New method for determining the thickness of non-edge-on disk galaxies

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1. Introduction

The thickness of disk galaxies is a very important parameter for understanding these objects, although all of which are very thin comparing with the length of the disk. Sancisi and Allen (1979) estimated the thickness of the edge-on Sb galaxy NGC 891, on the basis of observation of neutral hydrogen. Van der Kruit & Searle (1981a) proposed a model for light distribution in the disks of edge-on spiral galaxies, assuming that a galaxy has a locally isothermal, self-gravitating, truncated and exponential disk. The model has the feature of being isothermal in z-direction at all radii with a scale parameter of $z_0$ and has an exponential dependence of surface brightness on $r$ with scale length of $r_d$. The space luminosity of this model can be described by

$$L(r, z) = L_0 e^{-r/r_d} \sec h^2(z/z_0)$$

With this model, van der Kruit & Searle (1981a, 1981b, 1982a, 1982b, named KS hereafter) determined $r_d$ and $z_0$ for several edge-on galaxies without an appreciable bulge. Unfortunately, this method may not be suitable for general disk galaxies, e.g. face-on galaxies.

Peng (1988) put forward a method to measure the thickness of face-on galaxies on the basis of asymptotic formula of the disturbed gravitational potential. And a revised method based on the exact integral expression for disturbed gravitational potential has been presented by Zhao et al. (2004) to estimate the thickness of face-on disk galaxies. But the results obtained by these two correlative methods should be checked with the results gotten by other independent ways.

In this paper, we present the solution of the Jeans equation along z-direction first and then describe how to use this solution to determine the thickness of face-on disk galaxies. In Section 3, we show the applications of our method to two near face-on galaxies, NGC 1566 and NGC 5247, which have been extensively studied in spectroscopy. The results are also presented. And a brief discussion is given in the last section of this paper.

2. Method

2.1. The distribution of the vertical velocity dispersion

For a stable, axisymmetric galaxy, the Jeans equation along the z-direction is

$$\frac{\partial}{\partial z} (\rho \langle V_z^2 \rangle) = -\rho \triangledown^2 \phi$$

where $\langle V_z^2 \rangle$ is square of the z-direction velocity dispersion. As the first of the fundamental assumptions, we accept Parenago’s density distribution law along the z-direction

$$\rho(r, z) = \rho(r, 0) e^{-\alpha |z|} = \frac{\alpha}{2} \sigma(r) e^{-\alpha |z|}$$

where $\alpha$ is a parameter the reciprocal of which is half of the effective thickness, $H$, of a galaxy. $\alpha$ may be taken as a constant, for the scale height is basically independent of
the radius at least for the thick disk of our Galaxy (e.g., see Freeman, 1987). $\sigma(r)$ is its surface density. de Grijs & van der Kruit (1996) have also confirmed that the simple exponential fit of the vertical stellar light distribution is a good approximation. Hence, the potential caused by this density can be found by solving the Poisson’s equation of the potential

$$\nabla^2 \varphi(r, z) = 2\pi G \rho(r) e^{-\alpha |z|}. \quad (4)$$

The rigorous solution of the potential has been given by Peng et al. (1978)

$$\varphi(r, z) = -\pi G \int_0^\infty \frac{\rho}{r^2} \left( e^{-\alpha |z|} - \alpha e^{-\beta |z|} \right) J_0(\beta r) s(\beta) d\beta \quad (5)$$

where

$$s(\beta) = \int_0^\infty r J_0(\beta r) \sigma(r) dr. \quad (6)$$

And $J_0(\beta r)$ is the well known Bessel function of order 0.

Then using equation (5), we integrate equation (2) from $z$ to $\infty$ ($z \geq 0$) at both sides of the equal sign respectively, it follows (Huang, Huang & Peng 1979)

$$\left( \rho \langle V^2_z \rangle \right)_{z=0} = -2\pi G \rho(r, 0)$$

$$\cdot \int_0^\infty \frac{\beta}{\beta^2 - \alpha^2} \left[ \frac{1}{\alpha+\beta} e^{-(\alpha+\beta)z} - \frac{1}{\alpha} e^{-2\alpha z} \right] J_0(\beta r) s(\beta) d\beta \quad (7)$$

