Horizontal Branch Stars: The Interplay between Observations and Theory, and Insights into the Formation of the Galaxy

M. Catelan

Abstract We review and discuss horizontal branch (HB) stars in a broad astrophysical context, including both variable and non-variable stars. A reassessment of the Oosterhoff dichotomy is presented, which provides unprecedented detail regarding its origin and systematics. We show that the Oosterhoff dichotomy and the distribution of globular clusters in the HB morphology-metallicity plane both exclude, with high statistical significance, the possibility that the Galactic halo may have formed from the accretion of dwarf galaxies resembling present-day Milky Way satellites such as Fornax, Sagittarius, and the LMC—an argument which, due to its strong reliance on the ancient RR Lyrae stars, is essentially independent of the chemical evolution of these systems after the very earliest epochs in the Galaxy’s history. Convenient analytical fits to isochrones in the HB type–[Fe/H] plane are also provided. In this sense, a rediscussion of the second-parameter problem is also presented, focusing on the cases of NGC 288/NGC 362, M13/M3, the extreme outer-halo globular clusters with predominantly red HBs, and the metal-rich globular clusters NGC 6388 and NGC 6441. The recently revived possibility that the helium abundance may play an important role as a second parameter is also addressed, and possible constraints on this scenario discussed. We critically discuss the possibility that the observed properties of HB stars in NGC 6388 and NGC 6441 might be accounted for if these clusters possess a relatively minor population of helium-enriched stars. A technique is proposed to estimate the HB types of extragalactic globular clusters on the basis of integrated far-UV photometry. The importance of bright type II Cepheids as tracers of faint blue HB stars in distant systems is also emphasized. The relationship between the absolute V magnitude of the HB at the RR Lyrae level and metallicity, as obtained on the basis of trigonometric parallax measurements for the star RR Lyr, is also revisited. Taking into due account the evolutionary status of RR Lyr, the derived relation implies a true distance modulus to the LMC of \((m-M)_0 = 18.44 \pm 0.11\). Techniques providing discrepant slopes and zero points for the \(M_V(RRL) - [\text{Fe/H}]\) relation are briefly discussed. We provide a convenient analytical fit to theoretical model predictions for the period change rates of RR Lyrae stars in globular clusters, and compare the model results with the available data. Finally, the conductive opacities used in evolutionary calculations of low-mass stars are also investigated.

Keywords Galaxies: Local Group · Galaxy: formation · Galaxy: globular cluster: general · Stars: evolution · Stars: Hertzsprung-Russell diagram · Stars: horizontal-branch · Stars: variables: other

1 Introduction

1.1 A Bit of History

In her beautiful review of (hot) horizontal-branch (HB) stars, Moehler (2001) notes that Barnard (1900) was the first to detect the presence of (blue) horizontal-branch stars in globular clusters. The term horizontal branch appears to have been coined by ten Bruggencate (1927), to whom Moehler (2004) assigns the discovery of the horizontal branch—which he noticed when plotting the color-magnitude data obtained by Shapley (1915) in the latter’s study of NGC 5272 (M3). Of course, with the development of nuclear astrophysics
and the establishment of modern stellar evolution theory still several years away, it was not until three decades later that Hoyle & Schwarzschild (1955) first correctly identified HB stars as the progeny of low-mass red giant branch (RGB) stars, burning helium in their center and hydrogen in a shell around the core.

The first successful HB models were actually computed by Faulkner (1966), and Castellani & Renzini (1968) and Iben & Rood (1970) were the first to recognize that substantial mass loss on the RGB phase was needed to explain the observed colors of HB stars in globular clusters, with moreover a significant spread in mass loss amounts from star to star in any given globular cluster being needed to explain their observed color ranges—blue HB stars losing, on average, more mass than red HB stars. The distribution of masses along the HB often resembles a normal or Gaussian distribution (Rood & Crocker 1989; Dixon et al. 1996; Valcarce & Catelan 2008), and normal deviates are accordingly often adopted in the construction of “synthetic horizontal branches” (e.g., Rood 1973; Castellani & Tornambè 1981; Caputo et al. 1987; Catelan 1993; Lee 1990; Lee, Demarque, & Zinn 1990; Cassisi et al. 2004). The presence of mass distributions that resemble Gaussian deviates strongly suggests the presence of stochastic mass loss processes on the RGB. However, deviations from a Gaussian shape are also not uncommon among globular clusters, particularly in the cases of those having bimodal HBs and/or long blue tails with gaps (Catelan et al. 1998; Ferraro et al. 1998; Piotto et al. 1999; Momany et al. 2004).

1.2 The Complexity of the “HB Phenomenon”

It is virtually impossible to write a short review paper on HB stars covering “observations”, “theory”, and “implications for the formation of the Galaxy”: each one of these subjects covers so much material that one could rather write separate review papers for each one of them. Moreover, a review of HB stars cannot be complete without looking into their progenitors and their progeny. The task of a reviewer of HB stars is accordingly a daunting one, and it is virtually impossible to aim at completeness. In the present paper, while attempting to cover a broad spectrum of HB-related topics, we again hold no hope of providing a complete review of the literature on these subjects. Recent reviews focusing on several more or less specific topics related to HB stars have been provided by Cacciari (1999, 2003), Chaboyer (1999), de Boer (1999), Moehler (2001, 2004), Sweigart (1997b, 1999), Demarque (1999), Landsman (1999), Lee et al. (1999), Walker (2000), Green, Liebert, & Saffer (2001), Cacciari & Clementini (2003), Bono (2003), De Medeiros (2003), Piotto (2003), Maxted (2004a,b), Cassisi (2005), Storm (2006), Beers (1996), Stetson, Vandenberg, & Bolte (1996), Sarajedini, Chaboyer, & Demarque (1997), Fusi Pecci & Bellazzini (1997), Rood, Whitney, & D’Cruz (1997), and Rood (1998). Similarly, excellent sections focused on HB stars can be found in the reviews on the evolution of low-mass stars, Population II stars, globular clusters, and related topics by Renzini (1977, 1983), Iben & Renzini (1984), Castellani (1985, 1999), Caputo (1985, 1998), Renzini & Fusi Pecci (1988), Rood & Crocker (1989), Iben (1991), Zinn (1993a,b), D’Antona (1999), Feast (1999), Carney (2001), Harris (2001), and Gratton, Sneden, & Carretta (2004b), among others. Other recent reviews by the present author on the subject of HB stars include Catelan (2004b, 2006, 2008a,b, 2009).

In Figure 1 we show a visual color-magnitude diagram (CMD) for the Carina dwarf spheroidal (dSph) satellite of the Milky Way, with several different HB components indicated, including both a red clump and a red HB. The RR Lyrae “gap” and the blue HB are also indicated. Similarly, Figure 2 shows CMDs for the Galactic globular cluster NGC 6752, in the visual (left panel) and in the $U, U-V$ plane (right panel). These plots reveal the complexity of the blue tail phenomenon, with the positions of the HBA, HBB, and EHB components (see below) indicated, along with those of the Grundahl et al. (1999) and Momany et al. (2002, 2004) “jumps” and of possible blue HB gaps. The place where blue hook stars would be found, if present in the cluster (NGC 6752 actually appears to lack blue hook stars, according to Momany et al.), is also schematically indicated.

In the next section, we briefly discuss each of these components in turn.

2 The Different Constituents of the Horizontal Branch

2.1 The red clump

Red clumps originate from red giants that undergo the helium flash at the tip of the RGB while still having a total mass of more than $\sim 1 \, M_\odot$ (but less than $\sim 2 - 2.5 \, M_\odot$; see, e.g., Fig. 2 in Girardi 1999); accordingly, they are commonly present in intermediate-age systems and old open clusters (e.g., Cannon 1970; Brocato, Di Carlo, & Menna 2001; Grocholski & Sarajedini...
2.2 The red HB

As just stated, red HB stars are the lower-mass analogues of the red clump stars. They are commonly present in metal-rich (e.g., Armandroff 1988; Ortolani et al. 1995) as well as relatively young globular clusters (e.g., Stetson et al. 1989; Buonanno et al. 1993). However, red HB stars can also be found in metal-poor systems with “normal” ages, where they are usually interpreted as either the progeny of red giants that lost very little mass on their ascent of the RGB, or as former blue HB stars or RR Lyrae variables that have evolved to the right on the color-magnitude diagram, now approaching the end of their evolution as HB stars and the beginning of the asymptotic giant branch (AGB) phase. Another route towards producing (admittedly fewer) red HB stars is through the evolution of blue straggler stars (BSS; Renzini & Fusi Pecci 1988; Fusi Pecci et al. 1992). Fusi Pecci et al. suggest, in fact, that a red HB star “of BSS origin” should be present for every ~ 6 BSS present in a given globular cluster. (Note that, depending on the amount of mass lost by the BSS on the RGB phase, at least some of these might better classify as red clump stars.)
2.3 The RR Lyrae “gap”

This is the part of the HB that crosses the Cepheid instability strip. The term “gap” is very inadequate, but is still commonly used. The reason for this is that, in order to properly place an RR Lyrae variable in the color-magnitude diagram, one needs to follow its whole pulsation cycle and thereby obtain reliable mean colors and magnitudes. Since most color-magnitude diagram studies lack the sufficient time coverage, these stars are often simply omitted from the published CMDs, thereby leading to an artificial “gap” at the instability strip level. However, there are indeed several globular clusters—the so-called bimodal-HB globular clusters— which do seem to have relatively few RR Lyrae stars compared to the nonvariable HB stars to their right and left in the CMD (see Catelan et al. 1998 for a re- view and extensive references). The most famous such cluster is NGC 2808 (Harris 1974), which in spite of the recent discovery of a sizeable population of RR Lyrae stars, still remains firmly classified as having a bimodal HB (Corwin et al. 2004).

2.4 The blue HB

As clearly revealed by its name, blue HB stars are HB stars falling to the blue of the RR Lyrae instability strip. There have been numerous subdivisions of the blue HB, the most common including HBA (or A-type HB), HBB (B-type HB), and EHB (extreme or extended HB) stars (see Fig. 2). HBA stars are blue HB stars cooler than about 12,000 K; HBB stars include those with temperatures in the range between 12,000 K and 20,000 K; and EHB stars include HB stars hotter than 20,000 K. The latter cover a remarkably small range in envelope masses, generally less than 0.02 \( M_\odot \) (e.g., Dorman, Rood, & O’Connell 1993)—and therefore also in total masses, since the He-core mass is essentially the same for all low-mass stars with a given chemical composition (e.g., Caputo & Degl’Innocenti 1995). In a visual CMD, the blue HB component may contain a “horizontal” part—the canonical blue HB (see Fig. 1)—and an effectively “vertical” component, commonly referred to as the the blue HB tail (see Fig. 2).

Many authors have suggested that the blue HB proper and the blue HB tail are separated by a “gap” which is indeed seen in several globular clusters around \( (B-V)_0 \simeq 0 \) (e.g., Buonanno, Corsi, & Fusì Pecci 1985; Caloi 1999)—but perhaps not in all clusters containing blue tails (Catelan et al. 1998; Lee & Carney 1999a). On the other hand, several additional “gaps” may be present along the blue tail (Newell 1973; Newell & Graham 1976; Newell & Sadler 1978; Lee & Cannon 1980; Ferraro et al. 1998; Piotto et al. 1999; Momany et al. 2004); a recent, detailed description of these gaps has been provided by Momany et al. Several of these gaps, as discussed by Catelan et al., may still require more sophisticated statistical analyses to establish their reality beyond reasonable doubt, particularly when well-observed clusters seem to lack such gaps altogether, as in the case of M2 (NGC 7089; Lee & Carney 1999a).

Most impressively, Behr (2003b) has recently argued that the field “blue HB star” sample studied by Newell and Newell & Graham—and which gave rise to the very concept of gaps along the blue HB—is mostly comprised of stars in different evolutionary phases. We quote Behr:

We expected to find [based on detailed spectral analysis] a high fraction of HB candidates among the faint blue high-latitude stars listed by Newell (1973) and Newell & Graham (1976), especially in light of the ‘gaps’ in the color distribution of these stars... But fewer than half (11 of 27) of the Newell stars that we observed were clearly HB objects, with another 11 stars classified as Population I dwarfs, and the remaining five stars marked as pAGB [post-AGB], subgiants, and such.

The EHB component is the globular cluster analog of the field blue subdwarf (or sdB) stars (Caloi 1972). The majority of the (nearby) field sdB stars, according to Altman, Edelmann, & de Boer (2004) and Arifyanto et al. (2005), appear to have disk kinematics, unlike HBA stars, which they find to be mostly halo stars (see also Altman, Catelan, & Zoccali 2005). Accordingly, Altman et al. (2004) pose the question whether field HBA and EHB stars truly have a similar physical origin.

Detailed studies of the color-magnitude diagrams of globular clusters in several different bandpasses indicate the presence of additional “fine structure” on the blue HB, most notably the Grundahl jump (Grundahl et al. 1999) at a temperature around 11,500 K, which is seen as a sudden deviation, towards brighter magnitudes, of HB stars hotter than this point, particularly evident when using the Strömgren \( u \) band (Grundahl et al. 1999). However, it is also present when using the Johnson \( U \) passband (Newell 1970; Siegel et al. 1999; Bedin et al. 2000; Baev, Markov, & Spasova 2001; Markov, Spasova, & Baev 2001; Catelan et al. 2002a; Momany et al. 2002, 2004), as can be clearly seen from Fig. 2. This “jump” has been interpreted by Grundahl et al. as due to radiative levitation and diffusion effects, which lead to the metal-poor HB stars hotter than 11,500 K possessing strongly metal-enhanced and helium-depleted atmospheres.1

---

1Fortunately, and as emphasized by Landsman (1999) and Behr (2003a), Mg appears to be virtually unaffected by radiative levitation and diffusion effects, thus allowing a “window” into the original metallicity of the star—which is of extreme importance in the case of field stars in particular.
was extensively reviewed by Grundahl et al.; empirical evidence supporting their arguments was provided in spectroscopic studies of blue HB stars in globular clusters by Behr et al. (1999), Moehler et al. (1999b, 2000, 2003), Behr (2003a), and Fabbian et al. (2005). Diffusion calculations that showed how some of the observed spectroscopic and photometric patterns come about were provided by Michaud, Vauclair, & Vauclair (1983), Michaud, Richer, & Richard (2007, 2008), and Bon-Hoa, LeBlanc, & Hauschildt (2000). The “gap” at $(B-V)_0 \approx 0$ has been tentatively associated with this phenomenon (Caloi 1999), but such a color appears much redder than would be expected for the observed Grundahl jump temperature of 11,500 K. In addition, the theoretical calculations by Grundahl et al. suggest that other bandpasses in the Strömgren and Johnson systems besides $u$ and $U$ should not be dramatically affected by this phenomenon.

Most interestingly, recent observations have shown that the Grundahl jump phenomenon is also accompanied by a sharp drop in measured rotation velocities, blue HB stars hotter than 11,500 K presenting essentially no rotation, in contrast with cooler stars which may show quite significant rotation velocities, up to about 40 km s$^{-1}$ (Petersen, Tarbell, & Carney 1983; Peterson 1983, 1985; Peterson, Rood, & Crocker 1995; Behr, Cohen, & McCarthy 2000a; Behr et al. 2000b; Kinman et al. 2000; Recio-Blanco et al. 2002, 2004; Behr 2003a,b; Carney et al. 2003). While Carney et al. note that the presence of rotation in both red and blue HB stars presents a problem for models in which rotation is a candidate second parameter, it must be noted that the abrupt disappearance of rotation exactly at the Grundahl jump temperature indicates that diffusion and levitation patterns interfere with the star's observed surface rotation, somehow quickly damping the latter for temperatures higher than about 11,500 K (Sills & Pinsonneault 2000; Sweigart 2002). Therefore, we have at present no means to check whether stars hotter than this temperature may have arrived on the ZAHB with surface rotation velocities faster than those observed for red HB and cooler HB stars. Astroseismology may provide a very useful probe of the (internal) rotation velocities of hot HB stars in the near future (Kawaler & Hostler 2005). A most intriguing piece of the puzzle is provided by the RR Lyrae stars, which for which no evidence of rotation has been observed so far (see Carney et al. 2003 for extensive references). Why would cool blue HB stars and red HB stars show clear signatures of rotation, but not the intermediate-temperature RR Lyrae variables? More extensive spectroscopic observations of RR Lyrae stars are clearly needed to settle this issue.

What is the physical origin of the sudden onset of radiative levitation and diffusion patterns that lead to the Grundahl et al. (1999) jump at 11,500 K? The answer to this question is not entirely clear at present, but Sweigart (2002) noted that this temperature is very close to the one corresponding to the disappearance of surface convection in HB stars, thus strongly suggesting a link between the disappearance of convection and the onset of radiative levitation and gravitational diffusion effects in these stars. In addition, Sweigart suggests that the low rotation velocities of stars hotter than the Grundahl jump may be due to the spin down of the surface layers by a weak stellar wind induced by the radiative levitation of Fe. As shown by Vink & Cassisi (2002), such winds are indeed predicted by theory.

2.5 The extended (or extreme) HB

Before a star arrives on the HB, it must undergo a helium flash—the onset of helium burning, under partially degenerate conditions, which takes place at the tip of the RGB (Schwarzschild & Härm 1962). It is interesting to note that several early hydrodynamic studies of the He-flash indicated the occurrence of an explosive event and actual disruption of the star (e.g., Edwards 1969; Wickett 1977; Cole & Deupree 1980). Given that this prediction is in clear conflict with the very existence of HB stars, significant effort has also been devoted towards the computation of hydrostatic flashes (e.g., Mengel & Gross 1976; Despain 1980; Mengel & Sweigart 1981). The hydrostatic approximation, with some numerical and physical refinements, is still often used in modern calculations (e.g., Brown et al. 2001; Cassisi et al. 2003; Lanz et al. 2004; Piersanti, Tornambè, & Castellani 2004; Serenelli & Weiss 2005)—and these have indeed been vindicated by modern hydrodynamic studies (e.g., Deupree 1996; Dearborn, Lattanzio, & Eggleton 2006) which revealed that the aforementioned hydrodynamical predictions were indeed incorrect. It is perhaps not a conclusively settled matter whether any of the material that is nuclearly processed during the He-flash may reach the surface of the star, and also whether one or more mass loss episodes may be triggered by the primary and secondary core flashes; however, and to the extent that the latest He-flash calculations provide a realistic description of the process, no such phenomena should be expected. Indeed, Dearborn et al. have recently pointed out that, due to expansion velocities that are much lower than the local sound speed, hydrostatic modeling should indeed capture the essence of the helium flash process. On the other hand, the 3D models by Dearborn et al. intriguingly reveal motions “of an apparently convective nature” beyond the H-burning shell, which these authors
claim not to be directly associated with the flash, but which may bring the products of H-burning to the surface of the star.

On the other hand, the situation can become much more complex when the star loses so much mass on its ascent of the RGB that the He-flash ends up taking place not at the RGB tip, but rather during the helium white dwarf cooling curve—a so-called late hot flasher (Brown et al. 2001; Cassisi et al. 2003; Castellani, Castellani, & Prada Moroni 2006). In this case, extensive mixing between the envelope and regions that underwent significant hydrogen and helium burning is indeed expected, and these stars, when they finally manage to settle on the zero-age HB (ZAHB), may end up as the so-called blue hook stars (e.g., D'Cruz et al. 1996, 2000; Whitney et al. 1998; Rosenberg, Recio-Blanco, & García-Marín 2004; Busso et al. 2007; Ripepi et al. 2007), which have temperatures in excess of 35,000 K (Moehler et al. 2002, 2004, 2007). This is to be compared with the case in which the star undergoes an early hot flash—i.e., prior to arriving on the white dwarf cooling sequence—in which case Brown et al. find that no mixing takes place and a “canonical” EHB star results. In this scenario, the reason why the blue hook stars appear fainter than its peers on the EHB is twofold. First, they tend to have smaller helium-core masses, due to the fact that they leave the RGB before igniting helium in their core, as a consequence of extreme mass loss on the RGB (e.g., Castellani & Castellani 1993; D'Cruz et al. 1996; Brown et al. 2001). Second, and most importantly in the late flasher scenario, their atmospheres present large enhancements in both helium and carbon as a consequence of mixing, thus affecting the bolometric corrections (Brown et al. 2001).

Spectroscopic evidence largely favoring this “late flash mixing” scenario for the origin of blue hook stars has recently been presented by Lanz et al. (2004) and Moehler et al. (2002, 2004, 2007). An intriguing indication from these spectroscopic studies for at least some of the hotter EHB stars is that some hydrogen still remains in their atmospheres, which is not expected on the basis of the theoretical models. As argued by Moehler et al., some residual hydrogen may perhaps survive a late hot flash and later diffuse to the surface during the HB phase. Note also that, in analogy with the Grundahl et al. (1999) “jump,” another “jump” has recently been suggested to be present in blue-tail globulars, namely the Momany jump (see Fig. 2), at a temperature around 21,000 K (Momany et al. 2002, 2004)—which these authors conjecture to be related to the early helium flashers (but see Catelan 2008a for a critical discussion).

Recently, a sub-population of cluster stars with a large, primordial enhancement in the helium abundance has also been suggested as a possible channel producing the blue hook stars (Lee et al. 2005b; Yoon et al. 2008). However, it is unclear how the abundance patterns observed in blue hook stars would be accounted for in this scenario (e.g., Moehler et al. 2007). In like vein, Castellani et al. (2006) have recently advanced the intriguing suggestion that at least some of the very hot and underluminous blue hook stars might actually be more straightforwardly explained as photometric blends and/or binary stars.

3 Variable Stars on the HB

Until recently, it was thought that RR Lyrae stars were the only class of variable stars on the HB. The situation has changed recently, with the discovery (in a beautiful example of theoretical modelling preceding the observations) of non-radial pulsation among sdB stars. These non-radial pulsators are now divided into two different groups. Therefore, there are currently three known classes of variable HB stars, namely:

• RR Lyrae stars: These are radial pulsators with periods in the range between about 0.2 d and 1.0 d, known to pulsate primarily in the fundamental mode (RR Lyrae stars of type a and b, now lumped together as RRab or RR0 stars) and in the first overtone (RR Lyrae stars of type c, nowadays also referred to as RR1 stars). They have long been known to be present in large numbers in Galactic globular clusters (Pickering & Bailey 1895), and were first correctly identified as radially pulsating stars by Shapley (1914). The letters “a,” “b,” “c” were first used by Bailey (1902) (see his p. 132) to classify RR Lyrae stars as a function of their light curve shapes; Bailey also noted that the RRb subclass is ... similar to (the RR)a (subclass), of which it may be regarded as a modification.

Indeed, both are now known to be fundamental-mode pulsators; in fact, the identification of RRab’s with fundamental pulsation and of the RRc’s with first overtone pulsation seems to have first been clearly made by Schwarzschild (1940). The notation in which the numbers 0 and 1 are used as opposed to the letters “ab” and “c,” respectively, was first introduced by Alcock et al. (2000).

In addition, there are also double-mode pulsators (RRd or RR01 stars), pulsating simultaneously in the fundamental and first-overtone modes, as first recognized by Jerzykiewicz & Wenzel (1977) among field stars and by Sandage, Katem, & Sandage (1981) (see
Horizontal Branch Stars: Observations and Theory

their Sect. IIIa) in globular clusters (see also Gruberbauer et al. 2007 for an impressively detailed frequency analysis, using MOST [Microvariability Oscillations of Stars] satellite observations, of the RRd star AQ Leo, the prototype of this class). The notation “RRd” for these stars appears to first have been used by Nemec (1984, 1985). The ratio between the first-overtone and fundamental period for the RRd stars is quite well defined; indeed, on the basis of the data for M68 (NGC 4590), IC 4499, and M15 (NGC 7078) compiled in Table 8 of Kovács & Walker (1999), we obtain \( \langle P_1/P_0 \rangle = 0.7454 \) for 38 stars,\(^2\) with a minimum value of \( P_1/P_0 = 0.7433 \) and a maximum value \( P_1/P_0 = 0.7481 \). For the LMC RRd, Alcock et al. (2000) find comparable period ratios, but with upper and lower values shifted downward by \( \approx 0.0015 \) (compare with their Fig. 5). An impressive summary of period ratios for a variety of stellar systems (the LMC, the SMC, Draco, Sculptor, M15, M68, IC 4499, M3, and the Galactic field) is provided in Figure 1 of Popielski, Dziembowski, & Cassisi (2000), which basically confirms the above range in period ratios. Recently, Clementini et al. (2004) have reported on the discovery of two RRd variables in M3 whose period ratios, in the range \( 0.738 - 0.739 \), fall well below those for previously known RRd stars.

It has also been suggested that RR Lyrae stars may pulsate in the \textit{second} overtone (e.g., Demers & Wehlau 1977; Alcock et al. 1996; Walker & Nemec 1996; Kiss et al. 1999; Clement & Rowe 2000); accordingly, short-period, low-amplitude variables which are suspected of being second-overtone pulsators are now often classified as RRe or RR2 stars (but see Kovács 1998, Catelan 2004b, Bono et al. 1997a; 1997b, for arguments suggesting that at least some of them may rather represent the short-period end of the RRc distribution). In fact, Alcock et al. (2000) suggest that double-mode variables pulsating simultaneously in the first and \textit{second} overtones may also exist, and tentatively assign them an RR12 subclass.

