Threshold resummation for nonleptonic $B$ meson decays

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Abstract

We investigate the double logarithmic corrections $\alpha_s \ln^2 x$, $x$ being a parton momentum fraction, in two-body nonleptonic $B$ meson decays in collinear factorization theorem of perturbative QCD (PQCD). It is found that these corrections are universal for factorizable amplitudes, i.e., at the leading power, and that their threshold resummation smears the end-point singularities from $x \to 0$. The double logarithmic corrections, depending on the topologies of nonfactorizable amplitudes at the subleading power, are negligible due to the color-transparency argument and to the overlap of three meson distribution amplitudes. We show that the PQCD approach to two-body nonleptonic $B$ meson decays respects the factorization assumption in the heavy-quark limit.

I. INTRODUCTION

It has been shown that fixed-order evaluation of $B$ meson semileptonic decay amplitudes in the framework of collinear factorization theorem suffers the end-point singularities from a parton momentum fraction $x \to 0$ [1–3]. On the other hand, the double logarithms $\alpha_s \ln^2 x$ appearing in higher-order corrections to these decays have been observed [4–8]. We argue that when the end-point region is important, $\alpha_s \ln^2 x$ cannot be treated as a small expansion parameter, and should be summed to all orders. A systematic treatment of these logarithms has been proposed by grouping them into a quark jet function, whose dependence on $x$ is governed by an evolution equation [6]. A Sudakov form factor, obtained by solving the evolution equation, decreases fast enough at the end point. The above procedure is referred to as threshold resummation, whose technical details can be found in [9–11]. It turns out that in a self-consistent analysis, where the original factorization formulas [12–15] are convoluted with the Sudakov factor, the end-point singularities do not exist [16]. Therefore, it is not necessary to introduce arbitrary infrared cutoffs for momentum fractions $x$ even in the collinear factorization theorem, such as the QCD factorization (QCDF) approach [17] to exclusive $B$ meson decays. The same observation has been made recently in the framework of soft-collinear effective theory [18].

In this letter we shall examine the double logarithmic corrections to all the topologies of two-body nonleptonic $B$ meson decay amplitudes, including charmless and charmful modes. The topologies contain both emission and annihilation, which are further divided into factorizable and nonfactorizable types. Our results for the charmless and charmful decays are the same, and summarized as follows. The double logarithmic corrections are crucial for factorizable (regardless of charmless or charmful, emission or annihilation) contributions due to the presence of the potential linear end-point singularities. As to nonfactorizable contributions, the double logarithmic corrections exist, and carry a color factor different from that in the factorizable cases. However, they are less crucial for charmless decays, since the end-point singularities are at most logarithmic: there exists soft cancellation between a pair of nonfactorizable emission diagrams near the end point due to the color-transparency argument [19]. For charmful decays, the hierarchy that the $b$ quark mass is much larger than the $c$ quark mass is the necessary condition for perturbative QCD (PQCD) to be applicable [20]. Under this hierarchy, the soft cancellation between a pair of nonfactorizable emission amplitudes holds approximately. The factorization formulas for nonfactorizable (charmless and charmful) annihilation amplitudes involve the overlap integrals of three meson distribution amplitudes, such that the end-point singularities become milder.

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In Sec. II we identify the double logarithms involved in the various topologies of decay amplitudes. The numerical analysis is performed in Sec. III. We show that threshold resummation is not required for nonfactorizable amplitudes, since the logarithmic singularities can be easily smeared by the Sudakov effect from \(k_T\) resummation \[21\]–\[23\]. Note that threshold resummation deals with the double logarithmic corrections to hard amplitudes, while \(k_T\) resummation deals with those to hadron wave functions. Hence, the former may not be universal as stated above, but the latter must be, since wave functions are. Both effects are necessary, especially when the end-point singularity is linear. Considering either of them, though smears the singularity \[6\], [18], does not suppress the soft contribution strongly enough \[16\]. The results derived in this work provide a useful reference for the study of two-body nonleptonic \(B\) meson decays. At last, we explicitly demonstrate that the PQCD formalism for two-body nonleptonic \(B\) meson decays reduces to the factorization assumption \[24\] in the heavy-quark limit. Section IV is the conclusion.

