Orientational Kerr effect in liquid crystal ferroelectrics and modulation of partially polarized light

Alexei D. Kiselev\textsuperscript{1}, Vladimir V. Kesaev\textsuperscript{2} and Evgeny P. Pozhidaev\textsuperscript{2}

\textsuperscript{1} Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics, 49 Kronverksky Prospekt, St. Petersburg 197101, Russia
\textsuperscript{2} Lebedev Physical Institute RAS, 53 Leninskiy Ave., Moscow 119991, Russia

E-mail: alexei.d.kiselev@gmail.com

Abstract. We study modulation of partially polarized light governed by the orientational Kerr effect in subwavelength-pitch deformed-helix ferroelectric liquid crystals. In our experimental setup based on the Mach-Zehnder interferometer, it is found that the electric field induced shift of the interference fringes crucially depends on the degree of polarization of the incident light. We show that the experimental data can be theoretically described in terms of the electrically dependent Pancharatnam relative phase. The electric field dependence of both the Pancharatnam and geometric phases is evaluated by comparing the experimental and theoretical results.

Orientational structures characterizing orientationally ordered anisotropic liquids known as the liquid crystals are extremely sensitive to both external fields and boundary conditions. This sensitivity underlines the modes of operation of liquid crystal (LC) spatial light modulators (SLMs) that have been widely utilized to modulate amplitude, phase, or polarization of light waves in space and time [1]. Ferroelectric liquid crystals (FLCs) are known to be characterized by very fast response time and represent most promising chiral liquid crystal material. Most of the FLC modes, however, are not suitable for phase-only modulation devices because their optical axis sweeps in the plane of the cell substrate producing undesirable changes in the polarization state of the incident light.

In order to get around the optical axis switching problem the system based on the orientational Kerr effect in a vertically aligned deformed helix ferroelectric LC (DHFLC) with subwavelength helix pitch was suggested in [2, 3] (a recent review of the physics of the DHFLC effect can be found in [4]). In the geometry of a uniform lying helix, phase modulation of light in DHFLC cells was also studied using the experimental technique based on a Mach-Zehnder two-arm interferometer [5, 6].

An important point is that all the above mentioned studies deal with the case of linearly polarized illumination where the incident beam is fully polarized. In our recent paper [7], we have found that, for unpolarized incident light, the electro-optic response is insensitive to the effect of electric-field-induced rotation.

In this paper, we present the results of investigation into the effects of the degree of polarization in the electro-optic behavior of the planar aligned DHFLC cells illuminated with partially polarized light and extend our theoretical analysis of the Pancharatnam and geometric
(Pancharatnam-Berry) phases to the general case of non-unitary evolution of mixed polarization states.

We consider a DHFLC layer of thickness $D$ with the $z$ axis normal to the bounding surfaces: $z = 0$ and $z = D$ (see Fig. 1). The geometry of a uniform lying DHFLC helix with subwavelength pitch where the helix (twisting) axis $\hat{h}$ is directed along the $x$ axis will be our primary concern. In this geometry depicted in Fig. 1 the equilibrium orientational structure can be described as a helical twisting pattern where FLC molecules align on average along a local unit director $\mathbf{d} = \cos \theta \hat{h} + \sin \theta \hat{c}$, where $\theta$ is the smectic tilt angle; $\hat{h} = \hat{x}$ is the twisting axis normal to the smectic layers and $\hat{c} \perp \hat{h}$ is the $c$-director. The FLC director lies on the smectic cone depicted in Fig. 1(left) and rotates in a helical fashion about a uniform twisting axis $\hat{h}$ forming the FLC helix. This rotation is described by the azimuthal angle around the cone $\phi$ that specifies orientation of the $c$-director in the plane perpendicular to $\hat{h}$: $\hat{c} = \cos \phi \hat{y} + \sin \phi \hat{z}$.

Figure 1. Geometry of smectic cone (left) and DHFLC cell (right). 1 – cell substrates; 2 – incident light beam; 3 – smectic layers.

According to Refs. [8, 9, 5], optical properties of such cells can be described by the effective dielectric tensor of a homogenized DHFLC helical structure. The zero-field ($E = 0$) dielectric tensor is uniaxially anisotropic with the optical axis directed along the twisting axis $\hat{h} = \hat{x}$. The zero-field effective refractive indices of extraordinary (ordinary) waves, $n_h$($n_p$), generally depend on the smectic tilt angle $\theta$ and the optical (high frequency) dielectric constants characterizing the FLC material (see, e.g., equation (8) in Ref. [5] giving the expressions for $\epsilon_h = n_h^2$ and $\epsilon_p = n_p^2$).

