Asymptotic quasinormal modes of scalar field in a gravity’s rainbow

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Abstract

In the context of a gravity’s rainbow, the asymptotic quasinormal modes of the scalar perturbation in the quantum modified Schwarzschild black holes are investigated. By using the monodromy method, we calculated and obtained the asymptotic quasinormal frequencies, which are dominated not only by the mass parameter of the spacetime, but also by the energy functions from the modified dispersion relations. However, the real parts of the asymptotic quasinormal modes is still $T_H \ln 3$, which is consistent with Hod’s conjecture. In addition, for the quantum corrected black hole, the area spacing is calculated and the result is independent of the energy functions, in spite of the area itself is energy dependence. And that, by relating the area spectrum to loop quantum gravity, the Barbero-Immirzi parameter is given and it remains the same as from the usual black hole.

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I. INTRODUCTION

As is well known, the response of black hole background to the initial configuration is dominated by the quasinormal modes (QNMs) - a set of complex frequencies which only depend on the parameters of the black hole rather than the initial perturbation (see [1, 2] for reviews). The characterization of QNMs can be play a important role in the observation of black hole in the university. Meanwhile, the properties of QNMs can be used to investigate the black hole stability. In addition, QNMs has get interpretation in Conformal Field Theory [3, 4, 5, 6, 7, 8]. And that, the asymptotic QNMs can be related to the quantum theory of black hole and quantum geometry [9, 10]. That is, motivated by Hod’s conjecture [9], it is suggested that the value of asymptotic QNMs may carry important information about the quantization of black hole area and help to fix certain parameters in quantum geometry [10].

Based on the numerically results of QNMs [11], Hod suggested that the real parts of highly damped QNMs in the Schwazchild black hole can be expressed as \( \omega_R = T_H \ln 3 \) [9]. And that, by using Bohr’s correspondence principle, the real part could be identified as the characteristic transition frequency for the black hole and the fundamental quanta of black hole area as \( l_p^2 4 \ln 3 \) can be obtained [9, 10]. On the other hand, as a main candidate theory of quantum gravity, loop quantum gravity (LQG) (see [12, 13] for reviews) has a remarkable result-the discrete area spectra. However, there is an unknown 'natural constant' called the Barbero-Immirzi parameter and the area spectra have an ambiguity. Using the area quanta of black hole, Dreyer presented the free parameter and proposed that LQG should be based on \( SO(3) \) rather than \( SU(2) \) [10]. Thus, one hope that the asymptotic QNMs will have a role in the quest for quantum geometry, especially for quantum properties of black hole.

For analytically compute the asymptotic value of the QNMs and confirm the Hod’s conjecture, the monodromy method of investigating quasinormal modes was proposed [15]. In the method, the radial coordinate is analytically continued into the complex plane and the properties of the potential and the tortoise coordinate around the origin, the horizons and the infinity are used. The obtained asymptotic QNMs present an analytical proof on the Hod’s conjecture for scalar and some gravitational perturbations in Schwarzschild black hole. In the lectures [15, 16, 17, 18, 19, 20, 21, 22, 23, 24], the monodromy technique has been extend to many cases such as different perturbations like Dirac field and electromagnetic field in more general static black holes and the asymptotic QNMs have been obtained. However,
the researches imply that whether the argument $\omega_R = T_H \ln 3$ applies to universal cases is still a open problem and much less attention was paid to the case of analytically calculating the asymptotic QNMs in quantum corrected black hole [22].

It is believed that, with quantum gravity effects, a discrete picture of spacetime will emerge and a new physics will be necessary. In [25, 26], the Planck scale corrected spacetime named as gravity’s rainbow has been presented. The main feature of the gravity’s rainbow is that the geometry of spacetime depend on the energy of a particle moving in it. That is to say, for the spacetime with Planck scale correction effects, there are different geometries for probe particles with different energies. The modified geometries of spacetime can be described by one parameter family of metric as a function of particle’s energy observed by an inertial observer. Moreover, the modified Schwarzschild solution in the gravity’s rainbow has been given [26] and it’s some thermodynamics quantities and asymptotic flatness have been investigated [27, 28].

