Magnetic moiré surface states and flat chern band in topological insulators

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We theoretically study the effect of magnetic moiré superlattice on the topological surface states by introducing a continuum model of Dirac electrons with a single Dirac cone moving in the time-reversal symmetry breaking periodic potential. The Zeeman-type moiré potentials generically gap out the moiré surface Dirac cones and give rise to isolated flat Chern min minibands with Chern number ±1. This result provides a promising platform for realizing the time-reversal breaking correlated topological phases. In a $C_6$ periodic potential, when the scalar $U_0$ and Zeeman $\Delta_1$ moiré potential strengths are equal to each other, we find that energetically the first three bands of $\Gamma$-valley moiré surface electrons are non-degenerate and realize i) an $s$-orbital model on a honeycomb lattice, ii) a degenerate $p_x, p_y$-orbitals model on a honeycomb lattice, and iii) a hybridized $sd^2$-orbital model on a kagome lattice, where moiré surface Dirac cones in these bands emerge. When $U_0 \neq \Delta_1$, the difference between the two moiré potential serves as an effective spin-orbit coupling and opens a topological gap in the emergent moiré surface Dirac cones.

Introduction.-Recently, moiré superlattices in twisted two-dimensional (2D) materials provide a novel platform to study a variety of strong correlation effects in flat minibands. Two prime examples are twisted graphene and transition metal dichalcogenide (TMD) multilayers [1–5]. Motivated by the success of twisted van der Waals heterostructures, it is natural to study the effect of moiré superlattice of the Dirac cone on the surface of a 3D topological insulator (TI). Moiré superlattices in TI materials are ubiquitous, either in TI film grown on lattice mismatched substrate, or misalignment of topmost quintuple layer in bulk Bi$_2$Te$_3$ [6–13]. Previous studies have revealed the folded gapless Dirac cone within the bulk gap due to its topological nature from the time-reversal (TR) invariant moiré superlattice, where moiré surface states do not form isolated minibands [14–16]. Thus to introduce TR breaking is a natural step to obtain isolated and even topological moiré surface minibands. The incorporation of the magnetic proximity effect into topological surface states have significantly enriched the variety of quantum matter [17–19] exemplified in heterostructures with magnetic insulators [20–22] and intrinsic magnetic TI [23–29], in particular van der Waals MnBi$_2$Te$_4$, which is compatible with the Bi$_2$Te$_3$ family materials [30]. Therefore it is straightforward to twist the van der Waals heterostructure of TI and magnetic insulator, while their effect on topological surface states have not been studied theoretically.

In this paper, we study the band structure of magnetic moiré surface states of TIs. Until now, all of the experimental moiré systems are TR invariant at the single particle level, thus the total Chern number is always equal to zero. Therefore, even with flat bands, it is quite difficult to achieve TR breaking interacting topological states such as the fractional Chern insulator in these systems. This motivates us to consider moiré superlattice of magnetic topological surface states. The topological nature of moiré surface Dirac cones is protected by TR symmetry, we find a Zeeman-type moiré potential generically opens the gap in the moiré surface Dirac cones and give rise to isolated flat Chern minibands with Chern number ±1. In a $C_6$ periodic potential, the $\Gamma$-valley moiré surface electrons simulate 2D honeycomb lattice physics, leading to emergent moiré surface Dirac cones.

Moiré Dirac electron.-We introduce and study a model of TI surface Dirac fermions in periodic scalar potential with TR breaking and analyze its normal band structure and topology in moiré Brillouin zone (MBZ). Now we start with the massless Dirac fermion with a single Dirac cone at $\Gamma$ in 2D

$$\mathcal{H}_0(k) = v_F (k_x \sigma_y - k_y \sigma_x),$$

where $v_F$ is the Fermi velocity, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices, $k = (k_x, k_y)$ are the 2D momentum. A uniform exchange coupling between an out-of-plane magnetization and Dirac fermion opens a gap in the surface spectrum, and leads to the surface quantum Hall effect with a half-quantized Hall conductance. Now we put this Dirac fermion in a periodic TR breaking moiré potential $U(r)$ with a discrete translational symmetry, then to the lowest-order perturbation,

