HFGLDS: Hesitant Fuzzy Gained and Lost Dominance Score Method Based on Hesitant Fuzzy Utility Function for Multi-Criteria Decision Making

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ABSTRACT Hesitant fuzzy set (HFS) is a mighty mathematical tool to represent the hesitant fuzzy information, which can reveal the situation of hesitancy in practical problems. More recently, HFS has been generally utilized in multi-criteria decision-making (MCDM) problem. How to cope with the hesitant fuzzy information effectively is crucial to address the problem of MCDM under hesitant fuzzy environment. In this study, a novel hesitant fuzzy utility function is proposed to depict the hesitant fuzzy information contained in the hesitant fuzzy element (HFE) and realizes the transformation from the hesitant fuzzy information to the crisp value. Firstly, a novel hesitant degree measure is presented to eliminate the drawbacks of traditional methods. The new hesitant degree measure can effectively describe the reliability of the HFE. Afterward, a novel approach of the hesitant fuzzy utility function is constructed by combining the hesitant fuzzy score value and the hesitant degree of the HFE. To generate accurate decision results, we introduce the gained and lost dominance score (GLDS) approach in our work. Then, a new hesitant fuzzy GLDS method is presented based on the GLDS method and the hesitant fuzzy utility function. Furthermore, we utilize the hesitant fuzzy utility value of the alternative to construct the comparison matrix between the criteria, which is used to identify the weight value of the criteria. Ultimately, a practical application is provided to demonstrate the validity and rationality of the proposed method, and comparative analysis with the existing decision making approaches.

INDEX TERMS Hesitant fuzzy set, multi-criteria decision-making, hesitant fuzzy utility function, hesitant degree, hesitant fuzzy GLDS method.

I. INTRODUCTION
Real-world decision-making problems are very complex and usually contain various uncertainties [1]. In traditional methods, decision-makers (DMs) use accurate values to express the evaluation information of the alternative. However, uncertainty and fuzziness information in the decision-making problem cannot be reflected by exact value, and it is easy to lead to a decision making mistake by using traditional method. How to express and handle uncertain and fuzzy information effectively has always been the focus of attention. To precisely depict uncertain and fuzzy information, Zadeh [2] provided the definition of the fuzzy set in 1965. He utilized the membership degree of an element to the set to depict the fuzziness. Due to the advantages of the fuzzy set in managing uncertain and fuzzy information, fuzzy set theory has been broadly used in many different fields [3]–[6]. With the increasing complexity of objective things, human reasoning on objective things shows more uncertainty. Traditional fuzzy sets cannot describe the degree of uncertainty in human beings’ reasoning about objective things. In the light of this issue, many scholars have extended the fuzzy set theory in different forms [7]–[9]. Torra [10] proposed a novel extended form for fuzzy sets, namely, hesitant fuzzy set (HFS). HFS allows the membership degree of elements in the set to be composed of several different values, which is a valid method to depict the information of hesitant uncertainty. HFS is effective for both simple and complex decision-making problems, and is very
close to the reasoning process of people’s depiction of objective things [11]. Because of the great advantage in describing hesitant uncertainty information, HFSs have attracted the attention of many scholars, and are employed to address the problem of multi-criteria decision-making (MCDM) in practical [12]–[15].

One of the pivotal problems in employing the HFS is how to process the hesitant fuzzy information productively. Many researchers have investigated HFS from different aspects and put forward many approaches to handle the hesitant fuzzy information. Xia and Xu [16] proposed the mathematical model of HFS and gave the definition of the hesitant fuzzy element (HFE). Simultaneously, HFE is regarded as the basic unit of the HFS. To compare the size of two HFEs, Xia and Xu [16] presented a method to calculate the score value of HFE, and provided the comparison criterion of the HFE. However, Xia and Xu’s method is limited in validity due to it only takes into account the average value of the HFE. To overcome this defect, Farhadinia [17] defined a new hesitant fuzzy score function by introducing $\beta(l)$ index, and provided the comparison criterion of the HFE. However, when using Farhadinia’s method to calculate the score value of two HFEs, it is necessary to extend the shorter one until the length of two HFEs is equal, which is unreasonable. To solve the shortcomings of the traditional score function, Zhang and Xu [18] suggested a novel measure function and applied it to solve the MCDM issues. We find that the parameter $\delta$ is introduced in Zhang and Xu’s method. However, the value of parameter $\delta$ is difficult to be determined by objective methods. To measure the information volume of HFE, Xu and Xia [19] designed the measurement methods of hesitant fuzzy entropy and cross entropy, and devoted them to the MCDM problems. Zhao et al. [20] adopted the binary entropy model to characterize the uncertainty related to the HFE, and designed a new axiom framework for the hesitant fuzzy entropy measure. The new hesitant fuzzy entropy takes into account both the fuzziness and non-specificity of the HFE. Hu et al. [21] proposed the concept of interval bound footprint (IBF) and defined a novel hesitant fuzzy entropy based on IBF. These measures of hesitant fuzzy entropy only represent the uncertain information contained in the HFE, sometimes it is difficult to determine the relationship between two different HFEs through hesitant fuzzy entropy. To measure the similarity between two HFSs, many scholars have studied the distance measure and similarity measure in hesitant fuzzy environment. Xu and Xia [22] developed various distance measurement methods in hesitant fuzzy environment, and employed the distance measure to characterize the similarity between the HFSs. To overcome the shortcomings of Xu and Xia’s method, Singha et al. [23] improved the existing distance measurement methods, and devised a series of new distance measurement methods for the HFS. Rezaei and Rezaei [24] developed a novel distance measure and similarity measure by considering the hesitant fuzzy index of the HFS, and applied them in pattern recognition. When adopting the approach of hesitant fuzzy similarity measure, it is necessary to guarantee that the number of elements in the two HFEs is the same. If not the same, we need to extend the shorter one by adding some values until the length of all HFEs is the same. Due to the different experts have different risk preferences, it is difficult to unify the extended method for HFEs in practical problems. To aggregate hesitant fuzzy information, many researchers have presented various aggregation operators to deal with information in different hesitant fuzzy environments. Xia et al. suggested a set of aggregation operators to aggregate the hesitant fuzzy information in Ref. [25]. Zhu and Xu [26] studied the characteristics of the Bonferroni mean operators under hesitant fuzzy environment, and presented some new hesitant fuzzy aggregation operators based on Bonferroni mean operators. Zhang [27] generalized the power aggregation operator to the hesitant fuzzy environment, and designed a wide range of hesitant fuzzy power aggregation operators. Liao and Xu [28] extended the hesitant fuzzy hybrid weighted aggregation operators, and proposed a series of new generalized hesitant fuzzy hybrid weighted aggregation operators. These aggregation operators have many excellent properties. However, when the number of element in HFSs increases, the aggregation process will become more complex.

