Computability-logic web: an alternative to deep learning

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Abstract

Computability logic (CoL) is a powerful, mathematically rigorous computational model. In this paper, we show that CoL-web, a web extension to CoL, naturally supports web programming where database updates are involved. To be specific, we discuss an implementation of the AI ATM based on CoL (CL9 to be exact).

More importantly, we argue that CoL-web supports a general AI and, therefore, is a good alternative to neural nets and deep learning. We also discuss how to integrate neural nets into CoL-web.

Keywords: Computability logic; Web programming; Game semantics; AI;

1 Introduction

It is not difficult to point out the weaknesses of neural nets and deep learning. Simply put, neural nets are too weak to support general AI. They receive inputs (numbers), perform simple arithmetic operations and produce outputs (numbers). Consequently, they provide only primitive services such as object classifications. Although object classification has some interesting applications, the power of classification is in fact not much compared to all the complex services a human can provide. Complex services – making a coffee, withdrawing money from ATM, etc – are not well supported by neural nets. In addition, their classification services are not perfect, as they are only approximate.

A human can provide complex services to others. The notion of services and how to complete them thus play a key role for an AI to imitate a human. In other words, the right move towards general AI would be to find (a) a mathematical notion for services, and (b) how an AI automatically generates a strategy for completing the service calls.

Fortunately, Japaridze developed a theory for services/games involving complex ones. Computability logic (CoL) [1]-[4], is an elegant theory of (multi-)agent services. In CoL, services are seen as games between a machine and its environment and logical operators stand for operations on games. It understands interaction among agents in its most general — game-based — sense.

In this paper, we discuss a web programming model based on CoL and implement an AI ATM. An AI ATM is different from a regular ATM in that the former automatically generates a strategy for a service call, while the latter does not.

We assume the following in our model:

• Each agent corresponds to a web site with a URL. An agent’s knowledgebase(KB) is described in its homepage.

• Agents are initially inactive. An inactive agent becomes activated when another agent invokes a query for the former.

• Our model supports the query/knowledge duality, also known as querying knowledge. That is, knowledge of an agent can be obtained from another agent by invoking queries to the latter.
To make things simple, we choose CL9 – a fragment of CoL – as our target language. CL9 includes sequential operators: sequential disjunction (▽) and sequential conjunction (△) operators. These operators model knowledgebase updates. Imagine an ATM that maintains balances on Kim. Balances change over time. Whenever Kim presses the deposit button for $1, the machine must be able to update the balance of the person. This can be represented by

\[ \text{balance}(\$0) \uplus \text{balance}(\$1) \uplus \cdots. \]

In this paper, we present CL9Φ which is a web-based implementation of CL9. This implementation is straightforward and its correctness is rather obvious. What is interesting is that CL9 is a novel web programming model with possible database updates. It would provide a good starting point for future high-level web programming.

2 Preliminaries

In this section a brief overview of CL9 is given.

There are two players: the machine ⊤ and the environment ⊥.

There are two sorts of atoms: elementary atoms p, q, . . . to represent elementary games, and general atoms P, Q, . . . to represent any, not-necessarily-elementary, games.

Constant elementary games ⊤ is always a true proposition, and ⊥ is always a false proposition.

Negation ¬ is a role-switch operation: For example, ¬(0 = 1) is true, while (0 = 1) is false.

Choice operations The choice operations model decision steps in the course of interaction, with disjunction ⊔ meaning the machine’s choice, and conjunction ⊓ meaning choice by the environment.

Parallel operations A ∧ B means the parallel-and, while A ∨ B means the parallel-or. In A ∧ B, ⊤ is considered the winner if it wins in both A and B, while in A ∨ B it is sufficient to win in one of A and B.

Reduction → is defined by ¬A ∨ B.

Sequential operations A▽B (resp. A△B) is a game that starts and proceeds as a play of A; it will also end as an ordinary play of A unless, at some point, ⊤ (resp. ⊥) decides — by making a special switch move — to abandon A and switch to B. A▽B is quite similar to the if-then-else in imperative languages.

We reserve § as a special symbol for switch moves. Thus, whenever ⊥ wants to switch from a given component A_i to A_{i+1} in A_0△A_1△A_n, it makes the move §. Note that ⊤, too, needs to make switch moves in a △-game to “catch up” with ⊥. The switches made by ⊥ in a △-game we call leading switches, and the switches made by ⊤ in a △-game we call catch-up switches.

