ABSTRACT To improve the systematic bit error rate (BER) performance and power efficiency, an optimized antenna selection and adaptive power allocation scheme is proposed for physical layer network coding (PNC) with selective soft-message-forward (SSMF) cooperation. In this scheme, two adaptive power allocation (APA) schemes are proposed for the multiple access (MA) and broadcast (BC) stages, respectively. In the MA stage, the APA scheme aims to maximize the Euclidean distance of network coding symbols received by relay node. In the second BC stage, the relay node takes the maximization of the minimum mutual information of two source nodes as objective function. Then it uses the narrow range method (NRM) to select the optimal antenna set and equally allocates power to them. To further reduce the systematic complexity, an antenna set pre-selection is proposed. It minimizes the antenna number in the initial antenna set of the NRM based on the deletion standard which does not reduce the mutual information in the BC stage. Simulation results indicate that our APA schemes surpass the existing equal power allocation (EPA) scheme with a binary phase-shift keying modulation. Compared with current EPA scheme, the proposed APA schemes for the BC, MA and global stages obtain about 1 dB, 2 dB and 3.5 dB performance gains at BER of $10^{-3}$ with two antennas in the relay node, respectively.

INDEX TERMS Physical layer network coding, selective soft-message-forward, antenna selection, adaptive power allocation.

I. INTRODUCTION

With rapidly developed demand of high speed wireless networks, the fifth generation (5G) wireless communications are expected to improve throughput and spectral efficiency [1]–[3]. Network coding (NC) was a milestone to satisfy these requirement and thus it draw much attentions from wireless communication researchers [4], [5]. In traditional relay networks, two source nodes exchanged signals with each other through four time slots, which did not make full use of spectrum efficiency. Different from this method, in the NC, a relay node encoded signals from different source nodes into one NC symbol sequence, and then forwarded it through one time slot. It only needed three time slots for two source nodes to exchange signals. This three-time-slot NC was called high-layer network coding (HNC) [4], [6], [7]. However, the HNC had the defect of over-detection. In the multiple access (MA) stage, the HNC used two time slots to recognize the accurate signals sent by two source nodes. But for a relay node, it only needed to obtain the NC symbols forwarded from source nodes in the broadcast (BC) stage. Then, a physical layer network coding (PNC) was proposed to
cut unnecessary transmission for much higher efficiency [8], [9]. The relay node did not need to decode received signals separately. It directly mapped the superimposed signals into NC symbols, and then it turned the mutual interferences among signals into a certain part of the NC. Therefore, for the PNC, only two time slots were needed for two source nodes exchanging signals with each other, which greatly improved the network throughput and spectrum efficiency.

At present, well-known relaying schemes are the amplify-and-forward (AF), decode-and-forward (DF), soft-message-forward (SMF), and selective soft-message-forward (SSMF) schemes. In the AF scheme, the relay node simply amplified the superimposed signals and noises, causing a noise amplification problem [10]. In the DF scheme, a relay node decoded and then forwarded the superimposed signals to the destination. It effectively solved the noise amplification problem [11]. However, when the channel quality of the source-to-relay (S-R) link was poor, the relay node cannot decode signals correctly, thereby decreasing the bit error rate (BER).

In most studies of the DF scheme, the relay node mapped the superposition signals into GF(2) by specific mapping rules. Here, GF(2) represented the binary Galois Field (GF). Different from those, in the SMF scheme, after the relay node received the superposition signals, it firstly used the zero forcing (ZF) algorithm to correct the distortion introduced by the channel. Then, it mapped the superposition signals into the log-likelihood ratio (LLR) metrics. It retained the reliability metrics contained in the superposition signals, which were beneficial to the decoding in the BC stage [12]. However, the SMF scheme required much higher symbol synchronization and mapping algorithm complexity when compared with those of the AF one. To reduce the complexity of the SMF scheme, a selective soft-message-forward (SSMF) scheme was proposed. Instead of mapping all signals into LLR metrics, the relay node only needed to map one signal of the highest reliability into the LLR metrics.

In literatures, PNC was mainly discussed with an equal distance model. However, in practical wireless communications, the distances between two source nodes and a relay node were sometimes different, which caused the near-far effects (NFE) [13]. Moreover, even if the distances between two source nodes and the relay node were equal, due to the influence of path loss, shadow fading, and multipath fading, the signal strength received by the relay node from the two source nodes were still different. It reduced the systematic BER performance [14]. Therefore, the power allocation in both the MA and BC stage was essential. Currently, the research on power allocation for PNC was mainly based on the two most important schemes, namely, the AF and DF schemes. For the AF scheme, a power allocation algorithm applicable to a network of two MIMO terminals and a single-antenna relay network was proposed [15]. It took the ergodic sum rate as the objective function and gave the closed-form expression of the optimal power. Different from [15], an optimal power allocation scheme for untrusted relay scenarios was proposed, where the AF relay was both a necessary helper and a potential eavesdropper [16]. It took the secrecy sum rate as the objective function. For the DF scheme, Mylene, et al., analytically determined the optimal power values at both relay and source nodes, which led to a maximization of the sum rate under fairness constraints [17]. The SSMF scheme had better systematic performance than the AF and DF schemes. However, there was still no detailed investigation on power allocation of PNC based on SSMF schemes to the best of our knowledge.

