Revisiting Supply Chain System with Deteriorating Items and Transportation Cost

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Abstract. Supply chain system with deteriorating items and transportation cost with environmental consideration has recently become a popular research stream. This study revisits a supply chain system with deteriorating items and transportation cost. Processing the defective items, which increases cost, affects supply chain decisions. We present an integrated inventory model involving a three-stage supply chain and defective items with no shortage. We then derive the minimal total cost considering supply chain integration and deteriorating items. Numerical examples are provided to illustrate how these models can be applied in practice. Sensitivity analysis is performed to gain more insight on changing parameters in the numerical studies.

1. Introduction
Due to increasing globalization, firms face a highly rapidly changing industrial conditions. The objective of our study is to determine the optimal cycle time and the replenishment policy for the integrated system which minimizes the average total cost per unit time. The motivation for looking at such models comes from the competitive environment and greater information transparency between suppliers, manufacturers, and retailers in the supply chain. Some researches on three-stage supply chain model were done by the following researchers. Ben-Daya et al. [1] explored the joint economic lot sizing problem in the context of a three-stage supply chain. Sana et al. [2] investigated a three-stage supply chain consisting of multiple suppliers, multiple manufacturers, and multiple retailers. Neither of them considered deteriorating items and logistic cost. Chung et al. [3] developed an integrated two-stage production–inventory deteriorating product model, in which stock-dependent, imperfect items and just-in-time delivery were considered.

In this study, we developed a generalized mathematical model considering three-stage supply chain for deteriorating items considering transportation cost. Our objective is to minimize the total system cost per unit time. We illustrate the process with a numerical example and analyzed the sensitivity of crucial parameters to provide managerial insights.
2. Inventory Model Construction

The three-stage supply chain inventory model with transportation cost and defective items is presented as follows:

![Diagram of inventory relationship between manufacturer and retailer](image)

**Figure 1.** Inventory relationship between manufacturer and retailer (assume \( n = 3, a_2 = 6 \))

2.1. Total Cost of the Supply Chain

By adding the relevant costs of the retailer, manufacturer, and supplier, we obtained the total cost per unit time of the supply chain:

\[
TC(a_1, a_2, T, n) = \{\text{retailer total cost}\} + \{\text{manufacturer total cost}\} + \{\text{supplier total cost}\}
\]

\[
= \frac{O_y}{T} + \frac{S}{T} + \frac{h_s}{2} \cdot \frac{TD}{a_2} + \frac{O_m n}{a_1 T} + \frac{n S_m}{a_2 T} + \frac{h_s}{2} \cdot \left[ D \cdot (a_2 - 1) - \frac{D^2}{P_m} \cdot (a_2 + 2) \right] + \frac{C_m}{a_2 T} \cdot \left( n \cdot G_m - a_2 \cdot T \cdot D \right) + \frac{A_m}{a_1 a_2 T} + \frac{h_s}{a_1 a_2 T} \cdot \left[ \int_{t=0}^{T_1} I_{s1}(t) dt + \int_{t=0}^{T_2} I_{s2}(t) dt \right]
\]

\[
- \frac{G_m \cdot (a_1 n - 1)}{2 P_m} - \frac{G_m n \cdot [a_2 T \cdot (1 - D / P_m) \cdot (a_1^2 - a_1)]}{2} + \frac{C_m}{a_1 a_2 T} (P_s \cdot T_1 - a_1 \cdot n \cdot G_m)
\]

(1)
Where $a_2$ is the integer multiple of supplier cycle time parameter, $a_2$ is the integer multiple of manufacturer cycle time parameter, and $N$ is the integer number of manufacturer replenishment per cycle time.

2.2. Numerical Analysis

We assumed that $a_1, a_2$ and $n$ are positive integers and the related function formulas as a function of $T$ as follows:

$$TC_{a_1, a_2, n}(T) = k_1 \cdot T^2 + k_2 \cdot T + \frac{k_3}{T},$$

(2)

