I. INTRODUCTION

In recent years, research on the structure of complex interacting systems and the dynamical processes occurring on these systems has attracted a lot of attention. A majority of studies, focused on investigating the structure and properties of complex systems, model them as a single connected network, where the link between two nodes represent an interaction between two entities [1–4]. However, most real world systems are composed of networks which interact with each other. For example, the power distribution network and the communication network. The nodes in the power distribution network depend on communication nodes (routers) for control, while communication nodes depend on power stations for electricity [5,6]. Due to this interdependence, failure of a proportion of nodes in either network may result in complete collapse of both networks. This is confirmed by recent studies on the percolation behavior [8–10] of interdependent networks; percolation analysis revealed the presence of first order phase transitions, i.e., collapse of portion of nodes in one network may lead to catastrophic collapse of the entire system. Thus, the study of such interacting networks is key in furthering our understanding of real world systems.

Not all real world systems exhibit interdependent behavior, in this article we study one such system. Consider a bot-net (computers infected with malware) launching denial of service (DoS) attacks against SCADA (supervisory control and data acquisition) systems which control power stations. A typical denial of service attack (DoS) happens when the target is overwhelmed with service and resource requests. This prevents legitimate users from accessing the service and may even cause the target server to shutdown. A bot-net is a network of computers, infected with malware, which launch DoS attacks, due to the malware, against Internet servers, routers, or any other critical infrastructure such as SCADA systems. Simulation studies have shown that such a distributed denial of service attack (DDoS) on SCADA systems can result in failure of power stations [12, 13]. Also, the communication links providing connectivity to the bot-net may depend on the electricity supplied by the power distribution network. Thus, a DDoS attack by a bot-net may lead to the failure of power stations, which is likely to cause fragmentation of the bot-net, resulting in the reduction of DDoS attacks. The decrease in the number of DDoS attacks may allow the failed power stations to resume operation which may also cause the failed links to become operational and the cycle may continue. Clearly, such attacks have the potential to damage the entire power distribution network. Understanding the dynamics of the system under DDoS attacks is crucial for formulating strategies to counter such attacks. Motivated by this real world problem, we formulate a mathematical model of two interacting networks and study their robustness against such attacks. We do this by using tools from statistical mechanics, namely, percolation theory.

Since the nodes in the bot-net launch an attack on the nodes in the power distribution network, we call this interaction ‘antagonistic’, a term also used in [14]. The interaction between the power distribution network and the bot-net is termed as dependent, since the links of the bot-net are dependent on the nodes of the power distribution network. Since the bot-net antagonizes the power distribution network, we refer to the bot-net as antagonistic network and the power distribution network as victim network. To reiterate, nodes infected with malware in the antagonistic network causes failure of a node in the victim network, while failure of a node in the victim network may result in failure of a link in the antagonistic network.
network. This results in a negative feedback leading to the self regulation of attacks launched by the antagonistic network. Such self regulating mechanisms abound in biological systems such as: inter-cellular and intra-cellular machinery [15], the mammalian immune system [16], and ecological systems [17].

Recently, in the research community, the study of interdependent networks is slowly gaining traction. The robustness and phase transition properties of networks consisting of two or more interdependent network was studied in [8] [18] [20]. These studies discovered that networks of interdependent networks can exhibit first order discontinuous phase transitions making them susceptible to catastrophic failure under attacks. First order phase transitions were also observed in single networks consisting of connectivity and dependency links [27] [28]. Furthermore, unlike isolated scale free networks, it was found, that it is difficult to protect interdependent networks against an attack [29] [31] by protecting high degree nodes. However, these results may not be applicable for networks exhibiting antagonistic and dependent interactions, in fact our investigations show that these results are not observed in such systems.

A system of networks with mutual antagonistic interactions and interdependent networks with fraction of antagonistic nodes was recently studied in [14] and [32] respectively. In the purely antagonistic case, nodes functioning in one network cause failure of nodes in the other and vice versa. In the mixed case, [32], the two networks are interdependent with antagonistic nodes in both the networks. This is different from the system considered here, because, in our case antagonistic nodes are present in only one network. The difference may seem minor, but it has a significant effect on the phase transitions of the system. We show that the phase transitions observed in pure and mixed antagonistic interacting networks are very different from those observed in interacting antagonistic dependent networks.