Therefore, an optimized antenna selection and two adaptive power allocation (APA) schemes for PNC with SSMF cooperation are proposed and implemented to cover this scenario. For the MA and BC stages, the two APA schemes were proposed based on the maximum Euclidean distance (MED) and the max-min mutual information (M MMI) theories, respectively. In the MA stage, two source nodes use the channel estimation algorithm [18], [19] to obtain the channel state information (CSI). Subsequently, they adaptively calculate the transmission power to maximize the Euclidean distance of NC symbols received by relay nodes. In the BC stage, the relay node selects the optimal antenna set to transmit signals based on the MM MI theory and the narrow range method (NRM), so as to maximize the minimum mutual information of two source nodes in this time slot. Compared with the global exhaustion search algorithm, the NRM greatly reduces the systematic complexity. Then, an antenna set pre-selection (ASP) algorithm is proposed to further reduce the systematic complexity. When the NRM is used to select the optimal antenna set, the antenna number in the initial antenna set is expected to be as small as possible. To meet this requirement, all antennas of relay node are preselected based on mutual information. If the mutual information between the two source nodes in the BC stage does not decrease after the antenna is removed, the antenna can be eliminated from the initial antenna set. In conclusion, the main contributions of this study are summarized as follows.

- **An APA scheme for the MA stage based on the MED theory.**
  An APA scheme for the MA stage is proposed based on the MED theory. After two source nodes obtain the CSI through channel estimation, the transmission power is calculated by maximizing the Euclidean distance of the received NC symbols as the objective function.

- **An APA scheme for the BC stage based on the MM MI theory.**
  An APA scheme for the BC stage is proposed based on the MM MI theory. With the objective function of maximizing the minimum mutual information of two source nodes in this slot, the relay node uses NRM algorithm to select the optimal antenna set and equally allocates power to them. Compared with the global exhaustion search algorithm, the NRM greatly reduces the systematic complexity.

- **An ASP algorithm based on mutual information for lower complexity.**
An ASP algorithm is proposed based on the deletion standard which do not reduce the mutual information between two source nodes in the BC stage. By pre-selecting all antennas of relay node based on this deletion standard, the antenna number in the initial antenna set of the NRM algorithm is minimized. Hence, the systematic complexity is further reduced.

The remainder of the paper is organized as follows. Section II introduces a system model of a two-way MIMO relay network and the SSMF-based PNC. Section III and Section IV presents the proposed APA schemes for the MA and BC stage, respectively. Subsequently, Section V discusses the ASP algorithm based on the mutual information. Section VI compares the computational complexity between the NRM, global exhaustion search and ASP algorithms. Section VII presents the numerical simulations and result analyses to verify good performance of the proposed APA schemes and ASP algorithm. Finally, Section VIII concludes the entire study.

II. SSMF-BASED PNC SCHEME

Multiple-input-multiple-output (MIMO) technology can improve channel capacity, and the multiple-antenna are widely used in modern wireless communications. The system model is based on a two-way MIMO relay network and it is presented as follows. Section II introduces a system model of a two-way MIMO relay network and it is widely used in modern wireless communications. The system model is based on a two-way MIMO relay network and it is shown in Fig. 1. The source and relay nodes have 1 and N antennas, respectively.

![FIGURE 1. System model of a two-way MIMO relay network.](image)

The entire cooperative transmission of the SSMF-based PNC can be divided into two stages, namely, the MA and BC stages. In the MA stage, two source nodes S1 and S2 simultaneously transmit signals x1 and x2 to relay node R. In transmissions, all links are assumed to be independent, identically distributed (i.i.d.) quasi-static Rayleigh fading channels where the channel coefficients remain static in each frame period but change randomly in different frame periods. In addition, signals of x1 and x2 arrive at the relay node at a symbol level synchronization. The received signals at the relay node are presented as

\[ x_n = \sqrt{P_{s1}h_{11,n}x_1} + \sqrt{P_{s2}h_{21,n}x_2} + n_{i1}, \]

where \( x_n \) is the received signal at the i-th antenna of the relay node, \( P_{s1} \) is the transmission power of the source node S1, \( h_{ij,1} \) is the Rayleigh channel coefficient of the link between the source node \( S_j \) and the i-th antenna of the relay node in the MA stage with zero mean and variance \( \delta_{ij}^2 \), \( n_{i1} \) is the AWGN at the i-th antenna of the relay node with zero mean and variance \( \sigma_i^2 \), and \( x_j \) is the signal sent by the source node \( S_j \), which is modulated with the binary phase-shift keying modulation.

For generality, we first discuss the case where relay nodes have only two antennas, i.e., \( N = 2 \). For consciousness, (1) is rewritten in the following matrix form as

\[ X_R = HX + N, \]

\[ H = \begin{bmatrix} \sqrt{P_{s1}h_{11,1}} & \sqrt{P_{s2}h_{12,1}} \\ \sqrt{P_{s1}h_{21,1}} & \sqrt{P_{s2}h_{22,1}} \end{bmatrix}. \]

In the PNC, relay nodes only need to know the NC symbols, which will be forwarded to the destination nodes, rather than the exact signals sent by the source nodes. Then, in the SSMF-based PNC scheme, the relay node does not decode the two received signals separately, but it obtains the “sum of signals” and the “difference of signals” of them through linear transformation, and it directly detects them [20]. Then, the received signals at the relay node are rewritten as

\[ X_R = HX + N = (HD^{-1})(DX) = \hat{H}X + N, \]

where there are

\[ D = 2D^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \]

\[ \hat{X} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}. \]

Subsequently, the relay node uses the zero forcing (ZF) algorithm [21] to correct the distortions introduced by the channel as

\[ Y_R = GX_R = G(\hat{H}\hat{X} + N) = \hat{X} + \hat{G}N = \hat{X} + \hat{N}, \]

where \( G = (\hat{H}^H\hat{H})^{-1}\hat{H}^H \), represents the equilibrium matrix of the ZF algorithm,

\[ \hat{N} = [\hat{n}_1 \hat{n}_2]^T, \]

\[ \hat{n}_i(i = 1, 2) \] are the amplified noises, which satisfy the Gaussian distribution with zero mean and variance \( \sigma_i^2 \). And they are caused by the ZF algorithm at the i-th antenna of the relay node in the MA stage.