Non-radial modes have also been suggested to be present in a fraction of the RR Lyrae stars (e.g., Kovács 1995; Kolenberg 2002; Gruberbauer et al. 2007; and references therein), primarily as a means to explain the so-called Blazhko (1907) effect (e.g., Dziembowski & Mizerski 2004). The Blazhko effect consists in a periodic modulation, on a much longer timescale than the primary period, of the light curve shape.\(^3\) The modulation (or Blazhko) period falls in the range between 5.309 d (as found by Jurcsik et al. 2006 for the field star SS Cnc) and 530 d (as found by Nagy 1998 for the field star RS Boo). A variety of other multiperiodic phenomena may also take place in RR Lyrae stars (see, e.g., Alcock et al. 2000, 2004; Mizerski 2004; Gruberbauer et al. 2007).

We note, in passing, that the Fourier decomposition \( (D_m) \) method (Jurcsik & Kovács 1996) has recently been criticized by Cacciari, Corwin, & Carney (2005) as a diagnostic of the Blazhko effect in RR Lyrae stars.

- \textit{V361 Hya variables:} Also known as EC 14026 or sdBV stars, these are non-radial, short-period, low-order \( p \)-mode pulsators, whose periods fall in the range 80–400 s, and whose amplitudes bracket the interval between 4 and 25 mmag. Their temperatures are found in the range between 29,000 K and 36,000 K, and their gravities in the range \( 5.2 \leq \log g \leq 6.1 \) (Kilkenny 2002; Fontaine et al. 2004, 2006a,b). Note that this class of variable star had been predicted (Charpinet et al. 1996) \textit{before} it was first observed (Kilkenny et al. 1997).

- \textit{V1093 Her variables:} Also known as PG1716+426 or “Betsy” variables (after Elizabeth Green, the discoverer of the group), these are non-radial, relatively long-period, high-order \( g \)-mode pulsators (e.g., Green et al. 2003; Fontaine et al. 2003; Reed et al. 2004). Their periods fall in the range 2000–9000 s, and their amplitudes are very small, generally being smaller than 0.5 mmag. Their temperatures bracket the interval 25,000–30,000 K, and their gravities are in the range \( 5.1 \leq \log g \leq 5.8 \) (Fontaine et al. 2004, 2006a,b; Reed et al. 2004).

- \textit{Mixed-mode variables:} Variable stars showing both V361 Hya and V1093 Her characteristics have recently been discovered as well (Baran et al. 2005; Schuh et al. 2006).

For a recent discussion of the pulsation mechanism (the “Fe-bump opacity mechanism”) driving the pulsations in V361 Hya and V1093 Her stars, see Jeffery & Saio (2006). It is important to emphasize that the presence of variable stars on the HB phase provides us with a unique opportunity to utilize \textit{stellar pulsation observations and theory} to improve our understanding of

\(^2\)It is basically this ratio that allows one to compute the “fundamentalized” period of an RRc star by using the relation \( \log P_0 = \log P_1 + 0.128 \).

\(^3\)In their review of 100 years of observations of the star RR Lyrae, Szeidl & Kolláth (2000) point out that the phenomenon might more appropriately be called the \textit{Blazhko-Shapley effect}, since Shapley (1916) was actually the first to demonstrate that the oscillations in the maxima of an RR Lyrae star can be described as a periodic variation in the shape of the light curve and in the height of the maxima.
HB stars in general. As well known, the pulsation properties of stars are fundamentally related to their mean densities through the so-called period-mean density relation—an expression originally due to Ritter (1879). This extremely important equation specifies that the period is inversely proportional to the square root of the mean density of the star, which in turn can be expressed in terms of the star’s global physical parameters, such as mass $M$, luminosity $L$, and effective temperature $T_{\text{eff}}$ (e.g., van Albada & Baker 1971; Bono et al. 1997b; Caputo, Marconi, & Santolamazza 1998; Di Criscienzo, Marconi, & Caputo 2004). As a matter of fact, Ritter’s relation is applicable, to first order, over an impressively wide range of stellar parameters (e.g., Cox 1974). In addition, the observed periods—and especially so in the case of non-radial pulsators—depend on the star’s detailed structural profile. This is a natural consequence of the fact that the measured periods are directly related to the speed of travel of the sound waves across the stellar interior. In this sense, the importance of the non-radial variables on the HB to constrain the internal structures of HB stars, including the amount of internal rotation, has recently been emphasized by Kawaler (2006), Kawaler & Hostler (2005), and Fontaine et al. (2006a), and nicely demonstrated in practice by Charpinet et al. (2005, 2006), Randall et al. (2007), and Van Grootel et al. (2008). It is interesting to note that the high rotation velocity recently derived by Dixon, Landsman, & Brown (2004) for the post-AGB star ZNG 1 in M5 might be accounted for, as noted by those authors, if RGB and HB stars are able to maintain rapid rotating cores, as may also be required (Sills & Pinsonneault 2000) to explain the presence of fast rotators on the HB (see Sect. 2.4 for extensive references).

It is worth noting that several authors have called attention to the possible presence of non-variable HB stars inside the RR Lyrae instability strip (e.g., Sandage & Katem 1968; Wehlau 1990; Xiong, Cheng, & Deng 1998; Stetson, Catelan, & Smith 2005; and references therein). The recent discovery of Cepheid variables presenting pulsation amplitudes at the mmag level (Buchler et al. 2005) suggests that at least some of these “non-variable” stars may indeed be varying, though with much smaller amplitudes than would have been possible to detect with more traditional techniques—as nicely demonstrated by Clement & Rowe (2001) in the case of NGC 5897. According to Buchler et al., “ultralow amplitude” RR Lyrae stars are predicted to exist close to both the blue and red edges of the instability strip. Note, on the other hand, that the General Catalog of Variable Stars (Kholopov et al. 1998) defines RR Lyrae stars as having amplitudes in the range 0.2 to 2 mag in $V$, thus showing that RR Lyrae with $V$ amplitudes smaller than 0.2 mag, though known to exist even among the normally higher-amplitude RRab’s (e.g., Sandage 1981; Grenon & Waelkens 1986; Clement & Rowe 2001; Sokolovsky et al. 2002, 2003; Cacciari et al. 2005), are indeed quite rare. Note that variable stars falling on the red HB—i.e., outside the formal boundaries of the RR Lyrae instability strip—have also been suggested to exist (Xiong, Cheng, & Deng 1998 and references therein).

Still in regard to pulsation amplitudes, an interesting recent result worth mentioning are the very large amplitudes—up to $\sim 5$ mag—found for RR Lyrae stars in the far UV (Downes et al. 2004; Dieball et al. 2005, 2007; Welsh et al. 2005; Wheatley et al. 2005; see also Fig. 1 in Bonnell et al. 1982 for an example of an earlier RR Lyrae light curve in the far UV, already indicative of large amplitudes), and up to $\sim 2.6$ mag in the near UV (Browne 2005). Conversely, in the near-IR, RR Lyrae amplitudes become quite small (e.g., Jones, Carney, & Latham 1998; Liu & Janes 1990a). Thus, RR Lyrae stars usually reveal themselves more easily when time-series surveys are conducted using bluer bandpasses, whereas average magnitudes can be more easily computed using near-IR observations (see also §6.3.1 below).

Before closing this section, we would like to comment on the possibility that some stars in the rapid phase of evolution between the RGB tip and the ZAHB do in fact become variables. Evolutionary paths for these stars are shown, for instance, in Fig. 4 in Mengel & Sweigart (1981) and Fig. 4 in Brown et al. (2001), where it can be seen that many of these stars do cross the instability strip before settling on the ZAHB. While distinguishing them from more ordinary RGB, AGB, and RR Lyrae stars is obviously far from straightforward, both due to the very small expected number of stars in this phase and because of their overlapping with them on the CMD, their extremely fast evolutionary timescale ($\sim 10^6$ yr) hints at the possibility of identifying them by appropriate monitoring in wide-field surveys. This is because much more extreme period change rates might obtain for them than for most other variables in a similar position on the CMD. To the best of our knowledge, no other technique has been proposed in the literature for the detection of these elusive post-RGB tip/pre-ZAHB stars. The reader is referred to the recent paper by Silva Aguirre et al. (2008) for an application of this method to the case of the Galactic globular cluster M3.

\footnote{See http://www.sai.msu.su/groups/cluster/gcvs/gcvs/iii/vartype.txt.}
4 Detecting HB Stars in Distant Systems: Methods

While HB stars are relatively bright, their direct detection in distant systems (i.e., at the distance of M31 and beyond) is a challenge. This is particularly so in the case of blue HB stars, which can be several magnitudes fainter than the “horizontal” level in V (Fig. 2), due to the increase in the V-band bolometric correction at high temperatures. While the blue HB stars become brighter towards the far ultraviolet, which could make them candidates for detection from space, the required exposure times, at the distance of M31, are prohibitively large. Therefore, while extremely long-exposure HST observations have demonstrated that it is possible to reliably detect blue HB stars down to at least the main sequence turnoff level in M31 (e.g., Brown et al. 2003, 2004b, 2006), alternative approaches are needed at present to obtain a more complete assessment of the HB morphologies in extragalactic systems.

4.1 Far-UV Observations

The far-UV output from globular clusters is strongly dependent on the temperature of the sources. Since hot HB stars are important UV emitters (e.g., Dorman, O’Connell, & Rood 1995; see also Catelan 2008b for a recent review), this is expected to translate into a far UV (integrated) color−HB type correlation, especially when the contribution of bright, individual far-UV sources (such as post-AGB stars), which are present in clusters in non-statistically significant numbers, is removed from estimates of the integrated far UV colors.

Landsman et al. (2001) and Landsman & Catelan (2009, in preparation) tested this prediction, using integrated far UV fluxes for Galactic globular clusters as summarized by Dorman et al. (1995), and including revisions to the far UV photometry based on images taken with the Ultraviolet Imaging Telescope (UIT). The result is shown in Figure 3, which depicts the trend of variation in the (15−V)0 color (where “15” stands for a bandpass centered at 1500 Å) as a function of the HB morphology parameter \( (B2−R)/(B+V+R) \) (Buonanno 1993; Buonanno et al. 1997), where \( B2 \) is the number of blue HB stars bluer than \( B−V \) = −0.02, and \( B, V, R \) are the numbers of blue, variable (RR Lyrae), and red HB stars, respectively. The latter number counts are also widely used in constructing the so-called Lee-Zinn parameter, \( L = (B−R)/(B+V+R) \) (Zinn 1986; Lee 1990; Lee et al. 1990). The reason why we use the Buonanno as opposed to the Lee-Zinn parameter is that it is more successful at breaking the degeneracy that characterizes \( L \) in the case of clusters with completely blue HBs. More specifically, \( L \) is not able to distinguish between a long blue tail cluster such as M13 or NGC 6752 and a stubby blue HB cluster such as NGC 288 (all having \( L \approx +1.0 \)), whereas \( (B2−R)/(B+V+R) \) (as well as several other HB morphology parameters, including some of those defined by Fusi Pecci et al. 1993 and Catelan et al. 2001a) are better suited for the task. Indeed, Figure 3 shows that, apart from the clusters with bimodal HBs (NGC 6388, NGC 6441, M75), there is a good correlation between the Buonanno parameter and \( (15−V)0 \), characterized by a high correlation coefficient \( r = 0.93 \). Using \( L \), one finds an \( r = 0.82 \) instead, but this is primarily due to the large difference in far UV color between 47 Tucanae (NGC 104) and the group of blue HB clusters: removing 47 Tuc, the correlation coefficient, when using \( L \), drops to \( r = 0.57 \), whereas the one obtained using the Buonanno parameters still remains fairly high at \( r = 0.84 \). These correlations are in good agreement with the theoretical predictions by Landsman et al. for an \( α \)-enhanced composition, as can clearly be seen from the dotted line in Figure 3. In summary, one should be able to estimate, at least as a first approximation, the HB type of an old extragalactic globular cluster by measuring its integrated far UV light.

Indeed, that the far UV light is a powerful diagnostic of the presence of hot HB stars has been demonstrated by the observation of a peculiarly high far UV flux from the moderately metal-rich Galactic globular...
clusters NGC 6388 and NGC 6441 (Rich, Minniti, & Liebert 1993), which led the authors to suggest the existence of long blue tails in these clusters several years before these were first directly observed with HST (Piotto et al. 1997; Rich et al. 1997). Recently, Peterson et al. (2003) have performed a detailed analysis of the mid-UV flux from the extremely massive M31 globular cluster G1, finding that it likely presents both regular and extreme blue HB stars. Its recent detection in far-UV observations with GALEX (Rey et al. 2005, 2007) lends support to their conclusion. The HST color-magnitude diagram for the cluster (Rich et al. 2005) cannot reveal the presence of hot HB stars since it just reaches the HB level, but it is not clearly inconsistent with the presence of a few blue HB stars at the “horizontal” level.5

To close, it is important to note that the Hβ and Hδ integrated indices can be sensitive indicators of the presence and temperature of HB stars in unresolved stellar systems (e.g., Lee, Yoon, & Lee 2002; Maraston et al. 2003; Schiavon et al. 2004; Percival et al. 2009; and references therein). The dependence of these and several other photometric and spectroscopic indicators on HB morphology has recently also been discussed by Lee, Lee, & Gibson (2002), Proctor, Forbes, & Beasley (2004), Salaris & Cassisi (2007), and Koleva et al. (2008), among others.

4.2 Type II Cepheids: Bright Indicators of the Presence of Faint Blue HB Stars

Another technique to infer the presence of a blue HB component in extragalactic systems without the need to obtain exceedingly deep photometry could use fairly bright type II Cepheids (including BL Herculis, W Virginis, and RV Tauri stars) as an indicator. Indeed, type II Cepheids are widely believed to be the immediate progeny of HB stars with little envelope mass (but with still enough to reach the AGB stage)—i.e., blue HB stars. This is supported both by the observational record, which shows that type II Cepheids are present only when a sizeable blue HB component is also present (Wallerstein 1970; Smith & Wehlau 1985), seemingly irrespective of the metallicity (Pritzl et al. 2002b, 2003);6 and by theoretical models (Schwarzchild & Härn 1970; Gingold 1976; 1985; see also Di Criscienzo et al. 2007 for a recent discussion and additional references). Accordingly, detection of type II Cepheids should immediately imply the presence of a sizeable blue HB component. In this sense, it is worth noting that Dolphin et al. (2004) and Williams (2005) found possible type II Cepheids in variability studies of small M31 halo fields; if this classification is confirmed (the authors note that the stars could also be Anomalous Cepheids, for instance), this would be consistent with the deep M31 CMDs obtained by Brown et al. (2003, 2006), which clearly reveal the presence of a sizeable (though seemingly not very extended) blue HB component. Likewise, Pritzl et al. (2005a) suggest that the M31 dSph satellites And I and And III may also contain a small number of type II Cepheids, which should also imply the presence of blue HB components. Their HST CMDs for these galaxies do indeed suggest the presence of blue HB stars, particularly evident in the case of And I. Last but not least, we note that Fliri et al. (2006) and Villardeel, Jordi, & Ribas (2007) have recently reported the detection of type II Cepheids in M31, including many in the direction of the M31 bulge—which, if confirmed, implies the presence of a sizeable blue HB component in that direction as well.

5 Mass Loss on the RGB

Knowledge of how red giants lose mass is one of the indispensable conditions for understanding the properties of HB stars, including their color distributions, the production of RR Lyrae and sdB stars, the variation in HB morphology with metallicity, and the (in)famous second-parameter phenomenon. Several previous reviewers have emphasized the need for an advancement in our knowledge of RGB mass loss for a consensus on a variety of problems involving HB morphology to be achieved (e.g., Rood 1973, 1998; Rood & Crocker 1989; Rood et al. 1998). Unfortunately, even though more than three decades have passed since a very convenient mass loss formula was advanced by Reimers (1975a, b), his expression is still widely used as a “law,” even though it is now known that the Reimers mass loss formula does not properly account for the mass loss rates that have become available in the more recent literature. In fact, several other formulations have been proposed that do provide a better description of these empirical mass loss rates, some of which are summarized in Table 1. Additional formulations and references are provided in Origlia et al. (2007), Schröder & Cuntz (2007), and McDonald & van Loon (2007). Boyer et al. (2008) provide a critical discussion of recent results based on Spitzer Space Telescope infrared

5While Rich et al. (2005) provide L values for their observed M31 globular clusters, no completeness corrections were applied in deriving them, which may lead to important underestimates of the number of blue HB stars—and therefore of L—for those among them possessing well-populated blue HB tails.

6There is a single known exception to this rule, provided by the type II Cepheid in Palomar 3 (Borissova, Ivanov, & Catelan 2000).
Table 1  Formulae to Compute the Mass Loss Rate in Red Giant Stars

| Formula name     | Formula for $\frac{dM_{\text{RGB}}}{dt}$ (in $M_\odot \text{yr}^{-1}$) |
|------------------|-------------------------------------------------------------------|
| Reimers          | $5.5 \times 10^{-13} \left( \frac{L}{g R} \right)$               |
| Modified Reimers | $8.5 \times 10^{-10} \left( \frac{L}{g R} \right)^{1.4}$          |
| Mullan           | $2.4 \times 10^{-11} \left( \frac{g}{10 R} \right)^{-0.9}$        |
| Goldberg         | $1.2 \times 10^{-15} R^{+3.2}$                                    |
| Judge-Stencel    | $6.3 \times 10^{-8} g^{-1.6}$                                     |
| VandenBerg       | $3.4 \times 10^{-12} L^{+1.1} g^{-0.9}$                            |

*Reimers formula as given in Kudritzki & Reimers (1978). All other expressions derived by Catelan (2000; see his Appendix A for details) on the basis of the data provided by Judge & Stencel (1991). The gravity $g$ is in cgs units, and luminosity $L$ and radius $R$ in solar units.

Clearly, the metallicity dependence differs substantially among the different expressions. This can have important consequences for the prediction of the temperature distribution of hot HB stars in metal-rich systems, with implications not only for our understanding of the origin and nature of hot HB stars in the Galactic bulge (e.g., Busso et al. 2005; Busso & Moehler 2008), but also for our modelling and interpretation of the UV-upturn phenomenon in elliptical galaxies and bulges of spirals (e.g., Horch, Demarque, & Pinsonneault 1992; Bressan, Chiosi, & Fagotto 1994; Dorman et al. 1995; Yi, Demarque, & Kim 1997; Yi et al. 1999), as well as for our understanding of the relatively red HB morphologies of the most metal-poor globular clusters in our galaxy (see Fig. 7 below). In Sect. 7, we shall also address the possible impact of several different mass loss formulae upon analyses of specific pairs of second-parameter globular clusters.

On the other hand, the reader should bear in mind that it remains very unclear at present whether any analytical formula can be reliably used to compute the mass loss in red giant stars. For instance, Origlia et al. (2002) find, using ISOCAM near-IR data for six globular clusters with a range in metallicities, that: i) None of the formulae given in Table 1 reproduces their derived mass loss rates, the latter being in general larger than the former by at least one order of magnitude; ii) There is no clear dependence between mass loss rate and any of the basic stellar parameters $L$, $g$, or $R$; iii) Mass loss takes place near the RGB tip only (more specifically, within the very final $10^6$ years of evolution on the
RGB—compare with the timescale shown in Fig. 16 below), and is not constant but rather episodic; iv) The mass loss episodes must last longer than a few days, but less than $10^6$ yr; v) There is no clear correlation between mass loss and metallicity. We will discuss the implications of their results for the evolutionary interpretation of the second parameter problem in Sect. 7; see also Sect. 6.2 below for a discussion of the impact of different mass loss recipes upon theoretical isochrones in the $L - [\text{Fe/H}]$ plane.

Assuming their results are able to withstand the test of time (see Boyer et al. 2008 for some recently raised caveats), how does one reconcile theOriglia et al. (2002) study with the empirical results upon which the formulae in Table 1 are based? The answer to this question is not clear at present, but it is possible that the latter provide a better description of mass loss on the AGB, the former being instead more suitable for first-ascent giants. The connection between mass loss and stellar variability, which has frequently been addressed in the literature (e.g., Willson & Bowen 1984; Bowen 1988; Willson 1988, 2000; Ramdani & Jorissen 2001; Lebzelter & Wood 2005), may provide the key to the riddle. Lebzelter et al. (2005) concluded that, at a similar luminosity, red giants with higher pulsation amplitudes present higher mass loss rates. At the same time, while variability in AGB stars (which become more easily enshrouded in dust) has long been established, that in first-ascent red giants is a fairly recent result (Ita et al. 2002). Importantly, it appears rather clear now that AGB and RGB stars have different pulsation properties (e.g., Kiss & Bedding 2004; Soszyński et al. 2004), first-ascent variables having smaller pulsation amplitudes and falling on a different period-luminosity relation than AGB variables. On the other hand, Soszyński et al. suggest, based on the coincidence of the RGB pulsators belonging to the LMC, the SMC, and the Galactic bulge in a Petersen (1973) (or period ratio vs. period) diagram, that RGB pulsators with different metallicities do not have significantly different pulsation properties. If confirmed, these results may provide very important constraints on the extent to which mass loss on the RGB may vary with metallicity.

6 The Oosterhoff Dichotomy: Constraints on the Galaxy’s Formation History

6.1 The Oosterhoff Dichotomy: Systematics

It has recently been argued that the Oosterhoff (1939, 1944) dichotomy may very well hold the key to the formation history of the Galactic halo. The argument goes as follows: the Galactic halo shows a sharp division between Oosterhoff type I (OoI, average periods of the ab-type RR Lyrae variables $(P_{ab}) \approx 0.55$ d) and Oosterhoff type II (OoII, with $(P_{ab}) \approx 0.65$ d) globular clusters, with very few clusters in the range $0.58 \leq (P_{ab}) \leq 0.62$ (the “Oosterhoff gap”). 7 On the other hand, the dwarf spheroidal (dSph) satellite galaxies of the Milky Way, as well as their respective globular clusters, fall preferentially on the “Oosterhoff gap” region. One of the main scenarios for the formation of the Galactic halo envisages the build-up of the halo from the accretion of smaller “protogalactic fragments” not unlike the present-day Milky Way dSph satellite galaxies (e.g., Searle & Zinn 1978; Zinn 1993b). However, if this were the case, the present-day halo should not display the Oosterhoff dichotomy, since the dSph galaxies and their globular clusters are predominantly intermediate between the two Oosterhoff classes. Therefore, the Galactic halo cannot have been assembled by the accretion of dwarf galaxies resembling the present-day Milky Way satellites—including, in fact, the LMC dIrr (Catelan 2004b).

One criticism that might perhaps be drawn against this argument is related to the fact that not too many globular clusters satisfied the fairly strict selection criteria established by Catelan (2004b), which restricted his sample to globular clusters containing at least 10 known RRab variables with measured periods. With this selection criterion, Catelan found that 19 globular clusters were OoI, 9 were OoII, and two (or 6.7%) were Oosterhoff-intermediate. Two additional globular clusters, NGC 6388 and NGC 6441, were tentatively assigned to a new class, OoIII (Pritzl et al. 2000, 2001, 2002b, 2003). As one can easily see, only a relatively small fraction of the Galactic globular clusters, of order 20%, were included.

In order to improve the statistics, we add to the sample originally studied by Catelan (2004b) all Galactic globular clusters with at least 5 known RRab variables

---

7The question whether the Galactic halo field also presents the Oosterhoff dichotomy, as found by Suntzeff, Kinman, & Kraft (1991), or not, as suggested by the QUEST (Vivas & Zinn 2003) and ROTSE (Kinemuchi 2004) surveys (see also Sandage 2006), remains an open issue at present (see also Catelan 2004b). We note, however, that inspection of Figure 9 in Vivas et al. (2004) reveals that many of the candidate Oosterhoff-intermediate stars in the QUEST survey (e.g., their stars 1, 9, 27, 95) have extremely uncertain amplitudes, thus rendering their position in the Bailey diagram—and thus their Oosterhoff status—similarly uncertain (Catelan 2006). The recently released results of the LONEOS-I survey also strongly support the reality of the Oosterhoff dichotomy among field halo stars: in this sense, Figures 19 and 20 in Miceli et al. (2008) constitute especially striking evidence.
Horizontal Branch Stars: Observations and Theory

The globular cluster data used in Figure 5 are given in Table 2 (Galactic globulars) and Table 3 (globular clusters associated with the dwarf galaxy satellites of the Milky Way). The information contained therein comes from the following sources:

- For the Fornax dSph globular clusters 3, 4, and 5, we adopt the ⟨P_{ab}⟩ values from Greco et al. (2005; see also Greco et al. 2007 for a recent, detailed discussion of the case of Fornax 4). For clusters 1 and 2, we adopt the values from the entry “RRab with good P” in Table 4 of Mackey & Gilmore (2003). Metallicity values are the ones provided by Mackey & Gilmore (2004b).

