Two-loop renormalization of $\tan \beta$ and its gauge dependence

Youichi Yamada

Department of Physics, Tohoku University, Sendai 980-8578, Japan

Abstract

Renormalization of two-loop divergent corrections to the vacuum expectation values $(v_1, v_2)$ of the two Higgs doublets in the minimal supersymmetric standard model, and their ratio $\tan \beta = v_2/v_1$, is discussed for general $R_\xi$ gauge fixings. When the renormalized $(v_1, v_2)$ are defined to give the minimum of the loop-corrected effective potential, it is shown that, beyond the one-loop level, the dimensionful parameters in the $R_\xi$ gauge fixing term generate gauge dependence of the renormalized $\tan \beta$. Additional shifts of the Higgs fields are necessary to realize the gauge-independent renormalization of $\tan \beta$. 

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Several extensions of the standard model have more than one Higgs boson doublet. For example, the minimal supersymmetric (SUSY) standard model (MSSM) \[1, 2\] has two Higgs doublets

\[ H_1 = (H_0^1, H_1^-), \quad H_2 = (H_2^+, H_0^0). \]

Both $H_1^0$ and $H_2^0$ acquire the vacuum expectation values (VEVs) $v_i$ ($i = 1, 2$) which spontaneously break the SU(2) $\times$ U(1) gauge symmetry. $H_i^0$ are then expanded about the minimum of the Higgs potential as

\[ H_i^0 = \frac{v_i}{\sqrt{2}} + \phi_i^0. \]

$\phi_i^0$ are shifted Higgs fields with vanishing VEVs. I assume that CP violation in the Higgs sector is negligible and take $v_i$ as real and positive.

A lot of physical quantities of the theory depend on the Higgs VEVs. In calculating radiative corrections to these quantities, the VEVs have to be renormalized. In the minimal standard model with only one Higgs doublet, the renormalization of the Higgs VEV is usually substituted by that of the weak boson masses \[3, 4\]. However, this is not enough for extended theories with two or more Higgs doublets. For example, the renormalization of $v_i$ in the MSSM is usually performed \[3, 4, 5\] by specifying the weak boson masses, which are proportional to $v_1^2 + v_2^2$, and the ratio $\tan \beta \equiv v_2/v_1$. Since $\tan \beta$ itself is not a physical observable, however, a lot of renormalization schemes for $\tan \beta$ have been proposed in the studies of the radiative corrections in the MSSM. Some of them are listed in Ref. \[8\]. In this letter, I concentrate on process-independent definitions of $\tan \beta$, which are given by the ratio of the renormalized VEVs $v_i$. I discuss the renormalization of the ultraviolet (UV) divergent corrections to $v_i$ and $\tan \beta$, working in the modified minimal subtraction schemes with dimensional reduction \[9\] (\text{DR} scheme). The results are presented as the renormalization group equations (RGEs) for $v_i$ and $\tan \beta$. Since they are not physical observables, they may depend on the gauge fixing in general. I therefore investigate their gauge dependence in the general $R_\xi$ gauge fixing \[10\]. Although I show the results for the MSSM, the results for the gauge dependence can be generalized for other models with two or more Higgs doublets.

Even within the \text{DR} scheme, there still remains an ambiguity of the way how to cancel the radiative shifts of the Higgs VEVs, $\Delta v_i$, by the one-point functions of $\phi_i^0$ by tadpole diagrams. One way is to cancel $\Delta v_i$ entirely by the shift of $\phi_i^0$. As a result, the tadpole contributions have to be added to all quantities which depend on $v_i$. The renormalized $v_i$ give the minimum of the tree-level Higgs potential and are just tree-level functions of the gauge-symmetric quadratic and quartic couplings in the Higgs potential. These $v_i$ are therefore independent of the gauge fixing parameters \[11\]. This renormalization scheme for $v_i$ is sometimes used \[12, 13, 14, 15\] to show manifest gauge independence of physical quantities. However, since the running of $v_i$ in this scheme is very rapid \[16\], and the tadpole contributions appear in almost any corrections, this scheme is often inconvenient in practical calculations.
Another, more popular way [11, 2] is to absorb $\Delta v_i$ by the shift of quadratic terms in the Higgs potential. The renormalized $v_i$ then give the minimum of the loop-corrected effective potential $V_{\text{eff}}(H_1, H_2)$. This scheme is very convenient in practical calculation, because the explicit forms of the tadpole diagrams are necessary only for twopoint functions of the Higgs bosons. However, the effective potential is generally dependent on the gauge fixing parameters [17, 18, 19, 20]. The gauge dependence of the renormalized $v_i$ and their ratio $\tan \beta$ then might be a serious problem in calculating radiative corrections. I will therefore discuss the gauge dependence of the running $\tan \beta$ in this definition, in general $R_\xi$ gauges and to the two-loop order.

