Properties of Type-Ia Supernova Light Curves

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Abstract

I show that the characteristic diffusion timescale and the gamma-ray escape timescale, of SN Ia ejecta, are related with each other through the time when the bolometric luminosity, $L_{\text{bol}}$, intersects with instantaneous radioactive decay luminosity, $L_{\gamma}$, for the second time after the light-curve peak. Analytical arguments, numerical radiation-transport calculations, and observational tests show that $L_{\text{bol}}$ generally intersects $L_{\gamma}$ at roughly 1.7 times the characteristic diffusion timescale of the ejecta. This relation implies that the gamma-ray escape timescale is typically 2.7 times the diffusion timescale, and also implies that the bolometric luminosity 15 days after the peak, $L_{\text{bol}}(t_{15})$, must be close to the instantaneous decay luminosity at that time, $L_{\gamma}(t_{15})$. With the employed calculations and observations, the accuracy of $L_{\text{bol}} = L_{\gamma}$ at $t = t_{15}$ is found to be comparable to the simple version of “Arnett’s rule” ($L_{\text{bol}} = L_{\gamma}$ at $t = t_{\text{peak}}$). This relation aids the interpretation of SN Ia light curves and may also be applicable to general hydrogen-free explosion scenarios powered by other central engines.

Key words: supernovae: general

1. Introduction

 Supernovae of Type-Ia are believed to result from thermonuclear explosions of white dwarfs (Hoyle & Fowler 1960). While they play a major role as a cosmographic tool, the identity of the progenitors and the nature of the ignition process remain a mystery (for a general review, see Maoz et al. 2014). In this work, however, an agnostic stance is taken on the exact nature of the progenitor or explosion, and instead I focus on the generic properties of the light curves.

 It is well known that the optical display of a Type-Ia explosion is predominantly powered by the radioactive decay chain of $^{56}\text{Ni} \rightarrow^{56}\text{Co} \rightarrow^{56}\text{Fe}$ (Pankey 1962; Truran et al. 1967; Bodansky et al. 1968; Colgate & McKee 1969), whose power is a precisely known exponentially decaying function of time (e.g., Nadyozhin 1994,

$$L_{\gamma} = \frac{M_{\text{Ni}}}{M_{\odot}}(C_{\text{Ni}}e^{-t/\tau_{\text{Ni}}} + C_{\text{Co}}e^{-t/\tau_{\text{Co}}}) \text{ erg s}^{-1},$$

where $t$ and $M_{\text{Ni}}$ are time and $^{56}\text{Ni}$ mass, and $C_{\text{Ni}} \approx 6.45 \times 10^{13}$, $C_{\text{Co}} \approx 1.45 \times 10^{13}$, $\tau_{\text{Ni}} = 8.8$ days and $\tau_{\text{Co}} = 111.3$ days. A time-dependent fraction of this energy input is thermalized in the expanding ejecta, and thus the actual shape of the light curve is dictated by the competition between adiabatic degradation of internal energy into kinetic energy, and the loss of internal energy via radiation.

 Figure 1 provides a schematic Type-Ia light curve. Since the explosion starts from a compact star, the ejecta begins its life as an opaque ball of plasma and thus the bolometric luminosity ($L_{\text{bol}}$, black solid) rises at early times. Though the instantaneous radioactive power ($L_{\gamma}$, red solid line) is very high at this time, the time that it takes for radiation to diffuse out of the ejecta is much larger than the age. As the expansion reduces the density, eventually the age of the ejecta surpasses the diffusion time of

$$t_{d} = \left[\frac{3kM_{\text{ej}}}{4\pi\kappa cv}\right]^{\frac{1}{2}},$$

where $k$, $M_{\text{ej}}$, $c$, and $v$ are the opacity, ejecta mass, speed of light and ejecta velocity, respectively. Near this point, a significant amount of the deposited energy can be radiated rather than converted into kinetic energy for the first time, and the light curve reaches its peak. The well-known “Arnett’s rule” (Arnett 1979, 1982) defines this feature; he first showed that the light-curve peak must be close to the instantaneous decay power at that time (point A).