II. DOUBLE LOGARITHMS

We take the \(B \to K\pi\) decays as an example. The momenta of the \(B\) meson, kaon and pion, and of their valence quarks are chosen as

\[
P_1 = (M_B/\sqrt{2})(1, 1, 0_T), \quad k_1 = (M_B/\sqrt{2})(x_1, 0, 0_T),
\]

\[
P_2 = (M_B/\sqrt{2})(1, 0, 0_T), \quad k_2 = x_2 P_2,
\]

\[
P_3 = (M_B/\sqrt{2})(0, 1, 0_T), \quad k_3 = x_3 P_3,
\]

respectively, \(x_{1,2,3}\) being the momentum fractions. The parton momentum \(k_1\) \((k_3)\) is associated with the spectator quark in the \(B\) meson \((\text{pion})\). The momentum \(k_2\) is associated with the light quark \((\text{not the } s\text{ quark})\) in the kaon. The mass difference between the \(B\) meson and the \(b\) quark, \(\bar{\Lambda} = M_B - m_b\), is treated as a small scale in the following analysis. To concentrate on the double logarithm \(\alpha_s \ln^2 x\), we adopt the collinear factorization theorem, and do not include the transverse momenta \(k_T\) of the valence quarks.

If the end-point region is defined by \(x \to 0\), the double logarithm, being of the collinear origin, is produced through the loop integral in the covariant gauge,

\[
J^{(1)} = -ig^2 C \int \frac{d^4l}{(2\pi)^4} \frac{n_+ \cdot n}{n_+ \cdot (l - xP)n \cdot l^2},
\]

\[
= -\frac{\alpha_s}{4\pi} C \ln^2 x + \cdots,
\]

where \(C\) is a color factor depending on the topology, the dimensionless vectors \(n_+ = (1, 0, 0_T)\) and \(n_- = (0, 1, 0_T)\) on the light cone, \(n\) an arbitrary dimensionless vector, and \(xP\) the fractional momentum carried by a parton. The Feynman rules \(n^+_a/n_+ \cdot (l - xP)\) and \(n_a/n \cdot l\) arise from the eikonal approximation of quark propagators. Their appearance will be explained below. A double logarithm is associated with a system containing two scales \[11\]: one is of \(O(\Lambda_{QCD})\) and another is a large momentum transfer. With this principle, it is easy to identify the source of double logarithms in the different topologies of diagrams.

Our analysis of the double logarithmic corrections applies to charmful \(B\) meson decays, such as \(B \to D\pi\). It has been argued that the PQCD formalism for these decays holds under the hierarchy \[20\],

\[
M_B \gg M_D \gg \bar{\Lambda},
\]

with \(M_D\) being the \(D\) meson mass. The relation \(M_B \gg M_D\) justifies the perturbative calculation of the \(B \to D\) transition at large recoil and the definition of light-cone \(D\) meson wave functions. The relation \(M_D \gg \bar{\Lambda}\) justifies the power expansion in the parameters \(\bar{\Lambda}/M_D\) and \(\bar{\Lambda}/M_B\). Therefore, the \(B \to D\) hard amplitudes are basically the same as the \(B \to \pi\) ones at the leading power. It has been shown that the charm mass effect appears through the ratio \((M_D/M_B)^2 \sim 0.1\), which is indeed negligible.

A. Factorizable Diagrams

The lowest-order factorizable and nonfactorizable diagrams for the \(B \to K\pi\) decays are displayed in Fig. 1. We start with the factorizable emission diagram in Fig. 1(a), which gives a hard amplitude proportional to \(1/(x_1 x_2^3)\). Obviously, this amplitude, as convoluted with the pion distribution amplitude \(\phi_\pi(x_3)\), leads to a logarithmic divergence for \(\phi_\pi(x_3) \sim x_3\) \((\text{twist-2})\) and to a linear divergence for \(\phi_\pi(x_3) \sim 1\) \((\text{two-parton twist-3})\) at small \(x_3\). As \(x_3 \to 0\) (to be
precise, \( x_3 \sim O(\bar{\Lambda}/M_B) \), the internal quark, carrying the momentum \( P_3 - x_3 P_3 \), approaches the mass shell. This is the origin of the end-point singularity. The double logarithms in radiative corrections to Fig. 1(a) then become important. Part of the \( O(\alpha_s) \) corrections are shown in Fig. 2. The discussion of these diagrams is the same as that for the \( B \to \pi \) form factors \([6]\), and the results are summarized below.

