Identifying the optimum water flow and number of buckets in laboratory-scale Pelton turbine using Central Composite Design (CCD)

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Abstract. Pelton turbines were a type of water turbines which, principally, turned the water kinetic energy into electrical energy. The working principle of Pelton turbines was to utilize water power to produce turbine power. This paper reported the optimization of design parameters for laboratory-scale Pelton turbine using the central composite design (CCD) as the response surface methodology (RSM). This research obtained the quadratic polynomial equations for the turbine power. The independent variables were water flow (0.11 m³/s; 0.13 m³/s; 0.15 m³/s) and number of buckets (13; 15; 17 pieces). The results, based on the RSM, showed that the most influential parameter on the turbine power was the number of buckets. The optimum turbine power was 24 kW on 0.16 m³/s water flow and 18 buckets of RSM.

1. Introduction
To date, energy develops rapidly, both in developed and developing countries. Energy is a vital necessity for socio-economic continuity and prosperity; with electrical energy as the most used energy. Electrical energy does not pollute the environment and it converts into other forms of energy. Currently, the usage of Steam Power Plant and Diesel Power Plant to generate electricity that burn fossil fuels such as coal and petroleum created a problem of non-renewable energy source depletion and pollution as a result of burning coal and petroleum. To resolve this obstacle, there needs a new renewable energy source [1]. Water is one of the renewable energies with great potential as an energy source because it does not create pollutant. Utilizing water energy as an electricity source is beneficial, particularly in an area with no electricity but has large water resources. Therefore, there was a development of the Micro Hydro Power (MHP) Plant. MHP Plant is a small-scale hydropower plant that requires small power (10-150 kW), suitable for the Pelton turbine [2].

Lester A. Pelton, an American engineer, found the first Pelton turbine in 1880. Pelton turbine is an impulse turbine but unlikely developed widely for rural scale, especially area with hydropower potential such as large falling water. This turbine operates on the head up to 1800 m, relatively requires less amount of water. Pelton turbine has a basin blade, allowing impulses from the water flow to turn into a force which ultimately provides for the torque that rotates the turbine wheel. High water speed created a high impulse that requires a nozzle. Thus, the turbine blade must be able to withstand the force that occurred due to momentum changes of the water jet from the nozzle [3]. (Agar and Rasi, 2008) [4] conducted a research using laboratory-scale Pelton turbine with various water flow (0.14; 0.17; 0.20 l/s)
and 14 buckets. The results stated a mechanical efficiency of 47% created by the turbine with 0.171 ℓ/s water flow. Visibly, the laboratory-scale Pelton turbine demonstrated the hydropower principle well, and thus, it is suitable as a prop in the renewable energy field [5].

The various design parameters of the turbine determine the quantity and quality. This research used the RSM method to find which parameter contributes the most to the performance of the Pelton turbine. The RSM method is an important method in designing, formulating, developing, and analyzing a scientific study [6]. This application is customary in the engineering field to obtain the optimum value of a pre-determined parameter that resulted in an optimal response. This research used the varieties of bucket and water flow to find the turbine performance, and RSM to obtain the optimum design parameter for the best performance.

2. Methods

2.1. Experimental Apparatus

Figure 1 displays the installation test of the Pelton turbine. In the installation, there was a 103.5 cm³ draft tube to hold the water that flew to and from the turbine. The Pelton turbine in this research used a centrifugal pump with an AC motor as the activator to move or drain the water from the tube to the turbine and put pressure on the water. The pump in this research was from NOCCHI brand in DHR 44M type with 980 W power and 50 Hz frequency. There were three valves in the installation to control the water flow in the turbine system. Additionally, there were two nozzles with 8 mm diameter each at the top and right side of the turbine blade to direct the flow to the blade. The diameter and length of the shaft were 30 and 370 mm, respectively. There was a pressure tank and flow divider in the installation to stabilize the water pressure and dividing pressurized water flow from the pump to the nozzle. The pipe diameter was 1.5 inch or 48 mm that functioned to channel the water in the turbine. This research used double hemispherical cup blade in 45 mm width, 44 mm height, and 10 mm depth. Figure 2 shows the shape of the bucket and nozzle from the Pelton turbine.

