Dating individual quasars with the He II proximity effect

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ABSTRACT

Constraints on the time-scales of quasar activity are key to understanding the formation and growth of supermassive black holes (SMBHs), quasar triggering mechanisms, and possible feedback effects on their host galaxies. However, observational estimates of this so-called quasar lifetime are highly uncertain (tQ ≈ 104–109 yr), because most methods are indirect and involve many model-dependent assumptions. Direct evidence of earlier activity is gained from the higher ionization state of the intergalactic medium (IGM) in the quasar environs, observable as enhanced Lyα transmission in the so-called proximity zone. Due to the ~30 Myr equilibration time-scale of He II in the z ~ 3 IGM, the size of the He II proximity zone depends on the time the quasar had been active before our observation ton ≤ tQ, enabling up to ±0.2 dex precise measurements of individual quasar on-times that are comparable to the e-folding time-scale tQ ~ 44 Myr of SMBH growth. Here we present the first statistical sample of 13 quasars whose accurate and precise systemic redshifts allow for measurements of sufficiently precise He II quasar proximity zone sizes between ≈ 2 and ≈ 15 proper Mpc from science-grade Hubble Space Telescope (HST) spectra. Comparing these sizes to predictions from cosmological hydrodynamical simulations post-processed with one-dimensional radiative transfer, we infer a broad range of quasar on-times from ton ≤ 1 Myr to ton > 30 Myr that does not depend on quasar luminosity, black hole mass, or Eddington ratio. These results point to episodic quasar activity over a long duty cycle, but do not rule out substantial SMBH growth during phases of radiative inefficiency or obscuration.

Key words: intergalactic medium – quasars: absorption lines – quasars: general – quasars: supermassive black holes – dark ages, reionization, first stars

1 INTRODUCTION

Quasars are the most powerful sources of radiation that have emitted at an almost sustained high luminosity during the short ≤ 60 yr time-frame accessible to modern astronomical observations (Schmidt 1963). Most likely they are powered by accretion of baryons onto SMBHs (e.g. Salpeter 1964; Lynden-Bell 1969; Rees 1984), and it is believed that past quasar phases are required to explain the MBH = 10⁸–10¹⁰ M☉ SMBHs found in the centres of nearby quiescent bulge-dominated galaxies (Soltan 1982; Kormendy & Richstone 1995; Yu & Tremaine 2002; Kormendy & Ho 2013). In numerical models of galaxy and black hole co-evolution, SMBH growth is triggered by gas inflow from major galaxy mergers (e.g. Di Matteo et al. 2005; Springel et al. 2005; Hopkins et al. 2005a,b, 2006, 2008; Capelo et al. 2015; Steinborn et al. 2018) and/or secular disc instabilities (e.g. Hopkins & Quataert 2010; Novak et al. 2011; Bournaud et al. 2011; Gabor & Bournaud 2013; Hopkins et al. 2016; Anglés-Alcázar et al. 2013, 2017, 2020), but the physical processes on the relevant scales (sub-pc to a few pc) are still not fully understood. In both scenarios, kinetic and thermal feedback from stars and the SMBH self-regulate SMBH growth and obscuration. Once enough gas has been expelled, the SMBH shines as a short-lived UV-bright quasar until feedback quenches SMBH growth, and potentially also the star formation in the host galaxy (e.g. Sanders et al. 1988; Di Matteo et al. 2005; Springel et al. 2005; Hopkins et al. 2005b, 2006).

Although these models successfully reproduce many observed properties of galaxies and quasars, it remains challenging for them to explain the prevalence of MBH = 10⁸–10¹⁰ M☉ SMBHs in z < 6 quasars, i.e. only < 10⁴ yr after the Big Bang (Jiang et al. 2007; Kurk et al. 2007; Mortlock et al. 2011; Venemans et al. 2013; De Rosa et al. 2014; Wu et al. 2015; Mazzucchelli et al. 2017; Bañados et al. 2018; Shen et al. 2019; Wang et al. 2020; Yang et al. 2020). These early SMBHs require either quasi-continuous Eddington-limited accretion onto massive black hole seeds (Sijacki et al. 2009; Di Matteo et al. 2012; Johnson et al. 2013), super-Eddington accretion (Volonteri &
Constraining the characteristic time-scales governing quasar activity is key to understanding the existence of early SMBHs, quasar triggering mechanisms, and whether feedback from SMBH growth might quench black hole fuelling and star formation. There is a growing consensus from models of galaxy and black hole co-evolution that fuelling, feedback, and quenching are intimately intertwined, which conspire to produce episodic quasar activity on a wide range of time-scales ($10^3$–$10^8$ yr, Ciotti & Ostriker 2001; Di Matteo et al. 2005; Hopkins et al. 2005b, 2006; Hopkins & Hernquist 2009; Hopkins et al. 2016; Novak et al. 2011; Gabor & Bournaud 2013; Steinborn et al. 2018; Anglés-Alcázar et al. 2017, 2020), often with significant variability in the accretion rate down to the time resolution limit of the simulation (10–100 yr, e.g. Novak et al. 2011; Anglés-Alcázar et al. 2020). Such short-term changes in the accretion rate may explain why some quasars show strong variability in their luminosity and/or their emission lines on time-scales of days to decades (LaMassa et al. 2015; Runnoe et al. 2016; MacLeod et al. 2016; McElroy et al. 2016; Yang et al. 2018). However, for longer time-scales the constraints from observations are uncertain by several orders of magnitude (e.g. Martini 2004), because the methods (i) are necessarily more indirect, (ii) are sensitive to particular time-scales, (iii) often yield a population average, and (iv) involve many model-dependent assumptions.

Comparisons of the quasar number density to their host dark matter halo abundance inferred from quasar clustering studies constrain the quasar duty cycle $t_{dc}$, i.e. the total time over the age of the Universe that a galaxy hosts a quasar (Haiman & Hui 2001; Martini & Weinberg 2001). Due to varying assumptions on how quasars populate dark matter haloes applied to partially discrepant quasar clustering measurements at $z_{em} \sim 2$–4, the inferred quasar duty cycle spans a wide range $10^6$ yr $\lesssim t_{dc} \lesssim 10^9$ yr, and may depend on redshift and/or luminosity (Porciani et al. 2004; Croom et al. 2005; Adelberger & Steidel 2005; Shen et al. 2007; White et al. 2008, 2012; Efekekarzadeh et al. 2015). Alternatively, the quasar duty cycle can be estimated by extending the Solomon (1982) argument such that the quasar luminosity function traces the gas accretion history onto SMBHs in present-day early-type galaxies. For present-day $M_{BH} > 10^9 M_{\odot}$ SMBHs that shine as quasars at their Eddington limit the inferred duty cycle is $t_{dc} = (6-30) \times 10^7$ yr (Yu & Tremaine 2002; Marconi et al. 2004; Shankar et al. 2004). The quasar duty cycle is a population average that is insensitive to the duration of individual quasar episodes.

The time distribution of high-accretion events, often called the episodic quasar lifetime $t_Q$, can be predicted by current numerical models or observationally estimated based on light travel time arguments. It has been suggested that mismatches between the level of nuclear activity and the ionization conditions of gas in and around the host galaxies imply significant nuclear variability on time-scales $t_Q \sim 0.1$ Myr (Schawinski et al. 2015; Sartori et al. 2016; Schirmer et al. 2016; Keel et al. 2017; Oppenheimer et al. 2018). However, such short quasar lifetimes cannot explain the existence of giant ($\sim 400$ kpc) Ly$\alpha$ nebulae around $z_{em} \sim 2$–3 quasars (Cantalupo et al. 2014; Hennawi et al. 2015; Cai et al. 2017; Arrigoni Battaiia et al. 2018), which require sustained activity for a few Myr. Moreover, the measured equivalent widths of Ly$\alpha$ emitters, enhanced by quasar-powered fluorescence, suggest quasar lifetimes of 1–40 Myr depending on the emitter sample and the quasar opening angle (Adelberger et al. 2006; Cantalupo et al. 2012; Trainor & Steidel 2013; Borisowa et al. 2016; Marino et al. 2018). However, due to geometric dilution of the quasar flux, luminous Ly$\alpha$ emitters at distances of several Mpc are more likely to be powered intrinsically (Khrykin et al. 2016).

The quasar lifetime can also be constrained from the locally enhanced UV radiation field in the quasar vicinity, the so-called proximity effect, which manifests itself as a region of enhanced IGM Ly$\alpha$ transmission (e.g. Bajtlik et al. 1988; Scott et al. 2000; Dall’Aglio et al. 2008; Calverley et al. 2011). Because the IGM reacts to a change in the photoionization rate $\Gamma$ within a finite equilibration time-scale $t_{eq} \approx \Gamma^{-1}$, the existence of the proximity effect implies that quasars had been emitting continuously for $t_Q \gtrsim t_{eq}$ (e.g. Bajtlik et al. 1988).

With an $\text{H} \text{I}$ UV background photoionization rate $\Gamma_{\text{H} \text{I}} \approx 10^{-12}$ s$^{-1}$ measured in the $2 \lesssim z \lesssim 5$ $\text{H} \text{I}$ Ly$\alpha$ forest (Bekker & Bolton 2013) one obtains a weak lower limit $t_{Q} \gtrsim 0.03$ Myr for the quasar population.

During and shortly after $\text{H} \text{I}$ reionization at $z \gtrsim 5.7$, the low IGM $\text{H} \text{I}$ Ly$\alpha$ transmission enables measurements of well-defined sizes of $\text{H} \text{I}$ proximity zones around individual quasars (Fan et al. 2006; Carilli et al. 2010; Eilers et al. 2017). The observation of any particular quasar is only sensitive to the time the quasar had been active prior to our observation at a random point during its current luminous episode, henceforth called the on-time $t_{on} \lesssim t_Q$. From their very small proximity zones given their luminosity, Eilers et al. (2020) concluded that 5–10 per cent of all $z_{em} \sim 6$ quasars had recently turned on ($t_{on} \lesssim 0.1$ Myr), which is also supported by a lack of extended $\text{Ly} \alpha$ emission around them (Farina et al. 2019). However, because $\text{H} \text{I}$ quickly equilibrates after quasar turn-on, $z_{em} \sim 6$ $\text{H} \text{I}$ proximity zones are insensitive to turn-on times $t_{on} > 0.1$ Myr unless the IGM was significantly neutral (Keating et al. 2015; Eilers et al. 2017, 2018; Davies et al. 2020).

Stronger limits on the quasar lifetime may be inferred from the transverse proximity effect of a foreground quasar in a background line of sight via the additional light travel time. This effect has not been unambiguously detected in the $z \sim 2$–3 $\text{H} \text{I}$ Ly$\alpha$ forest due to overdense environments around quasar hosts, quasar obscuration, and the small quasar boost to the overall $\text{H} \text{I}$ photoionization rate (e.g. Liske & Williger 2001; Croft 2004; Hennawi & Prochaska 2007; Kirkman & Tytler 2008; Prochaska et al. 2013, but see Gonçalves et al. 2008). The low UV background in the post-reionization IGM increases the chance to discover the transverse proximity effect in the $\text{H} \text{I}$ ($t_{on} > 11$ Myr, Gallerani et al. 2008) and the $\text{He} \text{II}$ forest ($t_{on} > 10$–25 Myr, Jakobsen et al. 2003; Worseck & Wisotzki 2006; Worseck et al. 2007; Schmidt et al. 2017), but frequent non-detections can either be explained by a young age ($t_{on} < 10$ Myr) or obscuration (Schmidt et al. 2018).