Recently, HFS has been widely utilized in MCDM problems, and many researchers have studied the problem of MCDM under hesitant fuzzy environment. Liao and Xu [11] improved the classical VIKOR approach, and applied it in the hesitant fuzzy environment to handle the problem of MCDM. Wu and Liao [29] claimed that the VIKOR approach only taken into account the subordinate utility value, and neglect the subordinate ranking order, which makes the decision results less robust. According to the knowledge of prospect theory, Zhang and Xu [18] extended the TODIM method and presented a new hesitant fuzzy MCDM approach. However, this method ignores the individual regret value of the alternative, and the desirable alternative may perform poorly concerning some criteria. Wan et al. [30] applied the PROMETHEE approach to the problem of MCDM under hesitant fuzzy environment, and developed a new hesitant fuzzy PROMETHEE method. Then, the hesitant fuzzy PROMETHEE approach is utilized to address the problem of green supplier selection. Farhadinia and Xu [31] extended the HFS, defined a new ordered weighted HFS, and suggested a hesitant fuzzy MCDM approach according to the distance and aggregation operators. The above method solves the problem of MCDM in hesitant fuzzy environment from different aspects.

From the above analysis, it can be seen that both the method of hesitant fuzzy information processing and MCDM are limited in rationality. Motivated by these problems, a novel hesitant fuzzy utility function is presented to depict the hesitant fuzzy information contained in the HFE, and the gained and lost dominance score (GLDS) method is introduced to address the problem of MCDM under hesitant fuzzy environment. The new hesitant fuzzy utility function can realize the transformation from the hesitant fuzzy information to the crisp value. To circumvent the shortcoming of tradi-
tional hesitant degree measure, we devise a new approach for measuring the hesitant degree of the HFE. Afterwards, a novel hesitant fuzzy utility function is suggested by combining the hesitant fuzzy score value and the hesitant degree of the HFE. Finally, combining the hesitant fuzzy utility value of the HFE and the GLDS approach, we develop a novel hesitant fuzzy GLDS (HFGLDS) method under hesitant fuzzy environment.

In summary, we intend to make the following contributions in this study.

1. We devise a new hesitant degree measure to eliminate the drawbacks of traditional methods.
2. We develop a novel hesitant fuzzy utility function by combining the hesitant fuzzy score value and the hesitant degree of the HFE.
3. We use the hesitant fuzzy utility value to establish the comparison matrix between the criteria, which is employed to determine the weight value of each criterion.
4. Based on the hesitant fuzzy utility function and GLDS approach, we design a new HFGLDS method to address the problem of MCDM under hesitant fuzzy environment.
5. A practical application is provided to demonstrate the validity and rationality of the HFGLDS method. The advantages of the HFGLDS approach are emphasized by comparative analyses with other hesitant fuzzy MCDM methods.

We have organized this study in the following way: In Section 2, we briefly recall the concepts of HFS and introduce the GLDS method. In Section 3, we present some new approaches for processing the hesitant fuzzy information, including the hesitant degree measure and the hesitant fuzzy utility function. Some important properties of the hesitant fuzzy utility function are also proved in Section 3. After introducing the hesitant fuzzy utility function and the GLDS approach, we develop a new HFGLDS method in Section 4. Some comparative analyses with the existing hesitant fuzzy MCDM approach are provided in Section 4. A practical application is provided in Section 5 to demonstrate the validity and rationality of the HFGLDS method. Finally, we present our conclusions in Section 6.

II. PRELIMINARY

In this section, we briefly review some essential knowledge of HFS and HFE, and introduce the main steps of the GLDS method.

A. THE CONCEPTS OF HFS AND HFE
The classical fuzzy set theory describes the membership degree of an element to a fixed set. Nevertheless, in real life, people’s cognition of objective things is usually uncertain, and they often hesitate between multiple pieces of information. At this time, the traditional fuzzy set can not effectively express the uncertainty. The traditional fuzzy set theory has its limitations. Therefore, some different extended forms of fuzzy sets are proposed. Among them, the theory of the HFS is very suitable to describe the uncertain situation when the DM has several different evaluation values for the alternative [1]. HFS permits the membership of an element to the set represented by several possible values, and is an effective mathematical tool for expressing and processing uncertain information.

Definition 1 (Hesitant Fuzzy Set): Assume that $F$ is a complete finite set, a hesitant fuzzy set on $F$ is a function $h$ that maps elements in $F$ to a subset of $[0, 1]$, and defined below [16],

$$H = \{ f, h(f) > |f \in F \}$$

where $h(f)$ is a set of several different values defined in $[0, 1]$, representing the possible membership degree of the element $f \in F$ to the set $H$.

For convenience, Xia and Xu [16] refer to $h(f)$ as a HFE in the HFS, which is the basic unit of $H$. Usually the HFE is denoted as $h = \{ \eta^{(1)}, \eta^{(2)}, \ldots, \eta^{(\theta)} \}$, where $\eta^{(1)} < \eta^{(2)} < \ldots < \eta^{(\theta)}$. $\#l$ represents the number of elements in $h$. In particular, $h = \{1\}$ is called the full set, and $h = \{0\}$ is called an empty set. In practical problems, the number of elements in different HFE may be different. In some cases, we need to extend the shorter HFE to accurately handle the hesitant fuzzy information. In this study, we adopt an optimistic attitude to extend the HFE, that is, adding the maximum element of the shorter HFE into the shorter HFE until both of two HFEs have the same length.

Because of the unique expression form of the HFE, it is difficult to directly compare the size of two HFEs. To solve this problem, Xia and Xu [16] presented a method to calculate the score value of HFE, which is utilized to compare the size of two different HFEs. The hesitant fuzzy score function is described below.

Definition 2 (Hesitant Fuzzy Score Function): Suppose $h$ is a HFE in the HFS, the score function of $h$ is depicted as follows [16]:

$$s(h) = \frac{1}{\#l} \sum_{\eta \in h} \eta$$

where $\#l$ represents the number of elements in $h$. $\eta$ is the element in the HFE.

For two HFEs $h_1$ and $h_2$, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 \sim h_2$. The greater the score value of the HFE is, the larger the corresponding HFE is. We note that $s(h)$ gives only the average information of all the elements in the HFE. Sometimes the score function is invalid in comparing the size of two HFEs. For example, there are two HFEs $h_1 = \{0.3, 0.5, 0.7\}$ and $h_2 = \{0.4, 0.6\}$. The score values of $h_1$ and $h_2$ are calculated by Eq.(2), and the results are $s(h_1) = 0.5$ and $s(h_2) = 0.5$, respectively. From the intuitive judgment, $h_1$ and $h_2$ are two different HFEs. The counter-intuitive results are obtained by the calculation method of the score function. To settle the issue, Chen and Xu [32] proposed the calculation method of hesitant fuzzy variance function, whose definition is depicted below.
Definition 3 (Hesitant Fuzzy Variance Function): Assuming that $h$ is a HFE in the HFS, the definition of the variance function [32] of $h$ is depicted as:

$$d(h) = \sqrt{\frac{1}{\#h} \sum_{\eta \in h} (\eta - s(h))^2}$$

(3)

where $d(h)$ denotes the variance function of $h$, $\#h$ represents the number of elements in $h$. When the score value is the same, the smaller the $d(h)$ is, the larger the HFE is. Chen and Xu [32] give the comparison criterion between two HFEs by combining the score value and the variance value, which depicted as follows.

Definition 4 (Hesitant Fuzzy Element Comparison Criterion): Suppose $h_1$ and $h_2$ are two HFEs, $s(h_1)$ and $s(h_2)$ are their score values, respectively, and $d(h_1)$ and $d(h_2)$ are their variance values, respectively. Then the comparison rules of $h_1$ and $h_2$ are depicted as follows:

1. if $s(h_1) > s(h_2)$, then $h_1 \succ h_2$
2. if $s(h_1) = s(h_2)$
   a) if $d(h_1) > d(h_2)$, then $h_2 \succ h_1$
   b) if $d(h_1) = d(h_2)$, then $h_1 = h_2$
   c) if $d(h_1) < d(h_2)$, then $h_1 \succ h_2$

The following example is utilized to illustrate the comparison rules of HFE.

