Effect of the Weight reduction of a Gear Wheel on Modal Characteristics

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Abstract. The paper deals with the influence of gear weight reduction on its modal properties. The aim of the paper is to use the simulation tools to introduce the dependence of natural frequencies on the change in the shape of the gear, which is reflected in the change in weight. At the beginning of the paper, the theory and experimental bases of vibration testing and analysis are introduced to understand the principles of modal parameter specification. Verification of the boundary conditions used in the simulation was performed using the experimental method and it is described in the next part of the article. Tests confirmed the consistency of the values achieved by both approaches. Thus, the same boundary conditions could be used to specify the dependence of natural frequencies on the change in the geometry of the gear (which is associated with weight reduction). Numerical analysis was performed by FEM analysis using PTC Creo software. Experimental modal analysis was performed using a PULSE measurement system.

1 Introduction

The operation of the machines is accompanied by mechanical vibration. Slight periodically repeated movement causes fatigue of the material, which may result in destruction. The main causes of vibration are dynamic forces accompanying manufacturing inaccuracies of parts and components, the play of moving parts, the interaction of parts with friction and rolling, and the imbalance of parts and components with rotating, oscillating, oscillating and reciprocating movements. Weak vibrations can cause resonance of other parts and components and thus become a source of strong mechanical vibration and noise. In addition to transmitting forces, the machine design is also loaded due to fuel combustion, gas expansion, fluid flow, cyclic changes in the magnetic field, and various external vibration sources. The vibration measurement allows observing the direct dynamic action of the forces transmitted by the machine structure. Knowing the real load of the structure by means of vibration measurement allows operating the machine according to its actual technical condition while observing other conditions [1, 2].

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Oscillation of structures, machines and equipment in technical practice may be either required (vibrating mills, conveyors, vibrating grinders, etc.), but is usually an undesirable phenomenon which should be eliminated to the lowest possible level. Excessive vibration values greatly affect the service life of structures and equipment and are also a source of adverse noise. Their effect leads to loss of energy in systems. For this reason, it is necessary to be able to correctly describe the oscillating movement and, based on this knowledge, to adequately dimension the system. Therefore, we are looking for the response of a dynamic system when investigating oscillations [3].

2 Modal analysis of a gear wheel

Products have to be designed to have acceptable vibration and noise characteristics. There are two ways: an analysis of the measured response (signal) to an unknown excitation and a modal analysis of the sensitivity of the dynamic system as a ratio of the response to the excitation. The goal is to predict the response to expected excitation for linear (or linearized) systems as the product of the determined sensitivity and excitation force. Modal parameters are a measure of the dynamic characteristics of a dynamic system (driving comfort and driving safety). Natural frequencies are detectable by modal analysis. This is based on the task of free non-damped vibration, and its solution can be effectively converted to the problem of eigenvalues. Its output is natural frequencies and natural shapes. It is also the starting point for further dynamic analyses, e.g. for harmonic or transient [4, 5].

When the structures are excited by frequencies that are close or equal to their resonant frequencies, they oscillate in special shapes that are called mode shapes. By understanding the behaviour of the structure in the range of frequencies at which a resonance may occur, the structure can be designed to be stronger and more stable, so it is possible to avoid the destructions caused by unwanted oscillation [6].

2.1. Experimental and numerical analysis

Modal analysis (MA) is usually divided based on the method of deriving the modal model into theoretical and experimental. Experimental is further divided into classical and operational (operational). The result of a theoretical and experimental modal analysis is a modal model from which it is possible to derive modal parameters: natural frequencies, damping and natural modes, respectively. Generally, spectral matrices and modal matrices. In a theoretical modal analysis, the modal model is derived from the knowledge of the physical properties of the system, while in an experimental modal analysis it is derived from the measured frequency transfer functions [7].

The finite element method (FEM) is an approximate method that seeks to solve problems described by differential equations. It is dominated by variation formulation, deformation approach (unknown parameters are displacements) and numerical solution. Since the numerical solution requires a large number of mathematical operations, the development of this method was limited by the development of computer technology [8].

Experimental modal analysis (EMA) is an analysis that is performed on real mechanical structures (prototypes) and describes the mechanical system by a linear mathematical model with a finite number of degrees of freedom. EMA determines the modal parameters of the structure: modal frequency, mode shape and modal damping. These parameters are then used in a given frequency band to create a mathematical model that is described by linearly bound differential equations 2nd order. Their solutions are natural frequencies and natural shapes. The waveforms corresponding to the individual natural frequencies are called modes. Experimental modal analysis is based on the determination of the Frequency
Modal analysis (MA) is usually divided based on the method of deriving the modal model: theoretical or experimental. Theoretical analysis is based only on the system’s physical properties, while in an experimental modal analysis it is derived from the measured frequency transfer functions [7].

