The Quantum Superstring as a WZNW Model with N=2 Superconformal Symmetry

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We present a new development in our approach to the covariant quantization of superstrings in 10 dimensions which is based on a gauged WZNW model. To incorporate worldsheet diffeomorphisms we need the quartet of ghosts \((b_{zz}, c^z, \beta_{zz}, \gamma^z)\) for topological gravity. The currents of this combined system form an \(N=2\) superconformal algebra. The model has vanishing central charge and contains two anticommuting BRST charges, 

\[
Q_S = Q_W + \oint c^z (T_Z^W + \frac{1}{2} T_{Zz}^{\text{top}}) + \gamma^z (B_Z^W + \frac{1}{2} B_{Zz}^{\text{top}}),
\]

where \(\eta_z\) is obtained by the usual fermionization of \(\beta_{zz}, \gamma^z\). Physical states form the cohomology of \(Q_S + Q_V\), have nonnegative grading, and are annihilated by \(b_0\) and \(\beta_0\). We no longer introduce any ghosts by hand, and the formalism is completely Lorentz covariant.
1. Introduction and Summary

In a series of papers [1 - 6] we have presented a new approach to the classic problem of the quantization of the Green-Schwarz superstring preserving manifest super-Poincaré invariance in $D = (9, 1)$. We began with Berkovits’ formulation based on pure spinors [7 - 14], but we relaxed the constraints on these spinors by adding new ghost fields. Then we constructed a nilpotent BRST charge $Q$ by requiring nilpotency of the BRST transformation rules and invariance of the free-field action (the latter requirement is equivalent to imposing holomorphy – or anti-holomorphicity – on the BRST currents: $\bar{\partial} j_z = 0$ is equivalent to $[Q, H] = 0$ according to the Noether theorem). Each time nilpotency did not hold on a given field we added a new ghost. Finally, at some point we introduced by hand a ghost system $b, c_z$ which made the BRST charge nilpotent even though the number of fields was finite [1]. (In the past numerous approaches based on the BV formalism have led to an infinite set of ghosts [15].)

The action $S$, which was BRST invariant, did not yield a vanishing central charge $c$, but introducing by hand another ghost pair $\omega_m, \eta_z^m$ (which was taken to be BRST inert in order not to undo the result $Q^2 = 0$), we also obtained $c = 0$ [1][2]. However, the resulting conformal field theory (constructed in terms of the energy-momentum tensor $T_{zz}$, the BRST current $j^B_z$, the ghost current $J_{gh}^z$ and a composite antighost operator $B_{zz}$) could not be identified with an $\mathcal{N} = 2$ superconformal model, or with a generalization worked out by Kazama [16] (which contains two more generators $F_{zzz}$ and $\Phi_{zzz}$ with conformal spin $3$ and ghost number $-2$ and $-3$, respectively).

To obtain the correct cohomology, we required that vertex operators be BRST invariant. This implemented the constraints at the level of the cohomology. However, the ghost system $b, c_z$ which we had earlier introduced to obtain a nilpotent BRST operator, now turned out to be the cause that the cohomology was trivial. To remedy this defect, the concept of a grading was introduced, and vertex operators were required to have nonnegative grading [2]. These grading conditions were shown to be equivalent to equivariant cohomology [3]. Homological perturbation theory [17] leads to the same results, at least at the classical level (i.e., with only Poisson brackets, or with single contractions): if one removes (co)homology classes by adding new ghosts, one needs in general an infinite set of such ghosts [18], but one may again truncate this series by introducing the $b, c_z$ system. The grading number turned out to be the antifield number (as defined in homological perturbation theory) minus the ghost number [2,6].
The same approach was shown to yield correct results for the superparticle [4], and even for ordinary gauge field theory [5]. It was also shown how to extend this approach to the combined left- and right-moving sector of the superstring [4].

In this article, we first show that if one does not short-circuit the process of constructing a nilpotent BRST charge (and an invariant action) by introducing the ghosts $b, c_z$, but instead goes on implementing BRST nilpotency on each field by adding more ghosts when needed, one ends up with a very simple system: three current multiplets $(J_{g M}^g, J_{gh M}^g, J_{h M}^h)$ with $J_M = (J_m, J_\alpha, J^\alpha)$ which can be viewed as the currents of a WZNW model. Two of these multiplets, namely $(J_{g M}^g, J_{gh M}^g)$, contain the set of fields we found in our earlier work, whereas the third multiplet, $J_{h M}^h$, is associated with the gauging of these WZNW multiplets, and contains three more currents which close the BRST algebra. The algebra on which this model is based is the super-Poincaré algebra in 10 dimensions with a fermionic central extension.

The currents of this model form a Kazama algebra [16], the BRST current is nilpotent and the central charge vanishes. However, the corresponding BRST charge $Q_W$ has too much cohomology because vertex operators depend not only on $x^m, \theta^\alpha, p_{z\alpha}$, but also on $x^{h,m}, \theta^{h,\alpha}$ and $p_{z\alpha}^h$. Moreover, in all our work we have consistently ignored worldsheet diffeomorphisms up till now. Both problems are solved by introducing a quartet $(b_{zz}, c^z, \beta_{zz}, \gamma^z)$ of ghosts for topological gravity. The currents of this model form an $N = 2$ superconformal algebra. The need for such a quartet was discussed in lectures by Dijkgraaf, Verlinde and Verlinde [19] (see also [20]); it provides the parametrization of the moduli of the Riemann surfaces. Combining our WZNW model with the topological gravitational quartet, the properly modified currents of this combined system form an $N = 2$ superconformal algebra [21]. One of these currents is the BRST current $j_B^S = j_B^W + j_{top}^B$, where $j_B^W$ is the BRST current of the WZNW model while $j_{top}^B = \gamma^z b_{zz}$ is the BRST current of the topological gravity. In addition, as for any topological model, there is a second BRST charge which anticommutes with $Q_S$, given by [19]

$$Q_V = \oint c^z \left( T_{zz}^W + \frac{1}{2} T_{zz}^{top} \right) + \gamma^z \left( B_{zz}^W + \frac{1}{2} B_{zz}^{top} \right). \quad (1.1)$$

The main point of this article is the definition and construction of physical states. Physical states correspond to vertex operators which are polynomials in the fields and derivatives thereof, lie in the cohomology of $Q_S + \oint \eta_z + Q_V$, have nonnegative grading, and are annihilated by the zero modes $b_0 = \oint z b_{zz}$ and $\beta_0 = \oint z \beta_{zz}$. The field $\eta_z$ is obtained
by fermionizing the commuting ghosts $\beta_{zz}$ and $\gamma^z$, and the operator $\oint \eta_z$ is added to the BRST charge $Q_S + Q_V$ for the same reasons for which it is added in the RNS formalism [22] working in the large Hilbert space. Since $\{ b_0, Q_S + Q_V + \oint \eta_z \} = \oint (T_{zz}^W + T_{zz}^h)$ and $[\beta_0, Q_S + Q_V + \oint \eta_z] = b_0 + \oint z B_{zz}$, the requirement that $b_0$ and $\beta_0$ annihilate vertex operators puts them on-shell, and (as we shall show) eliminates the doubling in the WZNW model mentioned above.

Thus we have found a covariant formulation of the quantum superstring with the following properties

1) it is based on a WZNW model,

2) it is conformally invariant (it has vanishing central charge),

3) the currents form an $N = 2$ conformal superalgebra,

4) it yields the correct cohomology (checked for the open string in the sector with ghost number one and conformal spin zero, to be published elsewhere),

5) no ghosts are any longer introduced by hand.

Having shown that our previous work is based on a WZNW model suggests that covariant quantum computations in superstring theory may be easier than thought. In particular, the precise form of the measure, which is crucial for tree and loop level computations, may be easier to determine for this formulation with WZNW currents. Perhaps this very general and minimal model can finally shed light on the mysterious classical $\kappa$-symmetry.

Before concluding this introduction, we would like to mention related work. The Padua group has given a derivation of the pure spinor formalism using the complexified superembedding of the twistorial version of GS superstrings [23]. They showed that the pure spinor formalism originates from the superembedding approach to superbranes, namely it arises as a result of a conventional BRST gauge-fixing of a complexified and twisted $N = 2$ worldsheet supersymmetric superembedding of the Green-Schwarz superstring. In the light of our developments, it might be useful to compare the two formalisms to construct the underlying classical gauge invariant action.

