Two-photon cooling of a nonlinear quantum oscillator

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Abstract

The cooling effects of a nonlinear quantum oscillator via its interaction with an artificial atom (qubit) are investigated. The quantum dissipations through the environmental reservoir of the nonlinear oscillator are included, taking into account the nonlinearity of the qubit-oscillator interaction. For appropriate bath temperatures and the resonator’s quality factors, we demonstrate effective cooling below the thermal background. As the photon coherence functions behave differently for even and odd photon number states, we describe a mechanism distinguishing those states. The analytical formalism developed is general and can be applied to a wide range of systems.

Key words: two-photon effects, cooling, qubit.

1 Introduction

Simple models describing the main properties of various phenomena in physics are always of particular importance. The Jaynes-Cummings model, for instance, provides a simple description of the interaction of matter with an electromagnetic field [1]. It consists of a two-state particle interacting with a single quantized mode, applicable in general to cavity quantum electrodynamics. Now, there is an increased interest to apply such a simple model to more complex systems like superconducting electrical quantum circuits. This allows us to investigate them in an analogous way as a two-level atom interacting with a quantized electromagnetic cavity mode [2]. As an advantage, for instance, the strong coupling limit and enhanced lifetimes can be achieved in

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superconducting devices [3]. Remarkably, a Josephson qubit was designed that entangles qubit information [4]. Entanglement between a superconducting flux qubit and a superconducting quantum interference device was demonstrated in [5], while a procedure to directly measure the state of an electromagnetic field inside a superconducting transmission line, coupled to a Cooper-pair box, was proposed in [6], making them attractive to quantum computation processing. Of particular interest are the studies regarding the dephasing of the superconduction qubit induced by the photon noise [7]. Other systems refer to coupling of the superconducting qubit to a solid-state nanomechanical resonator. Various interesting effects related to the nanomechanical resonator state were demonstrated. In particular, squeezing of the nanomechanical resonator state occurs when coupling it to a Josephson quantum circuit [8]. The fidelity of a state transfer from the Josephson junction to a nanomechanical resonator was investigated in [9]. Entanglement from a nanomechanical resonator weakly coupled to a single Cooper-pair box [10], continuous measurement of the energy eigenstates of a nanomechanical resonator without a nondemolition probe [11] or signatures for a classical to quantum transition of a driven nonlinear nanomechanical resonator [12] were already discussed.

Via engineering superconducting elements as artificial atoms and coupling them to a photon field of a resonator or to vibrational states of a nanomechanical resonator one can demonstrate other interesting phenomena such as single artificial atom lasing or cooling. In particular, schemes for ground-state cooling of mechanical resonators were proposed in [13]. A flux qubit was experimentally cooled [14] by using techniques somewhat related to the well-known optical sideband cooling methods (see e.g. Ref. [15] and references therein). Continuous monitoring of Rabi oscillations in a Josephson flux qubit was reported in [16] while lasing effects of a Josephson-junction charge qubit, embedded in a superconducting resonator, was experimentally demonstrated in [17]. Single-qubit lasing and cooling at the Rabi frequency was proposed in [18], while a mechanism of simultaneously cooling of an artificial atom and its neighboring quantum system was analyzed in [19]. In some of these systems the nonlinear qubit-oscillator interaction was considered, i.e. the case when the qubit exchanges simultaneously more than one photon with the resonator mode. However, the dissipations of the nonlinear system due to interaction of the quantized mode with the environmental reservoir are more complex and require a further treatment.

Thus here we report additional results regarding the nonlinear matter-light interactions which are general and applicable to a wide range of systems. To this end, we investigate the properties of a quantum oscillator coupled nonlinearly with a driven qubit through two-photon effects and damped via the thermal environmental reservoir, and focus on cooling phenomena of the oscillator’s degrees of freedom. Due to a high degree of correlations between the particles generated in such a two-photon process, we consider the nonlinear
damping of the generated photons. This allows us to describe the system by using the properties of su(1,1) algebra. In the steady-state we obtain a mean photon number well below unity and thermal limit. Consequently, an effective cooling mechanism via nonlinear processes is discussed. Further, we propose a scheme which is able to distinguish between even and odd photon number states corresponding to su(1,1) algebra via measuring the second-order photon coherence function (and/or higher-order photon correlations). In addition, photon statistics may show quantum features, i.e. an important step towards single-photon sources.

