Motion Planning of Multi-Robots Object Transport with Deformable Sheet

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Abstract—Using a deformable sheet to handle objects is convenient and found in many practical applications. Modeling of deformable sheet-object interactions and robot formation are among the main challenges in multi-robot object manipulation through deformable sheets. Motion planning of robotic team is critical for object manipulation quality. We present a computational robots-sheet-object model and the dual-capability robotic motion planner for optimal trajectory. A computational virtual variable cables model (VVCN) is presented to simplify the modeling of the deformable sheet with an arbitrary number of used robots. The motion planner is first formulated as a formation optimization problem. A path generation algorithm is then employed for obstacle avoidance and crossing features. The motion planner successfully produces an obstacle crossing path in complex scenarios where other benchmark planners fail. The motion planner successfully produces an obstacle crossing path and demonstrates a robotic team of three robots. We further extend the results to a larger number of robots to illustrate the algorithms and analysis.

Index Terms—Multi-robot manipulation, Path planning for multiple mobile robots.

I. INTRODUCTION

Deformable sheets are commonly used as flexible carriers in many object manipulation applications, such as transferring patients with bed sheets in hospital [1]. Due to sheet deformability, multiple supporters (e.g., mobile robots) are often required to carry the object through the sheet. For the robot-sheet object transporting system, the deformability brings the advantage to use robot formation to change their relative positions and thus manipulate the object. However, this attractive feature also results in complex design for motion planning and control because of uncertainties of object motion and existence of multiple local equilibrium of the object on deformed sheet that is held by a robotic team. The goal of this paper is to propose a motion planner for the robotic team of the robots-sheet object transporting system in cluttered environment. Using a mobile robot team to transport or manipulate a rigid object has been reported in the past decades. By different contact methods, multi-robot manipulation can be classified into three types: pushing, grasping, and caging [2]. In these systems, robots are in direct contact with the transported or manipulated objects. The work in [3] proposed a distributed multi-robot control method to transport a rigid object and the results were extended with an adaptive method without the knowledge of payload parameters [4]. These direct robot-object contact configuration results in a fixed formation shape and it is difficult for the robot team to adjust its formation for obstacle avoidance in cluttered environment, such as narrow passes. Gong et al. [5] proposed to use a parallel mechanism on multi-mobile crawlers to achieve smooth transportation on uneven ground. The robots can even climb stairs through crawler chassis. For multi-mobile manipulators, Ren et al. [6] presented a fully distributed control scheme for a team of networked mobile manipulators to cooperatively transport an unknown object. However, these approaches using multi-mobile manipulators are expensive and the kinematically redundant manipulators also bring additional planning and control design challenges.

Multi-robot object transport with flexible sheets is a more challenging task than that under direct robot-contact due to the complex interactions of deformable sheet with the object [7]. Physical and geometric models are among the main modeling methods for deformable objects [8]. For example, the mass-spring model [9] and finite element method [10] of cloth in computer graphics are among the physical models, both of which involve complex meshing and computation. In the field of robotic manipulation, geometric models are often the common choice. In [11], [12], the geometric constraints of the deformable sheet are used for motion planning without the complete deformable model. Assuming a completely flexible sheet, the work in [13], [14] established the geometric model of the sheet-object kinematic relationship with a three-robot team. Artificial potential field is the most widely used method for obstacle avoidance for multi-robot formation [15]. Sampling-based planning algorithms have been demonstrated efficiency for searching high-dimensional C-space [16]. The main disadvantage of these methods is the difficulty to maintain the shape of the robot formation during planning. Alonso et al. [17] computed the locally optimal robotic formation through a formulated constrained nonlinear optimization problem for multi-mobile manipulator object transport. The robot team chose the optimal formation from several preset geometric shapes. For obstacle avoidance, the robot team was treated as a whole and only moved bypass the obstacles. Similar development was also reported in [18] for direct contact multi-robot object transportation. Unlike object transport with direct contact, the use of deformable sheet to handle objects by a multi-robot team allows flexible adjustment of relative motion among robots and therefore, the height of the object wrapped by the sheet can vary. This new feature enables to allow the transported object to possibly pass across any limited-height obstacles by carefully designing a robotic formation control.
Inspired by the above observations and discussions, we present a motion planner for deformable sheet-based object transportation by a multi-robot team to pass across obstacles with various heights. We first propose a computational algorithm for robots-sheet-object system to obtain the object position in a sheet that is held by arbitrary numbers of mobile robots. The algorithm is built on a virtual variable cables model (VVCM) in which the object is considered to be suspended by multiple virtual cables, similar to drones-cables-payload system [19]. Different from cable-driven parallel robots (e.g., [20]), the length of each virtual cable is related to the object-sheet contact point and the robotic holding point of the sheet. Using the VVCM, the proposed motion planner allows the robotic team to have the dual capabilities of obstacle avoidance and crossing, which are effective and efficient in cluttered environments (e.g., a narrow corridor with obstacles). We demonstrate the VVCM and motion planning algorithms through extensive experiments.