An approach to pure spinors which differs from that of Berkovits was begun by Aisaka and Kazama [24]. They factorized the pure spinor constraints into a reducible and irreducible part preserving the subgroup $U(5)$ of the Lorentz group. This factorization leads to a new first class algebra of constraints which yields the BRST charge by the usual construction. The set of new ghost fields needed for implementing the constraints has
vanishing conformal charge. It would be interesting to compare their formalism with the results of the present work where we use the gauged WZNW model to obtain first class constraints.

The paper is organized as follows: in section 2, we discuss the underlying WZNW structure, we derive the Maurer-Cartan forms and we introduce the h-sector of currents. In section 3, we construct the BRST invariant action and point out its relation to the classical Green-Schwarz action and the free field action on which our earlier work is based. In section 4, the underlying conformal field theory is analyzed. In section 5 the quartet for topological gravity is introduced, and its current algebra derived. Finally, in section 6 the definition of physical states is given. The relation to our previous approach is given in section 7. In the conclusions, we discuss open issues and possible future applications.

2. The underlying WZNW structure

We start from Berkovits’ BRST-like charge $Q_B = \oint dz j^B_z(z)$ with $j^B_z(z) = i \lambda^\alpha d_z \alpha$ and $\lambda^\alpha (\alpha = 1, \ldots, 16)$ a real commuting spinor-ghost, but we do not impose the constraints $\lambda^\gamma \gamma^m = 0$. We follow the notation and the definitions of [6]. The operator $d_z \lambda^\alpha = p_z \alpha + i \partial_z x^m (\gamma^m \theta) (\alpha) + \frac{1}{2} (\gamma^m \theta) (\alpha) \gamma_m \partial_z \theta$ (we restrict ourselves to the left-moving sector) yields BRST transformations on $x^m$ and $d_z \lambda^\alpha$ (using $x^m (z) x^n (w) \sim - \eta^{mn} \ln (z - w)$ and $p_z \alpha (z) \theta^\beta (w) \sim \delta^\alpha_\beta / (z - w)$) which are not nilpotent, but become nilpotent if one adds ghosts $\xi^m$ and $\chi^\alpha (and antighosts $\beta_z^m$ and $\kappa^z_\alpha$; the antighost for $\lambda^\alpha$ is $w_z \alpha$). The resulting antihermitian BRST charge reads

$$Q = \oint dz \left( i \lambda^\alpha d_z \alpha - \xi^m \Pi^z_m - \chi^\alpha \partial_z \theta^\alpha - 2 \xi^m (\kappa_z \gamma^m \lambda) - i \beta_z^m \lambda \gamma^m \lambda \right), \quad (2.1)$$

with $\Pi^z_m = \partial_z x^m - i \theta^\gamma \gamma^m \partial_z \theta$.

The first three terms in $Q$ can be written as $-J^q_M c^M$ with $c^M = (\xi^m, \lambda^\alpha, \chi^\alpha)$ and $J^q_M = (\Pi^z_m, -i d_z \alpha, \partial_z \theta^\alpha)$. We view the $J^q_M$ which appear in $Q$ as operators whose classical counterparts are first class constraints which determine the structure constants. From the OPE’s

$$(-id)_{z \alpha} (z)(-id)_{w \beta} (w) \sim -2i \frac{\gamma^m_{\alpha \beta} \Pi^z_{wm} (w)}{z - w}, \quad (-id)_{z \alpha} (z) \Pi^z_m (w) \sim -2 \frac{\gamma^m_{\alpha \beta} \theta^\beta (w)}{z - w},$$

$$\Pi^z_m (z) \Pi^z_{wn} (w) \sim - \frac{1}{(z - w)^2} \delta_m n, \quad (-id)_{z \alpha} (z) \theta^\beta (w) \sim - \frac{i}{(z - w)^2} \delta^\alpha \beta,$$  \quad (2.2)
we can extract an affine Lie algebra
\[ J_M^g(z) J_N^g(w) \sim \frac{J_P^f P_{MN}}{(z-w)} - \frac{\mathcal{H}_{MN}}{(z-w)^2}. \] (2.3)

Conversely, requiring closure of the affine Lie algebra fixes the trilinear term in \( d_{z\alpha} \).

Introducing abstract generators \( T_M = (P_m, Q_\alpha, K^\alpha) \) satisfying \( [T_M, T_N] = T_P f_{MPN} \), we find only two nonvanishing structure constants
\begin{equation}
\{Q_\alpha, Q_\beta\} = -2i\gamma^{m}_{\alpha\beta} P_m, \quad [Q_\alpha, P_m] = -2\gamma^m_{,\alpha\beta} K^\beta.
\end{equation}
Introducing the antighosts \( b_M = (\beta_{z\alpha}, w_{z\alpha}, \kappa_{z\alpha}) \) satisfying \( c^M(z)b_N(w) = \delta^M_N \frac{1}{z-w} \), the BRST charge in (2.4) can be written as
\[ Q = \oint dz \left( -J_M^g c^M - \frac{1}{2} b_M f_{MPN} c^P c^N (-)^N \right) = -\oint dz \left( J_M^g + \frac{1}{2} J_M^{gh} \right) c^M, \] (2.5)
where \((-)^N = +1 \) for \( P_m \) and \((-)^N = -1 \) for \( Q_\alpha, K^\alpha \). The ghost currents
\[ J_M^{gh} = b_M f_{MPN} c^P (-)^N = \left( 2\kappa_z \gamma_m \lambda, 2\xi^m (\gamma_m \kappa_z)_{,\alpha} + 2i\beta_{z\alpha} (\gamma^m \lambda)_{,\alpha}, 0 \right), \] (2.6)
satisfy (2.3) without double poles.

At this point, we make contact with work by Green and Siegel of a decade ago. In [25] it was shown that the Green-Schwarz action can be reformulated as a WZNW model based on a super Lie algebra with abstract generators \( P_m, Q_\alpha, K^\alpha \), which correspond to the left-invariant one-forms \( \Pi_z^m, \partial_z \theta^\alpha \), and \( d_{z\alpha} \) appearing in our work. This super Lie algebra is an extension of the usual \( D = (9, 1) \) super-Poincaré algebra with \( Q_\alpha \) and \( P_m \) to a super Lie algebra where \( K^\alpha \) is a central charge. It is nilpotent, but has a non-degenerate invariant metric [26]. The three current multiplets \( (J_M^g, J_M^{gh}, J_M^{gh}) \) form representations of this super Lie algebra. In the next paragraph we give some details.

A coset approach with unitary \( g = e^{P_m x^m} e^{Q_\alpha \theta^\alpha} e^{K^\alpha \phi_\alpha} \), containing antihermitian \( P_m \), hermitian \( Q_\alpha \) and \( K^\alpha \), satisfying (2.4), and real \( x^m, \theta^\alpha \) and \( \phi_\alpha \), leads to the usual left-invariant one-forms, Lie derivatives and covariant derivatives. In particular the left-invariant one-form \( g^{-1}dg \) corresponding to \( Q_\alpha \) is equal to \( \partial_z \theta^\alpha \), and the one-form corresponding to \( P_m \) is given by \( \Pi_z^m = \partial_z x^m - i\theta \gamma^m \partial_z \theta \). The one-form \( g^{-1}dg \) corresponding to \( K^\alpha \) yields the current \(-id_{z\alpha} = \partial_z \phi_\alpha + 2 \partial_z x^m (\gamma_m \theta)_{,\alpha} - \frac{2}{3} (\gamma_m \theta)_{,\alpha} (\gamma^m \partial_z \theta) \). Defining \( p_{z\alpha} = i\partial_z \phi_\alpha + i\partial_z x^m (\gamma_m \theta)_{,\alpha} + \frac{1}{6} (\gamma_m \theta)_{,\alpha} (\gamma^m \partial_z \theta) \) one obtains the operator \( d_{z\alpha} \) appearing in (2.1), \( d_{z\alpha} = p_{z\alpha} + i\partial_z x^m (\gamma_m \theta)_{,\alpha} + \frac{1}{2} (\gamma_m \theta)_{,\alpha} \gamma_m \partial_z \theta \) where \( p_{z\alpha}, \theta^\alpha \) and \( x^m \) are free fields.
(see below). The Lie derivative $K^\alpha = \partial/\partial \phi_\alpha$ can be represented as $K^\alpha = -i \oint dz \partial_z \theta^\alpha$ when $\theta^\beta(z) \phi_\alpha(w) \sim i \delta^\beta_\alpha \ln(z - w)$, but it vanishes (being an integral of a total derivative) when it acts in the space which contains only $\partial_z \phi_\alpha$. This is due to the fact that $K^\alpha$ generates constant shifts of the coordinate $\phi_\alpha$, but only $\partial_z \phi_\alpha$ (identified with the conjugate momentum of $\theta^\alpha$) appears in the theory.

