Experimental and theoretical uncertainties in top-quark mass measurements at the Tevatron and the LHC

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Abstract.
Since the discovery of the top-quark at the Tevatron collider in 1995, enormous efforts have been devoted to the precise determination of its properties and in particular of its mass, $m_{\text{top}}$. Together with precision electroweak measurements, $m_{\text{top}}$ can be used to check the Higgs boson mass ($m_H$) dependencies on $m_{\text{top}}$, and the W boson mass ($m_W$) within the Standard Model, and to constrain the allowed mass range for heavy particles predicted by Standard Model extensions. In this paper a review of the current experimental and theoretical uncertainties affecting the top-quark mass measurements at the Tevatron and the LHC is provided: strategies and techniques adopted to evaluate the main systematic uncertainties within the various experiments are summarised and compared, highlighting, when appropriate, complementarities and differences which are important ingredients to set the basis of a future world $m_{\text{top}}$ combination.

1. Introduction
The top-quark mass is a fundamental parameter of the Standard Model of particle physics (SM). Together with precision electroweak measurements, $m_{\text{top}}$ can be used to check the Higgs boson mass ($m_H$) dependencies on $m_{\text{top}}$, and the W boson mass ($m_W$), and to constrain the allowed mass range for heavy particles predicted by extensions of the SM. A large effort has been undertaken to the precise $m_{\text{top}}$ determination both at the Tevatron and the LHC colliders. A summary of the most recent results can be found in [1]. As overview, the time evolution of the Tevatron and the LHC measurements is displayed in Figures 1 and 2, respectively. In the figures, the horizontal bands represent the current $m_{\text{top}}$ combination values: $m_{\text{top}}^{\text{Tevatron}} = 173.2 \pm 0.6\text{(stat)} \pm 0.8\text{(syst)}$ GeV [2], $m_{\text{top}}^{\text{LHC}} = 173.3 \pm 0.5\text{(stat)} \pm 1.3\text{(syst)}$ GeV [3]. Except for the very early Tevatron measurements systematic uncertainties are dominating; these can be grouped into three main categories: theoretical, experimental and background related uncertainties. They will be briefly described in the following sections. The discussion will be based on information from direct $m_{\text{top}}$ measurements included either in the present Tevatron and LHC combinations, or presented for the first time at the TOP2012 conference [1, 2, 3].

The LHC combined value is based on a partial set of measurements from the 2011 datasets, and it is expected to be updated in the near future by including the latest results. Measurements included in the present combination are highlighted by the horizontal grey band in Figure 2.
Table 1. Summary of theoretical, experimental and background related uncertainties on \(m_{\text{top}}\) in different experiments at the Tevatron and the LHC colliders [1, 2, 3]. Typical uncertainty values and ranges are given in GeV. See text for details. (*) denotes cases in which the hadronisation systematics is included in the JES uncertainty, while (**) denotes cases in which the UE systematics is included in the hadronisation systematics.

