Spin-echo entanglement protection from random telegraph noise

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Abstract
We analyze local spin-echo procedures for protecting entanglement between two non-interacting qubits, each subject to pure-dephasing random telegraph noise. For superconducting qubits, this simple model captures the characteristic features of the effect of bistable impurities coupled to the device. An analytic expression for the entanglement dynamics is reported. Peculiar features related to the non-Gaussian nature of the noise already observed in the single-qubit dynamics also occur in the entanglement dynamics for proper values of the ratio $g = v/\gamma$, between the qubit–impurity coupling strength and the switching rate of the random telegraph process, and of the separation between the pulses $\Delta t$. We found that the echo procedure may delay the disappearance of entanglement, cancel the dynamical structure of entanglement revivals and dark periods and induce peculiar plateau-like behaviors of the concurrence.

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(Some figures may appear in colour only in the online journal)

1. Introduction
Precise control of multiple coupled quantum systems is a major goal toward the realization of quantum computation [1, 2]. Multi-pulse sequences developed in the field of nuclear magnetic resonance (NMR) have recently been applied to mitigate noise in various qubit implementations ranging from atomic ensembles to the solid-state platform [3–5]. When a multi-qubit system is considered, the main issue is to control the entanglement dynamics in order to maintain a sufficient level of entanglement long enough for efficiently performing two-qubit operations and entanglement storage. On a more fundamental level, the issue is the possibility of preventing entanglement disappearance, i.e. the phenomenon of entanglement sudden death (ESD) [6].

These problems, shared to a certain extent by all qubit implementations, are particularly severe for solid-state qubits, which suffer from material-inherent noise sources. In particular, superconducting qubits are sensitive to fluctuating impurities located in the insulating materials surrounding superconducting islands, or inside the junctions. Very often, impurities originate from random telegraph (RT) fluctuations of island polarizations [7, 8] or of magnetic fluxes in SQUID geometries [9]. The noise spectrum of the corresponding variables is a Lorentzian centered at zero frequency. An ensemble of impurities may originate from the $1/f$ low-frequency behavior of the noise spectrum observed in several nanodevices [10]. Large-amplitude noise at low frequencies gives rise to dephasing, which is due to the randomization of the dynamic phase difference...
between superpositions of the qubit computational states. This phenomenon, as a difference with irreversible energy relaxation, is in principle reversible and can be refocused dynamically through the application of coherent control-pulse methods [2]. The possibility of extending to superconducting nanocircuits decoupling techniques developed in NMR has been demonstrated in different laboratories [3, 5, 11].

The simplest decoupling procedure is the spin-echo sequence [12]. It was successfully implemented in a charge qubit already in 2002 [11], unambiguously proving that low-frequency energy-level fluctuations due to 1/f charge noise were the main source of dephasing. Since two-qubit correlations are more fragile than single-qubit coherence, it is natural to ask whether a generalization of the spin-echo technique can be exploited to maintain entanglement between two solid-state qubits in the presence of low-frequency noise. This analysis falls within the rapidly developing research area aiming to extend pulse-based dynamical decoupling procedures [13] to the entanglement dynamics [14]. Recently, dynamical decoupling techniques to mitigate noise and enhance the lifetime of entangled state that is formed in a superconducting flux qubit coupled to a microscopic two-level system were implemented [15].

In this paper, we address this issue considering a simple model system that captures peculiar features of the effects of impurities on superconducting qubits: each qubit is longitudinally coupled (pure dephasing), with a coupling strength \( \nu \), to a bistable fluctuator randomly switching between the two states with rate \( \gamma \) [16–18]. The non-Gaussian nature of the RT process clearly manifests itself when the Markovian approximation does not apply, that is, when the fluctuator is ‘slow’ enough. More precisely, the Markovian approximation breaks down when the ratio between the coupling strength and the switching rate, \( g = \nu / \gamma \), is sufficiently larger than 1. When this condition is met, the single-qubit dynamics displays non-exponential decay, beatings and dependence on the system initial conditions [16, 19, 20]. In [18], the effect of an echo procedure on a superconducting qubit affected by an RT noise source at pure dephasing has been studied. In the non-Gaussian regime the echo signal as a function of time shows a series of plateaus whose position and height depend separately on \( \nu \) and \( \gamma \). In general, when \( g \geq 1 \) the echo signal differs significantly from the prediction based on a Gaussian approximation of the stochastic process.

