Sensitivity physics expected to the measurement of the quartic $WW\gamma\gamma$ couplings at the LHeC and the FCC-he

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Abstract

We explore the physics expected sensitivity at the Large Hadron electron Collider (LHeC) and the Future Circular Collider-hadron electron (FCC-he) to search for the anomalous quartic $WW\gamma\gamma$ couplings in single $W$-boson production in association with a photon. We study the process $e p \rightarrow e^-\gamma^* p \rightarrow e W \gamma q' X$ via the subprocess $\gamma^* q \rightarrow W \gamma q'$. The center-of-mass energies and luminosities of the LHeC are assumed to be $\sqrt{s} = 1.30, 1.98$ TeV, $\mathcal{L} = 10 - 100$ fb$^{-1}$ and for the FCC-he $\sqrt{s} = 3.46, 5.29$ TeV and $\mathcal{L} = 100 - 1000$ fb$^{-1}$. With these energies and luminosities, we estimate bounds on the anomalous quartic $WW\gamma\gamma$ couplings at 95% C.L., which can be an order of magnitude stringent than the experimental limits report by ATLAS and CMS Collaborations at the LHC.

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I. INTRODUCTION

The $SU(2)_L \times U(1)_Y$ gauge invariant structure of the Standard Model (SM) \cite{1,3} specifies the form and strength of the self-interactions of the vector bosons fields, particularly the Quartic Gauge Couplings (QGC): $WW\gamma\gamma$, $WW\gamma Z$, $WWZZ$, $WWWW$, $ZZZZ$, $ZZZ\gamma$, $ZZ\gamma\gamma$, $Z\gamma\gamma\gamma$ and $\gamma\gamma\gamma\gamma$. Studying processes to which these QGC can contribute may yield further confirmation of the non-Abelian gauge structure of the SM or signal the presence of new physics Beyond the SM (BSM), at as yet unprobed energy scales. Such as the present and future colliders: the Large Hadron Collider (LHC), the High-Luminosity Large Hadron Collider (HL-LHC), the High-Energy Large Hadron Collider (HE-LHC) \cite{4}, the Large Hadron electron Collider (LHeC) \cite{5,8}, the Future Circular Collider-hadron electron (FCC-he) \cite{9}, the International Linear Collider (ILC) \cite{10}, the Compact Linear Collider (CLIC) \cite{11}, the Circular Electron Positron Collider (CEPC) \cite{12} and the Future Circular Collider $e^+e^-$ (FCC-ee) \cite{13}. All these colliders contemplate in their physics programs the study of the aQGC.

In the last few years, the aQGC production processes and single-$W$ and double-$W$ production in hadron-hadron, lepton-hadron and lepton-lepton colliders in its different collision modes, have attracted the attention because future colliders with high energies, high luminosities and cleaner environments may allow their experimental studies. Such studies are interesting for several reasons. They allow the further independent test of the SM, the quartic $WW\gamma\gamma$ vertex can be probed and the Higgs boson plays an important role in $WW$ channel.

In this paper, we are interested in estimating limits in the aQGC $f_{M,i}$ and $f_{T,i}$ with $i = 0,1,2,\ldots,7$ for the possible energies and luminosities of the LHeC and the FCC-he in its different stages, that is $\sqrt{s} = 1.30$ TeV, $1.98$ TeV, $\mathcal{L} = 10$ fb$^{-1}$, $30$ fb$^{-1}$, $50$ fb$^{-1}$, $100$ fb$^{-1}$ and $\sqrt{s} = 3.46$ TeV, $5.29$ TeV, $\mathcal{L} = 100$ fb$^{-1}$, $300$ fb$^{-1}$, $500$ fb$^{-1}$, $1000$ fb$^{-1}$, respectively. There are 13 Feynman diagrams at the tree level contributing to the process $ep \rightarrow e^-\gamma^* p \rightarrow eW\gamma q'X$ via the subprocess $\gamma^* q \rightarrow W\gamma q'$, where $q = u, c, \bar{d}, \bar{s}$ and $q' = d, s, \bar{u}, \bar{c}$ as shown in Figs. 1 and 2.

For an experimental and phenomenological review on the evolution in the measurements of the limits on the aQGC in the context of present and future colliders, such as the LEP at the CERN \cite{14–17}, D0 and CDF at the Tevatron \cite{18,19}, ATLAS and CMS at the LHC
and in the post-LHC era as the LHeC and the FCC-he \cite{22, 23}, the ILC, the CLIC, the CEPC and the FCC-ee, we invited the reader to review the Refs. \cite{24–57}, as well as Table I of Ref. \cite{23}.

This paper describes searches for the sensitivity physics expected to the measurement of the $WW\gamma\gamma$ aQGC at the LHeC and the FCC-he using the process $ep \rightarrow e^-\gamma^*p \rightarrow eW\gamma q' X$. In section II, we briefly describe the theoretical aspects of the operators in our effective Lagrangian. In Section III, we derive the bounds for the aQGC at the LHeC and the FCC-he. In Section IV, we summarize our conclusions.

II. BRIEF REVIEW OF THE THEORETICAL ASPECTS

In the absence of a specific model of new physics, the Effective Field Theory (EFT) approach is very useful. An EFT parameterizes, in a model-independent way, the low-energy effects of the new physics to be found at higher energies.

With this approach, we start from an EFT to probing model-independent limits on $W^+W^−\gamma\gamma$ quartic gauge boson vertex. The EFT approach is the natural way to extend the SM such that the gauge symmetries are respected. In addition, the EFT it is general enough to capture any BSM physics. The EFT yet also provides guidance as to the most likely place to see the effects of new physics.

The measurement of the $WW\gamma\gamma$ couplings can be made quantitative by introducing a more general $WW\gamma\gamma$ vertex. For our discussion of phenomenological sensitivities in Section III, we shall use the phenomenological effective Lagrangian which comes from several $SU(2)_L \times U(1)_Y$ invariant dimension-8 effective operators \cite{58}:

$$\mathcal{L}_{\text{eff}} = \sum_{i=1}^{2} \frac{f_{S,i}}{\Lambda^4} O_{S,i} + \sum_{i=0}^{9} \frac{f_{T,i}}{\Lambda^4} O_{T,i} + \sum_{i=0}^{7} \frac{f_{M,i}}{\Lambda^4} O_{M,i},$$

in this equation, the indices S, T and M of the couplings and the operators represent three classes of genuine aQGC operators \cite{57}. In addition, it should be mentioned that the $f_{T,i}/\Lambda^4$ associated operators characterize the effect of new physics on the scattering of transversely polarized vector bosons, and $f_{M,i}/\Lambda^4$ includes mixed transverse and longitudinal scatterings. A list of these operators is given below.

i) First class of independent scalar operators:
\[
O_{S,0} = [(D_{\mu} \Phi)^\dagger (D_{\nu} \Phi)] \times [(D_{\mu} \Phi)^\dagger (D_{\nu} \Phi)],
\]
\[
O_{S,1} = [(D_{\mu} \Phi)^\dagger (D_{\nu} \Phi)] \times [(D_{\nu} \Phi)^\dagger (D_{\mu} \Phi)],
\]

\[\text{ii) Second class of independent mixed operators:}\]

\[
O_{M,0} = Tr[W_{\mu\nu} W^{\mu\nu}] \times [(D_{\beta} \Phi)^\dagger (D^{\beta} \Phi)],
\]
\[
O_{M,1} = Tr[W_{\mu\nu} W^{\nu\beta}] \times [(D_{\beta} \Phi)^\dagger (D^{\mu} \Phi)],
\]
\[
O_{M,2} = [B_{\mu\nu} B^{\mu\nu}] \times [(D_{\beta} \Phi)^\dagger (D^{\beta} \Phi)],
\]
\[
O_{M,3} = [B_{\mu\nu} B^{\nu\beta}] \times [(D_{\beta} \Phi)^\dagger (D^{\mu} \Phi)],
\]
\[
O_{M,4} = [(D_{\mu} \Phi)^\dagger W_{\beta\nu} (D^{\mu} \Phi)] \times B^{\beta\nu},
\]
\[
O_{M,5} = [(D_{\mu} \Phi)^\dagger W_{\beta\nu} (D^{\nu} \Phi)] \times B^{\beta\mu},
\]
\[
O_{M,6} = [(D_{\mu} \Phi)^\dagger W_{\beta\nu} W^{\beta\nu} (D^{\mu} \Phi)],
\]
\[
O_{M,7} = [(D_{\mu} \Phi)^\dagger W_{\beta\nu} W^{\beta\mu} (D^{\nu} \Phi)],
\]

