PSR J0348 + 0432: a tool to constrain $f(R)$-theories

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There are several approaches to extend General Relativity in order to explain the phenomena related to the Dark Matter and Dark Energy. These theories, generally called Extended Theories of Gravity, can be tested using observations coming from relativistic binary systems as PSR J0348 + 0432. Using a class of analytical $f(R)$-theories, one can construct the first time derivative of orbital period of the binary systems starting from a quadrupolar gravitational emission. Our aim is to set boundaries on the parameters of the theory in order to understand if they are ruled out, or not, by the observations on PSR J0348 + 0432. Finally, we have computed an upper limit on the graviton mass showing that agree with constraint coming from other observations.

I. INTRODUCTION

Einstein’s theory of gravity, known as General Relativity (GR), is a well description of gravitational phenomena at astrophysical and cosmological scales. The theoretical predictions are in good agreement with the measurements in particular at the Solar System scale \cite{1}. Even if, the local gravity is very well described by GR, there are several observations at astrophysical and cosmological scales for which we need to add extra ingredients in the amount of matter and energy densities of the Universe to fit the data. Observations based on Supernovae Type Ia (SNeIa), Cosmic Microwave Background (CMB) temperature anisotropies, Baryon Acoustic Oscillations (BAO) and other observables, pointed out that the Universe is in a period of accelerated expansion favoring the concordance $\Lambda$CDM model \cite{2,3}. In this model, we need to input an extra component, known as Dark Energy (DE), that have the effect of a fluid with a negative pressure, to explain the acceleration of the Universe at cosmological scales \cite{4}. Furthermore, we also need to input an extra contribution in the amount of matter, known as Dark Matter (DM) to explain the clustering of the Large Scale Structure. There are observational evidences point out that the baryonic matter is not enough to explain phenomena at galaxy and cluster scales, like rotation curves, gravitational lensing, cluster profiles and others \cite{5}. However, we do not know the nature, at particle level, of this two dark ingredients, even if there are many explanations in literature: quintessence, string theory, holographic principle are examples of DE models, while Weakly Interacting Particles (WIMPS), Axions, or Massive Compact Halo Objects (MACHOs) was been proposed like candidates for DM. As an alternative, instead to introduce two unknown components to explain the dynamics and the evolution of Universe at all scales, it is possible change its the geometrical description. In this framework the Extend Theories of Gravity (ETGs) are been developed \cite{6}. By introducing the higher curvature terms in the Lagrangian allow us to reconstruct the DE and DM phenomenology at all scales, from planetary dynamics \cite{5,7}, to flat rotation curves of spiral galaxies \cite{8}, and the velocity dispersion of ellipticals \cite{9}, and from cluster of galaxies \cite{10,11} until the dynamics of the Universe as a whole \cite{12}.

However, to constraint and validate ETGs could be extremely important take also into account the GWs emission. In GR, the field equations linearized around a Minkowskian metric, show that small perturbations of the metric propagate following a wave equation \cite{13,14}. All astrophysical systems that emit GWs are very well described in the framework of GR, where the gravitational interaction is mediate by a massless boson known as graviton. But, the further degrees of freedom that come out considering ETGs lead to have also massive gravitational modes.

In particular, from the power spectrum of weak lensing one can estimate an upper limit of $7 \times 10^{-32}$ eV for the graviton mass. Then, from clusters of galaxies, it is possible to obtain an upper limit of $2 \times 10^{-29} h_{0}$ eV, where $h_{0}$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$. Finally, studying the emission of GWs from the binary systems, one can infer an upper limit of graviton mass of $7.6 \times 10^{-20}$ eV \cite{1}.

Since the mass of graviton have also effect of the waveform and on the polarization modes, the detection of GWs will point out that GR is validated or that it must be extended \cite{15}. At moment, the most important tool, regarding to GWs, to constrain both theories of gravity is related to the gravitational emission from binary systems of White Dwarfs (WDs), Neutron Stars (NS) and Black Holes (BHs) \cite{16}. Timing data analysis on the well-known binary pulsar B1913 + 16 have confirmed that the energy loss by the system can be explained with the emission of GWs. Let’s point out that those type of systems, due to the high precision of the mass estimation, and also for the strong field gravity regime in which they are (PSR B1913 + 16 or PSR J0348 + 0432 are two examples of system with very precise measurements), represent a very good laboratory to test theories of gravity using Post-Keplerian parameters \cite{18,22}. Let’s start to consider a generic class of ETGs, called $f(R)$-theories, where we replace the Einstein-Hilbert Lagrangian with a more general function of the Ricci curvature. For an analytic $f(R)$, it is possible evaluate the first time derivatives.
of the orbital period for a binary system and comparing the theoretical estimation with the observed one [23], we put bounds on theory parameters and then we compute an upper limit on the graviton mass.

