Modelling passenger service rate at a transport hub serviced by a single urban bus route as a queueing system

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Abstract. In this paper the daily service rate at a bus stop from the urban passenger transport network is reviewed. The bus stop is located close to a railway and a bus station. A mathematical model has been developed for the average daily incoming flows of passengers to the monitored bus stop. The service rate has been modelled using pulse Dirac delta functions. The daily irregularity of operation of a particular bus line at this bus stop has been evaluated and the operation of the system has been modelled as a queueing system, in order to assess the capacity of the bus line and the organisation of work. The type of incoming flow, as well as the service rate has been defined as a non-stationary Poisson flow. A system of differential-algebraic equations has been chosen as a model according to Kolmogorov’s model for stochastic processes. A specific numerical method for solving system differential equations has been devised with Dirac delta functions in the right part of the equations. Under non-stationary conditions, the basic values of the system parameters have been calculated and an application has been created in a MatLab environment. The results are suitable for accurately determining the routing speed and the average waiting time of the passengers at the bus stop. They can also determine the refusal of passengers to wait at the passenger stop.

1. Introduction
Public transport of passengers in modern urban conditions is a dynamically developing area in which, despite common objectives, many mutually exclusive interests often clash. However, the socio-economic status of the cities significantly depends on it.

In figure 1, the main stakeholders related to mass urban passenger transport in populated areas are indicated where: \( A \) is the set of carriers; \( B \) – the set of passengers; \( C \) – the set of state and local authorities; \( D \) – the society.

The combination of the four sets \( A \cup B \cup C \cup D \) defines the operation of mass urban passenger transport system and their cross section \( A \cap B \cap C \cap D \) represents the common interests of the stakeholders.

The main task of the carriers (set \( A \)) is to obtain maximum profit. Passengers (set \( B \)) strive for: the shortest time for transportation, the lowest price and maximum comfort and safety. The aspiration of the state and the local government (set \( C \)) is to make public transport as accessible and safe as possible, respecting the interests of all parties. The interests of society (set \( D \)) are related to providing ecological and energy efficient transport. In these relationships, a number of theoretical and practical problems arise, and the effectiveness of the system of mass urban passenger transport depends on their solution.
The use of mathematical modelling in solving these problems allows for modification of the parameters of the system in different combinations, in order to find the optimal one. In resolving such problems, the system is formalised as mathematical models that reflect the fundamental laws of its operation.

The analysis of the situation in question in the given area indicates a significant number of studies on the formation of criteria, indicators and methodologies that allow for assessment of the quality of the transport service provided to the population.

A number of studies have addressed issues related to urban traffic modelling and the impact of the route scheme of mass urban passenger transport [1, 2]. Assessment of the transport mobility of the population for a given populated location [3], determining and exploring areas for pedestrian accessibility [4, 5], modelling and evaluating urban routing schemes [6, 7], improving the quality of the transport service [8] are some studies in such fields. The models developed allow us to solve a number of application problems related to rolling stock, the regularity of transport, ensuring safety, forming multimodal systems, etc.

It is worth noting that the application of the models in question improves safety, reduces costs, ensures high quality of service, improves the environmental performance of cities, ensures energy efficiency of transport, etc.

Special attention shall be paid to the survey of mass urban passenger transport at bus stops providing transportation of passengers on the territory of a given city from a transport hub. Given the nature of the processes at these bus stops at certain periods of time (on arrival of trains and coaches) a large number of passengers gather there. There are also many changing passenger flows, which are serviced at specific moments of time with the ability to master passenger traffic to vehicle capacity.

An examination of the nature of interaction between passenger traffic and urban passenger transport (UPT) is essential for proper organisation of work. A number of studies assess the capacity and operation of mass urban passenger transport at bus stops on a given route. A model of microscopic simulation of bus stops with multiple connections is proposed in [9] while in [10] a simulation of bus stops in mixed traffic is made. Overloads and congestion at bus stops are reviewed in [11].