Direct estimates of prolonged quasar activity can be inferred from the line-of-sight $\text{He} \text{II}$ proximity zones of $z_{em} \sim 3$–4 quasars at the tail end of the $\text{He} \text{II}$ reionization epoch, thanks to the long $\text{He} \text{II}$ equilibration time-scale $t_{eq} \approx 10$ Myr (Khrykin et al. 2016). This is comparable to the $\epsilon$-folding time-scale of SMBH growth $t_{\ast} \approx 44$ Myr (Salpeter 1964), and may offer unique constraints on the range of episodic quasar lifetimes in models of galaxy and black hole co-evolution. In Khrykin et al. (2019, hereafter Paper I) we introduced a new statistical Bayesian method to infer on-times of individual quasars from their $\text{He} \text{II}$ proximity zones, accounting for the degeneracy between the initial ambient IGM $\text{He} \text{II}$ fraction and the quasar on-time that had affected previous analyses (Syphers & Shull 2014; Zheng et al. 2015). Applying our method to six $\text{He} \text{II}$-transparent quasars$^2$ at $z_{em} > 3.6$ we inferred 0.3 dex precise on-

$^2$ $\text{He} \text{II}$-transparent quasars are rare quasars with sufficient flux at $\text{He} \text{II}$ $\text{Ly} \alpha$ to secure a science-grade (signal-to-noise ratio $\gtrsim 3$) spectrum with HST (e.g. Worseck & Prochaska 2011; Syphers et al. 2012; Worseck et al. 2019).
times for two quasars ($\tau_\text{on} \approx 0.6$ and 5.8 Myr, respectively), while the remaining quasars allowed just for a joint constraint of a short $\sim 1$ Myr on-time due to uncertainties in the quasar systemic redshifts.

Here we build on results from Paper I, and apply a similar statistical algorithm to the sample of seventeen $2.74 < z_{\text{em}} < 3.51$ \HeII-transparent quasars, twelve of which have accurate and precise systemic redshifts. In Section 2 we describe our observations and the relevant parameters of our quasar sample. We present measurements of the \HeII proximity zone sizes in Section 3. In Section 4 we summarize our numerical model, before reporting on the inferred individual quasar on-times and their relation to quasar properties in Section 5. We discuss our results and remaining uncertainties in Section 6, before concluding in Section 7.

We assume a flat ΛCDM cosmology with dimensionless Hubble constant $h = 0.7$ ($H_0 = 100h\, \text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$), density parameters ($\Omega_m, \Omega_b, \Omega_\Lambda = (0.27, 0.046, 0.73)$ for total matter, baryons, and cosmological constant, a linear dark matter power spectrum amplitude on a scale of $8h^{-1}\,\text{Mpc}$, $c_8 = 0.8$, a spectral index of density perturbations $n_s = 0.96$, and a helium mass fraction $Y = 0.24$, consistent with Planck Collaboration et al. (2020). Proper distances are quoted explicitly in proper Mpc (pMpc).

## 2 OUR SPECTROSCOPIC DATA SET ON \HEII PROXIMITY ZONES

### 2.1 HST/COS spectra of \HeII proximity zones

We use the HST UV spectra of seventeen out of twenty $z_{\text{em}} < 3.6$ \HeII-transparent quasars from Worseck et al. (2019), to which we refer for a detailed description of the data reduction. Three $z_{\text{em}} \approx 3$ quasars (HS 1157+3143, SDSS J0924+4852, SDSS J1101+1053) from Worseck et al. (2019) were excluded due to geocoronal \HI Ly$\alpha$ contamination of the \HeII quasar proximity zone. For two $z_{\text{em}} \approx 2.94$ quasars (SDSS J0818+4908, SDSS J0936+2927) we used only data taken in HST’s orbital shadow to exclude contamination of their \HeII proximity zones by geocoronal N\text{I} 12100 Å emission, as discussed in Worseck et al. (2019). Likewise, geocoronal O\text{I} was excluded for four $z_{\text{em}} \approx 3.28$ quasars (HE2QS J1706+5904, HE2QS J2149–0859, Q 0302–003, HE2QS J0233–0149). Although the sightline to HE2QS J1706+5904 is not suitable for studying intergalactic \HeII due to an optically thick \HI Lyman limit system at $z = 0.4040$ (Worseck et al. 2019), its \HeII proximity zone is not impacted. Table 1 lists the relevant properties of our sample.

All 17 HST spectra were taken with the Cosmic Origins Spectrograph (COS; Green et al. 2012), employing the G140L grating (12 spectra) or the G130M grating (5 spectra). Their resolving power $R = \lambda/A\lambda$ varies with wavelength and with the spatial position on the detector. Table 1 lists the appropriate values in the spectral range of interest. The spectra were rebinned to 2–3 pixels per resolution element ($R < 3000: \approx 0.24\, \text{Å}\,\text{pixel}^{-1}, 10000 \leq R \leq 15000: \approx 0.04\, \text{Å}\,\text{pixel}^{-1}, R > 15000: \approx 0.03\, \text{Å}\,\text{pixel}^{-1}$), yielding a signal-to-noise ratio $S/N=3$–19 in the quasar continuum immediately redward of the \HeII quasar proximity zone.

After correction for Galactic extinction, the spectra were normalized by power laws $f_\lambda \propto \lambda^{\beta}$, fitted to spectral regions without strong emission and absorption features, but accounting for unidentified partial \HI Lyman limit breaks (Worseck et al. 2019). Since our sample lacks contemporaneous and continuous spectral coverage from the rest-frame extreme UV to the near UV, a detailed correction for cumulative \HI Lyman continuum attenuation is not possible. As such, the fitted power laws with a typical range in slope $-3 \leq \beta \leq -0.5$ do not represent the intrinsic quasar spectral energy distributions (SEDs). The typical continuum error of a few per cent does not affect our analysis. We do not account for quasar \HeII Ly$\alpha$ and metal emission that is difficult to predict in detail in quasar accretion disc models (e.g. Syphers et al. 2011a; Syphers & Shull 2013). Only two quasars (SDSS J0936+2927 and SDSS J2346–0016) show such features, and they are sufficiently weak that they do not change our results. Likewise, contamination of the proximity zone by low-redshift \HI Lyman series lines is expected to be weak compared to the \HeII Ly$\alpha$ absorption.

Together with the seven $z_{\text{em}} > 3.6$ quasars discussed in Paper I, we arrive at a total sample of 24 quasars with \HeII proximity zones (Table 1).
Table 1. Our combined sample of 24 quasars with He$^+_n$ proximity zones, comprising new measurements for 17 $z_{\text{em}} > 3.6$ quasars and results for seven $z_{\text{em}} > 3.6$ quasars from Paper I. We list the name, position, HST/COS resolving power and signal-to-noise ratio near He$^+_n$ Ly$\alpha$ in the quasar rest-frame, quasar redshift, velocity (redshift) uncertainty, emission line and instrument for redshift measurement, SDSS or Pan-STARRS1 magnitude corrected for Galactic extinction, absolute magnitude at 1450 Å rest-frame, He$^+$-ionizing photon production rate $Q_{\text{He}}$, measured proximity zone size $R_{\text{p}}$, and inferred quasar on-time $t_{\text{on}}$ for an assumed uniform prior on the He$^+_n$ fraction $0.1 \leq X_{\text{He}} \leq 1$ (Section 5.2).

| Quasar        | R.A.          | Decl.         | $R$   | S/N$^a$ | $z_{\text{em}}$ | $\sigma_v$ | Instrument | $m_{\text{I}}$ | $m_{\text{I}}$ | $M_{1450}$ | $\log(\frac{Q_{\text{He}}}{\text{pM}})$ | $t_{\text{on}}$ |
|--------------|---------------|---------------|-------|---------|-----------------|------------|-------------|--------------|-------------|----------|----------------------------------------|--------------|
| HE2347−4342  | 23:46:23.211  | +33:25:57.6   | 17000 | 19      | 2.8852         | 44.4       | [O iv] FIRE | 18.67        | 26.83       | 56.34    | −0.63 ± 0.04                               | < 0.46       |
| HE2Q5 2149+0859 | 21:49:27.177  | +08:59:35.2   | 1700  | 3       | 3.2538         | 656        | CAFOS       | 18.86        | 26.66       | 56.27    | −0.04 ± 2.03                               | < 0.66       |
| HE2Q5 1706+5904 | 17:06:07.154  | +59:04:06.7   | 1700  | 3       | 3.2518         | 656        | CAFOS       | 18.86        | 26.66       | 56.27    | −0.04 ± 2.03                               | < 0.66       |
| SDSS J2336+0126 | 23:36:05.265  | +01:06:10.0   | 2400  | 3        | 2.9368         | 273        | TripleSpec  | 18.78        | 26.66       | 56.27    | 0.77 ± 0.87                               | < 1.01       |
| SDSS J2346−0016 | 23:46:41.103  | −00:16:07.2   | 2600  | 8        | 3.5076         | 273        | TripleSpec  | 17.68        | 27.97       | 56.79    | 0.66 ± 0.17                               | < 0.31       |
| J0818+4008     | 08:18:11.978  | +40:00:45.7   | 2200  | 2        | 2.9258         | 656        | CIV BOSS    | 18.36        | 26.93       | 56.38    | 2.22 ± 0.25                               | < 2.42       |
| HE2Q5 2109+2408 | 21:09:20.955  | +24:00:07.6   | 1800  | 4        | 3.4231         | 656        | CAFOS       | 18.52        | 27.12       | 56.45    | 1.34 ± 0.19                               | < 2.77       |
| HS 1149+4809    | 11:49:10.101  | +48:56:57.3   | 10000 | 6        | 3.3500         | 400        | H$^+$ LUCI   | 17.77        | 27.84       | 56.74    | 4.21 ± 1.19                               | < 1.01       |
| HE2Q5 2033+0114 | 20:33:06.001  | +01:49:50.7   | 17000 | 7        | 3.3115         | 656        | CAFOS       | 18.41        | 27.17       | 56.47    | 4.71 ± 1.98                               | < 6.24       |
| Q 1602+576     | 16:02:53.559  | +57:30:54.2   | 15000 | 7        | 2.8608         | 273        | TripleSpec  | 17.22        | 27.99       | 56.80    | 6.10 ± 0.97                               | 1.98±1.61     |
| PC 0058+0215    | 00:58:00.539  | +02:31:31.4   | 2200  | 2        | 2.8842         | 273        | TripleSpec  | 18.77        | 26.46       | 56.19    | 7.10 ± 0.97                               | < 7.24       |
| HS 1700+6416    | 17:00:10.061  | +64:12:09.1   | 2100  | 15       | 2.7472         | 273        | TripleSpec  | 15.79        | 29.33       | 57.34    | 7.10 ± 1.01                               | 0.80±0.50     |
| SDSS J0936+2927 | 09:36:34.358  | +29:27:13.7   | 2200  | 2        | 2.9248         | 44.4       | [O III]       | 18.06        | 27.20       | 56.49    | 8.59 ± 0.15                               | 1.62±0.23     |
| HE2Q5 1024+1849 | 10:24:37.313  | +18:34:27.5   | 15000 | 5        | 3.2852         | 273        | Mg II LUCI   | 17.66        | 27.54       | 56.62    | 9.38 ± 0.97                               | 5.93±0.23     |
| SDSS J1253+6817 | 12:53:53.571  | +68:17:14.2   | 2600  | 7        | 3.7453         | 44.4       | [O III]       | 18.45        | 27.19       | 56.48    | 11.40 ± 0.12                               | < 23.55      |
| Q 0032+00       | 00:32:49.485  | +00:08:13.7   | 19000 | 3        | 3.2850         | 44.4       | [O III]       | 17.34        | 28.21       | 56.89    | 13.20 ± 0.13                               | < 11.36      |
| HE2Q5 2157+2330 | 21:57:43.367  | +23:37:33.3   | 2200  | 2        | 3.2854         | 44.4       | [O III]       | 17.67        | 27.77       | 56.72    | 17.40 ± 0.14                               | < 31.84      |

$^a$Signal-to-noise per pixel near He$^+_n$ Ly$\alpha$ ($R < 3000$: $0.24 \text{ pixel}^{-1}$, $10000 < R < 15000$: $0.04 \text{ pixel}^{-1}$, $R > 15000$: $0.03 \text{ pixel}^{-1}$).

