We demonstrate that perturbative QCD allows one to calculate the absolute cross section of diffractive exclusive production of photons at large $Q^2$ at HERA, while the aligned jet model allows one to estimate the cross section for intermediate $Q^2 \sim 2 \text{GeV}^2$. Furthermore, we find that the imaginary part of the amplitude for the production of real photons is larger than the imaginary part of the corresponding DIS amplitude by a factor of about 2, leading to predictions of a significant counting rate for the current generation of experiments at HERA. We also find a large azimuthal angle asymmetry in $ep$ scattering for HERA kinematics which allows one to directly measure the real part of the DVCS amplitude and hence the nondiagonal parton distributions.

Recent data from HERA has spurred great interest in exclusive or diffractive direct production of photons in $e^- p$ scattering (DVCS- deeply virtual Compton scattering) as another source to obtain more information about the gluon distribution inside the proton for nonforward scattering. In recent years studies of diffractive vector meson production and deeply virtual Compton scattering has greatly increased our theoretical understanding about the gluon distribution in nonforward kinematics and how it compares to the gluon distribution in the forward direction, see list of references in Ref. 1. We are interested in the production of a real photon compared to the inclusive DIS cross section. The exclusive process is nonforward in its nature, since the photon initiating the process is virtual ($q^2 < 0$) and the final state photon is real, forcing a small but finite momentum transfer to the target proton i.e forcing a nonforward kinematic situation as we would like.

Similar to the case of deep inelastic scattering, in real photon production it is possible to calculate within perturbative QCD the $Q^2$ evolution of the amplitude but not its value at the normalization point at $Q^2_0 \sim \text{few GeV}^2$ where it is given by nonperturbative effects. Hence we start by discussing expectations for this region. It was demonstrated in [1] that the aligned jet model coupled with the idea of color screening provides a reasonable semiquantitative description of $F_{2N}(x \leq 10^{-2}, Q^2_0)$. One can write $\sigma_{tot}(\gamma^* N)$ using the Gribov
disperion representation as

\[
\sigma_{\text{tot}}(\gamma^\ast N) = \frac{\alpha}{3\pi} \int_{M_0^2}^{\infty} \sigma_{\text{tot}}('AJM' - N) R e^{+} e^{-} (M^2) M^2 \frac{\delta^2}{M^2} dM^2 , \tag{1}
\]

Based on the logic of the local quark-hadron duality (see e.g. and references therein) we take the lower limit of integration \(M_0^2 \sim 0.5 \text{GeV}^2 \leq m_r^2\). In the case of real photon production the imaginary part of the amplitude for \(t = 0\) is obtained from Eq. 1 by replacing one of the propagators by \(1/M^2\). Using Eq. 1 we find

\[
R \equiv \frac{ImA(\gamma^\ast + N \rightarrow \gamma^\ast + N)_{t=0}}{ImA(\gamma^\ast + N \rightarrow \gamma + N)_{t=0}} = \frac{Q^2}{Q^2 + M_0^2} \ln^{-1}(1 + Q^2/m_r^2). \tag{2}
\]

For \(M_0^2 \sim 0.4 \div 0.6 \text{ GeV}^2\) and \(Q^2 \approx 2 - 3 \text{ GeV}^2\) Eq. 2 leads to \(R \approx 0.5\). A similar value of \(R\) has been found in. The process of exclusive direct production of photons in first nontrivial order can be calculated as the sum of hard contributions within the framework of QCD evolution equations and a soft contribution which we evaluated above within the aligned jet model. In order to calculate the imaginary part of the amplitude, we need to calculate the hard amplitude as well as the gluon-nucleon scattering plus the soft aligned jet model contribution. We find the following general expression for the imaginary part of the amplitude

\[
ImA(x, Q^2, Q_0^2) = ImA(x, Q_0^2) + 4\pi^2 \alpha \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \int_x^{1} \frac{dx_1}{x_1} \left[ P_{qq}(x/x_1, \Delta/x_1) g(x_1, x_2, Q'^2) + P_{qq}(x/x_1, \Delta/x_1) q(x_1, x_2, Q'^2) \right], \tag{3}
\]

