The Kinematics Analysis of a Novel Self-Reconfigurable Modular Robot Based on Screw Theory

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ABSTRACT: This paper introduces our newly designed self-reconfigurable modular robot named Larva-Bot, which has two DOFs with intersecting rotation axis normal to each other. Each module has autonomous mobility, and can assemble with other modules to form various configurations. In this study, a typical 6-DOF structure, composed of Larva-Bots, is proposed to analyze the kinematic characteristics of the modular robot. The common kinematics model of the robot is based on the Denavit-Hartenberg (D-H) parameter method, while its inverse kinematics has inefficient calculation and complicated solutions. To solve the problems, the screw theory method is applied in this paper. The forward kinematics model is constructed by using the product of exponentials (POEs) formula, while the inverse kinematics analysis is based on Paden-Kahan Sub-problems. Finally, the reliability of the method has been proved by computations and simulation.

KEYWORDS: Larva-Bot; Modular robot; Reconfigurable robot; Screw theory; POE; Paden-Kahan Sub-problem.

1 INTRODUCTION

In recent years, self-reconfigurable robots have been widely studied\(^1\)\(^2\). The scientists get motivated by the swarm robot behaviors of the insects, which, for example, can hold together to get across a gap\(^3\). As the insects, this kind of robot is constructed of several modules that have self-mobility and also can be assembled into complex configurations with better locomotion performance and sensory ability. It has a very strong adaptability while they can interact between them and the environments or among themselves\(^4\). Some modular robots have been already presented. They can be classified into two kinds. One of them have a very good metamorphism and reconfiguration performance with the help of the other modules, such as M-TRAIN\(^5\), PolyBot\(^6\), Conro\(^7\), Roombot\(^8\), Superbot\(^9\), M-Blocks\(^10\), M-cube\(^11\) etc. However, the mobility of each unit is limited and it can hardly accomplish some functions alone. For the other kind of modular robots, every module can move around, such as Swarm-Bot\(^12\), Sambot\(^13\), Millibot\(^14\) etc, and They can also connect to each other to work together. This kind of robots have better mobility, while the design of their mechanical structures imposes many restrictions on the metamorphism ability. The Swarm-Bot and the Millibot can only form planar construction, and the Sambot only have one DOF for every unit which limits its reconfiguration performance.

In this paper we propose a novel self-reconfigurable modular robot named Larva-Bot. Each unit has 2 DOFs, which insure its deformability, and have two driving wheels, which contribute to the self-mobility. The actuators, sensors, microprocessors communication unit and power are all embedded in the module. With these parts, the Larva-Bot unit has the autonomy to do some exploring job. When it comes across a barrier, it can send signals to other modules. Then the modules who have received the signal will get together with the emitter and assemble themselves into a configuration to deal with the trouble. The study of the kinematic characteristics is the basis for further research. Here we take a 6-DOFs structure that is made up of 3 modules to analyze its kinematics.

Screw theory is widely used in kinematics, dynamics and control of the robot. So in this study the POE formula and the Paden-Kahan Sub-problems are used to solve the kinematics problems of the 6-DOFs modular robot. The screw theory method has some advantages than the conventional Denavit-Hartenberg parameter method. Firstly, the screw theory allows a global description of rigid body motion which does not suffer from singularities due to the use of local coordinates\(^15\). Secondly, that screw theory provides a
very geometric description of rigid motion which simplifies the analysis of mechanisms to great extent\[16\]. In addition, the method generally reduces the whole inverse kinematics problem into appropriate sub-problems which has known solutions.

This paper is organized as follows. Section II introduces the mechanical structure and its function and implementation of our newly designed modular robot. Section III describes the Paden-Kahan sub-problems used in Section IV. Section IV gives forward and inverse Kinematics of the 6-DOF structure using the screw theory. The checking computations and simulation are presented in Section V. The last section summarizes the paper and gives some prospects for future work.

