Simulation of the filtration mechanism of hyaluronic acid in total knee prosthesis

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Abstract. Polyethylene (UHMWPE) wear in current knee prosthesis causes prosthesis loosening after no more than 15 years. In this work, a steady state one-dimensional lubrication model with non-Newtonian fluid, porous elastic layer on tibial component, ultra-filtration mechanism of fluid and some features of the surface roughness is studied through a numerical technique based on the Finite Element Method. The results show that the UHMWPE stiffness makes difficult the lubrication mechanism of the artificial joint and promotes abrasive and fatigue wear. Nevertheless, the use of compliant porous materials on the tibial component could reduce friction and wear. Moreover, the ultra-filtration mechanism promotes efficiency on the joint.

1. Introduction

Polyethylene (UHMWPE) wear in the tibial component is considered the main causative factor in the aseptic loosening of knee prostheses.

Some previous works \cite{1} showed that low elasticity and porosity of the articular cartilage favour an efficient lubrication in the joint contact area, preventing cartilage from wearing. At the same time, Di Paolo et al \cite{2} through a model of knee prosthesis with smooth tibial component observed that the pseudoplastic characteristic of the synovial fluid diminishes shear stress on the material even in narrower lubrication channels. The same work \cite{2} showed that a material with the stiffness of the articular cartilage would allow a wider load distribution and major gaps between tibial and femoral components, making abrasion and material fatigue lighter. Due to its high stiffness currently used UHMWPE lacks these advantages. Some hydrogels \cite{3} have been recently synthesized, showing great mechanic strength and a stiffness similar to that of the articular cartilage. Such hydrogels could be more efficient than the current polymers.

One restriction of the cited work \cite{2} is that the model does not take into account the surface roughness which noticeably influences on the variables affecting of the polyethylene wear \cite{4}. Furthermore, their work does not include the effects of hyaluronic acid (HA) in the joint lubrication. In this regard, Kawano et al \cite{5} gave intra-articular injections of HA to rabbits suffering from osteoarthritis. Their results showed that doses of HA reduce the friction coefficient on pathological joints to normal levels observed in a healthy joint.

In this work a model of lubrication for knee prosthesis with a compliant tibial component, which is capable of exuding and absorbing fluid, and with a harmonic rough surface is numerically solved. The
non-Newtonian synovial fluid is modeled by a power law and considering a viscosity dependant on the HA concentration.

2. Model
The equivalent model geometry is represented by a rigid cylinder in longitudinal contact with a rough flat surface (see figure 1). The governing equations of the problem arise from the principles of mass conservation and momentum balance under the following simplifying hypotheses:

1. Lubrication approximation.
2. Non-Newtonian fluid (pseudo-plastic) modeled according to a power law [2].
3. Viscosity varying with concentration of HA as a consequence of the filtration process of the porous material.
4. Laminar, unidirectional and incompressible flow. Steady state.
5. Molecules of hyaluronic acid bigger than the material pores.
6. Non-deformable metallic femoral element.
7. Tibial element covered with a thin layer of low stiffness material with a harmonically rough surface, with the capacity to exude and absorb fluid due to a plane deformation state.
8. Constant elastic material parameters.
9. Load area wider than the thickness of the deformable porous substrate.
10. Negligible internal flows.
11. Harmonically surface roughness.

Hypotheses from 1 to 4 applied to conservation laws for the fluid lead to a modified Reynolds’ lubrication equation [2]. The integral form of this dimensionless equation is:

\[ p(x) = 12 \pi \eta_0 \int \mu \left( \frac{h(x)}{h_0} \right)^n \frac{h(x)}{h_0} - 2 \frac{d_0}{h_0} \, dx \]  

(1)

where \( p(x) \) is the pressure, \( h(x) \) is the separation between the articular surfaces that performs the lubrication channel, \( q(x) \) is the flow rate in the lubrication channel, and \( n \) is the dimensionless power law exponent for synovial fluid.

