Measurement of dijet azimuthal decorrelations in $pp$ collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector and determination of the strong coupling

M. Aaboud et al.*
(ATLAS Collaboration)

(Received 15 May 2018; published 7 November 2018)

A measurement of the rapidity and transverse momentum dependence of dijet azimuthal decorrelations is presented, using the quantity $R_{\Delta\phi}$. The quantity $R_{\Delta\phi}$ specifies the fraction of the inclusive dijet events in which the azimuthal opening angle of the two jets with the highest transverse momenta is less than a given value of the parameter $\Delta\phi_{\text{max}}$. The measurement uses an event sample corresponding to an integrated luminosity of 20.2 fb$^{-1}$ collected with the ATLAS detector at the CERN Large Hadron Collider. Predictions of a perturbative QCD calculation at next-to-leading order in the strong coupling with corrections for nonperturbative effects are compared to the data. The theoretical predictions describe the data in the whole kinematic region. The data are used to determine the strong coupling $\alpha_S$ and to study its running for momentum transfers from 260 GeV to above 1.6 TeV. Analysis that combines data at all momentum transfers results in $\alpha_S(m_Z) = 0.1127^{+0.0063}_{-0.0037}$.

DOI: 10.1103/PhysRevD.98.092004

I. INTRODUCTION

In high-energy particle collisions, measurements of the production rates of hadronic jets with large transverse momentum $p_T$ relative to the beam direction can be employed to test the predictions of perturbative quantum chromodynamics (pQCD). The results can also be used to determine the strong coupling $\alpha_S$, and to test the pQCD predictions for the dependence of $\alpha_S$ on the momentum transfer $Q$ (the “running” of $\alpha_S$) by the renormalization group equation (RGE) [1,2]. Previous tests of the RGE through $\alpha_S$ determinations in hadronic final states have been performed using event data taken in $ep$ collisions ($5 < Q < 60$ GeV) [3–5], in $e^+e^-$ annihilation ($10 < Q < 210$ GeV) [6,7], in $pp$ collisions ($50 < Q < 400$ GeV) [8–10], and in $pp$ collisions ($130 < Q < 1400$ GeV) [11–15]. The world average value is currently $\alpha_S(m_Z) = 0.1181 \pm 0.0011$ [16].

Recent $\alpha_S$ results from hadron collisions are limited by theoretical uncertainties related to the scale dependence of the fixed-order pQCD calculations. The most precise $\alpha_S(m_Z)$ result from hadron collision data is $\alpha_S(m_Z) = 0.1161^{+0.0041}_{-0.0048}$ [9], obtained from inclusive jet cross-section data, using pQCD predictions beyond the next-to-leading order (NLO). However, when the cross-section data are used in $\alpha_S$ determinations, the extracted $\alpha_S$ results are directly affected by our knowledge of the parton distribution functions (PDFs) of the proton, and their $Q$ dependence. The PDF parametrizations depend on assumptions about $\alpha_S$ and the RGE in the global data analyses in which they are determined. Therefore, in determinations of $\alpha_S$ and its $Q$ dependence from cross-section data the RGE is already assumed in the inputs. Such a conceptual limitation when using cross-section data can largely be avoided by using ratios of multijet cross sections in which PDFs cancel to some extent. So far, the multijet cross-section ratios $R_{\Delta\phi}$ [10] and $R_{\Delta\phi}$ [11] have been used for $\alpha_S$ determinations at hadron colliders. In this article, $\alpha_S$ is determined from dijet azimuthal decorrelations, based on the multijet cross-section ratio $R_{\Delta\phi}$ [17]. The RGE predictions are tested up to $Q = 1.675$ TeV.

The decorrelation of dijets in the azimuthal plane has been the subject of a number of measurements at the Fermilab Tevatron Collider [18] and the CERN Large Hadron Collider (LHC) [19,20]. The variable $\Delta\phi_{\text{dijet}}$ investigated in these analyses is defined from the angles in the azimuthal plane (the plane perpendicular to the beam direction) $\phi_{1,2}$ of the two highest-$p_T$ jets in the event as $\Delta\phi_{\text{dijet}} = |\phi_1 - \phi_2|$. In exclusive high-$p_T$ dijet final states, the two jets are correlated in the azimuthal plane with $\Delta\phi_{\text{dijet}} = \pi$. Deviations from this ($\Delta\phi_{\text{dijet}} < \pi$) are due to additional activity in the final state, as described in pQCD by processes of higher order in $\alpha_S$. Due to kinematic constraints, the phase space in $2 \to 3$ processes is restricted to $\Delta\phi_{\text{dijet}} > 2\pi/3$ [21] and lower $\Delta\phi_{\text{dijet}}$ values are only accessible in $2 \to 4$ processes. Measurements of dijet production with $2\pi/3 < \Delta\phi_{\text{dijet}} < \pi$ ($\Delta\phi_{\text{dijet}} < 2\pi/3$)
The quantity $R_{\Delta \phi}$ is defined as the fraction of all inclusive dijet events in which $\Delta \phi_{\text{dijet}}$ is less than a specified value $\Delta \phi_{\text{max}}$. This quantity can be exploited to extend the scope of the previous analyses towards studies of the rapidity dependence of dijet azimuthal decorrelations. Since $R_{\Delta \phi}$ is defined as a ratio of multijet cross sections for which the PDFs cancel to a large extent, it is well suited for determinations of $\alpha_S$ and for studies of its running.

The quantity $R_{\Delta \phi}$ has so far been measured in $p \bar{p}$ collisions at a center-of-mass energy of $\sqrt{s} = 1.96$ TeV at the Fermilab Tevatron Collider [22]. This article presents the first measurement of $R_{\Delta \phi}$ in $pp$ collisions, based on data at $\sqrt{s} = 8$ TeV taken with the ATLAS detector during 2012 at the LHC, corresponding to an integrated luminosity of $20.2 \pm 0.4$ fb$^{-1}$ [23]. The data are corrected to “particle level” [24], and are used to extract $\alpha_S$ and to study its running over a range of momentum transfers of $262 < Q < 1675$ GeV.

## II. Definition of $R_{\Delta \phi}$ and the Analysis Phase Space

The definitions of the quantity $R_{\Delta \phi}$ and the choices of the variables that define the analysis phase space are taken from the proposal in Ref. [17]. Jets are defined by the anti-$k_T$ jet algorithm as implemented in FastJet [25,26]. The anti-$k_T$ jet algorithm is a successive recombination algorithm in which particles are clustered into jets in the $E$-scheme (i.e., the jet four-momentum is computed as the sum of the particle four-momenta). The radius parameter is chosen to be $R = 0.6$. This is large enough for a jet to include a sufficient amount of soft and hard radiation around the jet axis, thereby improving the properties of pQCD calculations at fixed order in $\alpha_S$, and it is small enough to avoid excessive contributions from the underlying event [27]. An inclusive dijet event sample is extracted by selecting all events with two or more jets, where the two leading-$p_T$ jets have $p_T > p_{T \text{min}}$. The dijet phase space is further specified in terms of the variables $y_{\text{boost}}$ and $y^*$, computed from the rapidities, $y_1$ and $y_2$, of the two leading-$p_T$ jets as $y_{\text{boost}} = (y_1 + y_2)/2$ and $y^* = |y_1 - y_2|/2$, respectively. In 2 to 2 processes, the variable $y_{\text{boost}}$ specifies the longitudinal boost between the dijet and the proton-proton center-of-mass frames, and $y^*$ (which is longitudinally boost invariant) represents the absolute value of the jet rapidities in the dijet center-of-mass frame. The dijet phase space is restricted to $|y_{\text{boost}}| < y_{\text{boost}}^\text{max}$ and $y^* < y_{\text{max}}^*$. The variable $H_T$ is defined as the scalar sum of the jet $p_T$ for all jets $i$ with $p_{T_i} > p_{T \text{min}}$ and $|y_i - y_{\text{boost}}| < y_{\text{max}}^*$. Furthermore, the leading-$p_T$ jet is required to have $p_{T_1} > H_T/3$. The values of the parameters $p_{T \text{min}}$, $y_{\text{boost}}^\text{max}$, and $y_{\text{max}}^*$ ensure that jets are well measured in the detector within $|y| < 2.5$ and that contributions from nonperturbative corrections and pileup (additional proton-proton interactions within the same or nearby bunch crossings) are small. The requirement $p_{T_1} > H_T/3$ ensures (for a given $H_T$) a well-defined minimum $p_{T_1}$ which allows single-jet triggers to be used in the measurement. It also reduces the contributions from events with four or more jets, and therefore pQCD corrections from higher orders in $\alpha_S$. The values of all parameters are specified in Table I. The quantity $R_{\Delta \phi}$ is defined in this inclusive dijet event sample as the ratio

$$R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}}) = \frac{\frac{d^2 \sigma_{\text{dijet}}(\Delta \phi_{\text{dijet}} < \Delta \phi_{\text{max}})}{dH_T dy^*}}{\frac{d^2 \sigma_{\text{dijet}}(\text{inclusive})}{dH_T dy^*}},$$

where the denominator is the inclusive dijet cross section in the phase space defined above, in bins of the variables $H_T$ and $y^*$. The numerator is given by the subset of the denominator for which $\Delta \phi_{\text{dijet}}$ of the two leading-$p_T$ jets obeys $\Delta \phi_{\text{dijet}} < \Delta \phi_{\text{max}}$. The measurement of the $y^*$ dependence of $R_{\Delta \phi}$ allows a test of the rapidity dependence of the pQCD matrix elements. The value of $\Delta \phi_{\text{max}}$ is directly related to the hardness of the jet(s) produced in addition to the two leading-$p_T$ jets in the event. The transverse momentum sum $\sum H_T$ is one possible choice that can be related to the scale at which $\alpha_S$ is probed. The measurement is made as a function of $H_T$ in three different

### Table I. The values of the parameters and the requirements that define the analysis phase space for the inclusive dijet event sample.

| Variable     | Value     |
|--------------|-----------|
| $p_{T \text{min}}$ | 100 GeV   |
| $y_{\text{max}}$ | 2.0       |
| $p_{T_1}/H_T$  | >1/3      |

### Table II. The $H_T$, $y^*$, and $\Delta \phi_{\text{max}}$ regions in which $R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}})$ is measured.

| Quantity                  | Value         |
|---------------------------|---------------|
| $H_T$ bin boundaries (in TeV) | 0.45, 0.6, 0.75, 0.9, 1.1, 1.4, 1.8, 2.2, 2.7, 4.0 |
| $y^*$ regions              | 0.0–0.5, 0.5–1.0, 1.0–2.0 |
| $\Delta \phi_{\text{max}}$ values | 7π/8, 5π/6, 3π/4, 2π/3 |
y regions and for four different values of $\Delta \phi_{\text{max}}$ (see Table II).

III. THEORETICAL PREDICTIONS

The theoretical predictions in this analysis are obtained from perturbative calculations at fixed order in $\alpha_s$ with additional corrections for nonperturbative effects.

The pQCD calculations are carried out using NLOJET++ [28,29] interfaced to FASTNLO [30,31] based on the matrix elements for massless quarks in the $\overline{\text{MS}}$ scheme [32]. The renormalization and factorization scales are set to $\mu_s = \mu_t = \mu_0 = H_T/2$. In inclusive dijet production at leading order (LO) in pQCD this choice is equivalent to other common choices: $\mu_0 = \tilde{p}_T = (p_{T1} + p_{T2})/2$ and $\mu_0 = p_{T1}$. The evolution of $\alpha_s$ is computed using the numerical solution of the next-to-leading-logarithmic (2-loop) approximation of the RGE.

The pQCD predictions for the ratio $R_{\Delta \phi}$ are obtained from the ratio of the cross sections in the numerator and denominator in Eq. (1), computed to the same relative order (both either to NLO or to LO). The pQCD predictions for the cross section in the denominator by NLOJET++ are available up to NLO. For $\Delta \phi_{\text{max}} = 7\pi/8, 5\pi/6, 5\pi/4 (2\pi/3)$ the numerator is a three-jet (four-jet) quantity for which the pQCD predictions in NLOJET++ are available up to NLO (LO) [21].

