The problem under consideration has, actually, two aspects. The first one concerns the case when a nucleon is in free space that is in vacuum, while the second one studies a nucleon embedded into a nuclear environment. Both of these two aspects have been considered in the framework of chiral topological models of QCD. In this sense the whole content of the thesis may be divided into two main parts.

In the first part the original Skyrme model (with $m_{\pi} \neq 0$) has been extended by inclusion of the light scalar - isoscalar $\sigma$ - meson. The lagrangian has following advantages:
- It satisfies the scale invariance and trace anomaly constraint of QCD;
- It gives a realistic nucleon - nucleon (NN) interaction potential, which is quite close to the phenomenological one.
The lagrangian has been further extended by explicit inclusion of $\sigma$, $\rho$ and $\omega$ - mesons as well. In order to get a more complete picture of NN potential the appropriate meson - nucleon vertex form - factors were obtained.

The second part of the thesis considers a nucleon immersed into a nuclear medium. For this purpose a medium modified Skyrme lagrangian has been proposed. The lagrangian describes well such well known medium effects as decreasing of nucleon mass and increasing of its size (swelling). We studied also a system with finite temperature ($T \neq 0$) also. The temperature effects were taken into account by using the method of termofield dynamics (TFD). The explicit calculations show that the temperature dependence of meson - nucleon vertex form - factors is not simple. When the temperature increases they remain almost unchanged until a certain temperature $T_C$ then start to decrease dramatically. The corresponding critical temperatures for each meson - nucleon system were calculated and the appropriate conclusion about their origin was brought.

The methods and results of this thesis may be used in the studies of heavy - ion collisions as well as of cosmology and of astrophysics.
Introduction

The main goal of this thesis is the investigation of hadron properties in free space as well as in nuclear medium. It has been generally believed that, properties of hadrons and their interaction should be described in framework the of Quantum Chromodynamics (QCD) which is the underlining fundamental theory of hadron physics. However, in this theory one is failed with the difficulty related to the running coupling constant $\alpha(q^2)$. In fact, for large momentum transfer $q$, as in the deep inelastic scattering of leptons by hadrons, $\alpha(q^2)$ is small, and hence one can use the perturbation theory to make theoretical predictions which turned out to be in good agreement with the experiment. On the other hand, for the processes of interest in nuclear physics or low energy hadron physics, the length scale is typically $\sim 1\, fm$, that corresponds to small momentum transfer $q$. The running coupling constant $\alpha(q^2)$ is then large and perturbation theory is useless. In this strong coupling regime, non-perturbative methods are indispensable, but so far, not much successes has been achieved in this respect. In this regime one is led to modelling non-perturbative QCD. So, the challenge to nuclear physicist is thus to find models which can bridge the gap between the fundamental theory and our wealth of knowledge about low energy phenomenology.

Our choice of models should be inspired by QCD, perhaps one day will be derived from it, and embody its important features, such as confinement and chiral symmetry, while avoiding the full complexity of a non-abelian gauge theory. The parameters of these models must, for now at least, be determined phenomenologically, although the hope is that eventually they should be calculated from QCD.

At present there is no systematic method for treating continuum QCD in the perturbative regime. With the exception of lattice Monte Carlo methods which still encounter both conceptual and technical problems, especially related to high fermion and understanding of the vacuum and its properties.
low-energy physics at the hadronic scale is mostly qualitative and relies heavily on the use of such phenomenological models as potential models [1], bag models [2], hybrid-bag models [3] and effective chiral models [4].

In the non relativistic potential models of Isgur and Karl [1] the quarks are treated as constituent of protons much like in the conventional parton model. The confinement character of QCD is put in by hand using a linearly rising potential between the constituent quarks at large distances. The remaining gluonic effects are treated perturbatively, leading to Breit-Wigner hyperfine interactions. While these models are justified for the heavy flavor hadrons such as charmonium and bottomonium, they are questionable for the light hadrons such as $N$ and $\Delta$, where we know that, the constituent quarks are highly relativistic.

In the usual bag models, for instance, in the MIT bag model, the quarks are confined into a region of hadronic size $V \approx \Lambda^3$, inside of which the quarks carry their current masses and are treated fully relativistically. The bag is assumed to be a bubble of perturbative vacuum immersed into a non perturbative environment characterized by a bulk parameter $B$ (the bag pressure) [5]. Because of asymptotic freedom the remaining gluonic effects are treated perturbatively inside the bag. Although rather successful in describing the spectroscopy of the lightest hadrons, the MIT bag model suffers from a serious drawback, namely, axial-vector conservation. The confining bag wall breaks explicitly chiral symmetry, allowing left and right-handed quarks to mix at the boundary. We know that, in the chiral limit ($m_f = 0$) QCD is chirally symmetric. The standard treatment of the MIT and cloudy bag model (CBM) lack translational invariance and they do not provide a dynamical description of hadron. This makes center of mass (c.m.) and recoil corrections difficult to calculate [6, 7, 8, 9]. Besides, these models are not convenient to describe nucleon-nucleon dynamics as they make explicit use of the quark degrees of freedom.

On the other hand, since the fundamental constituents in QCD, quarks and gluons are confined, it might be more suitable to eliminate their degrees of freedom which are not directly observable and incorporate their effects into an effective theory framed in terms of observable hadronic degrees of freedom including as many relevant features of QCD as possible.

This alternative theoretical route can be traced back to the works of 't Hooft [10] and Witten [11]. 't Hooft proposed to look for the parameter in QCD, other than the coupling "constant" $-\alpha(q^2)$ in terms of which one can perform expansions and arrived at the $1/N_c$ expansion, $N_c$ being the number of colors. Here one can note that, there is nothing fundamental in the value $N_c = 3$; this is required by phenomenological considerations as the fit to the $\pi_0 \rightarrow 2\gamma$ decay rate or to the ratio of $e^+e^-$ cross sections, $\sigma_{tot}(e^+e^-)/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$.

In a detailed analysis of QCD diagrams in the $1/N_c$ expansion Witten [11] was led to the conjecture that in the large $N_c$ limit, QCD is equivalent to an effective field theory involving only mesons (and glueballs). Soon after this result, Witten [12] also recognized that baryons can be regarded as solitons of this meson theory in the manner first proposed by Skyrme [13].

All this results give support to the idea that low energy hadron physics can be described by effective theories constructed from meson fields alone. Therefore nonlinear chiral models including only meson degree of freedom will be used to study hadron properties in free space and in nuclear medium. Before entering into more details, we would like first to clearly identify what we mean by nuclear medium. There are obviously two different regimes, leading to two different physical representations of the medium. At very high density one expects a phase transition in which quarks and gluons are deconfined and more or less at the same time chiral symmetry is restored. Above the critical density $\rho_c$ - which is expected to be of order of several normal nuclear matter density $\rho_0$ - one thus expects a uniform medium of weakly interacting (almost) massless quarks and gluons.

Below this critical density, quarks and gluons are confined and one is dealing with a
medium composed of more or less densely packed hadrons. Near the critical density it may be difficult to clearly identify nucleons in the medium dominated by all types of hadrons (mainly mesons and baryon resonances). At normal nuclear density or less the situation is however much simpler. We know that, the relevant degrees of freedom are nucleons interacting by the exchange of many correlated mesons [14].

We shall concentrate in this thesis on the low energy regime. One of the most exiting topics in this regime is the study of variation of hadron properties as the nuclear environment changes. Recent experiments from HELIOS-3 [15] and the CERES [16] at SPS/CERN energies have shown that there exists a large excess of $e^+e^-$ pairs in central $S+Au$ collisions. Those experimental results may give a hint of some change of a hadron properties in nuclei. Ultrarelativistic heavy - ion experiments (e.g. at RHIC) [17] are also expected to give significant information on the strong interaction (QCD) through the detection of changes in hadronic properties.

Theoretically lattice QCD simulations may eventually give the most reliable information on the density and or temperature dependence of hadron properties in matter. However current simulations have been performed only for finite temperature ($T \neq 0$). Therefore many authors have studied hadron masses in matter using effective theories and have reported that the mass decreases in nuclear medium. In the present thesis we shall make an attempt to modify Skyrme model to study the changes in mass of hadrons and in their interaction.

The present thesis is organized as follows. In Chapter 1 the basic ideas and techniques of the Skyrme model "for beginners" is reviewed. In Chapters 2 - 4 we shall consider nucleon properties and nucleon - nucleon (NN) interaction in free space. Here the original Skyrme model is extended by inclusion of scalar - isoscalar meson which plays a crucial role in central the nucleon - nucleon interaction.

It is believed that the nucleon properties and NN interaction may change in the nuclear medium. This modification is considered in Chapters 5-7. The main conclusion of the present thesis are summarized in the last chapter.
Chapter 1

The physics of Skyrme solitons.
(A brief review)

An important property of QCD is its chiral symmetry. This invariance must be preserved in the corresponding effective field theories.

1.1 The $\sigma$ model

The simplest chirally invariant effective field theory is the $\sigma$ model proposed by Gell Mann and Levy [22]. Its version without fermions falls precisely in the class of effective field theories contemplated by Witten. This version of the $\sigma$ model can be formulated in the following way. Consider a $2 \times 2$ matrix field

$$U(r) = \frac{1}{f_\pi} [\sigma(r) + i\vec{\pi}(r)]$$

(1.1)

where $\sigma(r)$ is a scalar and isoscalar field, $\vec{\pi}(r)$ is a pion field and $f_\pi$ is the pion decay constant. The Lagrangian density can then be written as

$$L_2(r) = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 = \frac{f_\pi^2}{4} \text{Tr} [\partial_\mu U(r) \partial^\mu U^+(r)]$$

(1.2)

In the non-linear realization of the $\sigma$ model, the $\sigma$ field is eliminated from the model via the chirally invariant constraint,

$$\sigma^2 + \vec{\pi}^2 = f_\pi^2$$

(1.3)

implying that $U$ is unitary. Defining $\sigma = f_\pi \cos \theta(r)$ and $\vec{\pi} = f_\pi \hat{\pi} \sin \theta(r)$, we can write $U = e^{i\hat{\pi} \hat{\pi} \theta(r)}$ with $\hat{\pi} = \vec{\pi}/|\vec{\pi}|$.

The Lagrangian (1.2) possesses chiral $SU(2)_V \otimes SU(2)_A$ symmetry. Left and right transformations are associated with left and right multiplication by elements of $SU(2)$ group. The chiral $SU(2)_R \otimes SU(2)_L$ group of transformations of the field $U$ will correspond to the direct product of left and right transformations of the elements of $SU(2)$ group: $U \rightarrow A U B^+$ with arbitrary constant $SU(2)$ matrices $A$ and $B$. The chirally invariant action is usually written by means of left (or right) invariant Cartan forms:

$$L_2 = -\frac{f_\pi^2}{4} \text{Tr} (L_\mu L^\mu),$$

$$L_\mu = U^+ \partial_\mu U, \quad R_\mu = U \partial_\mu U^+.$$ (1.4)

\footnote{A reader is referred to more extended review articles [18, 19, 20, 21]}
In terms of basic $\pi$ and $\sigma$ fields the corresponding vector $\vec{V}_\mu = [\vec{\pi}\partial_\mu\vec{\pi}]$ and axial currents $\vec{A}_\mu = \sigma\partial_\mu\vec{\pi} - \vec{\pi}\partial_\mu\sigma$ may be obtained by infinitesimal rotations in isospin space $\sigma \rightarrow \sigma$, $\vec{\pi} \rightarrow \vec{\pi} - [\vec{\varepsilon}\vec{\pi}]$ and $\sigma \rightarrow \sigma - (\vec{\varepsilon}\vec{\pi})$, $\vec{\pi} \rightarrow \vec{\pi} + \vec{\varepsilon}\sigma$ respectively. Clearly $\partial_\mu V^\mu = 0$ and $\partial_\mu A^\mu = 0$ in accordance with chiral invariance.

The trouble with the non-linear $\sigma$ model is that it leads to energetically unstable solutions; by dimensional arguments, one can see that the energy scales like $R$, where $R$ is the size of the system, and it vanishes when the system shrinks to zero size.

### 1.2 The Skyrme Model

To remedy this shortcoming, Skyrme [13] proposed the addition of higher-order terms in the derivative $\partial_\mu U$ to the form $L_2(r)$. He suggested to introduce additional term of the form

$$L_{4a}(r) = \frac{1}{32\epsilon^2} \text{Tr} [(\partial_\mu U)U^+, (\partial_\nu U)U^+]^2 = \frac{1}{32\epsilon^2} \text{Tr} [L_\mu, L_\nu]^2 ,$$

with the four derivative term. The resulting Lagrangian density

$$L_{sk}(r) = L_2(r) + L_{4a}(r)$$

yields a complicate non-linear Euler-Lagrange equation which is difficult to solve. However, if one makes the ansatz that the pion field is directed radially in configuration space, $\hat{\pi} = \hat{r}$, $\hat{r}$ being a unit vector in coordinate space $\hat{r} = \vec{r}/|\vec{r}|$, the $U$ field is of the "hedgehog" form [3, 13]

$$U = U_0 = \exp (i\vec{\tau}\hat{\pi}(r)).$$

In this case, the Lagrangian is

$$L = \int d^3r L_{sk}(r) = \frac{\pi f_\pi}{e} \int_0^\infty dx dx [x^2(\theta'^2 + \frac{2\sin^2 \theta}{x^2}) +$$

$$+ 4\sin^2 \theta (\frac{\sin^2 \theta}{x^2} + 2\theta'^2)]$$

with the choice of length scale $x = 2\epsilon f_\pi r$ and $\theta' = d\theta/dx$. Minimizing $L$ with respect to $\theta$, i.e. using $\frac{d}{dx} (\frac{\partial L}{\partial \theta'}) = \frac{\partial L}{\partial \theta}$ one gets the following Euler-Lagrange equation

$$\theta''(\frac{x^2}{4} + 2\sin^2 \theta) + \frac{x\theta'}{2} - \frac{\sin 2\theta}{4} +$$

$$+ \theta'^2 \sin 2\theta - \frac{\sin^2 \theta \sin 2\theta}{x^2} = 0.$$  

(1.9)

Soliton solutions can be found with the boundary conditions $\theta(\infty) = 0$ and $\theta(0) = n\pi$, $n$-integer. This solution is now energetically stable since the contribution of the term $L_{4a}$ of (1.6) to the energy is proportional to $1/R$ so that the total energy can be minimized with respect to $R$ to give a finite value. It can also be shown [13] that the solutions characterized by this boundary conditions are topological solitons whose winding number is $n$.

### 1.2.1 The Baryonic Current

Besides the usual Noether currents, such as $V_\mu$ and $A_\mu$, associated with various symmetries fulfilled by the Lagrangian $L_{sk}(r)$, Skyrme showed that one can construct an "anomalous" current

$$B_\mu = \frac{1}{e} \varepsilon_{\mu\nu\rho} \text{Tr} I^\alpha I^\beta I^\gamma$$

(1.10)
which is automatically conserved. A new quantum number can be therefore associated with $B_\mu$ since this current does not correspond to any initial symmetry of the Lagrangian. With the hedgehog solution, the time component of $B_\mu$ is

$$B_0(r) = \frac{(\cos 2\theta - 1) d\theta}{4\pi^2 r^2}.$$  \hspace{1cm} (1.11)

Integrating this density over all space $\vec{r}$ and taking into account the boundary conditions one gets $B = \int d^3 r B_0(r) = n$ where $n$ is winding number characterizing the solution $U$. At this point Skyrme identified $n$ with the byron number, or equivalently $B_\mu$ with the byronic current. Baryons therefore emerge from the model as topological solitons.

### 1.2.2 Quantization.

So far, the Lagrangian as well as the Euler-Lagrange equation are classical. $U$ is a classical field and therefore carries no definite spin and isospin values. A simple quantization method starting from the classical solution $U = U_0 = \exp (i \vec{r} \vec{a}(r))$ was proposed by Adkins, Nappi and Witten [12]. These authors introduce collective coordinates through the unitary transformation $A = a_4 + i \vec{r} \vec{a}$ where the $a_\mu$’s are independent of $\vec{r}$ but can depend on time $t$. The $a_\mu$’s also satisfy the constraint $a_4^2 + \vec{a}^2 = 1$ since $A$ is unitary. Under this transformation, $U_0$ becomes $U = A(t) U_0 A^+(t)$. Substituting $U$ into the Lagrangian yields

$$L = \int d^3 r L_{sk}[U = A(t) U_0 A^+(t)] = -M + \lambda \text{Tr} (\dot{A} \dot{A}^+) =$$

$$= -M + 2\lambda \sum_{\mu=1}^4 (\dot{a}_\mu)^2$$  \hspace{1cm} (1.12)

where e.g. $\dot{a} = da/dt$, $M$ is given by eq. (1.8) and

$$\lambda = \frac{\pi}{3 f_\pi e^3} \int_0^\infty dx x^2 \sin^2 \theta [1 + 4(\theta'^2 + \sin^2 \theta/x^2)]$$  \hspace{1cm} (1.13)

From the Lagrangian one gets the Hamiltonian

$$H = \Pi_\mu \dot{a}_\mu - L,$$  \hspace{1cm} (1.14)

where $\Pi_\mu$’s are the conjugate momenta $\Pi_\mu = \partial L/\partial \dot{a}_\mu = 4\lambda \dot{a}_\mu$. Substitution of this equation into the previous one yields

$$H = M + \frac{1}{8\lambda} \sum_{\mu=1}^4 \Pi_\mu^2.$$  \hspace{1cm} (1.15)

With the usual canonical quantization procedure $\Pi_\mu = -i \frac{\partial}{\partial a_\mu}$, the Hamiltonian takes the form

$$H = M + \frac{1}{8\lambda} \sum_{\mu=1}^4 (\frac{\partial^2}{\partial a_\mu^2})$$  \hspace{1cm} (1.16)

with the constraint $a_4^2 + \vec{a}^2 = 1$. Because of this constraint the term $\sum_{\mu=1}^4 \frac{\partial^2}{\partial a_\mu^2}$ is to be interpreted as the Laplacian on a 3-dimensional sphere. The wave functions of $H$ are polynomials in $a_\mu$. 
1.2. 3 The Spin and Isospin Operators

Under a rotation about the z-axis in configuration space by a small angle $\delta$, it is easy to show that

$$e^{i\vec{\tau}\vec{r}} \rightarrow e^{i\delta \tau_z/2}e^{i\vec{\tau}\vec{r}}e^{-i\delta \tau_z/2}$$  (1.17)

Hence, under this rotation the field configuration $AU_0A^+$ transforms into a new one with

$$A \rightarrow Ae^{i\delta \tau_z/2},$$

$$(a_4 + i\vec{\tau}\vec{a}) \rightarrow (a_4 + i\vec{\tau}\vec{a})(1 + i\delta \tau_z/2)$$  (1.18)

This gives after some algebra

$$a_4 \rightarrow a_4 - \delta a_z/2, \quad a_z \rightarrow a_z + \delta a_4/2,$$

$$a_x \rightarrow a_x - \delta a_y/2, \quad a_y \rightarrow a_y + \delta a_x/2.$$  (1.19)

Identifying this transformation with the initial rotation written in terms of the spin operators, i.e. $a_\mu \rightarrow (1 + i\delta J_z)a_\mu$ one finds

$$J_z = \frac{i}{2}[-a_4 \frac{\partial}{\partial a_z} + a_z \frac{\partial}{\partial a_4} - a_x \frac{\partial}{\partial a_y} + a_y \frac{\partial}{\partial a_x}]$$  (1.20)

Under a rotation about the z-axis of isospin space by a small angle $\delta A \rightarrow e^{-i\delta \tau_z/2}A$ and the same calculation as before leads to

$$I_z = \frac{i}{2}[a_4 \frac{\partial}{\partial a_z} - a_z \frac{\partial}{\partial a_4} - a_x \frac{\partial}{\partial a_y} + a_y \frac{\partial}{\partial a_x}]$$  (1.21)

It is easy to verify that the spin-isospin wave functions for various baryons are:

$$|p \uparrow\rangle = (a_x + ia_y)/\pi, \quad |p \downarrow\rangle = -i(a_4 + ia_z)/\pi,$$

$$|n \uparrow\rangle = i(a_4 + ia_z)/\pi, \quad |n \downarrow\rangle = -(a_x - ia_y)/\pi.$$  (1.22)

The calculation of observables is then reduced to the evaluation of appropriate matrix element of operators (constructed from the field configuration) with these spin-isospin wave functions. For example, the proton mass is

$$\langle p \uparrow |H|p \uparrow\rangle = \frac{1}{\pi^2} \int d^4a \delta(\sum_\mu a_\mu^2 - 1) (a_x - ia_y)H(a_x + ia_y) =$$

$$= M + \frac{3}{8\lambda}$$  (1.23)

where the expression (1.16) for $H(U = AU_0A^+)$ has been used.

1.2. 4 Results for the Static Properties of Baryons

The previous procedure has been applied by Adkins, Nappi and Witten [12] to the calculation of baryon masses (nucleon and delta resonance), charge radii, magnetic moments and coupling constants. The results are listed in Table 1.1. These authors adjust $f_\pi$ and $e$ to fit the nucleon and $\Delta$ masses obtaining $f_\pi = 64.5$ MeV and $e = 5.45$. The other quantities are predicted. They are in qualitative agreement with experiment (within 30% ) except for $g_A.$
Table 1.1: Baryon properties in the original Skyrme model [12].

| Quantity     | Prediction | Experiment |
|--------------|------------|------------|
| $M_N$        | input      | 939 MeV    |
| $M_\Delta$   | input      | 1232 MeV   |
| $f_\pi$      | 64.5 MeV   | 93 MeV     |
| $\langle r^2 \rangle_{I=0}$ | 0.59 fm | 0.72 fm |
| $\langle r^2 \rangle_{M,J=0}$ | 0.92 fm | 0.81 fm |
| $\mu_p$      | 1.87       | 2.79       |
| $\mu_n$      | -1.31      | -1.91      |
| $g_A$        | 0.61       | 1.23       |
| $g_{\pi NN}$ | 8.9        | 13.5       |

1.3 Two baryon system

The principal goal of studying the $B = 2$ sector of the Skyrme model is to apply it to the nucleon nucleon (NN) interaction. Characteristic features of the nuclear force shared by various semiphenomenological potentials are as follows:

a) a long range ($\geq 1.4 \text{ fm}$) interaction dominated by one-pion exchange potential (OPEP),

b) a medium-range ($\sim 1 \text{ fm}$) attraction in the central force, and

c) a strong short-range repulsion ($\leq 0.7 \text{ fm}$).

The OPEP is well established, while the microscopic origins of the medium- and short-range interactions are less well understood.

A simple study has already been made by Skyrme himself, which surprisingly contains many essential features, at least qualitatively. He proved that a matrix product of two $B = 1$ configurations, $U = U_1U_2$, belongs to the $B = 2$ sector. Then he proposed the "product ansatz" for two Skyrmions separated by $\vec{R}$ individually iso-rotated by $A$ and $B$:

$$U_{12} = AU_0(\vec{r} + \vec{R}/2)A^+ BU_0(\vec{r} - \vec{R}/2)B^+$$  \hspace{1cm} (1.24)

The classical energies calculated using this ansatz giving

$$E_{12} = 2E_0 + V(\vec{R}, C),$$  \hspace{1cm} (1.25)

where the static potential $V$ depends of the relative distance $\vec{R}$ and the relative orientation $C = A^+B$. A global rotation of $U$ does not change the total energy. Skyrme pointed out the following two facts:

i. For large $R = |\vec{R}|$, $V(\vec{R}, C)$ approaches the one-pion-exchange potential (OPEP):

$$V(\vec{R}, C) \approx T_{ab} \nabla_a \nabla_b \{ \exp(-m_\pi R)/R \}$$  \hspace{1cm} (1.26)

with $T_{ab} = \text{Tr} [\tau_a C \tau_b C^+]/2$. The Yukawa function in eq. (1.26) is replaced by $1/R$ for the massless pion. This is natural since the Skyrmion contains a one pion cloud in the tail region.

ii. At $R = 0$ and $A = B = 1$, the configuration $U_{12}$ takes the form of hedgehog with $B = 2$ and its energy is about 1 GeV higher than the two Skyrmion threshold. This excess of energy at $R = 0$ cannot be treated in a static ansatz.

As $A$ and $B$ are not restricted to the Pauli group, the potential $V(\vec{R}, C)$ can be written: $V(\vec{R}, C) \approx T_{ab} \nabla_a \nabla_b \{ \exp(-m_\pi R)/R \}$ with $T_{ab} = \text{Tr} [\tau_a C \tau_b C^+]/2$. The Yukawa function in eq. (1.26) is replaced by $1/R$ for the massless pion. This is natural since the Skyrmion contains a one pion cloud in the tail region.

