Accretion of Chaplygin gas upon black holes: formation of faster outflowing winds

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Abstract
We study the accretion of modified Chaplygin gas upon different types of black holes. Modified Chaplygin gas is one of the best candidates for a combined model of dark matter and dark energy. In addition, from a field theoretical point of view the modified Chaplygin gas model is equivalent to that of a scalar field having a self-interacting potential. We formulate the equations related to both spherical accretion and disc accretion, and respective winds. The corresponding numerical solutions of the flow, particularly of velocity, are presented and analysed. We show that the accretion–wind system of modified Chaplygin gas dramatically alters the wind solutions, producing faster winds, upon changes in physical parameters, while accretion solutions qualitatively remain unaffected. This implies that modified Chaplygin gas is more prone to produce outflow which is the natural consequence of the dark energy into the system.

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1. Introduction

In nature, the compact objects, particularly the black holes (BHs), are not visible but can be detected by the presence of the accretion disc around them. By analysing light rays off an accretion disc, one can speculate the properties of the central compact object. The formation of accretion discs can most commonly be understood in a binary system where accretion of matter into a compact object from the companion star forms a disc-like structure. The proto-planetary discs, discs around active galactic nuclei, etc are also examples of the accretion disc [1].
Although the accretion phenomena around compact objects (particularly BHs) have been extensively discussed over the last three decades (e.g. [1]), it was started long ago in 1952 by Bondi [2]. He studied the stationary spherical accretion problem by introducing formal fluid dynamical equations in the Newtonian framework. In the framework of general relativity, the study of accretion was initiated by Michel [3]. By choosing the Newtonian gravitational potential, Shakura and Sunyaev [4] formulated a very simplistic but effective model of the accretion disc. Some aspects of the accretion disc in a fully relativistic framework had been studied by Novikov and Thorne [5] and Page and Thorne [6]. Subsequently, various aspects related to the critical behaviour of general relativistic flows in spherical symmetry have been studied [7–10]. Although there are a few steps forward, still it is extremely difficult to simulate the full scale realistic accretion discs including outflows in a full general relativistic framework.

One may note that in BH accretion, an important issue is that the flow of accreting matter must be transonic in nature, i.e. there should be sonic point(s) [11–13] in the flow. On the other hand, accretion flow around a neutron star is not necessarily transonic (i.e. sonic point may or may not exist). It depends on the inner disc boundary conditions influenced by the neutron star. Note that, to simplify the nonlinearity arisen due to general relativity, often the pseudo-Newtonian approach is used by introducing the pseudo-Newtonian potential for the accretion disc. In this scheme, to study the accretion flow, the dynamical equations are written in the Newtonian theory with an effective gravitational force corresponding to the above pseudo-Newtonian potential. Paczynski and Wiita (PW) [14] first proposed such a pseudo-Newtonian potential for a non-rotating BH and it has been frequently used in simulations [15, 16]. Subsequently, there were several other proposals [17, 18] for pseudo-Newtonian potentials, but it has been shown [19] that the former one (by PW) is better than the others for non-rotating compact objects. Then Mukhopadhyay [20] proposed a pseudo-Newtonian potential for rotating compact objects directly from the spacetime metric. He was able to reproduce exactly the general relativistic values of last stable circular orbits and had shown in calculating the specific energies of last stable circular orbits that the difference between the general relativistic value and that corresponding to his potential was less than 10% in Kerr geometry. Also when the BH spin is set to zero, the potential of Mukhopadhyay [20] reduces to that of PW.

At present, there are various observed data (particularly from distant type-Ia supernova explosions) which strongly indicate that our universe is undergoing an acceleration phase starting at the cosmological redshift $z \approx 1$ [21, 22]. Now the reason for this accelerating expansion can be described in two ways: by introducing non-baryonic matter (known as dark energy (DE)) having negative pressure (violating the strong energy condition $\rho + 3p < 0$) either in the framework of general relativity or in the framework of modified gravity theory, such as $f(R)$ gravity, where the extra terms in the modified Friedmann equations are responsible for the acceleration. Also these recent observational results support various cosmological tests such as gravitational lensing, galaxy number counts, etc [23].

