Charge-Confining Gravitational Electrovacuum Shock Wave

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Abstract

In previous publications we have extensively studied spherically symmetric solutions of gravity coupled to a non-standard type of non-linear electrodynamics containing a square root of the ordinary Maxwell Lagrangian (the latter is known to yield QCD-like confinement in flat space-time). A class of these solutions describe non-standard black holes of Reissner-Nordström-(anti-)de-Sitter type with an additional constant radial vacuum electric field, in particular, a non-asymptotically flat Reissner-Nordström-type black hole. Here we study the ultra-relativistic boost (Aichelburg-Sexl-type) limit of the latter and show that, unlike the ordinary Reissner-Nordström case, we obtain a gravitational electrovacuum shock wave as a result of the persistence of the gauge field due to the “square-root” Maxwell Lagrangian term. Next, we show that this gravitational electrovacuum shock wave confines charged test particles (both massive and massless) within a finite distance from its front.

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1. Introduction

In his analysis in Ref.[1] 't Hooft has shown that in any effective quantum
gauge theory, which is able to describe linear confinement phenomena, the
energy density of electrostatic field configurations should be a linear function
of the electric displacement field $\vec{D}$ in the infrared region. In particular, a
consistent quantum approach has been developed in [1], where the electric
displacement field appears as an “infrared counterterm”.

The simplest way to realize these ideas in flat space-time was proposed
in Refs.[2] by considering a nonlinear gauge theory with an action:

$$S = \int d^4x \, L(F^2), \quad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2},$$

(1)

$$F^2 \equiv F_{\mu\nu}F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

where $f_0$ is a positive coupling constant. It has been shown in the first
reference in [2] that the “square root” Maxwell term $\sqrt{-F^2}$ naturally arises
as a result of spontaneous breakdown of scale symmetry of the original scale-
Invariant Maxwell action. For static field configurations the model (1) yields
an energy density that indeed contains a term linear w.r.t. $|\vec{D}|$, where $\vec{D} = \vec{E} - \frac{f_0}{\sqrt{2}}\frac{\vec{E}}{|\vec{E}|}$. The model (1) produces a confining effective potential containing
a Coulomb plus a linear one, which is of the form of the well-known “Cornell”
potential [3] in the phenomenological description of quarkonium systems (see
second reference in [2]). Let us remark that one could start with the non-
Abelian version of the action (1) as well, since the non-Abelian theory can
effectively be reduced to an Abelian one in the static spherically symmetric
case as pointed out in the second reference in [2].

Coupling of the nonlinear gauge field system (1) to ordinary Einstein
gravity as well as to $f(R) = R + \alpha R^2$ gravity was recently studied in [4, 5, 6],
where the following interesting new features of the pertinent static spherically
symmetric solutions have been found:

(i) Appearance of a constant radial vacuum electric field (in addition
to the Coulomb one) in charged black holes within Reissner-Nordström--
(anti-)de-Sitter space-times, in particular, in electrically neutral black holes

1Let us also mention the recent papers [7] where coupling of ordinary Einstein gravity
to the pure “square-root” gauge field Lagrangian is discussed. One of the new interesting
features in this model is the existence of dyonic solutions.
with Schwarzschild-(anti-)de-Sitter geometry. Let us particularly stress, that constant radial electric fields do not exist as solutions of ordinary Maxwell electrodynamics.

(ii) Novel mechanism of dynamical generation of cosmological constant through the nonlinear gauge field dynamics of the “square-root” gauge field term.

(iii) Even in case of vanishing effective cosmological constant the resulting Reissner-Nordström-type black hole, apart from carrying an additional constant radial vacuum electric field, turns out to be non-asymptotically flat – a feature resembling the gravitational effect of a hedgehog [8].

(iv) New “tube-like universe” solutions of Levi-Civita-Bertotti-Robinson [9] type.

(v) Coupling the gravity/non-linear gauge field system self-consistently to lightlike branes [3] produces charge-hiding and charge-confining “thin-shell” wormhole solutions displaying QCD-like charge confinement [5]. Similar effect is obtained also via coupling to ordinary Nambu-Goto branes [11].

Coupling of the nonlinear gauge field system (1) to \( f(R) = R + \alpha R^2 \) gravity plus a dilaton [6] produces in addition to the above properties (i)-(v) also an appearance of dynamical effective gauge couplings as well as confinement-deconfinement transition effect as functions of the dilaton vacuum expectation value. Finally, non-singular wormhole solutions that do not require exotic matter have been found in a scalar curvature square plus Ricci squared model [12].

