Theoretical and experimental investigation of spatial temperature distribution in active fiber under conditions of laser radiation generation

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Abstract. The longitudinal and transversal temperature distribution in active waveguide, doped with Yb/Er ions, under conditions of laser generation was calculated as the composition of two mathematical models: computation of longitudinal pump radiation distribution by solution of rate equations for Yb/Er doped active medium and computation of transversal temperature distribution by solution of heat equation for convectively cooled fiber using finite element method. The theoretical results were in a good agreement with experimental data, measured by the piezoelectric resonators used as temperature sensors, placed on the surface of the fiber. The pump radiation transferring length in active waveguide and the location of the most heated fiber area, as well as the maximum temperature value, were calculated using a created model of pump radiation transferring into GT-wave active fiber.

1. Introduction

High power fiber lasers are widely used in scientific research and different industry applications. Nowadays, the output power of the continuous-wave fiber lasers has reached 20 kW and 500 kW in single-mode and multi-mode regimes, respectively. Due to the presence of quantum defect, i.e. the energy difference between the pump and generated photon, part of the pumping power inevitably converts into heat, which causes heating of the active medium and changes its characteristics, such as the absorption and luminescence cross sections of active ions or the refractive index. This leads to a change in the output parameters of laser radiation: beam quality, laser efficiency, output power [1]. Therefore, it becomes necessary to measure active fiber temperature under lasing conditions.

There are several mathematical models where the temperature distribution in an active fiber has been calculated [2]. However, it is important to determine the correspondence between calculations and actual heating. In this work, the longitudinal temperature distribution of the active laser fiber is also measured experimentally. The temperature value was measured using a piezoelectric temperature sensor placed on the surface of the fiber. The equivalent temperature of sensor was determined from the frequency shift of the piezoelectric resonance [3], previously calibrated under uniform heating conditions. The frequency of piezoelectric resonance was measured as the frequency of electromagnetic oscillations in the circuit based on the Pierce generator with a crystal sensor placed between the capacitor plates [4]. The temperature value of the various active fiber cladding was calculated using the composition mathematical model of the radiation propagation in fiber laser and the solution of the heat equation. This allows one to compare the temperature of the crystal sensor with the heating of the most critical element of the fiber laser - the polymer cladding. We also investigated the region near the pump input with a higher spatial resolution. A mathematical model of transferring the pump radiation from a multimode fiber into a single-mode fiber core has been developed.
2. Theoretical study

2.1. Mathematical model of fiber laser
In this subsection the numerical simulation of a CW fiber laser with an active medium, doped with Yb\textsuperscript{3+}/Er\textsuperscript{3+} ions, pumped by multimode radiation from semiconductor diodes is described. The following parameters will be estimated from the model results:

- longitudinal distribution of the optical pumping power in the fiber under the conditions of laser radiation generation
- longitudinal distribution of the generated heat power
- dependence of the output power of the fiber laser on the pump power
- longitudinal temperature distribution in fiber

A schematic representation of the propagation of pump and lasing radiation in a simulated fiber laser in a stationary lasing mode is shown in the figure 1.

![Figure 1. Block diagram of signal and pump radiation propagation in the cavity of a fiber laser formed by fiber Bragg gratings (FBG).](image)

In this figure: \( P_s^+(z), P_s^-(z) \) - a function of the power of the signal radiation propagating in the forward and reverse directions. \( P_p(z) \) is a function of the pump radiation power.

