REVIEW OF HEAVY QUARK PHYSICS – THEORY

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Recent progress in the theory of B-meson decays is reviewed with emphasis on the aspects related to the B-factory data.

1 Introduction

The two B-meson factories operating at the KEK and SLAC $e^+e^-$ storage rings have outperformed their projected luminosities and have produced a wealth of data, in particular on CP asymmetries and rare B-decays. Impressive results were also presented at this conference by the two Fermilab experiments CDF and D0 in B physics and top-quark physics. These and other experiments have provided precision measurements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements establishing the unitarity of this matrix. The KM-mechanism of CP violation is now being tested with ever increasing precision in a large number of decay modes.

In my talk I will concentrate on the following three topics:

• Current determination of $|V_{cb}|$ and $|V_{ub}|$.

• Progress in flavor-changing-neutral current (FCNC) induced rare B-decays.

• Comparison of various theoretical approaches to non-leptonic B-decays with current data in selected two-body B-decays.

Aspects of charm physics are discussed in the plenary talks by Ian Shipsey, and the Lattice-QCD results related to flavor physics are taken up by Shoji Hashimoto.

2 Current determinations of $|V_{cb}|$ and $|V_{ub}|$

The CKM matrix is written below in the Wolfenstein parameterization in terms of the four parameters: $A$, $\lambda$, $\rho$, $\eta$:

$$V_{\text{CKM}} = \left( \begin{array}{ccc} 1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\ -\lambda (1 + i A^2 \lambda^4 \eta) & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 (1 + i \lambda^2 \eta) & 1 \end{array} \right).$$

Unitarity of the CKM matrix implies six relations, of which the one resulting from the equation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ is the principal focus of the current experiments in B-decays. This is a triangle relation in the complex plane, and the three angles of this triangle are called $\alpha$, $\beta$, and $\gamma$, with the BELLE convention being $\phi_2 = \alpha$, $\phi_1 = \beta$ and $\phi_3 = \gamma$. The unitarity relation in discussion can also be written as

$$R_b e^{i \gamma} + R_t e^{-i \beta} = 1,$$

where $R_b = \left( 1 - \frac{\lambda^2}{2} \right) \frac{1}{\lambda} \left| V_{cb} \right|$ and $R_t = \frac{1}{\lambda} \left| V_{td} \right| / \left| V_{cb} \right|$. Thus, precise determination of $|V_{cb}|$, $|V_{ub}|$ and $|V_{td}|$ and the three CP-violating phases $\alpha$, $\beta$, $\gamma$ is crucial in testing the CKM paradigm.

2.1 $|V_{cb}|$ from the decays $B \to X_c \ell \nu_\ell$

Determinations of $|V_{cb}|$ are based on the semi-leptonic decay $b \to c\ell\nu_\ell$. This transition can
be measured either inclusively through the process $B \rightarrow X_c \ell \nu$, where $X_c$ is a hadronic state with a net $c$-quantum number, or exclusively, such as the decays $B \rightarrow (D, D^*) \ell \nu$. In either case, intimate knowledge of QCD is required to go from the partonic process to the hadronic states. The fact that $m_b \gg \Lambda_{QCD}$ has led to novel applications of QCD in which heavy quark expansion (HQE) plays a central role and the underlying theory is termed as HQET. Concentrating on the inclusive decays, the semi-leptonic decay rate can be calculated as a power series

$$\Gamma = \Gamma_0 + \frac{1}{m_b} \Gamma_1 + \frac{1}{m_b^2} \Gamma_2 + \frac{1}{m_b^3} \Gamma_3 + \ldots,$$

(2)

where each $\Gamma_i$ is a perturbation series in $\alpha_s(m_b)$, the QCD coupling constant at the scale $m_b$. Here $\Gamma_0$ is the decay width of a free $b$-quark, which gives the parton-model result. The coefficient of the leading power correction $\Gamma_1$ is absent, and the effect of the $1/m_b$ correction is collected in $\Gamma_2$, which can be expressed in terms of two non-perturbative parameters called $\lambda_1$ - kinetic energy of the $b$-quark and $\lambda_2$ - its chromomagnetic moment. These quantities, also called $\mu_\pi^2$ and $\mu_2^2$, respectively, in the literature, are defined in terms of the following matrix elements $^{17,18,19,20}$:

$$2M_B \lambda_1 \equiv \langle B(v) | \bar{Q} v (iD)^2 Q | B(v) \rangle,$$

$$6M_B \lambda_2 \equiv \langle B(v) | \bar{Q} v \sigma_{\mu\nu} [iD^\mu, iD^\nu] Q | B(v) \rangle,$$

where $D_{\mu}$ is the covariant derivative and heavy quark fields are characterized by the 4-velocity, $v$. At $O(\Lambda_{QCD}^3/m_b^3)$, six new matrix elements enter in $\Gamma_3$, usually denoted by $\rho_{1,2}$ and $T_{1,2,3,4}$, discussed below.

Data have been analyzed in the theoretical accuracy in which corrections up to $O(\alpha_s^2 \beta_0)$, $O(\alpha_s \Lambda_{QCD}/m_b)$ and $O(\Lambda_{QCD}^2/m_b^2)$ are taken into account $^{21,22}$, with $\beta_0$ being the lowest order $\beta$-function in QCD. The choice of the parameters entering the fit depends on whether or not an expansion in $1/m_c$ is performed. In addition to this choice, a quark mass scheme has to be specified. Bauer et al. $^{21}$ have carried out a comprehensive study of both of these issues using five quark mass schemes: $1S$, $PS$, $MS$, kinematic, and the pole mass.

To extract the value of $|V_{cb}|$ and other fit parameters, three different distributions, namely the charged lepton energy spectrum and the hadronic invariant mass spectrum in $B \rightarrow X_c \ell \nu$, and the photon energy spectrum in $B \rightarrow X_c \gamma$ have been studied. Theoretical analyses are carried out in terms of the moments and not the distributions themselves. Defining the integral $R_n (E_{cut}, \mu) \equiv \int_{E_{cut}}^{\infty} dE_{\ell} |\Gamma (E_{\ell} - \mu)^n d\Gamma /dE_{\ell}|$, where $E_{cut}$ is a lower cut on the charged lepton energy, moments of the lepton energy spectrum are given by $\langle E_{\ell}^n \rangle = R_n (E_{cut}, 0)/R_0 (E_{cut}, 0)$. For the $B \rightarrow X_c \ell \nu$ hadronic invariant mass spectrum, the moments are defined likewise with the cutoff $E_{cut}$. Analyses of the $B$-factory data have been presented at this conference by the BABAR $^{23,24,25}$ and BELLE $^{26,27}$ collaborations. Studies along these lines of some of the moments were also undertaken by the CDF $^{28}$, CLEO $^{29,30}$ and DELPHI $^{31}$ collaborations.

