Multi-conductor Transmission Line Model of Split-winding Transformer for Frequency Response and Disk-to-disk Fault Analysis

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Abstract

For transformer designers, split-winding transformer modeling in different frequency ranges is of great importance. In this paper, for the first time, a multi-conductor transmission line model is proposed for high-frequency modeling of the split-winding transformer. In this model, all the turns in layers and disks have been considered and the model's parameters have been calculated using the finite element method. In order to validate the proposed model, the results are compared with the result of a model, which is based on finite element and coupled field-circuit. It is shown that the introduced model has good accuracy and it can be employed for split-winding transformer modeling in different frequency ranges. In addition, using the validated multi-conductor transmission line model, the frequency response of the split-winding transformer and disk-to-disk short circuit fault are analyzed.

NOMENCLATURE

| Symbol | Description |
|--------|-------------|
| U and I | voltage and current |
| R, L, C, and G | resistance, inductance, capacitance, and conductance |
| Z and Y | impedance and inductance matrix |
| \( U_s(i) \) and \( I_s(i) \) | sending end voltage and current of i-th transmission line |
| \( U_r(i) \) and \( I_r(i) \) | receiving end voltage and current |
| l | the average length of the line |
| \( Y_0 \) | the characteristic admittance matrix of the model |
| \( A, J \) | the magnetic vector potential, current density |
| \( \mu \) | Permeability |
| \( \rho \) | the volume charge density |
| \( \varepsilon_0 \) | the permittivity of free space |
| S | the conductor's cross-section |
| \( W_0 \) | the magnetic energy |
| \( B_1 \) | The magnetic flux density |

Subscripts

| Subscript | Description |
|-----------|-------------|
| FE | Finite element |
| MTL | Multi-conductor transmission line |
| CC | correlation coefficient |
| FRA | Frequency Response Analysis |

1. INTRODUCTION

Nowadays, transformers are employed in different applications and a transformer designer encounters several electromagnetic and insulating complex problems [1-2]. The growing trend in the use of traction systems has increased the need for different structures of multi-winding transformers.

In a transformer, impulse overvoltages can easily cause an insulation breakdown, internal short-circuits and mechanical displacement. The special structure of a split-winding transformer may cause problems in insulating system design under different loading conditions. Investigating transformer response to different disturbances, and controlling these conditions can lead to high reliability in a traction network. Therefore, providing suitable models is important for analyzing the behavior of this type of transformers especially under high-frequency transients.

High-frequency transients have been studied in two-winding oil-immersed and dry-type transformers [3-7]. Model parameters and inductance matrix for two-
winding transformers have been calculated for different purposes based on analytical, finite element (FE) [8 - 13] and heuristic [14] methods.

Aghmasheh et al. have used the gray box modeling to study the transients in power transformer windings and its parameters have been obtained by applying the mathematical transfer function and the measured frequency response [15]. In literature, FE models have been employed to study high-frequency electromagnetic transients [16], frequency response [9, 17] and winding fault diagnosis [18]. Additionally, in order to model the behaviour of transformer, 2D and 3D FE models have been coupled to external electrical circuits in literature [19-20]. The coupled field-circuit approach has also been applied for high-frequency modeling of the transformer [17, 20, 21], in which the windings have been modeled as integrated blocks that can cause a considerable error in modeling the ohmic behaviour of windings.

The multi-conductor transmission line (MTL) model has been investigated in various papers to study the transient behavior of different transformers. Also, using the optimized MTL model, the partial discharge has been studied in power transformer windings [8, 22]. Jafari et al. have introduced a modified MTL model for partial discharge localization in power transformers [23]. Zhang et al. have investigated on the distribution of very fast transient overvoltages using an analytical model based on the MTL model and finite difference time domain approach [24]. The hybrid models, which are a combination of multi and single-transmission line models, have been introduced to simulate the transients responses of transformer windings in literature [25, 26]; these models significantly reduce the number of linear equations.

The electromagnetic and thermal behaviour of the split-winding transformer has been studied in [27, 28]. Some models based on equivalent circuit and coupled field-circuit approach have been previously proposed to simulate the low-frequency behaviour of the split-winding transformer [29, 30]. Additionally, research on the design optimization of the split-winding traction transformer has been presented under normal loading conditions [31].

For the split-winding transformer, novel models based on the coupled field-circuit approach and detailed model, have been introduced and validated for high-frequency analysis by Sobouti et al. [32]. The coupled field-circuit approach is very accurate, but it needs more computational time. Additionally, detailed model is a fast method but with lower accuracy.

