Dynamic Renormalization Group and Noise Induced Transitions in a Reaction Diffusion Model.

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We investigate how additive weak noise (correlated as well as uncorrelated) modifies the parameters of the Gray-Scott (GS) reaction diffusion system by performing numerical simulations and applying a Renormalization Group (RG) analysis in the neighborhood of the spatial scale where biochemical reactions take place. One can obtain the same sequence of spatial-temporal patterns by means of two equivalent routes: (i) by increasing only the noise intensity and keeping all other model parameters fixed, or (ii) keeping the noise fixed, and adjusting certain model parameters to their running scale-dependent values as predicted by the RG. This explicit demonstration validates the dynamic RG transformation for finite scales in a two-dimensional stochastic model and provides further physical insight into the coarse-graining analysis proposed by this scheme. Through several study cases we explore the role of noise and its temporal correlation in self-organization and propose a way to drive the system into a new desired state in a controlled way.

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I. INTRODUCTION.

The dynamics of certain biological systems frequently follows some self-organization process where the development of new, complex structures takes place primarily in and through the system itself. This self-organization is normally triggered by internal variation processes, which are usually called "fluctuations" or "noise", that have a positive influence on the system. For instance, recent theoretical studies and experiments with cultured glial cells and the Belousov-Zhabotinsky reaction have shown that noise may play a constructive role on the dynamical behavior of spatially extended systems. Therefore, one can not ignore the role of noise in chemical and biological self-organization and its relationship with the environmental selection of emergent patterns.

In these systems there is a strong interplay between the equations of the microscopic dynamics and the environmental fluctuations acting on the smallest scales. In order to get a qualitative idea of the system behavior one can average out the influence of noise on the shortest scales by increasing, infinitesimally, the scale of observation. This can be done applying dynamic Renormalization Group (RG) techniques. One obtains then the so-called RG flow equations which are generally used to study the RG flow of the system parameters and to find the fixed points in the long wavelength limit (at the fixed points the system is invariant under the scaling transformation). But our interest in this paper is in the finite-size renormalization (at finite wavelengths or finite scales), i.e. on the influence of noise at the scale where the chemical or biological reactions take place.

Our final aim is to validate the RG procedure as a general tool which can be used at the scales where these biochemical reactions take place and predict the transitions induced by noise with different correlation properties. As an example we will investigate the influence of noise on a reaction diffusion system in the vicinity of a transition point. Performing numerical simulations and applying the RG analysis at finite scale ranges we can get a better understanding of how weak noise modifies the parameters of this dynamical system and the patterns it converges to.

We consider the Gray-Scott (GS) reaction diffusion model, which is one of the simplest models of biochemical relevance leading to spatial and temporal patterns when diffusion is included. Numerical simulations of this system in the deterministic case have revealed a surprisingly large set of complex and irregular patterns as a function of the system parameters. In addition we want to include the influence of the fluctuating environment. Fluctuations of internal (thermal) origin represent microscopic degrees of freedom, and since they evolve in spatial and temporal scales much shorter than those of the gross variables of the system they are assumed to be additive and uncorrelated in space and time. When the origin of the noise is external there may exist a coupling between the system and the fluctuations.

In this case there is no difference between the time and length scales of the noise and the field, and this
noise could be multiplicative and have some structure in space or time. We study the following stochastic system with additive noise:

\[
\begin{align*}
\frac{\partial V}{\partial t} &= \lambda U V^2 - \mu V + D_v \nabla^2 V + \eta_v(x, y, t) \\
\frac{\partial U}{\partial t} &= u_0 - \lambda U V^2 - \nu U + D_u \nabla^2 U + \eta_u(x, y, t)
\end{align*}
\]

where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \). We will consider Gaussian white zero-mean noises (uncorrelated) \(< \eta_{u,v}(x, y, t)\eta_{u,v}(x', y', t') >= 2A_{u,v} \delta(x-x')\delta(y-y')\delta(t-t') \) and Ornstein-Uhlenbeck (OU) correlated noise \(< \eta_{u,v}(x, y, t)\eta_{u,v}(x', y', t') >= \hat{A}_{u,v} \exp(-\frac{t-t'}{\tau}) \delta(x-x')\delta(y-y') \) (the Gaussian white noise case limit \(2A_{u,v}\delta(t-t')\) is obtained when the correlation time \(\tau \to 0\) while keeping \(A_{u,v} = \hat{A}_{u,v}\tau\) constant). In the absence of noise, Eq. \(1\) defines the Gray-Scott model, which is a variant of the autocatalytic Selkov model of glycolisis, corresponding to the following chemical reactions:

\[
\begin{align*}
U &\xrightarrow{2V} 3V \\
V &\xrightarrow{P} P \\
U &\xrightarrow{Q} Q \\
U &\xrightarrow{u_0} U.
\end{align*}
\]