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More quantitative analyses using the product ansatz have been performed by several authors and the precise form of the potential \( V(\vec{R}, C) \) was worked out. The NN interaction is then obtained by the semiclassical method from \( V(\vec{R}, C) \). Each Skyrmion is rotated by the collective coordinate \( A \) or \( B \), and the relevant spin-isospin component is extracted. One finds that the NN potential consists of three terms, given by

\[
V_{NN}(\vec{R}) = V_0(R) + (\vec{1}\vec{2})(\vec{3}\vec{4})V_\sigma(R) + (\vec{1}\vec{2})S_{12}V_T(R) \tag{1.27}
\]

where \( S_{12} \) is the standard tensor operator for nucleons:

\[
S_{12} = 3(\vec{3}\vec{4})(\vec{3}\vec{4}) - (\vec{3}\vec{4})(\vec{3}\vec{4}) \tag{1.28}
\]

The last two terms in (1.27) tend asymptotically to the OPEP, as expected, while the first term has a shorter range. Unfortunately, it is found that the spin-isospin independent part of the central force \( V_0 \) is strongly repulsive at medium distances so that \( V_{NN} \) in Eq. (1.27) does not account for nuclear binding.

### 1.4 Scaling behavior of the Skyrme model

The second shortcoming of the Skyrme model was revealed by Shechter [23]. He proved that the chiral effective Lagrangian such as the Skyrme model have different scaling behavior than QCD. Classically the QCD Lagrangian (with massless quarks) is invariant under scale transformations \( x_\mu \to e^{-\lambda}x_\mu \) with \( \lambda \) real. The corresponding conserved current is the dilaton current, \( S_\mu = T_{\mu\nu}x^\nu \) where \( T_{\mu\nu} \) is the energy-momentum tensor. At the quantum level, however \( S_\mu \) has nonvanishing divergence [24]

\[
\partial^\mu S_\mu = \left( \frac{\beta(g)}{2g} \right) F^a_{\mu\nu} F^{a\mu\nu} \tag{1.29}
\]

where \( g \) is the coupling constant, \( \beta(g) \) the QCD beta function and \( F^a_{\mu\nu} \) the gluonic field strength tensor. Since \( \partial^\mu S_\mu = T^\mu_\mu \) the trace of \( T_{\mu\nu} \) in Eq. (1.29) is called the ”trace anomaly”.

To restore the anomalous scaling behavior of QCD at the effective Lagrangian level one introduces a scalar field \( \chi(x) \) [23] such that \( \partial^\mu S_\mu = -b\chi^4 \). Fluctuations in \( \chi \) around \( \chi_0 \) may be interpreted as a scalar glueball or a quarkonium.

In the next chapter we shall take into account scalar dilaton field to restore the anomalous scaling behavior and consider NN interaction. Meanwhile, to make this point clearer and for further references we briefly outline the meaning of ”scale dimension” below. Let’s consider the kinetic terms of a fermion field (e.g. quark field in QCD)

\[
\mathcal{L}_{\text{kin}}^\psi(x) = i\bar{\psi}(x)\partial^\mu\gamma_\mu\psi(x) \tag{1.30}
\]

and a boson (pion) field

\[
\mathcal{L}_{\text{kin}}^\pi(r) = \frac{1}{2}\partial_\mu \pi \partial^\mu \pi. \tag{1.31}
\]

The scale invariance demands that the action \( S = \int d^4x \mathcal{L} \) should be invariant under scale transformations \( x_\mu \to e^{-\lambda}x'_\mu \):

\[
S = \int d^4x \mathcal{L}_{\text{kin}}(x) = \int d^4x' \mathcal{L}'_{\text{kin}}(x') \tag{1.32}
\]

Let the fermion and boson fields scale as \( \psi(x) = \exp(n\lambda)\psi'(x') \) and \( \pi(x) = \exp(m\lambda)\pi'(x') \) respectively. For fermions we get

\[
S = i \int d^4x\bar{\psi}(x)\partial^\mu\gamma_\mu\psi(x) = i \int d^4x'\bar{\psi}'(x')\partial^\mu\gamma_\mu\psi'(x')e^{\lambda^2m\lambda}e^{-4\lambda} = \int d^4x'\mathcal{L}'_{\text{kin}}(x') \tag{1.33}
\]
and hence \(-4\lambda + \lambda + 2n\lambda = \lambda(2n - 3) = 0\). Since \(\lambda\) is arbitrary then \(n = 3/2\). Similarly, for boson fields one can show that \(m = 1\). Therefore we may conclude that

\[
x_\mu \rightarrow e^{-\lambda}x_\mu, \quad \psi(x) \rightarrow e^{3\lambda/2}\psi(x), \quad \pi(x) \rightarrow e^{\lambda}\pi(x).
\]

(1.34)
i.e. the scale dimension \(d\) equals to \(d_f = 3/2\) and \(d_b = 1\) for fermions and bosons respectively. Obviously, the scale dimension of chiral nonlinear field \(U(x)\) is zero. For this reason the four derivative term of the Skyrme Lagrangian \(\mathcal{L}_{4a}\) is scale invariant by itself:

\[
\int d^4x\mathcal{L}_{4a}(x) \rightarrow \int d^4x e^{-4\lambda}\mathcal{L}'_{4a}(x)e^{4\lambda} = \int d^4x'\mathcal{L}'_{4a}(x')
\]

(1.35)
whereas the kinetic term

\[
\mathcal{L}_2(x) = \frac{f_\pi^2}{4} \text{Tr} \left[ \partial_\mu U(x) \partial^\mu U^+(x) \right]
\]

(1.36)
is not:

\[
\int d^4x\mathcal{L}_2(x) \rightarrow \frac{f_\pi^2}{4} \int d^4x' e^{-4\lambda}e^{2\lambda}\text{Tr} \left[ \partial_\mu U'(x') \partial^\mu U^+(x') \right] =
\]

(1.37)

How can the invariance be restored? According to Shehter [23] an additional boson field \(\chi(x)\) should be included into the Skyrme Lagrangian. As a result the minimal version of scale invariant Skyrme Lagrangian will be given by

\[
\mathcal{L}(U, \chi) = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \frac{f_\pi^2}{4} \left( \frac{\chi}{\chi_0} \right)^2 \text{Tr} \left[ \partial_\mu U \partial^\mu U^+ \right] + \\
+ \frac{1}{32e^2} \text{Tr} \left[ (\partial_\mu U)^+ (\partial_\mu U)^+ \right]^2
\]

(1.38)
where \(\chi_0\) is the vacuum expectation value of a dilaton field \(\chi(x)\).

But unfortunately this is not the whole story. The basic QCD Lagrangian whose main features should be reflected in an effective one has the "trace anomaly" mentioned above. To restore the anomalous scaling behavior of QCD at the effective Lagrangian level one has to introduce an effective potential \(V(\chi)\), describing the self interaction of dilaton fields. It was shown that, the effective potential could be uniquely determined by the vacuum energy density \(b\chi_0^4/4\) and the condition that the minimum should occur at \(\chi = \chi_0\) which yield [24]

\[
V(\chi) = \frac{b\chi_0}{16} \left( 1 + \frac{\chi}{\chi_0} \right) [4 \ln(\chi/\chi_0) - 1].
\]

(1.39)
The parameters \(b\) and \(\chi_0\) are obtained from the value of the gluon condensate \(C_g\)

\[
\langle 0 | \partial_\mu S_\mu | 0 \rangle = \langle 0 | (\frac{\beta(g)}{2g}) F^a_{\mu\nu} F^{a\mu\nu} | 0 \rangle \equiv C_g = b\chi_0^4
\]

(1.40)
and the dilaton mass

\[
m_\chi^2 = \frac{\partial^2 V}{\partial \chi^2} |_{\chi = \chi_0} = 4b\chi_0^2
\]

(1.41)
which together with \(f_\pi\) and \(e\) specify the parameters of the Lagrangian. It has been shown that, the suitable choice of parameters \(\chi_0 \approx 120MeV, C_g \approx (300MeV)^4, e \approx 4.5\) and \(f_\pi = 93MeV\) leads to a large dilaton mass \(m_\chi \approx 1500MeV\), which may be identified with a glueball. Although this scalar field has nothing to do with a scalar \(\sigma\) meson \((m_\sigma \approx 600MeV)\), its inclusion into the Skyrme Lagrangian proved to solve a long standing problem of missing of intermediate range attraction in the central \(NN\) interaction [25, 26].
Nevertheless, it would be quite desirable to include into the Skyrme Lagrangian a dilaton field with lower mass e.g. $m_{\chi} = m_{\sigma} \approx 600 MeV$. This goal has been achieved by using the method of bosonization in ref.s [21, 27]. As a result, a modified Lagrangian which differs from Shehters one [23] mainly due to the self interaction potential:

$$V(\chi) = \frac{b\chi_0}{24} \left\{ 1 - \left( \frac{\chi}{\chi_0} \right)^4 - \frac{4}{\varepsilon} \left[ 1 - \left( \frac{\chi}{\chi_0} \right)^\varepsilon \right] \right\}$$ (1.42)

has been proposed. We shall use this model to study $NN$ interaction in the next chapter.

### 1.5 PCAC and Goldberger - Treiman relations

The chiral invariance of Skyrme model leads to well known relations. Here we show how these relations can be derived in a rather model independent way. Considering pion - nucleon system we start with the interaction Hamiltonian:

$$H_{int} = -ig_{\pi NN}\bar{\psi}(x)\gamma_5\vec{T}\psi(x)\vec{\varphi}_\pi(x)$$ (1.43)

where $g_{\pi NN}$ is the pion - nucleon coupling constant, $\psi(x)$ and $\varphi_\pi(x)$ are nucleon and pion fields respectively. The latter satisfies Klein - Gordon equation:

$$(\partial_\mu \partial^\mu + m_\pi^2)\vec{\varphi}_\pi(x) = -ig_{\pi NN}\bar{\psi}(x)\gamma_5\vec{T}\psi(x)$$ (1.44)

Axial current of this system is the sum of the nucleonic, $\vec{A}_\mu^N$:

$$\vec{A}_\mu^N = g_A\bar{\psi}(x)\gamma_\mu\gamma_5\vec{T}\psi(x)/2$$ (1.45)

and pionic, $\vec{A}_\mu^\pi$,

$$\vec{A}_\mu^\pi = f_\pi\partial_\mu\vec{\varphi}_\pi(x)$$ (1.46)

axial currents:

$$\vec{A}_\mu = \vec{A}_\mu^N + \vec{A}_\mu^\pi$$ (1.47)

Using Dirac ($i\gamma_\mu \partial^\mu - M)\psi(x) = 0$ and Klein - Gordon equations one may obtain

$$\partial^\mu \vec{A}_\mu^N = Mg_A\bar{\psi}(x)i\gamma_5\vec{T}\psi(x),$$

$$\partial^\mu \vec{A}_\mu^\pi = -f_\pi m_\pi^2\vec{\varphi}_\pi(x)$$ (1.48)

Note that, the last second equation in (1.48) is called as PCAC (partial conservation of axial current) relation. Now, the requirement of conservation of the total axial current $\partial^\mu \vec{A}_\mu = 0$ gives following equation for the divergent of $\vec{A}_\mu$:

$$(\partial_\mu \partial^\mu + m_\pi^2)\vec{\varphi}_\pi(x) = -g_AM\bar{\psi}(x)i\gamma_5\vec{T}\psi(x)$$ (1.49)

Finally, comparing right hand sides of the Eq.s (1.44) and (1.49) one easily gets Goldberger - Treiman relation:

$$Mg_A = f_\pi g_{\pi NN}$$ (1.50)

This utmost important relation between axial coupling constant $g_A$ and pion - nucleon coupling constant $g_{\pi NN}$ is in good agreement with the experiment: $g_{\pi NN} = 13.4 \pm 0.1$, $Mg_A/f_\pi = 12.78 \pm 0.1$. The small deviation reflects the accuracy of the relation. We shall come back to this point considering in - medium modification of this relation in Chapter 5.
Chapter 2

Scalar Dilaton - Quarkonium Meson in Nucleon - Nucleon Interaction.¹

2.1 Introduction

Skyrme-like models allow for the study of the properties of individual baryons and offer a framework for investigating interactions between classical baryons. The simplicity of calculations and the existence of a transparent connection between NN and N\bar{N} interactions within the Skyrme model are distinct advantage over the models such as nonrelativistic quark models or the bag models. The most of calculations based on the product approximation within Skyrme-like models lead to projected potentials that are in qualitative agreement with empirically motivated NN potentials at short and long distances. However many of them does not take into account the scale anomaly of the QCD omitting scalar particles and fail to reproduce the intermediate range attraction in the scalar channel.

At the present time there are two different approaches to include scalar meson - dilaton into the Skyrme model. In both approaches the interaction of dilaton field with the chiral field is dictated by the scale invariance. The main difference is in the origin of dilaton. On the one hand, there is the approach in which dilaton is associated with the glueball [31]. In this approach the glueball field saturates the scale anomaly completely. In the other approach the dilaton is treated as a quarkonium arising due to fluctuations of the quark condensate [27], which is an order parameter for the chiral symmetry breaking. In this approach the dilaton - quarkonium saturates a part of the scale anomaly, namely by the part which is produced by the quark loops. This choice of the quarkonium as a dilaton is based on two principal observations:

a) The experimental studies of scalar resonances [32] show that the real lightest candidate for the glueball state is $f_0(1590)$ which does not appear in $\pi\pi$ and $K\bar{K}$ productions.

b) Consideration of the chiral anomaly [27] shows that the only gauge-invariant combination of the gluon field $G_{\mu\nu}G_{\lambda\rho}$ can contact only with a total antisymmetric tensor $\varepsilon^{\mu\nu\lambda\rho}$. Thus this combination has $J^{PC} = 0^{-+}$ quantum number and contributes to Wess - Zumino - Witten action producing $U(1)$ anomaly. It suggests that the $0^{++}$ glueball field as a fluctuation of $G_{\mu\nu}^2$ cannot interact directly with chiral field, but only through the mixing with the dilaton – quarkonium. The estimate gives small values for the mixing angle of glueball and quarkonium states.

In the present chapter we calculate the central $NN$ potential within a Skyrme model suggested by Andrianov and Novozhilov in ref. [27]. The model is based on effective low

¹The present chapter is based on following articles by the author and his collaborators: [28, 29, 30]
energy action for pseudoscalar and scalar (dilaton - quarkonium) meson which has been derived starting directly from the QCD generating functional by the joint chiral and conformal bozonization method.

2.2 The Lagrangian

The Lagrangian suggested by Andrianov and Novozhilov [27] is

\[ \mathcal{L}_{AN}(U, \sigma) = \mathcal{L}_2 + \mathcal{L}_{4a} + W_\sigma, \]

\[ \mathcal{L}_2 = -\frac{f_\pi^2}{4}[\text{Tr} L_\mu L^\mu - 2(\partial_\mu \sigma)^2]e^{-2\sigma}, \]

\[ \mathcal{L}_{4a} = \frac{1}{32e^2} \text{Tr} [L_\mu, L_\nu]^2, \]

\[ W_\sigma = -\frac{C_g}{24}[e^{-4\sigma} - 1 + \frac{4}{\varepsilon}(1 - e^{-\varepsilon\sigma})], \]

where \( f_\pi = 93 MeV \) is pion decay constant, \( e^- \) dimensionless parameter, \( C_g \) - gluon condensate parameter : \( C_g = ((300 - 400)MeV)^4 \), \( \sigma \) - scalar meson field, \( L_\mu = U^+ \partial_\mu U \), \( U \) is an \( SU(2) \) matrix chiral field. It is a generalization of the well-known original Skyrme model [12, 13] (OSM) and takes into account the conformal anomaly of the QCD. The term \( \mathcal{L}_2 \) includes the kinetic term of the chiral field and scalar fields. The \( \mathcal{L}_{4a} \) is a well-known Skyrme term. The effective potential for the scalar field is a result of the extrapolation of the low energy potential into the high energy region in one-loop approximation to the Gell-Mann-Low QCD \( \beta \)-function. The parameter \( \varepsilon \) depends on the number of flavors \( N_f \): \( \varepsilon = 8N_f/(33 - 2N_f) \). Note that in the limit of heavy \( \sigma \)-meson the potential becomes equal to the symmetric quartic term \( \mathcal{L}_{4s} = \frac{\gamma}{8e^2}[\text{Tr} L_\mu L^\mu]^2 \), which is necessary to reproduce the \( \pi\pi \) scattering data [33]. We extend the Lagrangian (2.1) by adding chiral symmetry breaking term:

\[ \mathcal{L}_{\chi sb} = \frac{e^{-3\sigma} m_\pi^2 f_\pi^2}{2} \text{Tr} (U - 1) \]

(2.2)

to provide the correct asymptotic behavior of the soliton field. Thus our model is given by:

\[ \mathcal{L} = \mathcal{L}_{AN} + \mathcal{L}_{\chi sb} \]

(2.3)

In the static approximation when some parts of the Lagrangian have simple form

\[ \mathcal{L}_2 = \frac{f_\pi^2}{4}[\rho^2 \text{Tr} L_\mu^2 - 2\rho^2], \]

\[ \mathcal{L}_{4a} = \frac{1}{32e^2} \text{Tr} [L_i, L_j]^2, \]

(2.4)

where \( \rho \equiv \exp(-\sigma(r)) \), we may use the most popular hedgehog ansatz: \( U = U_0 = \exp(\sqrt{r} \hat{n}\theta(r)) \), \( \hat{r} = \hat{n}/|\hat{n}| \) and propose spherical symmetry for the scalar field \( \sigma(r) \). Then
the mass functional for classical soliton is given by:

\[
M_H = \frac{8\pi f_\pi}{e} \int_0^\infty dx (\bar{M}_2 + \bar{M}_{4a} + \bar{M}_W + \bar{M}_\pi),
\]
\[
\bar{M}_2 = \rho^2(\theta'^2x^2/2 + s^2)/4 + \rho^2x^2/8,
\]
\[
\bar{M}_{4a} = s^2(d/2 + \theta'^2),
\]
\[
\bar{M}_W = D_{\text{eff}} x^2[\rho^4 - 1 + \frac{4}{\varepsilon}(1 - \rho^\varepsilon)]
\]
\[
\bar{M}_\pi = \rho^3(1-c)x^2\beta^2/4
\]

where \( x = 2ef_\pi r, c \equiv \cos(\theta), s \equiv \sin(\theta), d = (s/x)^2, \beta = m_\pi/(2f_\pi e), D_{\text{eff}} = C_g/384e^2f_\pi^4 \).

The Euler – Lagrange equations for the chiral angle \( \theta(x) \) and scalar meson shape function \( \rho(x) \) follow to be

\[
\theta''[\frac{1}{4}x^2\rho^2 + 2s^2] + \frac{1}{4}[2x^2\theta'\rho' - 2x\theta'^2 - \rho^2s_2] +
\]
\[
s_2(\theta'^2 - d) - \rho^2x^2\beta s/4 = 0,
\]
\[
x^2\rho'/4 - \rho x^2(2\theta'^2/2 + d)/2 + x\rho'/2 - 4D_{\text{eff}}x^2(\rho^3 -
\]
\[-\rho^\varepsilon - 1) - 3\rho^2(1-c)x^2\beta^2/4 = 0,
\]

where \( s_2 \equiv \sin(2\theta) \) and a prime corresponds to the derivative with respect to \( x \). For the asymptotic behavior of \( \theta(x) \) at large distances when \( \rho \to 1 \) we get from (2.6) and (2.7) a familiar formula: \[34\]

\[
\theta(x) \sim \frac{3g_{\pi NN}f_\pi e^2}{2\pi M_N x^2}(1 + \beta x) \exp\left(-\beta x\right) \equiv A\frac{(1 + \beta x)}{x^2} \exp\left(-\beta x\right)
\]

where \( M_N \) is the nucleon mass, \( g_{\pi NN} = 13.5 \) – the pion-nucleon coupling constant. This formula gives connection between \( g_{\pi NN} \) and Skyrme parameter \( e \) and reveals that \( e \) is not a free parameter of Skyrme model in some sense. The behavior of \( \rho(x) \) at large \( x \) exhibits a rapid decrease from the unity:

\[
\rho(x) \sim 1 - \gamma e^{-\tilde{D}x} / x, \quad \tilde{D} = \sqrt{16D_{\text{eff}}(4 - \varepsilon)}
\]

At small distances the behavior of \( \theta(x) \) and \( \rho(x) \) is given by:

\[
\theta(x) \sim \pi - \alpha x
\]
\[
\rho \sim \rho_0 - ax^2
\]

with \( a = [3a^2\rho_0 + 16D_{\text{eff}}(\rho_0^3 - \rho_0^\varepsilon - 1) + 6\rho_0^2\beta^2]/6. \)

Thus Eq.s (2.6) and (2.7) may be solved numerically using well known methods \[35, 36\] and the parameters \( A, \alpha, \gamma \) and \( \rho_0 \) in Eq.s. (2.6), (2.10) may be adjusted to yield continuity in \( \theta, \rho, \theta', \rho' \) at a merging point. In addition, the consequence of the virial theorem:

\[
\int_0^\infty dx[\bar{M}_2 - \bar{M}_{4a} + 3(\bar{M}_W + \bar{M}_\pi)] = 0
\]

may be used to control the accuracy of stable solutions of Eq.s (2.6), (2.7). The resulting solution, \( \theta(r) \), is similar to that one in the original Skyrme model (OSM) \[12\] while \( \rho(r) \) rapidly goes to the unity starting from \( \rho_0 \) at \( r = 0 \).

Masses of nucleon \( M_N \) and \( \Delta - \) isobar - \( M_\Delta \) may be calculated from the following expressions:

\[
M_N = M_H + 3/8\lambda_M
\]
\[
M_\Delta = M_H + 15/8\lambda_M
\]

where \( M_H \) - soliton mass (5), \( \lambda_M \) is momentum inertia of the rotating skyrmion:

\[
\lambda_M = \frac{4\pi}{3\sqrt{3}f_\pi} \int_0^\infty dx x^2 s^2\rho^2/4 + \theta'^2 + d]
\]
2.3 The nucleon – nucleon interaction

We investigate the skyrmion-skyrmion interaction by using the product approximation:

\[
U(\vec{x}, \vec{r}_1, \vec{r}_2, \hat{A}_1, \hat{A}_2) = \hat{A}_1 U_0(\vec{x} - \vec{r}_1) \hat{A}_1^+ \hat{A}_2 U_0(\vec{x} - \vec{r}_2) \hat{A}_2^+ \equiv U_1 U_2
\]

\[
\rho(\vec{x}, \vec{r}_1, \vec{r}_2) = \rho(\vec{x} - \vec{r}_1) \rho(\vec{x} - \vec{r}_2) \equiv \rho_1 \rho_2 \tag{2.14}
\]

where \(U_0(\vec{x} - \vec{r}_i)\) for \(i = 1, 2\) is the hedgehog solution \((U_0(\vec{r}) = \exp(i\vec{r}\vec{\theta}(r)), \vec{r} = \vec{r}/|\vec{r}|)\) located at \(\vec{r}_i\), and \(A_i\) the collective coordinate to describe the rotation. The skyrmion-skyrmion potential is defined by:

\[
V(\vec{r}) = - \int d\vec{x} [\mathcal{L}(U_1 U_2, \rho_1 \rho_2) - \mathcal{L}(U_1, \rho_1) - \mathcal{L}(U_2, \rho_2)]
\]

\[
\tag{2.15}
\]

where \(\vec{r}\) is the relative coordinate between two skyrmions \((\vec{r} = \vec{r}_1 - \vec{r}_2)\). The product approximation has reasonable properties:

a) the baryon number is additive,

b) it gives a correct behavior at the asymptotic region, i.e., the one-pion exchange potential,

c) at \(\vec{r} = 0\) the product form is very close to the hedgehog solution of the baryon number \(B = 2\).

