BENEFITS OF GROUND-BASED PHOTOMETRIC FOLLOW-UP FOR TRANSITING EXTRASOLAR PLANETS DISCOVERED WITH KEPLER AND COROT

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ABSTRACT
Currently, over 40 transiting planets have been discovered by ground-based photometric surveys, and space-based missions such as Kepler and CoRoT are expected to detect hundreds more. Follow-up photometric observations from the ground will play an important role in constraining both orbital and physical parameters for newly discovered planets, especially those with small radii ($R_p \sim 4 R_\oplus$) and/or intermediate-to-long orbital periods ($P \gtrsim 30$ days). Here, we simulate transit light curves (LCs) from Kepler-like photometry and ground-based observations in the near-infrared (NIR) to determine how jointly modeling space-based and ground-based LCs can improve measurements of the transit duration and planet–star radius ratio. We find that adding observations of at least one ground-based transit to space-based observations can significantly improve the accuracy for measuring the transit duration and planet–star radius ratio of small planets ($R_p \lesssim 4 R_\oplus$) in long-period ($\sim 1$ year) orbits, largely thanks to the reduced effect of limb darkening in the NIR. We also demonstrate that multiple ground-based observations are needed to gain a substantial improvement in the measurement accuracy for small planets with short orbital periods ($\sim 3$ days). Finally, we consider the role that higher ground-based precisions will play in constraining parameter measurements for typical Kepler targets. Our results can help inform the priorities of transit follow-up programs (including both primary and secondary transit of planets discovered with Kepler and CoRoT), leading to improved constraints for transit durations, planet sizes, and orbital eccentricities.

Key words: planetary systems – planets and satellites: general – techniques: photometric

1. INTRODUCTION
Over the past decade, ground-based photometry has discovered over 40 planets that transit their host star. A majority of these known transiting planets have radii and masses that are comparable to Jupiter, and most have orbital periods that are less than 5 days. Currently, space-based planet searches (e.g., CoRoT and Kepler) are poised to extend the reach of the transit method to small planets ($R_p \lesssim 4 R_\oplus$) and longer orbital periods ($P \gtrsim 30$ days). These systems present exciting opportunities for follow-up observations to characterize the physical properties of potentially rocky planets. As the number of known transiting planets grows, it will become increasingly important to make the best possible use of follow-up resources (Clarkson et al. 2007; O’Donovan et al. 2007; Pont et al. 2007; Latham et al. 2008). Indeed, merely confirming the planet candidates from CoRoT and Kepler will present a significant challenge, as the target stars will be fainter, the planets will be smaller, and the orbital periods will be longer than for the targets discovered in ground-based surveys. High-resolution imaging will be important for rejecting blends with background objects or wide binary companions. Moderate-resolution optical and infrared (IR) spectroscopy will be essential for rejecting spectroscopic binaries and hierarchical systems. Finally, at least several high-precision Doppler observations will be needed to confirm planet candidates and to measure planet masses and orbits (Gautier et al. 2007; Latham 2007; Brown & Latham 2008).

There will continue to be an important role for follow-up photometric observations. For ground-based transit searches, follow-up observations have routinely obtained higher quality photometry, provided more precise measurements of planet and orbital parameters, and served as the basis for transit timing variations. In this paper, we focus on yet another potential role for follow-up photometric observations. Due to an approximate degeneracy between the stellar limb darkening (LD) parameters and the impact parameter (the minimum distance from the center of the planet to the center of the stars when projected onto the sky plane; e.g., see also Pál 2008), white light observations provide only weak constraints on the impact parameter. The uncertainty in the impact parameter propagates to interfere with the measurement of the planet–star radius ratio and transit duration (Knutson et al. 2007). An accurate measurement of the radius ratio is critical for establishing the size, density, and composition of transiting planets. A precise measurement of the transit duration can constrain the stellar density (and hence the stellar size) for planets on circular orbits (e.g., Torres et al. 2008) and/or help to characterize the orbital eccentricity (e.g., Ford et al. 2008; Bakos et al. 2009). Thus, the near degeneracy between the impact parameter and stellar LD model will affect all Kepler observations (and early CoRoT discoveries) that use a single, broad, white broadband filter, which can be severely affected by stellar LD. Fortunately, ground-based follow-up photometry is routinely performed at near-infrared (NIR) wavelengths which are only minimally impacted by LD (e.g., observing with a Sloan i’ or z’ filter; see Holman et al. 2006, for example, as well as Section 3.2 in this paper). Thus, NIR photometry (or space-based IR photometry; e.g., Nutzman et al. 2009) can enable a more precise measurement of the impact parameter, planet–star radius ratio, and transit duration (as shown in Section 3). Ground-based follow-up photometry may offer additional advantages for very high cadence observations to search for transit timing variations (Agol et al. 2005; Holman & Murray 2005; Ford & Holman 2007) and/or changes in the transit duration or shape that could be caused due to moons, rings, or tidal bulge (Kipping 2009; Ragozzine & Wolf 2009).