Each of the transport nodes has its own specific characteristics in terms of location, infrastructure, as well as ensured interaction with mass urban passenger transport. This requires a study of the specific systems of interaction for each of the transport nodes. At the same time, the theoretical development of a common model for servicing passenger flows from UPT for a given bus stop is related to the exploration of all inbound passenger flows oriented towards certain routes. Similarly, the development of a specific inflow pattern to a given route can also be applied to other inbound flows to the specific routes.

The basis for the modelling in this work is a passenger transport hub in the city of Ruse and the need to ensure the movement of passenger traffic with the mass urban passenger transport. The transport hub includes bus and railway transport and the connection to the mass urban passenger transport is provided by trolley and bus transport.

2. Formalizing the system as a single channel queueing system
The number of passengers arriving at a stop from the MUPT can be seen as a stochastic process because it depends on many factors. What is important in this case is the service intensity, the maximum
allowable waiting time for the passengers at the bus stop, the number and type of individual routes passing through it. The analysis of passenger flow and the service of the UPT can be done by making a 24-hour model, a queueing system, [12, 13]. For the specific passenger transport unit in the city of Ruse, which includes passenger flow from railway $\lambda(t)$, bus $\lambda_2(t)$ terminals and passengers who have arrived at a specific station of the UPT route from the respective district of $\lambda(t)$ it can be assumed that the flow of passengers is ordinary (Poisson) and in general from arbitrary periods, the total $\lambda$ flow is non-stationary $\lambda = \lambda(t)$. Passengers arriving at the bus stop travel on different routes and use vehicles of different capacity. It is, therefore, appropriate that each route passing through the stop is considered as part of multiple independent single-channel systems, servicing corresponding routes from that stop, along with the corresponding inflows. The work analyses and models the total flow of queries on one route passing through one stop [14-16].

Service intensity $\mu(t)$ can be judged by the daily timetable of the respective route at the bus stop, as well as the capacity of the servicing unit (the capacity of the buses/trolleys that service that specific route). It is also known that at a time when the server is busy, the incoming order waits for a certain time $d$ (differential-algebraic one and is of the type “stiff system” (solid system equations). Special numerical application gives a numerical solution to the system after these transformations, it is taken into account that the system has a large dimensionality; it is differential equations with system error less than $3 \times 10^{-8}$ has to be calculated. In this case, the number of passengers for moment $t$ for the specific server (route), i.e.

$$P_k(t) = ?, \forall k = 0, \infty, \in [0, T], n = 1,$$

where one full working day $T = 0$ of 24 hours is considered to be unit period $T$. The start of the working day $T = 0$ coincides with the astronomical start of the day (0 hours and 0 min), the end of the workday $T = 24$, coincides with the end of the astronomical day (24 hours and 0 min).

The following summary can be made for the model as a queueing system: a non-stationary flow of queries with density $\lambda(t)$ is entered into a queueing system with $n = 1$ service channel. The service time of a random dimension with an indicative distribution and service rate $\mu(t)$, also non-stationary in time. Number $n$ of servicing channels is one. An order that arrives at a time when the channel is busy, waits for some time $T_0$ then leaves the system as not-serviced, i.e. a denial system. The flow of distressed queries is $\nu = 1/T_0$, which is a constant over time.

Consequently, the system is type $(M/M/n)$ in stationary mode. To describe a system of this type the following system of differential equations of Kolmogorov (Erlang-Kolmogorov) is used [17, 18]:

$$\frac{dP_0(t)}{dt} = -\lambda(t)P_0(t)+\mu(t)P_1(t);...;$$

$$\frac{dP_k(t)}{dt} = \lambda(t)P_{k-1}(t)-\left(\lambda(t)+k\mu(t)\right)P_k(t) + \mu(t)(k+1)P_{k+1}(t);...;$$

$$\frac{dP_n(t)}{dt} = \lambda(t)P_{n-1}(t)-\left(\lambda(t)+n\mu(t)\right)P_n(t) + (n\mu(t)+\nu)P_{n+1}(t);...;$$

$$\frac{dP_{n+1}(t)}{dt} = \lambda(t)P_{n+1}(t)-\left(\lambda(t)+\nu(t)+sv\right)P_n(t) + (\nu(t)+(s+1)v)P_{n+1}(t),$$

where $s$ is the number of passengers in the queue when the server is busy.

