New considerations on the separability of very noisy mixed states and implications for NMR quantum computing

J.D. Bulnes(a), R.S. Sarthour(a), E.R. de Azevedo(b), F.A. Bonk(b), J.C.C. Freitas(c), A.P. Guimarães(a), T.J. Bonagamba(b), I.S. Oliveira(a)

(a) Centro Brasileiro de Pesquisas Físicas
Rua Dr. Xavier Sigaud 150, Rio de Janeiro - 22290-180, Brazil
(b) Instituto de Física de São Carlos
Universidade de São Paulo, São Paulo - P.O. Box 369, 13560-970, Brazil
(c) Depto. Física, Universidade Federal do Espírito Santo
Vitória - 22060-900, Espírito Santo, Brazil

July 10, 2018

Abstract
We revise the problem first addressed by Braunstein and co-workers
(Phys. Rev. Lett. 83 (5) (1999) 1054) concerning the separability of
very noisy mixed states represented by general density matrices with
the form $\rho_{\varepsilon} = (1-\varepsilon)M_d + \varepsilon\rho_1$. From a detailed numerical analysis, it is
shown that: (1) there exist infinite values in the interval taken for the
density matrix expansion coefficients, $-1 \leq c_{\alpha_1,...,\alpha_N} \leq 1$, which give
rise to non-physical density matrices, with trace equal to 1, but at least
one negative eigenvalue; (2) there exist entangled matrices outside the
predicted entanglement region, and (3) there exist separable matrices
inside the same region. It is also shown that the lower and upper
bounds of $\varepsilon$ depend on the coefficients of the expansion of $\rho_1$ in the
Pauli basis. If $\rho_1$ is hermitian with trace equal to 1, but is allowed to
have negative eigenvalues, it is shown that $\rho_{\varepsilon}$ can be entangled, even
for two qubits.
The literature of Nuclear Magnetic Resonance quantum computing has been one of the most fruitful for the past few years, since the discovery of pseudo-pure states by Gershenfeld and Chuang [1] and Cory et al. [2]. Experiments performed by different groups reported various implementations of algorithms [3, 4, 5], simulations [6] and quantum entanglement [7, 9], including teleportation [8] and other quantum effects [10, 11]. These results contrast with the landmark paper published by Braunstein and co-workers in 1999 [12] in which it is argued that, with the present stage of NMR technology, all those experiments could be interpreted through usual classical correlations between spins. These conclusions were further extended by Linden and Pospescu [13]. The latest of such experiments (to the best of the authors knowledge) was performed in 2003 by Mehring et. al [14] who reported entanglement of nuclear and electron spins in a molecular single-crystal, using the combined techniques of NMR and EPR.

Reference [12] considered arbitrary density matrices for \( N \) qubits in the form

\[
\rho_\epsilon = (1 - \epsilon)M_d + \epsilon \rho_1
\]  

(1)

where \( d = 2^N \) is the dimension of the Hilbert space for \( N \) qubits and \( M_d = I/d \) the maximally mixed density matrix. \( I \) is the identity matrix in the space of \( N \) qubits, and \( \rho_1 \) an arbitrary density matrix. Matrices of this form are expanded in a basis of Pauli density matrices:

\[
\rho = \frac{1}{2^N} \epsilon_{\alpha_1...\alpha_N} \sigma_{\alpha_1} \otimes \ldots \otimes \sigma_{\alpha_N}
\]  

(2)

where \( \{\alpha_s\} \equiv \{0, i_s\} = \{0, 1, 2, 3\} \) and “\( s \)” indicates the \( s \)-th qubit with
the sum made over repeated indices. Normalization imposes $c_{0\ldots0} = 1$ and the other coefficients are in the interval $-1 \leq c_{\alpha_1\ldots\alpha_N} \leq 1$. From this, after a transformation for an overcomplete basis, it is established that, for the case $N = 2$, taking the minimum value of the coefficients, $c_{\alpha_1,\alpha_2} = -1$, the bound $\epsilon \leq 1/15$ limits the region below which $\rho_k$ is separable. Generalization for arbitrary $N$ leads to $\epsilon \leq 1/4^N$. Since typically $\epsilon \approx 10^{-5}$ in NMR room temperature liquid state experiments, the conclusion is that so far no entanglement has ever taken place in NMR experiments, a conclusion which has been revised by others [15, 16]. One important observation is that in Ref. [12] this bound is assumed to hold independently of $\rho_1$.

