Fuzzy Adaptive Backstepping Control of Nonlinear Uncertain Systems With Unmeasured States and Input Saturation

KAI-YU HU1,2, AALY YUSUF2, AND ZIAN CHENG1
1College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China
2School of Engineering Science, University of Chinese Academy of Sciences, Beijing 100101, China
Corresponding author: Kai-Yu Hu (hukaiyuluran@126.com; hkywuye@163.com)
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ABSTRACT  Considering the uncertainties caused by multiple concurrent complex environments, we investigate the robust adaptive control of multi-source uncertain systems under unmeasurable states and input saturation. For the unmeasurable states caused by sensor faults, a state observer is proposed to provide state estimation values for the controller; for the compound uncertain functions composed of nonlinear uncertain parameters and environmental disturbances, an optimal fuzzy approximation strategy is designed to reproduce these uncertain functions. Finally, the state observer with optimal approximation errors is proved to be stable. Subsequently, the low-pass filter, dynamic surface, and backstepping joint control scheme ensures the robust stability of the uncertain systems, while avoiding the differential explosion caused by disturbances. A series of hierarchical adaptive laws enable the controller to adjust parameters to achieve an optimal tracking of the entire process of time-varying reference output. The compensation algorithm designed with a Nussbaum function solves the problem of input saturation caused by actuator faults. Finally, the nonlinear uncertain systems based on fuzzy adaptive backstepping achieve robust tracking control of time-varying signals. The stability is proved by Lyapunov functions, and the effectiveness of the method simulation is verified using hypersonic vehicle model as an example.

INDEX TERMS  Uncertain systems, robust stability, approximation error, adaptive control, state observer.

I. INTRODUCTION
Most systems will face modeling control problems caused by multiple complex environmental factors simultaneously: stronger nonlinearity, uncertain parameters, and external disturbances [1]–[4]. Such systems have higher requirements for the robustness and adaptive performance of the control algorithm, designing the control algorithm is a challenge [5]. It is necessary to study the nonlinear robust control system with compound uncertainties that meets the requirements. Owing to the high cost of the real systems, the controller needs help to enhance reliability. When the sensors fail, and the states are not measurable, the controllers should be able to achieve stable tracking of the commands [6]. When performing extreme sports, the actuator will malfunction or saturate because of the physical structure limitation. Controller design should consider this input limited control requirement [7]. Therefore, To solve the robust control problem of nonlinear systems under complex disturbances and parameter uncertainties, and to solve the problem of reliable control when sensor and actuator faults cause unmeasurable state and input saturation, this study proposes a controller design that meets the above-mentioned requirements, then provides a robust and a highly reliable control system.

For the nonlinear system with strict feedback, the backstepping method solves the problem of system stability analysis and control design. Backstepping is a robust control method that can decompose a high-order nonlinear system into several low-order subsystems [8]. The backstepping method begins with the last-order subsystem after designing the virtual control law to stabilize the subsystem, then designing and stabilizing the previous-order subsystem including the last-order subsystem; thus, it is called the stepwise backward method [9]. This method has obvious advantages in
the robust control of uncertain systems. However, it also has following issues. First, the virtual controllers that repeatedly seek higher derivatives will cause computational expansion problems; the design process then requires accurate system functions, which may not be available in engineering [10]. To solve the problem of computational expansion, the existing literature proposes the dynamic surface control (DSC), which simplifies the computational complexity by passing the virtual control variable through a first-order filter [11]. With the development of the backstepping method, adaptive fuzzy backstepping theory has also been used in strictly feedback nonlinear systems [12]. This theory has the following advantages in nonlinear control system. First, it does not require accurate models of the systems and is robust to uncertain functions; second, it does not require the systems to meet the matching conditions; third, it does not need the linearization of unknown nonlinear functions [13]–[15]. Therefore, in the design of controllers for nonlinear uncertain systems, the adaptive fuzzy backstepping method is typically used.

Owing to the physical characteristics of the actuator’s mechanical structure, the actuator signal has a saturation constraint problem [16]. One method to solve this issue is to ignore the influence of saturation constraints and directly limit the amplitude of the control signal after it is solved; this method is complicated in ensuring system stability [17]. A more suitable method would be that considers the saturation limitation of the system in the controller design process. In [18], a wavelet network anti-saturation adaptive backstepping controller was designed for a class of nonlinear systems with limited inputs. In [19], a time-varying sliding mode control scheme for uncertain nonlinear systems with limited saturation was discussed. In [20], based on the composite state and disturbance observers, an output-feedback controller was designed by introducing an auxiliary system to eliminate the effect of input saturation. In [21], the robust semiglobal coordinated control was proposed for multiagent systems with input saturation and input additive disturbance. In [22], a radial basis function neural network controller approximated the system uncertainties, and the input saturation was controlled by designing an auxiliary system. To solve the problem of input saturation, this study uses the hyperbolic tangent function to reconstruct the input saturation and introduces the Nussbaum function to compensate for the input saturation limitation of the system in the controller design process. The aforementioned results provide a solution for the unmeasured states and saturation but lacks uncertain control scheme. Taking hypersonic flight vehicle (HFV) as an example, this study examines the uncertain robust control, solves the problem of unmeasured states and saturation. The main contributions are as follows:

1) A multi-state nonlinear uncertain system is established. For a compound uncertain function composed of nonlinear uncertain terms and disturbances, optimal fuzzy logic is used for online approximation and the system stability under the condition of approximation error is proved.

2) Targeting the control problem under compound uncertain conditions, a fuzzy adaptive backstepping algorithm is designed. By improving the virtual control and adaptive laws, the robust control of the nonlinear uncertain system is completed without using the precise model and matching conditions.

3) An adaptive observer is designed to estimate the unmeasured states. This provides premise variables for fuzzy approximation, virtual control, and adaptive laws. The hyperbolic tangent function and Nussbaum function reconstructs the input saturation and compensates the nonlinearity caused by saturation, respectively.

The rest of this paper is organized as follows. Section 2 introduces the multi-state attack angle control systems, input saturation and compound uncertainties; Section 3 shows the state observer and its stability proof; Section 4 shows the adaptive backstepping controller and its stability proof; Section 5 uses HFV model simulation to verify the effectiveness of the method; Section 6 summarizes the full text.

II. MODELS AND PROBLEMS

To study nonlinear control technology with disturbance and parameter uncertainties, the following multi-state single input single output (SISO) model is established:

\[
\begin{align*}
\dot{x}_1(t) &= f_1(x_1(t)) + g_1(x_1(t))u(t) + d_1(t) \\
\dot{x}_n(t) &= f_n(x_n(t)) + g_n(x_n(t))u(v(t)) + d_n(t) \\
y(t) &= x_1(t)
\end{align*}
\]

where \(x(t) = [x_1(t), \ldots, x_i(t)]^T \in \mathbb{R}^i \) is the system states; \(y \in \mathbb{R} \) is the system output; \(f_i(\cdot) \) and \(g_i(\cdot) \) are the smooth uncertain nonlinear functions; \(v(t) \) is the control input variable; \(d(t) \) is the compound disturbance caused by the coupling of the system uncertain parameters and external environment; \(u(v(t)) \) is a control signal with input saturation constraints expressed...
as:

$$u(v(t)) = \text{sat}(v(t)) = \begin{cases} \text{sign}(v(t))u_M, & |v(t)| \geq u_M \\ v(t), & |v(t)| < u_M \end{cases}$$

(2)

where $u_M$ is the constraint value of the controller.

