GRAVITATION, COSMOLOGY AND SPACE-TIME TORSION

A.V. Minkevich

Department of Theoretical Physics, Belarusian State University, Minsk, Belarus
Department of Physics and Computer Methods, Warmia and Mazury University in Olsztyn, Olsztyn, Poland
E-mail: MinkAV@bsu.by; awm@matman.uwm.edu.pl

Abstract. Poincaré gauge theory of gravity offers opportunities to solve some principal problems of general relativity theory and modern cosmology. In the frame of this theory the gravitational interaction can have the repulsion character in the case of usual gravitating matter with positive values of energy density and pressure satisfying energy dominance condition. Cosmological consequences of gravitational repulsion are considered in the case of homogeneous isotropic models in connection with the problem of cosmological singularity and dark energy problem of general relativity theory. Regular Big Bang inflationary scenario with accelerating stage of cosmological expansion at asymptotics and the principal role of space-time torsion in this scenario are discussed.

1 Introduction

Einsteinian general relativity theory (GR) is the base of modern theory of gravitational interaction and relativistic cosmology. GR allows to describe different gravitating systems and cosmological models at widely changing scales of physical parameters. At the same time GR possesses certain principal problems, which, in particular, appear in cosmology. At first of all, this is the problem of cosmological singularity (PCS): the stage of cosmological expansion according to Einstein gravitation equations has the beginning in the time in the past (the problem of the beginning of the Universe in the time) and in accordance with GR the singular state with divergent energy density and singular metrics appears at the beginning of cosmological expansion. It is because the gravitational interaction for usual gravitating matter with positive values of energy density and pressure satisfying energy dominance condition in the frame of GR as well as Newton’s theory of gravity has the character of attraction, but not repulsion always. The PCS is particular case of general problem of gravitational singularities of GR [1]. Note that in the frame of GR the gravitational interaction can have the repulsion character in the case of gravitating systems with negative pressure (for example, scalar fields in inflationary cosmology). However, the PCS can not be solved by taking into account such systems. According to widely known opinion, the solution of PCS has to be connected with quantum gravitational effects beyond Planckian conditions, when the energy density surpasses the Planckian one. A number of particular regular cosmological solutions was obtained in the frame of candidates to quantum gravitation theory.
- string theory/M-theory and loop quantum gravity. Radical ideas connected with notions of strings, branes, extra-dimensions, space-time foam etc are used in these theories (some features of these solutions are discussed in [2,3]).

As it was shown in a number of papers (see [2-4] and Refs herein) the gauge approach in theory of gravitational interaction offers opportunities to solve the PCS in the frame of usual field-theoretical description of gravity in 4-dimensional physical space-time, where constructive Einsteinian definitions of space-time notions are valid locally. The structure of physical space-time in the framework of gauge approach to gravitation, generally speaking, is more complicated in comparison with GR. So, in the frame of the Poincaré gauge theory of gravity (PGTG), which is the most important gauge theory of gravitation, the physical space-time possesses the structure of Riemann-Cartan continuum. Gravitational field in PGTG is described by means of interacting metric and torsion fields. The presence of space-time torsion can change the character of gravitational interaction by certain physical conditions in the case of usual gravitating matter. This fact plays the important role allowing to solve some principal problems of GR including the PCS.

The present paper is organized by the following way. In Section 2 the question “why we need the Poincaré gauge theory of gravity” is discussed. Principal moments concerning the solution of PCS in the frame of PGTG are given in Section 3, and in Section 4 recent results about possible solution of “dark energy problem” of GR without dark energy on the base of PGTG are briefly discussed. In Conclusion some possible physical consequences of space-time torsion in connection with considered problems of GR are discussed.

2 Local gauge invariance principle and Poincaré gauge theory of gravity

As it is known, the local gauge invariance principle is the basis of modern theory of fundamental physical interactions. The theory of electro-week interaction, quantum chromodynamics, Grand Unified models of particle physics were built by using this principle. From physical point of view, the local gauge invariance principle establishes the correspondence between certain important conserving physical quantities, connected according to the Noether’s theorem with some symmetries groups, and fundamental physical fields, which have as a source corresponding physical quantities and play the role of carriers of fundamental physical interactions. The applying of this principle to gravitational interaction leads, generally speaking, to generalization of Einsteinian theory of gravitation.