Where

$$k_1 = \frac{a_1 a_2 D^2 \theta}{2} \cdot [(C_s + h_s) \cdot (\frac{a_1 D}{P_i} + \frac{D}{nP_m P_i} + \frac{a_1 D}{nP_m P_i} - \frac{1}{P_i} + \frac{D}{nP_m P_i} - \frac{a_1 D}{P_i^2})]^n,$$

$$k_2 = \frac{1}{2} \left[ \frac{h_a a_2 D^2}{P_i} + \frac{h_a a_2 D^2}{P_m} + \frac{2 h_a a_2 D^2}{P_m} - \frac{h_a a_2 D^2}{nP_m} + \frac{h_2 a_2 D^2}{nP_m} - \frac{h_2 a_2 D^2}{P_m} + \frac{h_2 a_2 D^2}{nP_m} - h_a D \right],$$

$$+ \frac{C_a a_2 D^2}{nP_m} - \frac{h_a a_2 D}{nP_m} - \frac{(C_s + h_s) \cdot (a_1 - 1 - \frac{a_1 D}{nP_m} + \frac{D}{nP_m} - \frac{a_1 D}{P_i} + \frac{h_2 D}{4P_m} \cdot (a_1 + \frac{1}{n}))}{nP_m},$$

(3)

$$k_3 = \gamma O_r + S_r + \frac{A_x}{a_1 a_2} + \frac{A_z}{a_2^2} + \frac{n S_m}{a_2} + \frac{n O_m}{a_2^2}. $$

The $T$ value can be derived, and convexity can be proved by calculating the second-order derivatives of the total cost. One has:

$$\frac{\partial^2 TC(a_1, a_2, n, T)}{\partial T^2} = \frac{d^2 TC(T | a_1, a_2, n)}{dT^2} = \frac{d^2 TC_{a_1, a_2, n}(T)}{dT^2} = 2k_1 + \frac{2k_3}{T^2} > 0$$

(4)

When $k_2 > 0, a_1, a_2$ and $n$ are given positive integers; when $k_1, k_2$ and $k_3$ are all positive, this inventory model contains an optimal solution within a limited range, and $TC$ is convex.

$$\frac{dT C_{a_1, a_2, n}(T)}{dT} = 2k_1 T + k_2 + \frac{k_3}{T^2} = 0$$

(5)

Repeat until it converges into an optimal $T$ as follows:

$$T_{n+1} = \sqrt[2 + 2 \cdot k_1 \cdot T_n]{k_3}$$

(6)

After the first-order partial derivative of the total cost $TC_{a_1, n, T}(a_1)$ is determined one has:

$$a_1^* = 6 \cdot \{[54ZX^2 - Y^3 + 6X \sqrt[3]{3Z} \sqrt[3]{27ZX^2 - Y^3}]^2 + Y^2$$

$$- Y \cdot \sqrt[3]{54ZX^2 - Y^3 + 6X \sqrt[3]{3Z} \sqrt[3]{27ZX^2 - Y^3}}$$

$$/ X \cdot \sqrt[3]{54ZX^2 - Y^3 + 6X \sqrt[3]{3Z} \sqrt[3]{27ZX^2 - Y^3}}$$

(7)
where 
\[ X = \frac{a_2^2T^2D^2}{P_s} \cdot (2h_i + C_s) \]

\[ Y = a_2^2T^2D^1 \cdot \left( \frac{C_\theta}{P_s} + \frac{h_\theta}{P_m} - \frac{3C_\theta}{4nP_mP_s} - \frac{h_\theta}{4nP_m^2} - \frac{h_\theta}{nP_mP_s} \right) \]

\[ + a_2^2T^2D^2 \cdot \left( -\frac{C_\theta}{2P_s} - \frac{h_\theta}{2P_m} + \frac{C_\theta}{2nP_m} + \frac{h_\theta}{2nP_m} + \frac{a_2TDh_i}{2} + \frac{a_2Th_\theta}{2P_s} \right) \]

\[ Z = \frac{A_1}{a_2T} \]

When \( a_1, n \) and \( T > 0 \) are given and the inventory model possesses an independent solution, verify \( TC_{a_1,n,T}(a_2) \) as a convex function. The optimal condition involves \( a_2 \) value using the cost difference method is as follows:

\[ (a_2^*) \cdot (a_2^*) - (a_2^*)^2 \leq (a_2^*) \cdot (a_2^* + 1). \]  \( \text{(8)} \)

