Entangled States Generated via Two Superconducting Quantum Interference Devices (SQUIDs) in cavity QED

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We propose a scheme for generating entangled states for two superconducting quantum interference devices in a thermal cavity with the assistance of a microwave pulse.

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Quantum computers can solve some problems much faster than the classical computers, such as factorizing a large integer [1] and searching for an item from a disordered system [2]. Thus, finding out the practical qubits is the key problem in building the quantum computers. About seven years ago, superconducting quantum interference devices (SQUIDs) were

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proposed as candidates to serve as the qubits for a superconducting quantum computer \[3\]. In the following years, some schemes were been proposed to perform quantum logic by using superconducting devices such as Josephson-junction circuits \[4, 5, 6\], Josephson junctions \[7, 8, 9\], Cooper pair boxes \[10, 11, 12\], and (SQUIDs) \[13, 14, 15, 16\].

Yang and Cnu \[17\] proposed a scheme to generate entanglement and logical gates. In their scheme, they entangled two SQUIDs with two levels in a vacuum cavity. Zhang et al. \[18\] presented a protocol to generate an entangled state with two three-level atoms. In this paper, we will entangle two SQUIDs with three levels in a thermal cavity driven by a classical field.

![FIG. 1: The Λ-type lowest three levels of the SQUIDs.](image)

We consider two SQUIDs coupled to a single-mode cavity. The Hamiltonian of the system is written as

\[
H = H_s + H_c + H_{c-s} + H_{m-s},
\]

where \(H_c\) and \(H_s\) are the Hamiltonian of the cavity field and the Hamiltonian of the SQUIDs, respectively. \(H_{m-s}\) is the interactional energy between the SQUIDs and the microwave pulse, and \(H_{c-s}\) is the interaction Hamiltonian between the SQUIDs and the cavity. The cavity is only coupled to the Λ-type lowest three levels of the SQUIDs, which are denoted by \(|0\rangle\), \(|1\rangle\), and \(|2\rangle\) (FIG.1) . In the case where the cavity field is far-off resonance with a transition between a levels \(|0\rangle\) and \(|1\rangle\) and a transition between levels \(|1\rangle\) and \(|2\rangle\), we assume the frequency of the microwave pulse to be equal to \(\omega_{20}\) (the transition frequency between levels
Thus, the interaction Hamiltonian in the interaction picture is

\[ H_I = H_{Ic} + H_{Im}, \]

\[ H_{Ic} = g \sum_{i=1,2} [e^{i\delta t} a_i^+ S_i^- + e^{-i\delta t} a S_i^+], \]

\[ H_{Im} = \Omega \sum_{i=1,2} (S_i^+ + S_i^-), \]  

(2)

where \( g \) is the coupling constant between the SQUIDs and the cavity field, corresponding to the transitions between \( |0\rangle \) and \( |2\rangle \); \( \delta \) is the detuning between \( \omega_{20} \) and the cavity frequency \( \omega \); \( a^\dagger \) and \( a \) are the creation and the annihilation operators for the cavity mode, \( S_i^+ = |2\rangle_i \langle 0|, S_i^- = |0\rangle_i \langle 2|; \) \( \Omega \) is the Rabi frequency; \( H_{Ic} \) and \( H_{Im} \) are the cavity-SQUIDs interaction Hamiltonian and the microwave pulse-SQUIDs interaction Hamiltonian in the interaction picture, respectively.

Following the method in Ref. [19], when \( 2\Omega \gg \delta, g \) and \( \delta \gg g/2 \), we can obtain the effective Hamiltonian of the system

\[ H_e = \lambda \left[ \frac{1}{2} \sum_{i=1,2} (|0\rangle_i \langle 0| + |2\rangle_i \langle 2|) + (S_i^+ S_i^+ + S_i^+ S_i^- + H.c.) \right], \]  

(3)

where \( \lambda = g^2/2\delta \). The evolution operator \( U(t) \) is given by

\[ U(t) = e^{-iH_0t}e^{-iH_1t}, \]  

(4)

where \( H_0 = \Omega \sum_{i=1}^2 (S_i^+ + S_i^-) \) and \([H_0, H_e] = 0\). Because \( U(t) \) is independent of the cavity field state, we allow the cavity to be in the thermal state. In order to generate a maximally entangled state of two SQUIDs, we assume that the two SQUIDs are prepared in the state \( |0\rangle_1 |0\rangle_2 \). Next, let us consider the first SQUID driven by a classical microwave pulse (without cavity) whose frequency is equal to \( \omega_{10} \). The interaction Hamiltonian is written as

\[ H = \Omega (S_1^+ + S_2^-). \]  

(5)

Hence \( |0\rangle_1 \) becomes

\[ |0\rangle_1 \rightarrow \cos \Omega t |0\rangle_1 - i \sin \Omega t |1\rangle_1. \]  

(6)

If we let \( \Omega t = \arccos \sqrt{\frac{2}{3}} \), the first SQUID is in the state

\[ |\sqrt{\frac{2}{3}} 1\rangle_1 - i |\sqrt{\frac{2}{3}} 0\rangle_1. \]  

(7)
while the second one is still in the state $|0\rangle_2$. Then, both the SQUIDs are put into the cavity.

