Hyperon forward spin polarizability $\gamma_0$ in baryon chiral perturbation theory

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We present the calculation of the hyperon forward spin polarizability $\gamma_0$ using manifestly Lorentz covariant baryon chiral perturbation theory including the intermediate contribution of the spin 3/2 states. As at the considered order the extraction of $\gamma_0$ is a pure prediction of chiral perturbation theory, the obtained values are a good test for this theory. After including explicitly the decuplet states, our SU(2) results have a very good agreement with the experimental data and we extend our framework to SU(3) to give predictions to the hyperons’ $\gamma_0$ values. Prominent are the $\Sigma^-$ and $\Xi^-$ baryons as their photon transition to the decuplet is forbidden in SU(3) symmetry and therefore they are not sensitive to the explicit inclusion of the decuplet in the theory.

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I Introduction

From the experimental study of Compton scattering on a baryon target one can extract relevant information about the inner structure of baryons. With the help of the sum rules for integral characteristics of the cross sections, very important observables like polarizabilities can be assessed. The focus of this work is the forward spin polarizability $\gamma_0$, which represents the deformation of a hadron relative to its spin axis when scattering photons in the extreme forward direction. It is related to the photo-absorption $\gamma N$ cross sections $\sigma_{3/2,1/2}$ with total helicities 3/2 (for parallel photon and target helicities) and 1/2 (for antiparallel photon and target helicities) via the sum rule in Ref. [1]

$$\gamma_0 = -\frac{1}{4\pi^2} \int_{\omega_0}^{\infty} d\omega \frac{\sigma_{3/2}(\omega) - \sigma_{1/2}(\omega)}{\omega^3},$$

originally found in Ref. [2]. The energy $\omega_0$ is the threshold for an associated neutral pion in the intermediate state. Experimental results for the proton $\gamma_0$ were obtained in Ref. [3] and, more recently, in Ref. [4]. Furthermore, dispersion relation studies have been performed in Ref. [5] for both nucleon spin polarizabilities.

Concerning the theoretical approach, the nucleon’s structure has been thoroughly studied with the help of effective field theories on Compton scattering data in Refs. [6], [7] and [8]. The spin-dependent piece of the amplitude $e^\mu M^\text{SD}_{\mu\nu} e^\nu$ attracted particular interest. The term proportional to $\omega^3$ ($\omega$ is the photon’s energy) contains the whole information about $\gamma_0$, via the master formula

$$\gamma_0[\vec{\sigma} \cdot (\vec{\epsilon} \times \vec{\epsilon}^*)] = -\frac{1}{4\pi} \frac{\partial}{\partial \omega^2} \frac{e^\mu M^\text{SD}_{\mu\nu} e^\nu}{\omega} \bigg|_{\omega=0},$$

as described in Refs. [9], [1] and [10]. Here $\vec{\sigma}$ is the vector of Pauli matrices, $\vec{\epsilon}$ and $\vec{\epsilon}^*$ are the polarizations of incoming and outgoing photons, respectively, $\alpha$ is the fine-structure constant and $e$ the elementary charge.

Early calculations in models of chiral perturbation theory (ChPT) that include only nucleonic intermediate states have been performed both in a heavy-baryon approach as well as in fully covariant calculations as in Ref. [11]. In Refs. [12] and [13] the theory was extended such as to include isospin-3/2 intermediate states, namely the $\Delta(1232)$ resonance. It was found that the inclusion of the latter state greatly improved the convergence between theory and empirical evidence.

When considering ChPT in SU(3) models, valuable predictions about the hyperons’ polarizabilities can be calculated, where there are no experimental data available yet. First results with the help of heavy-baryon ChPT were obtained in Ref. [10] and later improved in our work, Ref. [14]. The predictions are expected to be more reliable when using a fully covariant model and introducing the $\Delta(1232)$.

Along this line in this work we perform a calculation of the amplitude $\epsilon^\mu M^\text{SD}_{\mu\nu} \epsilon^\nu$ including corrections induced both by a fully covariant version and intermediate spin 3/2 states in a full extension to SU(3) flavor. The leading
ChPT order for the quantity $\gamma_0$ is a $p^3$ calculation. When including the $\Delta(1232)$ resonance, the study of Ref. [12] uses the so-called small scale expansion introduced in Refs. [15] and [16]. We opt for a different power counting scheme, following Ref. [17] and, furthermore, we introduce the couplings in a consistent dynamics, using the full $\Delta(1232)$ propagator as in Refs. [18], [19], [20] and [21]. A study comparing different effective field theoretical models has been performed in Ref. [22].

