Gravitational radiation from a torus around a black hole

Maurice H.P.M. van Putten, MIT, Cambridge, MA 02139-4307

Long gamma-ray bursts (GRBs) from rapidly spinning black hole-torus systems may represent hypernovae or black hole-neutron star coalescence. We show that the torus may radiate gravitational radiation powered by the spin-energy of the black hole in the presence of non-axisymmetries. The coupling to the spin-energy of the black hole is due to equivalence in poloidal topology to pulsar magnetospheres. Results calculated in the suspended accretion state indicate that GRBs are potentially the most powerful LIGO/VIRGO burst-sources in the Universe, with an expected duration of 10-15s on a horizontal branch of 1-2kHz in the $\dot{f}(f)-$diagram.

Cosmological gamma-ray bursts (GRBs) are the most enigmatic events in the Universe. Their emissions are characteristically non-thermal in the range of a few hundred keV. The BATSE catalogue shows a bi-modal distribution of GRB durations, with short bursts of about 0.3s and long bursts of about 30s [1]. The afterglow phenomenon – broad band secondary emissions generally towards lower energies – has revolutionized our understanding of GRBs as internal shocks in baryon poor (leptonic) outflows and external shocks in interaction with the interstellar medium (see [2]). The notion of GRB-emissions from shocks a certain distance away from the source [3] gracefully circumvents the original compactness problem. The energetics and rapid temporal variability observed in GRBs suggest an association with energetic compact sources. Notable candidates are hypernovae in star-forming regions [4,5] and black hole-neutron star coalescence [6]. Hypernovae represent a variant on failed supernovae [7], produced in core-collapse of rapidly rotating, strongly magnetized
massive stars. Recently, it has been suggested [8] that relics of GRBs might be found in soft X-ray transients which show chemical abundances in $\alpha$-nuclei, such as GRO J1655-40 [10] and V4641 Sgr [11]. This potential GRB/SXT connection by hypernovae [8] could become a relevant factor in GRB-phenomenology, next to their cosmological origin and bi-modal distribution.

Catastrophic events from high-angular momentum compact sources such as hypernovae or black hole-neutron star coalescence are expected to result in black hole plus disk or torus systems (see [12] for a review). Observational support for black hole plus disk/torus-systems presently consists of the following. Short/long bursts may be identified with hyperaccretion/suspended-accretion onto slowly/rapidly spinning black holes [13]. This also points towards a positive correlation between fluence and the spin-rate of the black hole, in agreement with the distinct values of $<V/V_{\text{max}}>$ for long and short bursts [14]. This suggests that short bursts may feature afterglows (cf. the 2s event GRB 000301C [15]). Baryonic winds emanating from the torus may be powerful when derived in suspended accretion from the spin-energy of the black hole, which may be the input to collimation [16,17]. The high-mass ejections onto a companion star in the hypernova scenario of [8] can derive from the spin-energy of the central black hole by a safe margin.

In this Letter, we focus on gravitational radiation from the torus powered by the the spin-energy of the black hole. We shall find that this represents a major fraction of the black hole-luminosity, emitted as presently “unseen” emissions, whenever the torus becomes non-axisymmetric. True calorimetry of gamma-ray bursts, therefore, may be obtained from measuring the fluence in these gravitational-wave emissions [17]. The remainder will be emitted in various ways, such as Poynting flux-winds, baryonic collimating winds and, when sufficiently hot, neutrino emissions. Note that this prediction for as yet unseen emissions invalidates the often stated suggestion that the total energy budget is generally reduced for beamed emissions, in the case of black hole-inner engines with rapid spin. Furthermore, the emissions from the torus in low-frequency radio-waves (modulated) are of potential interest to the planned Low Frequency Radio Antenna (LOFAR) and the Square Kilometre Array.
(SKA), suggesting to consider correlated LIGO/VIRGO-LOFAR/SKA searches.

