Blow up of solutions of semilinear wave equations related to nonlinear waves in de Sitter spacetime

Kimitoshi Tsutaya · Yuta Wakasugi

Received: 17 July 2021 / Accepted: 3 December 2021 / Published online: 23 December 2021
© The Author(s), under exclusive licence to Springer Nature Switzerland AG 2021

Abstract
Consider a nonlinear wave equation for a massless scalar field with self-interaction in the spatially flat de Sitter spacetime. We show that blow-up in a finite time occurs for the equation with arbitrary power nonlinearity as well as upper bounds of the lifespan of blow-up solutions. The blow-up condition is the same as in the accelerated expanding Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime. We also show the same results for the space derivative nonlinear term.

Keywords Wave equation · Blow-up · Lifespan · de Sitter spacetime

Mathematics Subject Classification 35Q85 · 35L05 · 35L70

1 Introduction
This paper is subsequent to our recent work [13–16] concerned with the semilinear wave equations in the Friedmann–Lemaître–Robertson–Walker (FLRW) spacetimes.

We consider the following Cauchy problem:

\[
\begin{align*}
    u_{tt} - a(t) \Delta u + \mu u_t &= |u|^p \text{ or } |\nabla_x u|^p, & t > 0, \ x \in \mathbb{R}^n, \\
    u(0, x) &= \varepsilon u_0(x), \ u_t(0, x) = \varepsilon u_1(x), & x \in \mathbb{R}^n,
\end{align*}
\]

(1.1)
where \( a \in C^1([0, \infty)) \) satisfies
\[
a(t) > 0, \quad a'(t) \leq 0 \quad \text{for all } t \geq 0, \quad \text{and } \int_0^\infty a(t)^{1/2}dt < \infty,
\]
(1.2)
also \( \Delta = \partial_i^2 + \cdots + \partial_n^2, \) \( \nabla_x u = (\partial_1 u, \ldots, \partial_n u), \) \( \partial_j = \partial/\partial x^j, \) \( j = 1, \ldots, n, \) \( (x^1, \ldots, x^n) \in \mathbb{R}^n, \) \( \mu \) is a nonnegative constant, \( p > 1, \) and \( \varepsilon > 0 \) is a small parameter.

For the special case \( a(t) \equiv 1, \) there have been many results. Especially important among them are Todorova and Yordanov \([12]\) and Zhang \([20]\) which have proved that the critical exponent is given by the so-called Fujita exponent \( p_F(n) = 1 + 2/n. \) The critical exponent means the threshold condition on \( p \) between global existence and blow-up of solutions for small initial data. We refer to the introduction of \([7]\) for more details.

For general \( a(t), \) D’Abbicco and Lucente \([3]\) proved that if \( a(t) \) satisfies the condition
\[
a(t) \leq C(t/\mu)^{-\alpha}
\]
with some constants \( \alpha < \min\{1, \frac{2}{n}\} \) and \( C > 0, \) then no global weak solutions exist, provided that
\[
1 + \max\{0, \alpha\} < p \leq 1 + \frac{2}{n(1 - \alpha)}.
\]
In case \( a(t) = e^{-2t} \) and \( n = \mu = 3, \) Galstian \([5]\) showed a lower bound of the lifespan of solutions for \( 2 \leq p < 5. \) For this case, we can apply the result above by \([3]\) and see that no global weak solutions exist for \( 1 < p < 3 \) by taking arbitrary \( \alpha < 2/3. \) We remark that the result of \([3]\) cannot be applied to \( p \geq 3. \) To our best knowledge, there are no results of upper bounds of the lifespan of solutions.

For the time-dependent damping case \( \mu = \mu(t) \) or the Klein–Gordon equation, we refer to, e.g., \([1,4,18,19]\).

