On transverse spin sum rules

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Abstract

In this work we provide explicit calculations that support the conclusions stated in Ref. [12] regarding recent literature on transverse polarization. We also compare and contrast two methods of deriving spin sum rules.

Introduction

At present, understanding the helicity and transverse spin structure of the proton in the context of Deep Inelastic Scattering (DIS) is of great interest. Intense experimental and theoretical research activities have been going on in this field for more than a decade. It is well-known that since DIS is a light-cone dominated process, the most appropriate theoretical tool to study it is provided by Light Front Quantization (for a review, see Ref. [1]). In order to understand the spin structure of proton which is a composite object and investigate any sum rule associated with it, one should start from the intrinsic spin operators $J_i$, $i = 1, 2, 3$ which can be constructed from the Pauli-Lubansky operator. It is well-known that $J_i$'s are frame independent (see for example, Refs. [2, 3, 4]) whereas the usual rotation operators (which form part of the Poincare generators) are frame dependent. Any angular momentum sum rule, solely based on rotation operators that are part of Poincare generators, will have frame dependence. Same is also true, in general, if one starts with Pauli-Lubansky operators as we discuss below. As already stated, the solution to this problem is to start from intrinsic spin operators $J_i$.

Helicity operator $J^3$ (whose explicit construction and a perturbative analysis in light front QCD is carried out in Ref. [5] in the total transverse momentum zero frame)
is kinematical (interaction free). On the other hand, it is well known that the transverse rotation operators and hence the transverse spin operators in light front theory are dynamical (interaction dependent). Construction and analysis of $J^i$, ($i = 1, 2$) in light front QCD is carried out in Ref. [6, 7].

We have shown in Ref. [6, 7] that just like the helicity operator $J^3$, the transverse spin $J^i$, $i = 1, 2$ of the composite state can be separated in the gauge $A^+ = 0$ into orbital-like (explicit dependence on the coordinate $x^\perp$) contribution $J^i_1$ and coordinate-independent parts $J^i_II$ and $J^i_III$. What is the phenomenological relevance of this separation? Most interestingly, the proton matrix element of $J^i_II$ is shown to be directly related to the integral of the well-known transverse polarized structure function $g_T$ just as the proton matrix element of the coordinate-independent quark intrinsic part of $J^3$ is related to the polarized structure function $g_1$. Based on $J^i$, $i = 1, 2$ in light front QCD, in Ref. [7], a transverse spin sum rule was proposed and verified for a dressed quark to $O(g^2)$ in perturbation theory.

Recently, the matrix element of the transverse component of the Pauli-Lubanski operator has been formally analyzed in Refs. [8] and [9] following the approach of Ref. [10] and using the parameterizations of the off-forward matrix elements of the energy-momentum tensor. Refs. [8] and [9] appear to be partly inspired by Ref. [11] in which a relation between the expectation value of equal time transverse rotation generator $J_q^i$ and the form factors $A_q(0)$ and $B_q(0)$ is obtained using delocalized wave packet states that are transversely polarized in the rest frame of the nucleon.

We have pointed out in Ref. [12] that many of the statements in Refs. [8] and [9] appear unsupported by explicit calculations. In this work we present explicit calculations supporting our statements in Ref. [12]. We also compare and contrast our method [5, 6, 7] with the method used in Refs. [8] and [9] to derive sum rules.

**General outline of the calculation**

The starting point in Ref. [8] and [9] is the Pauli-Lubanski operator which is defined in terms of energy momentum tensor in a very standard way as follows.

$$ W^\mu = -\frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} J_{\nu\alpha} P_\beta $$

$$ M^{\mu\nu} = \frac{1}{2} \int dx^- d^2 x^\perp \left[ x^{\mu} T^{+\nu} - x^{\nu} T^{+\mu} \right]. \quad (1) $$

It is to be noted that in Refs. [8] and [9] the authors have calculated the matrix element of $W^i$ in the frame $P^\perp = 0$ (which is clear from Eq. (9) of [8]), although the results have been claimed to be frame independent. On the other hand, in Ref. [6, 7] where calculations are done completely within the framework of light-front QCD using Light-front gauge, intrinsic spin operators are used. Note that intrinsic helicity operator $\hat{J}^3$ and intrinsic transverse spin $\hat{J}^i, i = 1, 2$ among themselves obey the angular momentum algebra (see for example, Refs. [2, 3, 4]). For a massive particle like nucleon,
intrinsic spin operators and Pauli-Lubanski operators are related through the following relations.