The generator for rigid spacetime supersymmetry $\oint dz \, q_{z\alpha}$ can be determined by requiring that it leaves $\Pi^m_z, \partial_z \theta^\alpha,$ and $d_{z\alpha}$ invariant. It is given by $q_{z\alpha} = p_{z\alpha} - i \partial_z x^m (\gamma_m \theta)_\alpha - \frac{1}{6}(\gamma_m \theta)_\alpha (\theta \gamma^m \partial_z \theta) \, \mathbb{I}$, which takes on a very simple form in terms of $x, \theta$ and $\phi$, namely $q_{z\alpha} = i \partial_z \phi_\alpha$. Since we only use composite operators which are susy invariant, $N = (1, 1)$ spacetime susy is manifestly maintained at all stages.

The currents $J^g_M(z)$ in (2.5) are related to the left-invariant one-forms $g^{-1}dg = T_M J^g_M$ given by $J^g_M = (\Pi^m_z, \partial_z \theta^\alpha, -i d_{z\alpha})$. They satisfy $J^g_M = H_{MN} J^g_N$ where $H_{mn} = \eta_{mn}$, $H^0_\alpha = H^0_\alpha = \delta^\beta_\alpha$. The matrix $H_{MN}$ is not an invariant metric. The easiest way to see this is to take its inverse $H^{MN}$ and to construct the bilinear expression $T_N T_M H^{MN} = P_m P^m$; since this operator does not commute with all generators, the matrix $H^{MN}$ is not an invariant matrix. One can understand this by noting that the explicit expressions for the left-invariant one-forms depend on the basis for the group. Taking for example $g = e^{Q_\alpha \theta^\alpha} e^{P_m x^m} e^{K^\alpha \phi_\alpha}$ one finds$^4$

$$
 g^{-1}dg = P_m \Pi^m + Q_\alpha d\theta^\alpha + K^\alpha \left[ d\phi_\alpha - 2 x^m (\gamma_m \theta) - \frac{2i}{3} (\gamma^m \theta) (\theta \gamma_m \theta) \right]. \tag{2.7}
$$

This expression differs from $T_M J^g_M$, note the bare $x$. For given $J^g_M$ and different parametrization of $g$ also $J^g_M$ will be different, and in this sense $H_{MN}$ is basis-dependent. Taking different bases is like choosing a gauge: the physical results (cohomology) should be basis-independent.

In order to construct the WZNW action we do need an invariant metric. One can find an invariant metric $H^{MN}$ by constructing the Casimir operator$^5$ $C_2 = T_N T_M H^{MN}(-)^N$.  

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4 Rewriting this group element as $g = e^{P_m x^m} e^{Q_\alpha \theta^\alpha} e^{K^\alpha (\phi_\alpha + 2 x^m (\gamma_m \theta)_\alpha)}$, the result in (2.7) follows.

5 One may check that the sign $(-)^N$ in the Casimir operator is needed by working out the case of $Osp(1|2)$. The generators are the bosonic generators $(J_3, J_+, J_-)$ of $Sp(2) = SU(1, 1)$, and the two fermionic generators $(Q_+, Q_-)$. They satisfy $[J_3, J_+] = J_+, [J_3, Q_+] = \frac{1}{2} Q_+$, $[J_+, J_-] = 2 J_3$ and $[J_\pm, Q_\pm] = -Q_\pm$. The anticommutators are $\{Q_+, Q_- \} = \frac{1}{2} J_3$ and $\{Q_\pm, Q_\pm \} = \pm \frac{1}{2} J_\pm$. The Casimir operator is $C_2 = J_3^2 + \frac{1}{2} (J_+ J_- + J_- J_+) - (Q_+ Q_- - Q_- Q_+)$. Assuming that $C_2$ is given by the formula in the text with the factor $(-)^N$ one finds on the basis $(J_3, J_+, J_-, Q_+, Q_-)$ that
It is in our case given by \( P_m P_m + 2i Q_{\alpha} K^\alpha \). One can also use the metric \( H_{MN} \) which is given by the central charges in (2.3). This is also an invariant metric and it is the inverse of the metric in the Casimir operator as we now show. Defining an inner product for generators \((T_M, T_N) = H_{MN}\), invariance of this inner product under adjoint transformations leads to the relation \(-([T_M, T_P], T_N) + (T_M, [T_P, T_N]) = 0\). In terms of \( H_{MN} \) this reads \(-H_{RN} f^R_{MP} + H_{MR} f^R_{PN} = 0\). Raising indices with \( H_{MN} \) yields \(-f^A_{PM} H^{MB} + f^B_{PN} H^{NA} = 0\), and this equation is also the equation one gets if one requires that \( C_2 \) commutes with \( T_P \). The final result reads

\[
H_{MN}^{\alpha\beta} = \begin{pmatrix}
\eta^{mn} & 0 & 0 \\
0 & 0 & i\delta^\alpha_\beta \\
0 & -i\delta^\alpha_\beta & 0
\end{pmatrix}, \quad H_{MN} = \begin{pmatrix}
\eta_{mn} & 0 & 0 \\
0 & 0 & i\delta^\alpha_\beta \\
0 & -i\delta^\alpha_\beta & 0
\end{pmatrix}. \quad (2.8)
\]

For semisimple super Lie algebras one may use the supertrace to construct an invariant metric: \( \text{str}(\text{Jc}) = \text{str}(T_M T_N) J^N c^M \) where the supertrace \( \text{str}(T_M T_N) \equiv H_{MN} \) is nonvanishing. We have not found an anti-de Sitter extension for the set \( T_M \), so we could not have used a Wigner-Inönu contraction to obtain \( H_{MN} \).

For our considerations it is useful to keep track of reality properties. Note that \( J^g_M, J^{g,M}_M, c^M \), and \( J^g_M c^M \) are hermitian, \( T_M J^g M \) is antihermitian, but \( c = T_M c^M \) and \( b_M \) have no definite reality properties. The ghost currents \( J^{gh}_M \) have the same reality properties as the gauge currents \( J^g_M \).

Classically (i.e., taking only single contractions into account) \( Q \) is nilpotent. However, it fails to be nilpotent when acting on the antighosts \( b_M \). This is due to the double poles in the current algebra (due to derivatives of simple contractions: there are no double contractions) generated by \( J^g_M \), whereas the current algebra generated by the ghost currents \( J^{gh}_M \) is the block-diagonal matrix \((1, 2\tau_1, \tau_2, -i\tau_2)\), and thus \( H_{MN} \) is the block-diagonal matrix \((1, 2\tau_1, i\tau_2)\). This agrees with the Killing metric \( H_{MN} = c f^P_{MQ} f^Q_{NP} (-)^P \) with \( c = 2/3 \). (The factor \((-)^P \) in this expression is needed because the contraction of the indices \( P \) does not follow our northeast-southwest convention). The only simple superalgebras with nondegenerate Killing metric are \( SU(m|n) \) for \( m \neq n \), \( Osp(m|n) \) except \( Osp(2m|2m + 2) \), and \( F(4) \) and \( G(3) \) [27]. In our case we are of course dealing with a nilpotent algebra (all triple-(anti)commutators vanish), but we have explicitly exhibited an invariant metric.

More specifically, the BRST transformation of \( \kappa^\alpha_z \) is proportional to \( J^{g,\alpha}_z \) but the BRST transformation of \( J^{gh}_z \) is due to the double pole in \( J^{gh}_z(z) J^{g,\alpha}_z(w) \). Similarly for \( \beta^m_z \) and \( w_{z\alpha} \).
$J_M^h$ does not have double poles. Following the usual treatment of gauged WZNW models and the corresponding susy coset models \[28\], we introduce new hermitian currents\[8\] which correspond to $J_M^h = (\Pi_{zm}, -id_{z\alpha}, \partial_z \theta^\alpha)$. We determine their transformation rules in the following way: we add these new currents to the BRST transformation laws of the antighost fields and then we require nilpotency on the antighosts. From (2.11) and requiring nilpotency of (2.12) we deduce that the only nontrivial OPE’s\[10\] on all fields, and the iterative construction which we followed to obtain a nilpotent BRST charge, terminates at this early point. The result is\[11\] we obtain

\[
\begin{align*}
\{Q, J^h_{z\alpha}\} &= -i\partial_z \lambda^\alpha, \\
\{Q, J^h_{zm}\} &= -\Pi_{zm} - 2\kappa_{z\alpha} \gamma_m \lambda - J^h_{zm}, \\
\{Q, w_{z\alpha}\} &= i d_{z\alpha} - 2i\beta_{zm} (\gamma^m \lambda)_{\alpha} - 2\xi^m (\gamma_m \kappa_{z})_{\alpha} - J^h_{zm}.
\end{align*}
\]

