Potential landscape of a clean superconductor mapped by pm/s-moving vortices

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We investigated vortex dynamics in a pristine sample of type II superconductor NbSe2 by scanning tunneling microscopy at 4.2 K. The decay of the magnetic field at \( \sim nT/s \) in our superconducting magnet induced the motion of vortices at \( \sim pm/s \). Characteristics of the trajectories and speed variations of the moving vortices clearly revealed the potential landscape of vortices in the sample. Our vortex driving method may have wide application for the study of vortex matter on the nanometer scale by directly mapping an energy landscape.

Understanding vortex motion in superconductors plays a key role in developing useful superconductivity-based applications, and is crucial to answer many open questions about vortex matter. Although various techniques have yielded success in imaging vortices and have become instrumental in our microscopic understanding of vortex dynamics (1,2,3,4,5), it is still challenging to capture the motion of individual vortices when inter-vortex spacing is less
than 100 nm and a few studies on this length scale have been reported to date. We used an extremely slow (∼ nT/s) decay of a 0.5 T field (corresponding to inter-vortex spacing of about 70 nm) in our superconducting magnet as the driving source for ultra-slow (∼ pm/s) vortex motion in pristine NbSe₂, and observed this motion by scanning tunneling microscopy (STM). We tracked vortices within a field of view, 400 × 400 nm², over a week to extract highly time- and space-resolved data. Despite the use of pristine NbSe₂, non-uniform motion was observed. The non-uniformity can be explained by the instability occurring when vortices exit the sample by overcoming surface barriers. This instability governed vortex motion allowed for mapping the potential landscape via trajectories and spatial variations of speeds of the moving vortices.

When an external magnetic field between the lower and upper critical values is applied to a type II superconductor below its critical temperature, magnetic vortices self-arrange in the form of a triangular lattice, called a vortex lattice (VL), with its lattice constant \( a(B) \) (1)

\[
a(B) = \left( \frac{2}{\sqrt{3}} \right)^{1/2} \left( \frac{\Phi_0}{B} \right)^{1/2},
\]

where \( B \) is the magnetic field penetrating the superconductor, and the magnetic flux quantum \( \Phi_0 = 2.07 \times 10^{-15} \text{Tm}^2 \). We used the \( B \)-dependance of the lattice constant of the VL to drive the motion of vortices. A small expansion of the lattice constant can lead to a large motion if the vortex system is comprised of many vortices. The magnetic field stored in our commercial superconducting magnet decays due to residual resistances at spot-welded joints in the superconducting coil. We found that the field decayed from 0.500 to 0.491 T over 21 days and 17 hours, which gives the average decay rate of \( \Delta B/\Delta t \approx -4.2 \text{ nT/s} \) (∼ −0.36 mT/day).

Using a series inductor-resistor (L-R) circuit as a model (Fig. 1), the magnetic field becomes

\[
B(t) = B(0)e^{-t/\tau},
\]

where \( \tau = L/R \). Eq. 2 with the initial/final magnetic fields measured and \( L = 12.4 \text{ H} \) for our magnet, gives the decay time constant, \( \tau \approx 3.7 \text{ years} \), and the resistance, \( R = L/\tau \approx 0.1 \mu\Omega \).
This decay causes the triangular VL to expand (Fig. 1b), satisfying Eq. 1. As a consequence of the continuity relation of vortex density and vortex flow, the speed of vortex motion may be calculated:

\[ v_{\text{theory}} = \frac{r}{2B} \left| \frac{dB}{dt} \right| = \frac{r}{2\tau} = \frac{r \cdot R}{2L} \]  

(3)

at a distance \( r \) away from the center of the sample. In this study, we used roughly a round disk of pristine NbSe\(_2\) with a diameter of 5 mm (thickness = 0.5 mm). Since the STM tip was positioned near the center of the sample, a uniform motion, not exceeding \( \sim 10 \) pm/s, was expected. This model so far only predicts motion of a constant speed and straight trajectory. However, the motion could be affected by randomly distributed pinning sites (RDPS) \(^7\) in the bulk or the surface barriers (SB) \(^8\) along the edges.

