$n + p \to d + \gamma$ in Effective Field Theory

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Abstract

The radiative capture process $n + p \to d + \gamma$ provides clear evidence for meson exchange currents in nuclear physics. We compute this process at low energies using a recently developed power counting for the effective field theory that describes nucleon-nucleon interactions. The leading order contribution to this process comes from the photon coupling to the nucleon magnetic moments. At subleading order there are other contributions. Among these are graphs where the photon couples directly to pions, i.e. meson exchange currents. These diagrams are divergent and require the presence of a local four-nucleon-one-photon counterterm. The coefficient of this operator is determined by the measured cross section, $\sigma^{\text{expt}} = 334.2 \pm 0.5$ mb, for incident neutrons with speed $|\mathbf{v}| = 2200$ m/s.
I. INTRODUCTION

The radiative capture \( n + p \rightarrow d + \gamma \) is a classic nuclear physics process where meson exchange currents play a role. For protons at rest and incident neutrons, with speed \( |v| = 2200 \, \text{m/s} \), the cross section for this process has the experimental value, \( \sigma^{\text{expt}} = 334.2 \pm 0.5 \, \text{mb} \). Naively, one expects that an effective range calculation of this cross section would be very close to this. However, such a calculation gives a value which is approximately 10% smaller than \( \sigma^{\text{expt}} \). As first suggested by Brown and Riska, this discrepancy can at least partly be accounted for by the inclusion of meson exchange currents. More recent work by Park, Min and Rho using effective field theory with Weinberg’s power counting for the nucleon-nucleon interaction, a resonance saturation hypothesis for the coefficients of some operators and a momentum cutoff finds the value \( \sigma = 334 \pm 2 \, \text{mb} \). This prediction is relatively insensitive to the value of the cut-off and is compatible with \( \sigma^{\text{expt}} \).

Recently, a consistent power counting for the nucleon-nucleon interaction has been established. At leading order in Weinberg’s scheme, pion exchange is included in the \( NN \) potential and it is interated to all orders to predict the \( NN \) scattering amplitude. However, the work of shows that iterating the pions without including the effects of operators with explicit factors of the quark masses or derivatives does not give a systematic improvement in the prediction for the \( NN \) scattering amplitude. Using the power counting of, the bubble chain formed by multiple insertions of the momentum independent four-nucleon operator gives the leading scattering amplitude for systems with large scattering lengths. Higher derivative operators, operators involving insertions of the light quark mass matrix and pion exchange are of subleading order and are treated in perturbation theory. The expansion parameter used in is \( Q \sim |p|, m_\pi \), and one expands in \( Q/\Lambda \) while keeping all orders in \( aQ \) (see also ). Here \( \Lambda \) is a nonperturbative hadronic scale and \( a \) is the scattering length. At next-to-leading order (NLO) in the \( Q \) expansion simple analytic expressions can be derived for physical quantities. Various observables in the two-nucleon sector have been determined at NLO with this power counting, such as the electromagnetic form factors and moments of the deuteron, the polarizabilities of the deuteron, Compton scattering cross sections and parity violating observables.

In this work we compute the cross section for the radiative capture of extremely low momentum neutrons \( n + p \rightarrow d + \gamma \) at NLO in the effective field theory \( Q \) expansion. Capture from the \( ^3S_1 \) channel is suppressed in the expansion compared to capture from the \( ^1S_0 \) channel. At zero-recoil, the amplitude for capture from the \( ^3S_1 \) channel vanishes since it is simply the overlap of two orthogonal eigenstates of the strong Hamiltonian. Hence, at leading order, only the isovector \( ^1S_0 \) capture occurs and it arises from the isovector magnetic moment interactions of the nucleons. Since the amplitudes for capture from the \( ^1S_0 \) and \( ^3S_1 \) channels do not interfere, at NLO the cross section comes only from the amplitude for capture from the \( ^1S_0 \) channel. At this order there are contributions from a single insertion of four-nucleon operators with two derivatives, from a single insertion of four-nucleon operators with an insertion of the quark mass matrix, and from the exchange of a potential pion. The potential pion exchange occurs in graphs with a nucleon magnetic moment interaction and also in graphs where the potential pion is minimally coupled to the electromagnetic field. The latter contributions are historically called meson exchange currents. In addition to these contributions there is also a contribution from a four-nucleon-one-photon contact interaction.
The coefficient of this operator has not been previously determined and a major purpose of this paper is to fix its value.

