Novel Method for Designing Waveform With Good Correlation Properties Based on High-Order Norm Unified Representation

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ABSTRACT This study exploits high-order p-norm to uniformly represent the peak sidelobe level (PSL) and integrated sidelobe level (ISL) commonly used in waveform optimization, and presents a unified optimization objective function by considering the requirement that waveforms have good correlation properties in radar and communication systems. A novel and efficient waveform optimization algorithm is then proposed, which tackles the waveform and waveform set optimization of the PSL, the weighted sidelobe level, the ISL, and the weighted ISL. Applying the unified optimization objective function, the iteration factors are dynamically adjusted based on the quasi-Newton algorithm; in addition, the objective function is optimized with the gradient descent search algorithm and includes the optimization of autocorrelation and cross-correlation. Because certain application scenarios involve the Doppler sensitivity of waveforms, the problem is converted to cross-correlation optimization via channel division of Doppler frequency, and waveform with anti-Doppler sensitivity can be obtained. Multiple numerical simulations illustrate the positive performance of the proposed method. Furthermore, compared with certain traditional algorithms, the waveforms designed by this algorithm demonstrate improved correlation properties.

INDEX TERMS Autocorrelation, cross-correlation, waveform design, peak sidelobe level, integrated sidelobe level, high-order norm, Doppler sensitivity.

I. INTRODUCTION

The problem of obtaining a waveform design with good correlation properties has attracted considerable interest over the past several decades in radar systems, wireless communications, sonar research, and cryptography. In a code division multiple access (CDMA) communication system, it is necessary to design a waveform set with good correlation properties to meet the requirements of multiple users because it reduces the interference between signals and the complexity of the receiver [1], [2], [3], [4]. Often, the system requires a waveform set composed of multiple waveforms rather than a single waveform; furthermore, the design of the waveform set must simultaneously consider the properties of autocorrelation and cross-correlation. For radar signal processing, waveform design is important for performing the target detection. In the face of different working environments and detection scenarios, a waveform with good correlation properties signifies good parameter estimation and anti-jamming ability. In recent years, multiple-input multiple-output (MIMO) radar has become a research hotspot in radar signal processing, and waveform design is one of its important contents [5], [6], [7], [8], [9], [10], [11], [12]. The key goal of MIMO radar waveform design is to suppress the autocorrelation sidelobes and cross-correlation levels between waveforms, so as to avoid false alarms triggered by sidelobes, submerge weak targets in nearby distance units in detection, or enable the receive antenna to distinguish echo signals of different waveforms. Currently, multiple waveform design methods exist that have demonstrated good results for suppressing the range sidelobes and improving the cross-correlation properties between waveforms. The pseudo-random, Frank, P3 code, P4 code,
and Golomb codes have good autocorrelation properties. However, the code length, waveform number, and phase degree of freedom of these waveforms are limited; they are not totally applicable to the requirements of the MIMO radar and CDMA multiuser systems [13], [14], [15], [16], [17].

To obtain waveforms and waveform sets with good autocorrelation and cross-correlation properties, extensive research has been conducted in the past few decades. In [18], the integrated sidelobe level (ISL) was adopted as an evaluation metric to measure waveform optimization. Using the Golomb sequence, the Frank sequence, or a random sequence as the original waveform and by minimizing the ISL or an equivalent objective function as the optimization objective, the authors proposed a cyclic algorithm–new (CAN). In order to design waveform with a constant modulus and good autocorrelation properties, the weighted CAN (WeCAN) based on the original waveform was proposed in [19]. The WeCAN algorithm can be utilized to design constant modulus MIMO waveforms with low autocorrelation and cross-correlation levels in a specified lag interval; this solves the problem of local minization with the constant modulus constraint in a specific lag interval. For the design of a waveform set, both the autocorrelation of a single waveform and the cross-correlation between waveforms should be considered. Therefore, the design of a waveform set with low correlation was studied in [20]. The multi-sequence CAN (Multi-CAN) and multi-sequence WeCAN algorithms were proposed to realize the optimal design of a waveform set and can be applied to radar imaging, channel estimation, medical ultrasound systems, and covert underwater communication.

In view of the problem of fast large-scale waveform optimization, ISL and weighted ISL minimization were expressed as non-convex quartic optimization in the frequency domain in [21]. Moreover, they were simplified as quadratic problems by optimization-minimization technology, thus solving the limitation of high computational complexity faced by waveform optimization for non-convex. The real-time optimization of large-scale waveforms was realized using lower computational burdens and faster convergence rates. In [22], a new algorithm with high computational efficiency and more flexibility was proposed based on the general framework of optimization–minimization. It designed low autocorrelation constant modulus waveforms by directly minimizing the ISL and accelerated the overall convergence by fast Fourier transform (FFT). In [23], the authors proposed an efficient minimization–maximization algorithm and put forward the following three objective functions in optimization: two for the unified measurements and one for the ISL measurement. This approach does not optimize the ISL directly, but converts it to a number of simple objective functions and replaces the original objective function with an approximation function in iteration, which is called monotonic minimizer for integrated sidelobe level (MISL). In [24], for designing waveforms with a constant modulus, the authors took the waveform with the lowest sidelobe or specified spectral shape as the evaluation metric and exploited the alternating direction multiplier method to deal with the augmented Lagrange scheme of the separate objective function. In a study of waveform design with good correlation properties in the peak sidelobe level (PSL) and the ISL, the authors in [25] presented an iterative method based on the coordinate descent method to deal with the optimization problems that were generally non-convex and non-deterministic polynomial (NP)-hard. For continuous phase and discrete phase constraints, the algorithm can be designed to meet the requirements of various applications using a new method of partition or FFT. The authors in [26] examined the joint design of transmitting waveforms and receiving filters in MIMO radar and considered the maximization of the signal-to-noise ratio (SNR) under the constraints of the ISL and the PSL of the pulse compression output as the optimization objective; this effectively improved the target detection performance in the presence of interference.

