Insight into dynamic magnetic properties of YMnO$_3$/FM bilayer in a time-dependent magnetic field

Chun-lu Chang$^{1,2}$, Wei Wang$^{1,a}$, Dan Lv$^{2,b}$, Zhen-yu Liu$^1$, Ming Tian$^1$

$^1$ School of Sciences, Shenyang University of Technology, Shenyang 110870, China
$^2$ School of Environmental and Chemical Engineering, Shenyang University of Technology, Shenyang 110870, China

Received: 3 October 2020 / Accepted: 24 February 2021
© The Author(s), under exclusive licence to Società Italiana di Fisica and Springer-Verlag GmbH Germany, part of Springer Nature 2021

Abstract Based on Monte Carlo simulation, a mixed-spin (5/2, 2, 3/2) Ising model is constructed to investigate the dynamic magnetic properties of antiferromagnetic/ferromagnetic YMnO$_3$/FM bilayer under a time-dependent magnetic field. The effects of exchange interaction, magnetic field and temperature are involved in this work. Masses of numerical results of the dynamic order parameter, susceptibility, internal energy, and critical temperature are obtained under the influence of the diverse physical parameters. Moreover, the phase diagrams and the hysteresis loops of the system are discussed.

1 Introduction

Multifunctional ceramics materials have received extensive attention in the past decades [1, 2]. In particular, multiferroics, a member of the multifunctional material family, have been the protagonist of the studies because of the coexistence of electric and magnetic orders. They have many potential applications in many fields such as multiple-state memory devices, magnetoelectric transformers, and tunable microwave devices [3–5]. With the development of technology, a novel type of multiferroic compound YMnO$_3$ with perovskite crystal structure has been discovered. It can exhibit antiferromagnetism and ferroelectricity at room temperature in the magnetic field and the electric field, respectively [6–8]. Experimentally, the antiferromagnetic/ferromagnetic (AFM/FM) materials such as YMnO$_3$-based composites have been successfully prepared by applying various methods such as sol–gel method [9], solid-state reaction method [10], and dip coating [11].

On the other hand, it is worthwhile to mention that much theoretical effort has been devoted to better explaining the experimental results and providing a feasible direction for further experiments. As the important multiferroic materials, the AFM/FM composites have been widely studied in theory in recent years. Utilizing Monte Carlo (MC) simulation based on Metropolis algorithm, a theoretical calculation was implemented to study the ground-state phase diagrams and the hysteresis loops of the perovskite YMnO$_3$ under the influence of different physical parameters, such as external field, exchange interaction, and crystal

$^a$ e-mail: ww9803@126.com (corresponding author)
$^b$ e-mail: ludan1977918@163.com (corresponding author)
A quantum microscopic investigation was performed to investigate the magnetic entropy change of BiFeO$_3$ [13]. Other theoretical studies were also carried out to study the thermodynamic properties of the YMnO$_3$ and BiFeO$_3$ multiferroics [14, 15].

Recently, great interest has been aroused in the research on AFM/FM bilayers such as YMnO$_3$/FM and BiFeO$_3$/FM bilayers due to their superior applications in information storage, gas sensors, spintronic, and solar energy conversion [16–18]. The excellent electric and magnetic characteristics of the BiFeO$_3$/YMnO$_3$ bilayer were discussed by Masrour using MC simulation [19]. The hysteresis loops and critical behaviors caused by different exchange interactions and magnetic field were given. A MC simulation was also performed to investigate the magnetoelectric interactions of the FM/FE multiferroic bilayer system [20]. In Refs. [21, 22], Ainane et al. deduced the influence of the exchange interaction and the transverse field on the hysteresis loops and susceptibility of the bilayer system described by the transverse Ising model. In addition, various mixed-spin Ising models were used successfully to study the dielectric properties of the low-dimensional ferroelectric materials [23, 24].