Within the framework of Einstein’s gravity, due to this present accelerating phase, it is reasonable to believe that DE is the dominating part (74.5% of the energy content in the observable universe) [24] of the total energy of the universe. The candidates for DE [25, 26] can be classified as the cosmological constant, dynamical component like quintessence [27], k-essence [28], Chaplygin gas (CG) [29, 30], etc. Although the cosmological constant is by far the simplest and the most popular candidate for DE, from the point of view of fine tuning and the cosmic coincidence problem, it is not a suitable candidate for DE. On the other hand, dynamical DE models are favourable as they admit to construct ‘tracker’ [31] or ‘attractor’ [32] solutions. But most of the dynamical DE models are described by a scalar field (often called a quintessence field) which is unable to describe the transition from a universe filled
with matter to an exponentially expanding universe. However, DE can be represented by an exotic type of fluid known as CG [29] having the equation of state $p = -\frac{\beta}{\rho}$, where $p$ and $\rho$ are, respectively, the pressure and energy density and $\beta$ is a positive constant. Subsequently, this equation of state was generalized to $p = -\frac{\beta}{\rho^n}$, $0 \leq n \leq 1$, and is known as generalized CG (GCG) [30]. In recent past, there was further modification to this equation of state as [33]

$$p = \alpha \rho - \frac{\beta}{\rho^n}$$

(1)

and the model is known as modified CG (MCG). It gives the cosmological evolution from an initial radiation era (with $\alpha = \frac{1}{3}$) to (asymptotically) the $\Lambda CDM$ era (where fluid behaves as a cosmological constant). This cosmological model can be described from the field theoretical point of view by introducing a scalar field $\phi$ having the self-interacting potential $V(\phi)$ so that the effective Lagrangian becomes

$$L_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi).$$

Moreover, the MCG model is interacting from the phenomenological view point and it can be motivated by braneworld interpretation [34]. Further, the MCG model is in favour of the cosmological tests, namely gravitational lensing [35], gamma-ray bursts [36] in addition to the above-mentioned observational evidence. Unlike GCG, the velocity of sound with MCG is compatible with the cosmic evolution. Also the MCG model (as well as the GCG model) is naturally constrained through the cosmological observable [37] to explain the overwhelming energy density of the universe at the present era. At low energy density, the model behaves as polytropic gas [38] with the negative value of the index and hence it is possible to have astrophysical implications of the models with an alternate way of restricting the parameters.

In this work, we plan to study the flow of MCG around BHs. Due to the present accelerated expansion of the universe, the matter in the universe is dominated by DE (almost $\frac{2}{3}$ of the matter is in the form of DE). Therefore, it is reasonable to assume that the accreting matter is in the form of DE. Babichev et al [39, 40] were the pioneers to think about the DE accretion upon a BH, in the framework of Bondi accretion [2]. Further, as we have stated above, MCG is one of the most favourable candidates for DE filling our universe mostly. However, as matter approaches the BH, the centrifugal force increases rapidly compared to the gravitational force (particularly for the flow with slowly varying specific angular momentum). Hence, matter feels increasing centrifugal force around a radius little away from the BH, then slows down and piles up around the radius. This region refers to CENBOL (CENtrifugal pressure supported BOundary Layer) (see, e.g., [41]) acting as the effective boundary layer of the BH system which, like stellar surface, could produce outflowing winds. Therefore, due to the presence of the CENBOL, a BH system, like star, easily exhibits wind of matter along with inflow. However, once matter crosses the CENBOL, the dominant gravitational force solely controls dynamics and matter falls into the BH.

The size and shape of galaxies depend strongly on their angular momentum; spiral galaxies containing discs possess a net sense of rotation while elliptical galaxies, that are without discs, possess none. Hoyle [42] first proposed gravitational instability, arising from gravitational coupling with the surrounding matter (tidal interactions), as a possible explanation for galactic rotation. Alternatively, people have proposed [43, 44] that the origin of galaxy rotation could be due to primordial turbulence/vorticity. However, vortical modes decay with time and, therefore, a significant vorticity at the time of galaxy formation would typically require an unrealistic magnitude of vorticity in the early universe. At present, it is widely believed that the hierarchical clustering of cold dark matter [45] is the origin of structures in the universe. Consequently, the angular momentum of dark matter halos and eventually the rotation of galaxies is thought to be produced by gravitational tidal torque [46]. It has been suggested that
the halos obtain their spins through the cumulative acquisition of angular momentum from satellite accretion. DE that has accreted on a galaxy would be similarly torqued by such tidal interactions. Although it is difficult to estimate this effect without invoking specific models, it is reasonable to expect that some angular momentum might reside in the DE halos of galaxies. If such rotating DE were to accrete on a compact object, then it would carry a part of its angular momentum with it, thus leading to the formation of a DE accretion disc as opposed to Bondi accretion.

