Inverse solution of slave hand for single-port laparoscopic surgery robot based on hybrid Particle Swarm Optimization

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Abstract. Because of the particularity of the slave hand of the single hole laparoscopic surgery robot configuration, it cannot meet Pieper criterion, and its inverse kinematics solution cannot be obtained by traditional kinematics model. In order to solve the problems, the screw theory is used to establish the kinematics model for the slave hand of the single hole laparoscopic surgery robot. On this basis, the inverse kinematics is converted into a minimum solution problem. The optimal solution of each joint variable is obtained by using the hybrid particle swarm optimization (HPSO), and then the accuracy of pose matrix is analysed to verify the effectiveness of this method.

1. Introduction

With the development of robotics, robots are widely used in industry, medical treatment and education. Minimally invasive surgical robot is a great progress in the medical industry, although it cannot completely replace the doctor, the minimally invasive surgical robot can reduce the pressure of doctors [1], more accurately locate the lesions, reduce the risk of surgery, and reduce the pain of patients. Due to the particularity of the minimally invasive operation robot, the accuracy of inverse kinematics solution will greatly affect the operation efficiency, it is very important to obtain the inverse kinematics solution of the operation robot. Because of the complex mechanical structure of the single-hole laparoscopic operation robot, the condition of analytical solution is not satisfied. At present, the main methods to solve inverse kinematics of mechanical arms include: algebraic numerical solution, such as Jacobi pseudo-inverse method [2], gradient projection method [3], weighted minimum norm method [4]. The main problem is that errors will be accumulated in the iterative solution process, resulting in inaccurate solution. The geometric method requires the manipulator to conform to a specific configuration, and the solving process is complex and the generalization is not wide [5]. Neural network theory to solve the inverse kinematics research more [6], but because of the influence of neural network structure on the final result of uncertainty and needs a large number of known training samples, the application of this kind of method is limited.

Aiming at the complex problem of establishing kinematics model with D-H parameter method, forward kinematics analysis of slave hand for single-port laparoscopic surgery robot is carried out by using screw theory which can intuitively describe the geometric characteristics of slave hand for single-port laparoscopic surgery robot and simplify the analysis process. The pose matrix is obtained
based on the screw method, and the inverse kinematics solution is converted into the minimum value problem of the distance between two space vectors. The HPSO is used to solve this problem, which avoids the complexity of the solution process and ensures the optimal solution.

2. Kinematic model of slave hand for single-port laparoscopic surgery robot

The slave hand for single-port laparoscopic surgery robot is composed of a mobile joint and six rotating joints from the mobile phone arm, as shown in Fig.1. At this time, it is the robot state of the initial posture, and the tool coordinate system $T$ and the base coordinate system $S$ are established.

![Fig.1 Position and posture diagram of manipulator](image)

The initial pose $g_{st}(0)$ of the slave hand manipulator can be calculated as

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_1 + l_2 + l_3 + l_4 + l_5 + l_6 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (1)

The unit direction vector of the center axis of each joint is:

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \omega_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \omega_4 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \omega_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \omega_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ (2)

The position vector of each joint is:

$$p_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad p_k = \begin{bmatrix} 0 \\ \frac{\sum_{i=1}^{k-1} l_i}{l_k} \\ 0 \end{bmatrix}, \quad k = 2, 3, \ldots, 7$$ (3)

According to the screw theory, $\omega=0$ when the rigid body is in translational motion, and $\omega \neq 0$ when the rigid body is in rotational motion:

$$e^{\omega} = \begin{cases} e^{\omega \omega} (1 - e^{\omega \omega}) + \theta \omega \omega' \nu & \omega \neq 0 \\ 0 & \omega = 0 \end{cases}$$ (4)

From Rodrigue's rotation formula:

$$e^{\omega} = I + \hat{\omega} \sin \theta + \hat{\omega}^2 (1 - \cos \theta)$$ (5)

From formula (2) - (5), the exponential product of each joint is:
Where $d$ is the translation distance of joint 1 and $\theta$ is the rotation angle of the remaining joints.

