Accurate masses of very low mass stars: IV Improved mass-luminosity relations. *

X. Delfosse1, T. Forveille2,3, D. Ségransan3, J.-L Beuzit3,2, S. Udry4, C. Perrier3 and M. Mayor4

1 Instituto de Astrofísica de Canarias E-38200 La Laguna, Tenerife, Canary Islands, Spain
2 Canada-France-Hawaii Telescope Corporation, 65-1238 Mamalahoa Highway, Kamuela, HI 96743, U.S.A.
3 Observatoire de Grenoble, 414 rue de la Piscine, Domaine Universitaire de S3 Martin d’Hères, F-38041 Grenoble, France
4 Observatoire de Genève, 51 Ch des Maillettes, 1290 Sauverny, Switzerland

Received 21 July 2000; Accepted 28 September 2000

Abstract. We present improved visual and near-infrared empirical mass-luminosity relations for very low mass stars (M<0.6 M⊙). These relations make use of all stellar masses in this range known with better than 10% accuracy, most of which are new determinations with 0.2 to 5% accuracy from our own programme, presented in a companion paper.

As predicted by stellar structure models, the metallicity dispersion of the field populations induces a large scatter around the mean V band relation, while the infrared relations are much tighter. The agreement of the observed infrared mass-luminosity relations with the theoretical relations of Baraffe et al. (1998) and Siess et al. (2000) is impressive, while we find an increasingly significant discrepancy in the V band for decreasing masses. The theoretical mass-luminosity relation which is insufficiently steep, and has introduced significant errors in the local stellar mass functions derived from V band luminosity functions.

Key words: Stars: binaries, spectroscopic - Stars: binaries, visual - Stars: low mass, brown dwarfs - Stars: late-type - Technique: radial velocities

1. Introduction

The mass of a star is arguably its most basic characteristic, since most stellar properties have a very steep mass dependency. Yet, it can only be directly determined for some stars in multiple systems, and for most stars it has to be inferred from more directly observable parameters, the luminosity, the chemical composition, and the evolutionary status. An accurately calibrated Mass-Luminosity (hereafter M/L) relation, or in full generality a Mass-Composition-Age/Luminosity relation, is therefore an essential astrophysical tool (e.g. Andersen 1991, 1998). It is needed at many places to convert observable stellar light to the underlying mass, and perhaps most importantly, to derive stellar mass functions from more readily obtained luminosity functions. The latter in turn provide an essential diagnostic of star formation theories. They also represent a basic building block for galactic and stellar cluster dynamical models.

The M/L relation is fairly well constrained for solar-type and intermediate mass stars: a sizeable number of such stars in eclipsing systems have had their mass determined with better than 1% relative accuracy (Andersen 1991), and the theory of these stars approximately matches this excellent precision, when both evolutionary effects and metallicity are taken into account (Andersen 1998). For both smaller and more massive stars however, the M/L relation is significantly more uncertain, as theory and observations meet there with new difficulties.

Here we address the low-mass end of the HR diagram, below 0.6M⊙, where stellar models face two major hurdles (Chabrier & Baraffe 2000 for a recent review):

- the onset of low temperature electron degeneracy in the stellar core (Chabrier & Baraffe 1997, 2000);
- a complex cold and high gravity stellar atmosphere, dominated by molecular and dust opacity (Allard et al. 1997; Jones & Tsuji 1997; Allard 1998).

Much progress has been made in the past few years, with the realization that an accurate atmosphere description must be used as an outer boundary condition to the stellar interior equations (Baraffe et al. 1993), and with an increased sophistication of the atmospheric models (Allard et al., in prep). State of the art models (Baraffe et al. 1998; Chabrier et al. 2000) now produce good to fair agreement with most observational colour-magnitude diagrams (Goldman et al. 1999, for an example). This lends considerable credence to their general reliability. Their description of some of the input physics however remains in-
complete or approximate; some molecular opacity sources are still described by relatively crude approximations, or by line lists that remain incomplete (though vastly improved), the validity of the mixing-length approximation in the convective atmosphere is questionable, and atmospheric dust condensation and settling introduces new uncertainties at the lowest effective temperatures. The actual severity of these known shortcomings of the models is unclear, making an independent check of their M/L prediction most desirable.

