The Binary Black Hole Merger Rate from Ultraluminous X-ray Source Progenitors

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ABSTRACT

Ultraluminous X-ray sources (ULXs) exceed the Eddington luminosity for a \(\approx 10M_\odot\) black hole. The recent detection of black hole mergers by the gravitational wave detector ALIGO indicates that black holes with masses \(\gtrsim 10M_\odot\) do indeed exist. Motivated by this, we explore a scenario where ULXs consist of black holes formed by the collapse of high-mass, low-metallicity stars, and that these ULXs become binary black holes (BBHs) that eventually merge. We use empirical relations between the number of ULXs and the star formation rate and host galaxy metallicity to estimate the ULX formation rate and the BBH merger rate at all redshifts. This assumes the ULX rate is directly proportional to the star formation rate for a given metallicity, and that the black hole accretion rate is distributed as a log-normal distribution. We include an enhancement in the ULX formation rate at earlier epochs due to lower mean metallicities. With simplified assumptions, our model is able to reproduce both the rate and mass distribution of BBH mergers in the nearby universe inferred from the detection of GW 150914, LVT 151012, GW 151226, and GW 170104 by ALIGO if the peak accretion rate of ULXs is a factor \(\approx 1 — 300\) greater than the Eddington rate. Our predictions of the BBH merger rate, mass distribution, and redshift evolution can be tested by ALIGO in the near future, which in turn can be used to explore connections between the ULX formation and BBH merger rates over cosmic time.

Key words: gravitational waves — X-rays: binaries — stars: black holes — stars: massive — accretion, accretion disks

1 INTRODUCTION

Ultraluminous X-ray sources (ULXs) are off-nuclear X-ray point sources in nearby galaxies with X-ray luminosities greater than the Eddington luminosity for a \(\approx 10M_\odot\) black hole (e.g., Fabbiano 1988; Mizuno et al. 1999; Colbert & Mushotzky 1999; Colbert & Ptak 2002; Liu & Mirabel 2005). They are likely related to accretion onto high-mass black holes, although their exact nature is still in question; until recently, there was no reliable evidence for black holes with masses \(\gtrsim 10 M_\odot\) outside of the supermassive black holes as the centers of galaxies. It has been suggested that ULXs are a result of beamed, anisotropic emission (Georganopoulos, Aharonian, & Kirk 2002; Körding, Falcke, & Markoff 2002), although strong beaming is now disfavored due to observations of ionized nebulae around some ULXs that indicates their emission is isotropic (e.g., Pakull & Mirioni 2003; Gürtler, et al. 2006; Behera et al. 2010). The two remaining possibilities are super-Eddington accretion (e.g., Begelman 2003; Poutanen et al. 2007; Finke & Böttcher 2007).
capturing high-mass stars to explain the local population (e.g., Blecha et al. 2006).

The detection of gravitational waves by the Advanced Laser Interferometer Gravitational Wave Observatory (ALIGO) has given the first evidence that black holes with masses $\gtrsim 10 M_\odot$ exist. Previous mass estimates $\gtrsim 10 M_\odot$ of the black hole in IC 10 X-1 (Prestwich et al. 2007; Silverman & Filippenko 2008) are no longer considered reliable (Laycock et al. 2015; Abbott et al. 2016e). The first gravitational wave event detected, dubbed GW 150914, consisted of the merging of two black holes with masses $35^+4^- M_\odot$ and $29 \pm 4 M_\odot$ merging to form a black hole with mass $62 \pm 4 M_\odot$ at redshift $z = 0.09^{+0.03}_{-0.04}$ (Abbott et al. 2016e). Although the pre-merger black hole mass estimates were later updated to $35^{+4}_{-10} M_\odot$ and $30^{+7}_{-5} M_\odot$ (Abbott et al. 2016c). More merging black holes have been subsequently detected by ALIGO (Abbott et al. 2016b,c,d,e,f, 2017). In this paper, we re-evaluate the nature of ULXs in light of these discoveries. We investigate a scenario for ULX formation similar to that proposed by Zampieri & Roberts (2009), that ULXs include high-mass black holes formed by low-metallicity stars local to the ULX (i.e., not from stars in the early universe). In our scenario, the ULXs are born as binary high-mass field stars, before one of the stars explodes in a supernova and turn into a black hole. Thus, the black hole in a ULX system is born not long before the ULX system was created. The extremely high mass stars needed to make the high-mass black holes do indeed exist in the local universe; the η Carina system consists of binary stars whose masses combine to be $250 M_\odot$ (Kashi & Soker 2010). The fact that most high mass stars are found in binaries (García & Merrifield 2001; Sana et al. 2008) would also seem to make this scenario likely.

In Section 2, we develop a simple model for ULXs and compute the ULX formation rate as a function of redshift. Inoue, Tanaka, & Soker (2016) have suggested that ULXs are the progenitors of BBH mergers; that is, an accreting high-mass black hole and high-mass star become a binary black hole (BBH) system after the companion star explodes in a supernova; and the BBH eventually merges, and is potentially detectable by ALIGO (see also Bulik et al. 2011). They use this to estimate the rate of BBH mergers expected in the local universe. In Section 3, we use our knowledge of the ULX formation rate to estimate the BBH merger rate. Our calculation goes beyond the estimates by Inoue et al. (2016) and explores detailed redshift and metallicity dependences of the ULX and corresponding BBH merger rates for different binary BH mass combinations. Our modeling results are compared with the rate and mass distribution inferred from mergers that have been detected so far (GW 150914, LVT 151012, GW 151226, and GW 170104; Abbott et al. 2016a,c,d,e,f, 2017). Finally, we conclude with a discussion (Section 4). In Appendix A, we compute an enhancement in cosmological ULX production due to decreasing metallicity at higher redshifts.