One should notice that all of the above solutions are both static and spherically symmetric. The purpose of the present note is to study time-dependent “shock wave” type solutions in the gravity/non-linear gauge field system.

Gravitational shock waves are of particular relevance (starting with the classic paper [13]) in modern field and string theory, especially due to their role in the description of impulsive lightlike signals in general relativity [14], high-energy scattering at Planck energies and ultra-relativistic heavy ion collisions (see e.g. [15] and references therein), etc.

Gravitational shock wave solutions can be obtained from static spherically symmetric ones by an ultra-relativistic boost procedure originating from the

\[ \text{For a systematic reparametrization-invariant world-volume Lagrangian formulation of lightlike branes, see Refs. [10].} \]
paper by Aichelburg and Sexl [13] (boost of ordinary Schwarzschild metric, see also [16]), which was subsequently generalized to ultra-relativistic boosts of Reissner-Nordström, Reissner-Nordström-(anti-)de-Sitter and Kerr-Newman geometries [17, 18, 19].

When performing an ultra-relativistic boost of the ordinary Reissner-Nordström metric, in order to get a well defined limit one must appropriately rescale the mass ($m \sim 1/\gamma$) and the charge ($Q^2 \sim 1/\gamma$) with the Lorentz-boost factor $\gamma = (1 - w^2)^{-\frac{1}{2}} \to \infty$ when $w \to 1$. This implies the vanishing of the electromagnetic field in the ultra-relativistic limit, although a finite $\delta$-function contribution of the latter remains in the energy-momentum tensor, so that it contributes only to the shape of the gravitational shock wave profile.

The first task in the present note is to study the ultra-relativistic boost limit of the simplest non-trivial new solution obtained by coupling gravity to the non-linear gauge field system (1), namely, the above mentioned non-asymptotically flat Reissner-Nordström type black hole carrying an additional constant radial vacuum electric field apart from the Coulomb one. Unlike ordinary Reissner-Nordström case where the electromagnetic field vanishes in the boost limit, here the resulting shock wave we obtain is a non-trivial gravitational electrovacuum one with a non-zero electric and magnetic fields originating from the constant radial vacuum electric field in the rest-frame.

The second main task is to study the dynamics of charged test particles (both massive and massless) in the newly obtained gravitational electrovacuum shock wave background. We show that the latter exhibits a QCD-like charge-confining feature.

**2. Ultra-Relativistic Limit of Non-standard Black Hole with Confining Electric Potential**

The simplest coupling to gravity of the nonlinear gauge field system with a square root of the Maxwell term [11] known to produce QCD-like confinement in flat space-time [2] is given by the action (we use units with Newton constant $G_N = 1$):

$$S = \int d^4x \sqrt{-g} \left[ \frac{R(g) - 2\Lambda_0}{16\pi} + L(F^2) \right] , \quad L(F^2) = -\frac{1}{4} F^2 - \frac{f_0}{2} \sqrt{\varepsilon} F^2 , \quad F^2 = F_{\kappa\lambda} F^{\mu\nu} g^{\kappa\mu} g^{\lambda\nu} , \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} .$$
Here \( R(g) = R_{\mu\nu} g^{\mu\nu} \) is the scalar curvature of the space-time metric \( g_{\mu\nu} \), \( R_{\mu\nu} \) is the Ricci tensor and \( g \equiv \text{det} \| g_{\mu\nu} \| \); the sign factor \( \varepsilon = \pm 1 \) in the square-root term in (2) corresponds to “magnetic” or “electric” dominance – in what follows we consider the \( \varepsilon = -1 \) case; \( f_0 \) is a positive coupling constant measuring the strength of charge confinement.

Let us stress that we do not need to introduce any bare cosmological constant \( \Lambda_0 \) in (2) since the “square-root” Maxwell term dynamically generates a non-zero effective cosmological constant \( \Lambda_{\text{eff}} = 2\pi f_0^2 \). The role of the bare \( \Lambda_0 \) is just shifting the effective \( \Lambda_{\text{eff}} \) (see Eqs. (8) below).