In this paper, we investigate the possibility of preserving entanglement between two Josephson qubits each subject to an RT fluctuator at pure dephasing, by applying simultaneous echo pulses to the two qubits. We shall show that the echo procedure may delay the disappearance of entanglement and cancel the dynamical structure of entanglement revivals followed by dark periods, exhibiting also a peculiar plateau-like behavior.

2. The Hamiltonian model

We consider two non-interacting superconducting qubits, \( A \) and \( B \), each longitudinally coupled to a bistable impurity [21]. We are interested in the regime where the impurity splitting is smaller than the temperature, so that the impurity behaves like a classical RT fluctuator [16, 20, 22].

The total Hamiltonian of our system is given by

\[
H_\alpha = H_A + H_B, \quad \text{where the Hamiltonian of each qubit affected by the RT impurity, } H_A (\alpha = A, B), \text{ is}
\]

\[
H_A = H_{0,a} + \nu_a(t),
\]

\[
H_{0,a} = -(\Omega_a/2)\sigma_z - \left((v_a\xi_a(t))/2\right)\sigma_x,
\]

where \( \xi_a(t) \) instantly switches between 0 and 1 at random times with a switching rate \( \gamma_a \) and \( v_a \) is the coupling constant of qubit-\( a \) with a nearby impurity. The power spectrum of the unperturbed equilibrium fluctuations of \( \xi_a(t) \) is a Lorentzian,

\[
s_a(\omega) = v_a^2\gamma_a/[2(\gamma_a^2 + \omega^2)],
\]

and \( \nu_a(t) \) denotes an external control field.

Due to the longitudinal coupling, the diagonal elements of the reduced density matrix of each qubit (populations) are constant, whereas the off-diagonal elements (coherences) decay. The noisy dynamics has been studied in [16, 17, 20], where the single-qubit coherence, \( q_{0,a}(t) \), has been found in analytic form

\[
\frac{g_{0,a}(t)}{g_{0,a}(0)} = e^{-i\Omega t} e^{-i\pi/4} \left[ A_\alpha e^{-i\nu t} + (1 - A_\alpha) e^{-i\nu t} \right],
\]

where

\[
A_\alpha = \frac{1}{2}\left( 1 + \mu_\alpha - i g_\alpha \delta p_{0,a}\right), \mu_\alpha = \sqrt{1 - g_\alpha^2} \text{ and } \delta p_{0,a}\text{ is the initial population difference of impurity-}a\text{’s states.}
\]

Based on equation (2), two regimes can be identified, depending on the value of the parameter \( g_\alpha = v_\alpha / \gamma_\alpha \). When \( g_\alpha \ll 1 \) the impurity behaves like a weakly coupled and short-time correlated noise source affecting the qubit. The coherence decays exponentially with a decoherence rate given by the golden rule rate, \( \propto v_\alpha^2 / \gamma_\alpha \). In this regime the discrete process can be approximated as a Gaussian stochastic process completely characterized by the noise spectrum \( s_\alpha(\omega) \). It is common to refer to this situation as ‘weakly coupled’ impurity [16]. For \( g_\alpha \geq 1 \) instead the discrete nature of the process shows up in the qubit time evolution which shows beatings at frequencies \( \Omega_a \pm \nu_a \) and a long time decay with rate \( \gamma_a \). In this ‘strong coupling’ regime, in fact, the fluctuator splits the qubit’s levels and the qubit just experiences rare hops between these states with hopping rate \( \gamma_a \).

Control is operated as in [24], the external field \( \nu_a(t) \) being a sequence of \( \pi \)-pulses about \( \dot{x} \). We consider a modified spin-echo protocol consisting of two consecutive \( \pi \)-pulses separated by an interval \( \Delta t \). The external pulses are short enough for both relaxation and spectral diffusion during each of the pulses to be neglected; thus the pulse evolution operator is

\[
S_{P,a} \approx \exp(i\pi/2\sigma_x a) = i\sigma_x a.
\]

The evolution between the pulses reads

\[
S_a = T\left( \int_0^{\Delta t} \exp[i H_a(t')] dt' \right).
\]

