Toward a solution to the $R_{AA}$ and $v_2$ puzzle for heavy quarks

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(Dated:)

The heavy quarks constitute a unique probe of the quark-gluon plasma properties. Both at RHIC and LHC energies a puzzling relation between the nuclear modification factor $R_{AA}(p_T)$ and the elliptic flow $v_2(p_T)$ has been observed which challenged all the existing models, especially for D mesons. We discuss how the temperature dependence of the heavy quark drag coefficient is responsible to address for a large part of such a puzzle. In particular, we have considered four different models to evaluate the temperature dependence of drag and diffusion coefficients propagating through a quark gluon plasma (QGP). All the four different models are set to reproduce the same $R_{AA}(p_T)$ observed in experiments at RHIC and LHC energy. We point out that for the same $R_{AA}(p_T)$ one can generate 2-3 times more $v_2$ depending on the temperature dependence of the heavy quark drag coefficient. An increasing drag coefficient as $T \to T_c$ is a major ingredient for a simultaneous description of $R_{AA}(p_T)$ and $v_2(p_T)$.

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The ongoing nuclear collision programs at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies are expected to create a medium that behaves like a nearly perfect fluid, where the bulk properties of the matter are governed by the light quarks and gluons called Quark Gluon Plasma (QGP) [1, 2]. To characterize the QGP, penetrating and well calibrated probes are essential. In this context, the heavy quarks (HQs), mainly charm and bottom quarks, play a vital role since they do not constitute the bulk part of the matter owing to their larger mass compared to the temperature created in ultra-relativistic heavy-ion collisions.

There are presently two main observables related with heavy quarks that have been measured at both RHIC and LHC energies. The first one is the so-called nuclear suppression factor $R_{AA}$ that is the ratio between the $p_T$ spectra of heavy flavored hadrons (D and B) produced in nucleus + nucleus collisions with respect to those produced in proton + proton collisions. More specifically at RHIC until recently has not been possible to measure directly D and B but only the leptons through their semileptonic decays. The other key observable is the elliptic flow $v_2 = \langle \cos(2\phi_p) \rangle$, a measure of the anisotropy in the angular distribution that corresponds to the anisotropic emission of particles with respect to the azimuthal angle $\phi_p$. Despite their large mass, experimentally measured nuclear suppression factor $R_{AA}$ and elliptic flow $v_2$ of the heavy mesons are comparable to that of light hadrons [16, 20]. This is in contrast to the expectations drawn initially from the perturbative interaction of HQs with the medium which predicted a $R_{AA} \approx 0.6$ for charm quarks, $R_{AA} \approx 0.8-0.9$ for bottom quarks in the central collisions at intermediate $p_T$. Also the $v_2$ was predicted to be much smaller with respect to the light hadron ones [12].

Several theoretical efforts have been made in order to calculate the experimentally observed $R_{AA}$ and $v_2$ for the non-photonic single electron spectra within the Fokker-Planck approach [7-10, 13-21, 24, 26] and relativistic Boltzmann transport approach [16, 29, 33-35, 49, 50]. Furthermore, also in a pQCD framework supplemented by the hard thermal loop scheme several advances have been made to evaluate realistic Debye mass and running coupling constants [16, 20] and three-body scattering effects [10, 21, 22, 23] have been implemented. It has been show [38] that the inclusion of both elastic and inelastic collisions with a dynamical energy loss formalism reduces the gap between the theoretical and experimental results for $R_{AA}$ as $p_T \to 5$ GeV [39, 40]. Several other improvements have been made [41, 43] to advance the description of the data. Interaction from AdS/CFT [54] have also been implemented [24, 28, 50] to study the heavy flavor dynamics at RHIC and LHC. Essentially all the models show some difficulties to describe simultaneously both $R_{AA}(p_T)$ and $v_2(p_T)$ and such a trait is not only present at RHIC energy but also in the results coming from collisions at LHC energy [20].

In this letter we will address the impact of the temperature dependence of the interaction (drag coefficient) on $R_{AA}$ and $v_2$ relation simultaneously. For this we are considering four different models having different T dependent drag coefficients. For the momentum evolution of the HQ we are using 3+1 D Langevin dynamics. We notice that the several approaches and modelings of the HQ in-medium interaction differs significantly for the T dependence of the drag coefficient they entail. One can go from a $T^2$ dependence of the AdS/CFT approach to a drag...
coefficient that even increase with decrease $T$. The aim of this letter is to show that while generally a similar $R_{AA}(p_T)$ corresponds to larger $v_2(p_T)$ indeed the specific $T$ dependence of the drag can strongly modify the amount of $v_2(p_T)$ even if the models are tuned to reproduced the same $R_{AA}(p_T)$ observed experimentally. Our analysis shows that this is quite unlike that a drag with $T^2$ dependence can generate larger elliptic flow as the one observed experimentally at both RHIC and LHC. Instead a nearly constant drag or an increasing one as $T \rightarrow T_c$ strongly quenches the puzzling $R_{AA}(p_T) - v_2(p_T)$ relation.