However, since the density matrix in Eq. (1) is arbitrary we can apply the reasoning to the simplest case: $N = 1$ (of course, this involves no entanglement!). Taking $c_{\alpha_1} = -1$, this leads to:

$$\rho_{N=1} = \frac{1}{2} \begin{pmatrix} 0 & -1 + i \\ -1 - i & 2 \end{pmatrix}$$

(3)

We see that this matrix satisfies the condition $\text{Tr}(\rho) = 1$, but its eigenvalues are $\lambda_1 = \frac{1+\sqrt{3}}{2}$ and $\lambda_2 = \frac{1-\sqrt{3}}{2}$. Therefore, $\lambda_2 < 0$, and Eq. (3) cannot represent a density matrix of a physical system [17]. The same can be observed for the cases $N = 2$, with $c_{\alpha_1,\alpha_2} = -1$, and $N = 3$, with $c_{\alpha_1,\alpha_2,\alpha_3} = -1$, whose respective density matrices are:

$$\rho_{N=2} = \frac{1}{4} \begin{pmatrix} -2 & -2 + 2i & -2 + 2i & 2 \\ -2 - 2i & 2 & -2 & 0 \\ -2 - 2i & -2 & 2 & 0 \\ -2i & 0 & 0 & 2 \end{pmatrix}$$

(4)

also satisfying the normalization constraint $\text{Tr}(\rho) = 1$, but with eigenvalues $\lambda_1 = \frac{-2+2\sqrt{3}}{4}, \lambda_2 = \frac{-2-2\sqrt{3}}{4} < 0, \lambda_3 = \lambda_4 = 1$, and
\[ \rho_{N=3} = \frac{1}{8} \times \]

\[
\begin{pmatrix}
-6 & -4 + 4i & -4 + 4i & 4i & -4 + 4i & 4i & 4i & 2 + 2i \\
-4 - 4i & 2 & -4 & 0 & -4 & 0 & -2 + 2i & 0 \\
-4 - 4i & -4 & 2 & 0 & -4 & -2 + 2i & 0 & 0 \\
-4i & 0 & 0 & 2 & -2 - 2i & 0 & 0 & 0 \\
-4 - 4i & -4 & -4 & -2 + 2i & 2 & 0 & 0 & 0 \\
-4i & 0 & -2 - 2i & 0 & 0 & 2 & 0 & 0 \\
-4i & -2 - 2i & 0 & 0 & 0 & 0 & 2 & 0 \\
2 - 2i & 0 & 0 & 0 & 0 & 0 & 0 & 2
\end{pmatrix}
\]

with \( \text{Tr}(\rho) = 1 \), and eigenvalues \( \lambda_1 = \frac{-8+6\sqrt{3}}{8}, \lambda_2 = \frac{-8-6\sqrt{3}}{8} < 0, \lambda_3 = \lambda_4 = \lambda_5 = \frac{4+2\sqrt{3}}{8}, \lambda_6 = \lambda_7 = \lambda_8 = \frac{4-2\sqrt{3}}{8} \). Since a physical density matrix must be a positive operator [18], none of the above matrices can be regarded as representing a physical system.

In order to find valid intervals, consider an example where all the coefficients are equal to some constant \( c_{\alpha_1,\ldots,\alpha_N} = c \) in Eq.(2), and let us impose \( \lambda \geq 0 \) for the eigenvalues of the resulting matrix. This leads to the following intervals: for \( N = 1 \), \(-0.58 \leq c \leq 0.58 \); for \( N = 2 \), \(-0.15 \leq c \leq 0.33 \); for \( N = 3 \), \(-0.05 \leq c \leq 0.15 \), and so on. This case defines only one possible set of values for the coefficients, but will be useful to derive a number of results. Note that, for \( N = 2 \) and \( c_{\alpha_1,\alpha_2} = c \), the intervals \(-1 \leq c < -0.15 \) and \( 0.33 < c \leq 1 \) define infinite non-physical matrices within \(-1 \leq c \leq 1 \).

