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Prioritizing and queueing the emergency departments’ patients using a novel data-driven decision-making methodology, a real case study

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ABSTRACT

One of the principal problems in epidemic disruptions like the COVID-19 pandemic is that the number of patients needing hospitals’ emergency departments’ services significantly grows. Since COVID-19 is an infectious disease, any aggregation has to be prevented accordingly. However, few aggregations cannot be prevented, including hospitals. To the best of our knowledge, COVID-19 is a life-threatening disease, especially for people in poor health conditions. Therefore, it sounds reasonable to optimize the health care queuing systems to minimize the infection rate by prioritizing patients based on their health condition so patients with a higher risk of infection will leave the queue sooner. In this paper, relying on data mining models and expert’s opinions, we propose a method for patient classification and prioritizing. The optimal number of servers (treatment systems) will be determined by benefiting from a mixed-integer model and the grasshopper optimization algorithm.

1. Introduction

COVID-19 paralyzed all the world. When a world-scale disruption like this new pandemic accrues, all human societies commit a common responsibility against it far beyond race or political boundaries. In this situation, the best solution would be to cooperate to figure out how to overcome it and learn from it for the next possible disruptions. Numerous papers discussed how to get more prepared in defense of COVID-19 (in managerial aspects) and other pandemics like that alluded as follows.

Since the emergent of COVID-19, hospitals have postponed elective surgeries to the best of our knowledge for benign diseases such as not complicated cholelithiasis. But it’s not a secret that the blockage of elective surgeries for a long period will lead to a large number of high-risk patients. Therefore Manzia et al. (2020) proposed an international survey about managing a common and underrated surgical task during the COVID-19 disruption. They suggest a COVID-19 free pathway for patients waiting for elective surgery to prevent the mentioned problem. Due to the limits of hospitals’ equipment and human force in encountering COVID-19, Alipour Vaezi and Tavakkoli-Moghaddam (2020) suggested selecting several hospitals according to their qualifications in different areas of each city and accumulate all the resources there. So they proposed a multi-criterion decision-making (MCDM) based methodology for solving COVID-19 preparedness centers’ location-allocation problem. The best-worst method (BWM) and the weighted aggregated sum product assessment method (WASPAS) have been implemented in their study for this reason. COVID-19 is highly infectious. Several shopping centers and stores limited the number of shoppers inside the store to prevent infection. So Perlman and Yechiali (2020) designed two queue optimization models for determining the size of the queues in two phases, shopping (the first phase) and payment (the second phase). One of the industries that endured a massive impact from the COVID pandemic was the air-transportation industry. Budd et al. (2020) investigated this impact on dimensions and proposed valuable managerial insights. Hao et al. (2020) investigated the impact of COVID-19 on China’s hotel industry. They also proposed four different post-pandemic strategies: multi-business and multi-channels, product design and investment preference, digital and intelligent transformation, and market reshuffle to overcome the damages incurred. Ugur and Akbıyık (2020) study the impact of COVID-19 on the tourism industry worldwide. They provide a cross-regional comparison and some valuable managerial insights about flexibility, agility, travel insurance, etc.

Data mining is the process of discovering the hidden patterns in large data sets benefitting methods that are mainly composed of machine learning and statistical techniques (Alipour-Vaezi, Aghsami, et al.,
The data mining phrase cannot completely represent the meaning of this branch of science, so many references refer to it as knowledge mining. (Han et al., 2011)

Many researchers used data mining techniques for better decision-making in the COVID-19 pandemic situation. Radanliev et al. (2020) analyzed COVID-19 records with data mining methods and visualized the interrelationships between scientific research data records on COVID-19. Baralic et al. (2020) dedicated their research to assess the drug safety combination used for COVID-19 treatment. They used a toxic genomic data mining approach to discover the hidden patterns and provide valuable managerial insights. Buscema et al. (2020) used the Topological Weighted Centroid (TWC) algorithm to analyze the COVID-19 data in Italy. It gave the official Italian outbreak location and the trend in the geographical expansion of the disease. Sui et al. (2020) proposed a data mining study to analyze the changes in buses’ carbon emission patterns in the post-COVID-19 pandemic future. Harshavardhan et al. (2020) used an advanced machine learning-based analytic on COVID-19 pandemic data using generative adversarial networks (GAN) to diagnose and predict patients by their medical images such as X-Ray, CT Scan, etc. Ren et al. (2020) identified the potential of Chinese traditional medicines and treatments for COVID-19 treatment by a data-driven method. Barnes et al. (2020) proposed a text mining approach for understanding panic buying during the new pandemic by extending the compensatory control theory. Benefiting from big data, they integrated the text mining method with linear mixed models. Ismael and Şengür (2021) introduced a new application for deep learning in COVID-19 detection based on the patients’ chest X-ray images.

