Dynamic additional loading of the frame of a multi-story building after the failure of one of the structures

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Abstract. A calculated model of a building frame fragment is presented for calculating the dynamic additional loading of structural elements in the zone of possible local destruction after the failure of one of the structures. To model such a fragment, a substructure isolated from a structural system of the building frame was used in the form of a two-span statically indeterminate continuous beam under various boundary conditions on the extreme supports, loaded with a specified design load. The sudden removal of the middle supporting column of the substructure is modelled by reducing to zero for some short-time the internal force in this column, obtained when calculating the entire frame of the building. The use of level calculated schemes and analytical description of the movement of reinforced concrete substructure with variable stiffness due to cracking for determining the dynamic additional loading of frame elements of a multi-story building after the failure of one of the structures allows to define the parameters of the dynamic response of the substructure and simulate in more detail the zones of possible local destruction of the building under the considered special action. The developed algorithm for calculating physically and structurally nonlinear reinforced concrete systems can be used for the calculated analysis of the frame of multi-story buildings when designing their protection against progressive collapse.

1. Introduction

In scientific publications [1–4] and normative documents on the normalization of the protection of buildings and structures against progressive collapse [5–8], for calculated analysis with a special action, such a concept is used as the “zone of possible local destruction” of the structural system, outlined by a fragment of the building’s frame in places of possible failure of one of the supporting structures. The selection of such most stressed zones under the considered actions aims not only at ensuring engineering visibility, but also a more rigorous assessment of the stress-strain state of structures falling into such a zone from the position of criteria for a special limit state. The available experience of such analysis, judging by the publications, for buildings [9–12] or constructions [13] shows that such a calculation, even with the use of computer technologies and effective software systems, requires a lot of computational time and highly qualified specialists.

In this regard, the purpose of the present work was the development of a methodology and an effective algorithm for calculating the parameters of dynamic additional loading of the frame of a multi-story building after the failure of one of the supporting structures. To achieve this purpose, the
following tasks are formulated: to perform modeling of the zone of local destruction of the reinforced concrete frame of a multi-story building using level calculated models; to build a calculated model of the substructure to determine the dynamic additional loadings of the considered structural system and the failure time of the removed structure.

2. Methods

The reinforced concrete frame of a multi-story building, loaded with operational loads and emergency action in the form of the sudden removal of one of the columns of the building frame, is considered. The substructure in the zone of possible local destruction with a hypothetically removed column of the first floor is isolated in the building frames (Figure 1a, b).

As a first approximation, the zone of possible local destruction is modelled by a substructure in the form of a two-span reinforced concrete continuous beam loaded with operational load \( P_i \) (Figure 1c).

The sudden removal of the middle supporting column of the considered substructure is modelled by a decrease in the internal force \( R(t) \) in this column from \( P_0 \) to zero for some short time \( t_r \) (Figure 2 - scheme A). Where \( P_0 \) - the force in the removed column calculated when calculating the entire frame according to the primary scheme of the first level, simulating the entire frame of the building.

The force \( R(t) \) as a function of time is represented by the following expression:

\[
R(t) = \begin{cases} 
P_0 \left[ 1 - \left( \frac{t}{t_r} \right)^n \right] & \text{if } 0 < t \leq t_r; \\
0 & \text{if } t > t_r,
\end{cases}
\]  \hspace{1cm} (1)

where \( t_r \) - column failure time, depending on the nature of the special action [14–17], \( n \) - a positive exponent describing the force change function \( R(t) \) (Figure 2 - scheme A).
Figure 2. Schemes of dynamic additional loading of a substructure when the middle column is suddenly removed.

The force $R(t)$ in time can be represented as the sum of two-component forces: the force $P_0$ causing the initial static stress-strain state of the substructure and the force $-P(t)$ causing the movement of the system (Figure 2 - scheme B, C):

$$R(t) = P_0 - P(t),$$  \hspace{1cm} (2)

where $P(t)$ - the force applied in the opposite direction which causes the movement of the system:

$$P(t) = \begin{cases} P_0 \left(\frac{t}{t_r}\right)^n & \text{if } 0 < t \leq t_r; \\ P_0 & \text{if } t > t_r. \end{cases}$$  \hspace{1cm} (3)

Thus, the static-dynamic deformation of the considered substructure can be represented by the sum of two stages. Stage 1: static loading of the substructure to a given operational load, at which the internal force in the removed column reaches a value $P_0$ (Figure 2 - scheme B) and stage 2: dynamic additional loading with a force causing the movement of the system (Figure 2 - scheme C).

