Second principle approach to the analysis of unsteady flow and heat transfer in a tube with arc-shaped corrugation

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Abstract. Passive convective heat transfer enhancement techniques are well known and widespread tool for increasing the efficiency of heat transfer equipment. In spite of the ability of the first principle approach to forecast the macroscopic effects of the passive techniques for heat transfer enhancement, namely the increase of both the overall heat exchanged and the head losses, a first principle analysis based on energy, momentum and mass local conservation equations is hardly able to give a comprehensive explanation of how local modifications in the boundary layers contribute to the overall effect. A deeper insight on the heat transfer enhancement mechanisms can be instead obtained within a second principle approach, through the analysis of the local exergy dissipation phenomena which are related to heat transfer and fluid flow. To this aim, the analysis based on the second principle approach implemented through a careful consideration of the local entropy generation rate seems the most suitable, since it allows to identify more precisely the cause of the loss of efficiency in the heat transfer process, thus providing a useful guide in the choice of the most suitable heat transfer enhancement techniques.

1. Introduction
Convective heat transfer enhancement in ducts may be obtained by modifying the fluid-dynamics and then the thermal boundary layer. In many industrial sectors this goal is frequently achieved by suitably modifying the duct wall (mainly throughout periodic corrugation performed by means of cold working operations) or by inserting in the main flow devices which are able to modify the flow regime (turbulators, wire coil inserts, etc...)\cite{1,2}. These techniques, known as passive intensification techniques, mainly rely on their ability to anticipate the transition to the unsteady flow regime and hence to the turbulent one. Passive techniques are usually applied based on the experience of the designer and their effect is verified \textit{a posteriori}, through the determination of the heat actually exchanged.

Bejan\cite{3} pointed out that heat transfer and fluid flow mechanisms are intimately related to thermodynamics through entropy generation due to the non-ideality of the transfer process. He also pointed out that the entropy generation due to the convective heat transfer in internal flow is inversely
proportional to the convection coefficient. This result can be easily understood if one considers that, for the same amount of heat exchanged, to a higher convection coefficient a lower wall to fluid temperature difference is associated and, therefore, a lesser degree of irreversibility in the heat transfer process.

In spite of the extensive work done on the topic of entropy generation due to convection heat transfer, the knowledge of the overall entropy generation given by the entropy balance of the heat transfer section is of limited use in the engineering practice. Otherwise, the knowledge of the entropy generation on a local scale, could be a useful tool to identify the zones of the heat transfer device, possibly away from the wall-fluid interface, where the entropy generation may be high. Finally, the detailed map of the local entropy generation might suggest possible geometry or operational optimal changes intended to increase the overall system efficiency.

Sciacovelli et al. [4] have presented a critical review of the contributions to the local entropy generation analysis to systems involving convective heat transfer to which the early work by Pagliarini and Barozzi [5] can be added.

An example of the heat transfer efficiency optimization based on the local entropy generation rate analysis was provided by Giangasparo and Sciubba [6], who recently applied a heuristic approach to the optimization of a solar heat exchanger.

Calculating the local entropy generation rate requires the resolution of the mass, momentum and energy conservation equations and the subsequent post-processing of the velocity and temperature data as shown by De Groot and Mazur [7]. It is a challenging task, which requires significant computing resources, especially when the unsteady flow regime is considered.

The transition from the laminar to the unsteady flow regime in a two dimensional periodic arc-shaped profile channel has been studied by Ničeno and Nobile [8]. Rainieri et al. [9] have confirmed their critical Reynolds number value, and they have extended the analysis to the case of the axisymmetric duct, by simulating the heat and mass transfer phenomenon in three spatial dimensions.

In this paper the second principle approach, based on the analysis of the local entropy generation, is applied to the study of convective heat transfer in the transition from the laminar to the unsteady flow regime in a two dimensional channel with a periodic arc-shaped profile.

2. Problem statement
The two-dimensional channel shown in figure 1 has been hereby considered. A Newtonian fluid in steady flow condition exchanges heat with the duct wall, on which a uniform heat flux condition is applied.

