Some remarks on axion dark matter, dark energy and energy of the quantum vacuum

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In connection with the problem of dark matter, we discuss recent results on constraining the parameters of axion-to-nucleon interaction following from the experiment on measuring the difference of Casimir forces. It is shown that this experiment not only leads to competitive constraints, but provides stronger support to other constraints obtained in Casimir physics so far. The description of dark energy by means of cosmological constant originated from the quantum vacuum is considered in terms of the renormalization procedures in quantum field theory. It is argued that only the renormalized value of cosmological constant directly connected with the observed density of dark energy is of physical significance, so that some statements in the literature concerning the vacuum catastrophe may be considered as an exaggeration.

Keywords: Axions; dark matter; dark energy; cosmological constant.

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1. Introduction

At the moment the situation in fundamental physics is much in common with that in the end of the nineteenth century. At that time it was commonly believed that all space is full of hypothetical substance, ether, needed to ensure propagation of the electromagnetic waves. According to current concept, more than 95% of the energy of the Universe consists of the so-called dark energy and dark matter which we are presently not capable to observe directly. The single difference is that we already have several indirect evidences for the existence of dark energy and dark matter and hope to get the direct ones in the immediate future, whereas all attempts to find some experimental fact in favor of ether more than a centure ago failed.

It has long been known that stellar motion in the neighborhood of our galaxy cannot be explained by the gravitational theory if the mass of galaxy is not much larger than that of visible matter. The same is true for the clusters of galaxies. The missing dark matter contributes up to 27% of the energy of our Universe. There
are many attempts to understand the nature of dark matter, but the most realistic model suggests that it consists of light uncharged pseudoscalar particles, axions, whose interaction with familiar elementary particles, such as photons, electrons and nucleons, is very weak.

One more unseen substance is the dark energy which was introduced rather recently in order to explain the accelerated expansion of the Universe in the framework of General Relativity Theory. Unlike the dark matter, the dark energy is characterized by the negative pressure which makes possible the effect of accelerated expansion. The dark energy contributes approximately 68% of the Universe energy. The most plausible explanation for the dark energy is by means of the cosmological constant in Einstein’s equations which can be interpreted as originated from the quantum vacuum. In doing so, however, the so-called vacuum catastrophe arises because the value of the cosmological constant predicted by quantum field theory greatly exceeds the one needed to explain an observed acceleration of the Universe expansion.

In this paper, we discuss new constraints on the axion-to-nucleon coupling constant obtained from measuring the difference of Casimir forces, as well as the possibilities to obtain even stronger constraints by optimizing several other experiments of Casimir physics. We also consider the problem of dark energy and the possibility of its resolution by means of the cosmological constant originated from the quantum vacuum. It is argued that with a proper renormalization procedure only the energy density corresponding to a physical value of the cosmological constant should be considered as the source of gravitational interaction which settles the problem of vacuum catastrophe.

The paper is organized as follows. In Sec. 2 we discuss axions, axion-like particles and new constraints on their parameters obtained in the Casimir physics. Section 3 is devoted to the problem of cosmological constant and its renormalization. In Sec. 4 the reader will find our conclusions and a discussion. We use the system of units where $\hbar = c = 1$.

2. Novel constraints on axion-like particles from Casimir physics

The search for axion-like particles and, thus, for the axion dark matter is performed by exploiting their presumed interactions with photons, electrons and nucleons. There is a great number of such kind experiments already performed, in operation and planned using different laboratory techniques (see also Ref. 6 for experiments in atomic physics). Although rather strong constraints are already obtained on the coupling constants of axions to photons and electrons, the laboratory constraints on axion-to-nucleon interaction remain relatively weak, whereas the astrophysical constraints suffer from serious uncertainties in theory of dense nuclear matter.

It is well known that interaction of axions with nucleons is described either by pseudoscalar or pseudovector Lagrangian densities. Both Lagrangians lead to common spin-dependent effective potential between two nucleons due to exchange...
of one axion. After an averaging over the volumes of two unpolarized bodies, this leads to zero force, i.e., the process of one-axion exchange between nucleons does not manifest itself in measurements of small forces between two macroscopic bodies. If, however, one considers the exchange of two axions described by the pseudoscalar Lagrangian, the two nucleons interact by means of spin-independent effective potential

\[ V(r) = -\frac{g_{an}^4}{32\pi^3m_n^2} \frac{m_a}{r} K_1(2m_ar), \]  

where \( g_{an} \) is the axion-to-nucleon interaction constant, \( m_n \) and \( m_a \) are the nucleon and axion masses, \( r \) is the separation distance between two nucleons, and \( K_1(z) \) is the Bessel function of the second kind. It should be particularly emphasized that up to the present the form of effective potential between two nucleons due to exchange of two axions, described by the pseudovector Lagrangian, remains a mystery.