- For globular clusters that have been associated with the Sagittarius dSph and which have at least 5 RR Lyrae variables, we have adopted ⟨P_{ab}⟩ values from Cacciari, Bellazzini, & Colucci (2002) (M54 = NGC 6715), Salinas et al. (2005) (Arp 2, NGC 5634), and Stetson et al. (2005) (NGC 4147). While the association of M54 and Arp 2 to the Sagittarius dSph dates back to early studies of the system (e.g., Ibata et al. 1995; Da Costa & Armandroff 1995), that of the other two quoted globular clusters has only recently been advanced (NGC 4147: Bellazzini, Ferraro, & Ibata 2003a; Bellazzini et al. 2003b; NGC 5634: Bellazzini, Ferraro, & Ibata 2002).9 NGC 5634 is an interesting case, since the preliminary results of Salinas et al. (2005) indicate that, in

---

8http://www.astro.utoronto.ca/~cclement/read.html

---

9Note that the possibility has also been raised that the massive outer-halo globular cluster NGC 2419 has once been associated with the Sagittarius dSph (Newberg et al. 2003), or perhaps been the stripped nucleus of a dwarf galaxy (van den Bergh & Mackey 2004)—but we consider it a bona fide Galactic globular cluster in the present analysis (see also Ripepi et al. 2007). There are a few additional clusters whose association with Sagittarius has been proposed (e.g., Dinescu et al. 2001; Palma, Majewski, & Johnston 2002; Majewski et al. 2003; Carraro, Zinn, & Moni Bidin 2007). However, it is well known that the Fornax dSph, with its five globular clusters, has an anomalously high globular cluster specific frequency (e.g., van den Bergh 1998 and references therein); accordingly, Sagittarius, with a total luminosity that seems comparable to that of Fornax (van den Bergh 2000; Majewski et al. 2003), would have an even more anomalous specific frequency of globular clusters if all these candidates (in addition to the five fairly clear-cut cases of M54, Arp 2, Ter 7, Ter 8, and Pal 12) were indeed associated to it. Therefore, caution recommends not to attribute final membership status to all the new candidates before their association to the galaxy has been conclusively established. Likewise, we do not include NGC 6388 and NGC 6441 as extragalactic globular clusters, in spite of their suggested association with dwarf galaxies that were long ago captured by the Milky Way (Ree et al. 2002). Finally, we note that Lee, Gim, & Casetti-Dinescu (2007) recently raised the intriguing possibility that pretty much all globular clusters with extended HBs are of extragalactic origin (being the former nuclei of dwarf galaxies), unlike most other globulars in the halo; the latter would be more consistent with a dissipational collapse scenario.
Fig. 5  (Left panel) The Oosterhoff dichotomy among Galactic globular clusters. Clusters belonging to the bulge or disk are shown as squares; those belonging to the “young halo” in the Mackey & van den Bergh (2005) classification scheme are shown as filled gray circles; and clusters belonging to their “old halo” are shown as filled black circles. Clusters with at least 10 RRab stars with measured periods are shown as large symbols, while those with 5 to 9 RRab’s are shown with smaller symbols.  (Right panel) Same as in the previous plot, but now including dwarf satellite galaxies of the Milky Way and their associated globular clusters. The dSph galaxies and the Magellanic Clouds are shown as open inverted triangles; LMC globulars are shown as open triangles; globular clusters associated with the Fornax dSph are shown as open losanges; those associated with the Sagittarius dSph are shown as open stars; possible Canis Major dSph globulars are shown as black circles with gray contours; and ω Cen is shown as an open hexagon. It is obvious from this plot that the dwarf galaxies orbiting the Milky Way and their associated globular clusters preferentially occupy the Oosterhoff gap region, in stark contrast with the Galactic globular clusters.

spite of a ⟨Pab⟩ value indicative of OoII status, its Bailey (or period-amplitude) diagram suggests instead that the cluster is Oosterhoff-intermediate (see Catelan 2004b for a discussion of the importance of the Bailey diagram in defining Oosterhoff status). Metallicity values are from the Harris (1996) catalog (Feb. 2003 update).

- For globular clusters which have been associated with the CMa dSph (Frinchaboy et al. 2004; Martin et al. 2004), the data come from Corwin et al. (2004) (NGC 2808), Walker (1998) (NGC 1851, with the addition of five new confirmed RRab’s from Sumerel et al. 2004), and Zorotovic et al. (2009, in preparation) (NGC 5286). Metallicity values are from the Harris (1996) catalog (Feb. 2003 update).
- As to the LMC globular clusters, we retrieved OGLE data (Soszyński et al. 2003) from their online catalog, and derived mean periods therefrom. The clusters that were included in their survey turned out to be NGC 1835, NGC 1898, NGC 1916, NGC 1928, NGC 2005, and NGC 2019. For NGC 1466, we used the value from Walker (1992b). For the remaining clusters—namely, Reticulum, NGC 1786, NGC 1841, NGC 2210, and NGC 2257—we utilized the values summarized in Table 6 of Walker (1992a). Metallicity values for all LMC clusters were taken from the recent Mackey & Gilmore (2004b) compilation.

An interesting characteristic found in the OGLE data for NGC 1835 is the presence of a bimodal period distribution for the RRab variables, as if the

10 The widespread usage of the term “Bailey diagram” to refer to the period-amplitude diagram appears to be relatively recent: for instance, it is not used either in Sandage’s (1958) Vatican Conference paper or in Smith’s (1995) monograph. To be sure, Bailey (1913, 1919) did produce plots showing, among many other quantities, “range” (i.e., amplitude) vs. period (his Figs. 4 and 7, respectively), but no great emphasis appears to have been placed by him on this specific diagram, other than noting that “there appears to be... a fairly well marked relation between the period and the maximum magnitude, the minimum magnitude, and the range of variation” (Bailey 1913, his p. 87).

11 http://www.physics.mcmaster.ca/Globular.html

12 ftp://ftp.astrouw.edu.pl/ogle/ogle2/var_stars/lmc_rrlyr/
Table 2  Galactic Globular Clusters with at Least 5 Known RRab Variables

| Name, Cluster | Other | [Fe/H] | $\langle P_{ab} \rangle$ (d) | $N_{ab}$ | Type | $\mathcal{L}$ | Population
|---------------|-------|--------|----------------|--------|------|----------|----------------|
| NGC 362       |       | −1.16  | 0.564          | 13     | OoI  | −0.87    | YH             |
| NGC 1261      |       | −1.35  | 0.555          | 13     | OoI  | −0.71    | YH             |
| NGC 2419      |       | −2.12  | 0.655          | 24     | OoII | +0.86    | OH             |
| NGC 3201      |       | −1.58  | 0.554          | 72     | OoI  | +0.08    | YH             |
| NGC 4590 M68  |       | −2.06  | 0.622          | 14     | OoII | +0.17    | YH             |
| NGC 4833      |       | −1.88  | 0.708          | 7      | OoII | +0.93    | OH             |
| NGC 5024 M53  |       | −1.99  | 0.649          | 29     | OoII | +0.81    | OH             |
| NGC 5053      |       | −2.29  | 0.672          | 5      | OoII | +0.52    | YH             |
| NGC 5139 ω Cen|       | −1.62  | 0.651          | 76     | OoII | +0.89    | ω C            |
| NGC 5272 M3   |       | −1.57  | 0.555          | 145    | OoI  | +0.18    | YH             |
| NGC 5466      |       | −2.22  | 0.646          | 13     | OoII | +0.58    | YH             |
| NGC 5824      |       | −1.85  | 0.624          | 7      | OoII | +0.79    | OH             |
| NGC 5904 M5   |       | −1.27  | 0.551          | 91     | OoI  | +0.31    | OH             |
| NGC 5986      |       | −1.58  | 0.652          | 7      | OoII | +0.97    | OH             |
| NGC 6121 M4   |       | −1.20  | 0.553          | 31     | OoI  | −0.06    | OH             |
| NGC 6171 M107 |       | −1.04  | 0.538          | 15     | OoI  | −0.73    | OH             |
| NGC 6229      |       | −1.43  | 0.553          | 30     | OoI  | +0.24    | YH             |
| NGC 6266 M62  |       | −1.29  | 0.548          | 131    | OoI  | +0.55    | OH             |
| NGC 6284      |       | −1.32  | 0.588          | 6      | Oo-Int| +0.88    | OH             |
| NGC 6333 M9   |       | −1.75  | 0.638          | 8      | OoII | +0.87    | OH             |
| NGC 6341 M92  |       | −2.28  | 0.630          | 11     | OoII | +0.91    | OH             |
| NGC 6362      |       | −0.95  | 0.547          | 18     | OoI  | −0.58    | OH             |
| NGC 6388      |       | −0.60  | 0.676          | 9      | OoIII| −0.69    | BD             |
| NGC 6402 M14  |       | −1.39  | 0.564          | 39     | OoI  | +0.65    | OH             |
| NGC 6426      |       | −2.26  | 0.704          | 9      | OoII | +0.58    | YH             |
| NGC 6441      |       | −0.53  | 0.756          | 43     | OoIII| −0.73    | BD             |
| NGC 6558      |       | −1.44  | 0.556          | 6      | OoI  | +0.70    | OH             |
| NGC 6584      |       | −1.49  | 0.560          | 34     | OoI  | −0.15    | YH             |
| NGC 6626 M28  |       | −1.45  | 0.577          | 8      | OoI  | +0.90    | OH             |
| NGC 6642      |       | −1.35  | 0.544          | 10     | OoI  | −0.04    | YH             |
| NGC 6656 M22  |       | −1.64  | 0.632          | 10     | OoII | +0.91    | OH             |
| NGC 6712      |       | −1.01  | 0.557          | 7      | OoI  | −0.62    | OH             |
| NGC 6723      |       | −1.12  | 0.541          | 23     | OoI  | −0.08    | OH             |
| NGC 6864 M75  |       | −1.16  | 0.587          | 25     | Oo-Int| −0.07    | OH             |
| NGC 6934      |       | −1.54  | 0.574          | 68     | OoI  | +0.25    | YH             |
| NGC 6981 M72  |       | −1.40  | 0.547          | 24     | OoI  | +0.14    | YH             |
| NGC 7006      |       | −1.63  | 0.569          | 53     | OoI  | −0.28    | YH             |
| NGC 7078 M15  |       | −2.26  | 0.637          | 39     | OoII | +0.67    | YH             |
| NGC 7089 M2   |       | −1.62  | 0.725          | 17     | OoII | +0.92    | OH             |
| IC 4499       |       | −1.60  | 0.580          | 63     | Oo-Int| +0.11    | YH             |
| Ruprecht 106  |       | −1.67  | 0.617          | 13     | Oo-Int| −0.82    | YH             |

*aYH = “Young Halo”; OH = “Old Halo”; BD = “Bulge/Disk”; ω C = ω Centauri. From Mackey & van den Bergh (2005).*
Table 3  Milky Way Dwarf Galaxy Satellite Globular Clusters with at Least 5 RRab Variables

| Name          | Other | [Fe/H] | \(\langle P_{ab}\rangle\) (d) | \(N_{ab}\) | Type     | \(L\)  | Population |
|---------------|-------|--------|-------------------------------|-------------|----------|--------|------------|
| **LMC Globular Clusters**                           |       |        |                               |             |          |        |            |
| NGC 1466      |       | −2.17  | 0.581                         | 19          | Oo-Int   | +0.42  | LMC        |
| NGC 1786      |       | −1.87  | 0.677                         | 17          | OoII     | +0.39  | LMC        |
| NGC 1835      |       | −1.79  | 0.606                         | 55          | Oo-Int   | +0.57  | LMC        |
| NGC 1841      |       | −2.11  | 0.676                         | 18          | OoII     | +0.71  | LMC        |
| NGC 1898      |       | −1.37  | 0.565                         | 17          | OoI      | +0.03  | LMC        |
| NGC 1916      |       | −2.08  | 0.729                         | 6           | OoII     | +0.97  | LMC        |
| NGC 1928      |       | −1.27  | 0.634                         | 6           | OoII     | +0.94  | LMC        |
| NGC 2005      |       | −1.92  | 0.668                         | 6           | OoII     | +0.90  | LMC        |
| NGC 2019      |       | −1.81  | 0.606                         | 24          | Oo-Int   | +0.66  | LMC        |
| NGC 2210      |       | −1.97  | 0.598                         | 13          | Oo-Int   | +0.65  | LMC        |
| NGC 2257      |       | −1.63  | 0.578                         | 17          | OoI      | +0.42  | LMC        |
| Reticulum     |       | −1.66  | 0.559                         | 16          | OoI      | +0.00  | LMC        |
| **Fornax dSph Globular Clusters**                   |       |        |                               |             |          |        |            |
| Fornax GC1    |       | −2.05  | 0.611                         | 5           | Oo-Int   | −0.30  | Fornax    |
| Fornax GC2    |       | −1.83  | 0.574                         | 15          | OoI      | +0.50  | Fornax    |
| Fornax GC3    | NGC 1049 | −2.04  | 0.606                         | 13          | Oo-Int   | +0.44  | Fornax    |
| Fornax GC4    |       | −1.90  | 0.600                         | 11          | Oo-Int   | −0.42  | Fornax    |
| Fornax GC5    |       | −1.90  | 0.576                         | 7           | OoI      | +0.52  | Fornax    |
| **Sagittarius dSph (Candidate) Globular Clusters**   |       |        |                               |             |          |        |            |
| NGC 4147      |       | −1.83  | 0.525                         | 12          | OoI      | +0.55  | Sag?      |
| NGC 5634      |       | −1.88  | 0.660                         | 12          | OoII/Int | +0.91  | Sag/OH?   |
| NGC 6715      | M54   | −1.58  | 0.590                         | 55          | Oo-Int   | +0.54  | Sag       |
| Arp 2         |       | −1.76  | 0.584                         | 8           | Oo-Int   | +0.53  | Sag       |
| **CMa dSph (Candidate) Globular Clusters**           |       |        |                               |             |          |        |            |
| NGC 1851      |       | −1.22  | 0.571                         | 21          | OoI      | −0.32  | CMa/OH?   |
| NGC 2808      |       | −1.15  | 0.563                         | 10          | OoI      | −0.49  | CMa/OH?   |
| NGC 5286      |       | −1.67  | 0.630                         | 29          | OoII     | +0.80  | CMa/OH?   |
Horizontal Branch Stars: Observations and Theory

cluster actually presented an internal Oosterhoff dichotomy (see Fig. 4 in Soszyński et al. 2003). A similar phenomenon has recently been discovered in the Fornax dSph globular cluster 4 (Greco et al. 2005). There are preliminary indications (Vidal et al. 2009, in preparation) that NGC 6284 may, in spite of the cluster’s Oosterhoff-intermediate classification indicated by \( \langle P_{ab} \rangle \), lack RR Lyrae stars in the Oosterhoff gap region as well.

- In the special case of the OoIII globular clusters NGC 6388 and NGC 6441, the newly revised \( \langle P_{ab} \rangle \) values from Corwin et al. (2006) were adopted.
- For the dSph galaxies, the values come from Table 1 in Catelan (2004b).
- For the SMC, we adopted the \( \langle P_{ab} \rangle \) value from Soszyński et al. (2002), and a metallicity that is an average between the Smith et al. (1992) and Butler, Demarque, & Smith (1982) values.
- For the LMC, the adopted RR Lyrae metallicity represents an average between the spectroscopic measurements of Borissova et al. (2004) and Gratton et al. (2004a). The question regarding the LMC’s \( \langle P_{ab} \rangle \) value is more complicated: as it happens, the MACHO (Alcock et al. 1996) and OGLE (Soszyński et al. 2003) surveys provide results that differ by 0.01 d from one another, the MACHO result, \( \langle P_{ab} \rangle = 0.583 \) d, placing the galaxy inside the Oosterhoff gap region in Figure 5, but the OGLE result, \( \langle P_{ab} \rangle = 0.573 \) d, indicating an OoI classification instead. Given the large databases used in both studies—5455 RRab’s in the case of OGLE (Soszyński et al. 2003) and 6158 in the case of MACHO (Alcock et al. 2003)—this difference appears to be intrinsic. This could be due to population gradients in the LMC, since whereas the OGLE fields are concentrated along the bar of the LMC, the MACHO survey includes many LMC halo fields.\(^{13}\)

A similar segregation has been suggested to exist in our own galaxy, in the sense that OoI (i.e., longer-period) variables would be more confined to relatively small distances from the Galactic plane, whereas OoII variables would lie farther from the plane on average\(^{14}\) only that, in the case of the LMC, the longer-period variables would be located preferentially farther away from the bar.

For our present purposes, we decided to include two separate datapoints for the LMC RR Lyrae in Figure 5, to reflect the possible presence of two different populations. We note, in addition, that Alcock et al. (1996) already called attention to the Oosterhoff-intermediate nature of the RR Lyrae period distribution in that galaxy. For a recent discussion of the evidence for a genuine halo component in the LMC (as indicated primarily by the velocity dispersion of the RR Lyrae), see Minniti et al. (2003) and Borissova et al. (2004), but also Gallart et al. (2004), Zaritsky (2004), and Carrera et al. (2008) (and references therein) for possible alternative interpretations.

Figure 6 (right panel) compares the \( \langle P_{ab} \rangle \) distributions for Galactic globular clusters and nearby extragalactic systems. Simple eye inspection clearly reveals that the two distributions are remarkably different, with the one for extragalactic systems strongly peaking at a \( \langle P_{ab} \rangle \approx 0.6 \) d, which however corresponds to the minimum of the Galactic globular cluster distribution. A Kolmogorov-Smirnov test confirms that the difference is highly significant, with a \( D_{KS} = 0.3338 \), implying a probability that the two sets are derived from the same parent distribution of only \( P_{KS} = 1.8\% \). Removing the dwarf galaxies (but not their associated globular clusters) from the sample changes these figures slightly: we now find \( D_{KS} = 0.2856 \), implying a \( P_{KS} = 12.5\% \). (These figures do not change significantly if we keep only the “young halo” component among the Galactic globulars.) Removing NGC 6388 and NGC 6441 from the Galactic globular cluster sample so that the comparison is limited to globular cluster systems having comparable metallicities, we find \( D_{KS} = 0.3034 \), implying a \( P_{KS} = 9.2\% \). Finally, by assuming that \( \omega \) Cen, NGC 1851, NGC 2808, and NGC 5286 are all bona-fide members of the Galactic globular cluster system—which is equivalent to assuming that the CMa dSph does not possess its own globular cluster system, which might perhaps be supported by the fact that the suggested CMa galaxy appears to be much too young and metal-rich (e.g., Sbordone et al. 2005b; Carraro, Moitinho, & Vázquez 2008) to host several old and metal-poor globulars—we find \( D_{KS} = 0.3714 \), with a corresponding \( P_{KS} = 2.8\% \).

In summary, it appears quite clear that the distribution of mean RRab periods for the Galactic and nearby extragalactic systems have only a very small probability of being compatible with one another.

\(^{13}\)For a comparison between the two teams’ area coverage, see the maps available at [http://bulge.princeton.edu/~ogle/ogle2/rrlyr_lmc_map.html](http://bulge.princeton.edu/~ogle/ogle2/rrlyr_lmc_map.html) (OGLE team) and [http://www.macho.mcmaster.ca/Systems/Coords/LMC_Fields.gif](http://www.macho.mcmaster.ca/Systems/Coords/LMC_Fields.gif) (MACHO project).

\(^{14}\)This presumably implies that the HB morphology of the field is bluer near the plane than farther away, which is consistent with the results from Preston, Seldman, & Beers (1991) and Kinman, Suntzeff, & Kraft (1994). Interestingly, Lee & Carney (1999a) note that the results of Layden (1996) for RR Lyrae stars and Wilhelm et al. (1996) for HB stars in general support an accompanying change in kinematic behavior; from prograde systemic rotation to retrograde rotation as one moves away from the plane.
6.2 On the Origin of the Oosterhoff Dichotomy

In Figure 7, we show the variation in the $\mathcal{L}$ parameter with metallicity for Galactic globular clusters. To produce this figure, the main source of HB morphology parameters was the compilation by Mackey & van den Bergh (2005) for Galactic globulars, and the one by Mackey & Gilmore (2004b) for nearby extragalactic globular clusters. The [Fe/H] values came from the Feb. 2003 edition of the Harris (1996) catalog. We note, however, that some of the values provided in the Mackey & van den Bergh compilation appear to be incorrect. For NGC 6388 and NGC 6441, in particular, they quote values of $\mathcal{L} = -1.0$, which cannot be correct given the well-known presence of extended blue HB components (Piotto et al. 1997; Rich et al. 1997) and RR Lyrae variables (Silbermann et al. 1994; Layden et al. 1999; Pritzl et al. 2000, 2001, 2002b, 2003; Corwin et al. 2006) in both clusters. Accordingly, we computed new $\mathcal{L}$ values for these globulars, from the photometry provided in the quoted papers and in Piotto et al. (2002). Number counts for non-variable stars in the Piotto et al. study were kindly provided by M. Zoccali (2003, priv. comm.), whereas the number counts for RR Lyrae variables were taken from the Pritzl et al. and Corwin et al. studies. We obtain $\mathcal{L} \simeq -0.69$ for NGC 6388, and $\mathcal{L} \simeq -0.73$ for NGC 6441.

In the case of M3, Catelan, Ferraro, & Rood (2001b) and Catelan, Rood, & Ferraro (2002b) have shown that the HB morphology is not constant with radius, the HB type closer to the center being significantly bluer than the HB type farther away. Previous values of the $\mathcal{L}$ parameter for the cluster tended to be based on photometric measurements that were carried out for the outer parts, so that the HB type was accordingly underestimated. When properly taking into account HB stars from all radial regions in the cluster in the number counts, a value $\mathcal{L} = 0.18$ obtains (Catelan 2004a)—which is about 0.1 bluer than provided in the available compilations (e.g., Harris 1996; Mackey & van den Bergh 2005).

$\omega$ Centauri (NGC 5139) lacks an HB type measurement in both the Harris (1996) and Mackey & van den Bergh (2005) compilations. We accordingly adopt the HB type provided by Borkova & Marsakov (2000), namely $\mathcal{L} = 0.89$.

In Figure 7 are also displayed isochrones based on the work of Catelan & de Freitas Pacheco (1993). These are practically identical to the ones presented in their Figure 1a, the difference in age with respect to the middle (reference) isochrone being +0.8 Gyr (upper
isochrone) and $-2.0$ Gyr (lower isochrone).\textsuperscript{15} These isochrones were computed assuming that the total mass loss on the RGB does not depend on metallicity, as recently suggested by Origlia et al. (2002). On the other hand, an RGB mass loss dependence on metallicity, as suggested by the expressions shown in Table 1 (see Fig. 4), would lead to somewhat distorted isochrones in the $L - [\text{Fe/H}]$ plane, predicting redder HB types at lower $[\text{Fe/H}]$ (and vice-versa) than would be the case for a similar, constant-$\Delta M$ isochrone. This effect is clearly shown in Fig. 9 of Rey et al. (2001), which compares isochrones computed with constant mass loss and isochrones computed assuming a mass loss rate as given by the Reimers (1975a,b) mass loss formula.

Note that these isochrones can be well described by a modified Fermi-Dirac profile as follows:

$$L = \frac{2}{1 + \gamma \exp \left( \frac{[\text{Fe/H]} + \alpha}{\beta} \right)} - 1. \quad (1)$$

The middle isochrone provides a reasonable approximation to the inner-halo globular clusters, which in fact comprise a large fraction of the “old halo” globulars shown in Figure 5. It is also very similar to the isochrones labeled “$\Delta t = 0.0$” in Rey et al. (2001) (their Fig. 9). For this isochrone, the parameters used are $\alpha = 1.227$, $\beta = 0.130$, and $\gamma = 1$. For the lower isochrone, which in Catelan & de Freitas Pacheco...
corresponded to the LMC globular clusters, one has $\alpha = 1.707$, $\beta = 0.140$, and $\gamma = 1$; the relative age scale is fairly similar to that in Fig. 9b of Rey et al., except (as expected) for the most metal-poor clusters with the redder HB types. Finally, the upper isochrone represents the RR Lyrae stars in the Galactic bulge, and is well described by $\alpha = 1.027$, $\beta = 0.130$, and $\gamma = 1$.

Note that the plots in Figure 7 actually use an inverted form of eq. (1), which we also provide for convenience:

$$[\text{Fe/H}] = \beta \ln \left( \frac{2}{1 + L} - 1 \right) \frac{1}{\gamma} - \alpha. \quad (2)$$

Figure 7 (left panel) shows very clearly that there is a zone of avoidance for RR Lyrae-rich globular clusters in our galaxy, defined by a rectangle located in the region $-2.18 \lesssim [\text{Fe/H}] \lesssim -1.5$, $0.3 \lesssim L \lesssim 0.75$. However, in the right panel, when globular clusters associated with the neighboring galaxies are plotted, one sees that this region is actually preferentially occupied by the external globulars. The situation immediately brings to mind the similar phenomenon that was found in our analysis of Figure 5; in that case, the “zone of avoidance” for the Galactic globulars—the Oosterhoff gap—was also the zone preferentially occupied by the globulars associated with neighboring galaxies. Therefore, one cannot help but suspect that the noted “avoidance region” in the left panel of Figure 7 will also be related to the Oosterhoff-intermediate globulars, as was the Oosterhoff gap region in Figure 5. Indeed, Figure 7 (right panel) clearly shows that nearby extragalactic globulars do not only preferentially fall in the quoted region of this diagram: there is in fact a smaller, well-defined, triangular-shaped region in the plot where most Oosterhoff-intermediate globulars of extragalactic origin (7 out of 9) can be found. The vertices of this triangle, in the $[\text{Fe/H}] - L$ plane, are given by the following coordinates: $(-1.5, 0.525)$; $(-1.92, 0.76)$; and $(-2.26, 0.36)$.