Detached eclipsing M-dwarf binaries are rare, with only three known to date. Most mass determinations for Very Low Mass Stars (VLMS) are therefore instead obtained from visual and interferometric pairs, which until recently have not yielded comparable precisions. The current state of the art empirical M/L relation for M dwarfs (Henry & McCarthy 1993; Henry et al. 1999) as a result mostly rests on masses determined with 5-20% accuracy. The last two years have seen a dramatic evolution resulting mostly rests on masses determined with 5-20% accuracy. The last two years have seen a dramatic evolution in this respect, with two groups breaking through the former ~5% accuracy barrier. The first team used the 1 mas per measurement astrometric accuracy of the Fine Guidance Sensors (Benedict et al. 1999) on HST to determine system masses for three angularly resolved binaries with 2 to 10% accuracy (Franz et al. 1999; Torres et al. 1999; Benedict et al. 2000). Shortly thereafter our team demonstrated even better accuracies for masses of VLMS, of only 1-3% (Forveille et al. 1999; Delfosse et al. 1999), by combining very accurate radial velocities with precise angular separations from adaptive optics. In a companion paper (Ségransan et al. 2000) we determine a dozen new or improved masses, with the same method and with accuracies that now range between 0.5 and 5%. The same paper also presents improved masses for two of the three known eclipsing M-dwarf systems, with 0.2% accuracy. Here we take advantage of this wealth of accurate new data, and reassess the VLMS M/L relation on a much firmer ground.

In Sect. 2 we briefly describe the sample of accurately determined very low stellar masses. We then discuss the resulting empirical M/L relation in Sect. 3, and compare it with theoretical models in Sect. 4.

2. Sample

2.1. Accurate masses for M dwarfs

We adopt a 10% mass accuracy cutoff for inclusion in our new M dwarf M/L relations, as a compromise between good statistics and the quality of the individual measurements. To our knowledge 32 M dwarfs fulfill this criterion. They can be divided into four broad categories:

1. Four systems have orbits of a quality that hasn’t changed much since Henry & McCarthy (1993), but masses which are now somewhat better determined thanks to the availability of the Hipparcos parallaxes (Henry et al. 1999, and this paper): Gl 65, Gl 661, Gl 702, and Gl 860. One should note however that the Gl 661 masses derived by Martin et al. (1998) represent a ~3 $\sigma$ correction to the Henry & McCarthy (1993) values. The new masses are much more consistent with the average M/L relation, and we believe that Henry & McCarthy (1993) had underestimated their standard errors for that particular pair. These four systems have long to very long periods, up to 90 years for Gl 702AB. Their inclusion is a testimony to the care and dedication of binary star observers over many decades, but the accuracy of most of these masses is unlikely to be significantly improved over the next few years.

2. The three eclipsing systems have extremely accurate masses, with relative precisions of 0.5% for CM Dra (Metcalfe et al. 1999), and 0.2% for YY Gem and GJ 2069A (Ségransan et al. 2000). Their parallaxes are unfortunately less precisely known, in part due to their slightly larger distances than those of the visual systems. Their luminosities are therefore more uncertain than their masses. Their flux ratios in the near-IR JHK bands have also not yet been determined.

3. The masses of Gl 473 (8%; Torres et al. 1999) and Gl 791.2 (2%; Benedict et al. 2000) result from the effort of the FGS astrometry team on HST (Benedict et al. 1999). We have chosen to temporarily exclude the measurement of the Gl 748 system mass by the same group (5%; Franz et al. 1999), a M/L relation has to be used to estimate individual masses for the two stars in that system. This would introduce an undesirable circular aspect to our discussion. We have on the other hand retained their Gl 473 determination, which formally has the same weakness. The two components of that system are sufficiently similar that this contributes negligible additional uncertainties.

4. Most of the masses in Table 3 result from our own programme: since 1995 we have been monitoring a sample of solar neighbourhood M dwarfs with high precision radial velocity and adaptive optics imaging observations (Delfosse et al. 1999, for a complete presentation of the project). These observations have resulted in a new orbit for Gl 747, and in improved orbits and masses with 0.5 to 5% accuracy for Gl 234, Gl 644, Gl 831, Gl 866 (Ségransan et al. 2000), Gl 570B and Gl 623 (Ségransan et al., in prep). Some additional binaries, including discoveries from that programme (Delfosse et al. 1999, Beuzit et al., in prep.) are nearing the time when their masses will be known with similar accuracies.