The equation in (1.1) generalizes the nonlinear wave equations in the de Sitter spacetime. The metric of the de Sitter spacetime with zero space curvature is given by
\[
g : ds^2 = -dt^2 + e^{2Ht}d\sigma^2,
\]
where the speed of light is equal to 1, \( d\sigma^2 \) is the line element of \( n \)-dimensional Euclidean space and \( H \) is the Hubble constant. The scale factor \( e^{Ht} \) describes expansion of the spatial metric, and also is determined by solving the Einstein equation with the cosmological constant \( \Lambda \) and the energy-momentum tensor for the perfect fluid. Let \( \rho \) and \( p \) be the energy density and pressure, respectively, and let us add an equation of state as \( p = \omega \rho \) with a constant \( \omega. \) If \( \omega = -1, \) then we obtain the constant density \( \rho \) and the scale factor
\[
R(t) = ce^{Ht},
\]
with some constant \( c \) and \( H = \sqrt{(16\pi G \rho + 2\Lambda)/(n(n - 1))}, \) where \( G \) is the gravitational constant. See \([2, (3.1)]\) for the derivation of this scale factor and \([6]\) for another derivation.

For the spatially flat de Sitter metric, the semilinear wave equations \( \Box_g u = |g|^{-1/2}\partial_\alpha(|g|^{1/2}g^{\alpha\beta}\partial_\beta)u = -|u|^p \) or \(-|\nabla_x u|^p \) with \( p > 1 \) become
\[
u_{tt} - e^{-2Ht}\Delta u + nHu_t = |u|^p, \quad \text{or} \quad |\nabla_x u|^p.
\]
(1.3)
Our aim of this paper is to show that blow-up in a finite time occurs for the generalized equation (1.1) as well as upper bounds of the lifespan of the blow-up solutions.

Our previous work \([13–16]\) has treated the case \(-1 < \omega \leq 1, \) where the background metric is given by
\[
g : ds^2 = -dt^2 + t^{4/(n(1+w))}d\sigma^2,
\]
(1.4)
which is the FLRW metric with zero spatial curvature.

We have shown blow-up of solutions in a finite time and upper bounds of their lifespan for $\Box_g u = -|u|^p$ for case of decelerated expansion $2/n - 1 < w \leq 1$, $n \geq 2$ in [13,15] and for case of accelerated expansion $-1 < w \leq 2/n - 1$, $n \geq 2$ in [16]. Furthermore, we have studied equation $\Box_g u = -|u|^p$ or $-|\nabla_x u|^p$ in [14]. It should be noted here that if $-1 < w \leq 2/n - 1$, then blow-up in a finite time can happen to occur for all $p > 1$.

In this paper we show very similar results to the ones above in case of accelerated expansion $\Box_g u = -|u|^p$ or $-|\nabla_x u|^p$ in [14]. It should be noted here that if $-1 < w \leq 2/n - 1$, then blow-up in a finite time can happen to occur for all $p > 1$.

We now state our main result. Let $T_\varepsilon$ be the lifespan of solutions of (1.1), that is, $T_\varepsilon$ is the supremum of $T$ such that (1.1) have a solution for $x \in \mathbb{R}^n$ and $0 \leq t < T$.

**Theorem 1** Let $a(t)$ satisfy (1.2) and let $n \geq 1$, $\mu \geq 0$ and $p > 1$. Assume that $u_0 \in C^2(\mathbb{R}^n)$ and $u_1 \in C^1(\mathbb{R}^n)$, supp $u_0$, supp $u_1$ $\subset \{|x| \leq R\}$ with $R > 0$ and

$$\int (\mu u_0(x) + u_1(x))dx > 0.$$  

Suppose that problem (1.1) has a classical solution $u \in C^2([0, T) \times \mathbb{R}^n)$. Then $T < \infty$ and for arbitrary $\varepsilon > 0$, the lifespan $T_\varepsilon$ of the solution $u$ to (1.1) is estimated as

$$T_\varepsilon \leq C \varepsilon^{-(p-1)} \quad \text{if } \mu > 0,$$

$$T_\varepsilon \leq C \varepsilon^{-(p-1)/(p+1)} \quad \text{if } \mu = 0,$$

where $C > 0$ is a constant independent of $\varepsilon$.

**Remark** The upper bound of the lifespan for $\mu = 0$ is better than that for $\mu > 0$ in the theorem.

In order to prove Theorem 1, we first show the following proposition. We denote $u' = (u_t, \nabla_x u)$, and $u''$ represents the vector of the second derivatives of $u$ with respect to time and space variables.