(2.10)

(2.11)

It turns out that without further ghost fields and generators all BRST transformations are also nilpotent on the $h$-currents. Hence, the BRST transformations are now nilpotent on all fields, and the iterative construction which we followed to obtain a nilpotent BRST charge, terminates at this early point. The result is $Q = \oint dz \bar{J}^B_\bar{z}$ with

\[
\bar{J}_\bar{z}^B = -\lambda^\alpha \left(-id_{z\alpha} + J^h_{z\alpha}\right) - \xi^m \left(\Pi_{zm} + J^h_{zm}\right) - \chi^\alpha \left(\partial_z \theta^\alpha + J^h_{zm}\right) + 2\xi^m (\gamma_m \lambda)_{\alpha} - i\beta_{zm} \lambda^m \lambda + \gamma_{zm} \lambda^m \lambda.
\]

(2.12)

From (2.11) and requiring nilpotency of (2.12) we deduce that the only nontrivial OPE’s of the $h$-currents are given by

\[
\begin{align*}
J^h_{z\alpha}(z)J^h_{w\beta}(w) &\sim -2i\gamma^m_{\alpha\beta} J_{zm}(w) / (z - w), \\
J^h_{zm}(z)J^h_{wm}(w) &\sim -\frac{1}{(z - w)^2} \eta_{mn}, \\
J^h_{z\alpha}(z)J^\beta_{w\gamma}(w) &\sim \frac{i}{(z - w)^2} \delta^\alpha_{\beta}.
\end{align*}
\]

(2.13)

Note that the sign of the double poles in the OPE’s for $J_M^h$ is opposite to the sign of the double poles for the currents $J_M^h$. This will play a role below.

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7 For a generic WZNW model the ghost currents $J_M^h$ produce double poles due to double contractions and these are needed to cancel the double poles of the other currents $J_M^h + J_M^h$. For $J_M^h$ only the OPE $J_M^h(z)J_M^h(w) \sim -2i\gamma^m_{\alpha\beta} J_{zm}(w) / (z - w)$ is nonvanishing.

8 Recall that integrating out the gauge fields $A_z$ and $\bar{A}_z$ for the diagonal maximal subgroup, one obtains a Jacobian which can be exponentiated to yield another WZNW model for this subgroup \[28\].
3. The action

The BRST charge $Q$ obtained above corresponds for the left-moving sector to the BRST charge for a WZNW model on $G \times G/G$ where the maximal diagonal subgroup $G$ is gauged. $G$ is the superalgebra in $D = 3, 4, 6, 10$ dimensions generated by $(P_m, Q_{\alpha}, K^\alpha)$.

The action for this WZNW model is given by

$$S_g = \int d^2x \frac{1}{2} \eta^{\mu\nu} J^g_{\mu N} J^g_{\nu M} \mathcal{H}^{NM} + k \int d^3x \epsilon^{\mu\nu\rho} J^g_{\mu M} J^g_{\nu N} J^g_{\rho R} \mathcal{F}^{RNM} =$$

$$= \int d^2x \left( \frac{1}{2} \eta_{mn} J^g_{\mu} J^g_{\nu} + i J^g_{\alpha \mu} J^g_{\alpha \nu} \right) \eta^{\mu\nu}$$

$$+ 2i \int d^3x \epsilon^{\mu\nu\rho} \gamma_{m\alpha\beta} \left( J^g_{\mu} J^g_{\nu} J^g_{\rho} \right),$$

where the indices of the structure constants have been raised with the invariant metric and the constant $k$ has been chosen such that $\theta$ will be left-moving. Since the Maurer-Cartan equations $dJ^g = -\frac{1}{2} f^N_{MN} J^g_P J^g_N$ imply that $2 \gamma_{m\alpha\beta} J^g_{[\mu \nu]} \gamma_{\sigma\beta} = -\partial_{[\mu \nu \beta]}$, the second term in $S_g$ can be written as a two-dimensional integral, and the two terms with $\eta^{\mu\nu}$ and $\epsilon^{\mu\nu}$ combine into the chiral combination $i J^g_{z\alpha} J^g_{\bar{z} \alpha}$. We can, of course, always add a term, $-\frac{1}{2} \eta_{mn} J^g_{z\mu} J^g_{\bar{z} \nu} \epsilon^{\mu\nu}$ to the action as it vanishes, and obtain then also a chiral expression for the terms with $J^g_{z\mu}$. The result reads

$$\mathcal{L}_g = \frac{1}{2} \Pi_m^m \Pi_n^n \eta_{mn} + d^{(\phi)}_{z\alpha} \bar{\partial} \theta^\alpha,$$

where $d^{(\phi)}_{z\alpha} = i J^g_{\phi\alpha}$ still depends on $\phi$, see section 2.

We now replace $J^g_{z\alpha}$ by $-i d_{z\alpha}$ which amounts to replacing $\partial_z \phi_{\alpha}$ by $-i p_{z\alpha} + \ldots$, as explained in section 2. Substituting the explicit expressions for $d_{z\alpha}$ and $\Pi^m_z$, the action becomes the free-field action from which we started in our previous work

$$\mathcal{L} = \frac{1}{2} \partial_z x^m \bar{\partial}_z x^n \eta_{mn} + p_{z\alpha} \bar{\partial}_z \theta^\alpha,$$

with $\partial_z = \partial_\sigma - i \partial_\tau$ and $\bar{\partial}_z = -\partial_\sigma - i \partial_\tau$. We therefore discover at this point that our previous work [1 - 6] was based on a WZNW model. The original WZNW model in terms of $\phi, \theta$ and $x$ is a complicated interacting theory, but by introducing the variable $p_{z\alpha}$ (also a complicated expression in terms of $\phi, \theta$ and $x$), one obtains a free-field action. Thus $p, \theta$ and $x$ form a free-field realization of the affine Lie algebra.

The replacement of $\partial_z \phi_{\alpha}$ by $-i p_{z\alpha} + \ldots$ can be justified as follows. The fields in the BRST operator are on-shell and on-shell $\bar{\partial}_z \phi_{\alpha} = 0$. In that case one can solve $\phi_{\alpha}$ in terms
of $p_{z\alpha}$ and replacing $\phi_\alpha$ by $p_{z\alpha}$ amounts to a change of basis. In the WZNW action (3.1), on the other hand, $\bar{\partial}_z \phi_\alpha$ is nonvanishing, but $\phi_\alpha$ only appears in the combination $\partial_\mu \phi_\alpha \partial^\mu \theta^\alpha$. This expression may again be replaced by $p_\mu \phi_\alpha$ since $p_\mu$ can be decomposed into a gradient $\partial_\mu \phi_\alpha$ and a curl $\epsilon_\mu \nu \partial^\nu \phi_\alpha$, and the latter is pure gauge.

Our approach also gives a geometrical interpretation of Berkovits’ approach. Gauging the generator $Q_\alpha$ is classically equivalent to setting the current $J^g_\alpha = -id_\alpha$ in (3.1) equal to zero, and this yields the classical Green-Schwarz action. A more group theoretical way to set $J^g_\alpha = 0$ involves a group contraction. The Lie algebra generated by $P_m, Q_\alpha, K^\alpha$ has an outside automorphism

$$[G, Q_\alpha] = \frac{1}{2} Q_\alpha, \quad [G, P_m] = P_m, \quad [G, K^\alpha] = \frac{3}{2} K^\alpha.$$

(3.4)

It leads to the grading of the ghosts $\lambda^\alpha, \xi^m$ and $\chi_\alpha$ which we used in our earlier articles to define the cohomology [2]. In addition, one can introduce a contraction parameter $R$ as follows

$$\{Q_\alpha, Q_\beta\} = -2i \gamma^m_{\alpha\beta} P_m, \quad [Q_\alpha, P_m] = -2 R \gamma^m_{\alpha\beta} K^\beta.$$

(3.5)

The currents $J^{g,M}$ become now $R$-dependent, but evaluating the Wess-Zumino term $\mathcal{H}_{MN} J^{g,N} J^{g,M} J^{g,R}$ one finds that it is $R$-independent. In the kinetic term $\mathcal{H}_{MN} J^{g,N}_\mu J^{g,M}_{\nu} \eta^{\mu\nu}$, the one-form associated with $K^\alpha$ becomes $R$-dependent

$$dz^\alpha = i \partial_z \phi_\alpha + 2iR \partial_z z^m (\gamma^m \theta)_\alpha + \frac{2R}{3} (\gamma^m \theta)_\alpha (\theta \gamma^m \partial_z \theta),$$

(3.6)

while $\mathcal{H}_{MN}$ acquires a factor $1/R$ in the fermionic sector. If one defines $p_{z\alpha}$ such that $dz^\alpha$ becomes $R$-independent (for example by choosing for $p_{z\alpha}$ the expression given in section 1), one can take the limit $R \to \infty$ and obtains the classical Green-Schwarz action.

