Thermal characteristic of double-row cylindrical roller bearing of high-speed train using analytical solution and finite element analysis

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Abstract. The complicated thermal characteristics critically affect the reliability of double-row cylindrical roller bearing (DCRB) in the high-speed railway, which will cause the deformation and vibration and severely lead to bearing failure. Thus, research on the heat distribution and steady temperature field is essential for DCRB. In this paper, the Green’s function was established and programmed with MATLAB under certain boundary conditions to verify the temperature field of DCRB. It computes faster than the finite element analysis (FEA). According to the simulation of ANSYS, temperature in outer ring is higher than inner ring and it is the maximum in roller, while the rib reduces the temperature in bearing and temperature near rib is larger, which result in asymmetry in raceway. All these results are consistent with the accrual working. This paper provided a heat portion equation between the rollers and the raceway and calculated more accurate and comprehensive than other equation. All results in this paper made sure the more exact temperature field in the DCRB with more convenient calculation and verification.

1. Introduction

DCRB is the key component and the most vulnerable parts of high-speed train, especially under high speed, the temperature in the bearing rises quickly, which affects the reliability of bearing and dangers the safety of the train. Zhou [1] analyzed wear is the main failure form inside the bearing, and accompanied by heat generation. Nikas [2] studied the temperature effect, and showed that internal friction particles cause the "flash temperature". Therefore, it is imperative to study the temperature field inside the bearing.

So far, there are many methods to obtain the temperature field of the bearing. The common method is FEA. Aruns [3] introduced the application of numerical methods in this field. Xiu [4] used ANSYS to analyze the mechanism of internal temperature generation. Tao [5] used ANSYS to establish a three-dimensional single ball bearing model to obtain a steady-state temperature field. Kalek [6] established an equivalent thermal resistance model to predict the steady-state temperature field. Baiρi [7] provided an analytical method based on the heat transfer equation in a three-dimensional Cartesian coordinate system, but the boundary conditions were not flexible enough. Hannon [8] provided another semi-analytical method based on integral transformation, while the temperature field depend...
on integrated area and the ability of the integration. It has some limits when calculates. This paper offers a new way of obtaining and verifying the temperature.

2. Propose analytical model

There are many analytical methods, relatively speaking, the Green's Function is flexible, and it is rare to use Green's Function to solve temperature field in bearing. Cole [9] used the one-dimensional Green's Function solution to calculate the steady-state temperature. This paper proposed a high-dimensional Green's function to obtain the bearing’s temperature and the result compared with COMSOL to verify each other. The ANSYS is used to simulating the three-dimensional model, and contrast to the analytical result by simplifying model to two-dimension.

Model assumptions:
① Since it is a steady-state solution, the temperature does not change with time;
② Ignoring the heat transfer with the space, and not considering the radiation;
③ Assuming that the material stiffness is sufficient, and the corresponding ideal model will be maintained;
④ Approximate treatment of complex raceways and rollers.

2.1. Heat distribution

The power loss on the contact surface directly affects the final distribution of temperature. Many scholars have studied the portion. Evans [10] studied the heat source distribution under lubrication conditions. Waddad [11] proposed a complex heat source equivalent model to calculate the heat source distribution problem. Hannon [8] studied the heat distribution under circulating lubrication conditions and corrected it based on the Jaeger’s formula. Peng [12] obtained the ratio of convective heat generation under a certain oil film thickness based on Cann [13], but all these studies were not comprehensive for the actual working condition.

