Topics of monopole dynamics in gluodynamics

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Three topics of monopole dynamics in gluodynamics are presented. 1) Gauge (in)dependence of the monopole effective action (S. Ito, T.W. Park, T. Suzuki): Four different abelian projections are studied. MA and Laplacian abelian gauges show almost the same renormalization flow and renormalized trajectory. 2) The quantized dual abelian Higgs model (DAH) derived from SU(2) gluodynamics and its vacuum structure (Y. Koma, E.-M. Ilgenfritz, T. Suzuki, M.I. Polikarpov): Monte-Carlo analysis of DAH is done. The quantum average of the dual field strength shows a flux tube profile similar to the color-electric field profile measured in SU(2) gluodynamics. 3) Lattice instanton action from 3D SU(2) Georgi-Glashow model (GG) (T. Yazawa, T. Suzuki): (GG) 3 is studied on the lattice in the London limit. We determine an effective instanton action both in unitary and MA gauges. For some range of parameters, we obtain almost perfect actions which look the same in both gauges (gauge independence) and which reproduce well the string tension.

1. Gauge (in)dependence of the monopole effective action in SU(2)

Four different abelian projections are studied. Maximally abelian gauge (MAG) is given by diagonalizing the following operator

\[ X(x) = \sum_{\mu} \left[ U_{\mu}(x) \sigma_3 U_{\mu}^\dagger(x) + U_{\mu}(x - \hat{\mu}) \sigma_3 U_{\mu}(x - \hat{\mu}) \right]. \]

Polyakov gauge and F 12 gauge are defined with diagonalizing the operators

\[ X_{P\mu}(x) = \prod_{i=1}^{N_4} U_4(x + (i-1)\hat{4}), \]
\[ X_{F12}(x) = U_1(x) U_2(x + \hat{4}) U_1^\dagger(x + 2) U_2^\dagger(x), \]

respectively.

The gauge fixing matrix \( \Omega \) in the Laplacian abelian gauge (LAG) is determined as

\[ \Omega^\dagger(x) \sigma_3 \Omega(x) = \hat{\phi}_a(x) \sigma_a, \]

where \( \phi \) is the eigenvector belonging to the lowest eigenvalue of the covariant Laplacian \( \delta_{xy} \):

\[ -\delta_{xy} = \sum_{\mu} \left( 2 \delta_{xy} \delta_{\mu x + \hat{\mu}} - R_{\mu x}(x) \delta_{y, x + \hat{\mu}} - R_{\mu y}(y) \delta_{y, x - \hat{\mu}} \right), \]

where \( R_{\mu x}(x) = \frac{1}{2} Tr \left( \sigma_3 U_{\mu}(x) \sigma_3 U_{\mu}^\dagger(x) \right) \).

An effective monopole action \( S(k) \) is determined by the Swendsen’s inverse M-C method. We adopt 27 two-point and 6-point interaction terms as the form of action.

\[ S[k] = \sum_{i=1}^{g28} g_i S_i[k + g_28(k^2(x))^2 + g_29(k^2(x))]. \]

Running of the coupling constants are determined by the effective actions \( S(n)[k(n)] \) fixed from configurations of blocked monopole currents \( k(n) \):

\[ S[k] \rightarrow S^{(2)}[k(2)] \rightarrow \ldots \rightarrow S^{(n)}[k(n)] \rightarrow \ldots \]

Figure shows the most dominant self coupling versus \( b \) in all gauges adopted. In the case of MAG and LAG, the scaling behavior (a unique curve for different \( n \)) is seen and both coupling constants are very close to each other. However, the scaling behavior is not seen for small \( b \) in other two unitary gauges. We need more steps of the block-spin transformation.
2. The quantized dual abelian Higgs model derived from SU(2) gluodynamics and its vacuum structure

We have studied the quantitative relation between Abelian projected SU(2) gluodynamics and the quantum U(1) dual Abelian Higgs model (DAHM). The lattice action is

\[ S_{\text{DAH}}[B, \chi, \chi^*] = \sum_s \left[ \frac{\beta_g}{2} \sum_{\mu<\nu} *F_{s,\mu\nu}^2 + \gamma \sum_\mu \left| \chi_s - e^{iB_{s,\mu}} \chi_{s+\hat{\mu}} \right|^2 + \lambda \left( |\chi_s|^2 - 1 \right)^2 \right], \]

with the dual field strength \( *F_{s,\mu\nu} = B_{s,\mu} + B_{s+\hat{\mu},\nu} - B_{s+\hat{\nu},\mu} - B_{s,\nu} \), the dual gauge field \( B_{s,\mu} \) and \( \chi_s \) as a complex monopole field. We determined the bare couplings \( \beta_g, \gamma, \) and \( \lambda \) by matching the monopole action extracted from SU(2) lattice gauge theory in MAG with the effective monopole representation of DAHM. The latter is derived \[ \text{from DAHM by path integration, keeping only the monopole currents } k_\mu(s) \text{ fixed. The form of the two- and four-point couplings results from integration over the Higgs field’s modulus. Identifying the monopoles in both approaches, the values of the Coulomb, the 2- and 4-point couplings are obtained from monopole current networks (obtained by Abelian projection of SU(2) fields) using the extended Swendsen method.}\]