The BABAR collaboration have studied the dependence of the lepton and hadron moments on the cutoff $E_{cut}$ and compared their measurements with the theoretical calculation by Gambino and Uraltsev $^{22}$ using the so-called kinematic scheme for the $b$-quark mass $m_b^{kin}(\mu)$, renormalized at the scale $\mu = 1$ GeV. Excellent agreement between experiment and theory is observed, allowing to determine the fit parameters in this scheme with the results $^{23,25}$:

$$|V_{cb}| = \{(4.14 \pm 0.4_{exp} \pm 0.4_{HQE} \pm 0.6_{th}) \times 10^{-3},$$

$$m_b (1 \text{GeV}) = (4.61 \pm 0.05_{exp} \pm 0.04_{HQE} \pm 0.02_{th}) \text{ GeV},$$

$$m_c (1 \text{GeV}) = (1.18 \pm 0.07_{exp} \pm 0.06_{HQE} \pm 0.02_{th}) \text{ GeV}. \quad (4)$$
The global fit of the data from the BABAR, BELLE, CDF, CLEO and DELPHI collaborations in the so-called 1S-scheme for the $b$-quark mass undertaken by Bauer et al. \cite{Bauer:2002un} leads to the following fit values for $|V_{cb}|$ and $m_b$:

$$|V_{cb}| = (41.4 \pm 0.6 \pm 0.1_{\tau_B}) \times 10^{-3},$$

$$m_b^{1S} = (4.68 \pm 0.03) \text{ GeV}. \quad (5)$$

The two analyses yielding (4) and (5) are in excellent agreement with each other. The achieved accuracy $\delta|V_{cb}|/|V_{cb}| \simeq 2\%$ is impressive, and the precision on $m_b$ is also remarkable, $\delta m_b/m_b = O(10^{-3})$, with a similar precision obtained on the mass difference $m_b - m_c$.

2.2 $|V_{cb}|$ from $B \to (D, D^*)\ell\nu_\ell$ decays

The classic application of HQET in heavy $\to$ heavy decays is $B \to D^*\ell\nu_\ell$. The differential distribution in the variable $\omega = v_B.v_{D^*}$, where $v_B(v_{D^*})$ is the four-velocity of the $B(D^*)$-meson, is given by

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{4\pi^3} |V_{cb}|^2 m_B^2 (m_B - m_{D^*})^2$$

$$\times (\omega^2 - 1)^{1/2} \mathcal{G}(\omega) |\mathcal{F}(\omega)|^2,$$

where $\mathcal{G}(\omega)$ is a phase space factor with $\mathcal{G}(1) = 1$, and $\mathcal{F}(\omega)$ is the Isgur–Wise (IW) function \cite{Isgur:1989vq} with the normalization at the symmetry point $\mathcal{F}(1) = 1$. Leading $\Lambda_{QCD}/m_b$ corrections in $\mathcal{F}(1)$ are absent due to Luke’s theorem \cite{Luke:1994je}. Theoretical issues are the precise determination of the second order power correction to $\mathcal{F}(\omega = 1)$, the slope $\rho^2$ and the curvature $c$ of the IW-function:

$$\mathcal{F}(\omega) = \mathcal{F}(1) \left[ 1 + \rho^2 (\omega - 1) + c (\omega - 1)^2 + \ldots \right]. \quad (6)$$

Bounds on $\rho^2$ have been obtained by Bjorken \cite{Bjorken:1988as} and Uraltsev \cite{Uraltsev:1990ad}, which can be combined to yield $\rho^2 > 3/4$. Likewise, bounds on the second (and higher) derivatives of the IW-function have been worked out by the Orsay group \cite{Gubernari:1994vs}, yielding $c > 15/32$ \cite{Aubert:2002aj}. These bounds have not been used (at least not uniformly) in the current analyses of the $B \to D^*\ell\nu_\ell$ data by the experimental groups. This, combined with the possibility that the data sets may also differ significantly from experiment to experiment, results in considerable dispersion in the values of $\mathcal{F}(1)|V_{cb}|$ and $\rho^2$ and hence in a large $\chi^2$ of the combined fit, summarized by the HFAG averages \cite{HFAG:2008ab}:

$$\mathcal{F}(1)|V_{cb}| = (37.7 \pm 0.9) \times 10^{-3}, \quad (7)$$

$$\rho^2 = 1.56 \pm 0.14 \quad (\chi^2 = 26.9/14).$$

To convert this into a value of $|V_{cb}|$, we need to know $\mathcal{F}(1)$. In terms of the perturbative (QED and QCD) and non-perturbative (leading $\delta_{1/m^2}$ and sub-leading $\delta_{1/m^3}$) corrections, $\mathcal{F}(1)$ can be expressed as follows:

$$\mathcal{F}(1) = \eta_A [1 + \delta_{1/m^2} + \delta_{1/m^3}], \quad (8)$$

where $\eta_A$ is the perturbative renormalization of the IW-function, known in the meanwhile to three loops \cite{Braaten:1990pp}. One- and two-loop corrections yield $\eta_A \simeq 0.933$ and the $O(\alpha_s^2)$ contribution amounts to $\eta_A^{(3)} = -0.005$. Default value of $\mathcal{F}(1)$ used by HFAG is based on the BABAR book \cite{Aubert:2006sj} $\mathcal{F}(1) = 0.91 \pm 0.04$. A recent Lattice-QCD calculation in the quenched approximation yields \cite{Becirevic:2008ri} $\mathcal{F}(1) = 0.913^{+0.0238}_{-0.0173}^{-0.0302}$, which is now being reevaluated with dynamical quarks \cite{Becirevic:2008ri}.

With $\mathcal{F}(1) = 0.91 \pm 0.04$, HFAG quotes the following average \cite{HFAG:2008ab}

$$|V_{cb}|_{B \to D^*\ell\nu_\ell} = (41.4 \pm 1.0_{\exp} \pm 1.8_{\text{theo}}) \times 10^{-3}. \quad (9)$$

The resulting value of $|V_{cb}|$ is in excellent agreement with the ones given in (4) and (5) obtained from the inclusive decays. However, in view of the rather large $\chi^2$ of the fit and the remark on the slope of the IW-function made earlier, there is room for systematic improvements in the determination of $|V_{cb}|$ from the exclusive analysis.

The decay $B \to D^*\ell\nu_\ell$ still suffers from paucity of data. An analysis of the current...
data in this decay mode using the HQET formalism, which admits leading $1/m_b$ corrections, is $|V_{cb}| = (40.4 \pm 3.6_{\text{exp}} \pm 2.3_{\text{th}}) \times 10^{-3}$. The determination of $|V_{cb}|$ from $B \to D \ell \nu$ can be significantly improved at the B-meson factories.

