Probabilistic Thinking Based on Probabilistic Thinking Responses to Math Problem Solving

Yulia Maftuhah Hidayati¹ Nurul Afifah², Nurhidayati¹, Djalal Fuadi⁴, Apuanor⁵, Alivermana Wiguna⁶, Muhammad Waluyo⁷

¹-⁴ Faculty of Teacher Training and Education, Universitas Muhammadiyah Surakarta, Indonesia
⁵-⁶ Faculty of Teacher Training and Education, Sekolah Tinggi Keguruan Ilmu Pendidikan Muhammadiyah Sampit, Indonesia
⁷ Doctoral school of education, University of Szeged, Hungary

Corresponding email: ymh284@ums.ac.id

Abstract. The objective of the study is to describe the fourth-grade students’ probabilistic thinking response of SD State Elementary 4 Kaliwuluh, Karanganyar regency, Central Java, Indonesia to math solving the problem of geometry. It used a qualitative approach. The results of the study show that the students’ responses were in four categories. Their non-response responses were difficult to understand questions and so that they guessed their answers. In the non-statistic responses, they could solve their answers self-confidently. In the partial statistic response, they related the materials obtained, but they did not use a proper concept. In the statistic responses, they could relate the answers by using a mathematical concept and count systematically.

1. Introduction

The education in the 4.0 Era requires students to understand and adapt to the 21st century. According to [1] the 21st-century competency students have include communicators, creators, critical thinkers, and collaborators. Mathematics as an elementary school course is to develop logical, analytical, systematical, creative, and critical thinking [2]. They employ them for finding, obtaining, managing, and using information based on logic and scientific thinking to face an uncertain life [3]. Furthermore, elementary school becomes the most fundamental education for developing a true concept to actualize a qualified generation [4].

Building and developing students for mathematical problem-solving in a school or daily life are a fundamental part of them [5]. They need to understand a concept, reason, and communicate some suggestions for mathematical problem solving by developing a thinking competency. Thus, they not only have a deterministic thinking competency, but they also have a probabilistic thinking one.

In Indonesia, the education science of mathematics is part of science that studies a result estimation of statistic trial, called probabilistic. It requires students for mathematical problem solving by using probabilistic thinking [6]. The material of the 5th math learning outcome of elementary school is a kind of two-dimensional figure and space. It is a sort of geometrics from elementary school to higher education [7].

[8] Suggested that students could explore geometrics to develop a thinking skill for problem-solving. According to [9] one of the reasoning skills the students develop in the mental development theory of Test of Logical Thinking (TOLT) by Piaget is probabilistic reasoning. The research by [10]
showed that the students’ reasoning for geometry problem solving was low. Their problems were related to reasoning skills to estimate and making a conclusion based on reasoning skills and mathematical thinking in probabilistic material [11].

Students’ responses to mathematical problem solving are essential. A response is part of expression, term, or idea of students’ mathematics used for solving a mathematical problem of geometry and probabilistic material. In terms of probabilistic thinking response, [12] suggested four categories of non-response, non-statistic, partial statistic, and statistic responses.

In the research of the students’ probabilistic thinking by [13] showed that in solving a sample space problem, the students gave the statistic responses, but in the probability of an event, they gave a partial statistic response. The other research showed that the students’ mathematics was low. The research of the students’ probabilistic thinking by [14] that they were high in mathematics, but they gave different responses. The research result showed the male’s probabilistic thinking was higher than the female’s one. The problem statement is how is students’ probabilistic thinking to mathematical problem-solving?

2. Method

2.1 Research Subject
The study applied a qualitative approach. The objective is to describe probabilistic students’ thinking skills to geometry-mathematic problem solving related to probabilistic thinking response. The data were a written description of data and pictures in a text [15].

The research subjects were the 5th-grade students of SD Negeri State Elementary School 4, Sragen regency, Central Java. The subjects were based on different genders and high mathematic skills. The early survey interviewed class teachers of mathematics and observed the students’ mathematics skills based on the students’ outcomes of daily tests. The teachers explained the students who had high mathematic skills. After the process, there were two students as a research subject.

| Name       | Subject |
|------------|---------|
| MNA (pi)   | S1 (Subject 1/One) |
| RASH (pa)  | S2 (Subject 2/Two) |

2.2 Research Instrument
The study was based on the interview and competency test to mathematical problem solving of the probability of geometry material as a core instrument.

