Unification in Intersecting Brane Models

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We propose a unification scenario for supersymmetric intersecting brane models. The quarks and
leptons are embedded into adjoint representations of SO(32), which are obtained by type I
string and broken by compactification on orbifolds. Its single unified gauge coupling can give rise to
different gauge couplings below the unification scale, due to effects of magnetic fluxes. The crucial
mechanism is brane recombination preserving supersymmetry.

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Although the open string description of the gauge the-
ory in type I/II brane stack models provides a promis-
ing explanation of the origin of Minimal Supersymmetric
Standard Model (MSSM), there have been lack of under-
standing on unification, contrary to other compactifica-
tion schemes. If various vacua are to be understood as
spontaneously broken phases above the compactification
scale, of a string theory, there should be an unification of
gauge groups and couplings unify described
by brane recombination preserving supersymmetry.

I. EMBEDDING INTO SO(32) ADJOUTS

One may note that, for example, the X, Y gauge
bosons in conventional Georgi–Glashow SU(5) unifica-
tion [1] have the correct colors and weak isospins (3, 2),
as (s)quarks. Of course they do not have the desired U(1)
hypermultiplets, but provided that they are also charged
under additional U(1) symmetries, after diagonal sym-
metry breaking they may have the same quantum num-
bers. We first investigate this possibility: embedding all
the MSSM fields into the SO(32) adjoints, which are the
only representations predicted by type I string theory
[2]. Clearly, however, we require much bigger group than
Georgi–Glashow SU(5). The ingredients are extra
dimensions and magnetic fluxes, which is equivalent to
intersecting branes at angles. Masslessness and chirality
come from zero modes of Dirac operator.

A supporting evidence of this picture is suggested in
the dual M/F-theory compactification [2]. The intersec-
tion of D6 branes is purely geometrically described as
unwinding of U(M + N) singularity to U(M) and U(N)
and the bifundamental fermions come from the branching
of the adjoint

\[(M + N)^2 \rightarrow (M^2, 1) + (1, N^2) + (M, N)\]

where we should count only one bifundamental because
the other is CPT conjugate. In the intersecting brane
picture, a string stretched from M brane stack to N
brane stack corresponds to chiral bifundamental repre-
sentation (M, N), and the other string having the op-
posite orientation is CPT conjugate due to the opposite
GSO projection [3].

Consider a toy model. The gauge group \(H = U(3)_C \times
U(2)_L \times U(1)_R \times U(1)_N\) arises from breaking an unified
group \(G = U(7)\) whose adjoint have the following charge
assignment under the above subgroup \(H\),

\[
\begin{pmatrix}
8 & q & u \\
q^c & 3 & l \\
1 & e & c
\end{pmatrix}
\]

where the notations are self-explanatory; the block-diagonal numbers refer to the dimensions of gauge
bosons, and off-diagonal blocks correspond to complex
quarks and sleptons and hermitian conjugates. We will
see the fields corresponding to blank entries are massive.
They have the correct quantum numbers, as well as hyper-
charge defined as linear combinations

\[
Q_Y = \frac{1}{6}Q_C - \frac{1}{2}Q_L - \frac{1}{2}Q_R.
\]

This breaking is achieved by the following background
magnetic flux \(F = 2\partial_\xi A_5\) on the extra two-torus \(T^2\),

\[
2\pi\alpha' A_5 = \begin{pmatrix}
\frac{m_1}{n_1} I_{n_1} & \frac{m_2}{n_2} I_{n_2} & \frac{m_3}{n_3} I_{n_3} & \frac{m_4}{n_4} I_{n_4} \\
\frac{a_1}{n_1} I_{n_1} & \frac{a_2}{n_2} I_{n_2} & \frac{a_3}{n_3} I_{n_3} & \frac{a_4}{n_4} I_{n_4}
\end{pmatrix} x^4
\]

Here gcd\((n_a, m_a)\) are 3, 2, 1, 1, respectively, \(m_1/n_1 = m_2/n_2 = m_3/n_3, m_4/n_4, a_1 \neq a_3, a_2 \neq a_4\), and \(\alpha'\) will be
interpreted as Regge slope. In compactification with
more dimensions, we may put some of the blocks to dif-
ferent gauge field components than \(A_5\).

