Baryogenesis at the Electroweak Phase Transition for a SUSY Model with a Gauge Singlet

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Abstract

SUSY models with a gauge singlet easily allow for a strongly first order electroweak phase transition (EWPT). We discuss the wall profile, in particular transitional CP violation during the EWPT. We calculate CP violating source terms for the charginos in the WKB approximation and solve the relevant transport equations to obtain the generated baryon asymmetry.

1 Introduction

The ingredients of electroweak baryogenesis, a first order phase transition, CP violation and baryon number violation can be used to work out theoretically a large asymmetry in a very concrete way. They can be tested in experiments at the electroweak scale and in lattice simulations. The standard model does not provide a phase transition with the present bounds on the Higgs mass and it also does not contain strong enough CP violation. This is different in supersymmetric variants of electroweak models. In the MSSM there is a (rather small ) corner left - with the lightest Higgs mass above 100 GeV and stop mass slightly below m_{top} - there one can produce sizable baryon asymmetry. In NMSSM type supersymmetric models with an additional singlet there is much more parameter space for successful baryogenesis.

2 The model

How one should go “beyond” is a completely open question. Supersymmetric models are very promising but (still) not checked by experiments. We discuss a SUSY model which contains besides the fields of the MSSM a gauge singlet with the superpotential (“NMSSM”)

\[ W = \mu H_1 H_2 + \lambda S H_1 H_2 + \frac{k}{3} S^3 + r S \]  

and (universal) soft SUSY breaking terms

\[ \mathcal{L}_{\text{soft}} = \lambda A_\lambda S \tilde{H}_1 H_2 + \frac{k}{3} A_k S^3 + Y_{e} A_{e} \tilde{e}^c \tilde{H}_1 + Y_{d} A_{d} \tilde{d}^c \tilde{q} H_1 + Y_{u} A_{u} \tilde{u}^c \tilde{q} H_2 + \text{h.c.} \]  

Our final parameters are

\[ \tan \beta, x, \lambda, k, M_0, A_0, m_0^2, \]

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where \( \tan \beta \), \( x = < S > \), \( \lambda \) and \( k \) are fixed at the electroweak scale, and \( M_0 \), \( A_0 \) and \( m_0^2 \) at the GUT scale. The renormalisation group procedure is indicated in fig. 1(a).

Already at the tree level there are terms in the potential of \( \phi^3 \)-type,

\[
(\lambda \mu^2 S + h.c.)(|H_1^0|^2 + |H_2^0|^2) + (\lambda A_3 S H_1^0 H_2^0 + \frac{k}{3} A_3 S^3 + h.c.).
\] (3)

Adding the usual 1-loop temperature dependent terms we can discuss minima of the thermal potential and given the more general form of the NMSSM (1) we can find a bright range of parameters where \( < S > \sim < H_{1,2} > \), and where the effective \( \phi^3 \)-term is large enough to produce a strongly first order phase transition. In fig. 1(b) we show an example of a scan in the \( M_0-A_0 \) plane, where a strongly first order phase transition happens for Higgs masses up to 115 GeV.

### 3 CP-violating bubble walls

We solve the equations of motion for the Higgs and singlet fields

\[
H_{1,2}^0 = \tilde{h}_{1,2} e^{i \theta_{1,2}}, \quad \theta = \theta_1 + \theta_2, \quad h = \sqrt{|H_1|^2 + |H_2|^2}
\] (4)

\[
S = n + i c, \quad s = |S|
\] (5)

to obtain the profile of the bubble wall. (See also the contribution of P. John to these proceedings). CP violation leading to non-vanishing \( \theta \) and \( c \) can be induced explicitly in the parameters of the Higgs potential, or spontaneously.

In the NMSSM there is the possibility of CP violation which is only present during the phase transition (transitional CP violation). It provides large CP violation for baryon number production, without generating large electric dipole moments for the electron and neutron. Fig. 2 shows two examples of bubble wall shapes in the NMSSM for parameter sets given in ref. 4.

### 4 WKB approximation and dispersion relations

For thick bubble walls, \( L_w \gg 1/T \) the dispersion relations of particles moving in the background of the Higgs profile can be reliably calculated in the WKB approximation. In the NMSSM we find \( 3/T < L_w < 20 \). To order \( h \) the dispersion relations of charginos and stops contain CP violating terms. These enter as source terms in the Boltzmann equations for the (particle-antiparticle)
Figure 2: (a): Example of a trajectory in the $h$-$s$ plane (solid line) and the straight connection between the symmetric and the broken minimum (dashed line). (b): Transitionally CP-violating bubble wall profile for some parameter set. $x$ is the position variable. (All units in GeV.)

chemical potentials and fuel the creation of a baryon asymmetry through the weak sphalerons in the hot phase.