Figure 2(a) generates the double logarithm \( \alpha_s \ln^2 x_3 \) through Eq. (2) with the color factor \( C = C_F = 4/3 \). In this case the quark propagator gives \( n_\alpha/n_\perp \cdot (l - x_3 P_3) \) under the eikonal approximation in the collinear region with the loop momentum \( l \) being parallel to \( P_3 \). The light quark propagator gives \( n_\alpha/n \cdot l \) with \( n \) being almost in the direction of \( P_3 \). A small plus component has been included into \( n \) to regularize the infrared pole of Eq. (2). The self-energy correction in Fig. 2(b), and the vertex corrections in Figs. 2(c) and 2(d) produce only the single logarithm \( \alpha_s \ln x_3 \), since these diagrams do not involve a large momentum transfer. The double logarithms are produced from the vertex corrections in Figs. 2(c) and 2(f) with the gluons attaching the kaon. However, they cancel each other due to the opposite signs in the eikonal approximations for a quark and for an anti-quark. Note that Fig. 2(d) contains the double logarithm \( \alpha_s \ln^2 x_1 \) as shown in \([4]\). However, this double logarithm is not very relevant, because the convolution in \( x_1 \), with the corresponding hard amplitude \( 1/(x_1 x_2^2) \), diverges at most logarithmically.

Figure 1(b) gives an amplitude proportional to \( 1/(x_1^2 x_3) \). The factorization formula diverges linearly, if using the \( B \) meson distribution amplitudes derived in \([25,26]\). This end-point singularity indicates that collinear enhancements arise from the higher-order corrections associated with the internal light quark as \( x_3 \sim O(\bar{\Lambda}^2/M^2_B) \). The analysis is similar to that for Fig. 1(a): only the diagram, in which the radiative gluon attaches the quark and the internal quark carrying the momentum \( P_3 - k_1 \), generates the double logarithm \( \alpha_s \ln^2 x_3 \). In this case the internal quark propagator gives \( n_\alpha/n \cdot (l - k_1) \) in Eq. (2), \( n \) stands for the quark velocity \( v = P_1/M_B \), and the color factor is \( C = C_F \).

The above study of double logarithms applies to the factorizable annihilation diagrams straightforwardly. The lowest-order diagram in Fig. 1(c) gives a hard amplitude proportional to \( 1/(x_2 x_3^2) \), \( x_3 \equiv 1 - x_3 \). Hence, the end-point region is defined by \( x_3 \to 0 \), where additional collinear divergences are associated with the internal quark carrying the momentum \( P_2 + x_3 P_3 \). The loop correction to the weak decay vertex, in which the radiative gluon attaches the internal quark and the light spectator quark flowing into the pion, produces the double logarithm \( \alpha_s \ln^2 x_3 \). The internal quark propagator leads to \( n_\alpha/n_\perp \cdot (l + x_3 P_3) \) in Eq. (2), the vector \( n \) denotes the direction almost parallel to \( P_3 \), and the color factor is \( C = C_F \). The double logarithms in the pair of diagrams with the radiative gluon attaching the quark and the spectator quark cancel, since a \( B \) meson, dominated by soft dynamics, remains color transparent for collinear gluons. The discussion of the double logarithmic corrections from the other next-to-leading-order diagrams is similar: they generate only the single logarithms.