II. EFFECTIVE FIELD THEORY FOR NUCLEON-NUCLEON INTERACTIONS

The terms in the effective Lagrange density describing the interactions between nucleons, pions, and photons can be classified by the number of nucleon fields that appear. It is convenient to write

$$L = L_0 + L_1 + L_2 + \ldots,$$

where $L_n$ contains $n$-body nucleon operators.

$L_0$ is constructed from the photon field $A^\mu = (A^0, A^i)$ and the pion fields which are incorporated into an $SU(2)$ matrix,

$$\Sigma = \exp \left( \frac{2i\Pi}{f} \right), \quad \Pi = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix},$$

where $f = 131 \text{ MeV}$ is the pion decay constant. $\Sigma$ transforms under the global $SU(2)_L \times SU(2)_R$ chiral and $U(1)_{em}$ gauge symmetries as

$$\Sigma \to L\Sigma R^\dagger, \quad \Sigma \to e^{i\alpha Q_{em}}\Sigma e^{-i\alpha Q_{em}},$$

where $L \in SU(2)_L, R \in SU(2)_R$ and $Q_{em}$ is the charge matrix,

$$Q_{em} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

The part of the Lagrange density without nucleon fields is

$$L_0 = \frac{1}{2}(E^2 - B^2) + \frac{f^2}{8} \text{Tr} D^\mu \Sigma D^\nu \Sigma^\dagger + \frac{f^2}{4} \lambda \text{Tr} m_q (\Sigma + \Sigma^\dagger) + \ldots.$$

The ellipses denote operators with more covariant derivatives $D_\mu$, insertions of the quark mass matrix $m_q = \text{diag}(m_u, m_d)$, or factors of the electric and magnetic fields. The parameter $\lambda$ has dimensions of mass and $m_\pi^2 = \lambda(m_u + m_d) = (137 \text{ MeV})^2$. Acting on $\Sigma$, the covariant derivative is

$$D_\mu \Sigma = \partial_\mu \Sigma + i e [Q_{em}, \Sigma] A_\mu.$$

When describing pion-nucleon interactions, it is convenient to introduce the field $\xi = \exp (i\Pi/f) = \sqrt{\Sigma}$. Under $SU(2)_L \times SU(2)_R$ it transforms as

$$\xi \to L\xi U^\dagger = U\xi R^\dagger,$$

where $U$ is a complicated nonlinear function of $L, R$, and the pion fields. Since $U$ depends on the pion fields it has spacetime dependence. The nucleon fields are introduced in a doublet of spin 1/2 fields.
that transforms under the chiral $SU(2)_L \times SU(2)_R$ symmetry as $N \to U N$ and under the $U(1)_{em}$ gauge transformation as $N \to e^{i \alpha_Q_{em}} N$. Acting on nucleon fields, the covariant derivative is

$$D_\mu N = (\partial_\mu + V_\mu + ieQ_{em}A_\mu)N,$$

where

$$V_\mu = \frac{1}{2}(\xi D_\mu \xi^\dagger + \xi^\dagger D_\mu \xi) = \frac{1}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi + ieA_\mu(\xi^\dagger Q_{em} \xi - \xi Q_{em} \xi^\dagger)) .$$

The covariant derivative of $N$ transforms in the same way as $N$ under $SU(2)_L \times SU(2)_R$ transformations (i.e. $D_\mu N \to UD_\mu N$) and under $U(1)$ gauge transformations (i.e. $D_\mu N \to e^{i \alpha_Q_{em}} D_\mu N$).

The one-body terms in the Lagrange density are

$$\mathcal{L}_1 = N^\dagger \left( \frac{D^2}{2M} N + \frac{i g_A}{2} N^\dagger \sigma \cdot (\xi D \xi^\dagger - \xi^\dagger D \xi) N \right) + \frac{e}{2M} N^\dagger \left( \kappa_0 + \frac{\kappa_1}{2}(\xi^\dagger \tau_3 \xi + \xi \tau_3 \xi^\dagger) \right) \sigma \cdot B N + \ldots ,$$

(10)

where $M = 939$ MeV is the nucleon mass and $\kappa_0 = \frac{1}{2}(\kappa_p + \kappa_n) = 0.4399$ and $\kappa_1 = \frac{1}{2}(\kappa_p - \kappa_n) = 2.35294$ are the isoscalar and isovector nucleon magnetic moments in nuclear magnetons. The nucleon matrix element of the axial current is $g_A = 1.25$.

