NRQCD: Fundamentals and Applications to Quarkonium Decay and Production

Geoffrey T. Bodwin

HEP Division, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, IL 60439

I discuss NRQCD and, in particular, the NRQCD factorization formalism for quarkonium production and decay. I also summarize the current status of the comparison between the predictions of NRQCD factorization and experimental measurements.

1. Nonrelativistic QCD (NRQCD)

In a heavy quarkonium, a bound state of a heavy quark and antiquark, there are many important momentum scales. These include $m$, the heavy-quark mass; $mv$, the typical heavy-quark momentum in the quarkonium rest frame; and $mv^2$, the typical heavy-quark rest-frame kinetic energy and binding energy. Here $v$ is the typical heavy-quark velocity in the quarkonium rest frame. For charmonium, $v^2 \approx 0.3$, while for bottomonium, $v^2 \approx 0.1$.

In theoretical analyses, it is useful to treat the physics at each of these scales separately. Owing to asymptotic freedom, interactions at the scale $m$ can be treated perturbatively. Approximate symmetries (e.g. heavy-quark spin symmetry) can be exploited at some scales. Often, analytic calculations simplify when they involve only one scale at a time. Lattice calculations can encompass only a limited range of scales, and so become more tractable after scale separation.

Effective field theories provide a convenient way to separate scales. An effective theory describes the low-momentum degrees of freedom in the original theory. It is constructed by integrating out the high-momentum degrees of freedom in that theory. Nonrelativistic QCD (NRQCD) is an effective field theory that separates scales of order $m$ and higher from the other scales in QCD. NRQCD has a UV cutoff $\Lambda \sim m$. For processes with $p < \Lambda$, NRQCD reproduces QCD. Processes with $p > \Lambda$ are not manifest in NRQCD, but they affect the coefficients of local interactions. $\Lambda$ plays the role of a factorization scale between the hard and soft physics.

At leading order in $v$, the NRQCD action is just the Schrödinger-Pauli action for a heavy quark and a heavy antiquark. In order to reproduce QCD completely, one would need an infinite number of interactions of all orders in $v$. However, in practice one works to a given precision in $v$.

NRQCD and NRQED (the corresponding effective theory for QED) have had
a number of well-established successes. These include calculations of the properties of QED bound states, such as positronium and muonium, lattice calculations of quarkonium spectra, and lattice calculations of $\alpha_s$. However, here we will focus on recent applications to quarkonium decay and production.

2. Quarkonium Decay and Production

In heavy-quarkonium decays and hard-scattering production, large scales appear: Both the heavy-quark mass $m$ and the quarkonium transverse momentum $p_T$ are much larger than $\Lambda_{\text{QCD}}$. The hope is that NRQCD would allow one to separate these short-distance, perturbative scales from the long-distance quarkonium dynamics.

In the case of quarkonium decays, convincing arguments have been given that the short-distance decay process can be represented in NRQCD by point-like four-fermion interactions.\(^4\) This result leads to the factorization formula

$$\Gamma(H \to \text{light hadrons}) = \sum_n 2 \text{Im} F_n(\Lambda) \langle H|O_n(\Lambda)|H \rangle.$$  \(1\)

The $F_n$ are “short-distance coefficients,” which are determined by perturbative matching of amplitudes between QCD and NRQCD. The factors $\langle H|O_n(\Lambda)|H \rangle$ are inherently nonperturbative matrix elements of four-fermion operators in the quarkonium state. The sum in Eq. \(1\) is actually an expansion in powers of $v$ and is truncated at some finite order. A similar factorization formula has been conjectured to hold for inclusive quarkonium production at large $p_T$.\(^4\) The production matrix elements are the crossed versions of the decay matrix elements. Only the color-singlet production and decay matrix elements are simply related. Nayak, Qiu, and Sterman have found that, in next-to-next-to-leading order in $\alpha_s$, the NRQCD production matrix elements must be modified by the inclusion of eikonal lines in order to allow one to factor all of the IR-divergent contributions out of the short-distance coefficients.\(^5\) This finding does not affect existing phenomenology, which is at the tree level or one-loop level. However, it raises important issues with regard to the validity of the NRQCD factorization formula at all orders in $\alpha_s$.