3 Logic CL9Φ

In this section we review the propositional system CL9 [5] and slightly extend it. Our presentation closely follows the one in [5]. We assume that there are infinitely many nonlogical elementary atoms, denoted by p, q, r, s and infinitely many nonlogical general atoms, denoted by P, Q, R, S.

Formulas, to which we refer as CL9-formulas, are built from atoms and operators in the standard way.

Definition 3.1 The class of CL9-formulas is defined as the smallest set of expressions such that all atoms are in it and, if F and G are in it, then so are ¬F, F ∧ G, F ∨ G, F → G, F △ G, F △ G, F △ G, F △ G.
Now we define \( \text{CL9}^\#, \) a slight extension to \( \text{CL9} \) with environment parameters. Let \( F \) be a \( \text{CL9} \)-formula. We introduce a new \( \text{env-annotated} \) formula \( F^\omega \) which reads as ‘play \( F \) against an agent \( \omega \). For an \( \ominus \)-occurrence or an \( \triangle \)-occurrence \( O \) in \( F^\omega \), we say \( \omega \) is the \( \text{matching} \) environment of \( O \). For example, \( (p \ominus (q \ominus r))^\omega \) is an agent-annotated formula and \( \omega \) is the matching environment of both occurrences of \( \ominus \). Similarly for \( (p \triangle (q \triangle r))^\omega \). We extend this definition to subformulas and formulas. For a subformula \( F' \) of the above \( F^\omega \), we say that \( \omega \) is the \( \text{matching} \) environment of both \( F' \) and \( F \).

In introducing environments to a formula \( F \), one issue is whether we allow ‘env-switching’ formulas of the form \( (F[R^\omega])^\omega \). Here \( F[R] \) represents a formula with some occurrence of a subformula \( R \). That is, the machine initially plays \( F \) against agent \( \omega \) and then switches to play against another agent \( u \) in the course of playing \( F \). For technical reasons, we focus on non ‘env-switching’ formulas. This leads to the following definition:

**Definition 3.2** The class of \( \text{CL9}^\# \)-formulas is defined as the smallest set of expressions such that (a) For any \( \text{CL9} \)-formula \( F \) and any agent \( \omega \), \( F^\omega \) are in it and, (b) if \( H \) and \( J \) are in it, then so are \( \neg H, H \land J, H \lor J, H \rightarrow J \).

**Definition 3.3** Given a \( \text{CL9}^\# \)-formula \( J \), the skeleton of \( J \) – denoted by \( \text{skeleton}(J) \) – is obtained by replacing every occurrence \( F^\omega \) by \( F \).

For example, \( \text{skeleton}(p \ominus (q \ominus r))^\omega = p \ominus (q \ominus r) \).

We borrow the following definitions from [5]. They apply both to \( \text{CL9} \) and \( \text{CL9}^\# \).

An **interpretation** for \( \text{CL9} \) is a function that sends each nonlogical elementary atom to an elementary game, and sends each general atom to any, not-necessarily-elementary, static game. This mapping extends to all formulas by letting it respect all logical operators as the corresponding game operations. That is, \( \top^* = \top, (E \Delta F)^* = E^* \Delta F^*, \) etc. When \( F^* = A \), we say that \( \ast \text{ interprets } F \text{ as } A \).

A formula \( F \) is said to be **valid** iff, for every interpretation \( \ast \), the game \( F^\ast \) is computable. And \( F \) is **uniformly valid** iff there is an HPM \( \mathcal{H} \), called a **uniform solution** for \( F \), such that \( \mathcal{H} \) wins \( F^\ast \) for every interpretation \( \ast \).

A **sequential (sub)formula** is one of the form \( F_0 \Delta \ldots \Delta F_n \) or \( F_0 \lor \ldots \lor F_n \). We say that \( F_0 \) is the **head** of such a (sub)formula, and \( F_1, \ldots, F_n \) form its **tail**.

The **capitalization** of a formula is the result of replacing in it every sequential subformula by its head.

A formula is said to be **elementary** iff it is a formula of classical propositional logic.

An occurrence of a subformula in a formula is **positive** iff it is not in the scope of \( \neg \). Otherwise it is **negative**.

