Entropic force, running gravitational coupling and future singularities

Maryam Aghaei Abchouyeli† and Behrouz Mirza‡
Department of Physics, Isfahan University of Technology, Isfahan, 84156-83111, Iran
Zeinab Sherkatghanad‡
Department of Physics, Isfahan University of Technology, Isfahan, 84156-83111, Iran
Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, Canada, N2L 3G1

The effects of a running gravitational coupling and the entropic force on future singularities are considered. Although it is expected that the quantum corrections remove the future singularities or change the singularity type, treating the running gravitational coupling as a function of energy density is found to cause no change in the type of singularity but causes a delay in the time that a singularity occurs. The entropic force is found to replace the singularity type II by $\frac{7\pi}{4}$ (a = const., $H = \infty, p = \infty, \rho = \infty$) which differs from previously known type III and to remove the $w$-singularity. We also consider an effective cosmological model and show that the types I and II are replaced by the singularity type III.

I. INTRODUCTION

Both the theoretical cosmology and observational data indicate that our accelerated expanding universe can be described by an equation of state (EOS) parameter $w$ around $-1$. In general, when the universe passes through the $\Lambda$CDM epoch, $w$ is exactly $-1$. However, a phantom-dominated universe can be described by $w$ around 1 while if $w$ is slightly more than $-1$, the quintessence dark epoch occurs. In the region where barotropic index $w$ is lower than $-1$, a future singularity may occur in the form of an infinite scale factor, energy density, and pressure which has come to be called the "big rip". Other types of singularities have been explored, increasing the family of candidates. It is known that different types of singularities may arise during the expansion of the universe, which may be classified as follows: Type I (Big Rip): Infinite $a$, $\rho$, and $p$. Type II (Sudden): Finite $a$, $H$, and $p$; divergent $\dot{H}$ and $p$. Type III (Big Freeze): Finite $a$; infinite $H$, $\rho$, and $p$. This singularity type is a subcase of Finite Scale Factor singularity Type IV: Finite $a$, $H$, $\dot{H}$, $\rho$, and $p$ but infinite higher derivatives of $H$. Type V: Infinite $w$ (barotropic index): By expanding the scale factor around the time of the singularity, we get $a(t) = c + (t - t_s)^{n_1} + (t - t_s)^{n_2} + ...$, where $n_1 < n_2 < ...$ and $c > 0$. In this way, $w$-singularity may occur without any divergence in pressure or energy density with the following properties:
(i) $p_s \neq 0$: $n_1 = 2$ and $n_2 = [3, \infty)$,
(ii) $p_s = 0$: $n_1 = [3, \infty)$ and $n_2 = [n_1 + 1, \infty)$.

$w$-singularity was first proposed in [14] with a different form of the scale factor.

According to Tipler’s definition [15], the big rip (Type I) singularity is strong whereas types II, III, IV, and $V$ are weak. Based on Krolak’s definition [16], however, only types II, IV, and $V$ are weak [17]. Conformal anomaly effects can change the type of singularity but bulk viscosity merely decreases the time when singularities may happen [18, 21].

We may adopt an approach in which the concepts of information and holography play the central role. By holography is meant a situation in which all the information of a volume $V$ can be encoded on its boundary screen. In this situation, the effect of entropic force can produce the generalized Friedman equations to yield a new type of future singularity. The entropic force will change the singularity from Type II to $\frac{7\pi}{4}$ (a = const., $H = \infty, p = \infty, \rho = \infty$) which differs from previously known type III.

In general, the gravitational coupling is assumed as a universal constant, but the idea of variation of physical constants such as the tational coupling $G$, the charge of the electron $e$, the velocity of light $c$ has been investigated in both theoretical and experimental physics [29-41] and might be effective to avoid singularities [41]. In this paper we study an asymptotically safe scenario as a quantum effect to avoid some exotic behaviors of the universe near the singularity time. Considering Gravitational coupling as a function of energy density in the asymptotically safe scenario [30, 42] does not change the type of future singularity but only delay the time while a singularity appears.