Therefore, we have concentrated on MCG, the accretion–wind disc system around BHs, along with Bondi flow. We have examined the accretion and wind solutions extensively. The results are then compared with the standard accretion–wind phenomena with flow having the polytropic equation of state. The summary of the work is presented at the end.

2. Basic equations

Bondi [2] formulated simple spherical accretion and wind in the Newtonian framework around a non-rotating star. As gravity is very strong near compact objects, general relativistic treatment gives proper description of the accretion flow. However, the full general relativistic set of equations is so cumbersome that it is difficult to relate different terms with the corresponding physics behind them and as a result transparency of the description might be hidden. Thus, we describe the system by the pseudo-Newtonian approach, i.e. Newtonian theory with a modified gravitational potential (force). This helps to describe the general relativistic features approximately and useful to deal with a less understood system.

For an isolated compact object, matter falls onto it from all directions, resulting in a spherical accretion as defined by Bondi [2]. However, as described in section 1, infall onto a super-massive BH at the centre of a spiral galaxy forms an accretion disc. Here we consider the inviscid fluid so that there is no dissipative force into the system and then the specific angular momentum of disc fluid is constant throughout a particular flow. This importantly implies that the accretion of MCG onto a BH may be hindered for the same reasons as that the accretion of standard cold dark matter is hindered [47, 48]. Therefore, in steady state the radial momentum conservation equation describing disc dynamics is [1]

\[
 u \frac{du}{dx} + \frac{1}{\rho} \frac{dp}{dx} - \frac{\lambda^2}{x^3} + F_g(x) = 0,
\]

where all variables are expressed in dimensionless units as follows:

\[
 u = u_r = \frac{1}{c_s}, x = \frac{r}{r_g}, r_g = \frac{GM}{c^2}, \quad \text{where} \quad c_s, M \text{ and } c \text{ are the speed of sound, mass of the BH and the speed of light, respectively, and } r \text{ and } v \text{ are dimensionful radial coordinate and radial velocity, respectively. Here } p \text{ and } \rho \text{ are the dimensionless isotropic pressure and density of the flow. } F_g = \frac{(x^2-2)^2(x^2-j^2)^{3/2}}{x^{4/3}(x^2-j^2)^{5/2}} \text{ is the gravitational force corresponding to the pseudo-Newtonian potential [20] and } \lambda \text{ is the dimensionless specific angular momentum (in units of } \frac{GM}{c^2} \text{) of the flow. } j \text{ is the dimensionless specific angular momentum of the BH (Kerr parameter). In the case of Bondi flow, } \lambda = 0 \text{ throughout. The equation of continuity, i.e. vertically integrated mass conservation relation, in general for disc accretion, gives}
\]

\[
 \frac{d}{dx}(xu\Sigma) = 0,
\]

where \( \Sigma \), the vertically integrated density, is given by [38]

\[
 \Sigma = I_r \rho_e h(x),
\]

with \( I_r \) the constant (related to the equation of state of the accreting fluid) [1], \( \rho_e \) the density at the equatorial plane, and \( h(x) \) the half-thickness of the disc.
Assuming the vertical equilibrium, the expression for \( h(x) \) can be written as

\[
h(x) = c_s \sqrt{\frac{x}{F_g}}
\]  

(5)

where \( c_s^2 = \frac{\partial p}{\partial \rho} \sim \frac{\rho}{\rho} \) is the square of the sound speed. Note that for spherical accretion, equation (3) reduces to

\[
\frac{d}{dx}(x^2 u \rho) = 0.
\]

In this work, the accreting matter is chosen to be MCG. From the equation of state (given by equation (1)) of MCG, we obtain

\[
c_s^2 = \frac{\partial p}{\partial \rho} = \alpha + \beta n \rho^{n+1}
\]

such that

\[
\rho = \left( \frac{n \beta}{c_s^2 - \alpha} \right)^{\frac{1}{n+1}}
\]

(7)

and thus

\[
\frac{1}{\rho} \frac{d\rho}{dx} = -\frac{2 c_s^3}{(n+1) (c_s^2 - \alpha)} \frac{dc_s}{dx}
\]

\[
= -\frac{1}{n+1} \frac{d}{dx} (c_s^2) - \left( \frac{\alpha}{n+1} \right) \frac{d}{dx} \{ \ln(c_s^2 - \alpha) \}.
\]

(8)

Now integrating equations (2) and (3) and using the MCG equation of state (1), we obtain two conservation equations, namely

(a) energy conservation:

\[
\varepsilon = \frac{1}{2} u^2 - \frac{c_s^2}{n+1} - \frac{\alpha}{n+1} \ln(c_s^2 - \alpha) + \frac{1}{2} \lambda^2 \frac{x^2}{x^2} + V(x),
\]

