Varying Alpha Driven by the Dirac-Born-Infeld Scalar Field

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ABSTRACT

Since about ten years ago, varying $\alpha$ theories attracted many attentions, mainly due to the first observational evidence from the quasar absorption spectra that the fine structure “constant” might change with cosmological time. In this work, we investigate the cosmic evolution of $\alpha$ driven by the Dirac-Born-Infeld (DBI) scalar field. To be general, we consider various couplings between the DBI scalar field and the electromagnetic field. We also confront the resulting $\Delta \alpha/\alpha$ with the observational constraints, and find that various cosmological evolution histories of $\Delta \alpha/\alpha$ are allowed. Comparing with the case of varying $\alpha$ driven by quintessence, the corresponding constraints on the parameters of coupling have been relaxed, thanks to the relativistic correction of the DBI scalar field.

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I. INTRODUCTION

For many years, there are some unremitting speculations in the subject of the possible variations of fundamental constants. One of the earliest works is the famous large number hypothesis proposed by Dirac in 1937 [1]. In the fundamental “constants”, the most observationally sensitive one is the electromagnetic fine structure “constant”, \( \alpha = e^2/\hbar c \). Since about ten years ago, this subject attracted many attentions again, mainly due to the first observational evidence from the quasar absorption spectra that the fine structure “constant” might change with cosmological time [2, 3].

Subsequently, many authors obtained various observational constraints on the possible variation of the fine structure “constant” \( \alpha \). In the literature, it is convenient to introduce a quantity \( \Delta \alpha/\alpha = (\alpha - \alpha_0)/\alpha_0 \), where the subscript “0” indicates the present value of the corresponding quantity. Obviously, \( \Delta \alpha/\alpha \) is time-dependent. A brief summary of the observational constraints on \( \Delta \alpha/\alpha \) can be found in e.g. [4]. The most ancient constraint comes from the Big Bang Nucleosynthesis (BBN) [5, 6], namely, \( \Delta \alpha/\alpha \lesssim 10^{-2} \), in the redshift range \( z = 10^6 - 10^8 \). The next constraint comes from the power spectrum of anisotropy in the cosmic microwave background (CMB) [6], i.e., \( \Delta \alpha/\alpha < 10^{-2} \), for redshift \( z \approx 10^1 \). In the medium redshift range, the constraint comes from the absorption spectra of distant quasars [2, 3, 7, 8]. Since the results in the literature are controversial, it is better to consider the conservative constraint \( \Delta \alpha/\alpha \lesssim 10^{-6} \), in the redshift range \( z = 3 - 0.4 \). From the radioactive life-time of \(^{187}\)Re derived from meteoritic studies [9], the constraint is given by \( \Delta \alpha/\alpha \lesssim 10^{-7} \) for redshift \( z = 0.45 \). Finally, from the Oklo natural nuclear reactor [10], it is found that \( \Delta \alpha/\alpha \lesssim 10^{-7} \) for redshift \( z = 0.14 \). For convenience, we summarize the above constraints in Table I and label them by the gray areas in Figs. 2—5, 7 and 8.

| \( \Delta \alpha/\alpha \) | redshift | observation | Ref. |
|-----------------|---------|-------------|-----|
| \( \lesssim 10^{-2} \) | \( 10^6 - 10^8 \) | BBN | [5, 6] |
| \( < 10^{-2} \) | \( 10^4 \) | CMB | [6] |
| \( \lesssim 10^{-6} \) | \( 3 - 0.4 \) | quasars | [2, 3, 7, 8] |
| \( \lesssim 10^{-7} \) | 0.45 | meteorite | [9] |
| \( \lesssim 10^{-7} \) | 0.14 | Oklo | [10] |

A varying \( \alpha \) might be due to a varying speed of light \( c \) [11, 12, 13], while Lorentz invariance is broken. The other possibility for a varying \( \alpha \) is due to a varying electron charge \( e \). In 1982, Bekenstein proposed such a varying \( \alpha \) model [14], which preserves local gauge and Lorentz invariance, and is generally covariant. This model has been revived and generalized after the first observational evidence of varying \( \alpha \) from the quasar absorption spectra [2, 3]. This is a dilaton theory with coupling to the electromagnetic \( F^2 \) part of the Lagrangian, but not to the other gauge fields. One example of this type of models is the so-called BSBM model in the literature [15, 16, 17, 18].

