Modeling the Performance of Vortex Tube using Response Surface Methodology and Artificial Neural Networks

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Abstract. Vortex tube gives hot and cold streams of gas taking pressurized gas as input. Though this device is mainly used for spot cooling purposes, hot temperature of outlet gas as a response variable, is also of concern for experimentation. The present study deals with modelling and analyzing the effect of five input controllable parameters, each considered with two levels (Max. and Min) viz., internal Diameter of the hot tube in mm, D₁, Length of the hot tube in mm, L, inlet Pressure of air, Kgf / cm², P, nozzle Diameter, in mm, Dₙ, and Diameter of the orifice, in mm, Dₒ, on the responses - cold and hot temperatures of outlet air streams (T_c & T_h) in °C. Box Behnken design is used for experimentation. The interrelationship between responses and input variables is modelled using Response Surface Methodology (RSM) and Artificial Neural Networks (ANN) separately. Regression equations have been developed for the responses using RSM. ANN modelling is observed giving better prediction results compared to RSM modelling as is evidence that through R-square and Average Absolute Deviation (AAD) for both the models.

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1. Introduction
Compressed or pressurized gas / air are the input to the vortex tube. (Counter flow) Vortex tube separates the inlet gas / air into hot and cold streams exhausting from either ends of the vortex tube. Streams separation of hot and cold air is depicted in Fig. 1.

Figure 1. Working of Counter flow Vortex tube

Understanding of heat transfer phenomena, influence of various design parameters, have been the subject matters of various research studies on Vortex Tube. Employing Taguchi method, Pinar et al. [1], attempted to maximize the temperature gradient between the two ends of the Vortex Tubes. Statistical significance of the three factors and response has been established through this study. Prabakaran et al. [2] used Response Surface Methodology (RSM), and found that the selected parameters had significant effect on temperature difference. Cooling the vortex tube externally better the performance of vortex tube was the conclusion [3, 4]. Using Box- Behnken design, five controllable parameters taken at two levels (max. and min.), and experimentation is carried out and results for cold and hot outlet air temperatures (responses) for all the 46 trial conditions have been recorded. Modelling of the performance of the vortex tube is attempted using both RSM and Artificial
Neural Networks (ANN). The objective of the present study is to develop a prediction system that predicts the values of the responses at different combinations of five input parameters.

2. Methodology

2.1 Response Surface Methodology

Design layouts can be either Central Composite Designs (CCD) or Box Behnken Designs and the appropriate design can be chosen depending on the no. of variables. In the present work, Box Behnken Design is used for experimentation. For the five parameters, each at two levels, viz., internal Diameter of the hot tube (13 mm and 15 mm), \( D_t \), Length of the hot tube (150 mm and 180 mm), \( L \), inlet Pressure of air (1 Kgf/cm\(^2\), 3 Kgf/cm\(^2\)), \( P \), nozzle Diameter, (9 mm and 12 mm), \( D_n \), and Diameter of the orifice, (6 mm and 8 mm), \( D_o \), 46 experimental trials/test points were required to be conducted in all. The design layout, with the uncoded level values of the parameters and the two responses values noted after experimentation (cold and hot temperatures of outlet air streams \( T_c \) & \( T_h \) in \(^\circ\)C), is presented in Table 1.

Table 1. Box Behnken Design and Experimental Results

| Run | Factor 1 | Factor 2 | Factor 3 | Factor 4 | Factor 5 | Response 1 | Response 2 |
|-----|----------|----------|----------|----------|----------|------------|------------|
|     | L | Dt | Dn | Do | P | Tc | Th |
| 1   | 165 | 15 | 10.5 | 8 | 2 | 17 | 38 |
| 2   | 180 | 13 | 10.5 | 7 | 2 | 14 | 45 |
| 3   | 165 | 14 | 12 | 8 | 2 | 19 | 41 |
| 4   | 165 | 14 | 9 | 8 | 2 | 17 | 40 |
| 5   | 180 | 14 | 10.5 | 7 | 3 | 12 | 47 |
| 6   | 165 | 14 | 10.5 | 7 | 2 | 18 | 40 |
| 7   | 165 | 14 | 9 | 7 | 1 | 19 | 35 |
| 8   | 150 | 14 | 9 | 7 | 2 | 15 | 38 |
| 9   | 165 | 15 | 10.5 | 6 | 2 | 17 | 39 |
| 10  | 165 | 15 | 10.5 | 7 | 3 | 13 | 41 |
| 11  | 165 | 14 | 10.5 | 7 | 2 | 18 | 40 |
| 12  | 165 | 13 | 9 | 7 | 2 | 14 | 40 |
| 13  | 165 | 14 | 10.5 | 7 | 2 | 18 | 40 |
| 14  | 150 | 14 | 10.5 | 7 | 1 | 19 | 43 |
| 15  | 180 | 14 | 10.5 | 7 | 1 | 20 | 41 |
| 16  | 165 | 13 | 10.5 | 6 | 2 | 15 | 44 |
| 17  | 165 | 15 | 12 | 7 | 2 | 16 | 37 |
| 18  | 165 | 14 | 10.5 | 7 | 2 | 17 | 40 |
| 19  | 165 | 14 | 12 | 7 | 1 | 20 | 39 |
| 20  | 165 | 14 | 12 | 6 | 2 | 16 | 45 |
| 21  | 150 | 15 | 10.5 | 7 | 2 | 16 | 44 |
| 22  | 180 | 14 | 12 | 7 | 2 | 15 | 41 |
2.2 Artificial Neural Networks