(9)

where \( V(x) = \int F_g(x) dx \),

(b) conservation of mass:

\[
\dot{M} = \odot \rho c_s \frac{x^2}{F_g} u = \odot c_s u \left( \frac{x^3}{F_g} \right)^{\frac{1}{2}} \left( \frac{n \beta}{c_s^2 - \alpha} \right)^{\frac{n+1}{2}},
\]

(10)

where \( \odot \) is a geometric constant, depending on the exact geometry of the flow. The entropy accretion/wind rate (\( \dot{\mu} \)) is related to the mass accretion/wind rate (\( \dot{M} \)), given by \( \dot{\mu} = Kn \dot{M} \) (\( K \) is a constant carrying the information of entropy) so that \( [1] \)

\[
\dot{\mu} \simeq \odot c_s u \left( \frac{x^3}{F_g} \right)^{\frac{1}{2}} \left( \frac{n \beta}{c_s^2 - \alpha} \right)^{\frac{n+1}{2}}.
\]

(11)

Now differentiating the \( \rho \) replaced part of equation (10), we obtain

\[
\frac{(1-n) c_s^2 + \alpha (n+1) c_s}{(n+1) c_s (c_s^2 - \alpha)} \frac{dc_s}{dx} = \frac{3}{2x} - \frac{1}{2F} \frac{dF}{dx} + \frac{1}{u} \frac{du}{dx}
\]

(12)

and then combining equations (8) and (12) to replace \( \frac{1}{\rho} \frac{d\rho}{dx} \) in equation (2), we obtain

\[
\frac{du}{dx} = \frac{\frac{c_s^2}{x} - F_g(x) + \left( \frac{3}{2} - \frac{1}{F_g} \frac{dF}{dx} \right) \left[ (1-n) \frac{c_s^2}{(1-n) c_s^2 + \alpha (n+1)} \right]}{u - \frac{2c_s^2}{u (1-n) c_s^2 + \alpha (n+1)}} = \frac{N}{D}
\]

(13)
Equation (13) has two branches of solution, accretion (inflow) and wind (outflow). Note that, for accretion, far away from BH, \( u < c_s \) and very close to it \( u > c_s \), while for wind it is opposite. However, in either of the cases, there is an intermediate location where the denominator of equation (13) vanishes. Thus, for a smooth solution throughout, the numerator has to be zero at that location. This point/location is called the sonic point or the critical point (\( x_c \)). In studying accretion–wind phenomena, the existence of a critical point plays an important role. Further, the critical point always exists in the accretion disc around BHs and its global analysis helps to understand the stability of physical parameters. Now as \( N = 0 = D \) at the critical point \((x = x_c)\), using l’Hospital’s rule and after some algebra, the velocity gradient of the accreting matter at the critical point obeys the quadratic equation

\[
A \left( \frac{du}{dx} \right)_{x=x_c}^2 + B \left( \frac{du}{dx} \right)_{x=x_c} + C = 0, \tag{14}
\]

where

\[
A = 2 \left[ 1 - \frac{2 \left( c_{sc}^2 - \alpha \right) (n + 1) \left( (1 - n) c_{sc}^2 + 2 \alpha (n + 1) \right)}{(1 - n) c_{sc}^2 + \alpha (n + 1)} \right],
\]

\[
B = -2 \frac{c_{sc}^2 - \alpha}{c_{sc}^2 + \alpha (n + 1)} \left( (1 - n) c_{sc}^2 + 2 \alpha (n + 1) \right) \left[ \frac{F_g(x_c)}{c_s^2(x_c)} - \frac{\lambda^2}{x_c^2} \right],
\]

\[
C = \frac{3 \lambda^2}{x_c^4} \left( \frac{dF_g}{dx} \right)_{x=x_c} - \left( \frac{1}{F_g} \left( \frac{dF_g}{dx} \right)^2 \right)_{x=x_c} - \frac{3}{x_c^2} - \left( \frac{1}{F_g} \frac{d^2F_g}{dx^2} \right)_{x=x_c} \left( \frac{u_c^2}{2} \right)
\]

where \( c_{sc} \) denotes the speed of sound at the critical point. Solving \( N = 0 = D \) at the critical point, the Mach number and the flow velocity at that location, respectively, denoted as \( M_c \) and \( u_c \), are given by

\[
M_c = \frac{u_c}{c_{sc}} = \left[ \frac{2c_{sc}^2}{(1 - n) c_{sc}^2 + \alpha (1 + n)} \right]^{\frac{1}{2}}, \tag{16}
\]