In a very recent article, [33], researchers investigated networks which recover spontaneously after an attack. They observe a phenomenon where the mean number of active nodes undergo a phase transition. In our model, where both the networks recover due to the negative feedback, we observe that the size of the giant connected component (GCC) fluctuates only in a certain region of the parameter space.

Our contributions are summarized as follows. We analytically and numerically study the percolation behavior of the system. We analytically show that unlike the first order phase transition observed in interdependent networks, the phase transition is continuous. Although the antagonistic network depends on the victim network, our results suggests that, for Erdos-Renyi and scale free networks, the antagonistic network always percolates, while the attacked network may fail completely. Additionally, we show that, in comparison with an isolated network, networks with dependent and antagonistic interactions are more robust against random attacks.

More importantly, we find that such a system exhibits a threshold behavior. If the dependency and antagonism is high enough, the giant connected components in the two networks oscillate, while for low dependency and antagonism the giant connected component remains stable. Such phenomenon is neither observed in interdependent networks [10], nor in networks exhibiting antagonistic interactions [14] [32].

The article is organized as follows. The system model is detailed in Sec. II analytical results are discussed in Sec. III while numerical results are outlined in Sec. IV and the implications of the results are presented in Sec. V.

II. MODEL

The system consists of two interacting networks, let $A$ be the antagonistic computer network, and $B$ be the non-antagonistic power distribution network. Let $q_a, q_a > 0$, be the fraction of the communication links in network $A$ that depend on the power stations (nodes) in network $B$. Let $r, r > 0$, be the fraction of nodes (computers) in network $A$ infected by malware. We assume that a computer infected by malware attacks a node uniformly chosen at random from network $B$. Thus, a node in network $B$ fails when it is attacked by a compromised node from network $A$, or if it is disconnected from the giant connected component. We assume that communication links in network $A$, depend on a randomly chosen node in $B$. Thus a link in $A$ may fail when a node, on which it depends on, fails. Without an Internet connection it is not possible to launch a DDoS attack, hence the failure of communication links may isolate the compromised nodes resulting in cessation of attacks. Thus a compromised node is unable to launch a DDoS attack if it is not connected to the giant connected component. The fraction of nodes in network $A$, infected with malware, $(r)$, is assumed to be constant. However, the fraction of nodes that launch an attack depends on the GCC of the network, which may change with time.

Let $P_a(k)$ and $P_b(k)$ be the degree distribution of networks $A$ and $B$ respectively, and let $N_a$, $N_b$ be the number of nodes in $A$ and $B$. Let $G^0_a(f_a)$ and $G^0_b(f_b)$ denote the probability generating functions for distributions $P_a(k)$ and $P_b(k)$. Let $G^1_a(f_a)$ and $G^1_b(f_b)$ be the generating functions for excess degree distribution of network $A$ and $B$ respectively. The distribution of the number of links encountered by traversing a randomly chosen link (without including the randomly chosen link) is termed as the excess degree distribution. For networks generated by the configuration model [32], excess degree distribution $Q(k)$, is given by $\frac{1}{(k+1)^2} (k+1) P(k+1)$, where $< k >$ is the average degree and $P(k)$ is the degree distribution of the network. We assume that both the networks are generated using the configuration model.
III. ANALYTICAL RESULTS

