Dust ion-acoustic shocks in quantum dusty pair-ion plasmas

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Abstract

The formation of dust ion-acoustic shocks (DIASs) in a four-component quantum plasma whose constituents are electrons, both positive and negative ions and immobile charged dust grains, is studied. The effects of both the dissipation due to kinematic viscosity and the dispersion caused by the charge separation as well as the quantum tunneling due to the Bohm potential are taken into account. The propagation of small but finite amplitude dust ion-acoustic waves (DIAWs) is governed by the Korteweg-de Vries-Burger (KdVB) equation which exhibits both oscillatory and monotonic shocks depending not only on the viscosity parameters \( \eta_{\pm} = \mu_{\pm} \omega_{p-} / c_s^{2} \) (where \( \mu_{\pm} \) are the coefficients of kinematic viscosity, \( \omega_{p-} \) is the plasma frequency for negative ions and \( c_s \) is the ion-sound speed) but also on the quantum parameter \( H \) (the ratio of the electron plasmon to the electron Fermi energy) and the positive to negative ion density ratio \( \beta \). Large amplitude stationary shocks are recovered for a Mach number \( (M) \) exceeding its critical value \( (M_c) \). Unlike the small amplitude shocks, quite a smaller value of \( \eta_{+}, \eta_{-}, H \) and \( \beta \) may lead to the large amplitude monotonic shock structures. The results could be of importance in astrophysical and laser produced plasmas.

Keywords: Dust ion-acoustic waves, Shock waves, Quantum plasmas, Kinematic viscosity.

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1. Introduction

Physically, a pair-ion plasma is similar to an electron-positron plasma, in which particles have the same mass and opposite charges. Negative ions are found to be an extra component (which may occur naturally or may be injected from external sources) in most space and laboratory plasmas [1,2]. There are a number of works for investigating such pair ion plasmas. Recently, Kim and Merlino [3] have discussed the conditions under which dust, when injected into a laboratory negative ion plasma, becomes positively charged for very large values of negative ion density $\gtrsim 500$ times the electron density. Rapp et al [4] have discussed the possible role of negative ions in explaining their observations of positively charged nanoparticles in the mesosphere under nighttime conditions. Cooney et al [5] have investigated a two-dimensional soliton in a pair-ion plasma. Also, the role of negative ions in a laboratory dusty plasma have been discussed by Klumov et al [6]. An experimental investigation of the effects of negative ions on shock formation in a collisional Q-machine plasma has been made by Luo et al [7]. Moreover, It has been pointed out that such pair-ion plasmas have potential applications in the atmosphere of D-region of the Earth’s ionosphere, the Earth’s mesosphere, the solar atmosphere as well as in the microelectronics plasma processing reactors [8]. Takeuchi et al [9] in their work have reported the experimental observations of ion-acoustic shocks in an unmagnetized plasmas whose constituents are electrons as well as positive and negative ions. They observed wave steepening of positive or negative jumps for certain values of the positive to negative ion density ratio ($\beta = n_{+0}/n_{-0}$). They suggested that their experimental observations can be well explained by means of the Korteweg de-Vries (KdV) equation in which the nonliner coefficient ($A$) can change its sign when a significant fraction of negative ions is present in an electron-positive ion plasma. Various laboratory experiments [10] have been conducted over the last few years to study the formation of dust ion-acoustic shocks (DIASs) in dusty plasmas. Dust ion acoustic compressional pulses have been observed to steepen as they travel through a plasma containing negatively charged dust grains. Theoretical models [11] have been proposed to explain the formation of small amplitude DIASs in terms of the Korteweg-de Vries-Burger (KdVB) equation, in which the dissipative terms comes from the dust charge perturbations [12] and kinematic viscosity [13].

Quantum plasmas, where the finite width of the electron wave functions gives rise to collective effects [14,15], are currently an emerging field of research area. In view of its
potential applications in micro-electronic devices [16], nanoscale systems [17], in laser fusion plasmas [18], next generation high intensity light sources [19,20] as well as in dense astrophysical environments [21], various collective processes have been investigated in a number of research works [e.g. see Refs. 22-26].

