Drift waves with dust acoustic wave coupling

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Abstract

Drift wave is a prominent mode of a magnetized plasma of inhomogeneous density. It plays an important role in the transport of particles, energy and momentum perpendicular to the ambient magnetic field. The frequency of this mode is governed by the inhomogeneity scale length and is much lower than the typical homogeneous plasma modes involving ions and electrons. In this work the possibility of coupling of this particular mode with the low frequency modes of a dusty plasma medium is considered.

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The study of low frequency drift modes (frequency much lower than ion cyclotron frequency) has received widespread attention [1–6]. These waves are excited in the presence of plasma density inhomogeneity in a magnetized plasma and are responsible for transport in such a system [7]. In this work we seek a possible coupling of this low frequency mode with the modes of a dusty plasma medium [8–16]. As is well known the dusty plasma has, in addition to electron and ions, a third heavier dust species which typically gets negatively charged by the attachment of electron species from the background plasma. The presence of third heavier dust species in plasma leads to additional low frequencies modes in the system. These low frequency modes of the dusty plasma medium can couple with the low frequency drift wave modes under appropriate parameter regime. This paper explores such a possibility.

A classical Coulomb plasma with micron sized dust particles can lie in strongly coupled regime [17] because the coupling parameter $\Gamma = \frac{(Z_d e)^2}{4\pi\epsilon_0 a T_d}$ can easily be of the order 1 or larger ($Z_d e$ is the charge on the dust grain, $a$ is the inter-grain distance and $T_d$ is the dust temperature). The dust species in the plasma acquires a high charge ($Q \sim 1000e$) where $e$ is the electronic charge and has a very low temperature ($T_d \sim 0.1eV$) due to which it can be found in a strongly coupled regime. Recent experiments also confirm that dust particles can form crystal structures [18, 19]. There are many approximate models to represent the dynamics of strongly coupled fluid. One such model is Generalised Hydrodynamic (GHD) model, first reported by Berkovsky [20] for the electron-ion plasma where ion fluid was treated as strongly coupled. This model was later adopted by Kaw and Sen [21] in dusty plasma to study low frequency mode characteristics when dust particles are strongly coupled. This model predicted the existence of transverse shear mode in strongly coupled dusty plasma which has been later confirmed in many simulations [22, 23] as well as experiments [24]. The GHD model will be adopted by us here to depict the dynamics of dust species.

The manuscript has been organized as follows. Section II describes a model set of
equations for the coupling of drift waves to the dust acoustic wave. In section III, a
dispersion relation has been obtained. The dust species is treated both as a weakly and
strongly correlated fluid system. In section IV we numerically plot the dispersion relation
and discuss the effect of coupling of the drift wave with the dust species. Section V
contains the summary.

II. MODEL EQUATIONS FOR DRIFT WAVES IN DUSTY PLASMA

![Schematic of system configuration for the coupling of drift wave with dust acoustic wave](image)

FIG. 1: Schematics of system configuration for the coupling of drift wave with dust acoustic wave

We consider the plasma medium in a slab geometry, schematics has been depicted in
Fig.1. The applied external magnetic field is $\vec{B}$ is directed along $\hat{z}$ axis. The magnetic
field strength is such that while the lighter electron and ion species are magnetized, the
dust species remains unmagnetized. The plasma density inhomogeneity $\nabla n$ is chosen to
be along $\hat{x}$. The suffix $s = i, e, d$ refers to the ion, electron and dust species of the plasma
respectively. All the three species are assumed to have an equilibrium density profile
which is inhomogeneous and has an exponential form given by the expression:

$$n_{0s}(x) = n_{00s} \exp\left(-\frac{x}{L_n}\right)$$  \hspace{1cm} (1)
In equilibrium the charge balance is provided by the condition.

\[ n_{0i} = n_{0e} + Z_d n_{0d} \]  \hspace{1cm} (2)

For drift wave like perturbations it is assumed that the wavelength perpendicular to the magnetic field is much smaller than the parallel wavelength. The lighter electron species provides a balance of electric and pressure forces in the parallel direction to yield the following Boltzmann relationship for the electron density perturbations:

\[ n_{e1} = n_{e0} \exp \left( \frac{e\phi}{T_e} \right) \]  \hspace{1cm} (3)

Here, \( T_e \) corresponds to the electron temperature and \( \phi \) is the electrostatic potential. Similarly, we denote ion and dust temperatures by \( T_i \) and \( T_d \) respectively.

