THE ISOMORPHISM CONJECTURE FOR
3-MANIFOLD GROUPS AND $K$-THEORY OF
VIRTUALLY POLY-SURFACE GROUPS

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Abstract. This article has two purposes. In [15] we showed that the FIC (Fibered Isomorphism Conjecture for pseudoisotopy functor) for a particular class of 3-manifolds (we denoted this class by $\mathcal{C}$) is the key to prove the FIC for 3-manifold groups in general. And we proved the FIC for the fundamental groups of members of a subclass of $\mathcal{C}$. This result was obtained by showing that the double of any member of this subclass is either Seifert fibered or supports a nonpositively curved metric. In this article we prove that for any $M \in \mathcal{C}$ there is a closed 3-manifold $P$ such that either $P$ is Seifert fibered or is a nonpositively curved 3-manifold and $\pi_1(M)$ is a subgroup of $\pi_1(P)$. As a consequence this proves that the FIC is true for any $B$-group (see definition 4.2 in [15]). Therefore, the FIC is true for any Haken 3-manifold group and hence for any 3-manifold group (using the reduction theorem of [15]) provided we assume the Geometrization conjecture. The above result also proves the FIC for a class of 4-manifold groups (see [14]).

The second aspect of this article is to relax a condition in the definition of strongly poly-surface group ([13]) and define a new class of groups (we call them weak strongly poly-surface groups). Then using the above result we prove the FIC for any virtually weak strongly poly-surface group. We also give a corrected proof of the main lemma of [13].

1. Introduction

Throughout this article by ‘FIC’ we mean the Fibered Isomorphism Conjecture of Farrell and Jones ([5]) corresponding to the stable topological pseudoisotopy functor. The statement and the basic results

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related to this conjecture and the basics needed on 3-manifold topology have already appeared in several articles, for example see \cite{6}, \cite{14} and \cite{15}. We refer the reader to these sources for the preliminaries.

In this article we always consider orientable 3-manifolds. Note that the FIC for the fundamental group of nonorientable 3-manifolds follow easily from the orientable ones (see remark 3.1 in \cite{15}).

1.1. Geometry and the FIC for 3-manifold groups. In \cite{15} we called a group $G$ a $B$-group if it contains the fundamental group of a compact irreducible 3-manifold with all the boundary components of higher genus (that is, when genus $\geq 2$) and incompressible (nonempty) as a subgroup of finite index. We denoted by $\mathcal{C}$ the later class of 3-manifolds. In the proof of the main theorem of \cite{15} we showed that the $B$-groups are the key class of groups for which the FIC should be proved to prove the FIC for 3-manifold groups. If $M \in \mathcal{C}$ and $M$ contains no essential annuli (see definition 4.3 in \cite{15}) then we showed in theorem 4.3 of \cite{15} that the FIC is true for a $B$-group which contains $\pi_1(M)$ as a subgroup of finite index. This result was accomplished by showing that in such a situation $\pi_1(M)$ is isomorphic to a subgroup of the fundamental group of a closed 3-manifold $N$ so that either $N$ is a Seifert fibered space or is a nonpositively curved Riemannian manifold. In fact in this case $N$ was the double of $M$. In this article we prove the following.

**Theorem 1.1.1.** Let $M \in \mathcal{C}$. Then there is a closed 3-manifold $P$ so that $P$ is either a Seifert fibered space or is a nonpositively curved 3-manifold and $\pi_1(M)$ is a subgroup of $\pi_1(P)$.

**Remark 1.1.1.** Here we remark that if $M$ has at least one torus boundary component then it follows from theorem 3.2 of \cite{14} that $\pi_1(M)$ is a subgroup of a closed nonpositively curved 3-manifold. This can be seen by first taking the double $N$ of $M$ along higher genus ($\geq 2$) boundary components and then taking the double $P$ of $N$ and applying \cite{14}.

An important application to the above theorem is the following theorem.

