An analysis of students’ written mathematical communication in learning limits of functions through dependent field and independent field cognitive style at the eleventh grade of SMAN 1 Surakarta

S N Hayati¹, P Sujatmiko¹, I Kurniawati¹

¹ Mathematics Education Department, Faculty of Teacher Training and Education, Sebelas Maret University, Surakarta, Indonesia

Email: sitinurhayati.sn17@gmail.com

Abstract. This paper aims to describe students’ written mathematical communication through a dependent field (DF) and independent field (IF) cognitive style in limits of functions material at the eleventh grade of SMA Negeri 1 Surakarta in the academic year of 2018/2019. Students of XI MIPA 7 and XI MIPA 8 were involved in this study. By using a descriptive qualitative approach, the data were analyzed to observe students’ levels in written mathematical communication. The data were obtained by a written test. The triangulation of time was adopted to validate the data. Meanwhile, analysis techniques derived from Miles and Huberman were applied to analyze the data. The indicators of written mathematical communication were categorized into four indicators. The levels of that were also divided into four levels. The findings indicated that cognitive style affects students’ level in written mathematical communication. The majority of students with DF cognitive styles are in level I or II. Oppositely, those with IF cognitive style, the level of written mathematical communication are varied in each indicator.

1. Introduction
Mathematics is one of the subjects being taught in all levels of schools, from the beginning of primary school up to the end of high school. Some competencies or mathematical abilities which should be learned and comprehended by students in the learning process are mathematical consideration and reasoning, mathematical argumentation, mathematical communication, modeling, problem solving and arrangement, mathematical representation, mathematical symbol, as well as tools and technology[1]. Furthermore, NCTM (National Council of Teachers of Mathematics) determines mathematical standards in school consist of content standards and process standards [1]. Process standard comprises problem-solving, reasoning, proving, connection, communication, and representation. Therefore, communication plays a vital role in mathematics[2]. Communication will assist students in constructing meaning in a mathematical discussion. Communication is considered as the means by which teachers and student can share the processes of learning, understanding, and doing mathematics[3]. Based on those reflections, it may be seen that one of the competencies which should be owned in learning mathematics is mathematical communication.

Mathematical communication is an ability to communicate their mathematical thinking coherently and clearly to peers, teachers and others; to analyze and evaluate the mathematical thinking and strategies of others and to use the language of mathematics to express mathematical ideas precisely[4]. Student can explore and consolidate mathematical thinking and knowledge in solving problems through communication[5]. Mathematical communication can be classified into written and oral communication[6]. Written mathematical communication is the process of expressing mathematical ideas and understanding in writing[7]. The communication process will help students to construct meaning in the discussion. Everyone’s mathematical communication skill are different. As an example, mathematical communication skill of the students who had extrovert personality was better than those who had introvert personality[8].

One of the materials in mathematics subject of high school (SMA) grade XI is the limits of functions. The learning goals of limits of functions material are: students are expected to explain limits of functions in
algebra and its existence (whether or not the limits exist) and solve a problem related to limits of function in algebra. The observation field in SMA Negeri 1 Surakarta proves that in limits material, students can solve the limits of functions’ problems based on completion steps in their textbook, and some students still wrong in using mathematical symbols to draw or explain their ideas. Besides, from the teacher’s perspective, students have already understood what is meant by limits. Deciding which steps to solve limits of functions, drawing limits of functions point, and finishing contextual problems related to limits of functions become the difficulties faced by students. Those difficulties may retard students’ work to decide which completion is appropriate deals with limits of functions.

Some ways to make students more comfortable to solve the limits of function problems are, analyze what the problem is and decide which completion step is appropriate to be applied. When students are having difficulties in deciding the completion with direct substitution or specific strategies, students are also having difficulties to solve limits of functions. That is probably occurred because of the information on the problem cannot be accepted yet by students. Further, to resolve the contextual problem related to limits of functions, students should be able to analyze the problem, then develop it to limits of functions form. Students will have difficulties to resolve the contextual problem if they still have difficulties to analyze the problem and express the idea in the form of limits of functions. It occurs because students’ ability to do those is still weak. Students have already understood the meaning of limits of function, yet students still have difficulties in deciding the limits of function completion because of the lack of mathematical communication.

Every student has a different ability to perceive, analyze, and process information on any problem. It also occurs in learning mathematics, in which the context of this study is the limits of functions. It is by the teacher who has the perspective to students that with the same problem, the answer or the completion is different one to another. Its difference may come out because of their different cognitive styles.

