REALISTIC $\text{SO}(5) \times \text{U}(1)$ MODEL IN RS SPACE

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The gauge bosons and Englert-Brout-Higgs (EBH) boson are unified in the five dimensional RS spacetime. The EBH boson is identified with a part of the fifth dimensional component of the gauge potential. In the $\text{SO}(5) \times \text{U}(1)$ gauge-Higgs unification the EW symmetry is dynamically broken. The EBH boson, predicted with a mass around 130 GeV, naturally becomes stable so that it appears as missing energy and momentum in collider experiments. Collider signatures such as gauge couplings of quarks and leptons and production of KK $\gamma$ and $Z$ are also discussed.

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1 Introduction

The last particle yet to be discovered in the standard model (SM) of strong and electroweak (EW) interactions is the Englert-Brout-Higgs (EBH) boson. Possible signals for the EBH boson at the LHC experiments have been reported, but more data are necessary for the confirmation.

If the EBH boson is found around 125 GeV, but with non-SM couplings to $W$, $Z$, and fermions, or if the EBH boson is not seen at LHC, not because it does not exist, but because it is stable, then the gauge-Higgs unification scenario becomes plausible. In either case it becomes urgent matter to explore the gauge-Higgs unification.

2 Gauge-Higgs Unification

We start with gauge theory in higher dimensions where extra-dimensional space is not simply connected. Take a five-dimensional theory. Zero modes of four-dimensional components of the vector potentials $A_\mu$ contain photon, $W$, and $Z$, whereas zero modes of the extra dimensional component $A_y$ contain the 4D EBH boson. Thus the EBH boson becomes a part of the gauge bosons, leading to the gauge-Higgs unification (GHU)\(^{123}\)

When the extra dimensional space is not simply connected, the zero mode of $A_y$ appears as an Aharonov-Bohm (AB) phase in the extra dimension. Though its non-vanishing vacuum expectation value (vev) gives vanishing field strengths ($F_{\mu y}$), it becomes a physical degree of freedom, causing dynamical gauge symmetry breaking by the Hosotani mechanism at the quantum level.

Symbolically the phase appears as a Wilson line integral along a non-contractible loop $C$ in the fifth dimension

$$e^{i\hat{\theta}(x)} \sim P \exp \left\{ ig \int_C dy A_y \right\} , \quad \hat{\theta}(x) = \theta_H + \frac{H(x)}{f_H} . \quad (1)$$
are introduced on the Planck brane. The brane scalar, which is $(0,0,0)$, reduces the symmetry to $SO(5) \times U(1)_X$, where \( \theta \) is sandwiched by the Planck brane at \( \theta = 0 \) and makes all exotic fermions heavy. The resultant symmetry is $SU(2)_L \times U(1)_Y$. The $SO(4) \times U(1)_X$ chiral anomalies are cancelled with the brane fermions. After the symmetry breaking by $\theta_H$, the low energy spectrum is the same as in SM.

It distinguishes GHU from SM.

### 3 $SO(5) \times U(1)$ Gauge-Higgs Unification in RS space

Several features must be implemented in a realistic model. First of all quark-lepton content must be chiral. This feature is most easily realized if the extra dimensional space has the structure of an orbifold. Secondly the model must naturally contain the SM gauge structure $SU(2)_L \times U(1)_Y$. This is achieved by starting with the gauge group $SO(5) \times U(1)_X$ which incorporates the custodial symmetry. Thirdly the EW symmetry must be dynamically broken, which is achieved, with minimal fermion content, in the Randall-Sundrum (RS) warped spacetime:

$$ds^2 = e^{-2\sigma(y)} dx^\mu dx_\mu + dy^2$$

where $\sigma(y) = ky$ for $0 \leq y \leq L$ and $\sigma(y) = \sigma(-y) = \sigma(y + 2L)$. The warp factor is given by $z_L = e^{kL}$. The bulk spacetime $0 < y < L$ is AdS spacetime with the curvature $-6k^2$, which is sandwiched by the Planck brane at $y = 0$ and the TeV brane at $y = L$. It has topology of $M^4 \times (S^1/Z_2)$.

