Photon production from non-equilibrium
disoriented chiral condensates in a longitudinal expansion: A
theoretic framework

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Abstract

A theoretical framework is developed for treating the quantization of the photons in a spacetime with a longitudinal expansion. This can be used to study the production of the photons through the non-equilibrium relaxation of a disoriented chiral condensate presumably formed in the expanding hot central region in ultra-relativistic heavy-ion collisions. These photons can be a signature of the formation of disoriented chiral condensates in the direct photon measurements of heavy-ion collisions.

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Disoriented Chiral Condensates (DCCs), the correlated space-time regions where the chiral order parameter of QCD is chirally rotated from its orientation of the true vacuum state in the isospin space, might be formed in high energy hadronic or nuclear collisions. The search of such a DCC would provide a probe to the understanding of the chiral structure of the QCD vacuum and/or the chiral phase transition of strong interactions at high temperatures. Various experimental searches of this phenomenon include cosmic ray experiments, the MiniMAX experiment in proton-antiproton collisions at the Fermilab Tevatron, or event-by-event analysis of Pb-Pb collisions by the WA98 and NA49 collaborations at the CERN SPS. Although no clear experimental evidence for DCC formation has been reported so far, the search of this phenomenon is still part of the missions in the heavy-ions physics program in present or planned experiments at RHIC and LHC [1, 2].

In relativistic heavy ion collisions, the large energy deposit in the central rapidity region leaves behind a hot and dense plasma at a temperature above 200 MeV where the chiral symmetry is restored. Then, the plasma cools down in a rapid hydrodynamic expansion accompanied by the chiral phase transition. The subsequent out of equilibrium evolution triggers an exponential growth of long-wavelength fluctuations via the spinoidal instabilities [3–5], resulting in the formation of disoriented chiral condensates (DCCs) (see also Refs. [6, 7]). The decay of such DCCs to the true QCD vacuum is expected to radiate copiously soft pions that could be a potential observational signature of the chiral phase transition. However, the emitted pions will strongly interact with the background of other hadronic matter so that the hadronic signal may be severely masked. In contrast, electromagnetic probe such as photon and lepton with longer mean free path in the medium serves as a good candidate and can reveal more detailed non-equilibrium information on the DCCs with minimal distortion [8–11].

It has been proposed that anomalous radiation of low-momentum photon and/or low-mass dileptons can be produced from a DCC. In particular, in Ref. [10], Boyanovsky et al. have extensively studied the photon production from the DCC with a nonvanishing expectation value of the neutral pion through the $U_A(1)$ anomalous vertex. Later, the authors in Ref. [11] have taken into account another dominant contribution that also involves the dynamics of $\pi^0$ due to the decay of the vector meson through the electromagnetic vertex. They have found that for large initial amplitudes of the $\pi^0$ mean field the photon production is enhanced by parametric amplification, resulting in a distinct energy distribution of the produced photons. However, the above two works have ignored the hydrodynamical expansion and adopted the simple “quench” approximation for the chiral phase transition. This quenched phase transition has been widely used in the study of non-equilibrium phenomena of DCCs [3, 8–10]. Based upon the Bjorken’s scenario, in ultra-relativistic heavy-ion collisions, an approximate Lorentz boost invariant particle flow along the longitudinal direction might be created in the central region [12]. At late times following the heavy nuclei collisions, a transverse flow can be generated due to the multi-scattering between the produced particles, as such the expansion becomes three dimensional [13]. In Ref. [14], the authors have considered the effect of the hydrodynamical expansion of the plasma to the production of photons from the non-equilibrium relaxation of a DCC. It is found that the expansion smoothes out the resonances in the process of parametric amplification such that the non-equilibrium photons are dominant to the thermal photons over the range $0.2 – 2$ GeV. This work is to assume a spherical boost invariant hydrodynamical flow which for small values of the rapidity is conformally flat, thus greatly simplifying the treatment of the photon field in the expanding spacetime. In this work, we attempt to consider the nonequilibrium photon
production from a DCC in a more realistic longitudinal boost invariant hydrodynamical expansion, which certainly requires a quantization scheme for electromagnetic fields under this anisotropic expanding background.

The study of the quantized electromagnetic fields interacting with the gravitation fields or propagating in the curved spacetime is of interest in many different aspects, one of which is to consider the photon production in the early universe with its possible effect on the initial anisotropies [15]. Thus a consistent quantization scheme of electromagnetic theory under an anisotropic expanding background is required. In Ref. [16], quantization of electromagnetic fields in a diagonal Bianchi type I metric is proposed using the Fourier mode expansion of the fields developed in Ref. [17]. Applying the WKB particle concept in asymptotic in and out regions of spacetime where the background expansion is adiabatically slow gives the normalization of the mode functions. Then particle production of the free photons is considered by performing the standard Bogoliubov transformation between annihilation and creation operators in in and out regions. Here we instead try to provide an alternative quantization scheme directly on electromagnetic potentials. An application of this quantization method to interactive photon fields is straightforward, in particular when some other charged matter fields are also present with the interaction term depending upon electromagnetic potentials.

After introducing our model and summarizing the dynamical equations for describing the evolution of the expectation value of the fields within the DCCs, in this Letter we will propose the quantization method on quantizing electromagnetic potentials in an anisotropic expanding background and discuss its application to the interactive photons. The formalism for considering photon production will be developed. Summary and discussions on our future investigation will be presented at the end.

The longitudinal expansion, say along the $z$ axis, of the boost invariant hydrodynamical flow can be described by the proper time $\tau$ and the space-time rapidity $\eta$, defined as

$$\tau \equiv (t^2 - z^2)^{1/2}, \quad \eta \equiv \frac{1}{2} \ln \left( \frac{t + z}{t - z} \right),$$  

where $(t, x) = (t, x_{\perp}, z) = (t, x, y, z)$ are the coordinates in the laboratory,

$$t = \tau \cosh \eta, \quad z = \tau \sinh \eta. \quad (2)$$

The ranges of these coordinates are set to be $0 \leq \tau < \infty$ and $0 \leq \eta < \infty$, restricted to the forward light cone. The Minkowski line element is then given by

$$ds^2 = dt^2 - dx^2 = d\tau^2 - d\mathbf{x}_{\perp}^2 - \tau^2 d\eta^2. \quad (3)$$

Eq. (3) is the Kasner metric that serves as the background comoving frame under which non-equilibrium photons are to be produced from a DCC domain.