In order to address the problem that completely complementary waveforms are sensitive to Doppler frequency and have not been extensively used, a limited memory-Broyden–Fletcher–Goldfarb–Shanno (L-BFGS) iterative method was proposed in [27] to develop complementary waveforms with high Doppler tolerance that suppress the range of sidelobes with a certain Doppler frequency shift. For the MIMO radar waveform design, under the constraint of the peak-to-average ratio, the compromise problem between the SNR and the ISL was proposed in [28]; a non-decreasing algorithm for solving this problem was presented by combining the sequential optimization algorithm with the minimization–maximization method. The problem of robust orthogonal frequency division of multiplexing radar waveform design was examined in [29]; three different robust radar waveform design criteria based on power minimization were proposed that designed the waveform by optimizing the transmission power in the worst-case scenario. With similarity and norm constraints, the NP-hard quadratic optimization problem was studied in [30], and a computationally efficient continuous and discrete phase iterative algorithm was proposed. By introducing auxiliary variables, the problem was transformed into two sub-problems with closed solutions, avoiding the non-convergence problem of the alternating direction multiplier method in dealing with the NP-hard problem. Authors in [31] inspired the Doppler compensation processing, and a modified range Doppler processing method was proposed.

The abovementioned research analysis reveals that classical algorithms such as CAN and WeCAN do not directly solve the ISL or weighted ISL objective functions. Instead, these algorithms convert the optimization objective function to the frequency domain, replace it with an approximately equivalent objective function, and use FFT for a fast, efficient solution. Furthermore, when waveform optimization was performed in prior studies, either the ISL or the PSL was considered as the optimization objective according to the system requirements, and the optimization problems were solved by employing different algorithms. To design constant modulus waveform or waveform set with good correlation
properties, this study considers the autocorrelation sidelobes and the cross-correlation levels as the evaluation metric and exploits the high-order p-norm to uniformly and simultaneously represent the objective functions of the ISL and the PSL. On this basis, the iterative factors are dynamically adjusted based on the quasi-Newton algorithm [32], and the correlation properties of the waveform and the waveform set are optimized by the gradient descent search algorithm. Furthermore, by considering the weighted correlation level as the optimization objective, the local optimization of the specified lag interval and the adjustment of the autocorrelation and cross-correlation optimization proportion between the waveforms are realized. We also propose a waveform optimization method based on channel division to address the problem of anti-Doppler frequency. The aim of this method is to obtain waveforms with good Doppler tolerance. In conclusion, this study proposes a novel waveform optimization algorithm that takes the PSL or the ISL as the optimization objective to realize the optimization of a single waveform, a waveform set, a locally weighted waveform, and an anti-Doppler sensitivity waveform.

The main contributions of this paper are as follows:

(i) The common waveform evaluation metric peak sidelobe level and integrated sidelobe level are represented uniformly by high-order norm, and a new optimization objective function which can jointly characterize ISL and PSL is obtained.

(ii) On the basis of the optimization objective function, a novel optimization algorithm is proposed to accomplish waveform and waveform set optimization, as well as weighted waveform optimization.

(iii) The problem of Doppler sensitivity for the waveform is transformed into a cross-correlation problem by using the method of frequency channel division, and the waveform optimization of Doppler tolerance correction is realized.

(iv) The proposed method satisfies the constraint of constant modulus, and the good performance of the method is verified by a large number of numerical simulations.

The rest of this paper is organized as follows: In Section II, the signal model is described. The high-order p-norm based waveform optimization algorithm is developed in Section III. In part one of Section III, optimization of a waveform and a waveform set is introduced. Then, weighted waveform optimization is built in part two of Section III. In part three of Section III, waveform design with good Doppler tolerance is presented. The numerical simulations and discussions are provided in Section IV. Finally, the conclusions of this paper can be found in Section V.

II. SIGNAL MODEL AND PROBLEM DESCRIPTION

Assume that in MIMO radar or CDMA multi-user communication systems, the transmitter emits $M$ orthogonal waveforms with a length of $N$, and the whole waveform set constitutes a waveform matrix $S = [ s_1, s_2, \ldots, s_m, \ldots, s_M ]$ of size $MN$. Here, $s_m = [ a_{m0}(n), a_{m1}(n), \ldots, a_{mN}(n) ]$, $a_{m}(n)$ is the $m$-th waveform with a length of $N$, and $\theta_m(n)$ is the phase with uniformly distributed between 0 and $2\pi$. When $M = 1$, it is a single waveform, and the optimization objective only considers the autocorrelation without the cross-correlation between waveforms.