The outline of this paper is as follows. In Section [II] we give a theoretical introduction to the appropriate chiral Lagrangians and the power-counting used. The kinematical considerations and assumptions for the calculation of $\gamma_0$ are presented in Section [III] and the results are discussed in Section [IV]. Finally, we briefly summarize in Section [V].

II ChPT involving pseudoscalar mesons, baryons and photons

For the description of hyperon polarizabilities we use a manifestly Lorentz covariant SU(3) version of chiral perturbation theory involving pseudoscalar mesons, baryons and photons (see details in Refs. [23] and [24]). The lowest-order chiral Lagrangian involving pseudoscalar mesons $\phi$, baryons $B$ and photons $A_\mu$ reads

$$L = L^{(2)}_\phi + L^{(1)}_{\phi B},$$

where

$$L^{(2)}_\phi = \frac{F_0^2}{4} \text{Tr}(u_\mu u^\mu + \chi^+)$$

is the $O(p^2)$ meson Lagrangian and

$$L^{(1)}_{\phi B} = \text{Tr}\left(\bar{B}(iD - m)B\right) + \frac{D}{2} \text{Tr}\left(\bar{B}\gamma^\mu \gamma_5 \{u_\mu, B\}\right) + \frac{F}{2} \text{Tr}\left(\bar{B}\gamma^\mu \gamma_5 [u_\mu, B]\right)$$

is the $O(p^1)$ Lagrangian including baryons. The symbols $[ ]$ and $\{ \}$ occurring in Eq. 3 and in the following denote the commutator and anticommutator in flavour space, respectively. The vielbein $u_\mu$ is given by $i\{u^2, \nabla_{\mu} u\}$, with $u^2 = U = \exp\left(\frac{\chi}{F_0}\right)$, where $\nabla_{\mu} u = \partial_{\mu} u - i(v_\mu + a_\mu)u + iu(v_\mu - a_\mu)$ and $D_{\mu} B = \partial_{\mu} B + [\Gamma_{\mu}, B]$ are the covariant derivatives acting on meson and baryon octet fields, respectively, and $m$ denotes the baryon octet mass in the chiral limit. The chiral connection is given by $\Gamma_{\mu} = \frac{1}{2}[u^\dagger, \partial_{\mu} u] - \frac{1}{2}u^\dagger (v_\mu + a_\mu)u - \frac{1}{2}u(v_\mu - a_\mu)u^\dagger$. Since we are working with photon fields, we set $v_\mu$ to $\epsilon_{\mu} Q$ and $a_\mu$ to 0. The constant $F_0$ is the meson decay constant in the chiral limit and the low-energy constants $D$ and $F$ are determined from hyperon $\beta$ decays, where the combination $F + D$ corresponds to the low-energy constant $g_A$ in the SU(2) limit. The explicit form of the $3 \times 3$ charge matrix $Q$, meson $\phi$ and baryon $B$ matrices is given in Appendix A. The term $\text{Tr}(\chi^+)$ is responsible for the explicit breaking of the chiral symmetry due to the finite quark masses

$$\text{Tr}(\chi^+) = \text{Tr}\left(\chi U^\dagger + U^{\dagger} \chi\right),$$

where in our case $\chi = M^2$ and $M$ is the meson mass. The power-counting scheme followed here gives the order

$$N = 4N_L + \sum_{d=1}^{\infty} dN_d - 2P_\phi - P_B$$

(7)

to a diagram, where $N_L$ stands for the number of loops, $N_d$ for the number of vertices from Lagrangians of order $d$, and $P_\phi$ and $P_B$ for the number of meson and baryon propagators, respectively (see also Ref. [25]).

