Progressive Failure Analysis of Tianshansi Slope

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Abstract. The physical significance of the traditional slice block method analysis of a slope is presented in this paper. The strength-softening characteristics of the geomaterial are characterized by the strength reduction coefficient of the post-failure zone of the stress-strain curve of the complete process. In the study, the transfer coefficient method (TCM) hypothesis was generalized, and the partial strength reduction method (PSRM) and perfect elastoplastic model (PEPM) were employed. The progressive failure process of a slope was simulated by the proposed generalized slice block method; a critical-stress-state determination method was proposed. The safety factor of the extended slice block method was found to be closely related to deformation of the slope, and the characteristics of the critical stress state movement were revealed by the proposed generalized slice block method. The unbalanced thrust and driving sliding force gradually increase and the frictional resistance gradually decreases during deformation failure. When the last slice block is in the critical stress state, complete slope failure occurs. Taking the Tieshansi slope of Huangshi City as an example, the evolution characteristics of the physical and mechanical variables during progressive failure of the slope were revealed by the proposed generalized TCM using PSRM- and PEPM-based numerical simulations. The results show that the proposed method can describe the progressive failure process of the Tieshansi slope. The emphasis of this research is on extending the traditional strength reduction slice block method so that the results are closely related to the slope deformation.
1. Introduction

The evaluation of slope-safety factors is a comprehensive scientific research topic, which includes not only its mechanism, but also slope stability. More than 10 types of calculation methods for limit equilibrium stability have been proposed: examples are the Fellenius method [1], the simplified Bishop method [2], the Spencer method [3], the Janbu method [4], the transfer coefficient method (TCM), the Sarma method [5], the wedge method, and the finite element strength reduction method [6-10].

The limit equilibrium method is widely used in engineering design [11-12] Different limit equilibrium slice methods are based on different assumptions about the action point of normal force on the bottom edge of the slice and the action direction of the thrust between the slices for a slope with a given slip surface. Hence, the stability evaluation results obtained by different methods are not the same.

With advances in numerical analysis, an increasing number of calculation methods have been proposed [13-15]. Recently, the vector sum method [16] was applied to evaluate the stability of a slope and a high dam, and a three-dimensional strict equilibrium equation based on certain assumptions was established, and four criteria were proposed [17]. A partial strength reduction method (PSRM) was proposed [18] to simulate the progressive failure process.

The finite element method is a widely used numerical analysis method. The main software packages are ABAQUS, ANASYS, 3D-σ, and Geo-studio. Their corresponding theoretical foundation is based on the assumption of small deformation and is usually applied to continuum media mechanics. Other numerical methods such as the fast Lagrangian analysis of continua, discrete element method, discontinuous deformation analysis method, and popular element method have been increasingly applied to slope stability analysis.

In previous studies, the sliding surface was classified into a failure zone, critical zone, and stable zone. The transfer law of the landslide force and deformation was analyzed, and the characteristics of the critical block (or unit) force of the slope were proposed [19-22]. Based on the failure mechanisms, deformation modes and control standards of thrust-type, pull-type, and mixed landslides were proposed, and the deformation–stress analysis was combined with the possible failure modes of a slope. Accordingly, the following evaluation methods were defined: comprehensive sliding–resistance, main thrust, comprehensive displacement, and surplus displacement methods [23-24].

In this study, the slice block method was extended, and the perfect elastoplastic model (PEPM) was used in the TCM. The safety factor for the surplus frictional method was defined. The progressive failure process was described by the main thrust method (MTM), surplus frictional force method (SFM), comprehensive displacement method (CDM), and surplus displacement method (SDM). These methods were used for the slope progressive failure stability analysis. In fact, this analytical method provides a new idea for examining slope stability.

1.1. Transfer Coefficient Method

The limit equilibrium slice method was used in engineering, but it has shortcomings: for example, the interaction force between the slices is simplified, and the traditional slice method is not related to deformation. The TCM is explained below.

Hypothesis:
(1) The slice blocks are rigid, and there is no deformation. The blocks are divided vertically at certain intervals. (2) The force acting on a block because of a slice block behind it is parallel to the bottom edge of the back block, and the force is exerted at the center of the front block. (3) The rotation of each slice block and the shear force between the two slice blocks are ignored. (4) The shear stress of the bottom edge of the slice block is in a critical stress state.