\(V(x, y, t)\) and \(U(x, y, t)\) represent the concentrations of the chemical species \(U\) and \(V\), and are functions of the two-dimensional space and time variables. \(\lambda\) is the constant rate of the reactions, \(P\) and \(Q\) are inert products, \(\mu\) is the decay rate of \(V\) and \(\nu\) the decay rate of \(U\). The equilibrium concentration of \(U\) is \(u_0/\nu\), where \(u_0\) is the feed rate constant. The chemical species \(U\) and \(V\) can diffuse with independent diffusion constants \(D_u\) and \(D_v\). All the model parameters, including the noise amplitudes \(A_{u,v}\), are positive.

In the case of Gaussian white noise the stochastic system described in Eq. \(1\) has been numerically integrated for a particular set of parameters \(9\). The pattern to which the system converged changed drastically with small changes in the noise intensity, suggesting that this system can suffer noise induced transitions when subjected to random fluctuations \(10\).

Additionally, one can apply the RG transformation to the Gray-Scott model subject to additive random fluctuations \(11\). The dynamic RG procedure consists of two steps: an elimination or thinning out of the fast or short wave-length modes (\(\frac{1}{s} < |\vec{k}| < \Lambda\) where \(s = e^t \geq 1\) is the scale in wavenumber space above which the fluctuations have been integrated over, \(\Lambda = \frac{4\pi}{L}\) and \(L\) the minimal length where the equations under consideration are valid) followed by a rescaling of the remaining modes (\(\vec{k} \to s\vec{k}\) \(\vec{x}\)). Applying this infinitesimal transformation one gets the effective (i.e. renormalized) parameters of the system when going through an infinitesimal scale change. This analysis leads one to consider a set of coupled differential equations governing the RG flow in parameter space when this system is influenced by noise. At one-loop order in the noise amplitude, only two out of the total of eight model parameters run with scale, namely, the decay rate \(\nu\), of the \(U\) field and the nonlinear coupling \(\lambda\), and the corrections are driven by the \(v\)-noise (not the \(u\)-noise). In particular for the case of white Gaussian noise, in a two-dimensional space, the pertinent equations read \(11\)

\[
\begin{align*}
\frac{dv}{dl} &= zv + \lambda A_v K_2 \Lambda^2 \\
\frac{d\lambda}{dl} &= (2\lambda + z - \frac{4\lambda A_v K_2 \Lambda^2}{(\mu + D_v \Lambda^2)(\mu + \nu + D_v \Lambda^2 + D_u \Lambda^2)}) \lambda
\end{align*}
\]

and for the case of OU noise with long-range correlations read:
\[ \frac{d\nu}{dt} = z\nu + \csc \left[ \frac{(1+2\theta_v)\pi}{2} \right] \frac{\lambda A_v K_2\Lambda^2}{(\mu + D_v\Lambda^2)^{1+2\theta_v}} \]

\[ \frac{d\lambda}{dt} = (2\chi + z)\lambda - 4\csc \left[ \frac{(1-2\theta_v)\pi}{2} \right] \frac{\lambda A_v K_2\Lambda^2(\nu + D_v\Lambda^2)}{(\mu + D_v\Lambda^2)^{1-2\theta_v} - (\mu + D_v\Lambda^2)^{1-2\theta_v}} \]

with \( A_v = \dot{A}_v\tau \) and \( \theta_v = 1 \) [12]. Here \( dt \) is an infinitesimal change in the scale \( s \) of the system (from \( s = e^t \) to \( s = s + ds \) with \( ds = s dt \)). The scaling parameters \( z \) and \( \chi \) are the so-called dynamic exponent (for time) and roughness exponent (for the fields \( U \) and \( V \)) and are not needed for the present study.

These equations predict how the effective parameters of the system run with scale and noise amplitude and have been derived and analyzed in detail for the long-wavelength limit (i.e. in the large scale limit \( s \to \infty \)) in [11]. Our aim in this paper is to validate these equations in the intermediate scale range, far from the fixed points, where the chemical processes take place. To carry this out, we will use these effective parameters (and negligible noise), and compare the resultant patterns based on these running parameters with those that are obtained numerically when all the model parameters are held fixed and only the noise intensity is increased.

