PAST, PRESENT, AND POSSIBLE FUTURE LIMITS
ON THE PHOTON REST MASS

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ABSTRACT

In an historical context, present limits on the photon rest mass are reviewed. More stringent, yet speculative, limits which have been proposed are mentioned. Finally, new theoretical ideas and possible experimental improvements on the present limits are discussed, along with possible relationships between these two areas.

1. Historical Background

As recounted in reviews on the topic, studies of the force law between electric charges date back to the middle of the 1700's. The first experimental test of the inverse-square law was done in 1769 by Robison, who was partially motivated by the work of Benjamin Franklin. Although Robison's work, and also that of Cavendish, preceded that of Coulomb, their results were not published until decades later. Coulomb published his results in a timely manner and hence received recognition. The results of these authors were parametrized in terms of a force law of the form

\[ F \propto \frac{1}{r^{2+q}}. \]  

as was much later work.

However, with the discovery of the quantum-mechanical generalization of Maxwell's equations, we now know that the inverse-square law can be viewed as a consequence of the masslessness of the gauge particle which mediates the electromagnetic force, the photon. If the photon were to have a mass, then the "massive Maxwell equations," or Proca equations, are

\[ \nabla \cdot E = 4\pi \rho - \mu^2 \Phi, \]  

\[ \nabla \times E = -\frac{1}{c} \frac{\partial H}{\partial t}, \]  

\[ \nabla \cdot H = 0, \]  

\[ \nabla \times H = \frac{1}{c} \frac{\partial E}{\partial t} + \frac{4\pi}{c} J - \mu^2 A. \]  

One still has the definitions

\[ H = \nabla \times A, \]  

\[ E = -\nabla V - \frac{1}{c} \frac{\partial A}{\partial t}. \]
The charge conservation condition is
\[(\Box + \mu^2)A_\nu = \frac{4\pi}{c}J_\nu.\] (8)

One loses gauge invariance and instead must satisfy the Lorentz gauge condition,
\[\partial^\lambda A_\lambda = 0.\] (9)

In the above, \(\mu\) is the photon rest mass, in units of inverse length \((c/\hbar)\).

2. Consequences of \(\mu \neq 0\)

2.1. The Velocity of Light

What would be the consequences of there being a photon mass? The first is that, as with all other massive particles, the velocity of light would not be a constant, and \(c\) would only be the “limiting velocity.” (de Broglie pioneered this idea.)

In particular, the group velocity of light would be
\[\frac{v_g}{c} = \left[1 - \frac{\omega^2}{\omega_\mu^2}\right]^{1/2}, \quad \omega_\mu = \mu c.\] (10)

The best limit on the photon mass using this method comes from measuring the difference in arrival times of different-frequency radio waves from the Crab pulsar.\(^3\) A dispersion is seen. This dispersion could be taken to indicate a photon mass if it were not for the fact that the electron plasma in interstellar space produces the same type of dispersion relation as that of Eq. (10), but with \(\omega_\mu\) replaced by
\[\omega_p = \left(\frac{4\pi n e^2}{m}\right)^{1/2},\] (11)

where \(n\) is the free electron density. Because of other, better photon mass limits, the result is interpreted as being due to an average, interstellar density of \(n = 0.028\) electrons/cm\(^3\). But the dispersion so obtained implies, by itself, a photon mass limit of
\[\mu \leq 6 \times 10^{-12} \text{eV} = 10^{-44} \text{gm} = 3 \times 10^7 \text{cm}^{-1}.\] (12)

2.2. Coulomb’s Law

If the photon has a mass, then the scalar potential is defined by
\[\left(\nabla^2 - \mu^2\right)\Phi(r) = -4\pi \rho(r),\] (13)

meaning it has a Yukawa form:
\[\Phi(r) = \frac{e^{-\mu r}}{r}.\] (14)
Now consider two concentric, conducting, spherical shells of radii \(a\) and \(b\), that are first grounded, then decoupled from ground, and then have a potential applied to the outer shell of radius \(a\). Then the above paragraph means that the electrostatic potential difference between the two shells is no longer zero. Rather, it is

\[
\frac{\Delta V}{V} = \frac{\phi(a) - \phi(b)}{\phi(a)} \simeq \frac{1}{6} \mu^2 (a^2 - b^2) + O[(\mu a)^4],
\]

(15)

where

\[
\phi(r) = K \left[ \frac{e^{\mu r} - e^{-\mu r}}{2\mu r} \right].
\]

(16)

In a sophisticated multi-shell version of this basic idea, Williams, Faller, and Hill obtained the best laboratory limit to date on the photon mass,

\[
\mu \leq 10^{-14} \text{eV} = 2 \times 10^{-47} \text{gm} = 6 \times 10^{-11} \text{cm}^{-1}.
\]

(17)

Before continuing, note the result of both Eqs. (10) and (15), that physical effects of a photon mass first appear in order \((\mu L)^2\), where \(L\) is some scale size of the system. This is a theorem. In particular, it means that to get a good limit, you either have to make a very precise measurement with a small apparatus (as was done for Coulomb’s Law), or else, if the measurement is not as precise, one needs a very large apparatus (as was done with the Crab nebula and with the magnetic measurements of the next subsection).

