Emission times and opacities from interferometry in non-central Relativistic Nuclear Collisions

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The nuclear overlap zone in non-central relativistic heavy ion collisions is azimuthally very asymmetric. By varying the angle between the axes of deformation and the transverse direction of the pair momenta, the transverse HBT radii oscillate in a characteristic way. It is shown that these oscillations allow determination of source sizes, deformations as well as the opacity and duration of emission of the source created in any non-central high energy nuclear collisions. The behavior of the physical quantities with centrality of the collisions is discussed — in particular changes caused by a possible phase transition to a quark-gluon plasma.

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Understanding particle emission in relativistic nuclear collision is crucial in order to determine whether a phase transition to quark-gluon plasma has occurred. Particle interferometry was invented by Hanbury-Brown & Twiss (HBT) for stellar size determination and is now employed in nuclear collisions. It is a very powerful method to determine the 3-dimensional source sizes, lifetimes, duration of emission, flow, etc. of pions, kaons, etc. at freeze-out. Since the number of pairs grow with the multiplicity per event squared the HBT method will become even better at RHIC and LHC colliders where the multiplicity will be even higher. In this paper an extension of the HBT method is presented for asymmetric and opaque sources created in non-central collisions. It is shown that combining HBT with a determination of the reaction plane can be exploited to find not only the size and deformation of the source but also its opacity as well as the duration of emission separately.

The reaction plane, which breaks azimuthal symmetry, has been successfully determined in non-central heavy ion collisions from intermediate up to relativistic energies. The particle spectra are expanded in harmonics of the azimuthal angle φ event-by-event. The transverse particle distribution as a gaussian function of transverse radius. It is therefore convenient for asymmetric sources to employ gaussian parametrizations in both transverse directions

\[ S_\perp (x, y) \sim \exp \left( -\frac{x^2}{2R_x^2} - \frac{y^2}{2R_y^2} \right). \]  

Here \( R_x \) and \( R_y \) are the gaussian transverse sizes of the collision zones at freeze-out. This elliptic source neglects directed flow and we shall therefore be restricted to regions around cms midrapidity where \( v_1 \) vanish.

The azimuthal asymmetry or “deformation” of the source can be defined as the relative difference between the gaussian radii squared

\[ \delta = \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2}. \]  

A simple estimate can be obtained from the full transverse extent of the initial overlap of two nuclei with radius \( R_A \) colliding with impact parameter \( b \) (see Fig. 1). They are simply: \( R_x = R_A - b/2 \) and \( R_y = \sqrt{R_A^2 - b^2/4} \), and the corresponding deformation is

\[ \delta = \frac{b}{2R_A}. \]  

The rms radii of the nuclear overlap zone weighted with longitudinal thicknesses results in a deformation that is only slightly smaller at semicentral collisions. However, as the source expands the deformation \( \delta \approx (R_y - R_x)/(R_y + R_x) \) decreases for two reasons. Firstly, the expansion increase \( (R_x + R_y) \) and secondly, \( (R_y - R_x) \) decrease because the average velocities are larger in the x- than y-direction. The latter is a consequence of the experimentally measured positive elliptic flow \( (v_2 > 0) \) in Eq. (3) in relativistic nuclear collisions where shadowing is minor. Measuring the decrease of the deformation with centrality will reveal important information on the expansion up to freeze-out. For very peripheral collisions, where only a single nucleon-nucleon collision occurs, the source must be azimuthally symmetric, i.e., the
deformation must vanish and therefore Eq. (4), which assumes continuous densities, breaks down.

The standard HBT method for calculating the Bose-Einstein correlation function from the interference of two identical particles is now briefly discussed. For a source of size $R$ we consider two particles emitted a distance $\sim R$ apart with relative momentum $q = (p_1 - p_2)$ and average momentum, $P = (p_1 + p_2)/2$. Typical heavy ion sources in nuclear collisions are of size $R \sim 5$ fm, so that interference occurs predominantly when $q \sim R/R \sim 40$ MeV/c. Since typical particle momenta are $p_1 \gg P \sim 300$ MeV/c, the interfering particles travel almost parallel, i.e., $p_1 \approx p_2 \approx P \gg q$. The correlation function due to Bose-Einstein interference of identical spin zero bosons $\pi^\pm \pi^\pm$, $K^\pm K^\pm$, etc.) from an incoherent source is (see, e.g., [3])

$$C_2(q, P) = 1 + \left| \frac{\int d^4x S(x, P) e^{iqx}}{\int d^4x S(x, P)} \right|^2,
$$

(5)

where $S(x, P)$ is the source distribution function describing the phase space density of the emitting source.

