Teacher images on the derivatives concept

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Abstract. This study aims to figure out the image of high school math teachers on the concept of the derivative and how they experience in learning the concepts. This study is a phenomenology research. It focuses on revealing the teachers’ perceptions and meaning of derivative concepts. Data are collected from questionnaires and interviews on four high school teachers who had taught the derivatives concept. The results show that there is a gap between teachers’ image about the concept of the derivative with the scientific conception. The experiences gained by teachers in learning the concept of derivatives were more focused on procedural understanding rather than conceptual understanding.

1. Introduction

In essence, everyone has a perception of something. Unconsciously, perception arises since a baby is born and knows the world. It related to the notion of “concept image” and “concept definition”. The term “concept image” is used to describe the total cognitive structure associated with concept formation [1]. It contains all mental pictures and there, associated properties, and processes. For example, the students’ image on the derivatives concept. Students’ experience influences the meaning of the derivatives concept that has formed in their minds. The illustration shows that an experience influences the formation of concept images. Moreover, the characteristics of the concept image are also influenced by the capacity and the background knowledge possessed by a person [2].

Like students, teachers also have a concept image in themselves. A teacher was a student. The teachers’ image also influenced by the experience, the capacity, and the knowledge possessed [2]. The learning experiences influence how conception has been formed. The long process illustrates that a concept is not acquired just one step [3]. The complex learning process can be observed through didactic situations which created in a learning. It can be observed using the didactic triangle theory (it consist of teacher-student-content) [4]. Every teachers’ actions can affect the didactic situation series created. It can affect the mental action of the student. This condition indicates that the teachers’ conception of the subject matter and the teaching process plays an important role in creating effective learning [5]. Therefore, through an effective learning is expected to create meaningful learning.

The purpose of this study was to understand the meaning and the experiences of interpretation of high school teachers on the concept of derivatives. There were two basic questions of this article, namely: 1) What is the meaning of the derivative for the teachers?; 2) How do the teachers’ experiences on the derivatives concept?. We wanted to get the meaning of something more deeply about how the
concept image of the derivative concept from teachers’ perspective after going through various experiences that form the teachers’ image on the concept of derivatives.

2. Method

This study is a phenomenology research because the focus is to describe the essence of meaning and experiences that formed in a person. Data were collected from questionnaires and interviews on four high school teachers who had taught the derivatives concept. The questionnaire contained several questions developed to explore the concept images the teachers possess and the meaning of the derivative concept (for example, look [6]). Table 1 shows the sample of the questionnaire.

| Description of the question |
|----------------------------|
| What kinds of solving methods do you use when you solve a tangent gradient at a point with \( x = 1 \) in the following functions? How do you solve it? |
| 1. \( f(x) = 2x - 4 \) | 2. \( f(x) = x^2 - 4x + 3 \) | 3. \( f(x) = |x - 1| \) |

We created the respondent categories, i.e. F1 and F2 for female subjects, while M1 and M2 for male subjects. The four respondents are alumni of mathematics for education major who had graduated on the undergraduate level. The teaching experiences as math teachers were as follows: F1 for 1 years, F2 for 2 years, M1 for 3 years, and M2 for 6 months.

3. Result and discussion

In this part, we divided the result into two sub-themes, i.e. about the teachers’ image to the concept of derivative and the experiences that teachers have when learning derivative concepts. The experience was gained by tracing how they learn derivative concepts.

3.1. The meaning of the derivative concept

The meaning of derivative from the teachers’ image on the following categories. The first category was the teachers’ image when confronted with the word “derivatives of functions”. The second category was the teachers’ image of the derivative of the function \( f(x)=2x-4 \). The third category was the teachers’ image of the derivative of the function \( f(x)=x^2-4x+3 \). The fourth category was the teachers’ image of the derivative of the function \( f(x)=|x-1| \).

The first category was the teachers’ image when confronted with the word “derivatives of functions”. From the questionnaires and interviews given, we found what respondents think when faced with the word “derivatives functions”.

| F1 | “The gradient of the tangents of a curve, definition of derivative of a function, notation of derivation of a function, the derivative of a functions’ properties, application of derivatives function, function graph, curve, limit algebraic functions, and polynomial” |
| F2 | “The gradient of tangents at a point on the curve, to determine the rise or fall of the interval of a curve, to determine the stationary point, the upper or lower concave interval, the inflection point, to help to solve the optimization problem, the acceleration rate, and the theorem l’hopital” |
| M1 | “Function. ...Yes, any function. Some have derivatives at all points on the function, but some are not”. From the definition, the derivative is the limit (already written on the answer sheet on the previous day). A function has a derivative at a point if the limit exists (the limit provision exists, there are 3 conditions). The example \( f(x)=|x| \), \( f(x) \) has no derivative at \( (0,0) \) since the right limit and the left limit is different. Furthermore, if a derivative function exists at a point then its function is continuous at that point” |
M2 : “Limit, gradient, maximum or minimum area, velocity, stationary point, determine the function of rising or fall, limit, function domain, function range, and continuity”.

