Magnetization of neutron star matter

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The magnetization of neutron star matter in magnetic fields is studied by employing the FSUGold interaction. It is found that the magnetic susceptibilities of the charged particles (proton, electron and muon) can be larger than that of neutron. The effects of the anomalous magnetic moments (AMM) of each component on the magnetic susceptibility are examined in detail. It is found that the proton and electron AMM affect their respective magnetic susceptibility evidently in strong magnetic fields. In addition, they are the protons instead of the electrons that contribute most significantly to the magnetization of the neutron star matter in a relative weak magnetic field, and the induced magnetic field due to the magnetization can be appear to be very large. Finally, the effect of the density-dependent symmetry energy on the magnetization is discussed.

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I. INTRODUCTION

Neutron stars in the universe tend to contain matter of supranuclear density in their interiors, with typical mass $M \sim 1.4 M_\odot$ and radii $R \sim 10$ km. As one class of compact objects, neutron stars have been arousing tremendous interest amongst scientists because of many novel features. One of the features of neutron stars is their strong magnetic field that could be the largest one observed in nature. The typical magnitudes of a surface magnetic fields are as large as $10^{11} - 10^{13}$ G [1]. It is currently assumed that the soft gamma repeaters (SGR) and anomalous X-ray pulsars, candidates for the magnetars, have a strong surface magnetic fields up to $10^{14} - 10^{15}$ G [2]. The magnetic field in the interior could be as large as $10^{18}$ G according to the scalar virial theorem [3]. It is interesting that the strong magnetic fields were also created in heavy-ion collisions [4, 5], which may help us to understand the response of the dense matter under the presence of strong magnetic fields.

Over the past decades, many works have been dedicated to the effects of the magnetic field on neutron star properties, such as the equation of states [6], neutron star structure [7], transport properties and the cooling or heating of magnetized stars [8]. An unclear but interesting problem is the origin of such strong magnetic field. A simple analysis showed that a weak magnetic field in a progenitor star could be amplified during the gravitational collapse due to magnetic flux conservation. However, it can not explain the very strong surface magnetic field in magnetars [9]. Another explanation called the magnetohydrodynamic dynamo mechanism based on the rapidly rotating plasma of a protoneutron star [10] which is generally accepted as the standard explanation for the origin of the magnetar’s large magnetic fields, is unable to explain all the features of the supernova remnants surrounding these objects [11, 12]. An interesting mechanism being suggested for the origin is the possible existence of a phase transition to a ferromagnetic state, namely spontaneous magnetization. Such argument has been investigated widely within various theoretical approaches (without the background magnetic field) [13], but the results are still divergent. Even some authors showed that a possibility that a strong magnetic field is produced by color ferromagnetic quark matter in neutron stars [14]. Astronomical observations found that the SGR 1806-20 emitted a giant flare on 27 December 2004 with the total flare energy by $2 \times 10^{46}$ erg and the energy release probably occurred during a catastrophic reconfiguration of the neutron star’s magnetic field since the emitted energy significantly exceeds the rotational energy loss in the same period [15]. These phenomena are perhaps related to the magnetization of the neutron star matter. In addition, the anisotropic pressure is related to the magnetization for the magnetized matter [16]. Therefore, the magnetization is an important physical quantity for neutron stars.

Some calculations have been performed for the magnetization of nuclear matter or pure neutron matter in magnetic fields [16-18]. Seldom calculations were carried out for the $\beta$-stable matter. In Ref. [19], the magnetization of the $\beta$-stable matter was studied and it is shown that the magnetization never appears to become very large. However, this conclusion could be revised according to our calculations, as shown later. Because of the small mass and hence the small mageneton, the magnetization of electrons may be important compared with that of neutron. Therefore, in the present study, the magnetization of the $\beta$-stable neutron star matter, which consists of protons, neutrons, electrons and muons, will be investigated. Not only the AMM of nucleons but also the one of leptons are included here. The main purposes of the this study are as follows. Firstly, the contribution of each component as well as the effect of the anomalous magnetic moments (AMM) will be presented in detail. Secondly, we further explore whether the strong magnetic fields of the neutron stars originate from the highly degenerate relativistic electron gas. Finally, the symmetry energy effects on the magnetization will be presented.