In a neural network, the first important stage is the training step. In the training step, an input is introduced to the network accompanied by the desired output. Initially, the weights were set randomly. Since the output may not be what is expected, the weights may need to be altered [5, 6]. During the training phase, random weights are changed by the back-propagation algorithm to produce a satisfactory level of performance. Back Propagation algorithm is a learning technique that adjusts weights in neural network by propagating weight changes backward from the output to the input neurons. After training, the weights contain meaningful information. When a satisfactory level of the performance is reached, the training will stop. Then the network uses these weights to make decisions. In this paper, to evaluate model performance, absolute fraction of variance (R-Squared (R^2)) was computed from the results produced by the ANN model. R-Squared measures the proportion of the variation around the mean. R-square is 1 if the model fits perfectly.

3. Results and Discussion
The results obtained after experimentation are modelled using RSM and ANN.

3.1 Modelling using RSM
The mathematical relationships for the correlation of the output responses like Cold Temperature \((T_c)\) and Hot Temperature \((T_h)\) are obtained from the MiniTab 17 software using RSM. Eqs 1 & 2 are the regression equations for Cold Temperature \((T_c)\) and Hot Temperature \((T_h)\) respectively.

\[
T_c = -413.1 + 1.522 L + 30.67 Dt + 14.90 Dn + 2.90 Do - 2.12 P - 0.005093 L*L - 0.979 Dt*Dt - 0.3981 Dn*Dn + 0.271 Do*Do - 0.313 P*P + 0.0333 L*Dt - 0.0222 L*Dn - 0.0000 L*Do - 0.0333 L*P - 0.333 Dt*Dn + 0.250 Dt*Do + 0.250 Dt*P + 0.333 Dn*Do - 0.167 Dn*P + 0.500 Do*P
\]

(1)

\[
T_h = 84 - 4.517 L + 5.63 Dt + 62.71 Dn - 7.71 Do + 7.35 P + 0.02056 L*L + 0.125 Dt*Dt + 0.389 Dn*Dn + 0.292 Do*Do + 0.208 P*P + 0.0167 L*Dt - 0.2556 L*Dn + 0.0167 L*Do - 0.0167 L*P - 1.667 Dt*Dn + 0.500 Dt*Do - 0.250 Dt*P - 0.667 Dn*Do + 0.333 Dn*P - 0.250 Do*P
\]

(2)

3.2 Modelling using ANN

For training the neural network model, the inputs and outputs must be between 0 and 1 so the values are normalized. The mathematical models developed by RSM and ANN are evaluated for the prediction capability of the outputs for the cold and hot temperatures in vortex tube. The coefficient of determination \((R^2)\) and AAD (absolute average deviation) are determined by Eq. (3) and Eq. (4), respectively [7, 8]; and their values are calculated in order to compare the best model.

\[
R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_{i,\text{cal}} - Y_{i,\text{exp}})^2}{\sum_{i=1}^{n}(Y_{i,\text{exp}} - Y_{\text{avg,exp}})^2}
\]

(3)

\[
AAD = \left\lfloor\left(\frac{\sum_{i=1}^{n}(Y_{i,\text{exp}} - Y_{i,\text{cal}})^2}{n}\right)^{\frac{1}{2}}\right\rfloor \times 100
\]

