Motion of Spinning Particles in Gravitational Fields

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Abstract

A new path equation in absolute parallelism (AP) geometry is derived. The equation is a generalization of three path equations derived in a previous work. It can be considered as a geodesic equation modified by a torsion term, whose numerical coefficient jumps by steps of one half. The torsion term is parametrized using the fine structure constant. It is suggested that the new equation may describe the trajectories of spinning particles under the influence of a gravitational field, and the torsion term represents a type of interaction between the quantum spin of the moving particle and the background field.

Weak field limits of the new path equation show that the gravitational potential felt by a spinning particle is different from that felt by a spinless particle (or a macroscopic body).

As a byproduct, and in order to derive the new path equation, the AP-space is reconstructed using a new affine connexion preserving metricity. The new AP-structure has non-vanishing curvature. In certain limits, the new AP-structure can be reduced either to the ordinary Riemannian space, or to the conventional AP-space.
1 Introduction

It is well known that most of the astronomical informations are carried by massless spinning particles. Astronomers usually extract astronomical informations from photons, which are spin-1 massless particles. In 1987 astronomers succeeded in extracting informations carried by nutrinos (spin- one half massless particles) coming from SN1987A. In the near future we expect gravitons (spin-2 massless particles) to open a third window through which we look at the universe. These particles, during their trip from their sources to the detectors, are moving in different gravitational fields. Thus, it is of fundamental importance, for astronomy and space studies, to know the exact trajectories of such particles, and the interaction between the spin of these particles and the background gravitational field.

The problem of motion of spinning particles in a background gravitational field has been tackled by different authors. The trails of those authors can be classified into two main approaches. The first approach contains two classes: (i) quantization of a relativistic equation of motion of a rotating object (e.g. Papapetrou equation, Dixon equation), (cf. Melek (1988)) (ii) geometrization (or more specifically covariantization) of a quantum mechanical equation of motion of a spinning particle (e.g. Schrödenger equation, Pauli equation), (cf. DeWitt (1957), Galvao and Teitelboim (1980)). The second approach depends on a different philosophy in which paths (or curves) of an appropriate geometry are considered to represent trajectories of test particles. Einstein had followed this approach.

\footnote{In the present work we are going to distinguish between spin and rotation. We use the term spin for the quantum property of a microscopic particle, while the term rotation is used for the corresponding classical mechanical property of a macroscopic object.}
and used the geodesic and null geodesic curves of the Riemannian geometry to describe
the motion of test particles and of photons respectively.

Although the philosophy of the second approach is successful in describing motion
of test particles including massless particles (photons), yet it was neglected by many
authors in dealing with motion of other spinning particles. We believe that this philosophy
deserves further investigations especially to look for the equation of motion of spinning
particles in gravitational fields. It is to be noted that while the first approach is suitable
for describing short range motion of spinning particles i.e. motion on the laboratory
scale, it is not suitable for describing long range motion (e.g. motion on scales such as
solar system, galactic, intergalactic scales). The second approach is more suitable for
describing trajectories of test particles especially long range motion of massless particles
in gravitational fields (motion of photons using null-geodesics). We are mainly interested
in this approach.

To explore the capabilities of this approach we directed our attention to the AP-
geometry. The cause is that short range motion of spinning particles is described success-
fully using this geometry, since spinors can be defined and related to the structure of the
AP-spaces.

In a trial to look for possible paths in this geometry, three path equations were derived
(Wanas et. al. (1995a) by generalizing the method given by Bazanski (1977,1989). These
equations can be written in the form,

\[
\frac{dV^\mu}{dS} + \{^\mu_{\alpha\beta}\} V^\alpha V^\beta = -\Lambda^\mu_{(\alpha\beta)} V^\alpha V^\beta,
\]  

(1.1)
\[ \frac{dW^\mu}{dS^0} + \{_{\alpha\beta}^\mu\} W^\alpha W^\beta = -\frac{1}{2} \Lambda^{\mu}_{(\alpha\beta)} W^\alpha W^\beta, \]
(1.2)

\[ \frac{dU^\mu}{dS^-} + \{_{\alpha\beta}^\mu\} U^\alpha U^\beta = 0, \]
(1.3)

where \( S^+, S^0 \), and \( S^- \) are the evolution parameters characterizing the three paths respectively; and \( V^\alpha, W^\alpha \) and \( U^\alpha \) are the tangents to the corresponding paths and the brackets (\( () \)) are used for symmetrization. The torsion of the AP-space is defined by

\[ \Lambda^{\alpha}_{\mu\nu} \overset{\text{def.}}{=} \Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu}, \]

where \( \Gamma^{\alpha}_{\mu\nu} \) is a non-symmetric affine connexion defined as a consequence of the AP-condition (see section 2). These equations can be considered as generalization of the geodesic equation of Riemannian geometry.