This work is organized as follows. In Sec. II, a brief introduction of the relativistic mean field approach is presented. The magnetization of each component of the neutron star matter, along with the effects of the AMM and the symmetry energy, are analyzed in detail in Sec. III. Finally a summary is given in Sec. IV.
II. RELATIVISTIC MEAN FIELD WITH THE NEW INTERACTION—FSUGOLD

Nowadays the relativistic mean field (RMF) theory as a density-functional approach has become a very useful tool in nuclear physics [20]. In the RMF theory of nuclear matter that made of nucleons (p,n) and leptons (e, \(\mu\)) in a uniform magnetic field \(B\), the total interacting Lagrangian density is given by

\[
\mathcal{L} = \bar{\psi}_\mu(i\gamma^\mu\partial_\mu - M - g_\omega\sigma - \frac{g_\omega}{2}\gamma^\mu\gamma^\nu\sigma \cdot \rho_\nu + g_\omega\gamma^\mu\omega_\mu \\
- g_2\gamma^\mu\gamma^\nu \left( \frac{1}{4} \kappa_\sigma \sigma_{\mu\nu} F^{\mu\nu} \right) \psi_\mu - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\
+ \frac{1}{2} g_\rho \rho_\mu \gamma^\mu \sigma - \left( \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 e_{\sigma}^3 + \frac{1}{4} \kappa_\sigma \sigma^4 \right) \\
- \frac{1}{4} g_{\omega\omega} \omega_\mu \omega_\mu + \frac{\zeta}{4!} g_{\sigma\omega}(\omega_\mu \omega_\mu)^2 \\
- \frac{1}{4} R_{\rho\rho} R^{\rho\rho} + \frac{1}{2} m_{\rho}^2 \rho_\mu \rho_\mu + \kappa_p g_{\rho\rho}^1 \rho_\mu \rho_\mu + \kappa_p g_{\rho\rho}^2 \omega_\mu \omega_\mu \\
+ \bar{\psi}_\mu(i\gamma^\mu\partial_\mu - m_\tau - \gamma^\mu A_\mu - \frac{1}{4} \kappa_\sigma \sigma_{\mu\nu} F^{\mu\nu}) \psi_\mu
\]

with \(A^\mu = (0, B, x, 0)\) and \(\sigma^{\mu\nu} = \frac{1}{2} \left[ \gamma^\mu, \gamma^\nu \right]\). \(\kappa_p = 1.7928\mu_N\), \(\kappa_n = -1.9130\mu_N\), \(\kappa_\sigma = 1.15965 \times 10^{-3} \mu_B\) and \(\kappa_\rho = 1.16592 \times 10^{-3} \mu_B\) are the AMM for protons, neutrons, electrons and muons, respectively [22], where \(\mu_N (\mu_B)\) denotes the nuclear (Bohr) magneton of nucleons (leptons). \(M, m_\sigma, m_\omega\) and \(m_\rho\) are the nucleon-, the \(\sigma\)-, the \(\omega\)- and the \(\rho\)-meson masses, respectively. The nucleon field \(\psi_\mu\) interacts with the \(\sigma, \omega, \rho\) meson fields \(\sigma, \omega_\mu, \rho_\mu\) and with the photon field \(A_\mu\). The field tensors for the vector meson are given as \(\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu\) and by similar expression for \(\rho\) meson and the photon. The self-coupling terms with coupling constants \(g_2\) and \(g_3\) for the \(\sigma\) meson turned out to be crucial [22]. Compared with the previous RMF models, the RMF interactions employed in this work are FSUGold where two additional parameters \(\zeta\) and \(\Lambda_\sigma\) have been introduced: \(\omega\) meson self-interactions as described by \(\zeta\) which soften the equation of state at high density, and the nonlinear mixed isoscalar-isovector coupling described by \(\Lambda_\sigma\) that modifies the density-dependence of the symmetry energy. The FSUGold interaction gives a good description of ground state properties as well as excitations of finite nuclei [23]. In our previous work, this new interaction was used to study the properties of dense matter and symmetry energy in strong magnetic fields [24].