 But this is not the only time the bolometric luminosity equals the decay power. Shortly after the peak of the light curve, there is a significant amount of radiation still trapped and diffusing outward in the ejecta (Pinto & Eastman 2000). The bolometric luminosity remains greater than the decay power for some time while this “excess” energy is drained and the decay power falls onto the more slowly declining $^{56}\text{Co} \rightarrow^{56}\text{Fe}$ curve. Therefore the bolometric luminosity crosses the decay power again after the peak at point B.

 Note that the bolometric luminosity crosses the decay power for a second time, not the actual heating rate ($H$, pink dashed line). Due to the escape of $\gamma$-rays, the non-thermalized fraction of the decay power increases with time, so the heating rate crosses $L_{\text{bol}}$ only once near the peak, and at late times it asymptotes to $L_{\text{bol}}$. Also, the existence of point B may not be a universal feature of Type-Ia light curves. In extreme cases, the decay power may barely graze the bolometric luminosity at a single point near the peak, or it may not cross at all.

 The aim of this study is to understand how the time, $t_{B}$, of point B relates to the time, $t_{A}$, of point A. While the classical papers by Arnett (1982) and Pinto & Eastman (2000) provide semi-analytical model light curves, simple arguments are employed here to show why the time of point B is expected to be a constant multiple of the characteristic diffusion timescale of the ejecta. The claim is tested using both a set of Monte-Carlo radiation-transport calculations describing
various possible ejecta configurations, and also observational data, to find that \( t_B/t_d \approx 1.7 \). In the end, one of the main implications of this finding—the bolometric luminosity at 15 days after the peak is close to the radioactive decay power at that time, \( \dot{L}_{\text{bol}}(t_{15}) \approx \dot{L}_{\gamma}(t_{15}) \), is tested and I provide a discussion of how potentially this can be applied in the study of the observed width–luminosity relation (WLR; Phillips et al. 1999).

2. Semi-analytical Arguments

Energy conservation for the expanding ejecta implies that

\[
\frac{dE}{dt} + P\frac{dV}{dt} + L_{\text{bol}} = H, \tag{3}
\]

where \( E, P \), and \( V \) are internal energy, pressure, and volume, respectively. The energy deposited from radioactive decay is stored, spent doing work on the expanding ejecta, and lost through the photosphere. Next, we employ two excellent and commonly invoked assumptions. First, the ejecta are homologously expanding in time with an isotropic velocity gradient, such that for a uniform density profile \( dV/dt = 4\pi\rho V^2 \). Second, the plasma is radiation-dominated, with \( P = E/3V \). Under these assumptions,

\[
\frac{dE}{dt} + \rho\frac{dV}{dt} = \frac{1}{t} \frac{d(tE)}{dt} \tag{4}
\]

Writing the heating rate as \( H = \dot{L}_\gamma F_\gamma \), where \( F_\gamma \) is the time-dependent deposition fraction, Equation (3) becomes

\[
\frac{1}{t} \frac{d(tE)}{dt} = \dot{L}_\gamma F_\gamma - \dot{L}_{\text{bol}}. \tag{5}
\]

This is equivalent to Equation 10 of Kasen & Bildsten (2010) with a magnetar spin-down power \( \dot{L}_P \) instead of radioactivity, and also to Equation (2) of Katz et al. (2013), before time integration.

A critical piece in Equation (5) is \( F_\gamma \), where it must be a function that stays close to unity at early times, and then gradually asymptotes to zero at late times. First, consider early times, as they provide a simple way to derive the point A. When \( F_\gamma = 1 \), the condition \( \dot{L}_{\text{bol}} = \dot{L}_\gamma \) is satisfied only when \( d(tE)/dt = 0 \). Employing the diffusion equation, one can derive an approximate relation,

\[
\dot{L}_{\text{bol}} \approx \frac{4\pi RcE}{3 \kappa \rho V} = \frac{tE}{t_d^2}, \tag{6}
\]

between bolometric luminosity and internal energy, where \( R \) and \( \rho \) are the radius and density. Taking time derivatives of both sides in Equation (6), one sees that \( d(tE)/dt = 0 \) is true when \( dt_{\text{bol}}/dt = 0 \), which is satisfied at the peak of the light curve (near the point A).