2.2. Photometry

Table 1 lists the basic properties of the selected systems: parallaxes, spectral types and integrated photometry. Table 2 lists the magnitude differences for the V, R, I, J, H and K photometric bands. The near-IR flux ratios were obtained either from literature infrared speckle observations, or extracted from our adaptics images (Ségransan et al. 2000). For the three eclipsing systems the visi-
Table 1. Basic parameters for the M-dwarf systems with accurate masses. The parallaxes mostly originate from the Hipparcos catalog (ESA 1997), the Yale General Catalog of trigonometric Parallaxes (Van Altena et al. 1995), and from Ségransan et al. (2000). In that paper we derived optimal combinations of orbital and astrometric parallaxes, which often have a strong contribution from either an Hipparcos or a Yale catalog astrometric parallax. Some individual entries are additionally taken from Soderhjelm (1999), Forveille et al. (1999), and Benedict et al. (2004). All spectral types are from either Reid et al. (1997) or Hawley et al. (1997) and refer to the integrated light of the system. The photometry is taken from the extensive homogenized compilation of Leggett (1992), except for YY Gem, Gl 702AB and GJ2069A. The photometry of GJ2069A is from Weiss (1991). The optical photometry of YY Gem is from Kron et al. (1957) (RI), Eggen (1968) (UBV), Barnes et al. (1978) (UBVRI), and the infrared photometry from Johnson (1965), Glass (1972), and Veeder (1974). The optical photometry for Gl 702AB is from Bessel (1990) and the infrared photometry from Alonso et al. (1994). All photometry was converted to the Johnson-Cousins-CIT system adopted by Leggett (1992), using the colour transformations listed in that paper. Multiple measurements for the same band were averaged with equal weights.

### Table 1.

| Name          | $\pi$ (mas) | $\sigma(\pi)$ (mas) | Ref | Spectral type | $B$ | $V$ | $R_c$ | $I_c$ | $J$ | $H$ | $K$ | $L$ | $L'$ |
|---------------|-------------|---------------------|-----|---------------|-----|-----|-------|-------|-----|-----|-----|-----|-----|
| Gl 65AB       | 373.7       | 2.7                 | Yale95 | M5.5V         | 13.87 | 12.00 | 10.37 | 8.31 | 6.24 | 5.67 | 5.33 | 5.00 |
| Gl 234AB      | 243.7       | 2.0                 | Ség00  | M4.5V         | 12.80 | 11.08 | 9.77  | 8.06 | 6.40 | 5.78 | 5.49 | 5.33 |
| YY Gem        | 74.7        | 2.5                 | Yale95 | M0V           | 10.50 | 9.07  | 8.10  | 7.09 | 6.01 | 5.35 | 5.18 | 5.16 |
| Gl 2069A      | 78.05       | 5.69                | HIP    | M3.5V         | 11.89 | 10.68 | 9.09  |       |     |     |     |     |
| Gl 473AB      | 227.9       | 4.6                 | Yale95 | M5V           | 14.30 | 12.46 | 10.90 | 9.27 | 6.67 | 6.14 | 5.91 | 5.63 |
| Gl 570B       | 169.8       | 0.9                 | For99  | M1V           | 9.57  | 8.09  | 7.09  | 5.97 | 4.75 | 4.14 | 3.93 | 3.77 |
| Gl 623AB      | 124.34      | 1.16                | HIP    | M2.5V         | 11.76 | 10.26 | 9.27  | 7.96 | 6.67 | 6.14 | 5.91 | 5.63 |
| CM Dra        | 69.2        | 2.5                 | Yale95 | M4.5V         | 14.49 | 12.91 | 11.67 | 9.99 | 8.54 | 8.07 | 7.79 |       |
| Gl 644AB      | 154.8       | 0.6                 | Ség00  | M3V           | 10.60 | 9.02  | 7.92  | 6.55 | 5.28 | 4.64 | 4.39 | 4.14 |
| Gl 661AB      | 156.66      | 1.37                | Sod99  | M3.5V         | 10.89 | 9.40  | 8.30  | 6.89 | 5.56 | 5.04 | 4.82 | 4.60 |
| Gl 702AB      | 195.7       | 0.9                 | Sod99  | K0V           | 4.88  | 4.02  | 3.51  | 3.05 | 2.42 | 1.96 | 1.89 |       |
| Gl 747AB      | 120.2       | 0.2                 | Ség00  | M3V           | 12.93 | 11.25 | 10.11 | 8.65 | 7.25 | 6.69 | 6.43 |       |
| Gl 791.2AB    | 112.9       | 0.3                 | Ben00  | M4.5V         | 14.72 | 13.06 | 11.72 | 9.96 | 8.20 | 7.63 | 7.33 |       |
| Gl 831AB      | 117.5       | 2.0                 | Ség00  | M4.5V         | 13.68 | 12.01 | 10.71 | 9.02 | 7.29 | 6.70 | 6.42 |       |
| Gl 860AB      | 247.5       | 1.5                 | HIP    | M3V+M4V       | 11.25 | 9.59  | 8.40  | 6.91 | 5.56 | 4.97 | 4.71 | 4.48 |
| Gl 866ABC     | 293.6       | 0.9                 | Ség00  | M5.5V         | 14.29 | 12.33 | 10.66 | 8.62 | 6.50 | 5.91 | 5.57 | 5.22 |

