Transport of inertial particles in turbulent boundary layers

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Abstract. A direct numerical simulations (DNS) of a spatially evolving particle-laden turbulent boundary layer has been performed to study turbophoresis effects in presence of changing local Stokes number. The data show a preferential particle localization near the wall at the streamwise position where the local Stokes number $St^+$ assumes a value close to 25, similarly to that found in channel flow. Note that a complete steady state will never been reached for the particle concentration in this kind of flow. The effects of the seeding and of preferential sampling of the fluid velocity will be described as well.

1. Introduction

Particle-laden turbulent flows can be found in many technological devices and natural phenomena. The main parameter that controls the dynamics of particles in turbulent flow is the so-called Stokes number $St$ which is the ratio between a characteristic particle relaxation time ($\tau_p$) and a characteristic time scale of the flow (Balachandar & Eaton, 2010). The characteristic phenomenology that appears in wall-bounded flow is turbophoresis, the preferential particle accumulation at the wall. In recent years many experiments and direct numerical simulation (DNS) have been performed in order to study turbophoresis, see Soldati & Marchioli (2009) for a review, where the main configuration was usually channel or pipe flow. These parallel flows are statistically homogeneous in the flow direction and for this reason the characteristic Stokes number of the particle does not change during the particle evolution. Varying the local Stokes number can bring anomalous phenomena in particle transport as observed in particle-laden jet flows (Picano et al., 2010) where a preferential particle accumulation is found near the jet axis when this parameter is of order 1. In addition, the particle motion is typically assumed periodic along the channel/pipe length and so just the statistical steady state can be analyzed. Spatially evolving simulations can be useful in order to characterize typical lengths needed to achieve a statistical steady state and to perform a complete study of the transient phase, see Picano et al. (2009). In this context, the study of particle dynamics in spatial evolving boundary layers assumes a fundamental role to advance our understanding of multiphase flows because it combines these two characteristics: i) changing Stokes number in the main stream direction and ii) spatially evolving configuration. It is possible to define a nominal Stokes number $St_0$ based on external free-stream velocity $U_\infty$ and displacement thickness at the inflow $\delta_0^*$, $St_0 = \tau_p U_\infty / \delta_0^*$.
However, for the boundary layer, two further different Stokes numbers can be defined, one in local internal units $St^+$ and another in external units $St_{99}$,

$$St^+ = \frac{\tau_p U^2}{\nu}, \quad St_{99} = \frac{\tau_p U_\infty}{\delta_{99}},$$

(1)

where $U_\tau$ is the friction velocity, $\nu$ the kinematic viscosity, and $\delta_{99}$ the integral thickness of the boundary layer, respectively. This implies that particles tend to fluid particle limit moving downstream in the boundary layer. The behavior of these two characteristic Stokes numbers along the streamwise coordinate is shown in figure 1.

![Figure 1. a) Development of the viscous Stokes number in streamwise direction for two nominal Stokes number. b) Development of the external Stokes number in streamwise direction for two nominal Stokes number.](image)

Our aim is to study the particle dynamic inside a boundary layer up to Reynolds number $Re_\theta = 2500$, based on the momentum thickness $\theta$ in a large computational domain. Actually, there are no direct simulations of particle-laden turbulent boundary layer flows in the literature, while for bubbly turbulent boundary layer there are some DNS in the literature. Ferrante & Elghobashi (2007) studied the effects of micro-bubbles in a turbulent boundary layer up to $Re_\theta = 2900$ and, more recently, Mattson & Mahesh (2011) performed a one-way coupling simulation for bubble dynamics in turbulent boundary layers up to $Re_\theta = 1800$.

2. Numerical methodology

A direct numerical simulation (DNS) of a particle-laden turbulent boundary layer has been performed by means of the standard pseudo-spectral code SIMSON developed at KTH during the last twenty years (Chevalier et al., 2007). One-way coupling is assumed to model particle-flow interactions; in this context data from unladen simulations for the flow are already available based on previous experience at KTH in this field, see Schlatter & Örlü (2010); Schlatter et al. (2009). In order to reach a fully developed turbulent flow, the carrier phase needs to obtain values of the Reynolds number $Re_\theta \simeq 2000$. In the present case the $Re_\theta$ at the end of the domain is 2500. In addition is important to start with relatively low Reynolds number at the inflow $Re_\theta \simeq 200$. The flow at the beginning of the domain is tripped by a localized forcing random in time and in spanwise direction. The computational domain is $x_L \times y_L \times z_L = 3000\delta_0^* \times 100\delta_0^* \times 120\delta_0^*$ in the streamwise, wall-normal and spanwise directions based on displacement thickness $\delta_0^*$. A resolution of $n_x = 4096 \times n_y = 301 \times n_z = 384$, where $n_x$ and $n_z$ are the Fourier modes in
streamwise and spanwise directions and $n_y$ are the number of Chebyshev modes in wall-normal direction, was used. The geometrical and the flow parameters are essentially the same as in the simulation described by Schlatter et al. (2009).

