The use of a conical lens to find the refractive index of liquids

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Abstract. In this work, the basic idea is to determine the refractive index of liquids known using a conical lens. The measurement of the refractive index of liquids is an important work in engineering and science since it is one of the most important optical parameter. The adulteration problem is increasing day by day; therefore it is necessary to implement new and simple devices for measuring the refractive index of several materials.

There is a great variety of interferometric methods that may be used for determining the refractive index. However, these methods either need sophisticated equipment or have low accuracy. Our system consists of a conical lens coupled to a cylindrical container with a liquid whose composition can be changed easily or adulterated. The diameter of the emergent beam of the container is associated to the specific index of refraction of each substance. Any adulteration of the liquid will be reflected in the diameter of the beam, which will be detected by a charge-coupled device (CCD). Our hypothesis is supported by developed mathematical calculations and numerical simulations.

1. Introduction

The refractive index of materials plays a vital role in many branches of physics, biology, instrumentation and chemistry since it exhibits the optical properties of the material. The adulteration problem is increased day by day; this can be controlled using a technique for determining the change in the refractive index of chemical modifications.

Different interferometric methods have been used to measure the refractive index of liquids. Such as, Mach-Zehnder, Jamin, Michelson and Fabry-Perot interferometer, Rayleigh refractometer [1-4]. In these techniques, the optical path length of a laser
beam is modified by the presence of the sample under consideration. This modification is converted into a formula for the refractive index. Other method of measuring the refractive index is using the Talbot effect and a moire technique [5]. In this method, two plane parallel were used, one as standard, and the other as a sample under test. Chandra and Bhaiya [6] described a method based on the minimum deviation and Snell’s law for the determination of refractive index of various liquids. The minimum deviation method is by far the most accurate. However, requires the fabrication of a large aperture prism and has several other limitations. So method of ellipsometry can be used only for determination of refractive indices of thin film grown on substrates.

In this article we have discussed a technique for the measurement of refractive index of various solutions using a conical lens. This can be done by coupled the conical surface in a cylindrical container with the unknown refractive index liquids. Because, the patterns generated by conical lens have very large focal depth, the measurements do not depend closely to a detection plane particular.

2. Method

The Bessel beam propagates in free space with minimum spread in the transverse direction over distances of more than several meters. Experimentally, Bessel beams can be generated through the illumination of a conical lens with a monochromatic plane wave [7]. Figure 1 shows the geometry by generated Bessel beams using a conical lens.

![Figure 1](image)

**Figure 1.** Schematic behavior of a collimated beam through a conical lens of base angle $\alpha$.

The angle of intersection of geometrical rays with the optical axis is deduced from Snell’s law and takes the form:

$$n \sin \alpha = \sin (\alpha + \beta),$$

(1)

where $n$ is the refractive index of the conical lens and $\alpha$ is the angle formed by the conical surface with the flat surface of the conical lens. Assuming the paraxial approximation, one get:

$$\beta = (n-1)\alpha.$$

(2)

The maximum diffraction-free range $Z_{\text{Max}}$ of the Bessel beams may be written as:

$$Z_{\text{Max}} = \frac{r}{\beta} = \frac{r}{(n-1)\alpha}.$$

(3)
The transverse wave field generated by a conical lens is described by the Bessel function of the first kind \( J_0(k,r) \), where the constant \( k_r \) is related to the inclination angle of the normal to the cone surface with respect to the symmetry axis by:

\[
k_r = (n-1)k\alpha ,
\]

then, the Bessel beam can be expressed as a function of the refractive index of the conical lens \( n_g \):

\[
J_0\left( k\alpha \left[ n_g - 1 \right] r \right).
\]

We can note, that the equations (3) and (5) are a function of \( \alpha \). Then, this parameter can be modified, when a conical lens with a given refractive index is immersed in a medium, which can be filled with a liquid having an unknown or desired refractive index (see Fig. 2).

