Semi-classical gravity in de Sitter spacetime and the cosmological constant

Benito A. Juárez-Aubry*

Departamento de Física Matemática
Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas,
Universidad Nacional Autónoma de México,
A. Postal 70-543, Mexico City 045010, Mexico

12 March 2019

Abstract

We show that there exist solutions to the semi-classical gravity equations in de Sitter spacetime sourced by the renormalised stress-energy tensor of a free Klein-Gordon field. For the massless scalar, solutions exist for every possible value of the cosmological constant, provided that the curvature coupling parameter is chosen appropriately. In the massive case, the mass of the field and the curvature coupling severely constrain the allowed values of $\Lambda$. For a massive, minimally coupled field, a “small $\Lambda$” solution is found, fixed by the relation $m^2 \simeq 4.89707 \times 10^{12} \Lambda$. We emphasise that in this framework, the old cosmological constant problem dissipates, in the sense that there are no bare-vs-physical values of $\Lambda$, for only the physical $\Lambda$ appears in the semi-classical equations.

1 Introduction

The standard model of cosmology, $\Lambda$CDM, is currently the most successful theoretical model for explaining the evolution of the universe as a whole from the CMB epoch to the current era. While there is no doubt about its phenomenological success, it has left room for several puzzles in theoretical physics. Some of these puzzles include the origin of the small and positive value of $\Lambda$, in contrast with the (heuristically estimated) energy density of matter and the nature of dark matter; the problem of flatness; the origin of CDM and its rôle in the CMB data and galaxy rotation curves; the absence of tensor modes in the CMB as far as we have measured; the large-scale isotropy and homogeneity of spacetime and how structure forms at short-scales and, of course, the rôle that quantum theory plays as a foundation of cosmology itself.

In addressing some of these puzzles, it is widely agreed that one of the early stages of the evolution of the universe should be described by an inflationary cosmological

*benito.juarez@iimas.unam.mx
phase, which should resolve, for instance, the flatness puzzle and realise the cosmological principle of large-scale homogeneity and isotropy.

On the one hand, it is remarkable that the simplicity of the idea of inflation is in agreement with available cosmological data. On the other hand, the fact that inflation is a framework, rather than a concrete model, leaves the status of inflationary cosmology without precise guidelines of what the concrete realisation of inflation should be. To aggravate the situation, many simple, well-motivated models seem to be at odds with our available data \[1\]. For example, many inflationary models seem to be divorced from the standard model of particle physics, and there is no natural field available to play the rôle of the inflaton.

In line with these ideas, and motivated by observational cosmology, phenomenological alternative models to $\Lambda$CDM, ranging from modifications of General Relativity \[2\] to the above-mentioned addition of matter fields not present in the standard model, for example, in the context of inflation.

In this paper, we wish to put forward the idea that a potentially good approach to dealing with (at least some of) the problems mentioned above is to take semi-classical gravity seriously in the cosmological context.\[3\] For example, it is quite possible that higher-order curvature corrections can play a rôle during inflation\[4\] without necessarily adding new inflaton fields. Also, problems such as the cosmological constant puzzle are best framed in the setting of semi-classical gravity, without necessarily making reference to Minkowskian mode expansions or energy cut-offs (see e.g. the excellent references \[3, 4, 5\]), as we shall see below.

In this paper, we study the semi-classical gravity equations in de Sitter spacetime for a free Klein-Gordon field. On the one hand, this is an interesting problem in its own right in the sense that few exact semi-classical gravity solutions are known. The author is aware of \[6\], in which a massive Klein-Gordon field with $\xi = 1/6$ exhibits a de Sitter phase. Due to the isometries of de Sitter spacetime, we are able to show the existence of a large number of solutions parametrised by the mass parameter of the Klein-Gordon field, $m^2$, and the curvature coupling parameter, $\xi$. On the other hand, as we shall see, in this setting the old cosmological constant problem disappears.

The old cosmological constant problem is a naturalness one, which posits that some bare cosmological constant, $\Lambda_{\text{bare}}$, of the order of the very large energy density of the quantum fields of matter, should be renormalised with exquisite precision to the small, positive value that we observe, $\Lambda_{\text{ren}}$.