As a more general and realistic description I adopt \( F_\gamma = 1 - e^{-\tau} \) (e.g., Pinto & Eastman 2001). Here, \( \tau = (t_v/t_d)^2 \) is the mean optical depth for gamma-rays, representing the mostly absorptive nature of Compton opacity. Due to the homologous expansion, the optical depth scales as \( r^{-2} \) and the timescale \( t_d \) is chosen such that \( \tau(t_d) = 1 \).

Positrons from the decay of \(^{56}\text{Co}\) will only start escaping the ejecta at very late times (e.g., Milne et al. 1999), and therefore more formally \( F_\gamma \) should apply to only the gamma-component of \( \dot{L}_\gamma \). But for the sake of simplicity, the energetically less important positron contribution is ignored. A comparison of this prescription with a sample Monte-Carlo radiation-transport calculation (presented in Section 3.1) is illustrated in Figure 2, where \( t_v \sim 46 \) days reproduces this specific model well.

With this description of \( F_\gamma \), Equation (5) becomes

\[
\dot{L}_\gamma e^{-\tau} + \frac{1}{t} \frac{d(tE)}{dt} = \dot{L}_\gamma - \dot{L}_{\text{bol}}, \tag{7}
\]

The opacity changes with temperature and composition and may have significant time dependence (e.g., Khokhlov et al. 1993), which is ignored in this simplistic argument. For a more general derivation see Arnett et al. (2017), and Khatami & Kasen (2018).
where \( L^* = L_{\text{bol}} \) is satisfied on two conditions: (1) \( L^* e^{-\gamma t} = dt(E)/dt/t = 0 \), equivalent to the condition for point A discussed above for \( F_L \equiv 1 \); and (2) \[
\frac{dt(E)}{dt} = -L^* e^{-\gamma t}, \tag{8}
\]
which is the condition for point B, and is satisfied only when \( t = t_B \). Unfortunately Equation (8) is not analytically integrable to elementary functions without approximations to the term \( e^{-\gamma t} \).

But Equation (8) can be simplified using the fact that the internal energy changes in time roughly as \( dE/dt \approx -2E/t \), near point B. This can be seen from the relationship between \( E \) and \( dt(E)/dt \), illustrated in Figure 3 for a sample model. Note the quantity \( d(tE)/dt \) shares a common tangent with \(-E\) near the time \( t \approx t_B \). Considering a general form of \( dE/dt = xE/t \), where \( x \) is a negative real number for \( t > t_B \), the time derivative of the left-hand term in Equation (8) becomes

\[
\frac{d}{dt} \left( \frac{d(tE)}{dt} \right) = \frac{E}{t} (x^2 + x). \tag{9}
\]

For example, the rate of change in the internal energy at the minimum of \( dE/dt \) can be found by solving \( x^2 + x = 0 \) for its non-zero root as \( dE/dt = -E/t \). Since the curves \( d(tE)/dt \) and \(-E\) are tangents to each other near the point of interest, the time derivatives must also be similar at that point. Thus, the solution to \( x^2 + x = -x \) gives \( dE/dt = -2E/t \) near \( t \approx t_B \).

While this argument does not explain why the two curves are expected to share a common tangent near \( t \approx t_B \), numerical calculations (Section 3.1) demonstrate that it is a good assumption.

Taking \( dE/dt \approx -2E/t \), and replacing \( E \) with Equation (6), Equation (8) becomes

\[
L_{\text{bol}}(t_B) t_B^{-1} \tau_B^{-1} \approx L^* (t_B) e^{-\gamma t_B} t_B^{-3/2}. \tag{10}
\]

Recall that the condition for point B is \( L_{\text{bol}}(t_B) = L^* (t_B) \), and thus the time of point B satisfies

\[
t_B / t_B \approx e^{(t_B / t_B)^{3/2}}. \tag{11}
\]

This implies that if the time of point B, \( t_B \), is proportional or slowly varying with \( t \), across a wide range of models, then one should expect it to also be a constant multiple of diffusion timescale \( t_d \). Furthermore, since the energy deposition terms cancel, Equation (11) may also be applicable to other types of central power sources, such as magnetar spin-down (Kasen & Bildsten 2010; Woosley 2010) and black hole accretion (Dexter & Kasen 2013).

3. Numerical Models and Observations

In this section, the arguments presented in Section 2 are explored with simple numerical calculations and with a set of bolometric measurements available from the literature.