The bulk part of the action consists of the $SO(5)$ and $U(1)_X$ gauge fields $A_M$ and $B_M$ with gauge couplings $g_A$ and $g_B$, and bulk fermions. The bulk fermions $\Psi_a$ are introduced in the vector representation of $SO(5)$. In each generation two multiplets in the quark sector and two multiplets in the lepton sector are introduced. They satisfy the orbifold boundary conditions:

$$(A_\mu/x,y_j - y) = P_j (A_\mu/y) P_j^{-1},$$

$$(B_{\mu},B_y) (x,y_j - y) = (B_{\mu},-B_y) (x,y_j + y),$$

$$\Psi_a (x,y_j - y) = P_j^0 \Psi_a (x,y_j + y),$$

where $(y_0,y_1) = (0,L)$ and $P_j = P_j^\dagger = P_j^{-1}$. In particular we take $P_j = \text{diag}(-1,-1,-1,-1,1)$, which reduces the symmetry to $SO(4) \times U(1)_X$. In addition brane fermions and brane scalar are introduced on the Planck brane. The brane scalar, which is $(0,\frac{1}{2})$ representation of $SO(4) \simeq SU(2)_L \times SU(2)_R$, spontaneously breaks the symmetry $SU(2)_R \times U(1)_X$ to $U(1)_Y$ and makes all exotic fermions heavy. The resultant symmetry is $SU(2)_L \times U(1)_Y$. The $SO(4) \times U(1)_X$ chiral anomalies are cancelled with the brane fermions. After the symmetry breaking by $\theta_H$, the low energy spectrum is the same as in SM.

### 4 Dynamical EW Symmetry Breaking by the Hosotani Mechanism

One of the nicest features in this model is that the EW symmetry is dynamically broken to the electromagnetic $U(1)_{EM}$ by the Hosotani mechanism. For the dynamical EW symmetry breaking it is crucial that (i) the multiplet containing a top quark is in the vector representation of $SO(5)$, and (ii) the spacetime is Randall-Sundrum warped space, but is not flat.

The effective potential $V_{\text{eff}}$ for $\theta_H$ at the one-loop level is depicted in fig. 1. In the pure gauge theory the symmetry remains unbroken. In the presence of the top quark, whose mass
is larger than $m_W$, $V_{\text{eff}}$ is minimized at $\theta_H = \pm \frac{1}{2} \pi$ so that the EW symmetry breaks down to $U(1)_{\text{EM}}$.

The EBH boson mass $m_H$ is given by $m_H^2 = f_H^2 (d^2 V_{\text{eff}}/d \theta_H^2)$ at the minimum. It is found that $m_H = 135 (72)$ GeV for $z_L = 10^{15} (10^5)$. This does not contradict with the current experimental data, since the EBH boson becomes stable as is seen below.

5 Effective Low-Energy Interactions

The effective Lagrangian at low energies among the EBH boson, $W$, $Z$, quarks and leptons is approximately given by $^{[10][11][12]}$

$$\mathcal{L}_{\text{eff}} \sim - \left( \frac{1}{2} g f_H \sin \hat{\theta}_H \right)^2 \left\{ W_\mu W^\mu + \frac{1}{2 \cos^2 \theta_W} Z_\mu Z^\mu \right\} - y_f f_H \sin \hat{\theta}_H \bar{\psi}_f \psi_f$$  \hspace{1cm} (5)

where $\hat{\theta}_H$ is given by $^{[1]}$ and $\frac{1}{2} g f_H = m_{\text{KK}}/\pi \sqrt{KL}$. The expression is valid to good accuracy for large $z_L$. The Kaluza-Klein mass is given by $m_{\text{KK}} = \pi k z_L^{-1}$, which turns out to be around 1.4 GeV for $z_L = 10^{15}$. In SM one has $v + H$ in place of $f_H \sin \hat{\theta}_H$. The nature of $\theta_H$ as an AB phase forces the appearance of periodic, non-linear mass functions, a distinguishing feature of the gauge-Higgs unification. $f_H = 246$ GeV for $\theta_H = \pm \frac{1}{2} \pi$.

The masses are given by $m_W = \frac{1}{2} g f_H | \sin \theta_H |$, $m_Z = m_W / \cos \theta_W$, and $m_f = y_f | \sin \theta_H |$. The couplings of the EBH boson to $W$, $Z$, quarks and leptons are obtained by expanding the expression $^5$ in a Taylor series in $H$. The linear couplings are found to be

$$WWZ, ZZH, \text{Yukawa couplings} = (\text{SM values}) \times \cos \theta_H .$$ \hspace{1cm} (6)

They are suppressed, compared with the SM values, by a universal factor $\cos \theta_H$. This is a specific character of the gauge-Higgs unification.$^{[13]}$ In particular, the linear couplings vanish at $\theta_H = \frac{1}{2} \pi$. We stress that $\theta_H \neq 0$ gives masses to $W$, $Z$ and fermions as in SM, but gives vanishing linear couplings at $\theta_H = \frac{1}{2} \pi$.