In the semi-classical setting, as we shall see below, bare and renormalised quantities for $\Lambda$ simply do not appear. Instead, $\Lambda$ should take values that are consistent with solutions to the semi-classical Einstein field equations. The point that we shall make below, in the context of a Klein-Gordon simple model, is that in some cases the values of $\Lambda$ is highly restricted, and sometimes uniquely determined, in terms of the parameters of the Klein-Gordon field, $m^2$ and $\xi$. Moreover, as we shall see, while the value of $\Lambda$ will be necessarily proportional to $m^2$, the proportionality factor can account for several orders of magnitude in difference between the two parameters. In particular, in the case of a massive, minimally coupled field, the ratio $m^2/\Lambda \sim 10^{12}$.

---

1Of course, semi-classical gravity is indeed taken seriously for computing cosmological perturbations, but we wish to emphasise its rôle in other cosmological situations.

2$f(R)$ inflation, for example, hints in this direction.
This paper is organised as follows: After this introduction, in Sec. 2 we briefly review the elements of semi-classical gravity. In Sec. 3 we specify to the problem of a Klein-Gordon field in de Sitter spacetime. We show in Sec. 4 that there exists solutions to the problem stated in sec. 3, and that for such solutions the cosmological constant will (almost always) be restricted to specific values in terms of the \( m^2 \) and \( \xi \) parameters of the Klein-Gordon field. Our final remarks are made in Sec. 5.

Throughout this work we set \( \hbar = 1 \) and \( c = 1 \). Spacetime points are denoted by Roman characters (\( x, y, \ldots \)). Complex conjugation is denoted by an overline. Operators on a Hilbert space are surmounted with carets, e.g. \( \hat{O} \) and the adjoint is denoted by a \( ^* \), e.g. \( \hat{O}^* \) is the adjoint of \( \hat{O} \). Spacetime points are denoted by roman characters. In de Sitter spacetime, \( x \) denotes the space coordinates. Abstract index tensor notation is used throughout. \( O(x) \) denotes a quantity for which \( O(x)/x \) is bounded as \( x \to 0 \).

2 Semi-classical gravity preliminaries

We wish to obtain solutions to the semi-classical Einstein field equations for a quantum Klein-Gordon field in de Sitter spacetime. The semi-classical gravity equations are

\[
\begin{align*}
G_{ab} + \Lambda g_{ab} &= 8\pi G_N \langle \hat{T}_{ab} \Psi \rangle, \\
(\Box - m^2 - \xi R)\hat{\Phi} &= 0,
\end{align*}
\]

(2.1a) (2.1b)

where the classical geometry is sourced by the renormalised quantum stress-energy tensor of the matter field. Here, we consider \( \Lambda > 0 \) as the cosmological constant. The main reason for considering positive values is that we wish to restrict the analysis to spacetimes that are globally hyperbolic. In this case, no additional boundary conditions need to be prescribed for the field equations.

The quantum Klein-Gordon field is an operator-valued distribution, \( f \mapsto \hat{\Phi}(f) \), for \( f \in C^\infty_0(M) \), which is densely defined on the relevant Fock space of the theory, \( \hat{\Phi}(f) : \mathcal{B} \subset \mathcal{H} \to \mathcal{H} \). In our case of interest, the Fock space will be the one that is “built out” of the Bunch-Davies vacuum. The set of field observables forms a unital operator \(^*\)-algebra, \( \mathcal{A}_{KG} \), generated by smeared fields and subject to the following relations: for \( f, g \in C^\infty_0(M) \), (i) \( f \mapsto \hat{\Phi}(f) \) is linear, (ii) \( \hat{\Phi}(f)^* = \hat{\Phi}(f) \) the field is self-adjoint, (iii) \( \hat{\Phi}\left(\Box - m^2 - \xi R\right)f = 0 \), the field equation (2.1b) holds by integration by parts twice and (iv) \([\hat{\Phi}(f), \hat{\Phi}(g)] = -iE(f,g)1\) the field satisfies the canonical commutation relations, where \( E = E^- - E^+ \) is the advanced-minus-retarded fundamental Green operator of the Klein-Gordon operator \( \Box - m^2 - \xi R \).