| Uncertainty source                          | CDF | DØ | ATLAS | CMS |
|--------------------------------------------|-----|----|-------|-----|
| Parton distribution functions (PDF)        | 0.1 | 0.2| 0.1 – 0.6 | 0.1 – 0.5 |
| MC generator choice                        | 0.1 – 0.7 | 0.3 – 0.6 | 0.3 – 1.3 | < 0.1 – 0.4 |
| Hadronisation model                        | 0.2 – 0.3 | 0.6 | 0.2 – 0.9 | (*) |
| Underlying event (UE)                      | (**) | (**) | 0.2 – 0.6 | 0.2 – 1.4 |
| Colour reconnection (CR)                   | 0.2 – 0.5 | 0.3 – 0.5 | 0.6 – 1.2 | 0.1 – 0.5 |
| QCD radiation (ISR/FSR)                    | 0.1 – 2.7 | 0.3 – 0.4 | 0.5 – 2.5 | 0.3 – 1.2 |
| Pileup                                      | 0.1 – 0.2 | 0.1 – 1.2 | 0.05 – 0.7 | 0.1 – 1.0 |
| \(b\)-tagging and \(b\)-jet modelling      | 0.2 – 2.5 | 0.1 – 0.2 | 0.3 – 0.5 | 0.1 – 0.5 |
| Jet resolution and reconstruction          | –   | 0.2 – 0.3 | 0.1 – 0.9 | 0.1 – 0.5 |
| \(E_{\text{T}}^\text{miss}\) modelling     | –   | –   | 0.1 | 0.1 – 0.4 |
| in-situ jet energy scale stat. component   | 0.5 – 1.5 | 0.5 – 0.6 | 0.4 | 0.4 |
| Light-jet energy scale (JES)               | 0.5 – 4.4 | 0.6 – 3.4 | 0.7 – 2.1 | 0.2 – 2.1 |
| \(b\)-jet energy scale (bJES)             | 0.1 – 0.8 | 0.3 – 0.7 | 1.4 – 2.5 | 0.5 – 1.1 |
| Background from MC                         | < 0.01 – 1.7 | < 0.01 – 1.0 | 0.1 – 1.8 | 0.1 – 0.2 |
| Background from data                       | 0.1 – 0.6 | 0.2 | 0.6 – 1.9 | 0.2 – 0.4 |
| Statistics                                 | 0.5 – 10.3 | 0.8 – 12.3 | 0.6 – 4.0 | 0.3 – 4.6 |
| Total systematics                           | 1.0 – 5.7 | 1.2 – 3.9 | 2.3 – 4.6 | 1.0 – 4.6 |
| Total                                      | 1.1 – 11.5 | 1.5 – 12.8 | 2.4 – 6.3 | 1.1 – 6.5 |

The typical \(m_{\text{top}}\) uncertainty values and ranges, across the various analyses and for each of the sources listed below are reported in Table 1.

2. Theoretical uncertainties

Theory based uncertainties are related to the simulation of \(t\bar{t}\) signal events, to the event modelling and the description of the hard scattering environment. Choices to be made in the signal simulation are the (anti) proton distribution functions (PDF), the Monte Carlo generator (MC) and the hadronisation model. On the event modelling side, important ingredients are related to the description of the underlying event (UE), via MC tunes, and the settings adopted for the
modelling of colour reconnection (CR), extra QCD radiation and the description of additional interactions accompanying the hard scatter (pileup). A detailed description of each of these aspects can be found in [4].

2.1. Parton distribution functions, PDF
Uncertainties on the partonic composition of the (anti) proton impact the top-pair production cross section and the $t\bar{t}$ event kinematics. These effects are evaluated by varying PDF eigenvectors within their uncertainty and comparing the $m_{\text{top}}$ results obtained from different PDF sets [5]. At the Tevatron, the PDF uncertainties are evaluated by comparing $m_{\text{top}}$ results obtained using MRST98L and CTEQ6M sets (CTEQ6M only for D∅). At LHC, CMS investigates $m_{\text{top}}$ changes using MSTW08, CTEQ6.6 and NNPDF2.0: $m_{\text{top}}$ variations changing CTEQ6.6 eigenvectors are considered in ATLAS. In all cases, the generated events are reweighted to reflect the change with respect to the default PDF used in the simulation. Uncertainties on $m_{\text{top}}$ in the most recent analyses are sub-dominant, nevertheless, in the near future, PDF fits including additional LHC data will further help in reducing this uncertainty contribution.

