Spontaneous disordering and symmetry breaking in complex plasmas

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Abstract

Spontaneous symmetry breaking is an essential feature of modern science. We demonstrate that it also plays an important role in the physics of complex plasmas. Complex plasmas can serve as a powerful tool for observing and studying discrete types of symmetry and disordering at the kinetic level that numerous many-body systems exhibit.

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I. INTRODUCTION

In this Letter we address dynamical processes in highly ordered complex plasmas associated with spontaneous symmetry breaking.

Spontaneous Symmetry Breaking (SSB) plays a crucial role in elementary particle physics but is also very common in classical physics [1]. It happens whenever the system goes from a state which has a certain symmetry, e.g. rotational symmetry, into an ordered state, which does not have this symmetry anymore. In general, this state not necessarily has to be the ground (vacuum) state and the transition to the new state may or may not be associated with a phase transition. For example, in the case of magnetization the spins point all in one direction (ordered state) whereas above the Curie temperature there is no preferred direction. Another example from a mechanical system without phase transition is a vertical stick which bends under a sufficiently high force from above to one side breaking the rotational symmetry of the system without the force.

Different symmetries coexisting in the same phase, and symmetry transformations escorting phase transitions are widely spread in nature. For instance, the mechanisms of symmetry breaking are thought to be inherent in the molecular basis of life [2]. SSB is also an important feature of elementary particle physics [3]. The Universe itself is believed to have experienced a cascade of symmetry-breaking phase transitions which broke the symmetry of the originally unified interaction giving rise to all known fundamental forces [4–6].

Symmetry effects are crucial either in 3D and 2D systems. Chiral (mirror-isomeric) clusters [7], magic clusters of a new symmetry ‘frozen-in’ by a solid surface [8], or dynamical symmetry breaking by the surface stress anisotropy of a two-phase monolayer on an elastic substrate [9] are examples of the importance of 2D or quasi-2D systems in many applications.

Low pressure, low temperature plasmas are called complex plasmas if they contain microparticles as an additional thermodynamically active component. In the size domain of 1-10 \( \mu \text{m} \) (normally used in experiments with complex plasmas) these particles can be visualized individually, providing hence an atomistic (kinetic) level of investigations [10, 11]. The interparticle spacing can be of the order of 0.1-1 mm and characteristic time-scales are of the order of 0.01-0.1 s. These unique characteristics allow to investigate the microscopic mechanism of SSB and phase transitions at the kinetic level.

Common wisdom dictates that symmetry breaking is an inherent attribute of systems in
an active state. Hence these effects are naturally important in complex plasmas where the particle cloud-plasma feedback mechanisms underlying many dynamical processes are easy to vitalize. Also in complex plasmas where different kind of phase transitions exist, e.g. in the electrorheological plasmas [12], one can find examples for classical SSB. Another option, interesting in many applications, is the clustering of a new phase which is dissymmetric with regard to a background symmetry (as an example of fluid phase separation in binary complex plasmas see [13]).

It is important to mention that the microparticles, collecting electrons and ions from the plasma background, become charged (most often negatively [10]) and hence should be confined by external electric fields. The configuration of the confining forces might deeply affect the geometry and actual structure of the microparticle cloud. In rf discharge complex plasmas the particles are self-trapped inside the plasma because of a favorable configuration of the electric fields [14]. One of the interesting things is the possibility to levitate a monolayer of particles under gravity conditions. In this case the particle suspension has a flat practically two dimensional structure. This is, of course, a very attractive simplification (’from a theoretical point of view’), significantly lowering the description difficulties. Below we concentrate mostly on 2D complex plasmas.

Depending on the discharge conditions, the monolayer can have crystalline or liquid order. 2D configurations of dust particles either in crystalline or liquid state were successfully used to study phase transitions, dynamics of waves and many transport phenomena in complex plasmas [15–21]. A symmetry disordering escorting a crystalline-liquid phase transition has been investigated experimentally in [13–17]. Dislocation nucleation (a ’shear instability’) has been reported in [22, 23], albeit the importance of SSB for this phenomenon has not been explained.

