Suspensions with reduced violin string modes

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Abstract. We discuss the possibility of significantly reducing the number and Q-factor of violin string modes in the mirror suspension. Simulations of a bar-flexure suspension and an orthogonal ribbon have shown a reduction in the number of violin string modes when compared to a normal ribbon suspension. By calculating the expected suspension thermal noise, we find that the orthogonal ribbon provides a promising suspension alternative. A lower number of violin modes oscillating in the direction of the laser and a reduction in violin mode peak values of at least 23dB can be achieved with a slight increase in thermal noise above 40Hz.

1. Introduction
In order to achieve desired sensitivities in second generation interferometric gravitational wave detectors [1], thermal noise must be reduced below the standard quantum limit at frequencies higher than several hundred Hz. Consequently, materials of very low intrinsic loss such as fused silica [2, 3] and silicon [4] have been proposed for use as mirror suspension elements. The problem that can result from a low loss suspension material and hence very high Q-factor violin modes is that concerning unwanted excitation of the suspension. High amplitude vibrations with long ring down times can result in saturation effects and servo instabilities. Preventing instabilities induced from violin mode excitation requires the use of high Q notch filters at each resonant frequency. This further increases the complexity of the interferometer control system.

We discuss a possible solution to this problem whereby the geometry in the suspension element is chosen to reduce the Q and number of violin modes in the detection band. One can understand how this is achieved by considering the mechanics of a test mass suspended by normal ribbons. Given that the ribbons are loaded close to their breaking stress, bending of the ribbon is confined to a region very near to its two ends. The idea is then to alter the geometry of the central region where almost no pendulum mode bending of the ribbon occurs. Since the modification occurs in a region of very little pendulum mode bending, this modification will have only a small effect on the pendulum mode stiffness. It will however have a significant affect on the violin mode restoring force changing the violin mode frequencies and Q values.

The two suspension types that are introduced in the following section are the bar-flexure suspension and the orthogonal ribbon. Using an analytical computer program that extends a program written by Mark Barton [5, 6], we analyse the violin modes and simulate the suspension thermal noise for these two suspension types. The results of this analysis is presented in this paper.
2. New Suspensions
The suspensions that are discussed in this article differ from typical suspension wires or ribbons. The suspension elements consist of three sections; two flexures at each end and a central section. A diagram of a general triple sectioned suspension element is shown in Figure 1.

![Figure 1. Triple section suspension element](image)

The definitions for the ‘x direction’ and ‘y direction’ that is used to describe the direction of violin mode oscillations later in this article is illustrated in Figure 2. The x direction is defined as the direction in line with the laser, while the y direction is defined as the direction perpendicular to the laser in the plane of the interferometer. The two flexures at each end of the suspension are orientated such that their thin dimension is in the x direction.

When sufficiently loaded, almost all of the bending required for the pendulum mode of the suspended optic occurs in the two end flexures. This means that the pendulum mode Q factor is similar to that expected from a normal ribbon suspension of the same cross section dimensions as the end flexures. Since the geometry of the central section should have very little effect on the pendulum mode Q factor, we are free to alter these dimensions in order to improve the suspension thermal noise in terms of the violin string modes.

The two suspension types that will be compared to a normal ribbon suspension are the bar-flexure suspension and the orthogonal ribbon suspension. These are described below.

2.1. Bar - Flexure
A bar-flexure suspension element is illustrated in Figure 3a. In our modelling of this suspension type, we are considering four of these niobium bar-flexure elements supporting a 4.2kg test mass as will be used at AIGO. Niobium is chosen because it is the lowest loss metal. By using niobium, fabrication of the suspensions can be achieved through electric discharge machining or diffusion bonding. The total length of the suspension is 300mm. The end flexures are 3mm wide and 25µm thick and hence loaded at approximately 50% of the breaking stress of niobium. The central bar has a square cross section area of dimensions 3mm by 3mm.

2.2. Orthogonal Ribbon
Figure 3b illustrates an orthogonal ribbon. Again, we model four of these niobium orthogonal ribbons suspending a 4.2kg test mass. As for the bar-flexure suspension, niobium provides a
Figure 2. Definitions of the spatial coordinates used in this article. The x direction is defined as the direction in line with the laser, while the y direction is defined as the direction perpendicular to the laser in the plane of the interferometer.

good solution in terms of being low loss and relatively simple to machine. The total length of the suspension is 300mm while the end flexures are 3mm wide and 25µm thick. The aspect ratio of the central section is the same as that of the flexures, 120, except that the 25µm dimension of this ribbon section is orthogonal to the direction of the laser field and the end flexures.

