1. INTRODUCTION

A long-outstanding problem in cosmology is the synchronized evolution of the quasar population over the redshift range $0 \leq z \leq 5$. Observations in the optical (Pei 1995) and radio (Shaver et al. 1996) show a pronounced peak in the abundance of bright quasars at $z \approx 2.5$; recent X-ray observations (Miyaji, Hasinger, & Schmidt 2000) confirm the rapid rise from $z = 0$ toward $z \approx 2$ but have not shown evidence for a decline at still higher redshifts. Individual quasars are widely understood to consist of supermassive black holes (BHs) powered by accretion (Lynden-Bell 1967; Rees 1984). A plausible timescale for quasar activity is then the Eddington time, $4 \times 10^7 (\epsilon/0.1) \, \text{yr}$, the e-folding time for the growth of a BH accreting mass at a rate $M$, while shining at the Eddington luminosity with a radiative efficiency of $L = L_{\text{Edd}} = \epsilon M c^2$. The lifetime $t_Q$ of the luminous phase of quasars can be estimated directly, by considering the space density of quasars and galaxies. At $z \sim 2$, the ratio $n_Q/n_g \sim 3 \times 10^{-3}$ implies the reassuringly close value of $t_Q \sim t_{\text{folding}} \, n_Q/n_g \sim 10^7 \, \text{yr}$ (Blandford 1999 and references therein). These lifetimes are significantly shorter than the Hubble time, suggesting that the quasar population evolves on cosmic timescales by some mechanism other than local accretion physics near the BH.

It is tempting to identify quasars with halos condensing in a cold dark matter (CDM) dominated universe, as the halo population naturally evolves on cosmic timescales (Efstathiou & Rees 1988; Haiman & Loeb 1998; Kauffmann & Haehnelt 2000). Furthermore, quasars reside in a subset of all galaxies, while the redshift evolution of the galaxy population as a whole (qualitatively similar to that of bright quasars) has been successfully described by associating galaxies with dark halos (e.g., Lacey & Cole 1993; Kauffmann & White 1993). A further link between galaxies and quasars comes from the recent detection and measurements of the masses of massive BHs at the centers of nearby galaxies (Magorrian et al. 1998; van der Marel 1999).

These arguments suggest that the evolution of the quasar population can indeed be described by "semianalytic" models, associating quasars with dark matter halos. In this type of modeling, the quasar lifetime plays an important role. The quasar phase in a single halo could last longer ($t_Q \sim 10^8 \, \text{yr}$), with correspondingly small $M_{\text{bh}}/M_{\text{halo}}$ ratios, or last longer ($t_Q \sim 10^6 \, \text{yr}$), with larger BH formation efficiencies (Haiman & Loeb 1998; Haehnelt, Natarajan, & Rees 1998). Note that although recent studies have established a correlation between the bulge mass $M_{\text{bulge}}$ and BH mass $M_{\text{bh}}$, this correlation leaves a considerable uncertainty in the relation between $M_{\text{bh}}$ and the mass $M_{\text{halo}}$ of its host halo. If the initial density fluctuations are Gaussian with a CDM power spectrum, the clustering of collapsed halos is a function of their mass: rarer, more massive halos cluster more strongly (Kaiser 1984; Mo & White 1996). Hence, measurements of quasar clustering are a potentially useful probe of both BH formation efficiencies and quasar lifetimes (La Franca, Andreani, & Cristiani 1998; Haehnelt et al. 1998).

In this paper we assess the feasibility of breaking the above degeneracy and inferring quasar lifetimes from the statistics of clustering that will be available from the Sloan Digital Sky Survey (SDSS; York et al. 2000) and Anglo-Australian Telescope Two-Degree Field (2dF; Boyle et al. 1999). Previous papers (e.g., Stephens et al. 1997; Sabbey et al. 1999) have yielded estimates suggesting that quasars are clustered more strongly than galaxies. However, the current uncertainties are large, especially at high redshifts, where clustering has been found to decrease (Iovino & Shaver 1988; Iovino, Shaver, & Cristiani 1991), to stay constant (Andreani & Cristiani 1992; Croom & Shanks 1996), or to increase with redshift (La Franca et al. 1998). As a result, no strong constraints on the lifetime of quasars can be obtained yet. The
key advance of forthcoming surveys over previous efforts is twofold. Because of their sheer size, i.e., the large number of quasars covering a large fraction of the sky, both shot noise and sample variance can be beaten down, significantly reducing the statistical uncertainties. Furthermore, the large sample size will eliminate the need to combine data from different surveys with different selection criteria, hence allowing cleaner interpretation.

Recent measurements of the local massive black hole density have stimulated discussions of a radiative efficiency that is much lower than the usual $\sim 0.1$ (e.g., Haehnelt et al. 1998). A convincing constraint on the lifetime of quasars could be therefore highly interesting, as this might have implications for the local accretion physics near the BH.

This paper is organized as follows. In § 2 we summarize our models for the quasar luminosity function, and in § 3 we compute the quasar correlation function in these models. In § 4 we compare the model predictions with presently available data, and in § 5 we assess the ability of future optical redshift surveys to discriminate between the various models. In § 6 we repeat our analysis in the soft X-ray band and examine the contribution of quasars to the X-ray background and its autocorrelation. In § 7 we address some of the caveats arising from our assumptions, and in § 8 we summarize our conclusions and the implications of this work.

2. MODELS FOR THE QUASAR LUMINOSITY FUNCTION

In this section we briefly summarize our model for the luminosity function (LF) of quasars, based on associating quasar BHs with dark halos. Our treatment is similar to that of previous works (Haiman & Loeb 1998; Haehnelt et al. 1998), although it differs in some of the details. A more extensive treatment is provided in the Appendix. The main assumption is that there is, on average, a direct monotonic relation between halo mass $M_{\text{halo}}$ and average quasar luminosity $L_{M,z}$, which we parameterize using the simple power-law Ansatz:

$$L_{M,z} = x_0(z)M_{\text{halo}}^{\alpha(z)}M_\odot. \quad (1)$$

Here $x_0(z)$ and $\alpha(z)$ are “free functions,” whose values are found by the requirement that the resulting luminosity function agrees with observations. As explained in the Appendix, our model has one free parameter, the lifetime $t_q$, which uniquely determines $x_0(z)$ and $\alpha(z)$ in any given background cosmology. We assume the background cosmology to be either flat (ACDM) with $(\Omega_m, \Omega_b, \sigma_8, h, n) = (0.7, 0.3, 0.65, 1.0, 1.0)$ or open (OCDM) with $(\Omega_m, \Omega_b, \sigma_8, h, n) = (0, 0.3, 0.65, 0.82, 1.3)$. In ACDM we find $[-\log(x_0/L_{\odot}M_\odot^{-1}), \alpha] \approx (1, 0.4)$ and $\approx (-0.2, -0.1)$ for lifetimes of $t_q = 10^8$ and $10^9$ yr, respectively. Similarly, in OCDM we find $[-\log(x_0/L_{\odot}M_\odot^{-1}), \alpha] \approx (1, 0.25)$ and $\approx (-0.2, -0.25)$ for these two lifetimes.