Remark 1: To ensure the universality of the method, this paper constructs a general system model (1). After the pro-of is completed, the experiment (Section 5) will show the process of deriving system (1) from a traditional hypersonic cruise vehicle. System (1) will be given specific aerodynamic parameters, attitude angle and rudder deflection angle and other HFV physical coefficients in the experiment to verify the method is valid.

Using the approximate effect of the hyperbolic tangent function on the saturation function, the smooth function $h(v(t))$ is defined as follows: $v(t)$ is abbreviated as $v$, and the remaining variables are treated similarly:

$$h(v) = u_M \times \text{tanh} \left( \frac{v}{u_M} \right) = u_M \frac{e^{v/u_M} - e^{-v/u_M}}{e^{v/u_M} + e^{-v/u_M}}$$

(3)

Then the saturation function can be expressed as:

$$u(v) = h(v) + d(v)$$

(4)

where

$$d(v) = \text{sat}(v) - h(v)$$

(5)

The constraint value of (36) is written as follows:

$$|d(v)| = |\text{sat}(v) - h(v)| \leq u_M(1 - \tanh(1)) = D$$

(6)

The purpose of this study is to allow the system output $y$ track the reference signal $y_r$. The sign of $g_i(\cdot)$ in (1) is set to positive without any loss of generality. According to the nature of the smooth function, there exist functions $g_{low,i}, g_{up,i} \geq 0$, such that $g_{low,i} \leq g_i(\cdot) \leq g_{up,i}$ holds.

Some assumptions are clear before controller design:

Assumption 1: The reference signal $y_r$ is smooth, and its first and second derivatives exist and are bounded, that is, there is a defined normal number $cm \in R$, such that:

$$||y_r - \hat{y}_r|| \leq cm \quad (7)$$

where $|| \cdot ||$ represents the L2 norm of a matrix or vector.

Assumption 2: The compound disturbances $d_i$ is unknown but bounded, i.e., $|d_i| < c_i$ and $c_i$ is unknown.

Assumption 3: There is one constant $g_{bd} > 0$, such that $|g_i(\cdot)| \leq g_{bd}$ is established, and the constant $g_{bd}$ is unknown.

According to Equation (4), system (1) is equivalent to the following form:

$$\begin{cases} \dot{x}_i = f_i(x_i, x_{i+1}) + d_i \\ \dot{\hat{x}}_n = f_n(x_n, u) + d_{com} \\ y = x_1 \end{cases}$$

(8)

The system compound disturbances satisfy:

$$d_{com} = g_n d(v) + d_n$$

(9)

where $d_n$ is the additive disturbance, $g_n$ is the multiplicative uncertainty parameter, the combination is defined as the compound disturbances, which directly acts on the virtual control variable and indirectly affects the previous steps.

A further simplification of system (1) can be obtained as:

$$\begin{cases} \dot{x}_i = f'_i(x_i, x_{i+1}) + x_{i+1} \\ \dot{\hat{x}}_n = f'_n(x_n, u) + u \\ y = x_1 \end{cases}$$

(10)

where $f_i'(\cdot)$ is the compound uncertain functions with uncertain parameters $f_i(\cdot)$ and compound disturbances:

$$\begin{cases} f'_i(x_i, x_{i+1}) = f_i(x_i) + (g_i(x_i) - 1)x_{i+1} + d_i, \\ f'_n(x_n, u) = f_n(x_n) + (g_n(x_n) - 1)u + d_{com,n} \\ u = h(v) \end{cases}$$

(11)

The disturbances are expressed as follows:

$$d_{com,n} = d_{com} = d_n + g_n(x_n)d(v)$$

(12)

Assumption 4: $f'_i(\cdot)$ satisfies the Lipschitz condition, that is, a series of known positive integers $m_i$ that makes the following inequality true:

$$|f'_i(X_1) - f'_i(X_2)| \leq m_i \|X_1 - X_2\|$$

(13)

where $X_1, X_2 \in R^n$.

Assumption 5: The actuator is low-pass, i.e. $x_{i,f} = H_L(s)x_i \approx x_i$. Therefore, the existence of a constant $\beta_{i,0}$ makes the following inequality true:

$$\|x_i - x_{i,f}\| \leq \beta_{i,0}$$

(14)

Studying the control method of the uncertain systems under unmeasured states and input saturation will avoid the damage caused by faults. In (10), we assume that only output $y$ is measurable. Therefore, the control goal is to propose a state observer to estimate the unmeasured states, compensate for input saturation, and design an adaptive backstepping controller to make the system robustly track the reference signal $y_r$ under compound uncertainties. In Section 3, the design of the state observer is explained.

III. STATE OBSERVER DESIGN

Since only the output $y$ is measurable, and the state variables $x_2, \ldots, x_n$ are not directly measurable, so an observer is designed to estimate the states. Fuzzy state observer is:

$$\begin{cases} \hat{x}_i = \hat{x}_{i+1} + k_i(y - \hat{x}_i) + \hat{f}_i'(\hat{x}_i, \hat{x}_{i+1}, \theta_i) \\ \hat{x}_n = k_n(y - \hat{x}_i) + \hat{f}_n'(\hat{x}_n, u, \theta_n) + u \\ \hat{y} = \hat{x}_1 \end{cases}$$

(15)

where $\hat{x}_i$ is the observation states, and the fuzzy system function fitting the compound uncertainties can satisfy:

$$\hat{f}_i'(\hat{x}_i, \hat{x}_{i+1}, \theta_i) = \theta_i^T \xi_i(\hat{x}_i, \hat{x}_{i+1})$$

(16)

The basic configuration of a fuzzy logic system consists of a fuzzifier, some fuzzy IF-THEN rules, a fuzzy inference
engine and a defuzzifier. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector \( x = [x_1, \ldots, x_n] \in \mathbb{R}^n \) to an output adjustable parameter vector \( \theta = [\theta_1, \ldots, \theta_f] \in \mathbb{R}^f \). The \( j \)th fuzzy rule is written as Rule \( j \): If \( x_1 \) is \( A_1^j \), and \( x_2 \) is \( A_2^j \), \ldots, \( x_n \) is \( A_n^j \), then \( \theta_j^f \), where \( A_1^j, A_2^j, \ldots, A_n^j \) are fuzzy variables and \( \theta_j^f \) is an element of adjustable parameter vector. By using product inference, center-average and singleton fuzzifier, the output of the fuzzy system can be expressed as

\[
\theta_j^f \xi_i = \begin{bmatrix} \theta_{1j}^f \\ \vdots \\ \theta_{nj}^f \end{bmatrix}^T \begin{bmatrix} \xi_{1i} \\ \vdots \\ \xi_{ni} \end{bmatrix}
\]