2.2. MC generator choice
Various simulation programs [6] are available for the generation of $t\bar{t}$ (and background) events. They differ both in technical implementation aspects and in the perturbation order at which calculations are performed. Typical choices are the leading order (LO) PYTHIA and HERWIG programs, the LO multi-leg programs ALPGEN and MADGRAPH, for the simulation of the $t\bar{t}$ event in association with extra jets, and the next-to-leading order (NLO) programs MC@NLO and POWHEG. Typically, only a subset of the available MC programs are considered per experiment. At the Tevatron, differences between PYTHIA and MC@NLO, ALPGEN and MC@NLO are quoted by CDF and D∅, respectively. At the LHC, ATLAS quotes differences between MC@NLO and POWHEG, while CMS investigates $m_{\text{top}}$ changes within MADGRAPH, POWHEG, and ALPGEN (documented for di-lepton analyses only).

2.3. Hadronisation models
Hadronisation models (cluster or string fragmentation as implemented in HERWIG and PYTHIA, respectively) describe the transition from final state partons to colourless hadrons. The change in $m_{\text{top}}$ obtained by exchanging cluster and string models in a fixed MC setup is generally quoted as hadronisation uncertainty. This source of uncertainty is typically also considered as one of the components of the Jet Energy Scale uncertainty (JES) and as a contribution to the uncertainty stemming from the MC modelling. Sizeable double counting effects between these two sources are anticipated, but no quantitative information is available to date. For the time being, experiments choose either to quote (this is the case for CDF, D∅, and ATLAS) or to exclude (CMS) the hadronisation systematics related to the $t\bar{t}$ modelling in the MC. Given the relative large size of the uncertainty (Table 1), a harmonisation of the treatment of this systematics and an in-depth investigation of the level of double counting effects involved when considering both components is important for the next generation of measurements and $m_{\text{top}}$ combinations.

2.4. Underlying event, colour reconnection and MC tunes
Additional low $p_T$ processes can contribute to the total observed activity in hard hadron collisions. This is modelled using different settings for the description of interactions between proton remnants (underlying event) [7]. The cross-talk between the hard interaction and the beam remnants in different colour systems (colour reconnection) can change the event kinematics and introduce biases in the determination of $m_{\text{top}}$. These effects are in general evaluated by
changing Pythia tunes, and using tunes with and without colour reconnection effects included [8]. With the large \( t\bar{t} \) samples collected especially at the LHC, in the near future MC tunes will be improved and CR effects and their modelling can be further constrained using data.

2.5. Radiation systematics

Increased or decreased levels of initial- and final-state QCD radiation (ISR/FSR) accompanying the \( t\bar{t} \) system modify the event kinematics and topology, and alter the performances of the \( t\bar{t} \) reconstruction algorithms. Radiation levels can be constrained using \( t\bar{t} \) events directly, for example, by studying the jet multiplicity in the \( l+\)jets, or di-lepton channels. However, in general, these measurements suffer from large systematic uncertainties due to detector effects and from the uncertainty in the luminosity determination. As a consequence, alternative methods to constrain the parameters controlling ISR/FSR have been derived. At the Tevatron, Drell-Yan events (\( q\bar{q} \rightarrow \gamma^* / Z \rightarrow l^+ l^- \)) are used. These processes have the same initial state as most of \( t\bar{t} \) events (\( q\bar{q} \rightarrow t\bar{t} \)), but no final-state radiation. The mean \( p_T \) of the produced di-lepton pairs is sensitive to the initial-state radiation (ISR) settings. The values of \( \Lambda_{QCD} \) and \( Q^2 \) in Pythia that bracket the data define the ISR variations [9], and the same parameter ranges are used to vary the final-state radiation settings when evaluating the \( m_{top} \) systematics. At the LHC [10], as an alternative to the study of the jet multiplicity in \( t\bar{t} \) events, jet-veto analyses are exploited. In the \( t\bar{t} \) di-lepton channel, the fraction of events which do not contain an extra jet above a certain \( p_T \) threshold is compared for data and MC to constrain the modelling of quark and gluon radiation. The veto criterion can be extended to probe jet activity beyond the leading additional jet, adding sensitivity to all hard emissions accompanying the system. Variation of ISR/FSR parameters within Pythia, constrained by data, are used to evaluate \( m_{top} \) systematics due to the radiation effects at the Tevatron and by the ATLAS experiment. In CMS, where \( t\bar{t} \) events are generated using multi-leg MC, samples with varied \( Q^2 \) and parton-jet matching scales are used to address this systematics. Investigations from [3] and [10] indicate that the two approaches describe to a large extent the same physics effect. Larger datasets are anticipated to further constrain ISR/FSR effects and help reducing their impact on the determination of \( m_{top} \).

