Flow resistance in a variable cross section channel within the numerical model of shallow water

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Abstract. The resistance to a flow in open channels of river type depends on a variety of parameters. The coefficient of bottom roughness is an integral quantity accounting for the geometric and physical features of the river bottom. Definition of the channel cross-section variability contribution to the roughness coefficient is the aim of our research. We have applied the 2D numerical model of surface water dynamics in the shallow water approximation to our task. The Manning roughness coefficient for non-prismatic open channels has been estimated depending on the parameters of the variable cross-section of the water flow in the range \( n = 0 \div 0.02 \).

1. Introduction
Riverbeds have complex irregular structure of bottom and coastlines. Their morphometric features are the large- and small-scale bottom inhomogeneities, meandering [1, 2], and variations of the channel cross-section [3-5] and hydraulic radius. Each of these factors contributes to the channel flow resistance [6].

The hydrological model of Chezy is traditionally used to evaluate the hydrodynamic resistance through the Manning roughness coefficient \( n \) [7, 8]. There are several different approaches to determine the coefficient \( n \). Since the roughness coefficient is a phenomenological parameter accounting for a large number of dissimilar factors, there are some difficulties in its evaluation [8, 9]. Practically in channels transmission capacity calculation the coefficient \( n \) can be determined from special tables in accordance with the channel characteristics [9]. Numerical non-stationary models of shallow water allow us to estimate the effective Manning coefficient for meandering river beds [10]. In the current article we research the contribution of the channel cross-section geometry to the flow resistance using the numerical shallow water model. We have considered non-prismatic open channels and estimated the average flow velocity and roughness coefficient for them.

2. Models and methods
To calculate the channel flow dynamics, we have used the shallow water equations:

\[
\frac{\partial H}{\partial t} + \nabla \cdot (HV) = \sigma, \\
\frac{\partial (HV)}{\partial t} + \nabla \cdot (HV \otimes \dot{V}) = -gH \nabla \cdot \eta + HF + \sigma(\bar{U} - \bar{V}),
\]
where $H$ is the water depth, $\vec{V}$ is the water velocity vector, $\sigma = Q / S$ is the surface density of water sources and sinks (m/s), $Q$ is discharge, $S$ is area of water source, $g$ is the gravitational acceleration, $\eta(x, y, t) = H(x, y, t) + b(x, y)$ is the free water surface level, $b(x, y)$ is the relief function, $\vec{U}$ is the water velocity vector in the source or drain, $\vec{F}$ is the force vector. The force of bottom friction is determined by the formula

$$\vec{F}^{\text{fric}} = -\frac{\lambda}{2} \vec{V} |\vec{V}|,$$

where $\lambda = 2gn^2 / H^{4/3}$ is the coefficient of hydraulic resistance, $n$ is the Manning coefficient, which determines the bottom roughness (s/m$^{1/3}$).

![Figure 1. The open channel geometry with a variable cross-section. The water source of power $Q$ has been set at the left boundary.](image)

For numerical integration of shallow water equations, the Combined Smooth Particle Hydrodynamics – Total Variation Diminishing method has been applied [11]. The idea of the method consists in the joint use of the Lagrangian and Eulerian approaches [12]. To accelerate computational simulations significantly, we have used the parallel code for GPUs [13]. The scheme has several positive properties including a through calculation of the moving "water-dry bottom" boundaries, second-order accuracy by time and spatial coordinates, and monotony of solution for complex irregular relief. We obtain a solution on a regular grid $\Delta x = \Delta y = 10$ m with number of cells 256x4096 accounting for a water flow rate $Q = 1500$ m$^3$/s ($\sigma = 0.06$ m/s, $\vec{U} = 0$ m/s) at the input of the computational domain (Figure 1). On the right boundary (See Figure 1), we use the conditions type "waterfall", which are described in detail in the work [14]. Water does not reach the boundaries at $y = \pm 1280$ m in our numerical calculations because of the shape of the relief $b(x, y)$. We fill the channel with water by starting from the dry bottom ($H = 0$) at the time $t = 0$.

