Evaluation of COVID-19 pandemic spreading using computational analysis on nonlinear SITR model

S. E. Ghasemi¹ | Sina Gouran²

¹Department of Engineering Sciences, Hakim Sabzevari University, Sabzevar, Iran
²School of Mechanical Engineering, Babol University of Technology, Babol, Iran

Correspondence
S. E. Ghasemi, Department of Engineering Sciences, Hakim Sabzevari University, Sabzevar, Iran.
Email: s.ebrahim.ghasemi@gmail.com
Communicated by: M. Efendiev

The main purpose of present paper is to investigate the nonlinear model of COVID-19 (novel coronavirus) computationally. The SITR model is designed according to four classifications of Susceptible (S), Infectious (I), Treatment (T) and Recovered (R). Two convenient and effective numerical techniques namely the Adams–Bashforth Method (ABM) and Milne-Simpson Method (MSM) are employed to analyze the epidemic model. The influences of the contact rate parameter (β), recovery parameter (μ) and death parameter (α) on the variables including S, I and R are studied comprehensively. The obtained findings indicate that by increasing the contact rate parameter the infectious and recovered categories enhance but the susceptible mechanism decreases.

KEYWORDS
computational methods, COVID-19, epidemic model, infectious, susceptible

MSC CLASSIFICATION
65L05, 37N30, 01-08, 26C15

INTRODUCTION

In recent months, the novel Coronavirus (COVID-19) outbreak has claimed the lives of thousands of people around the world and has had irreparable consequences in various areas of human life. The origin of this virus is claimed to be from animals especially wild animals as Wuhan is one of the important cities in which people are quite interested in wild animals and animal market is popular in Wuhan.¹ Subsequently, it has been discovered that human to human transmission is also possible.² This novel coronavirus is spread through close contact and respiratory droplets while in serious cases causing severe atypical pneumonia.³ Other complications including sepsis, infection of the heart, liver, the digestive tract and multiple organ failure may arise and are especially dangerous in more susceptible groups such as the elderly and people with underlying comorbidities.⁴⁻¹⁰ Many positive cases are experiencing some problems related to respiratory system.¹¹ Transmission is easily expected once an infected individual coughs or sneezes so liquid droplets in respiratory system are responsible for transmission. So, staying away from infected people is the best approach to stay healthy.¹¹

Untill 18th August 2021, more than 209.83 million persons have been contaminated and more than 4.4 million persons have died by COVID-19 while 188.06 million individuals got recovered in the whole world. Figure 1 shows the data of infected, recovered and died persons by novel coronavirus for top 12 countries.¹² Also, the trend of transmission of COVID-19 has been demonstrated in Figure 2 graphically.

The accumulative number of infected (by number of days since 10,000 cases) and died (by number of days since 100 deaths) persons are shown in Figure 3A,B, respectively.¹²

Further, the infected and death persons of Europe countries are compared with those of USA in Figure 4.¹²
FIGURE 1  Infected, recovered and died individuals with COVID-19 for different countries\textsuperscript{12} [Colour figure can be viewed at wileyonlinelibrary.com]
The distribution of total cases for different countries is depicted in Figure 5.12

In most physical, engineering and medical phenomena, variables and parameters are simplified by mathematical modeling. Many researchers investigated the mathematical model of infectious diseases using numerical and analytical methods. Also, many researches have been performed theoretically on the models of epidemiology.13–16 A SEIR model with some required corrections for investigation on spreading rate of COVID-19 was suggested by Kucharski et al.17 Goufo et al.18 studied on fractional SIR epidemic model of measles. Different SEIR models by utilizing artificial intelligence were proposed by Yang et al.19 for predicting the increasing trend and peaks of novel corona virus in china. Ogren and Martin20 utilized the Newton method to achieve the optimum control schedule in the biologically SIR models. Also recently, many studies have been performed on models of infectious diseases and COVID-19.21–30

Based on above literature review, the prime aim of current study is to simulate and mathematically analyze the COVID-19 virus. Two efficient numerical methods are used in this paper to solve the problem. After solving the nonlinear equations of SITR model, the variation of three effective parameters including contact, recovery and death rates on evaluation criteria of COVID-19 are investigated in details.

2 | MATHEMATICAL MODEL

The SITR model of COVID-19 with fundamental specifications of the pandemic model is explained in present section. Figure 6 illustrates a composition of the current model with introducing various parameters in details. Table 1 presents the basic characteristics of the SITR model.

