Displacement of propagating squeezed microwave states

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Displacement of propagating quantum states of light is a fundamental operation for quantum communication. It enables fundamental studies on macroscopic quantum coherence and plays an important role in quantum teleportation protocols with continuous variables. In our experiments we have successfully implemented this operation for propagating squeezed microwave states. We demonstrate that, even for strong displacement amplitudes, there is no degradation of the squeezing level in the reconstructed quantum states. Furthermore, we confirm that path entanglement generated by using displaced squeezed states stays constant over a wide range of the displacement power.

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Propagating quantum microwave signals in the form of squeezed states are promising candidates for information processing and communication tasks in superconducting quantum networks. With respect to frequency range and material technology for transmission lines, quantum microwaves can directly interact with information processing platforms based on superconducting quantum circuits (SQC). Thus, additional circuitry for frequency interconversion and a possible technology mismatch are avoided. Long coherence times demonstrated for microwave photons in superconducting resonators translate into distances in the range of kilometers for propagating waves in superconducting environments. Therefore, SQC in combination with propagating microwaves could provide the basis for the implementation of a short- to medium-range quantum communication and information processing. One of the cornerstones of quantum communication is the paradigm of quantum teleportation which allows one to faithfully transmit an unknown quantum state between two spatially-separated parties using a quantum entangled pair and a classical communication channel. Various experiments in the past have demonstrated the feasibility of this idea in different quantum technologies such as quantum optics, superconducting quantum circuits, and atomic systems, among others.

Fundamental operations needed to implement the quantum teleportation protocol with continuous variables include the generation of two-mode squeezing, quadrature measurements, and a conditional displacement. While there are experimental advances for the first two operations, a controllable displacement has not yet been demonstrated in the microwave regime. It is important to note that displacement belongs to the universal set of quantum gates required for quantum information processing with continuous variables. Moreover, from a more fundamental point of view, displacement can allow one to study very general limits of quantum entanglement and coherence. Therefore, it is important to investigate capabilities and possible limitations of the displacement operation applied to non-classical propagating microwave states.
The task of the JPA is to perform squeezing of the displacement operation on propagating squeezed states \[18, 19\]. The JPA consists of a quarter-wavelength coplanar waveguide resonator shunted to ground with a dc-SQUID. The dc-SQUID is inductively coupled to an on-chip antenna used for the application of a pump signal. The strong pump tone allows one to modulate the Josephson inductance of the dc-SQUID at twice the JPA frequency \( f_{\text{pump}} = 2f_{\text{JPA}} \), thus, fulfilling the condition for parametric amplification \[19\]. Additional details, including the sample layout, are provided in Ref. 20. The quadrature components are used for orthogonal channels from 4 to 2. The path-entangled signals are digitized by the ADC card and processed by a measurement program which extracts orthogonal quadratures \[2, 3\], where we first equally distribute the incoming signals to Ref. 22. The complex displacement parameter \( \alpha \) is analogous to a 99% reflective beam splitter, in which the insertion loss of the coupler is \( \kappa \simeq \sim -0.18 \) dB (between ports P1 and P3). Here, the directional coupler is analogous to a 99% reflective beam splitter, in which the displacement signal is weakly coupled to the incident light.

In order to reconstruct propagating displaced squeezed states, we apply the dual-path reconstruction scheme \[2, 3\], where we first equally distribute the incoming signal along two paths with a hybrid ring. Both outputs are independently amplified with a chain of cryogenic and room temperature rf-amplifiers, downconverted in a two-stage process, and finally used for cross-correlation measurements. In the analog stage, a local oscillator (LO), see Fig. IV defines a phase reference for detected signals. An important modification (in comparison with Refs. 2 and 3) of our setup consists in using image reject mixers, which allow us to filter one of the signal sidebands already in the analog part, reducing the number of digitizer channels from 4 to 2. The path-entangled signals are mixed with the LO and produce intermediate frequency (IF) signals at \( f_{\text{IF}} = f_{\text{LO}} - f_{\text{JPA}} = 11 \) MHz. Then, the IF signals are digitized by the ADC card and processed by a measurement program which extracts orthogonal quadratures \( I_{1,2} \) and \( Q_{1,2} \) by performing down-conversion and digital filtering. Extended experimental schematics may be found in Ref. 20. The quadrature components are used to calculate correlation moments \( \langle I_n^m Q_k^l \rangle \) up to the fourth order, i.e., \( n + m + k + l \leq 4 \) for \( n, m, k, l \in \mathbb{N} \). In

\[
\hat{S}(\xi) = \exp(\frac{i}{2}\xi^{*}\hat{a}^{2} - \frac{i}{2}\xi(\hat{a}^\dagger)^2), \quad \xi = re^{i\phi}
\]

is a complex squeezing amplitude. Here, the phase \( \phi \) determines the squeezed quadrature, while the squeezing factor \( r \) parameterizes the amount of squeezing. An experimental realization of the squeezing operator requires a large non-linearity. Figure 2 illustrates such a nonlinear flux dependence (measured as the phase response of a reflected input signal from the sample) of the JPA resonance frequency. It also provides information about the nondegenerate gain as a function of the pump power \( P_{\text{pump}} \) at a chosen working point \( I_{\text{coi}} = +30 \) \( \mu \)A. For all subsequent measurements, we use a fixed pump power of \( P_{\text{pump}} = -25 \) \( \text{dBm} \), which corresponds to a nondegenerateshow of the incident vacuum state, i.e., \( \hat{S}(\xi)|0\rangle \), where

\[
\begin{align*}
\hat{S}(\xi) & = \exp \left( \frac{i}{2}\xi^{*}\hat{a}^{2} - \frac{i}{2}\xi(\hat{a}^\dagger)^2 \right), \\
\xi & = re^{i\phi}
\end{align*}
\]

