Efficient W state entanglement concentration using quantum-dot and optical microcavities

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We present an entanglement concentration protocols (ECPs) for less-entangled W state with quantum-dot and microcavity coupled system. The present protocol uses the quantum nondemolition measurement on the spin parity to construct the parity check gate. Different from other ECPs, this less-entangled W state with quantum-dot and microcavity coupled system can be concentrated with the help of some single photons. The whole protocol can be repeated to get a higher success probability. It may be useful in current quantum information processing.

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I. INTRODUCTION

Entanglement plays an important role in quantum information processing [1, 2]. It has many practical applications, such as quantum teleportation [3, 4], quantum cryptography [5, 6], quantum dense coding [7], quantum secure direct communication [8, 9], and other quantum information protocols [11–16]. In order to achieve such tasks, the legitimate uses, say the sender Alice and the receiver Bob should first share the entanglement in distant locations. In a long-distance quantum communication, quantum repeaters are unusually used to sent the quantum signals over an optical fiber or a free space [17, 18]. Unfortunately, in a practical transmission, the entangled quantum system cannot avoid the channel noise from the environment. It will make the quantum system decoherence. That is, a maximally entangled state will become a less-entangled state or a mixed entangled state.

Quantum concentration is to distill some maximally entangled state from an ensemble in a pure less-entangled state [19–37]. In 1996, Bennett et al. proposed an entanglement concentration protocol (ECP), which is called Schmidt projection method [19]. Bose et al. proposed an ECP based on the swapping [22]. ECPs based on linear optical elements were proposed by Yamamoto et al. and Zhao et al., respectively [23–26]. In 2008, an ECP based on cross-Kerr nonlinearity was proposed [27]. In 2011, an ECP based on the quantum-dot in an optical cavity was proposed. This kind of ECPs are all used to concentrate a less-entangled state $\alpha|000\rangle + \beta|111\rangle$ to a maximally entangled state $\frac{1}{\sqrt{2}}(|001\rangle + |110\rangle)$. Unusually, in each concentration step, they choose two similar copies of less-entangled states and after performing the ECP, at least one pair of maximally entangled state can be obtained with certain probability. The most advantage of this ECP is that they do not need to know the exact coefficients $\alpha$ and $\beta$. On the other hand, there is another kind of ECP which can be used to concentration a less-entangled W state into a maximally entangled W state. For example, in 2003, Cao and Yang proposed an ECP for W-class state using joint unitary transformation [38]. Zhang et al. proposed an ECP with the help of collective Bell-state measurement [39]. The ECP’s for a special less-entangled W state were proposed in both linear optical system and cavity QED system [40, 41]. In 2011, Yildiz proposed an optimal ECP for asymmetric W states of the form [42]

$$\frac{1}{\sqrt{2}}|001\rangle + \frac{1}{2}|010\rangle + \frac{1}{2}|100\rangle,$$

$$\frac{1}{2}|001\rangle + \frac{1}{2}|010\rangle + \frac{1}{\sqrt{2}}|100\rangle. \quad (1)$$

On the other hand, the ECPs described above are unusually based on the universal qubit say $|0\rangle$ and $|1\rangle$ or the optical system, which $|0\rangle = |H\rangle$ and $|1\rangle = |V\rangle$ [23–26]. Here $|H\rangle$ and $|V\rangle$ represent the horizontal and vertical polarization of the photon, respectively. Recently, there is a novel candidate for qubit which is a single spin coupled to an optical microcavity based on a charged quantum-dot [43–47]. For example, Hu et al. proposed an deterministic photon entangler using a charged quantum-dot inside a microcavity [44]. They also proposed an entanglement beam splitter and discussed the loss-resistant state teleportation and entanglement swapping.

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using a quantum dot spin in an optical microcavity \[44\]–\[46\]. Bonato et al. discussed the CNOT gate and Bell-state analysis in the weak-coupling cavity QED regime \[47\]. Wang et al. proposed two entanglement purification protocols based on the hybrid entangled state using quantum-dot and microcavity coupled system \[48\]–\[49\]. Recently, a efficient quantum repeater protocol was proposed \[50\]. Inspired by the novel works of Hu and Wang, we propose an ECP for the less-entangled W state exploiting the quantum-dot and microcavity coupled system. In this protocol, the less-entangled W state of the spin in the cavity QED system can be concentrated into a maximally entangled W state with some ancillary single photons. This protocol is quite different from the others. First, we can concentrate the arbitrary less-entangled W state. Second, we do not need two copies of less-entangled pairs. Third, the ECP can be performed between different degrees of freedoms, that is we use the single photons to concentrate the less-entangled state in spin. Fourth, by repeating this ECP, it can reach a higher success probability.

This paper is organized as follows: In Sec. II, we describe the theoretical model for our ECP. We call it a hybrid parity check gate based on photon and electron coupled systems. In Sec. III, we explain our ECP based on the parity check gate. In Sec. IV, we discuss the efficiency and errors on a practical implementation. In Sec. V, we present a discussion and summary.