When the ferromagnetic and ferroelectric materials are applied to a time-dependent magnetic or electric field, it is practical and interesting to investigate the dynamic properties of these systems theoretically. By means of the effective-field theory (EFT), Deviren et al. investigated the dynamic thermal properties and hysteresis behaviors of the cylindrical Ising nanotube under the influence of a time-dependent oscillating magnetic field [25]. A two-dimensional ferromagnetic kinetic Ising model was established to study the influence of next-nearest neighbor interactions on the dynamic magnetic properties [26]. The dynamic mean-field theory was applied to investigate the dynamic phase transition, compensation points as well as dynamic hysteresis behaviors of a two-dimensional ferrimagnetic system in an oscillating magnetic field [27]. The nonequilibrium dynamic properties of a mixed-spin ferrimagnetic system in a time-dependent magnetic field were studied. The phase transition and dynamic critical temperature of the system were discussed in detail [28]. Moreover, unique dynamic magnetic properties were discovered in other mixed-spin Ising systems driven by the time-dependent magnetic field [29–32]. In remarkable, M. Acharyya et al. devoted much energy to the study of the dynamic properties of many magnetic nanosystems based on the MC simulation. They clarified a profound introduction about the dynamic transition and investigated the nonequilibrium dynamic phase transitions in the Ising ferromagnets driven by an external time-varying magnetic field [33]. They also focused on the dynamic phase transitions of the many-body magnetic systems under an external time-dependent magnetic field [34–36]. In addition, the MC simulation was developed to study the compensation behaviors in a spin-1/2 diluted Ising ferrimagnet [37] and a Blume–Capel trilayer ferrimagnet [38] as well as the variation of residual magnetization with the ratio of coupling strengths and non-magnetic impurities in an Ising trilayer system [39].

In our previous work, much effort was made to explore the magnetic and dielectric properties of the low-dimensional mixed-spin nanosystems using Monte Carlo simulation [40–45]. Besides, such BiFeO$_3$/FM bilayer systems were also investigated [46–48]. However, to the best of our knowledge, few work has been done on the dynamic magnetic properties of the YMnO$_3$/FM bilayer systems driven by the oscillating magnetic field, such as dynamic order parameter, internal energy, susceptibility, and hysteresis behaviors. Therefore, in this work, we shall study the effects of various parameters on the dynamic magnetic properties of a mixed-spin (5/2, 2, 3/2) YMnO$_3$/FM bilayer system under the time-dependent oscillating magnetic field. The organization of the paper is as follows: In Sect. 2, the model and method are illustrated. In Sect. 3, the results and discussion are presented. Eventually, a conclusion is included in Sect. 4.

 Springer
2 Model and method

We should mention that there are lots of studies about the dynamic magnetic and dielectric properties of the low-dimensional nanosystems, which are constructed on various extended mixed-spin Ising systems [49–51]. Therefore, in the present work, we propose a mixed-spin (5/2, 2, 3/2) Ising model to describe the YMnO₃/FM bilayer system with the honeycomb lattices in Fig. 1. It consists of the AFM (YMnO₃) top layer and the FM bottom layer. The red balls represent the Mn atoms with spin-2 on the top layer, and the green and yellow balls denote the transition-metal ions with spin-3/2 (such as Co) and spin-5/2 (such as FeIII) in the bottom layer, respectively. The number of sublattices in each layer is described by \( N \) and \( L \)/equal 12 represents the thickness of the bilayer. Additional calculations were performed to determine the value of \( N \), but no significant difference was found in the simulation results when \( N \) is changed from 20 to 100. Therefore, in order to save computer time and ensure accuracy of the results, we decide to set \( 2 \times 20 \times 20 \times 2 \) as the number of total sublattices for the bilayer. The Hamiltonian of the system is given as follows:

\[
H = -J_{ab} \sum_{<i,j>} S^z_{ia} S^z_{jb} - J_{cd} \sum_{<m,n>} \sigma^z_{mc} \mu^z_{nd} - J_{ac} \sum_{<i,m>} S^z_{ia} \sigma^z_{mc} - J_{bd} \sum_{<j,m>} S^z_{jb} \mu^z_{nd} - h(t) \left( \sum_i S^z_{ia} + \sum_j S^z_{jb} + \sum_m \sigma^z_{mc} + \sum_n \mu^z_{nd} \right)
\]  

(1)

where \( S^z_{ia}, S^z_{jb}, \sigma^z_{mc} \) and \( \mu^z_{nd} \) denote the \( z \)-components of the Ising spins of sublattices a, b, c and d, respectively. They can take the spin values of \( S^z_{ia(jb)} = \pm 2, \pm 1, 0, \sigma^z_{mc} = \pm \frac{3}{2}, \pm \frac{1}{2}, \mu^z_{nd} = \pm \frac{5}{2}, \pm \frac{3}{2}, \pm \frac{1}{2} \), respectively. Here, \( \langle \cdots \rangle \) indicates the summation of nearest neighbor spins in the same kind sublattices and between the different sublattices located in different adjacent layers. Then, \( J_{ab} <0 \) denotes the intralayer AFM exchange interaction constant between sublattices a and b in the top layer, and \( J_{cd} >0 \) represents the intralayer FM exchange interaction constant between sublattices c and d in the bottom layer. \( J_{ac} >0 \) and \( J_{bd} <0 \) are the interlayer exchange interaction constants between the adjacent sublattices a and b, c and d, respectively. In this work, the AFM exchange interaction constant \( J_{ab} \) was used as the reduced unit of energy and temperature and set \( |J_{ab}| = 1 \). Finally, the time-dependent oscillating magnetic field is given as the following form.

