Vlasov-Fokker-Planck simulations of fast-electron transport with hydrodynamic plasma response

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Abstract. We report on kinetic simulations of the transport of laser-produced relativistic electron beams (REB) through solid-density plasma, including the hydrodynamic response of the plasma. We consider REBs with parameters relevant to fast-ignition of compressed inertial confinement fusion capsules. We show that over the 10–20ps timescales required for fast-ignition, thermal pressure (from Ohmic heating) can significantly modify the density which in turn strongly affects the propagation of injected fast-electrons; it allows them to re-collimate into a narrow, intense beam under conditions where they initially undergo beam-hollowing. Similar static-density calculations do not show re-collimation. The re-collimation effect is attributed to PdV cooling in the pressure-induced density-channel, which in turn suppresses defocusing magnetic fields generated by resistivity gradients. These simulations have been carried out using the new 2D-3V Vlasov-Fokker-Planck (VFP) code FIDO running in hybrid mode.

1. Introduction
The generation of energetic electron-beams during the interaction of ultra-intense laser pulses with solid targets, and the subsequent propagation of these beams into the target has received a lot of interest, both experimental and theoretical, in the last 15 years. This interest stems in large part from Tabak’s proposed “fast-ignition” concept for inertial confinement fusion (ICF) [1]. The HiPER project, currently in the preparatory phase, is a prime example of a laser-facility designed to achieve inertial fusion energy via fast ignition. To date, all experimental and most theoretical work on fast-electron transport has considered timescales of up to about 1ps. This is too short for solid-density plasma to move – ions are essentially frozen – and much shorter than the 10–20ps duration of a viable fast-ignition laser. On these 0.1–1ps timescales, signatures of collimated propagation [2] and propagation in an annular ring [3] have been observed under some conditions. A criterion for “resistive collimation” of an electron beam was proposed in [4] and an explanation of beam-hollowing (e.g. annular ring) can be found in [5]. Both used kinetic simulations to help tease out the underlying mechanisms. The basic description of these resistive phenomena is that the collisionless fast-electron current $j_f$ is balanced almost exactly by a return current of cold, collisional background electrons, $j_c$. Using a basic Ohm’s law, the E-field is $E = \eta j_c \approx -\eta j_f$ (where $\eta$ is resistivity) which yields a B-field induction equation $\partial B/\partial t = \eta (\nabla \times j_f) + \nabla \eta \times j_f$, with the first & second terms giving rise to collimating and beam-hollowing type B-fields, respectively. Results from 30ps duration, hybrid simulations of fast-electrons propagating into an imploded DT fuel core have recently been reported [6]. They include hydrodynamic response and show that expansion of the core is not important during energy deposition, but do not consider the gold cone-tip.

Back of the envelope calculations hint that significant cavitation of the plasma - through which the REB propagates - could occur over the 10–20ps beam duration required for ignition. Previously, the effect of static density-modulations of the plasma has been considered [7]. In this paper, we consider the effect of plasma density evolution on REB propagation. We show, for the first time, that density changes in the solid target, caused by the REB, greatly affect its propagation on timescales exceeding $\sim 5ps$.

The requirements for fast-ignition [8] are currently thought to be an electron-beam energy of at least 25kJ, duration 10–20ps, radius $\sim 20\mu m$ and electron kinetic energy of $T_f \approx 0.3–2 \text{MeV}$. A suitable laser needs an intensity $I \approx 2 \times 10^{20} \text{W/cm}^2$ at wavelength $\lambda_p = 0.53\mu m$ and power $P_l \approx 3 \text{PW}$ assuming a conversion efficiency (to fast electrons) of $\eta_{abs} \approx 0.3$. Ohmic heating of a solid-density target by the high current of fast electrons under these conditions can easily raise the temperature of the background electrons by several keV in a picosecond, in the vicinity of...
beam. We estimate a temperature evolution of $T_e \approx 12(t/\text{ps})^{2/5} \text{keV}$ based on [4] (eqn. on pg. 2) at solid density $n_e = 10^{24}\text{cm}^{-3}$, $Z\ln \Lambda = 40$ for a beam with $P_{abs} = 1\text{PW}$, $T_f = 1\text{MeV}$ and radius $20\mu$m. The strong pressure gradients in turn cause density modulations to develop on the timescale of the sound-speed transit time across the heated region, typically a few picoseconds. Linearising the fluid continuity equation and momentum equation, subject to a prescribed, fixed, temperature perturbation ($T_c = T_0 + \delta T \cos(2\pi x/\lambda)$) and assuming an initially uniform density, it is straightforward to estimate the density perturbation driven up by the pressure modulation; $\delta \rho/\rho_o = 2\pi^2 (\Delta C_s t/\lambda)^2 \approx 3.3(Z/A)\delta T_{keV} t_{ps}^2/\lambda_{\mu m}^2$ where $\Delta C_s$ is the sound-speed associated with the overheat $\delta T$. Hence the time for 10% modulation is estimated to be 1.1ps for a gold plasma (assuming $Z = 30$) with $\delta T = 4\text{keV}$ at $\lambda = 5\mu m$, for example.