With the new intervals, the eigenvalues for the case \( N = 2 \), for \( c = -0.15 \), are all positive and \( \rho \) satisfies \( \text{tr}(\rho) = 1 \):

\[ \{\lambda_i\} = \{0.007596, 0.267404, 0.362499, 0.362500\} \]
The same is true for $N = 3$ and $c = -0.05$:

$$\{\lambda_i\} = \{0.003798, 0.133702, 0.165401, 0.165401, 0.165401, 0.122099, 0.122099, 0.122099\}$$

Note that, except for $N = 1$, the intervals are asymmetric. If we write the intervals in the form $-1/A_N \leq c_{\alpha_1...\alpha_N} \leq 1/B_N$ (where, for instance, $(A_2, B_2) = (6.67, 3.03)$, $(A_3, B_3) = (20.00, 6.67)$), applying the procedure of [12] we find that the lower bound for $\epsilon$ in the case $N = 2$ would be $\epsilon \leq A_2/15 = 0.44$, larger than the previous $\epsilon \leq 1/15$ and, in the case $N = 3$, $\epsilon \leq A_3/63 = 0.32$, also larger than the previous $\epsilon \leq 1/64$. According to Ref. [12], for $N = 2$, a matrix $\rho_\epsilon$ with $\epsilon > 0.33$ would be entangled, but according to the above there exist matrices in this interval which are separable. It is important to recall that in that reference, the bounds are independent of $\rho_1$. This is not a problem of basis choice, for it also appears in the second case considered in Ref. [12], in which a continuous overcomplete basis is used. In this case, from Eq. (9) of [12] one can considers those $w(\vec{n}_1, \ldots, \vec{n}_N)$ which, for $a > 1$, satisfy the relation $w(\vec{n}_1, \ldots, \vec{n}_N) \geq -2^{2N-1}/a(4\pi)^N$, we see that $\rho_\epsilon$ is separable for $\epsilon \leq a/(a + 2^{2N-1})$. In particular, for $N = 2$ and $a = 6$ we see that $\rho_\epsilon$ will be separable for $\epsilon \leq 3/7$, but according to Ref. [12], matrices $\rho_\epsilon$ with $\epsilon > 0.33$ are entangled.

Next, let us present an explicit example of separable $\rho_\epsilon$ for $N = 2$ within the entangled region given in Ref. [12]. Let $\rho_1$ be a matrix which, in the Pauli basis, is defined by the coefficients $c_{\alpha_1,\alpha_2} = c = -0.15$. Thus, using Eq. (2), one has:
\[ \rho_1 = \begin{pmatrix}
0.1375 & -0.0750 + 0.0750i & -0.0750 + 0.0750i & 0.0750i \\
-0.0750 - 0.0750i & 0.2875 & -0.0750 & 0 \\
-0.0750 - 0.0750i & -0.0750 & 0.2875 & 0 \\
-0.0750i & 0 & 0 & 0.2875 \\
\end{pmatrix} \]

(6)

This matrix satisfies Tr(\(\rho\)) = 1 and has eigenvalues \(\lambda_1 = 0.0076, \lambda_2 = 0.2674, \lambda_3 = \lambda_4 = 0.3625\). Therefore, it is a true density matrix. Now, if we take \(\epsilon = 0.40\) (that is, entangled according to Ref. [12]) and replace in Eq. (1), one obtains

\[ \rho_\epsilon = \begin{pmatrix}
0.2050 & -0.0300 + 0.0300i & -0.0300 + 0.0300i & 0.0300i \\
-0.0300 - 0.0300i & 0.2650 & -0.0300 & 0 \\
-0.0300 - 0.0300i & -0.0300 & 0.2650 & 0 \\
-0.0300i & 0 & 0 & 0.2650 \\
\end{pmatrix} \]

(7)

which is also a true density matrix, since Tr(\(\rho_\epsilon\)) = 1, and has positive eigenvalues: \(\lambda_1 = 0.1530, \lambda_2 = 0.2569, \lambda_3 = \lambda_4 = 0.2950\). But, according to Peres criterium [19], this matrix is separable. To show that, let us expand \(\rho_\epsilon\) in the following basis of density matrices:

\[ \rho_1 = \frac{1}{2} \begin{pmatrix}
1 & 0 \\
0 & 1 \\
\end{pmatrix}; \quad \rho_2 = \frac{1}{2} \begin{pmatrix}
1 & -i \\
i & 1 \\
\end{pmatrix}; \quad \rho_3 = \frac{1}{2} \begin{pmatrix}
1 & 1 \\
1 & 1 \\
\end{pmatrix}; \quad \rho_4 = \begin{pmatrix}
1 & 0 \\
0 & 0 \\
\end{pmatrix} \]

(8)

