On the search of a potential displacement surface with the use of the local variation method

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Abstract. The purpose of the article is to develop some basic conceptual provisions, algorithm and method of the search of the surface of the smallest resistance to shift, based on the minimization of durability functionality. The algorithm uses the approach, based on the direct iteration of various trial displacement surfaces and the selection of the surface with the minimum functionality value from these surfaces with the method of local variations. The coefficients allowing to estimate the possibility of rocks destruction at various sites of this surface are offered. The results of the search of a potential destruction surface for a pit board are given as an example.

1 Introduction

The displacement of rocks, caused by carrying out excavations and formation of rather big developed spaces is determined by the deformations in the mass as in the continuous environment as well by the destructions which nature and scales generally depend on many mining-and-geological factors.

The authors consider the explosive rocks shift with the formation of the space structure of destruction, localized on the surface of the minimum shift resistance in the created field of tension as the result of some external impacts on the mass. Collapses of abrupt boards of the deep pits and slopes, sudden collapses of the undermined thick rock layers, formation of the main cracks between the extended developed spaces or between them as well as a day surface are connected with such type of destruction.

All this, including the formation of new extended destruction structures, is caused not by concentration of tension in separate zones, but by the general distribution of tension in the rock mass, including the aforesaid zones. In case of such space uncertainty of these structures there is a very complex problem of the search of a surface, which determines the location of possible explosive shift in the intense deformed mass of rocks with strength parameters, characteristic of it. Besides, it is important to define the assessment measure on the possibility of this surface destruction.

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This problem is almost unsolvable without additional hypotheses and the assumptions. Therefore, for example, during assessment of stability of the pits boards and determination of the corresponding weakest surfaces in the rocky breeds the hypothesis of the prism of possible collapses is generally used. The corresponding settlement scheme of the stability assessment of the board is based on the approximate creation of the weakest surface as well as on the separation of the collapse prism, limited to it, into blocks. Each of those blocks is in power interaction and counterbalanced by the corresponding concentrated forces, including the holding force according to the Mohr–Coulomb theory. [1].

Numerous algorithms are based on the ratios of the loose environment mechanics, which with some approximation can be used for the description of strongly jointed mass [2]. The overview of the approaches and corresponding methods is given in the work [3] in which they are generalized for the period of the early 1970th.

Besides, it is possible to specify some methods of calculation of the mining designs stability both in flat, and in space setting. Those methods are developed recently with the use of various numerical methods, such as finite-element method, SPH method, method of boundary elements, etc. [4-8].

2 Methods

Then an essentially new approach to the problem of the search of the surface of space extended destruction structures localization is offered. At the same time the corresponding hypotheses and the assumptions will be naturally used.

The complex of the geo-mechanical tasks, connected with the considered problem, includes two main directions. Firstly, it is the calculation and the corresponding interpretation of the intense deformed condition of the rock mass, both with mined-out spaces, and with natural slopes. As the result the zones of tension and unloading concentration, the zone of primary demultiplexing and destruction of the mass are revealed.

Secondly, the assessment of stability of the basic elements of the mining-and-geological structure or a mining design of the rock mass. This assessment is guided by the use of the intense deformed condition of rock formation, known from the corresponding calculations.

The described conceptual approach is implemented within the linear theory of elasticity. The author can notice that it is undoubtedly fair for the conditions of carrying out developments in rocky formation and at the mined-out spaces, extended in one direction. We do not find it necessary to stop on the detailed justification of justice of such model of the mass acceptance.

Let us consider the integrated criterion of the durability, associated with any rather extended surface in the rock mass. We write it as well as the related surfaces of exposure in the general coordinate plane, coinciding with the section plane in the form of the $ab$ line, which we will designate through $L$. Let us consider that both ends of the line come to the exposure surfaces, is rather smooth, the line has no self-crossings, the beginning is entered in the point $a$ towards $b$. We will call such lines admissible lines of realization of the destruction criterion (Figure 1).