We observed a massive influx of patients waiting in prolix queues in hospitals to visit a practitioner during this disaster. The vitality of managing these queues is undeniable. The queuing management systems’ idea is originally an operational research method that helps decision-makers (DMs) by modeling the system and calculating its performance measures (Abdel-Aal, 2020). There are several types of queuing systems, but the common thing is customers arriving at random times to facilities for receiving service, and after that, they will be departed. Queueing management scope of application is vast as human societies itself, but one of the most critical roles of queuing systems is in health care systems, where just a tiny difference in waiting time may result in one precious life. Here we mentioned some of the valuable studies in this field:

To minimize the patients’ queue time, Yeh and Lin (2007) proposed an integrated algorithm consisting of simulation and genetic algorithm for nurses’ schedule optimization. Sharif et al. (2014) considered the accumulating priority queue (APQ), where the customer’s priorities are a function of their waiting time. They prioritized system queues by considering a multi-server APQ with Poisson arrivals for each class and a common exponential service time distribution in the health care field. Belciug and Gorunescu (2015) made St. George’s Hospital, London, bed occupancy, and resource utilization optimized from queueing models and evolutionary computation. Loving et al. (2017) maintained a radiology department by improving its long queues through a four-step framework. First, they analyzed the factors contributing to queue formation to improve processes to reduce service times and variability and then address queues’ psychology. Based on uniformization, van Brummelen et al. (2018) proposed a computational method for queue length computation of a time-dependent queueing network in the Dutch blood collection site. Aghsami et al. (2021) applied a queueing-inventory model to a hospital blood inventory management case study. Due to enhancing the emergency health care systems, Cildoz et al. (2019) designed a novel queue policy, the accumulative priority queue with the finite horizon (APQ-h) based on APQ. Their method considers the acuity level of patients and their waiting time, and the stage of healthcare treatment. Joseph et al. (2020) proposed a queueing optimization model for emergency physicians’ active patients’ queues. They realized the emergency physicians mostly visit the new patients in the early hours of their shifts. This number tends to decrease when the shift goes one till the end. They suggest, for future researchers, to examine links between active queue size and care quality. COVID-19 treating systems needs to get modeled through queueing models either. Therefore Meares and Jones (2020) proposed a methodology based on queuing theory to predict the number of intensive care beds needed during the COVID-19 pandemic.

One of the boldest issues that emerged in this pandemic is incising the patients’ arrival rate to the emergency department. As we know, if the density in the queue system increases and due to improper allocation of resources or servers, the number of patients in the system grows, it reduces the social distance and the spread of the infectious disease (i.e., COVID-19) to other emergency department personnel and patients. Therefore, the server should be added as much as possible to reduce the queue length and density in the emergency department and reduce the risk of disease. According to the mentioned literature review, we found it vital to minimize the patients’ waiting time.

To the best of our knowledge, no article investigates the queueing system of an emergency department using data mining techniques integrated with mathematical modeling solved by metaheuristic algorithm, and that is the research gap we stabilized our study on it. The point is, in this research, we used data mining techniques to classify patients and organize them in the queues, and benefitting from queueing models minimized their waiting time in their queues. Fig. 1 shows the main ingredients of this article.

The rest of the article is organized as follows. The article’s problem would be clearly described in section two, and the proposed methodology will be illuminated. Section three addresses the solution methodology and introduces the used methods. In this section, each solver will be discussed thoroughly. Section five provides a real-life case study for a better understanding of the function of the methodology. Section six investigates the computational results and bolds the improvements achieved through this model. In section seven, the sensitivity of the model to different parameters will be examined. A helpful discussion and valuable managerial insights are proposed in section eight. The article ends with a comprehensive conclusion in section nine and a brief acknowledgment in section ten.

![Fig. 1. Summary of the article.](attachment:summary.png)
2. Problem description & mathematical formulation

2.1. Problem description

The necessity of healthcare systems’ optimization, especially in disruption situations as the COVID-19 pandemic, is undeniable. It’s a fact that a slight difference in service time or waiting time in medical systems can lead to saving or losing a person’s life (Masoumi et al., 2021). With this in mind, here we raise a research question: Is it reasonable to line up patients in the hospital’s emergency department based on their order of arrival? If a triage nurse finds a very ill patient in bad condition in several hospitals, perhaps he/she sends him to the physician as soon as possible. But still, it seems inadequate in the disruptive COVID-19 situation.

Every day we observe a massive influx of patients needing medical services in an emergency. There are two ways that patients can get into the emergency department and use the services in any regular hospital. A group of patients comes to the emergency department by themselves (group a). The others who cannot walk personally into the hospital (in most cases are unconscious) use ambulances (group b). The second group is at a higher priority of getting services towards the first group.

So, they go to the queue head and receive service after completing the current patient’s service. The group (a) enter the triage system where their general health features data are gathered with triage nurses’ help. After prioritizing, they visit the physicians and receive medical services by priority. Fig. 2 gives a schematic view of the patient flow in the hospital’s emergency department.