For a single waveform, the ISL is presented in [18] to describe the correlation properties, as follows:

$$ ISL = 2 \sum_{k=1}^{N-1} |r_a(k)|^2 $$

(1)

where $r_a(k) = \sum_{n=0}^{N-1} s_m(n)s_m^*(n-k), k \neq 0$. Therefore, the optimization of a single waveform can be considered as solving the minimization problem of the sum of the power of all the autocorrelation sidelobes. For a waveform set composed of $M$ waveforms, the metric factor is defined by the following equation:

$$ \varepsilon = \sum_{m=1}^{M} \sum_{n=0}^{N-1} |r_a(k)|^2 + \sum_{m_1=1}^{M} \sum_{m_2=1}^{M} \sum_{n=0}^{N-1} |r_c(k)|^2 $$

(2)

where $r_c(k) = \sum_{n=0}^{N-1} s_m(n)s_{m_j}(n-k)$ and $m$, $m_j \in [1, 2, \ldots, M]$. The first part of $\varepsilon$ represents the sum of the power of the autocorrelation sidelobes of the $M$ waveforms, and the second part is the sum of the power represented by the cross-correlation levels of the $M$ waveforms. On this basis, the optimization of a waveform set can be understood as solving the minimization problem of the sum of the power of the waveform sidelobes and the cross-correlation levels. In [18], the optimization objective function is expressed equivalently and solved by the CAN algorithm. For a waveform set, the corresponding Multi-CAN algorithm can be obtained in [20].

Similar to the ISL, the PSL is commonly adopted as an evaluation metric for waveform optimization. For a single waveform, it denotes the maximum value of the autocorrelation sidelobes; for a waveform set, it represents the maximum value of the waveform autocorrelation sidelobes and the cross-correlation levels. The PSL can be described as follows:

$$ PSL = \max_{k \neq 0} |r(k)| $$

(3)

For a single waveform, $r(k) = r_a(k)$. For a waveform set, $r(k)$ is a vector composed of the autocorrelation sidelobe $r_a(k)$ and the cross-correlation level $r_c(k)$. Consequently, the waveform optimization with the PSL as an evaluation factor can be considered as the problem of minimizing the peak level.

III. HIGH-ORDER NORM OPTIMIZATION METHOD

The waveform optimization algorithms reviewed in the literature [18], [19], [20] do not directly solve the ISL, the weighted ISL, or other objective functions. Instead, they transform the optimization objective function into a frequency domain and replace it with an approximately equivalent objective function to solve. To achieve the solution of the non-convex quartic function, the algorithms have the characteristics of high efficiency and can handle long waveform optimization. For
waveform optimization, the ISL and PSL are two common optimization objectives. The former indicates the total power of all sidelobes, while the latter represents the maximum sidelobe power of all sidelobes. In radar, communication, and sonar applications, the appropriate objective function between the ISL and the PSL should be selected according to the specific application scenario, and the Doppler tolerance is also considered in waveform optimization.

The research object of this paper is the optimization of waveform and waveform set with phase-coded sequence as original waveform. This study utilizes the high-order norm to uniformly represent the ISL and the PSL and adopts the gradient descent algorithm based on the quasi-Newton method to realize the optimization of waveform and waveform set. Meanwhile, the weighted waveform optimization is achieved by introducing weighting coefficients that satisfy the optimization requirements of certain specific scenarios. Finally, based on the division of frequency channels, the waveform optimization of anti-Doppler frequency sensitivity is presented.

A. OPTIMIZATION OF A WAVEFORM AND A WAVEFORM SET

An original waveform set comprises $M$ phase-coded sequences with a length of $N$, and the m-th waveform is represented by $s_m(n) = e^{j\omega_m(n)}$, $n \in [0, \ldots, N - 1]$. Because of the symmetry of aperiodic autocorrelation, only the single-side lobe is presented. Similarly, the k-th aperiodic cross-correlation between the sequences with a length of $s$ is presented.

Finally, based on the division of frequency channels, the optimization requirements of certain specific scenarios. So, the optimization objective can be written as follows:

$$ a_m(k) = \sum_{n=0}^{N-1} s_m(n)s^*_m(n-k) \quad (k = 1, 2, \ldots, N - 1) \quad (4) $$

So, $A_m = [a_m(1), a_m(2), \ldots, a_m(N-1)]$ represents the $N-1$ aperiodic autocorrelation sidelobes of the m-th waveform; similarly, the k-th aperiodic cross-correlation between the waveforms $s_m$ and $s_{m'}$ can be written as follows:

$$ b_{mn}(k) = \begin{cases} \sum_{n=k}^{N-1} s_m(n)s^*_{m'}(n-k) & (k = 0, 1, \ldots, N - 1) \\ \sum_{n=-N+1}^{-1} s_m(n)s^*_{m'}(n-k) & (k = -N+1, \ldots, -1) \end{cases} \quad (5) $$

where $m \in [1, 2, \ldots, M - 1], m' \in [m + 1, m + 2, \ldots, M]$. Then, $B_{nm'} = [b_{nm'}(-N + 1), \ldots, b_{nm'}(N - 1)]$ represents the aperiodic cross-correlation between the waveforms $s_m$ and $s_{m'}$.

