STATISTICAL HADRONIZATION OF CHARM IN HEAVY ION COLLISIONS

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Production of open and hidden charm hadrons in heavy ion collisions is considered within the statistical coalescence model (SCM). Charmed quark-antiquark pairs are assumed to be created at the initial stage of the reaction in hard parton collisions. The number of these pairs is conserved during the evolution of the system. At hadronization, the charmed (anti)quarks are distributed among open and hidden charm hadrons in accordance with laws of statistical mechanics. Important special cases: a system with a small number of charmed quark-antiquark pairs and charm hadronization in a subsystem of the whole system are considered. The model calculations are compared with the preliminary PHENIX data for J/ψ production at RHIC. Possible influence of the in-nuclear modification of the parton distribution functions (shadowing) on the SCM results is studied.

The thermal hadron gas (HG) model explains successfully the chemical composition of light hadrons produced by colliding nuclei in a wide range center-of-mass energies. A large body of experimental data can be well fitted with only three free parameters: the temperature $T$, baryonic chemical potential $\mu_b$ and volume $V$ at the point of chemical freeze-out. This suggested the idea to see, whether this model can be applied also to charm particles. A straightforward generalization, when particles containing heavy (anti)quarks are treated in the same way as light hadrons, could hardly be successful. The estimated relaxation time for charm production and annihilation in HG or even in a deconfined medium appeared to be much larger than the total lifetime of the thermal system. Therefore, it has been generally believed that the production mechanism of heavy-flavored hadrons is completely different from that of light ones. For instance, the standard picture of charmonium production in nucleus-nucleus collisions

\footnote{Sometimes one more parameter, the strangeness suppression factor $\gamma_\Lambda$ is introduced.}
assumes that charmonia are created exclusively at the initial stage of the reaction in primary nucleon-nucleon collisions. During the subsequent evolution of the system, the number of hidden charm mesons is reduced because of absorption of pre-resonance charmonium states in the nuclei (the normal nuclear suppression), interactions of charmonia with secondary hadrons (comovers), dissociation of $c\bar{c}$ bound states in the deconfined medium. The last mechanism was expected to be especially strong and charmonia were proposed to be used as a probe of the state of matter created at the early stage of the collision. It was found that the $J/\psi$ suppression with respect to Drell-Yan muon pairs measured in proton-nucleus and nucleus-nucleus collisions with light projectiles can be explained by the normal nuclear suppression alone\(^4\). In contrast, the NA50 experiment with a heavy projectile and target (lead-lead) revealed essentially stronger $J/\psi$ suppression for central collisions\(^5\). This anomalous $J/\psi$ suppression was attributed\(^6\) to formation of quark-gluon plasma (QGP). In the same time, purely hadronic scenarios still cannot be excluded\(^7\). Despite of quite successful agreement with the $J/\psi$ data, the standard scenario seems to be in trouble explaining the $\psi'$ yield. The recent lattice simulations\(^8\) suggest that the temperature of $\psi'$ dissociation $T_d(\psi')$ lies far below the deconfinement point\(^9\) $T_c$: $T_d(\psi') \approx 0.1-0.2 T_c$. Therefore, not only the quark-gluon plasma, but also a hadronic co-mover medium should completely eliminate $\psi'$ charmonia in central Pb+Pb collisions at SPS. However, the experiment revealed a sizable $\psi'$ yield (see, for instance\(^10\)). It was observed\(^11\) that $\psi'$ to $J/\psi$ ratio decreases with centrality only in peripheral lead-lead collisions, but saturates at sufficiently large number of participants $N_p \geq 100$. The value of the ratio in (semi)central collisions is approximately constant and equal to the ratio of the densities of these charmonium states in an equilibrium HG. Hence, on one hand, the production of charm cannot be thermal, because of the large relaxation time, on the other hand, the multiplicity ratio of different charmonium states is thermal. This paradox is resolved in the statistical coalescence model (SCM)\(^12\). In this model, the charm quarks $c$ and antiquarks $\bar{c}$ are created at the initial stage of A+A reaction in the hard parton collisions. This is similar to the standard approach. But like in the thermal model\(^2\), the formation of observed hadrons with open and hidden charm takes place later at the hadronization stage near the point of chemical freeze-out. Production and annihilation of charm quark-antiquark pairs at all stages after the initial are neglected. Therefore the number $c$ and $\bar{c}$ in the system may be very far from chemical equilibrium, but their distribution over different species of hadrons with open and hidden charm
is controlled by the laws of statistical mechanics.

