Factorization and Endpoint Singularities in Heavy-to-Light decays

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We prove a factorization theorem for heavy-to-light form factors. Our result differs in several important ways from previous proposals. A proper separation of scales gives hard kernels that are free of endpoint singularities. A general procedure is described for including soft effects usually associated with the tail of wavefunctions in hard exclusive processes. We give an operator formulation of these soft effects using the soft-collinear effective theory, and show that they appear at the same order in the power counting as the hard spectator contribution.

Exclusive hadronic form factors simplify dramatically at momentum transfers much larger than hadronic scales, \( Q^2 \gg \Lambda^2 \). Typically, they factor into non-perturbative light cone wavefunctions \( \phi_{a,b} \) for mesons \( a \) and \( b \), convoluted with a calculable hard scattering kernel \( T \).

\[
F(Q^2) = \frac{f_a f_b}{Q^2} \int dx dy T(x,y,\mu) \phi_a(x,\mu) \phi_b(y,\mu) + \ldots
\]  

Here \( f_i \) are meson decay constants, the hard scattering kernel \( T(x,y) \) is calculated perturbatively in an expansion in \( \alpha_s \), and the ellipses denotes terms suppressed by additional powers of \( 1/Q \). For example, the electromagnetic form factor of a pion has \( a = b = \pi \) and at \( \mu = Q \): \( T(x,y) = 8\pi\alpha_s(Q)/(9xy) \). For Eq. (1) to be well defined it is sufficient that \( \phi_i(x) \sim x^n, \phi_i(x) \sim (1-x)^m \) with any \( n, m > 0 \). A linear falloff is sometimes assumed, but we will not use this assumption.

Beyond leading order (LO) in \( 1/Q \) issues arise. There are soft contributions to the form factor, which arise from configurations where a single quark carries most of the meson momentum and leaves \( p^\mu \sim \Lambda \) for the remaining constituents, and these have been estimated using QCD sum rules. Furthermore, power suppressed hard exchange contributions tend to give contributions diverging as \( \int dx/x \). Examples include \( 1/Q \) corrections to the pion form factor, \( 1/m_b \) corrections in \( B \to \pi \pi, K \pi \) decays, and one-gluon exchange for heavy-to-light form factors.

The soft-collinear effective theory (SCET) reproduces the factorization in Eq. (1), and provides a framework to analyze power corrections based solely on QCD. This theory consists of collinear fields interacting with soft or ultrasoft (usoft) degrees of freedom.

The fields are categorized by the scaling of their momenta: collinear \( p_c = (p_c^+, p_c^-) = (n \cdot \vec{p}_c, \vec{n} \cdot \vec{p}_c) \sim Q(\lambda^2, 1, \lambda) \), soft \( p_s^+ \sim Q\Lambda \) and usoft \( p_{us}^+ \sim Q\Lambda^2 \), where \( n^2 = \vec{n}^2 = 0 \), \( n \cdot \vec{n} = 2 \), and \( \lambda \ll 1 \) is the expansion parameter.

In this paper we show how SCET can be used to understand factorization and soft-endpoint contributions in heavy-to-light form factors for decays such as \( B \to \pi\ell\nu, B \to K^{*+}e^+\nu \) and \( B \to \rho\gamma \), building on [4]. Here the large scales are \( Q = \{m_b, E\} \), where the final meson has \( E = m_B/2 - q^2/(2m_B) \).

- We prove a factorization formula for heavy-to-light decays involving the LO light-cone wavefunctions, a jet function, plus a reduced set of non-perturbative matrix elements which obey form factor relations.
- Calculable kernels are free of divergences. Endpoint singularities are fake and arise from improperly matching onto \( T(x,y) \). They appear in non-factorizable operators and can be parameterized without invoking suppression from Sudakov effects.
- A single collinear meson state can be used to categorize all contributions. Soft effects associated with the tail of wavefunctions are described by matrix elements of operators with a definite power counting. The categories “factorizable” and “non-factorizable” are more accurate than “hard” and “soft” contributions.
- There are two perturbative scales in the problem: \( Q \) and \( \mu_0 \sim Q\Lambda \). We separate these scales by matching in two stages, onto a SCET\(_I\) at \( \mu = Q \), and onto a SCET\(_II\) at \( \mu = \mu_0 \).
- The LO result for heavy-to-light decays comes from power suppressed operators in SCET\(_I\), which match onto LO operators in SCET\(_II\).

