Active Finite Reward Automaton Inference and Reinforcement Learning Using Queries and Counterexamples

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Abstract. Despite the fact that deep reinforcement learning (RL) has surpassed human-level performances in various tasks, it still has several fundamental challenges. First, most RL methods require intensive data from the exploration of the environment to achieve satisfactory performance. Second, the use of neural networks in RL renders it hard to interpret the internals of the system in a way that humans can understand. To address these two challenges, we propose a framework that enables an RL agent to reason over its exploration process and distill high-level knowledge for effectively guiding its future explorations. Specifically, we propose a novel RL algorithm that learns high-level knowledge in the form of a finite reward automaton by using the $L^*$ learning algorithm. We prove that in episodic RL, a finite reward automaton can express any non-Markovian bounded reward functions with finitely many reward values and approximate any non-Markovian bounded reward function (with infinitely many reward values) with arbitrary precision. We also provide a lower bound for the episode length such that the proposed RL approach almost surely converges to an optimal policy in the limit. We test this approach on two RL environments with non-Markovian reward functions, choosing a variety of tasks with increasing complexity for each environment. We compare our algorithm with the state-of-the-art RL algorithms for non-Markovian reward functions, such as Joint Inference of Reward machines and Policies for RL (JIRP), Learning Reward Machine (LRM), and Proximal Policy Optimization (PPO2). Our results show that our algorithm converges to an optimal policy faster than other baseline methods.

1 Introduction

Despite the fact that deep reinforcement learning (RL) has surpassed human-level performances in various tasks, it still has several fundamental challenges. First, most RL methods require intensive data from the exploration of the environment to achieve satisfactory performance. Second, the use of neural networks in RL renders it hard to interpret the internals of the system in a way that humans can understand.

To address these two challenges, we propose a framework that enables an RL agent to reason over its exploration process and distill high-level knowledge for effectively
guiding its future explorations. Specifically, we learn high-level knowledge in the form of finite reward automata, a type of Mealy machine that encodes non-Markovian reward functions. The finite reward automata can be converted to a deterministic finite state machine, allowing a practitioner to more easily reason about what the agent is learning [14]. Thus, this representation is more interpretable than frameworks that use neural networks.

In comparison with other methods that also learn high-level knowledge during RL, the one proposed in this paper actively infers a finite reward automaton from the RL episodes. We prove that in episodic RL, a finite reward automaton can express any non-Markovian bounded reward functions with finitely many reward values and approximate any non-Markovian bounded reward function (with infinitely many reward values) with arbitrary precision. As the learning agent infers this finite reward automaton during RL, it also performs RL (specifically, q-learning) to maximize its obtained rewards based on the inferred finite reward automaton. The inference method is inspired by the L* learning algorithm [4], and modified to the framework of RL. We maintain two q-functions, one for incentivizing the learning agent to answer the membership queries during the explorations and the other one for obtaining optimal policies for the inferred finite reward automaton (in order to answer the equivalence queries). Furthermore, we prove that the proposed RL approach almost surely converges to an optimal policy in the limit, if the episode length is longer than a theoretical lower bound value.

We implement the proposed approach and three baseline methods (JIRP-SAT [33], LRM-QRM [24], and PPO2 [22]) in the Office world [33] and Minecraft world [3] scenarios. The results show that, at worst the approach converges to an optimal policy 88.8% faster than any of the baselines, and at best the approach converges while the other baselines do not.

1.1 Related works

Our work is closely related to the use of formal methods in RL, such as RL for finite reward automata [17] and RL with temporal logic specifications [11, 19, 24, 27, 2]. The current methods assume that high-level knowledge, in the form of reward machines or temporal logic specifications, is known a priori. However, in real-life use cases, such knowledge is implicit and must be inferred from data.

Towards the end of inferring high-level knowledge, several approaches have been proposed [37, 33, 16, 20]. In these methods, the agent jointly learns the high-level knowledge and RL-policies concurrently. In [37], the inferred high-level knowledge is represented by temporal logic formulas and used for RL-based transfer learning. The methods for inferring temporal logic formulas from data can be found in [15, 11, 18, 7, 21, 36, 34, 25, 35, 23, 32, 35, 31, 18, 10].

In comparison with temporal logic formulas, the finite reward automata used in this paper are more expressive in representing the high-level structural relationships. Moreover, even if the inferred finite reward automaton is incorrect during the first training loop, the agent is still able to self-correct and learn more complex tasks. In order to learn finite reward automata, the authors in [33] proposed using passive inference of finite reward automata and utilizing the inferred finite reward automata to expedite RL.
In [16], the authors proposed a method to infer reward machines to represent the memories of Partially observable Markov decision processes (POMDP) and perform RL for the POMDP with the inferred reward machines.

In contrast, our method actively infers the finite reward automaton in environments with non-Markovian reward functions. The active inference is facilitated by L* learning. This algorithm assumes the existence of a teacher who can answer the membership and equivalence queries [4,28,29,30]. In our approach, an RL engine fulfills the role of the teacher, and the queries are answered through interaction with the environment through the RL engine.

During the submission of this paper, an interesting method was proposed by the authors of [12] which also used L* learning for non-Markovian Rewards. While superficially similar to our work, the two approaches differ in three ways:

1. The proposed approach in this paper uses finite reward automata, while [12] works with deterministic finite automata (DFAs) and general automata. This is a notable distinction because we prove that finite reward automata can express any non-Markovian bounded reward function with finitely many reward values in episodic RL.

2. The authors of [12] use only maintain one type of DFA for answering the equivalence queries. The proposed approach in this paper maintains two types of finite reward automata, using one to answer equivalence queries, and the other to answer membership queries. This additional finite reward automaton can incentivize the agent to answer membership queries during the exploration.

3. We provide a lower bound for the episode length such that the proposed RL approach almost surely converges to an optimal policy in the limit.

2 Finite Reward Automata

In this section we introduce necessary background on reinforcement learning and finite reward automata.

2.1 Markov Decision Processes and Finite Reward Automata

Let $\mathcal{M} = (X, x_{\text{init}}, A, p, \mathcal{P}, R, L)$ be a labeled Markov decision process (labeled MDP) with a non-Markovian reward function, where the state space $X$ and action set $A$ are finite, $x_{\text{init}} \in X$ is a set of initial states, $p : X \times A \times X \rightarrow [0, 1]$ is a probabilistic transition relation, $\mathcal{P}$ is a set of propositional variables (i.e., labels), $R : (2^\mathcal{P})^+ \rightarrow \mathbb{R}$ is a non-Markovian reward function where the reward depends on the history of the propositional variables (i.e., labels) that has been encountered, and $L : X \times A \times X \rightarrow 2^\mathcal{P}$ is a labeling function.

