Traditional formation scenarios fail to explain 4:3 mean motion resonances

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ABSTRACT
At least two multi-planetary systems in a 4:3 mean motion resonance have been found by radial velocity surveys.1 These planets are gas giants and the systems are only stable when protected by a resonance. Additionally the Kepler mission has detected at least four strong candidate planetary systems with a period ratio close to 4:3.

This paper investigates traditional dynamical scenarios for the formation of these systems. We systematically study migration scenarios with both N-body and hydrodynamic simulations. We investigate scenarios involving the in situ formation of two planets in resonance. We look at the results from finely tuned planet–planet scattering simulations with gas disc damping. Finally, we investigate a formation scenario involving isolation-mass embryos.

Although the combined planet–planet scattering and damping scenario seems promising, none of the above scenarios is successful in forming enough systems in 4:3 resonance with planetary masses similar to the observed ones. This is a negative result but it has important implications for planet formation. Previous studies were successful in forming 2:1 and 3:2 resonances. This is generally believed to be evidence of planet migration. We highlight the main differences between those studies and our failure in forming a 4:3 resonance. We also speculate on more exotic and complicated ideas. These results will guide future investigators towards exploring the above scenarios and alternative mechanisms in a more general framework.

Key words: methods: analytical – methods: numerical – planets and satellites: formation.

1 INTRODUCTION
To date, 7772 extra-solar planets have been discovered via numerous detection techniques, including pulsar timing (e.g. Wolszczan & Frail 1992), radial velocity (RV; see e.g. Mayor & Queloz 1995), transits (e.g. Charbonneau et al. 2000) and micro-lensing (e.g. Bond et al. 2004), while thousands of candidate systems from the Kepler transit mission await confirmation (Batalha et al. 2012).

A significant fraction (∼13 per cent) of the known planetary systems have been confirmed to possess systems of multiple planets. Multi-planet systems can provide valuable information on their history that single-planet systems cannot.

For example, multiple highly excited planets may indicate early dynamical instabilities (Rasio & Ford 1996). The existence of mean motion resonances (MMRs) on the other hand suggests that convergent migration occurred in the presence of dissipative forces. Numerous examples of resonant systems are known, both in extra-solar planetary systems and Solar system satellites such as the 1:2:4 Laplace resonance in the Io-Europa-Ganymede system. The most studied planetary system in an MMR is Gliese 876 (e.g. Lee & Peale 2001, 2002; Marcy et al. 2001; Snellgrove, Papaloizou & Nelson 2001; Nelson & Papaloizou 2003; Beaugé & Michtchenko 2003; Veras 2007; Rein & Papaloizou 2010). All these studies suggest that migration (albeit potentially driven by a variety of mechanisms) is an important factor in the sculpting of planetary systems.

Convergent migration into closely spaced resonances (e.g. 3:2) has been shown to be plausible, but difficult (Rein, Papaloizou & Kley 2010). The difficulties arise because planets that initially form far apart need to migrate through more widely spaced resonances (e.g. the 2:1 MMR) before subsequently being captured into the more closely spaced (e.g. 3:2) MMR. To avoid the requirement for fine-tuning of initial locations, this requires relatively high migration rates (Rein et al. 2010). Even more closely spaced 4:3 resonances are suspected for some systems such as HD 200964 (Johnson et al. 2011) and KOI 115 (Borucki et al. 2011), suggesting that even more violent migration histories must have occurred to allow such
systems to skip through the exterior 2:1 and 3:2 resonances before going on to be captured into the observed 4:3 resonance.

We investigate the formation of 4:3 resonances in massive systems, i.e. planets with masses up to several times that of Jupiter. We examine mechanisms which include smooth migration, in situ formation, scattering and damping. We demonstrate that the simple smooth migration mechanisms suspected to form the known 2:1 and 3:2 systems cannot plausibly form systems of massive planets in 4:3 resonances.

We also study the formation of lower mass planets and find that these can readily form from a series of isolation-mass embryos in tightly packed systems which then lead to 4:3 (and even closer) resonances. But these mechanisms underestimate the number of tightly packed systems in close resonances with multiple massive planets. The rate of detection of these multi-planetary systems (period ratios of 4:3 and closer) can therefore tell us valuable, not directly observable, information about the formation history of extra-solar planetary systems and the Solar system itself.

In Section 2, we summarize observational results of closely packed planetary systems. Then we discuss the phase space and stability of massive systems in Section 3. The results of many different formation scenarios are presented in Section 4. This is the main part of our paper. We then extend the study to lower mass systems in 4:3 resonance and show that there is no difficulty in forming these in Section 5. Finally we summarize and discuss these results in Section 6.

2 OBSERVATIONAL EVIDENCE FOR 4:3 RESONANCES

As noted in Section 1, while many systems are thought to be in (or to be close to) a variety of MMRs, the population of systems suspected to occupy the closely spaced 4:3 MMR is much smaller. In this section, we review the systems, both Solar system satellites and exoplanets, which are suspected of populating 4:3 resonances.

2.1 4:3 resonances in the Solar system

The best-known pair of Solar system satellites or planets which are locked in a 4:3 MMR with each other are Titan and Hyperion. With a mass of \(5.6 \times 10^{18}\) kg (Thomas et al. 2007), Hyperion is 168 times less massive than Ceres and almost five orders of magnitude less massive than Titan, meaning that Hyperion effectively acts as a test particle and has a negligible effect on Titan’s orbit. Hence, this resonance is dominated by a single term in the disturbing function (containing the longitude of pericentre of Hyperion and the mean longitudes of both satellites).

The satellites’ motion has been well characterized by observations over several decades (Taylor 1984) and the stability afforded by the resonance has been considered in great detail (Colombo, Franklin & Shapiro 1974; Bevilacqua et al. 1980; Stellmach 1999). Although recently Beaugé, Michtchenko & Ferraz-Mello (2006) suggested that Titan and Hyperion are examples of a system captured into resonance due to migration, Bevilacqua et al. (1980) came to a different conclusion. The latter suggested that Hyperion was formed at its present location and cast doubt on a scenario where Titan and Hyperion achieved their current configuration through smooth differential tidal evolution across the chaotic zone. They further argued that a possible reason why Hyperion was accreted together in the 4:3 libration zone instead of the 3:2 libration zone is because in the former, more restricted region, the relative velocities of surviving planetesimals were small enough for coagulation to occur.

Other cases of 4:3 resonances in the Solar system include the asteroid Thule which is in the 4:3 resonance with Jupiter (for more objects, see p. 49 in Barucci et al. 2008).

Unfortunately, none of the above considerations translate easily into a 4:3 configuration with two massive planets. In this case, multiple terms in the disturbing function must be considered and stable libration zones are not as well characterized. However, as demonstrated later, capture into this resonance due to migration still proves to be difficult.

