PROBING QUARK FRAGMENTATION FUNCTIONS FOR SPIN-1/2 BARYON PRODUCTION IN UNPOLARIZED $e^+e^-$ ANNIHILATION

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(MIT-CTP: #2365  HEP-PH: #9410337  Submitted to: Nucl. Phys. B  September 1994)

Abstract

We study the measurement of the quark fragmentation functions for spin-1/2 baryon production ($\Lambda$ and $\bar{\Lambda}$, in particular) in unpolarized $e^+e^-$ annihilation. The spin-dependent fragmentation functions $\hat{g}_1(z)$ and $\hat{h}_1(z)$ can be probed in the process as a result of quark-antiquark spin correlation and the weak decay of the baryons. The relevant cross section is expressed as a product of the two-jet cross-section, the fragmentation functions, and the differential width of the hyperon decay.

*This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative agreement #DF-FC02-94ER40818 and #DE-FG02-92ER40702.

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I. INTRODUCTION

Due to confinement, the basic building blocks of QCD, quarks and gluons, cannot emerge as asymptotic states of the theory. Rather the high energy quarks and gluons in a hard scattering process show up in a detector as jets of hadrons, and their axes register the momentum directions of the initial partons. The process of converting a colored parton into a shower of hadrons is called parton fragmentation, in which it has long been known that the soft QCD physics dominates. As a result, analyzing fragmentation directly from first principles is very difficult. Up to now the fragmentation process is mainly modelled with a set of empirical rules and a large Monte Carlo code \[1\]. Given the paucity of our knowledge, experimental information on various aspects of fragmentation is quite valuable for understanding the essential physics in the underlying process.

Semi-inclusive information about fragmentation is contained in fragmentation functions, which measure the probability for a quark to fragment to a specific hadron with a fixed longitudinal momentum. All the fragmentation functions experimentally measured so far are spin-independent. In Refs. \[2–4\], the novel spin-dependent fragmentation functions \[\hat{g}_1(z)\] and \[\hat{h}_1(z)\] are introduced for spin-1/2 baryon production. Both \[\hat{g}_1(z)\] and \[\hat{h}_1(z)\] are of leading twist and can produce significant effects in high-energy processes. The physics of these fragmentation functions is quite simple. The \[\hat{g}_1(z)\] fragmentation function represents the probability density to find a spin-1/2 baryon with its polarization in the direction of the longitudinal polarization of the original quark. Likewise, \[\hat{h}_1(z)\] represents the probability density to find a spin-1/2 baryon with its polarization in the direction of the transverse polarization of the original quark. Here, longitudinal or transverse is defined relative to the momentum of the quark. Undoubtedly, experimental data on these fragmentation functions contain important information about spin-transfer during quark fragmentation.

To measure the spin-dependent fragmentation functions, one normally needs to produce a polarized quark jet. This is possible with polarized \[e^+e^-\] collisions or polarized deep-inelastic scattering. On the other hand, in an unpolarized \[e^+e^-\] annihilation, the polarizations of the created quark and antiquark pair are correlated due to the chiral invariance of the electroweak interactions. Therefore if one can find a way to measure the spin correlation of the produced hadrons, one can obtain information on the spin-dependent fragmentation functions. This observation forms the foundation for our work.

In this paper, we study the measurement of the spin-dependent fragmentation functions in unpolarized \[e^+e^-\] annihilation. For definiteness, we consider correlated production of \(\Lambda\) and \(\bar{\Lambda}\) in quark and antiquark jets respectively. The \(\Lambda\) and \(\bar{\Lambda}\) are observed by their weak decay to nucleon-pion and anti-nucleon-pion respectively. The weak decay allows one to measure the complete spin-density matrix of the parent baryon. The spin-dependent information is extracted from the angular correlation of the decay products of \(\Lambda\) and \(\bar{\Lambda}\). Our result, of course, is valid for production of any spin-1/2 baryon which decays weakly to another spin-1/2 baryon and a pion.

This paper is organized as follows. In §II, we write down the differential cross section as a product of the density matrices for the jet production, the quark fragmentation, and the baryon decay. We then present the helicity formalism on which the subsequent calculation is based. In §III, we evaluate the jet production density matrix. In §IV, the quark fragmentation density matrix is defined and expressed in terms of the fragmentation functions
\( f_1(z), \hat{g}_1(z), \) and \( \hat{h}_1(z) \). In §V, \( \Lambda \) and \( \bar{\Lambda} \) decay density matrices are constructed. And finally in §VI, we put everything together to form the final cross section, and discuss its physical significance. We summarize our results in §VII. The reader who is interested primarily in our results and their physical interpretation might omit §III,§IV, and §V and skip directly to the formulas and discussions in §VI and §VII.

**II. SPIN DENSITY-MATRIX FORMALISM IN HELICITY BASIS**

The calculation of the cross section for \( e^-e^+ \to q\bar{q} \to \Lambda X \bar{\Lambda} \to p\pi^- \bar{p}\pi^+ \bar{X} \) can be carried out by following the standard procedure of the operator production expansion, or equivalently, by using the collinear expansion and “cut diagram” technique [5,6]. However, since we are interested in a result at the leading twist, there is a more physical approach which employs the language of the parton model and follows the process step by step. First, we calculate the quark-antiquark production; then, we deal with quark and antiquark fragmentations; next, we include the \( \Lambda \) and \( \bar{\Lambda} \) decay; and finally, we assemble the \( q\bar{q} \) production, fragmentation and decay processes together to obtain the experimentally observable particle distributions. The only subtlety in the case of a spin-dependent calculation is that all the intermediate quantities must be in a form of spin-density matrices. Then the cross section is expressed as a trace of product of spin-density matrices for separate subprocesses. In our case we have,

\[
\frac{d^8 \sigma}{d\Omega_h dz d\bar{z} d^2 P^\perp_p d^2 P^\perp_{\bar{p}}} = \left( \frac{d\sigma(e^-e^+ \to q\bar{q})}{d\Omega_q} \right)_{\tilde{h}h',\tilde{h}'h} \left( \frac{d\hat{M}}{dz} \right)^{\tilde{h}'h} \left( \frac{d\hat{M}}{dz} \right)^{\tilde{h}h'} \left( \frac{d^2 D}{d^2 P^\perp_p} \right)^{H'\tilde{H}} \left( \frac{d^2 D}{d^2 P^\perp_{\bar{p}}} \right)^{\tilde{H}'H}, \quad (1)
\]

where \( h(\tilde{h}) \) and \( H(H') \) are indices labelling spin states of quark (antiquark) and hyperon (antihyperon), respectively. By convention, repeated indices are summed over. The bulk of this paper is devoted to defining and calculating these spin-density matrices.

