Performance Comparison of Function Evaluation Methods

Leo Liberti*

February 1, 2008

Abstract

We perform a comparison of the performance and efficiency of four different function evaluation methods: black-box functions, binary trees, $n$-ary trees and string parsing. The test consists in evaluating 8 different functions of two variables $x, y$ over 5000 floating point values of the pair $(x, y)$. The outcome of the test indicates that the $n$-ary tree representation of algebraic expressions is the fastest method, closely followed by black-box function method, then by binary trees and lastly by string parsing.

Keywords: function evaluation, tree, algebraic expression, parser

Important warning. There is a mistake in the test code that invalidates the most important result of this paper, i.e. that $n$-ary tree based function evaluation is faster than black-box function evaluation. It is not true: it is slower by about an order of magnitude. However it is true that $n$-ary tree based evaluation is faster than the other methods discussed in this paper.

1 Introduction

In this article we describe a test designed to measure the comparative efficiency of four different function evaluation methods (see section 2 for details):

- black-box functions;
- binary tree representation;
- $n$-ary tree representation;
- string parsing.

Because of the huge number of parameters involved in such a test (efficiency of compiler, quality of test source code, type of hardware, type of test functions, number of variables, object code optimization level, and so on) it is evident that this test can be neither definitive nor unchallengeable. However, the results indicate a clear winner in the $n$-ary tree representation, closely followed by black-box functions and binary tree representation. Last (expectedly) comes string parsing. It is somewhat of a surprise to discover that $n$-ary tree representation gathers better results than the precompiled black-box functions method. This finding is discussed below (section 3).

*Centre for Process Systems Engineering, Imperial College of Science, Technology and Medicine, London, UK. E-mail: l.liberti@ic.ac.uk.
Existing literature in this topic is scarce or non-existent. Some of the early work focused on how to handle floating point computation efficiently, rather than on the actual method used for the evaluation\cite{Ash64}. In other instances, the evaluation of certain classes of functions (e.g. polynomials, see\cite{Fik67}) was investigated.

Most people tend to use the black-box functions method by exploiting the compiler’s capabilities in this sense. Further efforts for better evaluation methods are usually sought after only in connection to very specific functions and problems, e.g. in astronomy\cite{Sch00}, in number theory\cite{Odl90}, or when using discrete/boolean functions\cite{MMS95}.

A novel evaluation method, based on threaded binary trees, was proposed in\cite{KP97}; this method partially eliminates the cost of recursion by “threading” the binary expression tree before the evaluation. Because the operation of threading the binary tree is recursive in nature, the CPU time savings are only possible if the same tree is evaluated many times (as is the case in most engineering applications). However, the benefits of this approach should decrease with the use of $n$-ary trees, as threading a list of like operators in an $n$-ary tree has no effect.

The test consists in evaluating eight different functions of one and two variables $x, y$ over 5000 randomly generated pairs of values for $(x, y)$, all in the interval $[0, 1]$. The functions are:

1. $x$;
2. $x + y$;
3. $x^y$;
4. $(x + y)x^y$;
5. $\sin(x)$;
6. $\sin((x + y)x^y)$;
7. $x + y + 1$;
8. $2xy(x + y + 1)$.

The above functions have been chosen as a representative set for unary and binary operators, in the sense that both binary operators (sum, power) and unary operators (the sine function) are present. Little does it matter for the outcome of the test that not all operator types have been employed, for the time taken to carry out floating point computation would have been exactly the same in all cases.

The number of variables has been limited to two for simplicity. However, for reasons which will become apparent below (see section 2.3 about the description of the $n$-ary tree representation), adding more variables to the expressions would only have served to overemphasize the outcome of the tests, especially in the case where long sequences of like operands are employed (e.g. linear equations, or products of the kind $x_1x_2x_3\cdots x_n$).

The test code has been written in pure C in order to minimize the effect of compiler overhead, and compiled with the GNU C Compiler version 2.95.2. All optimization options have been tested ($-O$, $-O2$, $-O3$) as well as “debug” mode and “normal” (no flags) mode. In all cases the test results have been consistent with the order: $n$-ary trees, black-box functions, binary trees, string parsing.

The test has been run on a Pentium-III 450MHz with 192MB RAM with the Linux operating system. All results have been obtained by running the executables in single-user mode and by flushing RAM caches after each run. Enabling the caches and running in multi-user modes gathers
similar results but occasionally the black-box functions wins out (by very little indeed) over the n-ary tree method. However, situations where the black-box functions method wins over the other methods cannot be replicated; they depend very much on the behaviour of the caching code and on the general load of the machine at any given moment. In any case, this kind of outcome only occurs when repeatedly running the same executable over and over again as different processes, not when calling the same function many times within the same process.

Section 2 describes the four types of evaluation methods tested. Section 3 describes the implementation of the methods. Section 4 discusses the results of the test. The code used to run the test can be downloaded, inspected and reused under the GNU public license from

http://liberti.dhs.org/liberti/evaltest

2 Evaluation Methods

In this section we shall carry out a theoretical analysis of the evaluation methods tested in this article.