II. DYNAMIC RG THEORY WITH FINITE SCALING AND UNCORRELATED NOISE.

The RG consists of thinning the degrees of freedom followed by a re-scaling of length and time [5]. It is a general calculational scheme for treating problems where fluctuations at many length scales are important. The final aim of the RG is to describe how the dynamics of a system subject to random fluctuations evolves as one changes the scale of observation. In practice, the RG procedure permits one to establish rigorous correspondences between sets of parameters defining physically different states observed at different scales. Indeed, (3,4) express the fact that \( s \) and \( \lambda \) change with the scale \( l \), as functions of the noise and the other model parameters, as a consequence of the coarse-graining. In particular, in the case under consideration, this correspondence is due to the presence of noise and a finite change in the spatial scale.

Following Eq. (3) we observe, that for a scale variation of \( \Delta l \) the predicted running or modification of the \( \nu \) and \( \lambda \) parameters induced by noise alone, should be \( \Delta \nu = +\Delta l \frac{\lambda A_v K_2\Lambda^2}{(\mu + D_v\Lambda^2)^{1+2\theta_v}} \) and \( \Delta \lambda = -\Delta l \left( \frac{4\lambda A_v K_2\Lambda^2}{(\mu + D_v\Lambda^2)^{1-2\theta_v}} \right) \) respectively. For the two-dimensional case, \( K_2 = 1/(2\pi) \). In our numerical simulation the smallest scale of observation is that of the cell with \( \Delta x \times \Delta y \) and \( \Delta x = \Delta y \). This determines the minimal wavelength and maximal momentum \( \Lambda = \frac{2\pi}{\Delta x} \), which can be considered in the problem. Let us take the cell as our initial scale, our scaling factor is therefore \( s = 1 \).

The diffusion term allows us to average data over neighboring cells, this in turn defines a greater scale of approximately \( 3\Delta x \times 3\Delta y \) which leads us to make \( \Delta s = 2 = s\Delta l \) and \( \Delta l = 2 \).

We will explore the patterns that are formed within a certain range of the parameter space. Similar studies have been performed in the deterministic Gray-Scott model, keeping \( \lambda = 1 \) constant, varying \( \mu \) between 0.06 and 0.14 and \( \nu \) between 0.01 and 0.07 (in this region there is a transition from two stable steady states to one trivial state and a great variety of patterns) [8].

In Table II we show, for a given noise intensity \( A_v \), the predicted effective parameters \( \lambda + \Delta \lambda \) and \( \nu + \Delta \nu \) based on the one-loop RG calculations in [11], for \( \lambda = 1, D_v = 0.5, D_u = 1 \), \( u_0 = 0.05 = \nu \) and \( \mu = 0.1155 \). The labels in column 1 and 5 will be used later to name the study cases. In Table II we show the same calculations, in a different region of parameter space, when the unperturbed values are \( \nu = u_0 = 0.03 \) and \( \mu = 0.086 \). In both cases we have set \( \nu = u_0 \) so that the trivial state solution is always \( U = 1 \).

The numerical simulations of system evolution have been performed using forward Euler integration of the finite-difference equations following discretization of space and time in the stochastic partial differential equations. The spatial mesh consists of a lattice of \( 256 \times 256 \) cells of size \( \Delta x = \Delta y = 2.2 \), with periodic boundary conditions. Noise has been discretized as well. The initial conditions consisted of one localized square pulse with \( (U = 0.5, V = 0.25) \) perturbing the trivial steady state \( (U = 1, V = 0) \) plus random Gaussian noise. The perturbing pulse measured \( 22 \times 22 \), just wide enough to allow the autocatalytic reaction to be locally self-sustaining. The system has been numerically integrated up to 30000 time steps (step size \( \Delta t = 1 \)). In the figures, only the concentration of the substrate \( U \) is shown. When displayed in color, the blue represents a concentration between 0.2 and 0.4, where the
TABLE I: Effective or renormalized parameters (sixth and seventh columns) induced by a white noise of intensity \( A_v \) when the unperturbed or bare values are \( \mu = 0.1155, \nu = u_0 = 0.05 \) and \( \lambda = 1 \).

