About the longitudinal slopes of flat rivers

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Abstract. The article discusses the methods of determining the longitudinal slopes of rivers, their advantages and disadvantages are noted. Various methods of determining the longitudinal slopes of rivers are analyzed. It is revealed that the weighted average slope adopted by the hydrometeorological service, determined by the graphoanalytic method, does not fully reflect the essence of the influence of slopes on the speed of water movement. It is noted that it is expedient to use the differential equation of uneven motion to determine the slopes of the water surface in the lower reaches of lowland rivers. The criterion ratio of slopes characterizing the stability of the riverbed in the longitudinal direction is revealed. The hydraulics-morphological method of determining the level and longitudinal slope of the free flow surface, as well as quantitative criteria for the movement of river flows, has been continued.

1. Introduction

The study of the formation of riverbeds and the solution of various hydrological and hydraulic problems related to the calculation of the nature and speed of flow is almost impossible without knowledge of the quantitative characteristics of the longitudinal slopes of rivers. Many scientists of the CIS and abroad are engaged in methods of determining the averaged longitudinal slopes of rivers [1-2;4-5;7;9]. However, the weighted average slope adopted in the hydrometeorological service [10-11], determined by the graphoanalytic method, does not reflect the essence of the influence of slopes on the speed of water movement and is purely qualitative. Therefore, studies to obtain a quantitative characteristic of the longitudinal slope resulting from the hydraulic conditions of the water flow are of great practical importance.

In addition, information about the slope of the water surface is necessary to solve two main problems of channel hydraulics:

- To assess the channel capacity determined by water flow.
- To determine the value of the Shezi coefficient, taking into account hydraulic resistances.

2. Materials and methods

Currently, two longitudinal slopes of the river are distinguished and determined: average and weighted average or equalized. The average slope refers to the ratio of the total fall of the river to its length. Since the average slope of the longitudinal profile of the river, determined without taking into account its shape, does not always sufficiently characterize the hydraulic features of the water flow, therefore, a weighted average or equalized slope is determined.
Determining the average slope does not cause any difficulties. It is enough just to determine the excess of the source over the mouth or observation point ($\Delta H$) and the length of the river ($L$) and calculate the average slope using the formula:

$$I = \frac{\Delta H, m}{L, km}$$ (1)

Considerable attention is paid to the issue of developing methods for determining the weighted average slope of rivers due to the fact that a significant influence of the slope on the speed of water movement in the channel has been reliably established.

Weighted average slopes are understood as slopes obtained by graphically drawing an averaged straight line that cuts off the profile breaks in such a way that the conditions of minimum and equality of the cut and added areas are observed (figure 1). With this method of determining the slope, the area of the natural profile is first calculated:

$$F = \frac{H_1 + H_2}{2} l_1 + \frac{H_2 + H_3}{2} l_2 + \frac{H_{n-1} + H_n}{2} l_{n-1}$$ (2)

Where $H_1, H_2, ..., H_n$ - marks of the profile tipping points, $l_1, l_2, ..., l_{n-1}$ – the distance between the profile tipping points, in total equal to $L$.

The alignment line (figure 1) passes through the point C with coordinates:

$$x_c = \frac{L}{2}; \quad y_c = H_{ave} = \frac{F}{L}$$

This method was proposed by G.A. Alekseev for leveling terrain profiles primarily in road construction.

In order to reveal the physical meaning of slope alignment, let's take the longitudinal profile of the river, consisting of $n$ sections. The parameters of these sections will be: $I_1, l_1; I_2, l_2; ....; I_n, l_n; H_1, H_2; ....; H_n$.

The arithmetic mean slope for this river:

$$I_{ave} = \frac{I_1 + I_2 + .......I_n}{n}$$ (3)

Weighted average slope:

$$I_{weig} = \frac{I_1 l_1 + I_2 l_2 + .......I_n l_n}{L} = \frac{H_1 - H_n}{L}$$ (4)
Thus, the slope determined by dividing the total fall of the river by its length is already a weighted average, weighted by the length of the river.