2.2 4:3 resonances in exo-planetary systems

2.2.1 RV systems

There are currently two multi-planetary systems discovered by the RV method which are reported to be in or near a 4:3 MMR.

HD 200964 consists of two planets with masses \(m_1 = 1.8 M_{\text{Jup}}\) and \(m_2 = 0.9 M_{\text{Jup}}\) (Johnson et al. 2011). The period ratio is close to 1.33 which suggests that the system is in a 4:3 MMR. The results of a Monte Carlo fitting routine that includes a penalty for unstable systems strongly favour systems in resonance (Johnson et al. 2011, see also Section 3). Another planetary system is suspected to also be in or near a 4:3 MMR (Giguere et al., in preparation, private communication). Both systems consist of two massive (and most likely gaseous) planets on wide (a few au) orbits. Both systems are subject to a short dynamical instability if the planets are not protected by a MMR. Their parameters are listed in Table 1.

In the top panel of Fig. 1, we plot the cumulative distribution function of all the multi-planetary systems that have been discovered with the RV method. In systems with more than two planets, each pair is treated independently. To compile this data set, we made use of the Open Exoplanet Catalogue. One can clearly see the tendency for planets to pile up near integer ratios of the period ratio such as 2:1 and 3:1. This is usually attributed to resonant capture during the migration phase (Lee & Peale 2002). The number of planets near the 4:3 resonance is far too small to make a statistical argument at this time. We also colour code the total mass of the two planets. Note that mostly high mass planets get captured in the 2:1 and 3:2 resonances, whereas lower mass planets get preferentially captured into more closely spaced resonances.

In this paper, we investigate the conditions under which systems on the far left-hand side of this plot form.

The reported best-fitting solutions of the HD 200964 system puts it in a 4:3 resonance. However, there are other orbital solutions which are stable and cannot be ruled out with high confidence. For example, HD 200964 could also be in a 3:2 resonance (Johnson et al. 2011). With the currently published RV data points, this results in a higher \(\chi^2\) value, therefore not being the best, but still a possible fit.

| Star     | \(M_\star\) (M\(_\odot\)) | Planet | \(m\) (M\(_{\text{Jup}}\)) | \(P\) (d) | \(P_2/P_1\) |
|----------|--------------------------|--------|--------------------------|----------|------------|
| HD 200964| 1.44                     | b      | 1.99                     | 613.8    | 1.34       |
|          |                          |        | c                        | 0.90     | 825.0      |

Table 1. Details of the planetary system HD200964 which has been detected by RV variations and is in or near a 4:3 MMR. A second, very similar system has been detected but has not been announced by the time this paper was submitted (Giguere et al., in preparation).

\footnote{https://github.com/hannorein/open_exoplanet_catalogue}
This paper is concerned about the formation scenarios of the report systems. Fitting RV data is a notoriously difficult job. We do not attempt to redo the analysis of Johnson et al. (2011).

As we will show below, it is very difficult to get the system into the 4:3 resonance without fine-tuned initial conditions. One could therefore take our results and use it as a strong prior while fitting the RV light curve, rejecting systems in 4:3 resonance. We do not want to go that far and think this is actually dangerous. If there is a new formation mechanism that we did not take into account, one can easily draw a wrong conclusion.

### 2.2.2 Kepler systems

In Table 2, we list the four Kepler candidate systems which may occupy a 4:3 MMR (Batalha et al. 2012). The first column lists the Kepler Object of Interest Number of the candidate system. The second and third columns list the planet radius in units of Earth radii. The fourth and fifth columns list the orbital periods in days. The last column lists the ratio of the periods. We follow the procedure outlined in Veras & Ford (2011) and exclude any systems which are unlikely to be in resonance despite having period ratios close to 1.33. Note that all of the Kepler systems listed in Table 2 are smaller and closer-in than the RV systems listed in Table 1: the largest of the KOIs has a radius less than that of Uranus, and all have orbital periods less than 8 d. It is thus not unreasonable to assume that their formation mechanism differs significantly from that of the massive planets at larger semi-major axes.

We plot the cumulative distribution of all Kepler planet candidates (KOIs, Batalha et al. 2012) in the bottom panel of Fig. 1. As in Section 2.2.1, for systems with more than two planets, every pair in that system is treated independently. Although there are clear features in this distribution near the 3:2 and 2:1 resonances, no statistically significant accumulation of planets can be seen near the 4:3 resonance. Furthermore, in contrast to the RV systems, there is no strong correlation between mass and the proximity to a resonance.

It is worth pointing out the Kepler systems Kepler 36 and KOI 262. These systems are near the 6:7 MMR and 5:6 MMR, respectively (Carter et al. 2012; Fabrycky et al. 2012), i.e. even closer spaced than the 4:3 MMR that we study here. The masses of those planets are all below 9 M⊕.

### 3 PHASE SPACE AND STABILITY ANALYSIS

#### 3.1 Expansion of the disturbing function

Resonant theory has been applied to dynamical problems in the Solar system with great success (e.g. Murray & Dermott 2000; Morbidelli 2002). However, properties of extra-solar systems typically do not adhere to the approximations which can be adopted for the Solar system (Beaugé & Michtchenko 2003; Veras & Armitage 2007). In particular, predicting the evolution of two massive bodies on non-circular orbits poses a rich dynamical challenge. Although much analytical progress has been made characterizing regimes of motion when these bodies are in the strong 2:1 MMR (Beaugé, Ferraz-Mello & Michtchenko 2003; Michtchenko, Beaugé & Ferraz-Mello 2008a,b), few investigators have modelled in detail other first-order resonances, partly because of their close proximity to the chaotic resonant overlap region (Wisdom 1980; Mustill & Wyatt 2012).

Further, detailed analyses of particular systems benefit from well-constrained observational data, showcasing another benefit of analysing Solar system bodies; the majority of confirmed extra-solar planets have unknown masses, bounded only from below. Orbital parameters for transiting systems are even less constrained, making detailed dynamical modelling almost impossible. One approach to tackling these issues is to consider when a planetary system cannot be in resonance, by confirming that any potential librating angle must circulate (Veras & Ford 2011). This procedure can be carried out analytically by using a disturbing function with a sufficient number of terms to accurately sample the desired system.

Here, we carry out the same procedure as in Veras & Ford (2011) by using both the disturbing function4 from Ellis & Murray (2000) to fourth order in eccentricities and the analytical formulae from Veras (2007). We assume coplanarity and use all resonant terms up to fourth order (including the relevant 8:6, 12:9 and 16:12 terms) and secular terms up to fourth order. Additionally and separately

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4 This is the same disturbing function which later appeared in Murray & Dermott (2000).
we perform the same analysis for the 3:2 MMR, given its close proximity to the 4:3 MMR.

The results are plotted in Fig. 2. There are four different areas in these plots.