After partial transposing [19], one obtains the following eigenvalues: \(\lambda_1 = 0.1530, \lambda_2 = 0.2569, \lambda_3 = \lambda_4 = 0.2950\). Since for two qubits the Peres criterium is a necessary and sufficient condition for separability [20], we conclude that \(\rho_\epsilon\) is separable. The same result can be reached using the same procedure given in Ref.[12]. To see that, we need the coefficients of \(\rho_\epsilon\) in the
Pauli basis, $d_{\alpha_1,\alpha_2}$, which are equal to $c\epsilon$ for the case the coefficients of $\rho_1$ are all equal to $c$ (see the Appendix). Replacing into Eq. (4) of Ref. [12], one has:

$$w_iw_i + d_{i0}w_j + w_id_{0j} + d_{ij} = \frac{1}{9} - \frac{0.15\epsilon}{3} - \frac{0.15\epsilon}{3} - 0.15\epsilon > 0$$

from which one obtains the separability condition: $\epsilon \leq 0.44$, satisfied by the previous $\rho_\epsilon$, with $\epsilon = 0.40$. This is not the only case; there exist infinite separable matrices $\rho_\epsilon$ with $\epsilon > 0.33$. This occurs because the upper and lower bounds are not fixed, as in Ref. [12], but change with $\rho_1$. This is why in Eq. (9) $\epsilon > 0.44$ must be considered, and not $\epsilon > 0.33$, as the correct upper bound.

Let us now build a matrix $\rho_1$ with the following set of unequal coefficients:

$c_{01} = -0.07; c_{02} = -0.07; c_{03} = -0.07; c_{10} = -0.06; c_{11} = -0.83; c_{12} = -0.03; c_{13} = -0.03; c_{20} = -0.06; c_{21} = -0.03; c_{22} = -0.03; c_{23} = -0.03; c_{30} = -0.06; c_{31} = -0.03; c_{32} = -0.03; c_{33} = -0.03$ which has eigenvalues $\{\lambda_i = 0.000996, 0.078152, 0.448038, 0.472814\}$. Thus, for $\epsilon = 0.13$ one obtains:

$$\rho_\epsilon = \begin{pmatrix}
+0.2448 & -0.0032 + 0.0032i & -0.0029 + 0.0029i \\
-0.0032 - 0.0032i & +0.2513 & -0.0279 \\
-0.0029 - 0.0029i & -0.0279 & +0.2506 \\
-0.0260 - 0.0019i & -0.0010 - 0.0010i & -0.0013 - 0.0013i \\
\end{pmatrix}$$

(9)

with eigenvalues: $\lambda_1 = 0.217629$, $\lambda_2 = 0.227660$, $\lambda_3 = 0.275745$, $\lambda_4 = 0.278966$. Applying Eq. (4) of [12], one has:

$$w_1w_1 + d_{10}w_1 + w_1d_{01} + d_{11} = -0.0218 < 0$$

7
where $d_{i,j}$ are the coefficients of $\rho_\epsilon$ in the Pauli basis (see Appendix). Therefore, $\rho_\epsilon$ is entangled, but with $\epsilon$ outside the bounds established in Ref. [12].

The appropriate procedure to determine whether NMR can or cannot implement entanglement consist in finding the matrix $\rho_1$, for instance from quantum state tomography experiments [18], determine the expansion coefficients of Eq.(2) and, from those, derive the respective lower and upper bounds which will be valid only for a particular set of matrices $\rho_\epsilon$.

Finally, we will show that with the only two requirements of $\rho_1$ being hermitian and satisfying $Tr(\rho_1) = 1$, a matrix $\rho_\epsilon$ of two qubits can be entangled, for usual values of $\epsilon$. In order $\rho_\epsilon$ to be a true density matrix the only requirement on $\rho_1$ is that it must be hermitian and have trace equal 1.

Imposing non-negative eigenvalues (EV) for $\rho_\epsilon$, one has:

$$EV\{\rho_\epsilon\} = EV\{(1 - \epsilon)M_d + \epsilon \rho_1\} \geq 0$$  \hspace{1cm} (10)

Since $[M_d + \rho_1, \rho_1] = 0$, $[M_d + \rho_1, M_d] = 0$ and $[\rho_1, M_d] = 0$, the EV of the sum is equal to the sum of EV:

$$EV\{\rho_1\} \geq \frac{1 - \epsilon}{\epsilon 2^N}$$  \hspace{1cm} (11)

That is, for the usual values $\epsilon \approx 10^{-5}$, the absolute values of the EV of $\rho_1$ can actually be very large.