The dynamics of the DCC as well as the photon production from a DCC in a generally expanding background can be described by the phenomenological action given by

$$S = \int d^4x \sqrt{g} \left( L_{\sigma} + L_A + L_{\pi^0 A} \right), \quad (4)$$

where

$$L_{\sigma} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \vec{\Phi} \cdot \partial_\nu \vec{\Phi} + \frac{1}{2} \frac{M^2}{2} \vec{\Phi} \cdot \vec{\Phi} - \lambda \left( \vec{\Phi} \cdot \vec{\Phi} \right)^2 + h\sigma, \quad (5)$$
The effective Lagrangian above can be determined by the low-energy pion physics. With the vector meson \( \omega \) the pseudo tensor is multiplied by the factor \( \frac{1}{g} \) where \( g \) is the electromagnetic field tensor. The dominant couplings between the photon and the neutron pion are given by the anomalous U(1) interaction as well as the decay of the vector meson \( V \). \( V \) is identified as the \( \omega \) meson with its coupling \( \lambda \) that is obtained from the \( \omega \rightarrow \pi^0\gamma \) decay width [18]. The signature of the metric we choose is \(- + + +\) where \( g = -\text{det}[g_{\mu\nu}] = \tau^2 \). In particular, since \( \epsilon^{\alpha\beta\mu\nu} \) is a tensor density of weight \(-1\), each of this pseudo tensor is multiplied by the factor \( 1/\sqrt{g} \) in a general coordinate system [19].

The above effective interactions on the photon have been studied by us and are obtained from the perturbative theory without involving in-medium modifications. The in-medium effects will enter only through the non-equilibrium fluctuations for the pion fields due to their strong couplings.

Within the DCC, we assume that the \( \sigma \) and the \( \pi^0 \) acquire the respective expectation values that both are a function of the comoving time \( \tau \) only in such a boost invariant background. Here we first shift \( \sigma \) and \( \pi^0 \) by their expectation values with respect to an initial non-equilibrium states:

\[
\sigma(x, \tau) = \phi(\tau) + \chi(x, \tau), \quad (\sigma(x, \tau)) = \phi(\tau), \\
\pi^0(x, \tau) = \zeta(\tau) + \psi(x, \tau), \quad (\pi^0(x, \tau)) = \zeta(\tau),
\]

with the tadpole conditions:

\[
(\chi(x, \tau)) = 0, \quad (\psi(x, \tau)) = 0, \quad (\bar{\pi}(x, \tau)) = 0.
\]

Using the Schwinger-Keldysh closed-time-path formulation of non-equilibrium quantum field theory allows us to derive the evolution equation of the non-equilibrium expectation values as well as the correlation functions of quantum fields. This technique has successfully been employed elsewhere within many different contexts and we refer the readers to the literature for details [20]. To self-consistently incorporate quantum fluctuation effects from the strong \( \sigma - \pi \) interactions, we then implement the large-\( N \) approximation that provides a nonperturbative resummation scheme below (Eqs. (2.7)-(2.9) of Ref. [10]). Using the tadpole conditions [10], the large-\( N \) equations of motion for the mean fields that also involve the perturbatively electromagnetic corrections can be obtained as follows:

\[
\begin{align*}
\left[ \partial_\tau^2 + \frac{1}{\tau} \partial_\tau - \frac{M^2}{2} + 4\lambda \phi^2(\tau) + 4\lambda \zeta^2(\tau) + 4\lambda \langle \bar{\pi}^2 \rangle(\tau) \right] \phi(\tau) - h = 0, \\
\left[ \partial_\tau^2 + \frac{1}{\tau} \partial_\tau - \frac{M^2}{2} + 4\lambda \phi^2(\tau) + 4\lambda \zeta^2(\tau) + 4\lambda \langle \bar{\pi}^2 \rangle(\tau) \right] \zeta(\tau) \\
- \frac{e^2}{32\pi^2 f_\pi} \langle F \bar{F} \rangle(\tau) + \frac{e^2 \lambda^2}{4m^2 \pi^2} \tau^{-1} \partial_\tau \left[ \tau^{-1} \zeta \right] \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} g_{\sigma\delta} \langle F_{\mu\nu} F_{\alpha\beta} \rangle(\tau) \\
+ \frac{e^2 \lambda^2}{4m^2 \pi^2} \tau^{-2} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} g_{\sigma\delta} \langle \partial_\gamma F_{\mu\nu} F_{\alpha\beta} \rangle(\tau) = 0,
\end{align*}
\]
where the dot means $d/d\tau$. The backreaction from the quantum fluctuations of the pions can be expressed by their Fourier mode functions $U_k(\tau)$ defined as:

$$\bar{\pi}(x, \tau) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\pi k}(\tau_0)}} \left[ \bar{a}_k U_k(\tau) e^{ikx} + \text{h.c.} \right],$$  \hspace{1cm} (12)

where $k \cdot x = k_\perp x_\perp + k_\eta x_\eta$ and $d^3k = d^2k_\perp dk_\eta$. $\bar{a}_k$ is a destruction operator for the mode with frequency $\omega_{\pi k}(\tau_0)$ to be determined from solving the time independent gap equation given by the initial states. The mode equations for the pions can be read off from the quadratic part of the effective Lagrangian as

$$\left[ \partial_\tau^2 + \frac{1}{\tau} \partial_\tau + k^2 - \frac{M_\sigma^2}{2} + 4\lambda \phi^2(\tau) + 4\lambda \zeta^2(\tau) + 4\lambda \langle \bar{\pi}^2 \rangle(\tau) \right] U_k(\tau) = 0, \hspace{1cm} (13)$$

where $k^2 = g_{\mu\nu}k^\mu k^\nu = k_\perp^2 + k_\eta^2/\tau^2$. The expectation values with respective to the initial states are given by

$$\langle \bar{\pi}^2 \rangle(\tau) = (N - 1) \int_0^\Lambda \frac{d^3k}{2(2\pi)^3} \omega_{\pi k}(\tau_0) \left[ |U_k(\tau)|^2 \right] \coth \left[ \frac{\omega_{\pi k}(\tau_0)}{2T_\eta} \right], \hspace{1cm} (14)$$

where $\langle \bar{\pi}^2 \rangle(\tau)$ will be self-consistently determined by the above equation with an initial temperature set to $T_\eta$ presumably above the critical temperature of the chiral phase transition. Thus, the corresponding the gap equation for the initial adiabatic modes is

$$\omega_{\pi k}(\tau_0) = k^2 - \frac{M_\sigma^2}{2} + 4\lambda \phi^2(\tau_0) + 4\lambda \zeta^2(\tau_0) + 4\lambda \langle \bar{\pi}^2 \rangle(\tau_0). \hspace{1cm} (15)$$