The RGE for $v_i$ can be obtained from the UV divergent corrections to $v_i$-dependent masses or couplings of particles. For simplicity, I use the corrections to two quark masses $m_b$ and $m_t$, ignoring the masses of all other quarks and leptons. These mass terms are generated from the $b\bar{b}H_1$ and $t\bar{t}H_2$ Yukawa couplings, respectively, as

$$L_{\text{int}} = -h_b b_R b_L (v_1/\sqrt{2} + \phi_1^0) - h_t t_R t_L (v_2/\sqrt{2} + \phi_2^0) + \text{h.c.} \quad (3)$$

The $R_\xi$ gauge fixing term takes the form

$$L_{\text{GF}} = -\frac{1}{2\xi_Z}(\partial^\mu Z_\mu - \rho_Z G_Z)^2 - \frac{1}{\xi_W}|\partial^\mu W^+_{\mu} - i\rho_W G^+_W|^2$$
$$-\frac{1}{2\xi_\gamma}(\partial^\mu \gamma_\mu)^2 - \frac{1}{2\xi_g}\sum_{a=1}^8(\partial^\mu g^a_\mu)^2. \quad (4)$$

The would-be Nambu-Goldstone bosons $G_V$ for $V = (Z, W)$ appear in Eq. (4). The parameters $\rho_V \equiv \xi_V m_V$, where $m_V^2 = g_V^2(v_1^2 + v_2^2)/4$ ($g_W^2 = g_Z^2$, $g_Z^2 = g_1^2 + g_2^2$) are masses of $Z$ and $W^\pm$, are introduced in Eq. (4). This is to emphasize that the gauge symmetry breaking terms $\xi_V m_V$ in $L_{\text{GF}}$, and also in the accompanied Fadeev-Popov ghost term, has very different nature from $v_i$ generated by the shifts (2), as shown later. The terms $\rho_V G_V$ in Eq. (4) are expressed in the gauge basis (1) of the Higgs bosons as

$$\rho_Z G_Z = \xi_Z m_Z G_Z \equiv -\sqrt{2}\text{Im}(\rho_{1Z} \phi_1^0 - \rho_{2Z} \phi_2^0), \quad (5)$$
$$\rho_W G^+_W = \xi_W m_W G^+_W \equiv -(\rho_{1W} H^+_1 - \rho_{2W} H^+_2), \quad (6)$$

with parameters $\rho_V$. The usual form of the $R_\xi$ gauge fixing in the MSSM is recovered by the substitution [3, 4]

$$(\rho_{1V}, \rho_{2V}) = \xi_V g_V(v_1, v_2)/2 = \xi_V m_V(\cos \beta, \sin \beta). \quad (7)$$

The UV divergent corrections to $m_b$ contain one source for the SU(2)×U(1) gauge symmetry breaking. It is either $v_1$ originated from the shift (2) of $H_0^1$, or $\rho_{1V}$ in the $R_\xi$ gauge fixing term (4) and the Fadeev-Popov ghost term. The former contribution is obtained from that to the $b_R b_L \phi_1^0$ Yukawa coupling $h_b$ by replacing external $\phi_1^0$ by $v_1/\sqrt{2}$, except for the wave function correction of $H_1^0$ to $h_b$. Similar argument holds for the UV
divergent corrections to \( m_t \) and to the \( \bar{t} \rho_1 \phi_0^0 \) Yukawa coupling \( h_t \). As a result, if the \( \rho_{iV} \) contributions are absent, the runnings of \( v_i \) are the same as those of the wave functions of \( H_i^0 \), namely
\[
\frac{dv_1}{dt} = \frac{1}{h_b} \left[ \sqrt{2} \frac{d}{dt}(m_b) - \frac{dh_b}{dt} v_1 \right] = -\gamma_1 v_1,
\]
\[
\frac{dv_2}{dt} = \frac{1}{h_t} \left[ \sqrt{2} \frac{d}{dt}(m_t) - \frac{dh_t}{dt} v_2 \right] = -\gamma_2 v_2,
\]
where \( t = \ln Q_{\overline{\text{DR}}} \) is the \( \overline{\text{DR}} \) renormalization scale. The anomalous dimensions of \( H_i^0 \) are denoted as \( \gamma_i \), which generally depend on the gauge fixing parameters \( \xi \). The RGEs (8) for \( v_i \) have been widely used in the Landau gauge \( \xi = \rho_{iV} = 0 \).