Our investigation of the potential contributions of ground-based transit follow-up observations is further motivated by...
the improving precision of ground-based photometry for transit follow-up applications. Once the host star and transit times are known, nearly routine ground-based transit follow-up observations can provide photometry with a precision of $\sim 1$ millimagnitude (mmag) at a rate of $\sim 1$ observation per minute for 11th and 12th magnitude stars, even with a relatively small observatory (e.g., 1.2 m telescope at Fred L. Whipple Observatory (FLWO); Holman et al. 2006; Winn et al. 2007). Recent observations with larger telescopes have achieved even higher photometric precisions for favorable planet–host stars, for example, 0.39 mmag minute$^{-1}$ in the $z'$ band for a $V = 12$ star with the 2.2 m University of Hawaii (UH) telescope (rescaled from the quoted precision for a $V = 12.7$ star; Johnson et al. 2009) and 0.315 mmag minute$^{-1}$ in the $z$ band for a $V = 12$ star with the 6.5 m Magellan telescope (rescaled from the quoted precision for a $V = 12.5$ star; Winn et al. 2009). It is particularly noteworthy that differential photometry largely corrected for atmospheric effects, as demonstrated by high-precision ground-based photometry that has reached a precision very close to that of the theoretical limit (e.g., Winn et al. 2009). This raises the possibility that modern instrumentation at a large observatory might soon be able to routinely achieve a photometric precision comparable to that of CoRoT or Kepler (Gillon et al. 2008).

With the Kepler mission’s recent launch (2009 March) and CoRoT’s discovery of $\sim 6$ new transiting planets to date, the future looks promising for finding many transiting planets with a range of sizes and orbital periods. It will be necessary to use all of the tools at hand in order to describe the overall population of extrasolar planets in terms of both orbital and physical parameters. Combining ground-based photometric observations with space-based transiting planet searches is a valuable tool, as astronomers push to constrain properties of potentially habitable Earth-like planets.

This paper explores the potential benefits of adding ground-based NIR photometry to higher-precision transit photometry measurements in white light soon to be made by Kepler. In Section 2.1, we describe how we generate simulated transit light curves (LCs) and describe our analysis procedure for fitting a transit model to each LC in Section 2.2. In Section 3, we report the results in terms of the precision with which physical and orbital parameters (e.g., radius ratio, planet velocity projected onto the plane of the sky) can be measured. Section 3.1 describes the results from Kepler-like (ground-based) observations alone. We report results for combining Kepler-like and ground-based observations in Section 3.3. In Sections 3.4 and 3.5, we consider the potential of higher-precision ground-based observations and the impact of a fainter target star. Finally, we summarize our results and discuss the implications for future ground-based follow-up campaigns in Section 4.

2. METHODS

In Section 2.1 we describe our model for generating simulated transit LCs, intended to mimic Kepler-like photometry and ground-based follow-up observations (in the NIR; see also Section 3.2). In Section 2.2, we discuss our analysis of the LCs, including the fitting process and how we quantify measurement uncertainties.

2.1. Simulations of Transiting Planet Light Curves

We generated mock transit LCs for Kepler-like and ground-based observations, using the code from Mandel & Agol (2002) to incorporate realistic LD. The projected separation from the center of the planet to the center of the star ($z$) is calculated as

$$z = \sqrt{b^2 + v^2(t - t_0)^2},$$

where $b$ is the impact parameter in units of stellar radii, $v$ is the planet velocity projected onto the plane of the sky and $t_0$ is the time of mid-transit. In this approximation, the radial star–planet separation is assumed to be constant over the transit duration. Since the transit duration is much shorter than the orbital period, this is an excellent approximation for all but highly eccentric orbits. Here, we define the total transit duration ($D$) as

$$D = \frac{2R_*\sqrt{(1 + p)^2 - b^2}}{v},$$

where $p$ is the planet–star radius ratio, defined as $p \equiv R_p/R_*$. We use the Claret (2000) quadratic LD law, which has the following form:

$$I(r) = 1 - u_1(1 - \sqrt{1 - (r/R_*)^2}) - u_2(1 - \sqrt{1 - (r/R_*)^2})^2/(1 - u_1/3 - u_2/6)/\pi,$$

as given in Mandel & Agol (2002). The variables $u_1$ and $u_2$ are the linear and quadratic LD coefficients.

For the purposes of simulating Kepler-like photometry, we consider a range of planet radii (from 0.5 to 12 $R_\oplus$) and orbital periods (from 1 day to 1 year) to generate the appropriate LCs. For our primary analysis, we consider planets on circular orbits, solar-type stars with stellar radii of one solar radius, and a representative value of 0.5 for the impact parameter. For the cases where we consider a near-central transit, we adopt an input impact parameter of 0.01. Finally, we use LD coefficients of 0.4382 and 0.2924 ($u_1$ and $u_2$) based on ATLAS models for a solar-type (G2V) star in the $V$ band (Claret 2000). We considered the same specifications for simulating ground-based photometry, but different LD coefficients were used (see Section 3.2). Note that we chose these specifications considering the fact that the Kepler mission is designed to search for Earth-size planets in the habitable zone. Given the mission lifetime and abundance of bright stars, most of the Kepler target stars are expected to range from late F stars to early K stars. The nominal visual magnitude range is 9–15. The mission specifications were chosen such that Kepler could detect an Earth-size planet in a 1 year orbit around a solar-type star with $V = 12$. If small planets are common, then there is a chance that Earth-size planets could be found around the relatively modest number of even brighter stars in the Kepler field.\(^4\)

Finally, when generating mock LCs for Kepler-like targets, we inject realistic measurement errors. We assume uncorrelated Gaussian noise and add this to each observation in each LC. For space-based observations, we assume a photometric precision of 0.4 mmag for each 1 minute integration, based on the expected precision from Kepler observations for a $V = 12$ star (Basri et al. 2005). Note that the typical integration time employed by Kepler\(^5\)

\(^4\) http://nspires.nasaprs.com/external/solicitations/summary.do?method=initch&solId=DB53C123-58C9-49FF-D74B-9E7A15DD98E1&path=open