The electric-field-induced anisotropy is generally biaxial so that the dielectric tensor is characterized by the three generally different principal values (eigenvalues): $\epsilon_{\pm} = n_{\pm}^2$ and $\epsilon_z = n_z^2$. The in-plane principal optical axes (eigenvectors)

$$\hat{d}_+ = \cos \psi_d \hat{x} + \sin \psi_d \hat{y}, \quad \hat{d}_- = \hat{z} \times \hat{d}_+$$

Figure 2. Experimental setup based on a Mach-Zehnder two-arm interferometer: BS is the beam splitter; $M_1$ and $M_2$ are the mirrors; PMT is the photomultiplier tube. The insert illustrates the electrically induced shift of the interference fringes.
are rotated about the vector of electric field, $\mathbf{E} \parallel \hat{z}$, by the azimuthal angle $\psi_\parallel$. In the low electric field region, the electric field dependence of the angle $\psi_\parallel$ is approximately linear: $\psi_\parallel \propto E$, whereas the electrically induced part of the principal refractive indices, $n_\pm$ and $n_\parallel$, is typically dominated by the Kerr-like nonlinear terms proportional to $E^2$ (see, e.g., equations (10)–(18) in Ref. [5]). This effect — the so-called orientational Kerr effect — is caused by the electrically induced distortions of the helical structure. It is governed by the effective dielectric tensor of a nanostructured chiral smectic liquid crystal defined through averaging over the FLC orientational structure [2, 3, 8, 9]. In electro-optical studies of DHFLC cells, the orientational Kerr effect typically manifests itself in the electrically dependent transmittance of normally incident light passing through crossed polarizers [2, 3, 8, 9]. This effect was also experimentally studied in [5, 7] using the experimental setup shown in Fig. 2. This setup is based on a Mach-Zehnder two-beam interferometer where the FLC cell is placed in the path of the sample beam. Referring to Fig. 2, a beam splitter (BS) divides a collimated laser light into two beams, the reference and the sample beams, which, after reflection at the mirrors $M_1$ and $M_2$, are recombined at the semireflecting surface of the beam splitter (BS). The interfering beams emerging from the interferometer optionally pass through an output polarizer with the transmission axis azimuth $\psi_p$ and then are projected by the lens onto a screen with a pinhole. Figure 3 shows the experimental results for the electric field dependence of the output light field intensity measured at different values of the degree of polarization in the DHFLC cell of thickness $D \approx 52 \, \mu m$ filled with the FLC mixture FLC-587. Solid lines represent the theoretical curves. The parameters of the mixture are: $n_\perp \approx 1.47$ is the ordinary refractive index, $n_\parallel \approx 1.69$ is the extraordinary refractive index, $\theta \approx 33.5^\circ$ is the smectic tilt angle, and $r_2 \approx 1.05$ is the biaxiality ratio.

Figure 3. Normalized intensity of output wavefield measured as a function of the electric field at different values of the degree of polarization, $P_0$, for the DHFLC cell of thickness $D \approx 52 \, \mu m$ filled with the FLC mixture FLC-587. Solid lines represent the theoretical curves.

In our theoretical analysis, we consider the input lightwave field characterized by the $2 \times 2$ equal-time coherence matrix: $M_0 = \langle \mathbf{E}_0 \otimes \mathbf{E}_0^* \rangle = I_0 \rho_0$, where $I_0 = \text{Tr} M_0$ is the intensity; $\rho_0$ is the polarization density matrix. In circular basis $\hat{e}_\pm = (\hat{x} \pm i \hat{y})/\sqrt{2}$, the density matrix $\rho_0$ is given by

$$2 \rho_0 = \sigma_0 + P_0 (\hat{s}_0 \cdot \sigma), \quad \sigma_0 = \text{diag}(1, 1), \quad \sigma = (\sigma_1, \sigma_2, \sigma_3), \quad \text{(2)}$$

