Observation of quantum-limited heat conduction over macroscopic distances

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The emerging quantum technological apparatuses\textsuperscript{1,2}, such as the quantum computer\textsuperscript{3–5}, call for extreme performance in thermal engineering at the nanoscale\textsuperscript{6}. Importantly, quantum mechanics sets a fundamental upper limit for the flow of information and heat, which is quantified by the quantum of thermal conductance\textsuperscript{7,8}. The physics of this kind of quantum-limited heat conduction has been experimentally studied for lattice vibrations, or phonons\textsuperscript{9}, for electromagnetic interactions\textsuperscript{10}, and for electrons\textsuperscript{11}. However, the short distance between the heat-exchanging bodies in the previous experiments hinders the applicability of these systems in quantum technology. Here, we present experimental observations of quantum-limited heat conduction over macroscopic distances extending to a metre. We achieved this striking improvement of four orders of magnitude in the distance by utilizing microwave photons travelling in superconducting transmission lines. Thus it seems that quantum-limited heat conduction has no fundamental restriction in its distance. This work lays the foundation for the integration of normal-metal components into superconducting transmission lines, and hence provides an important tool for circuit quantum electrodynamics\textsuperscript{12–14}, which is the basis of the emerging superconducting quantum computer\textsuperscript{15}. In particular, our results demonstrate that cooling of nanoelectronic devices can be carried out remotely with the help of a far-away engineered heat sink. In addition, quantum-limited heat conduction plays an important role in the contemporary studies of thermodynamics such as fluctuation relations and Maxwell’s demon\textsuperscript{16,17}. Here, the long distance provided by our results may, for example, lead to an ultimate efficiency of mesoscopic heat engines with promising practical applications\textsuperscript{18}.

The quantum of thermal conductance, $G_Q$, provides the fundamental upper limit for heat conduction through a single channel\textsuperscript{7}. This limit applies to fermions and bosons as the carriers of heat as well as to so-called anyons obeying even more general statistics\textsuperscript{8}. Although a few observations of quantum-limited heat conduction have been reported, the studied distances were shorter than 100 $\mu$m in all previous experiments: phononic heat conduction through four parallel submicrometre dielectric wires each supporting four vibrational modes\textsuperscript{9}, electromagnetic heat conduction in a superconducting loop over a 50-$\mu$m distance\textsuperscript{10,19}, and electronic heat conduction through an extremely short quantum point contact engineered in a two-dimensional electron gas\textsuperscript{11}. Photons, unlike many other carriers of heat, can travel macroscopic distances without significant scattering, for example, in optical fibres or superconducting waveguides. Thus, photons seem ideal for long-distance thermal engineering and provide attractive opportunities for various quantum thermodynamics experiments\textsuperscript{16}. To our knowledge, however, itinerant photons have not been previously employed in experimental studies of the quantum of thermal conductance.

In this Letter, we experimentally study heat conduction through a single channel formed by photons travelling in a long superconducting waveguide in a single transverse mode. This scheme supports a photonic thermal conductance very close to $G_Q$. Since this heat transport does not directly depend on the temperature of the substrate phonons, it provides an efficient method for remote temperature control. The superconducting waveguide is terminated at both ends by resistors composed of mesoscopic normal-metal islands (Islands A and B in Fig. 1). We measure the temperatures of the Islands A and B, and vary the temperature of Island B. A characteristic signal of photonic heat transport in our experiments is the increasing response of the temperature of Island A to the controlled temperature changes of Island B with decreasing phonon bath temperature (Figs. 2 and 3). The measured temperatures well agree with the thermal model, implying that the heat conduction essentially reaches the quantum limit. Furthermore, the most important features of the measurement results can be quantitatively predicted from a theoretical model involving no free parameters (Fig. 3). These observations constitute firm evidence of quantum-limited heat conduction over macroscopic distances.

