NUCLEAR SCALES

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Nuclear scales are discussed from the nuclear physics viewpoint. The conventional nuclear potential is characterized as a black box that interpolates nucleon-nucleon (NN) data, while being constrained by the best possible theoretical input. The latter consists of the longer-range parts of the NN force (e.g., OPEP, TPEP, the \( \pi \)-\( \gamma \) force), which can be calculated using chiral perturbation theory and gauged using modern phase-shift analyses. The shorter-range parts of the force are effectively parameterized by moments of the interaction that are independent of the details of the force model, in analogy to chiral perturbation theory. Results of GFMC calculations in light nuclei are interpreted in terms of fundamental scales, which are in good agreement with expectations from chiral effective field theories. Problems with spin-orbit-type observables are noted.

1 Introduction

When I looked at the list of participants at this workshop, I realized that nuclear physicists were in the minority (in spite of the workshop title). Because I have nothing to say about the \( ^1S_0 \) partial wave of the nucleon-nucleon (NN) system, I thought that it might be a reasonable idea to discuss topics related to nuclear scales, but from the nuclear physics viewpoint. I will adopt as my purview those scales of nuclear physics that somehow reflect the underlying dynamics. I will examine in roughly equal measure the two- and few-nucleon systems, and say a few words at the end about the many-nucleon problem, where one critical test of naturalness has been made. Ultimately I hope to demonstrate that power counting based on the properties of chiral effective field theories of nucleons and pions is manifested in nuclear properties. This will be developed using a “hybrid” approach, where field-theoretic objects are imbedded in a traditional nuclear potential. Beyond that, this talk can be viewed as a “cultural” exercise.

The few-nucleon systems are arguably the area of nuclear physics that has shown the greatest progress in the past 15 years. Although the two-nucleon problem was solved decades ago, a convincing solution of the triton \(^3\text{H}\) problem at the 1% level was not achieved until 1984. In fairly rapid succession (at roughly the same level of accuracy) we have added \(^4\text{He}\) (the Coulomb interaction was initially a problem), the \( \alpha \) particle \(^4\text{He}\), \(^5\text{He}\) (two \( n-\alpha \) resonances), and the ground and low-lying excited states of \( A = 6, 7, \) and 8 nuclei. Scattering of neutrons and protons on deuterium \((n-d\)
and p - d) has become a sophisticated industry. Reactions (such as radiative capture) have also been treated.

This area of nuclear physics has always been separate and has had one primary raison d’être: studying the nuclear force. The modus operandi has been sophisticated computational methods (such as the Faddeev equations and Green’s Function Monte Carlo [GFMC] techniques) together with brute-force solutions using the biggest and fastest computers. The key computational goal in all of this is attaining 1% accuracy in observables, which is typically good enough.

2 The Nuclear Potential

Why do we do this? Nuclear physics has a predictive theory predicated largely on potential-based dynamics. Given a nuclear potential, V, we can incorporate it into a Hamiltonian and solve the Schrödinger equation (or its equivalent, such as the Faddeev equations). A single (form for) V can lead to hundreds of predictions that can be compared to data. In addition, the tools developed for unraveling nuclear dynamics (such as phase-shift analysis [PSA]) are now so sensitive that they are capable of “measuring” various building blocks in the dynamics that are of interest to both the nuclear- and particle-physics communities. Finally, with the ability to perform detailed calculations comes the capability of determining how the underlying scales are reflected in nuclear properties.

Before we take a detailed look at these topics, a few words of history are in order concerning why it took nuclear physics so long to get into this position. Unlike atoms, where the dominant (by far) interaction is a single type of central force, the nuclear force is much more complicated. This complexity can be gauged by noting that two interacting nucleons with spin-$\frac{1}{2}$ and isospin-$\frac{1}{2}$ have 4 possible spin values and 4 possible isospin values, for a total of 16 possible spin-isospin components. This is reflected in the new AV$_{18}$ and older AV$_{14}$ potentials, which have 18 and 14 components, respectively. Moreover, the dominant type of force in a nucleus is not central, but tensor in nature (like magnetic forces), and noncentral forces make a theorist’s life much, much harder. The ability to handle this complexity depends almost entirely on very large computational resources, and this came rather late (in my career, at least).

