Modelling of correlation of actual and program feeds in the automatic cycle

A V Akintseva, A V Prokhorov and S V Omelchenko

1Department of Engineering, Technology and Construction, South Ural State University, Chelyabinsk, 76, Lenin Avenue, 454080, Russian Federation
2Department of Modern Educational Technologies, South Ural State University, Chelyabinsk, 76, Lenin Avenue, 454080, Russian Federation

*akintsevaav@susu.ru

Abstract. The article presents a model for calculating the current values of the actual radial feed in the internal grinding process controlled in an automatic cycle on a CNC machine. The model establishes the correlation of actual and program feeds with cutting forces, elastic deformation of the technological system, cutting modes, geometrical parameters of the contact area between the wheel and the workpiece, wheel characteristics and other technological parameters and conditions of processing in a wide range of their variation. The obtained model is used for predicting radii changes of the machined hole surface starting from the workpiece to the final size for a given automatic cycle and various technological conditions of the internal grinding operation. The model described in this article is the basis for the development of methodology for multiparameter optimization of grinding cycles in a multidimensional space of control parameters.

1. Introduction

Internal grinding is one of the most common methods of hole-finishing operation (27% an average and in some industries up to 70% of all machine operations). Meanwhile, there are high requirements on the internal grinding operations for accuracy (diameter accuracy – 5…6 tolerance grade) and quality of the processed surface (low surface roughness – Ra 0,06…2,5). More and more internal grinding operations are performed on CNC machines with wide control ranges of operating parameters and unequal technological capabilities. On the vast majority of modern CNC grinding machines the processing is carried out in an automatic step-by-step cycle with the ability to program control of all main operating parameters. As a result, the most complete use of the potential capabilities of CNC machines in the operation designing is a difficult task for the technologist, since currently there are no effective methods of optimal cycles designing. Normative-reference, calculation and production methods can be referred to the main methods of cycles designing. Each of them has a number of significant flaws that make it impossible to design high-productive cycles effectively in the conditions of modern automated production (Table. 1).

The solution to the problem above is a developing of design methodology for optimal internal grinding cycles [9], which allows to perform complex optimization of all parameters of the cycle control (cutting modes, geometrical parameters and wheel characteristics, lubricant-cooling agent, etc)
with mathematical precision using the dynamic programming method [10]. The basis of methodology is a mathematical model of metal removal process, considering kinematics features of the internal grinding process and establishing a complex functional relationship between the actual radial feed and technological parameters that affect the stability of the allowance removal process (cutting modes, elastic deformations of technological system, cutting force, mechanical features of the processed material, geometrical parameters of the contact zone between the wheel and the workpiece, wheel characteristics, its blunting, etc.) [11]. The metal removal model for the internal grinding, developed by us, makes it possible to calculate the current values of the actual feeds and the section radii of the hole machined surface; as a result it becomes possible to impose restrictions on the machining accuracy [12].

Table 1. Comparison of internal grinding cycle designing methods.

| Description of the cycle designing method | Flaws |
|----------------------------------------|-------|
| Normative method of cycles designing based on engineering standards and various reference books [1-2] | – evolved on the basis of statistical data of 60-80-s, for universal machine tools with manual control; – covers a narrow range of technological parameters and processing conditions; – used for the designing the starting cycle by «mid-points», requiring adaptation to the initial processing conditions. |
| Calculation method of cycles designing based on engineering techniques of cycles designing [3-8, etc.]. | – mathematical methods of optimization are not used; – there is no one of the most important and mandatory models of productivity limitations for processing accuracy; – inconstancy of the technological process both during the cycle of the part processing and during the processing of billets batch is not considered. |
| Productive (experimental) method of cycles designing based on the selection of cycles by processing a number of test billets | – cycle efficiency depends on the skill of the setter-up; – requires additional material and time costs; – cutting modes are taken underrated that leads to a decrease in the processing productivity. |