II. HYBRID PARITY CHECK GATE

Before we start to explain this ECP, we first describe the basic element for this protocol. It is also shown in Refs. \[44\]–\[49\]. As shown in Fig. 1, the system is composed of a single charged quantum-dot in micropillar microcavites. The charge exciton consists of two electrons bound in one hole and the excitation with negative charges can created by the optical excitation of the system. Therefore, if we consider a photon enters into the cavity from the input mode and it will interact with the electron in the coupling cavity. Interestingly, the left circularly polarized photon \(|L⟩\) only couples with the electron in the spin up state \(|↑⟩\) to the exciton \(X^+\) in the state \(|↑↑⟩\) because of the Pauli’s exclusion principle for two electrons. On the other hand, the right circularly polarized photon \(|R⟩\) only couples with the electron of the spin down \(|↓⟩\) in the state \(|↓↑⟩\). Here the \(|↑⟩\) and \(|↓⟩\) are the spin direction of the heavy hole spin state. In Ref. \[44\], Hu et al. discussed that such system essentially is an entanglement beam splitter which directly splits an initial hybrid product state of photon and spin into two entangled states via transmission and reflection in a deterministic way. They denoted the transmission and reflection operators as

\[
i = |R⟩⟨R| \otimes |↑⟩⟨↑| + |L⟩⟨L| \otimes |↓⟩⟨↓|.
\]

From Fig. 1, we consider a photon is in the state \(|R⟩\) with \(s_z = +1\) and the electron spin is \(|↑⟩\). Here the superscript \(|↑⟩\) means the photon’s propagation direction is along the z axis. Both the polarization of the photon and propagation direction are flipped into \(|L⟩\). In the same way, the photon and electron interaction in quantum dot and microcavity coupled systems can be fully described as

\[
|S⟩ = |R⟩⟨R| \otimes |↑⟩⟨↑| + |L⟩⟨L| \otimes |↓⟩⟨↓|.
\]

So if the initial input state is the photon-spin product

\[
\frac{1}{\sqrt{2}}(|R⟩ + |L⟩) \otimes \frac{1}{\sqrt{2}}(|↑⟩ + |↓⟩),
\]

it will be converted into the two constituent hybrid entangled state \(\frac{1}{\sqrt{2}^2}(|R⟩ |↑⟩ + |L⟩ |↓⟩)\) in the transmission port, say output1 mode and \(\frac{1}{\sqrt{2}^2}(|R⟩ |↓⟩ + |L⟩ |↑⟩)\) in the reflection port, say output1 mode respectively, with the success probability of 100%, in principle. In the following, we denote the transmission port as output1 mode and the reflection port as output1 for simple, shown in Fig. 1. A parity check gate has been widely in current quantum processing. Entanglement purification and concentration all need such elements. An optical parity check gate, such as polarization beam splitter, can convert the product state \(\frac{1}{\sqrt{2}^2}(|H⟩ + |V⟩) \otimes \frac{1}{\sqrt{2}^2}(|H⟩ + |V⟩)\) into two constituent entangled state \(\frac{1}{\sqrt{2}^2}(|H⟩ |H⟩ + |V⟩ |V⟩)\) and \(\frac{1}{\sqrt{2}^2}(|H⟩ |V⟩ + |V⟩ |H⟩)\). Compared with the polarization beam splitter in optical system, it essentially acts the same role of the parity check gate, but with different degrees of freedom. So we call it a hybrid parity check gate.
III. ECP FOR LESS-ENTANGLED W STATE

Now we start to explain our protocol. From Fig. 2, the less-entangled W state are shared by Alice, Bob and Charlie. It can be written as:

\[ |\Phi\rangle_{123} = \alpha_1 |\downarrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 + \alpha_2 |\uparrow\rangle_1 |\downarrow\rangle_2 |\uparrow\rangle_3 + \alpha_3 |\uparrow\rangle_1 |\uparrow\rangle_2 |\downarrow\rangle_3. \]  

(4)

Here \(|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 = 1\) and subscripts 1, 2 and 3 mean spin 1, spin 2 and spin 3 respectively. Suppose that the three parties know the initial coefficients \(\alpha_1, \alpha_2, \) and \(\alpha_3\). Alice first prepare a single photon of the form

\[ |\Phi\rangle_{P1} = \frac{\alpha_1}{\sqrt{\alpha_1^2 + \alpha_2^2}} |R\rangle_1 + \frac{\alpha_2}{\sqrt{\alpha_1^2 + \alpha_2^2}} |L\rangle_1. \]  

(5)

The subscript \(P1\) means the photon coupled with the spin 1. She sends the state \(|\Phi\rangle_{P1}\) to the cavity from the input mode. The initial less-entangled W state combined with the single photon evolve as

\[ |\Psi\rangle = |\Phi\rangle_{123} |\Phi\rangle_{P1} = (\alpha_1 |\downarrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 + \alpha_2 |\uparrow\rangle_1 |\downarrow\rangle_2 |\uparrow\rangle_3 + \alpha_3 |\uparrow\rangle_1 |\uparrow\rangle_2 |\downarrow\rangle_3) \]

