Abstract

We parametrize lepton mixing matrix, known as PMNS matrix, in terms of three parameters which account deviations of three mixing angles from their bi-maximal or tri-bimaximal values. On the basis of this parametrization we can determine corresponding charged lepton mixing matrix in terms of those three parameters which can deviate bi-maximal or tri-bimaximal mixing. We find that the charged lepton mixing matrices which can deviate bi-maximal mixing matrix and tri-bimaximal mixing matrix exhibit similar structures. Numerical analysis shows that these charged lepton mixing matrices are close to CKM matrix of quark sector.

Key-words: Lepton mixing matrix, charged lepton correction, Bimaximal mixing and Tribimaximal mixing.

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1 Introduction

Over the last three years contributions from reactor [1–3], accelerator [4,5] and solar [6] neutrino experiments have provided precise values of three mixing angles and two mass squared differences under a three-neutrino mixing scenario. Global analysis [7–9] of 3ν oscillation data available from various experiments provides us an overall view on mixing parameters.

As neutrino experiments have been trying for more and more precision measurements of neutrino mixing parameters, meanwhile theorists have been trying to realize the flavour mixing pattern of leptons. Bimaximal mixing (BM) [10] and Tri-bimaximal mixing (TBM) [11] have been playing an attractive role in the search of flavour mixing pattern over a decade. Both these mixing schemes are $\mu - \tau$ symmetric [12] and predict maximal atmospheric mixing and zero reactor angle. They differ in their predictions of solar angle in such that BM mixing predicts maximal value of solar angle while TBM mixing leads to a value which equals $\arcsin\left(\frac{1}{\sqrt{3}}\right)$. Out of these two mixing schemes predictions of TBM mixing are more closer to global data [7–9] compared to the other. With the confirmation of non zero $\theta_{13}$ the deviation of lepton mixing from exact BM or TBM pattern is clear. It is therefore useful to study the deviations of lepton mixing from exact BM or TBM pattern. Deviations from BM or TBM mixing is in fact a natural idea frequently discussed in the literature [13–15].

In this paper we introduce three parameters which account for deviations of the three mixing angles, namely solar, atmospheric and reactor angle from their exact BM or TBM values. We then parametrize the lepton mixing matrix in terms of these three deviation parameters. Parametrization of lepton mixing matrix in terms of deviation parameters is also discussed in Ref. [16]. Our parametrization set up is however different from that. We mainly implicate the parametrization set up in predicting possible structure of charged lepton mixing matrix which in turn can generate the lepton mixing matrix from BM or TBM neutrino mixing via charged lepton correction. Charged lepton correction [17,18] is a very common tool to deviate special mixing schemes like BM or TBM mixing. Corrections to special mixing schemes can also be accounted in mass matrix formalism. We also analyse numerically the charged lepton mixing matrices with an interest to compare them with the CKM matrix [19] of quark sector. In Grand Unified Theory (GUT) based models [20] CKM like charged lepton corrections to special mixing schemes are naturally considered. Such models also incorporates Quark-Lepton Complementarity (QLC) [21].

Rest of the paper is organized as follows : in Section 2 we discuss the parametrization of the lepton mixing matrix in terms of deviation parameters. In Section 3 we discuss an implication of our model in charged lepton correction scenario. Finally Section 4 is devoted to summary and discussion.
2 Parametrization of lepton mixing matrix

In general, lepton mixing matrix, known as PMNS matrix, is parametrized in terms of three mixing angles, namely $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ which are commonly known as solar, atmospheric and reactor angle; and three CP violating phases- one Dirac CP phase $\delta$ and two Majorana phases $\alpha$ and $\beta$. In the standard Particle Data Group (PDG) parametrization it looks like

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P,$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ ($i, j = 1, 2$) and $P = \text{diag}(1, e^{i\alpha}, e^{i\beta})$ contains the Majorana CP phases. In the present work we however drop Majorana phase matrix $P$ assuming that neutrinos obey Dirac nature.