Here, we investigated the asymptotic QNMs of massless scalar field in the modified Schwarzschild spacetime from the gravity’s rainbow. Our main aim is to verify the influence of spacetime’s quantum effects on the asymptotic QNMs and to test the Hod’s conjecture and it’s implies in the quantum corrected spacetime. By using the monodromy method, we analytically calculate the asymptotic quasinormal frequencies in the quantum corrected spacetime. The results show that, when the Plank scale modification of spacetime is taken into account, the asymptotic QNMs depend on not only the mass parameter of the black hole but also the energy functions, by which the quantum effects of spacetime are reflected. However, the real part of the asymptotic QNMs can still be expressed as $T_H \ln 3$ and hence the Hod’s conjecture is valid for the gravity’s rainbow. In addition, in the quantum corrected spacetime, some quantum implies [9, 10] of the Hod’s conjecture are verified. The area spacing of the quantum corrected black hole is calculated and the free parameter of LQG is presented. The obtained results are independent of the energy functions and remain the same as from the usual Schwarzschild black hole [9, 10].

The paper is organized as follows. In Sec. II the modified Schwarzschild solution from the gravity’s rainbow is introduced briefly. Then in Sec. III in the monodromy method framework, the asymptotic QNMs of massless scalar field in the quantum corrected spacetime are calculated and obtained analytically. Section IV for the quantum corrected black hole, Hod’s conjecture and it’s implies in quantum theory of black hole and LQG are verified.
The last part is the summary and conclusions.

II. GRAVITY’S RAINBOW AND MODIFIED SCHWARZSCHILD BLACK HOLES

Let us firstly briefly introduce the modified Schwarzschild solution from the gravity’s rainbow. When keeping Planck energy as an invariant scale, namely a universal constant for all inertial observers, to preserve the relativity of inertial frames, the double special relativity (DSR) has been proposed \[29, 30, 31, 32, 33\]. The starting point and the main result of DSR is the modified dispersion relation (MDR) as

\[ E^2 f_1^2 (E; \lambda) - p^2 f_2^2 (E; \lambda) = m_0^2, \]  

where \( f_1 \) and \( f_2 \) are two energy functions from which rotational symmetry can be preserved, \( \lambda \) is a parameter of order the Planck scale. The equation Eq.(1) shows that, MDR is energy dependent. It is to say, particles with different energy \( E \) have different energy-momentum relations. However, in the low energy realm i.e. \( E/E_p << 1 \), \( f_1 \) and \( f_2 \) approach to unit and MDR can return to the usual energy-momentum relation, where \( E_p \equiv 1/\sqrt{8\pi G} \) is the Planck energy. This is consistent with Bohr’s correspondence principle.

In the context of DSR, the deformed spacetime geometry has been investigated and some proposals have been presented \[25, 26, 35, 36, 37, 38\]. By the \[25, 26\], it is put forwarded that the flat spacetime with Planck scale corrections has the invariant as

\[ ds^2 = -\frac{dt^2}{f_1^2} + \frac{dr^2}{f_2^2} + r^2 d\Omega^2. \]  

This equation indicates that, the DSR spacetime depend on the energy of particle moving in it. That is to say, particles with different energy will probe different DSR spacetimes. Thus, the DSR spacetime is endowed with an energy dependent quadratic invariant, that is, an energy dependent metric, namely rainbow metric. Just like the DSR, when particle’s energy is a little quantity comparing with the Planck energy, the rainbow metric will turn to the usual flat spacetimes. And that, it has been pointed that the DSR spacetimes has equality with the usual flat spacetimes \[28\].

By extending the Eq.(2) to incorporate curvature, in \[26\], a gravity’s rainbow has been presented and a few quantum corrected spacetimes from the gravity’s rainbow have been
given. Thereamong, the modified Schwarzschild solution can be expressed in terms of energy independent coordinates and the energy independent mass parameter as
\[
\label{eq:metric}
dS^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \frac{1}{f_2^2 \left(1 - \frac{2GM}{r}\right)} dr^2 + r^2 d\Omega^2.
\]

Obviously, the metric of the modified Schwarzschild black hole is also energy dependent. That is, if a given observer probes the spacetime using the quanta with different energies, he will conclude that spacetime geometries have different effective description. Here, the particle’s energy \(E\) denotes it’s total energy measured at infinity from the black hole. In addition, the energy dependence of the gravity’s rainbow probably may has relation to the property of background independent of LQG \[12, 13\]. In fact, as a semi-classical limit, the MDR has been suggested by LQG \[39, 40, 41, 42\]. Thus, as a gravity’s rainbow, the present spacetime is endowed with Plank-scale modifications, i.e., quantum effects.

