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Unitarity and stability conditions in a 4-Higgs
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Abstract. In order to obtain a dark matter candidate we propose an extension of the $S_3$ symmetric 3-Higgs doublet model, adding a new scalar doublet, occupying all the irreducible representations of the discrete symmetry. To ensure stability of the dark matter particle we impose an extra $Z_2$ symmetry. We find the analytical masses of the scalar particles and constrain their values using stability and unitarity conditions.

1. Introduction.

Finding what dark matter (DM) is made of is one of the main challenges in particle physics and cosmology. It has been proposed in [1] that DM is conformed by neutral scalar particles, a very interesting hypothesis as these particles have the characteristics expected for DM: Neutral, cold and weakly interacting (see for example [2, 3, 4, 5, 6]).

Among the numerous proposals to extend the scalar sector of the standard model, the 3-Higgs Doublet Model with an $S_3$-family symmetry (3H-$S_3$) presents interesting phenomenology, such as the prediction of a non zero reactor neutrino mixing angle $\theta_{13}$ and of a CKM matrix in accordance with experimental results (see e.g. [7, 8, 9, 10, 11, 12, 13, 14]).

A natural generalization embracing these two ideas suggests itself: enlarge the 3H-$S_3$ model with an additional doublet representing a dark matter candidate. In this letter we present an analysis of the $S_3$-symmetric 4-Higgs Doublet Model (4HDM) in which we occupy all irreducible representations of the $S_3$ symmetry: one symmetric singlet, one antisymmetric singlet and one doublet. The 3H-$S_3$ is constituted by the symmetric singlet and doublet representations, while for the fourth Higgs doublet (in the antisymmetric representation) we impose a $Z_2$ symmetry ensuring the stability of the potential dark matter candidates. In the following we present the analytical calculation of the masses of the scalar particles at tree level and constrain their values using unitarity and stability conditions. A complete analysis of the model with its dark matter phenomenology will be presented elsewhere [15].
2. The model.

The most general $SU(2)_L \times U(1)_Y$ invariant renormalizable scalar potential for a 4-Higgs doublet model with additional $S_3 \times Z_2$ symmetry is given by:

$$V = \mu_0^2 H_1^\dagger H_1 + \mu_1^2 (H_1^\dagger H_2 + H_2^\dagger H_1) + \mu_2^2 H_a^\dagger H_a + \lambda_1 (H_1^\dagger H_1 + H_2^\dagger H_2)^2 + \lambda_2 (H_1^\dagger H_2 - H_2^\dagger H_1)^2$$

$$+ \lambda_3 [(H_1^\dagger H_1 - H_2^\dagger H_2)^2 + (H_1^\dagger H_2 + H_2^\dagger H_1)^2]$$

$$+ \lambda_4 [(H_1^\dagger H_1) (H_2^\dagger H_2 + H_2^\dagger H_1) + (H_2^\dagger H_2) (H_1^\dagger H_1 - H_2^\dagger H_2) + \text{h.c.}]$$

$$+ \lambda_5 (H_a^\dagger H_a) (H_1^\dagger H_1 + H_2^\dagger H_2)$$

$$+ \lambda_6 [(H_1^\dagger H_1) (H_1^\dagger H_a) + (H_2^\dagger H_2) (H_2^\dagger H_a)]$$

$$+ \lambda_7 [(H_1^\dagger H_1) (H_a^\dagger H_1) + (H_2^\dagger H_2) (H_a^\dagger H_2) + \text{h.c.}]$$

$$+ \lambda_8 (H_a^\dagger H_a)^2$$

$$+ \lambda_9 (H_a^\dagger H_a) (H_a^\dagger H_1 + H_2^\dagger H_2)$$

$$+ \lambda_{10} [(H_a^\dagger H_1) (H_1^\dagger H_a) + (H_2^\dagger H_2) (H_a^\dagger H_a)]$$

$$+ \lambda_{11} [(H_a^\dagger H_1) (H_a^\dagger H_1) + (H_a^\dagger H_2) (H_a^\dagger H_2) + \text{h.c.}]$$

$$+ \lambda_{12} [(H_a^\dagger H_1) (H_a^\dagger H_1) + (H_a^\dagger H_2) (H_a^\dagger H_2) + \text{h.c.}]$$

$$+ \lambda_{13} (H_a^\dagger H_a)^2$$

$$+ \lambda_{14} (H_a^\dagger H_a) (H_a^\dagger H_1 + H_2^\dagger H_2).$$

Here we have arranged the $SU(2)_L$ doublets $H_1$ and $H_2$ into a column vector transforming in the doublet representation of $S_3$, while $H_a$ and $H_a$ are required to transform in the symmetric and antisymmetric singlet representations of $S_3$ respectively. In the following we will assume all the couplings $\lambda_i, i = 1...14$ to be real, CP conserving, and $|\lambda_i| < 4\pi$ in order to fulfill perturbativity. Note that there are no terms with odd powers of $H_a$, the only field assumed odd under $Z_2$.