A surface occurrence is an occurrence that is not in the scope of a choice connective and not in the tail of any sequential subformula.

The **elementarization** of a \( \text{CL9} \)-formula \( F \) means the result of replacing in the capitalization of \( F \) every surface occurrence of the form \( G_1 \cap \ldots \cap G_n \) by \( \top \), every surface occurrence of the form \( G_1 \cup \ldots \cup G_n \) by \( \bot \), and every positive surface occurrence of each general literal by \( \bot \).

Finally, a formula is said to be **stable** iff its elementarization is a classical tautology; otherwise it is **instable**.

The proof system of \( \text{CL9}^\# \) is identical to that \( \text{CL9} \) in that agent parameters play no roles. \( \text{CL9}^\# \) consists of the following four rules of inference.

**Definition 3.4 Wait:** \( \tilde{H} \mapsto F \), where \( F \) is stable and \( \tilde{H} \) is the smallest set of formulas satisfying the following two conditions:

1. whenever \( F \) has a surface occurrence of a subformula \( G_1 \cap \ldots \cap G_n \) whose matching environment is \( \omega \), for each \( i \in \{1, \ldots, n\} \), \( \tilde{H} \) contains the result of replacing that occurrence in \( F \) by \( G_i^\omega \);
2. whenever \( F \) has a surface occurrence of a subformula \( G_0 \Delta G_1 \Delta \ldots \Delta G_n \) whose matching environment is \( \omega \), \( \tilde{H} \) contains the result of replacing that occurrence in \( F \) by \( (G_1 \Delta \ldots \Delta G_n)^\omega \).

**Choose:** \( H \mapsto F \), where \( H \) is the result of replacing in \( F \) a surface occurrence of a subformula \( G_1 \cup \ldots \cup G_n \) whose matching environment is \( \omega \) by \( G_i^\omega \) for some \( i \in \{1, \ldots, n\} \).
Switch: $H \mapsto F$, where $H$ is the result of replacing in $F$ a surface occurrence of a subformula $G_0 \lor G_1 \lor \ldots \lor G_n$ whose matching environment is $\omega$ by $(G_1 \lor \ldots \lor G_n)^\omega$.

Match: $H \mapsto F$, where $H$ is the result of replacing in $F$ two — one positive and one negative — surface occurrences of some general atom by a nonlogical elementary atom that does not occur in $F$.

Example 3.5 The following is a CL9-proof of $(b0\triangle b1\triangle b2)^w \rightarrow (b0\triangle b1\triangle b2)^w$:

1. $b2^u \rightarrow b2^w$ (from $\{\}$ by Wait);
2. $b2^u \rightarrow (b\triangle b)^w$ (from 1 by Switch);
3. $(b\triangle b)^w \rightarrow (b\triangle b)^w$ (from 2 by Wait);
4. $(b\triangle b)^w \rightarrow (b0\triangle b1\triangle b2)^w$ (from 3 by Switch);
5. $(b0\triangle b1\triangle b2)^w \rightarrow (b0\triangle b1\triangle b2)^w$ (from 4 by Wait);

4 Logic CL9$^{\circ,\Phi}$

To facilitate the execution procedure, following [5], we modify CL9$^{\phi}$ to obtain CL9$^{\circ,\Phi}$. Unlike CL9$^{\phi}$, this new language allows hyperformulas which contain the following.

- Hybrid atom: each hybrid atom is a pair consisting of a general atom $P$, called its general component, and a nonlogical elementary atom $q$, called its elementary component. We denote such a pair by $P_q$. It keeps track of the exact origin of each such elementary atom $q$.

- Underlined sequential formula: It is introduced for us not to forget the earlier components of sequential subformulas when Switch or Wait are applied. We now require that, in every sequential (sub)formula, one of the components be underlined.

The formulas of this modified language we call hyperformulas. We borrow the following definitions from [5].

By the general dehybridization of a hyperformula $F$ we mean the CL9-formula that results from $F$ by replacing in the latter every hybrid atom by its general component, and removing all underlines in sequential subformulas.

A surface occurrence of a subexpression in a given hyperformula $F$ means an occurrence that is not in the scope of a choice operator, such that, if the subexpression occurs within a component of a sequential subformula, that component is underlined or occurs earlier than the underlined component.

An active occurrence is an occurrence such that, whenever it happens to be within a component of a sequential subformula, that component is underlined.