One can choose any spin-density matrix formalism to perform the calculation. The simplest is perhaps the one in which all the spin indices are just the ordinary Dirac indices. However, the drawback of this approach is that all the density matrices are \( 4 \times 4 \) and they do not have a clear physical interpretation. As is often the case, the physics is much clearer in the helicity basis [7]. Throughout this paper, we shall use this formalism, and thus \( h(h') \) and \( H(H') \) in Eq. (1) are to be interpreted as helicity indices with values \( \pm 1/2 \) (or \( \pm \) for short).

In the helicity formalism, one first choose the helicity basis, \( u(h) \) for fermion and \( v(\tilde{h}) \) for antifermion. Then a general polarization state for fermion (antifermion) can be expressed as,

\[
U = au(+) + bu(-); \quad V = cv(+) + dv(-)
\]

or simply in terms of the two-component spinors,

\[
U = \begin{pmatrix} a \\ b \end{pmatrix}; \quad V = \begin{pmatrix} c \\ d \end{pmatrix}.
\]

3
Any processes and subprocesses with fermions and/or antifermions as external particles can be calculated as a density matrix with pairs of helicity indices. Each pair corresponds to one particle, with one index representing the spin state of the particle in the amplitude and the other in the conjugate amplitude. Obviously, the spin-dependent probability or cross section can be obtained by contracting these indices with the two-component wave functions in Eq. (3).

The helicity formalism treats the polarization of each fermion (antifermion) independently, since the two-component wave function is expanded in basis states which are different for each particle. In particular, the spin quantization axis is the direction of the particle’s momentum. This is rather convenient because one can quickly write down a two-component wave function once one knows the relative orientation between the particle’s momentum and its spin vector. In fact, it will be useful for us to define a coordinate system for each particle with its momentum as the z-axis. The directions of the \( \hat{x} \) and \( \hat{y} \) axes are defined as the directions of the spin vector \( S^\mu = \bar{u} \gamma^\mu \gamma_5 u \) associated with the following spinors,

\[
    u(\hat{x}) = \frac{1}{\sqrt{2}} (u(+) + u(-)) ,
\]

\[
    u(\hat{y}) = \frac{1}{\sqrt{2}} (u(+) + i u(-)) .
\]

This way one is assured that the \( \hat{x} \) and \( \hat{y} \) axes are defined in the same way for all particles and antiparticles. The choice of helicity basis for each particle fixes the relative orientation of the different coordinate systems. For fermions, the two-component spin wave function can be chosen universally as,

\[
    \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \cos \theta & e^{i\phi} \\ \sin \theta e^{-i\phi} \end{pmatrix} .
\]

(5)

where \( \theta \) and \( \phi \) are polar and azimuthal angles of the spin vector relative to the momentum and the \( x \)-axis.

Another important advantage of the helicity formalism is that an antifermion can be treated exactly like a fermion. This should be the case because the definition of fermion and antifermion is itself arbitrary due to charge conjugation symmetry. However, in ordinary calculations, some aspects of the symmetry are not obvious because antifermions are treated as holes in the negative energy fermion sea. For instance, the spinor \( v \) is associated with antifermion creation, whereas \( u \) is with fermion annihilation. In other words in a Feynman diagram the momentum flow for antifermions is always against the fermion number flow, and the Dirac algebra follows the latter flow. In the helicity formalism, if one were to follow the fermion number flow in ordering the helicity indices, one would find that all the density matrices would be expressed naturally in terms of the transpose of Pauli matrices. Furthermore, if one were to use the charge conjugation relation \( v = C \bar{u}^T \) to define the spinor for the antifermion, then the two-component wave function would be the complex conjugate of that in Eq. (5),

\[
    \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} \cos \theta & e^{-i\phi} \\ \sin \theta e^{i\phi} \end{pmatrix} .
\]

(6)

Note again that this wave function is associated with the antifermion creation. All these observations point to a simple scheme for dealing with antifermion spin calculations: Order
the spin indices in terms of the momentum flow rather than the fermion number flow. Then the spin-density matrices are all naturally expressed in terms of Pauli matrices and the two-component wave functions of antifermions are exactly the same as those of fermions. The wave function in Eq. (6) now appears in a transposed form and is naturally associated with an antiparticle being created. One final note: the $\hat{x}$ and $\hat{y}$ axes for the antiparticle are defined by the spin vector $S^\mu = -\bar{v}\gamma^\mu\gamma^5 v$ associated with the spinors,

$$v(\hat{x}) = \frac{1}{\sqrt{2}}(v(+) + v(-)),$$

$$v(\hat{y}) = \frac{1}{\sqrt{2}}(v(+) - iv(-)),$$  \hspace{1cm} (7)

where the minus sign follows from Eq. (6).

The following sections provide concrete examples for illustrating the helicity formalism in detail.

### III. QUARK-ANTIQUARK PRODUCTION DENSITY MATRIX

Our calculation begins with quark-antiquark production via photons ($\gamma^*$) and $Z^0$s. Here we work in the rest frame of $\gamma^*$ or $Z^0$. Event by event, we choose the $z$ axis in the direction of the quark jet, so the anti-quark momentum is in the $-z$ direction. [The identification of the quark jet can be made through study of jet ensembles. However, in the present calculation we implicitly assume that the $\Lambda$ is a fragment of the quark. The possibility of $\bar{q} \rightarrow \Lambda X$ can be included in the final result in a straightforward way, but the $\Lambda$’s resulting from $\bar{q} \rightarrow \Lambda X$ will dominantly occur at low $z$ (the longitudinal momentum fraction of the quark carried by $\Lambda$) and usually will not satisfy jet isolation cuts. Henceforth we ignore the possibility of $\bar{q} \rightarrow \Lambda X$ fragmentation.] By convention, the $y$ axis is defined by $\hat{y} = \hat{k} \times \hat{z}$, where $\hat{k}$ is the direction of the electron beam. In this coordinate system, the 4-momenta of electron and positron are

$$k^\mu = (E, -E \sin \Theta, 0, E \cos \Theta), \quad k'^\mu = (E, E \sin \Theta, 0, -E \cos \Theta),$$  \hspace{1cm} (8)

respectively, where $\Theta$ is the polar angle of the electron.

In this section, we seek an expression for the quark-antiquark production density matrix $(d\sigma(e^-e^+ \rightarrow q\bar{q})/d\Omega_q)_{h^i \bar{h}^j}$ in the helicity basis, where the ordering of the indices follows the momentum flow of quark and antiquark, and is shown explicitly in Fig. 1. While the diagonal elements of the density matrix are unique, the off-diagonal ones depend on the phase convention for the helicity basis. In this paper, we adopt the quark helicity states and the $\gamma$-matrix representation of Bjorken and Drell [8]. The anti-quark helicity states are chosen to be $v(h, k) = C\bar{u}(h, k)^T$, where $C = i\gamma^2\gamma^0$ is the charge conjugation matrix (this differs from [8] convention). And thus, in the zero-mass limit, we have,

$$u_q(+) = \sqrt{E} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad u_q(-) = \sqrt{E} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad v_q(+) = \sqrt{E} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad v_q(-) = \sqrt{E} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$  \hspace{1cm} (9)
Once again, the antiquark is moving in the $-z$ direction.