The PDFs are taken from the global analyses MMHT2014 (NLO) [33,34], CT14 (NLO) [35], and NNPDFv2.3 (NLO) [36]. For additional studies, the PDF sets ABMP16 (NNLO) [38]3 and HERAPDF 2.0 (NLO) [39] are used, which were obtained using data from selected processes only. All of these PDF sets were obtained for a series of discrete $\alpha_s(m_Z)$ values, in increments of $\Delta \alpha_s(m_Z) = 0.001$ (or $\Delta \alpha_s(m_Z) = 0.002$ for NNPDFv2.3). In all calculations in this paper, the PDF sets are consistently chosen to correspond to the value of $\alpha_s(m_Z)$ used in the matrix elements. The extraction of $\alpha_s$ from the experimental $R_{\Delta \phi}$ data requires a continuous dependence of the pQCD calculations on $\alpha_s(m_Z)$. This is obtained by analytic extrapolation (linear extrapolation) for $\alpha_s(m_Z)$ values inside (outside) the ranges provided by the PDF sets. The central predictions that are compared to the data use $\alpha_s(m_Z) = 0.118$, which is close to the current world average, and the MMHT2014 PDFs. The MMHT2014 PDFs also provide the largest range of $\alpha_s(m_Z)$ values (0.108 $\leq \alpha_s(m_Z) \leq 0.128$). For these reasons, the MMHT2014 PDFs are used to obtain the central results in the $\alpha_s$ determinations.

The uncertainties of the perturbative calculation are estimated from the scale dependence (as an estimate of missing higher-order pQCD corrections) and the PDF uncertainties. The former is evaluated from independent variations of $\mu_R$ and $\mu_F$ between $0_0/2$ and $2 \mu_0$ (with the restriction $0.5 \leq \mu_R/\mu_F \leq 2.0$). The PDF-induced uncertainty is computed by propagating the MMHT2014 PDF uncertainties. In addition, a “PDF set” uncertainty is included as the envelope of the differences of the results obtained with CT14, NNPDFv2.3, ABMP16, and HERAPDF 2.0, relative to those obtained with MMHT2014.

The pQCD predictions based on matrix elements for massless quarks also depend on the number of quark flavors, in gluon splitting ($g \to q \bar{q}$), $n_t$, which affects the tree-level matrix elements and their real and virtual corrections, as well as the RGE predictions and the PDFs obtained from global data analyses. The central results in this analysis are obtained for a consistent choice $n_t = 5$ in all of these contributions. Studies of the effects of using $n_t = 6$ in the matrix elements and the RGE, as documented in Appendix A, show that the corresponding effects for $R_{\Delta \phi}$ are between $-1\%$ and $+2\%$ over the whole kinematic range of this measurement. Appendix A also includes a study of the contributions from the $t \bar{t}$ production process, concluding that the effects on $R_{\Delta \phi}$ are less than 0.5% over the whole analysis phase space.

The corrections due to nonperturbative effects, related to hadronization and the underlying event, were obtained in Ref. [17], using the event generators PYTHIA 6.426 [40] and HERWIG 6.520 [41,42]. An estimate of the model uncertainty is obtained from a study of the dependence on the generator’s parameter settings (tunes), based on the PYTHIA tunes AMBT1 [43], DW [44], A [45], and S-Global [46], which differ in the parameter settings and the implementations of the parton-shower and underlying-event models. All model predictions for the total nonperturbative corrections lie below 2% (4%) for $\Delta \phi_{\text{max}} = 7\pi/8$ and 5$\pi/6$ ($\Delta \phi_{\text{max}} = 5\pi/4$ and 2$\pi/3$), and the different models agree within 2% (5%) for $\Delta \phi_{\text{max}} = 7\pi/8$ and 5$\pi/6$ ($\Delta \phi_{\text{max}} = 5\pi/4$ and 2$\pi/3$).

For this analysis, the central results are taken to be the average values obtained from PYTHIA with tunes AMBT1 and DW. The corresponding uncertainty is taken to be half of the difference (the numerical values are provided in Ref. [47]). The results obtained with PYTHIA tunes A and S-Global as well as HERWIG are used to study systematic uncertainties.

IV. ATLAS DETECTOR

ATLAS is a general-purpose detector consisting of an inner tracking detector, a calorimeter system, a muon spectrometer, and magnet systems. A detailed description of the ATLAS detector is given in Ref. [47]. The main components used in the $R_{\Delta \phi}$ measurement are the inner detector, the calorimeters, and the trigger system.
The position of the $pp$ interaction is determined from charged-particle tracks reconstructed in the inner detector, located inside a superconducting solenoid that provides a 2 T axial magnetic field. The inner detector, covering the region $|\eta| < 2.5$, consists of layers of silicon pixels, silicon microstrips, and transition radiation tracking detectors.

Jet energies and directions are measured in the three electromagnetic and four hadronic calorimeters with a coverage of $|\eta| < 4.9$. The electromagnetic liquid argon (LAr) calorimeters cover $|\eta| < 1.475$ (barrel), $1.375 < |\eta| < 3.2$ (endcap), and $3.1 < |\eta| < 4.9$ (forward). The regions $|\eta| < 0.8$ (barrel) and $0.8 < |\eta| < 1.7$ (extended barrel) are covered by scintillator/steel sampling hadronic calorimeters, while the regions $1.5 < |\eta| < 3.2$ and $3.1 < |\eta| < 4.9$ are covered by the hadronic endcap with LAr/Cu calorimeter modules, and the hadronic forward calorimeter with LAr/W modules.

During 2012, for $pp$ collisions, the ATLAS trigger system was divided into three levels, labeled L1, L2, and the event filter (EF) [48,49]. The L1 trigger is hardware-based, while L2 and EF are software based and impose increasingly refined selections designed to identify events of interest. The jet trigger identifies electromagnetically and hadronically interacting particles by reconstructing the energy deposited in the calorimeters. The L1 jet trigger uses a sliding window of $\Delta \eta \times \Delta \phi = 0.8 \times 0.8$ to find jets and requires these to have transverse energies $E_T$ above a given threshold, measured at the electromagnetic scale. Jets triggered by L1 are passed to the L2 jet trigger, which reconstructs jets in the same region using a simple cone jet algorithm with a cone size of 0.4 in $(\eta, \phi)$ space. Events are accepted if a L2 jet is above a given $E_T$ threshold. In events which pass L2, a full event reconstruction is performed by the EF. The jet EF constructs topological clusters [50] from which jets are then formed, using the anti-$k_t$ jet algorithm with a radius parameter of $R = 0.4$. These jets are then calibrated to the hadronic scale. Events for this analysis are collected either with single-jet triggers with different minimum $E_T$ requirements or with multijet triggers based on a single high-$E_T$ jet plus some amount of $H_T$ (the scalar $E_T$ sum) of the multijet system. The trigger efficiencies are determined relative to fully efficient reference triggers, and each trigger is used above an $H_T$ threshold where it is more than 98% efficient. The triggers used for the different $H_T$ regions in the offline analysis are listed in Table III.

Single-jet triggers select events if any jet with $|\eta| < 3.2$ is above the $E_T$ thresholds at L1, L2, and the EF. Due to their high rates, the single-jet triggers studied are highly prescaled during data-taking. Multijet triggers select events if an appropriate high-$E_T$ jet is identified and the $H_T$ value, summed over all jets at the EF with $|\eta| < 3.2$ and $E_T > 45$ GeV, is above a given threshold. The additional $H_T$ requirement significantly reduces the selected event rate, and lower prescales can be applied. The integrated luminosity of the data sample collected with the highest threshold triggers is $20.2 \pm 0.4$ fb$^{-1}$.

The detector response for the measured quantities is determined using a detailed simulation of the ATLAS detector in GEANT 4 [51,52]. The particle-level events, subjected to the detector simulation, were produced by the PYTHIA event generator [53] (version 8.160) with CT10 PDFs. The PYTHIA parameters were set according to the AU2 tune [54]. The “particle-level” jets are defined based on the four-momenta of the generated stable particles (as recommended in Ref. [24], with a proper lifetime $\tau$ satisfying $\epsilon \tau > 10$ mm, including muons and neutrinos from hadron decays). The “detector-level” jets are defined based on the four-momenta of the simulated detector objects.

### V. Measurement Procedure

The inclusive dijet events used for the measurement of $R_{\Delta \phi}$ were collected between April and December 2012 by the ATLAS detector in proton-proton collisions at $\sqrt{s} = 8$ TeV. All events used in this measurement are required to satisfy data-quality criteria which include stable beam conditions and stable operation of the tracking systems, calorimeters, solenoid, and trigger system. Events that pass the trigger selections described above are included in the sample if they contain at least one primary collision vertex with at least two associated tracks with $p_T > 400$ MeV, in order to reject contributions due to cosmic-ray events and beam background. The primary vertex with highest $\sum p_T^2$ of associated tracks is taken as the event vertex.

Jets are reconstructed offline using the anti-$k_t$ jet algorithm with a radius parameter $R = 0.6$. Input to the jet algorithm consists of locally calibrated three-dimensional topological clusters [50] formed from sums of calorimeter cell energies, corrected for local calorimeter response, dead material, and out-of-cluster losses for pions. The jets are further corrected for pileup contributions and then calibrated to the hadronic scale, as detailed in the following. The pileup correction is applied to account for the effects on the jet response from additional interactions within the same proton bunch crossing (“in-time pileup”) and from interactions in bunch crossings preceding or following the one of interest (“out-of-time pileup”). Energy is subtracted from each jet, based upon the energy density

| $H_T$ range [GeV] | Trigger type | Integrated luminosity [pb$^{-1}$] |
|-------------------|--------------|----------------------------------|
| 450–600           | Single-jet   | 9.6 ± 0.2                        |
| 600–750           | Single-jet   | 36 ± 1                           |
| 750–900           | Multi-jet    | 546 ± 11                         |
| > 900             | Multi-jet    | ($20.2 \pm 0.4$) \times 10^3    |

The detector response for the measured quantities is determined using a detailed simulation of the ATLAS detector in GEANT 4 [51,52]. The particle-level events, subjected to the detector simulation, were produced by the PYTHIA event generator [53] (version 8.160) with CT10 PDFs. The PYTHIA parameters were set according to the AU2 tune [54]. The “particle-level” jets are defined based on the four-momenta of the generated stable particles (as recommended in Ref. [24], with a proper lifetime $\tau$ satisfying $\epsilon \tau > 10$ mm, including muons and neutrinos from hadron decays). The “detector-level” jets are defined based on the four-momenta of the simulated detector objects.

### V. Measurement Procedure

The inclusive dijet events used for the measurement of $R_{\Delta \phi}$ were collected between April and December 2012 by the ATLAS detector in proton-proton collisions at $\sqrt{s} = 8$ TeV. All events used in this measurement are required to satisfy data-quality criteria which include stable beam conditions and stable operation of the tracking systems, calorimeters, solenoid, and trigger system. Events that pass the trigger selections described above are included in the sample if they contain at least one primary collision vertex with at least two associated tracks with $p_T > 400$ MeV, in order to reject contributions due to cosmic-ray events and beam background. The primary vertex with highest $\sum p_T^2$ of associated tracks is taken as the event vertex.

Jets are reconstructed offline using the anti-$k_t$ jet algorithm with a radius parameter $R = 0.6$. Input to the jet algorithm consists of locally calibrated three-dimensional topological clusters [50] formed from sums of calorimeter cell energies, corrected for local calorimeter response, dead material, and out-of-cluster losses for pions. The jets are further corrected for pileup contributions and then calibrated to the hadronic scale, as detailed in the following. The pileup correction is applied to account for the effects on the jet response from additional interactions within the same proton bunch crossing (“in-time pileup”) and from interactions in bunch crossings preceding or following the one of interest (“out-of-time pileup”). Energy is subtracted from each jet, based upon the energy density

| $H_T$ range [GeV] | Trigger type | Integrated luminosity [pb$^{-1}$] |
|-------------------|--------------|----------------------------------|
| 450–600           | Single-jet   | 9.6 ± 0.2                        |
| 600–750           | Single-jet   | 36 ± 1                           |
| 750–900           | Multi-jet    | 546 ± 11                         |
| > 900             | Multi-jet    | ($20.2 \pm 0.4$) \times 10^3    |
in the event and the measured area of the jet [55]. The jet energy is then adjusted by a small residual correction depending on the average pileup conditions for the event. This calibration restores the calorimeter energy scale, on average, to a reference point where pileup is not present [56]. Jets are then calibrated using an energy- and \( \eta \)-dependent correction to the hadronic scale with constants derived from data and Monte Carlo samples of jets produced in multi-jet processes. A residual calibration, based on a combination of several \textit{in situ} techniques, is applied to take into account differences between data and Monte Carlo simulation. In the central region of the detector, the uncertainty in the jet energy calibration is derived from the transverse momentum balance in \( Z + \text{jet} \), \( \gamma + \text{jet} \) or multijet events \textit{in situ}, by propagating the known uncertainties of the energies of the reference objects to the jet energies. The energy uncertainties for the central region are then propagated to the forward region by studying the transverse momentum balance in dijet events with one central and one forward jet [57]. The energy calibration uncertainty in the high-\( p_T \) range is estimated using the \textit{in situ} measurement of the response to single isolated hadrons [58]. The jet energy calibration’s total uncertainty is decomposed into 57 uncorrelated contributions, of which each is fully correlated in \( p_T \). The corresponding uncertainty in jet \( p_T \) is between 1% and 4% in the central region (\( |\eta| < 1.8 \)), and increases to 5% in the forward region (\( 1.8 < |\eta| < 4.5 \)).

