Collision terms in Chiral Kinetic Theory
from the On-Shell Effective Field Theory

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We use the on-shell effective field theory (OSEFT) for the derivation of the collision terms of the chiral kinetic theory, up to the first subleading order in the energy expansion. We first prove that the OSEFT Lagrangian can also be obtained from a Foldy-Wouthuysen (FW) diagonalization of the QED Lagrangian associated to a very energetic massless fermion. OSEFT is thus the quantum field theory counterpart of the FW diagonalization in relativistic quantum mechanics for massless fermions. It is important to note that in the FW picture the associated fermions are known to interact non-minimally with the electromagnetic fields, acquire magnetic moments, and have a spatial extent of radius one half their Compton wavelength. These facts are essential to give a semi-classical interpretation of the chiral kinetic theory. We find that the leading order collision term in the energy expansion describes particle-particle and particle-antiparticle collisions, mediated by a soft-photon exchange, and the subleading correction reveals the fact that a chiral fermion interacts differently with the two transverse circular polarized photon states which are present in a medium with chiral imbalance.
I. INTRODUCTION

After the appearance in 1928 of the Dirac equation [1] to describe the quantum mechanics of relativistic fermions, it was soon realized that its interpretation would not be straightforward, due to the presence of negative energy solutions [2]. In 1930 Schrödinger found the famous Zitterbewegung (ZB) motion of the Dirac electron [3], which describes rapid oscillations between positive and negative energy states. Then, Foldy and Wouthuysen [4] realized that after a canonical transformation one could diagonalize the Dirac Hamiltonian, separating positive and negative energy eigenstates, and eliminating the ZB motion in the solutions of the relativistic equation. This transformation could be done exactly for non-interacting fermions, but only at an approximate level for interacting fermions. Relativistic fermions can be considered in the original Dirac picture as pure and localized states, with minimal coupling to electromagnetic fields, but with inextricable coupled dynamics between positive and energy eigenstates. Alternatively, one can work in the Foldy-Wouthuysen (FW) picture, the only one that admits a semi-classical interpretation [5], in which the particles interact with the electromagnetic fields non-minimally, they acquire magnetic moment interactions, and act as having a spatial extent of radius one half their Compton wavelengths [6]. Given some of the difficulties and ambiguities in the interpretation of relativistic quantum mechanics, quantum field theory has been most successfully used to study quantum effects on relativistic fermionic systems.

Different relativistic systems exhibit a classical behavior in some extreme conditions. This is the case for the high temperature and/or density of QED and QCD plasmas. It is well known that the most energetic modes of those plasmas can be described as particles obeying simple classical transport equations [7, 8]. This approach has been very successful in describing the long distance physics of those plasmas. However, one may wonder how this approach is modified after including quantum corrections, and when these become relevant.

A new semi-classical transport theory for chiral fermions, the chiral kinetic theory (CKT), has been formulated in Refs. [9–11], starting with the action of a point-particle modified by the Berry curvature, a pure quantum effect, together with a modified Poisson bracket structure. Other alternative approaches to derive the same transport equation can be found

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In the literature [12–24], some paradoxes were found in the semi-classical interpretation of the CKT when considering binary collisions. In particular, after imposing conservation of the total angular momentum, particles trajectories might seem to make discontinuous shifts, depending on the observer, the so-called “side jumps”. This fact and that the particle current is also frame dependent led to the authors of Ref. [25] to propose a reformulation of CKT once collisions are taken into account.

We consider here the approaches to the CKT of Refs. [14, 22]. In Ref. [14] it was shown that the semi-classical equations of motion used for the construction of CKT could be derived from a FW diagonalized Hamiltonian. An effective quantum field theory, the OSEFT, was then proposed to disentangle particles and antiparticles. In this Letter we prove the full equivalence of the two approaches to a certain order of accuracy, by showing that the OSEFT Lagrangian can be reproduced by carrying out a FW transformation at the Lagrangian level. We can naturally expect that OSEFT might describe properties of the FW picture of relativistic quantum mechanics as well. It is important to note that in the FW picture the fermions are not pointlike, but have an extent of radius $r_c \sim \frac{1}{2E}$, where $E$ is the fermion energy, which corresponds to the length associated to the ZB motion. In Ref. [22] we used OSEFT to derive the transport equation associated to massless fermions, and we showed that the non-standard properties under Lorentz transformations of the distribution function of CKT can be derived from the transformation rules of the OSEFT quantum fields. The distribution function of the OSEFT spinning particles is defined as a function of the mean value of the position of the fermion in the FW representation of relativistic quantum mechanics. This mean position can be considered as the center of an electrical charge distribution, which in the Dirac picture is the distribution charge that corresponds to the ZB motion. The center of mass of extended spinning objects is frame dependent, and suffers a side jump when observed in a different frame, as realized in Ref. [27]. The side jumps of the distribution function of CKT are a reflection of the same fact, the mean position of an extended spinning particle depends on the observer.