As it has shown in Fig. 2, each group of patients has a different arrival rate. According to the Poisson process, group (a) and (b) arrive at the hospital with rates \( \lambda \) and \( \lambda' \), respectively. Also, each one receives a treating service with a rate \( \mu \) that is distributed exponentially.

In this article, according to the above paragraphs and to fulfill the mentioned research gap, we propose a methodology for prioritizing and queuing the patients. Fig. 3 illuminates the steps of our methodology clearly.

Based on this methodology, group (b) patients are in the “emergent” category and in the highest priority. For group (a) patients, after the triage nurse receives their general health information, this method will categorize them as “urgent”, “non-urgent”, or “self-care” patients. Hence, we have four priorities in this study. Also, the priority discipline is non-preemptive. In this case, the highest priority patients go to the head of the queue. Still, they must wait for any patient in treatment to complete treatment, even if that patient has a lower priority (For more reading about the non-preemptive priority queueing system, refer to the survey of Shortle et al. (2018)). Therefore, the corresponding queueing system is a Non-preemptive queueing System with four priorities. Also, we assume that there are \( C \) servers in the treatment system for serving the patients.

2.2. Mathematical formulation

This section develops a mixed-integer non-linear programming model to minimize the patients’ total average waiting cost in the treatment queueing system and the employment cost of servers in the treatment system. Before presenting the mathematical model, we present the notations in Table 1.

\[
Z = \min \sum_{i=1}^{C} C_i L_i W_i + C f(\mu) \quad \text{(1)}
\]

Eq. (1) determines the objective function \( (Z) \), which minimizes the total average waiting cost of the patients in the treatment queueing system and the employment cost of servers in the treatment system.

\[
W_i = \left[ \frac{C_i [1 - \rho_i] (C_i \mu_i)^{i-1} \sigma_i}{(1 - \sigma_i)} \right]^{-1} \quad \text{for } 1 \leq i \leq 4 \quad \text{(2)}
\]

Eq. (2) shows the average waiting time in the queue for patients with \( i \) th priority, while Eq. (3) indicates the average number of patients with priority \( i \) in the queue.

\[
\rho_i = \frac{\lambda_i}{C \mu} \quad \text{(4)}
\]

\[
\rho = \frac{\sum \lambda_i}{C \mu} \quad \text{(5)}
\]

\[
\rho < 1 \quad \text{(6)}
\]
σ

Eq. (4) calculates the fraction of time the server is busy with the priority i patients. It has to be said for \( \rho < 1 \), the system is stationary, which is shown in Eq. (6).

\[
\sigma_i = \sum_j \rho_j = 1.2.3.4
\]  

(7)

\[
\sigma_0 = 0
\]  

(8)

Eq. (7) addresses the sum of the first \( i \) value of \( \rho_i \).

\[ C(\mu) \leq B \]  

(9)

\[
f(\mu) = a\mu + b
\]  

(10)

Eq. (9) delineates that the cost of having \( C \) treatment system with the rate of \( \mu \) has to be less or equal to the system’s budget. The \( f(\mu) \) is calculated in Eq. (10).

\[ 1 \leq C \leq S \]  

(11)

\[ L \leq \mu \leq U \]  

(12)

Eq. (11) delineates the upper bound and the lower bound for the number of servers in the queue. Also, Eq. (12) asserts that the treatment rate is limited between its upper bound and lower bond.

3. Solution methodology

This section introduces the methods and techniques used to solve the problem, as mentioned in Fig. 3. We use the Random Forest (RF) algorithm for the classification step and the Grasshopper Optimization Algorithm (GOA) algorithm for solving the queueing optimization problem. Both of the chosen algorithms (RF & GOA) have considerably high accuracy and are among the most reliable methods. It has to be mentioned that exact methods should be used for small-scale problems instead of GOA (or any other metaheuristic algorithm). In this article, we deployed generalized algebraic modeling system (GAMS) software to achieve the results of small-scale problems.

3.1. Random Forest classifier (RF)

The RF algorithm is one of the most famous machine learning algorithms categorized as one of the black box models. This ensemble model is based on a decision tree algorithm. To eliminate the overfitting problem of decision trees, RF has been established by constructing a multitude of decision trees. (Alipour-Vaezi, Tavakkoli-Moghadaam, et al., 2021; Amit & Geman, 1997)

The RF algorithm can operate both classification (by outputting the classes’ mode) and regression (by outputting the mean of the classes). This article benefits from RF classifier to categorize patients and determine their level of risk. The data frame used to train and test the algorithm is a real dataset retrieved from Sherkat Naft hospital. The first step after cleaning the data is to give each data a proper label to determine their risk level. This work is done by an expert team consisting of experienced physicians. To prevent overfitting, 9 attributes of the overall attributes are extracted to be used in the prediction model. These attributes are mainly based on combining some features. One Hot Encoding technique is used to convert each underlying disease into a new Boolean column. Table 1 illuminates used data frame attributes.

