Computer Simulation of Kinetics of Parallel Mechanisms

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Abstract. The paper considers the issues of kinematics and dynamics of a parallel mechanism and computer simulation of its movement. To solve the problems of dynamics of parallel mechanisms, the method of Lagrange equations of the second kind was used. In computer modeling, the Matlab and Simulink application were used.

1. Introduction
Now in engineering, the urgent task is to design mechanisms for mechanical treatment of the internal surfaces of cavities of complex shape, for example, in aviation and space technology, in disaster technology, mechanisms are needed to provide access to the internal volumes of destroyed buildings and structures, to carry out repair and restoration work in pipelines, to perform certain surgical operations, etc. [1-8]. There are many projects devoted to the development of such robots and many are working on solving the problems that arise in this process [9-16].

In this article, a structural, kinematic and dynamic analysis of a tripod - a manipulator based on parallel mechanics was conducted. The questions of kinematic and dynamic analysis of a plain parallel mechanism were investigated. The use of parallel mechanisms complicates control tasks and leads to more complex control systems for such machines. Therefore, it becomes important to develop appropriate algorithms for solving problems related to the movement of parallel mechanisms. The authors faced problems of this type in solving a number of theoretical and applied tasks [17-24]. Well-known methods were used for the solution [2, 25-30].

For example, after obtaining a solution in a model using the method of Lagrange equations of the second kind, according to [15, 16], a model was created using the Denavit—Hartenberg method [30]. A comparison of these two solutions made it possible to make certain of applicability and reliability of the obtained numerical solution.

2. Mathematical model
Manipulators can be built on the basis of parallel mechanisms such as tripods. An example of such a tripod, which consists of a fixed base, a movable platform, and 4 legs, each of which consists of 2 rods and an active translational kinematic pair is shown in Fig. 1. To estimate the number of degrees of freedom W of the platform, the well-known Grubler and Somov—Malyshev formulas are used. Tripod shown in Fig. 1, has 3 degrees of freedom (two rotational and one translational).

According to [15], the generalized coordinates $q_{1-3}$ were introduced, which are the lengths of the rods $Aa, Bb, Cc$, respectively (Fig. 1).
It was believed that the fixed platform (base) is horizontal and the points \( A_i \) also form a regular triangle with side \( a \) and center at point \( A \). The legs \( A_iB_i \) have lengths \( l_i > 0 \) and are inclined to the base plane at angles \( \gamma_i, (0 < \gamma_i < \pi/2) \). The distance between the point \( B_i \) and the base (plane \( AXZ \)) is denoted by \( h \). The coordinate system \( AXYZ \) was connected with the base, and the coordinate system \( Bxyz \) was connected with the platform (Fig. 1).

The position of the joints \( A1-3 \) in the coordinate system \( AXYZ \) and the joints \( B1-3 \) in the coordinate system \( Bxyz \) is determined by the vectors. The position of the platform relative to the base is set by Euler angles \( \phi_1,2 \) and vector \( B \). The geometric relations between the coordinate systems \( AXYZ, Bxyz \) can be represented as a 4×4 matrix of homogeneous transformations

\[
T = T(h, \varphi_1, \varphi_2) = \begin{pmatrix}
\cos \varphi_1 & -\sin \varphi_1 & 0 & 0 \\
\sin \varphi_1 \cos \varphi_2 - \cos \varphi_1 \sin \varphi_2 \sin \varphi_1 h & \sin \varphi_1 \sin \varphi_2 - \cos \varphi_1 \sin \varphi_2 \cos \varphi_2 h & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

Then the generalized coordinate \( l_i \), as a function of distance \( h \) and angles \( \varphi_{1,2} \), is determined by the expression

\[
l_i = l_i(h, \varphi_1, \varphi_2) = \sqrt{\sum (A_{ij} - [B_{ij}])^2}, \quad i, j = 1,2,3. \tag{1}
\]

where the formulas for \( A_{ij} \) and \( B_{ij} \) are given in [15, 16]. Expressions for the rates and accelerations of the ends of the legs \( l_i \) are easy to find if we take the derivatives on \( h, \varphi_{1,2} \) of the formulas obtained.

According to the same scheme, the equations of movement of the platform are composed using the Lagrange equations of the second kind:

\[
\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{\varphi}_i} \right) - \frac{\partial E}{\partial \varphi_i} = Q_i, \quad i = 1,2,3, \tag{2}
\]

where \( E \) is the kinetic energy of the system; \( q_1 = h, q_2 = \varphi_1, q_3 = \varphi_2 \) are generalized coordinates; \( Q_i \) is the generalized force on the \( i \) generalized coordinate.

If the mass of the platform is \( M \), then its kinetic energy

\[
E = \frac{1}{2} [M v_B^2 + J(\dot{\varphi}_1)^2 + J(\dot{\varphi}_2)^2],
\]
where $v_B = \dot{h}$ is the velocity of the B-platform CG.