The discussion on how to compute the renormalised stress-energy tensor that appears on the right-hand side of (2.1a) is a well-trodden path for the Klein-Gordon field. Our purpose is therefore to give a short, non-exhaustive review and we refer the reader to the classical literature [7] for the details. The starting point is computing the two-point function in a Hadamard state, \( \Psi \), which in a geodesically convex subset, \( O \subset M \), in which \( \sigma(x,x') \), the half-squared geodesic distance is well defined, takes the Hadamard

---

3The interested reader might also look at [8, 9, 10, 11].
form
\[ G^+(x, x') = \frac{1}{8\pi^2} \left[ \frac{\Delta^{1/2}(x, x')}{\sigma_c(x, x')} + v(x, x') \ln \left( \sigma_c(x, x')/\ell^2 \right) + w(x, x') \right]. \tag{2.2} \]

Here, \( \sigma_c(x, x') = \sigma(x, x') + 2iT(x, x')\epsilon + \epsilon^2 \) is the regularised half-squared geodesic distance, \( \Delta \) is the van Vleck-Morette determinant and \( v \) and \( w \) are smooth bi-functions computed as a covariant Taylor series in powers of \( \sigma \) through the Hadamard recursion relations, obtained by demanding that \( G^+(x, x') \) be a solution of the Klein-Gordon equation in the \( x \)-variable, provided that an initial value \( w_0 \) for the \( O(1) \) term in the \( w \)-series is prescribed. The initial values of \( u \) and \( v \) are determined geometrically, and independent of the quantum state. The datum \( w_0 \) is state dependent.

The renormalised stress-energy tensor is obtained by acting with a differential two-point operator on the singularity-subtracted two-point function, the smooth bi-function
\[ G_{\text{reg}}^+ = G^+(x, x') - H_\ell(x, x'), \]

where \( H_\ell(x, x') = \frac{1}{8\pi^2} \left[ \frac{\Delta^{1/2}(x, x')}{\sigma_c(x, x')} + v(x, x') \ln \left( \sigma_c(x, x')/\ell^2 \right) + w^\text{Had}(x, x') \right]. \tag{2.3} \]
is the Hadamard parametrix, with \( \ell \) a fixed length scale and \( w^\text{Had} \) as the \( w \) smooth bi-function obtained from the initial value \( w_0 = 0 \). We define the renormalised stress-energy tensor by

\[
\langle \Psi | T_{ab}(x) | \Psi \rangle = \lim_{x' \to x} T_{ab} \left[ G_{\text{reg}}^+ - \frac{1}{8\pi} g_{ab} v_1(x, x') \right] + \Theta_{ab}(x), \tag{2.4a} \]

\[
T_{ab} = (1 - 2\xi) g_a {\mathbf b}' (\nabla a) (\nabla b') + \left( 2\xi - \frac{1}{2} \right) g_{ab} g^{cd} (\nabla c) (\nabla d') \frac{1}{2} g_{ab} m^2 \]
\[ + 2\xi \left[ -g_a {\mathbf b}' g_b {\mathbf b}' \nabla d \nabla b' + g_{ab} g^{cd} \nabla c \nabla d + \frac{1}{2} G_{ab} \right], \tag{2.4b} \]

where derivatives with primed and unprimed indeces are evaluated at the points \( x' \) and \( x \) respectively. In eq. 2.4, \( g_{a'b'} \) denotes the bi-vector of parallel transport from \( x \) to \( x' \), with the condition \( \lim_{x' \to x} g_{a'b'} = g_{ab} \). The term \( [v_1]_c \) is given by the diagonal of the \( v_1 \) coefficient in the Hadamard series of \( v \), namely \[ [v_1]_c = \frac{1}{8} m^4 + \frac{1}{4} \left( \xi - \frac{1}{6} \right) m^2 R - \frac{1}{24} \left( \xi - \frac{1}{5} \right) \Box R \]
\[ + \frac{1}{8} \left( \xi - \frac{1}{6} \right)^2 R^2 - \frac{1}{720} R_{ab} R^{ab} + \frac{1}{720} R_{abcd} R^{abcd}, \tag{2.5} \]
and \( \Theta \) a geometric, covariantly conserved, symmetric tensor of dimension of lengh to the minus fourth power, built out of the metric and its derivatives. For conformally-coupled fields, \( [v_1]_c \) is responsible for the trace anomaly \[ \Box \].