(17)

where fuzzy basis function \( \xi_{ji} \) satisfies

\[
\xi_{ji} = ((\prod_{j=1}^{n} \mu_{A_j}^*(x_j))/\sum_{j=1}^{r} (\prod_{j=1}^{n} \mu_{A_j}^*(x_j)))
\]

(18)

\( \mu^*(\cdot) \) is membership function value of the fuzzy variable \( A_j^i \), \( r \) is the number of fuzzy rules, \( j = 1, \ldots, r \) is any fuzzy rule. \( \hat{x}_{i+1,f} \) and \( u_f \) are the signals processed by the filter (19):

\[
\begin{aligned}
\hat{x}_{i,f} &= H_L(s)\hat{x}_i \\
u_f &= H_L(s)u
\end{aligned}
\]

(19)

where \( H_L(s) \) is the Butterworth low-pass filter (LPF). The expression of \( H_L(s) \) is as follows:

\[
H_L(s) = a_0/(s^r + a_{-1}s^{r-1} + \ldots + a_1s + a_0)
\]

(20)

And the parameters are set as follows:

To maintain calculation speed and retain more choices, the filter order is limited to any order below 5, a total of 4 filters. In Table 1, all the parameter values of the 4 filters are given. Controller selects the optimal filter order through online evaluation. The cutoff frequency is set to 1 rad/s.

This filter can be combined with the DSC method to solve the calculation expansion problem in the backstepping method, which can significantly improve the speed and practical value of the system.

Set

\[
A = \begin{bmatrix} -k_1 & \vdots & 0 \\ \vdots & I & \vdots \\ -k_n & 0 & \cdots & 0 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}, \quad B_n = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}, \\
B_i = [0 \cdots 1 \cdots 0]^T, \quad C = [1 \cdots 0 \cdots 0].
\]

TABLE 1. Parameters of \( H_L(s) \).

| Filter order (i) | \( a_4 \) | \( a_3 \) | \( a_2 \) | \( a_1 \) | \( a_0 \) |
|------------------|---------|---------|---------|---------|---------|
| 2                | 2.000   | 1.414   | 1.000   |         |         |
| 3                | 2.000   | 2.000   | 1.000   |         |         |
| 4                | 2.613   | 3.141   | 2.613   | 1.000   |         |
| 5                | 3.236   | 5.236   | 5.236   | 3.236   | 1.000   |

Then, select the matrix \( K \) such that \( A \) is Hurwitz matrix. The fuzzy state observer can be transformed into:

\[
\begin{aligned}
\hat{x}_i &= A\hat{x}_n + Ky + \sum_{i=1}^{n} B_i f_i^o(\hat{x}_i, \hat{x}_{i+1,f}(\theta_i)) + B_n u \\
\hat{y} &= C\hat{x}_n
\end{aligned}
\]

(21)

When \( i = 1 \), the element of the fuzzy system space established in (18) is:

\[
\hat{f}_1^o(\hat{x}_1, \hat{x}_{2,f}) = \theta_1^T \xi_1(\hat{x}_1, \hat{x}_{2,f})
\]

(22)

The optimal estimation of the parameter vector \( \theta_1 \) is defined as

\[
\theta_1^* = \arg \min_{\theta_1 \in \mathbb{R}^1} \{ \sup_{x \in \mathbb{R}} [\hat{f}_1^o(\hat{x}_1, \hat{x}_{2,f}(\theta_1)) - f_1^o(\hat{x}_1, \hat{x}_{2,f})] \}
\]

(23)

where \( \Omega_1 = \{ \theta_1 ||\theta_1|| \leq M_1 \}, M_1 \) is the upper bound of the selected free parameter modulus. We can further define the minimum approximation error \( \varepsilon_1 \) of the parameter \( \theta_1 \) as

\[
\varepsilon_1 = f_1^o(\hat{x}_1, \hat{x}_{2,f}) - \hat{f}_1^o(\hat{x}_1, \hat{x}_{2,f}(\theta_1))
\]

(24)

The spatial elements and their corresponding parameters with other values of \( i \) can be obtained in a similar manner. Nonlinear \( f_i^o \) to be approximated is uncertain. After setting a certain set of basis functions, the most suitable \( \theta_i \) cannot be determined in advance offline. So it is necessary to optimize \( \theta_i \) in real time and obtain its optimal estimation \( \theta_i^* \), then replace the true value with the estimation.

Similarly, let \( \hat{x}_{n+1,f} = u_f \), system (10) can be transformed into the following form:

\[
\begin{aligned}
\dot{\hat{x}}_i &= H_L(s)\hat{x}_i \\
u_f &= H_L(s)u \\
\hat{y} &= C\hat{x}_n
\end{aligned}
\]

(25)

where

\[
\Delta f_i = f_i^o(x_i, x_{i+1,f}) - \hat{f}_i^o(\hat{x}_i, \hat{x}_{i+1,f})
\]

(26)

The stability of the established fuzzy state observer is analyzed below:

Set the observer error to:

\[
e = x_n - \hat{x}_n
\]

(27)

From (21) and (25), we can obtain:

\[
\dot{e} = Ae + \sum_{i=1}^{n} B_i[\delta_i + \Delta f_i]
\]

(28)

where

\[
\delta_i = f_i^o(\hat{x}_i, \hat{x}_{i+1,f}) - \hat{f}_i^o(\hat{x}_i, \hat{x}_{i+1,f}(\theta_i))
\]

(29)

\( \delta_i \) is the approximation error of the fuzzy system. In the above selected Hurwitz matrix, \( A \), for a positive definite matrix \( Q \), there is a positive definite matrix \( P \) such that the Raccati equation shown below holds.

\[
A^T P + PA = -2Q
\]

(30)

Select the Lyapunov function \( V_0 \) as follows:

\[
V_0 = \frac{1}{2} e^T Pe
\]

(31)
Then the derivative of $V_0$ is:

$$V_0 = \frac{1}{2} e^T P e + \frac{1}{2} \dot{e}^T P \dot{e}$$

(32)

Substitute (28) and (30) into (32), then

$$\dot{V}_0 \leq -\lambda_{\min}(Q) \|e\|^2 + e^T P (\delta + \Delta F)$$

(33)

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of the positive $Q$, $\delta = [\delta_1, \ldots, \delta_n]^T$, $\Delta F = [\Delta f_1, \ldots, \Delta f_n]^T$. According to the convention of fuzzy approximation, Assumption 6 is reasonable [31].

Assumption 6: There is an unknown constant $\delta_1^*$, and the approximation error $\delta$ of the fuzzy system satisfies $|\delta_i| \leq \delta_i^*$. After the optimal estimation of the parameter vector, there exists an unknown constant $\epsilon_i^*$ that makes the fuzzy system approximation error $\epsilon_i$ satisfy the inequality $|\epsilon_i| \leq \epsilon_i^*$.