Clearly the coordinate system for treating quark polarization is just the one we have defined. On the other hand, the coordinate system associated with the anti-quark polarization $(\bar{x}, \bar{y}, \bar{z})$ can be worked out from our choice for the antiquark helicity states and the definition for the axes in the last section. The $\bar{z}$ axis is clearly in the $-z$ direction. Using Eq. (7), we find that the $\bar{x}$ axis is in the direction of the $x$ axis and the $\bar{y}$ axis in the $-y$ direction. [To avoid a kinematic zero, one has to restore the quark mass in the helicity states.] This coordinate system will be used for anti-particles throughout this paper.

A straightforward calculation with the standard-model electroweak currents and the above helicity states yields,

$$
\left(\frac{d\sigma}{d\Omega_q}\right)_{++,-} = \frac{N_c \alpha^2_{em}}{8s} \left\{ Q^2_q (1 + \cos^2 \Theta) + \chi_2 (v_q - a_q)^2 \left[ (1 + \cos^2 \Theta) (v_e^2 + a_e^2) - 4v_e a_e \cos \Theta \right] - 2Q_q \chi_1 (v_q - a_q) \left[ v_e (1 + \cos^2 \Theta) - 2a_e \cos \Theta \right] \right\},
\left(\frac{d\sigma}{d\Omega_q}\right)_{--,+} = \frac{N_c \alpha^2_{em}}{8s} \left\{ Q^2_q (1 + \cos^2 \Theta) + \chi_2 (v_q + a_q)^2 \left[ (1 + \cos^2 \Theta) (v_e^2 + a_e^2) + 4v_e a_e \cos \Theta \right] - 2Q_q \chi_1 (v_q + a_q) \left[ v_e (1 + \cos^2 \Theta) + 2a_e \cos \Theta \right] \right\},
\left(\frac{d\sigma}{d\Omega_q}\right)_{--,-} = \frac{N_c \alpha^2_{em}}{8s} \sin^2 \Theta \left\{ Q^2_q - \chi_2 (v_e^2 + a_e^2) (a_q^2 - v_q^2) - 2Q_q \chi_1 v_e \left( v_q + \frac{i \Gamma Z M_Z}{s - M_Z^2} a_q \right) \right\},
\left(\frac{d\sigma}{d\Omega_q}\right)_{++,--} = \frac{N_c \alpha^2_{em}}{8s} \sin^2 \Theta \left\{ Q^2_q - \chi_2 (v_e^2 + a_e^2) (a_q^2 - v_q^2) - 2Q_q \chi_1 v_e \left( v_q - \frac{i \Gamma Z M_Z}{s - M_Z^2} a_q \right) \right\},
\left(\frac{d\sigma}{d\Omega_q}\right)_{\text{others}} = 0,
$$

(10)

where $v_e = 4 \sin^2 \theta_W - 1$ and $a_e = -1$ are the vector and axial vector couplings of the electron to the $Z$. The couplings of the quarks to the $Z$ are $v_u = 1 - \frac{s}{3} \sin^2 \theta_W$, $v_d = v_s = -1 + \frac{4}{3} \sin^2 \theta_W$, $a_u = 1$, and $a_d = a_s = -1$. $N_c = 3$ is the color number of the quark, and

$$
\chi_1 = \frac{1}{16 \sin^2 \theta_W \cos^2 \theta_W (s - M^2_Z)^2 + \Gamma^2_Z M^2_Z},
\chi_2 = \frac{1}{256 \sin^4 \theta_W \cos^4 \theta_W (s - M^2_Z)^2 + \Gamma^2_Z M^2_Z}.
$$

(11)

(12)

Eq. (11) indicates that the quark and antiquark from the same vertex have opposite helicity, a consequence of the massless limit. Therefore, as we will see in the next section, the polarizations of $\Lambda$ and $\bar{\Lambda}$ are correlated through the effects of spin-dependent quark fragmentation functions.

The cross-section density matrix can also be expressed in terms of the Pauli matrices,
\[
\left( \frac{d\sigma}{d\Omega_q} \right) = \frac{N_c \alpha_s^2}{8s} \left\{ \frac{1}{2} Q_q^2 (1 + \cos^2 \Theta) + \frac{1}{2} \chi_2 \left[ (1 + \cos^2 \Theta) v_q^2 + a_q^2 (v_q^2 + a_q^2) + 8 v_e a_e v_q a_q \cos \Theta \right] - Q_q \chi_1 \left[ v_e v_q (1 + \cos^2 \Theta) + 2 a_e a_q \cos \Theta \right] \right\} (I^q \otimes \bar{I}^q - \sigma_z^q \otimes \sigma_z^q) \\
+ \left\{ \chi_2 \left[v_q a_q (v_q^2 + a_q^2) (1 + \cos^2 \Theta) + 2 v_e a_e (v_q^2 + a_q^2) \cos \Theta \right] - Q_q \chi_1 \left[ v_e v_q (1 + \cos^2 \Theta) + 2 v_q a_e \cos \Theta \right] \right\} (I^q \otimes \sigma_y^q - \sigma_y^q \otimes I^q) \\
+ \left( \frac{1}{2} Q_q^2 + \frac{1}{2} \chi_2 (v_q^2 + a_q^2) (v_q^2 - a_q^2) - Q_q \chi_1 v_e v_q \right) \sin^2 \Theta (\sigma_y^q \otimes \sigma_y^q + \sigma_y^q \otimes \sigma_y^q) \\
- \frac{Q_q \chi_1 v_e a_q \Gamma Z M_Z}{s - M_Z^2} \sin^2 \Theta (\sigma_z^q \otimes \sigma_y^q - \sigma_y^q \otimes \sigma_z^q) \right].
\]

Any physical cross section can be obtained by taking the trace of the cross section density matrix with quark-antiquark polarization density matrices, which are defined as

\[
D^q = \psi_q \psi_q^\dagger, \quad D^\bar{q} = \psi_q^\dagger \psi_q.
\]

For example, if we want to calculate the cross section when both quark and antiquark are polarized along the \( x \) axis, we have,

\[
\frac{d\sigma(e^- e^+ \rightarrow q(\hat{x}) \bar{q}(\hat{x}))}{d\Omega_q} = \text{tr} \left[ \left( \frac{d\sigma}{d\Omega_q} \right) (D^q \otimes D^\bar{q}) \right].
\]

where \( D^q = I^q + \sigma_z^q \) and \( D^\bar{q} = I^q + \sigma_z^\bar{q} \) are the density matrices for quark and antiquark polarization, respectively.

