Resource Optimization with Flexible Numerology and Frame Structure for Heterogeneous Services

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Abstract—We explore the potential of optimizing resource allocation with flexible numerology in frequency domain and variable frame structure in time domain, in presence of services with different types of requirements. We analyze the computational complexity and propose a scalable optimization algorithm based on searching in both the primal space and dual space that are complementary to each other. Numerical results show significant advantages of adopting flexibility in both time and frequency domains for capacity enhancement and meeting the requirements of mission critical services.

I. INTRODUCTION

The fifth generation (5G) of wireless communications systems is required to support a large variety of services, given the fundamental trade-offs between requirements of these services \cite{1}. A promising solution for higher resource efficiency while providing lower latency is the scalable transmission time intervals (TTIs) \cite{2}–\cite{7}. These works fall within the general notion of flexible resource allocation in the time-frequency domain, leading to new types of optimization problems. Optimization along the frequency dimension yields similar structures to problems such as multi-dimensional K\textsc{Napsack} or weighted MATCHING \cite{8}–\cite{10}. Resource optimization adopting flexibility in both dimensions regarding frequency and time, named 2-dimensional (2-D) resource allocation, poses new challenges \cite{11}, \cite{12}. This transition calls for new ideas to be proposed for algorithm design, as well as analysis to be done for tractability. Although flexible resource allocation along both the time and frequency dimensions is not entirely new \cite{11}, \cite{12}, frequency selective resource optimization with flexible sizes of resource units in the frequency and time two-dimensional space in 5G, has not yet been addressed to the best of our knowledge.

Based on 3GPP release for scalable numerologies and frame structures \cite{13}, we consider the resulting 2-D resource allocation problem. We address tractability and propose an algorithm with scalability. Our results suggest that considering the primal space and dual space are complementary to each other for problem solving. We show numerically that a flexible numerology and frame structure significantly outperforms the conventional scheme in both throughput enhancement and meeting mission critical services. For convenience, we use the term “flexible structure” to refer to the architecture in which both numerology and TTI are flexible. The traditional resource allocation in LTE which employs a fixed size of resource units, is referred to as “non-flexible structure”.

II. SYSTEM MODEL

Consider a base station and two categories of services. The first category, denoted by $K_{(f)}$, has strict latency requirement. For any service $k \in K_{(f)}$, denote its data demand (in bit) by $q_k$. The second category of services is denoted by $K_{(c)}$, for which the target is to maximize the throughput. We define $K = K_{(f)} \cup K_{(c)}$ the set of all services.

The resource in time-frequency domain consists of minimum resource units (MRUs). An MRU is characterized by its numerology and frame structure. The former is defined by sub-carrier spacing (SCS) and cyclic prefix (CP) length (a.k.a. the “guard interval” between the symbols), while the latter includes TTI duration. One resource allocation to a service involves a set of adjacent SCSs and TTIs in the frequency and time domain respectively with a configured CP length. For simplicity, hereafter we refer to the resource configuration of numerology and frame structure as blocks, and consider a candidate set $B$ of blocks, each consisting of multiple MRUs, see Figure 1. For each $b \in B$, the achieved throughput on block $b$ if $b$ is assigned to service $k$ ($k \in K$) is denoted by $r_{b,k}$.

Comparing to LTE that applies a fixed SCS of 15 kHz and TTI of 0.5 ms, 5G/new radio (NR) \cite{14} permits a large number of block shapes with numerology varying from 15 kHz to 480 kHz, and TTI varying from 0.125 ms to 0.5 ms. Here, we consider the four most widely accepted block shapes, see Table I. Note that Shape 4 supports extended CP length to trade off overhead against delay spread.

Given the channel profile, the transmission power, and the noise power, the throughput $r_{b,k}$ depends on the configuration of block $b$, including the time span and frequency range (characterized by SCS and TTI duration), CP length, and

![Figure 1](image-url)
symbol duration. We assume a total number of nine multipath channel profiles [13] Table B.2.1-4], and we predefine the mapping from the configuration parameters to the throughput based on the model in [15]. Due to limited space, and the fact that capacity model is not in the scope of this paper, we omit the details but provide the tutorial and source code in a public repository on GitHub [16].