Our procedure is quite general and similar analyses apply to other exclusive processes.

To understand the origin of the endpoint divergences, we consider the spectator interaction for heavy-to-light decays at \( \mathcal{O}(g^2) \) in Fig [1]. Taking \( p_{1,2} \) collinear and \( k, r \) ultrasoft, the \( \lambda \)-expansion of these graphs gives \( i\lambda A = \)

![FIG. 1: Tree level QCD graphs for heavy-to-light decays with one perturbative gluon. Note that \( p_0^\gamma \sim Q\Lambda \).](attachment:fig1.png)
\[ p^2 \bar{u}_n(p_1) X T^A u_n(p_0) \bar{v}(r) V T^A v_n(p_2) / P_g \] with

\[ (X \otimes V)^{(a)} = \frac{\Gamma \otimes \gamma_5^\mu}{n \cdot p_2} + \frac{\Gamma \gamma_5^\mu \otimes \gamma_\mu^\mu}{2m_n} + \ldots, \tag{2} \]

\[ (X \otimes V)^{(b)} = \left\{ \frac{\bar{p}_1 \gamma_5^\mu n \cdot p}{P_1 n \cdot p_1} + \frac{\gamma_5^\mu (n \cdot p)_{\mu}}{2} + \Gamma \right\} \Gamma \otimes \gamma_\mu^\mu \]

\[ - \frac{2 \bar{n} \cdot p \Gamma \otimes \gamma_\mu^\mu}{P_2 n \cdot p_2} + \frac{\bar{n} \cdot p \Gamma \gamma_5^\mu \otimes \gamma_\mu^\mu p_2 \cdot \bar{q}}{2F_2 n \cdot p_2} + \ldots, \]

where \( P_g = \bar{n} \cdot p_2 \cdot n \cdot r \), \( P_q = \bar{n} \cdot p_n \cdot r + p^2 \) and \( p = p_1 - p_2 \). Eq. \( \text{(2)} \) agrees with Ref. \[11\]. The \ldots denotes terms \( \propto P \). If one interprets Eq. \( \text{(2)} \) with \( \Gamma \) as \( M \), \( b = B \), it is tempting to extract \( T(x,y) \) setting \( p_1^2 = 0 \), \( (p_1 - p_2)^2 = 0 \), \( \bar{n} \cdot p_2 = 2 \bar{x}E, n \cdot r = y \). However the result includes terms \( \propto 1/x^2 \) or \( 1/y^2 \) leading to singular integrals. Note that in full QCD there are no singularities since they are regulated by momenta of order \( \Lambda \).

Several proposals have been made for dealing with these divergences. One approach regulates these singularities by introducing transverse parton momenta and including Sudakov form factors \[12, 14\]. However this proposal does not include all the non-perturbative contributions, or deal with the possibility that Sudakov suppression may not be large at \( m_b \approx 5 \) GeV. In \[11\] it was shown that at LO these divergences can be reabsorbed into “soft” form factors which satisfy form factor relations \[3, 13\]. However, this analysis was only performed to order \( \alpha_s \). Furthermore neither a rigorous field theoretical definition for these soft contributions exists, nor does a first principle derivation of their power counting.

To fully understand these issues requires a factorization formula with the generality to account for non-perturbative contributions. In this paper we prove that at leading order in \( 1/Q \) and all orders in \( \alpha_s \) a generic heavy-to-light form factor \( F \) can be split into factorizable and non-factorizable components \( F = f^F(Q) + f^{NF}(Q) \) where

\[ f^F(Q) = N_0 \int_0^1 dz \int_0^1 dx \int_0^\infty dr_+ T(z,Q,\mu_0) \]

\[ \times J((z,x),r_+,Q,\mu_0,\mu) \phi_M(x,\mu) \phi_B(r_+,\mu), \]

\[ f^{NF}(Q) = C_k(Q,\mu) \zeta_k(Q,\mu), \tag{4} \]

and \( N_0 = f_{BF} \pi m_b / (4E^2) \). The hard coefficients \( C_k \) and \( T \) can be calculated in an expansion in \( \alpha_s(Q) \), the jet function \( J \) is dominated by momenta \( p^2 \approx Q\Lambda \) and calculable perturbatively in \( \alpha_s(\sqrt{Q\Lambda}) \). The functions \( \phi_M \) and \( \phi_B \) are standard non-perturbative light-cone wavefunctions c.f. \[11, 15\], where our \( \phi_B \) denotes \( \phi_B^H \) or \( \phi_B \). Only \( \phi_B^H \) appears if \( J \) is calculated at tree level. Endpoint singularities only arise in matrix elements which determine the soft, non-perturbative form factors \( \zeta_k(Q,\mu) \), leaving the convolution integrals in the factorizable terms finite. There are three soft form factors \( \zeta_k(Q,\mu) \); one for pseudoscalar, and two for vector mesons. We show that