The results of these recent experimental observations can not be properly addressed without a deep understanding of this important issue. We would like to highlight this in the paper and report on the physics of spontaneous disordering of a ’cold’ plasma crystal, simulated melting and crystallization process, including associated defect clusters nucleation, dissociation, and symmetry alternation. These options are realizable in experimental complex plasmas, and can be mimicked in simulations, as we demonstrate below.
II. SPONTANEOUS DISORDERING OF A 2D PLASMA CRYSTAL

It is well known that two broken symmetries distinguish the crystalline state from the liquid: the broken translational order and the broken orientational order. In two dimensions for ordinary crystals it is also well known that even at low temperatures the translational order is broken by spontaneous disordering mediated by thermal fluctuations [24]. As a result, the fluctuation deflections (disordering) grow with distance and translational correlations decay (algebraically, see [25]).

2D plasma crystals also obey this common rule. The character of disordering may be deeply affected by the confinement forces, though. Usually such an 'in-plane' confinement is due to the bowl-shaped potential well self-maintained inside the discharge chamber, which to first order is approximately parabolic (see, e.g. [26, 27]), that is \( U_{\text{conf}} = \frac{1}{2} M \Omega^2 r^2 \), where \( r \) is the distance, \( M \) is the particle mass, and \( \Omega \) is the confinement parameter [28]. (The 'out-of-plane' confining forces, controlling the position of the entire lattice, are normally much stronger; below we consider the 'pure' 2D-case, assuming, hence, an absolutely stiff out-of-plane confinement.)

The fluctuation spectra can be calculated in the following manner. The long-range phonon contribution to the free energy of a 2D system of particles interacting via the Yukawa potential and confined by a shallow isotropic parabolic well can be conveniently represented as [18]:

\[
\Delta U = \frac{M}{2} \sum_{\mathbf{k}} (c_{1t}^2 |V_k|^2 + c_{2t}^2 |D_k|^2),
\]

where \( c_{tr,t} \) are the transverse (shear wave) and the longitudinal (compressional wave) sound speed, and \( V_k, D_k \) are the Fourier components of the vorticity \( V = \nabla \times \mathbf{u} \) and the divergency \( D = \nabla \cdot \mathbf{u} \) of the particle displacements \( \mathbf{u} = \sum_{\mathbf{k}} \mathbf{u}_k \exp(i\mathbf{k} \cdot \mathbf{r}) \), and \( \mathbf{k} \) is the wave vector \( (k = |\mathbf{k}|) \). The unperturbed crystal is supposed to be hexagonal.

The relationship (1) provides (see, e.g. [25]) the probability of the fluctuation \( w \sim \exp(-\Delta U/T) \). Next, using it, we can calculate the averaged fluctuation spectral intensity per unit mass as

\[
< |\mathbf{u}_k|^2 > = \frac{v_T^2}{k^2 c_{tr,t}^2 + \Omega^2} + \frac{v_T^2}{k^2 c_{l,t}^2 + \Omega^2}, \quad v_T^2 = \frac{T}{M},
\]

where \( v_T \) is the particle thermal velocity.
FIG. 1: (color online) Cluster of 721 particles, interacting via the Yukawa type forces, confined inside the parabolic well (for simulation details see [28]). The cluster size is $R/a \simeq 40$. Shown are (to the left) the triangulated particle positions (defect 7- and 5-fold cells are colored in red and green) and (to the right) the cluster density profile. On average, the cluster density systematically decreases toward the edge. The local density maxima (minima) are easily identified as the positions of 5-fold (7-fold) cells. Note that the first circular row of the defects happens to appear at $0.46R$ from the center. This agrees with the theoretical estimate $\sim (0.44 - 0.48)R$ obtained by using (8). Note also that the left part and the right part of the simulated cluster image are perfectly chiral (mirror-symmetric).

It is known (see [28, 29]) that a lattice layer of a finite size $R$ is stably confined if roughly:

$$\frac{c_l^2}{\Omega^2 Ra} \equiv p \simeq \text{const} = 3 - 4.$$  \hspace{1cm} (4)

Here the parameter $p$ stands for an effective number of the nearest neighbors of any edge particle. Note that according to (1) formally $\Omega \to 0$ at $R \to \infty$.