Finally, \( \mu(x) \) is the synovial fluid viscosity, which depends on the concentration of HA. Negami [6] showed that there is a linear relationship between the synovial fluid viscosity and the concentration of HA. The application of a mass balance between the inlet section and a section located in an x

![Figure 1. Equivalent model for a knee prosthesis with porous and rough material on tibial component.](image)
position allows us to describe the variations of the viscosity due to filtration as a function of the flow rates between both sections:

\[
\mu(x) = 1 + NVIS \left( \frac{q(0)}{q(x)} - 1 \right)
\]  

(2)

where \( NVIS = \frac{K C_0}{\mu_0} \), being \( K \) a constant taken from the bibliography [6], \( C_0 \) is the concentration of HA and \( \mu_0 \) the synovial fluid viscosity in the inlet section of the lubrication channel.

Boundary conditions to the equation (1) are the follow:

\[
x = b, \quad p = 0
\]  

(3)

\[
x = \bar{x}, \quad p = \frac{dp}{dx} = 0
\]  

(4)

where \( \bar{x} \) is a position in the channel outlet whose localization is unknown and becomes the free boundary of the problem.

The assumed the lubricant film thickness in this work is a modified version of the previous ones [1, 2] which takes into account the surface roughness on the tibial component. Its dimensionless expression is:

\[
h(x) = 2h_0 + x^2 + \alpha sen(ax) + d(x)
\]  

(5)

where \( 2h_0 \) is the intercrossing between the elements in an undeformed state, \( d(x) \) is the deformation of the material at each point in the channel. The values of sine wave amplitude are related to the experimental measurements of roughness according to:

\[
A = \sqrt{2Ra}
\]  

(6)

where \( A \) is the dimensional wave amplitude (\( A = aL \)) and \( Ra \) the experimental value.

The material deformation due to hypotheses 6, 7 and 8 is proportional to load at each point (local pressure) according to the following [1, 2]:

\[
d(x) = N_e p(x)
\]  

(7)

\[
N_e = \frac{\mu E}{L E} \left( \frac{R}{L} \right)^{1/2} 10^5
\]  

(8)

\[
E^* = \frac{(1-v)}{(1-2v)(1+v)}
\]  

(9)

The material capacity to exude and absorb fluid is expressed by a flow rate equation, which in a dimensionless form, results:

\[
q_e(x) = q_e(-b) + \theta d(x)
\]  

(10)

where \( \theta \) is the so-called exudation factor and measures the exuded-absorbed fluid volume per unit of deformed material volume.

Normal load, the tangential force (friction) and the friction coefficient are obtained through the following expressions, respectively [1, 2]:

\[
w = \int_{-b}^{\bar{x}} p(x) \ dx
\]  

(11)
\[ f_{0,b} = \int_{-b}^{b} \tau_{0,b}(x) \, dx \]  \hspace{1cm} (12)

\[ \phi_{0,b} = \frac{f_{0,b}}{w} \]  \hspace{1cm} (13)

3. Method of solution

The pressure-deformation relationship and the presence of the free boundary becomes the model non-linear. Therefore, a robust computational method is needed to perform a numerical solution. This method is based on:

- Discretization of the equations by the Galerkin Finite Element Method.
- Simultaneous solution of the equations using the Newton’s method, involving the determination of the free boundary at each iteration.
- Parametric continuation process with step size control.

The use of the free boundary and the nodal discretization is a one-dimensional adaptation of the spines method, commonly used in flow problems with free surfaces. The computational code was implemented in FORTRAN.

The exudation factor and NVIS parameter of the HA concentration were used as parameters which can be modified in the process of parametric continuation.

Given that \( C_0 \) and \( K \) are supposed to be constant, the variations in the NVIS parameter only show changes in the \( C_0 \) concentration of HA. Finally, the frequency as well as the sine wave amplitude were considered as fixed parameters and their values were based on previous works [2].