\[\text{ii) Third class of independent transverse operators:}\]

\[
O_{T,0} = Tr[W_{\mu\nu} W^{\mu\nu}] \times Tr[W_{\alpha\beta} W^{\alpha\beta}],
\]
\[
O_{T,1} = Tr[W_{\alpha\nu} W^{\mu\beta}] \times Tr[W_{\mu\beta} W^{\alpha\nu}],
\]
\[
O_{T,2} = Tr[W_{\alpha\mu} W^{\mu\beta}] \times Tr[W_{\beta\nu} W^{\nu\alpha}],
\]
\[
O_{T,5} = Tr[W_{\mu\nu} W^{\mu\nu}] \times B_{\alpha\beta} B^{\alpha\beta},
\]
\[
O_{T,6} = Tr[W_{\alpha\nu} W^{\mu\beta}] \times B_{\mu\beta} B^{\alpha\nu},
\]
\[
O_{T,7} = Tr[W_{\alpha\mu} W^{\mu\beta}] \times B_{\beta\nu} B^{\nu\alpha},
\]
\[
O_{T,8} = B_{\mu\nu} B^{\mu\nu} B_{\alpha\beta} B^{\alpha\beta},
\]
\[
O_{T,9} = B_{\alpha\mu} B^{\mu\beta} B_{\beta\nu} B^{\nu\alpha},
\]

In the operators (2)-(19) \(D_{\mu}\) is the covariant derivative, \(\Phi\) denotes the Higgs double field and \(B^{\mu\nu}, W^{\mu\nu}\) are the field strength tensors. It is appropriate to mention that the \(O_{S,0}\) and \(O_{S,1}\) operators given by Eqs. (2) and (3) contain the quartic \(W^+ W^- W^+ W^-\), \(W^+ W^- ZZ\) and \(ZZZZ\) couplings, which do not concern us here. In Refs. [32, 40, 41, 56–58], an exhaustive
study on the mechanism to build the dimension-8 operators corresponding to the aQGC is presented.

III. CROSS SECTION MEASUREMENTS AT THE LHEC AND THE FCC-HE

The phenomenological investigations at $ep$ colliders generally contain usual deep inelastic scattering reactions where the colliding proton dissociates into partons. Even these reactions have been more examined in the literature, exclusive and semi-elastic processes that are $\gamma^*\gamma^*$ and $\gamma^*p$ have been much less studied. The exclusive and semi-elastic processes have simpler final states with respect to $ep$ processes. Hence, these processes compensate for the advantages of $ep$ processes such as having high center-of-mass energy and high luminosity. Here, $\gamma^*p$ processes have effective luminosity and much higher energy compared to $\gamma^*\gamma^*$ process. This may be significant because of the high energy dependencies of the cross-sections containing the new physics parameters. For this reason, $\gamma^*p$ processes are expected to have a high sensitivity to the aQGC.

$\gamma^*p$ processes can be discerned from usual deep inelastic scattering processes by means of two experimental signatures. First signature is the forward large rapidity gap. Quasireal photons have a low virtuality and scattered with small angles from the beam pipe. Since the transverse momentum carried by a quasireal photon is small, photon emitting the incoming electron should also be scattered with small angles and exit the central detector without being detected. This causes a decrease in the energy deposit in the corresponding forward region. As a result of this, one of the forward regions of the central detector has a significant lack of energy. This defines the forward large-rapidity gap and usual $ep$ deep inelastic processes can be rejected by applying a selection cut on this quantity. Second experimental signature is provided by the forward detectors. Forward detectors are capable to detect particles with a large pseudorapidity. When a photon emitting from electron is scattered with a large pseudorapidity, it exceeds the pseudorapidity coverage of the central detectors. The detection of this electron by the forward detectors provides a distinctive signal for $\gamma^*p$ processes. However, LHeC Collaboration has a program of forward physics with extra detectors located in a region between a few tens up to several hundreds of metres from the interaction point [59].

In this section, the cross section of the $ep \rightarrow e^-\gamma^*p \rightarrow eW\gamma q'X$ signal is evaluated for the
center-of-mass energies and luminosities of the LHeC and the FCC-he, with their respective energies and luminosities $\sqrt{s} = 1.30, 1.98$ TeV, $\mathcal{L} = 10 - 100$ fb$^{-1}$ and $\sqrt{s} = 3.46, 5.29$ TeV, $\mathcal{L} = 100 - 1000$ fb$^{-1}$. For $ep \rightarrow e^- \gamma^* p \rightarrow eWq'X$ signal, we consider leptonic and hadronic decays of the $W$-boson; $W \rightarrow \nu l$, $W \rightarrow qq'$ with $\nu_l = \nu_e, \nu_\mu$, $l = e^-, \mu$ and $q = u, c, \bar{d}, \bar{s}$, $q' = d, s, \bar{u}, \bar{c}$, respectively.

To optimize the measurement of the electroweak-induced $eWq'X$ signal and improve the electroweak signal significance, we further consider selections on the following variables to suppress backgrounds. A summary of the baseline selection criteria for the kinematics cuts on the final state particles is given below.

i) Selected cuts for the $p_T$:

- $p_T^q > 20$ GeV (minimum $p_T$ for the jets), \hspace{1cm} (20)
- $p_T^\gamma > 10$ GeV (minimum $p_T$ for the photons), \hspace{1cm} (21)
- $p_T^l > 10$ GeV (minimum $p_T$ for the charged leptons). \hspace{1cm} (22)

ii) Selected cuts for the $\eta$:

- $|\eta_q| < 5$ (max rap for the jets), \hspace{1cm} (23)
- $|\eta_\gamma| < 2.5$ (max rap for the photons), \hspace{1cm} (24)
- $|\eta_l| < 2.5$ (max rap for the charged leptons). \hspace{1cm} (25)

iii) Selected cuts for the $\Delta R$:

- $\Delta R_{qq} = 0.4$ (min distance between jets), \hspace{1cm} (26)
- $\Delta R_{ll} = 0.4$ (min distance between leptons), \hspace{1cm} (27)
- $\Delta R_{\gamma q} = 0.4$ (min distance between $\gamma$ and jet), \hspace{1cm} (28)
- $\Delta R_{q\ell} = 0.4$ (min distance between jet and lepton), \hspace{1cm} (29)
- $\Delta R_{\gamma \ell} = 0.4$ (min distance between $\gamma$ and lepton). \hspace{1cm} (30)

As we mentioned above, the kinematic cuts given by Eqs. (20)-(30) are applied to reduce the background and to reach higher expected significance for the possible aQGC signal in the...
process \( ep \rightarrow e^-\gamma^*p \rightarrow eW\gamma q'X \). The sensitivities are investigated using the Monte Carlo simulations with a leading order in MadGraph5_aMC@NLO \[60\]. The operators described in Eqs. (4)-(19) are implemented into MadGraph5_aMC@NLO through Feynrules package [61] as a Universal FeynRules Output (UFO) module [62].

