On magnetic and vortical susceptibilities of the Cooper condensate

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Abstract

We discuss the susceptibility of the Cooper condensate in the s-wave 2+1 superconductor in the external magnetic field and in the rotating frame. The extended holographic model involving the charged rank-two field is considered and it is argued that the susceptibility does not vanish. We interpret non-vanishing susceptibilities as the admixture of the p-wave triplet component in the Cooper condensate in the external field.
I. INTRODUCTION

The ground state of conventional superconductors (SC) involves the Cooper pairs in the different spin and orbital states forming the charged condensate. We shall be interested in the specific respond of the ground state of 2 + 1 dimensional s-wave superconductor to the external magnetic field and rotation namely if the p-wave component of the condensate proportional to the external field is generated which is absent otherwise. The known examples of such phenomena one could have in mind are the generation of the triplet component via the Rashba term [1] or via spin-orbit interaction [2] in the singlet s-wave SC.

The interesting possibility of coexistence of the triplet and singlet order parameters in the SC occurs in the specific materials admitting the coexistence of the SC and antiferromagnetic (AF) orders. In this case at least in the model description the following relation takes place

\[ |\Delta_t| \propto |\Delta_s| |M| \]  

where \( \Delta_t, \Delta_s \) are the SC order parameters for triplet and singlet states while \( M \) is the AF magnetization order parameter. A bit loosely it can be claimed that the AF component induces the triplet Cooper pairing. In what follows our consideration has some similarities with this situation.

In the ground state of the conventional SC the magnetic field is screened by the supercurrent and is expelled from the bulk penetrating through the Abrikosov strings only. However one can imagine that the triplet spin component of the condensate can be generated by the external field in the bulk of the singlet SC. We shall look at the magnetic susceptibility of the Cooper condensate in the weak magnetic field defined as

\[ <0|\psi\sigma_{\mu\nu}\psi|0> = g\chi_s <0|\psi\psi|0|F_{\mu\nu} \] 

where \( g \) is dimensionful gauge coupling in 2 + 1. We will be interested if \( \chi_s \neq 0 \) and discuss this issue from the holographic viewpoint.

There is the well-known analogy between the external magnetic field and the rotation which is encoded in the specific form of external metric, see for example, the recent discussion in [4]. Therefore it is natural to consider the responses of the ground state of superconductor at the external magnetic and gravimagnetic field in parallel. To this aim we shall also introduce and discuss the vortical susceptibility of the Cooper condensate in the rotating
The partial motivation for this study is as follows. Consider the hadronic phase in QCD where the chiral symmetry is broken by the condensate \( \langle \bar{\Psi} \Psi \rangle \). The linear response of chiral condensate to the external magnetic field is parametrized as follows

\[
\langle 0 | \bar{\Psi} \sigma_{\mu\nu} \Psi | 0 \rangle = \chi_F \langle \bar{\Psi} \Psi \rangle F_{\mu\nu},
\]

where \( \chi_F \) is the magnetic susceptibility of the condensate introduced in [5]. The value of \( \chi_F \) can be derived by the different means. In particular, its value can be obtained from the anomalous CS terms in the conventional holographic model [6, 8] and in the extended model with additional rank-two fields [7].

Recently, the vortical susceptibility for the quark condensate was introduced and evaluated in the dense QCD via specific anomaly in the dense matter [9]

\[
\langle 0 | \bar{\Psi} \sigma_{\mu\nu} \Psi | 0 \rangle = \chi_G \langle \bar{\Psi} \Psi \rangle G_{\mu\nu},
\]

where \( G_{\mu\nu} \) is the curvature of the graviphoton field. The vortical susceptibility is the response of the chiral condensate to the external gravitational field corresponding to the rotation frame.

Of course, there are some differences between QCD chiral condensate and superconducting condensates. The chiral condensate is neutral while the Cooper condensate is charged. In QCD, the analogue of the Cooper condensate occurs only at high density in the color-flavor locking superconducting phase while the chiral condensate corresponds to the neutral exciton condensate in the condensed matter context. Let us emphasize that in the superconductor case contrary to QCD we deal with the non-relativistic system.

In this Letter, we consider the susceptibility of the SC \( \chi_s \) combining the conventional and holographic means. To this aim, we consider the 2+1 SC described by the \( AdS_4 \) bulk geometry in the extended holographic model which involves complex scalar, U(1) gauge field and rank-two field. We argue that both magnetic and vortical susceptibilities do not vanish.