![FIG. 2: Levels of factorization. The gray area corresponds to gluons in SCET\(I \) which are integrated out in SCET\(II \).](image-url)
lating field which has the right quantum numbers and significant overlap with the physical state. For the B we pick the standard HQET state $|B_q\rangle$ ~[10], while for the light meson $M$ we pick a state $|M_n\rangle$ whose interpolating field is built out of two collinear quarks, and involves all interactions in the LO collinear Lagrangian. Thus, the $B/M$ states are generated by soft/collinear fields in SCET II which have $p_c^2 \sim \Lambda^2$. Time-ordered products account for corrections to these states. We do not define $|M_n\rangle$ with collinear quarks in SCET I since here the offshellness is still large.

Eqs. 34 separate the contributions from hard momenta ($p^2 \sim Q^2$), jet momenta ($p^2 \sim Q\Lambda$), and nonperturbative momenta ($p^2 \sim \Lambda^2$), as illustrated in Fig. 4. In Fig. 1 it is the gluon that connects to the spectator which scales like a jet momentum. To separate these scales we match QCD onto an intermediate effective theory SCET I, valid for $\sqrt{\alpha}\Lambda/\mu < \mu < Q$, which contains collinear particles with offshellness $p_c^2 \sim Q\Lambda$ and a power counting in $\lambda = \sqrt{\alpha\Lambda/\mu}$. Since the collinear particles in SCET I satisfy $p_c^2 \sim Q\Lambda$ this theory does not describe the complete $B \to M$ process in QCD. A second step of matching is required onto SCET II, containing collinear particles with offshellness $p_c^2 \sim \Lambda^2$ and power counting in $\lambda' = \Lambda^2/\Lambda/\mu$. Wilson coefficients in SCET II determine $T, C, \xi$ of 3, while those in SCET II determine $J$. The usoft fields in SCET I are identical to soft fields in SCET II. This two-step procedure provides a simple and more general method of determining the SCET II soft-collinear operators compared to the procedure in Ref. 7.

SCET I is defined by its Lagrangian and heavy-to-light currents. The terms in the expansion of the collinear Lagrangian we require are $\mathcal{L}_c = \mathcal{L}_c^{(0)} + \mathcal{L}_c^{(1)} + \mathcal{L}_c^{(2a)} + \mathcal{L}_c^{(2b)}$. The superscript denotes the order in $\lambda$ that these terms contribute in the power counting [17]. The LO action for collinear quarks and gluons is [7, 5]

$$\mathcal{L}_c^{(0)} = \bar{\xi} n \cdot D + i D^\mu \frac{1}{i \bar{n} \cdot D_c} i D^c \frac{i}{2} \bar{\xi} + \mathcal{L}_c^{(0)}$$ (5)

with $\bar{n} \cdot D_c = \bar{\mathcal{P}} + g \bar{n} \cdot A_c, D^\perp_c = \mathcal{P}^{\perp} + g A_c^{\perp}, \bar{n} \cdot D = \bar{n} \cdot \partial + g n \cdot A_{us} + g n \cdot A_c$. The gluon action $\mathcal{L}_c^{(0)}$ can be found in Ref. 7. For the subleading action we find [18, 13, 21]

$$\mathcal{L}_c^{(1)} = \bar{\xi}_{\perp} i D^\mu_c \frac{1}{i \bar{n} \cdot D_c} i D^c \frac{i}{2} \bar{\xi} + \mathcal{L}_c^{(1)}$$ (6a)

$$\mathcal{L}_c^{(1)} = \frac{1}{2 g^2} \text{tr} \left[ [i D^\mu, i D^c] [i D_\mu, i D^c] \right] + \text{g.f.}$$ (6b)