Figure 3. a) Bar-Flexure suspension. b) Orthogonal ribbon suspension.
3. Calculating the Pendulum mode Dilution Factor

The use of flexures at each end of the suspensions allow the use of stiffer central section geometries without seriously compromising the pendulum mode stiffness. Thinner flexures exhibit lower bending stiffness, and hence store a lower amount of energy compared to the stored gravitational energy. Gravitational potential energy results from the vertical displacement of the optic through its pendulum mode swing. The term ‘dissipation dilution factor’ is given to the ratio of elastic energy to total energy (elastic plus gravitational). Since it can be considered that the gravitational restoring force contributes no loss, less bending energy storage results in a lower dilution factor and hence a higher pendulum mode Q factor.

We investigate the effect these triple section suspension geometries have on the pendulum mode dissipation dilution factor by calculating the stored elastic potential energy. The stored elastic energy for a typical suspension element bending in a single plane is calculated by solving the bending equation shown in Eq. (1). Four boundary conditions are defined to simulate the end conditions that would occur during pendulum mode flexing.

\[
\lambda^2 Y'''(z) - Y''(z) = 0
\]  

(1)

Here \( z \) is the position along the suspension length, \( Y \) the transverse displacement perpendicular to \( z \), and \( \lambda \) is the characteristic bending length. The value \( \lambda \) is given by Eq. (2), where \( E \) is the Young’s Modulus, \( I \) the area moment of inertia and \( F_T \) the tension. The cross section area of the suspension element is used to determine the area moment of inertia and is dependant on the bending direction. For a rectangular cross section bending in the x direction, the area moment of inertia is given by \( I = \frac{1}{12} a^3 b \), where the values \( a \) and \( b \) are the x and y suspension element dimensions respectively.

\[
\lambda = \sqrt{\frac{EI}{F_T}}
\]  

(2)

The potential energy is calculated by integrating along the suspension length as shown in Eq. (3).

\[
P_{E_{bend}} = \frac{1}{2} EI \int_0^L Y(z)^2 dz
\]  

(3)

To determine the elastic potential energy of a triple section suspension element, a set of three bending equations must be solved. These are given in Eq. (4) where the subscripts 1, 2 and 3 represent the three sections of the suspension, i.e. the two end flexures and the central section. To solve these equations, 12 boundary conditions are required, two at each end to simulate pendulum mode bending, and four at each section interface to ensure displacement, angle, moment and force continuity.

\[
\begin{align*}
\lambda_1^2 Y_{1}'''(z) - Y_{1}''(z) &= 0 \\
\lambda_2^2 Y_{2}'''(z) - Y_{2}''(z) &= 0 \\
\lambda_3^2 Y_{3}'''(z) - Y_{3}''(z) &= 0
\end{align*}
\]  

(4)

The potential energy is calculated by integrating along the entire length of the suspension as shown in Eq. (5). \( I_1, I_2 \) and \( I_3 \) are the area moment of inertia values for each section.

\[
P_{E_{bend}} = \frac{1}{2} E \left( I_1 \int_0^{L_1} Y_1(z)^2 dz + I_2 \int_{L_1}^{L_2} Y_2(z)^2 dz + I_3 \int_{L_2}^{L} Y_3(z)^2 dz \right)
\]  

(5)

The dilution factor is determined by calculating the fraction of bending potential energy, \( P_{E_{bend}} \), to the total potential energy, \( P_{E_{grav}} + P_{E_{bend}} \). Typically dilution factors range from
0.002 to 0.005 for wire suspensions. The Q-factor is effectively scaled by the reciprocal of the dilution factor, thus for a dilution factor of 0.002, the measured Q-factor would by approximately 500 times greater than the Q-factor limited by energy dissipation processes from the suspension material.

4. Comparison of Pendulum mode Dilution Factors
For triple section suspension elements, short flexures and stiff central sections are desirable in order to reduce the violin mode density and Q-factors. However these dimensions have a bearing on the pendulum mode Q-factor, a value that we do not want to significantly degrade.