$$\mathcal{H} = \mathcal{H}_0(-i\partial_r) + U(r),$$

$$U(r) = U_0(r) \sigma_0 + \Delta(r) \sigma_z,$$

where $U(r) = U(r + L_{1,2})$, and $L_{1,2}$ are two primitive vectors of the moiré superlattice, $\sigma_0$ is the identity matrix. $U_0(r)$ is the scalar potential, and $\Delta(r)$ is the Zeeman-type potential from magnetic exchange interaction which contains both the moiré and uniform parts. This model can apply to bulk TI crystal with twisted surface states in the interface between a TI and a ferromagnetic insulator.

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Fig. 1. (a) Schematic diagram of a $C_6$ moiré potential. (b) Energy spectrum at potential $(U_0, \Delta_1, \Delta_0) = (1, 0, 0)$. The corresponding MBZ is shown in the center. The entire spectrum remains gapless due to TR symmetry. The Dirac points are at $\Gamma$ and $M$. The Zeeman moiré potential or uniform Zeeman term will generically gap out Dirac points as shown in (c,d) with $(U_0, \Delta_1, \Delta_0) = (1, 0, 1)$ and $(1, 0.5, 0)$, respectively. (e)-(g) When $U_0 = \Delta_1$, the first three conduction bands originate from $s$ orbital, $p_x$, $p_y$ orbitals at honeycomb lattice and $sd^2$ orbitals at kagome site. All these band structures have characteristic Dirac points at high symmetric points despite of finite $\Delta_0$, $\Delta_1$. (h) $U_0 \neq \Delta_1$ effectively introduce spin-orbit coupling and introduces a topological gap, and each bands gain non-trivial Chern numbers. All parameters are in unit of $v_F/L$.

Although all of the physical effects discussed in this paper are generic for any magnetic moiré superlattice on topological surface states. To be concrete, we would like to start from a simple model describing the TIs of Bi$_2$Te$_3$ family [18], where threefold rotations with respect to the $z$-axis ($C_{3z}$) require that $U(r) = U(C_{3z}r)$. We consider the periodic potential with a form

$$U(r) = 2(U_0 + \Delta_1 \sigma_z) \sum_{j=1}^{3} \cos(G_j \cdot r) + \Delta_0 \sigma_z,$$

where $G_j = (4\pi/\sqrt{3}L)(-\sin(2\pi j/3), \cos(2\pi j/3))$ are three reciprocal vectors. $(U_0, \Delta_1, \Delta_0)$ are the moiré potential strength, which are scalar potential, Zeeman moiré potential, and uniform Zeeman term. There are two sets of energy scales $v_F/L$ and $(U_0, \Delta_1, \Delta_0)$ in this system, and the low-energy physics is determined by the ratio between $v_F/L$ and $(U_0, \Delta_1, \Delta_0)$. In particular, as we will see in the following, the interplay of moiré potential $(U_0, \Delta_1, \Delta_0)$ would give rise to different band topology.