2. Modelling the interrelation of actual and program feeds in the automatic cycle

The radial component has the greatest influence on the formation of the actual radial feed of all the components of the cutting force. Analysis of currently known sources [13-15, etc.] revealed that at present time there are no wide-range analytical dependencies for calculating cutting forces which consider the kinematic features of internal grinding and cover most of the main technological parameters. Therefore, a force model of the internal grinding has been developed on the basis of the functional interrelation between the metal removal intensity and elementary metal volumes that are deformable in the shift zone. More detailed information about the force model of internal grinding, developed by us, can be found in the article [16]. The formula for the radial component of the cutting force for internal grinding is given below:

\[ P_{y,z} = M_1 S_{y,z} + M_2 \sqrt{S_{z,z}} \]  (1)
where $S_{z,i}$ – radial feed on $i$-th stroke $z$-th stage, mm/double stroke; $z$ – order number of cycle stage; $i$ – order number of grinding wheel stroke; $M_1$, $M_2$ – analytical coefficients characterizing the interrelation of various technological parameters of internal grinding; can be calculated by the following formulas:

$$M_1 = \frac{1.86\sigma_i \pi d V_{soc}}{(V_1 + V_2)^2 + V_{soc}^2}$$  \hspace{1cm} (2)$$

$$M_2 = \frac{\sigma_i \eta T}{3} \sqrt{\frac{dD}{d - D}}$$  \hspace{1cm} (3)$$

where $V_1$ – circumferential wheel speed, m/s; $V_2$ – rotational speed of the billet, m/min; $\sigma_i$ – average value of tension intensity, N/mm$^2$; $d$ – billet diameter, mm; $D$ – wheel diameter, mm; $T$ – total grinding wheel height, mm; $\eta$ – degree of wheel blunting;

The presence of gaps in the kinematic chains of the machine unit and the elastic compliance of the technological system leads to the fact that the actual law of wheel feed motion differs from the programmed, which is set for each cycle stage from the machine control panel by controlling the control impact on the drive motor of the machine feed. As a result, the actual radial feed is not equal to the program value. The graphic display of the actual radial feed for internal grinding causes great difficulty for several reasons: 1) process of allowance removal in the middle and extreme section of the billet differs significantly due to transients in the zone of wheel reverse (in this article the least loaded mid-section is considered); 2) discontinuity of radial feed due to constant switching of the axial feed direction (tool stroke); 3) presence of stepwise switching of axial and radial feeds speed during the whole cycle; 4) non-linear interrelation of actual and program radial feeds.

For the classic internal grinding cycle a double stroke consists of a forward working stroke (axial movement of the tool from right to left, assigned $P$) and reverse idle stroke (axial movement of the tool from left to right, assigned $X$), wherein on the idle stroke the program value of radial feed is equal to zero and the metal removal process is carried out only by tension. Therefore the actual radial feed on $P$ and $X$ strokes are significantly different. Actual radial feed is convenient to depict as a polygon of average values distribution (as it is shown in Fig.1) for the least loaded middle section. Further the «intermittent» actual radial feed will be depicted as a histogram for convenience of perception.

![Figure 1](image1.png)

- $S_{z,i}$ for working stroke
- $H^P_{z,i}$ for working stroke
- $H^X_{z,i}$ for idle stroke

**Figure 1.** Graphic display of actual and program radial feeds interrelation

Fig.2 (for convenience of perception fig.3 ab are the enlarged part of fig.2 – area marked with a dashed line) represents the graph of program and actual motion paths of wheel generating about initial
position of the billet axis which is taken as the coordinate origin (fixed reference point). Accumulated program and actual feeds in the automatic cycle of internal grinding of absolutely round billet which contains a forward working and reverse idle strokes are represented in the form of columns on graphs (fig. 2 and 3). The ABGH column is a feed graph for $P$, and BCKH – for $X$. Segment AB is a value of accumulated program feed at $P$, i.e. how far the wheel could move if there were no part. Since at $X$ the program feed is equal to zero, the value of program feed at $X$ (segment BC) is equal to the program feed at $P$ (segment AB). Therefore, these columns are equal in a double stroke. However, if there is a part, the elastic deformation of technological system by the value of AD at $P$, and by the value of BF at $X$ occurs. The size of DEHG column is equal to accumulated actual feed at $P$, and FKH column – at $X$, since metal removal occurs due to elastic deformation. Removal value at $X$ is equal to distance FE, and at $P$ it is equal to distance DL. It should be noted that the trajectory of allowance removal at processing of absolutely round billet on $i$-th stroke $z$-th stage is the spiral of Archimedes [11], which promotes to the accumulation of additional processing error. In this case the value of additional processing error depends on actual radial feed at the previous tool stroke. The more allowance removed for the last stroke, the more hereditary ellipse the billet has.