III. ASYMPTOTIC QUASINORMAL MODES

Now, in the monodromy method framework \[15\], we analytically calculate the asymptotic QNMs of massless scalar field in the modified Schwarzschild from the gravity’s rainbow. Firstly, as in the usual Schwarzschild black hole \[34\], by using the Klein-Gordon equation, we give the perturbation equation for the scalar field \(\phi\) in the quantum corrected spacetime. Then, substituting Eq.\ref{eq:metric} into the Klein-Gordon equation
\[
\label{eq:KG}
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x_\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x_\nu}\right) \phi = 0,
\]
and using the ansatz of separation of variables \(\phi = \frac{1}{r} e^{i\omega t} \psi(r) Y(\theta, \varphi)\), the radial perturbation equation can be given as
\[
\left(r^2 - 2GMr\right) \frac{\partial^2 \psi(r)}{\partial r^2} + 2GM \frac{\partial \psi(r)}{\partial r} + \left[r^2 f_2^2 \left(1 - \frac{2GM}{r}\right)^{-1} \omega^2 - \frac{2GM}{r^3} - \frac{l(l+1)}{r^2}\right] \psi(r) = 0,
\]

where \(l\) is the orbital angular momentum.

For convenience, we define the tortoise coordinate of the quantum modified black hole as
\[
x = \int \frac{dr}{h(r)} = \frac{f_1}{f_2} \left[r + 2GM \ln \left(\frac{r}{2GM} - 1\right)\right],
\]
with
\[
h(r) = \frac{f_2}{f_1} \left(1 - \frac{2GM}{r}\right).
\]
Obviously, at the infinity, we have \( x \rightarrow +\infty \). And that, in the vicinity of \( r = 0 \), we have
\[
h(r) \sim -\frac{f_2}{f_1} \frac{2GM}{r}
\]
and the tortoise coordinate takes the form
\[
x \sim -\int \frac{f_1}{f_2} \frac{r}{2GM} dr = -\frac{f_1}{f_2} \frac{r^2}{4GM}.
\]
(8)

In addition, from the metric Eq.(3), it is seen that the event horizon of the modified black hole is at
\[
r_+ = 2GM.
\]
(9)

Then, near the event horizon, we have
\[
h(r) \sim (r - r_+) h'(r_+) = \frac{f_2}{f_1} \frac{2GM}{r_+} \quad \text{and} \quad x \sim \frac{f_2}{f_2} 2GM \ln (r - 2GM).
\]

This shows that, at the horizon, we have \( x \rightarrow -\infty \).

Thus, substituting Eq.(6) into Eq.(5), the radial wave equation is translated into
\[
\frac{d^2\psi(r)}{dx^2} + [\omega^2 - V(r)] \psi(r) = 0,
\]
where \( V(r) \) is the effect potential with
\[
V(r) = \frac{f_2^2}{f_1^2} \left( 1 - \frac{2GM}{r} \right) \left[ \frac{l(l+1)}{r^2} + \frac{2GM}{r^3} \right].
\]
(11)

We see that, in the quantum corrected black hole, the effective potential depend not only the radial coordinate and the angular quantum number but also the energy functions. This shows that the effective potential has some Plank-scale modifications. But, as in the usual black hole [14], while \( x \rightarrow \pm \infty \), we have \( V(r) \rightarrow 0 \), then the Eq.(10) can be translated into the standard wave equation in terms of the tortoise coordinate Eq.(6). That is, the solutions of Eq.(10) behave as \( \psi(x) \sim e^{\pm i\omega x} \) at the horizon and infinity.

