A New Technique for solving Picture Fuzzy Differential Equation

J. Dhivya¹, K. Meena¹, M.N. Saroja²
¹Department of Mathematics, Kumaraguru College of Technology, Coimbatore, Tamil Nadu, India
²Department of Information Technology, Kumaraguru College of Technology, Coimbatore, Tamil Nadu, India
E-mail: jdhivyamaths@gmail.com ¹ & meenakandhan2018@gmail.com², saroja.mn.it@kct.ac.in³

Abstract. Picture Fuzzy set (PFS) is an extension of fuzzy set (FS) and intuitionistic fuzzy set (IFS) that can model the uncertainty by integrating the concept of positive, negative and neutral membership degree of an element. In this paper, the solution of Picture Fuzzy ordinary differential equation of first order by means of picture fuzzy number is exemplified and intend to define the picture fuzzy number for $(\alpha, \beta, \gamma)$- cut. Finally, we illustrate the numerical example for drug distribution in human body for different drug levels is discussed for determining its effectiveness and practicality of the first order differential equation involving picture fuzzy numbers.

Keywords
Picture Fuzzy Set (PFS), Picture Fuzzy Number (PFN), Triangular Picture Fuzzy Set (TPFS)

1. Introduction
Ambiguity is an inevitable constituent of daily life and to deal with the uncertainty Fuzzy set theory (FST) is explored. FST established by Zadeh [1], plays a vital part in decision making (DM) under ambiguous situation. Though, in some case FST is not much skilled to play decisive role. Various extensions of fuzzy set (FS) have been made successfully in most of the real-world uncertain problems. A significant generalization of FST is the intuitionistic fuzzy set (IFS) theory developed by Atanassov [2] describing two functions stating the degree of membership and the degree of non-membership distinctly the sum of two degrees must not exceed one. Although IFS has been useful in various problems of real-life situations, it has some lacking to deal with neutrality. Degree of neutrality concept can be found when we come across with human opinions which involves more answers of type: yes, no, abstain, refusal.

In the Literature, Cuong and Kreinovich [3], presented the concept PFS which is a direct extension of FS and IFS by integrating the concepts of positive, negative and neutral membership degree of an element. Cuong [4] proposed distance measure between PFS and discussed its properties. Compositions of picture fuzzy relations was studied by Phong and co-authors [5]. Cuong and Hai [6] constructed fuzzy inference processes for some operations in picture fuzzy systems. Cuong [7] presented De Morgan fuzzy triples on PFS, Viet [8] presented picture fuzzy inference system based on membership graph, Singh [9] presented
correlation coefficients of PFSs. Later, Palash [10] defined \((\alpha, \delta, \beta)\)-cut of PFS and studied some of its properties. There are some imprecise or uncertain parameters arises, by modeling science and engineering problems. For an uncertainty model with differential equations, the imprecise differential equation concept emerges. Differentiation plays a vital role in science and engineering with imprecise or uncertain parameters. Many researchers [11-19] introduces Fuzzy differential equation to model this uncertainty. Later, intuitionistic fuzzy differential equation [20-25] was emerged. But these three logics have not the refusal term. To change such refusal situation, Picture fuzzy set were developed. The different factors of PFS have been applied in the differential equations (DE).

The structure of the article is as follows. The pre-requisite concepts and definitions are given in the preliminary section. Followed by preliminaries, the solution of the first-order differential equation with triangular picture fuzzy numbers as initial conditions are derived. Finally, the numerical example for drug distribution in human body for different drug levels are illustrated and its graphical interpretations are also shown. The future research scope is discussed in the conclusion part.

2. Preliminaries
Some of the basic definitions are discussed in this section.

2.1 Fuzzy Set (FS) [1]
Let \(\mathcal{X}\) be a nonempty and a set of universes of discourse. A fuzzy set \(A\) drawn from \(\mathcal{X}\) is defined as\(A = \{x, \mu_A(x) : x \in \mathcal{X}\}\), where \(\mu_A(x) : X \to [0, 1]\) shows the degree of membership function of the fuzzy set \(A\).

2.2 Intuitionistic Fuzzy Set (IFS) [2]
An intuitionistic fuzzy set \(A\) in \(X\) is an object with the form \(A = \{x, \mu_A(x), \nu_A(x) : x \in X\}\), where the functions \(\mu_A(x), \nu_A(x) : X \to [0, 1]\) define respectively, the degree of membership and non-membership of the element \(x \in X\) to the set \(A\), which is a subset of \(X\), and for every element \(x \in X\), \(0 \leq \mu_A(x) + \nu_A(x) \leq 1\). The degree of hesitancy of \(x\) is denoted by \(\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))\).