### IV. FRAGMENTATION DENSITY MATRIX FORMALISM

Quark fragmentation functions were introduced to describe hadron production from the underlying hard-parton processes. Apart from the well-known, spin-independent and chiral-even fragmentation function \( \hat{f}_1(z) \), there exist various chiral-odd and spin-dependent fragmentation functions which are of particular interest because they describe novel spin effects in hadron production \[2,3\]. At the leading twist, there are two additional fragmentation functions for spin-1/2 baryon production, \( \hat{g}_1(z) \) and \( \hat{h}_1(z) \). All these fragmentation functions can be expressed as light-cone correlations in QCD,

\[
\hat{f}_1(z) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{-i\lambda z} \left\langle 0 | \hat{\psi}(0) \Lambda(PS) X \right| \left\langle \Lambda(PS) X | \bar{\psi}(\lambda n) | 0 \right\rangle,
\]

\[
\hat{g}_1(z) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{-i\lambda z} \left\langle 0 | \hat{\psi}_{75}(0) \Lambda(PS_{||}) X \right| \left\langle \Lambda(PS_{||}) X | \bar{\psi}(\lambda n) | 0 \right\rangle,
\]

\[
\hat{h}_1(z) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{-i\lambda z} \left\langle 0 | \hat{\psi}_{7\perp}(0) \Lambda(PS_{\perp}) X \right| \left\langle \Lambda(PS_{\perp}) X | \bar{\psi}(\lambda n) | 0 \right\rangle,
\]
where $\Lambda(PS)$ represents the $\Lambda$ hyperon with four-momentum $P^\mu$ and polarization $S^\mu$, normalized to $P^2 = M_\Lambda^2$ and $S^2 = -M_\Lambda^2$ respectively. We write $P^\mu = p^\mu + n^\mu M_\Lambda^2/2$, $S^\mu = S_\parallel + M_\Lambda S_\perp$, and $S^\mu = S \cdot np^\mu + S \cdot pn^\mu$ with $p^\mu$ and $n^\mu$ two null vectors ($p^2 = n^2 = 0$ $p \cdot n = 1$). The special component of $p$ ($n$) is along (opposite to) the direction of the $\Lambda$ momentum. These light-cone vectors will be defined separately for every observable hadron and will be labelled accordingly. The variable $z$, representing the momentum fraction of the quark carried by $\Lambda$, is defined in the usual way $z = 2P \cdot q/q^2$, where $q$ is the momentum carried by the virtual boson. The summation over $X$ is implicit and covers all possible states which can be populated by the quark fragmentation, and also the renormalization point ($\mu^2$) dependence is suppressed. [QCD radiative corrections generate a different $\mu^2$ dependence for each moment of these fragmentation functions, which is associated with the $Q^2$ evolution of the experimental data. Although important for comparison with experiment, QCD evolution does not disturb the classification of spin dependent effects, so we suppress it throughout this analysis.]

We turn to the density matrix formalism for quark fragmentation functions. Here we aim to construct quark and antiquark fragmentation density matrices that depend on helicity indices of quark (antiquark) and hyperon (antihyperon). Such a construction can proceed in three steps. First, we construct a density matrix in the Dirac representation. Then, we obtain a density matrix in the mixed representation, in which the helicity indices of quark (antiquark) and hyperon (antihyperon). Such a construction can proceed.

For the quark fragmentation $q \to \Lambda X$, we define

$$
\hat{M}_\Lambda(z, S, P)_{\alpha\beta} = \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0|\psi_\alpha(0)|\Lambda(PS)X\rangle\langle\Lambda(PS)X|\bar{\psi}_\beta(\lambda n)|0\rangle .
$$

(19)

Then, combining with Eqs. [16 – 18], we have,

$$
\hat{M}_\Lambda(z, S, p/z) = \frac{\hat{f}_1(z)}{z} \hat{p} + \frac{\hat{g}_1(z)}{z} (n \cdot S_\parallel) \gamma_5 \hat{p} + \frac{\hat{h}_1(z)}{z} \gamma_5 \hat{S}_\perp \hat{p} .
$$

(20)

to the leading twist. Another set of fragmentation functions can be defined to describe anti-quark fragmentation $\bar{q} \to \bar{\Lambda} \bar{X}$. However, they can be related to the above fragmentation functions by using charge conjugation. In fact, if we define the anti-quark fragmentation density matrix as

$$
\hat{M}_\Lambda(\bar{z}, \bar{S}, \bar{P})_{\alpha\beta} = \int \frac{d\lambda}{2\pi} e^{-i\lambda/\bar{z}} \langle 0|\bar{\psi}_\beta(0)|\bar{\Lambda}(\bar{P}\bar{S})\bar{X}\rangle\langle\bar{\Lambda}(\bar{P}\bar{S})\bar{X}|\bar{\psi}_\alpha(\lambda \bar{n})|0\rangle ,
$$

(21)

we find by using charge conjugation,

$$
\hat{M}_\Lambda(\bar{z}, \bar{S}, \bar{P}/\bar{z}) = -C^{-1} \hat{M}_\Lambda^T(\bar{z}, \bar{S}, \bar{P}/\bar{z}) C ,
$$

(22)

where $\hat{M}_\Lambda^T(z, S, p/z)$ means the transpose of matrix $\hat{M}_\Lambda(z, S, p/z)$. Consequently for the anti-quark fragmentation $\bar{q} \to \bar{\Lambda} \bar{X}$,

$$
\hat{M}_\Lambda(\bar{z}, \bar{S}, \bar{P}/\bar{z}) = \frac{\hat{f}_1(\bar{z})}{\bar{z}} \bar{\hat{p}} - \frac{\hat{g}_1(\bar{z})}{\bar{z}} (\bar{n} \cdot \bar{S}_\parallel) \gamma_5 \bar{\hat{p}} + \frac{\hat{h}_1(\bar{z})}{\bar{z}} \gamma_5 \bar{\hat{S}}_\perp \bar{\hat{p}} .
$$

(23)
Notice the sign change for the \( \hat{g}_1(z) \) term.