Once the couplings in \( S_{\text{DAH}}[B, \chi, \chi^*] \) are fixed in this way, for a given lattice spacing, one should be able to mimic SU(2) gluodynamics results simulating DAHM as an effective theory: (1) It would be interesting to recover the string tension by calculating dual ’t Hooft loop expectation values and fitting them to an area law; (2) One would like to reproduce the flux tube profile (dual field strength, distribution of monopole current, the Higgs profile). For both purposes, the external color-electric source is introduced as a Dirac string term \( \Sigma_{s,\mu\nu} \) living in the minimal surface spanned by the color-electric current loop, by replacing \( F_{s,\mu\nu} \rightarrow F_{s,\mu\nu} - 2\pi \Sigma_{s,\mu\nu} \) in the action.

![Figure 1. G1 vs b in various gauges for \( n = 2 \sim 4 \) steps of blocking on \( 24^4 \).](image1)

![Figure 2. The profile of the color-electric field in DAHM. The SU(2) result is also plotted for comparison.](image2)

We have chosen \( \beta_{SU(2)} = 2.5115 \) (adopting \( a = 0.086 \text{ fm} \) for which the profile of the SU(2) flux tube has been studied in Abelian projection). For the corresponding dual lattice the bare DAHM couplings are \( \beta_g = 0.04, \gamma = 0.46, \) and \( \lambda = 1.17 \). With external charges included, analyzing the change \( \Delta F(R,T) \) of free energy caused by rectangular ’t Hooft loops, a string tension is obtained which amounts to 86 % of \( \sigma_{SU(2)} \). The quantum average of the dual field strength shows a flux tube profile similar to the color-electric field profile measured in Ref.\[3\]. A clear signal of the monopole current encircling the flux tube could not be obtained, due to a
large vortex density found at the actual set of coupling parameters. The present status suggest that monopole dynamics is really providing the link relating SU(2) gluodynamics to DAHM. For a quantitative success, however, more complicated monopole actions must be envisaged. Alternatively, matching could have been arranged deeper in the infrared, on blocked lattices.

3. Lattice instanton action from 3D SU(2) Georgi-Glashow model

The three-dimensional Georgi-Glashow model (GG)$_3$ has a famous ’t Hooft-Polyakov instanton having a magnetic charge. Polyakov showed analytically \[8\] that the string tension of 3D Georgi-Glashow model has a finite value. He made a quasi-classical calculation using a dilute Coulomb gas approximation of ’t Hooft-Polyakov instantons:

\[
S = \text{const} \frac{1}{g^2} \sum_a q_a^2 + \frac{1}{4g^2} \sum_{a \neq b} q_a q_b \frac{2 \pi}{|x_a - x_b|}.
\]

We study SU(2) lattice (GG)$_3$ model in the London limit using Monte-Carlo simulation. Abelian and instanton dominance are observed after abelian projections in a unitary gauge ($\phi^a = \delta^3a$) and MAG.

When we restrict ourselves to some regions of parameters $\beta$ and $\kappa$ we find that the DeGrand-Toussaint instanton \[9\] configuration realizes the coulomb gas condition. The method is as follows. First, the lattice instanton action is derived from instanton configuration using inverse MC method \[10\] and the action adopted is composed of 10 quadratic interactions:

\[
S_m = \frac{G_0}{g^2} \sum_a k(x)^2 + G_1 \sum_{a,\mu} k(x)k(x + \mu)
+ G_2 \sum ... + G_9 \sum ...
\]

This action is well fitted with the lattice Coulomb propagator ($\Delta_{L}^{-1}$):

\[
S_m \sim \text{Self term} + \text{Const.} \sum_{x,x'} k(x)\Delta_{L}^{-1}(x-x')k(x') .
\]

Next, we perform the block-spin transformation using the scale $a = \sqrt{\sigma_L/\sigma_{\text{phys}}}$. The proportional coefficient of Coulomb term behave as $1/b^2$, where $b = na$ and $n$ is the number of block-spin transformation as seen in Fig. 3. Finally, we obtain an almost perfect instanton action of (GG)$_3$ and it reproduces well the string tension \[11\].

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