Area Spectrum of Extremal Reissner-Nordström Black Holes from Quasi-normal Modes

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Abstract

Using the quasi-normal modes frequency of extremal Reissner-Nordström black holes, we obtain area spectrum for these type of black holes. We show that the area and entropy black hole horizon are equally spaced. Our results for the spacing of the area spectrum differ from that of schwarzschild black holes.

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1 Introduction

The quantization of the black hole horizon area is one of the most interesting manifestations of quantum gravity. Since its first prediction by Bekenstein in 1974 [1], there has been much work on this topic [2]-[16]. Recently, the quantization of the black hole area has been considered [5], [6] as a result of the absorption of a quasi-normal mode excitation. The quasi-normal modes of black holes are the characteristic, ringing frequencies which result from their perturbations [17] and provide a unique signature of these objects [18], possible to be observed in gravitational waves. In asymptotically flat spacetimes the idea of QNMs started with the work of Regge and Wheeler [19] where the stability of a black hole was tested, and were first numerically computed by Chandrasekhar and Detweiler several years later [20]. The quasi-normal modes bring now a lot of interest in different contexts: in AdS/CFT correspondence [21]-[31], when considering thermodynamic properties of black holes in loop quantum gravity [6]-[8], in the context of possible connection with critical collapse [21, 32, 33].

Bekenstein’s idea for quantizing a black hole is based on the fact that its horizon area, in the nonextreme case, behaves as a classical adiabatic invariant [1], [4]. In the spirit of Ehrenfest principle, any classical adiabatic invariant corresponds to a quantum entity with discrete spectrum, Bekenstein conjectured that the horizon area of a non-extremal quantum black hole should have a discrete eigenvalue spectrum. Moreover, the possibility of a connection between the quasinormal frequencies of black holes and the quantum properties of the entropy spectrum was first observed by Bekenstein [34], and further developed by Hod [5]. In particular, Hod proposed that the real part of the quasinormal frequencies, in the infinite damping limit, might be related via the correspondence principle to the fundamental quanta of mass and angular momentum. The proposed correspondence between quasinormal frequencies and the fundamental quantum of mass automatically leads to an equally spaced area spectrum. Remarkably, the spacing was such as to allow a statistical mechanical interpretation for the resulting eigenvalues for the Bekenstein-Hawking entropy. Dreyer[6] also used the large damping quasi-normal mode frequency to fix the value of the Immirzi parameter, $\gamma$, in loop quantum gravity. He found that loop quantum gravity gives a correct prediction for the Bekenstein-Hawking entropy if gauge group should be SO(3), and not SU(2).

In this letter our aim is to obtain the area and entropy spectrum of extremal Reissner-Nordström (RN) black holes in four dimensional spacetime. Using the results of [35, 36] for highly damped quasi-normal modes we show how the horizon area and entropy would be quantized. The authors of [36] noted, the variation of the mass of a RN black hole is not enough to determine the variation of its area, since the corresponding variation of the charge must be known. These authors, then, assumed the same area quantum as in the Schwarzschild case and deduced the corresponding quantum of charge emission. It would seem that in the case of an extremeal RN black hole this issue does not arise, since $M$ and $Q$ are equal. We show that the results for the spacing of the area spectrum differ from schwarzschild black hole case. Conversely, if we assume that $\Delta A$ is indeed universal [36] and thus remains as in the Schwarzschild case $\Delta A = 4\hbar \ln 3$, then the real part of the quasinormal frequency for extremal RN black hole is different from schwarzschild black hole case.
2 Extremal Reissner-Nordström Black Holes

The RN black hole’s (event and inner) horizons are given in terms of the black hole parameters by

\[ r_\pm = M \pm \sqrt{M^2 - Q^2}, \]  

(1)

where \( M \) and \( Q \) are respectively mass and charge of black hole. In the extreme case these two horizons are coincides

\[ r_\pm = M, \quad M = Q. \]  

(2)

According a very interesting conclusion follows [35] (see also more recent paper [36]) , the real part of the quasinormal frequency for extremal RN black holes coincides with the Schwarzschild value

\[ \omega_{RN}^R = \frac{\ln 3}{4\pi R_H}, \]  

(3)

where

\[ R_H = 2M. \]  

(4)

We assume that this classical frequency plays an important role in the dynamics of the black hole and is relevant to its quantum properties [5, 6]. In particular, we consider \( \omega_{RN}^R \) to be a fundamental vibrational frequency for a black hole of energy \( E = M \). Given a system with energy \( E \) and vibrational frequency \( \omega \) one can show that the quantity

\[ I = \int \frac{dE}{\omega(E)}, \]  

(5)

is an adiabatic invariant [7], which via Bohr-Sommerfeld quantization has an equally spaced spectrum in the semi-classical (large \( n \)) limit:

\[ I \approx n\hbar. \]  

(6)