Finally, the relay node maps the signals with smallest \( \hat{n}_i \) into NC symbols. Specifically, the NC symbols are represented as

\[ x_{NC} = \begin{cases} \text{LLR}(x_1 + x_2|y_{r1}), & \hat{\sigma}_1^2 < \hat{\sigma}_2^2 \\ \text{LLR}(x_1 + x_2|y_{r2}), & \text{otherwise} \end{cases}, \]

where there are

\[ \text{LLR}(x_1 + x_2|y_{r1}) \approx -\frac{2}{\hat{\sigma}_1^2} + \log_2\frac{2y_{r1}}{\hat{\sigma}_1^2} - \frac{2y_{r1}}{\hat{\sigma}_1^2}, \]

\[ \text{LLR}(x_1 + x_2|y_{r2}) \approx -\frac{2}{\hat{\sigma}_2^2} + \log_2\frac{2y_{r1}}{\hat{\sigma}_2^2}. \]
at the relay node are affected by the transmitting power of each source node. Therefore, the constellation of signals received by the first antenna of relay node is shown in Fig. 2. In Fig. 2, $\gamma$ is the decision threshold, and there are

$$\gamma = (A + B)/2,$$

$$A = \sqrt{P_{s1}|h_{11,1}|} + \sqrt{P_{s2}|h_{12,1}|},$$

$$B = \sqrt{P_{s1}|h_{11,1}|} - \sqrt{P_{s2}|h_{12,1}|}.$$  

Fig. 2(a) illustrates the situation where the Euclidean distance between $A$ and $B$ is small. In this case, because of noise, $x_1 \oplus x_2$ is easily misjudged. However, in Fig. 2(b), when the Euclidean distance between $A$ and $B$ increases, the possibility of $x_1 \oplus x_2$ being misjudged is reduced. At this time, according to BER formula [22], the BER of the MA stage decreases accordingly. Hence, the two source nodes need to adaptively adjust the transmission power to maximize the Euclidean distance between $A$ and $B$, in other word, to make $B$ close to zero. That is the equivalent equation of

$$\sqrt{P_{s1}|h_{11,1}|} = \sqrt{P_{s2}|h_{12,1}|},$$

s.t. $P_{s1} + P_{s2} = P_s$  

(17)

where $P_s$ is the total transmitted energy of source nodes $S_1$ and $S_2$ in the MA stage.

Similarly, for $x_{22}$, $\sqrt{P_{s1}|h_{21,1}|}$ is required to approach to $\sqrt{P_{s2}|h_{22,1}|}$, namely,

$$\sqrt{P_{s1}|h_{21,1}|} = \sqrt{P_{s2}|h_{22,1}|},$$

s.t. $P_{s1} + P_{s2} = P_s$  

(18)

Because the two equations in (17) and (18) can not be satisfied at the same time, they can be rewritten as

$$\min \left| \sqrt{P_{s1}|h_{11,1}|} - \sqrt{P_{s2}|h_{12,1}|} \right| + \sqrt{P_{s1}|h_{21,1}|} - \sqrt{P_{s2}|h_{22,1}|},$$

s.t. $P_{s1} + P_{s2} = P_s$  

(19)

which is related to the sum of two absolute value functions. Therefore, the minimum solution of (19) is zero. When

$$\sqrt{P_{s1}|h_{11,1}|} > \sqrt{P_{s2}|h_{12,1}|}, \sqrt{P_{s1}|h_{21,1}|} > \sqrt{P_{s2}|h_{22,1}|},$$

then (20) can be rewritten as

$$\sqrt{P_{s1}(|h_{11,1}| + |h_{21,1}|)} - \sqrt{P_{s2}(|h_{12,1}| + |h_{22,1}|)} = 0.$$  

(20)

In this case, the transmitted energy of $S_1$ and $S_2$ are separately expressed as

$$P_{s1} = P_s \frac{(|h_{12,1}| + |h_{22,1}|)^2}{(|h_{12,1}| + |h_{22,1}|)^2 + (|h_{11,1}| + |h_{21,1}|)^2},$$

$$P_{s2} = P_s \frac{(|h_{11,1}| + |h_{21,1}|)^2}{(|h_{12,1}| + |h_{22,1}|)^2 + (|h_{11,1}| + |h_{21,1}|)^2}. $$

(21)

(22)

In other cases, for example of $\sqrt{P_{s1}|h_{11,1}|} < \sqrt{P_{s2}|h_{12,1}|}$, $\sqrt{P_{s1}|h_{21,1}|} > \sqrt{P_{s2}|h_{22,1}|}$, the transmitted energy of $S_1$ and $S_2$ can be expressed similarly as mentioned above.