The jet energy resolution has been measured in the data using the bisector method in dijet events [59–61] and the Monte Carlo simulation is seen to be in good agreement with the data. The uncertainty in the jet energy resolution is affected by selection parameters for jets, such as the amount of nearby jet activity, and depends on the \( \eta \) and \( p_T \) values of the jets. Further details about the determinations of the jet energy scale and resolution are given in Refs. [58,59,62].

The angular resolution of jets is obtained in the Monte Carlo simulation by matching particle-level jets with detector-level jets, when their distance in \( \Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \) is smaller than the jet radius parameter. The jet \( \eta \) and \( \phi \) resolutions are obtained from a Gaussian fit to the distributions of the difference between the detector-level and particle-level values of the corresponding quantity. The difference between the angular resolutions determined from different Monte Carlo simulations is taken as a systematic uncertainty for the measurement result, which is about 10%–15% for \( p_T < 150 \text{ GeV} \) and decreases to about 1% for \( p_T > 400 \text{ GeV} \). The bias in jet \( \eta \) and \( \phi \) is found to be negligible.

All jets within the whole detector acceptance, \( |\eta| < 4.9 \), are considered in the analysis. Data-quality requirements are applied to each reconstructed jet according to its properties, to reject spurious jets not originating from hard-scattering events. In each \( H_T \) bin, events from a single trigger are used and the same trigger is used for the numerator and the denominator of \( R_{\Delta \phi} \). In order to test the stability of the measurement results, the event sample is divided into subsamples with different pileup conditions. The \( R_{\Delta \phi} \) results for different pileup conditions are compatible within the statistical uncertainties without any systematic trends. The measurement is also tested for variations resulting from loosening the requirements on the event- and jet-data-quality conditions, and the observed variations are also consistent within the statistical uncertainties.

The distributions of \( R_{\Delta \phi}(H_T, \eta, \Delta \phi_{\text{max}}) \) are corrected for experimental effects, including detector resolutions and inefficiencies, using the simulation. To ensure that the simulation describes all relevant distributions, including the \( p_T \) and \( \eta \) distributions of the jets, the generated events are reweighted, based on the properties of the generated jets, to match these distributions in data, and to match the \( H_T \) dependence of the observed inclusive dijet cross section as well as the \( R_{\Delta \phi} \) distributions and their \( H_T \) dependence. To minimize migrations between \( H_T \) bins due to resolution effects, the bin widths are chosen to be larger than the detector resolution. The bin purities, defined as the fraction of all reconstructed events that are generated in the same bin, are 65%–85% for \( \Delta \phi_{\text{max}} = 7\pi/8 \) and \( 5\pi/6 \), and 50%–75% for \( \Delta \phi_{\text{max}} = 3\pi/4 \) and \( 2\pi/3 \). The bin efficiencies, defined as the fraction of all generated events that are reconstructed in the same bin, have values in the same ranges as the bin purities. The corrections are obtained bin by bin from the generated \texttt{Pythia} events as the ratio of the \( R_{\Delta \phi} \) results for the particle-level jets and the detector-level jets. These corrections are typically between 0% and 3%, and never outside the range from −10% to +10%. Uncertainties in these corrections due to the modeling of the migrations by the simulation are estimated from the changes of the correction factors when varying the reweighting function. In most parts of the phase space, these uncertainties are below 1%. The results from the bin-by-bin correction procedure were compared to the results when using a Bayesian iterative unfolding procedure [63], and the two results agree within their statistical uncertainties.

The uncertainties of the \( R_{\Delta \phi} \) measurements include two sources of statistical uncertainty and 62 sources of systematic uncertainty. The statistical uncertainties arise from the data and from the correction factors. The systematic uncertainties are from the correction factors (two independent sources, related to variations of the reweighting of the generated events), the jet energy calibration (57 independent sources), the jet energy resolution, and the \( \eta \) and \( \phi \) resolutions. To avoid double counting of statistical fluctuations, the \( H_T \) dependence of the uncertainty distributions is smoothed by fitting either linear or quadratic functions in \( \log(H_T/\text{GeV}) \). From all 62 sources of experimental correlated uncertainties, the dominant systematic uncertainties are due to the jet energy calibration. For \( \Delta \phi_{\text{max}} = 7\pi/8 \) and \( 5\pi/6 \) the jet energy calibration uncertainties are typically between 1.0% and 1.5% and always less than 3.1%.
smaller values of \( \Delta \phi_{\text{max}} \) they can be as large as 4% (for \( \Delta \phi_{\text{max}} = 3\pi/4 \)) or 9% (for \( \Delta \phi_{\text{max}} = 2\pi/3 \)).

VI. MEASUREMENT RESULTS

The measurement results for \( R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}}) \) are corrected to the particle level and presented as a function of \( H_T \), in different regions of \( y^* \) and for different \( \Delta \phi_{\text{max}} \) requirements. The results are listed in Appendix B in Tables VI–IX, and displayed in Fig. 1, at the arithmetic center of the \( H_T \) bins. At fixed \((y^*, \Delta \phi_{\text{max}})\), \( R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}}) \) decreases with increasing \( H_T \) and increases with increasing \( y^* \) at fixed \((H_T, \Delta \phi_{\text{max}})\). At fixed \((H_T, y^*)\), \( R_{\Delta \phi} \) decreases with decreasing \( \Delta \phi_{\text{max}} \).

Theoretical predictions based on NLO pQCD (for \( \Delta \phi_{\text{max}} = 7\pi/8, 5\pi/6, \) and \( 3\pi/4 \)) or LO (for \( \Delta \phi_{\text{max}} = 2\pi/3 \)) with corrections for nonperturbative effects, as described in Sec. III, are compared to the data. The ratios of data to the theoretical predictions are displayed in Fig. 2. To provide further information about the convergence of the pQCD calculation, the inverse of the NLO requirements. The results are listed in Appendix B in Tables VI–IX, and displayed in Fig. 1, at the arithmetic center of the \( H_T \) bins. At fixed \((y^*, \Delta \phi_{\text{max}})\), \( R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}}) \) decreases with increasing \( H_T \) and increases with increasing \( y^* \) at fixed \((H_T, \Delta \phi_{\text{max}})\). At fixed \((H_T, y^*)\), \( R_{\Delta \phi} \) decreases with decreasing \( \Delta \phi_{\text{max}} \).

Theoretical predictions based on NLO pQCD (for \( \Delta \phi_{\text{max}} = 7\pi/8, 5\pi/6, \) and \( 3\pi/4 \)) or LO (for \( \Delta \phi_{\text{max}} = 2\pi/3 \)) with corrections for nonperturbative effects, as described in Sec. III, are compared to the data. The ratios of data to the theoretical predictions are displayed in Fig. 2. To provide further information about the convergence of the pQCD calculation, the inverse of the NLO requirements. The results are listed in Appendix B in Tables VI–IX, and displayed in Fig. 1, at the arithmetic center of the \( H_T \) bins. At fixed \((y^*, \Delta \phi_{\text{max}})\), \( R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}}) \) decreases with increasing \( H_T \) and increases with increasing \( y^* \) at fixed \((H_T, \Delta \phi_{\text{max}})\). At fixed \((H_T, y^*)\), \( R_{\Delta \phi} \) decreases with decreasing \( \Delta \phi_{\text{max}} \).

The theoretical predictions, based on pQCD at NLO (for \( \Delta \phi_{\text{max}} = 7\pi/8, 5\pi/6, \) and \( 3\pi/4 \)) and LO (for \( \Delta \phi_{\text{max}} = 2\pi/3 \)) are shown as solid and dashed lines, respectively. The shaded bands display the PDF uncertainties and the scale dependence, added in quadrature.

VII. SELECTION OF DATA POINTS FOR THE \( \alpha_s \) EXTRACTION

The extraction of \( \alpha_s(Q) \) at different scales \( Q = H_T/2 \) is based on a combination of data points in different kinematic regions of \( y^* \) and \( \Delta \phi_{\text{max}} \), with the same \( H_T \). The data points are chosen according to the following criteria.

1. Data points are used only from kinematic regions in which the pQCD predictions appear to be most reliable, as judged by the renormalization and factorization scale dependence, and by the NLO \( K \)-factors.
2. For simplicity, data points are only combined in the \( \alpha_s \) extraction if they are statistically independent, i.e., if their accessible phase space does not overlap.
3. The preferred data points are those for which the cancellation of the PDFs between the numerator and the denominator in \( R_{\Delta \phi} \) is largest.
4. The experimental uncertainty at large \( H_T \) is limited by the sample size. If the above criteria give equal preference to two or more data sets with overlapping phase space, the data points with smaller statistical uncertainties are used to test the RGE at the largest possible momentum transfers with the highest precision.

Based on criterion (1), the data points obtained for \( \Delta \phi_{\text{max}} = 2\pi/3 \) are excluded, as the pQCD predictions in NLO are only available at LO. Furthermore, it is

![FIG. 1. The measurement of \( R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}}) \) as a function of \( H_T \) in three regions of \( y^* \) and for four choices of \( \Delta \phi_{\text{max}} \). The inner error bars indicate the statistical uncertainties, and the sum in quadrature of statistical and systematic uncertainties is displayed by the total error bars. The theoretical predictions, based on pQCD at NLO (for \( \Delta \phi_{\text{max}} = 7\pi/8, 5\pi/6, \) and \( 3\pi/4 \)) and LO (for \( \Delta \phi_{\text{max}} = 2\pi/3 \)) are shown as solid and dashed lines, respectively. The shaded bands display the PDF uncertainties and the scale dependence, added in quadrature.](092004-6)
observed that the points for $\Delta\phi_{\text{max}} = 3\pi/4$ have a large scale dependence, which is typically between $+15\%$ and $-10\%$. For the remaining data points with $\Delta\phi_{\text{max}} = 7\pi/8$ and $5\pi/6$ at larger $y^{*}$ ($1 < y^{*} < 2$), the NLO corrections are negative and (with a size of $5\%-23\%$) larger than those at smaller $y^{*}$, indicating potentially larger corrections from not yet calculated higher orders. The conclusion from criterion (1) is therefore that the pQCD predictions are most reliable in the four kinematic regions $0 < y^{*} < 0.5$ and $0.5 < y^{*} < 1$, for $\Delta\phi_{\text{max}} = 7\pi/8$ and $\Delta\phi_{\text{max}} = 5\pi/6$, where the NLO $K$-factors are typically within $\pm5\%$ of unity.

The requirement of statistically independent data points according to criterion (2) means that the data points from different $y^{*}$ regions can be combined, but not those with different $\Delta\phi_{\text{max}}$. The choice whether to use the data with $\Delta\phi_{\text{max}} = 7\pi/8$ or $5\pi/6$ (in either case combining the data for $0 < y^{*} < 0.5$ and $0.5 < y^{*} < 1$) is therefore based on criteria (3) and (4).

The cancellation of the PDFs, as addressed in criterion (3), is largest for those data points for which the phase space of the numerator in Eq. (1) is closest to that of the denominator. Since the numerator of $R_{\Delta\phi}$ is a subset of the denominator, this applies more to the data at larger values of $\Delta\phi_{\text{max}}$. For those points, the fractional contributions from different partonic subprocesses ($gg \rightarrow$ jets, $gq \rightarrow$ jets, $qq \rightarrow$ jets), and the ranges in the accessible proton momentum fraction $x$ are more similar for the numerator and denominator, resulting in a larger cancellation of PDFs in $R_{\Delta\phi}$. This argument, based on the third criterion, leads to the same conclusion as the suggestion of criterion (4), to use the data set with smallest statistical uncertainty.

Based on the four criteria, $\alpha_{S}$ is therefore extracted combining the data points in the rapidity regions $0 < y^{*} < 0.5$ and $0.5 < y^{*} < 1$ for $\Delta\phi_{\text{max}} = 7\pi/8$. Extractions of $\alpha_{S}$ from the data points in other kinematical regions in $y^{*}$ and $\Delta\phi_{\text{max}}$ are used to investigate the dependence of the final results on those choices.