The paper is organized as follows. In Sec. 2 we introduce the system of interest and derive the corresponding master equation. The next section 3 analyzes the results. Finally the summary is given in Sec. 4.

2 The model

Particularly, we consider a Josephson flux qubit coupled inductively to a slow LC oscillator. The frequency of the oscillator is much lower than the qubit’s tunnel splitting, i.e. $\omega_c \ll \Delta$. The qubit is driven with Rabi frequencies near resonance with the oscillator frequency that affect the oscillator, increasing its oscillation amplitude. Near the symmetry point (i.e. the energy bias $\epsilon$ between the flux states is negligibly small) and after transformation to the qubit’s eigenbasis, the Hamiltonian describing the systems is:

$$H = \Delta \sigma_z/2 + \Omega \cos(\omega t)\sigma_x + \omega_c a^+ a - g\sigma_x(a + a^+),$$  (1)

where the first term describes the qubit while the second one considers its driving by an applied AC magnetic flux with amplitude $\Omega$ and frequency $\omega$. The last two terms describe the oscillator with frequency $\omega_c = 1/\sqrt{LC}$ as well as the qubit-oscillator interaction, respectively. Here $g \approx MI_p I_c/\omega_c$, where $M$ is the mutual inductance, $I_p$ the magnitude of the persistent current in the qubit, and $I_c = \sqrt{\omega_c/2L}$ the amplitude of the vacuum fluctuations of the current in the LC oscillator. $a^+$ and $a$ are the creation and annihilation operators corresponding to the oscillator degrees of freedom, while $\sigma_i$ ($i \in \{x, y, z\}$) are the Pauli matrices operating in the dressed flux basis of the qubit subsystem. As $\Delta \gg \omega_c$, the transverse coupling in the Hamiltonian (1) is transformed into a second-order longitudinal coupling by employing a Schrieffer-Wolff type transformation, i.e. $\hat{U}_S = \exp(iS)$ with $S = (g/\Delta)(a^+ a + a_+ a)$. By further using the rotating wave approximation with respect to $\omega$ and diagonalizing the qubit term as well as applying the secular approximation, i.e. omitting terms oscillating with the generalized Rabi frequency, one arrives at the following Hamiltonian describing the nonlinear interaction between the qubit and the
oscillator:

\[
H = \Omega_R \sigma_z / 2 + \omega_c a^{+} a + g_2 (a^{+} \sigma^- + \sigma^{+} a^2) / 2 \\
- g_0 (a a^{+} + a^{+} a) \sigma_z / 4.
\]

(2)

Here \( g_2 = 2g^2 \sin 2\theta / \Delta \) gives the nonlinear qubit-oscillator coupling strength while \( g_0 = 4g^2 \cos 2\theta / \Delta \) accounts for a frequency shift of the qubit’s frequency. Further \( \cot 2\theta = \delta \omega / \Omega \), where \( \delta \omega = \Delta - \omega \) and where \( \Omega_R = \sqrt{(\delta \omega)^2 + \Omega^2} \) stands for the generalized Rabi frequency. The Hamiltonian (2) involves two-quantum processes, i.e. two-particle exchanges between the qubit and the nonlinear oscillator, which means that the quanta are created and annihilated simultaneously in pairs. The particles generated via such a quadratic process are known to be highly correlated, i.e. a single photon pair behaves like a quasiparticle [21].

The spontaneous emission damping of the qubit in this picture is given as [18,22]:

\[
\dot{\rho}_{sp} = -\gamma^{(0)} [\sigma_z, \sigma_z \rho] - \sum_{\alpha_1 \neq \alpha_2 \in \{+, -\}} \gamma^{(\alpha_1)} [\sigma^{\alpha_1}, \sigma^{\alpha_2} \rho] + \text{H.c.},
\]

(3)

where \( \gamma^{(+)} = \Gamma_0 \cos^4 \theta / 2, \gamma^{(-)} = \Gamma_0 \sin^4 \theta / 2 \) and \( \gamma^{(0)} = \Gamma_0 \sin^2 2\theta / 8 \).