**Proposition 2** Let $F(t, x, u, u', u'')$ be a function of class $C^1$ in $t, x, u, u'$ and $u''$ satisfying

$$F(t, x, 0, 0, u'') = 0 \quad \text{for all } t \text{ and } u''$$  

and let $u(t, x)$ be a $C^2$-solution of the equation

$$u_{tt} - a(t) \Delta u = F(t, x, u, u', u'')$$

with $a \in C^1([0, \infty))$ such that $a(t) > 0$ and $a'(t) \leq 0$ for all $t \geq 0$ in the region

$$\Lambda_{T, x_0} = \{(t, x) \in [0, T) \times \mathbb{R}^n : |x - x_0| < A(T) - A(t)\}$$
for some $T > 0$ and $x_0 \in \mathbb{R}^n$, where
\[
A(t) = \int_0^t \sqrt{a(s)} ds.
\] (2.3)

Assume that
\[
u(0, x) = u_t(0, x) = 0 \quad \text{for } |x - x_0| < A(T).
\] (2.4)

Then $u$ vanishes in $\Lambda_{T, x_0}$.

**Proof** We prove the proposition following [10,11]. We have only to consider the case $x_0 = 0$ since the equation is invariant under translation with respect to $x$. Let
\[
\psi(\lambda, x) = A^{-1} \left[ A(T) - \left( (A(T) - \lambda)^2 + A(T)^2(2\lambda A(T) - \lambda^2)|x|^2 \right)^{1/2} \right],
\]
where $A^{-1}$ is the inverse function of $A(t)$, and $\lambda$ is a parameter between 0 and $A(T)$. We then have
\[
\psi(0, x) = 0, \quad \lim_{\lambda \to A(T)} \psi(\lambda, x) = A^{-1}(A(T) - |x|).
\]

Define the region $R_\lambda$ by
\[
R_\lambda = \{(t, x) : 0 \leq t \leq \psi(\lambda, x), \ |x| < A(T) - A(t)\}.
\]

We have
\[
\Lambda_{T, 0} = \bigcup_{0 \leq \lambda < A(T)} R_\lambda
\]
and also
\[
|\nabla_x \psi(\lambda, x)| = \left| (A^{-1})' \left( A(T) - \left( (A(T) - \lambda)^2 + A(T)^2(2\lambda A(T) - \lambda^2)|x|^2 \right)^{1/2} \right) \right| \cdot \frac{(2\lambda A(T) - \lambda^2)|x|}{A(T)^2 \left( (A(T) - \lambda)^2 + A(T)^2(2\lambda A(T) - \lambda^2)|x|^2 \right)^{1/2}}
\] (2.5)

for $0 \leq \lambda < A(T)$. We here note that
\[
(A^{-1})' \left( A(T) - \left( (A(T) - \lambda)^2 + A(T)^2(2\lambda A(T) - \lambda^2)|x|^2 \right)^{1/2} \right)
= \left\{ \sqrt{a} \circ A^{-1} \left( A(T) - \left( (A(T) - \lambda)^2 + A(T)^2(2\lambda A(T) - \lambda^2)|x|^2 \right)^{1/2} \right) \right\}^{-1}
= \frac{1}{\sqrt{a(\psi(\lambda, x))}}.
\] (2.6)

Define the surface $S_\lambda$ by
\[
S_\lambda = \{(t, x) : t = \psi(\lambda, x), \ |x| < A(T)\}.
\]