The outline of the paper is the following: in Sec. II we briefly introduce the theoretical framework in which we describe binary systems computing the energy lost through GWs emission. In Sec. III we summarize how to compute the first time derivative on the orbital period for a binary system in in -gravity and, estimate the bounds on graviton mass. Finally, in Sec. IV we give our conclusions and remarks.

II. THEORETICAL FRAMEWORK: f( R)-GRAVITY

In ETGs, the field equations are computed, extending the Hilbert-Einstein Lagrangian with adding of higher-order curvature invariants and minimally or non-minimally coupled scalar fields. The simplest way is to consider a more general function of the curvature is

\[ f(R) = \sum_{n} \frac{f^n(R_0)}{n!} (R - R_0)^n \simeq f_0 + f_0' R + \frac{f_0''}{2} R^2 + \ldots \]  

(3)

In this context, using theoretical approach given in [29], the total average flux of energy emitted in GWs has the following form

\[ \frac{dE}{dt} = \frac{G}{60} \left( f_0' \left( \mathcal{Q}^{ij} \mathcal{Q}_{ij} \right) - f_0'' \left( \mathcal{Q}^{ij} \mathcal{Q}_{ij} \right) \right) \cdot \frac{f(R)}{f(R)} . \]

The ratio of \( f_0' \) and \( f_0'' \) defines an effective mass \( m_g^2 f_0' \) related to massive modes in GWs [30]. Until now, the only modes of gravitational waves that have been investigated are the massless ones, this exclude the possibility to observe further gravitational waveform coming out from massive terms, although, tests in this direction are already done on stochastic background of GWs [31–34].

From the equations of conservation we know that the gravitational radiation, predicted by GR, is proportional to the third derivative of the quadrupole momentum, while the terms due to the monopole and the dipole moment are zero. In \( f(R) \)-gravity we find a different situation, the further degrees of freedom of the gravitational fields in Einstein frame gave dipole and monopole contributions to gravitational emission (as shown in [35] in table 10.2, [36, 37]), that disappear in the Jordan frame, the physical frame where the observation are performed, where the \( f(R) \)- gravity is a straightforward extension of GR (see, e.g., [18]). Furthermore, the fact that we have extended the theory of gravity can be reflected by a mass of graviton not equal to zero. As well known, in ETG it is possible obtain massive gravitons in a natural way [30]. The main feature is that higher-order terms or induced scalar fields in the Lagrangian, give rise to massless, massive spin-2 gravitons and massive spin-0 gravitons. Such gravitational modes results in 6 polarizations, according to the Riemann Theorem stating that in a given n-dimensional space, \( n(n - 1)/2 \) degrees of freedom are possible. The fact that 6 polarization states emerge is in agreement with the possible allowed polarizations of spin-2 field [33]. In fact, the spin degenerations is

- \( d = (2s + 1), \ m_g \neq 0 \Rightarrow s = 2, \ d = 5 \)
- \( d = 2s, \ m_g = 0 \Rightarrow s = 1, \ d = 2 \)
- \( d = (2s + 1), \ m_g \neq 0 \Rightarrow s = 0, \ d = 1 \)

The massive spin-2 gravitational states, usually are ghost particles. The role of massive gravitons result relevant also in the case which we want define a cutoff mass at TeV scale. This limits allow us, both to circumvent the hierarchy problem that the detection of the Higgs boson. For

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1 For convenience we will use \( f \) instead of \( f(R) \). All considerations are developed here in metric formalism. From now on we assume physical units \( G = c = 1 \).
example, in such a case, the Standard Model of particles should be confirmed without recurring to perturbative, renormalizable schemes involving new particles \[m_{\text{DM}}\]. Furthermore, ETGs in post Newtonian regime gives rise at the Yukawa-like correction to the Newtonian potential. These corrections are dependent by a characteristic length of self-gravitating structures that is connected to massive graviton modes. Upper limits on graviton mass come out when ones try to solve the connection between masses and sizes of self-gravitating structures without invoking huge amounts of DM.