Both BM and TBM matrices predict $\theta_{13}^{bm/tb} = 0$ and $\theta_{23}^{bm/tb} = 45^\circ$ (suffixes $bm$ and $tb$ represent BM and TBM respectively). However their predictions for solar angle are different and are given by $\theta_{12}^{bm} = 45^\circ$ and $\theta_{12}^{tb} = \arcsin(\frac{1}{\sqrt{3}})$. Putting these predictions in Eq.(1), BM and TBM matrices can be obtained as

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

We now introduce three parameters which account for the deviations of three mixing angles from their corresponding BM or TBM values as follows :

$$\begin{align*}
\theta_{12} & = \theta_{12}^{bm/tb} + \delta\theta_{12}^{bm/tb}, \\
\theta_{23} & = \theta_{23}^{bm/tb} + \delta\theta_{23}^{bm/tb}, \\
\theta_{13} & = \delta\theta_{13}^{bm/tb},
\end{align*}$$

where the deviation parameters $\delta\theta_{12}^{bm/tb}$ and $\delta\theta_{23}^{bm/tb}$ can take positive as well as negative values whereas $\delta\theta_{13}^{bm/tb}$ takes only positive values. We present the best fit and $3\sigma$ values of mixing angles and Dirac CP phase in Table 1 [9]. Based on these global data we calculate the values of deviation parameters and are presented in Table 2.

For BM mixing we have from Eq.(4)

$$\begin{align*}
\theta_{12} & = 45^\circ + \delta\theta_{12}^{bm}, \\
\theta_{23} & = 45^\circ + \delta\theta_{23}^{bm}, \\
\theta_{13} & = 0.
\end{align*}$$
Substituting these values in Eq.(1) we have PMNS matrix as

\[
U_{PMNS} = \begin{pmatrix}
1 & \frac{1}{\sqrt{2}} \tilde{p} r e^{-i\delta} & \frac{1}{\sqrt{2}} \tilde{q} r e^{-i\delta} \\
0 & \frac{1}{\sqrt{2}} \tilde{p} q e^{i\delta} & \frac{1}{\sqrt{2}} \tilde{q} q e^{i\delta} \\
0 & -\frac{1}{2} (p q e^{i\delta} - \tilde{p} q e^{-i\delta}) & \frac{1}{2} (p q e^{i\delta} + \tilde{p} q e^{-i\delta})
\end{pmatrix},
\]

where

\[
p = \cos \delta \theta_{12} - \sin \delta \theta_{12}; \quad \tilde{p} = \cos \delta \theta_{12} + \sin \delta \theta_{12};
q = \cos \delta \theta_{23} + \sin \delta \theta_{23}; \quad \tilde{q} = \cos \delta \theta_{23} - \sin \delta \theta_{23};
\]

\[
r = \sin \delta \theta_{13}; \quad \tilde{r} = \cos \delta \theta_{13}.
\]

For TBM mixing we have from Eq.(4)

\[
\begin{align*}
\theta_{12} & = 35.26^\circ + \delta \theta_{12}; \\
\theta_{23} & = 45^\circ + \delta \theta_{23}; \\
\theta_{13} & = \delta \theta_{13}.
\end{align*}
\]

Table 1: Best fit and 3\(\sigma\) values of mixing angles and Dirac CP phase for normal and inverted hierarchy (NH and IH) from global data [9].

Substituting these values in Eq.(1) we have PMNS matrix as

\[
U_{PMNS} = \begin{pmatrix}
\sqrt{\frac{2}{3}} \tilde{p} r' e^{i\delta} & \frac{1}{\sqrt{3}} \tilde{p} r' e^{i\delta} & \frac{1}{\sqrt{2}} q r' e^{i\delta} \\
-\frac{1}{\sqrt{6}} (\tilde{p} q' + \sqrt{2} p q r' e^{i\delta}) & \frac{1}{\sqrt{3}} (p q' - \frac{1}{\sqrt{2}} \tilde{p} q r' e^{i\delta}) & \frac{1}{\sqrt{2}} q r' e^{i\delta} \\
\frac{1}{\sqrt{6}} (\tilde{p} q' - \sqrt{2} p q r' e^{i\delta}) & -\frac{1}{\sqrt{3}} (p q' + \frac{1}{\sqrt{2}} \tilde{p} q r' e^{i\delta}) & \frac{1}{\sqrt{2}} q r' e^{i\delta}
\end{pmatrix},
\]