![Diagram of niobium bar-flexure pendulum mode dilution factor](image)

Figure 4. Dissipation dilution factor for three niobium bar-flexure suspensions of total length 300mm supporting a 4.2kg test mass. This is compared to the dilution factor of a niobium normal ribbon of width length 300mm, width 3mm and thickness 25µm. The flexure length is not relevant to the normal ribbon suspension.

The dilution factor for a niobium normal ribbon of length 300mm, width 3mm and thickness 25µm compared to a niobium bar-flexure suspension with varying bar thicknesses of length 300mm is shown in Figure 4. The simulation involves a 4.2kg test mass being supported by four suspension elements as discussed earlier. In all bar-flexure cases, it can be seen that a larger bar thickness or a shorter flexure will increase the dilution factor and hence degrade the pendulum mode Q-factor compared to that expected from the ribbon suspension. This occurs due to a significant increase in the pendulum mode bending stiffness. However, the increase in dilution factor can be kept well below 1% for bar thickness up to 3mm if flexure lengths are 1mm long.

Figure 5 illustrates the dilution factor of a niobium orthogonal ribbon with varying central section width compared to a niobium normal ribbon suspension. The width of the orthogonal section is the dimension in line with the laser, as is illustrated in figure 5. The cross section area of the orthogonal region is kept constant at $7.5 \times 10^{-8} m^2$ for the various orthogonal section widths. As for the bar-flexure case, the simulation assumes a 4.2kg test mass being supported by four suspension elements 300mm in length. Again, the increase in dilution factor can be kept below 1% for a 3mm thick central section for flexures that are at least 1mm long.

5. Violin modes
The violin modes up to 5kHz for the niobium normal ribbon, the niobium orthogonal ribbon and the niobium bar-flexure suspension are shown in Figure 6. It can be seen clearly that the violin mode density is reduced when using the orthogonal ribbon or bar-flexure suspension as a replacement for the normal ribbon. The orthogonal ribbon and the normal ribbon exhibit a
Figure 5. Dissipation dilution factor for three niobium orthogonal ribbon suspensions of total length 300mm supporting a 4.2kg test mass. This is compared to the dilution factor of a niobium normal ribbon of width length 300mm, width 3mm and thickness 25µm. The flexure length is not relevant to the normal ribbon suspension.

similar total number of violin modes, (31 and 32 respectively), however an important difference is that for the normal ribbon, 23 of the modes oscillate in the direction of the laser field (x direction), while in the orthogonal ribbon only 9 oscillate in this direction. Oscillations in the direction orthogonal to the laser field (y direction) can be tolerated to a larger degree since y to x coupling is accepted to be approximately 0.1% in an advanced detector [7].

Figure 6. The violin mode frequencies of three different Nb suspension types; normal ribbon, orthogonal ribbon and the bar-flexure suspension. The x direction is the direction of the laser field, while the y direction is orthogonal to the direction of the laser field.

The increased stiffness in both the x and y direction of the bar-flexure suspension results in a lower density of violin modes in both directions, 8 violin modes in the x direction and 6 violin modes in the y direction. Unfortunately, the fundamental violin modes in both directions (x direction - 70Hz, y direction - 95Hz) are much lower in frequency compared to the normal ribbon
(x direction - 210Hz, y direction - 255Hz) or orthogonal ribbon (x direction - 218Hz, y direction - 213Hz). This is due to the larger linear mass density of the bar. The mode frequencies can be made higher with shorter or thicker end flexures, however this modification comes at a cost of decreased pendulum mode Q-factor. In fact the high linear mass density of the bar-flexure suspension results in a larger amount of suspension thermal noise coupling to displacement noise on the optic. For this reason, the bar-flexure suspension is not a useful replacement for a normal ribbon suspension, despite a much lower violin mode density. This is discussed in the following section.

6. Thermal Noise

A plot of the expected suspension thermal noise from a 4.2kg test mass supported by four bar-flexure suspensions shows a significant increase at frequencies higher than 10Hz. A comparison of the niobium bar-flexure suspension to the niobium normal ribbon is shown in Figure 7. The increase in thermal noise is caused by a higher linear mass density of the central section coupling heavily to x direction motion at the test mass. For this reason, the bar-flexure suspension is not an adequate method for test mass suspension.

