Joint Design of Physical-layer Network Coding and LDPC Code Modulated by 2FSK

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Abstract—In order to improve the transmission efficiency in wireless communication and reduce the bit error rate in the transmission process, a physical-layer network coding(PNC) system with 2FSK modulation and low-density parity check(LDPC) codes as the channel coding is established, and a relay detection scheme for the system is designed. This scheme first obtains the waveform diagram of the signal in the network encoding process. According to the characteristics of 2FSK modulation, and then we can extracts the prior probability soft information by the waveform characteristics, and finally we can combine the log likelihood ratio belief propagation(LLR-BP) algorithm to complete the decoding and relay mapping process. The simulation results show that the system adopts 2FSK modulation and has superior bit error rate performance compared with BPSK modulation. After introducing low-density parity check(LDPC) codes, the system can ensure high-speed transmission of information while effectively improving system Bit error ratio performance.

1. INTRODUCTION
In the traditional wireless communication system, two nodes require four time slots to complete one information exchange through single relay and the traditional relay only allows for the amplifying and retransmitting the information which results in very low data throughput capacity and efficiency of data transmission. The Network Coding, NC technique found by R.Alshwede in 2000 can increase the data throughput capacity and efficiency of data transmission greatly as compared to traditional relay forwarding, however, in the actual transmission process of wireless data communication, there exists many interference and blackout which all pose significant challenge for NC. Hence, S.Zhang introduced the concept of Physical-layer Network Coding, PNC in 2006. As compared to NC, the PNC receives the superposed signal after modulation and interference by noise at the relay node and then map to the results of Network Coding based on corresponding mapping rules designed for the type of modulation. In this way, the data throughput capacity is further increased. In comparison with the traditional double-direction relay channel model, PNC increased the throughput capacity by 50% on the basis of NC, and doubled the capacity of traditional transmission methods.

As of now, the global research fields related to PNC mainly includes, the combination of PNC and modulation techniques, the combination of PNC and channel coding techniques. Articles [8][9] focus on the research of the combination of PNC and channel coding techniques based on BPSK modulation, but in the actual communication system, the problem of inversion may occur for the BPSK modulation during the demodulation process which leads to lowering of the system’s bit error rate functions. Article [10] improves the decoding algorithm with regards to the joint system of PNC and LDPC. It offers another algorithm based on the BP algorithm, lowering the difficulty level of decoding, but this algorithm involves massive calculation and the system has a relatively low efficiency of decoding.
Article [11] mentions the double-direction relay system of the PNC technique based on FSK modulation, in which the demodulation method used by the relay node is non-coherent modulation while in fact, most of the actual communication systems use coherent modulation. With regards to the problems mentioned above, this article will make a joint design of the physical layer Network Coding, the 2FSK modulation technique and the LDPC channel coding, i.e. the 2FSK-LDPC-PNC system. This system adopts 2FSK modulation which will solve the inversion problem caused by BPSK modulation easily. It uses non-coherent demodulation which most of the current communication systems use. And the LLR-BP decoding used when relay decoding also lowers the amount of calculation required significantly.

2. SYSTEM MODEL
The 2FSK-LDPC-PNC system designed in this article includes uplink (Fig. 1) and downlink (Fig. 2), the originating node of the system work in half duplex mode, the channels are symmetrical and the signals are synchronized.

2.1 In the uplink phase (time slot 1), originating nodes A and B send out information streams $S_A$ and $S_B$ respectively, through LDPC coding for $S_A$ and $S_B$, information streams $C_A$ and $C_B$ can be received. Then, it applies 2FSK modulation for the coded information streams, the two signals after modulation will then be broadcasted to relay node R. After the superposed signal $Y_R$ is received at relay, it is LLR-BP coded and PNC mapped to obtain the result of Network Coding $S_R$. The signal received at relay can be represented by:

$$Y_R(t) = X_A(t) + X_B(t) + n_{AR}(t) + n_{BR}(t)$$

In which $X_A(t)$ and $X_B(t)$ are signals after the codes $C_A$ and $C_B$ have been modulated and is represented by:

$$X_i(t) = \begin{cases} A\cos(2\pi f t) & C_i = 1 \\ A\cos(2\pi f t) & C_i = 0 \end{cases} (i = A, B)$$

$n_{AR}$ and $n_{BR}$ represents the Gaussian noise from nodes A and B to the relay R channel, with an average value of 0 and variance $\sigma^2 = N_0$.