This paper is organized as follows: Sec. II investigates the singularities of the generalized Friedman equations with a running gravitational coupling. Sec. III examines the effect of entropic force on future singularities. The effects of both running gravitational coupling and entropic force on the different types of singularities are studied in Section IV. In Sec. V, an effective cosmological model is used to study future singularities. It is expected that a
quantum correction removes the predicted future singularities of the universe or at least changes the singularity type.

II. RUNNING GRAVITATIONAL COUPLING AND THE FUTURE SINGULARITIES

A running gravitational coupling may represent some essential features of an asymptotically safe gravitational model [30]. An asymptotically safe gravitational model is important because of its quantum background. It is expected that some quantum corrections on General Relativity can remove the predicted future singularities of the universe or at least change the singularity type. In an asymptotically safe model the gravitational coupling \( G \) should be a function of an energy scale, so the more usual choice energy scale is energy density and the energy density is a function of time. Thus the gravitational coupling can be taken as \( G(\rho) \equiv G(\rho(t)) \), where \( G(\rho(t)) \sim \rho(t)^{-\frac{\alpha}{2}} \) and \( \alpha \geq 2 \), which represent a kind of asymptotically safe model [30].

Therefore, the pressure \( p(t) \) and energy density \( \rho(t) \) for this form of running gravitational coupling are given by \( \rho = 1 \)

\[
\rho(t) = \left( \frac{3}{8\pi} H^2(t) \right)^{\frac{1}{1-\beta}},
\]

(4)

\[
p(t) = -\frac{1}{4\pi} \left( \frac{2H^2(t) + \frac{dH(t)}{dt}}{2} \right) \frac{3}{8\pi} H^2(t)^{\frac{1}{1-\beta}},
\]

(5)

where, \( \beta = \frac{2}{1-\alpha} \) and, for asymptotic safety \( \beta \geq 0.5 \).

Now let us consider the effect of running gravitational coupling on future singularities. Since the energy density is divergent in Type I and III singularities, we can consider the effect of a running gravitational coupling on these types of future singularities. In general, the scale factor \( a(t) \) associated with Type I for the standard Friedman equations can be expressed as follows [6]

\[
a(t) = a_0(t - t_s)^n,
\]

(6)

where, \( n \) is a positive constant and \( 0 < t < t_s \). In this situation, the scale factor diverges at a finite time \( (t \to t_s) \) where \( t_s \) is the instant of singularity. For the scale factor in Eq. (6), the behaviors of the energy density and pressure in Eqs. (3) and (4), respectively, show that Type I does not change in response to the effect of a running gravitational coupling (Fig. 1). This type of singularity is so strong that it cannot be affected by an asymptotically safe scenario; however, the larger values of \( \beta \) near the singularity causes a delay in the time the singularity appears.

The scale factor related to Type III singularity is given as follows [10, 26]:

\[
a(t) = 1 - (1 - \frac{t}{t_s})^n + (\frac{t}{t_s})^q(a_s - 1),
\]

(7)

For the special values \( n < 1 \) and \( 0 < q < 1 \), Type III singularity occurs and the strong and weak energy conditions are violated. Therefore, Type III singularity should remain intact by considering a running gravitational coupling in an asymptotically safe scenario. However, our results show that larger values of \( \beta \) or a weaker gravitational force for the singularity types I and III

FIG. 1: Pressure \( p \) and energy density \( \rho \) in an asymptotically safe scenario with respect to \( t \) for Type I singularity for \( n = 3 \), \( t_s = 10000 \), and \( \beta = 0.5 \) [red (dashed) line], 0.6 [blue (dashed-dotted) line], and 0.7 [black (solid) line].