2.3. Planetary Magnetic Fields

With a massive photon, magnetic multipole fields will change to a Yukawa form just as electric multipole fields do. In particular, a field from a magnetic dipole, \(D\), becomes

\[
\mathbf{H} = \frac{De^{-\mu r}}{r^3} \left[ \left( 1 + \mu r + \frac{1}{3} \mu^2 r^2 \right) (3\hat{z} \cdot \hat{r} \hat{z} - \hat{z}) - \frac{2}{3} \mu^2 r^2 \hat{z} \right]
\]

(18)

In addition to the general “Yukawa” contraction of the size of the field, there is a new last term in Eq. (18), the “external field effect”: \( - \frac{2}{3} \mu^2 r^2 \hat{z} \). On a sphere surrounding the dipole, it appears to be a constant field antiparallel to the dipole. The first person to look for such an effect was Schrödinger, who studied the Earth’s magnetic field. (He was interested in a finite photon mass in conjunction with his ideas to unify gravity and electromagnetism.)

The best application of this method to date, using higher multipoles in the analysis, was done by Davis, Jr., et al. They considered an even bigger magnet than the Earth, Jupiter. Using data from the Pioneer 10 flyby of Jupiter, a limit of

\[
\mu \leq 6 \times 10^{-16} \text{eV} = 8 \times 10^{-49} \text{gm} = 2 \times 10^{-11} \text{cm}^{-1}
\]

(19)

was obtained.

3. More Speculative Limits

For many years now, more speculative limits on the photon mass have been proposed based on considerations of distant astronomical objects with magnetic
fields spread over large volumes. They range from \( \sim 2 \times 10^{-20} \, eV \), from the properties of the galactic magnetic field, to \( \sim 10^{-27} \, eV \), from the properties of interstellar gas in the Small Magellanic Cloud. (Consult Ref. 6 for a discussion of these limits.)

However, the magnetohydrodynamics of these distant, large objects is in no way rigorously understood. Such simple questions as if the plasma is driving the field, or visa versa, remain subjects of debate. It is not understood why these huge magnetized bodies can maintain their coherence over time scales approaching the cosmological. Indeed, this observation leads to the next section.

4. New Theoretical Ideas

4.1. Strings

In normal point field theory, couplings such as \( S A^\lambda A_\lambda \), where \( S \) is a scalar, are zero by gauge invariance. However, Kostelecký, Potting, and Samuel have pointed out that this is no longer true in string theories.\(^7\) In particular, if \( S \) is the gravitational curvature, \( R \), then this coupling could lead to primeval magnetic fields of the sizes presently observed.

4.2. Spontaneous Symmetry Breaking

With the successes of the standard model, it has been natural to ask if the \( U(1) \) of electromagnetism might be broken at some low temperature, thereby yielding a photon mass in that regime. Although dynamical symmetry breaking is ruled out,\(^8\) much interest has been shown in possible spontaneous symmetry breaking.\(^9\)–\(^12\) This is true even though it is hard to find such a theory which would predict a new experimental signature but yet would not already be in conflict with other experiment. Further, there is no hard prediction of what the transition temperature might be. Nonetheless, the idea is fascinating, and has partially stimulated thoughts about one possible new experiment.

5. Possible New Experiments

5.1. Coulomb’s Law

The experiment of Williams, Faller, and Hill\(^4\) remains the best laboratory limit on the photon rest mass. Note that, because of the theorem we mentioned, any increased experimental sensitivity yields a better photon mass limit as \( (\mu L)^2 \). Therefore, one needs to improve the signal to noise ratio of an experiment by a factor of 100 to get an improved photon mass limit of a factor of 10. Thus, it will be a nontrivial task to go beyond the present laboratory limit.

Even so, Henry Hill\(^13\) is considering such an experiment, and has discussed it with his two earlier collaborators.\(^4\) To begin, a significant improvement over the previous experiment appears possible just by advances of standard experimental techniques. Further, another significant reduction in noise should be possible by using dilution refrigeration technology to reduce the temperature of the apparatus to \( mK \). These cryogenic techniques have been developed for Weber-bar gravitational
wave detectors.