At the quantum level the constraint $d_\alpha = 0$ is implemented by Berkovits’ BRST charge $Q_B = \oint i \lambda^\alpha d_\alpha$. The condition $Q_B|\psi\rangle = 0$ would be the natural condition for gauging $J_\alpha$, but since $Q_B$ is not nilpotent, one must impose the pure spinor constraint $\lambda \gamma^m \lambda = 0$ in his approach. In conventional gauged WZNW models one can only gauge a subalgebra. In the present case one can only gauge $K^\alpha, P_m$ or $\{K^\alpha, P_m\}$ in each sector of $G \times G$ since only they generate a proper Lorentz-invariant subalgebra, or the whole of the diagonal subgroup $G$ in $G \times G$. Taking the latter case the gauging of $G$ leads to the multiplet of currents $J^h_M$, and as action for these currents we take

$$S_h = - \int d^2 z \left( \frac{1}{2} J^h_{zm} J^{hm}_z + i J^h_{za} J^{ha}_z \right).$$

(3.7)
The minus sign in front of this action amounts to changing the level \( k = 1 \) into \( k = -1 \). The propagators of \( x^h, \theta^h \) and \( p^h_z \) have an extra minus sign, and if the \( h \)-currents in terms of \( h \)-coordinates differ from the corresponding \( g \)-currents by an extra overall minus sign, then one obtains (2.13) with the same structure constants as for the \( g \)-currents, but with an extra minus sign for the double poles. Without the currents \( J^h_M \) nilpotency of \( Q \) requires further terms depending on the ghosts \( b \) and \( c_z \), but with \( J^h_M \) the double poles due to \( J^g_M \) are cancelled, and no \( b,c_z \) terms are needed for nilpotency.

4. Conformal Field Theory

Having obtained a WZNW formulation, we can study its properties as a conformal field theory. One can construct the energy-momentum tensor \( T_{zz} \)

\[
T_{zz} = -\frac{1}{2} \partial_z x^m \partial_z x_m - p_{z\alpha} \partial_z \theta^\alpha - \beta_{zm} \partial_z \xi^m - \kappa_z^\alpha \partial_z \chi^\alpha - w_{z\alpha} \partial_z \lambda^\alpha
\]

\[
+ \frac{1}{2} J^h_z J^h, z + i J^h_{z\alpha} J^h, \alpha.
\]

The first two terms can be rewritten as \(-\frac{1}{2} \Pi_{mz} \Pi^m_z - d_{\alpha} \partial_z \theta^\alpha\). (Since the Killing-Cartan metric vanishes, the prefactor in the Sugawara construction of the energy-momentum tensor in the \( h \)-sector equals unity.) Since the action is a free action, it is easy to check that the conformal charge is zero. Explicitly: the \( c = 10 - 32 - 20 + 32 + 32 = 22 \) of the sector with \( J^g_M \) and \( J^h_M \) is cancelled by the \( c = 10 - 32 \) of the sector with \( J^h_M \). The ghost current is given by

\[
j_z^{gh} = -\beta_{zm} \xi^m - \kappa^\alpha_z \chi^\alpha - w_{z\alpha} \lambda^\alpha.
\]

Since the anomaly in the OPE \( j_z^{gh}(z) j_w^{gh}(w) = c_j/(z-w)^2 \) of the ghost current with itself is not zero but given by \( c_j = -22 \), while \( T_{zz}(z) j_w^{gh}(w) = 22/(z-w)^3 + j_z^{gh}(z)/(z-w)^2 \), this superconformal algebra seems to be twisted.

In addition to the BRST current \( j^B_z \) given in \( 2.12 \) and satisfying

\[
j^B_z(z) j^B_w(w) \sim 0,
\]

there is another fermionic operator \( B_{zz} \) dual to the BRST current, obtained by interchanging ghosts and antighosts \[29\] and taking the difference of the \( g \)-currents and \( h \)-currents \[30\]

\[
B_{zz} = -\frac{i}{2} \kappa^\alpha_z \left( -id_{z\alpha} - J^h_{z\alpha} \right) + \frac{1}{2} \beta_{zm} \left( \Pi_{zm} - J^h_{zm} \right) + \frac{i}{2} w_{z\alpha} \left( \partial_z \theta^\alpha - J^h_{z\alpha} \right).
\]
It is an antihermitian spin 2 operator, and satisfies
\[ j^B_z(z)B_{ww}(w) \sim \frac{-22}{(z-w)^3} + \frac{j^{gh}_w(w)}{(z-w)^2} + T_{ww}(w), \quad T_{zz}(z) = \{Q, B_{zz}(z)\}, \quad (4.5) \]
as well as
\[ B_{zz}(z)B_{ww}(w) \sim \frac{F_{www}}{(z-w)}, \quad (4.6) \]
where
\[ F_{zzz}(z) = -i\beta^m_z \kappa_z \gamma_m (\partial_z \theta^\alpha + J^h_\alpha_z) + \frac{i}{2} (\kappa_z \gamma_m \kappa_z) (\Pi^m_z + j^h_m). \quad (4.7) \]
The current \( F_{zzz} \) is not only BRST closed, \( j^B_z(z)F_{www}(w) = 0 \), but even BRST exact
\[ j^B_z(z)\Phi_{www}(w) \sim \frac{F_{www}(w)}{(z-w)}, \quad \Phi_{zzz} = -\frac{i}{2} \beta^m_z \kappa_z \gamma_m \kappa_z. \quad (4.8) \]
The six currents \( j^B_z, B_{zz}, j^{gh}_z, T_{zz}, F_{zzz}, \Phi_{zzz} \) generate a closed algebra which has the form of a Kazama algebra. This is expected: quantization of gauged WZNW models generically leads to Kazama algebras instead of N=2 superconformal algebras \cite{16} \cite{31}.

In our earlier work with \( b, c_z \) present, we only partially succeeded in constructing an operator \( B_{zz} \) with the correct properties \cite{1}, but the expression in (4.4) satisfies all the desired properties.

5. The gravitational topological Koszul quartet

Consider a quartet \((b_{zz}, c^z, \beta_{zz}, \gamma^z)\) containing the usual spin \((2, -1)\) gravitational ghosts, and spin \((2, -1)\) commuting counterparts \((\beta_{zz}, \gamma^z)\). The propagators are
\[ c^z(z)b_{ww}(w) \sim \frac{1}{z-w}, \quad \gamma^z(z)\beta_{ww}(w) \sim \frac{1}{z-w}. \quad (5.1) \]
From these fields we construct the energy-momentum tensor \( T_{zz} \), the BRST current \( j^B_z \), the ghost current \( J^{gh}_z \), and an anticommuting spin 2 current \( B_{zz} \)
\[ T_{zz} = -2\beta_{zz} \partial_z \gamma^z - \partial_z \beta_{zz} \gamma^z - 2b_{zz} \partial_z c^z - \partial_z b_{zz} c^z, \quad (5.2) \]
\[ j^B_z = -b_{zz} \gamma^z, \quad J^{gh}_z = -b_{zz} c^z - 2\beta_{zz} \gamma^z, \]
\[ B_{zz} = 2\beta_{zz} \partial_z c^z + c^z \partial_z \beta_{zz} + \mu b_{zz}. \]
The real constant $\mu$ will be fixed later. As usual $b_{zz}$ and $c^z$ have ghost number $-1$ and $+1$, respectively, but $\beta_{zz}$ and $\gamma^z$ have ghost number $-2$ and $+2$, respectively. Hence, $B_{zz}$ has ghost number $-1$.