with $D^\mu = n^\mu \bar{n} \cdot D_c/2 + D^\perp + n^\mu n \cdot D/2$ and g.f. denotes gauge fixing terms. In our proof the mixed collinear-usoft Lagrangian $\mathcal{L}_c^{(0)}$ will play a crucial role and was first considered in [21]. Using the label operator formalism [6] we obtain the gauge invariant QCD result:

$$\mathcal{L}_c^{(1)} = ig \bar{\xi}_{\perp} \frac{1}{i \bar{n} \cdot D_c} i D^c \frac{i}{2} \bar{\xi} + \mathcal{L}_c^{(1)}$$ (7)

$$\mathcal{L}_c^{(2a)} = ig \bar{\xi}_{\perp} \frac{1}{i \bar{n} \cdot D_c} i D^c \frac{i}{2} \bar{\xi} + \mathcal{L}_c^{(2a)}$$ (8a)

$$\mathcal{L}_c^{(2b)} = ig \bar{\xi}_{\perp} \frac{1}{i \bar{n} \cdot D_c} i D^c \frac{i}{2} \bar{\xi} + \mathcal{L}_c^{(2b)}$$ (8b)

where $i g \partial_{\perp} = [i \bar{n} \cdot D_c, i D^c]$ and $i g M = [i \bar{n} \cdot D_c, i D^c + g \bar{n} \cdot A_c]$. A possible four quark operator $(\bar{\xi_n} W^T A^g W^\dagger \xi_n) + 1/2 (\bar{\xi_n} W^T A^g \bar{\xi_n})$ has been eliminated using the collinear gluon equations of motion. Finally the SCET I currents we will need are [4, 18, 13, 21]

$$J^{(0)} = C_T(\omega_1)(\bar{\xi_n} W)_{\omega_1} \Gamma h_v$$ (8)

$$J^{(1a)} = B_1^{(2)}(\omega_1, \omega_2) (\bar{\xi_n} W)_{\omega_2} (W^{T}_1 i D^c_{\omega_1} W)_{\omega_2} \frac{\Gamma_1}{m_b} h_v$$ (8a)

$$J^{(1b)} = B_1^{(2)}(\omega_1, \omega_2) (\bar{\xi_n} W)_{\omega_1} (W^{T}_1 i D^c_{\omega_1} W)_{\omega_2} \frac{\Gamma_1}{m_b} h_v$$ (8b)

where we sum over $\omega_1, \omega_2$. Here $J^{(1a,1b)}$ correspond to the $K_1^{(d)}$ of [21], which are the most general allowed operators at any order in $\alpha_s$, taking $v_{\perp} = 0$.

In matching onto SCET II we need two collinear quarks to give non zero overlap with $|M_n\rangle$, so we only need operators with two collinear quarks in SCET I. For the graphs in SCET I it is necessary to have a $\mathcal{L}_c^{(n)}$ interaction to turn the usoft spectator in the B into a collinear quark. This is the generic reason that the form factors in the range of $q^2$ considered here, $q^2 \lesssim 10 \text{GeV}^2$, are suppressed relative to their size near $q^2_{\text{max}}$. More than one $\mathcal{L}_c^{(n)}$ insertion is forbidden at this order. The relevant time-ordered products are

$$T_0^F = T[J^{(0)}, i \mathcal{L}_c^{(1)}] = \int d^4 x T \left[ J^{(0)}(0) i \mathcal{L}_c^{(1)}(x) \right]$$ (9)

as well as

$$T_1 = T[J^{(1a)}, i \mathcal{L}_c^{(1)}], \quad T_2 = T[J^{(1b)}, i \mathcal{L}_c^{(1)}], \quad T_3 = T[J^{(0)}, i \mathcal{L}_c^{(2a)}], \quad T_4 = T[J^{(0)}, i \mathcal{L}_c^{(2a)}], \quad T_5 = T[J^{(0)}, i \mathcal{L}_c^{(3)}], \quad T_6 = T[J^{(0)}, i \mathcal{L}_c^{(3)}], \quad T_7 = T[J^{(0)}, i \mathcal{L}_c^{(3)}].$$ (10)

The time-ordered product $T_0^F$ is enhanced by one power of $\lambda$ in SCET I compared to the other terms, however its matching onto SCET II does not give rise to enhanced contributions to form factors. Higher order $T_i$’s do not contribute at the order we are working.