However, in general \( R_\xi \) gauges, \( \rho_{iV} \) in the gauge fixing terms (4) may give additional contributions to the quark mass running, as \( b\bar{b} \rho_{1V} \) and \( t\bar{t} \rho_{2V} \). Since they have no corresponding contributions to the \( \bar{b}\phi_1 \) and \( t\phi_2 \) couplings, the RGEs for \( v_i \) deviate [21, 6] from Eq. (8). Their general forms are then
\[
\frac{dv_i}{dt} = -\gamma_i v_i + Y_{iV} \rho_{iV},
\]
where \( Y_{iV} \) are polynomials of dimensionless couplings. Therefore, the RGE for \( \tan \beta \) becomes, using Eq. (7),
\[
\frac{d}{dt} \tan \beta = \tan \beta \left( -\gamma_2 + \gamma_1 + \frac{\xi_V g_V}{2} Y_{2V} - \frac{\xi_V g_V}{2} Y_{1V} \right).
\]

I then give explicit form of the RGE for \( \tan \beta \) in the MSSM, to the two-loop order. First, one-loop RGEs for \( v_i \) \( (i = 1, 2) \) are
\[
\left. \frac{dv_i}{dt} \right|_{1\text{loop}} = -\gamma_{i(1)} v_i + \frac{1}{(4\pi)^2} \left( g_Z \rho_{iZ} + 2 g_2 \rho_{iW} \right)
\]
\[
= v_i \left[ -\gamma_{i(1)} + \frac{1}{(4\pi)^2} \left( \frac{\xi_Z g_Z^2}{2} + \xi_W g_2^2 \right) \right],
\]
with the one-loop anomalous dimensions \( \gamma_{i(1)} \),
\[
(4\pi)^2 \gamma_{i(1)} = N_c h_q^2 - \frac{3}{4} g_2^2 \left( 1 - \frac{2}{3} \xi_W - \frac{1}{3} \xi_Z \right) - \frac{1}{4} g_1^2 (1 - \xi_Z),
\]
where \( h_q^2 = (h_b^2, h_t^2) \) for \( i = (1, 2) \), respectively, and \( N_c = 3 \). The \( \rho_{1Z} \) contribution to \( m_b \) is obtained from the diagram in Fig. 1. All other contributions of \( \rho_{iV} \) to \( m_q \) come from similar diagrams. Eq. (11) is consistent with the result in Refs. [3, 7] for \( \xi = 1 \). Since the gauge dependence of \( \gamma_i \), as well as the contribution from \( (\rho_{iZ}, \rho_{iW}) \) satisfying Eq. (7), cancels in the ratio (11), the one-loop running \( \tan \beta \) is gauge parameter independent in the \( R_\xi \) gauge.
I next proceed to the two-loop corrections. The two-loop anomalous dimensions \( \gamma_i^{(2)} \) are obtained from the general formula \([22]\) in the \( \overline{\text{MS}} \) scheme (the modified minimal subtraction schemes with dimensional regularization), after conversion into the \( \overline{\text{DR}} \) scheme \([23]\), as

\[
(4\pi)^4 \gamma_1^{(2)} = -N_c(3h_6^4 + h_6^2 h_7^2) + 2N_c h_6^2 \left( \frac{8}{3} g_3^2 - \frac{1}{9} g_Y^2 \right) + L(g),
\]

\[
(4\pi)^4 \gamma_2^{(2)} = -N_c(3h_4^4 + h_4^2 h_7^2) + 2N_c h_4^2 \left( \frac{8}{3} g_3^2 + \frac{2}{9} g_Y^2 \right) + L(g).
\]

The last term \( L(g) \) is a gauge-dependent \( \mathcal{O}(g^4) \) polynomial and is common both for \( \gamma_1^{(2)} \) and \( \gamma_2^{(2)} \). The \( \xi \) dependence of the \( \mathcal{O}(h_q^2 g^2) \) terms completely cancels out \([24]\). Note also that the \( \mathcal{O}(h_q^4) \) and \( \mathcal{O}(h_q^2 g^2) \) terms agree with the result in the \( \xi = 0 \) gauge \([24]\) and with the superfield calculation \([24]\) which uses manifestly supersymmetric gauge fixing.