Now by taking \( \omega_{RN}^R \) in this context, we have

\[ I = \int \frac{dE}{\omega_{RN}^R} = \int \frac{4\pi R_H dM}{\ln 3} = \frac{4\pi}{\ln 3} \int 2M dM = \frac{4\pi}{\ln 3} M^2 + c, \]  

(7)

where \( c \) is a constant. In the other hand, the black hole horizon area is given by

\[ A = 4\pi r_+^2. \]  

(8)

Which using Eq.(2) in extremal case is as following

\[ A = 4\pi M^2. \]  

(9)

The Boher-Sommerfeld quantization law and Eq.(7) then implies that the area spectrum is equally spaced,

\[ A_n = n\hbar \ln 3. \]  

(10)

By another method we can obtain above result. From Eq.(9 we get

\[ \Delta A = 8\pi M \Delta M = 8\pi M \hbar \omega_{RN}^R \]  

(11)
where we have associated the energy spacing with a frequency through \( \Delta M = \Delta E = \hbar \omega_R^{RN} \). Now using Eqs.\((3, 4)\) we have

\[
\Delta A = \hbar \ln 3,
\]

therefore the extremal RN black hole have a discrete spectrum as

\[
A_n = n\hbar \ln 3.
\]

Which is exactly the result of Eq.\((10)\). Using the definition of the Bekenstein-Hawking entropy we have

\[
S = \frac{A_n}{4\hbar} = \frac{n \ln 3}{4}.
\]

The above results for area spectrum and entropy is contradicted by results of Andersson and Howls \([36]\) for the extremal RN black holes. Andersson and Howls have assumed that \( \Delta A \) is universal and thus remains as in the Schwarzschild case \( \Delta A = 4\hbar \ln 3 \).

Now if we assume that \( \Delta A \) is indeed universal \([36]\) and thus remains as in the Schwarzschild case \( \Delta A = 4\hbar \ln 3 \), then the real part of the quasinormal frequency for extremal RN black hole is as

\[
\omega^{RN}_R = \frac{\ln 3}{\pi R_H},
\]

in this case we have

\[
I = \int \frac{dE}{\omega^{RN}_R} = \int \frac{\pi R_H}{\ln 3} dM = \frac{\pi}{\ln 3} \int 2M dM = \frac{\pi}{\ln 3} M^2 + c,
\]

now Eqs.\((5, 8, 9, 16)\) then implies that the area spectrum is equally spaced as following,

\[
A_n = 4n\hbar \ln 3.
\]

\section{Conclusion}

Bekenestein’s idea for quantizing a black hole is based on the fact that its horizon area, in the nonextremal case, behaves as a classical adiabatic invariant. It is interesting to investigate how extremal black holes would be quantized. Discrete spectra arise in quantum mechanics in the presence of a periodicity in the classical system Which in turn leads to the existence of an adiabatic invariant or action variable. Bohr-Sommerfeld quantization implies that this adiabatic invariant has an equally spaced spectrum in the semi-classical limit. In this letter we have considered the extremal RN black hole in four dimensional spacetime, using the results for highly damped quasi-normal modes, we obtained the area and entropy spectrum of event horizon. Here we accepts the proposed correspondence between the quasi-normal modes frequencies and a transition energy \( \Delta M \), we have finds that the quantum area should be \( \Delta A = \hbar \ln 3 \). Although the real parts of highly damped quasi-normal modes for schwarzschild and extremal RN black hole is equal \([36]\) \( \omega_R = \frac{\ln 3}{8\pi M} \) as one can see for example in \([7, 16, 36]\) \( \Delta A = 4\hbar \ln 3 \) for schwarzschild black hole. Therefore in contrast with claim of \([36]\) \( \Delta A \) is not universal for all black holes. Also Abdalla et al \([10]\) have been shown that the results for spacing of the area spectrum for near extreme Kerr and near extreme schwarzschild- de Sitter black holes differ from that for
schwarzschild, as well as for non-extreme Kerr black holes. Although such a difference for problem under consideration in [10] as the authors have been mentioned may be justified due to the quite different nature of the asymptotic quasi-normal mode spectrum of the near extreme black hole, in our problem the real parts of highly damped quasi-normal modes for schwarzschild and extremal RN black hole is equal. According to Eq.(2) the location of horizon for extreme RN black hole is in $r = M$, but schwarzschild black hole horizon located in $r = 2M$, therefore the factor 4 in area quantum of schwarzschild black hole $\Delta A = 4\hbar \ln 3$, come from the factor 2 in $r = 2M$.

In the other hand if we assume that $\Delta A$ is indeed universal [36] and thus remains as in the Schwarzschild case $\Delta A = 4\hbar \ln 3$, then the real part of the quasinormal frequency for extremal RN black hole is as $\omega_{RN}^{R} = \frac{\ln 3}{\pi R H}$, which is different from schwarzschild black hole case.

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