Shadowing and antishadowing of the electromagnetic nuclear structure functions are produced by the coherence of multiscattering quark nuclear processes. This picture leads to substantially different antishadowing for charged and neutral current processes, particularly in anti-neutrino reactions, thus affecting the extraction of the weak-mixing angle $\sin^2 \theta_W$.

1 Introduction

There are two experimental motivations for the present work: the results on the comparison of the nuclear versus nucleon electromagnetic structure functions, which show a depletion at very small $x_{Bj}$ and an enhancement at slightly larger values ($x_{Bj} \sim 0.1$), and the NuTeV measurements of the electroweak mixing angle $\sin^2 \theta_W$.

The electroweak mixing angle $\sin^2 \theta_W$ is an essential parameter in the standard model of electroweak interactions. Until recently, a consistent value was obtained from all electroweak observables. However, the NuTeV Collaboration has determined a value for $\sin^2 \theta_W$ from measurements of the ratio of charged and neutral current deep inelastic neutrino–nucleus and anti-neutrino–nucleus scattering in iron targets which has a $3\sigma$ deviation with respect to the fit of the standard model predictions from other electroweak measurements. Although the NuTeV analysis takes into account many sources of systematic errors, there still remains the question of whether the reported deviation could be accounted for by QCD effects. Here we shall investigate whether the anomalous NuTeV result for $\sin^2 \theta_W$ could be due to the different
behavior of leading-twist nuclear shadowing and antishadowing effects for charged and neutral currents.

The physics of the nuclear shadowing in deep inelastic scattering can be most easily understood in the laboratory frame using the Glauber-Gribov picture. The virtual photon, $W$ or $Z^0$, produces a quark-antiquark color-dipole pair which can interact diffractionally or inelastically on the nucleons in the nucleus. The destructive and constructive interference of diffractive amplitudes from Regge exchanges on the upstream nucleons then causes shadowing and antishadowing of the virtual photon interactions on the back-face nucleons. The coherence between processes which occur on different nucleons at separation $L_A$ requires small Bjorken $x_B: 1/M x_B = 2\nu/Q^2 \geq L_A$. An example of the interference of one- and two-step processes in deep inelastic lepton-nucleus scattering is illustrated in Fig. 1. In the case where the diffractive amplitude on $N_1$ is imaginary, the two-step process has the phase $i \times i = -1$ relative to the one-step amplitude, producing destructive interference (the second factor of $i$ arises from integration over the quasi-real intermediate state.) In the case where the diffractive amplitude on $N_1$ is due to $C = +$ Reggeon exchange with intercept $\alpha_R(0) = 1/2$, for example, the phase of the two-step amplitude is $\frac{1}{\sqrt{2}} (1 - i) \times i = \frac{1}{\sqrt{2}} (i + 1)$ relative to the one-step amplitude, thus producing constructive interference and antishadowing. Due to the different energy behavior, this also indicates that shadowing will be dominant at very small $x$ values, where the pomeron is the most important Regge exchange, while antishadowing will appear at a bit larger $x$ values.

![Figure 1: The one-step and two-step processes in DIS on a nucleus. If the scattering on nucleon $N_1$ is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\bar{q}$ flux reaching $N_2$. This causes shadowing of the charged and neutral current nuclear structure functions.](image)

2 Parameterizations of quark-nucleon scattering

We shall assume that the high-energy antiquark-nucleon scattering amplitude $T_{\bar{q}N}$ has the Regge and analytic behavior characteristic of normal hadronic amplitudes. Following the model of Ref. 6, we consider a standard Reggeon at $\alpha_R = 1/2$, an Odderon exchange term, a pseudoscalar exchange term, and a term at $\alpha_R = -1$, in addition to the Pomeron-exchange term.

The Pomeron exchange has the intercept $\alpha_P = 1 + \delta$. For the amputated $\bar{q} - N$ amplitude $T_{\bar{q}N}$ and $q - N$ amplitude $T_{qN}$ with $q = u, d$, $N = p, n$, we assume the following parameterization, including terms which represent pseudoscalar Reggeon exchange. The resulting
amplitude is:

\[
T_{\bar{u}-p} = \sigma \left[ s \left( i + \tan \frac{\pi \delta}{2} \right) \beta_1(\tau^2) - s\beta_0(\tau^2) - (1 - i)s^{1/2}\beta_{1/2}^{\theta^+}(\tau^2) \right. \\
+ (1 + i)s^{1/2}\beta_{1/2}^{\theta^0}(\tau^2) - (1 - i)s^{1/2}\beta_{1/2}^{\theta^1}(\tau^2) + W(1 - i)s^{1/2}\beta_{1/2}^{\text{pseudo}}(\tau^2) \\
+ (1 + i)s^{1/2}\beta_{1/2}^{\theta^1}(\tau^2) + is^{-1}\beta_{-1}^{-1}(\tau^2) \right].
\]