Under the above assumptions, the scheme of the slice blocks is shown in Figure 1. The basic equations of the TCM are as follows (see Eq. 1–9).

For the \( i \)-th slice block

- Normal pressure \( N_i \) is as follows:
  \[
  N_i = W_i \cos \alpha_i + P_{i-1} \sin(\alpha_{i-1} - \alpha_i) + \beta_i l_i \cos \alpha_i \cos \alpha_i + \Delta l_i \cos \alpha_i \sin \alpha_i
  \]
  (1)

- Normal stress \( \sigma_{i,n} \) is as follows:
  \[
  \sigma_{i,n} = \frac{N_i}{l_i}
  \]
  (2)

- Critical frictional stress \( \tau_{i,\text{peak}} \) is as follows:
  \[
  \tau_{i,\text{peak}} = c_i + \sigma_{i,n} \tan \varphi_i
  \]
  (3)

- Critical frictional force \( T_i^{\text{crit}} \) is as follows:
  \[
  T_i^{\text{crit}} = c_i l_i + N_i \tan \varphi_i
  \]
  (4)

- Frictional stress after the strength reduction (SR) \( \tau_{i,f} \) is as follows:
  \[
  \tau_{i,f} = (c_i + \sigma_{i,n} \tan \varphi_i) / f
  \]
  (5)

- Frictional force after the SR \( T_{i,f}^{\text{crit}} \) is as follows:
  \[
  T_{i,f}^{\text{crit}} = T_i^{\text{crit}} / f
  \]
  (6)

- Sliding force \( P_i^s \) is as follows:
  \[
  P_i^s = W_i \sin \alpha_i + P_{i-1} \cos(\alpha_{i-1} - \alpha_i) + \beta_i l_i \cos \alpha_i \sin \alpha_i + \Delta l_i \cos \alpha_i \cos \alpha_i
  \]
  (7)

- Shear stress \( \tau_i^u \) is as follows:
  \[
  \tau_i^u = P_i^s / l_i
  \]
  (8)

- Surplus thrust \( P_i \) is as follows:
  \[
  P_i = P_i^s - T_{i,f}^{\text{crit}}
  \]
  (9)
where $W_i$, $\beta_i$, $\Delta_i$, $l_i$, $\alpha_i$, $c_i$, $\varphi_i$ are the weight, vertical and horizontal uniform load of slope surface, length of bottom edge, angle between the bottom edge and horizontal axis, cohesion of the bottom edge, and frictional angle of the $i$-th slice block, respectively. Further, $f$ is the SR coefficient along the entire sliding surface in the critical stress state.

Eq. 1–9 are repeated to obtain the reduction coefficient $\hat{f}$. In the above analysis, it is difficult for the shear stress of the entire bottom edge to be in the peak stress state.

![Fig.1 Slice block layout scheme.](image)

2. Partial Strength Reduction Method

The scheme of the slice block method is shown in the Figure 1. The total safety factor of a slope is obtained by using Eq. 1–9 by the TCM. The surplus driving force of the last slice block is zero. The calculation steps of the PSRM proposed in previous studies (Yingfa et al., 2013, 2015, 2017) are described below:

Let the reduction coefficient be 1 for the first to the $m$-th slice block; the $m$-th slice block is in the critical stress state. Then, the calculations are performed from the first to the $m+1$-th slice block, ..., and from the first to the $n$-th slice block, and the reduction coefficients ($f_i$, $i \in (m,n)$) are obtained by the PSRM.

The physical meaning of the safety factor is explained as follows: The slice block located in the critical stress state moves forward from the $m$-th block to the $m+1$-th block, ..., to the $n$-th block. When the critical stress state reaches the $n$-th block, the safety factor is equal to that of the whole slope obtained by traditional calculation. The calculation results show that the safety factor of the $m$-th slice block is the least (equal to 1.0), while the $m$-th slice block is in the critical stress state.