Based on the exponential product of each joint and formula (1), the pose matrix from the slave hand for single-port laparoscopic surgery robot in tandem with the joints was calculated:

$$e^{\hat{\theta}a} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} e^{\hat{\theta}b} = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & l_1 \sin \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & l_1 (1 - \cos \theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\hat{\theta}c} = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 & (l_i + l_j) \sin \theta_3 \\ \sin \theta_3 & \cos \theta_3 & 0 & (l_i + l_j) (1 - \cos \theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} e^{\hat{\theta}d} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_4 & \sin \theta_4 & (l_i + l_j + l_k) (1 - \cos \theta_4) \\ -\sin \theta_4 & \cos \theta_4 & 0 & (l_i + l_j + l_k) \sin \theta_4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$e^{\hat{\theta}e} = \begin{bmatrix} \cos \theta_5 & 0 & \sin \theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta_5 & 0 & \cos \theta_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} e^{\hat{\theta}f} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_6 & \sin \theta_6 & (l_i + l_j + l_k + l_l) (1 - \cos \theta_6) \\ 0 & -\sin \theta_6 & \cos \theta_6 & (l_i + l_j + l_k + l_l) \sin \theta_6 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$e^{\hat{\theta}g} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_7 & \sin \theta_7 & (l_i + l_j + l_k + l_l + l_m) (1 - \cos \theta_7) \\ 0 & -\sin \theta_7 & \cos \theta_7 & (l_i + l_j + l_k + l_l + l_m) \sin \theta_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where $d$ is the translation distance of joint 1 and $\theta$ is the rotation angle of the remaining joints.

Based on the exponential product of each joint and formula (1), the pose matrix from the slave hand for single-port laparoscopic surgery robot in tandem with the joints was calculated:

$$T = e^{\hat{\theta}a} e^{\hat{\theta}b} e^{\hat{\theta}c} e^{\hat{\theta}d} e^{\hat{\theta}e} e^{\hat{\theta}f} e^{\hat{\theta}g} g_e(0)$$

$$= \begin{bmatrix} R_{3x3} & P_{3x1} \\ 0_{1x3} & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. HPSO algorithm solves inverse kinematics

3.1. Establishing objective function

By using the pose matrix $T$ obtained through forward kinematics, the problem of solving inverse kinematics is transformed into finding the minimum linear distance between pose matrix $T'$ and the expected pose matrix $T$.

$$f(t) = \| T - T' \|_2 = \sqrt{\lambda}$$

$\lambda$ is the maximum eigenvalue of $(T - T')^H (T - T')$.

By observing the position matrix $P$ and posture matrix $R$ of pose matrix $T$, it is found that the error of position matrix is much greater than that of posture matrix if the two are not of the same order of the magnitude. In the process of searching for the optimal solution, the algorithm may only find the optimal position, but the posture cannot find the optimal solution. In order to solve this problem, $T$ and $T'$ are first normalized, so that the position matrix and the posture matrix are located at the same order
of magnitude. After the normalization of the matrix, the optimization process of the optimal solution will become smoother and can converge to the optimal solution more quickly.

3.2. Introduction of HPSO algorithm

Particle Swarm Optimization (PSO) is a bionic algorithm that simulates a flock of birds searching for food in an unknown space [7]. In other words, PSO initializes a group of random particles and finds the optimal solution through continuous iteration. Each particle has its own position and velocity in space, and each particle has an N-dimensional direction. After each iteration, the position and velocity of particles are updated by pressing the following formula:

$$\begin{align*}
V_{id}(t+1) &= \omega V_{id}(t) + c_1 r_1 (p_{best_{id}} - X_{id}(t)) + c_2 r_2 (g_{best_{id}} - X_{id}(t)) \\
X_{id}(t+1) &= X_{id}(t) + V_{id}(t+1)
\end{align*}$$

\text{(8)}$$

$t$ is the number of iterations, $c_1$, $c_2$ is the learning factor, $p_{best_{id}}$ is the current optimal position, $g_{best_{id}}$ is the global optimal position, $r_1$, $r_2$ is the random number within the range $[0,1]$, and $\omega$ is the inertia weight.