To prove the factorization formula given in [8, 14], we decouple the collinear-usoft interaction in the LO Lagrangian $\mathcal{L}_c^{(0)}$ using the field redefinitions [7]

$$\xi_n^{(0)} = Y^\dagger \xi_n, \quad A^{(0)}_n = Y^\dagger A_n Y,$$ (11)

$$Y(x) = P \exp \left[ ig \int_{-\infty}^x ds \cdot n \cdot A_{us}(ns) \right].$$
While this introduces a factor of $Y^\dagger$ into the leading current, it only appears in the combination $\mathcal{H}_v = [Y^\dagger h_v]$

$$J^{(0)} = C_T(\omega_i)(\xi_n^0 W(0))_{\omega_i}^{\Gamma} \mathcal{H}_v .$$

The situation is similar in $L_{q\xi}^{(1)}$ and $L_{q\xi}^{(2)}$, where usoft fields/interactions now only appear in the combination $Q = [Y^{\dagger} q_{us}]$. On the other hand we have

$$L_{q\xi}^{(1)} = \xi_n^0 [Y^{\dagger} i D_{us}^\rho Y] \frac{1}{i\not{\nu} - D(0)} i D_{c,\xi}^\rho \frac{1}{2} \xi_n^0 + \text{h.c.},$$

$$L_{q\xi}^{(2)} = ig \xi_n^0 \frac{1}{i\not{\nu} - D_c(0)} [Y^{\dagger} M Y] W(0) Q + \text{h.c.}$$

Thus, the time-ordered products fall into two categories: “factorizable”, $T_f^{(0,1,2,3)}$, in which the usoft interactions all occur in $H$ and $Q$, and “non-factorizable”, $T_{nF}^{(4,5,6)}$, with an additional $[Y^{\dagger} D_{us}^\rho Y]$ or $[Y^{\dagger} M Y]$. It can be clearly seen that there is no double counting when the soft and hard contributions are defined this way. The matching onto SCETII for these two cases is discussed separately.

For the factorizable terms $T_f^F = T[J^{\dagger}, i Lf^F]$ each $J^{\dagger}$ and $L^F$ splits into collinear and usoft parts in SCETI, $J^{\dagger} = T^{\dagger}(\omega) J_{\omega}, \mathcal{H}_v, L^F = \mathcal{Q} J + \text{h.c.}$, where $J$’s denote products of collinear fields. To factorize these time-ordered products we follow Ref. [2]. From momentum conservation we have $\omega_1 + \omega_2 \to \bar{n} \cdot p_M$ of meson $M$, so we suppress this dependence and let $\bar{\omega} = \omega_1 - \omega_2$. With this notation we can write

$$T^F_{\omega} = T_{\omega}^F \int d^4 x T[\mathcal{J}(0) \Gamma H(0)] \mathcal{Q}(x) J(x) \]$$

$$= T_{\omega}^F \int d^4 x \{ T[\mathcal{J}(0) \Gamma_x J(x)] T[\mathcal{Q}(x) \Gamma_x H(0)] \},$$

where $T_{\omega}^F = \{ C_T(n \cdot p_M), B_{\xi}^F(n \cdot p_M), B_{c,\xi}^F(n \cdot p_M, \bar{\omega}), C_T(n \cdot p_M) \}$. In the second line we performed a Fierz transformation on the color and spin indices, absorbing prefactors to give $T(\bar{\omega})$, and dropping a $T^A \otimes T^A$ which gives no contribution in SCETI. We now lower the off-shellness of the external collinear particles to $p_\perp^2 \sim \Lambda^2$. The $T^F_{\omega}$ run exactly like their $J^F_{\omega}$ currents. Since we have explicitly kept the usoft part of the momentum of collinear particles, matching onto SCETIII amounts to setting $p_{\perp t}^2 = n \cdot p^2 = 0$ on external lines and expanding the $T^F_{\omega}$'s. Matching at $\mu_0 \approx \sqrt{Q^2}$ the usoft fields become soft (eg. $Y \to S$), and the collinear T-product matches onto a bilinear collinear quark operator in SCETI,

$$T[\mathcal{J}(0) J(x)] = \delta(x^+) \delta^2(x^+) \int d\bar{\omega} d\bar{k}^+ e^{i \bar{k}^+ \bar{q} \cdot x -}$$

$$\times J(\bar{\omega}, \bar{n}, \bar{k}^+) \{ \xi_n^F W_\perp \delta(\bar{n} - \bar{P}_+) W^{\dagger} \xi_n^F \}. $$

