Probing the Perturbative NLO Parton Evolution in the Small–x Region

M. Glück, C. Pisano, E. Reya

Universität Dortmund, Institut für Physik, D-44221 Dortmund, Germany

Abstract

A dedicated test of the perturbative QCD NLO parton evolution in the very small–x region is performed. We find a good agreement with recent precision HERA–data for $F_2^p(x, Q^2)$, as well as with the present determination of the curvature of $F_2^p$. Characteristically, perturbative QCD evolutions result in a positive curvature which increases as $x$ decreases. Future precision measurements in the very small $x$–region, $x < 10^{-4}$, could provide a sensitive test of the range of validity of perturbative QCD.
Parton distributions \( f(x, Q^2) \), \( f = q, \bar{q}, g \), underlie a \( Q^2 \)-evolution dictated by perturbative QCD at \( Q^2 \gtrsim 1 \text{ GeV}^2 \). It was recently stated \cite{1} that the NLO perturbative QCD \( Q^2 \)-evolution disagrees with HERA data \cite{2,3} on \( F_2^p(x, Q^2) \) in the small–\( x \) region, \( x \lesssim 10^{-3} \). In view of the importance of this statement we perform here an independent study of this issue. In contrast to \cite{1} we shall undertake this analysis in the standard framework where one sets up input distributions at some low \( Q^2_0 \), here taken to be \( Q^2_0 = 1.5 \text{ GeV}^2 \), corresponding to the lowest \( Q^2 \) considered in \cite{1}, and adapting these distributions to the data considered. In the present case the data considered will be restricted to

\[
1.5 \text{ GeV}^2 \leq Q^2 \leq 12 \text{ GeV}^2, \quad 3 \times 10^{-5} \lesssim x \lesssim 3 \times 10^{-3}
\]  

as in \cite{1} and will be taken from the corresponding measured \( F_2^p(x, Q^2) \) of the H1 collaboration \cite{2}. The choice of these data is motivated by their higher precision as compared to corresponding data of the ZEUS collaboration \cite{3}, in particular in the very small–\( x \) region.

We shall choose two sets of input distributions based on the GRV98 parton distributions \cite{4}. In the first set we shall adopt \( u_\nu, d_\nu, s = \bar{s} \) and \( \Delta \equiv \bar{d} - \bar{u} \) from GRV98 and modify \( \bar{u} + \bar{d} \) and the gluon distribution in the small–\( x \) region to obtain an optimal fit to the H1 data \cite{2} in the aforementioned kinematical region. We shall refer to this fit as the ‘best fit’. The second choice will be constrained to modify the GRV98 \( \bar{u} + \bar{d} \) and \( g \) distributions in the small–\( x \) region as little as possible. We shall refer to this fit as GRV\(_{\text{mod}}\). It will turn out that both input distributions are compatible with the data to practically the same extent, i.e. yielding comparable \( \chi^2/dof \). In view of these observations we do not agree with the conclusions of ref. \cite{1}, i.e. we do not confirm a disagreement between the NLO \( Q^2 \)-evolution of \( f(x, Q^2) \) and the measured \cite{2,3} \( Q^2 \)-dependence of \( F_2^p(x, Q^2) \).

The remaining flavor–singlet input distributions at \( Q^2_0 = 1.5 \text{ GeV}^2 \) to be adapted to
the recent small–$x$ data are expressed as

$$xg(x, Q^2_0) = N_g x^{-a_g} \left( 1 + A_g \sqrt{x} + 7.283x \right) (1 - x)^{4.759} \quad (2)$$

$$x(\bar{u} + \bar{d})(x, Q^2_0) = N_s a^{-a_s} \left( 1 + A_s \sqrt{x} - 4.046x \right) (1 - x)^{4.225} \quad (3)$$

where the parameters relevant for the large $x$–region, $x > 10^{-3}$, which is of no relevance for the present small–$x$ studies, are kept unchanged and are taken from, e.g. GRV98 [4]. The refitted relevant small–$x$ parameters turn out to be

‘best fit’ : $N_g = 1.70$, $a_g = 0.027$, $A_g = -1.034$

$N_s = 0.171$, $a_s = 0.177$, $A_s = 2.613$ \quad (4)