## 2 ULX FORMATION

In this section, we discuss our simple ULX model, and its formation rate, both in the local universe and extending to high redshift. We consider only those ULXs that are compact objects with high-mass companions, neglecting those with low-mass companions, which cannot become BBH systems. We make a number of simplifying assumptions about the evolution of high mass stars, binaries, and supernovae. We avoid discussion of poorly-understood tidal chemical mixing (e.g., Mandel & de Mink 2014) and common envelope binary evolution (e.g., Belczynski et al. 2007, 2016).

### 2.1 Simple ULX Model

Consider two massive ($\gtrsim 10 M_\odot$), low-metallicity stars. After some time, one of the stars explodes in a supernova leaving a neutron star or black hole remnant of mass $M_{BH,1}$. At some point in the binary’s life time, the black hole is accreting from its companion star, creating intense X-ray radiation with an X-ray luminosity $L_X \gtrsim 10^{39}$ erg s$^{-1}$. The binary is in its ULX phase. The X-ray luminosity of the ULX can be expressed as a fraction $\epsilon_{Edd}$ of the Eddington luminosity, so that

$$L_X = \epsilon_{Edd} L_{Edd} = \epsilon_{Edd} L_0 m_{BH,1},$$

where $L_0 = 1.26 \times 10^{38}$ erg s$^{-1}$ and $m_{BH,1} = M_{BH,1}/M_\odot$. We assume that the ULX emission is isotropic, consistent with observations of ionized nebulae around some ULXs. We take the ULX lifetime to be $\tau_{ULX} = 0.1$ Myr, consistent with several recent estimates (see the discussion by Mineo et al. 2013; Inoue et al. 2016). However, this value is uncertain, as discussed in Section 4.

Eventually, the massive companion star will exhaust its nuclear fuel and also end its life in a supernova that leaves behind a compact object of mass $M_{BH,2}$. We assume that $M_{BH,2} \leq M_{BH,1}$ since generally lower mass stars live longer and make lower mass compact objects than higher mass stars when in isolation, although this can be altered in the mass exchange in binary systems. We allow $m_{BH,1} = M_{BH,1}/M_\odot$ and $m_{BH,2} = M_{BH,2}/M_\odot$ to go down to 1.4, we actually are including neutron stars as well, since some ULXs are known to be accreting neutron stars (e.g., Bachetti et al. 2014; Israel et al. 2017). However, we will refer to them as black holes for most of this paper, since in our model black hole ULXs will outnumber neutron star ULXs, consistent with population synthesis modeling (Fragos et al. 2015), although if ULXs are anisotropic emitters, this may not be the case (Wiktorowicz et al. 2017; Middleton & King 2017). For the maximum mass of the compact object, we use $m_{BH,max} = 130$ based on the maximum possible black hole mass found from stellar evolution and supernova models of low-metallicity stars by Spera et al. (2015).

### 2.2 Local ULX Formation

We are interested in ULXs that eventually become BBHs, so these must be ULXs with high-mass stellar companions. There is evidence that most ULXs in spiral galaxies have high-mass companions, while ULXs in elliptical galaxies have low-mass companions (e.g., Swartz et al. 2004). We assume that all ULXs in spiral galaxies have high-mass companions, and all ULXs in elliptical galaxies have low-mass companions, so that we only explore those ULXs that are found in spiral galaxies (similar to Inoue et al. 2016). In a survey of nearby galaxies, Walton et al. (2011) found that the X-ray luminosity function for ULXs in spiral galaxies is

$$N_X(L_X) = \frac{dN}{dL_X} \propto L_X^{-\alpha}$$

with $\alpha = 1.85 \pm 0.11$ and $L_X$ ranging from $L_{min} = 10^{39}$ erg s$^{-1}$ to $L_{max} = 6 \times 10^{40}$ erg s$^{-1}$. Similarly, they find the number of ULXs per unit stellar mass of the spiral host galaxy is

$$N_X(M_{Gal}) = \frac{dN}{dM_{Gal}} = N_0 M_\star^{-\beta}$$
where $M_{\text{Gal}} = 10^3 \ M_\odot$ is the host galaxy stellar mass\footnote{\(M_{\odot} = 10^6 M_\odot\)}, $\beta = 0.64 \pm 0.07$, and

$$N_0 = N_X(M = 1) = 3.3 \times 10^{-4} \ M_\odot^{-1}. \quad (4)$$

If the Milky Way has a stellar mass of \(6.43 \pm 0.63 \times 10^4 \ M_\odot\)\footnote{\(M_{\odot} = 10^6 M_\odot\)} (McMillan 2011), then Equation (3) predicts \(\approx 1.5\) ULXs in our Galaxy, approximately consistent with 0 that are observed. Combining Equations (2) and (3) we expect that

$$N_X(L_X, M_{\text{Gal}}) = \frac{dN}{dL_X dM_{\text{Gal}}} = N_0'' M_3^{-\beta} L_4^{\alpha}, \quad (5)$$

where $L_4 = L_X/(10^{40} \text{ erg s}^{-1})$ and $N_0'' = N_X(L_{40} = 1, M_3 = 1)$. Here we assume that $N_X(L_X)$ and $N_X(M_{\text{Gal}})$ are independent. Using

$$N_X(M_{\text{Gal}}) = \int_{L_{\text{min}}}^{L_{\text{max}}} dL \ N_X(L_X, M_{\text{Gal}}) \quad (6)$$

one can solve for the normalization constant in Equation (5),

$$N_0'' = 2.4 \times 10^{-5} \ M_\odot^{-1} \ (10^{40} \text{ erg s}^{-1})^{-1}. \quad (7)$$

