Miron’s Generalizations of Lagrange and Finsler Geometries: a Self–Consistent Approach to Locally Anisotropic Gravity

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In honour of Academician Radu Miron 70th birthday

Modern gauge theories of high energy physics, investigations in classical and quantum gravity and recent unifications of superstring theories (the so–called M– F– and S– theories) are characterized by a large application of geometric and topological methods. There are elaborated a number of Kaluza–Klein models of space–time and proposed different variants of compactification of higher dimensions. One of the still unsolved important physical problem is the definition of the mechanism of (in general dynamical) such compactification with a rigorous ”selection” of the four dimensional space–time physics from the low energy dynamics of (super) string and supergravitational theories. Another question of challenge of the modern physics is the local anisotropy of background radiation and the development of a consistent scenarios of quantum and classical cosmology.

In our works [4, 5, 6, 7] we have concerned the mentioned topics in a more general context of modelling physical processes on (super)vector bundles provided with nonlinear and distinguished connections and metric structures (containing as particular cases both Kaluza–Klein spaces and various extensions of Lagrange and Finsler spaces). We based our investigations on the fundamental results of the famous R. Miron’s Romanian school of Finsler geometry and its generalizations (as basic references we cite here some monographs and recent works [4]).

Perhaps, in the special literature there are cited more than a thousand of works on Finsler geometry, its generalizations and applications. It is well known that a metric more general than a Riemannian one was proposed in 1854 by B. Riemann and studied for the first time, in 1918, by P. Finsler who was a post–graduate student of C. Carathéodory. The purpose of applications of such generalized metrics in thermodynamics and, more generally, in physics was obvious. At present time there are published tens of monographs containing Finsler–like physical theories. Nevertheless the bulk of physicists still persists on a broad implementation of Finsler geometry in modern physics. This skepticism consists not only on a conservatism caused by the predomination of Riemann geometry (with some extensions to Einstein–Cartan–Spaces with torsion and nonmetricity) or by the ”excessive complexity” of Finsler geometry for developing physical theories. At first site the problem of construction of a general
"Finsler–physics" is very cumbersome and even unsurmountable. For instance, the most fundamental physical concepts of energy, momentum and rotation momentum are correspondingly strongly related with the supposed isotropy of space–time with respect to time and space translations and rotations (there is a group of automorphisms of the flat Minkowski space, the so–called generalized Lorentz, or Poincaré group, which also acts in the tangent bundle of the (pseudo)–Riemannian space). There are not local symmetries on spaces with generic local anisotropy and it "was concluded" that not having even local (pseudo)–rotations it impossible to construct a consistent physical theory of Finsler–like space–time and to define a theory of fundamental locally anisotropic field interactions because, for example, electrons are described by spinor fields, but spinors are closely related with the group of rotational isotropies which could not be defined for a general Finsler or Lagrange space.

It should be noted that the development of physical theories in media and spaces with local anisotropy became a rather evident necessity if there are rigorously analyzed a number of recent experimental data from modern cosmology and astrophysics, which reflects substantial anisotropies of the space–time structure at different state of evolution of the Universe, and if there are studied various processes in anisotropic, unordered and nonhomogeneous media, with dislocations and disclinations and/or with fluctuations and anisotropic diffusion. Supposing that anisotropies are included only in the energy–momentum tensors of matter (in order to develop some cosmological scenarious), which directs as the source in the Einstein equations the dynamics of Universe, one does not obtain a self–consistent theory if the condition of isotropy of metric (being a solution of field equations) is imposed. The modern Cosmology requires a more general geometric background with anisotropies of both matter and space–time metric.

In our works we tried to demonstrate that all basic difficulties connected with the elaboration of locally anisotropic physics are bridged over if the problems are touched on by modelling both generalized Finsler geometries and physical theories on vector bundles provided with compatible nonlinear and distinguished connections, metric, gauge and spinor fields. Such idea and methods, being very important for a further development of a number of directions in theoretical and mathematical physics, have been manifested in an explicit geometric form in the R. Miron and co–authors works. In some initial and different forms the idea of modelling of physical interactions on spaces with different geometric structures was contained in the Yano and Ishihara monograph (lifts of geometric object on the total space of tangent bundles) and in the A. Z. Petrov and N. S. Sinyukov works (deformation of connections and metric structures by maps generalizing conformal transforms).

The first our contributions to extensions of Finsler geometry and applications were the formulation of the theory of nearly autoparallel maps of generalized Lagrange and Finsler spaces and the proposal of definition of conservation laws for locally anisotropic field interactions by using such generalizations of conformal maps. The next step was the differential geometry of Clifford and spinor fibrations and of affine and linear frame bundles provided with nonlinear connection structure and in consequence the formulation of the theory of locally anisotropic field interactions.

In order to demonstrate that the locally anisotropic physics is naturally contained in modern (super) string theories we investigated the low energy limit of string locally anisotropic perturbations and proved that there are alternative variants when the grav-
itational and matter field interactions could be locally isotropic or anisotropic [5]. We formulated the supergeometry of vector superbundles enabled with general nonlinear connections and extended the diagram methods of superstrings to the case of higher order anisotropies. Another directions of our investigations is connected with higher order supersymmetric stochastic processes and superdiffusion. The basic results are summarized in monographs [4].

The main conclusion of this note is that after the R. Miron and co–authors idea of modelling both geometric constructions and physical theories was applied in our works in a manner as to unify both Kaluza–Klein and generalized Finsler (super)spaces by imbedding them in modern string theories it was compelled that the present–day physics can be generalized as to include locally anisotropic interactions and spaces.

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