The static NN potential may be obtained by using a standard technique [37, 38, 39, 40] which gives the following expressions for the scalar-isoscalar part of central NN interaction:

\[
V_C(r) = \frac{8\pi f_\pi}{c} \int_0^1 dt \int_0^\infty dx x^2[V_2(x, x_1) + V_{4a}(x, x_1) + V_W(x, x_1) + V_{\chi_{sb}}(x, x_1) + 1 \to 2]
\]

\[
\tag{2.16}
\]

Here the contributions from \(\mathcal{L}_2, \mathcal{L}_{4a}, \mathcal{L}_{\chi_{sb}}\) and \(W_\sigma\) are denoted by \(V_2, V_{4a}, V_{\chi_{sb}}\) and \(V_W\) respectively:

\[
V_2(x, x_1) = \{\rho(x) \phi(x) \rho_1 \phi_1 z_1 + (\rho_1^2 - 1)\} \times[\rho^2(x)(\tilde{\theta}(x) + 3d(x)) + \phi^2(x)]/8
\]

\[
V_W(x, x_1) = D_{\text{eff}}\{[\rho^4(x)(\rho_1^4 - 2) + 1] - 4[\rho^\varepsilon(x)(\rho_1^\varepsilon - 2) + 1]/\varepsilon\}/2
\]

\[
V_{4a}(x, x_1) = \{d_1[3d(x) + 2\tilde{\theta}(x)] + \tilde{\theta}(x)\tilde{\theta}_1(1 - z_1^2)/2\}/3
\]

\[
V_{\chi_{sb}}(x, x_1) = -\rho^3(x)d^2(x)\{\rho_1^3[c(x)c_1 - 1]/2 - c(x) + 1\}/4
\]

where the following notations are introduced: \(\tilde{\theta}(x) = \theta^2(x) - d(x)\), \(\phi(x) = \rho'(x)\), \(x_{1,2} = x^2 + r^2\pm2xr\), \(z_{1,2} = (x \pm \tilde{r})/x_{1,2}\), \(\tilde{r} = 2ref_\pi\), \(r\) is a relative distance between the two nucleons, \(\rho_i \equiv \rho(x_i)\), \(d_i \equiv d(x_i)\), \((i = 1, 2)\) e.t.c.

2.4 Results and discussions.

As a starting point we make an attempt to describe the pion – nucleon coupling constant \(g_{\pi NN}\) and \(N - \Delta\) mass splitting: \(M_{N\Delta} = M_\Delta - M_N\) optimizing the only free parameter of the model - \(C_g\). In fact, the value of \(f_\pi\) should be fixed to its experimental value, and the Skyrme parameter \(c\) must be defined by Eqs. (2.6) - (2.8) with \(g_{\pi NN} = (g_{\pi NN})^{exp} = 13.5\) during the numerical solving procedure of corresponding equations of motion. The typical results could be seen from Fig. 2.1 where dashed and dotted curves display the profile functions...
Figure 2.1: The radial dependence of chiral angle $\theta(r)$ - solid line and shape function $\rho(r) = \exp(-\sigma(r))$ - the dotted lines for the soliton corresponding to the set of parameters $f_\pi = 93\,\text{MeV}$, $e = 4.69$, $C_g = (122.3\,\text{MeV})^4$.

$\theta(r)$ and the shape function $\rho(r) \equiv \exp(-\sigma(r))$ respectively. It is well known [27] that the Lagrangian (2.1) with $C_g = (300\,\text{MeV})^4$ gives a large value for the soliton mass. Our precise analysis [29] show that the gluon condensate should be reduced up to the $C_g = (122.3\,\text{MeV})^4$ to reproduce $g_{\pi NN}$ and the masses $(M_\Delta - M_N = 293\,\text{MeV}, M_N = 954.5\,\text{MeV})$ [29].

In this case the calculated value of axial coupling constant $g_A$, which is given by the following expression:

$$g_A = -\frac{\pi}{3e^2} \int_0^\infty dx \, x^2 \{\rho^2(\theta' + s_2/x) + 4[s_2(\theta'^2 + d)/x + 2\theta'd]\}$$ (2.18)

is equal to 1.01 i.e. much better then that in OSM [12] ($g_A^{OSM} = 0.61$). It is clear from Eq. (2.18) that $g_A$ is very sensitive to the Skyrme parameter $e$ which is connected to $g_{\pi NN}$ through the equations (2.6) - (2.8), since Goldberger Treiman relation does not work due to $\mathcal{L}_{\chi_{sb}}$ term.

Thus by using the following optimal set of parameters: $f_\pi = 93\,\text{MeV}$, $e = 4.69$, $C_g = (122.3\,\text{MeV})^4$ in Eqs. (2.16),(2.17) we calculated the scalar - isoscalar part of central nucleon - nucleon potential - the solid line in Fig. 2.2. It may be compared with corresponding
interaction component of the realistic phenomenological "Paris" potential [41], displayed there with the dashed line. It is seen from the figure that the attraction is shifted to larger separations and much stronger. The similar result was obtained in ref. [42] within the framework of realistic pseudoscalar - vector chiral Lagrangian where the \( \sigma \) - meson exchange was simulated by a correlated two-pion exchange between the two solitons.

To discuss this effect in a more detail we bring in figures Fig.2.3 (a) and Fig.2.3(b) the net contributions from each term. It is well known that in the OSM the second order derivative term \( \mathcal{L}_2 \) does not contribute to the central potential at all \( \rho(x) = 1 \) in Eqs. (2.16), (2.17), whereas the contribution from \( \mathcal{L}_{4a} \) is strongly repulsive (see Fig.2.3(b)). After the inclusion of scalar dilaton this repulsion may be compensated by an attractive contribution arising from the kinetic term \( \mathcal{L}_2 \) [see Fig.2.3(a)] in the region \( r \geq 1.5 \text{fm} \). As to the contributions from \( \mathcal{L}_{\chi_{sb}} \) and \( W_\sigma \) terms they are not large, especially, at short and medium distances [see Fig.2.3(a) and Fig.2.3(b)].

Note that the relevance of product approximation in the short range region of the interaction may be doubtful. However, the considering attraction appears at separations greater than \( 1.2 \text{fm} \) where the approximation is valid.
Figure 2.3: The net contributions to the central NN potential given by the Eqs. (2.16), (2.17). The $V_2$, $V_{sb}$, $V_4$, and $V_W$ curves are the contributions from $\mathcal{L}_2$, $\mathcal{L}_{\chi_{sb}}$, $\mathcal{L}_{4a}$ and $W_\sigma$ terms respectively.

We conclude that the model we have presented here leads to a correct values of $f_\pi$, $g_{\pi NN}$, $M_{N\Delta}$ and gives a desired attraction in the central part of $NN$ interactions. On the other side, by comparing this result with a result of Yabu et al. [25] where no attempt had been made to optimize the parameters we may affirm that the attraction appears in a natural way in both cases regardless the origin of dilaton. One may expect that the inclusion of $\omega$–meson coupling term to the Lagrangian [43] and taking into account of deformation effects [44, 45, 46] will give a better description of $NN$ and $N\bar{N}$ interactions. That will be the subject of the next chapter.
Chapter 3

The Deformation of the Interacting Nucleon in the Skyrme Model. ¹

3.1 Introduction

The possible changes in the radius of a nucleon in interaction with another nucleon was investigated in the skyrmion model by Kalbermann et. al. [49]. It was found that there is a swelling of the nucleon at intermediate distances between the nucleons which were assumed to preserve their spherical shape. On the other hand Hajduk and Schwesinger [50] considering the skyrmion to be soft for the deformation showed that there are several deformed states of rotating skyrmions. In particular, besides the ground state of a spherical shape there exist a degenerate doublet exotic states with the same quantum numbers of the nucleon $(s = t = 1/2)$ of oblate and prolate shapes. One may therefore wonder whether the nucleon can change its shape under the action of strong interactions.

Certainly, several studies have been made of the deformation effects on the Skyrmions [44, 45, 46], but these were carried out to obtain a better understanding of the possible sources of attraction in the central nucleon - nucleon potential. As a result it was shown that the deformation effect is very important and may reduce the central repulsion by about 40%.

In the previous chapter the problem of missing central attraction in the $NN$ interaction has been investigated within the model of Andrianov and Novozhilov [27] which starts with the Skyrme Lagrangian supplemented by a dilaton scalar field $\sigma(r)$ so as to satisfy the QCD trace anomaly constraint. It has been concluded that this model gives the desired attraction in the central part of the $NN$ interaction. The resulting central scalar - isoscalar part of the potential is in qualitative agreement with the phenomenological one.

In the present chapter we shall concentrate on the effects of modification of the shape of a nucleon at a quantitative level using skyrmions. We shall use the same Lagrangian (2.1)-(2.4) [27, 51] which has been used in the previous chapter.

3.2 The interaction of deformed nucleons

As it was shown in the previous chapter, the Lagrangian $\mathcal{L}_{AN}(U, \sigma)$ is a generalization of the well known original Skyrme model [12, 13] and takes into account the conformal anomaly of the QCD.

We have also shown that the Lagrangian produces in a natural and transparent way the intermediate - range attraction in the central part of baryon - baryon interaction even in the

¹The present chapter is based on following articles by the author and his collaborators: [47, 48]
product approximation [30]. Here we let the interacting nucleons deform. Assume that both skyrmions deform into an ellipsoidal shape as they come close together. To describe this we write the chiral field \( U \) and the dilaton field \( \sigma \) as the non-spherical hedgehog form given by:

\[
U_0(\vec{r}) = \exp \left( i \vec{q} \theta (q) \right) \quad \sigma(\vec{r}) = \sigma(q) \tag{3.1}
\]

where the spatial vector \( \vec{q} \) has the components \( \delta_x, \delta_y, \delta_z \) and \( \hat{q} \) is the unit vector: \( \hat{q} = \vec{q}/q \) with the deformation parameters \( \delta_x \) and \( \delta_z \). The profile functions \( \theta \) and \( \sigma \) are assumed to be the solutions of the Euler - Lagrange equations in the spherical case given in (2.6). The ansatz, in Eq. (3.1), leads to a modification of the static mass, \( M'_H \), and the moment of inertia, \( \lambda_M^* \), of the Skyrmion

\[
M_H' = [AM_2 + BM_{4a} + M_{\chi^sb} + M_W]/\eta \\
\lambda_M^* = [\lambda_2 + A\lambda_{4a}]/\eta \tag{3.2}
\]

where \( \eta = \delta_z^2 \delta_z \), \( A = (2\delta_x^2 + \delta_z^2)/3 \), \( B = \delta_x^2(\delta_x^2 + 2\delta_z^2)/3 \) and \( M_i \) and \( \lambda_i \) denote the relevant contributions from \( \mathcal{L}_i \) term in Eq.(2.1) for the spherical case \( \delta_x = \delta_z = 1 \). As we are mainly interested in the region where the medium range attraction takes place - \( R \sim 1.25fm \) (typical separation between nucleons in nuclei) we restrict ourselves to the familiar product ansatz:

\[
U = \hat{A}_1 U_0(\vec{X} - \vec{q}/2)\hat{A}_2 U_0(\vec{X} + \vec{q}/2)\hat{A}_2^+ U_0(\vec{X} - \vec{q}/2)\hat{A}_1^+ U_0(\vec{X} + \vec{q}/2) \equiv U_1 U_2 \\
\rho = \rho(\vec{X} - \vec{q}/2)\rho(\vec{X} + \vec{q}/2) \equiv \rho_1 \rho_2 , \ \rho \equiv \exp(-\sigma) \tag{3.3}
\]

where \( \hat{A}_1, \hat{A}_2 \) are the collective coordinates of skyrmions to describe their rotational motion, \( \vec{q} \) is the vector along z axis: \( q_x = 0, q_y = 0, q_z = q = R\delta_z \) and \( R \) is the distance between skyrmions. The static skyrmion - skyrmion potential is defined by

\[
V(\vec{R}, \delta_x, \delta_z) = - \int d\vec{X} [\mathcal{L}(U_1 U_2, \rho_1 \rho_2) - \mathcal{L}(U_1, \rho_1) - \mathcal{L}(U_2, \rho_2)] \tag{3.4}
\]

The application of the usual projection methods developed in [44, 45, 46, 52] to Eqs (3.1) - (3.4) yields the following representation for the central scalar - isoscalar part of the nucleon - nucleon interaction :

\[
V_c(R, \delta_x, \delta_z) = \frac{1}{\eta} \left[ V_{\chi^sb}(q) + V_W(q) + \delta_x^2(V_2(q) + \delta_z^2 V_{4a}(q)) + (\delta_z^2 - \delta_x^2)(V_2^{\text{def}}(q) + \delta_z^2 V_{4a}^{\text{def}}(q)) \right] \tag{3.5}
\]

where the terms \( V_{2a}^{\text{def}} \) and \( V_{4a}^{\text{def}} \) are the net contributions from the deformation effect of the terms \( \mathcal{L}_2 \) and \( \mathcal{L}_4a \) in Eqs (2.1)-(2.3) respectively. They are given by

\[
V_2^{\text{def}}(q) = \frac{(\pi f_\pi)}{e} \int_0^\infty dx \times \times \times dt [\rho_1^2(\rho_2^2 - 1)(\beta_1 + Y_1 t_1^2) + \phi_1^2 t_1^2(\rho_2^2 - 1) + + \rho_1 \rho_2 \phi_1 \phi_2 + (1 \leftrightarrow 2)], \tag{3.6}
\]

\[
V_4^{\text{def}}(q) = \frac{(8\pi f_\pi)}{3e} \int_0^\infty dx \times \times \times dt [(\gamma_1 x_1^2 + 3\beta_1)(Y_2 t_2^2 + \beta_2) - Y_1 Y_2 Z Z_3 - - \beta_1 \beta_2 - 2 \beta_2 Y_1 t_2^2 + (1 \leftrightarrow 2)].
\]
with
\[ \rho(x) = \exp(-\sigma(x)), \quad \phi(x) = d\rho/dx, \]
\[ \beta(x) = (\sin \theta(x)/x)^2, \quad Y(x) = [\theta^2(x) - \beta(x)]/x^2, \]
\[ Z = x^2 - q^2/4, \quad Z_3 = x^2t^2 - q^2/4, \quad q = \delta_x r_f \rho, \]
\[ t_{1,2}^2 = q^2/4 + x^2t^2 \pm xt, \quad x_{1,2}^2 = t_{1,2}^2 + x^2(1 - t^2), \]
\[ \rho_i \equiv \rho(x_i), \quad \phi_i \equiv \phi(x_i), \quad Y_i \equiv Y(x_i), \quad \beta_i \equiv \beta(x_i). \]

Other terms in Eq. (3.5) don’t include deformation effects explicitly. They are brought in previous chapter. We shall calculate the deformation parameters \( \delta_x(r) \) and \( \delta_z(r) \) by

Figure 3.4: Central isospin - independent potential in Eq.(3.5) calculated for the cases with the dilaton field (solid curve) and without one (dashed curve) as a function of the internucleon distance \( R \). The dotted curve is the corresponding interaction component in the "Paris" potential [41].

minimizing the total static energy of two nucleon system at each separation \( R \) using Eq.s (3.1) - (3.5).

The resulting values of the parameters will be used to study the changes in the shape of the nucleon. Now we illustrate this procedure for the case of the isoscalar mean square radius and the appropriate intrinsic quadrupole moment. The normalized isoscalar mean
square radius along each axis may be defined by

$$\langle r_i^2 \rangle_{I=0}^* = \frac{\int d\vec{r} r_i^2 B_0(\vec{r})}{\int d\vec{r} B_0(\vec{r})}$$

(3.8)

where \(i = x, y, z\) and \(B_0(\vec{r})\) is the baryon charge distribution

$$B_0(\vec{r}) = \frac{1}{24\pi^2} \epsilon^{ijk} \text{Tr} [L_i L_j L_k].$$

(3.9)

The inclusion of the deformation in a simple way: \(r_i \rightarrow q_i/\delta_i\) as in Eq. (3.1) yields the following relation between the radius of a free spherical nucleon \(\langle r^2 \rangle_{I=0}\) and a deformed one: \(\langle r_i^2 \rangle_{I=0}^* = \frac{1}{\delta_i^2} \langle r_i^2 \rangle_{I=0}\). Therefore the appropriate quadrupole moment characterizing the shape of the distribution of baryon matter is compared to that of an ellipsoid with axis \(1/\delta_i\) and \(1/\delta_z\):

$$Q_{I=0} = 3 \langle r_z^2 \rangle_{I=0}^* - \langle r_z^2 \rangle_{I=0} = 2 \langle r_z^2 \rangle_{I=0} (1/\delta_z^2 - 1/\delta_z^2).$$

(3.10)

The explicit formulas for \(Q_{I=1}\) defined by \(Q_{I=1} = 3 \langle r_z^2 \rangle_{I=1}^* - \langle r_z^2 \rangle_{I=1}^*\) are rather complicated and may be found elsewhere [48].

### 3.3 Results and discussions

In the numerical calculations we consider the following two cases: the Lagrangian with the dilaton and the pure Skyrme model when \(\sigma = 0\) in Eq. (2.1). In both cases the parameters \(f_\pi\), \(e\) and \(m_\pi\) were fixed at the values: \(f_\pi = 93\text{MeV}, e = 2\pi, m_\pi = 139\text{MeV}\). For the gluon condensate we use \(C_g = (283\text{MeV})^4\) as obtained from lattice QCD calculations [53]. The mass of the scalar meson, \(m_\sigma\), defined by \(m_\sigma = \sqrt{2C_g/2f_\pi}\) is then 610\text{MeV}.

This set of parameters produce the following static properties of the nucleon: \(M_N = 1054\text{MeV}, g_A = 0.65, \langle r^2 \rangle_{I=0}^{1/2} = 0.38\text{fm} \) and \(\langle r^2 \rangle_{I=1}^{1/2} = 0.66\text{fm}\) in the dilaton case. No attempt is made here to search for a realistic set of parameters since our interest was mainly in the link between properties of \(NN\) interaction and the shape of the nucleon.

In Fig. 3.4 the central scalar - isoscalar part of the nucleon - nucleon interaction has been presented for both cases including deformation effects. For a comparison of the corresponding interaction component with the realistic phenomenological "Paris" potential [41] is also displayed here ( the dotted line in Fig. 3.4). It is clear that the Lagrangian with the dilaton field is able to describe the nucleon - nucleon interaction in the intermediate region quite well.

We now turn to the changes in the shapes of interacting nucleons. The intrinsic quadrupole moments \(Q_{I=0}\) and \(Q_{I=1}\) are shown in Fig.3.5 and Fig.3.6 respectively. In the pure Skyrme model when there is no attraction \([V^2 = V^2_{\text{def}} = 0\) in Eq.(3.5)] between skyrmions it becomes oblate (dashed lines in the figures 3.5, 3.6) due to the strong repulsion caused by the \(V^4\) terms in Eq. (3.5). The inclusion of the dilaton leads to the following qualitative picture:

At large separations skyrmion is obviously in a spherical shape, becomes prolate at the intermediate region and deforms to an oblate shape at small distances where the repulsion dominates.

As the nucleons approach each other they change shapes from prolate into oblate at \(R \sim 1.2\text{fm}\). Comparing figures Fig.3.5 and Fig.3.6 it may be noticed that the isoscalar intrinsic quadrupole moment \(Q_{I=0}\) is much smaller than the isovector one \(Q_{I=1}\) at intermediate separations. The intrinsic quadrupole moment of the proton defined by:

$$Q_x = (Q_{I=0} + Q_{I=1})/2$$

(3.11)
reaches a maximum value of $Q_p = 0.016 \text{fm}^2$ at $r \sim 1.5 \text{fm}$. The corresponding eccentricity $\varepsilon$ (see Fig. 3.7) reaches the maximum value $\varepsilon \approx 0.02$. Note that for deuteron $\varepsilon_D \approx 0.12$, due to the small mixing of $D$-state. In this sense, one may conclude that the shape of a nucleon in the nuclei is "more spherical" than the deuteron. We expect new data from high-energy electron scattering on nuclei to make the situation clear.

As a concluding remark we have to underline that the deformed states of oblate (prolate) shapes may not necessarily belong to the $K = 1$ band found in ref. [50] since for a strongly deformed system the quantization procedure used here needs some modifications. It is a generalization of the well-known original Skyrme model [12, 13] and takes into account the conformal anomaly of QCD.
Figure 3.6: The same as in Fig. 3.5 but for the isovector quadrupole moment $Q_{I=1}$. 
Figure 3.7: The excentricity of the ellipsoidally deformed nucleon defined as \( \varepsilon = \frac{(a^2 - b^2)}{(a^2 + b^2)} \), where a and b are the tranverse and conjugate axes of the ellipse.
Chapter 4

The nucleon–nucleon interaction and properties of the nucleon in a $\pi\rho\omega$ soliton model including a dilaton field with anomalous dimension

4.1 Introduction

Recently Furnstahl, Tang and Serot (FTS) [55] have proposed a new model for nuclear matter and finite nuclei that realizes QCD symmetries such as chiral symmetry, broken scale invariance and the phenomenology of vector meson dominance. An important feature of this approach is the inclusion of light scalar degrees of freedom, which are given an anomalous scale dimension. The vacuum dynamics of QCD is constrained by the trace anomaly and related low–energy theorems of QCD. The scalar–isoscalar sector of the theory is divided into a low mass part that is adequately described by a scalar meson (quarkonium) with anomalous dimension and a high mass part (gluonium), that can be “integrated out”, leading to various couplings among the remaining fields. The application of the model to the properties of nuclear matter as well as finite nuclei gave a satisfactory description. Further developments of the model [56, 57] showed that the light scalar related to the trace anomaly can play a significant role not only in the description of bound nucleons but also in the description of heavy–ion collisions. It was also shown that the anomalous cannot be due to an effect of nuclear density on the trace anomaly of QCD [56].

Here a natural question arises: What is the role of this light quarkonium in the description of the properties of a single nucleon, when it is taken into account in topological nonlinear chiral soliton models, which are similar to the FTS effective Lagrangian on the single nucleon level? In the present Chapter we introduce a dilaton field with an anomalous dimension into the $\pi\rho\omega$–model proposed in ref. [58] and investigate some properties of single nucleon which emerges as a soliton in the sector with baryon number one ($B = 1$). Note that, analysis of pion - nucleon scattering phase shifts made in refs. [59, 60] indicate the existence of the light scalar meson.

It is well known that a scalar–isoscalar meson, the sigma, plays an important role in the nucleon–nucleon (NN) interaction especially within one–boson–exchange (OBE) models [61, 62]. Note also that, the missing medium range attraction was a long standing puzzle in Skyrme like models. In Chapter 2 we have shown that explicit inclusion of a scalar meson into the Skyrme model produces in a natural way the desired attraction. However, OBE model includes vector mesons also. Therefore, it would be quite interesting to investigate the central part of the NN interaction when the light scalar ($\sigma$), $\rho$ and $\omega$–mesons are taken into account explicitly. This is what will be considered in present Chapter. In particular, it is important that when one is to properly describe the intermediate range attraction in the

\footnote{The present chapter is based on following articles by the author and his collaborators: [29, 54]}
central NN interaction, the successful description of the single nucleon properties within the \( \pi\rho\omega \) model should not be destroyed.

We also note that in an soliton approach with explicit regulated two–pion loop graphs one is able to get the proper intermediate range attraction. In that case, however, one does not stay within a simple OBE approach any more (as done here) and also needs to calculate the modifications of the isovector two–pion exchange to the \( \rho \) and so on. For comparison, we mention that recent developments of the original Bonn OBE potential performed at Jülich also include multi–meson exchanges leading to a renormalization of various interactions, couplings and cut–off parameters [63, 64]. The model we investigate is related closely to the OBE approximation of the NN force.

### 4.2 The \( \pi\rho\omega\sigma \) model

Including a \( \sigma \)–meson by means of the scale invariance and trace anomaly of QCD into the \( \pi\rho\omega \)–model [65] can be done in terms of the following chiral Lagrangian of the coupled \( \pi\rho\omega\sigma \) system,

\[
\mathcal{L} = S_0^2 e^{-2\sigma/d} \frac{2}{2} \partial_\mu \sigma \partial^\mu \sigma - f_\pi^2 e^{-2\sigma/d} \frac{4}{4} \text{Tr} L_\mu L^\mu - 2 \text{Tr} \left[ l_\mu + r_\mu + ig\vec{\tau}\vec{\rho}_\mu + +ig\omega_\mu \right] ^2 + \frac{3}{2} g_\mu \omega_\mu - \frac{1}{4} \left( \omega_\mu \omega_\nu + \vec{\rho}_\mu \vec{\rho}_\nu \right) + \frac{f_\omega^2 m_\pi^2 e^{-3\sigma/d}}{2} \text{Tr} \left( U - 1 \right) - \frac{d^2 S_0^2 m_\sigma^2}{16} \left[ 1 - e^{-4\sigma/d} \left( \frac{4\sigma}{d} + 1 \right) \right],
\]

where the pion fields are parametrized in terms of \( U = \exp \left( i\vec{r} \cdot \vec{\pi} / f_\pi \right) \) and \( \xi = \sqrt{U} \), left/right–handed currents are given by \( L_\mu = U^+ \partial_\mu U \), \( l_\mu = \xi^+ \partial_\mu \xi \), \( r_\mu = \xi \partial_\mu \xi^+ \), and the pertinent vector meson \( (\vec{\rho}, \omega) \) field strength tensors are \( \vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu + g \left[ \vec{\rho}_\mu \times \vec{\rho}_\nu \right] \) and \( \omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \). Furthermore, the topological baryon number current is given by \( B^\mu = c^\mu \omega \gamma_\mu \text{Tr} L^\alpha L^\beta L_\gamma / (24\pi^2) \).