The two-body Lagrange density needed for NLO calculations is

$$\mathcal{L}_2 = - \left( C_0^{(3S_1)} + D_2^{(3S_1)} \lambda Tr m_q \right) (N^T P_i N)^\dagger (N^T \overline{P}_i N)$$

$$\quad + \frac{C_0^{(3S_0)}}{8} \left[ (N^T P_i N)^\dagger \left( N^T \left[ P_i \overline{D}^2 + \overline{D}^2 P_i - 2 \overline{D} P_i \overline{D} \right] \right) N \right] + h.c.$$ 

$$\quad - \left( C_0^{(3S_0)} + D_2^{(3S_0)} \lambda Tr m_q \right) (N^T \overline{P}_i N)^\dagger (N^T \overline{P}_i N)$$

$$\quad + \frac{C_2^{(3S_0)}}{8} \left[ (N^T \overline{P}_i N)^\dagger \left( N^T \left[ \overline{P}_i \overline{D}^2 + \overline{D}^2 \overline{P}_i - 2 \overline{D} \overline{P}_i \overline{D} \right] \right) N \right] + h.c.$$ 

$$\quad + \left[ eL_1 (N^T P_i N)^\dagger (N^T \overline{P}_i N) B_i - eL_2 i \epsilon_{ijk} (N^T P_i N)^\dagger (N^T P_j N) B_k + h.c. \right],$$

(11)

where $P_i$ and $\overline{P}_i$ are spin-isospin projectors for the spin-triplet channel and the spin-singlet channel respectively,

$$P_i \equiv \frac{1}{\sqrt{8}} \sigma_2 \sigma_i \tau_2 , \quad \text{Tr } P_i^\dagger P_j = \frac{1}{2} \delta_{ij}$$

$$\overline{P}_i \equiv \frac{1}{\sqrt{8}} \sigma_2 \tau_2 \tau_i , \quad \text{Tr } \overline{P}_i^\dagger \overline{P}_j = \frac{1}{2} \delta_{ij} .$$

(12)

The $\sigma$ matrices act on the nucleon spin indices, while the $\tau$ matrices act on isospin indices. The local operators responsible for $S-D$ mixing do not contribute at NLO. Terms in $\mathcal{L}_2$ involving the pion field have been neglected in eq. (11).
The values of the coefficients of the four-nucleon operators in $L_2$ depend on the regularization and subtraction scheme that is adopted. The power counting of [1] is manifest in the power divergence subtraction scheme, PDS, and we shall use it in this paper. (Momentum subtraction schemes can also have this power counting [18,19].) In PDS one works in $D$ dimensions and the poles at both $D = 4$ and $D = 3$ in the loop integrations associated with Feynman diagrams are subtracted. The $D = 4$ poles are from logarithmic ultraviolet divergences and the $D = 3$ poles are from linear ultraviolet divergences. There is some freedom in how the Lagrangian is continued to $D$-dimensions. We choose to keep the Pauli spin matrices three dimensional and continue the derivatives to $D$-dimensions. This is similar to the scheme proposed by t’Hooft and Veltman [20] for chiral gauge theories and ends up being convenient since the $n + p \rightarrow d + \gamma$ amplitude is proportional to the antisymmetric Levi-Civita tensor, $\epsilon_{ijk}$. At NLO two basic divergent integrals are encountered. The first is

$$
I_0 \equiv \left( \frac{\mu}{2} \right)^{4-D} \int \frac{d^{(D-1)} q}{(2\pi)^{(D-1)}} \left( \frac{1}{q^2 + a^2} \right) 
= (\sqrt{a^2})^{D-3} \Gamma \left( \frac{3 - D}{2} \right) \left( \frac{\mu/2}{(4\pi)^{(D-1)/2}} \right)^{4-D}. \tag{13}
$$