Before we state the theorem recall from \cite{15} that we called a group $\Gamma$ to satisfy the FICwF if the FIC is true for $\Gamma \wr F$ for any finite group $F$.

**Theorem 1.1.2.** (Key Lemma) The FICwF is true for $B$-groups.
The above Theorem has several consequences as described in [15]. We recall some of these consequences here.

**Corollary 1.1.1.** The hypothesis the FIC is true for $B$-groups can be removed from the statements of any result stated in [15].

Among several results the following results were proved in [15], under the assumption that the FIC is true for $B$-groups. For the other consequences we refer the reader to [15].

**Corollary 1.1.2.** (Proposition 2.2.1) Let $M^3$ be a closed 3-manifold fibering over the circle. Then the FICwF is true for $\pi_1(M)$.

Recall that the main lemma of [13] is a particular case of the above corollary. Hence we have completed the proof of the main lemma of [13].

Corollary 1.1.2 also has application in proving the FIC for mapping class groups of surfaces (see [3]).

Following the convention of [15], in the next corollary by ‘3-manifold’ we mean irreducible 3-manifold with infinite fundamental group. Note that in such a situation the 3-manifold is aspherical.

**Corollary 1.1.3.** ([15, theorem 4.5]) Let $M$ be either a compact 3-manifold with nonempty boundary or a noncompact 3-manifold. Then the FICwF is true for $\pi_1(M)$.

**Corollary 1.1.4.** ([15, theorem 4.5]) The FICwF is true for the fundamental group of any virtual Haken 3-manifold.

**Corollary 1.1.5.** ([15, corollary 4.2]) Assume that the Geometrization conjecture is true. Then the FICwF is true for any 3-manifold group.

**Remark 1.1.2.** The Farrell-Jones Isomorphism Conjecture in the case of algebraic $K$-theory functor for a class of 3-manifold groups is considered in a recent paper ([2]) by Bartels and Lück. It is shown there that the $K$-theoretic assembly map in the conjecture is rationally injective for these 3-manifold groups.

Now recall that in [14] we proved the FIC for the fundamental group of a class of 4-manifolds under a certain assumption (special) and we pointed out there that we do not need this hypothesis if the FIC were true for $B$-groups. Hence an application of Theorem 1.1.2 is that we
can remove this assumption from these results and hence we have the following more general result.

**Corollary 1.1.6.** The hypothesis special can be removed from the statements of any result in [14]. In particular, we have that the FIC is true for the fundamental group of the following 4-manifolds $M$.

- $M$ is a surface bundle over a surface. ([14], theorem 1.5).
- $M$ is a fiber bundle over the circle and $M$ has a complex structure. ([14], theorem 1.3).
- $M$ is a complex elliptic surface. ([14], theorem 1.2).

1.2. **The FIC for poly-surface groups.** Now we come to the application part of Theorem 1.1.1 for poly-surface groups. In [13] we defined strongly poly-surface groups. This definition was along the same lines as the definition of strongly poly-free groups in [1]. But the strongly poly-surface groups are more general and difficult to tackle. We describe the main problem which occur with strongly poly-surface groups. In strongly poly-free groups a certain kind of 3-manifolds appeared. These were mapping tori of compact surfaces with nonempty boundary. Leeb’s work ([11]) was directly applicable here to use non-positively curved geometry on such 3-manifolds. But in general a mapping torus of a closed surface does not support such a metric. In fact an arbitrary mapping torus of a closed surface need not even has a non-positively curved metric in the Alexandrov sense. On the other hand in the case of strongly poly-surface groups we allowed mapping tori of any surface (closed, compact or non-compact with one end). These are a large class of 3-manifolds and as we recalled above, the geometry of nonpositively curvature is not applicable here directly. In [15] we developed some new techniques to tackle this kind of situation. The main idea was to use direct limit argument and reduce it to compact 3-manifold with nonempty boundary case. In [15] we dealt with compact 3-manifolds which has at least one incompressible torus boundary component. In Theorem 1.1.1 we complete this program considering the other cases.