(i) In the dark-orange region to the bottom-right labelled ‘NO 3:2’, no 3:2 solutions are permitted (while 4:3 solutions may or may not exist).
(ii) In the dark-red region to the far left, neither 3:2 nor 4:3 solutions are permitted.
(iii) In the light-pink region labelled ‘NO 4:3’, no 4:3 solutions are permitted (while 3:2 solutions may or may not exist).
(iv) In the central light-orange region no definitive statement can be made to exclude either the 3:2 or 4:3 MMRs (but this does not equate to a statement that both can definitely exist).

Fig. 2(a) illustrates the excluded 4:3 and 3:2 MMR regions for the HD 200964 system, assuming \( m_1 = 1.44 \, M_\odot \), \( m_1 = 1.8 \, M_{\text{Jup}} \), \( m_2 = 0.9 \, M_{\text{Jup}} \) and fixed outer planet values of \( a_2 = 3.0 \, \text{au} \) and \( e_2 = 0.01 \). This plot helps constrain the prospects for the system evolving in MMR given particular orbital parameters. Alternatively, if an MMR is assumed, then the plot helps constrain the planets’ allowable orbital parameters. Fig. 2(b) over-plots a simple estimate of libration width from equation (4.46) of Veras & Armitage (2004) which assumes that just one disturbing function term is retained; this approach, often used in the Solar system, poorly reproduces the allowed resonant motions for this extra-Solar system. Fig. 2(c) demonstrates what the excluded regions look like in the limit of an inner planet mass of zero. In this case, both resonances become somewhat decoupled, and there is a clear structure around each nominal commensurability (see also Section 3.2). This plot may be compared to fig. 8.7 of Murray & Dermott (2000); differences arise because of the masses adopted here and the additional terms of the disturbing function used.

This set of plots illustrates the difficulty in restricting resonance phase space analytically for two massive exoplanets. But because the system is only stable when it is protected by a resonance (see also the following section) we can use such an analysis in the interpretation of orbital parameters which are weakly constrained from RV data.

### 3.2 Direct N-body simulations

In the previous section, we presented results of a resonant theory which was achieved by an expansion of the disturbing function. This allowed us to get an overview of the phase space structure of the HD 200964 system. We also run direct N-body simulations of systems with two planets to investigate their stability. The freely available code REBOUND (Rein & Liu 2012) is used for all integrations in this section. We choose the Wisdom–Holman-type integrator (Wisdom & Holman 1991) included in REBOUND. To verify the results, we implemented a 15th-order adaptive Radau integrator (Everhart 1985). We find that the results do not depend on the integrator or any numerical parameters such as the timestep.

For each run, we add two shadow particles to the simulation in addition to the two planets. This allows us to measure the long-term stability of the system on short time-scales by calculating the maximum Lyapunov exponent. To do that, the position and velocity of each shadow particle is initially set to those of the planets. They are then perturbed by a small amount. At regular intervals, the rate of divergence between the shadow particle and the planet is measured and rescaled to the initial displacement. By keeping track of the rescalings, we can estimate the maximum Lyapunov characteristic exponent. See Wisdom (1983) for more details on the numerical algorithm. If the numerically calculated Lyapunov time-scale is smaller or comparable to the run time, we call the system stable and unstable otherwise. The colour scale in all stability plots was chosen to reflect this definition. White regions are unstable, whereas dark regions are stable and have a Lyapunov time-scale that satisfies the above criteria.

The Lyapunov exponent gives us a good overview of the parameter space. In Fig. 3, we plot the results of \( 4 \times 10^5 \) N-body simulations, each running for approximately \( 2 \times 10^6 \) dynamical times. The mass of the central object is that of the star HD 200964. We show the results for three different planetary masses. For Fig. 3(a), we use the observed masses of the HD 200964 system (see Table 1). We repeat the same calculation with planets that each have \( 3 \, M_{\text{Jup}} \) in Fig. 3(b) and for Earth mass planets in Fig. 10. The initial semi-major axis and eccentricity of the inner orbit are \( a_1 = 2, e_1 = 0.01 \). These values are the same for all simulations. The initial semi-major
axis and eccentricity of the outer planet are varied and shown on the axis of the plot. All angles are chosen from a uniform distribution. The system is assumed to be coplanar.5

In Fig. 3(a), one can see bands and islands of stability for systems in MMRs. Examples are located at $a_2 = 2.62$ (3:2), $a_2 = 3.17$ (2:1) and $a_2 = 3.68$ (5:2). The location of the 4:3 MMR is also visible at $a_2 = 2.42$. This is a small stable island compared to the other resonances mentioned. Also note that the eccentricity for these stable solutions is very high $e_2 \sim 0.64$. However, this is also a function of the inner planet’s eccentricity which has been kept fixed. We present a slice of the parameter space in the $e_1, e_2$ plane in Appendix A.

The stable island that we attribute to the 4:3 MMR does not coincide with the best-fitting solution of Johnson et al. (2011). The eccentricities near the stable island are much higher than in the RV fit. Nevertheless, we verified that their solution is indeed stable. We conclude that their solution corresponds to fine-tuned initial conditions.

For systems with even higher mass planets than in the HD 200964 system, the stable regions get even smaller. This can be verified in Fig. 3(b). Note that the reported masses for HD 200964 are the minimum masses and could be significantly larger if the system is inclined with respect to the line of sight. However, this result suggests that the masses cannot be much larger in order for the system to be stable.

Two additional plots showing a slice in the $a_1, a_2$ plane of the parameter space are presented in Appendix A.

4 FORMATION MECHANISMS

In this section, we investigate potential methods for the formation of a pair of massive planets in a 4:3 resonance. In Section 4.1, we show that the long-standing idea of convergent migration fails to produce closely packed resonances for massive planets. We then consider the in situ formation of planets in Section 4.2, starting from small embryos in resonance which accrete mass from the protoplanetary disc. In Section 4.3, scattering and simultaneous damping is considered as one possible alternative to the cold formation scenarios mentioned above. Finally, we discuss alternative formation scenarios in Section 4.4.

We acknowledge that other plausible mechanisms may exist, some of which are mentioned in the discussion section.

4.1 Convergent migration in a disc

Migration of planets through a disc depends on numerous parameters such as the planet and disc mass, disc viscosity, surface-density profile, disc scale height and the equation of state, to just mention a few. This allows for the possibility of convergent migration of planetary orbits, during which pairs of planets can pass through orbital period commensurabilities. If the convergent migration rate is sufficiently low, the planets can capture into resonance (Goldreich 1965; Rein et al. 2010). In Section 4.1.1, we will simplify the migration process by assuming that we can describe it with only one semi-major axis and one eccentricity damping time-scale per planet. This allows us to understand the physical processes at work during resonance capture, study a wide parameter regime and not get lost in the complicated details of planetary migration. In Section 4.1.2, we will then relax some of these simplifications and study the formation of a planetary system near a MMR using hydro-dynamical simulations.

