Few interacting particles in a random potential

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Abstract: We study the localization length of few interacting particles in a random potential. Concentrating on the case of three particles we show that their localization length is strongly enhanced comparing to the enhancement for two interacting particles.

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Recently it had been shown that in a random potential two repulsing/attracting particles can propagate coherently on a distance $l_c$ which is much larger than one particle localization length $l_1$ in absence of interaction \cite{1}. In some sense interaction destroys quantum interference which leads to one-particle localization and creates an effective pair of two particles of size $l_1$ propagating on a large distance. For better understanding of this result Imry developped \cite{2} a scaling block picture of localization for interacting particles which can be applied in principle for a larger number of particles and higher dimensions. Intensive numerical investigations of Pichard and coworkers \cite{3} and von Oppen and coworkers \cite{4} confirmed the existence of the two interacting particles (TIP) effect. While some additional checks are still required the results \cite{3, 4} definitely show that in one-dimensional case the TIP length is $l_c \propto l_1^\alpha$ with $\alpha$ close to the theoretically predicted power \cite{1} $\alpha = 2$. These results are also in agreement with the previous studies of Dorokhov who analysed the case of two particles confined by strong attraction in a well with size much smaller than $l_1$ \cite{5}. The investigations of TIP effect in higher dimensions have been done in \cite{2, 3, 6} and they demonstrated that in dimension $d = 3$ the TIP pair can be delocalized below one-particle Anderson transition where all one-particle states are localized.

While now the properties of TIP propagation reached a level of qualitative understanding, the problem of a larger number of interacting particles is still not well understood. From the physical point of view the most interesting situation is the case of finite density of particles. However, the analysis in this case is quite complicated and at present only estimate \cite{2} and numerical studies in \cite{8} have addressed this problem. One of the ways to
approach this problem is to analyse the case of larger number of particles. The simplest case is three interacting particles where the situation is not so trivial since two particle interaction leads to the Breit-Wigner structure of eigenstates \[9, 10, 11, 12\]. In this paper we will concentrate on this three particle model.

However, before the analysis of three particle model let us first discuss some generalized model of TIP (see also \[13\]). In this model the first particle is moving on sites \(n_1\) in a one channel Anderson model with diagonal disorder changing in the interval \(\pm W_1\), the intersites hopping matrix element is \(V_1\) and localization length at the center of the band is \(l_1 \approx 25(V_1/W_1)^2 \gg 1\). The second particle is moving in a strip with \(M\) transverse channels with sites marked by index \(n_2\) along the strip and index \(\tilde{n}_2(1 \leq \tilde{n}_2 \leq M)\) in transverse direction. The disorder in the strip is independent of disorder in the chain with first particle and the localization length for the second particle is \(l_2 \propto M\). The hopping in the strip is \(V \approx V_1\), and we assume that \(l_2 > l_1\). Now we will analyse what will happen if the interaction of the form \(U\delta_{n_1,n_2}\) is switched on between two particles. Similarly to \[1\] one should first estimate the transition matrix elements \(U_s\) between eigenstates without interaction (\(U = 0\)). This gives

\[
U_s = U \sum_{n_1, n_2, \tilde{n}_2} R^{+}_{n_1, m_1} \tilde{R}_{\tilde{n}_2, m_2, \tilde{m}_2} R_{n_1, m_1} \tilde{R}_{n_2, \tilde{n}_2, m_2, \tilde{m}_2} \delta_{n_1, n_2} \tag{1}
\]

where \(R\) represents the transformation between the lattice basis and one-particle eigenstates so that \(R_{n_1, m_1} \approx \exp(-|n_1 - m_1|/l_1 - i\theta_{n_1, m_1})/\sqrt{l_1}\) and \(\tilde{R}_{\tilde{n}_2, \tilde{n}_2, m_2, \tilde{m}_2} \approx \exp(-|n_2 - m_2|/l_2 - i\theta_{n_2, \tilde{n}_2, m_2, \tilde{m}_2})/\sqrt{Ml_2}\) correspondingly for the first and second particle. The phase \(\theta\) randomly changes with indices. Due to the exponential decrease of \(R\) one should take into account only the states with \(|n_1, m_1| < l_1\). For the case \(l_2 > l_1\)
the sum in (1) contains approximately $l_1 M$ random terms so that $U_s \approx U/(l_2 \sqrt{l_1 M})$. The interaction induced transition rate is given by the Fermi golden rule $\Gamma \sim U_s^2 \rho_c$ where $\rho_c \approx l_1 l_2 M/V$ is the density of coupled states. As the result, $\Gamma \sim U^2/(V l_2)$ is independent of $l_1$ and $M$.

