Mixed symmetry states for $^{70}$Zn, $^{72}$Ge, $^{74}$Se, $^{76}$Kr isotones by using IBM-2

Mohammed Abdul Kadhim Hadi Al – Sadi¹, Qasim Shakir Kadhim² and Salwan Hassan Abdul Zahrah Yasi³

Environment Pollution Department, College of Environment, Al-Qasim Green University
Corresponding Email: moh_2005_ammed@yahoo.com
Physics Department, College of Basic Education, Babylon University
Corresponding Email: kasmshaker@yahoo.com
Education Ministry
Corresponding Email: salwan.Hassan@yahoo.com

Abstract. In present study we have calculated some nuclear properties for $^{70}$Zn, $^{72}$Ge, $^{74}$Se, $^{76}$Kr isotones, firstly definition some excited energy levels by regulate a small number of parameters in Hamiltonian operators and estimated these parameters, secondary display the energy levels $(2^+_1, 3^+_1, 5^+_1, 1^+_1)$ that have (MSS) property which gets a quick response when changing majorana term ζ2 as a result of mixture. Thirdly it has been calculated energy ratios $4^+_1/2^+_1$ and $6^+_1/2^+_1$, it was found that these isotones have a varied dynamic symmetries. As well as have been calculated the probability for electric quadrople transition B(E2), magnetic dipole B(M1) and mixing ratios $\delta(E2/M1)$, which All express a good Agreement compared with experimental data.

Key words: IBM-2, Mixed Symmetry state (MSS), electric quadrople transition B(E2), magnetic dipole B(M1), Mixing ratios $\delta(E2/M1)$, 70Zn, 72Ge, 74Se, 76Kr Isotones.

1. Introduction
The interacting boson model (IBM) is suitable for describing intermediate and heavy atomic nuclei. In the IBA-model the collective properties of a nucleus are described in terms of a system of bosons which can be either in an s ($l=0$) or in a d ($l=2$) state. In the formation of these collective pairs only the valence nucleons will be important. This means that the number of bosons is equal to the number of pairs of particles outside the closed shell [1]. Those low lying of the eager energy levels in some nuclei can be obtained by adjusting a small number of parameters in Hamiltonian operators and it is possible to solve this operator exactly for certain sets of parameters using group theoretical methods. The Hamiltonian can be regarded as a general rotation in a six dimensional space. The six dimensions are formed by the s-boson and the five components of the d-boson, $d_2, d_1, d_0, d_1, d_2$. This means that the general Hamiltonian can be discussed in terms of the group $U(6)$, of all unitary transformations in six dimensions[2].

2. The Interacting Proton and Neutron Boson Model
The Hamiltonian in IBM-2 is define as:[3]

$$H = \varepsilon_d(n_{d\nu} + n_{d\pi}) + \kappa(Q_{\nu} Q_{\pi}) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi}$$

(1)

Where $\varepsilon_d$ is the energy of d-bosons, $n_{d\nu}$ is the number of d bosons, where $\nu$ correspond to $\pi$ (proton) or $\nu$ (neutron) bosons, $Q_{\nu}$ and $Q_{\pi}$ is the quadrupole - quadrupole interaction between proton and...
neutron and $\kappa$ is a coefficient expresses the strength interaction between them. And the quadruple operator $Q_p$ is defined as:[4]

$$Q_p = [d^+_p s_p + s^+_p d_p]^{(2)} + x_p [d^+_p d_p]^{(2)}$$  \hspace{1cm} (2)

The third and fourth term ($V_{nn}, V_{np}$) which correspond to the interaction between like boson, Which define as:[5]

$$V_{np} \sum_{L=0,2,4} C^p_L (d^+_p \tilde{d}^+_p)^{(L)} \cdot [\tilde{d}^+_p \tilde{d}^-_p]$$  \hspace{1cm} (3)

A standout amongst those Characteristics from claiming degrees of freedom, may be on investigation the blended symmetry states, when mixture the middle of those wave capacities for Proton Also neutron, to IBM-2 those majorana haul $M_\nu(\nu \pi)$ incorporates three parameters are $\xi_1, \xi_2$ Furthermore $\xi_3$ used to consider blended symmetry states, be composed as: [6]

$$M_{\nu\pi} = \frac{1}{2} \xi_2 ([s^+_\nu s^+_\pi - d^+_\nu d^+_\pi]^{(2)}, [s^-\nu s^-\pi]^{(2)}) - \sum_{k=1,3} \xi_k ([d^+_\nu d^+_\pi]^{(k)}, [d^-\nu d^-\pi]^{(k)})$$  \hspace{1cm} (4)

3. Energy Spectrum

The $^{70}$Zn, $^{72}$Ge, $^{74}$Se and $^{76}$Kr isotones, have N=40 and Z are a variable between (30-36) this means that the number of neutron bosons are $N_\nu=5$ and numbers of proton bosons $N_\pi$ is equal (1, 2, 3, 4 ) respectively. The estimated parameters for the calculations of the low- lying excited energy levels for $^{70}$Zn, $^{72}$Ge, $^{74}$Se and $^{76}$Kr isotones are given in the table1.