Moreover, the load applied to the implant is three times the body weight of a person of 75 kg of mass.

| Denomination                        | Symbol | Magnitude          |
|-------------------------------------|--------|--------------------|
| Equivalent cylinder radius [m]      | R      | 0.70               |
| Tangential velocity [m/s]           | V      | 1.91 \times 10^{-2}|
| Viscosity [Pa s]                    | \( \mu(-b) = \mu_0 \) | 1.00               |
| Elastic Modulus [MPa]               | E      | 20 \times 500      |
| Power law exponent [3]              | N      | 0.60               |
| Poisson’s coefficient               | \( \nu \) | 0.40               |
| Porous layer thickness [m]          | L      | 1.00 \times 10^{-3}|
| Applied load per unit length [N/m]  | W      | 7.36 \times 10^{4} |
| Sine wave amplitude [\mu m]         | A      | 1.3                |

4. Results

4.1. Effects of the elastic modulus

Figure 2 shows the film thickness for two different elastic modulus 20 and 500 MPa. In both cases, the same surface roughness, which is characteristic of the polyethylene [2], as well as for some hydrogels [3], has been considered.

Note that the roughness amplitude (A=1.3 \( \mu m \)) of the non-deformed material is greater than the film thickness (see figure 2). However, the fluid pressure deforms the surface and significantly reduces the amplitude roughness. Such situation assures a full fluid film between components.

Nevertheless, a low stiffness material allows lubrication channels 80% higher and thus, avoids the direct contact between the materials.
In figure 3, the fluid pressure developed for the conditions given in figure 2 are shown. It can be seen that the pressure due to the use of the rigid material, exceeds 60 % the admissible stress suggested by UHMWPE manufacturers (10 MPa) [2]. The situation is worsened by the presence of rough surfaces, since maximum pressure reaches 18 MPa, a value 80 % higher than the admissible stress allowed as shown in figure 3.

Low stiffness materials such as the PAMPS–PDMAAm DN gel [3], allow a greatest deformation. This fact is an advantage since the pressure distributes in a surface three times wider than current UHMWPE does (see figures 2 and 3). Since the elastic limit of the PAMPS–PDMAAm DN gel is similar to the current UHMWPE [2, 3], the results shown in figure 3 indicate that the former would support tensions 40% lower than its admissible limit.

4.2. Effects of material exudation
Figure 4 shows the variation of the friction coefficient versus the exudation factor for materials with high and low stiffness. For both of them, the friction coefficient decreases with the increase of the exudation factor $\theta$. 

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**Figure 2.** Lubrication film thickness for rough and smooth materials of different stiffness.

**Figure 3.** Fluid pressure in the contact zone for channels depicted in figure 2.
This can be understood by analyzing figure 5. When surfaces are more separated, the velocity gradients are reduced and consequently the shear stress, directly related to the friction coefficient. Moreover, channel increase prevents the solid-to-solid contact and therefore abrasion.

4.3 Effects of hyaluronic acid concentration
The increase in the flow rate exuded by the porous material reduces the concentration of HA. Therefore, it leads to a decrease in the viscosity. In this way, as shown in Figure 6, the friction coefficient decreases because of a reduction of the shear stress over the deformable material.

An adequate concentration of HA can improve the mechanical response of a hydrogel. Furthermore, a polyethylene with exudation capacity would have beneficial mechanical effects due to HA filtration, similar to that of the natural cartilage.
5. Conclusions
The results of this work show that a low stiffness material with capacity to exude fluid has greater mechanical advantages than the current UHMWPE due to the following reasons:

- A greater deformation allows the articular surfaces to be more separated by a full fluid film. Therefore, a lubrication mechanism is guaranteed even with surface roughness, preserving the material from abrasion.

- Under the same load, the maximum pressure values remain below the admissible stress limit proposed by manufacturers. For current UHMWPE this limit is exceeded in 80 %, even more taking into account the surface roughness.

- The exudation and absorption capacity from a porous material would allow the friction coefficient to be reduced.

- The exudation produces changes in the concentration of HA in the fluid along the contact area. This reduces the lubricant viscosity and therefore the friction coefficient.

The results presented in this work could guide the experimentation with new materials taking into account the proposed mechanical characteristics. Some hydrogels with properties similar to that of the articular cartilage are the most promising, since certain exudation capacity could reduce their current wear problems.

6. References

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