2.3 $|V_{ub}|$ from the decays $B \to X_u \ell \nu$

HQET techniques allow to calculate the inclusive decay rate $B \to X_u \ell \nu$ rather accurately. However, the experimental problem in measuring this transition lies in the huge background from the dominant decays $B \to X_c \ell \nu$, which can be brought under control only through severe cuts on the kinematics. For example, these cuts are imposed on the lepton energy, demanding $E_\ell > (m_D^2 - m_{Xu}^2)/2m_B$, and/or the momentum transfer to the lepton pair $q^2$ restricting it below a threshold value $q^2 < q_{\text{max}}^2$ and/or the hadron mass recoiling against the leptons, which is required to satisfy $m_X < m_D$. With these cuts, the phase space of the decay $B \to X_u \ell \nu$ is greatly reduced. A bigger problem is encountered in the end-point region (also called the shape function region), where the leading power correction is no longer $1/m_b^2$ but rather $1/m_b \Lambda_{\text{QCD}}$, slowing the convergence of the expansion. Moreover, in the region of energetic leptons with low invariant mass hadronic states, $E_X \sim m_b$, $m_{Xu}^2 \sim m_D^2 \sim \Lambda_{\text{QCD}} m_b \ll m_b^2$, the differential rate is sensitive to the details of the shape function $f(k_+)$, where $k_+ = k_0 + k_3$ with $k_3 \sim O(\Lambda_{\text{QCD}})$.

The need to know $f(k_+)$ can be circumvented by doing a combined analysis of the data on $B \to X_u \ell \nu$ and $B \to X_s \gamma$. Using the operator product expansion (OPE) to calculate the photon energy spectrum in the inclusive decay $B \to X_s \gamma$, the leading terms in the spectrum (neglecting the bremsstrahlung corrections) can be re-summed into a shape function:

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts}V_{tb}|^2 |C_7^{\text{eff}}|^2 f(1-x),$$

where $x = \frac{2E_\ell}{m_b}$. In the leading order, $E_\ell$ and $M_{Xu}$ spectra in $B \to X_u \ell \nu$ are also governed by $f(x)$. Thus, $f(x)$ can be measured in $B \to X_s \gamma$ and used in the analysis of data in $B \to X_u \ell \nu$.

Following this argument, a useful relation emerges:

$$\frac{|V_{ub}|}{|V_{tb}|V_{ts}} = \left( \frac{3\alpha}{\pi} |C_7^{\text{eff}}|^2 \frac{\Gamma_u(E_c)}{\Gamma_s(E_c)} \right)^{1/2} (1 + \delta(E_c)),$$

where

$$\Gamma_u(E_c) = \int_{E_c}^{m_B/2} dE_\ell \frac{d\Gamma_u}{dE_\ell},$$

$$\Gamma_s(E_c) = \frac{2}{m_b} \int_{E_c}^{m_B/2} dE_\ell \frac{d\Gamma_s}{dE_\ell},$$

and $\delta(E_c)$ incorporates the sub-leading terms in $O(\Lambda_{\text{QCD}}/m_b)$, which can only be modeled at present. In addition, there are perturbative corrections to the spectra and in the relation (11) also.

Theoretical uncertainties in the extraction of $|V_{ub}|$ arise from the weak annihilation (WA) contributions which depend on the size of factorization violation. Also, the $O(\Lambda_{\text{QCD}}^2/m_b^2)$ contributions, which have been studied using a model for $f(x)$, are found to grow as $q^2$ and $m_{\text{cat}}$ are increased.

At this conference, the BELLE collaboration have presented impressive new analyses for the inclusive $B \to X_u \ell \nu$ decays with fully reconstructed tags. This has allowed them to measure the partial branching ratio (with $m_X < 1.7$ GeV, $q^2 > 8$ GeV$^2$)

$$\Delta B(B \to X_u \ell \nu) = (0.99 \pm 0.15_{\text{stat}} \pm 0.18_{\text{syst}}) \times (0.04(b \to u) \pm 0.07(b \to c)) \times 10^{-3}.$$

To get the full branching ratio, a knowledge of the shape function is needed which is obtained in a model-dependent analysis from the measured $B \to X_s \gamma$ spectrum. Using the expression $B(B \to X_u \ell \nu) = \Delta B(B \to X_u \ell \nu)/f_u$, the BELLE analysis estimates...
\( f_u = 0.294 \pm 0.044 \) \(^{51}\). Combined with the PDG prescription \(^9\)

\[
|V_{ub}| = 0.00424 \left[ \frac{B(B \to X_u \ell \nu)}{0.002 \tau_B} \right],
\]

yields \(^{51}\)

\[
|V_{ub}| = (5.54 \pm 0.42(\text{stat}) \pm 0.50(\text{syst.}) \pm 0.12(b \to u) \pm 0.19(b \to c) \pm 0.42(f_u) \pm 0.27(B \to |V_{ub}|)) \times 10^{-3}. \tag{15}
\]

The corresponding determination of \(|V_{ub}|\) by the BABAR collaboration using this method gives \(^{53}\)

\[
|V_{ub}| = (5.18 \pm 0.52 \pm 0.42) \times 10^{-3}, \tag{16}
\]

where the errors are statistical and systematic, respectively. The current determination of \(|V_{ub}|\) from inclusive measurements including the above BABAR and BELLE measurements is summarized by HFAG (Summer 2004 update) \(^{38}\), with the average

\[
|V_{ub}| = (4.70 \pm 0.44) \times 10^{-3}, \tag{17}
\]

having a \(\chi^2/\text{p.d.f} = 6.7/7\). This amounts to about 10\% precision on \(|V_{ub}|\). Recently, a new method to determine \(|V_{ub}|\) from the inclusive decays \(B \to X_u \ell \nu\) has been proposed \(^{54}\) which uses a cut on the hadronic light-cone variable \(P_+ = E_X - |P_X|\). The efficiency and sensitivity to non-perturbative effects in the \(P_+\)-cut method is argued to be similar to the one on the hadron mass cut, and the \(P_+\)-spectrum can be calculated in a controlled theoretical framework.

### 2.4 \(|V_{ub}|\) from exclusive decays

\(|V_{ub}|\) has also been determined from the exclusive decays \(B \to (\pi, \rho) \ell \nu\). Theoretical accuracy is limited by the imprecise knowledge of the form factors. A number of theoretical techniques has been used to determine them. These include, among others, Light-cone QCD sum rules \(^{55}\), Quenched- and Unquenched-Lattice QCD simulations \(^{12}\), and Lattice-QCD based phenomenological studies \(^{56}\). New measurements and analysis of the decay \(B \to \pi \ell \nu\) have been presented at this conference by the BELLE collaboration and compared with a number of Lattice-QCD calculations, and the extracted values of \(|V_{ub}|\) (in units of \(10^{-3}\)) are as follows \(^{57}\):

\[
|V_{ub}|_{\text{quenched}} = (3.90 \pm 0.71 \pm 0.23^{+0.62}_{-0.48});
\]

\[
|V_{ub}|_{\text{FNAL}^{04}} = (3.87 \pm 0.70 \pm 0.23^{+0.51}_{-0.51});
\]

\[
|V_{ub}|_{\text{HQQCD}} = (4.73 \pm 0.85 \pm 0.27^{+0.74}_{-0.50}).
\]

Hence, current Lattice-QCD results show considerable dispersion (about 20\%) in the extraction of \(|V_{ub}|\) from data.