The energy spectra of the proton, electron, muon and muon are given by

\[
E_{p,t>s}^p = \sqrt{k_1^2 + \left( \sqrt{M^2 + 2Bev - sk_p} \right)^2} + g_\omega \omega_0 + g_\rho \rho_0, \\
E_{e,t>s}^n = \sqrt{k_2^2 + \left( \sqrt{M^2 + k_2^2 + k_2^2 - sk_p} \right)^2} + g_\omega \omega_0 + g_\rho \rho_0, \\
E_{e,t>s}^e = \sqrt{k_3^2 + \left( \sqrt{m_\rho^2 + 2Bev - sk_p} \right)^2}, \\
E_{e,t>s}^\mu = \sqrt{k_4^2 + \left( \sqrt{m_\rho^2 + 2Bev - sk_p} \right)^2},
\]

where \(\nu = 0, 1, 2, 3...\) denotes the Landau levels for charged particles and \(s = 1(-1)\) is spin-up (spin-down). The chemical potentials \(\mu\) are obtained by replacing the \(k_\ell\) by \(k_{\ell,\nu,s}\), where \(k_\ell\) is the momentum along the \(z\)-axis and \(k_{\ell,\nu,s}\) is the Fermi momentum.

III. MAGNETIZATION OF NEUTRON STAR MATTER

The thermodynamical potential for the charged particle is given by

\[
\Omega = -\frac{eB}{2\pi^2} \sum_{\nu,s} \int_0^{\infty} dk_\ell \frac{1}{\beta} \ln \left[ 1 + e^{-\beta(E_{\nu,s} - \mu)} \right]
\]

where the contribution of antiparticles is not taken into account. The magnetization \(M = -\frac{\partial \Omega}{\partial B}\) takes the form

\[
M = \frac{eB}{2\pi^2} \sum_{\nu,s} \int_0^{\infty} dk_\ell \frac{1}{\beta} \ln \left[ 1 + e^{-\beta(E_{\nu,s} - \mu)} \right] \\
- \frac{eB}{2\pi^2} \sum_{\nu,s} \int_0^{\infty} dk_\ell \frac{e^{-\beta(E_{\nu,s} - \mu)}}{1 + e^{-\beta(E_{\nu,s} - \mu)}} \frac{\partial E_{\nu,s}}{\partial B}.
\]

In the zero-temperature limit the proton magnetization is

\[
M_p = -\frac{e_p}{B} + \frac{\rho_p E_p^p}{B} - \frac{eB}{2\pi^2} \sum_{\nu,s} \left( \frac{\sqrt{M^2 + 2Bev - sk_p}}{\sqrt{M^2 + 2Bev - sk_p}} - sk_p \right) \ln \left( \frac{E_p + k_{p,\nu,s}^0}{\sqrt{M^2 + 2Bev - sk_p}} \right)
\]

where \(\rho_p\) is the proton density and the energy density of the proton is

\[
e_p = \left( \frac{eB}{4\pi^2} \sum_{\nu,s} k_{p,\nu,s}^0 E_p^p + \left( \sqrt{M^2 + 2Bev - sk_p} \right)^2 \right) \times \ln \left( \frac{k_{p,\nu,s}^0 + E_p^p}{\sqrt{M^2 + 2Bev - sk_p}} \right).
\]

Here the feeble change of \(\sigma\)-field is neglected. Similar expression can be obtained for the electron and muon. For simplicity, we calculate the neutron magnetization with \(M_n = (\rho_{p,1} - \rho_{p,3})\mu_n\). The magnetic susceptibility is given as \(\chi = M/B\). We would like to stress that, due to their Landau diamagnetism, there are not such a simple relation between the magnetization and spin polarization for the charged particles.

