Spin down during quiescence of the fastest known accretion-powered pulsar

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ABSTRACT

We present a timing solution for the 598.89 Hz accreting millisecond pulsar, IGR J00291+5934, using Rossi X-ray Timing Explorer data taken during the two outbursts exhibited by the source on 2008 August and September. We estimate the neutron star spin frequency and we refine the system orbital solution. To achieve the highest possible accuracy in the measurement of the spin frequency variation experienced by the source in-between the 2008 August outburst and the last outburst exhibited in 2004, we re-analysed the latter considering the whole data set available. We find that the source spins down during quiescence at an average rate of \( \dot{\nu} = (-4.1 \pm 1.2) \times 10^{-11} \) Hz s\(^{-1}\). We discuss possible scenarios that can account for the long-term neutron star spin-down in terms of either magneto-dipole emission, emission of gravitational waves, and a propeller effect. If interpreted in terms of magneto-dipole emission, the measured spin down translates into an upper limit to the neutron star magnetic field, \( B \lesssim 3 \times 10^{10} \) G, while an upper limit to the average neutron star mass quadrupole moment of \( Q < 2 \times 10^{36} \) g cm\(^2\) is set if the spin down is interpreted in terms of the emission of gravitational waves.

Key words. gravitational waves — stars: neutron — stars: rotation — pulsars: individual (IGR J00291+5934) — X-rays: binaries

1. Introduction

The discovery of the first accreting millisecond pulsar (AMSP) in 1998, SAX J1808.4–3658 (Wijnands \& van der Klis 1998), confirmed the predictions of the recycling scenario, according to which millisecond radio pulsars are the end product of a long phase of accretion of matter and angular momentum onto a neutron star (NS) hosted in a low mass X-ray binary (see, e.g., Bhattacharya \& van den Heuvel 1991). In the twelve years since the first discovery, the class of AMSPs has grown to thirteen members, all X-ray transients. To perform a timing analysis of different outbursts of the same source allows one to estimate the system orbital evolution. The orbital period has been observed to increase at a rate of nearly two orders of magnitude larger than what is predicted by conservative mass transfer (Di Salvo et al. 2008; Burderi et al. 2009, see also Hartman et al. 2008; H08 hereafter). This has led the authors to argue that a large fraction of the mass transferred by the companion star is ejected by the system taking away the angular momentum needed to match the observed value. A regular NS spin down has also been measured by H08 (see also Hartman et al. 2009) leading to stringent upper limits on the various mechanisms that can brake down a pulsar during quiescence such as magneto-dipole emission, emission of gravitational waves and a propeller effect. These effects, and in particular the spin down torque associated with the emission of gravitational waves, \( N_{GW} \), crucially depend on the spin frequency of the NS (\( N_{GW} \propto \nu^5 \)). It is therefore very appealing to shed light on the long-term behaviour of the fastest AMSP discovered so far, the 598.89 Hz pulsar IGR J00291+5934 (J00291 in the following). In this paper, we present a timing analysis based on the two outbursts shown by the source in 2008, and observed by the Rossi X-ray Timing Explorer (RXTE). The results thus obtained are compared with the rotational state of J00291 at the end of the outburst exhibited on 2004 December, that is the only other outburst of this source for which high temporal resolution data are available.

2. Observations

The X-ray transient J00291, was discovered by INTEGRAL on 2004 December 2 (Shaw et al. 2005). The 598.89 Hz pulsations found in its light curve make it the fastest AMSP discovered so far (Galloway et al. 2005, G05 hereinafter).

