The weak gravity conjecture is proposed as a criterion to distinguish the landscape from the swampland in string theory. As an application in cosmology of this conjecture, we use it to impose theoretical constraint on parameters of the Chaplygin-gas-type models. Our analysis indicates that the Chaplygin-gas-type models realized in quintessence field are in the swampland.

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The dark energy problem is undoubtedly one of the most actively researched topics in the vast landscape of vacua, then as a low-energy effective field theory were proposed in Ref. [4]. Recently these semi-classically consistent effective field theories are essentially inconsistent at quantum level. These effective field theories are in the so-called swampland, whereas the essentially consistent ones are in the string landscape. [5] Some criteria of consistent effective field theory were proposed in Ref. [4]. Recently the weak gravity conjecture proposed in Ref. [6] further helps to rule out those effective field theories in the swampland. For a four-dimensional \textit{U}(1) gauge field coupled to gravity with coupling \( g \), there naturally exists a new cut-off scale below the Planck scale in asymptotic flat background: \( \Lambda \sim g M_p \), where \( M_p \) is the Planck scale. Above this cut-off the effective field theory breaks down and a more stringy approach is needed. This conjecture was generalized to asymptotic dS/AdS background in Ref. [7], leading to an upper bound for the cosmological constant \( \rho_\Lambda \leq g^2 M_p^4 \). This conjecture is also applied to inflationary cosmology.\[8\textendash 10\] For the dark energy problem, according to Ref. [11], if we believe that our universe is one of the vast landscape of vacua, then as a low-energy effective field theory to describe the vacuum energy, the quintessence should not be in the swampland, namely, the variation of the canonical scalar field should be less than the Planck mass \( M_p \). Following Ref. [11], we have

\[
\Delta \phi(z_m) = \int_0^{\phi(z_m)} \frac{d\phi(z)}{M_p} = \int_0^{z_m} \left[ 3(1 + w(z)) \rho \right]^{\frac{1}{2}} \frac{dz}{(1 + z)} < 1, \quad (1)
\]

where \( w(z) \) and \( \rho \) respectively denote the \textit{EoS} and the energy density of the quintessence field.

The requirement Eq. (1) can be applied to any phenomenological models that can be realized by a quintessence field. Here we focus on the Chaplygin-gas-type models, a class of phenomenological models characterized by its particular forms of \textit{EoS}, by which we can always reconstruct the corresponding quintessence field theory or other field theories. In the following, we assume a flat Friedman–Robertson–Walker (FRW) universe consisting of dark energy and dust-like matter with the Friedmann equation

\[
3 M_p^2 H^2 = \rho_m + \rho_{DE}. \quad (2)
\]

The Chaplygin gas (CG) is a perfect fluid with the \textit{EoS}

\[
\rho_{CG} = -A/\rho_{CG}, \quad (3)
\]

where \( A \) is a positive constant. It was introduced by Chaplygin\[12\] in the field of aerodynamics. The Chaplygin gas model was first proposed in Ref. [13] as an alternative to solve the dark energy problem. Then it was used as a unified description for dark matter and dark energy (UDME),\[14\] where the UDME fluid evolves from a state of pressureless dust in the past to the state like a cosmological constant in the future. Such \textit{EoS} can be originated from tachyon field described by the Born–Infeld action with a constant potential\[11, 13\] which can be related to a perturbed d-brane in \((d + 2)\)-dimensional spacetime. A universe consisting of only the Chaplygin gas can be realized
by a quintessence field with the potential

\[ V(\phi) = \frac{\sqrt{A}}{2} \left( \cosh \sqrt{3} \phi / M_p + \frac{1}{\cosh \sqrt{3} \phi / M_p} \right). \]  

(4)

When considering the existence of other component such as baryon or radiation, the exact form of the potential will change, but in principle, we can always reconstruct such a potential. The original Chaplygin gas model is in fact incompatible with the observations. As a generalization, the EoS (3) can be modified as\(^{[15]}\)

\[ p_{\text{GCG}} = -\frac{A}{\rho_{\text{GCG}}^s}, \]  

(5)

where \( A \) and \( \alpha \) are constants. In the case of the generalized Chaplygin gas (GCG), the universe evolves from a dust dominated phase, through a phase described by the EoS \( p = \alpha \rho \), to end up with a de Sitter phase. This EoS can be derived from the generalized Born–Infeld theory.\(^{[15]}\) The original EoS (3) can also be generalized to

\[ p_{\text{VCG}} = -\frac{A(a)}{\rho_{\text{VCG}}^s}, \]  

(6)

where \( A(a) \) is not a constant but a variable with respect to the scale factor \( a \). This variable Chaplygin gas (VCG) model is inspired by the Born–Infeld theories with potentials which are not constant.\(^{[16]}\) It differs from the original CG because it ends up in a qiiessence phase with the EoS \( w_{\text{VCG}} = -1 + 6/n \). The GCG and the VCG models seem compatible with some of current observations.\(^{[16–22]}\) However, there are also some controversies over the compatibility of these models with some other observational requirements.\(^{[23,24]}\)