An abandoned occurrence is an occurrence such that, whenever it happens to be within a component of a sequential subformula, that component is to the left of the underlined component of the same subformula.

An elementary hyperformula is one not containing choice and sequential operators, underlines, and general and hybrid atoms.

The capitalization of a hyperformula $F$ is defined as the result of replacing in it every sequential subformula by its underlined component, after which all underlines are removed.

The elementarization

$$\|F\|$$

of a hyperformula $F$ is the result of replacing, in the capitalization of $F$, every surface occurrence of the form $G_1 \lor \ldots \lor G_n$ by $\top$, every surface occurrence of the form $G_1 \lor \ldots \lor G_n$ by $\bot$, every positive surface occurrence of each general literal by $\bot$, and every surface occurrence of each hybrid atom by the elementary component of that atom.

A hyperformula $F$ is stable iff its elementarization $\|F\|$ is a classical tautology; otherwise it is unstable. A hyperformula $F$ is said to be balanced iff, for every hybrid atom $P_q$ occurring in $F$, the following two conditions are satisfied:

1. $F$ has exactly two occurrences of $P_q$, one positive and the other negative, and both occurrences are surface occurrences;
2. the elementary atom \( q \) does not occur in \( F \), nor is it the elementary component of any hybrid atom occurring in \( F \) other than \( P_q \).

An active occurrence of a hybrid atom (or the corresponding literal) in a balanced hyperformula is **widowed** iff the other occurrence of the same hybrid atom is abandoned.

We extend \( \text{CL9}^\Phi \) to \( \text{CL9}^{\Phi,\Phi} \). The language of \( \text{CL9}^\Phi \) allows any balanced hyperformulas, which we also refer to as \( \text{CL9}^{\Phi,\Phi} \)-formulas.

**Definition 4.1** Logic \( \text{CL9}^{\Phi,\Phi} \) is given by the following rules for balanced hyperformulas (below simply referred to as “(sub)formulas”):

**Wait**: \( \tilde{H} \rightarrow F \), where \( F \) is stable and \( \tilde{H} \) is the smallest set of formulas satisfying the following two conditions:

1. whenever \( F \) has an active surface occurrence of a subformula \( G_1 \sqcap \ldots \sqcap G_n \) whose matching environment is \( \omega \), for each \( i \in \{1, \ldots, n\} \), \( \tilde{H} \) contains the result of replacing that occurrence in \( F \) by \( G_i^\omega \);
2. whenever \( F \) has an active surface occurrence of a subformula \( G_0 \sqcap \ldots \sqcap G_m \sqcup G_{m+1} \sqcap \ldots \sqcap G_n \) whose matching environment is \( \omega \), \( \tilde{H} \) contains the result of replacing that occurrence in \( F \) by \((G_0 \sqcap \ldots \sqcap G_m \sqcup G_{m+1} \sqcap \ldots \sqcap G_n)^\omega\).

**Choose**: \( H \rightarrow F \), where \( H \) is the result of replacing in \( F \) an active surface occurrence of a subformula \( G_1 \sqcap \ldots \sqcap G_n \) whose matching environment is \( \omega \) by \( G_i^\omega \) for some \( i \in \{1, \ldots, n\} \).

**Switch**: \( H \rightarrow F \), where \( H \) is the result of replacing in \( F \) an active surface occurrence of a subformula \( G_0 \sqcap \ldots \sqcap G_m \sqcap G_{m+1} \sqcap \ldots \sqcap G_n \) whose matching environment is \( \omega \) by \((G_0 \sqcap \ldots \sqcap G_m \sqcap G_{m+1} \sqcap \ldots \sqcap G_n)^\omega\).

**Match**: \( H \rightarrow F \), where \( H \) has two — a positive and a negative — active surface occurrences of some hybrid atom \( P_q \), and \( F \) is the result of replacing in \( H \) both occurrences by \( P \).

An effective procedure that converts any \( \text{CL9}^\Phi \)-proof of any formula \( G \) into a \( \text{CL9}^{\Phi,\Phi} \)-proof of \( G \) is given in [5].

