Analysis and Optimization of Opioid Drug Transmission Based on Spatial-time-based Model

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Abstract. The abuse of opioids has become a serious problem in the world. In order to help control the use of opioids, this paper established a new spatial-time-based model based on Logistic model and SIR model to analyse the source and threshold of opioids, and then used factor vector association method to study the social factors of drug abuse in the data of NFLIS and the U.S. socio-economic census. Further we optimized the spatial-time-based model and make sensitivity analysis of parameters of model.

1. Introduction

At present, synthetic and non-synthetic opioids are widely used as a painkiller in medical treatment. However, opioid abuse is also increasing public health problem, which has become a serious problem. Many researches show that the number of addicts has increased dramatically over the decades [1-5].

During the decades, some mathematic models are established to study the patterns of transmission of infectious diseases such as SIR model [6], SIS model [7], epidemics–logistics model [8] and so on. These models are based on the different states of the population. Under some assumptions, the population is divided into several warehouse chambers and then the differential equations are used to depict the changes of population state. Network nodes in SIS model are in two states: Susceptible state (healthy state) and infection state. In the process of virus propagation, the susceptible state may be converted to the infection state with a certain probability. The nodes of the infection state will be restored to the susceptibility state with a certain probability [9]. SIR model divides the crowd into three warehouse rooms, namely infected, susceptible and healing. This model applies to certain diseases such as smallpox-where patients cured after infection will not be re-infected again. Many subsequent models, including the SIRS model [10] are extended based on the two classic models. In addition, these models make homogenization hypothesis for the population.

However, the potential factors affect the transmission of epidemics. Hence, there is necessary for further treatment for analyzing these factors. We use the NFLIS database to establish a mathematical model and perform qualitative and quantitative analysis, identifying the spread and characteristics of the reported incidents as well as possible locations where opioid will be used. Through data from the United States socio-economic census, we revise the model to look for the socio-economic characteristics of cities and those who abuse drugs. Finally, we explore the prevention measures to solve the problem of opioid abuse.
2. Transmission Analysis of Model-based Opioid Drugs

Considering the spatial-time rules of opioids transmission, we will establish a new spatial-time-based model to analyze the sources and threshold of opioid drugs and then study the social correlation influence factors of drug abuse by the factor vector association method.

2.1. Sources and Threshold of Spatial-time-based Model for Opioid Drugs

2.1.1. Construction of Spatial-time-based Model. We assume that the changes of infected people by opioids can be fitted by using the logistic model and establish the following time-based model:

\[
\frac{dN(t)}{dt} = r(l - \frac{N(t)}{N_0})N(t)
\]

\[N(t = 0) = N_0\]  \hspace{1cm} (1)

where \(N_0\) is the number of infected people by opioids, \(t\) is time and its unit is the year, \(r\) is the success rate of drug infection. Thus \(N(t)\) can be given by the following formula:

\[
N(t) = \frac{K}{N_0 + (1 - \frac{1}{e^r})} \hspace{1cm} (2)
\]

Moreover, because of the changes in drug overdose death rates and the possibility of detoxification, we improve the model (1) into the following model:

\[
\frac{dF(t)}{dt} = -\lambda P(t)F(t)
\]

\[
\frac{dP(t)}{dt} = \frac{\lambda P(t)}{N} F(t) - \mu P(t)
\]

\[
\frac{dR(t)}{dt} = \mu P(t) \hspace{1cm} (3)
\]

where \(F(t)\) denotes the population not being infected, \(P(t)\) denotes the population already being infected, \(R(t)\) denotes the drug overdose death tolls and rehabilitations for drugs. \(\lambda\) denotes the success rate of opioids diffusion. \(\mu\) denotes the replacement rate of a certain opioid. \(N\) denotes the total population of a county. Then we get

\[
F(t) + P(t) + R(t) = N \hspace{1cm} (4)
\]

Based on the logistic model, we find that the amount of opioids infection will increase gradually, but they will not spread out in the beginning. Next, based on the convection, conduction and radiation of heat transfer pattern in the physics [11]. When the amount of the infected is up to a certain value, the diffusion of opioids begins.

The infected surrounding areas will become a new trigger. The physical model is given as follows.

\[
F(t, x, y) = \frac{\partial F(t, x, y)}{\partial T} = 0 \hspace{1cm} (5)
\]

The time interval for infection is determined by two factors: distance from birthplace and the time up to thresholds of diffusion. The following formula is established:

\[
G(t) = \gamma \frac{N(t)}{N} \hspace{1cm} (6)
\]

where \(N(t)\) represents the amount of people infected in the county when it is the time \(t\), \(\gamma\) denotes the influencing factor which is determined by crowd flow and urban radiation.