Hence, on the galactic plane ($z = 0$), $\langle V^2_z \rangle$ could be written as

$$\langle V^2_z \rangle_{(z=0)} = \pi G \int_0^\infty \frac{\beta}{(\alpha+\beta) \pi} J_0(\beta r) s(\beta) d\beta$$

$$= \frac{\pi G}{\alpha} \sigma(r) [1 - P(r)] \quad (8)$$

where

$$P(r, \alpha) = \frac{1}{\sigma(r)} \int_0^\infty \beta \zeta(\alpha, \beta) J_0(\beta r) s(\beta) d\beta,$$

$$\zeta(\alpha, \beta) = 1 - \frac{1}{(1+\beta/\alpha)^2}. \quad (9)$$

2.2. The thickness parameter $\alpha$

By assuming that a galaxy has an infinitesimally thin disk, Freeman (1970) studied 36 spiral and S0 galaxies with surface photometry and showed that the radial light distribution in the disks of spiral galaxies can be described by an exponentially decreasing surface-brightness with increasing galactocentric radius. Along the vertical direction, de Grijs & van der Kruit (1996) have shown that a simple exponential fits turned out to be good approximations of the stellar light distribution. Thus, the model to account for the space-luminosity can be described by

$$L(r, z) = I_0 e^{-r/r_d} e^{-\alpha |z|}, \quad (10)$$

where $r$ is the position along the major axis, $I_0$ the central space luminosity in the plane of the galaxy, $r_d$ the scale length of the disk. From this we calculate the face-on distribution of surface-brightness

$$I(r) = I_0 e^{-r/r_d}, \quad (11)$$

where $I_0$ is the central surface brightness of the disk.

Making use of the observed constant color index (Van der Kruit & Searle 1981b, 1982a), Van der Kruit & Freeman (1986) pointed out that the mass-to-light ratio of the old disk, $Y_* = (M/L)^{old\ disk}$, is approximately constant with the radius. This means we’ll take the projected surface density as

$$\sigma(r) = Y_* I(r) = Y_* I_0 e^{-r/r_d} = \sigma_0 e^{-r/r_d}, \quad (12)$$

where $\sigma_0$ is the surface density at $r = 0$.

Therefore, equation (6) could be reduced by substituting $\sigma(r)$ with equation (12) to

$$s(\beta) = \int_0^\infty r J_0(\beta r) \sigma(r) dr = Y_* I_0 \frac{r_d^2}{(1 + r_d^2 \beta^2)^{3/2}}. \quad (13)$$

And the expression of $\langle V^2_z \rangle$ may be rewritten as

$$\langle V^2_z \rangle = \frac{\pi G}{\alpha} \frac{Y_*}{1 - P(r)} \quad (14)$$

where

$$P_1(r) = P(r) e^{-r/r_d} \quad (15)$$

$$= \int_0^\infty \frac{r_d^2}{(1 + r_d^2 \beta^2)^{3/2}} \beta \left[ 1 - \frac{1}{(1+\beta/\alpha)^2} \right] J_0(\beta r) d\beta.$$

Hence, if we know the $I_0, r_d$ from photometric study and $\langle V^2_z \rangle$ from spectroscopic observation, we can calculate the thickness parameter $\alpha$ through equation (14) and therefore obtain the thickness of galaxies.

3. Application

We found that there have existed extensively spectroscopic study on two nearly face-on galaxies NGC 1556 and NGC 5247. Therefore, we choose these two galaxies as examples to illustrate how our method works. The general information of these two galaxies are summarized in Table 1.

3.1. Photometric decomposition

Although there also existed photometric results for NGC 1566 (de Vaucouleurs 1973; Bottema 1992) and 5247 (van der Kruit & Freeman 1986), we can not use these data directly since the photometric decomposition is very important for our purpose. The photometric decomposition can disentangle the contributions to the total luminosity of the bulge and disk, and this will allow the identification of the disk region. Moreover, because the infrared images would suffer much less extinctions than optical ones do, we use the 2MASS H-band images to do photometric measurements.

To obtain photometric parameters, $r_d$ and $\mu_0$, we use the two dimensional image-fitting algorithm GALFIT (Peng et al. 2002) designed to extract structural parameters directly from the galaxy images. GALFIT assumes
Table 1. Parameters of NGC 1566 and NGC 5247.

| NGC  | Inclination  | Classification   | Distance$^a$ (Mpc) | $M_H$ (mag) | ref   |
|------|--------------|------------------|--------------------|-------------|-------|
| 1566 | 28° ± 5°     | Sc(s)I, Seyfert 1 | 17.4               | -24.0       | (1), (2), (4) |
| 5247 | 20° ± 3°     | Sc(s)I-II        | 16.0               | -23.2       | (1), (3), (4) |

$^a$ Obtained based on $H_0=75$ km s$^{-1}$ Mpc$^{-1}$.