4.1.1 N-body simulations

We use the N-body code REBOUND (Rein & Liu 2012) to model the orbital evolution of two massive planets. We apply non-conservative forces to the outer planets which damp its semi-major axis and eccentricity (see Lee & Peale 2002, for details on the implementation of the non-conservative forces). We also refer the reader to the work by Rein et al. (2010) on an investigation of the 3:2 resonance in the HD 45364 system, which uses a similar methodology. We experimented with applying non-conservative forces to the inner planet simultaneously, but did not see any qualitative difference and will not investigate this further.

Initially, the planets start far apart from each other on circular orbits. The assumption of circular orbits is reasonable in the currently...

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5 We discuss the formation of an inclined system in Appendix B but observe no qualitative difference to the coplanar case.
favoured core accretion model of planet formation. Furthermore, the eccentricity damping time-scale is much shorter than the migration time-scale in a standard disc unless the eccentricity is very large (Muto, Takeuchi & Ida 2011). The planets are also assumed to be coplanar. Even though convergent migration and resonant capture can theoretically excite inclination (Tremaine & Yu 2000), the inclination damping time-scale is again much shorter than the migration time-scale as long as the planets are embedded in the disc, i.e. $i < 10^7$ (Rein 2011). We also tried relaxing this condition and ran additionally three-dimensional simulations with finite relative initial inclination between the planets. The results are shown in Appendix B. No significant changes can be observed. We therefore do not investigate this further.

The outer planet migrates inwards on a time-scale $\tau_{\mu}$ and might get captured into a resonance with the inner planet. In Fig. 4, we plot the final period ratios seen after the migration reached a quasi-steady state and both planets migrate inwards self-similarly. The simulations are conducted with different semi-major axis damping rates and eccentricity damping rates which are the axes of the plot. Each pixel represents one $N$-body simulation. The colour scheme has been chosen so that blue corresponds to a final period ratio close to 4:3, red is close to 3:2 and green is close to 2:1. The colour black indicates that either planets are captured in another resonance or that at least one planet got ejected. The latter is more common for short migration time-scales, $\tau_{\mu}$.

We find that it is essentially impossible to form systems in a 4:3 resonance via a simple convergent migration scenario with the observed mass ratio. There are two fundamental problems. First, a very high migration rate is required to allow the planets to pass through the 2:1 and 3:2 resonances. Secondly, there is only a small stable island in parameter space around the 4:3 resonance and the migration scenarios tend not to put the planets into this specific location.

To understand the evolutionary track, recall the stability plot in Fig. 3(a). In the migration scenario, the outer planet is initially on a circular orbit far away from the inner planet, that is, the bottom-right on the plot. Convergent migration brings the planets closer together.

On the plot, this corresponds to moving to the left along a horizontal line. Depending on the migration speed, the planet feels the resonant interaction from the inner planet and starts gaining eccentricity, thus moving upwards in the plot. If the migration rate is slow enough, the planet will get captured into a resonance which will protect it from close encounters. However, if the migration rate is so fast that the planet slips through all resonances, it ends up in an unstable regime (red region in the bottom-left of the plot). In fact, it has to go through an unstable region to get to the stable island that corresponds to the 4:3 resonance. This is why such a smooth migration scenario cannot form tight resonances for massive planets.

### 4.1.2 Hydro-dynamical simulations

We now go beyond the simple $N$-body model and use the two-dimensional hydrodynamic code FARGO (Masset 2000) to perform simulations of two gravitationally interacting planets that also undergo interactions with an accretion disc. This setup is used to further test the rapid migration hypothesis investigated with $N$-body simulations and parametrized migration forces in Section 4.1.1. Again, the approach is conceptually similar to that of Rein et al. (2010). We are in particular looking for more complicated effects such as varying migration rates that might be overlooked by running simplified $N$-body simulations.

Our simulations use a cylindrical grid with $N_r = 628$ and $N_\phi = 512$. The radial extent of the computational domain goes from 0.5 to 5 au. We use non-reflecting boundary conditions to minimize the effects from the finite domain size. Both the disc and planet properties are varied in the simulations and are listed in Table 3. We run a total of over 50 different simulations. All planets except those in simulations labelled inresonance (see Section 4.2 for a discussion of those) start initially on circular orbits. Some of the simulations start with small mass planets and allow for accretion of mass from the proto-stellar disc.

None of the simulations listed in Table 3 results in a stable 4:3 resonance. The simulations either capture into a 2:1 or 3:2 resonance or scatter due to close encounters. The result is therefore in perfect agreement with the $N$-body simulations of Section 4.1.1.

In Fig. 5, we show the evolution of the semi-major axis of both planets as solid lines. The dashed line shows the nominal position of the 4:3 resonance of the inner planet and is thus the semi-major axis we are so desperately trying to reach. As can be seen easily, we do not achieve this. In Fig. 6, we also plot the surface density profile of the disc at the end of the simulation. Note that the final simulation time varies significantly between the simulations. Whenever a steady or adiabatic state was achieved, we decided to stop the simulation. There are several physical reasons why we cannot get the planets in a 4:3 resonance with these kinds of migration scenarios.

First, the planets are very massive and therefore the resonances are strong. This prefers more widely separated resonances as has already been shown by the $N$-body simulations above. It can be verified in the simulations labelled vanilla, h0.07, slope0 and many others.

Secondly, if we force the planets to have an extremely rapid (and most likely completely unphysical) migration rate, the planets do indeed get closer initially. This can be seen in the simulation sigma8 that has a very massive disc which leads to a fast migration rate. Once in resonance, the planets start migrating outwards again. The same happens in the Grand Tack Scenario (Walsh et al. 2011).

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6 Note that we use the minimum masses and ignore possibly even higher mass ratios if the system is inclined with respect to the line of sight.

7 Not all are presented in the table.
After a while, the outer planets start migrating out very quickly due to the heavy disc and a Type III migration regime. This breaks the resonance. Shortly after the resonance is lost, the planets start moving in again, recapture in resonance and the whole cycle repeats. We never observe the planets capturing in a 4:3 resonance in this process.

Thirdly, the inner planet is massive and will open a gap in any reasonable disc model. The location of the 4:3 resonance is a factor of 1.2 away from the inner planet in terms of its semi-major axis. Note that the gap cannot be smaller than a few scale heights or Hill radii (Crida, Morbidelli & Masset 2006). However, depending on the precise planet mass and disc model, the outer planet might not open a gap on its own. In that case, it might get trapped at the gap edge. Migration stops or at least slows down, preventing the outer planet getting closer.

