We analyze the oscillation signals generated by one extra sterile neutrino. We fully take into account the effects of the established oscillations among active neutrinos. We analyze the effects in solar, atmospheric, reactor and beam experiments, cosmology and supernovae. We analyze the impact of the LSND anomaly on cosmology showing, in the full 4 neutrino context, that it is not compatible with standard cosmology. We identify the still allowed regions of the parameter space, outlining which are the future experiments which can improve the bounds.

1 Introduction

The solar and atmospheric neutrino anomalies are today very well explained by oscillations among the three active Standard Model (SM) neutrinos. The contribution from a fourth sterile neutrino once was an alternative explanation of the observed anomalies, but today it can represent only a subleading contribution to the standard scenario of active-only oscillations. Many extensions of the Standard Model foresee the existence of fermions which might have a mass of the order TeV$^2/M_{Pl}$. A few candidates are, in alphabetic order, axino, branino, dilatino, familino, Goldstino, Majorino, modulino, radino. The relevant questions today are then the following: how large can be a subdominant sterile neutrino effect and where do we have to look for it? Of course the mixing of a sterile neutrino with the active ones affects directly the neutrino experiments (solar, atmospheric, reactor and beam). However neutral fermions with eV-scale mass, which is the definition of “sterile neutrinos” for us, are typically stable enough to give effects in cosmology (Big-Bang Nucleosynthesis, Cosmic Microwave Background, Large Scale Structure). Furthermore active-sterile neutrinos mixing would have modified the fluxes of active neutrinos coming from the SN1987A supernova with respect to the standard scenario (with no sterile neutrinos).

After presenting our non-standard parametrization of active-sterile mixing (Sec. 2) we will summarize the effects of a sterile neutrino on solar and KamLAND experiments (Sec. 3), on
atmospheric, reactor neutrinos, short and long-baseline neutrino beams (Sec. 4), on cosmology (Sec. 5) and on supernovae (Sec. 6). The results will be shown in a final set of plots which will identify the regions of parameter space which are excluded (in the case of experiments) or strongly disfavored (in the case of cosmology and supernovae). To avoid showing an unreadable plot, we show only a few lines and shade the whole region excluded by both solar and atmospheric (and reactor and beam) experiments. To better understand the separated bounds we refer to the plots of 1.

2 Parametrization of active-sterile mixing

Active-sterile neutrinos mixing is usually considered assuming that the initial active neutrino $|\nu_a\rangle$ oscillates with a single large $\Delta m^2$ into a mixed neutrino $\cos \theta_s |\nu'_a\rangle + \sin \theta_s |\nu_s\rangle$. We relax these simplifying assumptions and study the more general 4-neutrino context.

In absence of sterile neutrinos, we denote by $U$ the usual $3 \times 3$ mixing matrix that relates neutrino flavor eigenstates $\nu_e, \mu, \tau$ to active neutrino mass eigenstates $\nu_{a1}, \nu_{a2}, \nu_{a3}$ (i.e., $\nu_{\ell} = U_{\ell i} \nu_{a i}$ ($i = \{1, 2, 3\}$, $\ell = \{e, \mu, \tau\}$). In order to parametrize the mixing of an additional sterile neutrino we introduce a complex unit versor $\vec{n}$ which defines the combination of active neutrinos $\vec{n} \cdot \vec{\nu} = n_e \nu_e + n_\mu \nu_\mu + n_\tau \nu_\tau = n_1 \nu_1^e + n_2 \nu_2^\mu + n_3 \nu_3^\tau$ (1)

to which the sterile neutrino mixes with an angle $\theta_s$. With this parameterization the fourth mass eigenstate is a superposition of the sterile neutrino $\nu_s$ and of the combination $\vec{n} \cdot \vec{\nu}$: $\nu_4 = \nu_s \cos \theta_s + n_\ell \nu_\ell \sin \theta_s$. The connection between our parametrization and the usual ones can be found in 1. In Fig. 1 we exemplify the two limiting cases we focus on: mixing with a flavor eigenstate (fig. 1a) and mixing with a mass eigenstate (fig. 1b).

From now on we will consider a $3 + 1$ scheme, with normal hierarchy and we will set $\theta_{13} = 0$. We have checked that allowing $\theta_{13}$ to assume its maximum allowed value leads to at most minor corrections to our analysis.