The basic idea behind the echo procedure is that the sequence of two \( \pi \) pulses about \( \dot{x} \) reverses the sign of the qubit–fluctuator interaction during the two time intervals \( \Delta t \). This is clearly seen considering the short \( \Delta t \) limit of \( S_a \propto i\sigma_x \Delta t \) and the composition of Pauli matrices \( \sigma_x \sigma_x \sigma_x = -\sigma_z \). As a result, the effect of fluctuations slower than \( \Delta t \) is canceled at time \( 2\Delta t \), the residual signal decay being due to faster noise components. The echo signal for a qubit subject to an RT fluctuator at pure dephasing has been evaluated both in [23], starting from a quantum description of the impurity, and
in [18]. Here we report the final form of the single-qubit concurrence \( q_{c,u}(t) \)
\[
\frac{q_{c,u}(2\Delta t)}{q_{c,u}(0)} = e^{-\gamma_{u}\Delta t} \mu_{a}^{2} \times \left[ \frac{1+\mu_{a}}{2} e^{i\mu_{a}\gamma_{u}\Delta t} + \frac{1-\mu_{a}}{2} e^{-i\mu_{a}\gamma_{u}\Delta t} - (1-\mu_{a}^{2}) \right].
\]  
(3)

Note that for \( \gamma_{u}\Delta t \to 0 \) one obtains \( q_{c,u}(2\Delta t) \to 1 \), as expected when the pulse separation \( \Delta t \) is much smaller than the noise correlation time \( 1/\gamma_{u} \). In the following, we shall use the above analytic expressions of single-qubit coherences to study the time behavior of entanglement.

3. Dynamics of entanglement and entanglement echo

Based on the single-qubit coherence reported in the previous section, we investigate the time behavior of entanglement between the two qubits. We consider the simple situation where the two qubits \( A \) and \( B \), initially prepared in an entangled state, evolve independently each subject to a RT process, as described by the Hamiltonian (1). We suppose that each qubit is subject to an echo sequence, as described above. For the sake of simplicity, we suppose that the pulses are applied simultaneously to the two qubits and evaluate the concurrence at time \( 2\Delta t \). The entanglement echo is compared with dynamics of entanglement when no pulses are applied. To this end, we need the evolved two-qubit density matrix. Since the two qubits are non-interacting, their density matrix can be obtained following a standard procedure based on the knowledge of single-qubit dynamics [25, 26]. This procedure also holds when the qubits are subject to independent external control fields, for the case of our interest, to two echo procedures.

3.1. Concurrence for non-interacting qubits

We take as initial states the class of extended Werner-like (EWL) states [27]
\[
\rho_{1} = r|1_{a}\rangle\langle 1_{a}| + \frac{1-r}{4} \mathbb{1}_{A}, \quad \rho_{2} = r|2_{a}\rangle\langle 2_{a}| + \frac{1-r}{4} \mathbb{1}_{A},
\]  
(4)

whose pure parts are the one-excitation and two-excitation Bell-like states \( |1_{a}\rangle = a|01\rangle + b|10\rangle, \; |2_{a}\rangle = a|00\rangle + b|11\rangle \), where \( |a|^{2} + |b|^{2} = 1 \). The purity parameter \( r \) quantifies the purity of the state, given by \( P = (1+3r^{2})/4 \). The density matrix of EWL states, in the computational basis \( B = \{ |00\rangle, |11\rangle, |01\rangle, |10\rangle, |1\rangle, |2\rangle \} \), has an X form [28] and this structure is maintained at \( t > 0 \) during the pure-dephasing dynamics we are considering here. The initial entanglement is equal for both the EWL states of equation (4), \( C_{\rho_{1}}(0) = C_{\rho_{2}}(0) = 2 \max\{0, |ab| + 1/4\}\), where \( C \) is the concurrence [29]. Initial states are thus entangled for \( r > r^{*} = (1+4|ab|)^{-1} \).