3.1. Radiation-transport Calculations

The time-dependent Monte-Carlo radiation-transport code developed by Lucy (2005) is used. The code includes Compton-scattering and photoelectric absorption for \( \gamma \)-ray transport, and employs gray transport for the optical radiation. Despite its simplicity, the resulting bolometric light curves are in good agreement with more advanced tools (e.g., SEDONA; Kasen et al. 2006).

A small grid of 48 model light curves was generated based on a simple template. The ejecta are assumed to have a uniform density and the \( ^{56}\text{Ni} \) mass is confined to the innermost region. The ejecta mass is varied between \( 0.8 < M_{ej} < 2.2 M_\odot \), and the \( ^{56}\text{Ni} \) mass was varied between \( 0.2 < M_{Ni}/M_{ej} < 0.7 \), so as to sample the wide range of possibilities emerging from various progenitor and explosion scenarios. For simplicity, a constant gray opacity of 0.1 cm\(^2\)g\(^{-1}\) is employed, and the initial outer radius and velocity of the ejecta were kept constant at \( 10^8 \) cm and \( 10^8 \text{ cm s}^{-1} \), respectively, for each model. Each spherically symmetric model is computed with 100 spatial zones and \( 5 \times 10^4 \) radioactive matter packets. This choice is computationally cheap yet produces results with an acceptable level of noise. The final smooth light curves were obtained by applying a Savitzky–Golay filter with a second-order polynomial.

Generic features of the resulting light curves are presented in Figure 4 for \( M_{ej} = 1.4 M_\odot \) with varying \( ^{56}\text{Ni} \) mass, and for \( M_{Ni} = 0.55 M_\odot \) with varying ejecta mass. With increasing ejecta mass, the diffusion timescale is longer, so the light curve evolves more slowly, reaching lower peak luminosities. With increasing \( ^{56}\text{Ni} \) mass, the light curves are also broader, but they reach higher peak luminosities. These well-known general attributes are in good agreement with the results from many prior studies (e.g., Pinto & Eastman 2000; Woosley et al. 2007). Note that, with increasing \( M_{Ni}/M_{ej} \), the point A crossing is delayed with respect to the peak of the light curve, and \( L^* \) spends less time under \( L_{bol} \) (lower \( t_b / t_B \)). For extreme ratios, roughly when \( M_{Ni}/M_{ej} \geq 0.8 \) in these models, \( L^* \) never crosses \( L_{bol} \).

Using this grid of models, we can numerically justify one of the key assumptions used in the derivation of Section 2—that the change in time-weighted internal energy is always tangential to the negative of the internal energy near point B, so \( dE/dt_{t \approx t_B} \approx -2E/t_B \). Figure 5 illustrates the dimensionless quantity \( x \) defined in Equation (9), evaluated at \( t_B \) for all
As a simple sensitivity test, a subset of four Chandrasekhar mass models with varying $^{56}\text{Ni}$ masses were recomputed with a broken power-law density profile, instead of the uniform one. Following the setup from Kasen (2010), I adopt an inner segment of density, $\rho_i \propto r^{-\delta}$ for $\nu < \nu_i$, and an outer segment, $\rho_o \propto r^{-n}$ for $\nu \geq \nu_i$, with $\delta = 1$ and $n = 10$. Here, $r$ is radius and $\nu_i$ is a transitional velocity point connecting the two profile segments (Equation 1) of Kasen (2010). These power-law models have higher interior (innermost solar mass) and lower exterior density for a given kinetic energy and mass. The effect of higher central density is dominant, and generally this leads to longer diffusion timescales and slower evolving light curves. However, the dimensionless quantity $x$ is still close to $-2$ in all four models, which implies $-E$ and $dE/dt$ still share a common tangent near point B. The ratios of the timescales are systematically shifted by about $-0.1$, but otherwise they show the same clustered pattern. While this simple test demonstrates the generic nature of these results, it should be noted that the post-peak “hump” seen in bolometric light curves is not modeled here. As noted by many prior studies (e.g., Shen et al. 2018), the complex redistribution due to florescence requires models. It has a value close to $\sim 1.8$ for a wide range of $t_B$, justifying the assumption.

