Motivation

- QE dominates neutrino cross sections in the energy range relevant to neutrino oscillation experiments \( E_\nu = 200 \text{ MeV to } 2.5 \text{ GeV} \).
- Important source of systematic uncertainties is related to treatment of nuclear model dependence of QE cross sections.
- We study QE neutrino charged-current interactions within realistic model and nuclear model dependence of QE neutrino cross sections:
  - (⋆) Relativistic Fermi Gas Model (RFGM)
  - (⋆) Plane-Wave Impulse Approximation (PWIA) - no final state interaction (FSI)
  - (⋆) FSI effects within Relativistic Distorted-Wave Impulse Approximation (RDWIA)
Ingredients of approximations studied:

- **RFGM** with Fermi momentum \( p_F = 225 \) MeV/c and binding energy \( \epsilon_b = 27 \) MeV (specific for \(^{16}\)O).

- **PWIA** using nuclear shell model with NLSH bound nucleon wave functions [1] and nucleon-nucleon (NN) correlation effect [2, 3].
  
  \( \ast \) average occupancy of nuclear shells \( S_\alpha = 0.75 \)

  \( \ast \) average strength of NN correlated part of nuclear spectral function \( (1 - S_\alpha) = 0.25 \) (supported by recent JLab measurement).

- **RDWIA** with LEA code [4]. This code was successfully tested against \( A(e, e'p) \) data [5, 6, 7, 8]. In calculations we employed the \( \Gamma_2 \) off shell single nucleon current operator in Coulomb gauge, MMD form factors [9], the NLSH bound nucleon wave functions and EDAD1 relativistic optical potential [10].

- **FSI** effects are important for exclusive reactions \( A(e, e'p) \) and \( A(\nu, \mu p) \).
Reduced cross sections as test ground of neutrino-nuclear exclusive cross sections

Lepton CC QE exclusive reaction

\[ l(k_i) + A(p_A) \rightarrow l'(k_f) + N(p_x) + B(p_B), \]  

(1)

Reduced cross section (measure the strength of nuclear effects)

\[ \sigma_{\text{red}} = \frac{d^5 \sigma}{d\epsilon_f d\Omega_f d\Omega_x} / K \sigma_{lN}, \]  

(2)

where \( d^5 \sigma / d\epsilon_f d\Omega_f d\Omega_x \) is differential exclusive cross section, \( \epsilon_f, \Omega_f \) are energy and solid angle of the scattered lepton, \( \Omega_x \) is solid angle for ejectile nucleon momentum, \( K \) is kinematical phase space factors and \( \sigma_{lN} \) is elementary cross section for the lepton scattering from moving free nucleon.

\( \sigma_{\text{red}} \) is nucleon momentum distribution in PWIA.

\( \sigma_{\text{red}} \) should be similar for electron and neutrino scattering (apart from small differences due to FSI effects for electron and neutrino induced reactions).
Inclusive cross section and flux problem

- Because of FSI there is nucleon flux absorption in exclusive channels (described by imaginary part of optical potential).
- For inclusive reaction (all channels) probability flux must conserve.
- We calculate inclusive and total cross sections for nuclear shell contributions using only real part of optical potential.
- The inclusive and total cross sections with FSI and NN short-range correlations (high-momentum part of nuclear spectral function) is calculated as

\[
\frac{d^3\sigma}{d\varepsilon_f d\Omega_f} = \left( \frac{d^3\sigma}{d\varepsilon_f d\Omega_f} \right)_{\text{RDWIA}}^{\text{nuclear shells}} + \Lambda(\varepsilon_f, \Omega_f) \left( \frac{d^3\sigma}{d\varepsilon_f d\Omega_f} \right)_{\text{HM}}^{\text{NN correlations}},
\]

where

\[
\Lambda(\varepsilon_f, \Omega_f) = \left( \frac{d^3\sigma}{d\varepsilon_f d\Omega_f} \right)_{\text{RDWIA}}^{\text{nuclear shells}} \bigg/ \left( \frac{d^3\sigma}{d\varepsilon_f d\Omega_f} \right)_{\text{PWIA}}^{\text{nuclear shells}},
\]

describes the relative strength of FSI.