Without confinement ($\Omega = 0$) the fluctuation spectrum (3) apparently diverges $\propto k^{-2}$ at $k \to 0$, and, as a consequence, in agreement with [24, 25], the crystal ordering decays algebraically with the distance $r$, i.e. the density-density correlation behaves as

$$< \rho(\mathbf{r}_1)\rho(\mathbf{r}_2) > \propto r^{-n} \cos(b \mathbf{r}).$$  \hspace{1cm} (5)

Here $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $b$ is the vector of the reciprocal lattice, and $n = \frac{2\pi \sqrt{3} v_T^2}{\sqrt{3} c_{tr}}$. It is assumed that $r \gg a$ is large compared to the interparticle separation $a$. 
In the experiments $\Omega \neq 0$ is always finite (though noticeably small, one or two orders of magnitude less than the frequency of the local 'caged' oscillations of the individual particles \cite{26, 27}). From \cite{3} at non-vanishing $\Omega$ it immediately follows that the fluctuations remain finite even at $k \to 0$. This absence of a singularity alters the character of disordering from algebraic \cite{5} to exponential at a scale depending on the confinement parameter:

$$r \sim r_c = c_l/\Omega < R. \quad (6)$$

It is essential that both asymptotes – algebraic and exponential – must be treated as 'near-field' ($r \ll r_c$) and 'far-field' ($r \sim r_c$) approximations. Hence it would be logical to assume that the ordering decay alternates with distance from algebraic to exponential. This is indeed in qualitative agreement with observations \cite{30–32}.

Remarkably \cite{1}-\cite{3} are formally similar to the equations describing director fluctuations in nematic crystals in the presence of a magnetic field \cite{25, 33}. The action of the magnetic field is known as suppressing the large-scale director fluctuations in liquid crystals.

The length scale $\sim r_c$ seems to be of a fundamental importance. The particles, experiencing a horizontal confinement, are distributed non-uniformly. The steady-state displacements of the particles $u$ in the plasma crystal from their ideal locations in a uniform 2D crystal represent a growing function with distance $u = r^3/8r_c^2$ \cite{28}. The lattice breaks up when

$$\frac{u}{r} = \frac{1}{8} \frac{r^2}{r_c^2} > L, \quad (7)$$

where $L$ is the Lindemann parameter. Since $L=0.16-0.18$ (see, e.g. \cite{20}), it follows that the first row of defects most probably appears at $r \simeq r_c\sqrt{8L} \simeq (1.1 - 1.2)r_c$. Making use of \cite{4}, \cite{6} one can estimate the size of the domains (or equivalent correlation length) as:

$$\frac{r_{cor}}{R} \simeq \sqrt{8pL\frac{a}{R}}. \quad (8)$$

The correlation length \cite{8} does not depend explicitly on the temperature. In other words, for purely topological reasons the big crystal spontaneously splits, assembling an array of sub-domains, even at zero temperature. The estimated values of $r_{cor}$ agree well with those obtained in the simulation – see Fig. \cite{1} and in experiments. For instance, it has been observed in \cite{27, 31} that the crystal orientational order had a power law decay at distances $r/R < 1/4$ in fairly good agreement with $\approx 0.3$ following from \cite{8}.

The 'one-plus' correlation length \cite{8}, unavoidably introducing a network of sub-domains to a lattice layer, is of crucial importance, e.g., for observations of the so called hexatic state
in the plasma crystals that is still an outstanding and controversial issue in complex plasma studies [27, 30, 31].

III. DEFECT CLUSTERS IN PLASMA CRYSTALS

One of the possible scenarios for melting (recrystallization) in a 2D complex plasma is a precipitous increase (decrease) in the density of the dislocations and the dislocation aggregates (such as defect clusters, grain boundaries etc.) [32, 34]. To realize this scenario in simulations, it is desirable to avoid any aforementioned complications associated with the lattice layer sectioning 'from the very beginning'. A promising tool in that sense, allowing to create a defect-free initial lattice layer, is a hexagonal confinement cell proposed in [16].

We performed a series of simulations that revealed several peculiarities in symmetry that are worth to mention.