Table 1. Parameters used in the IBM-2 Hamiltonian for even-even $^{70}$Zn, $^{72}$Ge, $^{74}$Se and $^{76}$Kr isotones (in MeV ) except $\chi_v$ and $\chi_\nu$, of non-units

| Isotones | N_\pi | N_\nu | $\epsilon$ | $\kappa$ | $\chi_v$ | $\chi_\nu$ | CL_{v}(0,2,4) | CL_{\nu}(0,2,4) | $\xi_2$ |
|----------|-------|-------|----------|--------|--------|---------|------------|-------------|-------|
| $^{70}$Zn | 1     | 1.045 | -0.092   | -1.20  | -0.70  | -0.05   | -0.09      | 0.0, 0.0, 0.0 | -0.090 |
| $^{72}$Ge | 2     | 1.099 | -0.090   | -1.10  | -1.30  | 0.00    | 0.00       | 0.0, 0.0, 0.0 | -0.088 |
| $^{74}$Se | 3     | 0.888 | -0.086   | -0.180 | -1.10  | -1.06   | 0.00, -0.06 | 0.0, 0.0, 0.0 | -0.086 |
| $^{76}$Kr | 4     | 1.000 | -0.088   | -1.00  | -0.98  | 0.00    | 0.00       | 0.0, 0.0, 0.0 | -0.084 |

Starting with table 1: it might make watched that those parameter of bosons energy ($\epsilon$) will be diminishing over worth when proton numbers try dependent upon mid shell (N=39) et cetera it increments again, this implies that those bosons energy is proportional of the energy level particularly for those state $2^+_1$ by the bosons in the mid-shell need least energies Since they need aid under those impact of the two shells (28-50). The ($\kappa$) parameter declines proportionally with the increment of the protons amount contingent upon the collaboration quality of the electric quadruple to protons What’s more neutrons. Those ($\chi_\nu$) will be the deformity parameter for protons and the qualities would continuously expanding linearly with the expanding protons numbers. The qualities for majorana parameters terms ($\xi_1=3$, $\xi_2$) would chose for certain qualities Also are movable with the energy about mixed-symmetry states (MSS) for some energized vitality levels. Figure 1: indicates the IBM-2 Parameters Likewise An work of the protons numbers to every one isotones investigated
Figure 1. IBM-2 parameters ($\varepsilon$, $\kappa$, $\chi_\pi$, $\chi_\nu$, $\zeta_2$, $\zeta_{1,3}$) for isotones studied as a function of the protons numbers

The best fit to the excited energy levels has been observed specially for the ground band (i.e. the states $2^+_1$, $4^+_1$, and $6^+_1$), while the second and third bands are found to show acceptable agreement. These bands have some properties of the $\beta$ and $\gamma$ band so they are called (quasi beta and gamma band). All of the momentum and parity for some energy levels that had not been previously identified experimentally for each isotopes are identified throughout this study. Figure 2. hint at a correlation the middle of test vitality levels [7,8,9,10] and ascertained values for at isotones contemplated.
Figure 2. Comparison between calculated energy level and the available experimental data

\[ A = ^{70}\text{Zn}, \quad B = ^{72}\text{Ge}, \quad C = ^{74}\text{Se} \text{ and } D = ^{76}\text{Kr}. \]

The ratios \( \frac{4^+_1}{2^+_1} \) and \( \frac{6^+_1}{2^+_1} \) are the key of this problem, where this ratio is equal to 2, 2.5 and 3.3 for U(5), O(6) and SU(3) respectively. Figure 3 shows the \( \frac{4^+_1}{2^+_1} \) and \( \frac{6^+_1}{2^+_1} \) ratios for \(^{70}\text{Zn}, ^{72}\text{Ge}, ^{74}\text{Se} \) and \(^{76}\text{Kr} \) isotones as a function of protons number which indicates that these isotones have a varied dynamic symmetries started from rotational motion U(5) least deformed in \(^{70}\text{Zn} \) with increasing deformation of these isotones with increases protons numbers to become transition nuclei between U(5)-O(6) symmetries for \(^{72}\text{Ge} \), and \(^{74}\text{Se} \) respectively, and finally more closely to O(6) symmetry group for \(^{76}\text{Kr} \) isotones. Figure 3: the experimental \( \frac{E4^+_1}{E2^+_1} \) and \( \frac{E6^+_1}{E2^+_1} \) in comparison with IBM-2 result and IBM-1 limits.