$^b$For HE 2347−4342 we obtained the observed AB magnitude $m_{\text{AB}} = 16.83$ with our VLT/ForS2 spectrum calibrated to $R_{\text{p}} = 17.18$ (Worseck et al. 2008).

$^c$Quasar $z_{\text{em}} > 3.6$ quasars reported in Paper I. The inferred $t_{\text{on}}$ is based on the extended grid of radiative transfer models (Section 4.1) and our updated definition of upper and lower limits (Section 5.2).
for their average blueshifts, and precisions were assigned from the standard deviations $\sigma_\nu$ of the velocity distributions in large quasar samples (Richards et al. 2002; Boroson 2005; Richards et al. 2011; Shen et al. 2016). The redshift precision was our primary criterion for the eventually adopted emission line and redshift in Table 1:

(i) We preferred redshifts from detected narrow [O iii] 4507 emission (S/N$>10$ across the line, peak to continuum ratio $>0.3$, 5 quasars in Table 1). We assumed a luminosity-independent blueshift of $-27.1$ km s$^{-1}$ w.r.t. the systemic frame defined by low-ionization forbidden lines ([O ii], [N ii], [S ii]), and a velocity precision $\sigma_\nu = 44.4$ km s$^{-1}$, both based on the high-confidence sample in Boroson (2005). The values are in good agreement with Hewett & Wild (2010; Shen et al. 2016).

(ii) If [O iii] was not covered or not detected due to low S/N or the Baldwin effect (e.g. Baldwin 1977; Stern & Laor 2012; Shen & Ho 2014; Shen 2016), we took broad Mg ii if the line was detected at S/N$>10$ (6 quasars). We assumed the same luminosity-independent blueshift of $-109$ km s$^{-1}$ as for [O iii], and a velocity precision $\sigma_\nu = 273$ km s$^{-1}$ derived by Gaussian error propagation of the [O ii] velocity precision (44.4 km s$^{-1}$) and the Mg ii velocity dispersion w.r.t. [O iii] (269 km s$^{-1}$) from Richards et al. (2002). These values are consistent with more recent determinations (Hewett & Wild 2010; Shen 2016; Shen et al. 2016).

(iii) If neither [O iii] nor Mg ii was usable, but broad H$\beta$ had been covered in our $K$ band spectra, we took the H$\beta$ line, which reasonably traces the systemic frame. We adopted a blueshift of $-109$ km s$^{-1}$ and a velocity precision $\sigma_\nu = 400$ km s$^{-1}$ (Shen et al. 2016). Three quasars in our sample have H$\beta$ redshifts.

(iv) For the 10 quasars lacking near-infrared spectra of sufficient quality we measured the redshift from C iv covered in the optical spectra. We corrected for the known correlation of the C iv blueshift with continuum luminosity (Hewett & Wild 2010; Richards et al. 2011; Shen et al. 2016) using a sample of lower-redshift BOSS quasar spectra covering C iv and Mg ii (see Paper I for details). Our determined blueshift w.r.t. Mg ii

$$\Delta v = \left[-192.4 - 599.6\log \left(\frac{1450\,\text{Å} \times L_\lambda_{\text{rest}}}{10^{45}\,\text{erg s}^{-1}}\right)\right] \text{km s}^{-1} \quad (1)$$

is in reasonable agreement with the independent determination by Shen et al. (2016) for a distinct sample and line-centring algorithm. Considering the large standard deviation $\sigma_\nu = 656$ km s$^{-1}$ of the corrected C iv redshifts w.r.t. the Mg ii redshifts, we ignored the smaller spread of the Mg ii redshifts about the systemic frame (Paper I).

For all five He ii-transparent quasars with previous systemic redshift determinations (Mg ii, H$\beta$, H$\gamma$ and/or [O iii]) from the literature our results are broadly consistent, in spite of differences in the employed methods (Table 2). The main sources of discrepancy are the treatment of line asymmetries and the averaging of results for multiple emission lines.

### 2.4 Absolute magnitudes and photon production rates

Optical photometry of the quasars was mainly obtained from SDSS Data Release 12 (Alam et al. 2015), with the exception of HE2QS J2311–1417 that has been covered in Data Release 1 of the Panoramic Survey Telescope and Rapid Response System 1 (Pan-STARRS1; Chambers et al. 2016; Flewelling et al. 2020), and HE 2347–4342 that has been taken from Worseck et al. (2008). For all quasars except HE 2347–4342 we determined the absolute

| Quasar | $z_{\text{em}}$ | $z_{\text{other}}$ | Reference |
|--------|----------------|-------------------|-----------|
| HS 1700+6616 | 2.7412 ± 0.0034 | 2.7510 ± 0.0030 | Trauner & Steidel (2012) |
| HE 2347–4342 | 2.8852 ± 0.0006 | 2.886 ± 0.001 | Syphers et al. (2010) |
| Q 0302–003 | 3.2850 ± 0.0006 | 3.2860 ± 0.0005 | Syphers & Shull (2014) |
| SDSS J1253+6817 | 3.4753 ± 0.0007 | 3.4829 | Coatman et al. (2017) |
| SDSS J2346–0016 | 3.5076 ± 0.0004 | 3.510 ± 0.0030 | Zheng et al. (2015) |

AB magnitude at rest-frame wavelength 1450 Å from the extinction-corrected i band point-spread-function AB magnitude $m_i$ as

$$M_{1450} (\zeta_{\text{em}}) = m_i - 5 \log \left(\frac{d_L (\zeta_{\text{em}})}{\text{Mpc}}\right) - 25 - K (\zeta_{\text{em}}), \quad (2)$$

with the luminosity distance $d_L$ in our adopted cosmology, and the bandpass correction $K$ that was obtained by scaling the Lusso et al. (2015) quasar UV SED to the $i$ band flux (Kulkarni et al. 2019). The latter assumption was necessary due to inaccurate relative fluxing of many of our HE2QS discovery spectra, and we estimate an error of 0.1 mag for $M_{1450}$. We used the Lusso et al. (2015) SED to estimate the quasar flux density at the H i Lyman limit $f_{V,912}$. Assuming a power-law SED $f_\nu \propto \nu^{-\alpha_V}$ at frequencies $\nu > \nu_{912} = 3.287 \times 10^{13}$ Hz, the total He ii-ionizing photon production rate is

$$Q = \frac{4\pi d_L^2}{(1 + \zeta_{\text{em}})} \int_{\nu_{228}}^{\infty} f_\nu d\nu = \frac{4\pi d_L^2}{(1 + \zeta_{\text{em}})} \int_{\nu_{228}}^{\infty} f_\nu d\nu, \quad (3)$$

with Planck’s constant $h\nu$ and the He ii Lyman limit frequency $\nu_{228} = 4 \times 10^{12}$ Hz.

Due to cumulative IGM absorption by H i Lyman limit systems (e.g. Worseck & Prochaska 2011) and the He ii Lyman series, the He ii-ionizing power has to be inferred by extrapolation of the SED. As in Paper I we assumed a power-law SED slope $\alpha_V = 1.5$ at $\nu > \nu_{912}$, consistent with recent measurements in stacked and composite quasar spectra (Shull et al. 2012; Stevans et al. 2014; Lusso et al. 2015). Since the actual value of $\alpha_V$ depends on the chosen continuum windows, the total spectral coverage, and the exclusion of weak quasar emission lines (Stevans et al. 2014; Tilton et al. 2016), the large range in $\alpha_V$ values from the literature is not surprising. SED reconstructions for two He ii-transmitting quasars with complete spectral coverage indicate quasar-to-quasar variations in $\alpha_V$ around our chosen value (Syphers & Shull 2013, 2014). A very hard SED ($\alpha_V = 0.7$; Tilton et al. 2016) increases $Q$ by a factor 6.5, but may violate constraints on the He ii ionization history (Khair 2017; Kulkarni et al. 2019). Very soft SEDs ($\alpha_V \geq 2$; Lusso et al. 2018) are ruled out by the HST/COS spectra, and would lead to a modest $\sim 0.4$ dex increase in the inferred quasar on-times (Paper I). Changes in the slope at $\nu > \nu_{228}$ do not significantly change our results (Khrykin et al. 2016, 2019).

Equation (3) assumes that the escape fraction of He ii-ionizing photons is unity. Recently, Shull & Danforth (2020) have suggested that many quasars could have escape fractions $0.5$–$0.9$ based on the observed range in the IGM He ii/H i column density ratio. However, these observations are likely explained by radiative transfer in the IGM and the contribution of star-forming galaxies to the H i-ionizing background (e.g. Haardt & Madau 2012; Khair & Srianand 2019; Puchwein et al. 2019; Faucher-Giguère 2020). In these models the required sample-averaged H i Lyman continuum escape fraction of galaxies is $\sim 1$ per cent at $z \sim 3$, which is within recent observational

![Table 2. He ii-transparent quasars with measured systemic redshifts $\zeta_{\text{em}}$ from this work, which have previous redshift determinations $z_{\text{other}}$ (Mg ii, H$\beta$, H$\gamma$, [O iii] or combinations thereof) from the literature.](image-url)
constraints (Grazian et al. 2017; Steidel et al. 2018; Fletcher et al. 2019). Therefore, the He II/H I column density ratio does not uniquely constrain the He II ionizing escape fraction of unobscured quasars, such that our assumption of a unity escape fraction is justified.

2.5 Black hole masses and Eddington ratios

For the subset of 14 quasars with coverage of Mg II, Black hole masses and Eddington ratios were estimated from scaling relations

\[ \log \left( \frac{M_{\text{BH}}}{M_\odot} \right) = a + b \log \left( \frac{\lambda_e L_{\lambda_e}}{10^{44} \text{erg s}^{-1}} \right) + 2 \log \left( \frac{\text{FWHM}}{\text{km s}^{-1}} \right), \]  

(4)

with \((\lambda_e, a, b) = (3000 \text{Å}, 0.86, 0.5)\) for Mg II (Vestergaard & Osmer 2009) and \((\lambda_e, a, b) = (5100 \text{Å}, 0.91, 0.5)\) for Hβ (Vestergaard & Peterson 2006).

Due to the varying spectral coverage and quality, measurements of the full width at half maximum (FWHM) of Mg II (Hβ) were made for 10 (6) quasars. We adopt the Hβ measurements when available. Table 3 lists the measurements and the derived quantities. The spectra were iteratively fit with a combination of a local power-law continuum, the Tsuzuki et al. (2006) Fe II emission template, and Gaussian emission line profiles. Due to the modest S/N, a single Gaussian was considered sufficient for most Mg II lines. Each Hβ emission line was decomposed into one narrow and two broad Gaussians, and the nearby [O III] doublet was considered simultaneously with two Gaussians for each line of the doublet. Bayesian joint posterior distributions of the degenerate line parameters were estimated using the Goodman & Weare (2010) affine-invariant ensemble sampler for Markov Chain Monte Carlo (MCMC) as implemented in emcee (Foreman-Mackey et al. 2013). From the Mg II and Hβ broad line parameter posterior distributions we computed the posterior probability density functions (PDFs) of the total FWHM, and adopt their median values and their equal-tailed 68 per cent credible intervals as our measurements and statistical uncertainties, respectively. Due to the inaccurate fluxing of the near-infrared spectra, the continuum luminosities at \(\lambda_e\) were estimated from \(M_{1450}\) by extrapolating the Lusso et al. (2015) power-law continuum \(f_\nu \propto \nu^{-0.61}\) obtained between 912 Å and 2500 Å, and assuming an uncertainty of 0.2 dex.