| case | \( A_v \) | \( \lambda \) | \( \nu \) | \( \lambda + \Delta \lambda \) | \( \nu + \Delta \nu \) |
|------|----------|-------|-------|-----------------|-----------------|
| A    | \( 4.8 \times 10^{-6} \) | 1    | 0.05  | 1 (-10\(^{-6}\)) | 0.05 (+3 \times 10^{-6}) |
| B    | \( 1.61 \times 10^{-1} \) | 1    | 0.05  | b    | 0.99997          | 0.0561          |
| C    | \( 4.36 \times 10^{-7} \) | 1    | 0.05  | c    | 0.99991          | 0.05027         |
| D    | \( 4.84 \times 10^{-4} \) | 1    | 0.05  | d    | 0.999903         | 0.0503          |

TABLE II: Effective or renormalized parameters (sixth and seventh columns) induced by a white noise of intensity \( A_v \) when the unperturbed or bare values are \( \mu = 0.086, \nu = u_0 = 0.03 \) and \( \lambda = 1 \).

| case | \( A_v \) | \( \lambda \) | \( \nu \) | \( \lambda + \Delta \lambda \) | \( \nu + \Delta \nu \) |
|------|----------|-------|-------|-----------------|-----------------|
| I    | \( 4.8 \times 10^{-6} \) | 1    | 0.03  | 1 (-10\(^{-6}\)) | 0.03 (+3 \times 10^{-6}) |
| II   | \( 1.60 \times 10^{-1} \) | 1    | 0.03  | ii | 0.99997          | 0.0301          |
| III  | \( 4.33 \times 10^{-7} \) | 1    | 0.03  | iii | 0.999914         | 0.03026         |

substrate is being depleted by the autocatalytic production of \( V \), yellow represents an intermediate concentration of roughly 0.8 and red represents the trivial steady state \((U = 1)\).

A. Noise Induced Transition

We will now solve numerically the time evolution of this system \((\dot{u}, \dot{v}, \dot{w}, \dot{\mu})\) holding all parameters fixed and study the resulting patterns as a function of increasing noise intensity \( A_v \). First, for \( \lambda = 1, D_v = 0.5, D_u = 1, u_0 = 0.05, \mu = 0.1155, \nu = 0.05 \) and \( A_u = 0 \), we integrate this system with very weak noise \( A_v = 4.8 \times 10^{-6} \) and obtain the pattern shown in Fig. 1A. Then we increase only the noise intensity with the strength given in the second column of Table I (\( \lambda \) and \( \nu \) are held fixed to the values given in the next two columns) and obtain the patterns shown in Figs. 1B to D. The system evolves first forming long stripes. As we vary only the noise, some of the stripes split into spots and finally spots are the only stable structures. Noise induces a transition between different patterns or states.

Next we repeat this study at a different location in parameter space: \( u_0 = \nu = 0.03, \mu = 0.086 \) keeping \( \lambda = 1, D_v = 0.5, D_u = 1 \) and setting the noise intensity \( A_v \) to the values given in the second column of Table II (\( \lambda \) and \( \nu \) are held fixed to the values given in the next two columns). We obtain now the patterns shown in Figs. 3I to III. For weak noise, the system fills in the space with a pattern of short tubular structures and many spheres in clusters. Then, as we increase the noise intensity, the spheres clusters tend to disappear and the tubular structures become longer.

We have chosen two particular examples where weak noise induces an observable transition between two patterns (in some other cases weak noise or the equivalent small parameter variation does not modify significatively the resulting pattern with respect to the reference case).

B. Transitions induced by parameter variation.

Next we apply a fixed weak noise of intensity \( A_v = 4.8 \times 10^{-6} \) and numerically solve the system \((\dot{u}, \dot{v}, \dot{w}, \dot{\mu})\) with the renormalized parameters, \( \bar{\lambda} = \lambda + \Delta \lambda \) and \( \bar{\nu} = \nu + \Delta \nu \), that correspond to the coarse-grain scale \( \Delta \ell = 2 \) and the noise intensities studied above which are given in the sixth and seventh column of Tables I and II. In Fig. 2(a) to (d), we show the results for \( u_0 = 0.05 \) and \( \mu = 0.1155 \) and in Fig. 3(i) to (ii) for \( u_0 = 0.03 \) and \( \mu = 0.086 \). The results are equivalent to the patterns obtained before when only the noise was applied. Within the limits of perturbation theory, one can therefore obtain the same sequence of patterns and morphologies either by increasing the noise intensity alone or by holding the noise fixed and, applying a coarse-graining analysis, adjusting the \( \lambda \) and \( \nu \) parameters to their renormalized values as predicted by the RG. This is the main result of this paper.