The paper is organized as follows. In Section 2, we remind the simplest holographic models of s-wave superconductor. In Section 3, we introduce the polarization of the Cooper condensate in the magnetic field. Section 4 involves the arguments showing that the magnetic susceptibility does not vanish for 2+1 and 3+1 cases. In Section 5, we make a few comments concerning the vortical susceptibility of the condensate. The results and open questions are summarized in the Conclusion.
II. THE HOLOGRAPHIC MODEL

In this section we discuss the dual model for 2+1 superconductor. In the case of s-wave superconductivity the relevant dual model reads as (see [10–12] for reviews)

\[
S = \int d^4x \sqrt{-g} \left[ R + \frac{\sigma}{L^2} - F_{\mu\nu}^2 - |\partial_\mu \Psi - igA_\mu \Psi|^2 - m^2 |\Psi|^2 \right],
\]

(5)

\[
ds^2 = -fdt^2 + \frac{dr^2}{f} + r^2 (dx^2 + dy^2), \quad f = \frac{r^2}{L^2} \left(1 - \frac{r_0^2}{r^2}\right).
\]

(6)

We work in the rigid background space-time - AdS with black hole, and do not consider the feedback on the gravity by scalar \(\Psi\) and electromagnetic field \(A_\mu\). The charged scalar \(\Psi\) is dual to the condensate \(<\psi\bar{\psi}>\) in an usual s-wave superconductor and \(A_\mu\) is dual to the electric current. In the vicinity of the boundary we have the following asymptotic behavior of the fields.

\[
\Psi = \frac{\Psi_1}{r} + \frac{\Psi_2}{r^2} + \ldots
\]

(7)

\[
\Phi = A_0 = \mu - \frac{\rho}{r}, \quad A_x = By + \frac{J_x}{r}, \quad J_x = 0.
\]

(8)

where the zero-component of electromagnetic field provides the chemical potential \(\mu\) and charge density of dual theory on the boundary. The dimension of \([<\psi\bar{\psi}>]=3,[\Psi]=1\) and \(\Psi_2\) corresponds to the value of condensate \(<\psi\bar{\psi}>\).

We study the behavior of \(J_{\mu\nu} = <\psi\sigma_{\mu\nu}\bar{\psi}>\) in the presence of external magnetic field \(A_x = By\) and introduce an antisymmetric field \(B_{\mu\nu}\) that will be a source for charged tensor current \(J_{\mu\nu}\). The Lagrangian for antisymmetric field \(B_{\mu\nu}\) has the following form

\[
\Delta L = |dB - igA \wedge B|^2 - m^2 |B_{\mu\nu}|^2 + \lambda |\Psi|^4 B_{\mu\nu}^* F^{\mu\nu} + \lambda |\Psi|^4 B_{\mu\nu}^* F^{\mu\nu}.
\]

(9)

It is useful to compare the Lagrangian (9) with the Lagrangian for the antisymmetric tensor field considered in the extended holographic model for QCD [7, 13]. Remind that the minimal holographic QCD model defined in 5d AdS-like space involves the gauge fields \(A_L, A_R\) in \(U(N_F) \times U(N_F)\) supplemented by the Chern-Simons terms and massive self-dual rank-two field \(B_{mn} = B_{+,mn} + iB_{-,mn}\) and massive complex scalar \(X = X_+ + iX_-\) both in the bifundamental representation of the gauge group. The interaction terms in the Lagrangian involving the rank-two term looks as follows

\[
L_{int} = \lambda_{QCD} X_{\pm} F_{V,\pm mn} B_{\pm mn}^*,
\]

(10)
where $F_V$ is the gauge curvature for the vector gauge field $A_V = A_L + A_R$. The non-vanishing coefficient $\lambda_{QCD}$ in front of triple interaction term XBF implies the non-vanishing magnetic susceptibility $\chi_F$ of the quark condensate. It was shown in [6, 14] that the non-vanishing susceptibility also follows from the anomaly in the axial current which corresponds to the nontrivial Chern-Simons term in the holographic action. In the PCAC approximation $\chi_F = -\frac{N_c}{4\pi^2 f^2}$ [14] and one could say that the external field induces the spin polarization of the condensate via the Goldstone modes.