The propagated statistical uncertainties in \(M_{\text{BH}}\) are much smaller than the \(\approx 0.55\) dex uncertainty in the scaling relations, as estimated from the scatter of quasars with black hole masses from reverberation mapping (Vestergaard & Peterson 2006; Vestergaard & Osmer 2009; Shen 2013). Other scaling relations yield similar values to within 0.1–0.4 dex (McLure & Dunlop 2004; Ho & Kim 2015; Mejía-Restrepo et al. 2016; Woo et al. 2018; Bahk et al. 2019). We note that using the Tsuzuki et al. (2006) Fe II template instead of the Vestergaard & Wilkes (2001) template used to derive the Vestergaard & Osmer (2009) scaling relation may underestimate the black hole mass by \(\approx 0.2\) dex (Schindler et al. 2020). For the three quasars with Mg II and Hβ FWHM measurements we obtained comparable virial black hole masses.

Table 3. Estimated black hole masses \(M_{\text{BH}}\) and Eddington ratios \(L_{\text{bol}}/L_{\text{Edd}}\) for the 14 quasars with measured total FWHM of Mg II or Hβ decomposed into \(n_G\) Gaussian profiles. All quoted errors are 1σ statistical uncertainties.

| Quasar       | Line | \(n_G\) | FWHM km s\(^{-1}\) | \(L_{\text{bol}}/L_{\text{Edd}}\) | \(M_{\text{BH}}/M_\odot\) | \(L_{\text{bol}}/L_{\text{Edd}}\) |
|--------------|------|---------|-------------------|-----------------------|------------------|-----------------|
| HE 2347+3432 | Mg II | 2       | 4604 ± 19         | 9.44 ± 0.10           | 0.54 ± 0.15      |
| HS 1700+6416 | Mg II | 2       | 3837 ± 91         | 9.81 ± 0.10           | 0.05 ± 0.15      |
| HS 1024+1849 | Mg II | 1       | 5609 ± 415        | 9.79 ± 0.12           | 0.59 ± 0.16      |
| Q 1602+576   | Mg II | 1       | 2285 ± 91         | 9.09 ± 0.11           | 0.28 ± 0.15      |
| PC0588+0215  | Mg II | 1       | 3930 ± 320        | 9.26 ± 0.12           | 0.44 ± 0.16      |
| SDSS 1003+2927 | Mg II | 1 | 2650 ± 92         | 9.06 ± 0.10           | 0.02 ± 0.15      |
| SDSS 1123+0126 | Mg II | 1 | 2192 ± 274       | 7.89 ± 0.15           | 0.10 ± 0.18      |
| SDSS 1246+0016 | Hβ | 1 | 3474 ± 137       | 9.45 ± 0.11           | 0.09 ± 0.15      |
| Q 0002-003   | Hβ   | 2       | 3017 ± 320        | 9.38 ± 0.14           | 0.07 ± 0.17      |
| HE0251+1730  | Hβ   | 2       | 3659 ± 324        | 9.46 ± 0.13           | 0.17 ± 0.17      |
| HS 0911+4809 | Hβ   | 2       | 2872 ± 1002       | 9.27 ± 0.32           | 0.05 ± 0.34      |
| SDSS 1125+6817 | Hβ | 1 | 4845 ± 1006      | 9.59 ± 0.21           | 0.51 ± 0.23      |
| HE0251+1730  | Hβ   | 2       | 4128 ± 230        | 9.69 ± 0.11           | 0.18 ± 0.16      |
| SDSS J1119+5202 | Hβ | 2 | 5462 ± 867      | 9.85 ± 0.17           | 0.47 ± 0.20      |

is (Shapiro 2005; Madau et al. 2014)

\[ L_{\text{Edd}} = \frac{4\pi G m_p}{\sigma_T (1 - Y/2)} M_{\text{BH}} = 1.43 \times 10^{38} \left( \frac{M_{\text{BH}}}{M_\odot} \right) \text{erg s}^{-1}, \]  

(6)

where \(m_p\) is the proton mass, \(\sigma_T\) is the Thomson scattering cross section, and natural constants are written in their usual symbols. Expressing Equation (5) in magnitudes, the logarithmic Eddington ratio is

\[ \log \left( \frac{L_{\text{bol}}}{L_{\text{Edd}}} \right) = -0.364 M_{1450} \log \left( \frac{M_{\text{BH}}}{M_\odot} \right) - 0.82. \]  

(7)

Again, the statistical uncertainties in the Eddington ratios listed in Table 3 are smaller than the \(\approx 0.55\) dex systematic uncertainty induced by the virial black hole mass scaling relations.

3 MEASURED HE II PROXIMITY ZONE SIZES

In the quasar proximity zone the gas is more highly ionized, visible in the spectrum as excess Lyα transmission. One wishes to robustly quantify its extent while limiting the impact of small-scale density fluctuations and observational effects in heterogeneous samples, such as differences in spectral resolution and S/N. To that aim, we adopted a procedure similar to work on H I quasar proximity zones at \(z_{\text{em}} \approx 6\) (e.g. Fan et al. 2006; Carilli et al. 2010; Eilers et al. 2017), and smoothed the normalized \(HST/\text{CO}S\) spectra with a Gaussian filter with an FWHM of 1 pMpc at the respective quasar redshift. The smoothing FWHM corresponds to 4.5–8.5 pixels in our COS G140L spectra binned to 0.24 Å pixel\(^{-1}\). The proximity zone size \(R_{\text{pz}}\) is then defined as the cosmological proper distance to the first pixel where the smoothed normalized flux drops below 0.1. At \(z > 3\) the intergalactic He II Lyα transmission on similar scales rarely exceeds this threshold (Worseck et al. 2016, 2019), such that the He II proximity zones are well defined. At lower redshifts the He II proximity zone sizes become less distinct due to the emerging post-reionization He II Lyα forest. However, our radiative transfer simulations (Section 4.1) account for density fluctuations in a predominantly ionized IGM with an initial He II fraction as low as 1 per cent, consistent with the inferences from the He II Lyα forest (Worseck et al. 2019). Furthermore, we verified with realistic mock spectra that our HST/CO S spectra are of sufficient quality to yield robust proximity zone sizes (Appendix A). Consequently, our quoted uncertainties \(\sigma_{R_{\text{pz}}}\) on the

5 We note that in the literature, most expressions for the quasar Eddington luminosity incorrectly assume a pure hydrogen plasma.
Figure 1. Left: Normalized HST/COS spectra (grey; overplotted error bars are statistical 1σ Poisson errors) of nine $z_{\text{em}} < 3.6$ quasars from Table 1 with measured small H$\upmu$n proximity zone sizes $R_{pz} < 5$ pMpc (labelled). Distances are for the H$\upmu$n Ly$\alpha$ transition relative to the quasar at the estimated systemic redshift $z_{\text{em}}$. With this sign convention, absorption at distances $< 0$ is due to infalling He $\upmu$n gas clouds or foreground H, while absorption at large distances is dominated by intergalactic H$\upmu$n. The violet squares with error bars mark the quasar redshift uncertainties. The blue lines show the normalized flux smoothed with a Gaussian filter with FWHM of 1 pMpc. The red dots mark the measured $R_{pz}$, defined as the position where the smoothed flux falls below 0.1. Right: Spectral regions of the labelled quasar emission lines used to estimate the systemic redshifts. The flux density is shown in black, whereas the grey lines show the corresponding 1σ error array. Vertical dashed lines mark the measured mode of the emission lines, yielding the emission line redshifts $z_{\text{line}}$. The vertical solid lines mark the applied velocity offsets to estimate the systemic redshifts $z_{\text{em}}$. 

$z_{\text{line}} = 2.8844, z_{\text{em}} = 2.8852$ 
$z_{\text{line}} = 3.2189, z_{\text{em}} = 3.2356$ 
$z_{\text{line}} = 3.2354, z_{\text{em}} = 3.2518$ 
$z_{\text{line}} = 2.9430, z_{\text{em}} = 2.9598$ 
$z_{\text{line}} = 3.4044, z_{\text{em}} = 3.4235$ 
$z_{\text{line}} = 3.3093, z_{\text{em}} = 3.3300$ 
$z_{\text{line}} = 2.9311, z_{\text{em}} = 2.9322$ 
$z_{\text{line}} = 3.2094, z_{\text{em}} = 3.2294$ 
$z_{\text{line}} = 3.3500, z_{\text{em}} = 3.3500$ 
$z_{\text{line}} = 3.3464, z_{\text{em}} = 3.3550$ 
$z_{\text{line}} = 3.3500, z_{\text{em}} = 3.3500$ 

$N_{\text{MNRAS}}$ 000, 1–20 (2021)
proximity zone sizes are based on the individual quasar redshift uncertainties, ranging from ~ 0.1 pMpc ([O III]) to ~ 2 pMpc (C IV).

Figures 1 and 2 show the He II proximity zone spectra of the 17 \( z_{\text{em}} < 3.6 \) quasars from Table 1, ordered by their measured \( R_{pz} \). We also show the spectral regions of the UV-optical emission lines used to determine their systemic redshifts. The smoothed normalized flux drops significantly within several proper Mpc of each quasar, which a posteriori justifies our chosen smoothing scale. The flux shows some contamination from unrelated low-redshift absorption (e.g. at negative distances in Fig. 1 and 2), but this contamination is very unlikely to affect the measurements of He II proximity zone sizes. We see strong diversity in the proximity zone sizes and flux profiles. Most He II proximity zones show structure due to IGM density fluctuations and radiative transfer effects, both of which have been probed with high-resolution optical spectra of the coeval H I Lyα absorption (Reimers et al. 1997; Hogan et al. 1997; Anderson et al. 1999; Heap et al. 2000; Smette et al. 2002; Fechner & Reimers 2007; Shull et al. 2010; Syphers & Shull 2013, 2014; Zheng et al. 2019).

The accurate and precise redshifts of all quasars plotted in Fig. 2 suggest that large He II proximity zone sizes \( R_{pz} > 5 \) pMpc are common. Even with less precise redshifts their proximity zones would be much larger than the ones shown in Fig. 1. Measurements of small He II proximity zones are often hampered by the large C IV quasar redshift uncertainty, limiting the further use of ten quasars from our combined sample (Table 1). The five \( z_{\text{em}} < 3.6 \) quasars with C IV red-
The luminous quasar HE 2347–4342 lacks a He\(\beta\) proximity zone, which is likely explained by its peculiar associated absorption system (Reimers et al. 1997; Fechner et al. 2004). Many of its N\(\lambda\) and O\(\lambda\) bearing components are consistent with being photoionized by the quasar (Fechner et al. 2004). Because their He\(\beta\) column densities are insufficient to shield He\(\beta\)-ionizing photons, Shull et al. (2010) argued that HE 2347–4342 has recently turned on (\(t_{\text{em}} < 1\) Myr). However, the location of the absorbing gas is unconstrained. The accurate and precise [O\(\lambda\)] redshift \(z_{\text{em}} = 2.8852 \pm 0.0006\) implies large gas infall velocities of \(\sim 1500\) km s\(^{-1}\) onto the circumnuclear region (Fig. 3), and even higher velocities if the gas resides in the circumgalactic medium of the host galaxy. Redshift space distortions (Hui et al. 1997; Weinberg et al. 1997) significantly affect the He\(\beta\) transmission profile, possibly masking the He\(\beta\) proximity effect altogether. Conclusive evidence for or against HE 2347–4342 being a young quasar requires detailed numerical modelling of gas associated with the quasar or its host galaxy, which is outside the scope of this work. Because of the extreme peculiar gas velocities that are not represented in our simulations (Section 4.1), we exclude HE 2347–4342 from further discussion.

