ON THE INVERSION OF STOKES PROFILES WITH LOCAL STRAY-LIGHT CONTAMINATION
A. Asensio Ramos & R. Manso Sainz

Instituto de Astrofísica de Canarias, 38205, La Laguna, Tenerife, Spain
Departamento de Astrofísica, Universidad de La Laguna, E-38206 La Laguna, Tenerife, Spain

ABSTRACT

Obtaining the magnetic properties of non-resolved structures in the solar photosphere is always challenging and problems arise because the inversion is carried out through the numerical minimization of a merit function that depends on the proposed model. We investigate the reliability of inversions in which the stray-light contamination is obtained from the same observations as a local average. In this case, we show that it is fundamental to include the covariance between the observed Stokes profiles and the stray-light contamination. The ensuing modified merit function of the inversion process penalizes large stray-light contaminations simply because of the presence of positive correlations between the observables and the stray-light, fundamentally produced by spatially variable systematics. We caution that using the wrong merit function, artificially large stray-light contaminations might be inferred. Since this effect disappears if the stray-light contamination is obtained as an average over the full field-of-view, we recommend to take into account stray-light contamination using a global approach.

Subject headings: methods: data analysis, statistical — techniques: polarimetric — Sun: photosphere

1. INTRODUCTION

The quantitative investigation of the magnetism of structures in the solar atmosphere is done through the analysis of the observed Stokes profiles. Particularly difficult is to infer the properties of weakly magnetized regions of the solar surface such as the quiet intergranular network. The reason is that the observed polarization signals often stay at the detection limit of modern spectro-polarimeters because the spatial resolution is still not high enough to resolve the smallest magnetic structures. Therefore, the magnetic properties are inferred assuming unresolved structures, which is model dependent.

Due to lack of information, it is customary to apply relatively simple models to explain the observations. The model parameters are estimated using a least-squares (or, equivalently, maximum-likelihood) approach [e.g., Skumanich & Lites 1987; Ruiz Cobo & del Toro Iniesta 1992; Socas-Navarro et al. 2000; Frutiger et al. 2000; Lagg et al. 2004; Asensio Ramos et al. 2008, and many others] or using a fully Bayesian approach [Asensio Ramos et al. 2007; Asensio Ramos 2009]. For the inversion of unresolved structures, the idea of using several components that contribute to the observed Stokes profiles has been around since its introduction by Lites & Skumanich (1990). In its simplest form, the method consists in using a filling factor \( \alpha \) of the pixel that accounts for a non-magnetized component (either stray-light or a pure non-magnetic plasma) and the remaining \( 1 - \alpha \) fraction of the pixel is filled by a magnetic component.

This paper points out that, in case the stray-light contamination is obtained directly from the observations as an average over a given field-of-view, some modifications are necessary in the inversion procedure to take into account the eventual presence of correlation between the observed Stokes profiles and the stray-light. This modification has the remarkable property of penalizing large stray-light contaminations.

2. DISCUSSION

In order to obtain information on the magnetic and thermodynamical properties of the solar atmosphere, one proposes a model atmosphere that depends on a set of parameters \( \theta \) (e.g., the temperature \( T \) at one or several heights, hydrogen density, magnetic field strength and inclination, etc). This model is used to synthesize the Stokes vector \( \textbf{O}_{\text{mod}}(x, y, \lambda; \theta) \) for an arbitrary number of spectral lines at given spatial position \( (x, y) \) and wavelength \( \lambda \). Expressing the observed Stokes profiles as \( \textbf{O}_{\text{obs}}(x, y, \lambda) \), it is customary to obtain the “best” parameters \( \theta \) as those minimizing the following merit function [e.g., Press et al. (1986)]:

\[
\chi^2(x, y) = \frac{1}{4N} \sum_{i=1}^{4} \sum_{j=1}^{N} \left[ \frac{O_{i,\text{mod}}^{\text{mod}}(x, y, \lambda_j; \theta) - O_{i,\text{obs}}^{\text{obs}}(x, y, \lambda_j)}{\sigma_i^2(x, y, \lambda_j)} \right]^2,
\]

where the sum over \( j \) is extended to all \( N \) observed wavelength points. In this equation, \( \sigma_i^2(x, y, \lambda_j) \) represents the variance of the numerator, for each wavelength \( \lambda_j \), Stokes parameter \( i = I, Q, U, \) and \( V \), and at each location \( (x, y) \), due to possible uncertainties in the observations (measurement errors and noise). This is different from “real” variations due to space-time fluctuations (e.g., intensity contrast due to granule-intergranule fluctuations). Usually, the model is known with certainty — as when we fit a Gaussian profile to an observed spectral line —, and \( \sigma_i^2(x, y, \lambda_j) \) is just the noise variance \( \sigma_n^2 \) of the data. This noise variance can be estimated from the observations, ideally, by taking several observations of the same object under identical observational conditions. Unfortunately, this is often just not possible. It is then customary in spectroscopic observations to select a continuum window and, assuming that it should be spectrally flat, all fluctuations are due to noise. The flatness assumption is usually not fulfilled due to systematic effects and the estimated variance might be larger than the
occupying a fraction 1
model atmosphere representative for a magnetized region
problem that will be considered elsewhere.