\[ M^i \gamma^i = W^i - P^i \gamma^3 = \epsilon^{ij}(\frac{1}{2} F^j P^+ + K^3 P^i - \frac{1}{2} E^j P^-) - P^i \gamma^3 , \]

\[ \gamma^3 = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+} (E^1 P^2 - E^2 P^1) \]  

(2)

In the above expressions, \( F^i = M^{-i} \) are the light-front transverse rotation operators and are interaction dependent or dynamical; while \( E^i = M^{+i} \) are light-front transverse boost operators and are interaction independent or kinematical. Longitudinal boost \( K^3 = M^{+3} \) and helicity \( J^3 = M^{12} \) are also kinematical. Note that the light front transverse rotation and boost operators were mis-identified in Refs. [8] and [9]. This was already pointed out in Ref. [13]. Moreover, the authors of Refs. [8] and [9] did not consider longitudinal boost operator \( K^3 = M^{+3} \) for working explicitly in \( P^\perp = 0 \) frame and only for such a choice of frame, both the starting points appear to be the same. In the following, we kept this term to show an example in the course of our explicit calculations that, in general, for a frame with non-zero \( P^\perp \) both are not the same. Following Refs. [8] and [9] we also assume that the various Poincare generators can be separated to quark and gluon parts.

Next, to compare with the results of Refs. [9] and [8], we need to calculate the transverse component of the Pauli-Lubanski operator corresponding to species \( i \) formally defined as

\[ W^i_1 = \frac{1}{2} F^2_i P^+ + K^3_i P^2 - \frac{1}{2} E^2_i P^- \]  

(3)

and its matrix element in a transversely polarized state

\[ \frac{\langle PS^{(1)} | W^i_1 | PS^{(1)} \rangle}{(2\pi)^3 2P^+ \delta^3(0)} \]  

(4)

where \( i \) denotes either the quark or gluon part. Note that, in the rest of the paper, we always deal with only one component, namely, \( W^i_1 \), while calculation with \( W^i_2 \) is trivially same and unnecessary for our purpose.

The transverse rotation operator is

\[ F^2_i = \frac{1}{2} M_{i}^{-2} = \frac{1}{4} \int dx^- d^2 x^\perp \left[ x^- T_i^{+2} - x^2 T_i^{++} \right] . \]  

(5)

We note that,

\[ K^3_i = \frac{1}{2} M_i^{+-} = \frac{1}{4} \int dx^- d^2 x^\perp \left[ x^+ T_i^{+--} - x^- T_i^{++} \right] \]

\[ = \frac{1}{2} P^- + \tilde{K}_i^3 , \]
\[ E_i^2 = M_i^{+2} = \frac{1}{2} \int dx^- d^2 x^\perp \left[ x^+ T_i^{-+2} - x^2 T_i^{-+} \right] \]
\[ = x^+ p^2 + E_i^2. \]  

(6)

In writing the last equalities in both the above expressions, we note that light-front time \( x^+ \) can be taken out of the integral in the first terms and simplified. Putting them back in Eq. (3), we see that only the second terms in these expressions contribute to \( W_i^1 \). Thus we find that

\[ W_i^1 = \frac{1}{2} F_i^2 p^+ + K^3 p^2 - \frac{1}{2} \tilde{E}_i^2 p^- \]  

(7)

with no explicit \( x^+ \) dependence. Lastly, the light-front helicity operator is given by

\[ J_i^3 = M_i^{12} = \frac{1}{2} \int dx^- d^2 x^\perp \left[ x^1 T_i^{+2} - x^2 T_i^{+1} \right]. \]  

(8)

According to the procedure prescribed in Ref. [10] and followed in Refs. [8, 9], rest

of the calculation relies on defining the Fourier transform of the off-forward matrix elements of relevant component of energy momentum tensor and then consider the forward limit. Since \( W_i^1 \) is independent of \( x^+ \) explicitly, we consider three dimensional Fourier Transform of the off-forward matrix element. In general, we define

\[ \langle p'S^{(1)} | \hat{\delta}^\alpha (k_- , k_i) | pS^{(1)} \rangle = \frac{1}{2} \int dx^- d^2 x^\perp e^{i(k_- x^- + k_i x^\perp)} x^\alpha \langle p'S^{(1)} | \hat{\sigma}(x) | pS^{(1)} \rangle \]  