The next step is to change \( \hat{M}_\Lambda \), which depend functionally on the spin \( S^\mu \) of the \( \Lambda \), to a \( 2 \times 2 \) fragmentation density matrix carrying \( \Lambda (\bar{\Lambda}) \) helicity indices. The density matrix is defined as,

\[
\hat{M}(z, P^{HH'}) = \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0|\psi_\alpha(0)|\Lambda(PH')X\rangle \langle \Lambda(PH)X|\bar{\psi}_\beta(\lambda n)|0\rangle .
\]  

(24)

The transformation is possible because Eq. (20) with a general \( S \) contains all the information about the \( \Lambda \) spin dependence of the quark fragmentation. If we choose \( S \) to reproduce the helicity eigenstates, the diagonal elements of the spin density matrix are immediately obtained,

\[
\hat{M}_{++} = \hat{M}_{\Lambda}(z, S^\mu = e^\mu_x, p/z) = \frac{\hat{f}_1(z)}{z} \hat{p} + \frac{\hat{g}_1(z)}{z} \gamma_5 \hat{p},
\]

\[
\hat{M}_{-+} = \hat{M}_{\Lambda}(z, S^\mu = -e^\mu_x, p/z) = \frac{\hat{f}_1(z)}{z} \hat{p} - \frac{\hat{g}_1(z)}{z} \gamma_5 \hat{p},
\]

(25)

where we have chosen the direction of the \( \Lambda \) momentum as the \( z \) direction. [In the leading twist calculation, the direction of the quark and \( \Lambda \) can be taken to be collinear. The contributions from the \( \Lambda \) transverse momentum are among higher twists.] According to the superposition principle, we may extract the off-diagonal elements of the spin density matrix from Eq. (20). From

\[
|S^\mu = M_\Lambda e^\mu_x\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |S^\mu = M_\Lambda e^\mu_y\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix},
\]

we have

\[
\hat{M}_\Lambda(z, S^\mu = M_\Lambda e^\mu_x, p/z) = \frac{1}{2} (\hat{M}_{++} + \hat{M}_{-+} + \hat{M}_{-+} + \hat{M}_{-+}) ,
\]

(27)

\[
\hat{M}_\Lambda(z, S^\mu = M_\Lambda e^\mu_y, p/z) = \frac{1}{2} (\hat{M}_{++} + i\hat{M}_{-+} - i\hat{M}_{-+} + \hat{M}_{-+}) .
\]

(28)

From these relations we can read off the off-diagonal components of \( \hat{M} \) in the \( \Lambda \) helicity basis,

\[
\hat{M}_{+-} = \frac{\hat{h}_1(z)}{z} \gamma_5 (\hat{\psi}_x - i\hat{\psi}_y) \hat{p},
\]

\[
\hat{M}_{-+} = \frac{\hat{h}_1(z)}{z} \gamma_5 (\hat{\psi}_x + i\hat{\psi}_y) \hat{p}.
\]

(29)

Here, the four-vectors \( e^\mu_x, e^\mu_y \) and \( e^\mu_z \) are defined by \( e^\mu_x = (0, 1, 0, 0) \), \( e^\mu_y = (0, 0, 1, 0) \) and \( e^\mu_z = (P^z, 0, 0, P^0) \). In terms of the Pauli matrices \( \{\sigma_k^\Lambda\} \) and \( 2 \times 2 \) identity matrix \( \Pi^\Lambda \), Eqs. (25) and (29) can be summarized as,

\[
\hat{M}_\Lambda(z, p/z) = \frac{\hat{f}_1(z)}{z} \hat{p} \Pi^\Lambda + \frac{\hat{g}_1(z)}{z} \gamma_5 \hat{p} \sigma_z^\Lambda - \frac{\hat{h}_1(z)}{z} \gamma_5 \hat{p} \left( \hat{\psi}_x \sigma_x^\Lambda + \hat{\psi}_y \sigma_y^\Lambda \right).
\]

(30)
For the antiquark fragmentation, we define the density matrix analogous to Eq. (24),

\[ \hat{M}(\bar{z}, \bar{p}/\bar{z}) = \frac{d\lambda}{2\pi} e^{-i\lambda/z} \left< 0\left| \bar{\psi}_\beta(0)|\bar{\Lambda}(\bar{P}\bar{H}')\bar{X}\right> \left< \bar{\Lambda}(\bar{P}\bar{H})\bar{X}|\bar{\psi}_\alpha(\lambda\bar{n})|0 \right> . \] (31)

Then a totally parallel analysis shows that

\[ \hat{M}_A(\bar{z}, \bar{p}/\bar{z}) = \frac{\hat{f}_1(\bar{z})}{\bar{z}} \bar{p} \bar{\Pi}^\lambda - \frac{\hat{g}_1(\bar{z})}{\bar{z}} \gamma_5 \bar{p} \sigma_2^\lambda - \frac{\hat{h}_1(\bar{z})}{\bar{z}} \gamma_5 \bar{p} \left( \sigma_x^\lambda \phi_x + \sigma_y^\lambda \phi_y \right) , \] (32)

with similar definitions

\[ \hat{M}_{++} = \hat{M}_A(\bar{z}, \bar{S}^\mu = e_\mu^d, \bar{p}/\bar{z}) , \]
\[ \hat{M}_{--} = \hat{M}_A(\bar{z}, \bar{S}^\mu = -e_\mu^u, \bar{p}/\bar{z}) . \] (33)

Here, the four-vectors \( e_\mu^d \), \( e_\mu^u \), and \( e_\mu^e \) are defined by \( e_\mu^d = (0, 1, 0, 0) \), \( e_\mu^u = (0, 0, 1, 0) \) and \( e_\mu^e = (\bar{P}^e, 0, 0, \bar{P}^0) \) in the frame where the \( \bar{\Lambda} \) momentum is in the \( \bar{z} \)-direction.

Finally we calculate the density matrices entirely in the helicity basis. This reformulation is straightforward—the Dirac matrix form of \( \hat{M}_A(z, p/z) \) is inserted between the helicity basis states \( \bar{u}(h') \) and \( u(h) \). After taking care of the proper normalization, the resulting \( 2 \times 2 \) matrix, whose elements are labelled by the quark helicities, \( h'h \), can be expanded in a quark Pauli matrix basis, as was just done for the \( \Lambda \) indices. The result has the remarkably simple direct product form,

\[ \frac{d\bar{M}_A}{d\bar{z}}(z, p/z) = \hat{f}_1(z) \bar{\Pi}^q \otimes \bar{\Pi}^\lambda + \hat{g}_1(z) \sigma_2^q \otimes \sigma_2^\lambda + \hat{h}_1(z) \left( \sigma_2^q \otimes \sigma_2^\lambda + \sigma_2^q \otimes \sigma_2^\lambda \right) . \] (34)

It is clear from this form that \( \hat{g}_1 \) measures the probability that the longitudinal polarization of the quark transferred to that of \( \Lambda \) with fractional momentum \( z \). The \( \hat{h}_1 \) measures the probability that the transversity of the quark is transferred. There is an obvious invariance of the \( \hat{h}_1 \) term under rotations about the momentum direction, since transverse momenta have been integrated out of the fragmentation process. An analogous expression can be obtained for the anti-quark fragmentation into \( \bar{\Lambda} \), where the corresponding Pauli matrices will be defined in terms of the anti-particle’s helicities. The direct product form for the antiquark case is

\[ \frac{d\hat{M}_{\bar{A}}}{d\bar{z}}(\bar{z}, \bar{p}/\bar{z}) = \hat{f}_1(\bar{z}) \bar{\Pi}^q \otimes \bar{\Pi}^\lambda + \hat{g}_1(\bar{z}) \sigma_2^q \otimes \sigma_2^\lambda + \hat{h}_1(\bar{z}) \left( \sigma_2^q \otimes \sigma_2^\lambda + \sigma_2^q \otimes \sigma_2^\lambda \right) . \] (35)

The fact that Eqs. (34) and (35) are identical is a direct consequence of charge conjugation symmetry, which is manifest in the formalism developed in §II.