VIII. DETERMINATION OF $\alpha_{S}$

The $R_{\Delta\phi}$ measurements in the selected kinematic regions are used to determine $\alpha_{S}$ and to test the QCD predictions for its running as a function of the scale $Q = H_{T}/2$. The $\alpha_{S}$ results are extracted by using MINUIT [64], to minimize the $\chi^{2}$ function specified in Appendix C. In this approach, the experimental and theoretical uncertainties that are correlated between all data points are treated in the Hessian method [65] by including a nuisance parameter for each uncertainty source, as described in Appendix C. The only exceptions are the uncertainties due to the PDF set and the $\mu_{h,s}$ dependence of the pQCD calculation. These uncertainties are determined from the variations of the $\alpha_{S}$ results,
when repeating the $\alpha_S$ extractions for different PDF sets and for variations of the scales $\mu_R, \mu_F$ as described in Sec. III.

Results of $\alpha_S(Q)$ (with $Q = H_T/2$, taken at the arithmetic centers of the $H_T$ bins) are determined from the $R_{\Delta \phi}$ data for $\Delta \phi_{\max} = 7\pi/8$, combining the data points in the two $y^*$ regions of $0 < y^* < 0.5$ and $0.5 < y^* < 1.0$. Nine $\alpha_S(Q)$ values are determined in the range $262 < Q < 1675$ GeV. A single $\chi^2$ minimization provides the uncertainties due to the statistical uncertainties, the experimental correlated uncertainties, the uncertainties due to the nonperturbative corrections, and the MMHT2014 PDF uncertainty. Separate $\chi^2$ minimizations are made for variations of $\mu_R$ and $\mu_F$ (in the ranges described in Sec. III), and also for the CT14, NNPDFv2.3, ABMP16, and HERAPDF 2.0 PDF sets. The largest individual variations are used to quantify the uncertainty due to the scale dependence and the PDF set, respectively. The so-defined PDF set uncertainty may partially double count some of the uncertainties already taken into account by the MMHT2014 PDF uncertainties, but it may also include some additional systematic uncertainties due to different approaches in the PDF determinations. The $\alpha_S(Q)$ results are displayed in Fig. 3 and listed in Table IV.

In addition, assuming the validity of the RGE, all 18 data points in $0 < y^* < 0.5$ and $0.5 < y^* < 1.0$ for $\Delta \phi_{\max} = 7\pi/8$ are used to extract a combined $\alpha_S(m_Z)$ result. The combined fit (for MMHT2014 PDFs at the default scale) gives $\chi^2 = 21.7$ for 17 degrees of freedom and a result of $\alpha_S(m_Z) = 0.1127$ (the uncertainties are detailed in Table V). The fit is then repeated for the CT14, NNPDFv2.3, ABMP16, and HERAPDF 2.0 PDF sets, for which the $\alpha_S(m_Z)$ results differ by $-0.0001$, $+0.0022$, $+0.0026$, and $+0.0029$, respectively. Fits for various choices of $\mu_R$ and $\mu_F$ result in variations of the $\alpha_S(m_Z)$ results between $-0.0019$ and $+0.0052$.

Further dependence of the $\alpha_S$ results on some of the analysis choices is investigated in a series of systematic studies.

(i) Changing the $\Delta \phi_{\max}$ requirement.—Based on the criteria outlined in Sec. VII it was decided to use the data for $\Delta \phi_{\max} = 7\pi/8$ in the $\alpha_S$ analysis. If, instead, the data with $\Delta \phi_{\max} = 5\pi/6$ are used, the $\alpha_S(m_Z)$ result changes by $+0.0052$ to $\alpha_S(m_Z) = 0.1179$, with an uncertainty of $+0.0065$ and $-0.0045$ due to the scale dependence.

(ii) Extending the $y^*$ region.—For the central $\alpha_S$ results, the data points with $1 < y^* < 2$ are excluded. If $\alpha_S(m_Z)$ is determined only from the data points for $1 < y^* < 2$ (with $\Delta \phi_{\max} = 7\pi/8$) the $\alpha_S(m_Z)$ result changes by $-0.0018$, with an increased scale dependence, to $\alpha_S(m_Z) = 0.1109^{+0.0071}_{-0.0031}$ with $\chi^2 = 13.8$ for 7 degrees of freedom. If the data points for $1 < y^* < 2$ are combined with those

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$Q$ [GeV] & $\alpha_S(Q)$ & Total uncertainty & Statistical & Experimental correlated & Nonperturbative corrections & MMHT2014 uncertainty set & $\mu_{\alpha_S}$ variation \\
\hline
262.5 & 0.1029 & 6.0 & 2.8 & +1.6 & +0.4 & +0.4 & +5.3 \\
337.5 & 0.0970 & 8.0 & 2.6 & +1.5 & +0.4 & +0.3 & +7.0 \\
412.5 & 0.0936 & 4.0 & 2.3 & +1.3 & +0.3 & +2.6 & +2.5 \\
500.0 & 0.0901 & 1.5 & 2.7 & +1.2 & +0.3 & +1.3 & +1.0 \\
625.0 & 0.0890 & 2.4 & 2.6 & +1.1 & +0.3 & +0.3 & +1.3 \\
800.0 & 0.0850 & 5.9 & 2.4 & +1.0 & +0.4 & +0.4 & +3.5 \\
1000 & 0.0856 & 4.0 & 2.7 & +1.1 & +0.4 & +0.4 & +3.4 \\
1225 & 0.0790 & 6.0 & 3.4 & +1.3 & < ± 0.1 & +0.5 & +1.7 \\
1675 & 0.0723 & 8.6 & 6.1 & ± 0.1 & ± 0.5 & ± 1.7 & ± 2.8 \\
\hline
\end{tabular}
\caption{The results for $\alpha_S(Q)$ determined from the $R_{\Delta \phi}$ data for $\Delta \phi_{\max} = 7\pi/8$ with $0 < y^* < 0.5$ and $0.5 < y^* < 1.0$. All uncertainties have been multiplied by a factor of 10.}
\end{table}
TABLE V. Fit result for $\alpha_S(m_Z)$, determined from the $R_{\Delta \phi}$ data for $\Delta \phi_{\text{max}} = 7\pi/8$ with $0 < y^* < 0.5$ and $0.5 < y^* < 1.0$. All uncertainties have been multiplied by a factor of $10^3$.

| $\alpha_S(m_Z)$ | Total uncertainty | Statistical | Experimental | Nonperturbative corrections | MMHT2014 uncertainty | PDF set | $\mu_{R,F}$ variation |
|-----------------|-------------------|-------------|--------------|-----------------------------|----------------------|--------|----------------------|
| 0.1127         | $+6.3$            | $\pm 0.5$   | $+1.8$       | $+0.3$                      | $+0.6$             | $+2.9$ | $+5.2$              |

for $0 < y^* < 0.5$ and $0.5 < y^* < 1$, the result is $\alpha_S(m_Z) = 0.1135^{+0.0051}_{-0.0025}$.

(iii) **Smoothing the systematic uncertainties.**—In the experimental measurement, the systematic uncertainties that are correlated between different data points were smoothed in order to avoid double counting of statistical fluctuations. For this purpose, the systematic uncertainties were fitted with a linear function in $\log(H_T/$GeV). If, alternatively, a quadratic function is used, the central $\alpha_S(m_Z)$ result changes by $-0.0006$, and the experimental uncertainty is changed from $+0.0018$ to $+0.0017$ to $-0.0016$.

(iv) **Stronger correlations of experimental uncertainties.**—The largest experimental uncertainties are due to the jet energy calibration. These are represented by contributions from 57 independent sources. Some of the correlations are estimated on the basis of prior assumptions. In a study of the systematic effects these assumptions are varied, resulting in an alternative scenario with stronger correlations between some of these sources. This changes the combined $\alpha_S(m_Z)$ result by $-0.0004$, while the experimental correlated uncertainty is reduced from $+0.0016$ to $+0.0012$.

(v) **Treatment of nonperturbative corrections.**—The central $\alpha_S$ results are obtained using the average values of the nonperturbative corrections from PYTHIA tunes ABT1 and DW, and the spread between the average and the individual models is taken as a correlated uncertainty, which is treated in the Hessian approach by fitting a corresponding nuisance parameter. Alternatively, the $\alpha_S(m_Z)$ result is also extracted by fixing the values for the nonperturbative corrections to the individual model predictions from HERWIG (default) and PYTHIA with tunes ABT1, DW, S Global, and A, and to unity (corresponding to zero nonperturbative corrections). The corresponding changes of the $\alpha_S(m_Z)$ result for the different choices are between $-0.0004$ and $+0.0011$.

(vi) **Choice of $n_\ell = 6$ versus $n_\ell = 5$.**—The choice of $n_\ell = 6$ corresponds to the rather extreme approximation in which the top quark is included as a massless quark in the pQCD calculation. The effect of using $n_\ell = 6$ instead of $n_\ell = 5$ in the pQCD matrix elements and the RGE and the corresponding impact on $R_{\Delta \phi}$ are discussed in Appendix A. The effects on the extracted $\alpha_S$ results are also studied and are found to be between $+1.3\%$ (at low $H_T$) and $-1.1\%$ (at high $H_T$) for the nine $\alpha_S(Q)$ results. The combined $\alpha_S(m_Z)$ result changes by $-0.0006$ from 0.1127 (for $n_\ell = 5$) to 0.1121 (for $n_\ell = 6$).

(vii) **A scan of the renormalization scale dependence.**—Unlike all other uncertainties which are treated in the Hessian approach, the uncertainty due to the renormalization and factorization scale dependence is obtained from individual fits in which both scales are set to fixed values. To ensure that the largest variation may not occur at intermediate values, a scan of the renormalization scale dependence in finer steps is made. For each of the three variations of $\mu_R$ by factors of $x_{\mu_R} = 0.5, 1.0, 2.0$, the renormalization scale is varied by nine logarithmically equal-spaced factors of $x_{\mu_R} = 0.5, 0.596, 0.708, 0.841, 1.0, 1.189, 1.413, 1.679, 2.0$.

It is seen that the largest upward variation of $+0.0052$ is obtained for the correlated variation $x_{\mu_R} = x_{\mu_F} = 2.0$. The lowest variation of $-0.0027$ is obtained for the anti-correlated variation $x_{\mu_R} = 0.5$ and $x_{\mu_F} = 2.0$, which is, however, outside the range $0.5 < x_{\mu_R}/x_{\mu_F} < 2$. The lowest variation within this range ($-0.0014$) is obtained for $x_{\mu_R} = 0.5$ and $x_{\mu_F} = 1.0$.

(viii) **Effects of the Hessian method.**—In the Hessian approach, a fit can explore the multidimensional uncertainty space to find the $\chi^2$ minimum at values of the nuisance parameters associated to the sources of systematic uncertainties, that do not represent the best knowledge of the corresponding sources. While in this analysis the shifts of the nuisance parameters are all small, it is still interesting to study their effects on the $\alpha_S$ fit results. Therefore, the $\alpha_S(m_Z)$ extraction is repeated, initially including the uncorrelated (i.e., statistical) uncertainties only. Then, step by step, the experimental correlated uncertainties, the uncertainties of the nonperturbative corrections, and the PDF uncertainties are included. These fits produce $\alpha_S(m_Z)$ results that differ by less than $+0.0004$ from the central result.

These systematic studies show that the $\alpha_S$ results are rather independent of the analysis choices and demonstrate the stability of the $\alpha_S$ extraction procedure. These variations are not treated as additional uncertainties because their resulting effects are smaller than the other theoretical uncertainties. The largest variation of the $\alpha_S(m_Z)$ result,
by +0.0052, is obtained when using the data with $\Delta \phi_{\text{max}} = 5\pi/6$ instead of $\Delta \phi_{\text{max}} = 7\pi/8$. This difference may be due to different higher-order corrections to the NLO pQCD results for different $\Delta \phi_{\text{max}}$ values. This assumption is consistent with the observed scale dependence of the $\alpha_S(m_Z)$ results, within which the results for both choices of $\Delta \phi_{\text{max}}$ agree (0.1127 ± 0.0052 versus 0.1179 − 0.0045 for $\Delta \phi_{\text{max}} = 5\pi/6$ and 7\pi/8, respectively). It is therefore concluded from the systematic studies that no further uncertainties need to be assigned.

The final result from the combined fit is $\alpha_S(m_Z) = 0.1127^{+0.0063}_{-0.0027}$ with the individual uncertainty contributions given in Table V. This result and the corresponding RGE prediction are also shown in Fig. 3. For all $\alpha_S$ results in Tables IV and V, the uncertainties are dominated by the $\mu_R$ dependence of the NLO pQCD calculation.