The effective potential (1) corresponds to an attractive force. The presence of such forces can be tested in precise experiments on force measurements between closely spaced macroscopic bodies and, more specifically, on measuring the Casimir force. In so doing, the force arising due to two-axion exchange, \( F_a(d) \), is calculated in the experimental configuration by a pairwise summation of potentials (1) over the volumes of test bodies with subsequent negative differentiation with respect to separation \( d \) between them. Then, the constraints on \( g_{an} \) and \( m_a \) are obtained from the inequalities

\[ |F_a(d)| \leq \Delta F_C(d), \quad |F'_a(d)| \leq \Delta F'_C(d), \]  

where \( \Delta F_C \) or \( \Delta F'_C \) are the total experimental errors in the Casimir force or in its gradient according to which quantity is measured.

Following this methodology, the constraints on \( g_{an} \) at different \( m_a \) were obtained from experiments on measuring the Casimir-Polder force, the gradient of the Casimir force, the Casimir pressure, the lateral Casimir force, and in Ref. 17 from the Casimir-less experiment (see Ref. 19 for a review). From measuring the Casimir interaction, we eventually reached up to several orders of magnitude stronger constraints on an axion-to-nucleon interaction constant depending on the interaction range.

It is the subject of a considerable literature that there is a puzzle in comparison between the measured Casimir force and theoretical predictions of the fundamental Lifshitz theory. It was found that for metallic test bodies the theoretical predictions are consistent with the measurement data only under a condition that the relaxation properties of free electrons are disregarded in computations (for a review see Refs. 12, 20 and more recent results 21–26). Taking into account that two theoretical predictions diverged for only a few percent, some doubts could be casted upon the validity of constraints on an axion obtained so far from the measure of agreement between the data and one of two predictions.
In this connection, the recent difference force experiment\textsuperscript{25} is of great importance because in its configuration the theoretical predictions made with disregarded and included relaxation properties of free electrons differ by up to a factor of 1000. In Ref.\textsuperscript{25} the measured quantity is the difference of Casimir forces between a Cr and Ni-coated sapphire sphere and Au and Ni sectors of the rotating disc covered by the homogeneous Ti and Au overlayers. These overlayers greatly enhance the variation in the difference of Casimir forces when the relaxation properties of free electrons are disregarded or included in computations. As a result, one theoretical approach, taking into account the relaxation properties of free electrons, was unambiguously excluded and another one, disregarding these properties, was conclusively confirmed.

Taking into consideration a distinctive role of the experiment of Ref.\textsuperscript{25}, its measurement results have been used for obtaining constraints on the axion-to-nucleon coupling constants\textsuperscript{27}. The forces $F_{a}^{\text{Ni,NI}}(d)$ and $F_{a}^{\text{Ni,AN}}(d)$ arising due to two-axion exchange between a Ni-coated sphere and covered by Ti and Au overlayers Ni and Au sectors of the rotating disc, have been calculated analytically\textsuperscript{27}. Then the constraints on $g_{an}$ for different $m_{a}$ were found from the inequality

\begin{equation}
\left| F_{a}^{\text{Ni,NI}}(d) - F_{a}^{\text{Ni,AN}}(d) \right| \leq \Delta F_{\text{diff}},
\end{equation}

where predictions of the Lifshitz theory for the difference of Casimir forces with the relaxation properties of free electrons disregarded were confirmed to within $\Delta F_{\text{diff}} = 1 \, \text{fN}$ error.