2.3 Research procedure
The data gathering included as follows. The first stage is to gather the data by giving an assignment of geometry probability. After finishing the assignment, the next stage was the unstructured interview of how the students completed it for getting the data in more detail. The interview results were recorded by the digital camera so that the data could be saved. Then, the recording results were transcribed into a dialogue text. The data were classified and reduced and they were displayed or presented. The triangulation technique was used to compare the data obtained in the first and second stages. The method triangulation was used to compare the data of test results with the interview ones and the source one was employed to interview the teachers and students.
2.4 Data Analysis
The data were analyzed by presenting the description of data collection and the analysis was separately conducted. The data were analyzed by 1) making a transcription and coding, 2) classifying the data, 3) reducing the data, 4) displaying the data, 5) interpreting probabilistic thinking for a mathematical problem-solving base on probabilistic thinking responses, and 6) making a conclusion.

3. Research Results
The data were analyzed based on the description of the probabilistic thinking response developed by Sharma identifying the level of responses when interviewing the students in solving the questions given

| Responses   | Descriptions                                                   |
|-------------|----------------------------------------------------------------|
| Non-responses | Not being able to explain their thinking.                      |
|             | Frequently guessing the answers (I don’t know, I forget, I only guess). |
| Non-statistic | Relating their daily experiences, faith, advantage, superstition, and religion. |
| Partial statistic | Adapting rules and applying them improperly.                   |
| Statistic   | Explaining thinking by using classical and frequent interpretations, numerical probability, and listing a sample space. |

The students’ non-responses could not explain their thinking and frequently guess the answers, for example, I don’t know, I forget, I only guess. The non-statistic responses related to their daily experiences, faith, advantage, superstition, and religion. The partial statistic responses adapted rules and applying them improperly. They are usually related to proportional representation and misunderstanding and equiprobability bias. The statistic responses concerned the students who explained their thinking by using classic interpretation. Figure 1 is the interview results with S1 in solving a question of geometry by using probabilistic thinking.

![Figure 1](image)

Figure 1. The Interview Results with S1
In Figure, S1 listed the form of rectangle probable to seem in the pattern. S1 constructed the members of the sample space without using a certain pattern. Here are other review results with S1.
Then, explain how to get the answer?

S1: Well, formerly I drew, Mam, on the sheet. Then, I gave the numbers at each picture of a rectangle, Mam. I gave the numbers from 1 to 6, like this. So, there were 6 rectangles, Mam.

P: Thus, because it seemed that there were pieces, you concluded that there were 6 rectangles?

S1: Yes, Mam.

P: Formerly whether you knew or not, what is the form of a rectangle?

S1: I knew, Mam. Like this (then, S1 drew a rectangle on the sheet).

S1 employed personal thinking of the form of the rectangle based on early competency. In question number 2, then, S1 could not differentiate between circle and sphere. Also, S1 still made a conclusion by using an uncertain pattern and only wrote the answer based on subjective thinking without any sketch or a certain pattern. The following are the interview results.

P: Then, could you explain what were the steps you used for solving problem number 2?

S1: Yes, I could, Mam. This is I saw the picture that there were pieces of a sphere. Then, I counted 4 times 6 sides of cubes equal to 24 spheres, Mam.

In question number 3, S1 was in the subjective probabilistic thinking level. It seemed that the student listed the pictures probable attached in the frame by listing the experiment in one level incompletely. It is based on subjective reasoning and thinking as described in the interview results.

P: Why did you add a straight line in the middle of the circle?

S1: Well, in essence, I made a perpendicular straight line, Mam.

In question number 4, it seemed that in terms of Figure 1 S1 predicted an event based on subjective thinking of rotational symmetry of the square. In answering question number 4, S1 also made a probable event based on subjective thinking. The following are the interview results.

S1: There were two ways. Firstly, the picture was attached in the square. Then, the second picture was made a slope and attached on the frame, Mam.