Before going to detailed analysis, two points should be noted. First, in Ref. [8], the weak gravity conjecture results in \( \phi \leq \phi_0 \) for \( V(\phi) = \lambda \phi^4 \) and some other polynomial potentials. In fact, the minimum of such potential lies in \( \phi_0 = 0 \), therefore the value of \( \phi \) is essentially the variation with respect to \( \phi_0 \). Moreover, the potential \( V(\phi) \) can be shifted without affecting physical results, and in general we do not require the minimum position \( \phi_0 = 0 \). Thus the physically meaningful quantity is indeed the variation of the field with respect to some \( \phi_0 \), rather than its absolute magnitude. Second, we choose the upper limit of the integral in Eq. (1) as \( z_m = 1089 \), the recombination redshift, because in general, the variation of \( |\Delta \phi(z_m)|/M_p \) with respect to \( z_m \) is negligible for \( z_m \gtrsim 1000 \), as we will see later.

Let us start from the original Chaplygin gas model. By energy conservation, we get the evolution of the energy density of the CG

\[ \rho_{\text{CG}} = \sqrt{A + B/a^6}, \]  

(7)

where \( B \) is an integration constant. Setting \( a = a_0 \equiv 1 \) leads to the initial value \( \rho_{\text{CG0}} = \sqrt{A + B} \). Defining \( A_s = A/(A + B) \), the energy density can be recast into

\[ \rho_{\text{CG}} = \rho_{\text{CG0}} \sqrt{A_s + (1 - A_s)(1 + z)^6}, \]  

(8)

where \( 1 + z \equiv 1/a \) has been used. Then the EoS can be expressed by

\[ w_{\text{CG}}(z) = -\frac{A_s}{A_s + (1 - A_s)(1 + z)^6}. \]  

(9)

Note that the physical significance of \( A_s \) is just \( A_s = -w_{\text{CG0}} \), thus \( A_s \) must be less than 1 in order that the CG can be realized by a quintessence field and such that the criterion (1) can be applied. By Eq. (2) we have

\[ \frac{\rho_{\text{CG}}}{3M_p^2H^2} = \frac{\rho_{\text{CG}}}{\rho_{\text{CG}} + \rho_m} = \frac{\Omega_{\text{CG}}}{\Omega_{\text{CG}} + \Omega_m}, \]  

(10)

where \( \Omega_m = \Omega_{m0}(1 + z)^3 \) and \( \Omega_{\text{CG}} = \frac{\rho_{\text{CG}}}{3M_p^2H_0^2} \). Note that in the literature the parameter \( \Omega_{m0} \) often refers to the baryonic matter only. Even in some literature (for example, Refs. [17,18]), the contribution from baryon is just ignored. Here we make a loose assumption that the matter component is not confined to baryonic matter, it may also be a mixture of baryonic matter and the dark matter originated from sources other than the Chaplygin-gas-type fluid.

**Fig. 1.** \( \Delta \phi(1089) \) is far larger than \( M_p \). This implies that the original Chaplygin gas model realized in quintessence can be ruled out by the theoretical criterion.

Applying the criterion (1), we find that for all the region of the parameter space, \( \Delta \phi(1089) \) is far larger than \( M_p \), as we can see from Fig. 1. This implies that the original CG model is inconsistent with the theoretical requirement and therefore may not be a viable model. In fact, constraints from observations also rule out the original CG model.\(^{[17–20]}\)

In the case of the generalized Chaplygin gas model, the basic equations are similar to those of the original one. The energy density is

\[ \rho_{\text{GCG}} = \rho_{\text{GCG0}} [A_s + (1 - A_s)(1 + z)^{3(1 + \alpha)}]^{1/(1 + \alpha)}, \]  

(11)
and the EoS can be expressed by

$$w_{\text{GCG}}(z) = -\frac{A_s}{A_s + (1 - A_s)(1 + z)^3(1 + \alpha)}. \quad (12)$$

Again, $A_s = -w_{\text{GCG}0}$ should be less than unity. Note that when $\alpha = 1$ we recover the original Chaplygin gas model. Defining the fractional energy density the same way as in Eq. (10) and inserting them into Eq. (1), we obtain the limits on the GCG model. In this model, there are in fact three parameters: $A_s$, $\alpha$ and $\Omega_m0$. In Fig. 2, we present the theoretical boundary on the $A_s$-$\alpha$ plane. We choose $\Omega_m0$ to be 0.04 and 0.27, respectively. The first case corresponds to $\Omega_m0 = \Omega_b$. That is, $\Omega_m0$ represents baryonic matter only, and all the dark matter is described by the GCG. The value 0.04 is in agreement with observations such as SDSS, GCG. The value 0.27 is consistent with WMAP. The allowed part is bounded by the solid line. Unlike the case of CG, the GCG model can stand for the theoretical test within certain region of the parameter space as shown in the figure. It is confirmed that CG, corresponding to $\alpha = 1$ on the plot, is far outside the allowed region. We also illustrate in Fig. 3. that the variation of $|\Delta \phi(z_m)|/M_p$ is practically negligible for $z_m \gtrsim 1000$, in favour of the fact that the choice of $z_m = 1089$ is reasonable.