At first time the local gauge invariance principle was applied in order to build the gravitation theory by Utiyama in Ref.[5] by considering the Lorentz group as gauge group corresponding to gravitational interaction. Utiyama introduced the Lorentz gauge field, which has transformation properties of anholonomic Lorentz connection. By identifying this field with anholonomic connection of riemannian space-time, Utiyama obtained Einstein gravitation equations of GR by this way. The work by Utiyama [5] was criticized by many authors. At first of all, if anholonomic Lorentz connection is considered as independent gauge field,
it can be identified with a connection of Riemann-Cartan continuum with torsion, but not
riemannian connection [6-8]. Moreover, if a source of gravitational field includes the energy-
momentum tensor of gravitating matter, we can not consider the Lorentz group as gauge
group corresponding to gravitational interaction. Note that metric theories of gravitation
in 4-dimensional pseudo-riemannian space–time including GR, in the frame of which the
energy-momentum tensor is a source of gravitational field, can be introduced in the frame
of gauge approach by the localization of 4-parametric translation group [9, 10]. By localiz-
ing 4-translations and introducing gauge field as symmetric tensor field of second order, the
structure of initial flat space-time changes, and gauge field becomes to connected with metric
tensor of physical space-time. Because the localized translation group leads us to general
coordinate transformations, from this point of view the general covariance of GR plays the
dynamical role. At the same time the local Lorentz group (group of tetrad Lorentz transfor-
mations) in GR and other metric theories of gravitation does not play any dynamical role
from the point of view of gauge approach, because corresponding Noether’s invariant in these
theories is identically equal to zero [11]. The other treatment to localization of translation
group was presented in [12, 13], where gravitation field was introduced as tetrad field in 4-
dimensional space-time with absolute parallelism. This theory is not covariant with respect
to localized tetrad Lorentz transformations, and in fact it is intermediate step to gravitation
theory with independent gauge Lorentz field. If one means that the Lorentz group plays the
dynamical role in the gauge field theory and the Lorentz gauge field exists in the nature, in
this case we obtain with necessity the gravitation theory in the Riemann-Cartan space-time
as natural generalization of GR (see, for example, [14-16]). Corresponding theory is known
as Poincaré gauge theory of gravitation.

Gravitational gauge field variables in PGTG are the tetrad \( h^i_{\mu} \) (translational gauge field)
and the Lorentz connection \( A^{ik}_{\mu} \) (Lorentz gauge field); corresponding field strengths are the
torsion tensor \( S^i_{\mu\nu} \) and the curvature tensor \( F^{ik}_{\mu\nu} \) defined as

\[
S^i_{\mu\nu} = \partial_{[\nu} h^i_{\mu]} - h_{k[\mu} A^{ik}_{\nu]} ,
\]
\[
F^{ik}_{\mu\nu} = 2\partial_{[\mu} A^{ik}_{\nu]} + 2A^{il}_{[\mu} A^k_{l\nu]} ,
\]

where holonomic and anholonomic space-time coordinates are denoted by means of greek and
latin indices respectively. As sources of gravitational field in PGTG are energy-momentum
and spin tensors. The gravitational Lagrangian of PGTG is invariant built by means of grav-
itational field strengths. The simplest PGTG is the Einstein-Cartan theory based on gravita-
tional Lagrangian in the form of scalar curvature of Riemann-Cartan space-time [7,8,17]. In
a certain sense the Einstein-Cartan theory of gravitation is degenerate theory [18]. Like gauge
Yang-Mills fields, gravitational Lagrangian of PGTG has to include invariants quadratic
in gravitational field strengths - curvature and torsion tensors. The including of linear in

\footnote{Because in the frame of gauge approach the gravitational interaction is connected with space-time transformations, the gauge treatment to gravitation has essential differences in comparison with Yang-Mills fields connected with internal symmetries groups. As a result, there are different gauge treatments to gravitational interaction not detailed in this paper.}
curvature term (scalar curvature) to gravitational Lagrangian is necessary to satisfy the correspondence principle with GR.