This paper focuses on obtaining the generic heat distribution ratio between the raceway and the rollers in any moving state. Based on Muzychk [14], the average temperature and the Maximum temperature distribution formula of the elliptical moving heat source are:

\[ R \cdot k \cdot a = 0.323 \cdot a \cdot b^{-1} \cdot P_e^{-2} \]  
\[ R \cdot k \cdot a = 0.589 \cdot a \cdot b^{-1} \cdot P_e^{-2} \]  

The absorption capacity per unit area per unit time obtained when the temperature rises by one centigrade, the corresponding heat flow is Equations (3), (4), and the ratio \( \gamma \) can be calculated:

\[ Q_s = \frac{1}{R_s} = 6.192 \sqrt{\rho_s C_p k_s (t_s \sqrt{t_s})^{-1}} \]  
\[ Q_r = \frac{1}{R_r} = 6.192 \sqrt{\rho_r C_p k_r (t_r \sqrt{t_r})^{-1}} \]  
\[ \gamma = \frac{Q_s}{Q_r} = \frac{\sqrt{\rho_s C_p k_s (t_s \sqrt{t_s})^{-1}}}{\sqrt{\rho_r C_p k_r (t_r \sqrt{t_r})^{-1}}} \]  

According to Hu [15], the distribution is related to \( \sqrt{\rho k C_p} \). In this article, \( \gamma = \sqrt{\rho_s C_p k_s (\sqrt{\rho_r C_p k_r})^{-1}} \), when there is no relative sliding between the contact surface, while there is relative sliding, \( \gamma = \sqrt{\rho_s C_p k_s \sqrt{V_s} (\sqrt{\rho_r C_p k_r \sqrt{V_r}})}^{-1} \) is consistent with the result in Gui [16]. When the sliding speed of one surface is too fast than the other, the surface with the fast can obtain more heat by Equation (5), and the corresponding temperature is high. This conclusion is consistent with the results in Clarke [17]. When one side remains stationary and the other side slides, the \( \gamma \) is zero. Therefore, further research is needed. According to Bhushan [18], the heat distribution equation of the stationary surface is:
\[ \gamma = \left( 1 + \frac{k_s}{k_r} \sqrt{1 + P_{er}} \right)^{-1} \] (6)

The Equation (6) shows that when the speed is zero, the distribution coefficient only related to the heat transfer coefficient, which obviously is wrong. When both objects are moving,

\[ \frac{Q_s}{Q_r} = \frac{k_s}{k_r} \left( \frac{1 + P_{es}}{1 + P_{er}} \right) \] (7)

This Meets the result in Equation (5), in order to eliminate the effect of zero sliding speed, the correction coefficient \( s \) should be added.

\[ \gamma = \left( 1 + \frac{k_s}{k_r} \sqrt{\frac{s + P_{es}}{s + P_{er}}} \right)^{-1} \] (8)

From Equation (8), when \( P_{e1} \) and \( P_{e2} \) are relatively large, the ratio approximately equals to that in Equation (5).

2.2. Green’s function of model

We simplified the rib of outer ring to obtain a two-dimensional hollow cylinder, and establishing equation based on the radial and circumferential dimension. There is no internal heat source and no influence of initial condition. The Green's function obtained from the control Equation (10), and the first boundary condition in the inner ring, the second boundary condition in the outer ring, the initial temperature is zero.

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial G}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 G}{\partial \theta^2} + \frac{1}{k} \delta(r-r') \delta(\theta-\theta') \delta(t-t') = \frac{1}{\alpha} \frac{\partial G}{\partial t} \] (10)

Boundary conditions:

\[ T = f_1(r, \theta), \quad r = a, \theta \in [0, 2\pi] \]
\[ \frac{\partial T}{\partial r} = f_2(r, \theta), \quad r = b, \theta \in [0, 2\pi] \]

Initial conditions:

\[ T = 0 \quad t < t' \]

The partial differential equation of Green's function is:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial G}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 G}{\partial \theta^2} + \frac{1}{k} \delta(r-r') \delta(\theta-\theta') \delta(t-t') = \frac{1}{\alpha} \frac{\partial G}{\partial t} \] (11)

Boundary conditions of auxiliary problems:

\[ G = 0, \quad r = a, \theta \in [0, 2\pi] \]
\[ \frac{\partial G}{\partial r} = 0, \quad r = b, \theta \in [0, 2\pi] \]