According to the Young’s inequality and assumptions 4 to 6, the following inequality holds:

$$e^T P \delta \leq \frac{1}{2} \|e\|^2 + \frac{1}{2} \|P\|^2 \|\delta\|^2 \leq \frac{1}{2} \|e\|^2 + \frac{1}{2} \|P\|^2 \|\delta^*\|^2$$

(34)

$$e^T P \Delta F \leq \frac{1}{2} \|e\|^2 + \frac{1}{2} \|P\|^2 \|\Delta F\|^2$$

$$\leq \frac{1}{2} \|e\|^2 + \frac{1}{2} \|P\|^2 \sum_{i=1}^{n} |\Delta f_i|^2$$

$$\leq \frac{1}{2} \|e\|^2 + \frac{1}{2} \|P\|^2 \sum_{i=1}^{n} m_i^2 \|e\|^2$$

$$+ \|P\|^2 \sum_{i=1}^{n} m_i^2 \eta_{i,0}^2$$

(35)

where $\delta^* = [\delta_1^*, \ldots, \delta_n^*]^T$. Integrating (33), (34), and (35) we obtain:

$$\dot{V}_0 \leq -r \|e\|^2 + M_0$$

(36)

and

$$r = \lambda_{\min}(Q) - 1 - \|P\| \sum_{i=1}^{n} m_i^2$$

$$M_0 = \frac{1}{2} \|P\|^2 (\|\delta^*\|^2 + 2 \sum_{i=1}^{n} m_i^2 \eta_{i,0}^2)$$

(37)

(38)

According to the Lyapunov exponential stability theorem, choosing the appropriate parameters so that $r > 0$ can make the observer stable, that is, when error satisfies $\|e\| > \sqrt{M_0/r}$, the error of the observation system converges.

**IV. CONTROLLER DESIGN AND STABILITY**

This section combines the backstepping and DSC methods to design the controller of compound uncertain systems.

**Step 1:** Since the state in the system cannot be directly measured, the feedback control law in the system cannot be designed with the true states. In the previous section, we showed that the observation error of the fuzzy state observer is convergent and therefore the observer states can be used for the controller design, which is equivalent to designing the actual system. The definition is as follows:

$$\begin{align*}
\dot{z}_1 &= y - y_r \\
\dot{z}_2 &= \hat{y}_r - \alpha_{i-1}
\end{align*}$$

(39)

where $\alpha_{i-1}$ is the virtual control variable, and the specific expression is provided below.

The derivative of variable $z_1$ is:

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r$$

$$= \dot{x}_2 + f_i(x_1, x_2) - \dot{y}_r$$

$$= \dot{x}_2 + f_i(x_1, \hat{x}_2) - y_r + e_2 + \Delta f_1$$

$$= \hat{x}_2 + \theta_i^T \xi_i(x_1, \hat{x}_2) - y_r + e_2 + \epsilon_1 + \Delta f_1$$

(40)

where $e_2 = x_2 - \hat{x}_2$ is the state estimation error.

Using formula (39), (40) can be transformed into:

$$\dot{z}_1 = z_2 + \alpha_1 + \theta_i^T \xi_i(x_1, \hat{x}_2) + \theta_i^T \xi_i(x_1, \hat{x}_2) - y_r + e_2 + \epsilon_1 + \Delta f_1$$

(41)

It can be seen from [8] that there is a continuous function $N_1(\cdot)$ in a certain interval that makes the following inequality true:

$$-\tilde{\alpha}_{vir,1} \leq N_1(\cdot)$$

(44)

And according to the nature of the continuous function, (45) is obtained as follows:

$$\tilde{\kappa}_1 \leq \frac{-\tilde{\kappa}_1}{T_1} + N_{max,1}$$

(45)

where $N_{max,1} = \max[N_1(\cdot)]$.

Select the Lyapunov function $V_1$ as follows:

$$V_1 = \frac{1}{2} \tilde{e}_1^2 + \frac{1}{2} \tilde{\kappa}_1^2 + \frac{1}{2} \frac{\tilde{\theta}_1^T}{T_1} \tilde{\theta}_1 + \frac{1}{2} \frac{\tilde{\theta}_1^T}{T_1} \tilde{\theta}_1$$

(46)

where

$$\tilde{\theta}_1 = \epsilon_i^* - \hat{e}_1$$

(47)

and $\hat{e}_1$ is the estimation of $\epsilon_i^*$. Then the derivative of $V_1$ is:

$$\dot{V}_1 = z_1 \dot{z}_1 + \kappa_1 \dot{\kappa}_1 - \frac{1}{r_1} \tilde{\theta}_1^T \tilde{\theta}_1 - \frac{1}{T_1} \tilde{\theta}_1^T \tilde{\theta}_1$$

$$\leq z_1 \dot{z}_2 + \alpha_1 + \theta_i^T \xi_i(x_1, \hat{x}_2) + \theta_i^T \xi_i(x_1, \hat{x}_2) - y_r + e_2 + \epsilon_1 + \Delta f_1 + \kappa_1 \left(\frac{-\tilde{\kappa}_1}{T_1} + N_{max,1} \right) - \frac{1}{r_1} \tilde{\theta}_1^T \tilde{\theta}_1 - \frac{1}{T_1} \tilde{\theta}_1^T \tilde{\theta}_1$$

$$\leq z_1 \dot{z}_2 + \alpha_{vir,1} + \theta_i^T \xi_i(x_1, \hat{x}_2) + \theta_i^T \xi_i(x_1, \hat{x}_2) - y_r$$

$$+ z_1 (\kappa_1 + e_2 + \Delta f_1) + \kappa_1 \left(\frac{-\tilde{\kappa}_1}{T_1} + N_{max,1} \right) - \frac{1}{r_1} \tilde{\theta}_1^T \tilde{\theta}_1$$

(48)
\[
- \frac{1}{\bar{r}_1} \bar{e}_1 + |z_1| e_1^* \leq 0 \tag{48}
\]

**Lemma 1 [20]:** For any \( \lambda_1^* > 0 \), the following formula holds:

\[
|x| - x \tanh(x/\lambda_1^*) \leq 0.2785 \lambda_1^* = \lambda_0 \tag{49}
\]

According to Lemma 1, the replacement symbol can be obtained:

\[
|z_1| - z_1 \tanh(z_1/\lambda_1^*) \leq \lambda_0 \tag{50}
\]

that is

\[
- z_1 \tanh(z_1/\lambda_1^*) \leq \lambda_0 - |z_1| \leq \lambda_0 \tag{51}
\]