Following [29, 41], we consider a Running gravitational coupling varying by time. The generalized Friedman equations are given by

\[
3H^2 = 8\pi G(t)\rho,
\]

(1)

\[
\frac{\dot{a}(t)}{a(t)} = -4\pi G(t)(\rho + \frac{\rho}{3c^2}),
\]

(2)

where \( a(t) \) is the scale factor and the dots is the divertive with respect to time. In this condition the Bianchi identity with respect to the covariant derivative of each side of the Einstein field equation in this way

\[
\dot{\rho} + 3H(\rho + \frac{\rho}{c^2}) = -\rho \frac{\dot{G}(t)}{G(t)},
\]

(3)
correspond to a larger value of $t_s$, which means a delay in the appearance of the singularities. Thus, we may expect that an asymptotically safe running gravitational coupling (as a quantum effect) does not change the singularity types nor remove any of the singularities. Pressure $p(t)$ and energy density $\rho(t)$ are depicted in Fig. 2.

III. THE EFFECT OF ENTROPIC FORCE ON THE FUTURE SINGULARITIES

In this Section, we consider the effect of entropic force on future singularities. Easson, Frampton and Smoot obtained the modified Friedman equations by considering the entropic force scenario, which introduces entropic force as a result of surface effects [23–25]. Thus, the Friedman equations are generalized to the following form [23]

$$\ddot{a}(t) = -\frac{4\pi G}{3}(\rho + 3p) + c_1 H^2(t) + c_2 \dot{H}(t)$$

$$H^2(t) = \frac{8\pi G}{3}\rho + c_1 H^2(t) + c_2 \dot{H}(t),$$

Here, the coefficients $c_1$ and $c_2$ are determined by observations [24, 25]. In this way, energy density $\rho(t)$ and pressure $p(t)$ are given by

$$\rho(t) = \frac{3}{8\pi G}[H(t)^2 - c_1 H(t)^2 - c_2 \dot{H}(t)],$$

$$p(t) = \frac{1}{4\pi G}[-3 + 3c_1 H(t)^2 + \frac{-2 + 3c_2}{2} \dot{H}(t)].$$

Now let us consider the effect of entropic force on the future singularities. For the scale factor related to Type I in Eq. (6), we can plot $\rho(t)$ and $p(t)$ in Eqs. (10) and (11) with respect to $t$. The results indicate no change for Type I singularity. Also, the sudden singularity (Type II) occurs for the scale factor represented in Eq. (7) when $1 < n < 2$ and $0 < q < 1$ and the dominant energy condition is violated [18]. Using the scale factor for Type II singularity and the energy density $\rho(t)$ and pressure $p(t)$ in Eqs. (10) and (11) respectively, we find that the Hubble parameter $H$ is finite although $\dot{H}$, $\rho$, and $p$ are infinite. $\rho$ diverges as a result of $\dot{H}$ term in Eq. (11). Thus, for all positive values of $c_1$ and $c_2$ a new type of singularity is taking place which we call it type III (Fig. 3). It is similar to Type III but only with a different
TABLE I: Violated energy conditions for \(q = 0.5, n = 1.5\) in an entropic force scenario.

| \(c_1\) | \(c_2\) | \(\frac{c_1}{c_2}\) | Violated energy conditions | Singularity |
|--------|--------|-----------------|---------------------|--------------|
| 0 < \(c_1\) < 1 | 0 < \(c_2\) < 1 | > 1 | DEC and DNEC | \(II \rightarrow TTT\) |
| 0 < \(c_1\) < 1 | 0 < \(c_2\) < 1 | < 1 | No | \(II \rightarrow TTT\) |
| \(c_1\) > 1 | \(c_2\) > 1 | > 1 | No | \(II \rightarrow TTT\) |
| \(c_1\) > 1 | \(c_2\) > 1 | < 1 | No | \(II \rightarrow TTT\) |
| \(c_1\) > 1 | 0 < \(c_2\) < 1 | > 1 | No | \(II \rightarrow TTT\) |

FIG. 4: \(w\) with respect to \(t\) (entropic force scenario) for \(n = 2, q = 3.5, c = 4, c_1 = 0.2, c_2 = 0.01,\) and \(t_s = 10000.\)

behavior of \(H\). This new type can be characterized by finite \(a\) and \(H\) as well as infinite \(\dot{H}, \rho,\) and \(p\). Turned into Krolak’s definition, a weak singularity (Type \(II\)) turns into a strong singularity (Type \(III\)) \[15\]. Furthermore, the violated energy conditions are sensitive to the values of coefficients \(c_1\) and \(c_2\). The results are presented in Tables I and II.