The spread of opioids in some area fit with the increase of the logistic model and relates to the own infection. The following spatial-time-based model is established:
\[ f_i(t+1) = f_i(t) + N(t) + G_{i-1}(t) \]

\[
\begin{align*}
 & \quad \frac{dN(t)}{dt} = r\left(1 - \frac{N(t)}{N}\right)N(t) \\
 & s.t. \quad G_{i-1}(t) = \frac{\lambda N(t)}{N}
\end{align*}
\]

(7)

where \( f_i(t) \) denotes the opioids infection of \( i \) area at the \( t \) moment. \( f_i(t+1) \) denotes the opioids infection of \( i \) area at the \( t+1 \) moment. \( G_{i-1}(t) \) denotes the opioids infection of \( i-1 \) county at the \( t \) moment.

We accumulate the drugs data in NFLIS because of the limited small data. Suppose the original data sequence is as follows:

\[ x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \]

Accumulate \( x^{(0)} \) to generate a new data sequence:

\[ x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)) = (x^{(0)}(1), x^{(0)}(1)+x^{(0)}(2), \ldots, x^{(0)}(1)+x^{(0)}(2)+\cdots+x^{(0)}(n)) \]

(9)

\( x^{(1)}(k) \) denotes that the accumulation of first \( k \) data in the data sequence \( x^{(0)} \).

\[ x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \cdots, n \]

(10)

Construct a cumulative matrix \( B \) and data vector \( Y \):

\[ \hat{u} = (a, b)^T = (B^T B)^{-1} B^T Y = \begin{bmatrix} 0.07625 \\ 7200.13 \end{bmatrix} \]

(11)

**Figure 1.** the threshold level.

Then \( a = 0.07625, b = 7200.13 \). Solve the differential equation \( \frac{dx^{(1)}}{dt} + ax^{(1)} = b \), so

\[ x^{(1)}(k+1) = (x^{(1)}(1) - \frac{b}{a})e^{-at} + \frac{b}{a} = 16445.9e^{0.07625k} - 16244.9 \]

(12)

2.1.2. Sources and Threshold of opioids. We view all kinds of opioids as a whole and neglect any alternation in our model. Now we integrate heroin addicts as a separate group and analyze the change of drug reports. Because of the difference on population toll in each county, it is more reasonable to replace total population with growth rate.

\[ N(t) = \frac{K}{1 + \left(\frac{K}{P_0} - 1\right)e^{-rt}} \]

After derivation of this equation, we derive the following formula:

\[
\frac{dN(t)}{dt} = \frac{-rK^2 \left(\frac{K}{P_0} - 1\right)e^{rt}}{\left[1 + \left(\frac{K}{P_0} - 1\right)e^{-rt}\right]^2}
\]

(14)
If \( \frac{d^2 N(i)}{d t^2} = 0 \), then the point of local maximum is:

\[
    t_m = \frac{\ln \left( \frac{K}{P_0} \right)}{r K} \tag{15}
\]

So we can make a conclusion that if population or rate of drug cases continuously grows, \( t_m \) will decrease. Hence, the threshold level in our model is \( \frac{t_m}{3} \), which is shown in Fig. 1.

### 2.2. Influence of Social Correlation Factors for Drug Abuse

#### 2.2.1. Factor Vector Association method.
To consider the changes at various time points, we establish a matrix vector \( x_{ij} \), where \( x_{ij} \) indicates the actual value of drug reports in the state \( i \) at the year of \( j \), \( \bar{x}_{ij} \) indicates predictive value of drug reports in the state \( i \) at the year of \( j \). Let \( \Delta x_{ij} = (x_{ij} - \bar{x}_{ij}) \), then

\[
    \begin{cases}
    \Delta x_{ij} > 0 > \delta, \Delta x_{ij} = 1; \\
    \Delta x_{ij} < \delta < 0, \Delta x_{ij} = -1 \\
    |\Delta x_{ij}| < |\delta|, \Delta x_{ij} = 0
    \end{cases} \tag{16}
\]

The dataset of a state can be shaped into a \( 1 \times n \) vector \( x = (x_{11}, x_{12}, x_{13}, \cdots) \).