When the relay node has $N$ antennas, the transmission power of the two source nodes can also be calculated by
the above method. When \( \sqrt{P_1 |h_{11,1}|} > \sqrt{P_2 |h_{12,1}|} \), where \( i = 1, 2, \ldots, N \), the transmitted energy of \( S_1 \) and \( S_2 \) are respectively expressed as

\[
P_{s_1} = P_s \frac{(|h_{12,1}| + \ldots + |h_{N,1,1}|)^2}{(|h_{12,1}| + \ldots + |h_{N,1,1}|)^2 + (|h_{11,1}| + \ldots + |h_{N,1,1}|)^2},
\]

(23)

\[
P_{s_2} = P_s \frac{(|h_{11,1}| + \ldots + |h_{N,1,1}|)^2}{(|h_{12,1}| + \ldots + |h_{N,1,1}|)^2 + (|h_{11,1}| + \ldots + |h_{N,1,1}|)^2}.
\]

(24)

**IV. APA SCHEME BASED ON MMMI THEORY**

In the previous SSMF-based PNC, all antennas of relay nodes are allocated with same power to participate in cooperative forwarding in the BC stage. It results in low energy efficiency at relay node. In view of this defect, an APA scheme for the BC stage based on MMMI theory is proposed in this section.

When there are \( N \) antennas at a relay node, the received signals at two source nodes in the BC stage can be expressed as

\[
y_{s_1} = \sqrt{P_1} h_{11,2} x_{NC} + \sqrt{P_2} h_{21,2} x_{NC} + \ldots + \sqrt{P_N} h_{N,2,2} x_{NC} + n_{1,2},
\]

(25)

\[
y_{s_2} = \sqrt{P_1} h_{12,2} x_{NC} + \sqrt{P_2} h_{22,2} x_{NC} + \ldots + \sqrt{P_N} h_{N,2,2} x_{NC} + n_{2,2}.
\]

(26)

The mutual information obtained by the two source nodes in the BC stage can be expressed respectively as

\[
I_{s_1} = \log_2 (1 + \frac{P_1 |h_{11,2}|^2 + P_2 |h_{21,2}|^2 + \ldots + P_N |h_{N,1,2}|^2}{\sigma_1^2}),
\]

(27)

\[
I_{s_2} = \log_2 (1 + \frac{P_1 |h_{12,2}|^2 + P_2 |h_{22,2}|^2 + \ldots + P_N |h_{N,2,2}|^2}{\sigma_2^2}).
\]

(28)

To improve the systematic performance, the power of each antenna is excepted to be allocated properly to maximize the mutual information of the two source nodes. In other words, (27) and (28) are maximized. Meanwhile, to take into account the performance of both \( S_1 \) and \( S_2 \), the APA scheme for the BC stage is proposed by using the MMMI theory. And the power allocation problem in the BC stage can be converted into the following optimization problems as

\[
\max_{R_t} \left\{ \min(I_{s_1,i}, I_{s_2,i}) \right\},
\]

(29)

\[
s.t. P_{r_1} + P_{r_2} + \ldots + P_{r_N} = P_r
\]

where \( P_r \) is the total transmitting power of all antennas at relay node, \( R_t \) is the optimal antenna set under the time-slot channel condition, \( I_{s_1,i} \) and \( I_{s_2,i} \) represent the mutual information obtained by source node \( S_1 \) and \( S_2 \) in the BC stage under the optimal antenna set, respectively.

To reduce the systematic complexity, the relay node uses the NRM to select the optimal antenna set and equally allocates power to them. Since a relay node has \( N \) antennas, the initial antenna set can be expressed as \( R_0 = \{a_1, a_2, \ldots, a_N\} \), where \( a_i \) represents the \( i \)-th antenna of the relay node. Then, the initial minimum mutual information of the two source nodes can be calculated by (27) and (28) and they are presented as

\[
I_{min,0} = \min(I_{s_1,0}, I_{s_2,0}),
\]

(30)

where \( P_{r_1} = P_{r_2} = \ldots = P_{r_N} = P_r/N \).

To obtain the optimal antenna set, some new antenna sets are needed to be obtained by deleting one antenna from the initial antenna set. The minimum mutual information of the new antenna set is also needed to be calculated. For example, the antenna \( a_1 \) from the initial antenna set can be deleted to obtain a new antenna set \( R_1 = \{a_2, a_3, \ldots, a_N\} \), where \( R_1 \) is defined as the initial antenna set excluding one specific antenna \( a_i \). By this method, the minimum mutual information of the new antenna set \( R_1 \) can be expressed as

\[
I_{min,1} = \min(I_{s_1,1}, I_{s_2,1}),
\]

(31)

where \( P_{r_2} = P_{r_3} = \ldots = P_{r_N} = P_r/(N - 1) \), and

\[
I_{s_1} = \log_2 (1 + \frac{P_2 |h_{21,2}|^2 + P_3 |h_{31,2}|^2 + \ldots + P_N |h_{N,1,2}|^2}{\sigma_1^2}),
\]

(32)

\[
I_{s_2} = \log_2 (1 + \frac{P_2 |h_{22,2}|^2 + P_3 |h_{32,2}|^2 + \ldots + P_N |h_{N,2,2}|^2}{\sigma_2^2}).
\]

(33)

Repeat the above operations to obtain \( N \) minimum mutual informations, and then select the largest one as the minimum mutual information after deleting one antenna. And there is

\[
I_{min,1} = \max(I_{s_1,1}, I_{s_2,1}, \ldots, I_{min,1}).
\]

(34)

To maximize the minimum mutual information of the two source nodes in the BC stage, the minimum mutual information of the new antenna set should be greater than that of the initial mutual information. So there is

\[
I_{min,1}/I_{min,0} \geq 1.
\]

(35)

If (34) satisfies (35), the new antenna set and the minimum mutual information in this case are taken as the initial antenna set and initial mutual information. Then the above operations are continued until (34) no longer satisfies (35) or there is only one antenna in the antenna set. In general, the block diagram of selecting the optimal antenna set by the NRM is shown in Fig. 3.