GRV mod : $N_g = 1.443$, $a_g = 0.125$, $A_g = -2.656$

$N_s = 0.270$, $a_s = 0.117$, $A_s = 1.70$ \quad (5)

to be compared with the original GRV98 parameters [4]: $N_g = 1.443$, $a_g = 0.147$, $A_g = -2.656$ and $N_s = 0.273$, $a_s = 0.121$, $A_s = 1.80$. The resulting predictions are compared to the H1–data [2] in Fig. 1. These results are also consistent with the ZEUS–data [3] with partly lower statistics. The corresponding $\chi^2/\text{dof}$ are 0.50 for the ‘best fit’ ($\text{dof} = 48$) and 0.94 for GRV mod ($\text{dof} = 50$), respectively. Our treatment of the heavy flavor contributions to $F_2$ differs from that in [1]. We evaluate these contributions in the fixed flavor $f = 3$ scheme of [4], together with the massive heavy quark ($c, b$) contributions, rather than in the $f = 4$ (massless) scheme utilized in [4]. We have checked, however, that our disagreement with [1] does not result from our $f = 3$ plus heavy quarks vs. the $f = 4$ massless quark calculations in [1]: we have also performed a fit for $f = 4$ massless quarks and the results for $F_2$ and its curvature, to be discussed below, remain essentially unchanged.

In Figs. 2 and 3 we show our gluon and sea input distributions in (2) and (3), as well as their evolved shapes at $Q^2 = 4.5$ GeV$^2$ in the small–$x$ region. It can be seen that both
of our new small–$x$ gluon distributions at $Q^2 = 4.5$ GeV$^2$ conform to the rising shape obtained in most available analyses published so far, in contrast to the valence–like shape obtained in [1] where the gluon density $xg$ decreases as $x \to 0$. It is possible to conceive a valence–like gluon at some very–low $Q^2$ scale, as in [4], but even in this extreme case the gluon ends up as non valence–like at $Q^2 > 1$ GeV$^2$, in particular at $Q^2 = 4.5$ GeV$^2$, as physically expected.

Turning now to the curvature test of $F_2$ advocated and discussed in [1], we first present in Fig. 4 our results for $F_2(x, Q^2)$ at $x = 10^{-4}$, together with two representative expectations of global fits [5, 6], as a function of

$$q = \log_{10} \left( 1 + \frac{Q^2}{0.5 \text{ GeV}^2} \right).$$

(6)

This variable has the advantage that most measurements lie along a straight line [1] as indicated by the dotted line at $x = 10^{-4}$ in Fig. 4. The MRST01 parametrization [5] results in a sizable curvature for $F_2$ in contrast to all other fits shown in Fig. 4. This large curvature, incompatible with the data presented in [1], is mainly caused by the valence–like input gluon distribution of MRST01 at $Q_0^2 = 1$ GeV$^2$ in the small–$x$ region which becomes even negative for $x < 10^{-3}$ [5]. A similar result was obtained in [1] based on a particular gluon distribution $xg(x, Q^2)$ which decreases with decreasing $x$ for $x \lesssim 10^{-3}$ even at $Q^2 = 4.5$ GeV$^2$ (cf. fig. 7 in [1]). More explicitly the curvature can be directly extracted from

$$F_2(x, Q^2) = a_0(x) + a_1(x)q + a_2(x)q^2.$$  

(7)

The curvature $a_2(x) = \frac{1}{2} \partial^2_q F_2(x, Q^2)$ is evaluated by fitting the predictions for $F_2(x, Q^2)$ at fixed values of $x$ to a (kinematically) given interval of $q$. In Fig. 5(a) we present $a_2(x)$ which results from experimentally selected $q$–intervals [1]:

$$0.7 \leq q \leq 1.4 \quad \text{for} \quad 2 \times 10^{-4} < x < 10^{-2}$$

$$0.7 \leq q \leq 1.2 \quad \text{for} \quad 5 \times 10^{-5} < x \leq 2 \times 10^{-4}$$

$$0.6 \leq q \leq 0.8 \quad \text{for} \quad x = 5 \times 10^{-5}.$$  

(8)
Notice that the average value of $q$ decreases with decreasing $x$ due to the kinematically more restricted $Q^2$ range accessible experimentally. For comparison we also show in Fig. 5(b) the curvature $a_2(x)$ for an $x$–independent fixed $q$–interval

$$0.6 \leq q \leq 1.4 \quad (1.5 \text{ GeV}^2 \leq Q^2 \leq 12 \text{ GeV}^2) .$$