It must be noticed that this is an effective field theory with an ultraviolet momentum cutoff of the order of $\Lambda \approx m_V$. Without considering the perturbative backreaction effects from the photons, the above equations have been extensively studied in Ref. [4], aiming at finding the phase space of initial conditions that may lead to instabilities on long wavelength modes during the subsequent nonequilibrium chiral phase transition. If so, then the DCCs may be formed, and thus produce an anomalous transverse distribution of secondary pions when compared to a more conventional boost invariant hydrodynamic flow in local thermal equilibrium. Here these background solutions are of importance to provide a nonequilibrium environment from which the photons are to be produced.

The relevant Lagrangian densities for describing the dynamics of photon production under the time dependent expectation value of the neutral pion are given by $L_A$ and $L_{\zeta A}$ in particular where the pion field in $L_{\omega A}$ is replaced by its mean value $\zeta$. The underlying gauge symmetry embedded in electromagnetic theory reflects the fact that not all gauge potential are of physical relevance. Thus we can avoid potential gauge ambiguities by eliminating all possible redundant degrees of freedom with a choice of the gauge fixing.

The canonical momenta, $\Pi^r = (\Pi^r, \Pi^i)$, conjugate to the potential fields, $A_\mu = (A_\tau, A_i)$, are defined by

$$\Pi^r = \frac{\delta}{\delta A_\tau} \sqrt{g} (L_A + L_{\zeta A}) = 0, \hspace{1cm} (16)$$

$$\Pi^i = \frac{\delta}{\delta A_i} \sqrt{g} (L_A + L_{\zeta A}) = \sqrt{g} (g^{ij} F_{0j} + \frac{1}{\sqrt{g}} \frac{e^2}{8\pi^2 f_\pi} \zeta \epsilon^{ijk} F_{jk}). \hspace{1cm} (17)$$
However, since $A_\tau$ is not a dynamical variable as its conjugate momentum $\Pi_\tau$ vanishes, we can set $A_\tau = 0$, a temporal gauge. Then the further constraint equation can be obtained by taking the variation of the Lagrangian density with respect to $A_\tau$ that gives the Gauss law $\partial_i \Pi^i = 0$. Thus, the choice of the gauge fixing can be such as

$$A_\tau = 0 ; \quad g^{ij} \partial_i \hat{A}_j = 0 ,$$

the counterpart of the Coulomb gauge in a flat spacetime. The gauge conditions leave us two transverse vector potentials. It proves more convenient to parametrize the polarization vectors in a longitudinal expanding background as follows:

$$\varepsilon^{(1)}_{\theta \mathbf{k} i} = \frac{1}{k T} \left( \frac{k_x k_\eta}{k^2}, \frac{k_y k_\eta}{k^2}, -k_\perp \tau^2 \right) , \quad \varepsilon^{(2)}_{\phi \mathbf{k} i} = \left( -\frac{k_y}{k}, \frac{k_x}{k}, 0 \right) , \quad \varepsilon^{(3)}_{r \mathbf{k} i} = \frac{1}{k} (k_x, k_y, k_\eta) .$$

They obey

$$\varepsilon^{(1)}_{-\mathbf{k} i} = \varepsilon^{(1)}_{\mathbf{k} i} , \quad \varepsilon^{(2)}_{-\mathbf{k} i} = -\varepsilon^{(2)}_{\mathbf{k} i} , \quad \varepsilon^{(a)}_{\mathbf{k} i} \varepsilon^{(a)}_{\mathbf{k} j} = \frac{1}{\sqrt{g}} \epsilon^{lmn} \varepsilon^{(b)}_{\mathbf{k} m} \varepsilon^{(c)}_{\mathbf{k} n} ,$$

where $(a, b, c) = (1, 2, 3)$ and its cyclic order. Notice that

$$\varepsilon^{(1)}_{\mathbf{k} i} = \frac{k^2}{k^2 T} \left( \varepsilon^{(1)}_{\mathbf{k} i} - \frac{2 k_\eta}{k T} \varepsilon^{(3)}_{\mathbf{k} i} \right) ,$$

so the longitudinal expansion might lead to the change of the direction of the polarization vector $\varepsilon^{(1)}_{\mathbf{k} i}$, but keeps the other two polarization vectors $\varepsilon^{(2)}_{\mathbf{k} i}$ and $\varepsilon^{(3)}_{\mathbf{k} i}$ remaining their directions. Thus, the mode expansion of the vector potentials must involve not only the state with polarization vector $\varepsilon^{(2)}_{\mathbf{k} i}$, but also the mixed state with the respective polarization vectors $\varepsilon^{(1)}_{\mathbf{k} i}$ and $\varepsilon^{(3)}_{\mathbf{k} i}$, given by

$$A_i (\mathbf{x}, \tau) = \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\sqrt{2k (\tau_0)}} \left\{ b_{1k} \left( V_{1k} (\tau) \varepsilon^{(1)}_{\mathbf{k} i} + V_{3k} (\tau) \varepsilon^{(3)}_{\mathbf{k} i} \right) + b_{2k} V_{2k} (\tau) \varepsilon^{(2)}_{\mathbf{k} i} \right\} e^{i k \cdot \mathbf{x}} + h.c. \right\} .$$

The creation and annihilation operators satisfy the canonical commutation relations. The mode function $V_{3k} (\tau)$ certainly is not an independent function and its evolution depends on the choice of the gauge that gives

$$\dot{V}_{3k} (\tau) + \frac{k^2}{k^2 \tau^2} V_{3k} (\tau) = \frac{2 k_\perp k_\eta}{k^2 \tau^2} V_{1k} (\tau) .$$