Within the uncertainties, the $\alpha_S(m_Z)$ result is consistent with the current world average value of $\alpha_S(m_Z) = 0.1181$ ± 0.0011 [16] and with recent $\alpha_S$ results from multijet cross-section ratio measurements in hadron collisions, namely from the D0 measurement of $R_{\Delta \phi}$ [10] ($\alpha_S(m_Z) = 0.1191^{−0.0071}_{+0.0048}$), and from the CMS measurements of $R_{3/2}$ [11] ($\alpha_S(m_Z) = 0.1148$ ± 0.0055), the inclusive jet cross section [12,13] ($\alpha_S(m_Z) = 0.1185^{+0.0063}_{−0.0042}$, $\alpha_S(m_Z) = 0.1164^{+0.0060}_{−0.0043}$), and the three-jet cross section [14] ($\alpha_S(m_Z) = 0.1171^{+0.0074}_{−0.0049}$), and the ATLAS measurement of transverse energy–energy correlations [15] ($\alpha_S(m_Z) = 0.1165^{+0.0088}_{−0.0071}$, with comparable uncertainties. The compatibility of the results of this analysis, based on the measurements of $R_{\Delta \phi}$, with the world average value of $\alpha_S(m_Z)$ is demonstrated in Appendix D.

The individual $\alpha_S(Q)$ results are compared in Fig. 4 with previously published $\alpha_S$ results obtained from jet measurements [4–7,9–15] and with the RGE prediction for the combined $\alpha_S(m_Z)$ result obtained in this analysis. The new results agree with previous $\alpha_S(Q)$ results in the region of overlap, and extend the pQCD tests to momentum transfers up to 1.6 TeV, where RGE predictions are consistent with the $\alpha_S(Q)$ results, as discussed in Appendix E.

**IX. SUMMARY**

The multijet cross-section ratio $R_{\Delta \phi}$ is measured at the LHC. The quantity $R_{\Delta \phi}$ specifies the fraction of the inclusive dijet events in which the azimuthal opening angle of the two jets with the highest transverse momenta is less than a given value of the parameter $\Delta \phi_{\text{max}}$. The $R_{\Delta \phi}$ results, measured in 20.2 fb$^{-1}$ of $pp$ collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector, are presented as a function of three variables: the total transverse momentum $H_T$, the dijet rapidity interval $y'$, and the parameter $\Delta \phi_{\text{max}}$. The $H_T$ and $y'$ dependences of the data are well described by theoretical predictions based on NLO pQCD (for $\Delta \phi_{\text{max}} = 7\pi/8$, 5\pi/6, and 3\pi/4), or LO pQCD (for $\Delta \phi_{\text{max}} = 2\pi/3$), with corrections for non-perturbative effects. Based on the data points for $\Delta \phi_{\text{max}} = 7\pi/8$ with $0 < y' < 0.5$ and $0.5 < y' < 1$, nine $\alpha_S$ results are determined, at a scale of $Q = H_T/2$, over the range of $262 < Q < 1675$ GeV. The $\alpha_S(Q)$ results are consistent with the predictions of the RGE, and a combined analysis results in a value of $\alpha_S(m_Z) = 0.1127^{+0.0063}_{−0.0027}$, where the uncertainty is dominated by the scale dependence of the NLO pQCD predictions.

**ACKNOWLEDGMENTS**

We thank CERN for the very successful operation of the LHC, as well as the support staff from our institutions without whom ATLAS could not be operated efficiently. We acknowledge the support of ANPCyT, Argentina; YerPhI, Armenia; ARC, Australia; BMWFFW and FWF, Austria; ANAS, Azerbaijan; SSTC, Belarus; CNPq and FAPESP, Brazil; NSERC, NRC and CFI, Canada; CERN; CONICYT, Chile; CAS, MOST and NSFC, China; COLCIENCIAS, Colombia; MSMT CR, MPO CR and VSC CR, Czech Republic; DNRF and DNSRC, Denmark; IN2P3-CNRS, CEA-DRF/IRFU, France; SRNSFG, Georgia; BMBF, HGF, and MPG, Germany; GSRT, Greece; RGC, Hong Kong SAR, China; ISF, I-CORE and Benoziyo Center, Israel; INFN, Italy; MEXT and JSPS, Japan; CNRSRT, Morocco; NWO, Netherlands; RCN, Norway; MINSW and NCN, Poland; FCT, Portugal; MNE/IFA, Romania; MES of Russia and NRC KI, Russian Federation; JINR; MESTD, Serbia; MSSR, Slovakia; ARRS and MIZŠ, Slovenia; DST/NRF, South Africa; MINECO, Spain; SRC and Wallenberg Foundation, Sweden; SERI, SNSF and Cantons of Bern and Geneva, Switzerland; MOST, Taiwan; TAEK, Turkey; STFC,
APPENDIX A: EFFECTS OF TOP QUARK CONTRIBUTIONS ON THE pQCD PREDICTIONS

There are two ways in which contributions from top quarks affect the pQCD predictions for $R_{\Delta \phi}$. First, the pQCD predictions based on matrix elements for massless quarks also depend on the number of quark flavors in gluon splitting ($g \rightarrow q\bar{q}$), $n_f$, which affects the tree-level matrix elements and their real and virtual corrections, as well as the RGE predictions. The pQCD predictions for the central analysis are obtained for $n_f = 5$. The effects for the measured quantity $R_{\Delta \phi}$ for the choice $n_f = 6$ are computed in this Appendix. Second, since the decay products of hadronically decaying (anti)top quarks are sometimes reconstructed as multiple jets, the $O(\alpha_s^3)$ $t\bar{t}$ production process also contributes to three-jet topologies. Since this contribution is of lower order in $\alpha_s$ as compared to the pQCD $O(\alpha_s^2)$ three-jet production processes, it is a “super-leading” contribution, which is formally more important. This potentially large contribution and the corresponding effects for $R_{\Delta \phi}$ are also estimated in this Appendix.

In a pQCD calculation in which quark masses are properly taken into account, the contributions from the massive top quark arise naturally at higher momentum transfers, according to the available phase space. In calculations based on matrix elements for massless quarks, $n_f$ is a parameter in the calculation. For jet production at the LHC, the alternatives are $n_f = 5$, i.e., ignoring the contributions from $g \rightarrow t\bar{t}$ processes (which is the central choice for this analysis), or $n_f = 6$, i.e., treating the top quark as a sixth massless quark. The relative difference between the two alternatives is evaluated from the effects due to the RGE and the matrix elements. For this purpose, the 2-loop solution of the RGE for $n_f = 5$ is replaced by the 2-loop solutions for $n_f = 5$ and $n_f = 6$ with 1-loop matching [67] at the pole mass of the top quark $m_{\text{top}}$, assuming that $m_{\text{top}}$ is equal to the world average of the measured “Monte Carlo” mass of 173.21 GeV [16]. In addition, the matrix elements are recomputed for $n_f = 6$. For a fixed value of $\alpha_s(m_Z) = 0.118$, the corresponding effects for the pQCD predictions for $R_{\Delta \phi}$ are in the range of $-1\%$ to $+2\%$.

The effects on $R_{\Delta \phi}$ due to the contributions from hadronically decays of $t\bar{t}$ final states are estimated using POWHEG-BOX [68] (for the pQCD matrix elements) interfaced with PYTHIA (for the parton shower, underlying event, and hadronization) and CTEQ6L1 PDFs [69]. It is seen that the $t\bar{t}$ process contributes 0.003–0.2% to the denominator of $R_{\Delta \phi}$ (the inclusive dijet cross section), and 0.006%–0.5% to the numerator (with $\Delta \phi_{\text{max}} = 7\pi/8$). The effects for the ratio $R_{\Delta \phi}$ are 0%–0.5% in the analysis phase space, and there are no systematic trends in the considered distributions within the statistical uncertainties of the generated POWHEG-BOX event sample. Since this effect is about four to eight times smaller than the typical uncertainty due to the renormalization scale dependence, the corresponding effects on $\alpha_s$ are not investigated further.

APPENDIX B: DATA TABLES

The results of the $R_{\Delta \phi}$ measurements are listed in Tables VI–IX, together with their relative statistical and systematic uncertainties.
TABLE VI. The $R_{\Delta\phi}$ measurement results for $\Delta\phi_{\text{max}} = 7\pi/8$ with their relative statistical and systematic uncertainties.

| $H_T$ [GeV] | $y^*$ | $R_{\Delta\phi}$ | Statistical uncertainties [%] | Systematic uncertainties [%] |
|-------------|-------|------------------|-------------------------------|-----------------------------|
| 450–600     | 0.0–0.5 | $1.88 \times 10^{-1}$ | $\pm 2.2$                      | $+1.8$ $-1.7$               |
| 600–750     | 0.0–0.5 | $1.85 \times 10^{-1}$ | $\pm 2.2$                      | $+1.6$ $-1.5$               |
| 750–900     | 0.0–0.5 | $1.82 \times 10^{-1}$ | $\pm 1.3$                      | $+1.4$ $-1.4$               |
| 900–1100    | 0.0–0.5 | $1.67 \times 10^{-1}$ | $\pm 0.9$                      | $+1.3$ $-1.3$               |
| 1100–1400   | 0.0–0.5 | $1.56 \times 10^{-1}$ | $\pm 0.7$                      | $+1.2$ $-1.2$               |
| 1400–1800   | 0.0–0.5 | $1.36 \times 10^{-1}$ | $\pm 1.0$                      | $+1.2$ $-1.2$               |
| 1800–2200   | 0.0–0.5 | $1.25 \times 10^{-1}$ | $\pm 1.9$                      | $+1.2$ $-1.3$               |
| 2200–2700   | 0.0–0.5 | $1.02 \times 10^{-1}$ | $\pm 4.1$                      | $+1.3$ $-1.4$               |
| 2700–4000   | 0.0–0.5 | $0.82 \times 10^{-1}$ | $\pm 9.9$                      | $+1.5$ $-1.7$               |
| 450–600     | 0.5–1.0 | $1.97 \times 10^{-1}$ | $\pm 2.2$                      | $+1.5$ $-1.6$               |
| 600–750     | 0.5–1.0 | $2.04 \times 10^{-1}$ | $\pm 2.3$                      | $+1.3$ $-1.4$               |
| 750–900     | 0.5–1.0 | $1.94 \times 10^{-1}$ | $\pm 1.3$                      | $+1.2$ $-1.3$               |
| 900–1100    | 0.5–1.0 | $1.83 \times 10^{-1}$ | $\pm 0.8$                      | $+1.2$ $-1.2$               |
| 1100–1400   | 0.5–1.0 | $1.73 \times 10^{-1}$ | $\pm 0.8$                      | $+1.3$ $-1.2$               |
| 1400–1800   | 0.5–1.0 | $1.59 \times 10^{-1}$ | $\pm 1.1$                      | $+1.4$ $-1.3$               |
| 1800–2200   | 0.5–1.0 | $1.44 \times 10^{-1}$ | $\pm 2.3$                      | $+1.7$ $-1.5$               |
| 2200–2700   | 0.5–1.0 | $1.28 \times 10^{-1}$ | $\pm 5.4$                      | $+1.9$ $-1.7$               |
| 2700–4000   | 0.5–1.0 | $1.13 \times 10^{-1}$ | $\pm 1.6$                      | $+2.4$ $-2.0$               |
| 450–600     | 1.0–2.0 | $2.42 \times 10^{-1}$ | $\pm 2.3$                      | $+2.3$ $-1.0$               |
| 600–750     | 1.0–2.0 | $2.40 \times 10^{-1}$ | $\pm 2.5$                      | $+1.9$ $-1.1$               |
| 750–900     | 1.0–2.0 | $2.54 \times 10^{-1}$ | $\pm 1.5$                      | $+1.7$ $-1.2$               |
| 900–1100    | 1.0–2.0 | $2.40 \times 10^{-1}$ | $\pm 1.1$                      | $+1.6$ $-1.4$               |
| 1100–1400   | 1.0–2.0 | $2.33 \times 10^{-1}$ | $\pm 1.0$                      | $+1.6$ $-1.7$               |
| 1400–1800   | 1.0–2.0 | $2.18 \times 10^{-1}$ | $\pm 1.8$                      | $+1.6$ $-2.2$               |
| 1800–2200   | 1.0–2.0 | $2.22 \times 10^{-1}$ | $\pm 4.4$                      | $+1.6$ $-2.7$               |
| 2200–2700   | 1.0–2.0 | $1.96 \times 10^{-1}$ | $\pm 14$                       | $+1.7$ $-3.1$               |

TABLE VII. The $R_{\Delta\phi}$ measurement results for $\Delta\phi_{\text{max}} = 5\pi/6$ with their relative statistical and systematic uncertainties.