In Figure 1 we demonstrate the agreement between the LF computed in our models and the observational data at two different redshifts, $z = 2$ and $z = 3$. For reference, the upper labels in this figure show the apparent magnitudes in the SDSS $g'$ band, assuming that the intrinsic quasar spectrum is the same as the mean spectrum in the Elvis et al. (1994) quasar sample. The photometric detection threshold$^2$

2 See http://www.sdss.org/science/tech_summary.html.

![Figure 1](image)

**Fig. 1.**—Fits to the quasar luminosity function at redshifts $z = 2$ and $3$ in our models, with two different quasar lifetimes: $t_q = 10^8$ yr (solid curves) and $t_q = 10^9$ yr (dotted curves). Also shown are the data and the fitting function (dashed curves) for the LF from Pei (1995). The quality of our fits at different redshifts or in the OCDM model are similar. The upper labels show the corresponding apparent magnitudes in the SDSS $g'$ band, assuming that the intrinsic quasar spectrum is the same as the mean spectrum in the Elvis et al. (1994) quasar sample.
expressions for Jing 1999 and Sheth & Tormen 1999 for more accurate critical overdensity in the Press-Schechter formalism [see trum of Eisenstein & Hu 1999), and is the usual distribution, customarily expressed by $P_L$, assuming either a short or a long lifetime and a power-law can be obtained to the luminosity function of quasars, $kM_\bullet$ a given mass $P$ where and.

Assuming a conservative initial black hole mass of 1 $a$ White 1996) The black hole mass grows during the quasar phase as $D$.

As demonstrated in the previous section, equally good Ðts to the luminosity of quasars, $b(M, z)$ is the bias parameter for halos of $M \in$, which, however, do not affect our results here]. The bias associated with quasars with luminosity $L$ in our models is given by averaging over halos of different masses associated with this luminosity.

Following equation (A6), we obtain

$$b(L, z) = \left[ \frac{d\phi}{dL} (L, z) \right]^{-1} \int_0^\infty dM \frac{dN}{dM} (M, z)$$

$$\times b(M, z) \frac{d\phi}{dL} (L, L_{\text{b}}) f_{\text{od}}(M, z).$$

We show in Figure 2 the resulting bias parameter $b(L, z)$ in the models corresponding to Figure 1, with short and long lifetimes, and at redshifts $z = 2$ and 3. As expected, quasars are more highly biased in the long-lifetime model, by a ratio $b(\text{long})/b(\text{short}) \gtrsim 2$. In the OCDM case, at the detection threshold of SDSS, we find $b(\text{long}) \approx 3$ at $z = 3$ and $b(\text{long}) \approx 2$ at $z = 2$. Bright quasars with $g' = 17$ are predicted to have a bias at $z = 3$ as large as $b = 10$. For reference, we also show in this figure the bias parameters obtained in the OCDM cosmology, which are significantly lower than in the OCDM case. The number of quasars observed at a fixed flux implies an intrinsically larger number of sources if OCDM is assumed because the volume per unit redshift and solid angle in an open universe is smaller. This lowers the corresponding halo mass and therefore the bias.

4. COMPARISON WITH AVAILABLE DATA

As emphasized in § 1, the presently available data leave considerable uncertainties in the clustering of quasars. Nevertheless, it is interesting to contrast the results of the previous section with preliminary results from the already relatively large, homogeneous sample of high-redshift quasars in the 2dF survey (Boyle et al. 2000). Our predictions are obtained by relating the apparent magnitude limit to a minimum absolute luminosity at a given redshift: $\log \left[ L_{\text{min}}(z)/L_{b, \odot} \right] = 0.4 (5.48 - B + 5 \log [d_L(z)/\text{pc}] - 5)$. This relation assumes no K-correction, justified by the nearly flat quasar spectra ($\nu F_\nu = \text{constant}$) at the relevant wavelengths (e.g., Elvis et al. 1994; Pei 1995). In our model, the correlation length $r_0$ is given implicitly by

$$\xi_0(r) = b^2(z)D^2(z)\xi_m(r) = 1,$$ 

where $\xi_m(r)$ is the usual dark matter correlation function and $b(z)$ is the value of the bias parameter $b(L, z)$ as determined in the previous section but now averaged over all quasars with magnitudes brighter than the detection limit,

$$\bar{b}(z) = \left[ \int L_{\text{min}}(z) dL \frac{d\phi}{dL} \right]^{-1} \int L_{\text{min}}(z) dL \frac{d\phi}{dL} b(L, z).$$

In Figure 3 we show the resulting correlation lengths in the long- and short-lifetime models. Also shown is a preliminary data point with 1σ error bars from the 2dF survey, based on $\approx 3000$ quasars with apparent magnitudes $B < 20.85$ (Croom et al. 1999). The upper panel shows the results in our fiducial LCDM model with predictions for this magnitude cut. The published results for $r_0$ are cosmology dependent, and we have simply converted them for our cosmological models by taking the corresponding average

3 See http://www.mso.anu.edu.au/DunkIsland/Proceedings.
of the redshift distance and angular diameter distance; this crude treatment is adequate given the large measurement errors. Our models generically predict a gradual increase of the correlation length with redshift ("positive evolution"). The clustering is dominated by the faintest quasars near the threshold luminosity; as a result, the fixed magnitude cut of $B = 20.85$ corresponds to more massive and more highly clustered halos at higher redshifts. There are two additional effects that determine the redshift evolution of clustering: (1) quasars of a fixed luminosity are more abundant toward high-$z$, requiring smaller halo masses to match their number density; and (2) halos with a fixed mass are more highly clustered toward high-$z$. We find, however, that these effects are less important than the increase in $M_{\text{halo}}$ caused by fixing the apparent magnitude threshold, which gives rise to the overall positive evolution.