On the other hand, dark energy [19] has been one of the most active fields in modern cosmology since the discovery of accelerated expansion of our universe [20]. Most of dark energy models are described by a dynamical scalar field. It is possible to image that such a cosmological scalar field could be coupled with the electromagnetic field, and hence could drive the variation of \( \alpha \). So, one can generalize the Bekenstein-type varying \( \alpha \) model by replacing the dilaton with the scalar field dark energy. Further, the coupling between the scalar field and the electromagnetic field could also be generalized. Actually, the varying \( \alpha \) models driven by quintessence have been extensively investigated in the literature (e.g. [3, 21, 22, 23, 24, 23, 22, 27, 30]). In addition, we mention that varying \( \alpha \) driven by phantom has been considered in the BSBM model [15, 16, 17, 18] while its model parameter \( \omega \) is negative. The special case of varying \( \alpha \) driven by k-essence whose Lagrangian \( \mathcal{L}(X) = X^\omega - V(\phi) \) has also been considered in e.g. [21].

Recently, the Dirac-Born-Infeld (DBI) scalar field attracted many attentions. In type IIB string theory, the DBI action arises naturally in the D3-brane motion within a warped geometry or “throat”. It can give a variety of novel cosmological consequences. For instance, the DBI scalar field can be used to drive the inflation (see e.g. [28, 29, 31, 31]). More recently, the DBI scalar field has been proposed to play the
role of dark energy \[32, 33, 34, 35\]. Therefore, it is natural to consider the varying \( \alpha \) driven by the DBI scalar field in the present work.

This paper is organized as followings. In Sec. II, we will briefly review the varying \( \alpha \) driven by quintessence. In Sec. III, we consider the varying \( \alpha \) driven by the DBI scalar field, and confront it with the observational constraints. A brief conclusion is given in Sec. IV.

II. BRIEF REVIEW ON THE VARYING ALPHA DRIVEN BY QUINTESSENCE

Following \([4, 21, 27]\), the relevant action is given by

\[
S = \frac{1}{2m^2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} L_\phi - \frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} + S_m + S_r,
\]

where \( F_{\mu\nu} \) are the components of the electromagnetic field tensor; \( S_m \) and \( S_r \) are the actions of pressureless matter and radiation, respectively; \( m_p \equiv (8\pi G)^{-1/2} \) is the reduced Planck mass; \( L_\phi \) is the Lagrangian of the scalar field. For the case of quintessence, the corresponding \( L_\phi \) reads

\[
L_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi),
\]

where \( V(\phi) \) is the potential. Notice that \( B_F \) takes the place of \( e^{-2} \) in Eq. (1) actually \([22, 37]\), one can easily see that the effective fine structure “constant” is given by \([4, 21]\)

\[
\alpha = \frac{\alpha_0}{B_F(\phi(x, t))}.
\]

Thus, we find that

\[
\Delta \alpha = \frac{\alpha - \alpha_0}{\alpha_0} = \frac{1 - B_F(\phi)}{B_F(\phi)}.
\]

Notice that the present value of the coupling \( B_F \) should be 1. In general, \( \phi \) and hence \( \alpha \) are functions of spacetime. However, as is well known, we can safely neglect the spatial variation of \( \phi \) and \( \alpha \), which is usually a good approximation. Therefore, we only consider the homogeneous \( \phi \) and \( \alpha \) throughout this paper. The relevant equations governing the cosmological evolution in a flat universe read

\[
H^2 = \frac{1}{3m^2} (\rho_m + \rho_r + \rho_\phi),
\]

\[
\ddot{\phi} + 3H\dot{\phi} + V(\phi) = 0,
\]