In Eq.(4.1), \( S_0 \) is the vacuum expectation value of scalar field in free space matter, \( f_\pi \) is the pion decay constant \( (f_\pi = 93 \text{ MeV}) \) and \( g = g_{\rho\pi\pi} \) is determined through the KSFR relation \( g = m / \sqrt{2} f_\pi \). The model assumes the masses of \( \rho \) and \( \omega \) mesons to be equal, \( m_\rho = m_\omega = m \). The mass of the \( \sigma \) is related to the gluon condensate in the usual way \( [55, 66, 67] \) \( m_\sigma = 2 \sqrt{C G / (dS_0) \text{, where } d \text{ is the scale dimension of scalar field } (d > 1). \text{ Being “mapped” onto the states of a nucleon, the Lagrangian Eq.(4.1) will be similar to the FTS effective Lagrangian.} \)

Nucleons arise as soliton solutions from the Lagrangian Eq.(4.1) in the sector with baryon number \( B = 1 \) as it has been explained in Chapter I. To construct them one goes through a two step procedure. First, one finds the classical soliton which has neither good spin nor good isospin. Then an adiabatic rotation of the soliton is performed and it is quantized collectively. The classical soliton follows from Eq.(4.1) by virtue of a spherical symmetrical ansätze for the meson fields:

\[
U(r) = \exp \left( i\vec{r} \theta(r) \right), \quad \rho^a_i = \varepsilon_{iak} \frac{G(r)}{gr}, \quad \omega_\mu(r) = \omega(r) \delta_\mu^0, \quad \sigma(r) = \sigma(r). \tag{4.2}
\]

In what follows we call \( \theta(r), G(r), \omega(r), \text{ and } \sigma(r) \) the pion–, \( \rho_–, \omega_, \text{ and } \sigma_– \)meson profile functions, respectively. The pertinent boundary conditions to ensure baryon number one and

\[^2\text{We refer to further details of [54, 55] for details.}\]
finite energy are, \( \theta(0) = \pi, G(0) = -2, \omega'(0) = \sigma'(0) = 0, \theta(\infty) = G(\infty) = \omega(\infty) = \sigma(\infty) = 0 \). To project out baryonic states of good spin and isospin, we perform a time-independent SU(2) rotation

\[
U(\vec{r}, t) = A(t)U(\vec{r})A(t), \quad \xi(\vec{r}, t) = A(t)\xi(\vec{r})A(t)
\]

\[
\sigma(\vec{r}, t) = \sigma(r), \quad \omega(\vec{r}, t) = -\frac{\dot{\phi}(r)}{r}K \vec{r}
\]

\[
\vec{r} \cdot \vec{p}_0(\vec{r}, t) = \frac{2}{q}A(t)\vec{r} \cdot (K \xi_1(r) + \vec{r} K \cdot \vec{r} \xi_2(r))A(t),
\]

\[
\vec{r} \cdot \vec{p}_i(\vec{r}, t) = A(t)\vec{r} \cdot \vec{p}_i(\vec{r})A(t)
\]

with \( 2K \) the angular frequency of the spinning mode of soliton, \( i\vec{r} \cdot K = A^+ \dot{A} \). This leads to the time-dependent Lagrange function

\[
\mathcal{L}(t) = \int d\vec{r} \mathcal{L} = -M_H(\theta, G, \omega, \sigma) + \Lambda(\theta, G, \omega, \sigma, \phi, \xi_1, \xi_2) \text{Tr} (\dot{A} \dot{A}^+).
\]

Minimizing the classical mass \( M_H(\theta, G, \omega, \sigma) \) leads to the coupled differential equations for \( \theta, G, \omega \) and \( \sigma \) subject to the aforementioned boundary conditions. In the spirit of the large \( N_c \)-expansion, one then extremizes the moment of inertia \( \Lambda(\theta, G, \omega, \sigma, \phi, \xi_1, \xi_2) \) which gives the coupled differential equations for \( \xi_1, \xi_2 \) and \( \phi \) in the presence of the background profiles \( \theta, G, \omega \) and \( \sigma \). The pertinent boundary conditions are \( \phi(0) = \phi(\infty) = 0 \), \( \xi'_1(0) = \xi_1(\infty) = 0 \), \( \xi'_2(0) = \xi_2(\infty) = 0 \), \( 2\xi_1(0) + \xi_2(0) = 2 \). The masses of nucleon \( M_N \) and the mass of \( \Delta , M_\Delta \), are then given by \( M_N = M_H + 3/8\Lambda \) and \( M_\Delta = M_H + 3/15\Lambda \).

The electromagnetic form factors obtained in the usual way \([65]\) are:

\[
G_E^S(\vec{q}^2) = -\frac{4\pi m^2}{3g} \int_0^\infty j_0(qr)\omega(r)e^{-2\sigma/d}r^2 dr,
\]

\[
G_M^S(\vec{q}^2) = -\frac{2\pi M_Nm^2}{3g\Lambda} \int_0^\infty \frac{j_1(qr)}{qr} \phi(r)e^{-2\sigma/d}r^2 dr,
\]

\[
G_E^V(\vec{q}^2) = \frac{4\pi}{\Lambda} \int_0^\infty j_0(qr) \left\{ \frac{f^2}{3} [4s^4 + (1 + 2c)\xi_1 + \xi_2]e^{-2\sigma/d} + \frac{g\phi\theta\\s^2}{8\pi^2r^2} \right\} r^2 dr,
\]

\[
G_M^V(\vec{q}^2) = \frac{8\pi M_N}{3} \int_0^\infty \frac{j_1(qr)}{qr} \left\{ 2f^2[2s^2 - Gc]e^{-2\sigma/d} + \frac{3g}{8\pi^2\omega\theta\\s^2} \right\} r^2 dr,
\]

where \( s = \sin(\theta), c = \cos(\theta) \) and \( s_2 = \sin(\theta/2) \). The normalization is \( G_E^S(0) = G_E^V(0) = 1/2 \). Similarly, meson–nucleon vertex form factors may be calculated \([68, 69]\). In the Breit frame
one has:

\[
G_\pi(-q^2) = \frac{8\pi M_N f_\pi}{3q} (q^2 + m_\pi^2) \int_0^\infty j_1(qr) \sin(\theta) r^2 dr = \frac{8\pi M_N f_\pi}{3} \int_0^\infty \frac{j_1(qr)}{qr} \left[ -2\theta' c - \theta'' r c + \theta'^2 r s + \frac{2s}{r} + r m_\pi^2 s \right] r^2 dr,
\]

\[
G_E^p(-q^2) = \frac{2\pi}{g_\Lambda} \int_0^\infty j_0(qr) \left[ -\xi' - \frac{2\xi'}{r} + m^2 \xi - \frac{2\xi}{3r} + \frac{m^2 \xi}{3} \right] r^2 dr,
\]

\[
G_E^M(-q^2) = \frac{8\pi M_N}{3g} \int_0^\infty \frac{j_1(qr)}{qr} \left[ -G'' + 2G/r^2 + m^2 G \right] r^2 dr,
\]

\[
G_M^p(-q^2) = \frac{2\pi M_N}{\Lambda} \int_0^\infty \frac{j_1(qr)}{qr} \left[ \phi'' - \frac{2\phi}{r^2} - m^2 \phi \right] r^2 dr,
\]

where the “electric” and “magnetic” vector meson–nucleon form factors are connected to the Dirac \( F_1(t) \) and Pauli form factors \( F_2(t) \) through the following relations:

\[
G_E^i(t) = F_1^i(t) + t F_2^i(t)/4M_N^2,
\]

\[
G_M^i(t) = F_1^i(t) + F_2^i(t), \quad (i = \rho, \omega).
\]

### 4.3 Results and discussions.

#### 4.3.1 Static and electromagnetic properties of the nucleon.

Using the formulas given above we have calculated static and electromagnetic properties of nucleon. As can be seen from Eq.(4.1), the Lagrangian has no free parameters in the \( \pi \rho \omega \) sector. So, in actual calculations the parameters \( m_\pi, m, f_\pi \) are fixed at their empirical values, \( m_\pi = 138 \text{ MeV}, m = m_\rho = m_\omega = 770 \text{ MeV}, f_\pi = 93 \text{ MeV}, g = m/\sqrt{2}f_\pi = 5.85 \). In the \( \sigma \)–meson sector there are in general three free parameters: \( m_\sigma, S_0 \) and the scale dimension \( d \). The latter has been well studied for nuclear matter calculations \([55, 56]\). In particular, it was shown that for \( d \geq 2 \) the much debated Brown–Rho (BR) scaling may be recovered. Therefore, assuming that there is no dependence of \( d \) on the density, we shall use the best value \( d = 2.6 \) found in refs. \([55, 56]\). The values for \( S_0 \) were found to be \( S_0 = 90.6 \pm 95.6 \text{ MeV} \) \([55]\). So we put \( S_0 = f_\pi = 93 \text{ MeV} \). The mass of the \( \sigma \) or equivalently the gluon condensate \( C_g = m_\sigma^2 d d_0^2 / 4 = m_\sigma^2 d^2 f_\pi^2 / 4 \) is uncertain. We thus consider two cases: \( m_\sigma = 550 \text{ MeV} \) and \( m_\sigma = 720 \text{ MeV} \) in accordance with recent \( \pi \pi \) phase shift analyses \([59, 60]\) and with OBE values. We stress again that the precise nature of such a scalar–isoscalar field is not relevant here, only that it should not be a pure gluonium state. A summary of static nucleon properties obtained in both cases, i.e. with \( m_\sigma = 550 \text{ MeV} \) and \( m_\sigma = 720 \text{ MeV} \), is given in Table 4.2.

One immediately observes that the nucleon mass is again overestimated. This may not be regarded as a deficiency, since it is known that quantum fluctuations tend to decrease the mass substantially.
Table 4.2: Baryon properties in the $\pi\rho\omega$ and $\pi\rho\omega\sigma$ models

|                  | $\pi\rho\omega$ | $\pi\rho\omega\sigma$ | $\pi\rho\omega\sigma$ | Exp. |
|------------------|------------------|------------------------|------------------------|------|
| $m_\sigma$ [MeV] | $-$              | 550                    | 720                    |      |
| $C_g^{1/4}$ [MeV]| $-$              | 258                    | 295                    | 300±400 |
| $B^{1/4}$ [MeV]  | $-$              | 119                    | 121                    |      |
| $M_N$ [MeV]      | 1560             | 1492                   | 1511                   | 939  |
| $\Lambda$ [fm]   | 0.88             | 0.88                   | 0.88                   | -    |
| $M_\Delta - M_N$ [MeV] | 344           | 350                    | 350                    | 293  |
| $r_H = \langle r_B^2 \rangle^{1/2}$ [fm] | 0.5             | 0.5                    | 0.5                    | ~0.5 |
| $\langle r_E^2 \rangle^{1/2}$ [fm] | 0.92           | 0.94                   | 0.94                   | 0.86±0.01 |
| $\langle r_E^2 \rangle$ [fm] | -0.20          | -0.16                  | -0.16                  | -0.119±0.004 |
| $\langle r_M^2 \rangle^{1/2}$ [fm] | 0.84           | 0.85                   | 0.85                   | 0.86±0.06 |
| $\langle r_M^2 \rangle$ [fm] | 0.85           | 0.85                   | 0.85                   | 0.88±0.07 |
| $\mu_p$ [n.m.]  | 3.34             | 3.33                   | 3.33                   | 2.79 |
| $\mu_n$ [n.m.]  | -2.58            | -2.53                  | -2.53                  | -1.91 |
| $|\mu_p/\mu_n|$  | 1.29             | 1.30                   | 1.30                   | 1.46 |
| $g_A$            | 0.88             | 0.95                   | 0.95                   | 1.26±0.006 |
| $\langle r_A^2 \rangle^{1/2}$ [fm] | 0.63           | 0.66                   | 0.66                   | 0.65±0.07 |

To estimate the influence of the $\sigma$–meson we also show the results given by minimal version of $\pi\rho\omega$ model. As can be seen from Table 4.2, the inclusion of a light sigma meson into the basic $\pi\rho\omega$ model just slightly changes the nucleon mass and its electromagnetic properties. This may be explained by the fact that the role of intermediate scalar–isoscalar meson in gamma–nucleon interactions is negligible. In contrast, the presence of the sigma–meson leads to an enhancement of axial coupling constant as it was first observed in ref. [27]. Although, the physical mechanism of this change is not clear, it may be understood as mainly due to modification of meson profiles. This is in marked contrast to the inclusion of pion loop effects, which tend to lower the axial coupling even further [70]. In the present model $g_A = 0.88$ for the $\pi\rho\omega$ and $g_A = 0.95$ for the $\pi\rho\omega\sigma$ model, respectively. One may expect that an appropriate inclusion of $\sigma$ meson into a more complete version of the basic $\pi\rho\omega$ model might give the desired value $g_A = 1.26$.

4.3. 2 Meson–nucleon form factors.

One of the usual ways to calculate the meson–nucleon interaction potential within topological soliton models is the so–called product ansatz [25]. Within this approximation, the two–Skyrmion potential as a function of the relative angles of orientation between the Skyrmions has a compact form, and the extraction of the NN potential by projection onto asymptotic two–nucleon states is straightforward. This procedure gives only three nonvanishing channels: the central, spin–spin and tensor potential. At large and intermediate distances, the latter two compare well with e.g. the phenomenological Paris potential [41]. The major inadequacy found in such type of calculations is the lack of an intermediate range attraction in central potential. Although many remedies have been proposed, this result may not be genuine for Skyrme like models. In fact, the product ansatz, which is not a solution to the equations of motion, can only be considered accurate at large distances, and the failure of these calculations to reproduce the central range attraction may simply be
the failure of the product approximation to provide an adequate approximation to the exact solution. Although the lack of central attraction may be recovered by the inclusion of a scalar–isoscalar meson, the inherent ansatz dependence of the trial configuration remains as a major shortcoming of product approximation [52].

On the other hand there is another natural way which was first used by Holzwarth and Machleidt [72]. They proposed to calculate $V_{NN}$ within OBE model taking coupling constants and meson–nucleon form factors from a microscopical model such as the Cloudy Bag model or the Skyrme model. It was shown that the Skyrme form factor is a soft pion form factor that is compatible with the $\pi N$ and $NN$ systems. We shall use this strategy to investigate the $NN$ potential within present model.

The meson nucleon form factors given by well known procedure, proposed first by Cohen [69] are given in Eq.(4.6). Although they were derived in a microscopical and consequent way, these form factors could not be directly used in standard OBE schemes. The reason is that the OBE schemes [62] in momentum space use form factors defined for fields propagating on a flat metric, whereas the definition of form factors in Eq.(4.6) involve a nontrivial metric. Hence, before using the latters in OBE scheme one should modify the procedure by redifining meson fields. The modification for pion nucleon form factors in $\pi\rho\omega$ model is clearly outlined in refs. [72, 73]. Now, applying this procedure in the Lagrangian in Eq. (4.1) we get the following $\pi NN$ form factor:

$$G^\pi(-q^2) = \frac{8\pi M_N f_\pi}{3q^2}(q^2 + m_\pi^2) \int_0^\infty \frac{j_1(qr)}{r^2} \sqrt{M_T(r)} \sin(\theta) r^2 dr =$$

$$= \frac{8\pi M_N f_\pi}{3} \int_0^\infty \frac{j_1(qr)}{qr} [-2F'(r) - rF''(r) + \frac{2F(r)}{r} + rm_\pi^2 F(r)] r^2 dr,$$

where

$$M_T = [1 + 2 \tan^2(\theta/2)] e^{-2\sigma/d},$$

$$F(r) = \sqrt{3 + \cos^2(\theta(r))} - 4 \cos(\theta(r)) e^{-\sigma/d}. \quad (4.9)$$

The influence of metric factor $M_T$ to the pion–nucleon form factor is illustrated in Fig. 4.8.

It is seen that, without the inclusion of $M_T$ the form factor is softer than in OBE models (the dashed line in Fig. 4.8), while its inclusion via Eqs. (4.8), (4.9) gives a behavior closer to OBE models. In fact, a monopole approximation at small $q^2 = t$ of the normalised form factor $G^\pi(t)/G^\pi(0) \approx \Lambda_\pi^2/(\Lambda_\pi^2 - t)$ gives $\Lambda_\pi = 860$ MeV and $\Lambda_\pi = 1100$ MeV for $M_T = 1$ and $M_T \neq 1$ respectively, compared to its empirical fit: $\Lambda_\pi^{OBE} = 1300$ MeV (dotted line in Fig. 4.8). Note, however, that our result for $\Lambda_\pi$ including the metric is in line with recent coupled–channel calculations of the Jülich group [74]. There, a monopole form factor with $\Lambda_\pi \approx 800$ MeV is obtained.

Introducing a flat metric requires a canonical form for the kinetic part of the Lagrangian, which determines the dynamics of the field fluctuation. The kinetic term of the scalar meson in Eq. (4.1)

$$\mathcal{L}_\sigma^{kin} = S_0^2 e^{-2\sigma/d} \partial_\mu \sigma \partial^\mu \sigma/2 \quad (4.10)$$

can be easily rewritten in a usual way:

$$\mathcal{L}_\sigma^{kin} = \partial_\mu \sigma \partial^\mu \sigma/2 \quad (4.11)$$

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3 An early study giving credit to this line of reasoning can be found in ref.[71].
by the following redefinition of the basic sigma field:

$$\tilde{\sigma}(r) = S_0 d[1 - e^{-\sigma(r)/d}]$$  \hspace{1cm} (4.12)

Now the new field $\tilde{\sigma}$ may be identified with the real sigma field. Clearly this redefinition does not change the nucleon static properties given in Table 4.2. Note also that, using the above redinition in the last term of Eq. (4.1), one may easily conclude that $m_{\tilde{\sigma}} = m_{\sigma}$. The appropriate sigma–nucleon form factor is given by

$$G^{\sigma}(-q^2) = -4\pi \int_{0}^{\infty} j_0(qr) \left[ \tilde{\sigma}'' + \frac{2\tilde{\sigma}'}{r} - m_{\tilde{\sigma}}^2 \tilde{\sigma} \right] r^2 dr,$$  \hspace{1cm} (4.13)

and may be used in OBE models. We have not introduced any metric factors in the form factors of the heavier mesons since these should play a lesser role than in the case of the pion.
For small values of the squared four-momentum transfer $t$ each form factor can be parametrized in monopole form:

$$G_i(t) = g_i(\Lambda_i^2 - m_i^2)/(\Lambda_i^2 - m_i^2)$$  \hspace{1cm} (4.14)

($i = \pi, \rho, \omega, \sigma$). We present in Table 4.3 the range parameters (cut-offs) and the coupling constants of the resulting meson–nucleon dynamics.

**Table 4.3:** Meson–nucleon coupling constants and cut-off parameters of meson–nucleon form factors. The $\Lambda_i(i = \pi, \rho, \omega, \sigma)$ are cutoff parameters in equivalent monopole fits $1/(1 - t/\Lambda_i^2)$ to the normalized form factors $G_{iNN}(t)/G_{iNN}(0)$ around $t = 0$. The empirical values are from OBE potential fit [62].

| $G_{\pi NN}(0)$ | $G_{\sigma NN}(0)$ | $F_1^\rho(0)$ | $F_2^\rho(0)$ | $F_2^\rho(0)/F_1^\rho(0)$ | $F_2^\omega(0)/F_1^\omega(0)$ | $\Lambda_\pi$ (GeV) | $\Lambda_\sigma$ (GeV) | $\Lambda_1^\rho$ (GeV) | $\Lambda_2^\rho$ (GeV) | $\Lambda_1^\omega$ (GeV) | $\Lambda_2^\omega$ (GeV) |
|-----------------|-------------------|---------------|---------------|--------------------------|-----------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 14.74           | 13.97             | 14.17         | 13.53         |                          |                            | 1.2             | 0.59            | 0.62            | 0.92            | 0.95            | 1.12            |
| $G_{\pi NN}(0)$ | -                 | 6.2           | 6.19          | 9.1 (12.41)              |                            |                 |                 |                 |                 |                 |                 |
| $F_2^\rho(0)$   | 2.55              | 2.76          | 2.68          | 2.24                     |                            |                 |                 |                 |                 |                 |                 |
| $F_2^\omega(0)$ | 14.33             | 15.01         | 14.67         | 13.7                     |                            |                 |                 |                 |                 |                 |                 |
| $F_2^\omega(0)/F_1^\omega(0)$ | 5.6              | 5.43          | 5.47          | 6.1                      |                            |                 |                 |                 |                 |                 |                 |
| $F_2^\omega(0)/F_1^\omega(0)$ | -0.24            | -0.25         | -0.26         | 0                        |                            |                 |                 |                 |                 |                 |                 |
| $\Lambda_\pi$ (GeV) | 1.2              | 1.1           | 1.1           | 1.3\(\div\)2.0          |                            |                 |                 |                 |                 |                 |                 |
| $\Lambda_\sigma$ (GeV) | -                | 0.59          | 0.60          | 1.3\(\div\)2.0          |                            |                 |                 |                 |                 |                 |                 |
| $\Lambda_1^\rho$ (GeV) | 0.62             | 0.63          | 0.63          | 1.3                      |                            |                 |                 |                 |                 |                 |                 |
| $\Lambda_2^\rho$ (GeV) | 0.92             | 0.92          | 0.92          | 1.3                      |                            |                 |                 |                 |                 |                 |                 |
| $\Lambda_1^\omega$ (GeV) | 0.95             | 0.89          | 0.91          | 1.5                      |                            |                 |                 |                 |                 |                 |                 |
| $\Lambda_2^\omega$ (GeV) | 1.12             | 0.86          | 0.89          | -                        |                            |                 |                 |                 |                 |                 |                 |

One can see that there is no interference between $\sigma$–meson nucleon and e. g. the pion–nucleon coupling constants. In other words, the inclusion of $\sigma$–meson does not significantly affect meson–nucleon form factors that had been given by the $\pi\rho\omega$ model. As it is seen from Table 4.3 the values for meson–nucleon coupling constants are close to their empirical values (in some cases obtained by OBE model fits). This is one of the main advantages of the inclusion of a scalar–isoscalar meson as done in the present approach.

The corollary of the present model is that it gives significant information on the $\sigma$–nucleon interaction. As it is seen from the Table 4.3, the value for $g_{\sigma NN}$ and the cut–off parameter of sigma–nucleon vertex $\Lambda_\sigma$ are smaller than their OBE prediction $\Lambda_\sigma^{OBE} \approx 1300 \div 2000$ MeV. This contrast is evidently seen from the Fig. 4.9, where $G^\sigma(t)/G^\sigma(0)$ for two cases : $m_\sigma = 720$MeV and $m_\sigma = 550$MeV is presented with the solid and dashed lines respectively. The band enclosed by the dotted lines refers to the OBE monopole form factor with $\Lambda_\sigma^{OBE} = 1300 \ldots 2000$ MeV. One can conclude that the present model gives a softer $\sigma NN$ form factor than it obtained by OBE. As it had been noticed before, the $t$–plane for each form factor has a cut along the positive real axis extending from $t = t_0$ to $\infty$. The cut for the $\sigma$–nucleon vertex function starts at $t_0 = 4m_\pi^2$ reflecting the kinematical threshold for the $\sigma \to \pi\pi$ channel. More precisely this result follows from the asymptotic behavior of the meson profiles: For $r \to \infty$, we have $\theta(r) \sim \exp(-m_\pi r/m_\pi r)$, $\sigma(r) \sim \theta^2(r)$, which are derived from the equations of motion.
4.3. 3 Nucleon - Nucleon central interaction

Once the vertex function of the corresponding meson–nucleon interaction has been found, its appropriate contribution to the NN interaction may be easily calculated by using well known techniques from OBE. The detailed formulas are given elsewhere [42, 62]. In particular, the contribution of the $\sigma$–meson exchange to the central potential is given by

$$V_c^{\sigma}(r) = \int_0^\infty \frac{k^2 dk}{2\pi^2} \frac{G_{\sigma NN}(k^2)}{k^2 + m_\sigma^2} j_0(kr).$$  \hspace{1cm} (4.15)$$

The central $NN$ potential in the $T = 0, S = 1$ state (the deuteron state) is presented in Fig. 4.10 in comparison with Paris potential. Our prediction is in good agreement with the empirical one. Note that the desired attraction in the central $V_{NN}$ has before been obtained in the $\pi\rho\omega$ model by means of two–meson exchange [42].