III. CONDENSATE POLARIZATION IN MAGNETIC FIELD

To calculate the condensate of $< \psi \sigma_{\mu \nu} \psi >$ induced by the external magnetic field we will use the following procedure. First we consider the behavior of the field $B_{12}$ in the pure $AdS_4$ space without presence of external electromagnetic field. After that we switch on electromagnetic field $F_{12} = B$ and calculate how the solution for $B_{12}$ has changed. The susceptibility can be read off from the asymptotic behavior of this new solution. To make calculations easier we suppose that $r_0 = 0 = T$ and change to the variable $z = \frac{1}{r}$. From the dimensional analysis we choose $m^2 = -\frac{6}{L^2}$ and the equation of motion for $B_{12}$ is

$$\partial_z [z^2 \partial_z B_{12}] + m^2 B_{12} = \lambda \Psi F_{12}, \quad A_0 = \mu - \rho z.$$  \hspace{1cm} (11)

This equation has the following solution

$$B_{12} = C_1 z^2 + \frac{C_2}{z^3}.$$  \hspace{1cm} (12)

where $C_1 = < \psi \sigma_{12} \psi >$ and $C_2$ is a source for this operator in the boundary theory. After that we switch on an external magnetic field on the right-hand side of the equation [11]

$$\Psi = z^2 < \psi \psi > + O(z^3), \quad z \to 0$$

$$F_{12} = B + O(z^3), \quad z \to 0,$$

$$\Psi F_{12} = z^2 B < \psi \psi > + O(z^3), \quad z \to 0.$$  \hspace{1cm} (13)

This term modifies the solution in the following way

$$B_{12} = \left( C_1 - \frac{\lambda}{\rho} B \log z \right) z^2,$$  \hspace{1cm} (14)
that we can attribute the new additional term to the non-vanishing magnetic susceptibility of the condensate. It gives us the following expression for susceptibility

\[ < \psi \sigma_{\mu\nu} \psi > = -\frac{\lambda}{5} \log \frac{z_{UV}}{z_0} < \psi \psi > F_{\mu\nu}, \quad (15) \]

where \( z_{UV} \) is a UV cutoff and \( z_0 \) is some IR scale.

The IR scale \( z_0 \) entering the logarithm deserves some explanation. There is some natural scale in the holographic approach which yields the scale of the scalar condensate. It is related to the parameter of the gravity solution \( r_0 \) however to identify it more precisely it is necessary to perform more refined analysis. On the other hand it is possible to get some intuition concerning this scale if we consider the inhomogeneous external magnetic field.

Hence we look at plane wave for magnetic field \( F_{12} = B \exp(-i\omega t + i\vec{k} \cdot \vec{x}) \). The equation of motion for the field \( B_{12}(z) \) reads as

\[ \partial_z \left[ z^2 \partial_z B_{12} \right] + (m^2 + p^2 z^2) B_{12} = \lambda \Psi F_{12}, \]

where \( p^2 = \omega^2 - k_1^2 - k_2^2 \). \( (16) \)

and has the following solution in the limit \( z \to 0 \)

\[ B_{12}(z) = -\frac{\lambda}{5} F_{12} < \psi \psi > \log(pa) z^2, \quad z \to 0. \quad (17) \]

This yields the following formula for the condensate

\[ < \psi \sigma_{\mu} \psi > = -\frac{\lambda}{5} \log(pa) B_{\mu} < \psi \psi >, \quad (18) \]

where \( a \) is a microscopic scale for superconductor (e.g. an interatomic distance) and \( \sigma_{\mu} = \frac{1}{2} \epsilon_{\mu\nu\rho} \sigma_{\nu\rho}, \quad B_{\mu} = \frac{1}{2} \epsilon_{\mu\nu\rho} B_{\nu\rho} \).

