Reinforcement of $K^0_S$ regeneration in the model with Hermitian Hamiltonian

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Abstract

A simple model of the $K^0_S$ regeneration with a Hermitian Hamiltonian has been proposed. An increase of regeneration takes place.

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1 Introduction

In standard calculations of the $K^0_S$ regeneration [1-5] non-coupled equations of motion are used instead of the coupled ones. We note that in [5] a correct scheme of calculation is described, but the result (see Eqs. (7.83)-(7.89) of Ref. [5]) corresponds to non-coupled equations (see Eqs. (1) and (2) in Ref. [3], or Eq. (9.32) in Ref. [4]).

This is the principal error which entails a qualitative disagreement in the results [6,7]. In this regard a model based on the exact solution of coupled equations of motion has been proposed [6,7]. The basic element of this model is an optical potential. However, it was shown in [8] that for a system of equations in the strong absorption region the optical potential is inapplicable. In this paper we present a simple model of the direct calculation with a Hermitian Hamiltonian. The optical potential is not used. (For the $n\bar{n}$ transitions in the medium the calculations beyond the potential model were presented in [9-11].)

2 Model

We consider the process

$$K_L^0 \rightarrow K^0_S \rightarrow \pi\pi.$$ (1)

For definiteness, the decay into $2\pi$ is registered [12]. Let $K_L^0$ falls onto the plate at $t = 0$. Notations of Ref. [5] are used. Since $K^0$- and $\bar{K}^0$-interactions are known, we go into basis $(K^0, \bar{K}^0)$ using the relation

$$|K^0_L\rangle = (|K^0\rangle + |\bar{K}^0\rangle)/\sqrt{2}.$$ (2)

Here $|K^0\rangle$, $|\bar{K}^0\rangle$ and $|K^0_L\rangle$ are the states of $K^0$, $\bar{K}^0$ and $K_L^0$, respectively.

At the time $t$ we have [5]

$$|K^0_L(t)\rangle = (|K^0(t)\rangle + |\bar{K}^0(t)\rangle)/\sqrt{2} = (|K^0\rangle e^{-\lambda_{K^0} t} + |\bar{K}^0\rangle e^{-\lambda_{\bar{K}^0} t})/\sqrt{2}.$$ (3)

Here

$$\lambda_{K^0} = im_{K^0} + \Gamma_{K^0}/2,$$

$$\lambda_{\bar{K}^0} = im_{\bar{K}^0} + \Gamma_{\bar{K}^0}/2.$$ (4)

where

$$\Gamma_{K^0} = \Gamma_{K^0}^a + \Gamma_{K^0}^d,$$

$$\Gamma_{\bar{K}^0} = \Gamma_{\bar{K}^0}^a + \Gamma_{\bar{K}^0}^d.$$ (5)

Here $\Gamma_{K^0}^a$ and $\Gamma_{\bar{K}^0}^a$ are the widths of absorption and decay of $K^0$; $\Gamma_{K^0}^d$ and $\Gamma_{\bar{K}^0}^d$ are the widths of absorption and decay of $\bar{K}^0$, respectively.
The wave function of $K^0$ is

$$K^0(x) = \Omega^{-1/2} \exp(-ipx),$$

(6)

where $p = (E, \mathbf{p})$ is the 4-momentum of $K^0$; $E = p^2/2m + U_{K^0}$. To draw an analogy with the well-studied $n\bar{n}$ transitions we consider a non-relativistic problem. The potential $U_{K^0}$ is real. The unperturbed and interaction Hamiltonians are

$$H_0 = -\nabla^2/2m + U_{K^0},$$

$$H_I = H_{K^0\bar{K}^0} + H_W + V,$$

$$V = U_{\bar{K}^0} - U_{K^0},$$

$$H_{K^0\bar{K}^0} = \int d^3x (\epsilon \bar{\Psi}_{\bar{K}^0}(x)\Psi_{K^0}(x) + \text{H.c.}).$$

(7)

Here $U_{K^0}$ and $U_{\bar{K}^0}$ are the real potentials of $K^0$ and $\bar{K}^0$, $H_{K^0\bar{K}^0}$ and $H_W$ are the Hamiltonians of the $K^0\bar{K}^0$ conversion and decay of the $K$-mesons, respectively; $\bar{\Psi}_{K^0}$ and $\Psi_{K^0}$ are the fields of $\bar{K}^0$ and $K^0$, respectively.