2.3 Picture Fuzzy Set (PFS) [3]
A PFS \(A\) on a universe \(X\) is an object of the form \(A = \{x, \mu_A(x), \eta_A(x), \nu_A(x) : x \in X\}\), where \(\mu_A(x) : X \to [0, 1]\), \(\eta_A(x) : X \to [0, 1]\) and \(\nu_A(x) : X \to [0, 1]\) are called the degree of positive membership (PM), the degree of neutral membership (NeuM) and the degree of negative membership (NM) of \(x \in A\). \(\mu_A(x), \eta_A(x), \nu_A(x)\) must satisfy the condition \(\mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1, \forall x \in X\). Then \(\forall x \in X\), the degree of refusal membership \(\rho_A(x) = 1 - (\mu_A(x) + \eta_A(x) + \nu_A(x))\).

2.4 Picture Fuzzy Number [3]
For a fixed \(x \in A\), \((\mu_A(x), \eta_A(x), \nu_A(x), \rho_A(x))\) is called picture fuzzy number (PFN), where \(\mu_A(x) \in [0, 1]\), \(\eta_A(x) \in [0, 1]\), \(\nu_A(x) \in [0, 1]\) and \(\mu_A(x) + \eta_A(x) + \nu_A(x) + \rho_A(x) = 1\).

2.5 \((\alpha, \delta, \beta)\)-cut of PFS [10]
Let \((\alpha, \delta, \beta)\)-cut of PFS \(A\) of a universe set \(X\) is given as crisp subset defined by \(C_{\alpha, \delta, \beta}(A) = \{x: x \in X \text{ such that } \mu_A(x) \geq \alpha, \eta_A(x) \leq \delta, \nu_A(x) \leq \beta\}\) where \((\alpha, \delta, \beta) \in [0, 1]\) with \(\alpha + \delta + \beta \leq 1\).
That is, Positive membership function (PM) \( \alpha_{A_+} = \{ x \in X: \mu_A(x) \geq \alpha \} \), Neutral membership function (NeuM) \( \delta_{A_0} = \{ x \in X: \eta_A(x) = \delta \} \), Negative membership function (NM) \( \beta_{A-} = \{ x \in X: \upsilon_A(x) \leq \beta \} \) respectively.

2.6 Triangular Picture Fuzzy Sets [10]

A triangular picture fuzzy set is denoted by \( A = \langle [p_1, q_1, r_1; \alpha \gamma_1], [p''_1, q_1, r''_1; \theta_1], [p'_1, q_1, r'_1; \zeta_1] \rangle \) are defined by

\[
\alpha_{A}(x) = \begin{cases} 
\frac{x - p_1}{q_1 - p_1}, & p_1 \leq x \leq q_1 \\
\frac{r_1 - x}{r_1 - q_1}, & q_1 \leq x \leq r_1 \\
0, & \text{Otherwise}
\end{cases}
\]

\[
\gamma_{A}(x) = \begin{cases} 
\frac{(q_1 - x) + (x - p''_1)\theta_1}{q_1 - p''_1}, & p''_1 \leq x \leq q_1 \\
\frac{(x - q_1) + (r''_1 - x)\theta_1}{r''_1 - q_1}, & q_1 \leq x \leq r''_1 \\
1, & \text{Otherwise}
\end{cases}
\]

\[
\beta_{A}(x) = \begin{cases} 
\frac{q_1 - p'_1}{x - q_1}, & p'_1 \leq x \leq q_1 \\
\frac{(q_1 - x) + (r'_1 - x)\zeta_1}{r'_1 - q_1}, & q_1 \leq x \leq r'_1 \\
1, & \text{Otherwise}
\end{cases}
\]

3. First order linear homogeneous Picture Fuzzy Ordinary Differential Equation

**Definition 3.1**

Let \( A \) be a picture fuzzy number then \((\alpha, \delta, \beta)-cut\) of PFN \( A_{TN} = \langle (p, q, r); \gamma_A, \theta_A, \zeta_A \rangle \) is defined as \( A_{(\alpha, \delta, \beta)} = \{ [A_1(\alpha), A_2(\alpha)], [A'_1(\delta), A'_2(\delta)], [A''_1(\beta), A''_2(\beta)] \} \), where \( \alpha + \delta + \beta \leq 3 \).