**IV. WHY IS \( \lambda \) NOT EQUAL TO ZERO?**

In this Section we shall present the semi-qualitative arguments in favor of \( \lambda \neq 0 \). First, comment on the derivation of the similar constant in QCD. It has been evaluated from the correlator of the tensor and vector currents \( < VT > \) in the boundary theory \( [7, 13] \) which yields at large \( Q^2 \) in the massless QCD

\[ \int dx e^{iQx} < V_{\mu}(0)T_{\nu\rho}(x) > \propto < \bar{\Psi} \Psi > (\eta_{\mu\nu} Q_{\rho} - \eta_{\mu\rho} Q_{\nu}) Q^{-2} + O(Q^{-4}). \quad (19) \]
where the Euclidean OPE of two currents is taken into account. The correlator at low
virtualities is saturated by the $\rho$-meson state which has non-vanishing residues both for
vector and tensor currents.

On the other hand the same correlator can be evaluated holographically using the stan-
dard recipe that is varying the classical action in the bulk theory over the boundary values
of the vector and tensor fields

$$< V(0) T(x) > = \frac{\delta^2 S_{cl}}{\delta A(0) \delta B(x)}. \quad (20)$$

The bulk term $\lambda_{QCD} XBF$ contributes and taking into account the boundary behavior of
the neutral scalar $X(z) = < \bar{\Psi} \Psi > z^3 + \ldots$ one gets upon comparison of two expressions for
$< \bar{\Psi} \Psi >$ terms

$$\lambda_{QCD} = - \frac{3N_c}{4\pi^2}. \quad (21)$$

In our case similar calculation goes as follows. Consider the correlator $< VT_c >$ once
again and hunt for the $< \bar{\psi} \psi >$ term assuming that $Q^2$ is large enough. The charged tensor
current looks as $\psi \sigma \psi$ and two fermion legs can be send to the condensate yielding the
non-vanishing contribution from the tree diagram

$$\int dx e^{iQx} < V_\mu(0) T_\nu(x) > \propto < \psi \psi > (\eta_{\mu \nu} Q_\rho - \eta_{\mu \rho} Q_\nu) Q^{-2} + O(Q^{-4}) \quad (22)$$

At the bulk side we focus at the $\Psi BF$ term again and take into account the boundary
behavior of the charged scalar $\Psi(z) = z^2 < \bar{\psi} \psi > + O(z^3)$. Evaluating the bulk action with
this boundary condition we get the contribution proportional to the s-wave condensate as
well. Equating the leading terms in correlators evaluated in the bulk and in the boundary
superconductor we obtain that $\lambda \neq 0$. However at the holographic side we have the additional
factor $\log Q^2$ which obstructs the estimate of the numerical value of $\lambda$.

Here we present also similar calculation for $3 + 1$ superconductor when the dual action
reads as

$$S = \int d^5x \sqrt{-g} \left[ R + \frac{\sigma}{L^2} - \frac{1}{4} F_{\mu \nu}^2 + |D_\mu X|^2 - m_3^2 |X|^2 + |dB - iA \wedge B|^2 - m_4^2 |B|^2 \
+ \lambda X F_{\mu \nu} B^{\nu \mu} + \lambda X^\dagger F_{\mu \nu}^\dagger B^{\mu \nu} \right],
\quad ds^2 = \frac{1}{z^2} \left[ -dt^2 + dx_i^2 + dz^2 \right], \quad A_0 = \mu - \rho z^2. \quad (23)$$
where \( m_B \) and \( m_\Psi \) are chosen to satisfy tree-level dimensions for dual operators on the boundary

\[
z^5 \partial_z \left( z^{-3} \partial_z \right) X + m_\Psi^2 X = 0, \quad X \sim z^3 < \psi \bar{\psi} > + z J_\psi, \quad m_\Psi^2 = 3,
\]

\[
z \partial_z (z \partial_z) B_{12} + m_B^2 B_{12} = 0, \quad B_{12} \sim z^3 < \psi \sigma_{12} \psi > + \frac{J_B}{z^3}, \quad m_B^2 = -9, \quad (24)
\]

and we have set the radius of AdS to be \( L = 1 \). We can calculate the correlator of tensor and electric current \( < VT > \) imposing the following boundary conditions for the fields in AdS

\[
B_{12} = \frac{J_B(x)}{z^3}, \quad A_1 = J_1(x) z^2, z \rightarrow 0. \quad (25)
\]

If we assume that \( J_1, J_B \sim e^{\pm ipx} \) we get the following equations for \( B_{12} \) and \( A_1 \) fields

\[
z \partial_z \left( z^{-1} \partial_z \right) A_1 + p^2 A_1 = 0,
\]

\[
z \partial_z (z \partial_z) B_{12} + \left( m_B^2 + p^2 z^2 \right) B_{12} = 0. \quad (26)
\]