To reduce the form-factor related uncertainties in extracting \(|V_{ub}|\) from exclusive decays \(B \to (\pi, \rho) \ell \nu\), input from the rare \(B\)-decays \(B \to (K, K^*) \ell^+ \ell^-\) and HQET may be helpful. A proposal along these lines is the so-called Grinstein’s double ratio which would determine \(|V_{ub}|/|V_{ub}V_{cs}^*|\) from the end-point region of exclusive rare \(B\)-meson decays \(^{58,59}\), To carry out this program one has to measure four distributions: \(B \to \rho \ell \nu, B \to K^* \ell^+ \ell^-\), and \(D \to (\rho, K^*) \ell \nu\). With the help of this data and HQET, the ratio of the CKM factors \(|V_{ub}|/|V_{ub}V_{cs}^*|\) can be determined through the double ratio

\[
\frac{\Gamma(B \to \rho \ell \nu)}{\Gamma(B \to K^* \ell^+ \ell^-)} \cdot \frac{\Gamma(D \to K^* \ell \nu)}{\Gamma(D \to \rho \ell \nu)}. \tag{18}
\]

At the \(B\) factories, one expects enough data on these decays to allow a 10\% determination of \(|V_{ub}|\) from exclusive decays.

### 3 Radiative, semileptonic and leptonic rare \(B\) decays

Two inclusive rare \(B\)-decays of current experimental interest are \(B \to X_s \gamma\) and \(B \to X_d \ell^+ \ell^-\), where \(X_s\) is any hadronic state with \(s = 1\), containing no charmed particles. They probe the SM in the electroweak \(b \to s\) penguin sector. The CKM-suppressed decays \(B \to X_d \gamma\) and \(B \to X_d \ell^+ \ell^-\) are difficult to measure due to low rates and formidable backgrounds. Instead, the search for \(b \to d\) radiative transitions has been carried out in the modes \(B \to (\rho, \omega) \gamma\) providing interesting constraints on the CKM parameters. New
and improved upper limits have been presented at this conference on the branching ratio for $B^0_s \to \mu^+\mu^-$, testing supersymmetry in the large-$\tan\beta$ domain. We take up these decays below in turn.

3.1 $B \to X_s\gamma$: SM vs. Experiments

The effective Lagrangian for the decays $B \to X_s(\gamma, \ell^+\ell^-)$ in the SM reads as follows:

$$\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i . \quad (19)$$

The operators and their Wilson coefficients evaluated at the scale $\mu = m_b$ can be seen elsewhere. QCD-improved calculations in the effective theory require three steps:

(i) Matching $C_i(\mu_0) (\mu_0 \sim M_W, m_t)$: They have been calculated up to three loops. The three-loop matching is found to have less than 2% effect on $B(B \to X_s\gamma)$.

(ii) Operator mixing: This involves calculation of the anomalous dimension matrix, which is expanded in $\alpha_s(\mu)$. The anomalous dimensions up to $\alpha_s^2(\mu)$ are known and the $\alpha_s^3(\mu)$ calculations are in progress.

(iii) Matrix elements $\langle O_i \rangle (\mu_b) (\mu_b \sim m_b)$: The first two terms in the expansion in $\alpha_s(\mu_b)$ are known since (6). The $\mathcal{O}(\alpha_s^2 n_f)$ part of the 3-loop calculations has recently been done by Bieri, Greub and Steinhauser. The complete three-loop calculation of $\langle O_i \rangle$, which is not yet in hand, will reduce the quark mass scheme-dependence of the branching ratio $B(B \to X_s\gamma)$, and hence of the NLO decay rate for $B(B \to X_s\gamma)$. Finally, one has to add the Bremsstrahlung contribution $b \to s\gamma g$ to get the complete decay rate.

In the $\overline{\text{MS}}$ scheme, the NLO branching ratio is calculated as $B(B \to X_s\gamma)_{\text{SM}} = (3.70 \pm 0.30) \times 10^{-4} . \quad (20)$

Including the uncertainty due to scheme-dependence, this amounts to a theoretical precision of about 10%, comparable to the current experimental precision

$$B(B \to X_s\gamma)_{\text{Expt.}} = (3.52^{+0.30}_{-0.28}) \times 10^{-4} . \quad (21)$$

Within stated errors, SM and data are in agreement. In deriving (20), unitarity of the CKM matrix yielding $\lambda_t = -\lambda_c = -A\lambda^2 + \ldots = -(41.0 \pm 2.1) \times 10^{-3}$ has been used, where $\lambda_i = V_{tb} V_{t\ast i}$. The measurement (21) can also be used to determine $\lambda_t$. Current data and the NLO calculations in the SM imply

$$|1.69\lambda_u + 1.60\lambda_c + 0.60\lambda_t| = (0.94 \pm 0.07)|V_{cb}| , \quad (22)$$

leading to $\lambda_t = V_{tb} V_{t\ast b} = -(47.0 \pm 8.0) \times 10^{-3}$. As $V_{tb}$ is 1 to a very high accuracy, $B(B \to X_s\gamma)$ determines $V_{ts}$, both in sign and magnitude.

The current (NLO) theoretical precision on $B(B \to X_s\gamma)$ given in (21) has recently been questioned, using a multi-scale OPE involving three low energy scales: $m_b, \sqrt{m_b\Delta}$ and $\Delta = m_b - 2E_0$, where $E_0$ is the lower cut on the photon energy. With $E_0$ taken as 1.9 GeV and $\Delta = 1.1$ GeV, one has considerable uncertainty in the decay rate due to the dependence on $\Delta$.

3.2 $B \to X_s\ell^+\ell^-$: SM vs. Experiments

The NNLO calculation of the decay $B \to X_s\ell^+\ell^-$ corresponds to the NLO calculation of $B \to X_s\gamma$, as far as the number of loops in the diagrams is concerned. Including the leading power corrections in $1/m_b$ and $1/m_c$ and taking into account various parametric uncertainties, the branching ratios for the decays $B \to X_s\ell^+\ell^-$ in NNLO are

$$B(B \to X_se^+e^-)_{\text{SM}} \simeq B(B \to X_s\mu^+\mu^-)_{\text{SM}} = (4.2 \pm 0.7) \times 10^{-6} , \quad (23)$$

where a dilepton invariant mass cut, $m_{\ell\ell} > 0.2$ GeV has been assumed for comparison with data given below. These estimates make use of the NNLO calculation by Asatryan et al., restricted to $s \equiv q^2/m_b^2 < 0.25$. 