| $H_T$ [GeV] | $y^*$ | $R_{\Delta\phi}$ | Statistical uncertainties [%] | Systematic uncertainties [%] |
|-------------|-------|------------------|-------------------------------|-----------------------------|
| 450–600     | 0.0–0.5 | $1.22 \times 10^{-1}$ | $\pm 2.8$                      | $+2.0$ $-1.9$               |
| 600–750     | 0.0–0.5 | $1.13 \times 10^{-1}$ | $\pm 2.9$                      | $+1.7$ $-1.7$               |
| 750–900     | 0.0–0.5 | $1.10 \times 10^{-1}$ | $\pm 1.7$                      | $+1.5$ $-1.6$               |
| 900–1100    | 0.0–0.5 | $1.00 \times 10^{-1}$ | $\pm 1.3$                      | $+1.4$ $-1.5$               |
| 1100–1400   | 0.0–0.5 | $0.92 \times 10^{-1}$ | $\pm 1.0$                      | $+1.2$ $-1.5$               |
| 1400–1800   | 0.0–0.5 | $0.78 \times 10^{-1}$ | $\pm 1.4$                      | $+1.2$ $-1.5$               |
| 1800–2200   | 0.0–0.5 | $0.72 \times 10^{-1}$ | $\pm 2.6$                      | $+1.2$ $-1.7$               |
| 2200–2700   | 0.0–0.5 | $0.55 \times 10^{-1}$ | $\pm 5.7$                      | $+1.3$ $-1.9$               |
| 2700–4000   | 0.0–0.5 | $0.51 \times 10^{-1}$ | $\pm 13$                       | $+1.6$ $-2.3$               |
| 450–600     | 0.5–1.0 | $1.33 \times 10^{-1}$ | $\pm 2.9$                      | $+1.5$ $-1.8$               |
| 600–750     | 0.5–1.0 | $1.27 \times 10^{-1}$ | $\pm 3.1$                      | $+1.4$ $-1.5$               |
| 750–900     | 0.5–1.0 | $1.18 \times 10^{-1}$ | $\pm 1.8$                      | $+1.3$ $-1.3$               |
| 900–1100    | 0.5–1.0 | $1.11 \times 10^{-1}$ | $\pm 1.2$                      | $+1.3$ $-1.2$               |
| 1100–1400   | 0.5–1.0 | $1.03 \times 10^{-1}$ | $\pm 1.2$                      | $+1.4$ $-1.2$               |
| 1400–1800   | 0.5–1.0 | $0.93 \times 10^{-1}$ | $\pm 1.5$                      | $+1.6$ $-1.3$               |
| 1800–2200   | 0.5–1.0 | $0.85 \times 10^{-1}$ | $\pm 3.2$                      | $+1.9$ $-1.4$               |
| 2200–2700   | 0.5–1.0 | $0.74 \times 10^{-1}$ | $\pm 7.3$                      | $+2.2$ $-1.6$               |

(Table continued)
TABLE VII. (Continued)

| $H_T$ [GeV] | $y^*$ | $R_{\Delta \phi}$ | Statistical uncertainties [%] | Systematic uncertainties [%] |
|------------|------|-------------------|-------------------------------|-------------------------------|
| 450–600    | 1.0–2.0 | $1.58 \times 10^{-1}$ | ±2.9 | +3.1 | −1.0 |
| 600–750    | 1.0–2.0 | $1.54 \times 10^{-1}$ | ±3.3 | +2.5 | −0.9 |
| 750–900    | 1.0–2.0 | $1.62 \times 10^{-1}$ | ±2.3 | +2.1 | −1.1 |
| 900–1100   | 1.0–2.0 | $1.53 \times 10^{-1}$ | ±1.6 | +1.9 | −1.5 |
| 1100–1400  | 1.0–2.0 | $1.47 \times 10^{-1}$ | ±1.4 | +1.8 | −2.2 |
| 1400–1800  | 1.0–2.0 | $1.36 \times 10^{-1}$ | ±2.6 | +1.8 | −3.1 |
| 1800–2200  | 1.0–2.0 | $1.41 \times 10^{-1}$ | ±5.8 | +1.9 | −3.9 |
| 2200–2700  | 1.0–2.0 | $1.35 \times 10^{-1}$ | ±18  | +2.0 | −4.7 |

TABLE VIII. The $R_{\Delta \phi}$ measurement results for $\Delta \phi_{\text{max}} = 3\pi/4$ with their relative statistical and systematic uncertainties.

| $H_T$ [GeV] | $y^*$ | $R_{\Delta \phi}$ | Statistical uncertainties [%] | Systematic uncertainties [%] |
|------------|------|-------------------|-------------------------------|-------------------------------|
| 450–600    | 0.0–0.5 | $4.35 \times 10^{-2}$ | ±5.0 | +3.4 | −2.4 |
| 600–750    | 0.0–0.5 | $3.67 \times 10^{-2}$ | ±5.9 | +3.0 | −2.1 |
| 750–900    | 0.0–0.5 | $3.55 \times 10^{-2}$ | ±4.6 | +2.6 | −1.9 |
| 900–1100   | 0.0–0.5 | $3.24 \times 10^{-2}$ | ±3.9 | +2.3 | −1.8 |
| 1100–1400  | 0.0–0.5 | $2.84 \times 10^{-2}$ | ±2.5 | +2.0 | −1.8 |
| 1400–1800  | 0.0–0.5 | $2.27 \times 10^{-2}$ | ±3.2 | +1.8 | −2.0 |
| 1800–2200  | 0.0–0.5 | $1.89 \times 10^{-2}$ | ±5.5 | +1.8 | −2.2 |
| 2200–2700  | 0.0–0.5 | $1.43 \times 10^{-2}$ | ±12  | +1.9 | −2.5 |
| 450–600    | 0.5–1.0 | $4.68 \times 10^{-2}$ | ±5.5 | +2.2 | −2.6 |
| 600–750    | 0.5–1.0 | $4.01 \times 10^{-2}$ | ±6.1 | +1.8 | −1.9 |
| 750–900    | 0.5–1.0 | $3.92 \times 10^{-2}$ | ±4.1 | +1.6 | −1.6 |
| 900–1100   | 0.5–1.0 | $3.61 \times 10^{-2}$ | ±2.9 | +1.5 | −1.4 |
| 1100–1400  | 0.5–1.0 | $3.31 \times 10^{-2}$ | ±3.3 | +1.6 | −1.3 |
| 1400–1800  | 0.5–1.0 | $2.90 \times 10^{-2}$ | ±3.4 | +2.1 | −1.3 |
| 1800–2200  | 0.5–1.0 | $2.44 \times 10^{-2}$ | ±6.7 | +2.5 | −1.5 |
| 2200–2700  | 0.5–1.0 | $2.17 \times 10^{-2}$ | ±14  | +3.0 | −1.8 |
| 450–600    | 1.0–2.0 | $6.02 \times 10^{-2}$ | ±5.1 | +5.8 | −2.5 |
| 600–750    | 1.0–2.0 | $5.68 \times 10^{-2}$ | ±5.7 | +4.8 | −2.4 |
| 750–900    | 1.0–2.0 | $5.71 \times 10^{-2}$ | ±4.6 | +4.1 | −2.7 |
| 900–1100   | 1.0–2.0 | $5.19 \times 10^{-2}$ | ±3.4 | +3.7 | −3.2 |
| 1100–1400  | 1.0–2.0 | $4.95 \times 10^{-2}$ | ±2.7 | +3.5 | −4.0 |
| 1400–1800  | 1.0–2.0 | $4.56 \times 10^{-2}$ | ±5.0 | +3.7 | −5.0 |
| 1800–2200  | 1.0–2.0 | $5.25 \times 10^{-2}$ | ±11  | +4.1 | −6.1 |

TABLE IX. The $R_{\Delta \phi}$ measurement results for $\Delta \phi_{\text{max}} = 2\pi/3$ with their relative statistical and systematic uncertainties.

| $H_T$ [GeV] | $y^*$ | $R_{\Delta \phi}$ | Statistical uncertainties [%] | Systematic uncertainties [%] |
|------------|------|-------------------|-------------------------------|-------------------------------|
| 450–600    | 0.0–0.5 | $1.37 \times 10^{-2}$ | ±9.5 | +6.3 | −4.1 |
| 600–750    | 0.0–0.5 | $1.05 \times 10^{-2}$ | ±11 | +5.4 | −3.6 |
| 750–900    | 0.0–0.5 | $1.02 \times 10^{-2}$ | ±12 | +4.7 | −3.3 |
| 900–1100   | 0.0–0.5 | $0.87 \times 10^{-2}$ | ±8.9 | +4.1 | −3.2 |
| 1100–1400  | 0.0–0.5 | $0.70 \times 10^{-2}$ | ±6.0 | +3.5 | −3.2 |
| 1400–1800  | 0.0–0.5 | $0.48 \times 10^{-2}$ | ±7.8 | +3.2 | −3.3 |
| 1800–2200  | 0.0–0.5 | $0.38 \times 10^{-2}$ | ±13 | +3.2 | −3.7 |

(Continued)
### Table IX. (Continued)

| $H_T$ [GeV] | $\gamma$ | $R_{\Delta \phi}$ | Statistical uncertainties [%] | Systematic uncertainties [%] |
|-------------|---------|----------------|-----------------------------|-----------------------------|
| 450–600     | 0.5–1.0 | $1.45 \times 10^{-2}$ | ±11                        | +3.9                        |
| 600–750     | 0.5–1.0 | $1.07 \times 10^{-2}$ | ±12                        | +2.7                        |
| 750–900     | 0.5–1.0 | $1.14 \times 10^{-2}$ | ±11                        | +2.1                        |
| 900–1100    | 0.5–1.0 | $0.86 \times 10^{-2}$ | ±6.8                       | +2.2                        |
| 1100–1400   | 0.5–1.0 | $0.77 \times 10^{-2}$ | ±7.1                       | +2.8                        |
| 1400–1800   | 0.5–1.0 | $0.70 \times 10^{-2}$ | ±8.6                       | +3.8                        |
| 1800–2200   | 0.5–1.0 | $0.63 \times 10^{-2}$ | ±16                        | +4.8                        |
| 450–600     | 1.0–2.0 | $1.49 \times 10^{-2}$ | ±10                        | +9.0                        |
| 600–750     | 1.0–2.0 | $1.70 \times 10^{-2}$ | ±11                        | +7.4                        |
| 750–900     | 1.0–2.0 | $1.53 \times 10^{-2}$ | ±8.9                       | +6.5                        |
| 900–1100    | 1.0–2.0 | $1.29 \times 10^{-2}$ | ±7.5                       | +6.2                        |
| 1100–1400   | 1.0–2.0 | $1.12 \times 10^{-2}$ | ±6.6                       | +6.6                        |
| 1400–1800   | 1.0–2.0 | $1.02 \times 10^{-2}$ | ±12                        | +7.6                        |
| 1800–2200   | 1.0–2.0 | $1.61 \times 10^{-2}$ | ±20                        | +8.8                        |

### Appendix C: Definition of $\chi^2$

Given is a set of experimental measurement results in bins $i$ of a given quantity with central measurement results $d_i$ with statistical and uncorrelated systematic uncertainties $\sigma_i$, respectively. The experimental measurements are affected by various sources of correlated uncertainties, and $\delta_i(e_j)$, $\epsilon_j$, $\epsilon_j = 0$ corresponds to the central value of the measurement (i.e., $\delta_i(e_j) = 0$), while the relative uncertainties corresponding to plus/minus one standard deviation are given by $\delta_i(e_j = \pm 1) = \Delta d_i^\pm_j$. From the central measurement result and the relative uncertainties $\Delta d_i^\pm_j$, the continuous $e_j$ dependence of $\delta_i(e_j)$ can be obtained using quadratic interpolation

$$
\delta_i(e_j) = e_j \frac{\Delta d_i^+ - \Delta d_i^-}{2} + e_j^2 \frac{\Delta d_i^+ + \Delta d_i^-}{2}.
$$

The theoretical prediction $t_i(\alpha_S)$ for bin $i$ depends on the value of $\alpha_S$. Furthermore, the theoretical predictions are also affected by sources of correlated uncertainties; $\delta_i(\lambda_k)$ specifies the relative uncertainty of $t_i$ due to the source $k$. Like the $\epsilon_j$, the $\lambda_k$ are also treated as Gaussian distributed random variables with zero expectation value and unity width. It is assumed that the theoretical predictions can be obtained with statistical uncertainties which are negligible as compared to the statistical uncertainties of the measurements. The continuous dependence of the relative uncertainty $\delta_i(\lambda_k)$ can be obtained through quadratic interpolation

between the central result $t_i$ and the results $t_i^\pm_k$ obtained by variations corresponding to plus/minus one standard deviation due to source $k$

$$
\delta_i(\lambda_k) = \lambda_k \frac{t_i^+ - t_i^-}{2t_i} + \lambda_k^2 \left( \frac{t_i^+ + t_i^-}{2t_i} - 1 \right).
$$

The $\chi^2$ used in the $\alpha_S$ extraction is then computed as

$$
\chi^2(\alpha_S, \vec{\epsilon}, \vec{\lambda}) = \sum_i \left[ \frac{d_i - t_i(\alpha_S) \left(\sum_k \delta_i(\lambda_k)\right)}{\sigma_i + \sigma_i^\text{cor}} \right]^2
+ \sum_j \epsilon_j^2 + \sum_k \lambda_k^2,
$$

where $i$ runs over all data points, $j$ runs over all sources of experimental correlated uncertainties, and $k$ over all theoretical correlated uncertainties. The fit result of $\alpha_S$ is determined by minimizing $\chi^2$ with respect to $\alpha_S$ and the “nuisance parameters” $\epsilon_j$ and $\lambda_k$.

