CONSTRAINING LORENTZ VIOLATION USING SPECTROPOLARIMETRY OF COSMOLOGICAL SOURCES

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Spectropolarimetry of distant sources of electromagnetic radiation at wavelengths ranging from infrared to ultraviolet are used to constrain Lorentz violation. A bound of $3 \times 10^{-32}$ is placed on coefficients for Lorentz violation.

Lorentz symmetry is an important part of our current understanding of particle physics. A violation of this symmetry would be a signal of physics beyond the standard model. For example, Lorentz violation can arise in string theory. A general Lorentz-violating standard-model extension has been constructed. It allows for the possibility that the remnants of Lorentz violation occurring at the Planck scale may lead to small violations at energies attainable today. A number of experiments have been performed to test the fermion sector of the theory. The CPT-odd coefficients of the photon sector, which have been constrained experimentally to a high degree of precision, are expected to be zero from theoretical considerations. However, the CPT-even coefficients of the photon sector have received little attention. It is the goal of this work to understand the effects of these coefficients and to place bounds on them using existing spectropolarimetric measurements of distant cosmological sources.

A Lorentz-violating electrodynamics can be extracted from the standard-model extension. We neglect the CPT-odd coefficients for the reasons mentioned above. The relevant Lagrangian is $L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu}$ where $F_{\mu\nu}$ is the usual field strength, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. The second term is CPT even and Lorentz violating. The coefficient $(k_F)_{\kappa\lambda\mu\nu}$ is dimensionless. It has the symmetries of the Riemann tensor and a zero double trace. This leaves 19 independent components.

The equations of motion resulting from this Lagrangian are modified inhomogeneous Maxwell equations. With the aid of the usual homogeneous Maxwell equations, plane-wave solutions of the form $F_{\mu\nu}(x) = F_{\mu\nu}(p)e^{-i p_\nu x^\nu}$ can be found. The modified dispersion relation to leading order in $(k_F)_{\kappa\lambda\mu\nu}$ is

$$p_\pm^0 = (1 + \rho \pm \sigma)|\vec{p}|,$$

where $\rho = -\frac{1}{2} \bar{k}_\alpha^\alpha$ and $\sigma^2 = \frac{1}{2} (\bar{k}_{\alpha\beta})^2 - \rho^2$, with $\bar{k}_{\alpha\beta} \equiv (k_F)^{\alpha\mu\beta\nu} \hat{p}_\mu \hat{p}_\nu$ and $\hat{p}^\mu = p^\mu / |\vec{p}|$. At leading order in $(k_F)_{\kappa\lambda\mu\nu}$, the corresponding solutions for the
electric field, $\vec{E}_\pm$, are orthogonal and each is perpendicular to its group velocity $\vec{v}_\pm \equiv \nabla_{\vec{r}} p_\pm^0$. This implies the unit vectors $\hat{\vec{e}}_\pm \equiv \vec{E}_\pm / |\vec{E}_\pm|$ form a basis for the electric field. The general solution is of the form $\vec{E}(x) = (E_+ \hat{\vec{e}}_+ e^{-ip_0^+ t} + E_- \hat{\vec{e}}_- e^{-ip_0^- t})e^{i\vec{p} \cdot \vec{r}}$. The fact that the phase velocities $\vec{v}_\pm \equiv p_\pm^0 \hat{\vec{p}} / \hat{\vec{p}}^2$ of the two modes differ implies as the light propagates the relative phase between modes changes. The change in relative phase is

$$\Delta \phi = (p_+^0 - p_-^0) t \approx 2\pi \Delta v_p L / \lambda \approx 4\pi \sigma L / \lambda,$$

where $L$ is the distance the radiation traveled and $\lambda$ is its wavelength. This phase change results in a change in the polarization of the radiation. The $L/\lambda$ dependence suggests, for very distant sources producing light at short wavelengths, tiny differences in phase velocity may become detectable.

It is this $L/\lambda$ dependence that is exploited in this work in order to obtain a bound on $(k_F)_{\kappa\lambda\mu\nu}$. Recent spectropolarimetry of distant galaxies at wavelengths ranging from infrared to ultraviolet has made it possible to achieve values of $L/\lambda$ greater than $10^{31}$. Measured polarization parameters are typically order 1. Therefore, we expect an experimental sensitivity of $10^{-31}$ or better to components of $(k_F)_{\kappa\lambda\mu\nu}$.