Considering these three equations, it seems interesting to point out the following remarks:

1- Although four absolute derivatives are defined in the AP-space (see section 2), and used in deriving the above equations, only three equations were obtained including the geodesic equation (1.3).

2- In general, the other two equations ((1.1), (1.2)) cannot be reduced to the geodesic equation (1.3) unless the torsion vanishes. It has been shown (Wanas, and Melek (1995)) that the vanishing of the torsion of the AP-space will lead to the vanishing of the curvature tensors defined in that space. In this case the space will reduce to a flat one.

3- Equation (1.3) can be reduced to null-geodesic upon reparametrization.

4- The equations can be considered as geodesic equations modified by a torsion term. The important feature of this set of equations is that the numerical coefficient of the
torsion term jumps by a step of one half from one equation to the next. This is tempting to believe that paths in the AP-spaces are quantized in a certain sense.

From these remarks it seems that the AP-space possesses more fine structure than that appearing in its conventional picture (for more details about the conventional structure of the AP-spaces, (cf. Levi-Civita (1950), Mikhail (1952), Hayashi and Shirafuji (1979)). It contains in addition to the geodesic and null-geodesic equations, two more equations (1.1), (1.2).

Now the following groups of questions emerge:

1- Does the space possesses a general affine connexion that gives rise to family of paths in which the coefficient of the torsion term jumps by a step of \( \frac{1}{2} \) from equation to another in this family ? If so, what is the underlying geometric structure ?

2- What is the general form of the equations representing this family with a jumping torsion term ?

3- Is there any observational or experimental evidence for motion along such paths ? What is the order of magnitude of the deviation from the geodesic motion, if any? What is the intrinsic property of the moving particle that causes such deviation from the geodesic one?

The aim of the present work is to give answers to some of the above questions. In section 2, a trail to answer the first group of the above questions is given. In section 3 the general form of a path equation, giving the required family mentioned above, is derived using the geometry given in section 2. Section 4 contains a trail to attribute some physical meaning to the new path equation. The static weak field limits of the new equation are
2 The Underlying Geometry

In the conventional AP-geometry two affine connexions were defined. The first is Christoffel symbol defined as a consequence of the metricity condition,

\[ g_{\alpha\beta,\mu} = 0, \]  

(2.1)

where \( g_{\alpha\beta} \) is the metric tensor defined by,

\[ g_{\alpha\beta} \overset{def}{=} \lambda_{i}^{\alpha} \lambda_{i}^{\beta}, \]  

(2.2)

and \( \lambda^{a}_{i} \) are the tetrad vectors defining the structure of the AP-space in 4-dimensions. The semicolon is used to characterize covariant derivatives using Christoffel symbols. The second is the non-symmetric connexion \( \Gamma_{\alpha\beta}^{\mu} \) defined as a consequence of absolute parallelism condition:

\[ \lambda_{i}^{\alpha} + \Gamma_{\alpha\beta}^{\mu} \lambda_{i}^{\beta} = 0, \]  

(2.3)

the stroke and the (+) sign is used to characterize absolute derivatives using \( \Gamma_{\alpha\beta}^{\mu} \). Since this connexion is non-symmetric, one can define two more absolute derivatives: for the first we use a stroke and a (-) sign which indicates the use of the dual connexion \( \hat{\Gamma}_{\alpha\beta}^{\mu} (= \Gamma_{\beta\alpha}^{\mu}) \), and for the second we use a stroke without signs, which indicates the use of the symmetric part, \( \Gamma_{(\alpha\beta)}^{\mu} \). It can be shown that using (2.2), (2.3) we get,

\[ g_{\mu + + |\sigma} = 0, \]  

(2.4)

given in section 5. The work is discussed in section 6.
which means that metricity is also preserved using the non-symmetric connexion, recalling
that the metricity condition (2.1) is necessary but not sufficient to define Cristoffel symbol.

