Excited $B_c$ States via Continuum QCD

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(Dated: May 14, 2021)

We study the most recently observed excited $B_c$ states with the Dyson–Schwinger equation and the Bethe–Salpeter equation approach of continuum QCD. The obtained $M_{B_c^+(2S)} = 6.813(16)$ GeV, $M_{B_c^{*(+)}(2S)} = 6.841(18)$ GeV and the mass splitting $M_{B_c^+(2S)} - M_{B_c^{*(+)}(2S)} = 0.039$ GeV agree with the observations very well. Moreover we predict the leptonic decay constant $f_{B_c^+(2S)} = -0.165(10)$ GeV, $f_{B_c^{*(+)}(2S)} = -0.161(7)$ GeV respectively.

1. Introduction — Recently, two excited $B_c$ mesons, $B_c^+(2S)$ and $B_c^{*(+)}(2S)$, were observed in the mass spectrum of $B^+\pi^+\pi^-$ for the first time by the CMS experiment at $\sqrt{s} = 13$ TeV [1]. The mass of $B_c^+(2S)$ is determined to be $M_{B_c^+(2S)} = 6871.0 \pm 1.2(\text{stat.}) \pm 0.8(\text{syst.})$ MeV, while the mass difference $M_{B_c^+(2S)} - M_{B_c^{*(+)}(2S)} = 29.0 \pm 1.5(\text{stat.}) \pm 0.7(\text{syst.})$ MeV, where $M_{B_c^{*(+)}(2S)}$ is defined as $M_{B_c^{*(+)}(2S)} = M_{B_c^{*(+)}(2S)} - (M_{B_c^+(1S)} - M_{B_c^+(1S)})$. The above results are then confirmed by the LHCb experiment with $8.5 \text{ fb}^{-1}$ $pp$ collision data [2], being $M_{B_c^+(2S)} = 6872.1 \pm 1.3(\text{stat.}) \pm 0.1(\text{syst.}) \pm 0.8(B_c^+) \text{ MeV}$ and $M_{B_c^{*(+)}(2S)} = 31.0 \pm 1.4(\text{stat.}) \text{ MeV}$, respectively.

Investigating the open flavor states such as the $B_c^+$ family of $(c\bar{b})$ mesons could enrich our understanding of the strong interaction. There have been plenty of theory studies and we refer to Ref. [3] and the references therein for the contemporary statements. Exploring the excited states relies on the detailed understanding of long range behavior of strong interaction and encounters the difficulties due to the intrinsic complexity. The quark model has been thoroughly applied to study hadron spectrum and, by using a phenomenological nonrelativistic potential model, the mass spectrum and decay properties of $(c\bar{b})$ mesons have been explored (see, e.g. Ref. [4]). However, investigations based on $ab\ initio$ theory of strong interactions, Quantum Chromodynamics(QCD)(QCD), are still challenges. The precise predictions of charmed-bottom ground state from Lattice QCD(IQCD) [5] has been released recently with the the masses $M_{B_c^+} = 6276(3)(6)\text{MeV}$ and $M_{B_c^{**}} = 6331(4)(6)\text{MeV}$ respectively. Studying the masses of the excited states in IQCD are more difficult than determining those of the ground states accurately [6, 7] and the leptonic decay constants of excited $B_c^+$ states have not yet been touched. For details of the difficulties to study the decay constant in IQCD simulations please refer to Refs. [8, 9], where trials of calculating the decay constant of the first radial excited pion are carried out with the inspiration of a continuum theory prediction [10].

As a continuum functional method of QCD, the Dyson-Schwinger equation and Bethe-Salpeter equation (DS-BSE) [11–13] approach is complementary to IQCD and a covariant way to bridge the hadron physics and the fundamental degree of QCD. The difficulty of investigating the open flavor hadrons within this approach has been reported in Ref. [14] and that for exotic and radial excited states has been displayed in Ref. [15]. Then some efforts (for example Ref. [16–18]) have been made. Using an algebraic model, the mass of $B_c^+$, which is consistent with the world average value, has been predicted [19]. However it is not possible to predict the decay constants and the properties of the radial excited states in that framework, because the interaction lacks the relative momentum dependence. A novel extrapolation method has been developed in Ref. [20]. Therein the obtained masses and decay constants of the ground states mesons are comparable to experimental measurements and IQCD simulations, showing the success of the rainbow ladder (RL) approximation. What’s more, taking into account the flavor dependence of the quark-gluon interaction properly, we give a successful and unified description of the ground states of the open flavor mesons and the quarkonia [21]. Our results of the heavy mesons deviate from the experiment and IQCD results only about 1% for the ground state masses and less than 7% for the decay constants. The predicted masses of the $B_c$ mesons, $M_{B_c^+} = 6290(3)\text{MeV}$ and $M_{B_c^{**}} = 6357(4)\text{MeV}$, are comparable with the experimental and IQCD values.