The forces acting on the platform, as well as the radius vectors $R_B$, $R_i$ ($i = 1, 2, 3$), points of their application in the projections on the axis of the coordinate system $AXYZ$:

$$P = (0, -Mg, 0)^T, F_i = (F_i \cos \gamma_{1i}, F_i \cos \gamma_{2i}, F_i \cos \gamma_{3i})^T,$$

$$R_B = (0, h, 0)^T, R_i = ([B_{1i}], [B_{2i}], [B_{3i}])^T,$$

where $\cos \gamma_{ij} = \cos \gamma_{ij}(\phi_1, \phi_2) = \frac{[\beta_i]}{i}$.

Further, using the elements of the Jacobi matrix of vectors $R_i = R_i(h, \phi_1, \phi_2)$, the values of the generalized forces $Q_{1,2,3}$ were obtained. These values were substituted in (2), and the desired ODE system was obtained for solving the problems of dynamic tripod analysis.

$$M\ddot{h} = -Mg + F_1 \cos \gamma_{12} + F_2 \cos \gamma_{22} + F_3 \cos \gamma_{32},$$

$$J\ddot{\phi}_1 = \frac{b}{2} F_1 (-\cos \gamma_{21} \sin \phi_1 + \cos \gamma_{12} \cos \phi_1 \cos \phi_2) +$$

$$+ \frac{b}{2} F_2 (-\cos \gamma_{21} \sin \phi_1 + \cos \gamma_{22} \cos \phi_1 \cos \phi_2 + \cos \gamma_{23} \cos \phi_1 \sin \phi_2) +$$

$$+ r F_3 (\cos \gamma_{13} \sin \phi_1 + \cos \gamma_{32} \cos \phi_1 \cos \phi_2 - \cos \gamma_{23} \cos \phi_1 \sin \phi_2),$$

$$J\ddot{\phi}_2 = \frac{b}{2} F_1 [\cos \gamma_{12} (-\sin \phi_1 \sin \phi_2 + \cos \phi_2) + \cos \gamma_{13} (\sin \phi_1 \cos \phi_2 - \sin \phi_2)] +$$

$$+ \frac{b}{2} F_2 [\cos \gamma_{22} (\sin \phi_1 \sin \phi_2 + \cos \phi_2) + \cos \gamma_{23} (\sin \phi_1 \cos \phi_2 - \sin \phi_2)] +$$

$$+ r F_3 (\cos \gamma_{32} \sin \phi_1 \sin \phi_2 - \cos \gamma_{33} \sin \phi_1 \cos \phi_2),$$

where $F_{1,3}$ are external forces.

To solve the resulting system of equations numerically on a computer, standard mathematical software packages were used. A similar algorithm was applied to tripods with various structural and kinematic schemes.

3. **Computer model**

For computer simulation, a model was created using block diagrams of the SimMechanics, which is an extension of the MatLab. Fig. 2 shows a fragment of the $A_1B_1C_1$ leg of the tripod model under consideration.

![Figure 2](image)

**Figure 2.** Fragment of the block diagram of the model of a planar parallel mechanism with three identical legs, which simulates the leg $A_1B_1C_1$ with the R-pairs connecting platforms and links.

According to [15], the generalized coordinates $q_{1,3}$ were introduced, which are the lengths of the rods $Aa$, $Bb$, $Cc$, respectively (Fig. 1).

Figures 3 and 4 show the results of comparing the numerical solution and computer simulation for two different tripod block diagrams. Numerical calculations and computer modeling were carried out using the mathematical application program package Matlab. In each figure, the results of the angular acceleration calculations for link $A_1B_1$, $A_2B_2$, and $A_3B_3$ go from top to bottom respectively. The green color shows the results of a numerical solution of the Lagrange equations of the second kind. Red color shows the results of computer simulation.

The tripod block diagram in Fig. 3 shows three identical RRR-legs (each leg consists of two half-legs connected by three rotational pairs). The block diagram of the tripod in Fig. 4 also shows three identical RPR-legs of the (in the middle legs are interconnected by one prismatic pair). The central link $AB$ is absent in both schemes.
Figure 3. Results of computer simulation of the tripod RRR for the first ten seconds of movement.
Figure 4. Results of computer simulation of the tripod RPR for the first ten seconds of movement.

4. Conclusions
Using the Grubler and Somov—Malyshev formulas, an analysis of the number of degrees of freedom of these schemes is performed. Although the expressions for rate and accelerations of the ends of the legs $l_{i}$ (Bi points) of the tripod with 2 degrees of freedom, the tripod with 3 degrees of freedom is easy to find by differentiating and twice differentiating the corresponding expressions by generalized coordinates, however, these expressions are too cumbersome and unsuitable for practical use. The study of the velocities and accelerations of the ends of the $l_{1,3}$ legs is easier to do using mathematical modeling and using modern software.

