A new approach for predicting the shear capacity of FRCM strengthened RC beams in shear

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Abstract. In spite of the availability of different models proposed to predict the shear capacity of reinforced concrete (RC) beams strengthened in shear using fabric reinforced cementitious matrix (FRCM) system, accurately predicting this value remains a challenge. The simplified compression field theory has shown to accurately predict the shear capacity of RC beams. This paper presents an analytical model based on the simplified compression field theory for predicting the shear capacity of RC beams strengthened in shear using FRCM strengthening system. The model has been validated against a data series of over sixty RC beams strengthened in shear using different FRCM types with different strengthening configuration and orientation. The results showed that the model can reasonably predict the load carrying capacity of the beams. The ratio of theoretically predicted and experimental results for the load carrying capacity ranged between 0.67 and 1.33 while average of this ratio was 1.01 with a coefficient of variation of 0.16.

1. Introduction and background
Reinforced concrete (RC) members may become deficient in shear due to various reasons including reinforcement corrosion, design error, aggressive environmental conditions, overloading and poor maintenance [1]. To remedy these defects, fiber reinforced composites have been used widely as a strengthening material for different applications including strengthening of RC beams [2,3] and RC slab [4]. Despite their strengthening efficacy, FRPs are susceptible to failure in extreme weather conditions and unable to apply on wet surfaces[5,6]. Lately, the cementitious-based fabric reinforced cementitious matrix (FRCM) has shown to be a promising alternative to FRPs[5]. Existing literature revealed that the externally bonded (EB) fabric reinforced cementitious matrix can successfully be used for strengthening of RC beams both in shear and flexure [1,7–12]. However, the externally bonded FRCM technique is associated with debonding at the FRCM/concrete interface that limits the utilization of the strengthening material [1]. A recently introduced hybrid “near surface embedded/externally bonded” NSE/EB-FRCM system has shown to be promising in mitigating this type of failure and thus increases the FRCM utilization [13]. In spite of the availability of different models, accurately predicting the shear capacity of FRCM strengthened RC beams remains a challenge. The state of art on the comparison of the available models for predicting the contribution of the FRCM system on the shear strength of RC beams strengthened with externally bonded FRCM system has been recently presented by Gonzalez-Libreros et al. [14]. The existing models are based on 45-degree truss analogy neglecting the contribution of the concrete in the tension zone and thus gives a conservative result. Moreover, in each model the contribution of the strengthening system was computed as a difference of the shear capacity of the strengthened specimen and the reference specimen. This limits the use of the models as they all require to test and measure the shear capacity of the respective unstrengthen beam. An accurate
method of predicting the shear capacity of RC beams called modified compression field theory (MCFT) was developed by Vecchio and Collins [15] in which the concrete tensile stresses of the cracked section was taken into account. However, this model requires solving large number of equations in an iterative process. Bentz et al. [16] developed a simplified version of MCFT that reduce the number of parameters and iterations and yet can accurately predict the shear capacity. In simplified compression field theory, the shear strength of RC element is determined as a function of two different parameters; viz., inclination of the diagonal compressive stress ($\theta$) and the tensile stress factor in the cracked concrete ($\beta$).

In light of the aforementioned gaps, this paper presents an accurate method of predicting the load carrying capacity of RC beams strengthened in shear using FRCM composites, based on the simplified compression field theory. An experimental results of 62 FRCM strengthened RC beams, obtained from the literature, have been used to validate the proposed analytical model.

2. Proposed model
Consider a beam element strengthened with FRCM system in Figure 1a and 1b. The shear in the beam section is resisted by the diagonal tension stress, $f_1$ and the diagonal compression stress, $f_2$.

From the equilibrium of stresses, Figure 1c,

$$f_1 + f_2 = \nu (\tan \theta + \cot \theta) \quad (1)$$

Where $\theta$ is the inclination of the diagonal compressive stress.

From the equilibrium of the forces, Figure 1d,

$$A_{sv} f_{sv} + A_{fv} f_{fv} = (f_2 \sin^2 \theta - f_1 \cos^2 \theta) b_w S_v \quad (2)$$

From Equation (1) and Equation (2),

$$\nu = f_1 \cot \theta + \frac{A_{sv} f_{sv}}{b_w S_v} \cot \theta + \frac{A_{fv} f_{fv}}{b_w S_v} \cot \theta \quad (2)$$

Where,

$A_{sv}$ and $A_{fv}$ are the area of internal shear reinforcement and FRCM strengthening, respectively, $f_{sv}$ and $f_{fv}$ are the tensile strength of transverse reinforcement bars and FRCM system, respectively. $b_w$ the web width. $S_v$ is the stirrup spacing. $\rho_{sv}$ and $\rho_{fv}$ are the reinforcement ratio of stirrups and FRCM system, respectively.