The standard approach to HQ dynamics in the QGP is to follow their evolution by means of a Fokker-Plank equation solved stochastically by the Langevin equations. The relativistic Langevin equations of motion for the evolution of the momentum and position of the heavy quarks can be written in the form

$$dx_i = \frac{p_i}{E} dt,$$

$$dp_i = -\Gamma p_i dt + C_{ij} p_j \sqrt{dt} \quad (1)$$

where $dx_i$ and $dp_i$ are the shift of the coordinate and momentum in each time step $dt$. $\Gamma$ and $C_{ij}$ are the drag force and the covariance matrix in terms of independent Gaussian-normal distributed random variables $\rho_i, P(\rho) = (2\pi)^{-3/2} e^{-\rho^2/2}$, which obey the relations $<\rho_i \rho_j> = \delta_{ij}$ and $<\rho_i^2> = 0$, respectively. The covariance matrix is related to the diffusion tensor,

$$C_{ij} = \sqrt{2B_0} P^\perp_{ij} + \sqrt{2B_1} P^\parallel_{ij}, \quad (2)$$

where $P^\perp_{ij} = \delta_{ij} - p_i p_j / p^2$ and $P^\parallel_{ij} = p_i p_j / p^2$ are the transverse and longitudinal projector operators respectively.

Under the assumption, $B_0 = B_1 = D$, Eq (2) becomes $C_{ij} = \sqrt{2D} \rho \delta_{ij}$. Such an assumption strictly valid only for $p \rightarrow 0$, is usually employed at finite $p$ in application for heavy quark dynamics in the QGP \cite{5,11,15,23,31}.

Our objective is to demonstrate the effect of the temperature dependent interaction (drag coefficient) on the $R_{AA}$ and $v_2$ obtained from different models i.e for the same $R_{AA}$ how the $v_2$ is built up under various temperature dependence of the interaction. For this purpose we consider four different modelings to calculate the drag and diffusion coefficients which are the key ingredients to solve the Langevin equation. Such model have to be considered merely as an expedient-device to generate different $T$ dependence of the $\Gamma(T)$ but the results and conclusions deduced will be much more general because they do not depend on the way the $\Gamma(T)$ has been obtained. In this sense within a Fokker-Planck approach it is not relevant if the drag and diffusion coefficients has been evaluated considering only collisional or radiative loss.

Model-I (pQCD)

The elastic interaction of heavy quarks with the light quarks, anti-quarks and gluons in the bulk has been considered within the framework of pQCD to calculate the drag and diffusion coefficients. The scattering matrix $M_{gHQ}$, $M_{qHQ}$ and $M_{gHQ}$ are the well known Commbride matrix that includes $s, t, u$ channel and their interferences terms \cite{17}. The divergence associated with the $t$-channel diagrams due to massless intermediate particle exchange has been shielded introducing the Debye screening mass $m_D = \sqrt{4\pi a_s T}$. The temperature dependence of the coupling \cite{52}:

$$g^2(T) = 2\beta_0 \ln(\frac{2\pi T}{(T_c/0.77)}) + \frac{\beta_1}{\beta_0} \ln(\frac{2\pi T}{(T_c/0.77)}) \quad (3)$$

where $\beta_0 = (11 - 2N_f/3)/16\pi^2$ and $\beta_1 = (102 - 38N_f)/16\pi^2$. $N_f$ is the number of flavor and $T_c$ is the transition temperature.

Model-II (AdS/CFT)

We have also considered the drag force from the gauge/string duality \cite{53}, namely the conjectured equivalence between conformal N=4 SYM gauge theory and gravitational theory in Anti de Sitter space-time i.e. AdS/CFT. By matching the energy density of QCD and SYM, which leads to $T_{SYM} = T_{QCD}/3^{\frac{1}{4}}$, and the string prediction for quark-antiquark potential with lattice gauge theory which gives $3.5 < \lambda < 8$ \cite{54}. One finds:

$$\Gamma_{con} = C\frac{T^2_{QCD}}{M_c} \quad (4)$$

where $C = \frac{\pi \sqrt{\lambda}}{2\sqrt{3}} = 2.1 \pm 0.5$. The corresponding diffusion constant $D$ can be obtained from the fluctuation-dissipation relation. Studies of heavy flavor momentum evolution within the Langevin dynamics using AdS/CFT can be found in Ref. \cite{21,22}.