Based on the graphs $R(t)$ in Figure 2, it can be seen that the value $n=1$ can be considered as the average value for estimating the column removal rate. Then, assuming $n=1$, the dynamic additional loading of the elements of the considered substructure has two phases: the phase of removal of the column at $0 < t \leq t_r$, and the phase of stable oscillation of the substructure with the removed column at $t > t_r$. The force $P(t)$ causing the movement of the system is determined by the expression:

$$P(t) = \begin{cases} P_0 \left(\frac{t}{t_r}\right) & \text{if } 0 < t \leq t_r; \\ P_0 & \text{if } t > t_r. \end{cases}$$  \hspace{1cm} (4)

In the general case, with various structural solutions of beam-column connections in the building frame, the calculated scheme of the substructure in the form of a reinforced concrete beam having a
concentrated mass \( m \) in the middle of the span, depending on the boundary conditions, can be represented by three options (Figure 3).

![Figure 3](image)

**Figure 3.** The calculated scheme of the substructure under different boundary conditions: a – hinge support, b – elastic restraint, c – rigid restraint.

In the given calculated scheme, the influence of the upper floors of the building frame is modeled by a spring with rigidity \( K_3 \).

To calculate the dynamic parameters of the substructure, it is necessary to find its total stiffness, which will be determined by the load causing a unit displacement of the substructure in the direction of the spring \( K_3 \). The total stiffness of the considered reinforced concrete substructure depends on the boundary conditions, as well as the level of loading and the presence of cracks in it. In the case of rigid restraint, the total stiffness of the considered substructure, taking into account cracks, is calculated using the formula:

\[
K = \frac{192 \cdot B}{l^3} + K_3,  
\]

where \( B \) - the rigidity of the cross-section of a reinforced concrete beam, taking into account cracks at the first stage of static-dynamic loading, is determined from the expressions [18,19]:

\[
B = \varphi_1 E_s A_s h_0^2,  
\]

\[
\varphi_1 = \frac{z}{h_0} + 1.25 \frac{\psi_b \cdot \mu \cdot \alpha}{\nu (\varphi_f + \xi)}.  
\]

In formulas (6) - (7), the traditional designations of the reinforced concrete theory are accepted [18,19]:

- \( z \) - distance from the center of gravity of the cross-sectional area of the reinforcement to the point of application of the resultant force in the compressed zone of the section above the crack (an arm of the inner pair of forces);
- \( \psi_b \) - coefficient taking into account the uneven distribution of deformations of the extreme compressed concrete fiber along the length of the cracked area;
- \( \mu \) - percentage of reinforcement;
- \( \alpha \) - the ratio of elastic modulus of reinforcement and concrete;
- \( \nu \) - coefficient characterizing the elastoplastic state of concrete in a compressed zone;
- \( \varphi_f \) - coefficient taking into account the presence of compressed overhangs;
- \( \xi \) - the relative height of the compressed zone of concrete.

Equation of movement of the substructure (Figure 3) with one degree of freedom is written as [20–22]:

\[
m \cdot \ddot{u}(t) + c \cdot \dot{u}(t) + K \cdot u(t) = P(t),  
\]

or
\[
\ddot{u}(t) + 2\zeta \omega_n \dot{u}(t) + \omega_n^2 u(t) = \frac{P(t)}{m},
\]

where: \( u(t), \dot{u}(t), \ddot{u}(t) \) accordingly the displacement, speed, and acceleration of the point of application of the concentrated mass; \( \omega_n = \sqrt{\frac{K}{m}} \) - natural frequency of the system without taking into account energy dissipation; \( \zeta = \frac{c}{2m\omega_n} \) - damping coefficient.

Equation (9) is solved using the Duhamel integral:

\[
u(t) = \frac{1}{m\omega_D} \int_0^t P(\tau)e^{-\zeta \omega_D (t-\tau)} \sin \omega_D (t-\tau) d\tau,
\]

where: \( \omega_D = \omega_n \sqrt{1 - \zeta^2} \) - natural frequency of the system taking into account energy dissipation.