Experimentally the correlation functions are often parametrized by the gaussian form

$$C_2(q_i, q_o, q_l) = 1 + \lambda \exp[-q_i^2 R_s^2 - q_o^2 R_o^2 - q_l^2 R_l^2
\quad - 2q_i q_o R_{os} - 2q_i q_l R_{ol} - 2q_o q_l R_{sl}],
$$

(6)

Here, $q = q_1 - q_2 = (q_i, q_o, q_l)$ is the relative momentum between the two particles and $R_{i, o, l} = \delta R_{o, l}$ the corresponding sideward, outward, longitudinal, outsideward, out-long and sideward-longitudinal HBT radii respectively. We have suppressed the $P$ dependence. We will employ the standard geometry, where the longitudinal direction is along the beam axis, the outward direction is along $P_{\perp} \simeq p_{\perp,i}$, and the sideward axis is perpendicular to these. Usually, each pair of particles is lorentz boosted longitudinal to the system where their rapidity vanishes, $y = 0$. Their average momentum $P$ is then perpendicular to the beam axis and is chosen as the outward direction. In this system the pair velocity $\beta_P = P/E_P$ points in the outward direction with $\beta_o = p_o/\lambda_P$, where $\lambda_P = \sqrt{m^2 + p_P^2}$ is the transverse mass, and both the $R_o$ and $R_{ol}$ vanish at midrapidity (see Ref. [3] for further analyses). Also $R_{os}$ vanishes for a cylindrically symmetric source or if the azimuthal angle of the reaction plane is not determined and therefore averaged over — as has been the case experimentally so far. The reduction factor $\lambda$ in Eq. (4) may be due to long lived resonances [4], coherence effects, incorrect Coulomb corrections or other effects. It is $\lambda \sim 0.5$ for pions and $\lambda \sim 0.9$ for kaons.

The Bose-Einstein correlation function can now be calculated for a deformed source. Let us first investigate transparent sources and, as in [4], parametrize the transverse and temporal extent by gaussians

$$S(x, P) \sim S_\perp(x, y) \exp[-(\tau - \tau_f)^2/2\delta \tau^2] e^{p_u u/T},
$$

(7)

with longitudinal Bjorken flow, $u = (\cosh \eta, 0, 0, \sinh \eta)$. Effects of transverse flow will be discussed below. The transverse radii $R_s, R_o$ are the gaussian radii at freeze-out, $\tau_f$ is the freeze-out time and $\delta \tau$ the duration of emission.

In order to calculate the correlation function of [5] the gaussian approximation is employed (see, e.g., [3]) which results in a correlation function on the form as in Eq. (4). Inserting the source (4) in Eq. (6) and Fourier transforming we obtain the correlation function. Comparing to the experimental parametrization of Eq.(6), one calculates the HBT radii. For transparent sources the azimuthal dependence of the HBT radii has been calculated in detail by Wiedemann [14]. In the longitudinal center-of-mass system of the pair ($y = 0$), the HBT radii are around midrapidities

$$R_s^2 = R_s^2 [1 + \delta \cos(2\phi)],
$$

$$R_o^2 = R_o^2 [1 - \delta \cos(2\phi)],
$$

$$R_{os}^2 = R_s^2 \delta \sin(2\phi),
$$

(9)

$$R_{ol}^2 = \frac{T}{m} \tau_f^2,
$$

(11)

where $R^2 = (R_s^2 + R_o^2)/2$ is the average of the source radii squared. As in asymmetric flow, Eq. (6), $\phi$ is the azimuthal angle between the transverse momentum $p_\perp$ and the reaction plane. It is therefore the angle by which the $R_{o, s}$ axes are rotated with respect to the $(x, y)$ reaction plane (see Fig. 1). Near target and projectile rapidities the directed flow is appreciable and leads to $\cos \phi$ terms in Eqs. (8). The out- and sideward HBT radii show a characteristic modulation as function of azimuthal angle with amplitude of same magnitude but opposite sign. Measuring the amplitude modulation of $R_{o, s}$ determines five quantities and thus overdetermines the three source parameters which are the source size $R$, deformation $\delta$ and and duration of emission $\delta \tau$.