(\(\frac{\alpha_1}{\sqrt{\alpha_1^2 + \alpha_2^2}} |R\rangle_1 + \frac{\alpha_2}{\sqrt{\alpha_1^2 + \alpha_2^2}} |L\rangle_1\))

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\[ P^1_1 = \frac{|\alpha_1|^2 (|\alpha_3|^2 + 2|\alpha_2|^2)}{|\alpha_1|^2 + |\alpha_2|^2}, \]  

The success probability is

\[ P^1_1 = \frac{|\alpha_1|^2 (|\alpha_3|^2 + 2|\alpha_2|^2)}{|\alpha_1|^2 + |\alpha_2|^2}. \]  

In order to get \(|\Phi_1\rangle_{123}\), one of the three parties, says Alice, Bob or Charlie should perform a local operation of phase rotation on her or his spin. The success probability is

\[ P^1_1 = \frac{|\alpha_1|^2 (|\alpha_3|^2 + 2|\alpha_2|^2)}{|\alpha_1|^2 + |\alpha_2|^2}. \]  

On the other hand, after passing through the microm Cavity, if the photon is reflected and in the output 1, then the Eq. (10) collapses to

\[ |\Psi'> = \frac{\alpha^2_1}{\sqrt{\alpha^2_1 + \alpha^2_2}} |L\rangle_1 |\downarrow\rangle_2 |\uparrow\rangle_3 \]

\[ + \frac{\alpha^2_2}{\sqrt{\alpha^2_1 + \alpha^2_2}} |R\rangle_1 |\uparrow\rangle_2 |\downarrow\rangle_3. \]  

Following the same principle described above, if the D_1 fires, they will obtain

\[ |\Phi_3\rangle_{123} = \frac{\alpha^2_1}{\sqrt{\alpha^2_1 + \alpha^2_2 + 2\alpha^2_3}} |\downarrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 \]

\[ + \frac{\alpha^2_2}{\sqrt{\alpha^2_1 + \alpha^2_2 + 2\alpha^2_3}} |\uparrow\rangle_1 |\downarrow\rangle_2 |\uparrow\rangle_3 \]

\[ + \frac{\alpha^2_3}{\sqrt{\alpha^2_1 + \alpha^2_2 + 2\alpha^2_3}} |\uparrow\rangle_1 |\uparrow\rangle_2 |\downarrow\rangle_3. \]  

if the D_2 fires, they will obtain

\[ |\Phi_4\rangle_{123} = \frac{\alpha^2_1}{\sqrt{\alpha^2_1 + \alpha^2_2 + 2\alpha^2_3}} |\downarrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 \]

\[ + \frac{\alpha^2_2}{\sqrt{\alpha^2_1 + \alpha^2_2 + 2\alpha^2_3}} |\uparrow\rangle_1 |\downarrow\rangle_2 |\uparrow\rangle_3 \]

\[ + \frac{\alpha^2_3}{\sqrt{\alpha^2_1 + \alpha^2_2 + 2\alpha^2_3}} |\uparrow\rangle_1 |\uparrow\rangle_2 |\downarrow\rangle_3. \]  

In order to get \(|\Phi_3\rangle_{123}\), one of the parties, says Alice, Bob or Charlie should perform a local operation of phase rotation on her or his spin.

It is interesting to compare \(|\Phi_1\rangle_{123}\) with \(|\Phi_3\rangle_{123}\). \(|\Phi_1\rangle_{123}\) only has two different coefficients, \(\frac{\alpha_1}{\sqrt{\alpha^2_1 + \alpha^2_2}}\) and \(\frac{\alpha_2}{\sqrt{\alpha^2_1 + \alpha^2_2}}\), and the initial coefficient \(\alpha_1\) disappears. But \(|\Phi_3\rangle_{123}\) still has three different coefficients. We denote \(\alpha'_1 = \frac{\alpha_1}{\sqrt{\alpha^2_1 + \alpha^2_3}}\), \(\alpha'_2 = \frac{\alpha_2}{\sqrt{\alpha^2_1 + \alpha^2_3}}\) and \(\alpha'_3 = \frac{\alpha_3}{\sqrt{\alpha^2_1 + \alpha^2_3}}\). So if Alice obtains \(|\Phi_1\rangle_{123}\), it is successful. Then she asks Charlie to continue this ECP. Otherwise, she has to repeat this ECP in a second round. That is, she prepares another single photon of the form

\[ |\Phi\rangle_p^1 = \frac{\alpha'_1}{\sqrt{\alpha'^2_1 + \alpha'^2_2}} |R\rangle_1 + \frac{\alpha'_2}{\sqrt{\alpha'^2_1 + \alpha'^2_2}} |L\rangle_1. \]  