This last would allow a first search, however theoretically ill-defined, for a low-temperature phase transition. One would have to try to reduce all electrical signals to as low a level as possible, since signals could ruin a phase transition, just as a magnetic field can ruin superconductivity via the Meissner effect.\textsuperscript{12}

5.2. Solar System Magnetic Fields

Although there have been other missions to Jupiter since Pioneer 10, for our purposes no striking improvement in data has been obtained since these missions have also been flybys. However, that situation will change with the arrival of the Galileo probe to Jupiter in 1995.\textsuperscript{14} This craft will have enough fuel to maintain attitude control for approximately 10 eccentric orbits about Jupiter, with distances from the surface ranging from a single closest perijove of $4R_J$ to a varying distance of about $100R_J$. In principle this added data could allow a a better Jupiter photon mass limit by a factor of perhaps 2 to 4. However, the closest approach of $4R_J$ is further out than the Pioneer 10 distance of 2.84$R_J$.

Even more intriguing is the Ulysses mission to the sun.\textsuperscript{15,16} The probe has just encountered Jupiter\textsuperscript{16}, obtaining a gravity boost that has set it in a solar polar orbit. During June-November 1994 and June-September 1995 Ulysses will pass over the south and north polar regions at distances between 1.7 to 2.9 $AU$. A prime advantage of this orbit is that one expects the solar wind to be greatly reduced over the poles. On the other hand, the nature of the complications from the “Archimedes spiral,” caused by the magnetic field rotating with the sun, remain a subject of discussion.\textsuperscript{17}

Most significantly, the true time-dependent value of the solar dipole magnetic moment is uncertain, but it is believed to be approximately\textsuperscript{18} $6 - 12 \text{ Gauss } R_S^3$. Since an Astronomical Unit is about $200R_S$, this means that one would expect the solar dipole field in these regions to be approximately $0.01\gamma$ ($1\text{ Tesla} = 10^4\text{ Gauss} = 10^9\gamma$). This is just at the limit of what onboard magnetometers can measure.

Thus, although the the sun is clearly the biggest magnet we can “get our hands on,” we will have to wait to find out if Ulysses’ orbit will be close enough to the sun to obtain an improved solar-system limit on the mass of the photon.

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6. References

1. A. S. Goldhaber and M. M. Nieto, Rev. Mod. Phys. 43 (1971) 277.
2. A. S. Goldhaber and M. M. Nieto, Sci. Am. 234, No. 5 (1975) 86.
3. G Feinberg, Science 166 (1969) 879.
4. E. R. Williams, J. E. Faller, and H. Hill, Phys Rev. Lett. 26 (1971) 721.
5. L. Davis, Jr., A. S. Goldhaber, and M. M. Nieto, Phys Rev. Lett. 35 (1975) 1402.
6. J. D. Barrow and R. R. Burman, Nature 307 (1984) 14.
7. V. A. Kostelecký, R. Potting, and S. Samuel, in Proceedings of the 1991 Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics, Vol. II, ed. S. Hagarty, et al. (World Scientific, Singapore, 1992), p. 299; V. A. Kostelecký and S. Samuel, Phys. Rev. Lett. 66 (1991) 1811.
8. L. B. Okun, Phys. Lett. B 78 (1978) 597; A. Yu. Ignatiev, V. A. Kuzmin, and M. E. Shaposhnikov, ibid. 84 (1979) 315; M. B. Voloshin and L. B. Okun', JETP Lett. 28 (1978) 145 [Pis'ma Zh. Eksp. Teor. Fiz 28 (1978) 156].
9. N. Dombey, Nature 288 (1980) 643; J. R. Primack and M. A. Sher, ibid. 680, ibid. 299 (1982) 187; L. F. Abbott and M. B. Gavella, ibid. 299 (1982) 187; H. Georgi, P. Ginsparg, and S. L. Glashow, ibid. 306 (1983) 765.
10. S. Nussinov, Phys. Rev. Lett. 59 (1987) 2401; R. N. Mohapatra, ibid. 59 (1987) 1510.
11. M. Suzuki, Phys. Rev. D 38 (1988) 1544.
12. M. Bucher and A. S. Goldhaber, in Proceedings of the 26th International Conference on High Energy Physics (submitted).
13. H. Hill, private communication.
14. For a special volume on the Galileo mission see: Space Sci. Rev. 60 (1992).
15. For a special volume on the Ulysses mission see: Astron. Astrophys Suppl. Ser. 92 (1992).
16. For an issue, with many reports on the Ulysses encounter with Jupiter, see: Science 257 (11 Sept. 1992). An overview, by E. J. Smith, K.-P Wenzel, and D. E. Page, begins on p. 1503.
17. J. R. Jokipii and J. Kóta, Geophys. Res. Lett. 16 (1989) 1.
18. T. Hoeksema, private communication.