The main contributions of this paper are twofold. First, the proposed VVCM for deformable sheet-object interactions is new and provides an efficient method to rapidly capture the equilibrium point of the object in the sheet held by arbitrary number of robots. Second, the motion planning algorithm extends the existing planners for multi-robot object transportation with the new capability of obstacle crossing, which not only expands the workspace of the robot team, but also improves the efficiency.

The remainder of this paper is organized as follows. We first present the problem statement in Section II. Section III presents the computational model for object manipulation with deformable sheet. Section IV discusses the motion planner with obstacle crossing capability. Section V presents the experimental results and finally, Section VI summarizes the concluding remarks.

II. PROBLEM STATEMENT

We consider the multi-robots object manipulation through a deformable sheet as a quasi-three-dimensional problem, that is, the trajectory of the transported object, denoted as \( O \), is in three-dimensional (3D) space and the mobile robots only move on the planar terrain. Fig. 1(a) illustrates the system configuration for a three-robot case. A deformable sheet is held by an \( N \)-mobile robot team, \( N \in \mathbb{N} \). The coordinate frame of the deformable sheet is denoted as \( S \subset \mathbb{R}^2 \) and its vertices are denoted as \( v_i = [x_{vi}, y_{vi}]^T \in \mathcal{S} \), \( i = 1, \ldots, N \). The contact point between object \( O \) and the sheet is denoted as \( v_o = [x_{vo}, y_{vo}]^T \in \mathcal{S} \), as shown in Fig. 1(b). A world coordinate frame \( W \subset \mathbb{R}^3 \) is also used and the sheet is held by each robot at contact point \( p_i = [r_i^T, z_r]^T \), \( r_i = [x_i, y_i]^T \), \( i = 1, \ldots, N \), where \( z_r \) is a constant height for all robots. The coordinate of object \( O \) in \( W \) as \( p_o = [x_o, y_o, z_o]^T \); see Fig. 1(c). We denote the length of the \( i \)th ridge as the straight-line between \( p_i \) and \( p_o \). The planar position of the \( i \)th robot is captured by \( r_i \). The collection of all robots’ position is denoted as \( \mathcal{R}_N = \{ r_1, \ldots, r_N \} \).

The object motion of the robot-sheet system is closely related to the number of robots, the material and shape of the sheet and the object. To precisely describe the motion planning problem, we consider the following assumptions. First, The object \( O \) is assumed to be a mass point. The object transport and motion is quasi-static, that is, the robot team moves slowly and smoothly and their dynamic effects are negligible. The deformable sheet is soft and inelastic. Therefore, the object moves freely on the sheet under gravity. Under such condition with \( N \) robots, the sheet holding points form a convex polygon with \( N \) vertices and the object \( O \) is located inside the polygon; see Fig. 1. It is clear that in non-constrained conditions, the object \( O \) tends to move and stay at the position of minimum potential energy.

**Problem Statement:** The motion planning problem is to design the trajectory of robot team \( \mathcal{R}_N \) such that object trajectory follows a given profile in 3D configuration space with obstacles.

III. MULTI-ROBOT MANIPULATION MODELS

In this section, we present a computational approach to obtain the object position under the motion of the robotic team \( \mathcal{R}_N \). The goal of the multi-robot manipulation model is to compute object positions \( \{ p_o, v_o \} \) for given \( \mathcal{R}_N \). The basic idea is to consider the analogy between the proposed problem and the multi-cable model as described in the next section.