and

\[
c_{sc} = \left[ \frac{(1 - n) \left( \frac{2}{n} - F_g(x_c) \right)}{2 \left( \frac{dF_g}{dx} \right)_{x=x_c} - \frac{1}{x_c^2}} \right] \left[ 1 + \sqrt{1 + \frac{4 \alpha (n + 1) \left( \frac{1}{n} \left( \frac{dF_g}{dx} \right)_{x=x_c} - \frac{1}{x_c^2} \right)}{(1 - n)^2 \left( \frac{1}{n}(\frac{dF_g}{dx})_{x=x_c} - \frac{1}{x_c^2}) \right)} \right]^{\frac{1}{2}}. \tag{17}
\]

Thus, integrating the velocity gradient (i.e. equations (13) and (14)) with appropriate boundary conditions [1] we can obtain the flow properties. The solution procedure has been discussed in detail in previous work [49, 50], which is not repeated here.

It is to be noted that if we put \( \alpha = 0, \beta = -\kappa, n \rightarrow -\gamma \), then we obtain \( p = \kappa \rho \gamma \). In that case the resulting differential equation of \( u \), expressions for \( A, B, C \) of equation (15), Mach number, speed of sound, match with the Mukhopadhyay results [1].

3. Analysis of solutions for spherical accretion–wind

At the quintessence or phantom era, the whole universe is supposed to be filled up with the DE fluid. In this case, it will be very natural that DE will fall upon a BH from every possible direction forming a spherical accretion–wind system. So obviously \( \lambda \) will be zero.
In figures 1(a)–(d) we have two distinct curves—one represents the velocity of accreting matter and the other the velocity of the wind, i.e. the matter thrown outwards from the BH. As BH is a gravitationally attracting body, the velocity of accretion gradually increases as matter approaches to the BH and reaches almost to the speed of light near the BH event horizon. On the other hand, while the wind velocity is very small near the BH (it is almost impossible to escape out when matter is very near to the BH), it increases gradually as matter comes away off the BH. The point of intersection of these velocity profiles, i.e. the radius where accretion velocity is equal to wind velocity, is the same as the critical point. Now the position of the critical point depends upon many parameters. If matter is such that it has a tendency to easily increase its velocity, then the rate of change of velocity increases faster for accretion as matter approaches the BH and that for wind it decreases. Hence the critical point appears far away from the BH. On the other hand, if matter has a tendency to be flown outwards, then the rate of change of velocity increases faster for wind as matter comes away off the BH and that for accretion decreases. As the wind velocity becomes equal to the accretion velocity at a nearer to the BH, the critical point appears close to the BH, in the second case.
We have studied various possible solutions for \( j = 0 \). The ranges of parameters of MCG are chosen in order to obtain physical solutions such that the accretion extends from infinity to the BH event horizon. When \( \alpha = 0 \) and \( n \) is negative, i.e. the adiabatic case, the solutions match exactly with that of the adiabatic Bondi flow, as shown in figure 1(a). In figure 1(b), we have increased the value of \( \alpha \) and noted that the wind rate increases significantly. Note that this is the same as the isothermal Bondi flow, as the first term in the MCG equation of state dominates the second one when \( \rho < 1 \). In figures 1(c) and (d), \( n \) is increased and the increase of wind rate is revealed in figure 1(c) significantly. This is because at a high (positive) \( n \), the negative term in MCG in equation (1) dominates significantly the first term for \( \rho < 1 \) rendering a very high negative pressure, which results in a faster wind. However, once the values of \( \alpha \) and \( n \) change to their highest (physically) possible values, as chosen in figure 1(d), \( c_{\text{b}} \) becomes smaller than the chosen \( \sqrt{\alpha} \). From equation (7), this gives a physical solution only if \( \beta < 0 \). However, this does not correspond to a negative pressure rendering a slower wind in figure 1(d). This issue has been discussed again in section 6 in order to discuss sound speed in the flows.

4. Analysis of solutions for disc accretion–wind

Here we consider the DE accretion through spiral galaxies. Like Bondi solutions, two distinct curves in figures 2–5 represent the the velocities of accreting matter and wind. If the accretion disc has a higher specific angular momentum, then the corresponding centrifugal force tries to throw matter outwards. Therefore, wind increases which results in the shifting of the critical point nearer to the BH.

