The kinematic advantage of electric cars

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Received 15 June 2015, revised 31 August 2015
Accepted for publication 11 September 2015
Published 6 October 2015

Abstract

Acceleration of a common car with with a turbocharged diesel engine is compared to the same type with an electric motor in terms of kinematics. Starting from a state of rest, the electric car reaches a distant spot earlier than the diesel car, even though the latter has a better specification for engine power and average acceleration from 0 to 100 km h$^{-1}$. A three phase model of acceleration as a function of time fits the data of the electric car accurately. The first phase is a quadratic growth of acceleration in time. It is shown that the tenfold higher coefficient for the first phase accounts for most of the kinematic advantage of the electric car.

Keywords: kinematics, traffic, electric car, motion, acceleration

(Some figures may appear in colour only in the online journal)

1. Introduction

Electric cars (electric vehicles, EV) are considered a new technology to reduce environmental problems in cities, and in the entire atmosphere. Protagonists also claim superior driving characteristics, including an unsurpassed acceleration. Major car manufacturers offer a substantial variety of electric cars. Nevertheless, many people do not consider an electric motor as an alternative to a combustion engine when buying a new car. The advantages are too vague, and there are too many contradictory opinions.

In this contribution, the different acceleration of an electric and a diesel engine is analysed with the example of a common compact car, the Volkswagen Golf. Comprehensive performance data are available for this car. According to [1, 2], the acceleration of the 85 kW
electric motor and the 81 kW turbocharged diesel engine is very similar for a large variety of speed intervals, as summarized in table 1.

This matches the physicist’s preconception that the acceleration is identical where the power and mass are the same. It contradicts the notion of unsurpassed agility as claimed by electric car enthusiasts.

It turns out that higher agility does not contradict the conventional test data, because the latter is an inappropriate description. This statement could be considered provocative, since technical data of the form $\tau$ seconds for 0 km h$^{-1}$ to 100 km h$^{-1}$ have been standard for decades. However, in practice the most important question is: how long does it take to travel a certain distance starting from rest, for example to cross a road or to reach a gap on a frequented highway? Occasionally one finds data on the time it takes to travel 400 m, or a quarter mile (401.25 m), but to assess ordinary driving situations, 400 m is far too long.

The following measurements can be confirmed by students at the very beginning of physics instructions in university or college, since the main issue is pure kinematics. Later, the same set of data could be further interpreted in terms of force, energy and power, especially for speeds above 30 km h$^{-1}$. The only obstacle remaining is the legal aspect of driving a rental car on a public street in the desired mode.

### Table 1. Conventional acceleration specification for three different engines of a Volkswagen Golf: Electric motor, 85 kW [2]; 1.6 TDI bluemotion, 81 kW [1]; and GTD, 135 kW [3].

|            | 85 kW electric | 81 kW diesel | 135 kW diesel |
|------------|----------------|--------------|---------------|
| 0 to 40 km h$^{-1}$ | 2.6 s          | 2.4 s        | 2.0 s         |
| 0 to 80 km h$^{-1}$ | 7.1 s          | 7.3 s        | 5.4 s         |
| 0 to 100 km h$^{-1}$ | 10.8 s         | 10.7 s       | 7.5 s         |

### 2. Experiment

Acceleration measurements were taken for two versions of the Volkswagen Golf: first, the eGolf with an 85 kW electric motor, and secondly the GTD with a 135 kW turbocharged diesel engine and a six gear automatic drive. Two runs each were recorded on a straight and flat section of a highway (A 73 southbound, from Baiersdorf-Nord, near Erlangen, Germany.). Both cars were very slowly moving at constant velocity at neutral accelerator position, then the driver pressed the accelerator pedal to its limit and subsequently released it when the speed of 100 km h$^{-1}$ was reached. From the practical point of view, this is in favour of the diesel engine: while the electric car can be immediately started from rest, the diesel engine’s brake has to be released first, which would take several tens of seconds. From the scientific point of view, the kinematic performance at comparable initial conditions is a subject of investigation.