When the optimization objective is the PSL, the aim of which is to minimize the maximum value of the vector composed of all the waveform autocorrelation sidelobes and cross-correlation levels, the objective function can be expressed as:

$$ \text{minimize}[\max([|A|, |B|])] \quad (6) $$

where $\cdot$ denotes the absolute value, $A$ and $B$ are two row vectors, $A = [A_1, A_2, \ldots, A_M]$ and $B = [B_{12}, B_{13}, \ldots, B_{M}]$. When $M = 1$, only the autocorrelation sidelobes of a single waveform are present. For the solution of the minimization–maximization problem, the high-order p-norm is employed to represent the maximum value of the waveform vector in this paper. Consequently, the optimization problem of the minimization-maximization is converted to the problem of minimizing the high-order p-norm. Here, we solve a relaxed minimization problem and it can be written as follows:

$$ \text{minimize}[\| |A|, |B| \|_p] \quad (7) $$

where $\| \cdot \|_p$ represents the norm of order $p$. For the convenience of analysis, the optimization objective vector can be expressed as follows:

$$ [|A|, |B|] = [\beta_1, \beta_2, \ldots, \beta_K] = \Gamma \quad (8) $$

where $\beta_j$ is the sidelobe modulus or cross-correlation modulus of the waveforms, $k \in [1, 2, \ldots, K]$.

$$ K = M(N - 1) + M(M - 1)(2N - 1)/2 \quad (9) $$

is the total number of all the sidelobes of the $M$ waveforms and the cross-correlation between them. Therefore, the objective function with the PSL as the optimization objective can be written as follows:

$$ f_1(\Theta) = \| \Gamma \|_p = \left( \sum_{k=1}^{K} \beta^p_k \right)^{1/p} \quad (10) $$

$$ \Theta = [\theta_1(0), \ldots, \theta_1(N-1), \theta_2(0), \ldots, \theta_M(N-1)] \quad (11) $$

where $\Theta$ is the phase of the waveform, and the optimized waveform can be expressed by $e^{j\Theta}$.

If the ISL is selected as the optimization objective, minimizing the sum of the autocorrelation sidelobe power and the cross-correlation power of all the waveforms is required. The autocorrelation sidelobe power of the waveforms can be written as follows:

$$ p_a = \sum_{m=1}^{M} \sum_{k=1}^{N-1} |a_m(k)|^2 \quad (12) $$

The corresponding cross-correlation power can be written as follows:

$$ p_c = \sum_{m=1}^{M-1} \sum_{m'=m+1}^{M} \sum_{k=-N+1}^{N-1} |b_{mn'}(k)|^2 \quad (13) $$

The optimization objective is $\text{minimize}(p_a + p_c)$, that means to minimize the sum of the power of the autocorrelation sidelobes and the cross-correlation power of all the waveforms (there is no cross-correlation power for a single waveform). As a similar representation, the objective function with the ISL as the evaluation metric can be written as follows:

$$ f_2(\Theta) = \sum_{m=1}^{M} \sum_{k=1}^{N-1} |a_m(k)|^2 + \sum_{m=1}^{M-1} \sum_{m'=m+1}^{M} \sum_{k=-N+1}^{N-1} |b_{mn'}(k)|^2 \quad (14) $$
\[ f(\Theta) = \alpha f_1(\Theta) + (1 - \alpha) f_2(\Theta) \]
\[ = \alpha(\sum_{k=1}^{K} \beta_k^p)^{1/p} + (1 - \alpha) \sum_{k=1}^{K} \beta_k^2 \] (15)
where \( \alpha \) is 1 or 0 as the optimization control factor that is employed to select the peak sidelobe or integral sidelobe as the evaluation metric. When \( \alpha = 1 \), the optimization objective is the PSL, and \( \alpha = 0 \) for the ISL. It should be noted that the value between 0 and 1 is meaningless because there is no corresponding optimization objective function. Finally, the waveform optimization is converted to minimize the novel objective function expressed by \( f(\Theta) \), i.e.

\[
\text{minimize}[\alpha(\sum_{k=1}^{K} \beta_k^p)^{1/p} + (1 - \alpha) \sum_{k=1}^{K} \beta_k^2] \] (16)

With the new optimization objective function described in equation (15), the iterative factor is dynamically adjusted based on the quasi-Newton algorithm, and the novel algorithm proposed in this study is as follows.

**Algorithm 1 Waveform Optimization Method Based on New Objective Function**

**Input:** original waveform phase \( \Theta_0 \) and the iteration threshold \( \epsilon \)

**Output:** optimized waveform phase \( \Theta \)

1. let \( D_0 \) be the identity matrix (size \( MN \times MN \)), \( k = 0 \)
2. while flag=1 do
   3. calculate the gradient \( G_k \) of the objective function \( f(\Theta_k) \) and \( J_k = -D_k G_k \);
   4. determine the iteration factor \( \lambda_k \), and \( \Theta_k+1 = \Theta_k + S_k \), \( \delta_k = \lambda_k J_k \);
   5. calculate the gradient \( G_{k+1} \) of the objective function \( f(\Theta_{k+1}) \)
   6. if \( \|G_{k+1}\|_2 > \epsilon \) then
      7. \( y_k = G_{k+1} - G_k \), \( D_{k+1} = (I - \frac{y_k s_k^T}{y_k s_k}) D_k (I - \frac{s_k y_k^T}{s_k y_k}) \), \( k = k + 1 \) flag=1
   else
   8. Output: optimized waveform phase \( \Theta = \Theta_{k+1} \), flag=0
9. end if
10. end while
11. return result