Figure 1 shows the structure of the sample used in the experiments together with the measurement scheme. We study several samples with different parameters as presented in Table I. The length of the coplanar waveguide is either 20 cm or 1 m, and it has a double-spiral structure on a silicon chip with the size of $1 \times 1 \text{ cm}^2$ or $2 \times 2 \text{ cm}^2$, respectively. The electron temperatures of the normal-metal islands are measured and controlled with the help of normal-metal–insulator–superconductor (NIS) junctions (Methods). There are four nominally identical NIS junctions at each island. In all samples, the normal-metal islands terminating the waveguide have two galvanic contacts to superconducting lines: one to the centre conductor of the waveguide and the other to the ground plane. The waveguide has a centre conductor width of 10 $\mu$m, and a separation between the ground plane and the centre conductor of 5 $\mu$m. In the control
TABLE I | Main parameters of the measured samples. Columns show the waveguide lengths, normal-metal resistances $R_i$, $i \in \{A, B\}$, normal-metal materials, and normal-metal volumes (length × width × thickness). Furthermore, $G_T/G_Q$ provides the estimated ratio of the realised photonic thermal conductance and the quantum of thermal conductance at temperatures of approximately 150 mK.

| Sample | Length (m) | $R_\text{i}$ (Ω) | Material | Volume (nm³) | $G_T/G_Q$ |
|--------|-----------|----------------|----------|-------------|-----------|
| A1     | 0.2       | 65            | Cu       | $5000 \times 300 \times 20$ | 98 %      |
| A2     | 1.0       | 75            | AuPd     | $3200 \times 300 \times 40$ | 94 %      |
| A3     | 0.2       | 150           | AuPd     | $3200 \times 300 \times 20$ | 60 %      |
| Control| 0.2       | 100           | AuPd     | $3200 \times 300 \times 20$ | 0 %       |

Island B are used for temperature control whereas the current-biased ($I_{th,A}$, $I_{th,B}$) junctions at both islands are used for thermometry. The thermometer calibration data for Sample A1 is shown in Fig. 5b, c. Here, we focus mainly on the analysis of Sample A1, which exhibits the highest photonic thermal conductance according to the model. It also achieves the lowest electron temperature, below 90 mK at the 10-mK base temperature of the cryostat. The other samples show higher minimum electron temperatures, which increases the uncertainty in their thermometry at the very low temperatures.

We analyse the thermal conductance between the Islands A and B, $G_{AB}$, and those to the phonon bath, $G_A0$ and $G_B0$, as schematically presented in Fig. 1b. By linearising the heat flows of Island A at small temperature differences, energy conservation yields a differential temperature response: $dT_A/dT_B = G_{AB}/(G_{AB} + G_A0)$. If the conductance $G_{AB}$ between the islands is quantum limited and the heat conduction to the bath stems from qualitatively different phenomena, $G_{AB}$ dominates over $G_A0$ at low enough temperatures. Thus, the temperature response generally tends to unity with decreasing temperatures.

The photonic net power flow from normal-metal Island A to B is given by (Methods and Ref. 20)

$$P_T = \int_0^\infty \frac{d\omega}{2\pi} \hbar \omega |t(\omega)|^2 \left[ \frac{1}{\exp(\frac{-\hbar\omega}{k_B T_A}) - 1} - \frac{1}{\exp(\frac{-\hbar\omega}{k_B T_B}) - 1} \right]$$

(1)

where $\hbar$ is the reduced Planck constant, $k_B$ is the Boltzmann constant, and $|t(\omega)|^2$ is a transmission coefficient that depends on the photon angular frequency $\omega$, the characteristic impedance of the transmission line, and the resistances of the terminating normal-metal islands (see Eq. 9 for details). If the characteristic impedance of the transmission line equals the island resistances, we have $|t(\omega)|^2 = 1$. In this case, an analytical solution is obtained, $P_T = \frac{\pi k_B^2}{12}\left(T_A^2 - T_B^2\right)$, which can furthermore be expressed in terms of the quantum of thermal conductance as $P_T = G_Q(T_A - T_B)$, where $G_Q = \frac{\pi k_B^2 T}{6h}$ with $T = (T_A + T_B)/2$. The thermal conductance to the
phonon bath at temperature $T_0$ can be approximated by the electron–phonon conductance as $G_{A0} \approx G_{ep,A} = 5\Omega_N\Omega_A T_0^3$ in Island A (Methods). Here, $\Omega_N$ is a material parameter describing the strength of the electron–phonon coupling in the normal metal, and $\Omega_A$ is the volume of Island A. Thus, one obtains a simple theoretical prediction without any free parameters based on quantum-limited photonic heat conduction and electron–phonon coupling