The time $t_c$, defined as $\tau (t_c) = 1$, is evaluated by fitting the models with the adopted functional form of $F_{\nu}$, as illustrated in Figure 2. Finally, the time ratios in Equation (11) are shown in Figure 6 for all of our models. The apparent anti-correlation is not captured in the simplified arguments of Section 2. But as can be seen in Figure 4, in models with increasing $M_{Ni}/M_{ej}$, the decay power spends less time under the bolometric luminosity, which results in a shorter $t_B$. Also, some of the spread in the ratios of $t_c/t_B$ and $t_B/t_d$ are due to the assumptions of constant gray opacity and ejecta velocity, which results in a diffusion timescale that is not sensitive to $^{56}\text{Ni}$ mass. Nonetheless, the points cluster within a fairly confined region, with only a $30\%$ variation in $t_B/t_d$, despite the very wide range of ejecta parameters. The ratio $t_c/t_d$ also clusters in a narrow range, as expected from Equation (11). Taking central values of $t_B/t_d = 1.7$ and $t_c/t_B = 1.6$, we see that the ratio of $t_c/t_d \approx 2.7$ is also approximately constant.

As a simple sensitivity test, a subset of four Chandrasekhar mass models with varying $^{56}\text{Ni}$ masses were recomputed with a broken power-law density profile, instead of the uniform one. Following the setup from Kasen (2010), I adopt an inner segment of density, $\rho_i \propto r^{-\delta}$ for $\nu < \nu_i$, and an outer segment, $\rho_o \propto r^{-n}$ for $\nu \geq \nu_i$, with $\delta = 1$ and $n = 10$. Here, $r$ is radius and $\nu_i$ is a transitional velocity point connecting the two profile segments (Equation 1) of Kasen (2010). These power-law models have higher interior (innermost solar mass) and lower exterior density for a given kinetic energy and mass. The effect of higher central density is dominant, and generally this leads to longer diffusion timescales and slower evolving light curves. However, the dimensionless quantity $x$ is still close to $-2$ in all four models, which implies $-E$ and $dE/dt$ still share a common tangent near point B. The ratios of the timescales are systematically shifted by about $-0.1$, but otherwise they show the same clustered pattern. While this simple test demonstrates the generic nature of these results, it should be noted that the post-peak “hump” seen in bolometric light curves is not modeled here. As noted by many prior studies (e.g., Shen et al. 2018), the complex redistribution due to florescence requires
non-gray radiative transport without the assumption of local thermodynamic equilibrium.

### 3.2. Comparison with Observations

Approximate bolometric light curves of actual SNe Ia can provide an independent, practical check on whether the ratios \( t_B/t_d \) and \( t_{\gamma}/t_B \) are really constant across a wide range of explosions. Unfortunately, bolometric measurements are difficult and are not frequently published. In this work, a small but modern set of spectroscopically normal Type-Ia light curves compiled by the Nearby Supernova Factory project\(^6\) is employed. For details on the sample selection and construction of the bolometric luminosity see Scalzo et al. (2014), Childress et al. (2013), and Aldering et al. (2002).

The reconstructed \( M_{SNi}, M_{ej} \) values, and “joint” host galaxy reddening values from Tables 2 and 3 of Scalzo et al. (2014) are adopted. The radioactive decay power is assumed to cross the bolometric curve near the time of peak, which is estimated through the characteristic diffusion timescale with corresponding \( M_{ej} \), but with constant \( \kappa = 0.1 \text{ cm}^2\text{g}^{-1} \) and \( v = 10^3 \text{ cm s}^{-1} \). From the original list of 19 events, 3 cases, SNF20080717-000, SNF20080913-031, and SNF20080918-004, were discarded due to their irregular luminosity evolution such that \( L_e \), never crosses \( L_{bol} \), or crosses it more than twice.

Figure 7 shows the ratio \( t_B/t_d \) measured for the remaining 16 events. In all cases, the ratio is distributed in a narrow range around a central value of \( t_B/t_d \approx 1.65 \). This is in close agreement with the simple radiation-transport calculations presented in Figure 6. Note that these 16 events had a relatively narrow range in ejecta mass \( 0.9 < M_{ej} < 1.4 \text{ M}_{\odot} \), but \( 0.3 < M_{SNi}/M_{ej} < 0.65 \), similar to the range covered by the model light curves. Therefore the tighter distribution of \( t_B/t_d \) as compared to the models may be indicating a systematic difference between the reconstructed parameters from this observational sample and the simple radiation-transport calculations used in Section 3.1.