Although the use of potentials is conventional in nuclear theory, a reasonable question to ask is: why should anyone take them seriously? There are quite a few parameters in the potential (roughly 3 per spin-isospin component, which are fit to NN data) and the amount of physics present in the interior
region of various force models ranges from modest to none.

To me the conventional nuclear potential is a “black box”, an interpolating mechanism whose input mixes the best available theory with NN scattering data. The output is a set of predictions of nuclear properties, as indicated by the cartoon in Fig. 1. To the extent that the mass of predictions outweighs the number of parameters (it does), the procedure is predictive. To a large extent this workshop is predicated on substituting Chiral Perturbation Theory ($\chi$PT) for the traditional potential in the black box.

There are actually two closely-related black boxes. One is the phase-shift-analysis black box, and the other is the nuclear-potential black box. In the distant past they were not so closely related. Those who did PSA were typically not those who developed potential models, and the latter were usually not fit directly to the NN data base as they are now. Thus, the “first-generation” potentials were not especially good fits to the NN data base. Today’s “second-generation” potentials range from good to excellent fits. In other words, the “black box” parameterizes very well.

The typical strong-interaction physics input is illustrated in Fig. (2). The longest-range part of the strong force is mediated by the lightest meson: the pion. Figure (2a) illustrates the one-pion-exchange potential (OPEP), whose iterates (in the Schrödinger equation) dominate the potential energy, as we will see. This is also the origin of the bulk of the tensor force. An isospin-violating potential (of the same range) that incorporates both a virtual $\gamma$ and $\pi$ is shown in Fig. (2c). Part of the two-pion-exchange potential (TPEP), due to the exchange of uncorrelated (non-interacting) pions, is depicted in Fig. (2b), while the exchange of a typical heavy meson (correlated or interacting pions) is portrayed in Fig. (2d).

Figure (3) schematically illustrates the result. The tail of $V(r)$ is OPEP
Figure 2: Nuclear-force components, some of which (a,b,d,e) are represented (at least schematically) in nuclear potential models. Dashed lines depict pions.

(plus the much smaller $\pi$-$\gamma$-exchange potential). As we move inward TPEP plays a role, followed by heavy-meson exchange and the shortest-range cutoff. The short-range parts have always been problematic. All potentials contain OPEP, while most contain some form of two-pion exchange (although not the asymptotically correct one). Some potentials, such as the $AV_{18}$, have a purely phenomenological interior, while others incorporate known heavy mesons and other physics, yet both types are effective. How can this be so?

Nuclei are basically low-momentum systems. Consequently, short-range parts of potentials are not probed in detail. In essence the situation is like an effective-range expansion: only a few moments of the NN potential are actually needed in almost all applications. Thus any form of parameterization works, whether it is pure phenomenology or a detailed treatment of the physics. This synopsis of the situation (illustrated later) also reflects the philosophy of $\chi$PT: the short-range interactions are counter terms fit to data. If the organization of the problem is effective, only a few such counter terms are required for each spin-isospin component.

The innermost region does more than determine moments of $V$; it regulates the divergences present in any potential. Nuclear mechanisms such as OPE typically fall off slowly in momentum space (or not at all), and this leads to divergent loops when the Schrödinger equation is solved. Providing support in momentum space (smoothing short-distance behavior in configuration space)
is the function of these regulators (usually called form factors). Fitted form factors have a typical range of $\Lambda \sim 1$ GeV, which is probably not an accident given the range of convergence of the chiral expansion. Having regulated the loops, the fitting procedure then provides a kind of “renormalization” as well, if we view renormalization as a consistent scheme for relating different experimental observables. These observables are appropriate to the $\sim 1$ GeV scale of the regulators.

However fundamental this description of nuclear-potential methodology is viewed, the black box is very well tuned. Most few-nucleon quantities that are calculated (and there are many) agree well with experiment. I will generally ignore these successes (see, however, the impressive achievements reported in Refs. 1, 2). The few problems that exist are very interesting and appear to be correlated.