For Fig. 1(d) with the internal quark carrying the momentum \( P_3 + x_2 P_2 \), the end-point region corresponds to \( x_3 \to 0 \). The loop correction to the weak decay vertex, in which the radiative gluon attaches the quark and the internal quark, produces the double logarithm \( \alpha_s \ln^2 x_2 \) with the color factor \( C = C_F \). The internal quark propagator gives \( n_\alpha/n_\perp \cdot (l + x_2 P_2) \) in Eq. (2), and the vector \( n \) denotes the direction almost parallel to \( P_2 \). The discussion for the other diagrams is similar to that for Fig. 1(c). It is then concluded that the double logarithms \( \alpha_s \ln^2 x \) are universal for the factorizable amplitudes, and independent of the color structures of the four-fermion operators.

### B. Nonfactorizable Diagrams

In Fig. 1(e) the internal quark carries the momentum \( x_2 P_2 - k_1 + x_3 P_3 \), \( x_2 \equiv 1 - x_2 \). There are three kinematic regions, in which the quark approaches the mass shell, and additional infrared divergences appear:

\[
\begin{align*}
(1) & \quad x_1 \sim x_2 \sim x_3 \sim O(\bar{\Lambda}/M_B) , \\
(2) & \quad x_1 \sim x_3 \sim O(\bar{\Lambda}/M_B) , \quad x_2 \sim O(1) , \\
(3) & \quad x_1 \sim x_2 \sim O(\bar{\Lambda}/M_B) , \quad x_3 \sim O(1) . \\
\end{align*}
\]

In the first configuration no double logarithm is generated due to the lack of a large scale. Because the hard gluon propagator is proportional to \( 1/(x_1 x_2) \), the contribution to the hard amplitude from the third configuration is power-suppressed compared to that from the second one. Therefore, the end-point region is defined by the second configuration, in which additional collinear divergences from the loop momentum parallel to \( P_2 \) are generated.

The vertex corrections in Figs. 3(a) and 3(b) contain the double logarithms \( \alpha_s \ln^2 x_3 \). The internal quark propagator leads to \( n_\alpha/n_\perp \cdot (l + x_3 P_3) \) in Eq. (2), and the vector \( n \) stands for the quark velocity \( v \) in the former, and for the direction almost along \( P_3 \) in the latter. If there are two separate color flows between the kaon and the \( B \to \pi \) transition form factor, the two double logarithms cancel each other, since they have the same color factor, but are opposite in sign. If there is only a single color flow, the two double logarithms possess different color factors and add
together. The result is proportional to the net color factor \( C = C_F + 1/(2N_c) = N_c/2 \), \( N_c = 3 \) being the number of colors. The other diagrams give only the single logarithms. In Fig. 1(f), the internal light quark carries the momentum \( x_2 P_2 - k_1 + x_3 P_3 \). Hence, the end-point region is defined by the configuration,

\[
x_1 \sim x_3 \sim O(\bar{\Lambda}/M_B) , \quad x_2 \sim O(1) .
\]  

(5)

Additional collinear divergences and the double logarithm \( \alpha_s \ln^2 x_3 \) with the color factor \( C = N_c/2 \) occur, when the diagram involves only a single color flow. This result is the same as for Fig. 1(e).

We emphasize that the factorization formula for the nonfactorizable annihilation diagrams suffers an ambiguity in defining a light-cone \( B \) meson distribution amplitude. In the factorizable emission diagrams only the kinematics of the pion is relevant, whose momentum specifies a unique light-cone direction. The hard amplitudes then contain only the variable \( k_1^+ \) through the inner product \( k_1 \cdot P_3 \). An unambiguous light-cone \( B \) meson distribution amplitude \( \phi_B(x_1) \), depending on \( x_1 = k_1^+ / P_1^+ \), can be defined. For the nonfactorizable emission diagrams, the kaon kinematics is also relevant, and the \( k_1^- \) dependence appears in the hard amplitudes through \( k_1 \cdot P_2 \). Fortunately, \( k_1^- \) less essential than \( k_1^+ \) [17], is negligible, and \( \phi_B(x_1) \) can be defined. In the factorizable annihilation diagrams the \( B \) meson dynamics decouples. In the nonfactorizable annihilation diagrams, differing from the other topologies, the outgoing kaon and pion specify two different light-cone directions (one in the plus direction and another in the minus direction). Hence, both the components \( k_1^+ \) and \( k_1^- \) are important in the hard amplitudes. However, we argue that the current leading-power formalism lives with such an ambiguity present at the subleading level. For the discussion below, we simply assume that \( k_1 \) lies in the plus direction as indicated in Eq. (1).