Example 1: Let $H$ be a given HFS. $h_1 = \{0.2, 0.3, 0.5, 0.8\}$, $h_2 = \{0.4, 0.6, 0.8\}$, and $h_3 = \{0.3, 0.45, 0.6\}$ are three HFEs defined in $H$. Comparing the size of the three HFEs by the above comparison rules.

The score values of the three HFEs are calculated below:

$$s(h_1) = 0.45, \quad s(h_2) = 0.6, \quad s(h_3) = 0.45$$

According to the score values, we can get $h_2 \succ h_1$, and $h_2 \succ h_3$. However, we cannot distinguish the size between $h_1$ and $h_3$. Hence, we also require to compute the variance values of the $h_1$ and $h_3$.

$$d(h_1) = 0.2291, \quad d(h_3) = 0.1225$$

Due to $d(h_1) > d(h_3)$, then $h_3 \succ h_1$. Therefore, the ranking result of the three HFEs is $h_2 \succ h_3 \succ h_1$.

Definition 5 (Hesitant Fuzzy Deviation Function): To measure the deviation degree of each element in the HFE, Wei et al. [33] proposed the calculation method for the deviation function of the HFE, whose definition is described as follows:

Suppose $h = \{\eta^{(1)}, \eta^{(2)}, \cdots, \eta^{(#h)}\}$ is a HFE defined in the HFS $H$. The deviation function of the $h$ is defined as:

$$v(h) = \frac{2}{\#h \times (#h - 1)} \sum_{i=1}^{#h-1} \sum_{j=i+1}^{#h} (\eta^{(j)} - \eta^{(i)})$$

(4)

where $\#h$ represents the number of elements in $h$. In particular, when $\#h = 1$, $v(h) = 0$.

B. THE GLDS METHOD

The gained and lost dominance score (GLDS) method was first proposed by Wu and Liao [29], and applied to the problem of MCDM with probabilistic linguistic information. This approach is to determine the ranking of the alternative by calculating the dominance flow between two alternatives on each criterion. The optimal alternative should have a larger gained dominance score and a smaller lost dominance score concerning each criterion. The calculation process of the GLDS method is simple and has high robustness. Now we briefly depict the decision process of the GLDS approach as follows.

Suppose there are $n$ alternatives in the MCDM problem, and each alternative has $m$ criteria. A set of alternatives $P = \{p_1, p_2, \cdots, p_n\}$, and the criteria set $C = \{C_1, C_2, \cdots, C_m\}$. The corresponding criterion weight vector is $W = \{\omega_1, \omega_2, \cdots, \omega_m\}$. The evaluation value of the alternative $p_i$ ($i = 1, 2, \cdots, n$) under the criterion $C_j$ ($j = 1, 2, \cdots, m$) is expressed by $u_{ij}$.

- Step 1: Calculate the dominance flow $g_j(p_i, p_k)$ of the alternative $p_i$ ($i = 1, 2, \cdots, n$) to the alternative $p_k$ ($k = 1, 2, \cdots, n$) under criterion $C_j$ ($j = 1, 2, \cdots, m$) according to the following approach.

$$g_j(p_i, p_k) = \begin{cases} \max \{u_{ij} - u_{kj}\}, & \text{for benefit criterion } C_j \\ \max \{u_{kj} - u_{ij}\}, & \text{for cost criterion } C_j \end{cases}$$

(5)

where $u_{ij}$ represents the assessment value of the alternative $p_i$ regarding to the criterion $C_j$, and $u_{ij}$ denotes the evaluation value of the alternative $p_k$ under the criterion $C_j$.

- Step 2: Compute the gained dominance score value for the alternative $p_i$ ($i = 1, 2, \cdots, n$) concerning criterion $C_j$ ($j = 1, 2, \cdots, m$), and denotes as $Gd_j(p_i)$, which depicted as follows:

$$Gd_j(p_i) = \sum_{k=1}^{n} g_j(p_i, p_k)$$

(6)

- Step 3: Calculate the lost dominance score value of the alternative $p_i$ ($i = 1, 2, \cdots, n$) with respect to the criterion $C_j$ ($j = 1, 2, \cdots, m$), and called it as $Ld_j(p_i)$. The expression is presented below.

$$Ld_j(p_i) = \max_k g_j(p_k, p_i), \quad (k = 1, 2, \cdots, n)$$

(7)

- Step 4: Compute the net gained dominance score value for the alternative $p_i$ ($i = 1, 2, \cdots, n$) according to the following expression:

$$DS(p_i) = \sum_{j=1}^{m} \omega_j Gd_j(p_i)$$

(8)

where $\omega_j$ is the weight value of the criterion $C_j$. According to the net gained dominance score value of the alternative $p_i$, we can determine the ranking results.
of all alternatives in descending order, and the rank set is $R_1 = \{r_1(p_1), r_1(p_2), \ldots, r_1(p_n)\}$.

- Step 5: According to Eq. (9), the net lost dominance score value of the alternative $p_i$ ($i = 1, 2, \ldots, n$) is calculated, which shows as follows:

$$LS(p_i) = \max_j \omega_j Ld_j(p_i), \quad (j = 1, 2, \ldots, m) \tag{9}$$

where $\omega_j$ is the weight value of the criterion $C_j$.

We can obtain the ranking results of all alternatives in ascending order on the basis of the net lost dominance score value of the alternative $p_i$, and the rank set is $R_2 = \{r_2(p_1), r_2(p_2), \ldots, r_2(p_n)\}$.

- Step 6: Using the vector normalization method to normalize $DS(p_i)$ and $LS(p_i)$. The normalization approach is described as follows:

$$DS(p_i) = \frac{DS(p_i)}{\sqrt{\sum_{i=1}^{n}(DS(p_i))^2}}$$

$$LS(p_i) = \frac{LS(p_i)}{\sqrt{\sum_{i=1}^{n}(LS(p_i))^2}} \tag{10}$$

- Step 7: According to $\tilde{DS}(p_i)$ and $\tilde{LS}(p_i)$ of the alternative $p_i$ ($i = 1, 2, \ldots, n$), the collective score value ($CS(p_i)$) for the alternative $p_i$ is calculated below:

$$CS(p_i) = \frac{n - r_1(p_i) + 1}{n(n+1)/2} \tilde{DS}(p_i) - \frac{r_2(p_i)}{n(n+1)/2} \tilde{LS}(p_i) \tag{11}$$

- Step 8: The final ranking results are generated in descending order of $CS$ value.

In this paper, we utilize the GLDS approach to solve the multi-criteria decision making problem under hesitant fuzzy environment.