(9)

where \( \alpha = -, 1, 2 \). Using translational invariance, we find

\[ \langle p'S^{(1)} | \hat{\delta}^\alpha (k) | pS^{(1)} \rangle = -i(2\pi)^3 \frac{\partial}{\partial k_\alpha} \left[ \delta^3 (k + p' - p) \langle p'S^{(1)} | \hat{\sigma}(0) | pS^{(1)} \rangle \right] \]
\[ = -i(2\pi)^3 \delta^3 (k + p' - p) \frac{\partial}{\partial k_\alpha} \langle p'S^{(1)} | \hat{\sigma}(0) | pS^{(1)} \rangle \]  

(10)

ignoring the term containing the derivative on the delta function [10].

Thus, with \( \Delta = p' - p \), we find

\[ \langle pS^{(1)} | F_i^2 | pS^{(1)} \rangle = i(2\pi)^3 \delta^3 (0) \left[ \frac{\partial}{\partial \Delta_-} \langle p'S^{(1)} | T_i^{-+2} (0) | pS^{(1)} \rangle \right. \]
\[ \left. - \frac{\partial}{\partial \Delta_2} \langle p'S^{(1)} | T_i^{-+} (0) | pS^{(1)} \rangle \right]_{\Delta = 0}, \]  

(11)

\[ \langle pS^{(1)} | K_i^3 | pS^{(1)} \rangle = -\frac{i}{2} (2\pi)^3 \delta^3 (0) \left[ \frac{\partial}{\partial \Delta_-} \langle p'S^{(1)} | T_i^{++} (0) | pS^{(1)} \rangle \right]_{\Delta = 0}. \]  

(12)

and

\[ \langle pS^{(1)} | \tilde{E}_i^2 | pS^{(1)} \rangle = -i(2\pi)^3 \delta^3 (0) \left[ \frac{\partial}{\partial \Delta_2} \langle p'S^{(1)} | T_i^{++} (0) | pS^{(1)} \rangle \right]_{\Delta = 0}. \]  

(13)
Matrix elements of the Energy-Momentum tensor

We start from the following parameterization as used in Refs. [9, 8],

\[ \langle P', S' | T^{\mu \nu}_{i}(0) | PS \rangle = U(P', S') \left[ A_i(\Delta^2) \frac{1}{2} (\gamma^\mu \overline{P}{}^\nu + \gamma^\nu \overline{P}{}^\mu) \right. \\
+ B_i(\Delta^2) \frac{1}{2M_N} \left( \overline{P}{}^\mu i\sigma^{\nu\alpha} \Delta_{\alpha} + \overline{P}{}^\nu i\sigma^{\mu\alpha} \Delta_{\alpha} \right) \\
+ C_i(\Delta^2) \frac{1}{M_N} \left( \Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2 \right) + \overline{C}_i(\Delta^2) M_N g^{\mu\nu} \left. \right] U(P, S). \]

(14)

Here \( \overline{P} = \frac{1}{2}(P + P') \).

Note that, in the above parameterization, effects of QCD-interactions are buried in
the form factors while the associated Lorentz structures are given in terms of asymptotic
spin-half nucleonic states. We can either calculate the matrix elements in Eq. (14)
directly (which we denote by method I) or use the Gordon identity

\[ \overline{U}(P', S') \frac{i}{2M_N} \sigma^{\mu\nu} \Delta_{\nu} U(P, S) = \overline{U}(P', S') \gamma^\mu U(P, S) - \overline{U}(P', S') \frac{(P + P')^\mu}{2M_N} U(P, S) \]

(15)
to eliminate either the “\( \sigma \)” terms (method II) or the “\( \gamma \)” terms (method III which is used
in Ref. [8]) from Eq. (14).