There has also been much interests in investigating the structure and dynamics of shocks in quantum like systems, such as nonlinear optical fibers and Bose-Einstein condensates [27-29]. The structure of such shocks are quite different from the classical ones where the shocks are typically governed by the transport processes, i.e., the viscosity and thermal conduction. Unlike classical fluids, quantum plasmas typically exhibit dispersion due to the quantum tunneling associated with the Bohm potential instead of dissipation [22-26]. For this reason, even a quantum shock propagating with constant velocity in a uniform media does not exhibit a stationary structure. Transition from initial to compressed quantum media occurs in the form of a train of solitons propagating with different velocities and with different amplitudes [22-26]. Such a train of solitons also provides a non-monotonic transition from the initial to final state of the medium. However, there are various quantum plasma systems in which both dissipation and dispersion play roles. Such important roles have been studied in different quantum plasma systems for the formation of ion-acoustic shocks (where the dissipation is due to kinematic viscosity) recently by Sahu et al [30] and Misra et al [26].

It is therefore of interest to examine the effects of kinematic viscosity as well as the quantum mechanical effects on the propagation of dust ion-acoustic waves (DIAWs) and to show how the dispersion caused by the charge separation as well as the density correlation due to quantum fluctuation, and the dissipation due to kinematic viscosity play crucial roles in the formation of shock waves instead of solitary wave solutions. We also investigate how the presence of negative ions modify the wave structures in a multi-component dusty quantum plasma. This is the aim of the present investigation. By using the standard reductive perturbation technique (RPT), the small amplitude DIAWs is described by the Korteweg-de Vries-Berger (KdVB) equation, where the Burger term appears due to kinematic viscosity determined by both positive and negative ions. The equation is then numerically solved to show that either oscillatory (dispersion-dominant case) or monotonic (dissipation-dominant case) shock wave solutions are possible to exist depending on the nondimensional quantum diffraction parameter $H$, the viscosity parameter $\eta_+ \pm \eta_-$ and $\beta$, the equilibrium positive to negative ion density ratio.
We have also recovered the large amplitude shock solutions for values of the Mach number \((M)\) exceeding critical value \((M_c)\).

2. Basic equations and small amplitude shock solutions

We consider the propagation of DIAWs in an unmagnetized collisionless quantum plasma composed of electrons, positive and negative ions and immobile negatively charged dust grains. The dynamics of DIAWs in our quantum dusty plasma is governed by the following set of hydrodynamic equations:

\[
0 = \frac{e}{m_e} \frac{\partial \phi}{\partial x} - \frac{1}{m_e n_e} \frac{\partial p_e}{\partial x} + \frac{\hbar^2}{2m_e^2} \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{n_e}} \frac{\partial^2 \sqrt{n_e}}{\partial x^2} \right),
\]

\[
\frac{\partial n_+}{\partial t} + \frac{\partial (n_+ u_+)}{\partial x} = 0,
\]

\[
\frac{\partial u_+}{\partial t} + u_+ \frac{\partial u_+}{\partial x} = -Z_+ e \frac{\partial \phi}{\partial x} + \mu_+ \frac{\partial^2 u_+}{\partial x^2},
\]

\[
\frac{\partial n_-}{\partial t} + \frac{\partial (n_- u_-)}{\partial x} = 0,
\]

\[
\frac{\partial u_-}{\partial t} + u_- \frac{\partial u_-}{\partial x} = Z_- e \frac{\partial \phi}{\partial x} + \mu_- \frac{\partial^2 u_-}{\partial x^2},
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0} (n_e - Z_+ n_+ + Z_- n_-),
\]