![Figure 2. Diagram of energy levels and transitions in Yb\textsuperscript{3+} and Er\textsuperscript{3+} ions [2].](image)

For the given energy diagram of active medium doped with Yb/Er ions (figure 2), the rate equations [2] can be written as:

\[
\frac{\partial n_2}{\partial t} = 0 = -\frac{n_2}{\tau_{21}} + \frac{n_3}{\tau_{32}} + W_{12} n_1 - W_{21} n_1 - 2C_{up} N_{Er} n_2^2
\]
(1)

\[
\frac{\partial n_2}{\partial t} = 0 = -\frac{n_2}{\tau_{21}} + \frac{n_3}{\tau_{32}} + W_{12} n_1 - W_{21} n_1 - 2C_{up} N_{Er} n_2^2
\]
(2)

\[
\frac{\partial n_6}{\partial t} = 0 = -\frac{n_6}{\tau_{65}} + W_{56} n_5 - W_{65} n_6 - R_{61} N_{Er} n_1 n_5 + R_{55} N_{Er} n_3 n_5
\]
(3)

\[
n_1 = 1 - n_2 - n_3
\]
(4)

\[
n_5 = 1 - n_6
\]
(5)

\[
\frac{dP_p}{dz} = [n_6 c_{65}(\lambda_p) - n_5 c_{56}(\lambda_p) - n_1 c_{13}(\lambda_p) - \alpha_p] P_p
\]
(6)
\[ \pm \frac{dP_s^\pm}{dz} = [n_2c_{21}(\lambda_s) - n_1c_{12}(\lambda_s) - \alpha_s]P_s^\pm + \frac{2hc^2}{\lambda_s^5}m_sn_2c_{21}(\lambda_s)\Delta\lambda \]  

(7)

Equations (1-5) describe the distribution of the population inversion of Yb and Er ions, equations (6-7) describe the propagation of pump and signal radiation in the forward and reverse directions in the cavity. In this equations, \( n_i(z) \) is the concentration function of Er \((i = 1, 2, 3)\) and Yb \((i = 5, 6)\) ions in the corresponding energy states. \( W_{ij}(z) \) is a quantity describing the induced transitions probability, \( c_{ij} \) is a constant describing the degree of interaction of radiation with active ions during the energy transition \( i \rightarrow j \), \( C_{up} \) is the up-conversion coefficient, \( R_{ij} \) is the coefficient of cross-relaxation between ions at levels \( i, j \), \( N_{Yb,Er} \) - concentration of ytterbium and erbium ions:

\[
c_{ij}(\lambda) = \Gamma_s\sigma_{ij}(\lambda)N_{Er,i}, \quad ij = 12,21,13
\]

(8)

\[
c_{lm}(\lambda) = \Gamma_p\sigma_{ij}(\lambda)N_{Yb,lm}, \quad lm = 56,65
\]

(9)

\[
W_{ij} = \frac{c_{ij}(\lambda_s)[P_s^+(z) + P_s^-(z)]\lambda_s}{A_{core}hcN_{Er}}, \quad ij = 12,21,13
\]

(10)

\[
W_{lm} = \frac{c_{lm}(\lambda_p)[P_s^+(z) + P_s^-(z)]\lambda_p}{A_{core}hcN_{Yb}}, \quad lm = 56,65
\]

(11)

This system of rate equations is complemented by the boundary conditions determined by the reflection coefficients from FBG:

\[
P_p(z = 0) = P_{pump}^0
\]

(12)

\[
P_s^-(z = L) = P_s^+(z = L) \times r_{oc}
\]

(13)

\[
P_s^+(z = 0) = P_s^+(z = 0) \times r_{hr}
\]

(14)

\[
P_{out} = P_s^+(z = L) - P_s^-(z = L)
\]

(15)

The compiled system of equations (1-15) is a boundary value problem, for the solution of which the shooting method was used. The initial approximations were given for the left boundary of the resonator \((z = 0)\):

\[
P_p(z = 0) = P_{pump}
\]

(16)

\[
P_s^- = \frac{P_{pump}}{2}
\]

\[
P_s^+ = P_s^- \times R_{hr}
\]

The solution is a distribution of the population inversion, pump and lasing power along the active fiber at the nodes of the sampling grid.