Analyzing the graphoanalytic method adopted by the hydrometeorological service of G.A. Alekseev [1], K.F. Mikhnevich [3] comes to the conclusion that this method does not reflect the essence of the influence of slopes on the speed of water movement in the channel.

Criticizing the method, K.F. Mikhnevich offers his own original way of determining the "resulting" (weighted average) slope through the average flow velocity weighted along the length of the river. For his method, he uses the well-known Shezi-Manning formula:

\[ \nu = A \sqrt{I}, \]

Where \( A = \frac{I}{n} h \sqrt{h} \), taking the parameters (the roughness of the riverbed and the average depth of the river) constant throughout the river.

A.P. Kopylov [4] in order to simplify the process of determining weighted average slopes as much as possible, proposed another method, conventionally called logarithmic. In other words, the weighted average slope of the river is determined through the logarithm of the partial slopes of the profile weighted by the length of the sections. The formula for calculations has the form:

\[ \lg \bar{I} = \sum_{i=1}^{n} \frac{I}{L} \lg I_i \]

A.P. Kopylov, analyzing the methods they considered for determining the slope of rivers, noted the peculiar universality of the logarithmic method, expressed in ensuring the required accuracy, eliminating the need for a longitudinal profile of the river and simplicity of calculations. At the same time, he did not notice that the horizontal lines of uneven slopes cut by temporary watercourses become tortuous, they are depicted only in a horizontal projection.

However, all the methods discussed above for determining the longitudinal slope of rivers to one degree or another do not satisfy the requirement of quantifying the flow of water flowing from hydraulic conditions.

In the present work, a scientific attempt has been made: to obtain an objective quantitative assessment of observations of the slope of the water surface of river flows, and to show their role in assessing the stability of riverbeds and canals.

3. Results and Discussion

The slope of the free surface of the river flow is one of the most important hydraulic characteristics [6-9]. However, the recommendations on the organization of observations over the slope [10-11] are purely qualitative in nature.

Studies [6;12;14] have established that the longitudinal slope of river flows to one degree or another characterize the capacity and stability of riverbeds and channels, as well as the processes of reshaping the waterworks. Therefore, in order to fully reflect the role of the slope in solving the above and other problems of river flows, a hydraulic approach can be used within the framework of the equations of hydromechanics.

The equations of hydromechanics connect the velocity field with the pressure field and this leads to the fact that, in principle, the measured pressure drop (or piezometric slope) can be used to judge the flow of water passing through the section. In developed countries, most industrial flowmeters (for example, in England 90%) are based precisely on the measurement of differential pressure (slopes).

Considering that the movement of water in a river stream is uneven and non-stationary, therefore, the systems of Saint-Venant and Boussinesq equations are used in the study of these flows. As experience shows, the possibilities of practical application of these models are determined mainly by the accuracy of setting parameters reflecting the individual properties of each particular channel,
which arise from the tasks of parametric identification, that is, finding and clarifying the characteristics of open channels and flows in them. In recent years, these problems have been solved both experimentally and theoretically by many researchers: N.A. Kartvelish [9], G.A. Atanov [16], etc. Naturally, summarizing the results of these studies throughout the river will help to obtain a general model of uneven movement, which can be used for the above purposes.

For the first time, optimization methods for solving the identification problem are presented in the form of series of degrees x and z with constant coefficients. Using the continuity equation and the momentum equation, this hydrometric model is brought to this approximation:

$$i - \frac{\partial h}{\partial x} = \left[ \frac{1}{g*} \frac{\partial U}{\partial t} + \frac{\alpha U}{g*} \frac{\partial U}{\partial x} - \frac{\alpha - 1}{g*} \frac{U \partial F}{\partial t} \right]$$

$$+ \frac{I}{g*F} \left[ \frac{\partial^2}{\partial t dx} \left( \beta_1 FRU \frac{\partial h}{\partial x} + \beta_2 FR \frac{\partial h}{\partial t} \right) + \frac{\partial^2}{\partial x^2} \left( \beta_3 FRU^2 \frac{\partial h}{\partial x} + \beta_4 FRU \frac{\partial h}{\partial t} \right) \right]$$

$$\frac{\partial UF}{\partial x} + \frac{\partial F}{\partial t} = 0$$ (8)

Where j is the dissipative term; α, β₁, β₂, β₃, β₄ are coefficients depending on the velocity distribution over the live section, the remaining designations are generally accepted in hydrometry.