where \(n_\alpha, u_\alpha, m_\alpha\) are respectively the density (with equilibrium value \(n_{\alpha 0}\)), velocity and mass for electrons \((\alpha = e)\), positive ions \((\alpha = +)\) and negative ions \((\alpha = -)\); \(\hbar\) is the Planck’s constant divided by \(2\pi\); \(\phi\) is the electrostatic wave potential; \(p_e\) is the electron pressure; \(x\) and \(t\) are respectively the space and time variables, and \(\mu_{\pm}\) is the coefficient of kinematic viscosity due to positive (negative) ions. At equilibrium the overall charge neutrality condition reads

\[
n_{e 0} + Z_- n_{- 0} + Z_{d 0} n_{d 0} = Z_+ n_{+ 0},
\]

where \(Z_{\pm}, Z_{d 0}\) are the charge states for positive (negative) ions and dusts, \(n_{d 0}\) is the equilibrium dust number density. We assume that the ions are cold, and electrons obey the following pressure law [31].

\[
p_e = \frac{m_e v_F^2}{3n_{e 0}^2} n_e^3,
\]
where \( v_{Fe} = \sqrt{2k_BT_{Fe}/m_e} \) is the electron Fermi thermal speed, \( T_{Fe} \) is the particle Fermi temperature given by \( k_BT_{Fe} = \hbar^2(3\pi^2)^{2/3}n_{e0}^{2/3}/2m_e \), \( k_B \) is the Boltzmann’s constant. Now introducing the following normalizations

\[
x \to \omega_p x/c_s, \quad t \to \omega_p t, \quad n_\alpha \to n_\alpha/n_{\alpha 0}, \quad u_\alpha \to u_\alpha/c_s, \quad \phi \to e\phi/(k_BT_{Fe}),
\]

(9)

where \( \alpha = e, +, - \) and \( \omega_{p\alpha} = \sqrt{n_{\alpha 0}e^2/\varepsilon_0m_\alpha} \) is the \( \alpha \)-particle plasma frequency, \( c_s = \sqrt{k_BT_{Fe}/m_-} \) is the quantum ion-acoustic speed, we get the following the normalized set of basic equations as :

\[
0 = \frac{\partial \phi}{\partial x} - 2n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2\mu} \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{n_e}} \frac{\partial^2 \sqrt{n_e}}{\partial x^2} \right),
\]

(10)

\[
\frac{\partial n_+}{\partial t} + \frac{\partial (n_+u_+)}{\partial x} = 0,
\]

(11)

\[
\frac{\partial u_+}{\partial t} + u_+ \frac{\partial u_+}{\partial x} = -m \frac{\partial \phi}{\partial x} + \eta_+ \frac{\partial^2 u_+}{\partial x^2},
\]

(12)

\[
\frac{\partial n_-}{\partial t} + \frac{\partial (n_-u_-)}{\partial x} = 0,
\]

(13)

\[
\frac{\partial u_-}{\partial t} + u_- \frac{\partial u_-}{\partial x} = \frac{\partial \phi}{\partial x} + \eta_- \frac{\partial^2 u_-}{\partial x^2},
\]

(14)

\[
\frac{\partial^2 \phi}{\partial x^2} = \mu n_e - \beta n_+ + n_-,
\]

(15)

where \( \mu = n_{e0}/Z_- n_{-0}, \beta = Z_+ n_{+0}/Z_- n_{-0} \) connected through the charge neutrality condition [Eq.(7)] \( \mu = \beta - 1 - \delta \) with \( \delta = Z_{d0}n_{d0}/Z_- n_{-0}, m = m_-/m_+, \eta_\pm = \mu_\pm \omega_p/c_s^2 \) and the nondimensional quantum parameter \( H = \hbar\omega_{pe}/(k_BT_{Fe}) \) (the ratio between the electron plasmon energy and the electron Fermi energy) proportional to quantum diffraction. Now, integrating once the Eq. (10) with the boundary conditions viz. \( n_e \to 1, \frac{\partial n_e}{\partial x} \to 0 \) and \( \phi \to 0 \) at \( \pm\infty \) we have

\[
\phi = -1 + n_e^2 - \frac{H^2}{2\mu} \frac{1}{\sqrt{n_e}} \frac{\partial^2 \sqrt{n_e}}{\partial x^2}.
\]