The first step in our analysis is to choose a coordinate system in which to work. A natural choice is a celestial equatorial system with the 3-axis aligned along the celestial north pole at equinox 2000.0 at a declination 90°. The 1- and 2-axis are at declination 0° and right ascension 0° and 90°, respectively. The goal is to place bounds on components of $(k_F)_{\kappa\lambda\mu\nu}$ in this frame. However, for a source at an arbitrary position on the sky, this is not the most convenient coordinate system. Polarization is given by the behavior of $\vec{E}$ in the plane perpendicular to the direction of propagation. Therefore, for each source, we define a ‘primed’ frame where $\hat{\vec{e}}_1' = (1; 0, 0, 1)$ at leading order. Then the primed-frame basis vector $\hat{\vec{e}}_3'$ points from the source towards the Earth. To match standard polarimetric conventions we choose $\hat{\vec{e}}_1'$ so that it points south. The idea is to do much of the analysis in the primed frame where things are simple and then use observer covariance to write $(k_F)_{\kappa\lambda\mu\nu}$ in terms of $(k_F)_{\kappa\lambda\mu\nu}$. The two frames are related by a rotation.

In the primed frame $\rho = \frac{1}{2}(\hat{k}_1' + \hat{k}_2')$ and $\sigma^2 = (\hat{k}_1')^2 + \frac{1}{4}(\hat{k}_1' - \hat{k}_2')^2$. The form of $\sigma^2$ suggests defining an angle $\xi$ such that $\hat{k}_1' = \sigma \sin \xi$ and $\hat{k}_2' = \sigma \cos \xi$. Note that while $\rho$ and $\sigma$ are frame independent, $\xi$ is not. Solving the modified Maxwell equations in this frame gives $\hat{\vec{e}}_\pm \propto (\sin \xi, \pm 1 - \cos \xi, 0)$. This implies that the birefringent modes are linearly polarized. From the solutions $\hat{\vec{e}}_\pm$ and Eq. (2) it is evident that $\sigma$ and $\xi$ are the relevant parameters for polarimetry of a particular source. More precisely,
\[ \sigma \sin \xi \text{ and } \sigma \cos \xi \text{ represent the minimal linear combinations of } (k_F)_{\kappa \lambda \mu \nu} \text{ effecting the polarization of a given source. It can be shown that } \sigma \sin \xi \text{ and } \sigma \cos \xi, \text{ written in terms of the celestial equatorial } (k_F)_{\kappa \lambda \mu \nu}, \text{ depend on the right ascension and declination of the source and 10 independent components of } (k_F)_{\kappa \lambda \mu \nu}. \text{ It is these ten components that are bounded in this work. We denote these components as } k^a, a = 1, \ldots, 10. \text{ A suitable choose for } k^a \text{ in terms of } (k_F)_{\kappa \lambda \mu \nu} \text{ can be found in the literature.} \]

In general, plane waves are elliptically polarized. The ellipse can be characterized by two angles: \( \psi \), the angle between \( \hat{e}_1' \) and the major axis of the ellipse and \( \chi = \pm \arctan \left( \frac{\text{minor axis}}{\text{major axis}} \right) \), which describes the shape of the ellipse and the helicity. The change in relative phase, Eq. (2), results in a change in both these angles. Most published polarimetric data of astronomical sources do not include measurements of \( \chi \). Therefore, our analysis focuses on finding an expression for the change in \( \psi \). It will not only depend on \( k^a \), the wavelength \( \lambda \), and the distance to the source \( L \), but also on the values of \( \psi \) and \( \chi \) when the light is emitted. Our approach is to look for wavelength dependence in the observed polarization. This approach assumes that the polarization at the source is relatively constant over the wavelengths considered.

We seek an expression for \( \delta \psi = \psi - \psi_0 \), the difference between \( \psi \) at two wavelengths, \( \lambda \) and \( \lambda_0 \). We find

\[ \delta \psi = \frac{1}{2} \tan^{-1} \frac{\sin \xi \cos \zeta_0 + \cos \tilde{\xi} \sin \zeta_0 \cos(\delta \phi - \phi_0)}{\cos \xi \cos \zeta_0 - \sin \xi \sin \zeta_0 \cos(\delta \phi - \phi_0)}, \] (3)

where \( \delta \phi = 4 \pi \sigma (L/\lambda - L/\lambda_0), \tilde{\xi} = \xi - 2\psi_0 \) and \( \phi_0 \equiv \tan^{-1}(\tan 2\chi_0 / \sin \tilde{\xi}) \), \( \zeta_0 \equiv \cos^{-1}(\cos 2\chi_0 \cos \tilde{\xi}) \). The polarization at \( \lambda_0 \) is given by \( \psi_0 \) and \( \chi_0 \). Two of these parameters need to be fit to the data. This is equivalent to fitting the initial polarization. The third parameter can be fixed to a convenient value.