The NRQCD factorization formalism gains much of its predictive power from the fact that the nonperturbative matrix elements are universal, \textit{i.e.}, process independent. Although some decay matrix elements have been computed on the lattice,\(^9\) in general, the matrix elements must be extracted phenomenologically. The consistency of the phenomenological matrix elements from process to process is a key test of the NRQCD factorization formalism.

An important feature of the NRQCD factorization formalism is that quarkonium decay and production occur through color-octet, as well as color-singlet, $Q\overline{Q}$ states. If one drops all of the color-octet contributions, then the result is the color-singlet model (CSM).
NRQCD: Fundamentals and Applications

3. Comparison of NRQCD factorization with experiment

3.1. P-wave decays

A global fit of the NRQCD predictions for the inclusive decay rates of P-wave quarkonia (Ref. 7) is in good overall agreement with the data and yields values for the NRQCD matrix elements that are in good agreement with those from lattice calculations (Ref. 6). More recently, a number of next-to-leading-order NRQCD predictions for decay rates of P-wave quarkonia have been verified by more precise experimental measurements.8

3.2. Quarkonium production at the Tevatron

The CDF data for J/ψ production at the Tevatron are shown in Fig. 1 along with the NRQCD factorization result. As can be seen, the color-singlet contributions, whose normalizations are reasonably well known from decay processes, are smaller than the data by more than an order of magnitude. The color-octet contributions bring the theory into good agreement with the data. However, it should be remembered that the color-octet matrix elements are obtained by fitting to the Tevatron data. The shape of the data is consistent with NRQCD factorization, but there is a good deal of freedom to change the predicted shape by adjusting the relative values of the color-octet matrix elements. The Tevatron data for ψ′ and Υ production are also fit well by the NRQCD factorization expressions. In subsections that follow, we discuss more stringent tests of NRQCD factorization, in which the values of the color-octet matrix elements that were obtained in the fits to the Tevatron data are used to make predictions for other processes.
3.3. $\gamma\gamma \rightarrow J/\psi + X$ at LEP

As can be seen in Fig. 2, in the case of $J/\psi$ production in $\gamma\gamma$ collisions at LEP, comparison of theory with Delphi data clearly favors NRQCD factorization over the color-singlet model. The theoretical uncertainties that are shown arise from uncertainties in the color-octet matrix elements and in the choices of renormalization and factorization scales. The uncertainties in the color-octet matrix elements are quite large because different linear combinations of matrix elements appear in $J/\psi$ production at LEP than in $J/\psi$ production at the Tevatron.

3.4. Quarkonium production in DIS at HERA

The leading-order NRQCD factorization and CSM predictions (Ref. [13]) for the $J/\psi$ inclusive production cross sections $d\sigma/dp_T^2$ and $d\sigma/dQ^2$ in $ep$ deep-inelastic scattering (DIS) are in good agreement with the H1 data (Ref. [14]). Those data lie well above the CSM prediction. The cross section $d\sigma/dz$, which is differential in the energy fraction (inelasticity) $z$, is not fit well by either the NRQCD factorization or CSM predictions. The poor fit of the NRQCD factorization prediction may be a consequence of the breakdown of both the $v$ expansion and the $\alpha_s$ expansion near $z = 1$. This phenomenon will be discussed in conjunction with inelastic quarkonium photoproduction at HERA below. The ZEUS data for $d\sigma/dQ^2$ agree less well with the NRQCD prediction than the H1 data, but they have larger error bars [15].