Then she lets this single photon entrance into the microcavity and couple with the spin. After the photon passing through the microcavity, following the same principle, if this single photon is in the output 2 and detected by D_3 or D_4, the concentration is successful. Otherwise, if it is in the output 1 and detected by D_1 or D_2, the concentration is a failure. Alice should prepare a third single photon and restart to perform this ECP until it is successful. So the success probability in the second round is

\[ P^2_1 = \frac{|\alpha_1|^4 (|\alpha_2|^2 |\alpha_3|^2 + 2|\alpha_2|^4)}{(|\alpha_1|^2 + |\alpha_2|^2)(|\alpha_1|^2 + |\alpha_2|^2)}. \]  

the success probability in the third round is

\[ P^3_1 = \frac{|\alpha_1|^8 (|\alpha_2|^6 |\alpha_3|^2 + 2|\alpha_2|^8)}{(|\alpha_1|^2 + |\alpha_2|^2)(|\alpha_1|^4 + |\alpha_2|^4)(|\alpha_1|^4 + |\alpha_2|^4)}. \]  

If it is repeated for \(K\) times, the success probability is

\[ P^K_1 = \frac{|\alpha_1|^{2K} (|\alpha_2|^{2K-2} |\alpha_3|^2 + 2|\alpha_2|^{2K})}{(|\alpha_1|^2 + |\alpha_2|^2)(|\alpha_1|^4 + |\alpha_2|^4)\cdots(|\alpha_1|^{2K} + |\alpha_2|^{2K})}. \]  

The total success probability for Alice is

\[ P = P^1_1 + P^2_1 + \cdots = \sum_{K=1}^{\infty} P^K_1. \]  

If Alice is successful, then Charlie start to perform this ECP. His concentration step is analogy with Alice. In detail, he first prepares a single photon of the form

\[ |\Phi\rangle_{p3} = \frac{\alpha_3}{\sqrt{\alpha^2_3 + \alpha^2_2}} |R\rangle_3 + \frac{\alpha_3}{\sqrt{\alpha^2_3 + \alpha^2_2}} |L\rangle_3. \]  

Charlie lets his single photon entrance the microw cavity and couple with the spin. Then the state \(|\Phi_1\rangle_{123}\) combined with \(|\Phi\rangle_{p3}\) evolves as

\[ |\Psi\rangle_1 = |\Phi_1\rangle_{123} |\Phi\rangle_{p2} = \left( \frac{\alpha_2}{\sqrt{\alpha^2_3 + 2\alpha^2_2}} |\downarrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 \right) \]

\[ + \frac{\alpha_2}{\sqrt{\alpha^2_3 + 2\alpha^2_2}} |\uparrow\rangle_1 |\downarrow\rangle_2 |\uparrow\rangle_3 \]

\[ + \frac{\alpha_3}{\sqrt{\alpha^2_3 + 2\alpha^2_2}} |\uparrow\rangle_1 |\uparrow\rangle_2 |\downarrow\rangle_3 \]

\[ + \frac{\alpha_3}{\sqrt{\alpha^2_3 + 2\alpha^2_2}} |\uparrow\rangle_1 |\downarrow\rangle_2 |\uparrow\rangle_3 \]
If $D$ fires, it can be rewritten as

$$\Phi = \sum_{\alpha} (|\alpha\rangle |\alpha\rangle) + \sum_{\alpha} (|\alpha\rangle |\alpha\rangle) + \sum_{\alpha} (|\alpha\rangle |\alpha\rangle)$$

Finally, after the photon passing through the HWP and PBS, if $D_5$ fires, the remained state is essentially the maximally entangled W state

$$|\Phi_5\rangle_3 = \frac{1}{\sqrt{3}} (|\downarrow\rangle |\downarrow\rangle |\uparrow\rangle + |\uparrow\rangle |\downarrow\rangle |\uparrow\rangle + |\downarrow\rangle |\uparrow\rangle |\uparrow\rangle)$$

If $D_6$ fires, the remained state is

$$|\Phi_6\rangle_3 = \frac{1}{\sqrt{3}} (|\downarrow\rangle |\uparrow\rangle |\downarrow\rangle + |\uparrow\rangle |\downarrow\rangle |\uparrow\rangle - |\downarrow\rangle |\uparrow\rangle |\downarrow\rangle)$$

They can obtain $|\Phi_{123}\rangle$ by performing a local operation of the phase rotation on one of the spin.