where

\[
p' = \cos \delta \theta_{12} - \frac{1}{\sqrt{2}} \sin \delta \theta_{12}; \quad \tilde{p}' = \cos \delta \theta_{12} + \frac{1}{\sqrt{2}} \sin \delta \theta_{12};
q' = \cos \delta \theta_{23} + \sin \delta \theta_{23}; \quad \tilde{q}' = \cos \delta \theta_{23} - \sin \delta \theta_{23};
r' = \sin \delta \theta_{13}; \quad \tilde{r}' = \cos \delta \theta_{13}.
\]
Mixing Scheme | Model | Parameter | Best fit | 3 σ |
|-------------|--------|-----------|----------|-----|
| NH          | $\delta \theta_{12}$ | $-10.4^\circ$ | 13.2° - (-7.2°) |
|             | $\delta \theta_{23}$ | $3.9^\circ$ | -6.2° - 8.3° |
|             | $\delta \theta_{13}$ | $8.6^\circ$ | 7.9° - 9.3° |
| BM          | $\delta \theta_{12}$ | $-10.4^\circ$ | 13.2° - (-7.2°) |
|             | $\delta \theta_{23}$ | $4.2^\circ$ | -5.6° - 8.1° |
|             | $\delta \theta_{13}$ | $8.7^\circ$ | 8.0° - 9.4° |
| IH          | $\delta \theta_{12}$ | $-0.66^\circ$ | -3.46° - 2.53° |
|             | $\delta \theta_{23}$ | $3.9^\circ$ | -6.2° - 8.3° |
|             | $\delta \theta_{13}$ | $8.6^\circ$ | 7.9° - 9.3° |
| TBM         | $\delta \theta_{12}$ | $-0.66^\circ$ | -3.46° - 2.53° |
|             | $\delta \theta_{23}$ | $4.2^\circ$ | -5.6° - 8.1° |
|             | $\delta \theta_{13}$ | $8.7^\circ$ | 8.0° - 9.4° |

Table 2: Calculated values of deviation parameters from global data.

We want to emphasize that parametrization of lepton mixing matrix in terms of deviation parameters has also been discussed by King [16]. There also exists some interest in parametrizing the lepton mixing matrix in terms of Wolfenstein parameter $\lambda$ [22], where $\lambda$ accounts for the deviations of mixing angles from their values predicted by special mixing schemes.

3 An implication of the model: charged lepton mixing matrix

Deviations from BM or TBM mixing can be accounted in terms of charged lepton corrections [17,18]. In the basis where both charged lepton mass matrix ($m_l$) and left handed Majorana mass matrix ($m_\nu$) are non diagonal, lepton mixing matrix is given by the product of two mixing matrices as

$$U_{PMNS} = U_{lL}^\dagger U_\nu,$$  \hspace{1cm} (11)

where $U_{lL}$ diagonalizes $m_l$ and $U_\nu$ corresponds to the diagonalization of $m_\nu$. In the basis in which charged lepton mass matrix is itself diagonal PMNS matrix is directly given by $U_\nu$, $U_{lL}$ being identity matrix. The general idea of charged lepton correction is to work in the basis where both $m_l$ and $m_\nu$ are non diagonal and then considering $U_\nu$ be a special mixing matrix like BM or TBM a small perturbation to it is accounted from $U_{lL}$ leading to the desired PMNS matrix. Following this set up charged lepton corrections to special mixing patterns like BM, TBM, Hexagonal mixing etc. are done. For example charged lepton corrections to BM mixing are found in Refs. [23,24] and those to TBM mixing are discussed in Refs. [24,25]. With the same idea, in our work, we first find out $U_{lL}$ which can deviate BM neutrino mixing
matrix and yield the lepton mixing matrix in Eq.(6). In that case $U_\nu$ in Eq.(11) is given by $U_{BM}$ and corresponding $U_{IL}$ is then given by

$$U_{IL}^{bm} = \begin{pmatrix}
\frac{a}{\sqrt{2}} & -\frac{1}{\sqrt{2}}(b + z_1) & \frac{1}{\sqrt{2}}(c - z_2) \\
\frac{1}{\sqrt{2}}(d + z_3) & \frac{1}{2}(e + z_4) & \frac{1}{2}(f - z_5) \\
\frac{1}{\sqrt{2}}(d - z_3) & \frac{1}{2}(e - z_4) & \frac{1}{2}(f + z_5)
\end{pmatrix},$$

