We outline how the time-reversal symmetry $T$ can be systematically used to test the Kobayashi-Maskawa mechanism embedded in the CKM matrix using pairs of $B$ mesons created at the $\Upsilon(4S)$ and pairs of $D$ mesons from $\psi(3770)$. 

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Weak decays are known to violate the set of improper (i.e. discrete) transformations \(P\) (parity), \(C\) (charge conjugation), \(T\) (time-reversal), and \(CP\). It is well known that electromagnetism and the strong force obey these symmetries, as bound by existing experimental data. The combination \(CPT\) is related to the Lorentz group, and is conserved in locally gauge invariant quantum field theories. Models of quantum gravity may violate Lorentz invariance, and hence \(CPT\). These violations in turn could manifest themselves as a difference between the \(CP\) and \(T\) violations found in weak decays, hence \(CP\), \(T\), and \(CPT\) tests are complementary parts of a triplet of tests of quark flavour transitions. It has long been known that one can perform a Kabir asymmetry test that can be interpreted in a dual way, i.e. as a \(CP\) and \(T\) symmetry test [1]. In 2012 the \(\text{B}A\text{B}\text{A}\text{R}\) experiment, following a prescription laid down in [3], performed a test of \(T\) symmetry non-invariance (as well as \(CP\) and \(CPT\) tests) in \(B\) decays using pairs of mesons, one decaying via a flavour filter, and the other decaying into either the \(CP\)-even or \(CP\)-odd filter state corresponding to a golden mode decay \(B \to c\bar{c}K_{S,L}^0\) [3]. These proceedings summarise the work presented in [4], where we discuss applications of this methodology to other sets of filter bases in the context of entangled pairs of neutral \(B_{d,s}\) and \(D\) mesons. The remainder of these proceedings start with a recap of the methodology, followed by considerations of applications to \(B\) and \(D\) decays before concluding with a brief summary.

Entangled pairs of neutral pseudoscalar mesons (denoted by \(P_1\) and \(P_2\)) provide one with access to a physical system were the time ordering of events can be naturally reversed by comparing the two time orderings of the superposition \(\Phi\). The wave function associated with such an entangled state is \(\Phi = (|P_1P_2\rangle + |P_2P_1\rangle)/\sqrt{2}\), and this can be experimentally tested as there is a pair of mesons that collapse into either the first or the second time ordering. The next key point is to understand the \(T\) conjugate pairs of decay filters required to experimentally reconstruct and compare between the two time orderings. In order to unambiguously test \(T\) one requires two different orthonormal basis pairs. These can be any orthonormal pairs, but for convenience flavour i.e. \{particle, antiparticle\} and \(CP\) eigenvalue \{+, −\} are obvious choices. Hence there are four distinct comparisons that can be made for a given scenario. These are (i) \(\bar{P}^0 \to P_− \to \bar{P}^0\) vs \(P_+ \to P^0 \to P_+\), (ii) \(P_+ \to P^0 \to P_+\) vs \(\bar{P}^0 \to P_− \to \bar{P}^0\), (iv) \(P_− \to P^0 \to P_−\) vs \(\bar{P}^0 \to P_+ \to \bar{P}^0\), where \(P^0 (\bar{P}^0)\) refers to a neutral meson flavour filter decay identifying a particle or anti-particle, and those with subscripts \(+\) or \(−\) refer to the eigenvalue of the \(CP\) filter final state.

The flavour filter basis pair is defined by neutral meson decays to flavour specific final states, i.e. states that are only accessible to particle or anti-particle decays. The definition of this set could be extended to include Cabibbo allowed vs Cabibbo suppressed decays in the case of flavour tagging for \(D\) mesons, at the cost of dilution of the signal. The original proposal for the \(CP\) filter basis was to use the approximately orthonormal set given by the \(B\) decay to a charmonium (\(c\bar{c}\)) plus a \(K_S\) (\(CP = −1\)) or \(K_L\) (\(CP = +1\)). This can be extended to include a number of other final states as discussed below. As pointed out in [4], one should also recognise that decays of pseudoscalars to two spin one (vector or axial-vector) particles also constitute a set of exact \(CP\) filter basis pairs if one performs a full angular analysis to separate out the even and odd components. This broadens the range of \(T\) violation tests that one can perform in the Standard Model.