The derivative of \( f(\Theta) \) is the key step of the new waveform optimization algorithm, and the derivative of \( f(\Theta) \) can be expressed as follows:

\[ \frac{\partial f(\Theta)}{\partial \theta_n} = \left[ \alpha K \sum_{k=1}^{K} \beta_k^p \right]^{1/p-1} \sum_{k=1}^{K} \left( \frac{\beta_k^p - 2}{\beta_k^p} \right) \frac{\partial \beta_k}{\partial \theta_n} + 2(1 - \alpha) \sum_{k=1}^{K} \beta_k \frac{\partial \beta_k}{\partial \theta_n} \] (17)

where \( \theta_n \in [\theta_m(0), \ldots, \theta_m(N-1)] \). Then, the following result is obtained when \( \beta_k \) is autocorrelation sidelobe.

\[ \frac{\partial \beta_k}{\partial \theta_n} = \beta_k^{-1} \text{Re}(A_m^* \frac{\partial A_m}{\partial \theta_n}) \] (18)

where \( \text{Re} \) denotes the real part. The correlation calculation can be computed by convolution. Let \( q_m \) represents the flip conjugate of \( s_m \), or \( q_m(-n) = s_m^\ast(n) \). Then, the autocorrelation of the waveform can be calculated as \( a_m(k) = s_m(k) \otimes q_m(k) \). Thus, the derivative of \( A_m \) can be expressed as follows:

\[ \frac{\partial A_m}{\partial \theta_n} = \frac{\partial s_m}{\partial \theta_n} \otimes q_m + s_m \otimes \frac{\partial q_m}{\partial \theta_n} \] (19)

Therefore, the derivative of \( f(\Theta) \) can be expressed by

\[ \frac{\partial f(\Theta)}{\partial \theta_n} = \left[ \alpha K \sum_{k=1}^{K} \beta_k^p \right]^{1/p-1} \sum_{k=1}^{K} \left( \frac{\beta_k^p - 2}{\beta_k^p} \right) \frac{\partial A_m}{\partial \theta_n} + 2(1 - \alpha) \sum_{k=1}^{K} \text{Re}(A_m^* \frac{\partial A_m}{\partial \theta_n}) \] (21)

Similarly, when \( \beta_k \) is the cross-correlation of the waveforms \( s_m \) and \( s_m' \), the following result is obtained:

\[ \frac{\partial f(\Theta)}{\partial \theta_n} = \left[ \alpha K \sum_{k=1}^{K} \beta_k^p \right]^{1/p-1} \sum_{k=1}^{K} \left( \frac{\beta_k^p - 2}{\beta_k^p} \right) \frac{\partial B_{mm'}}{\partial \theta_n} + 2(1 - \alpha) \sum_{k=1}^{K} \text{Re}(B_{mm'}^* \frac{\partial B_{mm'}}{\partial \theta_n}) \] (22)

For the optimization algorithm proposed in this paper, the one-dimensional search algorithm for the iterative factor \( \lambda_k \) is realized by the golden section algorithm [33]. \( \lambda_k \) is essentially the search step for each iteration of the optimization process. Iteration minimizes the current optimization objective function with the iterative factor. Our algorithm utilizes the same gradient descent factor for all phase variables. Therefore, it has a fast convergence speed and requires less computation. The procedure is as follows.

In theory, the high-order norm in the proposed algorithm tends to the infinity. In practice, the determination of the high-order p-norm is related to the waveform length \( N \) and the
number of waveforms $M$. When $p$ is the high-order value, it can represent the maximum value of the element but an extremely large $p$ value will increase the computational complexity. Therefore, in iteration of the algorithm, an update must be performed to adapt to the new maximum value of the sidelobes. The procedure is as follows. Note that to avoid a calculation overflow caused by a norm value that is too large in the entire iteration; the maximum value of $p$ should be set according to the length and number of waveforms. It can be seen that algorithm one is the waveform optimization method proposed, while algorithm two and algorithm three are methods used to determine iteration factors and norm values in algorithm one.

**Algorithm 3 Method for Determining $p$**

**Input**: iteration threshold $\epsilon$ and $p = 1$

**Output**: norm value $p$

1. calculate the maximum value of the sidelobes $\max(\|\beta_k\|)$ and $f_1(\Theta) = \|\beta_k\|_p$
2. if $\|f_1(\Theta) - \max(\|\beta_k\|)\|_2 > \epsilon$ then
3. $p = p + 1$ and return to (1) repeat
4. else
5. Output: $p$
6. end if
7. return result

**B. WEIGHTED OPTIMIZATION OF A WAVEFORM AND A WAVEFORM SET**

In some radar and communication application scenarios, waveforms with low sidelobes in a specified lag inter-

**Algorithm 2 Method for Iteration Factor**

**Input**: iteration threshold $\epsilon$, $a=0$ and $b=1$

**Output**: iterative factor $\lambda_k$

1. set $\lambda_a = a + 0.382(b - a)$ and $\lambda_b = a + 0.618(b - a)$
2. calculate the objective function values $\eta_a = f(\Theta_k + \lambda_a k)$ and $\eta_b = f(\Theta_k + \lambda_b k)$
3. if $\eta_a \geq \eta_b$ then
4. $a = \lambda_a$, $\lambda_a = \lambda_b$ and $\lambda_b = 0.618(b - a)$
5. if $\|a - b\|_2 > \epsilon$ then
6. return to (2) repeat
7. else
8. Output: $\lambda_k = (a + b)/2$
9. end if
10. else
11. $b = \lambda_b$, $\lambda_b = \lambda_a$ and $\lambda_a = 0.382(b - a)$
12. if $\|a - b\|_2 > \epsilon$ then
13. return to (2) repeat
14. else
15. Output: $\lambda_k = (a + b)/2$
16. end if
17. end if
18. return result