$I_0$ has no pole at $D = 4$ but does have a pole at $D = 3$. Its value in the PDS scheme is,

$$
I_0^{PDS} = \left( \frac{1}{4\pi} \right) (\mu - \sqrt{a^2}). \tag{14}
$$

The second is the two loop integral,

$$
I_1 \equiv \left( \frac{\mu}{2} \right)^{2(4-D)} \int \frac{d^{D-1} q}{(2\pi)^{D-1}} \frac{d^{D-1} l}{(2\pi)^{D-1}} \frac{1}{q^2 + a^2} \frac{1}{l^2 + b^2} \frac{1}{(a - l)^2 + c^2}. \tag{15}
$$

$I_1$ has no pole at $D = 3$ but does have the pole, $-1/32\pi^2(D - 4)$, at $D = 4$. Therefore $I_1$ has the same value in minimal subtraction (MS) as in PDS [21],

$$
I_1^{PDS} = I_1^{MS} = -\frac{1}{16\pi^2} \left( \log \left( \frac{\sqrt{a^2} + \sqrt{b^2} + \sqrt{c^2}}{\mu} \right) + \delta \right), \tag{16}
$$

where

$$
\delta = \frac{1}{2} \left( \gamma_E - 1 - \log \left( \frac{\pi}{4} \right) \right) \tag{17}
$$

and $\gamma_E$ is Euler’s constant. Note that because of its logarithmic divergence, $[3I_1/(D - 1)]^{PDS} = I_1^{PDS} + 1/96\pi^2$. There is considerable freedom in the precise way the subtractions are handled. For example, if the poles in $D = 4$ are subtracted with MS then $\delta = -1/2$. Finally, we stress that one cannot blindly evaluate the integrals in $D$-dimensions and subtract the poles to get the required PDS value. For example, if $a$ and $b$ are set to zero then $I_1$ has a double pole at $D = 3$. However, this is associated with a logarithmic infrared divergence in three dimensions, not an ultraviolet divergence, and so it is not subtracted.

Most of the coefficients in $L_2$ have been determined. At NLO the deuteron magnetic moment [12] is
\[ \mu_d = \frac{\alpha}{2M} \left( \kappa_p + \kappa_n + L_2 \frac{2M\gamma(\mu - \gamma)^2}{\pi} \right), \]  

where \( \gamma = \sqrt{MB} \) with \( B = 2.225 \text{ MeV} \) the binding energy of the deuteron and \( \mu \) is the subtraction point. The coefficient \( L_2 \) depends on the subtraction point in such a way that the physical quantity \( \mu_d \) is \( \mu \) independent. The experimental value of the magnetic moment of the deuteron is \( \mu_d = 0.85741 \) nuclear magnetons and comparing this with the prediction above implies that the coefficient \( L_2 \) (renormalized at \( \mu = m_\pi \)) is,

\[ L_2(m_\pi) = -0.149 \text{ fm}^4. \]  

Note that,

\[ N^T P_i \sigma_j N = i \epsilon_{ijk} N^T P_k N, \]

and so the operator with coefficient \( L_2 \) in eq. (14) is the same as in [12].

The coefficients of the four-nucleon operators in eq. (11) that don’t involve the electromagnetic field have been fixed from comparison with experimental data on \( NN \) scattering. We will review this in the following section. The only unknown coefficient that contributes at NLO is \( L_1 \) and it will be determined in this work.

## III. S-WAVE \( NN \) SCATTERING

The \( ^1S_0 \) \( NN \) scattering amplitude, at center of mass momentum \( p \), has the expansion \( A^{(1S_0)}(p) = \sum_{n=-1}^{\infty} A_n^{(1S_0)}(p) \), where \( A_n^{(1S_0)}(p) \) is of order \( Q^n \). At leading order only the four nucleon operator with no derivatives need be included and

\[ A_{-1}^{(1S_0)}(p) = \frac{-C_0^{(1S_0)}}{1 + C_0^{(1S_0)} M(\mu + ip)/4\pi}. \]  

(21)