Renewed activity was detected by RXTE on 2008 August 13 (Chakrabarty et al. 2008). The 2.5–25 keV X-ray flux\textsuperscript{1} reaches a peak level of \((6.3\pm0.2)\times10^{-10}\) erg cm\(^{-2}\) s\(^{-1}\), which is \(\approx 0.5\) times the peak flux observed during the 2004 outburst (G05). The flux decreases on a timescale \(\tau \approx 3\) d and the source returns to quiescence \(\sim 5\) d after the first detection. The light curve recorded by the PCA of the Proportional Counter Array (PCA) aboard RXTE is plotted in Fig.\textsuperscript{1}. As the nearby source V709 Cas (17 arcmin away) contributes to the X-ray flux detected by RXTE in the direction of J00291 (Markwardt \& Swank 2008), the observed count-rate stays at a level of \(\sim 6\) c s\(^{-1}\) PCU\(^{-1}\) (corresponding to \((7.2\pm2)\times10^{-11}\) erg cm\(^{-2}\) s\(^{-1}\); 2.5–25 keV) even when the J00291 outburst is presumably over. J00291 is again detected in outburst

\textsuperscript{1} The spectrum of J00291, as observed by the PCA aboard RXTE, is evaluated by modelling data recorded by the top layer of the PCU with an absorbed power law. We fix the nH to \(0.43\times10^{22}\) cm\(^{-2}\) (Paizis et al. 2005). A 6.4 keV iron line is sometimes needed to model the spectrum.
on 2008 September, 21 and the fluence of this second episode is similar to that of the first one.

To perform a timing analysis on the 598.89 Hz pulsar signal, we consider events recorded by the PCA (Obsid P93013) both in good xenon (1µs temporal resolution), and event mode (125µs temporal resolution) configurations. All the arrival times were first corrected with respect to the Solar System barycentre, considering the position of the optical counterpart determined by Torres et al. (2008, T08 hereinafter), RA=00° 29′ 03″.05 ± 0′.01, DEC=59° 34′ 18″.93 ± 0′.05. A re-analysis of the data taken by RXTE during the 2004 outburst (ObsId P90052 and P90425), is also reported. Despite a temporal analysis of the 2004 outburst of J00291 having already been performed by G05, Falanga et al. (2005, F05 in the following) and Burderi et al. (2007, B07), such a re-analysis is aimed at deriving the most accurate estimate of the spin frequency at the end of the outburst that can then be compared to the spin frequency of the source measured in 2008, after ≃ 3 yr of quiescence.

3. Temporal analysis

3.1. The 2008 outbursts

To check the presence of pulsations during the 2008 outbursts of J00291, we first correct the photon-arrival times for the source orbital motion. As no eccentricity was detected as a result of the timing analysis performed on the data of the 2004 outburst (G05, see also Sect. 3.2), we consider a circular orbit to correct photon-arrival times, \( t_{\text{em}} - t_{\text{arr}} = x \sin[\ell(t_{\text{em}})] \). Here \( t_{\text{arr}} \) and \( t_{\text{em}} \) are the photon arrival and emission times, respectively, \( x = a \sin \theta / c \) the projected semi-major axis of the NS orbit, \( \ell(t_{\text{em}}) = 2\pi(t_{\text{em}} - T^*/P_{\text{orb}}) \) the mean orbital longitude, \( P_{\text{orb}} \) the orbital period, and \( T^* \) the epoch at which the mean orbital longitude is equal to zero (see Deeter et al. (1983), for a discussion of this choice of orbital epoch). After a set of values for the orbital parameters is considered, the corrected emission times are obtained by iterating the above relation until the difference between successive steps is of the order of the RXTE absolute timing accuracy (3.4µs, Jahoda et al. 2006). We first consider the values of \( x \) and \( P_{\text{orb}} \) given by G05 as orbital evolution is not expected to change them significantly during the time elapsed between the 2004 and 2008 outbursts (≃ 3.7 yr). The propagation of the error in the value of \( T^* \) quoted by G05 yields instead an uncertainty of ≃ 80 s. We then use the technique described by Papitto et al. (2005) to improve the estimate of \( T^* \). The difference of ≃ −200 s between the improved estimate we find and the value predicted according to the G05 solution indicates how a correction of ≃ −0.015 s (2.5σ from the G05 estimate) to the value of the orbital period has to be applied. Using this improved orbital solution and folding 500s-long data segments in 12 phase bins around the frequency \( \nu_{108} = 598.89213046 \) Hz, we detect pulsations at the 99% confidence level in the interval MJD 54691.9–54696.8, which we refer to as the 2008 August outburst. Pulses are again detected in the interval MJD 54730.5–54740.1 (2008 September outburst), after observations have been folded around the frequency \( \nu_{108} = 598.89213060 \) Hz. A detection is assessed according to the criterion stated by Leahy et al. (1983), rejecting the profiles that have a probability larger than 1% of being due to chance.