where \(P_{qq}\) and \(P_{gg}\) are the evolution kernels and are taken from Ref. We neglect the \(x_2\) dependence for the moment. The result for the ratio

\[
R = \frac{ImA(\gamma^\ast + p \rightarrow \gamma^\ast + p)/ImA(\gamma^\ast + p \rightarrow \gamma + p)}{ImA(\gamma^\ast + p \rightarrow \gamma^\ast + p)}
\]

is given in the \(x\) range from \(10^{-4}\) to \(10^{-2}\) and for a \(Q^2\) of 3.5, 12 and 45 GeV\(^2\) relevant at HERA. According to our previous discussion we chose the initial distribution for the imaginary part of the DVCS amplitude to be twice that of the initial distribution for the imaginary part of the DIS amplitude. We find \(R\) to be between 0.551, 0.573 and 0.57 for \(x = 10^{-4}\), 0.541, 0.562 and 0.557 for \(x = 10^{-3}\) and 0.518, 0.519 and 0.505 for \(x = 10^{-2}\) in the given \(Q^2\) range. The ratio \(R\) will approach
1/2 as $Q^2$ is decreased to the nonperturbative scale since this is our aligned jet model estimate. The reason for the deviation from $R = 1/2$ is due to the difference in the evolution kernels.

As far as the complete amplitude at small $x$ is concerned, we can reconstruct the real part via dispersion relations, which to a very good approximation is given by:

$$
\eta \equiv \frac{Re A}{Im A} = \frac{\pi}{2} \frac{d \ln (x l m A)}{d \ln x}.
$$

(4)

Therefore, our claims for the imaginary part of the amplitude also hold for the whole amplitude at small $x$.

To check the feasibility of measuring a DVCS signal against the DIS background, we will be interested in the fractional number of DIS events to DVCS events at HERA given by:

$$
R_\gamma = \frac{\sigma(\gamma^* + p \rightarrow \gamma + p)}{\sigma_{tot}(\gamma^* p)} \approx \frac{d\sigma(\gamma^* + p \rightarrow \gamma + p)}{dt}|_{t=0} \times \frac{1}{B}/\sigma_{tot}(\gamma^* p)
$$

$$
= \frac{\pi \alpha}{4R^2Q^2B} F_2(x, Q^2)(1 + \eta^2),
$$

(5)

where $\eta = Re A/Im A \simeq 0.09 - 0.27$ for the given $Q^2$ range, is given by Eq. 3. Note that only $d\sigma/dt(t = 0)$ is calculable in QCD. The $t$ dependence is taken from the data fits to hard diffractive processes. We computed $R_\gamma$ for $x$ between $10^{-4}$ and $10^{-2}$ and for a $Q^2$ of 2, 3, 5, 12 and 45 GeV$^2$ with the following results, where the numbers for $F_2$ were taken from Eq. 3. We find $R_\gamma \simeq 1.1 \times 10^{-3}, 4.9 \times 10^{-4}$ at $x = 10^{-4}, 10^{-3}$ and $Q^2 = 2$ GeV$^2$; $R_\gamma \simeq 1.07 \times 10^{-3}, 0.3 \times 10^{-4}$ at $x = 10^{-4}, 10^{-3}$ and $Q^2 = 3.5$ GeV$^2$; $R_\gamma \simeq 4.5 \times 10^{-4}, 3.78 \times 10^{-4}$ at $x = 10^{-4}, 10^{-3}, 10^{-2}$ and $Q^2 = 12$ GeV$^2$; and finally $R_\gamma \simeq 1.49 \times 10^{-4}, 1.04 \times 10^{-4}$ at $x = 10^{-3}, 10^{-2}$ and $Q^2 = 45$ GeV$^2$.