A home-built STM operating in vacuum at liquid helium temperature (4.2 K) was used. The sample was cleaved at room temperature under a pressure of \( \sim 6 \times 10^{-7} \) mbar and transferred to the STM. When thermal equilibrium was reached, a magnetic field parallel to the c-axis of the NbSe\(_2\) was raised from 0 to 0.500 T at a rate of 0.100 T/min. The measurement was delayed 4 hours after the initial field was stored in the superconducting magnet, to allow for transient relaxation of materials affecting the magnetic field. We operated the magnet in the persistent current mode during the measurement, subject to the slow decay.

Vortices in an area of \( 400 \times 400 \) nm\(^2\) (128 \times 128 pixels) were visualized in a \( dI/dV \)-spectroscopic image via a lock-in amplifier (Fig. 2a). At a scan-speed of 600 nm/s (this gave a 4 min completion time per frame), we did not observe significant distortions of the VL between adjacent frames, indicating that this scan-speed was sufficient to capture the motion of the VL. Over 17 days, we recorded consecutive spectroscopic images in the area, from which by automated data analysis we extracted the time-series of speed \( v(t) \) for all vortices \(^9\). In this Letter, we present and discuss specifically the data taken during the last 7 days and ignore earlier data for two reasons. One is to avoid as much as possible any transient effects \(^10\).
after the initial ramp-up of the magnetic field. The other is that this was the longest continuous observation without any interruption of data collection, such as refilling the liquid helium dewar. This 7-day data series contains 2560 images of VL in total for a total number of \( v \)-data points of 102810.

The gray-scaled image in Fig. 2a is the first spectroscopic image in the 7-day data series, showing a regular triangular VL with a lattice constant of 69 nm. As the magnetic field decayed, the VL moved mostly along one of its own principal axes (\( \theta_{\text{VL}} = 150^\circ \)), while maintaining its ordered structure (see Supplementary Video 1 for the complete motion of VL over 7 days). As the time-colored trajectories of four vortices (labeled by 1, 2, 3, and 4) indicate, the motion of the VL was non-uniform in terms of speed and direction. Despite the non-uniform motion, the inter-vortex spacing remained almost constant throughout the measurement. Even at the highest velocity motion, 45 times faster than the averaged motion (marked as green circles in Fig. 2a and Figs. 3a,b), the ordered structure of the VL was still maintained. This rigidity of the VL indicate that a strong vortex-vortex interaction dominates distortions of the VL due to local pinning forces. After analyzing the 2560 images of the VL, we found that the lattice constant expanded by \( \sim 0.3 \) nm over 7 days, which is consistent with the field decay of \( \sim \) nT/s. However, we consider that this amount of change is negligible compared to the 69 nm lattice constant. Therefore, a single vortex motion represents the dynamics of the whole VL to a good approximation. This allowed us to combine all the time-series of individual vortices into a single time-series by sorting the observed speeds by time (black in Fig. 3a). Then we decomposed \( v \) into \( v_{\text{par}} \) (red) and \( v_{\text{per}} \) (blue) (\( v_{\text{par}} \) and \( v_{\text{per}} \) are the speeds along and perpendicular to \( \theta_{\text{VL}} \) respectively). We found that \( \langle v \rangle, \langle v_{\text{par}} \rangle, \) and \( \langle v_{\text{per}} \rangle \) are 2.5, 0.9, and 0.1 pm/s respectively.

The observed motion at \( \sim \) pm/s agrees well with the estimated speed based on the \( L-R \) model. However, to better understand the non-uniform motion of VL, it is necessary to include the effects of weak pinning sites in bulk and surface barriers, as we will discuss later.
Strikingly, by overlaying the trajectories of all vortices over 7 days (color in Fig. 2b), we found that vortices repeatedly visited specific places (SPs) more often than other regions, although vortices arrived at the SPs along different paths. This characteristics also left the 2560-averaged VL image (gray-scaled), which represents the probability distribution of finding a vortex in space. The bright sites, where vortices were more likely to be found than in dark regions, clearly form a triangular lattice with its lattice constant close to that of the VL (Fig. 2a).