FIG. 2: Dislocation density vs. inverse mean particle kinetic energy. The dashed line is the least squares fit to the Arrhenius-type dependence \( \frac{N_{\text{disl}}}{N} = A \exp\left(\frac{-Q}{\langle K \rangle}\right) + B \) plotted with \( A = 0.12, B = 0.037, Q = 3.3 \text{ eV} \). The initial exponential drop in the number of dislocations is clearly seen at \( \langle K \rangle^{-1} < 2 \text{ eV}^{-1} \). Next, at higher \( \langle K \rangle^{-1} \), the exponential decay is replaced by a power law decay [16, 32]: in the figure \( N_{\text{disl}}/N \approx \text{const} \) due to the narrow range of the inverse mean kinetic energies involved. The simulation parameters were chosen to match the recrystallization experiment [16].
First, the order parameter of the paired defects – dislocations, – was systematically lower for 7-fold cells. This is not surprising actually from a purely geometric point of view because the 5-fold cell in a pair is more compact.

Second, simulations manifested that not only isolated pairs – dislocations \( (5^7) \), but also compact triplets like \( (5^7_5) \), quadruplets \( (5^7_7^5) \) etc., or even elongated defect chains were quite frequent. Actually they dominantly defined the symmetry of the entire particle suspension. It would certainly be promising to connect the cluster formation in ordered complex plasmas with the general percolation process known in many similar applications (see, e.g., [36, 37]).

Third, in such melted clusters, in agreement with recent experimental observations [16, 32], the defect density permanently decreased upon cooling. At higher temperatures in the beginning of the recrystallization process, while the mutual interparticle collisions were still frequent, the defect density dropped exponentially. Then, at lower temperatures, the decay rate significantly slowed down (see Fig. 2).

A sharp drop in the defect numbers followed by a quasi-saturation resembles the well-known situation [38] in which both thermal activation and tunneling events occur. Hence, by analogy, the fact that in our case the system of defects behaves in a similar way could be naturally explained by an annihilation scenario which is presumably of the \textit{dissipative tunneling} type [38, 39] at lower mean kinetic energies.

IV. COMPLEXITY OF NUCLEATION KINETICS

Nucleation of dislocations is another important example of spontaneous symmetry breaking on a scale of elementary cells.

Whatever the melting scenario would be true, still there would remain a question what mechanism explains nucleation of the primary dislocation clusters. Recently this issue has been studied experimentally: Spontaneous nucleation of the edge-dislocation pairs (followed by their dissociation) has been successfully observed at the kinetic level in the experiments with plasma crystals [22]. Since the Burgers vector of the entire lattice is kept constant (e.g. zero) spontaneously created dislocations must be paired forming defect quadruplets of the type \( (5^7_7^5) \). (The Burgers vector characterizes the magnitude and direction of the crystalline lattice distortion by a dislocation [40].) These dislocation clusters were created in the lattice
locations where the internal shear stress exceeded a threshold. It has also been shown that even an elementary act of nucleation is in fact a multi-scale process consisting of the latent ‘pre-phase’, prompt nucleation of a defect cluster, and dissociation of the cluster followed by the escape of free dislocations [23].

In the experiments [22, 23] it was suggested that the stress that finally caused nucleation was affected by the differential crystal rotation. The exact reason of nucleation, however, was difficult to determine consistently. In simulations the nucleation conditions are certainly easier to identify.

To demonstrate nucleation in simulations a ‘deformable’ hexagonal cell is used (Fig. 3). It confines a 2D cloud of equally charged particles interacting pairwise via the Yukawa (the screened Coulomb) force:

$$f_{i,j} = \frac{q^2}{R^2} \frac{R_{i,j}}{R} \left(1 + \frac{R}{\lambda}\right) \exp\left(-\frac{R}{\lambda}\right).$$

(9)

where $R_{i,j} = r_i - r_j$ is the relative coordinate and $R = |R_{i,j}|$ is the distance between the particles $i, j$ with the coordinates $r_i, r_j$; $q$ is the particle charge and $\lambda$ is the screening length.

The cell design is similar to that applied in [16] to simulate melting and recrystallization.