It is then expected that the physical degrees of freedom are those two transverse modes with the polarization vectors obeying the transversality condition,

$$\sum_{a=1,2} \varepsilon^{(a)}_{\mathbf{k} i} \varepsilon^{(a)}_{\mathbf{k} j} = g_{ij} - \frac{k_i k_j}{k^2} = P_{ij} (\mathbf{k}) .$$

It will be seen later that the equation (23) obtained from the gauge fixing then becomes crucial in the sense that the expression of the gauge invariant quantity of interest, for example electric and magnetic fields, have no $V_{3k}$ dependence.
can be straightforwardly computed as follows:

\[ \Pi^i = E^i + \frac{e^2}{4\pi^2 f_\pi} \zeta B^i, \]

where the electric and magnetic fields can be expressed in terms of two transverse degrees of freedom as follows:

\[ E^i(x, \tau) = \sqrt{g} g^{ij} F_{0j} \]
\[ = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2k(\tau_0)}} \sqrt{g} \left\{ b_{1k} \left( \dot{V}_{1k}(\tau) + \frac{k^2}{k^2 \tau} V_{1k}(\tau) \right) \varepsilon_k^{(1)i} + b_{2k} V_{2k}(\tau) \varepsilon_k^{(2)i} \right\} e^{ikx} + \text{h.c.} \}, \]

\[ B^i(x, \tau) = \frac{1}{2} \epsilon^{ijk} F_{jk} \]
\[ = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2k(\tau_0)}} \sqrt{g} \left\{ i k \left[ b_{1k} V_{1k}(\tau) \varepsilon_k^{(2)i} - b_{2k} V_{2k}(\tau) \varepsilon_k^{(1)i} \right] e^{ikx} + \text{h.c.} \right\} , \]

so that the fields always point to the direction perpendicular to the direction of their propagation \( k \).

The commutation relation between the vector potentials and their conjugate momenta can be straightforwardly computed as follows:

\[ [\Pi^i_k, A_{-kj}] = -i\hbar \delta^i_j(k) \]
\[ + \sqrt{g} \frac{1}{2k(\tau_0)} \left\{ \left[ \left( \dot{V}_{1k}(\tau) + \frac{k^2}{k^2 \tau} V_{1k}(\tau) \right) \varepsilon_k^{(1)i} + \frac{e^2}{4\pi^2 f_\pi} \zeta i k V_{1k}(\tau) \varepsilon_k^{(2)i} \right] V_{3k}^*(\tau) \varepsilon_{kj}^{(3)} \right\} \]
\[ - \left[ \left( \dot{V}_{1k}^*(\tau) + \frac{k^2}{k^2 \tau} V_{1k}^*(\tau) \right) \varepsilon_k^{(1)i} + \frac{e^2}{4\pi^2 f_\pi} \zeta i k V_{1k}^*(\tau) \varepsilon_k^{(2)i} \right] V_{3k}(\tau) \varepsilon_{kj}^{(3)} \}, \]

where the Wronskian conditions of the mode functions \( V_{1k}(\tau) \) and \( V_{2k}(\tau) \) and their time derivatives \( \dot{V}_{1k}(\tau) \) and \( \dot{V}_{2k}(\tau) \) are given by

\[ \sqrt{g} \frac{1}{2k(\tau_0)} [V_{1k}(\tau) \dot{V}_{1k}^*(\tau) - \dot{V}_{1k}(\tau) V_{1k}^*(\tau)] = -i\hbar, \]

\[ \sqrt{g} \frac{1}{2k(\tau_0)} [V_{2k}(\tau) \dot{V}_{2k}^*(\tau) - \dot{V}_{2k}(\tau) V_{2k}^*(\tau)] = -i\hbar. \]

The Wronskian equations are to fix the normalization conditions of the mode functions. The first term of the commutation relation above \([26]\) is expected whereas the additional terms with the \( V_{3k}(\tau) \) dependence are present to satisfy the gauge choice, that is, \([\Pi^i_k, k^j A_{-kj}] = 0 \) obtained by means of its equation \([23]\). Notice that since we are interested in the dynamics of photon production from a DCC that does not necessarily involve the intermediate photon states, it will be seen later that the expression of the number density of the produced photons depends only on the dynamics of the mode functions respectively, \( V_{1k}(\tau) \) and \( V_{2k}(\tau) \).

Although the quantization scheme can be formulated in the way that all redundant degrees of freedom can be eliminated consistently by choosing a proper gauge fixing, there might exist a subtlety on finding a suitable mode expansion to further diagonalize Lagrangian density of the interactive photon fields. This subtlety will become more clear later.
as the above quantization scheme is applied to free photons first and to interactive photons of interest later. Besides, we will mention the known alternative quantization scheme in Ref. [16] that is proposed for investigating the dynamics of photon production under the more general time dependent anisotropic spacetime, and comment on the advantage of our formulation.

The Lagrangian density for free electromagnetic fields $L_A$ can be obtained by imposing the gauge conditions (18) as

$$L_A = \frac{1}{2} \dot{A}_T^i \dot{A}_T^i - \frac{1}{2} \partial_i A_T^i \partial^i A_T^i,$$

where the equations of motion for the Fourier modes of the vector potentials $A_{Tk}$ are obtained as follows:

$$\partial^2 A_{Tk}^i(\tau) + \frac{1}{\tau} \partial \tau A_{Tk}^i(\tau) + k^2 A_{Tk}^i(\tau) = 0.$$  

The mode functions for the linear polarization states are thus

$$\ddot{V}_{1k}^0(\tau) + \frac{1}{\tau} \dot{V}_{1k}^0(\tau) + \left( \frac{2k_\perp^2}{k^2 \tau^2} - \frac{3k_\perp^4}{k^4 \tau^2} + k^2 \right) V_{1k}^0(\tau) = 0,$$

$$\ddot{V}_{2k}^0(\tau) + \frac{1}{\tau} \dot{V}_{2k}^0(\tau) + k^2 V_{2k}^0(\tau) = 0,$$

where the Wranksian conditions (29) and (30) are satisfied. In Ref. [16], the quantization scheme is based upon the method developed in Ref. [17] where the mode functions $S(\pm)k$ to be defined later are used in the Fourier expansion of the electric and magnetic fields. The mode functions $S(\pm)k$ can be related to the mode functions $V_{1k}^0$ and $V_{2k}^0$ as

$$S(\pm)k = \frac{1}{2} \left[ \frac{k_\perp^2}{k^2 \tau} V_{1k}^0 + \frac{1}{k} \dot{V}_{1k}^0 \mp V_{2k}^0 \right].$$