Note that the four Oosterhoff-intermediate Galactic globulars (NGC 6284; M75 = NGC 6864; IC 4499; Ruprecht 106), indicated by large “×” symbols in the right-hand plot, are located very far away from the “Oosterhoff gap” region in this diagram, and may accordingly represent different phenomena, or even be due to statistical fluctuations (see also Catelan 2004b). Note, in this sense, that NGC 6284 has a mere 6 known RRab’s (Clement et al. 2001), none of which with a period that falls in the Oosterhoff gap range (i.e., 0.58–0.62 d); that M75 has a multimodal HB; and that the RRab’s in Rup 106 (which entirely lacks RRc’s) all fall very close to the red edge of the instability strip. Finally, IC 4499 is at the very edge of the Oosterhoff gap region, with a mean ab-type period of exactly 0.580 d, and might as well have been classified as OoI.

This scenario provides unprecedented detail about the origin of the Oosterhoff dichotomy, going significantly beyond the original and insightful early analyses of the problem by Castellani (1983), Renzini (1983), Lee & Zinn (1990), and Bono, Caputo, & Stellingwerf (1994)—who previously suggested that the origin of the Oosterhoff dichotomy among Galactic globular clusters was the lack of RR Lyrae-rich globulars (or, more specifically, the presence of a majority of clusters with exclusively blue HBs) at metallicities intermediate between the bulk of the OoI and the OoII clusters. Indeed, theoretical predictions of an “Oosterhoff-intermediate area” in the $[\text{Fe/H}] - L$ plane have previously been provided by Lee & Zinn and Bono et al.; the results of these studies are summarized in Figure 8, which shows reasonable success in predicting which globulars should be Oosterhoff-intermediate. It is interesting to note that the two Oosterhoff-intermediate Fornax dSph globular clusters which lie farther away from the triangular-shaped region in Figure 8, as well as Rup 106, fall very close to the Oosterhoff-intermediate region located towards red HB types in this plane, in agreement with the theoretical predictions by Lee & Zinn. (A region equivalent to the area labeled “Bono et al.” in this plot was also predicted, though over a somewhat more limited $L$ range for a given $[\text{Fe/H}]$, by Lee & Zinn—see their Fig. 2.)

An important question that remains unanswered and which will certainly require additional work is the following: why did the Galactic globular cluster system avoid the triangular-shaped region in Figure 7, whereas the nearby extragalactic globulars were instead preferentially “attracted” to it? According to Figure 7, part of the explanation could be that the Galactic “old halo” and nearby extragalactic globular clusters differ in age by $\sim 2$ Gyr (on average). However, little (if any) evidence for an age difference has been found so far: in the case of the Fornax dSph, Buonanno et al. (1998) show that all globular clusters have similar ages, comparable to those found in Galactic globular clusters of similar metallicity. The exception appears to be cluster 4, which is $\sim 3$ Gyr younger than the other Fornax dSph globulars (Buonanno et al. 1999), and which indeed appears to have the redder HB type (see Table 3). In the case of the LMC, virtually all “old” clusters have ages comparable to those of the oldest Galactic globular clusters, a point which has been repeatedly emphasized in the recent literature (e.g., Brocato et al. 1996; Johnson et al. 1999, 2002; Mackey &
Among the Oosterhoff-intermediate globular clusters in the Sagittarius dSph, only Arp 2 was previously suggested to be “young” (but note that it has a predominantly blue HB), although the recent VandenBerg (2000) and Salaris & Weiss (2002) studies suggest that its age is probably not much lower than those of Galactic globular clusters of similar metallicity. Also, Layden & Sarajedini (2000) argue that the Sagittarius dSph globulars M54 and Arp 2 (as well as Ter 8) are basically coeval with the bulk of the Galactic halo globular clusters. Therefore, it appears difficult, given the available age determinations for Oosterhoff-intermediate globular clusters, to argue that they differ in age significantly with respect to OoII and OoI globulars. In a similar vein, it should be noted that the separation of the clusters into “young halo” and “old halo” groups by Mackey & van den Bergh (2005) is somewhat artificial, since there is clearly significant overlap in turnoff ages between the two groups, as can be seen from Figure 9 in Mackey & Gilmore (2004b): according to the quoted figure, the oldest “young halo” globular clusters are at least as old as the oldest “old halo” clusters, and likewise, the youngest “old halo” clusters have ages that are similar to those of the youngest “young halo” clusters. This clearly shows the perils of using HB morphology as an age indicator for individual globular clusters. We note, in passing, that De Angeli et al. (2005) find no clear correlation between the ages of Galactic globular clusters and their galactocentric distances.

6.3 Implications for the Formation History of the Galaxy

Several authors have suggested that the “young halo” Galactic globular cluster system at least may have been formed from the accretion of dwarf galaxies resembling the present-day dwarf satellites of the Milky Way (e.g., Searle & Zinn 1978; Zinn 1993b; Mackey & van den Bergh 2005). However, we have already seen (Sect. 6.1) that the distribution of RR Lyrae periods for the Galactic globulars, including the “young halo” component, is different from that of those systems. It is instructive to perform a similar quantitative comparison between the distribution of HB types for the Galactic and nearby extragalactic globular clusters.

Using only the “young halo” component, we find a Kolmogorov-Smirnov value of \( D_{KS} = 0.5425 \), implying a probability of only \( P_{KS} = 0.2\% \) that the “young halo” Galactic globulars and the nearby extragalactic globulars were drawn from the same parent population. If we remove the LMC globulars from the sample (given that many authors suggest that the protogalactic fragments that led to the formation of the Milky Way were actually dSph-like, not LMC-like), we find \( D_{KS} = 0.498 \), implying a probability of only \( P_{KS} = 2\% \) that the samples were drawn from the same parent distribution. Therefore, the Kolmogorov-Smirnov test rejects, with high statistical significance, the hypothesis that the Galactic globular cluster system was assembled from the capture of protogalactic fragments that resembled the present-day dwarf satellite galaxies of the Milky Way—Fornax and Sagittarius in particular.

This does not mean, of course, that there may not have been a (now extinct) primordial dwarf galaxy population with properties that were different from those of the surviving dwarf satellite galaxies of the Milky Way. We thus face the task of defining whether our current sample of dwarf galaxies is atypical in some respect. For instance, it is now well established that the detailed chemical patterns of even the more metal-poor populations of the Milky Way dwarf satellite galaxies bear little resemblance to that found among most stars in...
our satellite galaxies do not appear to be anomalous in this respect, since Bonifacio et al. (2004; see especially their Fig. 5) have shown that their abundance patterns are consistent with those for objects which are supposed to have given rise to dSph galaxies—namely, the Damped Lyman α (DLA) systems (e.g., Matteucci, Molaro, & Vladilo 1997; Hopkins, Rao, & Turnshek 2005). Also, the globular clusters which were unquestionably accreted by the Milky Way from dSph fragments and which have been carefully studied for chemical abundance patterns, such as M54 (Brown, Wallerstein, & Gonzalez 1999), Palomar 12 (Brown et al. 1997; Cohen 2004), and Terzan 7 (Tautvaišienė et al. 2004; Sbordone et al. 2005a, 2007), show clearly different abundance patterns with respect to bona-fide Galactic globulars. A careful and detailed discussion of this point has recently been presented by Pritzl, Venn, & Irwin (2005b). In fact, the presence of such peculiar abundance patterns, particularly a lowered abundance of the alpha-captured elements, has recently been viewed as a signature of a possible accretion origin; the reader is referred to the recent spectroscopic study of NGC 5694 by Lee, López-Morales, & Carney (2006) for a recent example. Note, on the other hand, that there are at least some outer-halo globular clusters with completely normal abundance patterns (Ivans et al. 2001; Cohen & Melendez 2005; see also footnote 19 below), thus suggesting that it is not only the inner Galactic halo that contains globular clusters that formed “in situ.” On the other hand, Mottini, Wallerstein, & McWilliam (2008) have recently claimed that Arp 2 and Terzan 8—which are also Sagittarius globulars—both have alpha-element abundances that are typical of Galactic halo clusters, which certainly confuses the chemical picture somewhat. As far as the newly discovered SDSS dSph galaxies (e.g., Belokurov et al. 2006, 2007), recent surveys have revealed that at least some of them are also Oosterhoff intermediate (e.g., Kuehn et al. 2008) and present unusual abundance patterns (Frebel et al. 2009, in preparation), although seemingly “regular” OoI dwarfs are also present (Siegel 2006; Dall’Ora et al. 2006; Greco et al. 2008; Musella et al. 2009, in preparation; see also Catelan 2009 for a critical discussion).

In conclusion, most of the recent evidence suggests that, even though we seem to be witnessing mergers between dSph satellite galaxies and our own galaxy “in real time” (e.g., Sagittarius, CMa), these are not truly representative of the events that led to the formation of the Milky Way, and only a relatively minor fraction of the Galactic halo may have been assembled from dSph-like protogalactic fragments resembling the present-day Milky Way dSph satellites (see also Geisler et al. 2007 for a recent discussion).

6.3.1 RR Lyrae Stars in M31 and the Origin of the Galactic Halo

It has recently been suggested that the “young” second-parameter globular clusters in our galaxy (as well as at least some of the Milky Way dSph satellites) were accreted from M31 when the latter was forming its Population II stars (Kravtsov 2002). Given that our galaxy’s globular clusters clearly present the Oosterhoff dichotomy, but not so its dSph satellite galaxies, the question naturally presents itself: how does M31 classify, in terms of Oosterhoff status?

Of course, this is a very difficult question to answer, given that the detection of RR Lyrae variable stars at the distance of M31 is very difficult from the ground. It was originally attempted by Pritchet & van den Bergh (1987) using the CFH 3.6m telescope. These authors claimed the detection of 30 probable RR Lyrae stars in their surveyed field, thus suggesting that the M31 halo has a specific frequency of RR Lyrae variables comparable to that in RR Lyrae-rich Galactic globular clusters. They also reported a mean period for the ab-type variables of $\langle P_{ab} \rangle = 0.548$ d, thus classifying the M31 halo field as OoI.

More recently, Dolphin et al. (2004) performed a variability survey, using the WIYN 3.5m telescope, of an M31 halo field that includes the Pritchet & van den Bergh (1987) field, finding a specific frequency of RR Lyrae variables dramatically lower than in the Pritchet & van den Bergh study. Using time-series observations with ACS onboard the HST, Brown et al. (2004a) have shown that, in fact, the specific RR Lyrae frequency of the M31 halo is intermediate between the two quoted studies. Interestingly, they also find a $\langle P_{ab} \rangle = 0.594$ d, which corresponds to the Oosterhoff-intermediate domain in Figure 5. On the other hand, as we have seen, the dSph satellites of the Milky Way are indeed almost exclusively Oosterhoff-intermediate—and so are probably the dSph satellites of M31 as well (Pritzl et al. 2002a, 2005a). Therefore, the incorporation of dSph satellites from the M31 system into our own galaxy’s would not fundamentally change the stellar pulsation properties of the Milky Way’s dSph system.

To be sure, extensive variability surveys of not only the M31 halo field and its dwarf satellite galaxies, but also (and most challenging of all) of its globular clusters, are badly needed to place constraints on its old halo’s (and indeed, as we have just seen, the whole Local Group’s) formation history. It would be very important to establish whether the Oosterhoff dichotomy is present in the M31 globulars, as well as in M31’s general field, since this would enable a systematic comparison between the very oldest stars in the M31 dSph...
satellites and their peers in the general M31 halo, thus posing constraints on the extent to which the latter may have been assembled from “building blocks” similar to the former. Unfortunately, such studies have so far remained rather limited (Clementini et al. 2001; Contreras et al. 2008), given that they require expensive time-series observations from space. Ground-based surveys could in principle be carried out using large telescopes equipped with adaptive optics instruments, but unfortunately, this technique remains limited to near-infrared observations—a wavelength regime where not only the red giants are brightest but also the amplitudes of RR Lyrae stars are smallest (e.g., Longmore et al. 1985; Liu & Janes 1990b; Jones, Carney, & Fulbright 1996), thus making their detection hardest.

7 The Second-Parameter Problem

The second parameter problem of globular cluster astronomy is commonly defined as the existence of globular clusters with very similar metallicity (the “first parameter”; Sandage & Wallerstein 1960) and yet different HB morphologies. It appears to have been first noted by Sandage & Wallerstein (see pages 607 and 608 in their paper), and later also by Sandage & Wildey (1967) and van den Bergh (1967). The phenomenon has now been recognized to exist also in the M31 halo (Mackey et al. 2006), as well as in the Fornax dSph galaxy (Buonanno et al. 1998, 1999).

The second parameter candidate that was first noted in the literature seems to have been age. We quote Sandage & Wallerstein (1960):

... the character of the horizontal branch is spoiled by the two clusters M13 and M22. [Individual stars in] M13 appear to be metal-rich, whereas the character of the horizontal branch simulates that of the very weak-lined group (M15, M92, NGC 5897). [...] M13 is younger than M2 or M5 (Arp 1959). Consequently, in addition to chemical composition, the second parameter of age may be affecting the correlations...

(Note that the sense of the claimed age difference between M13 and other clusters of similar metallicity but redder HB type is the opposite of what modern stellar evolution theory indicates to be necessary to account for their differences in HB types.)

The next candidate second parameter was the helium abundance (Sandage & Wildey 1967; van den Bergh 1967). Later on, with the advent of the Searle & Zinn (1978) scenario for the formation of the Galactic halo—which was based upon the hypothesis that age is the second parameter, outer-halo globular clusters with predominantly red HBs being younger than inner-halo globulars with bluer HBs (see their Fig. 10)—age has quickly become the most popular second parameter candidate. However, the age hypothesis has proven controversial, with different perspectives having been presented on both the presence of significant age differences among Galactic globular clusters (e.g., Stetson et al. 1996; Sarajedini et al. 1997; and references therein) and the amount that may be required to account for different second-parameter pairs entirely in terms of age (e.g., Catelan & de Freitas Pacheco 1993; Lee et al. 1994; Ferraro et al. 1997). In addition, other candidate second parameters abound in the literature; these include (or are at least related to, and often involve combinations of) mass loss (Petersen 1982; Catelan 2000), cluster ellipticity (Norris 1983, 1987), stellar rotation (Mengel & Gross 1976; Fusi Pecci & Renzini 1978; Peterson 1985), magnetic fields (Rood & Seitzer 1981; Castellani 1983; Castellani & Paternò 1984), the cyanogen distribution among red giants (Norris 1981; Norris et al. 1981; Smith & Norris 1983), [CNO/Fe] (Rood & Seitzer 1981), super-oxygen-poor RGB stars (Catelan & de Freitas Pacheco 1995), the Na-O correlation among cluster giants (Carretta & Gratton 1996; Carretta et al. 2007), the helium-core mass at the helium flash (Demarque, Mengel, & Sweigart 1972; Sweigart & Catelan 1998), helium mixing (VandenBerg & Smith 1988; Langer & Hoffman 1995; Sweigart 1997a,b; Sweigart & Catelan 1998; Cavallo & Nagar 2000; Aikawa, Fujimoto, & Kato 2001; Caloi 2001), planetary systems (Soker 1998; Siess & Livio 1999; Soker & Harpaz 2000; Soker & Hadar 2001; Soker, Rappaport, & Fregeau 2001; Livio & Soker 2002; Soker & Harpaz 2007), globular cluster core density or concentration (Fusi Pecci et al. 1993; Buonanno et al. 1997), and cluster mass (Recio-Blanco et al. 2006). A very informative and relatively recent review of the several “second parameters” that have been proposed in the literature is provided by Fusi Pecci & Bellazzini (1997).

Recently, Cavallo, Suntzeff, & Pilachowski (2004) have argued that neither the deep mixing nor the primordial channel are capable of satisfactorily accounting for the abundance patterns observed among globular cluster red giant stars. In the same vein, Sneden et al. (2004) present an extremely interesting conundrum in regard to the extreme abundance anomalies seen in stars close to the RGB tip in M13, and whose solution may hold the key to the extent to which deep mixing and/or primordial contamination may affect the evolution of the stars on the HB:

The correlation of O and the Mg isotopic abundance ratio suggests that M13 giants in a very small interval of initial mass preferentially underwent severe pollution or ablation or, alternatively, were preferentially formed from material secularly ejected earlier from more massive cluster giants. These stars only just at this moment arrived essentially at the red giant tip, exactly where the effect of deep mixing, if it indeed exists, would likely be most easily manifest. It is a most remarkable coincidence, if indeed that’s what it is.
As mentioned previously, the helium abundance was one of the first second parameter candidates to be proposed in the literature. Recently, with observations of faint stars in globular clusters having shown that many of the abundance patterns observed among bright giants that might have been ascribed to deep mixing are also present down to the main-sequence level (see Gratton et al. 2004b for an extensive review and references), thus severely constraining the extent to which deep mixing may contribute to the observed patterns among the brighter giants, the helium abundance scenario has regained impetus, now under the label of primordial contamination by a previous generation of (likely massive AGB) stars (e.g., D’Antona et al. 2002, 2005; D’Antona & Caloi 2004, 2008; Norris 2004; Caloi & D’Antona 2005, 2007; Lee et al. 2005b; Busso et al. 2007; Newsham & Terndrup 2007; Piotto et al. 2007; Yoon et al. 2008; and references therein). For critical discussions of different theoretical scenarios for the formation of helium-enhanced (sub-)populations in globular clusters, the reader is referred to the recent papers by Bekki & Norris (2006), Chuzhoy (2006), Maeder & Meynet (2006), Prantzos & Charbonnel (2006), Bekki et al. (2007), D’Antona & Ventura (2007), Choi & Yi (2007, 2008), and Decressin, Charbonnel, & Meynet (2007). The earlier paper by Shi (1995, see its §4), which unfortunately has received little attention, already contained an interesting discussion of several different scenarios that could generate helium-enhanced stars in globular clusters. In turn, modern views of the deep helium mixing scenario have recently been advanced as well, including papers by Suda & Fujimoto (2006), Suda et al. (2007), and Yamada, Okazaki, & Fujimoto (2008). These authors propose that, in the dense environments of globular clusters, such deep mixing can indeed take place along the RGB, as triggered by tidal interactions. According to them, these phenomena could explain not only some level of He enrichment on the HB, but also the high rotation velocities that are observed among sufficiently cool blue HB stars. The same authors also suggest that cluster main sequence stars can likewise get polluted, by encounters with AGB stars (Tsujimoto, Shigeyama, & Suda 2007). Soker & Harpaz (2007) also speculate that He mixing can take place during the pre-ZAHB phase, triggered by the engulfment of low-mass companions and the resulting spin-up of the star’s envelope.

What are the constraints that may be posed on the enhanced helium hypothesis, using HB stars? In Figure 9, we show the impact of an enhanced helium abundance upon the log $g-\log T_{\text{eff}}$ diagram for blue HB stars. These results are based on the evolutionary calculations by Sweigart & Catelan (1998). As one can clearly see, a large increase in the helium abundance is expected to lead to a marked signature in this plane, with lower gravities resulting due to the increased luminosities that are brought about, with an increase in $Y$, for stars with efficient H-burning shells. For stars cooler than the Grundahl et al. (1999) “jump” (at about $11,500$ K; see Sect. 2.4), comparison between the spectroscopically derived gravities and the theoretical predictions should be fairly straightforward, the only noteworthy complication being associated with the distortion of the Balmer lines that may be effected by the predicted presence of a stellar wind (Vink & Cassisi 2002). In the case of M13, an increase in $Y$ up to $0.28$ has been suggested (Caloi & D’Antona 2005). The spectroscopic results by Moehler et al. (2003) do not rule out a $Y_{\text{MS}} \simeq 0.28$ for these cooler stars (but note again that proper inclusion of stellar winds might bring the derived gravities up, and therefore the required $Y$ values down), but appear inconsistent with a $Y_{\text{MS}} \approx 0.33$ or higher (see their Fig. 8b); in addition, they do not favor a difference in $Y$ between M13 and M3, although more data would certainly be needed to firmly establish this point. Note that the high helium abundance suggested for blue HB stars in NGC 2808 could also be constrained in this way, since Lee et al. (2005b) suggest that even the cooler blue HB stars in this cluster have helium abundances as high as $Y_{\text{MS}} = 0.33$, which should leave an obvious mark in the log $g-\log T_{\text{eff}}$
plane (Fig. 9). The helium enhancement proposed for the same cluster by D’Antona & Caloi (2004) is significantly smaller, being at the level of $Y_{\text{MS}} = 0.27$ for the cooler blue HB stars; the difference should be testable with a sufficiently large sample of stars. The same technique can likely be used to constrain the suggestion by Caloi & D’Antona (2008) that even in the case of M3 the blue HB stars may present some degree of helium enhancement.

The comparison between evolutionary model predictions and spectroscopically derived gravities for temperatures higher than the Grundahl jump is a much more complicated affair, since the presence of radiative levitation/gravitational diffusion effects in this region require the computation of detailed model atmospheres properly taking into account the extremely complex observed abundance patterns (Bonifacio, Castelli, & Hack 1995; Castelli, Parthasaraty, & Hack 1997) in order for reliable gravities, bolometric corrections and color transformation to be derived. To the best of our knowledge, no such detailed calculations have been carried out yet. Accordingly, it remains unclear whether models with an overall enhancement in all metals to super-solar levels (which is not what the quoted abundance studies indicate), as commonly employed in the literature, are adequate to derive gravities of stars hotter than the Grundahl jump.

7.2 Why Are the Most Metal-Poor Globular Clusters Not the Ones with the Bluest HBs?

The influence of the “first parameter” (metallicity) upon HB morphology can be summarized as the observed trend of HB type getting redder as metallicity increases (Sandage & Wallerstein 1960).\footnote{For Pecci et al. (1993) (see their Sect. 2.1) provide an enlightening discussion of the dependence of HB temperature on metallicity and RGB mass loss, showing how it can become very difficult for a metal-rich system to produce blue HB stars, compared to metal-poor systems. Note, in addition, that the RGB mass increases with metallicity for a fixed age and helium abundance [see, e.g., eq. (1) in Catelan & de Freitas Pacheco 1993], which also favors the production of redder HB types at higher $Z$.} Interestingly though, the most metal-poor globular clusters are not the ones with the bluest observed HBs, contrary to what might be expected from basic theory (see the isochrones overplotted on Figure 7). In fact, the trend of HB type becoming bluer with decreasing [Fe/H] appears to be reversed at $[\text{Fe/H}] \approx -1.8$ (see Fig. 7, left panel). The reason for such a reversal of trend is not entirely clear; while some have suggested that it could be due to a decrease in mass loss efficiency as metallicity decreases (Rood 1973), others have suggested instead that a majority of the metal-poor globulars may in fact be somewhat younger than Galactic globular clusters of intermediate metallicity (Yoon & Lee 2002). According to the isochrones shown in Figure 7, the required age difference between the “old halo” globular clusters and the most metal-poor ones with $0.1 \lesssim L \lesssim 0.7$ would have to be fairly large. On the other hand, the HB morphology-based age difference would decrease using mass loss formulae which imply a strong mass loss dependence on metallicity, as indeed favored by the mass formulae listed in Table 1 (see Fig. 4) but contrary, it should be noted, to the recent results by Origlia et al. (2002) (see Sect. 5 above). In any case, inspection of recent papers in which globular cluster ages have been computed provides little support for an age difference, except perhaps in the case of M68 (NGC 4590) (Rosenberg et al. 1999; VandenBerg 2000; Salaris & Weiss 2002; De Angeli et al. 2005). Interestingly, M68 does have a redder HB type than most other Galactic globular clusters of similar [Fe/H]—and the possibility that it may be relatively young was raised early on by Chaboyer et al. (1996).

7.3 A Reanalysis of Specific Second-Parameter Pairs

In what follows, we will readdress some classical second-parameter pairs, including NGC 288/NGC 362 and M13/M3, with the goal of answering the following question: are the measured turnoff age differences between these clusters sufficient to explain the observed difference in their HB types? We will also briefly discuss the case of the “young” outer-halo globular clusters Pal 3, Pal 4, and Eridanus, and provide a summary of recent developments on the second-parameter effect in metal-rich globular clusters. Note that an RGB mass loss that presents no dependence on $L$, $g$, or $R$, as suggested by Origlia et al. (2002), would require larger age differences than computed in the subsections below in order to account for any given second-parameter pair in terms of age.