The parameters of investigated active fiber for modeling were taken from its datasheet. The values of the absorption and luminescence cross sections, the up-conversion coefficient, the lifetimes at different energy levels, and passive losses in the fiber were taken from [2]. The used parameters are presented in table 1. For simplicity of the solution, the dependence of the parameters on temperature was not taken into account.

| Parameter                        | Designation | Numerical value   |
|----------------------------------|-------------|-------------------|
| Signal wavelength                | \( \lambda_s \) | 1550\times10^{-9} m |
| Signal bandwidth                 | \( \Delta\lambda_s \) | 4\times10^{-9} m |
| Pump wavelength                  | \( \lambda_p \) | 960\times10^{-9} m |
| Yb ion concentration             | \( N_{Yb} \) | 4\times10^{-26} m^{-3} (6000 ppm) |
| Er ion concentration             | \( N_{Er} \) | 2.2\times10^{-25} m^{-3} (330 ppm) |
| Up-conversion coefficient        | \( C_{up} \) | 3.0\times10^{-24} m^3 s^{-1} |
| Passive losses at \( \lambda_{signal} \) | \( \alpha_s \) | 20\times10^{-3} dB m^{-1} |
| Passive losses at \( \lambda_{pump} \) | \( \alpha_p \) | 70\times10^{-3} dB m^{-1} |
| Yb absorption crosssection at \( \lambda_{pump} \) | \( \sigma_{Yb} \) | 0.26\times10^{-24} m^2 |
| Parameter                                                                 | Value                                      |
|--------------------------------------------------------------------------|--------------------------------------------|
| Yb emission crosssection at \( \lambda_{pump} \) \( \sigma_{65} \)        | \( 0.14 \times 10^{-24} \text{m}^2 \)      |
| Er absorption crosssection at \( \lambda_{signal} \) \( \sigma_{12} \)   | \( 0.3418 \times 10^{-24} \text{m}^2 \)    |
| Er emission crosssection at \( \lambda_{signal} \) \( \sigma_{21} \)     | \( 0.4681 \times 10^{-24} \text{m}^2 \)    |
| Erbium lifetime in 2 state \( \tau_{21} \)                             | \( 1.0 \times 10^{-3} \text{s} \)          |
| Erbium lifetime in 3 state \( \tau_{32} \)                             | \( 1.0 \times 10^{-9} \text{s} \)          |
| Ytterbiu lifetime in 6 state \( \tau_{65} \)                           | \( 1.5 \times 10^{-3} \text{s} \)          |
| Active fiber core radius \( r_{core} \)                                 | 9 \text{ mkm}                             |
| Quartz cladding diameter for passive and active fibers \( r_{clad} \)   | \( 125 \times 10^{-6} \text{m} \)          |
| Cross-relaxation coefficient \( R_{35}, R_{61} \)                      | \( 2.37 \times 10^{-22} \text{m}^3 \text{s}^{-1} \) |
| Refractive coefficient of OC FBG \( R_{oc} \)                           | 0.701                                     |
| Refractive coefficient of HR FBG \( R_{hr} \)                           | 0.10                                      |
| Overlap integral between pump radiation and active core \( \Gamma_p \)   | 5.4 \times 10^{-3}                         |
| Overlap integral between signal radiation and active core \( \Gamma_s \)  | 1.0                                       |
| Resonator length \( L \)                                               | 4 \text{ m}                               |

The obtained distributions for various pump powers are shown in figure 3. Comparison of the calculated output power with experimental data is shown in figure 4. It can be seen that the model, considering an accuracy of measurement errors, is in a good agreement with real fiber laser.

**Figure 3.** Pump power distribution inside the cavity at different initial powers.

**Figure 4.** Comparison of model results and experimental data for laser output power.

The obtained distribution of the pump radiation power makes it possible to calculate the distribution of the thermal heat power along the active fiber. Considering that the heating is only caused by the presence of a quantum defect, the thermal power is equal to:

\[
P_{\text{heat}}(z = i \times \text{step}) = \frac{(P_{pump}^i - P_{pump}^{i+1}) \times \zeta}{\text{step}}
\]

(17)

Where \( i \) is the number of the sampling grid node, \( \zeta \) is the quantum defect ratio and \( \text{step} \) is the grid step.