The differential equation system is infinite and for the numerical solution it is necessary to take a finite number of equations. The number of equations is determined by the number $N$ in the sum $\sum_{i=N+1}^\infty P_i(t)$, which must not exceed a predetermined number $e$. For that reason, a system of $N = 502$ differential equations with system error less than $e = 10^{-8}$ has to be calculated. In this case, the number of probabilities $P_k(t)$ exceeds the number of equations by one and this requires the introduction of an additional algebraic equation for normalisation $\sum_{i=0}^N P_i(t) =1$. In the calculations related to the system (2) after these transformations, it is taken into account that the system has a large dimensionality; it is differential-algebraic one and is of the type “stiff system” (solid system equations). Special numerical methods have been developed to overcome these difficulties. A program of Matlab [19] has been devised to solve system (2), using the built-in “Solver” ode15s, a method of Gir. When entering $\lambda(t), \mu(t), N$ the application gives a numerical solution to the $P_k(t)$. The initial state of the system $P_1(t_0)$ is unknown,
but it is known that such type of processes are persistent and after a long enough time go into regular mode of operation. Therefore, a random initial state can be taken. The integration of the system in the interval \([0, T]\) needs to be done not once, but a sufficiently large number of times, after each integration the values at the end of the period become initial values for the next integration. In this way, the probability functions \(P_k(t)\) begin to tilt to their regular values. After several years of integration, it was found out that only after 5-6 periods do functions \(P_k(t)\) enter a regular mode (for two contiguous periods they remain the same). The accuracy in this case is high because the integration is done over 20 periods, the difference of all \(P_k(t)\) in the last and penultimate integration is less than \(10^{-8}\) for each \(t\).

The input flow \(\lambda(t)\) consists of the sum of the three above-mentioned independent streams:

\[
\lambda(t) = \lambda_1(t) + \lambda_2(t) + \lambda_3(t).
\]  

(3)

An appropriate way to model the inflows is by using a sufficiently large statistical database to approximate the least-squares method or some other method, such as the minimax method, [20]. In order to model more precisely the aggregate inflow, it is necessary to study separately all three incoming streams, and then to unite them to model the total inflow \(\lambda(t)\), in the absence of a sufficient number of statistical data. The approximate average values, per day, of the number of passengers \(A_1, A_2, \ldots, A_k\) arriving at specific time ranges in the 24-hour day \([t_i, t_{i+1}]\) from the train station, bus station and passengers arriving from nearby areas waiting for the specific route at the station in question. For modelling of \(\lambda(t)\) it is appropriate to choose a relatively elementary function with periodicity. The integral amount of \(\lambda(t)\) shall correspond to the total average number of requests for the whole day, i.e.:

\[
\int_0^{24} \lambda(t)dt = \sum_{i=1}^{b} A_i.
\]  

(4)

In this case a 24-hour day is divided into time intervals \([t_i, t_{i+1}]\), \(t_0\) corresponds to 0 hours and 0 minutes for which the average number of passengers \(A_i\) is known. One such sample allocation is given in table 1.