3 Sterile neutrinos and solar (and KamLAND) experiments

The solar neutrino anomaly is today explained by $\nu_e \rightarrow \nu_{\mu, \tau}$ oscillations in the LMA-MSW region with mixing parameters $\tan^2 \theta_{sun} = 0.41 \pm 0.05$, $\Delta m^2_{sun} = (7.1 \pm 0.6) \cdot 10^{-5}$ eV$^2$. We looked for a subdominant sterile neutrino signal in the experimental data. The work we have done is the following. First of all we compute, in the full 4 neutrinos context, the survival probability $P_{ee}$ at the detection point for an electron neutrino produced inside the sun. The most complicated task is to follow the evolution of the neutrinos inside the sun, which is better achieved in terms of the $4 \times 4$ neutrino density matrix. Possible non-adiabaticities are considered with usual crossing probabilities $P_C$ which however are computed analytically in a non-standard
way. This allows to develop a faster numerical code. Then we fit all the experimental data: the SNO CC+NC+ES spectra; the total CC, NC and ES rates measured by SNO with enhanced NC sensitivity; the Super-Kamiokande ES spectra; the Gallium rate obtained averaging the most recent SAGE, Gallex and GNO data; the Chlorine rate; the KamLAND reactor antineutrino data. We calculate a $\chi^2$ as function of the 2 parameters that describe sterile oscillations, marginalizing the full $\chi^2$ with respect to all other sources of uncertainty including the LMA parameters $\Delta m^2_{\text{sun}}$ and $\theta_{\text{sun}}$. This is motivated by the fact that experiments allow only relatively minor shifts from the LMA point.

No statistically significant evidence of a sterile neutrino has been found. The results are shown in Fig. 3 where we shaded the region excluded at 99% C.L. Notice that in the same plot we shaded also the region excluded at the same C.L. by atmospheric, reactor and beam experiments (Sec. 4). For separated plots see In we also outlined a few promising signals. The most powerful one would be to discriminate a 2% shift from the LMA region of the survival probability $P_{\text{ee}}$ at sub-MeV energies. In fact it is at such energies that LMA oscillations allow a $\nu_1$ component in the solar neutrino flux: at higher energies matter effects are so strong that the solar flux is constituted by $\nu_2$ only. The sterile neutrino $\nu_s$ can mix with $\nu_1$ either directly mixing to it or, independently on the mixing, because the two levels cross.

4 Sterile neutrinos in atmospheric, reactor and beam experiments

The atmospheric anomaly is today explained by $\nu_\mu \to \nu_\tau$ oscillations with parameters $\sin^2 2\theta_{\text{atm}} = 1.00 \pm 0.05$, $\Delta m^2_{\text{atm}} = (2.0 \pm 0.4) \times 10^{-3}$ eV$^2$. We looked for subleading effects due to sterile neutrinos in the data. We followed the evolution of the $4 \times 4$ neutrino density matrix $\rho$ from the production to the detection point. In each medium (air, mantle, core) the evolution is given by a diagonal matrix of phases $\exp(-2i\delta)$ where $\delta_i = m^2_{\nu_iL}/4E_\nu$. We fitted all most recent results and built a global $\chi^2$ which we marginalized with respect to $\Delta m^2_{\text{atm}}$ and $\theta_{\text{atm}}$. This makes the computation much simpler and is motivated by the fact that the experimental data allow only minor shifts from the $\Delta m^2_{\text{atm}}, \theta_{\text{atm}}$ point. We do not include in our fits the LSND data, which have been analyzed separately. In Sec. 5 we analyze the cosmological impact of the LSND anomaly.

It is useful to distinguish two class of experiments. In the data which are not sensitive to the atmospheric anomaly (CHOOZ, BUGEY, CDHS, CCFR, KARMEN, NOMAD, CHORUS) one has essentially to handle with vacuum oscillations. Disappearance experiments provide the dominant constraints. Instead, the experiments which see the atmospheric anomaly (SK, MACRO and K2K) probe sterile neutrinos in a significant but indirect way, essentially by the zenith-angle spectra of $\mu$-like events with TeV-scale energies.