The calculated power value is shown in figure 5. The graph shows that the maximum value of the released power is reached at the beginning of the active fiber, i.e. in the section where the pump power is maximum, and, consequently, the value of the absorbed power is also maximum.
The GT-wave fiber profile with crystal sensor on it is shown in figure 6. Pumping radiation from multimode semiconductor diodes in this type of active fiber is initially enters into a passive multimode fiber, from which it is subsequently transfers into the active single-mode fiber core, where is absorbed by the rare-earth ions.

Due to relatively low thermal conductivity of quartz silica and polymer, it is important to determine transversal temperature distribution inside the fiber. To calculate the temperature of different sections of the active fiber, the steady-state thermal conductivity equations (18-20) using Newton-Richmann boundary conditions (21) will be solved.

\[ k(R)\Delta T(R) = -Q(R) \] in the fiber core

\[ k(R)\Delta T(R) = 0 \] inside the other claddings

\[ \frac{dT(R)}{n} = 0 \] in the cladding’s boundaries

\[ \frac{dT(R)}{n} = -h_0(T_{\text{crystal}} - T_{\text{air}}) \] (18)

\[ \frac{dT(R)}{n} = -h_4(T_{\text{polymer}} - T_{\text{air}}) \] (19)

where Q (r) is the released heat power, k is the thermal conductivity coefficient, h is the convective heat transfer coefficient, n is the external normal vector to the surface.

The required value of the heat transfer coefficient was measured experimentally by investigation of the heating kinetics of the crystal using the heat balance equation (22) with the approximation of uniform heating of the crystal. Its solution is shown in (23).

\[ cm \frac{dT(t)}{dt} = hS(T_{\text{ext}} - T(t)) + Q \] (22)

\[ \Delta T(t) = T_1 + (T_0 - T_1) \exp \left( -\frac{hS}{mc} \right) \] (23)

where S is the surface area of the crystals, m is the mass of the crystals, c is the heat capacity, Q is the power of the heat source, T_0 is the initial temperature of the crystals, T_1 is the final temperature of the crystals. To determine the heat transfer coefficient, the kinetics of crystal heating were measured with a sequential increase in the pump power, when they were placed on the fiber at a distance of ≈50 cm from the pump input. The coefficient of convective heat transfer at the crystal - air interface was determined: h = 16.4 ± 2.3 W m⁻² K⁻¹.

Figure 5. Distribution of the released thermal power in the active fiber.

Figure 6. GT-wave fiber profile with crystal temperature sensor on it.
Then, equations (18) - (21) were solved by the finite element method. A three-dimensional diagram of an active fiber with a crystal-sensor is shown in figure 7. The temperature distribution in this element is shown in figure 8.

Crystal sensor affects the heating of the fiber segment. The fiber contacting with the crystal has a more efficient heat exchange and its temperature is much lower than that of the fiber with passive convective cooling. In order to correctly estimate fiber heating this factor should be taken into account. The calculated temperature distribution along the fiber near the crystal is shown in figure 9.

The dependence of the heating of various fiber claddings on the pump power is shown on figure 10. The correspondence between the thermal power and measured pump power was established using a fiber laser model. It is especially important to know polymer cladding temperature since it has much lower heat resistance (about 120 °C).