| Interval \(^a\) | \(A_i\) \(^b\) | Interval \(^a\) | \(A_i\) \(^b\) | Interval \(^a\) | \(A_i\) \(^b\) | Interval \(^a\) | \(A_i\) \(^b\) |
|-------------|---------------|-------------|---------------|-------------|---------------|-------------|---------------|
| [0, 1] | round (23.4) = 23 | [6, 7] | 32.5 | [12, 13] | 36.4 | [18, 19] | 42.9 |
| [1, 2] | round (19.5) = 20 | [7, 8] | 44.2 | [13, 14] | 33.8 | [19, 20] | 39.0 |
| [2, 3] | 15.6 | [8, 9] | 41.6 | [14, 15] | 32.5 | [20, 21] | 32.5 |
| [3, 4] | 14.3 | [9, 10] | 39.0 | [15, 16] | 32.5 | [21, 22] | 28.6 |
| [4, 5] | 15.6 | [10, 11] | 42.9 | [16, 17] | 37.7 | [22, 23] | 27.3 |
| [5, 6] | 22.1 | [11, 12] | 39.0 | [17, 18] | 44.2 | [23, 24] | 23.4 |

\(^a\) Interval from 24-hour day \([t_i, t_{i+1}]\), with beginning and end respectively at \(t_i^{th}\) and \(t_{i+1}^{th}\).

\(^b\) Average number of passengers \(A_i\) arriving at \([t_i, t_{i+1}]\), rounded to an integer.

For a \(\bar{A}(t)\) model, a Fourier row is selected with seven members with a 24-hour periodicity:

\[
\bar{\lambda}(t) = a_0 + a_1 \cos \left(\frac{2\pi t}{24}\right) + b_1 \sin \left(\frac{2\pi t}{24}\right) + a_2 \cos \left(\frac{4\pi t}{24}\right) + b_2 \sin \left(\frac{4\pi t}{24}\right) + a_3 \cos \left(\frac{6\pi t}{24}\right) + b_3 \sin \left(\frac{6\pi t}{24}\right).
\]  

(5)

Coefficients \(a_0, a_1, b_1, a_2, b_2, a_3, b_3\) can be calculated by using a method of least squares or by some other method minimizing approximation errors (e.g. Minimax method). In this case coefficients \(a_0, a_1, b_1, a_2, b_2, a_3, b_3\) are calculated by using a method of least squares. The coefficient of determination in the approximation of the model is \(R^2 = 0.94\). This forms are very solid, statistically significant relationship, which means that the selected pattern describes the behaviour of the incoming flow well. Coefficients \(a_0, a_1, b_1, a_2, b_2, a_3, b_3\) in the model, as well as their confidential intervals, guaranteed with a probability of \(\gamma = 0.95\), are given in table 2. For the approximation of the data in table 1 on the abscissa are taken the intervals in the middle \(\frac{t_i + t_{i+1}}{2}\), and on the ordinate the values are not rounded.
Table 2 shows that there is no confidential interval containing 0. This means that all odds are statistically significant (an alternative to the Student’s T-test). The approximate flow $\mu(t)$ and the data are given in figure 2.

| Coefficient | Value       | Confidential interval of the coefficient guaranteed with probability of $\gamma = 0.95$ |
|-------------|-------------|------------------------------------------------------------------------------------------|
| $a_0$       | 31.69       | (30.47, 32.9)                                                                           |
| $a_1$       | -8.853      | (-10.57, -7.132)                                                                         |
| $b_1$       | -4.848      | (-6.568, -3.128)                                                                         |
| $a_2$       | -2.448      | (-4.168, -0.7275)                                                                        |
| $b_2$       | -6.777      | (-8.497, -5.057)                                                                         |
| $a_3$       | 2.292       | (0.5716, 4.012)                                                                          |
| $b_3$       | 2.433       | (0.7132, 4.154)                                                                          |

Thus, the built-in flow $\mu(t)$ well reflects the dynamics of change at the individual intervals of the day, but the integral sum of the total volume $\int_0^{24} \mu(t) \, dt$ is not equal to $\sum_{i=1}^{k} A_i$, which necessitates the selection of a norming coefficient $\tilde{C}$ with property:

$$\tilde{C} \int_0^{24} \mu(t) \, dt = \sum_{i=1}^{k} A_i. \quad (6)$$

Then the inflow after normalization takes the form of:

$$\mu(t) = \tilde{C} \mu(t). \quad (7)$$

In this case $\tilde{C} = 1.0071$.