To study the excited states of $B_c^+$ and $B_c^{**}$ in the continuum QCD approach directly, one should develop a scheme by extending those given in Refs. [20] and [21]. In the extension, one should maintain the parameters as the same as (without any fine tuning) the ones which produce the masses and decay constants of the ground states successfully. In this Letter, we produce the masses and the decay constants of the first excited states, $B_c^+(2S)$
and $B_s^+(2S)$, via the continuum QCD approach. Our obtained mass of the excited states agree with the experimental observations very well. The obtained decay constants are also quite reasonable.

2. DSBSE approach — Here we present the RL truncated DSBSE approach which takes into account the flavor dependence of the quark-gluon interaction properly [21]. The BS equation is

$$
\Gamma^{fg}(k; P) = -\frac{4}{3} [Z_2^2] \int_{dq}^{\Lambda} \left[ D^{fg}_{\mu
u}(k - q) \gamma_{\mu} \chi^{fg}(q; P) \gamma_{\nu} \right],
$$

(1)

where $f$ and $g$ label the quark flavor, $\Gamma^{fg}(k; P)$ is the Bethe-Salpeter amplitude (BSA), $k$, and $P$ are the relative and total momentum of the meson. $\chi^{fg}(q; P) = S_f(q) \Gamma^{fg}(q; P) S_g(q)$ is the BS wave function, $S_f(q)$ and $S_g(q)$ are the quark propagators, where $q^+ = q + iP/2$, $q^- = q - (1 - \epsilon)P/2$, $\epsilon$ is the partitioning parameter describing the momentum partition between quark and antiquark and doesn’t affect the physical observables. The quark propagators satisfy the BS equation,

$$
\Sigma_f^{-1}(p) = Z_2(i\gamma \cdot p + Z_m m_f)
$$

+ \frac{4}{3} [Z_2^2] \int_{dq}^{\Lambda} D^{fg}_{\mu
u}(p - q) \gamma_{\mu} \Sigma_f(q) \gamma_{\nu}.
$$

(2)

In Eq.(1) and Eq.(2), $\int_{dq}^{\Lambda} = \int_{P}^{\Lambda} d^4q/(2\pi)^4$ stands for a Poincaré invariant regularized integration, with $\Lambda$ the regularization mass-scale, $m_f$ is the current quark mass at renormalization scale $\zeta$, $Z_2$ and $Z_m$ are the renormalization constants of the quark field and the quark mass depending on $\Lambda$ and $\zeta$. We adopt a flavor independent renormalization scheme and choose $\zeta = 2$ GeV. $D^{fg}_{\mu
u}(l) = (\hat{\delta}_{\mu
u} - \frac{i\sigma_{\mu\nu}}{2m_f}) G^{fg}(l^2)$ is the gluon propagator including the effect of the mass dependence of the dressed quark-gluon-vertex. The dressed function $G^{fg}(s)$ is composed of a flavor dependent infrared(IR) part and a flavor independent ultraviolet(UV) part,

$$
\mathcal{G}^{fg}(s) = \mathcal{G}_{IR}^{fg}(s) + \mathcal{G}_{UV}(s),
$$

(3)

$$
\mathcal{G}_{IR}^{fg}(s) = 8\pi^2 \frac{D_f D_g}{\omega_f^2 m_f^2} e^{-s/(\omega_f \omega_g)},
$$

(4)

$$
\mathcal{G}_{UV}(s) = \frac{8\pi^2 \gamma_m F(s)}{\ln(\tau + (1 + s/\Lambda_{QCD}^2))},
$$

(5)

where $F(s) = [1 - \exp(-s^2/[4m_f^4])] / s$, $\gamma_m = 12/(33 - 2N_f)$, with $m_f = 1.0$ GeV, $\tau = e^{10} - 1$, $N_f = 5$, and $\Lambda_{QCD} = 0.21$ GeV.