#### 2.2.2. Analysis of Correlation Factors.
After vectors normalization, we get unit multidimensional vectors. Then we utilize the cosine similarity to measure the correlations between variables and get the correlations of main impact factors in each state. Finally, select all the values whose absolute values outweigh 0.5. The positive correlation denotes that the impact factor can boost the increase of drug reports, while the negative one denotes the opposite.

Table 1 shows that main people of drugs use are Female under 30 years old, especially divorced women, the elderly and college students. The used reasons are physiological reasons for women, diseases for the old people and mental reasons for teenagers.

| Main Impact Factors | Correlations |
|---------------------|--------------|
| Percent; RELATIONSHIP - Householder | -0.79839597 |
| GRANDPARENTS - Responsible for grandchildren - Years responsible for grandchildren - 3 or 4 years | -0.64100123 |
| Percent; SCHOOL ENROLLMENT - Kindergarten | 0.5050144 |
| Percent; SCHOOL ENROLLMENT - Elementary school (grades 1-8) | -0.5157953 |
| Percent; SCHOOL ENROLLMENT - High school (grades 9-12) | -0.60852865 |
| Percent; SCHOOL ENROLLMENT - College or graduate school | 0.526707606 |
| Percent; ANCESTRY - American | 0.689906566 |
| Percent; ANCESTRY - French Canadian | -0.78411407 |
| Percent; ANCESTRY - Subsaharan African | -0.61656047 |
| Percent; ANCESTRY - Ukrainian | -0.55678480 |
| Percent; ANCESTRY - Welsh | -0.50112997 |
| Percent; ANCESTRY - West Indian (excluding Hispanic origin groups) | 0.552917414 |
3. Optimization of Spatial-time-based Model and Analysis

In this section, we will improve the spatial-time-based model and give some strategy suggestions by sensitivity analysis of parameters.

3.1. Optimization of Spatial-time-based Model

To improve the spatial-time-based model in Section 2.1, we add the main factors in Section 2.2 into the model (7). The optimized spatial-time-based model is given as follows.

\[
f_i(t+1) = f_i(t) + N(t) + G_{r,i}(t) + \theta(t)
\]

\[
\theta(t) = \sum_{\omega} \alpha_{\omega} \varphi_{\omega}(t) + \epsilon
\]

\[
G_r(t) = \gamma \frac{N(t)}{N}
\]

\[
\frac{dN(t)}{dt} = r(1 - \frac{N(t)}{N})N(t)
\]

where \(\alpha\) is a parameter, \(\theta(t)\) denotes the impact factor caused by other variables. \(\epsilon\) denotes the error variable. In this section, \(\epsilon\) is equal to 1.0, because opioids would have no change in a stable society. After adding this impact factor, we fit the curve and make a comparison. Fig. 2 obviously shows that the model is more accurate after justification.

3.2. Sensitivity Analysis

Assuming that \(N_m\) is the maximum sample capacity, that is, the numbers of drug cases when police take actions to control the opioids use. The formula is as follows.

\[
N_m = \kappa R
\]

where \(\kappa\) denotes the Tolerance coefficient, \(R\) denotes the tolerance. \(N_m\) changes with the shift of \(R\), which is shown in Fig. 3. From Tables 2 and 3, we see that with the cut of \(N_m\), the maximum value of accumulated drug reports decreases. That is, if we control the opioids use, the total number of reports will diminish.

Figure 2. The curve after correction

Figure 3. Sensitivity analysis.
Table 2. Sensitivity of $N_n$.

| $N_n$ | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
|-------|------|------|------|------|------|------|------|------|
| Sensitivity | 0.0533 | 0.1076 | 0.2055 | 0.3567 | 0.5431 | 0.7173 | 0.8453 | 0.9213 |

Table 3. Sensitivity of $N_n \cdot N_0 - 1$.

| $N_n \cdot N_0 - 1$ | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
|---------------------|------|------|------|------|------|------|------|------|
| Sensitivity         | 0.05354 | 0.1063 | 0.2 | 0.35 | 0.45 | 0.5 | 0.34 | 0.21 |

4. Conclusion

In this paper, we firstly built a time-based model and a spatial-based model based logistic model and SIS model and then we further built a spatial-time-based model to analyze the sources and threshold of opioid drugs. In addition, we studied the influence of social correlation factors on opioid abuses: physiological factors, psychological factors and external environmental factors. Moreover, we improved the spatial-time-based model by adding the main factors such as divorced women, the elderly and college students.

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