As a final word for the section, notice that the presence of the term \( [v_1]_c \), cf. eq. 2.5, spoils the second order, hyperbolic form of the semi-classical system (2.2). This is a well-known problem in semi-classical gravity. However, as we shall see below, in the symmetry-reduced case of de Sitter spacetime, this problem does not occur.
3 Semi-classical gravity in de Sitter spacetime

The metric tensor for the (3 + 1)-dimensional de Sitter spacetime has the form $g = (\alpha/\eta)^2(-dt^2 + dx^2 + dy^2 + dz^2)$, with $\alpha^2 = 3/\Lambda$. Eq. (2.1b) has been studied in de Sitter spacetime in \[13\] by exploiting the spatial symmetries of the problem, whereby the wave equation reduces to an ODE for the temporal part that can be brought to a Bessel equation form. The quantum fields can be concretely represented as operators in the Hilbert space $H$. As such, we have

$$\hat{\Phi}(\eta, x) = (2\pi)^{-3/2} \int d^3k \left( \psi_k e^{ik\cdot x} \hat{a}_k + \overline{\psi}_k e^{-ik\cdot x} \hat{a}^*_k \right),$$

$$\psi_k(\eta) = \alpha^{-1}(18\pi)^{1/2} \eta^{3/2} H^{(2)}_\nu(k\eta), \quad \nu^2 = 9/4 - 12(m^2/R + \xi).$$ (3.1a)

Annihilation operators annihilate $\Omega_{BD} \in \mathcal{H}$, the Bunch-Davies vacuum, which is cyclic in the sense that $\mathcal{A}_{KC} \Omega_{BD} \subset \mathcal{H}$ is dense in the Fock space $\mathcal{H}$.

The two-point function, which characterises the Bunch-Davies vacuum (as it is quasi-free), can then be obtained directly as a sum over modes,

$$G^+(x, x') = (2\pi)^{-3} \int_{\mathbb{R}^3} d^3k \psi_k(\eta)\overline{\psi}_k(\eta') e^{ik\cdot(x-x')},$$ (3.2a)

and it admits a closed form expression in terms of hypergeometric functions

$$G^+(x, x') = (16\pi\alpha^2)^{-1} \sec \left[ \pi \nu(1/4 - \nu^2) \right] F \left[ \frac{3}{2} + \nu; 2; 1 + \frac{(\eta - \eta')^2 - |r - r'|^2}{4\eta\eta'} \right].$$ (3.3)

We should mention that, in the algebraic approach to quantum field theory, the (algebraic) state is defined by all its $n$-point functions, as well as a normalisation and a positivity requirements. The concrete operators on a Hilbert space representations are obtained via the GNS construction. For vacuum states, all $n$-point functions are reconstructed from the two-point function -- they are quasi-free. Hence, eq. (3.2a) can be taken as the definition of the Bunch-Davies vacuum and the starting point of quantum field theory in de Sitter spacetime, together with the abstract Klein-Gordon algebra.

In \[13\] the point-splitting and renormalisation of the stress-energy tensor in the Bunch-Davies vacuum has been performed (cf. Sec. 2), yielding

$$\langle \Omega_{BD}|\hat{T}_{ab}\Omega_{BD} \rangle = \frac{g_{ab}}{(8\pi)^2} \left\{ m^2 \left[ m^2 + (\xi - 1/6) R \right] \left[ \psi(3/2 + \nu) + \psi(3/2 - \nu) \right] - \ln \left( 12m^2/R \right) - \frac{1}{2}(\xi - 1/6)^2 R^2 + \frac{R^2}{2160} - m^2(\xi - 1/6) R - \frac{m^2 R}{18} \right\}. \quad (3.4)$$

