Micromagnetic simulation of spin wave propagation in a ferromagnetic film with different thicknesses

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We have shown by micromagnetic simulation that the wavelength of the spin wave (SW) is modulated when the SW propagates across a step between two ferromagnetic films with different thicknesses. Since the wavelength of the SW depends on the film thickness, the thickness difference works as the different medium in Snell’s law. The wavelength modulation obtained by the simulation showed a good agreement with Snell’s law.

**Key words:** spin wave, refraction, Snell’s law, micromagnetic simulation, mumax3

1 Introduction

Spin wave (SW) is a significant phenomenon in the magnetics on the aspects of not only basic physics but also device application. As the SW has the characteristics of waves, it shows various phenomena such as interference1–4 and refraction.5–7 SW is also expected for device application, such as SW-based logic circuit8–11 or storage12–17 with its advantage of ultralow-power consumption. Consequently, for further understanding and utilizing the SW, it is important to investigate its propagation process.

The refraction law (Snell’s law) is one of the most important laws in wave propagation. The refraction occurs when an incident wave is injected into the medium with different wave velocity. The wavelength is also modified in the refraction process. In general, Snell’s law is expressed as follows:

\[
\sin \theta_i \sin \theta_r = \frac{v_1}{v_i} = \frac{d_1}{d_r}, \quad (1)
\]

where \(\theta_i\), \(v_1\), and \(d_1\) correspond to the incident (refraction) angle, the phase velocity of the wave, and the wavelength, respectively. In ordinal light wave, relationship between the incident and the refraction wave follows Snell’s law. However, there are some exceptions, such as birefringence in anisotropic materials.18 In such materials, the light wave does not follow Snell’s law. Hence, it is curious whether SW follows Snell’s law. There are several reports on the refraction of the SW at the junction between materials with different anisotropies5 and different materials6 by means of analytical calculations and micromagnetic simulations. Meanwhile, it is anticipated that the experiment in such a system is difficult because it is necessary to fabricate the sample so as to propagate the SW into different material. On the other hand, another way to refract the SW is the thickness difference. It is easier to fabricate the sample with different thicknesses in the same material. The wavevector of the SW is dependent on the film thickness. For example, the dispersion relation of the magnetostatic surface spin wave (MSSW) is expressed as

\[
f = \frac{\mu_0 \gamma}{2\pi} \sqrt{H(H + M_S)} + \frac{M_S^2}{4} (1 - e^{-2\pi d}), \quad (2)
\]

where \(f\) is the frequency of the wave, \(\mu_0\) is the vacuum permeability, \(\gamma\) is the gyromagnetic ratio \(1.76 \times 10^{11} \text{s}^{-1} \text{T}^{-1}\), \(H\) is the external magnetic field, \(M_s\) is the saturation magnetization, \(k\) is the wavevector of the SW, and \(d\) is the thickness of the film. If the thickness \(d\) is halved, the wavevector \(k\) is doubled, since the frequency \(f\) is unchanged between the refraction processes. Thus we can modify the wavevector by simply changing the film thickness. Considering Eqs. (1) and (2), Snell’s law for MSSW can be written as

\[
\frac{\sin \theta_i}{\sin \theta_r} = \frac{v_1}{v_i} = \frac{d_1}{d_r} = \frac{k_i}{k_r}, \quad (3)
\]

Eq. (3) clearly shows that the phase velocity, the wavelength, and the wavevector are dependent on the film thickness. Note that the reason why we used MSSW is because the decay length of MSSW is longer than the other mode, and MSSW can be excited by in-plane magnetic field, which makes the experimental setup easier than the other mode.

For the system with different film thickness, Tanabe et al.71 experimentally showed that the SW wavelength was modified after propagating through the step. Note that they carried out the time-domain measurement using the wave packet of the SW. For further understanding of Snell’s law for SWs, it is important to carry out the quantitative evaluation of overall propagation process in the system with different film thickness. In this paper, we report the micromagnetic study of the wavelength modulation during propagating through the step consisting of the films with different thicknesses, and verify MSSW follows Snell’s law [Eq. (3)] under the condition that the incident angle is zero (\(\theta_i = 0\)).
2 Micromagnetic Simulation