| Attribute                  | Type      |
|----------------------------|-----------|
| ID                         | Numerical |
| Age                       | Numerical |
| Pregnancy status          | Boolean   |
| Blood pressure            | Boolean   |
| Heart disease             | Boolean   |
| Kidney disease            | Boolean   |
| Diabetes                  | Boolean   |
| Liver disease             | Boolean   |
| Lung disease              | Boolean   |
| Immune Deficiency disease | Boolean   |
| Risk factor               | Categorical |

3.2. Grasshopper optimization algorithm (GOA)

Saremi et al. (2017) proposed a metaheuristic optimization algorithm based on grasshopper swarms’ inspiration. Their algorithm called Grasshopper Optimization Algorithm (GOA) models the behavior of grasshopper swarms’ social interaction toward the food supplies in nature for solving optimization problems. In other words, GOA has been inspired by the behavior of grasshopper swarms in the real world and mimicked the repulsion among them (Momeni et al., 2019; Malik Khanouyan et al., 2021).

The social interaction of locusts towards food in nature consists of two stages:

- Exploration
- Exploitation

Two abovementioned stages alongside the target seeking are the essence of the metaheuristic algorithms performed by grasshoppers naturally. So, based on the mathematical model represented below, they defined GOA as a new nature-inspired algorithm.

\[ X_i = S_i + G_i + A_i \]  

(3.1)

Eq. (3.1) shows the mathematical model representing the grasshoppers’ behavior. Notably, the \( X_i \) define the \( i \)th grasshopper while \( S_i, G_i \) and \( A_i \) respectively indicate the social interaction, the gravity force on the \( i \)th grasshopper, and the wind advection.

\[ S_i = \sum_{j \neq i} N \sigma(d_{ij}) \]  

(3.2)

Table 2:

| Dataset’s attributes.     | Type   |
|----------------------------|--------|
| ID                         | Numerical |
| Age                       | Numerical |
| Pregnancy status          | Boolean |
| Blood pressure            | Boolean |
| Heart disease             | Boolean |
| Kidney disease            | Boolean |
| Diabetes                  | Boolean |
| Liver disease             | Boolean |
| Lung disease              | Boolean |
| Immune Deficiency disease | Boolean |
| Risk factor               | Categorical |
The problem described in Table 3 is solved both with GAMS software and the GOA algorithm. The exact results of this problem are proposed in Table 4 and the results achieved with the GOA algorithm are listed in Table 5.

There is no need to emphasize the similarity of the GOA and GAMS results for Table 3’s small-scale problem. Both solvers agreed that this problem needs 3 servers to work properly. The service rate is determined as 8 and both \( L_{q}^{(1)} \) and \( L_{q}^{(2)} \) are pretty same in both solvers’ results.

As we mentioned before, hospitals may prioritize their patients in a higher number of classes or define the more upper bound of service rate and more upper bound for the number of servers. These issues lead to a larger feasible region. In Table 6, the details of the second test problem, which is a large-scale problem, can be observed.

The results of the second test problem are achieved using both GAMS software and the GOA algorithm. Table 7 proposes the findings of GAMS software, while Table 8 is dedicated to the GOA algorithm results. Fig. 4 depicts the Convergence of GOA to the optimal solution for this problem.

4.2. Grasshopper optimization algorithm (GOA)

We consider the other eight test problems to validate the proposed metaheuristic approach. In Table 9, all problems are provided. Eq. (4.1) defines the gap rate of the results between the (GAMS) and the GOA algorithm.

\[
G_{\text{rate}} = \frac{\text{Bestresult} - \text{Worstresult}}{\text{Bestresult}} \times 100
\]  

Based on Tables 7–9, there isn’t a significant difference between the two solvers reflected results. According to Table 9, it’s crystal clear that the gap rate between GOA and GAMS results is extremely minor. But it is vital to be mentioned that based on Table 9, the processing time of GOA is considerably lower for larger-scale problems. Therefore, it seems reasonable to solve large-scale problems which are time-consuming with the GOA.

4.3. Random forest algorithm

As showed in Table 10, a few different machine learning models, including RF, support vector machine (SVM), linear regression (LR), and linear discriminate analysis (LDA), have been fit into Sherkat Naft’s dataset (more details about this dataset is positioned in Section 3.1.) and are compared to select the most proficient one. According to the models’ results, the RF algorithm has the best accuracy score. Also, the difference between RF and SVM algorithms is slight enough to overlook, but the RF algorithm proposed the most accurate classification.

