Analysis of rail acoustic model based on phased ultrasonic array

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Abstract. In the traditional ultrasonic flaw detection, two forms of A scan and B scan are mainly used to check the echo information. The traditional method of defect analysis can only detect the existence of defects, but can not locate the exact position and shape of defects. In this paper, we will present a method of locating the defects in ultrasonic detection for rail. In this method, the position information and incidence angle of ultrasonic probe are obtained, and then the propagation path of primary wave and secondary wave of ultrasonic wave in rail is calculated by mathematical modeling. Finally, the internal defects of rail can be located by combining the echo information collected by ultrasonic probe.

1. Introduction
In welding, installation and using, some damage may be caused to rail which can not be observed with naked eyes. With the passage of time, the damage will gradually worsen. When the damage reaches a certain level, it is likely to cause serious safety accidents. In the traditional inspection process, the inspection results depend on some subjective factors such as the working experience and personal responsibility, which may lead to missed inspection and bring harm to safe operation.

In this paper, the propagation path of ultrasonic beam inside rail will be analyzed by means of data modeling, and a 3D model of rail will be established according to the size diagram of the rail given by the railway industry standard TB/T 2341.3-93 of the People's Republic of China. The three-dimensional Cartesian coordinate system is established by using a 3D model, and the trajectory of primary wave and secondary wave of ultrasonic wave is solved in the coordinate system. Finally, by drawing the trajectory on the 3D model, we can clearly and intuitively see the coverage of ultrasonic waves inside the rail.

2. Coordinate establishment and solution
In order to make the calculation results more intuitive, the three-dimensional Cartesian coordinate system is adopted in this paper, and then the incident point, the intersection point of primary wave and the intersection point of secondary wave are solved in the coordinate play.
2.1 Establishment of three-dimensional Cartesian coordinate system
For the selection and establishment of the coordinate system, there is also a simplified calculation. According to the rail structure diagram, the rail can be regarded as a cylinder about the central axis. The three-dimensional Cartesian coordinate system is selected for the coordinate system, with the transverse rail as the Y axis, the longitudinal rail as the X axis, and the bottom of the rail as the XOY plane. The specific coordinate system is shown in Fig. 1.

![Figure 1. A three-dimensional coordinate diagram of rail](image)

2.2 Solution of ultrasonic incident point
According to the requirement of flaw detection, the probe should be rotated on the flaw detection plane to enlarge the flaw detection area. Because the rail structure is more complex, the inspection plane is usually a slope. Assuming that the dip Angle of the inspection slope is $\alpha$ and the rotation origin coordinate is $O(x_0, y_0, z_0)$, the distance between the ultrasonic transmitting point on the probe and the rotation origin is $l$. Next, we will calculate the corresponding position $C_I(x_r, y_r, z_r)$ of the incident point after the probe rotates $\beta$ Angle on the slope.

Fig. 2 is a right triangle formed after the probe rotates $\beta$ Angle on the slope around the origin of rotation.

![Figure 2. Schematic diagram of probe rotation $\beta$ angle on slope surface](image)

It can be deduced from the figure:

$$\Delta y = l \cdot \cos(\beta) \quad (1)$$
$$m = l \cdot \sin(\beta) \quad (2)$$

In Fig. 3, the frontal view from the direction of Y axis can be calculated according to the m obtained.
Figure 3. Front view of y-axis direction after probe rotating $\beta$ angle above:

$$\triangle x = m \ast \cos(\alpha) \quad (3)$$
$$\triangle z = m \ast \sin(\alpha) \quad (4)$$

Finally, we can calculate:

$$x_r = x_0 + \triangle x \quad (5)$$
$$y_r = y_0 + \triangle y \quad (6)$$
$$z_r = z_0 + \triangle z \quad (7)$$

2.3. Solution of reflection point of primary wave

To solve the intersection point of primary waves, we will derive it from the special to the general way. Assume that the ultrasonic incident point is $CI(x_r, y_r, z_r)$ and the ultrasonic incident Angle is $\theta$. Let the intersection point of primary wave and $xoy$ plane be $C1(x_{c1}, y_{c1}, z_{c1})$. So let’s start with the rotation Angle $\beta$ equals 0 degrees. Fig. 4 is a schematic diagram for the solution, from which it can be deduced:

Figure 4. The diagram of ultrasonic beam in x-axis when the rotation angle is 0

$$n = \frac{z_r}{\tan(\theta)} \quad (8)$$
$$\triangle y = n \ast \sin(\theta) \quad (9)$$
$$\triangle x = z_r \ast \tan(\alpha) \quad (10)$$

At this point, the calculated intersection point of primary wave is:

$$x_{c1} = x_r + \triangle x \quad (11)$$
$$y_{c1} = y_r + \triangle y \quad (12)$$
$$z_{c1} = z_r + \triangle z \quad (13)$$

Beta indicates further specialization, assuming the rotated Angle 0, the calculation process is relatively complex, so the introduction of beta = 0 this special situation to simplify the calculation, the practice of using beta = 0 is obtained under the condition of intersection point coordinates on the slope surface for alpha plane revolve around point $(x_0, y_0, 0)$ beta (beta indicates 0) Angle, wait until the middle point $C_T(x_t, y_t, z_t)$, point by point $C_I$ and $C_T$ form a straight line, an extension of the straight line and $xoy$ surface intersection, finally can draw $C_1$ to solve the general formula.