In this Letter we also provide the collision term of CKT as derived by OSEFT (details of the derivation will be given elsewhere), which is computed in a $1/E$ expansion, rather than describing the collisions in terms of classical trajectories of extended objects. This formulation can be proved to be respectful with reparametrization invariance, and ultimately
with Lorentz invariance, to a certain order of accuracy in the energy expansion \[22\].

This manuscript is organized as follows. In Sec. II we prove that the OSEFT Lagrangian up to order \(1/E^2\), the one used in Ref. \[22\] for the derivation of the CKT, can be recovered from the QED Lagrangian by carrying out subsequent FW diagonalizations. Higher-order terms could be derived as well, but we leave this for future projects. In Sec. III we present the collision terms derived from OSEFT up to order \(1/E\). We conclude in Sec. IV. We use natural units throughout, \(\hbar = c = k_B = 1\).

II. DERIVATION OF THE OSEFT FROM A FW DIAGONALIZATION

Let us assume the existence of a massless charged fermion in a given frame with energy \(E\) and light-like velocity \(v^\mu = (1, \mathbf{v})\), where \(\mathbf{v}\) is a unit 3-vector. Let us define \(\tilde{v}^\mu = (1, -\mathbf{v})\), which is also a light-like vector. Thus \(v^2 = \tilde{v}^2 = 0\), but \(v \cdot \tilde{v} = 2\). We also define the orthogonal projector,

\[
P^\mu_\perp = g^{\mu\nu} - \frac{1}{2} (v^\mu \tilde{v}^\nu + \tilde{v}^\mu v^\nu) \tag{1}
\]

The massless QED Lagrangian describing this fermion reads

\[
\mathcal{L} = \bar{\psi}^{(0)} i\not{D} \psi^{(0)} , \tag{2}
\]

where \(\psi^{(0)}\) is the standard Dirac spinor.

We first perform the change in the field

\[
\psi^{(1)} = \exp \left( iE v \cdot x \right) \psi^{(0)} \tag{3}
\]

and using the decomposition (1) the Lagrangian can be written as

\[
\mathcal{L} = \bar{\psi}^{(1)} \left( i\not{D}_\perp + \frac{\tilde{v}}{2} (iv \cdot D) + \frac{\psi}{2} (2E + i\tilde{v} \cdot D) \right) \psi^{(1)} \tag{4}
\]

We define particle and antiparticle projectors as

\[
P_v = \frac{1}{2} \gamma^\mu \gamma^\nu , \quad P_\tilde{v} = \frac{1}{2} \gamma^\mu \gamma^\nu \tag{5}
\]

respectively, where \(u^\mu = (1, 0)\). By noting that \(\psi P_v = \tilde{\psi} P_{\tilde{v}} = 0\), and that \(\psi P_v = 2\psi P_{\tilde{v}}\), and \(\psi P_{\tilde{v}} = 2\psi P_v\), one can check that Eq. (4) reproduces the Lagrangian Eq. (55) of Ref. \[14\] for a single fermion of energy \(E\) and velocity \(v^\mu\).

Unfortunately, Eq. (4) mixes up particle and antiparticle degrees of freedom due to the presence of the “odd” operator \(i\not{D}_\perp\). To disentangle these two degrees of freedom of the Dirac
field a couple of different techniques were used in Ref. [14]. First, a FW diagonalization at
the Hamiltonian level, performed as an expansion in $\hbar$. An effective field theory, the OSEFT,
was then also proposed to separate particle and antiparticle degrees of freedom of the Dirac
field at a quantum field theory level. While it is not \textit{a priori} obvious, the two approaches
are fully equivalent. To show this we present here a third equivalent way, which consists
of performing a FW diagonalization at the Lagrangian level, which allows us to recover the
OSEFT Lagrangian at a given order of accuracy. These three techniques have been proven
to be equivalent for relativistic massive fermions, when the diagonalization is carried out as
an expansion in $1/m$ [28, 29], the inverse of the fermion mass.