Interactions resulting in the decay of \(\Upsilon(4S) \to B^0\bar{B}^0\) and \(\psi(3770) \to D^0\bar{D}^0\) are equivalent in terms of quantum entanglement of the final state. This entanglement has been validated by Belle for the \(\Upsilon(4S)\) scenario [5] and is assumed to be valid for \(\psi(3770)\) decays (which can be tested at a suitable charm factory).
As detailed in [4] it is possible to measure the Unitarity Triangle angle $\beta$ ($\alpha$) using $b \to c, s, d$ ($u$) decays. In addition one can perform $T$ symmetry tests in $c \to u, d, s$ transitions at the $\psi(3770)$. For charm the goal is first to test $T$ violation in mixing, and ultimately one day to extend the interpretation to constraining the charm Unitarity Triangle angle $\beta_c$. Interpretation is in terms of the $T$ violating phase measured for $\lambda_f = (q/p)(\bar{A}/A)$. The step from mixing constraints $(q/p)$ to the weak structure of interference between mixing and decay ($\lambda_f$) requires precision that goes beyond the current generation of experiments, and an improved understanding of hadronic uncertainties in the charm sector. Constraints on $\gamma$ using $c \to u$ decays are not viable as, while these processes contribute to a number of decays, they appear as sub-dominant penguin amplitudes. A review of possible interesting modes to construct $CP$ filter bases from can be found in [4, 3].

A number of previously unthought of $CP$ filter basis pairs can be used to search for $T$ violation. These include the following $B$ decays: the $b \to s$ loop decays $B \to (\eta', \phi, \omega)K_{S,L}$ and $B \to \phi K^*$; $b \to d$ loop transitions $B \to D^{*+}D^{*-}$; the $b \to c$ transition $B \to J/\psi K^*$; and $B \to J/\psi \rho$ which is a colour suppressed $b \to c$ transition with a potential $b \to d$ penguin contamination, all these states measure $\beta_c$. Additionally one can measure $\alpha$ using $b \to u$ transitions such as $B \to \rho \rho$ and $\omega \rho$ decays. Table 1 summarises estimates of precisions attainable for $T$ symmetry parameters (related to $2\sin 2\beta$) for $B$ decays at the current $B$ Factories and at Belle II. Effects at the level of the Standard Model expectation should be observable in all modes at Belle II (with $50ab^{-1}$ of data). In terms of $D$ decays the $CP$ filter basis modes of interest include: $D^0 \to K_{S,L}^0(\omega, \eta, \eta', \rho, \phi, f^0, a_0)$ and kinematically allowed modes with pairs of (axial-)vector mesons in the final state.

In summary we outline a set of $T$ symmetry invariance tests of $b \to u, c, d$, and $s$ as well as $c \to d$, and $s$ filter basis transitions that would enable one to over-constrain our understanding of the unitarity of the CKM matrix in terms of $B_{d,s}$ and $D$ decays in the context of the Standard Model. After almost five decades of $CP$ violation measurements it is possible to embark upon an equivalent era of $T$ (and $CPT$) violation tests in weak interactions to probe possible new physics contributions in tree and loop decays.

### Table 1: Estimated sensitivities on $\sigma(2\sin 2\beta)$ for promising $CP$ filter pairs.

| Filter basis pair | $B$ Factories | Belle II |
|-------------------|---------------|----------|
| $\eta'K_{S,L}^0$ | 0.6           | 0.08     |
| $\phi K^*$       | 1.1           | 0.13     |
| $\eta K_{S,L}^0$ | 1.8           | 0.17     |
| $\omega K_{S,L}^0$ | 2.0       | 0.22     |
| $D^*D^*$         | 2.0           | 0.29     |

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