### III. A NOVEL HESITANT FUZZY UTILITY FUNCTION

Due to the information expressed by HFS is very close to the way of human thinking and reasoning, HFE is an intuitive and flexible method to evaluate qualitative variables [11]. HFS is frequently utilized in MCDM problems as its advantages in expressing fuzzy uncertain information. More recently, the measure of hesitant fuzzy information has aroused growing concerns in the hesitant fuzzy environment. One of the key issues for applying HFE is how to process the hesitant fuzzy information effectively. In essence, the processing of hesitant fuzzy information is to map fuzzy information into the crisp number by using measurement function. At present, most researchers use the score function to evaluate the HFE. However, the score function can only reflect one aspect of HFE, and cannot completely represent the information contained in the HFE. Because of the complexity of HFE, it is necessary to synthesize the attributes for many aspects to represent the information contained in the HFE effectively. Therefore, in our work, we study the information contained in HFE from the perspective of the HFE themselves, and adopt the hesitant fuzzy utility value to depict the information contained in the HFE. Considering that the current research work on the hesitant fuzzy utility value is still less, we propose an approach for measuring the hesitant fuzzy utility value, and discuss the related properties of the method in detail. The new hesitant fuzzy utility function comprehensively considers the hesitant fuzzy score value and the hesitant degree of the HFE. Firstly, we present an approach to calculate the hesitant degree.

### A. NEW MEASURE METHOD FOR HESITANT DEGREE

The hesitant degree is a measure that represents the intrinsic information of HFE and can reflect the reliability of HFE. When making a decision, for instance, one expert may provide his evaluation value as $h_1 = \{0.4\}$, and another expert may give his evaluation value as $h_2 = \{0.3, 0.4, 0.5\}$. According to the intuitive judgment, we can know that $h_1$ has no hesitation and $h_2$ has some degree of hesitation. Ergo, the reliability of $h_1$ is greater than that of $h_2$. However, we cannot compare the hesitant degree of two HFEs by using the hesitant fuzzy score function. Therefore, it is necessary to design a method to measure the hesitant degree of the HFE. To evaluate the hesitant degree of HFE, some scholars have studied the measure of hesitant degree for the HFE, and put forward the calculation approach for hesitant degree of the HFE. We briefly introduce the existing measurement methods for hesitant degree as follows.

**Definition 6:** Suppose $h = \{\eta^{(1)}, \eta^{(2)}, \ldots, \eta^{(#l)}\}$ is a HFE defined in the HFS $H$. The hesitant degree of the HFE is defined as follows [34].

$$u(h) = 1 - \frac{1}{#l} \tag{12}$$

where $#l$ denotes the number of elements in HFE.

Through analysis, we find that Zeng et al.’s method only considers the length of HFE, but ignores the differences among the elements of the HFE. Therefore, this approach cannot accurately calculate the hesitant degree of the HFE. For example, suppose there are two HFEs $h_1 = \{0.8, 0.9\}$ and $h_2 = \{0.2, 0.9\}$. From intuitive analysis, we know that $h_1$ and $h_2$ have different hesitant degree. However, the hesitant degree of $h_1$ and $h_2$ calculated by Zeng et al.’s method is $u(h_1) = u(h_2) = 0.5$. Obviously, the result contradicts the intuitive analysis. Based on the above analysis, we know that Zeng et al.’s method has some defects, and cannot get accurate decision results in some cases. To circumvent the shortcomings, we develop a novel measure method for hesitant degree.

To overcome the defects of existing methods, we propose a novel approach for measuring the hesitant degree by combining Zeng et al.’s method and hesitant fuzzy deviation function. The definition of the new measure of hesitant degree is described as follows.

**Definition 7:** Assuming that $h$ is a HFE defined in HFS $H$. The new hesitant degree of the HFE is defined as follows.

$$Hd(h) = \frac{u(h) \times \nu(h)}{u(h) + \nu(h)} \tag{13}$$

where $Hd(h)$ represents the hesitant degree of $h$. 

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The greater the \(Hd(h)\) is, the higher the hesitant degree of the \(h\) is, and the worse the credibility is. Hesitant degree can effectively express the relationship between the elements in the HFE. When all elements in the HFE are the same, the hesitant degree is the smallest, and the minimum value is 0.

The new hesitant degree satisfies the following properties:

1. \(0 < Hd(h) \leq 1\);
2. If \(\#l = 1\), then \(Hd(h) = 0\);

The following is a simple example to illustrate how to calculate the hesitant degree of HFE.

**Example 2:** Assume that \(h = \{0.4, 0.7, 0.9\}\) is a HFE defined in the HFS \(H\). The hesitant degree of \(h\) is calculated as follows:

Firstly, the hesitant degree measure of Zeng et al.’s method is computed, and the result is as follows.

\[
u(h) = 1 - \frac{1}{3} = 0.6667\]

Then, calculate the deviation value of \(h\), and the result is depicted below.

\[
v(h) = \frac{2}{3 \times (3 - 1)} \times (0.3 + 0.5 + 0.2) = 0.3333\]

Finally, according to Eq.(13), we can obtain the hesitant degree of \(h\) as follows.

\[
HD(h) = \frac{0.3333 \times 0.6667}{0.3333 + 0.6667} = 0.2222
\]

Let’s go back to the problem we analyzed above. Two HFEs are \(h_1 = \{0.8, 0.9\}\), and \(h_2 = \{0.2, 0.9\}\), respectively. Using Eq.(13) to calculate the hesitant degree of the two HFEs, then we can obtain \(HD(h_1) = 0.0833\), and \(HD(h_2) = 0.2917\), which is consistent with our intuitive analysis. This example implies that our proposed method can effectively describe the hesitant degree of the HFE. We will consider the hesitant degree of HFE when calculating the hesitant fuzzy utility value.

### B. A NOVEL HESITANT FUZZY UTILITY FUNCTION

HFE can effectively express the hesitant fuzzy information of qualitative variables. However, it is difficult to make decisions directly through HFE in practical decision making problems. In practical MCDM problem, effective processing of the hesitant fuzzy information is the key to address the problem of MCDM under hesitant fuzzy environment. Hence, it is necessary to find a way to transform the hesitant fuzzy information into the crisp value that can help the decision maker to make a decision. To accurately describe the information contained in the HFE, we design a novel hesitant fuzzy utility function to quantify the hesitant fuzzy information to generate the crisp value of the HFE. The propose approach considers the hesitant degree and the hesitant fuzzy score value of the HFE, which can effectively characterize the information content in the HFE and simplify the processing of the hesitant fuzzy information. In addition, the new hesitant fuzzy utility function is completely driven by data in describing the information contained in the HFE, and realizes the transformation from the HFE to the crisp value. The definition of the hesitant fuzzy utility function is depicted as follows:

**Definition 7:** Assume that \(h\) is a HFE defined in the HFS \(H\). The hesitant fuzzy utility function of \(h\) is defined as follows.

\[
U(h) = (1 - Hd(h)) \times s(h)
\]

where \(U(h)\) is the hesitant fuzzy utility function of \(h\), \(Hd(h)\) is the hesitant degree of \(h\), and \(s(h)\) is the score function of \(h\).

Next we debate some properties of the new hesitant fuzzy utility function.

**Proposition 1:** For the given HFE \(h\), when \(0 \leq s(h) \leq 1\), and \(0 \leq Hd(h) \leq 1\), the hesitant fuzzy utility function \(U(h)\) increases monotonically with the score value \(s(h)\).

**Proof:**

\[
\frac{\partial U(h)}{\partial s(h)} = 1 - Hd(h) > 0
\]

Therefore, \(U(h)\) increases monotonically with \(s(h)\).

**Proposition 2:** For the given HFE \(h\), when \(0 \leq s(h) \leq 1\), and \(0 \leq Hd(h) \leq 1\), the hesitant fuzzy utility function \(U(h)\) decreases monotonically with the hesitant degree \(Hd(h)\).