We also investigated how the speed of the VL varied in space by defining the displacement of the VL, \((X(t), Y(t))\) (see Fig. 3b caption for the definitions). The \((X(t), Y(t))\) allows all trajectories to collapse into a single trace (discrete colors). By changing the variable from \(t\) to \(X\) in \(v_{\text{par}}\) and \(v_{\text{per}}\) (red and blue), one conveniently sees the variation of speeds in the \((X, Y)\)-space. Down/up arrows mark the moments when vortices were at the SPs. When vortices were at the SPs, both \(v_{\text{par}}\) and \(v_{\text{per}}\) were suppressed such that \(\sqrt{[v_{\text{par}}(t)]^2 + [v_{\text{per}}(t)]^2} < 3.5 \text{ pm/s}\). However, while vortices were transiting from SPs to SPs, accelerations/deccelerations occurred, showing fast motion such that \(\sqrt{[v_{\text{par}}(t)]^2 + [v_{\text{per}}(t)]^2} > 5 \text{ pm/s}\). Therefore, the high probability and the suppressed speeds at the SPs indicate that the SPs are the minima of the potential landscape (PL) seen by the moving vortices, which has been preserved despite the continuous field decay over 7 days.

In our observation of moving vortices in pristine NbSe\(_2\), there are two distinct features: 1) the non-uniform motion and 2) the manifestation/preservation of the PL. These two features could be caused by weak pinning sites, surface barriers, or the combination of both. If the weak pinning sites had played a dominant role in causing these two features, one would observe collective flux creep motion \(5\), shortly after an external magnetic field is raised and set to a constant value. Even in a constant external magnetic field, the penetration of the magnetic field into the superconductor is not instantaneous – the motion of vortices is impeded due to weak pinning sites, and is observable until they fully penetrate deep inside the super-
conductor. To check this, we ramped up the magnetic field from 0 to 0.75 T, stabilized it by continuously supplying the current to the magnet in the persistent current mode (refer to the caption of Fig. 4a for the detailed procedure), and immediately started the measurement, which continued over 2 days. One clearly sees quite reduced motion of vortices based on the overlaid trajectories (Fig. 4b). The $\langle v_{\text{par}} \rangle$ and $\langle u_{\text{par}} \rangle$ were reduced by a factor of 10 compared to those of the moving VL. This suppressed vortex motion in a constant field right after the field ramp-up provides strong evidence that the two features were unlikely to be caused by weak pinning sites. Rather, this observation indicates that the crystal of NbSe$_2$ we used in the study was a practically pinning-free superconductor. Moreover, this experiment confirmed that the magnetic field decay was the main driving source of the vortex motion.

If the influence of weak pinning sites cannot be significant, the most intriguing question is whether the surface barrier alone can cause the non-uniform motion and the manifestation/preservation of the PL. Indeed the importance of the SB in vortex dynamics of high quality crystals of NbSe$_2$ has been emphasized in transport measurements $^{11,12,13}$. Here we propose a possible mechanism based on the interplay between the SB and the continuous magnetic field decay. As an external magnetic field decays, vortices exit the superconductor only when vortices overcome the SB along the edges $^{8}$. An exit event occurs at surface defects along the edges, as a bundle of vortices $^{14,15,16}$. As soon as vortices exit, the vortices at the edges start re-arranging. This instability propagates through the superconductor via strong vortex-vortex interaction. As the field continues decaying, the instability keeps driving the vortex system with bundles of vortices leaving the sample at different locations based on local geometry. Therefore, the external force exerted on the vortex system chaotically varies in its magnitude, direction, and time. However, although a large number of vortices, $\delta N$, were expected to leave the sample over 7 days, it is still a small fraction (0.5 %) of the total number of vortices, $N$, when the VL image (Fig. 2a) was taken; $\delta N = \pi r_{\text{sample}}^2 \delta B/\Phi_0 = 2.4 \times 10^7$, and $N = \pi r_{\text{sample}}^2 B/\Phi_0 = 4.7 \times 10^9$. 
where $r_{\text{sample}} = 2.5$ mm, $\delta B = 4.2 \text{nT/s} \times 7 \text{ day} \approx 2.5 \text{ mT}$, and $B = 0.496$ T. This negligible leakage of vortices should link to the preservation of the PL, as experimentally observed (Fig. 2b). Therefore, each exit of a bundle of vortices can be considered as a perturbation to the vortex system. The continuing perturbations to the system cause the complicated patterns of trajectories and speed variation, but reveal the most energetically favorable configuration of vortices.