The future lepton-hadron colliders, such as the LHeC and the FCC-he can be operated as \( \gamma^*p \) colliders, in this case the emitted quasi-real photon \( \gamma^* \) is scattered with small angles from the beam pipe of \( e^- \) \[63–68\]. These processes can be described by the Equivalent Photon Approximation (EPA) \[66, 69, 70\], using the Weizsacker-Williams Approximation. The main idea of EPA is that the electromagnetic interaction of an electron with the complicated field of the proton bunch is replaced by a more simple Compton scattering of this proton with the flux of EPA generated by the electron bunch. For our case, the schematic diagram to the process \( ep \rightarrow e^-\gamma^*p \rightarrow eW\gamma q'X \) is given by Fig. 1, and the Feynman diagrams of the subprocess \( \gamma^*q \rightarrow W\gamma q' \) are shown in Fig. 2. In this context, the spectrum of EPA photons is given by \[66, 71\]:

\[
f_{\gamma_1}(x_1) = \frac{\alpha}{\pi E_e} \left\{ \frac{1 - x_1 + x_1^2/2}{x_1} \log \left( \frac{Q_{\text{max}}^2}{Q_{\text{min}}^2} \right) - \frac{m_e^2 x_1}{Q_{\text{min}}^2} \left( 1 - \frac{Q_{\text{min}}^2}{Q_{\text{max}}^2} \right) - \frac{1}{x_1} \right\} \left[ 1 - \frac{x_1}{2} \right] \log \left( \frac{x_1^2 E_e^2 + Q_{\text{max}}^2}{x_1^2 E_e^2 + Q_{\text{min}}^2} \right) \tag{31}\]

where \( x_1 = E_{\gamma_1}/E_e \) and \( Q_{\text{max}}^2 \) is maximum virtuality of the photon. The minimum value of \( Q_{\text{min}}^2 \) is:

\[
Q_{\text{min}}^2 = \frac{m_e^2 x_1^2}{1 - x_1} \tag{32}
\]

With all these tools, the total cross sections of the \( ep \rightarrow e^-\gamma^*p \rightarrow eW\gamma q'X \) signal at the LHeC and the FCC-he are determined by:

\[
\sigma(ep \rightarrow eW\gamma q'X) = \int f_{\gamma^*}(x) \sigma(\gamma^*q \rightarrow W\gamma q') dx. \tag{33}\]

The total cross section of the process \( ep \rightarrow e^-\gamma^*p \rightarrow eW\gamma q'X \), that is \( \sigma(f_{M,i}/\Lambda^4, f_{T,i}/\Lambda^4, \sqrt{s}) \) as a function of \( f_{M,i}/\Lambda^4 \) and \( f_{T,i}/\Lambda^4 \) with \( i = 0, 1, 2, ..., 7 \) for the energies of the LHeC with \( \sqrt{s} = 1.30 \text{ TeV}, 1.98 \text{ TeV} \) and the FCC-he with \( \sqrt{s} = 3.46 \text{ TeV}, \) 5.29 TeV are reported in a region defined by the kinematics cuts given in Eqs. (20)-(30).
Cross sections of the process $ep \rightarrow e^{-} \gamma^{*}p \rightarrow eW\gamma'qX$ as a function of aQGC $f_{M,i}/\Lambda^4$ ($f_{T,i}/\Lambda^4$) are given in Figs. (3)-(10). For evaluation of the total cross sections, the leptonic and hadronic decays of the $W$-boson in the final state are considered. Furthermore, the total cross sections for each coupling are evaluated while fixing the other couplings to zero.

The corresponding expected cross sections after acceptance cuts for the process $ep \rightarrow e^{-} \gamma^{*}p \rightarrow eW\gamma'qX$ give the value $\sigma(f_{T,6}/\Lambda^4,\sqrt{s}) \simeq 10^{5}$ pb for $|f_{T,6}/\Lambda^4| = 1 \times 10^{-8}$ with the hadronic decay channel of the $W$-boson. While the cross section is $\sigma(f_{T,6}/\Lambda^4,\sqrt{s}) \simeq 10^{4}$ pb for the leptonic decay channel of the $W$-boson.

In Tables I and II, we illustrate the total cross sections in the fiducial region given by Eqs. (20)-(30) for the process $ep \rightarrow e^{-} \gamma^{*}p \rightarrow eW\gamma'qX$ with the different $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ couplings, and for the future energies of the LHeC and the FCC-he.

From Figs. 3-10, as well as from Tables I and II it is clear that the cross section projects a greater dependence with respect to the $f_{T,6}/\Lambda^4$ and $f_{T,5}/\Lambda^4$ couplings than with respect to the $f_{M,7}/\Lambda^4$, $f_{M,0}/\Lambda^4$, $f_{M,1}/\Lambda^4$, etc. There is also a difference in the cross section measured up to an order of magnitude between the leptonic and hadronic cases. We should mention that the cross sections are evaluated in a region defined by the kinematic cuts given by Eqs. (20)-(30).

In summary, the $ep \rightarrow e^{-} \gamma^{*}p \rightarrow eW\gamma'qX$ cross section in the presence of anomalous couplings increase rapidly with the electron-proton increasing center-of-mass energy.

IV. $\chi^2$ Analysis and Limits on the AQC $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ at the LHeC and the FCC-HE

We perform $\chi^2$ analysis to obtain the limits on the anomalous $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ couplings. $\chi^2$ is defined as follows:

$$\chi^2(f_{M,i}/\Lambda^4, f_{T,i}/\Lambda^4) = \left( \frac{\sigma_{SM}(\sqrt{s}) - \sigma_{BSM}(\sqrt{s}, f_{M,i}/\Lambda^4, f_{T,i})}{\sigma_{SM}\delta_{st}} \right)^2,$$ (34)

where $\sigma_{SM}(\sqrt{s})$ is the cross section of the SM and $\sigma_{BSM}(\sqrt{s}, f_{M,i}/\Lambda^4, f_{T,i}/\Lambda^4)$ is the BSM cross section. $\delta_{st} = \frac{1}{\sqrt{N_{SM}}}$ is the systematic error and $N$ is the number of events:

$$N_{SM} = L_{int} \times \sigma_{SM}.$$ (35)
Here, we assume the integrated luminosities $L_{\text{int}} = 10 - 100 \text{fb}^{-1}$ for the LHeC and $L_{\text{int}} = 100 - 1000 \text{fb}^{-1}$ for the FCC-he.

Tables III and IV summarize all of the limits on the dimension-8 aQGC obtained from $ep \rightarrow e^{-}\gamma^{*}p \rightarrow eW\gamma'q'X$ data with the leptonic decay of the $W$-boson in the final state at center-of-mass energies of $\sqrt{s} = 1.30 \text{ TeV}$ and $\sqrt{s} = 1.98 \text{ TeV}$. For these limits, all parameters except the one shown are fixed to zero. The results for leptonic final state at $\sqrt{s} = 3.46 \text{ TeV}$ and $\sqrt{s} = 5.29 \text{ TeV}$ given in Table IV are better values compared to those obtained for the $\sqrt{s} = 1.30 \text{ TeV}$ and $\sqrt{s} = 1.98 \text{ TeV}$ presented in Table III. A similar behaviour can be seen for the hadronic decay of $W$-boson given in Tables V-VI. Also, the limits of $f_{T,6}/\Lambda^{4} = [-1.10; 1.60]$ with $\sqrt{s} = 3.46 \text{ TeV}$ and $f_{T,6}/\Lambda^{4} = [-4.37; 5.51] \times 10^{-1}$ with $\sqrt{s} = 5.29 \text{ TeV}$ in Table VI are the most stringent. These limits are also approximately an order of magnitude more stringent than those obtained at the LHeC and the FCC-he through the main $ep \rightarrow e^{-}\gamma^{*}p \rightarrow eW\gamma'q'X \rightarrow e\nu q'q'X$ reaction. [22, 23].