**Proof:**

\[
\frac{\partial U(h)}{\partial Hd(h)} = -s(h) < 0
\]

Hence, \(U(h)\) decreases monotonically with \(Hd(h)\).

**Proposition 3:** Assume that \(h_1\) and \(h_2\) are two HFEs in the HFS. If \(Hd(h_1) = Hd(h_2)\) and \(s(h_1) = s(h_2)\), then \(U(h_1) = U(h_2)\).

**Proof:** This conclusion is obvious.

**Proposition 4:** For the given HFE \(h\), if \(\#l = 1\), then we have \(U(h) = s(h)\).

**Proof:** When \(\#l = 1\), according to Eq.(13), we obtain \(HD(h) = 0\). Therefore, we have \(U(h) = s(h)\).

In this study, we employ the hesitant fuzzy utility function to quantify the hesitant fuzzy information contained in the HFE to get the crisp value of the HFE. Then, a new hesitant fuzzy MCDM approach is presented by combining the hesitant fuzzy utility function and the GLDS method to solve the MCDM problem under hesitant fuzzy environment.

### IV. HFGLDS APPROACH BASED ON HESITANT FUZZY UTILITY FUNCTION

As a powerful tool to characterize the uncertain and fuzzy information, HFE plays a significant role in practical decision making and is widely used in the field of MCDM. Many scholars have studied the problem of MCDM under the hesitant fuzzy environment, and put forward many hesitant fuzzy MCDM methods. However, traditional MCDM methods have some limitations as we mentioned in the introduction, and they are not suitable for decision-making problems in all cases. In complex practical problems, it is easy to lead to decision-making mistakes. To circumvent the problems existing in traditional methods, the GLDS decision-making approach [29] is introduced into the hesitant fuzzy MCDM
problem. Then, a novel hesitant fuzzy GLDS (HFGLDS) method is proposed based on the hesitant fuzzy utility function and the GLDS decision-making method. In HFGLDS method, the hesitant fuzzy utility function is applied to describe the information contained in each hesitant fuzzy attribute value, and then the final ranking results of each alternative are generated by the GLDS method to process the information. The main steps of the HFGLDS method are depicted below.

A. THE IMPLEMENTATION PROCESS OF THE HFGLDS METHOD

In order to avoid the interaction between DMs, the alternative is usually evaluated anonymously in the MCDM problem. Because of the different knowledge background and experience of DMs, it is difficult for different DMs to agree on the evaluation information of the same alternative. To describe the evaluation information of different DMs, we regard the decision information of different DMs as the HFE. Assume that there is a MCDM problem that consists of finite alternatives set \( S = \{S_1, S_2, \ldots, S_n \} \), a corresponding set of criteria \( C = \{C_1, C_2, \ldots, C_m \} \). A set of assessment information of the alternative \( S_i \) \((i = 1, 2, \ldots, n) \) under the criterion \( C_j \) \((j = 1, 2, \ldots, m) \) is given by the DMs, denoted as \( h_{ij} \). \( h_{ij} \) denotes the possible membership degree of the alternative \( S_i \) satisfies the criterion \( C_j \). The problem of MCDM is to choose the desirable alternative from a set of alternatives.

1) CONSTRUCT THE HESITANT FUZZY UTILITY MATRIX

- Step 1: Construct the hesitant fuzzy decision matrix
  According to the decision information of DMs for each alternative, a hesitant fuzzy decision matrix \( H \) is constructed for MCDM problems. Each element in \( H \) is a HFE, and the hesitant fuzzy decision matrix is expressed as follows:

\[
H = \begin{bmatrix}
h_{11} & h_{12} & \cdots & h_{1m} \\
h_{21} & h_{22} & \cdots & h_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
h_{n1} & h_{n2} & \cdots & h_{nm}
\end{bmatrix} \tag{15}
\]

- Step 2: Calculate the hesitant fuzzy score value and the hesitant degree of all the HFE in the hesitant fuzzy decision matrix.
  According to Eqs. (2) and (13), the hesitant fuzzy score value and the hesitant degree of the HFE for the alternative \( S_i \) \((i = 1, 2, \ldots, n) \) under all criteria are calculated, respectively.

- Step 3: Compute the hesitant fuzzy utility value of the HFE by combining the hesitant fuzzy score value and the hesitant degree of the HFE.
  According to Eq.(14), the hesitant fuzzy utility value \( U \) of each alternative under all criteria is computed, which can characterize the information contained in the HFE.

- Step 4: Construct the hesitant fuzzy utility matrix
  The hesitant fuzzy utility matrix can be constructed based on the hesitant fuzzy utility value of each HFE, which shown as follows.

\[
E = \begin{bmatrix}
U(h_{11}) & U(h_{12}) & \cdots & U(h_{1m}) \\
U(h_{21}) & U(h_{22}) & \cdots & U(h_{2m}) \\
\vdots & \vdots & \ddots & \vdots \\
U(h_{n1}) & U(h_{n2}) & \cdots & U(h_{nm})
\end{bmatrix} \tag{16}
\]

2) DETERMINE THE WEIGHT OF EACH CRITERION BASED ON THE HESITANT FUZZY UTILITY VALUE

The weight value of the criteria is an important parameter in MCDM problems. How to effectively identify the weight value of the criteria is a problem worth discussing. In most decision making approaches, the weight value of the criteria is given by the decision maker according to his own experience and knowledge. However, these methods are greatly affected by the subjectivity of decision makers, and sometimes it is difficult to get accurate decision results. To decrease the influence of the subjective cognition of the decision makers on the decision results, it is necessary to determine the weight value of the criteria by objective methods. In this study, we determine the weight value of each criterion based on the evaluation information about the alternative. Specifically, applying the assessment information of the alternative to construct the comparison matrix between the criteria, then the weight values of each criterion are obtained by solving the comparison matrix between the criteria. The comparison matrix between the criteria is established by utilizing the method in Ref. [35]. The detailed calculation process is described below.

- Step 1: We adopt the vector normalization method to normalize the hesitant fuzzy utility value, shown as Eq.(17).

\[
\tilde{U}(h_{ij}) = \frac{U(h_{ij})}{\sqrt{\sum_{i=1}^{n}(U(h_{ij}))^2}} \tag{17}
\]

- Step 2: According to the normalized hesitant fuzzy utility value, the covariance of the normalized hesitant fuzzy utility value for all alternatives between the criterion \( C_j \) \((j = 1, 2, \ldots, m) \) and the criterion \( C_t \) \((t = 1, 2, \ldots, m) \) is computed as follows and represented as \( \text{cov}(C_j, C_t) \).

\[
\text{cov}(C_j, C_t) = \frac{1}{n} \sum_{i=1}^{n} \left( \tilde{U}(h_{ij}) - \tilde{U}(h_{ij}) \right) \left( \tilde{U}(h_{iti}) - \tilde{U}(h_{iti}) \right) \tag{18}
\]

where \( \tilde{U}(h_{ij}) = \frac{1}{n} \sum_{i=1}^{n} U(h_{ij}), j, t = 1, 2, \ldots, m \).