The clustering in the long-lifetime model is stronger and evolves more rapidly than in the short-lifetime case. As we can see, the present measurement error bars are large: the whole range of lifetimes from $10^{6.5}$ to $10^8$ yr is broadly consistent with the data, to within $\sim 2\sigma$. For the $\Lambda$CDM model, the 2dF data point is consistent with a lifetime of 10 years, whereas in the OCDM model the lifetime is 0.8 yr. As we approach the redshift of the microwave background, galaxy surveys, and the Ly$\alpha$ forest will hopefully pin down to the accuracy required here.

Finally, we note that observational results on $r_0$ are commonly obtained by a fit to the two-point correlation of the form $\xi(r) = (r/r_0)^{-7}$. Since $r_0$ is the correlation length where the correlation is unity, we expect our formalism to begin to fail on such a scale because neither linear fluctuation growth nor linear biasing holds. On the other hand, it is also unclear whether $r_0$ as presently measured from a two-parameter fit to the still rather noisy observed two-point correlation necessarily corresponds to the true correlation length. While our crude comparison with existing data in Figure 3 suffices given the large measurement errors, superior data in the near future will demand a more refined treatment, which is the subject of the next section.

5. Expectations from the SDSS and 2dF

Although existing quasar clustering measurements still allow a wide range of quasar lifetimes and do not provide tight constraints on our models, forthcoming large quasar samples from SDSS or the complete 2dF survey are ideally suited for this purpose. Here we estimate the statistical uncertainties on the derived lifetimes, using the three-dimensional quasar power spectrum $P_\delta(k)$ derived from these surveys.

The variance of the power spectrum is computed by

$$\langle \delta P_\delta(k) \rangle = n_k^{-1}[b^2P(k) + \bar{n}]^2, \quad (6)$$

where the large scale fluctuations are assumed to be Gaussian, $n_k$ is the number of independent modes, $\bar{n}$ is the mean number density of observed quasars, $P_\delta(k) = b^2 P(k)$ is the quasar power spectrum, and $P(k)$ is the mass power spectrum (Feldman, Kaiser, & Peacock 1994). For a survey of volume $V$ and a $k$ bin of size $\Delta k$, we use $n_k = k^2 \Delta k V/4\pi^2$. The fractional variance is therefore $n_k^{-1}[1 + 1/[n b P(k)]]^2$.

In terms of minimizing this error, increasing the luminosity cut of a survey has the advantage of raising the bias $b$ but has the disadvantage of decreasing the abundance $n$. In practice, $b$ changes relatively slowly with mass (slower than $b \propto M$), whereas $n$ varies with mass much more rapidly ($n \sim 1/M$, or steeper if $M > M_\ast$). As a result, we find that for our purpose of determining the clustering and the quasar lifetime, it is better to include more (fainter) quasars.

The power spectrum $P_\delta(k)$ of quasars in $\Lambda$CDM is shown in Figure 4 at two different redshifts near the peak of the comoving quasar abundance, $z = 2$ and $z = 3$. We assume that redshift slices are taken centered at each redshift with a width of $\Delta z = 0.5$ (which enters into the volume $V$ above). Results are shown in the long- and short-lifetime models, together with the expected $1\sigma$ error bars from SDSS (crosses). Also shown in the lower panel are the expected error bars from 2dF (open squares), which are slightly larger because of the smaller volume (for SDSS, we assume an angular coverage of $\pi$ sr, and for 2dF, an area of 0.23 sr). We do not show error bars for 2dF beyond $k \sim 0.01$ h$^{-1}$ Mpc because larger scales would likely be affected by the survey window. We also only show scales where the mass power spectrum and biasing are believed to be linear.

As these figures show, the long and short $t_0$ models are easily distinguishable with the expected uncertainties both from the SDSS and the 2dF data, out to a scale of $\sim 100$ Mpc. The measurement errors at different scales are independent (under the Gaussian assumption) and hence, when combined, give powerful constraints; e.g., formally, even models with lifetimes differing by a few percent can be distinguished with high confidence using the SDSS. However, systematic errors due to the theoretical modeling are expected to be important at this level, which we will discuss in § 7.

In Figure 4 we have used the magnitude cuts for spectroscopy, i.e., $B = 20.85$ for 2dF and $B = 20.4$ ($g \approx 19$) for
The formalism we presented in §§ 2 and 3 is quite general, and we here apply it to the soft X-ray luminosity function (XRLF) from Miyaji et al. (2000). The details of the fitting procedure are given in the Appendix. In analogy with the optical case, we find that the clustering of X-ray–selected quasars depends strongly on the lifetime. As an example, including all quasars whose observed flux is above $3 \times 10^{-14}$ ergs cm$^{-2}$ s$^{-1}$, we find the correlation length at $z = 2$ to be $\approx 4$ h$^{-1}$ Mpc in the short-lifetime case and $\approx 11$ h$^{-1}$ Mpc in the long-lifetime case (ACDM). Current data probe the clustering of X-ray quasars only at low redshifts ($z \leq 1$; see Carrera et al. 1998), where our models suffer from significant uncertainties due to the subhalo problem discussed in § 7. Constraints at $z \geq 2$ could be available in the future from CXO and XMM, provided that a large area of the sky is surveyed at the improved sensitivities of these instruments.

We next focus on the quasar contribution to the soft X-ray background and its autocorrelation, which, as we will see, is dominated by quasar contributions at somewhat higher redshifts. The mean comoving emissivity at energy $E$ from all quasars at redshift $z_t$, typically in units of keV cm$^{-3}$ s$^{-1}$ sr$^{-1}$, is given in our models by

$$ \bar{\eta}(E, z) = \frac{1}{4\pi} \int_0^\infty dL \frac{d\phi}{dL} L_X(E, L), $$ (7)

where $L_X(E, L)$ is the luminosity (in keV s$^{-1}$ keV$^{-1}$) at the energy $E$ of a quasar whose luminosity at $(1 + z)$ keV is $L$. We have used the mean spectrum of Elvis et al. (1994) to include a small $K$-correction when computing the background at observed energies $E \neq 1$ keV. The mean background is the integral of the emissivity over redshift,

$$ \bar{I}(E) = \int_0^\infty \frac{d\chi}{(1 + z)} \bar{\eta}(E, \chi), $$ (8)

where $\bar{I}$ is typically given in units of keV cm$^{-2}$ s$^{-1}$ sr$^{-1}$ keV$^{-1}$, $E_\chi = E(1 + z)$, and $\chi$ is the comoving distance along the line of sight.