Eliminating the “\( \sigma \)” terms (method II) we have

\[ \langle P', S' | T^{\mu \nu}_{i}(0) | PS \rangle = \overline{U}(P', S') \left[ - B_i(\Delta^2) \frac{\overline{P}{}^\mu \overline{P}{}^\nu}{M_N} \right. \\
+ (A_i(\Delta^2) + B_i(\Delta^2)) \frac{1}{2} (\gamma^\mu \overline{P}{}^\nu + \gamma^\nu \overline{P}{}^\mu) \right. \\
+ C_i(\Delta^2) \frac{1}{M_N} \left( \Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2 \right) + \overline{C}_i(\Delta^2) M_N g^{\mu\nu} \left. \right] U(P, S) \]

(16)

On the other hand, eliminating the “\( \gamma \)” terms (method III) we have

\[ \langle P', S' | T^{\mu \nu}_{i}(0) | PS \rangle = \overline{U}(P', S') \left[ A_i(\Delta^2) \frac{\overline{P}{}^\mu \overline{P}{}^\nu}{M_N} \right. \\
+ (A_i(\Delta^2) + B_i(\Delta^2)) \frac{1}{2M_N} \left( \overline{P}{}^\mu i\sigma^{\nu\alpha} \Delta_{\alpha} + \overline{P}{}^\nu i\sigma^{\mu\alpha} \Delta_{\alpha} \right) \\
+ C_i(\Delta^2) \frac{1}{M_N} \left( \Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2 \right) + \overline{C}_i(\Delta^2) M_N g^{\mu\nu} \left. \right] U(P, S). \]
All the three methods should yield the same results since Eq. (15) is simply an identity which is valid for on-shell spin-half states.

We present explicit calculation in Method II. Further details of the calculation are given in the appendix that are used to obtain the results given below.

In the following we keep only terms which are linear in $\Delta$, which are relevant for the computation of matrix elements of transverse spin. Then, the matrix elements of $T_{\mu\nu}(0)$ in the transversely polarized state (to be specific, taken to be polarized along $+ve$ $x$ direction) are (in the frame $P^\perp = 0$)

$$
\langle P', S^{(1)} | T^{++} i(0) | PS^{(1)} \rangle = -B_i(\Delta^2\frac{\mathbf{P}'\cdot \mathbf{P}}{M_N} - i\Delta^{(2)}) ,
$$

(18)

$$
\langle P', S^{(1)} | T^{+1} i(0) | PS^{(1)} \rangle = 0,
$$

(19)

$$
\langle P', S^{(1)} | T^{+2} i(0) | PS^{(1)} \rangle = \frac{1}{2}(A_i(\Delta^2) + B_i(\Delta^2)) (-i) M_N \Delta^+ ,
$$

(20)

$$
\langle P', S^{(1)} | T^{++} i(0) | PS^{(1)} \rangle = -A_i(\Delta^2)iM_N\Delta^{(2)}
+ \bar{C}_i(\Delta^2) M_N g^{++} \left(-i\Delta^{(2)}\right).
$$

(21)

From Eq. (18) we re-confirm that $A_i(\Delta^2)$ does not appear in the matrix element of $T^{++}$ in a transversely polarized state [16].

Methods I and III yield the same results as Eqs. (18)-(21) in their dependence on $A_i$, $B_i$ and $\bar{C}_i$. Note, however, that in these methods, $\Delta^-$ appears which we need to evaluate. Since $P$ and $P'$ are on mass shell, $\Delta^-$ is related to $\Delta^+$ and $\Delta^\perp$ by

$$
\Delta^- = -\Delta^+ \frac{\Delta^+}{(P^+)^2 - (1/4)(\Delta^\perp)^2} \left(M^2 + \frac{1}{4}(\Delta^\perp)^2\right) \Rightarrow -\Delta^+ \frac{M^2}{(P^+)^2}.
$$

(22)

We ignore the $(\Delta^\perp)^2$ term since we are interested only in terms linear in $\Delta$. We also need to use the Eq. (31).

We summarize the results obtained so far, as follows.

1) In $T^{++}$ matrix element, coefficient of $A_i$ form factor vanishes and hence it depends only on $B_i$ form factor.
2) $T^{+2}$ matrix element depends only on $\Delta^+$ explicitly and
3) $T^{+-}$ matrix element depends only on $A_i$ and $\bar{C}_i$ form factors.