An expression like Eq. (34) could have been written upon consideration of the rotation invariance and parity conservation of the parton fragmentation process. Aside from an explicit derivation, what is novel here is the identification of the coefficients with the leading-twist fragmentation functions, which have a formal connection with QCD.
V. Λ (¯Λ) DECAY DENSITY MATRIX

It is well-known that the polarization of Λ and ¯Λ can be measured through their weak decay. The relevant formalism and experimental information about their decay are included in the standard particle data book. Here, for our purpose, we need to obtain the spin density matrices for the decay.

For the hyperon nonleptonic decay, the most general decay amplitude is

\[ M = G_F m_\pi^2 \bar{u}_f (A - B\gamma_5)u_i, \]  

(36)

where \( A \) and \( B \) are constants and generally complex numbers, and \( u_i \) and \( u_f \) are Dirac spinors for the initial and final baryons, respectively. Analogous to Eq. (36), the most general amplitude for the anti-hyperon nonleptonic decay is

\[ M' = -G_F m_\pi^2 \bar{v}_i (A^* + B^*\gamma_5)v_f, \]  

(37)

with \( v_i \) and \( v_f \) Dirac spinors for the initial and final anti-baryons respectively. The spin density matrices for the Λ and ¯Λ decay can be defined as,

\[ D_{HH'} = \sum_{s_p} \langle p\pi^- | \Lambda(PH) \rangle \langle \Lambda(PH') | p\pi^- \rangle, \]

\[ \bar{D}_{\bar{H}'H} = \sum_{\bar{s}_p} \langle \bar{p}\bar{\pi}^+ | \bar{\Lambda}(\bar{P}\bar{H}) \rangle \langle \bar{\Lambda}(\bar{P}\bar{H}') | \bar{p}\bar{\pi}^+ \rangle. \]  

(38)

We have neglected the phase space integration which can be discussed separately. Substituting Eq. (36) into Eq. (38) and again using the superposition principle, we find the four elements of \( D_{HH'} \),

\[ D_{++} = 2 \left[ (|a|^2 + |b|^2)P_p \cdot P + (|a|^2 - |b|^2)m_p M_\Lambda + (a^*b + ab^*)\frac{P_p \cdot S_R}{P_p \cdot e_{x - i e_y}} \right], \]

\[ D_{--} = 2 \left[ (|a|^2 + |b|^2)P_p \cdot P + (|a|^2 - |b|^2)m_p M_\Lambda - (a^*b + ab^*)\frac{P_p \cdot S_R}{P_p \cdot e_{x - i e_y}} \right], \]

\[ D_{+-} = 2(a^*b + ab^*)M_\Lambda P_p \cdot (e_{x - i e_y}), \]

\[ D_{-+} = 2(a^*b + ab^*)M_\Lambda P_p \cdot (e_{x + i e_y}), \]  

(39)

where \( S_R \) is related to the light-cone variables through \( S_R = p - n M_\Lambda^2 / 2 \), \( P_p \) is the momentum of the proton, and

\[ a = G_f m_{\pi^+}^2 A, \]

\[ b = -G_f m_{\pi^+}^2 B. \]  

(40)

In terms of Pauli matrices, we obtain a compact form for the density matrix,

\[ D = 2 \left[ (|a|^2 + |b|^2)P_p \cdot P + (|a|^2 - |b|^2)m_p M_\Lambda \right] \Pi^\Lambda \]

\[ + 2(a^*b + ab^*)\left(\frac{P_p \cdot S_R}{P_p \cdot e_{x - i e_y}}\right) \sigma_2^\Lambda \]

\[ + 2(a^*b + ab^*)M_\Lambda \left[ \left(\frac{P_p \cdot e_x}{P_p \cdot e_x}\right) \sigma_x^\Lambda + \left(\frac{P_p \cdot e_y}{P_p \cdot e_y}\right) \sigma_y^\Lambda \right]. \]  

(41)

A similar calculation for ¯Λ gives
\[ \bar{\mathbf{D}} = 2 \left[ (|a|^2 + |b|^2) \mathbf{P}_p \cdot \bar{\mathbf{P}} + (|a|^2 - |b|^2) m_p M_\Lambda \right] \mathbf{I}^\Lambda \]
\[ -2(a^* b + ab) (\mathbf{P}_p \cdot \bar{\mathbf{S}}_R) \sigma_2^\Lambda \]
\[ -2(a^* b + ab) M_\Lambda \left[ (\mathbf{P}_\bar{p} \cdot e_\bar{z}) \sigma_2^\Lambda + (\mathbf{P}_\bar{p} \cdot e_\bar{y}) \sigma_2^\Lambda \right] , \quad (42) \]

where \( \mathbf{P}_p \) is the momentum of the antiproton and \( \bar{\mathbf{S}}_R = \bar{\mathbf{p}} - \bar{\mathbf{n}} M_\Lambda^2 / 2 \).

When performing the phase space integration, we keep the transverse momenta of the proton and the antiproton as differential variables. To take into account the particle decay width and the motion of the parent particle, it is convenient to define the boost-invariant quantities

\[ \frac{d^2 \mathbf{D}}{d^2 P_p^\perp} = \frac{1}{2 M_\Lambda \Gamma_\Lambda} \int \frac{dP_p^z}{(2\pi)^3 2E_p} \frac{d^3 P_\pi}{(2\pi)^3 2E_\pi} \mathbf{D} (2\pi)^4 \delta^{(4)} (\mathbf{P} - \mathbf{P}_p - \mathbf{P}_\pi), \]
\[ \frac{d^2 \bar{\mathbf{D}}}{d^2 P_\bar{p}^\perp} = \frac{1}{2 M_\Lambda \Gamma_\Lambda} \int \frac{dP_\bar{p}^z}{(2\pi)^3 2E_\bar{p}} \frac{d^3 P_\pi}{(2\pi)^3 2E_\pi} \bar{\mathbf{D}} (2\pi)^4 \delta^{(4)} (\bar{\mathbf{P}} - \bar{\mathbf{P}}_\bar{p} - \bar{\mathbf{P}}_\pi), \quad (43) \]