The equations of motion corresponding to (2) read:

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda_0 g_{\mu\nu} = 8\pi T_{\mu\nu} ,
\]

(3)

\[
T_{\mu\nu} = \left( 1 - \frac{f_0}{\sqrt{-F^2}} \right) F_{\mu\kappa} F_{\nu\lambda} g^{\kappa\lambda} - \frac{1}{4} \left( F^2 + 2 f_0 \sqrt{-F^2} \right) g_{\mu\nu} ,
\]

(4)

and

\[
\partial_\nu \left( \sqrt{-g} \left( 1 - \frac{f_0}{\sqrt{-F^2}} \right) F_{\kappa\lambda} g^{\mu\kappa} g^{\nu\lambda} \right) = 0 .
\]

(5)

In our preceding papers [4, 5] we have shown that the gravity/nonlinear-gauge-field system (2) possesses static spherically symmetric solutions with a radial electric field containing both Coulomb and constant “vacuum” pieces:

\[
F_{0r} = \frac{\varepsilon_f f_0}{\sqrt{2}} - \frac{Q}{\sqrt{4\pi r^2}} , \quad \varepsilon_f \equiv \text{sign}(F_{0r}) = -\text{sign}(Q) ,
\]

(6)

and the space-time metric:

\[
ds^2 = -A(r) dt^2 + \frac{dr^2}{A(r)} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) ,
\]

(7)

\[
A(r) = 1 - \sqrt{8\pi |Q|} f_0 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\text{eff}}}{3} r^2 , \quad \Lambda_{\text{eff}} = \Lambda_0 + 2\pi f_0^2 ,
\]

(8)

is that of a Reissner-Nordström-(anti-)de-Sitter-type black hole depending on the sign of the dynamically generated effective cosmological constant \( \Lambda_{\text{eff}} \).

- Solution (6)–(8) represents a non-standard black hole whose “vacuum” electric field persists even in the electrically neutral case \( Q = 0 \) in (6) and (8)).
• In the presence of the bare cosmological constant $\Lambda_0$ in the action (2) its only effect is shifting of the dynamically generated cosmological constant $2\pi f_0^2$ in second Eq. (8).

• Solution (6)–(8) becomes a non-standard Reissner-Nordström-type black hole with additional constant vacuum radial electric field in spite of the presence of negative bare cosmological constant $\Lambda_0 < 0$ with $|\Lambda_0| = 2\pi f_0^2$, i.e., $\Lambda_{\text{eff}} = 0$ in (5). Another noteworthy feature is its non-flat space-time asymptotics due to the “leading” constant term in the metric coefficient (5) being different from 1 when $Q \neq 0$. This resembles the effect on gravity produced by a spherically symmetric “hedgehog” configuration of a nonlinear sigma-model scalar field with $SO(3)$ symmetry (see Refs. [20]).

In what follows we will consider in more detail the latter special case – the non-asymptotically flat Reissner-Nordström-type black hole with additional constant vacuum radial electric field ($\Lambda_0 < 0$ with $|\Lambda_0| = 2\pi f_0^2$ in (2), i.e., $\Lambda_{\text{eff}} = 0$ in (5)).

To derive the ultra-relativistic boost limit of (6)–(8) we follow the approach of Aichelburg-Sexl [13] and Lousto-Sanchez [17]. The first step is to transform to isotropic-like coordinates $(t, r) \rightarrow (t', r')$:

\[ t = \frac{1}{\sqrt{c_0}} t' \sqrt{\sqrt{c_0}} , \quad c_0 \equiv 1 - \sqrt{8\pi |Q| f_0} , \]

\[ r = \sqrt{c_0} r' \sqrt{\sqrt{c_0}} \left[ 1 + \frac{m}{2c_0^{3/2} r' \sqrt{\sqrt{c_0}}} \right]^2 - \frac{Q^2}{4c_0^{3/2} r' \sqrt{\sqrt{c_0}}} , \]

\[ x' = r' \cos \varphi \sin \theta , \quad y' = r' \sin \varphi \sin \theta , \quad z' = r' \cos \theta , \]

so that (6)–(8) becomes:

\[ ds^2 = -\tilde{A}(t', r') dt'^2 + \tilde{B}(r') \left( dx'^2 + dy'^2 + dz'^2 \right) , \]

\[ \tilde{A}(t', r') = r'^2(\sqrt{c_0} - 1) A(r(r')) , \quad \tilde{B}(r') = \frac{r'^2(r')} {r'^2} , \]

\[ F = F_0 dt \wedge dr = \left[ \frac{\varepsilon f_0}{\sqrt{2}} - \frac{Q}{\sqrt{4\pi r'^2(r')}} \right] \left( \tilde{A} \tilde{B} \right)^{1/2} \frac{1}{r'} \left[ dt' \wedge (x' dx' + y' dy' + z' dz') \right] \]
The next step is to perform a Lorentz boost in e.g. $x'$-direction:

$$(t', x') \rightarrow (\tilde{t}, \tilde{x}) \ , \ t' = \gamma (\tilde{t} - w \tilde{x}) \ , \ x' = \gamma (\tilde{x} - w \tilde{t}) \ , \ \gamma \equiv (1 - w^2)^{-\frac{1}{2}} \ , \ (14)$$

and then take the ultra-relativistic limit $w \rightarrow 1$, i.e., $\gamma \rightarrow \infty$, by rescaling the black hole parameters $m = p/\gamma$, $Q^2 = q^2/\gamma$ (with $p, q$ -- finite constants) and using the distributional limits:

$$\frac{\gamma}{r' \rho} \rightarrow \delta(u) \rho^{1-s} \frac{\Gamma(\frac{s}{2}) \Gamma(\frac{s-1}{2})}{\Gamma(\frac{s}{2})} \quad \text{for} \ s > 1 \ , \ \frac{\gamma}{r' \rho} \rightarrow \frac{1}{|u|} \delta(u) \ln \rho^2 . \quad (15)$$

Here the following notations are introduced:

$$v = \tilde{t} + \tilde{x} \ , \ u = \tilde{t} - \tilde{x} \ ; \ \rho \equiv \sqrt{y_1^2 + y_2^2} \ , \ (y_i)_{i=1,2} \equiv \underline{y} \equiv (y', z') \ , \ (16)$$

$$r' = \sqrt{x'^2 + \rho^2} \approx \sqrt{\gamma^2 u^2 + \rho^2} \quad \text{for large} \ \gamma .$$

The choice of the $t$-coordinate transformation (9) (which does not have an analog in the ordinary Reissner-Nordstr"om case) has been made in such a way as to obtain a well-defined ultra-relativistic limit for the metric (11)-(12). This limit turns out to coincide with that of the ordinary Reissner-Nordstr"om metric [17]:

$$ds^2 = -dvdu - h(\rho) \delta(u) du^2 + dy_1^2 + dy_2^2 \ , \ h(\rho) = 8p \ln \rho + \frac{3\pi q^2}{2\rho} \quad (17)$$

with non-zero Christoffel symbols:

$$\Gamma^v_{uu} = h(\rho) \delta'(u) \ , \ \Gamma^v_{ui} = \partial_i h \delta(u) \ , \ \Gamma^i_{uu} = \frac{1}{2} \partial_i h \delta(u) . \quad (18)$$

As in the standard case [13, 17], in order to remove the $\frac{1}{|u|}$ term in the $g_{uu}$ metric component coming from second relation (15), one has to perform an additional coordinate transformation in the $(v, u)$ subspace: $v \rightarrow v + 4p \ln |u| \ , \ u \rightarrow u$.

Concerning the electromagnetic field, the Coulomb piece in (13) vanishes in the ultra-relativistic limit as it happens in the ordinary Reissner-Nordstr"om case [17], however now we are left with a non-trivial limit:

$$F_{vu} = \frac{\varepsilon_F f_0}{2\sqrt{2}} \text{sign}(u) \ , \ F_{ui} = y_i \frac{\varepsilon_F f_0}{\sqrt{2}} \left( \frac{1}{|u|} - \delta(u) \ln \rho^2 \right) . \quad (19)$$
The electromagnetic field potential corresponding to (19) is:

$$F_{uv} = -\partial_u A_v, \quad A_v = -\frac{\varepsilon F f_0}{2\sqrt{2}}|u|,$$  \hspace{1cm} (20)

$$F_{ui} = -\partial_i A_u, \quad A_u = \frac{\varepsilon F f_0}{2\sqrt{2}} \rho^2 \left[ -\frac{1}{|u|} + \delta(u) (\ln \rho^2 - 1) \right].$$  \hspace{1cm} (21)

In terms of ordinary definitions of electric and magnetic fields $\vec{E} = (E_x, E_y', E_z')$, $\vec{B} = (B_x, B_y', B_z')$ we have:

$$E_x = \frac{\varepsilon F f_0}{\sqrt{2}} \text{sign}(\tilde{t} - \tilde{x}) \hspace{1cm} B_x = 0,$$

$$E_y' = \frac{\varepsilon F f_0}{\sqrt{2}} y' \left[ \ln \rho^2 \delta(\tilde{t} - \tilde{x}) - \frac{1}{|\tilde{t} - \tilde{x}|} \right] = B_{z'},$$

$$E_{z'} = \frac{\varepsilon F f_0}{\sqrt{2}} z' \left[ \ln \rho^2 \delta(\tilde{t} - \tilde{x}) - \frac{1}{|\tilde{t} - \tilde{x}|} \right] = -B_{y'}.$$  \hspace{1cm} (22)