From the obtained results, it is possible to estimate the limiting pump power for a convectively cooled active fiber. Considering the maximum heating of the polymer cladding as 100 K, the maximum pump power is 33 W for one-side pumping or 58 W for two-side pumping (29 W from both sides), which corresponds to the released thermal power of 7.8 W/m in the hottest part of the fiber.
3. Experiment
Since theoretical modelling always has some assumptions and simplifications, comparison with experimental measurements is highly desirable. In this work, piezoelectric U-shaped resonators made of LiNbO$_3$ are used as sensors to determine the longitudinal distribution of active fiber temperature. The electrical scheme of the experimental setup based on Pierce generator is shown in figure 11.

The main advantage of these crystals is their transparency in the optical range of 0.5–5 µm [5] and, as a consequence, the absence of additional heating of the sensors caused by absorption of scattered radiation and spontaneous luminescence from the active fiber. Due to the reverse piezoelectric effect, acoustic vibrations arise in piezoelectric crystals under the influence of an external alternating electric field. If the frequency of the electric field coincides with the frequency of one of the natural modes of crystal vibrations, then piezoelectric resonance can be excited. Any piezoelectric crystal has a set of piezoelectric resonances, which frequencies depend on geometric dimensions of the sample, its elastic constants and external conditions, in particular its temperature. In this work, we used a piezoelectric resonance at a frequency of 994 kHz.

Under conditions of uniform heating up to 30 K, the dependence of the frequency of the resonances used is linear and equal to

$$R_f(T) = R_f(T_0) + K_{PRT} (T - T_0), \quad K_{PRT} = \frac{\Delta R_f}{dT} \tag{24}$$

where $K_{PRT}$ is the resonant thermal coefficient, $R_f(T)$ is the piezoelectric resonance frequency.

When crystals are heated by an active fiber, their temperature is nonuniform throughout the volume, therefore, the concept of an equivalent temperature was introduced [6]:

$$\Theta_{Eq} = T_0 + K_{PRT} \Delta R_f \tag{25}$$

where $\Delta R_f$ is the frequency shift of the piezoelectric resonance with the resonant thermal coefficient $K_{PRT}$.

Due to high thermal conductivity of crystal and it small size the value of temperature inhomogenity is significantly less than the equivalent temperature, so the use of the equivalent temperature principle is justified. To measure the temperature of the active fiber, a crystal sensor was placed on top of it figure 6. The crystal was placed between the plates of a capacitor connected to the Pierce generator circuit (figure 11) [4].

Heating of the fiber under the conditions of generation of laser radiation will lead to a shift in piezoresonance frequencies of sensor crystals and, therefore, in a frequency of the output voltage $U_{out}$ of the Pierce generator circuit. Moving such a sensor along the length of the active fiber will provide a longitudinal temperature distribution in the fiber laser.

In this work, we investigated a fiber laser with active waveguide doped with Yb$^{3+}$/Er$^{3+}$ ions. Active fiber was 4 meters long and a pump absorption coefficient was 3.5 dB/m. The output radiation wavelength was 1550 nm, the laser efficiency from optical pumping was 0.33, the optical pumping wavelength was $\lambda = 962$ nm, and the quantum defect was $\eta = 38\%$. The scheme of experimental setup is shown in figure 12:
In the first experiment, the dependence of the crystal heating on the distance from the pumping point was investigated for one-side and two-side pumping regimes. The results of the longitudinal temperature distribution for various pump powers $W$ are shown in figure 13, 14. Solid lines show the results of mathematical modeling of heating described earlier.

The obtained graphs showed that the results of the model are in good agreement with the experimental data. We note that pumping from both sides makes it possible to reduce the maximum heating of the active fiber by 38% for the same total pump power. Also, a decrease in heating can be achieved by decreasing the generated thermal power by reducing the concentration of active ions and increasing the length of active fiber.

It is very important to know the value of the maximum temperature in the fiber and its localization, since this point is the most critical for the thermal stability of the fiber. The small size of the crystal sensor make it possible to measure the heating of the fiber in the hottest place (near the pump injection point) with a high spatial resolution. The dependence of heating in this region at a pump power of 11.5 W is shown in figure 15.
It can be seen that in the initial section the temperature of the crystal differs greatly from the model data, which may be due to the fact that most of the pump radiation has not yet managed to transfer into active core of a single-mode waveguide from a multimode pump waveguide in GT-wave fiber.