The obtained constraints are shown by the line 1 in Fig. 1. For comparison purposes line 2 in the same figure indicates the constraints following from measuring the Casimir pressure\textsuperscript{15} and line 3 the constraints found\textsuperscript{17} from the Casimir-less experiment\textsuperscript{15}. The constraints of line 4 are derived\textsuperscript{29} from the Cavendish-type experiment\textsuperscript{29}. For all lines, the regions of the plane above the line are excluded by the results of corresponding experiment and below a line are allowed. As it follows from Fig. 1 the constraints of line 1 are up to a factor of 14.6 stronger than the ones following from measuring the Casimir pressure (line 2) and, thus, stronger than all other constraints derived from measurements of the Casimir interaction\textsuperscript{19}. Only the

![Graph](image-url)  

**Fig. 1.** Constraints on the axion-to-nucleon coupling constant obtained from measuring the difference of Casimir forces (line 1), the Casimir pressure (line 2) and from the Casimir-less (line 3) and Cavendish-type (line 4) experiments are shown as functions of the axion mass (see text for further discussion).
line 3 found\(^{17}\) from the Casimir-less experiment\(^ {18}\) gives up to a factor of 2 stronger constraints. This allows to conclude that the new constraints of line 1 are in good agreement with other constraints obtained from previously performed experiments.

In fact the potentialities of experiments on measuring the Casimir interaction for obtaining stronger constraints on axion-to-nucleon coupling constants are not exhausted. All these experiments were not intended for constraining hypothetical interactions. It is shown\(^ {20}\) that minor modifications in the experimental setups on measuring the lateral and normal Casimir forces between sinusoidally corrugated surfaces and in the Casimir-less experiment would allow to further strengthen the obtained constraints over wider regions of axion masses than in original experiments. It was also suggested to use polarized test bodies\(^ {31}\).

Quite recently, an improved experiment on measuring the Casimir pressure between two parallel metallic plates at separations from 3 to 15 µm has been proposed\(^ {32}\). The suggested modifications in already existing experimental setup (the Cannex test of quantum vacuum\(^ {33}\)) allow more exact measurement of several important parameters, including the separation distance between the plates, applied voltages, frequency shift of the plate oscillation due to the Casimir pressure, vibration amplitude, characteristics of patch potentials, and relative tilt angle of the plates. It is shown\(^ {32}\) that with the proposed improvements the Cannex test will be capable to directly measure the thermal effect in the Casimir interaction and become a useful tool for investigation of the dark matter and dark energy.

3. Dark energy, cosmological constant and the quantum vacuum

As mentioned in Sec. 1, the dark energy responsible for an accelerated expansion of the Universe is well described by the cosmological constant in Einstein’s equations. In so doing, the value of dark energy density \(\varepsilon_{\text{de}} \sim 10^{-9} \text{ J/m}^3\) required to explain the observed acceleration corresponds to the cosmological constant

\[ \Lambda = 8\pi G \varepsilon_{\text{de}} \approx 2 \times 10^{-52} \text{ m}^{-2}, \tag{4} \]

where \(G\) is the gravitational constant.

It has long been noticed\(^ {23}\) that the cosmological constant can be considered as originating from the vacuum energy density of quantized fields. This, however, leads to a problem that was named the vacuum catastrophe.\(^ {5}\) The point is that if one assumes the validity of local quantum field theory up to the Planckian energy \(E_{\text{Pl}} \sim 10^{19} \text{ GeV} \sim 10^9 \text{ J}\) and makes a cutoff in the divergent vacuum energy at the Planckian momentum, the obtained energy density of order \(10^{111} \text{ J/m}^3\) exceeds the above value of \(\varepsilon_{\text{de}}\) by the factor \(10^{120}\) (see Refs. \(^{1}\),\(^ {35}\) for a review).

It was argued\(^ {36}\) however, that the renormalization procedure of quantum field theory consisting in a transition from the nonobservable (bare) to physical values of different quantities may provide a plausible explanation for the enormous difference between the values of \(\varepsilon_{\text{de}}\) and the energy density of quantum vacuum. Actually, the divergent vacuum-vacuum expectation values of the stress-energy tensor of \(P\)
bosonic fields with masses $m_1, m_2, \ldots, m_P$ and $g_1, g_2, \ldots, g_P$ degrees of freedom and $Q$ fermionic fields with masses $M_1, M_2, \ldots, M_Q$ and $h_1, h_2, \ldots, h_Q$ degrees of freedom, which are usually dropped by means of the normal ordering procedure of creation and annihilation operators, are given by

$$\langle 0 | T_{ij}(x) | 0 \rangle = \frac{1}{2(2\pi)^3} \int d^3 p \delta_{ij} \left( \sum_{l=1}^{P} \frac{g_l}{\sqrt{m_l^2 + p^2}} - \sum_{l=1}^{Q} \frac{h_l}{\sqrt{M_l^2 + p^2}} \right).$$  \tag{5}$$