If $\delta(z)$ is the mass fluctuation at some position at redshift $z$, then the fluctuation of the emissivity at the same position and redshift is given by $b_X(z) \delta(z) \bar{\eta}(E_\chi, z)$, where we have defined the X-ray emission-weighted bias $b_X(z)$ as

$$ b_X(z) = \frac{1}{\bar{I}(E_\chi, z)} \frac{1}{4\pi} \int_0^\infty dL \frac{d\phi}{dL} L_X(E_\chi, L) b_X(L, z). $$ (9)

For simplicity, we compute the autocorrelation $w_\theta$ of the XRB using the Limber approximation, together with the small angle approximation, as

$$ C_\ell(E) = \bar{I}(E)^{-1} \int \frac{d\chi}{\chi} W^2(E_\chi, \chi) P_\delta(\ell/r), $$

$$ w_\theta(E) = \frac{\int \ell \frac{d\ell}{2\pi} C_\ell(E) J_0(\ell/\theta)}{2}, $$ (10)

where $C_\ell$ is the angular power spectrum, $J_0$ is the zero-order Bessel function, $P_\delta(\ell/r)$ is the linear mass power spectrum today, $r$ is the angular diameter distance ($= \gamma$ for a flat universe), and $W(E_\chi, \chi) = \bar{\eta}(E_\chi, z) b_X(z) D(z)/(1 + z)$. When the power spectrum is measured in practice, shot noise has to be subtracted or should be included in the theoretical prediction, whereas the same is not necessary for the angular correlation except at zero lag.

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4 See http://asc.harvard.edu and http://xmm.vilspa.esa.es, respectively.
Our model predicts the correct mean background spectrum $I(E)$, computed from equation (8), at $E = 1$ keV. We have included all quasars down to the observed 1 keV flux of $2 \times 10^{-17}$ ergs cm$^{-2}$ s$^{-1}$, i.e., we used our models to extrapolate the XRLF to 2 orders of magnitude fainter than the ROSAT detection threshold for discrete sources (Hasinger & Zamorani 1997), to make up the remaining $\sim 50\%$ of the XRB at 1 keV. Our models predict a faint-end slope that is steeper than the Miyaji et al. (2000) fitting formulae, allowing faint quasars to contribute half of the background. The emissivities peak at $z \approx 2$, coinciding with the peak of the XRLF, implying that our model produces most of the XRB, as well as its autocorrelation signal at $z \approx 2$. Note that the known contribution from nearby galaxy clusters is $\sim 10\%$ (Gilli, Risaliti, & Salvati 1999), which we ignore here.

In Figure 5 we show our predictions for the two-point angular correlation $w_\theta$ of the XRB from quasars at 1 keV. Most measurements at the soft X-ray bands have yielded only upper limits, which are consistent with our predictions (e.g., variance at $\leq 0.12$ at a scale of 10' and $E = 0.9-2$ keV; see Carrera et al. 1998 and references therein). Soltan et al. (1999) obtained angular correlations significantly higher than previous results (dashed curve), which, taken at face value, would imply quasar lifetimes $t_0 \approx 10^8$ yr. However, the results of Soltan et al. (1999) could be partially explained by galactic contamination (Barcons et al. 2000). We therefore view this measurement as an upper limit, which is consistent with models using both lifetimes we considered.

Figure 5 shows that $w_\theta$ predicted in the long- and short-lifetime models differ by a factor of $\sim 2$ on angular scales of $0.1^{-1} \degree$, offering another potential probe of quasar lifetimes, provided that $w_\theta$ can be measured more accurately in the future and that the contribution to the clustering signal from nearby nonquasar sources (e.g., clusters) is small or can be subtracted out. Finally, we note that there have been detections of clustering on several degree scales at the hard X-ray bands (2–10 keV) from the HEAO satellite (Treyer et al. 1998); while a prediction for such energies would be interesting (see also Lahav, Piran, & Treyer 1997), it would require an extrapolation of the X-ray spectrum, since we normalize by fitting to the soft X-ray luminosity function.

7. FURTHER CONSIDERATIONS

We have shown above that the quasar lifetime could be measured to high precision, using the soon available large samples of quasars at $z \lesssim 3$, from either the 2dF or the SDSS survey. This precision, however, reflects only the statistical errors in the simple model we have adopted for relating quasars to dark matter halos. The main hindrance in determining the quasar lifetime will likely be systematic errors; here we discuss how several potential complications could affect the derived lifetime.

Obscured sources.—Considerations of the hard X-ray background have led several authors to suggest the presence of a large population of “absorbed” quasars, necessary to fit the hard slope and overall amplitude of the background. Although not a unique explanation for the XRB, this would imply that the true number of quasars near the faint end of both the optical and soft X-ray LF is $\sim 10$ times larger than what is observed. 90% being undetected as a result of large absorbing columns of dust in the optical and neutral hydrogen in the soft X-rays (see, e.g., Gilli et al. 1999). Unless the optically bright and dust-obscured phases occur within the same object (Fabian & Iwasawa 1999), this increase would have a direct effect on our results, since we would then need to adjust our fitting parameters to match an $\sim 10$ times higher quasar abundance. We find that this is easily achieved by leaving $x_0$ and $a$ unchanged and instead raising the lifetime from $10^{6.5}$ to $10^{7.5}$ yr in the short-lifetime model and from $10^8$ to $10^{8.6}$ yr in the long-lifetime model. In the latter case, a tenfold increase in the duty cycle requires only an increase in $t_0$ by a factor of $\approx 4$, owing to the shape of the age distribution $dp/dt$ (Lacey & Cole 1993). Our results would then hold as before, but they would describe the two cases of $t_0 = 10^{7.5}$ and $10^{8.6}$ yr. Interestingly, this scenario would imply that the quasar lifetime could not be shorter than $t_0 \approx 10^{7.5}$ yr, simply based on the abundance of quasars (see § 2). Future infrared and hard X-ray observations should help constrain the abundance of obscured sources and reduce this systematic uncertainty.

Multiple BHs in a single halo.—A possibility that could modify the simple picture adopted above is that a single halo might host several quasar black holes. A massive (e.g., $10^{14} M_\odot$) halo corresponds to a cluster of galaxies, while the Press-Schechter formalism counts this halo as a single object. If quasar activity is triggered by galaxy-galaxy mergers, a massive Press-Schechter halo, known to contain several galaxies, could equally well host several quasars (e.g., Cavaliere & Vittorini 1998). There is some observational evidence of perhaps merger-driven double quasar activity (Owen et al. 1985; Comins & Owen 1991). One could therefore envision that quasars reside in the subhalos of massive “parent” halos, a scenario that would modify the predicted clustering. To address these issues in detail, one needs to know the mass function of subhalos within a given parent halo, as well as the rate at which they merge and turn on. In principle, this information can be extracted.
from Monte Carlo realizations of the formation history of halos in the extended Press-Schechter formalism (i.e., the so-called merger tree) together with some estimate of the timescale for mergers of subhalos based on, for instance, dynamical friction (e.g., Kauffmann & Haehnelt 2000). Here we consider two toy models that we hope can bracket the plausible range of clustering predictions.