For Type \(IV\) and \(w\)-singularity, the scale factor is given by \[8\]

\[
a(t) = c + (t_s - t)^n + (t_s - t)^q, \tag{12}
\]

where, \(c\) is a positive constant. The special values of \(2 \leq n < \infty\) and \(n < q < \infty\) correspond to the singularity Type \(IV\). Also the \(w\)-singularity occurs when: i) \(n = 2\) and \(q = (3, \infty),\) ii) \(n = (3, \infty)\) and \(q = [n + 1, \infty).\)

Our results also show that the singularity Types \(III\) and \(IV\) which are described by the scale factors in Eq. \[7\] and Eq. \[12\], respectively, remain intact. It is interesting that the effect of entropic force removes the \(w\)-singularity (Fig. \[3\]). It should be noted that the presence of \(\dot{H}\) in Eqs. \[10\] and \[11\] is responsible for the removal of the \(w\)-singularity in entropic force scenario.

IV. THE EFFECTS OF BOTH THE ENTROPIC FORCE AND RUNNING GRAVITATIONAL COUPLING ON FUTURE SINGULARITIES

In the previous sections, we showed that the running gravitational coupling does not change the type of singularity. However, the effect of entropic force changes the singularity type from \(II\) to \(TTT\). Now, we account for the effects of both running gravitational coupling and
entropic force on Friedman equations
\[ \frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi}{3} \rho^{-\beta}(\rho + 3p) + c_1 H^2(t) + c_2 \dot{H}(t) \]

Thus, Eqs. (10), (11) reduce to
\[ \rho(t) = \left[ \frac{3}{8\pi}(H(t))^2 - c_1 H(t)^2 - c_2 \dot{H}(t)) \right]^{\frac{1}{\gamma}}, \]
\[ p(t) = \frac{1}{4\pi} \left( \frac{-3 + 3c_1}{2} H(t)^2 + \frac{-2 + 3c_2}{2} \dot{H}(t) \right) \]
\[ \times \left[ \frac{3}{8\pi}(H(t))^2 - c_1 H(t)^2 - c_2 \dot{H}(t)) \right]^{\frac{2}{\gamma}}. \]

Our asymptotically safe scenario is not relevant to the singularity types II, IV, and V as the energy density remains finite at the time the singularity appears. Using the scale factor in Eq. (10) for the singularity type I and Eq. (7) for type III, we can plot the energy density \( \rho(t) \) and pressure \( p(t) \) in Eqs. (15) and (16) with respect to time. In this case, neither Type I nor Type III change as a result of the effects of either running gravitational coupling or entropic force. These types are classified as strong singularities along the lines of Krolak’s definition [13, 16].

Our results indicate that larger values of \( \beta \) that represent a weaker gravitational force for the singularity types I and III still correspond to a larger value of \( t_s \) or a delay in the time that the singularities appear and further that the entropic force increases this effect of running gravitational coupling (Fig. 5). This is a novel behavior and characterizes the effects of both running gravitational coupling and entropic force on singularity types I and III.

