Research on LDPC in an OFDM System

Chong-Yue Shi¹, Hui Li¹2*, You-Ling zhou¹*, Ping Wang¹, Qian Li¹, Liu-Xun Xue¹, Jie Xu¹

¹School of Information and Communication Engineering, Hainan University, Haikou 570228, China
²Binjiang College, Nanjing University of Information Science and Technology, Wuxi 214105, China

*Corresponding author’s e-mail 1908100210015@hainanu.edu.cn

Abstract. In this paper, LDPC codes and OFDM technologies are studied, and combines LDPC and OFDM to simulate and verify the performance. The encoding of LDPC and logarithmic domain BP decoding algorithm and the principle of OFDM are introduced. Through MATLAB, the simulation of OFDM-LDPC system in different channel, different code length, different iteration times, LDPC and OFDM-LDPC in the same conditions are compared. Through simulation analysis, we can know that the performance of OFDM-LDPC system will improve with the increase of the code length. When the SNR is small, the number of iterations has little effect on the performance of OFDM-LDPC system. When the SNR is large, the performance of the system will be improved with the increase of the number of iterations. The performance of OFDM-LDPC codes is better than that of LDPC codes. The performance of OFDM-LDPC code is better than LDPC code in multipath Rayleigh fading channel.

1. Introduction
LDPC code was proposed by Gallager of MIT in his doctoral dissertation in 1962. After the emergence of Turbo code in 1993, people re-studied LDPC code and found that it has excellent performance. LDPC code is also used in various digital communication systems because of its excellent performance[1]. The LDPC decoding algorithm has low complexity and performance close to the Shannon limit, which can be regarded as one of the best performance channel error correction coding methods[2]. OFDM is a key technology to achieve high-speed data transmission in the field of wireless communication. It uses multi-carrier technology, merging multiplexing technology and modulation technology[3]. OFDM takes FFT and IFFT as the core, which has a high frequency band utilization rate and can overcome the adverse effects well of frequency selective fading and multipath delay of the wireless channel. Nowadays, OFDM technology has been widely used in wireless communication. It has the advantages of high frequency band utilization and anti-multipath influence, which greatly reduces inter-symbol interference. LDPC code is currently one of the best performance channel error correction coding methods, and as a long code coding scheme in 5G mobile communication[4], it can greatly improve data transmission efficiency, reduce bit error rate, and improve communication reliability[5]. Combining LDPC coding technology with OFDM can greatly improve the performance of the system.

2. Basic Principles of LDPC
LDPC coding is one of the most popular encoding methods. The check matrix of LDPC contains only
have a few non-zero elements, and the sparsity of the check matrix ensures that the decoding complexity and minimum code distance only linearly increase with the code length. Except that the check matrix is a sparse matrix, the code itself is not different from any other block codes. The LDPC code whose check matrix $H$ is $m \times n$ dimensional can be expressed as $(n, k)$ LDPC code, where $k = n - m$ and code rate $= k / n$.

### 2.1 LDPC coding principle

LDPC coding is based on the check matrix $H$, and then the generator matrix is obtained from the check matrix. The generator matrix generates different codewords. The design of the check matrix $H$ is the key part of the LDPC code encoding [7]. $H$ directly affects the complexity of encoding and decoding and the performance of LDPC. The generator matrix $G$ is changed from $H$, the output information $E = S \times G$ ($S$ is the input information), and the relationship between the check matrix and $E$ is $H \times E^T = 0$.

$$H = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix}$$

Figure 1. (10,2,4) Check matrix of LDPC code

It can be seen from the above that the check matrix and LDPC code are in one-to-one correspondence. In addition, the tanner diagram can also uniquely define LDPC code [8]. The Tanner diagram is composed of three parts: the variable node, the check node, and the edge connecting the check node and the variable node. The Tanner diagram corresponds to the check matrix $H$, which is used to define the code words, and the variable nodes correspond to the column vector of $H$, the check node corresponds to the row vector of the matrix $H$, and the non-zero elements in the matrix corresponds to an edge on the graph.