The ULX formation rate as a function of $L_X$ and host galaxy mass can be estimated as

$$\dot{N}_X(L_X, M_{\text{Gal}}) = \frac{dN}{dL_X dM_{\text{Gal}}} = \frac{N_X(M_{\text{Gal}}, L_X)}{t_{\text{ULX}}}. \quad (8)$$

The cosmological ULX formation rate per unit comoving volume, per unit X-ray luminosity, in the local universe can be found by convolving $\dot{N}_X(L_X, M_{\text{Gal}})$ with the spiral galaxy mass function in the local universe, $\phi(M_{\text{Gal}}; z = 0) = dN/(d\log_{10} M_{\text{Gal}}) dV$, so that

$$\dot{n}_X(L_X; z = 0) = \frac{d}{dL} \frac{d}{dL} \frac{d}{dM_{\text{Gal}}} \dot{N}_X(L_X, M_{\text{Gal}})$$

$$= \int_{M_{\text{Gal,min}}}^{M_{\text{Gal,max}}} dM_{\text{Gal}} \times \phi(M_{\text{Gal}}; z = 0) \times \ln(10)$$

$$\times \dot{N}_X(L_X, M_{\text{Gal}}) \quad (9)$$

where we use $M_{\text{Gal,min}} = 10^2 \ M_\odot$ and $M_{\text{Gal,max}} = 10^7 \ M_\odot \ M_\odot$. The function $\phi(M_{\text{Gal}}; z = 0)$ was found by Moffett et al. 2016, which they represented by a Schechter function,

$$\phi(M_{\text{Gal}}; z = 0) = \phi^* \ln(10) \left( \frac{M_{\text{Gal}}}{M^*} \right)^{1+\delta}$$

$$\times \exp \left( -\frac{M_{\text{Gal}}}{M^*} \right). \quad (10)$$

They found it was well-fit with parameters $\phi^* = (8.55 \pm 1.00) \times 10^{-4} \ Mpc^{-3}$, $\delta = -1.39 \pm 0.02$, and $M^* = 10^{10.70 \pm 0.05} \ M_\odot$, which we use. The total ULX formation rate density for all ULXs can be found by integrating,

$$\dot{n}_X(z = 0) = \int_{L_{\text{min}}}^{L_{\text{max}}} dL \ \dot{n}_X(L_X; z = 0). \quad (11)$$

### 2.3 Cosmological ULX Formation

There is evidence that high-mass X-ray binaries are connected with star formation (e.g., Grimm et al. 2003; Ranalli et al. 2003). Since in our model ULXs are formed from short-lived, high mass stars, the ULX formation rate should follow the star formation rate. However, in our model ULXs are formed from low-metallicity stars, and the average metallicity decreases with redshift. There is in fact some weak evidence that lower metallicity galaxies host more ULXs for a given star formation rate (e.g., Mapelli et al. 2010; Prestwich et al. 2013). In Appendix 3 we use some of this evidence to estimate a metallicity enhancement in ULX formation, $\zeta(z)$.

Using the knowledge of the SFR, metallicity enhancement, and local ULX formation rate as found in Section 2.2, one can find the ULX formation rate density for any $z$,

$$\dot{n}_X(L_X; z) = \frac{\psi(z)}{\psi(z = 0)} \zeta(z) \dot{n}_X(L_X; z = 0), \quad (12)$$

where $\psi(z) = dM_{\text{SFR}}/(dV dt)$ is the star formation rate (SFR) density. Finke, Razzaque, & Dernier (2010) found that the combination of the Cole et al. (2001) SFR, with parameters found by Hopkins & Beacom (2006), combined with an initial mass function (IMF) from Baldry & Glazebrook (2003), reproduced the luminosity density data available at the time better than other combinations of SFR and IMF. Therefore, we use this $\psi(z)$ here.

The total ULX formation rate density for all ULXs can be found by integrating,

$$\dot{n}_X(z) = \int_{L_{\text{min}}}^{L_{\text{max}}} dL \ \dot{n}_X(L_X; z), \quad (13)$$

where recall $L_{\text{min}} = 10^{39} \text{ erg s}^{-1}$ to $L_{\text{max}} = 6 \times 10^{40} \text{ erg s}^{-1}$. This result is shown in Figure 1 with and without the metallicity enhancement factor (i.e., the latter has $\zeta(z) = 1$). Not surprisingly, the ULX formation rate follows the shape of the star formation rate without the metallicity enhancement; including the enhancement, the ULX formation rate is larger at higher redshifts.
3 BBH MERGERS

