Cosmology of a Friedmann-Lamaitre-Robertson-Walker 3-brane, Late-Time Cosmic Acceleration, and the Cosmic Coincidence

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A late epoch cosmic acceleration may be naturally entangled with cosmic coincidence – the observation that at the onset of acceleration the vacuum energy density fraction nearly coincides with the matter density fraction. In this Letter we show that this is indeed the case with the cosmology of a Friedmann-Lamaitre-Robertson-Walker (FLRW) 3-brane in a five-dimensional anti-de Sitter spacetime. We derive the four-dimensional effective action on a FLRW 3-brane, from which we obtain a mass-reduction formula, namely, $M^2_p = \rho_b/|\Lambda_5|$, where $M_p$ is the effective (normalized) Planck mass, $\Lambda_5$ is the five-dimensional cosmological constant, and $\rho_b$ is the sum of the 3-brane tension $V$ and the matter density $\rho$. Although the range of variation in $\rho_b$ is strongly constrained, the big bang nucleosynthesis bound on the time variation of the effective Newton constant $G_N = (8\pi M_p^2)^{-1}$ is satisfied when the ratio $V/\rho \gtrsim O(10^3)$ on cosmological scales. The same bound leads to an effective equation of state close to $-1$ at late epochs in accordance with astrophysical and cosmological observations.

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Introduction. – The paradigm that the observable Universe is a branelike four-dimensional hypersurface embedded in a five- and higher-dimensional spacetime is fascinating as it provides new understanding of the feasibility of confining standard-model fields to a D(ricchlet)-3-brane. This revolutionary idea, known as brane-world proposal, is supported by fundamental theories that attempt to reconcile general relativity and quantum field theory, such as string theory and $M$ theory. In string theory or $M$ theory, gravity is a truly higher-dimensional theory, becoming effectively four-dimensional at lower energies. This behavior is seen in five-dimensional brane-world models in which the extra spatial dimension is strongly curved (or “warped”) due to the presence of a bulk cosmological constant in five dimensions. Warped spacetime models offer attractive theoretical insights into some of the significant questions in particle physics and cosmology, such as why there exists a large hierarchy between the 4D Planck mass and electroweak scale, and why our late-time low-energy world appears to be four-dimensional.

For viability of the brane-world scenario, the model must provide explanations to key questions of the concurrent cosmology, including (i) why the expansion rate of the Universe is accelerating and (ii) why the density of the cosmological vacuum energy (dark energy) is comparable to the matter density – the so-called cosmic coincidence problem. In this Letter, we show that the cosmology of a Friedmann-Lamaitre-Robertson-Walker (FLRW) 3-brane in a five-dimensional anti-de Sitter (AdS) spacetime can address these two key questions as a single, unified cosmological problem. Our results are based on the exact cosmological solutions and the four-dimensional effective action obtained from dimensional reduction of a five-dimensional bulk theory.

Model. – A 5D action that helps explore various features of low-energy gravitational interactions is given by

$$S = \int d^5x \sqrt{|g|} M_5^3 (R_5 - 2\Lambda_5) + 2 \int d^4x \sqrt{|h|} (\mathcal{L}_m - V),$$

(1)

where $M_5$ is the fundamental 5D Planck mass, $\mathcal{L}_m$ is the brane-matter Lagrangian, and $V$ is the brane tension. The bulk cosmological constant $\Lambda_5$ has the dimension of (length)$^{-2}$, similar to that of the Ricci scalar $R_5$. As we are interested in cosmological implications of a warped spacetime model, we shall write the 5D metric ansatz in the following form

$$ds^2 = -n^2(t,y) dt^2 + a^2(t,y) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) + dy^2,$$

(2)

where $k \in \{-1,0,1\}$ is a constant which parametrizes the 3D spatial curvature and $d\Omega^2$ is the metric of a 2-sphere. The equations of motion are given by

$$G^A_B = -\Lambda_5 \delta^A_B + \frac{\delta(y)}{M_5^3} \text{diag} (-\rho_b, p_b, p_b, p_b, 0),$$

(3)

with $\rho_b \equiv \rho + V$ and $p_b \equiv p - V$, where $\rho$ and $p$ are the density and the pressure of matter on a FLRW 3-brane. The parameter $h$ that appeared in Eq. (1) is the determinant of four-dimensional components of the bulk metric, i.e., $h_{\mu\nu}(x^\mu) = g_{\mu\nu}(x^\mu, y = 0)$.