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The spectrum for \( \hat{s} > 0.25 \) has been obtained from the NLO calculations using the scale \( \mu_R \simeq m_b/2 \), as this choice of scale reduces the NNLO contributions. Subsequent NNLO calculations cover the entire dilepton mass spectrum and are numerically in agreement with this procedure, yielding \( B(B \to X_s \mu^+\mu^-)_{\text{SM}} = (4.6 \pm 0.8) \times 10^{-6} \). The difference in the central values in these results and (23) is of parametric origin.

The BABAR and BELLE collaborations have measured the invariant dilepton and hadron mass spectra in \( B \to X_s \ell^+\ell^- \). Using the SM-based calculations to extrapolate through the cut-regions, the current averages of the branching ratios are \(^{38}\):

\[
B(B \to X_s e^+e^-) = (4.70^{+1.24}_{-1.20}) \times 10^{-6}, \\
B(B \to X_s \mu^+\mu^-) = (4.26^{+1.18}_{-1.16}) \times 10^{-6}, \\
B(B \to X_s \ell^+\ell^-) = (4.46^{+0.98}_{-0.96}) \times 10^{-6}.
\]

Thus, within the current experimental accuracy, which is typically 25\%, data and the SM agree with each other in the \( b \to s \) electroweak penguins. The measurements (21) and (24) provide valuable constraints on beyond-the-SM physics scenarios. Following the earlier analysis to determine the Wilson coefficients in \( b \to s \) transitions \(^{74,70,75}\), it has been recently argued \(^{76}\) that data now disfavor solutions in which the coefficient \( C_7^{\text{eff}} \) is similar in magnitude but opposite in sign to the SM coefficient. For example, this constraint disfavors SUSY models with large \( \tan \beta \) which admit such solutions.

Exclusive decays \( B \to (K, K^*)\ell^+\ell^- \) \((\ell^\pm = e^\pm, \mu^\pm)\) have also been measured by the BABAR and BELLE collaborations, and the current world averages of the branching ratios are \(^{38}\):

\[
B(B \to K\ell^+\ell^-) = (5.74^{+0.71}_{-0.66}) \times 10^{-7}, \\
B(B \to K^*\ell^+\ell^-) = (14.4^{+3.5}_{-3.4}) \times 10^{-7}, \\
B(B \to K^*\mu^+\mu^-) = (17.3^{+3.9}_{-2.7}) \times 10^{-7}.
\]

They are also in agreement with the SM-based estimates of the same, posted as \(^{70}\)

\[
B(B \to K\ell^+\ell^-) = (3.5 \pm 1.2) \times 10^{-7}, \quad B(B \to K^*e^+e^-) = (15.8 \pm 4.9) \times 10^{-7}, \quad \text{and} \quad B(B \to K^*\mu^+\mu^-) = (11.9 \pm 3.9) \times 10^{-7}
\]

with the error dominated by uncertainties on the form factors \(^{77}\).

The Forward-backward (FB) asymmetry in the decay \( B \to X_s \ell^+\ell^- \) \(^{78}\), defined as

\[
\hat{A}_{FB}(q^2) = \frac{1}{dB(B \to X_s \ell^+\ell^-)/dq^2} \int_{-1}^{1} d\cos \theta \frac{d^2B(B \to X_s \ell^+\ell^-)}{dq^2} \text{sgn}(\cos \theta),
\]

as well as the location of the zero-point of this asymmetry (called below \( q_0^2 \)) are precision tests of the SM. In NNLO, one has the following predictions: \( q_0^2 = (3.90 \pm 0.25) \text{ GeV}^2 \) \([ (3.76 \pm 0.22_{\text{theory}} \pm 0.24_{m_b}) \text{ GeV}^2 ]\), obtained by Gherghetta et al. \(^{79}\) [Asatryan et al. \(^{80}\)]. In the SM (and its extensions in which the operator basis remains unchanged), the FB-asymmetry in \( B \to K\ell^+\ell^- \) is zero and in \( B \to K^*\ell^+\ell^- \) it depends on the decay form factors. Model-dependent studies yield small form factor-related uncertainties in the zero-point of the asymmetry \( \hat{s}_0 = q_0^2/m_b^2 \) \(^{81}\).

HQET provides a symmetry argument why the uncertainty in \( \hat{s}_0 \) can be expected to be small which is determined by \(^{77}\)

\[
C_9^{\ell f\ell f} \hat{\sigma}_9^{\text{eff}} = \frac{2m_B m_{\hat{\sigma}}^2}{M_B s_0} C_7^{\ell f\ell f}.
\]

However, \( O(\alpha_s) \) corrections to the HQET-symmetry relations lead to substantial change in the profile of the FB-asymmetry function as well as a significant shift in \( \hat{s}_0 \) \(^{82,83}\). The zero of the FB-asymmetry is not very precisely localized due to hadronic uncertainties, exemplified by the estimate \(^{82}\) \( q_0^2 = (4.2 \pm 0.6) \text{ GeV}^2 \). One also expects that the intermediate scale \( \Delta \)-related uncertainties, worked out in the context of \( B \to X_s \gamma \) and \( B \to X_s \ell \nu \ell \) in SCET \(^{69,84}\), will also renormalize the dilepton spectra and the FB-asymmetries in \( B \to (X_s, K^*, ...)\ell^+\ell^- \).

At this conference, BELLE have presented the first measurement of the FB-asymmetry in the decays \( B \to (K, K^*)\ell^+\ell^- \). Data is compared with the SM predictions and with a beyond-the-SM scenario in which
the sign of $C_7^{\alpha ij}$ is flipped, with no firm conclusions. However, the beyond-the-SM scenario is disfavored on the grounds that it predicts too high a branching ratio for $B \to X_s \ell^+ \ell^-$ as well as for $B \to K^* \ell^+ \ell^-$.  