On the other hand, if the photon is in the output2 and leads the $D_7$ fire, the Eq. (22) collapses to

$$|\Phi_{7\rangle_3} = \frac{a^2}{\sqrt{a^2 + 2a^2 + a^2}} |\downarrow\rangle |\uparrow\rangle |\uparrow\rangle$$

(22)

It can be written as

$$|\Phi_{7\rangle_3} = \frac{a^2}{\sqrt{2a^2 + a^3}} |\downarrow\rangle |\uparrow\rangle |\uparrow\rangle$$

(27)

Otherwise, if $D_8$ fires, the Eq. (22) collapses to

$$|\Phi_{8\rangle_3} = \frac{a^2}{\sqrt{2a^2 + a^3}} |\downarrow\rangle |\uparrow\rangle |\uparrow\rangle$$

(28)

They can also obtain $|\Phi_{123}\rangle$, by performing a local operation of phase rotation on one of the spin. Eqs. (28) and (29) are both lesser-entangled W states, which have the same form of $|\Phi_{123}\rangle$. That is, if Charlie obtains Eq. (28), he can repeat this ECP and obtain the maximally entangled W state in a second round. They can also obtain the success probability for Charlie as

$$P_1^2 = \frac{3|a_2|^2|a_3|^2}{(|a_3|^2 + |a_2|^2)(|a_3|^2 + 2|a_2|^2)}$$

$$P_2^2 = \frac{3|a_2|^4|a_3|^4}{(|a_3|^2 + |a_2|^2)(|a_3|^4 + |a_2|^4)(|a_3|^2 + 2|a_2|^2) + \cdots}$$

$$P_2^K = \frac{3|a_2|^{2K}|a_3|^{2K}}{|a_3|^2 + |a_2|^2)(|a_3|^4 + |a_2|^4)(|a_3|^2 + 2|a_2|^2) + \cdots (|a_3|^{2K} + |a_2|^{2K})} \times \frac{1}{(|a_3|^2 + 2|a_2|^2)}$$

The total success probability for Charlie is

$$P_2 = P_1^2 + P_2^2 + \cdots = \sum_{K=1}^{\infty} P_2^K.$$  

(31)
IV. SUCCESS PROBABILITY AND EXPERIMENT FEASIBILITIES

Thus far, we have briefly explained this ECP. We can calculate the success probability for both Alice and Charlie. Suppose that Alice and Charlie repeat this ECP for $K$ times, the total success probability is

$$P_t = \sum_{K=1}^{\infty} P_1^K \sum_{K=1}^{\infty} P_2^K.$$  \hspace{1cm} (32)

If both Alice and Charlie perform this protocol only one time, the total success probability

$$P_t^1 = P_1^1 P_1^1 = \frac{3|\alpha_1|^2|\alpha_2|^2|\alpha_3|^2}{(|\alpha_1|^2 + |\alpha_2|^2)(|\alpha_3|^2 + |\alpha_2|^2)}.$$  \hspace{1cm} (33)

![FIG. 3: Success probability $P'$ of obtaining a maximally entangled $W$ state after performing this ECP is altered with the initial coefficient $\alpha_1(0, \sqrt{\frac{r}{2}})$. We chose $\alpha_2 = \frac{1}{\sqrt{2}}$. Curve A is the idea case with no leakage. Curve B is the success probability with $\kappa_s = 0.1\kappa$, $g = 0.5\kappa$, and $\gamma = 0.1\kappa$.](image)

Actually, the realization of this ECP relies on the efficiency of hybrid parity check for electrons and photon described in Sec. II. By solving the Heisenberg equations of motion for the cavity-field operator and the trion dipole operator in weak excitation approximation, we can calculate the practical transmission and reflection coefficients. Similar to Ref. [44], we denote $\omega_0$, $\omega_c$ and $\omega_{X^-}$ as the frequencies of the input photon, cavity mode, and the spin-dependent optical transition, respectively. In the approximation of weak excitation, the reflection and transmission coefficients can be written as

$$r(\omega) = 1 + t(\omega),$$

$$t(\omega) = \frac{-\kappa [i(\omega_{X^-} - \omega) + \frac{g}{2}]}{[i(\omega_{X^-} - \omega) + \frac{g}{2}][i(\omega_c - \omega) + \kappa + \frac{\kappa_s}{2} + g^2]},$$  \hspace{1cm} (34)

where $g$ represents the coupling constant, $\frac{g}{2}$ is the $X^-$ dipole decay rate. $\kappa$ and $\kappa_s/2$ are the cavity field decay rate into the input and output modes and the leaky rate, respectively. If we consider the condition that the resonant interaction with $\omega_c = \omega_{X^-} = \omega_0$, and $g = 0$, we can get the reflection and transmission coefficients as

$$r_0(\omega) = \frac{i(\omega_0 - \omega) + \frac{\kappa_s}{2}}{i(\omega_0 - \omega) + \frac{\kappa_s}{2} + \kappa},$$

$$t_0(\omega) = \frac{-\kappa}{i(\omega_0 - \omega) + \frac{\kappa_s}{2} + \kappa}.$$  \hspace{1cm} (35)