We first review site and bond percolation on a single isolated network with degree distribution $P(k)$ and site (bond) occupation probability $p$, more details can be found in [34]. In site percolation, each node is active with probability $p$ independent of other nodes, while in bond percolation each link is active with probability $p$. Thus, in site percolation $1 - p$ fraction of nodes are removed, while in bond percolation $1 - p$ fraction of links are removed. The connected component remaining after the node (link) removal is termed as the giant connected component (GCC). If $p$ is sufficiently low GCC may not exist. Let $S$ denote the fraction of nodes in the GCC. Let $f$ be the probability that a randomly chosen link does not lead to the GCC. Therefore, for site percolation

$$S_{site} = p \left( 1 - \sum_{k=0}^{\infty} f^k P(k) \right) = p(1 - G_0(f))$$

and for bond percolation $S_{bond} = 1 - G_0(f)$, where $G_0(f)$ is the probability generating function of the degree distribution. A link does not lead to GCC if it is inactive, or if it is active and the node at the other end of the link does not belong to the GCC. Assuming that the network is generated by the configuration model, $f$ can be written as,

$$f = 1 - p + \frac{p}{<k> - 1} \sum_{k=0}^{\infty} (k + 1) P(k + 1)$$

$$= 1 - p + pG_1(f)$$

Where $G_1(f)$ is the generating function of the excess degree distribution $Q(k)$. The size of the GCC is non zero if the solution of the fixed point equation is less than unity, i.e., $f < 1$. This can happen if and only if $p > p_c$, where

$$p_c = \frac{1}{G_1(1)} = \frac{<k^2>}{<k^2> - 1}$$

and $<k^2>$ is the second moment of the degree distribution.

In a system consisting of interacting antagonistic and dependent networks, the size of the GCC in both networks may change with time. Let $x_n$ and $y_n$ be the fraction of active links in network $A$ and fraction of active nodes in $B$ respectively, at step $n$. Similar to the single network case we define $S_a(n)$, $S_b(n)$ and $f_a(n)$, $f_b(n)$ for networks $A$ and $B$ at step $n$. In each step, network $A$ can change causing network $B$ to change in the same step. Note that bond percolation occurs on $A$ while site percolation occurs on $B$. The probability that a randomly chosen node in $B$ fails due to an attack from network $A$ is given by $1 - (1 - \frac{1}{N_0})^{rN_a}$. Assuming $N_a = N_b = N$, and for a large $N$ the probability of attack is approximately $r$.

Assume that initially both the networks are fully connected, compromised nodes launching a DDoS attack on nodes in $B$ initiates a site percolation process in $B$ with site occupation probability $y_1 = 1 - r$. The size of the GCC in $B$, $S_b(1)$, in $B$ becomes $y_1(1 - G_0^{\text{b}}(f_b(1)))$ where $f_b(1) = 1 - y_1 + y_1 G_1^{\text{b}}(f_b(1))$. This initiates a bond percolation process in $A$ with bond occupation probability $x_2 = 1 - q_a(1 - S_a(1))$ causing a change in size of GCC in $A$ which then affects network $B$ and so on. The change in GCC in $A$ and $B$ is described by the following sequence.

\[
\begin{align*}
    x_1 &= 1, \quad S_a(1) = 1 - G_0^{\text{a}}(f_a(1)) \\
    f_a(1) &= G_1^{\text{a}}(f_a(1)) \\
    y_1 &= 1 - rS_a(1), \quad S_b(1) = y_1(1 - G_0^{\text{b}}(f_b(1))) \\
    f_b(1) &= 1 - y_1 + y_1 G_1^{\text{b}}(f_b(1)) \\
    x_2 &= 1 - q_a(1 - S_a(1)), \quad S_a(2) = 1 - G_0^{\text{a}}(f_a(2)) \\
    f_a(2) &= 1 - x_2 + x_2 G_1^{\text{a}}(f_a(2)) \\
    y_2 &= 1 - rS_a(2), \quad S_b(2) = y_2(1 - G_0^{\text{b}}(f_b(2))) \\
    f_b(2) &= 1 - y_2 + y_2 G_1^{\text{b}}(f_b(2)) \\
    \vdots \\
    x_n &= 1 - q_a(1 - S_a(n - 1)), \quad S_a(n) = 1 - G_0^{\text{a}}(f_a(n)) \\
    f_a(n) &= 1 - x_n + x_n G_1^{\text{a}}(f_a(n)) \\
    y_n &= 1 - rS_a(n), \quad S_b(n) = y_n(1 - G_0^{\text{b}}(f_b(n))) \\
    f_b(n) &= 1 - y_n + y_n G_1^{\text{b}}(f_b(n))
\end{align*}
\]