Since a \( b \) quark is heavy, there is no soft cancellation between Figs. 1(g) and 1(h). However, the factorization formulas are overlap integrals of three meson distribution amplitudes, so that the end-point singularities are at most logarithmic. In Fig. 1(g), the internal \( b \) quark carries the momentum \( P_1 - k_1 - x_2 P_2 - \bar{x}_3 P_3 \). To have this internal quark on the mass shell, we demand

\[
x_1 \sim x_2 \sim \bar{x}_3 \sim O(\bar{\Lambda}/M_B) .
\]  

(6)

In this case the \( B \) meson momentum \( P_1 \) provides the large scale. The two diagrams, in which the radiative gluon emitted from the \( b \) quark attaches the \( s \) quark and the spectator quark in the pion, contribute constructively. The double logarithm appears as \( \alpha_s \ln^2 (x_2 + \bar{x}_3) \) with the color factor \( C = N_c/2 \). Because \( x_2 \) and \( \bar{x}_3 \) must be small simultaneously in order to have a large double logarithm, the phase space is restricted, and the resummation effect is expected to be negligible.

The virtual spectator quark in Fig. 1 (h) carries the momentum \( k_1 - x_2 P_2 - \bar{x}_3 P_3 \). A similar investigation shows that the dominant region for the corresponding factorization formula is the one with \( x_2 \sim O(\bar{\Lambda}/M_B) \) and \( \bar{x}_3 \sim O(1) \) at the leading twist. This is consistent with our assumption of considering only the plus component \( k_1^+ \), which is selected by the pion momentum \( P_3 \). Therefore, the end-point region for Fig. 1(h) is defined by

\[
x_1 \sim x_2 \sim O(\bar{\Lambda}/M_B) , \quad \bar{x}_3 \sim O(1) .
\]  

(7)

In this region the diagram with the radiative gluon attaching the internal spectator quark and the \( s \) quark generates the double logarithm \( \alpha_s \ln^2 x_2 \) with the color factor \( C = 1/(2N_c) \). The diagram with the radiative gluon attaching the internal spectator quark and the spectator quark in the pion produces only the single logarithm. Since the double logarithm has a smaller color factor \( 1/(2N_c) \), the resummation effect can be dropped. It is obvious from the above discussion that the double logarithms for the nonfactorizable diagrams are not universal.

### III. Numerical Analysis

It has been stated that the double logarithmic corrections to the nonfactorizable amplitudes are less essential. Below we shall proceed a quantitative study. As shown in [6], the threshold resummation of the double logarithms introduces a Sudakov factor \( S_\lambda(x) \) into the PQCD factorization formulas near the end points,

\[
S_\lambda(x) = \int_{a-i\infty}^{a+i\infty} \frac{dN J(N)}{2\pi i N} (1-x)^{-N} ,
\]  

(8)

where \( a \) is an arbitrary real constant larger than all the real parts of the poles involved in the integrand. The function \( J(N) \) is given, to the leading logarithm, by

\[
J(N) = \exp \left[ \frac{1}{2} \int_0^1 dz \frac{1 - z^{N-1}}{1 - z} \int_{(1-z)}^{(1-z)^2} d\lambda \gamma_B(\sqrt{\lambda M_B^2/2}) \right] ,
\]  

(9)
with the anomalous dimension $\gamma_K = C\alpha_s/\pi$. After deriving the Sudakov factors from the threshold resummation and from $k_T$ resummation for two-body nonleptonic $B$ meson decays, we perform the numerical analysis. For this purpose, we propose the parametrization for $S_t(x)$ [16,27],

$$S_t(x) = 2^{1+2c}\Gamma\left(\frac{3}{2} + c\right)\sqrt{\pi}\Gamma(1 + c)[x(1 - x)]^c,$$

where the parameter $c$ is determined by fitting the Mellin transformation of the above expression to $J(N)$ in Eq. (9). The fitted parameters are about $c = 0.27$ for the factorizable diagrams, $0.31$ for the nonfactorizable emission diagrams, and $0.03$ for the nonfactorizable annihilation diagram in Fig. 1(h). For Fig. 1(g), the Sudakov factor associated with the double logarithm $\alpha_s \ln^2(x_2 + x_3)$ can not be parametrized by Eq. (10). We simply consider Fig. 1(h) as an representative example to demonstrate the tiny resummation effect for the nonfactorizable annihilation diagrams.