2.1 Simulation Setup

The setup of the simulation is shown in Fig. 1(a). The origin of the coordinate system is set at the center of the film. The film has 50 µm-width and 60 µm-length. The film is set to be wide enough to reproduce the plane wave. The thicknesses of the left \((x < 0)\) and right \((x > 0)\) half of the film are \(d_l = 60\) nm and \(d_r = 30\) nm, respectively, and there is the “step” at the center \((x = 0)\). The simulation cell size is \(50 \times 50 \times 30\) nm\(^3\), which is small enough to reproduce the SW with 5 µm-wavelength. Note that the film consists of 2 cells (1 cell) along the thickness direction at the left (right) half. We assume typical material parameters of Permalloy: the saturation magnetization \(M_S = 8.0 \times 10^5\) A/m, the exchange stiffness \(A = 1.0 \times 10^{6}\) J/m, and the Gilbert damping \(\alpha = 0.01\). The magnetic anisotropy is set to zero. The static magnetic field \(\mu_0 H = 30\) mT is applied along the y axis. The SW is excited at the SW antenna at the left edge, and the SW propagates to the \(x\)-direction. This configuration can excite the MSSW mode. We used two types of the SW antenna: “1-terminal antenna” for the calculation of the dispersion relation [Fig. 1(b)] and “3-terminal antenna” for the calculation with the step [Fig. 1(c)]. In the case of “1-terminal antenna”, we apply a radio-frequency (rf) magnetic field \(h = h_{rf}\sin(2\pi ft)\hat{x}\) along the \(x\) direction at “+” signed area \((-30 \leq x \leq -29\) µm\) in Fig. 1(b). “3-terminal antenna”, on the other hand, mimicked the coplanar waveguide, often used in the experiment [Fig. 1(c)]. “3-terminal antenna” can excite the SW whose wavelength is exactly defined because of precisely positioned three electrodes. In Fig.1(c), it can excite 5 µm-wavelength SW. At “+” signed area \((-28 \leq x \leq -26\) µm\), rf magnetic field \(h = h_{rf}\sin(2\pi ft)\hat{x}\) is applied, and at “−” signed area \((-30 \leq x \leq -29\) µm and \(-25 \leq x \leq -24\) µm), rf magnetic field \(h = -h_{rf}\sin(2\pi ft)\hat{x}\) is applied. The amplitude of the rf field is fixed: \(\mu_0 h_{rf} = 1\) mT. In order to prevent SW reflections at the right edge, we set a high damping \((\alpha = 1)\) in \(x \geq 25\) µm.\(^{20}\) We utilize mumax3 code\(^{21–23}\) for the micromagnetic simulation.

![Fig. 1](image1.png)

Fig. 1  (a) Schematic illustration of simulation setup. (b) Schematic of “1-terminal antenna” used in the calculation of the dispersion relation. (c) Schematic of “3-terminal antenna” used in the simulation with the step.

![Fig. 2](image2.png)

Fig. 2  (a) Waveform of the SW at \(f = 7.206\) GHz. This waveform is obtained at the center of the film \((y = 0)\) on the bottom layer, and the time is \(t = 10\) ns. Dots and solid line indicate the simulation result and the fitting using Eq. (4), respectively. Vertical axis corresponds to \(z\)-component of the magnetization \((M_z)\) normalized by the saturation magnetization \((M_S)\). Horizontal axis corresponds to the position \(x\). (b) The frequency dependence of the wavelength. Circles and solid line are the simulation result and guide line, respectively. (c) The wavevector dependence of the frequency (dispersion relation). Circles and solid line indicate the simulation results and the MSSW dispersion relation [Eq. (2)], respectively.
2.2 Dispersion Relation

Since the “3-terminal antenna” is set to excite the SW with 5 \( \mu \)m-wavelength, it is necessary to find the frequency which can excite 5 \( \mu \)m-wavelength SW. We carried out the simulation in the frequency range from 7 to 7.5 GHz. We used “1-terminal antenna” and the thickness was 60 nm throughout the film (namely, there was no step) in this simulation. Dots in Fig. 2(a) show the waveform of the SW at \( f = 7.206 \) GHz. This waveform is obtained at the center of the film (\( y = 0 \)) on the bottom layer, and the time is \( t = 10.00 \) ns. To evaluate the wavelength of the SW, we fit waveform in Fig. 2(a) with the product of sine curve and exponential decay as follows:

\[
g(x) = \frac{M_r}{M_s} = A \cdot \exp[-B(x - C)] \cdot \sin \left[ \frac{2\pi}{\lambda}(x - C) \right] + D, \quad (4)
\]