In order to have an accurate and fast solution, in this paper, a new model based on MTL is introduced for split-winding transformer modeling. In all previous models, only simple layers and disks have been modeled in the split-winding traction transformer. But in this paper, all the turns in layers and disks of windings have been considered and the model parameters per turns have been calculated using the FE method. The results are compared to a validated coupled field-circuit approach. It is shown that the introduced method has good accuracy and low computational time. Using the validated model, the frequency response of the split-winding transformer under normal and faulty conditions (disk-to-disk short-circuit fault) is studied. The obtained results show the higher capability of the proposed model against the older techniques.

2. PROPOSED MODEL BASED ON MTL

One of the models that have been used to simulate the electromagnetic transients in transformer windings is the detailed model [32]. The validity of the detailed model depends on the accuracy of determining parameters, the number of model sections, and the selected error function. When a large number of sections are chased, although the validity ranges of the model can increase, it would be more difficult to find appropriate values for the parameters. This causes the model solution to become more time-consuming due to the larger dimensions of system equations. Thus, similar to many engineering problems, considering a compromise between modeling accuracy and calculation speed is important [33].

In the MTL model, each conductor of the winding is considered as a parallel transmission line, and all capacitive and inductive couplings are taken into account. By considering all the conductors and their lengths according to the technical specifications, the limitations of finite element models are eliminated. Thus, the validity of the model is higher than the detailed model [8].

The N-dimensional vectors of voltage and current of each distinct x-point located on each of n transmission lines could be formulated by the following wave equations:

\[
\frac{d[u(x,t)]}{dx} = - [R][i(x,t)] + [Z]\frac{d[i(x,t)]}{dt} 
\] (1)

\[
\frac{d[i(x,t)]}{dx} = - [G][u(x,t)] + [C]\frac{d[u(x,t)]}{dt} 
\] (2)

where, \( u \) and \( i \) are the voltage and current vectors, respectively. In addition, \( R, L, C, \) and \( G \) are respectively the resistance, inductance, capacitance, and conductive matrices.

Because of symmetrical electrical parameters, the above equations in the frequency domain could be explained as follows [8]:

\[
\frac{d^2[u(x)]}{dx^2} = [Z][Y][u(x)] = [P]^2[u(x)] 
\] (3)

\[
\frac{d^2[i(x)]}{dx^2} = [Y][Z][i(x)] = ([P]^2)^T[i(x)] 
\] (4)
where $Z$ and $Y$ are impedance and inductance matrix respectively [8].

Figure 1 shows the MTL model of the transformer winding.

The boundary conditions of the MTL model can be defined as follows:

$$U_R(i) = U_R(i + 1) \quad \text{For } i = 1 \text{ to } i = N - 1 \quad (5)$$

$$I_R(i) = -I_S(i + 1) \quad \text{For } i = 1 \text{ to } i = N - 1 \quad (6)$$

By solving the equations using the boundary conditions, it is possible to calculate the currents based on voltages (7):

$$\begin{bmatrix} I_s \\ I_R \end{bmatrix} = \begin{bmatrix} Y_a \, \coth(\mu |l|) & -Y_a \, \cosh(\mu |l|) \\ -Y_a \, \coth(\mu |l|) & Y_a \, \coth(\mu |l|) \end{bmatrix} \begin{bmatrix} U_s \\ U_R \end{bmatrix} \quad (7)$$

where, $U_S (i)$ and $I_S (i)$ are sending end voltage and current of $i$-th transmission line, and $U_R (i)$ and $I_R (i)$ are its receiving end voltage and current, $[P]$ is the product of matrices $Z$ and $Y$, $l$ is the average length of the line and $Y_0$ is the characteristic admittance matrix of the model, which is obtained from the following relation:

$$Y_0 = [Z]^{-1} [P] \quad (8)$$

3. MTL PARAMETER CALCULATION

Figure 2 and Table 1 show the windings structure and Specifications of the dry-type split-winding transformer. The high voltage windings are paralleled to construct a single primary winding [27,32].

3. 1. FE Model

Three-phase electromagnetic modeling of a transformer is time-consuming and may be impossible, but it is acceptable to employ single-phase modeling [32].

The magnetic fields can be modeled using Poisson’s equation, as follows:

