Low-lying Λ Baryons from the Lattice

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In a lattice QCD calculation with two light dynamical Chirally Improved (CI) quarks we determine ground state and some excited state masses in all four Λ baryon channels $1/2^+$ and $3/2^+$. We perform an infinite volume extrapolation and confirm the widely discussed Λ(1405). We also analyze the amount of octet-singlet mixing, which is helpful in comparing states with the quark model.

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One of the central aims of hadron spectroscopy is to understand the spin-flavor-parity structure of the excitation spectra for different quantum numbers. It seems in particular that the nucleon and Λ spectra show significant differences which, if properly understood, should illuminate the influence of quark mass and flavor on hadron structure. The lowest Λ($\frac{1}{2}^-$) mass lies below the Roper-like Λ(1600, $\frac{3}{2}^-$) and the negative parity nucleon $N^*(1535)$; unlike the nucleon sector it does have standard level ordering lying between the positive parity ground state and the first positive parity excitation. A baryons can be flavor singlets or octets or, due to the difference in light and strange quark masses, mixtures of both. Various continuum model studies discuss that mixing. This is the first lattice QCD analysis of the Λ baryons for dynamical quarks, that includes all four, namely the $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$-channels. We obtain ground states compatible with experiment in three of those. We also study the infinite volume limit and give for the first time lattice results on singlet-octet composition for all four sectors, obtaining the mass of the Λ(1405) and confirming its flavor singlet nature.

Lattice studies in the quenched case generally had problems to find the low lying Λ(1405). Even a study with two dynamical light quarks $\underline{1}$ $\underline{2}$ found too large mass values. Only recent results $\underline{3}$ obtained with PACS-CS (2 + 1)-flavor dynamical quarks configurations $\underline{4}$ show a level ordering which is compatible with experiment.

We study the baryons for a set of seven ensembles with two dynamical Chiral Improved (CI) quarks $\underline{5}$ $\underline{6}$. The CI fermion action consists of several hundred terms and obeys the Ginsparg-Wilson relation in a truncated approximation. The pion mass ranges from 255 to 596 MeV, the lattice spacing lies between 0.1324 and 0.1398 fm. The bulk of our results were obtained for lattices of size $16^3 \times 32$. For two ensembles with light pion masses (255 and 330 MeV) we also used $24 \times 48$ lattices. Thus $m_x L$ which controls the finite size effects is 4.08 and 5.61 in these two situations. Further details on the action and our simulation, as well as results for mesons, can be found in $\underline{7}$ $\underline{8}$. The strange quark is introduced as a valence quark and its mass fixed by the Ω-mass.

The Dirac and flavor structure of the interpolating fields used in our study is motivated by the quark model $\underline{9}$ $\underline{10}$, see also $\underline{11}$. Within the relativistic quark model there have been many determinations of the hadron spectrum, based on confining potentials and different assumptions on the hyperfine interaction (see, e.g., $\underline{12}$ $\underline{13}$). The singlet, octet and decuplet attribution $\underline{11}$ of the states has been evaluated based on such model calculations, e.g., in $\underline{15}$ (see also the summary in $\underline{16}$).

For the Λ baryons we use sets of up to 24 interpolating fields in each quantum channel. The singlet and octet combinations of Table $\underline{1}$ are combined with three possible choices of Dirac matrices $(\Gamma_1, \Gamma_2) = (1, C\gamma_5), (\gamma_5, C)$ and $(1, C\gamma_5\gamma_8)$ (denoted by $\chi_1, \chi_2$ and $\chi_3$ for short) for $J = \frac{1}{2}$ and eight combinations of Gaussian smeared quarks $\underline{17}$ $\underline{18}$ with two smearing widths ($n, w$). The operator numbering is given in Table $\underline{11}$. All interpolators are projected to definite parity and all Rarita-Schwinger fields (spin $\frac{3}{2}$) interpolators in Table $\underline{11}$ are projected to definite spin $\frac{1}{2}$ using the continuum formulation of the Rarita-Schwinger projector $\underline{19}$. $C$ denotes the charge conjugation matrix, $\gamma_5$ and $\gamma_8$ the time and the spatial direction Dirac matrices.