Gravitational radiation from a torus features several aspects which suggest considering long GRBs as potential sources for LIGO/VIRGO. Namely, the torus is strongly coupled to the spin-energy of the black hole; lumpiness in the torus will produce gravitational radiation at twice the Keplerian angular frequency, i.e., in the range of 1-2kHz; the emission in gravitational radiation should dominate over emissions in radio waves (see young pulsars [21]); the true rate of GRBs should be frequent as inferred from their beaming factor $520 \pm 85 [22]$. Note that gravitational wave-emissions from a torus in suspended accretion is powered by the spin-energy of the black hole and, hence, is distinct from gravitational wave-emissions from spiral-in in neutron star-neutron star mergers [23,24] or by fragmentation in collapse towards supernovae [25].

The torus is likely to possess dynamical and, potentially, radiative instabilities. A geometrically thick torus is consistent [17] with the recent indication that GRBs may be standard [22]. A thick torus is generally unstable [26]. If the torus reaches an appreciable mass fraction of the central black hole, it will be unstable to self-gravity (see [27]). Similar to rapidly rotating neutron stars, the torus may be subject to the Chandrasekhar-Friedman-Shutz instability. Since lumps of matter radiate preferentially on inner orbits, a quadrupolar, radial deformation of the torus might also be radiatively unstable. It would be of interest to study these radiative instabilities in further detail. We note that some of the QPOs in accretion disks in X-ray binaries have been attributed to general relativistic effects in orbital motions [29].

An equivalence in poloidal topology to pulsar magnetospheres shows a high incidence of black hole-luminosity into the torus, when magnetized by the remnant flux from the progenitor star – a massive star in hypernovae or a neutron star in coalescence onto a black hole. The black hole thus surrounded by a torus magnetosphere will adjust to its lowest energy state by developing an equilibrium magnetic moment [12]

$$\mu_H^e \approx aBJ_H,$$  \hspace{1cm} (1)
where $B$ denotes the average poloidal magnetic field in the vicinity of the black hole, and $a = J_H/M$ the specific angular momentum of a black hole with angular momentum $J_H$ and mass $M$. This equilibrium magnetic moment maintains an essentially maximal horizon flux [18]. It serves to preserve strong coupling to the torus magnetosphere and, through it, to the surrounding matter. The latter follows by equivalence in poloidal topology to pulsar magnetospheres [13,12], wherein the inner and outer faces of the torus each correspond to a pulsar with an appropriate angular velocity. When the black hole spins sufficiently rapidly, a state of suspended-accretion may result ([13] and below), wherein the magnetic moment (1) can support open magnetic field-lines to infinity. The latter may account for the beamed outflows of baryon poor jets along the axis of rotation [20,12]. However, the equivalence in poloidal topology to pulsar magnetospheres indicates that the black hole-luminosity $L_T$ onto the torus far exceeds such luminosity $L_p$ into baryon poor jets [17].

Estimates of the various emissions from the torus can be obtained in a suspended-accretion state [13]. Here, the emissions from the torus are replenished by spin-up Maxwell stresses on the inner face, through the magnetic connection to the black hole. This operates by equivalence in poloidal topology to pulsar magnetospheres: the pulsar which is equivalent to the inner face has an angular velocity $-(\Omega_H - \Omega_+)$, where $\Omega_H$ denotes the angular velocity of the black hole and $\Omega_+$ denotes the angular velocity of the inner face. The inner face of the torus thus receives a spin-up torque (adapted from [28,30,13])

$$\tau_+ = (\Omega_H - \Omega_+)f_H^2A^2,$$

where $f_H$ denotes the fraction of flux which reaches the horizon of the black hole, of the net poloidal flux $2\pi A$ supported by the torus. In equilibrium with the radiative losses from the torus, a suspended-accretion state will result.

The motion of the torus subject to the powerful shear between the inner and the outer faces of the torus remains, to leading order, Keplerian. Some deviation away from Keplerian motion is expected, as the competing torques tend to bring the two faces in state of super- and sub-Keplerian motion, with positive radial pressure which promotes a radially slender
shape. The interface separating the two faces is expected to be unstable, which favors
turbulent mixing into a state of uniform specific energy across the torus. Mixing enhances
differential rotation, as may be illustrated in the Newtonian limit, which gives rise to the
angular velocity \( \Omega(r) \approx \Omega_K(1 - (r - a)/a)^{1/2} \) as a function of radius \( r \) for a torus of major
radius \( a \). Compression into a more slender shape tends to reduce differential rotation. The
net result should be that the characteristically Keplerian decrease of angular velocity with
radius is approximately preserved. The inner and other faces will have, respectively, angular
velocities \( \Omega_\pm \approx \Omega_K(1 \pm 3b/4a) \), where \( b \) denotes their radial separation. The same trend
should hold in the Kerr metric. In what follows, we will neglect such perturbations \( 3b/4a \)
to the Keplerian velocity distribution.

