On “Observational constraints on Power - Law Cosmologies”

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Abstract

"Power Law Cosmologies" are defined by their growth of the cosmological scale factor as $t^\alpha$ regardless of the matter content or cosmological epoch. Constraints from the current age of the Universe and from the high redshift supernovae data require “large” $\alpha$ ($\approx 1$). We reinforce this by latest available observations. Such a large $\alpha$ is also consistent with the right amount of helium and the lowest observed metallicity in the universe for a model with the baryon entropy ratio $\approx 8.1 \times 10^{-9}$.
A power law growth, $a(t) = t^\alpha$, for the cosmological scale factor $a(t)$, is a generic feature of a class of models that attempt to dynamically solve the Cosmological constant problem [1]. Another example of a power law cosmology is the linear scaling produced by Allen [6] in a model determined by an $SU(2)$ cosmological instanton dominated universe. As pointed out by Kaplinghat et al [2] (hereinafter referred to as (I)), constraints on all such “power law cosmologies” from the present age of the universe and from the high redshift data are consistent with large $\alpha \approx 1$. However, (I) considered the primordial light element abundances from early universe nucleosynthesis and concluded that $\alpha$ is forced to lie in a very narrow range with an upper limit $\approx 0.55$. It was thus concluded in (I) that power-law cosmologies are not viable. In this letter, while we reinforce the constraints for a large $\alpha \approx 1$ from the more recent data for type Ia supernovae reported by the supernovae cosmology project [3], we demonstrate that the nucleosynthesis constraints on $\alpha$ arrived at in (I) are seriously in error. A large value $\alpha \approx 1$ is consistent with the right amount of helium observed in the universe in a model with the baryon to entropy ratio $\approx 8.1 \times 10^{-9}$.

In general for a power law cosmology, the present hubble parameter $H_o$ is related to the present epoch $t_o$ by $H_o t_o = \alpha$. In what follows we shall restrict ourselves to the case where the scale factor evolves linearly with time, i.e. $\alpha = 1$. This would include a Milne cosmology for which $a(t) = t$, as well as a general coasting cosmology for which $a(t) = kt$ [5]. The Hubble parameter is precisely the inverse of the age $t_o = (H_o)^{-1}$. In the standard big bang model [SBB], $t_o \approx 2/3H_o$. Thus the age of the universe inferred from a measurement of the Hubble parameter is 1.5 times the age inferred by the same measurement in standard matter dominated model. With the best reported value for $H_o$ standing at $(H_o) = 100h \ km(sec)^{-1}(Mpc)^{-1}$, with $h = 0.65$ [7,8], the age of the universe turns out to be $\approx 15Gyr$. Such an age is comfortably consistent with age estimates for old clusters. (I) has put constraints on the value of $\alpha$ using Perlmutter et al [4] data on SNIa at $z=0.83$. The quoted value of the figure of merit for SNIa favours a large $\alpha \geq 1$.

For $\alpha = 1$ the apparent magnitude $m(z)$, the absolute magnitude $M$ and the redshift $z$ of an object are related by the exact expression:

$$m(z) = 5 \ast log(aT^2) + z + M - 5 \ast logH_o + 25$$

Figure ‘A’ sums up the Supernova Cosmology project data for supernovae with redshifts between 0.18 and 0.83 together with the set from the Calan / Tololo [9] supernovae, at redshifts below 0.1. Figure ‘B’ projects the data points with the above $m(z)$ curve. As noted by [3] the curve for $(\Omega_A = \Omega_M = 0$ is “practically identical to bestfit plot for an unconstrained cosmology”. This reinforces (I) as far as the concordance of an $\alpha = 1$ power law cosmology with age and the $m-z$ relations are concerned.

As regards nucleosynthesis, with the expansion scale factor evolving linearly with time, the temperature scales as $aT = tT = \text{constant}$ as long as we are in an era where the photon entropy is not changing much. [The small entropy change at the time of $e^+$, $e^-$ annihilation does not alter the following argument as well as the results substantially]. The hubble expansion rate at a given temperature is much smaller than its corresponding value at the same temperature in standard cosmology. Taking the present age as the inverse of the hubble parameter and the present effective cosmic microwave background
\( (\text{CMB}) \) \text{“temperature” as} 2.7 \, \text{K}, \text{it is easily seen that the universe would be some 50 years old at temperatures} \approx 10^9 \, \text{K}. \text{Such a universe would take some 5000 years to cool to} 10^7 \, \text{K}!! \text{With the neutron decay rate around 888 seconds at low enough temperatures, it would seem that all neutrons would have decayed by the time nucleosynthesis may be expected to commence at around} 10^9 \, \text{K}. \text{This is precisely the argument (I) have used to label nucleosynthesis as spelling “disaster” for such cosmologies - and thus ruling them out. However, if we consider weak interaction rates of neutrons and protons, it is easily seen that the inverse (proton’s) beta decay remains effective and is not frozen until temperatures even slightly less than} 10^9 \, \text{K}. \text{The weak interactions of the leptons too remain in equilibrium until temperatures even lower:} 10^8 \, \text{K} \text{[10]. This has interesting consequences. Firstly, the equality of photon and neutrino temperature} (T_\nu = T) \text{is ensured even after the electron-positron annihilation. With temperature measured in units} T_0 = 10^9 \, \text{K}, \text{this leads to an exact expression for the p going to n rate as} \approx \exp \left[ -15/T_0 \right] \text{times the n going to p rate. Figure ‘C’ exhibits the p} \rightarrow \text{n rate in comparison to the hubble parameter near} T_0 \approx 1. \text{It is clear that by inverse beta decay a proton’s conversion into a neutron is not decoupled at temperatures as low as} 10^9 \, \text{K}. \text{The n/p ratio is expected to follow its equilibrium value irrespective of the neutron decay rate as long as both n going to p, and p going to n rates are large in comparison to the expansion rate of the universe and the rate of nucleon leak into the nucleosynthesis channel. Although the n/p ratio is small at temperatures} T_0 \approx 1, \text{every time any neutron branches off into the nucleosynthesis channel, the n/p ratio will be replenished by the inverse beta decay of the proton. Simple chemical kinetics shows that if we remove one of the reactants or the products of a reaction in equilibrium at a rate slower than the relaxation period of the equilibrium buffer, reactions proceed in an equilibrium restoring direction. As long as we keep precipitating a product at a small enough rate, reversible reactions that maintain a solution in equilibrium would restore the buffer to an equilibrium configuration. This is just what is referred to as “the law of mass action” in chemistry.}