The pulse profiles have an rms amplitude of ≃ 8% and are successfully modelled by a sinusoid. We fit the phases thus evaluated with the relation

\[
\phi(t) = \varphi(0) + (\nu_0 - \nu_f)(t - T_{\text{ref}}) + \frac{1}{2} \nu_f(t - T_{\text{ref}})^2 + R_{\text{orb}}(t),
\]

where \( T_{\text{ref}} \) is the reference epoch for the timing solution, \( \nu_0 \) and \( \nu_f \) are the pulsar frequency at the reference epoch and its mean derivative across the outburst, respectively, and \( R_{\text{orb}}(t) \) describes the phase residuals due to a difference between the parameters used to correct photon arrival times and the actual orbital parameters of the system. Neglecting second-order terms in the ec-
the best estimate of the spin frequency at the beginning of that outburst, ν\(F_0\) = 598.89213060 Hz.

The results we obtained, applying the same procedure outlined in Sect. 3.1, are listed in the rightmost column of Table 1. Only the parabolic model (\(\dot{\nu} = 0\)) is presented because the quadratic term is highly significant (\(\Delta\chi^2 = 303\) over 429 degrees of freedom with respect to a constant frequency model, which has one degree of freedom more). The timing solution is referred to an epoch at the end of the outburst to estimate the spin frequency after the accretion-induced spin-up is over, which can therefore be compared to the frequency at the beginning of the 2008 August outburst. The orbital parameters we obtain are compatible with those previously published by G05, F05, and B07, and are somewhat more precise as they rely on a longer baseline. The addition of three days of data to the data set considered by F05 and B07 indicates an average spin-up term \(ν_{\text{tot}} = (+5.1 \pm 0.3) \times 10^{-15}\) Hz s\(^{-1}\), which is lower with respect to those there evaluated [(8.5 \pm 1.1) \times 10^{-15}\) Hz s\(^{-1}\)]. The phase evolution and residuals with respect to the best-fit model are plotted in Fig. 4.

### 3.2. The 2004 outburst

This work is mainly focused on the study of the spin down experienced by J00291 during the quiescent phase that lasted since the end of the 2004 outburst to the onset of the 2008 August outburst. To this aim, the most accurate estimate of the spin frequency at the end of the 2004 outburst is needed. Previous works relied only on a fraction of the available data; G05 considered only the first three days of data, ObsId P90052, while the solutions of F05 and B07 are valid for the subsequent seven days, ObsId P90425, the only set of which data were publicly available at that time. We re-analysed the 2004 outburst including all the available RXTE data (ObsId P90052 and P90425). Time series were corrected using the position of the optical counterpart (T08), while the position of the proposed radio counterpart (Rupen et al. 2004) was considered in previous works. The results we obtained, applying the same procedure outlined in Sect. 3.1, are listed in the rightmost column of Table 1. Only the parabolic model (\(\dot{\nu} = 0\)) is presented because the quadratic term is highly significant (\(\Delta\chi^2 = 303\) over 429 degrees of freedom with respect to a constant frequency model, which has one degree of freedom more). The timing solution is referred to an epoch at the end of the outburst to estimate the spin frequency after the accretion-induced spin-up is over, which can therefore be compared to the frequency at the beginning of the 2008 August outburst. The orbital parameters we obtain are compatible with those previously published by G05, F05, and B07, and are somewhat more precise as they rely on a longer baseline. The addition of three days of data to the data set considered by F05 and B07 indicates an average spin-up term \(ν_{\text{tot}} = (+5.1 \pm 0.3) \times 10^{-15}\) Hz s\(^{-1}\), which is lower with respect to those there evaluated [(8.5 \pm 1.1) \times 10^{-15}\) Hz s\(^{-1}\)]. The phase evolution and residuals with respect to the best-fit model are plotted in Fig. 4.