The new objective functions of the weighted PSL and ISL can be written as follows:

\[
\begin{align*}
    f'_1(\Theta) &= \left[ \sum_{k=1}^{K_1} (\beta_k \odot W_a)^p + \sum_{k=K_1+1}^{K} (\beta_k \odot W_b)^p \right]^{1/p} \\
    f'_2(\Theta) &= \sum_{k=1}^{K_1} (\beta_k \odot W_a)^2 + \sum_{k=K_1+1}^{K} (\beta_k \odot W_b)^2
\end{align*}
\]

where $\odot$ represents the Hadamard product, $K_1 = M(N - 1)$, $W_a = [w_1, \ldots, w_{M(N-1)}]$ is the autocorrelation weighting coefficient of the waveforms, and $W_b = [w_1, \ldots, w_{M(M-1)(2N-1)/2}]$ is the cross-correlation weighting coefficient of the waveforms. The corresponding minimization optimization objective function can be written as follows:

\[
\minimize [\alpha f'_1(\Theta) + (1 - \alpha) f'_2(\Theta)]
\]

The ratio of the weighted to the unweighted sidelobes can be set by the weighting coefficient $W_a$. The position and proportion to be optimized between the autocorrelation and the cross-correlation can be set by the coefficient $W_b$. The weighted waveform optimization algorithm proposed in this paper breaks through the limitation of the WeCAN algorithm in [19] on the semi-positive definite of the weighting coefficient matrix. Note that the coefficients are constant under the condition of definite weighting coefficients, and the derivation method of the objective function in optimization algorithm can still be used. Therefore, by determining the optimization objective function and iteratively solving it with the novel optimization algorithm proposed in this study, a waveform with the properties of a locally optimized PSL or ISL can be obtained.

**C. WAVEFORM OPTIMIZATION WITH DOPPLER TOLERANCE CORRECTION**

The Doppler tolerance property in waveform optimization is another key problem to be considered. For a radar system, it determines whether the optimized waveforms can be used for high-speed target detection at a certain carrier frequency and bandwidth. For example, for the radar with a frequency of 10 GHz, when target’s maximum speed is 900 m/s, the Doppler frequency offset is 60 KHz. The oblique spine ambiguity function of the linear frequency modulation signal commonly used in radar means that the waveform has good Doppler tolerance; however, the pushpin ambiguity function of the phase-coded waveform (without considering Doppler tolerance) is sensitive to Doppler frequency shift. The Doppler sensitivity of phase-coded waveform is that the autocorrelation sidelobe rises significantly when the waveform has a Doppler frequency shift, which considerably affects target detection. To address this limitation, the fre-
frequency sensitivity problem is considered in waveform optimization such that the optimized waveform has good correlation properties in certain Doppler frequency shift ranges. The normalized Doppler frequency can be written as follows:

\[
 f = \frac{f_d}{B}
\]

(25)

where \( f_d \) is the Doppler frequency shift, \( B \) is the bandwidth, and for the phase-coded sequence \( B = 1/T \), \( T \) is the code rate. After determining the Doppler frequency range to be optimized, the Doppler frequency vectors of \( L + 1 \) channels can be expressed as follows:

\[
 F = [0, f_1/B, 2f_1/B, \ldots, f_{\text{max}}/B]
\]

(26)

where \( f_{\text{max}} \) is the optimization upper limit of the Doppler frequency and \( f_1 = f_{\text{max}}/L \) is the frequency interval. When the original waveform is represented as \( e^{\theta} \), the waveform set to be optimized can be represented as follows:

\[
 C = [c_0, \ldots, c_1, \ldots, c_L]
 = [e^{\theta(n)f_0 F(0)/fs} \ldots, e^{\theta(n)+j2\pi F(L)(n/fs)}]
\]

(27)

where \( f_0 \) is the sampling frequency \( (n = 0, 1, \ldots, N-1) \), \( c_0 \) is the waveform without a frequency shift, and the others are the waveforms with the Doppler frequency shift. This optimization problem can be transformed into an optimization of the autocorrelation of \( c_0 \) and the cross-correlation between \( c_0 \) and other \( L \) waveforms. The corresponding optimization objective function can be written as:

\[
 \text{minimize}(\max(||A_0||, ||B_L||))
\]

(28)

where \( A_0 \) is the autocorrelation sidelobes of \( c_0 \) and \( B_L \) represents the cross-correlation between \( c_0 \) and the waveform set \( \{c_1, c_2, \ldots, c_L\} \). It can be referred to (1) and (2). Therefore, a waveform with good Doppler tolerance properties can be obtained using the novel waveform set optimization method proposed in this paper.

**IV. SIMULATION AND NUMERICAL ANALYSIS**

In this section, multiple simulation experiments are presented to demonstrate the performance of the proposed algorithm. In the simulations, the autocorrelation sidelobes and the cross-correlation levels are normalized to the autocorrelation mainlobe and expressed by logarithm. The optimization results of a single waveform and a waveform set are provided, respectively.