3.5. Polarization of quarkonia at the Tevatron

The polarizations of quarkonia produced at large $p_T$ at the Tevatron provide potentially definitive tests of the color-octet mechanism. $J/\psi$ production at large-$p_T$...
Fig. 3. $J/\psi$ polarization at the Tevatron. $\alpha = 1$ corresponds to 100\% transverse polarization; $\alpha = -1$ corresponds to 100\% longitudinal polarization. The band is the total NRQCD factorization prediction. The other curves give the contributions from feeddown from higher charmonium states. The data points are from the CDF measurement. From Ref. 18.

is expected to be dominated by gluon fragmentation into a color-octet $Q\bar{Q}$ pair. This mechanism leads to transversely polarized $J/\psi$'s\textsuperscript{16,17} The NRQCD factorization prediction for the $J/\psi$ polarization as a function of $p_T$ (Ref. 18) is shown, along with the CDF data, in Fig. 3 The observed $J/\psi$ polarization is generally smaller than the prediction and seems to trend in the wrong direction, decreasing with increasing $p_T$. However, the experimental error bars are large, and only the last data point truly disagrees with the prediction. There are large uncertainties in the theoretical predictions that arise from uncertainties in the NRQCD matrix elements, which are indicated in the prediction band in Fig. 3. There are also large corrections of higher order in $\alpha_s$ and $v$ to the quarkonium production rate, some of which have been calculated\textsuperscript{17}. To first approximation, such corrections merely change the normalizations of the fitted color-octet matrix elements without changing the polarization strongly. It has been suggested that nonperturbative spin-flip processes, which are suppressed as $v^3$ and are not taken into account in present calculations, might be important\textsuperscript{17}. However, a recent lattice calculation suggests that this is not the case\textsuperscript{20}.

3.6. Inelastic photoproduction at HERA

Theoretical calculations of the cross section for inelastic photoproduction of quarkonium at HERA have been carried out in the NRQCD factorization formalism by several groups\textsuperscript{17}. The compilation of predictions from Ref. 11 and the H1 and Zeus data are shown in Fig. 4 plotted as function of the energy fraction $z$. As can be seen, the color-octet contribution is poorly determined, owing to large uncertainties in the color-octet matrix elements. Even so, there is little room for a color-octet contribution. Furthermore, as is shown in Fig. 4, corrections of next-to-leading order in $\alpha_s$ (NLO) increase the color-singlet piece by about a factor of two at large $z$. The color-singlet piece is then, by itself, in good agreement with the data.\textsuperscript{23,24} The data differential in $p_T$ are also compatible with NLO color-singlet production.
Fig. 4. The rate for inelastic quarkonium photo-production at HERA as a function of the energy fraction $z$. In the left-hand figure, the curves give leading-order (LO) total, direct, and resolved color-octet (CO) and color-singlet (CS) contributions. The band shows the uncertainty in the total contribution that arises from the uncertainties in the color-octet matrix elements. In the right-hand figure, the curves show the prediction in NLO of the color-singlet model. The band shows the uncertainties that arise from $\alpha_s$ and $m_c$. The data points are from the H1 and Zeus results of Refs. 21 and 22. From Ref. 10.

alone at large $p_T$. It should be noted, though, that there are large uncertainties in the NLO color-singlet contribution, which arise primarily from uncertainties in $m_c$ and $\alpha_s$. The true color-singlet contribution could be lower than the central value by about a factor of two, leaving more room for a color-octet contribution.

Near $z = 1$, the leading-order color-octet contribution grows rapidly, in apparent disagreement with the data. However, in this region, soft-gluon emission leads to large logarithms of $1 - z$ and also to large corrections of higher order in $v$, both of which must be resummed. The resummation of the corrections of higher order in $v$ leads to a nonperturbative “shape function.” Both the shape function and the resummed logarithmic corrections significantly smear out the color-octet contribution near $z = 1$ and may lead to a considerable improvement in the agreement of the NRQCD factorization predictions with the data.

