Chirality-induced magnon transport in AA-stacked bilayer honeycomb chiral magnets

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Abstract

In this Letter, we study the magnetic transport in AA-stacked bilayer honeycomb chiral magnets coupled either ferromagnetically or antiferromagnetically. For both couplings, we observe chirality-induced gaps, chiral protected edge states, magnon Hall and magnon spin Nernst effects of magnetic spin excitations. For ferromagnetically coupled layers, thermal Hall and spin Nernst conductivities do not change sign as function of magnetic field or temperature similar to single-layer honeycomb ferromagnetic insulator. In contrast, for antiferromagnetically coupled layers, we observe a sign change in the thermal Hall and spin Nernst conductivities as the magnetic field is reversed. We discuss possible experimental accessible honeycomb bilayer quantum materials in which these effects can be observed.

Keywords: magnon Hall effect, magnon spin nernst effect, magnon edge states, magnon spintronics

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(Some figures may appear in colour only in the online journal)
In this Letter, we study the chirality-induced magnetic transport in AA-stacked bilayer honeycomb chiral magnet coupled either ferromagnetically or antiferromagnetically. For ferromagnetically coupled layers, we compute the thermal Hall conductivity $\kappa_{xy}$, which shows a similar trend to that of single-layer honeycomb ferromagnetic insulator [21, 22] with no sign change. In contrast, for antiferromagnetically coupled layers, we observe a sign change in the thermal Hall conductivity and spin Nernst conductivity [24] as the magnetic field is reversed. We show that fully antiferromagnetically coupled bilayer with ordered Néel state exhibits similar properties to that of antiferromagnetically coupled bilayer ferromagnets. These results suggest an experimental search for chirality-induced bilayer honeycomb chiral magnets. In fact, many experimental realizations of bilayer honeycomb-lattice systems have been reported. They include magnetic compounds such as CrBr$_3$ [25, 26] which is a spin-1/2 bilayer honeycomb ferromagnetic insulator, Na$_3$Cu$_2$SbO$_6$ [27] and $\beta$-Cu$_2$V$_2$O$_7$ [28] are spin-1/2 Heisenberg antiferromagnetic materials, in each of which the $S = 1/2$ Cu$^{2+}$ ions are situated on the sites of weakly coupled honeycomb-lattice layers. Besides, the iridates A$_2$IrO$_3$ (A = Na, Li) [29, 30] have magnetically ordered Mott phases in which the Ir$^{4+}$ ions are situated on the sites of weakly-coupled honeycomb-lattice layers. In these honeycomb-lattice materials, spin chirality or DMI can be induced using many experimental growth techniques and our results can be confirmed directly.

2. Bilayer ferromagnetic insulator

The Hamiltonian for AA-stacked bilayer honeycomb chiral magnet shown in figure 1(a) is given by

$$H = H_{\text{FM}} + H_{\text{DM}} + H_{\text{ext}} + H_{\text{inter}}$$

where

$$H_{\text{FM}} = -J \sum_{(ij)} S_i^+ \cdot S_j^- - J' \sum_{(ij)} S_i^- \cdot S_j^+$$

$H_{\text{DM}} = \sum_{(ij)} D_{ij} \cdot S_i^+ \times S_j^-$, $H_{\text{ext}} = -h \sum_i S_i^z$, $H_{\text{inter}} = -J_z \sum_i S_i^z$.

$\tau$ denotes the top (T) and bottom (B) layers. $S_i$ is the spin moment at site $i$, $J > 0$ is a nearest-neighbour (NN) ferromagnetic interaction on each layer, $J' > 0$ is a next-nearest-neighbour (NNN) ferromagnetic interaction on each layer, and $D_{ij}$ is the DMI vector between sites $i$ and $j$, allowed by the NNN triangular plaquettes on the honeycomb lattice, where $D_{ij} = \nu_{ij} \textbf{D}$, and $\nu_{ij} = \pm 1$. The Zeeman magnetic field is $h$ in units of $\mu_B$. The interaction $J_z > 0$ represents the ferromagnetic or antiferromagnetic ($J_z < 0$) interlayer coupling.