### 3.3. The positional uncertainty

The uncertainties quoted in Table 1 are 1\(\sigma\) error based on the modelling of the phase evolution. However, the error in the
source position introduces an uncertainty in the determination of the pulse phases

\[ \Delta \phi_{\text{pos}} = \nu y (\sin(M_0 + \epsilon) \cos \beta \delta \lambda - \cos(M_0 + \epsilon) \sin \beta \delta \beta), \]

where \( y \) is the Earth distance from the Solar System barycentre in lt-s, \( \lambda \) and \( \beta \) are the ecliptic longitude and latitude, respectively, \( \delta \lambda \) and \( \delta \beta \) are the respective uncertainties, \( M_0 = (2n(T_0 - T_i)/P_{\text{orb}} - \lambda, T_0 \) is the start time of observations considered, \( T_i \) is the nearest epoch of passage at the vernal point, \( P_{\text{orb}} \) is the Earth orbital period, and \( \epsilon = 2n(t - T_i)/P_{\text{orb}} \) (see, e.g., [Lyne & Graham-Smith 1990]). For time intervals that are small with respect to the Earth orbital period (i.e. \( \epsilon << 1 \)) such as the ones considered in this work, this expression can be expanded as a polynomial and only the lowest order terms retained. As the value of the spin frequency at the reference epoch of the timing solution depends on the linear term of the temporal evolution of the phases [see Eq. (1)], the systematic uncertainty introduced by the position error in this measure, \( \delta \phi_{\text{pos}} \), is

\[ \delta \nu_{\text{pos}} = \nu y \left( \frac{2\pi}{P_{\text{orb}}} \right) \left[ \cos M_0 \cos \beta \delta \lambda + \sin M_0 \sin \beta \delta \beta \right]. \]

Considering the uncertainties in the position quoted by T08 \( \sigma_{\nu_{\text{pos}}} \leq (3.9 \times 10^{-3}) \), \( \sigma_{\nu} \leq (2.4 \times 10^{-3}) \), and evaluating this relation for the 2008 and the 2004 outbursts translates into \( \sigma_{\nu_{\text{pos}}} \approx 1 \times 10^{-8} \) Hz and \( \sigma_{\nu} \approx 3 \times 10^{-8} \) Hz, respectively. To get reliable estimates of the uncertainties affecting each measured spin frequency, these systematic errors have to be summed in quadrature with the statistical errors quoted in Table 1. The systematic error in the frequency variation between the two outbursts can instead be estimated as \( \sigma_{\Delta \nu_{\text{pos}}} \approx 4 \times 10^{-8} \) Hz.

### 3.4. The spin evolution of J00291 during quiescence

The estimate of the spin frequency of J00291 at the beginning of the 2008 August can be compared with the spin measured at the end of the 2004 outburst to measure the frequency variation experienced by the source during quiescence. Considering the value measured using the \( \nu \neq 0 \) model at the onset of the 2008 August outburst \( \nu_{\text{onset}} = 598.89213046(13) \) Hz, we thus obtain \( \Delta \nu = \nu_{\text{onset}} - \nu_{\text{pos}} = -0.48 \pm 0.13 \pm 0.04 \) \( \mu \)Hz, where the first error quoted is the statistical error given by the difference of the values quoted in Table 1 and the latter reflects the uncertainty on the source position. The average spin-down rate during quiescence is therefore estimated as \( \dot{\nu}_{\text{sd}} = (-4.1 \pm 1.1 \pm 0.3) \times 10^{-15} \) Hz s\(^{-1}\). The large uncertainty affecting this estimate is due to the limited statistics available for the 2008 August outburst which imply loose estimates of the spin frequency at the beginning of that outburst and of the spin frequency derivative during that outburst (see also the discussion).