The shear strength or RC beams strengthened with inclined fabrics at an angle of $\alpha$ to the beam length can be determined based on Equation (3) below.

$$\nu = f_1 \cot \theta + \rho_{sv} f_{sv} \cot \theta + \epsilon_{fv} K_{fv} \cot \theta \ (\sin \alpha + \cos \alpha) \quad (3)$$

Where, $K_{fv}$ is the axial stiffness of the FRCM composite given by equation below.

$$K_{fv} = \rho_{fv} E_{fv}$$

For intermittent FRCM system, $\rho_{fv}$ can be determined by Equation (4) below,

$$\rho_{fv} = \frac{N w_{s} n a_{fv}}{L_{cr} b_w} \quad (4)$$

Where $w_{s}$ is the FRCM strip width, $L_{cr}$ is the shear span length, ‘N’ is the number of FRCM strips, ‘n’ is the number of FRCM layers and $a_{fv}$ is the area of fabrics in the warp (transverse) direction. For continuous FRCM strengthening configuration the value of $N w_{s}/L_{cr}$ is unity.
a) Principal stresses in concrete.  

b) Beam cross-section. 

c) Forces in stirrups and FRM 

d) Mohr’s circle for stresses 

Figure 1. Equilibrium conditions for MCFT based on Vecchio and Collins [15].

As per the simplified compression field theory proposed by Bentz et al. [16] the contribution of concrete to the shear capacity is determined as a function of β.

\[ \nu_c = \beta \sqrt{f'_c} \]  

(5)

Thus,

\[ \nu = \beta \sqrt{f'_c' + \rho_{SV} f_{sv} \cot \theta + \varepsilon_{Fv} K_{FV} \cot \theta \left( \sin \alpha + \cos \alpha \right)} \]  

(6)

Where \( \beta \) is given by Equation (7) below,

\[ \beta = \frac{0.4}{1 + 1500 \varepsilon_x} \frac{1300}{1000 + S_{xe}} \]  

(7)

Where \( S_{xe} \) is the crack spacing given by Equation (8) below,

\[ S_{xe} = \frac{35S_x}{a_g + 16} \geq 0.85S_x \]  

(8)

Where \( S_x \) is the vertical distance between the longitudinal reinforcement (in mm) and \( a_g \) is the maximum diameter of aggregates (in mm).

Based on the simplified version of MCFT the value of \( \theta \) is approximated as follows.
\[ \theta = (29 + 7000 \varepsilon_x) \times \left( 0.88 + \frac{S_{se}}{2500} \right) \leq 75^\circ \]  

Where \( \varepsilon_x \) is the strain in the longitudinal reinforcement,

\[ \varepsilon_x = \frac{\nu \cot \theta - \beta \sqrt{f_{c}'} \tan \theta}{E_s \rho_{sx}} \]  

Where \( E_s, \rho_{sx}, f_{c}' \) and \( \nu \) are the elastic modulus of flexural reinforcement, reinforcement ratio of flexural tensile bars, concrete shear stress and shear stress in a RC member, respectively.

The proposed model is summarized in Figure 2 below.

**Figure 2.** Flowchart for determining the shear capacity using simplified MCFT.
3. Verification of the model
The ultimate load carrying capacity of 62 RC beams, strengthened in shear using different types of FRCM system; namely, carbon (C), glass (G), basalt (B), and polyparaphenylene benzobisoxazole (PBO), has been predicted and the results compared with the experimental results reported in the literature [11,18–24]. Figure 3 shows the plot for the ratio of theory predicted ($P_{ult}$) to the experimental values ($P_{ex}$) of ultimate load carrying capacity. Moreover, the details of the experimental results and theoretically predicted values of the ultimate load carrying capacity of the beams is summarized in Tables 1a and 1b. As can be seen in Figure 3 and the last column of Tables 1a and 1b, the ratio of the theoretically predicted and experimental results for the ultimate load carrying capacity ranged between 0.67 and 1.33 while the average of this ratio was 1.01 with 0.16 coefficient of variation. The results showed that the model can reasonably predict the ultimate load carrying capacity of the strengthened beams.