More details about calculation of the cross sections can be found in arXiv:0705.1051[nucl-th]. The calculated exclusive, inclusive, and total cross sections are presented in Figs.1-10. It should be note that negative values of \( p_m \) correspond to \( \phi = \pi \) and positive ones to \( \phi = 0 \) (\( \phi \) is azimuth angle of the ejectile nucleon).
FIG.1 Measured differential exclusive cross-section data for the removal of protons from 1p-shell of $^{16}$O as a function of missing momentum. The upper panels show JLab data [7] for electron beam energy $E_{\text{beam}}=2.442$ GeV, proton kinetic energy $T_p=427$ MeV, and $Q^2=0.8$ GeV$^2$. The lower panels show Sacle data [11] for $E_{\text{beam}}=580$ MeV, $T_p=160$ MeV, and $Q^2=0.3$ GeV$^2$. The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations.
FIG. 2  Measured reduced exclusive cross-section data for the removal of protons from 1p-shell of $^{16}$O as a function of missing momentum. The upper panels show Sacle data [12] for electron beam energy $E_{beam}=500$ MeV, proton kinetic energy $T_p=100$ MeV, and $Q^2=0.3$ GeV$^2$. The lower panels show NIKHEF data [13] for $E_{beam}=521$ MeV, $T_p=96$ MeV, $Q^2$ is varied. The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations.
FIG. 3  Comparison of the RDWIA electron, neutrino and antineutrino reduced cross sections for the removal of nucleons from 1p-shell of $^{16}\text{O}$ for Sacle [12] (upper panels) and NIKHEF [13] (lower panels) kinematic as functions of $p_m$. The solid line is electron while the dashed and dashed-dotted lines are respectively neutrino and antineutrino cross sections.
FIG. 4  Comparison of the RDWIA and the RFGM calculations for electron, neutrino and antineutrino reduced (left panels) and differential (right panels) results for the removal of nucleons from 1p- and 1s-shells of $^{16}$O. The upper and lower panels show the results calculated respectively for JLab [7] and Sacle [11] kinematics. The dashed-dotted line is the RDWIA calculation for electron scattering while the dashed and dotted lines are respectively for neutrino and antineutrino scattering. The solid line on the left panels shows the RFGM result while the solid and dashed lines on the right panels are respectively neutrino and antineutrino cross sections calculated in the Fermi gas model. The dashed-dotted and dotted lines on right panels are respectively neutrino and antineutrino cross sections calculated in the RDWIA.
FIG. 5  Inclusive cross section versus the energy transfer $\omega$ or invariant mass $W$ (lower-right panel) for electron scattering on $^{16}$O. The data are from Ref.[14] (SLAC, filled circles) and Ref.[15] (Frascati, filled triangles). SLAC data are for electron beam energy $E_e=540, 730$ MeV and scattering angle $\theta=37.1^\circ$. Frascati data are for $E_e=540$ MeV and $\theta=37.1^\circ$, $E_e=700, 880$ MeV and $\theta=32^\circ$. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.
FIG. 6  Inclusive cross section versus the energy transfer $\omega$ or invariant mass $W$ (lower panel) for electron scattering on $^{16}\text{O}$. The data are from Ref.[15] for electron beam energy $E_e=1080, 1200, 1500 \text{ MeV}$ and scattering angle $\theta=32^{\circ}$. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with a complex optical potential.
FIG. 7 Inclusive cross section versus the muon energy for neutrino scattering on $^{16}\text{O}$ and for the four values of incoming neutrino energy: $E_\nu=0.3, 0.5, 0.7$ and 1 GeV. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.
FIG. 8 Inclusive cross section versus the muon energy for antineutrino scattering on $^{16}\text{O}$ and for the four values of incoming neutrino energy: $E_\nu=0.3$, 0.5, 0.7 and 1 GeV. The solid line is the RDWIA calculation while the dotted and dashed lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.
FIG. 9 Total cross section for the CC QE scattering of muon neutrino on $^{16}\text{O}$ as a function of the incoming neutrino energy. The RDWIA results with the real part of optical potential (upper panel) and complex optical potential (lower panel) are shown together with calculations from Ref.[16] (dashed-dotted line) and Ref.[17] (dashed line). The solid and dotted lines are respectively results obtained in this work with and without contribution of the high-momentum component. For comparison, also shown are the data from Refs.[18, 19] for the $\text{D}_2$ target.
FIG. 10  Total cross section for CC QE scattering of muon neutrino (upper panel) and antineutrino (lower panel) on $^{16}$O as a function of incoming (anti)neutrino energy. The solid and dashed lines are respectively the RDWIA results with the real and complex optical potential. The dashed-dotted and dotted lines are respectively the RFGM and PWIA results. Data points for different targets are from Refs.[18, 19, 20, 21]
Summary

QE CC $\nu(\bar{\nu})^{16}\text{O}$ differential and reduced exclusive cross sections were studied in different approaches.

- In RDWIA the reduced exclusive cross sections for $\nu(\bar{\nu})$ scattering are similar to those of electron scattering and in a good agreement with data.
- The inclusive and total cross sections were calculated neglecting the imaginary part of relativistic optical potential and taking into account the effect of NN-correlations in the target ground state and tested against $^{16}\text{O}(e, e')$ scattering data. The agreement with data is better than 10% in the peak region.
- FSI effect reduces the total cross section for about 30% for $E_{\nu} = 200$ MeV compared to PWIA and decreases with neutrino energy down to 10% at 1 GeV.
- Effect of NN-correlations further reduces the total cross section for about 15% for $E_{\nu} = 200$ MeV. This effect also decreases with neutrino energy, down to 8% at 1 GeV.
- The Fermi gas model was tested against $e^{16}\text{O}$ data:
  - In the peak region RFGM overestimates the value of inclusive cross section at low momentum transfer ($|q| < 500$ MeV/c). The discrepancy with data is about 20% at $|q| = 300$ MeV/c and decreases as momentum transfer increases.
  - RFGM fails completely when compared to exclusive cross section data.
- For total neutrino cross sections RFGM result is about 15% higher than the RDWIA predictions at $E_{\nu} \sim 1$ GeV.
- Our results show that nuclear-model dependence of the inclusive and total cross sections weakens with neutrino energy but still remains significant for energy $E_{\nu} \lesssim 1$ GeV.
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