3. Visible and infrared Mass/Luminosity relations

The masses are listed in Table 1 with the individual absolute magnitudes derived from Table 1 and Table 2 for the four photometric bands (V, J, H and K) which have significant numbers of measurements. Fig. 1 shows the M/L relations for these 4 photometric bands. As can be seen immediately in Fig. 1, ~20 stars define the V and K relations, while the J and H ones still have smaller numbers of stars. A number of systems still lack magnitude difference measurements in those two bands.

Figure 2 presents the relation between stellar mass and the V-K colour index. This relation probably has too large an intrinsic dispersions to be generally useful, and is provided here mostly for illustration, and as a warning to potential users of similar relations.

Figure 1 shows the piecewise-linear relations adjusted by Henry & McCarthy (1993) to the J, H and K band data then available to them, and their piecewise-quadratic relation for the V band, with its Henry et al. (1994) V band update for the lower masses. These relations provide a reasonable description of the new data, but they do show significant discrepancies, in particular around their break-points. Clearly the quality of the new masses warrants the
use of higher order polynomials. We have found that the following fourth degree polynomials provide good descriptions of the data in Figs. 1 and 2:

\[ \log(M/M_\odot) = 10^{-3} \times [0.3 + 1.87 \times M_V + 7.6140 \times M_V^2 - 1.6980 \times M_V^3 + 0.606958 \times M_V^4] \quad \text{for} \quad M_V \in [9, 17] \]

\[ \log(M/M_\odot) = 10^{-3} \times [1.6 + 6.01 \times M_J + 14.888 \times M_J^2 - 5.3557 \times M_J^3 + 2.8518 \times M_J^4] \quad \text{for} \quad M_J \in [5, 11] \]

\[ \log(M/M_\odot) = 10^{-3} \times [1.4 + 4.76 \times M_H + 10.641 \times M_H^2 - 5.0320 \times M_H^3 + 0.28396 \times M_H^4] \quad \text{for} \quad M_H \in [5, 10] \]

\[ \log(M/M_\odot) = 10^{-3} \times [1.8 + 6.12 \times M_K + 13.205 \times M_K^2 - 6.2315 \times M_K^3 + 0.37529 \times M_K^4] \quad \text{for} \quad M_K \in [4.5, 9.5] \]

\[ \log(M/M_\odot) = 10^{-3} \times [7.4 + 17.61 \times (V - K) + 33.216 \times (V - K)^2 + 34.222 \times (V - K)^3 - 27.1986 \times (V - K)^4 + 4.94647 \times (V - K)^5 - 0.27454 \times (V - K)^6] \quad \text{for} \quad V - K \in [4, 7] \]

One striking characteristic of Fig. 1 is the very different scatters in its V and K diagrams. The V plot displays considerable dispersion around a mean relation, and some of its best measurements are also some of the most discrepant. The K plot on the other hand shows a one to one relation between Mass and Luminosity, and its (mild) outliers are systems with larger than average errorbars. The J and H plots also have little dispersion, to the extent that this can be assessed from their smaller number of measurements. The V band scatter is much larger than the measurement errors, which on average are actually somewhat smaller for V than for K. This different behaviour of the visual and infrared bands, first seen so clearly here, is predicted by all theoretical models, as discussed for instance in the recent review by Chabrier & Baraffe (2000).

