Spontaneous CP violation and Higgs spectrum in a Next to Minimal Supersymmetric Model.

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Abstract

We explore the possibility of spontaneous CP violation within the Next to Minimal Supersymmetric Standard Model. In the most general form of the model, without a discrete $Z_3$ symmetry, we find that even at tree level spontaneous CP violation can occur, while also permitting Higgs masses consistent with experiment.

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1 NMSSM

CP violation, the Higgs spectrum and supersymmetry are at the forefront of experimental investigation and theoretical interest. We focus on spontaneous CP violation in the Next to Minimal Supersymmetric Standard Model (NMSSM) which contains a singlet $N$ in addition to the two doublets $H_1$ and $H_2$ of the MSSM. Spontaneous CP violation is achievable in both the MSSM and NMSSM with the usual $Z_3$ discrete symmetry, but only by invoking radiative corrections to raise a negative (mass)$^2$ mode to a small, experimentally unacceptable, real mass.

Our main result is that spontaneous CP violation is possible for the general NMSSM potential even at tree level, and that Higgs masses need not be small. Above some SUSY-breaking scale, $M_S$, unknown but hopefully not far beyond experimental reach, the most general superpotential for these fields is

$$W = \lambda N H_1 H_2 - \frac{k}{3} N^3 - r N + \mu H_1 H_2 + W_{\text{Fermion}}$$

(1)

$\mu$ has dimension of mass and $r$ of (mass)$^2$. At lower energy a general quartic form is adopted, with 8 couplings $\lambda_i$, which at $M_S$ may be expressed in terms of the gauge couplings and the superpotential coupling constants, and may be determined at the electroweak scale $M_{\text{Weak}}$ using renormalization group (RG) equations, if $M_S > M_{\text{Weak}}$. New SUSY-breaking soft cubic and quadratic terms are added. In much of the work on the NMSSM, RG equations are used to run down the soft couplings from a hypothesized universal form at the scale $M_{\text{GUT}}$, but we do not assume this, and regard $m_i, i = 1 \ldots 7$, below as arbitrary parameters.

The effective potential is thus

$$V_0 = \frac{1}{2} \lambda_1 (H_1^\dagger H_1)^2 + \frac{1}{2} \lambda_2 (H_2^\dagger H_2)^2 + (\lambda_3 + \lambda_4) (H_1^\dagger H_1) (H_2^\dagger H_2) - \lambda_4 \left| H_1^\dagger H_2 \right|^2 + (\lambda_5 H_1^\dagger H_1 + \lambda_6 H_2^\dagger H_2) N^\dagger N + (\lambda_7 H_1 H_2 N^* + h.c.) + \lambda_8 (N^* N)^2 + \lambda_9 (N + h.c.) (H_1^\dagger H_1 + H_2^\dagger H_2) + m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 N^* N - m_4 (H_1 H_2 N + h.c.) - \frac{1}{3} m_5 (N^3 + h.c.) + \frac{1}{2} m_6^2 (H_1 H_2 + h.c.) + m_7^2 (N^2 + h.c.)$$

(2)

A restricted version of this has been advocated to explain why $\mu$ is of electroweak scale, the ‘$\mu$-problem’ of the MSSM. The terms in the superpotential involving dimensionful couplings and the soft terms in $m_6$ and $m_7$ are dropped, leaving a $Z_3$ discrete symmetry which protects the hierarchy. The VEV $< N >$ replaces $\mu$, thereby introducing a domain wall problem. This restricted model provides only a limited improvement on the MSSM.
(a) Surveys of the parameter space tend to favour a low energy Higgs spectrum very similar to the MSSM [8, 9].

(b) As in the MSSM, spontaneous CP violation is possible, but only as a result of radiative corrections [10, 11, 12]. The lightest neutral Higgs has a mass less than 45 GeV, and typically $\tan\beta \leq 1$.

We do not impose the $Z_3$ symmetry, so do not have the above domain wall problem, but are left with the $\mu$-problem. We find that spontaneous CP violation is more easily achieved in this model than in the MSSM or the NMSSM with $Z_3$.

We consider real coupling constants, so that the tree level potential is CP conserving, but admit complex VEVs for the neutral fields, giving

\[
V_0 = \frac{1}{2} (\lambda_1 v_1^2 + \lambda_2 v_2^2) + (\lambda_3 + \lambda_4) v_1^2 v_2^2 + (\lambda_5 v_1^2 + \lambda_6 v_2^2) v_3^2 \\
+ 2\lambda_7 v_1 v_2 v_3^2 \cos(\theta_1 + \theta_2 - 2\theta_3) + \lambda_8 v_3^4 + 2\lambda_9 (v_1^2 + v_2^2) v_3 \cos(\theta_3) \\
+ m_1^2 v_1^2 + m_2^2 v_2^2 + m_3^2 v_3^2 - 2m_4 v_1 v_2 v_3 \cos(\theta_1 + \theta_2 + \theta_3) \\
- \frac{2}{3} m_5 v_3^3 \cos(3\theta_3) + 2m_6^2 v_1 v_2 \cos(\theta_1 + \theta_2) + 2m_7^2 v_3^2 \cos(2\theta_3)
\]

where, without loss of generality, $\theta_2 = 0$, and $0 \leq \theta_3 < \pi$.