In the last question, S1 could make the structure of sample space members with a certain pattern, described all the members of probable sample space and wrote the number of sample space members quantitatively with a systematic-certain pattern.

Figure 2. The Results of S2’s Assignment
In assignment number 1, S2 could make a picture sketch of the forming of sample space and wrote notation for a sample space of rectangle (Figure 2). The student could compare the probability of based on the quantitative statement limited. The following are the interview results.

S2: Well, I could, Mam. So, in the early, I made the same rectangle as the question. Then, I counted six forms of a rectangle. This is the rectangle (indicating it in Figure). After that, I made another form. This is that if the lines inside were not removed, there were two large rectangles, Mam. So, there were eight rectangles, Mam.

In Figure 2, question 2, S2 could write S2 the answer quantitatively. Besides, S2 made a picture sketch and wrote a notation for each sample space. However, some answers were based on subjective reasoning. It means that the students could not determine a standard formula to solving the question. Here are the interview results.

S-2: In the picture, there were two-sphere pieces. Then, I tried to think them, Mam. In the above parts, if these were combined into half of the sphere. In the under parts, if these were combined into half of the sphere. After that, these parts were combined into one sphere.

In question number 3, S2 could explain an event result with a certain pattern. Besides, the student could predict the answer to the event based on subjective thinking logically. The following are the interview results.

P: Why did not you put a perpendicular line? It is also a straight line, isn't it?

S2: If it was in the middle, it would amount to 6 lines, Mam.

Question number 4, S2 could list the trial results completely, predict the most probable event based on subjective thinking, and compare the probable event based on the limited quantitative statement. The following are the interview results.

S2: We put the picture number one (using the sheet); picture number two was turned; picture three was the same as the picture four. Then, five pictures were tilted to the right; six pictures were tilted to the left.

Question number 5, the student counted the number of cubes based on subjective thinking and could not differentiate an original probabilistic event from another one. Also, S2 constructed the members of the sample space based on an uncertain pattern.

Based on the assignments and interviews, it is found that some students’ levels of probabilistic thinking as developed by Sharma.

Table 3. The Students’ Probabilistic Thinking Responses to Solve a Geometry-Mathematics Problem

| Students | Question 1 | Question 2 | Question 3 | Question 4 | Question 5 |
|----------|------------|------------|------------|------------|------------|
| Subject k 1 | Partial statistic | Partial statistic | Partial statistic | Partial statistic | Statistic |
| Subject 2 | Partial statistic | Partial statistic | Partial statistic | Partial statistic | Non-response |

a. Probabilistic Thinking Responses to Solve a Geometry Problem Number 1

Based on the written answers and interview results, S1 gave a partial statistic response when solving question number 1. S1 adapted the rules of rectangle form, but S1 could not use it completely. It made the misunderstanding of determining the number of rectangles with the available patterns.

S2 used a partial statistic response because the response referred to the proportional misunderstanding of predicting the other problem patterns. Additionally, based on the pattern of small and large rectangles, it was found the representative pattern.

b. Probabilistic Thinking Responses to Solve a Geometry Problem Number 2
Referring to the question solving and interview results, S1 gave a partial statistic response. The subject thought that the question was easy to understand. Although using a systematic count, the answer was incorrect because the early concept developed by the subject was false. Besides, S1 felt that the sphere was the same as the circle.

Based on the mathematical problem question solving and interview results, S2 gave a statistic response. S2 explained the thinking by using classic interpretation. The analytic reasoning and thinking were used for understanding the number of spheres in the cube without counting them. Besides, S2 could explain the numeric evidence, but it could be systematic and proper to describe a mathematical problem solving of question 2.

c. Probabilistic Thinking Responses to Solve a Geometry Problem Number 3
Based on probabilistic thinking, S2 showed a partial statistic response to solve question 1. It seems that S1 adopted the rules that the straight lines were in the plum or vertical lines and they did not indicate another comparison line pattern horizontally and other lines to determine the maximal results of circle pieces. It made the proportional misunderstanding of different straight line forms in variety and the representative straight lines developed in the circle of a vertical straight line.

