Multi-objective voltage control and reactive power optimization based on multi-objective particle swarm algorithm

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Abstract. This paper presents multi-objective particle swarm optimization (MOPSO) for voltage control and reactive power optimization. The optimization objective is to minimize active power loss and improve power system voltage profiles can be achieved by controlling a number of control variables such as generator voltages, transformer tap ratios and shunt capacitors, etc. A Fuzzy membership function is used to obtain the best compromise solution out of the available Pareto-optimal solutions. The results MOPSO are compared to the multi-objective genetic algorithm (MOGA), genetic algorithm (GA) and Particle Swarm Optimization (PSO). The IEEE 14 bus and IEEE 30 bus standards systems were used as test systems to demonstrate the applicability and efficiency of the proposed method.

1. Introduction
An important aspect of power systems is the problem of reactive power optimization, which plays an important role in optimal operation. In order to obtain available voltages and minimal active power losses, the reactive power optimization is necessary [1,2]. The objective of reactive power optimization is to minimize the active power loss and improve voltage profiles, which can be achieved by adjusting controllable variables, such as generator voltages, shunt capacitors and inductors, transformer taps setting, etc. The reactive power optimization is a complex nonlinear, multi-constraints, non-differentiable and mixed-integer problem [3].

In recent years, a wide variety of the heuristic methods have been proposed to solve the reactive power optimization problem and voltage control such as genetic algorithms (GA), evolutionary programming (EP), evolutionary strategies (ES), and particle swarm optimization (PSO) [4–8].

The reactive power optimization problem was considered as a single-objective optimization problem [9–12]. Recently, the reactive power optimization problem is formulated as a multi-objective optimization problem. However, the problem is not treated as a true multi-objective problem [13,14]. But the multi-objective problem was converted into a single objective reactive power problem by using weighted sum of objectives [15–17]. Regrettably, there is no rational basis of determining adequate weights and the objective function so formed may lose significance due to combining non commensurable objectives [13,14]. In addition, this requires multiple runs as many times as the number of desired Pareto-optimal solutions. Furthermore, this method cannot be used to find Pareto-optimal solutions in problems like reactive power optimization, which have a non-convex Pareto-optimal front [13]. Therefore conventional optimization methods can at best find one solution in one
simulation run, thereby making those methods inconvenient to solve multi objective optimization problems [14]. In the last few years, the multi-objective evolutionary algorithms (MOEAs) and MOPSO are getting immense popularity, mainly because of their ability to find a widespread of Pareto-optimal solutions in a single simulation run [14].

In this paper, MOPSO is presented for solving the multi-objective reactive power optimization problem and voltage control. In the first case, PSO and GA used for individual optimization for active power loss and voltage deviation. In the second case, the multi-objective optimization problem is formulated as a nonlinear constrained problem wherever the active power loss and the voltage deviations are treated as competing objectives and optimized simultaneously in a single run. Where we obtained to Pareto-optimal front from MOPSO. Then fuzzy-based mechanism is used to extract the best compromise solution over the trade-off curve. The results MOPSO are compared to MOGA, PSO and GA. The IEEE 14 bus and IEEE 30 bus standards systems were used as test systems to demonstrate the applicability and efficiency of the proposed method.

2. Problem formulation
The optimal reactive power dispatch and voltage control problem to optimize the steady state performance of a power system in terms of one or more objective functions while satisfying several equality and inequality constraints. Generally the problem can be formulated as follows [13,15].

2.1. Objective functions
2.1.1. Minimization of active power loss (PL). One of the main objective of the reactive power optimization is to minimize the active power losses in the transmission lines, which can be expressed as follows:

\[
\min p_L = \min \sum_{k=1}^{NE} G_k \left[ V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j) \right]
\]

where NE is the number of transmission lines; \( G_k \) is the conductance of the \( k \)-th line that connects bus \( i \) to bus \( j \); \( V_i \angle \delta_i \) and \( V_j \angle \delta_j \) are the voltages at end bus \( i \) and bus \( j \) of the \( k \)-th line.

2.1.2. Voltage deviation. An effective way to improve voltage profile is to minimize the deviation of voltage magnitudes at load buses from the desired value as follow:

\[
VD = \sum_{i=1}^{N_L} \left| V_i - V_i^{ref} \right|
\]

where \( N_L \) is the number of load buses; \( V_i \) and \( V_i^{ref} \) are actual and reference voltage magnitudes at bus \( i \). \( V_i^{ref} \) is usually set to be 1 pu.

2.2. Problem Constraints
2.2.1. Equality constraints. The minimization of the objective functions is subjected to a number of equality and inequality constraints. The equality constraints are active and reactive power balance at each bus which can be formulated by power flow equations as follows

\[
P_i - P_{io} - \sum_{j=1}^{NB} V_j \left[ G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \right] = 0, \quad i = 1, \ldots, NB
\]

\[
Q_i - Q_{io} - \sum_{j=1}^{NB} V_j \left[ G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j) \right] = 0, \quad i = 1, \ldots, NB
\]
where NB is the number of buses; \( P_{Gi} \) and \( Q_{Gi} \) are the generator active and reactive power, respectively; \( P_{Di} \) and \( Q_{Di} \) are the load active and reactive power, respectively; \( G_{ij} \) and \( B_{ij} \) are the transfer conductance and susceptance between bus \( i \) and bus \( j \), respectively.

