We present a numerical program, CAESAR, that allows us to resum large logarithmic contributions to jet observables in a fully automated way. As an application we obtain the first next-to-leading logarithmic distributions for event shapes in hadronic dijet production.

1 Why hadronic collisions?

Hadronic collisions constitute one of the most exciting environments for high energy physicists, both theorists and experimentalists. Due to the large centre-of-mass energies that can be reached, they are ideal for the search of new particles. Moreover, they are incredibly rich from the point of view of QCD. In particular, measures of final-state energy flow are important for various aspects.

Besides for the ‘standard’ measurements of the coupling constant $\alpha_s$ [1] and the colour factors [2], final state observables are particularly suited for investigations of the yet poorly known infrared domain of QCD, since they are affected by large non-perturbative (NP) contributions, originating both from hadronisation corrections [3] and from the so-called ‘soft underlying event’ [4].

Event shapes and jet rates (both referred to as ‘jet observables’) are among the most studied of such measures. The study of jet observables in hadronic dijet production is at the very beginning. Theoretically only fixed order perturbative (PT) calculations are available [5], and there exist experimental data for just one event shape [6]. Both studies show clearly that in the region where the observables are small, large infrared and collinear logarithms arise that need be resummed to obtain a sensible answer. The involved kinematics of the process makes analytic calculations really cumbersome, so that such observables are the ideal testing ground for an automated resummation approach such as the one provided by the program CAESAR [7]. The program is based on a master
formula whose input is determined numerically in a preliminary stage. The
user needs only to provide a subroutine that computes the observable given
a set of four-momenta. The program then returns the observable’s resummed
distribution $\Sigma(v)$ (the fraction of events for which the observable’s value is less
than $v$) at next-to-leading logarithmic (NLL) accuracy. $^1$ All the details of the
approach have been explained by Giulia Zanderighi [9]. In the following we
describe through some examples what kind of observables can be studied with
CAESAR, and present some output distributions.

2 Observable definition

The first observable we introduce in hadronic collisions is the transverse thrust,
which represents the analogous of the thrust in $e^+e^-$ in the plane orthogonal
to the beam axis:

$$T_t = \max_{\hat{n}_t} \frac{\sum_i |\vec{p}_{ti} \cdot \hat{n}_t|}{\sum_i |\vec{p}_{ti}|},$$

(1)

where the sum runs over all possible final state hadrons of transverse momenta
$p_{ti}$, and $\hat{n}_t$ is a unit transverse vector. $^2$ The distribution in this observable
needs resummation in the region $\tau_t \equiv 1 - T_t \ll 1$.

It is clear that the sum in (1) cannot include all the final state particles,
since any actual measurement necessarily excludes a region around the beam.
Typically one measures only particles whose rapidity $\eta$ is in the range $|\eta| < \Delta$.
In such a situation a particular class of NLL contributions emerges, the so-
called ‘non-global logs’ [10], which arise whenever an observable is sensitive to
secondary particle emission only in a limited region of the phase space. Their
presence causes a loss of accuracy in NLL predictions, since their expression is
known only in the large $N_c$ limit.

Fortunately, any observable can be made global with just small modifica-
tions of its definition, as we show in the following two examples.

1. Directly global thrust $\tau_{t,g}$:

$$\tau_{t,g} = 1 - \max_{\hat{n}_t} \frac{\sum_i' |\vec{p}_{ti} \cdot \hat{n}_t|}{\sum_i' |\vec{p}_{ti}|},$$

(2)

$^1$We recall that NLL accuracy means resumming all $\exp\{\alpha_s \ln n \ln^2 1/v\}$ and
$\exp\{\alpha_s \ln n \ln^1 1/v\}$ terms in $\Sigma(v)$ [8].

$^2$Here transverse means orthogonal to the beam.
where the sum now runs over all hadrons with $|\eta| < \Delta$ and $\Delta$ is taken as large as possible. Observables of this kind are actually non-global, but non-global effects do not show up at NLL accuracy as long as $v \geq e^{-c\Delta}$, with $c$ an observable’s dependent coefficient (in this case $c = 1$) [7].

2. Indirectly global thrust $\tau_{t,\Delta}$:

$$\tau_{t,\Delta} = 1 - \max_{\vec{n}_t} \frac{\sum' |\vec{p}_{ti} \cdot \vec{n}_t|}{\sum' |\vec{p}_{ti}|} + R_t, \quad R_t = \left| \frac{\sum' |\vec{n}_t|}{\sum' |\vec{p}_{ti}|} \right|.$$  \hspace{1cm} (3)

Here again the sum runs over all particles with $|\eta| < \Delta$, but $\Delta$ can be taken of $O(1)$, since the recoil term $R_t$, due to transverse momentum conservation, makes the observable sensitive also to particles inside the beam region.

Indirectly global observables are known to cause consistency problems for NLL resummations [11]. Actually what any NLL answer assumes is that an observable is kept small by forbidding particle emission above a given momentum scale. In this case $\tau_{t,\Delta}$ can be small not only because all involved momenta are required to be small, but also because vectorial cancellations occur among larger transverse momenta. It also happens that while the probability of vetoing radiation decreases with the observable’s value, that of having vectorial cancellations is independent of that value. Therefore, below a given $\tau_{t,c}$ the second mechanism overcomes the first, and NLL predictions break down developing a singularity. However, as long as $\tau_{t,\Delta}$ gets not too close to $\tau_{t,c}$ NLL predictions are still meaningful. The particular choice of the recoil term in (3) ensures that $\tau_{t,c}$ is away from the range of values of $\tau_{t,\Delta}$ that are accessible through PT calculations.

Analogously we define the two version of the thrust minor

$$T_{m,g} = \frac{\sum' |\vec{p}_{ti} \times \vec{n}_t|}{\sum' |\vec{p}_{ti}|}, \quad T_{m,\Delta} = \frac{\sum' |\vec{p}_{ti} \times \vec{n}_t|}{\sum' |\vec{p}_{ti}|} + R_t,$$ \hspace{1cm} (4)

which are both measures of the energy flow out of the plane formed by the beam and the thrust axis $\vec{n}_t$. 

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3 Some worked out examples

We present results for the directly global thrust and thrust minor, obtained in a fully automated way with the program CAESAR. The program recognises first that the two observables belong to the class for which a NLL resummation is feasible and automatically determines the input needed by the master formula. It then exploits the master formula to produce resummed curves such as the ones shown in fig. 1. These plots show the resummed differential distributions (without matching with fixed order) for $\tau_{t,g}$ and $T_{m,g}$ at the Tevatron II centre-of-mass energy $\sqrt{s} = 1.96\text{TeV}$. The dijet events are selected by requiring two hard jets with $E_t > 50\text{GeV}$ and $|\eta| < 1$. We use the CTEQ6M parton distributions [12] corresponding to $\alpha_s(M_Z) = 0.118$ and set both the renormalisation and factorisation scale at the partonic centre-of-mass energy. From the two distributions clearly emerges the separation among the various partonic channels, information that can be exploited for fits of parton distributions.

![Figure 1: The resummed differential distributions $D(v) \equiv d\Sigma(v)/d\ln v$ for the global transverse thrust (left) and thrust minor (right).](image)

4 Conclusions and outlook

The study of event shapes and jet rates in hadron hadron collisions is particularly important for the understanding of QCD dynamics. We have now a computer code that in a fully automated way provides the resummed distribution of any suitable jet observable in an arbitrary hard process. Much work
remains still to be done, both to refine the existing code and to include automat ed matching with fixed order results. Nevertheless we believe that such a program will open the way to a vast amount of phenomenological studies.

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