The transmission and reflection operators can be rewritten as

$$\hat{t}(\omega) = t_0(\omega)(|R\rangle\langle R| \otimes \uparrow | \uparrow \rangle | \uparrow \rangle + |L\rangle\langle L| \otimes \downarrow | \downarrow \rangle | \downarrow \rangle),$$

$$\hat{r}(\omega) = r_0(\omega)(|R\rangle\langle R| \otimes \uparrow | \uparrow \rangle | \uparrow \rangle + |L\rangle\langle L| \otimes \downarrow | \downarrow \rangle | \downarrow \rangle + r(\omega)(|R\rangle\langle R| \otimes \uparrow | \uparrow \rangle | \uparrow \rangle + |L\rangle\langle L| \otimes \downarrow | \downarrow \rangle | \downarrow \rangle).$$  \hspace{1cm} (36)

In a practical experiment, we let the Alice and Charlie only perform this ECP for one time, so the success probability for Alice can be rewritten as

$$P_1' = \frac{|\alpha_1|^2|\alpha_3|^2 + 2|\alpha_2|^2}{|\alpha_1|^2 + |\alpha_2|^2} \frac{|t_0(\omega)|}{\sqrt{|t_0(\omega)|^2 + |t(\omega)|^2}}.$$  \hspace{1cm} (37)

The practical success probability for Charlie can be rewritten as

$$P_2' = \frac{3|\alpha_2|^2|\alpha_3|^2}{(|\alpha_3|^2 + |\alpha_2|^2)(|\alpha_3|^2 + 2|\alpha_2|^2)} \frac{|r(\omega)|}{\sqrt{|r(\omega)|^2 + |r(\omega)|^2}}.$$  \hspace{1cm} (38)

The total probability $P'$ can be rewritten as

$$P' = P_1' P_2' = \frac{3|\alpha_1|^2|\alpha_2|^2|\alpha_3|^2}{(|\alpha_1|^2 + |\alpha_2|^2)(|\alpha_3|^2 + |\alpha_2|^2)} \frac{|t_0(\omega)||r(\omega)|}{\sqrt{(|t_0(\omega)|^2 + |t(\omega)|^2)(|r(\omega)|^2 + |r(\omega)|^2)}}.$$  \hspace{1cm} (39)

![FIG. 4: Success probability $P_1'$ for Alice performing this ECP shown in Eq. (37). We chose $\alpha_2 = \frac{1}{\sqrt{2}}$ and $\alpha_1 \in (0, \sqrt{\frac{r}{2}})$. Curve A is the idea case with no leakage and Curve B represents the success probability with $\kappa_s = 0.5\kappa$, $g = 0.5\kappa$, and $\gamma = 0.1\kappa$.](image)
probability with \(\kappa\) when \(\kappa\). It is shown that in the idea case, the max value can reach 0.5 and it can reach \(P_2' \approx 0.4\) when \(\kappa = 0.5\kappa\). Interestingly, the success probability shown in Eqs. (30) and (38) for \(\alpha = 0\). But both \(P_2\) and \(P_2'\) decrease rapidly when \(\alpha \in (0, 0.6)\) with \(P_2 \approx 0.5\) and \(P_2' \approx 0.43\). Actually, when \(\alpha_2 = \frac{1}{\sqrt{3}}\), Eqs. (30) and (38) can be simplified as

\[
P_2 = \frac{\frac{3}{2} - |\alpha_1|^2}{(1 - |\alpha_1|^2)(\frac{3}{2} - |\alpha_1|^2)},
\]

and

\[
P_2' = \frac{\frac{3}{2} - |\alpha_1|^2}{(1 - |\alpha_1|^2)(\frac{3}{2} - |\alpha_1|^2)} \frac{|r(\omega)|}{\sqrt{|r_0(\omega)|^2 + |r(\omega)|^2}}.
\]

In Fig. 6 we calculated the success probability for obtaining a maximally entangled \(W\) state when \(\kappa = 0.5\kappa\). Compared with Fig. 3, the success probability decreases. The max value is about \(P' \approx 0.16\).

\section{V. DISCUSSION AND SUMMARY}

In this ECP, the system can be realized in a self-assembled GaAs quantum dot or an InAs interface quantum dot in micropillar microcavities. Therefore, the long coherence of quantum dots and the strong coupling of the quantum dots with the cavity are required. It is reported that the coupling strength is acceptable for \(g = 0.5\) in a microwavvity with a diameter of \(d = 1.5\mu\text{m}\) with the cavity leakage. In 2008, Press \textit{et al.} demonstrated the optical initialization, rotation by arbitrary angle and projective measurement of an electron spin in a quantum dot. They showed that coherence time is 3.0 \(\mu\text{s}\) and about 10\(^5\) operations can be achieved within the qubit's coherence time. Greilich \textit{et al.} also reported their experiment about ultrafast optical rotation of spins about arbitrary axes on a picosecond timescale using laser pulses as control fields. Current experiment also demonstrated that the spin coherent time \(T_2^h > 100\mu\text{s}\) due to the suppressed electron-photon interaction and the lack of hole-nuclear hyperfine interaction. The cooling and fast coherent control of hole-spin states were also reported in 2010. Press \textit{et al.} increased decoherence time of a single quantum dot electron spin from nanoseconds to microseconds using ultrafast all-optical spin echo technique.