In this work we also include isospin-3/2 resonances, which give significant corrections to the full amplitude. The relevant terms of the Lagrangians that couple these decuplet fields to the octets of baryons and mesons are partly given in Refs. [26], [27], [28], where the kinetic term reads

$$L^{(1)}_\Delta = \delta_{\alpha \beta \gamma} \mu \nu \sigma \tau \nabla_{\alpha} \nabla_{\beta} \Delta_{\gamma \nu} \Delta_{\epsilon \mu \tau}.$$ (8)

We added the missing couplings by extending the known vertices from SU(2) (see Refs. [29] and [21]) to SU(3):

$$L^{(1)}_{\Delta_{\phi B}} = -\frac{i\sqrt{2}C}{F_0 M_\Delta} B^{ab} \epsilon^{cda} \gamma_{\mu \nu \lambda} (\partial_{\lambda} \Delta_{\mu}) \delta_{bc} \text{Tr}(D_\chi \phi)^{ce} + \text{H.c.},$$

$$L^{(2)}_{\Delta B} = -\frac{3i e g_M}{\sqrt{2}m(m + M_\Delta)} B^{ab} \epsilon^{cda} Q^{ce} (\partial_{\lambda} \Delta_{\mu}) \delta_{bc} F^{\mu \nu} + \text{H.c.},$$

where $C = \frac{1}{4} M_\Delta f_0 F_0$ and $Q = \frac{1}{2} Q(0)$ in the SU(3) limit.
where $\Delta^{ijk}$ are the decuplet states (see details in Appendix A). $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} \partial_\alpha A_\beta$ is the self-dual stress tensor of the electromagnetic field, and the Dirac tensors $\gamma^{\mu\nu}$ and $\gamma^{\mu\nu\lambda}$ are specified in Appendix A.

The couplings $C$ and $g_M$ are low-energy constants and $M_\Delta$ corresponds to the decuplet mass in the chiral limit. Note that the constant $C$ corresponds to the low-energy constant $h_A$ of SU(2) in Refs. [29] and [21] by the conversion $C = -\frac{h_A}{2\sqrt{2}}$. This definition of $h_A$ differs by a factor 2 from the definition found in Ref. [12]. This low-energy constant is extracted from the strong decay of the decuplet into the baryon octet and has been determined to be $h_A = 2.85$ in Ref. [30] for SU(2) and $C = -0.85$ in Ref. [28] for SU(3). It is also important to mention that the numerical value for the coupling constant $g_M$ has not yet been studied when extending the model to SU(3). Therefore the quality of the predictions very much depends on its correct value. We follow the method of Ref. [12] and estimate the value of $g_M$ by calculating the width of the electromagnetic decay of the $\Delta(1232)$:

$$\Gamma^{\text{EM}}_\Delta = -2\text{Im}(\Sigma^{\text{EM}}_\Delta) = -\frac{g^{2}M_\Delta}{4M^4_\Delta m^2} (M_\Delta - m)^3 (M_\Delta + m)^3,$$

(11)

where $\text{Im}(\Sigma^{\text{EM}}_\Delta)$ is the imaginary part of the electromagnetic $\Delta(1232)$ self-energy amplitude. Therefore, using the relation $\Gamma^{\text{EM}}_\Delta/(\Gamma^{\text{EM}}_\Delta + \Gamma^{\text{Strong}}_\Delta) = 0.55\%\ldots0.65\%$ and the strong decay width $\Gamma^{\text{Strong}}_\Delta = (118 \pm 2)\text{MeV}$, we get the value $g_M = 3.16 \pm 0.16$. Since data on the electromagnetic decays of the full decuplet are sparse and contain large errors, a determination of $g_M$ in the SU(3) version is not viable. We therefore also fix $g_M$ in SU(3) to the $\Delta \rightarrow \gamma N$ decay, i.e. we also use the value of $g_M = 3.16 \pm 0.16$ here. We should keep in mind that the central value of $g_M$ will change when going from SU(2) to SU(3), but we expect that with the present sizable error on $g_M$ this value is included.

For the covariant derivative we use

$$(D_\lambda \phi)^{ab} = \frac{1}{\sqrt{2}} \left( \partial_\lambda \phi^{ab} - i e A_\lambda(Q, \phi)^{ab} \right).$$

(12)

The factor $\frac{1}{\sqrt{2}}$ comes from the definition of the meson-octet matrix. When introducing the decuplet fields into the chiral theory, an additional small parameter of the ChPT expansion appears, $\delta = M_\Delta - m \sim 300 \text{MeV}$. Therefore, the counting scheme has to be revised. Here we follow the $\delta$ counting scheme from Ref. [17], where $\delta^2$ is counted as $O(p)$. It is adequate for the low-energy range close to pion-production threshold. Hence, one obtains the rule

$$N = 4N_L + \sum_{d=1}^\infty dN_d - 2P_\phi - P_B - \frac{1}{2} P_\Delta,$$

(13)