It is convenient to break the next order contribution into several pieces, \( A_0^{(1S_0)} = A_0^{(I)} + A_0^{(II)} + A_0^{(III)} + A_0^{(IV)} + A_0^{(V)} \), and, using PDS, ref. [7] found,

\[
\begin{align*}
A_0^{(I)} &= -C_2^{(1S_0)} p^2 \left[ \frac{A_{-1}^{(1S_0)}}{C_0^{(1S_0)}} \right]^2, \\
A_0^{(II)} &= \left( \frac{g_\Lambda}{2f^2} \right) \left( -1 + \frac{m_\pi^2}{4p^2} \ln \left( 1 + \frac{4p^2}{m_\pi^2} \right) \right), \\
A_0^{(III)} &= \frac{g_\Lambda}{f^2} \left( \frac{m_\pi M A_{-1}^{(1S_0)}}{4\pi} \right) \left( -\frac{(\mu + ip)}{m_\pi} + m_\pi \frac{2p}{2p} \left[ \tan^{-1} \left( \frac{2p}{m_\pi} \right) + i \frac{1}{2} \ln \left( 1 + \frac{4p^2}{m_\pi^2} \right) \right] \right), \\
A_0^{(IV)} &= \frac{g_\Lambda}{2f^2} \left( \frac{m_\pi M A_{-1}^{(1S_0)}}{4\pi} \right) \left( -\frac{(\mu + ip)}{m_\pi} \right)^2 + i \frac{1}{2} \ln \left( \frac{m_\pi^2 + 4p^2}{\mu^2} \right) + \frac{1}{6} - \delta \right), \\
A_0^{(V)} &= -D_2^{(1S_0)} m_\pi^2 \left[ \frac{A_{-1}^{(1S_0)}}{C_0^{(1S_0)}} \right]^2. 
\end{align*}
\]  

(22)
Actually, the above expression for $A_0^{(IV)}$ is slightly different from that in [1] because there 

are terms that appear above were absorbed into a redefinition of $D_2^{(1S_0)}$.

The scattering length gets contributions from each order in the $Q$ expansion. To use these 

results for the scattering amplitude over a region that includes very low $p$ it is necessary 

that the leading order amplitude give almost the correct the scattering length. This can 

be achieved at NLO by reordering the expansion in the following way [18]. Write

$$C_0^{(1S_0)} = \tilde{C}_0^{(1S_0)} + \Delta C_0^{(1S_0)},$$

(23)

and for these a fit over the region $7 \text{ MeV} < p < 100 \text{ MeV}$ finds, at $\mu = m_\pi$ [18],

$$\tilde{C}_0^{(1S_0)}(m_\pi) = -3.529 \text{ fm}^2, \ C_2^{(1S_0)}(m_\pi) = 3.04 \text{ fm}^4.$$  (26)

Discussion of the results of different fitting procedures can be found in [12,18,22,23].

In the $^3S_1$ channel, identical formulae hold for the scattering amplitude and scattering 

length once the replacement $^1S_0 \rightarrow ^3S_1$ is made for the superscripts. However, the fit 

to the data is done a little differently. For processes involving the deuteron it is convenient 

to constrain $\tilde{C}_0^{(1S_0)}$ so that $A_{-1}^{(3S_1)}$ gives the correct deuteron binding energy, $B = 2.2255 \text{ MeV}$. 

This implies that

$$\tilde{C}_0^{(3S_1)}(\mu) = -\frac{4\pi}{M} \left( \frac{1}{\mu - \gamma} \right),$$

(27)

where $\gamma = \sqrt{MB}$. In this channel a constrained fit to the $NN$ phase shift yields

$$\tilde{C}_0^{(3S_1)}(m_\pi) = -5.708 \text{ fm}^2, \ C_2^{(3S_1)}(m_\pi) = 10.8 \text{ fm}^4.$$  (28)
FIG. 1. Graphs contributing to the amplitude for \( n + p \rightarrow d + \gamma \) at leading order in the effective field theory expansion. The solid lines denote nucleons and the wavy lines denote photons. The light solid circles correspond to the nucleon magnetic moment coupling to the electromagnetic field. The crossed circle represents an insertion of the deuteron interpolating field which is taken to have \( ^3S_1 \) quantum numbers.