3.1 Model

After the companion star ends its life in a supernova, and joins its companion as a black hole, the BBH system is created. Thus far, we have made no assumptions about the Eddington ratio of the ULX. However, to use the ULX formation rate to estimate the BBH merger rate, we must use a distribution of Eddington ratios to convert from the ULX X-ray luminosities to the BBH masses. A study of accreting stellar-mass black holes indicates that this distribution resembles a log-normal distribution,

\[ P_P(\ell_{\text{Edd}}) = \frac{A_P}{\ell_{\text{Edd}}} \times \exp \left\{ -\frac{[\ln(\ell_{\text{Edd}}) - \ln(\ell_{\text{Edd},pk})]^2}{2\sigma^2} \right\} , \]  

with peak \( \ell_{\text{Edd},pk} \approx 10^{-2.5} - 10^{-2} \) and \( \sigma \approx 1 \) (Reynolds & Miller 2013). A study of X-ray selected broad-line active galactic nuclei by Sub et al. (2015) found the Eddington ratios of these objects are distributed as a log-normal distribution, with \( \ell_{\text{Edd},pk} = 10^{0.6} \) and \( \sigma = 0.8 \). If the accretion flow of ULXs behaves similarly to accreting stellar-mass black holes and active galactic nuclei, it seems reasonable to assume that this distribution of \( \ell_{\text{Edd}} \) for ULXs is also a log-normal distribution. Here we will use a log-normal distribution for \( P_P(\ell_{\text{Edd}}) \) with \( \sigma = 1 \) and various values for \( \ell_{\text{Edd},pk} \), which we consider a free parameter. Since we have \( \ell_{\text{max}} = 6 \times 10^{39} \) erg s\(^{-1}\), and the maximum possible black hole mass was found to be 130\( M_\odot \) by Spera et al. (2015), the Eddington ratio must extend to at least \( \ell_{\text{Edd}} \geq 3.7 \). Thus, we will allow the value of \( \ell_{\text{Edd}} \) to be \( \ell_{\text{min}} \) to explain the brightest ULXs, although the log-normal distribution may be more questionable for ULXs with \( \ell_{\text{Edd}} \gg 1 \), since the distribution was found for objects with lower \( \ell_{\text{Edd}} \). In equation (14), the normalization constant is found by performing the integral

\[ 1 = \int_{\ell_{\text{min}}}^{\ell_{\text{max}}} d\ell_{\text{Edd}} P_P(\ell_{\text{Edd}}) \]  

and solving for \( A_P \), where \( \ell_{\text{min}} = \ell_{\text{min}}/(L_0 m_{\text{BH,1}}) \) and \( \ell_{\text{max}} = \ell_{\text{max}}/(L_0 m_{\text{BH,1}}) \).

The BBHs' orbits decay by emitting gravitational waves, and they merge after

\[ t_{\text{GW}} = 5.6 \times 10^7 \left( \frac{a}{10 R_\odot} \right)^4 \left( \frac{M_{\text{BH,1}}}{30 M_\odot} \right)^{-1} \]  
\[ \times \left( \frac{M_{\text{BH,2}}}{30 M_\odot} \right)^{-1} \left( \frac{M_{\text{BH,1}}}{30 M_\odot} + M_{\text{BH,2}} \right)^{-1} \]  
\[ \text{yr} \quad (\text{Peters 1964}) \]  

where \( a \) is the initial separation of the BBHs. If a binary black hole system is created at redshift \( z_X \) and merges at \( z_m \), then

\[ t_{\text{GW}} = \int_{z_m}^{z_X} \left| \frac{dt}{dz} \right| dz \]  

where

\[ \left| \frac{dt}{dz} \right| = \frac{1}{H_0(1+z)\sqrt{(1+z)^4 \Omega_m + \Omega_A}} \]  

in a flat \( \Lambda \)CDM universe. To find \( z_X \) for a given \( z_m, m_{\text{BH,1}}, m_{\text{BH,2}}, \) and \( a \), we solve Equations (16) and (17) numerically using standard cosmological parameters \( H_0 = 70 \text{ km s}^{-1}\text{ Mpc}^{-1} \), \( \Omega_m = 0.3 \), and \( \Omega_A = 0.7 \). Doing this requires knowledge of the distributions of \( m_{\text{BH,2}}, m_{\text{BH,1}}, \) and \( a \).

Abbott et al. (2016a) use a power-law distribution for \( m_{\text{BH,2}} \) and we follow their calculation, so that the distribution, normalized to unity, is

\[ P_m(m_{\text{BH,2}}) = A_m m_{\text{BH,2}}^{-\gamma} \]  

where

\[ A_m = \begin{cases} \frac{(\gamma + 1)/m_{\text{BH,1}}^{\gamma+1} - m_{\text{BH,1}}^{\gamma+1}}{1/ln(m_{\text{BH,1}}/m_{\text{BH,2min}})}, & \gamma \neq 1 \\ 1/\ln(m_{\text{BH,1}}/m_{\text{BH,2min}}), & \gamma = 1 \end{cases} \]  

Abbott et al. (2016a) use \( \gamma = 0 \), and consequently do so we, although \( \gamma \) is in principle a free parameter.

The separation of binaries can change during their lives due to mass exchange between the two stars due to Roche lobe overflow and winds, tidal interactions, magnetic braking, and emission of gravitational radiation. Further, when massive stars explode the resulting compact object can get a significant “kick”, which can substantially alter the binary separation. These processes are modeled in detail in sophisticated population synthesis codes (e.g. Hurley et al. 2002 Belczynski et al. 2003, 2008). For our purposes, we are only interested in the separation when binaries become BBHs. Population synthesis calculations by Belczynski et al. (2002) indicate that the initial separation of the BBHs is a distribution with a peak of about \( a = 10 R_\odot \) in their “standard model”.