The other four quasars shown in Fig. 3 are less extreme. SDSS J1237+0126 has a blended optically thin H\(\alpha\) system at \(v \approx 500\) km s\(^{-1}\) that is likely responsible for its small measured He\(\beta\) proximity zone. No high-velocity infall is detected. The He\(\beta\) transmission at 1500–2200 km s\(^{-1}\) may belong to the proximity zone. However, provided that our simulations approximately capture the density and velocity field around the quasar host halo, our strict and simple definition of \(R_{pz}\) ensures a one-to-one comparison to our models (Section 4.1) that include cases of misestimated proximity zone sizes due to the density field and peculiar velocities, similar to recent work on \(z_{\text{em}} \sim 6\) H\(\alpha\) proximity zones (Eilers et al. 2017, 2020). The strong N\(\lambda\) absorbers at 5312 and 5481 km s\(^{-1}\) may either indicate a much larger proximity zone of SDSS J2346–0016 or a high-velocity outflow. The He\(\beta\) spectra of SDSS J2346–0016 and HS 0911+4809 are modulated by the density field traced by H\(\alpha\) absorption. Their short He\(\beta\) proximity zones do not seem affected by peculiar velocities. Finally, SDSS J1319+5202 from Paper I shows a possibly infalling H\(\alpha\) complex at \(v \approx -900\) km s\(^{-1}\) with strong associated C\(\lambda\) and N\(\lambda\) in two components. The complex does not affect

Figure 3. Normalized HST/COS UV spectra (grey) and optical spectra (green) of the five quasars with sufficiently precise redshifts (\(\sigma_v \leq 400\) km s\(^{-1}\); violet square with error bar) to measure small He\(\beta\) proximity zones (\(R_{pz} < 5\) pMpc). Velocities are for He\(\beta\) and H\(\alpha\) with respect to the quasar, with \(v < 0\) indicating infalling gas. As in Fig. 1 we also show the smoothed normalized UV flux (blue) with the corresponding velocity width \(v_{pz}\) of the He\(\beta\) proximity zone. Note the different resolving powers for the H\(\alpha\) square with error bar) to measure small He\(\beta\) velocities (Reimers et al. 1997; Fechner et al. 2004). Many of its N\(\lambda\) simulations, hampering estimates of the quasar on-time.
the measurement of $R_{\text{pp}}$ because the H\textsc{i} absorption in the rest of the proximity zone is much lower than expected for the $z \approx 3.9$ H\textsc{i} Ly$\alpha$ forest. In summary, except for the peculiar quasar HE 2347–4342, gas inflows and outflows seen in absorption do not substantially affect our measurements of small He\textsc{i} proximity zone sizes. Excluding HE 2347–4342 and HE2QS J2354–2033 from Paper I that has an anomalously large C\textsc{iv} blueshift, 22 quasars remain in our sample. Thirteen of them have sufficiently precise systemic redshifts ($\sigma_v \leq 400\text{ km s}^{-1}$) and black hole masses from near-infrared spectra.

4 THEORETICAL METHODS

4.1 Radiative transfer models of He\textsc{ii} proximity zones

To explain the diversity in the measured proximity zone sizes and to infer the on-times of the quasars, we used a combination of hydrodynamical simulations and one-dimensional radiative transfer simulations of He\textsc{ii} quasar proximity zones, described in detail in Khrykin et al. (2016, 2017). For their cosmological setting, we used the output of a Gadget-3 (Springel 2005) smooth particle hydrodynamics simulation run in a cubic volume of $(25h^{-1})^3$ comoving Mpc$^3$ containing 512$^3$ baryonic and dark matter particles, respectively. Using periodic boundary conditions, we drew 1000 one-dimensional density, velocity, and temperature distributions (skewers) in random directions around the most massive halo in the $z_{\text{sim}} = 3.1$ snapshot of the simulation. Assuming that cosmic structure evolution is negligible between $z_{\text{sim}}$ and the quasar redshifts (Table 1), we accounted for density evolution by rescaling the gas densities by a factor $(1+z_{\text{sim}})^3/(1+z)^3$. The resulting skewers have a length of 160 comoving Mpc, sampled at $dr = 11.9$ comoving kpc, corresponding to $dv = 0.86$–0.93 km s$^{-1}$ at $z_{\text{sim}} = 2.74$–3.5.

The skewers were processed with a one-dimensional radiative transfer algorithm based on the C$^2$-Ray code (Mellema et al. 2006), which tracks the evolution of H\textsc{i}, He\textsc{ii}, $e^-$, and the gas temperature to generate He\textsc{ii} Ly$\alpha$ transmission spectra of quasar proximity zones (Khrykin et al. 2016, 2017). Analogous to Paper I, we created a set of radiative transfer models for each quasar at its respective redshift $z_{\text{em}}$ and photon production rate $Q$ (Table 1), varying the quasar on-time $t_{\text{on}}$ and the initial He\textsc{ii} fraction in the ambient IGM at quasar turn-on $x_{\text{He\textsc{ii}},0}$ (or equivalently the UV background photoionization rate $\Gamma_{\text{He\textsc{ii}}}$). We assumed for simplicity that the quasars emitted continuously at their inferred luminosity for a time $t_{\text{on}}$ prior to our observation, i.e. as a “light bulb”. We considered a base-10 logarithmically spaced grid of on-times $\log(t_{\text{on}}/\text{Myr}) \in [-2, 2]$ with a step size $\Delta \log(t_{\text{on}}/\text{Myr}) = 0.125$. For the initial He\textsc{ii} fraction we took the inhomogeneously spaced grid $x_{\text{He\textsc{ii}},0} \in \{0.01, 0.05, 0.1, 0.2, 0.3, 0.5, 0.6, 0.7, 0.9, 1.0\}$. Because the quasars are at lower redshifts than those from Paper I we included models with $x_{\text{He\textsc{ii}},0} = 0.01$ representative of the IGM at the end of He\textsc{ii} reionization (Khrykin et al. 2016; Worseck et al. 2019). This resulted in a grid of 330 radiative transfer models per quasar, each with 1000 He\textsc{ii} Ly$\alpha$ transmission spectra. Redshift error is incorporated in our Bayesian inference (Section 4.2).

Figure 4 shows an example of model He\textsc{ii} proximity zone spectra varying $t_{\text{on}}$ (left) and $x_{\text{He\textsc{ii}},0}$ (right) for the same density skewer. The model proximity zone size $R_{\text{sim}}$ measured analogously to the HST/COS spectra (Section 3) depends on the quasar on-time, as the IGM responds to changes in the radiation field on the He$\alpha$ equilibrium time-scale $t_{\text{eq}} \approx \Gamma_{\text{He\textsc{ii}}}^{-1} \approx 30\text{ Myr}$ in the $z \sim 3$ IGM (Khrykin et al. 2016). For $t_{\text{on}} \leq t_{\text{eq}}$ the proximity zone size increases with $t_{\text{on}}$, but stalls for longer on-times (Paper I), as illustrated in the lower left panel of Fig. 4. The proximity zone size only weakly depends on the initial IGM He\textsc{ii} fraction due to the thermal proximity effect (Khrykin et al. 2017) and the definition of the proximity zone size that does not probe the actual size of the ionized region (Khrykin et al. 2016). Only at the lowest initial He\textsc{ii} fractions $x_{\text{He\textsc{ii}},0} < 0.05,$
4.2 Bayesian inference of the quasar on-time

To estimate the on-times of individual quasars in our sample, we performed MCMC inference on their measured He II proximity zones $R_{\text{psz}}$ using the Bayesian statistical method introduced in Paper I. First, we incorporated the individual quasar redshift uncertainties into the radiative transfer models by adding a Gaussian-distributed random deviate with a standard deviation $\sigma_{R_{\text{psz}}}$ (Table 1) to each $R_{\text{psz}}$ realization, similar to Paper I. For each resulting distribution $R'_{\text{sim}}$, we can write a Bayesian likelihood $L$ given the combination of model parameters $\{t_{\text{on}}, x_{\text{HeII,0}}\}$ per quasar,

$$L(R_{\text{psz}}|t_{\text{on}}, x_{\text{HeII,0}}) = p(R'_{\text{sim}} = R_{\text{psz}}|t_{\text{on}}, x_{\text{HeII,0}}),$$

where $p(R'_{\text{sim}} = R_{\text{psz}}|t_{\text{on}}, x_{\text{HeII,0}})$ is the PDF of the modeled He II proximity zone sizes plus redshift error $R'_{\text{sim}}$, evaluated at the value of the measured He II proximity zone size $R_{\text{psz}}$. To construct this PDF we used kernel density estimation (KDE) on the respective set of 1000 model spectra. An example KDE on the distribution of proximity zone sizes in one radiative transfer model is illustrated in the left panel of Fig. 5. Then we computed the likelihood of each radiative transfer model in our $\{t_{\text{on}}, x_{\text{HeII,0}}\}$ grid via Equation (8), and constructed each quasar’s continuous two-dimensional likelihood by bivariate spline interpolation.

To infer $t_{\text{on}}$ for each individual quasar we sampled the respective likelihood with MCMC. Because the initial He II fraction is quite uncertain at the redshifts of interest due to large-scale UV background fluctuations at the tail end of He II reionization (Davies et al. 2017; Worseck et al. 2019), we chose to impose a uniform prior $0.01 \leq x_{\text{HeII,0}} \leq 1$. Similarly, we set a uniform prior on $\log(t_{\text{on}}/\text{Myr})$ in the range $0.01 – 100$ Myr. The lower limit is motivated by the ubiquitous H i proximity effect implying $t_{\text{on}} \gtrsim 10^{-1} \text{Myr}$ at $z \sim 3$ except for the few youngest $z_{\text{em}} \geq 6$ quasars in Eilers et al. (2020). The upper limit of 100 Myr is driven by estimates of the quasar duty cycle ($t_{\text{dc}} = 1–1000$ Myr, e.g. Yu & Tremaine 2002; White et al. 2008). Furthermore, approximations of our modelling break down at longer on-times for two reasons: i) $t_{\text{on}}$ becomes comparable to the cooling time meaning that cooling cannot be neglected; ii) our assumption of a static density field for the radiative transfer in post-processing is not valid anymore (Khrzykin et al. 2016).

As an example, we show the results of the MCMC inference for the quasar Q 1602+576 in the right panels of Fig. 5. Analogous to the results in Paper I, the flat posterior PDF of $x_{\text{HeII,0}}$ signals the lack of sensitivity of the He II proximity zone size to the initial He II fraction in the IGM. Yet, we are able to put tight constraints on the on-time of Q 1602+576 due to the small uncertainty in its proximity zone size facilitated by its accurate and precise systemic redshift (Table 1).