The jet function $J(\bar{\omega}, \bar{n}, \bar{k}^+)$ is the Wilson coefficient for this matching step. Inserting this in [19],

$$T^F_{\omega} = \int d\bar{\omega} d\bar{k}^+ T(\bar{\omega}) J(\bar{\omega}, \bar{n}, \bar{k}^+) \mathcal{O}(\bar{n}, \bar{k}^+),$$

$$\mathcal{O}(\bar{n}, \bar{k}^+) = [\xi_n^F W \delta(\bar{n} - \bar{P}_+) \Gamma_c W^{\dagger} \xi_n^F ][\bar{q}^4 S \Gamma_\sigma \delta(P_+ - \bar{k}^+) S^\dagger h_v].$$

FIG. 3: Tree level graphs in SCETI. The graphs in a) are from $T_{1,2,4}$, while those in b) are from $T_{0,1,3,5,6}$. Where $Q(\bar{n}, \bar{k}^+)$ is the full operator in SCETI. Now taking the SCETII matrix element gives

$$(M_n \mathcal{O}(\bar{n}, \bar{k}^+)) B_n = N f_M f_B \phi_B(x) \phi_B(\bar{k}^+) ,$$

where $N$ is a normalization factor and $x = \bar{n}/(4E) + 1/2$. Combining Eqs. [16] and [17] reproduces Eq. [3].

For the non-factorizable operators $T^{NF}_{\omega}$, it is not possible to write the matrix elements as in $f^F$. Instead when matched onto SCETII these terms give $f^{NF}$ in Eq. [19] and should be understood to define the soft nonperturbative effects for the form factors. It remains to show that they satisfy the form factor relations [16]. Since the relevant time-ordered products only contain the current $J_0$, the argument is the same as in [2]; any Dirac structure in heavy-to-light currents can be reduced to only three, $\bar{c}_q W h_v$, $\bar{c}_q W^\gamma h_v$, and $\bar{c}_q W^\gamma h_v$. These three operators contribute only to $B \to P$, $\bar{B} \to V_\perp$ and $B \to V_\perp$, respectively, where $P$, $(V_\parallel, V_\perp)$ denote pseudoscalar, (longitudinally, transversely) polarized vector mesons. For $J_0$ this is true even in arbitrary time-ordered products with Lagrangian insertions, since Lagrangians are parity even Lorentz scalars. The $f^F$ term breaks these relations, but is calculable. At higher order in $\lambda$ non-factorizable contributions will also break these relations, since subleading currents appear in time-ordered products with nonfactorizable Lagrangian insertions.

The matrix elements of $T^F_{\omega}$ contain only $\phi_B^+$ to all orders in $\alpha_s$, since inserting a projector next to $\xi_n$ in $L_{q\xi}^{(1)}$, the $q_{us}$ appears as $\bar{q}_{us} \bar{g}_{\bar{n}}$ in the Fierzed operators. On the other hand, $T^F_{\omega}$ (which may contribute at $O(\alpha_s^3)$) has only $\bar{q}_{us} \bar{g}$ and so is proportional to $\phi_B$. However, $T^F_{\omega}$'s matrix element involves $J_0$ and therefore satisfies the same symmetry relations as the nonfactorizable matrix elements in $f^{NF}$. Therefore it can be absorbed into a redefinition of the $\xi_n^F$'s to all orders in perturbation theory.

The last step is to understand the power counting of the two contributions in Eqs. [31][19]. When we expand to match onto SCETII the new operators and coefficients scale with $1/Q$ in the same way as those in SCETI, up to a global $1/Q$ from switching from the $\xi_n^F$ to $\xi_n^F$ fields. The one exception is $T^{NF}_{\omega}$, since it is odd in the number of $D_\perp$ derivatives and this extra $\perp$ gets suppressed by at least one power of $\lambda$. Therefore, $T^F_{\omega}$ and $T^{NF}_{\omega}$ contribute at the same order in $1/Q$ to the form factors. We find a generic form factor to scale as $(\Lambda/Q)^{3/2}$, which is $\Lambda^2/Q^2$ suppressed compared to the scaling in $m_h$ near $q_{\text{max}}$ derived from HQET [10].
We finally show that the endpoint singularities encountered in \( \mathcal{O}(g^2) \) do not occur in \( f^F \) in the second step of matching. The contributions of the time-ordered products at \( \mathcal{O}(g^2) \) are shown in Fig. 3 and expanding we find