Recently Wygoda et al. (2019) proposed a novel method of estimating \( t_{\gamma} \), from the bolometric light curve, and on the same sample of observations (Scalzo et al. 2014), they have found that it spans a narrow range between 30 and 45 days. The values of \( t_B \) measured for these events in this study span between 19 and 30 days. This implies \( 30 < t_{\gamma} < 48 \text{ days for } t_{\gamma}/t_B = 1.6 \) (Figure 6), and in excellent agreement with the range measured in Wygoda et al. (2019). This agreement suggests that \( t_{\gamma} \) can be estimated without integration on bolometric measurements; instead a simple estimate of the peak time (i.e., in \( B \)-band) is used, \( t_d \approx t_{peak} \), and thus \( t_{\gamma} = 2.7 t_d \).

### 4. Luminosity 15 Days after Peak

Simple analytic arguments, numerical calculations, and observational tests indicate that \( t_B \approx 1.7 t_d \) for wide range of Type-Ia explosion light curves. But how can this relation be useful?

Consider the following aspects of Type-Ia light curves. (1) Both theoretical and observational studies have shown that the simple version of “Arnett’s rule” works with an accuracy of about 10% in Type-Ia explosions (e.g., Stritzinger et al. 2006; Blondin et al. 2013)—in most cases \( L_e \), crosses \( L_{bol} \) near the peak of the light curve at \( t \sim t_B \). (2) The range of characteristic diffusion timescale, or the range of the observed light-curve rise time, is not that great. It is almost always between 10–25 days, which, according to \( t_B \approx 1.7 t_{\gamma} \), implies \( t_B \) will be roughly between 7 and 18 days after the peak. (3) The rate of change \( dL_{bol}/dt \) and \( dL_e/dt \) are not too different near \( t_B \), unless the light curve is evolving too fast.

These aspects suggest that the time \( t_B \) will typically happen a few days earlier than the time 15 days after the peak, \( t_{15} \), and therefore normally \( L_e(t_B) > L_{bol}(t_{15}) \). However, because \( L_e \) is a decreasing function of time, the decay power at \( t_{15} \) should be much closer to the bolometric luminosity at that time, i.e., the following is true no matter \( t_B > t_{15} \) or \( t_B < t_{15} \):

\[
|L_e(t_{15}) - L_{bol}(t_{15})| < |L_e(t_B) - L_{bol}(t_B)|. \tag{12}
\]

In order to test how close \( L_e(t_{15}) \) is to \( L_{bol}(t_{15}) \), the luminosities measured at \( t_{15} \) are shown as a function of peak luminosity, \( L_{peak} \), in Figure 8. The “true” values at \( t_{15} \) have been measured from the model light curves computed in Section 3 (gray circles), and compared to the corresponding instantaneous radioactive power at that time (black bars).

In general, the light curves that reach higher peak luminosities will have higher luminosities at \( t_{15} \), and for a given ratio of \( M_{SNi}/M_{ej} \), this trend correlates with the ejecta mass (gray circle sizes). Note that \( L_e(t_{15}) \) always overpredicts the luminosity for the lowest ejecta masses, and it underpredicts it for the highest ejecta masses. This reflects the fact that \( t_B > t_{15} \) for lower ejecta masses and the opposite at higher ejecta masses. Overall, excluding the handful of rapidly evolving models with the lowest ejecta and \( ^{56}\text{Ni} \) masses, the radioactive decay power at \( t_{15} \) closely matches the bolometric luminosity at that time (to within \( \lesssim 10\%) \).

The left panel of Figure 9 shows the correlation between \( L_{15} \) and \( L_e(t_{15}) \). All of the radiation-transport calculation values (circles) fall closely around the one-to-one correlation. Also shown are the values measured from the sample of 16 SNe Ia presented in Section 3.2 (stars). As mentioned earlier, the simple calculations from Section 3.1 do not model the post-

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\(^6\) https://snfactory.lbl.gov
peak “hump” due to florescence, and therefore the observational values are slightly less luminous for a given \( L_{\gamma}(t) \). Since most of the poorly behaving cases are due to overpredicted luminosities, the overall fit can be improved by introducing a small offset factor, \( L_{15} = f_{\text{off}} L_{\gamma}(t_{15}) \), with \( f_{\text{off}} = 0.85 \) (dashed gray).