IV. CROSS SECTION FOR RADIATIVE CAPTURE

The amplitude for the radiative capture of extremely low momentum neutrons \( n + p \rightarrow d + \gamma \) has contributions from both the \( ^1S_0 \) and \( ^3S_1 \) NN channels. It can be written as

\[
\begin{align*}
iA(np \rightarrow d\gamma) & = e \times N T_2 \sigma_2 \left[ \sigma \cdot k \epsilon(d)^* \epsilon(\gamma)^* - \sigma \cdot \epsilon(\gamma)^* k \cdot \epsilon(d)^* \right] N + ie Y \epsilon^{ijk} \epsilon(d)^i k^j \epsilon(\gamma)^k (N T_2 T_3 \sigma_2 N),
\end{align*}
\]

where \( e = |e| \) is the magnitude of the electron charge, \( N \) is the doublet of nucleon spinors, \( \epsilon(\gamma) \) is the polarization vector for the photon, \( \epsilon(d) \) is the polarization vector for the deuteron and \( k \) is the outgoing photon momentum. The term with coefficient \( X \) corresponds to capture from the \( ^3S_1 \) channel while the term with coefficient \( Y \) corresponds to capture from the \( ^1S_0 \) channel. For convenience, we define dimensionless variables \( \tilde{X} \) and \( \tilde{Y} \), by

\[
\begin{align*}
X & = i \frac{2}{M} \sqrt{\frac{\pi}{\gamma^3}} \tilde{X}, \quad Y = i \frac{2}{M} \sqrt{\frac{\pi}{\gamma^3}} \tilde{Y}.
\end{align*}
\]

Both \( \tilde{X} \) and \( \tilde{Y} \) have the \( Q \) expansions, \( \tilde{X} = \tilde{X}_0 + \tilde{X}_1 + ..., \) and \( \tilde{Y} = \tilde{Y}_0 + \tilde{Y}_1 + .... \), where a subscript \( n \) denotes a contribution of order \( Q^n \). The capture cross section for neutrons with speed \( |v| \) arising from eq. (29) is

\[
\sigma = \frac{8\pi\alpha \gamma^3}{M^2 |v|} \left[ 2|\tilde{X}|^2 + |\tilde{Y}|^2 \right],
\]

where \( \alpha \) is the fine-structure constant.

At leading order, \( \tilde{X} \) and \( \tilde{Y} \) are calculated from the sum of Feynman diagrams in Fig. (1) and from wavefunction renormalization associated with the deuteron interpolating field [12], giving

\[
\begin{align*}
\tilde{X}_0 & = \kappa_0 \left( 1 + \frac{\gamma M}{4\pi} A_{-1}^{(3S_1)}(0) \right), \quad \tilde{Y}_0 = \kappa_1 \left( 1 + \frac{\gamma M}{4\pi} A_{-1}^{(1S_0)}(0) \right),
\end{align*}
\]
where \( A^{(1S_0)}_1(p) \) is the leading, order \( Q^{-1} \), contribution nucleon-nucleon scattering amplitude in the \( 1S_0 \) channel at an center of mass momentum \( p \). The scattering length is related to the nucleon-nucleon scattering amplitude at zero momentum

\[
A^{(1S_0)}(0) = -\frac{4\pi}{M} a^{(1S_0)},
\]  

(33)

and the experimental value for the \( 1S_0 \) scattering length is, \( a^{(1S_0)} = -23.714 \pm 0.013 \text{ fm} \). An analogous expression holds in the \( 3S_1 \) channel. At leading order, \( A^{(1S_0)}_{1L}(0) = -4\pi a^{(1S_0)}/M \) and \( A^{(3S_1)}_{1L}(0) = -4\pi/M\gamma \). Using this in eq. (32) gives,

\[
\bar{X}_0 = 0 \quad , \quad \bar{Y}_0 = \kappa_1 \left( 1 - \gamma a^{(1S_0)} \right),
\]

(34)

which implies the radiative capture cross section,

\[
\sigma^{LO} = \frac{8\pi\alpha\gamma^5\kappa^2(a^{(1S_0)})^2}{|\mathbf{v}|M^5} \left( 1 - \frac{1}{\gamma a^{(1S_0)}} \right)^2 = 297.2 \text{ mb}.
\]

(35)

This agrees with the results of Bethe and Longmire [2,3] when terms in their expression involving the effective range are neglected. Eq. (35) is about 10% less than the experimental value, \( \sigma^{\text{expt}} = 334.2 \pm 0.5 \text{ mb} \) [4]. Because \( \bar{X}_0 \) vanishes we only need compute \( \bar{Y}_1 \) to obtain the cross section at NLO.