2.2 In the downlink phase (time slot 2), the relay R LDPC codes the result of the result of Network Coding, $S_R$, and the information stream $C_R$ received after coding is then 2FSK modulated and broadcasted to A and B. After the information has been received, it is then decoded to obtain information stream $S_{R2A}$ and $S_{R2B}$. Finally, the originating nodes A and B compare the bit difference or calculate the decoded information stream with the information they send out themselves to obtain new data and therefore one information exchange is completed.

Fig. 1 Uplink Model of 2FSK-LDPC-PNC system
3. 2FSK-LDPC-PNC RELAY TESTING METHOD

The signal \( Y_R(t) \) received at the relay node is split into two and processed as shown in Fig. 3. It first passes the band pass filters of frequencies \( f_1 \) and \( f_2 \), then multiplied by the local carrier through the multiplier and finally the high frequency part is filtered out using the low band filter to obtain DC signal \( Z_A(t) \) and \( Z_B(t) \). The two DC signals \( Z_A(t) \) and \( Z_B(t) \) are then processed together to get one DC signal \( M(t) \). A sample \( m(t) \) is taken from \( M(t) \) and the soft information needed for coding is calculated from the sample and then keyed into the decoder for LLR-BP decoding and finally the result is obtained through decision decoding. The LLR-BP algorithm uses the log-likelihood ratio to represent the relative probability information in the probability domain. In this way, the complex multiplication is simplified into addition and hence the amount of calculation required in the decoding process is reduced. This algorithm used the wave characteristics of channel output and adapts the iterative decoding idea of soft input and soft output.

\( C_A \) and \( C_B \) are obtained from the information streams \( S_A \) and \( S_B \) sent out by the originating nodes A and B through LDPC coding. Fig. 4 shows the baseband signals of \( C_A \) and \( C_B \) (partially wave-like), Fig. 4(a) shows part of the wave in \( C_A \), which has a signal sequence of 1001110010; Fig. 4(b) shows part of the wave in \( C_B \), which has a signal sequence of 0100101100.

The wave pattern of the information streams \( X_A \) and \( X_B \) after \( C_A \) and \( C_B \) have been 2FSK modulated. Fig. 5 (a) shows part of the wave pattern of \( X_A \) while Fig. 5 (b) shows part of the wave...
pattern of $X_B$.

Based on equations (1) and (2) and use $n(t)$ to represent the superposed signal of the two noises $n_{AR}$ and $n_{BR}$ from A and B, the signal received at relay R can then be represented as:

$$Y_R(t) = \begin{cases} 
2.0 \cos 2\pi ft + n(t) & C_A C_B = 00 \\
A \cos 2\pi ft + A \cos 2\pi ft + n(t) & C_A C_B = 01 \\
2.0 \cos 2\pi ft + n(t) & C_A C_B = 11
\end{cases}$$  \hspace{1cm} (3)

The Gaussian noise $n(t)$ can be vector resolved into:

$$n(t) = n_1(t) \cos 2\pi ft + n_2(t) \sin 2\pi ft$$  \hspace{1cm} (4)

When the noise interference is negligible, the wave pattern of superposed signal $Y_R(t)$ as shown in Fig. 6

$$Y_R(t) = \begin{cases} 
2.0 + n_1(t) + n_2(t) & C_A C_B = 00 \\
n_1(t) - n_2(t) & C_A C_B = 01 \\
-2.0 + n_1(t) - n_2(t) & C_A C_B = 11
\end{cases}$$  \hspace{1cm} (5)
Taking the lower envelope of the wave pattern of $M(t)$, the sample obtained when $t$ has a value between 0 to 1 is 0, the sample is 2 when $t$ is from 2 to 3, and -2 when $t$ is from 4 to 5. The same procedure may be adapted for subsequent patterns and $m(t)$ is used to represent the sample obtained from $M(t)$, which is the prior probability soft information needed before decoding.