The ion continuity equation is

\[ \frac{\partial n_i}{\partial t} + \vec{v}_{i\perp} \cdot \vec{\nabla} n_i + \vec{v}_{iz} \cdot \vec{\nabla} n_i = 0 \]  \hspace{1cm} (4)

The perpendicular ion momentum equation yields \( v_{i\perp} \) the ion drift velocity. Using the low frequency drift approximation wherein \( \omega/\omega_{ci} \ll 1 \) we retain the \( \vec{E} \times \vec{B} \) and polarization drift velocities for the ions

\[ \vec{v}_{i\perp} = \frac{\dot{z} \times \vec{\nabla} \phi}{B} + \frac{1}{\omega_{ci} B} \left[ -\frac{\partial}{\partial t} (\vec{\nabla} \perp \phi) - \frac{\dot{z} \times \vec{\nabla} \phi}{B} \cdot \vec{\nabla} \phi \right] \]  \hspace{1cm} (5)

Here the first term \( \vec{V}_E = \dot{z} \times \vec{\nabla} \phi / B \) is the \( \vec{E} \times \vec{B} \) drift velocity of ions and the second term in the polarization drift velocity \( \vec{V}_p \) of the ions. We now substitute the expression of \( \vec{v}_{i\perp} \) in ion continuity equation and employ the quasineutrality condition of \( Z_d n_{d1} + n_{e1} = n_{i1} \) to eliminate ion density perturbation. Here \( n_{d1}, n_{e1}, n_{i1} \) are the perturbed densities for dust, electrons and ions respectively. Furthermore, the electron density perturbation \( n_{e1} \), is replaced by the scalar potential using the linearized Boltzmann relation provided by Eq.(3). One thus obtains

\[ \frac{\partial}{\partial t} \left( n_{e0} \frac{e\phi}{T_e} + Z_d n_{d1} \right) + n_{i0} \left[ \vec{\nabla} \perp \cdot \left( \frac{1}{\omega_{ci} B} \frac{\partial}{\partial t} (\vec{\nabla} \perp \phi) \right) - \vec{\nabla} \perp \cdot \left( \frac{1}{\omega_{ci} B} \vec{V}_E \cdot \vec{\nabla} \perp (\nabla^2 \phi) \right) \right] + \vec{V}_E \cdot \vec{\nabla} n_{i0} + n_{i0} \vec{\nabla} \cdot \vec{v}_{iz} = 0 \]  \hspace{1cm} (6)

Here the smallness of polarization drift \( \vec{V}_p \) by \( \omega/\omega_{ci} \) compared to \( \vec{E} \times \vec{B} \) has been considered. Rearranging the above equation one obtains an expression as:

\[ Z_d \frac{\partial}{\partial t} (n_{d1}) + \frac{\partial}{\partial t} \left[ n_{e0} \frac{e\phi}{T_e} - \frac{n_{i0} \nabla^2 \phi}{\omega_{ci} B} \right] + \frac{\dot{z} \times \vec{\nabla} \phi}{B} \cdot \vec{\nabla} \left[ \ln(n_{i0}) - \frac{1}{\omega_{ci} B} \nabla^2 \phi \right] + n_{i0} \vec{\nabla} \cdot \vec{v}_{iz} = 0 \]  \hspace{1cm} (7)
Eq. (7) is a modified equation for the drift wave dynamics in the presence of dust species. The last term represents the ion motion in the direction parallel to the ambient magnetic field which is often ignored in the usual drift wave treatment as variations parallel to the magnetic field is assumed to be much smaller. The first term in Eq. (7) is dependent on the dust density and represents the coupling with the dust dynamics. The ion dynamics in the direction parallel to the ambient magnetic field is written as:

\[
\frac{d\vec{v}_{iz}}{dt} = -\frac{e}{m_i} \vec{\nabla} \phi \tag{8}
\]

where \( k_z \ll k_\perp \) has been considered.