6 H Parity and Stable EBH Bosons

The fact that the $WWH$, $ZZH$ and Yukawa couplings vanish at $\theta_H = \frac{1}{2} \pi$ is a consequence of the symmetry. There emerges $H$-parity, $P_H$, at $\theta_H = \frac{1}{2}$. Among low-energy particles the EBH boson is odd under $P_H$, while all other SM particles are even. It immediately follows that the EBH boson becomes absolutely stable.$^{[14][19]}$

The proof for the existence of the $H$-parity proceeds as follows. First the action of the model is invariant under the mirror reflection in the fifth dimension; $(x^\mu, y) \rightarrow (x^\mu, -y)$, $(A_\mu, A_y) \rightarrow$
(A_μ, -A_μ), and \( Ψ \rightarrow ± γ^5 Ψ \). Under the reflection \( \hat{θ}_H \rightarrow - \hat{θ}_H \), while wave functions of all other SM particles remain invariant. Secondly, there arises the enhanced large gauge symmetry when all bulk fermions belong to the vector representation of \( SO(5) \). The periodicity in \( θ_H \) in physical quantities is halved to \( π \); \( θ_H + π \sim θ_H \). If there were a fermion in the spinor representation of \( SO(5) \), the periodicity would remain as the original \( 2π \). Thirdly the effective potential \( V_{\text{eff}}(θ_H) \) is minimized at \( θ_H = \frac{1}{2}π \) thanks to the presence of the top quark. Around \( θ_H = \frac{1}{2}π \) we have, for physical quantities, equivalence relations

\[
\frac{π}{2} + \frac{H}{f_{H}} \leftrightarrow - \frac{π}{2} - \frac{H}{f_{H}} \leftrightarrow \frac{π}{2} - \frac{H}{f_{H}}. \tag{7}
\]

The theory is invariant under \( P_H \) to all orders in perturbation theory. The symmetry around \( \frac{1}{2}π \) has been seen, for instance, in \( V_{\text{eff}}(θ_H) \) depicted in fig. 1.

Another proof for the \( H \)-parity has been provided, by noticing the invariance of the \( SO(5) \) algebra under the interchange of \( SU(2)'_L \) and \( SU(2)'_R \) and flip \( T^4 \rightarrow - T^4 \). At \( θ_H = \frac{1}{2}π \) the brane fields couple to only bulk fields which are even under this operation. This symmetry suppresses radiative corrections to the \( T \) parameter and \( Zbb \) coupling as noticed by Agashe et al. \[15\]

With the \( H \)-parity all \( H^n \)-couplings \((n: \text{an odd integer})\) to other SM particles are forbidden. The LEP2 constraint for the EBH boson mass \((m_H \geq 114\text{GeV})\) is also evaded as the \( ZZH \) coupling vanishes.

### 7 Collider Signatures

The phenomenology at \( θ_H = \frac{1}{2}π \) is extremely interesting.\[16\] The \( H \)-parity forbids production of a single EBH boson. EBH bosons are produced in pairs at collider experiments. The production rate is normal. The \( WWHH, ZZHH \) couplings are \(-1\) times the SM couplings. The EBH boson becomes stable. It implies that produced EBH bosons do not decay so that EBH bosons appear as missing energies and momenta in collider events. As a pair of EBH bosons, two stable particles, are produced, confirming them at Tevatron/LHC/ILC becomes very difficult. There are large background events containing neutrinos with the same topology. If polarized right-handed electron and left-handed positron beams can be prepared at ILC, then identification of stable EBH bosons becomes feasible by suppressing neutrino backgrounds.

The \( χ^2 \) values for the forward-backward asymmetry \( A_{FB} \) on the \( Z \) resonance in the \( e^+e^- \) annihilation and for the branching fractions of the \( Z \) decay are tabulated in Table 1. Although the gauge-Higgs unification scenario gives good agreement for \( A_{FB} \) in a wide range of \( z_L \), the branching fractions of the \( Z \) decay are reproduced only for large \( z_L \geq 10^{15} \).