![Figure 3. Verification of the model against experimental data.](image)

Table 1a. Summary of the theoretical and experimental results.

| Beam ID        | Concrete | Internal reinforcement | Strengthening composite | $P_{ult}$ | $P_{ex}$ | $P_{ult}/P_{ex}$ |
|----------------|----------|------------------------|-------------------------|-----------|----------|-----------------|
| Tetta et al. [24] |          |                        |                         |           |          |                 |
| CON            | 21.6     | 102                    | 177                     | -         | 0.022    | -               | 43.0333         | 51.8          | 0.83            |
| CL1            | 23       | 102                    | 177                     | -         | 0.022    | C 0.2038       | 74.7094         | 102.3         | 0.73            |
| CL1 STRIPS     | 20       | 102                    | 177                     | -         | 0.022    | C 0.3122       | 89.1772         | 110.7         | 0.81            |
| CH1            | 23.8     | 102                    | 177                     | -         | 0.022    | C 0.3042       | 90.1423         | 78.2          | 1.15            |
| CH1_CL1        | 20       | 102                    | 177                     | -         | 0.022    | C 0.5079       | 117.604         | 117.4         | 1               |
| CH2            | 23.8     | 102                    | 177                     | -         | 0.022    | C 0.6084       | 133.407         | 120.2         | 1.11            |
| CL3            | 20.8     | 102                    | 177                     | -         | 0.022    | C 0.6113       | 132.335         | 118           | 1.12            |
| CH2_CL1        | 20       | 102                    | 177                     | -         | 0.022    | C 0.8121       | 158.768         | 129.3         | 1.23            |
| CH3            | 22.6     | 102                    | 177                     | -         | 0.022    | C 0.9126       | 172.765         | 131.1         | 1.32            |
| CON_3.6        | 20.5     | 102                    | 177                     | -         | 0.022    | C 0.2038       | 103.913         | 133.8         | 0.78            |
| CL1_3.6        | 22.6     | 102                    | 177                     | -         | 0.022    | C 0.6113       | 185.999         | 158.7         | 1.17            |
| CL3_3.6        | 22.6     | 102                    | 177                     | -         | 0.022    | B 0.2658       | 55.926          | 82.66         | 0.68            |
| BS2            | 20       | 150                    | 159                     | -         | 82.66    | B 0.2658       | 55.926          | 82.66         | 0.68            |
| BS3            | 20       | 150                    | 159                     | -         | 83.51    | B 0.2658       | 55.926          | 83.51         | 0.67            |
| BS4            | 20       | 150                    | 159                     | -         | 88.74    | B 0.5316       | 82.062          | 88.74         | 0.92            |
| BS5            | 20       | 150                    | 159                     | -         | 92.53    | B 0.5316       | 82.062          | 92.53         | 0.89            |

Al-Salloum et al. [18]

| Beam ID | Concrete | Internal reinforcement | Strengthening composite | $P_{ult}$ | $P_{ex}$ | $P_{ult}/P_{ex}$ |
|---------|----------|------------------------|-------------------------|-----------|----------|-----------------|
| BS2     | 20       | 150                    | 159                     | -         | 82.66    | B 0.2658       | 55.926          | 82.66         | 0.68            |
| BS3     | 20       | 150                    | 159                     | -         | 83.51    | B 0.2658       | 55.926          | 83.51         | 0.67            |
| BS4     | 20       | 150                    | 159                     | -         | 88.74    | B 0.5316       | 82.062          | 88.74         | 0.92            |
| BS5     | 20       | 150                    | 159                     | -         | 92.53    | B 0.5316       | 82.062          | 92.53         | 0.89            |
Table 1b. Summary of the theoretical and experimental results.