At NLO there are contributions from insertions of the \( D_2, C_2 \) operators and from the exchange of potential pions, with the photon coupling both minimally to the pions and to the nucleons via their magnetic moment. In addition there is a contribution from the \( L_1 \) four-nucleon-one-photon operator. These can be divided into two categories, those that build up the \( 1S_0 \) scattering amplitude at NLO, \( A^{(1S_0)}_0 \), and those that do not. Writing, \( \bar{Y}_1 = \bar{Y}^{\text{rescatt}} + \bar{Y}^{(C_2)} + \bar{Y}^{(\pi,B)} + \bar{Y}^{(\pi,E)} + \bar{Y}^{(L_1)} \), we find the graphs in Figs. (2), (3), (4) and (4) give contributions

\[
\bar{Y}^{\text{rescatt}} = \kappa_1 \frac{\gamma M}{4\pi} A^{(1S_0)}_1(0),
\]

\[
\bar{Y}^{(C_2)} = -\kappa_1 \frac{\gamma^2}{2} \left[ \frac{C^{(1S_0)}_2 + C^{(3S_1)}_2}{C^{(1S_0)}_0 C^{(3S_1)}_0} \right] A^{(1S_0)}_1(0) + \frac{2 C^{(3S_1)}_2(\mu - \gamma)}{\gamma} \left( 1 + \frac{\gamma M}{4\pi} A^{(1S_0)}_{1L}(0) \right),
\]

\[
\bar{Y}^{(\pi,B)} = \kappa_1 \frac{g_\Delta^2 M\gamma}{8\pi f^2} \left( \frac{m_\pi - 2\gamma}{m_\pi + 2\gamma} + \frac{M m_\pi}{4\pi} A^{(1S_0)}_1(0) \left( \frac{m_\pi}{\gamma} \ln \left( 1 + 2\gamma \frac{m_\pi}{m_\pi + \gamma} \right) - 2 \frac{m_\pi + \gamma}{m_\pi + 2\gamma} \right) \right),
\]

\[
\bar{Y}^{(\pi,E)} = \frac{g_\Delta^2 M\gamma^2}{12\pi f^2} \left( \frac{m_\pi - \gamma}{m_\pi + \gamma} + \frac{M}{4\pi} A^{(1S_0)}_1(0) \left( \frac{3m_\pi - \gamma}{2(m_\pi + \gamma)} + \ln \left( \frac{m_\pi + \gamma}{\mu} \right) - \frac{1}{6} + \delta \right) \right),
\]

\[
\bar{Y}^{(L_1)} = L_1 \gamma \frac{\kappa^2}{C^{(1S_0)}_0 C^{(3S_1)}_0} A^{(1S_0)}_{1L}(0).
\]

(36)

The NLO \( 1S_0 \) scattering amplitude \( A^{(1S_0)}_0 \) was given in the previous section. Notice that the meson exchange current contribution \( \bar{Y}^{(\pi,E)} \) depends upon the renormalization scale \( \mu \). The graphs contributing to this term have a pole at \( D = 4 \) and require a subtraction. This is the reason for the logarithmic \( \mu \)-dependence. It is canceled by the \( \mu \)-dependence of the constant.
FIG. 2. Graphs contributing to the amplitude for $n + p \rightarrow d + \gamma$ at subleading order due to the insertion of the $C_2$ and $D_2$ operators. The solid lines denote nucleons and the wavy lines denote photons. The light solid circles correspond to the nucleon magnetic moment coupling of the photon. The solid square denotes either a $C_2$ operator or a $D_2$ operator. The crossed circle represents an insertion of the deuteron interpolating field. The last graph denotes the contribution from wavefunction renormalization.

$L_1$ so that $\tilde{Y}_1$ is independent of $\mu$. It is interesting to note that the contribution from the $C_2$ operators, $\tilde{Y}^{(C_2)}$, is not $\mu$ independent either. This explicit $\mu$-dependence is also canceled by the $\mu$-dependence of $L_1$. The renormalization group equation for the subtraction point dependence of $L_1$ is

$$
\mu \frac{d}{d\mu} \left[ L_1 - \frac{1}{2} \kappa_1 \left( C_2^{(s_0)} + C_2^{(s_1)} \right) \right] \frac{C_0^{(s_0)} C_0^{(s_1)}}{C_0^{(s_0)} C_0^{(s_1)}} = \frac{g_A^2 M^2}{48 \pi^2 f^2}. $$

(37)

Note that this is quite different from the renormalization group equation,

$$
\mu \frac{d}{d\mu} \left[ \frac{L_2}{C_0^{(s_1)}} \right] = 0,
$$

(38)

that $L_2$ satisfies.