The fact that the renormalisation ambiguity term, $\Theta_{ab}$, is absent in the expression above is due to the fact that for maximally symmetric spacetimes, this term is identically zero, as can be seen by the following argument. By local covariance and the stress-energy conservation, $\nabla^a \langle \Omega_{BD}|\hat{T}_{ab}\Omega_{BD} \rangle = 0$, we must have that in constant curvature spacetimes

$$\Theta_{ab} = \alpha_1 \left( -\frac{1}{2}g_{ab} R^2 + 2RR_{ab} \right) + \alpha_2 \left( 2R_a^\cdot c R_{cb} - \frac{1}{2}g_{ab} R^{cd} R_{cd} \right) + \alpha_3 m^4, \quad (3.5)$$
where the $\alpha_i \in \mathbb{R}$, $i = \{1, 2, 3\}$, are renormalisation ambiguities. The terms that accompany the coefficients $\alpha_1$ and $\alpha_2$ can be seen to vanish algebraically because $R_{abcd} = (\Lambda/3)^2(g_{ac}g_{bd} - g_{ad}g_{cb})$. The term $\alpha_3 m^4$ can be removed by demanding that Wald’s fourth axiom [27], namely that for the Minkowski vacuum in Minkowski spacetime $\langle \Omega_M|\hat{T}_{ab}\Omega_M \rangle = 0$, be satisfied in the limit

$$\lim_{\Lambda \to 0^+} \langle \Omega_BD|\hat{T}_{ab}\Omega_BD \rangle = \langle \Omega_M|\hat{T}_{ab}\Omega_M \rangle = 0.$$  

\section{Existence of solutions in de Sitter spacetime}

We now seek solutions to eq. (2.1). Due to the large symmetry of the problem, the task is reduced to solving an algebraic relation. In turn, this relation will provide the admissible values for $\Lambda$ in terms of the parameters of the Klein-Gordon field theory, $m^2$ and $\xi$.

\subsection{The massless case}

In the massless case, $m^2 = 0$, there are solutions for any $\Lambda > 0$ provided that $\xi$ takes the values $\xi_+ = 1/6 + (1080)^{-1/2}$ or $\xi_- = 1/6 - (1080)^{-1/2}$.

\subsection{The massive case}

For the massive case, set $x = m^2/(4\Lambda)$. The solutions lie at the roots of the function $f_\xi : \mathbb{R}^+ \to \mathbb{R}$ defined by

$$f_\xi(x) = \psi(3/2 + \nu(x)) + \psi(3/2 - \nu(x)) - \ln(12x) + [x + (\xi - 1/6)]^{-1} \left[ \frac{1 - 1080(\xi - 1/6)^2}{2160x} - \xi + \frac{1}{9} \right], \quad (4.1)$$

where $\nu$ is the complex-valued function $\nu(x) = (9/4 - 12x + \xi)^{1/2}$. There are three relevant cases of interest: (i) For $(9/4 + \xi)/12 < x$, $\nu(x)$ is purely imaginary, (ii) for $(9/4 + \xi)/12 = x$, $\nu(x) = 0$ and (iii) for $(9/4 + \xi)/12 > x$, $\nu(x)$ is real.

\subsubsection{Case (i) $x > (9/4 + \xi)/12$}

In this case, we choose the square root branch such that $\nu(x) = i(12x - \xi - 9/4)^{1/2}$. Set $y = 12x - \xi - 9/4$, then we need to find the roots of

$$g_\xi(y) = \psi\left(3/2 + iy^{1/2}\right) + \psi\left(3/2 - iy^{1/2}\right) - \ln(y + \xi + 9/4) + \frac{48}{1 + 4y + 52\xi} \left[ \frac{1 - 1080(\xi - 1/6)^2}{180(y + \xi + 9/4)} - \xi + \frac{1}{9} \right]. \quad (4.2)$$
for \( y > 0 \). In order to do so, define

\[
g_1(y, \xi) = \psi\left(3/2 + iy^{1/2}\right) + \psi\left(3/2 - iy^{1/2}\right) - \ln(y + \xi + 9/4), \tag{4.3a}
\]

\[
g_2(y, \xi) = \frac{48}{1 + 4y + 52\xi}\left[ \frac{1 - 1080(\xi - 1/6)^2}{180(y + \xi + 9/4)} - \xi + \frac{1}{9} \right], \tag{4.3b}
\]

in the domain \( y > 0, \xi > -y - 9/4 \), such that \( g_\xi(y) = g_1(y, \xi) + g_2(y, \xi) \). It is clear that \( g_1(y, \xi) \) is a decreasing function of \( \xi \).

Let us consider for concreteness \( \xi \geq 0 \). At fixed \( \xi \geq 0 \), \( g_1(y, \xi) \) is a strictly increasing, negative function in \( y \), which asymptotes logarithmically fast to \( 0^- \) as \( y \to \infty \).