![Tanner diagram](image)

Figure 2. (10,2,4) Tanner diagram of the LDPC code

The LDPC encoding method selected in this paper is the approximate lower triangular method. In order to ensure the sparsity of matrix $H$, the rows and columns of matrix $H$ can be rearranged to obtain an approximate lower triangular matrix as shown in figure 3 [9], after rearrangement, the $H$ has the following form:

$$H = \begin{bmatrix}
A & B & T
C & D & E
\end{bmatrix}^T$$

Figure 3. The parity check matrix of the quasi lower trig

The dimension of $A$ is $(m - g) \times (n - m)$, $B$ is $(m - g) \times g$, $T$ is $(m - g) \times (m - g)$, $C$ is
\( g \times (m - g) \), \( D \) is \( g \times g \), \( E \) is \( g \times (m - g) \), all these matrices are sparse matrices, \( T \) is the lower triangular matrix, multiply the matrix \( H \) to the left by \( \begin{pmatrix} I & 0 \\ -ET^{-1} & 0 \end{pmatrix} \), you can get the following matrix:

\[
\begin{pmatrix}
A & B \\
-ET^{-1}A + C & -ET^{-1}B + D & T
\end{pmatrix}
\]

(2)

Make \( x = (s_i, p_1, p_2) \), \( x \) is the sequence after encoding, \( s_i \) is the input information sequence, the combination of \( p_1 \) and \( p_2 \) is the parity bit part of the code, the length of \( p_1 \) is \( g \), the length of \( p_2 \) is \( (m - g) \), decompose \( Hx^T = 0 \) into two equations:

\[
\begin{align*}
As^T + Bp_1^T + Tp_2^T &= 0 \\
(-ET^{-1}A + C)x^T + (-ET^{-1}B + D)p_1^T &= 0
\end{align*}
\]

(3)
(4)

\( p_1 \) and \( p_2 \) can be calculated from the above, and the encoded information sequence can be obtained by substituting the two into \( x \).

2.2 LDPC decoding

Iterative decoding is an advantage of LDPC decoding\(^{[10]}\). There are two main types of decoding methods for LDPC codes, one is the belief propagation decoding algorithm, referred to as BP decoding algorithm, and the other is the bit flip decoding algorithm, referred to as BF decoding algorithm. BP decoding algorithm has better performance, but higher complexity\(^{[11]}\). The decoding algorithm chosen in this paper is logarithmic domain BP decoding algorithm.

Set the sender code word to \( c = (c_1, c_2, \ldots, c_n) \), the output sequence \( y = (y_1, y_2, \ldots, y_n) \), the code word after decoding is \( \hat{c} \). We can calculate that the initial probability likelihood ratio of the channel to \( i \) (where \( i \) represents the variable node) is \( L(p_i) \) (\( L(p_i) = \ln \frac{p_i(0)}{p_i(1)} = \ln \frac{p\{x_i = 1 | y_i\}}{p\{x_i = -1 | y_i\}}, i = 1, 2, \ldots, n \), \( p_i(0) \) is the initial probability that the information transmitted by the channel to \( i \) is 0, \( p_i(1) = 1 - p_i(0) \)). For each \( i \) and its adjacent parity node \( j \) (\( j \in C_i, C_i \) is the combination of check nodes connected to \( i \) in Tanner graph). Let’s make:

\[
L^{(0)}(q_{ij}) = L(p_i)
\]

(5)

where the \( L \)'s superscript represents the number of iterations. Check node update: for all check nodes and the variable node \( i \) connected to it, the \( l \)th iteration can be obtained (\( i \in R_j \)):

\[
L^{(l)}(r_{ji}) = 2 \tanh^{-1} \left( \prod_{n \in R_{ji}} \tanh \left( \frac{1}{2} L^{(l-1)}(q_{nj}) \right) \right)
\]

(6)

In the formula: \( L^{(l)}(r_{ji}) \) represents the message sent by the check node to the variable node, \( R_{ji} \) represents the set of all variable nodes excluding \( j \) and \( i \) connections. Variable node update: for all \( i \) and their adjacent \( j \) (\( j \in C_i \)), we can be obtained:

\[
L^{(l)}(q_{ij}) = L(p_i) + \prod_{n \in C_{ij}} L^{(l)}(r_{ni})
\]

(7)

In the formula: \( L^{(l)}(q_{ij}) \) represents the message sent by the variable node to the check node, \( C_{ij} \) represents the set of all Check nodes excluding \( j \) and \( i \) connections.