Based on the above inequalities, (48) can be transformed into

\[
V_1 \leq z_1[z_2 + \alpha_{vir,1} + \theta_i^T \xi_1(\hat{x}_1, x_2,f) + \bar{\theta}_i^T \xi_1(\hat{x}_1, x_2,f) - \dot{y}_r + z_1(\kappa_1 + e_2 + \Delta f_i) + \kappa_2(\kappa_1 + e_2 + \Delta f_i) - \frac{1}{\bar{r}_1} \bar{\theta}_i \theta_i]
\]

\[
- \frac{1}{\bar{r}_1} \bar{e}_1 + |z_1| e_1^* + z_1 \tanh(z_1/\lambda_1^*) + z_1 e_1^* \tanh(z_1/\lambda_1^*)
\]

\[
\leq z_1[z_2 + \alpha_{vir,1} + \theta_i^T \xi_1(\hat{x}_1, x_2,f) + \bar{\theta}_i^T \xi_1(\hat{x}_1, x_2,f) - \dot{y}_r + \bar{e}_1 \tanh(z_1/\lambda_1^*)] + z_1(\kappa_1 + e_2 + \Delta f_i) + \kappa_2(\kappa_1 + e_2 + \Delta f_i) - \frac{1}{\bar{r}_1} \bar{\theta}_i \theta_i
\]

\[
+ \frac{1}{\bar{r}_1} \bar{e}_1^T (\bar{r}_1 z_1 \tanh(z_1/\lambda_1^*) - \bar{e}_1) + e_1^* \lambda_0 \tag{52}
\]

Simultaneously according to the Young inequality, the following inequalities are obtained:

\[
z_1 \kappa_1 \leq \frac{1}{2} \kappa_1^2 + \frac{1}{2} \kappa_1^2 \tag{53}
\]

\[
z_1 e_2 \leq \frac{1}{2} \kappa_1^2 + \frac{1}{2} \kappa_1^2 \leq \frac{1}{2} \kappa_1^2 + \frac{1}{2} \|e\|^2 \tag{54}
\]

\[
z_1 \Delta f \leq \frac{1}{2} \kappa_1^2 + \frac{1}{2} \|\Delta f\|^2 \leq \frac{1}{2} \kappa_1^2 + m_1^2 \|e\|^2 + m_2^2 \beta_2^2 \tag{55}
\]

\[
\kappa_1 N_{\text{max},i} \leq \frac{1}{2} \kappa_1^2 + \frac{1}{2} (N_{\text{max},i})^2 \tag{56}
\]

Substituting Equations (53), (54), (55) and (56) into (52), after sorting and merging, (57) is obtained:

\[
V_1 \leq z_1[z_2 + \alpha_{vir,1} + \theta_i^T \xi_1(\hat{x}_1, x_2,f) + \bar{\theta}_i^T \xi_1(\hat{x}_1, x_2,f) - \dot{y}_r + \bar{e}_1 \tanh(z_1/\lambda_1^*)]
\]

\[
+ \frac{3}{2} \kappa_1^2 + \frac{1}{2} \|e\|^2 + m_2^2 \beta_2^2 - (\frac{1}{\bar{r}_1}) \kappa_1^2 \tag{59}
\]

\[
+ \frac{1}{2} (N_{\text{max},i})^2 + \frac{1}{\bar{r}_1} \bar{\theta}_i^T (r_1 z_1 \xi_1(\hat{x}_1, x_2,f) - \dot{\theta}_i)
\]

\[
+ \frac{1}{\bar{r}_1} \bar{e}_1^T (\bar{r}_1 z_1 \tanh(z_1/\lambda_1^*) - \bar{e}_1) + e_1^* \lambda_0 \tag{57}
\]

Select the virtual control variable and parameter adaption law as follows:

\[
\alpha_{vir,1} = -c_1 z_1 - \theta_i^T \xi_1(\hat{x}_1, x_2,f) + \dot{y}_r - \bar{e}_1 \tanh(z_1/\lambda_1^*) \tag{58}
\]

\[
\dot{\theta}_i = \text{proj}(r_1 z_1 \xi_1) \tag{59}
\]

Taking Equations (58), (59) and (60) into (57), we obtain:

\[
\dot{V}_1 \leq -c_1 \frac{3}{2} \kappa_1^2 + z_1 z_2 + M_1 - (1 - \kappa_1^2) \tag{61}
\]

where

\[
M_1 = \frac{1}{2} (N_{\text{max},i})^2 + (m_1^2 + \frac{1}{2}) \|e\|^2 + m_2^2 \beta_2^2 + e_1^* \lambda_0 \tag{62}
\]

Step i (2 \leq i \leq n): The derivative of \( z_i \) is:

\[
z_i = \dot{z}_i - \dot{\alpha}_i - \dot{\bar{e}}_i + k_i e_1 + \theta_i^T \xi_1(\hat{x}_i, \bar{x}_{i+1,f}) - \dot{\alpha}_i - \dot{\bar{e}}_i
\]

\[
+ \bar{\theta}_i^T \xi_1(\hat{x}_i, \bar{x}_{i+1,f}) - \dot{\alpha}_i - \dot{\bar{e}}_i
\]

where \( w_i = e_i - \delta_i \) is the error of the fuzzy system.

Select the auxiliary virtual control variable \( \alpha_{vir,i} \) and use the following first-order filter to eliminate noise:

\[
\dot{\alpha}_i = \alpha_{vir,i}
\]

Filter error \( \kappa_i = \alpha_i - \alpha_{vir,i} \). Differentiate \( \kappa_i \):

\[
\dot{\kappa}_i = \dot{\alpha}_i - \dot{\alpha}_{vir,i} \tag{64}
\]

Select Lyapunov function \( V_i \) as follows:

\[
V_i = \frac{1}{2} \kappa_i^2 + \frac{1}{2} \|e_i\|^2 + \frac{1}{2} \|	heta_i^T \xi_1\| + \frac{1}{2} \|w_i^T \bar{w}_i\| \tag{65}
\]

where

\[
\bar{w}_i = w_i^* - \bar{w}_i \tag{67}
\]

Derivative \( V_i \), substitute (63) and (65) into this derivative:

\[
\dot{V}_i = z_i \dot{z}_i + \kappa_i \dot{\kappa}_i - \frac{1}{r_1} \theta_i^T \dot{\theta}_i - \frac{1}{r_1} \bar{w}_i \bar{\dot{w}}_i \tag{68}
\]

According to Lemma 1 and the Young’s inequality, (68) can be transformed into:

\[
\dot{V}_i \leq z_i \dot{z}_i + \kappa_i \dot{\kappa}_i + k_i e_1 + \theta_i^T \xi_1(\hat{x}_i, \bar{x}_{i+1,f}) - \dot{\alpha}_i - \dot{\bar{e}}_i
\]

\[
+ \bar{\theta}_i^T \xi_1(\hat{x}_i, \bar{x}_{i+1,f}) + |z_i| w_i^* - |z_i| w_i^* \tanh(z_i/\lambda_i^*)
\]

\[
+ z_i \kappa_i^2 \tanh(z_i/\lambda_i^*) - \kappa_i^2 \|e_i\|^2 + N_{\text{max},i} \kappa_i - \frac{1}{r_1} \theta_i^T \theta_i
\]

\[
- \frac{1}{r_1} \bar{w}_i \bar{\dot{w}}_i \tag{68}
\]

According to Lemma 1 and the Young’s inequality, (68) can be transformed into:
The auxiliary virtual control variable $\alpha_{vir,i}$ and the parameter-adaptation law are selected as follows:

$$\alpha_{vir,i} = -c_i z_i - z_{i-1} - k_i e_i - \theta_i^T \xi_i(\hat{x}_i, \hat{x}_{i+1,f}) + \hat{\alpha}_{i-1} - \frac{1}{T_i} \hat{w}_i\tan(z_i/\lambda_n^*)$$