The outward unit normal at $(\psi(\lambda, x), x) \in S_\lambda$ is
\[
\frac{1}{\sqrt{1 + |\nabla_x \psi(\lambda, x)|^2}} (1, -\nabla_x \psi(\lambda, x)).
\]
Then
\[ \int_{\mathbb{R}_\lambda} 2u_t F dtdx = \int_{\mathbb{R}_\lambda} 2u_t (u_{tt} - a(t) \Delta u) dtdx \]
\[ = \int_{\mathbb{R}_\lambda} \left\{ \partial_t(u_t^2 + a(t)|\nabla_x u|^2) - 2 \nabla_x \cdot (a(t)u_t \nabla_x u) \right\} dtdx \]
\[ - a'(t)|\nabla_x u|^2 \right\} dtdx \]
\[ \geq \int_{\mathbb{R}_\lambda} \left\{ \partial_t(u_t^2 + a(t)|\nabla_x u|^2) - 2 \nabla_x \cdot (a(t)u_t \nabla_x u) \right\} dtdx \]
by (1.2). Let \( \lambda_0 \) satisfy \( 0 < \lambda_0 < A(T) \). Note by (2.5) and (2.6) that
\[
\sqrt{a(t)}|\nabla_x \psi| = \frac{(2\lambda A(T) - \lambda^2)|x|}{A(T)^2 \left\{ (A(T) - \lambda)^2 + A(T)^{-2}(2\lambda A(T) - \lambda^2)|x|^2 \right\}^{1/2}} \]
\[ < A(T)^{-1} \sqrt{2\lambda_0 A(T) - \lambda_0^2} \equiv \theta(\lambda_0) < 1 \]
on \( \mathcal{S}_\lambda \) for \( 0 \leq \lambda \leq \lambda_0 < A(T) \). Using the divergence theorem, we obtain
\[
\int_{\mathbb{R}_\lambda} 2u_t F dtdx \geq \int_{\mathcal{S}_\lambda} \left\{ u_t^2 + a(t)|\nabla_x u|^2 + 2 \nabla_x \psi \cdot (a(t)u_t \nabla_x u) \right\} \]
\[ \cdot \frac{1}{\sqrt{1 + |\nabla_x \psi|^2}} d\sigma \]
\[ \geq \int_{\mathcal{S}_\lambda} \left\{ u_t^2 + a(t)|\nabla_x u|^2 - \sqrt{a(t)}|\nabla_x \psi|(u_t^2 + a(t)|\nabla_x u|^2) \right\} \]
\[ \cdot \frac{1}{\sqrt{1 + |\nabla_x \psi|^2}} d\sigma \]
\[ \geq (1 - \theta(\lambda_0)) \int_{\mathcal{S}_\lambda} \frac{u_t^2 + a(t)|\nabla_x u|^2}{\sqrt{1 + |\nabla_x \psi|^2}} d\sigma \]
(2.7)
for \( 0 \leq \lambda \leq \lambda_0 < A(T) \).
On the other hand, by assumption (2.1), we have
\[ |u_t F(t, x, u, u', u'')| \leq C(u^2 + |u'|^2) \]
in \( \Lambda_{T, 0} \). We note that
\[
\int_0^\psi(\lambda, x) u(t, x)^2 dt \leq \frac{1}{2} \psi(\lambda, x)^2 \int_0^\psi(\lambda, x) u_t(t, x)^2 dt \leq T^2 \int_0^\psi(\lambda, x) u_t(t, x)^2 dt
\]
and also by (2.6) that

\[ \psi_\lambda(\lambda, x) = (A^{-1})' \left( A(T) - \{(A(T) - \lambda)^2 + A(T)^{-2}(2\lambda A(T) - \lambda^2)|x|^2 \}^{1/2} \right) \]

\[ \cdot \frac{1}{\sqrt{a(T)}} \cdot \frac{(A(T) - \lambda)(1 - A(T)^{-2}|x|^2)}{\{(A(T) - \lambda)^2 + A(T)^{-2}(2\lambda A(T) - \lambda^2)|x|^2 \}^{1/2}} \]

\[ |\psi_\lambda(\lambda, x)| \leq \frac{1}{\sqrt{a(T)}}. \]

We then have

\[ \int_{R_\lambda} 2u_t F \, dt \, dx \leq C(1 + T^2) \int_{R_\lambda} |u'|^2 \, dt \, dx \]

\[ \leq C_T \int_{R_\lambda} \left( u_i^2 + a(t)|\nabla_x u|^2 \right) \, dt \, dx \]

\[ = C_T \int_0^\lambda \int_{S_\mu} \left( u_i^2 + a(t)|\nabla_x u|^2 \right) \frac{\psi_\mu}{\sqrt{1 + |\nabla_x \psi|^2}} \, d\sigma \, d\mu \]

\[ \leq C_T \int_0^\lambda \int_{S_\mu} \frac{u_i^2 + a(t)|\nabla_x u|^2}{\sqrt{1 + |\nabla_x \psi|^2}} \, d\sigma \, d\mu \]