For the amplitude of process (1) we have

$$M(K_L \rightarrow K_S \rightarrow \pi\pi) =<\pi\pi \mid H_I \mid K_L(t)> =$$

$$\frac{1}{\sqrt{2}}[<\pi\pi \mid H_I \mid K^0 > e^{-\lambda_{K^0}t} + <\pi\pi \mid H_I \mid \bar{K}^0 > e^{-\lambda_{\bar{K}^0}t}].$$

(8)

The matrix element $<\pi\pi \mid H_I \mid K^0 >$ is calculated by means of the perturbation theory. In the lowest order in $H_{K^0\bar{K}^0}$ we have

$$<\pi\pi \mid H_I \mid K^0 >= <\pi\pi \mid H_W \mid K^0 > + \epsilon G(\bar{K}^0) <\pi\pi \mid H_W \mid \bar{K}^0 >,$$

$$G(\bar{K}^0) = -1/V$$

(9)

(see Fig. 1). Here $G(\bar{K}^0)$ is the propagator of $\bar{K}^0$; $<\pi\pi \mid H_W \mid K^0 >$ and Fig. 1a correspond to the zero order in $H_{K^0\bar{K}^0}$. They describe the direct decay $K^0 \rightarrow \pi\pi$. Figure 1b and the term $\epsilon G(\bar{K}^0) <\pi\pi \mid H_W \mid \bar{K}^0 >$ correspond to the first order in $H_{K^0\bar{K}^0}$ and all the orders in $V$ and $H_W$.

The regeneration arises from the difference between the $K^0N$- and $\bar{K}^0N$-interactions. It is involved in $V$ and so we hold only the second term in (9):

$$<\pi\pi \mid H_I \mid K^0 >= \epsilon G(\bar{K}^0) <\pi\pi \mid H_W \mid \bar{K}^0 > .$$

(10)

Similarly, for the second term in (8) we obtain

$$<\pi\pi \mid H_I \mid \bar{K}^0 >= \epsilon G(K^0) <\pi\pi \mid H_W \mid K^0 > ,$$

$$G(K^0) = 1/V.$$
Figure 1: The decay $K^0 \rightarrow 2\pi$ in the medium: (a) and (b) correspond to the zero and first orders in $H_{K^0\bar{K}^0}$, respectively.

Substituting (10) and (11) into (8) we get

$$M(K_L \rightarrow K_S \rightarrow \pi\pi) = \frac{1}{\sqrt{2}}[\epsilon G(K^0) < \pi\pi | H_W | \bar{K}^0 > e^{-\lambda_{K^0} t} + \epsilon G(K^0) < \pi\pi | H_W | K^0 > e^{-\lambda_{\bar{K}^0} t}].$$

(12)

We return to $K_L, K_S$ representation.

$$M(K_L \rightarrow K_S \rightarrow \pi\pi) = \frac{\epsilon}{2} < \pi\pi | H_W | K_S > [G(K^0)e^{-\lambda_{K^0} t} - G(\bar{K}^0)e^{-\lambda_{\bar{K}^0} t}] + F,$n

$$F = \frac{\epsilon}{2} < \pi\pi | H_W | K_L > [G(\bar{K}^0)e^{-\lambda_{K^0} t} + G(K^0)e^{-\lambda_{\bar{K}^0} t}].$$

(13)

If $< \pi\pi | H_W | K_L > = 0$, then we have

$$M(K_L \rightarrow K_S \rightarrow \pi\pi) = \frac{\epsilon}{2V} < \pi\pi | H_W | K_S > (e^{-\lambda_{K^0} t} + e^{-\lambda_{\bar{K}^0} t}).$$