In this article we remove the ‘non-compact surface with one end’ hypothesis from the definition of strongly poly-surface groups allowing non-compact surfaces with any number of ends and define the following class of groups. Then using Corollaries 1.1.2 and 1.1.3 we prove the FIC in this general situation.
Definition 1.2.1. A discrete group $\Gamma$ is called \textit{weak strongly poly-surface} if there exists a finite filtration of $\Gamma$ by subgroups: $1 = \Gamma_0 \subset \Gamma_1 \subset \cdots \subset \Gamma_n = \Gamma$ such that the following conditions are satisfied:

- $\Gamma_i$ is normal in $\Gamma$ for each $i$.
- $\Gamma_{i+1}/\Gamma_i$ is isomorphic to the fundamental group of a surface $F_i$ (say).
- for each $\gamma \in \Gamma$ and $i$ there is a diffeomorphism $f : F_i \to F_i$ such that the induced automorphism $f#_\gamma$ of $\pi_1(F_i)$ is equal to $c_\gamma$ up to inner automorphism, where $c_\gamma$ is the automorphism of $\Gamma_{i+1}/\Gamma_i \simeq \pi_1(F_i)$ induced by the conjugation action on $\Gamma$ by $\gamma$.

In such a situation we say that the group $\Gamma$ has \textit{rank} $\leq n$.

The only difference between strongly poly-surface group and weak strongly poly-surface group is in the last condition, that is when $\pi_1(F_i)$ is infinitely generated. In strongly poly-surface group the condition was that in this case $F_i$ has ‘one end’ and here $F_i$ can have any number of ends. All other conditions are same.

We prove the following.

**Theorem 1.2.1.** Let $\Delta$ be a virtually weak strongly poly-surface group. Then FIC is true for $\Delta$.

Here recall that given a class of group $\mathcal{A}$, a group $\Gamma$ is called virtually-$\mathcal{A}$, if there is a normal subgroup $A \in \mathcal{A}$ of $\Gamma$ of finite index.

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2. **Proof of Theorem 1.1.1 and Theorem 1.1.2**

2.1. **Proof of Theorem 1.1.1** Let us first recall the Nielsen-Thurston classification of surface diffeomorphisms. Let $S$ be a closed orientable surface of genus $\geq 2$ and let $f : S \to S$ be an orientation preserving diffeomorphism. Then $f$ belongs to one of the following classes of diffeomorphisms.

- there is a positive integer $n$ so that $f^n$ is isotopic to the identity.
- $f$ is isotopic to a reducible diffeomorphism. that is, there is a finite class of mutually nonparallel, disjoint and essential simple closed curves $\mathcal{A}$ on $S$ so that an isotopy of $f$ leaves $\mathcal{A}$ invariant.
• \( f \) is pseudo-Anosov.

We refer the reader to the papers [4], [7] and [12] for details and definitions of the above classification of surface diffeomorphisms. But we recall the following two well-known facts we need.

**Fact 1.** If \( f \) is pseudo-Anosov then \( f^n \) is pseudo-Anosov for all nonzero integers \( n \). Also any isotopy of \( f \) is pseudo-Anosov.

**Fact 2.** The class of pseudo-Anosov diffeomorphisms is disjoint from the first two classes of diffeomorphisms in the above classification.

The following lemma is an useful application of the above classification. We need this lemma crucially for the proof of Theorem 1.1.1.

**Lemma 2.1.1.** Let \( S \) be a closed orientable surface of genus \( \geq 2 \) and let \( \mathcal{A} \) be a finite class of disjoint, mutually nonparallel and essential simple closed curves on \( S \). Let \( f : S \rightarrow S \) be a pseudo-Anosov diffeomorphism. Then there is an integer \( n_0 \) so that no member of \( \mathcal{A} \) is parallel to any member of \( f^{n_0}(\mathcal{A}) \).