3.7. Double $c\bar{c}$ production at Belle

For the exclusive double charmonium process $e^+e^- \to J/\psi + \eta_c$, the cross section times the branching ratio into at least two charged tracks has been measured by the Belle collaboration to be $25.6 \pm 2.8 \pm 3.4$ fb (Ref. 20) and by the BaBar collaboration to be $17.6 \pm 2.8^{+1.5}_{-2.1}$ fb (Ref. 27). In contrast, leading-order NRQCD factorization calculations predict a cross section of $3.78 \pm 1.26$ fb (Ref. 25). A similar disagreement between NRQCD factorization and experiment holds for production of $\chi_{c0}$ and $\eta_c(2S)$ mesons in conjunction with a $J/\psi$ meson. A recent calculation of corrections of next-to-leading order in $\alpha_s$ leads to an enhancement of the theoretical prediction of about a factor 1.8 (Ref. 30). Calculations in the light-cone formalism are in
reasonably good agreement with the data. The light-cone formalism takes into account effects from the relative motion of the $Q$ and $\bar{Q}$ in the quarkonium, which is neglected at leading order in NRQCD. However, it is not clear that the model wave functions that are used in these calculations are good approximations to the true quarkonium light-cone wave functions.

There are also Belle results on inclusive double-charmonium production. For the ratio $R_{J/\psi} = \frac{\sigma(e^+e^- \to J/\psi + c\bar{c})}{\sigma(e^+e^- \to J/\psi + X)}$, the most recent Belle analysis yields $R_{J/\psi} = 0.82 \pm 0.15 \pm 0.14$, with $R_{J/\psi} > 0.48$ (90% confidence level). Predictions based on NRQCD factorization give $R_{J/\psi} \approx 0.1$ (Ref. 17). In the case of the absolute cross section for $J/\psi + c\bar{c}$ production, the Belle result of 0.6–1.1 pb (Ref. 34) disagrees with the prediction of 0.10–0.15 pb (Ref. 17) by almost an order of magnitude. This prediction is based only on the color-singlet contribution. However, corrections of higher order in $v$, including color-octet contributions, are not expected to be large. Neither are corrections of higher order in $\alpha_s$. It is difficult to see how any perturbative calculation could give a value for $R_{J/\psi}$ as large as 80%.

4. Summary

The effective field theory NRQCD is a convenient formalism for separating physics at the scale of the heavy-quark mass from physics at the scale of quarkonium bound-state dynamics. The NRQCD factorization approach provides a systematic method for calculating quarkonium decay and production rates as a double expansion in powers of $\alpha_s$ and $v$. NRQCD factorization for production rates relies upon hard-scattering factorization and has not yet been established. The NRQCD factorization approach has enjoyed a number of successes in inclusive $P$-wave quarkonium decays, quarkonium production at the Tevatron, $\gamma\gamma \to J/\psi + X$ at LEP, and quarkonium production in DIS at HERA. Other processes, including production of polarized quarkonium at the Tevatron, inelastic quarkonium photoproduction at HERA, and double $c\bar{c}$ production at Belle and BaBar are more problematic and point to the fact that our theoretical understanding of quarkonium production is still incomplete.

Acknowledgements

Work in the High Energy Physics Division at Argonne National Laboratory is supported by the U. S. Department of Energy, Division of High Energy Physics, under Contract No. W-31-109-ENG-38.