We will consider the PGTG with gravitational Lagrangian $L_G$ given in general form containing different invariants quadratic in the curvature and torsion tensors

$$L_G = f_0 F + F^{\alpha\beta\mu\nu} (f_1 F_{\alpha\beta\mu\nu} + f_2 F_{\mu\alpha\beta\nu} + f_3 F_{\mu\nu\alpha\beta}) + F^{\mu\nu} (f_4 F_{\mu\nu} + f_5 F_{\nu\mu}) + f_6 F^2 + S^{\alpha\mu\nu} (a_1 S_{\alpha\mu\nu} + a_2 S_{\nu\mu\alpha}) + a_3 S^{\alpha}_{\mu\alpha} S^{\beta}_{\mu\beta},$$

where $F_{\mu\nu} = F^{\alpha}_{\mu\alpha\nu}$, $F = F^{\mu}_{\mu}$, $f_i (i = 1, 2, \ldots, 6)$, $a_k (k = 1, 2, 3)$ are indefinite parameters, $f_0 = (16\pi G)^{-1}$, $G$ is Newton’s gravitational constant (the light velocity in the vacuum $c = 1$). Although the gravitational Lagrangian (1) includes a number of indefinite parameters, gravitational equations of PGTG for homogeneous isotropic models (HIM) considering below depend weakly on the choice of quadratic part of gravitational Lagrangian by virtue of their high spatial symmetry.

3 Problem of cosmological singularity and PGTG

According to observational data concerning anisotropy of cosmic microwave background, our Universe was sufficiently homogeneous and isotropic beginning from initial stages of cosmological expansion. In connection with this fact, the investigation of HIM is of greatest interest for relativistic cosmology. In the frame of PGTG homogeneous isotropic models are described in general case by means of three functions of time: the scale factor of Robertson-Walker metrics $R(t)$ and two torsion functions $S_1(t)$ and $S_2(t)$ determining the following components of torsion tensor (with holonomic indices) [19]: $S^{1}_{10} = S^{2}_{20} = S^{3}_{30} = S_1(t)$, $S_{123} = S_{231} = S_{312} = S_2(t) R^3/k^2\sin\theta$, where spatial spherical coordinates are used and $k = +1, 0, -1$ for closed, flat and open models respectively. The functions $S_1$ and $S_2$ have different properties with respect to transformations of spatial inversions, namely, unlike $S_1$ the function $S_2(t)$ has pseudoscalar character.

At first we will consider HIM with vanishing pseudoscalar torsion function (see [19, 2-4] and references herein) filled by gravitating matter with energy density $\rho$ and pressure $p$ (the average of spin distribution is supposed to be equal to zero). In this case gravitational equations of PGTG lead to the following generalized cosmological Friedmann equations (GCFE)

$$\frac{k}{R^2} + \left\{ \frac{d}{dt} \ln \left[ R \sqrt{1 + \alpha (\rho - 3p)} \right] \right\}^2 = \frac{8\pi G}{3} \frac{\rho + \frac{\rho}{3} (\rho - 3p)^2}{1 + \alpha (\rho - 3p)},$$

$$R^{-1} \frac{d}{dt} \left[ \frac{dR}{dt} + R \frac{d}{dt} \left( \ln \sqrt{1 + \alpha (\rho - 3p)} \right) \right] = -\frac{4\pi G}{3} \frac{\rho + 3p - \frac{\rho}{3} (\rho - 3p)^2}{1 + \alpha (\rho - 3p)}.$$  

The second indefinite parameter $a = 2a_1 + a_2 + 3a_3$ connected with quadratic in the torsion part of Lagrangian (1) in gravitational equations for HIM has to be equal to zero, if one supposes that cosmological equations do not contain high derivatives with respect to the scale factor $R(t)$.
where indefinite parameter $\alpha = \frac{f}{3f_0^2} > 0$ ($f = f_1 + f_2 + f_3 + f_4 + f_5 + 3f_6$) has inverse dimension of energy density. According to gravitational equations the torsion function $S_1$ is

$$S_1 = -\frac{1}{4} \frac{d}{dt} \ln |1 + \alpha (\rho - 3p)|$$  \hspace{1cm} (4)

and conservation law for gravitating matter has usual form

$$\dot{\rho} + 3H (\rho + p) = 0,$$  \hspace{1cm} (5)

where $H = \dot{R}/R$ is the Hubble parameter and a dot denotes the differentiation with respect to time.