Calculating the integral (10) after the corresponding trigonometric transformations, we obtain:

\[
u(t) = \begin{cases} 
\left( u_n \right)_0 \left[ \frac{t}{\tau} - \frac{1 - 2\zeta^2}{\tau, \omega_D} e^{-\zeta \omega_D \sin \omega_D t} - \frac{2\zeta}{\tau, \omega_D} \left( 1 - e^{-\zeta \omega_D \cos \omega_D t} \right) \right] & \text{if } 0 \leq t \leq \tau, \\
\left( u_n \right)_0 \left[ 1 + \frac{2\zeta}{\tau, \omega_D} \left( e^{-\zeta \omega_D \cos \omega_D t} - e^{-\zeta \omega_D (t-\tau)} \sin \omega_D (t-\tau) \right) - \frac{1 - 2\zeta^2}{\tau, \omega_D} \left( e^{-\zeta \omega_D \sin \omega_D t} - e^{-\zeta \omega_D (t-\tau)} \sin \omega_D (t-\tau) \right) \right] & \text{if } t > \tau,
\end{cases}
\]

where \( \left( u_n \right)_0 = \frac{P_0}{m\omega_n^2} \) - static deflection of the substructure.

The general formula (11) allows taking into account the simultaneous influence of the column failure time and energy dissipation during oscillations. If energy dissipation is neglected, then formula (11) is simplified:

\[
u(t) = \begin{cases} 
\left( u_n \right)_0 \left[ \frac{t}{\tau} - \frac{\sin \omega_D t}{\omega_D \tau} \right] & \text{if } 0 \leq t \leq \tau, \\
\left( u_n \right)_0 \left[ 1 - \frac{1}{\omega_D \tau} \left( \sin \omega_D t - \sin \omega_D (t-\tau) \right) \right] & \text{if } t > \tau.
\end{cases}
\]

3. Results and discussion

Equation (11) can be represented as graphs in the "displacement-time" coordinates describing the effect of the column failure time \( \tau \) on the dynamic response of the substructure (Figure 4).

Analysis of the graphs allows to note the following:

Under the influence of high-speed dynamic additional loading by the load \( P(t) \), the substructure with one removal column oscillates relative to the position of a certain axis, which is the axis of static displacement \( n-1 \) times the statically indeterminate substructure formed after the failure of the column.

Column failure time significantly affects the dynamic response of the substructure. When the \( \tau \) value approaches 0, the oscillation amplitude increases to the maximum value. When the value of the column failure time increases, the oscillation amplitude of the substructure decreases.
Analyzing dependence (12), it is not difficult to see that for \( t_c \leq 0.1T \), the system can be considered under the influence of impact loading. In this case, the dynamic additional loading coefficient \( \theta_d \) calculated as the ratio of the maximum dynamic and static deflections will be \( \theta_d = \frac{u_{\text{max}}(t)}{(u_u)_0} \approx 2 \).

At \( t_c \geq 3T \), it can be considered that the supporting column collapses slowly as under static loading. In this case, the dynamic additional loading coefficient will be equal to \( \theta_d = \frac{u_{\text{max}}(t)}{(u_u)_0} \approx 1 \).

![Graphs of the dynamic response of the substructure at different values of the column failure time and without taking into account energy dissipation: a – when \( t_c = 0 \); b – when \( t_c = 0.6T \); c – when \( t_c = 1.7T \); 1, 2 – dynamic and static column removal respectively.](image)

**Figure 4.** Graphs of the dynamic response of the substructure at different values of the column failure time and without taking into account energy dissipation: a – when \( t_c = 0 \); b – when \( t_c = 0.6T \); c – when \( t_c = 1.7T \); 1, 2 – dynamic and static column removal respectively.

The influence of the overlying floors on the response of the substructure in the considered calculated scheme (Figure 1c) is modeled by a spring with a stiffness \( K_s \). Changing the \( K_s \) value will change the frequency and amplitude of the substructure oscillations (Table 1).