### Appendix D: On the Compatibility of the $R_{\Delta \phi}$ Data and the World Average of $\alpha_S(m_Z)$

The $\alpha_S(m_Z)$ result in Table V is lower than the world average by approximately one standard deviation. In this Appendix, the consistency of the world average of $\alpha_S(m_Z)$ and the $R_{\Delta \phi}$ data is investigated using the $\chi^2$ values. The $\chi^2$ values are computed according to Appendix C, using the 18 data points with $\Delta \phi_{\text{max}} = 7\pi/8$, and $0.0 < \gamma < 0.5$ and $0.5 < \gamma < 1.0$. The theoretical predictions are computed for the fixed value of $\alpha_S(m_Z) = 0.1181$. The computation of $\chi^2$ uses the Hessian method for the treatment of all uncertainties except for the PDF set uncertainty.
The $\chi^2$ values between the 18 data points and the theoretical predictions when $\alpha_s(m_Z)$ is fixed to the world average value of $\alpha_s(m_Z) = 0.1181$ (third column) and when it is a free fitted parameter (fourth column) for variations of the scales $\mu_R$ and $\mu_F$ around the central choice $\mu_R = \mu_F = \mu_0 = H_T/2$.

| $\mu_R/\mu_0$ | $\mu_F/\mu_0$ | $\chi^2$ for $\alpha_s(m_Z) = 0.1181$ | $\chi^2$ for $\alpha_s(m_Z)$ free fit parameter |
|---------------|---------------|--------------------------------|--------------------------------|
| 0.5           | 0.5           | 62.4                           | 50.9                           |
| 0.5           | 1.0           | 56.3                           | 39.6                           |
| 1.0           | 0.5           | 31.6                           | 23.6                           |
| 1.0           | 1.0           | 29.7                           | 21.7                           |
| 1.0           | 2.0           | 28.4                           | 20.8                           |
| 2.0           | 1.0           | 19.2                           | 19.0                           |
| 2.0           | 2.0           | 19.3                           | 19.3                           |

and the scale dependence, so the $\chi^2$ values do not reflect these theoretical uncertainties. Therefore, a series of $\chi^2$ values is computed for possible combinations of variations of $\mu_R$ and $\mu_F$ around the central choice $\mu_R = \mu_F = \mu_0 = H_T/2$. The results are displayed in Table X and compared to the $\chi^2$ values obtained when $\alpha_s(m_Z)$ is a free fit parameter.

When $\alpha_s(m_Z)$ is fixed to the world average, the $\chi^2$ value for the central scale choice is slightly higher than the one obtained for a free $\alpha_s(m_Z)$, and also higher than the expectation of $\chi^2 = N_{\text{dof}} \pm \sqrt{2N_{\text{dof}}}$, where $N_{\text{dof}} = 18$ when $\alpha_s(m_Z)$ is fixed or 17 when it is a free fit parameter. However, the $\chi^2$ definition does not take into account the theoretical uncertainty due to the scale dependence. When the renormalization scale is increased by a factor of two, to $\mu_R = 2\mu_0$, lower $\chi^2$ values are obtained, which are similar in size to the ones obtained for a free $\alpha_s(m_Z)$, and close to the expectation (the dependence on the factorization scale is rather small). Since these $\chi^2$ values are well within the range of the expectation, it is concluded that, within their uncertainties, the theoretical predictions for the world average value of $\alpha_s(m_Z)$ are consistent with the $R_{\Delta \phi}$ data.

APPENDIX E: ON THE COMPATIBILITY OF THE RGE AND THE SLOPE OF THE $\alpha_s(Q)$ RESULTS

It is natural to ask whether the observed $Q$ dependence (i.e., the running) of the $\alpha_s(Q)$ results shown in Fig. 3 is described by the RGE or instead exhibits significant deviations at the highest $Q$ values, possibly indicating signals of physics beyond the Standard Model. The consistency of the RGE predictions with the observed slope is investigated in this Appendix. The RGE prediction would be in agreement with the observed $Q$ dependence of the $\alpha_s(Q)$ results if the latter, when evolved to $m_Z$, give $\alpha_s(m_Z)$ values that are independent of $Q$. For this purpose, a linear function in $\log_{10}(Q/1\text{ GeV})$, $f(Q) = c + m \cdot \log_{10}(Q/1\text{ GeV})$, is fitted to the nine $\alpha_s(m_Z)$ points in Fig. 3 (bottom) and their statistical uncertainties. Here the correlated systematic uncertainties are not taken into account as their correlations are nontrivial since the individual $\alpha_s(Q)$ results are obtained in separate fits, with different optimizations for the nuisance parameters. The fit results for the slope parameter $m$ and its uncertainty are displayed in Table XI for a fit to the $\alpha_s(m_Z)$ points at all nine $Q$ values, and also for fits to different subsets of the $\alpha_s(m_Z)$ points, omitting points either at lower or higher $Q$.

As documented in Table XI, a fit to all nine $\alpha_s(m_Z)$ points gives a slope that differs from zero by more than its uncertainty. Fits to groups of data points, however, show that the significance of this slope arises from the two points at lowest $Q$. Omitting the $\alpha_s(m_Z)$ point at lowest $Q$ (fitting points Nos. 2–9), or the two points at lowest $Q$ (fitting points # 3–9), both give fit results for which the slope parameter is more consistent with zero, while the $\alpha_s(m_Z)$ results change by less than $\pm 0.0001$. On the other hand, omitting the $\alpha_s(Q)$ points at highest $Q$ (fitting points Nos. 1–8 or Nos. 1–7) does not affect the significance of the slope. It is therefore concluded that the high-$Q$ behavior of the $\alpha_s(Q)$ results is consistent with the RGE and that the small differences at lowest $Q$ do not affect the combined $\alpha_s(m_Z)$ result.
[1] H. D. Politzer, Reliable Perturbative Results for Strong Interactions?, Phys. Rev. Lett. 30, 1346 (1973).
[2] D. J. Gross and F. Wilczek, Asymptotically free gauge theories, I, Phys. Rev. D 8, 3633 (1973).
[3] H1 Collaboration, Measurement of jet production cross sections in deep-inelastic ep scattering at HERA, Eur. Phys. J. C 77, 215 (2017).
[4] H1 Collaboration, Determination of the strong coupling constant $\alpha_s(m_Z)$ in next-to-next-to-leading order QCD using H1 jet cross section measurements, Eur. Phys. J. C 77, 791 (2017).
[5] ZEUS Collaboration, Inclusive-jet photoproduction at HERA and determination of $\alpha_s$, Nucl. Phys. B 864, 1 (2012).
[6] S. Bethke, S. Kluth, C. Pahl, and J. Schieck, Determination of the strong coupling $\alpha_s$ from hadronic event shapes with $O(\alpha_s^3)$ and resummed QCD predictions using JADE data, Eur. Phys. J. C 64, 351 (2009).
[7] G. Dissertori, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, G. Heinrich, G. Luisoni, and H. Stenzel, Determination of the strong coupling constant using matched NNLO + NLLA predictions for hadronic event shapes in e+e− annihilations, J. High Energy Phys. 08 (2009) 036.
[8] CDF Collaboration, Measurement of the Strong Coupling Constant from Inclusive Jet Production at the Tevatron $p\bar{p}$ Collider, Phys. Rev. Lett. 88, 042001 (2002).
[9] D0 Collaboration, Determination of the strong coupling constant from the inclusive jet cross section in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, Phys. Rev. D 80, 111107 (2009).
[10] D0 Collaboration, Measurement of angular correlations of jets at $\sqrt{s} = 1.96$ TeV and determination of the strong coupling at high momentum transfers, Phys. Lett. B 718, 56 (2012).
[11] CMS Collaboration, Measurement of the ratio of the inclusive 3-jet cross section to the inclusive 2-jet cross section in pp collisions at $\sqrt{s} = 7$ TeV and first determination of the strong coupling constant in the TeV range, Eur. Phys. J. C 73, 2604 (2013).
[12] CMS Collaboration, Constraints on parton distribution functions and extraction of the strong coupling constant from the inclusive jet cross section in pp collisions at $\sqrt{s} = 7$ TeV, Eur. Phys. J. C 75, 288 (2015).
[13] CMS Collaboration, Measurement and QCD analysis of double-differential inclusive jet cross sections in pp collisions at $\sqrt{s} = 8$ TeV and cross section ratios to 2.76 and 7 TeV, J. High Energy Phys. 03 (2017) 156.
[14] CMS Collaboration, Measurement of the inclusive 3-jet production differential cross section in proton–proton collisions at 7 TeV and determination of the strong coupling constant in the TeV range, Eur. Phys. J. C 75, 186 (2015).
[15] ATLAS Collaboration, Determination of the strong coupling constant $\alpha_s$ from transverse energy–energy correlations in multijet events at $\sqrt{s} = 8$ TeV using the ATLAS detector, Eur. Phys. J. C 77, 872 (2017).
[16] C. Patrignani et al., Review of particle physics, Chin. Phys. C 40, 100001 (2016).
[17] M. Wobisch, K. Chakravarthula, R. Dhullipudi, L. Sawyer, and M. Tamsett, A new quantity for studies of dijet azimuthal decorrelations, J. High Energy Phys. 01 (2013) 172.
[18] D0 Collaboration, Measurement of Dijet Azimuthal Decorrelations at Central Rapidities in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV, Phys. Rev. Lett. 94, 221801 (2005).
[19] CMS Collaboration, Dijet Azimuthal Decorrelations in $pp$ Collisions at $\sqrt{s} = 7$ TeV, Phys. Rev. Lett. 106, 122003 (2011).
[20] ATLAS Collaboration, Measurement of Dijet Azimuthal Decorrelations in $pp$ Collisions at $\sqrt{s} = 7$ TeV, Phys. Rev. Lett. 106, 172002 (2011).
[21] M. Wobisch and K. Rabbertz, Dijet azimuthal decorrelations for $\Delta\phi_{\text{dijet}} < 2\pi/3$ in perturbative QCD, J. High Energy Phys. 12 (2015) 024.
[22] D0 Collaboration, Measurement of the combined rapidity and $p_T$ dependence of dijet azimuthal decorrelations in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, Phys. Lett. B 721, 212 (2013).
[23] ATLAS Collaboration, Luminosity determination in $pp$ collisions at $\sqrt{s} = 8$ TeV using the ATLAS detector at the LHC, Eur. Phys. J. C 76, 653 (2016).
[24] C. Buttar et al., Standard Model Handles and Candles Working Group, Tools and jets summary report, arXiv: 0803.0678.
[25] M. Cacciari and G. P. Salam, Dispelling the $N^3$ myth for the $k_t$ jet-finder, Phys. Lett. B 641, 57 (2006).
[26] M. Cacciari, G. P. Salam, and G. Soyez, The anti-$k_t$ jet clustering algorithm, J. High Energy Phys. 04 (2008) 063.
[27] G. Soyez, Optimal jet radius in kinematic dijet reconstruction, J. High Energy Phys. 07 (2010) 075.
[28] Z. Nagy, Three jet cross-sections in hadron hadron collisions, Phys. Rev. D 68, 094002 (2003).
[29] Z. Nagy, Three jet cross-sections in hadron hadron collisions at next-to-leading order, Phys. Rev. Lett. 88, 122003 (2002).
[30] T. Kluge, K. Rabbertz, and M. Wobisch, FastNLO: Fast pQCD calculations for PDF fits, arXiv:hep-ph/0609285.
[31] M. Wobisch, D. Britzger, T. Kluge, K. Rabbertz, and F. Stober, Theory-data comparisons for jet measurements in hadron-induced processes, arXiv:1109.1310.
[32] W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Deep-inelastic scattering beyond the leading order in asymptotically free gauge theories, Phys. Rev. D 18, 3998 (1978).
[33] L. A. Harland-Lang, A. D. Martin, P. Motylinski, and R. S. Thorne, Parton distributions in the LHC era: MMHT 2014 PDFs, Eur. Phys. J. C 75, 204 (2015).
[34] L. A. Harland-Lang, A. D. Martin, P. Motylinski, and R. S. Thorne, Uncertainties on $\alpha_s$ in the MMHT2014 global PDF analysis and implications for SM predictions, Eur. Phys. J. C 75, 435 (2015).
[35] S. Dulat, T.-J. Hou, J. Gao, M. Guzzi, J. Huston, P. Nadolsky, J. Pumpin, C. Schmidt, D. Stump, and C.-P. Yuan, New parton distribution functions from a global analysis of quantum chromodynamics, Phys. Rev. D 93, 033006 (2016).
[36] R. D. Ball, V. Bertone, F. Cerruti, L. Del Debbio, S. Forte, A. Guffanti, J. I. Latorre, J. Rojo, and M. Ubiali, Impact of heavy quark masses on parton distributions and LHC phenomenology, Nucl. Phys. B 889, 296 (2011).
[37] R. D. Ball et al., Parton distributions for the LHC Run II, J. High Energy Phys. 04 (2015) 040.
[38] S. Alekhin, J. Blümlein, S. Moch, and R. Placakyte, Parton distribution functions, $\alpha_s$ and heavy-quark masses for LHC run II, Phys. Rev. D 96, 014011 (2017).