(16)

In order to investigate the propagation of small but finite amplitude DIAWs and to derive the required governing equation in our quantum dusty pair-ion plasma, we stretch the independent variables as \( \xi = \epsilon^{1/2}(x - \lambda t), \tau = \epsilon^{3/2}t \) with \( \eta_\pm = \epsilon^{1/2}\eta_{\pm 0} \), while \( \eta_{\pm 0} \) is a finite quantity of the order of unity, while the dependent variables are expanded as
\[ n_\alpha = 1 + \epsilon n_\alpha^{(1)} + \epsilon^2 n_\alpha^{(2)} + \epsilon^3 n_\alpha^{(3)} + \cdots, \]

\[ u_\alpha = 0 + \epsilon u_\alpha^{(1)} + \epsilon^2 u_\alpha^{(2)} + \epsilon^3 u_\alpha^{(3)} + \cdots, \]

\[ \phi = 0 + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \cdots, \]

where \( \alpha = e, +, - \) and \( \epsilon \) is a small nonzero parameter proportional to the amplitude of the perturbation. Now, substituting the expressions from Eqs.(17)-(19) into the Eqs.(10)-(16) and collecting the terms in different powers of \( \epsilon \), we obtain in the lowest order of \( \epsilon \) the dispersion law:

\[ \lambda = \pm \sqrt{\frac{2(1 + m\beta)}{\mu}}. \]

Which shows that the DIAW can propagate outward or inward depending on the positive or negative sign of \( \lambda \), which increases (decreases) with \( \beta (\mu) \). Now, in the next higher order of \( \epsilon \) we eliminate the second order perturbed quantities from a set of equations to obtain the required KdVB equation for DIAWs in our quantum plasma.

\[ \frac{\partial n}{\partial \tau} + A n \frac{\partial n}{\partial \xi} + B \frac{\partial^3 n}{\partial \xi^3} + C \frac{\partial^2 n}{\partial \xi^2} = 0, \]

where \( n \equiv n_e^{(1)} \) and the coefficients of the nonlinear \( (A) \), dispersive \( (B) \) and dissipative \( (C) \) terms are

\[ A = \sqrt{\frac{1 + m\beta}{2\mu} - \frac{3\sqrt{\mu}(1 - m^2\beta)}{2(1 + m\beta)^3}}, B = \frac{1 + m\beta}{128\mu^3} (16 - H^2), \]

\[ C = -\frac{\eta_{-0} + m\beta \eta_{+0}}{2(1 + m\beta)}. \]

We find that in Eq.(21) all the coefficients \( A, B \) and \( C \) are modified by the inclusion of negative ions, while \( B \) and \( C \) are further modified by the effects of quantum diffraction and the effects of kinematic viscosity respectively. In order that the DIAWs propagate with finite velocity \( \lambda \) we must have \( \mu(\equiv \beta - 1 - \delta) > 0 \) or \( \beta > 1 + \delta \). Note that in absence of viscosity term, Eq.(21) reduces to a usual KdV equation for the propagation of DIAWs, whereas for \( H = 4 \) (for which \( B = 0 \)) it reduces to a purely Burger equation.
In order to investigate the nonlinear dynamics of DIAWs we have numerically solved the KdVB equation (21) for different sets of parameters. The equation (21) has a solution that can be represented as an oscillatory shock. However, when the dissipation overwhelms the DIAW dispersion and when the dissipative effect (kinematic viscosity) is in nice balance with the nonlinearity arising from the nonlinear mode coupling of finite amplitude DIAWs in a dusty pair-ion plasma, we indeed have the possibility of monotonic compressive or rarefactive shocks. The dynamics of the latter is governed by a Burger equation. In the numerical procedure the KdVB equation was advanced in time with a standard fourth-order Runge-Kutta scheme with a time step of $10^{-3}$ s. The spatial derivatives were approximated with centered second-order difference approximations with a spatial grid spacing of 0.2 m.