The dust density is provided by the continuity equation:

\[
\frac{\partial}{\partial t}(n_d) + \vec{\nabla} \cdot (\vec{v}_d n_d) = 0 \tag{9}
\]

The dust velocity is determined by the momentum equation for which we use the generalized hydrodynamic equation representing the visco-elastic behaviour of the dust fluid in the strong coupling limit

\[
\left[ 1 + \tau_m \frac{d}{dt} \right] \left[ \frac{d\vec{v}_d}{dt} - Z_d e n_d \vec{v}_d \right] = \eta \vec{\nabla}^2 \vec{v}_d \tag{10}
\]

The parameter \( \tau_m \) represent the elastic relaxation time corresponding to the memory associated with the strong coupling limit and \( \eta \) is the viscosity parameter. It can be observed that for a physical phenomenon lying in the frequency domain of \( \omega \tau_m \ll 1 \) the dust fluid behaves like an ordinary hydrodynamic fluid. However, viscoelastic effects associated with strong coupling manifest in the limit of \( \omega \tau_m \gtrsim 1 \). It should also be noted that we have considered the strength of applied magnetic field such that the dust response remains unmagnetized. We thus have Eqs. (7, 9, 10) as the complete set of equations representing the coupling of drift wave with the dust acoustic waves.

In the next section, we obtain the linear dispersion relation of the above set of equations in both limits of weakly and strongly coupled response of the dust fluid.

### III. LINEAR DISPERSION RELATION OF DRIFT WAVE COUPLED WITH DUST ACOUSTIC WAVE

Linearizing Eq. (7) yields

\[
Z_d n_d \frac{d}{dt} \left[ n e_0 \frac{e\phi}{T_e} + \frac{n_i_0}{\omega c_i B} k_\perp^2 \phi \right] + \frac{k_y \phi}{\omega} \nabla (n_i_0) - \frac{n_i_0 e k_z v_{iz}}{\omega} = 0 \tag{11}
\]
Here $k_2^2 = k_x^2 + k_y^2$ is the perpendicular wave number and $k_x, k_y$ are the wavenumbers in the $\hat{x}$ and $\hat{y}$ directions respectively. The ion momentum equation (Eq. (8)) in parallel direction can be linearized as:

$$v_{iz} = \frac{ek_z\phi}{m_i\omega}$$

where $k_z$ is the parallel wavenumber such that the wavenumber in the system can be given as

$$k^2 = k_2^2 + k_z^2.$$

Similarly the linearization of the dust continuity and momentum equation leads to

$$n_{d1} = n_{d0}\frac{k}{\omega}v_{d1}$$

$$\left(\omega + \frac{\eta k^2}{m_d n_{d0}(1 - i\omega\tau_m)}\right)v_{d1} = -\frac{Z_d n_{d0}k\phi}{m_d\omega}$$

The dust velocity $v_{d1}$ is directed along the electric field $\vec{E} = -\nabla\phi$ which has no component along $\hat{x}$ the inhomogeneity direction. Thus the term $\vec{v} = \vec{n}_d \cdot \nabla\ln(n_{i0})$ has not been included in the linearized continuity equation for the dust species. Combining the equations (11-14) leads to the following dispersion relation:

$$-\left[\omega^2 + \frac{\omega\eta k^2}{m_d n_{d0}(1 - i\omega\tau_m)}\right]^{-1}C_D^2k^2 + \left[\frac{n_{e0}}{n_{i0}} + k_2^2\rho_s^2\right] + \frac{k_y T_e}{\omega e B} \nabla(\ln(n_{i0})) - \frac{k_z^2 v_s^2}{\omega^2} = 0$$

Here $C_D = \omega_{pd}\lambda_{di}$.

Using the normalization $\frac{\omega}{\omega_{ci}} \rightarrow \omega$, $k\rho_s \rightarrow k$, $\omega_{ci} \rightarrow t$, $\frac{x}{\rho_s} \rightarrow x$, $y$, $L_n$, $\omega_{ci}\tau_m \rightarrow \tau_m$, $\frac{\eta}{n_{d0}\omega_{ci}\rho_s^2} \rightarrow \eta$, above dispersion relation expressed in Eq. (15) can be obtain as,

$$-\left[\omega^2 + \frac{\omega\eta k^2}{(1 - i\omega\tau_m)}\right]^{-1}C_D^2k^2 + \left[\frac{n_{e0}}{n_{i0}} + k_2^2\rho_s^2\right] - \frac{1}{L_n\omega} + \frac{k_z^2 v_s^2}{\omega^2} = 0$$