**Table 1.** The influence of overlying floors on the dynamic response of a substructure when it is rigidly fixed to supports.

| Overlying floor stiffness \( K_i \) | Total stiffness of the substructure \( K \) | Oscillation frequency \( \omega_n \) | Oscillation period \( T \) | Dynamic additional loading coefficient \( \theta_d \) |
|----------------------------------|---------------------------------|-----------------|-------------------|-------------------|
| 0 \( \frac{192 \cdot B}{l^3} \)  | \( \frac{192 \cdot B}{l^3} \) | \( \sqrt{1.5} \cdot \omega_n \) | \( \frac{T}{\sqrt{1.5}} \) | \( \frac{\theta_d}{1.5} \) |
| 0.5 \( \frac{192 \cdot B}{l^3} \) | \( 1.5 \cdot \frac{192 \cdot B}{l^3} \) | \( \sqrt{1.5} \cdot \omega_n \) | \( \frac{T}{\sqrt{1.5}} \) | \( \frac{\theta_d}{1.5} \) |
| 1 \( \frac{192 \cdot B}{l^3} \)  | \( 2 \cdot \frac{192 \cdot B}{l^3} \) | \( \sqrt{2} \cdot \omega_n \) | \( \frac{T}{\sqrt{2}} \) | \( \frac{\theta_d}{2} \) |

From the analysis of table 1, it can be seen that when \( K_i \) increases from 0 to a value equal to \( \frac{192 \cdot B}{l^3} \), the total stiffness of the substructure increases. This significantly reduces the period of oscillations of the substructure and the dynamic additional loading coefficient \( \theta_d \). It follows that increasing the stiffness of the floors above the removal column, for example, by installing a beam with an increased cross-section height above the first floor is one of the most effective ways to increase the stability of a multistory building to progressive collapse. In high rise buildings, this is facilitated by the
arrangement of outrigger floors which significantly increase the survivability of buildings under special actions.

From the analysis of the considered calculated scheme of the substructure (Figure 3), it follows that changing the boundary conditions will change the total stiffness $K$ of the beam system. Table 2 shows the results of evaluating the influence of boundary conditions on the parameters of the dynamic response of the substructure at a certain fixed value of stiffness $K = 48 \cdot B / l^3 = \text{const.}$

**Table 2.** To analyze the influence of boundary conditions on the dynamic response of the substructure.

| Boundary condition | Total stiffness of the substructure $K$ | Oscillation frequency $\omega_n$ | Oscillation period $T$ | Dynamic additional loading coefficient $\theta_d$ |
|--------------------|----------------------------------------|---------------------------------|------------------------|-----------------------------------------------|
|                     | $\frac{192 \cdot B}{l^3} + K_3$        | $\omega_n$                      | $T$                    | $\theta_d$                                   |
|                     | $\frac{48 \cdot B}{l^3} + K_3$         | $\sqrt{2.5} \omega_n$          | $\frac{T}{\sqrt{2.5}}$ | $\frac{\theta_d}{2.5}$                       |

From the analysis of the values shown in table 2, it can be seen that the total stiffness of reinforced concrete beams with rigid restraint is four times greater than that of beams with hinge support, and accordingly this reduces the dynamic additional loading of the considered substructure by 2.5 times. In the construction of monolithic reinforced concrete frames, closer to the real boundary condition is the hinge with elastic restraint of the support (Figure 3b), for which the coefficient is in the range $(1...1 / 2.5)$.

4. Conclusions

The proposed variant of the calculation of dynamic additional loading of the reinforced concrete frame of a multistory building after the failure of one of the supporting structures using the decomposition method, level calculated schemes, and constructed analytical dependencies allows to determine the dynamic response parameters in the elements of the frame and to model in more detail the stress state in the zones of possible local destruction of the building under the considered action.

Numerical studies have established that the failure time of the structure significantly affects the coefficient of dynamic additional loading coefficient, calculated as the ratio of the maximum dynamic and static deflections. If the value of this time is less than 0.1 of the natural oscillation period, the substructure can be considered under the action of impact loading. The dynamic additional loading of the frame elements of a multi-story building largely depends on the rigidity, taking into account the influence of the floors above the substructure, and the conditions for fixing it on supports.

The constructed calculation algorithm and the obtained analytical dependencies, taking into account the failure time of the removed structure for dynamic additional loading of the building frame, can be used for the calculated dynamic analysis of structural systems of multi-story buildings when designing their protection from progressive collapse.