D
Lites & Skumanich 1990; Orozco Suárez et al. 2007a,b):

mosphere, the following model has been proposed (e.g.,
of Stokes profiles from quiet regions of the solar pho-












one associated with random noise effects.

However, it is also possible that the model contribute
to the variance uncertainty by incorporating explicitly
some observable — for example, if we fit a Gaussian prof-
ile to an observed spectral line, but keeping the equiva-
 lent width of the observed line profile. We study here an
example of this type that arises naturally when consid-
ering stray light in spectropolarimetric observations.

The proper interpretation and inversion of spectropo-
arimetric observations with spatial resolution requires
some treatment of stray light. “Stray light” refers to the
unavoidable spread of light from different regions on the
source. It is an effect produced by the extended tails of
the telescope’s point spread function, which causes that
a significant fraction of the photons detected in one pixel
results from regions outside the Airy disk. As a conse-
quence, there is always some amount of contamination
from regions other than the region that was supposed to
be imaged by the instrument. This problem has become
especially pressing recently, with the advent of high-
resolution space-borne spectropolarimeters like the SP
(Lites et al. 2001) aboard Hinode (Kosugi et al. 2007).
To deal with this problem when carrying out inversions
of Stokes profiles from quiet regions of the solar pho-
tosphere, the following model has been proposed (e.g.,
Lites & Skumanich 1990; Orozco Suárez et al. 2007a,b):

\[ O_{\text{mod}}(x, y; \theta) = \alpha D(x, y) + (1 - \alpha) S(x, y; \theta), \]  

where \( S(x, y; \theta) \) are the Stokes profiles emerging from a
model atmosphere representative for a magnetized region
occupying a fraction 1 - \( \alpha \) of the pixel (from now on,
we drop the dependence of the variables on \( \lambda \)). The
term \( D(x, y) \) is a stray-light contamination profile that
is obtained from a local average of the \( M \) pixels around
the pixel of interest (of the order of 1 arcsec\(^2\) around the

\[ \sigma^2_i(x, y) = \left( 1 + \frac{\alpha^2}{M} \right) \sigma_n^2 - 2\alpha \operatorname{Cov}[O_{\text{obs}}^i(x, y), D_i(x, y)]. \]  

The presence of \( D_i(x, y) \), which depends on the ob-
servations \( O_{\text{obs}}^i(x', y') \), produces that the variance is not
just the noise variance because a certain degree of corre-
lation might exist between \( O_{\text{obs}}^i(x, y) \) and \( D_i(x, y) \). The
reason is that both quantities are affected by noise and
systematic effects and they are the result of a complex re-
duction process that might introduce some correlation.
A straightforward calculation allows us to simplify the
variance to:

\[ \sigma^2_i(x, y) = \left( 1 + \frac{\alpha^2}{M} \right) \sigma_n^2 - 2\alpha \operatorname{Cov}[O_{\text{obs}}^i(x, y), D_i(x, y)]. \]  

It is of interest to define \( \tilde{\rho} \) as the covariance measured
in units of the noise variance:

\[ \tilde{\rho}\sigma_n^2 = \operatorname{Cov}[O_{\text{obs}}^i(x, y), D_i(x, y)] = \frac{1}{M} \sum_{(x', y') \in \Omega} \operatorname{Cov}[O_{\text{obs}}^i(x, y), O_{\text{obs}}^i(x', y')], \]  

which explicitly shows the dependence of \( D_i(x, y) \) on the
observed Stokes profiles and also indicates that \( \tilde{\rho} \) behaves
like an average correlation coefficient. Plugging this ex-
pression into Eq. (5) results in:

\[ \sigma^2_i(x, y) = \left( 1 + \frac{\alpha^2}{M} - 2\alpha\tilde{\rho} \right) \sigma_n^2. \]  