For point like quark fields, Fierz identities reduce the actual number of independent operators (see, e.g., $\underline{20}$). In particular, there are no non-vanishing point-like interpolators for $\Delta(\frac{5}{2})$ and singlet $\Lambda(\frac{1}{2})$. We use different quark smearing widths such that the Fierz identities do not apply. Hence $\chi_1, \chi_2$ and $\chi_3$ are all independent, for $J = \frac{3}{2}$ all interpolators are non-vanishing.

From the cross-correlation matrix $C_{ik}(t) = \langle O_i(t) O_k(0) \rangle$ we obtain the energy levels with help of the variational method $\underline{21}$ $\underline{22}$. One solves the generalized eigenvalue problem $C(t) \tilde{u}_n(t) = \lambda_n(t) C(t_0) \tilde{u}_n(t)$ in order to approximately recover the energy eigenstates $|n\rangle$. The eigenvalues $\lambda_n(t) \sim \exp(-E_n t)$ allow us to get the energy values and the eigenvectors serve as fingerprints of the states, indicating their content in...
terms of the lattice interpolators. The quality of the results depends on the statistics and the provided set of lattice operators. The dependence on \( t_0 \) is used to study the systematic error; in the final analysis we use \( t_0 = 1 \) (with the origin at 0). The statistical error is determined with single-elimination jack-knife. For the fits we use single exponential behavior but check the stability with double exponential fits; we take the correlation matrix for the correlated fits from the complete sample [8].

For the extrapolation to the physical pion mass we fit to the leading order chiral behavior, which is linear in \( m_\pi^2 \). Two ensembles (at pion masses 255 MeV and 588 MeV) suffer from a slight mistuning of the strange quark mass, which are therefore discarded in the extrapolation to the physical pion mass, whenever the effects are found significant. This is the case for the lowest energy levels in each channel (three lowest ones in \( \frac{1}{2} \)). The quoted systematic errors for these levels include the corresponding deviation and the dependence of the energy levels on the choice of interpolators and fit ranges for the eigenvalues.

In the present study we are restricted to 3-quark operators for the baryon. Note that ideally one should take into account also meson-baryon interpolators (see, e.g., the discussion in [22]). This leads to many more contributing graphs and necessitates also the inclusion of backtracking quark loops. The related computational and algorithmic effort prevented such lattice calculations so far, although such studies are in progress [24]. Due to sea quarks, in principle, 3-quark operators have overlap with meson-baryon states as well. The corresponding coupling was however found to be weak in actual simulations [22]. We will argue below that in particular in the s-wave channels we find hints of such coupling even for our interpolators.

\[ J^P = \frac{1}{2}^+, \] Finite volume: In Fig. we show results for the four lowest eigenstates for interpolator set (1,2,11,20,25,26,33,34,43). After extrapolation to the physical point our lowest energy level agrees well with the experimental \( \Lambda(1116) \). The systematic error estimated from different combinations of interpolators and fit ranges is indicated in the summary Fig. [8]. Analyzing the eigenvectors, we find that the ground state is dominated by octet interpolators of Dirac structure \( \chi_1 \) and \( \chi_3 \) in agreement with a relativistic quark model calculation [13]. Our first excitation is dominated by singlet interpolators (first Dirac structure) matching the \( \Lambda(1810) \) (singlet in the quark model). The Roper-like \( \Lambda(1600) \) (octet in the quark model) seems to be missing. This resembles the situation in the \( N(\frac{1}{2}^+) \) channel [7]. The second and third excitations are again dominated by octet interpolators.