In conclusion it should be noted that we do not intend to describe (cover) all $NN$ phase shifts staying only in the framework of the present model. Besides other mesons, which
are usually included in OBE picture, the full model should also take in account e.g. $N\Delta\rho$ couplings. In addition the $2\pi$ exchange and its strong mixing with $\sigma$ meson exchange (see e.g. ref.[75]) should be considered. Another reason possible which limits the accuracy of $NN$ phase shifts in the present model is that the $\sigma NN$ coupling is not sensitive to the mass of sigma (see Table 4.3) as it is in the OBE phenomenology. In fact, even when the $2\pi$ exchange is disregarded, the pure OBE model has to consider two types of sigmas with nearly the same masses but with quite different coupling constants. So, we refrain from performing direct calculations of $NN$ phase shifts in the present model. Instead, we point out that the meson–nucleon form factors found in the present model could be useful in a wider context of calculations of nucleon–nucleon observables (phase shifts, deuteron properties etc) and may give more information on meson–nucleon and nucleon–nucleon dynamics.

4.4 Summary

To summarize this chapter, we have developed a topological chiral soliton model with two-meson exchange and have noted that the form factors found in the present model could be useful in a wider context of calculations of nucleon–nucleon observables (phase shifts, deuteron properties etc) and may give more information on meson–nucleon and nucleon–nucleon dynamics.
an explicit light scalar–isoscalar meson field, which plays a central role in nuclear physics, based on the chiral symmetry and broken scale invariance of QCD. We have shown that for the single nucleon properties the successful description of the electromagnetic observables of the \( \pi \rho \omega \) model is not modified and even the value for the axial–vector coupling is somewhat improved. In the two–nucleon sector, this extended \( \pi \rho \omega \sigma \) Lagrangian leads to the correct intermediate range attraction in the central potential and a soft \( \sigma NN \) formfactor for both values of sigma meson mass \( m_\sigma = 550 \text{ MeV} \) and \( m_\sigma = 720 \text{ MeV} \).
Chapter 5

Density Dependence of Nucleon Properties in Nuclear Matter. ¹

5.1 Introduction

It is well established that in-medium nucleon-nucleon (N-N) cross sections manifest in heavy ion collisions need not to be the same as their free space values [76, 77]. The origin of these changes must be reflected in modifications of the N-N interaction which is predominantly one-boson exchange. The parameters in meson exchange models are meson nucleon coupling constants and their masses. To avoid divergencies in loop integrals the meson-nucleon vertices are modified by form-factors which effectively provide cut-off parameters. All these parameters would change in the nuclear medium.

Even for the free N-N interaction the boson exchange models use a phenomenological ansatz for vertex form factors. Recently Holzwarth and Machleidt [72] have shown that among the QCD inspired pion-nucleon form factors the Skyrme form factor [69] is the most preferable: it can describe well both the pion-nucleon and nucleon-nucleon systems.

The purpose of this Chapter is to investigate the role of the medium in modifying the properties like meson nucleon coupling constant, meson-nucleon form factors and the axial vector form factor and the coupling constant. The calculations provide a consistent approach to modification of these properties in a modified Skyrme model that may be valid in a nuclear medium. The $\sigma$-meson is introduced as a dilaton field to satisfy scale invariance. Even though it is well known that scale invariance is badly broken, this provides a way to introduce the $\sigma$-meson which is so essential for an N-N interaction that is consistent with the two-body scattering data and the bound deuteron.

5.2 The modified Skyrme Lagrangian

In refs [78, 80] a medium modified Skyrme Lagrangian was proposed and applied to study the static properties of nucleons embedded in the nuclear medium. It gave a good description of changes in nucleon mass and its size in the medium. Here we shall outline the basic features of the Lagrangian and then extend it by including the dilated $\sigma$ meson field in order to satisfy the scale invariance. It is well-known that scale invariance is badly broken, this symmetry is retained here to investigate its consequences.

¹The present chapter is based on following articles by the author and his collaborators: [78, 79, 80, 81, 82]
Our basic assumption in modifying of the Skyrme Lagrangian

\[ \mathcal{L}_{sk} = \frac{F^2}{16} \text{Tr} (\partial_\mu U)(\partial^\mu U^+) + \frac{1}{32e^2} \text{Tr} [U^+ \partial_\mu U, U^+ \partial_\mu U]^2 - \]

\[ - \frac{F_\pi^2 m_\pi^2}{16} \text{Tr} (U^+ - 1)(U - 1), \] (5.1)

where \( F_\pi \) and \( e \) are the parameters of the model, was that, in the lowest order expansion of the chiral field \( U = \exp(2i\vec{\tau}\vec{\pi}/F_\pi) \approx 1 + 2i(\vec{\tau}\vec{\pi})/F_\pi - 2\vec{\pi}^2/F_\pi^2 + \ldots \) the appropriate medium modified Skyrme Lagrangian \( \mathcal{L}_{sk}' \) should give the well known equation for the pion field \([85]\):

\[ \partial_\mu \partial^\mu \pi + (m_\pi^2 + \hat{\Pi})\pi = 0, \] (5.2)

where \( m_\pi \) is the pion mass and \( \hat{\Pi} \) is the self energy operator. This may be achieved simply by including \( \hat{\Pi} \) in the pion mass term of the Skyrme Lagrangian:

\[ \mathcal{L}_{\chi^{sb}}' = - \frac{F_\pi^2 m_\pi^2}{16} \text{Tr} [(U^+ - 1)(1 + \hat{\Pi}/m_\pi^2)(U - 1)]. \] (5.3)

In general, the operator \( \hat{\Pi} = \hat{\Pi}_s + \hat{\Pi}_p \) in Eq. (5.3) acts both on the center of mass coordinate, \( \vec{R} \), and on the internal coordinate, \( \vec{r} \), of the Skyrmion. Only homogenous nuclear matter is considered. The translational invariance of the medium and that of the basic Skyrme Lagrangian (5.1) makes the coordinate dependence of \( \hat{\Pi} \) simpler: \( \hat{\Pi} \equiv \hat{\Pi}(\vec{r} - \vec{R}) \) which is also relevant for a moving Skyrmion. Further we assume that the Skyrmion is placed at the center of the nucleus i.e. \( \vec{R} = 0 \).

The \( P \) - wave part of the pion self energy \( \hat{\Pi}_\Delta \) is dominated by the \( P_{33} \) resonance. A simple model used in practical calculations is the delta - hole model which concentrates on the pion - nucleon - delta interaction ignoring the nucleon particle - hole excitations. In momentum space the \( \hat{\Pi}_\Delta \) is given by \( \hat{\Pi}_p(\omega, \vec{k}) = \hat{\Pi}_\Delta(\omega, \vec{k}) = -\vec{k}^2 \chi(\omega, \vec{k})/(1 + g_\rho^2 \chi(\omega, \vec{k})) \) \([85]\) where \( g_\rho^2 \) is the Migdal parameter which accounts for the short range correlations, also known as the Ericson - Ericson - Lorentz - Lorenz effect. The pion susceptibility \( \chi \) has nearly linear dependence on the nuclear density \( \rho \):

\[ \chi(\omega, \vec{k}) \approx \frac{8}{9} \left( \frac{f_{\pi N}\Delta}{m_\pi} \right)^2 \frac{\rho \omega_\Delta}{\omega_\Delta^2 - \omega^2} \exp(-2\vec{k}^2/b^2), \] (5.4)

where \( f_{\pi N}\Delta \approx 2f_{\pi N} \) (\( f_{\pi N}^2/4\pi \approx 0.08 \)) is the coupling constant, \( \omega_\Delta \approx \vec{k}^2/2M_\Delta + M_\Delta - M_N \) and \( b (b \approx 7m_\pi) \) is the range of the vertex form factor that is chosen to have a Gaussian form. For the pions bound in the nuclear matter \( \omega \) is small \( 0 \leq \omega \leq m_\pi \), so in the lowest order \( \hat{\Pi}_\Delta \) has the form:

\[ \Pi_\Delta(\omega, \vec{k}) \approx -\vec{k}^2 \chi_\Delta - \frac{\vec{k}^2 \omega_\Delta^2 \alpha_{rt}}{\omega_\Delta^2}, \] (5.5)

where

\[ \chi_\Delta = 4\pi c_0 \rho/(1 + 4\pi g_\rho^2 \rho), \]

\[ \alpha_{rt} = \chi_\Delta/(1 + 4\pi g_\rho^2 c_0 \rho), \] (5.6)

\[ c_0 = 8f_{\pi N}\Delta/9m_\pi^2 4\pi \omega_\Delta . \]

In the static case (\( \omega \to 0 \)) it coincides with the Kisslinger optical potential \([85]\) which was used in ref. \([80]\).

\( ^2 \)Here and below the asterisk indicates the medium modified operators in quantities.
Then in coordinate space the following symmetrized Lagrangian may be obtained

\[ L^*_{sk} = \frac{F^2}{16} \text{Tr} \left( \frac{\partial U}{\partial t} \right) \left( \frac{\partial U^+}{\partial t} \right) - \frac{F^2}{16} (1 - \chi \Delta) \text{Tr} (\vec{\nabla} U^+) (\vec{\nabla} U) + \]

\[ + \frac{F^2}{16} \alpha \rho \chi \left( \frac{\partial \varphi}{\partial t} \right) \left( \frac{\partial \varphi}{\partial r} \right) + \frac{F^2 m \pi^2}{16} \chi (U + U^+ - 2) + \]

\[ + \frac{1}{32e^2} \text{Tr} [U^+ \partial \rho U, U^+ \partial \rho U]^2, \]

where \( m^2 = m_\pi^2 (1 + \Pi_S(\rho)/m_\pi^2) \), the effective pion mass arises from the \( S \)-wave part of the self energy \( \Pi_S(\rho) \). Since the operators in nuclear matter do not satisfy the Lorentz invariance there are two different effective coupling constants in the kinetic terms of Eq. (5.7). The third term with "mixed" derivative in the Lagrangian in Eq. (5.7) clearly vanishes in free space (\( \alpha \rho \chi \varphi = 0 \) if \( \rho = 0 \)) and contributes mainly to the moment of inertia, \( I \), of the skyrmion. We shall consider this contribution in Sect. 5 of the present Chapter by estimating the N\( \Delta \) mass splitting.

### 5.3 Inclusion of scalar meson

Recently arguments have been given in favor of the empirical evidence for a scalar-isoscalar meson, \( \sigma \). For example the isosinglet resonance with a mass \( m_\sigma = 553.3 \pm 0.5 \) MeV and width \( \Gamma = 242.6 \pm 1.2 MeV \) [59, 60] found in the recent \( \pi \pi \) phase shift analyses is believed to have the properties of the \( \sigma \) meson that is essential for the N-N potential in OBE models to explain the NN-scattering data.

However the \( \sigma \) meson as a chiral partner of the pion was originally excluded from the Skyrme type Lagrangians. The only way of including \( \sigma \) meson in the Lagrangian is by means of a dilaton field appropriate to the scale transformations \( x^\mu \to \tilde{e}^\mu x^\mu \). This procedure has been outlined in Chapter 1.

Now a skyrmion imbedded in nuclear matter is considered. When a dilaton field is introduced into the effective Lagrangian the dilaton potential \( V(\sigma) \) would be modified by a \( \sigma \) field generated by the medium itself. While several attempts have been made [88], the correct nature of this modification is still poorly understood. On the other hand it is well known that the gluon condensate \( C_g \) as well as the quark condensate \( \langle \bar{q}q \rangle \) decrease in the medium due to the partial restoration of the chiral symmetry. The present study is restricted to considering the medium modification of the dilaton potential, Eq. (1.42), by taking into account mainly the change of \( C_g \) i.e. \( C_g \to C_g^* \).

Thus putting all these considerations together the following Lagrangian is proposed for homogeneous nuclear medium in the static case

\[ L^*_{sk} = \frac{F^2}{16} \alpha \rho \chi \text{Tr} \tilde{L}_i^2 + \frac{1}{32e^2} \text{Tr} [\tilde{L}_i, \tilde{L}_j]^2 + \frac{F^2 m \pi^2}{16} \chi^4 \text{Tr} (U + U^+ - 2) - \]

\[ - \frac{F^2}{8} (\vec{\nabla} \chi)^2 + \frac{C_g^*}{24} \left[ 1 - \chi^4 - \frac{4}{\varepsilon} (1 - \chi^2) \right], \]

where \( \chi(r) = e^{-\sigma(r)} \), \( \alpha \rho = 1 - \chi \Delta \), \( \tilde{L}_i = U^+ \partial \rho U \). Note that the Skyrme parameter \( \varepsilon \) coupled to the fourth derivative term remains unchanged since this term is related to the exchange of a very heavy \( \rho \) - meson with mass \( m_\rho^* = m_\rho \to \infty \) [34].
There may be two alternative approaches to applying this Lagrangian in nuclear physics. In Quantum Hadrodynamic models, the $\sigma$ field plays the role of an external field modifying the properties of a soliton [88] or a bag [89] which moves in the background generated by the medium. In the present model this approach would mean that $\sigma \equiv \sigma(R)$ and $U \equiv U(r)$ [88] that makes the scale invariance doubtful. In contrast, the mean field approximation is not used for the $\sigma$ field. Instead, the $\sigma$-field is strongly coupled to the nonlinear pion fields so as to generate the soliton. We assume that $\sigma \equiv \sigma(r - R)$ and $U = U(r - R)$ for a moving skyrmion. In ref. [80] a similar approximation (Eq. (5.8) with $\sigma = 0$) was used to estimate the medium modified static properties of the nucleon and found a well known behavior of the nucleon mass $M^*_N/M_N < 1$ and its size $R^*_N/R_N > 1$. In the next section we shall investigate in detail the dynamical properties of the meson - nucleon system.

5.4 Meson - nucleon form factors and Goldberger - Treiman relation

The semiclassical procedure for calculating the meson - nucleon vertex form - factors in a topological chiral effective Lagrangian [68, 69] is well-known. In fact, the results of more accurate methods [90, 91], based on the correct quantization of the fluctuating chiral fields nearly coincide with the original result that was given by Cohen et al. [68]. Using the ansatz $U(\vec{r},t) = A(t)U_0(\vec{r} - \vec{R}(t))A^+(t)$ and defining the pion field as

$$\pi_\alpha(\vec{r}) = -\frac{i F_\pi}{4} \text{Tr} \left[\tau_\alpha A U_0(\vec{r} - \vec{R})A^+ \right]$$

the following expressions are obtained

$$G^*_{\pi NN}(q) = \frac{4\pi M^*_N F_\pi \alpha_p \left(\vec{q}^2 + m^2/\alpha_p\right)}{3q} \int_0^\infty j_1(qr) \sin(\theta) r^2 dr =$$

$$= \frac{4\pi M^*_N F_\pi \alpha_p}{3} \int_0^\infty \frac{j_1(qr)}{qr} S_\pi(r) r^3 dr$$

(5.10)

for the pion nucleon form factor and

$$G^*_{\sigma NN}(q) = 2\pi F_\pi (\vec{q}^2 + m^2) \int_0^\infty j_0(qr) \sigma(r) r^2 dr =$$

$$= 2\pi F_\pi \int_0^\infty j_0(qr) S_\sigma(r) dr$$

(5.11)

for the sigma nucleon form factor respectively. The details and the explicit expressions for the source functions $S_\pi(r)$ and $S_\sigma(r)$ are given in the Appendix A. Here we note that, in the chiral soliton models formulas for meson nucleon form - factors are mainly determined by the quantization scheme rather than by details of the Lagrangian. The latter manifests itself through equations of motion whose solutions $\theta(r)$ and $\sigma(r)$ with spherically symmetric ansatz are $U_0 = \exp(i\vec{r}\vec{n}\tilde{\theta}(r)), \vec{n} = \vec{r}/r, \sigma(\vec{r}) \equiv \sigma(r)$ that should be used in Eq.s (5.10), and (5.11). The effective mass of the nucleon $M^*_N$ is given by

$$M^*_N = M^*_H + \frac{3}{8I^*}$$

(5.12)

where $M^*_H$ is the mass of the classical hedgehog soliton and $I^*$ is the moment of inertia of the pion isoscal.
The effective pion decay constant $f_\pi^*$ should be understood before considering the Goldberger - Treiman (GT) relation. Medium renormalized pion - decay constant $f_\pi^*$ can be naturally defined by the PCAC relation:

$$\nabla A_\alpha(x) = f_\pi^* m_\pi^2 \pi_\alpha(x)$$  \hspace{1cm} (5.13)

The medium modified axial coupling constant $g_A^*$ measures the spin - isospin correlations in a nucleon, embedded in a medium and is defined as the expectation value of the space component of the axial current $A^i_\alpha$ in the nucleon state at zero momentum transfer [83, 84]:

$$\lim_{q \to 0} \langle N(\vec{P})|A^i_\alpha(r)|N(\vec{P})\rangle = \frac{2}{3} \lim_{q \to 0} G^*_A(q^2) \langle N|\sigma_i \tau^\alpha \rangle \frac{N}{2} \exp(i \vec{q} \vec{r})$$  \hspace{1cm} (5.14)

where $\vec{q} = (\vec{P} - \vec{P})$ and $G^*_A(q^2)$ is the axial form factor of nucleon, $G^*_A(0) = g_A^*$. Here $\sigma_i$ is the component of the nucleon spin. Due to the semiclassical quantization prescription, Eq. (5.9), the matrix element of the pion field evaluated between nucleon states is given by:

$$\langle N(P')|\pi_\alpha(\vec{r} - \vec{R})|N(P)\rangle = \frac{F_\pi}{6} \exp(i \vec{q} \vec{r}) \int d\vec{x} \exp(-i \vec{q} \vec{x}) \sin(\theta) \langle \sigma_\alpha(\vec{r} \vec{x})|N\rangle$$  \hspace{1cm} (5.15)

Evaluation of the matrix elements between nucleon states for both sides of Eq. (5.13) yields

$$g_A^* = \frac{4\pi F_\pi f_\pi^* m_\pi^2}{9} \int_0^{\infty} r^3 \sin(\theta) dr$$  \hspace{1cm} (5.16)

that was originally derived in [92] for a free particle. By comparing this equation with the expression for pion nucleon coupling constant: $g_{\pi NN}^* = G_{\pi NN}^*(q^2)|_{q=0}$ given by Eq. (5.10) the following medium modified Goldberger - Treiman relation is realized

$$g_{\pi NN}^* f_\pi^* = g_A^* M_N^*$$  \hspace{1cm} (5.17)

Although this relation has been suggested earlier in ref.s [93, 101] it has not been proved yet. On the other hand $g_A^*$ and the axial form factor $G_A(q^2)$ may be calculated directly from the Lagrangian in Eq. (5.8) in terms of the Noether currents [83, 84]. This gives

$$G_A(q^2) = -4\pi \int_0^\infty [j_0(qr) A_1(r) + \frac{j_1(qr)}{qr} A_2(r)] r^2 dr,$$

with

$$A_1(r) = \frac{s_2}{8r} \left[ e^{-2\sigma} \alpha_p F_\pi^2 + \frac{4}{c^2} (\theta^2 + d) \right],$$

$$A_2(r) = -A_1(r) + \frac{\theta'}{4} \left( F_\pi^2 \alpha_p e^{-2\sigma} + \frac{8d}{c^2} \right)$$  \hspace{1cm} (5.18)

and

$$g_A^* = -\frac{\pi}{3} \int_0^\infty \left\{ e^{-2\sigma(r)} F_\pi^2 \alpha_p \left( \theta' + \frac{s_2}{r} \right) + \frac{4}{c^2} \left( \theta^2 + d \right) \frac{s_2}{r} + 2\theta' d \right\} r^2 dr$$  \hspace{1cm} (5.19)

where $d = \sin^2(\theta)/r^2$, $s_2 = \sin(2\theta)$, and $\alpha_p$ is defined in Eq. (5.8). The renormalized pion decay constant $f_\pi^*$ is obtained by combining the results given in Eqs (5.17) and (5.19).
Table 5.4: Density dependence of hadrons properties. All quantities are in MeV.

| $\rho/\rho_0$ | $m_\pi^*$ | $m_\sigma^*$ | $(C_g^*)^{1/4}$ | $M_N^*$ | $\Gamma_{\sigma \to \pi \pi}^*$ | $\delta M_{N,\Delta}^0$ | $\delta M_{N,\Delta}^b$ |
|---|---|---|---|---|---|---|---|
| 0.0 | 139.00 | 550.1 | 260.70 | 1413 | 251.2 | 283.7 | 283.7 |
| 0.5 | 144.90 | 513.8 | 251.06 | 1271 | 88.7 | 200.1 | 238.1 |
| 1.0 | 149.06 | 493.8 | 246.12 | 1157 | 34.6 | 161.9 | 205.4 |

5.5 Renormalization of hadron masses

Before going to a quantitative analyses of the medium effects we fix the following set of parameters for free space: $F_\pi = 2f_\pi = 186\text{MeV}$, $m_\pi = 139\text{MeV}$, $C_g = (260\text{MeV})^4$. This gives a good description of the sigma meson properties $m_\sigma = 550\text{MeV}$, $\Gamma_\sigma = 251.2\text{MeV}$, that may be compared with their experimental values obtained from the recent $\pi\pi$ phase shift analysis [59, 60]. The Skyrme parameter $e$ has been adjusted to reproduce the pion nucleon coupling constant: $g_{\pi NN} = 13.5$ for $e = 4.05$. The well-established fixed parameters of the $P$ - wave pion self energy in Eq. (5.5) are used in the pion sector: $g_{\sigma} = 0.6$, $c_0 = 0.13m_\pi^3$ [80, 85].

Now, the medium dependence of the input parameters may be considered. The possible renormalization of the Skyrme parameter $e$ cannot be studied in the present approach unless the $\rho$ meson is included in the Lagrangian explicitly. So we take $e^* = e$.

We adopt the following parametrization of $m_\pi^*$

$$m_\pi^* = m_\pi \sqrt{1 + \Pi_s(\rho)/m_\pi^2} = m_\pi \sqrt{1 - 4\pi b_0 \rho \eta/m_\pi^2},$$

(5.20)

where $\eta = 1 + m_\pi/M_N$ and $b_0$ is an effective S - wave $\pi - N$ scattering length. It is anticipated [80] that the results will not be too much sensitive to the value of $b_0$.

The only input parameter in the scalar meson sector is $C_g^*$. The medium renormalization of the gluon condensate $C_g^*$, in contrast with the renormalization of the quark condensate $\langle \bar{q}q \rangle$ [94] and meson masses [95], is poorly known. However in the present approach (5.8) $C_g^*$ may be determined by $m_\sigma^*$ through the equation

$$C_g^* = \frac{3F_\pi^2 m_\sigma^2 N_f}{4(4 - \varepsilon)},$$

(5.21)

Various approaches [89, 93, 101] show that $m_\sigma^*$, has a linear density dependence. The following parametrization

$$\frac{m_\sigma^*}{m_\sigma} = 1 - 0.12 \frac{\rho}{\rho_0}$$

(5.22)

is adopted here. It is consistent with the one obtained in the Quark coupling model (QCM) of framework [89].

The results for the static properties of hadrons are presented in Table 5.4. The second ($m_\pi^*$) and the third ($m_\sigma^*$) columns of the table should be considered as input data for they were taken from other models [85, 89]. The medium renormalized gluon condensate $C_g^*$ is calculated from Eq.s (5.21) and (5.22) with $N_f = 2$, $\varepsilon = 8N_f/(33 - 2N_f)$. The change in the gluon condensate is small $\sim 5\%$ at normal nuclear matter density, $\rho_0 = 0.5m_\pi^3$. The stiffness of the gluon condensate as a consequence of the lack of scale invariance of QCD has been shown by Cohen [96] who found that the fourth root of the condensate might be altered by no more than $4\%$ [94].

The main contribution to the $\sigma\pi\pi$ vertex, and hence, to the decay width of $\sigma$ meson at the tree level: $\Gamma_{\sigma \to \pi \pi} = \frac{\alpha^2}{\pi} \sqrt{2\pi M_N^2(1 - 2\varepsilon)^2/f_\pi^2}$, where $\alpha = m_\pi^* / m_{\pi}^*$, arises from
the first term of the Lagrangian in Eq. (5.8). The Table 5.4 shows that the width $\Gamma_{\sigma \rightarrow \pi \pi}$ is decreased significantly in the medium. This stimulates an interest to observe the $\sigma$ mesons in nuclei by experiments proposed recently by Kunihiro et al. [97].

Now medium effects on the mass of nucleon $M_N^*$ and $N\Delta$ mass splitting $\delta M_{N\Delta} = M_N^* - M_N^*$ will be considered. In general, it is almost impossible to reproduce simultaneously the experimental values of masses and coupling constants within the Skyrme model even for a free particle. Since dynamics is the main interest, the set of parameters, was chosen so as to reproduce the pion nucleon coupling constant: $g_{\pi N N} = 13.5$. It is clear from Table 5.4 that the free space value of the nucleon mass $M_N$ is slightly large $M_N = 1413\text{MeV}$, whereas $\delta M_{N\Delta}$ is reproduced rather well $\delta M_{N\Delta} = 284\text{MeV}$ ($\delta M_{N\Delta}^{\text{exp}} = 293\text{MeV}$). The effective mass of the nucleon $M_N^*$ in normal nuclear matter density is decreased by a factor of $M_N^*/M_N = 0.82$ which is in a good agreement with the estimates based on QCD sum rules $M_N^*(QCD) = 680 \pm 80\text{MeV}$ i.e. $M_N^*/M_N = 0.72 \pm 0.09$ [98]. We underline that the mass of the nucleon should be treated as the mass of the baryon which emerges as a soliton in the sector with baryon number one ($B = 1$).