The EWL states of equation (4) evolve with fixed diagonal elements and time-dependent anti-diagonal elements given by, respectively, \( \rho_{12}(t) = \rho_{12}(0)|q_{A}(t)q_{B}(t)\rangle \langle q_{A}(t)q_{B}(t)| \) for the initial state \( \rho_{1} \) and \( \rho_{32}(t) = \rho_{32}(0)|q_{A}(t)q_{B}(t)\rangle \langle q_{A}(t)q_{B}(t)| \) for \( \rho_{2} \), where \( q_{A}(t) \) are given either by equation (2) or by equation (3), depending on the qubit evolution. The general expressions of the concurrences at time \( t \) for the two initial states of equation (4) are easily obtained as \( C_{\rho_{1}}(t) = 2 \max\{0, |\rho_{12}(t)| - \sqrt{\rho_{00}(0)\rho_{33}(0)}\} \) and \( C_{\rho_{2}}(t) = 2 \max\{0, |\rho_{32}(t)| - \sqrt{\rho_{11}(0)\rho_{22}(0)}\} \) [21]. These expressions coincide for the initial states \( \rho_{1} \) and \( \rho_{2} \), \( C_{\rho_{1}}(t) = C_{\rho_{2}}(t) \), where
\[
C(t) = 2 \max\{0, r|a|\sqrt{1-|a|^{2}}|q_{A}(t)q_{B}(t)| - (1-r)/4\}.
\]  
(5)

For initial pure states, \( r = 1 \), it is readily seen from equation (5) that \( C(t) \propto |q_{A}(t)q_{B}(t)| \) so that entanglement qualitatively behaves like the single-qubit coherence. In a more realistic case, the initial state is not pure. Here we consider a realistic degree of purity in superconducting systems. We refer to the experiment [30] where entangled states of two Josephson qubits with purity \( \approx 0.87 \) and fidelity to ideal Bell states \( \approx 0.90 \) have been generated by two-qubit interaction mediated by a cavity bus in a circuit quantum electrodynamics architecture and tunable in strength by two orders of magnitude on the nanosecond timescales. These states may be approximately described as EWL states with \( r = r_{\text{exp}} \approx 0.91 \) and \( |a| = 1/\sqrt{2} \), giving a concurrence \( C = 0.865 [31] \). In the following analysis, we shall assume that the qubits are prepared in an initial entangled state having these characteristics.

3.2. Entanglement echo

The advantage of the entanglement echo procedure and the peculiarities resulting from the effect of discrete stochastic processes affecting the two qubits are pointed out by comparing with the entanglement evolution at pure dephasing in the presence of RT fluctuations, which has been studied in [21]. Under these conditions the concurrence is given by equation (5) with \( q_{0,a}(t) \) as in equation (2). In the following analysis, we put \( \delta p_{0,a} = 0 \).

As a difference with the single-qubit evolution, where the threshold separating Gaussian and non-Gaussian behaviors is at \( g \sim 1 \), the entanglement dynamics displays qualitative different features at a threshold value depending on the characteristics of the initial entangled state, \( g_{\text{th}}(r, a) \). For initial mixed states, \( r < 1 \), the time behavior of entanglement qualitatively changes in correspondence to a threshold value \( g_{\text{th}}(r, a) > 1 \) [21]. There is always ESD with exponential decay for \( g < g_{\text{th}} \) and a complete disappearance with revivals for \( g > g_{\text{th}} \) (thin curves of figure 1). For the considered EWL initial states the threshold is \( g_{\text{th}} = 2.3 \). We remark that the entanglement revivals here occur in a classical environment incapable of any back-action. The interpretation of this effect differs from the explanation valid in the presence of quantum environments [32] and it is an open issue [33, 34].

We now act on each qubit with two simultaneous echo procedures. For the sake of simplicity, we suppose that the two fluctuators have equal switching rates, \( \gamma_{u} \equiv \gamma \), but different couplings to the corresponding qubit, in order to address the regime \( g_{A} \neq g_{B} \). The concurrence \( C(t = 2\Delta t) \) is found in this case by equation (5) with \( q_{c,u,a}(t) \) of equation (3).