**Infinite volume extrapolation:** We performed a volume analysis for several sets of interpolators and various fit ranges. The results in finite volume and the infinite volume extrapolations for the ground state for specific interpolators are shown in Fig. [8] The extrapolation follows the method of [24]. A stable choice is the set of interpolators \( \Lambda \) and \( t_{\text{min}} = 5a \). The corresponding systematic error estimated from different interpolators and fit ranges is indicated in the summary Fig. [6] Our final result is 1126(17)(11) MeV (statistical and systematic error), which agrees nicely with the experimental \( \Lambda(1116) \) mass.

\[ J^P = \frac{1}{2}^+, \] Finite volume: We use different sets of interpolators and fit ranges. We stress that our basis is large compared to that of other studies with three types of Dirac structures for both singlet and octet interpolators. We can extract the lowest four energy levels, shown in Fig. [8] using interpolators (2,3,10,18,26,27,34,42). We find that the ground state energy level agrees well with the experimental \( \Lambda(1405) \). The dependence of the levels on the tuning of the strange quark mass appears to be sizeable, albeit it an accident of our simulation. This
is one reason of the large systematic error depicted in Fig. 6.

The chosen set of interpolators is particularly suitable for an analysis of the content of the states. The spatial support of the quark fields is equivalent in all interpolators and hence does not require additional renormalization when comparing the contribution of different interpolators to the eigenstate. In addition, we use several interpolators for each given combination of flavor and Dirac structures, which allows us to identify a possibly higher number of states with similar structure. Within the basis used, the ground state is dominated by singlet interpolators of all three Dirac structures. There is, however, a considerable mixing with octet interpolators (second Dirac structure) of 15-20% in ensemble A66, i.e., at \( m_\pi \approx 255 \text{ MeV} \) (see Fig. 1). This mixing is expected to increase towards the physical point, which may complicate the functional dependence of the energy levels on the pion mass. The first and second excitation are both dominated by octet interpolators, by the second and first Dirac structure, respectively.

The first and second excited energy level are both a bit low but compatible with the experimental \( \Lambda(1670) \) and \( \Lambda(1800) \). In general in the \( J^P = \frac{1}{2}^+ \) channel one may expect coupling to \( \pi \Sigma \) and \( \overline{K}N \) in s-wave. In \[27, 28\] the expected energy levels in finite volumes are discussed based on a continuum hadron exchange model. There (with physical hadron masses), the low-lying scattering state levels in the \( \frac{1}{2}^- \) channel are well separated for \( m_\pi L \lesssim 3 \). For the pion masses of our study, the non-interacting meson-baryon thresholds lie close but (except for one point) above the lowest energy level observed. E.g., for the lowest pion mass, the values are \( m_\Sigma + m_\pi \approx 1.52 \text{ GeV}, m_N + m_\pi \approx 1.62 \text{ GeV} \), above the lowest level. This resembles the situation in the \( N(\frac{1}{2}^-) \) channel. Earlier work argued that the coupling of single baryons to baryon-meson channels may be too weak to lead to observable effects \[22\]. However, in our case, in s-wave scattering, we cannot exclude that one or even two of the observed three lowest energy levels correspond to scattering states. Note that the measured ground state energy level is always (except for one point) below s-wave threshold, thus supporting the \( \Lambda(1405) \) identification.

It has been conjectured from Chiral Unitary Theory that the lowest state may have a double-pole \[23, 29\] and a identification strategy for lattice simulations is suggested in \[30\]. This would require asymmetric boxes or moving frames.

**Infinite volume extrapolation:** We study the volume dependence of the three lowest states for different sets of interpolators and various fit ranges. We emphasize that the stability of the signals of the excitations is comparable to the ones of the ground state. The volume dependence of all three low states appears fairly flat in our simulations. In a few cases showing even negative finite volume corrections. These features are compatible with significant contributions of an attractive s-wave scattering state. For interpolators \(\{2,3,10,18,26,27,34,42\}\) and \( t_{\text{min}} = 5a \), after infinite volume extrapolation, we show the result of the final extrapolation of the ground state energy level to the physical pion mass in Fig. 6. The final result for the ground state agrees very well with the experimental \( \Lambda(1405) \). Both the first and the second excitation appear to be a bit low but are compatible with the experimental \( \Lambda(1670) \) and \( \Lambda(1800) \) (see Figs. 5 and 6) and might also possibly be s-wave \( \pi \Sigma \) and \( \overline{K}N \) signals.
FIG. 3. Energy levels for $\Lambda$ spin $\frac{1}{2}^-$ in a finite box of linear size $L \approx 2.2$ fm, for legend see caption of Fig. 1. Fits are omitted for clarity.