This expression for the variance has several interesting
properties. It depends on the stray-light contamination
coefficient \( \alpha \). The term in \( \alpha^2 \) accounts for the
inclusion of additional noise variance coming from the
stray-light profile. Its influence is heavily reduced when
averaging over many pixels, decreasing in proportion to
the number of pixels added. The linear term depends on
the covariance between the observed Stokes profile at a
given wavelength and the average profile that is consid-
ered as stray-light contamination. This term also goes to
zero when many pixels are considered for the averaging,
so we recommend to obtain the stray-light contamination
profile as an average over the full field-of-view. A
non-zero contribution can be produced by any spatially
variable systematic effect present in the final focal plane
produced by the camera (flatfield systematics or fringes)
or by any optics before (fringes). Additionally, correc-
tions carried out during data reduction can introduce corre-
lations between surrounding pixels. Finally, data compres-
sion like theJPEG compression used by Hinode
(Lites et al. 2002) to optimize telemetry can also intro-
duce correlation artifacts. As a consequence of this de-
pendence on \( \alpha \), the \( \chi^2 \) merit function to be optimized
to get the maximum-likelihood parameters \( \tilde{\theta} \) is different

\[ \text{Figure 1. Value of the variance in units of } \sigma_n^2 \text{ as defined in Eq. (1) for different values of } \alpha \text{ the covariance between the observed profile and the stray-light. Note that } \tilde{\rho} \text{ depends on } M \text{ but, for simplicity, we have assumed that } \operatorname{Cov}[O_{\text{obs}}^i(x, y), O_{\text{obs}}^i(x', y')] \text{ is constant, so that } \tilde{\rho} \text{ is independent of } M. \text{ Although not shown to avoid crowding, when the covariance is maximum } (\tilde{\rho} = 1), \text{ values of the stray-light contamination above } \sim 1/2 \text{ are strictly forbidden. The exact value of this limit can be obtained from Eq. (3).} \]
which has been obtained as the solution of \( \sigma_i^2(x, y) = 0 \). This quantity rapidly tends to \((2\bar{\rho})^{-1}\) for increasing values of \( M \) and \( \bar{\rho} \), assuming that \( \text{Cov} \left( O_{\text{obs}}(x, y), O_{\text{obs}}(x', y') \right) \) is constant on \( \Omega \). Taking \( M = 24 \) is roughly equivalent to consider a neighborhood \( \sim 1.5^\circ \) around the pixel of interest at Hinode’s spatial sampling. As seen in Fig. 1 the dependence of the variance on \( M \) is relatively weak and the parabolic shape of the curves tends to a straight line as \( M \) increases. If \( \bar{\rho} > 0 \), stray-light contaminations above the following limit are forbidden:

\[
\alpha_{\text{max}} = \bar{\rho}M - \sqrt{(\bar{\rho}M)^2 - M}, \tag{8}
\]

which has been obtained as the solution of \( \sigma^2(x, y) = 0 \). This quantity rapidly tends to \((2\bar{\rho})^{-1}\) for increasing values of \( M \) and is smaller than 1, whenever \( \bar{\rho} > 1/2 \) and \( M \geq (2\bar{\rho} - 1)^{-1} \). If \( \bar{\rho} < 0 \), the behavior is the opposite and large values of the stray-light contamination are favored.

It is important to estimate the covariance between stray-light contamination and observed profiles. To this end, we use the quiet Sun map observed by Lites et al. (2008) to compute the stray-light contamination in a typical Hinode observation as an average over a window of \( \sim 1.5^\circ \) around every pixel of interest. The upper left panel of Fig. 2 shows the variance of the observed Stokes \( I \) continuum normalized to the average variance of the whole map. This is equivalent to the noise variance in case the continuum is assumed to be spectrally flat. Our experience is that the average value of the standard deviation for Stokes \( I \) is a factor 3-4 larger than the noise estimation in Stokes \( Q, U \), and \( V \) made by Lites et al. (2008). We assign this difference to the unavoidable presence of systematic effects in Stokes \( I \) due to flatfielding procedures. Since they change from pixel to pixel, they can be absorbed as part of the noise, though probably not normally distributed. Note the presence of conspicuous horizontal stripes that show the presence of pixels along the slit and at the points of the camera associated with continuum wavelengths which present a somewhat higher variance (higher systematic effects). The upper

from the one commonly used in the literature. We shall see shortly that this can affect the results significantly.