A. Virtual Variable Cables Model (VVCM)

Based on the previous assumptions, the sheet deformation is distance-preserving. As the robot formation changes, the object \( O \) might move on the sheet due to gravity. Thus, the
contact point $v_o$ might also change. Since the distance $l_i$ between $v_o$ and $v_i$ varies, the connection between them can be viewed as through a virtual cable with variable length; see Figs. 1(b) and 1(c). Therefore, the object can be regarded as being transported by $N$ variable-length cables that are fixed on the robot. It is clear that the distance between object and vertex of the sheet is less than or equal to the corresponding virtual cable, namely,

$$l_i = \| v_o - v_i \| \geq \| p_o - p_i \|, \quad i = 1, \ldots, N. \quad (1)$$

The position of object $O$ is directly related to the shape of the robot formation, which might change during the process of transportation. To quantify the formation properties, we first introduce a few geometric variables that will be used later for obstacle avoidance design in motion planning. Fig. 2 illustrates the multi-robot formation configuration for the geometric variables. For any configuration, the robot configuration indicators include formation sizes and obstacle crossing height.

![Fig. 2. A space decomposition for multi-robot system and the indicators. (1) Obstacle-free convex region of the object. (2) Multi-robot formation region. (3) Maximum traversable obstacle region.](image)

The object is considered to be transported by the robotic team in convex space with width $W_{\text{convex}}$ as shown in the left plot of Fig. 2. There is an obstacle that the object might pass cross and the distance between $O$ and the obstacle is $z_{\text{safe}}$, which is the safety distance. The highest traversable obstacle height is then obtained as

$$z_{\text{obsmax}} = z_o - z_{\text{safe}} \quad (2)$$

We also define the robotic formation size, denoted as $W$, as

$$W = D + 2\Delta r \quad (3)$$

where $D$ is the diameter of the minimum circumscribed circle of the robotic team and $\Delta r$ is the safety distance factor at each side of the robotic formation; see Fig 2. For presentation convenience, we define the shortest base of the formation, denoted as $L_{\min}$, and the largest traversable obstacle diameter $d_{\text{obsmax}}$ as

$$L_{\min} = \min_{i,j \in I_N, i \neq j} \| r_i - r_j \|, \quad d_{\text{obsmax}} = L_{\min} - 2\Delta r \quad (4)$$

where index set $I_N = \{1, \ldots, N\}$. From (2)-(4), variables $W$, $d_{\text{obsmax}}$, and $z_{\text{obsmax}}$ are the functions of formation shape that is determined by $R_N$ and we will use these variables to design motion planner in Section IV.

The multi-cable structure is an under-constrained system and positions $(v_o, p_o)$ are jointly determined by the straightened cables. Fig. 3 illustrates the various cable configurations with robot formation $R_N$. The object’s position is determined by the status of the slackness/tautness of virtual cables. We therefore define the status of the $i$th virtual cable as $I_i = 0$ if the cable is slack; otherwise, $I_i = 1$ when it is taut, $i \in I_N$. Furthermore, defining taut cable index set $I_t = \{i: I_i = 1, i \in I_N\}$, we denote $m = |I_t|$ as the cardinality of $I_t$. When the $i$th cable is taut ($I_i = 1$), it satisfies

$$l_i = \| v_o - v_i \| = \| p_o - p_i \| \quad (5)$$

The formation can be transformed into the local coordinate system to solve the algebraic solution of the direct kinematics of the system, as shown in Fig. 1(d). In the local coordinate system, with $r_1$ as the origin and $\overrightarrow{r_1r_2}$ as the x-axis, the robots are arranged in counterclockwise order, denoted as $\tilde{r}_i = [\tilde{x}_i, \tilde{y}_i]^T, i \in I_N$, and the projected coordinate of the transported object is $\tilde{r}_o = [\tilde{x}_o, \tilde{y}_o]^T$. The direct kinematics can be simplified as known $\tilde{r}_i(i \in I_N, \tilde{x}_1 = \tilde{y}_1 = \tilde{y}_2 = 0)$ to find $(\tilde{x}_v, y_v, \tilde{x}_o, \tilde{y}_o, z_o)$, which consists of five independent variables.

For all possibility of $N$-robot team, three configurations exist under which object $O$ reach the static equilibrium condition. The sub-formation formed by the projection of $p_i (i \in I_t)$ must be convex polygons. Fig. 3 illustrates these three situation. Even if there is a concave vertex in the sub-formation, the corresponding cable will be slack, resulting in a change in the object’s equilibrium state.