\(^5\) http://www.colloquium.eu/congres/09COROT/docs/slides/03mardi/11h_d_rouan/alancer.pdf
will be 30 minutes, but for planet candidates, a fast cadence mode (i.e., 1 minute integration) will be used (Borucki et al. 2009). For ground-based observations, we consider multiple precisions: one relatively modest precision (1.7 mmag) which is routinely achievable with 1 m class telescopes and for which a $1/\sqrt{N}$ scaling has been demonstrated on timescales of up to $\sim 30$ minutes (Winn et al. 2007), and two relatively high precisions of 2 and 4 times better than the low precision (i.e., 0.85 and 0.425 mmag) for which it is not known if errors will scale as $1/\sqrt{N}$. We also assume uncorrelated Gaussian noise for all the ground-based precision cases. See Section 3.2 for a more detailed discussion of the generation of mock LCs for ground-based observations conducted in the NIR. Once the LCs were generated, we fitted and analyzed them using the process described in the following section (Section 2.2).

### 2.2. Analysis of Transiting Planet Light Curves

In order to analyze our simulated LCs, we employed the Levenberg–Marquardt least-squares method, which identifies a “best-fit” model for each simulated LC by minimizing $\chi^2$. We used the publicly available code *mpfitfun*\(^6\) for the fitting process, which allowed us to vary each of the following parameters that define the LCs: $b$, $v$, $t_0$, $c_1$, $c_2$, and $p$. Here, we have employed an alternative parameterization of LD where $c_1 \equiv u_1 + u_2$ and $c_2 \equiv u_1 - u_2$. In the fitting process, we restrict the values of the parameters for $b$, $c_1$, $c_2$, and $p$ to the intervals $[0,0.99]$, $[-1.1.5]$, $[-1.1.5]$, and $[0,1]$, respectively. In principle, one could adopt fixed values for the LD parameters based on other observations (stellar spectral type, temperature, metallicity, surface gravity) and stellar atmosphere models. However, this would result in underestimating the uncertainties for the transit parameters and potentially introduce biases if the spectroscopy and/or models are not accurate (Southworth 2008). For sufficiently large planets, we expect that the transit photometry will provide stronger constraints on the LD parameters, so that there is little benefit to incorporating assumptions about LD. For the smallest planets ($\sim 1 \, R_\oplus$), measuring even basic features of the transit LC will be a challenge, so it may be advantageous to constrain the LD parameters based on stellar properties and atmosphere models, at least for stars where the models have been well tested. In order to test the sensitivity of our conclusions to assumptions about the LD coefficients, we reanalyzed a subset of our simulated observations in three ways: (1) with both LD parameters fixed at their true values; (2) the quadratic coefficient fixed and the linear coefficient allowed to vary; and (3) both LD coefficients allowed to vary during the fitting process. We find that the same basic conclusions can be made for each case despite any systematic shifts in the numerical results. Therefore, we treat both the linear and quadratic LD coefficients as free parameters during the fitting of our transit LCs. See Section 3.3.2 for a further discussion of how fixing or varying LD terms affects the conclusions of this paper.

Once a “best fit” was found, the fitting routine returned two estimates of the uncertainties: (1) the formal 1σ error estimates obtained from the covariance matrix and (2) the bootstrap error estimates. Here, we have defined the bootstrap errors as the median absolute deviations (MADs; median $|\delta v|_{\text{fit,true}}$) between the best-fit values and the true parameter value used to generate the given LC. We report MAD rather than standard deviation because in some cases, a small fraction of the data sets result in LC parameters that differ from the input parameters by significantly more than is typical. The MAD is a more robust statistic that characterizes the typical deviation between the input and best-fit values. Also note that for a normal distribution, the 1σ error is $\approx 1.48 \times$ MAD.

We focus on how accurately the planet velocity and planet–star radius ratio could be measured when LD is taken into account, and we report fractional error estimates for these parameters (i.e., the error estimates normalized by the true value for each of these parameters). For example, we use the normalized 1σ errors, $\sigma_v/v_{\text{true}}$ (hereafter simply $\sigma_v/v$) to quantify the quality of the velocity measurements. This provides a direct method of estimating the measurement uncertainties in the transit duration. We use similar definitions to quantify the radius ratio estimates. Note that in order to accurately compare uncertainty estimates, we used the MAD taken from the results of 50 sets of simulated LCs for each set of input parameters. Unless otherwise specified, we report error estimates based on the covariance matrix. This choice is based on an initial comparison of the error estimates. While the 1σ and bootstrap errors are otherwise roughly comparable, we find a large discrepancy between our two error estimates when analyzing LCs due to planets with small radii ($R_p \lesssim 2 R_\oplus$). This is because for general nonlinear models, these two uncertainty estimates are not always comparable, even for Gaussian noise (e.g., Ford 2005). While the bootstrap analysis quantifies how much the best-fit model parameters change due to different realizations of the random noise, the covariance matrix analysis describes how rapidly $\chi^2$ increases as one varies the model parameters from their best-fit values. Therefore, the bootstrap analysis can underestimate the uncertainties when one of the parameters is pushed to the limit of its allowed range (e.g., impact parameter of zero for central transits). Note that these conditions hold even if the LD parameters are held fixed. Accordingly, we believe that the 1σ errors from the covariance matrix are more representative of the measurement precision.