Let us restrict ourselves to the case $N = 2$ and pick the following particular set of values for the coefficients: $c_{\alpha_1,\alpha_2} = -666.66$ (note that since here $\rho_1$ does not have to be a density matrix, these are valid coefficients). From
this, we obtain:

\[
\rho_1 = \begin{pmatrix}
-499.745 & -333.33 + 333.33i & -333.33 + 333.33i & 333.33i \\
-333.33 - 333.33i & 166.915 & -333.33 & 0 \\
-333.33 - 333.33i & -333.33 & 166.915 & 0 \\
-333.33 & 0 & 0 & 166.915 \\
\end{pmatrix}
\]

which has eigenvalues \( \lambda_1 = -1077.089496, \lambda_2 = 77.599495, \lambda_3 = 500.244999, \lambda_4 = 500.245000 \). From this, and setting \( \epsilon = 0.0002 \) in Eq.(1) we find the matrix:

\[
\rho_\epsilon = \begin{pmatrix}
0.15 & -0.0666 + 0.0666i & -0.0666 + 0.0666i & 0.0667i \\
-0.0666 - 0.0666i & 0.2833 & -0.0666 & 0 \\
-0.0666 - 0.0666i & -0.0666 & 0.2833 & 0 \\
-0.0667i & 0 & 0 & 0.2833 \\
\end{pmatrix}
\]

which has eigenvalues \( \lambda_1 = 0.034532, \lambda_2 = 0.265469, \lambda_3=0.349989,\lambda_4 = 0.34999990, \) and therefore is a good density matrix. According to the criterium of Ref. [12] this matrix would be entangled.

Summarizing, the method of Ref. [12] to decide whether NMR can or cannot produce entanglement has been revised. A numerical analysis has shown that the interval \( \pm 1 \) for the expansion coefficients is not generally correct, and that the bounds for entanglement are dependent on the matrix \( \rho_1 \). The non observation of this fact leads to the following problems: (1) existence of density matrices with negative eigenvalues; (2) existence of entangled matrices outside the predicted entanglement region; (3) existence of separable matrices inside the same entanglement region. We also show that with the requirements of \( \rho_1 \) being hermitian with trace equal 1, but allowed to have negative eigenvalues (a possible experimental situation, as reported in Ref. [21]), \( \rho_\epsilon \) can be entangled, even for two-qubits. However, we were not
able to produce entangled two qubits $\rho_\epsilon$, for $\rho_1$ being a true density matrix (that is, a positive operator), what is in accordance with one conclusion of Ref. [12]. Cases for higher numbers of qubits must be analyzed in separate, and they may lead to further insights into this problem.

Acknowledgement The authors acknowledge the support from CAPES, CNPq and FAPESP.

Corresponding email: jdiazb@cbpf.br

1 Appendix

Any given two-qubit matrix $\rho$ possesses the following elements in the expansion in the Pauli basis:

$$
\begin{align*}
\rho_{11} &= \frac{1}{4}(1 + c_{03} + c_{30} + c_{33}) & \rho_{12} &= \frac{1}{4}(c_{01} - ic_{02} + c_{31} - ic_{32}) \\
\rho_{13} &= \frac{1}{4}(c_{10} + c_{13} - ic_{20} - ic_{23}) & \rho_{14} &= \frac{1}{4}(c_{11} - ic_{12} - ic_{21} - c_{22}) \\
\rho_{21} &= \frac{1}{4}(c_{01} + ic_{02} + c_{31} + ic_{32}) & \rho_{22} &= \frac{1}{4}(1 - c_{03} + c_{30} - c_{33}) \\
\rho_{23} &= \frac{1}{4}(c_{11} + ic_{12} - ic_{21} + c_{22}) & \rho_{24} &= \frac{1}{4}(c_{10} - c_{13} - ic_{20} + ic_{23}) \\
\rho_{31} &= \frac{1}{4}(c_{10} + c_{13} + ic_{20} + ic_{23}) & \rho_{32} &= \frac{1}{4}(c_{11} - ic_{12} + ic_{21} + c_{22}) \\
\rho_{33} &= \frac{1}{4}(1 + c_{03} - c_{30} - c_{33}) & \rho_{34} &= \frac{1}{4}(c_{01} - ic_{02} - c_{31} + ic_{32}) \\
\rho_{41} &= \frac{1}{4}(c_{11} + ic_{12} + ic_{21} - c_{22}) & \rho_{42} &= \frac{1}{4}(c_{10} - c_{13} + ic_{20} - ic_{23}) \\
\rho_{43} &= \frac{1}{4}(c_{01} + ic_{02} - c_{31} - ic_{32}) & \rho_{44} &= \frac{1}{4}(1 - c_{03} - c_{30} + c_{33})
\end{align*}
$$