$$\frac{dT_A}{dT_B} = \frac{1}{1 + aT_0^2} \tag{2}$$

where $a = 30\Omega_N\Omega_A h/(\pi k_B^2)$ is a predetermined constant. We also devise a full thermal model shown in Fig. 5a for a more accurate description of the heat flows (Methods).

Figure 2 shows the measured island temperatures for Sample A1 at various bath temperatures together with the corresponding results from the full thermal model. The experimental observations are in good agreement with the simulations over a wide range of bath temperatures and bias voltages. The maximum cooling of Island B is obtained at its bias voltages $V_B \leq 2\Delta/e \approx 0.45$ mV, where $\Delta$ is the superconductor energy gap and $e$ is the elementary charge. The clearly observed cooling of distant Island A cannot be attributed to phonons since the phonon bath temperature increases as a result of the net heating power $|V_B I_B|$ dissipated during cooling of Island B. In addition to phonons, quasiparticles may contribute to the heat conduction, but their effect is weak and qualitatively in contradiction with our observations at low temperatures. Furthermore, the essentially vanishing temperature response for the control sample in Fig. 2f indicates that the photonic channel dominates in the heat conduction between the islands in Sample A1.

To accurately analyse the photonic heat conduction, we show the temperature of Island A in Fig. 3a as a function of the electron temperature of Island B for different bath temperatures. The measured temperatures of Island A in Fig. 3a are insensitive to heat conduction mechanisms that only involve Island B and its reservoirs. Therefore, this analysis method provides a robust way of studying photonic heat conduction. In Sample A1, the curvatures of $T_A$ as a function of $T_B$ are negative, which is in stark contrast to the positive curvature observed for the control sample. This fundamental difference is due to the absence of the photonic heat conduction in the control sample. In Samples A2 and A3 (Table I), the curvatures resemble that of A1 (data not shown).

At high bath temperatures, $T_A$ is almost independent of $T_B$, which is a consequence of the strong electron–phonon coupling. Figure 3b shows the differential temperature response, $dT_A/dT_B$, extracted at the lowest $T_B$ obtained for each bath temperature. The steep increase in $dT_A/dT_B$ at low bath temperatures is a signature of the photonic heat conduction: the thermal conductance between the islands, $G_{AB}$, determined by the photonic heat conduction, dominates over the conductance to the bath, $G_{A0}$.

In addition, Fig. 3b shows a prediction of the simplified model according to Eq. 2. Despite its simplicity, it captures the essential features of the experimental data of Samples A1 and A2 which exhibit photonic heat conduction very close to the quantum limit, $G_{A0}$. The deviation between the data and the simplified model at high temperatures is due to the neglected quasiparticle heat conduction between the islands and their reservoirs which increases $G_{A0}$. At low temperatures, on the other hand, the discrepancy increases due to the saturation of the electron temperatures not present in the simplified model. These observations bring insight to the good agreement between the experimental observations and the full thermal model.

In summary, we experimentally demonstrate quantum-limited heat conduction over macroscopic distances. The on-chip design of the resistors enables their straightforward utilisation in a multitude of different applications, including the initialization of quantum bits by con-
trolled cooling and remote cooling of other quantum-technological components. The methods developed in this study may also be used in the future to implement efficient heat transfer between separate chips and temperature stages of the cryostat. For example, the remotely cooled quantum device may be operated at a typical base temperature whereas the cold reservoir may be located at a lower-temperature stage which is incompatible with the relatively large power consumption of the actual device. The cooling distance we demonstrate here is sufficient for practically all present-day applications.