It is useful to compare the sensitivity of the these two classes of experiments. Since there are no MSW resonances, all these experiments are sensitive only to relatively large sterile mixing, $\theta_s \gtrsim 0.1$. Sterile mixing with $\nu_e$ (and with the $\nu_1$ and $\nu_2$ mass eigenstates that contain a sizable $\nu_e$ fraction) is better probed by reactor experiments, although $e$-like events at SK extend the sensitivity down to smaller values of $\Delta m^2$. On the contrary atmospheric experiments give more stringent tests of $\nu_s/\nu_\mu$ mixing and of $\nu_s/\nu_\tau$ mixing.

The results are shown in Fig. 3 where we shaded the region excluded at 99% C.L. (as already said above, for separated exclusion plots for solar and atmospheric experiments see)

5 Sterile neutrinos and cosmology

Cosmology can test sterile neutrinos in the following ways.

**Big Bang Nucleosynthesis (BBN).** BBN probes the total energy density of the Universe at $T \sim (0.1 \div 1)$ MeV. Given a few input parameters (the effective number of thermalized rela-
tivistic species, the baryon asymmetry \( n_B/n_\gamma = \eta \), and possibly the \( \nu_\ell/\bar{\nu}_\ell \) lepton asymmetries) BBN successfully predicts the abundances of several light nuclei. Today \( \eta \) is best determined within minimal cosmology by CMB data to be \( \eta = (6.15 \pm 0.25) \cdot 10^{-10} \). Thus, neglecting the lepton asymmetries (which is an excellent approximation unless they are much larger than baryon asymmetry) one can use the observations of primordial abundances to test if the number of thermalized neutrinos is \( N_\nu \approx 3.8 \) as predicted by the SM.

In our analysis we fix the active-active oscillation parameters to their experimental values, and given the sterile-active mixing parameters we solve the \( 4 \times 4 \) neutrino density matrix kinetic equations and follow the relevant network of Boltzmann equations in order to compute the \( ^4\text{He},^3\text{He} \) abundances. At the end we convert the computed abundances in an effective number of neutrinos. We consider \( N_\nu \gtrsim 3.8 \) disfavored by the experimental data \(^3\), while \( N_\nu \gtrsim 3.2 \) could be tested if the experimental determinations of the \( ^4\text{He},^3\text{He} \) abundances improve.

**Cosmic Microwave Background (CMB).** CMB anisotropies are sensitive to the energy content of the Universe at temperatures \( T \sim 1\text{ eV} \), since it determines the pattern of fluctuations which is measured by WMAP (and other experiments). Neutrinos affect CMB in various ways \(^4\). Global fits at the moment imply \(^5\) \( N_\nu^{\text{CMB}} \approx 3 \pm 2 \) somewhat depending on which priors and on which data are included in the fit. Future data might start discriminating 3 from 4 neutrinos.

**Large Scale Structure (LSS).** Neutrinos can be studied looking at the distribution of the galaxies because massive neutrinos move without interacting, making galaxies less clustered. Observations can constrain the neutrino relic density \( \Omega_\nu h^2 = \text{Tr}[m \cdot \rho]/(93.5\text{ eV}) \), where \( m \) is the \( 4 \times 4 \) neutrino mass matrix and \( \rho \) is the \( 4 \times 4 \) neutrino density matrix. We approximate the present bound \(^6\) with \( \Omega_\nu h^2 < 0.01 \). A 0.001 sensitivity in \( \Omega_\nu h^2 \) could be reached in the future.

For the technical details see \(^7\). The results are shown in Fig. \(^8\). The relevant line is the red one, which corresponds either to a number of effective neutrinos for BBN equal to 3.8 or to a neutrino relic density equal to 0.01, which are both excluded in standard cosmology. The bound from BBN dominates at lower \( \delta m^2 \), while the bound from LSS is stronger at higher \( \delta m^2 \) and small mixing angles. For the \( \nu_1/\nu_e \) mixing the bound is stronger then for the \( \nu_{2,3}/\nu_s \) mixing since \( \nu_1 \) has the maximal \( \nu_e \) component (see Fig. \(^9\)). BBN is more sensible to \( \nu_e \) since it enters directly in the weak reactions which interconvert neutrons into protons. For the mixing with flavor eigenstates the effect does not vanish for \( \Delta m^2 \rightarrow 0 \) since the sterile component is present in all the mass eigenstates (see Fig. \(^10\)).