At equilibrium

\[
\begin{align*}
    f_a &= f_a(n) = f_a(n - 1), \quad f_b = f_b(n) = f_b(n - 1) \\
    \\
    S_a &= S_a(n) = S_a(n - 1), \quad S_b = S_b(n) = S_b(n - 1). \\
    \end{align*}
\]

Thus we obtain :

\[
\begin{align*}
    S_a &= 1 - G_0^{\text{a}}(f_a) \\
    f_a &= q_a(1 - G_0^{\text{a}}(f_a) + (1 - G_0^{\text{b}}(f_a)))(1 - G_0^{\text{b}}(f_b)) + G_1^{\text{a}}(f_a) \left[ 1 - q_a(1 - G_0^{\text{a}}(f_a) + (1 - G_0^{\text{b}}(f_a)))(1 - G_0^{\text{b}}(f_b)) \right] \\
    S_b &= [1 - r(1 - G_0^{\text{a}}(f_a))] [1 - G_0^{\text{b}}(f_b)] \\
    f_b &= r(1 - G_0^{\text{a}}(f_a)) + [1 - r(1 - G_0^{\text{a}}(f_a)))] G_1^{\text{b}}(f_b)
\end{align*}
\]

(2)

Theoretical calculations are verified using extensive Monte Carlo simulations. Fig[1] shows a good agreement between the theoretical predictions and simulation results.

A. Analysis of the Equilibrium Points

We know check the conditions required for the boundary $(f_a, f_b \in \{0, 1\})$ and non-boundary points $(f_a, f_b \in (0, 1))$ to be equilibrium points.
The derivation of this condition can be found in the appendix. This condition implies that network A does not contain a GCC from the very beginning, since we assume that both networks are fully connected at the start, this scenario is not possible.

In the other case: \(0 < f_b < 1\) and \(f_a = 1\), substituting \(f_a = 1\) in equation (3) we obtain \(f_b = G_1^b(f_b)\). Thus, the conditions required for this point to be an equilibrium point are:

\[
1 - q_a G_0^a(fb) < \frac{<k_a>}{<k_a^2> - <k_a>} - \frac{<k_b>}{<k_b>} > 1
\]

(4)

The derivation of the first condition can be found in the appendix. The second condition corresponds to \(p > p_c\) for network B. For the isolated network case the condition for non existence of the GCC is \(p < \frac{<k_a>}{<k_a^2>}\). Since \(G_0(f_b) < 1\), the value of \(q_a\) required to fragment network A is higher compared to the isolated network case. Thus A is more robust against random attack as compared to the isolated network scenario.

3. Complete collapse of network B

Complete collapse of network B is possible when \(f_b = 1\) and \(f_a = 0\) or \(0 < f_a < 1\). Substituting \(f_b = 1\) in equation (2) we get

\[
f_a = q_a + (1 - q_a)G_1^a(f_a)
\]

Clearly, \(f_a\) cannot be zero since \(q_a > 0\).

The condition required for \(0 < f_a < 1\) can be obtained by substituting \(f_b = 1\) in equation (2). \(f_b = 1\) is possible if and only if a giant connected component does not exist in network B. This translates to:

\[
1 - q_a > \frac{<k_a>}{<k_a^2> - <k_a>}
\]

\[1 - r(1 - G_0^a(f_a)) < \frac{<k_a>}{<k_a^2> - <k_a>} \]  

(5)

Since \(1 - G_0^a(f_a) < 1\), the value of \(r\) required for complete destruction of the GCC is higher than the isolated network case, which makes the system of networks with antagonistic and dependent interactions more robust against random attacks as compared to isolated networks.