Now we are looking for a general affine connexion that is capable of producing the
family of all paths with jumping torison term. Recalling that the three path equations
were derived using the above mentioned derivatives, thus the simplest way to find such a
connexion is to combine linearly the above connexion using some parameters. After some
manipulations the general expression of this connexion can be written in the following
form,

\[ \nabla^\mu_{\alpha\beta} = a_1 \{^\mu_{\alpha\beta} \} + (a_2 - a_3) \Gamma^\mu_{\alpha\beta} - (a_3 + a_4) \Lambda^\mu_{\alpha\beta}, \]  

(2.5)

where \( a_1, a_2, a_3 \) and \( a_4 \) are parameters to be fixed later. It can be easily shown that
\( \nabla^\alpha_{\mu\nu} \) transforms as an affine connexion under the group of general coordinate transfor-
mations. It is clear that this connexion is non-symmetric. If we characterize the general
absolute derivative, using the affine connexion (2.5), by a double stroke we get after some
manipulations:

\[ g_{\mu\nu||\sigma} = (1 - a_1 - a_2 - a_3 - 2a_4)(g_{\mu\alpha} \{^\alpha_{\sigma\nu} \} + g_{\nu\alpha} \{^\alpha_{\mu\sigma} \}) - (a_3 + a_4)(g_{\mu\alpha} \gamma^\alpha_{\sigma\nu} + g_{\nu\alpha} \gamma^\alpha_{\sigma\mu} ), \]  

(2.6)

where \( \gamma^\alpha_{\mu\nu} \) is the contorsion defined by

\[ \gamma^\alpha_{\mu\nu} \overset{\text{def}}{=} \frac{1}{2}(\Lambda^\alpha_{\mu\nu} - \Lambda^\alpha_{\mu\nu} - \Lambda^\alpha_{\nu\mu} ) \]  

(2.7)

If we need metricity to be preserved along the paths characterized by (2.5) we should take,

\[ a_3 + a_4 = 0. \]

\[ 1 - a_1 - a_2 + a_3 = 0, \]

7
Taking \( a = a_1, b = a_2 + a_4 \) then the metricity condition can be written in the form

\[
a + b = 1. \tag{2.8}
\]

After using this condition it is clear that (2.5) will remain non-symmetric.

Using the general affine connexion (2.5) and the general metricity condition (2.8), one can define the following curvature tensors.

(i) If we replace, in the definition of Riemann-Christoffel tensor \( R^\alpha_{\mu
u\sigma} \), Christoffel symbol by the new connexion we get the curvature,

\[
B^\alpha_{\mu\nu\sigma} \overset{\text{def.}}{=} \nabla^\alpha_{\mu\nu\sigma} = \nabla^\alpha_{\mu\nu,\sigma} + \nabla^\alpha_{\sigma\nu} \nabla^\epsilon_{\mu\sigma} - \nabla^\alpha_{\epsilon\sigma} \nabla^\epsilon_{\mu\nu} , \tag{2.9a}
\]

which can be written in the form

\[
B^\alpha_{\mu\nu\sigma} = R^\alpha_{\mu\nu\sigma} + b_1 \gamma^\alpha_{\mu[\sigma;\nu]} + b_2 \gamma^\alpha_{\gamma\epsilon[\nu} \gamma^\epsilon_{\mu\sigma]} , \tag{2.9b}
\]

the square brackets are used for antisymmetrization.