Comparing Fig.4 and 7, it can be found that the sample $m$ has the following relationship with $AC$ and $BC$.

$$m = \begin{cases} -2 & C_A C_B = 00 \\ 0 & C_A C_B = 01 \text{ or } C_A C_B = 10 \\ 2 & C_A C_B = 11 \end{cases}$$  \hspace{1cm} (6)

Based on equal probability of the source, the prior probability form is:

$$\begin{align*}
\text{Pr}(m=-2) &= \frac{1}{4} \\
\text{Pr}(m=0) &= \frac{1}{2} \\
\text{Pr}(m=2) &= \frac{1}{4}
\end{align*}$$  \hspace{1cm} (7)

The network coding result after decoding is $S_R = C_A \oplus C_B$, hence

$$\begin{align*}
\text{Pr}(S_R=0) &= \text{Pr}(m=-2) + \text{Pr}(m=2) \\
\text{Pr}(S_R=1) &= \text{Pr}(m=0)
\end{align*}$$  \hspace{1cm} (8)

When the soft information $m$ is remapped to obtain $S_R$, the mapping relationship is as shown in Table 1. When soft information $m$ is 2 and -2, the mapping result is 0, when the decision result is 0, the mapping result is 1.

| $C_A C_B$ | $m$  | $S_R (C_A \oplus C_B)$ |
|----------|------|------------------------|
| 11       | 2    | 0                      |
| 10       | 0    | 1                      |
| 01       | 0    | 1                      |
| 00       | -2   | 0                      |

Convert equation (8) using logarithm to obtain:
According to Article [8], in the Gaussian noise channel, when the input is $x = a$, the probability of obtaining an output $y$ can be represented as:

$$P(y|x = a) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{E_b}{2\sigma^2}(y-a)^2\right)$$

Then equation (9) can be represented as:

$$L(P_i) = \ln \left( \frac{P(S_i = 1|m)}{P(S_i = 0|m)} \right) = \ln \left( \frac{\frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2\sigma^2}m^2\right)}{\frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2\sigma^2}(m-2)^2\right) + \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2\sigma^2}(m+2)^2\right)} \right)$$

Then using the LLR-BP algorithm to decode, let $P_j^0$ and $P_j^1$ be the prior probability of the channel before decoding, i.e., sample value $m$, and let $Q_j^0$ and $Q_j^1$ be the probability that the variable node transmitted to the checking node before decoding, and let $R_j^0$ and $R_j^1$ be the probability information at the checking nodes that satisfies the checking relationship. The main steps of the LLR-BP decoding algorithm are as follows:

a) Initialization Information

$$\begin{cases} Q_j^0 = P_j^0 \\ Q_j^1 = P_j^1 \end{cases}$$

b) Renew checking information $L(R_j)$ based on equations (13) and (14) with regards to all checking nodes.

$$\begin{cases} R_j^0 = \frac{1}{2} \left[ 1 + \prod_{j \in N(i):j} (1 - 2Q_j^1) \right] \\ R_j^1 = 1 - R_j^0 \end{cases}$$

$$L(R_j) = \ln \frac{R_j^1}{R_j^0}$$

(14)

c) Renew variable information $L(Q_j)$ based on equations (15) and (16) with regards to all variable nodes.

$$\begin{cases} Q_j^0 = P_j^0 \prod_{i \in M(j):i} R_j^0 \\ Q_j^1 = P_j^1 \prod_{i \in M(j):i} R_j^1 \end{cases}$$

$$L(Q_j) = \ln \frac{Q_j^1}{Q_j^0}$$

(16)

d) Decision decoding. Perform decision decoding based on equations (17) and (18), if $L(q_j) > 0$, then the decision is 1, otherwise, it is 1. If the sequence $\hat{c}$ after decoding satisfies $\hat{c} \cdot H^T = 0$, or
reach the maximum number of iterations, the decoding stops, otherwise, the iteration continues returning to step (2).