FIG. 4. Eigenvectors for $\Lambda$ spin $\frac{1}{2}^-$ ground state and first excitation for ensemble A66 ($m_\pi \approx 255$ MeV). The ground state is dominated by singlet interpolators of all three Dirac structures. The first (and also the second excitation, not shown) is dominated by octet interpolators. Note that a considerable mixing of singlet and octet is observed (15-20% for the ground state).

$J^P = \frac{3}{2}^+$, Finite volume: In spin $\frac{3}{2}$ channels, for symmetric quark fields, singlet interpolators vanish exactly due to Fierz identities. We use different quark smearing widths and thus circumvent the Fierz identities constructing singlet interpolators nevertheless. We derive results for the lowest three energy levels of the variational analysis of interpolators (2,9,10,16). Only the ground state can be clearly identified and its extrapolation agrees with the experimental $\Lambda(1890)$ mass, but with large uncertainty.

$J^P = \frac{5}{2}^-$, Finite volume: We choose interpolators (2,7,9,10,15) and find a clear gap between ground state and excitations. The extrapolation of the ground state energy level lies clearly above the experimental ground state $\Lambda(1520)$ and is compatible with the $\Lambda(1690)$. The excitations extrapolate close to the $\Lambda(2325)$.

From the eigenvectors we find that the two lowest states are dominated by octet, the second excitation by singlet interpolators. The quark model shows for $\Lambda(1520)$ singlet dominance (like for its companion $\Lambda(1405)$). We conclude that we might miss a signal for the ground state altogether, or, alternatively, there is a strong deviation from the leading chiral behavior towards the physical point. The latter case is intriguing as it might be related to strong coupling to an $s$-wave $\pi\Sigma(1385)$ state, which is discussed, e.g., in [31, 32].

Infinite volume extrapolation: The volume dependence turns out to be fairly flat, in a few cases even compatible with negative finite volume corrections. The final result in the infinite volume limit at the physical point again misses the experimental $\Lambda(1520)$ and agrees with the $\Lambda(1690)$ mass.

Summary: We present a comprehensive study of spin $\frac{1}{2}$ and $\frac{3}{2}$ $\Lambda$ baryon ground states and excitations, utilizing a large basis of interpolators in the variational analysis including differently smeared quark sources (which allows to sidestep the Fierz identities), three Dirac structures, and singlet and octet forms. Fig. 6 shows the results for
ground states and excitations for lattices of linear size $L \approx 2.2$ fm (lhs) and low lying levels after infinite volume extrapolation (rhs). The horizontal lines or boxes represent experimentally known states [16], where the box size indicates the statistical uncertainty of our results is indicated by error bars. The observation of $\Lambda(1405)$ with the employed basis suggests that this state has a non-negligible singlet 3-quark content. The Roper-like (octet) state $\Lambda(1600)$, on the other hand, may not couple to our 3-quark interpolators.

We analyze the volume dependence and find that only the ground state of spin $\frac{1}{2}^+$ shows a clear exponential dependence as expected for bound states. For all other discussed states, the volume dependence is either fairly flat or obscured by the statistical error. For the $\frac{1}{2}^-$, $\frac{1}{2}^+$, and $\frac{3}{2}^+$ channels the remaining small deviations at the physical point can be easily caused by the continuum limit and/or dynamical strange quarks. The discrepancy for the $\frac{1}{2}^+$ ground state might point to some effect which is so far not properly accounted for.

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FIG. 6. Energy levels extrapolated to the physical pion mass in infinite volume $L \approx 2.2$ fm (lhs) and low lying levels after infinite volume extrapolation (rhs). The horizontal lines or boxes represent experimentally known states [16], where the box size indicates the statistical uncertainty of our results is indicated by error bars. The observation of $\Lambda(1405)$ with the employed basis suggests that this state has a non-negligible singlet 3-quark content. The Roper-like (octet) state $\Lambda(1600)$, on the other hand, may not couple to our 3-quark interpolators.

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