Given $\rho_1$ for $N = 2$ and given $\epsilon$, the matrix $\rho_\epsilon$ will be defined by Eq. (1), which can in turn also be expanded in the Pauli basis. Let $d_{\alpha_i\alpha_j}$ be the expansion coefficients of $\rho_\epsilon$. Thus, for instance, for the element $(1, 1)$ we find the following relation:

$$
\frac{1 - \epsilon}{4} + \frac{\epsilon}{4}(1 + c_{03} + c_{30} + c_{33}) = \frac{1}{4}(1 + d_{03} + d_{30} + d_{33})
$$

(14)

Letting $d_{\alpha_1\alpha_2} = \epsilon c_{\alpha_1\alpha_2}$ the above equation will be valid for any value of $\epsilon$, particularly for $c_{03} = c_{30} = c_{33} = c$ and $d_{03} = d_{30} = d_{33} = d$. It is easy to
show that by setting $c_{\alpha_1, \alpha_2} = c e d_{\alpha_1, \alpha_2} = d = c\epsilon$ one obtains the same result for any two corresponding elements.

References

[1] N.A. Gershenfeld and I.L. Chuang, Science, 275, 350 (1997).

[2] D.G. Cory, A.F. Fahmy and T.F. Havel, Proc. Natl. Acad. Sci. USA, 94, 1634 (1997).

[3] I.L. Chuang, N. Gershenfeld and M.G. Kubinec, Phys. Rev. Lett, 80 (15), 1054 (1998).

[4] J.A. Jones and M. Mosca, Phys. Rev. Lett., 83 (5), 1050 (1999).

[5] L.M.K. Vandersypen, M. Steffan, G. Breyta, C.S. Yannoni, M.H. Sherwood and I.L. Chuang, Nature, 414, 883 (2001).

[6] S. Samaroo, C.H. Tseng, T.F. Havel, R. Laflamme, and D.G. Cory, Phys. Rev. Lett, 82 (26), 5381 (1999).

[7] N. Boulant, E.M. Fortunato, M.A. Pravia, G. Teklemariam, D.G. Cory, and T.F. Havel, Phys. Rev. A, 65, 024302 (2002).

[8] M.A. Nielsen, E. Knill and R. Laflamme, Nature, 395, 52 (1998).

[9] E. Knill, R. Laflamme, R. Martinez and C.-H. Tseng, Nature 404 (2000) 368.

[10] J.E. Ollerenshaw, D.A. Lidar, and L.E. Kay, Phys. Rev. Lett, 91 (21), 217904 (2003).
[11] R.J. Nelson, D.G. Cory and S. Lloyd, *Phys. Rev. A*, **61**, 022106 (2000).

[12] S.L. Braunstein, C.M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack, *Phys. Rev. Lett*, **83** (5), 1054 (1999).

[13] N. Linden and S. Popescu, *Phys. Rev. Lett.*, **87** (4), 047901 (2001).

[14] M. Mehring, J. Mende and W. Scherer, *Phys. Rev. Lett*. **90** (2003) 153001-1.

[15] R. Laflamme, http://quickreviews.org/cgi/display.cgi?reviewID 5laf.q-p. 9 811 018.

[16] G.L. Long, H.Y. Yan, Y.S. Li, C.C. Tu, S.J. Zhu, D. Ruan, Y. Sun, J.X. Tao and H.M. Chen, *Commun. Theor. Phys*. **38** (2002) 305.

[17] B. d’Espagnat, *Conceptual Foundations of Quantum Mechanics*, second edition, W.A. Benjamin, Inc., 1976.

[18] Michael A. Nielsen and Isaac L. Chuang, *Quantum Computation and Quantum Information* (Cambridge Press, Cambridge 2002).

[19] A. Peres, *Phys. Rev. Lett.*, **77** (8), 1413 (1996).

[20] M. Horodecki, P. Horodecki, and R. Horodecki, *Phys. Lett. A*, **223**, 1 (1996).

[21] R. Laflamme, E. Knill, W.H. Zurek, P. Catasti and S.V.S. Mariappan, *Phil. Trans. R. Soc. Lond. A* **356** (1998) 1941.

12