Then, using Eqs. (33) and (34), $\dot{S}(\pm)k$ can also be expressed as

$$\dot{S}(\pm)k = -\frac{1}{2} \left[ k V_{1k}^0 \mp \dot{V}_{2k}^0 \right],$$

and the evolution equations of $S(\pm)k$ are found remarkably simple, that is,

$$\ddot{S}(\pm)k(\tau) + \frac{1}{\tau} \dot{S}(\pm)k(\tau) + k^2 S(\pm)k(\tau) = 0,$$

obtained in Ref. [17]. The corresponding Wranksian conditions of the mode functions $S(\pm)k$ can also be found. Here we propose the quantization scheme on the electromagnetic potentials directly. One immediate merit is that the normalization condition of the mode functions that will be used later to compute the number density of produced photons can be straightforwardly determined. It is particularly useful to set up the initial-valued problem of the nonequilibrium system where the initial mode functions can be specified consistently subject to the Wranksian conditions. However, this huge simplification works only for the free photons. Can neither $V_{1k}^0$ nor $S(\pm)k$ further diagonalize the Lagrangian for the interactive photon fields where the appropriate perturbation expansion will be employed later.
As for the interactive photon fields under consideration, the corresponding Lagrangian density can be written in terms of the transverse degrees of freedom as

\[ L_A + L_{\zeta A} = \frac{1}{2} \dot{A}_T \dot{A}_T^\dagger - \frac{1}{2} \left( 1 - \frac{e^2 \lambda V}{m_r^2 m_T^2} \right) \partial_\tau A_T \partial_\tau A_T^\dagger - \frac{e^2}{4 \pi^2 f_\pi} \zeta \frac{1}{\sqrt{\lambda}} \epsilon^{lmn} A_T^l \partial_m A_T^n. \]  

(38)

From observing the mode equations (33) and (34), we add and subtract the term

\[ L_I = \frac{1}{2} A_T \left( \frac{2 \partial^2}{\tau^2 \partial^2 - 3 \partial^2}{\partial^4} \right) A_T^i, \]  

(39)

in the Lagrangian density \( L_A + L_{\zeta A} \). The subtracted term will modify the kinetic energy of the transverse modes of the vector potentials with the polarization vector \( \epsilon^{(1)} \), so that the expansion modes in terms of the circularly polarized states to be defined later can diagonalize the unperturbative Lagrangian density given by \( L_A + L_{\zeta A} - L_I \) whereas the added term, \( L_I \), is taken as the counter term that will be treated perturbatively. The mode expansion of the transverse vector potentials with the circularly polarized states are given by

\[ A_{T,1}(x, \tau) = \int \frac{d^3k}{(2\pi)^2} \frac{1}{\sqrt{2k(\tau_0)}} \left\{ \left[ b_{(+)}k V_{(+)}^0(k)(\tau) \epsilon^{(+)}_{ki} + b_{(-)}k V_{(-)}^0(k)(\tau) \epsilon^{(-)}_{ki} \right] e^{ikx} + \text{h.c.} \right\}, \]  

(40)

where the circular polarization vectors can be constructed out of the linear polarization vectors \( \epsilon^{(1)}_{ki} \) and \( \epsilon^{(2)}_{ki} \) as follows:

\[ \epsilon^{(+)}_{ki} = \frac{1}{\sqrt{2}} \left( i \epsilon^{(2)}_{ki} + \epsilon^{(1)}_{ki} \right), \quad \epsilon^{(-)}_{ki} = \frac{1}{\sqrt{2}} \left( i \epsilon^{(2)}_{ki} - \epsilon^{(1)}_{ki} \right). \]  

(41)

The corresponding mode equations obtained with respect to the unperturbative Lagrangian density then become

\[ \dot{V}_{(+)}^0(k)(\tau) + \frac{1}{\tau} \dot{V}_{(+)}^0(k)(\tau) + \left( 1 - \frac{e^2 \lambda V}{m_r^2 m_T^2} \right) k^2 V_{(+)}^0(k)(\tau) + \frac{e^2}{2 \pi^2 f_\pi} \dot{k} k V_{(+)}^0(k)(\tau) = 0, \]  

(42)

\[ \dot{V}_{(-)}^0(k)(\tau) + \frac{1}{\tau} \dot{V}_{(-)}^0(k)(\tau) + \left( 1 - \frac{e^2 \lambda V}{m_r^2 m_T^2} \right) k^2 V_{(-)}^0(k)(\tau) - \frac{e^2}{2 \pi^2 f_\pi} \dot{k} k V_{(-)}^0(k)(\tau) = 0. \]  

(43)

Thus, as compared with the mode equations of free photons (33) and (34), the involved interaction gives the additional time dependent frequency terms in the mode equations that depend on the dynamics of the expectation value of the neutral pion field. In particular, when the mean field undergoes the oscillations around the equilibrium value to be determined dynamically, it is known that the solutions will exhibit the features of the unstable bands and the growth of the fluctuating modes. The growth of the modes in the unstable bands translates into the profuse particle production. Thus, the photon production mechanism is that of parametric amplification. Here the background expansion will bring to us the extra damping effect. This is an extension of our previous studies on parametric amplification of the photon production to the case with the longitudinal background expansion. A novel phenomenon can be observed. We also observe the polarization asymmetry in the produced circularly polarized photons as a result of the pseudo-scalar nature of the coupling that is of interest to study. It is conceivable that after the heavy ions collision, the produced photons
are mainly having large momentum along the beam direction and then their $k_1/k$ can be perturbatively small. The perturbation expansion will be adopted for this small parameter $k_1^2/k^2$. The interaction Hamiltonain density $H_I$ can be constructed from the associated Lagrangian density $L_I$ given by

\[
H_I(\tau) = \frac{1}{2k(\tau_0)}\sqrt{g} \frac{k_1^2}{2k^2\tau_2^2} \left[ b^{(+)}k V^{0}_{(+)}k(\tau) - b^{(-)}k V^{0}_{(-)}k(\tau) \right. \\
\left. + b^{\dagger}_{(+)}k V^{0*}_{(+)}k(\tau) - b^{\dagger}_{(-)}k V^{0*}_{(-)}k(\tau) \right] \times [k \rightarrow -k]. \quad (44)
\]

Besides the interaction Hamilton depends on the amount of polarization asymmetry of the produced photons obtained from the mode equations \([12]\) and \([13]\) that certainly give the next order results.