**Matrix elements of the Pauli-Lubanski operator $W_i^1$**

Substituting the results for individual matrix elements, in Eq. (4) we get

$$
\frac{\langle PS^{(1)} | W_i^1 | PS^{(1)} \rangle}{\langle PS^{(1)} | PS^{(1)} \rangle} = \frac{1}{2P^+} \left[ \frac{P^+}{2} \left(2A_i(0) + B_i(0) + 2\mathbf{C}_i(0)\right) M_N + \frac{P^-}{2} (P^+)^2 B_i(0) \right]
$$

6
\[
\frac{1}{2} M_N \left( A_i(0) + B_i(0) + C_i(0) \right). \tag{23}
\]

Thus the matrix elements of \( T_i^{+2} \) and \( T_i^{-} \) make comparable contributions to the matrix element of \( W_i^1 \) in a transversely polarized state. The matrix element of \( T^{++} \) does not contribute to the matrix element of total \( W^1 \).

**Comment on frame dependence of \( W_i^1 \) Matrix elements**

From the definitions of intrinsic spin operators, it is clear that in \( P^\perp = 0 \) frame irrespective of the polarization \( S \),

\[
\begin{align*}
M_N \langle PS | \vec{J}_1^1 | PS \rangle &= \langle PS | W_i^1 | PS \rangle \quad \text{(24)} \\
\langle PS | \vec{J}_3^3 | PS \rangle &= \langle PS | J_3^3 | PS \rangle. \quad \text{(25)}
\end{align*}
\]

Note that in the appendix D of Ref. [7], the calculation of the matrix element of the intrinsic transverse spin operator in a transversely polarized dressed quark state in an arbitrary reference frame is presented and the frame independence is explicitly demonstrated. To calculate the RHS of the above equations in the \( P^\perp = 0 \) only and simply claim that the results are frame independent, as the authors of Refs. [8] and [9] do, is naive and without any basis as we demonstrate in the following. Extending the calculation presented in the last section for a frame with non-zero \( P^\perp \) (i.e., not putting \( P^\perp = 0 \) from the very beginning) one could easily show that even though LHS of the above equations are frame independent, while RHS are not necessarily frame independent.

Explicit calculation shows that, for a transversely polarized nucleon,

\[
\frac{\langle PS^{(1)} | \vec{J}_1^1 | PS^{(1)} \rangle}{\langle PS^{(1)} | PS^{(1)} \rangle} = \frac{1}{2} (A_i(0) + B_i(0) + C_i(0)) \tag{26}
\]

and for a longitudinally polarized nucleon

\[
\frac{\langle PS | \vec{J}_3^3 | PS \rangle}{\langle PS | PS \rangle} = \frac{1}{2} (A_i(0) + B_i(0)). \tag{27}
\]

which are frame independent. We also get

\[
\begin{align*}
\frac{\langle PS | \vec{J}_1^1 | PS \rangle}{\langle PS | PS \rangle} &= 0, \\
\frac{\langle PS^{(1)} | \vec{J}_3^3 | PS^{(1)} \rangle}{\langle PS^{(1)} | PS^{(1)} \rangle} &= 0. \tag{28}
\end{align*}
\]

Results in Eq (28) are frame independent and correctly represent the fact that the expectation value of the helicity in a transversely polarized nucleon must be zero and the expectation value of intrinsic transverse spin in a longitudinally polarized nucleon must
be zero. On the other hand, RHS of the corresponding equations as obtained from Eq. (24) and Eq. (25) are frame dependent and do not reflect the correct results:

\[
\frac{\langle PS \mid W_1^i \mid PS \rangle}{\langle PS \mid PS \rangle} = \frac{1}{2} (A_i(0) + B_i(0)) P^i; \\
\frac{\langle PS^{(1)} \mid J_3^i \mid PS^{(1)} \rangle}{\langle PS^{(1)} \mid PS^{(1)} \rangle} = -B_i \frac{P^i}{2M_N}.
\]

(29)

We get vanishing RHS only in the frame \( P^\perp = 0 \).