In order to be fully general, and to recover the results of OSEFT in an arbitrary frame [22],
from now on we will allow the frame vector $u^\mu$ to be an arbitrary time-like vector
$u^2 = 1$, while $v^\mu$ and $\tilde{v}^\mu$ are light-like vectors such $v \cdot \tilde{v} = 2$ and fulfilling the condition

$$u^\mu = \frac{v^\mu + \tilde{v}^\mu}{2}. \quad (6)$$

To remove the odd operator in Eq. (4) we carry out the canonical transformation

$$\psi^{(2)} = \exp \left( \frac{\hat{\psi} S^{(1)}}{2E} \right) \psi^{(1)}, \quad S^{(1)} = i\slashed{D}, \quad (7)$$

while the Lagrangian acting on the new field reads

$$\mathcal{L} = \bar{\psi}^{(2)} \exp \left( \frac{\hat{\psi} S^{(1)}}{2E} \right) \left( i\slashed{D} + \frac{\hat{\psi}}{2} (i\vec{v} \cdot \slashed{D}) + \frac{\hat{\psi}}{2} (2E + i\vec{v} \cdot \slashed{D}) \right) \exp \left( -\frac{\hat{\psi} S^{(1)}}{2E} \right) \psi^{(2)}. \quad (8)$$

Using the formula

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2!} [A, [A, B]] + \frac{1}{3!} [A, [A, [A, B]]] + \cdots \quad (9)$$

one can work out explicitly every term in the Lagrangian in terms of a $1/E$ expansion

$$\mathcal{L} = \bar{\psi}^{(2)} \left( \frac{i\vec{v} \cdot \slashed{D}}{2} + \frac{\hat{\psi}}{2} (2E + i\vec{v} \cdot \slashed{D}) \right) \psi^{(2)} \quad (10)$$

$$+ \frac{1}{2E} \bar{\psi}^{(2)} \left( (i\slashed{D})^2 \hat{\psi} - i\slashed{D} \hat{\psi} i\vec{v} \cdot \slashed{D} \hat{\psi} - i\vec{v} \cdot \slashed{D} i\slashed{D} \hat{\psi} - i\vec{v} \cdot \slashed{D} i\slashed{D} \hat{\psi} \right) \psi^{(2)}$$

$$- \frac{1}{16E^2} \bar{\psi}^{(2)} \left( \{i\slashed{D} \}^2, i\vec{v} \cdot \slashed{D} \right) \hat{\psi} + 2i\slashed{D} \hat{\psi} i\vec{v} \cdot \slashed{D} i\slashed{D} \hat{\psi} + 2i\slashed{D} \hat{\psi} i\vec{v} \cdot \slashed{D} i\slashed{D} \hat{\psi}$$

$$+ \{i\slashed{D} \}^2, i\vec{v} \cdot \slashed{D} \} \hat{\psi} \right) \psi^{(2)} - \bar{\psi}^{(2)} \left( \frac{1}{3E^2} (i\slashed{D})^2 \frac{1}{3E^2} \psi^{(2)} + \mathcal{O} \left( \frac{1}{E^3} \right) \right).$$

Notice that we have eliminated the odd operator at leading order in Eq. (10), but additional odd operators connecting particles and antiparticles at orders $1/E$ and $1/E^2$ are still
present. To eliminate those at $\mathcal{O}(1/E)$ we need to carry out an additional transformation,
\[ \psi^{(3)} = \exp \left( \frac{\psi^* S^{(2)}}{2E} \right) \psi^{(2)} , \]
with
\[ S^{(2)} = \frac{1}{2E} \left[ \left( i \vec{D} \cdot i \vec{v} \cdot D + i \vec{v} \cdot D i \vec{D} \right) P_v + \left( i \vec{v} \cdot i \vec{D} + i \vec{D} \cdot i \vec{v} \cdot D \right) P_{\tilde{v}} \right] . \]
This transformation generates itself new odd terms at subleading orders, while keeping the even operators untouched. Yet another transformation,
\[ \psi^{(4)} = \exp \left( \frac{\psi^* S^{(3)}}{2E} \right) \psi^{(3)} , \]
with
\[ S^{(3)} = \frac{1}{4E^2} \left( i \vec{D} \cdot i \vec{v} \cdot D + 2i \vec{v} \cdot D i \vec{D} \right) P_v + \left( i \vec{D} \cdot i \vec{v} \cdot D + (i \vec{v} \cdot D)^2 i \vec{D} \right) P_{\tilde{v}} - \frac{1}{3E^2} (i \vec{D})^3 \]
will remove all the pieces that mix particles and antiparticles at order $1/E^2$ (while there will be odd operators at the following orders in the energy expansion). Successive canonical transformations should be done at every order in the energy expansion to achieve a full diagonalization.