\[
h(t) = h_b + h_0 \sin(\omega t)
\]  

(2)

where \( h_b \) is the bias field of the time-dependent field, which plays a dominant role in improving the thermal stability of the system. Furthermore, from the point of view of dynamics, it can be considered as the conjugate field of dynamic order parameter \( Q \), just as the external magnetic field \( h \) is the conjugate field of magnetization in the thermodynamic case [52–54]. In the second item, \( h_0 \) and \( \omega \) stand for the amplitude and the angular frequency of the oscillating magnetic field, respectively.

The simulation method we used is the standard importance-sampling MC simulation based on the Metropolis algorithm [55]. In the \( xy \) plane of the structure, the periodic boundary condition is taken into account, while the free boundary condition is applied along the \( z \)-axis direction. The quantities we care about are as follows.

The instantaneous sublattice magnetizations per spin \( M_a, M_b, M_c, \) and \( M_d \) are:

\[
M_a(t) = \frac{1}{N \times N} \langle \sum_i S^z_{ia} \rangle
\]  

(3)
Fig. 1 Schematic of an antiferromagnetic/ferromagnetic mixed-spin (5/2, 2, 3/2) Ising model with the YMnO$_3$/FM bilayer structure. Four exchange interactions are considered in the structure. $J_{ab}$ and $J_{cd}$ denote the intralayer antiferromagnetic and ferromagnetic exchange interactions of different spins in the top layer between sublattices a and b, and in the bottom layer between sublattices c and d, respectively. $J_{ac}$ and $J_{bd}$ represent the interlayer exchange interactions between sublattices a and c, b and d, namely

$$M_b(t) = \frac{1}{N \times N} < \sum_j S_{jb}^z > \quad (4)$$

$$M_c(t) = \frac{1}{N \times N} < \sum_m \sigma_{mc}^z > \quad (5)$$

$$M_d(t) = \frac{1}{N \times N} < \sum_n \mu_{nd}^z > \quad (6)$$

and the instantaneous magnetization of the total system per spin $M$ is:

$$M(t) = \frac{M_a(t) + M_b(t) + M_c(t) + M_d(t)}{4} \quad (7)$$

Thus, the average dynamic order parameter per spin can be calculated by:

$$Q = \frac{\omega}{2\pi} \oint M(t) dt \quad (8)$$

The dynamical average internal energy per spin $U$ is:

$$U = \frac{\omega}{2\pi} \oint H(t) dt \quad (9)$$

Finally, the susceptibility $\chi$ of the system per spin is:

$$\chi = N^2 \beta \left( \langle Q^2 \rangle - \langle Q \rangle^2 \right); \quad (10)$$

here, $\beta = \frac{1}{k_B T}$, where $k_B$ and $T$ represents the Boltzmann constant and the absolute temperature, respectively. In this work, we select $k_B = 1$ for the convenience of calculation.

For the purpose to ensure the reliability of data, we present in Fig. 2 the average dynamic order parameter $Q$ as a function of Monte Carlo steps (MCS) with fixed $J_{cd} = 1.0$, $J_{ac} = 0.8$, $J_{bd} = -0.5$, $h_b = 0.5$, $h_0 = 1.0$, $\omega = 0.02 \pi$ and $T = 2.0$. It is noted that our data were generated with $5 \times 10^4$ MCS per spin for calculating the value of $Q$, but it keeps almost constant ($Q = 0.98544$) after processing the first $3 \times 10^4$ MCS. Thus, we used the rest $2 \times 10^4$ MCS to
Fig. 2 The average dynamic order parameter ($Q$) versus Monte Carlo steps (MCS) for $J_{cd} = 1.0$, $J_{ac} = 0.8$, $J_{bd} = -0.5$, $h_b = 0.5$, $h_0 = 1.0$, $\omega = 0.02\pi$, and $T = 2.0$.

Fig. 3 The temperature dependences of $Q$, $U$, $\chi$, and the diagram of $T_C$ for different $J_{ac}$ with $J_{cd} = 1.0$, $J_{bd} = -0.5$, $\omega = 0.02\pi$, $h_0 = 1.0$, and $h_b = 0.5$. 