(70)

$$\hat{\theta}_i = \text{proj}(r_i z_i \xi_i(\hat{x}_i, \hat{x}_{i+1,f}))$$

(71)

$$\hat{w}_i = r_i z_i \tan(z_i/\lambda_n^*)$$

(72)

Substituting (70), (71), and (72) into (69), hence

$$\dot{V}_i \leq -(c_i - \frac{1}{2}\kappa_n^2) z_i^2 - z_i z_{i-1} + z_{i+1} + M^T - (\frac{1}{\tau_n} - 1) \kappa_n^2$$

(73)

where

$$M^T = \frac{1}{2}(N_{\text{max},n})^2 + w_i^T \lambda_0,i$$

(74)

**Step n:** Define

$$z_{n+1} = h(v) - \alpha_n$$

(75)

The derivative of $z_n$ is:

$$\dot{z}_n = \hat{x}_n - \hat{\alpha}_{n-1} = k_n e_1 + \theta_n^T \xi_n(\hat{x}_n, u_f) + z_{n+1} + \alpha_n - \hat{\alpha}_{n-1}$$

(76)

Select the auxiliary virtual control variable $\alpha_{vir,n}$, and use the first-order filter as follows for filtering:

$$\begin{cases} \tau_n \dot{\alpha}_n + \alpha_n = \alpha_{vir,n} \\ \alpha_n(0) = \alpha_{vir,n}(0) \end{cases}$$

(77)

The filtering error is: $\kappa_n = \alpha_n - \alpha_{vir,n}$, and the derivative can be obtained as follows:

$$\dot{\kappa}_n = \dot{\alpha}_n - \dot{\alpha}_{vir,n} = \frac{\alpha_{vir.n} - \alpha_n}{\tau_n} - \dot{\alpha}_{vir,n} \leq -\frac{\kappa_n}{\tau_n} + N_{\text{max},n}$$

(78)

Select the Lyapunov function $V_n$ as:

$$V_n = \frac{1}{2} z_n^2 + \frac{1}{2} \kappa_n^2 + \frac{1}{2} \theta_n^T \theta_n + \frac{1}{2} w_n^T \dot{w}_n$$

(79)

Substituting (76) and (78) into the derivative of the function $V_n$, we can obtain:

$$\dot{V}_n = z_n \dot{z}_n + \kappa_n \dot{\kappa}_n - \frac{1}{r_n} \theta_n \dot{\theta}_n - \frac{1}{r_n} \dot{w}_n \dot{w}_n$$

$$\leq z_n[k_n e_1 + \theta_n^T \xi_n(\hat{x}_n, u_f) + z_{n+1} + \alpha_{vir,n} - \hat{\alpha}_{n-1}] + z_n \kappa_n$$

(80)

$$+ |z_n| w_n^* + \kappa_n \left(\frac{\kappa_n}{\tau_n} + N_{\text{max},n}\right) + \frac{1}{r_n} \theta_n^T \theta_n + \frac{1}{r_n} \dot{w}_n \dot{w}_n$$

$$- z_n w_n^* \tan(z_n/\lambda_n^*) + z_n w_n^* \tan(z_n/\lambda_n^*) + |z_n| w_n^*$$

According to Lemma 1 and the Young’s inequality, Equation (80) can be transformed into:

$$\dot{V}_n \leq z_n[k_n e_1 + \theta_n^T \xi_n(\hat{x}_n, u_f) + z_{n+1} + \alpha_{vir,n} - \hat{\alpha}_{n-1}] + z_n \kappa_n$$

$$- z_n w_n^* \tan(z_n/\lambda_n^*) + z_n w_n^* \tan(z_n/\lambda_n^*) + |z_n| w_n^*$$

$$+ \kappa_n \left(\frac{\kappa_n}{\tau_n} + N_{\text{max},n}\right) + \frac{1}{r_n} \theta_n^T \theta_n + \frac{1}{r_n} \dot{w}_n \dot{w}_n$$

(81)

The auxiliary virtual control variable $\alpha_{vir,n}$ and the parameter adaptation law are selected as follows:

$$\alpha_{vir,n} = -c_n z_n - z_{n-1} - k_n e_1 - \theta_n^T \xi_n(\hat{x}_n, u_f) + \hat{\alpha}_{n-1}$$

$$- \frac{1}{r_n} \theta_n^T (r_n z_n \xi_n(\hat{x}_n, u_f) - \hat{\theta}_n) + \frac{1}{r_n} \dot{w}_n \tan(z_n/\lambda_n^*)$$

(82)

$$\hat{\theta}_n = \text{proj}(r_n z_n \xi_n(\hat{x}_n, u_f))$$

(83)

$$\hat{w}_n = r_n z_n \tan(z_n/\lambda_n^*)$$

(84)

Substituting (82), (83), and (84) into (81), through integration we can obtain:

$$\dot{V}_n \leq -(c_n - \frac{1}{2}\kappa_n^2) z_n^2 - z_n z_{n-1} + z_n z_{n+1} + M_n - (\frac{1}{\tau_n} - 1) \kappa_n^2$$

(85)

where

$$M_n = 0.5(N_{\text{max},n})^2 + w_n^* \lambda_0,n$$

(86)

**Step n + 1:** Define

$$\dot{v} = -cv + \omega$$

(87)

$$\omega = N(\chi) \tilde{\omega}$$

(88)

$$\tilde{\omega} = -c_{n+1} z_{n+1} - z_n + \zeta c v + \hat{\alpha}_n$$

(89)

$$\dot{\chi} = \chi_m z_{n+1}$$

(90)

where $c > 0$ and $\gamma > 0$ is the design parameters; $N(\chi)$ is the Nussbaum gain function satisfying:

$$N(\chi) = \chi^2 \cos(\chi)$$

(91)

**Remark 2:** The Nussbaum gain function $N(\chi)$ satisfies the continuity and

$$\lim_{k \to \infty} \sup_k \frac{1}{k} \int_0^k N(\chi) d\chi = \infty$$

$$\lim_{k \to \infty} \inf_k \frac{1}{k} \int_0^k N(\chi) d\chi = -\infty$$

(92)

This property of $N(\chi)$ indicates that it is a globally reachable divergence function, and any value in $(-\infty, \infty)$ can be obtained from the output of $N(\chi)$ by adjusting $\chi$. Therefore, when the control input $v$ is saturated, the auxiliary control variable $\omega$ can be designed using the divergence of $N(\chi)$ to compensate for the limited $v$, so that theoretically any limited amplitude can be repaired by the controllable $N(\chi)$, and finally achieving the same ideal control effect as without saturation.

Furthermore, $c_{n+1} > 0$, $\zeta$ satisfies:

$$\zeta = \frac{\partial h(v)}{h(v)} = \frac{4}{(e^v/u_m + e^{-v/u_m})^2} > 0$$

(93)
FIGURE 1. Controller inversion design steps.