3.3 $B \to V\gamma$: SM vs. Experiments

The decays $B \to V\gamma$ ($V = K^*, \rho, \omega$) have been calculated in the NLO approximation using the effective Lagrangian given in (19) and its analogue for $b \to d$ transitions. Two dynamical approaches, namely the QCD Factorization 85 and pQCD 86 have been employed to establish factorization of the radiative decay amplitudes. The QCD-F approach leads to the following factorization Ansatz for the $B \to V\gamma(\gamma')$ amplitude:

$$f_k(q^2) = C_{kl} \xi_\perp(q^2) + C_{kl} \xi_\parallel(q^2) + \Phi_B \otimes T_k \otimes \Phi_V,$$

(27)

where $f_k(q^2)$ is a form factor in the full QCD and the terms on the r.h.s. contain factorizable and non-factorizable corrections. The functions $C_i$ ($i = \perp, \parallel$) admit a perturbative expansion $C_i = C_i^{(\gamma)} + \alpha_s C_i^{(\gamma')}$, with $C_i^{(\gamma)}$ being the Wilson coefficients, and the so-called hard spectator corrections are given in the last term in (27). The symbol $\otimes$ denotes convolution of the perturbative QCD kernels with the wave functions $\Phi_B$ and $\Phi_V$ of the $B$-Meson & $V$-Meson. Concentrating first on the $B \to K^*\gamma$ decays, the branching ratio is enhanced in the NLO by a $K$-factor evaluated as 83,87,88 $1.5 \leq K \leq 1.7$. The relation between $\xi_\perp^{(K^*)}$ and the full QCD form factor $T_1^{K^*}$ has been worked out in $O(\alpha_s)$ by Beneke and Feldmann 82: $T_1^{K^*}(0) = (1 + O(\alpha_s)) \xi_\perp^{(K^*)}(0)$. Using the default values for the $b$-quark mass in the pole mass scheme $m_{b,\text{pole}} = 4.65$ GeV and the soft HQET form factor $\xi_\perp^{K^*} = 0.35$ results in the following branching ratios 89:

$$B_{\text{th}}(B^0 \to K^{*0}\gamma) \approx (6.9 \pm 1.1) \times 10^{-5},$$

$$B_{\text{th}}(B^\pm \to K^{*\pm}\gamma) \approx (7.4 \pm 1.2) \times 10^{-5}.$$  

The above theoretical branching ratios are to be compared with the current experimental measurements 38:

$$B(B^0 \to K^{*0}\gamma) = (4.14 \pm 0.26) \times 10^{-5};$$

$$B(B^\pm \to K^{*\pm}\gamma) = (3.98 \pm 0.35) \times 10^{-5}.$$  

Consistency of the QCD-F approach with data requires $T_1^{K^*}(0) = 0.27 \pm 0.02$. This is about 30% smaller than the typical estimates in QCD sum rules.

In contrast to QCD-F, the pQCD approach is based on the so-called $k_\perp$-formalism, in which the transverse momenta are treated in the Sudakov formalism. Also, as opposed to the QCD-F approach, in which the form factors are external input, pQCD calculates these form factors yielding the following branching ratios 90:

$$B(B^0 \to K^{*0}\gamma) = (3.5^{+1.1}_{-0.8}) \times 10^{-5},$$

$$B(B^\pm \to K^{*\pm}\gamma) = (3.4^{+1.2}_{-0.9}) \times 10^{-5}.$$  

The resulting form factor $T_1^{K^*}(0) = 0.25 \pm 0.04$ is in agreement with its estimate based on the QCD-F approach and data. The decays $B \to (\rho, \omega)\gamma$ involve in addition to the (short-distance) penguin amplitude also significant long-distance contributions, in particular in the decays $B^\pm \to \rho^\pm\gamma$. In the factorization approximation, typical Annihilation-to-Penguin amplitude ratio is estimated as 91: $\epsilon_A(\rho^\pm\gamma) = 0.30 \pm 0.07$. $O(\alpha_s)$ corrections to the annihilation amplitude in $B^\pm \to \rho^\pm\gamma$ is calculated in the leading-twist approximation vanish in the chiral limit 92. Hence, non-factorizing annihilation contributions are likely small which can be tested experimentally in the decays $B^\pm \to \ell^\pm\nu_\ell\gamma$. The annihilation contribution to the decays $B^0 \to \rho^0\gamma$ and $B^0 \to \omega\gamma$ is expected to be suppressed (relative to the corresponding amplitude in $B^\pm \to \rho^\pm\gamma$) due to the electric charges ($Q_d/Q_u = -1/2$) and the colour factors, and the corresponding $A/P$ ratio for these decays is estimated as $\epsilon_A(\rho^0\gamma) \approx \epsilon_A(\omega\gamma) \approx 0.05$. Theoretical branching ratios for $B \to (\rho, \omega)\gamma$ decays can be more reliably calcu-
lated in terms of the following ratios \(^{87,89}\):
\[
R(\rho(\omega)\gamma) = \frac{\mathcal{B}(B \to \rho(\omega)\gamma)}{\mathcal{B}(B \to K^*\gamma)} .
\] (28)

Including the \(O(\alpha_s)\) and annihilation contributions \(^{89}\) \(R(\rho^\pm/K^\mp) = (3.3 \pm 1.0) \times 10^{-2}\) and \(R(\rho^0/K^*) \approx R(\omega/K^0) = (1.6 \pm 0.5) \times 10^{-2}\). Using the well-measured branching ratios \(\mathcal{B}(B \to K^*\gamma)\), and varying the CKM parameters in the allowed ranges, one gets the following branching ratios \(^{89}\): \(\mathcal{B}(B^\pm \to \rho^\pm\gamma) = (1.35 \pm 0.4) \times 10^{-6}\) and \(\mathcal{B}(B^0 \to \rho^0\gamma) \approx \mathcal{B}(B^0 \to \omega\gamma) = (0.65 \pm 0.2) \times 10^{-6}\). To make comparison of the SM with the current data, the following averaged branching ratio is invoked
\[
\mathcal{B}[B \to (\rho, \omega)\gamma] = \frac{1}{2} \left\{ \mathcal{B}(B^+ \to \rho^+\gamma) + \frac{T_{B^+}}{T_{B^0}} \left[ \mathcal{B}(B^0_d \to \rho^0\gamma) + \mathcal{B}(B^0_s \to \omega\gamma) \right] \right\} .
\]
In terms of this averaged ratio, the current upper limits (at 90% C.L.) are:
\[
\mathcal{B}_{\text{exp}}[B \to (\rho, \omega)\gamma] < 1.4 \times 10^{-6} \quad [\text{BELLE}];
R[(\rho, \omega)/K^+] < 0.035; \quad |V_{td}/V_{ts}| < 0.22 ,
\] (29)
and
\[
\mathcal{B}_{\text{exp}}[B \to (\rho, \omega)\gamma] < 1.2 \times 10^{-6} \quad [\text{BABAR}];
R[(\rho, \omega)/K^+] < 0.029; \quad |V_{td}/V_{ts}| < 0.19 .
\] (30)

Constraints from the more stringent BABAR upper limit \(^{93}\) \(R[(\rho, \omega)/K^+] < 0.029\) on the CKM parameters exclude up to almost 50% of the otherwise allowed parameter space, obtained from the CKMfitter group \(^{94}\).

