Some Properties of Open - String Theories

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Abstract

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Introduction

The key features of (perturbative) spectra and interactions for models of oriented closed strings have been clarified by a number of groups at the end of the last decade. These works, all inspired by ref. [2], have the virtue of providing general, handy rules for constructing closed-string spectra from a variety of conformal field theories. Basically, any modular invariant combination of conformal field theories that respects the spin-statistics relation between bosonic and fermionic contributions to the vacuum amplitude and saturates the total conformal anomaly defines the perturbative spectrum of a model of oriented closed strings. The resulting plethora of solutions, a nice arena for string models of particle physics, is actually rather disturbing from the viewpoint of string unification. It is fortunate in this respect that all these constructions are tied to string perturbation theory, for which no genuine weak-coupling arguments are available. Recent work on string dualities, inspired to a large extent by general features of the low-energy supergravity and reviewed in a number of contributions to this volume, is bringing new, concrete evidence for the long-held feeling that all string theories are somehow different manifestations of a single underlying entity.

The purpose of this talk is to illustrate the state of the art of a program aimed at associating “open descendants” to suitable models of oriented closed strings and, more generally, to suitable conformal field theories. This started in the late eighties with the proposal of ref. [3] and is surely lagging behind the corresponding work for models of oriented closed strings, but has by now evolved into an algorithm capable of associating to suitable closed spectra additional open spectra with well-defined patterns for the breaking of the internal (Chan-Paton) symmetry [4]. I shall try to describe this setting by referring to a few simple cases capable of displaying some of its key features. Technical details may be found in the original papers [5] [6]
and in a forthcoming review article. These open-string models supplement the existing “oriented closed” zoo, while providing string-based descriptions for some additional classes of (super)gravity models. The recent work on string dualities reviewed in other contributions to these Proceedings suggests possible applications to the strong-coupling regime of other string models.

For simplicity, I shall confine my attention to a few key properties of genus-one partition functions that have been investigated in some detail. The simplest open-string models are the descendants of the Type-IIb superstring and of the two ten-dimensional tachyonic models first introduced in ref. [9]. In writing their vacuum amplitudes, I shall omit the modular integrations, while trading the theta constants for the four characters of level-one \( SO(8) \) representations.

\[
O_8 = \frac{1}{2\eta^4} \left( \theta^4 \left[ \begin{array}{c} 0 \\ 1/2 \\ 0 \\ 1/2 \end{array} \right] \right), \quad V_8 = \frac{1}{2\eta^4} \left( \theta^4 \left[ \begin{array}{c} 0 \\ 1/2 \\ 0 \\ 1/2 \end{array} \right] \right), \\
S_8 = \frac{1}{2\eta^4} \left( \theta^4 \left[ \begin{array}{c} 1/2 \\ 0 \\ 1/2 \\ 0 \end{array} \right] \right), \quad C_8 = \frac{1}{2\eta^4} \left( \theta^4 \left[ \begin{array}{c} 1/2 \\ 0 \\ 1/2 \\ 1/2 \end{array} \right] \right), \quad (1)
\]

with \( \eta \) the Dedekind function, but the \( SO(8) \) characters have the additional virtue of providing an orthogonal decomposition of the spectrum. In this notation, the interesting ten-dimensional models correspond to the partition functions

\[
T_{IIb} = |V_8 - S_8|^2 \\
T_{0a} = |O_8|^2 + |V_8|^2 + |S_8|^2 + |S_8|^2 + |C_8|^2 + |C_8|^2 \\
T_{0b} = |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2 . \quad (2)
\]

All other ten-dimensional models have a left-right asymmetric spectrum, including the type-IIa superstring, whose partition function in this notation reads

\[
T_{IIa} = (V_8 - S_8)(\bar{V}_8 - \bar{C}_8) . \quad (3)
\]

The Klein-bottle Projection and the Crosscap Constraint

The starting point in the construction of open descendants is a projection of the closed spectrum mixing left and right movers. This is attained supplementing the modular invariant (halved) torus partition function with a Klein-bottle contribution.

\[
\text{Figure 1. Projection in the closed sector}
\]

