Effect Of Leading-edge Geometry Parameters On The Performance Of Miniature Cryogenic Expansion Turbine

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Abstract. The numerical simulation included solid blades with four different leading-edge thicknesses and four different leading-edge geometries. One of the geometries was square, one was ellipse A (ellipse ratio is 1), one was ellipse B (ellipse ratio is 2), and the other was ellipse C (ellipse ratio is 3). The four thicknesses were 0.7mm, 0.6mm, 0.5mm, 0.4mm. The results show increased efficiency loss for increased leading-edge thickness for square geometry. For ellipse geometries, there was no significant difference when the leading-edge thickness changed at the positive incidence range. For the same leading-edge thickness (0.7mm), square leading-edge caused more loss than ellipse leading-edge. For square geometry, the optimal incidence angle was about -8 degree(0.7mm). For ellipse geometries, the optimal incidence angle was about -30 degree(0.7mm). And with the decrease of leading-edge thickness, the square's optimal incidence angle was toward to zero degree, the ellipse's optimal incidence angle was toward to larger negative angle.

1. Introduction
Effect of trailing-edge geometry and thickness on the performance of turbine has already haven a experimental verification\cite{1}. While leading edge are often established in an arbitrary and intuitive way although test results seem to indicate that they have a sensible effect on the form of pump or turbine characteristics\cite{2}. In order to study the specified effect of leading-edge geometry and thickness on turbine, we compared the thermal performance of the four leading edge geometries with four different thicknesses based on numerical simulation.
For avoiding the excessive loss caused by the separation flow from the suction surface near the leading-edge, the flow at stagnation point is approximately radial (figure 1)[2][3]. But we found that the leading-edge geometry is square in figure 1. For ellipse leading-edge, whether the direction of the streamline is radial at stagnation point should be further explored.

2. Numerical method

Figure 2 shows a impeller with an ellipse A leading-edge, and its thickness is 0.7mm. Blade number is 10, the inlet diameter of the turbine is 16mm, the working fluid is Helium-4. In order to ensure the principle of single variable in the calculation process, the mass flow rate at the inlet of turbine is constant as 18 g/s. Since the incidence angle at the inlet of turbine will change, the relative speed cannot be constant, and the range of the inlet flow relative mach number is 0.112-0.176. The positive incidence angle means that the relative flow at the inlet of turbine impacting the pressure surface of the rotor, while the negative incidence angle impacting the suction surface of the rotor. Figure 3 shows the four different leading-edge geometries. Figure 4 shows the 3D mesh of the impeller(leading-edge amplification), the total element number is 471293, and total number of faces is 57546.

2.1 Continuity equation

In the flow passage part of the turbine expander, it is generally considered to be a stable so-called shaped flow [4], so the continuity equation is:

\[ \int_A \rho C \cdot dA = 0 \] (1)

Expand in cylindrical coordinates:

\[ \frac{1}{r} \frac{\partial (\rho C_r)}{\partial r} + \frac{\partial (\rho C_\theta)}{\partial \theta} + \frac{\partial (\rho C_z)}{\partial z} = 0 \] (2)

2.2 Momentum equation

In the flow channel of the turbine expander, the volume force is not considered generally, so the momentum equation of the inviscid fluid is [5]:

\[ \frac{dC}{dt} = -\frac{1}{\rho} \nabla P \] (3)

Expand in cylindrical coordinates:
\[ \frac{\partial C_r}{\partial t} + C_r \frac{\partial C_r}{\partial r} + \frac{C_\theta}{r} \frac{\partial C_r}{\partial \theta} + C_\theta \frac{\partial C_r}{\partial \theta} - \frac{C_r}{r} \frac{\partial^2}{\partial \theta^2} = -\frac{1}{\rho} \frac{\partial P}{\partial \theta} \]  

(4)

**Figure 2.** Geometry of the impeller

**Figure 3.** Different leading-edge geometries

**Figure 4.** Impeller mesh for numerical calculation
2.3 Energy equation

The stable flow energy equation without friction loss in a turbine expander can be simplified to:

\[ \frac{\partial C_\theta}{\partial t} + C_r \frac{\partial C_\theta}{\partial r} + C_\theta \frac{\partial C_\theta}{\partial \theta} + C_Z \frac{\partial C_\theta}{\partial Z} = -1 \frac{1}{r \rho} \frac{\partial P}{\partial \theta} \]  

(5)

\[ \frac{\partial C_z}{\partial t} + C_r \frac{\partial C_z}{\partial r} + C_\theta \frac{\partial C_z}{\partial \theta} + C_Z \frac{\partial C_z}{\partial Z} = -1 \frac{1}{\rho} \frac{\partial P}{\partial Z} \]  

(6)

2.4 Boundary condition

Impeller inlet mass flow rate, static temperature and outlet static pressure were known, we changed the coordinate component of absolute velocity to alter the inlet velocity triangle.

3. Results and Analysis

Since the simulation was only for the rotor of the turbine expander, comparing the absolute value of the results makes no sense, so we compared the results between ellipse A leading-edge(0.7mm thickness) and other situations. So the relative isentropic efficiency is the efficiency ratio between the other situations and the ellipse A leading-edge(0.7mm thickness) result. And in some points, the relative isentropic efficiency is larger than 1 which means that the efficiency is higher than the ellipse A leading-edge(0.7mm thickness) situation.