4. Both networks percolate

This is possible only when: \((f_a = 0, 0 < f_b < 1)\), or \((f_a, f_b = 0)\), or \((0 < f_a < 1, f_b = 0)\), or \(f_a, f_b \in (0, 1)\). If \(0 < f_b < 1\) then \(f_a\) must satisfy (2). Clearly, \(f_a = 0\) is not possible since \(q_a > 0\). One can similarly show that if \(0 < f_a < 1\), then \(f_b = 0\) is not possible since \(r > 0\).
If \( f_a, f_b = 0 \) then substituting \( f_a = 0 \) in equation \( \text{(3)} \) we get

\[
 f_b = r + (1 - r)G^b_1(f_b)
\]

Thus, \( f_b = 0 \) is not possible since \( r > 0 \). Hence, both networks percolate if and only if \( f_a, f_b \in (0, 1) \). The conditions on \( q_a \) and \( r \) required for \( 0 < f_a, f_b < 1 \) can be obtained by numerically solving equations \( \text{(2)} \) and \( \text{(3)} \).

**B. Stability Analysis**

The equilibrium points \( f_a^*, f_b^* \) are given by:

\[
 f_a^*, f_b^* = \left\{ \begin{array}{ll}
 1 & 0 < f_a^* < 1, \quad f_b^* = 0 \\
 0 & 0 < f_a^* < 1, \quad f_b^* = 1 \\
 \end{array} \right.
\]

The equilibrium point \( f_a^*, f_b^* \) in \( (0, 1) \) may not be stable. Let

\[
 f_1(f_a, f_b) = G^a_1(f_a) \\
 + G^b_1(f_a) \left[ 1 - q_a \left( G^0_0(f_b) + r(1 - G^0_0(f_a))(1 - G^0_0(f_b)) \right) \right]
\]

\[
 f_2(f_a, f_b) = r(1 - G^0_0(f_a)) + [1 - r(1 - G^0_0(f_a))]G^b_1(f_b)
\]

Equations \( \text{(2)} \) and \( \text{(3)} \) can be written as \( f_a = f_1(f_a, f_b) \) and \( f_b = f_2(f_a, f_b) \). This may allow one to calculate the fixed point iteratively as follows:

\[
 f_a(n + 1) = f_1(f_a(n), f_b(n)) \\
 f_b(n + 1) = f_2(f_a(n), f_b(n))
\]

The above system is not same as the one described in \( \text{(1)} \), this is because, in \( \text{(1)} \), \( f_a(n + 1) \) is a function of \( f_a(n) \) and \( f_b(n + 1) \). However, the equilibrium point of \( \text{(6)} \) is also the equilibrium point of \( \text{(1)} \). If the equilibrium point of \( \text{(6)} \) is unstable, then under a small perturbation, the system diverges away from the equilibrium point.

If the equilibrium point of the above system \( (f_a^*, f_b^*) \) is unstable then the corresponding equilibrium point of system \( \text{(1)} \), \( (S^*_a, S^*_b) \), is also unstable. This is because if \( f_a(n), f_b(n) \) in \( \text{(6)} \) diverge under perturbation, then \( f_a(n), f_b(n) \) also diverge under perturbation in \( \text{(1)} \). However, the reverse may not be true, i.e., stability of \( (f_a^*, f_b^*) \) may not imply stability of \( S^*_a \) and \( S^*_b \). Thus, the stability of system \( \text{(6)} \) is a necessary condition for \( S^*_a(n) \) and \( S^*_b(n) \) to be stable, or in other words if system \( \text{(6)} \) is unstable at the equilibrium point then system \( \text{(1)} \) is also unstable at the same equilibrium point.

We analyze the stability of system \( \text{(6)} \) using linear stability analysis. The equilibrium point \( (f_a^*, f_b^*) \) is stable if and only if magnitude of all the Eigen values of the Jacobian matrix \( J \) is less than one.

\[
 J = \begin{bmatrix}
 \frac{\partial f_1}{\partial f_a} & \frac{\partial f_1}{\partial f_b} \\
 \frac{\partial f_2}{\partial f_a} & \frac{\partial f_2}{\partial f_b}
\end{bmatrix}
\]

In other words the roots of the characteristic equation given below must be negative.

\[
 (\lambda - \frac{\partial f_1}{\partial f_a}) (\lambda - \frac{\partial f_2}{\partial f_b}) - \frac{\partial f_1}{\partial f_b} \frac{\partial f_2}{\partial f_a} \bigg|_{f_a = f_a^*, f_b = f_b^*} = 0 \quad \text{(7)}
\]

In the following sections we calculate the necessary conditions for stability of equilibrium points \( (f_a^*, f_b^*) \) and \( (0 < f_a^* < 1, f_b^* = 0) \). Stability conditions for the equilibrium point \( f_a^*, f_b^* \in (0, 1) \) can be calculated numerically by computing the roots of equation \( \text{(7)} \).