2 CHARACTERS OF LARVA-BOT

2.1 Mechanical structure

A single Larva-Bot module is shown in Figure 1. It can be roughly divided into two parts: one main part that contains microcontroller, battery, motor-driven wheels and servo; accessory part that contains sensors and active grabbing devices. The size of one module is 142*108*90mm. The mechanical structure of Larva-Bot is manufactured by desktop 3-dementional printer. PLA (Poly Lactic acid) plastic material is applied to decrease the weight and guarantee the strength. Each shell is constructed by several separate-made boards which are joined by bolts.

![Figure 1. The general schematic diagram of the Larva-Bot.](image)

2 degrees of freedoms work when a single module is combined in a multi-module system, including the rotate of servo in the main part and rotate of the disk that supports the active grabbing claws. The former has a range of 180 degrees and the latter has 360 degrees but rotates slower. When a module works as an individual, another 3 DOFs appear. Four claws implement radial motion simultaneously and thus two modules can be tightly connected. The module also possesses two symmetrical wheels in order to move on the ground. An omnidirectional wheel is added to support the accessory part.

Figure 2(a) indicates the inner structure of main part. A module has 3 negative grabbing devices with one on backboard and another two are on each sides of accessory part. Each wheel is linked with a gear motor and the motor is fastened on the support board. There is a separate space used to place the battery. On the left side, servo motor actuates the left body via helm and on the other side, a bearing is placed to support the right body, thus the rotate axis is determined.

![Figure 2. The exploded view of Larva-Bot module.](image)

Structure of the accessory part is shown in Figure 2(b). A stepper motor is fixed on the box body and the gear on it mates with the tooth around the edge of disk. The disk, spiral part, motor and motor support rotate as a whole and are driven by the stepper. The motor in middle is used for rotating the spiral. As shown in Figure 2(c), the bottom of the claw can mate with the spiral, insuring the follow-up motion created by the rotation of spiral. Because of the lamination of sideways, the claws merely have radial motion. This structure has advantage of self-locking, while external force on the claws cannot move them.

2.2 Function and implementation

![Figure 3. Larva-Bots constructa quadruped structure.](image)

Each module has its sensors that can detect the environment. When it encounters a barrier which is
too hard to get across, the processor inside the module will decide a configuration to deal with. Then it transmits a certain signal to other modules to get them together with a formation. Our modules can determine the location and pose of others via communication and then automatically move and connect with others. To realize accurate positioning when connecting, the module has 2 infrared emission tubes on the disk (active connecting device) and 2 receivers on each of the negative devices. Taking the received signal as feedback, modules are able to slightly adjust their position when two modules waiting for connection are near enough. Finally modules reach the perfect position and the disk turns in the right angle for grabbing work. Then the 4 concaves on negative devices mate with convex part of the claws. After the radial motion of the claws, they can tightly grab the concaves and thus two modules are successfully connected. When they connect to each other, they can move with their configurations instead of the wheels, resulting in more powerful functions.

3 PADEN-KAHAN SUB-PROBLEMS

Now there exist three Paden-Kahansub-problems [17]. Then we will give an introduction of the first two sub-problems which are used in this paper.

![Figure 4. Paden-Kahan Sub-problem 1.](image)

(a) Screw motion  
(b) Projection normal to $\xi$

Figure 4. Paden-Kahan Sub-problem 1.

1) Sub-problem 1: Rotation about an axe

As is shown in figure 4, $\xi$ is screw motion axis with zero pitch and $\omega$ is unit vector in that direction. There exist two points $p, q \in \mathbb{R}^3$. The point $p$ rotates around $\xi$ by $\theta$, thus it coincides with the point $q$.

We take a point $r$ in the $\xi$ and define $u = (p - r), v = (q - r)$. $u', v'$ is the mapping point of $u, v$ in the projective plane perpendicular to $\xi$.

$$u' = u - \omega t u \tan \theta = v - \omega t v$$

The $\theta$ that meets the conditions can be gotten from (1):

If $u' \neq 0$,

$$u' \times v' = \omega \sin \theta \parallel u' \parallel v' \parallel$$

$$u' \cdot v' = \cos \theta \parallel u' \parallel v' \parallel$$

$$\theta = \arctan (\omega^T(u' \times v'), u^T v') \quad (1)$$

If $u' = 0, \theta$ has infinitely many solutions.