where \( H \equiv \dot{a}/a \) is the Hubble parameter; \( a = (1 + z)^{-1} \) is the scale factor (we have set \( a_0 = 1 \); \( z \) is the redshift; a dot denotes the derivative with respect to cosmic time \( t \); the energy densities of pressureless matter and radiation are given by \( \rho_m = \rho_{m0}a^{-3} \) and \( \rho_r = \rho_{r0}a^{-4} \), respectively; \( \rho_\phi \) is the energy density of scalar field \( \phi \) [for the case of quintessence, \( \rho_\phi = \dot{\phi}^2/2 + V(\phi) \)]; the subscript “\( \phi \)” denotes the derivative with respect to \( \phi \). In fact, due to the coupling between the scalar field and the electromagnetic field, there should be an additional term in the right hand side of the equation of motion for \( \phi \), namely Eq. (6). This additional term is proportional to \( F_{\mu\nu} F^{\mu\nu} \) and the derivative of \( B_F \) \([21]\). However, it can be safely neglected thanks to the following facts: (i) the derivative of \( B_F \) is in fact equivalent to the time derivative of \( \alpha \) [cf. Eq. (3)], which is very small (given equivalence principle constraints \([27]\)]; see e.g. \([21]\); (ii) the statistical average of the term \( F_{\mu\nu} F^{\mu\nu} \) over a current state of the universe is zero \([4]\).

One can numerically solve Eqs. (5) and (6) to obtain the cosmological evolution of \( \phi(t) \). Then, the corresponding \( \alpha(t) \) is ready. We can confront it with the observational constraints. In fact, the varying \( \alpha \) models driven by quintessence have been extensively investigated in the literature (e.g. \([4, 21, 22, 23, 24, 25, 26, 27]\)). In the next section, we turn to the case of DBI scalar field.
III. VARYING ALPHA DRIVEN BY THE DBI SCALAR FIELD

In this section, we consider the varying $\alpha$ driven by the DBI scalar field. We firstly give out the relevant equations and solve them to get $\phi(t)$ and hence $\alpha(t)$. Then, we confront the fine structure “constant” $\alpha$ with the observational constraints.

A. Equations

The Lagrangian of DBI scalar field is given by [28, 33, 34]

$$\mathcal{L}_\phi = -\frac{1}{g_{\gamma m}^2} \left[ (f(\phi))^{-1} \sqrt{1 + f(\phi) \partial_\mu \phi \partial^\mu \phi} - f(\phi)^{-1} + V(\phi) \right],$$

where $g_{\gamma m}^2$ is the Yang-Mills coupling; $V(\phi)$ is the potential; $T = f(\phi)^{-1}$ is the warped brane tension. The pressure and energy density of the DBI scalar field are given by [28, 33, 34]

$$p_\phi = \frac{\gamma - 1}{f} - V(\phi),$$

$$\rho_\phi = \frac{\gamma - 1}{f} + V(\phi),$$

where the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - f(\phi) \dot{\phi}^2}},$$

which measures the “relativistic” motion of the DBI scalar field. In the “non-relativistic” limit, $K/T \ll 1$, and $\gamma \rightarrow 1 + K/T$, where $K = \dot{\phi}^2/2$ is the canonical kinetic energy. In this case, the equation-of-state parameter (EoS) of DBI scalar field $w = p_\phi/\rho_\phi \rightarrow (K - V)/(K + V)$, i.e., the DBI scalar field reduces to an ordinary quintessence field [34]. In the “ultra-relativistic” limit, $\gamma \rightarrow \infty$. In the medium $\gamma$ range, the non-canonical behavior due to the relativistic corrections will be crucial [28, 33, 34].