S2 showed a statistic response to solve the problem of question 3. S2 explained the thinking by making classic thinking, namely drawing a circle with different patterns of line pieces to find the most probable opportunity. Then, S2 listed each member of the sample space in two-line patterns of circle. S2 made a conclusion that the probable line pattern to find the most patterns of pieces was dividing a circle into two circles by putting a straight line horizontally.

d. Probabilistic Thinking Responses to Solve a Geometry Problem Number 4
S1 showed a partial statistic response based on probabilistic thinking. S1 adopted the rule, one rule – the two-dimensional figure can be turned so that its position can change. However, S1 could not use the rule properly. It seems that in a second way, S1 turned a square at its frame.

Based on the question solving and interview results, S2 showed a partial statistic response. S2 can make a probable picture by putting it a square frame properly based on the rule if the two-dimensional figure can be turned and it can change its position. In solving question number 4, S2 could not use the rule properly. It is proved that S2 made a slanting pattern to put a picture in the frame.

e. Probabilistic Thinking Responses to Solve a Geometry Problem Number 5
S1 showed a statistic response to solve question number 5. S1 could explain thinking by using classic interpretation and used numeric probability. Based on structuring the answer by using the number and numeral, it was solved by addition. After that, S1 listed the sample space well based on the event.

S2 showed a non-response response. S- 2 only guessed that there might be 15 cubes. It was based on direct count without other probable opportunities. Besides, S2 did not understand the rule so that S2 only counted parts of the seeming cubes.

4. Discussion
S1's and S2's responses to solve question number 1 are similar to each other. They are in partial statistic thinking responses. The students adopt the rule of rectangle forms they have learned at school, but they do not use them properly. It made the representative form appear, but the opportunity does not appear. In terms of solving question number 2, S1 shows a partial statistic response. It seems that adopts the rule and used it. S2 shows a statistic response by using reasoning and thinking based on the classic interpretation of sample space.
Concerning question number 3, S1 is in a partial statistic response. S1 understand a straight line inappropriately. The subject (S1) understands the form of a straight line, but S1 uses it for solving the question inappropriately. S2 shows a statistic response, but the subject answers the question appropriately and gives in a concrete and clear reason why the student does the question by using the step (way).

In terms of question number 4, the student (S1) can’t understand the question well. Various reasons are conveyed when solving it. The subject thinks that the square picture can be put in a slanting position or a position similar to ‘rice cake cutting’. It seems that the subject can’t give any probable reason. So, the students show a partial statistic response. They adapt the rule of the rotational symmetry and use it inappropriately for solving question number 4.

Referring to solve question number 5, S1 is in a statistic response by listing the members of sample space and uses it with numeric counting although there is some error in the counting results. However, S1 has been able to list the probability and mathematic counting systematically. However, S2 is in a non-response category. The subject only guesses and predicts its answer without clear counting.

Base on the description above, it can be shown that each subject has a different probabilistic thinking response to solving mathematics based on geometry. The steps of the problem solving are different from each other. The students solve a problem based on the solving plan, information collection, or related data, and the problem they have learned [16].

Certainly, each student has a different thinking response to determine a problem. The steps of problem-solving used by the students are consistent between the results and thinking process. It seems that the students only guess the answer base on subjective thinking and reasoning [17]. It is influenced by their habits in a problem solving by directly answering the counting and making a conclusion [18].

The male students' probabilistic thinking response is better than the female's. It is relevant to the research by [19], stating that in terms of the significant difference in the students' probabilistic-cognitive level based on the gender, the male is higher in misconception than the female.

The research by [20], shows that the connections between the geometry concepts are low. It means that these concepts can’t relate knowledge to another one in geometry and even in other fields. The students' reasoning is also low. Most of the students can't use what they have known to prove a problem. Besides, it can be concluded that male students' thinking to solve a probabilistic is more analytic and flexible than the female one.

5. Conclusion

The students’ responses were in four categories. Their non-response responses were difficult to understand questions and so that they guested their answers. In the non-statistic responses, they could solve their answers self-confidently. In the partial statistic response, they related the materials obtained, but they did not use a proper concept. In the statistic responses, they could relate the answers by using a mathematical concept and count systematically.

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