Study on the behavior of a light sphere rising in a square tube using the lattice Boltzmann method

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Abstract. In this study the motion of a light sphere under gravity in a square tube was numerically investigated through a three-dimensional lattice Boltzmann method. The width of the square tube is fixed at five times the diameter of the sphere. Simulations were carried out in the Reynolds number range of 140 to 200. Three types of behavior of the rising sphere were revealed, namely the steady state, periodic state and chaotic state. In particular, the sphere exhibits a spiral path in the periodic regime. The effects of the Reynolds number on the oscillations of the sphere were also examined.

1. Introduction

Gravity-driven flows containing solid particles are very common in natural world as well as in many industrial applications, such as fluidized bed and gravity separator. Understanding the bulk features of gravity-driven flows is important for both academic and engineering fields. For a particulate flow system, the interaction between particle and fluid determines the general features of flow as well as the pattern of particle motion. When a solid particle immersed in a fluid is released from rest under gravity, it may fall or rise according to the density ratio of solid to fluid. As is known, the motion of particle reaches a steady state at low Reynolds numbers. However, the loss of flow stability may occur as the effect of fluid inertia becomes strong, leading to the oscillatory motion of particle. The underlying mechanism can be explained as follows. The vortex shedding takes place due to the onset of flow in stability, which gives birth to oscillations in the lift and drag forces of particle. The interaction between solid particle and fluid can be very complex when the fluid inertia is significant. This deserves more attention.

There are extensive studies on the gravity-driven motion of particles in two dimensions. For instance, Aidun and Ding [1] used the lattice Boltzmann method to simulate the motion and interaction of two circular particles under gravity in a narrow channel. They reported a variety of patterns of particle motion (i.e., periodic state, chaotic state, and periodic-doubling bifurcation) at low but finite Reynolds numbers. By extending the range of Reynolds number, Verjus et al. [2] revealed new features of the same sedimentation system and established a link between the terminal Reynolds number and the non-dimensional driving force using a global diagram that illustrates the dynamic features of particles in a direct way. Zhang et al. [3] reported two distinct symmetry-breaking phenomena for the same problem, that is, an abrupt lateral migration that gives rise to asymmetrical movement centers and a divergent oscillation that leads to an asymmetric oscillatory motion of particles with zero phase lag. Recently, Nie and Lin [4] simulated the sedimentation of two particles with different densities in a vertical channel and showed that a discontinuous change in the settling velocity of particles may occur under certain conditions.
In comparison with two-dimensional analysis, studies on the motion of particles at non-zero Reynolds numbers in three dimensions area. Much effort was limited to the Stokes flow. Some problems associated with the flow instability have not been investigated satisfactorily in spite of the importance. In particular, little attention has been paid to the study of gravity-driven flows with freely rising particles. As shown by Jenny et al. [5], light spheres (i.e. the density ratio of solid to fluid less than one) may exhibit zigzagging periodic motion under certain conditions, which, however, is not the case for heavy spheres. Moreover, both light and heavy spheres present an oblique and oscillating state, which is characterized by different frequencies according to the density ratio. In order to fundamentally understand the features of gravity-driven flows with rising particles, it is necessary to pay more attention to the interaction between light particles and fluid. This motivates the present work. As the first step of the study, a three-dimensional lattice Boltzmann method was adopted here to simulate the gravity-driven motion of a light sphere in a square tube. Note that the present study is different from that of Jenny et al. [5] in the sense that the wall effects were taken into account here. This might bring different results.

2. Method
The motion of fluid was solved through a three-dimensional lattice Boltzmann method in this work. The discrete lattice Boltzmann equations of a single-relaxation-time model are expressed as [6],

\[ f_i(x + e_i \Delta t, t + \Delta t) - f_i(x, t) = -\frac{1}{\tau} \left[ f_i(x, t) - f_i^{(eq)}(x, t) \right] \tag{1} \]

where \( f_i(x,t) \) is the distribution function for the microscopic velocity \( e_i \), \( f_i^{(eq)}(x,t) \) is the corresponding equilibrium distribution function, \( \Delta t \) is the time step of the simulation, \( \tau \) is the relaxation time associated with the fluid viscosity, and \( w_i \) are weights related to the lattice model. The fluid density \( \rho \) and velocity \( \mathbf{u} \) are obtained through the following formulations,

\[ \rho = \sum_i f_i, \quad \rho \mathbf{u} = \sum_i f_i e_i \tag{2} \]