(4)

| Te No | Experimental Results | ANN Predicted | Error |
|-------|----------------------|---------------|-------|
|       | Cold Temperature     | Hot Temperature | Cold Temperature | Hot Temperature | Cold Temperature | Hot Temperature |
| 1     | 17                   | 38            | 16.99744      | 37.99893      | 0.002558         | 1.07E-03       |
| 2     | 14                   | 45            | 14.00052      | 44.99836      | -0.00052         | 1.64E-03       |
| 3     | 19                   | 41            | 19.00091      | 41.0013       | -0.00091         | -1.30E-03      |
| 4     | 17                   | 40            | 16.99955      | 39.99845      | 0.000451         | 1.55E-03       |
| 5     | 12                   | 47            | 11.99945      | 47.00023      | 0.000546         | -2.30E-04      |
| 6     | 18                   | 40            | 18.00027      | 40.00039      | -0.00027         | -3.85E-04      |
| 7     | 19                   | 35            | 18.99895      | 34.99828      | 0.001046         | 1.72E-03       |
| 8     | 15                   | 38            | 15.00184      | 38.00374      | -0.00184         | -3.74E-03      |
| 9     | 17                   | 39            | 16.99849      | 38.99811      | 0.001513         | 1.89E-03       |
| 10    | 13                   | 41            | 13.00183      | 41.00279      | -0.00183         | -2.79E-03      |
| 11    | 18                   | 40            | 18.00027      | 40.00039      | -0.00027         | -3.85E-04      |
| 12    | 14                   | 40            | 14.00025      | 40.0021       | -0.00025         | -2.10E-03      |
| 13    | 18                   | 40            | 18.00027      | 40.00039      | -0.00027         | -3.85E-04      |
| 14    | 19                   | 43            | 18.99885      | 42.99669      | 0.001145         | 3.31E-03       |
| 15    | 20                   | 41            | 19.99855      | 41.00025      | 0.001452         | 1.72E-03       |
| 16    | 15                   | 44            | 14.99993      | 44.00057      | 6.51E-05         | -5.66E-04      |
| 17    | 16                   | 37            | 15.9998      | 37.00326      | 0.000203         | -3.26E-03      |
| 18    | 17                   | 40            | 16.99955      | 39.99845      | 0.000451         | 1.55E-03       |
| 19    | 20                   | 39            | 20.00098      | 39.00172      | -0.00098         | -1.72E-03      |
| 20    | 16                   | 45            | 16.00052      | 45.00225      | -0.00052         | -2.25E-03      |
| 21    | 16                   | 44            | 16.00044      | 44.00236      | -0.00044         | -2.36E-03      |
| 22    | 15                   | 41            | 14.99818      | 40.99844      | 0.001821         | 1.56E-03       |
| 23    | 15                   | 50            | 14.99244      | 49.98928      | 0.007557         | 1.07E-02       |
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|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 24 | 14 | 42 | 13.99844 | 41.99819 | 0.001556 | 1.81E-03 |
| 25 | 17 | 40 | 16.99955 | 39.99845 | 0.000451 | 1.55E-03 |
| 26 | 18 | 42 | 18.00165 | 42.00081 | -0.00165 | -8.11E-04 |
| 27 | 13 | 50 | 13.00057 | 50.00068 | -0.00057 | -6.81E-04 |
| 28 | 20 | 34 | 20.00055 | 33.99794 | -0.00055 | 2.06E-03 |
| 29 | 13 | 46 | 13.00013 | 45.99787 | -0.00013 | 2.13E-03 |
| 30 | 16 | 47 | 15.9987 | 46.99772 | 0.001303 | 2.28E-03 |
| 31 | 16 | 39 | 15.99865 | 38.99775 | 0.001353 | 2.25E-03 |
| 32 | 21 | 35 | 20.99979 | 35.00281 | 0.000207 | -2.81E-03 |
| 33 | 16 | 45 | 16.00052 | 45.00022 | -0.00052 | -2.25E-03 |
| 34 | 17 | 44 | 17.00058 | 44.00194 | -0.00058 | -1.94E-03 |
| 35 | 17 | 40 | 16.99955 | 39.99845 | 0.000451 | 1.55E-03 |
| 36 | 22 | 38 | 22.00012 | 37.99331 | -0.00012 | 6.94E-04 |
| 37 | 13 | 42 | 13.00032 | 41.99896 | -0.00032 | 1.04E-03 |
| 38 | 17 | 40 | 16.99955 | 39.99845 | 0.000451 | 1.55E-03 |
| 39 | 16 | 55 | 15.92466 | 54.88134 | 0.075342 | 1.19E-01 |
| 40 | 20 | 39 | 20.00098 | 39.00172 | -0.00098 | -1.72E-03 |
| 41 | 18 | 41 | 18.00132 | 41.00069 | -0.00132 | -6.94E-04 |
| 42 | 12 | 47 | 11.99945 | 47.00023 | 0.000546 | -2.30E-04 |
| 43 | 15 | 48 | 14.99959 | 47.99756 | 0.000412 | 2.44E-03 |
| 44 | 16 | 40 | 15.99872 | 39.99765 | 0.001285 | 2.35E-03 |
| 45 | 16 | 48 | 15.99615 | 47.99281 | 0.003845 | 7.19E-03 |
| 46 | 13 | 48 | 12.99988 | 48.00176 | 0.000116 | -1.76E-03 |