**Table 1:** \( χ^2 \) fit for \( A_{FB} \) and \( Z \) decay fractions. The values of \( m_{KK}, m_H \) and \( m_{\text{tree}} \) are also listed.

| \( \sin^2 θ_W \) | \# of data | \( z_L = 10^{15} \) | \( 10^{10} \) | \( 10^{5} \) | SM |
| --- | --- | --- | --- | --- | --- |
| \( χ^2 \) | 6 | 6.3 | 6.4 | 7.1 | 10.8 |
| \( χ^2 \) | 8 | 16.5 | 37.7 | 184.5 | 13.6 |
| Sum of two \( χ^2 \) | 14 | 22.8 | 44.1 | 191.6 | 24.5 |
| \( m_{KK} \) (GeV) | 1466 | 1193 | 836 |
| \( m_H \) (GeV) | 135 | 108 | 72 |
| \( m_{\text{tree}} \) (GeV) | 79.84 | 79.80 | 79.71 | 79.95 |

The signatures of the extra dimension itself are obtained by observing KK excited states of various particles. Relatively clear signals can be found for KK \( Z^{(1)} \) and \( γ^{(1)} \), which subsequently decay into \( e^+e^- \) or \( μ^+μ^- \). The masses and total decay widths of \( Z^{(1)} \) and \( γ^{(1)} \) are tabulated in
Table 2. Unlike other conventional models the current gauge-Higgs unification model predicts large production rates and decay widths for KK gauge bosons. This is because right-handed quarks and leptons have large couplings to the KK gauge bosons. KK $Z^{(1)}$ corresponds to what is referred to as $Z'$ in the analyses of Tevatron and LHC data. So far no signal of $Z'$ has been found. This may indicate the necessity for improving the current gauge-Higgs unification model.

| $Z^{(1)}$ | $\gamma^{(1)}$ |
|-----------|----------------|
| $z_L$     | $m$ (GeV)   | $10^5$ | $10^{15}$ |
|           | $\Gamma$ (GeV) | 104 | 422 |
|           | $m$ (GeV)   | 653 | 1130 |
|           | $\Gamma$ (GeV) | 446 | 1959 |

8 Stable EBH Bosons as Dark Matter v.s. Supersymmetry

EBH bosons become stable at $\theta_H = \frac{1}{2} \pi$. They are copiously produced in the early universe. As the universe expands and the annihilation rate of EBH bosons falls, the annihilation processes get frozen and the remnant EBH bosons become dark matter. The annihilation couplings are determined from the effective interaction $V_{\text{eff}}$. The present mass density of cold dark matter has been determined by WMAP collaboration as $\Omega_{\text{CDM}} h^2 = 0.1131 \pm 0.0034$.

Suppose that the EBH mass is sufficiently smaller than $m_W$. In this case the dominant annihilation process is $HH \rightarrow b\bar{b}$, and the abundance turns out much larger than the WMAP value. If the EBH boson is heavier than $W$, $HH \rightarrow W^+W^-$ dominates, and the relic abundance turns out much smaller than the WMAP value. The cold dark matter abundance observed by WMAP is reproduced with $m_H = 70 \sim 75$ GeV.

This is a very attractive scenario; the EBH boson responsible for the EW symmetry breaking constitutes the dark matter of the universe. However, the EBH boson mass $m_H = 70 \sim 75$ GeV is realized in the current model with the warp factor $z_L \sim 10^5$, which conflicts with the precision measurements of the gauge couplings at low energies and collider data at high energies as seen above.

Of course nothing is wrong with the scenario of the gauge-Higgs unification with $z_L \geq 10^{15}$ in which the dark matter is accounted for by other particles. More ambitiously we ask if it is possible to have the gauge-Higgs unification in which, without conflicting with the collider data, stable EBH bosons account for the cold dark matter abundance observed by WMAP.

We argue that it is possible, provided supersymmetry (SUSY) exists. If SUSY is exact and unbroken, then the EBH boson remains massless as boson and fermion contributions to $V_{\text{eff}}(\theta_H)$ cancel each other. SUSY is softly or dynamically broken so that the cancellation becomes incomplete, leading to a non-vanishing $m_H$.

Conversely one can determine SUSY breaking scales such that the EBH boson acquires a mass around $70 \sim 75$ GeV with a given warp factor, say, $z_L = 10^{15}$. At the one loop level only the spectra of SUSY partners of $W$, $Z$, top quark and their KK towers are relevant. It is found that the scenario of light neutralinos ($< 100$ GeV), heavy gluinos ($> 1$ TeV), and the stop with $m_{\text{stop}} = 300 \sim 320$ GeV yield the desired mass $m_H = 70 \sim 75$ GeV at $z_L = 10^{15}$. Finding a stop may give a hint for extra dimensions.

9 Summary

We have seen that the minimal $SO(5) \times U(1)$ gauge-Higgs unification model in RS leads to astonishing prediction that the EBH boson is stable. The EBH boson is identified with a part of
the gauge potentials in the extra dimension. It appears as four-dimensional fluctuations of the AB phase in the extra dimension. The nature as an AB phase gives significant deviation from SM, which can be tested at colliders. We may be about to see the extra dimension at LHC.

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