Regarding the dispersed phase, as stated above, one-way coupling is assumed to model particle-flow interactions based on the assumption that the particles are very diluted. The particles are assumed to be small, rigid spheres with density order one thousand the density of the carrier phase. Gravity is neglected in this context so the particles are just subjected to only Stokes drag (Maxey & Riley, 1983). The same pseudo-spectral code coupled with a Lagrangian solver has been already implemented and tested in a channel flow simulation in very large domain, see Sardina et al. (2011), but for the present case we improved the order of the interpolation of fluid velocity at particle position from linear to 4th order and the time integration is performed by means of an Adams-Bashforth scheme. Particles are introduced at a constant rate in the already turbulent flow at $Re_\theta \approx 800$ so they will not be directly influenced by the trip forcing. Eight different populations, defined by the nominal Stokes number $St_0$ will be examined. Particles are introduced in a cross-section randomly in spanwise directions and at fixed wall-normal positions between the wall and the coordinate $4\delta_{99}$ with the particle velocity equal to the local flow velocity. In this way, most of the particles are injected outside the boundary layer. It is thus interesting to see how the boundary layer captures the inertial particles. The wall particle concentration reaches the quasi-steady state after almost $70000\delta^*_{99}/U_\infty$ time units and after that we have collected statistics for another $10000\delta^*_{99}/U_\infty$ units. The simulation has been run on 1536 cores on a Cray XE6 system using a hybrid MPI-OpenMP parallelization which has been developed to improve the performance of the Eulerian-Lagrangian solver. In the MPI version of the code, the Eulerian fields in Fourier space are parallelized along the spanwise direction and in physical space along the wall-normal direction. For this reasons, global communication is needed to exchange data among the different MPI ranks. The Lagrangian particles are distributed onto different processors and a long vector storage is seen by every MPI processor in order to interpolate the fluid velocity at particle positions. The weak point of this implementation is the maximum number of MPI processors that is limited to the number of the spanwise Fourier modes. A more efficient version of the code is parallelized in both spanwise and streamwise directions, this 2-D parallelization is able to reduce the number of global communications resulting in an increase of the performance. However, the implementation of a dispersed phase in this version is still a challenge. Thus, in order to increase the speed of the 1-D parallelization and at the same time to run on large number of cores, we added OpenMP parallel sections in the internal loops of the subroutines. In this way, the number of MPI ranks is always limited to the number of modes in spanwise directions, but the internal loops are faster because of the presence of threads. Every MPI rank contains several OpenMP threads depending on the architecture and our tests showed that the best performances are obtained for 6 OpenMP thread per node. The performance of the code is plotted against the number of cores for different parallel implementations in figure 2. The most efficient is obviously the full MPI with a 2-D parallelization and its scaling is nearly linear. The MPI 1-D parallelization is very inefficient by increasing the number of cores, in this case the performance saturate because the maximum number of MPI processors has been reached and most of the cores are just idling. Instead the hybrid parallelization is clearly better than the only 1-D MPI parallelization. The speed scales linearly by using 4 or 6 OpenMP. For the simulation we have used 384 nodes containing 4 OpenMP threads for a total number of cores of 1536.

3. Results

In figure 3a, we show the Reynolds number $Re_\theta$ versus the streamwise coordinate $x$. The circle denotes the streamwise position where particles are injected; at the end of the domain
Figure 2. Performance measured as inverse time per time step of the code for different parallel implementations against number of cores (strong scaling).

this characteristic Reynolds number is about $Re_\theta = 2500$, while the friction Reynolds number $Re_\tau = U_\tau \delta_{99}/\nu \simeq 800$. An instantaneous particle configuration is shown in fig. 3b for a nominal

Figure 3. a) Reynolds number based on momentum thickness against streamwise direction. b) Instantaneous particle distribution for the population $St_0 = 30$.

Stokes number $St_0 = 30$; this is one of the most accumulating set. The blue line represents the integral thickness of the boundary layer $\delta_{99}$. The wall normal axis is logarithmic in order to highlight the turbophoretic behavior near the wall. As shown in the figure, the particle behavior depends on the wall-normal location of injection. In particular, particles introduced above the shear layer tend to follow the laminar streamline with zero relative velocity with respect to the
flow. Conversely, particles that are injected or captured inside the boundary layer are directly attracted by the wall by the turbophoretic drift. Therefore, following the particle dynamics, the boundary layer can be divided in two zones: the region between $\delta/1000$ and $\delta$ is very depleted, whereas the region near the wall is populated by a large number of particles. A second local preferential peak of concentration can be observe near the boundary layer edge.

**Figure 4.** Selected particle trajectories along the domain, the plotted population Stokes number is $St_0 = 25$. Contours in the lateral and streamwise-normal planes represent an instantaneous flow velocity field while the bottom plane represent the streamwise velocity fluctuations field. Particle trajectories are colored proportional to their streamwise velocity.

In figure 4, some particle trajectories are plotted inside the domain with a nominal Stokes number $St_0 = 25$. To qualitatively address the physical structures and dimensions of the carrier phase field, contours of an instantaneous flow configuration are superimposed. The trajectories are colored proportionally to the particle streamwise velocity, slower particles are close to the wall (white colors) while the fastest are in the free-stream region (black colors). In this figure it is more evident that the particles seeded in the free-stream tend to follow the laminar streamwise, as stated above, without any deviation of the trajectory in spanwise direction. The particles that are instead inside the turbulent and the wall region are characterized by a trajectory with a certain degree of randomness.