3. Magnon bands

3.1. Ferromagnetic interlayer coupling

For ferromagnetic interlayer coupling $J_z > 0$, the Fourier space Hamiltonian of the Holstein–Primakoff [31] boson operators is given by $H = \sum_k \psi_k^1 \cdot \psi_k^+ \cdot \mathcal{H}(k)$ where $\psi_k^1 = (a_{kA}^\dagger, a_{kA}^\dagger, a_{kB}^\dagger, a_{kB}^\dagger)$, and

$$\mathcal{H}_{\text{FM}}(k) = \epsilon_{\sigma} \sigma_0 \otimes \sigma_0 + \sigma_0 \otimes \sigma_0 m_k - \nu_z \sigma_0 \otimes f_k \sigma_z f_k \sigma_z - v_x \tau_x \otimes \sigma_x,$$

where $\sigma$ and $\tau$ are triplet pseudo-spin Pauli matrices for the sublattice and layer degrees of freedom respectively, $\nu_0$ and $\sigma_0$ are identity matrices in each space, and $\sigma_z = (\sigma_x \pm i \sigma_y)/2$; $v_0 = h + z \nu_+ + z' \nu_-$, $\nu(t') \nu(t_j) = J_S(S)S(DS)(DS)$, $v_i = \sqrt{v_x^2 + v_y^2}$, $z(z') = 3(6)$, and $\epsilon_{\sigma} = v_0 + v_1 - 2v_3 \sum_{\mu=0} \cos(k \cdot a_{\mu}) \cos \phi$. Here, $f_k = e^{i\Delta z/2} \cos(\sqrt{3}k_c a/2) e^{-i\Delta z a_i^2/2}$, and $m_k = a_{V} \sum \sin(k \cdot a_{\mu}) \sin \phi$.

where $a_1 = \sqrt{3} \hat{x}$; $a_2 = (-\sqrt{3} \hat{x}, 3\hat{y})/2$ $a_3 = -(-\sqrt{3} \hat{x}, 3\hat{y})/2$. We have assumed a DMI along the $z$-axis. The phase factor...
Figure 2. Magnon band structures of spin-1/2 bilayer honeycomb chiral magnet for $h = v_1 = 0.5$, $v_2 = v_1' = 0.05$, $v_1 = 1$, $\phi = \pi/4$. (a) Ferromagnetic coupling. (b) Antiferromagnetic coupling.

$\phi = \arctan(D/J')$ is a magnetic flux generated by the DMI on the NNN triangular plaquettes. The eigenvalues are given by

$$\epsilon_{\alpha \pm}(k) = \epsilon_\alpha + (1)^\alpha v_1 \pm \sqrt{m_0^2 + |v_1'|^2},$$

(6)

where $\alpha = 1, 2$ is the layer index. For $v_1 = 0$, the Hamiltonian decouples to two single layers [21]. The magnon band is shown in figure 2(a). At the Dirac points $K_\pm = (\pm 4\pi/3, a, 0)$, the eigenvalues reduce to $\epsilon_{\alpha \pm}(K_\pm) = \epsilon_\phi + (-1)^\alpha v_1 \pm |m_0|$, where $m_0 = 3\sqrt{3}v_1\sin \phi$. This is similar to AA-stacked spin-orbit-coupled bilayer graphene [32, 33].

3.2. Antiferromagnetic interlayer coupling

We now consider antiferromagnetically coupled layers, with the spins on the upper or lower layer pointing down-wards, and the interlayer coupling $J_z < 0$. The top and bottom layers are still ferromagnetic insulators described by $H_{\text{ext}}^\text{F}$ and $H_{\text{DM}}$. To study the magnetic excitations, we perform a $\pi$-rotation about the $S_z$-axis on the top layer,

$$S_z^+ \rightarrow S_z^-, S_z^- \rightarrow -S_z^+, S_z^\pm \rightarrow -S_z^\mp.$$  

(7)

This rotation keeps the upper ferromagnetic layer invariant but points the spins in the new $z$-direction, and changes the sign of $H_{\text{ext}}^\text{F}$ and $H_{\text{DM}}$ on the top layer. The Fourier space Hamiltonian is

$$H = \frac{1}{2} \sum_{k} \hat{\Psi}_k^\dagger H_{\text{AFM}}(k) \hat{\Psi}_k + \text{const.},$$

where $\hat{\Psi}_k = (\psi_+^k, \psi_-^k)$, and

$$H_{\text{AFM}}(k) = \begin{pmatrix} A(k) & B \\ B & A^\dagger(-k) \end{pmatrix},$$

(8)

The matrices $A(k)$ and $B$ are given by

$$A(k) = \epsilon_0 \tau_0 \otimes \sigma_0 + \tau_1 \otimes \sigma_\tau m_0 \tau_0 \otimes \nu_1 \tau_0 \otimes (f_k \sigma_+ + f_k^\dagger \sigma_-)$$

$$+ \nu_2 \tau_0 \otimes \sigma_\tau, B = [v_1 \tau_2 \otimes \sigma_0, \nu_1 \tau_2 \otimes \sigma_0,] \epsilon_0 + \nu_1 \sum_{\gamma} \cos(k \cdot a_\gamma) \cos \phi.$$  