The PQCD factorization formulas for the various topologies of decay amplitudes are referred to [23,28]. The double logarithms and their threshold resummation effects on the diagrams in Fig. 1 are summarized in Table I. It is observed that the Sudakov factor decreases the factorizable emission and annihilation amplitudes by at least 40%. The resummation effect can also be as large as 40% for the imaginary part of each nonfactorizable emission diagram, but reduces to about 20% for their sum. This confirms our argument that the threshold resummation is less important due to the soft cancellation at the end points. There is almost no effect on the nonfactorizable annihilation diagram as expected. Since the corresponding hard amplitudes oscillate between positive and negative values, the convolution with $S_t$ may not always decrease.

|   | double logs. | amplitudes without $S_t$ | amplitudes with $S_t$ |
|---|-------------|-------------------------|-----------------------|
| 1(a) | $-\alpha_s/4\pi C_F \ln^2 x_3$ | 1190.9 | 679.6 |
| 1(b) | $-\alpha_s/4\pi C_F \ln^2 x_1$ | 394.8 | 306.7 |
| 1(a)+1(b) | | 1585.8 | 986.4 |
| 1(c) | $-\alpha_s/4\pi C_F \ln^2 x_3$ | $-33.3 + 48.5 i$ | $-38.1 + 38.5 i$ |
| 1(d) | $-\alpha_s/4\pi C_F \ln^2 x_2$ | $51.4 - 51.2 i$ | $48.1 - 37.8 i$ |
| 1(c)+1(d) | | $18.1 - 2.7 i$ | $10.0 + 0.7 i$ |
| 1(e) | $-\alpha_s/4\pi (N_c/2) \ln^2 x_3$ | $-114.3 + 131.6 i$ | $-102.2 + 80.6 i$ |
| 1(f) | $-\alpha_s/4\pi (N_c/2) \ln^2 x_3$ | $128.2 - 161.3 i$ | $123.9 - 117.6 i$ |
| 1(e)+1(f) | | $13.9 - 29.7 i$ | $21.7 - 37.0 i$ |
| 1(g) | $-\alpha_s/4\pi (N_c/2) \ln^2 (x_2 + x_3)$ | $7.6 - 9.7 i$ | $-29.1 - 34.5 i$ |
| 1(h) | $-\alpha_s/4\pi [1/(2N_c)] \ln^2 x_2$ | $-29.8 - 33.4 i$ | $-29.1 - 34.5 i$ |

TABLE I. Numerical effects from the threshold resummation on the different amplitudes (in the unit of $10^{-4}$ GeV). Note that the associated Wilson coefficients have been set to unity.

5
Next we examine the factorization limit of the PQCD approach to two-body nonleptonic $B$ meson decays at large $M_B$. It is found that the factorizable emission amplitude decreases like $M_B^{-3/2}$ as displayed in Fig. 4(a), if the $B$ meson decay constant $f_B$ scales like $f_B \propto M_B^{-1/2}$. This power-law behavior is consistent with that obtained in [17,29]. Figure 4(b) exhibits the ratio of the magnitude of the leading-power nonfactorizable emission amplitude over the factorizable one as a function of $M_B$. The curve actually descends with $M_B$ despite of small oscillation. If parametrizing the ratio as

$$r = \frac{\text{|Nonfact.|}}{\text{Fact.}} \propto \frac{1}{\ln^\alpha(M_B/\Lambda)},$$  

the best fit to the curve gives the power $\alpha \sim 1.0$ for $\Lambda \sim 0.4$ GeV. We have confirmed this logarithmic decrease up to $M_B \approx 300$ GeV. It implies that the PQCD formalism approaches the factorization assumption logarithmically. Note that the argument in [28] based on the power behavior of the integrand, instead of the integral, just leads to the existence of the factorization limit, providing no information on how fast to arrive at this limit. We do not bother to examine the behavior of the annihilation amplitudes, since they are explicitly suppressed by $1/M_B$ compared to the factorizable emission.