A Vernier low-g accelerometer LGA–BTA [4] was used on a CMA ulab datalogger [5] with 10 ms temporal resolution. The one-axis accelerometer was mounted to a wood plate with adjustable legs, placed on the floor behind the co-driver’s seat and aligned perpendicularly to the gravitational field by a sprit level. The datalogger was programmed with the software coach 6 [6]. The raw data were processed with qtiplot [7]. This test equipment is typical for higher education institutions, and for some high schools. Acceleration sensors of smartphones are sensitive enough for such measurements [8] and could be used alternatively.
The acceleration as a function of time is shown in figure 1. The data were averaged over 200 ms in order to reduce sensor noise, but integration was based on the raw data. The characteristic differences between the diesel engine and the electric motor are the following: (a) the diesel reaches maximum acceleration a full second later than the electric motor; (b) it yields larger acceleration for most of the time due to the higher power, (c) the acceleration is interrupted by two gear shifts at 2.2 s and 4.2 s (smaller drops are caused by slipping of the tyres), and (d) the speed of 100 km h\(^{-1}\) is reached two seconds earlier. In section 5 it will be shown that the time delay (a) is the main cause of inferior overall performance of the diesel engine.

The professional test values (or manufacturer’s specifications) for the time to accelerate from zero to 100 km h\(^{-1}\) are 10.8 s (10.4 s) for the electric motor and 7.5 s (7.5 s) for the diesel. Our values are 10.3 s and 8.2 s, respectively. The relatively large difference of 0.7 s for the diesel engine could be ascribed to the all-weather tyres, which are unavoidable with a rental car.

Note the different negative values for free rolling with neutral accelerator towards the ends of the curves. The losses for the electric motor are mainly due to external friction of the car, while the diesel engine adds internal friction.

3. Kinematic data

3.1. Speed

The speed as a function of time is calculated by integration of the acceleration data; it is shown in figure 2. It takes almost five seconds for the diesel car to become significantly faster than the car with the electric motor, though the latter provides a higher acceleration in the very first second only. Nevertheless, the diesel engine outperforms the electric motor in conventional terms, i.e. the final speed of 100 km h\(^{-1}\) is reached 2 s earlier.
A second integration yields the distance as a function of time. The graphs are very similar on the whole scale. Therefore, figure 3 shows the difference of distances traveled in time. The electric car is ahead for seven seconds. Most traffic situations which require fast acceleration do not last that long.

The time needed to travel various distances at full acceleration starting from rest is presented in table 2. The electric car is ahead for 92 m, before the diesel overtakes. For most
of the distance except for the very beginning, the diesel car passes a given point at higher speed. All this fortifies the invalidity of the model of constant acceleration, which often is used implicitly.

4. Power

The gain of kinetic energy, or the power $P$ depends on the mass $m$, its velocity $v$ and the acceleration $a$:

$$P_{\text{kin}} = \frac{dE_{\text{kin}}}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = m\frac{dv}{dt} = mva. \quad (1)$$

The specified mass of the eGolf including the driver is 1595 kg, the mass of the Golf GTD is 1431 kg. Both cars are carrying a co-driver of 80 kg.

Since the mechanical power $P_{\text{kin}}$ is a function of velocity, it is suitable to take the velocity as the variable instead of the time. The graphs $P_{\text{kin}}(v)$ are shown in figure 4. The linear growth in the low speed region corresponds to a constant torque, or driving force,
which is limited by the grip of the tyres\(^1\). At higher speed, the power is mainly limited by the maximum power of the electric motor, and the diesel engine, respectively. The slight drop of power at a higher speed is attributed to friction. Since the time delay of the diesel engine does not appear in the velocity abscissa, both curves have a similar structure.

5. Model

Three phases of acceleration can be clearly distinguished: (1) nonlinear rise, (2) constant, (3) power limited drop. These are the basis of a quantitative model. The data of the first phases fit well into a simple quadratic function of the form \(a_1(t) = \beta_1 t^2\). The coefficient \(\beta_1 = 72\ \text{m s}^{-4}\) for the electric motor, and \(7\ \text{m s}^{-4}\) for the diesel engine. In the second phase \(a_2 = 4.55\ \text{m s}^{-2}\) for the electric motor, and \(a = 5.3\ \text{m s}^{-2}\) for the diesel engine. The time of transition \(t_{12}\) is calculated by

\[
t_{12} = \frac{a_1}{\beta_1}.
\]