The magnetic susceptibilities of the proton, electron, muon and neutron versus the density \(\rho\) for the \(\beta\)-stable neutron star matter under different magnetic fields are presented in Fig. 1. For the charged particles, the magnetic susceptibilities show the oscillations in particular in the case of a relative weak magnetic field. Besides, as shown in the top three panels of Fig. 1, they are positive in most cases and sometimes fall into their negative ranges in the case of rather weak magnetic fields. The 'oscillation period' of the magnetic susceptibility depends on the density of the Landau energy states.
This density of state reduces with the increase of the magnetic field strength, and hence the 'oscillation period'. It is found that the magnetic susceptibilities of the charged particles tend to be larger than that of the neutron, indicating that the neutron star matter can not be treated simply as the pure neutron matter when one studies its magnetization. Neutrons carry no charge so that they have no Landau levels to fill. Hence, the direct coupling of neutrons to magnetic field is just due to the neutron AMM. For the protons, electrons and muons, however, their charge strongly couples with the magnetic field forming the Landau levels, and this coupling is much stronger than the direct coupling between the AMM.
FIG. 2: Magnetic susceptibilities of $n$, $\rho$, $e$ and $\mu$ for the $\beta$-stable matter as a function of the magnetic field strength $B$. The red bold curves correspond to average over a long period. The density we selected is $\rho = 0.16$ fm$^{-3}$ as an example.

and magnetic field. Roughly speaking, the more the Landau levels are, the stronger the magnetization. In an extreme case that the particles occupy the Landau ground state (only one Landau level), the magnetization vanishes due to the Landau diamagnetism being counterbalanced by the Pauli paramagnetism if one ignores the AMM. Our calculations indicate that the magnetization of the electrons is only a few percent, which is not much larger than these of other components. Accordingly, in contradiction with the investigation of Ref. [25], the primal magnetic field of the neutron stars can not be greatly boosted up by the magnetization of the highly degenerate relativistic electron gas. The fundamental reason is that the Pauli paramagnetism is canceled out to a large degree by the diamagnetism for the electron.

To show the effects of the AMM of each component on the magnetic susceptibility $\chi$, we present the calculated $\chi_n$, $\chi_e$, $\chi_\mu$ without the inclusion of the their AMM in the middle six panels of Fig. 1 remarked by dash curves for comparison. The effect of the muon AMM can be neglected completely because of its quite small value (about 1/207 of electron AMM). The proton and electron AMM affect their respective magnetic susceptibility evidently. With the inclusion of the AMM, the doubly degeneracy with opposite spin projections is destroyed and hence the peaks and shapes of the curves are modified. On the whole, the proton AMM leads to an enhancement of $\chi_p$, while the electron AMM causes the $\chi_e$ reduce slightly, which has connection with the spin polarization—the positive polarizability for protons but negative one for electrons. Compared with the proton AMM, the effect of the electron AMM is weaker because the electron AMM is about thousandth of its normal magnetic moment while the proton AMM shares the same order of magnitude as its normal magnetic moment.