Figure (3) also shows the characteristic radius, $b (=1.4$ fm), used in the Nijmegen PSA. Outside that distance OPEP and long-range electromagnetic interactions are incorporated. Inside that distance the treatment is purely phenomenological. One can also relax this separation and fit the entire potential, incorporating as much (or as little) physics in the interior region as desired. The best of the Nijmegen-fitted second-generation potentials have the ($\chi^2$-determined) quality of their best PSA. These ideas are illustrated in Fig. (4), which depicts the $^3P_0$ NN phase shift (in degrees) as a function of lab-frame NN kinetic energy. The dashed curve is the result of no interaction inside $b = 1.4$ fm and OPEP (plus some additional two-pion-range interac-
This seemingly modest physics input reproduces the shape of the phase shift but not the position of its zero. Incorporating an inner region specified by a single parameter leads to the dotted curve, which is an excellent fit in all but the finest details. Two additional parameters produce the ultimate solid curve, which perfectly matches the single-energy phase-shift points with error bars. We note, however, that the Nijmegen PSA is a multi-energy phase-shift analysis (data at all energies are simultaneously fit), which is a superior procedure and the strength of the method. For all but the most sophisticated treatments, a single short-range parameter suffices in the $^3P_0$ partial wave.

![Figure 4: $^3P_0$ phase shift calculated with OPEP tail for $r > b$ (dashed line), and with one (dotted) and 3 (solid) short-range-interaction parameters added.](image)

The physics in the tail of the NN force can be quantitatively tested in a PSA. The Nijmegen PSA determines the strength of the various $\pi$-N-N coupling constants ($\pi^0pp$, $\pi^0nn$, and $\pi^\pm np$) while (as a check) determining the $\pi^0$ and $\pi^\pm$ masses that parameterize OPEP (i.e., in the combination $V_\pi(m_\pi r)$). They find that within $\sim 1\%$ errors the 3 coupling-constant combinations are the same, and if they are assumed to be identical their latest results lead to the composite value

$$f^2 = 0.0749(3).$$

(1)

This is equivalent to $G = 13.05(2)$ and $G^2/4\pi = 13.56(5)$, with the Goldberger-
Treiman relation (written as an identity using the discrepancy, $\delta_{GT}$)

$$\frac{G}{M}(1 - \delta_{GT}) = \frac{g_A}{f_\pi},$$

producing

$$\delta_{GT} = 1.9(4)\%,$$

which has a natural size ($\sim m_{\pi}^2/\Lambda^2$). The Nijmegen analysis also finds

$$m_{\pi^0} = 139.4(10)\text{ MeV},$$

$$m_{\pi^+} = 135.6(13)\text{ MeV}.$$  

The very small errors on $m_{\pi}$ underscore the importance of OPEP in the nuclear force. In the less-well-measured reaction $p\bar{p} \rightarrow \Lambda\bar{\Lambda}$, the same type of analysis led to $m_K = 475(30)\text{ MeV}$, which agrees with measured masses. Equation (1) is also consistent with recent $\pi N$ PSA’s. Given this remarkable sensitivity to OPEP, can other components of the nuclear force be determined, as well?

It might be viewed as unremarkable that OPEP can be shown to be an important part of the nuclear force; it has to be there and its size is known. The TPEP is another matter entirely. This potential has had a checkered history, because it is the first part of the nuclear potential that is affected in leading order by off-shell choices and the first part that is strongly model dependent. The model dependence is obvious: chiral symmetry plays an essential role in weakening this force.

The off-shell dependence is a very old story, but one that continues to be retold (including at this workshop). The physics is very simple: a potential is a subamplitude meant to be iterated in the Schrödinger equation. As such, it is not an observable (i.e., it is unphysical) and different definitions (viz., different off-shell subamplitudes) are possible that lead to identical on-shell amplitudes (which are observables). Thus, it makes little difference which definition is used, as long as the approach is consistent.