### 3. RESULTS

In this section, we present results for the expected measurement precision for the planet velocity and the planet–star radius ratio based on the analysis of simulated photometric LCs. Several different cases are considered. The precisions obtained from *Kepler*-like photometry and ground-based photometry are discussed in Sections 3.1 and 3.2. The fractional accuracy obtained as a result of combining space-based LCs with either one (multiple) ground-based LC(s) is presented in Section 3.3.1 (Section 3.3.2). The final two sections discuss the benefit of having increased ground-based photometric precision (Section 3.4) and the measurement accuracies obtained when fainter stars (e.g., $V = 14$) are observed (Section 3.5).

#### 3.1. Kepler Photometry

##### 3.1.1. Fractional Accuracy Curves

The *Kepler* mission is expected to detect hundreds of transiting planets that are predicted to range from sub-Earth size to giant planets, with orbital periods from days to a year (Basri et al. 2005). Here, we estimate the measurement precision for the planet velocity and the planet–star radius ratio assuming *Kepler*-like photometry for a few fiducial cases. In Table 1, we list the precisions obtained from our analysis of the simulated transit LCs of a Neptune-size planet ($R_{\text{Nep}} \approx 4 R_\oplus$) and of a Jupiter-size planet ($R_{\text{Jup}} \approx 11 R_\oplus$) orbiting a solar-type star.

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\(^6\) The code is available via Craig Markwardt’s Web site, http://www.physics.wisc.edu/~craigm/idl/idl.html.
with periods of 3 days (short period) or 1 year (long period). We assume observational parameters as described in Section 2.1 for a non-central transit \((b = 0.5)\). Despite the effect of LD, we find that the accuracy with which the velocity and radius ratio can be measured is better than 10% in all cases considered (for solar-like stars). The best-case scenario is the detection of a short-period Jupiter, with an accuracy of about 0.1% for velocity and 0.08% for the radius ratio. In general, we find that planets with shorter periods allow for the best constraints on LC parameters due to the high number of transit LCs available for analysis (and thus the larger amount of in-transit data available). We assume an observing span of \(\sim 2.5\) years, so that the number of LCs acquired (and included in the fitting process) for planets with 3 day, 30 day, and 1 year orbital periods is 304, 30, and 3, respectively. Since an individual transit duration scales as \(\sim P^{1/3}\), and the rate of transits scales as \(\sim P^{-1}\), the integrated time in transit scales as \(\sim P^{-2/3}\), so the planet-star radius ratio of short-period planets will be generally better constrained than otherwise identical long-period planets if only space-based observations are available (for similar host stars). The same scalings apply for the time spent in ingress/egress, so parameters such as the planet velocity and eccentricity will also be best constrained for short-period planets (until time sampling becomes inadequate). While it becomes increasingly difficult to obtain precise measurements for planets with longer orbital periods and smaller radii, adding ground-based observations can improve upon this (see Section 3.3).

Besides the fiducial cases, it is also useful to examine the precision over larger ranges of planet sizes and orbital periods. In Figure 1, we show estimated fractional accuracies for the planet velocity and radius ratio from \textit{Kepler}-like photometry of a non-central transit as functions of the planet radius and orbital period. Recall that the total number of LCs used in the fitting process is a function of the orbital period and the assumed lifetime of the \textit{Kepler} mission. In all cases, the radius ratio is determined with higher precision than the planet velocity. We find that the size of an Earth-size planet can be measured to about 20% accuracy at any orbital period, but the velocity is very poorly constrained with approximately 80% accuracy. While an accuracy of 80% is effectively a non-detection, even a weak constraint on the velocity of small transiting planets will be useful for characterizing the overall eccentricity distribution (Ford et al. 2008). The velocity and radius ratio of Neptune-size planets can be measured to accuracies of about 2% and 0.5% for short periods. At longer periods, the accuracy of measurements for Neptune-size planets is approximately 10% and 3% for the velocity and radius ratio, respectively.

For a near-central transit \((b = 0.01;\) not shown), constraints on the size of Earth-size planets are more sensitive to the orbital period, with measurement accuracies of approximately 1% at short periods and 5% at longer periods. For Neptune-size planets, the velocity and radius ratio are both constrained with accuracies of about 0.08% at short periods. At longer periods, the velocity and radius ratio are determined with uncertainties of about 0.5%. These results indicate that photometric observations of a central transit offer better constraints on parameters than observations of a non-central transit. Possible reasons why observations of a central transit offer more accurate measurements include there being smaller effects from LD (i.e., the effect of LD is more easily distinguished from ingress/egress) and more observations in transit (due to a longer transit duration). Despite the higher quality of measurements obtained from observing central transits, this paper primarily focuses on the more representative case of observing non-central transits.

### 3.1.2. Analytic Fits

For a semi-analytic method of estimating the expected fractional uncertainties based on \textit{Kepler}-like observations, we fit power laws to our results for the accuracies of both planet velocity and radius ratio. The accuracies were fitted as a function of planet radius (specifically for radii ranging from \(4R_\oplus\) to \(12R_\oplus\)) for fiducial periods of 3 days, 30 days, and 1 year. We also present the accuracies as a function of the orbital period for planets with radii of \(1R_\oplus, 2R_\oplus, 4R_\oplus,\) and \(11R_\oplus\). Results are given in Tables 2 and 3, and the analytic fits as a function of the orbital period for the non-central transit case are shown in Figure 1. The trend in the accuracy as a function of planet radius is steep for non-central transits, with power-law exponents of about \(-2.6\) to \(-2.7\) and \(-1.9\) for the velocity and radius ratio accuracies, respectively. Most of the accuracies given as a function of the orbital period have a power-law exponent near the

| Table 1 | Fractional Accuracy of Planet Velocity and Size |
|---------|---------------------------------------------|
| \(v_p(R_p)\) | \(P\) (days) | \(\sigma_v/v\) | \(\sigma_p/p\) |
| \(4\) | \(3\) | 0.0212 | 0.00619 |
| \(4\) | 365 | 0.0933 | 0.0263 |
| \(11\) | \(3\) | 0.00143 | 0.000824 |
| \(11\) | 365 | 0.00651 | 0.00380 |