The OPE’s of these currents constitute a closed superconformal algebra

$$T_{zz}(z)T_{ww}(w) \sim \frac{2T_{ww}(w)}{(z-w)^2} + \frac{\partial_z T_{zz}}{(z-w)},$$

$$T_{zz}(z)j^B_z(w) \sim \frac{j^B_z(w)}{(z-w)^2} + \frac{\partial_z j^B_z}{(z-w)},$$

$$T_{zz}(z)B_{ww}(w) \sim \frac{2B_{ww}(w)}{(z-w)^2} + \frac{\partial_z B_{zz}}{(z-w)},$$

$$T_{zz}(z)J^g_{gw}(w) \sim \frac{3}{(z-w)^3} + \frac{J^g_{gw}(w)}{(z-w)^2} + \frac{\partial_z J^g_{gw}}{(z-w)},$$

$$J^g_{gw}(z)J^g_{gw}(w) \sim \frac{-3}{(z-w)^2},$$

$$j^B_z(z)B_{ww} \sim \frac{j^B_z(w)}{(z-w)} + \frac{T_{ww}}{(z-w)^2},$$

$$j^B_z(z)J^g_{gw}(w) \sim 0, \quad B_{zz}(z)B_{ww}(w) \sim 0,$$

$$J^g_{gw}j^B_z(w) \sim \frac{j^B_z(w)}{(z-w)} - \frac{B_{ww}(w)}{(z-w)},$$

The absence of an anomaly in the OPE of $T_{zz}$ with itself indicates that we are dealing with a twisted $N = 2$ algebra. It is clear that $T_{zz}, j^B_z$, and $B_{zz}$ are primary fields for any value of $\mu$, but the ghost current $J^g_{gw}$ has an anomaly $+3$, which is opposite to the anomaly in $J^g_{gw}(z)J^g_{gw}(w)$ and $j^B_z(z)B_{ww}(w)$. Furthermore, the BRST current and the $B$ field are nilpotent, while $j^B_zB_{zz}(z)$ reproduces $T_{zz}$ and $J^g_{gw}$.

We now add the currents $T^W_{zz}, j^B_z,W, B^W_{zz}, J^g_{gw},W$ and $F^{W}_{zzz}, \Phi^W_{zzz}$ of the Kazama algebra for the WZNW model to the currents of the topological gravitational model. As shown in $[30]$, one then ends up with an $N = 2$ superconformal algebra, provided one modifies the $B_{zz}$ field suitably. The properly modified currents for the sum of both systems are given by

$$\dot{T}_{zz} = T^W_{zz} + T_{zz}^{top}, \quad \dot{j}^B_z = j^B_z,W + j^B_z^{top},$$

$$\dot{J}^g_{gw} = J^g_{gw},W + J^{g, top},$$

$$\dot{B}_{zz} = B^W_{zz} + B^{top}_{zz}(\mu = 1) - \frac{1}{2} c^z F^W_{zzz} - \frac{1}{2} \gamma^z \Phi^W_{zzz}.$$
The currents of the Koszul quartet in (5.2) are denoted here by the superscript \( \text{top} \), while the currents of the WZNW model are denoted by the superscript \( W \) and can be found in eqs. (2.12), (4.1), (4.2), and (4.4). In particular we recall the relations

\[
\tilde{j}^B_w(z)B^W_w(w) \sim \frac{-22}{(z-w)^3} + \frac{j^{gh}_w(w)}{(z-w)^2} + \frac{T^W_{ww}(w)}{(z-w)},
\]

\[
B^W_{zz}(z)B^W_{ww}(w) \sim \frac{F^W_{wwww}(w)}{(z-w)},
\]

\[
j^B_z(z)\Phi^W_{wwww}(w) \sim \frac{F^W_{zzzz}(w)}{z-w}, \quad \tilde{j}^B_z(z)F^W_{wwww}(w) \sim 0.
\]

It is now straightforward to verify that the currents in (5.4) satisfy the same algebra as the currents in (5.2). For example, the conformal anomaly in the WZNW model cancels between the \( g \)-currents, the ghost-currents and the \( h \)-currents, while the Koszul quartet has no conformal anomaly, being topological. Furthermore,

\[
\hat{T}^B_{zz}(z), \hat{j}^{gh}_w(w) \sim \frac{(22+3)}{(z-w)^3} + \frac{j^{gh}_w(w)}{(z-w)^2} + \frac{\partial_w j^{gh}_w(w)}{(z-w)},
\]

\[
\hat{j}^{gh}_z(z)\hat{j}^{gh}_w(w) \sim \frac{-22 - 3}{(z-w)^2},
\]

confirming that the algebra is twisted. Less obvious are the OPE’s involving \( \hat{B}_{zz} \), but they are of the form (5.2), too. For example, in

\[
\hat{j}^B_z(z)\hat{B}_{ww}(w) \sim \frac{-22 - 3}{(z-w)^3} + \frac{j^{gh}_z(w)}{(z-w)^2} + \frac{\hat{T}_{ww}(w)}{(z-w)},
\]

the \( \Phi^W_{zzzz}(z) \) and \( F^W_{zzzz}(z) \) terms cancel. The most interesting case is \( \hat{B}_{zz}(z)\hat{B}_{ww}(w) \) which should vanish and does vanish. Another good check on the \( F^W_{zzzz} \) and \( \Phi^W_{zzzz} \) terms in \( \hat{B}_{zz} \) is \( \hat{j}^B_z(z)\hat{B}_{ww}(w) \) which should be independent of \( F^W_{zzzz} \) and \( \Phi^W_{zzzz} \); this is indeed the case.

6. Definition of Physical States

Having constructed the BRST charge \( Q_W \) according to the quantization prescription for WZNW models, we must now define the physical states. It is easy to see that the cohomology of \( Q_W \) by itself does not yield the correct spectrum for the superstring. As we
shall discuss in more detail below, the field equations for the cohomology depend on the coordinates $x + x^h$, $\theta + \theta^h$ and $p_{z\alpha} + p_{z\alpha}^h$, while the dependence on the coordinates $x - x^h$, $\theta - \theta^h$ and $p_{z\alpha} - p_{z\alpha}^h$ is not fixed. Thus, we have to follow a different path.

In purely topological models, there exists a second BRST charge, $Q_V$ given in the introduction, namely

$$Q_V = \oint c^z \left( T^W_{zz} + \frac{1}{2} T^{top}_{zz} \right) + \gamma^z \left( B^W_{zz} + \frac{1}{2} B^{top}_{zz} \right).$$

(6.1)

where the currents $T^W_{zz}, T^{top}_{zz}, B^W_{zz}$ and $B^{top}_{zz}$ are given in the previous section. As can be easily checked the above BRST charge anticommutes with the BRST charge

$$Q_S = Q_W + \oint \eta_z e^\phi b_{zz} + \oint \eta_z ,$$

(6.2)

where the bosonic ghosts $\gamma^z$ and $\beta_{zz}$ are fermionized in the usual way, namely $\gamma^z = \eta_z e^\phi$ and $\beta_{zz} = \partial_z \xi e^{-\phi}$ with $\xi(z)\eta_w(w) \sim (z-w)^{-1}$ and $\phi(z)\phi(w) \sim -\ln(z-w)$ (see for example [32]). The operator $\oint \eta_z$ is added for the same reasons as in the RNS formalism working in the large Hilbert space containing the zero mode $\xi_0$. The BRST charge is the sum of the BRST charges in the matter sector and in the topological sector,

$$Q = Q_S + Q_V ,$$

(6.3)

The physical states are therefore identified by the BRST cohomology $Q$ and by the grading condition formulated in [2]. Neglecting the topological gravity sector, it is easy to show that one recovers the correct equations of motion for the massless sector of open and closed string theory. (Essentially, the formulas for the equations of motion are given in [2] and they are related to the present ones by a similarity transformation on the superfields, $\tilde{A}_\alpha = e^R A_\alpha$, etc... where $R = \theta^\alpha D^h_\alpha + (x^m - i\theta^{\alpha m} \theta^h) \partial^h_m$). The complete analysis of the cohomology in the topological gravity sector and in the matter sector follows the description given by [19] and it will be given a separate publication.

The physical states are obviously defined up to gauge transformations, given by exact BRST vertex operators. This gauge freedom allows us to impose some gauge fixing conditions to remove the redundancy in the definition of physical states. Two important examples in the literature are the bosonic string and the fermionic string in the RNS formulation. In the bosonic string one imposes the Siegel condition: $b_0|\text{phys}\rangle = 0$. In the RNS string one imposes the superpartner condition $\beta_0|\text{phys}\rangle = 0$. In the RNS string one
has also to remember that in order to have a non-trivial cohomology one has to require that vertices are polynomials in the zero mode of the superghost $\gamma_0$.