Cognitive style is individual’s habitual way of organizing and processing information[9]. Cognitive style typically are referred to as variables of cognitive style and believed to affect the process of perception, thinking, and problem solving[10]. Many cognitive style classifications have been studied over the years by experts; one of them is Witkin, who classifies cognitive style into DF and IF [11]. DF students usually find it difficult to process complex information and tend to rely on other students’ ideas when solving a problem. Otherwise, IF students quickly analyze the complex and unstructured information, then organize it to solve a problem. It shows that IF students have higher mathematical communication than DF students. However, DF students may inevitably have the same or even higher mathematical communication than IF students as long as the problem being granted is related to the previous learned-material.

Based on previously explained problems, thus the problem statements of this study are: (1) How is the mathematical communication level of students with DF cognitive style towards limits of function material? and (2) How is the mathematical communication level of students with IF cognitive style towards limits of function material??. The objectives are (1) To describe the mathematical communication level of students with DF cognitive style towards limits of function material and (2) To describe mathematical communication level of students with IF cognitive style towards limits of function material.

2. Research Method
This study was conducted in SMA Negeri 1 Surakarta with the eleventh-grade students in the academic year 2018/2019 as the subject. The subjects cover students of XI MIPA 7 and XI MIPA 8. A qualitative approach with descriptive was used in this study. Data were collected through mathematics test results by students of XI MIPA 7 and XI MIPA 8 SMA Negeri 1 Surakarta. Besides, an interview with Mrs. Suryanti, S.Si, mathematics teacher of XI MIPA 7 and XI MIPA 8 SMA Negeri 1 Surakarta, was also conducted to collect supporting data.

This study selected subjects by using purposive sampling. The subjects selected were expected to help the researcher understand the phenomenon because students have already received the material. Indicators
of written mathematical communication are as follow:

a. To be able to express written mathematics ideas in the mathematical model,
b. To be able to draw mathematics ideas visually,
c. To be able to comprehend and interpret ideas in written form,
d. To be able to apply mathematical symbols and notation in expressing mathematics ideas.

In this study, written mathematical communication refers to High School Math Communication GRC Rubric, which is divided into four levels; those are:

a. Level I, Does not Meet,
b. Level II, Approaching,
c. Level III, Meets the Standards,
d. Level IV, Advanced Understanding[12].

The characteristics of each mathematical communication have been suited to the indicators of each mathematical written communication that was used in this research. The characteristics of each mathematical communication level are shown below:

| Table 1. The Characteristic Mathematical Communication Level |
|-------------------------------------------------------------|
| Indicators | Does not meet I | Approaching II | Meets the Standard III | Advanced Understanding IV |
| Indicator 1: | Students are able to write limits form being questioned, yet students are not able to decide the completion of the form of that limit. | Students are able to write limits form being questioned and decide the completion of that limits form with incomplete or even wrong completion steps. | Students are able to write limits form being questioned and decide the completion of the form of that limit. | Students are able to write limits form being questioned, decide the completion of that limits form, and state a conclusion. |
| Indicator 2: | Students do not present the graphic, or the graphic being presented is not suitable for the problems. | Students present the visible graphic that is suitable for the problems, yet it is not obviously and easily to be understood. | Students present the visible graphic that is suitable for the problems, yet it does not contain an explanation. | Students present the visible graphic that is suitable for the problems, and it contains an explanation. |
| Indicator 3: | The explanation given by students is not suitable for the completion steps. | The explanation given by students is suitable to the completion steps, yet it is not obviously and easily to be understood. | The explanation given by students is suitable for the completion steps and easy to be understood, yet there is no explanation in some parts. | The explanation given by students is complete and suitable for the completion steps and easy to be understood. |
Meanwhile, the cognitive style comprises DF cognitive style and IF cognitive style, which comes from the GEFT test and is categorized as strongly DF, slightly DF, slightly IF, and strongly IF[13].

The researcher arranged three different questions to collect the data. Question 1, as the first test instrument, was for consideration to decide the research subject and for measurement to know the students’ level of written mathematical communication. Furthermore, question 1 was also used as an instrument in the second and third tests. The former one aimed to measure written mathematical communication level of selected subjects, and the later aimed to measure written mathematical communication level of selected subjects if the results of the first and second test were not valid. In the instrument, students are asked to decide the completion of limits form using a method of limits value from the right and left approach, and graphic method. The students’ ability is observed through writing and completing limits form being questioned, drawing completion graphic of limits form, explaining the completion in each step, as well as using mathematical notation and symbol.