**V. PROPOSED ASP ALGORITHM**

Compared with the global exhaustion search algorithm, the NRM one can greatly reduce the systematic complexity. However, if the antenna number in the initial antenna set is large, the systematic complexity are still very high. Therefore, an ASP algorithm is proposed to minimize the antenna number in the initial antenna set.

When the \( k \)-th antenna in the initial antenna set does not participate in cooperative forwarding in the BC stage,
I. INTRODUCTION

The antenna set. Substituting (27) and (36) into the inequality expressed respectively as

\[ I_{s1}^k = \log_2(1 + \frac{\sum_{i=1(i\neq k)}^N |h_{1i,2}|^2}{\sigma_1^2}), \quad (36) \]

\[ I_{s2}^k = \log_2(1 + \frac{\sum_{i=1(i\neq k)}^N |h_{1i,2}|^2}{\sigma_2^2}), \quad (37) \]

where there is \( P_n = P_e/(N - 1), \) with \( 1 \leq i \leq N \) and \( i \neq k. \)

To reduce the complexity of the NRM algorithm, the antenna number in the initial antenna set is expected to be as small as possible. But the antenna deletion should conform to the criterion, which do not reduce the mutual information between the two source nodes in the BC stage. If \( I_{s1}^k \geq I_{s1} \) and \( I_{s2}^k \geq I_{s2}, \) the \( k \)-th antenna can be deleted from the initial antenna set. Substituting (27) and (36) into the inequality \( I_{s1}^k \geq I_{s1} \), there is

\[ \log_2(1 + \frac{P_r}{N-1} \sum_{i=1(i\neq k)}^N |h_{1i,2}|^2) \geq \log_2(1 + \frac{P_r}{N} \sum_{i=1}^N |h_{1i,2}|^2). \quad (38) \]

Since \( \log_2(1 + x) \) is a monotonic increasing function, (38) can be rewritten as

\[ \frac{P_r}{N-1} \sum_{i=1(i\neq k)}^N |h_{1i,2}|^2 \geq \frac{P_r}{N} \sum_{i=1}^N |h_{1i,2}|^2. \quad (39) \]

Simplifying (39), if \( I_{s1}^k \geq I_{s1} \), the channel coefficient of the \( k \)-th antenna to source node \( S_1 \) in the BC stage meets the following inequality, namely,

\[ |h_{1k,2}|^2 \leq \frac{1}{N} \sum_{i=1}^N |h_{1i,2}|^2. \quad (40) \]

Similarly, if \( I_{s2}^k \geq I_{s2} \), the channel coefficient of the \( k \)-th antenna to source node \( S_2 \) in the BC stage meets the following inequality, i.e.,

\[ |h_{2k,2}|^2 \leq \frac{1}{N} \sum_{i=1}^N |h_{2i,2}|^2. \quad (41) \]

In conclusion, if the channel coefficients of the \( k \)-th antenna to the source nodes \( S_1 \) and \( S_2 \) in the BC stage satisfy the inequality of (40) and (41) at the same time, then the \( k \)-th antenna can be deleted from the initial antenna set.

VI. COMPLEXITY ANALYSES

After using the APS and the NRM algorithms, the antenna changes in the optimal antenna set of the relay node are shown in Fig. 4, where \( N_{APS} \) and \( N_{NRM} \) represent the antenna number removed from the initial antenna set by the APS and the NRM algorithms, respectively. Here, the antenna marked with “×” means it has been removed from the antenna set.

![FIGURE 3. Block diagram of selecting the optimal antenna set by the NRM.](image)

![FIGURE 4. Antenna changes in the optimal antenna set.](image)

For the global exhaustion search algorithm, when there are \( N \) antennas at the relay node, the initial antenna set has a total of \( S_g \) subsets, where \( S_g \) can be represented as

\[ S_g = C_N^1 + C_N^2 + \ldots + C_N^N = 2^N - 1. \quad (42) \]

For each antenna subset, the minimum mutual information needs to be calculated once. Therefore, the global exhaustion search algorithm needs to calculate the minimum mutual information for \( 2^N - 1 \) times. For the NRM algorithm, each narrowing removes one antenna from the antenna set. If \( I_{min,1}/I_{min,0} < 1 \), the algorithm no longer continues. Therefore, the NRM algorithm needs to calculate the minimum mutual information for \( S_{NRM} \) times at most, where there is

\[ S_{NRM} = N + (N - 1) + \ldots + 1 = \frac{(N + 1)N}{2}. \quad (43) \]