Next we consider opaque sources. In relativistic heavy ion collisions source sizes and densities are large and one would expect rescatterings. As a result particles are predominantly emitted near the surface and arrive from the (front) side of the source facing towards the detector. In [2] it was found that for opaque sources, where mean free paths are smaller than source sizes, $\sigma m_{fp} \ll R$, the sideward HBT radius increase whereas the outward is significantly reduced. The simple geometrical cause is that the emission region in the outward direction is the surface region which considerably narrower than the whole source. As in [2] Glauber absorption is introduced by adding an absorption factor $\exp(-\int x \sigma p(x') dx')$ where $\sigma$ is the interaction cross section, $p$ the density of scatterers and the integral runs along the particle trajectory from source point $x$ to the detector. Defining the mean free path as $\lambda_{mfp} = (\sigma p(0))^{-1}$, where $p(0)$ is the central density, the source is opaque when $\lambda_{mfp} \ll R$ and transparent when $\lambda_{mfp} \gg R$. Calculating the correlation function for an
opaque source from Eq. (2) and comparing to the definition of the HBT radii in Eq. (1), one generally obtains by expanding for small deformations,

\[ R_{\phi}^2 = g_o R^2 \left[ 1 + \delta \cos(2\phi) \right], \]
\[ R_{\perp}^2 = g_o R^2 \left[ 1 - \delta \cos(2\phi) \right] + \beta_o^2 \delta^2 \tau^2, \]
\[ R_{\phi o}^2 = g_{\phi o} R^2 \delta \sin(2\phi). \]

Here \( g_{\phi o, s o} \) are model dependent factors that are functions of opacity but independent of the deformation. For a gaussian source \( (\rho \propto S_o) \), which is moderately opaque \( (\lambda_{mp}/R = 1) \), a numerical calculation gives \( g_o \simeq 1.4 \) and \( g_o \simeq 0.9 \) (see also [15]). For a disk source with transverse radius twice the gaussian radius \( 2R \), that emits like a black body \( (\lambda_{mp} \ll R) \), one finds \( g_o = 4/3 \) and \( g_o = 4 \left( \frac{4}{3} - \left( \frac{R}{\lambda_{mp}} \right)^2 \right) \simeq 0.2 \). In all cases \( g_{o,s} \simeq g_o \). Generally, \( (g_o - g_s) \geq 0 \) and the difference increases with opacity. Only for a completely transparent sources is \( g_o = g_s \). In Fig. 2 the HBT radii of Eqs. (12-14) are shown for a near-central collisions \( (\delta = 0.2) \) with a moderate duration of emission \( \delta^2 \tau^2/g_o R^2 = 1/4 \) for various opacities \( \lambda_{mp}/R = 0.1, 0.5, 1.0, 2.0, \infty \). As the opacity increases, \( g_o/g_s \) decreases and therefore also the outward HBT radius and its amplitude.

Comparing the HBT radii from an opaque source Eqs. (12-14) with those of a transparent source Eqs. (3-5), one notices that the amplitudes in \( R_o \) and \( R_o \) differ by the amount \( (g_o - g_s) \). The modulation of the HBT radii with \( \phi \) provides five measurable quantities which over-determinates the four physical quantities describing the source: its size \( R \), deformation \( \delta \), opacity \( (g_o - g_s) \) and duration of emission \( \delta \tau \), at each impact parameter. The azimuthal dependence of the HBT radii thus offers an unique way to determine the opacity of the source as well as the duration of emission separately.

Experimentally, HBT analyses have not been combined with determination of the reaction plane yet. Consequently, the azimuthal angle \( \phi \) is averaged and information on three of five measurable quantities in Eqs. (12-14) is lost. From the angular averaged difference between the out- and sideward HBT radii

\[ (R_o^2 - R_o^2)_{\phi} = \beta_o^2 \delta^2 \tau^2 - (g_o - g_s) R^2, \]

one can only determine the sum of the positive duration of emission and negative opacity effect. Experimentally the difference is small: NA49 [6] and NA44 [3] data even differ on the sign. Detailed analyses of the \( p_\perp \) dependence of the HBT radii from NA49 data within opaque sources [10] indicate that the sources are transparent or at most moderately opaque. However, the NA44 data for which \( R_o \lesssim R_s \) necessarily requires an opaque source as seen from Eq. (4). Furthermore, the \( p_\perp \) dependence of the transverse HBT radii change if the source sizes, opacities, and duration of emission also are \( p_\perp \) dependent.