2.1 Black-box Functions

This method for function evaluation is by far the easiest to implement and the most commonly used within the scientific community, especially where test code has to be rigged up or once-only jobs need to be run. It basically lets the compiler do the work of parsing the expression into a binary tree which is hardwired in the object code at compile time. Evaluations are supposed to be very fast (mainly because most of the work is carried out once only at compile time); its main drawback is that changes to the function formulation entail recompilation of the source code, which for most pieces of software is not an acceptable solution.

The programming paradigm for black-box functions follows the lines of the pseudocode below:

```plaintext
main:
  float x, y, f;
  f = blackbox(x, y);
end

function blackbox(float x, float y):
  float z;
  z = sin((x + y)*x^y);
  return z;
end function
```

The reason why this method is called “black-box functions” is that from the main procedure point of view, the function is really a black box, in the sense that apart from knowing what argument it requires, there is no run-time control over it.

2.2 Binary Trees

This representation is based on the idea that operators, variables and constants are nodes of a digraph; binary operators have two outcoming edges and unary operators have only one; leaf nodes
have no outcoming edges (for graph-related terminology and definitions, see [Har71], [KV00]). For example, the function $x + y + 1$ would be represented as in fig. [1]. Where unary operators are employed, a dummy second operand is often used.

This type of function representation is the most commonly used where there is a need for some degree of run-time control over the definition of the mathematical function being represented. However, it should be noted that performing algebraic operations on this representation is not overly simple. Most software that does not do symbolic manipulation of algebraic expressions employs this kind of representation. Furthermore, most general-purpose compilers (including the GNU C compiler) use this representation too.

2.3 $n$-ary Trees

This technique is a combination of binary trees and lists. Operators can have any number of operands. This allows for much more efficient handling of sequences of like operands, e.g. in linear expressions or long products ($x_1x_2\cdots x_n$). For example, the function $x + y + 1$ would be represented as in fig. [2].

![Figure 1: Binary tree representation for $x + y + 1$.](image1)

![Figure 2: $n$-ary tree representation for $x + y + 1$.](image2)

This type of function representation is often employed when symbolic manipulation is required. The data structures used by languages like Prolog and especially LISP are very similar in concept to $n$-ary trees.
2.4 String Parsing

String parsing is a process by which a string containing an algebraic expression is evaluated directly without middle steps like tree representation. String parsers usually consist of a lexical analyser, which returns tokens (operators), symbols and constants, and a grammatical interpreter which drives the lexical analyser on the given string. It then performs the mathematical operations signalled by the tokens on the operands. When a symbol is returned, it is looked up on a symbol table to discover its numeric value. Because of the design complexity, it is to be expected that string parsing is slower than other methods. It is mainly employed where the parsing is to be carried out once only (possibly as a pre-processing step to some main algorithm).

By changing the grammatical interpreter, a string parser can be used to build binary or \( n \)-ary trees for algebraic expressions input as strings. This is how compilers transform source code (i.e. strings) into object code.

3 Implementation in the C Language

The implementation of the techniques described in section 2 above has been carried out in C in order to minimize the amount of compiler-generated overhead code, as the C language has very low requirements in this respect. Furthermore, no external library has been used as it would have invalidated the timing tests. Instead, all the code necessary to the test has been written from scratch.

The part of the test concerning black-box functions was the easiest to code. No particular coding technique was employed. Functions returning results of the test expressions were compiled into the executable and called from the main routine.

Tree handling, in both the binary and \( n \)-ary forms, required more work. A tree, for the purposes of this test, is defined as follows.

```c
struct tree {
    int optype; // operator type
    long varindex; // variable index if node is a variable
    double value; // value if node is a constant
    struct tree** nodes; // subnodes if node is not a leaf node
    long nodesize; // number of subnodes
};
```

The above definition is generic enough to be able to accommodate both binary and \( n \)-ary trees. For binary trees, `nodesize` is always set to 2. No “string to tree” parser has been included as the test code did not need that kind of generality; all the function trees (both binary and \( n \)-ary) have been manually coded in.

The string parser has been derived from the ideas given in [Str99]. The parser code given therein has been modified to support exponentiation and unary functions in the form \( f(x) \).

3.1 Code Validation

The validity of the code has been verified along with the test proper. All results from evaluations with the four methods described above coincide (up to at least three significant digits) for each of
Table 1: Test results. Values are expressed in seconds of “user” CPU time (time spent on system calls was 0.00s in all cases).

4 Results

As has been mentioned in the introduction, the test consists in evaluating the eight expressions above (see page 2) over 5000 randomly generated pair values for \((x, y)\) (all in the interval \([0, 1]\)) with each of the four described evaluation methods, trying all the possible compiler code optimization flags. The test is carried out in a single-tasking environment where memory cache has little or no effect. The results of the test are reported in table 1.