where \( \Gamma \) is the total width of the \( \Lambda \) decay and \( \mathbf{P}_\pi \) is the momentum of the pion. Particle masses have to be kept explicitly in the decay processes although they can be neglected in high-energy quark fragmentation. After integration, Eq. (43) becomes,

\[ \frac{d^2 \mathbf{D}}{d^2 P_p^\perp} = \frac{\mathbf{D}}{8(2\pi)^2 M_\Lambda \Gamma_\Lambda \left| \mathbf{P} - \mathbf{P}_p^+ - \mathbf{P}_p^- \right|}, \]
\[ \frac{d^2 \bar{\mathbf{D}}}{d^2 P_\bar{p}^\perp} = \frac{\bar{\mathbf{D}}}{8(2\pi)^2 M_\Lambda \Gamma_\Lambda \left| \bar{\mathbf{P}} - \bar{\mathbf{P}}_\bar{p}^+ - \bar{\mathbf{P}}_\bar{p}^- \right|}. \quad (44) \]

where \( P^\pm \) are defined in the usual light-cone coordinates, \( P^\pm = (P^0 \pm P^z) / \sqrt{2} \).

VI. CROSS SECTION FOR \( e^- e^+ \rightarrow q\bar{q} \rightarrow \Lambda \bar{\Lambda} \bar{X} \rightarrow p\pi^- X \bar{p} \pi^+ \bar{X} \) AND ITS PHYSICAL SIGNIFICANCE

Now we have the spin density matrices for all the subprocesses in \( e^- e^+ \rightarrow q\bar{q} \rightarrow \Lambda \bar{\Lambda} \bar{X} \rightarrow p\pi^- X \bar{p} \pi^+ \bar{X} \). Substituting Eqs. (13), (34), (35), and (44) to Eq. (1), we get the final cross section
\[
\frac{d^8 \sigma}{d\Omega_A dz d^2P_p^+ d^2P_{\bar{p}}^+} = \frac{1}{2(2\pi)^2M_A \Gamma_A} \left[ P^+ - P^+ - P^- P^- \right] \cdot \frac{1}{2(2\pi)^2M_A \Gamma_A} \left[ \bar{P}_{\bar{p}}^+ - \bar{P}_{\bar{p}}^+ - \bar{P}_p^+ \bar{P}_p^- \right]
\]
\[
\times \frac{N_e \alpha_{em}^2}{4s} \cdot \left[ \left( |a|^2 + |b|^2 \right) \left( P_p \cdot P \right) + \left( |a|^2 - |b|^2 \right) m_p \Lambda \right]^2 ,
\]
\[
\times \sum_{q = u,d,s} \left[ Q_q^2 \left( 1 + \cos^2 \Theta \right) + \chi_2 \left( 1 + \cos^2 \Theta \right) (v_q^2 + a_q^2) (v_\bar{q}^2 + a_\bar{q}^2) + 8v_e a_e v_q a_q \cos \Theta \right]
\]
\[
-2Q_q \chi_1 \left[ v_e (1 + \cos^2 \Theta) + 2a_e a_q \cos \Theta \right] \left( \hat{f}_1^q (z) \hat{f}_1^\bar{q} (\bar{z}) + C_{gg} \hat{g}_1^q (z) \hat{g}_1^\bar{q} (\bar{z}) \right)
\]
\[
-\left\{ \chi_2 \left[ 2v_e a_q (v_e^2 + a_e^2) (1 + \cos^2 \Theta) + 4v_e a_e (v_q^2 + a_q^2) \cos \Theta \right] - 2Q_q \chi_1 \left[ a_q v_e (1 + \cos^2 \Theta) + 2v_q a_e \cos \Theta \right] \left( C_{fg} \hat{f}_1^q (z) \hat{f}_1^\bar{q} (\bar{z}) + C_{gf} \hat{g}_1^q (z) \hat{g}_1^\bar{q} (\bar{z}) \right)
\]
\[
- \left\{ Q_q^2 + \chi_2 (v_e^2 + a_e^2) (v_q^2 + a_q^2) - 2Q_q \chi_1 v_e v_q \right\} \cos(\varphi + \varphi)
\]
\[
+ 2Q_q \chi_1 \frac{\Gamma_z M_Z}{s - M_Z^2} v_e a_q \sin(\varphi + \varphi) \right \} \sin^2 \Theta C_{hh} \hat{h}_1^q (z) \hat{h}_1^\bar{q} (\bar{z}) \right],
\]
(45)

with \( \varphi \) and \( \varphi \) the azimuthal angles of the proton and the antiproton, respectively, in the coordinate system that is defined in the beginning of §III: the \( z \) axis is chosen to be the direction of the quark jet (or \( \Lambda \)) and the \( x \) axis in the plane of the beams and jets. The \( C \)’s are given by

\[
C_{gg} = \alpha^2 (P_p \cdot S_R) (P_{\bar{p}} \cdot \bar{S}_R) = \frac{\alpha^2 |P_p||P_{\bar{p}}|}{m_p^2} \eta \bar{\eta},
\]

\[
C_{fg} = \alpha \left( \frac{P_p \cdot S_R}{M_A m_p} \right) = \frac{|P_p||\eta|}{m_p},
\]

\[
C_{gf} = \alpha \left( \frac{P_{\bar{p}} \cdot \bar{S}_R}{M_A m_{\bar{p}}} \right) = \frac{|P_{\bar{p}}||\bar{\eta}|}{m_{\bar{p}}},
\]

\[
C_{hh} = \alpha^2 \left( \frac{P_p}{m_p} \right) \left( \frac{P_{\bar{p}}}{m_{\bar{p}}} \right) \eta \bar{\eta},
\]

(46)

where \( P_p \cdot P = (M_\Lambda^2 + m_p^2 - m_\pi^2)/2 \) in \( C_{ff} \) and \( \eta (\bar{\eta}) \) is \( \pm 1 \) depending on whether the momenta of proton (antiproton) and \( \Lambda (\bar{\Lambda}) \) are parallel or antiparallel. \( P_p|| \) and \( P_{\bar{p}}|| \) are the projections of the proton and antiproton momenta in the directions of \( \Lambda \) and \( \bar{\Lambda} \), respectively, in the respective rest frames of the parent particles, and \( \vec{P}_p \perp \) and \( \vec{P}_{\bar{p}} \perp \) are projections of the proton and antiproton momenta onto the \( x-y \) plane. The \( \alpha \) is the standard hyperon-decay parameter defined in the particle data table [10].