In order to have roots for \( g_\xi \), we seek that \( g_2(y, \xi) \) at fixed \( \xi \) take positive values for \( y > 0 \). First, notice that \( g_2(y, \xi) \) has a root at \( y = (16 - 25\xi - 1260\xi^2)/(20(-1 + 9\xi)) \), which is positive for \( \xi \in ((-25 + (81265)^{1/2})/2520, 1/9) \). We exclude these values for the following analysis, and consider that for fixed \( \xi \in [0, (-25 + (81265)^{1/2})/2520) \cup (1/9, \infty) \), \( g_2 \) has fixed sign in the domain \( y > 0 \). For sufficiently large \( y \), it is clear that \( \text{sgn}(g_2) = \text{sgn}(-\xi + 1/9) \), and hence \( g_2(\xi, y) > 0 \) for \( \xi \in [0, (-25 + (81265)^{1/2})/2520) \).

Notice that \( g_2(y, \xi) = O(y^{-2}) \), and hence \( g_1 \) and \( -g_2 \) are not parallel curves, with \( g_2 \) approaching \( 0 \) more rapidly as \( y \to \infty \). Thus, one can generically seek to find the intersection of the curves \( g_1 \) and \( -g_2 \). We illustrate this with the minimally coupled field, for which \( \xi = 0 \), where one finds that \( y \simeq 1.46912 \times 10^{13} \), i.e.,

\[
m^2 \simeq 4.89707 \times 10^{12}\Lambda, \quad \text{for } \xi = 0. \tag{4.4}
\]

### 4.2.2 Case (ii) \( x = (9/4 + \xi)/12 \)

In case (ii), the curvature coupling is restricted to \( \xi_c = 12x - 9/4 \) and we have to find the roots of

\[
f_{\xi_c}(x) = 2\psi(3/2) - \ln(12x) + (13x - 29/12)^{-1}\left( \frac{1 - 1080(12x - 29/12)^2}{2160x} - 12x + \frac{85}{36} \right), \tag{4.5}
\]

for \( x \in (0, 29/156) \cup (29/156, \infty) \). There exists one root of \( f_{\xi_c} \), which can be computed numerically to yield \( x_c \simeq 0.19736 \). Hence, there exist semi-classical solutions for the parameters of the problem satisfying \( m^2 = 4x_c\Lambda \) and \( \xi = 12x_c - 9/4 \).

### 4.2.3 Case (iii) \( 0 < x < (9/4 + \xi)/12 \)

Set \( z = -12x + \xi + 9/4 \) and define the function for \( 0 < z < \xi + 9/4 \)

\[
h(z, \xi) = -\log\left( \xi - z + \frac{9}{4} \right) + \psi\left( \frac{3}{2} - \sqrt{z} \right) + \psi\left( \sqrt{z} + \frac{3}{2} \right) - \left( \xi + \frac{1}{12} \left( \xi - z + \frac{9}{4} \right) - \frac{1}{6} \right)^{-1}\left( -\xi + \frac{1 - 1080(\xi - \frac{1}{4})^2}{180(\xi - z + \frac{9}{4})} + \frac{1}{9} \right). \tag{4.6}
\]

It follows from the roots of the digamma function (at negative argument) that for
sufficiently large \( \xi \) there are several roots for \( h \) that define solutions to the semi-classical problem, which can be explored numerically, and in turn fix the admissible values of \( \Lambda \) in terms of \( m^2 \) and \( \xi \).

5 Final remarks

In this work, we have proposed that, when taking the semi-classical Einstein field equations coupled to quantum matter seriously, the value of the cosmological constant will be determined by the field equations themselves in terms of the parameters of the theory – the mass and curvature coupling in the case of the Klein-Gordon field. In this sense, the physical cosmological constant that we measure in the universe should be determined from the semi-classical field equations sourced by the stress-energy tensor of the standard model of particle physics, presumably on a FLRW background to a good approximation.