As the critical-stress-state slice block moves forward, the safety factor of the slope increases gradually. The overall safety factor of a slope is calculated, and its maximum value ($f_n$) is obtained, until the $n$-th slice block reaches the critical stress state. Obviously, the critical stress state moves forward slightly, and the safety factor of the slope continues to increase, which is inconsistent with the rational analysis of a slope in situ. However, the surplus safety factor gradually decreases. The surplus safety factor ($f_{zs}$) is defined as follows: the surplus safety factor of the $i$-th slice block in the critical stress state is equal to that the difference between the safety factor ($f_n$) and the partial SR coefficient of the $i$-th slice block ($\hat{f}_i$) (see Eq. 10).
Here, Eq. 10 shows that when the \( n \)-th block is in the critical stress state, the whole slope is in the failure state, and the surplus safety factor \( f_{zs} = 0 \) is equal to zero. The field practice of surplus safety factor by the PSRM tells scientific researchers that the location of critical sliding stress surface can be determined on site.

3. Slice Block Method based on PEPM

The traditional slice method was generalized based on the PSRM and PEPM. Some basic assumptions were as follows:

3.1. Basic Assumptions

(5) It is assumed that the stress–strain relationship of the geological material of each slice block satisfies the PEPM, and the slice block can produce enough self-deformation and can be divided into vertical sections at certain intervals. (6) Assumptions (2) and (3) are employed. (7) The shear strains between the \( i \)-th slice block and the \( i+1 \)-th slice block satisfy the vector-sum relationship in the parallel and vertical bottom-edge directions in the failure zone (see Fig. 2).

![Fig. 2](image)

(a) Shear relationship of the connected slice blocks. (b) \( i \)-th slice block moves forward, and the \( i+1 \)-th slice block provides the conditions.

On the basis of the above assumptions, the calculation formula for force is basically consistent with that of the TCM, and the shear strain between the connected slices in failure zone has the following relationship:

\[
\gamma_i^s = \gamma_{i+1}^s + \gamma_{i+1}^n
\]  

(11)

The physical significance of hypothesis (7) is that as the \( i+1 \)-th slice block advances, the following condition holds for the \( i \)-th slice block:

\[
\gamma_{i+1}^s = \gamma_i^s \cos(\alpha_i - \alpha_{i+1})
\]  

(12)
When the angles between the two slice blocks are equal, the shear strains of the two slice blocks are equal.

3.2. Perfect elasto-plastic model (PEPM)

Based on the progressive failure analysis of the PEPM, the peak stress is reduced by the PSRM. The decrease in stress post-failure is indicated by the reduction in strength, and the unbalanced thrust increases in the failure zone. The critical stress state moves forward one by one. Under certain assumptions, the deformation solutions can also be obtained for the slice block method. The PEPM is presented as follows:

The PEPM is used widely to describe the shear stress–strain of the geomaterial, and the linear relationship between the shear stress and strain is employed for the critical stress.

\[ \tau_i = G_i \gamma_i, \quad \text{when } \gamma_i \leq \gamma_{i, \text{peak}} \quad (13) \]
\[ \tau_i = \tau_{i, \text{peak}}, \quad \text{when } \gamma_i > \gamma_{i, \text{peak}} \quad (14) \]

where \( G_i \) is the shear modulus. At the critical shear stress value, it is difficult to calculate the shear strain by using the stress. There is no one-to-one correspondence between the shear stress and the strain, when the stress is located at the critical stress.

3.3. Critical stress state determination

The generalized slice method can be used to calculate the frictional resistance, sliding force, and surplus thrust along the sliding surface. When the surplus thrust of the \( m \)-th slice block is greater than 0 (\( P_{m-1} > 0 \)), and the surplus thrust of the \( m \)-th slice block is less than 0 (\( P_m < 0 \)), the critical stress state slice block (CSB) must be located in the \( m \)-th block. Because the bottom edge of the \( m \)-th slice block is longer and \( P_m < 0 \), it is necessary to re-divide the bottom edge of the \( m \)-th slice block. The slice block is generally a triangle, parallelogram, or trapezoid; since the trapezoid can represent the three shapes, the following research is carried out on the trapezoid.

Assuming that the coordinates of the four points of the \( m \)-th trapezoidal slice block are \( (x_1^m, y_1^m) \), \( (x_2^m, y_2^m) \), \( (x_3^m, y_3^m) \), and \( (x_4^m, y_4^m) \), and that the inclination angle of the bottom edge is \( \alpha_m \), the surplus thrust of the \( m-1 \)-th slice block is greater than 0 (\( P_{m-1} > 0 \)). The \( m \)-th slice block is decomposed into the \( m \)-th and \( m+1 \)-th slice blocks, and the new \( m \)-th slice block consists of points 1, 2, 5, and 6 (see Fig. 3). The coordinates of points 5 and 6 can be obtained.