With the rate $\Gamma$ we can determine the interaction induced diffusion rate for the first particle which is $D_1 \sim l_1^2 \Gamma \sim U^2 l_1^2/(V l_2)$ and appears as the result of collisions of the first particle with the second one oscillating in the block of size $l_2$. Knowing the diffusion rate, it is possible to determine the localization length for a pair in a way similar to that used for the kicked rotator (3) and based on the uncertainty relation between the frequency and time (see also (3)). Indeed, the number of excited states in the first chain grows with time $t$ as $\Delta n_1 \sim (D_1 t)^{1/2}$. Since two particles are propagating together so that $|n_1 - n_2| < l_2$ the total number of excited states in both chains is $\Delta N \sim \Delta n_1 (M l_2) \delta E/V$ where $\delta E$ takes into account the factor that the states are excited only in some energy interval inside the band width $V$. Generally, $\delta E < V$ and it is of the order of Breit-Wigner width $\Gamma$ (4), but we will see that $\delta E$ does not enter in the final expression for the localization length of pair, and therefore actual value of $\delta E$ is not very important (see also (4)). Indeed, all these $\Delta N$ levels are homogeneously distributed in the energy interval $\delta E$ and the average splitting between them is $\Delta \nu \sim \delta E/\Delta N$. According to the uncertainty relation between frequency and time at the moment $t$ we can resolve discrete lines with the splitting $1/t$. Therefore, at the moment $t^*$ defined by the equation $\Delta \nu \sim 1/t^*$ the discreteness of the spectrum is resolved and the diffusive propagation is stopped at $t^* \sim \Delta N(t^*)/\delta E$. This condition gives the localization time $t^*$ for TIP pair
and the localization length for the first particle $l_{c1}$:

$$t^* \sim U^2 l_1^2 M^2 l_2 / V; \quad l_{c1} \sim \Delta n_1 \sim (U/V)^2 l_1^2 M$$ \hspace{1cm} (2)

The interesting feature of this result is that $l_{c1}$ is independent of $l_2$. This indicates that the nature of motion in the second chain does not influence much the localization in the first chain. For $M = 1$ the length $l_{c1}$ is the same as in the case of TIP localization in one chain. However, the growth of the number of channels $M$ in the strip leads to the increase of $l_{c1}$. The localization length for the second particle is $l_{c2} \sim l_2$ if $l_2 \gg l_{c1}$ and $l_{c2} \sim l_{c1}$ if $l_2 \ll l_{c1}$. The similar approach can be used for analysis of TIP localization in higher dimensions [13].

Let us now consider three interacting particles in one-dimensional chain with on site interaction $U_{12}\delta_{n_1,n_2}$, $U_{23}\delta_{n_2,n_3}$ and $U_{13}\delta_{n_1,n_3}$ where $n_{1,2,3}$ marks the site position of corresponding particle in the chain. As above, the one-particle localization length is $l_1$ and the band width is $4V$. For simplicity we will assume that $U_{13} = 0$ and $U_{23} > U_{12}$. Then in first approximation the particles 2-3 form a pair of size $l_1$ which is localized on the length $l_{c2} \sim (U_{23}/V)^2 l_1^2$. When this pair approaches the first particle at a distance $l_1$ the interaction between three particles in a block of size $l_1$ gives mixing between $l_1^3$ 3-particle states. An effective matrix element $U_{s1}$ of interaction between 3-particle states in the block of size $l_1$ should been calculated in the second order perturbation theory, since direct interaction couples only 2-particle states. Therefore, the matrix element between initial state $|123>$ and final state $|1'2'3'>$ is given by diagram presented in Fig.1 with intermediate state $|1'\bar{2}3'>$. It is of the form