References  (1) Sandage & Tammann 1981; (2) Bottema 1992; (3) van der Kruit & Freeman 1986; (4) Jarrett et al. 2003.

Fig. 1. For both galaxies we show the original images (left), the smooth and symmetric model images fitted by GALFIT (middle) and the residual images (right). For NGC 1566 (top), the size of each field is $2'.4 \times 2'.9$; for NGC 5247 (bottom), the size is $6'.7 \times 6'.7$.

a two-dimensional model profile for the galaxy. The functional forms of the models we choose to fit include combinations of a Gaussian, a Sérsic $r^{1/n}$ law and an exponential disk profile. For the case of a Seyfert 1 galaxy, NGC 1566, we use all three functions to fit, while we use the last two functions for NGC 5247. We fit the following: the $(x, y)$ position of the center, $M_{tot}$ (the total magnitude of the component), $r_e$ (the effective radius), $n$ (the Sérsic index), $q$ (the axis ratio), $r_d$ (the scale length of the exponential disk), the major position angle (PA), and $c$ (the diskiness/boxiness index). The detailed parameters of all fitted components are listed in Table 2. The original images, the smooth and symmetric model images fitted by GALFIT and the residual images are shown in the left, middle and right panels in Figure 1, respectively.
Table 2. Bulge-disk decompositions for NGC 1566 and NGC 5247. The fitting functions are a Gaussian (for the nucleus), Sérsic $r^{1/n}$ law (for the bulge) and an exponent (for the disk).

| Component | $M_{\text{tot}}$ (mag) | FWHM/$r_e/r_d^a$ (kpc) | $\mu_e/\mu_0^b$ (mag arcsec$^{-2}$) | PA (°) | $n$ | $q$ | $c$ | $\chi^2_\nu$ |
|-----------|------------------------|-------------------------|--------------------------------------|--------|-----|-----|-----|-------------|
| **NGC 1566** |                         |                         |                                      |        |    |    |    |             |
| Nucleus   | 11.70±0.03             | 0.32±0.01               | ...                                  | 88.02±2.36 | ... | 0.89±0.01 | 0.26±0.11 |
| Bulge     | 9.25±0.02              | 0.92±0.03               | 17.05±0.06                           | 12.21±0.48 | 1.65±0.03 | 0.79±0.00 | -0.28±0.01 |
| Disk      | 7.65±0.00              | 3.29±0.04               | 17.20±0.04                           | -15.62±0.32 | ... | 0.73±0.00 | -0.29±0.02 | 0.310 |
| **NGC 5247** |                         |                         |                                      |        |    |    |    |             |
| Bulge     | 10.69±0.01             | 0.67±0.02               | 18.06±0.04                           | 42.20±1.44 | 1.24±0.01 | 0.87±0.00 | 0.01±0.04 |
| Disk      | 7.84±0.00              | 4.78±0.03               | 18.60±0.03                           | -18.72±0.70 | ... | 0.80±0.00 | -0.08±0.02 | 0.130 |

$^a$ For the nucleus it is FWHM, for the bulge it is $r_e$, and for the disk it is $r_d$;

$^b$ For the bulge it is $\mu_e$, and for the disk it is $\mu_0$.

Fig. 2. 2MASS H-band radial surface brightness profiles. Symbols show the radial profiles obtained from ELLIPSE, the dotted line shows the Seyfert core component fitted to the Gaussian function, the dashed lines show the bulge components fitted to the Sérsic $r^{1/n}$ law, and the dashed-dotted lines show the disk components fitted to the exponential law. The thick solid lines are the superposition of all the components. The lower portion of each panel shows the difference between the data (ELLIPSE) and the fits (GALFIT).

In order to check whether or not the fitted results are reliable, we examine the residual images obtained by subtracting the fitted models. In both cases we found no obvious evidence (except for the expectant spiral arms) for components beyond the fitted models. Furthermore, we also measure surface brightness profiles using the IRAF$^1$/ELLIPSE algorithm. In Figure 2 we show the radial profiles obtained from the isophote fit-

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$^1$ IRAF is distributed by the National Optical Astronomy Observatories, which is operated by Associated of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.
ting by ELLIPSE and the profiles of the chosen model for each galaxy. The differences between the observational data (ELLIPSE) and the models (GALFIT) range mostly over \( \pm 0.2\) mag arcsec\(^{-2}\), except for the regions distorted by the spiral arms at large radii. So we conclude that our global fits are reliable.