There are three distinct classes of terms in the cross section. They correspond to three different type of angular dependences and are sensitive to different combinations of fragmentation functions. The first class involves terms with products of fragmentation functions \( \hat{f}_1(z) \hat{f}_1(\bar{z}) \) and \( \hat{g}_1(z) \hat{g}_1(\bar{z}) \). They have no azimuthal dependence. The dependence on the polar angle arises entirely from the two-jet production cross section. To isolated the
\( \hat{g}_1(z) \hat{g}_1(z) \) term, one has to measure the correlation of the proton and antiproton momenta with respect to the momenta of \( \Lambda \) and \( \bar{\Lambda} \). The second class of terms contains the products of fragmentation functions \( \hat{f}_1(z) \) and \( \hat{g}_1(z) \); they arise from parity violation in electron or quark coupling with \( Z \). The novel spin effects appear in the third class of terms, which involves the product of the transversity distributions \( \hat{h}_1(z) \hat{h}_1(z) \). Let us discuss the physics associated with this term in some detail.

The origin of the \( \sin^2 \Theta \cos(\varphi + \bar{\varphi}) \hat{h}_1(z) \hat{h}_1(z) \) term is seen in the expression for the cross section into \( q\bar{q} \), Eq. (13), in the term proportional to \( (\sigma_x^q \otimes \sigma_y^q + \sigma_y^q \otimes \sigma_y^q) \). To appreciate the structure of that particular form, consider the annihilation through the pure photon channel. The intermediate state photon is produced with helicity of \( \pm \), the term being considered must be maximum in magnitude. That requires the \( q \) and \( \bar{q} \) spins to be preferentially parallel to one another and to align with the \( e^- \) or \( e^+ \) beam directions. Recall that the \( q \) and \( \bar{q} \) momenta are in the \( \pm \hat{z} \) direction and the \( \hat{x} \) axis is in the scattering plane. Given the overall sign of the \( Q_4^2 \) term, it is seen that the cross section favors the \( \sigma_x^q \otimes \sigma_y^q \) expectation value be positive. Since the anti-particle’s \( \hat{x} \)-axis is oriented parallel to the particle’s \( x \)-axis, this means that the \( x \)-components of \( q \) and \( \bar{q} \) spin tend to be aligned. The corresponding \( y \)-components tend to be anti-aligned, since the anti-quark’s \( \hat{y} \)-axis is antiparallel to the quark’s \( y \)-axis in our convention.

If the quark spin has an azimuthal orientation specified by \( \phi_q \) and the anti-quark, \( \bar{\phi}_q \), then the alignment of spins just specified leads to \( \cos(\phi_q + \bar{\phi}_q) \) being positive, which favors the argument near 0. This result for the transverse components of the \( q\bar{q} \) pair, can be visualized by having the \( \bar{q} \) spin vector reflected through the scattering plane. Then the transverse spin vector of the quark tends to be parallel to the transverse spin vector of the reflected \( q \), i.e., the quark transverse spin tends to line up with the mirror reflected anti-quark transverse spin.

Next suppose that the quark (anti-quark) spin orientation is passed on to the \( \Lambda \) (\( \bar{\Lambda} \)) fragmentation product. The asymmetry of the \( \Lambda \) decay into \( p\pi^- \) provides a measure of the spin orientation. As Eq. (11) shows, the proton momentum tends to be aligned with the \( \Lambda \) spin. The corresponding Eq. (12) for the \( \bar{\Lambda} \) decay into an antiproton yields the opposite distribution—the antiproton momentum tends to be antiparallel to the \( \bar{\Lambda} \) spin orientation. Hence, while the \( x \)-components (\( y \)-components) of the quark-antiquark spins tend to align (anti-align), the \( x \)-components of the decay proton and antiproton momenta tend to anti-align (align). This is the interpretation of the \( \cos(\varphi + \bar{\varphi}) \) term in the cross section of Eq. (15).

Note that for the \( Z^0 \) intermediate state the conclusion is opposite from the above discussion, because \( v_{q}^2 - a_{q}^2 \) is negative. So the transverse momentum of the proton tends to be aligned with that of the reflected antiproton.

This construction leads to a simple phenomenological procedure for determining the value (for fixed \( z \) and \( \bar{z} \)) of the product \( \hat{h}_1(z) \hat{h}_1(z) \). For the photon case, the above discussion is summarized by the statement that \( \vec{P}_p^\perp \) and \( \vec{P}_{\bar{p}}^\perp \) tend to be on the same side of the scattering plane. For the \( Z^0 \) case the tendency is for opposite sides of the scattering plane. So it is natural to define an asymmetry (for fixed \( z \) and \( \bar{z} \)) via the number of proton-antiproton pairs on the same side of the scattering plane minus the number on opposite sides of the plane. The asymmetry selects the desired term, and has a simple \( \Theta \) dependence, so that
results from all \( \Theta \) can be combined. The precise expression for this asymmetry follows from Eq. (45).

VII. CONCLUSION

In this paper, we have obtained a differential cross section for the process \( e^-e^+ \rightarrow q\bar{q} \rightarrow \Lambda\bar{\Lambda}X \rightarrow p\pi^-X\bar{\pi}^+\bar{X} \). The cross section depends on, and therefore allows us to extract, the three twist-two quark fragmentation functions to the spin-1/2 hyperon, \( \hat{f}_1(z) \), \( \hat{g}_1(z) \), and \( \hat{h}_1(z) \). These fragmentation functions contain important information about the soft QCD physics in quark fragmentation. Particularly interesting are the spin-dependent fragmentation functions which encode the behavior of the spin transfer. The cross section was obtained through a \( 2 \times 2 \) spin density formalism in the helicity basis, in which the physics is made crystal clear. The formalism is general and can be used for other similar spin processes.

The physical process that we discussed is accessible currently at LEP and SLAC. While up and down quark jets can produce \( \Lambda \) and \( \bar{\Lambda} \) abundantly, the polarized hyperons are mostly produced from \( s\bar{s} \) jets, as we expect from the constituent quark model. The production rate for \( s\bar{s} \) jets is the same as \( d\bar{d} \) and \( b\bar{b} \). The spin transfer from polarized \( s \) to \( \Lambda \) is expected to be large, especially in the large \( z \) region. The weak decay of \( \Lambda \) (\( \bar{\Lambda} \)) into a proton and charged pion can be reconstructed easily and provides an excellent polarization analyzer. To obtain \( \hat{h}_1(z)\hat{h}_1(\bar{z}) \) with enhanced statistics, an asymmetry can be defined by summing over all events with different \( \Omega \) and \( P_\perp \)'s in bins of \( z \) and \( \bar{z} \). Given these remarks, we are looking forward to a first measurement of the spin-dependent fragmentation functions!
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FIGURES

FIG. 1. Feynman diagram for the process $e^- e^+ \rightarrow q \bar{q} \rightarrow \Lambda X \bar{\Lambda} \bar{X} \rightarrow p \pi^- X \bar{p} \pi^+ \bar{X}$. The arrows denote directions of momentum flow (the fermion number flow for antiparticles is against the momentum flow). The helicity indices are explained in the text.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9410337v1