We first consider the case when five cables are taut, i.e., $m = 5$, as shown in Fig. 3(c). In this case, all five cables satisfy (5). Through these five independent equations, the required five independent variables can be obtained. When more than five cables are taut in a larger robot team, $m > 5$, the object is constrained by full redundancy and therefore, we can obtain $(v_o, p_o)$ by solving the case of $m = 5$.

For the remaining two cases, some cables are slack and the object manipulation is under-constrain. Fig. 3(a) shows the case when two cables are slack and the configuration is then considered as a three-robot team, $m = 3$, similar to the case...
as shown in Fig. 1. In this case, we obtain

\[\hat{x}_o = \frac{x_{2b} x_{vo} + \hat{x}_2^2 - x_{vo}^2}{2x_2},\]

\[\hat{y}_o = \left(\frac{x_3 - \hat{x}_3 x_{vo}}{y_3 - \hat{x}_3 x_2}x_{vo} + \frac{y_3 y_{vo} - \hat{x}_3^2 x_2 - x_{vo}^2}{3 x_2}\right),\]

and \(J_z(v_o) = -(z_o - z)^2 = \hat{x}_2^2 + \hat{y}_2^2 - x_2^2 - y_2^2\). Two additional equations are needed to determine \(v_o\). Thus, a convex optimization problem is then formulated to minimize \(J_z(v_o)\) when the object maintains the position of minimum gravitational potential energy, namely,

\[v_o^* = \arg \min_{v_o} J_z(v_o), \quad v_o \in \Delta v_1 v_2 v_3 \subset S \quad (7)\]

where \(\Delta v_1 v_2 v_3\) is the triangle formed by vertices \(v_1, v_2\), and \(v_3\); see Fig. 1b. The convex polygon formation assumption of the robot team allows us to solve (7) by convex optimization method. From the first-order necessary condition of (7), we obtain the gradient \(\nabla J_z(v_o) = 0\) and it reduces to

\[\begin{align*}
&\frac{a_1 x_{vo} + a_2 y_{vo}}{a_3 x_{vo} + a_4 y_{vo} + a_5} = b_1, \\
&\frac{a_6 x_{vo} + a_7 y_{vo}}{a_8 x_{vo} + a_9 y_{vo} + a_{10}} = b_2,
\end{align*}\]

where

\[\begin{align*}
a_1 &= \frac{x_2^2 - y_2^2}{x_2}, \\
a_2 &= \frac{x_2(x_2 y_2 - y_2 x_2)}{y_2}, \\
a_3 &= \frac{x_2^2 - y_2^2}{2}, \\
a_4 &= \frac{x_3^2 - y_3^2}{2}, \\
a_5 &= \frac{x_3^2 - y_3^2}{2}, \\
a_6 &= \frac{x_3 x_2 - x_2 x_3}{y_3}, \\
a_7 &= \frac{x_3 y_2 - y_3 x_2}{y_3}, \\
a_8 &= \frac{x_3^2 - y_3^2}{2}, \\
a_9 &= \frac{x_3^2 - y_3^2}{2}, \\
a_{10} &= \frac{x_3^2 - y_3^2}{2}.
\end{align*}\]

Thus, \(v_o^* = [x_{vo}^*, y_{vo}^*]^T\) is obtained by solving (8) and \(\tilde{r}_o, z_o^*\) are obtained by (9). Then, \(r_o^*\) is obtained by coordinate transformation from \(\tilde{r}_o\), and finally, \(\hat{p}_o^*\) is obtained. It is straightforward to verify that the Hessian matrix of \(J_z\) is positive definite and therefore, the obtained optimal resolution is the global minimum.

For the last case, \(m = 4\) and Fig. 3b) shows the configuration with four taut cables. The approach is similar to that of \(m = 3\) and we omit the details. Through VVCM, we transform robots-sheet-object system into robot-cable-object problem and three static equilibrium conditions are obtained. In the simplest case of three-robot configuration, the direct kinematics solution is obtained from [8]. More than three robots actively hold the object, the system has multiple local static equilibrium and results in multiple solutions. Algorithm 1 summarizes the above discussion and results to compute the object position under the \(N\)-robot team.

### IV. Motion Planner with Obstacle Crossing Capability

In this section, we present a motion planner that can both bypass and cross obstacles. The planner first prioritizes the option of crossing obstacles than bypassing obstacles. The planner to bypass obstacles is taken from literature such as that in [17].