Table 1. Observational constraints

| $A_s$      | $\alpha$ | Reference | Observation | $\Omega_m0$ |
|------------|----------|-----------|-------------|-------------|
| [0.81,0.85] | [0.2,0.6] | [17] | CMB | 0 |
| 0.70 $^{+0.16}_{-0.17}$ | -0.09 $^{+0.54}_{-0.33}$ | [20] | Galaxy cluster & SN1a+FRHb | $\Omega_b$ |
| 0.936 | 3.75 | [18] | SN1a | 0 |
| 0.88 $^{+0.08}_{-0.03}$ | 1.57 $^{+0.1}_{-0.94}$ | [19] | SN1a | $\Omega_b$ |

However, the allowed region is not consistent with observations. The results of observational constraints are shown in Table 1. They all fall outside the theoretically allowed region. This indicates that the GCG model realized in quintessence is in the swampland. Since the figure shows that the greater the value of $\Omega_m0$ is, the larger the allowed region is, maybe GCG should be considered as a model for only dark energy, instead of as UDME, so that the allowed range may become large enough to be compatible with observations. However, without the merit of unifying dark matter and dark energy, this model is not as worthy as the simple $\Lambda$CDM model, due to its introducing one more parameter.

![Fig. 2. Limit on the GCG parameter $A_s$ and $\alpha$ for $z_m = 1089$. Below the dashed line is the allowed region for $\Omega_m0 = 0.04$, and the part under the solid line is the allowed region for $\Omega_m0 = 0.27$.](image)

![Fig. 3. $|\Delta \phi(z_m)|/M_p$ as a function of $z_m$. Its variation is negligible for $z_m \gtrsim 1000$.](image)

One difference of the variable Chaplygin gas with CG and GCG is that it evolves into a quintessence (that is, its EoS is a constant, but not $-1$), rather than a cosmological constant as in CG and GCG. By the energy conservation equation, the VCG density evolves as

$$\rho_{\text{VCG}} = a^{-\frac{\alpha}{6}} \left[ 6 \int A(a)a^5 da + B \right]^{1/2}, \quad (13)$$

where $B$ is an integration constant. Following Ref.[16], we assume the form $A(a) = A_0 a^{-n}$, where $A_0$ and $n$ are the parameters of the model. Inserting this form into Eq.(13) leads to

$$\rho_{\text{VCG}} = \sqrt{\frac{6}{6 - n}} \cdot \frac{A_0}{a^n} + \frac{B}{a^5}. \quad (14)$$

From this equation we can see that for $n < 0$ the energy density increases with time, exhibiting phantom behaviour. Thus we impose $n \geq 0$. Note that we can recover the original Chaplygin gas model for $n = 0$. 


Setting $a = a_0 \equiv 1$ we get the initial value $\rho_{\text{VCG0}} = \sqrt{\frac{6}{n-1}} A_0 + B$. Then defining $B_s = B/\rho_{\text{VCG0}}$ we can recast Eq. (14) into a more useful form

$$\rho_{\text{VCG}} = \rho_{\text{VCG0}} \sqrt{B_s(1+z)^6 + (1-B_s)(1+z)^n},$$

from which we obtain the range $0 < B_s < 1$. Then the EoS can be expressed by

$$w_{\text{VCG}} = \frac{6-n}{6(1-B_s)(1+z)^n}.$$

By setting $z \to -1$ we can see that the VCG ends up with a quiescence phase with $w_{\text{VCG}} = -1 + n/6$. Following the same way stated above, we define the corresponding fractional energy densities and insert them together with Eq. (16) into Eq. (1). We find that when $\Omega_{m0} = 0.04$, the criterion is violated for all the given range of the parameters. This indicates that the VCG realized in quintessence as a UDME is in the swamp-land. We expect that the situation will be ameliorated for $\Omega_{m0} = 0.27$. However, as shown in Fig. 4, the variation of the field is still greater than $M_*$. Thus we conclude that the VCG can not be realized in a fully consistent quintessence field theory either.

In summary, we have used the criterion (1) originated from the weak gravity conjecture to investigate the feasibility of the canonical scalar field description for Chaplygin-gas-type dark energy models. Although the models like the GCG and the VCG may be compatible with observations (the CG has been already ruled out by observation), their theoretical foundation may be problematic. For these models realized in quintessence, the CG and the VCG are in the swamp-land. For the GCG, the criterion sets a very tight constraint on the parameter space. However, this part is out of the best fit range set by observations, indicating that the canonical scalar field description of the GCG is incompatible with the theoretical requirement (1) either. Therefore, we reach the conclusion that the Chaplygin-gas-type models can not be realized in quintessence when the weak gravity conjecture is taken into account. Whether these models can be described by the field theories like some generalized Born–Infeld theories is still an open question worthy of further investigation.

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