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In order to implement the displacement operation \( \hat{D}(\alpha) = \exp(\alpha \hat{a} - \alpha^{*} \hat{a}^\dagger) \) on propagating squeezed states, we use a cryogenic directional coupler designed according to Ref. 22. The complex displacement parameter \( \alpha \) is controlled via the amplitude and the phase of a strong coherent signal incident at the coupling port of the directional coupler (see Fig. I). The propagating squeezed state from the JPA is applied to the input port P1 of the coupler. The coupling between the transmitted port P3 and the displacement port P2 is \(-19.5 \) \( \text{dB} \) in the vicinity of \( f_{\text{JPA}} \). The insertion loss of the coupler is \( \kappa \simeq -0.18 \) dB (between ports P1 and P3). Here, the directional coupler is analogous to a 99% reflective beam splitter, in which the displacement signal is weakly coupled to the incident light.

In order to reconstruct propagating displaced squeezed states, we apply the dual-path reconstruction scheme \[2, 3\], where we first equally distribute the incoming signal along two paths with a hybrid ring. Both outputs are independently amplified with a chain of cryogenic and room temperature rf-amplifiers, downconverted in a two-stage process, and finally used for cross-correlation measurements. In the analog stage, a local oscillator (LO), see Fig. IV defines a phase reference for detected signals. An important modification (in comparison with Refs. 2 and 3) of our setup consists in using image reject mixers, which allow us to filter one of the signal sidebands already in the analog part, reducing the number of digitizer channels from 4 to 2. The path-entangled signals are mixed with the LO and produce intermediate frequency (IF) signals at \( f_{\text{IF}} = f_{\text{LO}} - f_{\text{JPA}} = 11 \) MHz. Then, the IF signals are digitized by the ADC card and processed by a measurement program which extracts orthogonal quadratures \( I_{1,2} \) and \( Q_{1,2} \) by performing down-conversion and digital filtering. Extended experimental schematics may be found in Ref. 20. The quadrature components are used to calculate correlation moments \( \langle I_n^m Q_k^l \rangle \) up to the fourth order, i.e., \( n + m + k + l \leq 4 \) for \( n, m, k, l \in \mathbb{N} \). In

In this Letter, we experimentally confirm the feasibility of the displacement operation on propagating squeezed microwaves. Specifically, we demonstrate that both a single mode squeezing and a frequency-degenerate continuous variable path entanglement remain unchanged over the range of 30 decibel in the displacement power. These results are a very encouraging and a crucial step on the way towards quantum microwave teleportation with continuous variables \[13\].

Figure IV shows a circuit schematic of our experimental setup. We use a flux-driven Josephson parametric amplifier (JPA) for the generation of squeezed microwave states \[18, 19\]. The JPA consists of a quarter-wavelength coplanar waveguide resonator shunted to ground with a dc-SQUID. The dc-SQUID is inductively coupled to an on-chip antenna used for the application of a pump signal. The strong pump tone allows one to modulate the Josephson inductance of the dc-SQUID at twice the JPA frequency \( f_{\text{pump}} = 2f_{\text{JPA}} \), thus, fulfilling the condition for parametric amplification \[19\]. Additional details, including the sample layout, are provided in Ref. 20. The JPA is placed in a magnetically shielded sample-holder (we use a combination of cryoperm and superconducting shields) inside a custom-made dry dilution refrigerator. During all experiments the JPA temperature is stabilized at 50 mK. An input circulator allows us to separate incoming and outgoing signals, as required for the JPA operation. We calibrate our setup via a temperature sweep coming and outgoing signals, as required for the JPA operation. We calibrate our setup via a temperature sweep coming and outgoing signals, as required for the JPA operation.
spanning the phase space. The squeezing angle is stabilized at a squeezed vacuum state, a squeezed vacuum state with a displacement angle of $\theta$. This way, we can retrieve all the moments of the annihilating and creation operators, $\hat{a}$ and $\hat{a}^\dagger$, of the signal and the noise modes by using the beam splitter relations and the independence of the noise contributions from the two detection paths. Finally, based on the operator moments $\langle (\hat{a})^n (\hat{a}^\dagger)^m \rangle$, we calculate the covariance matrix $\vec{\sigma}$ of the signal mode, and reconstruct the Wigner function of the propagating signal incident at the input of the hybrid ring. We verify that the reconstructed states comply with the Heisenberg principle for the 2-nd and the 4-th order moments $\langle (\hat{a})^n (\hat{a}^\dagger)^m \rangle$ and with a Gaussianity criterion based on cumulants [3, 24, 25].

