Ghost condensation and \( CPT \) violation in neutrino sector

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Motivated by the recent MINOS observation, \( \Delta m_{32}^2 \neq \Delta \bar{m}_{32}^2 \), we study \( CPT \) violation in neutrino sector caused by ghost condensation. As a concrete model, we consider an extra dimension model with two branes, which are spatially separated and communicate only through singlet ‘messenger’ fermion fields which contain right handed neutrinos in the bulk. Assuming ghost condensation on one brane and localization of the standard model sector on the other brane, \( CPT \) violation occurs only in neutrino sector by Yukawa couplings with the messengers, which, at the same time, lead small neutrino masses. In the parameter space which is consistent with the MINOS observation suggests possible observation of the twinkling cosmic microwave background radiation and blinking light-ray from a quasar at a large distance, with a timescale about 0.1 day.

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I. INTRODUCTION

Recently preliminary result from MINOS was released \cite{1}:

\begin{align}
\Delta m^{2}_{32} &= 2.35^{+0.11}_{-0.08} \times 10^{-3} \text{ eV}^2, \\
\Delta \overline{m}^{2}_{32} &= 3.36^{+0.45}_{-0.40} \times 10^{-3} \text{ eV}^2, \\
\sin^2(2\theta_{23}) &> 0.91 \\
\sin^2(2\bar{\theta}_{23}) &= 0.86 \pm 0.11
\end{align}

(1)

where apparent difference between particle and anti-particle has been observed with 90% C.L \cite{4}. This result, even though it is preliminary, is surprising since CPT theorem does not allow the difference between particle and anti-particle in the well established frame work of local relativistic quantum field theory \cite{2}. Indeed every known particle except neutrinos has the same mass as the CPT conjugate anti-particle within experimental resolutions. We take this observation as a chance to consider CPT violation in neutrino sector.

CPT violation implies violation of Lorentz invariance \cite{6}. As a possible source of Lorentz violation, in the present paper we consider ghost condensation \cite{7}. Ghost condensation is a mechanism for spontaneous Lorentz breaking and leads to the simplest Higgs phase of gravity. It is indeed the simplest in the sense that the number of associated Nambu-Goldstone boson is just one. The order parameter of the spontaneous Lorentz breaking is the vev of derivative of a scalar field $\langle \partial_\mu \phi \rangle$, which is supposed to be timelike. By properly choosing the time coordinate, we can set $\langle \partial_i \phi \rangle = 0$ and, thus, ghost condensation is characterized by a scale $M \equiv \sqrt{|\langle \partial_0 \phi \rangle|}$. As in the usual Higgs mechanism, ghost condensation modifies infrared behavior of the force law, in this case gravity. In the limit $M \to 0$, Lorentz symmetry and thus general relativity are safely recovered. For this reason, there is no phenomenological lower bound on $M$, unless additional assumptions are made. (See \cite{8} for an example of such

\footnote{No sidereal modulation is found in the MINOS far detector \cite{2}, which is expected as a signal of Lorentz and CPT violation in an extension of the SM (SME) framework \cite{3}. We follow the conventional notations for mass differences and mixing angles, $\Delta m^{2}_{32}, \theta_{23}$ for neutrinos and $\Delta m^{2}_{32}, \bar{\theta}_{23}$ for antineutrinos, respectively.}

\footnote{See e.g., Ref. \cite{5} for a trial to understand the data without CPT violation.
an additional assumption and the corresponding lower bound on $M$. On the other hand, there are universal upper bounds on $M$ and the strongest is $M < 100$ GeV [9].