2. Simulation details

Calculations were carried out using FIDO which solves the relativistic Vlasov-Fokker-Planck equation for electrons, together with Maxwell’s equations on a 2D Cartesian spatial grid. The momentum space for the electron distribution function (EDF) is discretized on a spherical polar grid, as in [9], so $f_p(p_x,p_y,p_z,x,y,t)$ and the fields $E_x$, $E_y$ and $B_z$ are included. FIDO predominantly uses explicit finite-difference methods for advancing the EDF but e-i and e-e collisions are implicit. FIDO can run in either full-kinetic or hybrid mode. The results reported here used hybrid mode where only the injected fast-electrons are treated kinetically. Magneto-hydrodynamic motion of the ion-fluid (background-plasma fluid) in full-kinetic (hybrid) mode in response to thermal & magnetic pressure-gradients is implemented using a method adapted from Ziegler [10]. FIDO incorporates a suite of EM-field solvers; an implicit moment method was used here. Spitzer resistivity and ideal-gas electronic heat-capacity are used in hybrid mode.

The “base” simulation consists of an initially uniform plasma – $T_{eo} = 100\text{eV}$ and $n_{eo} = 1.2 \times 10^{20}\text{cm}^{-3}$ – with $Z\ln \Lambda = 40$ and $m_i = 6m_P$. A laser of peak intensity $I = 5 \times 10^{19}\text{W/cm}^2$ and Gaussian transverse profile of $1/e$ width $R_s = 3\mu m$ ($P \approx 14\text{TW}$) is incident onto this plasma. Fast-electrons are injected along the bottom edge of the domain, of size $30 \times 30$ microns, in the +ve (positive) $y$-direction with an energy distribution characterised by $T_f = 1.8\text{MeV}$ and a Gaussian angular distribution with $1/e$ half-cone angle $\theta_{inj} = 28^\circ$. The spatial profile of the injected energy flux mimics the laser radial profile and an absorption efficiency of $n_{abs} = 0.3$ is used. ($T_f$ is kept the same across the injection region.) The spatial grid resolution is $\Delta x = \Delta y = 0.2\mu m$ and open boundary conditions are used. The momentum grid uses $np = 50$ ($p_{max} = 22m_e c$) $n\theta = 1(-5)$ and $n\phi = 96$. (Typically $n\theta = 1$ is sufficient to give good results since $B_z$ rotates particles in $\phi$.)

3. Results

Initially the injected electrons ($\theta_{inj} = 28^\circ$) collimate within 100fs, under the action of the resistive $\eta(\nabla \times \mathbf{j}_f)$ B-field source. This is consistent with the self-collimation criterion, $\Gamma > 1$, given in [4], where $\Gamma = R_s/\langle r_g \theta_{inj}^2 \rangle$ and $r_g$ is the electron gyro-radius in the self-generated B-field. (A full expression for $\Gamma$ in terms of density, $Z$, electron energy, etc., is given in [4].) Under our conditions $\Gamma \geq 1$ for $t \geq 40$fs. By 1ps, the beam hollows beyond $10\mu m$ into the target, consistent with eqn. (13) in [5] which suggests the onset of beam-hollowing in about 0.5ps for a $5 \times 10^{19}\text{W/cm}^2$ laser and the conditions considered here. With hydrodynamic background plasma response included, an intense, narrow beam of fast-electrons has already re-formed on axis by 5ps as shown in figure 1(a). By 10ps, virtually all the injected electrons are funnelled down this central filament [fig. 1(b)]. Formation of this central filament does not occur when the background density is static (no hydro); instead a hollowed beam persists, as shown in figure 1(c). At 5ps the peak plasma temperature is $T_e \approx 2.2\text{keV}$ and fig. 1(d) shows that pressure gradients under the REB have already significantly cavitated the background plasma density by up to 40%.
Figure 1. Re-emergence of central REB – beam density $n_f \text{[m}^{-3}\text{]}$ (a)–(c) & (e) ; % plasma density change $\delta \rho/\rho_o$ (d). 14TW, $R_s = 3 \mu m$ laser: (a) 5ps, (b) 10ps, (c) 10ps without hydro, (d) 5ps. 3PW, $R_s = 20 \mu m$ + gold: (e) 20ps. [Hydro is used unless otherwise indicated.]