When the ASP algorithm is used, the antenna number in the initial antenna set of the NRM one is reduced from \( N \) to \( N' \), and the latter is expressed as

\[ N' = \sum_{k=1}^N [1 - P(|h_{1k,2}|^2)] \leq \frac{1}{N} \sum_{i=1}^N |h_{1i,2}|^2 \times P(|h_{k,2}|^2) \leq \frac{1}{N} \sum_{i=1}^N |h_{2i,2}|^2. \quad (44) \]
where $|h_{ij}|^2$ is the exponential distribution of parameter $1/\delta_{ij}^2$. When all the $\delta_{ij}^2$ are equal, (44) can be rewritten as

$$N' = N[1 - P(|h_{k1,2}|^2 \leq \frac{1}{N} \sum_{i=1}^{N} |h_{ij}|^2)^2]$$

$$= N[1 - P(|h_{k1,2}|^2 \leq \delta_{ij}^2)^2] = N[1 - (e^{-1})^2] \approx \frac{3}{5} N.$$  

Therefore, it only needs to calculate the minimum mutual information for $S_{ASP}$ times at most, and $S_{ASP}$ is expressed as

$$S_{ASP} = \frac{N' + (N' - 1) + \ldots + 1}{2} = 0.18N^2 + 0.3N.$$  

According to (42) and (43), when the antenna number at the relay node is greater than 2, the complexity of the NRM algorithm is lower than that of the global exhaustion search algorithm. With the increase of relay antennas, the performance advantage of the NRM algorithm is more significant. Because $N' < N$, the systematic complexity is further reduced when the ASP algorithm is used.

**VII. NUMERICAL SIMULATIONS AND RESULT ANALYSES**

The numerical simulations and result analyses of the proposed APA schemes and the ASP algorithm about BER and throughput are used to verify the performance in a two-way MIMO relay network. The simulation parameters are mainly configured as follows. There are $L = 500$ bits, $P_s = 2$ W, $P_r = N$ W (where $N$ is the antenna number of a relay node), $\sigma_i^2 = 1$, and $\delta_{ij}^2 = 1$. Excepted for other specification, a relay node has two antennas and it is located in the middle of the two source nodes.

In the previous SSMF-based PNC, the transmission power of the two source nodes in the MA stage is equal and constant. However, in practical wireless communications, due to the influence of NFE, path loss, shadow fading and multipath fading, if the transmission power of the two source nodes is equal and constant, the signal strength received by the relay node from the two source nodes is different. It reduces the systematic BER performance. Therefore, an APA scheme applicable to the MA stage is proposed with the MED theory. According to the CSI of different time slots, the source nodes adaptively adjust the transmission power to maximize the Euclidean distance of the NC symbols received by the relay nodes. In Fig. 5, at BER of $10^{-3}$, the APA scheme for the MA stage outperforms the EPA scheme by around 2 dB. In the previous SSMF-based PNC, all antennas of relay nodes participated in the cooperative forwarding at the BC stage. It leads to improper energy allocation of relay nodes. Therefore, an APA algorithm suitable for the BC stage is proposed with the MMMI theory. To solve the objective function of maximizing the minimum mutual information between the two source nodes, the relay node uses the NRM algorithm to select the optimal antenna set participating in the BC stage. It equally allocates transmitting power to the antenna in the optimal antenna set. Compared with the original EPA scheme, antennas with better channel quality to the source node are allocated with more power, while those with worse channel quality to the source node even do not participate in the cooperative forwarding. In Fig. 5, at BER of $10^{-3}$, the APA scheme for the BC stage outperforms the EPA scheme by around 1 dB. At BER of $10^{-3}$, if the APA scheme is adopted in both the MA and BC stages, the system obtains about 3.5 dB performance gain.

Throughput is defined as the average number of bits transmitted correctly by relay node per second [23]. The end-to-end throughput of different PNC schemes is shown in Fig. 6. At low SNRs, when the APA scheme is adopted for the SSMF-based PNC, the systematic energy is reasonably utilized and the BER performance is improved accordingly. Hence, the throughput of the SSMF-based PNC with

![Figure 5](image-url)  
**FIGURE 5.** End-to-end BER performance of different PNC schemes.

![Figure 6](image-url)  
**FIGURE 6.** End-to-end throughput performance of different PNC schemes.
APA scheme is greater than that of the DF-based and the SMF-based PNC. At high SNRs, the end-to-end throughput under different PNC scheme is insignificantly different because the BER under different PNC scheme is all remarkably small. The throughout all close to the ceiling, i.e., 2.5 Mbps.

In the EPA scheme, with the change of the distance between each of the two source nodes and relay node, the transmission power of the source nodes are equal and constant. As a result, the signal strengths from the two source nodes are different at the relay node. The Euclidean distance between A and B in Fig. 2 can not be maximized, thus it reduces the systematic BER performance. Different from that, in Fig. 7, when the APA scheme is adopted and the distance between source node $S_1$ and relay node is fixed at 15 km, the average transmission power ratio of the two source nodes changes with the distance between source node $S_2$ and relay node. When $S_2$ is much closer to the relay node, its channel link quality to the relay node is better, and the BER of $x_2$ at the relay node is smaller. Because the BER of NC symbol in PNC is mainly determined by the BER of both $x_1$ and $x_2$. When the BER of $x_1$ is close to that of $x_2$, the BER of NC symbol is the smallest. Therefore, to minimize the BER in the MA stage, the average transmission power ratio of $S_1$ should be larger. Then the BER of $x_1$ at relay node is close to that of $x_2$. On the contrary, when $S_2$ is far away from the relay node, the average transmission power ratio of $S_2$ should be much larger.