Rotate the intersecting point obtained under $\beta = 0$ around the point $(x_0, y_0, 0)$ $\beta(\beta \neq 0)$:

$$x_t = l \ast \sin(\beta) \ast \cos(\alpha) + x_r \quad (14)$$
$$y_t = l \ast \cos(\beta) + y_0 \quad (15)$$
The coordinate $C_T$ of the middle point is derived from Equations 14-18, and the parametric equation of the line can be written by using $C_I$ and point $C_T$:

\[
\begin{align*}
{x} &= (x_r - x_t) \ast t + x_t \\
{y} &= (y_r - y_t) \ast t + y_t \\
{z} &= (z_r - z_t) \ast t + z_t
\end{align*}
\]  

(19)

Prolong the equation of the line and $xoy$ plane in detail, that is, set $z = 0$, solve the $t$ value, and then substitute it into Equation 19 to solve the intersection point $C_1$ of the primary wave.

At this point, we have obtained the linear equation of primary wave:

\[
\begin{align*}
{x} &= (x_r - x_{c1}) \ast t + x_{c1} \\
{y} &= (y_r - y_{c1}) \ast t + y_{c1} \\
{z} &= (z_r - z_{c1}) \ast t + z_{c1}
\end{align*}
\]  

(20)

2.4. Solution of quadratic wave linear equation

Ultrasonic wave satisfies the law of reflection, that is, the reflection Angle is equal to the incident Angle, and the incident ray and the reflection ray are symmetric on the normal line. Using this characteristic, as long as the symmetry point of any point on the ultrasonic ray on the normal line is solved, the equation of the straight line where the reflection ray is located can be obtained. Since the geometric structure of the rail is relatively complex, when the intersection point of the primary wave obtained falls on the $xoy$ plane, the quadratic wave and the primary wave equation at this time will be weighed on the $xoy$ plane, and then the equation of the quadratic wave line can be obtained:

\[
\begin{align*}
{x} &= (x_r - x_s) \ast t + x_s \\
{y} &= (y_r - y_s) \ast t + y_s \\
{z} &= (z_r - z_s) \ast t + z_s
\end{align*}
\]  

(21)

3. Matlab simulation

Two ultrasonic probes are used in the model to achieve the full coverage of the flaw detection area. The probe placement area is shown in Fig. 5. In the figure, probe A realizes the detection of the rail bottom and the triangle area of the rail bottom, while probe B realizes the detection of the rail head and the rail waist area.

Figure 5. Diagram of relationship between rail profile and probe position
3.1 Ultrasonic coverage of rail bottom

Fig. 6 shows the ultrasonic distribution in the range of L = 34 mm from the rotation origin of the ultrasonic probe to the edge of the rail and the rotation Angle $\beta = 30$ degrees. The number of ultrasonic beams is 32 and the incidence Angle is $45^\circ$ to $76^\circ$.

![Figure 6. Diagram 1 of rail bottom simulation verification](image)

Fig. 7 Ultrasonic distribution in the range of the rotation origin distance from the ultrasonic probe to the edge of the rail L = 40 mm, rotation Angle $\beta = 90$ degrees, the number of ultrasonic beams is 32 and the incidence Angle is $30^\circ$ to $61^\circ$.

![Figure 7. Diagram 2 of rail bottom simulation verification](image)

3.2 Ultrasonic distribution of rail head and rail waist

Fig. 8 Ultrasonic distribution in the range of the rotation origin distance of ultrasonic probe from the edge of rail L = 75 mm, rotation Angle $\beta =-45^\circ$, number of ultrasonic beam 32 and incidence Angle $30^\circ$ to $61^\circ$. 

![Image](image)
Fig. 9 shows the ultrasonic distribution in the range of L =70mm from the rotation origin of the ultrasonic probe to the edge of the rail and the rotation Angle β=5°, and the number of ultrasonic beams is 20 and the incidence Angle is 40° to 59°.

4. CONCLUSION
In this paper, we give a kind of rail internal by obtaining ultrasonic a second wave of the algorithm, and through MATLAB simulation can see clear the ultrasound in the inside of the rail cover, the simulation results can provide theoretical support for rail flaw detection probe move and rotate, can guarantee a complete coverage of the internal ultrasonic rail, reduce the residual inspection personnel.

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