As the same in the usual black hole, the QNMs in the modified black hole can be supposed to be the solutions of perturbation equation Eq.(10) with the boundary conditions of purely ingoing waves at event horizon and purely outgoing waves at infinity, namely
\[
\psi(x) \sim e^{+i\omega x}, \quad x \rightarrow -\infty,
\]
(12)
\[
\psi(x) \sim e^{-i\omega x}, \quad x \rightarrow +\infty.
\]
(13)

Then, to analytically obtain the asymptotic QNMs for scalar field in the modified Schwarzschild black hole, we calculate the differential equation Eq.(10) by using the monodromy method. In the technology, it is essential to extend analytically Eq.(10) from it’s physical region \( r_+ < r < \infty \) to the whole complex \( r \)-plane. By the extension, it is find that
The solutions $\psi(r)$ behave multivaluedness around the singular points $r = 0$ and $r = r_+$ and that the asymptotic QNMs will be obtained by computing the monodromy around a chosen contour in the complex plane. As in [15], we put a branch cut from $r = 0$ to $r = r_+$ and the monodromy of $\psi(r)$ can be defined by the discontinuity across the cut. Moreover, from Eq.(6), the Stokes line $\text{Re}(x) = 0$ in the modified black hole can be obtained and the contour $L$ can be chosen as in figure 1.

We see that, inside the contour, the only singularity of $\psi(r)$ is the event horizon. According to the boundary condition Eq.(12) and the tortoise coordinate Eq.(6), we can obtain that the monodromy of $\psi(r)$ around the contour $L$ is the same as the monodromy of $e^{i\omega x}$ and must be

$$e^{\frac{\pi \omega}{\kappa}},$$

where $\kappa$ is the surface gravity on the event horizon. For the modified black hole shown as Eq.(3), $\kappa$ can be obtained by

$$\kappa = -\frac{1}{2} \lim_{r \to r_+} \frac{1}{r} \frac{\partial g_{tt}}{\partial r} = \frac{f_2}{f_1} \frac{1}{4GM}. $$

Then, the Hawking temperature can be given as

$$T_H = \frac{\kappa}{2\pi} = \frac{f_2}{f_1} \frac{1}{8\pi GM}. $$

We see that the surface gravity and the Hawking temperature of the modified Schwarzschild black hole are all depend on the energy of probe particle. That is, using the quanta with different energy, an observer at infinity will probe different effective temperature and surface gravity for the quantum corrected black hole. It is easy to verify that, in the context of the gravity’s rainbow, the energy dependence of surface gravity and the temperature ought to arises from MDR and should be the exhibition of quantum effects of the spacetime. And that, this should have some modification effects on the black hole physics, for example the QNMs.

Next, we compute the local monodromy of $\psi(x)$ around the same contour. For $r \sim 0$, from Eqs.(8) and (11), we have

$$V(r) \sim -\frac{1}{4x^2}. $$

It is found that, in the vicinity of $r = 0$, the potential has the simply form as in the usual Schwarzschild black hole [15]. Here, we could consider that the energy dependence of $V(r)$ is involved in the tortoise coordinate.
Similarly to [15], for using Bessel’s equation, we first rewrite the potential to the form

$$V(r) \sim -\frac{1-j^2}{4x^2},$$

(18)

where $j$ is the spin of the perturbation field. For our interesting, the results for the scalar perturbation could be obtain from the case $j = 0$. Thus, near $r = 0$, we obtain the perturbation equation

$$\left( \frac{\partial^2}{\partial x^2} + \omega^2 + \frac{1-j^2}{4x^2} \right) \psi(x) = 0,$$

(19)

and its solutions can be expressed as [15]

$$\psi(x) \sim B_+ \sqrt{2\pi \omega x} J_{\frac{1}{2}}(\omega x) + B_- \sqrt{2\pi \omega x} J_{-\frac{1}{2}}(\omega x),$$

(20)

where $B_{\pm}$ are integral constants and $J_\nu$ represents the Bessel function of order $\nu$.