The equation of motion for the DBI scalar field reads [28, 33]

$$\ddot{\phi} + \frac{3f \dot{\phi}}{2f} \dot{\phi}^2 - \frac{f \dot{\phi}}{f^2} + \frac{3H}{\gamma} \dot{\phi} + \left(V,\phi + \frac{f \dot{\phi}}{f^2}\right) \frac{1}{\gamma^2} = 0.$$  (11)

It is worth noting that the additional term in the right hand side of Eq. (11) due to the coupling between the scalar field and the electromagnetic field can be safely neglected, as already mentioned in Sec. III. The Friedmann equation is given by [28, 33]

$$H^2 = \frac{1}{g_{\gamma m}^2} \frac{1}{3m_p^2} (\rho_m + \rho_c + \rho_\phi),$$

in which the corresponding $\rho_\phi$ is given in Eq. (9). For the AdS throat, $f(\phi)$ is given by [28, 33, 34]

$$f(\phi) = \frac{\lambda}{\phi^4},$$

where $\lambda$ is a dimensionless constant. As in [33, 34], here we consider a quadratic potential

$$V(\phi) = \frac{1}{2} m^2 \phi^2,$$
where $m$ is a constant. It is convenient to introduce the following dimensionless quantities

$$
\tilde{\phi} \equiv \frac{\phi}{m_p}, \quad \tilde{t} \equiv mt, \quad \tilde{m} \equiv \frac{1}{g_{YM}} \cdot \frac{H_0}{m}, \quad \tilde{\lambda} \equiv \frac{m^2}{m_p^2} \lambda.
$$

Thus, we can recast Eq. (12) as

$$
\dot{\tilde{H}}^2 = \left(\frac{\dot{a}}{a}\right)^2 = \tilde{m}^2 \left(\Omega_{m0} a^{-3} + \Omega_{r0} a^{-4} + \Omega_{\phi0} \frac{\tilde{p}_\phi}{\tilde{\rho}_0}\right),
$$

where $\Omega_i \equiv \rho_i/(3m_p^2H_0^2)$ are the fractional energy densities of pressureless matter, radiation and DBI scalar field for $i = m, r$ and $\phi$, respectively; a prime denotes the derivative with respect to the new time variable $\tilde{t}$; and

$$
\frac{\tilde{p}_\phi}{\tilde{\rho}_{\phi0}} = \frac{\rho_\phi}{\rho_{\phi0}} = \frac{(\tilde{\gamma} - 1) \tilde{\phi}^4 \tilde{\lambda}^{-1} + \tilde{\phi}^2/2}{(\tilde{\gamma}_0 - 1) \tilde{\phi}_0^4 \tilde{\lambda}^{-1} + \tilde{\phi}_0^2/2},
$$

in which

$$
\tilde{\gamma} = \gamma = \frac{1}{\sqrt{1 - \tilde{\lambda} \tilde{\phi}^{-4} \tilde{\phi}^2}}.
$$

Also, we recast Eq. (11) as

$$
\tilde{\phi}'' - \frac{6}{\tilde{\phi}^2} \tilde{\phi}^2 + \frac{4}{\tilde{\lambda}} \tilde{\phi}^3 + \frac{3\tilde{H}}{\tilde{\gamma}^2} \tilde{\phi}' + \left(\tilde{\phi} - \frac{4}{\tilde{\lambda}} \tilde{\phi}^3\right) \frac{1}{\tilde{\gamma}^4} = 0.
$$

FIG. 1: The solution $\tilde{\phi}$ versus log $a$, for the case of $\tilde{\gamma}_0 = \gamma_0 = 10$. See text for details.

We can find out $\tilde{\phi}(\tilde{t})$ and $a(\tilde{t})$ by numerically solving the coupled differential equations (16) and (19), with Eqs. (17) and (18). For convenience, as in [4, 34], we adopt $\tilde{m} = 1$, $\tilde{\lambda} = 1$, and $g_{YM} = 1$. The initial conditions are chosen to be $\Omega_{\phi0} = 0.72$, $\Omega_{m0} = 0.27$ and hence $\Omega_{r0} = 0.01$. As is well known, they
are fully consistent with cosmological observations. Of course, the initial condition for $a$ is $a_0 = 1$. The initial conditions for $\phi$ are determined by $\Omega_{\phi 0}$, through

$$\Omega_{\phi 0} \equiv \frac{\rho_{\phi 0}}{3m_H^2 H_0^2} = \frac{\frac{\gamma_0}{3}}{2 \left( (\frac{\gamma_0}{3} - 1) \phi_0^2 + \frac{1}{2} \right)}.$$  