**Band structure.**-The band structures are calculated by using the plane-wave expansion with a cutoff of 80 basis sets. First we estimate the energy scale in the system. For a typical TI such as Bi$_2$Te$_3$, the Dirac velocity is $v_F \approx 250$ meV·nm [31, 32]. If we set the moiré lattice length as $L = 10$ nm with the twisted angle $\theta = a/L$, then the energy scale of $v_F/L = 25$ meV. Thus, in the unit of $v_F/L$, we expect energy scale of the effective potential at moiré scale is at the order of tens of meV [33–36], namely $\Delta_0$ is around $[0, 2]$, $\Delta_1$ is $[0, 2]$, and $U_0$ from $[0, 2]$. Fig. 1(a) is a schematic diagram of the $C_6$ moiré potential, and the minimum of potential constitutes a honeycomb lattice. We therefore expect the physics of the moiré band are generated by orbitals sitting on the honeycomb sites. In Fig. 1(b) with $(U_0, \Delta_1, \Delta_0) = (1, 0.0)$, the moiré potential $U_0$ folds the surface Dirac bands, and due to TR symmetry, the moiré surface Dirac cones at $\Gamma$ are preserved and satellite Dirac points at $M$ points, which is consistent with previous study [15, 16]. Moreover, $U_0$ introduces the particle-hole asymmetry. Generically these minibands are non-degenerate, since they are from the spin-orbit coupled topological surface states. Now we study the effect of TR breaking potentials. By turning on a finite either Zeeman term $\Delta_0$ or moiré potential $\Delta_1$, all the moiré Dirac cones are generically gapped as shown in Fig. 1(c) and Fig. 1(d), respectively.
In particular, when $U_0 = \Delta_1$, we find the re-emergence of the gapless moiré Dirac cones at $K$ in conduction bands of different energies as shown in Fig. 1(e). As shown more clearly in Fig. 1(h), the first three conduction bands can be classified into three different groups. To reveal the nature of moiré band physics, we identify the symmetries and centers of the Wannier orbitals underlying the moiré bands by employing topological quantum chemistry [37]. We first compute the symmetry of the Bloch states and classifying them in terms of the irreducible representations of the little groups at the corresponding high symmetry points, and then compare the list of irreducible representations with the Elementary Band Representations (EBR) of the space group $P6mm$ listed on the Bilbao Crystallographic server [38–41]. The results are listed in Fig. 2. Consistent with the emergent honeycomb structure of the moiré potential, the first set of bands is formed by $s$ orbital on the honeycomb lattice. These bands form a Dirac point at $K$ and are topologically equivalent to the $\pi$ bands of graphene except there is no spin degeneracy. The second set of bands, is formed by $p_x \pm ip_y$ orbitals on a honeycomb [42, 43] that form a pair of almost dispersionless bands and also have a Dirac node at $K$. The third set of bands is formed by three bands which have one flat band and Dirac point at $K$. The symmetry analysis reveals that they are generated by orbitals centered on a kagome lattice, namely on the $3c$ Wyckoff positions lie at the mid bonds between two honeycomb sites. Such orbitals come from hybridization of different orbitals at honeycomb sites, which effectively transforms the hexagonal symmetry of the moiré lattice into the physics of the kagome lattice, namely the $sd^2$ graphene [44]. It is worth mentioning that similar results found in $\Gamma$-valley TMD moiré bands [36]. However, we point out there is an essential difference is that here the moiré bands are non-degenerate, and the Dirac points are emergent even in the presence of TR breaking at $U_0 = \Delta_1$ and are protected by $C_{3v}$. As we see in the following, when $\Delta_1$ deviates from $U_0$, the deviation effectively act as the spin-orbit coupling and opens a topological gap at Dirac points. While in $\Gamma$-valley TMD moiré bands, the Zeeman potential will not lead to gap opening but just split the spin up and down bands in energy. It is noted that for an opposite Zeeman potential, all similar physics will occur on highest in energy moiré bands when $U_0 = -\Delta_1$.

To illustrate how these orbitals emerges in the above moiré bands when $U_0 = \Delta_1$, we analyze the spectrum of Dirac fermion in a rotational invariant potential, namely $H_0(-i\partial_r) + \Delta_0 \sigma_z + \Delta_1 V(r)(\sigma_0 + \sigma_z)$. Thus the moiré potential only acts on spin up. We approximate the moiré potential near its minimum as a harmonic trap, $V(r) \propto (r/L)^2$ with a cutoff length of $R_0 \approx 0.2L$. Fig. 3(a) shows the energy spectrum from the numerical calculation performed on a disc geometry with radius $R = 4R_0$. There are bound states in the gap which are