Now, let us compare our findings with the other results in the literature which used different cuts and different channels. In Ref. [22], a detailed study of the LHeC and the FCC-he sensitivity to the anomalous $f_{M,i}/\Lambda^{4}$ and $f_{T,i}/\Lambda^{4}$ couplings in $\nu_{e}\gamma\gamma q$ production were carried out. Using a $\chi^{2}$ analysis and kinematic cuts for the final state particles in $\nu_{e}\gamma\gamma q$ production, they got limits on the thirteen different anomalous couplings arising from dimension-8 operators. In Ref. [23], limits were obtained from diboson production at both the LHeC and the FCC-he, they obtained limits on the anomalous $f_{M,i}/\Lambda^{4}$ and $f_{T,i}/\Lambda^{4}$ couplings considering the process $e^{-}p \rightarrow e^{-}\gamma^{*}\gamma^{*}p \rightarrow e^{-}W^{+}W^{-}p$ with the subprocess $\gamma^{*}\gamma^{*} \rightarrow W^{+}W^{-}$. Its limits are weaker by about a factor of 3 or 5 and up to an order of magnitude regarding our results. CMS Collaboration [72, 73] at the LHC with $\sqrt{s} = 8 \text{ TeV}$ and to an integrated luminosity of $19.7 \text{ fb}^{-1}$ searches for exclusive or quasi-exclusive $WW$ production, via the signal topology $pp \rightarrow p^{*}W^{+}W^{-}p^{*}$ where the $p^{*}$ indicates that the final state protons either remain intact (exclusive or elastic production), or dissociate into an undetected system (quasi-exclusive or proton dissociation production). Their research is translated into upper limits on the aQGC operators $f_{M,0,1,2,3}/\Lambda^{4}$ (dimension-8). From its investigations, CMS Collaboration measures the electroweak-induced production of $W$ and two jets, where the $W$ boson decays leptonically, and experimental limits on aQGC $f_{M,0-7}/\Lambda^{4}$, $f_{T,0-2,5-7}/\Lambda^{4}$ are set at 95% C.L. [72, 73]. On the other hand, ATLAS Collaboration at the LHC studied the production of $WV\gamma$ events in $e\mu\nu\gamma$, $e\nuqq\gamma$ and $\muqq\gamma$ final states with $L_{\text{int}} = 20.2 \text{ fb}^{-1}$ of
TABLE I: For leptonic decay channel, summary of the total cross sections of the process \( ep \to e^{-}\gamma^* p \to eW\gamma q X \) at the LHeC with \( \sqrt{s} = 1.30, 1.98 \) TeV and at the FCC-he with \( \sqrt{s} = 3.46, 5.29 \) TeV depending on thirteen anomalous couplings obtained by dimension-8 operators. The total cross sections for each coupling are calculated while fixing the other couplings to zero.

| Couplings (TeV\(^{-4}\)) | \( \sqrt{s} = 1.30 \) TeV | \( \sqrt{s} = 1.98 \) TeV | \( \sqrt{s} = 3.46 \) TeV | \( \sqrt{s} = 5.29 \) TeV |
|--------------------------|-----------------|-----------------|-----------------|-----------------|
| \( f_{M0}/\Lambda^4 \)   | \( 2.29 \times 10^{-2} \) | \( 3.98 \times 10^{-1} \) | \( 7.40 \times 10^{-2} \) | \( 2.80 \) |
| \( f_{M1}/\Lambda^4 \)   | \( 1.66 \times 10^{-2} \) | \( 2.29 \times 10^{-1} \) | \( 6.30 \times 10^{-2} \) | \( 2.07 \) |
| \( f_{M2}/\Lambda^4 \)   | \( 6.79 \times 10^{-1} \) | \( 1.62 \times 10^1 \) | \( 2.29 \) | \( 1.17 \times 10^2 \) |
| \( f_{M3}/\Lambda^4 \)   | \( 4.47 \times 10^{-1} \) | \( 9.14 \) | \( 1.83 \) | \( 8.61 \times 10^1 \) |
| \( f_{M4}/\Lambda^4 \)   | \( 5.84 \times 10^{-2} \) | \( 1.25 \) | \( 1.94 \times 10^{-1} \) | \( 9.00 \) |
| \( f_{M5}/\Lambda^4 \)   | \( 4.35 \times 10^{-2} \) | \( 7.27 \times 10^{-1} \) | \( 1.70 \times 10^{-1} \) | \( 6.70 \) |
| \( f_{M7}/\Lambda^4 \)   | \( 1.05 \times 10^{-2} \) | \( 7.69 \times 10^{-2} \) | \( 3.29 \times 10^{-2} \) | \( 5.71 \times 10^{-1} \) |
| \( f_{T0}/\Lambda^4 \)   | \( 9.55 \times 10^{-1} \) | \( 3.59 \times 10^1 \) | \( 5.39 \) | \( 5.85 \times 10^2 \) |
| \( f_{T1}/\Lambda^4 \)   | \( 2.61 \) | \( 8.10 \times 10^1 \) | \( 1.96 \times 10^1 \) | \( 1.97 \times 10^3 \) |
| \( f_{T2}/\Lambda^4 \)   | \( 3.25 \times 10^{-1} \) | \( 1.02 \times 10^1 \) | \( 2.39 \) | \( 2.36 \times 10^2 \) |
| \( f_{T5}/\Lambda^4 \)   | \( 1.01 \times 10^1 \) | \( 3.88 \times 10^2 \) | \( 5.79 \times 10^1 \) | \( 6.31 \times 10^3 \) |
| \( f_{T6}/\Lambda^4 \)   | \( 2.79 \times 10^1 \) | \( 8.71 \times 10^2 \) | \( 2.12 \times 10^2 \) | \( 2.10 \times 10^4 \) |
| \( f_{T7}/\Lambda^4 \)   | \( 3.45 \) | \( 1.10 \times 10^2 \) | \( 26.13 \) | \( 2.51 \times 10^3 \) |

proton-proton collisions with \( \sqrt{s} = 8 \) TeV [21].
TABLE II: Same as in Table I, but for hadronic decay

| Couplings (TeV$^{-4}$) | LHeC            | FCC-he          |
|------------------------|-----------------|-----------------|
| $f_{M0}/\Lambda^4$     | $5.79 \times 10^{-2}$ | $9.38 \times 10^{-1}$ | $1.79 \times 10^{-1}$ | $2.50$ |
| $f_{M1}/\Lambda^4$     | $6.05 \times 10^{-2}$ | $6.89 \times 10^{-1}$ | $7.34 \times 10^{-1}$ | $5.88$ |
| $f_{M2}/\Lambda^4$     | $1.84$          | $3.84 \times 10^1$ | $5.70$ | $1.02 \times 10^2$ |
| $f_{M3}/\Lambda^4$     | $2.08$          | $2.82 \times 10^1$ | $2.98 \times 10^1$ | $2.49 \times 10^2$ |
| $f_{M4}/\Lambda^4$     | $1.54 \times 10^{-1}$ | $2.98$          | $4.79 \times 10^{-1}$ | $7.88$ |
| $f_{M5}/\Lambda^4$     | $1.80 \times 10^{-1}$ | $2.22$          | $2.32$ | $1.92 \times 10^1$ |
| $f_{M7}/\Lambda^4$     | $2.92 \times 10^{-2}$ | $2.15 \times 10^{-1}$ | $2.26 \times 10^{-1}$ | $1.59$ |
| $f_{T0}/\Lambda^4$     | $2.27$          | $7.24 \times 10^1$ | $1.51 \times 10^1$ | $5.07 \times 10^2$ |
| $f_{T1}/\Lambda^4$     | $1.14 \times 10^1$ | $2.15 \times 10^2$ | $5.81 \times 10^2$ | $7.07 \times 10^3$ |
| $f_{T2}/\Lambda^4$     | $1.21$          | $2.50 \times 10^1$ | $5.11 \times 10^1$ | $6.54 \times 10^2$ |
| $f_{T5}/\Lambda^4$     | $2.42 \times 10^1$ | $7.76 \times 10^2$ | $1.62 \times 10^2$ | $5.45 \times 10^3$ |
| $f_{T6}/\Lambda^4$     | $1.22 \times 10^2$ | $2.31 \times 10^3$ | $6.28 \times 10^3$ | $7.65 \times 10^4$ |
| $f_{T7}/\Lambda^4$     | $1.30 \times 10^1$ | $2.68 \times 10^2$ | $5.50 \times 10^2$ | $7.02 \times 10^3$ |

V. CONCLUSIONS

In this paper, the calculations on the production cross section are derived for the $eW\gamma q'X$ final states at the LHeC with center-of-mass energies $\sqrt{s} = 1.30, 1.98$ TeV and the FCC-he with $\sqrt{s} = 3.46, 5.29$ TeV in the fiducial regions given by Eqs. (20)-(30). Our results are summarized through Figs. 3-10 and in Tables I-II. Furthermore, we show in Tables III-VI, individual upper limits obtained from the aQGC $f_{M,0-5,7}/\Lambda^4$ and $f_{T,0-2,5-7}/\Lambda^4$ at 95% C.L. both at leptonic and hadronic decay channel of the $W$-boson. As can be seen in the results, the process gives strong constraints on aQGC limits at high energy region and high luminosities.