![Figure 1. Laboratory-scale Pelton turbine testing installation](image)

The Pelton turbine is a turbine with a high head and able to convert the kinetic energy of pressurized water into mechanical energy in the form of shaft rotation. The blade mass, blade distance, nozzle diameter, and water discharge are factors that affect the magnitude of shaft rotation in the turbine [4-5]. The factors in this research were the number of blades (13; 15; 17 pieces) and water flow (0.11; 0.13; 0.15 ℓ/s). This research aimed to find, quantitatively, factors that influence the output power created by the Pelton turbine shaft [18,19]. The regression analysis produced predictive equations and complete images to find out how the two factors contribute to the output power and the interactions between the two factors. The predictive equation was useful to generate a design parameter of the Pelton turbine that created an optimum power.
2.2. Performance Analysis

The performance and stability of a turbine depend on the water flow (Q), head (H), and torque in the axis. Impulse turbine is a turbine suitable for low water flow and high head, with the example of a Pelton turbine. While the reaction turbine is a turbine that works well with high water flow and low head. This research used Equation 1 [9] to calculate the water jet power from the nozzle.

\[ P_w = \rho g H Q \text{ (watt)} \]  

\( P_w \) is water jet power, \( \rho \) is water density, \( g \) is gravitational acceleration \((9.81 \text{ m/s}^2)\), \( H \) is head, and \( Q \) is water flow. The nozzle jet has the velocity \((v_j)\) calculated from water flow and total area \((A_j)\) of the nozzle hole that has a diameter \((D_j)\), displayed by Equation 2.

\[ v_j = \frac{Q}{A_j} = \frac{4Q}{\pi D_j^2} \text{ (m/s)} \]  

Equation 3 calculated the mechanical power generated by the turbine axis influenced by angular velocity \((\omega)\) and torque in the axis \((\tau)\). Obtaining the torque required multiplication between braking force \((F)\) and moment arm length \((l)\) at turbine rotational speed \((r)\).

\[ P_s = \omega \tau = 2\pi rlF \text{ (watt)} \]  

Angular velocity \((\omega)\) is tangential velocity of turbine runner \((v_t)\) divided by pitch radius \((L)\) in the Pelton turbine, calculated using Equation 4.

\[ \omega = \frac{v_t}{L} \text{ (rad/s)} \]  

This research used the mechanical efficiency of the Pelton turbine to measure how effective the kinetic power created by water jet nozzle that turned into motion in the turbine. Equation 5 shows the formulation of mechanical efficiency by dividing the axis power \((P_a)\) and water jet nozzle power \((P_w)\).

\[ \eta_m = \frac{P_a}{P_w} \times 100\% = \frac{2\pi rlF}{\rho g H Q} \times 100\% \]  

This research aimed to obtain a new method in optimizing the design parameter from the Pelton turbine, such as the number of buckets and water flow to get the best turbine efficiency. The response surface methodology (RSM) is an optimization method to solve the optimization problem in this research.

2.3. Response Surface Methodology

The response surface methodology is a statistic and mathematic method in empirical modeling to observe the independent variable influence (input variables, that were water flow and total blades) towards response variable (output variable, that was axis power) and to optimize the response variable. To obtain the optimized value, this research used the second-order model, the central composite design
(CCD). Equation 6 presents the second-order model of CCD, where $x$ is independent variable, $y$ is response variable and $\beta$ is constant [10].

$$\hat{y} = \beta_0 + \sum_{i=0}^{k} \beta_i x_i + \sum_{i=0}^{k} \beta_i x_i^2 + \sum_{i=0}^{k} \sum_{j=0}^{k} \beta_{ij} x_i x_j, i < j$$  \hspace{1cm} (6)

$$\hat{y} = b_0 + b_1 x_{11} + b_2 x_{12} + b_3 x_{11}^2 + b_4 x_{12}^2 + b_5 x_{11} x_{12}$$  \hspace{1cm} (7)

This research used CDC model with two factors and three levels that generated the regression equation 7 where $x_1$ is water flow, $x_2$ is the total blades, $i$ indicates the repeated number, and $b$ is a coefficient from the test results. The independent variable codes were -1,0,1, and ±1.412. The value of ±1.412 was the result of the rotatability value $2^{-1/4} = (2^2)^{-1/4} = 1.412$, where $k$ is the total independent variable. Table 1 shows the independent variable code of $x_1$ and $x_2$. Table 2 displays the design of experiment (DOE) and the results. The DOE results were two turbine power, actual and predicted, in which the experiment resulted in the actual data, and the regression equation resulted in the predicted result [11].