**Comparison of two methods**

In Refs. [6] and [7] we have presented a transverse spin sum rule for the nucleon in QCD using light front dynamics and intrinsic (boost invariant) transverse spin operators. The analysis relies on the explicit structure of Poincare generators and hence depends on the details of QCD dynamics since transverse spin operators in the light front theory are interaction dependent. We were able to separate the operator into terms with and without explicit coordinate dependence. The latter could be further separated into quark (\( J_{\text{II}}^i \)) and gluon parts (\( J_{\text{III}}^i \)). We have demonstrated [6, 7] that the nucleon matrix element of \( J_{\text{II}}^i \) is directly related to the integral of the well-known transverse polarized structure function \( g_T \) and the nucleon matrix element of \( J_{\text{III}}^i \) is directly related to the integral of the gluon distribution function that appears in transverse polarized hard scattering [17]. In the case of helicity, we have demonstrated similar connections [5], namely nucleon matrix element of (\( J_3^{q(i)} \)) is directly related to the polarized structure function \( g_1 \) and the nucleon matrix element of (\( J_3^{g(i)} \)) is directly related to the gluon distribution relevant to nucleon helicity [18]. Thus the physical content of our sum rules are very transparent.

On the other hand, the sum rule discussed in Refs. [9] and [8] contain form factors that parameterize the off-forward matrix elements of the energy momentum tensor. The details of the dynamics remain hidden in this formalism. The separation into orbital and intrinsic spin parts is not visible and relation of the sum rules to the quark and gluon helicity and transverse spin distribution functions that appear in various deep inelastic processes remain obscure.

**Conclusions**

We have found that (i) both the form factors \( A_i \) and \( C_i \) contribute to the matrix element of \( T_i^{+-} \) in a transversely polarized state, (ii) there is no relative suppression factor between these two contributions and (iii) the contribution to \( W_i^\perp \) from \( T_i^{++} \) contains only the form factor \( B_i \) and not the form factor \( A_i \). First two observations differ from that in Ref. [9] and eventually invalidates their argument regarding the consequence of Lorentz invariance, while the last finding is already a well established result [16]. We
have also compared and contrasted the sum rules advocated by us with the sum rules following the method of Ref. \[10\].

Appendix

All our calculations are performed in light front field theory. We will follow the conventions of Ref. \[14\]. Let us take the state to be polarized in the +ve x direction. Explicitly, it is given by

\[
| P, S^{(1)} \rangle = \frac{1}{\sqrt{2}} \left( | P, \text{up} \rangle + | P, \text{down} \rangle \right)
\]

(30)

where $| P, \text{up} \rangle$ and $| P, \text{down} \rangle$ are helicity eigenstates. Then we need to evaluate the matrix elements for up down, down up, up up and down down helicity states.

To simplify the calculations further, we use

\[
\bar{U}(P, S^{(1)}) \sigma^{\mu\nu} U((P, S^{(1)}) = 2 \epsilon^{\mu\nu\alpha\beta} \frac{P_\beta S_\alpha}{M_N}. \tag{31}
\]

Note that in our convention, $\epsilon^{+-12} = -2$ and hence Eq. (31) differs from the equation (5.36) of Ref. \[15\] by a factor of 2. The components of the polarization vector $S^\mu$ are explicitly $S^+ = 0$, $S^1 = M_N$, $S^2 = 0$ and $S^- = 2M_N \frac{P^1}{P^+}$. We present explicit calculations in Method II in which we need to calculate the five matrix elements, namely, $\bar{U}(P', S') U(P, S)$ and $\bar{U}(P', S') \gamma^\mu U(P, S)$.

Explicit evaluation of these matrix elements gives the following (in the frame with non-zero $P^\perp$):

$S' = \text{up}, S = \text{down}$

\[
\bar{U}(P', S') U(P, S) = \frac{1}{\sqrt{P^+ P'^+}} \left[ \bar{P}^+ \left( \Delta^{(1)} - i\Delta^{(2)} \right) - \Delta^+ \left( \bar{P}^{(1)} - i\bar{P}^{(2)} \right) \right], \tag{32}
\]

\[
\bar{U}(P', S') \gamma^1 U(P, S) = \frac{M_N}{\sqrt{P^+ P'^+}} \Delta^+, \tag{33}
\]

\[
\bar{U}(P', S') \gamma^2 U(P, S) = -i \frac{M_N}{\sqrt{P^+ P'^+}} \Delta^+ \tag{34}
\]

\[
\bar{U}(P', S') \gamma^+ U(P, S) = 0, \tag{35}
\]

\[
\bar{U}(P', S') \gamma^- U(P, S) = 2 \frac{M_N}{\sqrt{P^+ P'^+}} \left( \Delta^{(1)} - i\Delta^{(2)} \right). \tag{36}
\]