In the simplest of possible settings, the bosonic string, this operation is what in the early days of String Theory was said to lead from the “extended” to the “restricted” Shapiro-Virasoro model. In the “extended” model, all states in, say, the light-cone description are obtained acting on the vacuum with polynomials in the transverse left-moving oscillators \( \alpha_n^i \) and in the transverse right-moving oscillators \( \bar{\alpha}_n^i \), with the familiar Virasoro constraint on the entity of the corresponding total excitations. On the other hand, in the “restricted” model only polynomials with an additional discrete symmetry under the interchange of all the \( \alpha_n^i \) with
the $\tilde{\alpha}_n$ are allowed. In Superstring Theory, anomalies and their manifestation in this context, tadpoles \[13\], grant the motivation for the Klein-bottle projection that, in a general conformal field theory, exposes the rich structure of open models, exhibiting their patterns of internal symmetry and their planar duality in full-fledged form.

Let me follow the steps leading to the construction of open descendants, while referring to the models described in the previous Section. First of all, one is to halve the torus contribution, but I shall refrain from doing so explicitly. Then one is to add the Klein-bottle contribution. This is the first crucial step, since it determines the symmetry of the various sectors of the “restricted” closed spectrum under the interchange of their “left” and “right” parts. In general, the choice of Klein-bottle projection is not unique \[14\] \[15\] while, technically, the various projections differ in the signs for the characters that appear diagonally in the torus amplitude, that determine the (anti)symmetry of the restricted spectrum. For instance, the type-IIb model of eq. (2) could in principle admit four Klein-bottle projections, corresponding to all available choices of signs for the two characters $V_8$ and $S_8$. Actually, the fusion algebra introduces strong restrictions. Since in general only (anti)symmetric combinations of all left and right Verma modules are compatible with a modular invariant torus amplitude, it is simple to understand how these restrictions arise by “fusing” pairs of states. Thus, in this case the fusion of two (space-time) spinors yields a vector, and the $NS \otimes NS$ sector must always be symmetrized. The lesson that may be drawn from a closer inspection of this simple example is actually quite general: the available choices correspond to the $Z_2$ automorphisms of the fusion algebra compatible with the torus $GSO$ projection. With this proviso, one can see that the ten-dimensional models of eq. (2) allow at most four types of Klein-bottle projection, though two of them are really equivalent. The first, most natural choice, corresponds to the natural basis of characters for the space-time Lorentz group $SO(1,9)$, namely $(O_8, V_8, -S_8, -C_8)$, and results in the projections

$$K_{IIb} = \frac{1}{2} (V_8 - S_8) \ ,$$

$$K_{0a} = \frac{1}{2} (O_8 + V_8) \ ,$$

$$K_{0b} = \frac{1}{2} (O_8 + V_8 - S_8 - C_8) \ ,$$

(4)

whereby $NS - NS$ sectors are symmetrized while $R - R$ sectors are antisymmetrized. The other choices correspond to the basis $(O_8, V_8, S_8, C_8)$, to the basis $(-O_8, V_8, -S_8, C_8)$ and to the basis $(-O_8, V_8, S_8, -C_8)$. Only the first Klein-bottle projection is allowed in the type-IIb superstring, while the last two, clearly equivalent after a parity transformation, change the relative weight of the $RR$ sectors, and are therefore incompatible with the $GSO$ projection of the $Oa$ model as well. On the other hand, in the $Ob$ model one is lead to the additional genuinely inequivalent Klein-bottle projections

$$K_{0b}' = \frac{1}{2} (O_8 + V_8 + S_8 + C_8) \ ,$$

$$K_{0b}'' = \frac{1}{2} (-O_8 + V_8 + S_8 - C_8) \ .$$

(5)