The damping of the quantized oscillator mode depends on the environmental reservoir. In order to have a two-photon damping of the nonlinear oscillator we consider that the quantum oscillator couples with the environmental bath via the following Hamiltonian

\[
H_f = h \nu b^{\dagger} b + 2h \widetilde{\chi} (b^{\dagger} \beta^- + \beta^+ b).
\]

(4)

Here the operators \( b^{\dagger} (b) \) belong to the broadband reservoir of carrier frequency \( \nu \) and represent the photon generation (annihilation) operator for the bath. Such a reservoir can be obtained by assuming that the \( LC \) oscillator couples additionally with another circuit the frequency of which \( \nu \) is equal or close to \( 2\omega_c \). Eliminating the bath operators in the Born-Markov approximation one can arrive at the master equation describing the damping of the nonlinear oscillator. For further convenience we introduce the field operators

\[
\beta^+ = a^2 / 2, \quad \beta^- = a^2 / 2 \quad \text{and} \quad \beta_z = (a^+ a + 1 / 2) / 2
\]

which obey the commutation relations for \( su(1,1) \) algebra, i.e. \([\beta^+, \beta^-] = -2\beta_z \) and \([\beta_z, \beta^\pm] = \pm \beta^\pm \). These operators act on the corresponding bases states of the \( su(1,1) \) algebra in the following way:
\begin{align}
\beta^+|j,m\rangle &= \sqrt{(m+1)(m+2j)}|j,m+1\rangle, \\
\beta^-|j,m\rangle &= \sqrt{m(m+2j-1)}|j,m-1\rangle, \\
\beta_z|j,m\rangle &= (m+j)|j,m\rangle.
\end{align}

Here \(m \in \{0, 1, 2, \cdots, \infty\}\), while for a single mode field, as considered in our approach, the allowed value of the Bargmann index (i.e., \(j\)) is \(1/4\) \((3/4)\) for an even (odd) photon number. The correspondence between the number state of the single mode field \(|n\rangle\) and the \(su(1,1)\) basis states \(|j,m\rangle\) is \(|n\rangle \leftrightarrow |j,m\rangle\) for \(n = 2(m+j) - 1/2\) \([23]\).

We have derived the master equation corresponding to the damping of the nonlinear oscillator via two-photon processes which can be written as follows (see Appendix):

\begin{equation}
\dot{\rho}_f = -i[H_0, \rho] - \kappa(1 + \bar{n})\{[\beta^+, \beta^\dagger - \rho, \beta^-] + [\rho\beta^+, \beta^-]\} \\
- \kappa\bar{n}\{[\beta^-, \beta^\dagger + \rho, \beta^+] + [\rho\beta^-, \beta^+]\},
\end{equation}

with \(H_0 = 2\bar{\chi}\bar{n}\beta_z - \bar{\chi}\beta^+\beta\) describing an additional shift of the oscillator mode frequency proportional to \(\bar{n}\) and the Lamb shift proportional to \(\bar{\chi}\), respectively, induced by the thermostat via an effective coupling constant \(\bar{\chi}\). Here \(\kappa\) is the two-photon damping rate of the quantized mode while \(\bar{n}\) is the mean thermal photon number at frequency \(2\omega_c\). In fact, for \(\bar{n} = 0\) one obtains the well-known nonlinear damping of a quantized cavity mode via two-quantum processes used in Cavity Quantum Electrodynamics (see for instance \([23,24,25]\)).

Finally, the master equation characterizing our model reads as follows:

\begin{equation}
\dot{\rho} = -i[\bar{H}, \rho] - \Lambda_{\rho_{sp}} - \Lambda_{\rho_f},
\end{equation}