1. **Stability of the equilibrium point** \( (f_a^*, f_b^* = 1, 0 < f_b^* < 1) \)

After substituting \( f_a = 1 \) and \( f_b = f_b^* \) in equation \( \text{(7)} \), the Eigen values are:

\[
 \lambda_1 = (1 - q_a G^0_0(f_b^*)) \left\{ \frac{k_a^2}{k_a} - \frac{k_b}{k_a} \right\} - 1 \\
 \lambda_2 = G^b_1(f_b^*) - 1
\]

From condition \( \text{(4)} \), \( \lambda_1 < 1 \). Thus, the point is stable if and only if \( G^b_1(f_b^*) < 1 \). The points \( q_a^* \) and \( r^* \) which satisfy this condition can be computed numerically by evaluating the above condition along with condition \( \text{(4)} \).

2. **Stability of the equilibrium point** \( (0 < f_a^* < 1, f_b^* = 1) \)

The roots of the characteristic equation after substituting \( f_a = f_a^* \) and \( f_b = 1 \) are:

\[
 \lambda_1 = (1 - q_a G^a_1(f_a^*)) - 1 \\
 \lambda_2 = [1 - r(1 - G^a_0(f_a^*))] \left\{ \frac{k_b^2}{k_b} - \frac{k_a}{k_b} \right\} - 1
\]

From condition \( \text{(5)} \), \( \lambda_2 < 1 \). Thus the point is stable if and only if \( (1 - q_a) G^a_1(f_a^*) < 1 \). The region can be computed by numerically evaluating the above condition along with condition \( \text{(5)} \).

**C. Nature of the phase transition**

Fig. 2 shows a phase transition in network \( B \) when both networks are Erdos-Renyi and have the same mean degree. Thus a phase transition may occur near the equilibrium points \( (f_a^* = 1, 0 < f_b^* < 1) \) and \( (0 < f_a^* < 1, f_b^* = 1) \) if they exist and are stable. For \( (f_a < f_a^*, f_b^* = 1) \) the GCC emerges in network \( A \), while for \( (f_a < f_a^*, f_b^* = 1) \) the GCC emerges in network \( B \).

Using the implicit function theorem we show that the phase transition, in both the cases, is continuous in \( q_a \) and \( r \).

Let

\[
 g(f_a, f_b) = G^a_1(f_a) - f_a \\
 + G^b_1(f_a) \left[ 1 - q_a \left( G^0_0(f_b) + r(1 - G^0_0(f_a))(1 - G^0_0(f_b)) \right) \right]
\]

\[
 h(f_a, f_b) = r(1 - G^0_0(f_a)) + [1 - r(1 - G^0_0(f_a))]G^b_1(f_b) - f_b
\]
Equations (2) and (3) can be written as \( g(f_a, f_b) = 0 \) and \( h(f_a, f_b) = 0 \).

1. Phase transition at \((f_a^*, f_b^*) < 1\)

A phase transition is possible only if the equilibrium point is stable, therefore we assume that the conditions for stability of \((f_a^*, 0 < f_b^* < 1)\) are satisfied. We first show that the derivative of \( f_a \) with respect to \( r \) and \( q_a \) near \( f_a^* = 1 \) exists using the implicit function theorem on \( g(f_a, f_b) \). Since \( g(f_a, r, q_a) \) is continuously differentiable, according to the implicit function theorem

\[
\frac{df_a}{dr} = -\frac{\partial g}{\partial f_a} \frac{\partial f_a}{\partial r}.
\]

The derivative exists at \( f_a = f_a^* \) if and only if \( \frac{\partial g}{\partial f_a} \neq 0 \). The same existence condition holds true for \( \frac{df_b}{dq_a} \).