Most of all, the theory is chiral [2, 7]. This means,
for example either quark \(q\) or antiquark \(q^c\) in [1] will be
exclusively massless. This can be seen by index theorem. The Dirac operator is decomposed as \( \nabla_n = \nabla_{\alpha} + \nabla_{\beta} \) and, for \( 4k + 2 \) extra dimensions, the chirality is correlated. The difference of the number of left and right mover zero modes, is the first Chern number,

\[
n_+ - n_- = \frac{1}{2\pi} \int_{T^2} \text{Tr} Q F,
\]

where the trace is over gauge charge \( Q \) of commutant group \( L \) to \( H \), completely determined by branching. Plugging \( \mathbf{3} \), we see that the each quark and lepton is chiral. Moreover, there are \( n_+ - n_- \) degenerate bifundamental solutions for each off-diagonal solution. One can check \( \mathbf{9} \) that the commuting charge conjugate, indicated in (1). The resulting unbroken symmetry is the one commutes with (5). By the mechanism discussed, only the fermions with one chirality survive, which is determined by the orientation of flux or brane rotation. In fact the continuous rotation is not possible, because the quantization condition of \( (n_a, n_b) \). By “rotation” we mean recombination which we will discuss shortly.

The example \( \mathbf{11} \) thus is obtained as follows, where \( T \)-dual picture is more transparent. Start with 7 slices of coincident branes, separate stacks with \( 3c + 2l + 1r + 1N \) branes. Then after rotating \( 2l \) and \( 1r \) branes, we obtain chiral spectrum. One can check \( \mathbf{3} \) that the commuting generators of \( U(7) \) represents unbroken gauge group and chiral quarks and leptons, exclusively not paired with the charge conjugate, indicated in \( \mathbf{11} \).

We will generalize this argument to \( 4k + 2 \) extra dimensions, among them our interest is six. We have seen that it is easy to obtain chiral fermion zero mode, but only with supersymmetry the massless boson exists as superpartner. The supersymmetry condition is moduli dependent. In fact, for the case of 2 extra dimensions, there can never be massless scalar. In what follows, we will consider higher dimensional theory and rotation preserving supersymmetry.

The above toy model is anomalous. The best explanation for an anomaly free theory is that it is spontaneously broken phase of a unified theory, where the absence of anomaly is natural. The most suggestive scenario will be ten dimensional supergravity with nonabelian gauge group \( SO(32) \), which is the only anomaly free theory if we want the perturbative open string description. Symmetries are broken by projections associated with the compactification. In the brane picture, consistency is given by Ramond–Ramond (RR) tadpole cancellation condition. It guarantees anomaly freedom of nonabelian gauge anomalies and of other mixed anomalies involving \( U(1)'s \) by the generalized Green–Schwarz (GS) mechanism from antisymmetric tensor fields.

For this we introduce an orientifold plane (O-plane) having opposite RR charges to D-branes. This will set an upper bound of the rank of the gauge group. This is obtained by compounding worldsheet parity reversal \( \Omega \) and spatial reflection, under which O-plane is fixed. This brings about mirror fermion having charge \( (\mathbf{M}, \mathbf{N}) \) for \( (\mathbf{M}, \mathbf{N}) \) corresponding to mirror cycles. When a brane and its image brane are on top of orientifold plane, the gauge group is \( SO \) type. The symmetry breaking \( SO(A) \times U(B) \subset SO(A + 2B) \) is again described by parallel separation. The \( 2 \times 2 \) block matrix \( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \) plays the role of imaginary number, determined by Chan–Paton projector. For example, the above \( SU(7) \) toy model is realized and embedded in \( SO(14) \subset SO(32) \), having almost same structure. Since the final group is \( SO(32) \), inevitably we have hidden sector \( \mathbf{10} \).

During the compactification, we may meet more than one gauge groups thus as many adjoints, depending on the number of O-planes, so all the quarks and leptons are not necessarily belong to the same adjoint.

### II. BRANE RECOMBINATION

The above gauge symmetry breaking is studied as deformations of branes, “brane recombinations”. Mainly it has been discussed in non-supersymmetric theory in the context of tachyon condensation. However we want supersymmetry preserving recombination. In the brane picture, the supersymmetry exists when the cycles belong to \( SU(3) \) holonomy special Lagrangian. The supersymmetry-preserving deformation is parameterized by Wilson lines, as a special case of McLean’s theorem \( \mathbf{3} \). In particular, turning on off-diagonal component changes the cycle and “rotates” the brane. In other words, although we cannot continuously change the linear entries in \( \mathbf{3} \) due to integral quantization condition of \( n_a, n_b \), we can change them “effectively” \( \mathbf{12} \). It is because the corresponding fields to off-diagonal com-
components are charged under the subgroup $L$, which breaks $G$ down to the commutant $H$. In this sense, the description as (6) is far from complete. The proper description comes with coherent sheaves in K-theory [15], and we just use it as the resulting “effective” phase of broken theory.