To simplify matters, we ignore the scatter in $L$-$M$ in the following discussion. Suppose one is interested in quasars of a luminosity $L$ at redshift $z_0$, which correspond to Press-Schechter halos of mass $M_0$, in our formalism as laid out in §2. This choice of $M_0$ matches the abundance of quasars, expressed approximately as $L(dN/dL)\approx M_0[dN(M_0, z_0)/dM_0](t_0/H_0)$ (the merger or activation rate of halos is approximated as $\sim t_{\text{H}_0}$, where $t_{\text{H}_0}$ is the Hubble time), and implies the bias $b_0(L)\approx b(M_0, z_0)$.

In model A, we suppose that the Press-Schechter halos are identified at some earlier redshift $z_1$; these would be subhalos of those Press-Schechter halos identified at $z_0$. Quasars of luminosity $L$ now correspond to subhalos of mass $M_1$. The abundance of these subhalos is given by the Press-Schechter mass function $dN(M_1, z_1)/dM_1$, which is related to the mass function at $z_0$ by $dN(M_1, z_1)/dM_1 = (M_1/2\pi)^{1/2} M'_{\text{vir}} dM'_{\text{vir}}/(M_{\text{vir}} + M_{\text{vir}})^{3/2}$, where $M'_{\text{vir}} = M_{\text{vir}} dM/dN(M', z_0)/dM$, and $M_{\text{vir}}$ is the average number of $M_1$ + $dM_1$/2 subhalos within parent halos of mass $M_1$, given by (e.g., Sheth & Lemson 1999)

$$dN(M_1, z_1 | M, z_0) = M_1 \frac{1}{M_1} \frac{\delta_{1} - \delta_{0}}{\sqrt{2\pi} [\sigma^{2}(M_1) - \sigma^{2}(M)]^{3/2}} \times \exp \left\{ - \frac{(\delta_1 - \delta_0)^2}{2[\sigma^2(M_1) - \sigma^2(M)]} \right\} \frac{dM_1^{2}}{dM_1^{2}},$$

(11)

where $\delta_1 = \delta/D(z_1)$ and $\delta_0 = \delta/D(z_0)$ (see eq. [2]).

To match the abundance of quasars at luminosity $L$, we impose the condition that $M_1[dN(M_1, z_1)/dM_1] \approx M_0[dN(M_0, z_0)/dM_0]$, which determines $M_1$ given $z_1$, $M_0$, and $z_0$. The bias of the quasars is no longer $b_0(L)\approx b(M_0, z_0)$ but is instead given by

$$b_{\text{eff}}^{A}(L, z_1) = \left[ \frac{dN(M_1, z_1)}{dM_1} \right]^{-1} \int_{M_1}^{\infty} dM \frac{dN(M, z_0)}{dM} \times dN(M_1, z_1 | M, z_0) \frac{b(M, z_0)}{dM_1}.$$  

(12)

We show in Figure 6 the ratio of $b_{\text{eff}}^{A}(L, z_1)/b_0(L)$ as a function of $z_1$, for $z_0 = 3$ and $z_0 = 2$, respectively, and for a range of masses $M_0$, which are representative of the halos that dominate our clustering signal in previous discussions. It is interesting how the bias $b_{\text{eff}}^{A}(L, z_1)$ is not necessarily larger than our original bias $b_0(L)$, despite the fact that the bias of subhalos should be boosted by their taking residence in bigger halos. This is because the relevant masses here (e.g., $M_0$) are generally large, and we find that the number of halos of mass $M_0$ at $z_1$ is always smaller than the number of halos of the same mass at $z_0 < z_1$. Hence, to match the observed abundance of quasars at the same $L$, $M_1$ must be chosen to be smaller than $M_0$. As Figure 6 shows, this could, in some cases, more than compensate the increase in clustering due to massive parent halos. Because of these two opposing effects, the bias does not change by more than about 50% even if one considers $z_1$ as high as 10. This translates into a factor of $\sim 2$ uncertainty in our predictions for the quasar power spectrum. Our clustering predictions for the short- and long-lifetime models differ by a factor of $\sim 5$, implying that the lifetime can still be usefully constrained at $z \geq 2$. As we can see from Figure 6, at lower redshifts, or equivalently lower $M_0/M_*$, our predictions for the quasar power spectrum should be more uncertain.

One might imagine modifying the above model by allowing mergers to take place preferentially in massive parents and therefore boosting the predicted bias. In model B, we adopt a more general procedure of matching the observed quasar abundance by $L(dN/dL) = M_1 \int_{M_1}^{\infty} dM \frac{dN(M, z_0)/dM}{dM} \times dN(M_1, z_1 | M, z_0) / dM_1$, where $(t_0/H_0)f(M_1, M)$ is the probability that an $M_1$ subhalo residing within a parent halo of $M_0$ harbors an active quasar of luminosity $L$. It is conceivable that $f$ increases with the parent mass $M_0$: a more massive parent might encourage more quasar activity by having a higher fraction of mergers or collisions. The following heuristic argument shows that one might expect $[dN(M_1, z_1 | M, z_0) / dM_1]/f(M_1, M)$ to scale approximately as $\sim M_0^{4/3}$. Let $N_k$ be the number of subhalos inside a parent halo of mass $M$. The rate of collisions is given by $N_k^2 v_\perp \sigma_R R^3$, where $v_\perp$ is the velocity of the subhalos, $\sigma_R$ is their cross section, and $R^3$ is the volume of the parent halo. Using $N_k \propto M$ (which can be obtained from eq. [11] in the large $M$ limit), $v_\perp \propto (M/R)^{1/2}$ (virial theorem), and $R^3 \propto M$ (fixed overdensity of $\sim 200$ at the redshift of formation), the rate of collisions scales with the parent mass as $M_0^{4/3}$, if one ignores the possibility that $\sigma_R$ might depend on the parent mass as well. A similar scaling of $M_0^{4/3}$ has been observed in simulations of the starburst model for Lyman break objects (Kolatt et al. 1999; Wesciler et al. 1999).
To model the enhanced rate of collisions inside massive parent halos, we can simply modify model A by using $f(M_1, M) = (M/M_1)^{1/3}$. The effective bias is given by

$$b_{\text{eff}}^\theta(L, z_1) = \left[ \int_{M_1}^\infty dM \frac{dN(M, z_0)}{dM} d, \zeta (M_1, z_1 | M, z_0) \left( \frac{M}{M_1} \right)^{1/3} \right]^{-1} \times \left[ \int_{M_1}^\infty dM \frac{dN(M, z_0)}{dM} d, \zeta (M_1, z_1 | M, z_0) \left( \frac{M}{M_1} \right)^{1/3} b(M, z_0) \right].$$

(13)

We find that the above prescription does not significantly alter our conclusions following from model A: the $M^{1/3}$ enhancement of the activation rate inside massive parent halos turns out to be relatively shallow and translates to a small effect in the bias. Finally, we note that $z_1$ above could in principle depend on $M_0$ and $M_1$, a possibility that would require further modeling and is not pursued here.