- **(a)** shows the temperature dependence of $Q$ with varying $J_{ac}$.
- **(b)** illustrates the temperature dependence of $U$.
- **(c)** depicts the temperature dependence of $\chi$.
- **(d)** presents the diagram of $T_C$ for different $J_{ac}$.
Fig. 4 The temperature dependences of $Q$, $U$, $\chi$ and the diagram of $T_C$ for different $J_{bd}$ on the condition of $J_{cd} = 1.0$, $J_{ac} = 0.8$, $\omega = 0.02\pi$, $h_0 = 1.0$, and $h_b = 0.5$

calculate the average values of thermodynamic quantities to make the system in equilibrium. In addition, the angular frequency $\omega$ is measured in the unit of MCS$^{-1}$. We should mention that all the dynamic order parameters and dynamic energy we obtained were averaged over the entire period of the oscillating magnetic field.

3 Simulation results and discussions

3.1 Dynamic order parameter, susceptibility, internal energy, and phase diagrams

In this subsection, the dynamic order parameter($Q$), susceptibility($\chi$), internal energy($U$), and the critical temperature ($T_c$) of the present AFM/FM bilayer system under the influence of the intralayer and interlayer exchange interactions as well as the parameters ($h_b$, $h_0$, $\omega$) of the time-dependent oscillating magnetic field $h(t)$ are presented in Figs. 3, 4, 5, 6, 7, 8.

Firstly, for fixed values of $J_{cd} = 0.1$, $J_{bd} = -0.5$, $\omega = 0.02\pi$, $h_0 = 1.0$ and $h_b = 0.5$, the influence of $J_{ac}$ on the $Q$, $\chi$, $U$, and $T_C$ is presented. In Fig. 3a, six smooth curves are plotted in the ($Q$, $T$) plane for different values of $J_{ac}$ changed from 0.1 to 2.1. Only one saturation ($Q = 1.0$) is observed when $T$ is near zero and every $Q$ curve has a similar trend, that is, for lower $T$, all of them decrease slowly, and then decrease sharply with $T$ increasing, finally, stabilize at a constant value because of the existence of the bias magnetic field $h_b$. 

Springer
Fig. 5 The temperature dependences of $Q$, $U$, $\chi$ and the diagram of $T_C$ for different $J_{cd}$ with $J_{ac} = 0.8$, $J_{bd} = -0.5$, $\omega = 0.02 \pi$, $h_0 = 1.0$, and $h_b = 0.5$

Moreover, at a certain $T$, the larger $J_{ac}$ is beneficial to the increase of $Q$. Similar behaviors of $Q$ curves can be seen in the mixed-spin-1/2 and spin-3/2 ferrimagnetic system [28] and the nanographene system [31]. The result indicates that $J_{ac}$ is less likely to cause a change in the saturation value of $Q$. From Fig. 3b, all $U$ curves also have analogous change from low to high. And at a certain $T$, as $J_{ac}$ goes up, $U$ goes down. This phenomenon can be physically explained that when $J_{ac}$ increases, the mutual binding force between the spins would be stronger, leading to that the spins would be more ordered. Accordingly, the internal energy is lowered and the system becomes more stabilized. Similar $U$ behavior can be seen in Refs. [43, 44, 46]. The $\chi$ curves of the system are displayed in Fig. 3c. Every curve has a peak that corresponds to a certain temperature, which is called the critical temperature $T_C$ [19, 23, 24]. Apparently, the peak of the curve moves to the right with $J_{ac}$ increasing. The phenomenon can be analyzed from the perspective of energy. For stronger $J_{ac}$, if the spins become disordered, the higher temperature should be needed against the stronger exchange interaction energy between spins, leading to a higher $T_C$. Finally, the phase diagram in the $(J_{ac}, T)$ plane is illustrated in Fig. 3d. The phase diagram is divided into two regions by the $T_C$ curve. The region below the $T_C$ curve corresponds to the ordered system with the ferrimagnetic phase, whereas above the $T_C$, the disordered state is shown in the system, exhibiting the paramagnetic phase. The relation between $T_C$ and $J_{ac}$ is almost linear, which is similar to those results found in Refs. [29, 44].