Let us stress that the ultra-relativistic boost limit of $F_{\mu\nu}$ obtained in (19) (or (22)) is a particular vacuum solution $F^2 = -f_0^2$ of the nonlinear gauge field Eqs. (5), which does not exist as a solution in ordinary Maxwell electrodynamics. According to Eq. (4) its contribution to the total energy-momentum tensor is $-\frac{1}{4} f_0^2 g_{\mu\nu}$ which exactly cancels the contribution of the “bare” cosmological constant term in Einstein Eqs. (3) we started with ($\Lambda_0 = -2\pi f_0^2$).

Thus, the ultra-relativistic boost of the non-asymptotically flat Reissner-Nordström-type black hole with an additional constant “vacuum” radial electric field (6)–(8) describes a gravitational electrovacuum shock wave given by (17)–(19).

3. Charged Test-Particle Dynamics in the Electrovacuum Gravitational Shock Wave

We now consider the motion of charged test particles in the background of the gravitational electrovacuum shock wave (17)–(21). Here we study the case of test particle charge $q_0$ opposite to the pre-boost Reissner-Nordström charge $Q$, i.e.:

$$\text{sign}(q_0) = \varepsilon_F = -\text{sign}(Q)$$  \hspace{1cm} (23)
according to (6). For definiteness we take \( \text{sign}(q_0) = \varepsilon_F = -\text{sign}(Q) = 1 \).

The pertinent point-particle action reads:

\[
S_{\text{particle}} = \int d\lambda \left[ \frac{1}{2e} g_{\mu \nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} - \frac{1}{2} em_0^2 + q_0 \frac{dx^\mu}{d\lambda} A_\mu \right] = \int d\lambda \left[ \frac{1}{2e} \left( -\frac{dv}{d\lambda} - h(\rho)\delta(u) \left( \frac{du}{d\lambda} \right)^2 + \frac{dy_i}{d\lambda} dy_i \right) - \frac{1}{2} em_0^2 + q_0 \frac{dv}{d\lambda} A_v(u) + \frac{du}{d\lambda} q_0 A_u(u, \rho) \right].
\]

(24)

Here \( \lambda \) denotes the world-line parameter, \( e \) is the world-line “einbein”, \( m_0 \) is the test-particle mass, in particular, \( m_0 \) could be zero, and \( \rho, h(\rho), A_v, A_u \) are given in (16), (17), (20), (21) respectively.

Upon introducing world-line proper-time-like parameter \( \tau \) with \( \frac{d\tau}{d\lambda} = e \), the equations of motion w.r.t. \( x^\mu \) (geodesic) and \( e \) (mass-shell constraint) read:

\[
\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda} \dot{x}^\nu \dot{x}^\lambda - q_0 F_{\lambda\nu} g^{\lambda\mu} = 0, \quad (25)
\]

\[
g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + m_0^2 = 0 \quad \rightarrow \quad -v \dot{u} - h(\rho)\delta(u) \dot{u}^2 + \dot{y}^2 + m_0^2 = 0, \quad (26)
\]

where the dots represent \( \frac{d}{d\tau} \), and \( \Gamma^\mu_{\nu\lambda} \) and \( F_{\lambda\nu} \) are given in (18)-(19).

We have two conservation laws due to invariance of (24) w.r.t. \( v \)-translation (“light-cone energy” \( \mathcal{E} \) conservation) and w.r.t. rotation in the \((y_1, y_2)\)-subspace (angular momentum \( \mathcal{J} \) conservation), which according to Noether theorem read:

\[
\mathcal{E} = \frac{1}{2} \dot{u} - q_0 A_v, \quad \mathcal{J} = y_1 \dot{y}_2 - y_2 \dot{y}_1 \equiv y \times \dot{y}, \tag{27}
\]

(here \( \times \) indicates two-dimensional vector product). The first “energy”-conservation Eq.(27) upon substituting the expression for \( A_v \) from (20) reads explicitly:

\[
\dot{u} = -a \frac{|u|}{u} + 2\mathcal{E}, \quad a \equiv \frac{q_0 f_0}{\sqrt{2}} \tag{28}
\]

---

\(^3\)Note that the sign factor \( \varepsilon_F \) in Eqs.(19)-(22) is the only legacy of the Reissner-Nordström charge \( Q \) in the boosted electromagnetic field after performing the ultrarelativistic boost limit.