#### A. Optimal Formation Generation

As discussed in Section III-B, the robot team has multiple possible transporting formations. We formulate a constrained nonlinear optimization to compute a optimal formation \(\mathcal{R}_N^*\). If no feasible formation is found to cross the obstacle, the planner

**Algorithm 1: Deformable sheet object computation**

**Input**: \(\mathcal{R}_N, z_o\), and \(\{I_i\}_1^N\)

**Output**: \((p_o, v_o)\)

if FormationFeasible then

\[m = 0, l_i = 0\]

for \(i = 1\) to \(N\) do

\[\text{if } I(i) = 1 \text{ then } m \leftarrow m + 1, l_i \leftarrow \{l_i, i\}\]

if \(m \geq 5\) then

\[(p_o, v_o) \leftarrow \text{VVCM-Pentagon}(\mathcal{I}_t, \mathcal{R}_N)\]

else if \(m = 4\) then

\[(p_o, v_o) \leftarrow \text{VVCM-Quadrilateral}(\mathcal{I}_t, \mathcal{R}_N)\]

else if \(m = 3\) then

\[(p_o, v_o) \leftarrow \text{VVCM-Triangle}(\mathcal{I}_t, \mathcal{R}_N)\]

return \(p_o, v_o\)

else

return **FALSE**
then finds paths that can bypass obstacles as demonstrated in [17]. Fig. 4 illustrates the overall design of the planner.

The optimization problem considers to use formation variables discussed in Section III-A to construct cost $J_f$ as

$$J_f(R_N) = J_{\text{trans}} + J_{\text{pass}} + J_{\text{cross}}$$  \hspace{1cm} (10)

where $J_{\text{trans}}$, $J_{\text{pass}}$, and $J_{\text{cross}}$ are costs to penalize transportation, passable area and obstacle crossing, respectively. $J_{\text{trans}}$ ensures the safety and stability of system during transportation. In order to ensure that each robot participates in the handling of the object, each virtual cable is taut and $J_{\text{trans}}$ is given by

$$J_{\text{trans}} = \lambda_1 \|v_o - v_0\|^2 + \lambda_2 \sum_{i \neq j} \left(\|r_i - r_j\| - \|r_i^0 - r_j^0\|^2\right)$$  \hspace{1cm} (11)

where $v_0$ is the current contact point and $r_i^0$ is the current robot position, and $\lambda_1$ and $\lambda_2$ are the weighting factors for changes in virtual cable length and robot relative position, respectively. Cost $J_{\text{pass}}$ keeps the robot team in a safe range and ensures that the formation can rotate freely during obstacle avoidance. Therefore, $J_{\text{pass}}$ is designed as

$$J_{\text{pass}} = -\lambda_3 (W_{\text{convex}} - W)^2$$  \hspace{1cm} (12)

where $\lambda_3$ penalizes the outline size of the robot team. Cost $J_{\text{cross}}$ ensures that the formation can cross the obstacle in any direction and keeps the object at a safe height and therefore, is given as

$$J_{\text{cross}} = -\lambda_4 (z_{\text{obsmax}} - z_{\text{safe}})^2 - \lambda_5 (d_{\text{obsmax}} - d_{\text{safe}})^2$$  \hspace{1cm} (13)

where $z_{\text{safe}}$ and $\Delta r$ are constant for safe maneuvers, weights $\lambda_4$ and $\lambda_5$ penalize the height of object crossing the obstacle and formation size, respectively.

From the above discussion, it is clear that $J_{\text{trans}}$ hinders the deformation of the team, $J_{\text{pass}}$ limits the size of the formation, while $J_{\text{cross}}$ is the key item to ensure the obstacle crossing. With these cost functions, the optimization problem is formulated as

$$\min_{R_N} J_f(R_N)$$ \hspace{1cm} (14a)

Subj. to $\|p_o - p_i\| = \|v_o - v_i\|$, $i \in I_N$, \hspace{1cm} (14b)

$W \leq W_{\text{convex}}$, $z_{\text{obsmax}} \geq z_{\text{obs}}$, $d_{\text{obsmax}} \geq d_{\text{obs}}$. \hspace{1cm} (14c)

Constraint (14c) enforces that $W$ should be smaller than the obstacle-free convex region width $W_{\text{convex}}$ and also the obstacles should smaller than the maximum traversable obstacle region of the formation. These constraints ensure the feasibility and reachability of the formation.