Initial conditions:

\[ G = 0 \quad t < t' \]

The heat source term in the equation represents the internal heat source distribution of body heating rate: \( (r, \theta, t) = \delta(r-r') \delta(\theta-\theta') \delta(t-t') \) to the conservation of energy, the heat absorption from \( t \) to \( t+t' \) at any spatial position is equal to the amount of internal heat source.
Obtaining the Green function solution across some transformation and the steady state solution obtained from the unsteady state from integration.

\[
G(r,\theta)=\frac{\pi}{2k} \sum_{m=1}^{\infty} \sum_{v=1}^{\infty} \beta_m^{-2} N(\beta_m)^{-1} r' R_v(\beta_m, r') R_v(\beta_m, r) \cos \left( v(\theta-\theta') \right) + \frac{\pi}{4k} \sum_{m=1}^{\infty} \sum_{v=0}^{0} \beta_m^{-2} N(\beta_m)^{-1} r' R_v(\beta_m, r') R_v(\beta_m, r) \cos \left( v(\theta-\theta') \right)
\]

By substituting the \(R_v(\beta_m, r')\), \(R_v(\beta_m, r)\), \(N(\beta_m)\), obtaining the complete two-dimensional Green's function solution:

\[
G(r,\theta| r',\theta') = \frac{\pi}{4k} \sum_{m=1}^{\infty} \sum_{v=0}^{\infty} z r' \left[ J_v(\beta_mr) Y_v'(\beta_mb) - J_v'(\beta_mb) Y_v(\beta_mr) \right] \times \left[ J_v(\beta_mr') Y_v'(\beta_mb) - J_v'(\beta_mb) Y_v(\beta_mr') \right] \cos \left( v(\theta-\theta') \right)
\]

The temperature distribution obtained from the general solution of Green's function,

\[
T(r,\theta)=k \left( \frac{1}{h_1} \int_0^{2\pi} \frac{\partial g(r,\theta| r',\theta')}{\partial r'} \bigg|_{r'=a} f_1(r',\theta') \, d\theta' + \frac{1}{h_2} \int_0^{2\pi} \frac{G(r,\theta| r',\theta')}{r'} \bigg|_{r'=b} f_2(r',\theta') \, d\theta' \right)
\]

According to Figure 1 and Figure 2, the related temperature field under unit heat source displays, which has consistent with each other. Based on this result, temperature in any position can obtain. The red line represents the result of Equation (15), the green one tells the simulation’s result of COMSOL.

**Figure 1.** Radial change of temperature.  
**Figure 2.** Circumferential temperature.
3. Finite element model

This paper took a research on the DCRB used on the high-speed train. We use the CREO to establish the structure, and omit the cages and chamfers. The dimension was 130 mm × 240 mm × 160 mm. Each row has 17 rolling elements, and in order to obtain the equivalent heat plow, the area of contact surfaces should calculate. The detail parameters show in the Table 1.

| Inside diameter | Outside diameter | Width (mm) | Number of rolling element each row | Rolling element diameter (mm) | Rolling element length (mm) |
|-----------------|------------------|------------|-----------------------------------|-------------------------------|-----------------------------|
| 130             | 240              | 160        | 17                                | 27                           | 48                          |

In this paper, the power loss between the raceway and the roller was from the BEARINX (can calculate the power loss under certain conditions), which total frictional power loss on the inner raceway contact is 293.518 W, the total frictional power loss on the outer raceway contact is 324.158 W. Under condition of the inner ring’s speed 1500 r/min and the equivalent load 57.657 KN. In order to simulate fast, this paper just considered one row, and there are two cases studied, one has no rib and the other has.

Import the structure model imported to ANSYS and it will establish the contact pairs automatically. The elements are made of structural steel and the contact region is fractional with the coefficient of 0.0025. There were 342121 nodes and 225899 elements in the model, and the structure or contact pairs show as Figure 3 and Figure 4. The Figure 5 and Figure 6 tell the conditions of simulation and the temperature coupling between the rib and the ends of rings adopt.