**Figure 2.** Regression Curve of the ANN model
Table 3. Comparison of $R^2$ and AAD values for the outcomes

|                | Cold Temperature ($T_c$) | Hot Temperature ($T_h$) |
|----------------|--------------------------|-------------------------|
| **R square**   | RSM 0.9899               | ANN 0.9999              |
|                | RSM 0.9609               | ANN 0.9999              |
| **Average Absolute Deviation** | RSM 0.819               | ANN 0.573               |
|                | RSM 0.462                | ANN 0.255               |

The coefficient of determination ($R^2$) and average absolute deviation are calculated for the responses and found that $R^2$ values for both the values are near to 1 in both the methods. But to elaborate more the $R^2$ values of ANN is very much closure to 1 when compared to RSM values, and also the AAD values of ANN shows lesser deviation than the RSM values.

**Conclusions**

Experimentation has been performed under various input conditions using Box-Benkhen design with 46 experimental trials. Using the experimental results, the performance (cold and hot temperatures of outlet air) of the Vortex tube is modeled using both Artificial Neural Networks and Response Surface Methodology. It has been evaluated that the ANN model is predicting better than the RSM model as is evident from the $R^2$ values (99.99, 99.99 for ANN ($T_c$ and $T_h$); 98.99, 96.09 for RSM ($T_c$ and $T_h$)) and AAD values (0.573, 0.255 for ANN ($T_c$ and $T_h$); 0.819, 0.462 for RSM ($T_c$ and $T_h$)). Compared to the hitherto works, this work, considering five input controllable parameters, is comprehensive in modeling the performance of Vortex tube.

**References**

[1] Ahmet Murat Pinar, Onuralp Uluer, Volkan Kirmaci, (2009), “Optimization of counter flow Ranque-Hilsch Vortex Tube performance using Taguchi method” *International Journal of Refrigeration*, vol. 32, 1487-1494.

[2] J., Prabakaran., S., Vaidyanathan, D., Kanagarajan, (2012) “Establishing empirical relation to predict temperature difference of vortex tube using Response Surface Methodology”, *Journal of Engineering Science and Technology*, vol. 7, no.6, 722–731.

[3] S., Eiamsa-ard, K., Wongcharee, P., Promvonge, (2010) “Experimental investigation on energy separation in a counter-flow Ranque-Hilsch vortex tube: Effect of cooling a hot tube”, *International Communications in Heat and Mass Transfer*, vol. 37, 156-162.

[4] Kirmaci, V. (2017). Experimental Investigation of Cooling - Heating Performance of Counter Flow Ranque-Hilsch Vortex Tubes Having Different Length Diameter Ratio. *Cumhuriyet Science Journal*, 38(4), 813-821.

[5] Dincer, K., Tasdemir, S., Baskaya, S., & Uysal, B. (2008). Modeling of the effects of length to diameter ratio and nozzle number on the performance of counterflow Ranque–Hilsch vortex tubes using artificial neural networks. *Applied Thermal Engineering*, 28(17-18), 2380-2390.

[6] Korkmaz, M. E., Gümüşel, L., & Markal, B. (2012). Using artificial neural network for predicting performance of the Ranque–Hilsch vortex tube. *International Journal of Refrigeration*, 35(6), 1690-1696.

[7] Uluer, O., Kirmaci, V., & Ataş, Ş. (2009). Using the artificial neural network model for modeling the performance of the counter flow vortex tube. *Expert Systems with Applications*, 36(10), 12256-12263.

[8] Pouraria, H., Kia, S., Park, W., & Meh dizadeh, B. (2016). Modeling the cooling performance of vortex tube using a genetic algorithm-based artificial neural network. *Thermal Science*, 20(1), 53-65.