(20)

As mentioned above, DBI scalar field reduces to quintessence when $\gamma = 10$ for DBI scalar field. Thus, we can find $\phi_0$ from Eq. (20). Then, we can obtain $\phi'(t)$ and $a(t)$ by numerically solving the differential equations (16) and (19) with the initial conditions given above. Once $\phi(t)$ and $a(t)$ are ready, it is easy to get $\phi(\log a)$, where log indicates the logarithm to base 10. In Fig. 2 we present the solution $\phi$ versus $\log a$, for the case of $\gamma_0 = \gamma_0 = 10$.

![Figure 2: We plot log $|\Delta a|/a|$ as a function of log $a$ for Case (I) with $\zeta = 0.9 \times 10^{-6}$ (solid line), $\zeta = 3 \times 10^{-6}$ (short-dashed line) and $\zeta = 0.3 \times 10^{-6}$ (long-dashed line). Right panel is the enlarged part of $\log a \geq -1$. Only the curves not overlapping the gray areas are phenomenologically viable. Here, we adopt $\gamma_0 = \gamma_0 = 10$.

B. Cosmic evolution of $\alpha$ driven by the DBI scalar field with $\gamma_0 = \gamma_0 = 10$

Once the cosmic evolution of DBI scalar field $\phi$ is on hand, we can easily figure out the corresponding cosmic evolution of $\alpha$ from Eqs. (3) and (4), if the coupling $B_F(\phi)$ is given. In the literature, most authors restricted themselves to the case of linear coupling for simplicity. Instead, to be general, here we consider various coupling $B_F$ following [4]. Also, we confront the varying $\alpha$ driven by the DBI scalar field with the observational constraints mentioned in Sec. I. In this subsection, we fix $\gamma_0 = \gamma_0 = 10$.

- Case (I) Linear coupling
  
  In this case, the coupling is given by

$$B_F(\phi) = 1 - \zeta \left( \phi - \phi_0 \right),$$  

(21)

where $\zeta$ is a constant. This is the mostly considered coupling in the literature. From Eq. (4), we obtain the resulting $\Delta a/\alpha$, and present it in Fig. 2. We tried various $\zeta$ to verify in which cases all the observational constraints mentioned in Sec. I could be simultaneously satisfied. We found that they can be all respected for $\zeta \leq 0.9 \times 10^{-6}$. Notice that in the varying $\alpha$ model driven by quintessence [4] the upper bound of $\zeta$ is $0.6 \times 10^{-6}$ for the same $B_F$. So, we see that the constraint on $\zeta$ has been relaxed, thanks to the relativistic correction of the DBI scalar field.
FIG. 3: We plot \( \log |\Delta \alpha/\alpha| \) as a function of \( \log a \) for Case (II) with \( \zeta = 10^{-4} \) and \( q = 3.15 \) (solid line), \( q = 4 \) (short-dashed line) and \( q = 2.5 \) (long-dashed line). Right panel is the enlarged part of \( \log a \geq -1 \). Only the curves not overlapping the gray areas are phenomenologically viable. Here, we adopt \( \gamma_0 = \gamma_0 = 10. \)

- Case (II) Polynomial coupling
  In this case, one can generalize Eq. (21) to
  \[
  B_F(\tilde{\phi}) = 1 - \zeta \left( \tilde{\phi} - \tilde{\phi}_0 \right)^q, \tag{22}
  \]
  which allows the exponent \( q \) to be free. In this case, we find that the observations cannot put any upper bound on the exponent \( q \). In Fig. 3, we present the resulting \( \Delta \alpha/\alpha \) for a fixed \( \zeta = 10^{-4} \) and various \( q \). We find that the observational constraints can be all respected for \( q \geq 3.15 \). Notice that in the varying \( \alpha \) model driven by quintessence \([4]\) the lower bound of \( q \) is 6 for the same \( B_F \) with the same \( \zeta = 10^{-4} \). So, we see that the constraint on \( q \) has been relaxed, thanks to the relativistic correction of the DBI scalar field. On the other hand, we find that the upper bound of \( \zeta \) can be relaxed by increasing \( q \). In fact, the fine tuning in \( \zeta \) can be reduced for the enough large \( q \). For example, as shown in Fig. 4, with \( q = 8.4 \) the observational constraints can be all respected even for \( \zeta = 1 \). Notice that in the varying \( \alpha \) model driven by quintessence \([4]\) the lower bound of \( q \) is 17 for the same \( B_F \) with the same \( \zeta = 1 \). Again, the constraint on \( q \) has been relaxed in the case of DBI scalar field.