Bulk Solution. – Using the restriction $G^0_0 = 0$ and choosing the gauge $n_0 \equiv a(t,y) = 1$, in which case $t$ is the proper time on the brane, one finds that the warp factor $a(t,y)$ that solves Einstein’s equations in the 5D bulk and equations on a FLRW 3-brane is given by

$$a(t,y) = \left( \frac{a_0^2}{2} \left( 1 + \frac{\bar{\rho}_b}{6\Lambda_5} \right) + \frac{3C}{\Lambda_5 a_0^2} \right)^{1/2}$$

$$+ \left[ \frac{a_0^2}{2} \left( 1 - \frac{\bar{\rho}_b}{6\Lambda_5} - \frac{3C}{\Lambda_5 a_0^2} \right) \cosh \left( \sqrt{-\frac{2\Lambda_5}{3}} y \right) \right]^{1/2}$$

$$- \frac{\bar{\rho}_b}{\sqrt{-6\Lambda_5}} a_0 \sinh \left( \sqrt{-\frac{2\Lambda_5}{3}} |y| \right)^{1/2},$$

(4)

where $a_0 \equiv a(t,y = 0)$ and $\bar{\rho}_b = \rho_b/M_5^2$. The form of $a(t,y)$ is obtained using $n = a/a_0$. The integration constant $C$ enters into the brane analogue to the first Friedmann equation

$$H^2 + \frac{k}{a_0^2} = \frac{\Lambda_5}{6} + \frac{\bar{\rho}_b^2}{36} + \frac{C}{a_0^2},$$

(5)
where $H \equiv \dot{a}/a$ is the Hubble expansion parameter. The brane analogue to the second Friedmann equation is

$$
\dot{H} + H^2 = \frac{\Lambda}{6} - \frac{\rho_b^2}{36} (2 + 3w_b) - \frac{C}{a_0^3},
$$

where $w_b \equiv p_b/\rho_b$ is the effective state equation of state on a FLRW 3-brane. The brane evolution equations are quite different from Friedmann equations of standard cosmology: the distinguishing features are (i) the appearance of the brane energy density in a quadratic form, (ii) the dependence of $H^2$ on $\Lambda$, and (iii) the appearance of the bulk radiation term $C/a_0^3$.

If the radiation energy from bulk to brane (or vice versa) is negligibly small, then it would be reasonable to set $C = 0$. In the following we assume that $C = 0$ unless explicitly shown.

**Cosmic acceleration.** With $V = \text{const}$, the brane energy-conservation equation, $\dot{\rho}_b + 3H(\rho_b + p_b) = 0$, reduces to

$$
\dot{\rho} = \rho_s a_0^{-\gamma}, \quad \gamma = 3 (1 + w),
$$

where $w = p/\rho$ is the EOS of matter on the 3-brane and $\rho_s$ is a constant. With Eq. (7), Eq. (5) takes the following form:

$$
\frac{\dot{a}_b^2}{a_0^2} + \frac{k}{a_0^2} = \frac{\Lambda}{3} + \frac{V}{18} \rho_s^{-\gamma} + \frac{\rho_b^2}{36} (a_0^{-2\gamma}),
$$

where $\Lambda = \frac{\Lambda_4^2}{12} + \frac{V^2}{12}, \rho_s = \rho_0 M^{-3}$, and $V \equiv V/M^2$. This admits an exact solution when $k = 0$, which is given by

$$
\bar{\rho} = \frac{\rho_s}{a_0} = \frac{6H_0}{\sinh (\gamma H_0 t) + \nu (\cosh (\gamma H_0 t) - 1)},
$$

where $H_0 = \sqrt{\Lambda_4}$ and $\nu \equiv \frac{\nu}{H_0}$. From this we find that the Hubble expansion parameter is given by

$$
H = \frac{\dot{a}_b}{a_0} = \frac{H_0}{\nu} \left[ \frac{\nu \sinh (\gamma H_0 t) + \cosh (\gamma H_0 t)}{\cosh (\gamma H_0 t) - 1} \right].
$$