It is now easy to see how these FW partial diagonalizations allow us to reproduce the OSEFT Lagrangian at a certain order of accuracy. If we define the particle/antiparticle components at any order $(n)$ in the FW diagonalizations:
\[ \chi^{(n)} \equiv P_v \psi^{(n)} , \quad \xi^{(n)} \equiv P_{\tilde{v}} \psi^{(n)} \]
then at $\mathcal{O}(1/E^2)$ the particle Lagrangian reads
\[ \mathcal{L} = \chi^{(4)} \left( i \vec{v} \cdot D + \frac{1}{2E} (i \vec{D})^2 \right) - \frac{1}{8E^2} \left\{ (i \vec{D})^2 , i \vec{v} \cdot D \right\} - \frac{1}{4E^2} i \vec{D} \cdot i \vec{v} \cdot D i \vec{D} \right) \frac{\psi^{(4)}}{2} , \]
plus the analogous term for the antiparticle field $\xi^{(4)}$.

We stress that the diagonalization we have carried out for massless fermions assumes that $E$ is the hard scale, larger than the values of the electromagnetic fields and their gradients, and also of the derivatives of the Dirac field. Note also that
\[ \chi^{(4)} = e^{iEv} \left( \chi^{(0)} + \frac{\psi^* i \vec{D}}{E} \xi^{(0)} - \frac{(i \vec{D})^2}{8E^2} \chi^{(0)} - \frac{\psi}{4E^2} (i \vec{v} \cdot D i \vec{D} + i \vec{D} \cdot i \vec{v} \cdot D) \xi^{(0)} \right) + \mathcal{O} \left( \frac{1}{E^3} \right) , \]
that is, the new particle field is a combination of the particle and antiparticle fields of the
original Dirac picture \[4\]. The covariant derivatives in the expansion also tells us about the
non-local relation between the original Dirac picture and the FW one.

Let us note that the Lagrangian for \( \chi^{(4)} \) contains temporal derivatives beyond the leading
order term. Exactly as in Ref. \[30\], we perform a local field redefinition to eliminate temporal
derivatives beyond the leading order term. Thus, after doing

\[
\tilde{\chi} = \left(1 + \frac{(i\not{D} \perp)^2 \not{\gamma}}{4E^2}\right) \chi^{(4)},
\]

we end up at order \(1/E^2\)

\[
\mathcal{L} = \tilde{\chi} \left( iv \cdot D + \frac{1}{2E} (i\not{D} \perp)^2 + \frac{1}{8E^2} \left\{ \left\{ (i\not{D} \perp)^2, (iv \cdot D - i\tilde{v} \cdot D) \right\} - [i\not{D} \perp, [i\tilde{v} \cdot D, i\not{D} \perp]] \right\} \right) \frac{\not{\gamma}}{2} \tilde{\chi},
\]

which is the OSEFT Lagrangian deduced in Ref. \[30\], and used in Ref. \[22\] for the derivation
of the chiral transport equation.

While here we have shown how to derive the OSEFT Lagrangian associated to a single
fermion, it is possible to generalize the method and perform the diagonalizations associated
to having several fermions. One can also perform similar diagonalizations to derive the
OSEFT Lagrangian for the on-shell antiparticles, simply exchanging \( E \rightarrow -E \), and \( \nu^\mu \leftrightarrow \tilde{\nu}^\mu \)[22] in all the preceding equations.