![FIG. 3: Dislocation nucleation affected by a simple shear strain induced externally. The hexagonal simulation cell contains 631 interacting particles. The forces, deforming the cluster, are applied from left to right on top of the cell, and from right to left on bottom. The central horizontal part of the cell is kept non-deformed. The deformation rate is $\tau \dot{\varepsilon} = 0.05$, where $\tau$ is the Coulomb time-scale [18], $\varepsilon$ is the shear strain [40]. Three panels show the triangulated particle positions for the following time instants (from left to right): shortly before nucleation starts, immediately after nucleation, shortly after dissociation of the dislocation pair. The newly nucleated dislocations glide along the lines with the maximal stress.](image-url)
process of the plasma crystal.

The hexagonal simulation cell has the evident advantage of flexible shape, compared to, e.g., a parabolic cell confinement. Deforming the boundary of the cell, it is simple to manipulate the particles in a tractable way. An additional option of variable geometry enables an opportunity to separate or consolidate pure shear and simple shear deformation [40] if desirable. The strain rate is controllable during deformation as well.

Fig. 3 shows a simple-sheared particle lattice layer. At a properly chosen loading rate deformation affects the shear instability that ends up with nucleation of defect clusters in the bulk of the lattice layer. After a while, when deformation becomes stronger, the components of the clusters decoupled and the newly born free dislocations glided away in a similar manner as the dislocations observed in experiments.

V. TOPOLOGY OF THE DISLOCATION CLUSTER

Symmetry alternation is of primary importance for understanding nucleation of dislocation clusters. The compact cluster design is magic in the sense that the hexagonal symmetry of the particle system neatly turns into a nearly tetratic symmetry of the cluster core (like lead turns into gold when touched by the Philosophers Stone), see Fig. 4.

Despite an apparent simplicity of the cluster interior – only four nearest neighbor particles (marked ABCD in Fig. 4), the centers of the 5- and 7-fold cells, are in the core, – to discover the cluster topology was certainly a challenge [41]. In our case the interparticle interaction potential is of the screened Yukawa type, hence more compact in contrast to the $\propto r^{-3}$ interaction potential in case of magnetically interacting super-paramagnetic colloid particles considered in [41]. Thus there is a unique opportunity to verify whether the core topology is universal.

Let us start with a simple model treating the cluster as constituted of two point-like dislocations, which are set apart at a distance $r$ and allowed to glide only along two fixed crystallographic planes separated by one lattice period $a$, so that $r \equiv a / \sin \varphi$, where $\varphi$ is the angle of mutual orientation of the cluster components with respect to the gliding plane. The interaction energy of the point dislocations having the counter-directed Burgers vectors is [41-44]:

$$u_{dd}(r) = \frac{2}{\pi \sqrt{3}} M c \nu \mu [\ln \left( \frac{r}{a} \right) + \frac{a^2}{r^2}] + \text{const.}$$

(10)
FIG. 4: Topology of a four-component defect cluster. (black dots) Position of the centers of the 5- and the 7-fold cells obtained in simulations. The simulation time is $\Delta t/\tau = 120$. The mean kinetic energy of the particles is kept constant at the level $<K> = 7.3 \pm 0.2$ eV. The mean size of the cell edges is $<s> = 0.96 \pm 0.04$, where $<p>$ is the mean cell perimeter. (As a scaling parameter the lattice constant is used.) Centers of the 5-fold (A, C) and the 7-fold (B, D) cells compose a nearly regular compact tetragonal structure. The gray quadrilaterals represent a few superimposed experimental clusters [22, 23] shown for comparison. (inserts) Patterns of orientational preference revealed a visible 'quasi-hexagonal' trend in orientations for (left insert) the 5-fold and (right insert) the 7-fold components of the clusters. The mean radii are $R_5 = 0.721 > R_7 = 0.629$. The bold and the dashed arrows indicate schematically the Burgers vectors of the individual dislocations. The components of the clusters (a) and (b) are shown in the bottom panels. The individual dislocations should have anti-parallel Burgers vectors since the Burgers vector of the entire defect group must be zero. This evidently results in the mirror-isomeric configurations of the escaping dislocations (the gliding planes are indicated by the dashed lines).
It has a minimum, a *stable ground state*, at $r/a = \sqrt{2} = 1/\sin \varphi$. It corresponds to $\varphi = 45^\circ$, hence tetragonal symmetry of the cluster core might be considered as preferred.