5 RESULTS

5.1 He II proximity zone sizes do not scale with luminosity or redshift

Many studies on $z_{\text{em}} \sim 6$ H i quasar proximity zones (Maselli et al. 2007, 2009; Bolton & Haehnelt 2007; Lidz et al. 2007; Keating et al. 2015; Eilers et al. 2017; Davies et al. 2020) and on $z_{\text{em}} \sim 4$ He II proximity zones (Khrzykin et al. 2016, 2019) have emphasized that by definition $R_{\text{psz}}$ probes fully reionized gas in the quasar vicinity, and is always smaller than the actual size of the ionized region. Moreover, simple scaling laws with luminosity $R \propto L_{144}$ with $\gamma = 1/3$ ($\gamma = 1/2$) for a neutral (ionized) IGM do not apply for smoothed spectra and a realistic IGM density field (Bolton & Haehnelt 2007;
Davies et al. 2020). Although the H\textsc{i} proximity zone sizes of $z_{em} \approx 6$ quasars generally increase with luminosity (Eilers et al. 2017), 5–10 per cent of the population has undersized proximity zones that likely indicate short on-times $t_{on} \lesssim 0.01–0.1$ Myr (Eilers et al. 2018, 2020).

Figure 6 shows the measured He\textsc{ii} proximity zone size $R_{pz}$ as a function of absolute magnitude $M_{1450}$ for the combined sample of 22 quasars, i.e. Table 1 excluding HE 2347–4342 and HE2QS J2354–2033. We see no clear relation between $R_{pz}$ and $M_{1450}$. For the eight $M_{1450} \sim 28$ quasars with precise systemic redshifts there is a factor $\sim 5$ spread in $R_{pz}$. We overplot predictions of the average $R_{pz}(M_{1450})$ from our radiative transfer simulations at the median redshift $z_{em} = 3.29$ of our combined sample.\footnote{We verified that these relations do not change dramatically due to IGM density evolution in the redshift range of interest.} The model predictions for extreme values of the IGM He\textsc{ii} fraction before quasar turn-on $x_{He\textsc{ii},0} \in \{0.01, 1\}$ and two representative on-times $t_{on} \in \{1$ Myr, 10 Myr$\}$ cover a similar range of $R_{pz}$ as the observations. IGM density fluctuations give rise to intrinsic scatter in the simulated $R_{pz}$ that increases toward low He\textsc{ii} fractions, i.e. in the emerging He\textsc{ii} Ly\textalpha forest. The He\textsc{ii} fraction and the on-time are often degenerate (Khryunin et al. 2016, 2019), as illustrated by the overlapping curves for $(x_{He\textsc{ii},0}, t_{on}) = (0.01, 1$ Myr$)$ and $(x_{He\textsc{ii},0}, t_{on}) = (1, 10$ Myr$)$. Nevertheless, the smallest precisely measured He\textsc{ii} proximity zone sizes require short on-times of $\lesssim 1$ Myr, whereas the largest ones require long on-times of $\gtrsim 10$ Myr. Therefore, our measurements are sensitive to $t_{on}$ up to the He\textsc{ii} equilibration time-scale $t_{eq} \approx 30$ Myr at $z \sim 3$ (Khryunin et al. 2016, 2019).

Our finding that He\textsc{ii} proximity zone sizes do not scale with luminosity unlike their $z_{em} \sim 6$ H\textsc{i} counterparts, can be explained by the lack of sensitivity of H\textsc{i} proximity zone sizes to $t_{on} \gtrsim 0.1$ Myr, which is rooted in the shorter H\textsc{i} equilibration time-scale. If most $z_{em} \sim 6$ quasars shine longer than 0.1 Myr, their similar H\textsc{i} proximity zone sizes result in an overall strong scaling with luminosity (Davies et al. 2020), with shorter on-times being apparent as outliers (Eilers et al. 2017, 2018, 2020). In contrast, He\textsc{ii} proximity zone sizes probe $t_{on} \lesssim 30$ Myr, so variations in the individual on-times can effectively remove any correlation with luminosity.

As shown in Fig. 7 the He\textsc{ii} proximity zone size does not depend on redshift. Some of the scatter in $R_{pz}$ vs. redshift is due to the $\pm 3$ mag range in $M_{1450}$, but due to the lack of a clear correlation with luminosity we do not take out this dependence. The large scatter in $R_{pz}$ for the eight quasars at $M_{1450} \sim 28$ with precise systemic redshifts confirms that there is no evidence for a scaling with redshift. Our measurements do not support previous claims of a significant decline of the luminosity-normalized $R_{pz}$ at $z_{em} > 3.3$ by Zheng et al. (2015) for a slightly different definition of the proximity zone size that does not track the quasar ionization front (Khryunin et al. 2016). At fixed luminosity our radiative transfer simulations show a shallow decrease of $R_{pz}$ with redshift for a large range of on-times and IGM He\textsc{ii} fractions (Fig. 7). This is a direct consequence of the observational definition of $R_{pz}$ and its insensitivity to the IGM He\textsc{ii} fraction (Khryunin et al. 2016, 2019). Only for long on-times and small He\textsc{ii} fractions is there a significant decrease with redshift that is driven by IGM density evolution. Similar shallow relations have been obtained for H\textsc{i} proximity zones at $z_{em} \sim 6$ (Eilers et al. 2017; Davies et al. 2020). For the model relations in Fig. 7 we chose $M_{1450} = -27$, somewhat fainter than the median $M_{1450}$ of our sample, in order to facilitate comparison with Eilers et al. (2017). For the three $M_{1450} \sim 27$ quasars from Paper I small but uncertain proximity zone sizes indicate $t_{on} \ll 10$ Myr irrespective of the initial IGM He\textsc{ii} fraction, consistent with our joint analysis in Paper I. On the other hand, the large He\textsc{ii} proximity zone of the $M_{1450} \sim 27$ quasar SDSS J1253+6817 at $z_{em} = 3.4753$ requires $t_{on} > 1$ Myr.

Figure 7. Similar to Fig. 6 for the He\textsc{ii} proximity zone size $R_{pz}$ as a function of quasar redshift $z_{em}$. The colour coding indicates the quasar absolute magnitude. Overplotted is the average $R_{pz}(z_{em})$ and its 16–84th percentile scatter from our radiative transfer simulations at $M_{1450} = -27$ for different initial He\textsc{ii} fractions $x_{He\textsc{ii},0}$ and quasar on-times $t_{on}$. 

Figure 6. He\textsc{ii} proximity zone size $R_{pz}$ as a function of quasar absolute magnitude $M_{1450}$ for the 22 quasars in our combined sample. Circles and diamonds mark $z_{em} < 3.6$ and higher-redshift quasars (Paper I), respectively. The colour coding indicates the quasar redshift. $R_{pz}$ errors have been calculated from the individual redshift errors. The dotted line marks the average calculated from the individual redshift errors. The dotted line marks the average proximity zone size of the average proximity zone of the best fit relations (Khrykin et al. 2016, 2019). Only for long on-times $> 1$ Myr, their similar H\textsc{i} proximity zone sizes result in an overall strong scaling with luminosity (Davies et al. 2020), with shorter on-times being apparent as outliers (Eilers et al. 2017, 2018, 2020). In contrast, He\textsc{ii} proximity zone sizes probe $t_{on} \lesssim 30$ Myr, so variations in the individual on-times can effectively remove any correlation with luminosity.
irrespective of the He II fraction. This again highlights the sensitivity of our measured He II proximity zone sizes to the quasar on-time.

5.2 Constraints on the quasar on-time

Figure 8 shows the MCMC estimates of the quasar on-time posterior PDFs marginalized over the initial He II fraction for the 16 remaining $z_{\text{em}} < 3.6$ quasars excluding HE 2347–4342. Most PDFs are based on $\sim 130,000$ posterior samples depending on the width of the PDF. Many PDFs are $\geq 0$ at the upper or lower limit of our flat logarithmic prior on the quasar on-time $0.01 \, \text{Myr} \leq t_{\text{on}} \leq 100 \, \text{Myr}$. Because the lower limit of our prior is physically motivated by the H I proximity effect, while the flatness of the posterior at $t_{\text{on}} \gtrsim 30 \, \text{Myr}$ is effectively determined by the equilibration time-scale, some of these posterior PDFs provide only limited constraints on $t_{\text{on}}$. Specifically, if the posterior at $t_{\text{on}} = 0.01 \, \text{Myr}$ is $> 10$ per cent of its maximum, we quote a 1σ upper limit on $t_{\text{on}}$ as the 84th percentile of the posterior. Likewise, if the posterior at $t_{\text{on}} = 100 \, \text{Myr}$ is $> 10$ per cent of its maximum, we define a 1σ lower limit on $t_{\text{on}}$ as the 16th percentile of the posterior. For the remaining quasars we quote the median of the posterior as a measurement of $t_{\text{on}}$ with a 1σ equal-tailed credibility interval derived from the 16th and the 84th percentile of the posterior, respectively.

The upper panel of Fig. 8 shows the $t_{\text{on}}$ posteriors of the five quasars from Table 1 whose $R_{pz}$ values are highly uncertain due to C iv redshift errors. Large redshift errors significantly broaden the model $R_{\text{sim}}$ distributions and result in weak constraints on $t_{\text{on}}$, similar to four quasars from Paper I. According to the above definition, we obtain 1σ upper limits on $t_{\text{on}}$. Quasars with more precise redshifts ($\sigma_v \leq 400 \, \text{km s}^{-1}$) have significantly narrower $t_{\text{on}}$ posteriors, except SDSS J1237+0126 whose He II proximity zone size may have been underestimated (Section 3). For four quasars we obtain lower limits on the quasar on-time (middle panel of Fig. 8) due to their combination of a large proximity zone and moderate to low luminosity (PC 0058+0215). The bottom panel in Fig. 8 shows the $t_{\text{on}}$ posteriors of the six quasars for which we obtain individual on-time measurements. Their posteriors span distinct ranges in $t_{\text{on}}$, indicating an intrinsically broad distribution of quasar on-times from $\lesssim 1 \, \text{Myr}$ to $\sim 10 \, \text{Myr}$. The precision of the measurements ranges from 0.28 dex (HS 1700+6416) to 0.48 dex (HS 0911+4809), which mainly reflects the redshift precision. On-times of up to $\sim 10 \, \text{Myr}$ can be measured securely, but for longer on-times the flat tail due to the finite equilibration timescale becomes stronger, eventually resulting in lower limits on the on-time. Both the middle and lower panels show that with sufficiently precise redshifts, He II proximity zones constrain individual on-times from $\sim 1 \, \text{Myr}$ up to the He II equilibration time of $\sim 30 \, \text{Myr}$, which sets the physical limit of the method.

In Fig. 9 we plot the inferred quasar on-times or limits thereof as a function of quasar absolute magnitude. We also show the results for the six $z_{\text{em}} > 3.6$ quasars from Paper I with updated inferences based on our extended model grid and adjusted to our different definition of limits on $t_{\text{on}}$. There is strong diversity in the quasar on-times with no obvious dependence on absolute magnitude. This is similar to the lack of a trend in $R_{pz}(M_{1450})$ in Fig. 6, but now we account for the redshift dependence and provide quantitative constraints on $t_{\text{on}}$ based on the observed scatter of He II proximity zone sizes.