For the D3Q19 (i.e. nineteen velocities in three dimensions) lattice model used here, the discrete velocity vectors are,

\[ e_i = \begin{cases} 
(0,0,0) & i = 0 \\
(\pm 1,0,0) & i = 1 \sim 6 \\
(0,\pm 1,0) & i = 7 \sim 12 \\
(\pm 1,\pm 1,0) & i = 13 \sim 18 \\
(0,0,\pm 1) & i = 19 
\end{cases} \tag{3} \]

where \( c = \Delta x / \Delta t \), and \( \Delta x \) is the lattice spacing. The speed of sound is determined through \( c_s = c / \sqrt{3} \). Following Qian [6], the equilibrium distribution function is chosen as,

\[ f_i^{(eq)}(x,t) = w_i \rho \left[ 1 + \frac{3 e_i \cdot u}{e^2} + \frac{9 (e_i \cdot u)^2}{2 e^4} - \frac{3 u^2}{2 e^2} \right] \tag{4} \]

where the weights are set to be \( w_0 = 1/3, w_{1-6} = 1/18 \) and \( w_{7-18} = 1/36 \).

By performing a Chapman–Enskog expansion, the macroscopic mass and momentum equations in the low-Mach-number limit can be recovered:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{5} \]

\[ \frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{uu}) = -\nabla p + \mu \nabla^2 \mathbf{u} \tag{6} \]
In this method, the fluid viscosity is computed using \( \nu = c_s^2 \Delta t (\tau - 0.5) \). For simplicity, both the lattice spacing and time step is set to be 1 in the simulations, i.e. \( \Delta x = \Delta t = 1 \), unless otherwise stated.

In LBM the boundary conditions need to be handled in a special way. For example, the simple bounce-back scheme was usually adopted to satisfy the non-slip conditions of solid surfaces. Based on the interpolation method, Lallemand and Luo [7] proposed an improved bounce-back scheme which has a higher order of accuracy as compared with the simple one and is suitable for the moving boundaries. Therefore, the non-slip conditions of the surface of moving sphere were treated by the improved bounce-back scheme in the present simulations. The momentum-exchange method [7] was used to calculate the force and torque exerted on the sphere resulting from the motion of fluid. In addition, some fluid nodes may become solid nodes and some solid nodes may become fluid nodes due to the motion of sphere. To account for the effects of covered or uncovered fluid nodes, the method developed by Aidun et al. [8] was used here to compute the added force and torque exerted on the sphere. The motion of sphere is then determined by solving the Newton’s equations.

3. Problem
The purpose of this work is to investigate the behavior of a light sphere immersed in fluid which rises in a square tube due to the action of gravity. The schematic diagram of the problem is shown in Figure 1. The square tube with dimensions of \( L \times L \times H \) is filled with a fluid of density \( \rho \) and viscosity \( \nu \). A sphere of density \( \rho_s (\rho_s < \rho) \) and diameter \( d \) is immersed in the fluid. The positions of the sphere are denoted as \( X \) (horizontal), \( Y \) (transversal) and \( Z \) (vertical).

![Figure 1. Physical model and notations of the present problem.](image)

Because the terminal velocity of the sphere is unknown, the velocity scale of the present system is taken as,

\[
U = \sqrt{\left( \frac{\rho_s}{\rho} - 1 \right) g d}
\]

where \( g \) is the gravitational acceleration. The time scale is chosen as \( T = d/U \). The Reynolds number, defined as \( Re = U d/\nu \), is used to adjust the effect of fluid inertia in this work. In addition, a moving computational domain is employed to simulate an infinite tube. The upstream (top) boundary is \( 12d \) from the sphere, whereas the downstream (bottom) boundary is \( 25d \) from it (i.e., the overall tube
height is $H = 37d$). The normal derivative of the velocity is zero at the downstream boundary, and the velocity at the upstream boundary is zero.

In the simulations, some parameters are fixed as follows: $d = 32\rho_s = 0.1, \rho = 1, L = 5d$ and $H = 37d$. For all cases the sphere is released from rest at a position of $(-d, 0, 25d)$.

4. Results
The same computational code has been validated in our previous work [9]. For simplicity, the validation is not presented in this work. The focus is on the motion of a light sphere in a square tube depending on the Reynolds number.