5. Case study

For better understanding, this section provides a real-life case study based on the data collected from Javad Al Aeme hospital emergency

### Table 3

Assumed value of model’s parameters for the small-scale problem.

| Parameter | \( x_1 \) | \( x_2 \) | \( C_1^d \) | \( C_2^d \) | \( a \) | \( b \) | \( B \) | \( U \) | \( L \) | \( S \) |
|----------|---------|---------|----------|----------|------|------|-----|------|-----|-----|
| Value    | 1       | 15      | 3000     | 1000     | 9.5  | 110  | 95,000 | 8    | 2   | 4   |
department, which is engaged with COVID-19 patients and others. This dataset is covering the same features discussed in section 3.1, which is listed in Table 2. The case study dataset covers the patients’ entered the Javad Al Ame hospital’s emergency department in one hour.

Patients with different chief complaints arrive at the emergency department and ask for health care services. As shown in Fig. 2, patients reach the hospital in two ways: they can come on their own (group a) or use ambulances (group b). Group a’s patients have to give their health information to triage nurses. Table 1 (presented in supplementary materials) provides comprehensive details about the proposed case study.

The hospital in question currently possesses two treatment servers in the system (\( C = 2 \)). Due to the lack of servers (according to the numerous patients), the high-density queues and the long waiting time of patients are inevitable in this hospital. One of the boldest problems in this case study is that there is no classification to be done to discern bad condition patients from others. This results that patients with bad conditions may not receive services at the proper time, leading to life-threatening issues.

In this hospital, the arrival rate of the group a’s patients (\( \lambda' \)) is determined 38 for one hour while the arrival rate of the ambulances (\( \lambda' \)) is determined 0.5 based on the hospital’s historical data. The treatment process is according to the Poisson distribution with a rate of 12 (\( \mu = 12 \)). It’s noteworthy that the upper bound of the servers in the system (\( S \)) is 7. The treatment system’s budget (\( B \)) is renounced 100,000 also, the variable cost of a server in the treatment system per time (\( a \)) and the fixed cost of establishing a server in the treatment system (\( b \)) is equal to 10 and 100 respectively. The treatment process is according to the Poisson distribution, and the treatment rate’s upper bound (\( U \) and lower bound (\( L \)) are considered as 15 and 3. Cost per unit time when a patient waits in the formed queue in part i is considered as follow: \( C_{2}^{1} = 3000, C_{2}^{2} = 2500, C_{6}^{1} = 1500, C_{6}^{2} = 500 \).

### 6. Computational results

#### 6.1. Classification results

Based on the proposed method and the triage station’s collected data, the group a’s patients classified as urgent, non-urgent, and self-care. As mentioned before, group b’s patients are classified as emergent, which means they are the highest priority to enter the treatment system. As we know, their arrival rate of the ambulances is equal to 0.5, which means \( \lambda_1 = 0.5 \). Benefitting from the RF algorithm, the classification step is done for group a’s patients, and the entrance rate of each class is determined as mentioned in Table 11.

### Table 5

Results of the first test problem using GOA algorithm.

| Variable | \( Z \) | \( C \) | \( \mu \) | \( \lambda'_{1} \) | \( \lambda'_{2} \) |
|----------|---------|---------|--------|----------------|----------------|
| Value    | 611.323 | 3       | 8      | 0.020          | 0.681          |

### Table 6

Assumed value of model’s parameters for large-scale problem.

| Parameter | \( \lambda_1 \) | \( \lambda_2 \) | \( \lambda_3 \) | \( \lambda_4 \) | \( \lambda_5 \) | \( \lambda_6 \) | \( \lambda_7 \) | \( \lambda_8 \) |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Value     | 1              | 3              | 16             | 18             | 20             | 19             | 10             | 7              |

| Parameter | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) | \( C_6 \) | \( C_7 \) | \( C_8 \) |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Value     | 3500      | 3000      | 2500      | 2000      | 1500      | 1000      | 500       | 300       |

| Parameter | \( a \) | \( b \) | \( U \) | \( L \) | \( S \) |
|-----------|--------|--------|--------|--------|--------|
| Value     | 15     | 150    | 500000 | 20     | 2      | 15     |

### Table 7

Results of the second test problem using GAMS.

| Variable | \( Z \) | \( C \) | \( \mu \) | \( \rho \) | \( \rho_1 \) | \( \rho_2 \) | \( \rho_3 \) |
|----------|--------|--------|--------|--------|--------|--------|--------|
| Value    | 2690.8287 | 6     | 18.897 | 0.829 | 0.099 | 0.026 | 0.141 |

| Variable | \( \rho_4 \) | \( \rho_5 \) | \( \rho_6 \) | \( \rho_7 \) | \( \rho_8 \) | \( W_1^{(1)} \) | \( W_1^{(2)} \) |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Value    | 0.159       | 0.176       | 0.168       | 0.088       | 0.062       | 0.005       | 0.005       |

| Variable | \( W_2^{(1)} \) | \( W_2^{(2)} \) | \( W_3^{(1)} \) | \( W_3^{(2)} \) | \( W_3^{(3)} \) | \( W_3^{(4)} \) | \( W_3^{(5)} \) | \( W_3^{(6)} \) | \( W_3^{(7)} \) | \( W_3^{(8)} \) | \( L_4^{(1)} \) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Value    | 0.006           | 0.009           | 0.016           | 0.032           | 0.068           | 0.128           | 0.005           |

| Variable | \( L_4^{(2)} \) | \( L_4^{(3)} \) | \( L_4^{(4)} \) | \( L_4^{(5)} \) | \( L_4^{(6)} \) | \( L_4^{(7)} \) | \( L_4^{(8)} \) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Value    | 0.016           | 0.102           | 0.167           | 0.313           | 0.617           | 0.681           | 0.895           |