2.6. Pileup

Depending on the instantaneous luminosity, additional interactions (pileup) accompanying the hard process occur during the same bunch crossing. On average about 2, 9, and 20 interactions per bunch crossing are registered at the Tevatron, and at the LHC during 2011 and 2012 running conditions, respectively. To simulate the impact of this, additional low-\( p_T \) events from MC (or data for the D\( \Phi \) experiment) are overlaid to the \( t\bar{t} \) process. These events contribute to building up the total amount of energy and cause colour exchanges between the remnants, thereby increasing the number of particles produced in the hadronisation. They affect the JES determination and its related uncertainty, the \( m_{top} \) resolution and the event kinematics. In all cases, a reweighting procedure in the MC simulation is applied to match the luminosity profile observed in data. The uncertainty due to the pileup modelling is obtained from the uncertainties in the reweighting procedure. Except for the early LHC measurements, the corresponding \( m_{top} \) uncertainty is sub-dominant. A separate JES uncertainty contribution is associated to jet-calibration methods implementing pileup energy suppression techniques.

3. Experimental uncertainties

Experimental uncertainties stem from the modelling of the physics objects used in the analysis for the \( t\bar{t} \) reconstruction and from the description of the detector response. These are related to

\(^2\) An additional jet besides the two \( b \)-jets from the top-quark decay.
the identification, reconstruction and calibration of leptons, jets, and missing transverse energy, $E_T^{\text{miss}}$, in the events.

3.1. $b$-tagging, jet energy resolution and reconstruction efficiency

The identification of $b$-jets from the top-quark decays ($b$-tagging) is an important handle for the physics background reduction in $m_{\text{top}}$ analyses. Algorithms, based on $B$-hadron properties, are used and calibrated to account for detector effects and possible mis-modelling of the jet, tracking and event variables in the MC with respect to the data. Uncertainties in these calibrations are propagated to the analyses in form of $p_T$ and $\eta$ dependent scale factors (the statistical component of the scale factor uncertainty, in some cases dominating, will decrease as new data is collected and analysed). Moreover, differences in the jet energy resolution between data and MC, which translate into broader or narrower invariant mass distributions, are accounted for in the analyses by applying smearing procedures to the simulated MC events. Finally, the jet reconstruction efficiency measured in the data, relative to track-based jets, is propagated in the simulation and thus to the analyses.

3.2. $E_T^{\text{miss}}$ modelling, and lepton energy and momentum scales

The $E_T^{\text{miss}}$ absorbs the scale and resolution uncertainty of all physics objects like $e$, $\gamma$, $\mu$, $\tau$ and jets. In addition, there are soft-terms including un-clustered energy or energy not associated to physics objects. The uncertainty in these components, typically of $O(10\%)$, is propagated to the analyses. Uncertainties affecting the lepton momentum and energy scales constitute in general a sub-dominant contribution to the $m_{\text{top}}$ uncertainty, and vary between $0.1$ GeV and $0.2 - 0.3$ GeV for $t\bar{t} \rightarrow l+\text{jets}$ and di-lepton analyses, respectively.