This prediction agrees noticeably well with the results of simulations of finite clusters: On average in Fig. 4 the edge-to-diagonal angle in the cluster core is $<\varphi> = 42^\circ \pm 2^\circ$.

It is worth noting that the cluster core is nearly cyclic. A measure of it immediately follows from the famous Ptolemy’s inequality valid for any quadrilateral:

$$Pt \equiv \frac{s_1s_3 + s_2s_4}{d_1d_2} \geq 1,$$

(11)

where $s_1,...,4$ denote the (ordered) sides, and $d_1,2$ are the diagonals of the quadrilateral. Over 80 % of the recognized clusters have $1 \leq Pt \leq 1.03$ for the simulation results shown in Fig. 4. For comparison a hexagonal four-side cell corresponds to $Pt^{hex} = 2/\sqrt{3} = 1.1547$.

Note also that a stable defect cluster could not be obtained only by shifting positions of four central particles from a hexagonal configuration to tetragonal one. Such deformation would be reversible, hence unstable. A weakly deformed environment, impeding relaxation of the core particles back to the stable hexagonal configuration, is indeed a necessary 'lock' making the deformation plastic, i.e. irreversible.

At sufficiently strong external stress even a stable cluster dissociates. Whatever is the orientation of the cluster as a whole, escaping dislocations can glide only along two crystallographic directions (along Burgers vectors, see Fig. 4 (a, b)). This naturally explains the asymmetry of the escape directions and chirality of the defect configurations revealed by the newly nucleated dislocations in experiments [23].

VI. SUMMARY

Spontaneous symmetry breaking is a common and inherent feature of many systems in physics as well as other fields, it plays an important role, for example, from classical one-component plasmas to modern string representations [45], from the evolution of the early universe [46] to the dynamics of a wide variety of small-scale systems [47]. Therefore it is not surprising that SSB is present also in the physics of plasma crystals. As example we have considered dislocations in plasma crystals which exhibit a spontaneous disordering, involved in the process of melting, and form clusters that are caused by a shear instability and show an interesting topological symmetry.
VII. ACKNOWLEDGEMENT