From the exact expressions of the electric and magnetic fields in Eqs. \((26)\) and \((27)\), to order of $k_1^2/k^2$ they can be obtained perturbatively as

\[
E^i_0k(\tau) = E^i_{0k}(\tau) + \delta E^i_0k(\tau) + 2i \int d\tau' \theta(\tau - \tau') \left[ E^i_{0k}(\tau'), H_I(\tau') \right] + O(k_1^4/k^4), \quad (45)
\]

\[
B^i_0k(\tau) = B^i_{0k}(\tau) + 2i \int d\tau' \theta(\tau - \tau') \left[ B^i_{0k}(\tau'), H_I(\tau') \right] + O(k_1^4/k^4), \quad (46)
\]

where

\[
E^i_{0k}(\tau) = \frac{1}{\sqrt{2k(\tau_0)}}\sqrt{g} \left\{ \left[ b^{(+)}k \hat{V}^{0}_{(+)}k(\tau) + b^{\dagger}_{(+)}k \hat{V}^{0*}_{(+)}k(\tau) \right] \hat{\epsilon}^{(+i)}_k \right\} + [(+) \rightarrow (-)]. \quad (47)
\]

\[
\delta E^i_0k(\tau) = \frac{1}{\sqrt{2k(\tau_0)}}\sqrt{g} \frac{k_1^2}{2k^2\tau_2} \left\{ \left[ b^{(+)}k \hat{V}^{0}_{(+)}k(\tau) - b^{(-)}k \hat{V}^{0}_{(-)}k(\tau) \right. \right. \\
\left. \left. + b^{\dagger}_{(+)}k \hat{V}^{0*}_{(+)}k(\tau) - b^{\dagger}_{(-)}k \hat{V}^{0*}_{(-)}k(\tau) \right] \hat{\epsilon}^{(+i)}_k \right\} + [(+) \rightarrow (-)]. \quad (48)
\]

\[
B^i_{0k}(\tau) = \frac{-1}{\sqrt{2k(\tau_0)}}\sqrt{g}k \left\{ \left[ b^{(+)}k \hat{V}^{0}_{(+)}k(\tau) + b^{\dagger}_{(+)}k \hat{V}^{0*}_{(+)}k(\tau) \right] \hat{\epsilon}^{(+i)}_k \right\} - [(+) \rightarrow (-)]. \quad (49)
\]

Then, the backreaction effects from the photon fields to the evolution of the mean field of the neutral pion in Eq. \((11)\) can be expressed by the electric and magnetic fields that in turn can be obtained perturbatively with all above results. They are

\[
\frac{1}{4} \epsilon^{\alpha\beta\mu\nu} \langle F_{\alpha\beta} F_{\mu\nu} \rangle(\tau) = \langle \left\{ E^i_{0k}(\tau), B_{-ki}(\tau) \right\} \rangle \\
= \langle \left\{ E^i_{0k}(\tau), B_{-ki}(\tau) \right\} \rangle + \langle \left\{ \delta E^i_{0k}(\tau), B_{-ki}(\tau) \right\} \rangle \\
+ 2i \int d\tau' \theta(\tau - \tau') \langle \left\{ E^i_{0k}(\tau), [B_{0-ki}(\tau), H_I-k(\tau')] \right\} \rangle \\
+ 2i \int d\tau' \theta(\tau - \tau') \langle \left\{ B^i_{0k}(\tau), [E_{0-ki}(\tau), H_I-k(\tau')] \right\} \rangle + O(k_1^4/k^4), \quad (50)
\]

\[
\frac{1}{4} \epsilon^{\mu\nu\sigma} \epsilon^{\alpha\beta\gamma\delta} g_{\delta\sigma} \langle F_{\mu\nu} F_{\alpha\beta} \rangle(\tau) = \langle \left\{ B^i_{0k}(\tau), B_{-ki}(\tau) \right\} \rangle \\
= \langle B^i_{0k}(\tau) B_{0-ki}(\tau) \rangle + 2i \int d\tau' \theta(\tau - \tau') \langle \left\{ B^i_{0k}(\tau), [B_{0-ki}(\tau), H_I-k(\tau')] \right\} \rangle + O(k_1^4/k^4),
\]

\[
\frac{1}{4} \epsilon^{\mu\nu\sigma} \epsilon^{\alpha\beta\gamma\delta} g_{\delta\sigma} \langle \partial_{\gamma} F_{\mu\nu} F_{\alpha\beta} \rangle(\tau) = \frac{d}{d\tau} \langle \left\{ B^i_{0k}(\tau), B_{-ki}(\tau) \right\} \rangle.
\]
where the above correlation functions can be obtained from Eqs. (47), (48), and (49), expressed in terms of the mode functions, as follows:

\[
\langle \left\{ E_{0\mathbf{k}}^{\dagger}(\tau), B_{-\mathbf{k}i}(\tau) \right\} \rangle = \frac{1}{2k(\tau_0)} g k \frac{d}{d\tau} \left[ | V_{(\mathbf{k})}^0(\tau) |^2 (\tau) - | V_{(\mathbf{k})}^0(\tau) |^2 (\tau) \right],
\]

\[
\langle \left\{ \delta E_{0\mathbf{k}}^{\dagger}(\tau), B_{-\mathbf{k}i}(\tau) \right\} \rangle = \frac{1}{2k(\tau_0)^2} g \frac{k_1^2}{k^2} \left[ | V_{(\mathbf{k})}^0(\tau) |^2 (\tau) - | V_{(\mathbf{k})}^0(\tau) |^2 (\tau) \right],
\]