We define the size of $\mathcal{M}$, denoted as $|\mathcal{M}|$, to be $|X|$ (i.e., the cardinality of the set $X$). A policy $\pi : X \times A \rightarrow [0, 1]$ specifies the probability of taking each action for each state. The action-value function, denoted as $q_\pi(x, a)$, is the expected discounted reward if an agent applies policy $\pi$ after taking action $a$ from state $x$. A finite sequence $x_0a_0 \ldots x_ka_kx_{k+1}$ generated by $\mathcal{M}$ under certain policy $\pi$ is called a trajectory, starting from $x_1 = x_{\text{init}}$ and satisfies $\sum_{a \in A} \pi(x_k, a)P(x_k, a, x_{k+1}) > 0$ for all $k \geq 1$. Its corresponding label sequence is $\ell_0\ell_1 \ldots \ell_k$ where $L(x_i, a_i, x_{i+1}) = \ell_i$ for each $i \leq k$. 
Similarly, the corresponding reward sequence is \( r_1 \ldots r_k \), where \( r_i = R(\ell_0 \ell_1 \ldots \ell_k) \), for each \( i \leq k \). We call the pair \( (\lambda, \rho) := (\ell_1 \ldots \ell_k, r_1 \ldots r_k) \) a trace.

**Definition 1.** Let \( M = (X, x_{\text{init}}, A, p, P, R, L) \) be a labeled Markov decision process. We define a sequence \( (\ell_1, r_1), \ldots, (\ell_k, r_k) \) to be attainable if \( k \leq \text{eplength} \) and \( p(x_i, a_i, x_{i+1}) > 0 \) for each \( i \in \{0, \ldots, k\} \).

**Definition 2.** A finite reward automaton \( A = (W, w_{\text{init}}, 2^P, \mathcal{R}, \delta, \eta) \) consists of a finite, nonempty set \( W \) of states, an initial state \( w_{\text{init}} \in W \), an input alphabet \( 2^P \), an output alphabet \( \mathcal{R} \), a (deterministic) transition function \( \delta : W \times 2^P \rightarrow W \), and an output function \( \eta : W \times 2^P \rightarrow \mathcal{R} \), where \( \mathcal{R} \) is a finite set of reward values \( (\mathcal{R} \subset \mathbb{R}) \). We define the size of \( A \), denoted as \( |A| \), to be \( |W| \) (i.e., the cardinality of the set \( W \)).

**Remark 1.** A finite reward automaton is actually a Mealy machine (Shallit 2008) where the output alphabet is a finite set of values. When the output alphabet is an infinite set \( \mathcal{R} \), it is called a reward machine in [33, 17].

The run of a finite reward automaton \( A \) on a sequence of labels \( \ell_1 \ldots \ell_k \in (2^P)^* \) is a sequence \( w_0(\ell_1, r_1)w_1(\ell_2, r_2)\ldots w_{k-1}(\ell_k, r_k)w_k \) of states and label-reward pairs such that \( w_0 = w_{\text{init}} \) and for all \( i \in \{0, \ldots, k\} \), we have \( \delta(w_i, \ell_i) = w_{i+1} \) and \( \eta(w_i, \ell_i) = r_i \). We write \( A[\ell_1 \ldots \ell_k] = r_1 \ldots r_k \) to connect the input label sequence to the sequence of rewards produced by the machine \( A \) [33].

**Definition 3.** We define that a finite reward automaton \( A \) encodes the reward function \( R \) of a labeled MDP \( M \) if for every trajectory \( x_0a_0 \ldots x_ka_kx_{k+1} \) of finite length and the corresponding label sequence \( \ell_1 \ldots \ell_k \), the reward sequence equals \( A[\ell_1 \ldots \ell_k] \).

**Definition 4 (Reward Product MDP).** Let \( M = (X, x_{\text{init}}, A, p, P, R, L) \) be a labeled MDP with a non-Markovian reward function and \( A = (W, w_{\text{init}}, 2^P, \mathcal{R}, \delta, \eta) \) a finite reward automaton encoding its reward function. We define the product MDP \( M_A = (X', x'_1, A', p', P', R', L') \) by

- \( X' = X \times W \);
- \( x'_1 = (x_{\text{init}}, w_{\text{init}}) \);
- \( A' = A \);
- \( p'((x, w), a, (x', w')) \)
  - \( = \begin{cases} 
  p(x, a, x') & \text{if } w' = \delta(w, L(x, a, x')); \\
  0 & \text{otherwise}; 
  \end{cases} \)
- \( P' = P \);
- \( R'((x, w), a, (x', w')) = \eta(w, L(x, a, x')) \) and
- \( L' = L \).

It can be seen from Definition 4 that the reward function of the product MDP \( M_A \) is actually Markovian.
2.2 Reinforcement Learning With Finite Reward Automata

Q-learning [26] is a form of model-free reinforcement learning (RL). Starting from state $x$, the system selects an action $a$, which takes it to state $x'$ and obtains a reward $R$. The $Q$-function values will be updated by the following rule:

$$q(x, a) \leftarrow (1 - \alpha)q(x, a) + \alpha(R + \gamma \max_a q(x', a)).$$

The $q$-learning algorithm can be modified to learn an optimal policy when the general reward function is encoded by a finite reward automaton [17]. Starting from state $(x, w)$ in the product space, the system selects an action $a$, which takes it to state $(x', w')$ and obtains a reward $R$. The $Q$-function values will be updated by the following rule:

$$q(x, w, a) \leftarrow (1 - \alpha)q(x, w, a) + \alpha(R + \gamma \max_a q(x', w', a)).$$

We consider episodic $Q$-learning in this paper, and we use $eplength$ to denote the episode length.

3 Expressivity of Finite Reward Automata

In this section, we show that any non-Markovian reward function in episodic RL which has finitely many reward values can be encoded by finite reward automata, while any non-Markovian bounded reward function in episodic RL can be approximated by finite reward automata with arbitrary precision.

**Theorem 1.** For a labeled MDP $M = (X, x_{init}, A, p, P, R, L)$ with a non-Markovian reward function $R : (2^P)^+ \to R_f$ in episodic RL, where $R_f$ is a finite set of values in $\mathbb{R}$, there exists at least one finite reward automaton $A = (W, w_{init}, 2^P, R, \delta, \eta)$ that can encode the reward function $R$.

**Proof.** We use $\mathcal{I}$ to denote the set of trajectories of length at most $eplength$ and $\mathcal{T}_i$ to denote the maximal time index for trajectory $i \in \mathcal{I}$. We construct a finite reward automaton $A = (W, w_{init}, 2^P, R, \delta, \eta)$,

- $W = w_{init} \cup \{w_{i,t}\}_{i \in \mathcal{I}, 1 < t \leq \mathcal{T}_i}, w_{i,1} = w_{init}, \forall i \in \mathcal{I};$
- $R = R_f;$
- $\forall t \leq \mathcal{T}_i, \delta(w_{i,t}, L(x_{i,t}, a_{i,t}, x_{i,t+1})) = w_{i,t+1};$ and
- $\eta(w_{i,t}, \ell_i) = R(\ell_{i,1}, \ldots, \ell_{i,t}),$ where $\ell_{i,t} = L(x_{i,t}, a_{i,t}, x_{i,t+1}).$

Then, it can be easily shown that for every trajectory $x_0a_0 \ldots x_{i,k}a_{i,k}x_{i,k+1}$ and corresponding label sequence $\ell_1 \ldots \ell_k$ ($k \leq eplength$), the reward sequence equals $A[\ell_1 \ldots \ell_k]$, i.e., $A$ encodes the reward function $R$.