After electroweak symmetry breaking all Higgs doublets acquire a vacuum expectation value (vev) which we denote respectively by $v_0$, $v_1$, $v_2$ and $v_a$. Nevertheless, to avoid the breaking of the $Z_2$ symmetry we fix $v_a$ identically to zero, and from the minimization conditions (a.k.a. tadpole equations) of the potential (1) the fourth equation $\partial V/\partial v_a = 0$ is therefore automatically satisfied. The three minimization equations left ($\partial V/\partial v_i = 0, i = 1, 2, 3$) reduce to those of the 3H-S3, from which we reproduce the following conditions to the quadratic couplings:

$$\mu_0^2 = - (\lambda_5 + \lambda_6 + 2\lambda_7) (v_1^2 + v_2^2) - 2\lambda_8 v_0^2 + \frac{\lambda_4 (v_2^2 - 3v_1^2) v_2}{v_0}$$

$$\mu_1^2 = - (\lambda_5 + \lambda_6 + 2\lambda_7) v_0^2 - 2(\lambda_1 + \lambda_3)(v_1^2 + v_2^2) - 6\lambda_4 v_2 v_0$$

$$\mu_2^2 = - (\lambda_5 + \lambda_6 + 2\lambda_7) v_0^2 - 2(\lambda_1 + \lambda_3)(v_1^2 + v_2^2) + 3\lambda_4 \frac{v_0 (v_2^2 - v_1^2)}{v_2}. \quad (2)$$

The self consistency of the above conditions require either $\lambda_4 = 0$ or the alignment of the vacuum expectation values $v_1$ and $v_2$:

$$v_1 = \sqrt{3} v_2. \quad (3)$$

Since the case $\lambda_4 = 0$ is phenomenologically unappealing in the context of the 3HDM, we will assume the alignment (3) in the rest of this work.
2.1. The masses.

We parametrize the Higgs doublets by

\[ H_s = \begin{pmatrix} h_n^+ \\ v_n + h_n^0 + i h_n^1 \end{pmatrix} \]  

and similarly for \( H_1, H_2 \) and \( H_a \). Here the indexes \( n \) and \( p \) refer to neutral (scalar) and (neutral) pseudoscalar and we use primes to distinguish from the mass eigenstates denoted by the same letters. The masses of the scalar particles are found by diagonalizing the corresponding mass matrices, e.g. for the neutral scalar fields the following matrix:

\[ (M_{h,n}^2)_{ij} = \left. \frac{1}{2} \frac{\partial^2 V}{\partial h_i^m \partial h_j^n} \right|_{\text{min}} \]

which is block diagonal such that the primed fields \( h_n^0, h_1^0 \) and \( h_2^0 \) mix into the mass eigenstates \( h_n^0, h_1^0 \) and \( h_2^0 \) and the \( Z_2 \) odd field remains unmixed \( h_a^0 = h_a^0 \).

The expressions for the masses presented below are separated by neutral scalar, neutral pseudoscalar and charged particles.

- **Masses of the neutral scalar particles:**

\[
\begin{align*}
m_n^2 &= -18\lambda_4 v_0 v_2 \\
m_n^2 &= \mu_4^2 + 14 v_0^2 + 4(\lambda_{10} + \lambda_{11} + 2\lambda_{12}) v_2^2 \\
m_n^2 &= \left( \frac{1}{v_0} \right) (2\lambda_8 v_0^3 + v_2 (3\lambda_4 v_0^2 + 8(\lambda_1 + \lambda_3) v_0 v_2 - 4\lambda_4 v_2^2) + \\
&\quad ((4\lambda_8^2 v_0^6 - 12\lambda_4 \lambda_8 v_0^5 v_2 + (9\lambda_4^2 + \\
&\quad 16((\lambda_5 + \lambda_6 + 2\lambda_7)^2 - 2(\lambda_1 + \lambda_3 + 8)) v_0^2 v_2^2 + \\
&\quad 16\lambda_4 (3(\lambda_1 + \lambda_3 + 2(\lambda_5 + \lambda_6 + 2\lambda_7)) - \lambda_8) v_0^2 v_2^2 + \\
&\quad 8(8(\lambda_1 + \lambda_3)^2 + 21\lambda_4^2 v_0^2 v_2^4 + 64(\lambda_1 + \lambda_3) \lambda_4 v_0 v_2^5 + \\
&\quad 16\lambda_4^2 v_2^6)^{1/2} \right) \\
m_n^2 &= \left( \frac{1}{v_0} \right) (2\lambda_8 v_0^3 + v_2 (3\lambda_4 v_0^2 + 8(\lambda_1 + \lambda_3) v_0 v_2 - 4\lambda_4 v_2^2) + \\
&\quad -(4\lambda_8^2 v_0^6 - 12\lambda_4 \lambda_8 v_0^5 v_2 + (9\lambda_4^2 + \\
&\quad 16((\lambda_5 + \lambda_6 + 2\lambda_7)^2 - 2(\lambda_1 + \lambda_3 + 8)) v_0^2 v_2^2 + \\
&\quad 16\lambda_4 (3(\lambda_1 + \lambda_3 + 2(\lambda_5 + \lambda_6 + 2\lambda_7)) - \lambda_8) v_0^2 v_2^2 + \\
&\quad 8(8(\lambda_1 + \lambda_3)^2 + 21\lambda_4^2 v_0^2 v_2^4 + 64(\lambda_1 + \lambda_3) \lambda_4 v_0 v_2^5 + \\
&\quad 16\lambda_4^2 v_2^6)^{1/2} \right) \\
\end{align*}
\]