**Notes.** All accuracies were determined based on simulated \textit{Kepler}-like photometry of a non-central transit \((b = 0.5)\) of a planet orbiting a 12th magnitude \((V\)-band) solar-like \((G2V)\) star. A Gaussian noise of 0.4 mmag was added to each 1 minute integration. LD coefficients of \(u_1 = 0.4382\) and \(u_2 = 0.2924\) were used based on ATLAS models for a solar-like star (Claret 2000) to generate the LCs, but we allowed them to vary when fitting the models. The accuracies for the 3 day (one year) orbital periods are based on fitting 364 (3) LCs.
et al. (2007), Winn et al. (2007), and Sozzetti et al. (2009) with metric precision of 1.7 mmag each minute for each band based on simulated *Kepler*-like photometry of a non-central transit (as described in Table 1). The radius of the planet, $R_p$, is in units of $R_\oplus$.

Table 2

Fractional Accuracy as a Function of the Planet Size

| $P$ (days) | $\sigma_{r_p}$ | $\sigma_{r_p}/p$ |
|------------|----------------|-----------------|
| 3          | $0.865 R_\oplus^{2.56}$ | $0.0841 R_\oplus^{1.92}$ |
| 30         | $2.08 R_\oplus^{1.71}$ | $0.190 R_\oplus^{1.94}$ |
| 365        | $3.46 R_\oplus^{2.61}$ | $0.357 R_\oplus^{1.90}$ |

Notes. The analytical models were fitted for planets with radii ranging from $4 R_\oplus$ to $12 R_\oplus$ based on simulated *Kepler*-like photometry of a non-central transit (as described in Table 1). The radius of the planet, $R_p$, is in units of $R_\oplus$.

Table 3

Fractional Accuracy as a Function of the Orbital Period

| $R_p$ ($R_\oplus$) | $\sigma_{r_p}$ | $\sigma_{r_p}/p$ |
|-------------------|----------------|-----------------|
| 1                 | $0.620 R_\oplus^{0.017}$ | $0.188 R_\oplus^{0.019}$ |
| 2                 | $0.856 R_\oplus^{0.257}$ | $0.0234 R_\oplus^{0.265}$ |
| 4                 | $0.0488 R_\oplus^{0.322}$ | $0.00432 R_\oplus^{0.325}$ |
| 11                | $0.00101 R_\oplus^{0.327}$ | $0.000509 R_\oplus^{0.328}$ |

Notes. The analytical models were fitted for planets with periods from 1 day to 1 year based on simulated *Kepler*-like photometry of a non-central transit (as described in Table 1). The orbital period of the planet, $P$, is in units of days.

Table 4

Fractional Accuracy of Velocity from Ground-based Observations

| $P$ | $R_p$ ($R_\oplus$) | $r'$ | $i'$ | $z'$ |
|-----|-------------------|-----|-----|-----|
| 3   | 4                 | 0.573 | 0.570 | 0.552 |
| 11  | 11 R_\oplus         | 0.0809 | 0.0780 | 0.0746 |

Notes. The accuracies were determined based on simulated ground-based photometry for a non-central transit ($b = 0.5$) of a planet orbiting a solar-like star. A Gaussian photometric noise of 1.7 mmag was added to each 1 minute observation based on the typical precision obtained in the *Transit Light Curve* project observations of TRAPPIST-1 (Winn et al. 2007). Linear ($u_1$) and quadratic ($u_2$) LD coefficients were determined from ATLAS models (Claret 2004). The coefficients ($u_1$, $u_2$) used for generating simulated LCs in the $r'$, $i'$, and $z'$ bands are (0.433, 0.30215), (0.3183, 0.3375), and (0.22785, 0.3573), respectively.

Figure 1, which is why we fit a power law only over the range of 1.2 m telescope at FLWO. In order to simulate a generic data set, we assume that the time sampling of 1 minute takes into account the exposure time, readout time, setup time, and any other dead time between observations. Recall that Winn et al. (2007) demonstrated that their photometric errors decrease as $1/\sqrt{N}$ when binned over timescales of up to $\sim$30 minutes, justifying our assumption of uncorrelated noise. We assumed a quadratic LD law, with coefficients for a solar-type star determined from ATLAS models (Claret 2004). The linear and quadratic coefficients ($u_1$, $u_2$) used are (0.433, 0.30215), (0.3183, 0.3375), and (0.22785, 0.3573) for the $r'$, $i'$, and $z'$ bands, respectively. Fractional accuracies for the fiducial cases of planets of $4 R_\oplus$ and $11 R_\oplus$ and with orbital periods of 3 days and 1 year are presented in Table 4, respectively. The differences in accuracies are primarily a result of the different LD coefficients used for each filter. Assuming a common measurement precision, we find that the $z'$ filter typically offers the best precision, largely because LD is less significant at longer wavelengths (also see Pál 2008). It should be noted that for relatively noisy data, the $i'$ and $z'$ filters yield comparable measurement accuracies. Also, recent observations by Winn et al. (2008) achieved a better photometric precision of 0.77 mmag minute$^{-1}$ for a $V = 12$ star (rescaled from the quoted precision for a $V = 12.4$ star) in the $i$ band with the FLWO 1.2 m telescope. If a factor of 2 improvement in precision was routinely obtained, then the $i$ band would be equally useful compared to the $z'$ band. Other recent observations in the $z'$ band resulted in an even better photometric precision of 0.39 mmag minute$^{-1}$ for a $V = 12$ star (rescaled from the quoted precision for a $V = 12.7$ star) using the 2.2 m UH telescope (Johnson et al. 2009), which indicates that the level of ground-based photometric precision achieved is improving even as we write this paper. (Note that while this technique results in errors that scale as $1/\sqrt{N}$ over $\sim$20 minute timescales, it has yet to be demonstrated that observations with this level of noise will consistently follow that scale over longer timescales.) The key point is that LD plays a large role in reducing the measurement precisions for a planet’s size and orbital parameters, making it especially important to minimize its effects. Therefore, we conclude that it is highly desirable to obtain some NIR observations (e.g., using an $i'$ or $z'$ filter) when doing transit photometry from the ground. In the following sections, we use simulated ground-based observations conducted in the $z'$ band to determine the benefits of adding them to *Kepler*-like observations.