Validity is the degree of accuracy between the data that occurs in the object of research with data that can be reported by the researcher[14]. The data validity technique in this study was triangulation technique. Triangulation technique has been one of the most used methods to ensure validity in research[15]. Triangulation technique are divided into several types. In this study, the triangulation technique used was time triangulation, which was conducted by checking the data to the same source, but different in time. In this study, the data obtained from test 1 would be compared to that of test 2, which focused on the students’ ability of written mathematical communication. If the result of test 1 is the same as the result of test 2, the data are valid; otherwise, if the result of test 1 is different from test 2, the data are invalid. Then, if the results of test 1 and test 2 are not valid, test 3 would be conducted. Additionally, the result of test 3 will be compared to the result of test 1 and 2. If it is found the similarity between the result of test 3 and test 1, or the result of test 3 and test 2, the data are valid. After checking the validity of the data, then the researcher analyzed the data using an analysis technique by Miles and Huberman. It was done through 3 steps: data reduction, data display, and conclusion drawing or verification[14].

3. Result

The findings, based on the GEFT test, showed that each student in a class has a different cognitive style. This study indicated that nine students of XI MIPA 7 were categorized as strongly DF, 7 of that were slightly DF, 8 of that were slightly IF, and 9 of that were strongly IF. Students of XI MIPA 8 also categorized as strongly DF, slightly DF, slightly IF, and strongly IF, each of which consisted of 3, 5, 19, and 7 students. From those classifications, students with strongly DF and strongly IF were taken. The level of written mathematical communication students with DF cognitive style in this research are in level I or II. Contrarily, those with IF, the level of written mathematical communication are varied in each indicator. Indicator 1, the students with DF cognitive style are in level I (Does not Meet) or level II (Approaching) and the students with IF cognitive styles are in level I (Does not Meet), level II (Approaching), or level IV (Advanced Understanding). Indicator 2, The students with DF cognitive style are in level I (Does not Meet) or level II (Approaching) and the students with IF cognitive styles are in level I (Does not Meet), level II (Approaching).
(Approaching), or level IV (Advanced Understanding). Indicator 3, The students’ level with DF cognitive style cannot be identified because the students give none of the explanations in the completion and the students with IF cognitive styles are in level II (Approaching) or level IV (Advanced Understanding). Indicator 4, The students with DF cognitive style are in level I (Does not Meet) or level II (Approaching) and the students with IF cognitive styles are in level I (Does not Meet), II (Approaching), III (Meets the Standards), or IV (Advanced Understanding).

4. Discussion
Students’ level in written mathematical communication are as follow:
4.1. The Level Communication of Students with DF Cognitive Style
Below shows the results test of students with DF cognitive style:

![Figure 1. The Answer of Student 1](image1)

![Figure 2. The Answer of Student 2](image2)

![Figure 3. The Answer of Student 3](image3)

![Figure 4. The Graphic of Student 1](image4)
4.1.1. Indicator 1

Figure 1 shows that the limits form being asked was wrongly written by student 1. The student wrote, but it should be. The student could not decide the completion of the form of that limit. The question with left and right limits value was correctly answered, yet the method to decide left and right limits value was wrongly answered. Besides, the student did not state the conclusion.

Figure 2 shows that the student was able to write and complete the limits form being asked correctly. Otherwise, the completion steps were not entirely written. The table of value being drawn by the student was slightly correct to illustrate values surrounding $g_{1876}$. Furthermore, the student gave the inappropriate-written conclusion. In summary, indicator 1 comprises two different students’ level of written mathematical communication. Student’s ability of written mathematical communication was in level 1 (Does not Meet), where student 1 (figure 3.1) could write limits form being questioned, yet the student was not able to decide the completion of the form of that limit. Based on figure 2 and 3, students’ ability of written mathematical communication was in level 2 (Approaching) where students were able to write limits form being questioned and decide the completion of that limits form with uncomplete and even wrong completion steps.