Figure 4a–d presents the temperature dependence of the $Q$, $U$, $\chi$, and phase diagram under the influence of $J_{bd}$ with fixed $J_{cd} = 1.0$, $J_{ac} = 0.8$, $\omega = 0.02 \pi$, $h_0 = 1.0$ and $h_b = 0.5$. 
In Fig. 4a, when $3 \leq T \leq 10$, six $Q$ curves are seen clearly and the values of them become greater as $J_{cd}$ increases at a certain $T$. Comparing the $U$ curves in Figs. 4b and 3b, the great difference between them is that $J_{bd}$ has a more obvious influence on $U$ than $J_{ac}$, which can be summarized from the relatively dispersed $U$ curves in Fig. 4b. The $\chi$ curves are presented in Fig. 4c, and the change in $T_C$ under the influence of $J_{bd}$ is shown in Fig. 4d. When $J_{bd} \leq -1.5$, the $T_C$ curve decreases sharply. Actually, with the gradual decrease of $|J_{bd}|$, most of the spins bound by the small exchange interaction are released and reverse in a higher energy state, so $T_C$ will decrease correspondingly. The results can be comparable with those obtained in Refs. [19–21, 50, 56].

Then, let us discuss the effect of $J_{cd}$ on the $Q$, $U$, $\chi$ and $T_C$ of the system with fixed values of $J_{ac} = 0.8$, $J_{bd} = -0.5$, $\omega = 0.02 \pi$, and $h_0 = 1.0$. From Fig. 5a, we can see that when $T$ is lower ($T \leq 1$), all $Q$ curves tend to be constant, and as $J_{cd}$ increases, the temperature range corresponding to a constant $Q$ increases continuously. For instance, when $J_{cd} = 2.0$, the temperature range is about $0 \leq T \leq 5$. From a physical point of view, the $Q$ behavior exists under the competition between temperature, exchange interaction, and magnetic field. The larger $J_{cd}$ is more dominant in the competition among the three, which can restrict the spins to stay a high spin state longer, thus leading to the longer temperature independence of $Q$ [40–46]. Figure 5c presents a maximum in the low-temperature zone in the $\chi$ curve labeled $J_{cd} = 0.1$, which comes from the rapid drop in the $Q$ curve in Fig. 5a.
Fig. 7 The temperature dependences of $Q$, $U$, $\chi$ and the diagram of $T_C$ for different $h_0$ with $J_{cd} = 1.0$, $J_{ac} = 0.8$, $J_{bd} = -0.5$, $\omega = 0.02 \pi$ and $h_b = 0.5$.

Fig. 5d, as $J_{cd}$ increases from 0.1 to 1.0, $T_C$ goes up continuously and slowly. It means that the larger $J_{cd}$ is more conducive to the improvement of the thermal stability of the system.

Figures 6, 7, 8 describe various magnetic properties of the system under the effects of the time-dependent oscillating magnetic field. First of all, Fig. 6a–d reveals the variation of the $Q$, $U$ and $\chi$ with $T$ under the influence of $h_b$ as well as the phase diagram in the $(h_b, T_C)$ plane when $J_{cd} = 1.0$, $J_{ac} = 0.8$, $J_{bd} = -0.5$, $\omega = 0.02 \pi$ and $h_0 = 1.0$. Obviously, one can see from Fig. 6a that the bias magnetic field $h_b$ can enhance the value of $Q$ at a certain $T$ and the residual value of $Q$, suggesting that the $h_b$ has an effective ability of improving the thermal stability of the system, which is similar to that of exchange interaction. The results have been also observed in Refs. [31, 57–60]. As shown in Fig. 6b, the increase in $h_b$ is advantageous for the decline in $U$. Figure 6c and d reflects the influence of the change of $h_b$ on $T_C$. One can notice from Fig. 6d that $T_C$ increases linearly as $h_b$ increases. The reason for this is that increasing $h_b$ is beneficial to magnetic order of the system so that a higher $T_C$ temperature can be achieved to make the system disorder. Our results are consistent with those of Refs. [22, 31, 32, 48].

Next, Fig. 7a–c shows the $Q$, $U$, $\chi$ as a function of $T$ for the selected values of $h_0$ with fixed $J_{cd} = 1.0$, $J_{ac} = 0.8$, $J_{bd} = -0.5$, $\omega = 0.02 \pi$, $h_b = 0.5$, and Fig. 7d presents the phase diagram in the $(h_0, T_C)$ plane. In Fig. 7a, it is clear that all $Q$ curves start with the same saturation value and also end up with the same constant, which is different from the $Q$ behaviors obtained in Fig. 6a under the influence of $h_b$. From Fig. 7b, we can notice the
small fluctuation in the $U$ curves at low $T$, because $h_0$ is dominant in the competition with the temperature. From Fig. 7c and d, we can observe that a continuous increase in $h_0$ will cause a gradual decrease in $T_C$. We can remark $h_0$ can destroy the magnetic order of the system, which has an opposite effect to $h_b$. Comparable results have also been obtained in the studies of dynamic magnetic properties [28, 31, 32, 35, 57].