where now $P_\Delta$ is the number of $\Delta$ propagators. Since the $\Delta(1232)$ is a spin-3/2 resonance, it does not have the normal Dirac-propagator form for spin-1/2 particles, but takes the Rarita-Schwinger form given by

$$S_\Delta^{\alpha\beta}(p) = \left\{ \begin{array}{ll} \frac{\not{p} + M_\Delta}{p^2 - M_\Delta^2 + i\epsilon} & -g^{\alpha\beta} + \frac{1}{d-1} \gamma^{\alpha} \gamma^{\beta} \\ + \frac{1}{(d-1)M_\Delta} (\gamma^{\alpha} p^\beta - \gamma^{\beta} p^\alpha) + \frac{d-2}{(d-1)M_\Delta^2} \gamma^{\alpha} \gamma^{\beta} \end{array} \right\},$$

(14)

where $d$ is the number of dimensions of the Minkowski space, which after dimensional regularization is set to $d = 4$.

The calculations in this work are done up to order $p^3$ for the isospin-1/2 counting scheme, which is the leading-order calculation for the $\gamma_0$ observable, and up to order $p^3/2$ in the isospin-3/2 counting scheme. This choice is due to the fact that in the isospin-1/2 sector, the first contributions appear at loop level, which corresponds to the order $p^3$ if all the coupling vertices are extracted from the lowest-order Lagrangian. Instead of going to higher-order couplings and therefore obtaining loops of order $p^4$, we included the isospin-3/2 sector, which due to its lower order — the leading-order diagrams are at order $p^{7/2}$ — is expected to dominate over those contributions. In addition, there is the advantage that fewer additional low-energy constants are needed than for the case of higher orders. All the constants’ values chosen for this work are given in Table I. In order to obtain the SU(2) results, we simply put all the channels with non-vanishing strangeness to 0 and keep only those channels involving nucleons, pions and the isospin-3/2 quadruplet.

### III The forward spin polarizability $\gamma_0$

The diagrams contributing to the forward spin polarizability $\gamma_0$ are shown in Figs. 1 and 2. The calculation of the amplitudes corresponding to each of these diagrams is performed in the rest frame of the baryon. To calculate the
Table I: Numerical values for the hadron masses and decay constants used in the $\gamma_0$ calculations. All the dimensionfull values are given in units of MeV. The physical choice values for SU(2) were taken as in Ref. [12] — notice the difference by a factor 2 in the Lagrangian definitions of $h_A$ —, whereas for the chiral limit and SU(3) we followed Ref. [28]. The value for the coupling $g_M$ is calculated through Eq. (11).

|          | $m$ | $M_\Delta$ | $M_\pi$ | $M_K$ | $M_\eta$ | $F_0$ | $g_A$ | $D$ | $F$ | $C$ | $h_A$ | $g_M$ |
|----------|-----|------------|---------|-------|----------|-------|-------|-----|-----|-----|-------|-------|
| SU(2) chiral limit choice | 880 | 1152       | 140     | --   | --       | --    | 87     | 1.27| --  | --  | 2.85  | 3.16  |
| physical choice | 938.9 | 1232       | 138.04  | --   | --       | --    | 92.21  | 1.27| --  | --  | 2.85  | 3.16  |
| SU(3) chiral limit choice | 880 | 1152       | 140     | 496  | 547      | 87    | --     | 0.623| 0.441| $-D$| --    | 3.16  |
| physical choice | 1149 | 1381       | 140     | 496  | 547      | 108   | --     | 0.8 | 0.47| --  | 0.85  | 3.16  |

forward spin polarizability one needs to assume conservation of the photon energy $\omega = \omega'$ — incoming and outgoing photons have the same momenta $\vec{q} = \vec{q}'$. In the following, the Minkowski-space vectors used are:

\[
p^\mu = p'^\mu = (m, 0, 0, 0) \quad \epsilon^\mu = (0, \vec{\epsilon}) \quad \epsilon'^\mu = (0, \vec{\epsilon}') \quad q^\mu = q'^\mu = (\omega, \vec{q}),
\]

where $q$ and $q'$ are the 4-momenta of the incoming and outgoing photons, $\epsilon$ and $\epsilon'$ their respective polarizations, while $p$ and $p'$ are the momenta of the incoming and outgoing baryons, respectively. We work in the Weyl gauge, which leads to the condition $p \cdot \epsilon = 0$.