For the practically interesting case when information is known about the level $H=f(x, t)$ and the morphology of a fixed hydraulic solution with a coordinate $x_0$, after simple transformations of equations (6) and (7) V.V. Kovalenko obtained the equation in full derivatives [9]:

$$\frac{dU}{dt} + f_1(x, t)u^2 + f_2(x, t)u + f'_1(x, t)j = f_3(x, t)$$ (9)

In formula (8), the functions $f_i, f'_i, f_2, f_3$ with a known morphometry of the hydrogate depend only on the water level and its derivatives in coordinate and time.

To determine the water flow rate according to formula (8), information is needed not only about the level and slope, but also about the curvature of the free surface.

When determining hydraulic resistances according to equation (8), in addition to the measured slope and flow velocity, piezometric and inertial slopes play a well-known role.

As a result of analyzing the data of hydrological yearbooks in various hydraulic solutions, V.V. Kovalenko [9] obtained a quantitative criterion that allows assigning the location of the hydraulic solution and the length of the base for observations of the slope of the water surface.

It should be noted that the movement of water in rivers is uneven and unsteady, so its differential equation of motion in hydraulics will be written as:

$$-dz = d\left( \frac{\alpha Q^2}{2g \omega^2} \right) + \frac{Q^2}{K^2} dl,$$ (10)

After dividing by $dl$ we get:

$$I = \frac{dz}{dl} = d\frac{\alpha Q^2}{2g \omega^2} + \frac{Q^2}{K^2}$$ (11)

Where α-is the Coriolis coefficient; K-is the flow characteristic in this section; I-is the slope of the water surface; z-is the mark of the water surface.
When solving equation (9) for the case of water movement in rivers, we write it in finite differences (figure 2):

\[-(z_u - z_v) = \frac{\alpha v_H^2}{2g} - \frac{\alpha v_B^2}{2g} + \int_0^l Q^2 dl \]

(12)

Where \( z_u, z_v \) - are the marks of the water surface in the lower and upper sections; \( v_H = Q/\omega_H, v_B = Q/\omega_B \) - average flow rates, respectively, in sections \( \eta-H \) and \( \epsilon-\delta \); \( \omega_H, \omega_B \) - are the areas of living sections in the lower and upper sections; \( \alpha, \alpha_e \) - are the Coriolis coefficients in sections \( \eta-H \) and \( \epsilon-\delta \).

The integral on the right side of equation (12) expresses the loss of specific friction energy from the upper to the lower section. Let’s imagine it in the form:

\[ \int_0^l Q^2 dl = \frac{Q^2}{K^2} \int_0^l dl = \frac{Q^2 l}{K^2} = h_l \]

(13)

Equation (12) is rewritten as:

\[ z_v - z_u = \frac{\alpha Q^2}{2g} \left( \frac{1}{\omega_H^2} - \frac{1}{\omega_B^2} \right) + \frac{Q^2 l}{K^2} \]

(14)

Equation (14) can be simplified by dropping the first term on the right side:

\[ z_v - z_u = \frac{Q^2 l}{K^2} \]

(15)

Or

\[ \Delta z = \frac{Q^2 l}{K^2} \]

(16)

This is the basic equation of uneven water movement, which is widely used to design the shape of a free surface in rivers using water flow curves \( Q = Q(z) \).

The ratio of the drop \( \Delta z \) on the length of the river section to the square of the water flow rate \( Q \) is called the modulus of resistance of the riverbed section.