The numerical solution of Eq.(21) is shown in Figs.1 and 2 for different values of the system parameters $\beta$, $\eta_{\pm}$, $\delta$ and $H$. Figure 1 shows the oscillatory shock profiles of the electron density perturbation at the end of $\tau = 0.3$ s for $m = 4$, $\beta = 4$, $\eta_{+0} = 0.3$, $\eta_{-0} = 0.2$, $\delta = 0.4$ and $H = 0.2$. Even for small but non-zero values of $H$ and $\eta_{\pm0}$ we observe a train of oscillations ahead of the shock front decaying at $\xi \to -\infty$. Increasing the role of quantum effects ($H = 3$) we observe a less number of oscillations ahead of the shock wave. The amplitude of the shock front seems to increase with the increasing values of $H$ and $\beta$, but decreases with $\delta$. The oscillations decay quite fast in case of relatively higher values of $H$ and $\beta$ and quite slow at higher $\delta$. Moreover, in case of large quantum effects ($H = 3$) and large dissipative effects ($\eta_{+0} = 1.0$, $\eta_{-0} = 0.9$) and $\beta = 10$, $\delta = 0.4$ we recover monotonic transition from the oscillatory shocks (Fig.2). For $H < 3$, we again find few oscillations ahead of the shock in which first few oscillations at the wave front will be close to solitons. The stationary solution of Eq.(21) can also be obtained analytically. We find that $C$ always negative, $B \leq 0$ according as $H \geq 4$. Since $H > 4$ corresponds to a lower density region (as can be found from the Fermi temperature-density relation mentioned earlier) we consider $H < 4$, so that $B > 0$. Also, since $m$, $\beta > 1$, $A > 0$. Thus, in our purpose $A > 0$, $B > 0$ and $C < 0$. In order to find a stationary solution of Eq.(21) we use the transformation $\zeta = \xi - U_0\tau$ in Eq.(21), where $U_0$ is the normalized velocity of the DIA shock waves and obtain the following equation

$$B \frac{d^2 n}{d\zeta^2} + C \frac{dn}{d\zeta} + \frac{1}{2} A n^2 - U_0 n + (U_0 - \frac{1}{2} A) = 0,$$  

(23)

where we have imposed the boundary conditions $n \to 1$, $dn/d\zeta$, $d^2n/d\zeta^2 \to 0$ as $\zeta \to \infty$. Eq.
(23) describes a shock wave [32] whose velocity in the moving frame of reference is \( U_0 \). Since the nature of the shocks depends on the system parameters \( \beta, \eta_{\pm}, \delta \) and \( H \), we consider the case where the dissipation term dominates over the dispersive term. In that case, Eq.(23) reduces to

\[
(U_0 - An) \frac{dn}{d\zeta} = C \frac{d^2n}{d\zeta^2}.
\]

Which yields upon integration the following monotonic compressive shock solution

\[
n = \frac{U_0}{A} [1 - \tanh \{ - \frac{U_0}{2C} (\xi - U_0 \tau) \}]
\]

with the shock speed \( U_0 \), the shock height \( U_0/A \) and the shock thickness \(-C/U_0\). Since, \( A \) increases with \( \beta \), and \(-C\) increases with \( \beta \) and \( \eta_{\pm} \), the shock height decreases as \( \beta \) increases, and the thickness increases with increasing values of \( \beta \) and \( \eta_{\pm} \). When the dissipative effects are small, the shock will have an oscillatory profile, whereas for large values of \( \eta_{\pm} \sim 1 \), the shock will have a monotonic behavior as is seen in the numerical solution of Eq.(21). To determine the values of \( \eta_{\pm} \) and \( \beta \) analytically (since \( C \) depends on both \( \eta_{\pm} \) and \( \beta \) corresponding to monotonic or oscillatory shock profiles, we investigate the asymptotic behaviors of the solution of Eq.(23) for \( \zeta \rightarrow -\infty \). We substitute \( n(\zeta) = 1 + N(\zeta), N << 1 \) in Eq.(23) and linearize to obtain

\[
B \frac{d^2N}{d\zeta^2} + C \frac{dN}{d\zeta} + (A - U_0)N = 0,
\]