(12)

where

$$\begin{align*}
a &= \cos \delta \theta_{12}^{bm} \tilde{r}, \\
b &= \sin \delta \theta_{12}^{bm} \tilde{q}, \\
c &= \sin \delta \theta_{12}^{bm} q, \\
d &= \sin \delta \theta_{12}^{bm} \tilde{r}, \\
e &= q \tilde{r}, \\
f &= \tilde{q} \tilde{r}, \\
z_1 &= \cos \delta \theta_{12}^{bm} qre^{-i\delta}, \\
z_2 &= \cos \delta \theta_{12}^{bm} \tilde{q}re^{-i\delta}, \\
z_3 &= r e^{i\delta}, \\
z_4 &= \cos \delta \theta_{12}^{bm} \tilde{q} - \sin \delta \theta_{12}^{bm} qre^{-i\delta}, \\
z_5 &= \cos \delta \theta_{12}^{bm} q - \sin \delta \theta_{12}^{bm} \tilde{q}re^{-i\delta}.
\end{align*}$$

(13)

The parameters $a-f$ and $z_1-z_5$ are used to express the matrix in Eq.(12) in convenient way.

For TBM mixing case $U_\nu$ in Eq.(11) is given by $U_{TBM}$ and corresponding $U_{IL}$ is then given by

$$U_{IL}^{tb} = \begin{pmatrix}
a' & -\frac{1}{\sqrt{2}}(b' + z_1') & \frac{1}{\sqrt{2}}(c' - z_2') \\
\frac{1}{\sqrt{2}}(d' + z_3') & \frac{1}{2}(e' + z_4') & \frac{1}{2}(f' - z_5') \\
\frac{1}{\sqrt{2}}(d' - z_3') & \frac{1}{2}(e' - z_4') & \frac{1}{2}(f' + z_5')
\end{pmatrix},$$

(14)

where the parameters $a'-f'$ and $z_1'-z_5'$ are given by Eq.(13) with the substitutions of $\delta \theta_{12}^{tb}$, $q$, $\bar{q}$, $r$ and $\tilde{r}$ by $\delta \theta_{12}^{tb}$, $q'$, $\tilde{q}'$, $r'$ and $\tilde{r}'$ respectively.

We note that both charged lepton mixing matrices $U_{IL}^{bm}$ and $U_{IL}^{tb}$ have similar structure due to $\mu - \tau$ symmetry of BM and TBM mixing matrices. We estimate the numerical values (in modulus) of the elements of these mixing matrices for best fit values of deviation parameters and are presented in Eqs. (15) and (16).

$$U_{IL}^{bm} = \begin{pmatrix}
0.972512 & 0.183349 & 0.143535 \\
0.185651 & 0.980189 & 0.062209 \\
0.140544 & 0.074912 & 0.980319
\end{pmatrix}. $$

(15)

$$U_{IL}^{tb} = \begin{pmatrix}
0.988657 & 0.114991 & 0.096260 \\
0.108234 & 0.991394 & 0.072972 \\
0.103806 & 0.062329 & 0.992184
\end{pmatrix}. $$

(16)

Naturally there exists naive interest in searching connection between quark sector and lepton sector. Grand unified theories (GUTs) generally provide the framework for quark-lepton
unification. Quark-lepton-complementarity (QLC), which signifies interesting phenomenological relations between the lepton and quark mixing angles supports the idea of grand unification. Derivation of QLC relations assumes the deviation of lepton mixing from exact BM pattern to be described by quark mixing matrix. In GUT based models \[14,20,25\] charged lepton corrections to special neutrino mixing schemes are considered as CKM like. From such points of view we make comparison of the charged lepton mixing matrices in Eqs. (15) and (16) with the CKM matrix. For convenience, we present the best fit values (in modulus) of the elements of CKM matrix in Eq. (17) \[26\].