Finally, Fig. 8a–d exhibits the effect of $\omega$ on the $Q$, $U$, $\chi$, and $T_C$ with selected values of $J_{cd} = 1.0$, $J_{ac} = 0.8$, $J_{bd} = -0.5$, $h_0 = 1.0$, and $h_b = 0.5$. Figure 8a shows that the $Q$ is sensitive to $\omega$ only over a very small temperature range ($4 \leq T \leq 7$), and increases with the increase of $\omega$; however, when $T$ is relatively low ($T < 4$) and relatively high ($T > 7$), the change of $\omega$ affects $Q$ very slightly. Apparently, for relatively small value of $\omega$ ($\omega = 0.008\pi$), the oscillating magnetic field has a long period acting on the system and relaxes the system to undergo the phase transition more easily, so the $T_C$ becomes lower. On the contrary, a relatively large $\omega$ means that a short period is applied to the present system, weakening the influence of the oscillating magnetic field on the system and making the system keep order longer, so the $T_C$ is higher. The similar phenomenon is also shown in the study of nanographene bilayer [28], two-dimensional square lattice [32], and double-layer core/shell graphene nanoisland [61]. Next, turning our attention to Fig. 8b, there is an apparent result that $\omega$ has a negligible effect on $U$. In order to see clearly, we enlarge the figure partially as an inset and find that the increase of $\omega$ is beneficial to the enlargement of $U$, and causes $T_C$ to increase continuously, as shown in Fig. 8c and d.
Fig. 9 The hysteresis loops of the antiferromagnetic/ferromagnetic bilayer for different values of (a) $J_{ac}$ with $J_{bd} = -0.5, J_{cd} = 1.0, h_0 = 1.0, \omega = 0.02\pi, T = 0.05$, (b) $J_{bd}$ with $J_{ac} = 0.8, J_{cd} = 1.0, h_0 = 1.0, \omega = 0.02\pi, T = 0.05$, and (c) $J_{cd}$ with $J_{ac} = 0.8, J_{bd} = -0.5, h_0 = 1.0, \omega = 0.02\pi, T = 0.05$

3.2 Hysteresis loops

Figure 9a–c presents the hysteresis loops of the system under the influence of various exchange interactions with other fixed parameters $h_0 = 1.0, \omega = 0.02\pi, T = 0.05$. Firstly, in Fig. 9a for fixed values of $J_{bd} = -0.5$ and $J_{cd} = 1.0$, four hysteresis loops with the same area are observed when increasing the values of $J_{ac}$ from 0.1 to 2.0, which illustrates that $J_{ac}$ has a little effect on the hysteresis behaviors, while in Fig. 9b, changing the values of $J_{bd}$ from $-0.1$ to $-2.0$ with fixed $J_{ac} = 0.8$ and $J_{cd} = 1.0$, the area of the loops gets bigger first and then stays unchanged for $J_{bd} = -1.4$, which means that a larger $J_{bd}$ does not cause a significant change in the loop. Figure 9c exhibits the effect of $J_{cd}$ on the hysteresis loop with the other two fixed values of $J_{ac} = 0.8$ and $J_{bd} = -0.5$. It should be pointed out that the area of the loop always increases continuously as $J_{cd}$ increases from 0.1 to 2.0. In these above subfigures, only single-loop hysteresis behavior can be found, which is comparable to those obtained in Refs. [27, 62, 63].
Fig. 10 The hysteresis loops of the antiferromagnetic/ferromagnetic bilayer for different values of (a) $h_0$ with
\[ \omega = 0.02\pi, J_{cd} = 1.0, J_{ac} = 0.8, J_{bd} = -0.5, \text{ and } T = 0.05 \],
(b) $\omega$ with $h_0 = 1.0, J_{cd} = 1.0, J_{ac} = 0.8, J_{bd} = -0.5, \text{ and } T = 0.05$.

Figure 10a, b shows the influence of the $h_0$ and $\omega$ of the oscillating field on the hysteresis loops with fixed $J_{cd} = 1.0, J_{ac} = 0.8, J_{bd} = -0.5, \text{ and } T = 0.05$. In Fig. 10a, for $\omega = 0.02\pi$, an interesting behavior is seen that the increase in $h_0$ will decrease the area of the loops, which is different from the results under the influence of exchange interaction exhibited in Fig. 9a–c. The impact of $\omega$ on the hysteresis behavior is presented in Fig. 10b when $h_0 = 1.0$. No matter how $\omega$ changes, the area, number, and pattern of the hysteresis loops will not change significantly.