All terms containing the expression $\vec{\epsilon}' \cdot \vec{\epsilon}$ contribute to $\gamma_0$, as can be seen when comparing Eq. (2) with

\[
\vec{\epsilon}' \cdot \vec{\epsilon} = i \vec{\sigma}(\vec{\epsilon} \times \vec{\epsilon}') - (\vec{\epsilon} \cdot \vec{\epsilon}').
\]

Terms like $\vec{\epsilon}' \cdot q \vec{\epsilon}$ yield a contribution of $-i\omega \vec{\sigma}(\vec{\epsilon} \times \vec{\epsilon}')$ when projected onto the baryon states. All the other expressions that arise can be reduced to this simple case.

While the full set of diagrams in the spin-1/2 sector is gauge invariant, special care has to be taken when including the spin-3/2 states. The diagrams which include a minimal coupling of the photon to the $\Delta(1232)$ would need terms of higher order to fully restore gauge invariance. In order to be fully gauge invariant the Lagrangian of Eq. (9) should include the covariant derivative $D_\mu \Delta_\nu$ as opposed to the partial derivative $\partial_\mu \Delta_\nu$ only. The difference between the two derivatives are higher-order terms. To solve this discrepancy, we follow the solution of Ref. [6], where this problem has already been addressed for the case of the proton. In fact, for the neutral octet baryons the diagrams of Fig. 2 are fully gauge invariant. As for the charged octet baryons, this is only the case for the diagrams with charged mesons. Therefore, for these baryons, the strategy is to study two sets of diagrams separately: on the one hand, we have the one-particle-irreducible diagrams of Figs. 2(b), 2(h) and 2(i), which are calculated summing over all isospin channels; on the other hand, the missing one-particle-reducible loop diagrams of Figs. 2(c) to 2(g) are first calculated only for the charged meson channels. For the other channels the isospin factor is chosen such that the ratio between the isospins of the one-particle-reducible and one-particle-irreducible diagrams is the same as for the charged meson channels. When doing so, gauge invariance is insured and the restoration procedure involves higher-order terms.

IV Results and discussion

The numerical results for our calculations, when including nucleons, pions and $\Delta$ resonances only (hadrons with no strangeness) are given in Table I, where a comparison with the numerical values found by other groups is also given. Our calculation for the isospin-1/2 sector completely agrees with the results of Ref. [12]. For completion, we also included how the $\gamma_0$ values vary when taking the chiral limit, where the masses were set to the best-fit chiral masses. We compare our results with the HBCChPT results from Refs. [11, 13]. The discrepancy between the results does not lie in the parameter choice but in the heavy-mass expansion one assumes for HBCChPT.

As for the isospin-3/2 sector, our results differ from those of Ref. [12]. The reason for this is that we use a different counting scheme, and therefore a different set of diagrams. We also have a different Lagrangian, which directly sorts out the spurious spin-1/2 contributions of the Rarita-Schwinger spin-3/2 spinor. In Ref. [8] the $\Delta(1232)$ was introduced in the same way as in the present work. One should remark that there a tree-level diagram of order $p^{9/2}$ was included, which we left out here for consistency. Without this diagram, the numerical results in Ref. [8] are
Figure 1: Diagrams contributing to $\gamma_0$ with isospin-1/2 intermediate states. The crossed diagrams are obtained by the substitutions $\omega \leftrightarrow -\omega$ and $\not\epsilon \leftrightarrow \not\epsilon^*$. In perfect agreement with ours. The decomposition of our results for the nucleon polarizabilities of Table II into their individual parts is listed in Table III. The main correction to the polarizability results comes from the tree-level diagrams with virtual spin-3/2 baryons, while their loop diagrams give only a small contribution.