We characterize the squeezing level of the reconstructed quantum state in decibels as $S = -10 \log_{10}[(\Delta X_{sq})^2/0.25]$, where $(\Delta X_{sq})^2$ is the variance of the squeezed quadrature and the chosen vacuum reference is $(\Delta X_{vac})^2 \equiv 0.25$. A state is squeezed below the vacuum level when $(\Delta X_{sq})^2 < 0.25$. We define the displacement angle $\theta$ as the angle between the displacement direction and the $p$-axis, and the squeezing angle $\gamma = \phi/2$ as the angle between the antisyndicated quadrature and the $p$-axis. The angles are controlled experimentally by the phases of the coherent signals of the pump and the displacement tones with respect to the LO (cf. Fig.1). Figure 3 illustrates experimentally reconstructed Wigner functions of a squeezed vacuum and displaced squeezed vacua. The results show that the directional coupler allows us to displace the propagating squeezed state with a high degree of control over the magnitude and the phase of the displacement parameter $\alpha$. Even for a large displacement powers up to hundreds of photons, the resulting squeezing level remains approximately the same as for the undisplaced state. Notice that the photon number is related to a measurement bandwidth of 400 kHz, defined by a digital low-pass filter in the data processing. Systematic measurements of both the squeezing level and the total photon number versus the displacement power $P_{\text{disp}}$ are shown in Fig.4(a). There, we show that the resulting squeezing level for $P_{\text{disp}} \leq -125$ dBm is the same as for the undisplaced state, $S \approx 6.4$, within error bars. For larger displacement powers, $P_{\text{disp}} > -125$ dBm (data not shown), we observe an increasing degradation of squeezing. Based on numerical simulations, we attribute this observation to a limited calibration precision of the cross-correlation gain which distorts the reconstruction of states with a very high photon number. It is important to note here, that the demonstrated range of the controlled displacement of propagating squeezed states is already sufficient for many quantum communication protocols with continuous-variable microwaves including quantum teleportation [13, 15].

As it was confirmed in previous experiments [3], the squeezing at the hybrid ring input is equivalent to a path entanglement of the output beams. Therefore, from a fundamental point of view, one must check whether the amount of bipartite path entanglement between the outputs of the entangling hybrid ring depends on the displacement power. To this end, we measure the negativity $N$. For Gaussian states [3, 26], it is defined as $N \equiv \max \{0, (1 - \nu)/(2\nu)\}$, where $\nu \equiv ((\Delta(\vec{\sigma}) - (\Delta^2(\vec{\sigma}) - 4 \det \vec{\sigma})^{0.5})/2)^{0.5}$ and $\Delta(\vec{\sigma}) = \det \vec{\alpha} + \det \vec{\beta} - 2 \det \vec{\gamma}$. The condition $N > 0$ witnesses the presence of entanglement for a general quantum state. The covariance matrix $\vec{\sigma}$ of

FIG. 3. Reconstructed squeezed states at the input of the hybrid ring. The quantities $p$ and $q$ are dimensionless variables spanning the phase space. The squeezing angle is stabilized at $\gamma = 45^\circ$. The color code represents the Wigner function of (a) a squeezed vacuum state, (b) a displaced squeezed vacuum state with a displacement angle of $\theta = 135^\circ$, (c) a displaced squeezed vacuum state with a displacement angle of $\theta = 45^\circ$. Each measurement is averaged over $1.3 \times 10^8$ samples. The total photon numbers and squeezing levels are indicated in each panel. The insets show $1/e$ contours for the ideal vacuum (red), and experimental squeezed states (blue).
activity based on the beam splitter model and is mathematically equivalent to the Wigner function reconstruction of the state incident at the input of the hybrid ring. In contrast, the RSM is an independent experiment and reconstructs the moments of the output state using a calibration against a known reference signal which, in our case, is the two-mode vacuum. Thus, the RSM provides a direct evidence of the entanglement between two spatially separated modes. In Fig. 4(b), we show the results for both methods, proving that they coincide in our experiments. The good agreement between both methods confirms the high degree of control and understanding we have on our experiment. Both methods confirm the independence of the path entanglement from the displacement amplitude.

In conclusion, we have experimentally studied the displacement of propagating squeezed microwave states. We have shown that even states displaced by hundreds of photons do not lose their original squeezing level. The implemented displacement mechanism is an important step towards quantum communication and information processing experiments with continuous-variable quantum microwaves [13, 15]. Furthermore, we have applied this operation to general studies of the path entanglement generated from displaced squeezed microwaves via a beam splitter. We have demonstrated that the path entanglement is preserved over a wide range of displacement power. Finally, we interpret our results as an experimental spotlight on a more general issue in quantum mechanics: the quantum properties of large photonic states with hundreds of photons can be associated with a relatively small subset of these photons. In our specific case, the squeezed photons translate into entanglement after the beam splitter while the displacement photons do not.

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