\[
\frac{\partial g}{\partial f_a} = G_0^{a'} \left[ 1 - q_a \left( G_0^{b'} + r(1 - G_0^{a})(1 - G_0^{b}) \right) \right] - 1
\]

\[
+ (1 - G_1^a)q_a \left( G_0^{b'} \frac{\partial f_b}{\partial f_a} (1 - r(1 - G_0^{a})) - rG_1^b (1 - G_0^{b}) \right)
\]

where \( \frac{\partial f_b}{\partial f_a} = -rG_0^{a'} + rG_0^{b'}G_1^b + (1 - r(1 - G_0^{a}))G_1^{b'} \frac{\partial f_b}{\partial f_a} \).

At \( f_a = f_a^* = 1 \),

\[
\frac{\partial f_b}{\partial f_a} = \frac{rG_0^{a'}(1 - G_0^{b}(f_b))}{1 - G_1^b} < \infty
\]

Thus \( f_a \) is differentiable and hence continuous with respect to \( r \) and \( q_a \) at \( f_a = 1 \). Since \( S_a \) is a continuous (polynomial) function of \( f_a \), \( S_a \) is continuous in \( q_a \) and \( r \). Thus the phase transition is continuous.

2. Phase transition at \((0 < f_a^* < 1, f_b^* = 1)\)

Assuming the equilibrium point is stable we use the implicit function theorem on \( h(f_a, f_b) \), to show that the derivative of \( f_b \) with respect to \( q_a \) and \( r \) exists. By the implicit function theorem

\[
\frac{df_b}{dr} = -\frac{\partial h}{\partial r} \frac{\partial f_b}{\partial f_a} \frac{\partial f_a}{\partial r}
\]

\[
\frac{\partial h}{\partial f_b} = -rG_0^{a'} \frac{\partial f_a}{\partial f_b} (1 - G_1^b) + [1 - r(1 - G_0^a)]G_1^{b'}
\]

At \( f_b = f_b^* = 1 \),

\[
\frac{\partial f_a}{\partial f_b} = \frac{q_a(1 - G_0^a(f_a))G_0^{b'}(1) - r(1 - G_0^a(f_a))}{1 - G_1^b(f_b)(1 - q_a)}
\]

As the point is stable, \( G_0^a(f_a)(1 - q_a) < 1 \), and therefore \( \frac{\partial f_a}{\partial f_b} < \infty \). Hence at \( f_b = f_b^* = 1 \)

\[
\frac{\partial h}{\partial f_b} = [1 - r(1 - G_0^a(f_a))] \left( \frac{k_2^b > k_b} {k_b} \right) \neq 0
\]

This is because, \( 1 - r(1 - G_0^a) < \frac{k_2^b}{k_b} \).

Thus, \( f_b \) is differentiable and hence continuous with respect to \( q_a \) and \( r \) at \( f_b = 1 \). This implies that \( S_b \) is also continuous at the phase transition point with respect to \( q_a \) and \( r \).

IV. NUMERICAL RESULTS

Using numerical computations we find that the solution of system (1) may display periodic oscillations. This is illustrated in Fig. 4 for the case when both networks are Erdos-Renyi. However, these oscillations do not occur for all parameter values; in fact for low \( r \) and \( q_a \) the system is stable, as also seen in Fig. 2. Thus, the parameter space, \( r, q_a \in (0, 1] \), can be divided in four regions.

1. Network A destroyed, B percolates, no oscillations.
2. Network A percolates, B destroyed, no oscillations.
3. Both networks percolate, no oscillations.
4. System oscillates.
V. DISCUSSION

In this article we considered a system consisting of two networks exhibiting dependent and antagonistic interactions, i.e., a network whose nodes are antagonistic towards the nodes of the other network, while a portion of its links are dependent on the other network. Such a situation may arise when a bot-net launches DDoS attacks against SCADA switches which control the power stations of a power distribution network. Failure of power stations may cause the links connecting the bot-net to fail resulting in cessation of the attack.