Judge all variable nodes to get:
\[
L^{(i)}(q_i) = L(p_i) + \prod_{j \in c_i} L^{(j)}(r_{ji}) \tag{8}
\]

when \( L^{(i)}(q_i) < 0 \), decoding result is \( \hat{c}_i = 1 \), when \( L^{(i)}(q_i) > 0 \), decoding result is \( \hat{c}_i = 0 \).

3. Basic Principles of OFDM

OFDM has the advantages of high bandwidth utilization and strong resistance to inter-symbol interference\cite{12}. The basic principle of OFDM is that the original signal is divided into \( N \) sub-signals through serial-parallel conversion, and then the \( N \) sub-signals are respectively modulated on \( N \) mutually orthogonal sub-carriers. Finally, the transmitted signal is obtained by adding the \( N \) modulated signals. At the receiving end, the input signal is divided into \( N \) branches, which are used for frequency mixing and integration of \( N \) subcarriers respectively to recover parallel data, and then the original data can be recovered after serial-parallel conversion and demodulation.

When transmitting a set of data, OFDM will first map this set of data to a complex sequence \( \{d_0, d_1, \ldots, d_{N-1}\} \), where \( d_N = a_N + b_Nj \) is the output of the digital modulator. Then use IDFT to transform the data into the time domain, and get a new sequence \( \{S_0, S_1, \ldots, S_{N-1}\} \), the sequence \( \{S_0, S_1, \ldots, S_{N-1}\} \) is converted by D/A with an interval of \( \Delta t \) to obtain the transmitted signal. The receiving end take A/D sampling of the received signals at an interval of \( \Delta t \), then restore the sequence \( \{d_0, d_1, \ldots, d_{N-1}\} \) through DFT operation, then you can get the original input sequence. IDFT and DFT are generally implemented by IFFT and FFT transformation\cite{13}.

![Figure 4. OFDM system block diagram](image)

The existing OFDM subcarrier modulation and demodulation methods are mainly MPSK and MQAM. MQAM is more susceptible to the channel environment than MPSK, in MPSK modulation, BPSK modulation has the lowest bit error rate under the same SNR, and the bit error rate increases with the increase of M. Due to the low spectrum efficiency of BPSK, QPSK is generally more advantageous than BPSK, so QPSK is generally selected.

4. System simulation and analysis

The software used in this simulation is MTALAB, and the performance of the system is compared in the case of 160, 320, 640,and 1280 LDPC code lengths. The performance of the system is compared under the condition of 10, 50,and 100 iterations. The simulation comparison between LDPC and OFDM-LDPC in multipath Rayleigh fading channel and other conditions being the same.
5. Conclusion
From the above simulation results, we can get that the combination of LDPC coding technology and OFDM can greatly improve the system's bit error rate performance. The performance of LDPC-OFDM system will continuously improve with the increase of LDPC code length, but with the increase of code length, the computation will also increase greatly. Therefore, the code length of LDPC code should not be too large, and the code length should be selected effectively to achieve the optimal compromise.
between system complexity and system performance. When the SNR is small, the number of iterations has little impact on the bit error rate; when the SNR is large, the number of iterations has a great impact on the bit error rate performance. Therefore, we can select a small number of iterations to balance system complexity and system bit error rate when the SNR is small. The performance of the OFDM and LDPC combined system is better than that of the LDPC without OFDM. OFDM can improve the impact of multipath effect on the system.

This design only considers the impact of the multipath fading channel on the LDPC-OFDM system. In future research, we will also consider the impact of the Doppler frequency shift on the bit error rate performance of the LDPC-OFDM system.

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