**Proof.** As \( f : S \rightarrow S \) is pseudo-Anosov, by Fact 1, \( f^n \) is also pseudo-Anosov for all nonzero integers \( n \).

Let \( \mathcal{A} = \{c_1, c_2, \ldots, c_k\} \). Fix \( l \in \{1, 2, \ldots, k\} \). Then we claim that for every nonzero integer \( n \), \( f^n(c_l) \) is not parallel to \( c_l \). Since otherwise \( f^n \) will be reducible, which is a contradiction. Hence clearly, for distinct nonzero integers \( n \) and \( m \), \( f^n(c_l) \) is not parallel to \( f^m(c_l) \). Therefore, there is a positive integer \( N_l \) so that \( f^s(c_l) \) is not parallel to any \( c_i \) for \( i = 1, 2, \ldots, k \) and for \( s > N_l \).

Let \( n_0 = \max\{N_1, N_2, \ldots, N_k\} \). This proves the Lemma.

**Proof of Theorem 1.1.1.** If \( M \) contains no essential annulus (see definition 4.3 in [15]) then let \( P \) be the double of \( M \). Assume \( P \) is not Seifert fibered. Then by lemma 6.2 of [15] \( P \) is not a graph manifold and hence has a hyperbolic piece in its’ JSJ (Jaco-Shalen-Johannson and Thurston) decomposition. Using [11] we get that \( P \) supports a nonpositively curved Riemannian metric.

The proof is a bit involved and is the main feature of this article when there are essential annuli embedded in \( M \). For instance in such a situation just by taking double of \( M \) will produce more incompressible tori in the double of \( M \).

So assume that there are essential annuli embedded in \( M \). The main idea is to identify two copies of \( M \) along their boundary components by some diffeomorphisms of surfaces so that the essential annuli do not
match to produce unwanted incompressible tori. The suitable candidates for this purpose are the pseudo-Anosov diffeomorphisms.

Let $\partial_1, \ldots, \partial_k$ be the components of $\partial M$. Then $\partial_i$ is a closed orientable surface of genus $\geq 2$ for each $i$. Since $M$ contains properly embedded essential annuli, these annuli produces a finite class of pairwise disjoint, nonparallel, essential simple closed curves on the boundary. Using the remark (Remark 2.1.1) below choose the maximal (finite) class of such annuli and let $A$ be the corresponding finite class of simple closed curves on $\partial = \bigcup_{i=1}^k \partial_i$. Let $A_i = A \cap \partial_i$. By Lemma 2.1.1 there are pseudo-Anosov diffeomorphisms $f_i : \partial_i \to \partial_i$ so that no member of $A_i$ is parallel to any member of $f_i(A_i)$.

We need the following remark for the remaining proof.

**Remark 2.1.1.** There are finitely many essential annuli embedded in $M$ which are unique up to isotopy. This follows from the uniqueness of the JSJ decomposition of $M$ along incompressible tori and essential annuli. (see the splitting theorem of [9], p. 157).

Let $M_1$ and $M_2$ be two copies of $M$ and let us denote by $A^j$ the simple closed curves on $\partial M_j$ for $j = 1, 2$ corresponding to $A$ on $\partial M$. Let $P = M_1 \cup M_2 / \sim$, where $\sim$ stands for the identification of $x$ with $f_i(x)$ for each $i$ and for $x \in \partial_i$.
Note that $P$ is a Haken 3-manifold and hence by the JSJT decomposition there are finitely many embedded incompressible tori (say, $T$) in $P$ so that the complementary pieces are either Seifert fibered or hyperbolic. We check that either $P$ is Seifert fibered or it contains a hyperbolic piece. This follows from the following two lemmas.

Note that we can think of $\partial$ as a submanifold of $P$.