Let us find the actual radial feed on $i$-th stroke $z$-th stage from a closed dimensional contour (for absolutely round billet). Closed dimensional contour for internal grinding cycle can be described by the following way (in fig. 3, a and b is highlighted with a red line): for $P$ – formula (4), for $X$– formula (5).

$$S_{z,i}^p + y_{z,i}^X = H_{z,i}^p + y_{z,i}^p$$  \hspace{2cm} (4)  
$$y_{z,i-1}^p = H_{z,i}^p + y_{z,i}^p$$  \hspace{2cm} (5)

where $S_{z,i}^p$ – program value of radial feed on forward (reverse) working stroke, mm/double stroke; $y_{z,i-1}^p$ – elastic deformation of wheel axis on forward (reverse) working stroke, mm; $H_{z,i}^p$ – actual radial feed on forward (reverse) working stroke, mm.
Figure 2. Graphic display of interrelation between accumulated values of actual and program radial feeds for internal grinding cycle: $\Sigma S_{z,i}$ ($\Sigma H_{z,i}$) – accumulated value of program (actual) radial feed.

Figure 3. Graphic display of interrelation between accumulated values of actual and program radial feeds for the forward working (a) and reverse idle (b) strokes in the internal grinding cycle.

Analysis of equations (4)-(5) reveals that the current value of actual program feed directly depends on the increment of elastic deformation between the current and the previous stroke. Therefore the following universal mathematical notation can be made (* program value of radial feed is considered when the stroke is working).

$$S_{z,i}^* + y_{z,i-1} = H_{z,i} + y_{z,i}$$  \hspace{1cm} (6)

Equation (6) constitutes a functional interrelation between actual radial feed with the program feed and elastic deformations of wheel axis. A formula for the elastic deformations of wheel axis is obtained from the equation of the grinding spindle elastic line.
where \( A_5 \) – parameter determining the rigidity of the technological system, \( \text{mm/N} \).

Substitute equation (7) into expression (6):

\[
S'_{z,j} = H_{z,j} + P_{y,z,j}A_5
\]

(8)

Transform it by substituting expression (9) into formula (8):

\[
S'_{z,j} = H_{z,j} + \Delta P_{y,z,j}A_5
\]

(10)

Equation (10) constitutes interrelation between program and actual feeds, radial component of the cutting force, compliance of the technological system.

Find the value of actual radial feed on \( i \)-th stroke of \( z \)-th stage by substituting the formula for the radial component of cutting force (1) into equation (10):

\[
S'_{z,j} = H_{z,j} + \Delta P_{y,z,j}A_5
\]

(11)

Calculation of current values of actual feeds during the cycle is carried out according to formula (11) from the first tool stroke with the following initial conditions: for \( i = 1 \) values of program and actual radial feeds on the previous stroke are equal to zero, as in the case when the reverse stroke is idle. Accumulated value of actual allowance removal on \( i \)-th stroke of \( z \)-th stage is determined as a sum of actual radial feed values for each wheel stroke on \( z \)-th stage. As a result, it becomes possible to calculate the exact number of wheel strokes required for removal a given allowance value and, accordingly, to determine the main time.

In order to consider the initial radial runout the considered section of the billet is taken as ellipse described by an array of radii. When processing a non-circular billet, the radial component of cutting force on each radius will change depending on the initial profile, and accordingly the actual radial feed. Note: the wheel is considered to be conditionally rotated for the coincidence of contact point of the wheel with the billet and axis of the graphs in fig. 2 and 4. By analogy with the previous part of the article we obtain a formula for calculating the actual radial feed on each radius of considered section on \( i \)-th stroke of \( z \)-th stage. The sum of radial feed on \( i \)-th stroke of \( z \)-th stage (closed contour in fig. 4 is highlighted with a red line) can be found by the formula (12).

\[
\sum S_{z,j} = H_{z,i,g,b} + y_{z,i,g,b} + A_{z,i,g,b}
\]

(12)

where \( H_{z,i,g,b} \) – actual radial feed on \( b \)-th radius of \( g \)-th section on \( i \)-th stroke of \( z \)-th stage, \( \text{mm} \); \( g \) – order number of billet section; \( b \) – order number of radius at \( g \)-th billet section; \( y_{z,i,g,b} \) – elastic deformation of wheel axis on \( b \)-th radius of \( g \)-th section on \( i \)-th stroke of \( z \)-th stage, \( \text{mm} \).