The study of $N\Delta$ mass splitting gives a chance to estimate the contribution from the "mixed derivative" term - the third term on the r.h.s of Eq. (5.7):

$$\mathcal{L}_{rt} = \frac{F_\pi^2}{16 \omega_\Delta^2} \text{Tr} \left( \frac{\partial}{\partial t} \frac{\partial U^+}{\partial r} \right) \left( \frac{\partial}{\partial t} \frac{\partial U}{\partial r} \right).$$

(5.23)

Actually, owing to the canonical quantization $\delta M_{N\Delta}$ is related to the moment of inertia $I$ by: $\delta M_{N\Delta} = M_N^* - M_N^* = 3/2(I_0^* + I_{rt})$, where $I_{rt}$ is the net contribution from $\mathcal{L}_{rt}$ (clearly $I_{rt} = 0$ in free space). The explicit expressions for $I_0^*$ and $I_{rt}$ are given in the Appendix A. In Table 5.4 are shown $\delta M_{N\Delta}$ that has been calculated with the inclusion of $I_{rt}$ (denoted here $\delta M_{N\Delta}^r$) and without the inclusion of $I_{rt}$ ($\delta M_{N\Delta}^l$). In the nuclear medium the $\mathcal{L}_{rt}$ term leads to an enhancement of the moment of inertia decreasing $\delta M_{N\Delta}$ significantly. Even without the term $\mathcal{L}_{rt}$ in the Lagrangian the shift of $\delta M_{N\Delta}$ from its free value $\delta M_{N\Delta}^\text{exp}/\delta M_{N\Delta} = 0.75$ is larger than that obtained by Meissner [93] in a medium modified chiral soliton model based on the Brown Rho (BR) scaling law ($\delta M_{N\Delta}^\text{exp}/\delta M_{N\Delta} = 0.87$).

## 5.6 Renormalization of coupling constants and form - factors

The axial - vector exchange currents must be considered in order to investigate the medium effects on $g_A$ and on the axial form factor $G_A(q^2)$. However it is known that the bulk of exchange current effects arise from the $\Delta$-hole contributions. Including such $\Delta$-h effects would imply that bulk of the exchange currents effects are included in the effective $g_A$. In a heavy nucleus it is meaningful to take these axial exchange operators into account as corrections to the effective axial current operator of a single nucleon $\vec{A}_\alpha = -g_A^* \bar{\sigma}\vec{\tau}_\alpha$. So $g_A^*$ in Eq. (5.19) may be considered as an effective axial coupling constant modified by the medium polarization and screening effects since we are considering an effective one body problem of a nucleon embedded in the nuclear medium.

The second column of Table 5.5 displays a well known quenching behavior of $g_A$ that is mainly caused by a factor $\alpha_0 = 1 - \chi_\Delta < 1$ in the first term of Eq. (5.8). Note that the same set of input parameters $F_\pi, e, c_0$ (but without dilaton field $\sigma = 0$) gives the desired ratio $g_A^*/g_A = 0.8$ [80]. The present calculations show that the inclusion of the scalar meson, which induces an additional attraction, prevents larger quenching: $g_A^*/g_A = 0.9$.

In nuclei, a nucleon polarizes the medium in its vicinity. This leads to a screening effect that reduces the effective isoscalar coupling constant $\sigma$ in the current approach.
Table 5.5: Coupling constants and cut-off parameters at finite density. (All values are normalized to their free space ones.)

| $\rho/\rho_0$ | $g^*_A/g_A$ | $g^*_{\pi NN}/g_{\pi NN}$ | $g^*_{\sigma NN}/g_{\sigma NN}$ | $f^*_\pi/f_\pi$ | $\Lambda^*_\pi/\Lambda_\pi$ | $\Lambda^*_\sigma/\Lambda_\sigma$ | $r^*_a/r_a$ |
|---------------|-------------|---------------------------|---------------------------|--------------|---------------------------|---------------------------|-------------|
| 0.5           | 0.96        | 0.91                      | 0.88                      | 0.98         | 0.70                      | 0.90                      | 0.93         |
| 1.0           | 0.92        | 0.80                      | 0.78                      | 0.94         | 0.56                      | 0.84                      | 0.55         |

the screening mechanism may be described as being due to virtual $\Delta h$ excitations that have been taken into account by the self-energy $\hat{\Pi}_\Delta$ term in Eq. (5.5). At normal nuclear matter density the renormalization of $g_{\pi NN}$ amounts to a reduction of 25% of the coupling strength. This is sufficient to explain the quenching of the Gamov-Teller strength in heavy nuclei [99]. Furthermore this is consistent with a general argument based on Ward-Takahashi relations [100].

The effective pion decay constant $f^*_\pi$ is obtained by using the GT relation (5.17). Considering the ratios $f^*_\pi/f_\pi$ (Table 5.5) and $M^*_N/M_N$ (Table 5.4) it may be concluded that they both decrease in the nuclear medium. It would be interesting to compare these ratios with well known Brown-Rho scaling law [101] $f^*_\pi/f_\pi = M^*_N/M_N$ predicted by a simple Skyrme model. As it is seen from Tables 5.4 and 5.5 in our model $f^*_\pi/f_\pi \neq M^*_N/M_N$. Clearly, this deviation from Brown-Rho scaling law is caused by the the additional terms including the dilaton field.

The masses of the omega and sigma mesons are supposed to decrease by the scaling law. There are experimental indications that this is true. In the same way the $\sigma-N$ coupling constant is expected to decrease. The ratio of $g^*_{\sigma NN}$ to its free value is presented in Table 5.5. The changes in $g_{\pi NN}$ and $g_{\sigma NN}$ are nearly the same. Both are reduced in the medium by $\sim 25\%$ at $\rho = \rho_0$.

In Figs. 5.11 and 5.12, the renormalized $\pi NN$ and $\sigma NN$ vertex form factors respectively at $\rho = \rho_0$ (dashed lines) in comparison with these in the free space (solid lines) are displayed. Appreciable quenching of both form factors is observed. At small momentum transfer these can be parametrized by a monopole form i.e. $G_{\pi NN}(\vec{q}^2) = g_{\pi NN}/(1 + \vec{q}^2/\Lambda^2_{\pi})$ and $G_{\sigma NN}(\vec{q}^2) = g_{\sigma NN}/(1 + \vec{q}^2/\Lambda^2_{\sigma})$. Table 5.5 shows that the cut off parameter $\Lambda^*_\pi$ is decreased significantly at $\rho = \rho_0$. Relatively small changes in $G_{\sigma NN}(\vec{q}^2)$ seem to be caused by a stiffness of the $\sigma$-field or equivalently $C^*_f$ [94].

The nucleon axial form factor $G_A(\vec{q}^2)/G_A(0)$ calculated for $\rho = 0$ and $\rho = \rho_0$ is presented in Fig. 5.13 with solid and dashed curves respectively. It is seen that, the modification of $G_A(\vec{q}^2)$ is not as simple as that of $G_{\pi NN}(\vec{q}^2)$. The medium leads to a quenching of the meson nucleon form factors over a range of $\vec{q}^2$, while the quenching of $G_A(\vec{q}^2)$ takes place at $\vec{q}^2 = 0$ and $\vec{q}^2 > 10 fm^{-2}$.

A further interesting quantity is the pion-nucleon sigma term $\Sigma_{\pi N}$. It is both the chiral symmetry breaking piece of the nucleon mass and a measure of the scalar density of quarks inside the nucleon. Due to the Hellman- Feynman theorem, it may be easily calculated [102] in the Skyrme model that

$$\Sigma_{\pi N} = m_q \frac{\partial M_N}{\partial m_q} = \frac{\partial M_N}{\partial m^2_{\pi}} m^2_{\pi}$$

(5.24)

For free space ($\rho = 0$) the Lagrangian including scalar mesons (5.8) gives $\Sigma_{\pi N} \approx 20.1 MeV$ that is much smaller than that obtained in the original Skyrme model [83, 84]. It may be attributed to the absence of the dilaton field in the original model.
Figure 5.11: The $\pi NN$ form factor. The solid and dashed curves give results for the free space ($\rho = 0$) and the nuclear matter ($\rho = \rho_0$) respectively.

argued that $\Sigma_{\pi N}$ may also undergo changes in the nuclear medium i.e. $\Sigma^*_{\pi N} \neq \Sigma_{\pi N}$. Actually, PCAC allows us to relate $\Sigma_{\pi N}$ to the soft-pion limit of $\pi N$ scattering [103] whose parameters may not be the same in free space and the medium. So, defining an effective value of the in-medium pion-nucleon sigma commutator as

$$\Sigma^*_{\pi N} = m^* \langle N \mid \int d^3\bar{\psi}\psi \mid N \rangle^* = \frac{\partial M_{\pi N}^*}{\partial m^*} m^*_\pi$$ \quad (5.25)

where in the used notations $\langle N \rangle^*$ - is the state of the nucleon bound in nuclear matter, we obtain $\Sigma^*_{\pi N} \approx 40.1 MeV$ at normal nuclear density. This means a large increase of the nucleon sigma term: $\Sigma^*_N/\Sigma_{\pi N} \approx 2$. Using appropriate solutions of Eqs. (A1.13), it is estimated that

$$\frac{\Sigma^*_N}{\Sigma_{\pi N}} \approx \frac{m^*_\pi}{m^*_\pi} \int_0^\infty e^{-3\sigma^* \left[1 - \cos(\theta^*)\right] x^2} dx \approx 1.88 \frac{m^*_\pi}{m^*_\pi} , \quad (5.26)$$

where $\sigma^*(x), \theta^*(x)$ are profile functions at normal, $\rho = 0$, and nuclear, $\rho = \rho_0$, densities.
Figure 5.12: The $\sigma NN$ form factor. Solid and dashed curves are for $\rho = 0$ and $\rho = \rho_0$ respectively.

the medium renormalization of $\Sigma_{\pi N}$ is caused mainly due to a large modification of the profile functions (see Figs. 5.14 and 5.15). On the other hand $\Sigma_{\pi N}$ gives a good estimate for the quark condensate at finite density:

$$\langle \bar{q}q \rangle_{\rho} = 1 - \frac{\Sigma_{\pi N}^* \rho}{m_\pi^2 f_\pi^2}$$  \hspace{1cm} (5.27)

Hence, assuming the last equation holds it is concluded that the in medium enhancement of $\Sigma_{\pi N}$ leads to further quenching of the scalar density of quarks $\langle \bar{q}q \rangle_{\rho}$ in nuclear matter.

5.7 Discussions and summary

We have proposed a medium modified Skyrme like Lagrangian which takes into account the distortion of the basic nonlinear meson fields by the nuclear medium. It has been extended by including the scalar - isoscalar sigma meson. The influence of the medium on pion fields is introduced by the self energy operators $\hat{\Pi}_p, (= \hat{\Pi}_\Delta)$ and $\hat{\Pi}_s$ while the effect of the medium on the dilaton field is limited to the renormalization of the gluon condensate.
The Lagrangian is applied to study changes in the hadron masses and meson - nucleon vertex form - factors.

In particular, the mass of the $\Delta$ - resonance decreases more than that of the nucleon in the nuclear medium. Consequently the pion requires lesser energy to excite the nucleon to the $\Delta$ in the nuclear medium than it does for a free nucleon. The mass difference between $\Delta$ and $N$ decreases to 42% of that for free particles at the nuclear matter density. This is quite consistent with earlier estimates [93, 104] but contradicts the recent theoretical results of Mukhopadhyay and Vento [105], who found $\delta M_{N\Delta}^*/\delta M_{N\Delta} \approx 1.25$ for $\rho = 0.8\rho_0$. So, it would be quite interesting to study $N\Delta$ mass splitting experimentally by an analyses of the $N\Delta$ transitions in heavy nuclei.

We have investigated the characteristic changes of the decay width $\Gamma_{\sigma\rightarrow\pi\pi}$ at zero temperature. One may expect that the temperature-dependence of the physical quantities is qualitatively similar to the $\rho$ dependence. In this sense, our results are in good agreement with predictions of the in - medium NJL model [106], that at sufficiently high temperatures the $\sigma$ meson becomes a sharp resonance and its width may even vanish. Clearly, more precise predictions on $\Gamma_{\sigma\rightarrow\pi\pi}(T, \rho)$ in the framework of the present Lagrangian should be made by numerical calculations.
studying thermal Green’s function of $\sigma$-meson e.g. within Thermo Field Dynamics.

Furthermore the in-medium version of GT relation (5.17) holds in the present approach. The renormalized pion decay constant $f_\pi^*$ and nucleon mass $M_N^*$ do not satisfy the BR scaling [101]: the change in the nucleon mass is larger than that in $f_\pi^*$.

The medium effects lead to a quenching of the meson-nucleon form factors as well as the coupling constants $g_{\pi NN}$ and $g_{\sigma NN}$. The latter should be compared with the results of Banerjii and Tjon [107] obtained in the framework of CCM in the mean field approximation. There $g_{\sigma NN}$ increased at low densities making a looping behavior of the binding energy at saturation as a function of the density. However we believe that the natural reduction of the meson masses and coupling strengths found in the present model are expected to give a good description of the saturation properties of nuclear matter. So, it would be quite interesting to study the saturation properties of nuclear matter in relativistic Brueckner approach by using density dependent meson masses and coupling constants obtained in the present model.

By introducing a formal definition of the in-medium pion-nucleon sigma term $\Sigma_{\pi N} = m_\sigma^2 \partial M_N^*/\partial m_\sigma^2$, it is found that in contrast to the meson-nucleon coupling strengths the...
Figure 5.15: The profile function $\sigma(r)$ for the $\rho = 0$ (solid curve) and $\rho = \rho_0$ (dashed curve). They are the solutions of equations of motion (A1.13) in the sector with $B = 1$.

$\Sigma_{\pi N}$ increased in the medium: $\Sigma^*_{\pi N}/\Sigma_{\pi N} \approx 2$. This enhancement could lead to a decrease of the quark condensate $\langle \bar{q}q \rangle_\rho$ in nuclear matter. However as it was recently pointed out by Birse [108] this change should not lead to a drastic and rapid restoration of the chiral symmetry in nuclear matter.

It is anticipated that ultrarelativistic heavy - ion collision experiments (e.g. at RHIC) will provide significant new information on the strong interactions through the detection of changes in hadronic properties [17, 109]. This would provide an impetus to consider refined models for strong interactions in nuclear matter.

In conclusion, the modified in - medium Skyrme Lagrangian with scale invariance provides a useful insight into the role of medium in changing various properties of the mesons and nucleons. Even though scale invariance is badly broken in strong interactions its inclusion gives important information on the role of this symmetry property in a many-particle system.
Chapter 6

Nucleon electromagnetic form - factors in a nuclear medium at zero temperature.  

6.1 Introduction

The number of studies devoted to in - medium renormalization of the properties of nucleons has ever been increasing in recent years. Despite the absence of precise data from experiments from heavy - ion collisions and deep inelastic nucleon scattering, there are reasons to believe that nucleons immersed in a nuclear medium undergo swelling. Theoretically it is clear that this internuclear effect must be due to the effect of strong meson fields on the quarks constituting a given nucleon and on its mesonic cloud.

It is natural to assume that the change in the nucleon dimensions due to the intranuclear meson field is of a more general character. Specifically nucleon elastic and inelastic form factors in a nuclear medium are expected to be different from those in a vacuum.

An effective Skyrme Lagrangian describing the properties of a nucleon immersed in a nuclear medium has been given in the previous Chapter. Here we analyze the nucleon electromagnetic form factors in a nuclear medium.

6.2 Electromagnetic properties of nucleons

The problem of calculating of electromagnetic form factors for the nucleons reduces to computing isoscalar and isovector currents in a model. In topological models, in particular, in the Skyrme model, the isoscalar current is proportional to the baryonic current

\[ B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr} L_\nu L_\alpha L_\beta , \tag{6.1} \]

which is seen to be model independent. This expression remains in force in the theory based on the Lagrangian in Eq. (5.8). Since the isoscalar mean square radius \( \langle r^2 \rangle_{l=0} \) is expressed in terms of the zero component of the baryon current as

\[ \langle r^2 \rangle_{l=0} = \int_0^{\infty} B^0 d^3r , \tag{6.2} \]

variations in it are not very important. Nontrivial modifications are due exclusively to the distortion of the skyrmion profile function in the nuclear matter. Similarly, the isovector

\[[78, 79, 80, 81] 1\]
mean-square radius \( \langle r^2 \rangle_{I=1} \) which is determined by the zero component of the vector current

\[
\vec{V}_\mu = -i \frac{F^2}{16} C_\mu \text{Tr} \vec{\tau}(L_\mu + R_\mu) + \frac{i}{16e^2} \text{Tr} \vec{\tau}\{[L_\nu, [L_\mu, L_\nu]] + [R_\nu, [R_\mu, R_\nu]]\},
\]

(6.3)

where

\[
L_\mu = U^+ \partial_\mu U; \quad R_\mu = U \partial_\mu U^+ \quad C_\mu = \begin{cases} 1, & \mu = 0 \\ (1 - \chi_\Delta), & \mu = 1, 2, 3 \end{cases}
\]

[the vector current is treated as a Noether current that corresponds to the transformation \( U(x) \rightarrow \exp(iQ_L)U(x)\exp(-iQ_R) (Q_{L,R} - 2 \times 2 \text{ matrices}) \), does not depend explicitly on the medium parameters either. Relevant expressions for \( \langle r^2 \rangle \) are presented in the Appendix B.

Proceeding to analyze the isoscalar and isovector magnetic moments, we note that the former,

\[
\mu_{I=0} = \frac{1}{2} \int d\vec{r} \cdot \vec{r} \times \vec{B}
\]

(6.4)

does not depend on the medium parameters either. In contrast, the latter, \( \mu_{I=1} \), which is expressed in terms of the spatial component of the vector current as

\[
\mu_{I=1} = \frac{1}{2} \int d\vec{r} \cdot \vec{r} \times \vec{V}_3
\]

(6.5)

involves the features of the nuclear medium explicitly. In our approach these stem from the contribution of the kinetic term in equation (5.8).

The expressions for the electric and magnetic form factors, \( G_E \) and \( G_M \) respectively, can easily be obtained from equations (5.8) and (6.3) by using the technique developed in [18, 84]. The results are

\[
G_E^S(q) = -\frac{2}{\pi} \int_0^\infty s^2 \theta' j_0(qr) dr,
\]

\[
G_E^V(q) = \frac{\int_0^\infty r^2 s^2 \chi^2 + 4(\theta^2 + d)]j_0(qr) dr}{\int_0^\infty r^2 s^2 \chi^2 + 4(\theta^2 + d)] dr},
\]

\[
G_M^S(q) = -\frac{2M_N}{\pi e F_\pi \lambda_\mu} \int_0^\infty x^2 \alpha_\pi \frac{j_1(qx)}{qx} dx,
\]

\[
G_M^V(q) = -\frac{2M_N \lambda_M}{\int_0^\infty x^2 s^2 \chi^2 + 4(\theta^2 + d)] \frac{j_1(qx)}{qx} dx}
\]

(6.6)

(6.7)

where

\[
\lambda_M = \frac{8\pi}{3e^2 F_\pi} \int_0^\infty dx \left\{ x^2 s^2 \left( \frac{\chi^2}{4} + (\theta' + d) \right) \right\},
\]

(6.8)
is the skyrmion moment of inertia, \( x \equiv eF_\pi r \) and \( j_0, j_1 \) are spherical Bessel functions. Note that, in nuclear matter, the form factors do not obey the well known relations \([18, 84]\)

\[
G_M^S(q) = -2M_N \frac{\partial}{\partial q^2} G_E^S(q),
\]

\[
G_E^V(q) = -\frac{1}{M_N \lambda_M} \left( \frac{3}{2} + q^2 \frac{\partial}{\partial q^2} \right) G_M^V(q)
\]

because the Lorentz invariance does not hold here.

### 6.3 Input Parameters

In our calculations we set the input parameters as \( F_\pi = 186 MeV \) and \( C_g = (260 MeV)^4 \), whereby the correct value is reproduced for the sigma meson mass \( m_\sigma = 550 MeV \). The Skyrme parameter \( e \) was chosen in such a way as to fit the nucleon and delta-isobar masses in vacuum (\( M_N = 938 MeV \) and \( M_\Delta = 1232 MeV \) respectively). The effect of nuclear medium was specified by fixing the parameters of the polarization operator \( \Pi_\Delta \) at \( g_0 = 0.6 \) and \( c_0 = 0.13 \sigma^{-3/2} \) \([85]\) as it was done in Chapter 5.

Let us discuss the density dependence of the input parameters. As it was indicated in the previous chapter the renormalization of the Skyrme parameter \( e \) cannot be considered here because the \( \rho \) mesons have not been included on the effective Lagrangian \((5.8)\) explicitly. For this reason we set \( e^* = e \). The renormalized pion mass \( m_\pi^* \) has the form \([86]\)

\[
m_\pi^* = \frac{\sqrt{1 - 0.22 \rho / \rho_0}}{\sqrt{1 - 0.29 \rho / \rho_0}} m_\pi,
\]

where \( \rho_0 = 0.5 \sigma_0 \) - is the normal density of nuclear matter. This parameterization also takes into account the correction to the contribution of the \( S \)-wave component of the pion self energy \( \Pi_S \).

The only input parameter in the dilaton sector is the gluon condensate \( C_g^* \), whose in-medium renormalization has to be clarified conclusively. In the proposed model, this quantity is given by Eq. \((5.21)\) and can be expressed in terms of sigma meson mass as

\[
C_g^* = \frac{3F_\pi^2(m_\sigma^*)^2 N_f}{4(4 - \varepsilon)}
\]

Unfortunately, no information about the renormalization of \( m_\sigma \) has been deduced so far within the present approach. In view of this, the parametrization

\[
m_\sigma^* = [1 - 0.12 \frac{\rho}{\rho_0}] m_\sigma
\]

which was obtained in \([89]\) within the quark-meson coupling model, is taken for the sigma meson mass.

### 6.4 Results and Discussions

Let us now proceed to discuss our results.
Table 6.6: Ratio of the nucleon mass, nucleon root-mean-square radii, and nucleon magnetic moments in a nuclear medium (labelled with an asterisk) to the corresponding vacuum values.

| $\rho/\rho_0$ | $M_N^*/M_N$ | $\sqrt{\langle r^2 \rangle_{I=0}}$ | $\sqrt{\langle r^2 \rangle_{I=1}}$ | $\sqrt{\langle r^2 \rangle_p^*}$ | $\sqrt{\langle r^2 \rangle_p}$ | $\mu_I^*/\mu_I^*$ | $\mu_I^*/\mu_I^*$ | $\mu_p^*/\mu_p^*$ | $\mu_n^*/\mu_n^*$ |
|--------------|--------------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| 0.0          | 1.000        | 1.000           | 1.000           | 1.000           | 1.000           | 1.000         | 1.000         | 1.000         | 1.000         |
| 0.5          | 0.900        | 1.101           | 1.069           | 1.079           | 1.044           | 0.856         | 1.081         | 1.052         | 1.119         |
| 1.0          | 0.819        | 1.196           | 1.122           | 1.145           | 1.061           | 0.817         | 1.145         | 1.103         | 1.200         |

Table 6.6 presents the calculated root-mean-square radii and magnetic moments of the nucleons in the nuclear medium. It can be seen that, in-medium modifications to the root-mean-square radii of the proton are greater than those to the root-mean-square radii of the neutron. The effect is inverse for the magnetic moments - that is, the neutron magnetic moment is more sensitive to the impact of the medium than the proton magnetic moment.

In nuclear matter of normal density, the proton charge radius $\langle r^2 \rangle_p^{1/2}$ increases by some 14%, which is in agreement with the result presented in [111, 113]. We note that, only 5% increase in this quantity was found by Lu et al. [114], who took into account solely the renormalization of the nucleon mass, disregarding the renormalization of the pion-nucleon coupling constant $g_{\pi NN}$. However, in previous Chapter it has been shown that, the in-medium modifications of meson-nucleon coupling constants are less sizeable than the corresponding modifications of the nucleon mass.

From Table 6.6, it can be seen that the effect of a nuclear medium leads to an increase in the nucleon magnetic moment. As it was first revealed in [111, 112], the isoscalar magnetic moment changes more pronouncedly than the isovector one. In our calculations, the former and the latter change by about 14% and 8% respectively, at $\rho = 0.5\rho_0$ and by 18% and 15% respectively, at $\rho = \rho_0$.