I am aware of three forms of off-shell choices for TPEP at the subleading-order level. The most important distinction is the Brueckner-Watson (BW) vs. Taketani-Machida-Ohnuma (TMO) approach. Schrödinger (old-fashioned) perturbation theory leads naturally to an energy-dependent potential, because energy denominators (from loops, etc.) contain the overall system energy [i.e., $(E - H)^{-1}$], while energy transfers ($q_0$) are contained in pion propagators: $(q^2 + m_{\pi}^2 - q_0^2)^{-1}$. Retaining the energy dependence in OPEP (implicitly) was the BW approach, while eliminating it was the TMO approach. Long ago a variety of formal approaches were developed to deal with removal of energy dependence (reviewed for the deuteron problem in Ref.). Each has its strong
and weak points. The choice of BW or TMO leads to a different TPEP at leading order (see Ref. 21 for a detailed discussion), although a very simple transformation relates the two. At subleading order, unitary equivalences and “form” equivalences (viz., using $H^2\Psi = E^2\Psi$, rather than $H\Psi = E\Psi$) enter. We emphasize that none of these approaches is inherently correct or incorrect. Applications of a particular approach will be either correct or incorrect. I personally prefer the TMO approach because it makes the nuclear many-body problem much more tractable.

The first consistent calculation of TPEP using a chiral Lagrangian was performed by Ordóñez and van Kolck 22, who developed the BW form of the potential (the TMO form was given later in Ref. 21). This result has been recently confirmed by the Nijmegen group 28 (whose methods require the TMO form), who added this potential to OPEP for $r > b$. As a check of their procedure, they verified that TPEP is a function of $(2m_\pi r)$, and that the strength is given by $f^4$. This is an important result that directly tests the chiral mechanism in nuclear forces. Not only does this force significantly improve the overall fits to the NN data (compared to using OPEP alone in the tail of the force), but it will facilitate the construction of “third-generation” potentials 29 that contain improved physics (TPEP plus the $\pi\gamma$ force). Finally, although we know of no direct evidence for heavy-meson exchange, this mechanism is universally accepted.

3 The Few-Nucleon Problem

The full machinery of the GFMC method was unleashed on light nuclei by the Urbana-Los Alamos-Argonne collaboration. Their Hamiltonian contained the AV$_{18}$ potential (incorporating OPEP, a schematic form of TPEP, and a purely phenomenological short-range part) as well as the Urbana IX three-nucleon force [3NF] (which contains a schematic form of two-pion-exchange 3NF [such as Fig. (2e)], but no 4NF, such as Fig. (2f)). Their results can be summarized as follows. The deuteron ($^2$H) and triton ($^3$H) bound states are fit to the NN and 3NF models. The alpha particle ($^4$He) is a prediction and agrees with experiment. The required 3NF is weak. The nucleus $^5$He is actually two $n-\alpha$ resonances ($p_{1/2}$ and $p_{3/2}$) and their calculated splitting is 20-30% too small, but the agreement with experiment is otherwise good. There is underbinding in $A = 6, 7$, and 8 nuclei that is isospin dependent (higher-isospin states are worse). The experimental binding-energy difference of the isodoublet $^3$He and $^3$H (764 keV) is in good agreement with calculations (750(25) keV). The too-small spin-orbit splitting is identical to the so-called “$A_y$ puzzle”, where the nucleon asymmetry in N-d scattering at low energy is too small by comparable
amounts. A recent study\cite{3} of the latter suggests that no modification of the NN force allowed by NN data can account for the puzzle. It therefore appears plausible that a better 3NF containing more physics should be employed. Such forces will contain operators that have a spin-orbit character (one such term\cite{27} is required by Lorentz invariance), although only detailed calculations will determine whether magnitudes (or even signs) of such terms are appropriate to resolve the problems.