We extended the calculations to the SU(3) sector, again distinguishing between the case where the isospin-3/2 resonances were included and where only octet baryons were taken into account as intermediate states. Here we also considered both cases: when taking the physical average values and when choosing the chiral limit, see Table II. We also obtain predictions for the hyperon forward spin polarizabilities. A full listing of the results for the octet baryons is given in Table IV and the decomposition of the results for the nucleons in Table III. When extending the model from SU(2) to SU(3), one takes into account additional virtual states and different values for the parameters, whose impact we discuss in more detail below. Another interesting feature in SU(3) is also the appearance of the SU(3)-flavour forbidden photon transitions of the negatively charged octet baryons to those of the decuplet. The results of Table IV are also compared to HBChPT results (for preliminary results see Ref. [10] and for a complete and improved analysis see Ref. [14]). For HBChPT the nucleon values for $\gamma_0$ change only slightly when going from SU(2) to SU(3), the results remain large and positive. When changing to the covariant version (without the decuplet contribution) the SU(3) case leads to a reduction of the $\gamma_0$ results which still are positive. An additional inclusion of
Figure 2: Diagrams contributing to $\gamma_0$ with isospin-3/2 intermediate states. The crossed diagrams are obtained by the substitutions $\omega \leftrightarrow -\omega$ and $\epsilon \leftrightarrow \epsilon^*$. Except for the tree diagram, which has vertices of a second-order Lagrangian, all the vertices that appear are couplings of lowest-order Lagrangians.

| Model                      | proton          | neutron         |
|----------------------------|-----------------|-----------------|
|                            | this work [12]  | [11] [13] [8]   |
| without $\Delta$           |                 |                 |
| HBChPT covariant chiral limit | 2.15            | 3.24            |
| covariant physical values  | 2.07 2.07       | 3.06 3.06       |
| with $\Delta$              |                 |                 |
| HBChPT covariant chiral limit | -1.59(38)      | -0.59(38)       |
| covariant physical values  | -0.76(28) -1.74 | -1.00 0.15(28) |
| experiment [5]             | -1.01 ± 0.08(stat) ± 0.10(syst) | -0.38          |
| dispersion relations [5]   |                 |                 |
|                            | -1.34           |                 |

Table II: Numerical values for $\gamma_0$ obtained in the SU(2) sector in present and in other works in units of $10^{-4}$ fm$^4$. The choice of the numerical values for the constants for our own results can be found in Table I. The error in our results when including the $\Delta(1232)$ resonance arises from the uncertainty in the value of the low-energy constant $g_M$.

the decuplet leads in both SU(2) and SU(3) cases to negative $\gamma_0$ values closer to the empirical value for the proton of $(-1.01 \pm 0.08\text{(stat)} \pm 0.10\text{(syst)}) \cdot 10^{-4}\text{fm}^4$ presented in Ref. [3].

It is also interesting to compare the $\gamma_0$ results for the nucleons to those from dispersion relation studies found in Ref. [5] to be $\gamma_0^p = -1.34 \cdot 10^{-4}\text{fm}^4$ and $\gamma_0^n = -0.38 \cdot 10^{-4}\text{fm}^4$. The inclusion of isospin-3/2 states, while already having an important effect in HBChPT, leads to an even better agreement with the empirical values in the case of fully covariant calculations, both when taking the chiral limit as well as when taking the average of the physical values for the constants. In fact, the difference between these two parameter sets is of higher chiral order for the polarizabilities. The main source of uncertainty of our results is the constant $g_M$, whose variation leads to an error estimate as shown in Table IV. We would like to stress that the results obtained here are not subject to uncertainties related to renormalization schemes; for the considered order there are no divergences or power-counting breaking terms entering into the value of $\gamma_0$.

As already discussed above, the inclusion of virtual decuplet states is crucial for an agreement of the nucleon B$\chi$PT polarizabilities with phenomenological values, which is dominantly because of the tree-level diagrams, as shown in Table [II]. However, this is not the case for the $\Sigma^-$ and $\Xi^-$ baryons since the photon transitions to the corresponding decuplet states $\Sigma^{++}$ and $\Xi^{++}$ are forbidden in SU(3) symmetry, and the tree-level diagrams do not appear. Hence,
Table III: Decomposition of the proton and neutron polarizability results in units of $10^{-4}$ fm$^4$ into the contributions coming from the different sets of diagrams, when using the chiral limit for the masses and low-energy constants. The difference in results when using physical values or the chiral limit can be seen as systematical uncertainty.