At different thicknesses, the isentropic efficiency of each leading edge geometry was approximately the same when altering the incidence angle, in order not to repeat the same conclusion, we just show the result that leading-edge thickness is 0.7mm. As shown in figure 5, the square leading-edge caused more loss than ellipse leading-edge, and we found that at the range of negative incidence, the efficiency difference between the square leading-edge and ellipse leading-edge were more obvious than that at the range of positive incidence.

Figure 6 and figure 7 shows the effect of thickness on the performance of turbine rotor. For ellipse A, at the range of positive incidence, there is no significant difference between each thickness. While at the range of negative incidence, it shows that the efficiency will decrease with increased leading-edge thickness, excepting the thickness 0.4mm, that means we could not improve the efficiency just decrease the leading-edge thickness blindly. For square leading-edge, at the whole range of incidence, the leading-edge loss to increase with increased leading-edge thickness. Noticing that at the optimal incidence angle, the efficiency of ellipse A leading-edge has a rapid change, while square leading-edge not. We thought the reason was
that the stagnation position of ellipse leading-edge is a line, when the incidence toward to optimal angle, the gradient of pressure is discontinuity along the thickness direction, a singularity will exist when we solve the equation at the stagnation position. While the stagnation position of square leading-edge is a plane, the gradient of pressure is continuity along the thickness direction, so the curves are smoother. The same reason could explain the trend of isentropic efficiency with thickness. As the stagnation position of ellipse leading-edge is a line, the thickness has a less effect on stagnation condition. The reason why discrimination happens at the range of negative incidence is that the pressure difference between pressure surface and suction surface of leading-edge turns small at negative incidence, reducing the thickness promotes this process, of course it could improve the efficiency of turbine[2][3], but the low pressure difference can lead to blade load failure, which will drop the turbine efficiency. For square leading-edge, reducing the thickness could decrease the stagnation area, then reducing impact loss, so the efficiency has an improvement.

![Figure 5](image5.png)

**Figure 5.** Comparison of relative isentropic efficiency with different leading-edge geometries

![Figure 6](image6.png)

**Figure 6.** Comparison of relative isentropic efficiency with different leading-edge thicknesses

Figure 8 through figure 11 shows the change of optimal incidence angle of turbine rotor at leading-edge when altering the leading-edge thickness. We found that with the decrease of leading-edge thickness, the square's optimal incidence angle was toward to zero degree, the ellipse's optimal incidence angle was toward to larger negative angle. With the increasing of the ellipse ratio, the optimal efficiency (or optimal incidence angle) tends to linear change with the thickness variation, and the difference between the greater thickness is also gradually
Figure 7. Comparison of relative isentropic efficiency with different leading-edge thicknesses

Figure 8. Optimum efficiency (optimal incidence) at different thickness

Figure 9. Optimum efficiency (optimal incidence) at different thickness

Figure 10. Optimum efficiency (optimal incidence) at different thickness

Figure 11. Optimum efficiency (optimal incidence) at different thickness

...reduced. In other words, the tip leading edge is not sensitive to the thickness change without...
leading-edge load failure, and the blunt leading edge is sensitive to the thickness change. For square leading-edge having infinite blades, that means the blade thickness tends to zero, the optimal incidence will be zero, and there is no need to offset the relative velocity vortex component caused by the rotation of the running wheel using negative incidence.

We will discuss the problem that whether the direction of the leading edge stream line at stagnation position should be the same with the direction of the blade using the relative velocity vector diagram (with thickness of 0.7 mm, and under optimum efficiency) obtained by simulation. Figure 12 shows the relative velocity vector diagram at leading-edge. For the square front, the relative velocity vector direction in the marking place is perpendicular to the stagnation plane, that is, the streamline direction is the same as the leading-edge blade direction. While for ellipse leading-edge, the direction of streamline at stagnation position still needs to maintain a certain negative degree, in order to make the fluid get cross the blade from suction surface to pressure surface. But the relative velocity should not be too high to lead the blade load failure. Therefore, the optimum criterion that streamline direction should be perpendicular to the stagnation position at blade leading-edge could only apply to the square leading edge, not applicable to the ellipse leading edge.

![Figure 12. Relative velocity vector at leading-edge](image)

**Figure 12.** Relative velocity vector at leading-edge (P is pressure surface, S is suction surface)

**4. Conclusion**

This paper studied the effect of leading-edge geometry and thickness on the performance of miniature cryogenic expansion turbine based on numerical simulation method. It showed that square leading-edge caused more loss than ellipse leading-edge at the same leading-edge thickness. And with the decrease of leading-edge thickness, the square's optimal incidence angle was toward to zero degree, the ellipse's optimal incidence angle was toward to larger negative angle. The optimum criterion at the stagnation position of blade leading-edge could only apply to the square leading edge, the ellipse leading-edge did not follow it. For ellipse leading-edge, it still needed to maintain a certain negative degree at stagnation position to offset the relative velocity vortex component caused by the rotation of the running wheel.
5. Reference

[1] Herman W P and Ronald M H 1972 Effect of trailing-edge geometry and thickness on the performance of certain turbine stator blading *NASA TN D-6637* p 1-21

[2] Ardizzon G and Pavesi G 1998 Optimum incidence angle in centrifugal pumps and radial inflow turbines *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy* vol 212 p 97-107

[3] Glassman A J 1973 Turbine design and application *NASA SP-290* p 284-285

[4] Guanghua J 1988 Turboexpander *Beijing: Mechanical Industry Press* p 39-46

[5] Novak R A 1967 Streamline curvature computing procedures for fluid-flow problems *Journal of Engineering for Power, Transaction of the ASME(October)* p 488-490