In Eq.(4), $D_f,g$ and $\omega_f,g$ are parameters expressing the flavor dependent quark-gluon interaction, which are fixed by physical observables. Three groups of parameters corresponding to a varying of the interaction width are given in Ref. [21]. The parameters of the charm and beauty system are listed in Table I. The current mass on the mass shell is defined by

$$
\bar{m}_f^{\gamma} = \bar{m}_f \left( \frac{1}{2} \ln \frac{\bar{m}_f^2}{\Lambda_{QCD}^2} \right)^{\gamma_m},
$$

(6)

$$
\bar{m}_f = \lim_{p^2 \to \infty} \left( \frac{1}{2} \ln \frac{p^2}{\Lambda_{QCD}^2} \right)^{\gamma_m} M_f(p^2),
$$

(7)

where $\bar{m}_f$ is the renormalisation-group invariant current-quark mass[23] and $M_f(p^2)$ is the quark mass function in the quark propagator $S_f(p) = \frac{Z_f(p^2, \zeta)}{\gamma_f - 4\pi^2 F_f(p^2)}$. We extract the value $\bar{m}_c = 1.31$ GeV and $\bar{m}_b = 4.27$ GeV, which are commensurate with those given by PDG [24].

3. Extrapolation — The quark propagators in Eq.(1) are functions of the complex momenta $q^2$ which lies in a parabolic region. Any singular structure in the quark mass depends on $\Lambda$ and $\zeta$. We use a Padé approximation

$$
\lambda^{fg}(P^2) = \left[ \frac{K(k; q; P)}{\mathcal{G}^{fg}(k - q; P)} \right]_{\sigma \beta} \chi^{fg}(q; P),
$$

(8)

where $[K(k; q; P)]_{\sigma \beta} = -\frac{4}{3} [Z_2^2] D_{\mu \nu}^{fg}(k - q) [\gamma_\mu, \gamma_\nu]_{\beta},$ and $\alpha, \beta, \sigma$ and $\delta$ are the Dirac indexes. The meson mass is determined by $\lambda^{fg}(P^2) = -M^{2}_{\text{max}}$. An extrapolation to the physical bound state mass should be implemented while the state mass is larger than the contour border $M^2_{\text{max}}$. We use a Padé approximation

$$
\frac{1}{M^{fg}(P^2)} = \frac{1}{1 + \sum_{n=1}^{\infty} a_n (P^2 + s)^n},
$$

(9)

to fit the $\lambda^{fg}(P^2)$, with $s$, $a_n$ and $b_n$ the parameters. The leptonic decay constant of the pseudoscalar meson ($0^{-}$)
and vector meson \((1^-)\) are defined by

\[
f^{f_0^*(P^2)} P_\mu = Z_2 N_c \frac{\lambda^{bc}}{3} \tr \left( \frac{1}{k^2} \right) \gamma_\mu (k^0 P^2 + i k),
\]

and

\[
f^{f_1^g(P^2)} P_\mu = Z_2 N_c \frac{\lambda^{bc}}{3} \tr \left( \frac{1}{k^2} \right) \gamma_\mu (k^0 P^2 + i k),
\]

with \(\tr\) the trace of the Dirac index. \(f^{f_0}(P^2)\) is generally fitted by

\[
f^{f_0}(P^2) = f_0 + \sum_{n=1}^{\infty} c_n (P^2 + s)^n \frac{1}{1 + \sum_{n=1}^{\infty} d_n (P^2 + s)^n},
\]

where \(f_0\), \(c_n\) and \(d_n\) are parameters, and \(s = M^2\) is the square of the mass. The physical decay constant is \(f^{f_0}(\lambda^2) = f_0\).

4. Results — The series Eq.(9) converges very fast, a good fitting is obtained for \(n = 1\). An illustration of the mass extrapolation is given by Fig. 1, which is the case of \(B^+_c\). The black circles show the \(1/\lambda^{bc}(P^2)\) of the ground state \(B^+_c(1S)\). The mass, \(M_{B^+_c(1S)}\), lies in the parabolic region defined by the singularities of the quark propagator, so it is obtained directly. The red diamonds show the \(1/\lambda^{bc}(P^2)\) of the first radial excited state \(B^{+*}(2S)\). \(M_{B^{+*}(2S)}\) lies outside the parabolic region, and its value is extrapolated and presented by the blue stars. The open circles and diamonds correspond to the varying of the parameters in Table I, which is the main uncertainty of our results. The other excited states are analysis by the similar method.

The masses of the first radial excited state of the charm-beauty system are listed in Table II.