$$\nabla^2 A - \mu \frac{\partial^2 A}{\partial t^2} = -\mu \cdot J. \quad (9)$$

$$U_s(1) \quad I_s(1) \quad U_s(1)$$

$$U_s(2) \quad I_s(2) \quad U_s(2)$$

$$U_s(N) \quad I_s(N) \quad U_s(N)$$

**Figure 1. MTL Model**

**Figure 2. Single-phase view of a dry-type split-winding transformer**

| TABLE 1. Specifications of the transformer shown in Figure 2 [32]. |
|----------------------|-----------------|-----------------|-----------------|-----------------|
|                      | HV1             | HV2             | LV1             | LV2             |
| Rated power (MVA)    | 2               | 2               | 2               | 2               |
| Line voltage (V)     | 20000           | 20000           | 750             | 750             |
| Connection           | D               | D               | y               | D               |
| Layer number         | 1               | 1               | 3               | 3               |
| Axial channel (mm)   | -               | -               | 3*10            | 1*12            |
| Disk number          | 11              | 11              | 1               | 1               |
| Radial channel between disks (mm) | 9*12 / 1*12 | 9*12 / 1*12 | -               | -               |
| Turns                | 693             | 693             | 26              | 15              |
| Height (mm)          | 1230            | 1230            | 1240            | 1240            |
| Internal diameter (mm) | 654            | 654             | 332             | 929             |
| External diameter (mm) | 799           | 799             | 454             | 982             |
| Core diameter (mm)   | 320             |                 |                 |                 |
| $E_s$ (mm)           | 1015            |                 |                 |                 |
| $H_s$ (mm)           | 1410            |                 |                 |                 |

where, $A$, $J$, $\varepsilon$ and $\mu$, are the magnetic vector potential, current density, permittivity and permeability, respectively. Also, the fundamental postulate of electrostatics in free space specifies the divergence of $E$ using Equation (10):

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}, \quad (10)$$

where $\rho$ is the volume charge density and $\varepsilon_0$ is the permittivity of free space. In practical applications, the total field of an aggregate or a distribution of charges is usually calculated. The volume integral of both side of Equation (10) over an arbitrary volume $V$ is given by Equation (11):

$$\int_V \nabla \cdot E \, dv = \frac{1}{\varepsilon_0} \int_V \rho \, dv. \quad (11)$$
Using the divergence theorem, we have [34]:
\[ \oint_{\partial V} \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\varepsilon_0} \]  
(12)

In this paper, the FE models are used to solve Poisson’s equation and to obtain magnetic fields in different positions of transformer windings. Several FE models can be employed as introduced by Sobouti et al. [32]. In the present research, a 2D FE model with an equivalent core model is selected for the electromagnetic field calculations, and a 2D FE model with a simple core is selected for electrostatic computations [28].

3.2. Winding Resistance Calculation
The winding losses in the transformer consist of a DC and an eddy current losses. Using the calculated magnetic field, the current and losses distribution in windings and consequently windings resistances can be calculated.

In this paper, winding losses and resistances are computed based on a semi-analytical method [32]. This quasi-numerical method employs the FE method to calculate the magnetic field distribution, and an analytical method to determine the windings losses per windings part. The obtained results are shown in Figures 3 and 4 for typical traction transformer specifications [29].

Note that the winding ohmic resistance increases with an increase in the frequency as described in literature [35, 36].

3.3. Winding Capacitors Calculation
In multi-winding traction transformer modeling, the capacitances between LV and HV winding turns should be calculated in the MTL model. In this paper, the FE method has been used to calculate these capacitors. To reduces the calculation burden, a method based on FE and electrical charge calculation of each conductor is utilized to calculate capacitances [32]. Figure 5 shows electric potential (V) and electric field while the capacitances have been calculated.

Employing voltage \( V_i \) to \( i \)-th conductor (and setting the others to zero), the electric field can be calculated using the electrostatic analysis, and the electric charge on each conductor can be calculated using Equation (13) [6, 9, 10]:
\[ Q = (\varepsilon_r \varepsilon_0) \frac{S}{d} \]  
(13)

where \( S \) is the conductor’s cross-section, \( \varepsilon_0 \) and \( \varepsilon_r \) are free space and relative permittivity, respectively and \( d \) is the distance between conductors. Now, by calculating the electrical charges of all the conductors and forming the vector \( Q \), the column \( i \) of the matrix \( C \) can be obtained as follows:
\[ [c_{1i}, c_{2i}, ..., c_{ni}]^T = \frac{Q}{V_i} \]  
(14)

Because of the change in behavior of the electrical potential and flux distributions, studying short circuits in layers or disks is important. In the past, ohmic resistance changes have been used to detect short circuits. In this paper, capacitors are computed in both normal and faulty conditions. A fault condition in a disk or a layer can be

\[ \text{Surface: Electric potential (V) Streamline: Electric field} \]

(a) (b)
modeled by short-circuiting a number of turns with very low resistance. Figures 6-7 show the calculated capacitances in the normal and faulty conditions.

By comparing the results in Figures 6 and 7, it can be seen that the capacitance values decrease due to the short circuit of the layers and disks.

3.4. Winding Inductances Calculation using Electromagnetic Field Modelling

The calculation of inductive parameters includes the determination of mutual and self-inductances that are very important for impulse voltage distribution and frequency response analysis. In this paper, the energy method is used to calculate the winding inductances.