In summary, we exploit the single photons to concentrate the less-entangled \(W\) state for the charge qubits confined in the quantum dots. This ECP is quite different from the other protocols because this ECP is performed between different physical qubits, i.e. the charge qubits and the photons, while other protocols unusually use the same physical qubits. It provides us a good way to realize such ECP. Moreover, during this ECP, we only

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{fig5}
\caption{Success probability \(P'_2\) for Charlie performing this ECP shown in Eq. (38). We chose \(\alpha_2 = \frac{1}{\sqrt{3}}\) and \(\alpha_1 \in (0, \sqrt{\frac{2}{3}})\). Curve A is the idea case with no leakage and Curve B represents the success probability with \(\kappa = 0.5\kappa, \ g = 0.5\kappa,\) and \(\gamma = 0.1\kappa\).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{fig6}
\caption{Success probability \(P'\) of obtaining a maximally entangled \(W\) state after performing this ECP is altered with the initial coefficient \(\alpha_1 \in (0, \sqrt{\frac{2}{3}})\). We chose \(\alpha_2 = \frac{1}{\sqrt{3}}\). Curve A is the idea case with no leakage. Curve B is the success probability with \(\kappa = 0.5\kappa, \ g = 0.5\kappa,\) and \(\gamma = 0.1\kappa\).}
\end{figure}

We calculated the success probability of this ECP for different \(\alpha_1\) in the coupled system. Fig. 3 illustrates the success probability \(P'\) shown in Eq. (39) for obtaining the maximally entangled \(W\) state. We chose \(\alpha_2 = \frac{1}{\sqrt{3}}\) and changed \(\alpha_1 \in (0, \sqrt{\frac{2}{3}})\). It is shown that in the idea case without leakage, the success probability can reach 0.25 when \(\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{\sqrt{3}}\). But in a practical condition that \(\kappa = 0.1\kappa\), the max value \(P' \approx 0.18\) when \(\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{\sqrt{3}}\). We also calculated the success probability when \(\kappa = 0.5\kappa\), shown in Fig. 4, Fig. 5 and Fig. 6. In Fig. 4, it is the success probability for Alice performing this ECP one time in both the idea case and with the leakage \(\kappa = 0.5\kappa\).
require one pair of less-entangled W states while the conventional ECPs should resorts two same copies of such states. Therefore, this ECP seems more optimal. On the other hand, this protocol can be repeated to obtain a high success probability by consuming some single photons. Our protocol may be useful and flexible in current quantum information processing.