The deceleration parameter

$$
q \equiv -\frac{\dot{a}_b a_0}{\dot{H}} = -\frac{H + H^2}{H^2}
$$

changes sign from positive to negative when $\gamma H_0 t \sim 1.1$ (cf. Fig. 1). This implies a transition from decelerating to accelerating expansion. The onset time of acceleration depends on $\nu$ but only modestly: generally, we expect that $\nu = V/6H_0 = \sqrt{V^2/12\Lambda_4} \lesssim O(1)$. In the Randall-Sundrum (RS) limit ($\Lambda_4 = 0$), we find that $a_0(t) \propto (2t + \gamma V H_0 t^2)^{1/\gamma}$, which shows that the scale factor scales as $t^{1/\gamma}$ at early epochs and as $t^{2/\gamma}$ as late epochs. The crossover takes place when $H_0 t \sim 2/(\gamma \nu)$. In the generic case with $\Lambda_4 > 0$, the scale factor grows in the beginning as $t^{1/\gamma}$ (as in the $\Lambda_4 = 0$ case), but at late epochs it grows almost exponentially, $a_0(t) \propto [\cosh (\gamma H_0 t) - 2\nu/(1 + \nu)]^{1/\gamma}$.

**Cosmic coincidence:** Consider the Friedmann constraint

$$
\Omega_\Lambda + \Omega_\rho + \Omega_{\bar{\rho}^2} = 1,
$$

where

$$
\Omega_\Lambda \equiv \frac{\Lambda}{3H^2}, \quad \Omega_\rho \equiv \frac{\rho V}{18H^2}, \quad \Omega_{\bar{\rho}^2} \equiv \frac{\rho_b^2}{36H^2}.
$$

As shown in Fig. 1, $\Omega_\bar{\rho}^2$ starts out as the largest fraction around $H_0 t \gtrsim 0$, but $\Omega_\rho$ quickly overtakes it when $H_0 t \gtrsim 0.15$. Gradually, $\Omega_\rho$, which measures the bare vacuum energy density fraction, surpasses these two components. Notice that $\Omega_\Lambda + \Omega_\rho \approx 1$ when $H_0 t \gtrsim 0.5$. We can see, for $\nu \gtrsim 2$, that $\Omega_{\bar{\rho}^2} \approx 0.26$ and $\Omega_\rho \approx 0.74$ when $H_0 t \approx 0.75$. The crossover time between the quantities $\Omega_{m_0} = \Omega_\rho + \Omega_{\bar{\rho}^2}$ and $\Omega_\rho$ depends modestly on $\nu$. This provides strong theoretical evidence that dark energy may be the dominant component of the energy density of the Universe at late epochs, and it is consistent with results from astrophysical observations [11,12]. Unlike some other explanations of cosmic coincidence, such as quintessence in the form of a scalar field slowly rolling down a potential [13], the explanation here of cosmic coincidence does not require that the ratio $\Omega_{m_0}/\Omega_\Lambda$ be set to a specific value in the early Universe. Because of the modification of the Friedmann equation at very high energy, namely, $H \propto \rho$, new effects are expected in the earlier epochs and that could help to address the challenges that the $\Lambda$CDM cosmology faces at small (subgalaxy) scales [12].