Figures 6.16 and 6.17 display the proton and neutron electric form factors respectively, at the nuclear density values of $\rho = 0$, $\rho = 0.5\rho_0$ and $\rho = \rho_0$ (the solid, dashed and dotted curves respectively).

It can be seen that with increasing $q^2$ the form factors decrease much more slowly in a nuclear medium than in a vacuum. Figures 6.18 and 6.19 show the normalized magnetic form factors $G_M^*(q^2)/G_M^*(0)$ for the proton and neutron respectively. At $q^2 \sim 0.3 GeV^2 (7.5 Fm^{-2})$ the proton and neutron charge form factors decrease by about 9% and 2% respectively at $\rho = 0.5\rho_0$ and by about 17% and 4% at normal nuclear density. Similarly, the proton and neutron magnetic form factors decrease by about 6% and 5%, respectively, at $\rho = 0.5\rho_0$ and by about 11% and 9% at normal nuclear density.

From a comparison of the data in Figs. 6.16-6.19 it can be seen that the charge form factors are more sensitive to medium effects than the magnetic form factors. Qualitatively this conclusion complies with the results obtained in [114] on the basis of the quark-meson coupling model.

We note, however, that at the above momentum transfer value that is at $q^2 \sim 0.3 GeV^2$ the proton and neutron charge form factors decrease by about 5% and 6%, respectively, at
Figure 6.16: Proton electric form factor $G^p_E(q^2)$. The solid, dashed and dotted curves correspond to the nuclear density value of $\rho = 0$, $\rho = 0.5\rho_0$ and $\rho = \rho_0$ respectively.

Table 6.7: Form factors versus the nuclear density ($\rho = \lambda\rho_0$) at some values of the momentum transfer. Here the magnetic form factors are normalized to their values at $q = 0$.

| $q^2$(Fm$^{-2}$) | $\lambda$ | $G^p_E(q^2)$ | $G^n_E(q^2)$ | $\frac{G^p_M(q^2)}{G^p_M(0)}$ | $\frac{G^n_M(q^2)}{G^n_M(0)}$ |
|------------------|--------|--------------|--------------|-----------------------------|-----------------------------|
| 0.0              | 0.0    | 0.680        | 0.080        | 0.737                       | 0.730                       |
| 5.25             | 0.5    | 0.635        | 0.078        | 0.709                       | 0.709                       |
|                  | 1.0    | 0.599        | 0.076        | 0.686                       | 0.688                       |
| 7.55             | 0.5    | 0.529        | 0.089        | 0.618                       | 0.620                       |
|                  | 1.0    | 0.481        | 0.087        | 0.586                       | 0.590                       |
\[ \rho = 0.5\rho_0 \text{ and by about 8\% at normal nuclear density. A similar situation is observed for the magnetic form factors as well. That Lui et al.[114] found so weak suppression of the form factors in nuclear matter stems from their disregarding of the renormalization of meson-nucleon coupling constants and masses in their study.} \]

In ref. [115] the form factors for bound nucleons were calculated on the basis of the bag model. By preliminary fitting the input parameters of the model, the experimental values of the root-mean-square radii and magnetic moments were reproduced there rather well for the free nucleons. Further, medium effects were taken into account in that study through the renormalization of meson and nucleon masses. The analysis performed in [115] revealed that the nucleon electromagnetic form factors are suppressed in the \(^{12}\)C, \(^{40}\)Ca and \(^{56}\)Fe nuclei. The nucleon magnetic moments in medium were enhanced by 2\% to 20\% for various nuclear species. These results agree with ours and the ones from refs. [111, 112] However, the proton charge radius in the \(^{40}\)Ca nucleus is enhanced in ref. [115] by nearly 40\%, which seems too large in comparison with our results (about \(\sim 15\% \) at the normal nuclear density). It should also be noted that the results reported in [115] do not show the suppression of the axial coupling constant in nuclei. The failure to reproduce this well known effect casts serious doubts on the concept behind the calculations from [115].
It is well known that, in a vacuum, the neutron and proton form factors calculated in topological models at low momentum transfers obey following relations [12, 18]

$$\frac{G^p_M(q^2)}{G^p_M(0)} \approx \frac{G^n_M(q^2)}{G^n_M(0)} \approx G^p_E(q^2) .$$

Our calculations have demonstrated that, because of violation of Lorentz invariance, this relation is fulfilled only partly:

$$\frac{G^{p*}_M(q^2)}{G^{p*}_M(0)} \approx \frac{G^{n*}_M(q^2)}{G^{n*}_M(0)} \neq G^{p*}_E(q^2) .$$

We have also established that, at low momentum transfers ($q^2 < 7.5 Fm^{-2}$), the form factors show a nearly linear decrease with increasing a nuclear density. This results are in line with the results obtained in refs. [111, 112]. The nuclear density dependence of the form factors is illustrated in Tabl. 6.7 for various values of the momentum transfer.
Figure 6.19: Normalized neutron magnetic form factor $G_M^{n*}(q^2)/G_M^{n*}(0)$. The solid, dashed and dotted curves correspond to the nuclear density value of $\rho = 0$, $\rho = 0.5\rho_0$ and $\rho = \rho_0$ respectively.

### 6.5 Conclusions and Summary

On the basis of chiral and scale invariant Lagrangian, we have investigated the possible modifications to the nucleon electromagnetic form factors in a nuclear medium. Our results demonstrate that this model faithfully reproduces the features of hadrons as functions of the medium density. The calculations have revealed that the isoscalar nucleon form factors are more sensitive to the medium effects than the isovector ones. On the whole, our predictions are consistent with the results of other authors. This proves that, qualitatively, the resulting behavior of the features of nucleons in a nuclear medium is model independent. Specifically the coupling constants, masses and form factors are suppressed by the effective fields of the nuclear medium, whereas the dimensions and magnetic moments are enhanced. By way of example we indicate that, at the normal nuclear density the root mean square radii for protons and neutrons increase by some $\sim 15\% \sim 6\%$ respectively. With increasing density the nucleon form factors exhibit a nearly linear decrease in the region $q^2 \leq 7Fm^{-2}$.

Here we have taken no account of recoil effects. Both prior to and after collision with
a photon, the nucleon involved is assumed here to be frozen at the center of nucleus where the density is constant. As it was shown by Bunatian [116] the contribution to $\langle r^2 \rangle_N$ from recoil effects is small, but it can be expected that modifications to the nucleon structure will greatly depend on the nucleon momentum within Fermi sphere. The inclusion of recoil effects in the calculation of the nucleon electromagnetic form factors for finite nuclei would be of great interest in the problems of electron nucleon scattering. Calculations of this type may also be helpful in processing data from future MIT Bates and TJNAF experiments [117] that will measure the neutron electromagnetic form factor. Since there is no neutron free target relevant information will be extracted from data on electron nucleus (say $e^3He$) scattering where it is necessary to take into account nuclear medium effects.

Note that, the modification of nucleon electromagnetic form-factors in finite nuclei has been considered recently in ref. [118].
Chapter 7

Meson - Nucleon Vertex Form Factors at Finite Temperature.  

7.1 Introduction

In-medium properties of hadrons and their interactions is a field of high current interest. For studying phase transitions and thermodynamical properties of the nuclear system under extreme conditions, it is essential to determine the temperature ($T$) and density ($\rho$) dependence of the nuclear force. Below a critical temperature $T_c$, meson exchange picture of nuclear physics provides the natural description for the nucleon - nucleon ($NN$) interaction. Quarks and gluons are certainly present in all $NN$ interactions, but it is not always necessary to take them explicitly into account especially below a critical temperature $T_c$, when the transition into a quark gluon phase takes place. At distances smaller than a fermi, the inner structure of hadrons is probed since it involves the short distance or the high momentum components of the wave function. This structure is typically taken into account by including vertex form factors. In hot and dense matter, the structure of hadrons undergoes changes which should lead to a modification of the meson - nucleon coupling constants or, in general, the form factors. The temperature [121] and density dependence of some coupling constants have been investigated recently (see Chapters 5, 6), while that of form factors is still fairly unknown and will be studied in the present Chapter for the first time. For this purpose we shall use the formalism of Thermo Field Dynamics (TFD) in order to obtain the required temperature and density dependence.

The Thermo Field Dynamics (TFD) which was first suggested by Takahashi and Umezawa [122] - [126] is a real time operator formalism of finite temperature quantum field theory. In this framework, all the operator formalism of quantum field theory at zero temperature can be extended directly to finite temperature and density. Therefore in TFD it is possible to continue using Wick’s theorem and the Feynman diagrammatic approach as in the case of zero temperature theory. An auxiliary field is introduced which leads to a field doublet; the propagators become $2 \times 2$ matrices, the Feynman rules are now algebraic operational rules in the space of $2 \times 2$ matrices. Recently, some attempts to study in medium properties of hadrons at finite temperature using TFD [127, 128] have been made.

A study of relativistic heavy-ion collisions [129] is expected to lead us to a proper equation of state of hot and dense nuclear matter. The behaviour of nuclear matter as a function of temperature and density is relevant for investigation in nuclear physics, astrophysics,
cosmology and particle physics. The phase transition from hadronic system to a quark-gluon plasma phase and the subsequent hadronisation of the quark-gluon plasma phase can provide information on the asymptotic freedom as well as confinement behaviour in Quantum Chromodynamics (QCD). Therefore it is important to study theoretically the behaviour of the hadronic system at high temperatures. In this Chapter we shall make an attempt to study such a behaviour. In Sec.2, a brief review of the TFD is given. In Sec.3 we calculate the renormalization of meson nucleon vertices using the Feynman propagators from TFD. The results of calculations and their discussions will be presented in Sec.4.

7.2 Finite temperature formalism for meson and nucleon propagators.

The Thermo Field Dynamics is a real time operator formalism of quantum field theory at finite temperature. The main feature of the TFD is that thermal average of operator $\hat{A}$ is defined as the expectation value with respect to a temperature dependent vacuum, $\langle 0(\beta) | \hat{A} | 0(\beta) \rangle$, which is obtained from the regular vacuum by a Bogoliubov transformation. Therefore, we have

$$< \hat{A} > \equiv \text{Tr} \left( \hat{A} e^{-\beta(H-\mu)} \right) / \text{Tr} \left( e^{-\beta(H-\mu)} \right) = \langle 0(\beta) | \hat{A} | 0(\beta) \rangle, \quad (7.1)$$

where $\beta \equiv 1/k_BT$ with $k_B$ being the Boltzmann constant, $H$ is the total Hamiltonian of the system, and $\mu$ is the chemical potential. Due to the doubling of all degrees of freedom, every field is represented in the thermal doublet form in TFD. For example, the thermal doublet for the nucleon field is

$$\psi^{(a)}(x) \equiv \begin{pmatrix} \psi(x) \\ i \tilde{\psi}^\dagger(x) \end{pmatrix}, \quad (7.2)$$

where $\psi(x)$ is the ordinary nucleon field and $\tilde{\psi}$ is the doublet partner of $\psi(x)$. Here the superscript represents the transpose operation on the vector index. As a consequence, the thermal propagator of a fermion field is a $2 \times 2$ matrix defined as

$$iS_F^{(ab)}(x_1, x_2) = < 0(\beta) | T[\psi^{(a)}(x_1) \tilde{\psi}^{(b)}(x_2)] | 0(\beta) > \quad (7.3)$$

The free thermal propagator $S_F$ to be used in perturbation theory can be separated into two parts, i.e. the usual Feynman part $S_0$, propagator at zero temperature, and the temperature dependent part $S_T$ such that $S_F^{(ab)} = S_0^{(ab)} + S_T^{(ab)}$. For a fermion with mass $M$, the propagators have the form

$$S_0^{(ab)} = (\not{\hat{p}} + M)_{(ab)} \begin{pmatrix} G_0(p^2) & 0 \\ 0 & G_0^*(p^2) \end{pmatrix} \quad (7.4)$$

$$S_T^{(ab)} = 2\pi i \delta(p^2 - M^2)(\not{\hat{p}} + M)_{ab} \begin{pmatrix} \sin^2\theta_{p_0} & \frac{1}{2} \sin 2\theta_{p_0} \\ \frac{1}{2} \sin 2\theta_{p_0} & -\sin^2\theta_{p_0} \end{pmatrix} \quad (7.5)$$

with

$$G_0(p^2) = \frac{1}{p^2 - M^2 + i\epsilon},$$

$$\cos \theta_{p_0} = \frac{\theta(p_0)}{(1 + e^{-x})^{1/2}} + \frac{\theta(-p_0)}{(1 + e^x)^{1/2}},$$

$$\sin \theta(p_0) = \frac{e^{-x/2}\theta(p_0)}{(1 + e^{-x})^{1/2}} - \frac{e^{x/2}\theta(-p_0)}{(1 + e^x)^{1/2}}.$$
where $x = \beta(p_0 - \mu)$, $a$ and $b$ are Dirac indices, and $\theta(x)$ is the step function. The propagator of the scalar particle with mass $m_B$ is given by

$$\Delta^B(p) = \Delta^B_0(p) + \Delta^B_T(p),$$

(7.7)

$$\Delta^B_0 = \begin{pmatrix} D_0(p^2) & 0 \\ 0 & -D_0(p^2) \end{pmatrix}$$

(7.8)

$$\Delta^B_T = -2\pi i \delta(p^2 - m_B^2) \begin{pmatrix} \sinh^2 \phi_{p_0} & \frac{1}{2} \sinh 2\phi_{p_0} \\ \frac{1}{2} \sinh 2\phi_{p_0} & \sinh^2 \phi_{p_0} \end{pmatrix}$$

(7.9)

with

$$D_0(p^2) = \frac{1}{p^2 - m_B^2 + i\epsilon},$$

(7.10)

$$\cosh \phi_{p_0} = \frac{1}{(1 - e^{-|y|})^{1/2}},$$

(7.11)

$$\sinh \phi_{p_0} = \frac{e^{-|y|/2}}{(1 - e^{-|y|})^{1/2}},$$

where $y = \beta p_0$. Note that the propagator of vector mesons would be similar to that of the scalar mesons except for an additional factor of $(-g_{\mu\nu} + k_{\mu}k_{\nu}/m^2)$.

### 7.3 Vertex form factors

The OBE model of the N-N interaction [62] includes exchange of several mesons. Here we consider the meson-nucleon interactions of $\pi$, $\sigma$, $\omega$- and $\rho$-mesons:

$$\mathcal{L}_{\pi NN} = -ig_{\pi NN} \bar{\psi}\gamma_5 \vec{\tau}\psi \bar{\varphi}; \quad \mathcal{L}_{\sigma NN} = g_{\sigma NN} \bar{\psi}\varphi \psi \sigma;$$

$$\mathcal{L}_{\omega NN} = -g_{\omega NN} \bar{\psi}\gamma_\mu \varphi \omega^\mu - \frac{f_{\omega NN}}{4M} \bar{\psi}\sigma_{\mu\nu}\varphi \left[ \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \right];$$

(7.12)

$$\mathcal{L}_{\rho NN} = -g_{\rho NN} \bar{\psi}\gamma_\mu \varphi \rho^\mu - \frac{f_{\rho NN}}{4M} \bar{\psi}\sigma_{\mu\nu}\varphi \left[ \partial^\mu \rho^\nu - \partial^\nu \rho^\mu \right].$$

Here $g_{BNN}$ are the meson-nucleon coupling constants and $f_{\omega NN}$ and $f_{\rho NN}$ are the tensor coupling constants for $\omega$- and $\rho$- mesons respectively.

The corresponding meson nucleon form factors are usually defined as:

$$\langle N(p')|\Gamma_\pi^a|N(p)\rangle = ig_{\pi NN}(t)\bar{u}(p')\gamma_5\tau_a u(p)$$

$$\langle N(p')|\Gamma_\sigma|N(p)\rangle = -g_{\sigma NN}(t)\bar{u}(p')u(p)$$

$$\langle N(p')|\Gamma_\omega^\mu|N(p)\rangle = \bar{u}(p') \left[ \gamma^\mu G_{\omega NN}(t) + \frac{i}{2M} F_{\omega NN}(t)\sigma^{\mu\nu} q_\nu \right] u(p)$$

(7.13)

$$\langle N(p')|\Gamma_\rho^{\alpha,\mu}|N(p)\rangle = \bar{u}(p')\tau_\alpha \left[ \gamma^\mu G_{\rho NN}(t) + \frac{i}{2M} F_{\rho NN}(t)\sigma^{\mu\nu} q_\nu \right] u(p),$$
where $g_{\pi NN}$, $g_{\pi NN}$, $G_{\omega NN}$, $(F_{\omega NN})$, $G_{\rho NN}$ and $(F_{\rho NN})$ are form factors of meson-nucleon vertices; $t = q^2 - q_0^2 = (p - p')^2$ is the 4-momentum transfer and $M$ is the nucleon mass.

In order to investigate the $T$ dependence we calculate three line vertex correction, as is illustrated in Fig. 7.20 e.g. for $g_{\pi NN}(t, T)$. In accordance with Feynman rules this may be written as:

$$
\Gamma_A(t, T) = \Gamma_A(t) + i \sum_B \Lambda_{AB}(t, T),
$$

$$
\Lambda_{AB}(t, T) = \int \frac{d^4k}{(2\pi)^4} \Gamma_B(k^2) S_F(a) \Gamma_A(t) S_F(b) \Gamma_B(k^2) \Delta_B(k^2)
$$

(7.14)

where $A, B$ - denote $\pi, \sigma, \omega, \rho$ mesons, e.g. $\Gamma_\pi = g_{\pi NN}(t)$, $a = p' - k, b = p - k$. Here $S_F(a)$ and $\Delta_B(k^2)$ are the in-medium nucleon and meson propagators respectively. In the framework of TFD the thermal propagator has a $2 \times 2$ matrix structure, but only the 11-component refers to the physical field. So, we may use the following representation:

$$
S^{11}_F(a) \equiv S_F(a) = (\hat{a} + M)[G_0(a) + G_T(a)],
$$

$$
\Delta^{11} \equiv \Delta(k^2) = D_0(k^2) + D_T(k^2),
$$

$$
D_0(k^2) = \frac{1}{k^2 - m^2 + i\varepsilon}, \quad D_T(k^2) = -2\pi i n_B(k) \delta(k^2 - m^2),
$$

$$
G_0(a) = \frac{1}{a^2 - M^2 + i\varepsilon}, \quad G_T(a) = 2\pi i \delta(a^2 - M^2) N_F(a),
$$

(7.15)

where $\hat{a} = a_\mu \gamma^\mu$, $N_F(a) = \theta(a_0)n_F(a) + \theta(-a_0)\bar{n}_F(a)$, $m$ is the mass of corresponding meson.
and

\[ n_F(a) = \frac{1}{e^{\beta(|a_0|-\mu)} + 1}, \quad \bar{n}_F(a) = \frac{1}{e^{\beta(|a_0|+\mu)} + 1} \]  

\[ n_B(k) = \frac{1}{e^{\beta|k_0|} - 1} \]

are the fermion, antifermion and boson distribution functions respectively. It is clear that when one substitutes Eq. (7.15) into Eq. (7.14), \( \Lambda_{AB}(t, T) \) will be separated into two parts: one refers to the naive zero temperature contribution, while the other one depends on the density and temperature [127]. We shall concentrate on the latter part, which may be rewritten as follows:

\[ \Lambda_{AB}(t, T) = \int \frac{d^4k}{(2\pi)^4} \Gamma_B(k^2)(\hat{a} + M)\Gamma_A(t)(\hat{b} + M)\Gamma_B(k^2) \times \]

\[ \times [G_0(a)G_0(b)D_T^B(k^2) + 2G_0(a)G_T(b)D_0^B(k^2)] \]

Note that we have neglected terms that are quadratic in the temperature dependent distribution function. Now, using (7.17) and (7.15) in (7.14) we get the following expressions for the form factors:

\[ G_{ANN}(t, T)/G_{ANN}(t, T = 0) = 1 + i \sum_B \int \frac{d^4k}{(2\pi)^4} W_{AB}(t, k^2, T) \times \]

\[ \times [G_0(a)G_0(b)D_T^B(k^2) + 2G_0(a)G_T(b)D_0^B(k^2)] \]

where the explicit formulas for \( W_{AB}(t, k^2, T) \) may be found in the Appendix C. Here we present the expressions for \( W_{\omega\pi}(t, k^2, T) \) and \( W_{\rho\pi}(t, k^2, T) \) for illustration:

\[ W_{\omega\pi}(t, k^2, T) = \frac{3G_{\pi NN}(k^2)}{4} \left\{ 2[2M^2 - (ab)] - \frac{3F_{\omega NN}(t)t}{2G_{\omega NN}(t)} \right\}, \]

\[ W_{\rho\pi}(t, k^2, T) = \frac{-G_{\pi NN}(k^2)}{4} \left\{ 2[2M^2 - (ab)] - \frac{3F_{\rho NN}(t)t}{2G_{\rho NN}(t)} \right\}, \]

where \( W_{\omega\pi}(t, k^2, T) \) and \( W_{\rho\pi}(t, k^2, T) \) denote the contribution from the \( \pi \)-exchange diagram to the \( G_{\omega NN}(t, T) \) and \( G_{\rho NN}(t, T) \) respectively.

### 7.4 Results and discussions

In order to start the calculations, a set of free space meson - nucleon form factors are chosen. We choose the OBE monopole form factors [62] (Bonn A): \( G_{\text{BNN}}(t) = g_{\text{BNN}}(\Lambda_B^2 - m_B^2)/(\Lambda_B^2 - t) \), where, for example, \( g_{\pi NN}^2/4\pi = 14.09, \Lambda_{\pi NN} = 1005\text{MeV}, m_\rho = 550\text{MeV} \). Let us first discuss the \( T \) dependence of meson - nucleon coupling constants - \( g_{\text{BNN}}(T) \equiv G_{\text{BNN}}(t = m_B^2, T) \). The variation of the coupling constants \( g_{\pi NN}(T)/g_{\pi NN}(T = 0), g_{\sigma NN}(T)/g_{\sigma NN}(T = 0), g_{\omega NN}(T)/g_{\omega NN}(T = 0) \) and \( g_{\rho NN}(T)/g_{\rho NN}(T = 0) \) with temperature are displayed in Fig. 7.21. It is clear that they are nearly independent of the temperature below \( T_c^B \), then change rapidly for \( T > T_c^B \). A similar behavior was also predicted by Zhang et al. [127] and by Dominguez et al. [121] for pion - nucleon coupling constant
Figure 7.21: The ratio of meson-nucleon coupling constants at finite temperature $T$ at $T=0$ as a function of temperature; $\rho = 0$.

$g_{\pi NN}(T)/g_{\pi NN}(T = 0)$. But here in Fig. 7.21, an unexpected result is that, the $\omega - N$ coupling constant, $g_{\omega NN}(T)$ increases while the coupling constants of all other mesons decrease! Let’s consider a possible origin of this controversy, by comparing the medium modification of $\omega NN$ and $\rho NN$ coupling constants. In the present model the modifications arise from the triangle diagrams (Fig. 7.20). Actual calculations show that, for any $G_{BNN}(t, T)$ the triangle diagram with pion exchange gives a dominant contribution especially at small density. It is clear from Eq. (7.19) that at small $t$ the explicit expressions for $g_{\omega NN}(T)$ and $g_{\rho NN}(T)$ formally coincide. However, $W_{\rho\pi} = -\frac{W_{\omega\pi}}{3}$. and hence the two have an opposite sign. The factor $(-1/3)$ arises from the isotopic spin structure. In fact, using Eq. (7.14) and Fig.7.20, $g_{\omega NN}(T)$ and $g_{\rho NN}(T)$ may be written in a schematic way:

$$g_{\omega NN}(T) \approx g_{\omega NN}(T = 0) + g_{\pi NN}^2 \sum_{\beta} \tau_\beta g_{\omega NN} \tau_\beta + \ldots$$

$$g_{\rho NN}(T) \tau_\alpha \approx g_{\rho NN}(T = 0) \tau_\alpha + g_{\pi NN}^2 \sum_{\beta} \tau_\beta \tau_\alpha g_{\rho NN} \tau_\beta + \ldots$$

(7.20)

where $\alpha$ and $\beta$ are isospin indices.
Figure 7.22: The ratio of mean square radius at finite temperature $T$ and at $T = 0$ as a function of temperature; $\rho = 0$.

We omit the spin variables, since $\omega$ and $\rho$ have the same spin structure. Now, using $\sum_\beta \tau_\beta^2 = 3$ and $\sum_\beta \tau_\beta \tau_\alpha \tau_\beta = -\tau_\alpha$, it is clear that the contribution of the leading triangle diagram with pion exchange to $g_{\omega NN}(T)$ and $g_{\rho NN}(T)$ has different signs. In other words, the term $\Lambda_{\rho\pi}$ in Eq. (7.14) is negative, while $\Lambda_{\omega\pi}$ is positive.