### Table II. Masses of the first radial excited states of charm-beauty system (in GeV). The experimental data for \(M_{B^*_c(2S)}\), \(M_{B^{*+}(2S)}\), \(M_{B^{*+}(1S)}\) and \(M_{B^{+*}(2S)}\) are taken from Ref.[24], \(M_{B^{*+}(2S)}\) and \(M_{B^{*+}(1S)}\) from Ref.[2]. The mass splitting, \(M_{B^{*+}(1S)} - M_{B^{*+}(2S)}\), is quoted from Ref.[21]. The uncertainties of our results correspond to the varying of the parameters in Table I.

| \(M_{B^*_c(2S)}\) (expt.) | \(M_{B^{*+}(2S)}\) (expt.) | \(M_{B^{*+}(1S)}\) (expt.) | \(M_{B^{+*}(2S)}\) (expt.) | \(M_{B^{*+}(2S)} - M_{B^{*+}(1S)}\) (expt.) |
|--------------------------|--------------------------|--------------------------|--------------------------|------------------------------------------|
| 6.613(16)                | 6.841(18)                | 0.039                    | 0.048                    |
| 6.872(2)                 | 7.023(1)                 | 0.031                    |

The mass splitting, \(M_{B^{*+}(1S)} - M_{B^{*+}(2S)}\), is consistent with the recent measurement \[2\]. There is no experimental measurements of \(M_{B^{*+}(1S)}\) and \(M_{B^{*+}(2S)}\) hitherto, our predication waits for the future experimental verification.

To first order in the violation of unitary symmetry, the masses obey the equal spacing rule \[28, 29\]:

\[
\begin{align*}
(M_{B^*_c(2S)} + M_{B^{*+}(2S)})/2 &= M_{B^{*+}(1S)} = M_{B^{*+}(2S)}, \\
(M_{B^{*+}(2S)} + M_{B^{+*}(2S)})/2 &= M_{B^{*+}(1S)} = M_{B^{*+}(2S)}.
\end{align*}
\]

Our results show that the two sides of Eq.(14) and Eq.(15) differ by only 0.05 GeV which is also consistent to the proposal of the mass inequality in Ref. \[30\].

The series Eq. (12) for the leptonic decay constants also converges very fast, a good fitting is obtained also for \(n = 1\). An illustration of the extrapolation of the decay constants is given in Fig. 2, which is the case of \(B^+_c\). The physical value is extrapolated and presented by the blue stars. Our predication of the decay constants of the first radial excited beauty charmed mesons are listed in Table III. We estimate the uncertainty by the
similar method as the mass extrapolation. There is some suppression for the absolute value of decay constant of excited state comparing to ground state which agrees with the previous findings [10, 31–33] and the difference between the excited and ground states decreases with the increasing of the meson mass.

5. Conclusion — Very recently, CMS and LHCb reported the observation of two excited $B_c$ states with high precision [1, 2]. Although they are the normal states within the quark model language, the authors claim that the precision measurements open up an opportunity for the study of hadron physics based on the $ab\ initio$ theory of strong interactions. In this work, making use of a scattering kernel expressing the flavor dependent quark-gluon interaction properly which describes the ground scattering kernel, the authors claim that the splitting $M_{B_c^+(2S)} - M_{B_c^+(2S)}^{\text{cay}}$ may shed light on the future experimental detection. The obtained masses of the beauty-charm system also satisfy the equal spacing rule relation approximately. Furthermore the predicted leptonic decay constants may shed light on the future experimental result. The obtained masses are consistent with the recent observations of CMS and LHCb collaborations and the mass splittings $M_{B_c^+(2S)} - M_{B_c^+(2S)}^{\text{cay}}$ is comparable with the experimental result. The obtained masses of the beauty-charm system also satisfy the equal spacing rule relation approximately. Furthermore the predicted leptonic decay constants may shed light on the future experimental detection.

TABLE III. Our predications of the decay constants of the first radial excited beauty charmed mesons (in GeV). The uncertainties correspond to the varying of the parameters in Table I.

| $f_{B_c}^<(2S)$ | $f_{B_c}^<(2S)$ | $f_{B_c}^<(2S)$ | $f_{B_c}^<(2S)$ | $f_{B_c}^<(2S)$ | $f_{B_c}^<(2S)$ |
|----------------|----------------|----------------|----------------|----------------|----------------|
| -0.097(2)      | -0.119(6)      | -0.165(10)     | -0.161(7)      | -0.310(5)      | -0.320(6)      |

We acknowledge helpful conversations with Pianpian Qin, Sixue Qin, Craig Roberts and Minggang Zhao. This work is supported by: the Chinese Government Thousand Talents Plan for Young Professionals and the National Natural Science Foundation of China under contracts No. 11435001, and No. 11775041, the National Key Basic Research Program of China under contract No. 2015CB856900.
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