where \(\bar{H} = 2(\omega_c + \bar{\chi}\bar{n})\beta_z - \bar{\chi}\beta^+\beta^+ + (\Omega_R - 2g_0\beta_z)\sigma_z/2 + g_2(\sigma^-\beta^+ + \beta^-\sigma^+)\), while \(\Lambda_{\rho_{sp}}\) and \(\Lambda_{\rho_f}\) are given by Eq. (3) and Eq. (6), respectively. Note here that the form of the master equation (7) would be similar to the corresponding one describing a wide range of problems involving two-quantum processes as for instance the quantum dynamics of a single two-state particle or a collection of two-state particles possessing dipole-forbidden transitions, pumped with an intense laser field in two-photon resonance and damped at resonance via two-photon effects by an optical cavity containing a two-photon absorber. Similar nonlinear damping, as in Eq. (5), can be applied to a cavity mode crossed by an excited flux of such dipole-forbidden emitters. The laser/maser phenomenon via two photons can be developed here as well. Other applications refer to quantum effects in the present scheme as, for example, the first- and second-order squeezing of the oscillator’s quantum fluctuations. These studies will be presented elsewhere.
3 Results and discussions

A general analytical solution of Eq. (7) is not evident. However, one can obtain its solution for different regimes of interest, namely in the bad or good cavity limit. Therefore, we proceed by investigating the properties of Eq. (7) when the qubit’s quantum dynamics is faster than that of the nonlinear quantum oscillator, i.e. in the good cavity limit. Below the photon saturation number \( n_0 = \left( \frac{\Gamma_\parallel \Gamma_\perp}{2g_0^2} \right)^{1/2} \), with \( \Gamma_\perp = 4\gamma(0) + \Gamma_\parallel \) and \( \Gamma_\parallel = \gamma^{(+)} + \gamma^{(-)} \), one can integrate the qubit’s degrees of freedom to arrive at a master equation characterizing the quantum oscillator only:

\[
\dot{\rho} - i\chi[\beta^+\beta^-, \rho] = -\left( \kappa(1 + \bar{n}) + \Gamma_- \right)[\beta^+\beta^- \rho] - \left( \kappa\bar{n} + \Gamma_+ \right)[\beta^-\beta^+ \rho] + \text{H.c.}
\]  

(8)

Here \( \Gamma_\pm = g_0^2(1 \pm \langle \sigma_z \rangle_0)/(2\Gamma_\perp) \), with \( \langle \sigma_z \rangle_0 = (\gamma^{(-)} - \gamma^{(+)})/\Gamma_\parallel \) being the qubit inversion in the absence of the resonator mode. The two-photon resonance was assumed, i.e. \( \Omega_R - 2g_0\langle \beta_z \rangle = 2(\omega_c + \chi\bar{n}) \), as well as the relation: \( \kappa(1 + \bar{n}) \ll g_2 < \Gamma_0 \).

The steady-state solution for the diagonal elements of Eq. (8) is

\[
\rho_s = Z^{-1} \exp[-\alpha\beta_z],
\]  

(9)

where \( Z \) is determined by the requirement \( \text{Tr}(\rho_s) = 1 \) and \( \alpha = \ln \eta \), with \( \eta = \left( \kappa(1 + \bar{n}) + \Gamma_- \right)/(\kappa\bar{n} + \Gamma_+) \). The expectation values of the operators needed for evaluating the properties of the nonlinear oscillator are obtained from Eq. (5) and Eq. (9). In particular, the nonlinear oscillator mean photon number, i.e. \( \langle n \rangle = 2\langle \beta_z \rangle - 1/2 \), and its second- and fourth-order correlations can be determined from the following expressions:

\[
\langle \beta_z \rangle = j + \frac{1}{\eta - 1},
\]

\[
\langle \beta^+\beta^- \rangle = \frac{2(1 + j(\eta - 1))}{(\eta - 1)^2},
\]

\[
\langle \beta^{+2}\beta^{-2} \rangle = \frac{12(1 + \eta) + 4(\eta - 1)(5 + \eta)j}{(\eta - 1)^4} + \frac{8j^2}{(\eta - 1)^2}.
\]  

(10)