For the asymptotic QNMs, $\omega$ is approximately a purely imaginary and hence the line $\text{Im}(\omega x) = 0$ is almost the same as the line $\text{Re}(x) = 0$. Then, on the contour $L$, $x \to +\infty$ can actually rotated to $\omega x \to +\infty$ and the boundary condition Eq.(13) can be expressed as

$$\psi \sim e^{-i\omega x}, \omega x \to +\infty.$$

(21)

And that, along the contour $L$, it is convenient to obtain the asymptotic forms of $\psi(x)$ away from the origin $r = 0$ to $\omega x \to +\infty$. For $z \gg 1$, considering the asymptotic behavior

$$J_\nu(z) = \sqrt{\frac{2}{\pi z}} \cos \left( z - \frac{\nu \pi}{2} - \frac{\pi}{4} \right),$$

(22)

we obtain

$$\psi(x) \sim 2B_+ \cos (\omega x - \alpha_+) + 2B_- \cos (\omega x - \alpha_-)$$

$$= \left( B_+ e^{-i\alpha_+} + B_- e^{-i\alpha_-} \right) e^{i\omega x} + \left( B_+ e^{i\alpha_+} + B_- e^{i\alpha_-} \right) e^{-i\omega x},$$

(23)

where the phase shifts $\alpha_{\pm} = \frac{\pi}{4}(1 \pm j)$. Thus, from the boundary condition Eq.(21), it is seen that, as $\omega x \to +\infty$, the $B_{\pm}$ must satisfy

$$B_+ e^{-i\alpha_+} + B_- e^{-i\alpha_-} = 0,$$

(24)

and the solution $\psi(x)$ take the form

$$\psi(x) \sim \left( B_+ e^{i\alpha_+} + B_- e^{i\alpha_-} \right) e^{-i\omega x}.$$

(25)
Now, along the contour \( L \), let us from point \( A \) approach to point \( B \). In the process, we must turn an angle \( 3\pi/2 \) around the origin, i.e., \( 3\pi \) around \( x = 0 \). As well known, for \( z \sim 0 \), the Bessel functions satisfy

\[
J_\nu (z) = z^\nu w(z),
\]

(26)

where \( w(z) \) is an even holomorphic function. Then, after the rotation, we have

\[
\sqrt{2\pi}e^{i3\pi\omega x}J_{\frac{3}{2}} \left(e^{3\pi \omega x}\right) \sim 2e^{i6\alpha\pm} \cos (-\omega x - \alpha_{\pm}),
\]

(27)

and the solutions for \( \omega x \to -\infty \) can be obtained as

\[
\psi(x) \sim 2B_+e^{6i\alpha+} \cos (-\omega x - \alpha_{+}) + 2B_-e^{6i\alpha-} \cos (-\omega x - \alpha_{-})
= (B_+e^{7i\alpha+} + B_-e^{7i\alpha-})e^{i\omega x} + (B_+e^{5i\alpha+} + B_-e^{5i\alpha-})e^{-i\omega x}.
\]

(28)

Next, we continue along the contour \( L \) in the right half-plane. In the large semicircle of \( L \), the term \( \omega^2 \) dominates the potential \( V \) and hence the solutions \( \psi(x) \) of the perturbation equation Eq.(19) can be approximated as plane waves. By this way, when we complete the contour, the coefficient of \( e^{-i\omega x} \) in the \( \psi(x) \) can be remain unchanged and the coefficient of \( e^{i\omega x} \) makes only an exponentially small contribution to the solutions. Thus, when we return to the point \( A \) from \( B \), the coefficient of \( e^{-i\omega x} \) gets multiplied by

\[
\frac{B_+e^{5i\alpha+} + B_-e^{5i\alpha-}}{B_+e^{i\alpha+} + B_-e^{i\alpha-}}.
\]

(29)

In addition, similarity to Eq.(14), the monodromy of \( e^{-i\omega x} \) around the contour is \( e^{-\frac{\pi \omega}{k}} \). Thus, for the solutions \( \psi(x) \), to have the required monodromy of Eq.(14), we get

\[
\frac{B_+e^{5i\alpha+} + B_-e^{5i\alpha-}}{B_+e^{i\alpha+} + B_-e^{i\alpha-}}e^{-\frac{\pi \omega}{k}} = e^{\frac{\pi \omega}{k}}.
\]

(30)

Considering Eq.(24), the above equation can be translated into

\[
e^{\frac{2\pi \omega}{k}} = \frac{e^{6i\alpha+} - e^{6i\alpha-}}{e^{2i\alpha+} - e^{2i\alpha-}} = -(1 + 2 \cos \pi j).
\]