The third phase starts at \(t_{23}\), when the total power, i.e. the sum of \(P_{kin}\) and friction losses, reaches the available engine power \(P_{eng}\) specified by the manufacturer. More precisely:

\[
P_{eng} = P_{kin} + P_r + P_w + P_{rot} + P_{res} = P_{mod} + P_{res}.
\]

\(P_r\) represents the rolling resistance, \(P_w\) the air resistance and \(P_{rot}\) the rotational motion of the wheels. The residual power \(P_{res}\) accounts for unspecified losses, like tyre slip, etc, and is assumed to be independent of velocity. The sum of the predictable powers is named \(P_{mod}\). In the third phase, \(P_{mod}\) is constantly at its maximum value \(P_3\). It is to be estimated in advance; the following results show that \(P_3 = 0.9P_{eng}\) is a good value to start with.

The components of \(P_{mod}\) are calculated as follows. The rolling resistance force is proportional to the weight of the car.

\[
F_r = c_r mg
\]

with the coefficient \(c_r = 0.0065\) [9] and the gravitational field \(g = 9.81\ \text{m s}^{-2}\). The force of air resistance \(F_w\) is

\[
F_w = \frac{1}{2} \rho c_w A v^2
\]

with air density \(\rho = 1.217\ \text{kg m}^{-3}\) at \(8\ ^\circ\text{C}\) and \(969\ \text{hPa}\), coefficient of air resistance \(c_w = 0.281\) and cross section area \(A\) with \(c_w A = 0.615\ \text{m}^2\) [9]. The corresponding powers are \(P_r = F_r v\) and \(P_w = F_w v\).

A wheel consists of a rim, a tyre and a braking disk. The mass of a wheel is of the order 25 kg and hence the total mass of four wheels is \(M = 100\ \text{kg}\). Roughly, the set of wheels can be regarded as a solid cylinder with a moment of inertia

\[
\Theta = \frac{1}{2} Mr^2.
\]

\(^1\) Another limit could be the torque of the engine, but this is not the case here. Tyre slipping is noticeable for the poorly controlled diesel engine even on a clean, dry street.
The rotational energy is

\[ E_{\text{rot}} = \frac{1}{2} J \omega^2. \] (7)

The angular frequency \( \omega \) is determined by the car’s speed \( v \) according to

\[ \omega = \frac{v}{r} \] (8)

so the rotational energy depends only on the mass of the wheel, but not on its diameter:

\[ E_{\text{rot}} = \frac{1}{4} M v^2. \] (9)

Analogous to equation (1), the power associated with angular acceleration of the wheel is

\[ P_{\text{rot}} = \frac{1}{2} M a. \] (10)

The moment of inertia of the axle and the rotor are neglected.

The summands of

\[ P_{\text{mod}} = P_{\text{kin}} + P_{\tau} + P_{w} + P_{\text{rot}} \] (11)

are calculated as a function of time:

\[ P_{\text{mod}}(t) = ma(t)v(t) + c_{\tau} m g v(t) + \frac{1}{2} \rho c_w A (v(t))^3 + \frac{1}{2} M a(t)v(t) \] (12)

\( P_{\text{mod}}(t) \) is monotonic increasing in the first and second phase, and \( P_{\text{mod}}(t_{23}) = P_3 \) defines the transition time \( t_{23} \) from the second to the third phase of acceleration. The transition time \( t_{23} \) can directly be obtained from a spreadsheet of the data \( t \) and \( P_{\text{mod}}(t) \). The acceleration \( a(t > t_{23}) \) has to be calculated by recursion, because the components of \( P_{\text{mod}}(t) \) are functions of \( a(t) \) and \( v(t) \). Equation (12) is rearranged with \( P_{\text{mod}}(t \geq t_{23}) = P_3 \):

\[ ma(t)v(t) + \frac{1}{2} M a(t)v(t) = P_3 - c_{\tau} m g v(t) - \frac{1}{2} \rho c_w A (v(t))^3, \] (13)

\[ a(t) = \frac{P_3 - c_{\tau} m g v(t) - \frac{1}{2} \rho c_w A (v(t))^3}{(m + \frac{1}{2} M) v(t)}. \] (14)

The recursion rules are:

\[ a(t + \Delta t) = \frac{P_3 - c_{\tau} m g v(t) - \frac{1}{2} \rho c_w A (v(t))^3}{(m + \frac{1}{2} M) v(t)} \] (15)

and

\[ v(t + \Delta t) = v(t) + a(t) \Delta t. \] (16)