## 5 Execution Phase

The machine model of \( \text{CL9}^{\Phi,\Phi} \) is designed to process only one query/formula at a time. In distributed systems, however, it is natural for an agent to receive/process multiple queries from different users. For this reason, we introduce multiple queries to our machine. To do this, we assume that an agent maintains two queues: the *income queue* \( Q_I \) for storing a sequence of new incoming queries of the form \((Q_1, \ldots, Q_n)\) and the *temporarily solved queue* \( Q_S \) for storing a sequence of temporarily solved queries of the form \((KB_1 \rightarrow Q_1, \ldots, KB_n \rightarrow Q_n)\). Here each \( Q_i \) is a query and each \( KB_i \) is a knowledgebase. A query \( Q \) with respect to some knowledgebase is *temporarily solved* if \( Q \) is solved but \( \top \) has a remaining switch move in \( Q \). Otherwise \( Q \) is said to be *completely solved*.

As expected, processing real-time multiple queries causes some complications. To be specific, we process \( Q_I \) of the form \((Q_1, \ldots, Q_m)\) and \( Q_S \) of the form \((KB_1 \rightarrow Q'_1, \ldots, KB_n \rightarrow Q'_n)\) in the following way:

1. First stage is to initialize a temporary variable \( \text{NewKB} \) to \( KB \),
2. The second stage is to follow the *loop* procedure:

   ```
   procedure loop:
   ```

   - Case 1: \( Q_I \) is not empty:
     The machine tries to solve \( Q_1 \) by calling \( \text{Exec}(\text{NewKB} \rightarrow Q_1) \).
– If it fails, then report a failure, remove $Q_1$ from $QI$ and repeat loop.

– Suppose it is a success and $NewKB$ and $Q_1$ evolve to $NewKB'$ and $Q_1'$ after solving this query. We consider two cases.

(a) If it is completely solved, then report a success, remove $Q_1$ from $QI$, update $NewKB$ to $NewKB'$ and repeat loop. (b) If it is temporarily solved, then report a success, remove $Q_1$ from $QI$, insert $NewKB' \to Q_1'$ to $QS$, update $NewKB$ to $NewKB'$ and repeat loop.

• Case 2. $QI$ is empty and $QS$ nonempty: The machine tries to solve the first query $KB_1 \to Q_1'$ in $QS$.

– If $KB = NewKB$, it means nothing has changed since the last check. Hence the machine waits for any change such as the environment’s new move.

– Otherwise, the machine tries to solve $Q_1'$ with respect to $NewKB$. It thus removes the above query from $QS$, adds $Q_1'$ to $QI$, and repeat loop.

• Case 3. $QI$ is empty and $QS$ is empty: wait for new incoming service calls.

Below we will introduce an algorithm that executes a formula $J$. The algorithm is a minor variant of the one in [5] and contains two stages:

**Algorithm Exec**(*J*): % $J$ is a CL$\diamondsuit,\wp$-formula

1. Fix an interpretation *. First stage is to initialize a temporary variable $E$ to $J$, a position variable $\Omega$ to an empty position (\(\emptyset\)). Activate all the resource agents specified in $J$ by invoking proper queries to them. That is, for each negative occurrence of an annotated formula $F^\omega$ in $J$, activate $\omega$ by querying $F^\mu$ to $\omega$. Here $\mu$ is the current machine; On the other hand, we assume that all the querying agents – which appear positively in $J$ – are already active.

2. The second stage is to play $J$ according to the following mainloop procedure (which is a minor variant of [5]):

procedure loop(*Tree*): % $Tree$ is a proof tree of $J$

If $E$ is derived by Choose$^\diamondsuit$ from $H$, the machine makes the move $\alpha$ whose effect is choosing $G_\alpha$ in the $G_1 \sqcup \ldots \sqcup G_n$ subformula of $E$. So, after making move $\alpha$, the machine call loop on $\langle \Omega \rangle H^\alpha$. Let $\omega$ be the matching environment. Then inform $\omega$ of the move $\alpha$.

If $E$ is derived by Switch$^\diamondsuit$ from $H$, then the machine makes the move $\alpha$ whose effect is making a switch in the $G_0 \nabla \ldots \nabla G_m \nabla G_{m+1} \nabla \ldots \nabla G_n$ subformula. So, after making move $\alpha$, the machine calls loop on $\langle \Omega, \top \rangle H^\alpha$.