### V. AN EFFECTIVE COSMOLOGICAL MODEL

In this section, we consider an effective cosmological model which is a map from the generalized equations to the standard Friedman equations. Thus, Eqs. (13) and (14) can be expressed as
\[ H^2_{\text{eff}}(t) = \frac{8\pi}{3} \rho, \]
in compare with Friedmann Equations in general relativity, where,
\[ H^2_{\text{eff}}(t) = \frac{8\pi}{3} \left[ \frac{3}{8\pi} (H(t))^2 - c_1 H(t)^2 - c_2 \dot{H}(t) \right]^{\frac{1}{\gamma}}. \]

For the effective scale factor, we have
\[ a_{\text{eff}}(t) = \exp \left[ \int H_{\text{eff}}(t) dt \right], \]

### TABLE II: The effects of entropic force and running gravitational coupling on future singularities.

| Entropic Force     | Type  | An effective |
|--------------------|-------|--------------|
| \( c_1 = 0.1, c_2 = 0.01 \) | \( II \rightarrow III \) | \( c_1 = 0.1, c_2 = 0.01, \beta \geq 0.5 \) | \( I \rightarrow III \) |

Aditionally, using Eq. (17) and the conservation law \( \dot{\rho}_{\text{eff}} + 3H_{\text{eff}}(t)(\rho_{\text{eff}} + p_{\text{eff}}) = 0 \), we have
\[ \frac{\ddot{a}_{\text{eff}}(t)}{a_{\text{eff}}(t)} = -\frac{4\pi}{3} (\rho_{\text{eff}} + 3p_{\text{eff}}), \]
where
\[ \rho_{\text{eff}} = \frac{3H^2_{\text{eff}}(t)}{8\pi} \]
\[ p_{\text{eff}} = -\frac{1}{4\pi} \left( \frac{3}{2} H^2_{\text{eff}}(t) + \dot{H}_{\text{eff}}(t) \right). \]

Here, effective energy density and pressure have their original definitions \( \rho(t) \equiv \rho_{\text{eff}}(t) \) and \( p(t) \equiv p_{\text{eff}}(t) \), respectively in Eqs. (21) and (22). This means that by a redefinition for Hubble parameter the original form of Friedmann Equations remain unchanged. In order to study which singularities may change under this new definitions of cosmological parameters we consider Eqs. (19), (21) and (22). Using, the scale factor associated with Type I singularity in Eq. (6), we have shown that, for \( c_1, c_2 \geq 0 \), the effective scale factor in Eq. (19) is finite although \( H_{\text{eff}}, \rho \) and \( p \) are infinite. Thus, Type I singularity is replaced by Type III. The singularity type III is defined as finite \( a_{\text{eff}} \) and infinite \( H_{\text{eff}}, \rho \) and \( p \).

In other words, considering an asymptotically safe gravity and the entropic force lead to the replacement of the strong singularity of Type I with the weaker one of type III in the effective cosmological model.

Let us consider the effective scale factor in Eq. (19) for the other future singularities. For the singularity type II the scale factor \( a(t) \) described by Eq. (7) and the effective scale factor by Eq. (19), our results indicate that \( H_{\text{eff}}, \rho \) and \( p \) are infinite for \( \beta = 0 \) and \( c_1, c_2 \geq 0 \). So, Type II singularity is replaced by Type III. Furthermore, the singularity type III in Eq. (4) for \( \beta \geq 0.5 \) as well as types IV and V in Eq. (12) for \( \beta = 0 \) remain the same as before. The results are summarized in Table II.

### VI. CONCLUSION

In this paper, we studied the effects of running gravitational coupling and entropic force on future singularities. Since the energy density is divergent for types I and III in the singularity time, we can consider the effect of asymptotically safe running gravitational coupling for these types of future singularities. The effect of running gravitational coupling does not change the type of singularity. However, for singularity types I and III, a weaker
gravitational force (larger values of $\beta$) corresponds to a delay in the time that the singularity appear.

Furthermore, the entropic force changes the singularity from Type $II$ to a new type called Type $\mathbb IT$ ($a=$const., $H =$const., $H \to \infty$, $p \to \infty$, $\rho \to \infty$) which differs from previously known type $III$. The type of singularity is similar to type $III$ but only with a different behavior of $H$. Although the singularity types $I$, $III$ and $IV$ do not change as a result of considering the effect of entropic force, Type $V$, however, is removed in this case.

