Research on Frequency Domain Filtering Method Based on Frequency Response Zero Control Technology

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Abstract; Airspace adaptive beamforming has been widely used in radar anti-jamming. In this paper, by applying the appropriate model, the spatial adaptive beamforming method is applied to the time domain filtering to form an adaptive filter. For the filter, only the single frequency can form a null trap. This paper proposes a frequency response zero control technology. The broadband signal is filtered and achieved good results.

1. Introduction
Adaptive signal processing technology [1] is an important branch of signal processing. Adaptive beamforming has been widely used in spatial filtering, such as suppressing sidelobe interference, suppressing mainlobe interference, etc., in mainlobe anti-dry resistance. In order to solve the problems of increased sidelobe level and beam deformation, SJYu et al. proposed the blocking matrix method (BMP) and the feature projection preprocessing (EMP) algorithm [2-5]. In the case that the noise and interference cannot be completely filtered out in the spatial filtering, the space-time joint two-dimensional processing method is proposed [6-7], and the time delay tap is added after the radar antenna to form a time domain filtering processing structure, which is anti-noise. And the anti-interference has achieved good results.

In order to apply the many theories in spatial filtering in time domain filtering, this paper adds a certain number of time taps after the receiving antenna elements, each time constitutes a finite-length shock response filtering, each of which will The time tap is treated as array element, and weighted and summed to achieve the purpose of filtering. Furthermore, the spatial filtering method can be used in time domain processing, and a new method is opened up in adaptive signal processing.

In the ordinary SMI, BMP, and EMP methods, in the process of processing interference, it is usually only possible to form a null in a specific single direction, that is, when applied in the time domain, only single-frequency interference can be blocked. In order to deal with wideband signals, this paper proposes a frequency response zero control technique [8][9], which can form a certain frequency bandwidth recess near any frequency. It solves the problem that only the single-frequency interference can be processed when the spatial domain filtering method is applied in the time domain.

2. The Processing Structure
In order to apply the relevant theory of spatial beamforming to the time domain, time taps are added after the receiving antenna elements, each time tap forms a finite-length impulse response filter, and
each time tap is treated as an array element for each time. The signals after tapping are weighted and summed to perform time domain analysis on the model to achieve the purpose of time domain filtering. The processing structure is shown in Figure 1:

**Figure 1** Adaptive time domain filtering processing structure

### 3. The Signal Model

Let the input on the radar antenna be \( s(n) \), then the input signal of the \( p \)th delay node is:

\[
s_p = s[n - (p - 1)\Delta \tau] \quad (p = 1, 2, \ldots, N)
\]

Define the \( N \times 1 \) dimensional vector \( s \):

\[
s = [s_1 \ s_2 \ \cdots \ s_N]^H
\]

Redefine the \( N \times 1 \) dimensional interference vector \( j \) and the noise vector \( n \):

\[
 j = [j_1 \ j_2 \ \cdots \ j_N]^H \quad (3)
\]

\[
 n = [n_1 \ n_2 \ \cdots \ n_N]^H \quad (4)
\]

Then the input model of the time domain filtering process can represent:

\[
x = s + j + n = [x_1 \ x_2 \ \cdots \ x_N]^H
\]

As with array beamforming, define the \( N \times 1 \) dimensional vector \( w \):

\[
w = [w_1 \ w_2 \ \cdots \ w_N]^H
\]

Then the output signal of the time domain filtering process can be expressed as:

\[
y(f, n) = w^Hx = \sum_{p=1}^{N} w_p^H(f)x_p = \sum_{p=1}^{N} w_p^H(f)x[n - (p - 1)\Delta \tau]
\]

The frequency domain expression of its output is:

\[
y(f) = \sum_{p=1}^{N} w_p^H(f)x(f)\exp\{-j2\pi f[(p - 1)\Delta \tau]\}
\]

It can be seen from equation (8) that the time domain filtering process can distinguish the interference signals and can generate nulls of different depths and widths at the frequencies of the interference signals.

The time domain filtering process is the same as the spatial beamforming, and the dry signal ratio of the output signal is:

\[
\text{SINR} = \frac{w^H R_s w}{w^H R_j n w}
\]

Where \( R_s \) is the signal covariance matrix and \( R_j + n \) is the covariance matrix of the interference noise sum.

Like the normal line array, the optimal beamforming for time domain filtering has three optimal criteria: the minimum mean square error criterion (MMSE), the maximum signal to noise ratio criterion (MSNR), and the linear constrained minimum variance criterion (LCMV), under common application conditions, the three criteria are equivalent. The LCMV guidelines are used here.

The time domain filtering process based on the LCMV criterion can be described as a constrained optimization problem:

\[
\begin{cases}
\min w^H R_s w \\
s. t. \ w^H a_t = 1
\end{cases}
\]

Where \( a_t \) is a time domain steering vector whose expression is:

\[
a_t = [1 \ e^{j2\pi f_0 \Delta \tau} \ e^{j2\pi f_0 2\Delta \tau} \ \cdots \ e^{j2\pi f_0 (N-1)\Delta \tau}]
\]
Applying the Lagrangian algorithm to solve the above equation, the optimal weight of the time domain filtering process can be obtained:

$$\mathbf{w}_{\text{opt}} = \mu \mathbf{R}_\Delta^{-1} \mathbf{a}_t$$  \hspace{1cm} (12)

In the formula, $\mu = \frac{1}{\mathbf{a}_t^H \mathbf{R}_\Delta^{-1} \mathbf{a}_t}$ is a scalar quantity.