Let us estimate the characteristic length of pumping radiation transfer from the multimode to the single-mode waveguide. Let's denote the probability of the transition of the pump radiation photon from the multimode to the single-mode waveguide as \( \beta \) [1/m], and reverse transition probability as \( \alpha \) [1/m]. \( N_p / S_p, N_s / S_s \) are the photon flux densities inside two waveguides, and \( S_s, S_p \) are the cross-sectional areas of the corresponding waveguides. Then the rate equation for the photon flux density in the single-mode waveguide:

\[
\frac{\partial N_s}{\partial l} = -N_s \alpha + N_p \beta = -N_s \alpha + (N_0 - N_s) \beta
\] (26)

where \( N_0 \) is the total photon flux. In the stationary case at a large distance from the pump injection point, the equality of two photon fluxes is fulfilled:

\[
0 = -N_s x + N_p y \rightarrow \frac{N_s}{N_p} = \frac{d_s^2}{d_p^2} = \gamma
\] (27)

where \( d_p, d_s \) are the diameters of the multimode and single-mode waveguides.

The solution of equation (25) (\( N_p + N_s = \text{const} \)) is a monotonically increasing function with the limit \( \gamma N_0 (1 + \gamma) \):

\[
N_s(l) = \frac{\gamma N_0}{\gamma + 1} \left(1 - \exp(-(\gamma + 1) \alpha l)\right)
\] (28)

We assumed that the pump power distribution function in single-mode waveguide is the combination of the function of pump power distribution in GT-wave fiber \( P_{\text{pump}}(l) \) (figure 3) and the fraction of pump radiation in the single-mode fiber (28). We found the parameter \( \alpha \) from the approximation of the experimental data by this function.

\[
\frac{(\gamma + 1)N_s(l)}{\gamma N_0} \times P_{\text{pump}}(l)
\] (29)

The results of approximation are shown in figure. 16. The obtained value is \( \alpha = 0.32 \pm 0.03 \) m\(^{-1}\), the characteristic distance of pumping radiation is 5.7±0.6 cm (at a level of 90% of the value \( \gamma N_0 (1 + \gamma) \)).

From the analysis of function (28) it can be seen that a decrease in the ratio of the area of the single-mode active waveguide to the area of the multimode waveguide slows down the pump radiation transition into the active medium, which reduces the peak temperature value and shifts the maximum temperature point aside the pump input point.

4. Conclusions
In this work, a fiber laser operation with Yb/Er doped waveguide fiber was simulated. Computation of longitudinal pump radiation distribution was based on solution of rate equations and computation of transversal and longitudinal temperature distribution was made by solution of heat equation for convectively cooled fiber with crystal sensor. The theoretically calculated data were in a good agreement with the experimental measurements of fiber heating performed using a piezoelectric crystal as a temperature sensor. The usage of lithium niobate crystal sensors has many advantages: the accuracy of the measured heating, high spatial resolution and transparency for laser radiation \([5]\) and, as a result, the absence of additional heating due to absorption of spontaneous luminescence and scattered radiation.

In this work experimental dependences of the temperature distribution were obtained at various powers and pumping methods. The maximum values of crystal heating for one side pumping was 40 K and for two side pumping - 25 K for 23 W of total pump power. Corresponding calculated polymer heating was 80 K and 50 K, respectively.

The dependence of active fiber heating near the pump input, which is the most heated area of the active fiber, was also studied in more details. A probabilistic model has been developed for transferring
pump radiation from a multimode fiber to a single mode signal fiber, which makes it possible to estimate the typical length of pump radiation transfer in GT-wave active fiber, which was equal to 5.7 cm.

The work was carried out within the framework of the state task.

5. References

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