(ii) If we define the curvature \( W^\alpha_{\mu\nu\sigma} \) as a measure of the non-commutativity of the general absolute derivatives, we get

\[
\lambda^\alpha_{\mu||\nu\sigma} - \lambda^\alpha_{\nu||\sigma\mu} = \lambda^\alpha_{\mu\nu\sigma} = W^\alpha_{\mu\nu\sigma} \tag{2.10}
\]

where

\[
W^\alpha_{\mu\nu\sigma} = B^\alpha_{\mu\nu\sigma} - b(b - 1) \gamma^\alpha_{\gamma\epsilon\mu} \Lambda^\epsilon_{\nu\sigma} \tag{2.11}
\]

Now we have the following remarks: (1) It is to be considered that non of the curvature tensors defined by (2.9) and (2.11) vanishes. This means that the new structure of the
AP-space is, in general not flat. This result will be discussed in section 6. (2) Taking the metricity condition into consideration, we can show that two important special cases could be obtained:

The first case $a=1$ (i.e. $b=0$): In this case the geometry will be reduced to Riemannian geometry. This is obvious by substituting the values of the parameters into (2.5), (2.9)b, and (2.11), which give,

$$\nabla_{\mu\nu} = \{_{\mu\nu}\}$$

$$W_{\mu\nu\sigma} = B_{\mu\nu\sigma} = R_{\mu\nu\sigma}$$

The second case $a=0$ (i.e. $b=1$): In this case it can be easily shown, after some manipulations, that the geometry reduces to the conventional AP-geometry with

$$\nabla_{\mu\nu} = \Gamma_{\mu\nu}$$

$$B_{\mu\nu\sigma} = \Gamma_{\mu\sigma,\nu} - \Gamma_{\mu,\nu,\sigma} + \Gamma_{\epsilon\nu,\sigma} \Gamma_{\mu,\epsilon} - \Gamma_{\epsilon,\sigma} \Gamma_{\mu,\epsilon} = 0$$

$$W_{\mu\nu\sigma} = 0.$$

The curvature (2.15) vanishes as a consequence of the AP-condition.

These two cases will be discussed in the last section. Now to complete the structure of the space, one should look for a path equation corresponding to the new affine connexion, which can be considered as a generalization of equations (1.1), (1.2) and (1.3). This is done in the following section.
3 The General Form of the Path Equation

Generalizing the Bazanski (1977, 1989) Lagrangian, using the affine connexion (2.5) and the condition (2.8), we get

\[ L \overset{\text{def.}}{=} \lambda_i \lambda_{i\nu} Z^\mu \frac{\nabla \chi^\mu}{\nabla \tau} \]  
(3.1)

\[ Z^\mu \overset{\text{def.}}{=} \frac{dx^\mu}{d\tau}, \]  
(3.2)

\[ \frac{\nabla \chi^\mu}{\nabla \tau} \overset{\text{def.}}{=} \chi_{i||\alpha} Z^\alpha \]  
(3.3)

\[ \chi_{i||\alpha} \overset{\text{def.}}{=} \chi_{,\alpha} + \chi^\beta \nabla_{\beta \alpha}, \]  
(3.4)

where \( \tau \) is the evolution parameter along the new general path associated with (2.5), \( \chi^\mu \) is the deviation vector, and \( Z^\mu \) is the tangent to the path. Applying the variational formalism using the lagrangian (3.1), and noting that the general absolute differential operation commutes with raising and lowering indices (because of (2.8)), we get after necessary manipulation:

\[ \frac{dZ^\mu}{d\tau} + \left\{ \begin{array}{c} \mu \\ \nu \sigma \end{array} \right\} Z^\nu Z^\sigma = -b \lambda_{(\nu \sigma)} Z^\mu \frac{dZ^\nu}{d\tau} \]  
(3.5a)

Using (3.3), the last equation can be written in the form

\[ \frac{\nabla Z^\mu}{\nabla \tau} = 0. \]  
(3.5b)

Equation (3.5)a, or (3.5)b, represents a generalization of the path equations given in section 1. Moreover, if \( b = 0 \) this equation will reduce to geodesic of the metric (or null-geodesic upon reparametrization). This is consistent with the first special case given in
the above section. It can be easily shown that (3.5) will give rise to,

\[ \frac{d(g_{\alpha\beta}Z^\alpha Z^\beta)}{d\tau} = 0 \]

consequently, its first integral is given by,

\[ Z^\alpha Z_\alpha = Z^2 \]  \hspace{1cm} (3.6)

which means that Z is a constant along the path (3.5), and since Z is scalar under general coordinate transformation, then it will be constant in general.

4 Physical Meaning of the Torsion Term

Equation (3.5) represents a new path in the AP-space. As stated above, this equation can be reduced to a geodesic equation (as \( b=0 \)) which represents the trajectory of a test particle in a background gravitational field (it can also be reduced to a null-geodesic). So, what is the role of the torsion term on the R.H.S. of (3.5)a ? May it represent a type of interaction between the torsion of the background gravitational field and some intrinsic property of the moving particle ? If so, what is this intrinsic property ?