Finally, one can use these calculations to estimate the size of various contributions to the energy. Using results for $A = 2, 3, \text{ and } 4$, one finds

$$\frac{\langle V_\pi \rangle}{\langle V \rangle} \sim 70 - 80\%,$$

$$\langle V_\pi \rangle \sim -15 \text{ MeV/pair},$$

$$\langle V_{sr} \rangle \sim -5 \text{ MeV/pair},$$

$$\langle V_{3NF} \rangle \sim -1 \text{ MeV/triplet},$$

$$\langle T \rangle \sim 15 \text{ MeV/nucleon},$$

$$\langle V_C \rangle \sim \frac{2}{3} \text{ MeV/pp pair}.$$

(4a)\(4b\)\(4c\)\(4d\)\(4e\)\(4f\)

The quantities $V_\pi, V_{sr}, V_{3NF}, V_C$, and $T$ are the OPEP, short-range, 3NF, Coulomb, and kinetic-energy parts of the Hamiltonian. Our final task is to see whether these sizes can be understood in terms of nuclear scales.

### 4 Chiral Scales

The building blocks of $\chi PT$ are pion fields, derivatives, and nucleon fields coupled to form zero-range interactions. According to Manohar and Georgi\cite{31}, the generic Lagrangian can be written schematically as

$$\Delta L = c_{\ell mn} \left( \frac{\bar{N}(\ldots)N}{f_\pi^2 \Lambda} \right)^\ell \left( \frac{\pi}{m_\pi} \right)^m \left( \frac{\partial^\mu, m_\pi}{\Lambda} \right)^n f_\pi^2 \Lambda^\Delta,$$

(5)

where $\Delta = n + \ell - 2 \geq 0$ is the chiral constraint that makes $\chi PT$ work, $f_\pi = 92.4$ MeV, $\Lambda \sim 1$ GeV (as before), and $c_{\ell mn}$ $\sim 1$. The latter condition, “naturalness”, makes it possible to check scales in a convincing manner. Various matrices can be inserted between the nucleon fields. Note that the dependence on $\Lambda$ is given by $\Lambda^{-\Delta}$. Moreover, if we set $m = n = 0$ and look at the resulting class of zero-range interactions $[(\bar{N}N)^\ell]$, we see that $\ell = 2$ corresponds to a two-body interaction, $\ell = 3$ to a three-body interaction, $\ell = 4$ to a four-body interaction, etc., and they behave as $\Lambda^0, \Lambda^{-1}, \Lambda^{-2}, \ldots$, respectively. Since $\bar{N}N$ is like a nuclear density, and the density of nuclear matter is
\[ \sim 1.5 f_\pi^3, \text{ the relative size of these many-body operators is } \sim \left( \frac{1.5 f_\pi}{\Lambda} \right)^\ell \sim \left( \frac{1}{\Lambda} \right)^\ell \]

in normal nuclei (or smaller), and this converges fairly rapidly.

Is there a way to be more quantitative? Nuclear physics is not usually formulated in terms of zero-range forces. However, in 1992 a group at Los Alamos performed relativistic-Hartree-approximation calculations using zero-range forces of the same generic form as Eq.(5), but with no pions. Their motivation was to find an easy way to do Hartree-Fock calculations, where two nucleons are exchanged in an interaction. For zero-range interactions this can be rewritten using Fierz identities as a linear combination of the original Hartree terms. Thus, a single calculation is at the same time both Hartree and Hartree-Fock. Since the Fierz coefficients are numbers on the order of 1, if the Hartree coefficients \( c_{\ell mn} \) are natural, the Hartree-Fock results (simply a rearrangement for each class of Hartree term) will also be natural.

The calculation of Ref. fit the properties of 3 nuclei with 9 parameters [4 types of \( c_{200} \)'s, \( c_{300} \), two \( c_{400} \)'s, and two \( c_{202} \)'s] and predicted (quite accurately) the properties of 57 other nuclei. A large number of possible operators of orders \( \Lambda^0, \Lambda^{-1}, \text{ and } \Lambda^{-2} \) were not included (the authors were not trying to perform a chiral expansion), but the exercise was very successful. During Bryan Lynn’s visit to Los Alamos in 1994, we realized that the coupling constants of

![Figure 5: Negative logarithm of dimensionful parameters, \( |x_{\ell mn}| \). Negative values of \( x_{\ell mn} \) are shaded.](image-url)
that calculation could be checked for naturalness. The results delighted us. Although their coupling constants had been fitted using a $\chi^2$ procedure, their published values (used by Bryan and me) were, however, not the minimum-$\chi^2$ solution, which seemed to them not to be as predictive as the one chosen. The best-$\chi^2$ solution is shown graphically in Figs. (5) and (6). Figure (5) shows the coefficients (called $|x_{\ell mn}|$) with the dimensional factors of $f_\pi$ and $\Lambda$ (in units of MeV$^{-n}$) included. The 5 negative coefficients are ordered to