For \( \Delta t = 0.01 \text{ s} \), the relative accuracy \( (P_3 - P_{\text{mod}})/P_3 \) is of the order two per cent. Since \( a(t) \) is a function of \( v(t) \), two columns of data have to be processed simultaneously. For certain software, this is a difficult or even impossible task. Alternatively, one can set \( a(t \geq t_{23}) = a_{23} \), then calculate \( v(t) \) by integration, and use the new data column \( v(t) \) to recalculate \( a(t) \) from equation (14). Repeating this six to eight times yields accurate results with an error of \( (P_3 - P_{\text{mod}})/P_3 < 10^{-4} \).

For the experimental data obtained with the eGolf, the acceleration model with \( P_3 = 75 \text{ kW} \) (\( P_{\text{res}} = 10 \text{ kW} \)) fits the data very well, as shown in figure 5. After the two-fold integration
of \( a(t) \), the path difference is below 0.6 m for the entire time range. The increase of kinetic energy, \( P_{\text{kin}} \) as a function of speed is shown in figure 6. Both the model graph and the data slightly fall with increasing speed due to rising \( P_r \) and \( P_w \). The internal power of the rotating wheels is of similar magnitude, as shown in figure 7; it only changes the scale, but not the shape of \( P_{\text{kin}}(v) \).

The model can be used to illustrate how the three phases contribute to the difference in kinematics, as in the following example. The model of the electric car is taken as a reference. The maximum acceleration \( a_2 = 4.55 \text{ m s}^{-2} \) remains unaltered. The coefficient for the first phase is set to the value for the diesel engine, \( \beta_1 = 7 \text{ m s}^{-4} \), and the maximum power \( P_3 \) is increased by 20% and by 40%. The resulting differences of distance is shown in figure 8. Increasing the power does not help for the initial phase of acceleration, but it only extends the

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**Figure 5.** Acceleration model compared to data for the electric motor.

**Figure 6.** Increase of kinetic energy \( P_{\text{kin}} \) as a function of speed.
second phase in time before the power-limited third phase sets in. The disadvantage of a slow first phase inevitably remains as long as both cars under comparative analysis are in the second phase of constant acceleration.

Without a power limit, the car remains in the second phase of constant acceleration even at speed $100 \text{ km h}^{-1}$. With the data of the eGolf model ($\beta = 72 \text{ m s}^{-1}$, $a = 4.55 \text{ ms}^2$ and $m = 1675 \text{ kg}$), $P_{\text{mod}} = 229 \text{ kW}$ at $100 \text{ km h}^{-1}$, is reached in $6.3 \text{ s}$ from zero. Such a high

Figure 7. Power components of $(P_{\text{mod}} - P_{\text{kin}})$ for the eGolf model. $P_{n}$ (rolling resistance) and $P_{w}$ (air resistance) cause the falling of $P_{\text{kin}}$ in figure 6, while the change of rotational energy $P_{\text{rot}}$ is proportional to $P_{\text{kin}}$.

Figure 8. Margin of an electric car with $\beta_1 = 72 \text{ m s}^{-1}$, $P_s = 75 \text{ kW}$ over cars with combustion engine-type increase of acceleration, $\beta_1 = 7 \text{ m s}^{-1}$. Three different values for the power $P_3$ are shown.
electric power is not speculative, since it is less than half of today’s benchmark of 515 kW for regular electric cars [10]. Lower values than 6.3 s for 0 to 100 km h\(^{-1}\) can only be obtained with \(a_2 > 4.55 \text{ m} \text{s}^{-2}\). This is technically possible with tyres optimized for grip rather than low friction, four wheel drive, and other technologies from motorsports.

The model was also applied to the data of the diesel engine, as shown in figure 9. The first and second phase fit well with \(\beta_1 = 7 \text{ m} \text{s}^{-4}\) and \(a_2 = 5.30 \text{ m} \text{s}^{-2}\). The third phase is very non-uniform. An average \(P_3 = 95 \text{ kW}\) yields the same time for 0 to 100 km h\(^{-1}\), but the model car advances more than ten metres before reaching the final speed. This result again demonstrates the limited conclusiveness of specifying average acceleration. The low value of the best fitting \(P_3\) implies that a kW of diesel engine power is less useful than a kW of electric motor power: While \(P_{3/\text{eng}} = 0.9\) for the electric motor, \(P_{3/\text{eng}} = 0.7\) for the diesel motor. This is easy to understand qualitatively: the power of the electric motor is always at its maximum in the third phase, while the power of the diesel engine depends on its rotational speed, and is interrupted by gear shifts.