Galaxies without BHs.—Another possibility that could modify our picture is that only a fraction $f < 1$ of the halos harbor BHs; the duty cycle could then reflect this fraction, rather than the lifetime of quasars. Although there is evidence (e.g., Magorrian et al. 1998) that most nearby galaxies harbor a central BH, this is not necessarily the case at redshifts $z = 2-3$: the fraction $f$ of galaxies hosting BHs at $z = 2-3$ could in principle have merged with the fraction $1 - f$ of galaxies without BHs, satisfying the local constraint.

Using the extended Press-Schechter formalism (Lacey & Cole 1993), one can compute the rate of mergers between halos of various masses. On galaxy mass scales, the merger rates at $z = 2-3$ are comparable to the reciprocal of the Hubble time (see Fig. 5 in Haiman & Menou 2000), implying that a typical galaxy did not go through numerous major mergers between $z = 2-3$ and $z = 0$, i.e., that the fraction $f$ cannot be significantly less than unity at $z = 2-3$. A more detailed consideration of this issue is beyond the scope of this paper; we simply note that the lifetimes derived here scale approximately as $1/f$, where $f$ is likely of order unity.

Larger scatter in $L/M$.—The scatter $\sigma$ we assumed around the mean relation between quasar luminosity and halo mass is motivated by the scatter found empirically for the $M_{\text{bul}} - M_{\text{halo}}$ relation (Magorrian et al. 1998). It is interesting to consider the sensitivity of our conclusions to an increased $\sigma$. In general, scatter raises the number of quasars predicted by our models, by an amount that depends on the slope of the underlying mass function $dN/dM$. As a result, increasing $\sigma$ raises and flattens the predicted LF. We find that an increase of $\sigma$ from 0.5 to 1 (an additional order of magnitude of scatter) can be compensated by a steeper $L_{\text{bul}}$ relation, typically replacing $\alpha$ with $\approx \alpha - 0.5$. As a result of the increase in $\sigma$, quasars with a fixed $L$ are, on average, associated with larger and more highly biased halos.

Nevertheless, we find that the mean bias $b$ of all sources above a fixed flux (see eq. [5]), and therefore the correlation length $r_0$, is unchanged by the increased scatter (at the level of $\sim 3\%$). The reason for the insensitivity of $r_0$ to the amplitude of the scatter can be understood as follows. The mean bias $b$ of all sources with $L > L_{\text{min}}$ is dominated by the bias $b(L)$ of sources near the threshold $L_{\text{min}}$. The latter is obtained by averaging $b(M)$ over halos of different masses (see eq. [4]), and it is dominated by the bias of the smallest halos within the width of the scatter, i.e., of halos with mass $M_{\text{min}} = M/10^4$, where $\bar{M}$ defines the mean relation between $L_{\text{min}}$ and halo mass, i.e., $L_{\text{min}} = \bar{L}(\bar{M})$, and $\sigma$ quantifies the scatter (see eq. [A3]). As mentioned before, increasing the scatter makes the luminosity function flatter, which means to match the observed abundance of halos at a fixed luminosity $L_{\text{min}}$, one has to choose a higher $\bar{M}$. In other words, $\bar{M}$ scales up with the scatter, and it turns out to scale up approximately as $10^4$, making $M_{\text{min}}$ and hence the effective bias roughly independent of scatter.

We note that the relation between quasar luminosity and halo mass can, in principle, be derived from observations, by measuring $M_{\text{halo}}$ for the hosts of quasars (e.g., by weak lensing, or by finding test particles around quasars, such as nearby satellite galaxies).

Mass- and redshift-dependent lifetime.—In all of the above, we have assumed that the quasar lifetime is a single parameter, independent of the halo mass. This is not unreasonable if the Eddington time, the timescale for the growth of black hole mass, is indeed the relevant timescale, $4 \times 10^7(\epsilon/0.1) \text{yr}$. Implicit in such reasoning is that the active phase of the quasar is coincident with the phase where the black hole gains most of its mass. This is not the only possibility (see Haehnelt et al. 1998 for more discussions). One can attempt to explore how $t_\phi$ depends on halo mass by applying our method to quasars grouped into different absolute luminosity ranges, but the intrinsic scatter in the mass-luminosity relation should be kept in mind. We emphasize, however, since we fitted the luminosity function and clustering data at the same redshift, there is no need within our formalism to assume a redshift-independent lifetime. In fact, performing our exercise as a function of redshift could give interesting constraints on how $t_\phi$ evolves with redshift.

8. CONCLUSIONS

In this paper we have modeled the quasar luminosity function in detail in the optical and X-ray bands using the Press-Schechter formalism. The lifetime of quasars $t_\phi$ enters into our analysis through the duty cycle of quasars, and we find that matching the observed quasar LF to dark matter halos yields the constraint $10^6 \lesssim t_\phi \lesssim 10^{10} \text{yr}$; smaller lifetimes would imply overly massive BHs, while longer lifetimes would necessitate overly massive halos. This range reassuringly brackets the Eddington timescale of $4 \times 10^7(\epsilon/0.1) \text{yr}$.

The main conclusion of this paper is that the lifetime (and hence $\epsilon$, if the Eddington time is the relevant timescale for quasar activity) can be further constrained within this range using the clustering of quasars: for quasars with a fixed luminosity, longer $t_\phi$ implies larger host halo masses and higher bias. We find that as a result, the correlation length $r_0$ varies strongly with the assumed lifetime. Preliminary data from the 2dF survey already set mild constraints on the lifetime. Depending on the assumed cosmology, we find $t_\phi = 10^{7.7 \pm 0.8 \text{yr (ACDM)}}$ or $t_\phi = 10^{8.8 \pm 0.8 \text{yr (CDM)}}$ to within $1 \sigma$ statistical uncertainty. These values are also found to satisfy upper limits on the autocorrelation function of the soft X-ray background.

