Using spectator distributions to measure the initial geometry fluctuation

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A study of eccentricity ($\varepsilon_n$) fluctuations and its possible impact on final state momentum anisotropy ($v_n$) for symmetric collisions are presented in the framework of Glauber model. Effect of fluctuations of nucleon’s position on the initial geometry has been studied using a new method, where the difference between oppositely moving spectators is taken as a measurement of eccentricity fluctuations. This study shows that higher harmonics ($n=3, 4$ and $5$) of eccentricity are less sensitive to fluctuations in transverse plane compared to the $2^{nd}$ harmonic. Position fluctuations in transverse plane will increase $\varepsilon_2$ and hence possibly $v_2$ for the most central nucleus-nucleus collisions. For semi-central and peripheral collisions, the fluctuations have opposite effect, it deceases the eccentricity $\varepsilon_2$. The fluctuation of initial geometry can be studied in collider experiments by studying the spectator distribution on the both sides of the beam.

PACS-key : 25.75.Ld, 25.75.-q

1. Introduction

One of the main goals of the high energy heavy-ion collision experiments is to study the QCD phase diagram. To achieve this goal, one has to understand the properties of the system formed in such collisions. The momentum azimuthal angular anisotropy parameter $v_n$ has been considered as a good tool for studying the system formed in the early stages of high energy collisions at Super Proton Synchrotron (SPS), Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC). It describes the $n^{th}$ harmonic coefficient of the azimuthal Fourier decomposition of the momentum distribution with respect to the reaction plane angle ($\Psi$). The final state momentum anisotropy ($v_n$) reflects the hydrodynamic response of initial spatial anisotropy ($\varepsilon_n$). According to hydrodynamical description, $v_n$ is sensitive to the geometry of initial state of the system formed in the collision as well as the hydrodynamic evolution governed by the equation of state of the matter. Knowing the initial geometry and fluctuations in heavy-ion collisions has recently been shown to have important consequences on interpreting the experimental data from various experiment at RHIC and LHC. Experimentally measured non zero odd harmonic ($n\geq3$) has been interpreted as the result of statistical fluctuations in the transverse positions (according to uncertainty principle) of nucleons undergoing hadronic scattering. Moreover, measured $v_2$ cannot be described by an smooth

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initial energy density distribution, unless one includes flow fluctuations arising due to the eccentricity fluctuations in the calculations \(^{15,16}\). In addition to \(v_n\), there are other experimental observables which cannot be explained without including eccentricity fluctuations. For example, dihadron correlations in azimuthal angle \(^{17}\) and pseudorapidity \(^{18,19}\). The contribution from the odd harmonics associated with the particle azimuthal angle distribution to dihadron correlations is found to be an important factor.

Several phenomenological studies have been carried out on initial geometry anisotropy and fluctuations to understand its influence on experimental data \(^{20,21,22,23,24,25}\). The aim of this paper is to discuss the centrality dependence of various harmonics of initial spatial anisotropy and its sensitivity to the fluctuation in position of nucleons using a new method. In this paper, \(\varepsilon_n\) are calculated within a framework of Monte Carlo Glauber (MCG) model \(^{26}\), which allows the generation of collisions with event-by-event-fluctuating initial condition. The paper is organized in the following way. In section 2, Glauber model has been briefly discussed. Section 3 describes the study of \(\varepsilon_n\) and its sensitivity to the fluctuations using the MCG model. Finally, the summary has been given in section 4.

2. Model Description

In MCG model, the nuclear distribution function inside a nucleus is taken to be of the Woods-Saxon form,

\[
\rho_A(r) = \frac{\rho_0}{1 + \exp[(r - R)/d]},
\]

where the radius \((R)\) and the diffuse constant \((d)\) are taken as \(R = 6.38\) fm, \(d = 0.535\) fm for Au nucleus. In this model, nuclei are assembled by positioning the nucleons randomly in a three-dimensional coordinate system, on an event-by-event basis, according to the Woods-Saxon density profile. A collision between two nuclei is considered as a sequence of independent nucleon-nucleon collisions. In a nucleus-nucleus collision, two nucleons with transverse distance \(d \leq \sqrt{\sigma_{NN}/\pi}\) will collide with each other. Here \(\sigma_{NN}\) is the total nucleon-nucleon cross-section.