If $E$ is derived by Match$^\diamondsuit$ from $H$ through replacing the two (active surface) occurrences of a hybrid atom $P_\gamma$ in $H$ by $P$, then the machine finds within $\Omega$ and copies, in the positive occurrence of $P_\gamma$, all of the moves made so far by the environment in the negative occurrence of $P_\gamma$, and vice versa. This series of moves brings the game down to $\langle \Omega' \rangle E^* = \langle \Omega' \rangle H^*$, where $\Omega'$ is result of adding those moves to $\Omega$. So, now the machine calls loop on $\langle \Omega' \rangle H^*$.

Finally, suppose $E$ is derived by Wait$^\diamondsuit$. Our machine keeps granting permission (“waiting”).

Case 1. $\alpha$ is a move whose effect is moving in some abandoned subformula or a widowed hybrid literal of $E$. In this case, the machine calls loop on $\langle \Omega, \top \rangle E^*$.

Case 2. $\alpha$ is a move whose effect is moving in some active surface occurrence of a general atom in $E$. Again, in this case, the machine calls loop on $\langle \Omega, \top \rangle E^*$.

Case 3. $\alpha$ is a move whose effect is making a catch-up switch in some active surface occurrence of a $\nabla$-subformula. The machine calls loop on $\langle \Omega, \top \rangle E^*$.

Case 4. $\alpha$ is a move whose effect is making a move $\gamma$ in some active surface occurrence of a non-widowed hybrid atom. Let $\beta$ be the move whose effect is making the same move $\gamma$ within the other active
surface occurrence of the same hybrid atom. In this case, the machine makes the move $\beta$ and calls loop on $\langle \Omega, \bot \alpha, \top \beta \rangle E^*$.

**Case 5:** $\alpha$ is a move whose effect is a choice of the $i$th component in an active surface occurrence of a subformula $G_1 \sqcap \ldots \sqcap G_n$. Then the machine calls loop on $\langle \Omega \rangle H^*$, where $H$ is the result of replacing the above subformula by $G_i$ in $E$.

**Case 6:** $\alpha$ signifies a (leading) switch move within an active surface occurrence of a subformula $G_0 \triangle \ldots \triangle G_m \triangle G_{m+1} \triangle \ldots \triangle G_n$. Then the machine makes the same move $\alpha$ (signifying making a catch-up switch within the same subformula), and calls exec on $\langle \Omega, \bot \alpha, \top \alpha \rangle H^*$, where $H$ is the result of replacing the above subformula by $G_0 \triangle \ldots \triangle G_m \triangle G_{m+1} \triangle \ldots \triangle G_n$.

### 6 Examples

As an example of web system, we will look at the ATM of some bank. It is formulated with the user, an ATM, a database, and a credit company. We assume the following:

- There are two kinds of agents: super agents and regular agents. Super agents are prefixed with $. For example, $\$kim$ is a super agent. While regular agents behave according to the Exec procedure, super agents behave unpredictably.

- For simplicity, we assume the bank has only one customer named Kim. Further, the balance is restricted to one of the three amounts: $0, \$1$ or $\$2$.

- The database maintains balance information on Kim.

- Both the credit company and the ATM request balance checking to the database.

- The ATM has a ($\$1$) deposit button. Whenever pressed, it adds $1 to the account.

The above can be implemented as follows:

```plaintext
agent credit. % credit company
(b0\triangle b1 \triangle b2)^db. % b0 means the balance is $0, and so on.

% Here, we assume that ATM usage charge is zero, meaning deposit = balance.
agent db. % database
(d0\triangle d1 \triangle d2)^m. % d0 means the accumulated deposit is $0, and so on.
d0 \rightarrow b0.
d1 \rightarrow b1.
d2 \rightarrow b2.

agent m. % ATM machine
(d0\triangle d1 \triangle d2)^{skim}. % request deposit checking to kim
(b0\triangle b1 \triangle b2)^db. % request balance checking to DB

agent $skim. % $skim is a super agent.
(b0\triangle b1 \triangle b2)^m. % request balance checking to ATM
```

Now let us consider the agent $kim$ and the agent credit. They both want to know the balance of Kim’s account. The initial balance checking will return $b0$, meaning zero dollars. Later, suppose Kim deposits $\$1$. In this case, the balance information on $db$ will be updated to one dollar and, subsequently, the response to balance checking by ATM, kim and the credit company will be updated to $b1$, as desired.
7 Adding neural networks

The integration of neural nets and symbolic AI is often beneficial. There are several ambitious approaches such as DeepProblog[6] and these approaches try to combine both worlds within a single agent. Unfortunately, these approaches considerably increase the complexity of the machine.