Here the solution of PFS is

(i) \( \frac{dA_1(\alpha)}{d\alpha} > 0, \frac{dA_2(\alpha)}{d\alpha} < 0, \forall \alpha \in [0,1], A_1(1) \leq A_2(1) \)

(ii) \( \frac{dA'_1(\delta)}{d\delta} < 0, \frac{dA''_1(\delta)}{d\delta} > 0, \forall \delta \in [0,1], A''_1(0) \leq A'_1(0) \)

(iii) \( \frac{dA''_1(\beta)}{d\beta} < 0, \frac{dA'_1(\beta)}{d\beta} > 0, \forall \beta \in [0,1], A'_1(0) \leq A''_1(0) \)

**Definition 3.2**

The solution of the picture fuzzy differential equation be \( y(x) \) with its \((\alpha, \delta, \beta)-cut\) is \( y(\alpha, \delta, \beta) = \{ [y_1(x, \alpha), y_2(x, \alpha)], [y'_1(x, \delta), y'_2(x, \delta)], [y''_1(x, \beta), y''_2(x, \beta)] \} \).

The solution is a strong if

(i) \( \frac{dy_1(x, \alpha)}{d\alpha} > 0, \frac{dy_2(x, \alpha)}{d\alpha} < 0, \forall \alpha \in [0,1], y_1(x, 1) \leq y_2(x, 1) \)

(ii) \( \frac{dy'_1(x, \delta)}{d\delta} < 0, \frac{dy'_2(x, \delta)}{d\delta} > 0, \forall \delta \in [0,1], y'_1(x, 0) \leq y'_2(x, 0) \)

(iii) \( \frac{dy''_1(x, \beta)}{d\beta} < 0, \frac{dy''_2(x, \beta)}{d\beta} > 0, \forall \beta \in [0,1], y''_1(x, 0) \leq y''_2(x, 0) \)

Otherwise the solution is weak.
If \( A = \langle (p, q, r); \gamma_A, \theta_A, \zeta_A \rangle \) then \((\alpha, \delta, \beta)\)-cut is given by

\[
A_{(\alpha, \delta, \beta)} = \left\{ (a + \alpha (b - a))y_A, (c - \alpha (c - b))y_A \right\}, \quad \left\{ (b - \delta(b-a)) \theta_A, (b + \delta(c-b)) \theta_A \right\}, \quad \left\{ (b - \beta(b-a)) \zeta_A, (b + \beta(c-b)) \zeta_A \right\}
\]

4. Solution of First Order DE with TPFN

The first-order linear homogeneous Picture Fuzzy DE:

\[
\frac{dy}{dx} = cy \text{ with the initial condition } y(x_0) = A_{TN}(\langle p, q, r \rangle; y_A, \theta_A, \zeta_A). \quad \text{-------- (1)}
\]

**Case 4.1**

When \( \alpha \) is negative constant, i.e., \( \alpha < 0 \). Taking \( \alpha = -m \), where \( m \) is a positive real number.

Then \((\alpha, \delta, \beta)\)-cut of the equation (1) is

\[
\frac{d}{dx} \left[ y_1(x, \alpha), y_2(x, \alpha) \right] = -p \left[ y_1(x, \alpha), y_2(x, \alpha) \right], \quad \left[ y_1''(x, \alpha), y_2''(x, \alpha) \right]
\]

With the initial condition

\[
y(x_0; \alpha, \delta, \beta) = \left[ (p_1(\alpha), p_2(\alpha)), (p_1''(\delta), p_2''(\delta)), (p_1''(\beta), p_2''(\beta)) \right], \quad \alpha + \delta + \beta \leq 3, \quad \alpha, \delta, \beta \in [0, 1]
\]

That is,

\[
\frac{dy_1(x, \alpha)}{dx} = -my_1(x, \alpha), \quad \frac{dy_2(x, \alpha)}{dx} = -my_2(x, \alpha)
\]

\[
\frac{dy_1''(x, \delta)}{dx} = -my_1''(x, \delta), \quad \frac{dy_2''(x, \delta)}{dx} = -my_2''(x, \delta)
\]

\[
\frac{dy_1'(x, \beta)}{dx} = -my_1'(x, \beta), \quad \frac{dy_2'(x, \beta)}{dx} = -my_2'(x, \beta)
\]

With the initial condition

\[
y_1(x_0, \alpha) = p_1(\alpha); \quad y_2(x_0, \alpha) = p_2(\alpha)
\]

\[
y_1''(x_0, \delta) = p_1''(\delta); \quad y_2''(x_0, \delta) = p_2''(\delta)
\]

\[
y_1'(x_0, \beta) = p_1'(\beta); \quad y_2'(x_0, \beta) = p_2'(\beta)
\]

