New Signals in Precision Gravity Tests and Beyond

Q.G. Bailey\textsuperscript{1}, J.L. James\textsuperscript{2}, and J.R. Slone\textsuperscript{1}

\textsuperscript{1}Physics and Astronomy Department, Embry-Riddle Aeronautical University, Prescott, AZ 86301, USA

\textsuperscript{2}Physics and Astronomy Department, Vanderbilt University, Nashville, TN 37235, USA

We review the status of tests of spacetime symmetries with gravity. Recent theoretical and experimental work has involved gravitational wave signals, precision solar-system tests, and sensitive laboratory tests searching for violations of spacetime symmetries. We present some new theoretical results relevant for short-range gravity tests, with features of multiple length scales, and possible large non-Newtonian forces at short distances.

1. Background

Effective field theory has long been used to describe physics beyond General Relativity (GR) and the Standard Model (SM) of particle physics.\textsuperscript{1} In particular, any speculated deviations from spacetime symmetry coming from an unknown fundamental theory\textsuperscript{2,3} can be described in a model-agnostic way by adding to GR and the SM, generic Lagrangian density terms with background coefficients and known field operators.\textsuperscript{4} This framework has been widely used and can compare tests in distinctly different regimes.\textsuperscript{5–7}

The gravity sector of this effective field framework has been studied for more than 16 years and has revealed many intriguing signals for new physics in tests such as gravimetry, lunar laser ranging, and gravitational waves.\textsuperscript{8,9,14}

The gravity sector in the Riemann geometry limit is a general expansion with a Lagrange density in the form of

$$L = \frac{1}{\kappa} \sqrt{-g} \left( R + s_{\alpha\beta} R^{\alpha\beta} + ... + k_{\alpha\beta\gamma\delta} R^{\alpha\beta} R^{\gamma\delta} + ... \right), \quad (1)$$

where $g$ is the determinant of the metric $g_{\mu\nu}$ and the first term with the Ricci scalar $R$ is GR, while the remaining terms form a series with coefficients for symmetry breaking (e.g., $s_{\alpha\beta}$) coupled to curvature tensors. The series may include dynamical terms for the coefficients or the symmetry breaking may be explicit.\textsuperscript{15–17}
2. Recent measurements and theory

The phenomenology of the terms in (1) has been studied in a number of works. Observable effects in weak-field gravity tests have been established for minimal and nonminimal terms, and some work has been done on strong-field gravity regimes like cosmology. Effects on gravitational waves have been studied, showing that dispersion and birefringence occur generically as a result of CPT and Lorentz violation. Analysis has been performed in tests such as lunar laser ranging, gravimetry, pulsars, and using the catalog of GW events.

On the theory side, explicit local Lorentz and diffeomorphism symmetry cases have been explored various contexts. A “3+1” or ADM formulation of the EFT framework has been explored in Refs. 12, 22. Also, explicit breaking solutions have been explored for phenomenology. Other recent work includes much attention to vector models of spontaneous symmetry breaking, with black hole solutions being obtained, and the systematic construction of dynamical terms for the spontaneous symmetry breaking scenario in the gravity sector. Finally we note some recent theoretical work has identified general properties of backgrounds in effective field theory, and new types of tests are possible that search for non-Riemann geometry.

3. Short-range gravity signals

Presently, the nature of gravity is unknown on length scales less than micrometers. New types of forces, stronger than the Newtonian gravitational force, could exist and be consistent with experimental limits.

In references 10 and 26, Lorentz-symmetry breaking solutions for short-range gravity tests were found using an approximation of first order in the coefficients for the modified Newtonian force. This approximation makes searches in some short-range tests challenging, as some tests are designed to probe very small length scales at the cost of sensitivity to the Newtonian force. We comment here on progress towards “exact” solutions, i.e., to all orders in coefficients for Lorentz-symmetry breaking, that could be interesting for all short-range gravity tests.

One particular model that contains the interesting features of exact solutions is the following Lagrange density, a subset of the general action in (1),

\[ \mathcal{L} = \frac{1}{\kappa} \sqrt{-g} \left( R + k_{\alpha\beta} R^{\alpha\beta} \right). \]
The quantities $k_{\alpha\beta}$ are the coefficients for Lorentz violation for this case.

The action in (2), taken in the linearized gravity limit, yields field equations for the metric fluctuations $h_{\mu\nu}$. If we further restrict attention to the static limit and only isotropic coefficients $k_{00}$ and $k_{jj}$, in a special coordinate system, we obtain the following coupled equations for the metric components $h_{00}$ and $h_{jj}$:

$$\nabla^2(h_{00} + h_{jj}) - 3(k_{00} - \frac{1}{9}k_{jj})\nabla^4h_{00} + (k_{00} - k_{kk})\nabla^4h_{jj} = -32\pi G_N \rho,$$

$$\nabla^2(3h_{00} + h_{jj}) + 4(k_{00} - \frac{1}{3}k_{kk})\nabla^4h_{00} + \frac{8}{3}k_{kk}\nabla^4h_{jj} = 0.$$  

(3)

Note that $k_{jj} - k_{00} = k_{\mu\nu}\eta^{\mu\nu}$ is a Lorentz invariant scalar combination.