**Remark 2.** For a labeled MDP $M = (X, x_{init}, A, p, P, R, L)$ with a finite horizon, there can only be finitely many reward values, each corresponding to finitely many possible trajectories for the reward value.
Theorem 2. For a labeled MDP $\mathcal{M} = (X, x_{init}, A, p, \mathcal{P}, R, L)$ with a non-Markovian bounded reward function $R : (2^P)^+ \rightarrow \mathbb{R}$ in episodic RL, there exists at least one finite reward automaton $\mathcal{A} = (W, w_{init}, 2^n, \mathcal{R}, \delta, \eta)$ that can approximate the non-Markovian reward function $R$ with arbitrary precision.

Proof. For a bounded reward function taking values in $[r_{\min}, r_{\max}]$, where $r_{\min}, r_{\max} \in \mathbb{R}$, we construct a finite set $\mathcal{R}_\epsilon = \{r_{\min}, r_{\min} + \epsilon, r_{\min} + 2\epsilon, \ldots, r_{\min} + n\epsilon\}$, where $\epsilon \in \mathbb{R}, \epsilon > 0, n_{\max} = \max\{n \mid r_{\min} + n\epsilon \leq r_{\max}\}$. We use $\mathcal{I}$ to denote the set of trajectories of length at most $\epsilon_{\text{length}}$ generated from the labeled MDP $\mathcal{M}$ and $\mathcal{T}_i$ to denote the maximal time index for trajectory $i \in \mathcal{I}$. We construct a finite reward automaton $\mathcal{A} = (W, w_{init}, 2^n, \mathcal{R}, \delta, \eta)$, where

- $W = w_{init} \cup \{w_{i,t}\}_{i \in \mathcal{I}, 1 \leq t \leq T_i, w_{i,1} = w_{init}, \forall i \in \mathcal{I}}$
- $\mathcal{R} = \mathcal{R}_\epsilon$;
- $\forall t \leq T_i, \delta(w_{i,t}, L(x_{i,t}, a_{i,t}, x_{i,t+1})) = w_{i,t+1}$; and
- $\eta(w_{i,t}, \ell_{i,t}) = \arg\min_{r \in \mathcal{R}_\epsilon} |r - R(\ell_{i,1}, \ldots, \ell_{i,t})|$, where $\ell_{i,t} = L(x_{i,t}, a_{i,t}, x_{i,t+1})$.

Then, it can be easily shown that for any $\epsilon > 0$ and every trajectory $x_0a_0 \ldots x_{i,k}a_{i,k}x_{i,k+1}$ ($k \leq \epsilon_{\text{length}}$), we have $|A(\ell_{i,1}, \ldots, \ell_{i,t}) - R(\ell_{i,1}, \ldots, \ell_{i,t})| < \epsilon$, i.e., $\mathcal{A}$ approximates the reward function $R$ with arbitrary precision.

4 Active Finite Reward Automaton Inference and Reinforcement Learning (AFRAI-RL)

In this section, we introduce the Active Finite Reward Automaton Inference and Reinforcement Learning (AFRAI-RL) algorithm. Figure 1 shows the block diagram of the AFRAI-RL approach, and Algorithm 1 shows the procedures of the AFRAI-RL approach. The AFRAI-RL approach consists of an active finite reward automaton inference engine and an RL engine. In the following two subsections, we will introduce the two engines and their interactions for obtaining the optimal RL policy for tasks with non-Markovian rewards.

4.1 Active Finite Reward Automaton Inference Engine

In this subsection, we introduce the active finite reward automaton inference engine which is based on L* learning [4], which is an algorithm that learns a minimal deterministic finite automaton (DFA) that accepts an unknown regular language $\mathcal{L}$ by interacting with a teacher. In the setting of this paper, the role of teacher is fulfilled by the RL engine. For simplicity, we only consider RL tasks with non-Markovian reward functions with finitely many reward values (hence there exists at least one finite reward automaton that can encode the non-Markovian reward function).

We first show that a finite reward automaton can be converted to a DFA.

Definition 5. A deterministic finite automaton (DFA) is a five-tuple $\mathfrak{A} = (V, v_I, \Sigma, \delta, F)$ consisting of a nonempty, finite set $v$ of states, an initial state $v_I \in V$, an input alphabet $\Sigma$, a transition function $\delta : V \times \Sigma \rightarrow V$, and a set $F \subseteq V$ of final states. The size of an DFA, denoted by $|\mathfrak{A}|$, is the number $|V|$ of its states.
A run of an DFA $A = (V, v_I, \Sigma, \delta, F)$ on an input word $\tau = \tau_1 \ldots \tau_n$ is a sequence $v_0 \ldots v_n$ of states such that $v_0 = v_I$ and $v_i = \delta(v_{i-1}, \tau_i)$ for each $i \in \{1, \ldots, n\}$. A run $v_0 \ldots v_n$ of $A$ on a word $u$ is accepting if $v_n \in F$, and a word $u$ is accepted if there exists an accepting run. The language of an DFA $A$ is the set $L(A) = \{u \in \Sigma^* | A$ accepts $u\}$. As usual, we call two DFAs $A_1$ and $A_2$ equivalent if $L(A_1) = L(A_2)$.

We show that every finite reward automaton over the input alphabet $2^P$ and output alphabet $R$ can be translated into an “equivalent” DFA as defined below. This DFA operates over the combined alphabet $2^P \times R$ and accepts a word $(\ell_0, r_0) \ldots (\ell_k, r_k)$ if and only if $A$ outputs the reward sequence $r_0 \ldots r_k$ on reading the label sequence $\ell_0 \ldots \ell_k$.

**Lemma 1.** Given a finite reward automaton $A = (W, w_{init}, 2^P, R, \delta, \eta)$, one can construct a DFA $A_A$ with $|A| + 1$ states such that

$$L(A_A) = \{(\ell_0, r_0) \ldots (\ell_k, r_k) \in (2^P \times R)^* | A[\ell_0 \ldots \ell_k] = r_0 \ldots r_k\}.$$ 

**Proof.** Let $A = (W_A, w_{init}^A, 2^P, R, \delta_A, \eta_A)$ be a finite reward automaton. Then, we define a DFA $A_A = (V, v_I, \Sigma, \delta, F)$ over the combined alphabet $2^P \times R$ by

- $V = W_A \cup \{\bot\}$ with $\bot \notin W_A$;
- $v_I = w_{init}^A$.
\[ \Sigma = 2^P \times R; \]

\[ \delta(w, (\ell, r)) = \begin{cases} w' & \text{if } \delta_A(w, \ell) = w' \text{ and } \eta_A(w, \ell) = r; \\ \perp & \text{otherwise}; \end{cases} \]

\[ F = W_A. \]

In this definition, \( \perp \) is a new sink state to which \( A \) moves if its input does not correspond to a valid input-output pair produced by \( A \). A straightforward induction over the length of inputs to \( A \) shows that it indeed accepts the desired language. In total, \( A \) has \( |A| + 1 \) states.