\(^4\)This allows simultaneous treatment of both the massive and massless case; the genuine proper-time parameter in the massive case is \( \bar{\tau} \) with \( \frac{d\bar{\tau}}{d\lambda} = em_0 \).
The “mass-shell” constraint Eq.(26) yields a first-order equation for $v(\tau)$:

$$\dot{v} = -h(\rho) \dot{u} \delta(u) + \frac{1}{u} \left( \frac{y^2}{u} + m_0^2 \right),$$

(29)

where $y_i(\tau)$ is a solution of Eqs.(25) for $\mu = i$ (using expression (21) for $F_{ui}$):

$$\ddot{y}_i + a \frac{\dot{u}}{|u|} y_i + \delta(u) \left[ \frac{1}{2} \partial_i h(\rho) \dot{u}^2 - a \dot{u} \ln \rho \right] = 0 .$$

(30)

The form of the test particle trajectory depends significantly on the initial condition for $u(\tau)$: $u_0 = u(0)$. There are three types of initial conditions:

- (i) $|u_0| < 2\mathcal{E}/a$ – this is the case which will be treated in some detail below. In this case the particle trajectory “pierces” the shock wave, i.e., $u(\tau_0) = 0$ at some value $\tau_0$ of the proper time (world-line) parameter. We find $|u(\tau)| < 2\mathcal{E}/a$ for all $\tau$ (see Eq.(31) below) implying confinement (trapping) of the charged particle (both massive and massless) within finite distance $2\mathcal{E}/a$ from the shock wave.

- (ii) $|u_0| = 2\mathcal{E}/a$ – this initial condition by consistency applies only for massless charged test particles. Now we have $|u(\tau)| = 2\mathcal{E}/a = \text{const}$ and $y_i(\tau) = \text{const}$ for all $\tau$, $v(\tau)$ is an arbitrary linear function of $\tau$, i.e., this is free lightlike motion parallel to the shock wave at a fixed “critical” distance $2\mathcal{E}/a$ from the latter.

- (iii) $|u_0| > 2\mathcal{E}/a$ – in this case $|u(\tau)| > 2\mathcal{E}/a$ for all $\tau$ (see Appendix), i.e., the particle trajectory never “pierces” the shock wave and, since now the proper time (world-line) parameter $\tau$ flows opposite the “laboratory” time $t$, this motion in fact corresponds to a repulsion of an antiparticle (cf. Ref. [21]).

We start with case (i). With the initial condition $|u(0)| \equiv |u_0| < 2\mathcal{E}/a$ the solution of Eq.(28) reads:

$$u(\tau) = \text{sign}(\tau - \tau_0) \frac{2\mathcal{E}}{a} \left( 1 - e^{-a|\tau - \tau_0|} \right),$$

(31)

where:

$$u(\tau_0) = 0 \quad \tau_0 = \text{sign}(u_0) \frac{1}{a} \ln \left( \frac{2\mathcal{E}/a - |u_0|}{2\mathcal{E}/a} \right).$$

(32)
Eq. (31) implies:

$$|u(\tau)| = \frac{2\mathcal{E}}{a} (1 - e^{-a|\tau - \tau_0|}) \leq \frac{2\mathcal{E}}{a} \text{ for all } \tau \in (-\infty, +\infty).$$

(33)

Henceforth, for simplicity we take \(\tau_0 = 0\), i.e., \(u_0 = 0\).

Eq. (29) yields for \(v(\tau)\):

$$v(\tau) = -h(\rho_0)\theta(\tau) + \frac{1}{2\mathcal{E}} \int d\tau' e^{a|\tau'|} \left(\frac{y'^2}{2} + m_0^2\right), \quad \rho_0 = |y(0)|,$$

(34)

whereas Eqs. (30) for \(y_i\) acquire the form (taking into account (31) and using \(\delta(u) = |u|^{-1}\delta(\tau)\)):

$$\ddot{y} + \omega^2(\tau)y(\tau) + \delta(\tau) y(0) \left[\frac{\mathcal{E}}{\rho_0} \frac{dh}{d\rho}|_{\rho = \rho_0} - 2a \ln \rho_0\right] = 0, \quad \omega^2(\tau) \equiv \frac{a^2}{e^{a|\tau|} - 1}.$$

(35)

Asymptotically for large \(|\tau|\), \(y_i(\tau)\) is linear function:

$$y(\tau) \approx \alpha(\pm)\tau + \beta(\pm) \text{ for } \tau \to \pm\infty, \quad \alpha(\pm), \beta(\pm) = \text{const}.$$  