![Figure 6: Magnitude of dimensionless parameters $|c_{\ell mn}|$ (if greater than 1), and $1/|c_{\ell mn}|$ (shaded, if less than 1).](image)

the right. The coefficients group according to the dimensionful factors (e.g., the lowest values are the 4 $c_{200}$'s), which vary over 14 orders of magnitude. Figure (6) shows the dimensionless parameters grouped in the same way as in Fig. (5). If a $|c_{\ell mn}|$ is greater than one (in magnitude) it is plotted with no shading, while if it is less than one, $1/|c_{\ell mn}|$ is plotted instead and the result is shaded. Most of the values are small and natural by anyone’s criterion. The two large values (one positive and one negative) multiply operators with the same nonrelativistic limit, and in that limit the effective coefficient (the sum of the two) is natural (2.0), while the difference (corresponding in a nonrelativistic expansion to higher-order (in $1/\Lambda$) operators) is very large. I suspect that this latter result reflects missing operators in the chiral expansion, but we may never know. Better calculations by Furnstahl and Rusnak have reached the
same conclusion: nuclei reflect fundamental scales and display naturalness. As an aside, we note that the coupling constants of heavy bosons (to nucleons) exchanged between two nucleons can be shown\[11\] to be $\sim \Lambda/f_\pi \sim 10$ if they are natural. Indeed, this is the generic size of all such coupling constants.

5 Nuclear Scales

Having remarked earlier that pions play an exceptional role in nuclei, it is necessary to reintroduce them into the power-counting scheme. First, however, we need to set the momentum scale inside nuclei. This scale can be set\[11\] by an inverse correlation length ($R$), by an uncertainty-principle argument, or by examining Fig. (7), which illustrates energy accrual in the triton. If one fixes the distance between any two nucleons to be $x$, one can easily calculate the potential or kinetic energy accruing inside that distance (which requires integrating over the coordinates of the third nucleon). As $x \to \infty$, clearly all of the energy is calculated, so that after dividing the accrual inside $x$ by the total, the percentage accrual can be plotted, as in Fig. (7). As $x$ increases from the origin, the short-range repulsion is evident (the net potential energy is attractive, so negative values on this plot are repulsive). Most of the action takes place between 1 and 2 fm, corresponding to 100-200 MeV/c, and we choose to take $Q$ (the characteristic momentum) on the order of a pion mass (a convenient mnemonic):

$$Q \sim 1/R \sim m_\pi.$$  

(6)

We note that $Q$ is about half the Fermi momentum, $k_F$, which sets a scale in nuclear matter. We also note that the kinetic and potential energies closely track each other.

In terms of this correlation length or typical momentum, we can estimate the expectation values in a nucleus for various energy operators by rewriting their well-known expressions\[11\] in terms of $Q, f_\pi,$ and $\Lambda$. Doing this we find

$$\langle V_\pi \rangle \sim \frac{Q^3}{f_\pi \Lambda} \sim 25 \text{ MeV/pair},$$  

(7a)

$$\langle T \rangle \sim \frac{Q^2}{\Lambda} \sim 20 \text{ MeV/nucleon},$$  

(7b)

$$\langle V_{\text{nr}} \rangle \sim \frac{Q^3}{f_\pi \Lambda},$$  

(7c)

$$\langle TPEP \rangle \sim \frac{Q^5}{f_\pi^2 \Lambda^3} \sim \frac{3}{4} \text{ MeV/pair},$$  

(7d)

$$\langle V_{\text{NF}} \rangle \sim \frac{Q^6}{f_\pi^2 \Lambda^3} \sim \frac{1}{2} \text{ MeV/triplet}.$$  

(7e)
Figure 7: Percentages of accrual of kinetic energy (solid line), potential energy (short dashed line), and probability (long dashed line) within an interparticle separation, $x$, for any pair of nucleons in the triton.