(9)

where $\phi = \arctan(D/J')$ is the layer index. For $v_1 = 0$, the Hamiltonian decouples and the eigenvalues are given by

$$\epsilon_{\alpha \pm}(k) = (1)^\alpha h + \sqrt{\epsilon_0 \pm \sqrt{m_0^2 + |v_1'|^2}} - v_1^2.$$  

(10)

For $h = 0$, the energy bands are doubly degenerate—one of the major differences between ferromagnetically and antiferromagnetically coupled layers. Also notice that antiferromagnetically coupled layers have a linear dispersion near the Fermi surface (see figure 2(b)) as opposed to a quadratic dispersion in the ferromagnetic case. For $v_1 = 0$, equation (10) decouples and reduces to equation (6) with opposite magnetic field on each layer.

3.3. Bilayer antiferromagnetic insulator

In the fully antiferromagnetic case, each layer is modeled by the Heisenberg antiferromagnet, with $J, J_z, J_x < 0$. Due the $J_z$ term, the Heisenberg antiferromagnet is frustrated as opposed to the ferromagnetic counterpart. With zero DMI $H_{\text{DM}} = 0$, the system is considered to describe bilayer honeycomb antiferromagnetic material Bi$_3$Mn$_3$O$_{12}$ (NO$_3$) [34–38]. The ground state phase diagram of this model has been studied extensively [34–38]. It consists of an ordered Néel state for $J'/J < 1/6$ and a nonmagnetic state for $J'/J > 1/6$ [34]. For large values of $J_z$, the ground state is an interlayer valence-bond crystal in which the spins from both layers form dimers [37].

We are interested in the topological effects of the ordered Néel state for $J'/J < 1/6$ shown in figure 1(b). Such Néel state order exists in the bilayer honeycomb iridates A$_3$IrO$_3$ (A = Na, Li) [29, 30]. In this phase, the band structure in the absence of the chiral DMI exhibits Dirac points at $K_{\pm} = (\pm 4\pi/3, a, 0)$ [35–38]. A nearest-neighbour DMI does not introduce chirality and an external magnetic field introduces canting up to the saturated field when fully polarized ferromagnetic states
are recovered. These terms do not open a gap at $K_{\perp}$. As in the ferromagnetic case, chirality is introduced by a next-nearest-neighbour DMI. As we now show, this is very similar to antiferromagnetically coupled bilayer ferromagnets studied above at zero magnetic field. The only difference is that the NNN coupling is restricted to $1/\parallel \cdot \perp$.

We begin by performing the $\pi$-rotation described above on sublattice $A_1$ and $B_2$ such that the spins point along the new rotated $z$-axis. The SU(2)-invariant NN and NNN interactions on each layer are invariant under this rotation, but the U(1)-invariant out-of-plane DMI changes sign as in the previous case. In the bosonic representation, the Hamiltonian has the form as equation (8) with

$$\mathcal{A}(k) = \epsilon_0 \sigma_0 + m_k \sigma_z \sigma_0,$$

$$B(k) = \gamma_0 \sigma_z + \sum_{\nu} \cos(k \cdot v_\nu) \cos \phi.$$ 

As before $\mathcal{A}(k) = \mathcal{A}(-k)$, but $B(k) = B^*(k)$. The Hamiltonian is diagonalized as usual. The positive eigenvalues are given by

$$\epsilon_{\alpha \pm}(k) = \sqrt{\epsilon_0^2 + \epsilon_0^2 - \nu_1^2} \pm \nu_1 \text{R}_{\parallel} \sigma_0,$$

where $\epsilon_k = v_\nu - \epsilon_{\alpha \pm}$ and $\nu_1 = \sum_{\nu} \cos(k \cdot v_\nu) \cos \phi$. As before $\mathcal{A}(k) = \mathcal{A}(-k)$, but $B(k) = B^*(k)$. The Hamiltonian is diagonalized as usual. The positive eigenvalues are given by

$$(i) \quad \epsilon_{\alpha \pm}(k) = \sqrt{\epsilon_0^2 + \epsilon_0^2 - \nu_1^2} \pm \nu_1 \text{R}_{\parallel} \sigma_0,$$

where $\epsilon_k = v_\nu - \epsilon_{\alpha \pm}$ and $\nu_1 = \sum_{\nu} \cos(k \cdot v_\nu) \cos \phi$. The band structures depicted in figure 3 are very similar to that of bilayer ferromagnet with antiferromagnetic coupling for $h = 0$. As mentioned above, a finite magnetic field introduces spin canting. In this case, both the out-of-plane and in-plane DMIs contribute to the magnon excitations. This scenario is analyzed in the supplementary material.