It can be seen from formula (8) that the inertia weight represents the influence of the speed of the previous iteration on the current iteration speed. When the inertia weight is small, the local searching ability is strong, while the global searching ability is weak. Conversely, the local search ability is weak, while the global search ability is strong, so the dynamic adjustment of inertia weight is conducive to improving the running speed of the algorithm. In this paper, the form of exponential function is adopted to improve the inertia weight. The inertia weight decreases with the number of iterations, but it decreases slowly in the early stage, which is conducive to strengthening the global search ability and improving the efficiency of the algorithm.

$$\omega = \omega_{\text{max}} - (\omega_{\text{max}} - \omega_{\text{min}}) \frac{1}{1 - e^{-kT}}$$

\text{(9)}$$

SA (Simulated annealing) algorithm simulates the process of slow cooling of physical solid after high temperature melting [8], in which particles will become disorderly and orderly, and finally reach the lowest state of energy. SA algorithm will start from a high initial temperature $T$, and with the continuous decrease of temperature, repeat sampling will be carried out according to the rule of Metropolis with probability jump, and finally the global optimal solution can be obtained. SA can make the algorithm runout, avoid the disadvantage that the algorithm is easy to fall into the local optimal solution in the iterative process, and ensure the global optimal performance of the algorithm. PSO combined with SA has a faster convergence speed and higher precision. When the PSO falls into the local optimal solution, SA can jump out of the local trap and find a better solution.

3.3. The steps of algorithm implementation

Step 1: Initialize particle swarm. According to each joint range of motion, set the search space of particles, initialize the various parameters and random particle's position and speed.

Step 2: The fitness of particles is calculated by the fitness function $f(t)$ to obtain the optimal value pbest for individual particles and the global optimal value gbest for groups.

Step 3: Determine whether the global optimal value is unchanged or reaches the specified number of iterations. If it is, then go to Step 6.

Step 4: Calculate the probability value of the particle fitness value at the current temperature, and determine a value of gbest’ of a global optimal value, then update the particle's inertia weight according to formula (8), and update the particle's velocity and position through formula (9).

Step 5: Set the initial temperature and temperature cooling coefficient $k$ of simulated annealing.

Step 6: Calculate the fitness value of each particle at the current temperature, and update it according to the Metropolis criterion.

Step 7: determine whether meet the operation precision of a default, if meet the program to stop the iteration, and back to Step 6.
Step 8: Drop the temperature and return to Step 6 until the optimal solution is obtained.

4. Simulation experiment and analysis
A terminal pose matrix is taken at random as the expectation matrix $T$, and its inverse kinematics is solved by MATLAB’s HPSO. Substituting the obtained inverse solution into formula (6) gives the matrix $T'$. Compare the matrices $T$ and $T'$ and observe the error between them. The parameters of the connecting rod from the slave hand for single-port laparoscopic surgery robot are $l_0=20$, $l_1=50$, $l_2=40$, $l_3=30$, $l_4=20$, $l_6=50$, $l_7=20$, The random terminal pose is

$$T = \begin{bmatrix}
-0.645197533 & 0.176487995 & -0.743351917 & -133.285285754 \\
0.193145227 & -0.903672104 & -0.382193234 & 35.888855706 \\
-0.739198934 & -0.390165067 & 0.548959184 & -29.393398297 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

HPSO is used to solve the extreme value of fitness function and compare it with PSO and SAPAO respectively. The basic parameters of particle swarm are set as follows: the spatial dimension is 7, the maximum number of iterations is 500, the number of individuals is 200, the initial temperature of simulated annealing is $T=20$, $\omega_{max}=0.9$, $\omega_{min}=0.9$, and the annealing constant is 0.95. The comparison of convergence curve is shown in Fig.2.