(31)

Therefore, letting \( j = 0 \) and considering Eq.(15), for scalar perturbation in the modified Schwarzschild black hole from the gravity’s rainbow, the asymptotic QNMs are obtained as

\[
\omega = \frac{k}{2\pi} \ln 3 + ik \left(n + \frac{1}{2}\right) = \frac{f_2}{f_1} \frac{\ln 3}{8\pi GM} + i\frac{f_2}{f_1} \frac{1}{4GM} \left(n + \frac{1}{2}\right),
\]

(32)

where \( n \to \infty \).
Comparing the asymptotic QNMs with the results from the usual Schwarzschild black hole \cite{15}, it is find that the present asymptotic quasinormal frequencies depend not only on the mass parameters of the black hole, but also on the energy functions $f_1$ and $f_2$. This show that, the result obtained here has get quantum corrections exhibited as the dependence on the energy of probe particle. However, in the quantum corrected black hole, the real part of asymptotic quasinormal frequencies is still $T_H \ln 3$ and hence consistent with Hod’s conjecture. And that, for the case of low energy, the asymptotic quasinormal frequencies can return to the value in the usual Schwarzschild black hole \cite{15}.

IV. AREA SPECTRUM

In the section, as for the usual black hole \cite{9, 10}, we verify the area spectrum of the quantum corrected black hole and it’s implies in LQG. Based on Bohr’s correspondence, Hod argue that \cite{9}, the process of highly damped QNMs of black hole should be identical to the quantum transition of the corresponding system and the real part of the asymptotic QNMs should be equal to the quantum transition frequency. Then, in the modified black hole, with the radiated wave shown as Eq. (32), the energy of the corresponding radiation quanta is obtained as

$$E = \hbar \text{Re}(\omega) = \frac{f_2}{f_1} \frac{\hbar}{8\pi G M} \ln 3.$$  \hspace{1cm} (33)

In other hand, from Eq. (3), we can see the modified Schwarzschild solution from the gravity’s rainbow is asymptotically DSR. And that, the asymptotically DSR spacetimes has equality with the asymptotically flat spacetimes \cite{28}. Then, using the Komar integrals, we define the total Arnowitt-Deser-Misner (ADM) mass for the quantum corrected spacetime as

$$M_{\text{ADM}} = -\frac{1}{8\pi G} \int \varepsilon_{abcd} \nabla^c \xi^d = \frac{M}{f_1 f_2}.$$ \hspace{1cm} (34)

We find that, for the modified Schwarzschild black holes, the ADM mass is not equal to the mass parameter $M$. This shows that the quantum corrected spacetime have topological defects. Moreover, the total energy of the spacetime depend on the energy of the probe particle. Obviously, this is different from the usual Schwarzschild black hole, in which $M_{\text{ADM}} = M$. And that, the energy property for the modified black hole should arise
from the quantum effects of the spacetime, which is the gravity’s rainbow with the energy dependence.

Then, in the quantum modified black hole, after the radiation quanta shown in Eq. (33), the change of the mass parameter of the black hole is obtained by

$$\Delta M = f_1 f_2 E = f_2^2 \frac{\hbar}{8\pi GM} \ln 3. \quad (35)$$

In addition, from Eqs. (3) and (9), the event horizon area of the quantum corrected black hole can be obtained as

$$A = \int (g_{\theta\theta} g_{\phi\phi})_{r_+} d\theta d\varphi = \frac{16\pi G^2 M^2}{f_2^2}. \quad (36)$$

It should been pointed that, despite the event horizon Eq. (9) is universal for all observers and at the same place as the usual Schwarzschild black hole, the horizon area of the gravity’s rainbow is energy dependence and different from the value of the usual black hole.

Thus, for the quantum modified black hole, the area change corresponding to the radiation quanta from the asymptotic QNMs is obtained as

$$\Delta A = \frac{1}{f_2^2} 32\pi G^2 M \Delta M = 4 \ln 3 l_p^2, \quad (37)$$

with the Planck length $l_p = \sqrt{\frac{\hbar}{G}}$. We find that, for the quantum modified black hole, the area spacing is the same as the usual black hole [9, 10] and is all the same for all the observers, despite the horizon area itself is different from the value of the usual black hole and for different observers. This should be investigated future.