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[1] M. A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information(Cambridge University Press, Cambridge, UK, 2000).
[2] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Rev. Mod. Phys. 74, 145 (2002).
[3] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).
[4] A. Karlsson, M. Bourennane, Phys. Rev. A 58, 4394 (1998); F. G. Deng, C. Y. Li, Y. S. Li, H. Y. Zhou, and Y. Wang, Phys. Rev. A 72, 022338 (2005).
[5] A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991).
[6] C. S. Hong, Z. Y. Ou, L. M. Gang, Phys. Rev. Lett. 59, 2046 (1987).
[7] C. H. Bennett, G. B. Brassard, and N. D. Mermin, Phys. Rev. Lett. 68, 557 (1992).
[8] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992).
[9] G.-L. Long, and X.-S. Liu, Phys. Rev. A, 65, 032302 (2002).
[10] F.-G. Deng, G.-L. Long, and X.-S. Liu, Phys. Rev. A, 68, 042317 (2003).
[11] C. Wang, F. G. Deng, Y. S. Li, X. S. Liu, and G. L. Long, Phys. Rev. A 71, 042305 (2005).
[12] R. Cleve, D. Gottesman, H.K. Lo, Phys. Rev. Lett. 83, 648, (1999).
[13] A.M. Lance, T. Symul, W.P. Bowen, B.C. Sanders, P.K. Lam, Phys. Rev. Lett. 92, 177903, (2004).
[14] F. G. Deng, X. H. Li, C. Y. Li, P. Zhou, and H. Y. Zhou, Phys. Rev. A 72, 044301 (2005); F. G. Deng, X. H. Li, C. Y. Li, P. Zhou, and H. Y. Zhou, Europ. Phys. J. D 39, 459 (2006); X. H. Li, P. Zhou, C. Y. Li, H. Y. Zhou, and F. G. Deng, J. Phys. B 39, 1975 (2006).
[15] M. Hillery, V. Bužek, and A. Berthiaume, Phys. Rev. A 59, 1829(1999).
[16] A. Karlsson, M. Koashi, and N. Imoto, Phys. Rev. A, 59, 462 (1999).
[17] L. Xiao, G.-L. Long, F.-G. Deng, and J.-W. Pan, Phys. Rev. A 69, 052307 (2004).
[18] H. J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 81, 5932 (1998).
[19] L. M. Duan, M. D. Lukin, J. T. Cirac and P. Zoller, Nature(London) 414, 413 (2001).
[20] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A 53, 2046 (1996).
[21] S. Bose, V. Vedral, and P. L. Knight, Phys. Rev A 60, 194 (1999).
[22] B. S. Shi, Y. K. Jiang, and G. C. Guo, Phys. Rev. A 62, 054301 (2000).
[23] N. Paunković, Y. Omar, S. Bose, and V. Vedral, Phys. Rev. Lett. 88, 187903 (2002).
[24] Z. Zhao, J. W. Pan, and M. S. Zhan, Phys. Rev. A 64, 042301 (2001).
[25] Z. Zhao, T. Yang, Y. A. Chen, A. N. Zhang, and J. W. Pan, Phys. Rev. Lett. 90, 207901 (2003).
[26] T. Yamamoto, M. Koashi, and N. Imoto, Phys. Rev. A 64, 012304 (2001).
[27] T. Yamamoto, M. Koashi, S. K. Ozdemir, and N. Imoto, Nature 421, 343 (2003).
[28] W. Xiang-bin, and F. Heng, Phys. Rev. A 68, 060302 (2003).
[29] Z. L. Cao, L. H. Zhang, and M. Yang, Phys. Rev. A 71, 044302 (2005).
[30] M. Yang, Y. Zhao, W. Song, and Z. L. Cao, Phys. Rev. A 71, 044302(2005).
[31] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, Phys. Rev. A 77, 062325 (2008).
[32] Y. B. Sheng, L. Zhou, S. M. Zhao, and B. Y. Zheng, Phys. Rev. A 85, 012307 (2012).
[33] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, Phys. Lett. A 373, 1823 (2009).
[34] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, Quant. Inf. Process. 10, 272 (2010).
[35] Y. B. Sheng, L. Zhou, S. M. Zhao, and B. Y. Zheng, Phys. Rev. A 85, 012307 (2012).
[36] F. G. Deng, Phys. Rev. A 85, 022311 (2012).
[37] Y. B. Sheng, L. Zhou, and S. M. Zhao, Phys. Rev. A 85, 044305 (2012).
[38] C. W. Zhang, Quant. Inf. Process. 4, 196 (2004).
[39] Z. L. Cao and M. Yang, J. Phys. B 36, 4245 (2003).
[40] L. H. Zhang, M. Yang, and Z. L. Cao, Phys. A 374, 611 (2007).
[41] H. F. Wang, S. Zhang, and K. H. Yeon, Eur. Phys. J. D 56, 271 (2010).
[42] H. F. Wang, L. L. Sun, S. Zhang, and K.-Y. Yeon, Quant. Inf. Process. DOI 10.1007/s11128-011-0255-9, (2011).
[43] A. Yildiz, Phys. Rev. A 82, 012317 (2010).
[44] C. Y. Hu, W. J. Munro, and J. G. Rarity, Phys. Rev. B 78, 125318 (2008).
[45] C. Y. Hu, A. Young, J. L. ÓBrien, W. J. Munro, and J.
[46] C. Y. Hu, and J. G. Rarity, Phys. Rev. B 83, 115303 (2011).
[47] C. Bonato, F. Haupt, S. R. Oemrawsingh, J. Gudat, D. Ding, M. P. van Exter, and D. Bouwmeester, Phys. Rev. Lett. 104, 160503 (2010).
[48] C. Wang, Y. Zhang and G. S. Jin, Phys. Rev. A 84, 032307 (2011).
[49] C. Wang, Y. Zhang, and R. Zhang, Optics Express 19, 25685 (2011).
[50] T. J. Wang, S. Y. Song, and G. L. Long, Phys. Rev. A 85, 062311 (2012).
[51] J. P. Reithmaier, G. Sek, A. Löffler, C. Hofmann, S. Kuhn, S. Reitzenstein, L. V. Keldysh, V. D. Kulakovskii, T. L. Reinecke, and A. Forchel, Nature, 432 197 (2004).
[52] D. Press, T. D. Ladd, B. Zhang, and Y. Yamamoto, Nature 456, 218 (2008).
[53] Greilich, S. E. Economou, S. Spatzek, D. R. Yakovlev, D. Reuter, A. D. Wieck, T. L. Reinecke, and M. Bayer, Nat. Phys. 5, 262 (2009).
[54] D. Brunner, B. D. Geradot, P. A. Dalgarno, G. Wüst, K. Karrai, N. G. Stoltz, P. M. Petroff, and R. J. Warburton, Science 325, 70 (2009).
[55] B. D. Geradot, D. Brunner, P. A. Dalgarno, P. Öhberg, S. Seidl, M. Kronere, K. Karrai, N. G. Stoltz, P. M. Petroff, and R. J. Warburton, Nature (London) 451, 441 (2008).
[56] D. Press, L. De Greve, P. L. McMahon, T. D. Ladd, B. Friess, C. Schneider, M. Kamp, S. Höfling, A. Forchel, and Y. Yamamoto, Nat. Photo. 4, 367 (2010).