In conclusion, we explore the phenomenological aspects of the anomalous $WW\gamma\gamma$ couplings via the process $ep \rightarrow e^{-}\gamma^{*}p \rightarrow eW\gamma q'X$ at the LHeC and the FCC-he. These couplings are defined through a phenomenological effective Lagrangian. The major goal of
TABLE III: Limits on aQGC at the 95% C.L. via $e p \rightarrow e^- \gamma^* p \rightarrow e W \gamma q' X$ for $\sqrt{s} = 1.30, 1.98$ TeV

| Couplings (TeV$^{-4}$) | 10 fb$^{-1}$ | 30 fb$^{-1}$ | 50 fb$^{-1}$ | 100 fb$^{-1}$ |
|------------------------|-------------|-------------|-------------|-------------|
| $f_{M0}/\Lambda^4$    | $[-3.27;3.28] \times 10^3$ | $[-2.48;2.49] \times 10^3$ | $[-2.18;2.19] \times 10^3$ | $[-1.84;1.85] \times 10^3$ |
| $f_{M1}/\Lambda^4$    | $[-3.51;4.57] \times 10^3$ | $[-2.56;3.62] \times 10^3$ | $[-2.20;3.26] \times 10^3$ | $[-1.78;2.85] \times 10^3$ |
| $f_{M2}/\Lambda^4$    | $[-5.02;5.01] \times 10^2$ | $[-3.81;3.80] \times 10^2$ | $[-3.36;3.35] \times 10^2$ | $[-2.82;2.81] \times 10^2$ |
| $f_{M3}/\Lambda^4$    | $[-5.39;6.99] \times 10^2$ | $[-3.93;5.54] \times 10^2$ | $[-3.38;4.99] \times 10^2$ | $[-2.74;4.35] \times 10^2$ |
| $f_{M4}/\Lambda^4$    | $[-1.81;1.82] \times 10^3$ | $[-1.37;1.38] \times 10^3$ | $[-1.21;1.22] \times 10^3$ | $[-1.01;1.02] \times 10^3$ |
| $f_{M5}/\Lambda^4$    | $[-2.56;1.92] \times 10^3$ | $[-2.03;1.39] \times 10^3$ | $[-1.84;1.19] \times 10^3$ | $[-1.61;0.97] \times 10^3$ |
| $f_{M7}/\Lambda^4$    | $[-0.91;0.70] \times 10^4$ | $[-0.72;0.51] \times 10^4$ | $[-0.65;0.44] \times 10^4$ | $[-0.57;0.35] \times 10^4$ |
| $f_{T0}/\Lambda^4$    | $[-4.18;4.26] \times 10^2$ | $[-3.17;3.24] \times 10^2$ | $[-2.78;2.86] \times 10^2$ | $[-2.33;2.41] \times 10^2$ |
| $f_{T1}/\Lambda^4$    | $[-2.43;2.66] \times 10^2$ | $[-1.82;2.05] \times 10^2$ | $[-1.59;1.82] \times 10^2$ | $[-1.32;1.55] \times 10^2$ |
| $f_{T2}/\Lambda^4$    | $[-0.66;0.79] \times 10^3$ | $[-0.49;0.62] \times 10^3$ | $[-0.42;0.55] \times 10^3$ | $[-0.35;0.48] \times 10^3$ |
| $f_{T5}/\Lambda^4$    | $[-1.27;1.30] \times 10^2$ | $[-9.64;9.88] \times 10^1$ | $[-8.47;8.71] \times 10^1$ | $[-7.10;7.35] \times 10^1$ |
| $f_{T6}/\Lambda^4$    | $[-7.57;7.94] \times 10^1$ | $[-5.71;6.08] \times 10^1$ | $[-5.00;5.37] \times 10^1$ | $[-4.18;4.55] \times 10^1$ |
| $f_{T7}/\Lambda^4$    | $[-2.08;2.34] \times 10^2$ | $[-1.55;1.81] \times 10^2$ | $[-1.35;1.61] \times 10^2$ | $[-1.12;1.38] \times 10^2$ |

\[
\sqrt{s} = 1.98 \text{ TeV}
\]

| Couplings (TeV$^{-4}$) | 10 fb$^{-1}$ | 30 fb$^{-1}$ | 50 fb$^{-1}$ | 100 fb$^{-1}$ |
|------------------------|-------------|-------------|-------------|-------------|
| $f_{M0}/\Lambda^4$    | $[-8.86;8.89] \times 10^2$ | $[-6.73;6.76] \times 10^2$ | $[-5.92;5.95] \times 10^2$ | $[-4.97;5.00] \times 10^2$ |
| $f_{M1}/\Lambda^4$    | $[-1.07;1.28] \times 10^3$ | $[-0.79;1.00] \times 10^3$ | $[-0.68;0.90] \times 10^3$ | $[-0.56;0.78] \times 10^3$ |
| $f_{M2}/\Lambda^4$    | $[-1.35;1.34] \times 10^2$ | $[-1.03;1.02] \times 10^2$ | $[-0.90;0.89] \times 10^1$ | $[-0.75;0.76] \times 10^1$ |
| $f_{M3}/\Lambda^4$    | $[-1.66;1.92] \times 10^2$ | $[-1.24;1.49] \times 10^2$ | $[-1.07;1.33] \times 10^2$ | $[-0.88;1.14] \times 10^2$ |
| $f_{M4}/\Lambda^4$    | $[-4.86;4.88] \times 10^2$ | $[-3.69;3.70] \times 10^2$ | $[-3.25;3.26] \times 10^2$ | $[-2.73;2.75] \times 10^2$ |
| $f_{M5}/\Lambda^4$    | $[-0.71;0.59] \times 10^3$ | $[-0.55;0.44] \times 10^3$ | $[-0.50;0.38] \times 10^3$ | $[-0.43;0.31] \times 10^3$ |
| $f_{M7}/\Lambda^4$    | $[-2.57;2.14] \times 10^3$ | $[-2.01;1.58] \times 10^3$ | $[-1.80;1.37] \times 10^3$ | $[-1.55;1.12] \times 10^3$ |
| $f_{T0}/\Lambda^4$    | $[-9.00;9.03] \times 10^1$ | $[-6.83;6.86] \times 10^1$ | $[-6.01;6.04] \times 10^1$ | $[-5.05;5.08] \times 10^1$ |
| $f_{T1}/\Lambda^4$    | $[-5.91;6.09] \times 10^1$ | $[-4.47;4.65] \times 10^1$ | $[-3.92;4.10] \times 10^1$ | $[-3.28;3.47] \times 10^1$ |
| $f_{T2}/\Lambda^4$    | $[-1.60;1.78] \times 10^2$ | $[-1.20;1.38] \times 10^2$ | $[-1.04;1.23] \times 10^2$ | $[-0.86;1.05] \times 10^2$ |
| $f_{T5}/\Lambda^4$    | $[-2.69;2.80] \times 10^1$ | $[-2.03;2.14] \times 10^1$ | $[-1.78;1.89] \times 10^1$ | $[-1.49;1.60] \times 10^1$ |
| $f_{T6}/\Lambda^4$    | $[-1.77;1.89] \times 10^1$ | $[-1.33;1.45] \times 10^1$ | $[-1.17;1.28] \times 10^1$ | $[-0.97;1.09] \times 10^1$ |
| $f_{T7}/\Lambda^4$    | $[-5.05;5.29] \times 10^1$ | $[-3.81;4.05] \times 10^1$ | $[-3.34;3.58] \times 10^1$ | $[-2.79;3.03] \times 10^1$ |
TABLE IV: Limits on aQGC at the 95% C.L. via $e p \rightarrow e^{-}\gamma^{*}p \rightarrow eW\gamma q'X$ for $\sqrt{s} = 3.46, 5.29$ TeV