(14)

We put $m_{K^0} = m_{\bar{K}^0} = m, \Gamma_{K^0}^d = \Gamma_{\bar{K}^0}^d = \Gamma^d$. Now the process amplitude is as follows

$$M(K_L \rightarrow K_S \rightarrow \pi\pi) = \frac{\epsilon}{2V} < \pi\pi | H_W | K_S > (1 + e^{-\Delta\Gamma t/2})e^{-i(m+\Gamma^t/2)t}.$$ 

(15)

where

$$\Delta\Gamma = \Gamma_{K^0}^a - \Gamma_{\bar{K}^0}^a,$n

$$\Gamma^t = \Gamma_{K^0}^d + \Gamma_{\bar{K}^0}^a.$$ 

(16)

Finally

$$| M(K_L \rightarrow K_S \rightarrow \pi\pi) |^2 = J |< \pi\pi | H_W | K_S >|^2,$n

$$J = \frac{\epsilon^2}{4V^2}(1 + e^{-\Delta\Gamma t/2})e^{-\Gamma^t t}.$$ 

(17)
The width of process (1) is
\[ \Gamma(K_L \rightarrow K_S \rightarrow \pi\pi) = N \int d\Phi | M(K_L \rightarrow K_S \rightarrow \pi\pi) |^2 = J\Gamma_d(K_S \rightarrow \pi\pi), \]
\[ \Gamma_d(K_S \rightarrow \pi\pi) = N \int d\Phi | < \pi\pi | H_W | K_S > |^2, \]
(18)

where \( \Gamma_d(K_S \rightarrow \pi\pi) \) is the width of the decay \( K_S \rightarrow \pi\pi \).

Let the probability of finding \( \pi\pi \) be given by the exponential decay law. In the lowest order in \( \Gamma(K_L \rightarrow K_S \rightarrow \pi\pi) \)
\[ W_S(K_L \rightarrow K_S \rightarrow \pi\pi) = 1 - e^{-\Gamma(K_L \rightarrow K_S \rightarrow \pi\pi)t} \approx \Gamma(K_L \rightarrow K_S \rightarrow \pi\pi)t. \]
(19)
(By means of the values given below it is easy to verify that \( \Gamma(K_L \rightarrow K_S \rightarrow \pi\pi)t \ll 1. \))

![Figure 2: Probability of finding \( K_S^0 \).](image)

In the standard calculations [1-5] \( K_S^0 \) in the final state is considered whereas in (19) a probability of finding \( \pi\pi \) in the final state is given. To compare the results we modify our model. Let us assume that
\[ W_S(K_L \rightarrow K_S \rightarrow \pi\pi) = W_S(K_L \rightarrow K_S)W, \]
\[ W = \Gamma_d(K_S \rightarrow \pi\pi)/\Gamma^d, \]
(20)
where \( W_S(K_L \rightarrow K_S) \) and \( W \) are the probabilities of \( K_LK_S \) transition and decay \( K_S \rightarrow \pi\pi \), respectively. Substituting (19), we obtain
\[ W_S(K_L \rightarrow K_S) = J\Gamma^d t. \]
(21)

3 Results and discussion

For a copper absorber the probability of finding \( K_S^0 \) is shown in Fig. 2. The curve and squares depict the calculation performed using equation (21) and the old results given in [2], respectively.
Like in Ref. [2], we use $\sigma(K^0 N) = \frac{1}{3}\sigma(\bar{K}^0 N)$. We take $\sigma(K^0 n) = \sigma(K^0 p) = 15$ mb [13]. Our calculation gives the reinforcement of $K_S^0$ regeneration.

In the old calculation non-coupled equations of motion were considered, but that is wrong [6,7]. A model based on the exact solution of coupled equations of motion was proposed in [6,7]. The comparison of the results of this paper and Ref. [7] will be given in the next paper. The applicability of the optical potential in the particle oscillation problem (the coupled equations of motion) will be studied as well.

The main results of this paper are given by Eqs. (19) and (21). They are simple and transparent.
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