Our analysis showed that unlike interdependent networks and interacting networks exhibiting antagonistic interactions, which can display first order phase transition, the phase transition observed in the system considered here is continuous. Also, unlike interacting antagonistic networks [14 52], we do not find the existence of bistability in the solutions. Interdependent networks are more fragile against random attack than isolated networks [29 36], while the system discussed here is more robust against random attacks in comparison with an isolated network. This shows that such systems behave very differently from interdependent and purely interacting antagonistic networks. Also, for Erdos-Renyi and scale free networks, we do not find a region in the parameter space where the antagonistic network collapses. In other words, the attacking network does not collapse even though its links are dependent on the victim network.

Numerical calculations revealed a region in the parameter space where the giant connected component of both the networks starts oscillating. Outside this region the giant connected component of both the networks is stable. Numerical results suggest that oscillations occur when the antagonism and the dependence is very high, i.e., a large proportion of nodes in the antagonistic network launch an attack on the victim network, while large proportion of links in the antagonistic network are dependent on the nodes of the victim network.

In this article we studied a system where bond percolation process occurs on one network while site percola-
tion happens on the other. A system where bond or site percolation happens on both networks with antagonistic and dependent interactions is likely to exhibit similar results. We believe that this article provides valuable insights on the percolation behavior in systems consisting of networks exhibiting antagonistic and dependent interactions. Such studies are key for developing an understanding of real world interconnected and interdependent systems.

APPENDIX

The complete collapse of network $A$ is possible if and only if $f_a = 1$ and either $f_b = 0$ or $0 < f_b < 1$. We first consider the second case: $0 < f_b < 1$. If $f_b \in (0, 1)$, $f_a$ must satisfy

$$f_a = 1 - T(f_a) + T(f_a)G_a^b(f_a)$$

where

$$T(f_a) = 1 - q_a \left( G_a^b(f_b) + r(1 - G_a^b(f_a))(1 - G_a^b(f_b)) \right)$$

The necessary and sufficient condition for the existence of a giant connected component is $f_a < 1$. Let, $u \in [0, 1]$, be given by

$$u = 1 + \frac{f_a - 1}{T(f_a)}$$

Hence, $f_a = 1 + (u - 1)T(f_a)$. Substituting this in the above equation of $f_a$ we obtain

$$u = G_a^b(1 + (u - 1)T(u))$$

$$S_a = 1 - G_a^b(1 + (u - 1)T(u))$$

This is equivalent to bond percolation with bond occupation probability $T(u)$. Proceeding in a manner detailed in Ref. [2], let $H^a_0(u; T(u))$ be the generating function for the size of a cluster in $A$ starting from a randomly chosen node, while $H^a_1(u; T(u))$ be the size of cluster reached by following a randomly chosen edge. For a large $N$, the clusters are tree like which allows one to write

$$H^a_0(u; T(u)) = uG^a_0(1 + (H^a_0(u; T(u)) - 1)T(u))$$

$$H^a_1(u; T(u)) = uG^a_1(1 + (H^a_0(u; T(u)) - 1)T(u))$$

The mean size of the cluster, $< s_a >$, is given by,

$$< s_a > = \frac{\partial}{\partial u} H^a_0(u; T(u)) \bigg|_{u=1}$$

$$= 1 + G^a_0(1)H^a_1(1; T(1))T'(1)$$

Now,

$$H^a_1(1; T(1))T'(1) = 1 + G^a_1(1)[T(1)H^a_1(1; T(1)) + T'(1)]$$

$$H^a_1(1; T(1))T'(1) = 1 + G^a_1(1)T'(1)$$

The above equation diverges when $T(1) = T_c = \frac{1}{G^a_1(1)}$, hence for $T(1) > T_c$ we have a giant connected component or $f_a < 1$. The condition for $f_a = 1$ is given by

$$1 - q_a G^b_0(f_b) < \frac{< k_a >}{< k_a^2 >}$$

$$< k_a > < k_a^2 > < k_a >$$

If $f_b = 0$, then the above condition becomes

$$< k_a > < k_a > < k_a >$$

This condition implies that a giant connected component does not exist in $A$ at the very beginning.

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