The description of the teachers’ perception illustrated that the teachers viewed the derivative concept as a mathematical object. Related to this, the most teachers viewed mathematics as a mathematical object including definitions, theorems, evidence, problems, and solutions rather than as a conceptual tool used to construct such mathematical objects [7].

The second category was the teachers’ image of the derivative of the function $f(x)=2x-4$. We presented this question on the questionnaire as shown in Table 1. For the next description, we define m as the gradient of the line. Through the questionnaire, the teachers’ responses were as follows.

F1 : “The derivative of the function $f(x) = 2x - 4$ is 2. So, \( m = 2 \).”

F2 : “The gradient of the tangents at \( x = 1 \) equals of the gradient of \( f(x) \), that is \( m = 2 \).”

M1 : “$f(x) = 2x - 4$ is the equation of the line, then the gradient of the tangents at any point will be equal to the gradient \( f(x) \) i.e. 2. ….”

M2 : “$f(x) = 2x - 4$ is a straight line and the gradient on a straight line has the same gradient (So, at \( x = 1 \) the gradient is constant, that is 2). However, there is an exception if the line parallel to Y-axis, therefore it has no gradient. In this case, the line gradient $f(x) = 2x - 4$ is 2”.

On another occasion, we interviewed and asked a question, “What is the meaning of the first derivative of the function $f(x) = 2x - 4$ at the point with $x = 1$?”. F1 and F2 interpreted the gradient of the tangents of the curve at the point (1, −2) which has a slope value 2. F2 added an explanation that $2x - 4$ is a line, so its first derivative was not tangents but the gradient of the line itself. M1 defined the question as the slope of the tangents at that point and as the rate of change $f(x) = 2x - 1$ at $x = 1$. M1 also added that the tangents of line $f(x)$ at the point $x = 1$ coincided with $f(x)$ itself or is said to have no tangents, and its derivatives were constant at all points. Furthermore, M1 asserted that $f(x)$ had no tangents if any then it coincided. However, M1s’ reasoned that the definition of tangents was a line that intersects exactly one point on a curve. From the image of the concept of the tangents of line, M1 concluded that $f(x) = 2x - 4$ had no tangent. Then, M2 held that the linear derivative of a function was a constant. The linear function was a line with gradient 2. Along with its nature, the straight line had the same gradient at each point, so in $x = 1$ gradient remained the same or constant, that is 2.

These findings suggest that teachers do not fully develop the structures on the concept of tangent of line or modify existing concepts when processing new information [8]. M1s’ response illustrated a gap between the image of the concept of tangent lines embedded in the teacher with the scientific conceptions. It resembles understanding the idea of Euclides that the tangent line only cuts the curve at one point. The idea is true for a circle, but less support for the tangent line intersects more than one point [9].

The third category was the teachers’ image of the derivative of the function $f(x) = x^2 - 4x + 3$. Through the questions presented in the questionnaire, the teachers’ response was as follows.

F1 : “The derivative of the function $f'(x) = 2x - 4$. \( m = f'(1) = 2 - 4 = -2 \)”.

F2 : “$f'(x) = 2x - 4$. The tangent gradient at $x = 1$ is \( m = f'(1) = 2(1) - 4 = -2 \)”.

M1 : “$f'(x) = 2x - 4$. \( m = f'(1) = 2.1 - 4 = -2 \)”.

M2 : “$f'(x) = 2x - 4$. $f'(1) = 2(1) - 4 = -2$. Thus, the gradient of the tangent of the curve $f(x) = x^2 - 4x + 3$ at the point $x = 1$ is $-2$ ”.

The description illustrated no irregularity in the teachers’ understanding of tangents. In addition, examples like these are regular examples frequently given when learning in school.

The fourth category was the teachers’ image of the derivative of the function $f(x) = |x - 1|$ at the point with $x = 1$. The same response from three respondents stated that the tangential gradient at the point with $x = 1$ is 0. Here (Figure 1 and Figure 2), we showed examples of F2s’ response and M2s’ response when solving problem 3.
Figure 1 shows that there is a difference in concept images embedded in teachers’ minds about the definition of the gradient of the tangents. This is because teachers understand the concept partially. They do not notice that the gradient of the tangent is related to the concept of the derivative. In contrast to the response given by M1 that the derivative at the point with $x = 1$ does not exist because the value of the left and right limits is different. Therefore, $f'(1)$ does not exist. Through the description presented, we summarize the findings of the difference between teachers’ image and scientific conception in Table 2.