III. PROBLEM FORMULATION AND TRACTABILITY

Consider the problem of maximizing the total throughput for $K^{(c)}$, subject to latency and the demand constraints of $K^{(c)}$. Denoting by $I$ the set of MRUs, we use a binary parameter $a_{b,i}$ to map an MRUs $i$ to a block $b$, namely, $a_{b,i} = 1$ if and only if the block $b$ includes MRU $i$. We use the variable $x_{b,k} \in \{1, 0\}$ to indicate whether block $b (b \in B)$ is assigned to the service $k (k \in K)$. A block $b$ is infeasible for $k (k \in K^{(c)})$, if the ending time of $b$ exceeds $\tau_k$. This is modeled by setting $r_{b,k} = 0$. The problem is formulated as follows, where the two sets of constraints impose the bits demand requirement for $K^{(c)}$ and state non-overlapping allocation of blocks, respectively.

\[\text{[P0]} \quad \max \sum_{b \in B} \sum_{k \in K^{(c)}} r_{b,k} x_{b,k} \quad (1a)\]
\[\text{s.t.} \quad \sum_{k \in K^{(c)}} r_{b,k} x_{b,k} \geq q_k, \quad k \in K^{(c)} \quad (1b)\]
\[\sum_{k \in K^{(c)}} a_{b,i} x_{b,k} \leq 1, \quad i \in I \quad (1c)\]
\[x_{b,k} \in \{0, 1\}, \quad b \in B, \quad k \in K \quad (1d)\]

\textbf{Theorem 1:} P0 is $\mathcal{NP}$-hard.

\textbf{Proof:} We construct a polynomial-time reduction from the PARTITION PROBLEM (PP) for a set of integers $\{d_1, \ldots, d_n\}$. The task is to determine whether or not there is a partition such that the two subsets have equal sum of $\sum_{i=1}^{n} d_i/2$, where the numerator is assumed to be even. We define a single TTI size and multiple blocks that take the shape of one MRUs. There are two services, denoted by $k^t$ and $k^c$, in $K^{(t)}$ and $K^{(c)}$, respectively. The latency parameter of $k^t$ equals that of the TTI size, and the demand equals $\sum_{i=1}^{n} d_i/2$. Moreover, $r_{b,1} = r_{b,2} = d_b, b = 1, \ldots, n$. By construction, (1c) has no effect. Next, one can observe that partitioning the $n$ MRUs into two subsets, each providing a total throughput of $\sum_{i=1}^{n} d_i/2$, is equivalent to a feasible solution to PP. In addition, this can occur if and only if the objective function defined for $k^c$ reaches $\sum_{i=1}^{n} d_i/2$. Hence the conclusion.

IV. PROBLEM SOLVING

Our solution approach consists in performing assignment of blocks to services, based on utility values generated from linear programming (LP) relaxation and the Lagrangian dual (LD). For each candidate block-service assignment, the utility value is expected to reflect its priority of being considered in the resource allocation solution. We first discuss the assignment of blocks to services. Then, we discuss the derivation of the utility values by using LP and LD in Sections IV-B and IV-C, respectively. The overall algorithm selects the best output from the two methods.

A. Block Assignment

We denote by matrix $u$ of size $|B| \times |K|$ the utility matrix for all pairs of blocks and services. Namely, an element $u_{b,k}$ represents the utility of a block-service pair $(b, k)$ ($b \in B$ and $k \in K$). Note that $u$ is of the same dimensions as $x$. Block assignments for $K^{(t)}$ and $K^{(c)}$ are treated separately. The former is performed first because of the latency requirement.

Consider the permutation of all elements in $K^{(c)}$. For a given permutation, the blocks are allocated to the services sequentially according to the permutation. For each service $k \in K^{(t)}$ under consideration, the block $b$ with the highest value of $u_{b,k}$ is allocated. Next, all blocks sharing any MRU with $b$ are discarded. This is repeated for service $k$ until its demand is met. The procedure then repeats for the remaining element of $K^{(c)}$.

In next step, resource allocation is carried out for $K^{(c)}$. Here, two strategies are considered. Similar to the resource allocation for $K^{(t)}$ shown above, the non-overlapping constraint is accounted for both the strategies in $K^{(c)}$. In the first strategy, given a permutation of $K^{(c)}$, blocks are sequentially allocated to each service according to the utilities. A block is allocated to a service only if the corresponding utility is larger than a threshold $\rho$. After completing checking all blocks for the first service in the permutation, we move to the next service, and so on. When this procedure completes for all services in $K^{(c)}$, there may still be blocks not being allocated yet. Each of these blocks is then allocated to the service with highest possible utility. In the second strategy, we generate a permutation of $B$. According to the permutation, each block is allocated to the service with the highest utility. These two strategies will be used by the LP and LD based solution methods, respectively.