It is instructive to take a closer look at the lowest-mass states of the $Ob$ descendants resulting from the three types of projection. Standard properties of the ten-dimensional Lorentz group \[17\] then imply that $K_{0b}$ leaves a tachyon, a graviton and a dilaton in the $NS \otimes NS$ sectors, as well as two more scalars and two more antisymmetric tensors in the $R \otimes R$ sectors, and thus projects out all “chiral” closed-string fields. On the other hand, $K_{0b}'$ also leaves a tachyon,
a graviton and a dilaton in the $NS \otimes NS$ sectors, but now the the $R \otimes R$ sectors contain both a self-dual and an antiself-dual four-form, with a resulting projected closed spectrum that is again not chiral. The last projection, $K''_{ab}$, is actually the most interesting one. Indeed, the corresponding closed spectrum does not contain a tachyon, but only a graviton and an antisymmetric tensor in the $NS \otimes NS$ sectors, and is chiral, since in the $R \otimes R$ sectors it contains a self-dual four-form, as well as an antisymmetric tensor and a scalar. As we shall see, there are chiral open descendants of the $Ob$ model where the Green-Schwarz mechanism eliminates all gauge and gravitational anomalies. On the other hand, according to our rule, both the $Oa$ and the $IIb$ models allow only one Klein-bottle projection. The former leads to the models of ref. [5], while the latter leads to the Type-I $SO(32)$ superstring. Many of these remarks apply to arbitrary conformal models symmetric under the interchange of their left and right parts. Ref. [6] contains a discussion of the open descendants of $AA$ minimal models with a totally symmetric Klein-bottle projection, while refs. [14] and [15] contain several details on the general case for $SU(2)$ WZW models.

There is actually a nice geometrical setting for the various phase choices in the Klein-bottle projections. Indeed, a Klein bottle may be pictured as a self-intersecting surface or, alternatively, as a tube terminating at two crosscaps. A crosscap, or real projective plane, is a simple instance of non-orientable surface free of boundaries, and may be regarded as a disk where all pairs of opposite points lying along the boundary are identified.

Given a two-point amplitude as in fig. 3, moving one of the two punctures probes the fundamental group of the simply-punctured surface. According to the familiar properties of closed-string models, no price is to be paid when transporting one puncture around the other (path $\alpha$). However, here there is a more elementary move, namely the puncture may be displaced up to a point to emerge from the other identified point (path $\beta$). Since this move squares to the previous one, one is left with some free signs that reflect the behavior of the various two-point functions upon transport around the crosscap. The free signs are actually to be compatible with the fusion rules, and correspond to the free signs in the Klein-bottle projection [14] [15].

Before turning to the open spectrum, let me discuss in some detail the other available choice of “time” for the Klein bottle. Referring to fig. 2, the previous choice may be termed

---

**Figure 2.** Different choices of "time" on the Klein bottle

**Figure 3.** The crosscap
“vertical”, while this new choice may be termed “horizontal”. The two are related by the familiar modular transformation $\tau_2 \to 1/\tau_2$, and the second type of amplitude, that I shall denote $\tilde{K}$, exhibits the propagation of the closed spectrum between a pair of crosscaps. One should therefore demand that, in terms of the proper basis of characters, all coefficients in the transverse Klein-bottle amplitude be positive. This is actually guaranteed rather nicely by our previous rule. Thus, for instance, for the descendants of the ten-dimensional $Ob$ model,

$$
\tilde{K}_{0b} = \frac{2^6}{2} V_8 ,
$$
$$
\tilde{K}_{0b}' = \frac{2^6}{2} O_8 ,
$$
$$
\tilde{K}_{0b}'' = -\frac{2^6}{2} C_8 ,
$$

and, as anticipated, all coefficients are indeed positive if referred to the natural basis of space-time characters, $(O, V, -S, -C)$. It should be appreciated that these coefficients have a nice physical interpretation: they are squared moduli of the normalizations for the one-point functions of primary fields in front of a crosscap. Their values are determined to a large extent by the crosscap constraint \cite{18} \cite{14} \cite{15}, an eigenvalue equation expressing the equivalence of the different sewings for the amplitude of fig. 3, and fully so for all $SU(2)$ $WZW$ models in the $ADE$ series.

In our ten-dimensional models, the Klein-bottle projections $K$ and the corresponding vacuum-channel amplitudes $\tilde{K}$ are related by the $S$ matrix

$$
S = \frac{1}{2} \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 \\
\end{pmatrix},
$$

determined by the familiar properties of theta functions. The overall powers of two in the vacuum-channel amplitudes of eqs. \cite{8} deserve some additional comments since, as we shall see, they are related to the size of the open-string Chan-Paton groups. They draw their origin from a natural prescription to deal with divergent infrared contributions to the vacuum channel, whereby all vacuum amplitudes are expressed in terms of the moduli of their double covers. This prescription \cite{19} \cite{4} allows a convenient comparison between the contributions of the three surfaces of zero Euler character that determine the spectrum of open-string models, namely the Klein bottle, the annulus and the Möbius strip, and is equivalent \cite{20} to other methods of dealing with tadpoles and divergences \cite{21}.