### Table 8

Results of the second test problem using GOA algorithm.

| Variable | \( Z \) | \( C \) | \( \mu \) | \( \rho \) | \( \rho_1 \) | \( \rho_2 \) | \( \rho_3 \) |
|----------|--------|--------|--------|--------|--------|--------|--------|
| Value    | 2691.2396 | 6     | 18.901 | 0.829 | 0.099 | 0.027 | 0.141 |

| Variable | \( \rho_4 \) | \( \rho_5 \) | \( \rho_6 \) | \( \rho_7 \) | \( \rho_8 \) | \( W_1^{(1)} \) | \( W_1^{(2)} \) |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Value    | 0.159       | 0.176       | 0.167       | 0.088       | 0.062       | 0.005       | 0.005       |

| Variable | \( W_2^{(1)} \) | \( W_2^{(2)} \) | \( W_3^{(1)} \) | \( W_3^{(2)} \) | \( W_3^{(3)} \) | \( W_3^{(4)} \) | \( W_3^{(5)} \) | \( W_3^{(6)} \) | \( W_3^{(7)} \) | \( W_3^{(8)} \) | \( L_4^{(1)} \) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Value    | 0.006           | 0.009           | 0.016           | 0.033           | 0.068           | 0.128           | 0.005           |

| Variable | \( L_4^{(2)} \) | \( L_4^{(3)} \) | \( L_4^{(4)} \) | \( L_4^{(5)} \) | \( L_4^{(6)} \) | \( L_4^{(7)} \) | \( L_4^{(8)} \) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Value    | 0.016           | 0.103           | 0.168           | 0.314           | 0.618           | 0.682           | 0.896           |
6.2. Queue optimization results

After the classification step, the patients will be organized in the queueing system based on their priority. Table 12 provides the proposed model’s exact results, which are retrieved with the GAMS software.

6.3. System improvements

According to Table 12, this hospital has to increase the number of servers to four to reach the least waiting time for the emergent and urgent class patients. Also, the servers’ service rates have to be fixed at 13.072. Table 13 compares each variable of the hospital’s state before and after the optimization through the proposed model.

As it’s obvious in Table 13, one of the most important improvements of the results after implementing the proposed model is that the patients get categorized and, therefore, the average waiting time for bad condition patients ($W_{bi}^j$) is reduced brilliantly. This improvement in the COVID-19 situation also prevents the spreading of the disease between vulnerable patients. With this method’s findings, hospital’s DMs can optimize the queueing system and diminish queue density with minimum cost.

7. Sensitivity analysis

In this section, we will survey the effects of different parameters on the model. Here the impact of $a$, $b$, $U$, $B$, $S$, and $\lambda_i$ on $Z$, $\mu$, and $C$ will be investigated.

7.1. The variable cost of a server in the treatment system per time unit

The first parameter that will be investigated in this section is $a$ parameter representing the variable cost of a server in the treatment system per time unit. According to Fig. 5, the increment of $a$ parameter would result in a higher objective function.

Based on Fig. 6, if the variable costs of the treatment system server increase, first, the model tends to reduce the service rate and keep the number of servers constant, but from a point onwards, because this cost increases significantly, reduces the number of servers but instead maximizes service speed.

7.2. Fixed cost of establishing a server in the treatment system

As it has illuminated in Fig. 7, as the fixed cost of establishing a server in the treatment system grows, the objective function will grow either.

Fig. 8 indicates the effect of the fixed cost of establishing a server in the treatment system on the number of servers in the system and the service rate. It has had been shown in this figure that the number of servers and service rate parameters are to be affected stepwise.

7.3. Upper bound of treatment rate

The impact of the upper bound of treatment rate on the objective function has been depicted in Fig. 9. It has to be mentioned that the parameter $U$ and $Z$ have the opposite effect on each other. It’s noteworthy that the feasible area grows larger with this increase, and it is more possible to achieve better results.

With due respect to Fig. 10, it can be concluded that parameter $U$ has the opposite effect on parameter $Z$. But higher levels of the $U$ may have resulted in a higher service rate. As it is known, if the service rate’s upper bound reduces, the system will have to hire more servers, and the total cost will increase (see Fig. 10).

7.4. The budget of the treatment system

Fig. 11 exhibits the opposite effect of the treatment system’s budget on the objective function. Nevertheless, the budget increase after a specific point makes no difference in the objective function, and the system stays stationary.

