Mass Prediction for the Last Discovered Member of the Axial-Vector Nonet with Quantum Numbers $J^{PC} = 1^{+-}$

M. V. Chizhov* and M. N. Naydenov*

*Department of Atomic Physics, Faculty of Physics, Sofia University, Sofia, 1164 Bulgaria
*E-mail: mih@phys.uni-sofia.bg

Received June 1, 2020; revised June 14, 2020; accepted June 14, 2020

In this paper, we have shown that the novel mass relation among the vector states, $\phi(1020)$ and $\phi(1680)$, with quantum numbers $J^{PC} = 1^{+-}$ and axial-vector strangeonium state $h(\bar{s}s)$ with quantum numbers $1^{+-}$ is also valid for nonzero current quark mass. This relation predicts the mass of recently discovered $h(\bar{s}s)$ state by the BESIII Collaboration within experimental accuracy.

DOI: 10.1134/S0021364020150011

1. INTRODUCTION

It is known that there exist the two lowest-lying axial-vector nonets of meson states with quantum numbers for their neutral components $1^{+-}$ and $1^{+-}$. The first nonet $A$ includes isotriplet of $a_1$ mesons, two isodoublets of $K_{1A}$ strange mesons and two isosinglet states $f_1$. The second nonet $B$ consists of isotriplet of $h_1$ mesons, two isodoublets of $K_{1B}$ strange mesons and two isosinglet states $h_1$.

The two isosinglets of spin-1 states have nearly pure $u\bar{u} + d\bar{d}$ and $s\bar{s}$ structures. There is still discussion about identification of the two $f_1$ states among three observed resonances $f_1(1285)$, $f_1(1420)$, and $f_1(1510)$. While identification of the last from two $h_1(1170)$ and $h_1(1415)$ states has been established in the last two years. The BESIII Collaboration discovered axial-vector strangeonium state $h(\bar{s}s)$ [1–3] and confirmed its previous observations by the LASS [4] and Crystal Barrel [5] Collaborations.

There is interesting situation with the measured and the predicted mass of the axial-vector strangeonium state with quantum numbers $1^{+-}$. For the first time this state has been observed by the LASS Collaboration in 1988 with mass $m_{h(\bar{s}s)}^{\text{LASS}} = (1380 \pm 20)$ MeV. Therefore, this state got the name $h_1(1380)$. The latest measurements by the BESIII Collaboration $m_{h(\bar{s}s)}^{\text{BESIII}} = (1423.2 \pm 2.1 \pm 7.3)$ MeV and PDG average $m_{h(\bar{s}s)}^{\text{PDG}} = (1415 \pm 8)$ MeV [6] required changing the name of this resonance to $h_1(1415)$ in the last (2019) year. However, there was no accurate prediction of the mass of this state among cited theoretical papers in the first version of BESIII publication [2]. The precise prediction of $h(\bar{s}s)$ mass $m_{h(\bar{s}s)}^{\text{theor}} = (1415 \pm 13)$ MeV has been given in 2003 in [7].

The detail consideration of this prediction and new investigation will be presented below.

2. MODEL

Explanation of the spontaneous chiral symmetry breaking [8] and the introduction of quarks [9, 10] give us a principal possibility to describe the whole variety of the light hadron states, in particular the quark-antiquark meson states. The most theoretically and experimentally well studied is the nonet of pseudoscalar mesons, which arise as pseudo-Goldstone bosons in result of spontaneous and explicit chiral symmetry breaking. The properties of the pseudoscalar nonet can be accurately investigated using Nambu–Jona-Lasinio (NJL) model [11] or chiral perturbation theory [12]. Meanwhile, theoretical and experimental situation with identification and explaining properties of members of scalar nonet as quark-antiquark states is still unsatisfactory.

In this paper, we consider a model for spin-1 nonets in the framework of paper [7]. It is well known that besides the two considered lowest-lying axial-vector nonets, a nonet of vector mesons exists. The latest consists of isotriplet $\rho$ mesons, two isodoublets $K^*$ mesons and two isosinglets $\phi$ and $\phi$ with quantum numbers $1^{+-}$, which have nearly pure $u\bar{u} + d\bar{d}$ and $s\bar{s}$ structures, correspondingly. Therefore, there is an
obvious asymmetry between the numbers of axial-vector and vector nonets.