With the change of the distance between the source node $S_2$ and relay node, the distances between each antenna and the two source nodes are always equal. Then the channel qualities from each antenna to the two source nodes are similar. In Fig. 8, the channel qualities between each antenna and the two source nodes are different in a single forwarding process and the energy allocated is different from each other. However, in the process of multiple forwarding, the proportion of total energy allocated to each antenna is similar, which is close to 1/6.

In Fig. 9, with the increase of the antenna at relay nodes, the BER performance of both the EPA and the APA schemes is improved. The reasons are explained as follows. With the increase of antenna, the system obtains more spatial diversity gain in both the MA and BC stages. At BER of $10^{-3}$, the APA scheme obtains approximately 3.5 dB performance gain compared with the EPA scheme at the two-antenna relay system. However, at the eight-antenna relay system, the APA scheme obtains about 9.5 dB performance gain. This phenomenon can be explained as follows. In the BC stage, as the antenna increases, the APA scheme allocates more power to the antennas with higher channel quality than that in the counterpart EPA scheme. Therefore, as the increasing of antenna number, the advantage of the APA scheme shows more significant.

At low SNR, the end-to-end throughput of communication system increases as antenna number of the relay node increase.
increases. However, in Fig. 10, the end-to-end throughput with different numbers of antennas is insignificantly different at high SNR. They are all close to the ceiling, i.e., 2.5 Mbps. This phenomenon lies in the fact that the end-to-end BER with different numbers of antennas is all remarkably small at high SNR.

FIGURE 10. End-to-end throughput performance with different numbers of relay antennas.

The ASP algorithm is mainly based on the deletion standard which does not reduce the mutual information between two source nodes in the BC stage. According to (40) and (41), the algorithm essentially removes the antennas with channel quality lower than the average value from the initial antenna set. In Fig. 11, under different number of antennas, the ASP algorithm reduces the systematic complexity but it has little influence on BER performance. Fig. 12 shows the average number of antennas in the initial antenna set after the ASP algorithm. Since the ASP algorithm takes the average channel quality of all antennas at the relay node as the deletion standard, the average number of remaining antennas after the ASP algorithm basically remains unchanged under different SNRs. Hence, the experimental result is consistent with the fact indicated in (45).

FIGURE 11. Performance comparison of end-to-end BER before and after the ASP algorithm.

FIGURE 12. Average number of remaining antennas after the ASP algorithm under different SNR.

VIII. CONCLUSION

To improve BER performance and energy efficiency, two APA schemes are proposed based on the MED and the MMMI criteria. An ASP algorithm is adopted to further reduce the systematic complexity based on the mutual information theory. In addition, the influence of antenna number in a relay node, the distance between the source and relay nodes, and other factors on the proposed APA schemes and ASP algorithm are also investigated. Theoretical analyses and simulation results all indicate that the proposed APA schemes have better performance and lower complexity than existing PNC schemes. So they can be efficiently applied in practice of upcoming 5G wireless communications.

REFERENCES

[1] M. Li, “Performance analysis of wireless network maximum throughput based on network coding,” in Proc. 4th Int. Conf. Inf. Sci. Control Eng. (ICISCE), Changsha, China, Jul. 2017, pp. 1582–1586.
[2] H. Utatsu, K. Osawa, J. Mashino, S. Suyama, and H. Otsuka, “Throughput performance of relay backhaul enhancement using 3D beamforming,” in Proc. Int. Conf. Inf. Netw. (ICOIN), Kuala Lumpur, Malaysia, Jan. 2019, pp. 120–124.
[3] C. Zhang, J. Ge, J. Li, F. Gong, Y. Ji, and M. A. Farah, “Energy efficiency and spectral efficiency tradeoff for asymmetric two-way AF relaying with statistical CSI,” IEEE Trans. Veh. Technol., vol. 65, no. 4, pp. 2833–2839, Apr. 2016.
[4] P. Jesy and P. Deepthi, “Joint source channel network coding using QC LDPC codes,” in Proc. Int. Conf. Commun. Signal Process., Melsurvatthur, India, 2014, pp. 81–85.
[5] S.-Y.-R. Li, Q. T. Sun, and Z. Shao, “Linear network coding: Theory and algorithms,” Proc. IEEE, vol. 99, no. 3, pp. 372–387, Mar. 2011.
[6] C. Ibars, L. Giupponi, and S. Addepalli, “Distributed multiple access and flow control for wireless network coding,” in Proc. IEEE 71st Veh. Technol. Conf., Taipei, Taiwan, May 2010, pp. 1–6.
[7] K. Rajawat, N. Gatsis, S.-J. Kim, and G. B. Giannakis, “Cross-layer design of coded multicast for wireless random access networks,” IEEE J. Sel. Areas Commun., vol. 29, no. 10, pp. 1970–1980, Dec. 2011.
[8] S. Zhang, S. C. Liew, and P. P. Lam, “Hot topic: Physical-layer network coding,” in Proc. ACM Mobicom, 2006, pp. 358–365.
[9] P. Popovski and H. Yomo, “The anti-packets can increase the achievable throughput of a wireless multi-hop network,” in Proc. IEEE Int. Conf. Commun., Istanbul, Turkey, Jun. 2006, pp. 3885–3890.

[10] M. Moradikia, H. Bastami, A. Khuhestani, H. Behroozii, and L. Hanzo, “Cooperative secure transmission relying on optimal power allocation in the presence of untrusted relays, a passive eavesdropper and hardware impairments,” IEEE Access, vol. 7, pp. 116942–116964, Aug. 2019.