According to actual working condition, the Equivalent treatment to the raceway and the roller is necessary. In this text, the heat distribution ratio is 1:1 for neglecting the sliding in the contact surface calculated from Equation (8). Thus, the power flow on the inner raceway is 293.518 W and the power flow on the outer raceway is 324.158 W under the parameters display in Table 2.
Table 2. The area of different element.

| Inner raceway/mm² | Outer raceway/mm² | Roller/mm² |
|-------------------|-------------------|------------|
| 22282.7           | 29946.8           | 3832.1     |

3.1. Calculation and discussion

Compare Figure 7 and Figure 8, the bearing with rib has the lower maximum temperature, for the heat moves from roller to rib lead to the outer ring’s temperature higher, in contrast, the maximum temperature in the roller is lower. According to Figure 9 and Figure 10, the temperature near the rib is higher than the other side of the raceway with Asymmetry. From the Figure 11 and Figure 12, rollers have the maximum temperature and the lower temperature in rollers with rib.

![Figure 7. Temperature filed without rib of DCRB.](image_url)

![Figure 8. Temperature filed with rib of DCRB.](image_url)

![Figure 9. Temperature filed without rib of outer ring.](image_url)

![Figure 10. Temperature filed with rib of outer ring.](image_url)

All these figures of temperature field show that the tendency of is reasonable and the lower temperature in rollers with rib. Significantly, rib has no effect on temperature of inner ring. Thus, the simulation needs more verification with the Green’s function. Take the inner ring as example, the simulation result as Figure 13 and Figure 14.

![Figure 11. Temperature field of rollers without rib.](image_url)

![Figure 12. Temperature field of rollers with rib.](image_url)
Figure 13. Temperature field of inner ring without rib.

Figure 14. Temperature field of inner ring with rib.

For inner ring, substituting the Geometry values into the program, and taking the order of eigenvalue 500, the number of eigenvalue 500, the result is 0.044 °C under unit power flow. When the order is 1500 and the number is 500, the corresponding temperature is 0.045°C. Thus, when the number of order increase 1000, the value only changed 2.27%. Under this condition, the result is relative correct. According to the equivalent power flow on inner raceway (6212 W/m²) and the above analysis, taking the result 0.045, then the temperature on inner raceway is 70.2°C. From the Figure 13 and Figure 14, the high temperature on inner raceway is 67.2°C. The deviation of two results is 4.46%. Thus, this simulation is line with the actual situation.

4. Conclusions

In this paper, the Green’s function for calculating temperature of the hollow cylindrical bearing was established and programed with MATLAB. The program was exact when compared with the COMSOL or ANSYS. This paper provided a convenient way of Comparison and verification when do some simulation. According to simulation of ANSYS, rib reduces the maximum temperature of bearing due to heat conduction, and the highest temperature is on the surface of roller in DCRB and the lowest is on the inner raceway and rib has no influence on temperature of inner ring. Moreover, the heat portion of bearing obtained in this paper, which was comprehensive for studying the distribution of heat sources under any working conditions. These studying may be useful for obtaining the temperature in DCRB.

Nomenclature

$R$ = thermal resistance
$k$ = heat transfer coefficient
$a$ = short radius of Hertz contact surface
$P_e$ = Peclet number
$Q_s$ = heat density of stationary
$Q_r$ = heat density of moving source
$\gamma$ = coefficient of heat partion
$k_s$ = heat transfer coefficient of stationary
$k_r$ = heat transfer coefficient of moving part
$P_{er}$ = Peclet number of moving part
$P_{es}$ = Peclet number of stationary
$\alpha_s$ = convection heat transfer coefficient
$\alpha_r$ = convection heat transfer coefficient
$\rho$ = density of materials
$C_p$ = specific heat at constant pressure
$r$, $r'$ = radius
$\theta$, $\theta'$ = circle angle
$\beta_m = $eigenvalue

v= order of eigenvalue

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