We can write down the durability functionality at this line as follows

$$F_L = \frac{\int [C(l) + \tan \rho(l) \sigma_n(l)] dl}{\int \tau_n(l) dl}$$

where $\sigma_n(l)$, $\tau_n(l)$ are normal and tangent components of tension on the surface platforms along the line $L$, $l \in L$ ; $C(l)$, $\rho(l)$ are сцепления и углы внутреннего трения на тех же площадках. couplings and angles of the internal friction on the same platforms. It is
obvious that the numerator represents the integrated extreme value of the confining forces, and the denominator represents integrated the value of shear forces on line L. The comparison of these integrated forces also makes the physical essence of the functionality (1) as the integrated durability criterion on line L. It is the generalization of the local durability functionality according to Mohr–Coulomb, which can be written down for each point on L as follows:

\[
G_L = \frac{C(l) + \mu \rho(l)\sigma_n(l)}{\tau_n(l)}.
\]  

(2)

**Fig.1.** The trial line in the zone of estimated explosive shift of the rock formation

According to (1) we can state the following. If \( F_L < 1 \) at the chosen line, that the mass cannot obviously keep the integrity and there has to be a violation of the continuity in it on some surface with an outlet to the corresponding exposures.

If \( F_L > 1 \) at the chosen line, that does not mean that there is no another line \( L \), at which \( F_L \) can be less than 1.

In other words, it is necessary to iterate all the admissible lines at the controlled site and to find out whether there are no such \( L \) among them, where \( F_L < 1 \). If at least one such line is found, then there will be a continuity violation which is localized on some surface in the form of extended destruction structure in the mass.

It is obvious that the iteration of all the admissible lines with the same point of the beginning of \( a \) is equivalent to the search of the only line \( L^* \) on which the minimum of functionality is implemented (1). If this minimum \( F^* > 1 \), that the \( F_L \) value cannot be less than 1 at any other line with the same beginning. Thus, the task comes down to the search of the minimum of \( F_L \) functionality and the line on which it is implemented.

During implementation of this search we will take into account that the functionality \( F_L \) continuously depends on \( L \), i.e. a small variation \( F_L \) corresponds to the small variation of the line position. It is almost obvious if tension fields in the mass are continuous in the explored area. The continuity gives the chance to create the numerical multistep algorithm of the search of such line based on a small variation of some trial line. At the same time the trial line at the initial moment is understood as any admissible line \( L \), and then as each subsequent line, which is constantly changing step by step, appearing as a result of the algorithm application.

At the same time each change of the trial line position should be followed by the reduction of \( F_L \) value. Action of the algorithm is as follows:

1 - the \( F_L \) value is calculated on the trial line \( L \);
2 - the trial line varies (the way of the variation just defines the algorithm essence);
3 - the value corresponding to this new line \( L \) is calculated at the line and we will designate that through \( \tilde{F}_L \);
4 - if \( \tilde{F}_L < F_L \), then this line itself becomes trial as \( L \) now and all the procedure (1-4) repeats, but already concerning that;
5 - если \( \tilde{F}_L > F_L \), then this variation is cancelled and there is the return to the trial line \( L \);
6 - other variation of line \( L \) is set and the procedure (1-5) repeats.

Numerical calculations prove that in case of correct variations of the trial lines this algorithm unambiguously leads to an arbitrarily small neighborhood of the line on which the minimum is implemented \( F_L \).

Let us consider in more detail some parts of the numerical algorithm. It is implemented in the form of a set of programs, which are carrying out both the preparatory work, connected with formalization of a task and actually search and analysis of the intense deformed condition of the design areas. At the same time the first trial line, with which the search of potentially possible surface of explosive shift starts is set by the means of \( N \) ordered points and straight lines pieces, connecting these points. Thus, the chosen trial line in the algorithm is the polygonal line at which the initial point \( a \) is "fixed" and does not participate in the variation procedures.

At the same time, it is obviously necessary to avoid ingress to this line of points in which the tension is singular and, naturally, cannot participate in any numerical calculations.

The holding and shear forces are defined for each \( i \)-th link of the polygonal lines, are found as the result of the procedures (1-6) of the algorithm, the extreme, i.e.

\[
T_{Ci} = \int_{l_{i-1}}^{l_i} (C_i + tg\rho_i\sigma_n^i) dl, \quad T_i = \int_{l_{i-1}}^{l_i} \tau_n^i dl, \quad i = 1, \ldots, N. \tag{3}
\]

Since the first trial line and for all the subsequent ones, the values are calculated

\[
F_L = \left( \frac{\sum_{i=1}^{N} T_{Ci}}{\sum_{i=1}^{N} T_i} \right)_{j}, \quad j = 1, 2, \ldots \tag{4}
\]

where \( j \) – sequence number of the trial line.

Meanwhile \( F_L \) make the sequence of the decreasing numbers striving for the extreme value with increase in number of variations \( F_L^* \).

Thus, as a result of numerical implementation of the procedures (1-6) of the algorithm become known the potential surface of possible destruction which is defined by the line \( L^* \), and the corresponding value of the functionality minimum (1). Let us designate it through \( K^* \) and we will call it coefficient of firmness of a potential destruction surface.