![Comparison of convergence curves](image1)
(a) Comparison of convergence curves of PSO and HPSO
![Comparison of convergence curves](image2)
(b) Comparison of convergence curves of SAPSO and HPSO

Fig.2 Fitness value convergence curve

By comparing the Fig.2, it can be found that when the HPSO falls into the local optimum, the simulated annealing will jump out of the local minimum as far as possible, and then continue the global optimal search, which enhances the global optimization ability of the PSO algorithm, speeds up the evolution speed of the algorithm, and improves the convergence accuracy. The simulation results show that the convergence algorithm and search precision of the HPSO are superior to PSO and SAPSO.

After the fitness function converges, the optimal solution of each joint is obtained as follows:

$$\theta = \begin{bmatrix} 5.456953763898884 & 1.051405457657173 & 0.78428548825327 & 0.523004055678085 \\
0.784398630911503 & 1.044880522309139 & 1.044255309822576 \end{bmatrix}$$

According to the above optimal solution of each joint is put into the formula (6), and the terminal pose matrix obtained by the particle swarm can be obtained through forward kinematics:
By observing $T$ and $T'$, it can be found that the error of both position matrix $P$ and attitude matrix $R$ is very small, and it can be concluded that this method can achieve the positioning accuracy requirement.

5. Conclusion
In this paper, the screw theory is adopted to establish the motion model of slave hand for single-port laparoscopic surgery robot. Compared with the previous D-H method to establish the kinematics model, the geometric characteristics of rigid body motion can be described more intuitively. Only the global coordinate system and tool coordinate system need to be established to describe the rigid body motion as a whole. Aiming at solving the inverse kinematic position problem of the slave hand for single-port laparoscopic surgery robot, based on the pose matrix obtained from the forward kinematics, the inverse kinematic solution problem is transformed into the minimum value problem of the distance between two space vectors. The HPSO algorithm is adopted to solve the minimum problem, which reduces the computation, has good convergence, solves the problem quickly, and meets the requirement of precision.

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References
[1] Kwoh Y S, Hou J, Jonckheere E A, Hayati S. A robot with improved absolute positioning accuracy for CT guided stereotactic brain surgery [J]. IEEE transactions on bio-medical engineering, 1988, 35(2): 153-160.
[2] Tevatia G, Schaal S. Inverse kinematics for humanoid robots [C]. IEEE International Conference on Robotics and Automation. Piscataway, NJ, USA: IEEE, 2000: 294-299.
[3] Zu Di, Wu Zhenwei, Tan Dalong. An Effective Method for Solving Inverse Kinematics of Redundant Robots [J]. Journal of Mechanical Engineering, 2005(06): 71-75.
[4] Tan Fung Chan and R. V. Dubey, A weighted least-norm solution based scheme for avoiding joint limits for redundant joint manipulators [C]. IEEE International Conference on Robotics and Automation. Piscataway, NJ, USA: IEEE, 1993: 395-402.
[5] Huang Xiguang, Liu Bing, Li Qicai. A Conformal geometric algebra method for inverse kinematics of 6-DOF joint Robot [J]. Journal of Shanghai Jiao Tong University (S1): 86-89.
[6] Dong Yun, Yang Tao, Li Wen. Inverse kinematics solution of manipulator based on analytical method and Genetic Algorithm [J]. Computer Simulation, 2012, 029(003): 239-243.
[7] Koenderink J J, Van Doorn A J. Affine structure from motion [J]. Journal of Optical Society of America, 1991, 8(2): 377-385.
[8] Kirkpatrick S, Gelatt J C D, Vecchi M P. Optimization by simulated annealing [7]. Science, 1983, 220(4598): 671-680.