To start with, we consider the case \( g_{A} = g \) and evaluate \( C \) for two values \( g_{1} < g \) and \( g_{2} > g \). In figure 1, we plot the concurrences with and without echo as a function of \( g \Delta t \). To point out clearly the qualitative effects we take very
different values, \( g_1 = 0.7 \) and \( g_2 = 7 \). We observe that the echo procedure leads to a larger degree of entanglement with respect to the free evolution for both values of \( g \) considered. In particular, for \( g_1 \leq \bar{g} \) the ESD time is delayed (dashed lines in figure 1(a)). For \( g_2 > \bar{g} \) the dynamical structure of entanglement revivals and dark periods is canceled out by the echo procedure. Moreover, plateau-like features appear in the time behavior of entanglement (continuous lines in figure 1(a)). This is a typical non-Gaussian behavior linked to the free evolution for both values of \( g \). From equations (3) and (4), we observe a non-monotonic behavior of \( C \) as a function of \( \gamma \Delta t \) for \( g \approx 1 \), instead we observe a non-monotonic behavior of \( C \). The oscillatory behavior of \( C \) at fixed \( \gamma \Delta t \) when \( g \gg 1 \) is simply understood considering the large \( g \) limit of the coherence equation (2). Interestingly, the echo preserves entanglement even when \( g > \bar{g} \), when it would vanish in the absence of the echo procedure (continuous lines in figure 2). However, the quantitative value of the preserved entanglement might not be sufficient for an efficient realization of quantum error correction tasks. Finally, for large values of the pulse intervals, \( \gamma \Delta t \gg 1 \), the echo procedure is not able to efficiently reduce the detrimental effects of noise for any \( g \).

4. Conclusions

In this paper, we have investigated the effect of the simultaneous application of a modified version of the spin-echo protocol to two non-interacting qubits each subject to pure-dephasing RT noise. This simple system is inspired by superconducting qubits, where selected impurities producing RT noise are frequently observed and techniques from NMR have recently been implemented, showing the possibility of limiting defocusing due to low-frequency noise and to strongly coupled impurities [3, 5, 11].

Figure 1. Concurrence \( C(2\Delta t) \) as a function of \( \gamma \Delta t \). In (a) \( g_a = g \), with \( g_1 = 0.7 \) (green dashed lines) and \( g_2 = 7 \) (orange continuous lines), for the cases without echo (thin lines) and with echo (thick lines). For \( g_2 = 7 \) note the plateaus at \( \gamma \Delta t = 2\pi/g_2 \approx 0.9 \). In (b) \( g_A = 0.3 \) and \( g_B = 4 \) for the cases without echo (thin line) and with echo (thick line).

Figure 2. Concurrence \( C \) as a function of \( g \) for two values of \( \gamma \Delta t \) equal to 0.1 (purple dashed lines) and 1.1 (blue solid lines) for the cases without echo (thin lines) and with echo (thick lines). The dotted vertical line corresponds to \( \bar{g} = 2.3 \).
In this paper, we have presented an analytic expression for the concurrence in the presence of simultaneous echo protocols on the two qubits for a general class of initial entangled states. The main result is that the echoes either delay the ESD time or cancel the dynamical structure of entanglement revivals followed by dark periods, depending on the qubit–fluctuator coupling strength $g_a$. In particular, if at least one qubit has $g_a > \bar{g}$, the entanglement exhibits a novel dynamical structure consisting of plateaus occurring at selected values of pulse separation $\Delta t$. This effect is entirely due to the non-Gaussian nature of the noise. For identical qubits and impurities the plateaus times are at $v_a \Delta t_{\text{plateaus}} \approx 2\pi k$. In general, when the pulse lengths are very short, $1/\Delta t \gg \gamma_a$, the two echoes considerably slow down the entanglement decay up to relatively large values of $g_a \gtrsim \bar{g}$. Interestingly, for intermediate pulse lengths ($\gamma \Delta t \sim 1$) entanglement can be at least partly preserved even for $g > \bar{g}$ when it vanishes in the absence of the echo procedure.

This very simple analysis was aimed at pointing out relevant qualitative effects, starting from a physically relevant model. Relevant issues such as the feasibility of simultaneous pulses, the effect of timing imperfections and the interplay with additional low-frequency components leading to $1/f$ noise have not been addressed. At the single-qubit level the echo protocol can limit defocusing due to $1/f$ noise [3, 5, 11]. An analysis of the effect of $1/f$ noise on the entanglement both of uncoupled [31] and of coupled qubits [35] has recently been performed. This preliminary analysis suggests that the echo protocol and dynamical decoupling extensions may be conveniently exploited to protect entanglement between a solid-state qubit, and possibly other kinds of correlations like those quantified by the quantum discord [36], against RT and $1/f$ noise [37].

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