Note that most of the overpredicted cases are of lower ejecta mass, thus the overall fit can be further improved with a simple ejecta-mass-dependent offset, \( f_{\text{off}} = 0.15M_{\text{ej}} - 0.7 \). The right panel of Figure 9 illustrates the absolute fractional difference between \( f_{\text{off}} L_{\gamma}(t_{15}) \) and \( L_{\text{bol}}(t_{15}) \), for an ejecta-mass-dependent offset of \( f_{\text{off}} = 0.15M_{\text{ej}} - 0.7 \) (colored circles and stars) and without offset \( f_{\text{off}} = 1 \) (gray circles and stars).

Given the simplistic nature of the numerical models and the uncertainties in parameters estimated from the observations, it is hard to determine which comparison deserves more weight as a test. But it is encouraging to see that \( L_{\gamma}(t_{15}) \) very closely matches with \( L_{15} \) from a set of models that span a wide range of parameter space, and that a simple offset can greatly improve the overall agreement.

5. Conclusion

This study investigates the properties of SN Ia light curves through energy conservation arguments, radiation-transport calculations, and observational tests. The main finding is that for a wide range of parameters, the time when the instantaneous radioactive decay power crosses the bolometric luminosity for the second time, after the peak of light curve, appears to be a constant multiple of the characteristic diffusion timescale of the ejecta. For these sets of simulations and observed SNe Ia, this constant turns out to be \( \sim 1.7 \), i.e.,

\[
t_{\text{b}} \approx 1.7t_{\text{d}}.
\]  

It has been shown that the gamma-ray escape timescale is also related with the diffusion timescale roughly as, \( t_{\text{b}} \approx 2.7t_{\text{d}} \).

The primary implication of this finding is that this relation suggests the bolometric luminosity 15 days after the peak must be very close to the instantaneous radioactive decay luminosity.
at that time, i.e.,
\[ L_{\text{bol}}(t_3) \approx L_c(t_3). \]  
(14)

It may serve as a simple to tool that connects the observables of the WLR to a physical description of the ejecta. A calibrated version of this relation that works for individual band-absolute magnitudes is needed for a more practical application on WLR. For instance, the B-band magnitudes will evolve significantly faster than the bolometric magnitude after the peak, so without any calibration, it is likely that the B-band magnitude 15 days after the peak will be systematically overpredicted in all cases. These types of effects may be explored by employing more advanced radiation-transport tools, e.g., SEDONA (Kasen et al. 2006), STELLA (Blinnikov et al. 2006), CMFGEN (Hillier & Dessart 2012), and JKYLL (Ergon et al. 2018), to see if reliable calibrations can be built for specific bands. A proper radiative transfer treatment is crucial in modeling the post-peak “hump” in bolometric light curves as well.

There may also be other subtle uses of this relation, which it could be employed in the interpretation of certain poorly sampled light-curve measurements. For instance, if the peak of the light curve is missed, but the \(^{56}\)Ni mass is estimated from the nebular spectra, then \(t_0\) can be measured from the light curve. This relation implies \(t_4 \approx f_B/1.7\) and the peak luminosity would be \(L_{\text{peak}} \approx L_c(t_4)\).

As was emphasized originally in Arnett (1982), the Type-Ia explosion light curves are physically simpler than core-collapse Type-Ib/c, where there is a much weaker association between the main heating agent and the kinetic energy source. However, since the energy source terms cancel in the derivation of Equation (11), the proposed relation will also approximately hold for Type-Ib/c explosions. It would be an interesting future project to explore this relation in Type-Ib/c light curves, including the cases that are dominantly powered by energy sources other than radioactivity.

In general, a larger sample of observations with good constraints on the explosion date, host reddening and preferably with independently measured \(^{56}\)Ni masses (e.g., nebular spectra; Childress et al. 2015), would go a long way in demonstrating the usefulness of this relation.

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