### 3.2. Fitting the observed dispersion data

The vertical velocity dispersions of NGC 1566 were gotten from Table 4, column 4 in Bottema (1992); NGC 5247 from Table 1, column 4 in van der Kruit & Freeman (1986). As these two galaxies are not exactly face-on (see Table 1), this will affect the observational data at two aspects: (1) the observed velocity dispersion is not exactly equal to the vertical velocity dispersion. The observed dispersion is approximately 4\% higher than the real vertical dispersion for NGC 1566 (Bottema 1992), \( \sim 5\% \) larger for NGC 5247 (van der Kruit & Freeman 1986). (2) the observed surface brightness only amounts to about \( \langle V^2 \rangle \) for H-band, \( \sim 5\% \) larger for NGC 5247 (van der Kruit & Freeman 1986).

The thickness parameter, \( \alpha \), is the only unknown parameter in this equation if we have assumed the mass-to-light ratio. Alternatively, we can use the surface density-weighted average velocity dispersion, \( \langle V^2 \rangle \), to estimate the thickness parameter \( \alpha \) through the following equation,

\[
\langle V^2 \rangle = \int_0^{r_d} \frac{\pi G}{r} \sigma^2 (r)[1 - P(r)] rd r
\]

When equation (17) is used for measurements, equation (12) would be substituted for \( \sigma (r) \).

### 4. Discussion

So far, we have proposed an method to determine the thickness of the non-edge-on disk galaxies \( H \), based on the photometric result, scale length of disk \( (r_d) \) and extrapolated central surface brightness \( (I_0) \); and spectroscopic result, the vertical stellar velocity dispersion \( \langle V^2 \rangle \). The fundamental assumption required to derive the conclusions presented here is that the disk mass profile is exponential, both along the vertical direction (i.e., \( z \)-direction, see equation (3)), which is different from the isothermal model used by KS, and along the radial direction (i.e., \( r \)-direction, see equation (1)). It has been shown by de Grijs & van der Kruit (1996) that the best fitting vertical model is more peaked than expected for an isothermal sheet distribution and the simple exponential fits turned out to be good approximations of the vertical surface brightness profile. Peletier & de Grijs (1997) also find that the \( K \)-band light (often assumed to trace mass) indeed has a very peaky, almost exponential, vertical distribution in edge-on spirals. By contrast, Gould, Bahcall & Flynn (1996) find that the vertical distribution of Galactic M dwarfs (which probably trace the stellar disk) is somewhat less peaky.

Because our method is intended to work for a pure disk galaxy, the photometric decomposition is another important ingredient affecting the fitted result. As described in Section 3.1, photometric decomposition allows us to identify the real disk region and the range that the bulge/nucleus (if the galaxy has) affects. We also use the data just outside the effective radius \( r_e \) to fit \( \alpha \) for
Table 3. Photometric parameters and least-square fitted results for NGC 1566 and NGC 5247.

| NGC  | $l_0$  | $Y_*$  | $\alpha$  | $H^a$  | Band |
|------|--------|--------|-----------|-------|------|
| 1566 | 1193.6±29.7 | 0.49±0.03 | 0.48      | 1.75±0.56 | 1.14±0.36 | H    |
| 5247 | 328.7±4.5   | 0.91±0.03 | 0.22      | 1.32±0.52 | 1.52±0.60 | H    |

$^a$ Calculated according to $H = 2/\alpha$.

NGC 1566, and we obtain a much bigger $\chi^2$ of 1.8 and a smaller $\alpha$ of 0.96 kpc$^{-1}$. For NGC 5247, we use the data out to the center since there are no data just near the effective radius. This also gives a bigger $\chi^2$ of 0.35 and a smaller $\alpha$ of 1.0 kpc$^{-1}$. This suggests that we need to select the radial range according to the total mass of the bulge/nucleus. The inclination is another ingredient affecting the fitting results, particularly true for the mass-to-light ratio. Applied corrections for different inclinations will result in different mass-to-light ratio, but almost the same thickness parameter. For the same velocity dispersion data, a smaller inclination gives a larger mass-to-light ratio.