METHODS

Sample fabrication
The samples are fabricated in a multi-step process on 0.5-mm-thick silicon wafers with 300-nm-thick thermally grown silicon oxide layers. The transmission lines are fabricated in an optical-lithography process using a mask aligner and an electron beam evaporator. The wafers are cleaned with reactive ion etching before the metal deposition. The Al film has a thickness of 200 nm, on top of which films of Ti and Au are deposited with thicknesses of 3 and 5 nm, respectively, to prevent oxidation.

The nanostructures are fabricated with electron beam lithography. The mask consists of poly(methyl methacrylate) and poly[(methyl methacrylate)-co-(methacrylic acid)] layers, which enable a large undercut necessary for three-angle shadow evaporation. Prior to the metal deposition, the samples are cleaned with argon plasma in the electron beam evaporator. As the first metal, we deposit an Al layer which is oxidised in situ introducing the insulator layer for the NIS junctions. Subsequently, a layer of normal metal is deposited followed by a layer of Al. The normal metal is either AuPd (mass ratio 3:1) or Cu. Lift-off of the excess metal is performed with acetone followed by cleaning with isopropanol.

Measurements
The electrical measurements are performed at millikelvin temperatures achieved with a commercial cryogen-free dilution refrigerator. The chip is attached to a sample holder containing a printed circuit board (PCB), to which the sample is connected by Al bond wires. The PCB is connected to room-temperature measurement setup with lossy coaxial cables. To suppress electrical noise, the power-line-powered devices are connected to the sample through opto-isolators. Battery-powered amplifiers and voltage and current sources are connected to the sample without opto-isolation. The voltage \( V_B \) is swept slowly (down to 1 \( \mu \)V/s) to avoid apparent hysteresis. Furthermore, the measurements are repeated several times, and the data points with clear disturbance from random external fluctuations are excluded.

Photonic heat conduction
Here, we derive Eq. (1) for the photonic heat conduction starting from the first-principles circuit quantum electrodynamics. Previously, our case of two islands coupled with a transmission line has been studied with the help of classical circuit theory\textsuperscript{20}. These results can also be obtained using path integrals\textsuperscript{21}. In contrast, we analyse the system using methods discussed in ref. 22. In particular, the Heisenberg equations of motion are given by Kirchhoff’s circuit laws. Furthermore, a terminating resistor can be treated as a semi-infinite transmission line with a characteristic impedance equal to its resistance\textsuperscript{22}.

We express the boundary conditions originating from the Kirchhoff’s laws for the photon annihilation operators defined in Fig. 4 as

\[
\frac{1}{\sqrt{Z_0}} (\hat{b}_L - \hat{b}_R) = -\frac{1}{\sqrt{Z_A}} (\hat{a}_R - \hat{a}_L)
\]

\[
\frac{1}{\sqrt{Z_0}} (\hat{c}_R - \hat{c}_L) = -\frac{1}{\sqrt{Z_B}} (\hat{d}_L - \hat{d}_R)
\]

\[
\sqrt{Z_0} (\hat{b}_L + \hat{b}_R) = \sqrt{Z_A} (\hat{a}_R + \hat{a}_L)
\]

\[
\sqrt{Z_0} (\hat{c}_R + \hat{c}_L) = \sqrt{Z_B} (\hat{d}_L + \hat{d}_R)
\]

\[
\hat{c}_R = e^{i\phi} \hat{b}_R
\]

\[
\hat{b}_L = e^{i\phi} \hat{c}_L
\]
where $\phi = \omega s/v$ is the phase shift obtained by a wave with angular frequency $\omega$ and velocity $v$ when travelling over distance $s$. Assuming no photons coming from the right, $d_L = 0$, we can solve the transmission coefficient $t$ as defined as $d_R = t(\omega)\hat{a}_R$. Thus, we obtain

$$|t(\omega)|^2 = \frac{2}{1 + \frac{R_A^2 + R_B^2}{2R_A R_B} + \frac{r_A^2 r_B^2 + r_A^2 r_B^2 + r_A^2 r_B^2 + r_A^2 r_B^2}{2R_A R_B} \sin^2(\phi)}$$

The transmission coefficient is symmetric with respect to the exchange of resistances $R_A$ and $R_B$. In a matched case, $R_A = R_B = Z_0$, Eq. (9) simplifies to $|t(\omega)|^2 = 1$.