**Lemma 2.1.2.** $T$ can be isotoped off $\partial$.

*Proof.* On the contrary, assume that there exists a $T \in T$ such that any isotopy of $T$ intersects $\partial$. We can make this intersection in general position after isotoping $T$. Hence $T \cap M_j$ is a collection of annuli and 2-discs in $M_j$ for $j = 1, 2$. Now using the loop theorem (see [8], chapter 4, the loop theorem) and applying standard tricks from 3-manifold topology we can isotope $T$ further to make sure that the components of $T \cap M_j$ are essential annuli. Now by the uniqueness of essential annuli in $M$ it follows that $T \cap \partial M_j$ is parallel to a subcollection of $A^j$ for $j = 1, 2$. Since $T$ is obtained by gluing the boundaries $(T \cap \partial M_1)$ of the annuli $T \cap M_1$ with the boundaries $(T \cap \partial M_2)$ of the annuli $T \cap M_2$ under the maps $\{f_i\}$, it follows that $f_i$ sends a member of $A_i$ parallel to a member of $f_i(A_i)$. This is a contradiction to the choice of $\{f_i\}$. Hence $T$ can be isotoped off $\partial$. □

**Lemma 2.1.3.** Either $P$ is Seifert fibered or there is a hyperbolic piece in the JSJT decomposition of $P$.

*Proof.* Assume $P$ is not Seifert fibered. Let $S \in \{\partial_1, \ldots, \partial_k\}$. Using Lemma 2.1.2 isotope $T$ so that it does not intersect $\partial = \bigcup_{i=1}^{k} \partial_i$. Hence, there is a component of $P - T$, say $K$, containing $S$. We claim that $K$ is not Seifert fibered. On the contrary, assume $K$ is Seifert fibered. Note that $\pi_1(K)$ contains $\pi_1(S)$ as a subgroup. Now since $P$ is not Seifert fibered, $K$ has nonempty boundary and hence it has a finite index subgroup of the form $F^r \times \mathbb{Z}$, where $F^r$ is a (nonabelian) free group. Thus $\pi_1(S) \cap (F^r \times \mathbb{Z})$ is of finite index in $\pi_1(S)$ and therefore is a closed surface group of a surface of genus $\geq 2$. This is a contradiction because $F^r \times \mathbb{Z}$ cannot contain a nonabelian closed surface group. □

Now using [11] it follows in the non-Seifert fibered case that $P$ supports a nonpositively curved metric. This completes the proof of the Theorem. □
2.2. **A Key Lemma.** The key lemma to prove the FIC for discrete groups of this article is the following and it is an application of Theorem 1.1.1.

**Key Lemma.** Let $M \in \mathcal{C}$. Then the FICwF is true for $\pi_1(M)$. Consequently, the FIC is true for $B$-groups.

**Remark 2.2.1.** Here we recall that in the statement of the Key Lemma if one of the boundary components of $M$ is an incompressible torus then the same conclusion is true. See corollary 4.1 of [15].

The main application of the Key Lemma required for the proof of Theorem 1.2.1 is the following proposition.

**Proposition 2.2.1.** Let $M^3$ be a closed 3-manifold fibering over the circle. Then the FICwF is true for $\pi_1(M)$.

**Proof.** The Key Lemma and theorem 4.5 of [15] together completes the proof of the Proposition. \[\square\]

**Proof of the Key Lemma.** By Theorem 1.1.1 $\pi_1(M)$ is isomorphic to a subgroup of $\pi_1(P)$, where $P$ is either a Seifert fibered space or is a closed nonpositively curved Riemannian manifold. Let $G$ be a finite group. Now it follows that $\pi_1(M) \wr G$ is a subgroup of $\pi_1(P) \wr G$. Then the FIC for $\pi_1(P) \wr G$ is a consequence of theorem 4.1 of [15] when $P$ is nonpositively curved. The Seifert fibered spaces case follows from theorem 4.6 of [15]. Using the hereditary property (lemma 2.1 of [15]) of the FIC we complete the proof of the Key Lemma. \[\square\]

3. **Proof of Theorem 1.2.1**

The proof goes along the same line as the proof of the main theorem of [13]. The new ingredients in the following proof are Proposition 2.2.1 and Corollary 1.1.3.