These results are surprising because normally we would expect black-box functions to be the most efficient evaluation method, whereas the actual “test winner” is the \(n\)-ary tree representation (although, as has been noted in the introduction, this test is far from definitive — the parameters that can affect performance are too many to be controlled all at once). This result is strengthened by the consideration that black-box functions are so easy to program that the test cannot be invalidated because of “programming errors” or inefficient coding. On the other hand, it may be true that with more careful coding, the results referring to tree evaluation could be made even better.

4.1 \(n\)-ary Trees: the Best Evaluation Method

In order to explain this result, one has to consider the similarities between black-box functions and binary trees. Although from the programmer’s point of view the two methods are far from similar, the resulting machine code need not be all that different. As has been explained earlier, most compilers work in such a way as to hard-code binary trees representing the expressions within the object code. This binary tree structure may be hidden behind a simpler logic flow than that generated by the binary tree method, but the operations are carried out much in the same order in both methods. Thus, it comes to no surprise that the binary tree method gathers results which are worse, but not by much, than those of black box functions. The two methods are very similar, but in the black-box function case the code is created directly by the compiler and can be better optimized.

The \(n\)-ary tree method performs in the same way as the binary tree method, except where sequences of like operands with length greater than 2 appear in the expressions (e.g. in linear expressions with at least 3 nonzero coefficients), where it performs much faster. The evaluation algorithm is as follows:

```c
double eval(struct tree* expression, double varvalues) {
```
int i;
double ret;
switch(expression->optype) {
    case CONSTANT:
        return expression->value; // leaf node
    case VARIABLE:
        return varvalues[expression->varindex]; // leaf node
    case SUM:
        ret = 0;
        for (i = 0; i < expression->nodesize; i++) {
            ret += eval(expression->nodes[i], varvalues); // recursive call
        }
        return ret;
    case PRODUCT:
        ret = 1;
        for (i = 0; i < expression->nodesize; i++) {
            ret *= eval(expression->nodes[i], varvalues); // recursive call
        }
        return ret;
    // ... all other cases
}

Consider the cases depicted in fig. 1 and 2. In the first case, where the nodesize is always 2, a recursive function call has to be performed for each of the two ‘+’ operators; in the second case, however, only one recursive function call needs to be carried out (for the only ‘+’ operator).

This explains the advantage of using n-ary trees in evaluation of algebraic expressions. The computational cost of generating a recursive function call is high for most compilers. It becomes evident that the longer “like operand sequences” are, the better this evaluation method becomes.

4.2 Results on String Parsing

String parsing, although definitely the worst method, was also somewhat surprising in that it was not as bad as a superficial analysis would suggest. After all the code complexity of a lexical analyser and a grammatical interpreter is far greater than the other evaluation algorithms presented above. However, especially when memory caching was allowed, the timings of the string parsing method got better and better. The best result we obtained was close to 1.00s; so even though it still worse than the other methods, it was the technique that benefited most from caching (in different processes, however, not in the same process). However, the same test carried out using the popular UNIX utility bc (in most implementations based around the lex and yacc compiler tools) gathered an appalling result of over 26s of user CPU time, notwithstanding the fact that Stroustrup’s adapted parser is highly recursive in nature whereas lex and yacc usually generate non-recursive (and hence theoretically faster) code.

5 Conclusion

In this paper we have analysed the performance of four common evaluation methods: black-box functions, binary trees, n-ary trees and string parsing. The result of the test indicates that the n-ary tree expression representation is the best function evaluation method.
References

[Ash64] R.L. Ashenhurst. Function evaluation in unnormalized arithmetic. *Journal of the ACM*, 11(2):168–187, April 1964.

[Fik67] C.T. Fike. Methods of evaluating polynomial approximations in function evaluation routines. *Communications of the ACM*, 10(3):175–178, March 1967.

[Har71] Frank Harary. *Graph Theory*. Addison-Wesley, Reading, MA, 2nd edition, 1971.

[KP97] B.R. Keeping and Constantinos C. Pantelides. Novel methods for the efficient evaluation of stored mathematical expressions on scalar and vector computers. *AIChE Annual Meeting*, Paper #204b, nov 1997.

[KV00] Bernhard Korte and Jens Vygen. *Combinatorial Optimization, Theory and Algorithms*. Springer Verlag, Berlin, 2000.

[MMS+95] Patrick C. McGeer, Kenneth L. McMillan, Alexander Saldanha, Alberto L. Sangiovanni-Vincentelli, and Patrick Scaglia. Fast discrete function evaluation using decision diagrams. In *Proceedings of the Conference on Computer Aided Design*, pages 402–407, San Jose, CA, 1995. IEEE/ACM.

[Odl90] A.M. Odlyzko. Primes, quantum chaos, and computers. In *Number Theory*, pages 35–46. National Research Council, 1990.

[Sch00] Wayne Schlitt. The xstar n-body solver: Theory of operation. [http://www.midwestcs.com/xstar/n-body](http://www.midwestcs.com/xstar/n-body), February 2000.

[Str99] Bjarne Stroustrup. *The C++ Programming Language*. Addison-Wesley, Reading, MA, third edition, 1999.