Model-III (QPM)

The third model recently applied to estimate the heavy flavor transport coefficients is inspired by the quasi-particle model (QPM) \cite{55,61}. The QPM approach is a way to account for the non-perturbative dynamics by T-dependent quasi-particle masses, $m_q = 1/3g^2 T^2$, $m_g = 3/4g^2 T^2$, plus a T-dependence background field known as bag constant. Such an approach is able to successfully reproduce the thermodynamics of IQCD \cite{57} by fitting the coupling $g(T)$. To evaluate the drag and diffusion coefficients we have employed QPM tuned to the thermodynamics of the lattice QCD \cite{58}. Such a fit lead to the following coupling \cite{57}:

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln[\lambda(T_c - T_c)]]} \quad (5)$$
where $\lambda=2.6$ and $T/T_s=0.57$.

Model-IV ($\alpha_{QPM}(T), m_{q} = m_{g} = 0$)

To have a different set of drag and diffusion coefficients we are considering a case where the light quarks and gluons are massless but the coupling is from the QPM which obtained from the fit to the lattice data. This case has to be mainly considered as an expedient to have a drag which decreasing with $T$ as obtained for example in the T-matrix approach [3, 15, 37].

The variation of the drag coefficient with respect to temperature at $p=100$ MeV obtained within the four different models discussed above has been shown in Fig. 1. The behaviors remain quite similar also at high momentum but with different magnitude. These rescaled drag coefficients can reproduce almost the same $R_{AA}$ at RHIC energy. In AdS/CFT case the drag coefficient is proportional to $T^2$ where as in $\alpha_{QPM}(T), m_{q} = m_{g} = 0$ case the drag coefficient decrease with $T$ due to the strongly coupling at low temperature. It may be mentioned here that the drag coefficient obtained from the T-matrix is almost constant or slightly decrease with temperature.

We mention that the drag coefficient increases with temperature when the system behaves like a gas. For a molecular liquid the drag coefficient decreases with increasing temperature (except in a very few cases) because a significant part of the thermal energy goes into making the attraction between the interacting particles weaker, allowing them to move more freely and hence reducing the drag coefficient. The drag force of the partonic medium with non-perturbative effects may decrease with increasing temperature as shown in Ref. [3, 15] because in this case the medium interacts strongly more like a liquid.

In order to study the impact of the temperature dependence of the drag coefficient presented in the previous sections on the experimental observables, we have calculated the nuclear suppression factor, $R_{AA}$, using our initial charm and bottom quark distributions at initial time $t = \tau_i$ and final time $t = \tau_f$ at the freeze-out temperature as $R_{AA}(p) = \frac{\int(p, \tau_f)}{\int(p, \tau_i)}$.

Along with $R_{AA}$ we evaluate the anisotropic momentum distribution induced by the spatial anisotropy of the bulk medium and defined as

$$v_2 = \left(\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2}\right),$$

which measures the momentum space anisotropy.

We have performed simulation of $Au + Au$ collisions at $\sqrt{s} = 200$ AGeV for the minimum bias using a 3+1D transport approach [44–46]. The initial conditions for the bulk evolution in the coordinate space are given by the Glauber model condition, while in the momentum space we use a Boltzmann-Juttner distribution function up to a transverse momentum $p_T = 2$ GeV and at larger momenta mini-jet distributions as calculated within pQCD at NLO order [28]. At RHIC energy, $Au + Au$ at $\sqrt{s} = 200$, the maximum initial temperature of the fireball at the center is $T_i = 340$ MeV and the initial time for the fireball simulations is $\tau_i = 0.6$ fm/c (according to the criteria $\tau_i \cdot T_i \sim 1$). The heavy quarks distribution in momentum space are distributed in accordance with the charm distribution at pp collisions that have been taken from ref [4, 5], where in the coordinate space they are distributed according to $N_{coll}$.
The solution of the Langevin equation has been convoluted with the fragmentation functions of the heavy quarks at the quark-hadron transition temperature $T_c$ to obtain the momentum distribution of the D and B mesons. For the fragmentation, we use Peterson fragmentation function:

$$f(z) \propto \frac{1}{[z[1 - \frac{1}{2} - \epsilon_c z]^2]}$$

(7)

where $\epsilon_c = 0.04$ for charm quarks and $\epsilon_c = 0.005$ for bottom quark.

In Fig. 2 we have plotted $R_{AA}$ as a function of $p_T$ for the four different cases obtained within the Langevin dynamics at RHIC energy. As we mentioned, we try to reproduce the same $R_{AA}$ in all the cases by rescaling the drag and diffusion coefficients. The $v_2$ for the same $R_{AA}$ has been displayed in Fig 3 for all cases as a function of $p_T$. Our main striking point is that even if the $R_{AA}$ is very similar for all the four different cases, the $v_2$ built up is quite different depending on the temperature dependence of the drag coefficients (see Fig 4). This is because the $R_{AA}$ is more sensitive to the early stage of the evolution where as the $v_2$ is more sensitive to near $T_c$.