Therefore, we use a log-normal distribution for the initial separation of the BBHs, normalized to unity over \( a = 0 \) to \( a = \infty \),

\[ P_a(a) = \frac{1}{\sigma_a \sqrt{2\pi}} \times \exp \left\{ -\frac{[\ln(a) - \ln(a_{pk}) - \sigma_a^2]^2}{2\sigma_a^2} \right\} , \]  

with peak \( a_{pk} = 10 R_\odot \) and \( \sigma_a = 0.6 \). In general, however, \( a_{pk} \) can be considered another free parameter.

Putting everything together, the merger rate density

\[ \dot{n}_m(m_{\text{BH,1}}; z_m) = \frac{dN}{dV dt dm_{\text{BH,1}}} \]  
\[ = \left\{ \int_{\ell_{\text{min}}}^{\ell_{\text{max}}} d\ell_{\text{Edd}} P_P(\ell_{\text{Edd}}) \right\} \times \frac{L_X}{m_{\text{BH,1}}} \dot{n}_X(L_X; z = 0) \]  
\[ \times \int_{m_{\text{BHimin}}}^{m_{\text{BH,1}}} dm_{\text{BH,2}} P_m(m_{\text{BH,2}}) \]  
\[ \times \int_0^{\infty} da P_a(a) \times \psi(z_X(z_m, m_{\text{BH,1}}, m_{\text{BH,2}}, a)) \]  
\[ \times \zeta(z_X(z_m, m_{\text{BH,1}}, m_{\text{BH,2}}, a)) \]  

where \( \ell_{\text{min}} = \ell_{\text{min}}/(L_0 m_{\text{BH,1}}) \), \( \ell_{\text{max}} = \ell_{\text{max}}/(L_0 m_{\text{BH,1}}) \), \( L_X/m_{\text{BH,1}} \) and \( dL_X/dm_{\text{BH,1}} \) are calculated from Equation (1), and \( \dot{n}_X(L_X; z = 0) \) is given by Equation (9). The total merger rate

\[ \dot{n}_{m,tot}(z_m) = \int_{m_{\text{BH,imin}}}^{m_{\text{BH,imax}}} dm \dot{n}_m(m; z_m) \]  

where recall \( m_{\text{BH,imin}} = 1.4 \) and \( m_{\text{BH,imax}} = 130 \).

The total merger rate is plotted in Figure 2 again, with and without the metallicity enhancement factor \( \zeta(z) \). The overall shape of the curves roughly follows the ULX formation rate, delayed by a time \( t_{\text{GW}} \). The lower the Eddington ratio \( \ell_{\text{Edd,pck}} \), the more BBH formation.
mergers will originate from the observed ULX progenitor population.

The BBH merger rate per unit logarithmic primary black hole mass bin, \( \dot{n}_{\text{BH},1} \times \dot{n}(m; z_m) \), for \( \ell_{\text{Edd}, \text{pk}} = 10 \) is plotted as a function of \( m_{\text{BH},1} \) for various values of \( z_m \) in Figure 5. At low values of \( m_{\text{BH},1} \) mergers are fewer, since there is not enough time for many of them to occur within the age of the universe. At higher redshifts, low-mass mergers become fewer in number, since the universe is younger and there is less time for mergers to occur. Thus, overall the peak increases at high \( z \). This means this model produces few binary neutron star systems such as PSR 1913+16 (Taylor, Fowler, & McCulloch 1979) and PSR J0737−3039 (Burgay et al. 2003) that will merge within the age of the universe. However, we note that the population synthesis calculations of Belczynski et al. (2002) find a separate peak at \( a = 0.3 R_\odot \) for the initial separation of binary neutron stars. So another population, which will not go through a ULX phase, can explain the binary neutron stars. Also note the overall normalization increases up to \( z \approx 3 \) and decreases at higher \( z \), in agreement with Figure 2 in Figure 4. \( m_{\text{BH},1} \times \dot{n}(m; z_m = 0) = 0 \) is plotted as a function of \( m_{\text{BH},1} \) and \( m_{\text{BH},2} \). This was computed from Equation (22) using \( P(m) \rightarrow \delta(m_{\text{BH},1} - m_{\text{BH},2}) \). The peak in merger rate is where \( m_{\text{BH},1} = m_{\text{BH},2} \approx 6 \), in agreement with Figure 3.

The BBH merger rate per unit logarithmic primary black hole mass bin at \( z_m = 0 \) as a function of \( m_{\text{BH},1} \) is plotted in Figure 5 for different values of the peak initial BBH separation \( a_{\text{pk}} \), instead of our “default” value \( a_{\text{pk}} = 10 R_\odot \). Clearly, both the low-mass cutoff, and peak are strongly dependent on \( a_{\text{pk}} \). This leads us to the conclusion that future observations of mergers by ALIGO could build up enough statistics to observe this enhancement, which would then constrain the initial BBH separation.