(2.8)

for \( \lambda \leq \lambda_0 \). Set

\[ I(\lambda) = \int_{S_\lambda} \frac{u_i^2 + a(t)|\nabla_x u|^2}{\sqrt{1 + |\nabla_x \psi|^2}} \, d\sigma. \]

Using Gronwall’s inequality for (2.7) and (2.8), we see that \( I(\lambda) = 0 \) for \( 0 \leq \lambda \leq \lambda_0 < A(T) \). Therefore, since \( \lambda \) and \( \lambda_0 \) are arbitrary, we see that \( u' = 0 \) in \( A_{T,0} \), and thus by (2.4) also that \( u = 0 \) in \( A_{T,0} \). This completes the proof of the proposition.

From the proposition above, we easily see that the following corollary holds:

**Corollary 3** Let \( F \) in Proposition 2 satisfy (2.1). If \( u \) is a \( C^2 \)-solution of (2.2) and if \( u(0, x) = u_t(0, x) = 0 \) for \( |x| > R \) with \( R > 0 \), then \( u(t, x) = 0 \) for \( |x| > R + A(t) \).

**Proof of Theorem 1** By Corollary 3, we see that \( C^2 \)-solution \( u \) of (1.1) has the property of finite speed of propagation, and satisfies

\[ \text{supp } u(t, \cdot) \subset \{ |x| \leq A(t) + R \}, \quad \text{(2.9)} \]

where \( A(t) \) is given by (2.3).

We next introduce a test function \( \eta \in C_0^\infty([0, \infty)) \) satisfying

\[ 0 \leq \eta(t) \leq 1, \quad \eta(t) = \begin{cases} 1 & (0 \leq t \leq \frac{1}{2}), \\ 0 & (t \geq 1), \end{cases} \]
and let
\[
\eta^*(t) = \chi_{\text{supp } \eta'}(t) = \begin{cases} \eta(t) & (1/2 < t < 1), \\
0 & \text{otherwise,} \end{cases}
\]
where \(\eta' = d\eta/dt\). Set
\[
\psi_\tau(t) = \eta\left(\frac{t}{\tau}\right)^{2p'}, \quad \psi_\tau^*(t) = \eta^*\left(\frac{t}{\tau}\right)^{2p'}
\]
for \(0 < \tau < T_\varepsilon\). Then we see that
\[
|\psi_\tau'| \lesssim \tau^{-1}(\psi_\tau^*)^{1/p}, \quad |\psi_\tau''| \lesssim \tau^{-2}(\psi_\tau^*)^{1/p}.
\]

Multiplying equation in (1.1) by the test function \(\psi_\tau(t)\), and integrating over \(\mathbb{R}^n\), we have
\[
\frac{d^2}{dt^2} \int u\psi_\tau dx - 2\frac{d}{dt} \int u\psi_\tau' dx + \int u\psi_\tau'' dx + \mu \frac{d}{dt} \int u\psi_\tau dx - \mu \int u\psi_\tau' dx
= \int |u|^p \psi_\tau dx \quad \text{or} \quad \int |\nabla_x u|^p \psi_\tau dx.
\]