Figure 3. The experimental A= \( \frac{E4^+_1}{E2^+_1} \) and B= \( \frac{E6^+_1}{E2^+_1} \) in comparison with IBM-2 result and IBM-1 limits.
4. Mixed symmetry state:

Studying the influence of Majorana parameters ($\zeta_{1,3}$, $\zeta_2$) on the excitation energy level for $^{70}$Zn, $^{72}$Ge, $^{74}$Se and $^{76}$Kr isotones, we fixed the value of $\zeta_{1,3} = +0.05$ for all isotones and vary the $\zeta_2$ between (-0.090, -0.084) around the best-fitted to experimental data. It is found that the energy values for the states $(2^+_5, 3^+_1, 5^+_1, 1^+_1)$ are responded rapidly to the changes of the $\zeta_2$ parameters this property called Mixed symmetry state (MSS). Figures 4: explain the energy variation of these states as a function of the Majorana parameter $\zeta_2$.

**Figure 4.** The energy states variation as a function of the Majorana parameter $\zeta_2$. A=$^{70}$Zn, B=$^{72}$Ge, C=$^{74}$Se and D=$^{76}$Kr
5. Electric Transition

The E2 transitions calculated using the E2 operator is defined as:[7]

\[ T^{(E2)} = e_\pi Q_\pi + Q_\nu e_\nu \]  

(5)

The Q_\pi and Q_\nu operators are defined in equation (2), e_\pi and e_\nu are boson effective charges for protons and neutrons, depending on the boson number N_\nu, these parameters are free and can take any value to fit the experimental data. In present work the effective charge of proton e_\pi = 0.102(eb) and neutron e_\nu = -0.105 (eb). The reduced electric quadruple probabilities B(E2) is very sensitive to effective charge for proton and neutron. The B(E2) of 70Zn, 72Ge, 74Se, 76Kr isotones are shown in the Table 2. It was found that the good agreement between available experimental data and IBM-2 results [7,8,9,10].

| Transition | 70Zn | 72Ge | 74Se | 76Kr |
|------------|------|------|------|------|
| 2_1^+ → 0^+ | 0.0284 | 0.0285 | 0.0419 | 0.0419 | 0.0775 | 0.0774 | 0.1650 | 0.1658 |
| 2_2^+ → 0^+ | 0.0043 | 0.0345 | 0.0002 | 0.0002 | 0.0015 | 0.0002 | 0.0089 | 0.0026 |
| 0_2^+ → 2_1^+ | 0.0639 | 0.0137 | ---- | 0.0159 | 0.1421 | 0.1145 | ---- | 0.1499 |
| 2_2^+ → 2_1^+ | 0.1182 | 0.1112 | 0.1103 | 0.0018 | 0.0888 | 0.0299 | 0.0038 | 0.0137 |
| 2_1^+ → 2_1^+ | ---- | 0.0220 | ---- | 0.0035 | 0.0523 | ---- | ---- | 0.0187 |
| 2_1^+ → 2_1^+ | ---- | 0.0062 | ---- | 0.0247 | 0.0076 | ---- | ---- | 0.0088 |
| 2_1^+ → 2_1^+ | 0.0075 | 0.0002 | ---- | 0.0352 | 0.0074 | ---- | ---- | 0.0008 |
| 3_1^+ → 2_1^+ | ---- | 0.0399 | ---- | 0.0147 | 0.0199 | 0.0296 | 0.00191 | 0.0038 |
| 4_1^+ → 2_1 | ---- | 0.1046 | ---- | 0.1489 | 0.1476 | 0.1422 | ---- | 0.2745 |
| 5_1^+ → 4_1^+ | ---- | 0.0242 | ---- | 0.0070 | ---- | 0.0018 | 0.0058 | 0.0015 |
| 6_1^+ → 4_1^+ | ---- | 0.1977 | ---- | 0.2749 | 0.1331 | 0.1756 | 0.1794 | 0.3256 |