It can be observed here that when \( \eta \) approaches unity, the result is a substantial increase in the photon number and photon correlations. This will lead to lasing instability phenomena so that Eq. (8) and its solution are not valid anymore. However, \( \eta \gg 1 \) corresponds to the cooling of the nonlinear oscillator where
Fig. 1. (color online) The mean photon number of the nonlinear oscillator $\langle n \rangle$ as a function of detuning $\delta \omega$. The solid blue curve stands for $\bar{n}=4$, the long-dashed line for $\bar{n}=2$, while the short-dashed one corresponds to $\bar{n}=1$. The solid green curve shows the saturation photon number $n_0$. Here, $\omega_c/2\pi=27.5\text{MHz}$, $\kappa/2\pi=2\text{kHz}$, $\Delta/2\pi=3\text{GHz}$, $g/2\pi=18\text{MHz}$, $\Gamma_0/2\pi=0.5\text{MHz}$, $\Omega = \sqrt{\Omega^2_R - (\delta \omega)^2}$, and $j = 1/4$.

the application of solution (9) is justified below the photon saturation number $n_0$, because the mean photon number as well as second- and fourth-order photon correlations tend to lower values in this case. Note that the control parameter $\eta$ can be modified by adjusting the qubit’s parameters as well as the detuning of the external driving field.

Fig. (11) depicts the mean photon number in the nonlinear oscillator mode, i.e. $\langle n \rangle$ when $j = 1/4$, as a function of various parameters governing steady-state behaviors. As can be observed here, lower photon numbers can be achieved via a suitable choice of the parameters involved and below the thermal limit. By increasing the coupling coefficient $g$ such that $g_2 \ll \Omega_R$, the cooling efficiency can be further improved. Evidently, the qubit is more in its ground dressed-state, i.e. $\langle \sigma_z \rangle_0 < 0$ ($\delta \omega > 0$), when the cooling occurs. On the other hand, inversion of the qubit population can be created via modifying the detuning $\delta \omega$, that is for $\delta \omega < 0$. Thus, the cooling of the nonlinear oscillator occurs when controlling the qubit’s population quantum dynamics. Although we get lower photon numbers for the nonlinear oscillator mode, it will be not easy, in general, to achieve $\langle n \rangle \approx 0$. Due to approximations used in our approach, we cannot increase the coupling $g$ ($g_2$) further since we have performed the rotating wave approximation in the Hamiltonian (2). The counter-rotating terms have to be taken into account when proceeding to larger $g$ ($g_2$). Neither in this case can the degrees of freedom related to the qubit’s quantum dynamics be adiabatically eliminated because $g_2 \sim \Gamma_0$. Other limiting factors may appear due to fluctuations of external parameters. However, improving the oscillator quality factor one can achieve better cooling in general.

We focus further on the properties of photon coherences. The second-order coherence function, i.e. $g^{(2)}(0) = 4\langle \beta^+ \beta^- \rangle / \langle n \rangle^2$, and the fourth-order one, i.e. $g^{(4)}(0) = \langle \beta^+ \beta^- \rangle^2 / \langle \beta^+ \beta^- \rangle^2$, can be evaluated by using Eq. (5) and Eq. (9).
Fig. 2. (color online) The second-order photon coherence function $g^{(2)}(0)$ as a function of detuning $\delta \omega$ and for $j = 1/4$ (solid line), and $j = 3/4$ (long-dashed curve), respectively. The other parameters are the same as in Fig. (1) with $\bar{n} = 2$.

and represented as follows:

$$g^{(2)}(0) = \frac{32(1 + (\eta - 1)j)}{(5 + 4j(\eta - 1) - \eta)^2},$$
$$g^{(4)}(0) = 2 + \frac{1 + 3\eta + j(\eta^2 - 1)}{(1 + j(\eta - 1))^2}. \quad (11)$$

An interesting result here is that the above correlation functions behave differently for even ($j = 1/4$) or odd ($j = 3/4$) photon numbers. For instance, $g^{(2)}(0) = (3 + \eta)/2$ and $g^{(4)}(0) = 2 + 4(3 + 12\eta + \eta^2)/(3 + \eta)^2$ when $j = 1/4$, while $g^{(2)}(0) = 2(1 + 3\eta)/(1 + \eta)^2$ and $g^{(4)}(0) = 2 + 4(3 + 12\eta + 3\eta^2)/(1 + 3\eta)^2$ when $j = 3/4$. Particularly, for even or odd photon number states, the second-order coherence function $g^{(2)}(0)$ will be linearly or inversely proportional to $\eta$ when $\eta$ increases. Depending on the steady-state behaviors of the photon correlation functions, one can distinguish between the nonlinear oscillator’s odd and even photon number states. Thus, the photon coherence functions are a convenient tool to determine the parity of the photon number of the state $|j, m\rangle$ which corresponds to su(1,1) algebra.