Our results indicated that both the entropic force and a running gravitational coupling cause a delay in the singularity types $I$ and $III$. Finally, by introducing a dual cosmological model, we investigated different types of singularity. It was shown that types $I$ and $II$ change to type $III$. It is interesting that Type $I$ is replaced by a weaker one along the lines defined by Tipler. It should be noted that the singularity types $III$, $IV$ and $V$ do not change in this dual cosmological model.

[1] S. Hannestad and E. Mortsell, Probing the dark side: Constraints on the dark energy equation of state from CMB, large scale structure, and type Ia supernovae, Phys. Rev. D 66, 063508 (2002).

[2] S. Nojiri, S. D. Odintsov, Inhomogeneous Equation of State of the Universe: Phantom Era, Future Singularity and Crossing the Phantom Barrier, Phys. Rev. D 72, 023003, (2005), arXiv:hep-th/0505215.

[3] S. Capozziello, V.F. Cardone, E. Elizalde, S. Nojiri, S. D. Odintsov, Observational constraints on dark energy with generalized equations of state, Phys. Rev. D 73, 043512, (2006), arXiv:astro-ph/0508550.

[4] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phantom Energy: Dark Energy with $\omega_{-1}$ Causes a Cosmic Doomsday e, Phys. Rev. Lett. 91, 071301 (2003).

[5] S. Nojiri and S. D. Odintsov, Final state and thermodynamics of a dark energy universe, Phys. Rev. D 70, 103522 (2004).

[6] S. Nojiri, S. D. Odintsov and S. Tsujikawa, Properties of singularities in the (phantom) dark energy universe, Phys. Rev. D 71, 063004 (2005).

[7] L. F. Jambrina, and R. Lazkoz, Cosmological singularities and modified theories of gravity, Journal of Physics: Conference Series314, 012061, (2011), arXiv:0903.4775 [gr-qc].

[8] L. F. Jambrina, $w$-singularities in cosmological models, International Journal of Modern Physics A 27, (2012), arXiv:1012.3159 [gr-qc].

[9] Dabrowski, M.P. and Denkiewicz T., AIP, Conf. Proc., 1241, 561, (2010), arXiv:0910.0023.

[10] Denkiewicz et al., Cosmological tests of sudden future singularities, Phys. Rev. D 85, 083527, (2012), arXiv:1201.6661.

[11] K. Bamba, S. Capozziello, S. Nojiri, Sergei D. Odintsov, Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests, Astrophysics and Space Science, 342, 155-228, (2012), arXiv:1205.3421.

[12] L. F. Jambrina, $w$-cosmological singularities, Phys. Rev. D 82, 124004 (2010).

[13] L. F. Jambrina and R. Lazkoz, Geodesic behavior of sudden future singularities, Phys. Rev. D 70, 121503 (2004).

[14] M. P. Dabrowski and T. Denkiewicz, Barotropic index $w$-singularities in cosmology, Phys. Rev. D 79, 063521 (2009).

[15] F. J. Tipler, Singularities in conformally flat spacetimes, Phys. Lett. A 4, 8 (1977).

[16] A. Krolak, Towards the proof of the cosmic censorship hypothesis, Class. Quant. Grav. 3, 267 (1986).

[17] L. F. Jambrina and R. Lazkoz, Classification of cosmological milestones, Phys. Rev. D 74, 064030 (2006).

[18] S. J. M. Houndjo, Conformal anomaly around the sudden singularity, Europhys. Lett. 92, 10004 (2010), arXiv:1008.0664 [hep-th].

[19] S. Nojiri, S. D. Odintsov, Quantum escape of sudden future singularity, Phys. Lett. B 595, 1-8, (2004), arXiv:hep-th/0405078.

[20] C. W. Misner, The Isotropy of the Universe, Astrophys. J. 151, 431 (1968).

[21] I. Brevik, O. Gorbunova, Viscous Dark Cosmology with Account of Quantum Effects, Eur. Phys. J. C 56, 425 (2008).