Assuming constant fluid properties, the heat transfer problem is mathematically formulated as follows:

\[ \nabla \cdot \mathbf{v} = 0 \]  \hspace{1cm} (1)

\[ \rho \frac{Dv}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{v} \]  \hspace{1cm} (2)

\[ \rho c_p \frac{DT}{Dt} = \lambda \nabla^2 T + \mu \Phi \]  \hspace{1cm} (3)

being \( \mathbf{v} \) the velocity field, \( \rho \) the fluid density, \( t \) the time, \( p \) the pressure, \( \mu \) the fluid dynamic viscosity, \( T \) the fluid temperature, \( \alpha \) the thermal diffusivity and \( \Phi \) the viscous dissipation function.

Because of the periodicity of the channel geometry, the integration domain of the previous equation set was limited to a single channel module (figure 2), on whose inlet and outlet sections a periodic boundary condition is set, for both velocity and temperature distribution, while the bulk fluid temperature is incremented from the inlet to the outlet section based on the overall energy balance of the heat transfer module.
Equations (1) to (3) were solved by adopting the finite element method, implemented within the COMSOL Multiphysics environment. For Reynolds number values higher than the critical value, for which the flow becomes unsteady, the simulation stop time was set to 22,000 seconds, in order to obtain meaningful mean value for all the local variables.

![Figure 1. Sketch of the arc-shaped profile.](image1)

![Figure 2. Single module of the corrugated channel under investigation.](image2)

At the end of each simulation, the local entropy generation, due to both heat transfer and viscous dissipation, was computed as follows:

\[ s_T = -\frac{1}{T^2} \mathbf{q} \cdot \nabla T \]  
\[ s_v = \frac{\mu}{T} \Phi \]  

\[ \Delta S_T = \int_{A_1} \rho s (\mathbf{v} \cdot \mathbf{n}) dA - \int_{A_2} \rho s (\mathbf{v} \cdot \mathbf{n}) dA - \int_{A_w} \frac{q}{T_w} dA_w \]  

where \( s \) indicates the specific entropy, \( A \) the cross-sectional area, \( A_w \) the heat transfer surface area, \( q \) the wall heat flux and \( T_w \) the wall temperature. The subscript 1 and 2 refer to the inlet and outlet sections, respectively.

The entropy fluxes, through the inlet and outlet sections and through the solid wall as well, were computed by post-processing the results of the numerical simulations.
The comparison between the overall entropy generation rate given by eq. (6) and the value obtained by integrating the entropy generation distributions, $s_r$ and $s_u$, over the channel module volume, showed deviations included within ± 1%.

The accuracy of the numerical model was checked by performing several simulations on different grid sizes; the grid independence analysis was carried out by monitoring both the averaged Nusselt number and the overall entropy generation rate.

It was observed that for a number of grid element higher than 6000, a mesh-independent solution can be achieved.

3. Results

The numerical simulations were performed for Reynolds number values ranges between 30 and 403 and for Prandtl number equal to 0.7, keeping $T_1$ (i.e. the bulk temperature at the inlet section) fixed at 300 K. The channel under investigation is characterized by $L/D_{max}=1.4$ and $D_{min}/D_{max}=0.23$.

It has to be pointed out that the Reynolds number was evaluated by considering the average cross-section diameter as characteristic length ($D_{ave}$), as shown in figure 2:

$$\text{Re} = \frac{\rho W_{ave} D_{ave}}{\mu} \quad (7)$$

being $W_{ave}$ the average value of the fluid velocity evaluated at the average cross-section.

The results confirmed that the flow regime becomes unsteady for $Re$ higher than about 60, as highlighted by Niçeno and Nobile [8] and by Rainieri et al. [9].

For all the simulations the Bejan number, $(S_r + S_u)/S_r$, was higher than 0.999. Therefore, the analysis of the entropy generation rate was limited to the heat transfer effects, while the effects of viscous dissipation were disregarded.

Figure 3 shows the non-dimensional overall entropy generation rate, as a function of the Reynolds number, which can be defined as follows:

$$\Sigma_r = \frac{S_r}{Q / T_m} \quad (8)$$

where $T_m$ is the thermodynamic mean temperature, which, neglecting pressure variations, is expressed as follows:

$$T_m = \frac{(h_2 - h_1)}{(s_2 - s_1)} = \frac{(T_2 - T_1)}{\ln(T_2/T_1)} \quad (9)$$

being $h_1$ and $h_2$ the specific averaged enthalpy evaluated on the inlet and on the outlet sections, respectively. $T_1$ and $T_2$ indicate the inlet and outlet bulk temperature, respectively.