Energy dissipation at the resistor $R_B$ can be obtained from the average photon flux to the right in the transmission line with a characteristic impedance $R_B$ multiplied by the energy carried by each photon. Here, the zero-point energy does not appear in the dissipated power. Thus, the power per unit frequency can be expressed as

$$P_{\omega}(\omega) = \frac{h\omega |t(\omega)|^2 (\hat{a}_R^\dagger \hat{a}_R)}{1 - \exp(-\frac{h\omega}{k_B T_A})}$$

since the number of photons emitted by the left resistor is given in thermal equilibrium by the Bose–Einstein distribution. Due to symmetry, the power transfer to the opposite direction is given by

$$P_{\omega}(-\omega) = \frac{h\omega |t(\omega)|^2 (\hat{a}_L^\dagger \hat{a}_L)}{1 - \exp(-\frac{h\omega}{k_B T_B})}$$

The net photonic heat transport from $R_A$ to $R_B$ is, therefore, by

$$P_{\omega} = \int_0^{\infty} \frac{d\omega}{2\pi} (P_{\omega} - P_{\omega}(-\omega))$$

In the special case of a vanishing waveguide length, $s \to 0$, Eq. (9) yields $|t(\omega)|^2 = 4R_A R_B/(R_A + R_B)^2$ which is identical to the result considered in ref. 23 for two resistors in a loop. On the other hand, if one sets $Z_0$ to be inversely proportional to $s$ and takes the limit $s \to 0$, one obtains $|t(\omega)|^2 = 4R_A R_B/[((R_A + R_B)^2 + X^2)]$ with a reactance $X = Z_0 \omega s/v$. This result reproduces that of two resistances connected in a loop with a series reactance.

**NIS thermometry**

The quasiparticle current through an NIS junction with tunnelling resistance $R_T$ is given in the sequential-tunnelling theory by

$$I(V, T_N) = \frac{1}{e R_T} \int_0^\infty n_S(E) [f(E - eV, T_N) - f(E + eV, T_N)] dE$$

where $T_N$ is the normal-metal electron temperature, and $V$ the voltage across the junction. Here, the Fermi–Dirac distribution is given by

$$f(E, T) = \frac{1}{e^{E/(k_B T)} + 1}$$

and the superconductor density of quasiparticle states assumes the form

$$n_S(E) = \left| \text{Re} \frac{E/\Delta + i\gamma}{\sqrt{(E/\Delta + i\gamma)^2 - 1}} \right|$$

Above, $\gamma$ is the Dynes parameter accounting for the sub-gap current, and $\Delta$ is the superconductor energy gap. Experimentally, $\gamma$ is obtained as the ratio of the asymptotic resistance at large voltages and the resistance at zero voltage. We note that Eq. (13) has a very weak dependence on the temperature of the superconductor through the temperature dependence of $\Delta$. Thus, an NIS junction can be used as a thermometer probing the electron temperature of the normal-metal. We apply a constant current, and deduce the temperature from the measured voltage according to a calibration curve shown in Fig. 5.

**Thermal model**

In the full thermal model illustrated in Fig. 5a, we consider several heat transfer mechanisms: Firstly, the NIS junctions produce heat flows between the normal-metal islands and the superconducting leads. Secondly, the electrons in the normal metal exchange heat with the phonon bath. Thirdly, the islands exchange heat with each other by photons travelling in the transmission line. Finally, the model takes into account geometrical properties of the samples as well as properties specific to the measurement setup.

The NIS junctions can be used for cooling and heating of the normal metal, and the power out of the normal
metal can be computed from
\[ P_{\text{ideal}} = \frac{1}{e^2 R_f} \int_{-\infty}^{\infty} \kappa_N(E) (E-eV)[f(E-eV,T_N)-f(E,T_S)]dE \] (16)

We model the nonidealities in the NIS power by assuming a constant fraction, \( \beta \), of the power flowing to the superconductor to flow back to the normal metal. Thus, the back-flow power can be written as
\[ P_{\text{bf}} = \beta(IV + P_{\text{ideal}}) \] (17)

where \( IV \) gives the total power. Consequently, the total cooling power of an NIS junction is given by
\[ P_{\text{NIS}} = P_{\text{ideal}} - P_{\text{bf}} \] (18)

The physical background for the back flow has been studied in ref. 24. A factor of 2 is included in the power when two NIS junctions are connected to form an SINIS structure. Since the thermometers are based on similar NIS junctions, their powers are calculated with the same equations as for the actual power used to control the temperature of Island B. However, the voltages across the thermometer junctions must first be solved using Eq. (13), the island temperature and the thermometer bias current.