In the ‘pure’ SCM, which will be a subject of the present discussion, it is assumed that the hot strongly-interacting medium destroys all initially formed (‘primordial’) charmonia or even prevents their formation. A combined scenario, assuming that some of primordial charmonia do survive suppression in the medium, but additional hidden charm meson can be as well formed at hadronization, has been also considered.

Within the grand canonical approach, the deviation of the amount of (anti)charm in the system from its equilibrium value can be taken into account simply by multiplying the multiplicities of all open single (anti)charm species in equilibrium hadron gas by some factor $\gamma_c$:

$$\langle X \rangle = \gamma_c N_X(T, \mu_b, V) = \gamma_c \exp \left( c_X \frac{\mu_c}{T} \right) \tilde{N}_X(T, \mu_b, V).$$

(1)

Here $c_X = \pm 1$ is the charm of the species $X$. The charm chemical potential $\mu_c$ is needed to keep the total balance of open charm and anticharm in the thermal system. At zero $\mu_c$ but nonzero baryonic chemical potential $\mu_b$, the total number of open charm in the equilibrium HG

$$\tilde{N}_1 = \sum_X \tilde{N}_X(T, \mu_b, V)$$

(2)
is not equal to that of anticharm

$$\tilde{N}_1 = \sum_X \tilde{N}_\bar{X}(T, \mu_b, V).$$

(3)

This is because, for instance, a positive value of $\mu_b$ enhances the number of open charm baryons, that contribute to (2), and suppresses the number of antibaryons in (3). (The sums in (2) and (3) run over all species of single open charm and anticharm hadrons, respectively.) The value of $\mu_c$ should be chosen to restore the balance:

$$\exp(\mu_c/T)\tilde{N}_1 = \exp(-\mu_c/T)\tilde{N}_1.$$\hspace{1cm}(4)

A hidden charm meson contains a charm quark as well as an antiquark. Therefore, their thermal numbers are not influenced by the charm chemical potential. To take into account the deviation from the chemical equilibrium, one should multiply the thermal numbers of charmonia by $\gamma_c^2$:

$$\langle Y \rangle = \gamma_c^2 \tilde{N}_Y(T, \mu_b, V).$$

(5)

Provided that the average number of charm quarks and antiquarks $\langle C \rangle = \langle \bar{C} \rangle$ in the system is known, the factor $\gamma_c$ can be found from the equation:

$$\langle C \rangle = \gamma_c \exp(\mu_c/T)\tilde{N}_1 + \gamma_c^2 \tilde{N}_H .$$

(6)
Here $\tilde{N}_H$ is the total number of hidden charm particles in the system at chemical equilibrium:

$$\tilde{N}_H = \sum \tilde{N}_Y(T, \mu_b, V).$$

(The sum runs over all charmonium states.) The thermal number of hidden charm is much smaller than that of open charm. Therefore, if the number of $c\bar{c}$ pairs in the system is not extremely large (does not exceed the chemical equilibrium value by a factor of several hundreds), the hidden charm term in (6) can be neglected and $\gamma_c$ is simply given by

$$\gamma_c = \langle C \rangle \exp(\mu_c/T) \tilde{N}_1.$$ 

(8)

The grand canonical approach does not exactly correspond to the physical reality. It keeps balance only between the average quantities of charm and anticharm, while the partition function includes also configurations with nonequal numbers of charm quarks and antiquarks in the system. This does not influence the result, if the average number of $c\bar{c}$ pairs in the system is large $\langle C \rangle, \langle \bar{C} \rangle \gg 1$. In the opposite case, however, it should be taken into account that strong interactions conserve charm (weak interactions can be neglected), therefore the number $C$ of charm quarks in the system is always exactly equal to the number $\bar{C}$ of antiquarks.