| Couplings (TeV$^{-4}$) | 100 fb$^{-1}$ | 300 fb$^{-1}$ | 500 fb$^{-1}$ | 1000 fb$^{-1}$ |
|------------------------|--------------|--------------|--------------|--------------|
| $f_{M0}/\Lambda^4$    | $[-6.70;6.81] \times 10^{2}$ | $[-5.08;5.19] \times 10^{2}$ | $[-4.47;4.57] \times 10^{2}$ | $[-3.75;3.85] \times 10^{2}$ |
| $f_{M1}/\Lambda^4$    | $[-0.62;0.88] \times 10^{3}$ | $[-0.44;0.71] \times 10^{3}$ | $[-0.38;0.64] \times 10^{3}$ | $[-0.30;0.57] \times 10^{3}$ |
| $f_{M2}/\Lambda^4$    | $[-1.03;1.00] \times 10^{2}$ | $[-7.87;7.63] \times 10^{1}$ | $[-6.95;6.71] \times 10^{1}$ | $[-5.86;5.62] \times 10^{1}$ |
| $f_{M3}/\Lambda^4$    | $[-1.11;1.13] \times 10^{2}$ | $[-8.42;8.65] \times 10^{1}$ | $[-7.40;7.63] \times 10^{1}$ | $[-6.21;6.43] \times 10^{1}$ |
| $f_{M4}/\Lambda^4$    | $[-3.75;3.77] \times 10^{2}$ | $[-2.85;2.87] \times 10^{2}$ | $[-2.51;2.52] \times 10^{2}$ | $[-2.11;2.12] \times 10^{2}$ |
| $f_{M5}/\Lambda^4$    | $[-4.17;4.08] \times 10^{2}$ | $[-3.18;3.09] \times 10^{2}$ | $[-2.80;2.71] \times 10^{2}$ | $[-2.36;2.27] \times 10^{2}$ |
| $f_{M7}/\Lambda^4$    | $[-1.78;1.39] \times 10^{3}$ | $[-1.40;1.01] \times 10^{3}$ | $[-1.26;0.88] \times 10^{3}$ | $[-1.10;0.71] \times 10^{3}$ |
| $f_{T0}/\Lambda^4$    | $[-6.71;6.93] \times 10^{1}$ | $[-5.08;5.30] \times 10^{1}$ | $[-4.45;4.67] \times 10^{1}$ | $[-3.73;3.95] \times 10^{1}$ |
| $f_{T1}/\Lambda^4$    | $[-3.49;3.52] \times 10^{1}$ | $[-2.64;2.68] \times 10^{1}$ | $[-2.33;2.36] \times 10^{1}$ | $[-1.95;1.98] \times 10^{1}$ |
| $f_{T2}/\Lambda^4$    | $[-0.89;1.14] \times 10^{2}$ | $[-0.65;0.90] \times 10^{2}$ | $[-0.56;0.81] \times 10^{2}$ | $[-0.45;0.71] \times 10^{2}$ |
| $f_{T5}/\Lambda^4$    | $[-2.02;2.03] \times 10^{1}$ | $[-1.53;1.55] \times 10^{1}$ | $[-1.35;1.36] \times 10^{1}$ | $[-1.13;1.15] \times 10^{1}$ |
| $f_{T6}/\Lambda^4$    | $[-0.94;1.19] \times 10^{1}$ | $[-0.69;0.94] \times 10^{1}$ | $[-0.60;0.84] \times 10^{1}$ | $[-0.48;0.73] \times 10^{1}$ |
| $f_{T7}/\Lambda^4$    | $[-0.28;0.33] \times 10^{2}$ | $[-0.20;0.26] \times 10^{2}$ | $[-0.18;0.23] \times 10^{2}$ | $[-0.15;0.20] \times 10^{2}$ |

$\sqrt{s} = 5.29$ TeV

| Couplings (TeV$^{-4}$) | 100 fb$^{-1}$ | 300 fb$^{-1}$ | 500 fb$^{-1}$ | 1000 fb$^{-1}$ |
|------------------------|--------------|--------------|--------------|--------------|
| $f_{M0}/\Lambda^4$    | $[-1.19;1.24] \times 10^{2}$ | $[-9.00;9.45] \times 10^{1}$ | $[-7.90;8.34] \times 10^{1}$ | $[-6.61;7.05] \times 10^{1}$ |
| $f_{M1}/\Lambda^4$    | $[-1.28;1.53] \times 10^{2}$ | $[-0.95;1.20] \times 10^{2}$ | $[-0.82;1.07] \times 10^{2}$ | $[-0.67;0.92] \times 10^{2}$ |
| $f_{M2}/\Lambda^4$    | $[-1.84;1.82] \times 10^{1}$ | $[-1.40;1.38] \times 10^{1}$ | $[-1.23;1.22] \times 10^{1}$ | $[-1.04;1.02] \times 10^{1}$ |
| $f_{M3}/\Lambda^4$    | $[-2.06;2.22] \times 10^{1}$ | $[-1.54;1.70] \times 10^{1}$ | $[-1.35;1.51] \times 10^{1}$ | $[-1.12;1.28] \times 10^{1}$ |
| $f_{M4}/\Lambda^4$    | $[-6.62;6.68] \times 10^{1}$ | $[-5.02;5.09] \times 10^{1}$ | $[-4.42;4.48] \times 10^{1}$ | $[-3.71;3.77] \times 10^{1}$ |
| $f_{M5}/\Lambda^4$    | $[-0.83;0.72] \times 10^{2}$ | $[-0.65;0.59] \times 10^{2}$ | $[-0.58;0.47] \times 10^{2}$ | $[-0.49;0.39] \times 10^{2}$ |
| $f_{M7}/\Lambda^4$    | $[-2.98;2.76] \times 10^{2}$ | $[-2.29;2.07] \times 10^{2}$ | $[-2.03;1.81] \times 10^{2}$ | $[-1.73;1.50] \times 10^{2}$ |
| $f_{T0}/\Lambda^4$    | $[-8.18;8.23] \times 10^{1}$ | $[-6.21;6.26] \times 10^{1}$ | $[-5.46;5.51] \times 10^{1}$ | $[-4.59;4.64] \times 10^{1}$ |
| $f_{T1}/\Lambda^4$    | $[-4.37;4.60] \times 10^{1}$ | $[-3.29;3.52] \times 10^{1}$ | $[-2.88;3.11] \times 10^{1}$ | $[-2.41;2.64] \times 10^{1}$ |
| $f_{T2}/\Lambda^4$    | $[-1.23;1.36] \times 10^{1}$ | $[-0.92;1.05] \times 10^{1}$ | $[-0.81;0.93] \times 10^{1}$ | $[-0.67;0.79] \times 10^{1}$ |
| $f_{T5}/\Lambda^4$    | $[-2.41;2.60] \times 10^{1}$ | $[-1.81;1.99] \times 10^{1}$ | $[-1.58;1.77] \times 10^{1}$ | $[-1.31;1.50] \times 10^{1}$ |
| $f_{T6}/\Lambda^4$    | $[-1.22;1.53] \times 10^{1}$ | $[-0.89;1.21] \times 10^{1}$ | $[-0.77;1.08] \times 10^{1}$ | $[-0.63;0.94] \times 10^{1}$ |
| $f_{T7}/\Lambda^4$    | $[-3.83;4.07] \times 10^{1}$ | $[-2.88;3.13] \times 10^{1}$ | $[-2.52;2.77] \times 10^{1}$ | $[-2.10;2.35] \times 10^{1}$ |
TABLE V: Limits on aQGC at the 95% C.L. via $e p \rightarrow e^{-}\gamma p \rightarrow e W \gamma q X$ for $\sqrt{s} = 1.30, 1.98$ TeV