As long as the flow regime is laminar ($Re < Re_c$) the overall entropy generation slightly increases; when the flow becomes unsteady, the entropy generation rate decreases monotonically with the Reynolds number.

In the same figure the Nusselt number averaged on all over the interface, is shown; it is obtained, by surface integration, from the local Nusselt number which is defined as follows:

$$Nu = \frac{q}{(T_m - T_b)} \frac{D_{ave}}{\lambda} \quad (10)$$

being $T_b$ the arithmetic mean of the inlet and the outlet bulk temperature, $\lambda$ the fluid thermal conductivity and $D_{ave}$ is the characteristic length defined in figure 2.
As expected, it has an opposite trend to the entropy generation, since it increases with $Re$ in the unsteady flow regime.

Indeed, from the entropy balance equation (eq. (6)), the relation between $\Sigma_T$ and $Nu$ is as follows:

$$\Sigma_T = 1 - \frac{1}{Re Pr} \frac{1}{\ln \left( \frac{T_2}{T_1} \right)} \int_a^b Nu \frac{T_w - T_i}{T_w} dA_w^*$$

in which $A_w^*$ is the dimensionless heat transfer surface area.

The non-dimensional entropy generation ranges between 0 and 1; the lower limit can be reached for $T_w \to T_1$, while the upper one for $T_w \to \infty$.

**Figure 3.** Overall entropy generation rate caused by heat transfer and averaged Nusselt number as a function of the Reynolds number.

### 3.1. Velocity and temperature distribution

The flow pattern and the temperature distribution are visualized by the streamlines and the temperature contours in figures 4 and 5, respectively. The maps show the solution obtained for the simulation stop time. As expected, owing to the symmetry conditions, the solution is symmetrical with respect to the channel symmetry axis in the laminar flow regime, while no longer being symmetrical in case of unsteady flow.

The streamline pattern points out that in the laminar flow regime two large vortices between the mean flow and the wall are formed. When the flow regime becomes unsteady the vortices increase and they produce a strong interaction between the mean flow and the wall. It results a very thick thermal boundary layer in the laminar flow regime, which occupies the entire flow recirculation zone.

On the contrary in the unsteady flow regime the thickness of the thermal boundary layer reduces as the size of the vortices adhering to the wall reduces.

Furthermore, since the value of the inlet temperature is the same for all values of $Re$ here considered, the wall temperature considerably reduces as the Reynolds number increases.
Figure 4. Streamlines for different values of $Re$.  

Figure 5. Temperature contours for different values of $Re$.  

$Re = 30$  
$Re = 51$  
$Re = 100$  
$Re = 403$
3.2. Local entropy generation

The local entropy generation due to heat transfer only is shown in figure 6, for the same values of the Reynolds number shown in the previous figures.

In the laminar flow regime \((Re < Re_c)\) the entropy generation is particularly high away from the wall, where the temperature gradients are useless to the convective heat transfer. When, instead, the flow becomes unsteady \((Re > Re_c)\), the entropy generation is concentrated near the wall.

At the same time the area near the channel axis, in which entropy generation rate almost vanishes, amplifies. Despite the maximum entropy generation value grows by one order of magnitude from the laminar to the unsteady flow regime, the total generation rate reduces, as shown by the graph in figure 3. Due to the relation between entropy generation and heat transfer efficiency, figure 6 highlights the critical zones, where a useless entropy generation rate affects the heat transfer efficiency.

![Figure 6](image)

**Figure 6.** Entropy generation due to heat transfer for different values of \(Re\).

4. Conclusions

The convective heat transfer problem for a \(Pr = 0.7\) fluid in a two dimensional periodic arc-shaped profile channel has been considered in a Reynolds number range which includes both the laminar and the unsteady flow regime. The local entropy generation due to heat transfer has been calculated by post-processing the temperature data, to assess the heat transfer efficiency of the considered wall profile within a second principle approach.

On the basis of the relationship between entropy generation rate and heat transfer coefficient in internal flow, the maps of entropy generation in the flow have allowed to identify the zones which are the main responsible for the loss of heat transfer efficiency.

In conclusion, the second principle approach has demonstrated that optimizing the convective heat transfer at the wall-fluid interface is equivalent to optimize the local entropy generation in the system volume, by reducing the excess entropy generation rate where useless for the heat transfer process.
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