The more \( K^* \) in comparison with \( I \), the is more the safety margin. In case if \( K^* \leq I \) the integrated shear force on the potential destruction surface is more than the corresponding shear strength.

Now, when the potential destruction surface is known, and in a flat case there is a corresponding line \( L^* \), it is possible to define in what points and on what sites or links of this line values and corresponding (2) and (3), are more or less than \( I \), and what is the difference.
Proceeding from the definition of \( K^* \), we can state that \( T_{C_i}^*/T_i^* \) - coefficient of the site firmness. Let us designate it through \( K_{C_i}^* \).

Thus, we have an opportunity to estimate the contribution of each part of the line \( L^* \) in the value of the functionality minimum (1) as well as in and in the general condition of the possible destruction surface. At some sites \( K_{C_i}^* \) can be less than 1, and, therefore, they should be exposed to destruction. On the contrary, at other sites, \( K_{C_i}^* \) is more than 1, therefore there is a margin of safety. In such a case the general durability assessment consists of the set of those and other sites.

In general, there is no need that sites with \( K_{C_i}^* < 1 \) blocked the whole extent of line \( L^* \) from one exposure to another for the loss of bearing capacity of the mass. A part of the possible destruction surface can have a safety margin according to the calculations, but it is not sufficient for compensation of loss of bearing capacity at the other part of the surface. Explosive shift at the whole surface will result.

![Fig.2. Convergence of the iterative process](image)

The screen on which the process of approximation to a required surface with a functionality minimum by iteration, starting with some initial provision is shown in Figure 2.

### 3 Results

The results of the numerical realization of the considered approach to the search of the potential destruction surface and the corresponding algorithm are given as an example in Figure 3. The vertical section of a pit wall with the bottom at the depth of 300 m and the decrease in a mined-out space up to the depths of 325, 350 and 375 m is schematically represented in it. 4 lines are built for these depths according to procedures (1-6) of the algorithm in the mass section \( L^* \), on which functionality minima \( F_{C_i}^* \) are implemented with firmness coefficients \( K^* \), equal 1.24, 1.098, 1.035 and 0.941, respectively. Thus, the
potential destruction surfaces with which the lines $L^*$ are connected up to the depth of 350 m have safety margin ($K^* > 1$). At the depth of 375 m which include $L^*$ with the firmness coefficient $K^*$, equal 0.941 the potential destruction surface has no safety margin ($K^* < 1$) and on it, there will be an explosive shift considerably defining the loss of stability of the board in general including decrease in works up to 375 m., beginning from the lower bound of exposure, through all rock mass.

![Schematic representation of a pit wall with a bottom at the depth of 300 m and the decrease in mining operations up to the depths of 325, 350 and 375 m.](image)

$L^*$ - the lines of a minimum of functionality of durability defining the potential surfaces of destruction;
1, 2, 3 – sites of different levels of destruction and safety margin.

In conclusion we will pay attention on the fact that the functionality minimum $F^*$ is connected with the fixed initial point of trial line $L$ in the offered algorithm. The choice of this point is based on the analysis of stress state of rocks on the development analyses as well as on the experience of mining practice. Some uncertainty arising is easily eliminated as a result of comparison of values $K^*$, connected with different, but probable initial points [4].

## 4 Conclusion

During the development of mining operations in the mass and formations of new mined-out spaces there is a continuous redistribution of tension. In many cases the array passes into a
state when in its separate parts there can be destructions which at unstable development of process are localized, forming extended structures in the form of the displacement surfaces. It is especially characteristic in case of loss of stability of a pit wall.

The method of definition of a potential surface of destruction localization in the loaded rock mass is considered. The developed algorithm is received within the traditional idea of one-stage realization of durability on the whole displacement surface; in this connection it is possible to consider fair performance of the generalized destruction criterion which the basis for the algorithm.

The discrete algorithm of the trial surfaces iteration of displacement surfaces is the basis of the offered method. The Change of the is surface is connected with the consecutive small movements of all the points, setting the surface and with tracking of change of the current stability coefficient.

The algorithm of this sort allows to define the surface with the minimum value of stability coefficient of the geo-technical design. The carried-out calculations allowed to establish some stability of the algorithm for the initial configuration of a trial surface which is set randomly enough.

The disadvantage of the algorithm is the fact that the variation of a required surface is made with one motionless point which is usually located close to a special point on tension.

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