where $\sigma_i$ are the Pauli matrices; $\hat{s}_0 = (\sin(2\theta_0) \cos(2\phi_0), \sin(2\theta_0) \sin(2\phi_0), \cos(2\theta_0))$ is the normalized Stokes vector; $\phi_0$ is the polarization azimuth; $-\pi/4 \leq \pi/4 - \theta_0 \equiv \chi_0 \leq \pi/4$ is the ellipticity angle; and $\epsilon_\text{ell} = \tan \chi_0$ is the ellipticity. The transmission matrix of the DHFLC cell can be written in the similar form as follows

$$2 T_s (h) = (t_+ + t_-) \sigma_0 + (t_- - t_+) (\hat{s}_d \cdot \sigma), \quad t_\pm = |t_\pm| e^{i \Phi_\pm} = 1 - \rho_\pm^2 \frac{1}{1 - \rho_\pm^2} \exp(2im_\pm h) \exp(in_\pm h), \quad \text{(3)}$$
where \( \rho_\pm = (n_+ - n_m)/(n_+ + n_m) \) is the Fresnel reflection coefficient; \( n_m \) is the refractive index of the ambient medium; \( h = k \cos D = \omega/cD \) is the thickness parameter; \( \hat{s}_d = (\cos 2\psi_d, \sin 2\psi_d, 0) \) can be regarded the normalized Stokes vector of a light beam linearly polarized along the optic axis \( \hat{d}_x \). The transmission and density matrices can be used as an input to derive the expression for the intensity of the total output lightwave and fit the experimental data presented in Fig. 3, where the resulting theoretical curves are shown as solid lines.

We can now apply the interferometry based approach formulated in Refs. [10, 11] to the case of plane wave which is normally incident on the DHFLC cell and is characterized by the polarization density matrix (2) and derive the expressions for the Pancharatnam and geometric phases. The result reads

\[
\Phi_P = \Phi + \text{arg}[\text{Re} \tilde{F}_P + i\tilde{P} \text{Im} \tilde{F}_P] = \text{arg} \text{Tr}[\mathbf{T}_s(h)|\rho_0]\]
\[
\tilde{F}_P = \cos \Delta \Phi \cos[\theta_\lambda(h) - \theta_\lambda(0)] + i \sin \Delta \Phi \cos[\theta_\lambda(h) + \theta_\lambda(0)],
\]
\[
\Phi_g = \text{arg}[\text{Re} \tilde{F}_g + i\tilde{P} \text{Im} \tilde{F}_g], \quad \tilde{F}_g = \tilde{F}_p e^{-i\hat{d}_d},
\]
\[
\hat{d}_d = \int_0^h \cos(2\theta_\lambda(s))\Delta \Phi_s(s)ds, \quad \cos(2\theta_\lambda) = q_3/|\mathbf{q}|, \quad |\mathbf{q}| = \sqrt{q_3^2 + q_0^2[1 - (\hat{s}_d \cdot \hat{s}_0)^2]}
\]

where \( \Phi = (\Phi_+ + \Phi_-)/2 \) is the averaged phase shift; \( \Delta \Phi = (\Phi_+ - \Phi_-)/2 \) is the phase retardation; \( \tilde{P} = \sqrt{p_+(0)p_+(h)} - \sqrt{p_-(0)p_-(h)} \), \( p_+(0) = 1 + \mu P_0/2 \), \( p_+(h) = \text{Tr}(\rho) + \mu|\mathbf{q}| \), \( q_0 = P_0[r_2^2 - \tau_2^2], q_3 = 2\tau_- + P_0[r_2^2 + \tau_2^2](\hat{s}_d \cdot \hat{s}_0), \text{Tr}(\rho) = r_2^2 + \tau_2^2 + 2P_0\tau_- (\hat{s}_d \cdot \hat{s}_0), \tau_\pm = (|t_+| \pm |t_-|)/2. \)

In conclusion, we have studied electric-field-induced modulation of partially polarized light in the DHFLC with subwavelength helix pitch using the experimental technique based on the Mach-Zehnder interferometer. Such modulation occurs under the action of the voltage applied across the cell and manifests itself in the electrically dependent shift and contrast of the interference pattern. The effect of the degree of polarization is investigated theoretically and the theoretical results are in good agreement with the experimental data. Our analytical results show that, in contrast to the total Pancharatnam phase, the geometric phase is determined by the phase retardation and is independent of the averaged phase shift. In addition, the electric field dependence of the phases enters through the angle between the Stokes vectors \( \hat{s}_0 \) and \( \hat{s}_d \) describing the effect of electrically induced rotation of the optical axes. This effect is supressed in the limiting cases of unpolarized and circular polarized incident light.

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