| Model | used values | $p$ | $n$ | $\Sigma^+$ | $\Sigma^-$ | $\Sigma^0$ | $\Lambda$ | $\Xi^-$ | $\Xi^0$ |
|-------|-------------|-----|-----|-----------|-----------|----------|--------|--------|--------|
| without decuplet, HBChPT | 10, 14 | 4.69 | 4.53 | 2.77 | 2.54 | 2.44 | 2.62 | 0.52 | 0.68 |
| without decuplet, covariant | chiral limit | 1.53 | 2.28 | 0.90 | 0.89 | 1.60 | 1.09 | 0.08 | 0.15 |
| | physical values | 1.68 | 2.33 | 0.93 | 0.91 | 1.32 | 1.28 | 0.15 | 0.25 |
| with decuplet, covariant | chiral limit | -2.14(38) | -1.43(33) | -2.72(33) | 0.89 | 0.67(9) | -1.69(28) | 0.07 | -3.51(38) |
| | physical values | -1.64(33) | -1.03(33) | -2.30(33) | 0.90 | 0.47(8) | -1.25(25) | 0.13 | -3.02(33) |

Table IV: Numerical values for $\gamma_0$ obtained in our calculations, in units of $10^{-4}$ fm$^4$ in the SU(3) sector. The choice of the numerical values for the constants in the covariant case can be found in Table I, both for the chiral limit and for the physical average case. As for the HBChPT limit, we cite the results in Ref. 10, which were later corrected in our work in Ref. 14. The errors in the results with the decuplet arise from the uncertainty in the value of the low-energy constant $g_{M}$.

their values for the polarizabilities change only slightly by the small loop contributions with virtual decuplet baryons. To study the polarizabilities in B$\chi$PT, these two baryons might therefore be better suited than the proton and neutron since nearly all of the uncertainties coming from the inclusion of the decuplet drop out. Experimentally, it will be very hard to measure their polarizabilities, but it is feasible in lattice QCD. Furthermore, we want to emphasize that the main differences in numerical values for the proton and neutron polarizabilities in SU(2) and SU(3) come from the choice of the parameters in Table I. All contributions coming from $K$ or $\eta$ mesons are negligible. Choosing the masses and constants in SU(3) as in SU(2), which are equivalent parameter sets in terms of chiral counting up to the order $p^3$, will give nearly the same results.

The addition of $p^4$ contributions would be the next step to further refine the calculation.

V Summary

We have presented an extended calculation for the spin polarizability $\gamma_0$ of the baryon octet. The framework we choose is based on manifestly Lorentz covariant baryon chiral perturbation theory, both in the SU(2) as well as SU(3) versions. Furthermore, we explicitly include intermediate spin 3/2 states. The novel results of the present work concern both the SU(3) extension of fully covariant ChPT to the order $p^3$ and the inclusion of the spin 3/2 decuplet up to order $p^{7/2}$. Empirical results exist only for the nucleon case, where in both versions the inclusion of explicit decuplet states is crucial to find an agreement between phenomenology and B$\chi$PT. In particular, it is the tree-level diagram with virtual decuplet baryons that gives the dominant extra contribution. This also carries over to the SU(3) case, where all contributions from $K$ and $\eta$ loops turn out to be negligible. However, in SU(3) the two baryons $\Sigma^-$ and $\Xi^-$ are prominent as their photon transitions to the corresponding decuplet states are forbidden in SU(3) symmetry. As a result, the decuplet tree-level contributions are not present and their polarizabilities in pure B$\chi$PT remain nearly unchanged, meaning that also most of the uncertainties connected to the decuplet inclusion drop out. Since experimental polarizability measurements for these baryons are unprobable, comparisons to results from lattice QCD would be very interesting, as polarizabilities to chiral order $p^3$ are leading order predictions of B$\chi$PT. The $\gamma_0$ results for the hyperons, especially the ones for $\Sigma^-$ and $\Xi^-$, can therefore serve as a benchmark for other calculations.
in this sector. At this point it seems a necessity to extend the present calculation to the cases of the electric and magnetic polarizabilities $\alpha_E$ and $\beta_M$ of the nucleon and the baryon octet, especially the $\Sigma^-$ and $\Xi^-$, where probably a similar situation as above occurs with respect to the inclusion of decuplet states.
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Appendices

A Basic notations of ChPT

The octet matrices of pseudoscalar mesons $\phi$, photons $Q$ and baryons $B$ are given by

$$\phi = \sum_{a=1}^{8} \lambda_a \phi^a = \sqrt{2} \left( \begin{array}{ccc} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^- & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta \end{array} \right), \quad (A1)$$

$$Q = \frac{1}{2} \left( \lambda_3 + \frac{\lambda_8}{\sqrt{3}} \right) = \left( \begin{array}{ccc} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{array} \right), \quad (A2)$$

and

$$B = \frac{1}{\sqrt{3}} \sum_{a=1}^{8} \lambda_a B^a = \left( \begin{array}{ccc} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\ -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^- & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{array} \right). \quad (A3)$$