**The Open Sector and Chan-Paton Symmetry Breaking**

The next step in the construction has to do with the new, “twisted”, sector. This involves open strings, and a new projection obtained adding to a (halved) annulus amplitude a Möbius contribution.

$$
\frac{1}{2} \left( \begin{array}{c}
\text{Ba} \\
\text{Bb}
\end{array} \right) + \left( \begin{array}{c}
\text{Ba} \\
\text{Ba}
\end{array} \right)
$$

**Figure 4. Projección in the open sector**
These last two surfaces have vanishing Euler character and thus contribute to the same order of perturbation theory but, differently from the Klein bottle, have respectively two boundaries and one boundary and one crosscap. The boundaries may be pictured as drawn by the ends of open strings, the site of their Chan-Paton charges [4]. Therefore, it should come as no surprise that these additional contributions are polynomials of degree two and one in the multiplicities of the various charge spaces. The structure of these polynomials is subject to strong constraints, since it is to be compatible with the factorization of disk amplitudes.

Moreover, since a “horizontal” time in the annulus amplitude exhibits a vacuum channel where the closed spectrum propagates between a pair of holes (fig. 5), the coefficients in $\tilde{A}$ must satisfy positivity constraints with respect to a proper basis of characters, in a similar fashion to what we have seen for $\tilde{K}$. These coefficients are then properly interpreted as squared moduli of the normalizations of one-point functions on the disk.

In building the annulus amplitude, it is very useful to begin from the vacuum channel. This corresponds to the “horizontal time” in fig. 5 and accommodates the propagation of the closed spectrum, that is by now under control for a given model. One is thus faced with the choice of a number of (squared moduli of) reflection coefficients, and in some cases, including the simple ones I am dealing with here, the proper choice is fully determined by symmetry. The basic rule is simple to state. Given a closed spectrum described by a (quasi)diagonal partition function of the type

$$T = \sum_{i,j} N_{ij} \chi_i \bar{\chi}_j,$$  

the characters $\chi_i$ that may flow in vacuum channel of the annulus are those paired with their conjugates with respect to the given symmetry by the closed-string GSO projection [5]. This corresponds to the intuitive idea that a state flowing, say, to the right is to be turned, upon reflection, into its conjugate (in the sense of the closed-string GSO projection). If the boundary is to respect a given (extended) symmetry, a non-trivial reflection is possible only if the GSO-conjugate happens to be also conjugate with respect to the given symmetry. All this is perhaps simpler to appreciate by referring to our examples. Thus, in the $IIb$ case there is a single (super)sector, corresponding to $V_8 - S_8$, and

$$\tilde{A}_{IIb} = \frac{2^{-5}}{2} \alpha_{IIb}^2 (V_8 - S_8),$$  

where the overall normalization is chosen for later convenience. On the other hand, in the $O_8$ model only $O_8$ and $V_8$ are paired with their conjugates by the projection of eq. (8), since both $S_8$ and $C_8$ are self-conjugate in $SO(8)$, and therefore

$$\tilde{A}_{0a} = \frac{2^{-5}}{2}(\alpha_{0a}^2 V_8 + \beta_{0a}^2 O_8).$$  

(10)
Finally, in the $Ob$ model all characters are paired with their conjugates by the closed-string $GSO$ projection, and the annulus vacuum channel contains four reflection coefficients:

$$\tilde{A}_{ob} = \frac{2^{-6}}{2} (\alpha_{ob}^2 V_8 + \beta_{ob}^2 O_8 - \gamma_{ob}^2 S_8 - \delta_{ob}^2 C_8) . \quad (11)$$