| Beam ID | Concrete | Internal reinforcement | Strengthening composite | $P_{a,th}$ | $P_{a,ex}$ | $P_{a,ex}/P_{a,th}$ |
|---------|----------|------------------------|-------------------------|------------|------------|---------------------|
| Azam & Soudki [20] |          |                        |                         |            |            |                      |
| C-N     | 38       | 150                    | 307.5                   | -          | 123.5      | -                   |
| SB-GT   | 38       | 150                    | 307.5                   | -          | 146.3      | G 0.0238            |
| UW-GT   | 38       | 150                    | 307.5                   | -          | 180.2      | G 0.0238            |
| SB-CT1  | 38       | 150                    | 307.5                   | -          | 155.5      | C 0.0563            |
| UW-CT1  | 38       | 150                    | 307.5                   | -          | 151.8      | C 0.0563            |
| SB-CT2  | 38       | 150                    | 307.5                   | -          | 245.4      | C 0.1354            |
| UW-CT2  | 38       | 150                    | 307.5                   | -          | 253.4      | C 0.1354            |
| Ombres [19] |          |                        |                         |            |            |                      |
| TRA0    | 38.45    | 150                    | 225                     | 446.06     | 0.0023     | 75.35               |
| TRA1    | 38.45    | 150                    | 225                     | 446.06     | 0.0023     | 94.37               |
| TRA2    | 38.45    | 150                    | 225                     | 446.06     | 0.0026     | 85.2                |
| TRB0    | 56.275   | 150                    | 224.82                 | 446.06     | 0.0032     | 105.7               |
| TRB1    | 56.275   | 150                    | 224.82                 | 446.06     | 0.0032     | 139.53              |
| TRB2    | 36.45    | 150                    | 224.82                 | 446.06     | 0.0032     | 95.83               |
| TRB3    | 36.45    | 150                    | 224.82                 | 446.06     | 0.0032     | 95.93               |
| TRB4    | 47.825   | 150                    | 224.82                 | 446.06     | 0.0032     | 99.98               |
| TRB5    | 47.825   | 150                    | 224.82                 | 446.06     | 0.0032     | 99.88               |
| Tetta et al. [21] |          |                        |                         |            |            |                      |
| CON     | 21.6     | 102                    | 177                    | -          | 0.02227    | -                   |
| SB_M1   | 21.6     | 102                    | 177                    | -          | 0.02227    | C 0.0419            |
| UW_M1   | 23.8     | 102                    | 177                    | -          | 0.02227    | C 0.2287            |
| SB_M2   | 22.6     | 102                    | 177                    | -          | 0.02227    | C 0.3204            |
| UW_M2   | 23.8     | 102                    | 177                    | -          | 0.02227    | C 0.5933            |
| SB_M3   | 22.6     | 102                    | 177                    | -          | 0.02227    | C 0.4806            |
| UW_M3   | 22.6     | 102                    | 177                    | -          | 0.02227    | C 0.8899            |
| Loreto et al. [22] |          |                        |                         |            |            |                      |
| L_0_Ave | 29.13    | 152                    | 248                    | 276        | 0.0027     | 0.0304             |
| L_1_Ave | 29.13    | 152                    | 248                    | 276        | 0.0027     | 0.0304             |
| L_4_Ave | 29.13    | 152                    | 248                    | 276        | 0.0027     | 0.0304             |
| H_0_Ave | 42.91    | 152                    | 248                    | 276        | 0.0027     | 0.0304             |
| H_1_Ave | 42.91    | 152                    | 248                    | 276        | 0.0027     | 0.0304             |
| H_4_Ave | 42.91    | 152                    | 248                    | 276        | 0.0027     | 0.0304             |
| Escrig et al. [23] |          |                        |                         |            |            |                      |
| V-BR3-01 | 33.78   | 300                    | 254                    | -          | 0.00792   | B 0.017            |
| V-CXM25-01 | 33.78   | 300                    | 254                    | -          | 0.00792   | C 0.0251            |
| V-CXM25-02 | 34.07   | 300                    | 254                    | -          | 0.00792   | C 0.0501            |
| V-PXM750-01 | 34.07   | 300                    | 254                    | -          | 0.00792   | B 0.0388            |
| V-PXM750-02 | 34.07   | 300                    | 254                    | -          | 0.00792   | B 0.0777            |
| V-GPHDM-02 | 34.07   | 300                    | 254                    | -          | 0.00792   | G 0.0304            |
| V-CONTROL | 34.82   | 300                    | 254                    | -          | 0.00792   | -                  |
| Gonzalez-Libreros et al. [11] |          |                        |                         |            |            |                      |
| S1-CONTROL | 23.3  | 150                    | 230                    | 527        | 0.0022    | 0.06156           |
| S1-FRCM-F3-UN | 23.3  | 150                    | 230                    | 527        | 0.0022    | 0.06156           |
| S1-FRCM-F3-UA | 23.3  | 150                    | 230                    | 527        | 0.0022    | 0.06156           |
| S1-FRCM-F4-UN | 21.3  | 150                    | 230                    | 527        | 0.0022    | 0.06156           |
| S1-FRCM-F4-UA | 21.3  | 150                    | 230                    | 527        | 0.0022    | 0.06156           |
| S2-CONTROL | 24.7  | 150                    | 230                    | 527        | 0.0034    | 0.06156           |
| S2-FRCM-F3-UN | 24.7  | 150                    | 230                    | 527        | 0.0034    | 0.06156           |
| S2-FRCM-F3-UA | 24.7  | 150                    | 230                    | 527        | 0.0034    | 0.06156           |
| S2-FRCM-F4-UN | 21.3  | 150                    | 230                    | 527        | 0.0034    | 0.06156           |
| S2-FRCM-F4-UA | 21.3  | 150                    | 230                    | 527        | 0.0034    | 0.06156           |

Note: The table includes beam IDs, concrete properties, internal reinforcement, and experimental results.
4. Conclusion
A model based on simplified MCFT has been proposed to predict the ultimate load carrying capacity of FRCM strengthened RC beams. Over sixty test results of RC beams obtained from the literature have been used to validate the proposed model. The results indicated that the model can correctly predict the ultimate load carrying capacity of the FRCM strengthened beams. The ratio of theoretically predicted and experimental results for the ultimate load carrying capacity ranged between 0.67 and 1.33 while the average of this ratio was 1.01 with 0.16 coefficient of variation.

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