**A. OPTIMIZATION SIMULATION OF A WAVEFORM AND A WAVEFORM SET**

1) SIMULATION OF A SINGLE WAVEFORM OPTIMIZATION

In the simulation of a single waveform, a random phase-coded sequence is set as the original waveform. The phase of original waveform is the uniform distribution data from 0 to \( 2\pi \), and the waveform length \( N \) are 128, 256, 512, and 1024, respectively. It should be noted that the random phase-coded sequence is different from the Zadoff-chu sequence [34] and their phase distribution and correlation properties are different. The optimization control factor is 1 or 0, corresponding to the evaluation metric of PSL and ISL respectively.

Fig. 1 shows the waveform optimization results with a length of 128. The ISL is set as the optimization objective \((\alpha = 0)\) and the result is \(-12.5412\) dB after optimization. When \( \alpha = 1 \), the PSL is set as the optimization objective, and the optimization result is \(-33.5591\) dB. Fig. 2 shows the waveform optimization results with a length of 256. When \( \alpha = 0 \), the ISL is \(-12.9812\) dB; when \( \alpha = 1 \), the PSL is \(-36.6355\) dB. Fig. 3 and 4 show the optimization results with a length of 512 and 1,024, respectively. Through the optimization of the proposed algorithm, the ISL is \(-13.1689\) dB and \(-13.0636\) dB, and the PSL is \(-39.7999\) dB and \(-42.6551\) dB. The simulation results demonstrate that the sidelobes of the waveforms significantly declined after the optimization with the proposed algorithm.

For convenience of the analysis, the waveform optimization results with different lengths and optimization control factors are presented in Tables 1 and 2, respectively. When the ISL is considered as the evaluation metric, the performance is improved by 12-13 dB through optimization. Similarly,
when the PSL is considered as the evaluation metric, the performance improvement is 18-20 dB.

To further demonstrate the performance of the proposed algorithm, a comparison between the algorithm proposed in this study and the CAN and MISL algorithms is presented in Fig. 5. Since the latter two algorithms take the ISL as the optimization objective, the optimization control factor $\alpha$ is set to 0. Each simulation runs 10 times, and the average value is taken for analysis. The original waveforms are random phase-coded sequences with lengths of 128, 256, 512, and 1,024. The simulation results show that the performance of the proposed algorithm is better than those of the CAN and MISL algorithms. For example, for the waveform with a length of 1,024, the ISL value of the proposed algorithm is $-13.3365$ dB; the values of the CAN and MISL algorithms with the same waveform length are $-12.2585$ dB and $-12.5803$ dB, respectively.

2) SIMULATION OF A WAVEFORM SET OPTIMIZATION
For the waveform set optimization, the phase-coded sequences are considered as original waveforms with a length of 128 and a number of 4 ($M = 4$ and $N = 128$). First, $\alpha = 1$ is set to consider the PSL as the evaluation metric. The peak sidelobe of the original waveform is $-13.7161$ dB and the cross-correlation peak is $-13.0290$ dB (Fig. 6 (a) and (b)). Fig. 7 (a) and (b) show that the optimization results of the proposed algorithm are $-22.3025$ dB and $-22.2895$ dB, and that the peak sidelobe significantly decreased.
TABLE 1. ISL optimization results.

| Waveform length | 128   | 256   | 512   | 1,024 |
|-----------------|-------|-------|-------|-------|
| Original        | 0.4626 dB | 0.0738 dB | -0.1509 dB | -0.2691 dB |
| $\alpha = 0$    | -12.5412 dB | -12.9812 dB | -13.1689 dB | -13.0636 dB |

TABLE 2. PSL optimization results.

| Waveform length | 128   | 256   | 512   | 1,024 |
|-----------------|-------|-------|-------|-------|
| Original        | -15.6020 dB | -17.9813 dB | -21.9299 dB | -22.3816 dB |
| $\alpha = 1$    | -33.5591 dB | -36.6355 dB | -39.7999 dB | -42.6551 dB |

In order to further compare and analyze with the Multi-CAN algorithm, the ISL is considered as the evaluation metric. The number of waveforms is 4, and the lengths are 32, 64, 128, 256, and 512, respectively. The results in Fig. 8 demonstrate that for a waveform set optimization, the ISL performance of the proposed algorithm is better than that of the Multi-CAN algorithm.

**B. WEIGHTED OPTIMIZATION OF A WAVEFORM AND A WAVEFORM SET**

1) WEIGHTED OPTIMIZATION SIMULATION OF A SINGLE WAVEFORM

To verify the weighted optimization performance of the proposed algorithm for a single waveform, the phase-coded sequences with lengths of 128 and 256 are taken as the original waveform, the weighted positions are the 1/4 waveform length delay next to the mainlobe, and the weighting coefficient value is 1,024. Therefore, for the waveform with a length of 128, the weighting coefficient is $W_{1} = [0, \ldots, 0, 1024, \ldots, 1024]$ and the coefficient length is 127 (as the number of single sidelobes is 127). The weighting coefficient with a length of 256 is $W_{2} = [0, \ldots, 0, 1024, \ldots, 1024]$.