Forthcoming large quasar samples from SDSS or the complete 2dF survey are ideally suited for a study of quasar clustering, and they can, in principle, constrain the quasar lifetime to high accuracy, with small statistical errors. We
expect the modeling of the quasar-halo relation as well as the possible presence of obscured quasars to be the dominant sources of systematic uncertainty. Not discussed in depth in this paper is the possibility of using higher moments (such as skewness), which our models also make definite predictions for and will be considered in a future publication. Remarkably, our best determination of the lifetime of quasars might come from the statistics of high-redshift quasars, rather than the study of individual objects.

Near the completion of this work, we became aware of a similar, independent study by P. Martini & D. Weinberg.

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APPENDIX

In this appendix we describe our models for the luminosity function of quasars, based on associating quasar BHs with dark halos. Our main assumption is that there is, on average, a direct monotonic relation between halo mass and quasar light. Our treatment is similar to that of previous works (Haiman & Loeb 1998; Haehnelt et al. 1998) but differs in some of the details. We adopt the parameterization of the observational LF in the optical $B$ band, given in the redshift range $0 < z < 4$ by Pei (1995). We assume the background cosmology to be either flat ($\Lambda$CDM) with $(\Omega_M, \Omega_{\Lambda}, h, \sigma_8, n) = (0.7, 0.3, 0.65, 1.0, 1.0)$ or open (OCDM) with $(\Omega_M, \Omega_{\Lambda}, h, \sigma_8, n) = (0, 0.3, 0.65, 0.82, 1.3)$. The LF quoted by Pei (1995) is scaled appropriately with cosmology by keeping $(d\phi/dL)dVd\Omega = \text{constant}$ (where $dV$ is the volume element and $dL$ is the luminosity distance), so that $(d\phi/dL)dL$ is the comoving abundance in $\text{Mpc}^{-3}$ of quasars with $B$-band luminosity $L$ (in solar units $L_{\odot}$). The comoving abundance $dN/dM(z)$ of dark halos is assumed to follow the Press-Schechter (1974) formalism. We assume that each halo harbors a single quasar that turns on when the halo forms, i.e., triggered by merger (e.g., Percival & Miller 1999), and shines for a fixed lifetime $t_Q$ (relaxing these assumptions is discussed above in §7). The duty cycle $f_{\text{on}}$ of halos with mass $M$ at redshift $z$ is then given by the fraction of these halos younger than $t_Q$. The distribution of ages for halos of mass $M$ existing at redshift $z$ is obtained using the extended Press-Schechter formalism, which assumes that the halo formed at the epoch when it acquired half of its present mass (Lacey & Cole 1993). The duty cycle, which is the probability that a dark matter halo of a given mass harbors an active quasar, is simply

$$f_{\text{on}}(M, z) = \int_0^{t_Q} dt \frac{dp}{dt}(M, z, t).$$  \hspace{1cm} \text{(A1)}$$

A model in which the quasar turns on/off more gradually (as expected if the mass of the BH grows significantly during the luminous quasar phase) is equivalent to one having additional scatter in the ratio $L/M$, which is discussed in §7. We next relate the quasar luminosity to the mass of its host halo. We define $dp/dL(M, L, z)$ to be the probability that a halo of mass $M$ at redshift $z$ hosts a quasar with luminosity $L$ and express this quantity as

$$\frac{dp}{dL}(L, M, z) = \frac{dg}{dL}(L, M_{\text{tot}}, z)f_{\text{on}}(M, z).$$  \hspace{1cm} \text{(A2)}$$

Here $dg/dL(M_{\text{tot}}, \bar{L}_M, z)$ is the probability distribution of luminosities associated with the subset of halos of mass $M$ harboring a live quasar (normalized to $\int_0^\infty dL dg/dL = 1$), and $M_{\text{tot}}$ is the mass quasar luminosity for these halos. In the limit of a perfect intrinsic correlation, we would have $dg/dL(L, M_{\text{tot}}, z) = \delta(L - L_{M_{\text{tot}}})$; more realistically, this correlation will have nonnegligible scatter. Lacking an a priori theory for this scatter, we here simply assume that it follows the same functional form as the scatter found empirically for the $M_{\text{bulge}} - M_{\text{bulge}}$ relation (Magorrian et al. 1998), and we set

$$\frac{dg}{dL}(L, \bar{L}_M) \propto \exp \left[ -\frac{(\log L - \bar{L}_M)^2}{2\sigma^2} \right].$$  \hspace{1cm} \text{(A3)}$$

For reference, we note that the empirical scatter between $M_{\text{bulge}}$ and $M_{\text{bulge}}$ gives $\sigma \sim 0.5$; it is not yet clear, however, what fraction of this scatter is intrinsic versus instrumental (van der Marel 1999). One might expect the scatter in the $L_{\text{halo}}$-$M_{\text{halo}}$ relation not to be significantly larger, since (1) for a sufficiently high fueling rate, the luminosity $L$ corresponding to $M_{\text{halo}}$ is likely to always be near the Eddington limit; and (2) at least for disk galaxies, the bulge luminosity correlates well with the total luminosity $L_{\text{bulge}}$ ($\sigma \sim 0.5$; see, e.g., Andreakis & Sanders 1994); $L_{\text{bulge}}$ is tightly correlated with the velocity dispersion $\sigma_v$ through the Tully-Fisher relation (e.g., Raychaudhury et al. 1997) as is the total halo mass to $\sigma_v$ (Eisenstein & Loeb 1996). Nevertheless, in §7 we investigated the consequences of an increased scatter. We note that an extension of the models presented here, by following the merger histories of halos and their BHs, can, in principle, be used to estimate the scatter in $L/M_{\text{halo}}$. Cattaneo, Haehnelt, & Rees (1999) have used this approach to fit the observed relation $M_{\text{bulge}}/M_{\text{bulge}}$, including its scatter.
Fitting parameters $a(z)$ and $x_0(z)$ for the mean relation between $B$-band quasar luminosity $L_B$ and halo mass $M_{\text{halo}}$ given in eq. (A7), for two different quasar lifetimes: $t_Q = 10^{6.5}$ yr (solid curves) and $t_Q = 10^8$ yr (dotted curves). The filled dots correspond to a ΛCDM and the open dots to an OCDM cosmology. In all cases, we assumed a scatter with $\sigma = 0.5$ (see eq. [A3]) around the mean $L$-$M_{\text{halo}}$ relation.