\[
\langle \left\{ B_{0\mathbf{k}}^{\dagger}(\tau), \left[ E_{0\mathbf{k}}(\tau), H_{I,-\mathbf{k}}(\tau') \right] \right\} \rangle = \frac{1}{2k(\tau_0)^2} g^2 \frac{k_1^2}{4k^2} \left\{ \left[ \left( V_{(\mathbf{k})}^0(\tau)V_{(\mathbf{k})}^{0*}(\tau') + c. c. \right) \times \left( \dot{V}_{(\mathbf{k})}^0(\tau)V_{(\mathbf{k})}^{0*}(\tau') - c. c. \right) - \left[ (-) \rightarrow (+) \right] \right\},
\]

\[
\langle \left\{ E_{0\mathbf{k}}^{\dagger}(\tau), \left[ B_{0\mathbf{k}}(\tau), H_{I,-\mathbf{k}}(\tau') \right] \right\} \rangle = \frac{1}{2k(\tau_0)^2} g^2 \frac{k_1^2}{4k^2} \left\{ \left[ \left( \dot{V}_{(\mathbf{k})}^0(\tau)V_{(\mathbf{k})}^{0*}(\tau') + c. c. \right) \times \left( V_{(\mathbf{k})}^0(\tau)V_{(\mathbf{k})}^{0*}(\tau') - c. c. \right) - \left[ (-) \rightarrow (+) \right] \right\},
\]

\[
\langle \left\{ B_{0\mathbf{k}}^{\dagger}(\tau) B_{0\mathbf{k}}(\tau) \right\} \rangle = \frac{1}{2k(\tau_0)} g k^2 \sum_{a=\pm} V_{(a)\mathbf{k}}^0(\tau),
\]

\[
\langle \left\{ B_{0\mathbf{k}}^{\dagger}(\tau), \left[ B_{0\mathbf{k}}(\tau), H_{I\mathbf{k}}(\tau') \right] \right\} \rangle = \frac{1}{2k(\tau_0)^2} g^2 \frac{k_1^2}{4k^2} \sum_{a=\pm} \left[ V_{(a)\mathbf{k}}^0(\tau)V_{(a)\mathbf{k}}^{0*}(\tau') + c. c. \right]
\times \left[ V_{(\mathbf{k})}^0(\tau)V_{(\mathbf{k})}^{0*}(\tau') - c. c. \right]. 
\]

The curly bracket in above means the anticommutator. The results from the terms of order \(k_1^2/k^2\) give an estimate on the effects from their higher order contributions.

According to the Bjorken’s scenario the boost invariant hydrodynamical flow might be created after the heavy ion collisions. The corresponding initial vacuum state may already contain particles with respect to their asymptotical states to be observed in the detector [4].

Besides the relaxation of the DCC can give an additional effect to produce the photons. Let us now consider the initial state at time \(\tau_0\) given by the adiabatic modes in the comoving spacetime time that will be specified below. The corresponding initial particle number density can be expressed as

\[
\langle N_{\mathbf{k}}(\tau_0) \rangle = \frac{1}{2k(\tau_0)} \sqrt{g(\tau_0)} \left[ \langle E_{\mathbf{k}}^{\dagger}(\tau_0) E_{-\mathbf{k}i}(\tau_0) \rangle + \langle B_{\mathbf{k}}^{\dagger}(\tau_0) B_{-\mathbf{k}i}(\tau_0) \rangle \right] - 1. 
\]

The expectation value of the number operator with respect to an initial vacuum state evolves in time and has the following form:

\[
\langle N_{\mathbf{k}}(\tau) \rangle = \frac{1}{2k} \sqrt{g} \left[ \langle E_{\mathbf{k}}^{\dagger}(\tau) E_{-\mathbf{k}i}(\tau) \rangle + \langle B_{\mathbf{k}}^{\dagger}(\tau) B_{-\mathbf{k}i}(\tau) \rangle \right] - 1,
\]

that is, to order \(k_1^2/k^2\),

\[
\langle N_{\mathbf{k}}(\tau) \rangle = \langle E_{0\mathbf{k}}^{\dagger}(\tau) E_{0\mathbf{k}}(\tau) \rangle + \langle B_{0\mathbf{k}}^{\dagger}(\tau) B_{0\mathbf{k}}(\tau) \rangle + \{ \langle E_{0\mathbf{k}}^{\dagger}(\tau), \delta E_{0\mathbf{k}}(\tau) \rangle \} + 2i \int d\tau' \theta(\tau - \tau') \{ \langle E_{0\mathbf{k}}^{\dagger}(\tau), [E_{0\mathbf{k}}(\tau), H_{I\mathbf{k}}(\tau')] \rangle \} \\
+ 2i \int d\tau' \theta(\tau - \tau') \{ \langle B_{0\mathbf{k}}^{\dagger}(\tau), [B_{0\mathbf{k}}(\tau), H_{I\mathbf{k}}(\tau')] \rangle \} + \mathcal{O}(k_1^4/k^4). 
\]
Except for the correlation functions that we have found above, the correlation functions between the electric fields are obtained as

\[
\langle E^i_{0k}(\tau) E_{0-k}(\tau) \rangle = \frac{1}{2k(\tau_0)} \sum_{a=\pm} V^0_{(a)k}(\tau) V^0_{(a)k}^*(\tau),
\]

\[
\langle \{ E^i_{0k}(\tau), \delta E_{-ki}(\tau) \} \rangle = \frac{1}{2k(\tau_0)} \sum_{a=\pm} \frac{d}{d\tau} | V^0_{(a)k} |^2 (\tau),
\]

\[
\langle \{ E^i_{0k}(\tau), [E_{0ki}(\tau), H_{Ik}(\tau')] \} \rangle = \frac{1}{2k(\tau_0)} \sum_{a=\pm} \left[ V^0_{(a)k}(\tau) V^0_{(a)k}^*(\tau') + c.c. \right]
\]

\times \left[ V^0_{(a)k}(\tau) V^0_{(a)k}^*(\tau') - c.c. \right],
\]

(55)

Thus the phase space number density in the comoving frame is given by [10, 11]

\[
\frac{dN}{d\eta d^2k_\perp} = \langle N_k \rangle.
\]

(56)

We now need to relate this quantity to the invariant spectra of the produced photons measured in the laboratory frame. From the coordinate transformations (2), we find that

\[
k_\tau = p_\perp \cosh(\eta - w),
\]

\[
k_\eta = -p_\perp \tau \sinh(\eta - w),
\]

\[
k_y = p_y,
\]

\[
k_x = p_x,
\]