7.3.1 The Pair NGC 288–NGC 362

This is probably the second-parameter pair which has attracted the most attention in the literature, given the very similar metallicities of the two clusters but their strikingly different HB types—NGC 362 with a predominantly red HB, and NGC 288 with an almost entirely blue HB. Reviews and extensive references to early work have been recently provided by Bellazzini et al. (2001) and Catelan et al. (2001a). Here we present new model calculations computed along the same veins as in Catelan et al., using the several different prescriptions for mass loss on the RGB summarized in Table 1
above, and compare the results with the age difference for this pair, as carefully derived by Bellazzini et al. on the basis of the so-called “bridge method” (Stetson et al. 1996). The essence of this method is that the HB of NGC 1851 looks very similar to the sum of the NGC 288 and NGC 362 HBs, so that, by carefully superposing the latter CMDs with that for NGC 1851, the relative positions of their turnoffs—and hence their relative ages—can be derived.\footnote{Underlying this method is of course the assumption that NGC 1851 is chemically similar to both NGC 288 and NGC 362, as discussed in detail by Bellazzini et al. (2001). The reader should keep in mind the possibility that this may not be strictly true, given the abundance peculiarities recently detected in NGC 1851 by Yong & Grundahl (2008), the discovery of a bimodal subgiant branch in the cluster by Milone et al. (2008), and the results of the theoretical analyses by Cassisi et al. (2008) and Salaris, Cassisi, & Pietrinferni (2008). Note, on the other hand, that fairly tight constraints on the possibility of a spread in $Y$ in the cluster were recently provided by Catelan (2009).}

Figure 10 shows the result of this comparison. In this plot, the lines indicate the age difference that is required, from the standpoint of canonical stellar evolution theory, to account for the observed difference in HB morphology between the clusters, under the assumption that the second parameter is age, and for the different indicated recipes for the RGB mass loss. The gray bands illustrate, in turn, the measured age difference. (These bands are not horizontal because the age difference depends on the absolute age value itself, the log of the age difference remaining instead more approximately constant as a function of age.) The upper plot corresponds to an adopted metallicity $Z = 0.001$, whereas the bottom plot to a metallicity $Z = 0.002$. This figure is similar to Figure 7 in Catelan et al. (2001a), but one sees that the new results require slightly larger age differences between NGC 288 and NGC 362 to account for their relative HB types. The bottom plot clearly shows that, if the metallicity of these clusters is of order $Z = 0.002$ and NGC 288 is younger than about 12 Gyr, the pair can be accounted for in terms of age as the second parameter, irrespective of the mass loss formula used. Such a metallicity corresponds to an $[\text{Fe}/\text{H}] \simeq -1.2$ for an $[\alpha/\text{Fe}] = +0.3$ (Salaris, Chieffi, & Straniero 1993). Such an $[\text{Fe}/\text{H}]$ value is very similar to the values provided in the Feb. 2003 edition of the Harris (1996) catalog, and only slightly lower than obtained in the Carretta & Gratton (1997) scale—namely, $[\text{Fe}/\text{H}] = -1.07$ for NGC 288, and $[\text{Fe}/\text{H}] = -1.15$ for NGC 362. Therefore, it appears fair to say that, in the Carretta & Gratton scale, this pair is indeed consistent with age as the sole second parameter.

On the other hand, the case $Z = 0.001$, which corresponds to an $[\text{Fe}/\text{H}] \simeq -1.5$ for an $[\alpha/\text{Fe}] = +0.3$, does not provide equally satisfactory consistency with the hypothesis that age is the second parameter. As can be seen from the plot, only for NGC 288 ages lower than about 9 Gyr is it that consistency with the relative turnoff age difference is recovered, irrespective of the mass loss formulation used. For ages higher than 10 Gyr, several mass loss formulae lead to results that allow one to question the hypothesis that age is the (only) second parameter for this pair. The best consistency with the age hypothesis obtains when using the “Judge-Stencel” formula; the worst, when using the “Goldberg” formula (see Table 1). Note that an $[\text{Fe}/\text{H}] \simeq -1.5$ falls within the uncertainty range of the Zinn & West (1984) metallicity value, since these authors give $[\text{Fe}/\text{H}] = -1.40 \pm 0.12$ for NGC 288, and $[\text{Fe}/\text{H}] = -1.27 \pm 0.07$ for NGC 362; similar values are provided by Kraft & Ivans (2003) and Rutledge et al. (1997) (in the Zinn & West scale). Therefore, we conclude that it is more difficult to account for the pair NGC 288/NGC 362 entirely in terms of age if the Zinn & West scale better describes the actual abundances of globular cluster stars.

It should be noted, in this sense, that the recent results by Asplund et al. (2004) (see also Meléndez 2004) for the solar metal abundance imply a major downward revision in $Z$ with respect to the canonical value (i.e., from $Z/\odot \approx 0.02$ down to $Z/\odot = 0.0126$). If this propagates to the metallicity scale used for metal-poor stars, a major downward revision in the $[\text{Fe}/\text{H}]$ and $Z$ values for Galactic globular cluster stars may be in store as well. According to the above discussion, this downward revision would further complicate the explanation of second-parameter pairs in terms of age. The same applies, as already stated, if the mass loss results from Origlia et al. (2002) are used, as opposed to the mass loss recipes given in Table 1 (see Sect. 5 above for a critical discussion).

To close, we note that Grundahl (2003) has recently investigated the ages of NGC 288 and NGC 362 on the basis of Strömgren photometry, finding that the two clusters differ in age by less than 1 Gyr. If so, Figure 10 clearly shows that age cannot be the (sole) second parameter for this pair.

### 7.3.2 The Pair M13–M3

This pair was first seriously and quantitatively considered in the context of the second-parameter phenomenon by Catelan & de Freitas Pacheco (1995), who suggested that the age difference required to account for the difference in HB type between M3 (uniformly populated HB) and M13 (blue HB with a long blue tail)
The relative age that is required to explain the difference in HB type between NGC 288 and NGC 362 entirely in terms of age (lines) is plotted as a function of the NGC 288 absolute age for two different metallicities: $Z = 0.001$ (upper panel) and $Z = 0.002$ (lower panel). For each panel, each line corresponds to a different mass loss formula, as indicated in the insets. The shaded area corresponds to the turnoff age difference for the pair, from Bellazzini et al. (2001). (This plot represents an update with respect to the similar one presented in Catelan et al. 2001a.)

exceeded the constraints from turnoff stars. A similar conclusion was later reached by Ferraro et al. (1997). More recently, Rey et al. (2001) have reanalyzed the problem, incorporating the Reimers (1975a,b) mass loss “law” into their analysis, and finding an age difference of $1.7 \pm 0.7$ Gyr between the two clusters—which, according to them, “can produce the difference in HB morphology between the clusters.” Here we revisit the problem, investigating the impact of the several different mass loss formulae for red giant stars summarized in Sect. 5.

First we proceed to compute synthetic HB models using the global sample of M3 HB stars discussed in Catelan et al. (2001b) and Catelan (2004a), and assuming the canonical metallicity for the pair—namely, $Z = 0.001$. For M13, we have retrieved the HST photometry from Piotto et al. (2002) and performed number counts along the HB of the cluster. When we attempted to reproduce the observed M13 HB morphology in terms of canonical synthetic HBs, we quickly arrived at the conclusion that a unimodal distribution was inadequate, since it was unable to explain the large number of very low-mass stars at the extreme hot end of the M13 HB. Therefore, we incorporated a second, low-mass mode to our simulations, thus obtaining a much better agreement with the overall HB morphology of the cluster. It should be noted that the EHB of the cluster is extremely populous, containing about 30% of all HB stars in M13. We then computed the average total mass loss required to explain the derived masses of the HB stars for each mass loss formula in Table 1 and over a range in ages, which then allowed us to compute the age difference that was needed, with respect to M3, to account for the observed difference in HB type.
Fig. 12 Same as in the previous plot, but now ignoring M13’s EHB component in the calculations.

The result is shown in Figure 11. In the upper plot, we compare the theoretical results with the empirical age difference for the pair, as recently derived by Salaris & Weiss (2002). (Unless otherwise stated, in this section we use the Salaris & Weiss results obtained in the Zinn & West 1984 scale.) In the lower plot, the same theoretical results are compared with the age difference derived by Rey et al. (2001). We have also compared the model results with the VandenBerg (2000) age difference values, but these are intermediate between the other two studies so that we omit further discussion of this case in what follows. Note, in addition, that, within the errors, Johnson & Bolte (1998) and Rosenberg et al. (1999) find M13 and M3 to be essentially coeval, whereas Stetson (1998) claims that M13 may even have the younger turnoff of the two (see below). Grundahl (1999) provides an impressive comparison between the two clusters in the Strömgren $u$, $(u-y)_0$ plane (his Fig. 2, left panel), where one can clearly see that there is not much room for a significant age difference between the two clusters—Grundahl himself finding an age difference of only $0.7 \pm 0.2$ Gyr between M13 and M3, the former being older.

As can clearly be seen, the required age difference appears too large in comparison with the Salaris & Weiss (2002) age difference measurements, irrespective of the mass loss formula used. The larger age difference measured by Rey et al. (2001), on the other hand, may be compatible with the turnoff age difference, particularly for lower M3 ages and if the “Goldberg” formula provides a less reliable description of mass loss rates on the RGB than do the others.

Some authors have suggested, on the other hand, that the origin of EHB stars may not be directly linked to the age of a globular cluster, being more likely instead to be due to other physical processes, including helium enrichment, binarity, and any other processes that may trigger enhanced mass loss on the RGB (e.g., Green et al. 2001; Brown et al. 2001). If so, it follows that at least 30% of the HB stars in M13 do not owe their present-day color to age, a different second parameter being required. In fact, Rey et al. (2001) favor this option, although this is in conflict with the scenario of Park & Lee (1997) and Lee et al. (2002) for the origin of the “UV upturn phenomenon” and for the photometric evolution of galaxies, according to which even the hottest HB stars owe their existence to age (high ages naturally being implied for giant elliptical galaxies in this case). Note that this age scenario forms the basis upon which the recently measured GALEX ultraviolet spectra of early-type galaxies and extragalactic globular clusters is currently being interpreted (e.g., Lee et al. 2005a; Rey et al. 2005). On the other hand, in our own galaxy EHB stars are now known to be present even in open clusters (Kahúzny & Udalski 1992; Liebert, Saffer, & Green 1994; Green et al. 1997), again reinforcing the impression that old ages cannot be solely responsible for the origin of EHB stars—and hence suggesting that age evolution alone cannot fully account for the UV upturn phenomenon and for the evolution of the photometric properties of galaxies as a function of redshift.

To check whether the pair M13/M3 might be explained by assuming a completely different formation channel for the M13 EHB stars, we have repeated the above exercise but now removing all M13 EHB stars from the sample. Since there are so many EHB stars in the cluster, this clearly leads to a significantly higher mean mass for the M13 HB stars, and therefore to a smaller expected age difference between this cluster and M3. Indeed, Figure 12 confirms that, if one admits that the EHB stars owe their origin to a different physical process, one is able to account for the different HB types of M13 and M3 entirely in terms of the reported turnoff age differences, irrespective of the adopted mass loss recipe, provided (in the Salaris & Weiss 2002 case) M3 is younger than 12 Gyr. More stringent constraints
on the M3 absolute age would derive in the case of using the Origlia et al. (2002) results for the (lack of a) dependence of RGB mass loss rate on $L$, $g$, or $R$.

Finally, it is important to note that, according to Stetson’s (1998) report on ultra-precise photometry for M13 and M3 obtained with the CFHT, the intrinsic position of the turnoff point in M13 is actually bluer than in M3, which is of course completely unexpected if the former is indeed older than the latter. The author strongly argues that there are very few (if any) sources of systematic errors that could have led him to underestimate the turnoff color for M13 compared to M3 in his study. One might suspect reddening uncertainties to be the culprit, but it should be noted that both clusters have very low reddening: the Feb. 2003 edition of the Harris (1996) catalog lists reddening values $E(B-V) = 0.02$ mag and 0.01 mag for M13 and M3, respectively (the same values as used by Stetson). From Stetson’s study, one finds that, for both clusters to have the same turnoff color (and hence the same ages), the relative redenings of the two clusters, compared with the Harris catalog value, would have to be incorrect by $\Delta E(B-I) = 0.04$ mag, which amounts to $\Delta E(B-V) = 0.015$ mag according to the Rieke & Lebofsky (1985) standard extinction law—in the sense that the M13 reddening must have been overestimated, and/or the M3 reddening underestimated. While this may seem like a small change, it is worth noting that, on the basis of the Schlegel, Finkbeiner, & Davis (1998) dust maps, one finds $E(B-V) = 0.017$ mag for M13, and $E(B-V) = 0.013$ mag for M3: while the shifts are correct in sign, they are clearly insufficient in size to bring M13 to even the same turnoff age as M3. An even larger change would be needed, of course, to make M13 significantly older than M3. Stetson argues, in fact, that a more likely explanation for the differences in CMD positions and shapes between the two clusters would be provided by a difference in helium abundance, as has indeed been suggested by other authors as well (e.g., Caloi & D’Antona 2005; Cho et al. 2005). Even stronger constraints on the relative reddening values between the two clusters would certainly prove helpful in clarifying the situation.

7.3.3 Red HB Globular Clusters in the Extreme Outer Halo

It is worth revisiting the case of the outer halo globular clusters with predominantly red HB types which have had their ages measured with $HST$, such as Palomar 3, Palomar 4, and Eridanus. According to Stetson et al. (1999) and VandenBerg (2000), these clusters are slightly younger than the bulk of Galactic globular clusters with similar metallicity. Is the detected age difference between these clusters and inner halo clusters, such as M3 and M5, consistent with the observed difference in HB morphology between them?

The Pair M3–Pal 3

In terms of metallicity, Pal 3 provides a good match to M3. Accordingly, Catelan et al. (2001b) performed a study of the age difference between Pal 3 and M3 that is required to account for their relative HB types, and found that the age difference based on analyses of the turnoff points could easily account for the difference in mean HB color between the two clusters. We have repeated their exercise, finding a slightly larger age difference being needed to account for their different HB types, but basically confirming their results. On the other hand, Catelan et al. call attention to the fact that the mass dispersion along the HB of Pal 3 appears entirely consistent with zero, thus being significantly different from M3’s. If differences in mass loss among individual red giants is responsible for the presence of mass dispersion along the HBs of globular clusters, then the mass loss process clearly operated in a different way in Pal 3 than it did in M3. Accordingly, Catelan et al. (2001a) suggest that while age may be the “global” second parameter driving the mean HB color for at least some globular clusters, for many globular clusters environmental effects might be a “local” second parameter responsible for generating a dispersion in color around this mean value.

The “Pair” M5–Pal 4/Eridanus

In terms of chemical composition, both Pal 4 and Eridanus appear to have metallicities similar to M5’s. In addition, since Pal 4 and Eridanus appear to have similar chemical composition and CMD morphology but coarsely populated CMDs, it seems reasonable to combine the data for the two and perform a single analysis of the “pair” comprised by M5, on the one hand, and Pal 4/Eridanus, on the other (Catelan 2000).

Again, we repeat the analysis carried out by Catelan (2000), but now comparing the theoretical results with the age differences derived on the basis of the turnoff points by Stetson et al. (1999), VandenBerg (2000), and Salaris & Weiss (2002). The results are shown in Figure 13, where the upper panel shows the comparison between the HB morphology-based analysis and

---

19In fact, proper motion studies (Scholz et al. 1996; Cudworth 1997) indicate that M5 is an outer halo globular which just happens to be close to its perigalacticon at this point in time, the cluster actually spending much of its life at galactocentric distances larger than 50 kpc.
the turnoff age difference from Stetson et al. and VandenBerg, whereas the lower panel shows a comparison between the same results for the HB stars in these clusters and the turnoff ages from Salaris & Weiss.

These plots (which again give slightly higher HB type-based age differences between the clusters than originally reported by Catelan 2000) reveal that the turnoff age difference measured by Stetson et al. (1999) and VandenBerg (2000) is insufficient to account for the difference in HB types between M5 and Pal 4/Eridanus, irrespective of the mass loss formula used. On the other hand, the turnoff age difference reported by Salaris & Weiss is clearly more consistent with the hypothesis that the age difference between M5 and Pal 4/Eridanus is sufficient to account for their relative HB types. It is clearly very important to establish what the actual turnoff age difference between M5 and Pal 4/Eridanus is before we are in a position to decide whether age can be the (“global”) second parameter for this pair.

7.4 The Second-Parameter Effect at High Metallicities

While the second-parameter problem is traditionally thought to affect mostly intermediate-metallicity globular clusters, the fairly recent discovery of large (and peculiarly bright) RR Lyrae populations (Layden et al. 1999; Pritzl et al. 2000) and prominent blue HB tails (Piotto et al. 1997; Rich et al. 1997) in the moderately metal-rich Galactic globular clusters NGC 6388 and NGC 6441 has brought the phenomenon to the realm of $[\text{Fe/H}] \sim -0.5$ globular clusters as well.

Several hypotheses have been discussed in the literature to explain the observed HB morphology and peculiar RR Lyrae periods in these clusters. These include tidal collisions (Rich et al. 1997), helium mixing on the RGB, a primordially increased helium abundance, and increased core masses at the RGB tip as a result of internal rotation (Sweigart & Catelan 1998). Other explanations include a large spread in metallicities (Piotto et al. 1997; Sweigart 2002), a selective metal depletion scenario (Sweigart 1999), and a range in internal helium abundances (D’Antona & Caloi 2004). In addition, and also as an attempt to explain their peculiar HB morphologies, Ree et al. (2002) have suggested that NGC 6388 and NGC 6441 might be similar to $\omega$ Cen in nature, with a (small) internal metallicity spread and a (fairly large) internal range in ages; in their scenario, the blue HB and bright RR Lyrae components of NGC 6388 and NGC 6441 would be ascribed to a combination of lower metallicity and old ages, RR Lyrae stars being somewhat more metal-poor stars evolved away from a position on the blue ZAHB.

Unfortunately, most—if not all—of these scenarios face strict observational and theoretical constraints. Rich et al. (1997) show that tidal collisions cannot produce the observed HB morphology—and we add that it cannot lead to peculiarly bright RR Lyrae stars, either. A primordially increased helium abundance does not appear consistent with the position of the RGB bump in these clusters (Raimondo et al. 2002). A large spread in metallicities, as also pointed out by Raimondo et al., is not supported by observations of the cluster CMDs in the RGB region, since the latter do not show the large spread in colors that would be expected in this case. Moreover, Clementini et al. (2005) have recently found, based on VLT spectra, only a small (though significant) spread in $[\text{Fe/H}]$ among the RR Lyrae stars in NGC 6441—although this result could not be confirmed by Gratton et al. (2007) among the cluster’s red giants. Evolution away from a position on the blue ZAHB does
not produce enough bright RR Lyrae variables to explain the observed RR Lyrae period distributions, and neither is the sloping nature of the HB quantitatively reproduced in the Ree et al. (2002) scenario (Pritzl et al. 2002b).

Most puzzling of all is the evidence that the blue HB stars in these clusters cooler than the Grundahl et al. (1999) jump do not appear to have peculiarly low gravities (Moehler, Sweigart, & Catelan 1999a), which effectively rules out any scenario that requires the blue HB and RR Lyrae components to be brighter than in canonical models. However, a brighter HB at both the blue HB and the RR Lyrae levels seem to be required by i) The glaring evidence that the tip of the blue HB is ∼ 0.5 mag brighter in V than the red HB component; ii) The very long periods of the RR Lyrae stars in both clusters. Given the apparently irreconcilable evidence, we suggest that a reassessment of the spectroscopic gravities for a larger sample of blue HB stars in both clusters would prove well worth the effort.20

Assuming that the spectroscopic gravities can indeed be reconciled with the requirement that the blue HB component be unusually bright, we discuss the possibility that a internal spread in helium abundances (D’Antona & Caloi 2004) may help account for the HB morphology and RR Lyrae pulsation properties in these clusters (see Sweigart & Catelan 1998 for numerical simulations in the case of helium mixing).

Consider the case of NGC 2808, and assume that, as suggested by D’Antona & Caloi (2004), there is an internal helium abundance range among the stars in the cluster. In their scenario, NGC 2808’s red HB and RR Lyrae components have a “normal” helium abundance, blue HB stars at the “horizontal” level have a mild helium enhancement, and hotter blue HB stars have a much higher (initial) helium content, up to about 35%. Lee et al. (2005b) suggest a similar scenario for the cluster, only invoking an even higher helium enhancement for the blue HB stars. Now imagine how NGC 2808 might have looked in the not too remote past (say, a couple of Gyr ago): its present-day helium-enhanced blue HB stars would necessarily be redder as a consequence of the younger age, and therefore helium-enhanced stars would be present in relatively high numbers inside the RR Lyrae instability strip—thus leading to overluminous, long-period RR Lyrae stars. The stars at the tip of the blue HB would have an even higher helium abundance, leading to a marked sloping HB when compared with the red HB and RR Lyrae components in the same cluster. Clearly, the NGC 2808 envisaged by D’Antona & Caloi and Lee et al. would have looked a lot like the present-day NGC 6388 and NGC 6441 in the past! Accordingly, a possible explanation for the observed peculiarities in these clusters could involve both an enhanced helium abundance among a fraction of the cluster stars (note that the blue HB plus RR Lyrae components in both clusters constitute relatively small fractions of the overall HB populations, so that the RGB bump constraint might not be violated in this case) and a younger age with respect to the bulk of the Galactic globular clusters—the latter not necessarily being required, depending on metallicity (i.e., first parameter) and mass loss effects: indeed, the deep HST photometry presented by Catelan et al. (2006) for NGC 6388 reveals that the cluster is comparable in age to 47 Tuc.

A potential problem with this scenario is the fact that, for every single helium abundance value, one should expect, by analogy with what is observed in “single-population globular clusters,” a spread in mass as well. Therefore, at any given color, a spread in helium abundance should also be present, and it is unclear whether the implied HB luminosity distribution would match in detail the observed one, which appears fairly tight. This problem is especially evident in the recent simulated CMD by Busso et al. (2007; see their Fig. 7), but is not as apparent in the simulated CMDs of Caloi & D’Antona (2007)—which again may be due to the lack of a mass spread for the populations with enhanced Y in their simulations. Note, in this sense, that the simulations presented by Sweigart & Catelan (1998) in their helium mixing and rotation scenarios also ignore the effect of a spread in masses for each individual Y. In any case, Piotto (2008) has recently reported that NGC 6388 does show a double subgiant branch, thus reinforcing the evidence for the presence of more than one stellar population in the cluster.

We conclude that stringent tests of such a scenario could be provided by deep HST photometry (so that the turnoff ages, and possible splits indicative of multiple stellar populations, can be reliably established), spectroscopic gravities for a large sample of moderately cool blue HB stars in both clusters, and HB simulations in which both a spread in Y and in RGB mass loss are simultaneously taken into account.

8 On the RR Lyrae Luminosity-Metallicity Relation

Much has been written over the past several years in regard to the luminosities of HB stars at the RR Lyrae level, particularly in the V band, and its dependence on

20After this paper had been completed, Moehler & Sweigart (2006) revisited the problem and concluded that those previous results were indeed incorrect, most likely due to problems with the background subtraction from the spectra (blends).
metallicity; recent reviews of the subject include papers by Chaboyer (1999), Cacciari (1999, 2003), Cacciari & Clementini (2003), and Storm (2006). Bono (2003) and Cassisi (2005) have provided recent reviews in which theoretical uncertainties affecting the predicted properties of HB stars, including their luminosities, have been discussed in considerable detail. Several different theoretical results have recently been compared by Cacciari & Clementini (2003; see their Fig. 1) and Gallart, Zoccali, & Aparicio (2005; see their Fig. 9). Accordingly, in what follows we will content ourselves with providing an updated estimate of the HB luminosity-metallicity relation as based on the trigonometric parallax of RR Lyrae itself, only briefly mentioning recent progress in understanding some discrepancies that have prevented the establishment of a universally accepted $M_V$(RRL) – [Fe/H] relation.

It should be noted that this crucial relation has traditionally provided the very basis for the Population II distance scale, thereby constituting one of the most important techniques used to help nail down the first step in the cosmological distance ladder—namely, the distance to the LMC (e.g., Benedict et al. 2002; Kovács 2003; Storm 2006). Moreover, its slope and zero point have long been recognized as crucial ingredients in the determination of the absolute ages of globular clusters and their variation with metallicity (e.g., Sandage & Cacciari 1990; Walker 1992c). On the other hand, it should also be noted that this relation can only be used in a very approximate way to estimate the average absolute magnitudes of RR Lyrae stars of a given metallicity.

In order to properly evaluate the absolute magnitudes of individual RR Lyrae stars, more precise techniques, frequently involving a period-luminosity relation, are required. Unlike the case of classical Cepheids, however, good period-luminosity relations are not available for RR Lyrae stars in the visual bandpasses, for reasons that have been discussed in detail by Catelan, Pritzl, & Smith (2004). On the other hand, it has long been known that good RR Lyrae period-luminosity relations are present in the near-infrared (Longmore, Fernley, & Jameson 1986; Longmore et al. 1990). Empirical results have recently been critically discussed by Sollima, Cacciari, & Valenti (2006) and Feast et al. (2008), and Sollima et al. (2008) have recently provided a detailed analysis of the $J$, $H$, $K$ light curves of RR Lyrae itself. Theoretical calibrations using the near-infrared bandpasses $J$, $H$, $K$ have been provided by Cassisi et al. (2004), Catelan et al., and Del Principe et al. (2006), among others. As pointed out by Soszyński et al. (2003) and Catelan et al., a period-luminosity relation is also present in $I$, although in this case one finds somewhat more scatter and a stronger metallicity dependence than in the near infrared. Finally, Cortés & Catelan (2008) and Cáceres & Catelan (2008) have recently shown that very precise period-luminosity and period-color relations may also be defined for RR Lyrae stars in the Strömgren and SDSS filter systems, respectively.