Table 1 lists 16 sources with published polarimetric data with observed wavelengths ranging from 400 to 2200 nm. For each source, we choose \( \psi_0 \) as the mean polarization angle and use Eq. (3) to create a \( \chi^2 \) distribution. Each distribution is a function of \( \psi, \xi, \lambda_0, \) and \( \chi_0 \). They are then minimized with respect to \( \lambda_0 \), and \( \chi_0 \).

Figure 1 shows the minimized distribution for the source 3CR 68.1. The features of this contour are common to all sources in Table 1. The contour corresponds to a confidence level of about \((100 - 10^{-9})\%\). We see from Fig. 1 that the parameter space away from \( \sigma = 0, \xi = 0^\circ, \pm 90^\circ \) are eliminated by this source. These are only the regions where the theory predicts no change in \( \psi \). The regions near \( \xi = 0^\circ, \pm 90^\circ \) correspond to the radiation being in a specific combination of birefringent modes. For example, \( \xi = 0^\circ \) occurs if the light is
emitted in only one mode. We assume the probability of this happening for all 16 sources is small. With this assumption, the $\chi^2$ can be used to place a conservative constraint on $\sigma$. In Fig. 1, the bound is shown as a horizontal line.

| Source          | $L$ (Gpc) | $10^{10}L/\lambda$ | log$_{10}\sigma$ |
|-----------------|-----------|---------------------|------------------|
| IC 5063         | 0.04      | 0.56 - 2.8          | -30.8            |
| 3A 0557-383     | 0.12      | 2.2 - 8.4           | -31.2            |
| IRAS 18325-5925 | 0.07      | 1.0 - 4.9           | -31.0            |
| IRAS 19580-1818 | 0.13      | 1.8 - 9.1           | -31.0            |
| 3C 324          | 1.69      | 58 - 130            | -32.2            |
| 3C 256          | 1.92      | 70 - 140            | -32.2            |
| 3C 356          | 1.62      | 57 - 120            | -32.2            |
| F J084044.5+363328 | 1.71  | 62 - 120            | -32.2            |
| F J155632.8+351758 | 1.82  | 67 - 110            | -32.2            |
| 3CR 68.1        | 1.70      | 59 - 130            | -32.2            |
| QSO J2359-1241 | 1.48      | 87 - 90             | -31.1            |
| 3C 234          | 0.55      | 51 - 75             | -31.7            |
| 4C 40.36        | 2.02      | 73 - 160            | -32.2            |
| 4C 48.48        | 2.04      | 75 - 160            | -32.2            |
| IAU 0211-122    | 2.04      | 74 - 160            | -32.2            |
| IAU 0828+193    | 2.08      | 78 - 160            | -32.2            |

Table 1. Source Data.

Figure 1: Contours of $\chi^2$ for the source 3CR 68.1.

The bounds for each source are listed in the last column of Table 1. To estimate the constraint on $k^a$, we assume, for each source, the data are consistent with $\sigma = 0$. The bounds can be thought of as an estimate of the error $\delta \phi$ in a null measurement. We construct a second $\chi^2$ distribution, $\chi^2 = \Sigma_j (\sigma_j)^2 / (\delta \sigma_j)$, where the sum ranges over the 16 sources. This $\chi^2$ is a quadratic form in the $k^a$ coefficients. A constant value of $\chi^2$ correspond to a ten-dimensional ellipsoid in the $k^a$ space. We place a bound on the magnitude $|k^a| = \sqrt{k^a k^a}$ by...
minimizing $\chi^2$ with respect to the other nine degrees of freedom. This yields a bound of $|k^a| < 3 \times 10^{-32}$ at the 90% confidence level.

These are the first bounds on the coefficients $(k_F)_{\kappa\lambda\mu\nu}$. They are comparable to the best existing bounds in the fermion sector of the standard-model extension. An improvement in this bound can be expected if more measurements similar to those used here are made. Similar measurements of $\chi$ could also be used to improve the bound. In the future it may be possible to include X-ray polarimetry\[14\] which may lead to an improvement of several orders of magnitude.

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