| Couplings (TeV$^{-4}$) | 10 fb$^{-1}$ | 30 fb$^{-1}$ | 50 fb$^{-1}$ | 100 fb$^{-1}$ |
|-------------------------|-------------|-------------|-------------|-------------|
| $f_{M0}/\Lambda^4$     | [-2.37;2.41] × 10$^3$ | [-1.80;1.84] × 10$^3$ | [-1.59;1.62] × 10$^3$ | [-1.32;1.36] × 10$^3$ |
| $f_{M1}/\Lambda^4$     | [-1.95;2.59] × 10$^3$ | [-1.41;2.06] × 10$^3$ | [-1.21;1.86] × 10$^3$ | [-0.98;1.62] × 10$^3$ |
| $f_{M2}/\Lambda^4$     | [-3.68;3.61] × 10$^2$ | [-2.81;2.73] × 10$^2$ | [-2.47;2.40] × 10$^2$ | [-2.09;2.01] × 10$^2$ |
| $f_{M3}/\Lambda^4$     | [-2.93;3.98] × 10$^2$ | [-2.12;3.17] × 10$^2$ | [-1.82;2.87] × 10$^2$ | [-1.47;2.51] × 10$^2$ |
| $f_{M4}/\Lambda^4$     | [-1.31;1.33] × 10$^3$ | [-0.92;1.03] × 10$^2$ | [-0.72;0.93] × 10$^2$ | [-0.71;0.80] × 10$^2$ |
| $f_{M5}/\Lambda^4$     | [-1.44;1.07] × 10$^3$ | [-1.15;0.77] × 10$^3$ | [-1.04;0.66] × 10$^3$ | [-0.91;0.54] × 10$^3$ |
| $f_{M7}/\Lambda^4$     | [-5.18;3.85] × 10$^3$ | [-4.12;2.79] × 10$^3$ | [-3.73;2.39] × 10$^3$ | [-3.26;1.93] × 10$^3$ |
| $f_{T0}/\Lambda^4$     | [-3.28;3.29] × 10$^2$ | [-2.49;2.50] × 10$^2$ | [-2.19;2.20] × 10$^2$ | [-1.84;1.85] × 10$^2$ |
| $f_{T1}/\Lambda^4$     | [-1.31;1.63] × 10$^2$ | [-0.96;1.28] × 10$^2$ | [-0.83;1.15] × 10$^2$ | [-0.68;1.00] × 10$^2$ |
| $f_{T2}/\Lambda^4$     | [-4.06;4.98] × 10$^2$ | [-2.98;3.91] × 10$^2$ | [-2.58;3.50] × 10$^2$ | [-2.11;3.03] × 10$^2$ |
| $f_{T5}/\Lambda^4$     | [-9.54;10.51] × 10$^1$ | [-7.14;8.11] × 10$^1$ | [-6.23;7.20] × 10$^1$ | [-5.17;6.13] × 10$^1$ |
| $f_{T6}/\Lambda^4$     | [-3.85;5.15] × 10$^1$ | [-2.79;4.09] × 10$^1$ | [-2.40;3.70] × 10$^1$ | [-1.94;3.24] × 10$^1$ |
| $f_{T7}/\Lambda^4$     | [-1.23;1.52] × 10$^2$ | [-0.90;1.20] × 10$^2$ | [-0.78;1.08] × 10$^2$ | [-0.64;0.93] × 10$^2$ |

| Couplings (TeV$^{-4}$) | 10 fb$^{-1}$ | 30 fb$^{-1}$ | 50 fb$^{-1}$ | 100 fb$^{-1}$ |
|-------------------------|-------------|-------------|-------------|-------------|
| $f_{M0}/\Lambda^4$     | [-6.76;6.89] × 10$^2$ | [-5.13;5.25] × 10$^2$ | [-4.50;4.63] × 10$^2$ | [-3.78;3.90] × 10$^2$ |
| $f_{M1}/\Lambda^4$     | [-7.10;8.91] × 10$^2$ | [-5.21;7.02] × 10$^2$ | [-4.49;6.30] × 10$^2$ | [-3.66;5.47] × 10$^2$ |
| $f_{M2}/\Lambda^4$     | [-1.05;1.04] × 10$^2$ | [-7.96;7.86] × 10$^1$ | [-7.02;6.91] × 10$^1$ | [-5.91;5.80] × 10$^1$ |
| $f_{M3}/\Lambda^4$     | [-1.07;1.38] × 10$^2$ | [-0.78;1.09] × 10$^2$ | [-0.67;0.98] × 10$^2$ | [-0.54;0.86] × 10$^2$ |
| $f_{M4}/\Lambda^4$     | [-3.76;3.79] × 10$^2$ | [-2.85;2.89] × 10$^2$ | [-2.51;2.54] × 10$^2$ | [-2.10;2.14] × 10$^2$ |
| $f_{M5}/\Lambda^4$     | [-4.90;3.93] × 10$^2$ | [-3.86;2.89] × 10$^2$ | [-3.46;2.49] × 10$^2$ | [-3.00;2.03] × 10$^2$ |
| $f_{M7}/\Lambda^4$     | [-1.79;1.42] × 10$^3$ | [-1.41;1.04] × 10$^3$ | [-1.27;0.90] × 10$^3$ | [-1.10;0.73] × 10$^3$ |
| $f_{T0}/\Lambda^4$     | [-7.54;7.62] × 10$^1$ | [-5.72;5.80] × 10$^1$ | [-5.03;5.11] × 10$^1$ | [-4.23;4.30] × 10$^1$ |
| $f_{T1}/\Lambda^4$     | [-3.99;4.85] × 10$^1$ | [-2.94;3.80] × 10$^1$ | [-2.54;3.40] × 10$^1$ | [-2.08;2.94] × 10$^1$ |
| $f_{T2}/\Lambda^4$     | [-1.17;1.43] × 10$^2$ | [-0.86;1.12] × 10$^2$ | [-0.74;1.01] × 10$^2$ | [-0.61;0.87] × 10$^2$ |
| $f_{T5}/\Lambda^4$     | [-2.31;2.32] × 10$^1$ | [-1.75;1.76] × 10$^1$ | [-1.54;1.55] × 10$^1$ | [-1.29;1.31] × 10$^1$ |
| $f_{T6}/\Lambda^4$     | [-1.30;1.39] × 10$^1$ | [-0.98;1.07] × 10$^1$ | [-0.86;0.94] × 10$^1$ | [-0.71;0.80] × 10$^1$ |
| $f_{T7}/\Lambda^4$     | [-3.50;4.42] × 10$^1$ | [-2.57;3.49] × 10$^1$ | [-2.21;3.13] × 10$^1$ | [-1.80;2.72] × 10$^1$ |
### TABLE VI: Limits on aQGC at the 95% C.L. via $e^p \rightarrow e^{-}\gamma^*p \rightarrow eW\gamma q'X$ for $\sqrt{s} = 3.46, 5.29$ TeV