Fourthly, as Podlewska-Gaca, Papaloizou & Suszkiewicz (2012) point out, the interaction of the outer planet with the waves launched by the inner planet can cause additional torques if the outer planet does not open a gap on its own. This surfing effect tends to move the outer planet further away. This can be seen in simulations labelled sigma4_outer0.1_h0.03, sigma8_outer0.1, sigma4_m0.1, and a few others.

Of course this cannot be a complete survey of all possible scenarios even though we ran over 50 hydro-dynamical simulations. Also there are significant limitations and errors associated with such a numerical scheme. The resolution has been kept fixed and we did not check for convergence in every simulation. However, it seems very unlikely that this would lead to a completely different picture as not even one of our simulations comes even close to capturing the two planets in the 4:3 resonance for a significant amount of time.

To conclude, the results of our hydro-dynamical simulations are in agreement with the N-body simulations. They do not allow the capture of planets in the 4:3 resonance. In fact, due to gap opening and the surfing effect the problem of forming the resonance seems even harder to overcome than we would have estimated from N-body simulations.

4.2 In situ formation

While the presence of an MMR is generally taken as being the signature of convergent migration (see Section 4.1), perhaps the most conceptually simple scenario to consider is that the two planets in the 4:3 resonance formed directly from embryos with 4:3 period ratios. At early times with much lower masses, their mutual interactions would have been much weaker and some period of relatively unperturbed growth could have taken place. There is then the possibility that the pair of 4:3 planets which formed in situ could have migrated together through the disc to the currently observed locations.
Figure 5. Results of hydro-dynamical simulations. The solid lines show the semi-major axes of the inner and outer planet. The dashed line shows the semi-major axis corresponding to an outer 4:3 MMR with the inner planet.

We test this idea in a similar framework to that used in the last section. However, we do not start planets far apart from each other, but start them directly in the 4:3 MMR. To do this, we run an N-body simulation using REBOUND with two 10 $M_\oplus$ embryos. We choose a migration time-scale which results in a capture in the 4:3 MMR (see Section 4.1.1). After the planets have reached an equilibrium state and migrate together adiabatically, we switch to a full hydrodynamic simulation as in Section 4.1.2. We let the embryos interact with the disc and accreted mass. The accretion not only changes the mass but adds an additional component to the torque that is felt by the planet. The interaction with the disc is turned on adiabatically over several orbits. We vary the accretion rate, the mass of the outer planet and the slope of the initial surface density profile. The full list of initial conditions for these simulations is shown at the bottom of Table 3 under seed_inresonance, seed_inresonance_slow, seed_inresonance_slope0, seed_inresonance_outer4 and seed_inresonance_slope0_veryslow.

We plot the evolution of the semi-major axis and the mass of both planets in these simulations in Fig. 7. One can see that the 4:3 MMR is lost a few hundred to a few thousand years after the planets are inserted into the disc. The path to loosing the resonance is as follows. While the planets are acrretion, they are also migrating. They migrate initially in Type I, and later in Type II when their mass is sufficiently large. As the planets grow in mass, their dynamical interaction becomes stronger. The eccentricities of both planet rise. Eventually the resonance is lost. None of the planets gets ejected, but the interactions push them several Hill radii apart. Note that slowing down the process, as in simulation seed_inresonance_slope0_veryslow, does not prevent the resonance breaking. It merely delays it.

There are clearly many more parameters that one could vary. However, it seems unlikely that we can keep the planets locked in resonance while they grow in mass by more than one order of magnitude. This is because the mass that the embryos accrete has to come from the protoplanetary disc. The planets always interact with the disc, exchange angular momentum and start migrating.

4.3 Scattering and simultaneous damping

It has been shown in previous studies of planet–planet scattering (Chatterjee et al. 2008; Raymond et al. 2008) that the formation of MMRs in the aftermath of scattering between planets is possible, but that this is a relatively rare event (less than 1 per cent level). The low probability of capture into resonance in these investigations is likely a simple consequence of the fact that these simulations are conducted in the absence of any disc-gas damping (or other dissipative forces) and hence there is no means for the system to damp into resonance.
Traditional formation scenarios and the 4:3 resonance

Figure 6. Results of hydro-dynamical simulations. The plots show the gas surface density and the position of the two planets at the end of the simulation. Note that the final time varies significantly between different simulations.

Figure 7. Results of hydro-dynamical simulations of in situ formation. Top row: the solid lines show the semi-major axes of the inner and outer planet. The dashed (middle) line shows the semi-major axis corresponding to an outer 4:3 MMR with the inner planet. Bottom row: the mass of the planets as a function of time. The dashed line corresponds to the outer planet.

When planet–planet scattering simulations are conducted in the presence of a disc gas (e.g. Matsumura et al. 2010; Moeckel & Armitage 2012), the dissipative gas component means that a much higher fraction of systems are observed to form MMRs of a wide variety of orders (see their table 4). However, Matsumura et al. (2010) did not explicitly observe the creation of 4:3 resonances. We therefore investigate the scattering scenario further to understand whether such systems are simply rare (and thus statistically unlikely to be seen in their simulations), or whether the formation of a 4:3 resonance in this manner is essentially impossible.
We start from the assumption that an inner planet exists in an unperturbed, approximately circular orbit at a semi-major axis $a_1 \sim 1.6$ au. External to this, we assume that there existed an outer ($a \gg 2$ au) population ($N \geq 2$) of massive planets which became unstable and scattered a Jupiter-mass planet on to an orbit with a pericentre at $q_2 \sim 1.9$ au (the radial location of the 4:3 resonance with the inner planet at 1.6 au). We do not perform the lengthy, chaotic scattering simulations of this outer population, but rather assume that a scattering event has occurred with the outcome being that a 0.9 $M_{\text{Jup}}$ planet has been placed on to an orbit with $q_2 \sim 1.9$ au.

We consider that the outer body interacts with a remnant gas disc of some form, causing its orbit to damp, reducing in eccentricity and semi-major axis. In some fraction of systems with suitable damping terms, while the outer body is simultaneously circularizing and migrating inwards, the two planets may become trapped into a 4:3 MMR.

Figure 8. Evolution of two planets in the aftermath of a scattering event with a third planet (not modelled). The top plot shows the semi-major axes, the pericentres and apocentres as well as the period ratio as a function of time. The bottom plot shows the 4:3 resonance angles.

To investigate this scenario, we perform $N$-body integrations which include the effect of gas damping to investigate whether the system described above can be captured into a 4:3 resonance. The full details of the manner in which the simulations are specified are detailed in the following section.