Based on Fig. 12, the system’s budget increase makes the system increase the service rate. As the service rate reaches its upper bound, it’s
7.5. Upper bound for the number of servers in the treatment system

Higher levels of the parameter $S$ which represents the upper bound for the number of servers in the treatment system will lead to lower levels of $Z$. It has been depicted in Fig. 13.

Fig. 14 assured that if the $S$ grows, the parameter $C$ will be increased stepwise, and the service rate will experience reduction.

### Table 12
Results of the case study.

| $Z$  | $C$ | $\mu$ | $\rho$ | $\rho_i$ |
|------|-----|-------|-------|---------|
| Optimal solution | 964.1596 | 4 | 13.072 | 0.736 | 0.010 0.105 0.316 0.306 |

| $L_i^q$  | $W_i^q$ |
|---------|---------|
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Optimal solution | 0.005 | 0.059 | 0.306 | 0.997 | 0.009 | 0.011 | 0.019 | 0.062 |

### Table 13
The comparison of problem’s variables before and after the proposed model.

| Variable |
|----------|
| Number of servers in the treatment system |
| Treatment rate |
| The overall fraction of time the server is busy |
| Arrival rate of Patients |
| The expected total number of patients in the formed queue in the treatment system |
| the expected time a patient spends in the formed queue in the treatment system |

| Before | After |
|--------|-------|
| 2 | 4 |
| 12 | 13.072 |
| 1.604 | 0.736 |
| 0.5 | 0.5 |
| 0.5 | 5.5 |
| 0.5 | 16.5 |
| 0.5 | 16 |
| $L_1^q$ = 0.038 | $L_1^{(1)}$ = 0.005 |
| $L_2^q$ = 9.621 | $L_2^{(1)}$ = 0.059 |
| $L_3^q$ = 0.305 | $L_3^{(1)}$ = 0.997 |
| $L_4^q$ = 0.976 | $L_4^{(1)}$ = 0.009 |
| $W_1^q$ = 0.076 | $W_1^{(1)}$ = 0.011 |
| $W_2^q$ = 0.25 | $W_2^{(1)}$ = 0.019 |
| $W_3^q$ = 0.062 | $W_3^{(1)}$ = 0.009 |
| $W_4^q$ = 0.062 | $W_4^{(1)}$ = 0.011 |

Rational to add servers and decrease the service rate.

Fig. 5. The impact of the parameter $a$ on $Z$.

Fig. 6. The effect of the parameter $a$ on the $\mu$ and $C$.

Fig. 7. The impact of the parameter $b$ on $Z$.

Fig. 8. The effect of the parameter $b$ on the $\mu$ and $C$. 

Fig. 9. The comparison of problem’s variables before and after the proposed model.
7.6. The arrival rate of patient

The arrival rates are among the most important parameters for evaluating their impact on objective function and variables. In this section, the effect of $\lambda_i$ on $Z$, $C$ and $\mu$ where $i \in \{1,2,3,4\}$ will be investigated.

Fig. 15 shows the impact of the different arrival rates on the objective function. As its illuminated, the arrival rate of patients with higher priorities ($i = 1,2$) has a greater influence on the objective function. Hence, it’s reasonable that DMs pay more attention to this class of patients.

As shown in Fig. 16, the number of optimal servers for higher priorities is equal to or higher, and as a result, the number of optimal servers is more sensitive to the higher priorities’ arrival rate. In other words, at the same arrival rate for all priorities, the number of servers for the first priority is equal to or higher than the others. For example, for an arrival rate of 13, the number of servers is 5 for the first priority, 4 for the second priority, and 3 for the other two priorities. Therefore, the number of server’s graph for higher priorities is always higher than other priorities.

According to Fig. 17, with increasing the arrival rate, first, the service rate increases, and when it reaches its maximum, it decreases, and instead, the number of servers accelerates based on Fig. 16.

8. Discussion and managerial insights

This article possesses a considerable improvement toward previous researches. Applying the proposed method, benefiting from patients’ classification and queue optimization, waiting time (especially for bad condition patients) will considerably decrease. As a matter of fact, in the COVID-19 pandemic disruption, this method prevents vulnerable patients from infecting with the disease. According to the investigated case study, the system’s service rate increased and queue density reduced with the slightest cost. It’s notable for DMs that the hospital’s cost is
negligible toward the benefits that bring afterward, including optimized queue and lower COVID-19 spread rate.

This article helps DMs determine precisely how many servers (including physicians, nurses, beds, etc.) have to employ to reach the optimal waiting time and with which level of service rate. Therefore, based on sensitivity analysis, several managerial insights for DMs can be expressed as follows:

DMs are advised that to minimize the total cost of waiting time for patients and the service system, when the cost of adding servers to the system increases, according to Fig. 6, first reduce the service rate, i.e., reduce the number of physicians, nurses, or assistants and If this cost increases too much, it is better to keep the service rate high but reduce the number of servers.