Apart from the rather large values of $a_2(x)$ specific for the MRST01 fit as discussed above (cf. fig. 4), our ‘best fit’ and GRV$_{\text{mod}}$ results, based on the inputs in (4) and (5), respectively, do agree well with the experimental curvatures as calculated and presented in [1] using H1 data. It should be noted that perturbative NLO evolutions result in a positive curvature $a_2(x)$ which increases as $x$ decreases. This feature is supported by the data shown in fig. 5(a); since the data point at $x < 10^{-4}$ is statistically insignificant, future precision measurements in this very small $x$–region should provide a sensitive test of the range of validity of perturbative QCD evolutions.

Furthermore, the H1 collaboration [2] has found a good agreement between the perturbative NLO evolution and the slope of $F_2(x, Q^2)$, i.e. the first derivative $\partial_{Q^2} F_2$.

To conclude, the perturbative NLO evolution of parton distributions in the small–$x$ region is compatible with recent high–statistics measurements of the $Q^2$–dependence of $F_2^p(x, Q^2)$ in that region. A characteristic feature of perturbative QCD evolutions is a positive curvature $a_2(x)$ which increases as $x$ decreases (cf. fig. 5). Although present data are indicative for such a behavior, they are statistically insignificant for $x < 10^{-4}$. Future precision measurements and the ensuing improvements of the determination of the curvature in the very small $x$–region should provide further information concerning the detailed shapes of the gluon and sea distributions, and perhaps may even provide a sensitive test of the range of validity of perturbative QCD.

This work has been supported in part by the ‘Bundesministerium für Bildung und Forschung’, Berlin/Bonn.
References

[1] D. Haidt, *Eur. Phys. J.* C35, 519 (2004)

[2] C. Adloff et al., H1 Collab., *Eur. Phys. J.* C21, 33 (2001)

[3] S. Chekanov et al., ZEUS Collab., *Eur. Phys. J.* C21, 443 (2001)

[4] M. Glück, E. Reya, A. Vogt, *Eur. Phys. J.* C5, 461 (1998)

[5] A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne, *Eur. Phys. J.* C23, 73 (2002)

[6] J. Pumplin et al., *JHEP* 7, 12 (2002) [hep-ph/0201195]
Figure 1: Comparison of our ‘best fit’ and GRV\textsubscript{mod} results based on (4) and (5), respectively, with the H1 data \cite{2}. To ease the graphical representation, the results and data for the lowest bins in $Q^2 = 1.5 \text{ GeV}^2$ and $2 \text{ GeV}^2$ have been multiplied by 0.75 and 0.85, respectively, as indicated.
Figure 2: The gluon distributions at the input scale $Q^2_0 = 1.5$ GeV$^2$ corresponding to (2) with the ‘best fit’ and GRV$_{\text{mod}}$ parameters in (4) and (5), respectively, and at $Q^2 = 4.5$ GeV$^2$. For comparison, the original GRV98 results [4] are shown as well by the dotted curves.
Figure 3: The sea distribution $x(\bar{u} + \bar{d})$ at the input scale $Q_0^2 = 1.5$ GeV$^2$ in (3) with the 'best fit' and GRV$_{\text{mod}}$ parameters in (4) and (5), respectively, and at $Q^2 = 4.5$ GeV$^2$. For comparison, the original GRV98 results [4] are shown as well by the dotted curves.
Figure 4: Predictions for $F_2(x, Q^2)$ at $x = 10^{-4}$ plotted versus $q$ defined in (6). Representative global fit results are taken from MRST01 [5] and CTEQ6M [6]. Most small-$x$ measurements lie along the straight (dotted) line [1].
Figure 5: The curvature $a_2(x)$ as defined in (7) for (a) the variable $q$–intervals in (8) and (b) the fixed $q$–interval in (9). Also shown are the corresponding MRST01 results [5]. The experimental curvatures (squares) shown in (a) are taken from [1].