$S' = \text{down}, S = \text{up}$

\[
\bar{U}(P', S') U(P, S) = \frac{1}{\sqrt{P^+ P'^+}} \left[ -\bar{P}^+ \left( \Delta^{(1)} + i\Delta^{(2)} \right) + \Delta^+ \left( \bar{P}^{(1)} + i\bar{P}^{(2)} \right) \right], \tag{37}
\]

\[
\bar{U}(P', S') \gamma^1 U(P, S) = - \frac{M_N}{\sqrt{P^+ P'^+}} \Delta^+, \tag{38}
\]
\[ \mathcal{U}(P', S') \gamma^2 U(P, S) = -i \frac{M_N}{\sqrt{P^+ P'^+}} \Delta^+ , \]
\[ \mathcal{U}(P', S') \gamma^+ U(P, S) = 0 , \]
\[ \mathcal{U}(P', S') \gamma^- U(P, S) = -2 \frac{M_N}{\sqrt{P^+ P'^+}} \left( \Delta^{(1)} + i\Delta^{(2)} \right) . \]

\( S' = \text{up}, S = \text{up} \)

\[ \mathcal{U}(P', S') U(P, S) = \frac{1}{\sqrt{P^+ P'^+}} \mathcal{P}^+ \left( 2M_N \right) , \]
\[ \mathcal{U}(P', S') \gamma^1 U(P, S) = \frac{1}{\sqrt{P^+ P'^+}} \left[ \mathcal{P}^+ \left( 2\mathcal{P}^{(1)} - i\Delta^{(2)} \right) - \Delta^+ \frac{2}{2} \left( \Delta^{(1)} - 2 i \mathcal{P}^{(2)} \right) \right] , \]
\[ \mathcal{U}(P', S') \gamma^2 U(P, S) = \frac{1}{\sqrt{P^+ P'^+}} \left[ \mathcal{P}^+ \left( i \Delta^{(1)} + 2 \mathcal{P}^{(2)} \right) - \Delta^+ \frac{2}{2} \left( 2 i \mathcal{P}^{(1)} + \Delta^{(2)} \right) \right] , \]
\[ \mathcal{U}(P', S') \gamma^+ U(P, S) = 2 \sqrt{P^+ P'^+} , \]
\[ \mathcal{U}(P', S') \gamma^- U(P, S) = \frac{2}{\sqrt{P^+ P'^+}} \left[ M_N^2 + (\mathcal{P}^\perp)^2 - \frac{1}{4} (\Delta^\perp)^2 \right. \]
\[ \left. + i \left( \mathcal{P}^{(2)} \Delta^{(1)} - \mathcal{P}^{(1)} \Delta^{(2)} \right) \right] . \]

\( S' = \text{down}, S = \text{down} \)

\[ \mathcal{U}(P', S') U(P, S) = \frac{1}{\sqrt{P^+ P'^+}} \mathcal{P}^+ \left( 2M_N \right) , \]
\[ \mathcal{U}(P', S') \gamma^1 U(P, S) = \frac{1}{\sqrt{P^+ P'^+}} \left[ \mathcal{P}^+ \left( 2\mathcal{P}^{(1)} + i\Delta^{(2)} \right) - \Delta^+ \frac{2}{2} \left( \Delta^{(1)} + 2 i \mathcal{P}^{(2)} \right) \right] , \]
\[ \mathcal{U}(P', S') \gamma^2 U(P, S) = \frac{1}{\sqrt{P^+ P'^+}} \left[ \mathcal{P}^+ \left( - i \Delta^{(1)} + 2 \mathcal{P}^{(2)} \right) \right. \]
\[ \left. - \Delta^+ \frac{2}{2} \left( - 2 i \mathcal{P}^{(1)} + \Delta^{(2)} \right) \right] , \]
\[ \mathcal{U}(P', S') \gamma^+ U(P, S) = 2 \sqrt{P^+ P'^+} , \]
\[ \mathcal{U}(P', S') \gamma^- U(P, S) = \frac{2}{\sqrt{P^+ P'^+}} \left[ M_N^2 + (\mathcal{P}^\perp)^2 - \frac{1}{4} (\Delta^\perp)^2 \right. \]
\[ \left. + i \left( \mathcal{P}^{(1)} \Delta^{(2)} - \mathcal{P}^{(2)} \Delta^{(1)} \right) \right] . \]

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