Monte Carlo sampling is used for generating multiple permutations following a uniform distribution. The number of generated permutations is denoted by $N$. The best block-service assignment of all the permutations is selected.

B. Utility Estimation by LP Relaxation

A straightforward way to compute the utility matrix $u$ is to solve the LP relaxation of P0 and to use the LP optimum to set the utility values.

\[\text{[P0-LP]} \quad \max \sum_{b \in B} \sum_{k \in K^{(c)}} r_{b,k} x_{b,k} \quad (2a)\]
\[\text{s.t.} \quad (1b), (1c), \quad 0 \leq x_{b,k} \leq 1, \quad b \in B, \quad k \in K^{(c)} \quad (2b)\]
C. Utility Estimation by LD

By relaxing the constraints (1c) of P0 with Lagrangian multiplier $\lambda_i$ ($i \in I$), the Lagrangian is defined as follows:

$$L(x, \lambda) = \sum_{b \in B} \sum_{k \in K^{(c)}} r_{b,k} x_{b,k} + \sum_{i \in I} \lambda_i (1 - \sum_{b \in B} \sum_{k \in K} a_{b,i} x_{b,k})$$

The LD function is defined in (3).

$$[P1] \quad g(\lambda) = \max_x L(x, \lambda) \text{ s.t. } (1b) \text{ and } (1d).$$

Accordingly, we have the LD problem:

$$[P0-LD] \quad \min \lambda \geq 0 g(\lambda).$$

We define $\alpha_b = \sum_{i \in I} \lambda_i a_{b,i}$. Problem P1 decomposes for $K^{(c)}$ and $K^{(f)}$: The constraints (1b) are only for $K^{(f)}$. Therefore, solving P1 amounts to solving the two problems P2 and P3 for $K^{(c)}$ and $K^{(f)}$ respectively, shown below.

$$[P2] \quad \max_{x} \sum_{b \in B} \sum_{k \in K^{(c)}} (r_{b,k} - \alpha_b) x_{b,k} \quad \text{s.t.} \quad \sum_{k \in K^{(c)}} x_{b,k} \leq 1, \quad b \in B$$

$$x_{b,k} \in \{0, 1\}, \quad b \in B, \quad k \in K^{(c)}$$

$$[P3] \quad \min_{x} \sum_{b \in B} \sum_{k \in K^{(f)}} \alpha_b x_{b,k} \quad \text{s.t.} \quad \sum_{b \in B} r_{b,k} x_{b,k} \geq q_k, \quad k \in K^{(f)}$$

$$x_{b,k} \in \{0, 1\}, \quad b \in B, \quad k \in K^{(f)}$$

Note constraints (5b) are not present in P0, though these are implied by (1c) for services in $K^{(c)}$. The effect of (5b) is to partially restore the fact that (1c) is relaxed by Lagrangian multiplier, thus achieving the strengthening effect without affecting the complexity of P2.

Computing the optimum of P2 is straightforward. Each block $b$ is allocated to the service $\arg \max_k r_{b,k} - \alpha_b$ with $r_{b,k} - \alpha_b > 0$.

Problem P3 further decomposes to $|K^{(f)}|$ problems P3[k] ($k \in K^{(f)}$), each with objective max $\sum_{b \in B} \alpha_b x_{b,k}$, constraints $\sum_{b \in B} r_{b,k} x_{b,k} \geq q_k$, and binary variables $x_{b,k}$ ($b \in B$). Since there is only one service considered in each of these problems, we omit the index $k$ in the formulation of P3[k] ($k \in K^{(f)}$).

$$[P3[k]] \quad \min_{x} \sum_{b \in B} \alpha_b x_{b,k} \quad \text{s.t.} \quad \sum_{b \in B} r_{b,k} x_{b,k} \geq q_k$$

$$x_{b,k} \in \{0, 1\}, \quad b \in B$$

Each P3[k] can be reformulated as a KNAPSACK PROBLEM, by replacing every $x_{b,k}$ with $1 - \kappa_{b,k}$ ($\kappa_{b,k} \in \{0, 1\}$). Since $r_{b,k}$ and $q_k$ are integers (as they denote the amount of bits), the problem can be optimally solved by dynamic programming.