There is a NLO contribution that we have not explicitly included, a one-loop correction to the magnetic moments of the nucleons that is of order $m_\pi/(4\pi f^2)$. However, by using
the value $\kappa_1 = 2.35294$, which follows from the measured values of the neutron and proton magnetic moments, in $\tilde{Y}_0$, this effect has been taken into account. Similarly, using the measured value for $a^{(S_0)}$ in eq. (34) includes the effects of $\tilde{Y}^{(\text{rescatt})}$. Demanding that the NLO expression for $\tilde{Y}$ give the measured cross section implies that,

$$\tilde{Y}^{(C_2)} + \tilde{Y}^{(\pi,B)} + \tilde{Y}^{(\pi,E)} + \tilde{Y}^{(L_1)} = 0.92. \quad (39)$$

In eq. (39) using $\mu = m_\pi$, $\delta$ given by eq. (17) and $A^{(S_0)}_{-1} = -4\pi a_{(S_0)}/M$ yields, $\tilde{Y}^{(C_2)} = 0.38$, $\tilde{Y}^{(\pi,B)} = -0.33$, and $\tilde{Y}^{(\pi,E)} = 0.60$. Note that for $\tilde{Y}^{(C_2)}$ there is a significant cancellation between the two terms in the square brackets of eq. (39). About two thirds of the discrepancy between the measured cross section and the leading order expression is made up from the meson exchange current contribution. Most of the rest comes from $\tilde{Y}^{(L_1)}$. Eq. (39) implies that

$$L_1(m_\pi) = 1.63 \text{ fm}^4. \quad (40)$$
The value of $L_1$ is quite sensitive to the precise way that the poles at $D = 4$ are handled. For example if $\overline{MS}$ is used (i.e. $\delta = -1/2$) then $\tilde{Y}^{(\pi,E)} = 0.37$ which gives $L_1(m_\pi) = 3.03$ fm$^4$.

With $L_1$ determined, all the counterterms in the strong and electromagnetic sector that occur at next-to-leading order in the effective field theory $Q$ expansion are known. It is interesting to see that in this framework there is nothing special about meson exchange currents. They are simply one of the several contributions at NLO, occurring along with the strong interaction corrections to diagrams where the photon couples to the nucleon magnetic moments.

V. CONCLUDING REMARKS

We have computed the cross section for the radiative capture process $n + p \to d + \gamma$. At leading order we recover the effective range theory result (when the terms involving $r_0$ are neglected) which is about 10% smaller than the measured cross section. At NLO there are contributions from perturbative insertions of the $C_2$ operators, the $D_2$ operators, potential pion exchanges and from a previously unconstrained four-nucleon-one-photon counterterm with coefficient $L_1$. In order to reproduce the measured cross section, $\sigma^{\text{expt}}$, we find that $L_1(m_\pi) = 1.65$ fm$^4$. In more traditional approaches, meson exchange currents are required to explain the value of $\sigma^{\text{expt}}$. In effective field theory, the meson exchange current graphs are divergent and require regularization. As a result, their contribution to the cross section is not unique and depends upon the choice of regularization scheme. In addition, a local counterterm is required to absorb these divergences and its value is a priori unknown and scheme dependent. Having determined the value of $L_1$ from the radiative capture cross section, other processes arising from electromagnetic interactions such as deuteron breakup...
FIG. 5. Local counterterm contribution to the amplitude for $n + p \rightarrow d + \gamma$ at NLO. The solid lines denote nucleons and the wavy lines denote photons. The solid circle corresponds to an insertion of the $L_1$ operator. The crossed circle represents an insertion of the deuteron interpolating field.

$e + d \rightarrow e' + n + p$ can be computed at NLO. Work on this is in progress.

We would like to thank Jiunn-Wei Chen and David Kaplan for several discussions. This work is supported in part by the U.S. Dept. of Energy under Grants No. DE-FG03-97ER4014, DE-FG02-96ER40945 and DE-FG03-92-ER40701. KS acknowledges support from the NSF under a Graduate Research Fellowship.
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