Surprisingly, the behavior of the ratio $r$ with $M_B$ in PQCD is close to that in QCDF. However, the reasonings for achieving the same power counting are quite different. In the latter approach the factorizable contribution is assumed to be uncalculable in perturbation theory, and identified as being of $O(\alpha_s^0)$. The nonfactorizable contribution, being calculable, starts from $O(\alpha_s)$. Because of the soft cancellation at $x_3 \sim O(\Lambda/M_B)$, the nonfactorizable emission amplitude is dominated by the contribution from the region of $x_3 \sim O(1)$. In this region there is no further power suppression, and one has the ratio,

$$r_{\text{QCDF}} \sim \alpha_s(M_B) \propto \frac{1}{\ln(M_B/\Lambda_{\text{QCD}})}.$$  

In the former approach based on $k_T$ factorization theorem [30,31], both the factorizable and nonfactorizable contributions, being calculable, start from $O(\alpha_s)$. However, the Sudakov factor modifies the factorization formulas in the way that a pair of nonfactorizable diagrams exhibits a stronger cancellation as $M_B$ increases [28]. It turns out that the ratio $r$ also vanishes logarithmically as shown in Eq. (11).

IV. CONCLUSION

In this letter we have analyzed the double logarithmic corrections to two-body nonleptonic $B$ meson decays. The results are the same for charmless and charmful modes at the leading power. It has been observed that the double logarithms are universal for the factorizable emission and annihilation topologies. Their effect is crucial due to the presence of the potential linear end-point singularities. For the nonfactorizable emission diagrams, the logarithms carry a different (slightly larger) color factor. In the end-point region with a small parton momentum fraction, the nonfactorizable contributions cancel by pair, such that the end-point singularities are at most logarithmic. The threshold resummation effect is then expected to be minor. For the nonfactorizable annihilation diagrams, the factorization formulas involve the overlap integrals of three meson distribution amplitudes, and the singularities become also logarithmic. Furthermore, the double logarithms are either suppressed by phase space, or negligible with a tiny color factor. That is, the double logarithms for the nonfactorizable amplitudes depend on the topologies, and introduce only small effects in the current leading-power PQCD formalism.

We have also examined the heavy-quark limit of the PQCD approach to two-body nonleptonic $B$ meson decays numerically. The factorizable emission amplitude shows the desired power behavior, proportional to $M_B^{-3/2}$. PQCD reduces to the factorization assumption [24] as $M_B$ goes to infinity: the ratio of the nonfactorizable emission contribution over the factorizable one diminishes logarithmically. The annihilation contribution is explicitly power-suppressed. Our careful study justifies that the PQCD approach has a valid factorization limit, and the criticism in [17] is false. It is an interesting observation that the power counting rule for the above ratio, though originating from different reasonings, turns out to be identical in QCDF and in PQCD. With the gauge invariance proved in [15] and the correct heavy-quark limit demonstrated here, the PQCD approach stands as a solid theory.

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FIG. 1. Lowest-order diagrams which contribute to the $B \to K\pi$ decays, with (a) and (b) for the factorizable emission, (c) and (d) for factorizable annihilation, (e) and (f) for nonfactorizable emission, and (g) and (h) for nonfactorizable annihilation.
FIG. 2. $O(\alpha_s)$ corrections to Fig. 1(a).

FIG. 3. $O(\alpha_s)$ corrections to Fig. 1(e) which give rise to the double logarithms.
FIG. 4. (a) The factorizable emission amplitude as a function of $M_B$. (b) The ratio $r$ of the nonfactorizable emission amplitude over the factorizable one as a function of $M_B$. 