Let us start a trial to answer the last question. Recalling that, the aim of the work in the section 3 is to get an equation that represents the complete family of the equations given in section 1. Thus the parameter "b" of (3.5)a should consist of a half integer part. In this case we can write

\[ b = \frac{n}{2} \beta, \]  \hspace{1cm} (4.1)

where n is a natural number and \( \beta \) is another parameter.
It is well known that an intrinsic property of the particle which depends on half integers is its quantum spin. Several authors have pointed out that spinning particles feel space-time torsion (cf. Hehl (1971)). Others believe that spinning particles are the only probes which detect telleparallel geometry (cf. Nitch and Hehl (1980), Ross (1989)). Although most of these authors have used the term spin to mean rotation and have believed that only rotating sources can generate the space-time torsion, the situation here is different. It has been shown by the author (1990) that space-time torsion is generated whether the source of the field is rotating or not. Considering these arguments, we may suggest that the R.H.S. of (3.5) represents a type of interaction between the quantum spin of the moving particle and the torsion of the background gravitational field. Consequently, equation (3.5) may represent the trajectory of a spinning particle in a gravitational field. Consequently we take $n = 0, 1, 2, ...$ for particles with spin $0, \frac{1}{2}, 1, ...$ respectively. For macroscopic objects and spinless particles $n = 0$. This will reduce equation (3.5) to the geodesic (or null-geodesic) equation, and the geometric structure to a Riemannian one.

It is well known that the geodesic motion implies the validity of the weak equivalence principle (WEP). So, equation (3.5) implies that motion of spinning particles violates the WEP. If we take the parameter $\beta$ to be of order unity, then the torsion term will be of the same order of magnitude as the Christoffel symbol term which will be considerably large. But since there are no experimental or observational evidences for such large violation of (WEP) for the motion of spinning particles, thus the parameter $\beta$ should be of less order. From observational point of view the WEP is verified to an accuracy of about $10^{-2}$ on the galactic scale (cf. Longo (1988)), so $\beta$ should be less than $10^{-2}$.
From the previous discussion it is acceptable that the parameter $\beta$ may have the following properties:

1. It should be a dimensionless quantity to preserve the dimensions on both sides of (3.5)a.
2. It should be small compared to unity ($\beta \leq 10^{-2}$) to be consistent with relevant observations and experiments.
3. It should be connected to the intrinsic properties of elementary particles, especially those affecting motion of the spinning particles.
4. It should include, in its structure, Planck’s constant $\hbar$ or $h$.

To the knowledge of the author a quantity satisfying the above requirements is the fine structure constant $\alpha$ ($=\frac{e^2}{\hbar c} = \frac{1}{137}$). We can replace $\beta$ in (4.1) by the fine structure constant $\alpha$. But to be more conservative we are going to write (4.1) in the form

$$b = \frac{n}{2} \alpha \gamma$$

where $\gamma$ is a dimensionless parameter of order unity to be fixed by experiment. Now the parameter $b$ constitutes of three parts: the first” $\frac{n}{2}$” is the part that makes (3.5)a gives rise to (1.1), (1.2), (1.3), and the second part ”$\alpha$” acts as a reduction factor, while the third part”$\gamma$” is introduced for matching with experimental results. The appearance of the fine structure constant in this treatment will be discussed in the last section.

## 5 Weak Static Limits of the New Equation

The path equation (3.5) can be used as the equation of motion for any field theory, constructed using the AP-geometry, provided that the theory has good Newtonian limits. In
such theories, (e.g. Mikhail and Wanas(1977), Møller (1978), Hayashi and Shirafuji(1979)),
the tetrad vectors $\lambda_{i\mu}$ are considered as field variables. So, if we write

$$\lambda_{i\mu} = \delta_{i\mu} + \epsilon h_{i\mu}$$  \hspace{1cm} (5.1)