During the learning process, the inference engine maintains an observation table \( O = (S, E, T) \) where \( S \subseteq \Sigma^* \) is a set of prefixes \((\Sigma = 2^P)\), \( E \subseteq \Sigma^* \) is a set of suffixes and \( T : (S \cup S \cdot \Sigma) \times E \rightarrow \{0, 1\} \). \( \Sigma^* \) denotes finite traces from alphabet set \( \Sigma \). For \( s \in S \cup S \cdot \Sigma \), \( e \in E \), if \( s \cdot e \in \mathcal{L} \), then \( T(s, e) = 1 \) and if \( s \cdot e \notin \mathcal{L} \), then \( T(s, e) = 0 \). Membership queries assign the correct value (0 or 1) to \( T(s, e) \). For simplicity, we denote \( \text{row}(s) = (T(s, e_1), ..., T(s, e_n)) \in \{0, 1\}^n, |E| = n \). The inference engine will always keep the observation table closed and consistent as defined below.

**Definition 6.** An observation table \( O = (S, E, T) \) is closed if for each \( t \in S \cdot \Sigma \), we can find some \( s \in S \) such that \( \text{row}(s) = \text{row}(t) \).

**Definition 7.** \( O \) is consistent if whenever for \( s_1, s_2 \in S \), \( \text{row}(s_1) = \text{row}(s_2) \), then for any \( \sigma \in \Sigma \), we have \( \text{row}(s_1 \sigma) = \text{row}(s_2 \sigma) \).

**Remark 3.** If an observation table \( O = (S, E, T) \) is closed and consistent, it is possible to construct a DFA \( M(O) = (Q, \Sigma, \delta, Q_0, F) \) as the acceptor, where

\[ Q = \{ \text{row}(s) | s \in S \}; \]
\[ q_0 = \text{row}(\epsilon); \]
\[ \delta(\text{row}(s), \sigma) = \text{row}(s\sigma), \forall \sigma \in \Sigma; \]
\[ F = \{ \text{row}(s) | s \in S, T(s) = 1 \}. \]

**Definition 8.** For a closed and consistent observation table \( O = (S, E, T) \), we define a corresponding hypothesis finite reward automaton \( A^h(O) = (W^h, w^h_{init}, 2^P, R, \delta^h, \eta^h) \) as follows:

\[ W^h = \{ \text{row}(s) : s \in S \}; \]
\[ w^h_{init} = \text{row}(\epsilon); \]
\[ \delta^h(\text{row}(s), \sigma) = \text{row}(s\sigma); \]
\[ \eta^h(w, \lambda) = 1, \text{if } T(\lambda) = 1; \text{ and } \eta^h(w, \lambda) = 0, \text{ otherwise}. \]
Algorithm 1 AFRAI-RL

1: Initialize $O = (S, E, T)$, sample, $N_{sample}$, $C$, $q_m, q_h$
2: while there exists counterexample $(\lambda, \rho)$ do
3:   Add $(\lambda, \rho)$ and its prefixes to $S$
4:   $\text{Change}T \leftarrow 0$
5:   while $O$ is neither closed nor consistent ∧ ($\text{Change}T = 0$) do
6:     $\chi \leftarrow \text{CheckObsTable}(O)$
7:     $T$, sample, $N_{sample}$, $q_m \leftarrow M_{query}(T, \chi, \text{sample}, N_{sample}, C, q_m)$
8:   end while
9:   $(\lambda, \rho), O, \text{sample}, N_{sample}, q_h \leftarrow E_{query}(O, \text{sample}, N_{sample}, q_h)$
10: end while

Algorithm 1 shows the AFRAI-RL algorithm. Algorithm 1 starts by checking whether there are any counterexample traces that need to be added to the observation table. These traces and their suffixes are added to the observation table (Algorithm 1, Line 3), and the subroutine $\text{CheckObsTable}$ (see Algorithm 4) is used to find membership query traces (noted as $\chi$ in all algorithms).

Then, Algorithm 1 proceeds to answer two types of queries, namely the membership query (Algorithm 1, Line 7) and the equivalence query (Algorithm 1, Line 9).

**Answering Membership Queries:** The detailed procedures to answer membership queries are shown in Algorithm 2. The subroutine sets $T(\zeta)$ to 1 if the trace $\zeta$ can be accepted by the DFA converted from a finite reward automaton (that can encode an unknown non-Markovian reward function), and sets $T(\zeta)$ to 0 otherwise. We maintain a set $\text{sample}$ of accepted traces and use the subroutine $\text{CheckSample}$ (see Algorithm 5) to check whether a membership query can already be answered by the set $\text{sample}$ (Algorithm 2, Lines 6 to 8). If it can, we provide the answer to $T(\zeta)$ in the observation table $O$ (Algorithm 2, Line 8); otherwise, we perform RL to answer the membership query (see Algorithm 7 for details). If the membership query trace $\zeta$ is inconsistent with a trace in RL from the environment (e.g., the membership query trace $\zeta = (\ell_1, 0), (\ell_2, 1), (\ell_3, 1)$, while a trace $\zeta = (\ell_1, 0), (\ell_2, 0)$ is observed from the environment), then a flag is set (Algorithm 2, Line 14) to stop the loop, $T(\zeta)$ is set to zero immediately (Algorithm 2, Line 25), and move on to the next trace. Otherwise we add the trace to $\text{sample}$ (Algorithm 2, Line 16). To boost efficiency, we set a limit on how many episodes we perform to answer a membership query. This limit is $C \in \mathbb{Z}_{>0}$. We answer the membership query as 0 if after $C$ episodes the membership query still cannot be answered (Algorithm 2, Lines 24, 25). Such traces are recorded in the set $N_{sample}$ (Algorithm 2, Line 26). Afterwards, if the trace for the membership query is encountered in the environment, we use the subroutine $\text{CheckNSample}$ (see Algorithm 6) to change the original answers in the observation table $O$ accordingly. This is performed during both membership (Algorithm 2, Line 17) and equivalence queries (Algorithm 3, Line 7). If the answer was changed during a membership query (i.e., $\text{Change}T = 1$), we exit the Algorithm 2 (Line 19) to generate the additional membership query traces created from changing the table (Algorithm 1, Line 5).
Algorithm 2 MQuery
1: **Input**: $T, \chi, \text{sample}, \text{Nsample}, C, q_m$
2: $Query \leftarrow$ membership
3: for each $\zeta \in \chi$ do
4: Construct a query finite reward automaton $A^m(\zeta)$
5: $counter \leftarrow 0$
6: $PrevAnswer \leftarrow \text{check}(\zeta, \text{sample})$
7: if $PrevAnswer \neq \text{Null}$ then
8: $T(\zeta) = PrevAnswer$
9: else
10: $Inconsistent \leftarrow 0$
11: while $\text{check}(\zeta, \text{sample}) = \text{Null} \land (counter < C) \land Inconsistent = 0$ do
12: $\lambda, \rho, q_m \leftarrow \text{RL-Engine}(Query, A^m(\zeta), q_m)$
13: if $\zeta$ is inconsistent with $(\lambda, \rho)$ then
14: $Inconsistent \leftarrow 1$
15: else
16: Add $(\lambda, \rho)$ to sample
17: $ChangeT, T \leftarrow \text{CheckNSample}((\lambda, \rho), \text{Nsample}, T)$
18: if $ChangeT = 1$ then
19: return $T, \text{sample}, \text{Nsample}, q_m$
20: end if
21: counter $\leftarrow$ counter + 1
22: end if
23: end while
24: if $(counter > C) \lor (Inconsistent = 1)$ then
25: $T(\zeta) \leftarrow 0$
26: Add $\zeta$ to $\text{Nsample}$
27: else
28: $T(\zeta) \leftarrow 1$
29: end if
30: end if
31: end for
32: return $T, \text{sample}, \text{Nsample}, q_m$