The finite element method (FEM) is an approximate method that seeks to solve the differential equations that describe the system's behavior. Its development was limited by the development of computer technology [8]. Since the numerical solution requires a large number of mathematical operations, the deformation approach (unknown parameters are displacements) and numerical solution are used. It is dominated by variation formulation, which is natural for systems described by differential equations. It is also the starting point for further dynamic analyses, e.g. for harmonic or transient [4, 5].

The set of equipment used during the measuring is shown in Figure 1.

![Fig. 1. A set for measuring natural frequencies of the wheel.](image)

On the model, the measuring points (81 points) were highlighted as shown in Fig. 2 left. The distance between the points at which the measurements were realized was specified by the software where the geometry of the model has been created. Next activities were focused on the sensors positioning and on the specification of the location, where the wheel will be excited. The designation of the so-called “reference point” followed in the next step. The excitation of the gear was performed using the PULSE model 2827 - 002 analyser with its additional modules. The 3109-measuring module is used for a geometric model generation and for the measuring points’ design. The 7533-communication module is used to export and import data, while the 2627-A amplifiers have been applied within the measurements, the first due to amplifying the sensor’s signal and the second one in response to amplify the signal in the modal hammer [11].

The excitation of the gear was carried out in this point. The point with number 10 (Fig. 2 left) was stated. In Fig. 2 right is presented the FRF function.

![Fig. 2. The reference point position on the gear wheel (left) and the FRF function (right).](image)

Response Function (FRF). This function expressing dynamic acceleration is given by the relation [9, 10].

\[
H(\omega) = \frac{X(\omega)}{F(\omega)}
\]

where \(F(\omega)\) is a maximum value of a harmonic function \(f(t) = F(\omega)e^{i\omega t}\); \(X(\omega)\) is a complex amplitude of harmonic dependency \(x(t) = X(\omega)e^{i\omega t}\), if \(x(t)\) signifies a movement of oscillating signal as a feedback of the set on excitation, \(\omega\) is an angular velocity and \(t\) is time.

The aim of the preliminary study was to use experimental testing to verify the boundary conditions applied in the FEM analysis of modal gear. The carbon steel C45 was used as the material for tested gear wheel production. In the experimental measurements, the modal analysis of the gear was performed using the PULSE model 2827 - 002 analyser with its additional modules. The 3109-measuring module is used for a geometric model generation and for the measuring points’ design. The 7533-communication module is used to export and import data and A 4374 piezoelectric accelerometer was used as a vibration sensor. The wheel was excited by means of a Brüel & Kjær modal hammer, type 8203, which had a plastic tip. Two identical amplifiers 2627-A have been applied within the measurements, the first due to amplifying the sensor’s signal and the second one in response to amplify the signal in the modal hammer [11].
In parallel with experimental MA, a numerical model of gear has been prepared. The calculation was done by means of FEM (Finite Elements Method) analysis in the software PTC Creo. To complete the boundary conditions, it was necessary to define the basic material properties within the software that are: the specific density of the material $7.82708 \times 10^{-6}$ kg/mm$^3$, the value of Young's modulus $210 \times 10^9$ N/mm$^2$, Poisson's value 0.27, the maximal size of an element within FEM grid was 2 mm.

Experimentally obtained values of natural frequencies of the investigated gear wheel have been specified using the RFP (Rational Fraction Polynomial) function based on the recorded data. They are noticed in Table 1. In the same table, the values of natural frequencies that were obtained by Finite Element Method through the CAD/CAM system PTC Creo are also offered.

**Table 1. Natural frequencies of the gear wheel.**

| Natural frequencies | $f_1$ [Hz] | $f_2$ [Hz] | $f_3$ [Hz] | $f_4$ [Hz] |
|---------------------|------------|------------|------------|------------|
| Numerical method    | 4 207.6    | 8 109.0    | 12 535.3   | 13 640.4   |
| Experimental method | 4 289.3    | 8 212.2    | 12 751.4   | 13 861.3   |

Within the research, the first four mode shapes that correspond to the first four natural frequencies have been also assessed using both experimental and numerical approach. They are presented in Table 2.