4.3.1 Simulation methodology

We initialize each simulation with conditions approximating some precursor of the HD 200964 system. The central star has a mass of $M_* \sim 1.44 M_\odot$. The inner planet has a mass of $m_1 = 1.8 M_{\text{Jup}}$ and is located at $a_1 = 1.6$ au on a circular orbit, $e_1 = 0$. The outer planet has a mass of $m_2 = 0.9 M_{\text{Jup}}$. The pericentre $q_2$ is randomly chosen from a flat distribution $1.6 < q_2 < 2.2$ au. Note that the pericentre lies close to the location of the 4:3 resonance with the inner planet. The outer planet’s eccentricity is also randomly drawn from a flat distribution $e_{\text{out}} < e < 0.9$, where $e_{\text{crit}} = 1 - q_2/2.54$. Thus, the outer planet is constrained to start with a semi-major axis outside the location of the 2:1 resonance with the inner planet (at 2.54 au).

We parametrize the damping in the same manner as in Section 4.1.1, i.e. following the approach of Lee & Peale (2002) by calculating the Jacobi orbital elements. The semi-major axis $a$ and the eccentricity $e$ are then directly damped in the parametrized form $\dot{T} = K \frac{a}{a}$, where the damping rate $\dot{a}/a$ and the ratio of eccentricity damping to semi-major axis damping, $K$, are varied as free parameters. We implement this damping in a modified version of the Mercury integration package (Chambers 1999). Each simulation uses the hybrid-symplectic integrator in Mercury, and integrates the systems for a total time period $T_{\text{final}} = 100 a / \dot{a}$.

We perform 100 versions for each of the following 64 parameter combinations, thus performing a total of 6400 individual simulations. The damping rates $\dot{a}/a$ that we choose are $10^{-6}$, $10^{-5}$, $10^{-4}$ and $10^{-3}$ yr$^{-1}$. We investigate four values of $K$, 0.01, 0.1, 1 and 10. We perform simulations in which the damping occurs on either only the outer planet or both planets. We also vary the location within which the gas damping operates: either over a wide range $r < 50$ au, or in a small disc $r < 2.5$ au. The latter implies that the outer planet will initially only be damped when very close to pericentre.

4.3.2 Simulation results

We plot an example of a successful damping simulation in which an initially highly eccentric planet is subsequently made to damp and migrate into a 4:3 MMR with the inner planet in Fig. 8. In the example shown, the outer planet initially had orbital parameters such that $a_2 = 3.59$, $e_2 = 0.53$ and therefore $q_2 = 1.66$. The damping model was such that $\dot{a}/a = 10^{-5}$ yr$^{-1}$ and $K = 1$, with this damping operating on the outer body only for all $r < 50$ au. Over the first $t \sim 6 \times 10^7$ yr of the simulation, the outer planet can be seen to continuously suffer close encounters with the inner planet (kicking both semi-major axes and eccentricities to new values), while also experiencing rapid damping and inward migration due to the gas interactions. Finally at $t \sim 6 \times 10^7$ yr the damping component wins and the planet definitively captures into a 4:3 resonance with the inner planet. We plot the resonant angles $\theta_3, \theta_4$ and $\theta_{3+4}$ in the bottom of Fig. 8. They are defined as

\begin{equation}
\begin{align*}
\theta_3 &= 4(\lambda_2 - \sigma_2) - 3(\lambda_1 - \sigma_1) + 3(\sigma_2 - \sigma_1) \\
\theta_4 &= 4(\lambda_2 - \sigma_2) - 3(\lambda_1 - \sigma_1) + 4(\sigma_2 - \sigma_1) \\
\theta_{3+4} &= \theta_3 - \theta_4,
\end{align*}
\end{equation}

where $\lambda_i$ and $\sigma_i$ are the mean longitude and longitude of periapse of the $i$th planet, respectively. We observe that the resonant angle $\theta_3$ is librating.

The results of all 6400 individual simulations performed are shown in Fig. 9. The plots show the fraction of systems in a certain resonance as a function of the damping time-scales. We marginalize over the other parameters. Approximately 1 per cent of all systems are captured into the 4:3 resonance. Shorter damping time-scales in both semi-major axis and eccentricity tend to result in higher capture fractions into the 4:3 resonance.

However, we note that if this model is at all realistic, then we also predict that the fraction of systems occupying a 1:1 resonance would be higher than the fraction of systems in a 4:3 resonance. One can see from Fig. 9 that the fraction of systems in a 1:1 resonance is more than three times as large as the fraction of systems in a 4:3 resonance. An example of a capture into a 1:1 resonance is illustrated in Appendix C. Such resonances were examined by Laughlin & Chambers (2002) and shown to be detectable by Kepler (Ford & Gaudi 2006). This is consistent with simulations of
Figure 9. The fraction of scattering simulations which were subsequently captured into various MMRs as a function of the damping parameters $K$ and $r_a$.

There are few investigations dedicated to measuring four-body resonances outside of the well-studied Laplace resonance involving Io, Europa and Ganymede. Nesvorný & Morbidelli (1998a,b) considered the possible formation of resonant chains due to the presence of a disc. Raymond, Armitage & Gorelick (2010) find that a planetesimal disc can easily produce 4:2:1 four-body resonances. Like Matsumura et al. (2010), Moeckel & Armitage (2012) model planet–planet scattering during dissipation of a gas disc. The latter find that almost 45 per cent of their three-planet simulations which remain stable achieve a 9:6:4, 6:3:2, 3:2:1 or 4:2:1 four-body resonance. Although no pair of planets in these simulations reduces to a 4:3 MMR, this prevalence of resonant chains suggests that a 4:3 MMR may be formed in this manner.

Another mechanism not directly considered in this paper is that of a chaotic migration scenario. Here the continued inward migration of planets in a 3:2 MMR causes the eccentricity of the planets to increase to such an extent that the planets pass out of the portion of parameter space in which stable libration is possible, and into a region of chaotic scattering. In such a scenario, it is generally envisaged that without the protection afforded by being in a MMR, the planets will scatter one another during subsequent close approaches and the stable resonant configuration will be destroyed. However, as the planets move into the region of chaotic scattering, an external perturbation (conceptually due to interactions with a disc or scattering from a small planetary embryo) might act to kick the planet from the chaotic overlap region down into the 4:3 MMR region. It is clear that in order for this scenario to realistically occur, the time-scale for a stochastic scattering event must be shorter than the time-scale over which chaotic scattering can start to disrupt the system. Detailed investigation would be required to understand whether such a scenario is at all possible, and more likely than the proposed scattering and damping scenario (see Section 4.3).

5 SMALL MASS PLANETS

In this section, we extend the previous calculations and consider systems with small mass planets. The motivation for this is the majority of systems in the Kepler sample which have a much smaller mass than those discovered by RV (see Section 2.2.2). We will show that such systems can easily form via a variety of mechanisms and the problems discussed in the last section do not arise.

We first look at the stability map of small mass planetary systems in Section 5.1. Then we go on and study the traditional migration-capture scenario with N-body simulations in Section 5.2. In Section 5.3, we look at the possibility of growth of isolation mass embryos through direct collisions.