5.3 No dependence of quasar on-time on black hole mass and Eddington ratio

Figure 10 shows the quasar on-time as a function of black hole mass for the 13 quasars for which both quantities are available (Table 3 excluding HE 2347–4342). In the range of black hole mass spanned by our sample ($M_{\text{BH}} \approx 10^8–10^{10} M_\odot$) both quantities do not correlate, although the $\pm 0.55 \, \text{dex}$ systematic uncertainty in the virial black hole masses significantly contributes to the observed scatter. Figure 11 shows the quasar on-time as function of the Eddington ratio $L_{\text{bol}}/L_{\text{Edd}}$. The 13 quasars roughly emit at their Eddington limit considering the systematic uncertainty induced by the virial black hole mass measurements (Equation 7). The quasar on-time also does not correlate with the Eddington ratio. The dashed lines in Fig. 11 indicate the $\epsilon$-folding time-scale for SMBH growth for different radiative efficiencies $\epsilon$. For $\epsilon = 0.1$ (e.g. Yu & Tremaine 2002; Ueda et al. 2014; Shankar et al. 2020) and $L_{\text{bol}} = L_{\text{Edd}}$ one obtains $t_\gamma \approx 44 \, \text{Myr}$. This is significantly longer than the on-times $t_{\text{on}} \lesssim 5 \, \text{Myr}$ we infer for half of our sample. Such small $t_{\text{on}}$ values may indicate episodic quasar lifetimes, i.e. that their SMBHs grew
in short bursts with \( t_Q \ll t_S \). Unless most of our observed quasars are radiatively inefficient \( (\epsilon \ll 0.1, \text{e.g. Volonteri et al.} \text{2015; Davies et al.} \text{2019}) \), their short on-times imply a small mass growth by \( \lesssim 10 \) per cent in the current quasar episode. However, the occurrence of lower limits \( t_{\text{on}} \gtrsim 10 \text{Myr} \) indicates that not all quasar episodes are that short, so the \( \text{He} \, \eta \) proximity zones may sample a possibly very broad distribution of quasar lifetimes (Khrykin et al. 2021).

Figure 9. Inferred quasar on-time \( t_{\text{on}} \) as a function of absolute magnitude \( M_{1450} \) for the 22 quasars in our combined sample. Green symbols show \( t_{\text{on}} \) measurements (posterior median with 16–84th percentile range from Fig. 8), while blue and red symbols mark 1\( \sigma \) upper limits (84th percentile of the posterior) and 1\( \sigma \) lower limits (16th percentile of the posterior) for quasars with precise systemic redshifts \( (\sigma_v \leq 400 \text{ km s}^{-1}) \), respectively. Limits derived for quasars with larger redshift errors are shown in grey.

Figure 10. Inferred quasar on-time \( t_{\text{on}} \) as a function of black hole mass \( M_{\text{BH}} \) for 13 \( \text{He} \, \eta \)-transparent quasars (Table 3 excluding HE 2347–4342). Quasar on-time measurements and limits are labelled as in Fig. 9. Individual error bars are 1\( \sigma \) statistical errors, while the bottom bar indicates the \( \pm 0.5 \)\( \text{dex} \) systematic uncertainty in \( M_{\text{BH}} \).

Figure 11. Inferred quasar on-time \( t_{\text{on}} \) as a function of Eddington ratio \( L_{\text{bol}}/L_{\text{Edd}} \) for 13 \( \text{He} \, \eta \)-transparent quasars (Table 3 excluding HE 2347–4342). Quasar on-time measurements and limits are labelled as in Fig. 9. Individual error bars are 1\( \sigma \) statistical errors, while the bottom bar indicates the \( \pm 0.5 \)\( \text{dex} \) systematic uncertainty in \( L_{\text{bol}}/L_{\text{Edd}} \). The dashed lines show the \( e \)-folding time-scale of SMBH growth for different radiative efficiencies \( \epsilon \).

6 DISCUSSION

6.1 Dependence on the \( \text{He} \, \eta \) fraction prior

The inferred quasar on-times depend on the assumed prior on the initial \( \text{He} \, \eta \) fraction in the surrounding IGM. Similar to Paper I, we adopted a uniform prior \( 0.01 \leq x_{\text{He} \, \eta, 0} \leq 1 \) to reflect the considerable uncertainty in the local ionization conditions during \( \text{He} \, \eta \) reionization. The measured large-scale \( \text{He} \, \eta \) absorption indicates a low median \( \text{He} \, \eta \) fraction of 2–3 per cent at \( z \approx 3.1 \) (Khrykin et al. 2016; Worseck et al. 2019) with spatial variations that are consistent with semi-numerical models of a fluctuating \( \text{He} \, \eta \)-ionizing background after \( \text{He} \, \eta \) reionization (Davies & Furlanetto 2014; Davies et al. 2017). However, the parameters of these models are not well constrained by measurements of the \( \text{He} \, \eta \) effective optical depth, such that the spatial distribution of the \( \text{He} \, \eta \) fraction is still uncertain.

In order to explore how our results change with the \( x_{\text{He} \, \eta, 0} \) prior, we repeated the analysis with a restricted uniform prior \( 0.01 \leq x_{\text{He} \, \eta, 0} \leq 0.05 \) that is consistent with the inferences from the \( \text{He} \, \eta \) Ly\( \alpha \) absorption at 2.8 < \( z \) < 3.3 (Worseck et al. 2019). In Figure 12 we compare the resulting posterior PDFs for \( t_{\text{on}} \) to the ones from Section 5.2 for the 13 quasars with precisely measured \( \text{He} \, \eta \) proximity zone sizes based on their precise systemic redshifts (\( \sigma_v \leq 400 \text{ km s}^{-1} \)). Table 4 compares the on-times for all 22 quasars whose \( \text{He} \, \eta \) proximity zone sizes are not significantly negative, and therefore allow us to constrain \( t_{\text{on}} \) (Table 1 excluding HE 2347–4342 and HE2QS J2354–2033). Due to the fewer ionizing photons required to create a large proximity zone, the posterior PDFs shift to lower values, in agreement with the credible regions for the unrestricted prior at low \( x_{\text{He} \, \eta, 0} \) (Fig. 5). Upper and lower 1\( \sigma \) limits on \( t_{\text{on}} \) decrease by \( \approx 0.4 \)\( \text{dex} \) and \( \approx 0.5 \)\( \text{dex} \), respectively. The eight \( t_{\text{on}} \) measurements also shift to lower values by \( \approx 0.5 \)\( \text{dex} \), resulting in three more limits due to the decreasing ratio between the peak of the posterior and its tail at 100\( \text{Myr} \) according to the definition in Section 5.2. Nevertheless, we still infer a broad range in \( t_{\text{on}} \) from \( \lesssim 0.3 \) to \( \gtrsim 10 \)\( \text{Myr} \). Our constraints can be improved either by more...
precise systemic redshifts from CO or [C II] 158 \mu m emission from the quasar host galaxies covered at mm to sub-mm wavelengths, or via better priors on $x_{\text{He} \ i}^{-0.1$ from semi-numerical models of the fluctuating UV background at the end of He ii reionization (Davies et al. 2017) which is left to future work.

Table 4. On-times $t_{\text{on}}$ for the 22 quasars considered in this work for two different uniform priors on the initial IGM He ii fraction $x_{\text{He} \ i}^{-0.1$.

| Quasar         | 0.01 \leq x_{\text{He} \ i}^{-0.1} \leq 1 | 0.01 \leq x_{\text{He} \ i}^{-0.1} \leq 0.05 |
|----------------|------------------------------------------|------------------------------------------|
| HE2QS J1214+0859 | \leq 0.46                                   | \leq 0.18                                 |
| HE2QS J1706+5904  | \leq 0.66                                   | \leq 0.28                                 |
| SDSS J1257+0126  | \leq 1.01                                   | \leq 0.36                                 |
| SDSS J2346+0016  | 0.31^{+0.41}_{-0.21}                        | \leq 0.22                                 |
| SDSS J0818+4908  | \leq 2.42                                   | \leq 0.88                                 |
| HE2QS J0916+2408 | \leq 2.77                                   | \leq 1.02                                 |
| HS 0811+0809     | 1.01^{+1.20}_{-0.69}                        | \leq 0.34 to 0.44                         |
| HE2QS J0233-0149 | \leq 6.24                                   | \leq 2.36                                 |
| Q 1602+576       | 1.98^{+1.14}_{-0.54}                        | \leq 0.30                                 |
| PC 0058+0215     | \geq 7.24                                   | \leq 2.05                                 |
| HS 1700+6416     | 0.80^{+0.50}_{-0.35}                        | \leq 0.12                                 |
| SDSS J0956+2927  | 11.62^{+7.37}_{-4.57}                       | \leq 2.11                                 |
| HS 1024+1849     | 9.53^{+6.83}_{-2.04}                        | \leq 1.52                                 |
| SDSS J1253+0817  | \geq 23.55                                  | \leq 8.22                                 |
| Q 0302–003       | \leq 11.36                                  | \leq 3.33                                 |
| HE2QS J2157+2330 | \geq 31.84                                  | \leq 10.87                                |
| HE2QS J2331+3147 | \leq 0.86                                   | \leq 0.29                                 |
| SDSS J1614+4859  | \geq 7.98                                   | \leq 2.89                                 |
| SDSS J1711+6052  | \leq 7.48                                   | \leq 3.66                                 |
| SDSS J1319+5202  | 0.80^{+0.83}_{-0.50}                        | \leq 0.34 to 0.44                         |
| SDSS J1357+6237  | \geq 1.34                                   | \leq 0.71                                 |
| HE2QS J1630+6935 | 2.77^{+2.13}_{-0.82}                        | \leq 0.40                                 |

Figure 13. SMBH growth histories of the 13 He ii-transparent quasars with on-time constraints and measured black hole masses (circles). The lines show their exponential growth history assuming $e = 0.1$ and a constant Eddington ratio $L_{\text{bol}}/L_{\text{Edd}} = 1$.

6.2 Implications for SMBH growth

Here we consider the implications of our inferred quasar on-times for the growth histories of their SMBHs to their measured masses. In our modelling we assumed that each quasar shone at a constant “light bulb” luminosity $L_{\text{bol}}$ for the time $t_{\text{on}}$ prior to our observation. Because the inferred on-times are generally much smaller than the Salpeter time, the light bulb model is still a good approximation to the standard model of exponential mass growth\footnote{For exponential growth at constant Eddington ratio, Equations (7) and (10) imply that during $t_{\text{on}}$ the quasar absolute magnitude increases by $\Delta M_{1450} = 1.19 t_{\text{on}}/t_{\text{off}}$, which is small for most quasars in our sample.}

\[ M_{\text{BH}}(t) = M_{\text{seed}} e^{(t-t_{\text{seed}})/\Gamma} \]  

for a black hole with a constant radiative efficiency and Eddington ratio that had a seed mass $M_{\text{seed}}$ at some cosmic time $t_{\text{seed}}$. Figure 13 illustrates the exponential growth scenario for the 13 He ii-transparent quasars with on-time constraints and estimated black hole masses. The growth time $t_{\text{gr}} = t - t_{\text{seed}}$ from a stellar remnant seed mass $\sim 100 M_\odot$ are $t_{\text{gr}} \sim 700$ Myr, and for continuous unobserved SMBH growth these would be equal to the on-times $t_{\text{on}}$. For about half of our sample (7 out of 13) the on-times inferred from the He ii proximity zone sizes ($t_{\text{on}} \leq 5$ Myr) are considerably shorter than the Salpeter time $t_{\text{sg}} \approx t_{\text{gr}}$, implying SMBH growth during episodic quasar activity and/or in obscured phases.