### 4. Discussion and conclusions

We have presented a detailed timing analysis concerning the two outbursts shown by J00291 in 2008 August and September as observed by RXTE, as well as a re-analysis of the 2004 outburst using the whole RXTE dataset available.

Our analysis of the 2004 data confirms the significant spin up the source has undergone while accreting as already reported by F05 and B07. The estimate of the spin up rate we have presented here \( \dot{\nu}_{\text{sd}} = (+3.1 \pm 0.3) \times 10^{-13} \) Hz s\(^{-1}\) is evaluated on a longer temporal baseline with respect to that considered by those authors and is accordingly more accurate. The magnitude of the spin up is lower by a factor \(-0.4\) with respect to that quoted by those works, making its interpretation easier in terms of the NS accretion of the supposedly Keplerian disc angular momentum at the flux emitted by the source (see discussion in B07).

In contrast, no derivative is detected significantly during the 2008, August and September, outbursts, with 3\( \sigma \) upper limits on the spin up component of \( |\dot{\nu}_{\text{su}}| < 2 \times 10^{-12} \) and \( |\dot{\nu}_{\text{su}}| < 4.5 \times 10^{-11} \) Hz s\(^{-1}\), respectively. These estimates reflect the limited statistics available. An accretion-induced spin-up is expected to depend almost linearly on the mass accretion rate \( \dot{\nu} \propto M^{1/2} \), where \( \alpha \) is the index of the dependence of the inner disk radius on the mass accretion rate, \( R_{\text{in}} \propto M^{\alpha/2} \), \( \alpha = 2/7 \) if the inner disc radius is approximated by the Alfven radius). As the peak X-ray flux shown by the source during both 2008 outbursts is roughly half that of the 2004 outburst, the spin up rate is expected to scale accordingly, provided that the flux is a good tracer of the mass accretion rate. We thus expect \( \dot{\nu} \approx 2.5 \times 10^{-13} \) Hz s\(^{-1}\) during each of the outbursts shown by the source in 2008. While the upper limit to the September 2008 outburst spin up is of the same order as this value, the upper limit to the August 2008 data is one order of magnitude larger and can therefore not be considered as a tight constraint. This can also be viewed by considering the

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**Table 1.** Spin and orbital parameters of J00291.

| Parameter | 2008 August | 2008 September | 2004 December |
|-----------|-------------|----------------|--------------|
| \( \Delta \nu \) | 54691.9 - 54696.8 | 54730.5 - 54740.1 | 55342.3 - 55352.0 |
| \( T_{\text{obs}} \) (MJD) | 54691.939 | 54730.500 | 55352.0 |
| \( \text{asinc/c (lt-ms)} \) | 64.988(8) | 64.982(6) | 64.990(4) |
| \( P_{\text{orb}} \) (s) | 8844.07(2) | 8844.078(9) | 8844.079(1) |
| \( T^* \) (MJD) | 54691.938749(5) | 54730.529222(5) | 53345.1619264(5) |
| \( \nu = 0 \) model | | | |
| \( \nu \) (Hz) | 598.89213082(4) | 598.89213082(2) | |
| \( \chi^2/\text{dof} \) | 80.7/48 | 42.2/41 | |
| \( \nu \neq 0 \) model | | | |
| \( \nu \) (Hz) | 598.89213046(13) | 598.89213060(8) | 598.89213094(1) |
| \( \chi^2/\text{dof} \) | 68.9/47 | 34.7/40 | 497.3/429 |