The parameter $H$ should be regarded as about twice the scale height of the best fit of an exponential to the actual distribution. We’ve compared our some relevant results, e.g. $r_d/H$ (the scale length to the effective height ratio), $H$ (the effective height) of these three galaxies with those of eight edge-on spirals of KS, which are directly measured by fitting the photometric data. As shown in Table 4, our results are very consistent with KS. This makes us believing that our new method for determining the thickness of non-edge-on disk galaxies is reasonable and feasible. In Figure 5 we plot the thickness versus the Hubble type. The continuous line is the fitted result for the crosses, which represent the median value of the thickness for each type. The trend that the thickness of disks decreases along the Hubble sequence is also found by Ma (2002) and Zhao, Peng & Wang (2004). However, it needs further study on this correlation because the sample used here is rather small.
Fig. 4. Observed (filled triangles) distribution and our fitted results (dotted line) with H-band photometric data for the disk of NGC 5247. The observations are obtained by van der Kruit & Freeman (1986). But the data belonging to the bulge are excluded from this figure and not taken into account for our fit.

Table 4. Parameters of the near face-on galaxies from our study compare with those of edge-on galaxies from the study by van der Kruit & Searle.

| NGC | RC3 Type | $M_\mu^a$ | $r_d^b$ (kpc) | $H^c$ (kpc) | $r_d/H$ | ref |
|-----|----------|-----------|---------------|-------------|---------|-----|
| 1566| Sbc      | -24.0     | 3.3           | 1.1         | 3.0     | our study |
| 5247| Sbc      | -23.2     | 4.8           | 1.5         | 3.2     | our study |
| 891 | Sb       | -23.3     | 4.9           | 0.99        | 5.0     | (1)   |
| 4013| Sb       | -22.4     | 3.4           | 1.1         | 3.1     | (2)   |
| 4217| Sb       | -22.9     | 3.5           | 1.7         | 2.1     | (2)   |
| 4244| Scd      | -20.7     | 2.6           | 0.58        | 4.5     | (3)   |
| 4565| Sb       | -23.7     | 5.5           | 0.79        | 7.0     | (3)   |
| 5023| Scd      | -19.5     | 2.0           | 0.46        | 4.3     | (2)   |
| 5907| Sc       | -23.1     | 5.7           | 0.83        | 6.9     | (2)   |
| 7814| Sab      | -23.5     | 8.4           | 2.0         | 4.2     | (4)   |

$^a$ The magnitudes are all gotten from Jarrett et al. 2003;
$^b$ $r_d$ in KS is represented by $h$;
$^c$ $H$ is represented by $z_0$, which is obtained by fitting the light distribution with $\mu(R, z) = \mu(0, 0)(R/h)K_1(R/h)\text{sech}^2(z/z_0)$, $K_1$ is the modified Bessel function, in KS.

References: (1) van der Kruit & Searle 1981b; (2) van der Kruit & Searle 1982a; (3) van der Kruit & Searle 1981a; (4) van der Kruit & Searle 1982b.

The method presented in this paper is rather difficult to be used for measurements of large galactic samples because the distributions of the vertical stellar velocity dispersions are inconvenient to be obtained in practice. Fortunately, the most essential purpose of the present paper is to test the availability of the method proposed by Zhao et al. (2004), which is on the basis of the density wave theory. Hence, we measured the thickness of the disks of NGC 1566 and NGC 5247 used the method put forward by Zhao et al. (2004) and obtained 0.6 ± 0.4 kpc and 0.7 ± 0.5 kpc, for NGC 1566 and NGC 5247, respectively. These results are somewhat smaller than the results obtained
by using the method shown in this paper. However, these values are also in the range of the measurement error. Therefore, the method presented in Zhao, Peng & Wang (2004), which is more easily to be carried out, might be available and might be used to measure the thickness of the disks of nearly face-on galaxies for large samples.

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Appendix A: Mathematical basis