Some studies in this direction have been done also in the light flavor sector as shown in Ref. [62, 63]. The larger drag coefficient is at low temperature the larger is the $v_2$ even for the same $R_{AA}$. For example in the region of the peak for $v_2(p_T)$ we see a difference of about a factor 2.5 going from a $T^2$ dependence, like AdS/CFT to a inverse T dependence as it can occurs in a liquid. The last case or at list a constant (about) drag appears to be very much favored by the comparison with the data.

This study suggests the correct temperature dependence of drag coefficient has a crucial role for a simultaneous reproduction $R_{AA}$ and $v_2$. It can be here mentioned that the drag coefficient is almost constant with respect to temperature in T-matrix case [3, 15, 37]. However also a QPM can be considered quite close to the data given that we have not included the coalescence mechanism that would shift the $v_2(p_T)$ all cases considered by about a 15-20% upward. In Fig 4 we have introduced a new plot $R_{AA}$ vs $v_2$ at a given momentum ($p_T= 1.3$ GeV) to promote the importance of simultaneous reproduction of $R_{AA}$ and $v_2$. Fig 4 highlights how the $v_2(p_T)$ built up can differ up to a factor of around 2.5 (in the region of peak), for the same $R_{AA}(p_T)$, depending on the temperature dependence of the drag coefficient.

We have also extended our calculation to study $R_{AA}$ and $v_2$ at LHC performing simulations of $Pb + Pb$ at $\sqrt{s} = 2.76$ ATeV energy. In this case the initial maximum temperature at the center of the fireball is $T_0 = 510$ MeV and the initial time for the simulations is $\tau_0 \sim 1/T_0 = 0.3$ fm/c. In Fig 4 we show the $R_{AA}$ as a function of $p_T$ for the four different cases obtained within the Langevin dynamics at LHC energy. As we mentioned, we reproduce similar $R_{AA}$ in all the cases by rescaling the drag and diffusion coefficients. The elliptic flow $v_2$ for the same $R_{AA}$ has been plotted in Fig 5 for all cases as a function of $p_T$ . Similarly to the case of RHIC energy we get a similar trend for the $R_{AA}$ vs $v_2$ depending on the T dependence drag coefficients.

However, it has to be mentioned that in Ref [38] it has been pointed out that charm quarks having a moderate value of $M/T$ ratio can lead to significant deviation with respect to the Brownian Langevin dynamics. The full solution of the Boltzmann integral i.e. without the assumption of small collisional ex-
FIG. 5: Comparison of the nuclear suppression factor, $R_{AA}$, as a function of $p_T$, obtained within the Langevin (LV) evolution for the four different cases, with the experimental data at LHC energy.

FIG. 6: Comparison of the elliptic flow, $v_2$, as a function of $p_T$, obtained within the Langevin (LV) evolution for the four different cases, with the experimental data at RHIC energy.

changed momenta, leads in general a large $v_2(p_T)$. Such an effect depends on the anisotropy of the microscopic scattering and can not be studied in term of only the drag coefficient. It is however an effect that in general can be expected to be of the order of about 20% and does not change the systematic studied here. A further effect that is involved in the study of HQ observable is related to the hadronization process. If the possibility of the coalescence process is included their is a further enhancement of the $v_2(p_T)$ of about a 15% [9, 12, 64]. The impact of Boltzmann dynamics and hadronization by coalescence are larger at LHC and can be lead to a better agreements with the data for the case $\alpha_{QP M}(T)$ and QPM but does not modify the impact of the $T$-dependence of the drag coefficient discussed in this letter.

In summary, we have evaluated the drag and diffusion coefficients of the heavy quarks within four different models. With these transport coefficients and heavy quark initial distributions we have solved the Langevin equation. The solution of Langevin equation has been used to evaluate the nuclear suppression factor, $R_{AA}$, and elliptic flow, $v_2$. Results have been compared with the experimental data both at RHIC and LHC energies. Our primary intent is to highlight how the temperature dependence of the interaction (drag coefficient) provides an insights to understand an essential ingredient for the simultaneous reproduction of the nuclear suppression factor, $R_{AA}$, and elliptic flow, $v_2$ which is a current challenge almost for all the existing model. Our work shows that the reproduction of the data on $R_{AA}(p_{T})$ cannot be used to determine the drag coefficient $\Gamma(T)$ of heavy quarks. We find that typical $T$-dependence of the drag coefficients can lead to difference in $v_2$ by 2-3 times even if leading to the same $R_{AA}$. Our study suggests the correct temperature dependence of the drag coefficient may not be larger power of $T$ (like $T^2$ as in pQCD or AdS/CFT) rather a lower power of $T$ or may be constant in $T$. We reminded that $\Gamma(T)$ nearly constant or decreasing with $T$ would be more typical of a liquid and not of a gas.

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