![Figure 5. Paden-Kahan Sub-problem 2.](image)

2) Sub-problem 2: Rotation about two axe in order

As is shown in figure 5, $\xi_1$ and $\xi_2$ are two intersecting screw motion axis with zero pitch. $\omega_1, \omega_2$ are two unit vectors in their directions. There exist two points $p, q \in \mathbb{R}^3$. The point $p$ rotates around $\xi_2$ by $\theta_2$, then around $\xi_1$ by $\theta_1$, thus coincides with the point $q$.

From figure (5) we can know

$$\omega_2^T u = \omega_2^T z, \omega_1^T v = \omega_1^T z, \parallel u\parallel = \parallel z\parallel \quad (2)$$

To obtain $\theta_1$ and $\theta_2$, the coordinate value of point $c$ should be gotten. We define $z = c - r$, so

$$z = \alpha \omega_1 + \beta \omega_2 + \gamma (\omega_1 \times \omega_2) \quad (3)$$

$$\parallel z\parallel = \alpha^2 + \beta^2 + 2\alpha \beta \omega_1^T \omega_2 + \gamma^2 \parallel \omega_1 \times \omega_2 \parallel \quad (4)$$

From (2), (3) and (4), we can obtain

$$\alpha = \frac{(\omega_1^T \omega_2) \omega_2^T u - \omega_1^T T v}{(\omega_1^T \omega_2)^2 - 1}$$

$$\beta = \frac{(\omega_1^T \omega_2) \omega_1^T u - \omega_2^T T u}{(\omega_1^T \omega_2)^2 - 1}$$

$$\gamma = \pm \sqrt{\frac{||u||^2 - \alpha^2 - \beta^2 - 2\alpha \beta ||(\omega_1^T \omega_2)||}{||\omega_1 \times \omega_2||^2}} \quad (5)$$

where $y$ can have no real root, one root or two roots.

Then the solution can be obtained according to sub-problem 1.
4 KINEMATICS OF THE 6-DOF STRUCTURE

4.1 Kinematic modeling

Firstly we construct the twist of each joint.

\[
\begin{align*}
\omega_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\omega_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\omega_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
r_1 &= r_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
r_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
r_4 &= r_5 = r_6 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
\xi_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\xi_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\xi_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\xi_4 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\xi_5 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\xi_6 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{align*}
\]

The initial configuration of Figure 6 is

\[
g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2l \\ 0 & 0 & 0 & 1 \end{bmatrix}
\] (7)

Then the forward kinematics of the 6-DOF structure is

\[
g_{st}(\theta) = e^{\xi_1 \theta_1} e^{\xi_2 \theta_2} e^{\xi_3 \theta_3} e^{\xi_4 \theta_4} e^{\xi_5 \theta_5} e^{\xi_6 \theta_6}
\] (8)

while

\[
e^{\xi_1 \theta_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_1 & -\sin \theta_1 & 0 \\ 0 & \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
e^{\xi_2 \theta_2} = \begin{bmatrix} 0 & \cos \theta_2 & -\sin \theta_2 & 0 \\ 0 & \sin \theta_2 & \cos \theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
e^{\xi_3 \theta_3} = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & l \cdot \sin \theta_3 & 0 \\ \sin \theta_3 & \cos \theta_3 & l \cdot (1 - \cos \theta_3) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
e^{\xi_4 \theta_4} = \begin{bmatrix} \cos \theta_4 & -\sin \theta_4 & 0 & 0 \\ \sin \theta_4 & \cos \theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
e^{\xi_5 \theta_5} = \begin{bmatrix} \cos \theta_5 & -\sin \theta_5 & 2l \cdot \sin \theta_5 & 0 \\ \sin \theta_5 & \cos \theta_5 & 2l \cdot (1 - \cos \theta_5) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
e^{\xi_6 \theta_6} = \begin{bmatrix} \cos \theta_6 & -\sin \theta_6 & 0 & 0 \\ \sin \theta_6 & \cos \theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\] (9)

4.2 Inverse kinematics

In order to achieve certain motion for the modular robot, the end-effector of the 6-DOF structure should be in a corresponding position and pose in (8). Then converting the position and pose to angles of each joint via inverse kinematics is necessary. Then the following part will focus on the inverse kinematics analysis of the 6-DOF structure.