This expression is the dispersion relation for the ion drift wave in presence of dust dynamics. Now Eq. (16) can be analysed in its two asymptotic limits namely: (1) Hydrodynamic limit i.e $\omega\tau_m \ll 1$ in which dust particles are weakly correlated to each other and, (2) Kinetic limit $\omega\tau_m \gg 1$, in which dust particles are strongly correlated.
A. Hydrodynamic limit, $\omega \tau_m \ll 1$

In the limit where $\omega \tau_m \ll 1$ which is the case for weakly coupled dust particles, Eq. (16) can be written as,

$$-\frac{C_D^2}{v_s^2} \frac{k^2}{(\omega^2 + i\omega \eta k^2)} + \left[\frac{n_e}{n_i} + k^2 \rho_s^2\right] - \frac{1}{L_n \omega} k_y - \frac{k_z^2}{\omega^2} = 0 \quad (17)$$

Eq. (17) is the modified Hasegawa-Mima equation in weakly coupled dusty plasma. The presence of viscosity term $\eta k^2$ would lead to the damping of drift wave coupled to dust acoustic wave. However, in the low viscosity limit i.e. $\omega \gg \eta k^2$, the above equation (Eq. [17]) reduces to a simple quadratic equation as

$$\left(\frac{n_e}{n_i} + k^2 \rho_s^2\right)\omega^2 - \omega_m \omega - \left[\frac{C_D^2}{v_s^2} k^2 + k_z^2\right] = 0 \quad (18)$$

where $\omega_m = k_y/L_n$ is the classical drift wave frequency. The roots of the above equation are

$$\omega = \frac{\omega_m \pm \sqrt{\omega_m^2 + 4 \left(\frac{n_e}{n_i} + k^2 \rho_s^2\right) \left[\frac{C_D^2}{v_s^2} k^2 + k_z^2\right]}}{2 \left(\frac{n_e}{n_i} + k^2 \rho_s^2\right)} \quad (19)$$

The variation in $k_x$ is assumed to be very slow such that the wavelength in $\hat{x}$-direction to be very small compared to the scale length of the density gradient so that we can still linearize in the $\hat{x}$-direction in spite of the inhomogeneity in that direction.

B. Kinetic limit, $\omega \tau_m \gg 1$

The dust particles being at low temperature and having high charge, can readily go in the strongly coupled regime, while electrons and ions having comparatively very high temperature, can still be considered as light fluid. The dynamics of strongly coupled dust is no longer similar to that described by the ordinary fluid in the hydrodynamic limit $\omega \tau_m \ll 1$. Now its dynamics resembles to that shown by viscoelastic fluid. In the kinetic regime i.e $\omega \tau_m \gg 1$, Eq. (16) can be written as,

$$-\frac{C_D^2}{v_s^2} \frac{k^2}{(\omega^2 - \frac{1}{\tau_m} k^2)} + \left[\frac{n_e}{n_i} + k^2 \rho_s^2\right] - \frac{1}{L_n \omega} k_y - \frac{k_z^2}{\omega^2} = 0 \quad (20)$$
This equation can be further reduced to a fourth order equation in $\omega$ as,

$$\left[ \frac{n_{e0}}{n_{i0}} + k_{\perp}^2 \right] \omega^4 \left[ \frac{k_y}{L_n} \right] \omega^3 - \left[ \frac{C_D^2}{v_s^2} k^2 + \left( \frac{n_{e0}}{n_{i0}} + k_{\perp}^2 \right) \frac{\eta}{\tau_m} k^2 \right] \omega^2 + \frac{k_y \eta}{L_n \tau_m} k^2 \omega + \frac{\eta}{\tau_m} k_{\perp}^2 k^2 = 0 \quad (21)$$

Eq. (21) is the modified Hasegawa-Mima equation in strongly coupled dusty plasma.