Transverse flow may affect the out- and sideward HBT radii as opacity, i.e., the factors \( g_{o,s} \) may depend on both. For transparent sources transverse flow has been studied in [4] through the flow \( u = (\gamma \cosh(\eta), \gamma \sinh(\eta), u_1) \), where \( \gamma = \sqrt{1 + u_1^2} \), assuming that the transverse flow scales with transverse distance, \( u_o \simeq u_0 \tau_\perp /R \). Otherwise the same transparent gaussian source as in Eq. (2) was employed. To lowest order in the transverse flow both transverse HBT radii decrease by the same factor \( (1 + u_0^2 m_\perp /T) \) to leading order in \( u_0 \). This transverse flow correction is independent of the source size and therefore also the deformation. Consequently, spatial deformations reduce the amplitudes by the same amount in this model for both transparent (see [4]) and opaque sources. There are, however, box shape models where the transverse flow reduce \( R_o \) more than \( R_o \). A similar flow effect is found in hydrodynamic models also [17] although it is considerably less than the opacity effect, i.e., when the Cooper-Frye freeze-out condition is modified by removing the back side of the source. Preferrably, the \( p_\perp \) dependence of the HBT radii should be measured as well as the transverse flow from apparent temperatures in order to determine the magnitude of the flow and opacity separately. The transverse flow might also be azimuthally dependent, i.e., \( g_{o,s} \) depend on \( \phi \) and thereby change the amplitude. This effect can be estimated from the measured elliptic flow and we find that for semicentral \( Pb + Pb \) collisions at SPS energies \( u_2^2 - u_2^2 \sim v_2 \), which is only a few percent. Therefore, the azimuthally dependence of the flow and its effect on the amplitude in the HBT radii is also of that order only which is much less than \( \delta \simeq 0.5 \) for semi-central collisions. The conclusion is that besides opacity also transverse flow may affect the factors \( g_{o,s} \) but independently of azimuthal angle. Therefore Eqs. (12-14) are still valid and can be used to extract the duration of emission unambiguously.

Measuring the centrality or impact parameter dependence of the source sizes, deformation, opacity, emission times and duration of emission is very important for determining how the source change with initial energy density. If no phase transition takes place one would expect that source sizes and emission times increase gradually with centrality whereas the deformation decrease approximately as \( 3 \). In peripheral collisions source sizes and densities are small and few rescatterings occur. Therefore, the source is transparent and the HBT radii are given by Eqs. (3-4). For near central collisions sources sizes and densities are higher which leads to more rescatterings. Thus the source is more opaque and the amplitudes should differ. It would be interesting to observe this gradual change in amplitude with centrality. At the same time it would provide a direct experimental determination whether the source is transparent or opaque as well as extracting the magnitudes of the opacity and duration of emission.

If a phase transition occurs at some centrality, where energy densities exceed the critical value, one may observe sudden changes in these quantities. The emission time and duration of emission has in hydrodynamic cal-
culations [17] been predicted to increase drastically leading to very large \( R_l \) and \( R_o \). A long lived mixed phase would also emit particles as a black body and thus the opacity should also become large. If droplet formation occurs leading to rapidity fluctuations, one may be able to trigger on such fluctuations and find smaller longitudinal and sideward HBT radii [18]. Furthermore, if an interesting change in these quantities should occur at some centrality, it would also be most interesting to look for simultaneous \( J/\Psi \) suppression, strangeness enhancement, decrease in directed, elliptic or transverse flow [19], or other signals from forming a quark-gluon plasma.

In summary, the importance of measuring the reaction plane and HBT radii simultaneously has been stressed. The modulation of the HBT radii with azimuthal angle between the reaction plane and particle transverse momenta can be exploited to obtain source sizes, deformations, life-times, duration of emission and opacities separately. Tracking these physical quantities with centrality will provide detailed information about the source created in relativistic nuclear collisions and may reveal the phase transition to quark-gluon plasma.

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FIG. 1. Reaction plane of semi-central $Pb + Pb$ collision for impact parameter $b = R_{Pb} \simeq 7$fm. The overlap zone is deformed with $R_x \leq R_y$. The reaction plane ($x, z$) is rotated by the angle $\phi$ with respect to the transverse particle momentum $p_\perp$ which defines the outward direction in HBT analyses.

FIG. 2. HBT radii vs. angle between reaction plane and transverse particle momenta. The HBT radii are normalized to the angle averaged sideward HBT radius squared, $g_s R^2$, for slightly deformed $\delta = 0.2$ source with duration of emission $\beta^2 \delta \tau^2 / g_s R^2 = 1/4$ and with various opacities. The sideward HBT radius (full curve) is then the same for both transparent (Eq. (8)) and opaque (Eq. (12)) sources and likewise for the out-sid HBT radii (Eqs. (10) and (14), dashed curve). The outward HBT radii are shown with chain-dashed curves for a gaussian source (Eq. (13) and (9) for various opacities (from below and up): $\lambda_{nf}/R = 0.1, 0.5, 1.0, 2.0, \infty$. 