In Figure 4, points 1 and 4 are related as follows:

\[ y_{14} = k_{14} x_{14} + b_{14} \quad (15) \]

where \( k_{14} = \frac{y_4^m - y_1^m}{x_4^m - x_1^m} \), \( b_{14} = \frac{y_4^m - y_1^m}{x_4^m - x_1^m} x_1^m \).

Further, the equation relating points 2 and 3 is as follows:

\[ y_{23} = k_{23} x_{23} + b_{23} \quad (16) \]
where \( k_{23} = \frac{y^{m}_{1} - y^{m}_{2}}{x^{m}_{1} - x^{m}_{2}}, b = \frac{y^{m}_{2} - y^{m}_{3}}{x^{m}_{2} - x^{m}_{3}}x^{m}_{2} \).

The coordinates of points 5 and 6 are \( (x^{m}_{5}, k_{4}x^{m}_{5} + b_{4}) \) and \( (x^{m}_{6}, k_{23}x^{m}_{6} + b_{23}) \), respectively, and the unique unknown variable is \( x^{m}_{6} \). The weight and length of the bottom edge of trapezoid 1256 can be obtained. For the new \( m \)-th slice block, the parameters are calculated as follows:

Normal force \( N_{m}^{N} \) is as follows:

\[
N_{m}^{N} = W_{m}^{N} \cos \alpha_{m} + P_{m-1} \sin(\alpha_{m-1} - \alpha_{m}) + \beta_{m}l_{m}^{N} \cos \alpha_{m} \cos \alpha_{m} + \Delta_{m}l_{m}^{N} \cos \alpha_{m} \sin \alpha_{m} \tag{17}
\]

Critical frictional force \( T_{m}^{crit,N} \) is calculated as follows:

\[
T_{m}^{crit,N} = c_{m}m_{m}^{N} + N_{m}^{N} \tan \varphi_{m} \tag{18}
\]

Sliding force \( P_{m}^{s,N} \) is given below:

\[
P_{m}^{s,N} = W_{m}^{N} \sin \alpha_{m} + P_{m-1} \cos(\alpha_{m-1} - \alpha_{m}) + \beta_{m}l_{m}^{N} \cos \alpha_{m} \sin \alpha_{m} + \Delta_{m}l_{m}^{N} \cos \alpha_{m} \cos \alpha_{m} \tag{19}
\]

Fig.3 Scheme of slice-block decomposition.

According to the definition of the critical-stress-state slice block, the sliding force is equal to the frictional resistance.

\[
T_{m}^{crit,N} = P_{m}^{s,N} \tag{20}
\]

Here, Eq. 20 is a quadratic equation with one variable, and it can be solved. The slice block is shaped as a triangle when \( y^{m}_{6} = y^{m}_{5} \) and as a parallelogram when \( \left| y^{m}_{6} - y^{m}_{5} \right| = \left| y^{m}_{4} - y^{m}_{1} \right| \).

So far, the frictional stress strength at the bottom edge from the first to the \( m \)-th slice block reduced (see Eq. 5). The \( m+1 \)-th to the \( n \)-th slice blocks are located before the peak stress, and the frictional stress is equal to the sliding force divided by the length of the bottom edge. The normal stress is calculated by Eq. 2, and the driving shear stress is calculated by Eq. 8 from the first to the \( n \)-th slice block. The current frictional shear stress (\( \tau_{X}^{m} \)), driving shear stress (\( \tau_{i}^{s} \)), and normal stress (\( \sigma_{in}^{X} \))
from the first to the $n$-th slice blocks were completely determined and recorded as $\tau_i^X, \tau_i^u, \sigma_{i,s}^X, i \in (1, n)$.