3.3. Combining Ground- and Space-based Observations

In this section, we present results obtained from analyzing a combination of simulated ground-based and space-based observations to determine the planet velocity and radius ratio. We specifically discuss the improvements in measurement accuracy obtained when either one or several simulated ground-
Figure 2. Estimated fractional accuracies of the planet velocity and radius ratio as a function of the planet radius for various observational programs. The left column shows velocity accuracies and the right column shows radius ratio accuracies. Curves are based on observations from the ground only (\(c^\prime\) filter; dot-dashed curve), space only (Kepler; dashed curve), space plus one ground (dotted curve), and space plus multiple ground (solid curve). The number of multiple ground curves added varies based on the orbital period. Sixteen, eight, and two ground-based LCs were added to space-based LCs for orbital periods of 3 days, 30 days, and 1 year. The x-coordinate of the crosses in panel (a) indicate the actual radii used in the simulations.

3.3.1. Addition of One Ground-based Light Curve

The fractional accuracy curves for ground-based observations only, space-based observations only, and space plus either one or multiple ground-based LCs (for a non-central transit) are shown in Figures 2 and 3. The estimated fractional accuracies of planet velocity (left columns) and radius ratio (right columns) as a function of both planet radius (Figure 2) and orbital period (Figure 3) are shown. While the individual ground-based curves (dot-dashed) are clearly less accurate than the space-based curves (dashed) in all cases, adding just one ground-based LC to the space-based LCs does in fact increase the measurement precision (e.g., dotted curves). As emphasized in Figures 2 and 3, the results are dependent on both planet size and orbital period. First, note that only small planets (\(R_p \lesssim 4R_\oplus\)) gain significant measurement improvements from the simulated follow-up ground-based observations. This implies that ground-based photometric follow-up of the confirmed primary transits of Jupiter-size planets detected around relatively bright stars (\(V \sim 12\)) by Kepler should be a low priority. (Note that it is beyond the scope of this paper to investigate the benefit of ground-based observations for confirming the planetary nature of candidates or other types of observations, such as occultation.) Second, the space-only and space-plus-one-ground accuracy curves are barely distinguishable at short periods, but at long periods, the difference between the two curves is significant for planetary radii of up to approximately \(3R_\oplus\).

Figure 3 specifically illustrates the change in accuracy as a function of the orbital period for planets of 1 and 2 \(R_\oplus\). As discussed in Section 3.1, Kepler-like photometry only allowed the velocity of an Earth-size planet to be determined with a very weak constraint (about 80% accuracy) and the radius ratio to be determined to an accuracy of about 20%. With the addition of one ground-based LC, the accuracy with which the velocity and radius ratio are measured becomes about 70% and between 15% and 20% at long periods, respectively. If Kepler detects many short-period planets, then even loose constraints on the planet velocity can be useful for constraining the eccentricity distribution (Ford et al. 2008). Despite the fact that the ground-based LCs have about 4 times worse precision than the simulated Kepler observations, combining ground and space data helps constrain the impact and LD parameters in addition to improving the measurements of a long-period planet’s parameters because the total time of in-transit observations from the ground for a long-period planet is a larger fraction of the total in-transit observations available for that same planet over the lifetime of the Kepler mission. In essence, adding one ground-based LC to only three space-based LCs (for a long-period planet) results in a larger overall improvement compared to adding one ground-based LC to 304 space-based LCs (for a short-period planet).

For the case of a central transit (not shown), we find that adding one ground-based observation to Kepler observations barely increases the accuracy with which the radius ratio is
measured. For velocity measurements, there is a slight benefit obtained for smaller planets \((R_p \lesssim 2R_\oplus)\). Therefore, the primary cases that benefit from a combination of at least one ground-based observation with space-based ones are non-central transits of small planets \((R_p \lesssim 4R_\oplus)\) that have longer orbital periods. In all cases, the radius ratio is measured more accurately than the planet velocity, but the velocity measurements have a larger absolute improvement upon fitting a ground-based LC in combination with space-based observations. Finally, it is important to realize that the transit duration of a 1\(R_\oplus\) planet in a 1-year orbit is about 11.5 hr (for a non-central transit), which means that it is not possible to observe an entire transit from one location. Therefore, collaborations between observers in different parts of the world are key to obtaining complete LCs for long-period planets. Given the substantial improvement in measurement accuracy from the addition of one ground-based LC alone, observing a transit of a long-period planet is extremely valuable and worth the observing time.