\[
V_{CKM} = \begin{pmatrix}
0.97428 & 0.2253 & 0.00347 \\
0.2252 & 0.97345 & 0.0410 \\
0.00862 & 0.0403 & 0.999152
\end{pmatrix}.
\]  

We see that both the mixing matrices are close to CKM matrix. Like CKM matrix the diagonal elements in these mixing matrices are close to unity and non diagonal elements exhibit an approximate symmetric nature. One significant point, we note, is that the corner elements, namely \((U_{lL})_{13}\) and \((U_{lL})_{31}\) in both the mixing matrices are relatively larger compared to those of \(V_{CKM}\) matrix.

4 Summary and discussion

BM and TBM are two special neutrino mixing schemes. To accommodate non zero \(\theta_{13}\) and deviations of solar mixing and atmospheric mixing from maximality these special mixing schemes should be modified. We have three parameters, viz. \(\delta\theta_{12}^{bm/\text{TBM}}, \delta\theta_{23}^{bm/\text{TBM}}\) and \(\delta\theta_{13}^{bm/\text{TBM}}\), which account the deviations of lepton mixing angles from their BM or TBM values. Numerical values of these deviation parameters can be obtained from global 3\(\nu\) oscillation data. We then parametrize PMNS matrix in terms of these parameters. Such parametrization of lepton mixing matrix may help authors in phenomenological works which incorporate deviation of special mixing schemes. We implicate our parametrization set up in predicting possible structure of charged lepton mixing matrices which can generate the desired lepton mixing matrix from BM or TBM mixing matrices. We have found that charged lepton mixing matrices \(U_{lL}'s\) in both cases (BM and TBM) exhibit similar structures. Numerical analysis shows that these mixing matrices \((U_{lL}^{\text{BM}}\) and \(U_{lL}^{\text{TBM}}\), necessary to deviate BM mixing and TBM mixing in obtaining mixing parameters consistent with global data, are close to the CKM matrix of quark sector. This result is in agreement with the assumption, generally made in GUT based model, that charged lepton correction to neutrino mixing can be considered as CKM like.

References

[1] F. P. An et al. [DAYA-BAY Collaboration] Phys. Rev. Lett. 108 171803 (2012) [arXiv:1203.1669 [hep-ex]]; Phys. Rev. Lett. 112 061801 (2014).
[2] J K Ahn et al. [RENO Collaboration] Phys. Rev. Lett. 108 191802 (2012) arXiv:1204.0626 [hep-ex].

[3] Y Abe et al. 2012 [DOUBLE-CHOOZ Collaboration] Phys. Rev. Lett. 108 131801 arXiv:1112.6353 [hep-ex].

[4] P. Adamson et al. [MINOS Collaboration] Phys. Rev. Lett. 110, 251801 (2013), Phys. Rev. Lett. 110, 171801 (2013).

[5] K Abe et al. [T2K Collaboration] Phys. Rev. Lett. 112, 061802 (2014), Phys. Rev. Lett. 112, 181801 (2014)

[6] A Renshaw et al. [SK Collaboration] Physics Procedia 00 (2014) 110.

[7] G Fogli, E Lisi, D Montanino, A Palazzo and A Rotunno Phys. Rev. D 86 013012 (2012).

[8] D Forero, M Tortola and J Valle Phys. Rev. D 86 073012 (2012).

[9] D Forero, M Tortola and J Valle, arXiv:1405.7540 [hep-ph].

[10] V. D. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, Phys. Lett. B 437, 107 (1998).

[11] P F Harrison, D H Perkins and W G Scott 2002 Phys. Lett. B 530 167 arXiv:hep-ph/0202074; P F Harrison and W G Scott 2002 Phys. Lett. B 535 163 arXiv:hep-ph/0203209.