- **Masses of the neutral pseudo scalar particles:**

\[
\begin{align*}
m_n^2 &= 0 \\
m_n^2 &= \mu_4^2 + 14 v_0^2 + 4(\lambda_{10} + \lambda_{11} - 2\lambda_{12}) v_2^2 \\
m_n^2 &= \left( \frac{2(2\lambda_7 v_0^3 + 5\lambda_4 v_0^2 v_2 + 8\lambda_2 v_0 v_2^2 + 8\lambda_3 v_0 v_2^2)}{v_0} \right) \\
m_n^2 &= \left( \frac{2(2\lambda_7 v_0 + 14 v_2)(v_0^2 + 4v_2^2)}{v_0} \right).
\end{align*}
\]
• Masses of the charged particles:

\[
\begin{align*}
    m_{h^±} &= 0 \\
    m_{h^±_3} &= \mu^2 + 4\lambda_1 v^2_0 \\
    m_{h^±_1} &= - (\lambda_6 + 2\lambda_7)v^2_0 - 10\lambda_4 v_0 v_2 - 16\lambda_3 v^2_0 \\
    m_{h^±_2} &= - \frac{(\lambda_6 v_0 + 2\lambda_7 v_0 + 2\lambda_4 v_2)(v^2_0 + 4v^2_2)}{v_0}.
\end{align*}
\]

(8)

2.2. Stability conditions.

The stability conditions, ensure that the vacuum is stable i.e. that the potential has a minimum. They were found for the potential (1), following the procedure in [14] for the 3H-S\(_3\):

\[
\begin{align*}
    \lambda_8 &> 0 \\
    \lambda_1 + \lambda_3 &> 0 \\
    \lambda_5 &> -2\sqrt{(\lambda_1 + \lambda_3)\lambda_8} \\
    \lambda_5 + \lambda_6 - 2|\lambda_7| &> \sqrt{(\lambda_1 + \lambda_3)\lambda_8} \\
    \lambda_1 - \lambda_2 &> 0 \\
    \lambda_13 &> 0 \\
    \lambda_{10} &> -2\sqrt{(\lambda_1 + \lambda_3)\lambda_{13}} \\
    \lambda_{10} + \lambda_{11} - 2|\lambda_{12}| &> \sqrt{(\lambda_1 + \lambda_3)\lambda_{13}} \\
    \lambda_{14} &> -2\sqrt{\lambda_8\lambda_{13}}.
\end{align*}
\]

(9)

2.3. Unitarity constraints.

Unitarity constraints over the quartic parameters can be obtained from the elegant LQT method [16]. On account of renormalizability, scattering amplitudes cannot exhibit unphysical growth in the limit of high energies. Thus, one is led to impose (tree level) unitarity on different sets of scattering processes, in particular those of two particle states. Defining the matrix \( M_{ij} = M_{i \rightarrow j} \) where the indices stand for all possible two particle processes, it suffices to consider processes involving Higgs scalars and longitudinal vector bosons. The eigenvalues of the matrix \( M_{ij} \), denoted \( a^\pm_i \) and \( b_i \) for charged and neutral two particle channels, will then be constrained according to \( |a^\pm_i|, \ |b_i| < 16\pi \), reflecting the physical fact that the coefficient \( a_0 \) of the s-wave term in a partial wave expansion of the scattering amplitude is bounded (e.g. \( |a_0| < 1/2 \)) in the limit of high energy exchange.