B. Local Motion Planning

After robots reach optimal formation $R_N$ in Section IV-A we consider that the robots pass through one obstacle at a time. We keep the transporting formation unchanged and utilize the inner space of the formation to cross obstacles. While the robots cross an obstacle, we also can think that the obstacle passes the robot team. In multi-robot formation region, trajectory $\tau_o$ is viewed as a “virtual” path of the obstacle moving through the formation which can be obtained by a variety of path search methods (e.g., A* algorithm). The reverse path of $\tau_o$, denoted by $\tau_1$ from robot viewpoint is then regarded as the actual path of the robot team. If the environment is cluttered such as corridors, planned $\tau_1$ could collide with the boundary by directly avoiding obstacle. Thus, it is necessary to design the rotation of the formation to find a formation trajectory $\tau_2$ that goes straight across the obstacle.

![Fig. 5. Local motion planning for formation crossing the obstacle. (a) The entering and exiting sides of the formation when crossing the obstacle. (b) The four steps of movement after the robot formation enters the obstacle influence region: rotation, translation, rotation, translation.](image-url)
the rotation angle is $\theta_1$, the angle of vector $\vec{n}_{in}$ with the $x$-axis. The formation then translates to cross the obstacle from time $T_1$ to $T_2$. In the third step, $t \in (T_2, T_3]$, the formation rotates and vector $\vec{n}_{out}$ aligns with the $x$-axis, that is, the rotation angle is $\theta_2$, where $\theta_2$ is the relative angle between $\vec{n}_{in}$ and $\vec{n}_{out}$; see Fig. 5(a). Finally, the formation travels to pass the obstacle at time $T_3$. During the entire process, the robot formation translational speed $v$ is assumed to be constant, and the time durations $T_i$, $i = 1, \ldots, 4$, are determined by the moving distance.

We denote the robot formation centroid path as $\pi(t) = (x_{r_2}(t), y_{r_2}(t), \theta_{r_2}(t))$. From the above discussion, we have $y_{r_2}(t) = 0$ and a path generation is then designed as

$$(x_{r_2}(t), \theta_{r_2}(t)) = \begin{cases} (0, \frac{d_t}{T_1}) & t \in (0, T_1], \\ (v t, \theta_1) & t \in (T_1, T_2], \\ (v \delta_T, \frac{\theta_2 t - T_2}{T_2} + \theta_1) & t \in (T_2, T_3], \\ (v (t + \delta_T - T_3), \theta_1 + \theta_2) & t \in (T_3, T_4], \end{cases}$$

where $\delta_T = T_2 - T_1$, $\theta_1$ and $\theta_2$ are determined according to different entering and exiting side, which are positive in the counterclockwise direction.

Algorithm 2 summarizes the path generation design for the robot team, which can scale to any number of robots. Compared with the existing obstacle bypassing paths, the path generated by Algorithm 2 is to let the transported object cross over the obstacle in a straight line.

Algorithm 2: Local Path Generation

Input : $R_N^\star$, $d_{obs}$, $W_{\text{convex}}$
Output: $\pi(t)$ and $r_i(t)$, $(i = 1, \ldots, N)$.

$G \leftarrow \text{ObstacleInfluenceRegion}(R_N^\star, d_{obs}, W_{\text{convex}})$
$\tau_o \leftarrow \text{ObstaclePathSearch}(G, R_N^\star, d_{obs}, W_{\text{convex}})$
$\tau_1 \leftarrow \text{ReversePathConversion}(\tau_o)$
if IsFeasible($\tau_1$) then
    $\pi(t) \leftarrow \tau_1$, $r_i(t) \leftarrow \text{FormationToRobots}(\pi(t))$
    return $r_i(t)$.
else
    $\tau_2 \leftarrow \text{ObstacleCrossingPath}(G, R_N^\star, d_{obs}, W_{\text{convex}})$
    $\pi(t) \leftarrow \tau_2$, $r_i(t) \leftarrow \text{FormationToRobots}(\pi(t))$
    return $r_i(t)$.

V. EXPERIMENTAL RESULTS

In this section, we demonstrate the planning algorithms through experiments. We first show that the validation of the VVCM algorithm. By the VVCM, three robots forms the smallest object transport team and additional robots can be generalized on the three-robot configuration. Thus, a three-robot team is used in the following experiments.