In general, we conclude from the results (Fig. 7.21) that at a certain critical temperature $T_c^B$, the couplings $g_{BNN}$ change drastically. The thermal behavior of $g_{\pi NN}(T) = g_{\pi NN}(t, T)|_{t \to 0}$ has been investigated earlier [121] and we get here a similar behaviour of $g_{\pi NN}(T)$ as a function of temperature. The temperature $T_c$, where $g_{\pi NN}(T)$ changes dramatically, was interpreted in [121] as a signal for the quark - gluon deconfinement phase transition. Indeed, near $T_c$ the associated mean square radius $<r_{BNN}^2> = 6[\frac{\partial}{\partial t}\ln G_{BNN}(t, T)]|_{t \to 0}$ is a monotonically increasing function of $T$ and in fact diverges at the critical temperature. A similar behavior of $<r_{BNN}^2>$ is also found in our calculations and is illustrated in Fig. 7.22. As the critical temperature, $T_c^B$, is approached the strength of coupling of $\pi, \sigma$ and $\rho$ mesons to nucleons is quenched, at the same time, the size of nucleons as probed by the appropriate meson gets bigger. However, it is to be stressed that $T_c^B$ is not the same for all...
Table 7.8: Parameters of vertex form factors in Eq.s (7.21)-(7.22) at half of normal nuclear matter density \( \rho = 0.5\rho_0 \).

| Meson | \( \alpha_g \) | \( \beta_g \) | \( \alpha_\lambda \) | \( \beta_\lambda \) | \( T_c \) |
|-------|----------------|----------------|----------------|----------------|-------|
| \( \pi \) | 0.9925 | -1.3538 | 0.6789 | -1.0397 | \( \approx 180 \) |
| \( \sigma \) | 0.2572 | -1.2507 | 1.0413 | -1.6472 | \( \approx 90 \) |
| \( \rho \) | 0.3678 | -0.8579 | -0.4821 | -0.4655 | \( \approx 180 \) |
| \( \omega \) | 2.1001 | -1.8110 | 1.1595 | -4.44165 | \( \approx 175 \) |

Table 7.9: The same as in Table 7.8, but for \( \rho = \rho_0 \).

| Meson | \( \alpha_g \) | \( \beta_g \) | \( \alpha_\lambda \) | \( \beta_\lambda \) | \( T_c \) |
|-------|----------------|----------------|----------------|----------------|-------|
| \( \pi \) | 1.5047 | -1.7415 | 1.5816 | -2.2862 | \( \approx 185 \) |
| \( \sigma \) | 0.8238 | -1.8623 | 1.5911 | -2.0814 | \( \approx 90 \) |
| \( \rho \) | 0.6556 | -0.9325 | -0.1778 | -0.5858 | \( \approx 180 \) |
| \( \omega \) | 3.0774 | -2.6030 | 3.0351 | -9.8837 | \( \approx 175 \) |

mesons: \( T_\pi^c \approx 360\text{MeV}, T_\sigma^c \approx 95\text{MeV}, T_\omega^c \approx 175\text{MeV} \) and \( T_\rho^c \approx 200\text{MeV} \). In OBE picture this means that, in the temperature region e.g. \( 200\text{MeV} < T < 300\text{MeV} \), the \( \rho \) meson exchange is no longer important while the pion exchange is still important. On the other hand, the \( \sigma \) and \( \pi \) mesons are mainly responsible for the attraction between nucleons. So, the quenching of the \( \sigma NN \) coupling constant at \( T > T_\sigma^c \) leads to the vanishing of the bound state, as it was predicted earlier [127].

The density dependence of vertex form factors has been studied in greater detail [81, 93]. It was found that most of the vertex form factors are quenched at high densities and it was anticipated that the temperature dependence is likely to yield results that are qualitatively similar to those of density dependence.

Now, let’s consider their temperature dependence in some detail. The form factors as a function of momentum transfer at several temperatures are displayed in Figs. 7.23 - 7.25. It is seen that, the in-medium effects lead to the suppression of \( g_{\pi NN}(t,T), g_{\sigma NN}(t,T) \) and \( G_{\rho NN}(t,T) \). The temperature dependence of \( G_{\omega NN}(t,T) \), is opposite to that of \( G_{\rho NN} \) due to it’s isotopic structure as outlined above. The temperature dependence of tensor couplings of vector mesons \( F_{VNN}(t,T) \) are quite similar to that of vector couplings \( G_{VNN}(t,T) \). Particularly, the ratio \( \kappa_v = F_{VNN}/G_{VNN} \), where \( \kappa_\rho = 6.1 \) and \( \kappa_\omega = 0 \) in free space, remains constant in a wide range of temperature. For practical calculations a parametrization of these \( G_{BNN}(t,T,\rho) \) form factors is needed. At small momentum transfer we can parametrize them by a monopole form:

\[
G_{BNN}(t,T,\rho) = g_B(T,\rho)(A_B^2(T,\rho) - m_B^2)/(A_B^2(T,\rho) + t) \tag{7.21}
\]

Table 7.10: The same as in Table 7.8, but for \( \rho = 3\rho_0 \).

| Meson | \( \alpha_g \) | \( \beta_g \) | \( \alpha_\lambda \) | \( \beta_\lambda \) | \( T_c \) |
|-------|----------------|----------------|----------------|----------------|-------|
| \( \pi \) | 3.5869 | -3.4716 | 5.5941 | -8.5846 | \( \approx 220 \) |
| \( \sigma \) | 3.0715 | -4.2687 | 3.8743 | -4.0881 | \( \approx 90 \) |
| \( \rho \) | 2.3344 | -2.5660 | 1.3642 | -2.3675 | \( \approx 210 \) |
| \( \omega \) | 6.4629 | -5.0795 | 9.2984 | -26.3338 | \( \approx 175 \) |
where in general the effective mass of a meson, $m_B$, is also temperature dependent. Consideration of this dependence is beyond the scope of the present Chapter. But here, for simplicity, one may consider this as just a parameterization and choose the parameters $g_B(T, \rho)$ and $\Lambda_B^2(T, \rho)$. Their temperature dependence is still unknown. Here the $T$ dependence may be represented, for $T << T^B_c$, in a polynomial form as:

$$\frac{g_B(T, \rho)}{g_B(T = 0, \rho = 0)} = \Phi(\rho)[1 + \alpha^B_0(\rho)(T/T^B_c)^2 + \beta^B_0(\rho)(T/T^B_c)^4],$$

$$\frac{\Lambda_B(T, \rho)}{\Lambda_B(T = 0, \rho = 0)} = \Phi(\rho)[1 + \alpha^B_0(\rho)(T/T^B_c)^2 + \beta^B_0(\rho)(T/T^B_c)^4],$$

(7.22)

where $\Phi(\rho) = 1/(1 + C_0 \rho/\rho_0)$. The calculated form factors are fitted to this form and the parameters $C_0$, $\alpha$ and $\beta$ are determined. The results are presented in Tables 7.8, 7.9 and 7.10 for densities $\rho = 0.5 \rho_0$, $\rho = \rho_0$ and $\rho = 3 \rho_0$, respectively (with $\rho_0 = 0.17 fm^{-3}$ - the density of normal nuclear matter). The results show that the density dependence of parameters, and hence, the form factors is not so drastic (sharp) as their temperature dependence in

![Figure 7.23: Meson nucleon form factors at several temperatures as a function of $q^2$ using parametrisation of BONN group [62] for $\pi NN$ vertex at $\rho = 0$.](image)
the range of moderate densities. So, at $T \approx 0$ the Eq. (7.22) with $C_0 = 0.26$ is in good agreement with Brown Rho scaling law [131] (where $C_0 = 0.28$).

Summarizing this Chapter, we have considered the temperature dependence of meson - nucleon form factors and coupling constants in TFD formalism. It is shown that at a critical temperature, where the coupling constant changes drastically and the associated mean square radius diverges is different for different mesons. The temperature dependent vertex form factors are parametrized in a simple monopole form and the T-dependence of these parameters is clarified. These form factors may be used in the calculation of the in-medium NN cross sections [130] and in investigations of the properties of hot dense matter.

In the present calculations, the monopole form factors with the parameters (at $T = 0$) given by the Bonn group [62] are used. But the next question which may arise is: are the results sensitive to the shape of the input form factors? To answer this question a different set of form factors obtained in Chapter IV in the framework of a topological soliton model have been used. The thermal behaviour of such form factors is shown in Figs. 7.27 - 7.30.

Now, by comparing Figs. 7.23 - 7.26 and Figs.7.27 - 7.30 it is clear that only the $\sigma NN$ form factor is affected the most. The reason is that even in free space and at $T = 0$ the $\sigma NN$ form factor in the topological soliton model [54] is much harder ($\Lambda_\sigma \approx 600\,MeV$) than those

![Figure 7.24: The same as in Fig 7.23 but for $\sigma NN$ vertex.](image-url)
of the Bonn potential ($\Lambda_\sigma \approx 2000\text{MeV}$). Nevertheless, our main conclusions about thermal behavior of different meson nucleon vertices remain valid.

The variation of the meson-nucleon coupling constant with temperature (Fig. 7.21) clearly indicates that the attractive NN force due to $\sigma$-exchange decreases quite rapidly while the repulsive N-N force due to $\omega$-exchanges increases. This would lead to the fact that the nuclear matter is quite likely un-bound at high temperature. In fact it will look more like a hard sphere gas since the repulsive interaction at short distances will dominate. The attractive interactions in such a case play only a small perturbative role. The hot and dense nuclear matter could be approximated as a free gas with an excluded volume around each nucleon. Perhaps the extensive studies [132]-[135] along this line can be justified on the basis of the results obtained in this Chapter. The nuclear matter would be hard to compress.

It would be quite interesting to know, if the critical temperature, $T^{B}_{c}$, maybe uniquely considered as a point of transition from the hadron state to a quark gluon state, as it was suggested [121]. The usual way of estimating $T_{c}$ is based on QCD lattice calculations [136] or temperature dependent quark - gluon potentials [137]. In such calculations it is usual to start from high temperatures and then decreases the temperature, looking for a phase transition from the quark gluon state to a hadron state i.e., hadronization of the quarks.
Clearly this method gives a unique $T_c$, since at any stage there is either a hadron or a quark gluon plasma phase. Contrary to such studies the present calculations explore the transition from a hadron to a quark-gluon phase and find that the critical temperature for hadron $\rightarrow$ quark-gluon transition depends on the kind of hadrons under consideration. A similar result, where hadrons and quark - gluon plasma coexist have been obtained in lattice calculations [136]. In conclusion it is important to emphasize that the dynamics of hadrons and of quark-gluon plasma at finite temperature is poorly understood. The question of critical phenomenon, in particular critical temperature, in the hadronic systems is not well-known. Studies in nuclear matter are needed urgently to clarify the critical transition from a hadronic system to the quark gluon phase which will depend on both density and temperature. The experiments with relativistic heavy ion colliders would need such an understanding to clarify the production of quark-gluon plasma from hadrons and the eventual hadronisation of the quark-gluon system. It is possible that there will be clear and unambiguous signals for the formation and subsequent hadronisation of the quark-gluon plasma [138, 139].

**Appendix 1**

In the present Appendix following T. Cohen [69] we derive explicitly expressions for
$G_{\pi NN}(q^2)$ and $G_{\pi NN}^*(q^2)$ used in Chapter 5. The small fluctuations around the vacuum value are related to the pion field by

$$U = \exp(2i\vec{r}\pi/F_\pi) \approx 1 + 2i\vec{r}\pi/F_\pi - 2\vec{\pi}^2/F_\pi^2 + \ldots$$  

which gives the following approximation for the Lagrangian in Eq. (5.8):

$$\mathcal{L} \approx -\frac{1}{2}(\vec{\nabla}^2\pi)^2\alpha_p - \frac{1}{2}m_{\pi}^*\vec{\pi}^2$$  

and for the equation of motion:

$$-(\vec{\nabla}^2\pi)\alpha_p + m_{\pi}^*\vec{\pi} = 0$$

The factor $\alpha_p = 1 - \chi_\Delta$ in the last equation is obtained by renormalization of the pion propagator in the medium i.e. by the $\Delta$ - hole self energy term $\Pi_\Delta$. We define the in-medium $\pi NN$ coupling constant and the vertex form factor by introducing the source term

$$\vec{j} = iG_{\pi NN}^*\bar{\psi}\gamma^5\psi$$  

Figure 7.27: Meson-nucleon form factors at several temperatures as a function of $q^2$ using parametrisation of Meissnar et al. [54] for $\pi NN$ vertex at $\rho = 0$. 

$T = 0$ $- - T = 82$ MeV $\triangle\cdot\cdot\cdot T = 193$ MeV
Figure 7.28: The same as in Fig. 7.27 but for $\sigma NN$ vertex.

into the equation of motion:

$$\nabla^2 \bar{\pi} p + m^2 \pi = iG_{\pi NN}^* \bar{\psi}\gamma_5 \bar{\tau} \psi, \quad (A1.5)$$

where

$$\psi|N(\bar{P})\rangle = \frac{e^{i\bar{P}x}}{(2\pi)^{3/2}} \left( \frac{E^* + M_N^*}{2E^*} \right)^{1/2} \left( \frac{1}{(\bar{\sigma}\bar{P})/(E^* + M_N^*)} \right) \chi_{S}, \quad (A1.6)$$

where $E^2 = \bar{P}^2 + M^2$. In the Breit frame ($\bar{P}' = -\bar{P}, \bar{q} = \bar{P} - \bar{P}'$) the matrix element of the source evaluated between nucleon states is given by:

$$\langle N(\bar{P}')|\bar{j}(r)|N(\bar{P})\rangle = \frac{e^{i\bar{q}r}}{(2\pi)^{3/2} 2M_N} \langle N|\bar{\sigma}\bar{q}|N\rangle. \quad (A1.7)$$

Using the quantization rules

$$\pi_\alpha(r) = - \frac{iF_\pi}{4} \text{Tr} [\tau_\alpha AU_0(r-R)A^+] , \quad (A1.8)$$

$$\langle N|\text{Tr} \tau_\alpha A\tau_\beta A^+|N\rangle = - \frac{2}{3} \langle N|\sigma_\alpha \tau_\beta|N\rangle$$
we have

\[
\langle N'(\vec{P}')|\pi_\alpha(\vec{r}, \vec{R})|N(\vec{P})\rangle = \frac{e^{i\vec{q}\cdot\vec{r}}}{(2\pi)^3} \int \langle N'|\pi_\alpha(x)|N\rangle e^{-i\vec{q}\cdot\vec{x}} d\vec{x} =
\]

\[
= \frac{F_\pi e^{i\vec{q}\cdot\vec{r}}}{6(2\pi)^3} \int \langle N'|\sigma_\alpha(\vec{r}^2)|N\rangle e^{-i\vec{q}\cdot\vec{x}} \sin(\theta) d\vec{x}
\]

where \(\theta\) is defined by the hedgehog ansatz \(U_0(r) = e^{i(\vec{r}\cdot\vec{r})\theta(r)}\). Now the matrix element of Eq. (A1.5) is evaluated between collective wave functions for spin-up proton \(|p \uparrow\rangle\) with momentum \(\vec{P}\) and \(\vec{P}'\) in the Breit frame and using equations (A1.6) - (A1.9) to obtain

\[
G_{\pi NN}(q^2) = -\frac{iF_\pi \alpha_p M_N^*}{3q} \int (-\nabla^2 + m_\pi^2 / \alpha_p) \hat{x}_3 \sin(\theta) e^{-i\vec{q}\cdot\vec{x}} d\vec{x} =
\]

\[
= \frac{4\pi F_\pi \alpha_p M_N^*}{3} \int_0^\infty \frac{j_1(qx)}{qx} S_\pi(x) x^3 dx
\]

Figure 7.29: The same as in Fig. 7.27 but for \(\omega NN\) vertex.
Figure 7.30: The same as in Fig. 7.27 but for $\rho NN$ vertex.

with $c = \cos(\theta)$, $s = \sin(\theta)$.

Similarly, the coupling constant at the $\sigma NN$ vertex is defined by the equation:

$(-\vec{\nabla}^2 + m^*_\sigma)\sigma = G^*_\sigma NN \bar{\psi}\psi$. Evaluating matrix elements of both sides of this equation it is easy to obtain the $\sigma NN$ form factor:

$$G^*_\sigma NN(\vec{q}^2) = 2\pi F_\pi \int_{-\infty}^{\infty} j_0(qx)S_\sigma(x)dx, \quad (A1.12)$$

with $S_\sigma(x) = -x^2\sigma'' + 2x\sigma' + x^2m^*_\sigma^2\sigma$.

Note that the profile functions $\theta(r)$ and $\sigma(r)$ in $S_\pi(x)$ and $S_\sigma(x)$ are the solutions of the equations of motion:

$$\begin{align*}
\theta'' + 2x^2\chi^2\alpha_p + 4s_2\theta'^2 + 8s^2\theta'' + 2x\theta'\chi^2\alpha_p + 2x^2\theta'\chi\alpha_p - \\
-\chi^2\alpha_ps_2 - 4s_2d - x^2\beta^2\chi^3s = 0 \\
x^2\chi'' + 2\chi' - 2\chi\alpha_px^2(\theta^2/2 + d) - 16x^2D_{eff}(\chi^3 - \chi^{-1}) - \\
-3x^2\beta^2(1-c)\chi = 0.
\end{align*} \quad (A1.13)$$

where $x \equiv eF_\pi r$, $s_2 \equiv \sin(2\theta)$, $d \equiv s^2/x^2$, $\beta = m^*_\pi/eF_\pi$, $D_{eff} = C_9/24e^2F^4_\pi$, $\chi \equiv \exp(-\sigma(x))$. The boundary conditions are:

$\theta = c\theta + \delta \text{ for } x = 0, \alpha = (1 + \beta)\alpha_p \text{ for } x = 1.$
For completeness we write also the explicit expressions for $I = I^*_0 + I_{rt}$:

\begin{align*}
I^*_0 &= \frac{2\pi}{3\epsilon^3 F^*_\pi} \int_0^\infty s^2 \{ e^{-2\sigma} + 4(\theta'^2 + d) \} x^2 dx, \\
I_{rt} &= \frac{\pi\alpha_{rt}}{3\omega^2 \epsilon F^*_\pi} \int_0^\infty \{ \theta'^2 c^2 + 2d \} x^2 dx. \\
\end{align*}

\begin{equation}
(A1.14)
\end{equation}

**Appendix 2**

Here some auxiliary formulas for Chapter 6 are presented. The isoscalar and isovector mean square radii are given by

\begin{align*}
\langle r^2 \rangle_{I=0} &= -2 \int_0^\infty r^2 \theta' s^2 dr, \\
\langle r^2 \rangle_{I=1} &= \int_0^\infty r^4 s^2 [\chi^2 + 4(\theta'^2 + d)] dr, \\
\end{align*}

(A2.1)

where the notations $\theta$, $s$ etc are given in Appendix A. The nucleon mean square radii $\langle r^2 \rangle_{p,n}$ are expressed in terms of $\langle r^2 \rangle_{I=0}$ and $\langle r^2 \rangle_{I=1}$ as:

\begin{equation}
\langle r^2 \rangle_{p,n} = \frac{1}{2} \left[ \langle r^2 \rangle_{I=0} \pm \langle r^2 \rangle_{I=1} \right].
\end{equation}

We note that the expression for mean square radius in a medium is similar to that in a vacuum, a solution to the equation (A1.13) with medium parameters must be taken in former case.

The proton and neutron magnetic moments are given by

\begin{align*}
\mu_{p,n} &= \frac{1}{2} \left[ \mu_{p,n}^{I=0} + \mu_{p,n}^{I=1} \right], \\
\mu_{p,n}^{I=0} &= \frac{2}{9} M_N (M^*_\Delta - M^*_N) \langle r^2 \rangle_{I=0} \mu_B, \\
\mu_{p,n}^{I=1} &= -\mu_{p,n}^{I=1} = \frac{1}{3} \cdot \frac{8\pi}{3\epsilon^2 F^*_\pi} \int_0^\infty dx \left\{ x^2 s^2 \left( \frac{\alpha_p}{4} + (\theta' + d) \right) \right\}, \\
\end{align*}

(A2.2)

where $\mu_B$ is the Bohr magneton. For the nucleon form factors we have

\begin{align*}
G^p_E &= \frac{1}{2} (G^S_E + G^V_E), \\
G^n_E &= \frac{1}{2} (G^S_E - G^V_E), \\
G^p_M &= \frac{1}{2} (G^S_M + G^V_M), \\
G^n_M &= \frac{1}{2} (G^S_M - G^V_M).
\end{align*}

(A2.3)

**Appendix 3**
Here the explicit expressions for $W_{AB}(t, k^2, T)$ introduced in Chapter 7 are given.

$$W_{\pi\pi} = G_{\pi}(k^2)[M^2 - (ab)] , \ W_{\pi\sigma} = W_{\pi\pi}|_{G_{\pi} \rightarrow G_{\sigma}} ,$$

$$W_{\pi\omega} = A_{\pi}^\omega[M^2 - (ab)] + 3F_1^\omega(k^2)F_2^\omega(k^2)z , \quad (A3.1)$$

$$W_{\pi\rho} = -W_{\pi\omega}|_{(F_1^\omega \rightarrow F_1^\rho, \ A_\pi^\omega \rightarrow A_\rho^\omega)}$$

$$A_\pi^\omega = [F_1^\omega(k^2)]^2[4 - \frac{k^2}{m_\omega^2}] + \frac{3k^2[F_2^\omega(k^2)]^2}{4M^2} , \quad (A3.2)$$

$$W_{\sigma\pi} = -3g_\pi^2(k^2)[M^2 + (ab)] , \ W_{\sigma\sigma} = -\frac{1}{3}W_{\sigma\pi}|_{G_{\sigma} \rightarrow G_{\sigma}} ,$$

$$W_{\sigma\omega} = [M^2 + (ab)]A_\omega^\sigma ; \quad W_{\sigma\rho} = 3W_{\sigma\omega}|_{A_\omega^\sigma \rightarrow A_\rho^\sigma} ;$$

$$A_\sigma^\omega = -[F_1^\omega(k^2)]^2[4 - \frac{k^2}{m_\omega^2}] + \frac{3k^2[F_2^\omega(k^2)]^2}{4M^2} ;$$

$$W_{\omega\pi} = \frac{3g_\pi^2(k^2)}{4} \left\{ 2[2M^2 - (ab)] - \frac{3F_2^\omega(t)t}{2F_1^\omega(t)} \right\} ; \quad W_{\omega\sigma} = \frac{1}{3}W_{\omega\pi}|_{G_{\omega} \rightarrow G_{\sigma}} ; \quad (A3.3)$$

$$W_{\omega\omega} = \frac{1}{4} \left\{ 2\Gamma_1^\omega[2M^2 - (ab)] + \frac{\Gamma_2^\omega}{M^2}k^2[M^2 - (ab)] + 3F_1^\omega t + 3F_2^\omega(k^2t - z^2)] +$$

$$+3z\Gamma_3^\omega - \frac{F_2^\omega(t)}{2M^2F_1^\omega(t)}[3\Gamma_1^\omega t + \Gamma_2^\omega(k^2t - z^2)] +$$

$$+ \frac{\Gamma_3^\omega}{M^2}[z[(ab) + 3M^2] - 2(ak)(bk) - 2(aq)(bk)] \right\} ;$$

$$\Gamma_1^\omega = [F_1^\omega(k^2)]^2[2 - \frac{k^2}{m_\omega^2}] + \frac{k^2[F_2^\omega(k^2)]^2}{4M^2} ,$$

$$\Gamma_2^\omega = \frac{2M^2}{m_\omega^2}[F_1^\omega(k^2)]^2 - [F_2^\omega]^2 ;$$

$$\Gamma_3^\omega = F_1^\omega(k^2)F_2^\omega(k^2) , \quad W_{\omega\rho} = 3W_{\omega\omega}|_{\Gamma_1^\omega \rightarrow \Gamma_\rho^\omega} , \quad W_{\rho\pi} = -\frac{1}{3}W_{\omega\pi}|_{\omega \rightarrow \pi} ; \quad (A3.4)$$

$$W_{\rho\sigma} = -W_{\rho\pi}|_{G_{\rho} \rightarrow G_{\sigma}} ; \quad W_{\rho\omega} = W_{\omega\omega}|_{\Gamma_1^\omega \rightarrow \Gamma_\rho^\omega} ; \quad W_{\rho\rho} = -W_{\omega\omega}|_{\omega \rightarrow \rho} ;$$

where $G_\pi \equiv G_{\pi NN}(k^2), \ G_\sigma \equiv G_{\sigma NN}(k^2), \ F_1^\omega \equiv F_{\omega NN}, \ F_2^\omega \equiv F_{\omega NN}$, $z = (kq)$ and $q^2 = t$. 

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