### 3.4 Current bounds on \(\mathcal{B}(B^0_s \to \mu^+\mu^-)\)

New and improved upper limits have been presented at this conference by CDF and D0 collaborations \(^3\) for the decays \(B^0_s \to \mu^+\mu^-\) and \(B^0_d \to \mu^+\mu^-\):
\[
\mathcal{B}(B^0_s \to \mu^+\mu^-) < 3.8 \ [5.8] \times 10^{-7} \quad \text{CDF} ,
\]
\[
\mathcal{B}(B^0_d \to \mu^+\mu^-) < 1.5 \times 10^{-7} \quad [\text{CDF}] .
\] (31)

The CDF and DO upper limits have been combined to yield \(\mathcal{B}(B^0_s \to \mu^+\mu^-) < 2.7 \times 10^{-7}\), to be compared with the SM predictions \(^{95}\) \(\mathcal{B}(B^0_{s(d)} \to \mu^+\mu^-) = 3.4 \times 10^{-5}(1.0 \times 10^{-10})\) within \(+15\%\) theoretical uncertainty. Hence, currently there is no sensitivity for the SM decay rate. However, as the leptonic branching ratios probe the Higgs sector in beyond-the-SM scenarios, such as supersymmetry, and depend sensitively on \(\tan \beta\), the Tevatron upper limit on \(\mathcal{B}(B^0_{s(d)} \to \mu^+\mu^-)\) probes the large \(\tan \beta\) (say, > 50) parameter space, though the precise constraints are model dependent \(^{96,97}\).

### 4 \(B \to M_1 M_2\) Decays

Exclusive non-leptonic decays are the hardest nuts to crack in the theory of \(B\)-decays! Basically, there are four different theoretical approaches to calculate and/or parameterize the hadronic matrix elements in \(B \to M_1 M_2\) decays:

1. SU(2)/SU(3) symmetries and phenomenological Ansätze \(^{98,99,100,101}\)

2. Dynamical approaches based on perturbative QCD, such as the QCD Factorization \(^{85}\) and the competing pQCD approach \(^{86,102}\). These techniques are very popular and have a large following in China as well \(^{103}\).

3. Charming Penguins \(^{104,105}\) using the renormalization group invariant topological approach of Buras and Silvestrini \(^{106}\).

4. Soft Collinear Effective Theory (SCET), for which several formulations exist. At this conference, SCET and its applications are reviewed by Bauer, Pirjol, and Stewart \(^{107}\) to which we refer for detailed discussions.

These approaches will be discussed on the example of the decays \(B \to \pi\pi\) and \(B \to K\pi\) for which now there exist enough...
data to extract the underlying dynamical parameters.

4.1 $B \to \pi\pi$: SM vs. Experiments

There are three dominant topologies in the decays $B \to \pi\pi$ termed as Tree (T), Penguin (P) and Color-suppressed (C). In addition, there are several other subdominant topologies which will be neglected in the discussion below. Parameterization of the T, P, and C amplitudes is convention-dependent. In the Gronau-Rosner c-convention $T$, these amplitudes can be represented as

$$\sqrt{2} A^{+0} = -|T| e^{i\gamma} e^{i(\Delta + 0)} \left[ 1 + |C/T| e^{i\delta} \right],$$

$$A^{+-} = -|T| e^{i\delta} \left[ e^{i\gamma} + |P/T| e^{i\delta} \right],$$

$$\sqrt{2} A^{00} = -|T| e^{i\delta} \left[ |C/T| e^{i(\Delta - 0)} - |P/T| e^{i\delta} \right].$$

The charged-conjugate amplitudes $A^{ij}$ differ by the replacement $\gamma \to -\gamma$. There are 5 dynamical parameters $|T|$, $\delta$, $|C/T|$, $\Delta$, with $\delta_T = 0$ assumed for the overall phase. Thus, the weak phase $\gamma$ can be extracted together with other quantities if the complete set of experimental data on $B \to \pi\pi$ decays is available, which is not the case at present.

Several isospin bounds have been obtained on the penguin-pollution angle $\theta$ (or $\alpha_{\text{eff}} = \alpha + \theta$) $108,109,110$, with the Gronau-London-Sinha-Sinha bound $110$ being the strongest. These bounds are useful in constraining the parameters of the $B \to \pi\pi$ system and have been used to reduce their allowed ranges. The experimental branching ratios and the CP asymmetries $A_{CP}(\pi^+\pi^0)$, $A_{CP}(\pi^+\pi^-)$ and $A_{CP}(\pi^0\pi^0)$, as well as the value of the coefficient $S_{\pi^+\pi^-}$ in time-dependent CP asymmetry have been fitted to determine the various parameters. An updated analysis by Parkhomchenko $111$ based on the paper $112$ yields the following values:

$$|P/T| = 0.51^{+0.10}_{-0.09}, \quad |C/T| = 1.11^{+0.09}_{-0.10},$$

$$\delta = (-39.4^{+10.3}_{-8.9})^\circ; \quad \Delta = (-55.7^{+13.5}_{-12.3})^\circ;$$

$$\gamma = (65.3^{+4.7}_{-0.2})^\circ.$$

The range of $\gamma$ extracted from this analysis is in good agreement with the indirect estimate of the same from the unitarity triangle. However, the strong phases $\delta$ and $\Delta$ come out large; they are much larger than the predictions of the QCD-F approach $85$ with pQCD $86,102$ in better agreement with data, but neither of these approaches provides a good fit of the entire $B \to \pi\pi$ data. Similar results and conclusions are obtained by Buras et al. $113$ and Pivk $114$.

Data on $B \to \pi\pi$ decays are in agreement with the phenomenological approach of the so-called charming penguins $105$, and with the SCET-based analysis of Bauer et al. $115$ which also attributes a dominant role to the charming penguin amplitude. However, a proof of the factorization of the charming penguin amplitude in the SCET approach remains to be provided. In addition, SCET makes a number of predictions in the $B \to \pi\pi$ sector, such as the branching ratio $B(B^0 \to \pi^0\pi^0)$:

$$\mathcal{B}(B^0 \to \pi^0\pi^0) \bigg|_{\gamma=64^\circ} = (1.3 \pm 0.6) \times 10^{-6},$$

which is in agreement with the current experimental world average $1.2$

$$\mathcal{B}(B^0 \to \pi^0\pi^0) = (1.51 \pm 0.28) \times 10^{-6}. \quad (33)$$

In contrast, predictions of the QCD-F and pQCD approaches are rather similar: $B(B^0 \to \pi^0\pi^0) \sim 0.3 \times 10^{-6}$, in substantial disagreement with the data.