In Fig. (2) we show the dependence of the second-order coherence function $g^{(2)}(0)$ versus the detuning $\delta \omega$ and different values of $j$. These behaviors are explained as follows: for an even number of photons describing the state $|j, m\rangle$, i.e. $j = 1/4$, the mean-photon number $\langle n \rangle$ will be below unity (for $\bar{n} = 2$) and the normalized second-order coherence function increases accordingly, showing super-Poissonian photon statistics ($g^{(2)}(0) > 1$). Conversely, for an odd number of photons, i.e. $j = 3/4$, the mean-photon number $\langle n \rangle$ will be above unity (or near unity) and the second-order coherence function decreases, revealing near Poissonian ($g^{(2)}(0) \approx 1$) or even sub-Poissonian ($g^{(2)}(0) < 1$) photon statistics and a single-photon state can be created here. Note that the
fourth-order coherence function $g^{(4)}(0)$ approximately behaves as $g^{(2)}(0)$, but with a different magnitude.

4 Summary

In summary, we described a scheme capable of cooling an oscillator coupled to an externally pumped artificial atom (a Josephson flux qubit) and damped nonlinearly through interaction with its environmental thermal reservoir. Under certain conditions, the oscillator and the qubit exchange two-photons, allowing us to describe their quantum dynamics using the su(1,1) algebra. If the qubit’s dynamics is faster than that of the nonlinear oscillator, the cooling of the oscillator’s degrees of freedom occurs when controlling the qubit quantum dynamics. Evaluating the second-order photon correlation function (or higher-order correlations), one can distinguish between even and odd photon number states characterizing the oscillator. By adjusting the parameters involved, one can create a nonclassical field state with sub-Poissonian photon statistics. This will allow us to obtain a single-photon state of the nonlinear oscillator.

A Appendix

In this Appendix we obtain the equation (6). We start by indicating the Hamiltonian $H_f$ describing the interaction of the environmental bath with the nonlinear oscillator, i.e. the Eq. (4):

$$H_f = \hbar \nu b^\dagger b + 2\hbar \tilde{\chi}(b^\dagger \beta^- + \beta^+ b).$$

In the Born-Markov approximation one can eliminate the bath operators. For doing this we define an operator $Q_f$ which belongs to the oscillator’s subsystem and satisfy the following equation of motion:

$$\frac{d}{dt} \langle Q_f \rangle = 2i \tilde{\chi}\{b^\dagger [\beta^-, Q_f]] + \langle [Q_f, \beta^+] b \rangle\}. \quad (A.1)$$

The formal solution of the Heisenberg equation for $b^\dagger$ is:
\[ b^\dagger(t) = b^\dagger(0) e^{i(\nu + i\chi)t} - \frac{\beta(t)}{\nu - 2\omega_c + i\chi} \]
\[ = b^\dagger_v(t) - \frac{\beta(t)}{\nu - 2\omega_c + i\chi}, \quad (A.2) \]

with \( b(t) = [b^\dagger(t)]^+ \). Substituting Eq. (A.2) in Eq. (A.1) and using the Bogoluibov lemma [26]

\[ \langle b^\dagger_v(t) U(t) \rangle = -\frac{2\bar{\chi}}{\nu - 2\omega_c + i\chi} \bar{n} \langle [\beta^\dagger(t), U(t)] \rangle, \quad (A.3) \]

where \( U \) is an arbitrary operator belonging to the oscillator subsystem together with the identity \( Tr\{\frac{d}{dt}Q_f(t)\rho_f(0)\} = Tr\{\frac{d}{dt}\rho_f(t)Q_f(0)\} \) one arrives at Eq. (6).

There \( \kappa = \frac{\chi(2\bar{\chi})^2}{(\nu - 2\omega_c)^2 + \chi^2} \) and \( \bar{\chi} = \frac{(\nu - 2\omega_c)(2\bar{\chi})^2}{(\nu - 2\omega_c)^2 + \chi^2} \).

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