The lengths of the river sections are assigned in such a way that the shapes of the living sections, the roughness of the channel and the slopes of the water surface are approximately uniform on them.

The water in the riverbed, according to V.G. Glushkov[13], moves because every liter of it is able to generate energy under the influence of gravity and slope. This energy of the flow is manifested in...
the fact that forces arise in the flow that set the flow in motion. At the same time, a very important characteristic of the riverbed is the indicator of the stability of the riverbed in the longitudinal direction. This characteristic is the ratio of the slope of the longitudinal steady flow ($i$) to the actual household slope of the flow ($i_0$):

$$\sigma_i = \frac{i_0}{i}$$

(17)

It turns out that this indicator for many sections of rivers is much less than one, which indicates the instability of their channel. Only rivers with extremely stable channels will have $\sigma_i = 1.0$.

A good criterion for the direction of vertical channel deformations on the delta section of the river as a whole is the ratio of the actual ($I$) and "stable" ($I_0$) slopes of the water surface [12;14]. The values of $I_0$ are determined by the channel-forming water flow and the content of sediment in the stream:

$$I_0 = K_i Q_{wp}^{-1/9}$$

(18)

Where $K_i$ is the slope coefficient, determined experimentally, for the Amu Darya river is $K_i=31.7 \cdot 10^5$.

At $I > I_0$, erosion, cutting of the channel and lowering of the longitudinal profile should occur; at $I < I_0$, on the contrary, sediment deposition and an increase in the longitudinal profile should occur; at $I = I_0$, there are no irreversible vertical channel deformations, the position of the longitudinal profile is stable.

Violation or non-fulfillment of the condition $I = I_0$ is possible either due to a change in $I$, or due to a change in $I_0$.

Deviations from the state of stability, as shown by observations [14], cause irreversible channel deformations, that is: siltation in the areas of the lower reaches of the river at $I < I_0$; erosion in these areas at $I > I_0$.

Analyzing all the methods, one can note the peculiar versatility of the proposed hydraulic-morphological method, expressed in ensuring the required accuracy in determining the longitudinal slope, the location and length of the slope, as well as the objectivity of constructing the free surface curve observed in various channel processes. This versatility, in our opinion, gives grounds to recommend it for practical use in determining the slope and direction of deformation of channels along the length of delta sections of lowland rivers.

4. Conclusion

Based on the conducted research and analysis of known methods for determining the slopes of rivers with different longitudinal profiles in shape, the following general conclusions can be drawn:

- In road construction, the alignment of slopes according to the graphoanalytic method of Alekseev is a very successful technique that allows you to visually determine the minimum of earthworks. But, when applied to hydrological calculations, it does not reflect the essence of the influence of slopes on the speed of water movement.
- The above-considered known methods for determining the weighted average longitudinal slope of rivers to one degree or another do not meet modern requirements for the accuracy of determination.
- В данном разделе представлены результаты исследования.
- The considered logarithmic method for determining the weighted average slope of rivers, based on the principle of bringing the longitudinal profile to a "normal" shape, is both the simplest, labor-intensive and the most reasonable method. However, this method is not acceptable for hydro-hydraulic calculations, since it does not reflect the essence of the influence of slopes on channel deformations.
• Officially, regulatory documents on the observation of slopes are of a qualitative nature, which may reduce the representativeness of measurements.
• The Manual [10-11] recommends a formula for assigning the length of the base, based on the accuracy of measuring the water level and leveling posts, that is, determining \( L \) is related only to the capabilities of the technical equipment of the hydrological network. By the instructions, the length of the base is fixed not depending on the filling and roughness of the channel, but only in inverse dependence on the slope, which should lead to a decrease in the correlation of the measured slopes and expenses.
• As a result of the analysis of the quantitative characteristics of the longitudinal slopes of rivers by the equations of uneven movement of hydraulics, a hydraulic-morphological method for determining the level and slope corresponding to the actual natural values and criteria showing the role of slopes in the processes of channel deformation occurring in the lower reaches of lowland rivers are proposed.

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