In the fig. 5a, the wall concentration is plotted against the streamwise position. All the populations with $St_0$ greater than 10 are still characterized by an increase of wall accumulation also at the end of the domain while particles with $St_0 = 10$ seem to have reached the local concentration peak at $Re_\theta \approx 2200$ and after they start to slightly decrease their wall concentration. To capture the concentration peaks also for the heaviest particles a longer computational domain is required. Theoretically, in the hypothesis of an infinitely long streamwise domain all the particle populations will reach a concentration peak after a transient accumulation phase, the position of the peak is obviously dependent on the particle inertia.
Downstream of the location of maximum accumulation, the wall concentration will tend to diminish because the particles tend to the Lagrangian limit caused by decreasing their viscous Stokes number. The particles characterized by a nominal Stokes number order 10 reach a value of the viscous Stokes number order one only at very large $Re_\theta$ of about 10 million which is impossible in simulations or laboratory experiments. The main difference between particle-laden channel flows and particle-laden boundary layers is that in the case of boundary layers an equilibrium concentration in wall-normal direction can never be reached except for the trivial case of Lagrangian tracer behavior. At the end of the computational domain the most accumulating particles are characterized by a nominal Stokes number order 25. It is known from channel simulations that the particles most subjected to turbophoresis are characterized by a viscous Stokes number order $St^+ = 25$ and therefore we report in fig. 5b the wall concentration as a function of the local viscous Stokes number. Note that for every population $St^+$ diminishes by 25% in the streamwise direction between the injection point and the end of the domain. For the most accumulating particles $St_0 = 25 \div 30$, the largest concentration values are found corresponding to $St^+ = 19$ and 22 respectively. These values are close to those found in channel flows, which suggests that turbophoresis is dominated by the near-wall dynamics, essentially independent of the outer region of the boundary layer.

In order to quantify the capability of the boundary layer to capture the inertial particles injected in the originally laminar region, the wall concentration of the particles injected outside the boundary layer is plotted in figure 6a. The values of the concentration are almost 10 times less than the concentration obtained considering all the inertial particles and the particles characterized by $St_0 = 10 \div 30$ are the most accumulating at the end of the domain. Obviously the wall accumulation starts in a streamwise position almost 1000 units more downstream than the injection point. We define this position $Re_\theta^w$ as the position in which the wall concentration is for the first time greater than zero and this observable is shown in figure 6b against particle inertia. Surprisingly, the particle population that first reaches the wall is not the Lagrangian tracer but the population with nominal Stokes number order 10, this is an indication that these kind of particles tend to be easily captured by the wall, the position of the heaviest Stokes number population seems to follow a linear law as plotted in the figure.

Figure 5. a) Particle concentration close to the wall along the streamwise direction. b) Particle concentration close to the wall against local viscous Stokes number $St^+$. 
In order to highlight the preferential sampling of the flow experienced by particles, in figure 7a the wall-normal fluid velocity sampled by the particles is plotted against wall-normal direction. While for the fluid and the Lagrangian tracers the velocity is always positive, for the inertial particles this velocity changes sign becoming negative (towards the wall) inside the boundary layer. On the other hand, far from the wall, the wall-normal fluid velocity sampled by particles is positive due to the vocation of the particle to stay in the slow outward motions (Picano et al., 2009). In figure 7b the same observable is plotted but for the particles initially injected in the...
irrotational region. Here it mainly shows the effects and the importance of the seeding position. Also Lagrangian tracers behave very differently from the Eulerian flow, the wall-normal velocity sampled by Lagrangian tracers is negative, this is due to the fact that the tracers from the laminar region enter to the turbulent region only with the mechanism of entrainment characterized by negative flow velocities. The implications of the seeding position can be fundamental to the experimental techniques based on Lagrangian tracers dynamic such as PIV, PTV and LDA.

4. Conclusions

For the first time, a direct numerical simulation of a particle-laden spatially evolving turbulent boundary layer has been performed, reaching up to $Re_\theta = 2500$. The wall particle dynamics is dominated by the phenomenology of turbophoresis that is controlled by a viscous Stokes number varying in the streamwise direction. The particle concentration in the wall-normal direction never reaches a statistical equilibrium until the inertial particles reach the Lagrangian tracers regime but this happens only at astronomical Reynolds numbers. Particles initially injected in the laminar region are captured inside the boundary layer by entrainment motions characterized by wall-normal velocity towards the wall. These particles starts to accumulate at the wall after a certain length dependent essentially on particle inertia. The population with nominal Stokes number $St_0 = 10$ is most subjected to wall accumulation dynamics. Inside the turbulent boundary layer but far from the wall, particles tend to experience fluid velocity directed mainly towards the wall, completely different than the Eulerian velocity field. The same situation is achieved by Lagrangian tracers injected initially just in the laminar region, this implies a certain attention to seed the flow correctly in experiments with light Lagrangian inertial particles.

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