Can the Miraculous Mass Relation in the $P$-wave Spectroscopy of Charmonia and Bottomonia be extended to $B_c$ Mesons

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The mass relation $M_{l+} + 3M_{1+} + 5M_{2+} = 9M_{l+}$ miraculously holds for the charmonium and bottomonium systems with very high precision. Thus, this empirical fact may imply the existence of some theoretical and/or symmetry principles which have not been recognized yet. Based upon Quark Models, we propose $M_{l+} + 5M_{2+} = 3(M_{1+,1} + M_{1+,2})$ as an extension to the $P$-wave $B_c$ case. Our numerical analysis confirms its validity up to very small deviations. In fact, the soundness of this extension strengthens our confidence about existence of an underlying hidden principle in the field of hadron physics.

I. INTRODUCTION

As it is well recognized, there are many puzzles and anomalies in hadron physics which have not received satisfactory explanations so far: e.g. predicted states not yet observed, their computed masses, mass relations and decay patterns, etc. On a primary level, it hints that our knowledge on Quark Models (QM), which have played important roles in predicting and describing the hadron spectrum for decades [1–4], may possesses loopholes. Modern hadron physics suggests that the answers may emerge from our better understanding of non-perturbative Quantum Chromodynamics (QCD) [5–7]. Nevertheless, simpler but reasonable explanations could come from an adequate recognition of hidden symmetries, which should determine the characters of the mass spectrum of hadrons and possibly their production and decay patterns. Stabilising relations between the mass spectra of hadrons with different quantum numbers may shed some light on the underlying symmetries which are hidden and being missed.

The $B_c$ states are quite special because they contain two heavy quarks with different flavors, existing in an intermediate mass regime between the charmonium and bottomonium systems. Hence, their study could provide a window to test our knowledge of hadron physics, specially concerning non-perturbative QCD. The ground-state pseudoscalar $B_c$ meson was first observed in 1998 by the CDF collaboration at the Tevatron [8]. Recently, two excited states of $B_c$ were reported by the CMS [9] and, later confirmed, by the LHCb collaborations [10] at CERN; this success pushes the study on $B_c$ mesons forward. Optimistically, with the joint effort of theorists and experimentalists, the LHC provides an opportunity to explore the charm-beauty ($B_c$) states based on new observations.

II. DERIVATION AND ANALYSIS

Experimentally, the masses of the spin-singlet $P$-wave states almost coincide with the spin-averaged centroid of the triplet [11, 12] for the $c\bar{c}$ and $b\bar{b}$ systems. The hyperfine splitting or the singlet-triplet mass splitting is almost zero [13, 14], hence one can safely write

$$\Delta M_{hf} := \langle M(3P) \rangle - M(1P) = 0,$$

with $\langle M(3P) \rangle = (M_{0++} + 3M_{1++} + 5M_{2++})/9$ and $M(1P) = M_{1++}$. This is further supported by the experimental data [15], from which $\langle M(3P) \rangle/M(1P) = 1.0001$ for both the $c\bar{c}$ and $b\bar{b}$ systems; so, amazingly, Eq. (1) is well preserved. This observation indicates that even though the masses of $\chi_{c0}, \chi_{c1}$ and $\chi_{c2}$ are different, as we sum over the masses of the corresponding $P$-wave states, with proper weights, the contributions of the spin-spin interaction among the states seems to be negligible or mutually compensated with each other [16]. Now, let us extend the above equality to the case of $B_c$ mesons.