[39] H1 and ZEUS Collaborations, Combination of measurements of inclusive deep inelastic $e^+p$ scattering cross sections and QCD analysis of HERA data, Eur. Phys. J. C 75, 580 (2015).

[40] T. Sjöstrand, P. Edén, C. Friberg, L. Lönnblad, G. Miu, S. Mrenna, and E. Norrbin, High-energy physics event generation with PYTHIA 6.1, Comput. Phys. Commun. 135, 238 (2001).

[41] G. Corcella, I. G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M. H. Seymour, and B. R. Webber, HERWIG 6: An event generator for hadron emission reactions with interfering gluons (including supersymmetry processes), J. High Energy Phys. 01 (2001) 010.

[42] G. Corcella et al., HERWIG 6.5 release note, arXiv:hep-ph/0210213.

[43] ATLAS Collaboration, Charged particle multiplicities in $pp$ interactions at $\sqrt{s} = 0.9$ and 7 TeV in a diffractive limited phase-space measured with the ATLAS detector at the LHC and new PYTHIA6 tune, CERN Report No. ATLAS-CONF-2010-031, 2010, https://cds.cern.ch/record/1277665.

[44] M. G. Albrow et al., Tevatron-for-LHC Report of the QCD Working Group, arXiv:hep-ph/0610012.

[45] R. Field, Min-bias and the underlying event at the Tevatron and the LHC, Proceedings of Fermilab ME/EC Tuning Workshop, Fermilab, 2002 (unpublished).

[46] H. Schulz and P. Z. Skands, Energy scaling of minimum-bias tunes, Eur. Phys. J. C 71, 1644 (2011).

[47] ATLAS Collaboration, The ATLAS experiment at the CERN Large Hadron Collider, J. Instrum. 3, S08003 (2008).

[48] ATLAS Collaboration, Performance of the ATLAS trigger system in 2010, Eur. Phys. J. C 72, 1849 (2012).

[49] ATLAS Collaboration, The performance of the jet trigger for the ATLAS detector during 2011 data taking, Eur. Phys. J. C 76, 526 (2016).

[50] ATLAS Collaboration, Topological cell clustering in the ATLAS calorimeters and its performance in LHC Run I, Eur. Phys. J. C 77, 490 (2017).

[51] S. Agostinelli et al., GEANT4: A simulation toolkit, Nucl. Instrum. Methods Phys. Res., Sect. A 506, 250 (2003).

[52] ATLAS Collaboration, The ATLAS simulation infrastructure, Eur. Phys. J. C 70, 823 (2010).

[53] T. Sjöstrand, S. Mrenna, and P. Z. Skands, A brief introduction to PYTHIA 8.1, Comput. Phys. Commun. 178, 852 (2008).

[54] ATLAS Collaboration, Summary of ATLAS Pythia 8 tunes, CERN Report No. ATL-PHYS-PUB-2012-003, 2012, https://cds.cern.ch/record/1474107.

[55] M. Cacciari and G. P. Salam, Pileup subtraction using jet areas, Phys. Lett. B 659, 119 (2008).

[56] ATLAS Collaboration, Performance of pile-up mitigation techniques for jets in $pp$ collisions at $\sqrt{s} = 8$ TeV using the ATLAS detector, Eur. Phys. J. C 76, 581 (2016).

[57] ATLAS Collaboration, Monte Carlo calibration and combination of in-situ measurements of jet energy scale, jet energy resolution and jet mass in ATLAS, CERN Report No. ATLAS-CONF-2015-037, 2015, https://cds.cern.ch/record/2044941.

[58] ATLAS Collaboration, A measurement of the calorimeter response to single hadrons and determination of the jet energy scale uncertainty using LHC Run-1 $pp$-collision data with the ATLAS detector, Eur. Phys. J. C 77, 26 (2017).

[59] ATLAS Collaboration, Data-driven determination of the energy scale and resolution of jets reconstructed in the ATLAS calorimeters using dijet and multijet events at $\sqrt{s} = 8$ TeV, CERN Report No. ATLAS-CONF-2015-017, 2015, https://cds.cern.ch/record/2088678.

[60] ATLAS Collaboration, Jet energy resolution in proton-proton collisions at $\sqrt{s} = 7$ TeV recorded in 2010 with the ATLAS detector, Eur. Phys. J. C 73, 2306 (2013).

[61] ATLAS Collaboration, Determination of the jet energy scale and resolution at ATLAS using $Z/\gamma$-jet events in data at $\sqrt{s} = 8$ TeV, CERN Report No. ATLAS-CONF-2015-057, 2015, https://cds.cern.ch/record/2059846.

[62] ATLAS Collaboration, Jet global sequential corrections with the ATLAS detector in proton-proton collisions at $\sqrt{s} = 8$ TeV, CERN Report No. ATLAS-CONF-2015-002, 2015, https://cds.cern.ch/record/2011682.

[63] G. D’Agostini, A Multidimensional unfolding method based on Bayes’ theorem, Nucl. Instrum. Methods Phys. Res., Sect. A 362, 487 (1995).

[64] F. James and M. Roos, Minuit: A system for function minimization and analysis of the parameter errors and correlations, Comput. Phys. Commun. 10, 343 (1975).

[65] T. Carli et al., Experimental determination of Parton Distributions, in Proceedings of HERA and the LHC: A workshop on the implications of HERA for LHC physics: Proceedings Part A, edited by G. Altarelli et al., 2005, arXiv:hep-ph/0601012.

[66] ATLAS Collaboration, ATLAS computing acknowledgements, CERN Report No. ATL-GEN-PUB-2016-002, https://cds.cern.ch/record/2202407.

[67] K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Strong Coupling Constant with Flavor Thresholds at Four Loops in the MS Scheme, Phys. Rev. Lett. 79, 2184 (1997).

[68] P. Nason, A new method for combining NLO QCD with shower Monte Carlo algorithms, J. High Energy Phys. 11 (2004) 040.

[69] J. Pumplin, D. R. Stump, J. Huston, H.-L. Lai, P. Nadolsky, and W.-K. Tung, New generation of parton distributions with uncertainties from global QCD analysis, J. High Energy Phys. 07 (2002) 012.
MEASUREMENT OF DIJET AZIMUTHAL DECORRELATIONS ...

PHYS. REV. D 98, 092004 (2018)
MEASUREMENT OF DIJET AZIMUTHAL DECORRELATIONS … PHYS. REV. D 98, 092004 (2018)
Division of Physics and Tomonaga Center for the History of the Universe, Faculty of Pure and Applied Sciences, University of Tsukuba, Tsukuba, Japan

Department of Physics and Astronomy, Tufts University, Medford, Massachusetts, USA

Department of Physics and Astronomy, University of California Irvine, Irvine, California, USA

Department of Physics and Astronomy, University of Uppsala, Uppsala, Sweden

Instituto de Física Corpuscular (IFIC), Centro Mixto Universidad de Valencia - CSIC, Valencia, Spain

Department of Physics, University of British Columbia, Vancouver, British Columbia, Canada

Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia, Canada

Fakultät für Physik und Astronomie, Julius-Maximilians-Universität Würzburg, Würzburg, Germany

Department of Physics, University of Warwick, Coventry, United Kingdom

Waseda University, Tokyo, Japan

Department of Particle Physics, Weizmann Institute of Science, Rehovot, Israel

Department of Physics, University of Wisconsin, Madison, Wisconsin, USA

Fakultät für Mathematik und Naturwissenschaften, Fachgruppe Physik, Bergische Universität Wuppertal, Wuppertal, Germany

Department of Physics, Yale University, New Haven, Connecticut, USA

Yerevan Physics Institute, Yerevan, Armenia

\*Deceased.
\#Also at Department of Physics, King’s College London, London, United Kingdom.
\^Also at Institute of Physics, Azerbaijan Academy of Sciences, Baku, Azerbaijan.
\_Also at TRIUMF, Vancouver, British Columbia, Canada.
\~Also at Department of Physics and Astronomy, University of Louisville, Louisville, Kentucky, USA.
\ Straight at Department of Physics, California State University, Fresno, California, USA.
\/Also at Department of Physics, University of Fribourg, Fribourg, Switzerland.
\_Also at Departament de Física de la Universitat Autònoma de Barcelona, Barcelona, Spain.
\~Also at Tomsk State University, Tomsk, and Moscow Institute of Physics and Technology State University, Dolgoprudny, Russia.
\~Also at The Collaborative Innovation Center of Quantum Matter (CICQM), Beijing, China.
\^Also at Universita di Napoli Parthenope, Napoli, Italy.
\~Also at Institute of Particle Physics (IPP), Canada.
\^Also at II. Physikalisches Institut, Georg-August-Universität Göttingen, Göttingen, Germany.
\_Also at Dipartimento di Fisica E. Fermi, Università di Pisa, Pisa, Italy.
\*Also at Horia Hulubei National Institute of Physics and Nuclear Engineering, Bucharest, Romania.
\~Also at CPPM, Aix-Marseille Université, CNRS/IN2P3, Marseille, France.
\_Also at Department of Physics, St. Petersburg State Polytechnical University, St. Petersburg, Russia.
\~Also at Borough of Manhattan Community College, City University of New York, New York, USA.
\_Also at Department of Financial and Management Engineering, University of the Aegean, Chios, Greece.
\_Also at Centre for High Performance Computing, CSIR Campus, Rosebank, Cape Town, South Africa.
\_Also at Louisiana Tech University, Ruston, Louisiana, USA.
\_Also at Instituto Catalana de Recerca i Estudis Avançats, ICREA, Barcelona, Spain.
\^Also at Department of Physics, University of Michigan, Ann Arbor, Michigan, USA.
\_Also at Graduate School of Science, Osaka University, Osaka, Japan.
\~Also at Physikalisches Institut, Albert-Ludwigs-Universität Freiburg, Freiburg, Germany.
\~Also at Institute for Mathematics, Astrophysics and Particle Physics, Radboud University Nijmegen/Nikhef, Nijmegen, Netherlands.
\~Also at Near East University, Nicosia, North Cyprus, Mersin, Turkey.
\_Also at Institute of Theoretical Physics, Ilia State University, Tbilisi, Georgia.
\_Also at CERN, Geneva, Switzerland.
\_Also at Department of Physics, Stanford University, Stanford, California, USA.
\_Also at Ochadai Academic Production, Ochanomizu University, Tokyo, Japan.
\_Also at Manhattan College, New York, New York, USA.
\_Also at Novosibirsk State University, Novosibirsk, Russia.
\~Also at LPNHE, Sorbonne Université, Paris Diderot Sorbonne Paris Cité, CNRS/IN2P3, Paris, France.
\_Also at The City College of New York, New York, New York, USA.
\_Also at Departamento de Física Teorica y del Cosmos, Universidad de Granada, Granada (Spain), Spain.
\_Also at Department of Physics, California State University, Sacramento, California, USA.
\_Also at Moscow Institute of Physics and Technology State University, Dolgoprudny, Russia.
\~Also at Département de Physique Nucléaire et Corpusculaire, Université de Genève, Genève, Switzerland.
\_Also at Institut de Física d’Altes Energies (IFAE), Barcelona Institute of Science and Technology, Barcelona, Spain.