Finally, for $\mu(t)$ we have:

$$\mu(t) = 1.0071(31.69 - 8.853 \cos \frac{2\pi t}{24} - 4.848 \sin \frac{2\pi t}{24} - 2.448 \cos \frac{4\pi t}{24} - 6.777 \sin \frac{4\pi t}{24} + 2.292 \cos \frac{6\pi t}{24} + 2.433 \sin \frac{6\pi t}{24}). \quad (8)$$

When modelling service intensity $\mu(t)$, it is necessary to know the traffic schedule and the capacity of the vehicles servicing the route. In this case a route, which is serviced by one type of vehicle with a maximum service capacity of 35 passengers is considered. The service takes place at times $t_j^*$ (time of arrival of the bus/trolley) from the time of service to the $C_j^*$ (maximum vehicle capacity) of the passenger at one time. Namely, for a time interval $\Delta t$, theoretically $\Delta t \rightarrow 0$. Normally this interval is never 0, but it is negligible as compared to a 24-hour day (20-30 sec). More precise modelling takes place with the $\delta$ function on the right hand side of (2). For time interval $[t_j^*-\delta, t_j^*+\delta]$, where $\delta$ is negligible (theoretically

*Fig. 2. Approximate flow $\mu(t)$ and average number of passengers $A_i$, arriving in the interval $[t_i, t_{i+1}].$*
ε → 0), the vehicle services \( C_j^* \) passengers at once. For this small amount of time, the integral sum of the service rate must be equal to capacity \( C_j^* \). This is expressed by the basic properties of the \( \delta \) in the following way:

\[
\int_{t_j^*-\varepsilon}^{t_j^*+\varepsilon} \mu(t) \, dt = \int_{t_j^*-\varepsilon}^{t_j^*+\varepsilon} C_j^* \delta(t-t_j^*) \, dt = C_j^*.
\]

Then the service rate \( \mu(t) \) is calculated by the dependence:

\[
\mu(t) = \sum_{k=1}^k C_j^* \delta(t-t_j^*),
\]

(9)

To demonstrate the capabilities of the model, specific input data values are adopted, tailored to the work of the transport unit under consideration, including the railway station and the bus station and its interaction with mass urban passenger transport at a particular stop.

In this case, everywhere it is assumed that all vehicles at any time of arrival \( t_j^* \) have the same capacity of up to 35 people, i.e. \( C_j^* = 35, \forall j \). An example of the distribution of periods of time in hours of arrival \( t_j^* \) after 0:00h in hours, per working day, is as follows: 0.5; 1.5; 2.5; 3.5; 4.5; 5.33; 5.66; 6.25; 6.5; 6.75; 7; 7.25; 7.50; 7.75; 8; 8.2; 8.4; 8.6; 8.8; 9; 9.25; 9.5; 9.75; 10; 10.2; 10.4; 10.6; 10.8; 11; 11.25; 11.50; 11.75; 12; 12.25; 12.50; 12.75; 13; 13.25; 13.50; 13.75; 14; 14.2; 14.4; 14.6; 14.8; 15; 15.25; 15.50; 15.75; 16; 16.2; 16.4; 16.8; 17; 17.2; 17.4; 17.8; 18; 18.2; 18.4; 18.6; 18.8; 19; 19.33; 19.66; 19.99; 20; 20.33; 20.66; 21.5; 22.5; 23.5.