If the covariance value is less than zero, take its absolute value. Then, the covariance matrix between the criteria can be established below.

\[
C = \begin{bmatrix}
\text{cov}(C_1, C_1) & \text{cov}(C_1, C_2) & \cdots & \text{cov}(C_1, C_m) \\
\text{cov}(C_2, C_1) & \text{cov}(C_2, C_2) & \cdots & \text{cov}(C_2, C_m) \\
\vdots & \vdots & \ddots & \vdots \\
\text{cov}(C_m, C_1) & \text{cov}(C_m, C_2) & \cdots & \text{cov}(C_m, C_m)
\end{bmatrix} \tag{19}
\]
Step 3: Transform the covariance matrix to obtain the relative covariance matrix. The elements in the relative covariance matrix are obtained by dividing each column element $\text{cov}(C_j, C_i)$ ($j, t = 1, 2, \ldots, m$) by the element $\text{cov}(C_j, C_j)$, and represented as $R(C_j, C_i)$.

$$R = \begin{bmatrix}
1 & R(C_1, C_2) & \cdots & R(C_1, C_m) \\
R(C_2, C_1) & 1 & \cdots & R(C_2, C_m) \\
\vdots & \vdots & \ddots & \vdots \\
R(C_m, C_1) & R(C_m, C_2) & \cdots & 1
\end{bmatrix}$$  \hfill (20)

Step 4: According to the relative covariance matrix, we can obtain the comparison matrix between the criteria by Eqs.(21) and (22). The element in the comparison matrix is denoted as $B(C_j, C_i)$ ($i, t = 1, 2, \ldots, m$).

$$B(C_j, C_i) = \frac{R(C_j, C_i)}{\sqrt{R(C_j, C_i) \times R(C_i, C_i)}}$$  \hfill (21)

$$B(C_i, C_j) = \frac{R(C_i, C_j)}{\sqrt{R(C_i, C_j) \times R(C_j, C_j)}}$$  \hfill (22)

Then, the comparison matrix between the criteria can be constructed as follows.

$$B = \begin{bmatrix}
1 & B(C_1, C_2) & \cdots & B(C_1, C_m) \\
B(C_2, C_1) & 1 & \cdots & B(C_2, C_m) \\
\vdots & \vdots & \ddots & \vdots \\
B(C_m, C_1) & B(C_m, C_2) & \cdots & 1
\end{bmatrix}$$  \hfill (23)

Step 5: The weight value of each criterion can be generated by solving the comparison matrix between the criteria. The calculation process of the weight value of each criterion is described as follows [36]. Firstly, calculate the product of all elements in each row of the comparison matrix, denotes as $P_j$ ($j = 1, 2, \ldots, m$).

$$P_j = \prod_{i=1}^{m} B(C_j, C_i)$$  \hfill (24)

Then, compute the $m$-th root for the product of all elements in each row of the comparison matrix, represented as $G_j$.

$$G_j = \sqrt[m]{P_j}$$  \hfill (25)

Ultimately, the weight values of each criterion are obtained by normalizing $G_j$, which expressed as $\omega_j$.

$$\omega_j = \frac{G_j}{\sum_{j=1}^{m} G_j}$$  \hfill (26)

3) HFGGLDS METHOD BASED ON HESITANT FUZZY UTILITY VALUE

In this study, we present a novel hesitant fuzzy MCDM approach by combining the GLDS method and the hesitant fuzzy utility value. The new MCDM method is described below.

Step 1: According to the utility value of the HFE, the dominance flow $g_j(S_i, S_k)$ of the alternative $S_i$ ($i = 1, 2, \ldots, n$) over the alternative $S_k$ ($k = 1, 2, \ldots, n$) under the criterion $C_j$ ($j = 1, 2, \ldots, m$) is calculated, shown as Eq.(27).

$$g_j(S_i, S_k) = \begin{cases}
\max((U(h_{ij}) - U(h_{kj})), 0), & \text{for benefit criterion } C_j \\
\max((U(h_{kj}) - U(h_{ij})), 0), & \text{for cost criterion } C_j
\end{cases}$$  \hfill (27)

where $U(h_{ij})$ and $U(h_{kj})$ are the hesitant fuzzy utility values of $h_{ij}$ and $h_{kj}$, respectively. $h_{ij}$ represents the evaluation value of the alternative $S_i$ with respect to the criterion $C_j$.

Step 2: On the basis of the dominance flow $g_j(S_i, S_k)$ among the alternatives under the criterion $C_j$ ($j = 1, 2, \ldots, m$), the gained dominance score value of the alternative $S_i$ ($i = 1, 2, \ldots, n$) under the criterion $C_j$ is calculated, which expressed as follow:

$$Gd_j(S_i) = \sum_{k=1}^{n} g_j(S_i, S_k)$$  \hfill (28)

Step 3: According to Eq.(29), we can obtain the lost dominance score value of the alternative $S_i$ ($i = 1, 2, \ldots, n$) under the criterion $C_j$ ($j = 1, 2, \ldots, m$). The expression is presented below.

$$Ld_j(S_i) = \max_g g_j(S_k, S_i), \quad (k = 1, 2, \ldots, n)$$  \hfill (29)

Step 4: Compute the net gained dominance score value for the alternative $S_i$ ($i = 1, 2, \ldots, n$) according to the following expression:

$$DS(S_i) = \sum_{j=1}^{m} \omega_j Gd_j(S_i)$$  \hfill (30)

where $\omega_j$ is the weight value of the criterion $C_j$. According to the net gained dominance score value of the alternative $S_i$, we can determine the ranking results of all alternatives in descending order, and the rank set is $R_1 = \{r_1(S_1), r_1(S_2), \ldots, r_1(S_n)\}$.

Step 5: Calculate the net lost dominance score value of the alternative $S_i$ ($i = 1, 2, \ldots, n$) by Eq.(31).

$$LS(S_i) = \max_j \omega_j Ld_j(S_i), \quad (j = 1, 2, \ldots, m)$$  \hfill (31)

where $\omega_j$ is the weight value of the criterion $C_j$. We can obtain the ranking results of all alternatives in ascending order on the basis of the net lost dominance score value of the alternative $S_i$, and the rank set is $R_2 = \{r_2(S_1), r_2(S_2), \ldots, r_2(S_n)\}$.

Step 6: Using the vector normalization method to normalize $DS(S_i)$ and $LS(S_i)$. The normalization approach...
TABLE 1. The hesitant fuzzy decision matrix.

|   | $C_1$          | $C_2$          | $C_3$          | $C_4$          |
|---|----------------|----------------|----------------|----------------|
| $S_1$ | {0.2, 0.4, 0.7} | {0.2, 0.6, 0.8} | {0.2, 0.3, 0.6, 0.7, 0.9} | {0.3, 0.4, 0.5, 0.7, 0.8} |
| $S_2$ | {0.2, 0.4, 0.7, 0.9} | {0.1, 0.2, 0.4, 0.5} | {0.3, 0.4, 0.6, 0.9} | {0.5, 0.6, 0.8, 0.9} |
| $S_3$ | {0.3, 0.5, 0.6, 0.7} | {0.2, 0.4, 0.5, 0.6} | {0.3, 0.5, 0.7, 0.8} | {0.2, 0.5, 0.6, 0.7} |
| $S_4$ | {0.3, 0.5, 0.6} | {0.2, 0.4} | {0.5, 0.6, 0.7} | {0.8, 0.9} |

is depicted below:

$$
\begin{align*}
\tilde{DS}(S_i) &= \frac{DS(S_i)}{\sqrt{\sum_{i=1}^{n}(DS(S_i))^2}} \\
\tilde{LS}(S_i) &= \frac{LS(S_i)}{\sqrt{\sum_{i=1}^{n}(LS(S_i))^2}}
\end{align*}
$$

(32)

• Step 7: According to $\tilde{DS}(S_i)$ and $\tilde{LS}(S_i)$ of the alternative $S_i$ $(i = 1, 2, \cdots, n)$, the collective score value ($CS(S_i)$) of the alternative $S_i$ is calculated as follows:

$$
CS(S_i) = \tilde{DS}(S_i) \times \frac{n - r_1(S_i) + 1}{n(n + 1)/2} - \tilde{LS}(S_i) \times \frac{r_2(S_i)}{n(n + 1)/2}
$$

(33)

• Step 8: According to the collective score value of the alternative, we can obtain the ranking results for all alternatives, and the desirable alternative is determined. The main step of our proposed method is depicted in Fig.1.