As before the ranges of MCG parameters are fixed in order to obtain the physical solutions. In figures 2(a)–(d), we show the velocity profiles around a Schwarzschild BH with \( \lambda = 3.3 \) for MCG. The critical point is at \( x_c = 4.45 \) with \( \alpha \) varying from 0 to 0.07 and \( n \) from -1.6 to 0.6 for a physical accretion–wind system. As MCG has a tendency to escape away and to increase the wind, matter has to come very near to the BH to form the critical point, so that the stronger gravitational attraction therein can make accretion and wind velocities equal. However, when \( \alpha = 0 \) and \(-\frac{3}{5} < n < -\frac{3}{4} \), MCG is particularized into adiabatic gas for which the solutions obtained by earlier work [1] are recovered, as shown in figure 2(a). We note that if \( \lambda \) decreases, then the critical point shifts far from the BH. This is because the decrease of the rotational speed of accretion flow implies the decrease of centrifugal force. Hence, the wind velocity increases slowly and becomes equal to the accretion velocity at a larger radius which results in formation of the critical point away from the BH. However, an increase in \( \alpha \) keeping \( n \) unchanged results in a shift of the critical point nearer to the BH. Note that \( \beta \), which is responsible for negative pressure, is constrained by \( \alpha \) and must be positive in order to have physical density. See section 6 for further discussions in the issue. This suggests that the MCG has a natural tendency of being thrown out from the BH system. On the other hand, higher \( \alpha \) might render the first term in equation (1) dominating the second, making MCG isothermal-like. This again results in a faster wind. Therefore, as \( \alpha \) increases, matter from accretion flow will be thrown outwards more rapidly. Further, as \( n \) increases, the wind dominates with a very steeper velocity profile, as shown in figure 2(c), as discussed in order to explain figure 1(c). Moreover, with the increase of \( \alpha \), as in figure 1(d), \( \beta \) in figure 2(d) has to be negative rendering a slower wind.

In the rest of the figures we have considered the rotating BHs, i.e. the Kerr BHs. Figures 3(a)–(d) are for the Kerr parameter \( j = 0.5 \), when the specific angular momentum of the accretion disc is fixed at \( \lambda = 2.4 \) and the critical point \( x_c = 6 \). Note that \( \lambda \) decreases with the increase of \( j \). This is because the higher gravitational force acting radially at a high \( j \) does
not physically allow the disc to rotate faster and hence for a natural solution disc centrifugal force and thus $\lambda$ has to decrease. When the BH starts to rotate, its attractive power increases, and hence the critical point, i.e. the radius where the accretion and wind velocities are the same, forms away from the BH. The physically meaningful ranges of the values of $\alpha$ and $n$ turn out to be $0.038 \leq \alpha \leq 0.05575$ and $-1.6 \leq n \leq 0.1$. In figure 3(a), $\alpha$ and $n$ are chosen to be very small and the solutions are very similar to that of the adiabatic ones for the Kerr BH as shown in previous work [1]. Figure 3(c) shows that for a small $\alpha$ but large $n$ the wind velocity approaches the speed of light at a finite distance, close to the BH. This can be explained, recalling previous discussions for nonrotating BHs, as follows: increase in $n$ implies the abrupt increase in the negative term of the matter pressure $-\frac{\beta}{\rho}$ for a very small $\rho$ of the DE-dominated universe. Now negative pressure creates repulsion. Therefore, ultimately the increase in $n$ increases the repulsion in such a way that at a finite distance from the BH the tendency of matter being thrown out increases and the matter velocity tends to become speed of light. As in the case of accretion around nonrotating BHs, at a high $n$, once $\alpha$ changes to a highest (physically) possible value, gas pressure becomes positive rendering a slower outflow. In the realistic regime, for minimum '$n$' and maximum '$\alpha$', the accretion–wind
velocity profiles in figure 3(b) appear to be similar to that of the adiabatic ones. Low $n$ or negative $n$ implies (almost) adiabatic gas or (almost) isothermal gas equation of state, if $\rho$ is very small, particularly for $\rho < 1$: $p = \alpha \rho - \beta \rho^n \sim -\beta \rho^N \sim \beta \rho^N$ for $\beta > \alpha$, otherwise $\sim \alpha \rho$, where $N = -n$ and $\beta = -\beta$. On the other hand, for large values of $n$ and $\alpha$ (within the admissible range), both the profiles of wind and accretion velocities exhibit decreasing slope far away from the BH (see figure 3(d)) because of the reason explained earlier.

Figures 4(a)–(d) are for $j = 0.9$ and $\lambda = 2$. The admissible ranges for $\alpha$ and $n$ are $0.04 \leq \alpha \leq 0.055$ and $-1.2 \leq n \leq 0.4$ and the critical point is at $x_c = 6$. These figures are quite similar to figures 3(a)–(d).