It results from the metallicity dispersion of the solar neighbourhood populations, through the interplay of two physical mechanisms:

- a larger metallicity increases the atmospheric opacity in the visible range, which is dominated by TiO and VO molecular bands. For a given bolometric luminosity it therefore shifts the flux distribution towards the infrared;
- a larger metallicity decreases the bolometric luminosity for a given mass.

In the visible bands both effects work together, to decrease the visible luminosity of the more metal-rich stars at a given mass. In the near-infrared on the other hand, the redward shift of the flux distribution of the metal-rich stars counteracts their lower bolometric luminosity. The models therefore predict that infrared absolute magnitudes are largely insensitive to metallicity, and our empirical M/L relations confirm this.

At visible wavelengths on the other hand, metallicity determinations now become a crucial limiting factor in accurate comparisons with stellar models, as has long been the case for more massive stars (e.g. Andersen 1991). Quantitative metallicity measurements of M dwarfs are unfortunately difficult in the optical range (e.g. Valenti et

| Name     | $\Delta V$ Ref | $\Delta R$ Ref | $\Delta J$ Ref | $\Delta K$ Ref | $\Delta H$ Ref |
|----------|----------------|----------------|----------------|----------------|----------------|
| Gl 65AB  | 0.45 ± 0.08 Hen99 | 0.38 ± 0.03 Hen93 | 0.30 ± 0.02 Hen93 | 0.40 ± 0.07 Hen93 |
| Gl 234AB | 3.08 ± 0.05 Hen99 | 1.79 ± 0.30 C94  | 1.63 ± 0.11 C94  | 1.62 ± 0.02 D00 |
| YY Gem   | 0.51 ± 0.20 L78  | 0.34 ± 0.19 L78  | 0.32 ± 0.27 L78  | 1.8 ± 0.37 L78  |
| Gl 2069A | 0.57 ± 0.10 D99a | 1.19 ± 0.05 Tor99 | 0.20 ± 0.07 Tor99 | 0.44 ± 0.09 D00 |
| Gl 473AB | 0.01 ± 0.05 Hen99 | 0.13 ± 0.04 Tor99 | 0.20 ± 0.07 Tor99 | 0.44 ± 0.09 Tor99 |
| Gl 570BC | 1.66 ± 0.16 S    | 1.19 ± 0.05 Tor99 | 1.15 ± 0.05 Tor99 | 1.18 ± 0.03 Tor99 |
| Gl 623AB | 5.28 ± 0.10 B96  | 3.28 ± 0.3 Hen93  | 2.65 ± 0.03 Hen93 | 2.87 ± 0.14 Hen93 |
| CM Dra   | 0.14 ± 0.03 S    | 0.15 ± 0.01 L77  | 0.48 ± 0.06 D00  | 0.46 ± 0.03 D00 |
| Gl 644A-Bab | 0.08 ± 0.05 S  | -0.48 ± 0.06 D00 | -0.48 ± 0.06 D00 | -0.48 ± 0.06 D00 |
| Gl 644A-Bb | 0.49 ± 0.10 S   | 0.41 ± 0.01 Hen93 | 0.46 ± 0.02 Hen93 | 0.42 ± 0.07 Hen93 |
| Gl 661AB | 0.05 ± 0.05 TYC  | 1.51 ± 0.04 Hen93 | 0.74 ± 0.03 Hen93 | 0.10 ± 0.03 D00 |
| Gl 702AB | 1.86 ± 0.02 TYC  | 1.19 ± 0.10 Hen93 | 1.26 ± 0.03 D00  | 1.28 ± 0.02 D00 |
| Gl 747AB | 0.22 ± 0.05 S    | 1.14 ± 0.05 Hen93 | 1.37 ± 0.08 Hen93 | 1.37 ± 0.08 Hen93 |
| Gl 791.2AB | 3.27 ± 0.10 Ben99   | 0.52 ± 0.03 D00  | 0.54 ± 0.03 D00  | 0.54 ± 0.03 D00 |
| Gl 831AB | 2.10 ± 0.06 Hen99 | 1.97 ± 0.10 HIP   | 1.26 ± 0.03 D00  | 1.28 ± 0.02 D00 |
| Gl 860AB | 1.70 ± 0.09 HIP   | 1.14 ± 0.05 Hen93 | 1.37 ± 0.08 Hen93 | 1.37 ± 0.08 Hen93 |
| Gl 866AC | 0.40 ± 0.10 Hen99 | 0.52 ± 0.03 D00  | 0.54 ± 0.03 D00  | 0.54 ± 0.03 D00 |
| Gl 869AC | 2.04 ± 0.40 S    | 0.52 ± 0.03 D00  | 0.54 ± 0.03 D00  | 0.54 ± 0.03 D00 |