For the unequal flavor systems the charge conjugation parity is no longer a good quantum number. However, we can still define the spin-averaged mass of the corresponding triplet states as

$$\langle M(3P) \rangle = (M_{0++} + 3M_{1++} + 5M_{2++})/9,$$

where the singlet mass is $M(1P) = M_{1P}$. Note that, up to the first order of violating the unitary symmetry for the unequal flavor bound states, the masses obey the equal spacing rule [17, 18]:

$$\langle M_{cc} + M_{bb} \rangle/2 = M_{cb}.$$

Under this assumption, it is clear that Eq. (1) still holds for the unequal flavor system. The states with the same
TABLE I. Masses of the $P$-wave $B_c$ mesons (in MeV), quoted from quark models [19–26].

| $J^P$ | [20] | [21] | [22] | [23] | [24] | [25] | [26] |
|-------|------|------|------|------|------|------|------|
| $2^+$ | 6747 | 6743 | 6773 | 6772 | 6762 | 6768 | 6787 |
| $1^{++}$ | 6736 | 6729 | 6757 | 6760 | 6749 | 6750 | 6776 |
| $1^+$ | 6730 | 6717 | 6737 | 6734 | 6734 | 6741 | 6757 |
| $0^+$ | 6700 | 6683 | 6688 | 6701 | 6699 | 6706 | 6714 |

To qualitatively understand the origin of the deviations from $r = 1$, obtained from the QM inputs, we follow the approach of J. Rosner [27]. There, for $P$-wave mesons, the Hamiltonian in charge of the splitting and mixing of the multiplets is given by:

$$
\Delta H = c_1 \vec{L} \cdot \vec{S}_1 + c_2 \vec{L} \cdot \vec{S}_2 + c_T \left( \vec{S}_1 \cdot \vec{S}_2 \cdot \frac{\vec{r} \cdot \vec{r}}{r^2} - \vec{S}_1 \cdot \vec{S}_2 \right) + c_H \vec{S}_1 \cdot \vec{S}_2 \ ,
$$

thus the ratio ($r$) can be expressed as

$$
r = 1 + \frac{6c_H}{4M + c_1 + c_2 - c_T - 5c_H} .
$$

The above result indicates that any deviation from $r = 1$ depends directly on the hyperfine interaction, $\vec{S}_1 \cdot \vec{S}_2$, which gives mild contributions in the case of $P$-waves.

We also analyze how well Eq. (5) is preserved for the light mesons, charmonium and bottomonium systems, where the charge conjugation states $(1^{++}, 1^+)$ are taken into account. For the light mesons, we choose $f_0(980), h_1(1170), f_1(1285)$ and $f_2(1270)$ to estimate the relative error. The empirical values produce the ratios shown in Table II. In every case, the deviation from $r = 1$ is small; furthermore, it decreases with increasing current quark mass.

| TABLE II. |
| light system | charmonium | bottomonium |
| $r$ | 1.002(11) | 1.00412 | 1.00072 |

A simple analysis indicates that a mass relation for the $B_c$ systems is by no means trivial. Our proposed equality, Eq. (5), manifests a possible hidden symmetry or an underlying principle which induces the miraculous disappearance of the contribution of the spin-spin interaction when one sums up the masses of the $P_J$ states, with their proper weights. If we attribute this phenomenon to a conservation law, its deviation from $r = 1$ is due to the quantum corrections to the Noether quantity. It is also suggested that this deviation arises from relativistic corrections and/or non-perturbative effects, which tend to affect more the light systems and become suppressed when moving towards the heavy sector [28]. This fact explains why the heavier the quark constituents are, the smaller the deviation from $r = 1$ is.

III. SUMMARY

In summary, based upon QM approaches, we derived the mass relation Eq. (5) and found that, for the $B_c$ spectrum, the relation holds to a high precision. Also, our derivation implies that Eq. (1) is still true for the $B_c$ systems. Based on that fact and our computations, we expect that there should be a fundamental principle that makes Eq. (1) true for the $P$-level of charmonium and bottomonium systems, but also for the case of unequal flavor case, the $B_c$ system. Remarkably, the obtained values of $r$ show that these ratios deviate from $r = 1$ by order of $10^{-3}$. The mass relation for the $B_c$ systems proposed in this note can be further checked by Lattice QCD simulations [29] and future, more accurate, experimental measurements.
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