To find the membership function of the solution:

\[
\frac{d}{dx} \left[ y_1(x, \alpha) + \frac{1}{\lambda} y_2(x, \alpha) \right] = -\lambda m \left[ y_1(x, \alpha) + \frac{1}{\lambda} y_2(x, \alpha) \right] \quad \text{--------- (3)}
\]

Let \( \frac{1}{\lambda} = \lambda \) and \( y_1(x, \alpha) + \frac{1}{\lambda} y_2(x, \alpha) = z \)

Therefore, the solution is, \( z = Ke^{-\lambda mx} \) and \( \lambda = \pm 1 \)

Then,

\[
y_1(x, \alpha) + y_2(x, \alpha) = K_1e^{-mx}
\]

\[
y_1(x, \alpha) - y_2(x, \alpha) = K_2e^{mx}
\]

Solving we get,

\[
y_1(x, \alpha) = \frac{K_1e^{-mx} + K_2e^{mx}}{2} \quad \text{--------- (3)}
\]

\[
y_2(x, \alpha) = \frac{K_1e^{-mx} - K_2e^{mx}}{2} \quad \text{--------- (4)}
\]

Applying the initial conditions, we get,

\[
y_1(x_0, \alpha) = \frac{K_1e^{-mx_0} + K_2e^{mx_0}}{2}
\]
That is, 

\[ y_2(x_0, \alpha) = \frac{K_1 e^{-m x_0} + K_2 e^{m x_0}}{2} \]  

\[ y_2(x_0, \alpha) = \frac{K_1 e^{-m x_0} - K_2 e^{m x_0}}{2} \]

\[ y_2(x_0, \alpha) = K_1 e^{-m x_0} - K_2 e^{m x_0} = 2p_2(\alpha) \]  

\[ y_2(x_0, \alpha) = \frac{K_1 e^{-m x_0} - K_2 e^{m x_0}}{2} \]

From equations (5) and (6),

\[ K_1 = (p_1(\alpha) + p_2(\alpha)) e^{m x_0} \]

\[ K_2 = (p_1(\alpha) - p_2(\alpha)) e^{-m x_0} \]

The solution of \( y_1(x, \alpha) \) is

\[ y_1(x, \alpha) = \frac{p_1(\alpha) + p_2(\alpha)}{2} e^{-m(x-x_0)} + \frac{p_1(\alpha) - p_2(\alpha)}{2} e^{m(x-x_0)} \]

Similarly, the solution of the first order DE is given by

\[ y_2''(x, \delta) = \frac{p_1''(\delta) + p_2''(\delta)}{2} e^{-m(x-x_0)} + \frac{p_1''(\delta) - p_2''(\delta)}{2} e^{m(x-x_0)} \]

\[ y_2'(x, \delta) = \frac{p_1'(\delta) + p_2'(\delta)}{2} e^{-m(x-x_0)} + \frac{p_1'(\delta) - p_2'(\delta)}{2} e^{m(x-x_0)} \]

\[ y_2'(x, \beta) = \frac{p_1'(\beta) + p_2'(\beta)}{2} e^{-m(x-x_0)} + \frac{p_1'(\beta) - p_2'(\beta)}{2} e^{m(x-x_0)} \]

Suppose \( \frac{d}{dx} |y_1(x, \alpha)| > 0, \frac{d}{dx} |y_2(x, \alpha)| < 0, \frac{d}{d\delta} |y_1''(x, \delta)| < 0, \frac{d}{d\delta} |y_2''(x, \delta)| > 0 \) and \( \frac{d}{d\beta} |y_1'(x, \beta)| < 0, \frac{d}{d\beta} |y_2'(x, \beta)| > 0 \).

Then the solution is strong.

**Case 4.2**

When \( c \) is positive constant, i.e., \( c > 0 \), then \((\alpha, \delta, \beta)-cut of the equation (1) we get\)

\[ \frac{d}{dx} \left[ y_1(x, \alpha), y_2(x, \alpha), y_1''(x, \delta), y_2''(x, \delta), y_1'(x, \beta), y_2'(x, \beta) \right] \]

\[ = c \left[ y_1(x, \alpha), y_2(x, \alpha), y_1''(x, \delta), y_2''(x, \delta), y_1'(x, \beta), y_2'(x, \beta) \right] \]

With the initial condition

\[ y(x_0, 0, \alpha, \delta, \beta) = (p_1(\alpha), p_2(\alpha), p_1''(\delta), p_2''(\delta), p_1'(\beta), p_2'(\beta)), \alpha + \delta + \beta \leq 3, \alpha, \delta, \beta \in [0,1] \]