[22] E. Verlinde, On the Origin of Gravity and the Laws of Newton, JHEP 1004, 029 (2011), arXiv:1001.0785 [hep-th].

[23] D. A. Easson, P. H. Frampton and G. F. Smoot, Entropic Accelerating Universe, Phys. Lett. B 696, 273 (2011), arXiv:1002.4278 [hep-th].

[24] D. A. Easson, P. H. Frampton and G. F. Smoot, Entropic Inflation, arXiv:1003.1528 [hep-th].

[25] R. Casadio and A. Gruppuso, CMB acoustic scale in the entropic-like accelerating universe, Phys. Rev. D 84, 023503 (2011), arXiv:1005.0790 [gr-qc].

[26] J. D. Barrow, Sudden Future Singularities, Class. Quant. Grav. 21, L 79 (2004).

[27] Y. -F. Cai and E. N. Saridakis, Inflation in Entropic Cosmology: Primordial Perturbations and non-Gaussianities, Phys. Lett. B 697, 280, (2011) arXiv:1011.1245 [hep-th].

[28] J. D. Barrow, Cosmologies with Varying Light-Speed, Phy. Rev. D 59, 043515 (1998), arXiv:astro-ph/9811022.
[30] R. Casadio, S. D. H. Hsu and B. Mirza, Asymptotic Safety, Singularities, and Gravitational Collapse, Phys. Lett. B 695, 317 (2011), arXiv:1008.2768 [gr-qc].
[31] A. Albrecht and J. Magueijo, A time varying speed of light as a solution to cosmological puzzles, Phys. Rev. D. 59, 043516 (1998), arXiv:astro-ph/9811018.
[32] J. D. Barrow, Varying G and Other Constants, arXiv:gr-qc/9711084.
[33] J. D. Barrow and P. Parsons, Phys. Rev. D 55, (1997).
[34] M. Hindmarsh, D. Litim and Ch. Rahmede, Asymptotically Safe Cosmology, JCAP 1107, (2011), arXiv:1101.5401.
[35] A. Contillo, M. Hindmarsh and Ch. Rahmede, Renormalization group improvement of scalar field inflation, Phys. Rev. D 85, 043501, arXiv:1108.0422.
[36] A. Bonanno and M. Reuter, Cosmology of the Planck Era from a Renormalization Group for Quantum Gravity, Phys. Rev. D 65, 043508 (2002), arXiv:hep-th/0106133.
[37] M. Reuter and H. Weyer, Renormalization group improved gravitational actions: A Brans-Dicke approach, Phys. Rev. D 69 104022 (2004).
[38] P.A.M. Dirac, The cosmological constants, Nature 139, 323 (1937);
[39] J.D. Barrow and J. Magueijo, Varying- theories and solutions to the cosmological problems, Phys. Lett. B 443, 104 (1998); J.D. Barrow, Ann. Phys. (Berlin), 19, 202 (2010).
[40] M. Hindmarsh, I. D. Saltas, f(R) gravity from the renormalisation group, Phys. Rev. D 86, 064029, (2012), arXiv:1203.3957.
[41] M. P. Dabrowski and K. Marosek, Regularizing cosmological singularities by varying physical constants, JCAP, 1302, 012, (2013), arXiv:1207.4038 [hep-th].
[42] C. Wetterich, Exact evolution equation for the effective potential, Phys. Lett. B 301, 90 (1993); M. Reuter, Nonperturbative evolution equation for quantum gravity, Phys. Rev. D 57, 971 (1998), arXiv:9605030 [hep-th]; W. Souma, Non-Trivial Ultraviolet Fixed Point in Quantum Gravity, Prog. Theor. Phys. 102, 181 (1999), arXiv:9907027 [hep-th].
[43] S. Weinberg, Asymptotically Safe Inflation, Phys. Rev. D 81, 083535 (2010), arXiv:0911.3165 [hep-th].