5.1 Stability map with direct N-body simulations

The stability plots in Section 3.2 were dominated by large unstable regions and a small stable island associated with the 4:3 resonance. For two Earth mass planets, the stability plot looks very different, as shown in Fig. 10. Here, the parameter space of stable regions is much larger. More structure is visible due to the presence of higher order resonances. In principle, we can now capture into each of these resonances. In practice, the capture probability depends of course on the migration rate. We study this in the following section.

5.2 Convergent migration in a disc

In Fig. 11, we plot the same quantity as in Fig. 4 but the mass of the outer planet has been reduced by a factor of 10. This leads to smaller critical migration rates required to pass through the 2:1 and 3:2 resonance. It also increases the stability of the system and thus enables us to form some stable systems which stay in the...
4.3 resonance. It is again helpful to recall the stability plots in Figs 3(a) and 10 to understand the differences in the evolutionary track. The outer planet starts on the far right bottom of Fig. 10. It migrates inwards (moving to the left) and encounters various commensurabilities. Depending on the migration rate it captures into any of these resonances and gains eccentricity in the process (moves up). In contrast to the high mass example in Fig. 3(a), the resonances are stable (dark colour) and the planet does not have to cross an unstable part of the parameter space to get there. Note that the migration rates required are still very high, $\tau_d \sim 200–800$ yr. This corresponds to a Type III migration regime and is most likely not a realistic outcome of planet–disc interaction for these (reduced!) mass ratios. This is the opposite to the reasoning of Rein et al. (2010), where the mass ratios are such that Type III migration is considered to be the most likely formation scenario.

5.3 Growth of isolation-mass embryos

The end stage of the runaway (Wetherill & Stewart 1989; Kokubo & Ida 1996) and oligarchic growth (Ida & Makino 1993; Kokubo & Ida 2002) phases of planetary embryo growth is envisaged to be a series of isolation-mass embryos. These are a series of embryos each having accreted all solid material within an annulus of width $c_{iso}r_H$ of their location. Here, $c_{iso}$ is a dimensionless constant of order 10 and $r_H$ is the Hill radius of the embryo. For a power-law surface density profile $\alpha \Sigma_0 a^{-\alpha}$, where $\alpha$ denotes the slope of the power law, the mass of the isolation mass embryos can be written as (e.g. Ida & Lin 2004)

$$m_{iso} = 5 \times 10^{-3} c_{iso}^{3/2} \left( \frac{\Sigma_0}{10 \text{ g cm}^{-2}} \right)^{3/2} \times \left( \frac{a}{1 \text{ au}} \right)^{2-\alpha} \left( \frac{M}{M_\odot} \right)^{-1/2}.$$

We will use this as a starting point of our integrations.

5.3.1 Methods to simulate the growth of isolation-mass embryos

We follow a method conceptually similar to that employed in Zhou, Lin & Sun (2007) and Hansen & Murray (2011). We consider that a series of isolation-mass embryos has formed as described above. We then simulate their subsequent evolution as they excite, scatter and collide over $\sim 10^7$ yr.

The initial conditions used in equation (2) are set to be $\Sigma_0 = 10 \text{ g cm}^{-2}$, $c_{iso} = 7$ and $\alpha = 3/2$. We distribute the embryos over annuli with widths either $0.55 < a < 1.75 \text{ au}$ (Set A) or $0.1 < a < 2.2 \text{ au}$ (Set B).

The embryos are damped for the first $10^5$ yr of their evolution assuming an interaction with a gas disc of surface density of $\Sigma_0 = 2400 \text{ g cm}^{-2}$ using the same eccentricity damping model as that employed in Mandell, Raymond & Sigurdsson (2007). The gas disc is then allowed to dissipate on a time-scale, $\tau_d$, with this simply being modelled as a reduction in the damping force by a factor $e^{-t/\tau_d}$. We use $\tau_d = 10^5$, $10^6$ and $10^7$ yr for different subsets.

Typically, this results in the number of embryos $N$ remaining approximately constant for the first $10^5$ yr as the strong damping prevents many crossing-orbits developing. In the case of Set A, $N$ is approximately 40, and in the case of Set B, $N \sim 130$. As the damping begins to decrease (the gas disc dissipates), the embryos begin to excite and collide with one another, reducing the number of bodies present in the simulation. The final number of planets varies across the simulations but is typically between 5 and 7.

5.3.2 Results from N-body simulations of the growth of isolation-mass embryos

The results of a simulation run from Set A with $\tau = 10^6$ yr are given in Fig. 12. Fig. 12(a) shows the evolution of the periods as a function of time. At $\sim 10^6$ yr, the fourth and fifth planets from the star (highlighted in the example) have period ratios $\sim 1.33$, i.e. close to the 4:3 MMR period ratio. In Fig. 12(b), we show the resonant angles $\theta_3, \theta_4$ and the secular resonance angle $\theta_3,4$ defined in equation (1) for the fourth and fifth bodies from the star. The resonant angles are observed to circulate over the full $360^\circ$ range, while the secular angle librates for extended periods (although over very long time periods it too can be seen to circulate). Understanding in detail the strengths of these resonances and the fraction of time that the planets spend in (or adjacent to) these MMRs as a function
of (e.g.) the overlap of three-body resonances requires much more detailed analysis in the manner of Quillen (2011).

It is also of interest to note that in the simulations we performed, more closely spaced planetary configuration also arose. In Fig. 13, we illustrate the results of a simulation run from Set A with $\tau = 10^7$ yr. In this system, 10 low mass bodies survive at $t \sim 5 \times 10^5$ yr. These have period ratios 1.34, 1.24, 1.23, 1.32, 1.37, 1.17, 1.33, 1.32 and 1.19. Hence, multiple pairs are close to the 4:3 (1.33), 5:4 (1.25) and 7:6 (1.17) MMR period ratios. We have checked (but do not plot) the various possible two-body resonant angles for the respective pairs of planets close to the listed MMRs. We find that none of the bodies occupy exact two-body resonances despite their period ratios being suggestively close.

We thus find that the formation of rather closely spaced systems (i.e. with period ratios $1.15 \rightarrow 1.4$) is easily accomplished through the collisional evolution of isolation-mass embryos in an extended catastrophic collision phase of planet formation. But we emphasize that period commensurabilities do not equate to two-body resonances.

The total embryo mass considered in these simulations was low. For Set A, we have $\sim 3.0 \, M_\oplus$ and for Set B $\sim 6.0 \, M_\oplus$. This corresponds to solid surface densities consistent with a minimum mass nebular model. The absolute masses of the planets formed at $t \sim 10^9$ yr are subsequently low ($0.5 \, M_\oplus$ and $0.55 \, M_\oplus$ for the example in Fig. 12, 0.1--0.54 $M_\oplus$ for the bodies in Fig. 13). While these masses are likely smaller than those listed in Table 2, they are still within the range of detectable masses for Kepler (e.g. KOI-961; see Muirhead et al. 2012). All of the systems that we were able to form have multiple planets. We also note that Hansen & Murray (2011) found nothing conceptually different in their simulations performed at much higher absolute surface density normalizations. These simulations resulted in mass-period distributions approximately resembling those of the observed Kepler systems.