3.4. Displacement determination of the sliding surface

The stress distribution along the sliding surface can be obtained for the post-failure zone by using the PSRM with PEPM. The shear–strain distribution is also calculated from the first to the last slice block as follows:

The slice blocks from the $m+1$-th to the $n$-th slice block are within the elastic stress state, and the linear shear strain can be obtained by Eq. 13. The shear strain along the sliding surface in the failure zone can be also obtained on the basis of hypothesis (7). The relationship between the shear strains of the bottom edge of the $i$-th and the $i+1$-th slice block can be analyzed. The shear strain of the $i$-th slice block is the sum of the critical shear strain ($\gamma_{i, \text{peak}}$), the shear strain produced by the reduced stress ($\gamma_{i, \text{peak}}^*$), and the contribution ($s_i$) of the $i+1$-th to the $i$-th failure slice block.

$$\gamma_i = \gamma_{i, \text{peak}} + \gamma_i^* - \gamma_{i, \text{peak}}^*$$  \hspace{1cm} (21)

After strength reduction, the frictional shear stress ($\tau_i^*$) is as follows:

$$\tau_i^* = \tau_{i, \text{peak}} / f_i$$  \hspace{1cm} (22)  \hspace{1cm} \gamma_{i, \text{peak}}^* = \tau_i^* / G_i$$  \hspace{1cm} (23)

The stress–strain relationship in the post-failure zone is fixed to perform stability evaluation based on displacement.

From the information provided in sections 3.1–3.4, the solutions based on the above assumptions can be obtained for shear strain. Using the above solutions, the progressive failure stability of a slope can be evaluated. The obtained stress and displacement are the current shear stress and displacement and can be compared with the in situ monitoring stress and displacement to modify the model parameters.

3.5. Progressive failure analysis

Failure analysis is used to determine the final failure state of a slope. The possible failure mode of a slope must be obtained in order to evaluate and analyze the safety factor. The failure path of a slope can be determined via in situ investigation and detection, such as investigations dealing with the soil–rock interfaces, or the joints and fissures of a rock mass. When the failure path is known, the frictional stress of a slice block is gradually reduced according to the PSRM with the PEPM. Each slice block undergoes the critical stress state until the last slice block is in the critical stress state, and the stress–strain distribution along the sliding surface is obtained in the entire failure state. The frictional shear stress ($\tau_{i,p}^*$), driving shear stress ($\tau_{i,u}^p, \tau_{i,p}^b$), normal stress ($\sigma_{i,s}^p$), and shear strain ($\gamma_{i,p}^b$) are determined in the final failure state, and are denoted as $\tau_{i,p}, \tau_{i,u}, \sigma_{i,s}, \gamma_{i,p}, i \in (1, n)$.

4. Stability analysis

For a specific slope, the traditional safety factor of the whole slope can be calculated by using Eq. 1–9. To describe the characteristics of a slope at different times and evaluate its stability, the field frictional
stress, driving shear stress, normal stress, and shear strain are obtained according to the PSRM and PEPM and are recorded as $\tau_i^x, \tau_i^b, \sigma_i^x, \gamma_i^b, i \in (1,n)$, respectively. The frictional stress, driving shear stress, normal stress, and shear strain can be denoted as $\tau_i^{p,b}, \tau_i^{n,b}, \sigma_i^{p,b}, \gamma_i^{p,b}, i \in (1,n)$ under the failure mode for two-dimensional engineering. The complete stability evaluation of the sliding body can be performed. The evaluated safety factors for the main thrust method ($F_{MTM}$), comprehensive displacement method ($F_{CDM}$), and surplus displacement method ($F_{SDM}$) were presented in conferences (Ying-fa et al., 2017, 2015) The proposed safety factor for the surplus frictional force method ($F_{SFM}$) is as follows:

The sum of the differences ($T^{am}$ and $T^{bm}$) between the frictional stresses in the failure state ($\tau_i^{p,b}, i \in (m+1,n)$) and in the current state ($\tau_i^x, i \in (m+1,n)$) in the horizontal and vertical directions are obtained from the $m+1$-th to the $n$-th slice block, and the vector sum of their difference ($T^m$) can be also obtained.

$$T^{am} = \sum_{i=m+1}^{n} (\tau_i^{p,b} - \tau_i^x) \cos \alpha_i$$

$$T^{bm} = \sum_{i=m+1}^{n} (\tau_i^{p,b} - \tau_i^x) \sin \alpha_i$$

$$T^m = \sqrt{(T^{am})^2 + (T^{bm})^2}$$

The angle ($\alpha_f^m$) between the comprehensive vector sum and the horizontal axis is as follows:

$$\alpha_f^m = \arctan(T^{bm}/T^{am})$$

Then, the sum of the sliding force ($P_{xf}$ and $P_{yf}$) along the X- and Y-axes, and their vector sum ($P_f$) is calculated for the failure mode.