where \( A \) is the amplitude, \( B \) is the decay constant, \( C \) and \( D \) are the offset for \( x \) and \( g \) direction, respectively. Fitting result is shown in Fig. 2(a) with solid line. The wavelength of the SW given by the fitting is \( \lambda_i = 5.00 \) \( \mu \)m at \( f = 7.206 \) GHz. Fig. 2(b) shows the frequency dependence of the wavelength. It is found in this calculation that the frequency which can excite 5 \( \mu \)m-wavelength SW is 7.206 GHz. Using the relation \( k = 2\pi/\lambda \), the wavevector dependence of the frequency (namely, the dispersion relation) is calculated in Fig. 2(c). Circles and solid line indicate the simulation results and the MSSW dispersion relation [Eq. (2)], respectively. The simulation result shows good agreement with MSSW dispersion relation.

2.3 Simulation of the film with the step

We carried out the simulation with the step, using “3-terminal antenna”. Figure 3 shows the waveform of the SW in the film with the step at \( f = 7.206 \) GHz. This waveform is obtained at the center of the film (\( y = 0 \)) on the bottom layer, and the time is \( t = 9.06 \) ns. The wavelength of the SW in the left half (\( d_l = 60 \) nm) is \( \lambda_i = 5.00 \) \( \mu \)m. On the other hand, the wavelength of the SW in the right half (\( d_r = 30 \) nm) is \( \lambda_i = 2.59 \) \( \mu \)m. The wavelength of SW is almost halved after propagating through a half thick layer. Small discrepancy between expected value (2.50 \( \mu \)m) and the simulation result (2.59 \( \mu \)m) is due to roughness of the cell size in \( x-y \) direction. Figure 4 shows the snapshot of magnetization at \( f = 7.206 \) GHz and at \( t = 9.06 \) ns. It clearly shows the modulation of wavelength by passing through the step. Note that the length of the wave front appears to shrink as the SW propagates. The reason of this phenomenon can be described as follows: T. Schneider et al.\(^{24}\) pointed out that the SW excited from a point source propagates for the specific oblique direction under certain condition. The SW antenna is the collection of the point sources. The SW from each point source propagates in the oblique direction. In the center part of the film, waves are superposed and construct a plane wave. However, at the edges, they no longer form the plane wave. Thus the length of the wave front appears to shrink.

Time evolution of the waveform is shown in Fig. 5. The arrow in Fig. 5 follows a specific crest of the wave. It seems that there is discontinuity in the wave amplitude at the step (\( x = 0 \)). This can be explained as follows: Figure 5 shows the magnetization at the bottom layer only. Thickness of the left half is 60 nm, and that of the right half is 30 nm. The energy of the SW concentrates on the bottom layer in the right half when the SW propagates into the step. Hence, the amplitude of the SW increases at the right half. Note that the phase does not change in this process, as the wave crest remains unchanged after propagating through the step. In order to quantitatively evaluate the phase shift during propagating through the step, we fit the waveform with Eq. (4) at the left and the right area individually. Oscillation component is described as \( \sin[2\pi(x - C)/\lambda] \), and the amplitude of oscillation at the step (\( x = 0 \)) is given by \( \sin[-2\pi C/\lambda] \). If \( \sin[-2\pi C/\lambda] \) obtained from the left area and that obtained by the right area takes the same value, one can argue that there is no phase shift during propagating along the step. Figure 6(a) shows the time develop-
Fig. 5 Time development of the waveform. This waveform is obtained at the center of the film \((y = 0)\) on the bottom layer, and frequency is \(f = 7.206\) GHz. The arrow follows a specific crest of the wave.

Next, we investigated the phase velocity of the SW from the simulation result. The phase velocity of the SW in the left and right half are \(v_l = 3.63 \times 10^6\) m/s and \(v_r = 1.86 \times 10^6\) m/s, respectively. The phase velocity of SW is almost halved when propagating to a half thick layer. The wavelength and phase velocity of SW are halved in the half thick layer, whose results are in good agreement with Eq. (3).

3 Summary

We have shown by micromagnetic simulation that the wavelength and phase velocity of the SW is modulated when the SW propagates across a step between two ferromagnetic films with different thicknesses under the condition that the incident angle is zero \((\theta_i = 0)\). The result well follows the Snell’s law. We have also shown that there is no phase shift during propagating across a step. We propose that the SW refraction can be observed experimentally in such an easy system with different film thickness. We investigated the SW propagation in the real space using the continuous wave. The result will serve as a guide for experiments in real space measurements, such as Brillouin Light Scattering Spectroscopy.

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