Under the above assumptions, the cumulative probability that a halo of mass $M$ hosts a quasar with luminosity equal to or greater than $L$ is given by

$$p(L, M, z) = f_{\text{on}}(M, z) \int_L^\infty dL \frac{dq}{dL}(L, L_{M,z}) ,$$

(A4)

and matching the observed cumulative quasar LF requires

$$\int_L^\infty dL \frac{d\phi}{dL}(L, z) = \int_0^\infty dM \frac{dN}{dM}(M, z)p(L, M, z) ,$$

(A5)

or alternatively, matching the differential LF gives

$$\frac{d\phi}{dL}(L, z) = \int_0^\infty dM \frac{dN}{dM}(M, z) \frac{dq}{dL}(L, L_{M,z})f_{\text{on}}(M, z) .$$

(A6)

Equation (A5) or (A6), together with equations (A1), (A3), and (A4), implicitly determines the function $L_{M,z}$, once the quasar lifetime $t_Q$ and magnitude of scatter $\sigma$ are specified. In general, these equations would need to be solved iteratively. In practice, we have found that sufficiently accurate solutions (given the error bars on the observational LF in Pei 1995; see Fig. 1) can be found by using the simple power-law Ansatz:

$$L_{M,z} = x_0(z) M_{\text{halo}} \left( \frac{M_{\text{halo}}}{M_0} \right)^{a(z)} ,$$

(A7)

where the coefficients $x_0(z)$ and $a(z)$ depend on $t_Q$, $\sigma$, and the underlying cosmology (Haiman & Loeb 1998; Haehnelt et al. 1998). In summary, assuming a fixed scatter $\sigma$, our model has only one free parameter, the quasar lifetime $t_Q$.

We emphasize that our parameterization in equation (A7) is purely phenomenological: it gives us a convenient way to relate the quasar luminosity to the host halo mass ($L_{M,z}$). In reality, the quasar luminosity likely depends on the details of its immediate physical environment (e.g., gas supply, magnetic fields, angular momentum distribution, etc.), in addition to the halo mass. Our description includes these possibilities only in allowing a nonnegligible scatter around the mean relation $L_{M,z}$.

The rationale behind this choice is that the average properties of the physical environment should ultimately be governed by the halo mass (or circular velocity), as expected within the picture of structure formation via hierarchical clustering.

A useful check on the physical implications of equation (A7) is obtained by assuming that the luminosity $L_{M,z}$ is produced by a BH of mass $M_{\text{bh}}$, shining at the Eddington limit $L_{\text{Edd}} = (4\pi G m_p c/\sigma_V) M_{\text{bh}}$. In the mean spectrum of a sample of quasars with detections from radio to X-ray bands (Elvis et al. 1994), $\approx 7\%$ of the bolometric luminosity is emitted in the rest-frame $B$ band, resulting in $L = 0.07 L_{\text{Edd}} = 5 \times 10^3 L_{B,\odot} (M_{\text{bh}}/M_{\odot})$. Equation (A7) then translates into a relation between the mass of a BH and its host halo,

$$M_{\text{bh}} = 10^{-3.7} x_0(z) M_{\text{halo}} \left( \frac{M_{\text{halo}}}{M_0} \right)^{a(z)} ,$$

(A8)
As an example, Haehnelt et al. (1998) argue that the central BH mass is determined by a radiative feedback from the central BH that would unbind the disk in a dynamical time. Their derived scaling corresponds to $x = \frac{2}{3}$ and $x_0 \propto (1 + z)^{5/2}$, not far from what we find for the long-lifetime case (see Fig. 7 and discussion below).

In Figure 7 we show the values of the parameters $x_0(z)$ and $x(z)$ obtained in our models when two different quasar lifetimes are assumed, $t_Q = 10^{6.5}$ yr (solid curves) and $t_Q = 10^8$ yr (dotted curves). The filled dots show the parameters in $\Lambda$CDM, and the empty dots in the OCDM cosmology. We have set the arbitrary constant $M_\odot = 10^{12} M_\odot$ in both cases. Note that $t_Q$ determines both $x$ and $x_0$, and therefore the values of $x$ and $x_0$ are correlated. In general, the fitting parameters show little evolution in the range $2 < z < 4$, around the peak of the quasar LF. According to equation (A8), the corresponding BH masses in, e.g., a $10^{12} M_\odot$ halo at $2 < z < 4$ are $M_{bh} \approx 4 \times 10^{-4} M_{\text{halo}} = 4 \times 10^8 M_\odot$ and $M_{bh} \approx 2 \times 10^{-5} M_{\text{halo}} = 2 \times 10^7 M_\odot$ in the short- and long-lifetime models, respectively.

The fitting procedure described above can be repeated in the X-ray bands. We therefore fit the XRLF using equation (A7) analogously to the optical case, except $L_{M,x}$ now denotes the X-ray luminosity at 1 keV, quoted in units of erg s$^{-1}$. Note that the XRLF in Miyaji et al. (2000) is quoted as a function of luminosity at observed 1 keV, i.e., no $K$-correction is applied (alternatively, the XRLF can be interpreted as the rest-frame luminosity function of sources with an average intrinsic photon index of 2). Figure 8 shows the resulting fitting parameters $x_0$ and $x$ in the $\Lambda$CDM cosmology, analogous to those shown in Figure 8 for the optical case. It is apparent that both parameters have a somewhat different behavior from that in the optical. This reflects the fact that the mean quasar spectrum must evolve with redshift or is at least black hole/halo mass dependent: if every quasar had the same spectrum or at least a similar X-ray/optical flux ratio, the fitting parameters derived from the optical and X-ray LF would differ only by a constant in $x_0$. For our purpose of deriving clustering, it is sufficient to treat $x_0$ and $x$ as phenomenological fitting parameters, and we do not address the physical reason for the apparent spectral evolution (see Haiman & Menou for a brief discussion).

It is important to note that the simple power-law Ansatz in equation (A7) with the parameters shown in Figure 8 adequately fits only the faint end of the XRLF. In the optical case, the entire range of observed luminosities is well matched by our models (see Fig. 1). In comparison, the well-fitted range in X-rays typically extends from the detection threshold to up to 2–3 orders of magnitude in luminosity (i.e., typically up to $\sim 3 \times 10^{44}$ ergs s$^{-1}$), depending on redshift, and our models underestimate the abundance of still brighter quasars. One might then consider searching for a different Ansatz to replace equation (A7) that fits the entire range of the observed LF. However, we have verified that the rare quasars with these high luminosities would contribute negligibly both to the clustering signals and to the XRB investigated here. Therefore, we did not consider further improvements over equation (A7), since this would not change our results.

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