\text{What actually happens is that, depending on the baryon entropy ratio, helium starts precipitating out at temperatures around} 7 \times 10^9 \, \text{K}. \text{The rate of precipitation of helium is exhibited in Figure ‘D’ whence it is clear that the amount of nucleon precipitation into helium synthesis channel is negligible in comparison to the neutron formation and destruction due to inverse and forward beta decay respectively. This is sufficient to maintain n/p to its equilibrium value. Even in (SBB), at such temperatures much higher than the so called deuterium “bottleneck” temperature, there is a tiny amount of helium always forming. However, the universe keeps to such temperatures in SBB for less than 100’s of seconds only and so the amount formed before the “bottleneck” temperature is negligible. In the case at hand, the universe is at such temperatures for some 100 years and the tiny amounts of helium steadily builds up. This is conclusively demonstrated by resorting to a numerical integration of Boltzmann equations incorporating the entire network of reactions. We have done the required modifications in the standard nucleosynthesis code outlined by Kawano, to suit the linear expansion of the scale factor. Our code integrates 227 reactions between 64 nuclei and takes care of the slow change in baryon entropy ratio during \( e^+, e^- \) annihilation. Runs for different values of baryon to entropy ratio (\( \eta \)), and with the currently favoured value of 65 km/\text{sec}/\text{Mpc} for the hubble parameter, yield the}
result that an $\eta \approx 8.1 \times 10^{-9}$ gives just the right amount of Helium [23.8\%] as observed in the Universe [10].

As shown in [10], nucleosynthesis in a power law cosmology yields a metallicity quite close to the lowest observed metallicities. The only problem that one has to contend with is the significantly low yields of deuterium in such a cosmology. However, as pointed out in [10], the amount of Helium produced is quite sensitive to $\eta$ in such models. In an inhomogeneous universe, therefore, one can have the helium to hydrogen ratio to have a large variation. Deuterium can be produced by a spallation process much later in the history of the universe [11]. If one considers spallation of a helium deficient cloud onto a helium rich cloud, it is easy to produce deuterium as demonstrated by Epstein - but without overproduction of Lithium.

We conclude that nucleosynthesis does not rule out a power law cosmology as claimed in (I). As a matter of fact it may well turn out to do better - at least as far as metallicity is concerned.

Copies of our numerical code can be downloaded by anonymous ftp into iucaa.ernet.in. The executable file /in.coming/dlohiya/a.out must be run on an architecture supporting quadruple precession.

**Figure captions**

Fig. A: Hubble diagram, the magnitude residual and the uncertainty - normalized residual plots taken from the supernova cosmology project. “The curve for $(\Omega_\Lambda,\Omega_M) = (0,0)$ is practically identical to the best fit unconstrained cosmology”[3].

Fig. B: The Hubble diagram with the data points (taken from [3]) for linear coasting cosmology.

Fig. C: The inverse beta decay rate $p \rightarrow n$ and hubble expansion rate as a function of temperature in units of $10^9$ K. Inverse beta decay decouples only at $T_9 \approx 1.08K$.

Fig. D: Comparison of Helium precipitation and neutron production rates as a function of temperature. Helium production rate, which is identically equal to the nucleon precipitation rate out of the $n$ - $p$ equilibrium buffer at these temperatures is some 1000 times smaller than $n \leftrightarrow p$ conversion rates by beta decay.
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Calan/Tololo
(Hamuy et al,
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Supernova
Cosmology
Project

Fig. A.—
Freeze out of Inverse beta decay for $R(t)=kt$ coasting cosmology

Hubble expansion rate

$p \rightarrow n$ rate
Freeze out of Inverse beta decay for $R(t)=kt$ coasting cosmology

Rates $\rightarrow$

Temperature in units of $10^9$ K ($t9$) $\rightarrow$

- $p \rightarrow n$ rate
- Hubble expansion rate
Rates

Temperature in units of $10^9$ K

$p \rightarrow n$ rate

$<----1000 \times dHe/dt$

Hubble expansion rate
Rates --------> 

Temperature in units of $10^9$ K -------->

Hubble expansion rate

$\text{p} \rightarrow \text{n rate}$

$\langle \cdot \cdot \cdot \cdot \cdot 1000 \times \text{dHe/dt} \rangle$