3.3.2. Addition of Multiple Ground-based Light Curves

The benefits of adding multiple ground-based LCs to space-based observations are illustrated in Figures 2 and 3 as the solid curves. Note that different numbers of ground-based LCs were fitted in combination with space-based LCs depending on the orbital period (16, 8, and 2 ground-based LCs were fitted for periods of 3 days, 30 days, and 1 year, respectively). We selected the number of LCs to use in each analysis based on the number of ground-based observations that could realistically be completed over the lifetime of the Kepler mission. For non-central transits, we find that adding multiple ground-based LCs of short-period planets with radii less than about 3\(R_\oplus\) results in a larger overall improvement in both the velocity and the radius ratio when compared to adding only one ground-based LC (see Section 3.3.1). For longer periods, there is still a slight improvement from adding multiple ground-based LCs, but adding even one suffices to increase the measurement accuracy. There is still no significant benefit seen for planets with radii larger than \(\sim 4R_\oplus\).

For central transits of small planets at longer periods (not shown), there is no significant improvement from adding multiple ground-based LCs, just as there was none seen upon adding just one ground-based LC. However, for smaller planets \((R_p \lesssim 2R_\oplus)\) at short periods, there is a noticeable improvement in the measurement accuracy for the planet velocity when multiple ground-based LCs are fitted. For example, fitting only space-based observations of an Earth-size planet resulted in an accuracy of about 2%. When multiple ground-based LCs are fitted in conjunction with the space-based simulations, the measurement accuracy improves to about 0.8%. There is still no significant improvement seen in measuring the radius ratio for small planets with short periods.

In summary, measurements of the planet velocity and the radius ratio from non-central transits of small planets with short orbital periods benefit most from combining multiple ground-based observations with space-based observations. At long periods, adding just one high-quality ground-based LC provides the majority of the benefit. For central transits of small planets, the only improvement that we find is for the velocity measurement and solely for multiple ground-based observations of short-period planets. Note that the general conclusions are supported by simulations using each of the three different LD scenarios, as discussed in Section 2.2. In the simulations with both LD parameters fixed at their true values, we find that the benefit of adding ground-based observations is substantially reduced and only significant for small, long-period planets. We caution that assuming a fixed value for both LD parameters can result in significantly underestimating the uncertainties in other transit parameters (Southworth 2008). Our simulations in which only the quadratic LD parameter was held fixed (following the recommendation of Southworth 2008) show an improvement in the measurement precision for planets with radii \(\lesssim 2–4R_\oplus\) (for orbital periods of 3 days to 1 year). There is still a similar benefit to incorporating ground-based observations, as when fitting both LD coefficients. For small long-period (\(\sim 1\) year) planets, even a single ground-based transit observation may be sufficient to realize most of this benefit, while several transit observations may be necessary for small short-period planets (\(\sim 3\) days). Based on these results, we further conclude that our approach of fitting both LD parameters is reasonable.

3.4. Increased Ground-based Precision

As discussed in Section 3.2, recent observations by Winn et al. (2008) and Johnson et al. (2009) have obtained a level of photometric precision down to 0.77 mmag minute\(^{-1}\) in the \(i\) band and 0.39 mmag minute\(^{-1}\) in the \(z^\prime\) band (each rescaled from their respective quoted precisions). These are factors of over 2 and 4 improvement compared to our assumed precision of 1.7 mmag minute\(^{-1}\) in the \(z^\prime\) band. Considering that ground-based photometric precisions to these levels have already been obtained, it is plausible to assume that ground-based observations may routinely achieve such a level of precision in the near future. Therefore, we consider the improvement in the accuracy of estimating the planet velocity based on increased ground-based photometric precisions of 2 (0.85 mmag minute\(^{-1}\)) and 4 (0.425 mmag minute\(^{-1}\)) times our assumed routine precision of 1.7 mmag minute\(^{-1}\). The improvement in measuring the radius ratio has not been considered here, as it is already quite well constrained.
The fractional benefit for measuring the planet velocity with increased ground-based precision is illustrated in Figure 4. Here, we consider the accuracy obtained from only space observations divided by the accuracy obtained from space plus multiple ground observations (for a non-central transit). We define the benefit to be that ratio minus unity. The contours in Figure 4 represent the percentage increase in the accuracy, based on the fractional benefit defined above. As an example, the 1% contour defines the region in which the velocity has been measured with 1% more accuracy by including multiple ground-based observations in the fitting process. The improvement in the velocity measurement accuracy is shown over a range of orbital periods and planet radii for photometric precisions of 1.7 mmag minute$^{-1}$ (Figures 4(a) and (b)), 0.85 mmag minute$^{-1}$ (Figures 4(c) and (d)), and 0.425 mmag minute$^{-1}$ (Figures 4(e) and (f)). The top panels of Figures 4 (a) and (b) confirm that measurements of small planets (i.e. with radii less than 4$R_\oplus$) benefit by as much as 100% (i.e., a factor of 2 improvement) from the addition of multiple ground-based LCs that have a photometric precision of 1.7 mmag minute$^{-1}$. At this precision, larger planets with any orbital period have measurement improvements of only about 1%–5%. However, when we consider an increased precision of 0.85 mmag minute$^{-1}$ (Figures 4(c) and (d)), we find that there is a large region containing intermediate-to-long-period planets with radii larger than 4$R_\oplus$ where an improvement between 10% and 20% can be obtained. At this increased photometric precision, the measurement accuracy for small planets also increases, with a maximum improvement of over 150%. Finally, if we assume a further improved photometric precision of 0.425 mmag minute$^{-1}$ (Figures 4(e) and (f)), we find that measurement improvements of up to 50% can be obtained for larger planets with intermediate-to-long periods.
Small planet measurements continue to improve by as much as 200%. In summary, as the photometric precision of ground-based observations continues to improve, there is an increasingly large range of planets that will benefit. The present-day routine photometric precision of ground-based observations is not overly useful for characterizing large planets, but increasing precisions will allow for better constraints on almost any size planet in any size orbit. However, there will continue to be a minimal benefit from adding ground-based observations of larger planets with shorter orbital periods.