The solution of Eq.(26) are proportional to \( \exp(p\zeta) \), where

\[
p = \frac{-C}{2B} \pm \sqrt{\frac{C^2}{4B^2} - \frac{A - U_0}{B}}.
\]

It turns out that the DIA shock wave has a monotonic or oscillatory profile according as \( C^2 \gtrless 2(A - U_0)B \). The stationary oscillatory solution of Eq.(21) is obtained as

\[
n = 1 + D \exp \left( -\frac{\zeta C}{2B} \right) \cos \left( \frac{\zeta}{\sqrt{\frac{A - U_0}{B}}} \right),
\]

where \( D \) is a constant and \( \zeta = \zeta - U_0 \tau \).

3. Large amplitude shock solutions
We now consider large amplitude planar stationary shock. In the moving frame of reference \( \varsigma = x - Mt \), the basic normalized equations (10)-(15) may be integrated as

\[
\phi = -1 + n_e^2 - \frac{H^2}{2\mu} \frac{1}{\sqrt{n_e}} \frac{d^2 \sqrt{n_e}}{d\varsigma^2}
\]  

(29)

\[
n_\pm = \frac{M}{M - u_\pm}
\]  

(30)

\[-Mu_\pm + \frac{1}{2}u_\pm^2 = (-\mu, 1)\phi + \eta_\pm \frac{du_\pm}{d\varsigma}
\]  

(31)

\[
\frac{d^2 \phi}{d\varsigma^2} = \mu n_e - \beta n_+ + n_-
\]  

(32)

Eliminating the variables \( n_e \) and \( \phi \) and using the quasi-neutrality condition \( \mu n_e = \beta n_+ + n_- \) we obtain the following desired set of equations

\[
\frac{dn_+}{d\varsigma} = N_+,
\]  

(33)

\[
\frac{dn_-}{d\varsigma} = \left( f_1 - \frac{M \eta_+ N_+}{n_+^2} \right) \frac{n_-^2}{\mu Mn_-},
\]  

(34)

\[
\frac{dN_+}{d\varsigma} = f_2 + f_3 \left( f_1 - \frac{M \eta_+ N_+}{n_+^2} \right) \frac{n_-^2}{mMn_-} + f_4 N_+,
\]  

(35)

where

\[
f_1 = -M^2 \left[ \left( 1 - \frac{1}{n_+} \right) + m \left( 1 - \frac{1}{n_-} \right) \right] + \frac{M^2}{2} \left[ \left( 1 - \frac{1}{n_+} \right)^2 + m \left( 1 - \frac{1}{n_-} \right)^2 \right],
\]  

(36)

\[
f_2 = -4\mu \frac{\beta n_+ - n_-}{H^2 m (f + \beta)} \left[ \frac{m + M^2 \left( 1 - \frac{1}{n_+} \right) - \frac{M^2}{2} \left( 1 - \frac{1}{n_+} \right)^2 + \frac{M \eta_+ N_+}{n_+^2}}{n_-^2} \right] \left[ \frac{m n_-}{\mu n_-} \left( f_1 - \frac{M \eta_+ N_+}{n_+^2} \right) \frac{n_-^2}{n_+^2} \right],
\]  

(37)

\[
f_3 = \left[ \frac{2n_-}{mMn_-} \left( f_1 - \frac{M \eta_+ N_+}{n_+^2} \right) - \frac{M}{n_- \eta_-} \right] / (f + \beta),
\]  

(38)

\[
f_4 = \frac{n_-^2}{m \eta_- n_+^3} (2n_+ + M) / (f + \beta), \quad f = \frac{\eta_+ n_-^2}{m \eta_- n_+^3}
\]  