\[
\begin{align*}
q_j^0 &= k_j \prod_{i \in M(j)} R_{ij}^0 \\
q_j^i &= k_j \prod_{i \in M(j)} R_{ij}^i \\
L(q_j) &= \ln \frac{e_j}{\bar{e}_j}
\end{align*}
\]

(17)  

4. SIMULATION ANALYSIS

All simulation experiments in this article are performed by MATLAB software. It is assumed that the information the system sends out is synchronized and not considering the fading of the signal or the distribution of power. The channel noise inference used is Gaussian noise, and the total simulation data for each system is 3200 bit. When using LDPC codes, the bit rate is 0.5, the code length is 256 and the maximum number of iteration is 25.

4.1 Fig. 8 shows the simulated comparison of the bit error rate between the multiple joint communication system based on BPSK modulation and that based on 2FSK modulation. It can be seen from Fig.16 that when the BER has a magnitude of $10^{-2}$, the Network Coding system based on BPSK modulation (BPSK-NC) has an approximately 15dB loss in its BER as compared to the 2FSK-NC system and the physical layer Network Coding system based on BPSK modulation (BPSK-NC) has an approximately 8dB loss in its BER as compared to the 2FSK-PNC system. From above information, it can be concluded that in the same system, compared to article [6]which uses BPSK modulation, using 2FSK modulation can better improve the BER function of the system.

![Fig. 8 Bit Error Rate of 2FSK-NC, BPSK-NC, 2FSK-PNC and BPSK-PNC from node to node](image)

4.2 Fig. 9 shows the simulated comparison of the bit error rate between the traditional relay transmission communication system based on 2FSK modulation (2FSK-TS), the Network Coding system based on 2FSK modulation (2FSK-NC) and the 2FSK-PNC system. When BER has a magnitude of $10^{-3}$, the 2FSK-TS and 2FSK-NC have basically the same BER, but the BER for 2FSK-TS is still slightly better than that for 2FSK-NC. At the same BER magnitude, the BER for 2FSK-PNC has an approximately 10dB loss compared to 2FSK-TS. 2FSK-PNC has the worst BER data among the three. Hence, adopting PNC technique will worsen the system’s BER function, but it only takes two time slots for the 2FSK-PNC system to complete one information exchange which improves the system throughput and the system transmission efficiency greatly.
4.3 Fig. 10 shows the simulated comparison of the bit error rate from node to node between the 2FSK-NC, 2FSK-PNC and 2FSK-LDPC-PNC systems. When BER has a magnitude of $10^{-3}$, the 2FSK-PNC system has a 9.5 dB loss as compared to the 2FSK-NC system. After substituting LDPC as the channel code, the 2FSK-LDPC-PNC shows a 5.5 dB increase compared to the 2FSK-PNC system. Therefore, the 2FSK-LDPC-PNC system shows better BER performance compared to 2FSK-PNC, the adaptation of LDPC channel coding can effectively reduce the system bit error rate and thus increase the reliability of system transmission.

5. Conclusion
The 2FSK-LDPC-PNC system designed in this article adopts 2FSK modulation, uses LDPC for channel coding and designed corresponding relay checking strategies. This method extracts the prior probability soft information that the LLR-BP decoding algorithm requires from straightforward signal wave pattern graphs and then complete the decoding and mapping process. The simulation results show that as compared to the physical layer Network Coding method with BPSK modulation adopted in article [5][6], 2FSK modulation has a better bit error rate compared to the physical layer Network Coding system without channel coding. The 2FSK-LDPC-PNC system with LDPC code effectively lowers the bit error rate of system transmission while ensuring high throughput rate and high speed transmission.

Based on the research in this article, future research will also adopt high order modulation, different channel coding, the out-of-step issue of the signal and distribution of power.

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