Algorithm 3 EQuery
1: **Input**: $O, \text{sample}, \text{Nsample}, q_h$
2: Construct a hypothesis finite reward automaton $A^h(O)$
3: $Query \leftarrow$ equivalence
4: **Do**
5: $\lambda, \rho, q_h \leftarrow \text{RL-Engine}(Query, A^h, q_h)$
6: Add $(\lambda, \rho)$ to sample
7: $ChangeT, T \leftarrow \text{CheckNSample}((\lambda, \rho), \text{Nsample}, T)$
8: Until Find counterexample $(\lambda, \rho)$
9: **Return** $(\lambda, \rho), O, \text{sample}, \text{Nsample}, q_h$
Answering Equivalence Queries: The detailed procedures to answer equivalence queries are shown in Algorithm 3. We perform RL with the hypothesis finite reward automaton, updating sample along the way (Algorithm 3, Line 6), until a counterexample is found (Algorithm 3, Line 8). A counterexample is a trace where the rewards given by environment are different from the rewards given by hypothesis finite reward automaton. Specifically, there are two types of counterexamples. A positive counterexample is a trace that is accepted by the DFA converted from the current hypothesis finite reward automaton, but is not accepted by the DFA converted from any finite reward automaton that can encode the unknown non-Markovian reward function. A negative counterexample is a trace that is not accepted by the DFA converted from the current hypothesis finite reward automaton, but is accepted by the DFA converted from any finite reward automaton that can encode the unknown non-Markovian reward function. The RL-engine returns counterexamples to the inference engine for another round of inference (Algorithm 3, Line 5 and 9).

Algorithm 4 CheckObsTable

1: Input: $\mathcal{O}$
2: if $\mathcal{O}$ is not consistent then
3:     Find $s_1, s_2 \in S$, $\sigma \in \Sigma$ and $e \in E$ such that $\text{row}(s_1) = \text{row}(s_2)$ and $T(s_1\sigma e) \neq T(s_2\sigma e)$
4:     add $\sigma e$ to $E$
5:     $\chi \leftarrow (S \cup S\Sigma)\sigma e$
6: else if $\mathcal{O}$ is not closed then
7:     Find $s \in S$ and $\sigma \in \Sigma$ such that $\forall s \in S, \text{row}(s\sigma) \neq \text{row}(s)$
8:     add $s\sigma$ to $S$
9:     $\chi \leftarrow (s\sigma \cup s\sigma\Sigma)E$
10: end if
11: Return $\chi$

Algorithm 5 CheckSample

1: Input: $\zeta, \text{sample}$
2: for each trace $(\lambda, \rho)$ in sample do
3:     if $\zeta$ is prefix of $(\lambda, \rho)$ then
4:         Return 1
5:     end if
6:     if $\zeta$ is inconsistent with $(\lambda, \rho)$ then
7:         Return 0
8:     end if
9: end for
10: Return $Null$
Algorithm 6 CheckNSample

1: \textbf{Input}: $(\lambda, \rho), N_{sample}, T$
2: \textbf{for} each $\zeta' \in N_{sample}$ \textbf{do}
3: \hspace{1em} \textbf{if} $\zeta'$ is a prefix of $(\lambda, \rho)$ \textbf{then}
4: \hspace{2em} $\text{Change}T = 1$, $T(\zeta') = 1$
5: \hspace{1em} \textbf{end if}
6: \textbf{end for}
7: \textbf{Return} $\text{Change}T, T$

Algorithm 6 generates membership query traces based on whether or not the observation table $O$ is closed or consistent. If the table is not consistent, then we add $\sigma e$ to $E$ and each $\zeta \in (S \cup S\Sigma)\sigma e$ forms a membership query trace. If the table is not closed, then we add $\sigma e$ to $S$ and each $\zeta \in (S\sigma \cup S\Sigma)E$ forms a membership query trace.

Algorithm 5 shows the subroutine CheckSample. It returns 1 or 0 if the membership query for $\zeta$ has already been answered, and $\text{Null}$ otherwise. For each trace $(\lambda, \rho)$ in sample, if a membership query trace $\zeta$ is prefix of $(\lambda, \rho)$, then $\zeta$ must be accepted by the DFA converted from the finite reward automaton that encodes the unknown non-Markovian reward function; hence CheckSample returns 1. If $\zeta$ is inconsistent with a trace in sample, then $\zeta$ cannot be accepted by the DFA mentioned above and CheckSample returns 0.

Algorithm 6 shows the subroutine CheckNSample. Each trace $\zeta'$ in $N_{sample}$ is checked to see if it is a prefix of $(\lambda, \rho)$ (the recent answer from the RL-Engine). If the trace is a prefix, then its answer in the observation table is changed (Line 4). A flag, $\text{Change}T$, is set so that the observation table is rechecked for being closed and consistent in Algorithm 1.

By answering the membership and equivalence queries, $L^*$ algorithm is guaranteed to converge to the minimum DFA accepting the unknown regular language $L$ using $O(|\Sigma|n^2 + n \log c)$ membership queries and at most $n - 1$ equivalence queries, where $n$ denotes the number of states in the final DFA and $c$ is the length of the longest counterexample from the RL engine when answering equivalence queries.

4.2 Active Reinforcement Learning Engine

In this subsection, we introduce the active reinforcement learning engine. We first define a \textit{query finite reward automaton} corresponding to a membership query trace $\zeta = (\ell_1, r_1), \ldots, (\ell_k, r_k)$ as follows.

\textbf{Definition 9.} For a membership query trace $\zeta = (\ell_1, r_1), \ldots, (\ell_k, r_k)$, we define a corresponding query finite reward automaton $A^m(\zeta) = (W^m, w^m_{init}, 2^P, R, \delta^m, \eta^m)$ as follows:

- $W^m = \{w^m_0, w^m_1, \ldots, w^m_k\}$,
- $w^m_{init} = w^m_0$,
- for any $i \in [0, k - 1]$, $\delta^m(w^m_i, \ell_{i+1}) = w^m_{i+1}$, $\delta^m(w^m_i, \ell) = w^m_m$ for any $\ell \neq \ell_i$, and
- for any $w^m \in W^m$ and any $\ell \in 2^P$, $\eta^m(w^m, \ell) = 1$, if $\delta^m(w^m, \ell) \neq w^m$; and $\eta^m(w^m, \ell) = 0$, otherwise.
Intuitively, the query finite reward automaton corresponding to a membership query trace $\zeta = (\ell_1, r_1), \ldots, (\ell_k, r_k)$ is a finite reward automaton that outputs a reward of one every time a new label $\ell_i$ ($i \in [1, k]$) is achieved (and the state of the finite reward automaton moves from $w_i^m$ to $w_{i+1}^m$). Therefore, in performing RL with the query finite reward automaton, the rewards obtained from the query finite reward automaton serve as incentives to encounter the label sequence in the membership query trace and hence to answer the membership query.