2.2. Inequality constraints. The inequality constraints represent the system operating constraints as follows.

\[
\begin{align*}
V_{Gi}^{min} & \leq V_{Gi} \leq V_{Gi}^{max}, \quad i = 1, \ldots, NG; \\
Q_{Gi}^{min} & \leq Q_{Gi} \leq Q_{Gi}^{max}, \quad i = 1, \ldots, NG \\
V_{Li}^{min} & \leq V_{Li} \leq V_{Li}^{max}, \quad i = 1, \ldots, NLB; \\
Q_{ci}^{min} & \leq Q_{ci} \leq Q_{ci}^{max}, \quad i = 1, \ldots, NC \\
T_i^{min} & \leq T_i \leq T_i^{max}, \quad i = 1, \ldots, NT 
\end{align*}
\] (5)

Where \( V_{Gi} \) and \( Q_{Gi} \) are voltage magnitude and reactive power output at the \( i \)-th generator bus, respectively; \( V_{Li} \) is the voltages at load buses; \( Q_{ci} \) is reactive power compensation of the \( i \)-th switched VAR source; \( T_i \) is the tap setting of the \( i \)-th transformer. \( NG, NLB, NC \), and \( NT \) are the numbers of generators, load buses, switchable VAR sources, and transformers, respectively.

3. Best Compromise Solution

Fuzzy set theory has been implemented to derive efficiently a candidate Pareto optimal solution for the decision makers. Upon having the Pareto-optimal set of non-dominated solution, the proposed approach presents one solution to the decision maker as the best compromise solution. Due to imprecise nature of the decision maker’s judgment, each objective function of the \( i \)-th solution is represented by a membership function \( \mu_{Fi} \) defined as[18,19]:

\[
\mu_{Fi} = \begin{cases} 
1 & F_i \leq F_i^{min} \\
\frac{F_i^{max} - F_i}{F_i^{max} - F_i^{min}} & F_i^{min} \leq F_i \leq F_i^{max} \\
0 & F_i \geq F_i^{max} 
\end{cases} 
\] (6)

where \( F_i^{max} \) and \( F_i^{min} \) are the maximum and minimum values of the \( i \)-th objective function respectively. For each non-dominated solution \( k \), the normalized membership function \( \mu^k \) is.

\[
\mu^k = \frac{\sum_{i=1}^{N_{obj}} \mu_{Fi}}{\sum_{k=1}^{M} \sum_{i=1}^{N_{obj}} \mu_{Fi}} 
\] (7)

where \( M \) is the number of non-dominated solutions while \( N_{obj} \) is the number of the problem objectives. The best compromise solution is that having the maximum value of \( \mu^k \).

4. Simulation results

The IEEE 14 bus and 30 bus standards systems were used as test systems to demonstrate the applicability and efficiency of the proposed method [20]. Table 1 gives details of the test systems. The initial network active power loss for IEEE 14 bus and 30 bus are 13.393 MW and 17.557 MW respectively in the base case. For the test systems, all PV bus voltages are required to be maintained within the range of 0.95–1.1 pu. And for PQ bus 0.95–1.05 pu. The lower and upper limits of the transformer tapings are 0.95 and 1.05 pu. The MVA base is taken as 100 for both test systems.

To demonstrate the effectiveness of the proposed approach, two different cases have been considered as follows:

**Case 1:** The active power loss and voltage deviation objectives are optimized individually by using PSO and GA to explore the extremes of the trade-off surface and assess the diversity characteristics
of the Pareto-optimal solutions obtained through the proposed approach and taking into consideration all problem constraints.

**Case 2:** The problem was handled as a problem of multi-objective optimization where both active power loss and voltage deviation were optimized at the same time by using MOPSO and MOGA.

4.1. The results of IEEE 14-bus system for case 1

4.1.1. Reduce active power loss. Figure 1 shows the reduce PL after adjusting the reactive power variables such as generator voltages, transformer tap-settings and capacitor banks by using PSO and GA. Where initial active Power loss -13.3933 MW. After optimization with using PSO the minimum PL -12.494 MW and after optimization the minimum PL by using GA is12.492 MW.

4.1.2. Optimal voltage profile. Figure 2 shows optimal voltage profile after adjusting generator voltages, transformer tap-settings and capacitor banks by using PSO and GA. Voltage deviation after optimization by using PSO - 0.0377 pu and after optimization by using GA is 0.0331 pu. Where initial Voltage deviation - 0.4036 pu. Table 2 shows Comparison between PSO and GA for optimized the problem individually. This table shows that number of function evaluations for PSO less than GA. Thus, the time needed by PSO is less than the time needed by GA for function evaluations.