Using the transverse position coordinates of each colliding nucleon, various moments of participant eccentricity \(^{24}\) have been calculated as:

\[
\varepsilon_n = \frac{\langle r^2 \cos(n\varphi_{part}) \rangle^2 + \langle r^2 \sin(n\varphi_{part}) \rangle^2}{\langle r^2 \rangle},
\]

where \(r\) and \(\varphi_{part}\) are the polar coordinate positions of participating nucleons.

\[
r = \sqrt{x^2 + y^2}
\]

and

\[
\varphi_{part} = \tan^{-1}(y/x).
\]

In this study, approximately 7 million events for each configuration with fixed
impact parameter $b = 1\text{ fm}$, $5\text{ fm}$, and $9\text{ fm}$, are generated for Au+Au collisions with $\sigma_{NN} = 40\text{ mb}$. Variation in $\sigma_{NN}$ does not change results qualitatively. A standard MCG model code which is used as an input in AMPT model $^{27,28}$ has been used to generate events in this study. The distribution of nucleons in the transverse plane for a single event is shown in Fig. 1.

3. Results and Discussion

Fig. 2 shows magnitude of spatial initial eccentricity in transverse plane for different harmonics (from $n=2$ to $n=5$) in Au+Au collisions at $b = 1\text{ fm}$, $b = 5\text{ fm}$ and $b = 9\text{ fm}$. The large value of $\varepsilon_2$ for $b = 5\text{ fm}$ and $b = 9\text{ fm}$ is due to initial elliptic shape of the overlapping region in a collision of large impact parameter. All odd higher harmonics ($n > 2$) are generated due to fluctuations in transverse positions of nucleons. For a nucleus with smooth density distribution, all odd higher harmonics ($n > 2$) of $\varepsilon_n$ will be zero. In case of nucleus-nucleus collisions at $b = 1\text{ fm}$, the initial overlapping geometry is almost isotropic, hence magnitude of $\varepsilon_2$ is small and comparable with values of $\varepsilon_3$, $\varepsilon_4$ and $\varepsilon_5$. Centrality dependence of $\varepsilon_2$ can be understood, since it reflects the anisotropy of overlapping region of two nuclei. But we observed, as shown in Fig. 2, that all higher harmonics of $\varepsilon_n$ reveal similar centrality dependence, like to that $\varepsilon_2$. The fluctuations behave differently for collisions with different impact parameter. Although the non-zero $\varepsilon_2$ is originated due to initial elliptic shape, it can be modified by the nucleon density fluctuations. Therefore, it is very important to understand the role of initial geometry fluctuations in heavy-ion collisions. The main purpose of this paper is to investigate how various harmonics of eccentricity change with nucleon density fluctuations.
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The fluctuation varies on event-by-event basis. Therefore, the number of spectators \( (S) \) and participants \( (N_{\text{part}}) \) will also vary on event-by-event basis. Moreover, in a single event, number of spectators from target nucleus (labeled as \( A \)) and projectile nucleus (labeled as \( B \)) can be different due to the fluctuation of nucleon’s position in the transverse plane \(^{29}\). In this study, the difference in the number of spectator between two colliding nuclei \( (|S^A - S^B|) \) has been used to quantify the fluctuation, more fluctuation means large difference and vice-versa. Total number of events for a fixed impact parameter are divided in several sub-groups based on \( |S^A - S^B| \).

Fig. 3 shows, event-by-event distribution of number of spectators in \( A \)-nucleus \( (S^A) \) and in \( B \)-nucleus \( (S^B) \) from MCG model at \( b = 5 \ fm \). The maximum difference between \( S^A \) and \( S^B \) can be of the order of 50.

Values of \( \varepsilon_2, \varepsilon_3, \varepsilon_4, \) and \( \varepsilon_5 \) as function of \( |S^A - S^B| \) are shown in Fig. 4. Panel (a), (b) and (c) corresponds to events with fixed \( b = 1, 5 \) and \( 9 \ fm \), respectively. In each case, magnitude \( \varepsilon_3, \varepsilon_4, \) and \( \varepsilon_5 \) are scaled to match their values with \( \varepsilon_2 \) at \( |S^A - S^B| = 5 \). For central events with \( b = 1 \ fm \) (i.e. panel(a)), all harmonics of \( \varepsilon_n \) increases with increase in \( |S^A - S^B| \). This indicates that, fluctuations enhance the anisotropy for the most central collisions. As we have fixed the value of impact parameter, changes in \( \varepsilon_n \) is entirely due to fluctuations. What is striking in this observation is that, fluctuations making the system more elliptic, while also makes the system more triangular, quadratic and pentagonal.

Now for semi-central events with \( b = 5 \ fm \) (panel(b)), we can see that there is small change in eccentricity for \( n \geq 3 \), their values are increasing with increase in \( |S^A - S^B| \). But we observed that the magnitude of \( \varepsilon_2 \) decreases with increase in \( |S^A - S^B| \), unlike central events. This shows that the fluctuations in the transverse plane are decreasing initial elliptical geometry a nucleus-nucleus collision.