In the RL engine, we maintain two different types of q-functions: query q-functions for answering membership queries, denoted as $q_m$; and hypothesis q-functions for maximizing the cumulative rewards from the environment (also answering equivalence queries), denoted as $q_h$.

We update the query q-functions as follows:

$$q_m(x, w^m, a) \leftarrow (1 - \alpha)q_m(x, w^m, a) + \alpha(r_m + \gamma \max_a q_m(x', w'^m, a)),$$

Similarly, we update the hypothesis q-functions as follows:

$$q_h(x, w^h, a) \leftarrow (1 - \alpha)q_h(x, w^h, a) + \alpha(r_h + \gamma \max_a q_h(x', w'^h, a)),$$

Algorithm 7 RL-Engine

1: Hyperparameters: learning rate $\alpha$, discount factor $\gamma$, episode length $eplength$
2: Input: Variable Query, a reward automaton $A$, q-function $q$
3: $x \leftarrow InitialState(); (\lambda, \rho) \leftarrow []$
4: $w \leftarrow w_{init}$
5: for $0 \leq t < eplength$ do
6: $x', w', q \leftarrow Step(Query, A, q, x, w)$
7: append $(L(x, a, x'), r)$ to $(\lambda, \rho)$
8: $x \leftarrow x', w \leftarrow w', t \leftarrow t + 1$
9: end for
10: return $(\lambda, \rho, q)$

Algorithm 7 shows the procedure to run each RL episode to answer an membership query or equivalence query. Algorithm 7 first initializes the initial state $x$ of the MDP, the initial state of the (query or hypothesis) finite state automaton $w$ and the trace (as the empty trace). Specifically, $w$ is initialized as $w_{init}$ if it is in the membership query phase ($A = A_m$), and initialized as $w_{init}$ if it is in the equivalence query phase ($A = A_h$) (Line 6). Algorithm 8 is used to run one step through the environment (see next paragraph for details). We feed the query q-function and finite reward automaton into algorithm 8 if a membership query is being asked, and the hypothesis q-function and automaton otherwise. Algorithm 7 returns the trace, the query q-function and the hypothesis q-function (Line 10).
Algorithm 8 Step

1: Input: Variable Query, a finite reward automaton \( \mathcal{A} \), a q-function \( q \), an MDP state \( x \), and an FRA state \( w \).
2: \( a = \text{GetEpsilonGreedyAction}(q, w, x) \)
3: \( x' = \text{ExecuteAction}(x, a) \)
4: if Query = membership then
5: \( r \leftarrow \eta(w, L(x, a, x')) \);
6: else
7: Observe \( r \) from the environment;
8: end if
9: \( w' = \delta(w, L(x, a, x')) \)
10: \( q(x, w, a) \leftarrow (1 - \alpha)q(x, w, a) + \alpha(r + \gamma \max_a q(x', w', a)) \)
11: for \( \hat{w} \in W \setminus \{w\} \) do
12: \( \hat{w}' = \delta(\hat{w}, L(x, a, x')) \)
13: \( \hat{r} = \eta(\hat{w}, L(x, a, x')) \)
14: \( q(x, \hat{w}, a) \leftarrow (1 - \alpha)q(x, \hat{w}, a) + \alpha(\hat{r} + \gamma \max_a q(x', \hat{w}', a)) \)
15: end for
16: return \( x', w', q \)

Algorithm 8 runs one step through the environment, updating the q-function for either a membership or equivalence query. First, at state \( x \) an action \( a \) is selected according to the q-function using the epsilon-greedy approach and executed to reach a new state \( x' \) (Line 2). Then rewards are collected based on the type of query being asked. If a membership query is being asked, then the automaton’s reward function is used; otherwise the reward is observed from the environment. The next mealy state for the automaton is calculated (Line 9) and the q-function supplied is updated (Line 10).

Lemma 2. Let \( M \) be a labeled MDP and \( \mathcal{A} \) the finite reward automaton encoding the reward function of \( M \). Then, AFRAI with eplength \( \geq 2^{|M|+1}(|\mathcal{A}|+1)-1 \) almost surely learns a finite reward automaton in the limit that is equivalent to \( \mathcal{A} \) on all attainable label sequences.

Proof. Given an eplength \( = 2^{|M|+1}(|\mathcal{A}|+1)-1 \), we are almost sure to experience every possible attainable trace from the environment. The proof of this is similar to the proof provided for Lemma 2 in [33] (Proof is located in Appendix C of [33]). Given every attainable trace, AFRAI-RL will answer all membership and equivalence queries correctly with probability 1 in the limit, because the observation table \( O \) can be changed as the algorithm runs (Algorithm Lines 7 and 9). Furthermore, the author in [4] shows that if all membership and equivalence queries can be answered, a DFA can be formed from the observation table (as in Remark 3). This DFA will be the smallest DFA that can accept the language defined in Lemma 1. In this RL context, this language will match the language of the DFA converted from the finite reward automaton; i.e., the language will encode the finite reward automaton that is equivalent to \( \mathcal{A} \). Therefore, the observation table encodes the finite reward automaton that is equivalent to \( \mathcal{A} \).

With Lemma 2 we can proceed to prove Theorem 3.
Theorem 3. Let $\mathcal{M}$ be a labeled MDP and $A$ the reward machine encoding of the reward function of $\mathcal{M}$. Then, AFRAI-RL with $\text{eplength} \geq 2^{|\mathcal{M}|+1} \cdot (|A| + 1) - 1$ almost surely converges to an optimal policy in the limit.

Proof. Lemma 2 shows that, eventually, the reward machine learned by AFRAI-RL, will be equivalent to $A$ on all attainable label sequences. Let $\mathcal{H}$ be the reward machine learned by AFRAI-RL and $\mathcal{M}_H$ be the product MDP.

Thus an optimal policy for $\mathcal{M}_H$ will also be optimal for $\mathcal{M}$. When running episodes of QRM (Algorithm 7) under the reward machine $\mathcal{H}$, an update of a q-function connected to a state of $\mathcal{H}$ corresponds to updating the q function for $\mathcal{M}_H$. Since $\text{eplength} \geq |\mathcal{M}|$, the fact that QRM uses the epsilon-greedy strategy and that updates are done in parallel for all states of $\mathcal{H}$ implies that every state-action pair of the $\mathcal{M}_H$ will be seen infinitely often. Hence, the convergence of q-learning for $\mathcal{M}_H$ to an optimal policy is guaranteed by [26]. Therefore, as the number of episodes goes to infinity, with $\text{eplength} \geq 2^{|\mathcal{M}|+1} \cdot (|A| + 1) - 1$, AFRAI-RL converges towards an optimal policy.

5 Case Studies

In this section, we apply the proposed approach to the office world scenario adapted from [17] and the craft world scenario from [3]. We perform the following four different methods:

- AFRAI-RL: We use the libalf [6] implementation of active automata learning as the algorithm to infer finite reward automata.