By determining the first-order partial derivative of the total cost \( TC_{a_1,n,T}(a_2) \) in \( a_2 \) function, we obtain the following solution:

\[ a_2^* = \frac{\sqrt[3]{108FE^2 - 8G^3} + 12E \cdot \sqrt[3]{3F(27FE^2 - 4G^3)} + 4EG^2}{6E^2 \cdot \sqrt[3]{108FE^2 - 8G^3} + 12E \cdot \sqrt[3]{3F(27FE^2 - 4G^3)} - 2G} \]

where

\[ E = T^2D^3 \cdot \left( -\frac{C_\theta}{n^2P_m} - \frac{a_1C_\theta}{2nP_mP_s} + \frac{C_\theta}{n^2P_m} + \frac{h_\theta}{nP_m} - \frac{h_\theta}{n^2P_m} + \frac{a_1C_\theta}{P_mP_s} \right) \]

\[ + \frac{2a_1h_\theta}{P_mP_s} - \frac{a_1h_\theta}{2nP_m} - \frac{h_\theta}{2nP_m} \]

\[ F = \frac{1}{T} \cdot \left( nO_m + A_m + nS_m + \frac{A}{a_1} \right) \]

\[ G = \frac{1}{2} \cdot \left( TD^3 \cdot \left( \frac{h_s}{nP_m} + \frac{C_s}{nP_m} \right) + TD^2 \cdot \left( -\frac{C_s}{nP_m} + \frac{h_s}{nP_m} + \frac{h_s}{P_m} - \frac{h_s}{P_m} + \frac{a_1h_s}{P_s} \right) \right) \]

\[ + TD \cdot \left( h_m - h_i + a_1h_i \right) \]

In this study, our objective is to minimize \( TC(a_1, a_2, n) \). Since \( a_1, a_2 \) and \( n \) are restricted to integer values, we can adopt the iteration method to develop a simple procedure to derive the optimal cycle time, \( T \) and the minimum total system cost. Maple 8 was used to verify that the total cost function \( TC \) is a convex function. The methodology is presented as follows:

1. Start with \( n = 1 \).
2. Initialize \( a_1 \) and \( a_2 \), set \( a_1 \) and \( a_2 \) equal to 1 as a possible feasible solution, substitute the aforementioned parameter values into Equation 4 to determine \( k_2 \), then derive \( T \):
   - (a) When \( k_2 > 0 \), substitute \( k_2 \) into Equation 9 to determine the \( T \) value.
3. Replace \( n \) and \( T \) with the derived values and respectively substitute them into Equations 11 and 15 to derive new \( a_1 \) and \( a_2 \) values and verify the following conditions:
   - (a) If \( a_1^* \neq a_1 \) and \( a_2^* \neq a_2 \), then select either \( a_1 \) or \( a_2 \); go to Step 1 to initialize the setting.
   - (b) If \( a_1^* = a_1 \) and \( a_2^* \neq a_2 \), then use \( a_2 \) value and go to Step 1 to initialize the setting.
   - (c) If \( a_1^* \neq a_1 \) and \( a_2^* = a_2 \), then use \( a_1 \) value and go to Step 1 to initialize the setting.
(d) If \( a_1^* = a_1 \) and \( a_2^* = a_2 \) are fulfilled, then the derived \( n \) and \( T \) values have minimum \( TC(a_1, a_2, n, T) \), minimize total cost per unit time.

3. Sample Numerical Example

The parameter values in Table 1 are used as our example to solve the problem using Maple 14.

### Table 1. List of numerical parameters values

| Parameter symbols | Explanations                                      | Numerical values |
|-------------------|---------------------------------------------------|------------------|
| \( P_s \)         | Supplier annual production rate                   | 399,000          |
| \( P_m \)         | Manufacturer annual production rate               | 140,000          |
| \( D_m \)         | Manufacturer annual demand rate                   | 133,000          |
| \( D_{r1} \)      | First retailer annual demand rate                 | 9,000            |
| \( D_{r2} \)      | Second retailer annual demand rate                | 34,000           |
| \( D_{r3} \)      | Third retailer annual demand rate                 | 52,000           |
| \( D_{r4} \)      | Fourth retailer annual demand rate                | 18,000           |
| \( D_{r5} \)      | Fifth retailer annual demand rate                 | 20,000           |
| \( D \)           | Retailer annual demand rate                       | 133,000          |
| \( \theta \)      | Annual deterioration rate                         | 0.01             |
| \( h_s \)         | Supplier product carrying cost and manufacturer raw material carrying cost (NT$/item) | 1                |
| \( h_m \)         | Manufacturer product carrying cost per item (NT$/item) | 2                |
| \( h_r \)         | Retailer carrying cost per item (NT$/item)       | 5                |
| \( C_s \)         | Deterioration cost (NT$/item)                     | 2                |
| \( O_m \)         | Manufacturer ordering cost (NT$/order)            | 300              |
| \( O_r \)         | Single retailer ordering cost (NT$/order)        | 50               |
| \( A_s \)         | Supplier setup cost (NT$/setup)                   | 800              |
| \( A_m \)         | Manufacturer setup cost (NT$/setup)               | 200              |
| \( S_m \)         | Manufacturer logistic cost for replenishment (NT$/trip) | 60               |
| \( S_{r1} \)      | First retailer logistic cost (NT$/trip)           | 15               |
| \( S_{r2} \)      | Second retailer logistic cost (NT$/trip)          | 35               |
| \( S_{r3} \)      | Third retailer logistic cost (NT$/trip)           | 55               |
| \( S_{r4} \)      | Fourth retailer logistic cost (NT$/trip)          | 20               |
| \( S_{r5} \)      | Fifth retailer logistic cost (NT$/trip)           | 25               |