![Graph](image)

**Figure 2.** Time evolution of the horizontal and transversal positions of the sphere at $Re = 140$. Note that $t' = t/T, X' = X/d$ and $Y' = Y/d$. The same as below.

Figure 2 shows the time history of the horizontal ($X'$) and transversal ($Y'$) positions of the sphere at $Re = 140$. Note that the time is normalized through $t' = t/T$ and the positions are normalized through $X' = X/d$ and $Y' = Y/d$. As shown in Figure 2, the value of $X'$ reaches zero quickly and remains unchanged for $t' > 150$, indicating that the sphere moves to the tube axis eventually at low $Re$. In order to provide a better understanding of the motion, Figure 3-a and b presents the front view and top view of the velocity field around the sphere at steady state, respectively. It is clearly seen that two recirculation zones are generated on both side of the sphere (Figure 3-a). In addition, the narrow wake behind the sphere is clearly characterized by a strong upward stream. As compared with Figure 3-a, the fluid velocity shown in the top view is much smaller (Figure 3-b). Note that the reference velocity vector is the same for both Figure 3-a and b.

![Images](image)

**Figure 3.** Two-dimensional flow fields for the rising sphere at steady state at $Re = 140$: (a) front view and (b) top view.

As the Reynolds number increases, the motion of the sphere exhibits a periodic state which results from the onset of flow in stability. As seen in Figure 4, both horizontal and transversal positions of the sphere at $Re = 160$ show periodic oscillations after the initial transients die down (i.e. $t' > 300$). It is
also seen that $X'$ have the same amplitude and frequency of oscillations as $Y'$. In particular, the value of $X'$ becomes zero when the value of $Y'$ reaches its maximum or minimum. The reverse is also true. This indicates that the phase difference between $X'$ and $Y'$ is exactly 90 degree. Given that the sphere is rising in the tube because of $\rho_s/\rho < 1$, Figure 4 suggests a spiral trajectory of the sphere at $Re = 160$. Obviously, this spiral trajectory is around the tube axis.

**Figure 4.** Time history of the horizontal ($X'$) and transversal ($Y'$) positions of the sphere at $Re = 160$. The normalized time period is $t_p' = t_p/T = 78.9$.

Accordingly, Figure 5 shows the instantaneous velocity fields around the sphere (top view) at four representative positions at the same $Re$ as Figure 4. Note that the fluid velocity in the vicinity of the sphere is significantly larger than elsewhere. In addition, Figure 4 clearly indicates that the sphere exhibits a spiral path when it moves under gravity in the tube.

**Figure 5.** Instantaneous flow fields (top view) for the rising sphere at different times during one period at $Re = 160$.

Figure 6 shows the time history of the horizontal and transversal positions of the sphere at $Re = 180$, which also presents a spiral trajectory. However, in comparison with Figure 4, the amplitude of the oscillations of $X'$ or $Y'$ is nearly twice that of $Re = 160$. Furthermore, the time period seen at $Re = 180$
is smaller than that at Re = 160. This indicates that the sphere oscillates more strongly and more quickly in the tube as Re increases.

However, further increasing the Reynolds number may result in a completely different behavior of the motion, as shown in Figure 7. Instead of periodic oscillations, both horizontal and transversal positions of the sphere present chaotic oscillations at Re = 200, suggesting that a chaotic regime is reached at a Reynolds number between 180 and 200. In addition, the oscillations of $Y'$ begins to dominate the motion of the sphere when $t' > 700$, as seen in Figure 7. This suggests that the sphere does not exhibit a spiral path at Re = 200.

![Figure 6](image)

*Figure 6.* Time history of the horizontal ($X'$) and transversal ($Y'$) positions of the sphere at Re = 180. The normalized time period is $t_p' = 70.1$.

![Figure 7](image)

*Figure 7.* Time history of the horizontal and transversal positions of the sphere at Re = 200.

5. Conclusion
In this work a three-dimensional lattice Boltzmann method was used to simulate the gravity-driven motion of a light sphere in a $L=5d$ square tube. Much attention was paid to the effects of the Reynolds number on the behavior of the rising sphere. It has been shown that a Hopf bifurcation takes place at a Reynolds number between 140 and 160. Due to the loss of flow stability, the sphere exhibits a spiral trajectory when it is rising in the tube. The oscillations of the sphere are seen to be stronger and faster at a higher Re in the periodic regime. Moreover, results show that a chaotic regime is reached at a Reynolds number between 180 and 200.

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