In the simulation, $\alpha = 1$ and $\alpha = 0$ are set, respectively. Fig. 9 shows the optimization results of the waveform with a length of 128. The ISL of the original waveform at the weighted positions is $-3.2460$ dB, and the optimization result...
of the proposed algorithm is $-181.9896$ dB; meanwhile, the PSL of the original waveform at the weighted positions is $-16.0672$ dB, and the optimization result is $-196.2885$ dB.

Fig. 10 demonstrates the optimization results of a waveform with a length of 256. The ISL = $-186.0010$ dB and PSL = $-202.4746$ dB.

Simulations of the WeCAN algorithm are performed in comparison with the performance of the proposed algorithm. Fig. 11 and 12 show the weighted optimization results with lengths of 128 and 256. In these Figures, the original waveforms are the same as those in Fig. 9 and 10. The results indicate that the performance of the proposed algorithm is obviously better than that of the WeCAN algorithm.

Table 3 lists the comparative analysis of the ISL at the weighted positions of the WeCAN algorithm and the proposed algorithm with different lengths. The weighted length is 1/4 of the waveform length. As shown in the simulations, the proposed algorithm demonstrates a good weighted optimization performance at the local delay positions.

2) WEIGHTED OPTIMIZATION SIMULATION OF A WAVEFORM SET

In the weighted optimization simulation of a waveform set, the waveform set comprise two waveforms with a length of 128 (i.e., $M = 2$, $N = 128$). The autocorrelation weighting coefficients of the waveforms are...
TABLE 3. ISL optimization results.

| Waveform length | 32         | 64         | 128        | 256        | 512        |
|-----------------|------------|------------|------------|------------|------------|
| Original        | -4.6539 dB | -4.2342 dB | -3.2460 dB | -3.2695 dB | -4.2784 dB |
| WeCAN           | -16.8952 dB| -21.1690 dB| -21.0744 dB| -22.9815 dB| -22.6538 dB|
| Proposed        | -192.4597 dB| -191.8358 dB| -181.9896 dB| -186.0010 dB| -185.4765 dB|

C. WAVEFORM OPTIMIZATION WITH DOPPLER TOLERANCE

This study shows the waveform optimization results based on a Doppler tolerance correction for a single waveform by setting \( \alpha = 1 \) (or \( \alpha = 0 \), taking the PSL (or the ISL) as the evaluation metric, and considering the Doppler frequency sensitivity of the waveform. The original waveform is a phase-coded sequence with a length of 256 and a bandwidth of 100 MHz. The correction Doppler frequency

\[
W_1 = [0, 0, 0, \ldots, 8, 8, 8, 8, \ldots, 94, 95, 126] \quad \text{and} \quad W_1 = W_2,
\]

and the cross-correlation weighting coefficient of the waveform set is

\[
W_c = [0, 0, 0, \ldots, 8, 8, 8, 8, \ldots, 31, 32, 63, 192, 233, 224, 254].
\]

First, the optimization factor is set as \( \alpha = 1 \) and the PSL is considered as the optimization objective. Fig. 13 (a) shows the autocorrelation optimization results. The PSL of waveform 1 and waveform 2 at the weighted positions are \(-43.0259 \text{ dB} \) and \(-42.8333 \text{ dB} \), respectively. Fig. 13 (b) shows the cross-correlation optimization result, and the PSL of the weighted positions is \(-42.1442 \text{ dB} \).

Second, the optimization factor \( \alpha \) is set to 0 and the ISL is considered as the evaluation metric. As shown in Fig. 14 (a), the ISL of the two optimized waveforms are \(-30.8415 \text{ dB} \) and \(-30.3790 \text{ dB} \), respectively. Fig. 14 (b) shows that the ISL of the weighted position is \(-23.4695 \text{ dB} \) after the cross-correlation optimization.

FIGURE 14. Weighted optimization of a waveform set (\( \alpha = 0 \)).

FIGURE 15. Original waveform properties.

(a) Autocorrelation with frequency shift

(b) Original waveform ambiguity function (local)
range is 0.2 MHz, divided into 10 channels. As shown in Fig. 15 (a), the PSL of the original waveform with Doppler frequency is $-17.4920$ dB. The ambiguity function of the original waveform is shown in Fig. 15 (b). Note that the waveform is sensitive to the Doppler frequency shift, which is the peak side-lobe rises considerably on the frequency channel. As shown in Fig. 16 (a), the PSL of the optimized waveform is $-27.9547$ dB, and the ambiguity function of the optimized waveform is shown in Fig. 16 (b). The ambiguity function shows that the side lobes of the optimized waveform on the channels with the Doppler frequency shift are well suppressed; therefore, the optimized waveform has better correlation properties in a wider frequency range.

V. CONCLUSION

To deal with the optimization problem of a waveform as well as a waveform set, the ISL and the PSL are considered as the optimization objectives. In this paper, the high-order p-norm is deployed to uniformly represent the objective functions, and the quasi-Newton algorithm is adopted for iterative calculation. Improved optimization results are obtained via the proposed method compared with the traditional algorithms. Furthermore, this study defines the optimization objective function with weight. The proposed method completes the local weighting of the sidelobe at any position next to the mainlobe. The proposed method also considers the Doppler sensitivity of the waveform in the optimization, and good waveform optimization results are obtained. Extensive numerical simulation results are provided to validate the effectiveness of the proposed method in terms of PSL and ISL performance.

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