Under above presumption, the presented SITR model can be expressed by ordinary differential equations as follows31:
\[ S_1(t) = B - \beta I(t)S_1(t) - \delta T(t) - \alpha S_1(t) + \varphi(t) \]  
\[ S_2(t) = B - \beta I(t)S_2(t) - \delta T(t) - \alpha S_2(t) + \varphi(t) \]  
\[ I'(t) = -\mu I(t) + \beta I(t)[S_1(t) + S_2(t)] - \alpha I(t) + \beta \delta T(t) + \sigma I(t) + \varphi(t) \]  
\[ T'(t) = \mu I(t) - \rho T(t) - \alpha T(t) + \psi T(t) + \epsilon T(t) + \varphi(t) \]  
\[ R'(t) = -\alpha R(t) + \rho T(t) + \varphi(t) \] 

With initial boundary conditions:

\[ S_1(0) = 0.65, \ S_2(0) = 0.15, \ I(0) = 0.1, \ T(0) = 0.2, \ R(0) = 0.1 \]
Table 2 shows the specified values for different parameters of the governing equations. The function of \( \varphi(t) \) in the governing equations denotes the sudden variation caused by several reasons consist of communal meeting, trips and generic interactivity, that can generate any abrupt increment in the quantity of infected persons. It should be noted that the rates of recovery (\( \mu \)) and death (\( \alpha \)) can be interdependent to each other as increasing recovery rate by medical care, vaccine injection and quarantine of infected people leads to decrease of the death rate. In current mathematical model,
the effectuality of rising rate of recovery and reducing rate of death is evaluated mathematically. Also, the impact of other momentous parameters on rate of recovery is modeled and computed.

### 3 | THE PRINCIPLE OF METHODS

#### 3.1 | Adams–Bashforth method (ABM)

The basic idea of Adams methods is on integrand approximation using a polynomial with \((t_n, t_{n+1})\) utilizing \(k\) order polynomial and \(k + 1\) order scheme. The explicit type of Adams methods is named the Adams Bashforth technique which was blueprinted for solving differential equations.\(^{32}\) The Adams-Bashforth Method is described as follows:\(^{32}\)
3.1.1 Integrating

\[ y' = f(x, y) \] (7)

With the interval \([x_i, x_{i+1}]\).

3.1.2 Gives

\[
\int_{x_i}^{x_{i+1}} y' \, dx = \int_{x_i}^{x_{i+1}} f(x, y) \, dx
\] (8)

Equation (8) is equivalently written as

\[
y(x_{i+1}) = y(x_i) + \int_{x_i}^{x_{i+1}} f(x, y) \, dx
\] (9)

The following are the explicit method through the \((x_i, f_i), (x_i+1, f_{i+1}), (x_i+2, f_{i+2}), \ldots, (x_i+k, f_{i+k})\).

The interpolation polynomial with \((k-1)\) degree for backward difference of Newton's method is expressed as:

\[
p_{k-1}(x) = f_i + \frac{x - x_i}{h} \nabla f_i + \frac{(x - x_i)(x - x_{i-1})}{(2!)h^2} \nabla^2 f_i + \frac{(x - x_i)(x - x_{i+1})(x - x_{i-2})}{(3!)h^3} \nabla^3 f_i + \ldots + \frac{(x - x_i)(x - x_{i-1}) \ldots (x - x_{i-k+2})}{((k-1)!h^{k-1})} \nabla^{k-1} f_i
\] (10)

Setting: \(x-x_i = hs\).

Then Equation (10) gives

\[
p_{k-1}(x) = f_i + s \nabla f_i + \frac{1}{2}s(s+1) \nabla^2 f_i + \frac{1}{6}s(s+1)(s+2) \nabla^3 f_i + \ldots + \frac{s(s+1)(s+2) \ldots (s+k-2)}{(k-1)!} \nabla^{k-1} f_i
\] (11)

Noting that: \(s = \frac{x-x_i}{h} < 0\).

Replacing \(f(x, y)\) with \(p_{k-1}\) in Equation (9) yields:

\[
y_{i-1} = y_i + \int_0^1 \left[ f_i + s \nabla f_i + \frac{1}{2}s(s+1) \nabla^2 f_i + \frac{1}{6}s(s+1)(s+2) \nabla^3 f_i + \ldots \right] \, ds
\] (12)

Integrating Equation (12) gives:

\[
y_{i-1} = y_i + h \left[ f_i + \frac{1}{2} \nabla f_i + \frac{5}{12} \nabla^2 f_i + \frac{9}{24} \nabla^3 f_i \right]
\] (13)

Applying the backward difference formula into Equation (13), the result is then introduced back into Equation (13)
### 3.2 Milne-Simpson method (MSM)

Another computational scheme is known as the Milne or Milne-Simpson technique (Milne, 1953), the foundation of its procedure is on integrating the slope function \( f(t, y(t)) \) with the distance \( (x_{n-3}, x_{n+1}) \) and then applying the Simpson rule. This method utilizes the approximation of Lagrange polynomial for \( f(t, y(t)) \) with four mesh points: \((x_{n-3}, t_{n-3}), (x_{n-2}, t_{n-2}), (x_{n-1}, t_{n-1}), (x_n, t_n)\). It integrated over the interval \([x_{n-3}, x_{n+1}]\) the formula was discovered by William Edmund Milne in 1953; Milne method has second order accuracy and is stable. This method is described as follows:

#### 3.2.1 Integrating

\[
y' = f(x, y) \tag{15}
\]

With the interval \([x_i-3, x_i+1]\).