For a rapidly rotating BH with $j = 0.998$, the critical point is fixed at $x_c = 6$ and $\lambda$ is chosen to be 1.8. The possible ranges for ‘$\alpha$’ and ‘$n$’ are $0.042 \leq \alpha \leq 0.055$ and $-1.1 \leq n \leq 0.4$. The velocity profiles are presented in figures 5(a)–(d). In figures 5(a) and (b), it is shown that for small ‘$n$’ the solutions are very similar to that in the adiabatic cases irrespective of the choice of $\alpha$. In figures 5(c)–(d), it has been shown that for large ‘$n$’ wind velocity increases abruptly and tends to become the speed of light close to the BH, at
Figure 4. The variation of accretion and wind velocities in disc flows as functions of the radial coordinate for $j = 0.9$. The solid lines represent the accretion, whereas the dotted lines are for wind. Note that the magnitude of velocities is plotted.

a finite distance from the critical point, as in the cases of Bondi and some of the disc flows due to dominance of negative pressure, while the accretion flow has the same nature as in figures 5(a)–(b).

5. Accretion–wind solutions for viable MCG parameters: constraints from the current observed data

In sections 3 and 4, in drawing the accretion and wind curves we have emphasized the changes in the solutions for different extreme values of $\alpha$ and $n$. We have essentially considered the ranges of parameters which give rise to the physical solutions, i.e. the solutions extending from infinity to the BH horizon. In order to obtain such solutions, even we have chosen $n < 0$, which does not support the observed data. The main motivation to choose $n$ negative is to recover the results obtained for adiabatic gas\(^3\) in the present framework.

Based on the dimensionless age parameter $(H_0 - t_0)$ [52] and observed $H(z) - z$ [53] data for both cold dark matter (CDM) and unified dark matter energy (UDME) models, the

\(^3\) The choice of $\alpha = 0$ and $-\frac{1}{3} < n < -\frac{2}{3}$ renders MCG to adiabatic gas.
values of parameters $\alpha$ and $n$ are constrained [51]. In order to obtain a viable cosmology with MCG, $\alpha$ should be restricted to positive values. Besides this, there are best-fit values of the parameters for CDM and UDME models which correspond to $H_0 - t_0$ data. The permissible values of the parameters are given in table 1.

In sections 3 and 4, all the solutions, except for the ones with negative $n$ (reproducing adiabatic cases, particularly for $\alpha = 0$), are in accordance with observed constraint on $\alpha$ and $n$. In figures 6(a)–(d), we show the accretion–wind solutions for best-fit $\alpha$ and $n$ given in
Figure 6. The variation of accretion and wind velocities in disc flows as functions of the radial coordinate with $\alpha = 0.01$ and $n = 0.01$ (best-fit parameters), for (a) Bondi flow, (b), (c), (d) for disc flows around the BH with $j = 0, 0.5, 0.9$, respectively. The solid lines represent the accretion, whereas the dotted lines are for wind. Note that the magnitude of velocities is plotted.

Table 1. We consider different combinations of $\lambda$ and $j$. Figure 6(a) is for spherical accretion, i.e. $\lambda = j = 0$. As $\alpha = n = 0.01$, for $\rho < 1$ (which is the case in our solutions) the magnitude of the first term in equation (1) ($\alpha \rho$) is negligible compared to that of the second term ($\beta \rho^{-\lambda}$). Hence, flow will have negative pressure, but less negative compared to that in figure 1(c) which is for 40% higher $n$. Therefore, in the absence of centrifugal barrier, the critical point forms away off the BH compared to that in figure 1(c). Here we see that no proper wind solution is found for $x_c \lesssim 11.55$. Therefore, the solutions are for $x_c = 11.55$.

Figure 6(b) is for disc accretion around a Schwarzschild BH. Due to the presence of the centrifugal barrier, the radial velocity of matter decreases away from the BH compared to that in figure 6(a), and hence the critical point forms at an inner radius. We choose $x_c = 6$ for figure 6(b). Although the nature of accretion branch is similar to that in figure 6(a), the accretion velocity is very small far from the BH. On the other hand, slightly away from the critical point, the wind velocity increases steeply and becomes almost equal to the speed of light at a smaller radius than that in the Bondi flow shown in figure 6(a). This is because the centrifugal force counteracts the gravitational force until the very inner region of the disc. An
increase in $\lambda$ would make the wind even stronger having velocity close to the speed of light at a radius even nearer to the BH. However, compared to the wind profile in the presence of a stronger negative pressure (e.g. that in figure 2(c)) counteracting gravitational force, the steepness of the wind velocity profile remains small and hence the maximum possible $\lambda$ is restricted to a smaller value.