For a full understanding of vortex dynamics, it is important to know the PL of vortex system. This is determined by many parameters – the superconducting properties ($T_c, H_c$) of a superconductor, the geometry, the details of pinning sites, temperature, and magnetic field. By this new method, the PL of pristine NbSe$_2$ in 0.5 T was clearly revealed for the first time by large statistical groups of vortices moving at very slow speeds. Our method will be useful in studying various vortex-pinning interactions – quenched disorders, periodic pinning sites, high-$T_c$ superconductors – by directly mapping out an energy landscape, rather than by looking at instant motion of vortices. Moreover, in principle, the speed of the vortex motion can be controlled by adjusting the magnitude of the resistance $R$ (Eq. 3). This tunability of vortex motion combined with fast-scanning capability (17, 18) will allow for direct systematic investigation of the dynamic phases (19, 20) and phase transitions (21, 22) of vortex matter in various superconductors on the nanometer scale.

References and Notes

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9. For each vortex, ~77 pixels were used to determine its center position by calculating the “center of mass”. Each “mass” corresponds to the scaled value of a pixel. This improved the spatial resolution from 3.125 to 0.356 nm (= 3.125/√77). We also calculated the time at which the center of a vortex was reached by the STM tip, using the scan-speed. The speed \( v \) of a vortex in the \( n^{th} \) frame was calculated by the travel distance and the detected time difference of the vortex between \( (n - 1)^{th} \) and \( (n + 1)^{th} \) frames. This method eliminated the systematic errors which occur when calculating \( v \) using two adjacent frames \( (n^{th} \) and \( (n + 1)^{th} \)) due to the STM scanner creep. We achieved the \( v \) resolution of 0.74 pm/s (= 0.356 nm/(2 × 4 min)) in a single measurement. This limit was further improved by averaging more data points and continuous observation.

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Figure 1: **Model of driving source and vortex motion.** a, $L$-$R$ circuit corresponding to a field-decaying magnet due to non-zero $R$. $L = 12.4 \text{ H}$, $R \approx 0.1 \mu \Omega$, and $\tau = L/R \approx 3.7$ years (see text for the detail). b, Vortex motion model subject to the decay of a magnetic field. As a field $B$ decays, vortices leave the sample. Observation within a field of view (rectangular area) is made at a distance $r$ away from the center of the sample. The size of the field of view, vortices, and $r$ are drawn for illustration, not to scale.
Figure 2: **Trajectories of magnetically driven vortices in NbSe$_2$.** a, Collective and coherent motion of an ordered vortex lattice. The gray-scaled image, the first spectroscopic image of the 7-day observation, shows the configuration of the vortices in an area of $400 \times 400$ nm$^2$ at 0.496 T. Subject to the magnetic field decay, all vortices moved together as the trajectories of four vortices (color) indicate. Vortices 1, 2, 3, and 4 remained within the field of view for 5.1, 5.4, 4.5, and 4.1 days respectively. Trajectories are color-coded in time. The earliest time starts from red (1$^{\text{st}}$ day), and ticks mark every 24 hour on the color bar. The $\theta_{\text{VL}}$ denotes the angle of one of principle axes of the VL. An $ac$ modulation voltage $\Delta V_{\text{bias}} = 1$ mV$_{\text{rms}}$ with $f = 1973$ Hz, and a tunneling condition of $I_t = 0.1$ nA and $V_{\text{bias}} = 3$ mV between a STM tip and the sample were used. b, Overlaid trajectories (time-color scheme same to that of a) and the 2560-averaged spectroscopic image (gray-scaled) over 7 days. Notice that the places where the vortices more frequently passed by than other region form a triangular lattice.
Figure 3: **Speeds of the VL motion.**