- JIRP-SAT: We use the libalf [6] implementation of SAT-solving (see Section 3.2 of [33]) as the algorithm to infer finite reward automata.

- LRM-QRM: We use the QRM implementation from [17], adapted to test the agent at the end of each episode.

- PPO2: We use the Stable Baselines implementation of PPO2 [22]. The state space is a history of the past states the agent has been in. This was added because PPO2 doesn’t have any way of remembering it’s previous states.

In [33], the authors have shown that JIRP-SAT and JIRP-RPNI outperform q-learning in augmented state space (QAS), hierarchical reinforcement learning (HRL), and deep reinforcement learning with double q-learning (DDQN) in three case studies. Therefore, if we can show that AFRAI-RL outperforms JIRP-SAT and JIRP-RPNI, then we can deduce that AFRAI-RL outperforms QAS, HRL and DDQN as well.

5.1 Office World Scenario

We consider the office world scenario in the 9×12 grid-world. Figure 2 shows the map in the office world scenario. We use the triangle to denote the initial position of the agent. The agent has four possible actions at each time step: move north, move south,
move east and move west. After each action, the robot may slip to each of the two adjacent cells with probability of 0.05. In Algorithm 1, we set $C = 500$.

We consider the following three tasks:

**Task 1**: first go to a, then go to b and a in this order, and finally return to c. Episode Length was set to 200, and total training time was set to 1,000,000.

**Task 2**: first go to b, then go to c and a in this order, then repeat the sequence. Episode length was set to 800, and total training time was set to 2,000,000.

**Task 3**: first go to c, then go to b and a in this order, then go to b and c in this order and return to a. Episode length was set to 800, and total training time was set to 6,000,000.

Figure 3 shows the attained rewards of 10 independent simulation runs for each task, averaged every 10 training steps. For task 1, it can be seen that, on average, the proposed AFRAI-RL approach converges to an optimal policy in about 0.2 million training steps, while JIRP-SAT, LRM-QRM, and PPO2 do not converge to an optimal policy. For task 2, on average the proposed AFRAI-RL approach converges to an optimal policy in about 1.8 million training steps, while JIRP-SAT, LRM-QRM, and PPO2 do not converge to an optimal policy. For task 3, on average the proposed AFRAI-RL approach converges to an optimal policy in about 4.0 million training steps, while JIRP-SAT converges to an optimal policy in about 4.5 million training steps and LRM-QRM, and PPO2 do not converge to an optimal policy.

5.2 Minecraft world scenario

We consider the Minecraft world scenario in the $21 \times 21$ grid-world [3]. The four actions and the slip rates are the same as in the office world scenario. We train on two tasks: making a hammer and spear. In Algorithm 1, we set $C = 500$.

We consider the following two tasks, noting the symbol used for each object in parenthesis:

**Task 1**: Build a Hammer. Agent must collect string (b), stone (e), iron (f), stone (e) (in this order) and travel to the workbench (c) to make the hammer. Episode length was set to 400, and total training time was set to 400,000.

**Task 2**: Build a Spear. The agent must collect string, stone, wood (a), string and travel to the workbench to make the spear. Episode length was set to 400, and total training time was set to 250,000. Figure 4 shows the attained rewards of 10 independent simulation runs for each task, averaged every 10 training steps. For task 1, it can be seen that, on
Fig. 3: Attained rewards of 10 independent simulation runs of the office world scenario, averaged for every 10 training steps in AFRAI-RL (First row), JIRP-SAT (Second Row), LRM-QRM (Third Row), and PPO2 (Fourth Row): (a) Task 1; (b) Task 2; (c) Task 3.
Fig. 4: Attained rewards of 10 independent simulation runs of the Minecraft world scenario, averaged for every 10 training steps in AFRAI-RL (First row), JIRP-SAT (Second Row), LRM-QRM (Third Row) and PPO2 (Fourth Row): (a) Task 1; (b) Task 2.
average, the proposed AFRAI-RL approach converges to an optimal policy in about 190,000 training steps, while JIRP-SAT, LRM-QRM, and PPO2 do not converge to an optimal policy. For task 2, on average the proposed AFRAI-RL approach converges to an optimal policy in about 170,000 million training steps, while JIRP-SAT, LRM-QRM, and PPO2 do not converge to an optimal policy.

6 Conclusions

We propose an active reinforcement learning approach that infers finite reward automata during the RL process through interaction with the environment. The algorithm can actively guide the RL towards finding answers to the queries needed for inferring the finite reward automata, thus making the finite reward automaton learning process more efficient. The case studies show that this algorithm is more efficient than recent baseline algorithms for the environments used.

This work has potential in multiple future directions. First, we assume that the same state-action sequences lead to the same reward values in this paper (i.e., the method depends on the correct labeling of the trajectories). We will investigate the scenarios where incorrect labels can occur and the same state-action sequences may lead to different reward values. Second, current active RL work usually adopts a model-based framework and search for states that are less visited, thus improving the sample efficiency of RL. Our proposed approach adopts a model-free framework and actively recovers the reward structure, which turns out to be effective in the non-Markov RL scenarios. To make connections with the other work in active RL, we will investigate model-based RL with active finite reward automaton inference. Finally, as we consider RL for a single agent in this paper, we will extend the work to multi-agent settings for collaborative or non-collaborative games.

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References

1. D. Aksaray, A. Jones, Z. Kong, M. Schwager, and C. Belta, “Q-learning for robust satisfaction of signal temporal logic specifications,” in IEEE CDC’16, Dec 2016, pp. 6565–6570.
2. M. Alshiekh, R. Bloem, R. Ehlers, B. Könighofer, S. Niekum, and U. Topcu, “Safe reinforcement learning via shielding,” in AAAI’18, 2018.
3. J. Andreas, D. Klein, and S. Levine, “Modular multitask reinforcement learning with policy sketches,” in Proceedings of the 34th International Conference on Machine Learning-Volume 70. JMLR. org, 2017, pp. 166–175.
4. D. Angluin, “Learning regular sets from queries and counterexamples,” *Information and computation*, vol. 75, no. 2, pp. 87–106, 1987.

5. N. Baharisangari, J.-R. Gaglion, D. Neider, U. Topcu, and Z. Xu, “Uncertainty-aware signal temporal logic inference,” 2021. [Online]. Available: https://arxiv.org/abs/2105.11545

6. B. Bollig, J. Katoen, C. Kern, M. Leucker, D. Neider, and D. R. Piegdon, “libalf: The automata learning framework,” in *Computer Aided Verification, 22nd International Conference, CAV 2010, Edinburgh, UK, July 15-19, 2010. Proceedings*, 2010, pp. 360–364.

7. G. Bombara, C.-I. Vasile, F. Penedo, H. Yasuoka, and C. Beta, “A decision tree approach to data classification using signal temporal logic,” in *Proc. HSCC’16*, 2016, pp. 1–10.

8. M. Cai, M. Hasanbeig, S. Xiao, A. Abate, and Z. Kan, “Modular deep reinforcement learning for continuous motion planning with temporal logic,” 2021.

9. J. Fu and U. Topcu, “Probably approximately correct MDP learning and control with temporal logic constraints,” *Robotics: Science and Systems*, vol. abs/1404.7073, 2014.