The decuplet states $\Delta^{ijk}$ are specified as

$$\Delta^{111} = \Delta^{++}, \quad \Delta^{112} = \frac{1}{\sqrt{3}} \Delta^+, \quad \Delta^{122} = \frac{1}{\sqrt{3}} \Delta^0, \quad \Delta^{222} = \Delta^-, \quad (A4)$$

$$\Delta^{113} = \frac{1}{\sqrt{3}} \Sigma^{*+}, \quad \Delta^{123} = \frac{1}{\sqrt{6}} \Sigma^{*0}, \quad \Delta^{233} = \frac{1}{\sqrt{3}} \Sigma^{*-}, \quad (A4)$$

$$\Delta^{133} = \frac{1}{\sqrt{3}} \Xi^{*0}, \quad \Delta^{233} = \frac{1}{\sqrt{6}} \Xi^{*-}, \quad \Delta^{333} = \Omega^-.$$  

The Dirac tensors $\gamma^{\mu\nu}$ are defined as

$$\gamma^{\mu\nu} = \frac{1}{2} \left[ \gamma^\mu, \gamma^\nu \right] \quad \text{and} \quad \gamma^{\mu\nu\lambda} = \frac{1}{4} \left\{ \left[ \gamma^\mu, \gamma^\nu \right], \gamma^\lambda \right\}. \quad (A5)$$

B Loop integrals and dimensional regularization

The integrals for Figs. 1 and 2 are taken in $d$ dimensions and later dimensionally regularized to the normal 4-dimensional Minkowski space. To calculate the crossed diagrams, the simple substitutions

$$\omega \leftrightarrow -\omega \quad \text{and} \quad \epsilon^* \leftrightarrow \epsilon \quad (B1)$$
have to be performed. To obtain the final numerical results for the forward spin polarizability, the integrands of the structure constants of the $\ell^4$-terms are expanded up to the order $\omega^3$. The coefficients of the third order of the expansion are then used to evaluate the integrals.

The following loop integrals are of interest in this work:

\[ \int \frac{d^dz}{(2\pi)^d} \frac{1}{(2^2 - \Delta)^n} = (-1)^n i \frac{\Gamma \left( n - \frac{d}{2} \right) \Gamma \left( \frac{d}{2} \right)}{(4\pi)^{d/2} \Gamma(n) \Delta^{n-\frac{d}{2}}} \]

As a result, one has to dimensionally regularize the integrals, obtaining the expressions

\[ \lambda_1(\Delta) = \frac{\Gamma \left( 1 - \frac{d}{2} \right)}{(4\pi)^{d/2} \Delta^{1-\frac{d}{2}}} = -\frac{\Delta}{16\pi^2} \left( \frac{2}{\epsilon} - \log \left( \frac{\Delta}{\mu} \right) + \log(4\pi) - \gamma_E + 1 + \mathcal{O}(\epsilon) \right) \]

\[ \lambda_2(\Delta) = \frac{\Gamma \left( 2 - \frac{d}{2} \right)}{(4\pi)^{d/2} \Delta^{2-\frac{d}{2}}} = \frac{1}{16\pi^2} \left( \frac{2}{\epsilon} - \log \left( \frac{\Delta}{\mu} \right) + \log(4\pi) - \gamma_E + \mathcal{O}(\epsilon) \right) \]

\[ \lambda_3(\Delta) = \frac{\Gamma \left( 3 - \frac{d}{2} \right)}{(4\pi)^{d/2} \Delta^{3-\frac{d}{2}}} = \frac{1}{16\pi^2} \Delta \]

\[ \lambda_4(\Delta) = \frac{\Gamma \left( 4 - \frac{d}{2} \right)}{(4\pi)^{d/2} \Delta^{4-\frac{d}{2}}} = \frac{1}{16\pi^2} \Delta^2 \]

where $\epsilon = 4 - d$ and $\mu$ is the scale parameter set to the proton mass in this work. For regularization, the minimal subtraction (MS) would have to be performed in the EOMS scheme, where terms proportional to

\[ \frac{2}{\epsilon} + \log(4\pi) - \gamma_E + 1 \]

are subtracted. It is interesting to note that in this work no diagram had to be renormalized, as at order $p^{7/2}$ no divergent or power counting breaking terms contribute to $\gamma_0$.

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