**Effective Equation of State.** Eq. (6) can be written as

$$
w_b = -\frac{2}{3} + \frac{12H_0^2}{(\bar{\rho} + V)^3} \left[ 1 - \nu^2 + \frac{qH^2}{H_0^2} \right].
$$

As $H_0 t \rightarrow \infty$, $H \rightarrow H_0$, $q \rightarrow -1$, and when $V \gg \bar{\rho}$, which generally holds on large cosmological scales, we obtain

$$
w_b \approx -\frac{2}{3} + \frac{1}{3\nu^2} (-\nu^2) \approx -1.
$$

This is consistent with the result inferred from WMAP7 data: $w_b = -0.980 \pm 0.053$ (from $\Omega_k = 0$) and $w_b = -0.999^{+0.057}_{-0.056}$ (from $\Omega_k \neq 0$) [12]. In the earlier epochs with $\gamma H_0 t \lesssim 1.2$, we have $w_b > -1/3$, showing that a transition from matter to dark-energy dominance is naturally realized in the model.

In Fig. 2 we exhibit the parameter space for $\{\nu, H_0 t\}$ with a specific value of $w_b$ at present. If any two of the variables...
\{\nu, H_0^t, w\}$ are known, then the remaining one can be calculated. Typically, if $\nu \simeq 2$ and $H_0^t \simeq 0.75$, then $w_b \simeq -0.985$. In particular, the effective equation of state $w_b$ is given by

$$w_b = \frac{p_b}{\rho_b} = \frac{p - V}{\rho + V} = \frac{w - \zeta}{1 + \zeta},$$

(16)

where $\zeta \equiv V/\rho$. For brevity, suppose that the brane is populated mostly with ordinary (baryonic) matter plus cold dark matter, so $w \simeq 0$ ($\gamma \simeq 3$). In this case, cosmic acceleration occurs when $\zeta > 1/2$ (or $w_b < -1/3$). This result is consistent with the behavior of the 4D effective potential.

**Dimensionally reduced action.**— The gravitational part of the action (1) is

$$I \equiv \int d^4x \sqrt{-g} M_5^3 \left[ \frac{6}{a^2} \left( \frac{\dddot{a}}{n} + k - a^2 \frac{\dot{a}^2}{a^2} - a'' a \right) \right.$$  

$$+ \frac{6}{an} \left( a' n - \frac{\ddot{a} n}{n''} - 6 a' n' - 2 a'' n - 2 \Lambda_5 \right],$$

(17)

where the prime (dot) denotes a derivative with respect to $y$ ($t$). In order to derive from this a dimensionally reduced 4D effective action, we may separate $a''$ and $n''$ into nondistributional (bulk) and distributional (brane) terms

$$a'' = \dot{a}'' + [a']' \delta (y).$$

(18)

Using $n = \dot{a}/\dot{a}_0$ and the solution (4), the nondistributional part of the action (17) is evaluated to be

$$I_1 = \int d^4x \sqrt{-h} M_5^3 \left[ \frac{\dot{\rho}_b}{2 H \rho_b} \left( \Lambda_5 \rho_b \right) \frac{\dot{\rho}_b}{2 H \rho_b} \frac{\dot{\rho}_b}{2 H \rho_b} \right],$$

(19)

where $R_4 = 6 \left( \dot{a}_0/\dot{a}_0 + \dot{a}_0^2/a_0^2 + k/a_0^2 \right)$. In the above we have employed the background solution (4) and integrated out the $y$-dependent part of the 4D metric. The distributional part of the action (17) is evaluated to be

$$I_2 = \int d^4x \sqrt{-h} \frac{2}{3H} \left( \rho_b + 4H \rho_b \right).$$

(20)

The sum of $I_1$ and $I_2$ gives a dimensionally reduced action

$$S_{\text{eff}} = \int \sqrt{-h} d^4x \left[ \frac{M_5^3 \rho_b}{\left( -\Lambda_5 \right)} \left( \frac{R_4}{2} - \Lambda_{\text{eff}} \right) + \mathcal{L}_m \right],$$