4. Frequency Response Zero Control Technology

According to the second section analysis, this method can pass the frequency of the desired signal and prevent the interference signal frequency. However, it is limited to single-frequency interference. In order to prevent bandwidth interference, it can be implemented by frequency response zero control technology.

A time domain filtering processing structure consisting of $N$ time taps, and a spectral function for performing a perturbation control on the tap weights to form a null at a specified frequency can be expressed as:

$$\mathbf{G}(f) = \mathbf{W}^H \mathbf{a}(f_0)$$  \hspace{1cm} (13)

Where $f_0$ is the desired signal frequency, which can through the filter. The principle of frequency response zero control is as follows: On the basis of the original weight vector, the time tap weight is slightly perturbed so that the sum of the frequency responses of the time taps is close to zero in the specified frequency range. Starting from the above formula, let the time tap value of the perturbation be $\Delta \mathbf{W}$, so that the time domain filter after the perturbation forms a null at the $f$ frequency, the pattern $\mathbf{G}_1(f)$ after the time tap weight perturbation should satisfy:

$$\mathbf{G}_1(f) = \mathbf{W}^H : \mathbf{a}(f_0) = 0$$  \hspace{1cm} (14)

Let $\Delta \mathbf{W}$ be the disturbance amount of the weight, and the time tap original weight is $\mathbf{W}_0$, then $\mathbf{W}_1 = \mathbf{W}_0 + \Delta \mathbf{W}$. Decomposed (13) can get:

$$\mathbf{G}_1(f) = \mathbf{G}_0(f) + \Delta \mathbf{W}^H : \mathbf{a}(f_0)$$  \hspace{1cm} (15)

Where $\mathbf{G}_0(f)$ is the spectral function of the original filter processing structure.

When the $M$ frequencies $f = \{f_1, f_2, \cdots, f_M\}$ form a zero trap, a small value vector $\mathbf{\varepsilon} = [\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_M]$ can be introduced to control the depth of each recess on the frequency. $\mathbf{\varepsilon}$ as the filter processing structure at the zero-frequency power expectation value, $\varepsilon_i (i = 1, 2, \cdots, M)$ can not achieve the value of the ith control zero frequency on the depth of the depression, as follows:

$$\mathbf{G}_1(f) = \mathbf{G}_0(f) + \Delta \mathbf{W}^H : \mathbf{a}(f_0) = \mathbf{\varepsilon}$$  \hspace{1cm} (16)

on the basis of the original filter processing structure spectrogram, after the weight perturbation is controlled at the specified frequency, in order to maintain the excellent performance of the original spectrogram, the power response of the perturbed spectrogram at other frequencies should be changed as small as possible, that is, $\mathbf{G}_1(f)$ and $\mathbf{G}_0(f)$ are required to be as close as possible. Considering that the change of the spectrogram is caused by the disturbance of the weight, the minimum value of the weight disturbance can be optimized.

According to the expression of the spectrogram function after the weight value perturbation (15), the square of the gain of the spectrogram of the filter processing structure at the frequency $f_m$ is:

$$|\mathbf{G}_1(f_m)|^2 = \mathbf{W}_1^H \mathbf{a}(f_m) \cdot \mathbf{a}(f_m) \mathbf{W}_1^H$$  \hspace{1cm} (17)

The constraint condition that specifies the frequency control zero is equivalent to:

$$\sum_{m=1}^{M} |\mathbf{G}_1(f_m)|^2 = \sum_{m=1}^{M} \mathbf{W}_1^H \mathbf{a}(f_m) \cdot \mathbf{a}(f_m) \mathbf{W}_1^H$$  \hspace{1cm} (18)

When a wide null trap with a $\Delta f_m$ width is formed at $f_m (m = 1, \cdots, M)$, equation (18) should be written as:

$$\sum_{m=1}^{M} |\mathbf{G}_1(f_m)|^2 = \sum_{m=1}^{M} f_m + \frac{\Delta f_m}{2} \mathbf{W}_1^H \mathbf{a}(f_m) \cdot \mathbf{a}(f_m) \mathbf{W}_1^H df = \mathbf{\varepsilon}$$  \hspace{1cm} (19)

After leaving the perturbation weight $\mathbf{W}_1$, the constraint matrix is introduced according to the
interchangeable position of the summation formula and the integral number:

\[ Q = \sum_{m=1}^{M} \int \frac{\Delta f_m}{f_m} a(f_m) \cdot a(f_m)^H \, df \]  

(20)

The equation (19) should be written as:

\[ W_1^H Q W_1 = \varepsilon \]  

(21)