References

1. W. E. Caswell and G. P. Lepage, Phys. Lett. B167, 437 (1986).
2. B. A. Thacker and G. P. Lepage, Phys. Rev. D43, 196 (1991).
3. G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, and K. Hornbostel, Phys. Rev. D46, 4052 (1992) [arXiv:hep-lat/9205007].
4. G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D51, 1125 (1995) [arXiv:hep-ph/9407339]; 55, 5855(E) (1997).
5. G. C. Nayak, J. W. Qiu, and G. Sterman, *Phys. Lett.* **B613**, 45 (2005) [arXiv:hep-ph/0501235].
6. G. T. Bodwin, D. K. Sinclair, and S. Kim, *Phys. Rev. Lett.* **77**, 2376 (1996) [arXiv:hep-lat/9605023]; *Phys. Rev.* **D65**, 054504 (2002) [arXiv:hep-lat/0107011].
7. F. Maltoni, [arXiv:hep-ph/0007003]
8. A. Vairo, *AIP Conf. Proc.* **756**, 101 (2005) [arXiv:hep-ph/0412331].
9. F. Abe et al. [CDF Collaboration], *Phys. Rev. Lett.* **79**, 572 (1997).
10. M. Krämer, *Prog. Part. Nucl. Phys.* **47**, 141 (2001) [arXiv:hep-ph/0106120].
11. M. Klasen, B. A. Kniehl, L. N. Mihaila, and M. Steinhauser, *Phys. Rev. Lett.* **89**, 032001 (2002) [arXiv:hep-ph/0112259].
12. S. Todorova-Nova, in *Proceedings of the XXXI International Symposium on Multiparticle Dynamics*, Datong, China, 1–7 September, 2001 (World Scientific, Singapore, to appear) [arXiv:hep-ph/0112050].
13. B. A. Kniehl and L. Zwirner, *Nucl. Phys.* **B621**, 337 (2002) [arXiv:hep-ph/0112199].
14. A. Meyer, DESY-THESIS-1998-012; A. Meyer (unpublished); S. Mohrdieck (unpublished).
15. I. Katkov [ZEUS Collaboration], [arXiv:hep-ex/0309046]
16. P. L. Cho and M. B. Wise, *Phys. Lett.* **B346**, 129 (1995) [arXiv:hep-ph/9411303].
17. See N. Brambilla et al., [arXiv:hep-ph/0412158] for references.
18. E. Braaten, B. A. Kniehl, and J. Lee, *Phys. Rev.* **D62**, 094005 (2000) [arXiv:hep-ph/9911436].
19. T. Affolder et al. [CDF Collaboration], *Phys. Rev. Lett.* **86**, 3963 (2001).
20. G. T. Bodwin, J. Lee, and D. K. Sinclair, *Phys. Rev.* **D72**, 014009 (2005) [arXiv:hep-lat/0503032].
21. H1 Collaboration, Contributed paper 157aj, International Europhysics Conference on High Energy Physics (EPS99), Tampere, Finland, 1999.
22. Zeus Collaboration, Contributed Paper 851, International Conference on High Energy Physics (ICHEP2000), Osaka, Japan, 2000.
23. M. Krämer, J. Zunft, J. Steegborn, and P. M. Zerwas, *Phys. Lett.* **B348**, 657 (1995) [arXiv:hep-ph/9411372].
24. M. Krämer, *Nucl. Phys.* **B459**, 3 (1996) [arXiv:hep-ph/9508409].
25. M. Beneke, I. Z. Rothstein, and M. B. Wise, *Phys. Lett.* **B408**, 373 (1997) [arXiv:hep-ph/9702986].
26. P. Pakhlov [Belle Collaboration], [arXiv:hep-ex/0412041].
27. B. Aubert et al. [BABAR Collaboration], [arXiv:hep-ex/0506062]
28. E. Braaten and J. Lee, *Phys. Rev.* **D67**, 054007 (2003) [arXiv:hep-ph/0211085].
29. K. Y. Liu, Z. G. He, and K. T. Chao, *Phys. Lett.* **B557**, 45 (2003) [arXiv:hep-ph/0211181].
30. Y. J. Zhang, Y. j. Gao and K. T. Chao, [arXiv:hep-ph/0506076].
31. J. P. Ma and Z. G. Si, *Phys. Rev.* **D70**, 074007 (2004) [arXiv:hep-ph/0405111].
32. A. E. Bondar and V. L. Chernyak, *Phys. Lett.* **B612**, 215 (2005) [arXiv:hep-ph/0412335].
33. K. Abe et al. [Belle Collaboration], BELLE-CONF-0331, contributed paper, International Europhysics Conference on High Energy Physics (EPS 2003), Aachen, Germany, 2003.
34. K. Abe et al. [Belle Collaboration], *Phys. Rev. Lett.* **89**, 142001 (2002) [arXiv:hep-ex/0205104].