The presence of \( \delta \) functions in the part on the right of equation system (2) leads to serious computational difficulties. Standard numerical methods are not appropriate, so specific numerical methods have been constructed. One way to overcome these difficulties is through the so-called immersed interface method (IIM), [21, 22]. Another common approach to the numerical solution of differential equations similar to (2) is using differential schemes in which \( \delta \) function is replaced by its discrete analogues. Some of these analogues are as follows:

- basic “hat” function

\[
d_h^1(x) = \begin{cases} \frac{h^2 - |x|^2}{h^2}, & |x| \leq h \\ 0, & |x| > h \end{cases}
\]

(11)

- wide “hat” function

\[
d_h^2(x) = \begin{cases} \frac{2h^2 - |x|^2}{4h^2}, & |x| \leq 2h \\ 0, & |x| > 2h \end{cases}
\]

(12)

- smooth variant

\[
d_h^3(x) = \begin{cases} \cos\left(\frac{\pi |x|}{4h}\right), & |x| \leq 2h \\ 0, & |x| > 2h \end{cases}
\]

(13)

- function with a higher approximation level

\[
d_h^4(x) = \begin{cases} \left(1 - \frac{|x|^2}{h^2}\right), & |x| \leq h \\ 2 - \left(1 + \left(\frac{|x|}{h}\right)^2\right), & h \leq |x| \leq 2h \\ 0, & |x| > 2h \end{cases}
\]

(14)

where \( h \) is the step of numerical integration.

Other properties and variants of the \( \delta \) function are discussed in [23]. In the present work, the calculation of the \( \delta \) function is a variant (12). The waiting time of a passenger before deciding to leave is accepted to be 20 minutes, i.e. \( T_w = \frac{1}{3} h \).
3. Mathematical model application results

The “ode15s” solver was used when integrating system (2) into a Matlab programming environment, taking into account the integration step and constructing the discrete analogue of $\delta$. After integrating system (2), probabilities $P_k(t)$ are calculated. On the basis of these probabilities, the major system characteristics are also calculated. Figure 3 shows the probability of waiting for 1 to 5 passengers; in figure 4 from 6 to 10 passengers; in figure 5 from 11 to 20. As the traffic intervals between 6:30 am to 7:30 pm are short and the likelihood for a large number of passengers to wait at the stop is small, especially with an increase in the number of passengers, in figure 5, the probability is below 20% and bends to zero.

With the so-called waiting times and arrival intervals of the vehicles at the stop, according to figure 6 it follows that most often there will be about 6-8 passengers in the queue. Regarding the average waiting time at the stop, it is in the range of 0.15 to 0.17 hours, mainly in the period after 6.30 to 7.30 pm, as shown in figure 7 and it is higher for the rest of the time for objective reasons. This naturally affects the average number of failures, as shown in figure 8, which are at most 3 and more passengers mainly after 19:30. During the remaining periods the failures are for up to 2 people. The results obtained enable the planning and organization of vehicle traffic on routes and stops from the UPT in the case of the passenger transport unit involving passengers from three railway stations, a bus station and a pedestrian area. For detailed analysis and accurate planning of UPT work in any particular city, it is necessary to conduct surveys to collect real data related to the specifics of population movement.

Figure 3. Probability for 1 to 5 passengers in the queue.

Figure 4. Probability for 6 to 10 passengers in the queue.

Figure 5. Probability for 11 to 20 passengers in the queue.

Figure 6. Average number of passengers in a queue.
4. Conclusion

The developed model allows for real input data (passenger flow rates for a specific route) to describe the input flow density. The model is suitable for application to a bus and railway transport node.

Modelling the speed of service through the Dirac delta functions allows for a detailed description of the standard approximation. While the standard approximation assumes continuity of the entire service, in this case it is assumed that the service is at discrete moments of time. Input data are the service moments and the capacity of the vehicle.

The developed numerical method for solving differential algebraic equation systems takes into account the characteristic of Dirac delta functions. A file function that calculates the probabilities of the model is programmed in Matlab.

The solution of the given model allows us to determine the probabilities of a given state of the system at any point of time. The probability determination also allows us to calculate the system characteristics.

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