Lastly, we consider the temperature dependence of the hysteresis loop when other parameters are fixed $J_{cd} = 1.0, J_{ac} = 0.8, J_{bd} = -0.5, \omega = 0.02\pi$, and $h_0 = 1.0$. In Fig. 11a–f, the area of the loops decreases with $T$ increasing from 0.05 to 3.9. We can remark that $T$ has a negative effect on the hysteresis loops. Eventually, the loop vanishes when $T \geq 3.9$ as shown in Fig. 10f, which indicates that the system turns into a paramagnetic state. As we all know, the increase of $T$ can promote the disorder of the system, resulting in the diminution in the area of loops. Similar results have been found in some other researches of the ferroelectric or ferrielectric materials [23, 46–48, 51] and other magnetic materials [45, 64–72].

4 Conclusion

By applying Monte Carlo simulation, we study the dynamic magnetic properties of the mixed-spin $(5/2, 2, 3/2)$ Ising YMnO$_3$/FM bilayer in a time-dependent oscillating magnetic field. The results show that the system can display interesting $Q$, $U$, $\chi$ behaviors, depending on the competition among the exchange interaction, temperature, and time-dependent oscillating magnetic field. $T_C$ can be improved by increasing $J_{ac}$, $|J_{bd}|$, $J_{cd}$, $h_b$, and $\omega$, but the increase of $h_0$ will make $T_C$ go down. Under the effects of the exchange interaction and temperature, the hysteresis behaviors are obtained. We should mention that, for such the bilayer system, the finite size effect can be found when considering the effect of layer thickness, which is apparently distinct from the bulk system. It would be just an interesting study that we will
Fig. 11 The hysteresis loops of the antiferromagnetic/ferromagnetic bilayer for different values of $T$ with $J_{cd} = 1.0$, $J_{ac} = 0.8$, $J_{bd} = -0.5$, $\omega = 0.02\pi$, and $h_0 = 1.0$ make a great deal of energy to investigate the difference between the bilayer and the bulk system in future.

Acknowledgements This project was supported by the General Project of Liaoning Provincial Department of Education, China (Grant No. LJGD2019013), and National College Students Innovation and Entrepreneurship Training Program (Grant No. 202010142012).
Data Availability Statement  The raw data that support the findings of this study cannot be shared because it is related to some of the research we are currently conducting, so we will not disclose these raw data based on privacy considerations.