(57)

where \( k_\mu = (k, k) \) is the photon four-momentum in the comoving frame and \( p_\mu = (p, p) \) is that measured in the laboratory frame. We here also introduce the outgoing photon particle rapidity defined by the four momentum in the center of the mass coordinate system given by

\[
p_\mu = (p_\perp \cosh w, p_\perp, p_\perp \sinh w).
\]

(58)

Hence, from the momentum transformation laws, one can change the momentum variables from \( k_\mu \) into \( p_\mu \) in the spectral number density given by

\[
p \frac{dN}{d^3p} = \frac{dN}{d\eta d^2k_\perp} = \int dz d^2x_\perp \left| \frac{\partial k_\eta}{\partial \eta} \right| \frac{dN}{d\eta d^2x_\perp d\eta d^2k_\perp}.
\]

(59)

Thus, carrying out the Jacobian, the end result of the invariant spectra in the laboratory frame is obtained as [4]

\[
p \frac{dN}{d^3p} = A_\perp \int dk_\eta \frac{dN}{d\eta d^2x_\perp d\eta d^2k_\perp},
\]

(60)

where \( A_\perp \) is the transverse dimension of the effective transverse size of the colliding ions. This quantity is independent of \( w \) as a consequence of the assumed boost invariance.

Here we will postpone the full dynamical study on the photon production to our future work and consider free photons instead. With respect to an initial vacuum state of the photons given by the adiabatic modes, the particle number can also be measured by their asymptotical states to be observed in the detector under an expanding background. Exact
analytical solutions to the mode equations Eqs. (37) in the free photon case can allow to give an order-of-magnitude estimate on this effect, and they also provide an interesting example to illustrate the above formalism. To do so, we express the expectation value of the number operator as follows:

$$\langle N_k \rangle(\tau) = \frac{\tau}{2k(\tau_0)k} \left[ \hat{S}_{(+)} k(\tau) \hat{S}_{(+)}^* k(\tau) + \hat{S}_{(-)} k(\tau) \hat{S}_{(-)}^* k(\tau) \right] + k^2 \left( S_{(+)} k(\tau) S_{(+)}^* k(\tau) + S_{(-)} k(\tau) S_{(-)}^* k(\tau) \right) - 1. \quad (61)$$

The solutions $S_{(\pm)k}$ to the equations (37) can be found analytically in below [17]:

$$S_{(\pm)k}(\tau) = C_{(\pm)1} e^{-ik(\tau_0)\tau} H^{(1)}_{i\nu}(k_\perp \tau) + C_{(\pm)2} e^{-ik(\tau_0)\tau} H^{(2)}_{i\nu}(k_\perp \tau). \quad (62)$$

Here $H^{(1)}_{i\nu}$ and $H^{(2)}_{i\nu}$ denote the Hankel functions of the first and second kind with purely imaginary order, $\nu = k_\eta$. Notice that the two solutions are complex conjugates of each other. Thus, the above constants $C_1$ and $C_2$ are to be determined by the initial conditions. They can be obtained from Eqs. (35) and (36) in which the initial value of the mode functions $V_{(1)k}$ and $V_{(2)k}$ are specified by the adiabatic modes as [4]:

$$V_{(1)k}(\tau_0) = \frac{e^{-ik(\tau_0)\tau_0}}{\sqrt{\tau_0}}, \quad \tilde{V}_{(1)k}(\tau_0) = \frac{e^{-ik(\tau_0)\tau_0}}{\sqrt{\tau_0}} \left( -ik(\tau_0) - \frac{k^2_\perp}{k^2(\tau_0)\tau_0} \right), \quad (63)$$

$$V_{(2)k}(\tau_0) = \frac{e^{-ik(\tau_0)\tau_0}}{\sqrt{\tau_0}}, \quad \tilde{V}_{(2)k}(\tau_0) = \frac{e^{-ik(\tau_0)\tau_0}}{\sqrt{\tau_0}} \left( -ik(\tau_0) \right), \quad (64)$$

consistent with the Wronskian conditions (29) and (30). Fig. 1 displays the result of the spectra of the photons per unit effective transverse area of the collisions associated with its initial state measured with the reference of the asymptotical states. Typically, less one particle per unit effective transverse area is found [16]. It would be of interest in both theoretically and experimentally to learn how the photon production can be further amplified from the relaxation of a DCC under a longitudinal background expansion.

These derived dynamical equations display several important physical processes: i) nonlinear relaxation of the mean field within a DCC, ii) particle creation due to parametric amplification of the fluctuations driven by the time dependent mean field, iii) the enhancement from the boost effect under a longitudinal hydrodynamical expansion. This must be studied self-consistently from the coupled set of Eqs. (11), (14), and (50) by inserting the mode functions in Eqs. (13), (12), and (43). The invariant spectra of the observed photon production can be obtained from Eqs. (54), (55), and (60). In particular, the expressions of corrections give an estimate on the ignored terms with higher order $k_\perp^2/k^2$. Although we do not intend to offer a numerical analysis of the resulting equations in this paper, the nonequilibrium equation of motion has the potential for providing an enhancement of the photon production. This enhancement could lead to an experimentally observable signal in the direct photon measurements of heavy-ion collisions that can be a potential test of the formation of disoriented chiral condensates. We postpone to a forthcoming paper the full numerical study of these equations and an assessment of the potential phenomenological impact of the nonequilibrium dynamics.

In conclusion, what we have done in this work is to propose the quantization scheme on electromagnetic potentials under a longitudinal background expansion, which suits the nonequilibrium initial-valued problem. This is a necessary first step toward the study of
FIG. 1: The respective spectra of the photons per unit effective transverse area of the collisions in its initial state measured by the asymptotical states as a function of the transverse momentum (in units of $\tau_0$) at times (in units of $\tau_0$) $\tau = 2$ (long-dashed), 5 (short-dashed), 10 (solid).

photon production through the non-equilibrium relaxation of a disoriented chiral condensate formed in ultra-relativistic heavy-ion collisions. The next calculation to do would be a numerical study of the above equations with an eye on obtaining the spectra of the produced photons that can be compared with thermal photons from quark-gluon plasma and hadronic matter as done in Ref. [14]. This would have fascinating phenomenological consequences.

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