Let us first consider the situation, when the numbers of charm quarks and antiquarks are fixed (not necessary equal to each other). In this case, the partition function for particles with charm can be written as

$$Z_{CC\bar{C}}(T, \mu_b, V) = \tilde{N}_1^{\bar{C}} \tilde{N}_1^{C} \frac{\tilde{N}_H^{C-1} \tilde{N}_1^{\bar{C}-1}}{C! (C-1)! (\bar{C}-1)!} + \cdots$$

(9)

The dots stand for terms corresponding to more than one hidden charm meson in the system and also for terms including double and triple charm baryons. These terms, as it was already mentioned above, can be safely neglected, unless $C$ and $\bar{C}$ are extremely large.

As far as $\tilde{N}_H \ll \tilde{N}_1 \tilde{N}_1$ (which is true in all practically interesting situations), the numbers of open charm and anticharm hadrons are approximately equal to $C$ and $\bar{C}$, respectively, i.e. almost all charm quarks and antiquarks hadronize into open charm particles. Only a very small fraction form charmonia. The total number of hidden charm can be easily calculated:

$$\langle H \rangle_{CC\bar{C}} = \tilde{N}_H \frac{\partial \log Z_{CC\bar{C}}}{\partial \tilde{N}_H} \approx C\bar{C} \frac{\tilde{N}_H}{\tilde{N}_1 \tilde{N}_1}$$

(10)
In reality, however, the number \( K \) of \( c\bar{c} \) pairs is not fixed. It fluctuates from one event to another. The pairs are produced in independent collisions of nucleons whose number is large and the probability to produce a \( c\bar{c} \) pair in a single collision is small. Therefore, the fluctuations should follow the Poisson law:

\[
P(C = \bar{C} = K) = \exp\left(-\langle c\bar{c}\rangle_{AB}(b)\right) \frac{\left(\langle c\bar{c}\rangle_{AB}(b)\right)^{K}}{K!},
\]

(11)

where \( \langle c\bar{c}\rangle_{AB}(b) \) stands for average number of \( c\bar{c} \) pairs produced by nuclei \( A \) and \( B \) colliding at impact parameter \( b \).

The number of hidden charm averaged over all events at fixed centrality is given by the convolution of Eq.(10) with the probability (11). The result reads:

\[
\langle H \rangle_{AB}(b) = \langle c\bar{c}\rangle_{AB}(b)(\langle c\bar{c}\rangle_{AB}(b) + 1)\frac{\tilde{N}_H}{\tilde{N}_1\tilde{N}_1^T}.
\]

(12)

If one interested in the multiplicity of a particular charmonium species, one should replace the \( \tilde{N}_H \) in the numerator of (12) by the thermal multiplicity of this species. It must be also taken into account that an additional contribution to multiplicities of low-lying charmonium states comes from decays of excited ones.

Note that the ratios of the numbers of different charmonium states in SCM are exactly the same as in equilibrium HG:

\[
\frac{\langle \psi' \rangle_{AB}(b)}{\langle J/\psi \rangle_{AB}(b)} = \frac{\tilde{N}_{\psi'}}{\tilde{N}_{J/\psi}}.
\]

(13)

It agrees with the behavior of \( \psi' \) to \( J/\psi \) ratio in central Pb+Pb collisions at CERN SPS. This is not the case for peripheral events \( (N_p < 100) \). It can be explained in the following way: SCM is valid if the momenta of charm (anti)quarks (not their number!) are thermalized. This is possible only in a sufficiently large system.

The equation (12) gives the total \((4\pi)\) multiplicity of hidden charm. In real experimental situation, however, measurements are made in a limited rapidity window. In the most simple case, when the fraction of charmonia that fall into the relevant rapidity window does not depend on the centrality, one can merely use Eq.(12) multiplied by some factor \( \xi < 1 \). This situation is likely to be relevant to charmonium production at CERN SPS, where the multiplicity of light hadrons, that determine the freeze-out volume of the system, are approximately proportional to the number of nucleon participants \( N_p \) at all rapidities. Indeed, SCM fits well the measured\(^5,6,15\) \( J/\psi \)
to Drell-Yan ratio at CERN SPS (see Fig. 1, the details of the fit can be found in Ref. 16).

At BNL RHIC, the situation is different: the total (4π) multiplicity of light hadrons are still approximately proportional to the number of participants, while at midrapidity, it grows faster. The centrality dependence of charmonium production at different rapidities should in this case be also different. The formula (12) should be generalized, in such a way that the charmonium multiplicity not only in the whole system, but in a part of it, a subsystem (like a limited rapidity interval), could be calculated.