A. Experimental Setup

Fig. 6 shows the experimental setup and communication network of the system. Three robots (Turtlebot3 Waffle-Pi) were used in experiment. The transported object was a billiard ball. The position of robots and object were acquired at a rate of 60 Hz by the NOKOV motion capture system with 8 cameras. The deformable sheets are fabric cloth and plastic cloth and the latter was used primarily for the camera system to capture the position of the robot in real time. Each side length of the non-deformed sheet was 1.6 m, and the height of each vertex was 0.79 m. The experimental environment was a corridor (2 meters wide, 6 meters long) with two different-height obstacles in the middle. The first obstacle (Obstacle #1) was a circular shape with a radius of 0.1 m and a height of 5 cm, and the other (Obstacle #2) had a radius of 0.2 m and a height of 0.2 m. The robot controller was implemented in robot operating system (ROS) with embedded system (NVIDIA JETSON NANO and OpenCR controller). A remote laptop (Intel i7-4710HQ CPU with four cores and 8 GB RAM) was set up for state monitoring and robot controlling.

B. Experimental Results

We first show the effectiveness of the VVCM and Algorithm 1. According to VVCM, three robots are the smallest configuration. The accuracy of the object height is the key to verifying the validity of the model. The first experiment was conducted with the equilateral triangle formation of three robots as an example to verify the model. The actual heights of the object under different equilateral triangle formations were measured and compared with the theoretical results by VVCM. Fig. 7(a) shows the experimental results for the VVCM. The main cause of the error may be the elasticity of the cloth. For obstacles crossing, the influence of the model errors can be avoided by increasing the safety factor. The result shows that the $z_o$ (VVCM) is consistent with $z_o$ (real) in this case. Fig. 7(b)-(d) show the calculation examples of direct kinematics under different equilibrium states. These results
confirm that the position of the object can be obtained for any feasible formation.

A narrow corridor environment with obstacles was used in experiments and this case, the normal planner such as that in [17] cannot obtain the feasible trajectory successfully. We implemented the proposed motion planner and Fig. 8 shows the snapshots of the experiments and actual plots of the robot motion are illustrated in Fig. 9(a). The object went straight over the two obstacles. For crossing Obstacle #1, the average side lengths of the formation were 1.04, 1.04 and 1.05 m. For crossing Obstacle #2, the average side lengths of the formation were 1.30, 1.29 and 1.24 m (base side). Fig. 9(b) shows the actual height of the object and the height calculated by VVCM during the transportation process. The RMSE of two heights during the whole transportation process is 5.3 mm. The result further shows that the $z_o$ (VVCM) is consistent with $z_o$ (real) in real transporting tasks. The average height of the object in the process of crossing Obstacles #1 and #2 is 9.0 cm and 23.4 cm, respectively, which were both safe. Fig. 9(c) shows the angles of the normal vectors at the entering and exiting sides. The entering angles for the robot formation to cross two obstacles were both 0°, which is consistent with the local path generation method proposed in Section III-B. Fig. 9(d) shows the relative distances among the robots and it is clear that the relative distance changes corresponded to the object height changes as shown in Fig. 9(b). The main source of the fluctuation in these figures was from the robot speed variations due to the nonholonomic nature of the used robots under the formation rotation.

We further ran a simulation study of four robots crossing obstacles in different scenario to verify the scalability of the motion planner. Fig. 10 shows the results for the robot
team to pass a turned corridor carrying the object, in which different exiting edges were selected and when there is no need to rotate, the formation can pass directly. The results clearly demonstrated that the formation successfully followed the centerline of the trajectory while rotating was used to cross these obstacles. Indeed, the planner can be widely used to various numbers of robots and the algorithm is scalable. To apply this planner to complex environments, the robots should be equipped with diverse sensors to detect obstacles and build the map in real time. Distributed sensing and real-time control algorithms are required in these applications.

VI. CONCLUSION

The virtual variable cables model was first proposed to describe the object position of robots-sheet transportation system. Based on the VVCM, a motion planner with obstacle crossing capability was then proposed for object transportation by multi-mobile robots with a deformable sheet. The implementation of the planner was also presented to validate the model and planning algorithms. Compared with existing multi-robot object transport approaches, the proposed planner gave priority to crossing obstacles instead of bypassing obstacles. This property improved the efficiency to traverse through cluttered environments (e.g., narrow corridors with obstacles) in which the traditional methods cannot achieve. Future research directions include to generalize the design with online sensor-based detection and mapping function in an unknown environment.

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