References

1. S.H. Baek, H.W. Jang, C.M. Folkman, Y.L. Li, B. Winchester, J.X. Zhang, Q. He, Y.H. Chu, C.T. Nelson, M.S. Rzchowski, X.Q. Pan, R. Ramesh, L.Q. Chen, C.B. Eom, Nat. Mater. 9, 309 (2010)
2. T. Choi, Y. Horibe, H.T. Yi, Y.J. Choi, W. Wu, S.W. Cheong, Nat. Mater. 9, 253 (2010)
3. R. Ramesh, N.A. Spald, Nat. Mater. 6, 21 (2007)
4. G. Catalan, J.F. Scott, Adv. Mater. 21, 2463 (2009)
5. Y. Zhang, Z. Li, C.Y. Deng, J. Ma, Y.H. Lin, C.W. Nan, Appl. Phys. Lett. 92, 152510 (2008)
6. S.B. Cheng, M.L. Li, S.Q. Deng, S.Y. Bao, P.Z. Tang, W.H. Duan, J. Ma, C. Nan, J. Zhu, Adv. Funct. Mater. 26, 3589 (2016)
7. S.C. Abrahams, Acta Cryst. 57, 485 (2001)
8. R. Safi, H. Shokrollahi, Prog. Solid State Chem. 40, 6 (2012)
9. O. Nirmala, P.S. Reddy, V.D. Reddy, Mater. Today 23, (2019).
10. Y. Ma, X.T. Wang, Z.F. Wang, Q. Zhou, C.Y. Yang, Rare Metal Mat. Eng. 44, 52 (2015)
11. H. Kitahata, K. Tadanaga, T. Minami, N. Fujimura, T. Ito, J. Am. Ceram. Soc. 81, 1357 (1998)
12. I.E. Housni, R. Khalladi, H. Labrim, S. Ziti, L. Bahmad, Appl. Phys. A 9, 582 (2019)
13. Y. Benhouria, I. Bouziani, I. Essaoudi, A. Ainane, A. Oubelkacem, M. Saber, F. Dujardin, Phys. Scr. 86, 045704 (2012)
14. A. Feraoun, A. Zaim, M. Kerouad, J. Phys. Chem. Solid. 96–97, 75 (2016)
15. B. Deviren, M. Keskin, Phys. Lett. A. 376, 1011 (2012)
16. W.D. Baez, T. Datta, Phys. Procedia. 4, 15 (2010)
17. M. Ertaş, Superlatt. Microstruct. 85, 734 (2015)
18. E. Vatansever, H. Polat, J. Magn. Magn. Mater. 392, 42 (2015)
19. M. Keskin, E. Kantar, J. Magn. Magn. Mater. 322, 2789 (2010)
20. M. Ertaş, M. Keskin, Phys. B. 470–471, 76 (2015)
21. Y. Benhouria, I. Bouziani, I. Essaoudi, A. Ainane, A. Oubelkacem, M. Saber, F. Dujardin, Phys. Scr. 86, 045704 (2012)
48. W. Wang, L. Sun, R.D. Li, Z.Y. Gao, F. Wang, M. Tian, Res. Phys. 19, 103573 (2020)
49. A. Jabar, R. Masrou, Sol. Stat. Commun. 268, 38 (2017)
50. Y. Benhouria, I. Essaoudi, A. Ainane, R. Ahuja, F. Dujardin, Ferroelectrics 507, 58 (2017)
51. A. Feraoun, A. Zaim, M. Kerouad, Sol. Stat. Commun. 248, 88 (2016)
52. P. Riego, P. Vavassoriac, A. Bergera, Phys. B. 549, 13 (2018)
53. D.T. Robb, P.A. Rikvold, A. Berger, M.A. Novotny, Phys. Rev. E 76, 021124 (2007)
54. D.T. Robb, A. Ostrander, Phys. Rev. E 89, 022114 (2014)
55. N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller, E. Teller, J. Chem. Phys. 21, 1087 (1953)
56. W. Wang, H.J. Wu, P.N. Xie, J.H. Lv, Z. Peng, Eur. Phys. J. Plus 135, 605 (2020)
57. J.D. Alzate-Cardona, H. Barco-Ríos, E. Restrepo-Parra, Phys. Lett. A 382, 792 (2018)
58. Y. Benhouria, N. Khossossi, M. Houmad, I. Essaoudi, A. Ainane, R. Ahuja, Phys. E 105, 139 (2019)
59. H.S.S.R. Matte, K.S. Subrahmanya, C.N.R. Rao, J. Phys. Chem. C 113, 9982 (2009)
60. C.N.R. Rao, K.S. Subrahmanya, H.S.S.R. Matte, B. Abdulhakeem, A. Govindaraj, B. Das, Sci. Technol. Adv. Mater 11, 054502 (2010)
61. H.J. Wu, W. Wang, D. Lv, C.L. Chang, B.C. Li, M. Tian, J. Magn. Magn. Mater. 515, 167306 (2020)
62. L. Wang, B.H. Teng, Y.H. Rong, Y. Lv, Z.C. Wang, Sol. Stat. Commun. 152, 1641 (2012)
63. D. Lv, Y. Ma, X.H. Luo, W. Jiang, F. Wang, Q. Li, Phys. E 116, 113721 (2020)
64. W. Wang, Q. Li, D. Lv, R.J. Liu, Z. Peng, S. Yang, Carbon 120, 313 (2017)
65. L. Sun, W. Wang, C. Liu, B.H. Xu, D. Lv, Z.Y. Gao, Superlatt. Microstruct. 149, 106775 (2021)
66. L. Sun, W. Wang, Commun. Theor. Phys. 72, 115703 (2020)
67. L. Sun, W. Wang, Q. Li, F. Wang, H.J. Wu, Superlatt. Microstruct. 147, 106701 (2020)
68. W. Wang, L. Sun, Q. Li, D. Lv, Z.Y. Gao, T. Huang, J. Magn. Magn. Mater. 527, 167692 (2021)
69. D. Lv, W. Jiang, Y. Ma, Z.Y. Gao, F. Wang, Phys. E 106, 101 (2019)
70. D. Lv, Y. Yang, W. Jiang, F. Wang, Z.Y. Gao, M. Tian, Phys. A 514, 319 (2019)
71. D. Lv, F. Wang, R.J. Liu, Q. Xue, S.X. Li, J. Alloys Compd. 701, 935 (2017)
72. D. Lv, D.Z. Zhang, M. Yang, F. Wang, J. Yu, Superlatt. Microstruct. 151, 106833 (2021)