#### 3.2.2 Gives

\[
\int_{x_{i-3}}^{x_{i+1}} y' dx = \int_{x_{i-3}}^{x_{i+1}} f(x, y) dx \tag{16}
\]

Equation (16) is equivalently written as

\[
y(x_{i+1}) = y(x_{i-3}) + \int_{x_{i-3}}^{x_{i+1}} f(x, y) dx \tag{17}
\]

The following are the explicit method through the \((x_i, f_i), (x_{i+1}, f_{i+1}), (x_{i+2}, f_{i+2}), ..., (x_{i+k+1}, f_{i+k+1})\).

Interpolation polynomial with \((k-1)\) degree for backward difference of Newton’s method is expressed as:

\[
p_{k-1}(x) = f_i + \frac{x - x_i}{h} \nabla f_i + \frac{(x - x_i)(x - x_{i-1})}{(2!)h^2} \nabla^2 f_i + \frac{(x - x_i)(x - x_{i-1})(x - x_{i-2})}{(3!)h^3} \nabla^3 f_i + ... + \frac{(x - x_i)(x - x_{i-1})... (x - x_{i-k+2})}{((k-1)!)h^{k-1}} \nabla^{k-1} f_i \tag{18}
\]

Setting: \(x - x_i = hs\).

Then Equation (18) gives

\[
p_{k-1}(x) = f_i + s \nabla f_i + \frac{1}{2} s(s + 1) \nabla^2 f_i + \frac{1}{6} s(s + 1)(s + 2) \nabla^3 f_i + ... + \frac{s(s + 1)(s + 2)...(s + k - 2)}{(k - 1)!} \nabla^{k-1} f_i \tag{19}
\]

By substituting the backward difference polynomial we have:
Integrating Equation (20) gives

\[ y_{i+1} = y_{i-3} + \int_{-3}^{1} \left[ f_i + s\nabla f_i + \frac{1}{2}s(s+1)\nabla^2 f_i + \frac{1}{6}s(s+1)(s+2)\nabla^3 f_i + \cdots \right] ds \] (20)

Applying the backward difference formula into Equation (21), the result is then introduced back into Equation (21)

\[ y_{i+1} = y_{i-3} + h \left[ f_i - 4\nabla f_i + \frac{8}{3}\nabla^2 f_i \right] \] (21)

\[ y_{i+1} = y_{i-3} + \frac{4h}{3} \left[ 2f_i - f_{i-1} + 2f_{i-2} \right] \] (22)

4 | RESULTS AND DISCUSSION

After applying the Adams-Bashforth Method (ABM) and Milne-Simpson Method (MSM) to solve the nonlinear SITR model, the behavior of susceptible, infectious and recovered mechanisms under influence of varying the contact rate (β), rate of recovery (μ) and rate of death (α) will be discussed in details.
Figure 7 depicts the effect of contact rate on susceptible, infective and recovered mechanism. Figure 7A demonstrates that by increasing the rate of contact the susceptible people initially enhances however after a while it decreases. The reason for this is that by increasing rate of contact more individuals get infected and shifts to infectious category hence the quantity of persons in susceptible category reduces. Figure 7B shows that rise in rate of contact leads to enhance of the infected people. It can be concluded the novel corona virus has a very high transfer rate from one individual to other persons via communal meetings with cough or sneeze containing little droplets. Also, the increase in recovered persons is seen in Figure 7C as the contact rate increases.

The effect of recovery rate on susceptible, infective and recovered mechanism are displayed in Figure 8A–C. In Figure 8A, the enhancing trend of susceptible individuals by increasing rate of recovery is obvious. From Figure 8B, a slow rise in infectious category can be seen by increasing the recovery rate. It can be realized that by reducing the recovery rate the recovered persons form COVID-19 decrease, therefore many of the infected individuals will be shifted to death category. Also, as shown in Figure 8C, it is obvious that the number of recovered people increases with increasing the recovery rate.

Figure 9 exhibits the susceptible, infective and recovered profiles under influence of death rate. The reduction in susceptible profile by increasing the death rate is seen Figure 9A obviously. Figure 9B demonstrates the variation of infected people with different rate of death. It can be seen that by increasing the rate of death, the infected people reduces because individuals from infectious category shifts to the category of death. The effect of various death rate values on the number of recovered people is shown in Figure 9C. From this figure, it can be found that with increase in
death rate, the quantity of recovered individuals reduces. The reason for this, is that by rising rate of death, many of people from infectious and recovered categories dies.

5  |  CONCLUSION

In current study, two computational approaches called Adams-Bashforth and Milne-Simpson have been successfully employed to acquire the numerical solutions for SITR model of COVID-19. The effect of several effective parameters on overspread of novel coronavirus were evaluated which will be helpful to plan, monitor and prevent the prevalence of COVID-19 pandemic. The numerical outcomes showed that increasing the death rate can makes a decrease in infected profile. Furthermore, as a very important result, it can be realized that applying social distance and reducing the contact rate between people reduces the infected persons and ultimately reduces death.

ORCID
S. E. Ghasemi https://orcid.org/0000-0003-4267-5041
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How to cite this article: Ghasemi SE, Gouran S. Evaluation of COVID-19 pandemic spreading using computational analysis on nonlinear SITR model. Math Meth Appl Sci. 2022;45(17):11104-11116. doi:10.1002/mma.8439