![Expected Suspension Thermal Noise](image.png)

Figure 7. Expected suspension thermal noise from a niobium bar-flexure suspension compared to a normal ribbon. Both cases involve four suspension elements supporting a 4.2kg AIGO size test mass.

The orthogonal ribbon however provides a much more promising result. The expected suspension thermal noise from a 4.2kg test mass supported by four niobium orthogonal ribbons is illustrated in Figure 8. As expected, there is virtually no noticeable increase in pendulum mode thermal noise below 40Hz. The spectrum illustrates the lower number of x direction violin modes for the orthogonal ribbon, 9, compared to 23 for the normal suspension. The violin mode peak values of thermal noise are also lower, and correspond to a reduction in amplitude of more than 23dB. A small increase in thermal noise of a factor of approximately 5 is noticeable above 40Hz and is due to a lower fundamental violin mode Q-factor. This increase in thermal noise is the cost of obtaining lower amplitude violin mode peaks using the orthogonal ribbon.

In Figure 9, the expected suspension thermal noise spectrum has been modelled for a 30kg test mass being supported by 100µm thick fused silica ribbons and orthogonal ribbons. Again, a reduction in x direction violin modes, from 12 to 5, can be observed when comparing the orthogonal ribbon to the normal ribbon. A reduction in peak amplitude of about 21dB is
Figure 8. Expected suspension thermal noise from a niobium orthogonal ribbon suspension compared to a normal ribbon. Both cases involve four suspension elements supporting a 4.2kg AIGO size test mass.

evident along with a small increase in thermal noise above 100Hz of less than a factor of 2.7. For this suspension, it is likely that the small increase in thermal noise will still fall below the shot noise level for an advanced detector. The expected quantum noise for Advanced LIGO is also shown in Figure 9 for comparison with the thermal noise spectrums. At 100Hz, the quantum noise limited strain sensitivity for the 4km Advanced LIGO arm is expected to be higher than $h(f) = 10^{-24}/\sqrt{\text{Hz}}$ [8]. This is equivalent to a displacement sensitivity of $4 \times 10^{-21}\text{m}/\sqrt{\text{Hz}}$, a factor of four higher than the fused silica orthogonal ribbon suspension thermal noise at this frequency.

Figure 9. Expected suspension thermal noise from a fused silica orthogonal ribbon suspension compared to a fused silica normal ribbon. For this case, the test mass has been scaled up to 30kg. The expected quantum noise spectrum in an advanced detector is also illustrated.
7. Conclusion

We have discussed the use of two suspension elements comprising of three sections to suspend test mass in interferometric gravitational wave detectors. One of these, the orthogonal ribbon shows promise as an alternative to normal ribbon suspensions. By constructing a ribbon such that the central section is rotated 90 degrees around the vertical axis, a lower violin mode density and lower violin mode peak amplitudes can be achieved. The analysis of appropriate niobium orthogonal ribbons for supporting a 4.2kg test mass, as will be used at AIGO, has been modelled. The simulated suspension thermal noise spectrum illustrates a reduction in the number of x direction violin modes to 9, compared to 23 for the normal niobium ribbon suspension. A reduction in violin mode amplitude by at least 23dB is also observed. The spectrum also supports the prediction that these new suspension designs will not significantly increase pendulum mode thermal noise below 40Hz. The lower level of peak violin mode thermal noise presented by the orthogonal ribbon geometry is likely to reduce the risk of saturation or instabilities resulting from unwanted violin mode oscillation.

These improvements come at a cost of increased thermal noise above 40Hz, which is due to the lower Q-factor of the violin modes. By scaling the suspended test mass to 30kg, and modifying the niobium orthogonal ribbons to fused silica orthogonal ribbons capable of supporting this mass, we find similar advantages as for the niobium case. A lower violin mode density and lower violin mode Q factors accompany a slight increase in thermal noise above 100Hz. It is likely however, that the slight increase in thermal noise for this advanced orthogonal ribbon suspension, will still remain below the shot noise level.

Acknowledgments

This work is supported by the Australian Research Council and is part of the Australian Consortium for Interferometric Gravitational Astronomy (ACIGA). We also acknowledge the help from Mark Barton with regards to the suspension modelling program.

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