![Figure 2. Light path in a conical lens- liquid system. A large angle axicon is immersed in index-matching liquid, which is held in a cylindrical container.](image)

Accordingly, Snell’s law yields:

\[
n_g \sin \alpha = n_L \sin(\alpha + \gamma) ,
\]

where \( n_L \) is the refractive index of the solution, and in paraxial approximation, the equation (6) lead to:

\[
\gamma = \left( \frac{n_g}{n_L} - 1 \right) \alpha ,
\]

substituting \( \gamma \) in the following expression:

\[
n_L \gamma = n_a \theta ,
\]

we obtain the tunable convergence angle given by the following expression:

\[
\theta = \left( n_g - n_L \right) \frac{\alpha}{n_a} .
\]

We find that the central spot radius is related to the first zero of the zero-order Bessel function, where the radial distance from the core to the first intensity minimum is given as follows:
\[ \rho_0 = \frac{0.383\lambda}{\theta} = \frac{0.383\lambda n_a}{(n_g - n_L)\alpha}. \]  (10)

Then, the refractive index of the investigated liquid can be determined from the formula:

\[ n_L = \left( n_g - \frac{0.383\lambda}{\rho_0\alpha} \right), \]  (11)

where, \( n_a \) has been taken equal to 1 as the refractive index of air. Since \( n_g \) and \( \lambda \) are known, only \( \rho_0 \) is needed to determine the refractive index of the unknown solution. Can be seen from equation 11 that the refractive indices of the materials depend on the wavelength of incident light on the sample, in our case \( \lambda = 632\text{nm} \).

3. Results

The conical lens used in the present work has the following parameters: \( n = 1.5, \alpha = 1^\circ \). We then immerse this axicon in an index-matching liquid, with refractive index smaller than that of the axicon material. Initially, we record the distribution by free-propagation. Figure 3 displays experimental results of the distribution in free-propagation and a liquid well-known.

![Figure 3](image)

The radial distance from the core to the first intensity minimum by free propagation was 27.78 micrometers. After having measured the beam core size, the container is filled by a liquid. Again, we proceed to measure the radial distance. In this case, the size of the radius is increased and the corresponding value was 81.71 micrometers. Using the equation 11, we determine that the refraction index is 1.323, this it corresponds to distilled water. Figure 4 illustrates the recorded profiles of both cases; clearly we can observe the difference of magnitudes.
Figure 4. Intensity profiles by free-propagation (lines continuous) and by distilled water (lines dotted).

The liquid can be removed from the cell, replaced by another liquid as acetone. By changing the solution, the size of the core of the Bessel beam was increased. The measured values of refractive indices at 25 °C for various liquids are shown in Table I.

| Table I. Refractive index using a He-Ne laser light |
|-----------------------------------------------|
| Wavelength (nm) | Refractive index (water) | Refractive index (acetone) | Refractive index (glycerin) |
|-----------------|--------------------------|---------------------------|-----------------------------|
| 632             | 1.323                    | 1.355                     | 1.473                       |

The table II and figure 5 show some recorded of the refractive index of the solution as function of the beam core size. The experimental results show an increase in the beam size with refractive index (circles in figure). The continuous line represents the theoretical calculations. The effect of the liquid is to decrease the transverse spatial phase gradient and consequently to reduce the effective cone angle of the resulting Bessel-like beam.

| Table II. Beam radius using a He-Ne laser light |
|-----------------------------------------------|
| Wavelength (nm) | Beam radius (µm) (water) | Beam radius (µm) (acetone) | Beam radius (µm) (glycerin) |
|-----------------|----------------------------|----------------------------|----------------------------|
| 632             | 81.71                      | 99.24                      | 514.54                     |

Figure 5. Plots measurements of the beam core size as function of the refractive index of the solution: air, distilled water, acetone and glycerin (of left to right).
We can see that using a liquid with index of refraction very close to that of the axicon glass \((n_L \rightarrow n_s)\), the theoretical curve for the beam core width is asymptotic. In reality, when the refractive indexes are closely matched, the angle \(\alpha\) formed by the conical surface with the flat surface of the conical lens has little effect and the emergent beam is reduced to a plane wave. We measured the transverse intensity profile at different propagation distances by using a CCD camera. The beam generated by means of this system has an excellent quality: maintain a narrow beam width over a long propagation distance (termed “nondiffracting”). In the case of the distilled water it maintains the same diameter around 8 m and for the case of acetone 9 m. The very large focal depth is a great advantage of our system, since the measurement range is very wide generating a reduced measurement error.

4. Conclusions

We have demonstrated an interferometric technique for measuring the refractive index using a conical lens. This method can be implemented right away to identify purity of some substances in alimentary control. We generated line foci longer than what could be achieved with regular conical lens.

References

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