Episodic quasar activity on a wide range of time-scales has been predicted by many models of quasar and black hole co-evolution (e.g. Ciotti & Ostriker 2001; Hopkins et al. 2006; Novak et al. 2011; Anglés-Alcázar et al. 2017). Consider for simplicity a “blinking light bulb” model in which every quasar episode of $t_Q = 20$ Myr is followed by a quiescent phase of $t_{\text{off}} = 30$ Myr. In this case, our observations sample random times $t_{\text{on}} \leq t_Q$, and after each quasar episode the surrounding IGM will re-equilibrate to the He ii fraction implied by the UV background ($t_{\text{equ}} \approx \Gamma^{-1} \text{He} \ i > 10$–30 Myr at the redshifts of interest), causing the proximity zone to disappear. If the SMBHs grow exponentially, but not in the quiescent phases, the required time $t_{\text{blink}} \approx t_{\text{gr}}(t_{\text{on}} + t_{\text{off}})/t_Q \approx 1750$ Myr is barely sufficient to explain the SMBH masses in Fig. 13. Quasars would have to blink over the...
age of the Universe to acquire their black hole masses. The problem is exacerbated by shorter quasar lifetimes implied at lower initial He\(\,\,\nu\) fractions in the end stages of He\(\,\,\nu\) reionization (Section 6.1). This indicates that either the off-times must be shorter than the equilibration time, such that the proximity zones do not disappear (see Davies et al. 2020 for the similar case of \(z \approx 6\) H I proximity zones) or that black holes continue to grow in obscured phases, i.e. during \(t_{	ext{off}}\). X-ray-selected samples of Active Galactic Nuclei (AGN) revealed that the fraction of Compton-thin (equivalent line-of-sight hydrogen column density \(N_{\text{H}} = 10^{22} - 10^{24} \text{ cm}^{-2}\) ) obscured AGN depends on X-ray luminosity and redshift (e.g. Merloni et al. 2014; Ueda et al. 2014; Buchner et al. 2015; Aird et al. 2015). While at \(z \approx 2\) the obscured fraction decreases with luminosity from \(\approx 70\) per cent to \(\approx 30\) per cent (Ueda et al. 2014; Aird et al. 2015), at \(z \approx 3\) the obscured fraction is \(\approx 60\) per cent independent of luminosity (Buchner et al. 2015; Aird et al. 2015), possibly increasing further toward higher redshifts (Vito et al. 2018). It is therefore plausible that much of the mass growth occurred when the SMBH was obscured by gas and dust, such that SMBH mass and the duration of the UV-luminous quasar phase are uncorrelated. We will consider more complex lightcurves than the simple light bulb model in future work.

7 CONCLUSIONS

We have used a sample of \(17 \, 2.74 < z_{\text{em}} < 3.51\) He\(\,\,\nu\)-transparent quasars with science-grade \((S/N \gtrsim 3)\) HST/COS spectra (Worseck et al. 2019) to measure the sizes of their highly ionized He\(\,\,\nu\) proximity zones. Given that these zones typically span only a few pMpc, precise measurements are often hampered by quasar redshift error. Therefore, we obtained ancillary near-infrared spectroscopy to measure accurate and precise systemic redshifts of 12 quasars from low-ionization UV and optical emission lines (\(\text{Mg}\,\,\nu\), \(H\beta\), \([\text{O}\,\,\nu]\)) that also allow for estimates of the quasar black hole masses and Eddington ratios. Together with two \(z_{\text{em}} > 3.6\) quasars from Paper I and excluding the peculiar quasar HE 2347–4342 (e.g. Reimers et al. 1997), we have compiled the first statistical sample of 13 quasars with accurate and precise He\(\,\,\nu\) proximity zone sizes (\(\sigma_{R_{\text{p}}} \leq 1\) pMpc). Our main results are the following:

(i) He\(\,\,\nu\) proximity zone sizes span a large range \(2\) pMpc \(\lesssim R_{\text{pz}} \lesssim 15\) pMpc. Nine out of 13 quasars with precise systemic redshifts have \(R_{\text{pz}} > 5\) pMpc, suggesting that large proximity zones are common, and that quasar redshift error significantly limits further use of the remaining He\(\,\,\nu\)-transparent quasars.

(ii) He\(\,\,\nu\) proximity zone sizes do not correlate with quasar UV luminosity or redshift (Figs. 6 and 7). Given their weak sensitivity to the He\(\,\,\nu\) fraction in the ambient IGM (Khrykin et al. 2016, 2019), the factor \(\approx 5\) spread of \(R_{\text{pz}}\) at similar luminosity is mainly due to variations in the individual quasar on-time, but variations in the He\(\,\,\nu\) fraction due to patchy He\(\,\,\nu\) reionization and IGM density fluctuations contribute as well.

(iii) Exploiting the sensitivity of \(R_{\text{pz}}\) to quasar on-times \(t_{\text{on}}\) shorter than the equilibration time of He\(\,\,\nu\) in the ambient IGM \(t_{\text{eq}} = 10–30\) Myr at these redshifts (Khrykin et al. 2016; Worseck et al. 2019) we have inferred individual quasar on-times using our Bayesian framework developed in Paper I. The 13 quasars with precise He\(\,\,\nu\) proximity zone sizes span a large range in on-time from \(t_{\text{on}} \lesssim 1\) Myr to \(t_{\text{on}} > 30\) Myr, larger than the typical \(\pm 0.3\) dex statistical uncertainty due to remaining redshift error, the IGM density field, and the initial He\(\,\,\nu\) fraction prior to quasar activity.

(iv) The quasar on-time neither correlates with quasar luminosity (Fig. 9), nor with black hole mass (Fig. 10) or Eddington ratio (Fig. 11).

The predominantly short quasar on-times \(t_{\text{on}} < 10\) Myr and the lack of correlation with the black hole properties suggest that our observations sample the distribution of episodic quasar lifetimes. Unless these quasars are radiatively highly inefficient (Davies et al. 2019), their black holes must have grown in bursts significantly shorter than the \(\varepsilon\)-folding time-scale \(t_{\varepsilon} \approx 44\) Myr. Such short quasar lifetimes suggest a long quasar duty cycle that is, however, not well constrained given the age of the Universe at \(z < 4\) (\(\approx 1.6\) Gyr). This is different to the situation for \(z \approx 6\) H I proximity zones that are insensitive to quasar on-times \(t_{\text{on}} \gtrsim 0.1\) Myr (Eilers et al. 2017, 2018; Davies et al. 2020), but probe the growth of SMBHs \(\lesssim 1\) Gyr after the Big Bang. If SMBHs of quasars at \(z \approx 6\) accreted similarly to their counterparts at \(z \approx 3\), most of their mass must have been built up during phases of obscuration or radiative inefficiency.

The sensitivity of individual He\(\,\,\nu\) proximity zones to the time-scale of prior quasar activity of up to \(\approx 30\) Myr offers a unique opportunity to constrain the underlying distribution of episodic quasar lifetimes (Khrykin et al. 2021), which can be compared to predictions from models of galaxy and black hole co-evolution. Moreover, we anticipate to more than double the sample of He\(\,\,\nu\)-transparent quasars with precise systemic redshifts in our ongoing joint programme with HST/COS and Gemini/GNIRS (PI Worseck) to further resolve quasar activity on time-scales of several tens of Myr.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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APPENDIX A: MOCK HST/COS He II PROXIMITY ZONE SPECTRA

In our radiative transfer models, the He II proximity zone sizes were determined from high-resolution ($d_r = 11.9$ comoving kpc, $d_V = 0.86-0.93$ km/s$^{-1}$ at $z_{em} = 2.74-3.5$) noise-free He II proximity zone Lyman transmission spectra that had not been degraded to the spectral resolution and quality of the actual HST/COS spectra. We explored the consequences of this simplification with fully forward-modelled mock HST/COS He II proximity zone spectra of two quasars from our sample (Q 1602+576 taken with the G130M grating and SDSS J1253+6817 taken with the G140L grating), analogously to Worseck et al. (2016, 2019). For twelve different combinations of initial He II fraction $x_{HeII,0}$ in $[0.01, 0.05, 0.5, 1.0]$ and quasar on-time $t_{on}$ in $[1, 10, 100]$ Myr twenty model spectra were convolved with the respective HST/COS line-spread functions. Given the actual quasar continuum flux, grating sensitivity, exposure time, background conditions and spectral binning, expected COS counts per pixel were computed from the convolved He II transmission spectra. Realistic COS Poisson counts were simulated as Poisson deviates of the expected counts, and then converted back to He II transmission. Finally, $R_{pz}$ was determined in the same way as for the observed spectra. Redshift error was not included here.

Figure A1 shows the observed HST/COS He II proximity zone spectra and representative mock spectra of Q 1602+576 and SDSS J1253+6817. We chose the combinations of $x_{HeII,0}$ and $t_{on}$ to match the measured values. Apart from the small-scale structure in the proximity zone sourced by the density field, the mock spectra resemble the observed spectra very well. The bottom panels show the relative deviations of the proximity zone sizes determined in the mock spectra ($R_{pz}$) with respect to the ones in the high-resolution noise-free model spectra ($R_{sim}$). For most of the 240 mock spectra per quasar the values are very similar, in particular for the range of quasar on-times our method is sensitive to ($t_{on} < 30$ Myr). For long quasar on-times $t_{on} \sim 100$ Myr, $R_{pz}$ is sometimes significantly larger than $R_{sim}$ due to subtle deviations between the smoothed He II transmission profiles when accounting for the broad wings of the COS line-spread functions and the coarser binning of the COS spectra. In G140L spectra, proximity zone sizes $\lesssim 5$ Mpc are overestimated by up to 10 per cent, primarily due to the sharp drop of the He II transmission profile for short quasar on-times $\lesssim 1$ Myr. However, this bias is smaller than the error in $R_{pz}$ induced by quasar redshift error (Table 1). For larger $R_{sim}$ the bias...
Figure A1. Comparison of observed HST/COS He\textsuperscript{\textsc{i}} quasar proximity zone spectra (top panels, labelled) to representative realistic mock spectra (middle panels). The left (right) panels show COS G130M (G140L) spectra, plotted in grey with statistical 1\sigma Poisson errors. Distances are for the He\textsuperscript{\textsc{i}} Ly\textsubscript{\alpha} transition relative to the quasar at redshift $z_{em}$, with negative distances indicating pixels in the quasar continuum. The violet squares with error bars mark the quasar redshift uncertainties. The blue lines show the flux smoothed with a Gaussian filter with FWHM 1pMpc. The red dots mark the measured $R_{pz}$. The green solid lines in the middle panels show the smoothed high-resolution noise-free He\textsuperscript{\textsc{i}} transmission from our radiative transfer model employed in our MCMC analysis, yielding a different proximity zone size $R_{sim}$ (green dashed). The bottom panels show the relative deviation of $R_{pz}$ with respect to $R_{sim}$ in the radiative transfer models for three values of the quasar on-time $t_{on}$ (labelled). The dashed lines mark zero deviation.

- The proximity zone size is $R_{pz} = (6.10 \pm 0.97)$ pMpc for Q\,1602+576, with a z \textsubscript{em} = 2.8608 \pm 0.0035.
- For SDSS J1253+6817, the proximity zone size is $R_{pz} = (11.40 \pm 0.12)$ pMpc, with a z \textsubscript{em} = 3.4753 \pm 0.0007.

monotonically decreases to $\lesssim 2$ per cent, as the He\textsuperscript{\textsc{i}} proximity zone transmission profile drops more gradually. In COS G130M spectra the proximity zone size is generally not significantly overestimated even at small $R_{sim}$, as expected. The intrinsic scatter of $R_{pz}$ around $R_{sim}$ is small, i.e. $R_{pz}$ is robustly estimated in the COS spectra used in our analysis (see the middle panels of Fig. A1). We verified the robustness of $R_{pz}$ in the lowest-quality spectra (S/N~3).

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