![FIG. 2. (a) Energy spectrum with $(U_0, \Delta_1, \Delta_0) = (4, 4, 8)$. The three sets of bands originate from $s$ Wannier orbitals on the honeycomb lattice, $p_x \pm ip_y$ Wannier orbitals on the honeycomb lattice, and hybridized $sd^2$ orbitals on the kagome lattice. (b)-(d) Wave function density distribution of three selected Bloch states encircled in (a), which clearly show the different orbital characters. Here we only present spin up part for the moiré potential only acts on spin up when $U = \Delta_1$.](image)

![FIG. 3. (a) The bound states spectrum of Dirac fermion in the moiré potential valley. (b)-(f) The wave function density distributions of the bound states in energetic order show the $s, p_x, d_x^2, d_{xy}, d_{x^2-y^2}$ orbitals characteristics. ($p_x \pm ip_y$), ($d_{xy} \pm id_{x^2-y^2}$) are degenerate.](image)
classified into \(s, p\), and \(d\) orbitals in energetic order, where their wave function density distributions for corresponding bound states are clearly demonstrated in Fig. 2(b)-(f). Thus the tight-binding model from hopping of the bound states at the honeycomb lattice naturally give rise to the above moiré bands.

Then we study the physics away from \(U_0 = \Delta_1\). A typical Dirac spectrum is shown in Fig. 1(h) where the emergent Dirac points (as in Fig. 1(e-g)) are gapped, and one gets isolated flat bands. Such a gap opening must be topological as we can expect from EBR. We further calculate Chern number of the minibands which is well defined here, for the moiré potential regularizes the Dirac fermion into MBZ. As expected, these bands feature nontrivial Chern numbers. Furthermore, the \(p_x \pm ip_y\) and \(d_{xy} \pm id_{x^2-y^2}\) bound states are no longer degenerate when \(U_0 \neq \Delta_1\). Therefore, the magnetic moiré potential \(\delta = U_0 - \Delta_1\) (deviated from \(U_0 = \Delta_1\)) is an effective spin-orbit coupling and act as a topological mass term in the emergent Dirac bands [43], and the sign of Chern numbers is determined by the sign of \(\delta\). The bandwidth of the flat Chern band from \(p_x, p_y\) orbitals is about \(W \approx 0.2v_F/L\). Interestingly, the first pair of conduction bands essentially realize the Haldane mode on the honeycomb lattice [45], where the bandwidth is on the order of \(W \approx v_F/L\) and tuned by moiré lattice constant.

Now we understand the physics in magnetic moiré surface states is tuned by the twisting angle and is significantly smaller than the Coulomb repulsion energy, which make it an ideal platform for realizing interacting topological states [47–53]. For an estimation, taking the dielectric constant of TI surface states \(\epsilon_r \approx 5\), one obtains a Coulomb interaction energy \(U = e^2/\epsilon_r L \approx 30\) meV. Thus \(U/W \gtrsim 1\) for filling in the first sets of conduction/valence bands, while \(U/W \gtrsim 6\) for filling in the second sets of conduction band. Furthermore, even with either Chern number 0 or \(\pm 1\), the nondegenerate flat band allows a single Fermi surface with large density of states when partially filled, leading to a chance of realizing an intrinsic TR breaking superconductivity.

The band topology of lowest in energy magnetic moiré conduction bands is essentially rooted in the \(C_6\) periodic potential for a simple realization of honeycomb lattice, which is the case for \(\text{Bi}_2\text{Te}_3\). In a \(D_4\) periodic potential, one could also get flat Chern band but these interesting emergent Dirac cone from orbital-active models in biparticle lattice will not occur. Here we emphasize the results are different from previous study on twisted magnetic TI bilayer, there strong hybridization between top and bottom gapped surface states occurs [54].