Although \(-1 \leq \bar{\rho} \leq 1\), strong autocorrelations (\( \bar{\rho} < 0 \)) can be effectively discarded in our case. Figure 2 shows how \( \sigma^2_i/\sigma^2_\eta \) [see Eq. (7)] varies with \( M \) and \( \bar{\rho} \), assuming that \( \text{Cov} \left( O_{\text{obs}}(x, y), O_{\text{obs}}(x', y') \right) \) is constant on \( \Omega \). Taking \( M = 24 \) is roughly equivalent to consider a neighborhood \( \sim 1.5^\circ \) around the pixel of interest at Hinode’s spatial sampling. As seen in Fig. 1 the dependence of the variance on \( M \) is relatively weak and the parabolic shape of the curves tends to a straight line as \( M \) increases. If \( \bar{\rho} > 0 \), stray-light contaminations above the following limit are forbidden:

\[
\alpha_{\text{max}} = \bar{\rho}M - \sqrt{(\bar{\rho}M)^2 - M}, \tag{8}
\]

which has been obtained as the solution of \( \sigma^2(x, y) = 0 \). This quantity rapidly tends to \((2\bar{\rho})^{-1}\) for increasing values of \( M \) and is smaller than 1, whenever \( \bar{\rho} > 1/2 \) and \( M \geq (2\bar{\rho} - 1)^{-1} \). If \( \bar{\rho} < 0 \), the behavior is the opposite and large values of the stray-light contamination

\[\text{Note that this modifies the likelihood function used by Asensio Ramos (2009) in the Bayesian framework too.} \]
right panel of Fig. 2 shows the value of $\bar{\rho}$ for the whole map, which has been calculated using a small window in the continuum from 6302.915 Å to 6303.002 Å. Strictly speaking, the covariance should have been calculated using many realizations of the measurement process with the underlying solar profile fixed. Assuming some kind of ergodicity, we estimated the covariance using a wavelength window in the continuum. For consistency, we also estimated the covariance along the time using the time series of Lites et al. [2008], with very similar results. Again, the presence of horizontal stripes is clear, pointing to the systematic character of such defects. On average, $\bar{\rho} \sim 0.45$, although the distribution of covariances is clearly heavy-tailed, with the presence of large values of $\bar{\rho}$ much more frequently than in the case of a Gaussian distribution. The lower panels of Fig. 2 present histograms of the maps. A characteristic is that it is possible to find unphysical $|\rho| > 1$, a consequence of the small sample with which this quantity is computed.

It is customary in standard inversion codes to introduce different weights for each Stokes parameter so that Stokes $I$ does not dominate the merit function. One may think that this alleviates the effect of covariance on the inferred parameters. However, the inclusion of weights does not mimic properly the form of $\chi^2$ because the variance term in Eq. (7) depends explicitly on one of the model parameters, although the intuition is partially right. Therefore, one would expect differences with respect to the parameters obtained with a standard $\chi^2$. This is indeed the case, as shown in Tab. 1. This table shows the results of a least-squares fit to the Stokes profiles shown in Fig. 3 for different values of $\bar{\rho}$. The model proposed is that of Eq. (2) where the magnetic component is a Milne-Eddington atmosphere and the stray-light contamination is obtained from a local average of $M = 24$ pixels around the one of interest. The table presents the magnetic field strength $B$ and inclination $\theta_B$, together with the stray-light contamination $\alpha$ that minimizes the $\chi^2$ for different values of $\bar{\rho}$ and the value of the reduced $\chi^2$ at the minimum. A striking effect of the inclusion of stray-light is that the value of $\alpha$ decreases as soon as $\bar{\rho}$ increases, a direct consequence of the presence of positive correlations between the observed profile and the stray-light. Therefore, the magnetic field strength has to be adjusted accordingly in order to maintain the magnetic flux density, much in the direction of what has been explored by Martínez González et al. [2006]. Note that the cases with $\bar{\rho} > 0.5$ are not so common in the observations. As a consequence, since such large values are not representative of the sample profile, the fits are not good and they should be taken as indicative of what would happen in such a limiting case. Finally, it is important to point out that this behavior would be reduced if the field is strong enough to produce a substantial Zeeman splitting because the magnetic field strength can be inferred directly from the splitting.