### III. THE FIRST TIME DERIVATIVE OF THE ORBITAL PERIOD OF A BINARY SYSTEM: \( f(R) \) PARAMETERS AND BOUNDS ON GRAVITON MASS

Following the scheme that is generally used to compute GWs emission, and assuming a Keplerian motion of the stars in the binary system, we can define \( m_p \) as the pulsar mass, \( m_c \) as the companion mass, and \( \mu = m_c + m_p \) as the reduced mass. The motion is reduced at the \((x - y)\)-plane, so that averaging on the orbital period, \( \bar{P}_b \), and using the eqs. \[23\], we get the first time derivative of the orbital period \[23\]

\[
\dot{P}_b = -\frac{3}{20} \left( \frac{P_b}{2\pi} \right)^{\frac{5}{2}} \mu G^\frac{5}{2} (m_c + m_p)^{\frac{3}{2}} \epsilon^5 (1 - \epsilon^2)^{\frac{5}{2}} \times \\
\times \left[ f_0' (37\epsilon^4 + 292\epsilon^2 + 96) - \frac{f_0'' \mu^2 T^{-1}}{2(1 + \epsilon^2)^3} \times \\
\times (891\epsilon^8 + 28016\epsilon^6 + 82736\epsilon^4 + 43520\epsilon^2 + 3072) \right].
\]

where \( \epsilon \) is the eccentricity of the orbit, \( G \) is the Newtonian gravitational constant, and the quantities \( f_0' \) and \( f_0'' \) are the ones that we need to constrain. Once we have a theoretical prediction of \( \bar{P}_b \) in \( f(R) \)-theories we can, according with the prescription given in \[22\], compute an upper limit for the graviton mass.

#### A. Application to PSR J0348+0432

The binary system PSR J0348+0432, recently studied by Antoniadis et al. (2013) \[22\], gives a new possibility to understand which range of ETG’s parameters is allowed. It is a binary system composed by a pulsar spinning at 39 ms with mass 2.01 ± 0.04 \( M_\odot \) (\( m_p \)), and a White Dwarf (WD) companion with mass 0.172 ± 0.003 \( M_\odot \) (\( m_c \)). The orbital period of the system is \( P_b = 0.102 \) (days), and the eccentricity \( e = 2.36008 \times 10^{-6} \). For a correct estimation of the observed orbital decay, it was considered several kinematic effects that have to be subtracted by variation of the orbital period. The first one is the Shklovskii effect \[43\], that is an expression of the effect due to the proper motion of the the binary system. Its estimation for this system is

\[
\dot{P}_b^{\text{Shk}} = P_b \frac{\mu^2 d}{c} = 0.0129^{+0.0025}_{-0.0021} \times 10^{-13}. \quad (6)
\]

Another effect is due to the difference of Galactic accelerations between the binary system and the Solar System

\[
\dot{P}_b^{\text{Acc}} = P_b \frac{\mu c}{G} = 0.0037^{+0.0006}_{-0.0005} \times 10^{-13}. \quad (7)
\]

The last term is due to a possible variation of the gravitational constant \( G \) \[47\]:

\[
\dot{P}_b^G = -2P_b \frac{\dot{G}}{G} = (0.0003 ± 0.0018) \times 10^{-13}. \quad (8)
\]

Finally, the observed value of the first time derivative of the orbital period is \( \dot{P}_b = (-2.73 ± 0.45) \times 10^{-13} \). All those terms are quoted in \[22\].

As pointed out in \[23\], the ETGs are not ruled out when the orbital parameters of the binary systems are very well estimated, and the range of the \( f(R) \) parameters is 0.04 ≤ \( |f''_0| \) ≤ 38.

In Fig. 1 we report the result of our numerical analysis on the binary system PSR J0348+0432. We use the following notation: the black line shows the behavior of the GR prediction for the first derivative of the orbital period for a binary system; the red line represents the observed orbital period variation \( \dot{P}_b^{\text{Obs}} \) and its errors, in particular the dashed lines show the experimental error on the observation; the blue line shows the \( f(R) \)-theory prediction as computed in eq. \[5\].

![Figure 1. It is shown the result of our numerical analysis on the binary system PSR J0348+0432.](image_url)
which ETGs are able to explain the observed period variation of the GWs signal with respect to the plus (+) and cross (×) polarizations predicted by GR [18]. Indeed, the additional polarization modes predicted by GR [18], together with other results from observations in the Solar System and Cosmology, indicates that the study and the possible detection of massive modes of GWs could be the real discriminating between the theory of GR and its extensions.

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