Of course, the eigenvalues corresponding to physical processes already present in the 3H-S\(_3\) are the same in this 4H-S\(_3\), so here we only report on the new constraints. Their expressions in
Figure 1. Mass scan for neutral scalar particles, where $h_n^a$ is the Higgs boson of the SM.

terms of $\lambda_i$ are:

$$a_7^\pm = \pm \sqrt{\lambda_{10}^2 + 2\lambda_{10}(\lambda_{11} - 4\lambda_{12}) + 2(\lambda_{11}^2 - 6\lambda_{11}\lambda_{12} + 10\lambda_{12}^2)}$$

$$a_8^\pm = \pm \sqrt{(\lambda_{10} - 2\lambda_{12})(\lambda_{10} + 2\lambda_{11} - 6\lambda_{12})}$$

$$a_{10}^\pm = \lambda_{10} \pm \lambda_{11}$$

$$a_{11}^\pm = \pm \lambda_{14}$$

$$a_{12}^\pm = \pm \frac{\lambda_{14}}{2}$$

$$b_7 = \lambda_{10} + 2\lambda_{11} + 6\lambda_{12}.$$ 

Eigenvalues $a_i^\pm$ and $b_i$ where $i = 1, 2, ..., 6$, were found by Das and Dey for the 3HDM in [12], whereas $a_i^\pm$ where $i = 7, 8, ..., 12$ and $b_7$ correspond to 4H-$S_3$ interactions decoupled from the 3H-$S_3$ matrices, e.g. $h_n^a h_n^b h_n^c h_n^d$. The dispersion matrices involving fields from 4 Higgs Doublet, e.g. $h_n^a h_n^b h_n^c h_n^d$ were solved numerically during the scan with pseudorandom values for the $\lambda_i$ couplings.

3. Numerical analysis.

To perform the analysis is convenient to parametrize the vevs in spherical coordinates, so

$$v_0 = v \cos \theta$$

$$v_1 = v \sin \theta \cos \phi$$

$$v_2 = v \sin \theta \sin \phi,$$
Figure 2. Mass scan for neutral pseudoscalar particles, where $h^n_n$ is the Higgs boson of the SM.

Figure 3. Mass scan for charged particles, where $h^n_+^n$ is the Higgs boson of the SM.

where $\theta \in (0, \pi)$ and $\phi \in (0, 2\pi)$. With this parametrization and the relation found in Equation 3 we get

$$\tan^2 \phi = \frac{1}{3}.$$  \hspace{1cm} (12)
Figure 4. Mass scan for neutral scalar particles, where $h_n^2$ is the Higgs boson of the SM.

Figure 5. Mass scan for neutral pseudoscalar particles, where $h_n^2$ is the Higgs boson of the SM.

Hence the vevs only depend on $\theta$:

\[ v_0 = v \cos \theta \]
\[ v_2 = \frac{1}{2} v \sin \theta. \]  

(13)
Figure 6. Mass scan for charged particles, where $h^\nu_2$ is the Higgs boson of the SM.

We computed the dependence on $\tan \theta$ of the mass spectrum of the Higgs bosons. The figures 1, 2 and 3 correspond to the neutral scalar, neutral pseudoscalar and charged higgses where $h^\nu_n$ corresponds to the SM Higgs boson, while the figures 4, 5 and 6 correspond to the neutral scalar, neutral pseudoscalar and charged higgses where $h^\nu_2$ corresponds to the SM Higgs boson. All the figures above mentioned are scatter plots of mass vs. $\tan \theta$ and generated varying the $\lambda_i$ parameters randomly, where we choose only the points that satisfy the unitarity and stability conditions previously calculated.

4. Conclusions.
The S3 flavor symmetry has been very successful accommodating fermion masses and mixings; in this work we use four $SU(2)$ doublets, using all irreducible S3 representations, which provides a rich phenomenology where one or several dark matter candidates can be feasible. We found analytical expression for the masses of the scalar particles and constrained their numerical values applying stability and unitarity conditions. The mass spectra was computed for two cases: 1) the SM Higgs boson corresponds to $h^\nu_n$ and 2) the SM Higgs boson corresponds to $h^\nu_2$. In the plots 1, 2 and 3 corresponding to the first case, we found that the masses of $h^\nu_1$, $h^\nu_2$ and $h^\nu_2$ are heavier than the rest for large $\tan \theta$. While, in the plots 4, 5 and 6 all masses have nearly the same range, with an upper limit close to 1TeV, the exception being $m_{h^\nu_2}$ and $m_{h^\nu_2}$ with a 500GeV upper limit. As a perspective of these results we are currently working in the complete analysis of the model and on the dark matter phenomenology in order to constrain the parameter space of the model by relic density measurements.

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