The complete cross section of exclusive photon production includes the Bethe-Heitler and DVCS process plus an interference term stemming from both processes. We find the cross section for DVCS, Bethe-Heitler and the interference term to be

$$
\frac{d\sigma_{DVCS}}{dxdy|t|d\phi_r} = \frac{\pi \alpha^3g}{4R^2Q^6} (1 + (1 - y)^2)e^{-B|t|} F_2^2(x, Q^2)(1 + \eta^2),
$$

(6)

$$
\frac{d\sigma_{BH}}{dxdy|t|d\phi_r} = \frac{8\alpha^3g^2y^4}{\pi |t|Q^4(1 - y)} \left[ G_E^2(t) + \frac{|t|}{4m_N^2} G_M^2(t) \right] \cos^2(\phi_r),
$$

(7)

$$
\frac{d\sigma_{DVCS+BH}}{dxdy|t|d\phi_r} = \pm\frac{\alpha^3g^2y^2}{RQ^3} F_2(x, Q^2) \cos(\phi_r) e^{-B|t|/2} \sqrt{2(1 + (1 - y)^2)} |t|(1 - y)
$$
with \( y = 1 - E'/E \), where \( E' \) is the energy of the electron in the final state, \( \phi_r = \phi_N - \phi_e \), where \( \phi_N \) is the azimuthal angle of the final state proton with respect to the reaction axis and \( \phi_e \) is the azimuthal angle of the final state electron and where \( G_E(t) \) and \( G_M(t) \) are the electric and nucleon form factors. We describe them using the well known dipole fit. The + sign in the interference formula corresponds to an electron scattering off a proton and the - sign corresponds to the positron. The total cross section is then just the sum of the three terms in Eq. 8.

At this point it is important to determine how large the Bethe-Heitler background is as compared to DVCS for HERA kinematics, hence, in the following discussion, we will estimate the ratio \( D = \frac{< d\sigma_{DVCS+BH} >}{< d\sigma_{BH} >} \) allowing a background comparison with  4.

\[
D = \frac{G_E(t) + \frac{|t|}{4m_N^2} G_M(t)}{1 + \frac{|t|}{4m_N^2}}
\]  

(8)

Figure 1: a) \( D \) is plotted versus \(-t\) for \( x = 10^{-4}, 10^{-3}, 10^{-2}, Q^2 = 12 \text{ GeV}^2, B = 5 \text{ GeV}^{-2} \) and \( y = 0.4 \). The solid curve is for \( x = 10^{-4} \), the dotted one for \( x = 10^{-3} \) and the dashed one for \( x = 10^{-2} \). b) The asymmetry \( A \) is plotted versus \(-t\) for \( x = 10^{-4} \) (solid curve), \( x = 10^{-3} \) (dotted curve) and \( x = 10^{-2} \) (dashed curve) again for \( Q^2 = 12 \text{ GeV}^2, B = 5 \text{ GeV}^{-2} \) and \( y = 0.4 \).
Fig. 1b shows $A$ for the same kinematics as above and we find that the asymmetry is fairly sizeable already for small $t$. Due to this fairly large asymmetry, one has a first chance to access nondiagonal parton distributions through it. Also, we find that $A$ is very sensitive to the energy dependence of the scattering amplitude since it is proportional to $\eta$. As a result measurements of $A$ would provide a window to the energy behaviour of $F_{2N}(x, Q^2)$ beyond the energy range currently available at HERA.

Conclusion: In the above said we have shown that pQCD is applicable to exclusive photoproduction. We wrote down an equation for the imaginary part of the amplitude, which can be generalized to the complete amplitude at small $x$. We also found that the imaginary part of the amplitude of the production of a real photon is larger than the one in the case of DIS in a broad range of $Q^2$. We also found the same to be true for the full amplitude at small $x$. We also make experimentally testable predictions for the number of real photon events and suggest that the number of events are small but not too small to be detected at HERA. Finally, we demonstrated that first the Bethe-Heitler process does not overshadow the DVCS cross section and secondly that measuring the asymmetry $A$ at HERA would allow one to determine the real part of the DVCS amplitude, i.e., gain a first experimental insight into nondiagonal parton distributions.

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