For peripheral events with \( b = 9 \ fm \) (panel(c)), \( \varepsilon_2 \) changes sharply with change
in $|S^A - S^B|$, and shows a decreasing trend with increasing fluctuations, like semi-central events. The $\varepsilon_3$ and $\varepsilon_5$ increases with fluctuations and almost negligible change for $\varepsilon_4$ with respect to other harmonics. To quantify the sensitivity of $\varepsilon_n$ to fluctuations, ratios between maximum ($\varepsilon_n^{\max}$) and minimum ($\varepsilon_n^{\min}$) value of eccentricity has been calculated. The maximum change in $\varepsilon_2$, $\varepsilon_3$, $\varepsilon_4$ and $\varepsilon_5$ are $\sim 15\%$, $12\%$, $3\%$ & $7.5\%$; $\sim 8.3\%$, $4.5\%$, $2\%$ & $3.5\%$; and $\sim 52\%$, $14\%$, $3\%$ & $8.5\%$; for $b = 1$, 5 and 9 fm, respectively. This indicates that of all the harmonics, $\varepsilon_2$ is more sensitive to the fluctuation and then $\varepsilon_3$. This observation is consistent with the previous study done using different asymmetric collision in AMPT model $^{25}$, where it was shown that $v_2$ is more sensitive than $v_3$. On the other hand, we observed that the $\varepsilon_4$ is less sensitive to the fluctuations compared to $\varepsilon_5$.

We know that the final state momentum anisotropies are driven by the initial spatial anisotropy and flow coefficients ($v_n$) are proportional to $\varepsilon_n$. Therefore the change in $\varepsilon_n$ due to fluctuation will affect $v_n$ in similar manner. Sensitivity of the $v_n$ to the fluctuation can be different compared to $\varepsilon_n$ and that depends how the $\varepsilon_n$ evolve through different stages of the fireball history and translate into final-particle momentum anisotropies. But qualitatively, sensitivity of the $v_n$ could be similar like $\varepsilon_n$. Therefore, from Fig. 4 we expect that the initial fluctuations in transverse plane will generate more $v_2$ in most central collisions, whereas for semi-central to peripheral collision magnitude of $v_2$ will be reduced due to the fluctuations. This observation is in agreement with previous study carried out using 3+1 Viscous Hydrodynamics in Ref $^{16}$. In real collider experiment, one can easily measure number of spectator in both direction of beam and hence the difference between them. Therefore, using this
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Figure 4: (Color online) Spatial eccentricity $\varepsilon_2, \varepsilon_3, \varepsilon_4,$ and $\varepsilon_5$ versus $|S^A - S^B|$ for collisions at (a) $b = 1$ fm, (b) $b = 5$ fm and (c) $b = 9$ fm

method one can identify the events with different amount of fluctuations in one centrality bin and can measure $v_n$ to understand the effect of fluctuation. Narrow centrality bins will be more appropriate for this study. In experiment, centrality is usually estimated by number of particles produced in the collision. The number of spectators is anti-correlated with the number of particle participating nucleons in the collisions. We have studied how the average $N_{part}$ changes with $|S^A - S^B|$, as shown in Fig. 5, to estimate the effect of event mixing from different centrality group. We can see from Fig. 5 that the change in $<N_{part}>$ is less than 3%.

4. Summary

A study on initial collision geometry fluctuations for a symmetric system using MCG model has been presented. It has been observed that all other higher harmonics of $\varepsilon_n$ show centrality dependence like $\varepsilon_2$. A new method using number of spectator nucleons has been used to separate events with different amount of fluctuations. Due to fluctuations in the transverse plane of colliding nuclei, $\varepsilon_n$ (and
possibly \( v_2 \) increases in most central collision. For semi-central and peripheral collisions, \( \varepsilon_2 \) (and possibly \( v_2 \)) is minimised by the fluctuations, on the other hand all other higher harmonics are found to be higher due the fluctuations in the transverse plane. Moreover, we observed that 2\(^{nd}\) harmonic is more sensitive to the collision geometry fluctuation compared to higher harmonics although higher harmonics are generated due to fluctuations. This new proposed method can be applied to more realistic transport model and in real experiment to study the fluctuations in \( v_n \), which is very crucial to understand various properties like transport coefficient of the system created in heavy-ion collisions.

Only the fluctuation of nucleon’s position in the transverse plane has been discussed in this paper. For ultra-relativistic nucleus-nucleus collisions, nuclei are contracted along beam axis (Z-axis) and looks like as thin plates in lab-frame. As a result, fluctuations of nucleons position along longitudinal directions are negligibly small. But in case of collisions at a low energy, like AGS energy, where the out-of-plane squeeze-out phenomena in elliptic flow was observed, fluctuations along longitudinal directions may not be negligibly small. Future investigation can be done in this direction using various transport model to see the effect of longitudinal fluctuations in the final states momentum anisotropies using this new proposed methods.

Acknowledgments : This work is supported by the DOE Grant of Department of Physics and Astronomy, UCLA, USA.

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