$$U_{s1} = \sum_2 <12|U_{12}|1'\bar{2}> <\bar{2}3|U_{23}|2'3'> \left( \frac{1}{E_1 + E_2 + E_3 - E_1' - E_2' - E_3} \right) \sim \frac{U_{12}U_{23}}{l_1^3 \Delta_1}$$ \hspace{1cm} (3)
It is important that the summation is carried out only over single particle states, hence $\Delta_1 \sim V/l_1$ is single particle level spacing. Finally this gives the mixing rate in a block of size $l_1$

$$\Gamma_1 \sim U_{s1}^2 \rho_3 \sim (U_{12}U_{23}/V^2)^2 V/l_1$$

(4)

where $\rho_3 \sim l_1^3/V$ is the density of 3-particle states in the block. This $\Gamma_1$ gives the mixing rate during the collision of the first particle with the pair 2-3 in the block $l_1$. The frequency of such collisions is of the order of $l_1/l_{c2}$ since from ergodicity the ratio of time of the collision to the time between collisions is proportional to the ratio of volumes. Therefore, the average transition rate for 1-particle per unit time is $\tilde{\Gamma}_1 \sim \Gamma_1 l_1/l_{c2}$. Such transitions give the diffusion rate of the first particle $D_1 \sim \tilde{\Gamma}_1 l_1^2 \sim U_{12}^2/V$ since the size of transition is $l_1$. Similarly to the previous case with two chains the total number of excited states after time $t^*$ is $\Delta N \sim (D_1 t^*)^{1/2}(l_{c2}l_1)\delta E/V$ where $\delta E$ is an energy width in which the levels are mixed. The localization time $t^*$, as previously, is determined from the condition $\Delta N \sim \delta Et^*$ which gives

$$t^* \sim D_1(l_{c2}l_1)^2/V^2; \quad l_{c1}/l_1 \sim D_1 l_{c2}/V \sim (U_{12}U_{23}/V^2)^2 l_1^2$$

(5)

For $U_{12} \sim U_{23} \sim U$ the localization length for the first particle is enhanced only if there is an enhancement for two-particle localization length, namely $(U/V)^2 l_1 > 1$. This result is quite natural since for $(U/V)^2 l_1 < 1$ two-particle interaction is too weak and it is not able to mix three-particle levels. Another limiting case in (5) corresponds to $U \sim V$. For such interaction $l_{c1} \sim l_1^3$ which is similar to the case of three particles trapped in a bag of size $l_1$. Indeed, one can consider 3-particle bag model like TIP one.
with effective number of transverse channels $M_{ef} \sim l_1$, therefore for the 3-particle bag $l_{b3} \sim M_{ef}l_1^2 \sim l_1^3$. The same estimate for $l_{b3}$ had been also obtained in [13] basing on the approaches developed in [5, 2]. In some sense the result [3] shows that similar to the TIP case the ”size” and ”form” of the bag is not important for the effect. Let us also mention that the case $U_{23} \sim V$ is similar to previously analysed model of TIP in the chain and the strip [3]. Indeed, here the third particle gives the effective number of channels $M \sim l_1$ so that [3] becomes equivalent to [2]. Generalization of the result [3] for $k$ particles gives the enhancement $l_{ck}/l_1 \sim ((U/V)^2l_1)^{k-1}$.

For the 3-dimensional case $l_1$ in the enhancement factor $(U/V)^2l_1$ should be replaced by $l_1^3$ [3, 13] so the delocalization takes place if $((U/V)^2l_1^3)^{k-1} > 1$. This means that the delocalization border for few particles coincides approximately with that for TIP and therefore it is not possible to have propagating cluster with $k > 2$ repulsive particles. In some sense only TIP pairs are well defined.

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Figure captions

Fig. 1: Diagram for effective three particle matrix element $U_{s1}$ in (3).
Fig. 1