In paper [7] in the framework of extended U(1) massless quark NJL model a new approach to restoring the symmetry has been suggested. In this way new mass relations among spin-1 mesons from different nonets have been derived, which are confirmed experimentally. The basic idea was in the consideration of all possible Lorentz invariant local Yukawa interactions between quark currents: \( \bar{\psi} \gamma \gamma \psi \), \( \bar{\psi} \gamma^5 \gamma \psi \), \( \bar{\psi} T_{\mu \nu} \psi \), \( \bar{\psi} \sigma_{\mu \nu} \psi \) and the corresponding meson fields \( S \), \( P \), \( V_\mu \), \( A_\mu \), \( T_{\mu \nu \lambda} \) with quantum numbers \( 0^+ \), \( 0^- \), \( 1^- \), \( 1^+ \), \( 2^+ \). The axial-vector current \( \bar{\psi} \sigma_{\mu \nu} \psi \) and the corresponding second-rank antisymmetric tensor field \( T_{\mu \nu \lambda} \) possess two types of different quantum numbers: \( 1^- \) and \( 1^+ \). On the mass shell, \( T_{\mu \nu \lambda} \) can be decomposed into vector, \( R_\mu \), and axial-vector, \( B_\mu \), fields

\[
T_{\mu \nu \lambda} = (\hat{\partial}_\mu R_\nu - \hat{\partial}_\nu R_\mu) - \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} (\partial^\alpha B^\beta - \partial^\beta B^\alpha),
\]

with corresponding quantum numbers \( 1^- \) and \( 1^+ \).

Here, we introduce the definition \( \hat{\partial}_\mu = \partial_\mu / \sqrt{-\hat{g}} \). Vice versa, the \( R_\mu \) and \( B_\mu \) fields can be expressed in terms of \( T_{\mu \nu \lambda} \):

\[
R_\mu = \delta^\nu T_{\nu \mu}, \quad B_\mu = \frac{1}{2} \epsilon_{\nu \lambda \beta} \delta^\nu T_{[\lambda \beta]},
\]

which due to antisymmetry of \( T_{\mu \nu \lambda} \) obey the obvious identities

\[
\partial^\mu R_\mu = 0, \quad \partial^\mu B_\mu = 0.
\]

Therefore, the existing \( h(5\bar{\psi}) \) meson with quantum numbers \( 1^+ \) from axial-vector \( B \) nonet can be described by axial-vector field \( B_\mu \) and requires the inclusion in the model the quark current \( \bar{\psi} \sigma_{\mu \nu} \psi \) and the corresponding second-rank antisymmetric tensor field \( T_{\mu \nu \lambda} \). Since the two vector fields \( V_\mu \) and \( R_\mu \) have the same quantum numbers \( 1^- \), they could mix, which leads to the two physical states, \( \phi \) and \( \phi' \) for \( U(1) \) \( \bar{\psi} \psi \) quark structure. So, extending this suggestion to \( U(3) \) model, it can be proposed that the lowest-lying vector nonet of physical states with quantum numbers \( 1^- \) is produced from a mixing of vector and tensor nonets, similar to the two \( K_1(1270) \) and \( K_1(1400) \) physical states, which are superpositions of \( K_1A \) and \( K_{1B} \) states from the corresponding axial-vector nonets.

The new relation between the masses of \( \phi(1020) \), \( \phi' = \phi(1680) \) and \( h(5\bar{\psi}) \) physical states [7]

\[
2m_{\phi}^2 - m_\mu m_\eta + 2m_{\phi'}^2 = 3m_{h(5\bar{\psi})}^2,
\]

which predicts the mass of undiscovered then \( h(5\bar{\psi}) \) state, has been obtained in approximation of zero current quark mass. However, it can be rather applied to \( m(782) \), \( m = m(1650) \), and \( h(1170) \) physical states, which consist of the light \( u \) and \( d \) quarks. Therefore, to confirm the relation (4), here we investigate the case of nonzero current quark mass \( m_0 \).

In order to obtain relation (4) we consider only interactions of vector, \( V_\mu \), tensor, \( T_{\mu \nu \lambda} \), and scalar, \( S \), fields with the field of strange quark, \( \psi \). Linearized NJL Lagrangian with auxiliary (without kinetic terms) boson fields has the form

\[
\mathcal{L}_0 = \psi(\hat{\partial}^\mu - m_0)\psi + g_S \bar{\psi} \gamma^\mu S \psi - \frac{\mu_S^2}{2} S^2 \\
+ g_{\bar{\psi}} \bar{\psi} \gamma^\mu \psi V_\mu + \frac{\mu_{\bar{\psi}}^2}{2} V_{\mu \nu} V^\mu V_\nu \\
+ 4g_{\bar{\psi}} \bar{\psi} \sigma_{\mu \nu} \psi T_{\mu \nu \lambda} + \frac{\mu_T}{4} (\bar{\partial}^\nu T_{\mu \nu \lambda} \hat{\partial}^\lambda T^{[\mu \nu]} - T_{[\mu \nu \lambda]} T^{\lambda \nu \mu}),
\]

where \( \mu_S^2 \), \( \mu_{\bar{\psi}}^2 \), and \( \mu_T \) are bare mass terms. Using relations (1)–(3), the last two terms in Eq. (5) can be rewritten as

\[
g_{\bar{\psi}} \bar{\psi} \sigma_{\mu \nu} \psi \hat{\partial}_\mu R_\nu + ig_{\bar{\psi}} \bar{\psi} \sigma_{\mu \nu} \psi \hat{\partial}_\mu B_\nu + \frac{\mu_T}{2} (R^2 + B^2).
\]