4.1.2. Indicator 2

Functions form in the question was $h(x) = x$ for $x > 3$ and $h(x) = x^2 - 2x - 1$ for $x \leq 3$. The graphic of that function was parabola and straight line that non-continuously. Figure 3.4 shows that the student was wrongly drawing the graphic. The student drew the limits form in the dashed line. Figure 5 and 6 indicate that the students were drawing the graphics correctly, yet they were less understandable because of their some uncomplete parts. The dot in figure 5 could not be read obviously. Moreover, the value of $y$ when $x = 3$ also could not be read because the peak dot of parabola graphic and the straight line were the same, thus there was no sign showing the value of $y$ when $x = 3$. Figure 3.6 had an unclear discontinuity; both peaks seemed not in a line. In brief, indicator 2 comprises two different levels. Based on figure 4 the student had level I (Does not Meet), meanwhile based on figures 5 and 6 the students had level II (Approaching).

4.1.3. Indicator 3

Figures 1, 2, and 3 show that the students did not explain the completion steps. Most of the students just wrote tables, graphics, and limits value without giving a more in-depth explanation. Consequently, the students’ ability of written mathematical communication in indicator three could not be identified.
4.1.4. Indicator 4

Figures 1 and 2 show that the students made a mistake in writing mathematical notation of limits of functions, and they were correctly writing notation of function \( f(x) \). Moreover, the equation and inequality symbol was correctly written. Therefore, the students had been used mathematical symbols and notation with an inappropriate notation of limits, hence that figures show that the students were in level I (Does not Meet). Figure 3 shows that the student had a mistake in writing the notation of limits. The error was writing \( x \) approximates to 5 under \( \lim \) symbol \( f(x) \) instead of writing \( x \) approximates to 5 under the symbol of limits. The writing of function \( f(x) \) and the equation and inequality symbol were correctly written. In short, the students wrote notation of limits with the notation elements correctly, yet its position was incorrect. High school students should be write notation of limits, both elements and position. Hence, it can be derived that the student was in level II (Approaching) where the student used suitable mathematical notation and symbol, yet there was an error of the level in some parts.

4.2. The Level Communication of Students with IF Cognitive Style

Below shows the results test of students with IF cognitive style:

Figure 7. The Answer of Student 4

Figure 8. The Answer of Student 5

Figure 9. The Answer of Student 6

Figure 10. The Answer of Student 7
4.2.1. Indicator 1
As in figure 7 points out correct and complete limits form, and it was attached by the conclusion in each completion step, the student, therefore, was classified in level IV (Advanced Understanding) in indicator 1 where student can write form of limits, decide the completion of that form of limits, and state a conclusion. Figure 8 shows that student was correctly writing the form of limits and decide the completion of that form of limits, yet the given completion was incomplete since the student did not write the value of limits from right and left. Further, the table of value also was less appropriate because $x$ values being taken were inappropriate. Thus, the student was classified in level II (Approaching), where the student uses a suitable form of limits and can decide that completion form of limits, yet there is an incompleteness or even an error of completion steps.

Figure 9 indicates that the student wrote incorrect and incomplete forms of limits. The student was not able to complete the limits using limits value from the right-hand and left-hand approach. Those problems made the student classified to level I (Does not Meet) where the student can write the form of limits, yet cannot decide that form of limits completion.

Figure 10 displays the student was correctly writing the form of limits, able to complete that form of limits, and state a conclusion. However, in completing the form of limits, there was inappropriate completion. The table of value and the way of deciding the form of limits form right and left were slightly appropriate. Therefore the student was categorized in level II (Approaching).
4.2.2. **Indicator 2**
The student could draw a graph of functions clearly and ultimately, as stated in figure 11. The dots in the graphic could be read obviously. The graphic in figure 12 and 14, although it was drawn correctly, it was complex to be understood. The student did not assert a mark in $x$-coordinate and $y$-coordinate; hence, the dots were unclearly read.

The student has already asserted a note of numbers in $x$-coordinate and $y$-coordinate; hence, there was still an unclear dot in the graphic. The student drew the same discontinue peak dot; further, the graphic also did not contain a mark. It is shown in figure 13 that the student made an error in drawing a graphic of functions form. The graphic being drawn was continue, not parabola or straight line; otherwise, it was dashed line. Regarding figure 11 and 13, consecutively, the students were in level IV (Advanced Understanding) and level I (Does not Meet). Meanwhile, based on figure 12 and 14, the students were classified in level II (Approaching).

4.2.3. **Indicator 3**
It is stated in figure 7 that the student gave complete and understandable explanations on the completion steps. Those explanations comprised the steps to decide the auxiliary dots and the value of limits, as well as information about the graphic. Thus, the student was classified in level IV (Advanced Understanding), where the explanation being given was suitable for the completion steps; further, it was also understandable and ultimately stated.