Next we briefly discuss the superconducting proximity effect. Without the moiré potentials, the proximitized surface state is always a topological superconductor when \(\Delta_0 < \Delta_s\). If \(\mu = 0\) or \(\mu > \Delta_s\), with \(\Delta_s\) is the s-wave pairing amplitude. We find by adiabatically turning on the moiré potential, the band structure changes but without gap closing. Namely, the system is always in the topological superconducting state with above condition [46]. Therefore, Majorana zero mode in the vortex core and chiral Majorana edge modes are expected [55, 56].

Finally, we discuss the feasibility to realize our model of magnetic Dirac fermion in periodic potential. Mechanically robust single septuple layer of \(\text{MnBi}_2\text{Te}_4\) has been obtained experimentally [57], making it possible to implement twisted superlattice on \(\text{Bi}_2\text{Te}_3\) surfaces. The wave
function density of Dirac surface state resides in both MnBi₂Te₄ layer and topmost layer of Bi₂Te₃ [58], therefore both $\Delta_0$ and $\Delta_1$ are present. With the exchange coupling at the order of hundreds of meV, we expect the magnetic potential at moiré scale is at the order of tens of meV, namely $(U_0, \Delta_1, \Delta_0)$ is at the same order.

Summary.—We find the magnetic potentials generically gap out the moiré surface Dirac cones and lead to isolated flat Chern minibands with Chern number ±1, thus the fractional filling in it makes the strongly correlated topological states possible. The moiré surface electrons in a $C_6$ periodic potential simulate the physics of orbital-active honeycomb lattice, and the magnetic moiré potential acts as an effective spin-orbit coupling. Our model provides a convenient condensed matter platform to engineer the Haldane model with narrow bandwidth and strong interaction.