where $\epsilon$ is a small parameter, $\delta_{i\mu}$ is Kronecker delta and $h_{i\mu}$ represents deviations from flat space, then the weak field condition can be fulfilled by neglecting quantities of the second and higher orders in $\epsilon$ in the expanded field quantities. For a static field assumption, we are going to assume the vanishing of time derivatives of the field variables. The vector components $Z^\mu$ defined by (3.2) will have the values

$$Z^1 \approx Z^2 \approx Z^3 \approx \epsilon, \quad Z^0 \approx 1 - \epsilon$$  \hspace{1cm} (5.2)

where $\epsilon(\approx \frac{1}{c})$ is a parameter. If we want to add the condition of slowly moving particle to the previous conditions we should neglect quantities of second and higher orders of the parameter $\epsilon$. Thus, in expanding the quantities of the path equation (4.3) we are going to neglect quantities of orders $\epsilon^2$, $\epsilon^2$, $\epsilon \epsilon$ and higher, and also time derivatives of the field variable are to be neglected. To the first order of the parameters, the only field quantities that will contribute to the path equation (4.3) are given by

$$A_{00, i} = -\epsilon h_{00, i}, \quad (i = 1, 2, 3)$$  \hspace{1cm} (5.3)

$$\{0_{i0}\} = \frac{\epsilon}{2} Y_{00, i}, \quad (i = 1, 2, 3)$$  \hspace{1cm} (5.4)

where $Y_{\mu\nu}$ is defined by

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon Y_{\mu\nu}.$$
$g_{\mu\nu}$ is given by (2.2) and $\eta_{\mu\nu}$ is the Minkowski metric tensor. Substituting from (5.3),(5.4) into (4.3) we get after some manipulations:

$$\frac{d^2 x^i}{d\tau^2} = -\frac{1}{2} \epsilon \left( 1 - \frac{n}{2} \alpha \gamma \right) Y_{00,i} Z^0 Z^0. \quad (5.5)$$

In the present case, the metric of the Riemannian space, associated to AP-space, can be written in the form,

$$\left( \frac{d\tau}{dt} \right)^2 = c^2 \left( 1 + \epsilon Y_{00} \right). \quad (5.6)$$

Substituting from (5.6) into (5.5) we get after some manipulations:

$$\frac{d^2 x^i}{dt^2} = -\frac{c^2}{2} \epsilon \left( 1 - \frac{n}{2} \alpha \gamma \right) Y_{00,i} \quad (i = 1, 2, 3)$$

which can be written in the form,

$$\frac{d^2 x^i}{dt^2} = -\frac{\partial \Phi_s}{\partial x^i} \quad (i = 1, 2, 3), \quad (5.7)$$

where

$$\Phi_s = \frac{c^2}{2} \epsilon \left( 1 - \frac{n}{2} \alpha \gamma \right) Y_{00}. \quad (5.8)$$

Equation (5.7) has the same form as Newton’s equation of motion of a particle in a gravitational field having the potential $\Phi_s$ given by (5.8), which differs from the classical Newtonian potential. In the case of macroscopic particles ($n = 0$), we get from (5.8):

$$\Phi_s = \frac{c^2}{2} \epsilon Y_{00} = \Phi_N \quad (5.9)$$

where $\Phi_N$ is the Newtonian gravitational potential obtained from a similar treatment of the geodesic equation. Thus (5.8) can be written in the form

$$\Phi_s = \left( 1 - \frac{n}{2} \alpha \gamma \right) \Phi_N. \quad (5.10)$$
This last expression shows that the gravitational potential felt by the spinning particle is less than that felt by a spinless particle or a macroscopic test particle. In other words, the Newtonian potential is reduced for spinning particles, by a factor \( (1 - \frac{n^2}{2} \alpha \gamma) \).

6 Summary and Discussion

The marriage between the two philosophical ideas of the present century, quantization and geometrization, has never been successful so far. It is well known that quantization is successful in the domain of microphysics, while geometrization is successful in the domain of macrophysics, astrophysics and cosmology. For example, the dynamics of microscopic particles is well described within the framework of quantization, while the dynamics of macrophysical systems is described successfully in the framework of geometrization. The question now is: what is the best description of the dynamics of microscopic particle in a background gravitational field? Several authors have tried to answer this question, as mentioned in section 1. The problem is that their trails neither represent quantization of geometry, nor represent geometrization of quantum mechanics.