The two-loop \( \rho_{IV} \) contributions to \( dv_i/dt \) have \( \mathcal{O}(h_q^2 g_{IV}) \) and \( \mathcal{O}(g^3 \rho_{IV}) \) terms. The latter is common for both \( i = 1 \) and \( 2 \), and cancels out in the ratio \( \tan \beta \) if Eq. (7) is satisfied. Therefore, only the former \( \mathcal{O}(h_q^2 g_{IV}) \) contributions are explicitly calculated. For example, the \( \mathcal{O}(h_q^2 g_2 \rho_{1Z}) \) contribution to \( v_1 \) comes from the diagram (a) in Fig. 2, while other diagrams (b,c) cancel each other. The RGEs for \( v_i \) are finally

\[
\left. \frac{dv_i}{dt} \right|_{2\text{loop}} = -\gamma_i^{(2)} v_i - \frac{N_c h_q^2}{(4\pi)^2} (g_z \rho_{1Z} + 2g_W \rho_{1W}) + P_i(g) \rho_{IV},
\]

where again \( h_q^2 = (h_q^2, h_q^2) \) for \( i = (1, 2) \), respectively. \( P_i(g) \) are possibly gauge-dependent \( \mathcal{O}(g^3) \) functions which are common for both \( \rho_{IV} \) and \( \rho_{2V} \). It is therefore seen that, due to the \( \rho_{IV} \) contributions in Eq. (14), the running \( \tan \beta \) has the \( \mathcal{O}(h_q^2 g_2, h_q^2 g_Y) \) gauge parameter dependence. Although existing higher-order calculations of the corrections to the MSSM Higgs sector \([26, 27, 28, 29]\) have not included the contributions of these orders yet, the gauge dependence of \( \tan \beta \) may cause theoretical problem in future studies of the higher-order corrections in the MSSM.

One way to restore the gauge independence of renormalized running \( \tan \beta \) is to introduce gauge-dependent shifts of \( \phi_i^0 \) such as to cancel the \( \rho_{IV} \) contributions to the effective action. This modification corresponds to the addition of extra shifts of \( v_i \) to all diagrams. The running \( v_i \) in this new definition then obey the same RGEs as those for \( H_i \), namely Eq. (8). The modified renormalized \( \tan \beta \) becomes gauge independent to the two-loop order. However, an extra two-loop shift \( \delta(v_2/v_1) \) has to be added to any quantities which depend on \( \tan \beta \).

Before leaving, I briefly comment on two related issues in the process-independent on-shell renormalization of \( v_i \) and \( \tan \beta \) which is used in Refs. [3, 4]. First, they cancel the one-loop \( \rho_{IV} \) contributions by extra counterterms for \( v_i \), \( \delta v_i \), and determine their finite parts by imposing the condition \( \delta v_1/v_1 = \delta v_2/v_2 \). It is clear from Eq. (14) that this condition has to be modified beyond the one-loop. Second, the gauge dependence already appears in the one-loop finite part of the on-shell counterterm \( \delta(\tan \beta) \). This is similar
to the gauge dependence of the on-shell renormalized mixing matrices for other particles \[10\].

In conclusion, I discussed the UV renormalization of the ratio \(\tan \beta = v_2/v_1\) of the Higgs VEVs in the MSSM, to the two-loop order and in general \(R_\xi\) gauges. When renormalized \(v_i\) are given by the minimum of the loop-corrected effective potential, the contributions of \(\rho_{iV}\) in the \(R_\xi\) gauge fixing term cause two-loop gauge dependence of the RGE for \(\tan \beta\). To avoid this gauge dependence, the contributions of \(\rho_{iV}\) have to be cancelled by extra shift of the Higgs boson fields \(\phi_i^0\).

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Figure 1: One-loop divergent contribution of $\rho_{1Z}$ to $m_\ell$. There is another diagram obtained from this one by the interchange ($b_L \leftrightarrow b'_L$).
Figure 2: Two-loop divergent $\mathcal{O}(h_q^2 g \rho_{1Z})$ contributions to $m_b$. The diagrams obtained from (a–c) by the interchange ($b_L \leftrightarrow b'_L$) also contribute. The blobs in diagrams (b, c) denote the $\mathcal{O}(h_q^2)$ quark-Higgs and squark-higgsino subloops. One-loop subdivergences are subtracted in the calculation. The divergences of (b) and (c) completely cancel out.