Note that there is some discrepancy between the study cases (d)-(D) and (iii)-(III): in case (d), where only the deterministic parameters are changed, no self-replicating structure survives, whereas in (D), with noise, the pattern persists and in case (iii) the spheres have almost disappeared whereas in (III) the structures are erased. This difference may indicate that we are reaching the threshold where the higher order corrections (beyond one-loop) of the RG can
FIG. 1: Results obtained with all parameters held fixed and increasing only the noise intensity $A_v$ with the values given in column two of Table I. (A) and (B) are shown at $t = 20000$, (C)-(D) at $t = 10000$.

FIG. 2: Results obtained with renormalized parameters $\lambda + \Delta \lambda$ and $\nu + \Delta \nu$ as given in Table I, and holding the weak noise fixed $A_v = 4.8 \times 10^{-6}$. Figures (a)-(d) are shown at $t = 30000$.

no longer be ignored, or the point where the GS system is not renormalizable (new effective reactions are generated which cannot be described by a simple renormalization of the original parameters as was explained in [11]). We have observed that generally, for $A_v \approx 4.5 \times 10^{-4}$ and beyond, the patterns can become too noisy and/or non-equivalent to the patterns based on the renormalized parameter calculation. The dimensionless perturbation parameter of the RG analysis performed in [11] is $g = \frac{\lambda A_v K_2 \Lambda^2}{(\nu+D_v \Lambda^2)^2 - (\nu+D_u \Lambda^2)^2}$ which when evaluated at this limit gives $g \approx 4.5 \times 10^{-3}$.

Although the equivalence given by the RG equations breaks down at a certain noise intensity, the change that a pattern will suffer when applying weak noise (for instance, from long stripes into spots or from many spheres into long tubes) is easily predicted.

III. DYNAMIC RG THEORY WITH FINITE SCALING AND CORRELATED NOISE.

Next, we include a study case of this system with temporally correlated noise. Following Eq. (4) we observe, that for a scale variation of $\Delta l$ the predicted modification of the $\nu$ and $\lambda$ parameters induced by noise alone, should be $\Delta \nu = +\Delta l \csc \left(\frac{(1+2\nu)\pi}{2}\right) \frac{\lambda A_v K_2 \Lambda^2}{(\nu+D_v \Lambda^2)^2 - (\nu+D_u \Lambda^2)^2} \lambda$ and $\Delta \lambda = -\Delta l \csc \left(\frac{(1-2\nu)\pi}{2}\right) \frac{\lambda A_v K_2 \Lambda^2}{(\nu+D_v \Lambda^2)^2 - (\nu+D_u \Lambda^2)^2} \lambda$, respectively (where $\theta_v = 1$).

Our starting point now is equivalent to the previous study case (c), $\lambda = 0.99991$ and $\nu = 0.05027$, which (with the weakest white Gaussian noise $A_v = 4.3 \times 10^{-4}$) produces spots, see Fig. $\alpha$. Next we add some OU noise with $A_v = \hat{A}_v \tau = 2.61 \times 10^{-3}$ and $\tau = 2000$, see Fig. $\beta$, under this condition the system produces stripes and spots. Increasing the OU noise intensity further to $A_v = \hat{A}_v \tau = 7.67 \times 10^{-3}$ and $\tau = 5876$, see Fig. $\eta$, the pattern shows only long stripes. Adding OU noise of increasing strength to a system operated with the parameters of (c) we have reproduced the sequence (c) to (a).

We can explain this behaviour as follows: this correlated noise has induced a variation in $\lambda$ and $\nu$ of opposite sign to the case of white Gaussian noise $A_v = 4.3 \times 10^{-4}$ produces spots, see Fig. $\alpha$. Next we add some OU noise with $A_v = \hat{A}_v \tau = 2.61 \times 10^{-3}$ and $\tau = 2000$, see Fig. $\beta$, under this condition the system produces stripes and spots. Increasing the OU noise intensity further to $A_v = \hat{A}_v \tau = 7.67 \times 10^{-3}$ and $\tau = 5876$, see Fig. $\eta$, the pattern shows only long stripes. Adding OU noise of increasing strength to a system operated with the parameters of (c) we have reproduced the sequence (c) to (a).