8.1 The Variable Star RR Lyr and the HB Luminosity-Metallicity Relation

The star RR Lyr is the closest of its class, and accordingly has proven of great interest for trigonometric parallax studies. Its variability was noted by Williamina P. Fleming at the Harvard College Observatory prior to July 1889, but the discovery was not announced until a few years later by Pickering (1901).

Benedict et al. (2002) obtained, using the Hubble Space Telescope, a much more accurate (and significantly smaller) value for the absolute parallax of RR Lyrae than had previously been provided by Hipparcos, namely $\pi_{abs} = 3.82 \pm 0.20$ mas (compared to $\pi_{Hip} = 4.83 \pm 0.59$ mas; Perryman et al. 1997). Recently, van Leeuwen (2007) revised the trigonometric parallaxes provided by Hipparcos, and arrived at a value $\pi_{abs} = 3.46 \pm 0.64$ mas for RR Lyr. A weighted average of ground-based studies (van Altena, Lee, & Hoffleit 1995) indicated a parallax $\pi_{abs} = 3.0 \pm 1.9$ mas for RR Lyrae (see Fig. 6 in Benedict et al. 2002). Taking a weighted average of these results, we obtain a final value of $\pi_{abs} = 3.78 \pm 0.19$ mas for RR Lyr. This implies a revised distance modulus of $(m-M)_0 = 7.11 \pm 0.11$ mag for the star.

Benedict et al. (2002) argue in favor of a relatively low extinction towards RR Lyr, namely $A_V \approx 0.07 \pm 0.03$ mag. In recent work, an intensity-weighted mean magnitude of $\langle V \rangle = 7.76$ mag (Fernley et al. 1998a)21 has been adopted for RR Lyr. However, we note that this value is based on Hipparcos photometry, which may require a non-trivial transformation to the standard system. For comparison, Layden (1994) determines a $\langle V \rangle = 7.66$ mag, and Layden et al. (1996) find instead $\langle V \rangle = 7.74$ mag. On the other hand, Gould & Popowski (1998) argue strongly in favor of the Hipparcos-based magnitudes of Fernley et al., only proposing an additional, reddening-related correction: $V_{GP99} = V_{F98} - 0.2 E(B-V)$. Using a standard extinction law, $A_V \approx 3.1 E(B-V)$, and the reddening value obtained by Catelan & Cortés (2008) on the

---

21 This value is provided in Table 1 of their paper, which is only available in electronic format, from http://vizier.u-strasbg.fr/viz-bin/VizieR?-source=J/A+A/330/515.
basis of Strömgren photometry, namely $E(B-V) = 0.015 \pm 0.020$ mag, leads to a final, extinction-corrected RR Lyr mean magnitude of $(V_0) \simeq 7.71 \pm 0.06$ mag.

It is important to note that the intensity-mean magnitude (and even more so the corresponding magnitude-weighted average) does not necessarily correspond to the magnitude of the “equivalent static star” (i.e., the magnitude the star would have if it were not pulsating): an amplitude-dependent correction has to be applied. Such a correction has been obtained, both for fundamental (RRab) and first-overtone (RRc) variables, by Bono, Caputo, & Stellingwerf (1995) on the basis of detailed hydrodynamical pulsation models of RR Lyrae stars. RR Lyrae has long been known to be an overluminosity of $(V_0) \simeq 7.71 \pm 0.06$ mag, leads to a final, extinction-corrected RR Lyr mean magnitude of $(V_0) \simeq 7.71 \pm 0.06$ mag.

It is important to note that the intensity-mean magnitude (and even more so the corresponding magnitude-weighted average) does not necessarily correspond to the magnitude of the “equivalent static star” (i.e., the magnitude the star would have if it were not pulsating): an amplitude-dependent correction has to be applied. Such a correction has been obtained, both for fundamental (RRab) and first-overtone (RRc) variables, by Bono, Caputo, & Stellingwerf (1995) on the basis of detailed hydrodynamical pulsation models of RR Lyrae stars. RR Lyrae has long been known to be an overluminosity of $(V_0) \simeq 7.71 \pm 0.06$ mag, leads to a final, extinction-corrected RR Lyr mean magnitude of $(V_0) \simeq 7.71 \pm 0.06$ mag.

What do these results imply, in terms of the traditionally employed RR Lyrae luminosity-metallicity relation in the V band, usually taken in the form $M_V(RRl) = \alpha [Fe/H] + \beta$?

To answer this question, we shall first adopt a slope $\alpha = 0.23 \pm 0.04$ for this relation, as found and/or favored in several recent reviews of the subject, including Chaboyer (1999), Cacciari (1999, 2003), and Cacciari & Clementini (2003). Several recent analyses do provide additional support for this result: for instance, Clementini et al. (2003) and Gratton et al. (2004a) obtain $\alpha = 0.214 \pm 0.047$ from analysis of RR Lyrae variables in the LMC, whereas Olech et al. (2003) obtained $\alpha = 0.21 \pm 0.28$, depending on their treatment of presumably well-evolved RR Lyrae variables with periods around 0.7 d, from analysis of the RRab stars in $\omega$ Cen.

Using the Clementini et al. (1995) metallicity for RR Lyr in the Zinn & West (1984) scale, one then finds a value of $\beta = 0.98 \pm 0.13$, implying a final relationship of the following form:

$$M_V(RRL) = (0.23 \pm 0.04) [Fe/H]_{ZW} + (0.98 \pm 0.13). \quad (3)$$

If one transforms the Clementini et al. metallicity to the Carretta & Gratton (1997) scale and then repeats the analysis, one finds instead

$$M_V(RRL) = (0.23 \pm 0.04) [Fe/H]_{CG} + (0.93 \pm 0.13). \quad (4)$$

These relations, while based on a detailed reassessment of the absolute magnitude and evolutionary status of RR Lyr, turn out to be similar to the relation derived in the recent review papers by Chaboyer (1999), Cacciari (1999, 2003), and Cacciari & Clementini (2003), based on a critical analysis of several calibration techniques (but ignoring the evolutionary status of the star). This notwithstanding, some methods have provided somewhat discrepant slopes and/or zero points, and we shall momentarily address two such cases. Before doing so, however, we will immediately proceed to deriving the all-important distance modulus of the LMC that is implied by these relations (see also Alves 2004 for a recent review).

8.2 The Distance Modulus of the LMC

Eq. (3) implies an absolute magnitude $M_V(RRL) = 0.64 \pm 0.14$ mag at $[Fe/H] = -1.48 \pm 0.07$. The latter is the mean metallicity derived for LMC RR Lyrae variables by Gratton et al. (2004a), in the Zinn & West (1984) scale. Using a value $(V_0) = 19.068 \pm 0.102$ mag from Gratton et al., one then finds an updated true distance modulus for the LMC of $(m-M)_0^{LMC} = 18.42 \pm$
0.17. If one uses instead the average values for LMC RR Lyrae stars independently determined by Borissova et al. (2004), namely [Fe/H] = −1.46 ± 0.09 dex and ⟨V⟩ = 19.45 ± 0.04 mag, with their favored reddening of \(E(B-V) = 0.11\) mag, one finds \(⟨V_0⟩ = 19.11 ± 0.04\) mag for a \(M_V = 0.65 ± 0.14\) mag, thus implying a true distance modulus \((m-M)_0^{\text{LMC}} = 18.46 ± 0.15\). Taking a weighted average over these two results, we arrive at the following distance modulus for the LMC, based on our updated analysis of the star RR Lyr:

\[
(m-M)_0^{\text{LMC}} = 18.44 ± 0.11.
\] (5)

8.3 Are We Converging on a \(M_V(\text{RRL}) - [\text{Fe/H}]\) Relation Yet?

In what follows, we discuss a few discrepant calibrations of the HB luminosity-metallicity relation, and describe how the problem has recently been solved or what suggestions may have been advanced to reconcile the discrepant calibrations.

8.3.1 The Shallow Slope Obtained from HST Photometry of M31 Globular Clusters

Almost a decade ago, a very shallow slope, \(\alpha = 0.13 ± 0.07\), was derived by Fusi Pecci et al. (1996) on the basis of HST observations of M31 globular clusters. However, the determination of the HB level in their CMDs was far from straightforward, since instead of seeing a horizontal branch at the RR Lyrae level, whenever a blue or intermediate HB component was present their CMDs revealed instead surprisingly sloped “horizontal” branches, not unlike what one sees when plotting an \(I, (V-I)\) CMD for Galactic globulars. Such sloping HBs are not seen among Galactic globular clusters, and even though differences between Galactic and extragalactic globulars (related, for instance, to the different chemical evolution histories of the different galaxies, or to the amount of angular momentum available in the different protogalactic clouds) may indeed exist, theoretical models of HB stars do not predict similar CMD morphologies.

A possible solution to this puzzle has recently been provided by Rich et al. (2005), who found that the Fusi Pecci et al. (1996) CMDs were strongly affected by photometric blends, not accounted for in their original analysis. Investigating the impact of these blends upon the morphology of the HB in their CMDs, Rich et al. have found that the strange sloping nature of the HB in Fusi Pecci et al. is likely due to the presence of photometric blends. Accordingly, Rich et al. derived new CMDs for an enlarged sample of M31 GCs in which the effects of blends were taken into account. As a result, they provide a revised slope of \(\alpha ≃ 0.20 ± 0.09\), clearly in much better agreement with eq. (3). It should be noted, however, that careful inspection of the CMDs published by these authors still reveal unrealistic HB shapes, thus raising the possibility that additional corrections will be needed before a final relation between HB magnitude and metallicity can be derived on the basis of observations of M31 globulars.

8.3.2 The Faint Zero Point of the Method of Statistical Parallaxes

As far as the zero point of eq. (3) is concerned, the recent review by Cacciari & Clementini (2003) shows that there is reasonable agreement among the several different methods that are used to infer it. Importantly, the Baade-Wesselink method, which used to favor a faint zero point (i.e., fainter than provided by the above calibration by at least 0.1 mag; see review of earlier work by Fernley et al. 1998b), is now seen to be in good agreement with eq. (3) and with a brighter zero point for the HB luminosity-metallicity relation. Indeed, Kovács (2003), by applying the same Baade-Wesselink algorithm to both Galactic RR Lyrae and Cepheid variables, has recently shown that a consistent, “long” distance modulus for the LMC obtains, namely \((m-M)_0 = 18.55\) mag—about 0.05 mag brighter than implied by eq. (3).

This notwithstanding, at least one method remains that does keep repeatedly providing a faint zero point to the LMC: the statistical parallaxes method. For instance, Gould & Popowski (1998) favor a \(M_V = 0.77\) mag at \([\text{Fe/H}] = −1.6\) dex from analysis of a sample of 147 RR Lyrae stars, which is 0.21 mag fainter than implied by eq. 3; or \(M_V = 0.80\) mag at \([\text{Fe/H}] = −1.7\) dex after adding to the sample 716 non-variable metal-poor stars, which is 0.26 mag fainter than eq. (3). While Cacciari & Clementini (2003) hint that the presence of disk stars may affect the Gould & Popowski results, Dambis & Rastorguev (2001) recently provided a new application of the method in which thick disk stars were carefully separated from halo stars using both kinematic and metallicity criteria, but essentially confirming the Gould & Popowski results. Popowski & Gould (1998a,b) have provided a very careful and detailed analysis of the possible systematic uncertainties affecting the method, without succeeding in identifying a likely cause for the difference with respect to several other methods—in fact, they hint that the problem lies with the latter (see also Popowski & Gould 1999). It is indeed unclear what the solution to this problem
will be, but Cacciari & Clementini suggest that the Dambis & Rastorguev results may be affected by inhomogeneities in the distribution of their halo stars. Indeed, that the distribution of Galactic halo stars is not quite uniform has recently been noted, using blue HB stars, by Altmann et al. (2005) and Clewley et al. (2005) (but see also Brown et al. 2004a); using RR Lyrae stars, by Ivezić et al. (2000), Vivas et al. (2001), Vivas & Zinn (2003, 2006), Kinman et al. (2004), Zinn et al. (2004), and Keller et al. (2008); and, using several other types of tracers, by, for instance, Newberg et al. (2002), Ibata et al. (2003), Majewski et al. (2003, 2004), Martínez-Delgado et al. (2004), Newberg & Yanny (2005), and Rocha-Pinto et al. (2006), among many others.  

8.3.3 Differences between HB Stars in Globular Clusters and in the Field

The similarity between metal-poor field and cluster RR Lyrae stars appears reasonably established (Catelan 1998; De Santis & Cassisi 1999; Carretta et al. 2000), but there may be differences in regard to at least the more extreme HB stars, both in regard to their luminosity distribution (Brown et al. 2005 claim that the field contains significantly fewer stars at the blue end than do such globular clusters as M15 and NGC 6229) and their physical origin: Moni Bidin et al. (2006a,b) and Moni Bidin, Catelan, & Altmann (2008) find that the EHB stars in NGC 6752 and M80 (NGC 6093) do not appear to be associated to binary systems, unlike what seems to happen most frequently among field sdB stars (Green et al. 2001). Moni Bidin et al. (2008) interpret this phenomenon as likely due to a binary fraction-age relation (globular cluster stars being on average much older than field stars), a hypothesis which has recently been supported by Han (2008). In addition, the smaller envelope masses of the older cluster red giants make it much easier for the single-star channel to produce EHB stars, as also noted by Moni Bidin et al. (2008). It should also be noted that the abundance anomalies which are commonly seen in globular cluster red giants (e.g., Sneden et al. 2004; Gratton et al. 2004b; and references therein) appear not to be present in field red giants (Gratton et al. 2000; see also Sect. 5.2 in Grundahl et al. 1999 for extensive references to work prior to the year 2000). Therefore, if these abundance anomalies affect somehow the evolution of RGB and HB stars (through the “primordial” and/or “deep mixing” channel), as has indeed been often suggested in the literature (see §7.1 for extensive references), they should naturally be expected to have an impact upon the observed properties of HB stars—including color distribution, luminosities, gravities, and pulsation characteristics. While these differences have not been frequently detected, it is worth recalling the cases of NGC 6388 and NGC 6441, whose RR Lyrae stars are very different from field RR Lyrae stars with similar metallicity (Sweigart & Catelan 1998; Layden et al. 1999; Pritzl et al. 2000, 2001, 2002b; Clementini et al. 2005; Catelan et al. 2006; Matsunaga et al. 2006; Matsunaga 2007; see also Layden 1995 and Catelan 2004b for additional references to metal-rich globular clusters harboring relatively long-period RR Lyrae stars).  

8.3.4 The $M_V(RRL)$ – [Fe/H] Relation: Linear, Quadratic, or Even More Complex?

Steep slopes than provided by eq. (3) may result from the inclusion of RR Lyrae stars more metal-rich than [Fe/H] $\approx -1$, as originally suggested by Castellani, Chieffi, & Pulone (1991) from theoretical computations. In fact, for field stars in our galaxy, a quadratic relation between $M_V(RRL)$ and [Fe/H] is likely to be superior to a linear approximation when such metal-rich variables are included, in the sense that the metal-rich variables are fainter than would be expected using a linear fit to the more metal-poor data—a result which is strongly supported by the theoretical models (e.g., Castellani et al. 1991; Caputo et al. 2000; Bono et al. 2003; Catelan et al. 2004; and references therein). On the other hand, it should also be noted that the *helium abundance* is also expected to increase with $Z$, by amounts which may differ in different environments as a consequence of different chemical enrichment laws (as early noted by van den Bergh 1967). Accordingly, it is extremely important to realize that there is no strong physical basis for a universal $Y – [Fe/H]$ relation. As an example, Catelan & de Freitas Pacheco (1996) analyzed three different chemical enrichment scenarios which, though providing essentially the same helium abundance at low metallicities ([Fe/H] $<-1$), led to differences in $Y$ by $\Delta Y \approx 0.035$ at [Fe/H] $\approx -0.5$, and by up to 0.1 at solar metallicity (see also Catelan 2008b). Indeed, it would perhaps be surprising if galactic disks and spheroids presented precisely the same $\Delta Y - \Delta Z$ “law,” given the evidence that they present different element enrichment patterns—variations in the latter being expected to be accompanied by variations in the $\Delta Y - \Delta Z$ relation as well (see Figs. 1 and 2 in Catelan 2008b).
This can impact HB luminosities in a quite dramatic way. Since the HB luminosity depends on the helium abundance as

\[
\frac{dM_{\text{bol}}^{\text{HB}}}{dY} \approx 4.5
\]

(Catelan 1996), the quoted differences in helium abundance may lead to changes in the HB luminosity, from one chemical enrichment scenario to the next, by as much as 0.15 mag at $[\text{Fe}/\text{H}] = -0.5$, and by 0.45 mag (!) at $[\text{Fe}/\text{H}] = 0$. The helium enrichment law clearly plays a crucial role in defining the slope of the HB luminosity-metallicity relation at high metallicities, and this should be properly taken into account when studying different stellar populations.

In the same token, we believe it is a risky procedure to derive relative globular clusters ages, using the HB luminosity level as a standard candle, for globular clusters of different metallicities belonging to different populations, as has been done by Ortolani et al. (1995) in the case of 47 Tuc and NGC 6528, since they may have undergone difference helium enrichment laws and therefore have significantly different HB luminosities.

Another important implication of this result is that the production of hot HB sources at high metallicities, which are believed to be the main sources of the “UV upturn” (or UVX) light coming from elliptical galaxies and the bulges of spirals (see O’Connell 1999 for a review and extensive references), is importantly affected by their precise helium enrichment laws (e.g., Yi et al. 1997). In the Galactic bulge, evidence of the presence of hot HB stars has been provided and discussed by Bertelli et al. (1996), Terndrup et al. (1999, 2004), Peterson et al. (2001), Busso et al. (2005), and Busso & Moehler (2008). It should also be noted that the precise RGB mass loss recipe adopted may also significantly impact the predicted production of UV sources (see Sect. 5).

Therefore, a calibration such as the one provided by eq. (3) may not be straightforwardly applicable to just any galaxy, especially when more metal-rich components are considered. Also, from Ritter’s (1879) relation, one finds that the pulsation properties of RR Lyrae stars are directly inherited from, or rather reflected upon, their luminosities. However, Figure 5 clearly shows that the RR Lyrae that belong to our galaxy present a rather peculiar behavior, at least compared to those belonging to its neighboring galaxies. But if this is indeed so, what guarantees do we have that $M_Y (\text{RRL}) - [\text{Fe}/\text{H}]$ relations based upon Galactic halo RR Lyrae stars are universally applicable to other galaxies? Arguably, one should make sure that, if not the Oosterhoff dichotomy itself, at least the Oosterhoff-Arp-Preston-Sandage period-metallicity progression (Oosterhoff 1939, 1944; Arp 1955; Preston 1959; Sandage 1981, 2004; and references therein) is verified in the sample under scrutiny (Catelan 2004b)—which is only possible by using time-series observations.

Even for bona-fide Galactic RR Lyrae stars, one should recall that the $M_Y (\text{RRL}) - [\text{Fe}/\text{H}]$ relation can be strongly affected by evolutionary effects (e.g., Lee 1990; Demarque et al. 2000); these are more properly avoided when using the RR Lyrae period-luminosity relation to infer distances. In any case, one must keep in mind that there appear to be “pathological outliers” for which the available calibrations of any of these relations may not be straightforwardly applied, due to the fact that they may not be properly described by canonical models (Sweigart & Catelan 1998; Pritzl et al. 2001, 2002b; Raimondo et al. 2002). Also, if the helium abundance is enhanced in at least a fraction of the stars in some globular clusters, whether due to primordial or to evolutionary effects, models assuming $Y \approx 0.23 - 0.25$ will not provide a correct distance to the RR Lyrae stars in the systems which partook a primordial enhancement, or in which some RGB stars may have undergone helium enrichment and ended up as overluminous RR Lyrae. For critical discussions of several different channels for the production of high-Y stars in globular clusters, the reader is referred to the extensive list of references provided in §7.1.

Even if present-day Galactic globular clusters have only their blue HB components affected by a high Y, a few Gyr ago these blue HB components were actually (overluminous) RR Lyrae stars. Therefore, even if not in our present-day galaxy, there is no guarantee that in other (somewhat younger) galaxies and their respective globular cluster systems we might not run into overluminous RR Lyrae that are their analogues of the present-day, helium-enhanced, Galactic blue HB stars. The effect of a change in $Y$ upon the RR Lyrae stars. The effect of a change in $Y$ upon the RR Lyrae stars...
period-luminosity relation has been recently discussed by Catelan et al. (2004). Again, over/underluminosity should be reflected upon the pulsation periods of RR Lyrae stars, so that proper monitoring of their pulsation cycles remains the only way to safely apply local (or standard theoretical) HB luminosity calibrations to external systems. For nonvariable HB stars, unfortunately, such a consistency diagnostic is not available.

For the sake of argument, consider the Yoon & Lee (2002) scenario, whereby the Galactic OoII component is not a genuine component of our galaxy, having instead been accreted from an external galaxy at some point in the past. In this scenario, had such an accretion event not taken place, our Galactic globular cluster system would be characterized by the OoI component shown in Figure 5 (left panel) plus the OoIII component shown in the same panel. In this case, one would find for our Galactic globulars an opposite trend to what is seen in the Oosterhoff-Arp-Preston-Sandage progression, with the more metal-rich globulars presenting much longer periods—and therefore presumably having brighter HBs—than the OoI clusters. There is no guarantee that in external galaxies similar to our own but which have not (again in the Yoon & Lee scenario) incorporated an “OoII protogalactic fragment” in the course of its history, such a seemingly far-fetched behavior is not precisely what happened in practice.

8.4 A Caveat: The Evolutionary Status of the Star RR Lyr

One possible caveat with employing the star RR Lyr to constrain the average HB luminosity-metallicity relation at the instability strip level is the fact that we do not know a priori the evolutionary status of the star. Therefore, it could be significantly brighter (or fainter) than the average field RR Lyrae at the same metallicity, in which case our results could be systematically off by 0.1 mag or more, given the intrinsic spread in RR Lyrae luminosities (e.g., Sandage 1990). Indeed, Catelan & Cortés (2008) have recently argued, based on Strömgren photometry and theoretical modelling, that RR Lyr may be brighter than other field RR Lyrae stars of similar metallicity by 0.064 ± 0.013 mag in V, whereas Feast et al. (2008), comparing distances derived using type II Cepheids and RR Lyrae stars, similarly advocate that RR Lyr is over luminous by 0.16 ± 0.12 mag in V. Therefore, it may not be entirely safe to assume, as frequently done, that RR Lyr is truly representative of other RR Lyrae-type stars of similar metallicity. A definitive solution to this problem will probably have to wait for new, accurate parallax determinations for large numbers of RR Lyrae stars, such as will be afforded by the GAIA and SIM missions.

In the meantime, however, it could also be useful to look further into another diagnostic that is available to us at this point in time: the period change rate of RR Lyr. More specifically, a large, positive period change rate for the star could raise a yellow flag indicating the possibility of its being in a fast, advanced evolutionary stage, and therefore likely brighter than most other stars of similar metallicity. The presence of a near-zero or negative period change rate would be more in line with the star being found around the slowest part of the HB evolution. Of course, the caveat should be kept in mind that it is very often the case that RR Lyrae stars are seen to exhibit erratic period changes which are in some cases even orders of magnitude different from what is predicted by stellar evolution theory (see Smith 1995 for a review and discussions), so that the present argument should be taken with due caution.

Automatically computed and updated values of observed minus computed (O-C) times of maxima for RR Lyr can be found in the GEOS RR Lyr Database web page (see Le Borgne, Klotz, & Boër 2008). In Figure 14 we show one such curve, obtained with respect to a reference period of 0.5668378 d. As recently reviewed by Sterken (2005), a variable star with a period that is linearly changing with time will show a quadratic curve in the period-epoch diagram, with the value of the quadratic term given by

\[ \frac{\Delta P}{P^2} = \frac{O-C}{P} \]

25In fact, van den Bergh (1993) and Lee & Carney (1999b), among others, suggest precisely the opposite, namely, that it is the OoI component of the Galaxy that is of external origin, the OoII clusters having formed very early in the Galaxy’s infancy.