The solutions can be written as linear combinations of Bessel functions with proper boundary conditions

\[
B_{12} = -\frac{\pi p^3}{16} J_B e^{ipx} Y_3(p z), \quad A_1 = \frac{2z}{p} J_B J_1(p z) e^{-ipx}. \quad (27)
\]

That yields the following expression for the correlator

\[
< V(-p)T(p) > = -\frac{\lambda p_2}{8} p^2 \int_0^\infty dz J_1(p z) Y_3(p z) X(z) = -\frac{\lambda p_2}{8} p^2 \int_0^\infty dw J_1(w) Y_3(w) X(w/p) \approx_{p \gg 1}
\]

\[
\approx_{p \gg 1} -\frac{\lambda p_2}{8 p^2} < \psi \bar{\psi} > \int_0^\infty dw w^3 J_1(w) Y_3(w). \quad (28)
\]

Comparing this answer with \([22]\) we get that \( \lambda \neq 0 \) for \( 3 + 1 \) case as well.

V. ON THE VORTICAL SUSCEPTIBILITY OF THE COOPER CONDENSATE

Let us make a few comments concerning the similar response of the s-wave superconductor on the rotation. We introduce the corresponding susceptibility postponing the holographic study for the separate publication.
There is a well-known analogy between an external magnetic field and the rotating frame, manifested in the non-relativistic case by the substitution $e_f \vec{B} \leftrightarrow m\vec{\Omega}$ (see, for example [4]). It is thus natural to introduce a response of the Cooper condensate in superconductor to the rotation which can be parametrized as follows

$$<0|\Psi \sigma_{\mu\nu}\Psi|0> = \chi_{s,G} <\Psi\Psi> G_{\mu\nu}. \quad (29)$$

We treat the rotation via the curvature of an external graviphoton field $G_{\mu\nu}$ and denote the corresponding vortical susceptibility of the s-wave Cooper condensate as $\chi_{s,G}$.

Let us recall that the graviphoton field is introduced as the specific form of the background metric

$$ds^2 = (1 + 2\phi_g)dt^2 - (1 - 2\phi_g)dx^2 + 2A_g dx dt. \quad (30)$$

The gravimagnetic field corresponds to the angular velocity of rotation at small velocity

$$\vec{B}_g \propto \vec{\Omega}, \quad (31)$$

however at large velocities the relation between the gravimagnetic field and the angular velocity is more complicated.

In the rotating superconductor the magnetic field in the bulk is generated

$$\vec{B} = -\frac{2m\vec{\Omega}}{e}, \quad (32)$$

As we have shown in the previous Section this magnetic field induces the triplet component in the material with non-vanishing susceptibility. Hence this argument suggests that p-wave component is also generated in the s-wave condensate under rotation. This can be considered in the 2+1 case where such component can be generated in the plane everywhere besides the droplets where the condensate vanishes.

We postpone the holographic analysis of the rotating case when the angular velocity is introduced via the rotating black hole. The analysis is expected to be parallel to the discussion in [9].

VI. CONCLUSION

In this Letter we discuss in the holographic framework the effect of the external magnetic field on the s-wave superconductor at small temperature. We argue that apart from
the conventional Meissner effect there is the possibility of generation of the homogeneous p-wave component in the volume of superconductor due to the polarization of the Cooper condensate. The magnetic and rotational susceptibilities of the Cooper condensate are introduced. We consider the linear approximation when the triplet component is proportional to the external field and argue that the susceptibilities do not vanish.

In the usual setting the external magnetic field influences the volume of superconductor via the Abrikosov vortices and the condensate vanishes at their cores. The effect of condensate polarization we consider seems to have nothing to do with the vortices since we do not assume the vanishing of the s-wave condensate anywhere. The appearance of the p-wave admixture in s-wave superconductor caused by the Rashba term or by antiferromagnetic component seems to be the most related phenomena. It would be also important to fit our observation with discussion in [15–17].

Certainly it is interesting to make the numerical estimates of the magnetic and vortical susceptibilities of s-wave SC and take into account the temperature dependence. It would be also interesting to investigate the susceptibilities of the exciton condensate in the external fields which is more close analogue to QCD case.

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