| Couplings (TeV$^{-4}$) | 100 fb$^{-1}$ | 300 fb$^{-1}$ | 500 fb$^{-1}$ | 1000 fb$^{-1}$ |
|------------------------|---------------|---------------|--------------|---------------|
| $f_{M0}/\Lambda^4$    | [-5.12;5.13] $\times 10^2$ | [-3.89;3.90] $\times 10^2$ | [-3.42;3.44] $\times 10^2$ | [-2.88;2.89] $\times 10^2$ |
| $f_{M1}/\Lambda^4$    | [-1.93;2.58] $\times 10^2$ | [-1.40;2.05] $\times 10^2$ | [-1.20;1.85] $\times 10^2$ | [-0.97;1.62] $\times 10^2$ |
| $f_{M2}/\Lambda^4$    | [-0.87;0.70] $\times 10^2$ | [-0.68;0.52] $\times 10^2$ | [-0.61;0.45] $\times 10^2$ | [-0.53;0.37] $\times 10^2$ |
| $f_{M3}/\Lambda^4$    | [-2.94;3.93] $\times 10^1$ | [-2.13;3.12] $\times 10^1$ | [-1.83;2.82] $\times 10^1$ | [-1.48;2.47] $\times 10^1$ |
| $f_{M4}/\Lambda^4$    | [-2.78;2.89] $\times 10^2$ | [-2.10;2.21] $\times 10^2$ | [-1.84;1.95] $\times 10^2$ | [-1.54;1.65] $\times 10^2$ |
| $f_{M5}/\Lambda^4$    | [-1.53;1.02] $\times 10^2$ | [-1.24;0.73] $\times 10^2$ | [-1.13;0.62] $\times 10^2$ | [-1.00;0.49] $\times 10^2$ |
| $f_{M7}/\Lambda^4$    | [-5.29;3.76] $\times 10^2$ | [-4.24;2.71] $\times 10^2$ | [-3.85;2.32] $\times 10^2$ | [-3.39;1.86] $\times 10^2$ |
| $f_{T0}/\Lambda^4$    | [-4.62;5.00] $\times 10^1$ | [-3.47;3.85] $\times 10^1$ | [-3.03;3.41] $\times 10^1$ | [-2.52;2.90] $\times 10^1$ |
| $f_{T1}/\Lambda^4$    | [-0.74;0.81] $\times 10^1$ | [-0.56;0.62] $\times 10^1$ | [-0.49;0.55] $\times 10^1$ | [-0.40;0.47] $\times 10^1$ |
| $f_{T2}/\Lambda^4$    | [-2.26;2.99] $\times 10^1$ | [-1.64;2.37] $\times 10^1$ | [-1.41;2.14] $\times 10^1$ | [-1.14;1.87] $\times 10^1$ |
| $f_{T5}/\Lambda^4$    | [-1.42;1.51] $\times 10^1$ | [-1.07;1.16] $\times 10^1$ | [-0.94;1.02] $\times 10^1$ | [-0.78;0.87] $\times 10^1$ |
| $f_{T6}/\Lambda^4$    | [-2.12;2.62]        | [-1.56;2.05]        | [-1.34;1.84]        | [-1.10;1.60]        |
| $f_{T7}/\Lambda^4$    | [-7.18;8.88]        | [-5.27;6.98]        | [-4.55;6.26]        | [-3.72;5.42]        |

\[ \sqrt{s} = 5.29 \text{ TeV} \]

| Couplings (TeV$^{-4}$) | 100 fb$^{-1}$ | 300 fb$^{-1}$ | 500 fb$^{-1}$ | 1000 fb$^{-1}$ |
|------------------------|---------------|---------------|--------------|---------------|
| $f_{M0}/\Lambda^4$    | [-1.53;1.58] $\times 10^2$ | [-1.15;1.21] $\times 10^2$ | [-1.01;1.07] $\times 10^2$ | [-0.85;0.90] $\times 10^1$ |
| $f_{M1}/\Lambda^4$    | [-0.90;1.12] $\times 10^2$ | [-0.66;0.88] $\times 10^2$ | [-0.57;0.79] $\times 10^2$ | [-0.46;0.68] $\times 10^2$ |
| $f_{M2}/\Lambda^4$    | [-2.41;2.33] $\times 10^1$ | [-1.84;1.77] $\times 10^1$ | [-1.62;1.55] $\times 10^1$ | [-1.37;1.30] $\times 10^1$ |
| $f_{M3}/\Lambda^4$    | [-1.30;1.84] $\times 10^1$ | [-0.94;1.47] $\times 10^1$ | [-0.80;1.34] $\times 10^1$ | [-0.64;1.18] $\times 10^1$ |
| $f_{M4}/\Lambda^4$    | [-8.44;8.67] $\times 10^1$ | [-6.39;6.62] $\times 10^1$ | [-5.61;5.84] $\times 10^1$ | [-4.70;4.93] $\times 10^1$ |
| $f_{M5}/\Lambda^4$    | [-6.23;4.81] $\times 10^1$ | [-4.93;3.51] $\times 10^1$ | [-4.44;3.02] $\times 10^1$ | [-3.87;2.45] $\times 10^1$ |
| $f_{M7}/\Lambda^4$    | [-2.19;1.83] $\times 10^2$ | [-1.71;1.35] $\times 10^2$ | [-1.53;1.17] $\times 10^2$ | [-1.32;0.96] $\times 10^2$ |
| $f_{T0}/\Lambda^4$    | [-1.04;1.10] $\times 10^1$ | [-0.78;0.84] $\times 10^1$ | [-0.68;0.74] $\times 10^1$ | [-0.57;0.63] $\times 10^1$ |
| $f_{T1}/\Lambda^4$    | [-2.53;3.21]        | [-1.85;2.53]        | [-1.60;2.27]        | [-1.30;1.97]        |
| $f_{T2}/\Lambda^4$    | [-0.82;1.10] $\times 10^1$ | [-0.59;0.88] $\times 10^1$ | [-0.51;0.79] $\times 10^1$ | [-0.41;0.69] $\times 10^1$ |
| $f_{T5}/\Lambda^4$    | [-3.17;3.30]        | [-2.39;2.52]        | [-2.10;2.23]        | [-1.75;1.89]        |
| $f_{T6}/\Lambda^4$    | [-8.17;9.32] $\times 10^{-1}$ | [-6.08;7.23] $\times 10^{-1}$ | [-5.29;6.44] $\times 10^{-1}$ | [-4.37;5.51] $\times 10^{-1}$ |
| $f_{T7}/\Lambda^4$    | [-2.35;3.48]        | [-1.68;2.81]        | [-1.43;2.56]        | [-1.14;2.27]        |
these measurements will be the confirmation of the new physics BSM. If the energy scale of the new physics responsible for the non-standard gauge boson couplings $f_{M,i}/\Lambda^4$ and $f_{T,i}/\Lambda^4$ is the center-of-mass energy of 5.29 TeV and the integrated luminosity of 1000 fb$^{-1}$, these couplings are expected to be no larger than $\mathcal{O}(10^{-1})$. Our results indicate that with cleaner environments, appropriate fiducial regions, high energies and high luminosities for future colliders it will be possible to obtain stronger upper limits on the anomalous $WW\gamma\gamma$ couplings.

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FIG. 1: A schematic diagram for the processes $e^- p \rightarrow e^- \gamma^* p \rightarrow e^- W \gamma q' X$. 

FIG. 2: Feynman diagrams contributing to the subprocess $\gamma^* q \rightarrow W \gamma q'$. 

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FIG. 3: For pure-leptonic channel, the total cross sections of the process $e^- p \rightarrow e^- \gamma^* p \rightarrow e^- W^+\gamma qX$ as a function of the anomalous couplings for center-of-mass energy of $\sqrt{s} = 1.30$ TeV at the LHeC.

FIG. 4: Same as in Fig. 3, but for $\sqrt{s} = 1.98$ TeV at the LHeC.
FIG. 5: Same as in Fig. 3, but for hadronic decay.

FIG. 6: Same as in Fig. 4, but for hadronic decay.
FIG. 7: For pure-leptonic channel, the total cross sections of the process $e^- p \rightarrow e^- \gamma^* p \rightarrow e^- W^\pm qX$ as a function of the anomalous couplings for center-of-mass energy of $\sqrt{s} = 3.46$ TeV at the FCC-he.

FIG. 8: Same as in Fig. 7, but for $\sqrt{s} = 5.29$ TeV at the FCC-he.
FIG. 9: Same as in Fig. 7, but for hadronic decay.

FIG. 10: Same as in Fig. 8, but for hadronic decay.