6. Magnetic transition

In order to calculate M1 transition probability must be estimate the effective g-factor for proton and neutron, through used Samba taro relation which can be written as: [11,12]:

\[ g = (g_\pi N_\pi + g_\nu N_\nu) / (N_\pi + N_\nu) \]  

(6)

where g_\pi, g_\nu are g-factor of nuclear proton and neutron respectively. The total g-factor associated with magnetic momentum, is \( \mu = 2g \), in this work we used the experimental value of magnetic momentum for the 2^+_1 state to estimate the g factor, where \( \mu(2^+_1) = 2g(2^+_1) \), it was found that the predicted values are g_\pi = 0.552 (\mu N) and g_\nu = 0.58 (\mu N), and (g_\pi - g_\nu) = 0.371 (\mu N). The M1 operator is obtained by making 1 = 1 in the single boson operator of the IBM-2 and can be written as [13,14].

\[ T^{(M1)} = 0.77 \left[ (d^+_1 d^-_1)^{(1)}_\pi - (d^+_1 d^-_1)^{(1)}_\nu \right] (g_\pi - g_\nu) \]  

(7)

The reduced magnetic dipole Probabilities B(M1) 70Zn, 72Ge, 74Se, 76Kr isotones are calculated and presented Table 3.
Table 3. The B(M1) transition of $^{70}$Zn, $^{72}$Ge, $^{74}$Se, $^{76}$Kr isotopes, units ($\mu$N)$^2$

| Transition   | $^{70}$Zn | $^{72}$Ge | $^{74}$Se | $^{76}$Kr |
|--------------|-----------|-----------|-----------|-----------|
| $1^+_1 \rightarrow 0^+_1$ | Exp. | IBM-2 | Exp. | IBM-2 | Exp. | IBM-2 |
|               | ---- | 0.000222 | ---- | 0.00031 | ---- | 0.00042 | ---- | 0.00134 |
| $2^+_2 \rightarrow 2^+_1$ | 0.000103 | 0.000272 | 0.000003 | 0.00029 | 0.0000079 | 0.000037 | 0.000046 | 0.000013 |
| $3^+_2 \rightarrow 2^+_1$ | ---- | 0.000411 | ---- | 0.00112 | ---- | 0.00049 | ---- | 0.00022 |
| $4^+_2 \rightarrow 2^+_1$ | ---- | 0.00073 | ---- | 0.00006 | ---- | 0.00128 | ---- | 0.00283 |
| $3^+_1 \rightarrow 2^+_1$ | 0.000311 | 0.000007 | 0.000097 | 0.00117 | ---- | 0.00005 | ---- | 0.00005 |
| $5^+_1 \rightarrow 4^+_1$ | 0.000082 | 0.000111 | 0.000054 | 0.000032 | ---- | 0.000012 | ---- | 0.000012 |

7. Mixing Ratios $\delta$ (E2/M1)

After calculating the matrix element of $B(E2)$ and $B(M1)$ of gamma transition it is conceivable to look at the strength of E2 and M1 transition in terms of the Multipole mixing ratio ($\delta$) which can be written as follows: [15,16].

$$\delta(E2/M1) = 0.835 \text{ E}_\gamma \Delta(E2/M1)$$  \hspace{1cm} (8)

$E_\gamma$ the transition energy between the two states units (MeV) and $\Delta(E2/M1)$ is the ratio between reduced matrix element for E2 and M1 transition which can be written as [17,18]

$$\Delta \frac{(E2/M1)}{\text{E}_\gamma} = |\langle I_f | T(E2) | I_i \rangle \rangle^2 |\langle I_f | T(M1) | I_i \rangle \rangle^2|$$  \hspace{1cm} (9)

In Table (4) we presented the values of the mixing ratio for $^{70}$Zn, $^{72}$Ge, $^{74}$Se, $^{76}$Kr isotones in comparison with the available experimental values[7,8,9,10]. It is seen that both the magnitude and sign of $\delta$ are correctly obtained for most of the selected isotones.