B. NUMERICAL EXAMPLES

The following example demonstrates the decision process of our proposed method in detail, and compares it with several current decision making methods. This numerical example is derived from Ref. [32].

Example 3: The board of directors of a company decides to plan the development strategy of a large project for the company. We need to choose the most appropriate alternative from the four alternatives. $S = \{S_1, S_2, S_3, S_4\}$ is the alternative set composed of four alternatives. Four criteria that have a great impact on the alternative are considered, namely $C_1$: financial prospects, $C_2$: customer satisfaction, $C_3$: international business prospects, and $C_4$: learning and growth prospects. A set of criteria is $C = \{C_1, C_2, C_3, C_4\}$, which composed of four criteria. Board members were invited to assess the four alternatives. To eliminate the mutual influence between the board members, the board members evaluate the alternative anonymously. Considering that there may be different opinions among the board members, the HFE is used to express the evaluation information of board members on the alternative. According to the evaluation information of the board members, we can construct the hesitant fuzzy decision matrix $H$ as listed in Table 1.

The hesitant fuzzy score values of each HFE are computed by Eq.(2), which are displayed in Table 2.

TABLE 2. The hesitant fuzzy score value of each HFE.

|   | $C_1$          | $C_2$          | $C_3$          | $C_4$          |
|---|----------------|----------------|----------------|----------------|
| $S_1$ | 0.4333 | 0.5333 | 0.5400 | 0.5400 |
| $S_2$ | 0.5500 | 0.3000 | 0.5500 | 0.7000 |
| $S_3$ | 0.5250 | 0.4250 | 0.5750 | 0.5000 |
| $S_4$ | 0.4667 | 0.3000 | 0.6000 | 0.8500 |

The hesitant degree of each HFE is calculated by Eq.(13), and the results are displayed in Table 3.

TABLE 3. The hesitant degree of each HFE.

|   | $C_1$          | $C_2$          | $C_3$          | $C_4$          |
|---|----------------|----------------|----------------|----------------|
| $S_1$ | 0.2222 | 0.2500 | 0.2483 | 0.1962 |
| $S_2$ | 0.2609 | 0.1780 | 0.2308 | 0.1780 |
| $S_3$ | 0.1681 | 0.1681 | 0.2056 | 0.1967 |
| $S_4$ | 0.1538 | 0.1429 | 0.1111 | 0.0833 |

By combining the hesitant fuzzy score value and the hesitant degree of the HFE, the hesitant fuzzy utility value of each HFE is derived by Eq.(14), which is shown in Table 4.

TABLE 4. The hesitant fuzzy utility value of each HFE.

|   | $C_1$          | $C_2$          | $C_3$          | $C_4$          |
|---|----------------|----------------|----------------|----------------|
| $S_1$ | 0.3370 | 0.4000 | 0.4059 | 0.4340 |
| $S_2$ | 0.4065 | 0.2466 | 0.4231 | 0.5754 |
| $S_3$ | 0.4367 | 0.3536 | 0.4568 | 0.4016 |
| $S_4$ | 0.3949 | 0.2571 | 0.5333 | 0.7792 |

The hesitant degree of each HFE is calculated by Eq.(13), and the results are displayed in Table 3.

By combining the hesitant fuzzy score value and the hesitant degree of the HFE, the hesitant fuzzy utility value of each HFE is derived by Eq.(14), which is shown in Table 4.

By Eq.(17), we can get the normalized hesitant fuzzy utility value of each HFE, which listed in Table 5.
According to Eqs. (18)-(22), we can obtain the comparison matrix between the criteria, shown as

\[
B = \begin{bmatrix}
1 & 0.4537 & 0.8554 & 0.3486 \\
2.2041 & 1 & 1.8853 & 0.7685 \\
1.1691 & 0.5304 & 1 & 0.4076 \\
2.8682 & 1.3013 & 2.4534 & 1
\end{bmatrix}
\]

The weight values of the criteria can be derived by Eqs. (24)-(26), and the results are presented in Table 6.

According to the hesitant fuzzy utility value of each HFE, we can compute the gained and lost dominance score values by Eqs. (27)-(31), and the results are displayed in Table 7 and Table 8.

The normalized values of the net gained dominance score and net lost dominance score are calculated by Eq. (32), the results are listed in Table 9.
The normalized values of the $\tilde{D}(S_i)$ and $\tilde{L}(S_i)$ are shown in Table 9.

|     | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|-----|------|------|------|------|
| $\tilde{D}(S_i)$ | 0.2461 | 0.2914 | 0.2085 | 0.9006 |
| $\tilde{L}(S_i)$ | 0.6146 | 0.3628 | 0.6726 | 0.1956 |

Based on the results in Table 9, the collective score value of each alternative is generated by Eq. (33), which shows as follows:

- $CS(S_1) = -0.1352$
- $CS(S_2) = 0.0149$
- $CS(S_3) = -0.2482$
- $CS(S_4) = 0.3407$

According to the collective score value of each alternative, we can rank the alternative. The ranking result is $S_4 > S_2 > S_1 > S_3$, thus $S_4$ is the desirable alternative.

### C. COMPARATIVE ANALYSIS

To further demonstrate the rationality and validity of our proposed method, we compare it with some current hesitant fuzzy MCDM methods. The decision results for different decision making methods are shown in Table 10. From Table 10, we note that Xia and Xu’s method [25], ELECTRE II method [32], TOPSIS method [37] and our proposed method get the same decision results, which shows the effectiveness and rationality of our proposed method. Although Xia and Xu’s method, ELECTRE II method, and TOPSIS method can generate reasonable decision results in this example, there are some defects in these approaches which have attracted the attention of many scholars. Xia and Xu’s method integrates all the HFE directly by aggregation operator, and then the ranking result is obtained according to the score value. When the number of criteria in decision-making problem increases, aggregation process becomes more complex, which increases the burden on DMs. Therefore, Xia and Xu’s approach will be restricted by some factors in practical application. The ELECTRE II method has a strong dependence on the attitude of the DM, and the attitude of the DM has a great influence on the decision result. Hence, it is difficult to get the objective decision results in practical problems. TOPSIS method does not consider the relative importance of the distance between the alternative and the ideal or negative ideal solution, which is unreasonable. The proposed method can overcome the shortcomings which described above. The main reason is that the GLDS approach takes into account both the group utility value and individual regret value, and has strong robustness [29]. In the HFGLDS method, we adopt the hesitant fuzzy utility function to depict the information contained in the HFE, which can yield more reasonable and accurate results. The calculation process of our proposed method is simple, and takes full advantage of the evaluation information of the DM on the alternative, which enhances the reliability of our proposed method. Moreover, our proposed method can handle the problem of MCDM with unknown criteria weights. We use the hesitant fuzzy utility value to construct the comparison matrix between the criteria, and then the weight values of the criteria are obtained by solving the comparison matrix. This approach is only related to the evaluation information of the alternative, and can decrease the influence of subjective uncertainty on the decision result. In conclusion, the proposed method makes full use of the advantages of both the GLDS approach and the hesitant fuzzy utility function, and can generate more persuasive decision results. Compared with other methods, our proposed approach can effectively solve the problem of MCDM under hesitant fuzzy environment, and is suitable for practical decision making problem to a great extent.