According to Fig. 8, DMs might increase the number of servers and reduce the service rate if the fixed cost of establishing a server in the treatment system increases. In other words, if the parameter $b$ was considerably high, it’s recommended to reduce the number of servers and add the number of physicians, nurses, etc.

If it is not possible to hire a nurse and support staff in the emergency department, DMs should increase the number of servers. But if there are no restrictions to add the support staff to the system, based on Fig. 10, it will be more favorable for the system to reduce the number of servers while other system parameters remain the same.

According to Fig. 12, if the hospital’s budget increases, DMs should increase the service rate first, but if the rise continues, it’s better to employ more servers and reduce the service rate. Based on Fig. 14, if DMs cannot increase the server for technically, space, etc., they should increase nurses’ number. According to Fig. 15, as the arrival rate of different priorities increases, the objective function increases, and the total cost becomes higher for higher priorities. In other words, because of the higher importance of higher priorities, more expenses have to be paid. Concerning Fig. 16, DMs should pay more attention to higher priorities to increase different priorities’ arrival rates. Because by increasing the arrival rate of the higher priority patients compared to the arrival rate of lower priorities, DMs have to increase the number of the servers sooner. In other words, the threshold for increasing the

Fig. 15. The effect of the parameter $\lambda_i$ on $Z$.

Fig. 16. The effect of the parameter $\lambda_i$ on $C$. 
number of servers based on the arrival rate is lower for higher priorities. Based on Table 9, since the gap between results of GAMS and GOA can be condensed, and according to the considerable difference in their processing time, we recommend DMs to use GAMS in small scale problems such as optimizing a hospital’s queue, while they should benefit from GOA algorithm in massive-scale problems like optimizing all of the hospitals’ queues in a nation.

9. Conclusion

To the best of our knowledge, one of the challenges the hospital managers are facing in the emergency departments is the optimal amount of resource allocation so that the cost of waiting for patients in the queue and the cost of service are balanced. Resources in the emergency department (including physicians, nurses, paramedics, medical interns, medical devices, hospital beds, etc.) play the role of servers in queuing systems that patients refer to and wait in the queue to receive services from these servers and leave the system. A considerable cost is imposed on the system without reducing the waiting time by allocating additional and excessive resources. On the other hand, with the allocation of insufficient resources, patients’ waiting time in the emergency department increases. Therefore, a model was needed to be able to determine the correct allocation of resources so that the total expected cost and resources get minimized. This article proposed a methodology integrating data-mining techniques and mathematical formulation to determine the optimal number of servers and the optimal service rate so that the sum of the waiting and service costs get minimized.

In the proposed methodology, we assumed that patients with very bad health conditions use ambulances to get to the hospital. Other patients who are in lower priority of receiving treatment services have to declare their health condition information to the triage nurses. The first step of the methodology is to classify and prioritize the second group of patients (the ones who do not use ambulances) with data mining algorithms and attain each class’s arrival rate. In the second step using the arrival rates of each group of patients, the optimal number of servers and their optimal service rate is determined using the proposed mathematical model. According to section 3.3, for small and medium-scale problems we can use exact solvers (i.e., GAMS), and for large-scale problems, it’s better to use metaheuristic algorithms (i.e., GOA).

In this article, a real-life case study (Javad Al Aeme hospital, Bojnord, Iran) has been investigated with this method. By minimizing patients’ total waiting time in bad health conditions (emergent and urgent class) and determining the optimal number of servers, this hospital is much closer to optimality.

The results of this article are considerably useful for hospital managers. This method also can be efficiently applied to other medical institutions (i.e., vaccination centers, home nursing, etc.) and escalate their productivity level. We highly recommend that future researchers apply this method for others to optimize their queues and servicing systems. They can also consider other priority models or consider a queuing system for the triage station. The other suggestion that we make for future researchers is to apply the treatment system’s service time using Erlang distribution in the proposed method.

We assumed the servers in the treatment system are reliable while they may fail at any point in time, which may stop the service completely or lead to the service continuing at a lower rate (Aghsami & Jolai, 2020). Hence, server breakdowns would be an interesting topic for future research.

Moreover, it has to be discussed that in a few cases of real-life applications, patients censor their health information. It is vital that patients feel free to explain their health information. Therefore, the triage nurses who are gathering data have to make sure about patients’ confidence. Also, another way to avoid this kind of missing data is that the hospitals upload their patients’ records electronically in a comprehensive and integrated system to shorten the overall data gathering time in addition to preventing patient data censorship. This would be a perfect research problem for future researchers.

CRediT authorship contribution statement

Mohammad Alipour-Vaezi: Conceptualization, Methodology, Visualization, Data curation, Writing – original draft, Software, Data curation, Validation. Amir Aghsami: Conceptualization, Visualization, Methodology, Investigation, Writing – review & editing, Supervision. Fariborz Jolai: Investigation, Writing – review & editing, Validation, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
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