| Independent Variable | Name            | Units | Code        | -1 | 0  | 1  | -α (-1.414) | +α (+1.414) |
|----------------------|-----------------|-------|-------------|----|----|----|-------------|-------------|
| $x_1$                | Water Flow      | m$^3$/s | 0.11 | 0.13 | 0.15 | 0.10 | 0.16       |             |
| $x_2$                | Number of Buckets | pieces | 13 | 15 | 17 | 12 | 18         |             |

| Run | $x_1$ | $x_2$ | Water Flow | Number of Buckets | Turbine Power |
|-----|-------|-------|------------|------------------|--------------|
|     |       |       | Actual     | Predicted        |              |
| 1   | -1    | -1    | 0.11       | 13               | 9123.6       |
| 2   | 1     | -1    | 0.15       | 13               | 8616.8       |
| 3   | -1    | 1     | 0.11       | 17               | 20735.7      |
| 4   | 1     | 1     | 0.15       | 17               | 22118        |
| 5   | -1.414 | 0    | 0.10       | 15               | 14077.2      |
| 6   | 1.414 | 0     | 0.16       | 15               | 14976        |
| 7   | 0     | -1.414 | 0.13   | 12               | 9123.6       |
| 8   | 0     | 1.414 | 0.13       | 18               | 20736        |
| 9   | 0     | 0     | 0.13       | 15               | 14377        |
| 10  | 0     | 0     | 0.13       | 15               | 14377.3      |
| 11  | 0     | 0     | 0.13       | 15               | 14376.5      |
| 12  | 0     | 0     | 0.13       | 15               | 14376.2      |
| 13  | 0     | 0     | 0.13       | 15               | 14376.8      |

### 3. Results and Discussion

The ANOVA table, generated from the 13 tests, shows the influence of independent variables on the response variable. Significance level or fault tolerance was 5% ($\alpha=5\%$) with the correct level of 95%. The ANOVA table on Table 3 presents that the biggest independent variable contribution on influencing the response variable was at 94.5% number of blades, followed by the contribution of the interaction of the number of blades*the number of blades with 0.4%, the contribution of the interaction of the water flow*the number of blades with 0.4%, the contribution of water flow with 0.3%, the contribution of water flow*water flow with 0.1%, and an error rate of 4.3%. The $P$-value with less than 5% is the water flow and interaction between water flow*water flow with 0.000. Since $P$-value 0.000<0.050, then the independent variable of water flow significantly influences the response variable that was turbine axis. Table 3 displays the regression value (R2) that is 95.7%, and because the value is closer to 100%, it means the independent variable greatly influence the response variable. Meanwhile, the remaining 4.3% were the other influencers other than the independent variables.
Table 3. Analysis of Variance (ANOVA)

| Source                        | DF | Adj SS       | Adj MS       | F-Value | P-Value | Contribution |
|-------------------------------|----|--------------|--------------|---------|---------|--------------|
| Model                         | 5  | 218253597    | 43650719     | 31.13   | 0.000   |              |
| Linear                        | 2  | 216228468    | 108114234    | 77.11   | 0.000   |              |
| Water Flow \( (x_1) \)       | 1  | 576011       | 576011       | 0.41    | 0.542   | 0.3 %        |
| Number of Bucket \( (x_2) \) | 1  | 215652457    | 215652457    | 153.81  | 0.000   | 94.5 %       |
| Square                        | 2  | 1133002      | 566501       | 0.4     | 0.682   |              |
| Water Flow*Water Flow \( (x_1 \times x_3) \) | 1  | 225387       | 225387       | 0.16    | 0.000   | 0.1 %        |
| Number of Bucket*Number of Bucket \( (x_2 \times x_3) \) | 1  | 1012989      | 1012989      | 0.72    | 0.423   | 0.4 %        |
| 2-Way Interaction             | 1  | 892127       | 892127       | 0.64    | 0.451   |              |
| Water Flow*Number of Bucket \( (x_1 \times x_3) \) | 1  | 892127       | 892127       | 0.64    | 0.451   | 0.4 %        |
| Error                         | 7  | 9814430      | 1402061      | 4.3     | 0.423   |              |
| Total                         | 12 | 228068027    |              | 100     | 100     |              |

\[ R^2 = 95.7\% \]

\[ Y = 25785-280681(X_1)-1801(X_2)+449995(X_1)^2 + 95(X_2)^2 +11807(X_1)(X_2) \]  