The dynamic characteristics of the intermediate variable \(z_{n+1}\) are:
\[
\dot{z}_{n+1} = \frac{\partial h(v)}{\partial y} - \dot{\alpha}_n = \zeta(-cv + \omega) - \dot{\alpha}_n
\]
\[
= -\zeta cv + (\zeta N(\chi) - 1)\tilde{\omega} - \dot{\alpha}_n + \dot{\omega}
\]
\[
= -c_{n+1}z_{n+1} - z_n + (\zeta N(\chi) - 1)\tilde{\omega}
\]  

(94)

Select the Lyapunov function \(V_{n+1}\) as:
\[
V_{n+1} = \frac{1}{2}z_{n+1}^2
\]  

(95)

Solve the derivative of \(V_{n+1}\):
\[
\dot{V}_{n+1} = \dot{z}_{n+1}z_{n+1} - c_{n+1}z_n^2 - z_nz_{n+1} + \frac{1}{\gamma_X}(\zeta N(\chi) - 1)\dot{\chi}
\]  

(96)

Considering the boundedness of all signals of the closed-loop system, the Lyapunov function \(V_{\text{sum}}\) is selected as follows:
\[
V_{\text{sum}} = \sum_{i=0}^{n+1} V_i = V_0 + \sum_{i=1}^{n+1} \frac{1}{2}z_i^2 + \sum_{i=1}^{n} \frac{1}{2}(k_i^2 + 1)\theta_i^T \theta_i + \frac{1}{2R_i} \tilde{\theta}_i^T \tilde{\theta}_i)
\]  

(97)

where \(\tilde{\theta}_i = \tilde{\epsilon}_i\). Derivate the function \(V_{\text{sum}}\) and combine (61) (73), (85), and (87); through integration, we can obtain:
\[
\dot{V}_{\text{sum}} \leq -(r - m_1^2 - \frac{1}{2}) \|e\|^2 - (c_1 - \frac{3}{2})\xi_1^2 - \sum_{i=2}^{n} (c_i - \frac{1}{2})\xi_i^2
\]
\[
- c_{n+1}z_{n+1}^2 - \sum_{i=1}^{n} \frac{1}{\tau_i} - 1)\xi_i^2 + \sum_{i=0}^{n} M_i^i
\]  

(98)

Let
\[
C = \min\{(r - m_1^2 - 0.5)/\lambda_{\text{max}}(P), 2c_1 - 3, 2c_i - 1, 2c_{n+1}, 2/\tau_i - 2\}
\]  

(99)

\[
M = \sum_{i=0}^{n} M_i^i
\]  

(100)

Then (98) can be transformed into:
\[
\dot{V}_{\text{sum}} \leq -CV_{\text{sum}} + M + \frac{1}{\gamma_X}(\zeta N(\chi) - 1)\dot{\chi}
\]  

(101)

Integrating (101) will obtain:
\[
V_{\text{sum}} \leq V_{\text{sum}}(0)e^{-Ct} + \frac{M}{C}(1 - e^{-Ct})
\]  

(102)

Lemma 2 [32]: \(V(\cdot)\) and \(\chi(\cdot)\) are smooth functions defined on \([0, t_f]\), for any \(t \in [0, t_f]\), \(V(t) \geq 0\), and \(N(\chi)\) is the Nussbaum gain function. If the inequality is true as shown in (102), the functions \(V(\cdot)\) and \(\chi(\cdot)\) are bounded on \([0, t_f]\).

According to Lemma 2, \(V_{\text{sum}}(\cdot)\) and \(\dot{\chi}(\cdot)\) are bounded, and the tracking error satisfies the following inequality:
\[
|y - y_d|^2/2 = \xi_t^2/2 \leq V_{\text{sum}} \Rightarrow |y - y_d| \leq \sqrt{2V_{\text{sum}}}
\]  

(103)

The design steps of the above controller can be summarized as shown in Figure 1.

Therefore, according to the design process of the above controllers, the following theorem can be obtained:

Theorem 1: For input-constrained nonlinear uncertain systems (10) satisfies Assumptions 3-6, design state observers (15), controller (87), and system-assisted virtual control variables (58), (70), (82), select the parameter adaptive laws (59), (60), (71), (72), (83), (84), then all signals of the closed-loop system are semi-global uniformly bounded, and the observation error and tracking error are bounded.

Proof: From the aforementioned proofs, we can obtain the conclusion of Theorem 1. The detailed proof is omitted.

Except for the \(n\)th step algorithm related to the virtual control variable \(v\) and the \(n+1\)th algorithm with preset auxiliary control variables, which can freely set clear expressions, the 1 to \(n-1\)th step algorithms depend on the next step state \(x_{i+1}\) to solve \(\alpha_i\), then obtains an explicit expression.

Remark 3: In the experiment, we determine the physical parameters according to the product indicators of the controlled object. Approximation parameters include free adjustable parameters and basis functions. In an ideal environment, the observer can be designed when the system does not have unmeasured states. And the uncertain function is not unknown at any time. A less accurate uncertain function is first fitted through a number of known coordinate points, then the basis functions and free adjustable parameters are selected to approximate the fitting function. Under the offline experimental calibration controller, it can be optimized based on the observation error and obtain the basis functions and free adjustment parameters. The design of adaptive control parameters at each step must satisfy Young’s inequality and
Lyapunov theory. Then, the parameters are adjusted repeatedly to obtain the more accurate tracking curves, finally the optimal parameters are obtained.

V. SIMULATION VERIFICATION EXPERIMENT

This section uses the cruise HFV to verify the above controllers. Figure 2 is the test site. Flight platform has nonlinearity and uncertainty, vibration table simulates interference.

\[ f_1 = \frac{1}{2I_{yy}} \rho V^2 S \hat{c} (C_M(\alpha) + C_M(q)) \]

\[ f_2 = \frac{1}{2I_{yy}} \rho V^2 S \hat{c} \]

Assuming that the measurement sensor of the pitch angle is faulty and cannot achieve accurate measurement, it is necessary to use the proposed fuzzy state observer to design the control algorithm. The design parameters of the fuzzy state observer are: \( k_1 = 10 \), \( k_2 = 18 \). The control parameters are: \( c_1 = 13 \), \( c_2 = 18 \), \( c_3 = 7 \), \( c = 6 \), \( \gamma_f = 0.008 \), \( \tau_1 = 0.015 \), \( \tau_2 = 0.045 \), \( \chi(0) = 0.4 \). Select the following fuzzy set functions:

\[ \mu_{1,1}(\alpha, \dot{y}_r) = \exp(-x + 0.1849 - 0.1675 \times l^2) / 0.03 \]

\[ \times \exp(-y + 0.56 - 0.16 \times l^2 / 0.02) \]

\[ \mu_{2,1}(\alpha, q) = \exp(-x + 0.1849 - 0.1675 \times l^2) / 0.03 \]

\[ \times \exp(-y + 0.6541 - 1.3080 \times l^2 / 0.04) \]

where \( l = 1, \ldots, 6 \). If \( \dot{\alpha}_1 < -15 \), then \( \mu_{3,1} = 1 \); if \( \dot{\alpha}_1 > 15 \), then \( \mu_{3,6} = 1 \); In other cases: \( \mu_{3,1} = \exp(-(\dot{\alpha}_1 + 21 \times l^2) / 3) \).