Table 2. Magnitude differences for M-dwarf systems with accurate masses. Reference codes are: TYC for the Tycho catalogue HIP for the Hipparcos catalogue L77 for Lacy (1977), L78 for Leung & Schneider (1978), Hen93 for Henry & McCarthy (1993), C94 for Coppenbarger et al. (1994), B96 for Barbieri et al. (1996), Ben00 for Benedict et al. (2000), Tor99 for Torres et al. (1999), Hen99 for Henry et al. (1999), D99a for Delfosse et al. (1999a), For99 for Forveille et al. (1999), and D00 for the present paper. S stands for spectroscopic magnitude differences, inferred from the relative line depths for double-lined spectroscopic binaries.
Fig. 1. V, J, H and K band M/L relations. The circles are data from Henry & McCarthy (1993), Torres et al. (1999), Henry et al. (1999), Benedict et al. (2000) and Metcalfe et al. (1996). The triangles represent our recent measurements (Ségransan et al. 2000; Ségransan et al., in prep.; and Forveille et al. 1999). The masses and luminosities used in this figure are also listed in Table 3. The two curves represent the piecewise linear relation of Henry & McCarthy (1993; dotted line) and our polynomial fit (solid line).

4. Comparison to theoretical models

Figure 3 compares the empirical M/L data with the corresponding 5 Gyr theoretical isochrones (for which even the lowest-mass stars on the plot have settled on the main sequence) of Baraffe et al. (1998, hereafter BCAH) and Siess et al. (2000, hereafter SDF). For very low mass stars these two sets of models are up to now the only ones to use realistic model atmospheres as outer boundary conditions to the stellar interior equations. This has been found necessary for accurate results in this mass range (Chabrier et al. 1996). BCAH use atmospheres from Hauschildt et al. (1999), while SDF use the older Plez (1992) models. Besides this, and the different input physics that they use, the two sets of models differ by the technique used to compute observational quantities. SDF use the empirical bolometric correction tables of Kenyon & Hartmann (1995) to deduce absolute magnitudes in the various photometric bands from their theoretical bolometric luminosities and...
Table 3. Masses and absolute magnitudes for the M dwarfs used in the M/L relation. The mass references are Met96 for Metcalfe et al. (1996), Hen99 for Henry et al. (1999), Hen93 for Henry & McCarthy (1993), Mar98 for Martin et al. (1998), Tor99 for Torres et al. (1999), Ben00 for Benedict et al. (2000), Seg00a for Ségransan et al. (2000), Seg00b for Ségransan et al. (in prep.) and For99 for Forveille et al. (1999). When relevant we have modified the masses from Henry & McCarthy (1993) and Henry et al. (1999) to reflect a more accurate parallax in Table 1 than was available to these authors. The individual absolute magnitudes are determined from the system magnitudes and parallaxes listed in Table 1, with the magnitude differences of Table 2.

effective temperatures. BCAH on the other hand adopt a purely ab initio approach, and compute the absolute magnitudes from the stellar radii, the model atmosphere spectra, and the transmission profile of the photometric filters.

At the scale of Fig. 3 the BCAH and SDF models are nearly indistinguishable for the infrared bands, and both produce an impressive agreement there with the observational data. This is particularity striking for the K band, where many measurements define the M/L relation. The same is apparently true for J and H, where more data would nonetheless be welcome to confirm this behaviour.

In the V band (Fig. 3) on the other hand, neither of the two sets of models reproduces the observations perfectly. The BCAH models and the observations agree well above $\sim 0.5 \, M_\odot$, though with significant dispersion, but they diverge somewhat for lower masses. Below $0.2 \, M_\odot$, the solar metallicity models are systematically too luminous by $\sim 0.5$ magnitude. As most of the bolometric flux of such stars emerges in the near-IR bands, where the agreement is excellent, the models necessarily provide a good account of the relation between mass and bolometric magnitude. Their overall description of the stars is therefore most likely correct, and the V band discrepancy probably points to a relatively localized problem in the models. The two leading explanations for this discrepancy (Baraffe & Chabrier, private communication) are either a V band opacity source that would be missing in the atmospheric models, or some low level problem in the physical description.
Fig. 3. Comparison between V, J, H and K band M/L observational relation and theoretical ones. The three curves are 5 Gyr theoretical isochrones from Baraffe et al. [1998] for two metallicities and our polynomial fit. The asterisks represent 5 Gyr solar metallicity models from Siess et al. [2000].