26http://dbhrr.astr.obs-mip.fr/
Fig. 15  Average period change rate for RR Lyrae stars in Galactic globular clusters. The plusses, stars, crosses, and filled gray symbols represent new theoretical results for four different metallicity values (as shown in the inset); these can be very well represented by the analytical formula provided in eq. (8) and indicated by a solid line in this plot.

\[
\beta' = \frac{1}{2} \frac{dP}{dt} (P) E^2, \tag{7}
\]

where \( (P) \) is the average period value during the timespan of the observations, and \( E \) is the epoch. A formal least-squares fit to the data shown in Figure 14 provides a period change rate of \( \sim -0.1 \) d/Myr—although it should be emphasized that there is no a priori reason why the period should change linearly with time, and therefore no strong reason to believe that the assumed quadratic law provides an adequate representation of the data for RR Lyrae stars (e.g., Rathbun & Smith 1997 and references therein). As a matter of fact, according to this diagram and the similar one presented by Szeidl & Kolláth (2000), the rate of period change in RR Lyr has not at all been constant; one sees instead an apparent increase in the period at JD 2,418,000 or JD 2,420,000, only to be followed by a sudden decrease in the period that took place some time around JD 2,432,000, with a seemingly oscillatory behavior thereafter (interpreted by Szeidl & Kolláth as being due to the accumulation of random fluctuations in the period). One way or another, there is certainly no strong evidence of a sharply increasing period, as might be expected in the case of a very bright, extremely evolved RR Lyrae star that is not strongly affected by random period variations. While this by no means represents conclusive evidence that RR Lyr is not well evolved and therefore overluminous, it is at least consistent with the hypothesis that the star presents a luminosity that is not dramatically different from that of most other RR Lyrae stars of similar metallicity.

9 Period Change Rates of RR Lyrae Stars:
The Evolutionary Connection

As we have just seen in the case of RR Lyrae (Fig. 14), more often than not RR Lyrae variables show erratic rates of period change, presumably due to mixing events in the stellar core (Sweigart & Renzini 1979) or to the presence of hydromagnetic effects related to the conjectured existence of a magnetic cycle similar to the Sun’s (Stothers 1980). Interestingly, transient magnetic fields in the H and He ionization zones have recently been suggested to be responsible for the Blazhko effect as well (Stothers 2006). Note that the presence of a strong magnetic field in RR Lyr (as also required in other theoretical scenarios for the Blazhko effect) has recently been ruled out by the high-precision spectropolarimetric observations (carried out over an almost 4-yr timespan) by Chadid et al. (2004). Stothers (2006) speculates, again drawing an analogy with the Sun, that these measurements may simply indicate that a strong magnetic field may be deeply embedded in the star, becoming rather weak only at its surface (see §3 of his paper for extensive references to earlier work on this subject). Relatively recent reviews of the Blazhko effect in RR Lyrae stars have been provided by Smith (1997, 2006) and Kolenberg (2002).

This notwithstanding, given sufficient time coverage and large enough samples of stars, one may be able to detect, superimposed on the random period changes that characterize many RR Lyrae variables, the slow period changes that are due to the star’s evolution across the CMD. That the period of the star should change with time is a very basic prediction of stellar evolution and pulsation theory: as a star slowly changes its luminosity and temperature, its mean density also changes; according to Ritter’s (1879) period-mean density relation of pulsation theory, \( P \sqrt{\langle \rho \rangle} \approx \text{const.} \), the period should accordingly change in inverse proportion with the density, implying that a star with decreasing density—which may be due either to a decreasing temperature and/or an increasing luminosity, assuming the stellar mass is constant—should have an increasing period. Since stars evolving to the red and towards higher luminosities on the HR diagram are thought to be present mainly in globular clusters with blue HB types, it follows that positive period change rates that can be ascribed to evolutionary effects are expected mostly on blue HB clusters. Clusters with predominantly red or intermediate HBs, on the other hand,
The empirical error bars. In the case of NGC 4147, these evolutionary models and the observations, within Table 4. As can be seen, there is good agreement between Smith (1995)—the latter corresponding to the data-

we have added to the sample several additional glob-
ular clusters, including M2 (Lee & Carney 1999a) and NGC 5272 (Stetson et al. 2005), as well as earlier data

of simulations for both extremely blue and extremely red HBs, period change rates were computed for the metallicity values Z = 0.0005, 0.001, 0.002, and 0.006, as shown in Figure 15. Note that these theoretical results can be extremely well described by an exceedingly simple analytical formula (solid line in Fig. 15), namely

\[
\left\langle \frac{\Delta P}{\Delta t} \right\rangle = \frac{0.0053}{1 + 0.99 \mathcal{L}} \text{ (d/Myr),} \tag{8}
\]

where \( \mathcal{L} = (B - R)/(B + V + R) \).

The empirical data are also shown in Figure 15, overplotted on the theoretical results. Using the recent Nemec (2004) compilation as a starting point, we have added to the sample several additional globular clusters, including M2 (Lee & Carney 1999a) and NGC 4147 (Stetson et al. 2005), as well as earlier data for several globulars as summarized in Table 5.1 of Smith (1995)—the latter corresponding to the data-points without error bars. These data are listed in Table 4. As can be seen, there is good agreement between these evolutionary models and the observations, within the empirical error bars. In the case of NGC 4147, the timespan of the available observations has presumably been insufficient to reliably detect the evolutionary changes in the pulsation periods.

10 Conductive Opacities and the He-Core Mass at the Helium Flash

In an earlier review, Catelan, de Freitas Pacheco, & Horvath (1996) argued that uncertainties in the conductive opacities remained that might still lead to significant changes in the computed properties of HB stars, especially their luminosities. Since over 10 years have passed since the Catelan et al. study, a new look at the problem seems especially worthwhile—and this is the purpose of the present section.

To be sure, our approach does not mean that there are not other physical processes which are uncertain enough as to potentially lead to important changes in our understanding of HB evolution; quite the contrary, in fact. For instance, the treatments of diffusion, convection, and mixing, as well as such a key nuclear reaction rate as \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\), remain subject to considerable uncertainty, and future developments may accordingly affect HB evolutionary predictions in quite significant ways. Unfortunately, it would not be practical for us here to provide an extensive review of all of the many different physical processes that play an important role in defining the predicted properties of HB stars. The interested reader is referred to the recent reviews by Salaris et al. (2002), Cassisi (2005), and Catelan (2007) for critical discussions of several of the
more salient uncertainties affecting the computation of HB models.

10.1 Overview

One of the main ingredients affecting the luminosity level of HB stars are the conductive opacities in the cores of their progenitor RGB stars. The cores of RGB stars are electron-degenerate, so that electron conduction is the main form of energy transport. Naturally, the more efficient the transport of energy away from the He-core, the more difficult it becomes for the core to reach high enough temperatures for the onset of helium burning, with the end result that the He-flash is delayed for higher electron conductivities (i.e., smaller conductive opacities), the star reaching a higher He-core mass and a brighter location on the RGB tip—thus also being expected to lose more mass during its first ascent of the RGB and thereby settle on a bluer post-flash location on the ZAHB.

The He-core mass $M_c$ is also known to have a marked, direct impact upon the HB structures. The shapes of HB evolutionary tracks are strongly affected by the precise value of $M_c$, with a larger $M_c$ leading to less marked blueward loops on the HB phase (Sweigart & Gross 1976) and thereby affecting the predicted fraction of well-evolved RR Lyrae stars in globular clusters with predominantly blue HBs (Catelan 1992). Most importantly, higher $M_c$ values lead to brighter HBs as

$$\frac{dM_{\text{bol}}^{\text{HB}}}{dM_c} \approx 7.3 - 9.0 \text{ (mag}/M_\odot)$$

(Sweigart et al. 1987; Catelan 1996; Catelan et al. 1996); therefore, an uncertainty in the value of $M_c$ by only $\pm 0.01 \, M_\odot$ (or about $\pm 2\%$) is capable of affecting the HB luminosity level by $\pm 0.07 - 0.09$ mag. The numerical experiments by Sweigart & Gross (1978) indicate that a change in $M_c$ by about $0.01 \, M_\odot$ obtains when one reduces the conductive opacities used in their calculations by a (uniform) factor of 2.

The implied level of uncertainty in the HB models is important not only for the sake of comparing the predicted and empirically calibrated HB luminosity-metallicity relationships, but also (and perhaps most importantly) in the context of the self-consistent determination of globular cluster ages from evolutionary models of globular cluster stars; of the establishment of the RGB tip distance scale; of the determination of the helium abundance in globular clusters using Iben’s (1968) $R$-method; and of the placement of astrophysical constraints on fundamental (especially particle) physics parameters. As to the latter, constraints on the neutrino magnetic moment using bright RGB and HB stars were discussed by Raffelt & Weiss (1992) and Castellani & Degl’Innocenti (1993); those on the axion mass, by Raffelt & Dearborn (1987); and those on the number of extra dimensions in the Universe by Cassisi et al.
Fig. 17 As in Figure 16, but for the run of the Coulomb Γ parameter as a function of mass in the RGB core. The vertical line shows the mass coordinate where He ignition first takes place.

(2000). Reviews of the subject and extensive additional references have been provided by Raffelt (1996, 2000) and Catelan et al. (1996).

Until the mid-90’s, the two main sources of conductive opacities were the calculations by Hubbard & Lampe (1969) and Itoh et al. (1983). To date, most authors still use one of these two, as can be seen from the recent summary of physical ingredients in different evolutionary models compiled by Gallart et al. (2005). Yet, as pointed out by Catelan et al. (1996), the theoretical calculations of Itoh et al. in particular are not applicable to the physical conditions characterizing RGB cores, since they are restricted to the regime of relatively strong Coulomb interactions (Coulomb parameter Γ > 2), while in the RGB core Γ < 0.81—an intermediate regime between the liquid and crystal phases considered by Itoh et al. and an ideal gas. Since then, however, new conductive opacity calculations have been presented by Potekhin (1999) and Potekhin et al. (1999).

Figure 16 shows the run of temperature (left panel) and density (right panel) in the interior of a metal-poor, low-mass star as it climbs up the RGB. These models, which refer to a 0.8 M⊙ star with [Fe/H] = −2.3, [α/Fe] = 0.3, and Y = 0.2484, were kindly provided by D. A. VandenBerg (2005, priv. comm.). Nine “snapshots” are taken over an interval covering the final 56 × 10⁶ yr of the star’s evolution in the RGB phase: as can be seen from this plot, during this time interval the mass of the He core increases from about 0.28 M⊙ all the way up to its final value, of order 0.5 M⊙. Note that the He-flash actually takes place at a mass value just below 0.26 M⊙, due to the fact that the density-dependent neutrino energy losses lead to an efficient cooling of the innermost regions and therefore to a temperature inversion in the RGB core, as first shown by Thomas (1967). For the sake of simplicity, in what follows we shall assume a pure helium plasma.

Figure 17 shows the variations in the Coulomb Γ parameter [see, e.g., eq. (21) in Catelan et al. 1996], which measures the strength of the electrostatic interactions in the plasma, throughout the He core as a function of the star’s evolutionary status. Clearly, this diagram fully supports the assertion that Γ ≲ 0.81 in the RGB interior. This accordingly confirms the caveat that one should avoid using the Itoh et al. (1983) results in evolutionary calculations of low-mass stars.

Since the more recent Potekhin (1999) and Potekhin et al. (1999) calculations have superseded the Itoh et al. (1983) results, the preceding argument leads naturally to the following question: are the Potekhin results themselves fully applicable to the conditions characterizing RGB interiors?

In fact, not quite: Potekhin et al. (1999) explain that a fit to the static structure factor that is valid over the range 1 ≲ Γ ≲ 225 was employed in their calculations. Therefore, the possibility that a problem similar to the one affecting the Itoh et al. (1983) results is present also in the case of Potekhin’s calculations cannot be excluded; a fully self-consistent calculation for 0.1 ≲ Γ ≲ 0.8, which is the relevant range for the typical RGB interior (Fig. 17), would prove of great interest in the context of precision models of RGB stars and their HB progenitors. Potekhin et al. do note that Coulomb logarithms in the transition domain from weak (Γ ≪ 1) to strong (Γ ≥ 1) ion coupling could be calculated using the Boerker, Rogers, & DeWitt (1982) formalism, but unfortunately such a formalism was not applied in their calculations. In fact, it appears that the reason why such a range of Γ values has not been given great emphasis in such calculations as those by Itoh et al. or Potekhin et al. is that such studies are generally carried out primarily with applications to the more extreme conditions characterizing neutron stars and white dwarf cores in mind, the interiors of RGB stars having received comparatively little attention.

This, in fact, also brings to mind the question whether other approximations to physical situations that are rather more extreme in neutron stars and white dwarf cores may not have been adopted in these latest calculations that could render uncertain their application to the conditions characterizing RGB cores. In this sense, the most obvious ingredient is the electron
degeneracy level. Figure 18 shows the run of the ratio between the actual temperature and the Fermi temperature for the same models shown in the previous plots. As well known, for $T \ll T_{\text{Fermi}}$, strong degeneracy is present, whereas $T \gg T_{\text{Fermi}}$ implies a classical regime where degeneracy effects are unimportant. This plot indicates that RGB interiors fall in an intermediate regime between strongly degenerate and classical. In other words, partial degeneracy better describes the status of the RGB interior. Note, in particular, that at the mass coordinate where the He-flush takes place, the temperature ranges from about 10% to 45% the Fermi temperature. Clearly, the effects of partial degeneracy should be fully taken into account for conductive opacities that are applicable to the conditions characterizing RGB cores to obtain. How does this compare with the Potekhin (1999) and Potekhin et al. (1999) analyses?

As clearly stated by Potekhin et al. (1999), their analysis “is limited by the condition $T \ll T_{\text{Fermi}}$”—in other words, strong degeneracy is assumed in their calculations. Unfortunately, this cannot be expected to provide particularly reliable results for the RGB interior.\textsuperscript{27}

The above discussion refers to the contribution of electron-ion ($ei$) interactions to the conductive opacity. An apparently more subtle effect is related to the electron-electron ($ee$) interactions. Since these are usually of little importance in the more extreme conditions characterizing neutron stars and white dwarfs, the Potekhin (1999) and Potekhin et al. (1999) calculations have not treated this component in as much detail as in the case of the $ei$ interactions. The $ee$ component was indeed computed assuming complete degeneracy, with no corrections whatsoever due to partial degeneracy effects—and are accordingly expected to be accurate only to within a factor of $\sim 2$ (Potekhin 2005, priv. comm.). However, it must be noted that $ee$ effects are a major component of the conductive opacity in a He plasma in the conditions characterizing the RGB interior (Hubbard & Lampe 1969; Catelan et al. 1996). This is shown in Figure 19, where the ratio between the component of the conductive opacity due to $ee$ interactions and the total conductive opacity, according to the Potekhin calculations, is displayed. One sees that the $ee$ contribution reaches a minimum of about 28% of the total conductive opacity close to the mass coordinate where the He-flush takes place. The $ee$ contribution increases further as the He-flush is approached. Evidently, an error by a factor of two in the $ee$ contribution to the conductive opacity implies that the total opacity is uncertain by a comparable factor. Accordingly, greater attention should be devoted to this crucial

\textsuperscript{27}On the other hand, Potekhin (2005, priv. comm.) points out that the technique of “Fermi-Dirac averaging” has been adopted in the region of partial degeneracy, which at least improves somewhat the situation compared to what would obtain from straight application of the strongly degenerate results to partially degenerate conditions (see Potekhin 1999 for more details).
physical ingredient in future calculations of conductive opacities for RGB stars.

10.2 Laboratory Experiments

It is obviously of great interest to check how the Potekhin (1999) and Potekhin et al. (1999) opacities compare with experiments conducted in the laboratory. Such a comparison, in the case of electrical conduction, has recently been carried out by Stygar, Gerdin, & Fehl (2002), using laboratory data from Mintsev, Fortov, & Griaznov (1980), Benage, Shanahan, & Murillo (1999), and Benage (2000). Their results are shown in Figure 20, where the electrical conductivity is plotted as a function of the Coulomb $\Gamma$ parameter. These comparisons clearly reveal a very complex pattern, with the models systematically overestimating the conductivity (i.e., underestimating the conductive opacity) for $\Gamma \gtrsim 1.9$; presenting good agreement with the empirical results for $1.3 \lesssim \Gamma \lesssim 1.7$; and again overestimating the conductivity for $0.85 \lesssim \Gamma \lesssim 1.25$. It is unclear at present what the reasons for this behavior are, though Potekhin (2005, priv. comm.) points out that the laboratory plasmas, when heavy elements are present (the above experiments were carried out using xenon and aluminum), are almost always partially ionized, and it is very difficult to determine, and therefore account for, the actual degree of ionization, because of the plasma nonideality. Accordingly, more empirical tests of the Potekhin results using low-Z gases, especially He, would be highly desirable.

10.3 Comparison between Different Prescriptions

It is instructive to compare the Potekhin (1999) and Potekhin et al. (1999) results with those from the Itoh et al. (1983) and Hubbard & Lampe (1969) analyses. This is done in Figures 21 and 22, respectively, which show the relative differences between the conductive opacities computed by Itoh et al. and by Hubbard & Lampe with respect to the Potekhin results for the physical conditions characterizing the RGB interior calculations. In the Itoh et al. case, we have included the quantum corrections by Mitake, Ichimaru, & Itoh (1984), which however are rather small over the region of interest. The agreement between the Itoh-Mitake prescriptions and Potekhin’s is quite poor, with the former systematically overestimating the conductive opacity except very close to the core, where the conductive opacity is instead underestimated by about 25%. On the other hand, the Hubbard & Lampe calculations reveal a better agreement with the Potekhin results, the maximum overestimate of the conductive opacity by Hubbard & Lampe compared to Potekhin not exceeding about 55%. In the central core, again the older

---

28 According to Potekhin (2005, priv. comm.), this discrepancy can largely be ascribed to the fact that the Itoh et al. (1983) calculations are limited not only to strong coupling, but also to strong electron degeneracy—compare with the discussion in the previous footnote.
calculations underestimate the conductive opacity compared to the more recent results, albeit by a maximum of less than 20%. We conclude that the Potekhin results are in much better agreement with the canonical prescriptions by Hubbard & Lampe than they are with the results by Itoh et al.

While updated conductive opacities that are entirely suitable for the conditions characterizing RGB interiors are not available, it is clear that one should avoid using the Itoh et al. (1983) results in calculations of low-mass stellar evolution.

10.4 Latest Developments

After the first version of this paper had been completed, new conductive opacities were published by Cassisi et al. (2007)—in essence, a revised and updated installment of the Potekhin (1999) opacities. These calculations, while still not perfect, already do take several of the shortcomings noted in the previous sections into due account, and should accordingly be strongly preferred in state-of-the-art evolutionary computations.

§2.4, and especially Figures 3 and 4, in Cassisi et al. provide the results of a comparison between the new conductive opacities computed by Cassisi et al. and the earlier studies by Hubbard & Lampe (1969), Itoh et al. (1983), and Potekhin (1999). Important differences are found with respect to the results from all previous studies, especially those from the Itoh et al. team. According to Cassisi et al., the differences are mainly due to the following: i) Deficiencies in the earlier treatment of strongly coupled and relativistic plasmas (Hubbard & Lampe); ii) An inadequate extension towards the $T > T_F$ regime (Itoh et al.); iii) A neglect of $ee$ scattering (Potekhin).

When the new conductive opacities are used in evolutionary computations, Cassisi et al. find that the changes, with respect to the results based on the earlier Potekhin (1999) prescriptions, to be relatively modest. More specifically, they find: i) A reduction in the core mass at the He-flash by about $0.006 M_\odot$, irrespective of metallicity; ii) An increase in $M_V(ZAHB)$ at the RR Lyrae level, by an amount ranging from 0.06 mag at $Z = 0.0001$ down to 0.04 mag at solar metallicity; iii) An increase in the HB lifetime, by about 5-6%.

While relatively small, these systematic effects must obviously be properly taken into account in future precision studies involving HB stars.

11 Epilogue

The study of horizontal branch stars encompasses and/or has direct implications for many different branches of astrophysics. Without first looking into several of these areas, one might rush into the conclusion that the study of HB stars is a frustratingly limited affair. Nothing could be farther from the truth. As we have seen, in order to properly understand HB stars and appreciate their astrophysical implications, one must dwell on the physical processes (both canonical and non-canonical) that control the helium core flash in RGB stars; the physical processes that lead to mass loss in RGB stars; whether (and how) deep mixing takes place in RGB stars, both before and during the helium flash; how angular momentum evolves with time in low-mass stars; where, and how fast, radiative levitation and diffusion effects in moderately high-gravity stars “kick in”; how dramatically non-solar abundance ratios (again as a result of radiative levitation and gravitational settling) may affect model atmospheres and the photon output as a function of wavelength; how both radial and non-radial (both p and g) pulsation modes can be excited, and then evolve with time; how theory and observations of radial pulsators may place constraints on the physical parameters of stars, and thereby help determine the formation history of galaxies and the extragalactic distance scale; how asteroseismology of non-radial pulsators may help reveal the interior structure and thereby constrain the evolutionary history of stars; how the ultraviolet flux from giant elliptical galaxies and bulges of spirals come to being; how the stellar populations of resolved and unresolved
galaxies evolve with time; and so on and so forth. In summary, the study of HB stars is a challenging and far-reaching intellectual adventure, and we hope that the present review will have helped unveil some of the reasons why this is so.

The author warmly thanks A. Bonacic, G. Clementini, C. Moni Bidin, B. J. Pritzl, A. Reisenegger, and M. Zoccali, and most especially S. Cassisi, A. Y. Potekhin, H. A. Smith, A. V. Sweigart, and D. A. VandenBerg, for very helpful discussions and information. The staff and personnel of the Bologna Observatory and University, where part of this text was written, is warmly thanked for their hospitality. Support for this work was provided by Proyectos FONDECYT Regulares No. 1030954 and 1071002, by Proyecto Basal PFB-06/2007, by the FON-DAP Center for Astrophysics 15010003, and by a John Simon Guggenheim Memorial Foundation Fellowship. This research has made use of the GEOS RR Lyr Database.
References

Aikawa, M., Fujimoto, M. Y., & Kato, K. 2001, Astrophys. J., 560, 937
Alcock, C., et al. 1996, Astron. J., 111, 1146
Alcock, C., et al. 2000, Astrophys. J., 542, 257
Alcock, C., et al. 2003, Astrophys. J., 598, 597
Alcock, C., et al. 2004, Astron. J., 127, 334
Altman, M., Catelan, M., & Zoccali, M. 2005, Astron. Astrophys., 439, L5
Altman, M., Edelmann, H., & de Boer, K. S. 2004, Astron. Astrophys., 414, 181
Alves, D. R. 2004, NewA Rev., 48, 659
Arifyanto, M. I., Fuchs, B., Jahreiß, H., & Wielen, R. 2005, Astron. Astrophys., 433, 911
Armandroff, T. E. 1988, Astron. J., 96, 588
Arp, H. C. 1955, Astron. J., 60, 317
Arp, H. C. 1959, Astron. J., 64, 441
Asplund, M., Grevesse, N., Sauval, A. J., Allende Prieto, C., & Kispé, D. 2004, Astron. Astrophys., 417, 751
Baev, P. V., Markov, H., & Spassova, N. 2001, Mon. Not. R. Astron. Soc., 328, 944
Bailey, S. I. 1913, Harv. Coll. Observ. Annals, 78, 1
Bailey, S. I. 1919, Harv. Coll. Observ. Annals, 78, 1
Baran, A., Pigulski, A., Koziel, D., Ogloza, W., Silvotti, R., & Zola, S. 2005, Mon. Not. R. Astron. Soc., 360, 737
Barnard, E. E. 1900, Astrophys. J., 12, 176
Bellazzini, M., Ferraro, F. R., & Ibata, R. 2003b, Mon. Not. R. Astron. Soc., 344, 1097
Benage, J. F., Jr. 2000, Phys. Plasmas, 7, 2040
Benedict, G. F., et al. 2002, Astron. J., 123, 473
Bertielli, G., Bressan, A., Chiosi, C., & Ng, Y. K. 1996, Astron. Astrophys., 310, 115
Blazhko, S. 1956, Astron. Nachr., 175, 325
Boerker, D. B., Rogers, F. J., & DeWitt, H. E. 1982, Phys. Rev. A, 25, 1623
Bon-Hoa, A. H., LeBlanc, F., & Hauschildt, P. H. 2000, Astrophys. J. Lett., 535, L43
Bonifacio, P., Castelli, F., & Hack, M. 1995, Astron. Astrophys. Suppl. Ser., 110, 441
Bonifacio, P., Sbordone, L., Marconi, G., Pasquinelli, L., & Hill, V. 2004, Astron. Astrophys., 414, 503
Bonnell, J., Wu, C.-C., Bell, R. A., & Hutchinson, J. L. 1982, Publ. Astron. Soc. Pac., 94, 910
Bono, G. 2003, in Stellar Candles for the Extragalactic Distance Scale, Lecture Notes in Physics, Vol. 635, ed. D. Alloin & W. Gieren (Berlin: Springer), 85
Bono, G., Caputo, F., Cassisi, S., Incerpi, R., & Marconi, M. 1997a, Astrophys. J., 483, 811
Bono, G., Caputo, F., Castellani, V., & Marconi, M. 1997b, Astrophys. Space Sci., 121, 327
Bono, G., Caputo, F., & Stellingwerf, R. F. 1994, Astrophys. J., 423, 294
Bono, G., Caputo, F., & Stellingwerf, R. F. 1995, Astrophys. J. Suppl. Ser., 99, 263
Bono, G., Caputo, F., Castellani, V., & Marconi, M. 1997b, Astrophys. Space Sci., 121, 327
Borkova, J., Ivanov, V. D., & Catelan, M. 2000, IBVS, 665
Borissova, J., Minniti, D., Rejkuba, M., Alves, D., Cook, K. H., & Freeman, K. C. 2004, Astron. Astrophys., 423, 97
Borkova, T. V., & Marsakov, V. A. 2000, Astr. Reports, 44, 665
Bowen, G. H. 1988, in Pulsation and Mass Loss in Stars, ed. R. Stalio & L. A. Wilson (Dordrecht: Kluwer), 3
Boyer, M. L., McDonald, I., van Loon, J. Th., Woodward, C. E., Gehrz, R. D., Evans, A., & Dupree, A. K. 2008, Astron. J., 135, 1395
Bragaglia, A., Gratton, R. G., Carretta, E., Clementini, G., Di Fabrizio, L., & Marconi, M. 2001, Astron. J., 122, 207
Bressan, A., Chiosi, C., & Fagotto, F. 1994, Astrophys. J. Lett., 535, L43
Brocato, E., Castellani, V., Ferrari, F. R., Piersimon, A. M., & Testa, V. 1996, Mon. Not. R. Astron. Soc., 282, 614
Brocato, E., Di Carlo, E., & Mena, G. 2001, Astron. Astrophys., 374, 523
Brown, J. A., Wallerstein, G., & Gonzalez, G. 1999, Astron. J., 118, 1245
Brown, J. A., Wallerstein, G., & Zucker, D. 1997, Astron. J., 114, 180
Brown, T. M., Ferguson, H., Smith, E., Kimble, R. A., Sweigart, A. V., Renzini, A., & Rich, R. M. 2004a, Astron. J., 127, 2738
Brown, T. M., Ferguson, H., Smith, E., Kimble, R. A., Sweigart, A. V., Renzini, A., & Rich, R. M., & VandenBerg, D. A. 2003, Astrophys. J. Lett., 592, L17
Brown, T. M., Ferguson, H., Smith, E., Kimble, R. A., Sweigart, A. V., Renzini, A., Rich, R. M., & VandenBerg, D. A. 2004b, Astrophys. J. Lett., 613, L125