A model of a variable cross-section channel can be determined in the following form:

$$b(x, y) = -b_{\text{max}} \exp(-y^2/(A + B \cos(kx))^2) - Ix,$$

where $b_{\text{max}} = 5$ m, $I = 0.07$ m/km is a slope of the watercourse bottom along the $x$-coordinate, parameters $A$ and $B$ define the inhomogeneity of the cross section. The transverse channel scale $L(x)$ is a periodic function with $L_{\text{min}} = A - B$, $L_{\text{max}} = A + B$. Comparison of flows velocities in an inhomogeneous channel with $n = 0$ has a channel of a constant cross-section ($B = 0$) for different values of $n > 0$ allows obtaining a relation between the cross-section inhomogeneity parameters ($L_{\text{min}}$, $L_{\text{max}}$, $k$) and Manning coefficient $n$.

3. Results and Discussion

Figure 2 shows the velocity profiles for a heterogeneous channel with parameters $L_{\text{min}} = 500$ m, $L_{\text{max}} = 1000$ m and a straight channel with $L_{\text{min}} = L_{\text{max}}$. For a given set of parameters, in the inhomogeneous channel the average velocity is $V_x^{av} = 1.27$ m/s in the region of $5000$ m $\leq x \leq 15000$ m, and in the homogeneous channel the same value of velocity is turn out at $n = 0.013$. Asymmetry of the profiles along the $y$-coordinate is caused by the presence of the flow weak instability at $n < 0.02$ (Fig. 3a).
Figure 2. The x-velocity component profiles in the channel cross-section. Curve 1 corresponds to the velocity in the channels cross-section with maximum width; curve 2 shows the velocity profile in the narrow part of channel; curve 3 is the velocity profile in a homogeneous channel for $n = 0.013$.

At small values of $k$, an accelerated flow forms near the fairway with average velocity exceeding 4 m$^2$/s. Such regime is caused by the developed instability of the flow (Fig. 3b).

Figure 3. The distributions of the velocity modulus: a) quasi-laminar flow, b) turbulent flow

The nonstationary solution appears at $k < 0.002$ m$^{-1}$ that corresponds to the values of roughness coefficient $n < 0.005$. To analyze the stability of the flow, we have compared the time dependence of the water depth at several fixed points of the channel $(x_p, y_p)$ (see Figure 4). In the numerical simulation with $k = 0.0016$ m$^{-1}$ and $n = 0$, the amplitude of depth oscillations varies within 0.5 m (Fig. 4a). The model at $k = 0.006$ m$^{-1}$ and $n = 0$ gives the solution with small amplitude of depth oscillations (approximately 0.15 m) (Fig. 4b, curve 1), and it gives the stationary solution at the same $k$ and $n = 0.02$ (Fig. 4b, curve 2).
Figure 4. The time dependence of the depth at given points $(x_p, y_p)$: a) curve 1 at (10000 m; 0 m), curve 2 at (14000 m; 0 m), curve 3 at the point (12000 m; 100 m); b) curve 1 at $n = 0$, curve 2 for $n = 0.02$ at (10000 m; 0 m).

The results of a series of hydrodynamic simulations are summarized in Figure 5 which demonstrates the distribution of the average velocity as a dependence on the channel cross-section parameters. In Figure 5 we have introduced the following dimensionless parameters: $\delta_1 = \frac{2\pi}{kL_{av}}$, $L_{av} = \frac{L_{\text{min}} + L_{\text{max}}}{2}$, $\delta_2 = \frac{L_{\text{max}}}{L_{\text{min}}}$. 
To estimate the roughness coefficient in terms of velocity, the dependence of $n$ on the average velocity for a straight-line channel has been are calculated (Fig. 6). As a result, for the channel with a weak change in the cross-section ($\delta_1 > 4$ and $\delta_2 < 1.5$) the values of $n$ are small ($n < 0.01$), while for the strongly inhomogeneous channel characterized by a higher flow resistance, the Manning coefficient is in the range of $0.01 < n < 0.015$. According to the tables compiled by V.T. Chow, the contribution of the channel cross-section change is 0.00–0.015 depending on the channel inhomogeneity degree (gradual, random or multiple changes in the cross section). Our estimates are consistent with Chow's data [15].

![Figure 5](image5.png)

**Figure 5.** The average cross-sectional velocity versus the channel parameters

![Figure 6](image6.png)

**Figure 6.** The dependence of the Manning coefficient on the average velocity

4. **Conclusion**

We have studied the effect of open channel geometry on flow resistance using the numerical unsteady shallow water model. The main idea of our approach is the comparison of the average water velocities in inhomogeneous channel without the bottom roughness and straight channel with different values of the bottom roughness coefficient. The result of concordance of these two models allows estimating of the Manning roughness coefficient.

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