\[
i\mathcal{A}_i = g^2 \xi_\alpha(p_1) T^A h_v(p_2) \bar{v}(r) V_i T^A \xi_\beta(p_2) / P_\beta \,
\]

with

\[
X_0 \otimes V_0 = X_3 \otimes V_3 = X_6 \otimes V_6 = 0
\]

\[
X_1 \otimes V_1 = \frac{\gamma^\mu_1 \bar{\Gamma} \otimes \gamma^\mu_2}{2 n \cdot p}, \quad X_2 \otimes V_2 = \frac{\Gamma \theta \gamma^\mu \otimes \gamma^\mu_2}{2 m_b},
\]

\[
X_4 \otimes V_4 = \frac{1}{n \cdot p_2} \Gamma \otimes \left[ \mathfrak{g} - \frac{2 p_{us}^{ab}}{n \cdot r} \right],
\]

\[
X_5 \otimes V_5 = \left[ \frac{\gamma^\mu_1 \bar{f}_+ \gamma^\mu_2}{n \cdot p n \cdot r} + \frac{p_{us}^{ab} \gamma^\mu_1}{n \cdot p_1 n \cdot r} \right] \Gamma \otimes \gamma^\mu_1.
\]

The \( 1/x^2 \), \( 1/r^2 \) singularities only exist in the non-factorizable \( T_4 \) and \( T_5 \), while the factorizable \( T_{1,2} \) give non-singular jet functions. This is not surprising, since in full QCD all endpoint singularities are regulated by \( \Lambda \). Thus, if \(|u|\) soft operators are properly included to account for this region of momenta endpoint singularities will not arise.

As an example, for the form factor \( f_+ \) at leading order in \( 1/Q \) and all orders in \( \alpha_s \) we find

\[
f_+ = N_0 \int dx \, dz \, dr \, \frac{E - m_B}{m_B} T_a(\mu_0) J_a(z, x, r, \mu_0, \mu) \cdot \frac{2E - m_B}{m_B} T_b(z, \mu_0) J_b(z, x, r, \mu_0, \mu) \cdot \phi_M(x, \mu) \phi_\beta^\perp(r, \mu) + C(Q, \mu) \zeta(Q, \mu),
\]

where \( N_0 = f_+ f_+ m_B / (4E^2) \) and the \( Q \) dependence of \( T_{a,b} \) and \( J_{a,b} \) is implicit. Here \( T_{a,b} \) are the Wilson coefficients of the currents \( J^{(1a,1b)} \), the jet functions \( J_{a,b} \) are computed from the \( T_{1,2} \) time ordered products, and we have reabsorbed possible \( \phi_\beta^\perp \) contributions from \( T_3^F \) into \( \zeta \). For the jet functions at order \( \alpha_s \) we find

\[
J_a = J_b = \frac{\pi C_F}{N_c} \frac{\alpha_s(\mu_0)}{x r^+} \delta(x - z).
\]

At tree level the coefficients satisfy \( C = T_a = T_b = 1 \) and using Eq. (20) the first term in Eq. (19) then agrees with the non-singular hard contribution in \( \mathcal{O}(g) \). This simple approximation misses double logarithms in \( T_{a,b}(\mu_0) \) which may be larger than the single logarithms resummed in the \( \alpha_s(\mu_0) \) for \( \mu_0 \approx \sqrt{QX} \).

The one loop expression for \( C(Q, \mu) \) can be found in Eqs. (33), (60) of [4]. The non-perturbative matrix element \( \zeta(E) \) is the reduced soft form factor describing decays to pseudoscalar mesons.

In this paper we proved a factorization formula for heavy-to-light decays including spectator effects. The factorizable pieces are finite and determined by one-dimensional convolutions. The nonfactorizable pieces include non-perturbative gluon effects and satisfy form factor relations. They are not determined by the standard \( k_t \) dependent light-cone meson wave functions, which is different from the conclusion in \([12]\). Our leading order analysis needed the currents \( J^{(1a,1b)} \), unlike the analysis in Refs. \([18, 20]\) where these currents first enter at subleading order.

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