IV. RESULTS AND ANALYSIS

The coupling of drift waves with dust acoustic wave happens when the condition $\omega \sim \omega_* \sim k C_D \sim k z v_s$ is satisfied. The frequency $\omega = \omega_r + i \gamma$ can be complex where $\omega_r$ and $\gamma$ are the real and imaginary part. It has been shown that the coupling of drift wave with dust acoustic wave does not lead to instability. However, the presence of dust dynamics has been shown to strongly modify the dispersion properties of drift waves.

A. Weakly correlated dusty plasma ($\omega \tau_m \ll 1$)

It is evident from the roots [Eq. (19)] that there is a finite value of $\omega_r$ even at $k_y = 0$. Also we observe that mode saturates to $\omega_r = 1 \times 10^{-4}$ the large $k_y$ region where the mode frequency is independent of $k_y$ [see Eq. (19)]. It is due to the shielding by polarization drift in the perpendicular direction. The dispersion relation is valid in the regime where $k_{\perp}^2 \rho_s^2 \gg 1$ because we have taken $T_i \ll T_e$. Therefore even if $k_{\perp}^2 \rho_i^2 \ll 1$ (condition for ions to be magnetized, where $\rho_i = \frac{1}{\omega ci} \sqrt{\frac{T_i}{m_i}}$ is the ion Larmor radius at ion temperature), we can have the condition $k_{\perp}^2 \rho_s^2 \gg 1$ valid.

The presence of dust dynamics has been found to modify the dispersion relation significantly. The Fig. (3) compares the dispersion relation of the drift wave proposed by
FIG. 2: Dispersion Relation \( (k_z = 10^{-6}, L_n = 10^4, \frac{n_0}{n_{i0}} = 0.15, \frac{C_D}{v_i} = 10^{-4}) \) where we show that the mode saturates to 1 for large value of \( k_y \).

FIG. 3: Comparison of dispersion relations between Hasegawa-Mima drift wave and the drift wave coupled with dust acoustic wave for parameters \( (k_z = 10^{-6}, L_n = 10^4, \frac{n_0}{n_{i0}} = 0.15, \frac{C_D}{v_i} = 10^{-4}) \).

Hasegawa-Mima (red color) with the drift wave coupled to the acoustic mode (blue color).
FIG. 4: The effect of $k_z$ on the dispersion relation of the drift wave coupled to dust acoustic wave ($L_n = 10^4$, $\frac{n_{e0}}{n_{i0}} = 0.15$, $\frac{C_D}{v_s} = 10^{-4}$)

FIG. 5: The effect of depletion ratio of electrons $b = n_{e0}/n_{i0}$ on the dispersion relation of the drift wave coupled to dust acoustic wave ($L_n = 10^4$, $k_z = 10^{-6}$, $\frac{C_D}{v_s} = 10^{-4}$)

when the dust dynamics has also been considered. The dispersion relation does not depend on the value of $k_z$ as shown in Fig. 4. The mode frequency also depends on the
FIG. 6: The effect of gradient scale length $L_n$ on the dispersion relation of the drift wave coupled to dust acoustic wave ($b = n_0/n_{i0} = 0.15, k_z = 10^{-6}, C_D/n_e = 10^{-4}$)

value of depletion ratio $n_0/n_{i0}$ [Fig. (5)]. The larger the depletion ratio, lesser is the mode frequency in the region where $k_y$ is smaller. The scale length of the density gradient plays a vital role in deciding the coupling of drift wave with dust acoustic wave. The sharp peak does not appears at all the values of the scale length $L_n$ but emerges at a particular value of it. After this particular value, peak of the mode increases with increase in the length scale to a certain value as shown in Fig. (6). But the saturated mode which observed for the larger value of $k_y$ does not changes with the change in the parameters $k_z, n_0/n_{i0}$ and $L_n$.