The magnetic energy can be calculated using Equation (15) by which two conductors are coupled [8, 28]:

$$ W_{ij} = \frac{1}{2} \int_{vol} B_i \cdot H_j \, dv $$

(15)

where, $B_i$ is resulting magnetic flux density due to $i$-th conductor, and $H_j$ is resulting magnetic field due to $j$-th conductor.

By employing currents $I_i$ and $I_j$ to a pair of intended windings and calculating the magnetic energy ($W_{ij}$), the mutual inductance between them is calculated using Equation (16) [8, 28]:

$$ L_{ij} = \frac{2W_{ij}}{I_i I_j} $$

(16)

In addition, the self-inductance can be calculated by the following equation [8]:

$$ L_{ii} = \frac{2W_{ii}}{I_i^2} $$

(17)

As presented in literature [32, 37], the winding inductances decrease exponentially with an increase in the frequency.

The inductances per turn for the first disk of HV1 and the fourth layer of LV1 in two different frequencies and under normal and short-circuit conditions are shown in Figures 8 and 9, respectively. Figure 10 shows the magnetic flux density distribution in normal and faulty conditions.

As can be seen in Figures 8 and 9, the inductance values increase as a result of the short-circuit condition. Therefore, calculated parameters have been used in the...
MTL model and the frequency response of the split-winding transformer and disk-to-disk short circuit fault are analyzed. Figure 11 shows the flowchart of the general procedure for the current research.

4. SIMULATION RESULTS

The detailed 3D FE model with full consideration of both electromagnetic and electrostatic phenomena is the most accurate method. But, the detailed modeling of all layers and disks in the 3D model is very time-consuming, and due to the restrictions in computer hardware, it may be impossible [28, 29]. Thus, a designer may be forced to simplify the windings structure in the 3D model. This approach causes an error in high-frequency analysis. For the same structure modeling, the 3D FE model has higher accuracy. Thus, the 2D FE results are validated versus the 3D FE model with a simplified structure of the split-winding transformer as shown by Sobouti et al. [32]. In addition, the simulation time in the 2D FE model is less than the 3D model with sufficient accuracy. Therefore, a 2D model with an equivalent core can be used to model the split-winding transformer with a complete geometry for high and low-frequency electromagnetic transients studies.

To increase the accuracy in the 2D model, the 2D coupled field-circuit method has been used to model the split-winding transformer. As shown by Sobouti et al. [32], the resonant frequencies are acceptable while they are compared to the FE models. Therefore, the coupled field-circuit approach can model the transformer behavior during transients more accurately.

Also, in this paper, the MTL model is simulated using the electrical parameters determined in Section 3 in normal and faulty conditions. In this model, all the conductors of the disks and layers and their average length are modeled according to the technical specifications, and validated with the FE and the coupled field-circuit method. To obtain the frequency response of the transformer, an impulse voltage is applied to the transformer input terminals (HV) and the output impulse response is obtained (LV1 windings). Then, by dividing the output signal to the input signal and using the fast Fourier transform, the transfer function (TF) is obtained in the frequency domain [16]. Figures 12 and 13 show the results of the voltage distribution of the high-frequency MTL model in HV disks, the comparison of the results of the MTL model with the 2D FE, and the coupled-field circuit model in normal conditions. This comparison shows that the proposed method is able to model the high-frequency transient behavior of the split-winding transformer in normal conditions.

After short-circuiting the eleventh and tenth disks of the HV1 winding, the frequency response is determined. Figure 14 shows the difference of resonant frequencies in the MTL model in faulty conditions.
As it was mentioned in literature [38], a value smaller than 0.9996 for CC means that a faulty condition is occurred. By applying Equation (18) to different disk-to-disk fault conditions such as the data presented in Fig. 14, it is seen that the CC has a value between 0.6 and 0.95. Thus, the introduced high-frequency MTL model is an efficient (fast and accurate) method for using in frequency response analysis of transformer due to different fault conditions.

5. CONCLUSIONS

In this paper, the transient behaviour of the split-winding traction transformer has been studied at high frequencies in faulty and normal conditions using an MTL model. Given that modeling all turns of disks and layers in the windings is not possible in 3D models, the 2D FE models with detailed modeling of windings was introduced to model the electromagnetic transients more accurately.

In the proposed model, all the conductors of the disks and layers and their average length have been modeled. The 2D and coupled-field circuit models have been used to model and validate the transient electromagnetic behaviour of the split-winding traction transformer in the frequency domain, and the accuracy of the methods has been confirmed. It has been discussed that the MTL model not only is fast and simple, but also can be employed to model high-frequency transient behaviour of split-winding traction transformers with much higher accuracy compared to the detailed model. In addition, its frequency range has been increased compared to the detailed model. Also, it has been shown that the FRA result of transformer winding for each short-circuit fault is different from others.

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