Gravitational radiation from a torus surrounding a black hole tends to dominate radio
waves of the same frequency. This is generally the case for compact systems (of the order of
their Schwarzschild radius) in the presence of gravitationally weak magnetic fields. Consider
a torus with ellipticity \( \epsilon \), a magnetic moment \( \mu_T \) and mass \( m \) in rotation about its center
of mass. Its quadrupole moments in magnetic moment and mass are, respectively, \( \epsilon \mu \) and
\( \epsilon m \), which produce luminosities (adapted from [21]):
\[
L_{EM} \approx \pi^{-1}(\Omega_TM)^4(\mu_T/M^2)^2\epsilon^2 \quad \text{and} \quad L_{GW} \approx (32/5)(\Omega_TM)^{10/3}(m/M)^2\epsilon^2
\]
in geometrical units. These emissions may be compared with, respectively, the luminosity in radio emission
\( \sim \Omega_p^4\mu_p^2/\pi \) from an orthogonal pulsar and in gravitational-wave emissions
\( \sim (32/5)(\Omega_{orb}M)^{10/3} \) from neutron star-neutron star binaries with angular velocity \( \Omega_{orb} \) and chirp mass \( \mathcal{M} = (M_1M_2)^{3/5}/(M_1 + M_2)^{1/5} \) (for circular orbits). The ratio of radio-to-gravitational wave emissions can be evaluated as
\[
L_{EM} : L_{GW} \sim (\Omega M)^{2/3}(E_B/M)(M/m)^2 < 1, \quad (3)
\]
e.g., when \( E_B/M \sim 10^{-6} \) for the relative energy in the magnetic field and \( M/m \leq 10^2 \).

The suspended accretion state is described by equilibrium conditions for torque and
energy:
\[
\begin{align*}
\tau^+ & = \tau^- + \tau_{rad}, \\
\Omega^+\tau^+ & = \Omega^-\tau^- + \Omega\tau_{rad} + P_d,
\end{align*}
\]
where \( P_d \) denotes dissipation, \( \Omega \approx \Omega_K \) a mean orbital angular frequency and \( \tau_- = A^2 f_w^2 \Omega_- \) denotes the torque on the outer face of the torus. In (4), we neglect surface stresses due to radiation derived from \( P_d \), notably so in thermal and neutrino emissions. The net magnetic flux \( 2\pi A \) supported by the torus will partially connect to the black hole and to infinity (by Poynting-flux winds), respectively, with fractions \( f_H \) and \( f_w \). Thus, \( A \approx ab < B_\theta > \) in terms of the average poloidal component \( B_\theta \) in the torus. Generally, \( f_H + f_w = 1/2 - 1 \) with \( f_H \propto (M/a)^2 \) for a radially slender torus (which may be thick in the poloidal direction) of major radius \( a \). A remainder \( 1 - f_H - f_w \) is inactive in closed field-lines, whose endpoints are both on either face of the torus. These field-lines extend to the inner light surface and the outer light cylinder, and form toroidal “bags.” Note that for small differential rotation, we have \( (\Omega_K \tau_{rad} + P_d)/\Omega_K \tau_{rad} \approx (\Omega_+ \tau_+ - \Omega_- \tau_-)/\Omega_K (\tau_+ - \tau_-) \approx 2 \), in which limit the efficiency of the radiation is 50%.