We emphasize that these simulations have run for $t \sim 10^9$ yr. For the majority of this time period, the planets have been undamped. It is therefore realistic to consider these systems as being in an old, evolved state as might be observable by the (e.g.) Kepler mission. This mechanism may therefore explain the KOIs listed in Table 2.

The growth of planets from isolation-mass embryos seems to provide a natural means by which low mass planets can be either captured into closely spaced MMRs (i.e. 4:3), or have period commensurabilities approximately similar to such closely spaced MMRs while also remaining long-term stable. We emphasize that while such close spacing (period ratios less than 1.4) does not happen in the majority of systems simulated, it was common enough for us to observe it in 2 out of 60 systems simulated.

6 CONCLUSION

In this paper, we explored numerous methods of forming a 4:3 MMR in systems of both high mass planets. We found that it is extremely difficult to form stable massive systems in a 4:3 resonance. The discovery of multiple such systems in RV surveys leads us to conclude that traditional formation methods are failing to reproduce the observed fraction of systems that are in or close to this resonance. Moreover, we found that convergent migration due to torques imposed by a gas disc is not a viable mechanism for the formation of massive planets in a 4:3 resonance. In Section 4.1, we showed that it can be ruled out to form the system HD 200964 in this way. There are four reasons for this. First, the systems tend to lock into higher order resonances. Secondly, even with reduced masses the required migration time-scale to pass through resonances such as 2:1 and 3:2 is unphysically short. Thirdly, gap opening tends...
to stall migration at gap edges which prevents subsequent capture into close-in resonances. Fourthly, the surfing effect pushes planets away from each other unless both planets are able to open a clean gap. We conducted a large survey of hydro-dynamical simulations. But we did not find a way to realistically form a 4:3 resonance in any of these hydro-dynamical models.

It is important to note that these are planets detected by the RV method and thus the masses are only minimum masses. However, we found that masses higher than the minimum masses make it even harder to form a stable resonance.

In situ formation where planet embryos start out in the 4:3 resonance is also not a viable formation mechanism. In Section 4.2, we conducted hydro-dynamical simulations of such a scenario. There is always a phase when the planets migrate divergently during their mass accretion phase. As soon as the planets move apart from each other, outside of the 4:3 resonance, they cannot recapture into the resonance at a later stage because they preferentially capture in wider resonances. Gap opening and the tidal surfing effect are further preventing planets from staying or recapturing in a tight resonance while being embedded in a gas disc.

A combined scattering and damping mechanism does seem to be a plausible means of forming giant planets in closely spaced MMRs as we showed in Section 4.3. We found that for a wide range of damping parameters, close-in resonance can form. However, the initial conditions are finely tuned to allow such a capture. We estimate that the fraction of planetary systems that might end up in such a configuration is much smaller than the currently observed fraction of approximately 4 per cent. This scenario also predicts the formation a large number of planets in a 1:1 co-orbital resonance which is not consistent with current observations.

We extended the study to include small mass planets in the foresight of several Kepler planet candidates which are close to a 4:3 period ratio. We found that there is no problem in forming any of these smaller mass planets with a traditional migration scenario. We therefore expect that several of these systems will be confirmed by follow-up observations.

The main result in our paper is a negative one, i.e. we cannot explain the formation of the observed systems. One could use our results as evidence against the existence of massive planets in close-in resonances and search for other explanations of the observed RV signals. But this requires a great amount of caution. Using the same argument, we could have ruled out the first discovered Hot Jupiter. Rather, we hope that this study will guide future investigations and spur interest in these systems among the community.

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on 26 July 2018
Traditional formation scenarios and the 4:3 resonance

APPENDIX A: ADDITIONAL STABILITY PLOTS

In this appendix, we show additional stability plots of the phase space of two massive planets. The procedure is described in Section 3.2.

Fig. A1 shows two additional stability plots of the phase space in the $a_1$–$a_2$ plane. The initial eccentricities are 0.1 and 0.5 for the top and bottom plots, respectively. All other angles are drawn from a uniform distribution. The system is assumed to be coplanar. One can see that there is clear evidence of resonances as the stability only depends on the period ratio (not the individual periods) and the eccentricities of the planets.

Fig. A2 shows an additional stability plot of the phase space in the $e_1$–$e_2$ plane. The planets’ semi-major axes are initially at $a_1 = 2$ and $a_2 = 2.42$, i.e. close to a 4:3 period ratio. We find stable islands for a wide variety of eccentricities as long as $e_1 < 0.55$. One can see a clear anti-correlation of the eccentricity of the inner and outer planets.

APPENDIX B: FORCED MIGRATION FOR INCLINED PLANETS

We show the final period ratio of two convergently migrating planets in Fig. B1. The simulations are identical to those presented in Fig. 4 but are fully three dimensional and include a finite initial inclination of $i = 5^\circ$ between the planets.
There is no inclination damping present in this simulation. However, in contrast to Tremaine & Yu (2000), we include explicit eccentricity damping which prohibits substantial eccentricity and inclination growth.

One can see no qualitative or quantitative change in the results compared to the coplanar case. This allows us to restrict ourselves to the coplanar formation scenarios presented above which reduces the number of free parameters in the initial conditions.

**APPENDIX C: ILLUSTRATION OF CAPTURE INTO A 1:1 RESONANCE**

In Fig. C1, we plot an example of the capture of two giant planets into a 1:1 resonance. The simulation methodology is exactly as described in Section 5.3.1. The initial conditions for the inner planet are \( m_1 = 1.8 M_{\text{Jup}} \), \( a_1 = 1.6 \text{ au} \) and \( e_1 = 0 \), and for the outer planet \( m_2 = 0.9 M_{\text{Jup}} \), \( a_2 = 3.6 \text{ au} \) and \( e_2 = 0.46 \) (\( q_2 = 1.92 \)).

The damping parameters are the same as those applied for Fig. 12. The migration time-scale is \( \tau = 10^5 \text{ yr} \) and \( K = 1 \). The disc removal time-scale is \( \tau_d = 10^7 \text{ yr} \). The damping is operating on the outer body only for all \( r > 1 \text{ au} \).

The outer planet migrates inwards over the first \( 10^6 \text{ yr} \) of the simulation, while simultaneously suffering occasional strong perturbations from the inner planet. At \( t \sim 6 \times 10^5 \text{ yr} \), the pericentre of the outer planet drifts inside the apocentre of the inner planet and a period of strong interaction and rapid inward migration commences, leading to capture into the 1:1 resonance. The resonance remains stable until the end of the simulation.

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