The cumulative merger rate as measured by the observer can be found from

\[
\dot{N}_{m, \text{tot}}(z_m) = 4\pi \int_0^z dz_m \frac{\dot{n}_{m, \text{tot}}(z_m)}{1+z} \frac{dV_c}{dz_m} \tag{24}
\]

![Figure 2. Total BBH merger rate density, \( \dot{n}_{m, \text{tot}}(z_m) \), computed from Equation (23), with and without the metallicity enhancement factor, \( \zeta(z) \). Different values of the peak Eddington ratio, \( \ell_{\text{Edd}, \text{pk}} \), were used, as indicated on the plot. The ULX formation rate, as in Figure 1, is also shown as indicated.](image)

![Figure 3. BBH merger rate density per unit logarithmic primary black hole mass bin, \( m_{\text{BH},1} \times \dot{n}(m_{\text{BH},1}; z_m) \), plotted as a function of primary black hole mass, \( m_{\text{BH},1} \), for various values of \( z_m \). This calculation uses \( \ell_{\text{Edd}, \text{pk}} = 10 \) and includes the metallicity enhancement factor.](image)

![Figure 4. BBH merger rate density per unit logarithmic black hole mass bin plotted as a function of \( m_{\text{BH},1} \) and \( m_{\text{BH},2} \), for \( z_m = 0 \). This calculation uses \( \ell_{\text{Edd}, \text{pk}} = 10 \) and includes the metallicity enhancement factor.](image)
culculations by Mandel & de Mink (2016). Those authors simulate d one they use by a factor of 2, below the exponential cutoff the y use fact that our luminosity function is actually slightly lower than the Inoue et al. (2016) also used ULXs to estimate the merger ratein 3.2 Comparison with Other BBH Merger Models at high
L greater than theirs (see their Figures 1 and 2). This is despi te the local universe. Our calculations generally predict a me rger rate the formation of BBH systems from chemically homogeneous e-
olution of closely-interacting massive binary stars. All our models shown in Figure 2 are above theirs. The evolution at higher redshift is quite different. Mandel & de Mink (2016) find a peak of ≈ 20 Gpc⁻³ yr⁻¹ at z ≈ 0.5, then a merger rate declining sharply after that, so that there are hardly any mergers above z = 1.5; see their Figure 7.

By contrast, estimates by Dominik et al. (2013) find large numbers of BBH mergers (≥ 10–100 Gpc⁻³ yr⁻¹) out to redshift z ≳ 16, mainly depending on whether or not they include “high kicks”. Their “high kick” scenario seems to be ruled out by rate inferred by the detection of BBH mergers by ALIGO, since this scenario predicts ≲ 1 yr⁻¹ Gpc⁻³ at low z. In their scenarios without high kicks, they find 10–100 Gpc⁻³ yr⁻¹ at z = 0, consistent with our models with ℓ_{Edd, pk} ≥ 1.0. Their results for the evolution of the merger rate at high z is also different from ours; while our merger rates peak at z = 2–3, in all of their scenarios the merger rate peaks at z ≳ 4.

The predictions for the evolution of BBH merger rate with z by Mandel & de Mink (2016) and Dominik et al. (2013) not only differ substantially from our predictions, but differ substantially from each other as well. Both these authors take a more “first principles” approach, while our calculation is more empirically- motivated. We hope this makes it more accurate, although we note that there are many unknowns in our calculation (discussed further in Section 4). ALIGO will be able to detect BBH mergers out to no higher than z ≈ 1 when it reaches its peak sensitivity in ~2020 (Abbott et al. 2016d); the differences in the merger rate predictions by ourselves, Mandel & de Mink (2016), and Dominik et al. (2013) at higher z are unlikely to be observed in the near future. However, the striking rapid evolution in the merger rate with z predicted by Mandel & de Mink (2016) at z < 1 should be confirmed or refuted with further ALIGO observations.

3.3 Comparison with ALIGO Observations

Based on the ALIGO detection of three BBH mergers: GW 150914, GW 151226, the lower significance LVT 151012, Abbott et al. (2016a) constrained the BBH merger rate to be between 9 and 240 Gpc⁻³ yr⁻¹ in the local universe, with the most likely value...
being \( \approx 70 \text{ Gpc}^{-3} \text{ yr}^{-1} \) with an event-based analysis, and \( \approx 100 \text{ Gpc}^{-3} \text{ yr}^{-1} \) with a power-law fit to the distribution of \( m_{BH,1} \) (more on this below). The additional detection of GW 170104 tightens this range a bit, to \( 12-213 \text{ Gpc}^{-3} \text{ yr}^{-1} \) \cite{Abbott:2017oio}. All of our model curves in Figure 8 are consistent with \( \ell_{Edd, pk} > 1.0 \) within this range, with the closest model being the one with \( \ell_{Edd, pk} = 10.0 \). We conclude, based on the inferred ALIGO merger rate, that our model is a good representation if \( \ell_{Edd, pk} \gtrsim 1.0 \) for ULXs.

From ALIGO detections of BBH mergers, one can infer more information than just the total merger rate. Based on detections in ALIGO’s first observing run, and assuming that \( \dot{n}_m(m_{BH,1} ; z_m = 0) \propto m_{BH,1}^{\theta/5} \), \cite{Abbott:2016nhf} infer \( \theta = 2.5 \pm 1.8 \) for mergers with \( m_{BH,1} \) between \( 14.2-8.3 \) (GW 151226) and \( 36.2-5.2 \) (GW 150914). With the addition of GW 170104, this was revised to \( \theta = 2.3^{+1.4}_{-1.3} \). In our model, on a plot of the merger rate as a function of \( m_{BH,1} \), the slope is weakly dependent on \( \ell_{Edd, pk} \), as Figure 9 demonstrates. Here we also plot various power-laws consistent with the inferred ALIGO distribution for \( 14 < m_{BH,1} < 36 \).