The proof is by induction on the rank of the finite index weak strongly poly-surface group.

**Induction hypothesis** $I(n)$. For any weak strongly poly-surface group $\Gamma$ of rank $\leq n$ and for any finite group $G$, FIC is true for the wreath product $\Gamma \wr G$.

If the rank of $\Gamma$ is $\leq 0$ then $\Gamma \wr G = G$ finite and hence $I(0)$ holds.

Now assume $I(n - 1)$. We will show that $I(n)$ holds.
Let $\Gamma$ be a weak strongly poly-surface group of rank $\leq n$ and is a normal subgroup of $\Delta$ with $G$ as the finite quotient group. So we have a filtration by subgroups

$$1 = \Gamma_0 \subset \Gamma_1 \subset \cdots \subset \Gamma_n = \Gamma$$

with all the requirements as in the definition of weak strongly poly-surface group.

We have another exact sequence obtained after taking wreath product of the exact sequence $1 \to \Gamma_1 \to \Gamma \to \Gamma/\Gamma_1 \to 1$ with $G$. Let $p$ be the surjective homomorphism $\Gamma \rtimes G \to (\Gamma/\Gamma_1) \rtimes G$. Note that $\Gamma_1/\Gamma_1$ is a weak strongly poly-surface group of rank less or equal to $n - 1$.

By induction hypothesis FIC is true for $(\Gamma_1 \rtimes G) \rtimes G$. We would like to apply lemma 2.2 of [15]. Let $Z$ be a virtually cyclic subgroup of $(\Gamma_1 \rtimes G) \rtimes G$. Then there are two cases to consider.

**Z is finite.** In this case we have $p^{-1}(Z) < \Gamma_1 \rtimes Z < \Gamma_1 \rtimes (G \times Z)$. Since $\Gamma_1$ is a surface group, theorem 4.1 of [15] completes the proof in this case.

**Z is infinite.** Let $Z_1 = Z \cap (\Gamma_1 \rtimes G)$. Then $Z_1$ is an infinite cyclic normal subgroup of $Z$ of finite index. Let $Z_1$ be generated by $u$. We get $p^{-1}(Z) < p^{-1}(Z_1) \rtimes K$ where $K$ is isomorphic to $Z/Z_1$.

Also,

$$p^{-1}(Z_1) \rtimes K \simeq (\Gamma_1^G \rtimes \langle u \rangle) \rtimes K < \left( \prod_{g \in G} (\Gamma_1 \rtimes \langle \alpha_g \rangle \langle u \rangle) \right) \rtimes K$$

$$< \prod_{g \in G} ((\Gamma_1 \rtimes \alpha_g \langle u \rangle) \rtimes K).$$

Now we describe the notations in the above display. Let $t \in \Gamma^G$ which goes to $u$. Then $\alpha_g(\gamma) = t_g\gamma t_g^{-1}$ for all $\gamma \in \Gamma_1$ and $t_g$ is the value of $t$ at $g$. By definition each of these actions is induced by a diffeomorphism of the surface $F_0$ whose fundamental group is isomorphic to $\Gamma_1$.

Now there are two cases: (a) $\Gamma_1$ is finitely generated and (b) $\Gamma_1$ is infinitely generated.

The proof of Case (a) follows from Proposition 2.2.1 and Corollary 1.1.3 and by noting that if the FIC is true for two groups then it is true for their direct product also (see lemma 5.1 of [15]).
(b). $\Gamma_1$ is infinitely generated and hence free. We get that each factor on the right hand side of the above display is a wreath product of the fundamental group of a noncompact 3-manifold with the finite group $K$. Again using Corollary 1.1.3 we complete the proof of the theorem.
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