(8)

Based on the above analysis, this research obtained model as equation 8. Equation 8 shows the second-order regression equation from the conducted experiment. Table 3 displays the P-value from the equation model, lesser than 5% significance level. It means that the independent variables meaningfully contributed to the model. Figure 3 presents the data comparison, from the experiment result and from the second-order regression model. The figure indicates that the gap/distance between the actual and the predicted on run 1–12 was close. The closer the gap between actual and predicted graphics means high error level. Therefore, the second-order equation is suitable to predict the output power from the Pelton turbine even if there is a change of value in the independent variable, without a need for data retrieval through experiment.

Another way to check the model suitability is by using residual analysis in Figure 4. In Figure 4(a), the plots in the Versus Order graph do not form a certain pattern, indicating that, visually, the experimental results above are independent. In principle, the distribution of the plots in Figure 4(b) is a sign of normality. Figure 4(b) shows that if the data spread around the diagonal line and following the diagonal direction, then, the regression model fulfils the normality assumption. Figure 4(c) is a histogram graph to show the normality of data distribution by observing the residual. The graph also displays a normal distribution pattern, similar to Figure 4(b), with plots distribution on the verge of the diagonal line. Figure 4(d) shows that in the Versus Fits graph, the plots do not form a certain pattern and several residual plots randomly spread around the number zero, indicating that, visually, the data resulting from the experiment is homogenous.

Figure 3. Data comparison between actual and predicted
whereas the water flow gave no significance to the output power. The dark red color contour, additional buckets influenced the response variable. Meanwhile, increasing the water flow in the same bucket did not give in a significant increase in the response variable. The dark red contour indicates the lowest output power, less than 10 kW, with less than 13 buckets. O

Figure 5.

Figure 4. Residual Plots of Turbine Power (a) Versus Order (b) Normal Probability Plot (c) Histogram and (d) Versus Fit

Figure 5(a) shows the color contour plot as the effect of the number of buckets and water flow (independence variables) influenced the Pelton turbine axis output power (response variable). Black contour indicates the lowest output power, less than 10 kW, with less than 13 buckets. Observed from the color contour, additional buckets influenced the response variable. Meanwhile, increasing the water flow in the same bucket did not give in a significant increase in the response variable. The dark red contour in Figure 5(a) shows that 17 buckets and water flow of fewer than 0.13 m³/s created an output power of more than 22.5 kW. Figure 5(b) displays the surface plot in three dimensions that form the optimum peak. The most optimized power, as seen in Figure 5(b), was in the sample with 17 buckets whereas the water flow gave no significance to the output power.

Figure 5. (a) Response Contour Plot of Turbine Power (b) Response surface plot of Turbine Power
Figure 6 shows the optimized parameters (independent variables) to achieve a maximum response variable. It presents that the optimum independent variable of 0.1583 (0.16 m$^3$/s) water flow and 17.8284 (18) buckets generated the output of 2.417x10$^4$ (24 kW). From there, in conclusion, increasing the water flow and the number of buckets generated larger output. Figure 7(a) displays the interaction plot graph and shows that 17 buckets are more effective than 13 or 15 buckets. Besides, there is no interaction between the three total buckets, shown by no intersected line between them. Figure 7(b) indicates that the number of buckets significantly influence the turbine output. More buckets mean larger output power, whereas increasing the water flow only increasing little output power.

**Figure 6.** The optimized response between the number of buckets and water flow

| Optimal | W. Flow | Bucket |
|---------|---------|--------|
| D: 1.000 | 0.1583 | 17.3084 |
| Predict | [0.158] | 17.3084 |

- Figure 7. (a) Interaction Plot of Turbine Power (b) Main Effect Plot of Turbine Power

4. **Conclusion**

Based on the ANOVA analysis and regression in response surface methodology, independent variables significantly influence the output of the Pelton turbine, shown with the regression (R$^2$) value of 95.7% with the remaining 4.3% influenced by others. The number of buckets as the independent variable gave a great contribution of 94.5%. The optimization of response surface methodology showed that the most optimum parameters were 18 buckets and 0.16 m$^3$/s water flow that generated 24 kW turbine power.

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