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[1] Y. Nambu, Rev. Mod. Phys. 81, 1015 (2009).
[2] V. I. Goldanskii and V. V. Kuzmin, Usp. Phys. 157, 3 (1989).
[3] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).
[4] W. H. Zurek, Nature 317, 505 (1985).
[5] T. W. B. Kibble, Nature 317, 472 (1985).
[6] Yu. M. Bunkov and O. D. Timofeevskaya, J. of Low Temp. Phys. 110, 45 (1998).
[7] M. Lahav and L. Leiserowitz, Angew. Chem. Int. Ed. 38, 2535 (1999).
[8] M. Y. Lai and Y. L. Wang, Phys. Rev. Lett. 81, 164 (1998).
[9] W. Lu and Z. Suo, Phys. Rev. B 65, 085401 (2002).
[10] H. M. Thomas and G. Morfill, Nature (London) 379, 806 (1996).
[11] V. E. Fortov, A. V. Ivlev, S. A. Khrapak, A. G. Khrapak, and G. E. Morfill, Phys. Rep. 421, 1 (2005).
[12] K. R. Sütterlin, A. Wysocki, A. V. Ivlev, C. Räth, H. M. Thomas, M. Rubin-Zuzic, W. J. Goedheer, V. E. Fortov, A. M. Lipaev, V. I. Molotkov, O. F. Petrov, G. E. Morfill, and H. Löwen, Phys. Rev. Lett. 102, 085003 (2009).
[13] A. V. Ivlev, S. K. Zhdanov, H. M. Thomas and G. E. Morfill, Europ. Phys. Lett. 85, 45001 (2009).
[14] J. H. Chu and L. I, Phys. Rev. Lett. 72, 4009 (1994).
[15] D. Samsonov, S. K. Zhdanov, R. A. Quinn, S. I. Popel, and G. Morfill, Phys. Rev. Lett. 92, 255004 (2004).
[16] C. A. Knapek, D. Samsonov, S. Zhdanov, U. Konopka, and G. Morfill, Phys. Rev. Lett. 98, 015004 (2007).
[17] V. Nosenko, S. Zhdanov, A. V. Ivlev, and G. Morfill, Phys. Rev. Lett. 100, 025003 (2008).
[18] S. Zhdanov, S. Nunomura, D. Samsonov, and G. Morfill, Phys. Rev. E 68, 035401 (2003).
[19] S. Nunomura, D. Samsonov, S. Zhdanov, and G. Morfill, Phys. Rev. Lett. 95, 025003 (2005).
[20] S. Nunomura, D. Samsonov, S. Zhdanov, and G. Morfill, Phys. Rev. Lett. 96, 015003 (2006).
[21] V. Nosenko, J. Goree, and F. Skiff, Phys. Rev. Lett. 73, 016401 (2006).
[22] V. Nosenko, S. Zhdanov, and G. Morfill, Phys. Rev. Lett. 99, 025002 (2007).
[23] V. Nosenko, S. Zhdanov, and G. Morfill, Phyl. Mag. 88, 3747 (2008).
[24] V.L. Berezinskii, Sov. Phys. JETP 32, 493 (1971).
[25] L. D. Landau and E. M. Lifshitz, Statistical Physics, v. 5 (Pergamon Press, 1959).
[26] U. Konopka, Wechselwirkungen geladener Staubteilchen in Hochfrequenzplasmen, PhD Thesis (Ruhr- University, Bochum, 2000).
[27] T. E. Sheridan, Phys. Plasmas 16, 083705 (2009).
[28] S. Zhdanov, R. A. Quinn, D. Samsonov and G. E. Morfill, New J. Phys. 5, 74 (2003).
[29] H. Totsuji, Ch. Totsuji, and K. Tsuruta, Phys. Rev. E64, 066402 (2001).
[30] R. A. Quinn, C. Cui, J. Goree, and J. B. Pieper, Phys. Rev. E53, R2049 (1996).
[31] T. E. Sheridan, Phys. Plasmas 15, 103702 (2008).
[32] V. Nosenko, S. K. Zhdanov, A. V. Ivlev, C. A. Knapek, and G. E. Morfill, Phys. Rev. Lett. 103, 015001 (2009).
[33] P. G. de Gennes, C. R. Acad. Sci. Paris 266, 15 (1968).
[34] M. Li, W. L. Johnson, and W. A. Goddard III, Phys. Rev. B 54, 12067 (1996).
[35] G. E. Morfill and A. V. Ivlev, Rev. Mod. Phys. 81, 1354 (2009).
[36] H. Satz, Nucl. Phys. A 642, 130 (1998).
[37] D. Bonn, D. Ross, S. Hachem, S. Gridel, and J. Meunier, Europhys. Lett. 58, 74 (2002).
[38] P. Haenggi, P. Talkner, and M. Borkovec, Rev. Mod. Phys. 62, 251 (1990).
[39] H. Hofmann, G.-L. Ingold, and M. H. Thoma, Phys. Lett. B 317, 489 (1993).
[40] Kittel Ch., Introduction to Solid State Physics, 5th edn (Wiley, New York, 1976).
[41] C. Eisenmann, U. Gasser, P. Keim, and G. Maret, Phys. Rev. Lett. 95, 185502 (2005).
[42] F. M. Peeters and X. Wu, Phys. Rev. A 35, 3109 (1987).
[43] M. Peach and J. S. Koehler, Phys. Rev. 80, 4362 (1950).
[44] L. D. Landau and E. M. Lifshitz, v. 7 Theory of Elasticity (Pergamon Press, 1959).
[45] M. H. Thoma and G. E. Morfill, Europ. Phys. Lett. 82, 65001 (2008).
[46] E. W. Kolb and M. S. Turner, The Early Universe (Addison-Wesley, Redwood City, 1990).
[47] T. Kibble, Phys. Today 9, 47 (2007).