Figure 7. Rotation motion of the first 3 joints.

- Step1: solve $\theta_1, \theta_2, \theta_3$
The last 3 joints axes meet at point $q_w$, then we choose $q_w$ as the study object to solve $\theta_1, \theta_2, \theta_3$. The rotation movements of the last three joint do not affect the position of point $q_w$. So the movement can be simplified as Figure 7. In this case, we can set the known parameters as follows:

$$q_w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad q = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix}$$

(10)

From (8) we can obtain:

$$e^{\xi_1^\theta_1} e^{\xi_2^\theta_2} e^{\xi_3^\theta_3} e^{\xi_4^\theta_4} e^{\xi_5^\theta_5} e^{\xi_6^\theta_6} = g_{st}(0) g_{st}(0)^{-1} = g_1$$

(11)

Multiply (9) by point $q_w$, which is on $\xi_4, \xi_5, \xi_6$. According to the principle of rigid body, its position remains constant when rotating around the axis,

$$q_w = g_1 q_w = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix}$$

(12)

As for axis $\xi_2$ and $\xi_3$, we can apply Paden Kahan sub-problem 2,

$$u_1 = p - r_3$$

$$v_1 = c_2 - r_3$$

$$\alpha_1 = \frac{(\omega_2^T \omega_3) u_1 - \omega_2^T v_1}{(\omega_2^T \omega_3)^2 - 1} = q_w(z) - 1$$

$$\beta_1 = \frac{(\omega_3^T \omega_1) v_1 - \omega_3^T u_1}{(\omega_2^T \omega_3)^2 - 1} = 0$$

$$\gamma_1 = \pm \sqrt{l^2 - (q_w(z) - 1)^2}$$

(13)

$$q_w(z) = \begin{bmatrix} \alpha_1 \omega_2 + \beta_1 \omega_3 + \gamma_1 (\omega_2 \times \omega_3) + r_3 \\ 0 \\ 0 \end{bmatrix}$$

(14)

As for axis $\xi_1$ and $\xi_2$, we can solve this case directly by applying Paden Kahan sub-problem 2,

$$u_2 = q_w(z) - r_1$$

$$v_2 = q - r_1$$

$$\alpha_2 = \frac{(\omega_1^T \omega_2) u_2 - \omega_1^T v_2}{(\omega_1^T \omega_2)^2 - 1} = 0$$

$$\beta_2 = \frac{(\omega_2^T \omega_1) v_2 - \omega_2^T u_2}{(\omega_2^T \omega_1)^2 - 1} = q_w(z)$$

$$\gamma_2 = \pm \sqrt{l^2 - (q_w(z))^2}$$

$$q_w = \alpha_2 \omega_1 + \beta_2 \omega_2 + \gamma_2 (\omega_1 \times \omega_2) + r_1$$

(15)

$$q_w = \begin{bmatrix} \mp \sqrt{-q_w(z)^2 + 2q_w(z)} + q_w(z) \\ 0 \\ 0 \end{bmatrix}$$

(16)

According to the principle of sub-problem 1, the distance between the point and the axis remains unchanged.

$$e^{\xi_1^\theta_1} (q_{w_2} - p_1) = e^{\xi_2^\theta_2} (q_{w_2} - p_1) = e^{\xi_3^\theta_3} (q_{w_2} - p_1) = g_2$$

(17)