A solution of this problem may be, either starting by a certain geometry and using an appropriate procedure to see whether we can discover quantum features in this geometry; or starting in the quantum domain and using an appropriate procedure to look for geometric features. We believe that any appropriate geometry describing nature should contain paths that are quantized naturally, i.e. without using any quantization scheme, but how to discover such paths? The trial given in the present work represents a step in this direction. Although this trial is still far beyond quantization of geometry, it gives a
strong evidence that paths in the AP-geometry are, in some sense, quantized. We believe that this result is valid for any non-symmetric geometry (geometry with torsion), but this statement needs confirmation. Furthermore one can consider the present work as a step towards unification of the dynamics of microscopic and macroscopic particles, since the new path equation (3.5) could be applied for, macroscopic or microscopic, massive or massless, and for spinning or spinless particles.

AP-spaces are defined as spaces whose structure, in four dimensions, is defined completely by a tetrad vector field subject to the condition (2.3) (cf. Robertson (1932)). There is no complete agreement between authors on whether the AP-spaces are, in general, curved or flat. Because of (2.15) many authors believe that these spaces are flat (cf. Hayashi and Shirafuji (1979)). Others believe that these spaces are curved (cf. Mikhail and Wanas (1977)). The cause is that for a non-symmetric geometry (with a non-symmetric affine connexion) the curvature is not uniquely defined. However, all curvature tensors, in this space, are defined in terms of the torsion tensor, and consequently the vanishing of the torsion will lead to a flat space (Wanas and Melek(1995)). It is clear that the present work solves this controversial problem. The geometric structure established in the present work has the following features:

1. The structure of the space is defined completely (in 4-dimensions) by using a tetrad subject to the condition (2.3) and thus, by definition, we are still using an AP-geometry. So all what is done, from the geometric point of view, is that some hidden structures in this geometry are illuminated. We are going to call the structure developed in section 2 the Parametric Absolute Parallelism (PAP)-Space.
(2) It is an affinely connected space endowed by a general non-symmetric connexion (2.5). Thus the space possesses sufficient structure to carry out the operations of tensor analysis.

(3) At any point of the PAP-space we can define a metric tensor (2.2) which can be used to carry out the operations of raising and lowering indicies.

(4) The space is certainly curved since the curvature tensors (2.9) and (2.11), corresponding to the non-symmetric connexion (2.5), are all non-vanishing tensors. It is to be considered that other curvature tensors could be defined in the PAP-space by generalizing the AP-derivatives. All these tensors are non vanishing and reduce to Riemann-Christoffel curvature tensor, of the Riemannian geometry, as $b=0$.

(5) Paths in this geometry, corresponding to the connexion (2.5), are characterized by equation (3.5), which is considered as a generalization of the geodesic equation of Riemannian geometry.

(6) The new structure covers both the Riemannian geometry ($a = 1, b = 0$) and the conventional AP-geometry ($a = 0, b = 1$). Thus it is more general than both geometries. The new structure can be used to solve the problem (raised by Wanas and Melek(1995)) of constructing field theories, in non-symmetric geometries, that may be reduced to GR for $b = 0$ without any need for vanishing torsion.

The torsion of the space-time is assumed to be generated by rotating sources. Many authors believe in this statement (cf. Ross (1989)) . We beleive that torsion and metric tensors are both generic features of the gravitational field whether or not its source is rotating. Calculations show that, a sphecially symmetric solution for a version of general relativity written in AP-space, torsion has non-vanishing components while the source of
the field is non-rotating (cf. Wanas (1990)).

It is widely accepted that scalar particles (or macroscopic objects) cannot feel the torsion. As stated by some authors, scalar test particles detect the metric of the space while rotating test particles detect the torsion (cf. Nitch and Hehl (1980)). This is similar to the situation that neutral particles cannot feel the existence of an electromagnetic field. The question now is what is the intrinsic property of the test particle that interacts with the background field? In case if background electromagnetic field the property is the electric charge. Similarly if the background field is gravitational then the intrinsic property is the mass-energy of the particle which interacts with the metric and/or the spin of the particle which interacts with the torsion.

It has been shown (Wanas et al. (1995)b) that the results of applying the new path equation (3.5), in the solar system, are consistent with the observational bases of GR.

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