There is an immediate question [10]: why $CPT$ violation is seen only in neutrino sector. There are various stringent constraints on $CPT$ violation in other sectors. The most severe one is from $K^0 - \bar{K}^0$ mixing where $(m_{K^0} - m_{\bar{K}^0})/m_{\text{average}} < 8 \times 10^{-18}$ at CL=90% [11, 12]. In lepton sector, the bound is less severe but still significant as $(m_{e^+} - m_{e^-})/m_{\text{average}} < 8 \times 10^{-9}$ at CL=90% [11]. Taking all these severe constraints into account, it is suggested that only neutrinos may directly couple to ghost condensation. We notice that this situation is naturally realized in well motivated models where singlet neutrinos propagate in the bulk of extra dimension and the standard model particles are confined on a brane. The setup was originally suggested to explain small neutrino mass (see e.g., Refs.[13–15]). Now, in addition to the original setup, we suggest that ghost condensation takes place in a brane which is spatially separate from the standard model brane. In this case, the singlet neutrinos in the bulk naturally play the role of messenger fields through which the standard model sector communicates with ghost condensation. Then it is natural that a sizable $CPT$ violation can occur only in neutrino sector. We will explicitly suggest a minimal model where all the required properties are realized.

The paper is organized as follows. In Sec.II we describe the model in 5D and derive the 4D effective action where the neutrino and anti-neutrino masses are calculated. In Sec. III we consider various constraints on the model from cosmology and possible detection of “twinkling CMB” in the future. We then summarize the paper in Sec. IV.

II. MODEL

The model extra dimension is a flat$^3$ orbifold $S^1/Z_2$ with radius $R$ which can be equivalently described by an interval $I = [0, \pi R]$ where $y = 0$ and $y = \pi R$ correspond to the fixed points of the orbifold. We assume that the standard model sector is localized at $y = \pi R$ and ghost condensation takes place at $y = 0$. Singlet fermions, which contain right-handed neutrinos, are in the bulk so that they can mediate ghost condensation and the standard model sector. As a singlet does not couple to the standard model gauge bosons, it can nat-

$^3$ The setup can be built on warped extra dimension [16] as in [14].
urally propagate through the bulk as graviton does. Essentially the present setup extends the one in [13] by introducing ghost condensation so that we can enjoy all the advantages of previous studies and allow the necessary CPT violation.

The action is given as:

\[
S = \int d^4x \int_0^{\pi R} dy \left[ \frac{1}{\pi R} L_{\text{bulk}} + \delta(y) L_0 + \delta(y - \pi R) L_\pi \right]
\] (2)

where \( L_\pi \) contains all the SM particles, i.e., quarks, leptons, gauge bosons of \( SU(3)_c \times SU(2)_W \times U(1)_Y \) and interactions among them, \( L_0 \) has ghost condensation so that Lorentz violation takes place. The bulk Lagrangian \( L_{\text{bulk}} \) contains singlet neutrinos, \( n(x^\mu, y) \), propagating in the bulk, \( 0 \leq y \leq \pi R \) (See Fig. [1]). More explicitly,

\[
L_{\text{bulk}} = i \bar{n} \Gamma^M \partial_M n - m \bar{n} n + \text{h.c.},
\] (3)

\[
L_0 \ni \partial_\mu \phi \bar{n} \gamma^\mu n + \text{g.c.},
\] (4)

\[
L_\pi \ni \lambda \bar{\ell} H n + \text{h.c.} + \cdots,
\] (5)

where g.c. describes the dynamics of ghost condensation, \( \Gamma^M = (\gamma^\mu, i\gamma^5) \) and \( \cdot \cdot \cdot \) contains the rest of SM sector and \( M_5 \) is Gravity scale in five dimension which is related to Planck scale as \( M_{Pl}^2 = M_5^3 \pi R \). The bulk mass \( m \) is consistent with orbifold symmetry as it is \( m \propto \text{sign}(y) \).