Then we can get

$$e^{\xi_4^\theta_4} q_{w_2} - p_1 = e^{\xi_5^\theta_5} q_{w_2} - p_1 = e^{\xi_6^\theta_6} q_{w_2} - p_1 = g_2$$

(18)

which means,

$$q - p_1 = e^{\xi_1^\theta_1} (q_{w_2} - p_1) = e^{\xi_2^\theta_2} (q_{w_2} - p_1) = e^{\xi_3^\theta_3} (q_{w_2} - p_1) = g_2$$

(19)

Combine (14)(15)(19) we can get the coordinates of point $q_{w_1}$ and $q_{w_2}$. According to the coordinates, we apply Paden Kahan sub-problem 1 to obtain $\theta_1, \theta_2$ and $\theta_3$. Suppose $\tau_1 = q_w(z) - r_1$ and $\tau_2 = q_w(z) - r_1$, we can obtain:

$$\theta_1 = \arctan(\omega_1^T (u_2 \times \tau_2), u_2^T \tau_2)$$

(20)

$$\theta_2 = \arctan(\omega_2^T (\tau_2 \times u_2), \tau_2^T \tau_2)$$

$$\theta_3 = \arctan(\omega_3^T (\tau_2 \times u_2), \tau_1^T \tau_2)$$

From (9) we can know

$$e^{\xi_1^\theta_1} e^{\xi_2^\theta_2} e^{\xi_3^\theta_3} e^{\xi_4^\theta_4} e^{\xi_5^\theta_5} e^{\xi_6^\theta_6} = e^{-\xi_4^\theta_4} e^{-\xi_5^\theta_5} e^{-\xi_6^\theta_6} g_1 = g_2$$

(21)

Take a point $P_6$, which is on $\xi_6$,

$$P_6 = \begin{bmatrix} 0 \\ 0 \\ 3l \end{bmatrix}$$

(22)

Multiply (22) by $P_6$ on each side,

$$e^{\xi_4^\theta_4} e^{\xi_5^\theta_5} e^{\xi_6^\theta_6} P_6 = g_2 P_6 \quad q_6 = \begin{bmatrix} q_{x6} \\ q_{y6} \\ q_{z6} \end{bmatrix}$$

(23)

Solve the equation by applying Paden Kahan sub-problem 2:
\[ u_3 = P_6 - r_5 \]
\[ v_3 = q_6 - r_5 \]
\[ a_3 = \frac{(a_3^T \omega_3^T) u_3 - a_4^T v_3}{(a_3^T \omega_3^T)^2 - 1} = q_{z6} - 2l \]
\[ \beta_3 = \frac{(a_4^T \omega_3^T) v_3 - a_3^T u_3}{(a_4^T \omega_3^T)^2} = 0 \]
\[ \gamma_3 = \pm \sqrt{\|u_3\|^2 - a_3^2 - \beta_3^2 - 2a_3\beta_3 \omega_3^T} \]
\[ \omega_3 = \pm \sqrt{(q_{z6} - 2l)^2} \]
\[ q_{w3} = \alpha_3 \omega_4 + \beta_3 \omega_5 + \gamma_3 (\omega_4 \times \omega_5) + r_5 \]
\[ = \pm \sqrt{(q_{z6} - 2l)^2} \]

Following equations can be obtained
\[ e^{\xi \theta_5} P_6 = q_{w3} \]
\[ e^{\xi \theta_4} q_{w3} = q_6 \]

Apply Paden Kahan sub-problem to (26) to solve \( \theta_4, \theta_5 \). We suppose \( \tau_3 = q_{w3} - r_5 \)
\[ \theta_4 = \arctan (\frac{\omega_4^T (\tau_3 \times u_3), \tau_3^T u_3}{q_{w3} - 2l}) \]
\[ \theta_5 = \arctan (\frac{\omega_5^T (\tau_3 \times u_3), \tau_3^T u_3}{q_{w3} - 2l}) \]

Step 2: solve \( \theta_6 \)