The mass distributions for all of our models shown in Figure 8 are consistent with the ALIGO observations, which have admittedly large error bars. Our model is thus fairly robust with respect to the free parameter \( \ell_{Edd, pk} \), and we conclude that our model with \( \ell_{Edd, pk} \approx 1 - 300 \) can reproduce both the inferred rate and mass distribution of BBH mergers.

### 4 DISCUSSION

We have explored the scenario where ULXs consist of a massive black hole and companion star, become BBH systems, where the BBHs finally merge and create gravitational waves potentially detectable by ALIGO. By assuming that the ULX formation rate is directly proportional to the star formation rate (modulo metallicity effects that are taken into account as described in Appendix A), we computed the ULX formation rate as a function of redshift. We then assumed that every ULX becomes a BBH, and took into account the time for a BBH merger to occur, to compute the BBH merger rate. As discussed in Section 2, we made a number of simplifying assumptions about the evolution of high mass stars, binaries, and supernovae. For instance, \cite{Dominik:2013gra} have a fairly large suite of compact object merger rate models with various assumptions about whether Hertzsprung Gap stars can be common-envelope donor stars, whether or not delayed supernovae occur, and other variations.

Our derivation of the metallicity enhancement factor (Appendix A) makes use of a number of tentative results: namely, the relation between the metallicity of a host galaxy and the enhancement in the number of ULXs it produces, and in the evolution of the average metallicity of galaxies with redshift. For the latter, we note that a similar correlation from \cite{Kewley:2005} was used by \cite{Mandel:2016} and it is very similar to the “high-end” metallicity evolution used by \cite{Dominik:2013gra}. Further studies on the relationship of the number of ULXs and host galaxy metallicity, and the evolution of metallicity in the universe, could greatly improve our estimates.

We find that our model is consistent with the rate and distribution of BBH mergers inferred from the ALIGO detections if \( \ell_{Edd, pk} = 1 - 300 \). Our scenario requires super-Eddington accretion on to black holes with masses \( \lesssim 130 M_\odot \). This means that if our model is accurate and the BBH mergers observed by ALIGO originated as ULXs, a large fraction, possibly even the great majority, of ULXs have super-Eddington accretion. There is evidence for super-Eddington accretion in some ULXs; for example the discovery of pulsar ULXs requires super-Eddington accretion \cite{Bachetti:2014,Fuerst:2016,Israel:2017}. The X-ray spectrum of a ULX in NGC 5907 in various states led \cite{Walton:2015} to conclude that this source was accreting at a super-Eddington rate. ALIGO could see 100s of BBH mergers in the near future (e.g., \cite{Mandel:2016}); as it detects more mergers, the large errors on the inferred rate and distribution will get smaller, and our model parameters, such as \( \ell_{Edd, pk} \) and \( \alpha \) could be constrained.

One major unknown is the ULX timescale, \( t_{ULX} \). We use the value 1.0 Myr for all ULXs, the value used by Muller et al. (2012) and Heida et al. (2016). This value is consistent with the thermal timescale of high-mass main sequence stars. King et al. (2001) considered the thermal timescale to be the most likely relevant timescale for the lifetime of a ULX. However, the companion stars of ULXs may not be main sequence stars; indeed 11/62 ULXs examined by Heida et al. (2014) show infrared excesses consistent with being red supergiants; spectroscopic follow-up for one of these indicates it is indeed a red supergiant (Heida et al. 2018). Red supergiants have considerably shorter thermal timescales, and thus ULXs with red supergiant companions would have considerably shorter lifetimes. If all sources had shorter \( t_{ULX} \), our predicted merger rate would be lower for a given \( \ell_{Edd, pk} \), and higher \( \ell_{Edd, pk} \) would be required to be consistent with the observed BBH merger rate from ALIGO. However, most high-mass X-ray binaries do not have supergiant companions, and if this is also true for ULXs with high mass companions (the only ones we are interested in here), on average ULXs would have \( t_{ULX} \sim 0.1 \text{ Myr} \) \cite{Minezaki:2012}, thus we use this value. If the nuclear burning timescale is the relevant timescale for the lifetime of a ULX, the timescale could be as long as \( t_{ULX} = 1 \text{ Myr} \) \cite{Patruno:2008}, there is some evidence for these longer \( t_{ULX} \) from the kinematics of the
nebula around the ULX NGC 1313 X-2 (Pakull & Mirioni 2002). These longer lifetimes are consistent with the ages of the star clusters where ULXs reside, which have been constrained to be $\lesssim 4-20$ Myr (Ramsey et al. 2006, Grisi et al. 2011, Poutanen et al. 2013). If we had used a larger lifetime, $t_{\text{ULX}} = 1$ Myr, lower values of $\varepsilon_{\text{Edd, pk}}$ would be consistent with the ALIGO constraints on the merger rate.