The number of independent coefficients is a very important datum in the construction since, as we shall see, it determines the number of charge sectors in the models. Moreover, since the Möbius strip may be seen as a tube with a hole and a crosscap at the ends, the vacuum Möbius channel $\tilde{M}$ is also determined by the two amplitudes $\tilde{K}$ and $\tilde{A}$. Thus, in our models,

$$\tilde{M}_{IIb} = \frac{2}{2} \alpha_{IIb} (V_8 - S_8) ,$$

$$\tilde{M}_{0a} = \frac{2}{2} (\alpha_{0a} V_8 + \beta_{0a} O_8) , \quad (12)$$

$$\tilde{M}_{0b} = \frac{2}{2} \left( \alpha_{0b} V_8 + \beta_{0b} O_8 - \gamma_{0b} S_8 - \delta_{0b} C_8 \right) .$$

In these expressions the overall coefficients are geometric means of those in $\tilde{K}$ and $\tilde{A}$, with an additional factor of two that reflects the combinatorics of the Möbius vacuum channel. Once more, this choice may be justified by a careful analysis of the measure, and draws its motivation from the prescription of referring all amplitudes to their double covers.

One may now turn the $\tilde{A}$ amplitudes into direct-channel open-string partition functions using the $S$ matrix of the previous section. On the other hand, a similar operation for the Möbius amplitudes requires the use of a different matrix, $P$, related to the two more familiar matrices $S$ and $T$ by

$$P = T^{1/2} S T^2 S T^{1/2} . \quad (13)$$

Rather than performing these transformations directly, I would like to turn momentarily to the direct channel, in order to exhibit an ansatz for the Chan-Paton charge assignments. This choice drew its motivation from previous work of Cardy [22]. Aiming at a more direct derivation of the Verlinde formula [23], he had considered the annulus amplitude with fixed boundary conditions in a generic diagonal rational conformal field theory, providing a novel interpretation of the fusion-rule coefficients $N_{ij}^k$: they determine the bulk content ($k$) corresponding to boundary conditions $i$ and $j$. The observation of ref. [4] is that, given the one-to-one correspondence between bulk and boundary (charge) sectors of these rational models, the ansatz may be turned into one for an annulus amplitude with broken Chan-Paton symmetry, namely

$$A = \frac{1}{2} \sum_{i,j,k} N_{ij}^k n_i n_j \chi_k . \quad (14)$$

Actually, the Verlinde formula makes this expression particularly appealing, since it implies that

$$\tilde{A} \sim \frac{1}{2} \sum_i \chi_i \left( \sum_{j,k} S_{jk} n_k \right)^2 , \quad (15)$$

and therefore all coefficients in $\tilde{A}$ are perfect squares, as demanded by the structure of this contribution. We have developed a foolproof procedure to construct the charge assignments in general models [14] [15], and indeed in the non-diagonal $WZW$ models eq. (14) does not apply,

1In more complicated models the coefficients may turn out not to be independent. More details on this issue may be found in ref. [13].
though the simple models considered in this talk may be constructed in way directly suggested by this observation. Before displaying the corresponding amplitudes, let me introduce another, related, concept. This has to do with “complex” Chan-Paton charges, and is a generalization of a long-known property of boundaries, that in oriented open strings (or, in our case, in oriented sectors of open spectra) carry arrows that account for their orientation. Technically, “complex” boundaries carry charges valued in fundamental representations of unitary groups, whereas “real” boundaries carry charges valued in fundamental representations of symplectic or orthogonal groups [4]. In these models, this peculiarity presents itself whenever a naive transverse-channel amplitude would violate positivity, as will be seen in a simple example below. The further restrictions that restore consistency are, basically, the numerical constraints $n = \bar{n}$ for pairs of conjugate charges. “Complex” charges, however, appear also when some characters are not self conjugate [5].

Turning to the explicit form of the amplitudes, let me proceed by considering the descend-

dants of the type-IIb model. These are type-I models, and it has long been known that only one of them is anomaly free [16]. In order to see how the restrictions manifest themselves, let me write explicitly all the amplitudes:

$$
A_{I Ib} = \frac{n^2}{2} (V_8 - S_8) , \\
M_{I Ib} = -\frac{n^2}{2} (V_8 - S_8) , \\
\tilde{A}_{I Ib} = 2^{-5} \frac{n^2}{2} (V_8 - S_8) , \\
\tilde{M}_{I Ib} = -2 \frac{n^2}{2} (V_8 - S_8) .
$$

(16)