(36)

In the vicinity of \(\tau = 0\) the solution of (35) is given in terms of Bessel functions (since \(\omega^2(\tau) \approx a/|\tau|\) for \(\tau \to 0\)):

$$y(\tau) = \sqrt{|\tau|} \left[\frac{1}{\sqrt{a}} c_{(\pm)} J_1(2\sqrt{a|\tau|}) + \sqrt{a} d_{(\pm)} Y_1(2\sqrt{a|\tau|})\right],$$

(37)

where \(c_{(\pm)}, d_{(\pm)} = \text{const} and the subscripts } (\pm) correspond to \(\tau > 0 and \tau < 0\, \text{respectively.}

Using the series expansions of the Bessel functions and requiring continuity of \(y(\tau)\) at \(\tau = 0\), which implies \(y_0 \equiv y(0) = -\frac{1}{\pi} d_{(\pm)}\), we have for small \(\tau \, (\gamma_0 denotes the Euler constant):

$$y(\tau) \approx y_0 + \tau \left[\theta(\tau)c_{(\pm)} - \theta(-\tau)c_{(\pm)} - \text{sign}(\tau) 2a y_0 \left(\gamma_0 - \frac{1}{2} + \frac{1}{2} \ln a|\tau|\right)\right],$$

(38)

$$y(\tau) \approx \theta(\tau)c_{(\pm)} - \theta(-\tau)c_{(\pm)} - \text{sign}(\tau) 2a y_0 \left(\gamma_0 + \ln a|\tau|\right),$$

(39)

$$y(\tau) \approx -y_0 \frac{a}{|\tau|} + \delta(\tau) \left[c_{(\pm)} + c_{(\mp)} - 2a \left(2\gamma_0 + \ln a + \hat{c}\right)y_0\right].$$

(40)
where \( \hat{c} \) is a regularization ("cut-off") dependent constant in the renormalization of the singular distribution product

\[ "\ln |\tau| \delta(\tau)" = \hat{c} \delta(\tau) \]

Substituting (38)–(40) back into (35) we obtain (recall \( \rho_0 = |y_0| \)):

\[
\xi^+ + \xi^- = y_0 \left[ 2a \left( 2\gamma_0 + \ln a + \hat{c} \right) - \left( \frac{\xi}{\rho_0} \frac{dh}{d\rho} \bigg|_{\rho=\rho_0} - 2a \ln \rho_0 \right) \right],
\]

whereas substituting (36) and (38)–(39) into the expression for \( J \) (27) yields:

\[
J = \beta_{\pm} \times \alpha_{\pm} = \pm y_0 \times \xi_{\pm}.
\]

The compatibility of the last equalities in (42) follows from (41).

In ordinary boosted Schwarzschild case \([16]\) the transverse space coordinates \( y(\tau) \) experience a refraction (finite discontinuity of \( y(\tau) \)) at the shock wave (i.e., at \( \tau = 0 \)). In the present case \( y(\tau) \) experience reflection when meeting the shock wave due to the logarithmic singularity at \( \tau = 0 \) of \( y(\tau) \) according to Eq.(39):

\[
y(\tau) \approx -a y_0 \text{sign}(\tau) \ln |\tau| \to \pm \infty \text{ for } \tau \to \pm 0.
\]

Since the logarithmic singularity (43) is an integrable one, inserting (39) into Eq.(34) we find the same type of finite discontinuity in \( v(\tau) \) at the shock wave as in the ordinary boosted Schwarzschild or Reissner-Nordström case \([16, 17] \):

\[
v(\tau) \approx v_0 - h(\rho_0) \theta(\tau) + \frac{a^2 \rho_0^2}{2\xi} (\ln |\tau|)^2 \text{ for small } \tau, \text{ } v_0 = \text{const},
\]

\[ [v]_0 \equiv \lim_{\epsilon \to 0} (v(\epsilon) - v(-\epsilon)) = -h(\rho_0). \]

Using Eq.(36) implies for large \( \tau \):

\[
v(\tau) \approx \begin{cases} 
\frac{1}{2a} \left( m_0^2 + \beta^2 \right) e^{a\tau} \to +\infty, & \text{for } \tau \to +\infty \\
-\frac{1}{2a} \left( m_0^2 + \beta^2 \right) e^{a|\tau|} \to -\infty, & \text{for } \tau \to -\infty
\end{cases}
\]