Other quark amplitudes are obtained from this one. The isovector piece changes sign going to the \(d\) quark amplitude, and the odd \(C\) terms also change sign when one deals with the corresponding antiparticle amplitude. We also include strange quarks, with similar expressions. Although the phases of all these Regge exchanges are fixed, their strengths are taken as parameters, whose values are obtained from proton and neutron structure functions data and known quark distribution parameterizations, which in the model are related to the above amplitudes through:

\[
xq^{N_0}(x) = \frac{2}{(2\pi)^3} \frac{Cx^2}{1 - x} \int dsd^2k \Im T_{N_0}(s, \mu^2).
\]

3 Nuclear shadowing and antishadowing effects due to multiple scattering

Now let us turn to the scattering on a nuclear (\(A\)) target. The \(\bar{q} - N\) scattering amplitude can be obtained from the \(\bar{q} - N\) amplitude according to Glauber’s theory as follows,

\[
T_{\bar{q}A} = \sum_{k_1, k_2} \sum_{k_1 + k_2} \frac{1}{k_1 + k_2} \left( \frac{Z + N}{k_1 + k_2} \right) \frac{1}{M} \alpha^{k_1 + k_2 - 1} (T_{\bar{q}p})^{k_1} (T_{\bar{q}n})^{k_2} \theta(k_1 + k_2 - 1)
\]

where \(M = \text{Min}\{k_1 + k_2, Z\} - \text{Max}\{k_1 + k_2 - N, 0\} + 1\) and \(\alpha = i/(4\pi p_{c.m.} s^{1/2}(R^2 + 2b))\), with \(R^2 = \frac{2}{3} R_0^2\), \(R_0 = 1.123A^{1/3}\) fm, and \(b = 10 (\text{GeV}/c)^{-2}\). Then the nuclear quark distributions are readily obtained, and we get predictions for the ratio of structure functions \(F_2^A/F_2^N\), which gives an excellent description of the experimental data, showing both shadowing and antishadowing. Furthermore, it agrees remarkably well with the data for the ratio \(R_{\mu\nu}^\text{neutrino}/R_{\mu\nu}^\text{muon}\). For details see Ref. 5.

We can now apply this same nuclear distribution functions to neutrino deep inelastic scattering in nuclei. Our results show that shadowing is similar for electromagnetic and weak currents, but that there is a much stronger antishadowing effect for antineutrinos, and that neutral and charge currents give different antishadowing. Since in our nucleon quark distributions parametrization there is still some freedom, especially in the strange quark case, the details of these results could change, but certainly not the overall picture, which shows a substantially different antishadowing for charged and neutral current reactions.

4 Nuclear effects on extraction of \(\sin^2 \theta_W\)

The observables measured in neutrino DIS experiments are the ratios of neutral current to charged current events; these are related via Monte Carlo simulations to \(\sin^2 \theta_W\). In order to examine the possible impact of nuclear shadowing and antishadowing corrections on the extraction of \(\sin^2 \theta_W\), we will consider the Paschos and Wolfenstein relation

\[
R_N^\pm = \frac{\sigma(\nu_{\mu} + N \rightarrow \nu_{\mu} + X) - \sigma(\bar{\nu}_{\mu} + N \rightarrow \bar{\nu}_{\mu} + X)}{\sigma(\nu_{\mu} + N \rightarrow \mu^- + X) - \sigma(\bar{\nu}_{\mu} + N \rightarrow \mu^+ + X)} = \rho_0^2 \left( \frac{1}{2} - \sin^2 \theta_W \right),
\]
and a similar expression for the case in which there is a nuclear target \((N \rightarrow A)\). We estimate the nuclear effects on the weak mixing angle in the following way. First, we use the cross sections to calculate the Paschos-Wolfenstein ratios \(R_A^-(\sin^2 \theta_W)\) and \(R_N^-(\sin^2 \theta_W)\) for various values of \(\sin^2 \theta_W\), and using the NuTeV cutoffs. Second, we extract \(\rho^2\) by means of Eq. (4). We find a weak dependence of \(\rho^2\) on \(\sin^2 \theta_W\) and \(\rho^2\) very close to 1. Finally, we use the obtained \(\rho^2\) to extract the shadowing/antishadowing effect on the weak-mixing angle \(\Delta \sin^2 \theta_W\) from the modified relation:

\[
R_A^-(\sin^2 \theta_W) = \rho^2 \left( \frac{1}{2} - (\sin^2 \theta_W + \Delta \sin^2 \theta_W) \right).
\]

We have found that the nuclear modification to the weak-mixing angle is approximately \(\delta \sin^2 \theta_W = 0.001\). The value of \(\sin^2 \theta_W\) determined from the NuTeV experiment, without including nuclear shadowing/antishadowing due to multiple scattering, is in absolute value 0.005 larger than the best value obtain from other experiments. The model used here to compute nuclear shadowing/antishadowing effect would reduce the discrepancy between the neutrino and electromagnetic measurements of \(\sin^2 \theta_W\) by about 20%, although this could be made larger by modifying the strange quark distribution, which at present is not very well known.

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