The overall sign of the Möbius terms is conventional at this stage. A positive $n$ would imply an orthogonal gauge group, with $n(n-1)/2$ gauge bosons, while a negative $n$ would imply a symplectic gauge group, with $|n|(|n|+1)/2$ gauge bosons, and the proper choice is determined by a tadpole condition. The precise relation between tadpole conditions and anomaly cancellations was first pointed out for the type-I superstring in ref. [13]: the anomalies are linked to tadpoles of massless unphysical states. In our setting it is simple to track such states, and there is one in these models: it corresponds to the sector $S_8$ in the vacuum channel, that would start with a massless RR scalar projected out of the closed spectrum by the Klein-bottle of eq. (4). Setting to zero its total one-point function at genus one-half, determined by eqs. (6) and (16), yields a quadratic equation for $n$, that by construction is guaranteed to have two coincident roots, namely (fig. 6)

$$
2^{10} + n^2 - 2^6 n = 0 ,
$$

(17)

where the three terms originate from $\tilde{K}$, $\tilde{A}$ and $\tilde{M}$. As anticipated, the unique solution, $n = 32$, leads to a unique anomaly-free $SO(32)$ model, a long-known result of Green and Schwarz [16].

Figure 6. Tadpole condition for the Type-I superstring

Let me now turn to the $Oa$ models, that have a unique Klein-bottle projection and are thus simpler to deal with. In this case the direct-channel annulus has two charge sectors, and
one has the consistent set of assignments

\[ A_{0a} = \frac{n_B^2 + n_F^2}{2} (O_8 + V_8) - n_B n_F (S_8 + C_8) , \]
\[ M_{0a} = \frac{n_B + n_F}{2} (O_8 - V_8) , \]
\[ \tilde{A}_{0a} = \frac{2^{-5}}{2} \left( (n_B + n_F)^2 V_8 + (n_B - n_F)^2 O_8 \right) , \]
\[ \tilde{M}_{0a} = -2 \frac{n_B + n_F}{2} (O_8 + V_8) , \]

that determine the coefficients in eq. (18). This simple class of models is interesting, since it exhibits a new phenomenon, Chan-Paton symmetry breaking.

![Figure 7. Chan-Paton symmetry breaking](image)

Referring to fig. 7 one can appreciate that allowing, say, \( n_F \neq 0 \), has the effect of emptying the Chan-Paton charge matrices of the gauge vectors while moving their charges to new sectors of the spectrum. Moreover, this is all manifestly compatible with factorization. Thus, for instance, if \( n_B \) and \( n_F \) are both positive there are gauge bosons (corresponding to \( V_8 \)) in the adjoint representation of \( SO(n_B) \otimes SO(n_F) \), tachyons (corresponding to \( O_8 \)) in the symmetric two-tensor and scalar representations of the same group, and Majorana fermions in the \((n_B, n_F)\) representation. This spectrum is manifestly non-chiral, both in the closed sector projected according to eq. (4) and in the open sector. Correspondingly, the only massless tadpole in the vacuum channel corresponds to the \( V_8 \) sector, a physical one, and determines the dilaton tadpole at genus 1/2. Thus here the tadpole condition, that would fix the total size of the (orthogonal) gauge group to \( n_B + n_F = 32 \), is not related to gauge and gravitational anomalies as in the type-IIb case. To date, the role of these additional tadpole conditions is not understood to my own satisfaction.

Let me now turn to the \( Ob \) models, beginning with the usual ansatz, that in this case reads

\[ A_{0b} = \frac{n_o^2 + n_v^2 + n_s^2 + n_c^2}{2} V_8 + (n_o n_v + n_s n_c) O_8 \]
\[ - (n_o n_c + n_v n_s) S_8 - (n_o n_s + n_v n_c) C_8 , \]
\[ M_{0b} = -\frac{n_o + n_v + n_s + n_c}{2} V_8 , \]
\[ \tilde{A}_{0b} = \frac{2^{-6}}{2} \left( (n_o + n_v + n_s + n_c)^2 V_8 + (n_o + n_v - n_s - n_c)^2 O_8 \right) \]
\[ - (n_o + n_v + n_s - n_c)^2 S_8 - (n_o + n_v - n_s + n_c)^2 C_8 \right) , \]
\[ \tilde{M}_{0b} = -2 \frac{n_o + n_v + n_s + n_c}{2} V_8 . \]

Now the open descendants are non-chiral in their closed sector but they are chiral in their open sector and thus, not surprisingly, there are tadpole conditions (see also eq. (18)) related
to the two “unphysical” scalars corresponding to $S_8$ and $C_8$. The two resulting conditions, $n_o = n_v$ and $n_e = n_c$, dispose of all gauge anomalies. The $V_8$ tadpole is again a physical one, corresponding to the dilaton. Setting it to zero would fix to 64 the total size of the gauge group.