In the simple model that we have studied, that of a Klein-Gordon field in de Sitter spacetime, we can see that the common folklore stating that the “bare” value of the cosmological constant must have a very large contribution from quantum fields, and then cancelled by a fine-tuned counterterm to yield a small “renormalised” cosmological constant plays no rôle in the present calculations. Indeed, if one wishes to interpret our results in terms of bare and renormalised quantities, one could interpret semi-classical gravity as providing the renormalised \( \Lambda = \Lambda_{\text{ren}} \) directly. Hence, from this viewpoint, the old cosmological constant problem is not present. Further, this paper provides counter-evidence that one could estimate the value of \( \Lambda \) to be of the order of the mass of the fields. Indeed, we have exemplified in Sec. 4 that for massless fields, the cosmological constant can take any positive value, while for a massive, minimally-coupled field we have the ratio \( m^2/\Lambda \sim 10^{12} \).

We have not mentioned anything so far about the new cosmological constant problem (see e.g. [4, Sec. 2.3]), as this is an interacting theory problem. For addressing such a matter, perhaps modern techniques of perturbation theory in curved spacetimes should be useful [14].

Acknowledgments

The author thanks Claudio Dappiaggi for a useful e-mail exchange, in which ref. [6] was pointed to the author. The author acknowledges Daniel Sudarsky for stimulating conversations. This work is supported by a DGAPA-UNAM Postdoctoral Fellowship.

References

[1] A. Ijjas, P. J. Steinhardt and A. Loeb, “Inflationary paradigm in trouble after Planck2013”, Phys. Lett. B 723, 261 (2013) doi:10.1016/j.physletb.2013.05.023 [arXiv:1304.2785 [astro-ph.CO]].
[2] T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, “Modified Gravity and Cosmology”, Phys. Rept. 513 (2012) 1 doi:10.1016/j.physrep.2012.01.001 [arXiv:1106.2476 [astro-ph.CO]].

[3] S. Weinberg, “The Cosmological Constant Problem”, Rev. Mod. Phys. 61 (1989) 1. doi:10.1103/RevModPhys.61.1

[4] A. Padilla, “Lectures on the Cosmological Constant Problem”, arXiv:1502.05296 [hep-th].

[5] J. Martin, “Everything You Always Wanted To Know About The Cosmological Constant Problem (But Were Afraid To Ask)”, Comptes Rendus Physique 13 (2012) 566 doi:10.1016/j.crhy.2012.04.008 [arXiv:1205.3365 [astro-ph.CO]].

[6] C. Dappiaggi, K. Fredenhagen and N. Pinamonti, “Stable cosmological models driven by a free quantum scalar field”, Phys. Rev. D 77 (2008) 104015 doi:10.1103/PhysRevD.77.104015 [arXiv:0801.2850 [gr-qc]].

[7] R. M. Wald, Quantum Field Theory in Curved Space-Time and Black Hole Thermodynamics (University of Chicago Press, 1994).

[8] V. Moretti, “Comments on the stress energy tensor operator in curved spacetime”, Commun. Math. Phys. 232 (2003) 189 doi:10.1007/s00220-002-0702-7 [gr-qc/0109048].

[9] S. M. Christensen, “Vacuum Expectation Value of the Stress Tensor in an Arbitrary Curved Background: The Covariant Point Separation Method”, Phys. Rev. D 14 (1976) 2490. doi:10.1103/PhysRevD.14.2490

[10] R. M. Wald, “Trace Anomaly of a Conformally Invariant Quantum Field in Curved Space-Time”, Phys. Rev. D 17 (1978) 1477. doi:10.1103/PhysRevD.17.1477

[11] Y. Decanini and A. Folacci, “Hadamard renormalization of the stress-energy tensor for a quantized scalar field in a general spacetime of arbitrary dimension”, Phys. Rev. D 78 (2008) 044025 doi:10.1103/PhysRevD.78.044025 [gr-qc/0512118].

[12] B. S. DeWitt and R. W. Brehme, “Radiation damping in a gravitational field”, Annals Phys. 9 (1960) 220. doi:10.1016/0003-4916(60)90030-0

[13] T. S. Bunch and P. C. W. Davies, “Quantum Field Theory in de Sitter Space: Renormalization by Point Splitting”, Proc. Roy. Soc. Lond. A 360 (1978) 117. doi:10.1098/rspa.1978.0060

[14] K. Rejzner, Perturbative Algebraic Quantum Field Theory (Springer, 2016).