The following results were obtained for a tripod with 2 degrees of freedom: the inverse kinematics problem was solved; the direct problem of kinematics is solved - equations are obtained that determine the length of the legs as a function of the generalized coordinates; A mathematical model of the
dynamics of the mechanism is developed in the form of a system of ordinary differential equations with respect to generalized coordinates. Similar results were obtained for a tripod with 3 degrees of freedom.

Based on the analysis of the movements of the “trunk” type manipulator, a dynamic analysis of the tripod with 2 and 3 degrees of freedom was performed. For a tripod with 2 degrees of freedom, equations are obtained that determine the length of the legs as a function of the generalized coordinates; a model of the dynamics of the mechanism is developed in the form of a system of ordinary differential equations for generalized coordinates. Similar results were obtained for a tripod with 3 degrees of freedom. The results obtained are of theoretical and practical interest, and can also be used to build mathematical models.

References
[1] Merlet J P 2000 Parallel Robots. Solid mechanics and its applications (Merlet-Kluwer Academic Publishers).
[2] Glazunov V A, Kolesnik A Sh and Kraynev A F 1991 Spatial Mechanisms of Parallel Structure (Science, Moscow) (in Russian)
[3] Parallel mechanisms information center (http://www.parallelic.org/)
[4] Volkomorov S V, Kaganov Yu T and Karpenko A P 2010 Information technology, Application 5 1 (in Russian)
[5] Yaminikova O A, Yaminkov A S and Solyanik D Yu 2011 Science intensive technologies in mechanical engineering 5 3 (in Russian)
[6] Sementsev A M 2011 Science intensive technologies in mechanical engineering 5 14 (in Russian)
[7] Govorov I V 2011 Science intensive technologies in mechanical engineering 5 21 (in Russian)
[8] Swirad S 2018 MATEC Web of Conferences 249 03002
[9] Kosbolov S, Yeleukulov S Y, Atalykova A, Zhauyt A, Yestemessova G and Yusupova S 2018 MATEC Web of Conferences 22 01020
[10] Goryanina K I, Lukyanov A D and Katin O I 2018 MATEC Web of Conferences 226 02015
[11] Furch J and Tran C V 2018 MATEC Web of Conferences 234 020002
[12] Hongqiang Wan, Peiying Han, Shuai Ge, Fancong Li and Simiao Zhang 2018 MATEC Web of Conferences 237 01014
[13] Lankin A M, Lankina M Y and Lankin M V 2018 MATEC Web of Conferences 226 02021
[14] Garcia A, Cuan-Urquizo E, Roman-Flores A and Vazquez E 2018 MATEC Web of Conferences 249 03004
[15] Kaganov Yu T, Karpenko A P. 2009 Science and education 10 (in Russian)
[16] Kaganov Yu T, Karpenko A P. 2009 Science and education 11 (in Russian)
[17] Mkrtchyan O V 2012 Theory of mechanisms and machines 10 1 (in Russian)
[18] Mkrtchyan O V 2013 Theory of mechanisms and machines 11 1 (in Russian)
[19] Mkrtchyan O V 2012 Bulletin of Belgorod V. G. Shukhov State Technology University 2 211 (in Russian)
[20] Mkrtchyan O V 2014 Bulletin of Belgorod V. G. Shukhov State Technology University 1 217 (in Russian)
[21] Mkrtchyan O V 2012 Modern engineering. Science and education 2 96 (in Russian)
[22] Kartygin A V, Mkrtchyan O V and Starchik Yu Yu 2018 Fundamentals of mechanics 3 69 (in Russian)
[23] Mkrtchyan O V 2018 Mechanics and mechanical engineering. Science and practice 1 14 (in Russian)
[24] Mkrtchyan O V, Gerasimov M D and Kartygin A V 2017 Construction and road building machinery 12 17 (in Russian)
[25] Poduraev Yu V Mechatronics: basics, methods, applications 2007 (Mashinostroenie, Moscow)
[26] Volkomorov S V, Karpenko A P and Leletko A M 2008 Science and education 10 (in Russian)
[27] Zenkevich S L and Yushchenko A S 2000 Robot Control. Fundamentals of manipulation robots management (Publishing House of Moscow N. E. Bauman State Technical University, Moscow) (in Russian)

[28] Korendyasev A I, Salamander B L, Tyves L I et al. 1989 Robotized manipulation systems (Mashinostroenie, Moscow) (in Russian)

[29] Smirnov V A, Petrova L N and Fedorov V B 2006 Bulletin of SUSU 11 24 (in Russian)

[30] Denavit J and Hartenberg R S 1955 J. Appl. Mech. 77 215