In order to study the effect of finite thickness of the disk on the dynamical properties of disk galaxies, it is convenient to adopt cylindrical coordinates \((r, \theta, z)\) with the galactic center chosen at the origin of the coordinate system. The galactic plane is depicted by \(z = 0\) and \(\theta\) represents the azimuthal coordinate. For disk galaxies with zero-thickness, the self-gravitational potential \(\Phi\) is governed by Poisson’s equation

\[
\nabla^2 \Phi = 4\pi G\sigma(r, \theta)\delta(z), \tag{A.1}
\]

where \(G\) is the gravitational constant, \(\nabla^2\) denotes the Laplacian operator, \(\delta(z)\) is Dirac’s delta function that confines all the disk mass to the galactic plane and \(\sigma(r, \theta)\) represents the surface density. In regions outside the galactic plane \(z \neq 0\), equation (A.1) then becomes

\[
\nabla^2 \Phi(r, \theta, z) = 0. \tag{A.2}
\]

since there is no distribution of matter outside the disk plane. It is particularly convenient at this point to introduce the Laplace transform for the vertical coordinate \(z\) and the Fourier transform for the azimuthal angle \(\theta\), namely

\[
\Phi(r, \theta, z) = e^{-im\theta} \int_0^\infty U_\beta(x)e^{-\beta|z|}d\beta. \tag{A.3}
\]

Applying these transformations to equation (A.2) we then have

\[
x \frac{d}{dx} \left(x \frac{dU_\beta(x)}{dx}\right) + (x^2 - m^2)U_\beta(x) = 0, \tag{A.4}
\]

where \(x = \beta r\), and the solution to equation (A.4) is the well known Bessel function of order \(m\)

\[
U_\beta(x) \equiv U(\beta r) = -J_m(\beta r). \tag{A.5}
\]
Using equation (A.5) in equation (A.3), we obtain
\[
\Phi(r, \theta, z) = e^{-im\theta} \int_0^\infty [-J_m(\beta r)] e^{-\beta |z|} d\beta. \tag{A.6}
\]
On the other hand, we may integrate Poisson's equation (equation (A.1)) with respect to \(z\) to obtain the following expression for the surface density \(\sigma(r, \theta)\)

\[
\sigma(r, \theta) = \frac{1}{4\pi G} \{ \frac{\partial \Phi}{\partial z} |_{\beta=0} - \frac{\partial \Phi}{\partial z} |_{\beta=\infty} \}. \tag{A.7}
\]

Substituting equation (A.6) in equation (A.3) and equation (A.7) we can recast Poisson equation for disk galaxies with zero thickness in the form

\[
\nabla^2 \{ \int_0^\infty e^{-im\theta} [-J_m(\beta r)] e^{-\beta |z|} d\beta \} = \int_0^\infty e^{-im\theta} 2\beta J_m(\beta r) \delta(z) d\beta, \tag{A.8}
\]

\[
\nabla^2 [J_m(\beta r)e^{-im\theta-\beta|z-z'|}] = -2\beta J_m(\beta r)e^{-im\theta}\delta(z-z'). \tag{A.9}
\]

**Appendix B: Solution of Poisson’s equation for an axisymmetric disk**

On the basis of the mathematical treatment just depicted together with the standard method of Green functions, it is straightforward to derive the appropriate Poisson equation for the three-dimensional disk galaxies with finite thickness

\[
\nabla^2 \varphi(r, z) = 4\pi G \rho(r, z) \tag{B.1}
\]

where the vertical distribution of matter is depicted by Parenko's law, \(\rho(r, z) = \frac{\sigma(r)}{2\pi} e^{-|z|^2}\) (see equation (3) in the text). To proceed, we apply the Bessel-Fourier transform to \(\sigma(r)\) to obtain

\[
\sigma(r) = \int_0^\infty \beta J_0(\beta r) S(\beta) d\beta, \tag{B.2}
\]

\[
S(\beta) = \int_0^\infty r J_0(\beta r) \sigma(r) dr. \tag{B.3}
\]

where \(S(\beta)\) is the Bessel-Fourier transform for \(\sigma(r)\). Substituting equations (A.1) and (B.2) into equation (B.1), the three-dimensional Poisson equation can be rewritten as

\[
\nabla^2 \varphi(r, z) = 2\pi G \alpha \cdot \int_{-\infty}^{\infty} e^{-|z'|^2} d\beta \int_0^\infty \beta J_0(\beta r) S(\beta) \delta(z - z') d\beta. \tag{B.4}
\]

Compare equation (B.4) with equation (A.9) we then find the formal solution of the gravitational potential for a galactic disk with scale height \(H = 2/\alpha\)

\[
\varphi(r, z) = -2\pi G \alpha \int_0^\infty J_0(\beta r) S(\beta) F(\alpha, \beta, z) d\beta, \tag{B.5}
\]

where

\[
F(\alpha, \beta, z) = \frac{1}{\beta^2 - \alpha^2} \left[ \beta e^{-\alpha|z|} - \alpha e^{-\beta|z|} \right]. \tag{B.6}
\]

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