If the parameter $\alpha$ tends to zero, the torsion function (4) vanishes and GCFE (2)–(3) coincide with Friedmann cosmological equations of GR. The difference of (2)–(3) from Friedmann cosmological equations of GR is connected with terms containing the parameter $\alpha$. The value of $\alpha^{-1}$ determines the scale of extremely high energy densities. Solutions of GCFE coincide practically with corresponding solutions of GR, if the energy density is small $|\alpha (\rho - 3p)| \ll 1$ ($p \neq \frac{1}{3} \rho$). The difference between GR and PGTG can be significant at extremely high energy densities $|\alpha (\rho - 3p)| \gtrsim 1$, where the dynamics of HIM depends essentially on space-time torsion

The structure of GCFE (2)–(3) ensures regular behavior of cosmological solutions. In order to demonstrate this fact in the case of inflationary cosmological models, we will consider below HIM filled with scalar field $\phi$ minimally coupled with gravitation and gravitating matter with equation of state in the form $p_m = p_m(\rho_m)$ (values of gravitating matter are denoted by means of index "m"). Then the energy density $\rho$ and the pressure $p$ take the form

$$\rho = \frac{1}{2} \dot{\phi}^2 + V + \rho_m \ (\rho > 0), \quad p = \frac{1}{2} \dot{\phi}^2 - V + p_m,$$  \hspace{1cm} (6)

where $V = V(\phi)$ is a scalar field potential. Because the energy density $\rho$ is positive and $\alpha > 0$, from equation (2) in the case $k = +1$, 0 follows the relation:

$$Z = 1 + \alpha (\rho - 3p) = 1 + \alpha \left(4V - \dot{\phi}^2 + \rho_m - 3p_m \right) \geq 0.$$  \hspace{1cm} (7)

The condition (7) is valid not only for closed and flat models, but also for cosmological models of open type ($k = -1$) [2]. The domain of admissible values of scalar field $\phi$, time derivative $\dot{\phi}$ and energy density $\rho_m$ determined by (7) is limited in space $P$ of these variables by bound $L$ defined as

$$Z = 0 \quad \text{or} \quad \dot{\phi} = \pm \left(4V + \alpha^{-1} + \rho_m - 3p_m \right)^{\frac{1}{2}}.$$  \hspace{1cm} (8)

3Ultrarelativistic matter ($p = \frac{1}{3} \rho$) and gravitating vacuum ($p = -\rho$) with constant energy density are two exceptional systems, because GCFE (2)–(3) are identical to Friedmann cosmological equations of GR in these cases independently on values of energy density.
Unlike GR at compression stage the time derivative $\dot{\phi}$ does not diverge, and by reaching the bound $L$ the transition to the second part of cosmological solution containing the expansion stage takes place. From cosmological equation (2) by using the conservation law (5) follows that in space $P$ there are extremum surfaces, in points of which the Hubble parameter vanishes [2]. Extremum surfaces play the role of "bounce surfaces", because the time derivative of the Hubble parameter is positive on the greatest part of these surfaces in the case of scalar field potentials applying in chaotic inflation [2,4]. All cosmological solutions have bouncing character and are regular with respect to metrics, Hubble parameter and its time derivative. If gravitating matter satisfies standard conditions (energy density is positive, energy dominance condition is valid), any cosmological solution is not limited in the time, and before the expansion stage cosmological solution contains the compression stage and regular transition from compression to expansion. If the value of scalar field at the beginning of cosmological expansion is sufficiently large ($\phi \geq 1M_p$, where $M_p$ is the Planckian mass), like GR cosmological solution contains quasi-de-Sitter inflationary stage and post-inflationary stage with oscillating scalar field. Because the de-Sitter solution is exact solution of GCFE [20], characteristics of inflationary stage in our theory (in particular, the duration of this stage by given initial conditions for scalar field at the beginning of expansion) are close to that of GR. As numerical analysis of inflationary solutions of GCFE shows [4], the duration of transition stage from compression to expansion is several order less than duration of inflationary stage. By taking into account that duration of inflationary stage is extremely small [21], we can conclude that discussed regular cosmological solutions correspond to regular Big Bang scenario or Big Bounce. Note that if the scale of extremely high energy densities defined by $\alpha^{-1}$ is essentially less than the Planckian one, the behavior of cosmological solution at the end of inflationary stage differs from that of GR (in particular, the Hubble parameter oscillates by changing its sign) [2,4]. After transformation of oscillating scalar fields into ultrarelativistic particles and transition to radiation dominated stage, the further evolution of HIM (nucleosynthesis, transition to matter dominated stage) practically coincides with that of GR.