If a large fraction of ULXs are neutron stars, their X-ray emission could be beamed. This could alter our results; see e.g., King et al. (2001); Wiktorowicz et al. (2017), King et al. (2017); or Middleton & King (2017) for an exploration of this possibility. If beaming decreased the actual luminosity a factor $b$ relative to the observed luminosity, there would be a factor $1/b$ more sources that are not seen. Thus, the actual number of ULXs would be a factor $1/b$ higher. If $b$ is inversely proportional to accretion rate or intrinsic X-ray luminosity (see e.g. King 2009) then naturally the enhancement would be greater for brighter sources. The effect of a $b < 1$ on the total BBH merger rate is less obvious, but we expect it would alter the distribution of masses (e.g., Fig. 3) so that there would be more low-mass mergers and fewer high-mass mergers. Israel et al. (2017a) found that a beaming factor $b \approx 1/7$ for the neutron star ULX in NGC 5907 is consistent with expectations for a thin disk and avoidance of the “propeller” mechanism.

It is possible that BBHs occur from dynamical interactions in star clusters, and that this channel dominates the BBH merger rate (e.g., Sadowski et al. 2008; O’Leary et al. 2016, Chatterjee et al. 2017). Since we neglect BBH mergers that occur from other formation channels, and since not every BBH may originate as a ULX, the BBH merger rates we compute could be viewed as lower limits. This is also true because we neglect mergers that originated as accreting binaries with luminosities $L_X < 10^{39}$ erg s$^{-1}$, as described at the end of Section 3. However, again, our model reproduces the inferred rate and distribution of BBH mergers from ALIGO (Abbott et al. 2016a, 2017), so that mergers from other origins are not needed. It has been suggested that the spin orbit alignment of the detected mergers make a dynamical origin for them more likely (Rodriguez et al. 2016; Abbott et al. 2017). However, a population synthesis simulation that incorporates stellar rotation indicates that an isolated binary evolution scenario can explain the ALIGO measurements (Belczynski et al. 2013). The distribution of masses observed by ALIGO (i.e., $\theta$ in Section 3.3) could be an important clue to determining whether the binary evolution of dynamical channel dominates the formation of BBH mergers. In the coming years, we will learn much about BBHs and ULXs through the detection of gravitational waves.

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APPENDIX A: METALLICITY ENHANCEMENT FACTOR

A relationship between galaxy stellar mass and metallicity was found by Tremonti et al. (2004) using a study of 53,000 galaxies at $z \sim 0.1$ from the Sloan Digital Sky Survey. Ma et al. (2016) use cosmological simulations to derive the galaxy mass-metallicity relation and its evolution with redshift, with the simulation results matching observations at $z = 0 - 3$. We use the relationship from Ma et al. (2016), which is

$$ F_{\text{OH}} = 4.45 + 0.35 \log_{10}(M_{\text{Gal}}/M_\odot) + 0.93 \exp(-0.43z) $$  \hspace{1cm} (A1)

where we define $F_{\text{OH}} \equiv 12 + \log_{10}(O/H)$, and $O/H$ is the average gas-phase oxygen to hydrogen number ratio in the galaxy. A study by Mapelli et al. (2010) found that the number of ULXs per star formation rate in a sample of spiral, irregular, and peculiar galaxies was anti-correlated with the galaxies’ metallicities. Taking the solar value to be $F_{\text{OH}}\odot = 8.81$, their result can be written as an enhancement factor $A_{\text{OH}}$ in ULX formation relative to the solar value as

$$ \log_{10}[A_{\text{OH}}(F_{\text{OH}})] = 4.85 - 0.55F_{\text{OH}} . $$  \hspace{1cm} (A2)

The significance of this correlation was low, but at present it seems to be the best that can be done. We then define a metallicity enhancement as a function of redshift by

$$ \zeta(z) \equiv \frac{\int dM_{\text{Gal}} \phi(M_{\text{Gal}}; z)A_{\text{OH}}(F_{\text{OH}})}{\int dM_{\text{Gal}} \phi(M_{\text{Gal}}; z)} \times \frac{\int dM_{\text{Gal}} \phi(M_{\text{Gal}}; z = 0)A_{\text{OH}}(F_{\text{OH}})}{\int dM_{\text{Gal}} \phi(M_{\text{Gal}}; z = 0)} , $$  \hspace{1cm} (A3)

where $F_{\text{OH}}$ is found from $M_{\text{Gal}}$ from Equation (A1). This function $\zeta(z)$ gives the enhancement in ULX formation per SFR at redshift $z$, integrated over all galaxies at that redshift, relative to the ULX production rate at $z = 0$. Here the galaxy stellar mass function as a function of $z$ from the GOODS survey is used (Fontana et al. 2006). It is still parametrized as a Schechter function, Equation (10), but this time the parameters as a function of $z$ are given by

$$ \phi^* = 0.0035(1+z)^{-2.2} \text{ Mpc}^{-3} , $$  \hspace{1cm} (A4)

$$ \log_{10}(M^*/M_\odot) = 11.16 + 0.17z - 0.07z^2 , $$  \hspace{1cm} (A5)

$$ \delta = -1.18 - 0.82z . $$  \hspace{1cm} (A6)

In Equation (A3) all integrals are performed from $10^9 M_\odot$ to $10^{13} M_\odot$, the same values used by Fontana et al. (2006) to calculate the total stellar mass as a function of $z$. The calculation of $\zeta(z)$ is plotted in Figure [A1].

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Figure A1. The metallicity enhancement factor, $\zeta(z)$.

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