**Notes.** Numbers in parentheses are 1\( \sigma \) errors in the last significant digit. Upper limits are evaluated at 3\( \sigma \) confidence level. The uncertainties have been scaled by a factor \( \sqrt{2} \) to take into account a reduced \( \chi^2 \) of the best-fit model larger than 1. The uncertainties quoted in the estimates of \( \nu \) and \( \dot{\nu} \) do not include the systematic errors due to the positional uncertainty. \( ^{(a)} \) Time interval covered by the presented solution \( ^{(b)} \) Reference epoch for the timing solution.
The spin down torque associated with the emission of gravitational radiation has been proposed to explain the non-detection of accreting pulsars with frequencies higher than \( \approx 730 \text{ Hz} \) (Chakrabarty et al. 2003; see Wasater 1984; Bildsten 1998; Melatos & Payne 2005 for models describing mechanism that can lead to a non-zero mass quadrupole for an accreting pulsar). In this case, the spin-down torque is, \( \mathbf{\Omega}_{GW} = -(32/3)G(2\nu/c^3)^3 \sin^2 \alpha \) is the usual expression for the torque acting on a magnetised rotator in vacuum. The estimate of \( \dot{\nu}_{sd} \) we have given translates into a value of the magnetic dipole of \( \mu \approx 1.1(2) \times 10^{36} \, \mu_{I5} \) (1 + \( \sin^2 \alpha \)) G cm\(^2\), where \( \mu_{I5} \) is the moment of inertia in units of 10\(^{35}\) G cm\(^2\). This translates into a magnetic field, \( B \approx 2.2(4) \times 10^{8} \, \mu_{I5} \) (1 + \( \sin^2 \alpha \)) G at the magnetic poles of a 10 km NS. Considering \( \alpha = 0 \), an upper limit of \( \approx 3 \times 10^{8} \) G (3\( \sigma \) confidence level) on the magnetic field is obtained. This estimate fits well into the expected range of magnetic field strengths for the AMSPs to be the progenitors of recycled radio millisecond pulsars (\( \approx 10^{9} - 10^{10} \) G). It is also compatible with the requirements set on the dipole strength by the maximum and minimum accretion rate experienced by the source while showing pulsations. For pulsations to be observed, the magnetospheric radius has to lie between the NS radius, \( R_{NS} \), and the corotation radius, \( R_{C} = (GM/4\pi^2c^3)^{1/3} \) (23.6 m\(^{1/3}\)) km for J00291, where \( R_{NS} \) is the NS mass in units of 1.4 M\(_{\odot}\). The minimum flux at which we observe pulsations during the 2004 outburst is \( F_{\nu} < \approx 2 \times 10^{-15} \, \text{erg cm}^{-2} \text{s}^{-1} \) (MJD 54696, 751). Assuming that the bolometric correction factor that derived by G05 (2.54) and that the observed X-ray flux reflects the mass accretion rate, this translates into \( M_{\text{min}} = 0.9 \times 10^{-10} \, m_{I5} \, R_{10} \, d_{15} \, M_{\odot} \, \text{yr}^{-1} \), where \( R_{10} \) is the radius of the NS in units of 10 km, and \( d_{15} \) is the distance to the source in units of 4 kpc. Considering the value quoted by G05 for the peak flux during the 2004 outburst, the maximum accretion rate at which pulsations were observed can be estimated as \( M_{\text{max}} \approx 4.7 \times 10^{-10} \, m_{I5} \, R_{10} \, d_{15} \, M_{\odot} \, \text{yr}^{-1} \). Using the expressions derived by Psaltis & Chakrabarti (1999), the presence of pulsations at these two limiting accretion rates indicates that the magnetic dipole has to lie in the range, \( (0.2 - 21) \times 10^{26} \, d_{15} \, G \, cm^3 \), fully compatible with our estimate. As the minimum flux at which pulsations are observed is likely overestimated by a factor of \( \approx 2 \) because of the contribution of V709 Cas, the upper limit to the magnetic dipole is likely to be a factor \( \sqrt{2} \) smaller. Considering also the dynamical estimate of the maximum mass-accretion rate derived by B07 from the spin up rate observed during the 2004 outburst, the lower limit to the dipole strength increases to \( 0.6 \times 10^{26} \, G \, cm^3 \), still compatible with the estimate derived here. Our estimate of the magnetic field strength is also compatible with the upper limit