To get the four dimensional effective action, we do the Kaluza-Klein (KK) decomposition as

\[
n(x, y) = \sum_k n_k^L(x) f_k^L(y) + n_k^R f_k^R(y),
\] (6)

where \( f_k^L \) and \( f_k^R \) are the dimensionless KK basis functions satisfying the equation of motion:

\[
m_k f_k^L \mp \partial_y f_k^R - m f_k^L = 0.
\] (7)

For zero mode, \( k = 0 \), with \( m_k = 0 \), the Eq. (7) is separable

\[
\partial_y f_0^{R/L} \pm m f_0^{R/L} = 0,
\] (8)

so is easily solved as

\[
f_0^{R/L}(y) = C_{R/L} e^{\pm my}.
\] (9)

\[4\] A double kink mass can be introduced so that Kaluza-Klein parity is conserved [17] but we do not pursue this direction here.
For $m > 0$, right-handed zero mode is localized toward $y = 0$ and left-handed zero mode is localized toward the other end point, $y = \pi R$. Here the normalization constant $C_{R/L}$ for $R/L$-handed zero mode are determined by the normalization condition $\int_0^{\pi R} |f_{R/L}^0|^2 dy = 1$:

$$
C_{R/L} = \left( \frac{\pm 2m \pi R}{1 - e^{\pm 2m \pi R}} \right)^{1/2}.
$$

(10)

If Dirichlet boundary conditions to $n_L$ is imposed, $f_L^k(0) = 0 = f_L^k(\pi R)$ at boundaries and right-handed states $n_R^k$ survive at boundaries and can interact with boundary localized fields $^5$.

$^5$ The five dimensional Lorentz invariance determines the relative sign of $Z_2$-parity of $n_L$ and $n_R$ or equivalently the boundary conditions to them. If odd-parity is imposed to one chiral state (or satisfying Dirichlet boundary condition), the other state has to have opposite parity, i.e., even parity (or satisfying (modified) Neumann boundary condition).
\[ S_{\text{eff}} = \sum_{k=0}^{\infty} \ln R_\nu \delta n_R^k - m_k \ln R_\nu n_L^k + (R \leftrightarrow L) \]
\[ + \sum_k \lambda_\nu f_R^k(\pi R) H n_R^k + \text{h.c.} \]
\[ + \sum_{k,\ell} \frac{\partial \phi}{M_5} n_R^{k\nu} n_R^{\mu} f_R^{k\nu} f_R^{k\mu} + \text{h.c.} \]
\[ + \text{g.c.} \]  

(11)

where \( m_0 = 0, m_{k \geq 1} = \sqrt{m^2 + (k/R)^2} \). Using the collective notations for the left-handed and right-handed states, \( N_L = (\nu_L, n^1_L, n^2_L, n^3_L, \cdots) \) and \( N_R = (n^0_R, n^1_R, n^2_R, n^3_R, \cdots) \), respectively, we can construct the mass term \( \mathcal{N}_L \mathcal{M}_R + \text{h.c.} \) with \( (N \times N)|_{N \to \infty} \) mass matrix

\[
\mathcal{M} = \begin{pmatrix}
\lambda_0 v & \lambda_1 v & \lambda_2 v & \ldots & \lambda_N v \\
0 & m_1 & 0 & \cdots & 0 \\
0 & 0 & m_2 & \cdots & 0 \\
0 & 0 & 0 & m_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & 0 \\
0 & 0 & 0 & 0 & m_N 
\end{pmatrix},
\]

(12)

where \( v = \langle H \rangle \) after electroweak symmetry breaking and the effective Yukawa coupling for the n-th KK state is \( \lambda_n = \lambda_\nu f_R^n(\pi R) \). The physical mass eigenstates and their masses could be obtained after diagonalizing \( \mathcal{M} \mathcal{M}^\dagger \rightarrow U^\dagger \mathcal{M} \mathcal{M}^\dagger U \) by \( \mathcal{N}_L \rightarrow U^\dagger \mathcal{N}_L \). If \( \lambda_0 \equiv \lambda_\nu f_R^0(\pi R) \) were zero, the matrix \( \mathcal{M} \mathcal{M}^\dagger \) would have a zero eigenvalue with the corresponding eigenvector

\[
u_0 = \cos \theta_\nu \begin{pmatrix}
1 \\
-\lambda_1 v/m_1 \\
-\lambda_2 v/m_2 \\
\vdots \\
-\lambda_N v/m_N
\end{pmatrix}, \quad \tan^2 \theta_\nu = \sum_{n=1}^{N} \frac{\lambda_n^2 v^2}{m_n^2},
\]