Table 2. The difference between the teachers’ image and scientific conception.

| No | Meaning of derivative concept according to the teacher |
|----|-------------------------------------------------------|
| 1  | The curve $f(x) = 2x - 4$ has no tangent              |
| 2  | The curve $f(x) = |x - 1|$ has the value $f'(1) = 0$    |
| 3  | The tangent line $f(x) = |x - 1|$ at $x = 1$ is the x-axis |

3.2. Experience of the derivative concept

Through the questionnaire, it was found that three out of four respondents learned the concept of derivatives using the usual package of books circulating in schools. While the respondent M1 studied the concept of derivative referring to the calculus book. We then browsed how the presentation of the material displayed on the textbook.

We analysis of one of the textbooks used by teachers. The textbook used was a book that refers to the curriculum 2013. Presentation of textbooks began by providing: 1) Concept definition; 2) Examples of the problems; and 3) Exercises. The order of presentation when examined from the perspective of the
didactic situation theory shows the loss of an important process of action in constructing easily understood knowledge in a person [10]. This is because the definition of derivative concept was given instantly without inviting the reader to feel first-hand how the concept construction process begins.

In the example of the problem, the most serving displays instances of instances derivative from quadratic functions. Based on the definition, that the derivative is the slope of the tangent to the curve at a point on the curve. When this definition is applied to a quadratic function, a tangent is obtained that cuts the curve at a certain point along the curve. The less varied presentation of the examples leads to a partial understanding of ones’ mind on the concept of derivative.

As the first example, the discussion of the teachers’ understanding regarding the first derivative of the function \( f(x) = 2x - 4 \). We express the meaning of the tangent line \( f(x) \) at the point with \( x = 1 \) according to the respondent. M1 states that no tangent of the line can be made through \( f(x) = 2x - 4 \). This was certainly different from the fact that the tangent line \( f(x) = 2x - 4 \) was another line coinciding with the \( f(x) \). The difference of the concept image with the formal definition of the concept was caused by not embedding the concept well in M1’s mind. He believed a tangent of the line as a line that intersects exactly one point on a curve. So, he concluded that \( f(x) = 2x - 4 \) did not have a tangent of line because it was contrary to his image. The second example, the teacher was confronted with a function whose curves were not smooth, like the absolute value function \( f(x) = |x - 1| \). The response was the tangent line \( f(x) = |x - 1| \) at point with \( x = 1 \) was the x-axis and had a gradient \( m = 0 \). Those illustration shows that the embedded concept differed from the formal definition of the concept, where the function \( f(x) = |x - 1| \) had no derivative at \( x = 1 \) since the value of the left derivatives \( (f'(\text{c})) = \lim_{x \to \text{c}^+} \frac{f(x)-f(c)}{x-c} \) and right derivatives \( (f'(\text{c})) = \lim_{x \to \text{c}^-} \frac{f(x)-f(c)}{x-c} \) was different.

In the example of the exercise, it was found that the problems presented tend to direct the readers to procedural understanding. One example of the exercise question is shown in the task on the textbook (See Figure 3 below).

![Carilah gradien garis tangen dan garis normal pada kurva dengan persamaan \( y = \frac{2}{x-2} \) di titik (0, -1).](image)

**Figure 3.** Example task on the textbook

Indirectly, Figure 3 shows that student activity is limited to solving mathematical problems such as examples of problems given. It would be considered complicated. It refers to the difficulty to imagine the given situation logically. It tends to direct someone to become a mathematician by compelling. It certainly affects someone who does not like math. Consequently, it is not uncommon to find a trained student to copy the procedure on the exercise in the textbook without correctly interpreting what he was learning [11].

4. Conclusion

The results of the study show that there is a gap between the teachers’ image on the derivatives concept and the scientific conception. The teachers’ experience on the derivatives concept is more focused on procedural understanding rather than conceptual understanding [12]. Teachers have not developed their conceptual understanding of a condition when the function has no derivatives. For example, they did not use the concept of left derivatives and right derivatives when looking for a gradient of tangents \( f(x) = |x - 1| \). It shows that there are different the concept image with the formal definition. Nevertheless, due to the limited time and opportunity, this study is still lacking in many things. It may cause many questions who may take it as the source of the problem for further research. As for some questions asked for ourselves that might be a reflection. Questions include: 1) Does the teachers’ conception of the derivative concept influences the learning they create in the classroom?; 2) How is the
teachers’ pedagogical content knowledge about the derivative concept?; 3) How does the teacher introduce the derivative concept to students?

Acknowledgments
The first author would like to thank educational practitioners including teachers who have already to be respondents thesis supervisors who always provide support, and the environment of the Departemen Pendidikan Matematika, Sekolah Pascasarjana, Universitas Pendidikan Indonesia.

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