A Green function solution can be constructed where we use a point source $4\pi G_N \rho = \delta^{(3)}(\vec{r} - \vec{r}')$, and the point source solutions for $h_{00}$ and $h_{jj}$ are denoted $G_1$ and $G_2$. With $R = |\vec{r} - \vec{r}'|$, and with guidance from standard catalogues of Green functions, the general solutions take the form:

$$G_1 = \frac{1}{R} \left( A_1 e^{-q_1 R} + A_2 e^{-q_2 R} + A_3 \right),$$

$$G_2 = \frac{1}{R} \left( B_1 e^{-q_1 R} + B_2 e^{-q_2 R} + B_3 \right).$$  

(4)

Here the $A_n$'s and $B_n$'s are constants to be solved for as well as the $q_1$ and $q_2$. In constructing this solution we are assuming boundary conditions where the metric components vanish far from the source. Insertion of (4) into the point source version of (3), followed by using the well-known properties of the functions of $R = |\vec{r} - \vec{r}'|$, allows one to solve for the 8 parameters $A_1$, $A_2$, $A_3$, $B_1$, $B_2$, $B_3$, $q_1$, and $q_2$ from 8 resulting algebraic equations.

First, we find that for nontrivial solutions, both $q_1$ and $q_2$ must satisfy a quartic equation. To display this, it is convenient to write a short-hand $q^2 = u \pm v$, where $u$ and $v$ are given by:

$$u = \frac{-(k_{00} - \frac{2}{3}k_{jj})}{2(k_{00} + \frac{1}{3}k_{jj})^2},$$

$$v = \frac{\sqrt{(k_{00} - \frac{2}{3}k_{jj})^2 - 4(k_{00} + \frac{1}{3}k_{jj})^2}}{2(k_{00} + \frac{1}{3}k_{jj})^2}. \quad (5)$$

Note that $u$ is real and $v$ can be complex. The positions of the roots $\{q = z^{1/2} = \pm(u \pm v)^{1/2}\}$ in the complex plane depend on the values of the coefficients $k_{00}$ and $k_{jj}$. If $q$ is entirely real and positive, then the solutions in (4) will exhibit exponential damping in $R$ or short-range Yukawa-like behavior. The case where $q$ is negative and real will result in runaway
exponential increase and is not physically viable. When $q$ has an imaginary piece or is entirely imaginary, the solution will have oscillations in $R$.

As a sample for this proceedings, we assume the condition $q_1^2 \neq q_2^2$. This condition ensures that the coefficients $k_{00}$ and $k_{jj}$ are treated $a$ $priori$ independent. In (5), $q_1^2$ then takes one sign in the $\pm$, and $q_2^2$ takes the other sign. For this case we obtain the solutions for the Green function $G_1$ as follows:

$$
G_1 = \frac{1}{2\pi R} \left( 1 + \frac{k_{00} + \frac{1}{2}k_{jj}}{\sqrt{(k_{00} - \frac{2}{3}k_{jj})^2 - 4(k_{00} + \frac{1}{3}k_{jj})^2}} \right) \frac{e^{-R/\lambda_+}}{R} - \frac{1}{4\pi} \left( 1 - \frac{k_{00} + \frac{1}{2}k_{jj}}{\sqrt{(k_{00} - \frac{2}{3}k_{jj})^2 - 4(k_{00} + \frac{1}{3}k_{jj})^2}} \right) \frac{e^{-R/\lambda_-}}{R},
$$

where the $\lambda_\pm$ constants are defined by

$$
\frac{1}{(\lambda_\pm)^2} = u \pm v,
$$

and they act like two distinct length scales. The other Green function $G_2$ has a similar solution but it is $G_1$ that is proportional to the Newtonian potential.

We note the contrast of this result (6) with previous results. First, unlike the standard Yukawa potential, we have 2 length scales. Second, the amplitudes of the two terms vary depending on the values of the coefficients. In particular, we find that these amplitudes could take on large values (i.e., much greater than unity) for a narrow range of coefficient ratios $k_{jj}/k_{00}$, even if the coefficients themselves are “small”. This is in contrast to standard assumptions of the smallness of Lorentz-violating effects. Note that the length scales would also be small, so such large Lorentz-breaking forces could escape detection in long-range tests.

For practical evaluation over distributions of matter, such as those used in experiment, one would take the integral of the Green functions over the smooth matter distributions $\rho(\vec{r})$ as usual. A more complete treatment of the these types of solutions is forthcoming.

**Acknowledgments**

We acknowledge the support of the NSF, grant numbers 1806871 and 2207734, Embry-Riddle Aeronautical University, and the Arizona NASA Space Grant Consortium.
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