(13)

where the angle \( \theta_\nu \) describes the mixing between the zero mode and the higher KK modes. Thus, to the leading order in the small parameter \( \lambda_0 \ll 1 \), we can get the lowest eigenvalue which gives the physical neutrino mass squared:

\[
m_\nu^2 = u_0^\dagger \mathcal{M} \mathcal{M}^\dagger u_0 = \lambda_0^2 v^2 \cos^2 \theta_\nu.
\]

(14)
As the wave function of R-handed state at $y = \pi R$ is exponentially suppressed as $f^0_R(\pi R) \approx \sqrt{2m\pi Re^{-m\pi R}} \ll 1$, we naturally get a small neutrino mass, $m_{\nu} \sim 0.1$ eV, with a largish $mR \approx 10$ and $\lambda_{\nu} \sim 1$. As it is clearly seen in Eq. (13), the zero mode can mix with KK excitation modes but the mixing angle is highly suppressed $O(\lambda_{\nu}v/m_{KK})$. However the mass splitting between particle and antiparticle in the zero mode states, which we can take as a perturbation, cannot be neglected and actually is important to account the MINOS anomaly.

Once Ghost condensation takes place on $y = 0$ brane, $\langle \partial_0 \phi \rangle \neq 0$, the second term becomes another neutrino bilinear term

$$C_R^2 \frac{M^2}{M_5} \bar{\nu_R} \gamma^0 \nu_R,$$

where $M^2 = \langle \partial_0 \phi \rangle$. A convenient parameter, $\epsilon \equiv C_R^2 M^2/M_5$, measures the Lorentz violation. When $\epsilon \neq 0$ the masses of particle (neutrino, in our case) and anti-particle (anti-neutrino, in our case) are different by $\epsilon$:

$$m_\nu - m_{\bar{\nu}} \simeq \epsilon.$$  \hspace{1cm} (16)

The MINOS data in Eq. (2) implies that the mass difference in neutrino and anti-neutrino would be in the range of 0.01 eV or

$$\epsilon = C_R^2 \times \frac{M^2}{M_5} \sim 0.01 \text{ eV},$$

$$\Rightarrow M \sim 15 \text{ GeV} \times \left( \frac{R^{-1}}{1 \text{ TeV}} \right)^{\frac{1}{6}} \times \left( \frac{\epsilon}{0.01 \text{ eV}} \right)^{\frac{1}{2}}.$$  \hspace{1cm} (17)

Here we used $C_R^2 \simeq 2m\pi R \sim 60$ to fit the correct neutrino mass scale with $\lambda_{\nu} \sim 1$. Note that $\epsilon \sim 0.01$ eV is consistent with the bound $\epsilon < 10^3$ eV from neutron $\beta$ decay \cite{18}. Again as the splitting is small enough we can neglect the mixing between zero mode and other KK modes.

\footnote{The easiest way to see this difference is as follows. With the nonzero $\epsilon \neq 0$, the Dirac equation in momentum space is $(\gamma^\mu p_\mu - m_0 - \epsilon \gamma^0)\psi(p) = 0$ where $m_0$ is the Dirac mass coming from conventional Higgs mechanism. In the rest frame, $p^\mu = (m, \vec{0})$, the equation is reduced to $((m - \epsilon)\gamma^0 - m_0)\psi(p) = 0$ and the mass eigenvalues are $m = \pm m_0 + \epsilon$ or $|m| = |m_0 \pm \epsilon|.$}
III. IMPLICATIONS TO COSMOLOGY