4.2 $B \to K\pi$: SM vs. Experiments

The final topic covered in this talk is the $B \to K\pi$ decays. First, we note that the direct CP-asymmetry in the $B \to K\pi$ decays has now been measured by the BABAR and BELLE collaboration:

$$A_{CP}(\pi^+K^-) = (-10.1 \pm 2.5 \pm 0.5)^\circ \quad \text{[BELLE]},$$

$$(-13.3 \pm 3.0 \pm 0.9)^\circ \quad \text{[BABAR]}, \quad (34)$$
to be compared with the predictions of the two factorization-based approaches: \( A_{\text{CP}}(\pi^+K^-) = (-12.9 \pm 21.9)\% \) [pQCD] \(^{86,102}\) and \( A_{\text{CP}}(\pi^+K^-) = (-5.4 \pm +13.6)\% \) [QCD – F] \(^{85}\), with the latter falling short of a satisfactory description of data.

The charged and neutral \( B \to \pi K \) decays have received a lot of theoretical attention. In particular, many ratios involving these decays have been proposed to test the SM \(^{116,117,118,119}\) and extract useful bounds on the angle \( \gamma \), starting from the Fleischer-Mannel bound \(^{116}\) \( \sin^2 \gamma \leq R \), where the ratio \( R \) is defined as follows:

\[
R \equiv \frac{\tau_{B^+} B(B_d^0 \to \pi^- K^+)}{\tau_{B_d^0} B(B^+ \to \pi^+ K^0) + B(B^- \to \pi^- K^0)}
\]

The current experimental average \( R = 0.820 \pm 0.056 \) allows to put a bound: \( \gamma < 75^\circ \) (at 95\% C.L.). This is in comfortable agreement with the determination of \( \gamma \) from the \( B \to \pi\pi \) decays, given earlier, and the indirect unitarity constraints. Thus, both \( R \) and \( A_{\text{CP}}(\pi^+K^-) \) are in agreement with the SM. The same is the situation with the Lipkin sum rule \(^{118}\):

\[
R_L = 2 \frac{\Gamma(B^+ \to K^+\pi^0) + \Gamma(B^0 \to K^0\pi^0)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^0 \to K^+\pi^-)} = 1 + \mathcal{O}(\frac{P_{\text{EW}} + T_{\text{HF}}}{P}) \tag{36}
\]

implying significant electroweak penguin contribution in case \( R_L \) deviates significantly from 1. With the current experimental average \( R_L = 1.123 \pm 0.070 \), this is obviously not the case. This leaves then the two other ratios \( R_c \) and \( R_n \) involving the \( B \to \pi K \) decays of \( B^\pm \) and \( B^0 \) mesons:

\[
R_c = \frac{2 B(B^\pm \to \pi^0 K^\pm)}{B(B^\pm \to \pi^\mp K^\mp)} \quad \text{and} \quad R_n = \frac{1 B(B_d \to \pi^\mp K^\pm)}{2 B(B_d \to \pi^\pm K^0)} \tag{37}
\]

Their experimental values \( R_c = 1.004 \pm 0.084 \) and \( R_n = 0.789 \pm 0.075 \) are to be compared with the current SM-based estimates \(^{113}\) \( R_c = 1.14 \pm 0.05 \) and \( R_n = 1.11^{+0.04}_{-0.06} \). This implies \( R_c(\text{SM}) - R_c(\text{Exp}) = 0.14 \pm 0.10 \) and \( R_n(\text{SM}) - R_n(\text{Exp}) = -0.32 \pm 0.09 \). We conclude tentatively that SM is in agreement with the measurement of \( R_c \), but falls short of data in \( R_n \) by about 3.5\( \sigma \). Possible deviations from the SM, if confirmed, would imply new physics, advocated in this context, in particular, by Yoshikawa \(^{120}\), Beneke and Neubert \(^{121}\) and Buras et al. \(^{113}\).

Finally, a bound on \( B(B^0 \to K^0\overline{K}^0) \) based on \( SU(3) \) and \( B \to \pi \pi \) data, obtained recently by Fleischer and Recksiegel \(^{122}\), yielding \( B(B^0 \to K^0\overline{K}^0) < 1.5 \times 10^{-6} \) is well satisfied by the current measurements \(^{1,2}\)

\[
B(B^0 \to K^0\overline{K}^0) = (1.19 \pm 0.38 \pm 0.13) \times 10^{-6}.
\]

5 Summary

Dedicated experiments and progress in heavy quark expansion techniques have enabled precise determination of the CKM matrix elements entering in the unitarity triangle (1). In particular, \( |V_{cb}| \) is now determined quite precisely: \( \frac{\delta [V_{cb}]}{|V_{cb}|} \sim 2\% \) comparable to \( \frac{\delta [V_{ud}]}{|V_{ud}|} \). Current precision on \( |V_{ub}| \) from inclusive decays is about 10\% and a factor 2 worse for the exclusive decays. There are several theoretical proposals to improve the knowledge of \( |V_{ub}| \) requiring lot more data from the \( B \) factories which will be available in the near future.

The decay \( B \to X_s \gamma \), which serves as the standard candle in the FCNC \( B \)-decays, is in agreement with the SM with the current precision on the branching fraction at about 10\%. A major theoretical effort is under way to complete the NNLO calculations in \( B \to X_s \gamma \); at the same time digging deeper brings to the fore new hadronic uncertainties which will have to be controlled to reach the goal of 5\% theory precision in \( B(B \to X_s \gamma) \). Improved and new measurements in \( B \to X_s \ell^+\ell^- \) have been reported including a first shot at the forward-backward asymmetry in \( B \to K^\ast\ell^+\ell^- \). Data in the electroweak \( b \to s\ell^+\ell^- \) sector is in agreement
with the SM and this rapport will be tested with greatly improved precision in the future. Current upper limit on $B(B_s \rightarrow \mu^+\mu^-)$ from CDF/D0 probes interesting SUSY parameter space, impacting on the large-tan $\beta$ regime of SUSY.

Concerning non-leptonics, the largest current discrepancy between the data and the SM is in the decays involving QCD penguins. These include CP violation in the $b \rightarrow s \bar{s}s$ penguins, where data show a deviation of about 3 $\sigma$ from the SM $^{1,2}$ Also the ratio $R_n$ in the $B \rightarrow K\pi$ decays is out of line with the SM-estimates by slightly more than 3 $\sigma$. These deviations are not yet significant enough to announce physics beyond the SM; neither can they be wished away. Experimental evidence is now mounting that not only are the weak phases $\alpha$, $\beta$ $\gamma$ large, as anticipated in the SM, but so also are the strong (QCD) phases, unlikely to be generated by perturbative QCD alone. In addition, color-suppressed decays are not parametrically suppressed, as opposed to their estimates in the QCD-F and pQCD approaches. SCET— the emerging QCD technology— holds the promise to provide a better theoretical description of non-leptonics than existing methods. We look forward to theoretical developments as well as to new and exciting data from the ongoing and planned experiments at the $B$ factories and hadron colliders.

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