B. Strongly correlated dusty plasma ($\omega \tau_m \gg 1$)

It evident from (Fig. 7) that for large value of $k_y$, the frequency $\omega$ in strongly coupled regime vary as $\omega \sim k_y$ which is nothing but the shear mode as the terms $\left[\frac{n_0}{n_{i0}} + k^2_\perp\right] \omega^4$ and $\left[\frac{n_0}{n_{i0}} + k^2_\perp\right] \eta \xi_2$ balances each other which give rise to shear mode $\omega_2^2 = \frac{\eta}{\xi_m} k^2$. There also exists a mode which is similar to the old drift wave mode which goes to zero for large value of $k_y$. For large value of $\frac{\eta}{\xi_m}$, the term $\left[\frac{n_0}{n_{i0}} + k^2_\perp\right] \eta \xi_2 \omega^2$ balances the term
\( \frac{k_y \eta}{L_n \tau_m} k^2 \omega \) which give rise to a modified drift mode \( \omega_r = \frac{k_y/L_n}{(n_{e0}/n_{i0} + k_i^2)} \) which is deviated from that proposed by Hasegawa-Mima by a factor \( \frac{n_{e0}}{n_{i0}} \) in the denominator. Furthermore, for a small value of \( \frac{n_{e0}}{n_{i0}} \), the equation reduces back to what we obtained for the drift wave coupled to the dust acoustic wave in weakly coupled limit as expected as shown in Fig.(8).

![Image of dispersion relation](image.png)

**FIG. 7:** Dispersion relation of drift wave coupled to dust acoustic wave in strongly coupled dusty plasma \( (a = \frac{\eta}{\tau_m} = 10^{-8}, k_z = 10^{-6}, L_n = 10^4, \frac{C_p}{v_s} = 10^{-4}, \frac{n_{e0}}{n_{i0}} = 0.15) \)

We do not observe any growing mode in the strongly coupled regime (Fig.8). The dependence of dispersion relation on the parameters \( \frac{\eta}{\tau_m} \), gradient scale length \( L_n \), \( k_z \) and \( b = n_{e0}/n_{i0} \) has been shown in figures (8), (9), (10) and (11) respectively.
FIG. 8: Dependence of Dispersion relation of drift wave coupled to dust acoustic wave in strongly coupled dusty plasma on \( a = \frac{n}{\tau_m} \) \((k_z = 10^{-6}, \frac{C_D}{v_s} = 10^{-4}, L_n = 10^4, \frac{n_u}{n_i} = 0.15)\)

FIG. 9: Dependence of dispersion relation on \( L_n \) \((k_z = 10^{-6}, \frac{C_D}{v_s} = 10^{-4}, \frac{n_u}{n_i} = 1 \times 10^{-8})\)
FIG. 10: Dependence of dispersion relation on gradient scale length \( k_z \) \( (L_n = 10^4, \frac{C_n}{v_e} = 10^{-4}, \frac{n}{\tau_m} = 1 \times 10^{-8}, \frac{n_{e0}}{n_{i0}} = 0.15) \)

FIG. 11: Dependence of dispersion relation on \( b = n_{e0}/n_{i0} \) \( (L_n = 10^4, \frac{C_n}{v_e} = 10^{-4}, \frac{n}{\tau_m} = 1 \times 10^{-8}, k_z = 10^{-6}) \)
V. SUMMARY AND CONCLUSIONS

In weakly coupled nonuniform, magnetized dusty plasma, when the dust dynamics are also considered, there exists a possibility of coupling of drift wave with the dust acoustic wave in strong magnetic field approximation ($\omega \ll \omega_{ci}$). Now the drift mode has been significantly modified as compared to the earlier Hasegawa-Mima drift wave in the presence of the dust dynamics. For the large value of $k_y$, we observe a saturated mode which is independent of $k_\perp$. In this region, shielding is done by the polarization drift in the perpendicular direction. In addition, the dispersion properties of the drift mode is strongly modified with the parallel wave vector $k_z$, the factor $n_e n_i$ and the length scale of the density gradient $L_n$. However, saturated mode obtained for larger value of $k_y$ does not changes with the change in the parameters $k_z$, $n_e n_i$ and $L_n$.

In strongly coupled nonuniform magnetized dusty plasma, fourth order in $\omega$ for drift wave coupled with dust acoustic wave in the kinetic regime ($\omega \tau_m \gg 1$) has been obtained. Drift mode in strongly coupled regime converts to transverse shear mode at large $k_y$. For smaller $a = \frac{n_e}{n_i}$, the equation reduces back to that obtained in weakly coupled limit. The dust shear mode coupled to ion drift mode has been found to be the natural mode of plasma.

In conclusion, the dust dynamics (in both weakly and strongly correlated) has been shown to significantly influence the ion drift in an inhomogeneous, magnetized plasma and can be observed in laboratory experiments as well as in astrophysical plasma.

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