![Figure 1. Convergence of PL objective using PSO and GA for IEEE 14.](image1)

![Figure 2. Voltage profile after using PSO and GA for IEEE 14.](image2)

| Table 1. Description of the test systems. |
|----------------------------------------|
|                                      |
| IEEE14 | IEEE30 |
| Number of buses | 14 | 30 |
| Number of generators | 5 | 6 |
| Number of load buses | 9 | 24 |
| Number of transformers | 3 | 4 |
| Number of shunts capacitor | 1 | 2 |
| Number of control variables | 9 | 12 |

| Table 2. Comparison between PSO and GA for PL and VD optimized individually. |
|--------------------------------------------------------------------------------|
| Best PL(MW) | Best VD (pu) | number of function evaluations |
| PSO | 12.494 | 0.0377 | 10050 |
| GA | 12.492 | 0.0331 | 40200 |

4.2. The results of IEEE 30-bus system for case 1

4.2.1. Reduce active power loss. Figure 3 shows the reduce PL after adjusting the reactive power variables such as generator voltages, transformer tap-settings and capacitor banks by using PSO and
GA. Where initial PL -17.557 MW. After optimization with using PSO the minimum PL -16.44 MW and after optimization the minimum PL by using GA is 16.455 MW.

4.2.2. Optimal voltage profile. Figure 4 shows optimal voltage profile by using PSO and GA. Voltage deviation after optimization by using PSO - 0.1569 pu and after optimization by using GA is 0.1511 pu. Where initial Voltage deviation - 0.6256 pu.

Figure 3. Convergence of PL objective using PSO and GA for IEEE 30.

Figure 4. Voltage profile after using PSO and GA for IEEE 30.

4.3. The results of IEEE 14-bus system for case 2
Figure 5 shows Pareto-optimal front of MOPSO and MOGA. The membership functions given in (6) and (7) are used to evaluate each member of the Pareto-optimal set. Then it is possible to extract the best compromise solution that has the maximum membership function value. Table 3 gives a comparison between the results of individual optimization and MOPSO and MOGA. Table 4 shows comparison between the results of MOPSO and MOGA for Best compromise solution (BCS) and minimum value for each objective function.

Table 3. Comparison of best solutions of PL and VD.

|               | PSO   | GA    | MOGA   | MOPSO  |
|---------------|-------|-------|--------|--------|
| Min PL (MW)   | 12.494| 12.492| 12.5413| 12.5493|
| Min VD (pu)   | 0.0377| 0.0331| 0.0461 | 0.0386 |

Table 4. Best compromise solution (BCS) and minimum value for each objective function.

|               | MOPSO | MOGA |
|---------------|-------|------|
| BCS           | 12.7973| 0.1072 | 12.7179| 0.1132 |
| Min PL        | 12.5493| 0.1563 | 12.5413| 0.1693 |
| Min VD        | 13.7040| 0.0386 | 13.7500| 0.0461 |

4.4. The results of IEEE 30-bus system for case 2
Figure 6 shows Pareto-optimal front for MOPSO and MOGA. Table 5 gives a comparison between the results of individual optimization and MOPSO and MOGA. Table 6 shows comparison between the results of MOPSO and MOGA for Best compromise solution (BCS) and minimum value for each objective function. It is clear that the results of the proposed approach are almost identical to that of individual optimization. It can be concluded that the proposed approach is capable of exploring more efficient and non-inferior solutions. The close agreement between the results clearly shows the ability of the proposed approach to treat the multi-objective optimization problems as the best solution.
of each objective along with a manageable set of non-dominated solutions can be obtained in one single run.

Figure 5. Convergence of PL objective using PSO and GA for IEEE 14.

Figure 6. Voltage profile after using PSO and GA for IEEE 30.

| individual optimization | PSO | GA | MOGA | MOPSO |
|--------------------------|-----|----|------|-------|
| Min PL(MW)               | 16.44 | 16.4551 | 16.495 | 16.5071 |
| Min VD (pu)              | 0.1569 | 0.1511 | 0.1748 | 0.1660 |

Table 6. Best compromise solution (BCS) and minimum value for each objective function.

|                | MOPSO | MOGA |
|----------------|-------|------|
| PL(MW)         | 16.7117 | 16.6434 |
| VD(pu)         | 0.2618  | 0.2798 |
| MinPL          | 16.5071 | 16.4956 |
| MinVD          | 17.4768 | 17.5539 |
| BCS            | 0.1660  | 0.1748 |

5. Conclusions
In this paper, a multi-objective particle swarm optimization (MOPSO) for reactive power optimization and voltage control has been proposed. The problem has been formulated as multi-objective optimization problem where the active power loss and the voltage deviations for load buses are treated as competing objectives. Moreover, a Fuzzy membership function is used to obtain the best compromise solution out of the available Pareto-optimal solutions. The proposed approach has been successfully applied to solve problem of reactive power optimization and voltage control on IEEE 14-bus and 30-bus systems. The performance of the proposed approach is compared with PSO, GA, and MOGA. The all simulation results show that MOPSO could find high-quality solutions with more reliability and efficiency. Moreover, the computational time required for MOPSO algorithm is faster than MOGA. In addition, PSO algorithm based a multi-objective optimization method is capable of achieving global optimal solution.

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