We can explain this behaviour as follows: this correlated noise has induced a variation in $\lambda$ and $\nu$ of opposite sign to the case of white Gaussian noise leaving the system with new effective parameters $\lambda = 1.0001$ and $\nu = 0.05$ for the study case ($\eta$). If we solve the system with these renormalized parameters we recover the same situation as in the (a) study case which produces only stripes. For the intermediate case ($\beta$) the OU noise has induced a variation to the
FIG. 3: Results obtained with fixed parameters and increasing the noise intensity $A_v$ with the values given in column two of Table II. Figures (I)-(III) are shown at $t = 20000$.

FIG. 4: Results obtained with renormalized parameters $\lambda + \Delta \lambda$ and $\nu + \Delta \nu$ as given in Table II and with a fixed weak noise $A_v = 4.8 \times 10^{-6}$. Figures (i)-(iii) are shown at $t = 20000$.

new effective parameters $\hat{\lambda} = 0.99991$ and $\hat{\nu} = 0.05027$. We have included for completeness a simulation with these values (and the weakest white Gaussian noise $A_v = 4.3 \times 10^{-4}$), see Fig 5-(γ), the resulting pattern with stripes and spots is equivalent to the (β) case as expected.

Next we choose as starting point the study case (iii), $\nu = 0.03026$ and $\lambda = 0.999914$, see Fig 6-(1), which produces long tubes and few spheres. We add some OU noise with $A_v = \hat{A}_v \tau = 5.22 \times 10^{-3}$ and $\tau = 2000$, see Fig 6-(2), under this condition the system produces more spheres and shorter tubes. This correlated noise induces a variation in the effective system parameters: $\hat{\lambda} = 1.00001$ and $\hat{\nu} = 0.030072$. If we now solve the system with the renormalized parameters we obtain again a pattern with more spheres and shorter tubes as in the study case (2), see Fig 6-(3).

Applying correlated noise one can also find noise induced transitions. The RG prediction for the parameter variation is also valid for the case of correlated noise. Note that noise does not always drive the system in the same pattern-sequence direction since the sign of parameter variation/renormalization depends on the type of noise.

IV. CONCLUSION

We have studied the dynamics of the GS system with uncorrelated and correlated noise. The environmental fluctuations, the spatial diffusion and the non-linearities of the system create cross-coupling terms and the new effective system parameters depend on the noise amplitude and correlation time. We have shown that for weak noises the lowest order one-loop RG analysis can be applied at intermediate scale ranges, and be used to estimate the effective parameters of the system. In other words, by combining analytic and numerical work, we have established an equivalence between a sequence of patterns generated by varying the noise amplitude but keeping all other parameters fixed and a companion sequence generated by keeping the noise fixed and varying (i.e., renormalizing) instead some of the model parameters according to the RG flow equations.

For suitable noise intensities the perturbed values give rise to new patterns. Noise does not always drive the system in the same pattern-sequence direction since the sign of parameter variation/renormalization depends on the type of noise. Based on this knowledge and using weak noise, one can lead the system into a new state in a controlled way. Notice also that adding noise to a system which is expected to produce structures of a certain spatial correlation length (such as for
instance the spots in the study case ($\alpha$) do not necessarily drive the system into the production of patterns with shorter spatial correlations. On the contrary, it can drive the dynamics of the system into a new situation which produces patterns with longer spatial correlations (such as the stripes in the study case ($\eta$) with OU noise).

This raises important questions on the role of noise in chemical and biological self-organization and the environmental selection of emergent patterns. When describing a biological system not only should we estimate the relevant parameters of the system, but also the magnitude of the stochastic influences, the spatial scales involved and the correlation function. The interdependence of most biological systems with the environment makes this result specially relevant. In the case of non-linear systems and in the vicinity of transition points, the role of noise can be non-trivial, allowing the system to explore new, well organized situations which might be favored by the external natural selection processes. Applying the dynamic RG theory at finite scales one can have an intuitive idea of how the system will be driven by the environmental fluctuations.

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[12] These equations have been derived in [11] for a general case of noise with memory and a power law spectral density $S(w) = A_v w^{-2\theta_v}$. In particular, for the OU noise with spectral density $S(w) = \frac{A_v}{1+w^2}e^{-\frac{1+\omega^2}{\tau^2}}$, the value of the decay exponent $\theta_v$ is obtained in the limit of long correlation time $\tau \rightarrow \infty$ (but such that $A_v = \hat{A}_v \tau$ is, again, kept constant) rescaling the time to $\hat{t} = t/\tau$ and therefore rescaling the frequency to $\hat{w} = w\tau$ so that $S(\hat{w}) = A_v\hat{w}^{-2}$. 

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