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[1] Eva Y. Andrei and Allan H. MacDonald, “Graphene bilayers with a twist,” Nature Mat. 19, 1265 (2020).
[2] Leon Balents, Cory R. Dean, Dmitri K. Efetov, and Andrea F. Young, “Superconductivity and strong correlations in moiré flat bands,” Nature Phys. 16, 725 (2020).
[3] Stephen Carr, Shiang Fang, and Efthimios Kaxiras, “Electronic-structure methods for twisted moiré layers,” Nature Rev. Mater. 5, 748 (2020).
[4] R. Bistritzer and A. H. MacDonald, “Moiré bands in twisted double-layer graphene,” Proc. Natl. Acad. Sci. U.S.A. 108, 12233–12237 (2011).
[5] Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, “Unconventional superconductivity in magic-angle graphene superlattices,” Nature 556, 43 (2018).
[6] Can-Li Song, Yi-Lin Wang, Ye-Ping Jiang, Yi Zhang, Cui-Zu Chang, Lili Wang, Ke He, Xi Chen, Jin-Feng Jia, Yayu Wang, Zhong Fang, Xi-Dai, Xin-Cheng Xie, Xiao-Liang Qi, Shou-Cheng Zhang, Qi-Kun Xue, and Xucun Ma, “Topological insulator bi2se3 thin films grown on double-layer graphene by molecular beam epitaxy,” Appl. Phys. Lett. 97, 143118 (2010).
[7] Yilin Wang, Yeping Jiang, Mu Chen, Zhi Li, Canli Song, Lili Wang, Ke He, Xi Chen, Xucun Ma, and Qi-Kun Xue, “Scanning tunneling microscopy of interface properties of bi2se3on FeSe,” J. Phys. Condens. Matter 24, 475604 (2012).
[8] Jeong Heum Jeon, Won Jun Jang, Jong Keon Yoon, and Se-Jong Kahng, “Metal-supported high crystalline bi2se3 quintuple layers,” Nanotechnology 22, 465602 (2011).
[9] Y. Liu, Y. Y. Li, S. Rajput, D. Gilks, L. Lari, P. L. Galindo, M. Weinert, V. K. Lazarov, and L. Li, “Tuning dirac states by strain in the topological insulator bi2se3,” Nature Phys. 10, 294 (2014).
[10] Shuiuang Xu, Yu Han, Xiaolong Chen, Zefei Wu, Lin Wang, Tianyi Han, Weiguang Ye, Huanhuan Lu, Gen Long, Yingying Wu, Jiangxiazi Lin, Yuan Cai, K. M. Ho, Yuhe He, and Ning Wang, “van der waals epitaxial growth of atomically thin bi2se3 and thickness-dependent topological phase transition,” Nano Lett. 15, 2645 (2015).
[11] Jin-Peng Xu, Mei-Xiao Wang, Zhi Long Liu, Jian-Feng Ge, Xiaojun Yang, Canhua Liu, Zhi An Xie, Dandan Guan, Chun Lei Gao, Dong Qian, Ying Liu, Qiang-Hua Wang, Fu-Chun Zhang, Qi-Kun Xue, and Jin-Feng Jia, “Experimental detection of a majorana mode in the core of a magnetic vortex inside a topological insulator-superconductor bi₂te₃/nbse₂ heterostructure,” Phys. Rev. Lett. 114, 017001 (2015).
[12] Koen Schouteden, Zhe Li, Taishi Chen, Fengqi Song, Bart Partoens, Chris Van Haesendonck, and Kyungwha Park, “Moiré superlattices at the topological insulator bi2te3,” Sci. Rep. 6, 20278 (2016).
[13] Zacharias Hennighausen, Christopher Lane, Ioana Gianina Buda, Vincent K. Mathur, Arun Bansil, and Swastik Kar, “Evidence of a purely electronic two-dimensional lattice at the interface of tmd/bi2se3 heterostructures,” Nanoscale 11, 15029 (2019).
[14] Anthony Vargas, Fangze Liu, Christopher Lane, Daniel Rubin, Ismail Bilgin, Zacharias Hennighausen, Matthew DeCapua, Arun Bansil, and Swastik Kar, “Tunable and laser-reconfigurable 2d heterocrystals obtained by epitaxial stacking of crystallographically incommensurate bi2se3 and mos₂ atomic layers,” Sci. Adv. 3 (2017), 10.1126/sciadv.1601741.
[15] Taige Wang, Noah F. Q. Yuan, and Liang Fu, “Moiré surface states and enhanced superconductivity in topological insulators,” Phys. Rev. X 11, 021024 (2021).
[16] Jennifer Cano, Shiang Fang, J. H. Pixley, and Justin H. Wilson, “Moiré superlattice on the surface of a topological insulator,” Phys. Rev. B 103, 155157 (2021).
[17] M. Z. Hasan and C. L. Kane, “Colloquium: Topological insulators,” Rev. Mod. Phys. 82, 3045–3067 (2010).
[18] Xiao-Liang Qi and Shou-Cheng Zhang, “Topological insulators and superconductors,” Rev. Mod. Phys. 83, 1057–1110 (2011).
[19] Yoshinori Tokura, Kenji Yasuda, and Atsushi Tsukazaki, “Magnetic topological insulators,” Nat. Rev. Phys. 1, 126–143 (2019).
[20] Peng Wei, Ferhat Katmis, Badih A. Assaf, Hadas Steinberg, Pablo Jarillo-Herrero, Donald Heiman, and Jagadeesh S. Moodera, “Exchange-coupling-induced symmetry breaking in topological insulators,” Phys. Rev. Lett. 110, 186807 (2013).
[21] Ferhat Katmis, Valeria Laute, Flavio S. Nogueira, Badih A. Assaf, Michelle E. Jamer, Peng Wei, Biwurup Satpati, John W. Freeland, Ilya Eremim, Don Heiman, Pablo Jarillo-Herrero, and Jagadeesh S. Moodera, “A high-temperature ferromagnetic topological insulating phase by proximity coupling,” Nature 533, 513–516 (2016).
[22] Chi Tang, Cui-Zu Chang, Gejian Zhao, Yawen Liu,
Zilong Jiang, Chao-Xing Liu, Martha R. McCartney, David J. Smith, Tingyong Chen, Jagesh S. Moodera, and Jing Shi, “Above 400-k robust perpendicular ferromagnetic phase in a topological insulator,” Sci. Adv. 3 (2017), 10.1126/sciadv.1700307.