\[
\langle V_C \rangle \sim \alpha Q \sim 1 \text{ MeV/pp pair}.
\]  

(7f)

This exercise has been conducted in configuration space, and we have used $\Lambda \sim 4\pi f_\pi$. Some creativity was needed for the short-range potential, $V_{sr}$, and (7c) is more like an upper limit. The factor of $Q^3$ in (7a) and (7c) is easy to understand, since it just reflects the phase-space factor needed to convert a momentum-space expression ($\sim Q^0$) to configuration space, while a residual $1/4\pi$ from that exercise produces the $1/\Lambda$ (from $4\pi f_\pi$). We note that these results are consistent with what we inferred earlier from the GFMC calculations of $\Lambda = 2-4$ nuclei.

A more systematic treatment was developed in Ref.11, which leads to

\[
\langle \hat{E}_{\text{int}} \rangle \sim \frac{Q^\nu}{f_\pi^{2n_c} \Lambda^{2L+n_c+\Delta}},
\]

(8a)

\[
\nu = 1 + 2(L + n_c) + \Delta,
\]

(8b)

\[
\Delta = \sum_i (d_i + f_i/2 - 2) \geq 0,
\]

(8c)

where $L$ is the number of loops, and $n_c$ is a topological parameter that equals
the number of interacting nucleons minus the number of clusters (of interacting nucleons). The quantity \( \Delta \) is basically \((n + \ell - 2)\) from Eq. (5) summed over all vertices used to construct the irreducible energy operator \( \hat{E}_{\text{irr}} \). This neat formula summarizes the previous ones, and is equivalent to Weinberg’s result \( \left( Q^\nu \right) \) with factors of \( f_\pi \) and \( \Lambda \) put in.

We note that leading-order calculations \((\Delta = 0, L = 0)\) depend only on \( n_c \), whose smallest value is \( N - 1 \) for \( N \) interacting nucleons. \( N \)-body forces in a nucleus therefore scale as \((Q/\Lambda)^{N-1}\) and decrease as \( N \) increases. This is also a minimal condition for nuclear tractability, since very large \( N \)-body forces would nullify our calculational ability in nuclei. Each new nucleus (as we increase \( N \)) would require large, new, and very complicated forces for its description.

Naive applications of Weinberg power counting were shown to be ineffective in several talks \cite{36, 37, 38} at this workshop, and yet we have seen that nuclei exhibit power counting. How can both be true? It is in treating the reducible graphs that the original power counting has difficulty. The reducible graphs are necessary in order to build nuclear wave functions (in the language of traditional nuclear physics) obtained from iterating the Schrödinger equation. Equation (8a) summarizes and uses that wave-function information (however obtained) in calculating the expectation value of an irreducible operator. Hence the two seemingly contradictory statements are not in disagreement. The power counting we have seen here refers to how various \( N \)-body building blocks work inside a nucleus. This was called a “hybrid” approach earlier. Summing reducible graphs with a credible power-counting scheme is much more difficult, as we have seen at this workshop.

6 Summary

In summary, we have shown that OPEP dominates in few-nucleon systems. It has been “measured” in the Nijmegen PSA program. Very recently TPEP has been similarly “measured”, and the chiral form of this force has been verified. We have argued that the short-range parts of the nuclear potential enter the physics at low energy in the form of moments, only a few of which optimally incorporate the constraints of the \( NN \) data for any spin-isospin combination in the nuclear force. Most calculated few-nucleon observables agree very well with the experimental data, with a notable exception being spin-orbit-type observables, such as \( A_y \) in \( N - d \) scattering and the \( p_{1/2} - p_{3/2} \) splitting in \(^6\text{He}\). There is also a problem with higher-isospin states for \( A \geq 6 \). Isospin violation in \(^3\text{He} - ^3\text{H}\) is well understood and is consistent with natural mechanisms. Finally, three-nucleon forces are weak \((\sim 1\%)\) but necessary when used with
conventional $NN$ interactions. The weakening of $N$-body forces is a necessary condition for nuclear (calculational) tractability.

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