The electrons in the normal metal are coupled to the phonon bath, and the heat flow is given by
\[ P_{\text{ep},i} = \Omega_i \Sigma_N (T_i^5 - T_0^5) \] (19)

Here, \( \Omega_i \) is the volume of normal-metal block \( i \in \{A, B, AR, BR\} \). For Cu and AuPd, the parameter \( \Sigma_N \) is typically between \( 2 \times 10^9 \) and \( 4 \times 10^9 \) \( \text{WK}^{-5} \text{m}^{-3} \).

In the simulations, we use values \( 2.0 \times 10^9 \) \( \text{WK}^{-5} \text{m}^{-3} \) and 3.0 \( \times 10^9 \) \( \text{WK}^{-5} \text{m}^{-3} \) for Cu and AuPd, respectively, unless otherwise mentioned. The normal metal under the superconductors at the ends of the islands are excluded from the volume in the simulations due to the superconductor proximity effect. For small temperature differences, \( T_i \approx T_0 \), one obtains
\[ P_{\text{ep},i} = G_{\text{ep},i} (T_i - T_0) \] (20)

where \( G_{\text{ep},i} = 5 \Sigma_N \Omega_i T_0^4 \).

We account for heat leaks from a high-temperature environment by including constant heating powers to both islands, \( P_{\text{leak},A} \) and \( P_{\text{leak},B} \). They are fixed by the saturation of the electron temperature observed in Fig. 5 at low bath temperatures.

In the thermal model, we consider quasiparticle heat conduction only from the islands to their near-by normal-metal reservoirs. The reservoirs are a consequence of the three-angle evaporation method, and they provide an additional channel for thermalisation to the phonon bath. At both islands, there are actually two reservoirs which are presented as one in Fig. 5a for simplicity. The extremely weak quasiparticle heat conduction from one island to the other over a distance longer than 5 mm is included in the parasitic heat conduction as discussed below. The power flow at the normal-metal block \( i \) due to the quasiparticles is given by
\[ P_{\text{qp},i} = \kappa_S A T'(x_i) \] (21)

where \( T'(x_i) \) is the derivative of the temperature in the superconductor with respect to the position coordinate \( x_i \), and \( A \) is the cross section of the line. The superconductor heat conductivity, \( \kappa_S \), is related to the normal-state heat conductivity, \( \kappa_N \), at a temperature \( T \) by
\[ \kappa_S = \gamma(T) \kappa_N \] (22)
where \( \tilde{\gamma} \) is a suppression factor

\[
\tilde{\gamma}(T) = \frac{3}{2\pi^2} \int_{\Delta/(k_B T)}^{\infty} \frac{t^2}{\cosh^2(t/2)} \, dt
\]  

(23)

The normal-state heat conductivity of the line is obtained from the Wiedemann–Franz law as

\[
\kappa_N = \frac{L_0 T(x)}{\rho}
\]  

(24)

where \( \rho \) is the normal-state electric resistivity of the line, and \( L_0 = 2.4 \times 10^{-8} \text{ W}\Omega\text{K}^2 \) is the Lorenz number. The temperature profile in the superconducting lines can be calculated using a heat diffusion equation\(^{19}\). However, the electron–phonon coupling in a superconducting state is greatly suppressed with respect to that of a normal-state\(^{26}\). Thus, we neglect the electron–phonon coupling in the leads and assume here a linear temperature profile.

Andreev current plays a minor role in our experiments since the induced temperature changes at the islands are small in the subgap voltage regime where it may dominate\(^{37}\). Therefore, we do not consider it in the thermal model.