$$P_{xf} = \sum_{i=1}^{n} \tau_i^{p,b} l_i \cos \alpha_i$$

$$P_{yf} = \sum_{i=1}^{n} \tau_i^{p,b} l_i \sin \alpha_i$$

$$P_f = \sqrt{(P_{xf})^2 + (P_{yf})^2}$$

The angle ($\alpha_{pf}$) between the comprehensive vector sum ($P_f$) and the x-axis is as follows:

$$\alpha_{pf} = \arctan(P_{yf}/P_{xf})$$

The safety factor of SFM in the horizontal direction is

$$F_{SFM}^x = \left| T^{am}/P_{xf} \right|,$$

while that in the vertical direction is

$$F_{SFM}^y = \left| T^{bm}/P_{yf} \right|,$$

and that in the sliding direction is
\[ F_{SF_M} = \left| T^m \cos(\alpha^m - \alpha_{pf}) / P_f \right|. \]  

(34)

Note that \( m \) is the current critical-stress-state slice block.

5. Case Studies

5.1. Geological Survey

The Tieshansi slope is located on the eastern side of the entrance of Longdong iron mine in the Jianlin mountain. The geological coordinates of central point of the slope are longitude E114°53′21.83″, latitude N30°36′02″. The southern boundary is controlled by the accumulation platform of Daye Iron Mine, and the north boundary ends at a natural gully. The back edge of the slope is a cement road (see Fig. 4 and 5). The elevation of the back edge is 140 m, and that of the front edge is 100 m. The area of the slope is about 3280 m², and the main sliding direction of the slope is 287°. The thickness and volume of the sliding mass are 6–8 m and \( 1.14 \times 10^4 \) m³, respectively. The sliding surface consists of highly weathered structural strata.

5.2. Characteristics of the sliding mass, slip surface, and slide bed

According to ground investigation and borehole analysis, the slope consists of a rock and soil mass. The sliding mass can be classified into two layers. The upper part contains residual deposits and consists of brown and brown–yellow silty clay with gravel; its thickness is 3.0–4.0 m, and the soil-to-stone ratio is 7:3. The lower part consists of grayish-brown strongly weathered diorite with heavy joint fissures and loose structure. This layer is about 7 m thick, and the sliding body is about 6–8 m thick.

According to the analysis of the slip characteristics on the north side of the slope, the main slip surface is the interface between the weathered broken and complete diorite (thickness: 4–10 cm). The sliding bed consists of the diorite, and here, the diorite is relatively intact and has high strength.

5.3 Analysis

The calculation scheme (see Fig.7) of the slice block method is formulated according to the I–I’ profile (see Fig.6), and the weight of the sliding body is 17 kN/m³. The slice block angle and length of the bottom edge of each slice block are shown in Fig. 7.

According to the laboratory test and field experience, the model parameters of the saturated sliding surface are as follows:

Cohesion force: \( c = 19 \) kPa, frictional angle: \( \varphi = 23° \), and shear modulus: \( G = 6000 \) kPa.
According to TCM, the safety factor of Tieshansi slope under the saturated condition is 1.3493. The relationships between the driving force ($P_{i}^{STCM}$), friction resistance ($T_{i}^{TCM}$), surplus sliding force ($P_{i}^{TCM}$), and slice number (SN) of each slice block are shown in Figure 8. When the safety factor is 1.3493, the maximum driving force and frictional resistance correspond to the fifth and eighteenth slice blocks, respectively, and the surplus sliding force is zero for the last slice block.

According to the PSRM, the CSB is the eleventh slice block, when the safety factor is equal to 1. As the critical stress state advances sequentially, the safety factor of the PSRM in different CSBs gradually increases. When the last slice block is in the critical stress state, the safety factor of the PSRM is equal to that of the TCM (1.3493), and the relationship between the safety factor of the PSRM and the CSB number (i.e., CSN) is shown in Fig. 9. The surplus safety factors ($f_{zsf}$) of the PSRM for different CSBs were obtained (see Fig. 10). The increase in the total safety factor and the decrease in the surplus safety factor are shown in Fig. 9 and 10, respectively, corresponding to the sequential advance of the CSBs. That is, as the critical stress state advances, the degree of stability of the slope gradually decreases.
Progressive failure was analyzed by using the PSRM with the PEPM. The driving force ($P_i^{STCM}$), friction resistance ($T_i^{TCM}$), and surplus sliding force ($P_i^{TCM}$) under five CSBs are shown in Fig. 11–15. The variation law of the safety factors for four methods (MTM, SFM, CDM, and SDM) with the progressive movement of the CSBs is shown in Fig. 16–19, respectively.