[12] C S Lam 2001 Phys. Lett. B 507 214; P F Harrison and W G Scott 2002 Phys. Lett. B 547 219; C S Lam 2005 Phys. Rev. D 71 093001.

[13] M. Jezabek and Y. Sumino, Phys. Lett. B 457, 139 (1999); Z. Z. Xing, Phys. Rev. D 64, 093013 (2001), Phys. Lett. B 533 85 (2002) arXiv:hep-ph/0204049 v1.

[14] C. Giunti and M. Tanimoto, Phys. Rev. D 66, 053013 (2002).

[15] S K Kang and C. S. Kim Phys. Rev. D 90 (2014) 7, 077301; S T Petcov Nucl. Phys. B 892 (2015) 400-428; J Barry and W Rodejohann 2010 Phys. Rev. D 81 093002; X G He and A Zee 2011 Phys. Rev. D 84 053004; S Morisi, K M Patel and E Peinado 2011 Phys. Rev. D 84 053002 arXiv:1107.0690v2 [hep-ph].

[16] S. F. King arXiv:0710.0530 [hep-ph].

[17] S Antusch and S F King 2005 Phys. Lett. B 631 42 arXiv:hep-ph/0508044v2; A. Romanino Phys. Rev. D 70, 013003 (2004); K A Hochmuth, S T Petcov and W Rodejohann 2007 Phys. Lett. B 654 177 arXiv:0706.2975v2 [hep-ph]; S Boudjemaa and S F King 2009 Phys. Rev. D 79 033001.
[18] D Marzocca, S. T. Petcov, A Romanino and M. C. Sevilla J. High Energy Phys. 1305 (2013) 073; D Marzocca, A Romanino J. High Energy Phys. 1411 (2014) 159; S. Dev, R R Gautam and L Singh Phys. Rev. D 89 (2014) 1, 013006; S Roy and N N Singh Indian J. Phys. 88 (2014) 5, 513-519; K Siyeon [arXiv:1208.2645[hep-ph]]; J. A Acosta, A Aranda, M A. Buen-Abad, A D. Rojas Phys. Lett. B 718 (2013) 1413-1420; A Rashed Nucl. Phys. B 874 (2013) 679-697

[19] N. Cabibbo Phys. Rev. Lett. 10, 531 (1963), M. Kobayashi and T. Maskawa Prog. Theor. Phys. 49, 652 (1973).

[20] T. Ohlsson Phys. Lett. B 622, 159 (2005); D. Marzocca, S. T. Petcov, A. Romanino, M. Spinrath [arXiv:1108.0614v2 [hep-ph]]; S Antusch and V Maurer 2011 Phys. Rev. D 84 117301 [arXiv:1107.3728v2[hep-ph]]; S Antusch, C Gross, V Maurer and C Sluka [arXiv:1205.1051v2[hep-ph]].

[21] M. Raidal, Phys. Rev. Lett. 93, 161801 (2004); H. Minakata and A. Y. Smirnov, Phys. Rev. D 70, 073009 (2004); S. Antusch, S. F. King and R. N. Mohapatra, Phys. Lett. B 618, 150 (2005); Y Zheng Phys. Rev. D 81, 073009 (2010)

[22] S F King 2012 Phys. Lett. B 718 136 [arXiv:1205.0506v2 [hep-ph]]; B Hu Phys. Rev. D 87 (2013) 5, 053011 ; S. Roy, S. Morisi, N. N. Singh and J. W. F. Valle arXiv:1410.3658v1 [hep-ph].

[23] P H Frampton, S T Pet cov and W Rodejohann 2004 Nucl. Phys. B 687 31 [arXiv:hep-ph/0401206v2].

[24] F Bazzocchi and L Merlo [arXiv:1205.5135v1[hep-ph]]; C Duarah, A Das and N N Singh 2012 Phys. Lett. B 718 147 [arXiv:1207.5225[hep-ph]].

[25] C. Duarah, A. Das, N. N. Singh, Indin J. Phys. 2013, 87(12):1269-1274

[26] K Nakamura et. al. (Particle Data Group) J. Phys. G: Nucl. Part. Phys. 37 (2010) 075021.