Dongjin Zhang, Minji Shi, Tongshuai Zhu, Dingyu Xing, Haijun Zhang, and Jing Wang, “Topological axion states in the magnetic insulator mni2te4 with the quantized magnetoelastic effect,” Phys. Rev. Lett. 122, 206401 (2019).

Mikhail M. Otroko, Ilya I. Klimovskikh, Hendrik Bantmann, Alexander Zeugner, Ziya S. Aliev, Sebastian Gass, Anja U. B. Wolter, Alexander Ra. V. Koroleva, Dmitry Esyunin, Alexander M. Shikin, Maria Blanco-Rey, Martin Hoffmann, Alexander Ra. Yu. Vyzavoskaya, Sergey V. Eremeev, Yuri M. Koroteev, Imamaddin R. Amiralsanov, Mammad B. Babanly, Nazim T. Mamedov, Nadir A. Abdullayev, Vladimir N. Zverev, Bernd Büchner, Elke F. Schweir, Shiqiao Du, Bing-Lin Jiazhen Wu, Fucai Liu, Masato Sasase, Koichiro Ienaga, Yan Gong, Jingwen Guo, Jiaheng Li, Kejing Zhu, Meng-Zilong Jiang, Chao-Xing Liu, Martha R. McCartney, “A-type crystals with both magnetic and topological properties,” Science 325, 178–181 (2009).

Xia, D. Qian, D. Hseih, L. Wray, A. Pal, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, “Observation of a large-gap topological-insulator class with a single Dirac cone on the surface,” Nature Phys. 5, 398–402 (2009).

Kerelsky, L. J. McGilly, D. M. Kennes, L. Xian, Y. Mankowitz, S. Chen, K. Watanabe, T. Taniguchi, J. Hone, C. Dean, A. Rubio, and A. N. Pasupathy, “Maximized electron interactions at the magic angle in twisted bilayer graphene,” Nature 572, 96 (2019).

Fu, W. T. Lovorn, E. Tutic, I. Martin, and A. H. MacDonald, “Topological insulators in twisted transition metal dichalcogenide homobilayers,” Phys. Rev. Lett. 122, 086402 (2019).

Yang Zhang, Noah F. Q. Yuan, and Liang Fu, “Moiré quantum chemistry: Charge transfer in transition metal dichalcogenide superlattices,” Phys. Rev. B 102, 201115 (2020).

Mattia Angeli and Allan H MacDonald, “Valley transition of orbital-active bipartite lattices,” Proc. Natl. Acad. Sci. U.S.A. 118, e201826118 (2021).

Barry Bradlyn, L Elcoro, Jennifer Cano, MG Vergniory, Zhijun Wang, C Feber, MI Aroyo, and B Andrei Bernevig, “Topological quantum chemistry,” Nature 547, 298 (2017).

J. Perez-Mato, D Orozobengoa, Enre Tasci, Gemma De la Flor Martin, and A Kirov, “Crystallography online: Bilbao crystallographic server,” Bulg. Chem. Commun. 43, 183–197 (2011).

M I Aroyo, J M Perezmato, C Capillas, E Kroumova, Svetoslav Ivantchev, G Madariaga, A Kirov, and Hans Wondratschek, “Bilbao crystallographic server: I. databases and crystallographic computing programs,” Z. Krist. 221, 15–27 (2006).

Asen Kirov, Cesar Capillas, J Perez-Mato, and Hans Wondratschek, “Bilbao crystallographic server: ii. representations of crystallographic point groups and space groups,” Acta Cryst. 62, 115–28 (2006).

