Finite Rate of Innovation Theory Applied to Terahertz Signal Processing

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Abstract—Methods based on finite rate of innovation have been demonstrated to perform better than the relevant standard methods at other frequencies and for other imaging modalities. We apply this theory to terahertz imaging, outlining some of the necessary steps for specializing the method to best suit terahertz signals. To demonstrate the effectiveness of our method we apply it to simulated terahertz signals, comparing the amplitudes and time locations used to create the simulated signal with those extracted through our method from the simulated signal. The close agreement between the amplitudes and time locations of the modelled reflections illustrates the impressive accuracy of results produced from our method.

I. INTRODUCTION

Signal processing methods can achieve extraction of additional useful information, faster data processing speeds and quicker data acquisition. Methods based on finite rate of innovation (FRI) theory [1] have improved on the standard methods in other areas such as ultrasound [2] and radio signals [3], through various metrics including the number of data points required for accurate results. We are most interested in this metric, as it would allow for faster data acquisition as fewer data points would be needed to be measured. This would be particularly beneficial for our research group’s terahertz (THz) in vivo measurements on human skin [4], as this requires the patients to remain still for the duration of the measurement. By reducing this duration, we limit the time dependent factors, such as occlusion of the skin, and minimize the difficulty of the measurement for the patient, as they will need to remain still for a shorter period of time.

Here, we demonstrate key parts for the creation of a THz specific method based on FRI. Firstly we introduce an important signal property, that being the signal having a finite number of degrees of freedom per unit time [1]. In the context of our THz signal in reflection geometry, the degrees of freedom would be the amplitudes and time positions of the reflections. Therefore, we have two degrees of freedom per reflection per unit time. To simplify the matter, we can consider our whole measurement time range as the one unit of time. Another key step to making this method THz specific is to create a sampling kernel which accurately represents the form of our THz pulse whilst being mathematically described in a way useable in FRI. One which satisfies all these requirements is the sum-of-sinc sampling kernel, mathematically defined, in general terms, by:

\[ H(\omega) = \sum_{p \in \Pi} d_p \text{sinc} \left( \frac{\omega}{2\pi} - p \right), \]

where \( p \) is an integer in the chosen set of integers \( \Pi \), \( \omega \) is the frequency and \( \tau \) is the period containing an entire repetition of the SoS, as we will be using this period to cover our whole time range. The chosen integer number set \( \Pi \) and \( d_p \) are free parameters. Here \( \Pi \) is chosen to be \( \Pi = \{-P, \ldots, P\} \) with \( P = 25 \), and \( d_p \) are taken from a symmetric Hamming window. This kernel can be seen in figure 1, in time domain. The amplitude and time location of the peaks were selected to approximate our standard THz pulse shape, but it can be easily changed to suit any THz pulse shape.

II. RESULTS

We simulated a THz measurement in reflection geometry by using the sampling kernel shown in figure 1 as a template for each reflection. Our simulated data contained 5 reflections, each with their amplitude and time location randomly selected from an appropriate range. These amplitudes and time locations are represented by the blue Dirac peaks in figure 2. These simulated data were then fed through our code and processed using our THz specific FRI method and resulting in estimated amplitudes and time locations of the modelled reflections. Processed Dirac peaks, shown in orange, give the amplitudes and time locations of the reflections estimated by our method, after inputting the modelled data.

Fig. 1. A sum-of-sinc based sampling kernel for approximating THz signals in our finite rate of innovation method.

Fig. 1. Original Dirac peaks, shown in blue, represent the time locations and amplitudes of the modelled reflections. Processed Dirac peaks, shown in orange, give the amplitudes and time locations of the reflections estimated by our method, after inputting the modelled data.
amplitudes and time locations of the reflections, represented by the orange Dirac peaks. By comparing the width of our THz like kernel shown in figure 1 to the close time spacing between some of the reflections in figure 2, we see that there is an overlap between some reflections. Even with this overlap, the close agreement between the known amplitudes and time locations used to create the modelled data and the estimated values determined by our method shows the accuracy of this method. These results were achieved with a relatively low sampling rate, compared to the usual rate for other standard methods. The uniform sampling rate was selected to be low enough that approximately only 3 or 4 sampling points were used to describe each reflected pulse.

To conclude, we have outlined some of the key stages for applying FRI theory to THz. By applying our method to simulated THz reflection data, we successfully extracted amplitudes and time locations of the reflections which were in strong agreement to the chosen values used to create the simulated data. This was achieved whilst using a relatively low sampling rate which would allow for faster data acquisition, a key benefit FRI can provide.

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