3. CONCLUSIONS

We have demonstrated that the inversion of Stokes profiles with models that include stray-light contaminations obtained from the same observations has to be carried out with care. In general, the $\chi^2$ merit function is modified and has to include the effect of the variance of the stray-light profile and the covariance between such pro-

![Figure 3](image_url). Observed Stokes profiles (black solid line) of a pixel in the field-of-view observed by Lites et al. [2008]. Inversions with $\bar{\rho} = 0$ are shown in red, $\bar{\rho} = 0.5$ in blue, $\bar{\rho} = 0.7$ in green and $\bar{\rho} = 1$ in magenta. All of them give reasonably good results but the model parameters change dramatically as shown in Tab. 1. Note that the cases of large $\bar{\rho}$ are not realistic in this situation since we adopt a covariance that is too large with respect to the correct one.

| Table 1 | Maximum-likelihood parameters |
|---------|-------------------------------|
| $\hat{\rho}$ | $B$ | $\theta_B$ | $\alpha$ | $\chi^2_{\text{min}}$ |
| 0.0 | 844 | 155 | 0.91 | 2.0 |
| 0.5 | 558 | 162 | 0.88 | 3.8 |
| 0.7 | 114 | 176 | 0.46 | 11.3 |
| 1.0 | 62 | 169 | 0.02 | 13.8 |

We thank C. Beck, M. J. Martínez González and J. A. Rubiño Martín for useful suggestions. Financial sup-

file and the one being inverted. The most important consequence is that the noise variance now depends on the stray-light contamination parameter, $\alpha$.

A first contribution adds quadratically with the noise variance and is inversely proportional to the number of pixels that contribute to the stray-light profile. When the number of pixels is large enough, this term turns out to be almost negligible. The fundamental reason is that the signal-to-noise ratio of the stray-light profile increases and becomes much larger than that of the local profile, which dominates then the variance. If the stray-light contamination profile is obtained as an average over the full field-of-view, the effect of correlation diminishes considerably and the modified $\chi^2$ converges to the standard $\chi^2$ used in the past.

A second contribution adds linearly and is much more delicate. It accounts for all the systematic effects (fringes, flatfield systematics, data reduction effects, etc.) that plague the observations. The presence of systematics that are well above the noise level make both profiles vary similarly. The covariance term takes this into account by disfavoring models with large stray-light contaminations. In the absence of this term, any inversion method prefers to use a large stray-light contamination because the profiles become much more similar using the standard and incorrect $\chi^2$. A simulated exercise have shown that neglecting the presence of this covariance might lead to artificially strong magnetic field strengths and large stray-light contaminations.

We thank C. Beck, M. J. Martínez González and J. A. Rubiño Martín for useful suggestions. Financial sup-


port by the Spanish Ministry of Science and Innovation through projects AYA2010-18029 (Solar Magnetism and Astrophysical Spectropolarimetry) and Consolider-Ingenio 2010 CSD2009-00038 is gratefully acknowledged.

REFERENCES

Asensio Ramos, A. 2009, ApJ, 701, 1032
Asensio Ramos, A., Martínez González, M. J., & Rubiño Martín, J. A. 2007, A&A, 476, 959
Asensio Ramos, A., Trujillo Bueno, J., & Landi Degl’Innocenti, E. 2008, ApJ, 683, 542
Frutiger, C., Solanki, S. K., Fligge, M., & Bruls, J. H. M. J. 2000, A&A, 358, 1109
Kosugi, T., Matsuzaki, K., Sakao, T., et al. 2007, Sol. Phys., 243, 3
Lagg, A., Woch, J., Krupp, N., & Solanki, S. K. 2004, A&A, 414, 1109
Lites, B., Shine, R. A., López Ariste, A., & Tarbell, T. D. 2002, AGU Fall Meeting Abstracts, A471+
Lites, B. W., Elmore, D. F., Streander, K. V., et al. 2001, in Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, Vol. 4498, Proc. SPIE Vol. 4498, p. 73-83, UV/EUV and Visible Space Instrumentation for Astronomy and Solar Physics, ed. O. H. Siegmund, S. Fineschi, & M. A. Gummin, 73

Lites, B. W., Kubo, M., Socas-Navarro, H., et al. 2008, ApJ, 672, 1237
Lites, B. W. & Skumanich, A. 1990, ApJ, 348, 747
Martínez González, M. J., Collados, M., & Ruiz Cobo, B. 2006, A&A, 456, 1159
Orozco Suárez, D., Bellot Rubio, L. R., & del Toro Iniesta, J. C. 2007a, ApJ, 662, L31
Orozco Suárez, D., Bellot Rubio, L. R., del Toro Iniesta, J. C., et al. 2007b, ApJ, 670, L61
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1986, Numerical Recipes (Cambridge: Cambridge University Press)
Ruiz Cobo, B. & del Toro Iniesta, J. C. 1992, ApJ, 398, 375
Skumanich, A. & Lites, B. W. 1987, ApJ, 322, 473
Socas-Navarro, H., Trujillo Bueno, J., & Ruiz Cobo, B. 2000, ApJ, 530, 977