**Table 2. Natural modes of the gear wheel.**

| Natural shape | 1$^{\text{st}}$ | 2$^{\text{nd}}$ | 3$^{\text{rd}}$ | 4$^{\text{th}}$ |
|---------------|----------------|----------------|---------------|---------------|
| Numerical method | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) | ![Image](image4.png) |
| Experimental method | ![Image](image5.png) | ![Image](image6.png) | ![Image](image7.png) | ![Image](image8.png) |

It is clear from Tables 1 and 2 that achieved values of natural frequencies are close, so it could be said the boundary conditions were correctly set up. Frequency differences between the values achieved by both approaches were probably due to the fact that the actual body was not freely supported (placed on a soft foam) as well as due to a different number of finite elements and their sizes. While the evaluation of the experimental analysis was a finite element grid; that is, these elements were larger compared to numerical modal analysis in PTC Creo software.

### 2.2 Dependence of natural frequencies on the weight reduction

In the second phase, the modal parameters of gear wheels have been investigated at which the weights were reduced through the variation of the rib geometry of the gear wheel. As the basis, the gear wheel filled by material in the whole volume with the characteristics listed in Table 3 has been designed:

**Table 3. Gear wheel characteristics.**

| Parameter               | Value     |
|-------------------------|-----------|
| Base-circle diameter    | 105 mm    |
| Root diameter           | 112.76 mm |
In parallel with experimental MA, a numerical model of gear has been prepared. The calculation was done by means of FEM (Finite Elements Method) analysis in the software PTC Creo. To complete the boundary conditions, it was necessary to define the basic material properties within the software that are: the specific density of the material $7.82708 \times 10^{-6}$ kg/mm$^3$, the value of Young's modulus $210 \times 10^9$ N/mm$^2$, Poisson's value 0.27, the maximal size of an element within FEM grid was 2 mm.

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| Natural frequencies $f$ [Hz] | Numerical method | Experimental method |
|----------------------------|-----------------|-------------------|
| $f_1$ | 4207.6 | 4289.3 |
| $f_2$ | 8810.8 | 8212.2 |
| $f_3$ | 12653.4 | 12751.7 |
| $f_4$ | 13640.3 | 13861.4 |

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| Natural modes of the gear wheel |
|-------------------------------|

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| Gear wheel characteristics |
|----------------------------|
| Parameter                  | Value                  |
| Base-circle diameter       | 105 mm                 |
| Root diameter              | 112.76 mm              |
| Pitch circle diameter      | 120 mm                 |
| Tip diameter               | 132 mm                 |
| Module                     | 6 mm                   |
| Number of teeth            | 20                     |
| Pressure angle             | 20°                    |
| Gear wheel width           | 30 mm                  |
| Central hole diameter      | 17.5 mm                |

The weight of the wheel has been reduced through the change in geometry while four ribs were designed. During the next investigation, the number of ribs was fixed, while the width of ribs in radial direction has been varied from 12° to 48° with step 6°. The virtual models of individual gear wheels are presented in Fig. 3.

![Virtual models of the gear wheels with a gradual rib width change.](image1)

The same boundary conditions were set up as it was within the preliminary research described in section 2.1. They are as follows: the material of the gear wheel - steel; the specific density $\rho = 7.82708 \times 10^{-6}$ kg/mm$^3$, the value of Young's modulus $E = 210 \times 10^9$ N/mm$^2$ and Poisson's value 0.27. The dependencies of natural frequencies on the rib width and several examples of natural shapes of gear wheels are presented in Fig. 4.

![Dependencies of natural frequencies on the rib width and some natural modes of gear wheels.](image2)

3 Conclusions

Structural vibration problems present significant risks and constraints in the design of a wide range of engineering products. They may be responsible for structural integrity, or they may reduce the performance of the machinery. At the same time excessive vibrations always cause at least excessive noise and disturb the quiet operation of the machinery. Most
of the difficulties related to extreme oscillation are instigated by so-called "noise" modal properties of the mechanical system itself. These modal properties are determined and subsequently evaluated by the modal analysis itself. Thanks to these acquired parameters, we are able to assume system properties [12, 13].

In the article, the experimental and numerical modal analysis of the gear wheel has been carried out within the preliminary research to confirm the boundary conditions of FEM analysis in the CAD/CAM system PTC Creo. Consequently, the dependence of natural frequencies on a weight reduction of a gear wheel has been studied. It is clear based on the theory that the natural frequencies are in close relation to the rigidity of a solid body. However, from the graph presented in Fig. 4 can be noticed that values of natural frequencies increase more slowly at lower natural modes than in the case of higher natural modes. It can be also stated that the shapes of the deformed wheels with various rib width in the same natural mode are almost the same.

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