The maximum driving force ($P_i^{STCM}$) changes from the seventh slice block to the seventh, eighth, fifteenth, and fifteenth slice block; the maximum frictional resistance ($T_i^{TCM}$), from the eighth slice block to the eighth, eighth, eighth, and eighteenth slice block; and the maximum surplus sliding force ($P_i^{TCM}$), from the sixth slice block to the sixth, eighth, fifteenth, and fifteenth slice block. When the SR coefficient increases, the maximum frictional resistance decreases (from 870 kN to 678 kN), and the maximum driving force increases (from 1342 kN to 3233 kN) and so does the surplus sliding force (from 504 kN to 2608 kN). When the critical stress state changed from the tenth to the last slice block for all the safety factor evaluation methods (MTM, SFM, CDM, and SDM) of the slope, the result of the MTM changed from 1.69, 0.51, and 1.40 (in the directions of the X-axis and Y-axis, and the comprehensive value, respectively) to 0.0, 0.0, and 0.0, respectively; that of the SFM changed from 0.093, 0.038, and 0.085 to 0.0, 0.0, and 0.0, respectively; that of the CDM changed from 4.25, 4.35,
and 4.27 to 1.0, 1.0, and 1.0, respectively; and that of the SDM changed from 0.69, 0.48, and 0.64 to 0.0, 0.0, and 0.0, respectively.

From the solutions of the TCM, the safety factor obtained by the traditional slice block method is the overall stability of a slope. The surplus safety factor obtained by the PSRM is the evolution law of the surplus degree of the overall stability of the sliding body with changes in the critical stress state. The MTM, SFM, and SDM present the surplus degrees of the force and the displacement of a slope. The CDM describes the evaluation law of the overall displacement of a slope.

![Fig. 12 Curve between the force distribution and SN under the fifth CSB.](image1)

![Fig. 13 Curve between the force distribution and SN under the 18th CSB.](image2)

![Fig. 14 Curve between the force distribution and SN under the 21st CSB.](image3)

![Fig. 15 Curve between the force distribution and SN under the 23rd CSB.](image4)

![Fig. 16 Scheme between the MTM and the CSN](image5)

![Fig. 17 Scheme between the SFM and the CSN](image6)
The TCM mainly describes the force stability according to the evolution mechanism of the slope. The new methods (i.e., PSRM with PEPM) characterized the force stability of the slope by using the MTM and SFM. From a comparison between the results obtained by the TCM and the new methods, the following observations are made: the total safety factor of Tieshansi slope as obtained by TCM is 1.3493, and its surplus degree can be considered as 0.3493. The surplus stability obtained by the PSRM is also 0.3493. However, the two safety factors (MTM and SFM) for the PSRM with PEPM along the X and Y axes and the comprehensive value vary with the deformations.

6. Conclusion

The progressive process from the initial cracking to the complete failure of the sliding surface was analyzed by using the PSRM with PEPM. The traditional slice block method was extended so that the safety factor was closely related to deformation. The following are the contributions of this study:

(1) The calculation steps and physical significance of the PSRM were described in detail. The force evaluation law was described by the MTM and SFM based on the PSRM, which can provide the force characteristics during progressive failure.

(2) The hypothesis of the traditional slice method was generalized, and the PEPM was introduced. The PSRM with the PEPM was used to describe the complete process of the progressive failure of a slope. The variation characteristics of the force and displacement during progressive failure too were well described. The four safety factors (MTM, SFM, CDM, and SDM) varied with the deformation.

(3) The characteristics of the critical stress state were proposed, and a method for CSB determination was developed.

(4) Based on the analysis of the deformation relationship between the two slice-connected blocks in the failure zone, shear–strain relations for the two connected blocks were proposed. A method to determine the shear strain in the failure zone was developed for the PSRM with PEPM.

(5) The proposed generalized slice block method for the progressive failure of a slope can be extended to the other methods, such as the Janbu method, Sarma method, and simplified Bishop method.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (Grant No. 41372363, No.41641027 and No. 50879044). This work was also supported by the Geological Hazard Prevention Project in the Three Gorges Reservoirs (Grant No. 0001212015CC60005).
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