That is,

\[ \frac{dy_1(x, \alpha)}{dx} = c y_1(x, \alpha) \]

\[ \frac{dy_2(x, \alpha)}{dx} = c y_2(x, \alpha) \]

\[ \frac{dy_1''(x, \delta)}{dx} = c y_1''(x, \delta) \]

\[ \frac{dy_2''(x, \delta)}{dx} = c y_2''(x, \delta) \]

\[ \frac{dy_1'(x, \beta)}{dx} = c y_1'(x, \beta) \]

\[ \frac{dy_2'(x, \beta)}{dx} = c y_2'(x, \beta) \]

With the initial condition

\[ y_1(x_0, \alpha) = p_1(\alpha); y_2(x_0, \alpha) = p_2(\alpha) \]

\[ y_1''(x_0, \delta) = p_1''(\delta); y_2''(x_0, \delta) = p_2''(\delta) \]

\[ y_1'(x_0, \beta) = p_1'(\beta); y_2'(x_0, \beta) = p_2'(\beta) \]

Then the solution of the above equation is given by

\[ y_1(x, \alpha) = p_1(\alpha) e^c (x-x_0); y_2(x, \alpha) = p_2(\alpha) e^c (x-x_0) \]
\[ \frac{dy}{dt} \]

The solution for the first order differential equation is

\[ y_1(x, \alpha) = (1.4 + 0.7 \alpha)e^{\frac{\alpha}{2}}; \quad y_2(x, \alpha) = (2.8 - 0.7 \alpha)e^{\frac{\alpha}{2}} \]

\[ y_1''(x, \beta) = (0.6 - 0.2\beta)e^{\frac{\beta}{2}}; \quad y_2''(x, \beta) = (0.6 + 0.2\beta)e^{\frac{\beta}{2}} \]

Hence the solution is strong.

When \( t = 3 \), the values for the above equations for different values of \( (\alpha, \delta, \beta) \) is given in the following table.

| \( \alpha \) | \( y_1(x, \alpha) \) | \( y_2(x, \alpha) \) | \( \delta \) | \( y_1''(x, \delta) \) | \( y_2''(x, \delta) \) | \( \beta \) | \( y_1'(x, \beta) \) | \( y_2'(x, \beta) \) |
|-------------|-----------------|-----------------|------|-----------------|-----------------|------|-----------------|-----------------|
| 0           | 6.2744          | 12.5487         | 0    | 1.3445          | 1.3445          | 0    | 2.6890          | 2.6890          |
| 0.1         | 6.5881          | 12.2350         | 0.1  | 1.2997          | 1.3893          | 0.1  | 2.5994          | 2.7786          |
| 0.2         | 6.9018          | 11.9213         | 0.2  | 1.2549          | 1.4341          | 0.2  | 2.5097          | 2.8683          |
| 0.3         | 7.2155          | 11.6076         | 0.3  | 1.2101          | 1.4790          | 0.3  | 2.4201          | 2.9579          |
| 0.4         | 7.5292          | 11.2939         | 0.4  | 1.1652          | 1.5238          | 0.4  | 2.3305          | 3.0475          |
| 0.5         | 7.8430          | 10.9801         | 0.5  | 1.1204          | 1.5686          | 0.5  | 2.2408          | 3.1372          |
| 0.6         | 8.1567          | 10.6664         | 0.6  | 1.0756          | 1.6134          | 0.6  | 2.1512          | 3.2268          |
| 0.7         | 8.4704          | 10.3527         | 0.7  | 1.0308          | 1.6582          | 0.7  | 2.0616          | 3.3164          |
| 0.8         | 8.7841          | 10.0390         | 0.8  | 0.9860          | 1.7030          | 0.8  | 1.9719          | 3.4061          |
| 0.9         | 9.0978          | 9.7253          | 0.9  | 0.9412          | 1.7479          | 0.9  | 1.8823          | 3.4957          |
| 1.0         | 9.4115          | 9.4115          | 1.0  | 0.8963          | 1.7927          | 1.0  | 1.7927          | 3.5854          |

Fig 1: The graphical interpretation for table 1 for \( t = 3 \)
6. Conclusions
In this paper the first order DE has been solved by using picture fuzzy numbers. Also $(\alpha, \delta, \beta)$-cut methods were used for the Picture Fuzzy numbers. Further the proposed method is applied in the field of medical diagnosis to find the drug level of the patient in different dosage. The solution of the proposed method is proved as strong and the application model shows the effectiveness and practicality of the proposed method. MATLAB is used for the graphical representation and the solution is given for the positive, neutral and negative functions. In future, we will extend the study on the higher order DE with PFNs.

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