ascent (SGDA) to update the policy and reward function alternatively. It is well-known even the full-batch version of SGDA (i.e., GDA) may not converge properly [Benaïm and Hirsch, 1999, Lin et al., 2020]. As such, GAIL has no theoretical guarantee about the convergence or sample complexity.
To study the sample complexity of GAIL under the known transition setting, we make a small modification. In particular, we use the closed-form solution to the inner loop problem in (H.1):

\[ D_h^* (s, a) = \frac{P^{\pi_h^{(t)}} (s, a)}{P^{\pi_h^{(t)}} (s, a) + P^{\pi^E (s, a)}}. \]

Then the recovered reward function is

\[ w_h^{(t+1)} (s, a) = -\log(D_h^* (s, a)). \]

As for the policy, we use the mirror descent update [Shalev-Shwartz, 2012], which is widely applied to solving a saddle point problem:

\[ \pi_{h}^{(t+1)} (a|s) = \frac{\pi_h^{(t)} (a|s) \exp(\eta Q_h^{(t)} (s, a))}{\sum_{a' \in A} \pi_h^{(t)} (a'|s) \exp(\eta Q_h^{(t)} (s, a'))}, \]

where \( \eta \) is the stepsize and \( Q_h^{(t)} (s, a) \) is the action value function of \( \pi_{h}^{(t)} \) with reward \( w_h^{(t)} \).

The results of GAIL on Standard Imitation and Reset Cliff are plotted in Figure 7. Compared with results in Figure 2, we see that the performance of GAIL is comparative with other conventional AIL methods such as FEM and GTAL. In particular, there is no much difference in the order dependence of the planning horizon and the expert sample size between GAIL and conventional AIL methods. This is reasonable since all of them follow the state-action distribution matching principle and use the naive estimation in (4.3).