Obviously, \( Q \) is an \( N \times N \)-dimensional Hermitian matrix. The eigenvalue decomposition of \( Q \) can be expressed as:

\[ Q = \Gamma \Lambda \Gamma^H \]  

(22)

In the formula, \( \Gamma \) is the monsieur beaucaire matrix composed of the eigenvectors of \( Q \), and \( \Lambda \) is the eigenvalue diagonal matrix of \( Q \):

\[ \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_M) \]  

(23)

The minimum weight perturbation optimization model can be understood as: under the constraint that the frequency power integral of the null region is less than a certain constant, a weight vector \( W_1 \) is found to be closest to the initial weight vector \( W_0 \). In fact, the constraint of equation (21) can be replaced by a constraint on a set of eigenvectors of matrix \( Q \). In an ideal case, the perturbed weight of the filter processing structure should be constrained so that the frequency filter has a zero response in the frequency range of a certain bandwidth of interest, so the constraint condition (21) can be equivalent to:

\[ W_1^H e_i = \varepsilon_i \quad (i = 1, 2, \ldots, M) \]  

(24)

Where \( e_i \) is the \( i \)-th eigenvector of the constraint matrix \( Q \). In fact, the larger eigenvalue of the matrix \( Q \) is related to the number \( M \) of nulls. If we choose a suitable \( n \) such that \( M \leq n \leq N \), we can ensure that the time domain filtering structure has a range of interference frequencies of interest. Approximate zero response, through simulation verification, take \( n = M + 2 \) to meet the zero control requirement, then equation (24) can be equivalent to:

\[ W_1^H e_i = \varepsilon_i \quad (i = 1, 2, \ldots, n) \]  

(25)

In this way, the problem translates into a minimal optimization model of linear constraints:

\[ \text{obj: } \min f(W_1) = \|W_1 - W_0\|^2 \]  

(26)

S.T. \( W_1^H e_i = \varepsilon_i \quad (i = 1, 2, \ldots, n) \)  

(27)

This method can be called constrained matrix eigenvector, abbreviated as CME method. When the perturbation weight \( W_1 \) is a complex weight value, the complex model is needed to solve the above model, and \( W_1 \) is decomposed into the real part \( \text{real}(W_1) \) and the imaginary part \( \text{imag}(W_1) \) according to the formula (2.39) to calculate the filter perturbation. Weight \( W_1 \).

5. Simulation Analysis

Simulation conditions: 19 delay taps, delay time is 1/450s, expected signal frequency is 10kHz, and its signal-to-noise ratio is 10dB.

Simulation experiment 1: Using the optimal weight \( w_{\text{opt}} = \mu R^{-1} a_t \) processed by time domain filtering, the delay taps are weighted to form a static frequency response diagram of the adaptive filter. The simulation results are as follows:
It can be seen from the simulation results in Fig. 2 that the spatial adaptive beamforming algorithm is applied in the time domain filtering, and the frequency response map can also be formed at the desired frequency. Only the desired frequency can pass through the filter, and other frequencies are suppressed. However, there are side lobes in the static frequency response diagram, and the filter can also be used when the frequency power corresponding to the side lobes is strong.

Simulation experiment 2: There are other frequency interference signals passing through the filter, forming a depression at the interference frequency through the adaptive weight vector, setting the desired signal frequency to 10 kHz, and the interference signal frequency to be 10.1 kHz and 9.8 kHz. The signal-to-noise ratio is 10 dB and the drying ratio is 30 dB. The simulation results are shown in Figure 3.

It can be seen from the simulation results of Fig. 3 that in the case of an interference signal, the adaptive filter forms a depression at the interference signal frequency, and the normalized amplitude at 10.1 kHz is -166.5 dB, normalization at 9.9 kHz. The amplitude is -151.6dB. No depression is formed at the desired signal.

Simulation Experiment 3: Formation of a wide depression at a specific frequency. A depression of 50 Hz bandwidth is formed at 10.1 kHz and 9.9 kHz. The simulation results are as follows:
Figure 4 frequency response zero control technology

It can be seen from the simulation results of Fig. 4 that the adaptive filter forms a concave shape with a bandwidth of 50 Hz at the interference frequency by the frequency response zero control technique, which solves the problem that the conventional adaptive filtering can only filter out the single frequency signal.

6. Conclusion

In this paper, by adding time taps after the radar receiving antenna, the spatial adaptive beamforming processing method is applied to the time-frequency domain to form an adaptive time-domain filter, which provides a new idea and a new method for adaptive signal processing. In order to solve the problem that the ordinary adaptive time domain filter can only form a zero trap at a specific single-frequency interference, this paper proposes a frequency response zero control technology, which can form a null trap on the wideband signal and achieve good results. In theory, by selecting an appropriate time delay $\Delta \tau$, the adaptive filter can process signals in any frequency band, and as the frequency of the processed signal increases, the relative frequency difference between the interference signal and the desired signal can be smaller and smaller. It has a good practical application in high frequency anti-interference. However, the time delay $\Delta \tau$ in this paper is not easy to choose in practical applications, and it has a great influence on the method proposed in this paper, which needs further research.

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