(21)

with the effective potential given by

$$\Lambda_{\text{eff}} \equiv \frac{\dot{\rho}_b}{2H \rho_b} \left( \frac{5\Lambda_5}{6} + \frac{\dot{\rho}_b^2}{12} - \frac{C}{a_0^4} + \frac{\dot{\rho}_b^2}{9} + \frac{8\Lambda_5}{3} - \frac{2\Lambda_5 V}{\rho_b} \right).$$

(22)

The finiteness of Newton constant is required at low-energy scale where one ignores the effects of ordinary matter field on the brane. In this limit, the extra dimensional volume is finite in the same way as in canonical Randall-Sundrum models. In the presence of matter fields, we must consider a normalized Planck mass which generically depends on 4D coordinate time, since $\rho_b$ is time dependent. From Eq. (21) we read off the normalized Planck mass

$$M_5^2 \equiv M_5^3 \left( -\Lambda_5 \right) = \frac{\rho_b}{\left( -\Lambda_5 \right).}$$

(23)

In the limit that $\Lambda_4 = 0$ and $V \gg \rho$, Eq. (23) reduces to the formula or identification $8\pi G_N(0) \simeq V/(6M_5^3)$ used in [10], where $G_N(0)$ is the bare Newton’s constant identified in the low-energy limit (or when the matter density is much lower than the brane tension). The mass reduction formula for RS flat-brane models [4], $M_5^2 = M_5^3 \sqrt{-6/\Lambda_5}$, is obtained as a special limit of our result, namely, $M_5^3 \Lambda_5 \equiv \Lambda_4 \rho_0 = 0$, and $\Lambda_4 = 0$.

We make a remark here in regard to the scenario with $\Lambda_3 = 0$. The Dvali-Gabadadze-Porrati model [14] corresponds to a flat 5D bulk. In their model, it is argued that $R_4$ is generated from loop-level coupling of brane matter to the 4D graviton. At least at a classical level, $R_4$ is not generated in the dimensional reduction of the 5D action if $\Lambda_5 = 0$, and this is exactly what we found.

**AdS/FLRW-cosmology correspondence.**— In the limit $\zeta \equiv V/\rho \gg 1$, the 4D effective potential is approximated by

$$\Lambda_{\text{eff}} = -\frac{\gamma}{2x} \left( \Lambda_4 + \frac{\Lambda_5}{3} \right) + \frac{4\Lambda_4}{3},$$

(24)

Note that $\Lambda_4 \rightarrow \frac{3}{4} \Lambda_{\text{eff}}$ as $\zeta \rightarrow \infty$. This result, which relates the bare cosmological constant to the 4D effective potential in the limit $\rho \rightarrow 0$, is a direct manifestation of AdS/FLRW-cosmology correspondence. In a general case with finite $x$,

$$\Lambda_{\text{eff}} = -\frac{\gamma}{2(x + 1)} \left[ \Lambda_4 + \frac{\Lambda_5}{3} + \frac{\dot{\rho}_b^2}{12} (2\zeta + 1) \right] + \frac{\dot{\rho}_b^2}{9} (2\zeta + 1) + \frac{4\Lambda_4}{3} + \frac{2\Lambda_5}{\zeta + 1},$$

(25)

* We computed the integral indefinitely and then evaluated the result at $y = 0$. This approach is valid for the purpose of deriving the time-dependent part of the 4D gravitational coupling or effective Newton constant.
For any value of 3D curvature constant \( k \). The boundary action \( \mathcal{L}_{eff} \) is crucial to correctly reproduce the RS limit, i.e., \( \Lambda_{eff} \to 0 \) as \( \rho \to 0 \) and \( \Lambda_{4} \to 0 \). If \( \rho > 0 \), then \( \Lambda_{eff} \neq 0 \) even if \( \Lambda_{4} = 0 \). This shows that the vacuum energy on the brane or brane tension need not be directly tied to the effective cosmological constant on a FLRW 3-brane.