In the NMSSM (like in the MSSM) the dominant source for baryogenesis comes from the charginos with the mass matrix

$$\mathcal{L} = \cdots + (i \tilde{W}^-, \tilde{h}_1^-) \begin{pmatrix} M_2 & g_2(H_2^0)^* \\ g_2(H_1^0)^* & \mu + \lambda S \end{pmatrix} (i \tilde{W}^+)$$  \hspace{1cm} (6)$$

Diagonalizing this mass matrix via $M = VM_DU^\dagger$, and solving the chargino Dirac equation in the WKB approximation, we find two CP violating contributions for the dispersion relations of chargino particles and antiparticles

$$E = (\tilde{p}^2 + m^2)^{1/2} \pm \Delta E$$

$$\Delta E = \text{sgn}(p_z) \frac{\theta' + \delta' \sin^2(b) - \gamma' \sin^2(a)}{2\sqrt{\tilde{p}^2 + m_2^2}} \left( \sqrt{\tilde{p}_z^2 + m_2^2} - |p_z| \right) \frac{p_z \gamma' \sin^2(a)}{\sqrt{\tilde{p}^2 + m_2^2}}$$  \hspace{1cm} (7)$$

with

$$\tan(2\alpha) = \frac{2g_2(|M_2H_2^0|^2 + H_1^0(\mu + \lambda S)|}{|M_2|^2 + g_2^2|H_1^0|^2 - g_2^2|H_2^0|^2 - |\mu + \lambda S|^2},$$  \hspace{1cm} (8)$$

$$\gamma = \arg \left( (M_2H_2^0)^* + H_1^0(\mu + \lambda S) \right),$$  \hspace{1cm} (9)$$

and $\tan(2\beta)$ and $\delta$ obtained by exchange of $H_1$ and $H_2$ being the parameter of the diagonalization. As pointed out very recently in ref\cite{9} one should better use the kinetic momentum $\tilde{p}_{\text{kin}} = m \partial E/\partial p$ instead of the canonical momentum in the semiclassical limit leading to the Boltzmann equations. We then obtain a dispersion relation like (7) but with the last term omitted and the factor accompanying the ($\theta'$...) bracket changed to $m^2/2(\tilde{p}_{\text{kin}}^2 + m^2)$. $\Delta E$ is now totally symmetric under the exchange of $H_{1,2}$. This would destroy the most prominent term proportional to $(H_1^0H_2 - H_2^0H_1)$ in the older work on the MSSM.

5 Diffusion Equations and Application to the NMSSM

We treat the Boltzmann equations for the transport of quasi-classical particles with dispersion relations discussed above,

$$d_tf_i = (\partial_t + \vec{x} \cdot \partial_x + \vec{p} \cdot \partial_p) f_i = \mathcal{C}_i[f],$$  \hspace{1cm} (10)$$

in the fluid approximation

$$f_i(\vec{x}, \vec{p}, t) = \frac{1}{e^{\beta(E_i - v_0p_z - \mu_i) \pm 1}}.$$  \hspace{1cm} (11)$$
looking for a stationary solution, where \( \bar{z} = z - v_w t \), expanding in the perturbations and in \( v_w \), averaging over \( p_z \) with \( p_z, 1 \), taking the difference of particle and anti-particle chemical potentials, one finds

\[
-k_i(D_i\mu_i'' + v_w\mu_i') + \sum_p \Gamma_p^d \sum_j \mu_j = S_i,
\]

\[
S_i = \frac{D_i v_w}{(p_z^2/E_0)_0} (p_z\Delta E_i)' - \sum_p \Gamma_p^d (\Delta E_{sp,p}),
\]

with diffusion constants \( D_i = k_i(p_z^2/E_0)^2/(p_z^2\Gamma_i^p) \), statistical factors \( k_i \), interaction rates \( \Gamma \), and CP-violating source terms \( S_i \).

In the NMSSM, the relevant interactions are

\[
\mathcal{L}_{\text{int}} = y_3 c_t q_3 H_2 + y_3 c_q q_3 \bar{h}_2 + y_3 c_t \bar{q}_3 \bar{h}_2 - y_3 \mu c_q q_3^* H_1 + y_3 A_t \bar{c} \bar{q}_3 H_2 + \lambda s h_1 H_2 + \lambda s h_3 H_1 + \text{h.c.}
\]

and the supergauge interactions (in equilibrium!), the higgsino helicity flips (from \( \mu h_1 \bar{h}_2 \)), the Higgs and axial top number violation in the broken phase, and the strong spherons. The resulting interaction terms in the diffusion equations are

\[
(\Gamma_y + \Gamma_{y\lambda})(\mu H_2 + \mu Q_3 + \mu T), \quad \Gamma_{yy}(\mu H_1 - \mu Q_3 - \mu T), \quad \Gamma_{\lambda}(\mu \bar{z} + \mu H_1 + \mu H_2), \quad \\
\Gamma_{ss}(2\mu Q_3 + 2\mu Q_2 + 2\mu Q_1 + \mu T + \mu B + \mu C + \mu S + \mu U + \mu D), \quad \\
\Gamma_{hf}(\mu H_1 + \mu H_2), \quad \Gamma_{m}(\mu Q_3 + \mu T), \quad \Gamma_{H_1 \mu H_1}, \quad \Gamma_{H_2 \mu H_2}.
\]