In any recombinations, in fact in any interactions of D-branes, RR charges should be conserved. It means every recombination takes place preserving the anomaly content. When we survey $F \wedge F$ term of Chern class as in [41], it is invariant under brane recombinations. Since we are considering $4k + 2$ dimensions, an adjoint (before recombination), as well as chiral fermions, contributes chiral anomaly. After symmetry breaking, this number is invariant. As long as tadpole is cancelled, there is no chiral anomalies in the low energy limit.

To be specific, let us consider D6 branes in type IIA theory. We consider a “1/4” (BPS)-cycle, where only two cycle is wrapping nontrivially on four torus $T^4$ in, say, 4567 direction and the remaining one cycle is along the O-plane in 89 direction. Now consider a recombination (6)

\[(n_1, m_1)(n_2, m_2)(1, 0) + (n_1, -m_1)(n_2, -n_2)(1, 0)\]
\[\rightarrow 2n_1n_2(1, 0)(1, 0) + 2m_1m_2(0, 1)(0, 1)(1, 0).\]

In the presence of O-planes, we can choose two cycles as being always the images with respect to an O-plane, here $(1, 0)(1, 0)(1, 0)$, and this reflection symmetry is always preserved. Supersymmetry (seen by worldsheet spectrum) requires the intersecting angles to be same $\theta_2 = \theta_3$, which translates to

\[F_{45} = F_{67} \propto \left(\frac{m_1}{n_1}I_{n_1, n_2} - \frac{m_2}{n_2}I_{m_1, m_2}\right), F_{89} = 0,\]

which the above cycles satisfy. We observe the preserved supersymmetry component $\epsilon$

\[\Gamma^{\mu\nu}F_{\mu\nu}\epsilon = 0\]

is same for every intermediate step. It is because this relation is local, so that at every point of intermediate brane state this condition is satisfied. Since supersymmetry is preserved, there is no energy cost on “marginal” deformation. We can verify it by explicit DBI energy relation from the above supersymmetry condition

\[\text{Tr} \tau_9 V_2 V_3 V_4 \sqrt[4]{1 + (2\pi \alpha' F_{45})^2}(1 + (2\pi \alpha' F_{67})^2)\]
\[= n_1 n_2 \tau_9 V_2 V_3 V_4 + m_1 m_2 \tau_9 V_4\]

with $\tau_p \propto (2\pi \alpha')^{-p}$ being D$p$ brane tension and $V_i$ volume of $i$th two-torus. In $T$-dual picture in $x^5, x^7, x^9$ directions, the second line shows that the setup, before and after recombinations, has exactly the same energy as D5 branes on top of D9 branes, aligned on top of O-planes, which is again in the original picture: all the intersecting D6s along the O-planes. Therefore the energy of branes, before and after recombinations, are same. The interpolating dynamics is $T$-dual to D0-D4 bound state, or instanton obeying periodicity “toron.” The deformation freedom in this example corresponds to the two $SU(n_{112})$ instanton moduli space [10, 16].

Although we have seen that there is a recombination process that deforms intersecting branes to parallel ones, gauge coupling changes during this process.

The above D0-D4 bound state is $U$-dual to (F,D$p$) bound state which is interpreted as the electric flux on D-branes [12, 18]. The above BPS equation becomes string junction condition [17] and we can clearly see the supersymmetry condition at each local point, with the same supersymmetry components preserved. In this picture we can also see the no-energy-cost property under marginal deformation and the specific shape of final state [18]. The DBI energy is $V \sqrt{-\det (1 + 2\pi \alpha' F_{\mu})}$ for each stack, with a background flux $F$ provided by $T$-dual picture of branes, which can be seen by expanding fluctuation around these flux $A_{\mu} = \langle A_{\mu} \rangle + \delta A_{\mu}$ and reading off YM coupling by canonical normalization [17]. This is an unique property of DBI action which is not present in YM description. Thus below the compactification scale the intersecting branes does not give unified coupling, in general. This does not conflicts with neither energy costless deformation since it is the dynamics of extra dimensions, nor unification picture above unification scale.