Fig. 2. Mass-colour (V-K) relation for M dwarfs. The three curves are 5 Gyr theoretical isochrones from Baraffe et al. [1998] for two metallicities and our polynomial fit. The Siess et al. [2000] model are represented for 5 Gyr and solar metallicity with asterisks.

...tion of the shallower atmospheric levels which emit the visible flux.

The SDF models by contrast are ~0.5 mag too luminous in the V band above 0.3M⊙, produce an excellent agreement with the observations for 0.2M⊙, and look sub-luminous for 0.1M⊙. Here again, the excellent near-IR agreement with the observations indicates that these models nicely reproduce the relation between mass and bolometric magnitude, and the discrepancy rests in the V band bolometric correction. Indeed, the SDF models mostly target PMS stars and the Kenyon & Hartmann [1995] bolometric correction that they use applies for T Tauri stars, which have lower gravity than main sequence stars and hence somewhat different colours. The use of observational bolometric correction, which by definition are unaffected by missing opacity sources, on the other hand most likely explains why the SDF models agree better with the V band observations at ~0.2M⊙.

The characteristics of very low mass stars are frequently derived from photometry in the redder CCD bands, R and especially I, where these objects are brighter...
than in the V band. The validity of the theoretical M/L relation for these red bands is thus of significant interest, but too few VLMSs with accurate masses have known luminosities in the R, I, or z filters to provide a fully empirical verification. One can note however that Baraffe et al. (1995) observe that below $T_{\text{eff}} \sim 3700$ K the BCAH model $V - I$ and $V - R$ colours are too blue by 0.5 mag at a given luminosity, while colours which don’t involve the V band are much better predicted. Since the BCAH models of Baraffe et al. (1998) and Siess et al. (2000), empirical masses of 0.2 to 10% accuracy validate the near-IR Mass/Luminosity relations predicted by the recent stellar models of Baraffe et al. (1998) and Siess et al. (2000), down to $\sim 0.1 \, M_\odot$. They also point out some low level ($\sim 0.5$ mag) deficiencies of these models in the V band. Perhaps more importantly however, the V band M/L diagram represents direct evidence for an intrinsic dispersion around the mean M/L relation. This had previously remained hidden in the measurement noise, but there is, as theoreticians have kept telling us, no such thing as one single M/L relation for all M dwarfs. This is particularly true for the visible bands, while the dispersion in the near-IR JHK bands is much lower. Comparisons between measured masses and theoretical models will therefore increasingly depend on metallicity measurements for individual systems, which are not easily obtained.

The $\sim 0.5$ magnitude discrepancy between observational and model masses derived from visible photometry has some consequences for mass functions determination. As mass cannot be determined for volume-limited samples, the mass function is always obtained from a luminosity function, by writing that

$$\frac{dN}{dM} = \frac{dN}{dL} \times \frac{dL}{dM}$$

and the slope of the M/L relation therefore plays a central role in its derivation. Below $0.5 M_\odot$, the $dL/dM$ slope of the empirical M/L relation is steeper than that of the BCAH models and shallower than for the SDF ones, by 10 to 20%. Their use will therefore respectively underestimate and overestimate the number of lower mass stars by this amount. Probably more seriously, the large dispersion around the V band M/L relation will introduce large Malmquist-like biases in the derived mass function, which would need an excellent characterization of this dispersion to be corrected. The infrared relations have both better agreement with the observations and much lower dispersion. We strongly recommend that they be used rather than the V band relations, whenever possible.

Acknowledgements. We thank the technical staffs and telescope operators of both OHP and CFHT for their support during the long-term observations which have led to these results. We are grateful to Gilles Chabrier, Isabelle Baraffe, France Allard and Maria Rosa Zapatero Osorio for many useful discussions. We are also indebted to Lionel Siess for providing his models prior to publication.