6. Accuracy of measurement

From a scientific point of view, the measurements could be improved. As mentioned above, the initial speed was not exactly zero as claimed in the graphs, but instead the cars were moving very slowly as in a parking situation in order to give both cars comparable starting conditions. The final speed obtained by integration of acceleration data is 98 km h\(^{-1}\) rather than the anticipated 100 km h\(^{-1}\), as close inspection of figures 4 and 6 reveals. This deviation can be caused by the weakly defined initial condition, or by imperfect calibration of the speedometer, or a mixture of both. A more thorough analysis with independent velocity measurements by radar or electronic speedometer data and position measurement by GPS or even differential GPS is technically possible. The accuracy would be further enhanced by using the same type of tyres, compensating for non-standard air pressure and temperature, wind speed etc. The flatness of the test driveway can be assured more precisely than with the reading of the car’s built in navigation system. All this would turn the present science project for undergraduate students into an advanced engineering project. On the other hand, the very simple approach of taking acceleration data with an educational data acquisition system and making simplifying assumptions yields valid results.

7. Summary and outlook

The present data clearly confirm the claim that an electric motor outperforms a combustion engine in terms of agility, despite its inferior technical specifications. The cause is the faster emergence of acceleration. The common technical specification zero to 100 km h\(^{-1}\) is an inappropriate physical quantity to evaluate the kinematic performance, because the acceleration is very non-uniform in time. Measurements on the time it takes to run distances of 10 m, 20 m, 50 m, 100 m would be feasible and practically relevant.

The model of three acceleration phases relies on the three parameters \(\beta_1, a_2\) and \(P_3\). The parameter \(\beta_1\) describes the rise of acceleration when starting from rest; it is ten times larger for the electric motor than for the diesel engine tested. The maximum acceleration \(a_2\) is limited by the grip of the tyres rather than the engine, at least for high-powered cars. The maximum power \(P_3\) is only important at high speed and has no influence on the agility of the car. The model fits the data of an electric car accurately. The first and second phase also fit a car with a diesel engine, especially for the first gear.
The presented method is suitable to draw comparisons between different types of combustion engines in order to clarify some myths about diesel versus gasoline engines, turbochargers, and so on. Acceleration from different values of non-zero speed is of high practical interest. These are possible tasks for professional car journalism.

Acknowledgments

I thank Thore Meyn for driving the cars while I was busy taking the data, and Gesine Murphy for taking care of the language.

References

[1] Peters M 2014 Vergleichstest Skoda Rapid Spaceback und VW Golf Auto Motor Sport 6 52–56
[2] Lingner H 2014 Vergleichstest BMW i3, VW e-Golf Auto Motor Sport 10 152–5
[3] Gulde D 2013 Vergleichstest Opel Astra 2.0 CDTI Biturbo gegen VW Golf GTD Auto Motor Sport 16 30–34
[4] Vernier Software & Technology www.vernier.com
[5] ULAB datalogger. Dealers for individual countries are listed at http://cma-science.nl/english/dealers/index.html
[6] Centre for Microcomputer Applications University of Amsterdam http://cma-science.nl/english/software/coach6/coach6.html
[7] Qtiplot data analysis and scientific visualization software www.qtiplot.com
[8] Fahsl C, Vogt P, Wilhelm T and Kasper L 2015 Physics on the road: smartphone-Experimente im Straßenverkehr PhyDid-B—Didaktik der Physik, Frühjahrstagung accepted
[9] http://www.topspeed.com/cars/volkswagen/2015-volkswagen-e-golf-ar160292.html
[10] Technical data for Tesla S P85D www.teslamotors.com

Figure 9. Model applied to diesel engine acceleration data. Despite strong variation due to gear shifts, the principal structure for a linear growth (phase 2) followed by a nearly constant power (phase 3) can be recognized.