10. D. Furelos-Blanco, M. Law, A. Russo, K. Broda, and A. Jonsson, “Induction of subgoal automata for reinforcement learning,” *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 34, no. 04, pp. 3890–3897, Apr. 2020. [Online]. Available: https://ojs.aaai.org/index.php/AAAI/article/view/5802

11. J.-R. Gaglion, D. Neider, R. Roy, U. Topcu, and Z. Xu, “Learning linear temporal properties from noisy data: A maxsat-based approach,” in *ATVA 2021, Gold Coast, Australia, October 18-22*, ser. Lecture Notes in Computer Science. Springer, 2021.

12. M. Gaon and R. Brafman, “Reinforcement learning with non-markovian rewards,” *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 34, no. 04, pp. 3980–3987, Apr. 2020. [Online]. Available: https://ojs.aaai.org/index.php/AAAI/article/view/5814

13. A. Holzinger, B. Malle, A. Saranti, and B. Pfeifer, “Towards multi-modal causability with graph neural networks enabling information fusion for explainable AI,” *Information Fusion*, vol. 71, pp. 28–37, 2021. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1566253521000142

14. J. E. Hopcroft, R. Motwani, and J. D. Ullman, *Introduction to Automata Theory, Languages, and Computation (3rd Edition)*. Boston, MA, USA: Addison-Wesley Longman Publishing Co., Inc., 2006.

15. B. Hoxha, A. Dokhanchi, and G. Fainekos, “Mining parametric temporal logic properties in model-based design for cyber-physical systems,” *International Journal on Software Tools for Technology Transfer*, pp. 79–93, Feb 2017. [Online]. Available: http://dx.doi.org/10.1007/s10009-017-0447-4

16. R. A. T. Icarte, E. Waldie, T. Klassen, R. Valenzano, M. P. Castro, and S. A. McIlraith, “Learning reward machines for partially observable reinforcement learning,” in *NeurIPS*, 2019.

17. R. T. Icarte, T. Q. Klassen, R. A. Valenzano, and S. A. McIlraith, “Using reward machines for high-level task specification and decomposition in reinforcement learning,” in *ICML 2018, Stockholmsmässan, Stockholm, Sweden, July 10-15, 2018*, 2018, pp. 2112–2121.

18. Z. Kong, A. Jones, and C. Beta, “Temporal logics for learning and detection of anomalous behavior,” *IEEE TAC*, vol. 62, no. 3, pp. 1210–1222, Mar. 2017.

19. X. Li, C.-I. Vasile, and C. Beta, “Reinforcement learning with temporal logic rewards,” in *Proc. IROS’17*, Sept 2017, pp. 3834–3839.

20. D. Neider, J.-R. Gaglione, I. Gavran, U. Topcu, B. Wu, and Z. Xu, “Advice-guided reinforcement learning in a non-markovian environment,” in *AAAI’21*, 2021.

21. D. Neider and I. Gavran, “Learning linear temporal properties,” in *Formal Methods in Computer Aided Design (FMCAD)*, 2018, pp. 1–10.

22. J. Schulman, F. Wolski, P. Dhariwal, A. Radford, and O. Klimov, “Proximal policy optimization algorithms,” 2017.
23. A. Shah, P. Kamath, J. A. Shah, and S. Li, “Bayesian inference of temporal task specifications from demonstrations,” in *NeurIPS*, S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, Eds. Curran Associates, Inc., 2018, pp. 3808–3817. [Online]. Available: [http://papers.nips.cc/paper/7637-bayesian-inference-of-temporal-task-specifications-from-demonstrations.pdf](http://papers.nips.cc/paper/7637-bayesian-inference-of-temporal-task-specifications-from-demonstrations.pdf)

24. R. Toro Icarte, T. Q. Klassen, R. Valenzano, and S. A. McIlraith, “Teaching multiple tasks to an RL agent using LTL,” in *AAMAS’18*, Richland, SC, 2018, pp. 452–461.

25. M. Vazquez-Chanlatte, S. Jha, A. Tiwari, M. K. Ho, and S. A. Seshia, “Learning task specifications from demonstrations,” in *NeurIPS*, 2018, pp. 5372–5382.

26. C. J. C. H. Watkins and P. Dayan, “Q-learning,” *Machine Learning*, vol. 8, no. 3, pp. 279–292, May 1992. [Online]. Available: [https://doi.org/10.1007/BF00992698](https://doi.org/10.1007/BF00992698)

27. M. Wen, I. Papusha, and U. Topcu, “Learning from demonstrations with high-level side information,” in *Proc. IJCAI’17*, 2017, pp. 3055–3061.

28. B. Wu and H. Lin, “Counterexample-guided permissive supervisor synthesis for probabilistic systems through learning,” in *2015 American Control Conference (ACC)*. IEEE, 2015, pp. 2894–2899.

29. B. Wu, X. Zhang, and H. Lin, “Permissive supervisor synthesis for markov decision processes through learning,” *IEEE Transactions on Automatic Control*, vol. 64, no. 8, pp. 3332–3338, 2018.

30. ——, “Supervisor synthesis of pomdp based on automata learning,” *Automatica*, 2021, to appear. [Online]. Available: [https://arxiv.org/abs/1703.08262](https://arxiv.org/abs/1703.08262)

31. Z. Xu, M. Bittwistle, C. Belta, and A. Julius, “A temporal logic inference approach for model discrimination,” *IEEE Life Sciences Letters*, vol. 2, no. 3, pp. 19–22, Sept 2016.

32. Z. Xu, C. Belta, and A. Julius, “Temporal logic inference with prior information: An application to robot arm movements,” in *Proc. Anal. and Des. of Hybrid Syst.*, vol. 48, no. 27, Atlanta, GA, USA, Oct., 2015, pp. 141 – 146.

33. Z. Xu, I. Gavran, Y. Ahmad, R. Majumdar, D. Neider, U. Topcu, and B. Wu, “Joint inference of reward machines and policies for reinforcement learning,” in *Proceedings of the International Conference on Automated Planning and Scheduling*, vol. 30, 2020, pp. 590–598.

34. Z. Xu and A. Julius, “Census signal temporal logic inference for multiagent group behavior analysis,” *IEEE Trans. Autom. Sci. Eng.*, vol. 15, no. 1, pp. 264–277, Jan. 2018.

35. Z. Xu, M. Ornik, A. Julius, and U. Topcu, “Information-guided temporal logic inference with prior knowledge,” in *Proc. IEEE Amer. Control Conf.*, 2019. [Online]. Available: [https://arxiv.org/abs/1811.08846](https://arxiv.org/abs/1811.08846)

36. Z. Xu, S. Saha, B. Hu, S. Mishra, and A. Julius, “Advisory temporal logic inference and controller design for semiautonomous robots,” *IEEE Trans. Autom. Sci. Eng.*, pp. 1–19, 2018.

37. Z. Xu and U. Topcu, “Transfer of temporal logic formulas in reinforcement learning,” in *IJCAI-19*. International Joint Conferences on Artificial Intelligence Organization, 7 2019, pp. 4010–4018. [Online]. Available: [https://doi.org/10.24963/ijcai.2019/557](https://doi.org/10.24963/ijcai.2019/557)