For the derivative nonlinearity, we have by Poincaré's inequality,
\[
\int |\nabla_x u|^p \psi_\tau dx \geq \frac{\psi_\tau(t)}{(A(t) + R)^p} \int |u|^p dx \geq C \int |u|^p \psi_\tau dx.
\]
Hence, it suffices to show the theorem for the nonlinearity \(|u|^p\). Integrating equality (2.11) in \(t\) over \([0, \infty)\), we have
\[
\int_0^\infty \int u\psi_\tau'' dx dt - \mu \int_0^\infty \int u\psi_\tau' dx dt
= \varepsilon \int_0^\infty \left[ (\mu \psi_\tau(0) - \psi_\tau'(0))u_0(x) + \psi_\tau(0)u_1(x) \right] dx + \int_0^\infty \psi_\tau(t)|u|^p(t, x) dx dt
= \varepsilon \int_0^\infty (\mu u_0(x) + u_1(x)) dx + \int_0^\infty \psi_\tau(t)|u|^p(t, x) dx dt.
\]
Let
\[
I_\tau \equiv \int_0^\infty \psi_\tau(t)|u|^p(t, x) dx dt,
\]
\[
J \equiv \varepsilon \int_0^\infty (\mu u_0(x) + u_1(x)) dx > 0
\]
by assumption. Then we have
\[
I_\tau + J = \int_0^\infty \int u\psi_\tau'' dx dt - \mu \int_0^\infty \int u\psi_\tau' dx dt
\equiv K_1 + K_2.
\]
By Hölder’s inequality and (2.10), we have
\[ |K_1| \lesssim \tau^{-2} \left( \int_0^\infty \int |u|^p \psi_t^\ast dxdt \right)^{1/p} \left( \int_{\tau/2}^\tau \int_{|x| \leq A(t) + R} dxdt \right)^{1/p'} \]
\[ \lesssim \tau^{-2+1/p'} I_\tau^{1/p}, \]
similarly,
\[ |K_2| \lesssim \mu \tau^{-1+1/p'} I_\tau^{1/p}. \]
Thus we have
\[ I_\tau + J \lesssim (\tau^{-2+1/p'} + \mu \tau^{-1+1/p'}) I_\tau^{1/p}. \] (2.12)
We set
\[ E(\tau) = \tau^{-2+1/p'} + \mu \tau^{-1+1/p'}. \]
By Young’s inequality, we have
\[ E(\tau) I_\tau^{1/p} \leq \frac{1}{2} I_\tau + CE(\tau)^{p'}. \]
It follows from (2.12) that
\[ J \leq CE(\tau)^{p'}, \]
hence,
\[ \varepsilon \leq CE(\tau)^{p'} \]
holds for \( 0 < \tau < T_\varepsilon \). Thus we have
\[ \varepsilon \leq \begin{cases} C \tau^{-1/(p-1)} & \text{if } \mu > 0, \\ C \tau^{-(p+1)/(p-1)} & \text{if } \mu = 0, \end{cases} \]
which imply that
\[ \tau \leq \begin{cases} C \varepsilon^{-(p-1)} & \text{if } \mu > 0, \\ C \varepsilon^{-(p-1)/(p+1)} & \text{if } \mu = 0. \end{cases} \]
Since \( \tau \) is arbitrary in \((0, T_\varepsilon)\), we obtain the desired estimates of the lifespan. This completes the proof of the theorem.

Finally, we remark that it is possible to obtain similar blow-up results to the theorem for weak solutions by using only test function methods in [8,9,17]. However, an upper bound for the \( p \)-blow-up range is required. We thus refrain from going into the details.

3 Conclusion

We consider the original equation (1.3) in the de Sitter spacetime. We see from Theorem 1 that blow-up of solutions in a finite time occurs for all \( p > 1 \). This blow-up condition is exactly the same as that for \( \Box_g u = -|u|^p \) or \( -|\nabla_u|^p \) in the accelerated expanding FLRW.
spacetime with metric \((1.4)\) \((-1 < w \leq 2/n - 1)\) as stated in Sect. 1. Moreover, we have shown in \([14,16]\) the following upper bound of the lifespan for \(\Box_g u = -|u|^p\) or \(-|\nabla_x u|^p\):

\[
T_\varepsilon \leq C\varepsilon^{-(p-1)/2} \quad \text{if } p > 1 \text{ and } -1 < w < 2/n - 1.
\]

This estimate is similar to the one in Theorem 1. Therefore, we conjecture that in the accelerated expanding universe the nonlinear wave equation with the term \(|u|^p\) or \(|\nabla_x u|^p\) admits blow-up solutions for all \(p > 1\).

Acknowledgements The authors would like to thank the referee for his/her valuable comments and suggestions on the first version of this paper. They would also like to thank Professor Katayama whose question has led to an improvement of the estimates of the lifespan in Theorem 1. K. Tsutaya thanks the JSPS for financial support through the KAKENHI Grant Number JP18K03351.

Author Contributions All authors contributed equally to this work. All authors read and approved the final manuscript.

Funding K. Tsutaya thanks the JSPS for financial support through the KAKENHI Grant Number JP18K03351.

Availability of data and materials Not applicable.

Declarations

Conflict of interest The authors declare that they have no competing interests.

Code availability Not applicable.