From (21), we can obtain
\[ e^{\xi \theta_6} = -e^{\xi \theta_5} e^{\xi \theta_4} g_2 = g_3 \]

Pick point \( P_t \), which is not on \( \xi_6 \),
\[ P_t = \begin{bmatrix} 500 \\ 0 \\ 2l \end{bmatrix} \]

Multiply (28) by \( P_t \) in right:
\[ e^{\xi \theta_6} P_t = g_3 P_t = q_t = \begin{bmatrix} q_{xt} \\ q_{yt} \\ q_{zt} \end{bmatrix} \]

Obtain \( \theta_6 \) by applying the first Paden-Kahan sub-problems
\[ u_4 = P_t - r_6 \]
\[ v_4 = q_t - r_6 \]
\[ \theta_6 = \arctan (\omega_6^T (u_4 \times u_3), u_4^T u_3) \]
\[ = \arctan (\frac{q_{yt}}{q_{zt}}) \]

In sum, the full inverse-kinematic solutions are solved by applying the Paden-Kahan sub-problems. In the process of solving, we can choose the more suitable and simple reference points, such as (10), (22) and (29), resulting in a simplification of the calculating process and an improvement of the computation efficiency. In addition, the obvious geometric meaning allows us to select the desired and reasonable solutions easily.

5 SIMULATION AND RESULT

To verify the correctness of the result, we choose an arbitrary group of joint angles to have a test. The feasible angle range of the 1st, the 3rd and the 5th joints is set within range \((-\pi/2, \pi/2)\), while the 2nd, 4th and the 6th joints is within \((-\pi, \pi)\). We did the verification work through the following steps.

Step1: set a group of angles arbitrarily:
\[ \theta_1 = 0, \theta_2 = \frac{\pi}{4}, \theta_3 = \frac{\pi}{2}, \theta_4 = \frac{\pi}{3}, \theta_5 = \frac{\pi}{6} \]

Step 2: get the position and pose of the end effector through the POE formula of forward kinematics:
\[ g_{st}(0) = \begin{bmatrix} 0.12941 & 0.22413 & 0.96593 & 98.287 \\ -0.48297 & -0.83651 & 0.25883 & -98.287 \\ 0.86605 & -0.50000 & 0 & 139 \end{bmatrix} \]

Step 3: get inverse solutions of the 6 angles when the end effector is at the position and pose.

Table 1. Eight solutions of inverse kinematics.

| NO. | Joint \( \theta \)(rad) | Joint \( \theta_4 \)(rad) |
|-----|-------------------------|-------------------------|
| 1   | 1.0072 4 1.5657        |
| 2   | 0.7905 5 1.0459        |
| 3   | 1.5657 6 0.5225        |
| 2   | 0.0072 4 -1.5657       |
| 3   | 1.5657 6 -2.6191       |
| 1   | 0.0072 4 -1.5739       |
| 3   | -1.5657 6 0.5214       |
| 1   | 0.0072 4 1.5661        |
| 4   | -2.3511 5 -1.0999      |
| 3   | -1.5657 6 -2.6200      |
| 1   | 1.2237 4 1.9518        |
| 5   | -0.7905 5 0.7286       |
| 3   | -1.5657 6 -0.9822      |
| 1   | 1.2237 4 -1.1897       |
| 6   | -0.7905 5 -0.7286      |
| 3   | -1.5657 6 -0.9826      |
| 1   | 1.2237 4 -1.1897       |
| 7   | 2.3511 5 -0.7286       |
| 3   | 1.5657 6 -0.9826       |
| 1   | 1.2237 4 1.9518        |

We got 8 sets of solutions, listed in table 1. The relevant simulation results are shown in figure 9. As we can see, the position and pose of the end effector is the same with the first result shown in formula. So the method we use in this paper is steady and accurate. We can choose the most suitable solution according to the optimal conditions such as the shortest movement.
principle or lowest energy principle.

In the future works, the kinematics characteristics of complex configuration should be studied. In addition, velocity and dynamic analysis based on screw theory should be studied.

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