estimated by T08 from the X-ray quiescent luminosity, \( < 3 \times 10^{8} \, G \), using the criteria stated by [Burderi et al. (2002) and Di Salvo & Burderi (2003)]. The spin down torque associated with the emission of gravitational radiation has been proposed to explain the non-detection of accreting pulsars with frequencies higher than \( \approx 730 \text{ Hz} \) (Chakrabarty et al. 2003; see Wasater 1984; Bildsten 1998; Melatos & Payne 2005 for models describing mechanism that can lead to a non-zero mass quadrupole for an accreting pulsar). In this case, the spin-down torque is, \( \mathbf{\Omega}_{GW} = -(32/3)G(2\nu/c^3)^3 \sin^2 \alpha \) is the usual expression for the torque acting on a magnetised rotator in vacuum. The estimate of \( \dot{\nu}_{sd} \) we have given translates into a value of the magnetic dipole of \( \mu \approx 1.1(2) \times 10^{36} \, \mu_{I5} \) (1 + \( \sin^2 \alpha \)) G cm\(^2\), where \( \mu_{I5} \) is the moment of inertia in units of 10\(^{35}\) G cm\(^2\). This translates into a magnetic field, \( B \approx 2.2(4) \times 10^{8} \, \mu_{I5} \) (1 + \( \sin^2 \alpha \)) G at the magnetic poles of a 10 km NS. Considering \( \alpha = 0 \), an upper limit of \( \approx 3 \times 10^{8} \) G (3\( \sigma \) confidence level) on the magnetic field is obtained. This estimate fits well into the expected range of magnetic field strengths for the AMSPs to be the progenitors of recycled radio millisecond pulsars (\( \approx 10^{9} - 10^{10} \) G). It is also compatible with the requirements set on the dipole strength by the maximum and minimum accretion rate experienced by the source while showing pulsations. For pulsations to be observed, the magnetospheric radius has to lie between the NS radius, \( R_{NS} \), and the corotation radius, \( R_{C} = (GM/4\pi^2c^3)^{1/3} \) (23.6 m\(^{1/3}\)) km for J00291, where \( R_{NS} \) is the NS mass in units of 1.4 M\(_{\odot}\). The minimum flux at which we observe pulsations during the 2004 outburst is \( F_{\nu} < \approx 2 \times 10^{-15} \, \text{erg cm}^{-2} \text{s}^{-1} \) (MJD 54696, 751). Assuming that the bolometric correction factor that derived by G05 (2.54) and that the observed X-ray flux reflects the mass accretion rate, this translates into \( M_{\text{min}} = 0.9 \times 10^{-10} \, m_{I5} \, R_{10} \, d_{15} \, M_{\odot} \, \text{yr}^{-1} \), where \( R_{10} \) is the radius of the NS in units of 10 km, and \( d_{15} \) is the distance to the source in units of 4 kpc. Considering the value quoted by G05 for the peak flux during the 2004 outburst, the maximum accretion rate at which pulsations were observed can be estimated as \( M_{\text{max}} \approx 4.7 \times 10^{-10} \, m_{I5} \, R_{10} \, d_{15} \, M_{\odot} \, \text{yr}^{-1} \). Using the expressions derived by Psaltis & Chakrabarti (1999), the presence of pulsations at these two limiting accretion rates indicates that the magnetic dipole has to lie in the range, \( (0.2 - 21) \times 10^{26} \, d_{15} \, G \, cm^3 \), fully compatible with our estimate. 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Observations of future outbursts from this source will be used to monitor the constancy of the long-term spin down, and to derive tighter constraints on the parameters of the NS in J00291.

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Soon after this paper was first submitted, other two papers discussing the rotational evolution of this source during quiescence appeared on arXiv.org (Patruno 2010; Hartman et al. 2011). Even if the analysis presented by these authors slightly differs with respect to that presented here [in particular Patruno (2010) derived a timing solution under the preliminary assumption that pulse phases linearly correlate with the X-ray flux], the values they obtain for the spin-down rate of the source during quiescence are entirely compatible with that presented here. We thank Jacob M. Hartman for useful discussions and comments on this paper.

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