**a,** Speeds \(v, v_{\text{par}}, \) and \(v_{\text{per}}\) vs. time \((t)\). \(v_{\text{par}}\) and \(v_{\text{per}}\) are the components of the speed \(v\) of the VL, parallel and perpendicular to one of principal axes \((\theta_{\text{VL}} = 150^\circ)\) of the VL. The data of \(v, v_{\text{par}}, \) and \(v_{\text{per}}\) were smoothed by averaging over 9 points to reduce noise. \(v\) and \(v_{\text{per}}\) are shifted vertically by \(\pm 10\) pm/s respectively for clarity.

**b,** Displacements and speeds. The displacements \((X(t), Y(t))\) of the VL are defined by
\[
X(t) \equiv \int_{t_0}^{t} v_{\text{par}}(t) \, dt, \quad Y(t) \equiv \int_{t_0}^{t} v_{\text{per}}(t) \, dt.
\]
The plot of \((X(t), Y(t))\) is discretely color-coded every 23 hour \((= 1/f_0)\). The down/up arrows indicate when vortices were at the minimum of the potential landscape (bright sites in Fig. 2b). Dotted grid lines are drawn at every 69 nm, corresponding to the lattice constant of the VL at 0.496 T. The red and blue curves are the parametric plots of \((X(t), v_{\text{par}}(t))\) and \((X(t), v_{\text{per}}(t))\) respectively, where \((X(t), v_{\text{per}}(t))\) is shifted vertically by \(-10\) pm/s for clarity.
Figure 4: **Turning off the vortex motion.** a, Experimental procedure to maintain a constant magnetic field in a field-decaying superconducting magnet (Osheroff, D.; Private communication). First, while the heat switch is turned on (the residual resistance $R$ becomes $R_{\text{on}} \sim 20 \, \Omega$), a current $I_0$ flows through $L$, because the path across $R_{\text{on}}$ is now effectively open ($\tau_{\text{on}} = L/R_{\text{on}} = 12.4 \, \text{H}/20 \, \Omega \sim 0.5 \, \text{s}$). Second, by turning off the heat switch to store the magnetic field $B(I_0)$ in the magnet, $R_{\text{on}}$ reduces to $R_{\text{off}} \sim 0.1 \, \mu\Omega$. In other words, $\tau_{\text{on}}$ increases to $\approx 4 \, \text{years}$ ($\tau_{\text{off}} = 12.4 \, \text{H}/0.1 \, \mu\Omega$). Therefore, $I_0$ from the external current source flows through $R_{\text{off}}$ instead of $L$. Lastly, one continues supplying $I_0$ from the external current source to the magnet, even after the heat switch is off and $B(I_0)$ is stored. Then two currents, one from the external current source and the other in the magnet, flow in the opposite direction with the same magnitude $I_0$ across $R_{\text{off}}$, which leads to quite a reduced power dissipation at $R_{\text{off}}$, compared to that in the field-decaying case. b, Stationary vortices. According to the overlaid trajectories (color) over 2 days, vortices were prevented from moving while the procedure in a was employed. The continuous measurement time was significantly reduced, compared to the 7-day measurement. This is because the liquid helium boil-off rate increased with the magnet leads submerged in liquid helium to supply current during the measurement. The grayscale image was averaged over 704 VL images. The observation was made in an area of $400 \times 400 \, \text{nm}^2$ under $0.75 \, \text{T}$. The tunneling was achieved using $I_t = 0.1 \, \text{nA}$ and $V_{\text{bias}} = 3 \, \text{mV}$, while scanning at a scan-speed of $551 \, \text{nm/s}$.