After substituting the numerical values into the inventory model, we obtained the following equation:
\[ TC(a_1, a_2, T, n) = T^2 \cdot \left( \frac{1079.2392a_1a_2^2}{n} - \frac{1500.4062a_2^2}{n^2} - \frac{94.7625a_2^4}{n} + 886.6667a_1^2a_2^2 - 44.3334a_1a_2^2 \right) \\
+ T \cdot \left( 47333.3333a_1a_2 - \frac{9476.25a_2^4 + 3325a_2}{n} \right) \\
+ \frac{1}{T} \cdot \left( \frac{800}{a_1a_2} + \frac{360n}{a_2} + \frac{200}{a_2} + 400 \right) \] (10)

We adopted a spiral convergence to obtain the total cost per unit time by deriving the optimal value of the cycle time \( T \), the replenishment frequency \( n \), and integer multiple in a cycle \( a_1 \) and \( a_2 \).

First, initialize \( a_1 = 1, a_2 = 1 \) and \( n = 1 \); substitute them into Equation 4 to obtain \( K_2 \); then, adopt the given \( a_1, a_2 \) and \( n \) to obtain \( T \) by taking the first-order partial derivative of \( TC_{a_1,a_2,n}(T) \).

\[ k_2 = \frac{1}{2} \left\{ \frac{h_a a_1 D^2}{P_m} + \frac{h_a a_2 D^2}{P_m} + \frac{2h_a a_1 D^2}{P_m} + \frac{h_a a_1 D^2}{P_m} + h_a D + \frac{h_a a_2 D^2}{n P_m} - h_a D \right\} \\
+ \frac{C_a D^3}{n P_m} - \frac{C_a D^3}{n P_m} - h_a a_2 D \cdot \left( \frac{a_1 D}{P_m} + \frac{D}{P_m} + \frac{a_1 D}{P_x} \right) + h_a a_2 D - h_a a_2 D^2 \]

\[ = 44333.3333 \cdot a_1 \cdot a_2 - \frac{9476.25 \cdot a_2^4 + 3325 \cdot a_2 + 452200}{n} \]

\[ = 490382.0833 > 0 \] (11)

From Equation 11, \( k_2 > 0 \) can be substituted into Equation 6 to obtain \( T \); subsequently, let \( T(0) = 0 \) and adopt the convergent iterative process to obtain the optimal \( T \) value:

\[ T(1) = \sqrt{\frac{1760}{490382.0833 - 421.1667 \cdot T(0)}} = 0.05622099816. \] (12)

Repeat the process until the values spirally converges into the optimal solution, the \( T \) value is

\[ T(3) = T(4) = 0.05991012284 \] (13)

By substituting the \( T \) value into Equations 7 and 9, we obtained \( a_1^* = 2.235497146 \approx 2 \) and \( a_2^* = 3.153497053 \approx 3 \); we identified that \( a_1^* \neq a_1 \) and \( a_2^* \neq a_2 \). Therefore, substituting the new \( a_2 \) value into the previous step, we initialize its setting, and recalculate it until the spiral convergence yields the optimal cycle time \( T \) when the total cost per unit time is minimum. The optimal \( a_1 \) and \( a_2 \) of the cycle integer multiple and optimal replenishment frequency \( n \) can then be derived.

4. Conclusion and Future study

In this study, we revisit the collaborative inventory model involving a three-stage supply chain for deteriorating items considering transportation cost. From the numerical example and sensitivity analysis, we have shown that the primary parameters influence the retailer cycle time and the total system cost. It is illustrated that demand rate is the primary influence on the total system cost. The retailers can strategically plan the demand rate to control the cycle time and improve the total cost per unit time. The study provides insights for decision makers to determine how product deterioration and transportation
costs can influence the overall supply chain. The study can be extended to consider trade credit, quantity discount and partial backordering strategies.

Reference
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