In Ref. \[14\] the OSEFT was derived using the modern language of effective field theories,
where to describe on-shell particles one integrates out the off-shell modes. Like in the QED
Lagrangian these two set of modes are inherently coupled through the equations of motion.
When the off-shell components are integrated out, only particles remain in the effective theory
at the expense of having an infinite series of operators in the Lagrangian, but suppressed
by successive powers of \(1/E\). The FW diagonalization allows for a similar decoupling of
particles and antiparticles order by order in \(1/E\). As we arrive at the same result with FW
diagonalizations, we can therefore say that this and the OSEFT original approach describe
the same physics.
III. COLLISION TERMS AS DERIVED FROM OSEFT

In this Section we simply provide the result of the computation of the collision term of CKT, as derived from OSEFT. Details of the derivation will be presented in a forthcoming publication.

The collisionless transport equation of CKT was computed in Ref. [22] to $1/E^2$ accuracy. It is expressed by the left hand side of

$$\left( v_{1}^{\mu} - \frac{e^2}{2E_{1}^2} S_{\chi,1}^{\mu \nu} F_{\nu \rho}(X)(2u^\rho - v_1^\rho) \right) \left( \frac{\partial}{\partial X^\mu} - eF_{\mu \nu}(X) \frac{\partial}{\partial K_{1,\nu}} \right) f^{\chi}(X, K_1) = C[f, f] + \bar{C}[f, \bar{f}]$$

(20)

where $u^\mu$ is the frame vector, $K_1^\mu = (K_1^0, k_1)$ denotes the 4-momentum of the particle 1, $v_1^\mu = K_1^\mu / (u \cdot K_1)$, and

$$S_{\chi,1}^{\mu \nu} = \chi \frac{\epsilon^{\alpha \beta \mu \nu} u_\beta K_{1,\alpha}}{2 u \cdot K_1}, \quad \chi = \pm 1,$$

(21)

is the spin tensor, and $F_{\mu \nu}$ is the electromagnetic field. We have already taken into account the on-shell relation $K_1^2 - e S_{\chi,1}^{\mu \nu} F_{\mu \nu} = 0$. A similar equation holds for the on-shell antiparticle distribution function $\bar{f}^{\chi}$.

Our equation differs from that derived in Ref. [15], see also Ref. [16]. In Ref. [23] it is claimed that the discrepancy is due to the fact that the two different equations act on different Wigner functions. The equation deduced in Ref. [15] acts on a Wigner function associated to the full density, and thus contains the sum of the particle and antiparticle sectors, while our Wigner function is projected over the particle sector exclusively. It is unclear to us how the antiparticle sector drops out in Ref. [15], avoiding the mixing with the particles.

We present the collision term computed only to $1/E$ accuracy with OSEFT, and in the frame defined by the plasma $u^\mu = (1, 0)$, as it simplifies. The collision term is composed by two pieces, the first one corresponding to particle-particle collisions, $C[f, f]$, and a second one, $\bar{C}[f, \bar{f}]$, which describes collisions of the particle with the antiparticles. The two incoming fermions, 1 and 2, have arbitrary helicities $\chi$ and $\chi'$. Both are conserved in the collision so that the two outgoing fermions, 3 and 4, carry helicities $\chi$ and $\chi'$, respectively. As in (20), we have integrated over the energies the transport equation, so that it is taken
on shell. Then

\[
C[f, f] = \frac{1}{2E_1} \int \frac{d^3k_2}{(2\pi)^3} \frac{d^3k_3}{(2\pi)^3} \frac{d^3k_4}{(2\pi)^3} \sum_{\chi'} |M_{\chi, \chi'}|^2 (2\pi)^4 \delta^{(4)}(K_1 + K_2 - K_3 - K_4)
\times \left\{ f_3^\chi f_4^\chi' [1 - f_1^\chi][1 - f_2^\chi'] - f_1^\chi f_2^\chi' [1 - f_3^\chi][1 - f_4^\chi'] \right\},
\]

where we used \( f_i^\chi \equiv f^\chi(X, K_i) \), and all the energies \( E_i \) are on-shell. The scattering amplitude squared reads