Applying the method of dimensional regularization, it is easy to rewrite Eq. (5) in the space-time of $(4 + 2\epsilon)$-dimensions where $\epsilon$ is a complex number. Then, by considering $\epsilon \rightarrow 0$, one obtains

$$\langle 0 | T_{ij}(x) | 0 \rangle = I_\epsilon g_{ij},$$  \tag{6}$$

where

$$I_\epsilon = \frac{1}{64\pi^2} \left[ \sum_{l=1}^{P} \frac{g_l m_l^2}{\epsilon} \left( \frac{1}{\epsilon} - \frac{3 - 2\gamma}{2} + \ln \frac{m_l^2}{4\pi m_f^2} \right) + \sum_{l=1}^{Q} \frac{h_l M_l^2}{\epsilon} \left( \frac{1}{\epsilon} - \frac{3 - 2\gamma}{2} + \ln \frac{M_l^2}{4\pi m_f^2} \right) \right],$$  \tag{7}$$

$\gamma \approx 0.577$ is the Euler constant and $m_f$ is a fictitious mass introduced to preserve the same dimension of the stress-energy tensor as in 4-dimensional space-time.

From Eq. (6) it becomes clear that subtraction of the quantity (5) in order to make equal to zero the stress-energy tensor of the quantum vacuum in Minkowski space-time is equivalent to the renormalization of the bare cosmological constant $\Lambda^{(b)}_\epsilon$ with its physical (renormalized) value $\Lambda^{(\text{ren})}$ equal to zero. In curved space-time the value of

$$\Lambda^{(\text{ren})} = \Lambda = \Lambda^{(b)}_\epsilon + 8\pi G I_\epsilon \approx 2 \times 10^{-52} \text{ m}^{-2}$$  \tag{8}$$
is determined from the observed acceleration of the Universe expansion. As a result, only the finite value $\Lambda^{(\text{ren})}$ enters the Einstein equations after renormalization.

Within this approach, an infinitely large bare cosmological constant determined by the 00-component of the stress-energy tensor (5) and (6) should be viewed as of little physical importance. Specifically, the infinitely large energy density of the quantum vacuum should not be considered as a source of the gravitational field as well as the bare charge in quantum electrodynamics is not a source of physical Coulomb interaction. The source of physical gravitational interaction is described by $\Lambda^{(\text{ren})}$ and results in the finite energy density $\varepsilon_{\text{dc}}$. This is in close analogy to the Casimir effect where the finite (Casimir) energy density and pressure in a quantization region restricted by some material boundaries are obtained after subtraction of the infinitely large stress-energy tensor (5) defined in an unbounded Minkowski space-time. In doing so, only the resulting finite and measured in many experiments energy density is the source of gravitational interaction.
4. Conclusions and discussion

In the foregoing, we have considered some new results on constraining the parameters of axion-like particles as the most probable constituents of dark matter. It was noted that recent experiment on measuring the difference of Casimir forces not only leads to competitive constraints on axion-to-nucleon interaction, but provides stronger support to the constraints obtained from previous measurements of the Casimir interaction. One can conclude that the Casimir physics has potentialities for deriving even stronger constraints on the axion-to-nucleon interaction.

Concerning the problem of dark energy and the concept of cosmological constant, it was argued that a contradiction between the observed values and theoretical predictions of quantum field theory is probably exaggerated. Enormously or even infinitely large energy density and pressure of the quantum vacuum and related values of bare cosmological constant should not be treated as catastrophic because only the experimentally determined renormalized quantities are of physical significance like it is the case in QED and other quantum field theories of the Standard Model. The existence of vacuum condensates does not necessarily mean large energy density of the vacuum due to smallness of respective masses. In this regard, an enormously large energy density of the quantum vacuum cannot not be considered as a source of gravitational interaction keeping in mind that only the density of dark energy related to a renormalized cosmological constant should gravitate. The above argumentation treating the cosmological constant as one more fundamental constant of nature may appear somewhat incomplete in the absence of conclusive quantum theory of gravitation. It seems, however, that already developed quantum field theory in curved space-time \cite{38, 39} provides enough reason in favor of this approach.

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