- Case (III) Power-law coupling
  In this case, the coupling under consideration reads
  \[
  B_F(\tilde{\phi}) = \left( \frac{\tilde{\phi}}{\tilde{\phi}_0} \right)^\epsilon, \tag{23}
  \]
  where \( \epsilon \) is a constant. In Fig. 5, we present the resulting \( \Delta \alpha/\alpha \) for various \( \epsilon \). We find that all the observational constraints can be respected for \( \epsilon \leq 5.5 \times 10^{-7} \). Notice that in the varying \( \alpha \) model driven by quintessence \([4]\) the upper bound of \( \epsilon \) is \( 4 \times 10^{-7} \) for the same \( B_F \). Again, we see that the constraint on \( \epsilon \) has been relaxed, thanks to the relativistic correction of the DBI scalar field.

- Case (IV) Exponential coupling
  In this case, the coupling is given by
  \[
  B_F(\tilde{\phi}) = e^{-\zeta (\tilde{\phi} - \tilde{\phi}_0)}. \tag{24}
  \]
Notice that $\ddot{\phi} - \ddot{\phi}_0$ is of order unity (see Fig. 1), if $\zeta$ is of order unity or even larger, $B_F$ deviates from 1 considerably, and it is impossible to satisfy all the observational constraints at the same time [cf. Eqs. (3) and (4)]. If $\zeta \ll 1$, we see that $B_F(\ddot{\phi}) = e^{-\zeta(\ddot{\phi} - \ddot{\phi}_0)} \simeq 1 - \zeta (\ddot{\phi} - \ddot{\phi}_0)$, and hence Case (IV) reduces to Case (I) considered above.

![Graph](image1)

**FIG. 4:** We plot $\log |\Delta \alpha/\alpha|$ as a function of $\log \alpha$ for Case (II) with $\zeta = 1$ and $q = 8.4$ (solid line). Right panel is the enlarged part of $\log \alpha \geq -1$. Only the curves not overlapping the gray areas are phenomenologically viable. Here, we adopt $\bar{\gamma}_0 = \gamma_0 = 10$.

![Graph](image2)

**FIG. 5:** We plot $\log |\Delta \alpha/\alpha|$ as a function of $\log \alpha$ for Case (III) with $\epsilon = 5.5 \times 10^{-7}$ (solid line), $\epsilon = 3 \times 10^{-6}$ (short-dashed line) and $\epsilon = 1 \times 10^{-7}$ (long-dashed line). Right panel is the enlarged part of $\log \alpha \geq -1$. Only the curves not overlapping the gray areas are phenomenologically viable. Here, we adopt $\bar{\gamma}_0 = \gamma_0 = 10$.

**C. Cosmic evolution of $\alpha$ driven by the DBI scalar field with various $\bar{\gamma}_0$**

In the previous subsection, we considered the cosmic evolution of $\alpha$ driven by the DBI scalar field with a fixed $\bar{\gamma}_0 = \gamma_0 = 10$. In this subsection, to see the relevance of the relativistic correction of the DBI scalar field (which is measured by the Lorentz factor $\gamma$), we consider the cases with various $\bar{\gamma}_0$. 
For simplicity, we only consider the linear coupling $B_F$ given in Eq. (21). At first, we adopt $\tilde{\gamma}_0 = \gamma_0 = 5$. Following the procedure described in the end of Sec. III A, we can obtain the numerical solution $\tilde{\phi}$ versus $\log a$, and present it in the left panel of Fig. 6. Then, from Eq. (4), we get the resulting $\Delta \alpha / \alpha$, and present it in Fig. 7. We tried various $\zeta$ to verify in which cases all the observational constraints mentioned in Sec. I could be simultaneously satisfied. We found that they can be all respected for $\zeta \leq 0.63 \times 10^{-6}$. Although the upper bound of $\zeta$ is still larger than the upper bound $0.6 \times 10^{-6}$ in the varying $\alpha$ model driven by quintessence [4], they are fairly close in fact. This is not surprising. As mentioned in Sec. III A, the DBI scalar field becomes closer to quintessence when $\gamma$ is smaller [28, 33, 34].