In this class of models there are two more options for the Klein-bottle projection, and thus one may investigate the corresponding open sectors. I shall discuss in some detail only the models corresponding to $K'$ of eq. (3), since they are more interesting. In general, when one moves from one projection to another, complex charges appear. The need for this is easily seen by noticing that in this case only $C_8$ can flow in the M"obius amplitude. This has an important consequence, namely the gauge vectors correspond to unitary groups, and thus carry pairs $\v n\bar{n}$ of conjugate charges. Let me display the amplitudes, since this will make my remarks clearer:

$$
A''_{0b} = (n_1\bar{n}_1 + n_2\bar{n}_2) \ V_8 + (n_1\bar{n}_2 + n_2\bar{n}_1) \ O_8 \\
- (n_1n_2 + \bar{n}_1\bar{n}_2) \ S_8 - \frac{n_1^2 + n_2^2 + \bar{n}_1^2 + \bar{n}_2^2}{2} \ C_8 ,
$$

$$
M''_{0b} = \frac{n_1 + \bar{n}_1 - n_2 - \bar{n}_2}{2} \ C_8 ,
$$

$$
\bar{A}''_{0b} = \frac{2^{-6}}{2} \left( (n_1 + \bar{n}_1 + n_2 + \bar{n}_2)^2 \ V_8 - (n_1 - \bar{n}_1 + n_2 - \bar{n}_2)^2 \ O_8 \\
+ (n_1 - \bar{n}_1 - n_2 + \bar{n}_2)^2 \ S_8 - (n_1 + \bar{n}_1 - n_2 - \bar{n}_2)^2 \ C_8 \right) ,
$$

$$
\bar{M}''_{0b} = \frac{2}{2} \frac{n_1 + \bar{n}_1 - n_2 - \bar{n}_2}{2} \ C_8 .
$$

(20)

It should be appreciated that the coefficients of $O_8$ and $S_8$ must both vanish, since their sign is necessarily incorrect! As anticipated, however, this condition is directly implied by their relation to the dimensions of the fundamental representations of unitary groups. Comparing with eq. (3) shows that there is an additional tadpole condition for the “unphysical” scalar mode in the $C_8$ sector. This yields a relation between the dimensions of the two unitary groups, namely

$$
n_1 + \bar{n}_1 - n_2 - \bar{n}_2 = 64 ,
$$

sufficient to ensure the cancellation of all gauge and gravitational anomalies, as may be seen making use of the results of ref. [21]. In this case one does not have the option to set to zero the dilaton tadpole, and the size of the gauge group is therefore not determined. The option of relaxing the dilaton tadpole condition, considered in ref. [22], may lead to interesting progress in open-string string model building. The last class of open descendants may be constructed in a similar fashion, and is left as an exercise for the interested reader. Details of the construction of open descendants for large classes of conformal models, including the factorization properties of disk amplitudes for minimal and $SU(2)$ WZW models, may be found in refs. [1] [14] [13].

Lower-dimensional models may be constructed in a similar fashion. In six-dimensions there are large classes of chiral supersymmetric models with interesting gauge groups that exhibit a novel feature: several antisymmetric tensor dispose together of gauge and gravitational anomalies by a generalized Green-Schwarz mechanism [8]. To date, however, chiral four-dimensional models are not understood to my own satisfaction, and apparently lead to redundant tadpole conditions that limit their gauge groups to a small size.

In conclusion, I have described in some detail the construction of open descendants for some simple closed orientable models. Some general features of this procedure apply to arbitrary (left-right symmetric) conformal models, and endow them with one or more classes of open descendants. Hopefully this explicit derivation will help interested readers to get acquainted with some of the amusing properties of open-string models that, so far, we have been able to unveil.
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