[11] Q.-Y. Yu, Y.-T. Li, W.-X. Meng, and W. Xiang, “Uniquely decodable codes for physical-layer network coding in wireless cooperative communications,” IET Syst. J., vol. 13, no. 4, pp. 3956–3967, Dec. 2019.

[12] B. Okyere, L. Musavian, and R. Mumtaz, “Multi-user massive MIMO and physical layer network coding,” in Proc. IEEE Globecon Workshops (GC Wkshps), Waikoloa, HI, USA, Dec. 2019, pp. 1–6.

[13] Y.-F. Huang, T.-H. Tan, S.-Y. Peng, and C.-H. Cheng, “A total adaptive power allocation for physical layer network coding in wireless networks,” in Proc. 7th Int. Conf. Complex., Intell., Softw. Intensive Syst., Taichung, Taiwan, Jul. 2013, pp. 320–324.

[14] K. Lu, S. Fu, Y. Qian, and H.-H. Chen, “SER performance analysis for physical layer network coding over AWGN channels,” in Proc. GLOBECOM-IEEE Global Telecommun. Conf., Honolulu, HI, USA, Nov. 2009, pp. 1–6.

[15] K. P. Roshandeh, A. Khuhestani, M. Ardakani, and C. Tellambura, “Ergodic sum rate analysis and efficient power allocation for a massive MIMO two-way relay network,” IET Commun., vol. 11, no. 2, pp. 211–217, Jan. 2017.

[16] A. Khuhestani, P. L. Yeoh, and A. Mohammadi, “Optimal power allocation and secrecy sum rate in two-way untrusted relaying,” in Proc. GLOBECOM-IEEE Global Commun. Conf., Singapore, Dec. 2017, pp. 1–6.

[17] M. Pischella and D. Le Ruyet, “Optimal power allocation for the two-way relay channel with data rate fairness,” IEEE Commun. Lett., vol. 15, no. 9, pp. 959–961, Sep. 2011.

[18] R. Chayot, M.-L. Boucheret, C. Pouliiat, N. Thomas, N. Van Wambeke, and G. Lesthievnet, “Joint channel and carrier frequency estimation for M-ary CPM over frequency-selective channel using PAM decomposition,” in Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP), New Orleans, LA, USA, Mar. 2017, pp. 3789–3793.

[19] H. Yuan and P.-Y. Kam, “Soft-Decision-aided, smoothness-constrained channel estimation over time-varying fading channels with no channel model information,” IEEE Trans. Wireless Commun., vol. 16, no. 1, pp. 73–86, Jan. 2017.

[20] S. Zhang and S. C. Liew, “Physical layer network coding with multiple antennas,” in Proc. IEEE Wireless Commun. Netw. Conf., Sydney, NSW, Australia, Apr. 2010, pp. 1–6.

[21] G. H. Golub and C. F. Van Loan, Matrix Computations, 3rd ed. Baltimore, MD, USA: Johns Hopkins Univ. Press, 1996.

[22] A. F. Molisch, Wireless Communications, Hoboken, NJ, USA: Wiley, 2005.

[23] S. Lin, L. Fu, J. Xie, and X. Wang, “Hybrid network coding for unbalanced slotted ALOHA relay networks,” IEEE Trans. Wireless Commun., vol. 15, no. 1, pp. 298–313, Jan. 2016.

JIANRONG BAO (Senior Member, IEEE) received the B.S. degree in polymeric materials and engineering and the M.S.E.E. degree from the Zhejiang University of Technology, Hangzhou, China, in 2000 and 2004, respectively, and the Ph.D. degree in electrical engineering from the Department of Electronic Engineering, Tsinghua University, Beijing, China, in 2009.

He was a Postdoctoral Researcher with Zhejiang University, from 2011 to 2013, and with Southeast University, from 2014 to 2017, and then a Visiting Scholar with Columbia University, New York, NY, USA, in 2015. He is currently a Professor at the School of Communication Engineering, Hangzhou Dianzi University, Hangzhou. He is also a Visiting Scholar at the Key Laboratory of Electromagnetic Wave Information Technology and Metrology of Zhejiang Province, College of Information Engineering, China Jiliang University, Hangzhou. His main research interests include modern wireless communications, cognitive radio, information theory and coding, communication signal processing, and wireless sensor networks.

JIANHAI HE received the B.S.E.E. degree from Hangzhou Dianzi University, in 2002, and the M.S.E.E. degree from Zhejiang University, Hangzhou, China, in 2011.

He is currently an Associate Professor at the School of Electronic Information Engineering, Hangzhou Dianzi University, Hangzhou, China. His research interests include wireless communications, physical layer network coding, and SWIPT.

YUNXUAN LIN (Student Member, IEEE) received the B.S.E.E. degree from the School of Communication Engineering, Jiangsu University, Zhenjiang, China, in 2018. He is currently pursuing the master’s degree with the School of Communication Engineering, Hangzhou Dianzi University, Hangzhou, China. His research interests include cooperative communications, physical layer network coding, and SWIPT.

JIANHAI HE received the B.S.E.E. degree from Hangzhou Dianzi University, in 2002, and the M.S.E.E. degree from Zhejiang University, Hangzhou, China, in 2011.

He is currently an Associate Professor at the School of Electronic Information Engineering, Zhejiang University, Zhenjiang, China. He is currently a Lecturer at the School of Communication Engineering, Hangzhou Dianzi University, Hangzhou, China. His main research interests include wireless digital communications, iterative signal processing, the IoT, and embedded systems.