3.3. Light-quark jet energy scale

The Jet Energy Scale (JES) accounts for the transition from particle-level jets (or partons in the CDF case) to reconstructed jets. Its determination is highly non-trivial and involves both MC and in-situ techniques (i.e. $\gamma/Z+\text{jet}$, di-jet balance). The corresponding uncertainty is derived from in-situ measurements along with systematic variations of the MC simulation, and typically it is at the level of a few percent, varying with jet properties ($p_T, \eta$). Uncertainties on the JES can easily introduce $O(\text{few GeV})$ uncertainties on $m_{\text{top}}$. Most of the recent $m_{\text{top}}$ measurements in the $l+\text{jets}$, $E_T^{\text{miss}}+\text{jets}$, and all-hadronic channels have developed techniques to constrain in-situ the light-quark jet energy scale using information from the $W \rightarrow jj$ decays from top-quarks, exploiting a simultaneous fit to the data to determined $m_{\text{top}}$ along with a global jet energy scale factor. This procedure allows for a large reduction of the JES related systematics on the $m_{\text{top}}$ determination, at the price of a relatively small additional contribution to the statistical uncertainty from the increased dimensionality of the fit to the data. Although the method generally is very effective, at the level of precision characteristic of the latest measurements (and JES determinations) it can become less powerful due to its potential sensitivity to additional relative differences in the light- and $b$-jet energy response introduced by variations of the event modelling setting like UE, and CR (see CMS all-hadronic analysis in [1]).

3.4. $b$-jet energy scale

The detector response to heavy-flavour jets can differ to that of light-quark or gluon originated jets due to $b/c$ semileptonic decays, and to differences in the shower and particle content of $b$-jets with respect to the inclusive jets. Complementary strategies are in place for different experiments to reduce and evaluate the impact of the $b$-jet energy scale uncertainty (bJES) on the measurements. CDF evaluates the $b$-to-light-quark energy scale difference by applying single particle responses measured in data or predicted by MC to samples of jets in $t\bar{t}$ events.
In the latest measurements, DØ applies flavour dependent jet energy corrections to the MC to reduce the data/MC response difference and thus minimise the uncertainty. At the LHC, in ATLAS the bJES uncertainty receives contributions from uncertainties in the calorimeter single particle response, the b-jet fragmentation, and the limited knowledge of the detector material. In CMS, the maximum PYTHIA-Herwig jet energy response differences, for pure quark flavour at low $p_T$ and for pure gluon flavour at high $p_T$, are used as a flavour uncertainty applicable to any jet flavour mixture. Detector effects are considered to be negligible due to the extensive use of tracking to reconstruct individual particles in the jets (Particle-Flow algorithm) [3]. The bJES uncertainty is among the largest uncertainty sources in current LHC $m_{top}$ measurements. Improved analysis techniques aimed at constraining in-situ the bJES, or exploiting flavour dependent jet energy corrections are being investigated.

4. Background related uncertainties

Uncertainties on the background normalisation and shape introduce an uncertainty on $m_{top}$. The effects are analysis, kinematical selection, and decay channel dependent. The related uncertainties are classified into two main categories, based on their origin: MC-based or data-driven. The latter category apply for example to the data-driven estimate of the contribution of the fake leptons to the signal and to the normalisation of the $W$+jets background component.

5. Conclusions

The $m_{top}$ uncertainty is currently dominated by the systematic contributions stemming from the modelling of signal events, and the knowledge of the jet energy scale for light- and b-quark originated jets. Experiments at the Tevatron and at the LHC have been and are doing a great job in reducing the $m_{top}$ uncertainty related to the physics objects and the detector description. As an example, the development of in-situ techniques to constrain the light-jets energy scale allowed the mitigation of the traditional dominant JES uncertainty on $m_{top}$. Further improvements are expected for the next generation of measurements investigating techniques for the reduction of the flavour dependent JES uncertainty. As outline for the near future, and to reach and consolidate the sub-GeV $m_{top}$ precision, experimental efforts need to be complemented by a consistent reduction of the signal modelling uncertainties, in close collaboration with the theory community, and by extensive studies aimed at reducing source- by-source possible double counting effects.

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