Fortunately, in the multi-agent setting, this integration can be achieved in a rather simple way by introducing a new kind of agents called \(\eta\)-agent(neural-net agents).

There are now three kinds of agents:

- regular agents who perform deductive reasoning, and
- \(\eta\)-agents who perform inductive reasoning.
- super agents who are able to create resources.

An \(\eta\)-agent is an agent which is designed to implement low-level perceptions (visual data, etc) via training. That is, its knowledgebase is in neural-net form for easy training. In the sequel, \(\eta\)-agents are prefixed with \(\eta\).

We assume the following:
1. the input/output of a neural network is encapsulated in the form of an atomic predicate,
2. the output of a neural network is deterministic. Thus, we do not consider probability here, and
3. For simplicity, neural nets are specified in logical form (\(\text{CL9}^\Phi\) to be exact), instead of in functional form.

An \(\eta\)-agent with its knowledgebase \(N\) proceeds in two modes:

- When there is a query \(A\), it proceeds in deductive mode by processing \(A\) using \(N\) via \(\text{CL9}^\Phi\) deduction.
- When it is idle, it trains itself on sample data by updating \(N\).

As an example, we will look at the program which, given image of an animal, identifies its habitat.

We assume the following:

- The regular agent \(a\) implements the predicate \(\text{habitat}(j, h)\) where \(j\) is an image of an animal and \(h\) is the main habitat of the animal. We consider two kinds of animals: lion and tiger. We assume that \(j\) belongs to \(S = \{i_1, \ldots, i_3\}\) where \(S\) is a set of three images of animals.
- The \(\eta\)-agent \(\eta d\) implements the predicate \(\text{animal}(j, n)\) where \(j\) is image of an animal and \(n\) is the corresponding animal.

The above can be implemented as follows:

\[
\text{agent } a.\ %\ \text{animal habitats} \\
\%\ \text{Below is a query to } \eta d. \\
((\text{animal}(i_1, \text{lion}) \sqcup \text{animal}(i_1, \text{tiger})) \sqcap \\
(\text{animal}(i_2, \text{lion}) \sqcup \text{animal}(i_2, \text{tiger})) \sqcap \\
(\text{animal}(i_3, \text{lion}) \sqcup \text{animal}(i_3, \text{tiger})))^{\eta d}. \\
\%\ \text{Rules that maps animals to the corresponding habitats.} \\
\text{animal}(i_1, \text{tiger}) \rightarrow \text{habitat}(i_1, \text{india}). \\
\text{animal}(i_2, \text{tiger}) \rightarrow \text{habitat}(i_2, \text{india}). \\
\text{animal}(i_3, \text{tiger}) \rightarrow \text{habitat}(i_3, \text{india}). \\
\text{animal}(i_1, \text{lion}) \rightarrow \text{habitat}(i_1, \text{senegal}). \\
\text{animal}(i_2, \text{lion}) \rightarrow \text{habitat}(i_2, \text{senegal}). \\
\text{animal}(i_3, \text{lion}) \rightarrow \text{habitat}(i_3, \text{senegal}).
\]
Given image \( j \), the agent \( \eta d \) produces the corresponding animal via deep learning. We do not show the details here.

\[
\text{agent } \eta d. \text{ % } \eta \text{-agent}
\]

When the agent \( \eta d \) is idle, it trains itself and updates its knowledgebase by adjusting weights. Now let us invoke a query \( \text{habitat}(i_3, \text{india}) \sqcup \text{habitat}(i_3, \text{senegal}) \) to the agent \( a \) where \( i_3 \) is the image of some animal. To solve this query, the agent \( a \) invokes another query shown above to the agent \( \eta d \). Now \( \eta d \) switches from training mode to deduction mode to solve this query. Let us assume that the response from \( \eta d \) is \( \text{animal}(i_3, \text{lion}) \). Using the rules related to animal’s habitats, the agent \( a \) will return \( \text{habitat}(i_3, \text{senegal}) \) to the user. Note that our agents behave just like real-life agents. For example, a doctor typically trains himself when he is idle. If there is a service request, then he switches from training mode to deduction mode.

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