![Graphs showing the solution $\tilde{\phi}$ versus $\log a$ for $\tilde{\gamma}_0 = \gamma_0 = 5$ (left panel) and $100$ (right panel).](image)

FIG. 6: The solution $\tilde{\phi}$ versus $\log a$, for the case of $\tilde{\gamma}_0 = \gamma_0 = 5$ (left panel) and $100$ (right panel).

Other other hand, we consider the case of $\tilde{\gamma}_0 = \gamma_0 = 100$. We plot the corresponding numerical solution $\tilde{\phi}$ versus $\log a$ in the right panel of Fig. 6 and also present the resulting $\Delta \alpha / \alpha$ in Fig. 8. We find that all the observational constraints can be respected for $\zeta \leq 2.75 \times 10^{-6}$. Obviously, the constraint on $\zeta$ has been significantly relaxed, comparing with both the cases of DBI scalar field with $\tilde{\gamma}_0 = \gamma_0 = 10$ and quintessence [4]. This is due to the ultra-relativistic effect which is measured by the large $\gamma_0 = 100$. The DBI scalar field significantly deviates from quintessence when $\gamma$ is fairly large [28, 33, 54].

Together with the results of $\tilde{\gamma}_0 = \gamma_0 = 5$, $10$ and $100$, we can clearly see that the relaxation of the constraints on the parameters of coupling is mainly due to the relativistic correction of the DBI scalar field, which is measured by the Lorentz factor $\gamma$. The DBI scalar field deviates from quintessence more significantly when $\gamma$ is larger [28, 33, 54]; and hence as we have shown above, the constraints on the parameters of coupling is looser.

IV. CONCLUSION

Since about ten years ago, varying $\alpha$ theories attracted many attentions, mainly due to the first observational evidence from the quasar absorption spectra that the fine structure “constant” might change with cosmological time [2, 3]. In this work, we investigated the cosmic evolution of $\alpha$ driven by the DBI scalar field. To be general, we considered various couplings between the DBI scalar field and the electromagnetic field. We also confronted the resulting $\Delta \alpha / \alpha$ with the observational constraints, and found that various cosmological evolution histories of $\Delta \alpha / \alpha$ are allowed. Comparing with the case of varying $\alpha$ driven by quintessence [4], the corresponding constraints on the parameters of coupling have been relaxed, thanks to the relativistic correction of the DBI scalar field.
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FIG. 7: We plot log $|\Delta \alpha/\alpha|$ as a function of log $a$ for Case (I) with $\zeta = 0.63 \times 10^{-6}$ (solid line), $\zeta = 2 \times 10^{-6}$ (short-dashed line) and $\zeta = 0.2 \times 10^{-6}$ (long-dashed line). Right panel is the enlarged part of log $a \geq -1$. Only the curves not overlapping the gray areas are phenomenologically viable. Here, we adopt $\gamma_0 = \gamma_0 = 5$.

FIG. 8: We plot log $|\Delta \alpha/\alpha|$ as a function of log $a$ for Case (I) with $\zeta = 2.75 \times 10^{-6}$ (solid line), $\zeta = 8 \times 10^{-6}$ (short-dashed line) and $\zeta = 1 \times 10^{-6}$ (long-dashed line). Right panel is the enlarged part of log $a \geq -1$. Only the curves not overlapping the gray areas are phenomenologically viable. Here, we adopt $\gamma_0 = \gamma_0 = 100$.

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