We observe a weak island-to-island heat transport also in the control sample, in which the centre conductor is shunted as shown in Fig. 6. We model this parasitic heat transport by letting a constant proportion, \( \alpha \), of the total input power at Island B to flow into Island A,

\[
P_p = \alpha I_B V_B
\]  

(25)

The exact mechanism of the parasitic channel remains unknown, and the heat flow may depend on the sample geometry. The parasitic heat conduction extracted from the control sample includes all the heat conduction channels from one island to the other except the photonic heat conduction which is essentially absent due to the shunt. This heat flow may be attributed to quasiparticles since they can travel long distances before recombination, especially at low bath temperatures. Furthermore, although the electric contact of the shunting metal block between the ground plane and the centre conductor is of very low impedance, small residual photonic heat conduction cannot be fully excluded. Nevertheless, the parasitic heat conduction is much weaker than the total heat conduction in the actual devices. We note that the parasitic heat conduction is only added to the model for more accurate description at high heating powers.

We solve the heat balance equations for both islands and both reservoirs simultaneously. The equations can be expressed as (Fig. 5a)

\[
\begin{align*}
P_T + P_{\text{th},A} - P_{\text{leak}, A} - P_p + P_{\text{ep}, A} + P_{\text{qp}, A} &= 0 \\
P_{\text{NIS}} - P_T + P_{\text{th}, B} - P_{\text{leak}, B} + P_p + P_{\text{ep}, B} + P_{\text{qp}, B} &= 0 \\
P_{\text{ep}, \text{AR}} - P_{\text{qp}, A} &= 0 \\
P_{\text{ep}, \text{BR}} - P_{\text{qp}, B} &= 0
\end{align*}
\]  

(26) \hspace{1cm} (27) \hspace{1cm} (28) \hspace{1cm} (29)

These equations yield the temperatures \( T_i \), \( i \in \{A, B, \text{AR}, \text{BR}\} \) for a given phonon bath temperature, \( T_0 \), and bias voltage, \( V_B \), both of which are accurately controlled.

The parameters used in the full thermal model are shown in Table II. In the simulations, we slightly adjust the quasiparticle heat conductivity for improved agreement between the model and the experiments. More specifically, we set the temperature in Eq. (23) to be equal to the island temperatures increased by a small constant value and, in addition, we set the suppression factor to saturate at low temperatures. Hence, we introduce a replacement \( \tilde{\gamma}(T) \rightarrow \tilde{\gamma}(T + T_{\text{const}}) + \tilde{\gamma}(T_{\text{satur}}) \). This approximation can be justified by several arguments. Firstly, the superconductor heat conductivity depends on the purity of the sample\(^{29}\). Secondly, the superconductor energy gap has been observed to increase at small film thicknesses\(^{80}\). We use for all superconductors the same value, which is obtained from the current–voltage measurements of the NIS junctions, although the leads are thicker. The possibly smaller actual energy gap effectively corresponds to higher temperatures. Thirdly, the neglected electron–phonon coupling in the superconducting leads may result in nonlinear temperature profile increasing the quasiparticle heat conduction. The impurities in the sample may increase the electron–phonon coupling. Fourthly, the heat leakage through the measurement cables from a high-temperature environment and other possible heat leak mechanisms may increase the temperature of the superconductors. Increased quasiparticle densities have been observed previously, and they can be suppressed by effective shielding and enhanced relaxation\(^{31}\). Weak quasiparticle recombination can induce elevated quasiparticle temperatures. However, we increase only the heat conductivity and consider a linear temperature profile in the lead between the island and the near-by reservoir. In the simulations, the reservoirs have effective volumes somewhat larger than their physical volumes, thus, taking into account the quasiparticles thermalising in the reservoirs and the ones recombining.

\[\text{FIG. 6 | Control sample. SEM image of the control sample.}\]