3.5. Observing Fainter Stars

Because the Kepler mission will observe stars as faint as 15th magnitude (Bard et al. 2005), we find it useful to explore the improvement in the measurement accuracy for derived parameters obtained when observing transits of 14th magnitude stars (in the V band). We considered a photometric noise of 1.0 mmag for each 1 minute observation of a V = 14 star, based on rescaling the expected Kepler precision for a V = 12 star. The assumptions made in Section 3.2 for the ground-based precision are also implemented here. We considered a (low) ground-based photometric precision of 1.7 mmag each minute, which is comparable to the precision obtained by, for example, Charbonneau et al. (2007), Winn et al. (2007), and Sozzetti et al. (2009) for a V = 12 star with a 1 m class telescope. Here, we assume that a precision of 1.7 mmag minute\(^{-1}\) will be obtainable for a V = 14 star with a 2 m class telescope. Considering the recent improvements made in observing transits of V = 12 stars (e.g., Johnson et al. 2009), we also implement a better ground-based photometric precision of twice the lower precision, or 0.85 mmag minute\(^{-1}\). To justify this as a realistic choice, we assume that a medium-to-large-size telescope would be able to achieve this level of precision. We also acknowledge the anonymous referee for pointing out the possibility that it may be relatively easy to reach the photon noise limit for a 14th magnitude star compared to a brighter star, as fainter stars should have many more comparison stars within the observational field of view.

Figure 5 compares the benefit of adding multiple ground-based LCs to space-based observations for V = 14 stars with (non-central) transiting planets with 3 day, 30 day, and 1 year orbital periods. We consider the addition of one low-precision (1.7 mmag minute\(^{-1}\)) ground-based LC (solid), multiple low-precision ground-based LCs (dashed), one high-precision (0.85 mmag minute\(^{-1}\)) ground-based LC (dotted), and multiple high-precision ground-based LCs (dot-dashed). We assume a precision of 1.0 mmag minute\(^{-1}\) for Kepler-like observations of a V = 14 star.
with an improvement in velocity and radius ratio measurements of about 350% and 250% for $2R_\oplus$ planets with long periods. Overall, all cases benefit most from including ground-based observations for small planets, but the benefits are minimal for larger planets, similar to results in Section 3.3.2.

For near-central transits (not shown), we find that the overall benefits from adding ground-based observations are smaller than for the non-central transit cases, but are still appreciable. Overall, this indicates that using ground-based observations in conjunction with space-based observations can be increasingly beneficial for constraining the velocity and radius ratio of small planets that transit stars fainter than the 12th magnitude.

4. DISCUSSION

Previous studies have provided analytic estimates of the precision of transit parameters for a single set of observations (Carter et al. 2008). In this paper, we investigate the precision of transit model parameters based on heterogeneous observational data sets. In particular, we consider the case of high-quality space-based photometry using a broadband optical filter and lower quality ground-based photometry in the NIR. We have described how ground-based photometric observations of planets in transit can be useful for further constraining the planet velocity and planet–star radius ratio when they are combined with space-based observations obtained from missions such as Kepler and CoRoT. We considered cases for both a near-central transit ($b = 0.01$) and a non-central transit ($b = 0.5$), where the effects of LD are more significant. Our results revealed that in all cases considered, the parameters measured for central transits are constrained better than identical parameters measured for non-central transits. We find that the radius ratio is measured with higher accuracy than the planet velocity in accordance with analytic estimates that neglect LD (Carter et al. 2008). Accordingly, the measurement of the planet velocity benefited most from adding ground-based photometric observations in conjunction with space-based ones, and the benefits were largest for planets with non-central transits and radii less than $4R_\oplus$. We find a substantial increase in the measurement precision for the velocity of small non-centrally transiting planets with long orbital periods when at least one ground-based observation is added to space-based ones. Multiple ground-based LCs added to space-based ones further improve the measurement precision for the velocity of planets and provide a substantial benefit for a wider range of planet sizes and orbital periods. When higher ground-based precisions are considered, we find that any size planet in any orbital period can benefit from the addition of ground-based LCs. Furthermore, even fainter stars (e.g., $V = 14$) can benefit from the addition of relatively low-precision ground-based observations.

Our results are important for those planning ground-based photometric follow-up of transiting planets that are initially discovered with space-based missions such as Kepler and CoRoT. For example, we have shown that routine ground-based follow-up can be particularly useful for planets with radii smaller than about $4R_\oplus$. As ground-based observations push toward improved photometric precision, even the characterization of larger planets could benefit from the inclusion of NIR ground-based observations. Thus, ground-based follow-up can improve the characterization of a large range of transiting planets, particularly for the planet velocity, the transit duration, and the orbital eccentricity. The constraints on the transit duration and eccentricity can further constrain planet formation models as well as help with the planning of follow-up observations of primary transit or occultation with warm-Spitzer, James Webb Space Telescope (JWST), or ground-based observatories.

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