The equilibrium conditions (4) are closed, by specifying the internal stresses in the torus. We shall assume that the two faces are coupled by magnetohydrodynamical stresses due to radial components \( B_r \) of the magnetic field. These stresses are dissipative, by Ohmic heating and magnetic reconnection, which will heat the torus and brings about thermal and, possibly, neutrino emissions - with no surface stresses. By dimensional analysis

\[
P_d \approx A_r^2 (\Omega_+ - \Omega_-)^2, \quad A_r = ah < B_r^2 >^{1/2},
\]

where the second equation denotes the root-mean-square of the radial flux averaged over the interface between the two faces with contact area \( 2\pi ah \).

The magnetic stresses on and inside the torus depend differently on the magnetic field. While internal angular momentum transport between the two faces is mediated by \( < B_r^2 >^{1/2} \), the angular momentum transport from the black hole to the torus is by the average \( < B_\theta > \). The first comprises the spectral density average over all azimuthal quantum numbers \( m \), whereas the second only involves \( m = 0 \). Indeed, the net flux through the black hole is generated by the corotating horizon charge \( q \approx < B_n > J \) in magnetostatic equilibrium (1) with the mean external poloidal magnetic field. This averaging process is due to the no-
hair theorem. While the exact ratio depends on the details of the magnetohydrodynamical turbulence in the torus, a conservative estimate is that $A_r/A$ is about the square root of the number of azimuthal modes in the approximately uniform infrared spectrum, which should reach up to the first geometrical break at $m = a/b$, i.e.: $A_r/A \approx (a/b)^{1/2}$. Substitution of the first into the second equation of (4) gives a luminosity

$$L_{GW} \simeq \Omega_T \tau_{rad} \approx \Omega^2 A^2 \left[ 3(A_r/A)^2(b/a) - 2f_w^2 \right] \sim \Omega_T^2 A^2,$$

(6)

where $\Omega_T = (\Omega_+ + \Omega_-)/2$ and $\Omega_+ - \Omega_- \approx (3/2)(b/a)\Omega$, and using $f_w < 1$. Substitution of the right hand-side of (3) in the first equation of (4) gives

$$\frac{\Omega_T}{\Omega_H} \approx \frac{f_H^2}{1 + f_H^2 + f_w^2}.$$

(7)

The luminosity in gravitational waves, therefore, satisfies

$$L_{GW} \simeq L_H/2;$$

(8)

the luminosity in Poynting flux-winds is smaller by a factor $f_w^2$. In the above, the suspended-accretion state is facilitated by magnetohydrodynamical stresses; the results do not depend on the details of the instabilities which give rise to the required non-axisymmetries in the torus. It would be of interest to study the type of instability by numerical simulations.

On the secular time-scale of spin-down of the black hole, the Keplerian frequency of the torus will evolve in time. This indicates a horizontal branch of the frequency dynamics in the $\dot{f}(f)$-diagram [31]. Lumpiness in the torus will radiate at twice the Keplerian frequency of the torus, and hence produces gravitational wave-frequency of

$$f_{gw}(t) \sim 1 - 2kH/(1 + z), \quad df_{gw}(t)/dt = \text{const.}$$

(9)

for canonical GRB values for a black hole-torus system at redshift $z$. Here, the sign of the constant follows the change in Keplerian frequency of the torus; it depends sensitively on the details of the magnetic black hole-to-torus coupling provided by the inner torus magnetosphere. In particular, it depends on the detailed radial dependence of the horizon
flux as a function of the major radius of the torus, which is beyond the scope of the present analysis. If the torus shows violent behavior, the gravitational waves may be episodical, and be correlated with sub-bursts in long GRBs through modulations of the equilibrium magnetic moment. In this event, the linear chirp in (9) is more likely to be indicative of an ensemble average over bursts, rather than to hold for individual bursts. The duration should be the intrinsic duration of the gamma-ray burst event, i.e., about 10-15s as inferred from the mean value of 30s in the BATSE catalogue, corrected for redshift.