In our formalism, in order to gauge completely the model, we introduced in the previous section a replica of our the coordinates $x^m, \theta^\alpha$ and $p_{z\alpha}$. The physical states will therefore depend on the combination not annihilated by the BRST charge $Q_S$. In order to remove this redundancy one has to impose a gauge fixing condition and following the suggestion of the bosonic string, we impose the condition $B_0|\text{phys}\rangle = 0$ where $B_0 = \oint z B_{zz}$. The requirement $B_0|\text{phys}\rangle = 0$ is imposed by hand at this point, and one also expects that one should impose $b_0|\text{phys}\rangle = 0$ and $\beta_0|\text{phys}\rangle = 0$ as in the RNS framework. Imposing all three conditions seems too much. Fortunately, as we now show, one of these conditions follows from the others.

First of all, we observe that

$$\left\{ b_0, Q_S + Q_V + \oint \eta_z \right\} = \hat{L}_0 , \quad (6.4)$$

where $\hat{L}_0 = \oint (T_{zz}^W + T_{zz}^{\text{top}})$ is the Virasoro generator of the combined system. Therefore, imposing the gauge fixing condition $b_0|\text{phys}\rangle = 0$ we obtain the usual constraint on the physical states $\hat{L}_0|\text{phys}\rangle = 0$. The gauge fixing might be formulated in string field theory context as the Siegel gauge.

On the other hand, we can also fix the gauge symmetry (notice that one gauge invariance is generated by $Q_V$ and the other by $Q_S$) by the gauge choice $\beta_0|\text{phys}\rangle = 0$. Using the fact that

$$\left[ \beta_0, Q_S + Q_V + \oint dz \eta_z \right] = b_0 + B_0 , \quad (6.5)$$

we finally deduce the condition $B_0|\text{phys}\rangle = 0$. The latter condition removes the dependence on the combinations $J^g_M - J^h_M$, not fixed by the BRST charge $Q_W$.

The result of our analysis is the definition of physical states given in the introduction.

7. Equivalence with our former approach

The present new formulation of the superstring in terms of a WZNW model prompts us to ask how it is related to our earlier work [1 - 6] with the $b, c_z$ multiplet. Consider the composite operator which contains the $h$-currents in (2.12)

$$J_z = -\lambda^\alpha J^h_{z\alpha} - \xi^m J^h_{zm} - \chi_\alpha J^h_{z,\alpha} . \quad (7.1)$$
Due to the statistics of the ghosts $\xi^m, \lambda^\alpha, \chi_\alpha$, the OPE of two of these currents contains only first-order poles
\[
\mathcal{J}_z(w)\mathcal{J}_w(w) \sim -\frac{\xi^m \partial_w \xi_m + i\chi_\alpha \partial_w \lambda^\alpha - i\lambda^\alpha \partial_w \chi_\alpha - 2i(\lambda \gamma^m \lambda)J^h_m + 4\xi^m (\lambda \gamma_m J^h)}{z - w}. \tag{7.2}
\]
Furthermore, the new current
\[
S_z = \xi^m \partial_z \xi_m - i\chi_\alpha \partial_z \lambda^\alpha + i\lambda^\alpha \partial_z \chi_\alpha + 2i(\lambda \gamma^m \lambda)J^h_m - 4\xi^m (\lambda \gamma_m J^h) \tag{7.3}
\]
has no OPE’s with itself but with $\mathcal{J}_z(z)$ it yields
\[
\mathcal{J}_z(z)S_w(w) \sim \frac{3\mathcal{P}(w)}{(z - w)^2} + \frac{\partial_w \mathcal{P}(w)}{(z - w)}, \tag{7.4}
\]
with $\mathcal{P}(z) = 2i \xi^m \lambda \gamma_m \lambda$.

We first construct a new BRST operator $Q'$, starting from the generators $(\mathcal{J}_z, S_z, \mathcal{P})$ and adding new ghosts $(\gamma, b, \tilde{\beta}_z)$ whose antighosts are $(\beta_z, c_z, \tilde{\gamma})$
\[
Q' = \oint dz \left( \gamma \mathcal{J}_z + \frac{1}{2} b S_z + \tilde{\beta}_z \mathcal{P} + \gamma^2 c_z + y\gamma \beta \partial \tilde{\gamma} + z\gamma \partial \beta \tilde{\gamma} + t \partial \beta \gamma \tilde{\gamma} \right). \tag{7.5}
\]
The last four terms are the usual ghost-ghost-antighost term in the BRST charge. For $y = 1/2, z = 1, t = -1/2$ the BRST charge $Q'$ is nilpotent. Since $Q'$ does not contain the antighost $\beta_z$, and since the ghost number of $\gamma$ as well as its conformal spin vanishes and, finally, since $\gamma$ is a commuting field, it can be set to unity.

Consider now the BRST charge $Q_W$ in (2.12). It can be rewritten by adding to the terms with $g$-currents and ghost-currents the terms that yield the BRST charge $Q_{old}$ of our earlier work [1 - 6], while the terms with the $h$-currents can be extended to yield $Q'$
\[
Q_W = Q_{old} + Q' + \oint dz \left[ -2 c_z - \text{terms with } \tilde{\beta}_z \text{ and } \tilde{\gamma} \right] \tag{7.6}
\]
\[
Q_{old} = \oint dz \left[ j^E_z \big|_{j^h=0} + c_z - \frac{1}{2} b S_z \right]
\]
Because the double poles in the OPE’s of the $h$-currents differ by a sign from the corresponding poles of the $g$-currents, the anomaly (proportional to $S_z$) in $Q_{old}$ has the opposite sign to the anomaly in $Q'$.

Our original BRST charge can thus be written as
\[
Q_{old} = Q_W - Q' + 2 \oint c_z + \oint \left( \text{terms with } \tilde{\beta}_z \text{ and } \tilde{\gamma} \right). \tag{7.7}
\]
It contains the difference of the two nilpotent charges $Q_W$ and $Q'$. The anticommutator of $Q_W$ and $Q'$ is cancelled by the anticommutator of $2 \oint c_z$ with $Q'$, and the contributions with $\tilde{\beta}_z$ and $\tilde{\gamma}$ cancel in the square of the right-hand side. The remaining charge, $Q_{old}$ is thus nilpotent, as indeed it was found to be.

We see that the ghost pair $b,c_z$ has the surprising interpretation that $b$ is actually a ghost and $c_z$ is an antighost. This explains why $c_z$ has conformal spin 1 as all the other antighosts.

8. Conclusions and Outlook

We have obtained a covariant quantum superstring with manifest spacetime Lorentz invariance. It is based on a WZNW model, which itself is based on a particular non-semisimple Lie superalgebra, namely the super-Poincaré algebra with a fermionic central extension. No ghosts were any longer added by hand as in our earlier work; rather, the ghost structure directly follows from the requirement that our theory is invariant under superdiffeomorphisms. For this reason we added a quartet of ghosts $(b_{zz}, c^z, \beta_{zz}, \gamma^z)$ which is needed for the gravitational sector. The currents for the combined system satisfy an ordinary $N = 2$ superconformal algebra, which raises the hope that quantum computation may be easier than originally thought, and that the classical geometrical meaning of $\kappa$-symmetry may become clear.

Physical states are defined as follows: they lie in the cohomology $Q_S + Q_V$ defined in section 6, have vanishing grading, and are annihilated by $b_0$ and $\beta_0$. The latter two conditions are gauge choices and select particular representatives of the cohomology; they are the superextension of the well-known Siegel gauge $b_0|\text{phys}\rangle = 0$ of the bosonic string. The deeper meaning of the grading condition still eludes us, but we are studying this problem. The need for a grading condition is, as in our earlier work, that one obtains a nontrivial cohomology. The ghosts $\beta_{zz}$ and $\gamma^z$ are fermionized into the fields $\xi, \eta_z$ and $\phi$, with $\xi$ having grading 2, just like the field $b$ in our earlier work, and $\eta_z$ having grading $-2$. A complete discussion of the cohomology will be published elsewhere, but it is clear that it will contain the cohomology we obtained in our earlier work for the matter sector, and the cohomology of the gravitational sector as already discussed by many authors.

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At the 2002 Amsterdam Summer String Workshop E. Verlinde suggested to go on with adding ghosts in order to obtain BRST nilpotency, instead of adding by hand the $b, c_z$ system. At the 2003 Amsterdam Summer String Workshop H. Verlinde suggested to introduce the topological gravity quartet. This article contains the result of their suggestions.