The corresponding Feynman diagrams are presented in Fig. 1.

### 3. Quantum Corrections and Symmetry Breaking

Kinetic terms for the meson fields can be obtained through radiative quantum corrections. We calculate one-loop self-energy contributions for all the boson fields. For example, the contribution from the self-energy diagram of the scalar field (Fig. 2a) has the form

\[
\Pi^{SS}(q) = ig_S^2 N_C \\
\times \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[ \left( \frac{1}{p - m_0 + i\epsilon} \right) \left( \frac{1}{p - \not{\psi} - m_0 + i\epsilon} \right) \right]
\]

\[
\text{[The form of the mass term for the antisymmetric tensor field is suggested in [13].]}
\]
where

$$(7)$$

is the quadratically divergent integral and

$$(8)$$

is the logarithmically divergent integral.

The first two terms in the last equality of Eq. (7) give contribution into the mass term of the scalar field. The third term indicates the appearance of the kinetic term. To normalize the scalar field wavefunction correctly, we must require fulfillment of the condition

$$(10)$$

Applying the similar procedure to all boson fields introduced in (5) and (6)

$$\Pi^{\mu
u}_{\text{VR}} = 4g_{\nu}g_{\gamma}N_c I_0 \frac{m^2_0}{\sqrt{q}} (q_{\mu}q_{\nu} - q^2 g_{\mu\nu})$$

(11)

$$\Pi^{\mu
u}_{\text{TR}} = \frac{2}{3} g^2 g_{\gamma}N_c I_0 \frac{m^2_0}{q^2} (q_{\mu}q_{\nu} - q^2 g_{\mu\nu})$$

(13)

the following useful relations between various Yukawa coupling constants from normalization of kinetic terms are obtained:

$$3g^2_{\nu} = 2g^2 f = \frac{3}{2N_c I_0}.$$  

(15)
In other words, due to the dynamic origin of the kinetic terms, all interactions in the model are described by only one coupling constant, for example, $g = g_S$.

The self-interactions in Figs. 2b and 2c lead to the following effective potential for the scalar field

$$V_{\text{eff}} = \frac{m_s^2}{2}S^2 - 2g_{\mu\nu}S^3 + \frac{g^2}{2}S^4.$$ (16)

The extremum condition

$$\frac{dV_{\text{eff}}}{dS}|_{S=(S)} = m_s^2(S) - 6g_{\mu\nu}S^2 + 2g^2S^3 = 0,$$ (17)

at negative $m_s^2$ leads always to the absolute minimum of the effective potential with nontrivial solution $g(S) = (3m_0 - \sqrt{9m_0^2 - 2m_s^2})/2 < 0$. It corresponds to the spontaneous symmetry breaking, which provides the strange quark with positive constituent mass

$$m_s = m_0 - g(S) > 0.$$ (18)

It also leads to additional contributions to mass terms and mixing between vector bosons (Fig. 3).

There are no additional contribution to the mass of the vector field $V_{\mu}$, $m_{\mu} = \mu_{\mu} - 2g_{\mu}N_{C}(m_0I_0 + I_2)$, while its mixing (12) with the vector field $R_{\mu}$ gets such contribution after symmetry breaking

$$\Delta\Pi_{\mu\nu}^{VR} = -4g_{\gamma}g_{\tau}N_{C}g_{\mu}g_{\nu}g(S)(q_{\mu}q_{\nu} - q^2g_{\mu\nu})$$

+ finite terms, (19)

in such a way that it depends only on physical constituent mass of the strange quark:

$$\Pi_{\mu\nu}^{VR} + \Delta\Pi_{\mu\nu}^{VR} = \sqrt{\frac{18m_{\mu}^2}{q^2}(q_{\mu}q_{\nu} - q^2g_{\mu\nu})} \times \frac{1}{2}B_{\mu} \left( q^2 - m_{B}^2 \right) B^{\mu}.$$ (20)