As mentioned in Sec. II, the gravity’s rainbow may has some consistency with LQG. Next, as in [10], we relate the area spectrum of the gravity’s rainbow to quantum geometry. In LQG, the area of a black hole is quantized and the area element is given as [12, 13, 43]

$$A(j_{\text{min}}) = 8\pi l_p^2 \sqrt{j_{\text{min}}(j_{\text{min}} + 1)}, \quad (38)$$

where $\gamma$ is the so-called Barbero-Immirzi parameter and $j_{\text{min}}$ the smallest spin of the spin network intersecting the event horizon. Then, under the argument [10]

$$A(j_{\text{min}}) = \Delta A, \quad (39)$$

the $\gamma$ can be expressed as

$$\gamma = \frac{\ln 3}{2\pi \sqrt{j_{\text{min}}(j_{\text{min}} + 1)}}. \quad (40)$$
In addition, black hole entropy from loop quantum gravity is \[ S = N \ln(2j_{\text{min}} + 1), \] where \( N = \frac{A}{A_{\text{min}}} \) is the number of area element on the event horizon. Then, considering Eq. (39), we have
\[ S = \frac{A \ln(2j_{\text{min}} + 1)}{4l_p^2 \ln 3}. \]
Thus, letting the black hole entropy equal to the Bekenstein-Hawking entropy as
\[ S_{bh} = \frac{A}{4l_p^2}, \]
the smallest spin value is presented as
\[ j_{\text{min}} = 1. \]
The result is the same as obtained from the usual black hole and suggested that LQG should be based on \( SO(3) \) \cite{10}. And that, substituting the spin value into Eq.(40), the Barbero-Immirzi parameter can be fixed as
\[ \gamma = \frac{\ln 3}{2\pi \sqrt{2}}. \]
It is seen that, as the smallest spin value, the Barbero-Immirzi parameter has not the dependence on the probe particle and as the same as the results from the usual black hole \cite{10}. Obviously, as a elementary parameter of LQG, it’s independence on observer and spacetime background is satisfactory.

V. SUMMARY AND CONCLUSION

In the present work, the asymptotic QNMs in the quantum modified Schwarzschild black holes from the gravity’s rainbow are investigated. By using the monodromy method, the asymptotic QNMs for massless scalar field in the quantum corrected spacetime are analytically calculated and obtained. The results show that the highly damped quasinormal frequencies depend not only on the mass parameter of the black hole, but also on the energy of probe particle. Comparing with the results from the usual Schwarzschild black hole, in which the asymptotic QNMs only depend on the mass parameter \cite{15}, the energy dependence of the presented asymptotic QNMs should be a quantum effect and come from the quantum
corrections of the background spacetimes. This is a novel property of the asymptotic QNMs in the gravity’s rainbow and should be proved by other cases, such as different field in different gravity’s rainbow. It is also find that, in the context of gravity’s rainbow, the quantum effect of the asymptotic quasinormal frequencies is involved in the Hawking temperature and the real part of asymptotic QNMs is still $T_H \ln 3$. Thus, in support Hod’s conjecture[9], the present reach provide further evidence from a quantum corrected spacetime. In addition, by relating Hod’s conjecture to quantum theory of black hole [9, 10], the area quantization of the quantum modified black hole is investigated and the area spacing is obtained. The result is energy independent and is the same as coming from the usual Schwarzschild black hole [9, 10], despite the area itself is energy dependent and different from the value from the usual black hole. And that, in the present quantum corrected spacetime, by relating the area quantization to LQG [10], the smallest spin value and the free parameter in LQG are given. Also, the obtained results are energy independent and are the same as in the usual black holes [10]. The feature of the obtained area spectrum and the elementary parameter being independent of observer and background metric is in good agreement with expectation and should be verified in future.

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FIG. 1: The Stokes line $\text{Re}(x) = 0$ and the contour $L$ for the modified Schwarzschild black hole from the gravity's rainbow. The area of $\text{Re}(x) < 0$ is denoted by the regions with the hachures.