We also analyzed the impact of the LSND anomaly on cosmology. Extending previous analysis to the full 4 neutrino context, we show our results in Fig. \(^11\). We can conclude that
the region which explains the LSND anomaly through a sterile neutrino is strongly disfavored by Standard Cosmology (BBN primarily) since it would correspond to a number $N_\nu = 4$ of thermalized neutrinos which seems to be disfavored by $^4\text{He}$ data.

6 Sterile neutrinos in supernovae

Supernovae arise from the gravitational collapse of stars. The collapse begins when the iron core of the star reaches the Chandrasekhar limit. The collapse abruptly stops when nuclear densities are reached: the falling material bounces on the surface of the inner core and turns the implosion of the core in an explosion of the outer layers. Although there is not yet a definite theory of supernova explosion a few key features are quite robust. The beta reactions effectively act as a continuous pumping of energy and lepton number from the core matter (which gets neutronized) into the neutrinos, that carry them away and will give rise, at the end of the game, to a neutron star. Independently on the details which give rise to the supernova explosion, about 99% of the gravitational binding energy of the progenitor star (typically $10^{53}$ erg) is released through neutrino emission in a few tens of seconds. Despite the low statistics, observations from SN1987A[6] show an agreement between these estimates and the observed neutrino flux duration and intensity. If one adds channels where the active neutrinos can escape (sterile neutrinos in this case, but also for example extra dimensions[7]) one has to handle with the so called “energy constraint”: the total gravitational binding energy of the progenitor star is pretty well estimated and cannot be drastically overcome. All these considerations allow to use supernovae to put constraints on the mixing between active and sterile neutrinos. Other bounds could be considered. For example in the neutrino-driven picture of supernova explosion the active neutrinos flux is essential in order to provide the necessary energy to the outer material to successfully explode. However the “energy constraint” seems significantly more robust and it is the only one we use.

We compute the reduction of the flux of electron antineutrinos (which have been measured by[6]) with respect to the standard case (no sterile neutrinos and active neutrinos mixed with solar and atmospheric mixing parameters). The techniques to be used are a generalization of the solar ones, with a significant complication due to the fact that in a SN the matter density grows from zero up to the nuclear densities, making matter effects very strong. A crucial point is the matter density and the electron fraction profiles in the mantle of the star. We used the results of[8]. The results are shown in Fig. 3 where we plotted the region which produces a suppression of 70% or more of the SN1987A $\bar{\nu}_e$ rate (dashed blue contour). If a new supernova like SN1987A is observed in the future much more statistics will be available. Furthermore, the theoretical understanding of the supernovae explosion mechanism could significantly improve. Then, supernovae are a promising tool to test the physics of sterile neutrinos.

7 Conclusions

Neutrino physics is maybe the best field where the interplay between experiments and cosmology can be seen. We reviewed the effects of the mixing between active and sterile neutrinos taking into account the established active-active mixing. We analyzed effects on solar, atmospheric, reactor and beam experiments, cosmology (BBN, CMB and LSS) and supernovae. We found no statistically significant evidence of sterile neutrinos and identified the still allowed region of parameter space. We also outlined the future promising signals: sub-MeV solar experiments, more precise determinations of primordial abundances, new data for CMB, new data and improvement of theoretical understanding of supernovae.
Figure 3: Summary of sterile neutrino effects. The dashed region is excluded at 99% C.L. (2 dof) by solar or atmospheric or reactor or short base-line experiments. The red continuous line corresponds either to 3.8 neutrinos for BBN (for lower $\Delta m^2$) or to a neutrino relic density $\Omega_{\nu} = 0.01$ (for higher $\Delta m^2$), both excluded by standard cosmology. The dashed blue line corresponds to a suppression of the SN1987A $\bar{\nu}_e$ rate by more than 70%.

Acknowledgments

I warmly thank Marco Cirelli, Alessandro Strumia and Francesco Vissani for precious collaboration. It is also a pleasure to thank the Organizing Committee of the XXXIXth Rencontres de Moriond.

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