### V. PRACTICAL APPLICATION

The effectiveness of our proposed method is further demonstrated by a practical application. Examples are derived from Ref. [11] and briefly modified.

The development of transportation can promote the growth of the local economy. In recent years, the rapid development of high-speed railway has provided great convenience for people to travel. The development of high-speed railway has changed people’s travel concept, which has brought great challenges to the survival of airlines. In order to change this situation, many airlines have launched preferential policies. For example, they try to attract passengers by lowering prices. However, this approach has not been profitable for airlines. Gradually, they found that passengers pay more attention to the quality of airline services than the prices. To improve the service quality of airlines, a civil aviation bureau intends to evaluate the service quality of four famous airlines, select the best service quality airlines, and call for other airlines to learn from it. Suppose the alternative set consisting of four airlines is $T = \{T_1, T_2, T_3, T_4\}$. The service quality of

### TABLE 9. The normalized values of the $\tilde{D}(S_i)$ and $\tilde{L}(S_i)$.

|     | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|-----|------|------|------|------|
| $\tilde{D}(S_i)$ | 0.2461 | 0.2914 | 0.2085 | 0.9006 |
| $\tilde{L}(S_i)$ | 0.6146 | 0.3628 | 0.6726 | 0.1956 |

### TABLE 10. The comparison of decision results with different methods.

| Method                        | Ranking order | Optimal alternative |
|-------------------------------|---------------|---------------------|
| Xia and Xu’s method [25]      | $S_4 > S_2 > S_1 > S_3$ | $S_4$ |
| ELECTRE II method [32]        | $S_4 > S_2 > S_1 > S_3$ | $S_4$ |
| TOPSIS method [37]           | $S_4 > S_2 > S_1 > S_3$ | $S_4$ |
| Our proposed method          | $S_4 > S_2 > S_1 > S_3$ | $S_4$ |

### TABLE 11. The hesitant fuzzy decision matrix.

|     | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|-----|------|------|------|------|
| $T_1$ | {0.6, 0.7, 0.9} | {0.6, 0.8} | {0.3, 0.6, 0.9} | {0.4, 0.5, 0.9} |
| $T_2$ | {0.7, 0.8, 0.9} | {0.5, 0.8, 0.9} | {0.4, 0.8} | {0.5, 0.6, 0.7} |
| $T_3$ | {0.5, 0.6, 0.8} | {0.6, 0.7, 0.9} | {0.3, 0.5, 0.7} | {0.5, 0.7} |
| $T_4$ | {0.6, 0.9} | {0.7, 0.9} | {0.2, 0.4, 0.7} | {0.4, 0.5} |
TABLE 12. The gained dominance score value for each alternative.

|        | $C_1$ | $C_2$ | $C_3$ | $C_4$ | Net gained dominance score value |
|--------|-------|-------|-------|-------|-------------------------------|
| $T_1$  | 0.0957| 0.0063| 0.1582| 0.0542| 0.0912                        |
| $T_2$  | 0.3675| 0     | 0.2083| 0.2064| 0.2179                        |
| $T_3$  | 0     | 0.0473| 0.0678| 0.1494| 0.0676                        |
| $T_4$  | 0.0735| 0.2429| 0     | 0     | 0.0573                        |

TABLE 13. The lost dominance score value for each alternative.

|        | $C_1$ | $C_2$ | $C_3$ | $C_4$ | Net lost dominance score value |
|--------|-------|-------|-------|-------|-------------------------------|
| $T_1$  | 0.0906| 0.0857| 0.0167| 0.0666| 0.0321                        |
| $T_2$  | 0     | 0.0920| 0     | 0     | 0.0143                        |
| $T_3$  | 0.1752| 0.0652| 0.0619| 0.0190| 0.0467                        |
| $T_4$  | 0.1017| 0     | 0.1297| 0.1208| 0.0416                        |

TABLE 14. The collective score value of the service quality for the airline.

|        | $T_1$ | $T_2$ | $T_3$ | $T_4$ |
|--------|-------|-------|-------|-------|
| Collective score value | 0.0381| 0.3246| -0.2190| -0.1594|

TABLE 15. Comparison of decision results with different methods.

| Method                      | Ranking order | Optimal alternative |
|-----------------------------|---------------|---------------------|
| TOPSIS method [37]          | $T_2 \succ T_1 \succ T_3 \succ T_4$ | $T_2$ |
| Our proposed method         | $T_2 \succ T_1 \succ T_3 \succ T_4$ | $T_2$ |

From Table 15, we can know that $T_2$ is the airline with the best service quality, which is consistent with the TOPSIS method. This further illustrates the effectiveness of our proposed method. It has good applicability in practical application.

VI. CONCLUSION

The information expressed by HFS is very close to the human reasoning process and is widely used in the field of MCDM. However, it is difficult to make decisions directly through HFE in practical decision making problems. How to handle the hesitant fuzzy information effectively is the key to address the problem of MCDM. To address this issue, we develop a novel hesitant fuzzy utility function to quantify the hesitant fuzzy information contained in the HFE. The new hesitant fuzzy utility function is constructed by employing the hesitant fuzzy score value and the hesitant degree of the HFE. Such a new hesitant fuzzy utility function can precisely characterize the hesitant fuzzy information contained in the HFE, and realize the transformation from the hesitant fuzzy information to the crisp value. In addition, we present a novel hesitant degree measure to eliminate the defects of traditional methods. The new hesitant degree measure can effectively describe the hesitant degree of the HFE. For solving the problem of MCDM under hesitant fuzzy environment more effectively, we introduce the GLDS approach in this study. Based on the hesitant fuzzy utility function and the GLDS method, we develop a new HFGLDS approach to solve the problem of MCDM under hesitant fuzzy environment. The advantage of the HFGLDS method is that it takes into account the group utility values and the individual regret values simultaneously, which can generate more persuasive decision result. The ranking result of the group utility values and the individual regret values are used in the calculation of the collective score values, which makes the decision result more robust. Moreover, our proposed approach can address the problem of MCDM with unknown criteria weights. We utilize the evaluation information of the alternative to determine the weight value of each criterion, which can reduce the influence of decision maker’s subjective uncertainty on the decision result. In conclusion, HFGLDS method can provide a promising approach to solve the problem of MCDM under hesitant fuzzy environment. Eventually, the rationality and effectiveness of the HFGLDS method are illustrated through a practical application. Some comparative analyses are given to highlight the advantages of the HFGLDS approach.

In future work, we will systematically analyze the rationality of the hesitant fuzzy utility function through the practical decision-making problem. Besides, we will extend the HFGLDS approach to the interval-valued hesitant fuzzy environment to address more complex decision-making problem.

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