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References

[1] Pham A T, Lim N S and Tan K H 2017 Investigations of tensile membrane action in beam-slab systems under progressive collapse subject to different loading configurations and boundary
conditions Eng. Struct. 150 520–36
[2] Ren P, Li Y, Lu X, Guan H and Zhou Y 2016 Experimental investigation of progressive collapse resistance of one-way reinforced concrete beam–slab substrata under a middle-column-removal scenario Eng. Struct. 118 28–40
[3] Fedorova N V, Korenkov P A and Vu N T 2018 Experimental method of research of deformation of monolithic reinforced concrete building under accidental actions Building and reconstruction 4 42–52
[4] Fedorova N V and Ngoc V T 2019 Deformation and failure of monolithic reinforced concrete frames under special actions J. Phys.: Conf. Ser. 1425 012033
[5] SP 385.1325800.2018 Protection of buildings and structures against progressive collapse. Design code. Basic statements (Moscow: STANDARTINFORM)
[6] SP 296.1325800.2017 Buildings and structures. Accidental actions (Moscow: STANDARTINFORM)
[7] GSA 2013 General services administration alternate path analysis and design guidelines for progressive collapse resistance (Washington D.C.: U.S. General Services Administration)
[8] UFC 2016 Unified facilities Criteria: Design of Buildings to Resist Progressive Collapse, UFC 4-023-03 (Washington D.C.: U.S. Department of Defence)
[9] Kabantsev O and Mitrovic B 2018 Deformation and power characteristics monolithic reinforced concrete bearing systems in the mode of progressive collapse MATEC Web Conf. 251 02047
[10] Tamrazyan A G, Fedorov V S and Kharun M 2019 The effect of increased deformability of columns on the resistance to progressive collapse of buildings IOP Conf. Ser.: Mater. Sci. Eng. 675 012004
[11] Zenin S A, Kudinov O V, Shapiro G I and Gasanov A A 2016 Methods of Calculating of Large-Panel Buildings: How to Prevent Progressing Collapse Acad. Archit. Constr. 4 109-13
[12] Kolchunov V I and Androsova N B 2019 Nonequilibrium processes of power and enviromental resistance of reinforced concrete structures to progressive collapse J. Phys.: Conf. Ser. 1425 012053
[13] Belostotsky A M, Akimov P A, Aul A A, Dmitriev D S, Dyadchenko Y N, Nagibovich A I, Ostrovskiy K I and Pavlov A S 2018 Analysis of Mechanical Safety of Stadiums for the World Cup 2018 Acad. Archit. Constr. 3 118–29
[14] Tsai M H 2018 An Approximate Analytical Formulation for the Rise-Time Effect on Dynamic Structural Response under Column Loss Int. J. Struct. Stab. Dyn. 18 1850038
[15] Khuyen H T and Iwasaki E 2016 An approximate method of dynamic amplification factor for alternate load path in redundancy and progressive collapse linear static analysis for steel truss bridges Case Stud. Struct. Eng. 6 53–62
[16] Kokot S, Anthoine A, Negro P and Solomos G 2012 Static and dynamic analysis of a reinforced concrete flat slab frame building for progressive collapse Eng. Struct. 40 205–17
[17] Fu Q N, Tan K H, Zhou X H and Yang B 2017 Numerical simulations on three-dimensional composite structural systems against progressive collapse J. Constr. Steel Res. 135 125–36
[18] Kolcunov V I, Tuyen V N and Korenkov P A 2020 Deformation and failure of a monolithic reinforced concrete frame under accidental actions IOP Conf. Ser.: Mater. Sci. Eng. 753 032037
[19] Veryuzhskiy Yu V, Kolchunov V I, Barabash M S and Genzerskiy Yu V 2006 Komp’yuternye tekhnologii proektirovaniya zhelezobetonnykh konstruktsiy (Kiev: NAU)
[20] Tian Y and Su Y 2011 Dynamic response of reinforced concrete beams following instantaneous removal of a bearing column Int. J. Concr. Struct. Mater. 5 19–28
[21] Colomer Segura C, Hamra L, D’Antimo M, Demonceau J F and Feldmann M 2017 Determination of Loading Scenarios on Buildings Due to Column Damage Structures 12 1–12
[22] Tsai M H 2012 A performance-based design approach for retrofitting regular building frames with steel braces against sudden column loss J. Constr. Steel Res. 77 1–11