L Elcoro, Barry Bradlyn, Z. Wang, M. G. Vergniory, Jennifer Cano, C. Felser, B. Bernevig, D Orozobengoa, G. D. L. Flor, and M. Aroyo, “Double crystallographic groups and their representations on the bilbao crystallographic server,” J. Appl. Crystallogr 50, 1457 (2017).

Congjun Wu, Doron Bergman, Leon Balents, and S. Das Sarma, “Flat bands and wigner crystallization in the honeycomb optical lattice,” Phys. Rev. Lett. 99, 070401 (2007).

Huan Wang and Jing Wang, “Topological bands in two-dimensional orbital-active bipartite lattices,” Phys. Rev. B 103, L081109 (2021).

Miao Zhou, Zheng Liu, Wenmei Ming, Zhengfei Wang, and Feng Liu, “sd2 graphene: Kagome band in a hexagonal lattice,” Phys. Rev. Lett. 113, 236802 (2014).

F. D. M. Haldane, “Model for a quantum hall effect without landau levels: Condensed-matter realization of the ‘parity anomaly’,” Phys. Rev. Lett. 61, 2015–2018 (1988).

See Supplemental Material for more details.
[47] M. Levin and A. Stern, “Fractional topological insulators,” Phys. Rev. Lett. 103, 196803 (2009).
[48] Joseph Maciejko, Xiao-Liang Qi, H. Dennis Drew, and Shou-Cheng Zhang, “Topological quantization in units of the fine structure constant,” Phys. Rev. Lett. 105, 166803 (2010).
[49] X.-L. Qi, “Generic wave-function description of fractional quantum anomalous hall states and fractional topological insulators,” Phys. Rev. Lett. 107, 126803 (2011).
[50] E. Tang, J.-W. Mei, and X.-G. Wen, “High-temperature fractional quantum hall states,” Phys. Rev. Lett. 106, 236802 (2011).
[51] K. Sun, Z. Gu, H. Katsura, and S. Das Sarma, “Nearly flatbands with nontrivial topology,” Phys. Rev. Lett. 106, 236803 (2011).
[52] T. Neupert, L. Santos, C. Chamon, and C. Mudry, “Fractional quantum hall states at zero magnetic field,” Phys. Rev. Lett. 106, 236804 (2011).
[53] E. M. Spanton, A. A. Zibrov, H. Zhou, T. Taniguchi, K. Watanabe, M. P. Zaletel, and A. F. Young, “Observation of fractional chern insulators in a van der waals heterostructure,” Science 360, 62–66 (2018).
[54] Biao Lian, Zhaochen Liu, Yuanbo Zhang, and Jing Wang, “Flat chern band from twisted bilayer mbI₂te₄,” Phys. Rev. Lett. 124, 126402 (2020).
[55] Liang Fu and C. L. Kane, “Superconducting proximity effect and majorana fermions at the surface of a topological insulator,” Phys. Rev. Lett. 100, 096407 (2008).
[56] Jing Wang, Quan Zhou, Biao Lian, and Shou-Cheng Zhang, “Chiral topological superconductor and half-integer conductance plateau from quantum anomalous hall plateau transition,” Phys. Rev. B 92, 064520 (2015).
[57] Yujun Deng, Yijun Yu, Meng Zhu Shi, Zhongxun Guo, Zihan Xu, Jing Wang, Xian Hui Chen, and Yuanbo Zhang, “Quantum anomalous hall effect in intrinsic magnetic topological insulator mbI₂te₄,” Science 367, 895–900 (2020).
[58] M. M. Otrokov, T. V. Menshchikova, M. G. Vergniory, I. P. Rusinov, A. Yu. Vyazovskaya, Yu. M. Koroteev, G. Bihlmayer, A. Ernst, P. M. Echenique, A. Arnau, and E. V. Chulkov, “Highly-ordered wide bandgap materials for quantized anomalous Hall and magnetoelectric effects,” 2D Mater. 4, 025082 (2017).