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For odd $P$ by order 2 element $ber$ of $O_5$ is always 1, and it sits on the plane generated we want four dimensional Lorentz invariance. The number of $O_5$ can be up to 3. Other systems containing $O_pD_p$ and/or $O(p-4), D(p-4)$ can be $T$-dualized to the above models.

When more than one adjoint Higgs assume VEVs, in general the rank of the gauge group reduces. An important application is the electroweak Higgs mechanism [14]. It occurs when the bifundamental Higgs assumes constant VEV, which corresponds to assigning off-diagonal VEVs to original adjoint representation [1] in the position of $l$.

III. TYPE I COMPACTIFICATIONS

Finally we argue that all the SUSY type I/II orientifold models constructed so far be continuously connected, by above deformations and $T$-duality, to type I theory compactified on orbifold. The vital constraint is RR tadpole cancellation condition.

To cancel tadpole, it is crucial that every construction requires at least one orientifold plane $O_P$ as a negatively RR charged object. This can always be converted to $O_9$ plane by $T$-duality. In general, an orientifold group is generated by orbifold actions $g \in P$ and worldsheet parity reversals $\Omega \prod R_m(-1)^{F_L}$, where $R_m$ are spatial reflections of compact $x^m$ directions and $F_L$ is the spacetime fermion number. This $O_P$ plane is lying on the fixed plane under $\prod R_m$. By the $T$-dual $\prod R_m(-1)^{F_L}$, we can always convert it to $\Omega$, while orbifold group $P$ is untouched. The type IIB theory with orientifold element $\Omega$ is type I theory, so that every theory containing orientifold is regarded as compactification of type I string theory with $P$. It follows that the orbifold group $P$ completely determines additional orientifolds and thus gauge group.

Fixing one orientifold as $O_9$, let us then count the lower dimensional orientifolds. Consider a $T^n/Z_N$ orbifold [22]. For odd $N$, the only possible orientifold is $O_9$ since there is no even order element in $P$ compatible with lower dimensional orientifold. For even $N$, the only possibility other than $O_9$ is $O_5$, because in order not to have a tachyon, the difference of dimensions of D-branes should be a multiple of 4. We cannot put an additional $O_1$, if we want four dimensional Lorentz invariance. The number of $O_5$ is always 1, and it sits on the plane generated by order 2 element $g^{N/2}$. Note that if this orientifold group element $\Omega g^{N/2}$ exists, introducing $O_5$ is compulsory, which would be absent in the untwisted theory. The same is also applied to $T^n/(Z_M \times Z_N)$ orbifolds, where the number of $O_5$ can be up to 3. Other systems containing $O_pD_p$ and/or $O(p-4),D(p-4)$ can be $T$-dualized to the above models.

One can see, for example, the model in [27] is deformable to the one in [24], both of which are based on $T^9/(Z_2 \times Z_2)$ orbifold. In the latter picture, orbifold actions determine three 5 + 1 dimensional hyperplanes and harbor $O$-planes. With Chan–Paton projectors, the resulting gauge group consists of $Sp(k)$, which is embedded in $SO(32)$, for each $O_7$ plane. $SO(32)$ because the product of the RR charge of $O_p$ and the number of fixed points is always 16. In the maximal group case (without Wilson lines, or when all the D-branes are on top of one orientifold planes) we have only gauge group $Sp(8)^4$ which will be the final unification group obtainable from brane deformations. Nevertheless they are to be regarded as broken $SO(32)^4$ because the resulting 99 and 5,5i spectra in [24] can be explained by branching rules [496]→[136]+[3]→[256]. Note also we have $T$-dual symmetries exchanging $O_9$ with any of the $O_5$, Therefore every gauge group arising from $Dp$-$Dp$ branes can be embedded into $SO(32)$, for each orientifold plane. As a bonus, we see the $Sp(k)$ group describing small instants [27] is embedded into $SO(32)$, too.

Applying $T$-dualities in $x^5, x^7, x^9$ directions we have four $O_6$s. By brane deformations, we can see the spectrum of [23] can be obtained from this. We can always find the directions in which $T$-duality maps $O_9, D_9, O_5, D_5$ to six dimensional objects: $O_6$ and $D_6$. Therefore we can lift of the model to $M$-theory on $G_2$ manifold picture, where the desirable objects are six dimensional objects in Type IIA theory, which become geometric objects, i.e. singularities.

One may note that not every vacuum might be connected, since the deformation of orbifold/orientifold images should be always deformed together. However it is strongly restricted by anomaly cancelation of representations of $Spin(32)/Z_2$ [20].

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