Figure 6(c) is for disc solutions around a rotating BH with $j = 0.5$ and maximum possible $\lambda$. An increase in $j$ increases the gravitational power of the BH, resulting in a shift of the critical point away from the BH compared to that in figure 6(b) and forms at $x_c = 9$. Finally, the solution in figure 6(d) is for $j = 0.9$ with $x_c = 9$ and $\lambda = 0.4$, which are very similar to that in 6(c).

**6. Variation of sound speed in accretion–wind flows**

Equation (7) implies that $c_s^2 > \alpha$, provided $\beta, n > 0$. However, only positive $\beta$ assures negative pressure when $\alpha \geq 0$ (which is constrained from observed data). Any solution with $c_s^2 < \alpha$ does not imply MCG flow. Therefore, in realistic MCG flows, $c_s$ must have a lower bound which is $\sqrt{\alpha}$. Below we discuss a few typical sound speed profiles in the flows described above.

Figures 7(a) and (c) represent the variation of sound speed square in typical Bondi and disc accretion flows of adiabatic/adiabatic-like/isothermal-like gas. On the other hand, figures 7(b) and (d) represent the same for MCG showing clear lower bounds as respective values of $\alpha$. It is clear from equation (6) that $c_s$ decreases with the increase of $\rho$ for MCG. Hence $c_s^2$ in figures 7(b) and (d) decreases as a function of the radial coordinate, unlike that in flows shown in figures 7(a) and (c). Note that $\rho$ always increases as matter approaches the BH. If $c_s^2 \to \alpha$, then $\rho \to \infty$. This is only possible when flow is very close to the BH event horizon, which indeed is the case in figures 7(b) and (d) when $c_s^2 \to \alpha$ at the $r \to \text{event horizon}$.

Figure 7(a) represents a Bondi flow corresponding to the case shown in figure 1(a). Here the square of sound speed along the wind branch is less than that in accretion branch when $x > x_c$. As $x < x_c$ and matter approaches nearer to the BH, $c_s^2$ in wind increases rapidly, whereas $c_s^2$ in accretion increases much slowly. Figure 7(b) corresponds to the flow in figure 1(c). Here both the accretion and wind sound speeds are very high far away from $x_c$ compared to that in the vicinity of the BH. However, the very high sound speed in wind in figure 7(b) resembles that of close to the BH in figure 7(a). Similar features appear in figures 7(c) and (d) where the sound speed, corresponding to figures 4(a) and (c) showing disc flows around rotating BHs, is shown.

**7. Summary**

We have studied the accretion–wind solutions for MCG around BH. As the universe is expanding and is supposed, in many models, to be filled up with matter having negative pressure, it is very interesting to understand what happens to the BH accretion and wind when infalling matter is MCG. By nature, MCG has a positive pressure at an early stage of the universe and then the pressure becomes negative at latter epochs. Therefore, at late stages the universe experiences an accelerated expansion and as a result a normal accretion procedure must be impacted deeply by the infall of this matter. Indeed our results show that DE in the form of MCG is prone to be thrown outwards from the accretion flows. As the negative pressure increases with the increase of $n$, the wind velocity approaches to the speed of light.
at a near vicinity of the BH when $\rho < 1$, which is generically true for flows around BHs. In general the accretion–wind system of MCG dramatically alters the wind solutions, producing faster winds, upon changes in physical parameters, while accretion solutions qualitatively remain unaffected. This is, however, a natural consequence of the DE into the system.

Note, importantly, that our velocity profiles do not depend on $\dot{M}$ explicitly due to the inviscid assumption when there is no energy equation to be considered. However, it is of considerable interest to estimate the actual amount of MCG accretion onto the super-massive BHs to address the issue of observational consequences of our results which could explain the observed rapid growth of BHs at high redshift [54]. Babichev et al [39] considered the scenario where DE is accreted from the inter-galactic medium. Therefore, the accretion rate in their work is determined by the cosmological evolution of the averaged DE density in the universe. In our view, since the BHs we consider are siting already in the gravitational potential of galaxies (not in that sense isolated BHs), we expect that even before BHs start accreting, host galaxy would have already accreted some amount of DE. Hence the actual rate of accretion of DE onto BHs should really be computed from the ambient value of DE density.
inside host galaxies. We shall consider a detailed analysis of this problem/issue in a later publication.

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