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9. Appendix A: Massless Vertex for Open Superstrings with $\phi_\alpha$

One can in principle work with the field $\phi_\alpha$ present in the theory. In this appendix, we discuss how to recover the correct cohomology and, therefore, the correct spectrum of the theory in presence of the field $\phi_\alpha$. We follow technique of our previous derivations [1 - 6]. Instead using the auxiliary currents, we obtain a straightforward derivation of the massless spectrum of by using the $b, c_z$ system and the grading. From the construction of the previous appendix, we use the derivatives $D_\alpha, \partial_m$ and $D^\alpha$ which satisfies the commutation relations

$$\{D_\alpha, D_\beta\} = -2i\gamma^m_{\alpha\beta}\partial_m, \quad [D_\alpha, \partial_m] = -2\gamma^m_{\alpha\beta}D^\beta, \quad \{D_\alpha, D^\beta\} = 0,$$  \hspace{1cm} (9.1)

which depend on the field $\phi_\alpha$. In addition, we define the usual derivatives $D^\alpha_\alpha$ and $\partial^\alpha_m$, independent of $\phi_\alpha$ which satisfy the usual superspace relations $\{D^\alpha_\alpha, D^\beta_\beta\} = -2i\gamma^m_{\alpha\beta}\partial^\alpha_m$ and $[D^\alpha_\alpha, \partial^\alpha_m] = 0$.

If we keep $\phi_\alpha$ in the theory, the superfields $A_\alpha, \ldots, F^{\alpha\beta}$ in the vertex operator depends on the coordinate $x^m, \theta^\alpha, \phi_\alpha$ and the corresponding $h$-partners. The vertex $U^{(1|0)}(z)$ belongs to the space of zero graded polynomials (following the grading assignment given in [2]) and the cohomology is defined by

$$\{Q, U^{(1|0)}(z)\} = 0, \quad \delta U^{(1|0)} = [Q, \Omega(z)],$$  \hspace{1cm} (9.2)

where the gauge parameter superfields $\Omega$ is a function of $x^m, \theta^\alpha$ and $\phi_\alpha$. Computing (9.2), we obtain the following equations (we neglect the contributions of the $\omega$-dependent terms
since they are cohomologically trivial anyway). The condition \( \{ Q, U^{(1|0)}(z) \} = 0 \) implies the following equations

\[
\begin{align*}
D(\alpha A\beta) - \frac{1}{2} \gamma_{\alpha\beta} A_m &= 0, \\
\partial_m A_\alpha - D_\alpha A_m + \gamma_{m\alpha\beta} W_\beta &= 0, \\
\partial_{[m} A_{n]} + F_{mn} &= 0, \\
D_\beta W_\alpha + F_\beta^\alpha &= -D_\alpha A_\beta, \\
\partial_m W_\alpha + F_\alpha^m = D_\alpha A_m, \\
F_\alpha^\beta &= -D(\alpha W_\beta),
\end{align*}
\]

where the terms on the right side are due to the \( \phi_\alpha \)-dependence of \( U^{(1|0)} \). These field equations are invariant under the transformations \( \delta U^{(1|0)} = [Q, \Omega(z)] \)

\[
\delta A_\alpha = D_\alpha \Omega, \quad \delta A_m = \partial_m \Omega, \quad \delta W_\alpha = D^\beta \Omega, \quad \delta F_{mn} = 0, \quad \delta F_\alpha^\beta = 0, \quad \delta F^\alpha_m = 0.
\]

In order to remove the field dependence \( \phi_\alpha \), we have to impose an extra condition. This can be done easily by using the charge \( K_\alpha = \oint i\partial_z \theta^\alpha \) we discussed in the introduction. The charge \( K_\alpha \) commute with the BRST charge, it is an anticommuting nilpotent operator, and the physical states are defined by

\[
\{ Q, U \} = 0, \quad \{ K_\alpha, U \} = 0, \quad \delta U = [Q, \Omega], \quad [K_\alpha, \Omega] = 0.
\]

The physical states are defined as the equivariant cohomology of \( Q \) with respect to the gauge transformations generated by \( K_\alpha \) which correspond to constant shifts of the field \( \phi_\alpha \). This allows us to remove the zero modes of \( \phi_\alpha \) from the theory and the vertex operators will depend only on derivatives of \( \phi_\alpha \).

The conditions \( \{ K_\alpha, U \} = 0 \) and \( [K_\alpha, \Omega] = 0 \) imply that \( D_\alpha A_\beta = 0, \ldots, D_\alpha F^{\beta\gamma} = 0 \) and \( D_\alpha \Omega = 0 \) removes the dependence on the zero mode of \( \phi_\alpha \). Therefore, all derivatives in eqs. (9.3) become the usual derivatives \( D_\alpha \) and \( \partial_m \). Moreover, the resulting field equations coincide with the usual equation obtained in [7 - 14] and [1 - 6].

10. Appendix B: The Relation between \( \phi_\alpha \) and \( p_{z\alpha} \)

One can clarify the relation between \( \phi_\alpha \) and \( p_{z\alpha} \) by evaluating how for example \( \theta^\alpha \) and \( x^m \) transform under susy. The susy generator can be written in two different ways

\[
Q_\alpha^{\text{susy}} = \oint dz (i\partial_z \phi_\alpha) = \oint \left[ p_{z\alpha} - i\partial_z x^m (\gamma_m \theta)_\alpha - \frac{1}{6} (\gamma^m \theta)_\alpha (\theta \gamma_m \partial_z \theta) \right].
\]
From the right-hand side one obtains straightforwardly
\[\delta \theta^\alpha = [\epsilon^\beta Q^{susy\beta}, \theta^\alpha] = \epsilon^\alpha \]
and
\[\delta x^m = i \epsilon \gamma^m \theta,\]
which of course leave \(\Pi^m_z\) invariant. To obtain the same results from
the left-hand side we use the WZNW action (3.1) and apply perturbation theory in
the interaction picture. The action is the
\[\int d^2 u \text{ integral of } \]
\[L_g(u) = \frac{1}{2} \Pi^m_z \Pi^n_z \eta_{mn} + d_{za} \partial \theta^\alpha + \hat{d}_{za} \partial \hat{\theta}^\alpha = \]
\[\frac{1}{2} \left( \partial_z x^m - i \theta \gamma_m \partial_z \theta - i \hat{\theta} \gamma_m \partial_z \hat{\theta} \right) \left( \partial_{\bar{z}} x^m - i \theta \gamma^m \partial_{\bar{z}} \theta - i \hat{\theta} \gamma^m \partial_{\bar{z}} \hat{\theta} \right) + \]
\[i \left( \partial_z \phi_\alpha + 2 \partial_z x^m (\gamma_m \theta)_\alpha - \frac{2i}{3} (\gamma_m \theta)_\alpha (\theta \gamma^m \partial_z \theta) \right) \partial_{\bar{z}} \theta^\alpha + \]
\[i \left( \partial_{\bar{z}} \hat{\phi}_\alpha + 2 \partial_{\bar{z}} x^m (\gamma_m \hat{\theta})_\alpha - \frac{2i}{3} (\gamma_m \hat{\theta})_\alpha (\theta \gamma^m \partial_{\bar{z}} \hat{\theta}) \right) \partial_z \hat{\theta}^\alpha.\]
The \(\phi-\theta\) propagator reads
\[\phi_\alpha(z, \bar{z}) \theta^\beta(w, \bar{w}) \sim -i \ln |z - w|^2 \delta^\beta_\alpha,\]
and using it in (10.1) reproduces \(\delta \theta^\alpha = \epsilon^\alpha\), but for \(\delta x^m\) we need interaction vertices. The vertices with \(\hat{\theta}^\alpha\) cannot
contract with \(\phi_\alpha\) hence we need only the vertices \(x \theta\). This yields
\[\left[ Q^{susy, x^m}_\alpha, x^m \right] = \left[ \int_{\gamma_w} dz (i \partial_z \phi_\alpha) \right] \left[ \int d^2 u \left( - \frac{i}{2} \partial x^m (\theta \gamma_m \partial \hat{\theta}) - \frac{i}{2} \partial x^m (\theta \gamma_m \partial \theta) \right) \right] \left[ x^m(w) \right], \]
where the \(z\)-integral is taken along the curve \(\gamma_w\) which encircles \(w\). The vertices with \(\partial x^m\) yield total \(u\)-derivatives and integration over \(d^2 u\) yields a vanishing result. However, the
vertices with \(\partial x^m\) contribute because
\[i \partial \phi_\alpha(z) \partial \theta^\beta(u) = \delta_{\bar{u}} \left( \frac{1}{z - u} \right) = -\pi \delta^2(z - u).\]
Both \(\theta\)'s in \(\partial x^m \gamma_m \partial \theta\) contribute (one needs one partial integration) and after ordinary
integration over \(u\) and contour integration of \(z\), one obtains the correct result. At tree
graph level, diagrams with more than one interaction vertex do not contribute because they are disconnected graphs.
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