This is done along the lines of the configuration space renormalization of ultraviolet-divergent Feynman diagrams in standard quantum field theory employing the original ideas of Stueckelberg-Peterman-Bogoliubov. They formulate renormalization of products of distributions with coinciding singularities by first defining them (upon appropriate regularization) on the configuration space with the singularity subset removed, and then extending the result to the whole space in the spirit of the general theory of distributions (for recent developments see Ref.[22] and references therein).
Taking into account Eqs. (31), (33), (36), (38), (44), (46) describing the test particle trajectory, we conclude that the gravitational electrovacuum shock wave (17)–(19) confines charged particles with charges $q_0$ opposite to the pre-boost Reissner-Nordström charge $Q$. Namely, according to (33) $|u(\tau)| \leq \frac{2\mathcal{E}}{a}$ for all $\tau \in (-\infty, +\infty)$, and thus we deduce the size of the confining distance to be equal to $2\mathcal{E}/a \equiv 2\sqrt{2}\mathcal{E}/q_0f_0$ for both massive and massless charged test particles.

4. Conclusion

In the present note we studied the result of applying the Aichelburg-Sexl ultra-relativistic boost procedure to the static spherically symmetric solutions of a non-standard Reissner-Nordström type obtained from gravity coupled to a special kind of nonlinear gauge field system containing a “square-root” Maxwell term (2), which is known to produce charge confinement in flat space-time.

Unlike the case of the ultra-relativistic boost of ordinary Reissner-Nordström black hole solution [17], where the electromagnetic field disappears in the boost limit leaving only a residual term in the energy-momentum tensor, in the present case due to the “confinement”-inducing “square-root” Maxwell term the electromagnetic field persists in the ultra-relativistic limit as a non-trivial “vacuum” solution (22) of the nonlinear electromagnetic field equations of motion – a solution, which does not exist in ordinary Maxwell electrodynamics. The resulting “boosted” solution describes a gravitational electrovacuum shock wave which possesses the remarkable property of confining (trapping) charged test particles (both massive and massless) with charges opposite to the pre-boost-limit black hole charge.

The above obtained gravitational electrovacuum shock wave plus the trapped charged particles could give us a picture of a hadron in the ultra-relativistic limit and one interesting generalization of these calculations could be solving for a test Dirac particle instead of for a spinless point particle in the background of such electrovac-gravity shock wave.

Another possible extension of our results would be the study of collisions of two electrovac-gravity shock waves.

Finally, a very interesting issue that deserves further attention concerns the possible processes of pair creation and of pair annihilation in the presence of the gravitational electrovacuum shock wave solution.
Appendix A.

Unlike confinement of charged particles (massive and massless ones) when initially they are close enough to the shock wave ($|u_0| < 2\mathcal{E}/a$ implying $|u(\tau)| < 2\mathcal{E}/a$ for all $\tau$), the initial condition $|u_0| > 2\mathcal{E}/a$ produces the following solution of Eq.(28) (here we take for definiteness positive $u_0$, the trajectory for negative $u_0$ is obtained by changing $(u,v) \to (-u,-v)$):

$$u(\tau) = \frac{2\mathcal{E}}{a} + \left(u_0 - \frac{2\mathcal{E}}{a}\right)e^{-a\tau}, \quad (A.1)$$

$$v(\tau) > \frac{2\mathcal{E}}{a} \text{ for all } \tau, \quad u(\tau) \to \begin{cases} +\infty, \tau \to -\infty \\ \frac{2\mathcal{E}}{a}, \tau \to +\infty \end{cases} \quad (A.2)$$

In particular, the trajectory never comes close to the shock wave at $u = 0$, i.e., no delta-function terms appear in (35) and (29), therefore, there are no “refraction” of $y(\tau)$ and no discontinuity in $v(\tau)$ at $\tau = 0$. The large $|\tau|$-asymptotics of $v(\tau)$ is:

$$v(\tau) \to \begin{cases} +\infty \text{ when } \mathcal{J} \neq 0 \\ v_0 = \text{const} \text{ when } \mathcal{J} = 0 \text{ for } \tau \to -\infty \end{cases}, \quad (A.3)$$

$$v(\tau) \to -\infty \text{ for } \tau \to +\infty \quad (A.4)$$

where $\mathcal{J}$ indicates the conserved angular momentum. The last relations show that the proper time (world-line parameter) $\tau$ flows opposite to the “laboratory” time $t$, therefore, according to [21] we have to interpret the resulting trajectory given by (A.1)-(A.4) as a motion of an anti-particle which is completely repelled by the shock wave.

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