The center conductor of the coplanar waveguide is shunted to the ground plane at the location indicated by the black arrow.
Table II | Simulation parameters for Sample A1. Sample dimensions are based on the actual sample. \( I_{th,A} \) and \( I_{th,B} \) are externally controlled, \( \Delta, \gamma \) and resistances are extracted from independent current–voltage characteristics, \( \rho \) assumes a typical value for evaporated aluminium, \( L_l \) is calculated analytically. \( C_f \) is based on a finite-element simulation. \( P_{\text{leak},A} \) and \( P_{\text{leak},B} \) are obtained from island temperature saturation at low phonon bath temperatures in Fig. 5, \( \alpha \) is extracted from the control sample, \( \beta \) is obtained from the nonideal cooling power of the NIS junctions, \( \Omega_{AR} \) and \( \Omega_{HR} \) are the actual volumes multiplied by 2 to account also for the quasiparticle recombination in the superconductors, and \( T_{\text{satur}} \) and \( T_{\text{const}} \) are fitting parameters assuming realistic values.

| Parameter                                                      | Symbol         | Value                  | Unit       |
|----------------------------------------------------------------|----------------|------------------------|------------|
| Island volume                                                 | \( \Omega_A, \Omega_B \) | 4500 \times 300 \times 20 | nm\(^3\)   |
| Effective reservoir volume                                     | \( \Omega_{AR}, \Omega_{HR} \) | 10 \times \Omega_A |            |
| Cross section of lines from island to reservoir                | \( A \)       | 300 \times 100          | nm\(^2\)   |
| Distance from island to reservoir                              | \( l_{IR} \)   | 24                      | \( \mu m \) |
| Waveguide length                                               | \( s \)        | 0.193                   | m          |
| Superconductor energy gap                                      | \( \Delta \)   | 224                     | \( \mu eV \) |
| Inductance per unit length                                     | \( L_l \)      | 4.14 \times 10\(^{-7}\) | H/m        |
| Capacitance per unit length                                    | \( C_l \)      | 1.51 \times 10\(^{-10}\) | F/m        |
| Thermometer bias current                                       | \( I_{th,A}, I_{th,B} \) | 18                      | pA         |
| Resistivity of Al lines in normal state                         | \( \rho \)     | 1.0 \times 10\(^{-8}\) | \( \Omega m \) |
| Dynes parameter                                                | \( \gamma \)   | 1.05 \times 10\(^{-4}\) |            |
| Material parameter for Cu                                       | \( \Sigma_N \)  | 2.0 \times 10\(^{9}\) | W K\(^{-5}\) m\(^{-3}\) |
| Normal state junction resistance                                | \( R_T \)      | 15.5                    | k\( \Omega \) |
| Island resistance                                              | \( R_A, R_B \)  | 65                      | \( \Omega \) |
| Characteristic impedance                                       | \( Z_0 \)      | \( \sqrt{l_l/C_l} \)   |            |
| Phase velocity                                                 | \( v \)        | \( 1/\sqrt{C_lL_l} \)  |            |
| Lorenz number                                                  | \( L_0 \)      | 2.4 \times 10\(^{-8}\) | W \( \Omega \) K\(^{-2}\) |
| Saturation quasiparticle temperature for heat conductivity     | \( T_{\text{satur}} \) | 0.184                   | K          |
| Additional quasiparticle temperature for heat conductivity     | \( T_{\text{const}} \) | 0.036                   | K          |
| Parasitic heat conduction parameter                            | \( \alpha \)   | 3 \times 10\(^{-4}\)   |            |
| NIS back flow constant                                         | \( \beta \)    | 0.056                   |            |
| Heat leak                                                      | \( P_{\text{leak},A}, P_{\text{leak},B} \) | 1.6          | fW         |

in the superconductors. The requirement of the effective volume may also be explained by the uncertainty in the employed literature value of the electron–phonon coupling constant.

**Additional control samples without resistors**

We also fabricated and measured control samples without the normal metal resistors terminating the transmission line. Instead, the transmission line is connected to input and output ports through coupling capacitors. The capacitors and the transmission line form a resonator, the quality factor of which can be measured. Using an LCR model, we extract the internal quality factor of the system to be of the order of 60,000 indicating a negligibly weak effect in the heat conduction experiments. In fact, the energy losses due to the observed finite quality factor would only limit the photonic heat conduction beyond distances of the order of a kilometer.

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