Table 4. Mixing ratio (eb/$\mu$N) for $^{70}$Zn, $^{72}$Ge, $^{74}$Se, $^{76}$Kr isotones

| Nucleus | Transition | $E_\gamma$ (MeV) | Exp. | IBM-2 |
|---------|------------|-----------------|------|-------|
| $^{70}$Zn | $2^+_1 \rightarrow 2^+_1$ | 0.8743 | +3.8 | +1.54 |
|         | $2^+_1 \rightarrow 2^+_1$ | 1.0728 | +22.71 |
|         | $2^+_1 \rightarrow 2^+_1$ | 1.4911 | +13.15 |
| $2^+_1 \rightarrow 2^+_1$ | 1.5634 | +0.69 |
| $5^+_1 \rightarrow 4^+_1$ | 1.8120 | -5.11 |
| $^{72}$Ge | $2^+_1 \rightarrow 2^+_1$ | 0.6299 | -10.31 | -10.25 |
|         | $2^+_1 \rightarrow 2^+_1$ | 1.5683 | -1.91 |
|         | $2^+_1 \rightarrow 2^+_1$ | 2.2002 | -11.86 |
|         | $3^+_1 \rightarrow 2^+_1$ | 1.2309 | -13.84 |
| $5^+_1 \rightarrow 4^+_1$ | 1.1855 | -8.36 |
| $^{74}$Se | $2^+_1 \rightarrow 2^+_1$ | 0.6343 | -5.62 | -5.47 |
|         | $2^+_1 \rightarrow 2^+_1$ | 1.2039 | -14.87 |
|         | $2^+_1 \rightarrow 2^+_1$ | 1.6793 | +1.68 |
### Conclusion

In present work we have studied some microscopic properties for $^{70}$Zn, $^{72}$Ge, $^{74}$Se, $^{76}$Kr, isotones and determination some excited energy levels by regulate a small number of parameters in Hamiltonian operators and estimated these parameters, to give the best fit for the energy levels specially for the ground state bands compared with experimental data, while other bands (quasi $\gamma$ and $\beta$ band) gave acceptable results because of existence some levels that are undergo mixed symmetry states property (MSS) when rise or drop some energy states compared with corresponding experimental values, due to the mixing wave functions between protons and neutrons, these states are $(2^+_5, 3^+_1, 5^+_1, 1^+_1)$ gets a quick response when changing majorana term $\zeta^2$ as a result of mixture. Moreover it has been calculated energy ratios $4^+_1/2^+_1$ and $6^+_1/2^+_1$, it was found that these isotones have a varied dynamic symmetries started from rotational motion $U(5)$ least deformed in $^{70}$Zn with increasing deformation of these isotones with increases protons numbers to become transition nuclei between $U(5)$-O(6) symmetries for $^{72}$Ge, and $^{74}$Se respectively, and finally more closely to O(6) symmetry group for $^{76}$Kr isotones. However have been calculated the probability for electric quadrupole transition B(E2), magnetic dipole B(M1) and mixing ratios which All express a good Agreement compared with experimental data.

### References

[1] Olaf Scholten, "The interacting boson model approximation and application", Nucl. Phys. 14, 189 (1985).
[2] Walter Pfeifer, "An Introduction to the Interacting Boson Model of the Atomic Nucleus", 1998.
[3] Zhan Z., JieY., Xin W., YongL., Theo. Phys., 2001, 35(60), 711.
[4] Ramos J., HeydeK., RobledoL., GuzmanR., Phys. Rev., 2014, C 89, 34313.
[5] Ramos J., HeydeK., Nucl. Phys., 2009, 825, 39.
[6] Morales I., Frank A., Vargas C., Isacker P., Phys. Rev., 2008, C78, 24303.
[7] J. K. Tuli Pekerand, Nuclear Data Sheets for A = 70, 103, (2004), 389–514
[8] D. Abriola, A.A. Sonzogni, Nuclear Data Sheets for A = 72, 111, (2010), 1–140
[9] Balraj Singh, Ameenahr. Farhan, Nuclear Data Sheets for A = 74, 107, (2006), 1923–2102.
[10] M. J. Martin, Nuclear Data Sheets for A = 152*, sciencedirect., 2013, 114, 1197–1487.
[11] AboodS., NajimL., Advances in Applied Science Research, 2013, 4(1), 444.
[12] Al-KhudairF., Jin Z., Hong B., Chinese Phys., 2009, 33(1), 46.
[13] Al-Khudair F., Subber A., Jaafer A., J. Basra Science, 2014, 40 (1), 46.

[14] Jin Z., Al-Khudair F., Gui L., Jiang Z., Dong R., Theo. Phys., 2002, 37(3), 335.

[15] Al-Khudair F., Gui L., Theo. Phys., 2005, 33(6), 677.

[16] Coquard L., Ph.D. Thesis, Darmstadt University, Germany, 2010, pp 19.

[17] Young S., Lee L., Korean Phys., 2006, 49(2), 501.

[18] Subber A., Al-Khudair F., "δ(E2/M1) and X(E0/E2) mixing ratios in 134Ba by means of IBM-2", Turk. J. Phys., 36, 368-376, 2012.