(39)
We have numerically solved the system of equations (33)-(35) by Runge-Kutta scheme starting from the initial conditions $n_+ = n_- = 1.01, N_+ = 0.001$ with a step size $\Delta \zeta = 0.001$. It is found that the perturbations develop into shock waves provided the Mach speed $(M)$ exceeds its critical value $M_c = 5$. For $m = 4, \beta = 4, \delta = 0.1, \eta_{++} = 0.02, \eta_{--} = 0.01$ and $H = 0.4$, the oscillatory shock solution is shown to exist for $M = 6$ in Fig.3. As the value of $H$ decreases, the number of oscillations ahead of the shock decreases together with the increase of the amplitude at the upstream side. The effects of $\delta$ is to increase both the amplitude and width of the oscillatory shock profile. Also, increasing the value of $\beta$(say, 10), we can clearly see a less number of peaks and troughs at the upstream side ($\zeta = 0$). Unlike the small amplitude case, the large amplitude DIA monotonic shocks can exist even for comparatively small values of $\eta_{\pm 0}$. As for example, for $M = 6, m = 4, \beta = 4, \delta = 0.1, \eta_{++} = 0.2, \eta_{--} = 0.1$ and $H = 0.4$ the monotonic shock profile is shown in Fig.4. Further increasing the value of the quantum parameter $H$, e.g., for $H = 1$, the monotonic profile again transits into the oscillatory one (Fig.5). We would like to stress that we have obtained both small and large amplitude stationary and nonstationary DIA shock structures in our quantum plasma model; the train of oscillations propagate along with the shock with the same velocity.

4. Discussions and Conclusions

In this paper, we have investigated the nonlinear propagation of DIAWs in a four-component quantum plasma composed of electrons, positive and negative ions and stationary negatively charged dust grains. Both the dissipative (due to kinematic viscosity) and dispersive (due to Bohm potential) effects are taken into consideration for the formation of DIA shock structures. Both the stationary and nonstationary shock solutions are recovered in our quantum plasma model. The small amplitude nonstationary and stationary DIA shock structures are obtained numerically and analytically from the KdVB equation. The transition from oscillatory to monotonic shocks strongly depends not only on the quantum parameter $H$ and the viscosity parameter $\eta_{\pm}$, but also on the positive to negative ion density ratio $\beta$ (which is mainly due to the inclusion of negative ions). The shock structures are also modified by the effects of $H, \beta$ and $\delta$. Numerical simulation reveals that the small amplitude nonstationary shock (monotonic) transition occurs comparatively at larger values of $H(\geq 3), \beta(\geq 8)$ and $\eta_{\pm} \sim 1$, whereas for the large amplitude shocks comparatively lower values of $H(\sim 0.4), \beta(\sim 4)$ and $\eta_{\pm}(\sim 0.2)$ are required. Such significant modifications of the shock
wave front structures in our quantum plasma model could be of interest in astrophysical and laser produced plasmas. Furthermore, it is suggested that the experiments should be designed to look for dust ion-acoustic shocks (DIASs) and solitons in such quantum dusty pair-ion plasmas.

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Figure captions

Figure 1: Small amplitude oscillatory shock profile [nonstationary solution of Eq.(21)] for \( m = 4, \beta = 4, \eta_+ = 0.3, \eta_- = 0.2, \delta = 0.4 \) and \( H = 0.2 \).

Figure 2: Monotonic shock structure [nonstationary solution of Eq.(21)] for \( m = 4, \beta = 10, \eta_+ = 1.0, \eta_- = 0.9, \delta = 0.4 \) and \( H = 3 \).

Figure 3: Large amplitude oscillatory shock profiles for \( M = 6, m = 4, \beta = 4, \delta = 0.1, \eta_+ = 0.02, \eta_- = 0.01 \) and \( H = 0.4 \).

Figure 4: Large amplitude monotonic shock transition from the oscillatory profile in Fig.3 for \( M = 6, m = 4, \beta = 4, \delta = 0.1, \eta_+ = 0.2, \eta_- = 0.1 \) and \( H = 0.4 \).

Figure 5: The monotonic shock profile in Fig. 4 transits into the oscillatory one for \( H = 1 \). Other parameter values remain the same as Fig.4.
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