Additionally, figures 8 and 9 indicate that the students did not give any explanation in the completion. It was proven by the student only write the table of value $(x,y)$, function $f(x)$, and value of limits, as well as drew graphic (figure 8). Thus, it can be seen that the students were in level I (Does not Meet), where the explanations were not suitable for the completion steps. Regarding figure 10, the student was in level II (Approaching) since the student gave an explanation that was suitable for the completion steps. Yet, the word choices were difficult to be understood. For instance, the student wrote making a table $\lim_{s\to4} ( )$ in the first step whereas $\lim_{s\to4} f(x)$ was a value, hence what the table should be intended was unclear.

4.2.4. **Indicator 4**
The notation of limits and function in figure 7 were written correctly; besides, the student was also correctly using equation and inequality symbol. Thus, the work in figure 7 made the student classified in level IV (Advanced Understanding), where the notation and mathematic symbol are correct and consistent in each answer. Figure 8 shows that the student used notation and mathematic symbols correctly, yet there was one that unsuitable to its level. The student wrote an inequality symbol ($\neq$) followed by the word ‘inequality’ and the student wrote $f(x)$ followed by the word ‘function’. Therefore, the student was classified in level II (Approaching), where the notation and mathematic symbol are suitable, yet there is one that unsuitable to its level.

Figure 9 shows that the student used unsuitable or even wrong notation and mathematic symbols. The notation of function, as well as equation and inequality symbol, was written appropriately, yet not with the notation of limits. Since the student used slightly appropriate notation and mathematic symbols, the classification is in level I (Does not Meet). Meanwhile, the student’s work in figure 10 indicates the appropriate notation and mathematic symbol yet inconsistently. It comprised the writing of notation of limits that were correct and incorrect. Thus, the student had level III (Advanced Understanding), where the student used correctly yet inconsistent in each completion.

5. **Conclusion**
Based on the results explained previously, the conclusion can be drawn that IF/DF cognitive style affects
students’ level in written mathematical communication. Majority of students with DF cognitive style are in level I or II for each indicator. Indicator 1, The students with DF cognitive style are in level I (Does not Meet) or level II (Approaching). It shows that some students do not write appropriate notation of limits and are not able to complete the limit form being asked. Furthermore, some students write appropriate notation of limits and correct completion with slightly proper completion steps. Indicator 2, The students with DF cognitive style are in level I (Does not Meet) or level II (Approaching). It shows that some students are not able to illustrate the graphic or even able too illustrate the graphic unclearly. Indicator 3, The students’ level with DF cognitive style cannot be identified because the students give none of the explanations in the completion. Indicator 4, The students with DF cognitive style are in level I (Does not Meet) or level II (Approaching). It shows that some students use notation and mathematic symbols yet in unsuitability. Contrarily, those with IF, the level of written mathematical communication are varied in each indicator. Indicator 1, The students with IF cognitive styles are in level I (Does not Meet), level II (Approaching), or level IV (Advanced Understanding). It shows that some students do not write appropriate notation of limits and are not able to complete limit form being asked. Moreover, some students write appropriate notation of limits and correct completion with slightly proper completion steps, as well as some students write notation of limits, complete correctly limits form, and state a conclusion. Indicator 2, The students with IF cognitive styles are in level I (Does not Meet), level II (Approaching), or level IV (Advanced Understanding). It shows that some students can illustrate the graphic, illustrate the graphic yet with unclear drawing, as well as illustrate the clear and understandable graphic. Indicator 3, The students with IF cognitive styles are in level II (Approaching) or level IV (Advanced Understanding). It shows that some students state unclear explanation as well as state understandable and complete explanation which are suitable to the completion steps. Indicator 4, The students with IF cognitive styles are in level I (Does not Meet), II (Approaching), III (Meets the Standards), or IV (Advanced Understanding). It shows that some students use notation and mathematic symbols yet inappropriate in some parts. Additionally, some students also use correct notation and mathematic symbols yet inconsistently, as well as use correct notation and mathematic symbols consistently. Those findings prove that the ability of written mathematical communication plays an important role for students to solve the mathematics subject problem. Designing appropriate lesson which may help students to enhance written communication level is expected to be implemented in a lesson. For instance, the teacher should allow students to express their ideas verbally before write it down as well as to revise and correct their writing. At the end of the lesson, the teacher also should make a reflection on what they have learned. In addition, written mathematical communication skills can be enhanced by small group discussions[16] or using digital writing tool[17].

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