FIG. 3: (Color online) Magnetic susceptibilities of the neutron star matter versus the magnetic field strength $B$. The density we selected is $\rho = 0.16$ fm$^{-3}$ as an example. The calculations are performed with the modified FSUGold interactions [33] providing stiff ($\Lambda_v = 0.00$) to soft ($\Lambda_v = 0.04$) symmetry energy.
The magnetic susceptibility versus the magnetic field strength are presented in Fig. 2 taking the $\beta$-stable matter at $\rho = 0.16 \text{ fm}^{-3}$ as an example. The detailed structure of the magnetization exhibits strong de Haas–van Alphen oscillations. The amplitudes of the oscillations become increasing small as the magnetic field strength increases, and the magnetic susceptibilities of the charged particles tend toward zero when the magnetic field is very strong. The reason lies in the reduction of the Landau levels as the magnetic field strength increases. Though the neutron has no Landau levels to occupy, its magnetic susceptibility also fluctuates with the magnetic field owing to the fact that the magnetic field affects the neutron density at a given nucleon density. One conspicuous phenomenon is that the absolute value of the proton magnetization $M_p$ tends to be much larger than the $M_e$, $M_\mu$ and $M_n$ in a relative weak magnetic field—which is, the proton is much stronger magnetized compared with other components. One can easily realize from the relevant discussions about the Fig. 1. When the magnetic field is weak, the induced magnetic field due to the magnetization can be much stronger than the original field but fluctuated wildly. The irregularity oscillations can be averaged to smooth out the wild oscillations to a large extent, being analogous to the averaged viscosities in the presence of strong magnetic fields that discussed in Ref. [26]. The strong magnetization perhaps has something to do with the origin of the magnetic field in neutron stars: The original seed field is gradually amplified by the magnetization. Of course, it needs further investigation.

The density-dependent symmetry energy plays a crucial role in understanding a variety of issues in nuclear physics as well as astrophysics [22,35]. Fig. 3 displays the total magnetic susceptibility $\chi$ as a function of the magnetic field strength with the modified FSUGold interactions which yield stiff to soft symmetry energy, where $\Lambda_v$ is varied while $g_\rho$ is adjusted so that for each $\Lambda_v$ the asymmetry energy remains fixed at a given density and this prescription ensures that the binding energy as well as the proton density of a heavy nucleus, such as $^{208}\text{Pb}$, are within the measured values [13]. The interaction with a stiff symmetry energy tends to yield a large $\chi$ at a strong magnetic field $B > 10^4 B_c^e$ and the 'peaks' shift forward compared with that yields a soft symmetry energy. These stem from the fact that a stiffer symmetry energy gives a lower neutron fraction. As a consequence, the effects of the symmetry energy on the magnetization are distinct at strong magnetic fields.

IV. SUMMARY

The magnetization of neutron star matter in magnetic fields has been studied within the FSUGold interaction. The present analysis is based on the zero-temperature limit for simplicity since the Fermi temperature is much larger than the real temperature in normal neutron stars. The main conclusion are summarized as follows. (1) The magnetic susceptibilities of the neutron is not dominant, indicating the neutron star matter can not be treated as the pure neutron matter for simplicity when one studies its magnetization. (2) Being inconsistent with the conclusion in Ref. [25], the small electron magnetic susceptibility indicates the observed super-strong magnetic field of neutron stars does not originate from the induced Pauli paramagnetism of the highly degenerate relativistic electron gas in the neutron star interiors. (3) The proton and electron AMM affect their respective magnetic susceptibility evidently whereas the muon AMM can be neglected completely. The role of the AMM of the neutron, proton and electron suggested they can not be discarded arbitrarily. (4) The proton is found to be much stronger magnetized compared with other components when the magnetic field is relatively weak ($B < 10^2 B_c^e$). The magnetization of the matter can be appear to be very large, which differs from the conclusion in Ref. [19]. The calculation in Ref. [19] was correct, but it did not include the case of the very low magnetic fields so that it concluded the magnetic susceptibility is only a few percent. The magnetization perhaps is related to the origin of the strong magnetic field in neutron stars, but it needs to be explored further. (5) The magnetization of neutron star matter is affected distinctly by the density-dependent symmetry energy.

At low temperature and weak fields, pairing correlations may dominate the magnetic susceptibility. Pairing in the $1S_0$ and $3PF_2$ channels may have a large impact on the magnetic response of the system, which needs to be further investigated.

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