The evolution operator is described by Eq. (4) which has no effect on the state $|1\rangle_1|0\rangle_2$. After an interaction time $t_1$, the state of the system is

$$|\psi(t_1)\rangle = \sqrt{\frac{1}{3}}|1\rangle_1|0\rangle_2 - i\sqrt{\frac{2}{3}}e^{-i\lambda t_1} \{\cos(\lambda t_1)[\cos \Omega t_1|0\rangle_1 - i \sin \Omega t_1|2\rangle_1]$$

$$\times [\cos \Omega t_1|0\rangle_2 - i \sin \Omega t_1|2\rangle_2] - i \sin(\lambda t_1)[\cos \Omega t_1|2\rangle_1 - i \sin \Omega t_1|0\rangle_1]$$

$$\times [\cos \Omega t_1|2\rangle_2 - i \sin \Omega t_1|0\rangle_2]\}.$$  (8)

We choose $\Omega$ and the interaction time $t_1$ appropriately so that $\lambda t_1 = \pi/2$ and $\Omega t_1 = k\pi$, with $k$ being an integer. Then, we have

$$|\psi(\pi/2\lambda)\rangle = \sqrt{\frac{1}{3}}|1\rangle_1|0\rangle_2 + i\sqrt{\frac{2}{3}}|2\rangle_1|2\rangle_2.$$  (9)

SQUID 2 is then addressed by using a classical microwave pulse tuned to the transition $|0\rangle \leftrightarrow |1\rangle$ in the cavity. After this operation, the state, Eq. (9) becomes

$$|\psi'(\pi/2\lambda)\rangle = \sqrt{\frac{1}{3}}|1\rangle_1|1\rangle_2 + i\sqrt{\frac{2}{3}}|2\rangle_1|2\rangle_2.$$  (10)

Then we switch off the microwave pulse field, and the system will interact for another time $t_2$. Thus, the system’s time evolution operator has transformed the state in Eq. (10) into the state

$$|\psi(t_1 + t_2)\rangle = \sqrt{\frac{1}{3}}|1\rangle_1|1\rangle_2 + i\sqrt{\frac{2}{3}}e^{-i\lambda t_2} \{\cos(\lambda t_2)[\cos \Omega' t_2|2\rangle_1 - i \sin \Omega' t_2|0\rangle_1]$$

$$\times [\cos \Omega' t_2|2\rangle_2 - i \sin \Omega' t_2|0\rangle_2] - i \sin(\lambda t_2)[\cos \Omega' t_2|0\rangle_1 - i \sin \Omega' t_2|2\rangle_1]$$

$$\times [\cos \Omega' t_2|0\rangle_2 - i \sin \Omega' t_2|2\rangle_2]\\},$$  (11)

where $\Omega'$ is the Rabi frequency of the classical field during the interaction time $t_2$. If we choose the interaction time $t_2$ and the Rabi frequency $\Omega'$ appropriately so that $\lambda t_2 = \pi/4$ and $\Omega' t_2 = 2k'\pi$, with $k'$ being an integer, we have

$$|\psi(t_1 + t_2)\rangle = \sqrt{\frac{1}{3}}(|1\rangle_1|1\rangle_2 + i e^{-i\pi/4}|2\rangle_1|2\rangle_2 + e^{-i\pi/4}|0\rangle_1|0\rangle_2).$$  (12)
After that, we apply a classical field whose phase is chosen appropriately, then SQUID 2 undergoes the transition $|2\rangle_2 \rightarrow e^{-i\frac{\pi}{4}}|2\rangle_2$, $|0\rangle_2 \rightarrow e^{i\frac{\pi}{4}}|0\rangle_2$. Thus, the state in Eq. (12) becomes

$$|\psi\rangle = \sqrt{\frac{1}{3}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2 + |2\rangle_1|2\rangle_2).$$

Equation (13)

It should be noted that the level space of the SQUIDs can be changed by using an external flux $\Phi_x$ or critical current $I_c$. Thus, the interaction time between the SQUIDs and cavity can be controlled by $\Phi_x$. In summary, we have entangled two SQUIDs with three levels in a thermal cavity with the help of a microwave pulse.

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