Let, if $c$ (or $\bar{c}$) is present in the system, the probability to find it in the subsystem is $\xi < 1$. Then, if the number of $c\bar{c}$ pairs in the whole system is $K$, the probability to find $C$ charm quarks in the subsystem is given by

![Figure 1. The dependence of the $J/\psi$ to Drell-Yan ratio on the transverse energy at SPS. The vertical line shows the boundary of the applicability domain of the statistical coalescence model (SCM).](image)
the binomial law:

\[ w(C|K) = \frac{K!}{C!(K-C)!} \xi^C (1-\xi)^{K-C}. \tag{14} \]

We assume that the distributions of quarks and antiquarks are uncorrelated,\(^b\) and the probability distribution of the number of antiquarks \(\bar{C}\) is given by the same binomial law.

If the number of pairs \(K\) is distributed in accordance with the Poisson law \((11)\), the average multiplicity of charmonia in the subsystem is

\[
\langle H \rangle_{AB(b)}^{(\xi)} \approx \sum_{K=0}^{\infty} P(K) \sum_{C=0}^{K} w(C|K) \sum_{\bar{C}=0}^{K} w(\bar{C}|K) \frac{C\bar{C}}{N_H} \frac{\tilde{N}_H}{N_1N_1} \tag{15}
\]

Here the meaning of \(\langle c\bar{c} \rangle_{AB(b)}\) is the same as in Eq.(12): the average total \((4\pi)\) number of \(c\bar{c}\) pairs in an event with given centrality. In contrast, \(\tilde{N}_H\), \(\tilde{N}_1\) and \(\tilde{N}_1\) are related to the subsystem. These are the thermal multiplicities of charmonia, open charm and anticharm calculated with the thermodynamic parameters fitted within the HG model to the chemical composition and multiplicity of light hadrons in the rapidity window under interest.

In Fig.2, the SCM result is compared to the preliminary PHENIX data\(^{19}\) for \(J/\psi\) production in the central rapidity interval at the top RHIC energy \((\sqrt{s} = 200 \text{ GeV})\). The values of freeze-out parameters were taken from Ref.\(^{21}\): \(T = 177 \text{ MeV}\) and \(\mu_B = 29 \text{ MeV}\). The dependence of volume on the centrality was chosen to reproduce the measured multiplicity of charged hadrons.\(^{17}\) The value of \(\langle c\bar{c} \rangle_{AB(b)}\) was calculated in the Glauber approach. The charm production cross section in nucleon-nucleon collisions was fixed at\(^{20}\) \(\sigma_{c\bar{c}}^{NN} = 650 \mu\text{b}\), while \(\xi\) was considered as a free parameter. The minimum value of \(\chi^2/dof = 0.75\) is reached at \(\xi \approx 0.18\).

As is seen, the SCM dependence of the \(J/\psi\) multiplicity at midrapidity per binary collision on the centrality is almost flat for \(N_p \gtrsim 100\) in contrast to the total \(J/\psi\) yield, which is expected to grow in the same centrality region.\(^{22}\) This is because the multiplicity of light hadrons\(^{17}\) and, consequently, the hadronization volume at midrapidity grows faster with the

\(^b\)At this point, our approach differs from that of Ref.\(^{18}\), where a strong correlation \(C = \tilde{C}\) between the numbers of \(c\)-quarks and \(\bar{c}\)-antiquarks is assumed.
centrality than the total volume. If the influence of the in-nuclear modification of the parton distribution functions (shadowing) on charm production at RHIC is essential, a decrease should be observed instead of saturation (see Fig.2).

The centrality dependence of $J/\psi$ multiplicity in the QGP suppression scenario\textsuperscript{23} is also shown in Fig.2. The parameters of the model were fixed from SPS data\textsuperscript{5,6,15} and then extrapolated to RHIC. As is seen, the observed yield can hardly be explained by survived primordial charmonia only. It seems that $c\bar{c}$ coalescence at late stages of the reaction is here a dominant if not the only charmonium production mechanism.

In conclusion, the statistical coalescence model is consistent with the data on $J/\psi$ production in heavy nucleus collisions at SPS and RHIC energies, while the standard scenario of $J/\psi$ dissociation in QGP predicts too strong suppression at RHIC that is not favored by the experimental data.
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