This phenomenon - formation of a strong, central filament when hydrodynamic density changes are included - has been seen under other conditions, including tighter injection ($\theta_{inj} = 7^\circ$) with lower background charge state ($Z \ln \Lambda = 4$). The possibility that the central filament is purely a numerical phenomenon has been ruled out by tilting the direction of the injected electrons by $20^\circ$. The central filament still occurs and only when hydro is included. The emergence of the tightly collimated REB is also seen under conditions needed for fast-ignition. Figure 1(e) shows the fast-electron density at 20ps with hydrodynamic background-plasma response included. There is virtually no beam without hydro. The conditions here are $R_s = 20 \mu m$, $Z \ln \Lambda = 70$, $T_{eo} = 200eV$, $n_{eo} = 2.1 \times 10^{24} \text{cm}^{-3}$, $I = 2 \times 10^{20} \text{W/cm}^2$, $\theta_{inj} = 22^\circ$ and $\Delta x = \Delta y = 1 \mu m$.

4. Explanation of refocusing mechanism

The emergence of the central filament once density cavitation develops is linked with PdV cooling of the expanding plasma in the cavity. This in turn permits suppression of the resistivity-gradient driven decollimating/hollowing B-field source $\nabla \eta \times j_f$ and re-establishment of the dominance of the focusing $\eta(\nabla \times j_f)$ resistive source. With a static density profile and no thermal conduction, a rigid beam model of resistively driven B-field generation in an Ohmically-heated plasma predicts that the two sources largely cancel each other out leaving a small net defocusing $\dot{B}$ in the beam body. This is seen in our hybrid-VFP “base” simulations with static density [fig. 2(a)]. With hydro included, this delicate balance is disrupted. The PdV term in the electron-fluid energy equation contributes to evolving the temperature and therefore influences the resistivity profile (since $\eta \propto T_e^{-3/2}$ assuming Spitzer). In the limit where PdV effects dominate, the temperature evolves adiabatically (ignoring thermal conduction) so that $n_e/T_e^{3/2} = \text{const}$. which acts to reduce the temperature under the REB, with the effect initially scaling as $\delta T_{\text{PdV}}/T \propto \delta n/n \propto -t^2$. Figure 2(b) shows the modulated density & temperature...
profiles across the plasma at \( y = 2.5\, \mu m \), \( t = 5ps \) which clearly shows the lowering & flattening of \( T_e(x) \) by PdV effects. The flatter temperature profile in turn decreases \( \nabla \eta \) so that \( \partial_x \eta \) is reduced from 0.03 to 0.018Ω at \( x = 3.5\, \mu m \). This ensures that with hydro, the net B-field source generates a collimating B-field across the body of the REB. As shown in fig. 2(c), the beam-hollowing field [-ve (negative) \( B_z \)] is completely removed by 5ps, replaced with a \( \sim 100 \, T \) collimating field in the region \( 0 \leq x \leq 3\, \mu m \). It is thought that this field focuses the bulk of the injected electrons into the tight beam seen further into the plasma. FIDO currently neglects thermal conduction in the background in hybrid mode. We estimate that thermal conduction has less effect than PdV effects for transverse features of wavelength exceeding about 10\( \mu m \). Hence thermal conduction should not greatly affect the temperature dynamics in the crucial beam-focusing region just beyond the injection region. If present, it should further flatten the temperature profile thereby helping to inhibit beam-hollowing.

**Figure 2.** B-field sources + PdV effect for 14TW laser – (a) \( \nabla \eta \times j_f \) (-ve) and \( \nabla \times j_f \) (+ve) sources, (b) plasma density & elec. temp., and (c) B-field. All at \( y = 2.5\, \mu m \), \( t = 5ps \). Solid (dashed with • markers) curves are with (without) hydro.

5. Conclusions

We have directly seen the influence of hydrodynamic density evolution on the propagation of intense 10–20ps duration REBs in solid targets. It permits the reversal of beam-hollowing and re-collimation of the beam after some picoseconds. Pressure gradients in the Ohmically heated background plasma significantly modify the density within \( \sim 5ps \). Lowering of the plasma density just beyond the fast-electron injection point is accompanied by a PdV cooling effect which is sufficient to inhibit and reverse the formation of beam-hollowing fields. Whether or not this re-collimation mechanism persists into the highly compressed core of a fast-ignition target needs to be addressed. The results indicate that hydrodynamic motion must be included when addressing fast-electron transport through solid-density regions of a fast-ignition target.

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