Consider the P0-LD. The dual problem P0-LD can be solved using a sub-gradient method (17). Denote by $M$ the number of iterations in the sub-gradient method (one can also use convergence check as a candidate stopping rule). The utilities matrix $u$ is set as follows. For $K^{(f)}$, the optimal solutions to P3 in all iterations are averaged. The averaged values are used as utilities for corresponding block-service pairs. We remark that constraints (5d) impose that each block is assigned to up to one service. Therefore, for $k \in K^{(c)}$, $r_{b,k} - \alpha_b$ reflects the utility of block $b$ to service $k$. The normalized value of $r_{b,k} - \alpha_b$ (ranging from 0 to 1) at the final sub-gradient iteration is then used as utility for $(b, k)$.

V. NUMERICAL RESULTS

This section numerically shows the benefit of using flexible structure, compared against the non-flexible one. In addition, we show the deviation of the solution found by our algorithm from the global optimality. The advantage of combining LP with LD for problem solving is illustrated as well. The demands and latency tolerance (deadlines) of services in $K^{(f)}$ are set to be uniform, denoted by $q$ and $\tau$, respectively. Parameter settings are given in Table II.

| Parameter | Value |
|-----------|-------|
| Number of users | $|K^{(f)}| = |K^{(c)}| = 5$ |
| Time-frequency domain | 2 ms and 2 MHz |
| SNR range | [5, 30] (dB) |
| Demand $q$ | [10, 32, 64, 128, 256, 512] (kbps) |
| Latency tolerance $\tau$ | [0.125, 0.25, 0.5, 1.0, 1.5, 2] (ms) |
| Sub-gradient iterations | $M = 200$ |
| Generated permutations | $N = 400$ |
| Threshold (Section IV-A) | $\rho = 0.5$ |

Figure 2 compares the flexible structure and the non-flexible structure in terms of the throughput performance. It shows the average bit rate of services in $K^{(c)}$ depending on the latency tolerance $\tau$ of $K^{(f)}$. For the non-flexible structures, Shape 1, Shape 2, and Shape 3 in Table I are used separately. Each of these structures is referred to in the format of “TTI-SCS” (e.g. 0.25ms-30kHz means a shape of a fixed TTI of 0.25 ms and a fixed SCS of 30 kHz). We use the global optimum obtained by solving the integer programming problem (1) as the baseline for the flexible structure.

The flexible structure significantly outperforms the non-flexible ones. When the latency tolerance $\tau$ is lower than 1 ms, the bit rate achieved by our proposed algorithm is at least 4 times as high as the non-flexible cases. Via flexible structure, the system tends to benefit more on throughput when the latency tolerance becomes more stringent. On average, our block assignment algorithm with flexible structure increases the throughput by 170%.

Among the three non-flexible schemes, 0.25ms-30kHz outperforms the other two. Interestingly, in the case of short latency tolerance, the block with long TTI and narrow bandwidth (i.e. 0.5ms-15kHz) performs better than the one with shorter TTI and wider bandwidth (i.e. 0.125ms-60kHz). This is because the block 0.125ms-60kHz has a large span in frequency dimension, such that the flexibility in spectrum resources usage is significantly impacted. This inefficiency in
spectrum utilization can be hidden when the latency tolerance becomes relatively longer, allowing higher flexibility of the resource allocation in time domain. The results suggest that employing a flexible structure may enhance the throughput dramatically by combining the use of various block shapes. Though not shown by the figures here, we remark that the problem feasibility of the three non-flexible schemes is very sensitive to the latency tolerance. This issue is alleviated by the flexible structure.

Figure 3 shows the optimality gaps of the algorithms proposed in the paper, as functions of the demand of $K^{(c)}$. We compare three cases: using only LP, using only LD, and combining both together. The three cases are named by “LP-Only”, “LD-Only”, and “Combined”, respectively. One can observe that the optimality gap increases with the user demand. Meanwhile, searching in the dual space for computing block-service utilities in most cases leads to significantly better results than considering the LP relaxation. With high user demand, solving the LP relaxation is clearly inferior in terms of utility estimation. On the other hand, the accuracy is not affected too much by the demand in LD-Only. We remark that this happens in low demand cases that the LP relaxation leads to better evaluation of block-pair utilities than LD. Thus, these two are complementary to each other. Overall, the gap is below 20% for our proposed algorithm.

VI. CONCLUSION

The paper concludes that combining a flexible numerology and frame structure in resource allocation serves as a promising option for increasing spectral efficiency. Both mission critical service scheduling and capacity enhancement benefit from flexible structure. The paper suggests that, though resource allocation under such a flexible structure exhibits computational hardness theoretically, searching in primal space as well as dual space enables efficient algorithms for problem solving: LP and LD are complementary together for improving the objective value towards optimality.

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