In its turn $R_{\mu}$ and $B_{\mu}$ fields get additional contributions

$$\Delta\Pi_{\mu\nu}^{RR} = 4g_{\gamma}g_{\tau}N_{C}I_0 g^2(S)^2 - 2m_0g(S)$$

$\times (q_{\mu}q_{\nu} - q^2g_{\mu\nu}) + \text{finite terms},$ (21)

$$\Delta\Pi_{\mu\nu}^{BB} = -4g_{\gamma}g_{\tau}N_{C}I_0 g^2(S)^2 - 2m_0g(S)$$

$\times (q_{\mu}q_{\nu} - q^2g_{\mu\nu}) + \text{finite terms},$ (22)

at to their mass terms $m_{R}^2 = \mu_{R}^2 - 4g_{\gamma}g_{\tau}N_{C}I_0(m_0 + g^2(S)^2 - 2m_0g(S)) = \mu_{R}^2 - 6m_r^2$ and $m_{B}^2 = \mu_{B}^2 + 4g_{\gamma}g_{\tau}N_{C}I_0(m_0 + g^2(S)^2 - 2m_0g(S)) = \mu_{B}^2 + 6m_r^2$, which also depend on the physical constituent mass of strange quark.

As a result, the effective Lagrangian for the spin-1 bosons has the form

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}(V_{\mu}R_{\mu}) \left( \frac{q^2 - m_{\mu}^2}{\sqrt{18m_{\mu}^2q^2}} \right) \times \left( \frac{1}{2}B_{\mu} \left( q^2 - m_{B}^2 \right) B^{\mu} \right).$$ (23)

In paper [7] it has been shown that the masses of spin-1 isovector states $\rho$, $\rho' = \rho(1450)$, $b_1$ and spin-1 light isosinglets $\omega$, $\omega'$, $h_1(1710)$ correspond to the hypothesis of the maximal mixing between $V_{\mu}$ and $R_{\mu}$ states. It leads to the additional constraint

$$m_{\mu}^2 = m_{B}^2 - 12m_r^2.$$ (24)

Zeros of determinant of the matrix between $(V_{\mu}R_{\mu})^{T}$ doublets correspond to the masses for the physical $\phi$ and $\phi'$ bosons, while $h_1(\bar{S})$ meson has $m_B$ mass. Using Vieta’s formulas

$$m_0^2 + m_{\mu}^2 = 2m_0^2 + 6m_r^2, \quad m_0m_{\mu} = m_0^2,$$ (25)
for the quadratic equation \((q^2)^2 - 2(m_q^2 + 9m_q^2)q^2 + m_q^4 = 0\) and relation (24), we have reproduced the mass formula (4) without the assumption of initial massless quarks.

4. CONCLUSIONS

In this paper, we have shown that novel mass relation (4) for \(U(1)\) case of the strange quark with non-zero current mass, \(m_0\), has the same form as for initially massless quark. Therefore, the prediction of the mass of axial-vector strangeonium state \(h(s\bar{s})\) with quantum numbers \(1^{+-}\) in [7] is valid. This is also confirmed experimentally by the BESIII Collaboration [3].

REFERENCES

1. M. Ablikim, M. N. Achasov, X. C. Ai, et al. (BESIII Collab.), Phys. Rev. D 91, 112008 (2015).
2. M. Ablikim, M. N. Achasov, S. Ahmed, et al. (BESIII Collab.), arXiv:1804.05536v1 [hep-ex].
3. M. Ablikim, M. N. Achasov, S. Ahmed, et al. (BESIII Collab.), Phys. Rev. D 98, 072005 (2018).
4. D. Aston, N. Awaji, T. Bienz, et al. (LASS Collab.), Phys. Lett. B 201, 573 (1988).
5. A. Abele, J. Adomeit, C. Amsler, et al. (Crystal Barrel Collab.), Phys. Lett. B 415, 280 (1997).
6. M. Tanabashi, K. Hagiwara, K. Hikasa, et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018); 2019 update.
7. M. Chizhov, JETP Lett. 80, 73 (2004); hep-ph/0307100.
8. Y. Nambu, Report on a Conference, Purdue, 1960; in Broken Symmetries, Selected Papers by Y. Nambu, Ed. by T. Eguchi and K. Nishijima (World Scientific, Singapore, New Jersey, London, Hong Kong, 1995).
9. M. Gell-Mann, Phys. Lett. 8, 214 (1964).
10. G. Zweig, preprint CERN-TH-401 (CERN, 1964).
11. Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); Phys. Rev. 124, 246 (1961).
12. G. Ecker, Prog. Part. Nucl. Phys. 35, 1 (1995).
13. M. V. Chizhov, Mod. Phys. Lett. A 8, 2753 (1993).