References

Allard F., 1998, in “Brown dwarfs and extrasolar planets”, Eds. Rebolo R., Martín E.L., Zapatero Osorio M. R., ASP Conference Series 134, 370
Allard F., Hauschildt P.H., Alexander D.R., Starrfield S., 1997, ARA&A 35, 137
Alonso A., Arribas S., Martínez-Roger C., 1994, A&A 307, 365
Andersen J., 1991, A&AR 3, 91
Andersen J., 1998, p. 99 in Fundamental Stellar Properties: The Interaction between Observations and Theory, IAU Colloquium 189, T. R. Bedding et al., Eds., Kluwer: Dorrecht
Baraffe I., Chabrier G., Allard F., Hauschildt P.H., 1995, ApJ 446, L35
Baraffe I., Chabrier G., Allard F., Hauschildt P.H., 1998, A&A 337, 403
Barbieri C., DeMarchi G., Nota A., et al., 1996, A&A 315, 418
Barnes T.G., Evans D.S., Moffet T.J. 1978, MNRAS 183, 285
Bessel M.S., 1990, A&AS 83, 357
Benedict G.F., McArthur B., Chappell D.W., et al., 1999, AJ 118, 1086
Benedict G.F., McArthur B., Franzi O.G., Wasserman L.H., Henry T.J., 2000, AJ in press (August 2000 issue)
Chabrier G., Baraffe I., 1997, A&A 327, 1039
Chabrier G., Baraffe I., 2000, ARA&A in press; available on astro-ph/0006383
Chabrier G., Baraffe I., Plez B., 1996, ApJ 459, 91
Chabrier G., Baraffe I., Allard F., Hauschildt P.H., 2000, ApJ, in press
Coppenbarger D.S., Henry T.J., Mc Carthy Jr D.W., 1994, AJ 107, 1551
Delfosse X., Forveille T., Mayor M., Burnet M., Perrier C., 1999a, A&A 341, L63
Delfosse X., Forveille T., Udry S., et al., 1999b, A&A 350, L39
Delfosse X., Forveille T., Beuzit J.-L., et al., 1999c, A&A 344, 897
Eggen O., 1968, ApJS 16, 49
ESA, 1997, The HIPPARCOS Catalogue, ESA SP-1200
Forveille T., Beuzit J.-L., Delfosse X., et al., 1999, A&A 351, 619
Franz O.G., Henry T.J., Wasserman L.H., et al., 1998, AJ 116, 1432
Goldman B., Delfosse X., Forveille T., et al., 1999, A&A 351, L5
Glass I.S., 1975, MNRAS 171, 19P
Hauschildt P.H., Allard F., Baron E., 1999, ApJ 512, 377
Hawley S.L., Gizis J.E., Reid I.N., 1997, AJ 113, 1458
Henry T.J., McCarthy Jr D.W., 1993, ApJ 350, 334
Henry T.J., Franz O.G., Wasserman L.H., et al., 1999, ApJ 512, 864
Johnson H.J., 1965, ApJ 141, 170
Jones H. R. A., Tsuji T., 1997, ApJ 480, L39
Kenyon S.J., Hartmann L., 1995, ApJS 101, 117
Kron G.E., Gascoigne S.C.B., White H.S., 1957, AJ 62, 205
Lacy C.H. 1977, ApJ 218, 444
Leggett S.K., 1992, ApJS 82, 351
Leung K.C., Schneider D.P., 1978, AJ 94, 712
Martin C., Mignard F., Hartkopf W.I., McAlister H.A., 1998, A&AS 133, 149
Metcalfe T.S., Mathieu R.D., Latham D.W., Torres G., 1996, ApJ 456, 356
Plez B., 1992, A&AS 94, 527
Reid I.N., Hawley S.L., Gizis J.E., 1995, AJ 110, 1838
Ségransan D., Delfosse X., Forveille T., et al., 2000 (paper I), A&A in press
Siess L., Dufour E., Forestini E., 2000, A&A 358, 593
Soderhjelm S., 1999, A&A 341, 121
Torres G., Henry, T.J., Franz O.G., Wasserman L.H., 1999, AJ 117, 562
Valenti J. A., Piskunov N., Johns-Krull C. M., 1998, ApJ 498, 851
Van Altena W.F., Lee J.T., Hoffleit D., 1995, The General Catalogue of Trigonometric Stellar Parallaxes, Fourth edition, Yale University Observatory
Veeder G.J. 1974, AJ 79, 1056
Weiss E.W., 1991, AJ 101, 1882