We obtain a reduced set of diffusion equations for the chemical potentials \( \mu Q_3, \mu T, \mu H_1, \mu H_2 \) and \( \mu \bar{z} \); e.g. for \( \mu H_1, \mu H_2 \) and \( \mu \bar{z} \) they read

\[
-k_H D_H \mu H_1 + 6\Gamma_{yy}[\mu H_1 - \mu Q_3 - \mu T] + 2\Gamma_{\lambda}[\mu \bar{z} + \mu H_1 + \mu H_2] + 2\Gamma_{hf}(\mu H_1 + \mu H_2) + 2\Gamma_{H_1 \mu H_1} = S_{H_1},
\]

\[
-k_H D_H \mu H_2 + 6(\Gamma_y + \Gamma_{y\lambda})[\mu H_2 + \mu Q_3 + \mu T] + 2\Gamma_{\lambda}[\mu \bar{z} + \mu H_1 + \mu H_2] + 2\Gamma_{hf}(\mu H_1 + \mu H_2) + 2\Gamma_{H_2 \mu H_2} = S_{H_2},
\]

\[
-k_{\bar{z}} D_{\bar{z}} \mu \bar{z} + 2\Gamma_{\lambda}[\mu \bar{z} + \mu H_1 + \mu H_2] + \Gamma_{\bar{z}H_1} = S_{\bar{z}}
\]

where \( D_i \equiv D_i \frac{\partial}{\partial \mu_i} + v_w \frac{\partial}{\partial \mu_i} \). The transport equations can be further simplified if the top Yukawa interactions are assumed to be in equilibrium, which implies \( \mu H_2 + \mu Q_3 + \mu T = 0 \) and \( \mu H_1 - \mu Q_3 - \mu T = 0 \). We then find

\[
-k_{Q_3} D_{Q_3} \mu Q_3 - k_H D_H \mu H + (6\Gamma_m + 2\Gamma_H)\mu H + 6\Gamma_{ss}[cQ \mu Q_3 - c_H \mu H] = S_{Q_3} + S_H
\]

\[
-(k_{Q_3} + k_T) D_{Q_3} \mu Q_3 + k_T D_T \mu H + 3\Gamma_{ss}[cQ \mu Q_3 - c_H \mu H] = 0
\]

In this approximation the chargino source terms enter only via \( S_H = S_{H_1} - S_{H_2} \). Therefore, the dominating, \( \theta \)-dependent part of the chargino source term ("helicity part") cancels. The singlino, with a potentially large source term, decouples from the transport equations. With the dispersion relation (the "flavor" part survives. Thus, in the MSSM case the \( \delta \beta \) suppression of the baryon asymmetry is recovered. In the kinetic momentum approach also this contribution vanishes. Giving up the top Yukawa coupling equilibrium one also obtains a \( S_{H_1} + S_{H_2} \) contribution. In our (preliminary) studies this still leads to a sizable baryon asymmetry.
We solve set of diffusion equations by the Greens function method. The weak sphalerons, which are not in equilibrium, generate the baryon to entropy ratio in the hot phase
\[ \eta_B \equiv \frac{n_B}{s} = \frac{135 \Gamma_w s}{2 \pi^2 g_s v_w T} \int_0^\infty d\bar{z} \mu_{B_L}(\bar{z}), \] (19)
where \( \mu_{B_L} = 7 \mu_Q - 2 \mu_H \) is the chemical potential for the left-handed quark number (in the massless approximation).

The generated baryon asymmetry is rather sensitive to the squark spectrum. For universal squark masses there is a large suppression by strong sphalerons. The baryon asymmetry increases for with \( 1/v_w \) (at \( v_w \sim 0.01 \) this behavior would be cut off by the (neglected) effects of weak sphalerons). Thinner bubble walls enhance the baryon asymmetry, \( \eta \sim 1/L_w^2 \). The chargino contribution dominates the baryon production in the NMSSM. It is especially large for \( M_2 \sim \mu \). In fig. 3 we present two examples for the chargino contribution to the baryon asymmetry. In the case of explicit CP-violation small complex phases of the order \( 10^{-2} \) can account for the observed baryon asymmetry only for very small wall velocities, \( v_w \lesssim 0.01 \) (and only the right-handed stop is light). However, wall velocities in this range have recently been found in the MSSM. In the case of transitional CP violation a sufficient baryon number can be easily produced, also for larger wall velocities. Hence, transitional CP violation is particularly interesting for electroweak baryogenesis.

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