This potential has 3 extra terms as compared with a $Z_3$ invariant potential: a cubic term arising from the $\mu$ and $\lambda$ terms in the superpotential, and two quadratic terms with coefficients $m_6^2$ and $m_7^2$. If $\mu = 0$ the effective potential loses this cubic term, and the quadratic terms alone account for the difference between our results and previous ones in the literature.

The scalar mass matrix gives rise to 1 charged and 5 neutral particles. If the angles $\theta_1$ and $\theta_3$ are non-zero, the neutral matrix does not decouple into sectors with CP = +1 and -1.

2 Searches and results

We search the parameter space of this potential to see if spontaneous CP violation is compatible with experimental bounds on the Higgs spectrum.

There are 11 parameters, but we impose some restrictions.

(a) 4 superpotential parameters $\mu, \lambda, k$, and $h_t$, the top Yukawa coupling: Running $(\lambda, k)$ up from the electroweak scale using RG equations [13] requires them to be small to avoid blow-up at high energy. In the examples below we fix $\lambda = 0.5, k = 0.5$. We take the fixed point value $h_t = 1.05$ at the SUSY scale, bearing in mind that at the electroweak scale the running top mass is $h_t v_0 \sin\beta$.

(b) Soft breaking mass parameters $m_i, i = \ldots 7$:
Five of these can be traded for VEV magnitudes and phases $v_1, v_2, v_3, \theta_1, \theta_3$. 

\[
\langle H^0_i \rangle = v_i e^{i\theta_i} (i = 1, 2), \quad \langle N \rangle = v_3 e^{i\theta_3}.
\]
We eliminate one by the condition $v_0 = \sqrt{v_1^2 + v_2^2} = 174$ GeV and replace 2 others by the conventional $\tan \beta \equiv v_2/v_1$ and $R \equiv v_3/v_0$. A sixth mass, $m_4$ say, can be exchanged for the mass of the charged Higgs, $M_{H^+}$, which $b \to s + \gamma$ experiment suggests to be greater than 250 GeV [14]. We can obtain an analytic form for this:

$$M_{H^+}^2 = -\lambda_4 v_0^2 - \frac{2(\lambda_7 v_3^2 \sin(3\theta_3) + m_5^2 \sin \theta_3)}{\sin(2\beta) \sin(\theta_1 + \theta_2 + \theta_3)} \quad (5)$$

This shows how the parameter $m_5^2$, not necessarily positive, introduces extra freedom to raise the charged Higgs mass. This leaves one parameter $m_5$, with no particular interpretation.

In the case $\mu = 0$, $\lambda$ and $k$ cannot be too small. In the simplest case, where the Higgsino-gaugino mixing is small, the (unbroken SUSY) charged Higgsino mass is $|\mu + \lambda v_3 \cos \theta_3|$. The experimental lower bound, conservatively $\frac{1}{2} M_Z$, then disallows small $R (\equiv v_3/v_0)$. There is no upper bound on $R$, and indeed the cubic and quartic terms in the field $N$ can in some cases provide deep global minima of the potential for large $R$, thus excluding some otherwise acceptable values of the other parameters.

Our modus operandi was to scan over a grid of parameters chosen to make the first derivatives of the potential vanish at prescribed VEVs. Numerical minimization was performed to ensure that these were minima, giving positive (mass)$^2$, and to reject local minima. In many apparently reasonable cases the spontaneous CP violating minimum was metastable, with a lower electroweak and CP conserving vacuum lying at $v_1 = v_2 = 0$ and $v_3$ the order of TeV. We present two indicative examples from preliminary searches.

| Parameters giving spontaneous CP violation |
|---------------------------------------------|
| CASE | $\tan \beta$ | $R$ | $\theta_1$ | $\theta_3$ | $M_{H^+}$ | $m_5$ | $\mu$ |
| A    | 2.0          | 2.0 | 1.20$\pi$  | 0.65$\pi$  | 250 GeV   | 60 GeV | -20 GeV |
| B    | 2.0          | 2.0 | 0.60$\pi$  | 0.35$\pi$  | 250 GeV   | -100 GeV| 0       |

Case (A) is for a SUSY scale of 1 TeV, with quartic couplings radiatively corrected using RG equations, assuming that all squarks and gauginos lie at the SUSY scale. This has neutral Higgs masses from 89 to 318 GeV, a lightest neutralino of 48 GeV and a chargino of 99 GeV. Larger neutralino masses can be obtained by increasing $R$.

Case (B) is for $M_S = M_{Weak}$ i.e. a SUSY quartic potential. It gives neutral Higgs masses of 81 to 372 GeV, a lightest neutralino of 47 GeV, and a chargino of 79 GeV, ignoring gaugino mixing which should be included if indeed the SUSY scale is as low as the electroweak scale.

This case is presented as a go-theorem, a counter example to no-go examples [10]. As $\mu = 0$ here, the quartic potential has the standard $Z_3$ invariant form and avoids the conclusion of Romao [10] due to the two $Z_3$ violating soft terms. Nor do the conditions of the Georgi-Pais [13] no-go theorem apply. It relates to the
situation where CP is conserved at tree level and is broken only by perturbative radiative corrections. The soft terms here are not in this category.

We are encouraged to explore further the phenomenology of such models.

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