References

1. Bui, T.B.N., Reissig, M.: Global existence of small data solutions for wave models with sub-exponential propagation speed. Nonlinear Anal. 129, 173–188 (2015)
2. Chen, S., Gibbons, G. W., Li, Y., Yang, Y.: Friedmann’s equations in all dimensions and Chebyshev’s theorem. J. Cosmol. Astropart. Phys. 35 (2014)
3. D’Abbicco, M., Lucente, S.: A modified test function method for damped wave equations. Adv. Nonlinear Stud. 13, 867–892 (2013)
4. Ebert, M.R., Reissig, M.: Regularity theory and global existence of small data solutions to semi-linear de Sitter models with power non-linearity. Nonlinear Anal. Real World Appl. 40, 14–54 (2018)
5. Galstian, A.: Semilinear wave equation in the de Sitter spacetime with hyperbolic spatial part. In: Dang, P., Ku, M., Qian, T., Rodino, L.G. (eds.) New Trends in Analysis and Interdisciplinary Applications. Trends in Mathematics: Research Perspectives, pp. 489–498. Springer International Publishing, Cham (2017)
6. Galstian, A., Yagdjian, K.: Finite lifespan of solutions of the semilinear wave equation in the Einstein-de Sitter spacetime. Rev. Math. Phys. 32(2050018), 31 (2020). https://doi.org/10.1142/S0129055X2050018X
7. Ikeda, M., Inui, T., Wakasugi, Y.: The Cauchy problem for the nonlinear damped wave equation with slowly decaying data. NoDEA Nonlinear Differ. Equ. Appl. 24(2), 53 (2017)
8. Ikeda, M., Sobajima, M.: Sharp upper bound for lifespan of solutions to some critical semilinear parabolic, dispersive and hyperbolic equations via a test function method. Nonlinear Anal. 182, 57–74 (2019)
9. Ikeda, M., Wakasugi, Y.: A note on the lifespan of solutions to the semilinear damped wave equation. Proc. Am. Math. Soc. 143, 163–171 (2015)
10. John, F.: Nonlinear Wave Equations. Formation of Singularities. Amer. Math. Soc, Providence (1990)
11. Sogge, C.D.: Lectures on Non-Linear Wave Equations, 2nd edn. International Press of Boston Inc, Somerville (2008)
12. Todorova, G., Yordanov, B.: Critical exponent for a nonlinear wave equation with damping. J. Differ. Equ. 174, 464–489 (2001)
13. Tsutaya, K., Wakasugi, Y.: Blow up of solutions of semilinear wave equations in Friedmann-Lemaître-Robertson-Walker spacetime. J. Math. Phys. 61, 091503 (2020). https://doi.org/10.1063/1.5139301
14. Tsutaya, K., Wakasugi, Y.: On Glassey’s conjecture for semilinear wave equations in Friedmann-Lemaître-Robertson-Walker spacetime. Boundary Value Problems 2021, 94 (2021). https://doi.org/10.1186/s13661-021-01571-0
15. Tsutaya, K., Wakasugi, Y.: On heatlike lifespan of solutions of semilinear wave equations in Friedmann-Lemaître-Robertson-Walker spacetime. J. Math. Anal. Appl. 500, 125133 (2021). https://doi.org/10.1016/j.jmaa.2021.125133
16. Tsutaya, K., Wakasugi, Y.: Blow up of solutions of semilinear wave equations in accelerated expanding Friedmann-Lemaître-Robertson-Walker spacetime. Rev. Math. Phys. 33(2250003), 16 (2022). https://doi.org/10.1142/S0129055X22500039
17. Wakasugi, Y.: Critical exponent for the semilinear wave equation with scale invariant damping. In: Fourier Anal.: Trends Math. Springer, pp. 375–390 (2014)
18. Yagdjian, K.: The semilinear Klein-Gordon equation in de Sitter spacetime. Discrete Contin. Dyn. Syst. 2, 679–696 (2009)
19. Yagdjian, K.: Global existence of the scalar field in de Sitter spacetime. J. Math. Anal. Appl. 396, 323–344 (2012)
20. Zhang, Q.S.: A blow-up result for a nonlinear wave equation with damping: the critical case. C. R. Acad. Sci. Paris 333, 109–114 (2001)