Initial radial runout of the billet (\( A_{z,i,g,b} \), \( \text{mm} \)) for \( b \)-th radius of \( g \)-th section on \( i \)-th stroke of \( z \)-th stage can be obtained by formula:

\[
A_{z,i,g,b} = R_{z,i-1,g,b} - R_{\text{min}}
\]

(13)

where \( R_{\text{min}} \) – minimal value of billet radius, \( \text{mm} \); \( R_{z,i-1,g,b} \) – value of \( b \)-th radius of \( g \)-th section on the previous stroke of \( z \)-th stage, \( \text{mm} \).

Substituting expression (1) in formula (7) we will obtain the elastic deformation of the wheel on \( b \)-th of \( g \)-th section on \( i \)-th stroke of \( z \)-th stage:
Find the actual radial feed on $b$-th radius of $g$-th section on $i$-th stroke of $z$-th stage by substituting expressions (13) and (14) into formula (12).

\[
y_{z,i,g,b} = A_i M_1 H_{z,i,g,b} + A_b M_2 \sqrt{H_{z,i,g,b}}
\]  

(14)

\[
H_{z,i,g,b} = -\frac{A_b M_2}{2(1 + A_i M_1)} + \sqrt{\frac{A_b M_2}{2(1 + A_i M_1)} + \frac{\sum S_{z,i,g,b} - R_{z,i,g,b} + R_{\text{min}}}{1 + A_i M_1}}
\]  

(15)

Thus, formula (15) establishes the regularities of changes in the values of the actual radial feed and the radial component of the cutting force during the processing cycle for the given program feed values considering the compliance of technological system and initial radial runout of the billet.

\[7\]

\[51\ 52\ 2 \sum (1 ) 2 (1 ) 1\]

\[51\ 51\ 51\ 2\]

\[zigb\]

\[SR\ RAM\ AMH\ A\]

\[M \) A\]

\[-+\]

\[\sum\]

\[2\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]
– is the basis for creating a methodology for designing optimal internal grinding cycles, which allows to perform multi-parameter optimization in a multidimensional space of control parameters of the cycle with the required accuracy.

References

[1] General engineering standards of cutting modes for the technical regulation of work on metal-cutting machines. Part III. Broaching, grinding and lapping machines (Mechanical Engineering, Moscow, 1978)

[2] Cutting modes for work carried out on grinding and lapping Machines with manual control and semi-automatic (Publishing house ATKOCO, Chelyabinsk, 2007)

[3] Malkin S, Guo C, Grinding Technology: Theory and Applications of Machining with Abrasives (Industrial Press, New York, USA, 2008)

[4] Alagumurthi N, Panairadja K and Soundararajan V 2006 J. Materials and Manufacturing Processes 21 19

[5] Lur’e G B 1979 J.Mashinostroitel’ 3 12

[6] Dong S, Danai K, Malkin S and Deshunukh A 2004 J. Manuf. Sci. Eng., Trans. ASME 126 327

[7] Amitay G, Malkin S and Koren Y 1981 J. Eng. Ind. 103 103

[8] Phan A M, Summers M P and Parmigiani J P 2011 Int. Mechan. Eng. Congr. Expos (IMECE) 3 915

[9] Pereverzev PP and Akintseva AV 2016 J. Russian Engineering Research 36 974

[10] Bellman R Dynamic programming (Publishing House Foreign Literatures, Moscow, 1960)

[11] Pereverzev PP and Akintseva AV 2016 J. Russian Engineering Research 36 888

[12] Pereverzev PP and Akintseva AV 2016 J. Russian Engineering Research 36 1048

[13] Xuekun Li Modeling and simulation of grinding processes based on a virtual wheel model and microscopic interaction analysis (Dissertation, Worcester polytechnic institute, 2010)

[14] Korchak S.N. The Productivity of the Grinding of Steel Parts (Maschinostroenie, Moscow, 1974).

[15] Danilenko MV and Nosenko VA 2017 J. News of the Volgograd State Technical University 12 31

[16] Pereverzev PP and Akintseva AV 2016 J Procedia Engineerin 150 1113