The black hole-luminosity is partly directed to the torus, and partly in baryon poor outflows as input to the observed GRBs. Most of the luminosity is into the torus, since only a small fraction is into the jet through an open flux-tube along the axis of rotation. The open flux-tube is described by an opening angle $\theta_H$ on the horizon and an opening angle $\theta_j$ on the celestial sphere. Generally, $\theta_j < \theta_H$ by collimation, e.g., by baryonic collimating winds. A model dependent estimate gives an estimate for the mean geometrical beaming $f_b = \theta_j^2 / 2 \simeq 1/520$ (for bi-polar outflows) and an average GRB fluence of $5 \times 10^{50}$ [22]. The horizon opening angle $\theta_H$ should be sufficient to account for the GRB fluence, yet may leave a dominant fraction of the black hole-luminosity incident on the torus. For a bi-polar output, we have [17]:

$$L_p : L_T \simeq f_o^2,$$

where $f_o = \theta_H^2 / 4$ denotes the beaming factor of the flux-cone on the horizon. Identifying a long GRB with the spin-down of a rapidly spinning black hole, we have $L_p : L_T \simeq E_j : E_T$, where $E_j = E_{GRB} / \epsilon$ denotes the energy in the jet inferred from the observed GRB-fluence at a (model dependent) efficiency $\epsilon$, and $E_T \simeq \Omega_T / \Omega_H E_{rot}$ denotes the energy input to the torus derived from the rotational energy of the black hole. This gives an estimate $\theta_H \simeq 35^\circ$ [17], which may be standard if the torus is geometrically thick [17]. Thus, the ratio (10) is small.

The stability of the gravitational wave-frequency is somewhat uncertain, as it may be variable by the magnetohydrodynamical turbulence in the torus. Nonetheless, it is of interest
to consider the possibility of encountering a well-defined secular frequency sweep (upwards or downwards) in case the frequency behavior is quasi-periodic. In this event, a Fourier analysis suffices. The effective amplitude then correlates with the fluence in gravitational waves - derived from an enhancement in gain by a factor $\sqrt{n}$, where $n$ is the number of cycles in the emission. The effective amplitude of the gravitational radiation of a cosmologically nearby source distance $D$ satisfies

$$h_{\text{eff}}^{\text{GRB}} \sim \left( \frac{M}{D} \right) \left( \frac{E_{\text{GW}}}{M} \right)^{1/2}$$

for a net fluence $E_{\text{GW}}$ in gravitational waves. By (8), $E_{\text{GW}}$ is about one-half the fraction $\Omega_T/\Omega_H$ of the spin-energy of the black hole, i.e., $E_{\text{GW}} \simeq 0.1M_\odot (M/10M_\odot)$. A geometrical beaming factor of about 500 [22] gives rise to multiple events per year within a distance $D \sim 100\text{Mpc}$ with $h_{\text{eff}} \sim 10^{-21}$. Combined, this points towards GRBs as potential sources for LIGO/VIRGO.

In summary, black hole-torus systems representing hypernovae or black hole-neutron star coalescence are candidates of LIGO/VIRGO sources of gravitational radiation. The calorimetry of their non-thermal emissions is dominated by gravitational radiation from the torus, which derives from the spin-energy of the black hole. The gravitational wave-frequency is expected to be 1-2kHz on a horizontal branch in the $\dot{f}(f)$–diagram, by secular evolution on the time-scale of a long burst, for a duration of about 10-15s. If observed, these aspects discriminate these sources from, e.g., binary coalescence or prompt emission during a core-collapse. Current estimates of the geometrical beaming factor indicate that the true rate of GRBs may reach a few events per year within a distance of 100Mpc.

Optimal strategies for detecting these gravitational wave-sources appear to be by Fourier analysis, should be the signal be quasi-periodic. Alternatively, LIGO/VIRGO searches might be combined with future radio searches, such as LOFAR/SKA.

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Figure captions

Figure 1. Cartoon of a rapidly rotating black hole-torus system in suspended accretion. The black hole assumes an equilibrium magnetic moment in its lowest energy state. The torus magnetosphere is supported by a surrounding torus. Equivalence in poloidal topology to pulsar magnetospheres indicates a high incidence of the black hole-luminosity on the inner face of the torus. The torus reradiates this input in gravitational radiation, Poynting flux-winds and, possibly, neutrino emissions. [Reprinted from van Putten, M.H.P.M., Physics Reports, 345, 1-59 ©2001, Elsevier B.V.]
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FIGURE 1

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Torus

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