Regular character of all cosmological HIM describing by GCFE is connected with gravitational repulsion effect, which takes place in the case of usual gravitating matter with positive energy density at extreme conditions (extremely high energy densities and pressures), where principal role plays the space-time torsion [3].

4 Dark energy problem of GR and PGTG

Unlike the PCS, which is an old cosmological problem of GR, the dark energy problem (DEP) of GR is new problem appeared together with discovery of the acceleration of cosmological expansion at present epoch. By using Friedmann cosmological equations of GR in order to explain accelerating cosmological expansion, the notion of dark energy (or quintessence) was introduced in cosmology. According to obtained estimations, approximately 70% of energy in our Universe is related with some hypothetical form of gravitating matter with negative
pressure — dark energy — of unknown nature. Previously a number of investigations devoted to DEP were carried out (see review [22]). According to widely known opinion, the dark energy is associated with cosmological term. If the cosmological term is related to the vacuum energy density, it is necessary to explain, why it has the value close to critical energy density at present epoch (see for example [23]). Note that by including cosmological term of corresponding value to GCFE, we can build regular cosmology with observable accelerating expansion stage in the frame of PGTG. However, like GR, the DEP is not solved by this way.

As it was shown in Refs. [24,25], the PGTG offers opportunities to solve the DEP without using the notion of dark energy. It is because the space-time torsion in PGTG can change the character of gravitational interaction and lead to gravitational repulsion effect not only at extreme conditions, but also at very small energy densities. With this purpose the HIM with two torsion functions were built and investigated in the frame of PGTG. Cosmological equations for such HIM include the pseudoscalar torsion function $S_2$ with its first time derivative and contain besides $\alpha$ also two others indefinite parameters: $b = a_2 - a_1$ with dimension of parameter $f_0$ and dimensionless parameter $\varepsilon$, which is function of coefficients $f_i$ at quadratic in the curvature terms of gravitational Lagrangian. The pseudoscalar torsion function $S_2$ satisfies differential equation of second order, and according to gravitational equations the function $S_1$ can be expressed as function of the Hubble parameter, the torsion function $S_2$ with its first time derivative and parameters characterizing gravitating matter. If one supposes that $S_2 = 0$, then the equation for $S_2$-function vanishes and cosmological equations and the expression for $S_1$-function take previous form given in Section 3. However, there is other solution with not vanishing function $S_2$. As it was shown in Refs. [24,25], by certain restrictions on indefinite parameters obtained cosmological equations lead to accelerating expansion stage at asymptotics, when physical parameters characterizing cosmological models are sufficiently small. It is because the pseudoscalar torsion function contains at asymptotics some constant not vanishing value, which in the case $|\varepsilon| \ll 1$ is:

$$S_2^2 = \frac{f_0(f_0 - b)}{4fb} + \frac{\rho - 3p}{12b}.$$  \hfill (9)

As a result cosmological equations at asymptotics take the form of cosmological Friedmann equations with effective cosmological constant induced by pseudoscalar torsion function:

$$\frac{k}{R^2} + H^2 = \frac{1}{6b} \left[ \rho + \frac{3(3f_0 - b)^2}{4f} \right],$$  \hfill (10)

$$\dot{H} + H^2 = \frac{1}{12b} \left[ \rho + 3p - \frac{3(3f_0 - b)^2}{2f} \right].$$  \hfill (11)

By using at asymptotics the equation of state for dust matter, we obtain that cosmological equations (10-11) lead to observable accelerating cosmological expansion, if indefinite parameters $b$ and $\alpha$ are connected by the following way $b = [1 - (2.8\rho_c\alpha)^{1/2}]f_0$, where critical
energy density is \( \rho_{cr} = 6f_0H_0^2 \) (\( H_0 \) is the value of the Hubble parameter at present epoch). If we suppose that the scale of extremely high energy densities defined by \( \alpha^{-1} \) is larger than the energy density for quark-gluon matter, but less than the Planckian one, then we obtain the corresponding estimation for \( b \), which is very close to \( f_0 \).