From Eq. (9) we can see, particularly at late epochs or when \( \rho \) is time varying, this suggests that a mass normalized gravitational constant is time-dependent. This is acceptable since analysis of primordial nucleosynthesis has shown that \( G_{N} \) can vary, although the range of variation is strongly constrained. The BBN bound is satisfied when the ratio \( V/\rho \) is larger than \( O(10^{2}) \) or when the effective equation of state \( -1 < w_{b} \sim -0.99 \). At cosmological scales the background evolution of a FLRW 3-brane becomes increasingly similar to \( \Lambda \)CDM but the model is essentially different from \( \Lambda \)CDM at earlier epochs. With precise determination of the present deceleration parameter or the effects of a time varying equation of state, we can hope to explore the late-time role of high-energy field theories in the form of brane worlds and many new physical ideas.

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**Further constraints.** Next we consider perturbations about the background metric given by Eq. (2), along with the solution (4). The transverse traceless part of graviton fluctuations \( \delta g_{ij} = h_{ij}(x^{\mu}, y) = \sum \varphi_{m}(t)f_{m}(y)e^{ikx} \) leads to a complicated differential equation for the spatial and temporal functions, which take remarkably simple forms at \( y = 0^{+} \), namely,

\[
(20y_{i}^{2} - \rho_{0}\partial_{y} + 2m^{2})f_{i}(0_{+}) = 0, \tag{29a}
\]

\[
\left[ \frac{\partial_{l}^{2}}{l^{2}} + 3H\partial_{l} + \left( m^{2} + \frac{k^{2}}{a_{0}^{2}} \right) \right] \varphi_{m}(0_{+}) = 0, \tag{29b}
\]

where \( m^{2} \) is a separation constant. Equation (29b) is equivalent to a standard time-dependent equation for a massive scalar field in 4D de Sitter spacetime. The masses of Kaluza-Klein excitations are bounded by \( m^{2} > 4H^{2}/9 \), in which case the amplitudes of massive KK excitations rapidly decay away from the brane. This, along with a more stringent bound coming from Eq. (29a), implies that \( H \sim \sqrt{3V/(8M_{p}^{4})} \). This is similar to the bound coming from the background solution, namely \( \Lambda_{4} \sim \sqrt{V}/12 \) or \( H_{0} < \sqrt{V}/(6M_{p}^{4}) \).

**Conclusion.** Brane-world cosmology with a small deviation from RS fine-tuning (\( \Lambda_{4} = 0 \)) is able to produce a late-time cosmic acceleration. The model puts the constraints

\[
0 \lesssim \frac{\Lambda_{4}}{V^{2}} \lesssim \frac{1}{12}, \quad \nu = \frac{\bar{V}}{6H_{0}} \gtrsim 1.
\]

The smaller the deviation from the RS fine-tuning the larger the duration of cosmic deceleration, prior to the late-epoch acceleration. For the background solution (9), the brane tension is not fine-tuned but only bounded from below. However, once the ratio \( \Lambda_{4}/M_{p}^{2} \) is fixed in accordance with the observational bound \( \Lambda_{4}/M_{p}^{2} \sim 10^{-120} \), the ratio \( \bar{V}/6H_{0} \) also gets fixed, in which case there is a fine-tuning between the bulk cosmological constant and brane tension.

The method of dimensional reduction gave a simple formula, \( M_{P}^{2} = \rho_{b}/|\Lambda_{4}| \), which relates the normalized Planck mass \( M_{P} \) to the matter-energy density on the brane and the bulk cosmological constant. As \( \rho_{b} \) is time varying, this suggests that a mass normalized gravitational constant is time-dependent. The BBN bound is satisfied when the ratio \( V/\rho \) is larger than \( O(10^{2}) \) or when the effective equation of state \( -1 < w_{b} \sim -0.99 \). At cosmological scales the background evolution of a FLRW 3-brane becomes increasingly similar to \( \Lambda \)CDM but the model is essentially different from \( \Lambda \)CDM at earlier epochs. With precise determination of the present deceleration parameter or the effects of a time varying equation of state, we can hope to explore the late-time role of high-energy field theories in the form of brane worlds and many new physical ideas.
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