M. Catelan
Brown, T. M., Smith, E., Ferguson, H., Rich, R. M., Guhathakurta, P., Renzini, A., Sweigart, A. V., & Kimble, R. A. 2006, Astrophys. J., 652, 323
Brown, T. M., Sweigart, A. V., Lanz, T., Landsman, W. B., & Hubeny, I. 2001, Astron. J., 120, 368
Brown, W. R., Beers, T. C., Wilhelm, R., Allende Prieto, C., Geller, M. J., Kenyon, S. J., & Kurtz, M. J. 2008, Astron. J., 135, 564
Brown, W. R., Geller, M. J., Kenyon, S. J., Beers, T. C., Kurtz, M. J., & Roll, J. B. 2004a, Astron. J., 127, 1555
Brown, W. R., Geller, M. J., Kenyon, S. J., Kurtz, M. J., Allende Prieto, C., Beers, T. C., & Wilhelm, R. 2005, Astron. J., 130, 1097
Browne, S. E. 2005, Publ. Astron. Soc. Pac., 117, 726
Buonanno, R., Corsi, C. E., Bellazzini, M., Ferraro, F. R., & Fusi Pecci, F. 1997, Astron. J., 113, 706
Buonanno, R., Corsi, C. E., Castellani, M., Marconi, G., Fusi Pecci, F., & Zinn, R. 1999, Astron. J., 118, 1671
Buonanno, R., Corsi, C. E., & Fusi Pecci, F. 1985, Astron. Astrophys., 145, 97
Buonanno, R., Corsi, C. E., Fusi Pecci, F., Richer, H. B., & Fahlman, G. G. 1993, Astron. J., 105, 184
Buonanno, R., Corsi, C. E., Zinn, R., Fusi Pecci, F., Hardy, E., & Suntzeff, N. B. 1998, Astrophys. J. Lett., 501, L33
Busso, G., et al. 2007, Astron. Astrophys., 474, 105
Busso, G., Moehler, S., Zoccali, M., Heber, U., & Yi, S. 2005, Astrophys. J. Lett., 633, L29
Busso, G., & Moehler, S. 2008, in Hot Subdwarf Stars and Related Objects, ASP Conf. Ser., 392, ed. U. Heber, C. S. Jeffery, & R. Napiwotzki (San Francisco: ASP), 39
Butler, D., Demarque, P., & Smith, H. A. 1982, Astrophys. J., 257, 592
Cacciari, C. 1999, in Harmonizing Cosmic Distances in a Post-Hipparcos Era, ASP Conf. Ser., 167, ed. D. Egret & A. Heck (San Francisco: ASP), 140
Cacciari, C. 2003, in New Horizons in Globular Cluster Astronomy, ASP Conf. Ser., 296, ed. G. Piotto, G. Meylan, S. G. Djorgovski, & M. Riello (San Francisco: ASP), 329
Cacciari, C., Bellazzini, M., & Colucci, S. 2002, in Extragalactic Star Clusters, IAU Symp. Ser., 207, ed. D. Geisler, E. K. Grebel, & D. Minniti (San Francisco: IAU), 168
Cacciari, C., & Clementini, G. 2003, in Stellar Candles for the Extragalactic Distance Scale, Lecture Notes in Physics, Vol. 635, ed. D. Alloin & W. Gieren (Berlin: Springer), 105
Cacciari, C., Corwin, T. M., & Carney, B. W. 2005, Astron. J., 129, 267
Cáceres, C., & Catelan, M. 2008, Astrophys. J. Suppl. Ser., 179, 242
Caloi, V. 1972, Astron. Astrophys., 20, 357
Caloi, V. 1999, Astron. Astrophys., 343, 904
Caloi, V. 2001, Astron. Astrophys., 366, 91
Caloi, V., & D’Antona, F. 2005, Astron. Astrophys., 435, 987
Caloi, V., & D’Antona, F. 2005, Astron. Astrophys., 463, 949
Caloi, V., & D’Antona, F. 2008, Astrophys. J., 673, 847
Cannon, R. D. 1970, Mon. Not. R. Astron. Soc., 150, 111
Caputo, F. 1985, Rep. Prog. Phys., 48, 1235
Caputo, F. 1998, A&A Rev., 9, 33
Caputo, F., Castellani, V., Marconi, M., & Ripepi, V. 2000, Mon. Not. R. Astron. Soc., 316, 819
Caputo, F., De Stefanis, P., Paez, E., & Quarta, M. L. 1987, Astron. Astrophys. Suppl. Ser., 68, 119
Caputo, F., & Degl’Innocenti, S. 1995, Astron. Astrophys., 298, 833
Caputo, F., Marconi, M., & Santolamazza, P. 1998, Mon. Not. R. Astron. Soc., 293, 364
Carney, B. W. 2001, in Star Clusters, ed. L. Labhardt & B. Binggeli (Berlin: Springer), 1
Carney, B. W., Latham, D. W., Stefanik, R. P., Laird, J. B., & Morse, J. A. 2003, Astron. J., 125, 293
Carraro, G., Moitinho, A., & Vázquez, R. 2008, Mon. Not. R. Astron. Soc., 385, 1597
Carraro, G., Zinn, R., & Moni Bidin, C. 2007, Astron. Astrophys., 466, 181
Carrera, R., Gallart, C., Hardy, E., Aparicio, A., & Zinn, R. 2008, Astron. J., 135, 836
Carretta, E., Recio-Blanco, A., Gratton, R. G., Piotto, G., & Bragaglia, A. 2007, Astrophys. J. Lett., 671, L125
Carretta, E., & Gratton, R. 1996, in Formation of the Galactic Halo....Inside and Out, ASP Conf. Ser. 92, ed. H. Morrison & A. Sarajedini (San Francisco: ASP), 359
Carretta, E., & Gratton, R. 1997, Astron. Astrophys. Suppl. Ser., 121, 95
Carretta, E., Gratton, R., & Clementini, G. 2000, Mon. Not. R. Astron. Soc., 316, 721
Cassisi, S. 2005, preprint (astro-ph/0506161)
Cassisi, S., Castellani, M., Caputo, F., & Castellani, V. 2004, Astron. Astrophys., 426, 641
Cassisi, S., Castellani, V., Degl’Innocenti, S., Fiorentini, G., & Ricci, B. 2000, Phys. Lett. B, 481, 323
Cassisi, S., Potekhin, A. Y., Piotrinski, A., & Salaris, M. 2007, Astrophys. J., 661, 1094
Cassisi, S., Salaris, M., Piotrinski, A., Piotto, G., Milone, A. P., Bedin, L. R., & Anderson, J. 2008, Astrophys. J. Lett., 672, L115
Cassisi, S., Schlattl, H., Salaris, M., & Weiss, A. 2003, Astrophys. J. Lett., 582, L43
Castellani, M., & Castellani, V. 1993, Astrophys. J., 407, 649
Castellani, M., Castellani, V., & Prada Moroni, P. G. 2006, Astron. Astrophys., 457, 569
Castellani, V. 1983, MSAIt, 54, 141
Castellani, V. 1999, in Globular Clusters, ed. C. Martinez Roger, I. Pérez Fournon, & F. Sánchez (Cambridge: Cambridge University Press), 109
Castellani, V., Chieffi, A., & Pulone, L. 1991, Astrophys. J. Suppl. Ser., 76, 911
Castellani, V., & Degl’Innocenti, S. 1993, Astrophys. J., 402, 574
Castellani, V., Iannicola, G., Bonon, G., Zoccali, M., Cassisi, S., & Buonanno, R. 2006, Astron. Astrophys., 446, 569
Castellani, V., & Paterno, L. 1984, AN, 305, 251
Castellani, V., & Renzini, A. 1968, Astrophys. Space Sci., 2, 310
Lee, Y.-W. 1990, Astrophys. J., 363, 159
Lee, S.-W., & Cannon, R. D. 1980, J. Korean Astr. Soc., 21, 155
Lee, J.-W., López-Morales, M., & Carney, B. W. 2006, Astrophys. J. Lett., 646, L119
Lee, S.-W., & Cannon, R. D. 1980, J. Korean Astr. Soc., 13, 15
Lee, Y.-W. 1990, Astrophys. J., 363, 159
Lee, J.-W., & Carney, B. W. 1999a, Astron. J., 117, 2868
Lee, J.-W., & Carney, B. W. 1999b, Astron. J., 118, 1373
Lee, J.-W., López-Morales, M., & Carney, B. W. 2006, Astrophys. J. Lett., 646, L119
Lee, Y.-W., & Zinn, R. 1990, in Confrontation between Stellar Pulsation and Evolution, ASP Conf. Ser. 11, ed. C. Cacciari & G. Clementini (San Francisco: ASP), 185
Lee, Y.-W., & Zinn, R. 1990, in Confrontation between Stellar Pulsation and Evolution, ASP Conf. Ser. 11, ed. C. Cacciari & G. Clementini (San Francisco: ASP), 26
Liebert, J., Saffer, R. A., & Green, E. M. 1994, Astron. J., 107, 1408
Liu, T., & Janes, K. A. 1990a, Astrophys. J., 354, 273
Liu, T., & Janes, K. A. 1990b, Astrophys. J. Suppl. Ser., 70, 227
Lanz, T., Brown, T. M., Sweigart, A. V., Hubeny, I., & Landsmans, W. B. 2004, Astrophys. J., 602, 342
Layden, A. C. 1995, Astron. Astrophys., 295, 693
Kovács, G. 1998, in A Half Century of Stellar Pulsation Interpretations, ASP Conf. Ser. 135, ed. P. A. Bradley & J. A. Guzik (San Francisco: ASP), 52
Kovács, G. 2003, Mon. Not. R. Astron. Soc., 342, L58
Kovács, G., & Walker, A. R. 1999, Astrophys. J., 512, 271
Kraft, R. P., & Evans, I. I. 2003, Publ. Astron. Soc. Pac., 115, 143
Kraus, M., & Stecher, T. P. 2001, Bull. Am. Astron. Soc., 199, 70
Kuehn, C., et al. 2008, Astrophys. J. Lett., 674, L81
Landsman, W. 1999, in Spectrophotometric Dating of Stars and Galaxies, ASP Conf. Ser. 192, ed. I. Hubeny, S. R. Heap, & R. H. Cornett (San Francisco: ASP), 235
Landsman, W. B., Catelan, M., O’Connell, R. W., Pereira, D., & Stecher, T. P. 2001, Bull. Am. Astron. Soc., 199, 56.11
Langer, G. E., & Hoffman, R. D. 1995, Publ. Astron. Soc. Pac., 107, 1117
Lanz, T., Brown, T. M., Sweigart, A. V., Hubeny, I., & Landsmans, W. B. 2004, Astrophys. J., 602, 342
Layden, A. C. 1994, Astron. J., 108, 1722
Layden, A. C. 1995, Astron. J., 110, 2312
Layden, A. C. 1994, Astron. J., 108, 1016
Layden, A. C. 1996, in Formation of the Galactic Halo.... Inside and Out, ASP Conf. Ser. 92, ed. H. Morrison & A. D., & van den Bergh, S. 2005, Mon. Not. R. Astron. Soc., 360, 631
Maeder, A., & Meynet, G. 2006, Astron. Astrophys., 448, L37
Majewski, S. R., Osterheimer, J. C., Rocha-Pinto, H. J., Patterson, R. J., Guhathakurta, P., & Reitzel, D. 2004, Astrophys. J., 615, 738
Majewski, S. R., Skrutskie, M. F., Weinberg, M. D., & Osterheimer, J. C. 2003, Astrophys. J., 599, 1082
Maraston, C., Greggio, L., Renzini, A., Ortolani, S., Saglia, R. P., Puzia, T. H., & Kissler-Patig, M. 2003, Astron. Astrophys., 400, 823
Markov, H. S., Spassova, N. M., & Buev, P. V. 2001, Mon. Not. R. Astron. Soc., 326, 102
Martin, N. F., Ibata, R. A., Bellazzini, M., Irwin, M. J., Lewis, G. F., & Dehnen, W. 2004, Mon. Not. R. Astron. Soc., 348, 12
Martínez-Delgado, D., Gómez-Flechoso, M. Á., Aparicio, A., & Carrera, R. 2004, Astrophys. J., 601, 242
Matsunaga, N. 2007, in Why Galaxies Care About AGB Stars: Their Importance as Actors and Probes, ASP Conf. Ser., Vol. 378, ed. F. Kerschbaum, C. Charbonnel, & R. F. Wing (San Francisco: ASP), 86
Matsunaga, N., et al. 2006, Mon. Not. R. Astron. Soc., 370, 1979

Horizontal Branch Stars: Observations and Theory
Piotto, G., Zoccali, M., King, I. R., Djorgovski, S. G., Sosin, P., et al. 2002, Astron. Astrophys., 391, 945

Piotto, G., et al. 1997, in Advances in Stellar Evolution, ed. R. C. Peterson, R. C., Tarbell, T. D., & Carney, B. W. 1983, Astrophys. J., 265, 972

Piotto, G., et al. 1997, in Hot Stars in the Galactic halo, ed. S. J. Adelman, A. R. Upgren, & C. J. Adelman (Cambridge: Cambridge University Press), 41

Pickering, E. C. 1901, Harvard College Observatory Circular, 54, 1

Pickering, E. C., & Bailey, S. I. 1895, Astrophys. J., 2, 321

Piersanti, L., Tornambè, A., & Castellani, V. 2004, Mon. Not. R. Astron. Soc., 353, 243

Piotto, G. 2008, Mem. Soc. Astr. Italiana, 79, 334

Piotto, G., et al. 1997, in Advances in Stellar Evolution, ed. R. T. Rood & A. Renzini (Cambridge: Cambridge University Press), 84

Piotto, G., et al. 2002, Astron. Astrophys., 391, 945

Piotto, G., et al. 2007, Astrophys. J. Lett., 661, L53

Piotto, G., Zoccali, M., King, I. R., Djorgovski, S. G., Sosin, C., Rich, R. M., & Meylan, G. 1999, Astron. J., 118, 1727

Popielski, B. L., Dziembowski, W. A., & Cassisi, S. 2000, A&A, 50, 491

Popowski, P., & Gould, A. 1998a, Astrophys. J., 506, 259

Popowski, P., & Gould, A. 1998b, Astrophys. J., 506, 271

Popowski, P., & Gould, A. 1999, in Post-hipparcos Cosmic Candles, ed. A. Heck & F. Caputo (Dordrecht: Kluwer), 53

Potekhin, A. Y. 1999, Astron. Astrophys., 351, 787

Potekhin, A. Y., Baiko, D. A., Haensel, P., & Yakovlev, D. G. 1999, Astron. Astrophys., 345, 345

Prantzos, N., & Charbonnel, C. 2006, Astron. Astrophys., 458, 135

Preston, G. W. 1959, Astrophys. J., 130, 507

Preston, G. W., Schectman, S. A., & Beers, T. C. 1991, Astrophys. J., 375, 121

Pritchard, C. J., & van den Bergh, S. 1987, Astrophys. J., 316, 517

Pritzl, B. J., Armandroff, T. E., Jacoby, G. H., & Da Costa, G. S. 2002a, Astron. J., 124, 1464

Pritzl, B. J., Armandroff, T. E., Jacoby, G. H., & Da Costa, G. S. 2005, Astron. J., 129, 2232

Pritzl, B., Smith, H. A., Catelan, M., & Sweigart, A. V. 2000, Astrophys. J. Lett., 530, L41

Pritzl, B. J., Smith, H. A., Catelan, M., & Sweigart, A. V. 2001, Astron. J., 122, 2260; erratum: 2003, Astron. J., 125, 2750

Pritzl, B. J., Smith, H. A., Catelan, M., & Sweigart, A. V. 2002b, Astron. J., 124, 949; erratum: 2003, Astron. J., 125, 2752

Pritzl, B. J., Smith, H. A., Stetson, P. B., Catelan, M., Sweigart, A. V., Layden, A. C., & Rich, R. M. 2003, Astron. J., 126, 1381

Pritzl, B. J., Venn, K. A., & Irwin, M. J. 2005b, Astron. J., 130, 2140

Proctor, R. N., Forbes, D. A., & Beasley, M. A. 2004, Mon. Not. R. Astron. Soc., 355, 1327

Raffelt, G. G. 1996, Stars as Laboratories for Fundamental Physics: The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles (Chicago: University of Chicago Press)

Raffelt, G. G., & Weiss, A. 1992, Astron. Astrophys., 264, 536

Raimondo, G., Castellani, V., Cassisi, S., Brocato, E., & Piotto, G. 2002, Astrophys. J., 569, 975

Randall, A. J., & Jorissen, A. 2001, Astron. Astrophys., 372, 85

Randall, S. K., et al. 2007, Astron. Astrophys., 476, 1317

Rathbun, P. G., & Smith, H. A. 1997, Publ. Astron. Soc. Pac., 109, 1128

Recio-Blanco, A., Aparicio, A., Piotto, G., De Angeli, F., & Djorgovski, S. G. 2006, Astron. Astrophys., 452, 875

Recio-Blanco, A., Piotto, G., Aparicio, A., & Renzini, A. 2002, Astrophys. J. Lett., 572, L71

Recio-Blanco, A., Piotto, G., Aparicio, A., & Renzini, A. 2004, Astron. Astrophys., 417, 597

Ree, C. H., Yoon, S.-J., Rey, S.-C., & Lee, Y.-W. 2002, in Omega Centauri, A Unique Window into Astrophysics, ASP Conf. Ser. 265, ed. F. van Leeuwen, J. D. Hughes, & G. Piotto (San Francisco: ASP), 101

Reed, M. D., et al. 2004, Astrophys. J., 607, 445

Reimers, D. 1975a, in Problems in Stellar Atmospheres and Envelopes, ed. B. Aschek, W. H. Kegel, & G. Traving (Berlin: Springer), 229

Reimers, D. 1975b, in Problems in Stellar Atmospheres and Envelopes, ed. B. Aschek, W. H. Kegel, & G. Traving (Berlin: Springer), 229

Reimers, D. 1975a, in Problems in Stellar Atmospheres and Envelopes, ed. B. Aschek, W. H. Kegel, & G. Traving (Berlin: Springer), 229

Renzini, A. 1977, in Advanced Stages in Stellar Evolution, ed. P. Bouvier & A. Maeder (Geneva: Geneva Observatory), 151

Renzini, A. 1983, MSAI, 54, 335

Renzini, A., & Fusi Pecci, F. 1988, Annu. Rev. Astron. Astrophys., 26, 199

Rey, S.-C., et al. 2005, Astrophys. J. Lett., 619, L119

Rouenda, M., & Meylan, G. 2005, Acad. R. Sci., 109, 1128

Recio-Blanco, A., Piotto, G., Aparicio, A., & Renzini, A. 2004, Astron. Astrophys., 372, 85

Raffelt, G. G. 2000, Phys. Rep., 333, 593

Raffelt, G. G., & Dearborn, D. S. 1987, Phys. Rev. D, 36, 2211

Raddi, R. T. 1973, Astron. Astrophys., 184, 815

Raffelt, G. G., & Rood, R. T. 2003, Astrophys. J., 588, 299

Raffelt, G. G. 1996, Stars as Laboratories for Fundamental Physics: The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles (Chicago: University of Chicago Press)

Raffelt, G. G., & Rood, R. T. 2003, Astrophys. J., 588, 299

Raffelt, G. G., & Rood, R. T. 2003, Astron. J., 123, 1940

Raffelt, G. G., & Rood, R. T. 2003, Mon. Not. R. Astron. Soc., 355, 1327

Raffelt, G. G., & Rood, R. T. 2003, Astrophys. J., 588, 299

Raffelt, G. G., & Rood, R. T. 2003, Astron. J., 123, 1940

Raffelt, G. G., & Rood, R. T. 2003, Mon. Not. R. Astron. Soc., 355, 1327

Raffelt, G. G. 1996, Stars as Laboratories for Fundamental Physics: The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles (Chicago: University of Chicago Press)

Raffelt, G. G., & Rood, R. T. 2003, Astrophys. J., 588, 299

Raffelt, G. G., & Rood, R. T. 2003, Astron. J., 123, 1940

Raffelt, G. G., & Rood, R. T. 2003, Mon. Not. R. Astron. Soc., 355, 1327
Walker, A. R., & Nemec, J. M. 1996, Astron. J., 112, 2026
Wallerstein, G. 1970, Astrophys. J., 160, 345
Welsh, B. Y., et al. 2005, Astron. J., 130, 825
Wehlau, A. 1990, Astron. J., 99, 250
Wheatley, J. M., et al. 2005, Astrophys. J. Lett., 619, L123
Whitney, J. H., et al. 1998, Astrophys. J., 495, 284
Wickett, A. J. 1977, in Problems in Stellar Convection, ed. E. A. Spiegel & J. P. Zahn (Berlin: Springer), 284
Wilhelm, R., Beers, T. C., Kriessler, J. R., Pier, J. R., Sommer-Larsen, J., & Layden, A. C. 1996, in Formation of the Galactic Halo...Inside and Out, ASP Conf. Ser. 92, ed. H. Morrison & A. Sarajedini (San Francisco: ASP), 171
Williams, B. F. 2005, Astron. J., 129, 2663
Willson, L. A. 1988, in Pulsation and Mass Loss in Stars, ed. R. Stalio & L. A. Willson (Dordrecht: Kluwer), 285
Willson, L. A. 2000, Annu. Rev. Astron. Astrophys., 38, 573
Willson, L. A., & Bowen, G. H. 1984, Nature, 312, 429
Xiong, D. R., Cheng, Q. L., & Deng, L. 1998, Astrophys. J., 500, 449
Yamada, S., Okazaki, A. T., & Fujimoto, M. Y. 2008, Astrophys. J., 678, 922
Yi, S., Demarque, P., & Kim, Y.-C. 1997, Astrophys. J., 482, 677
Yi, S., Lee, Y.-W., Woo, J.-H., Park, J.-H., Demarque, P., & Oemler, Jr., A. 1999, Astrophys. J., 513, 128
Yong, D., & Grundahl, F. 2008, Astrophys. J. Lett., 672, L29
Yoon, S.-J., Joo, S.-J., Ree, C. H., Han, S.-L., Kim, D.-G., & Lee, Y.-W. 2008, Astrophys. J., 677, 1080
Yoon, S.-J., & Lee, Y.-W. 2002, Science, 297, 578
Zaritsky, D. 2004, Astrophys. J. Lett., 614, L37
Zinn, R. 1986, in Stellar Populations, ed. C. A. Norman, A. Renzini, & M. Tosi (Cambridge: Cambridge University Press), 73
Zinn, R. 1993a, in The Globular Cluster-Galaxy Connection, ASP Conf. Ser. 48, ed. G. H. Smith & J. P. Brodie (San Francisco: ASP), 38
Zinn, R. 1993b, in The Globular Cluster-Galaxy Connection, ASP Conf. Ser. 48, ed. G. H. Smith & J. P. Brodie (San Francisco: ASP), 302
Zinn, R., Vivas, A. K., Gallart, C., & Winnick, R. 2004, in Satellites and Tidal Streams, ASP Conf. Ser. 327, ed. F. Prada, D. Martínez-Delgado, & T. J. Mahoney (San Francisco: ASP), 92
Zinn, R., & West, M. J. 1984, Astrophys. J. Suppl. Ser., 55, 45