The investigation of inflationary HIM with pseudoscalar torsion function at extreme conditions at the beginning of cosmological expansion shows that the PGTG allows to build totally regular inflationary Big Bang scenario [25]. Like HIM discussed in Section 3, there are extremum surfaces in space of independent variables \( \phi, \dot{\phi}, S_2, \dot{S}_2, \rho_m \), in the points of which the Hubble parameter vanishes \( H = 0 \). Extremum surfaces depend on indefinite parameters \( \alpha, \varepsilon \) and in the case of open and closed models also on the scale factor \( R \) (as was noted above, the value of \( b \) depends on \( \alpha \) and is close to \( f_0 \)). Unlike HIM with vanishing pseudoscalar torsion function, the bounce (\( \dot{H}_0 > 0 \)) takes place only in limited domain of extremum surfaces with negligibly small values of the function \( S_2 \). Properties of regular inflationary solutions with pseudoscalar torsion function differ from that without pseudoscalar torsion function and depend essentially on indefinite parameters \( \alpha \) and \( \varepsilon \).

The regular Big Bang scenario was built in the frame of PGTG by classical description of gravitational field. If the energy density and values of torsion functions at transition stage from compression to expansion are less than the Planckian ones, quantum gravitational era was absent by evolution of the Universe. If the Planckian conditions were realized at the beginning of cosmological expansion, quantum gravitational corrections have to be taken into account; however, classical cosmological equations of PGTG ensure the regular character of the Universe evolution.

5 Conclusion

As follows from our consideration, the PGTG leads to certain principal differences in comparison with GR concerning the character of gravitational interaction for usual gravitating matter that offers opportunities to solve some principal problems of GR. Although the direct interaction of the torsion with minimally coupled spinless matter is absent, corresponding physical consequences of PGTG are connected essentially with space-time torsion by virtue of interaction between metric and torsion fields. According to PGTG, the domain of applicability of GR is limited, namely in the case of cosmological HIM the domain of admissible energy densities has upper limit determined by \( \alpha^{-1} \) and lower limit equal to \( \frac{3(f_0 - b)^2}{4f} \). The following question appears: by what way obtained physical consequences of PGTG can be verified? As was noted above, the behavior of regular inflationary cosmological models at the end of inflationary stage, generally speaking, differs from that of GR and depends on indefinite parameter \( \alpha \), and in the case of HIM with pseudoscalar torsion function also on parameter \( \varepsilon \). It can be possible cause of differences of perturbations of scalar fields at the end of inflationary stage in comparison with GR, that has direct physical interest in connection with observable anomalies in anisotropy of cosmic microwave background [26]. This means that the building of perturbations theory in inflationary HIM in the frame of PGTG is of
direct physical interest and possibly can test obtained cosmological consequences. Obtained results can be important also for other gravitating systems in astrophysics. In particular, the conclusion about existence of limiting (maximum) energy density for gravitating systems can be significant for so-called primordial black holes limiting their admissible minimum mass. Together with dark energy problem, the problem of the origin of not baryonic component of dark matter is principal problem of relativistic cosmology and astrophysics. From our consideration of DEP given in Section 4 follows that Newton’s law of gravitational attraction has limits of its applicability and space-time torsion can be essential in Newtonian approximation. If the torsion can lead to physical consequences in the frame of HIM as dark energy in GR, possibly the space-time torsion in the case of inhomogeneous matter distribution could be important for the solution of dark matter problem.

From our analysis given above follows that the PGTG can have the principal meaning for theory of gravitational interaction. Note that supergravity theory built in connection with the problem of unification of fundamental physical interactions, corresponds, strongly speaking, to PGTG, but not metric theory of gravitation, because the gauge group of supergravity theory includes the Lorentz group. As it is known, the simplest supergravity theory corresponds to the simplest PGTG – Einstein-Cartan theory. If the PGTG is correct gravitation theory, in this case quantum gravitation theory must have as quasi-classical approximation the gravitation theory in the Riemann-Cartan, but not pseudo-riemannian space-time.

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Note that the vacuum Schwarzschild solution for metrics with vanishing torsion is exact solution of PGTG independently on indefinite parameters of gravitational Lagrangian (1).
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