Taking $\epsilon \sim 0.01$ eV and the bound on the size of extra dimension\(^7\), we get the lower bound on $M$ from the neutrino sector, $M \gtrsim 10$ GeV. On the other hand, from nonlinear dynamics of the gravity sector, we obtain the upper bound on $M$ \([9]\). Thus, there is only a small window for $M$:

$$10 \text{ GeV} \lesssim M \lesssim 100 \text{ GeV}. \tag{18}$$

Since $M$ should be pretty close to 100 GeV we expect to see twinkling of the CMB \([9]\) and blinking of the light from a quasar at a large distance with the twinkling timescale

$$T_{\text{twinkling}} \sim \frac{M}{100 \text{ GeV}} \cdot \frac{300 \text{ km/s}}{v} \times 0.1 \text{ day}, \tag{19}$$

where $v$ is the overall velocity of the rest frame of ghost condensation relative to the CMB and the quasar rest frame, respectively.

There are other bounds which we should take into account but, as shall see, they do not set additional constraints. Due to the coupling of neutrinos to ghost condensation, an ultrarelativistic neutrino emits a Nambu-Goldstone boson \([19]\). This is an analogue of Čerenkov radiation. Demanding that neutrinos from SN1987A should not be deflected by this process, we obtain the bound \([20]\)

$$\epsilon < 10^4 \text{ eV} \times \left( \frac{M}{100 \text{ GeV}} \right)^{3/2} \times \left( \frac{0.1 \text{ eV}}{m} \right). \tag{20}$$

This is automatically satisfied if (18) is satisfied. If we demand that the total relative energy lost from cosmological neutrinos to Nambu-Goldstone bosons during the epoch between neutrino decoupling to the last scattering surface of the CMB, then we obtain the bound \([20]\)

$$\epsilon < 10^9 \text{ eV} \times \left( \frac{M}{100 \text{ GeV}} \right)^2 \times \left( \frac{0.1 \text{ eV}}{m} \right). \tag{21}$$

Again, this is satisfied under (18).

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\(^7\) Since the right-handed singlet neutrino does not couple to the gauge bosons, it can only be probed by processes involving Higgs field. As Yukawa coupling for KK-neutrino is not small, KK neutrinos can contribute to the Higgs decay ($H \to \ell \nu \nu$) if they are light enough. Taking current mass bound for the Higgs into account, we conclude that $m_1 \gtrsim 100$ GeV or $R^{-1} \gtrsim 10$ GeV is allowed by LEP-II and Tevatron experiments.
IV. CONCLUSION

In this paper we have considered ghost condensation as a source of CPT violation. Taking the coupling with neutrino current into account, we could provide a possible difference between neutrino and anti-neutrino, namely $\Delta m^2 \neq \Delta \bar{m}^2$ which has been recently hinted by MINOS experiment even though the result is preliminary. To avoid a sizable CPT violation in other sectors, we suggest an extra dimension model where the right-handed neutrinos in the bulk mediate the standard model sector and the ghost condensate which takes place at a distance from the standard model brane. The model leads to interesting observational consequences: twinkling CMB and blinking quasars with the timescale around tens to hundreds of minutes which might be within the reach of future CMB observations (e.g. Planck [21]) and astronomical point source observations.

Finally discussion on flavor non-diagonal CPT violation is in order. Flavor non-diagonal dimension five operators directly contribute to neutrino oscillation as in the standard matter effects and so are highly constrained by solar, atmospheric and other ground based neutrino experiments. To forbid flavor non-diagonal terms we assume flavor symmetry on the ghost condensate brane so that only flavor diagonal dimension five operators are induced at the leading order. On the other hand, flavor transition induced by Yukawa couplings are small $\epsilon_{i \neq j} \sim \sum_k y_{ik} y_{kj}^* |f_R^0(\pi R)|^2 \epsilon \sim 10^{-24} \epsilon$.

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[21] According to the “Bluebook”, Planck’s instruments have angular resolution at the level of (5-33) arcmin or (7.2 – 47) × 10^{-4} rad. The Bluebook can be downloaded at the official website of Planck project (http://www.rssd.esa.int/index.php?project=planck).