The deflection range of the rudder angle is \( \pm 20 \) deg, and the magnitude of the harmonic interference torque on the pitch axis is: \( 2 \times 10^6 \sin(2t) \). The traditional method (TM) in [17] shows strong robustness when only considering the presence of interference on the pitch axis. In this paper, the uncertainty of aerodynamic parameters is fully considered. To verify the effectiveness of our proposed new method (NM), a comparative test is carried out with TM.

TABLE 2. Performance comparison between TM and NM.

|                  | 0–5s, 5–11s, 11–16s | 0–5s, 5–11s, 11–16s |
|------------------|---------------------|---------------------|
| \( e_1 \)       | 1.56, 1.55, 1.55    | 0.01, 0.01, 0.009   |
| \( e_2 \)       | 1.04, 1.04, 1.03    | 0.001, 0.001, 0.001|
| \( e_{q,\max} \)| 2.41, 2.87, 2.66    | 0.03, 0.02, 0.03    |
| \( e_{q,r,\max} \)| 3.29, 4.64, 3.34    | 0.16, 0.26, 0.08    |
| \( e_{a,\max} \)| 1.64, 1.89, 1.74    | 0.02, 0.03, 0.02    |
| \( t_{q,\max} \) | —                   | 0.99, 0.89, 0.98    |
| \( t_{q,r,\max} \)| —                   | 0.01, 0.01, 0.004   |
| \( t_{a,\max} \)| —                   | 0.92, 1.12, 1.88    |

Table 2 lists the results of the approximation, estimation, and angle of attack tracking performance obtained after NM and TM are used in the HFV model in this section. Table 3 shows the optimization range of NM vs. TM. The calculation formula for the optimization percentage is:

\[ \text{Pec.} = |\text{TM} - \text{NM}| / \text{TM} \times 100\% \]

\( e_{q,\max} \), \( e_{q,r,\max} \), \( e_{a,\max} \), \( t_{q,\max} \), \( t_{q,r,\max} \), and \( t_{a,\max} \) in Table 2, respectively represent the maximum state estimation error of pitch angle and angular rate, maximum steady-state tracking error of attack angle, maximum estimation response time of pitch angle and angular rate, and maximum tracking response time of attack angle. The comparison shows that TM cannot ensure the stability of HFV under compound uncertainties,
the approximation and tracking errors are significantly large, and the response time cannot be collected owing to instability; the percentages in Table 3 further shows that NM greatly improves TM, NM in this study has excellent performance, the optimal approximation errors are significantly small, which make HFV stable estimation as well as a tracking status and output.

**TABLE 3. Optimization range of NM vs. TM.**

|   | 0-5s  | 5-11s | 11-16s |
|---|-------|-------|--------|
| $e_1$ | 99.36% | 99.35% | 99.42% |
| $e_2$ | 99.904% | 99.904% | 99.903% |
| $e_{q_{\infty}}$ | 98.76% | 99.3% | 98.87% |
| $e_{q_{\infty}}$ | 95.14% | 94.4% | 97.6% |
| $e_{q_{\infty}}$ | 98.78% | 98.41% | 98.85% |

**TABLE 4. Rudder deflection command comparison.**

|   | 0-5s | 5-11s | 11-16s |
|---|------|-------|--------|
| NM $\delta$/deg | 5.3 | 15.6 | -9.0 |
| TM $\delta$/deg | 13.1 | 34.1 | -20.9 |

Table 4 shows the input comparison of the proposed method and method of [17] that do not consider input saturation. NM has a significantly smaller rudder command at different periods of time; while TM requires a larger rudder angle, which is difficult to achieve under restricted input conditions. Therefore, the proposed method can adapt to a stricter input environment.

Figure 3 shows the minimum approximation error of the compound uncertain functions optimized by (23) for two loops ($\alpha$ and $q$). Both errors are small and bounded to meet the control requirements. Optimal approximation is a prerequisite for accurate state estimation and attack angle tracking control.

Figure 4 presents the estimation of the unmeasurable states: pitch angle and pitch rate. Although there are disturbances and the change of pitch angular velocity is fast, the fuzzy state observer designed in this study is still robust and accurate in tracking the pitch angle and angular rate.

Figure 5(a) shows the tracking result of the output angle of attack. The response is fast and stable. This shows that the combination of DSC and adaptive backstepping achieve real-time precise and robust control of time-varying output, as well as effectively avoids the problem of differential explosion under uncertain conditions. This enables fast response of HFV. Figure 5(b) shows the response curve of the rudder deviation. The solid line is the input command, and the broken line is the rudder deviation angle, which is within the limits and meets the control requirements under saturation. Therefore, the designed control algorithm can achieve the control goal with constrained rudder.
VI. CONCLUSION
In this study, we solved the problems of robust control with compound uncertainties, unmeasured states, and saturated control input. The optimal fuzzy theory approximates the uncertain functions online. Subsequently, for the saturation problem occurring in the controller, the hyperbolic tangent function with smooth characteristics perfectly described the relationship between control inputs and saturation functions. The introduced Nussbaum function compensated the nonlinearity caused by the input saturation. Moreover, this study helped in solving the differential explosion problem of the backstepping method, and used the LPF to eliminate the virtual control variables generated by the DSC, which effectively simplifies the calculation. Targeting the unmeasured states caused by sensor faults, a fuzzy state observer was used for online estimation. Finally, an adaptive backstepping controller designed based on this enables the HFV system to accurately track the expected output attack angle. In HFV, the proposed method provided a technical support for solving the undetectable states and input saturation problems caused by the sensor-rudder compound faults and showed a strong application prospect.

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KAI-YU HU received the B.E. degree in automation from Jilin University, Changchun, China, in 2012, and the M.S. degree in astrophysics from the University of Chinese Academy of Sciences, Beijing, China, in 2015. He is currently pursuing the Ph.D. degree with the Nanjing University of Aeronautics and Astronautics, Nanjing, China.

His research interests include adaptive control, non-Gaussian systems, and self-repairing control.

AALY YUSUF received the B.E. degree from Peking University.

He is currently a Researcher and a Chief Engineer of the Xinjiang Astronomical Observatory of National Astronomical Observatories, Chinese Academy of Sciences. He attends and manages a lot of important projects, such as China Lunar Exploration, Mars Exploration, 973, and NSFC. He is also hosting the project of upgrading for 25m antenna. His main research interests include adaptive control, intelligent power management system for more electric aircraft, modeling and control of active chilled beam for HVAC systems, switching and impulsive systems, and model-based online learning.

ZIAN CHENG was born in Xuzhou, Jiangsu. He graduated from Southwest Forestry University, Kunming, Yunnan. He is currently pursuing the Ph.D. degree with the Nanjing University of Aeronautics and Astronautics, Nanjing, China.

He has extensive experience in software design. His research interests include adaptive control, guaranteed performance control, and fault tolerant tracking control for hypersonic flight vehicle.

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