\[
|M_{\chi, \chi'}|^2 \equiv 4e^4 E_1^2 E_2^2 D_{R}^{\mu\nu}(X, q) D_{A}^{\alpha\beta}(X, q)
\times \left\{ v_1^\mu \left( v_1^\nu - \frac{q_{1,1}^\nu}{E_1} \right) + v_1^\nu \left( v_1^\mu - \frac{q_{1,1}^\mu}{E_1} \right) - \frac{2i}{E_1} v_1^\mu q_\alpha S_{\chi,1}^{\alpha\mu} + \frac{2i}{E_1} v_1^\nu q_\alpha S_{\chi,1}^{\alpha\nu} \right\}
\times \left\{ v_2^\sigma \left( v_2^\rho + \frac{q_{2,2}^\rho}{E_2} \right) + v_2^\rho \left( v_2^\sigma + \frac{q_{2,2}^\sigma}{E_2} \right) - \frac{2i}{E_2} v_2^\rho q_\sigma S_{\chi',2}^{\rho\sigma} + \frac{2i}{E_2} v_2^\sigma q_\sigma S_{\chi',2}^{\rho\sigma} \right\}.
\]

written in terms of the momentum transfer \( q_\mu = K_1^\mu - K_3^\mu \). Here \( q_{1,1/2}^\mu \) are the orthogonal components to the \( v_{1/2}^\mu \) vectors, respectively.

The retarded/advanced photon propagator is denoted by \( D_{R/A}^{\mu\nu} \) and depends on the momentum transfer of the collision. In a medium where parity \( P \) and \( CP \) (where \( C \) denotes charge conjugation) are broken by the presence of a chiral chemical potential, the photon propagators can be written in Coulomb gauge in terms of a longitudinal component, and two circular polarized transverse states \[31, 32\]

\[
D_{\mu\nu}(X, q) = \delta_{\mu0}\delta_{\nu0}D_L(X, q) + \sum_{h=\pm} P_{\mu\nu}^{T,h}D_T^h(X, q),
\]

where \( h = \pm \) labels the two circular polarised transverse photon states, left and right, and we introduced the projectors

\[
P_{\mu\nu}^{T,h} = \frac{1}{2} \left( \delta_{ij} - \hat{q}_i \hat{q}_j - ih\epsilon^{ijk} \hat{q}_k \right) \delta_{\mu i} \delta_{\nu j},
\]

with \( \hat{q}^i = q^i/|q| \).

At leading order in the energy expansion, Eq. (23) describes the collision of two energetic fermions with the exchange of a soft photon. The corrections of order \( 1/E \) in Eq. (23) include also the effect that a chiral fermion interacts differently with the two transverse circular photon polarized states in a medium with chiral imbalance, as previously found out in Ref. [22] for a thermal plasma with chiral imbalance.
IV. CONCLUSIONS

Foldy and Wouthuysen designed a formulation to derive the correct non-relativistic limit of QED by carrying out subsequent diagonalizations of the Dirac Hamiltonian in a $1/m$ expansion. Their approach also made possible a semi-classical interpretation of relativistic quantum mechanics, even if the resulting framework described particles with different properties than in the original Dirac picture. Later on, some effective field theories, such as non-relativistic QED (NRQED) or heavy quark effective theory (HQET) were formulated to describe the field theoretical counterpart of the FW formulation. The two different approaches were shown to be equivalent.

In this manuscript we have shown that the OSEFT can be obtained from a FW diagonalization of the massless QED Lagrangian, showing thus that it is the generalization of the ideas that have been widely applied for massive fermions to the massless case. While in the first case one requires the fermion mass to be the large scale, in the second it is the fermion energy itself. This then allows us to recognize the limitations of OSEFT, and also of CKT, as for example the low energy modes in a plasma cannot be described as quasiparticles in the way that the high energetic modes are.

The advantages of the effective field theory formulation are clear. OSEFT has already been used to describe power corrections to the hard thermal loops of QED in a thermal plasma. It is also possible to check that the formulation is respectful with reparametrization invariance, and thus also, with Lorentz invariance. However it is important to realize that in order to give a semi-classical interpretation of the FW/OSEFT formulations, the associated particles cannot be considered as pure localized states, but rather having a spatial extension of the order of the Compton wavelength. This allows us to understand the non-standard properties under Lorentz symmetry of CKT.

Procedures to derive collision terms from quantum field theories are standard. We have used them in this manuscript with the OSEFT, thus completing the formulation of CKT that was started in Ref. [22]. Details of the derivation, or applications of our results will be discussed in forthcoming publications.
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