4. Magnon transports

4.1. Magnon edges states

Magnetic transports in topological magnon insulator materials are encoded in the protected chiral edge states of the system induced by the DMI. Figure 4(a) shows the evolution of the chiral protected edge states of the ferromagnetically coupled layers in different parameter regimes. The chiral edge states propagate in the same direction as depicted schematically in figure 5(a). For antiferromagnetically coupled case, the same situation is observed with different parameters as depicted in figure 4(b). However, the chiral edge states propagate in opposite directions for the top and bottom layers because of opposite DMI as shown schematically in figure 5(b).

4.2. Magnon Hall effect

The most interesting property of chiral magnetic systems is the observation of magnon Hall effect [2, 4, 18, 19]. In magnon Hall effect [1], as well as magnon spin Nernst effect [24], the non-vanishing Berry curvature induces an effective magnetic field in the system, upon the application of a temperature gradient. The propagation of magnons in the bilayer system is deflected by the chiral DMI. Magnon Hall effect is characterized by a transverse thermal Hall conductivity, given by

$$\kappa_{xy} = -2k_B T V^{-1} \sum_{\mathbf{k}} c_2(n_\nu) \Omega_{\parallel}(k),$$

where $V$ is
the volume of the system, $k_B$ is the Boltzmann constant, $T$ is the temperature, $n_\mu = n_B[\epsilon_\mu(k)] = [e^{\epsilon_\mu(k)/k_B T} - 1]^{-1}$ is the Bose function, $c_2(x) = (1 + x)(\ln(1 + x))^2 - (\ln x)^2 - 2Li_2(-x)$, and $Li_2(x)$ is a dilogarithm. Magnon spin Nernst conductivity has a similar definition \[ \alpha_{xy} = \sum_{\mu} \partial \Omega_{\mu}(k)/\partial k_{xy} \], where $\psi_{\mu k}$ are the eigenstates of the Hamiltonian and $\mu$ labels the bands; $v_{\mu x} = \partial H(k)/\partial k_{xy}$ defines the velocity operators.

Figures 6 and 7 show the dependence of thermal Hall conductivity on the magnetic field and the temperature for the ferromagnetically coupled layers. As the temperature approaches zero, $\kappa_{xy}$ vanishes due to lack of thermal excitations, but it never changes sign as the temperature increases or the magnetic field changes sign. This is what is observed theoretically in the single layer honeycomb chiral ferromagnet [22]. However, for antiferromagnetically coupled layers shown in figures 7(a) and (b), we see that $\kappa_{xy}$ changes sign as the magnetic field is reversed and vanishes at zero field. The sign change in $\kappa_{xy}$ is encoded in the magnon bulk bands, the Berry curvatures, and the propagation of the chiral edge states. The sign change in $\kappa_{xy}$ is very similar to what was observed on the pyrochlore chiral magnets upon reversing the direction of the applied magnetic field [2, 3]. Due to the Berry curvature, $\alpha_{xy}$ shows similar trends (not shown). We also observe that for the chirality-proximity effect, where only one layer contains a chiral DMI, topological effects are induced in the bilayer system and thermal conductivity $\kappa_{xy}$ is suppressed (see the supplementary material).

5. Conclusion

We have studied chirality-induced magnon transport in AA-stacked bilayer honeycomb chiral magnets. We observe remarkable
distinctive features for ferromagnetic and antiferromagnetic couplings. In particular, the band structure and the chiral edge states have different topological properties. As a result thermal Hall and spin Nernst conductivities show a sign change for antiferromagnetic coupling in contrast to ferromagnetic coupling. As far as we know, chirality-induced transports and thermal Hall effect still await experimental observation on the honeycomb lattice. As mentioned above, there are many accessible AA-stacked bilayer honeycomb quantum magnets in which chirality can be induced and these theoretical results can be confirmed. Experiments can also probe the observed magnon edge states, by noticing that spin-1/2 quantum magnets map to hardcore bosons. Thus, the magnon edge states correspond to bosonic edge states, which can be studied experimentally in ultracold atoms on optical lattices similar to the realization of Haldane model [39]. Our results can also be applied to magnon spintronics in chiral bilayer quantum magnetic systems.

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