Reconstruction of modified gravity with ghost dark energy models

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Abstract

In this work, we reconstruct the $f(R)$ modified gravity for different ghost and generalized-ghost dark energy (DE) models in FRW flat universe, which describe the accelerated expansion of the universe. The equation of state and deceleration parameter of reconstructed $f(R)$ gravity have been calculated. The equation of state and deceleration parameter of reconstructed $f(R)$-ghost/generalized-ghost DE, have been calculated. We show that the corresponding $f(R)$ gravity of ghost/generalized-ghost DE model can behave like phantom or quintessence. Also the transition between deceleration to acceleration regime is indicated by deceleration parameter diagram for reconstructed $f(R)$ generalized-ghost DE model.

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I. INTRODUCTION

The acceleration expansion of the present universe has been accepted by many observational evidence [1]. Although the dark energy (DE) scenario with a negative pressure can drive this expansion, the modified gravity, so called $f(R)$ gravity, has been introduced as a self consistent and nontrivial alternative way to explain early time inflation and present acceleration expansion. In $f(R)$ gravity, the Einstein-Hilbert action is modified and generalized by incorporating a new phenomenological function of Ricci scalar, $f(R)$.

On the other hand the dynamical DE models are proposed in order to solve DE problems (cosmic coincidence and fine tuning problems). Recently, the Veneziano ghost DE has been attracted a deal of attention in the dynamical DE category. The Veneziano ghost is proposed to solve the $U(1)$ problem in low-energy effective theory of QCD [2] and has no contribution in the flat Minkowski spacetime. In curved spacetime, however, it makes a small energy density proportional to $\Lambda_{QCD}^3 H$, where $\Lambda_{QCD}$ is QCD mass scale and $H$ is Hubbel parameter. This small vacuum energy density can be considered as a driver engine for evolution of the universe. It should be mentioned that the Veneziano ghost DE model does not violate unitarity, causality, gauge invariance etc [3–5]. In fact the description of DE in terms of the Veneziano ghost is just a matter of convenience to describe very complicated infrared dynamics of strongly coupled QCD. In other words, the veneziano ghost is not a new physical propagating degree of freedom. The same dynamics can be described without using the ghost, with some other approaches (e.g. direct lattice simulations). Also note that this model is totally arisen from standard model and general relativity. Therefore one needs not to introduce any new parameter or new degree of freedom and this fact is the most advantages of ghost DE. With $\Lambda_{QCD} \sim 100Mev$ and $H \sim 10^{-33}ev$, the right order of observed DE density can be given by ghost DE. This numerical coincidence also shows that this model gets ride of fine tuning problem [4, 5]. Many authors have already suggested DE model with energy density as $\rho = \alpha H$ [6, 7] and $\rho = \alpha H + O(H^2)$ [8]. Generally it is very difficult to accept such a power like behavior as QCD is a theory with a mass gap determined by the scale $\sim 100 Mev$. The power like scaling $\sim H$ is due to the complicated topological structure of strongly coupled QCD, not related to the physical massive propagating degrees of freedom. In fact the linear in Hubble constant “$H$” scaling is not a baseless assumption, but rather has a strong theoretical support tested in a number of models, and tested in
the lattice QCD simulations. Recently the ghost-scalar field models have been investigated. Further more ghost DE model has been fitted with current observational data including SNIa, BAO, CMB, BBN.

On the other hand, a consistent \( f(R) \) function in modified gravity context, has been reconstructed by many dynamical DE models. It is worthwhile to mention that the modified \( f(R) \) gravity model considered as an effective description of the underlying theory of DE, and considering the ghost DE scenario as pointing in the direction of the underlying theory of DE, it is interesting to study how the \( f(R) \) gravity can describe the ghost DE density as effective theory of DE models. This motivated us to establish the different models of \( f(R) \) gravity according to the ghost and generalized-ghost DE model. In this paper we want to reconstruct a consistent modified gravity so that it gives the cosmological evolution of ghost DE model.

II. GENERAL FORMALISM OF \( F(R) \) GRAVITY

The general action of \( f(R) \) gravity in Planck mass unit in which \( h = c = 8\pi G = 1 \), is given by

\[
S = \int \sqrt{-g} \, d^4x \left[ \frac{R + f(R)}{2} + \mathcal{L}_m \right],
\]

where \( g, R \) and \( \mathcal{L}_m \) are the determinant of metric \( g_{\mu\nu} \), Ricci scalar and matter contribution of the action, respectively. Here \( G \) is gravitational constant and \( f(R) \) is an arbitrary function of \( R \). In metric formalism, Where the action is varied only by the metric, by taking the variation of action with respect to the metric \( g_{\mu\nu} \), the following field equation is obtained

\[
\left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) - \left[ \frac{1}{2} g_{\mu\nu} f(R) - R_{\mu\nu} f'(R) + \left( \nabla_\mu \nabla_\nu - g_{\mu\nu} \Box \right) f'(R) \right] = T^{(m)}_{\mu\nu}.
\]

Here the prime denotes a derivative with respect to \( R \) and \( R_{\mu\nu} \) and \( T^{(m)}_{\mu\nu} \) are the Ricci tensor and the energy-momentum tensor of the matter, respectively.

In a spatially flat-FRW background universe, with following line element

\[
ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2 \right],
\]

by taking the matter in the prefect fluid form, in which \( T^{(m)}_{\mu\nu} = \text{diag}(-\rho_m, p_m, p_m, p_m) \), the field equation reduces to following modified Friedmann equations

\[
H^2 = \frac{1}{3} \rho_{eff},
\]
\[2\dot{H} + 3H^2 = -p_{\text{eff}}.\]  
(5)

Here \( H = \dot{a}/a \) is the Hubble parameter and

\[
\rho_{\text{eff}} = \left[ -\frac{1}{2} f(R) + 3(\dot{H} + H^2) f'(R) - 18(4H^2\dot{H} + H\ddot{H}) f''(R) \right] + \rho_m, \]
(6)

\[
p_{\text{eff}} = \left[ \frac{1}{2} f(R) - (\dot{H} + 3H^2) f'(R) \\
+ 6(8H^2\dot{H} + 6H\ddot{H} + 4H^2 + \ddot{H}) f''(R) + 36(\dot{H} + 4H\dot{H})^2 f'''(R) \right] + p_m. \]
(7)

The Ricci scalar is obtained as

\[ R = 6(\dot{H} + 2H^2), \]
(8)

where the dot denotes a derivative with respect to cosmic time \( t \).

The energy conservation law gives

\[ \dot{\rho}_{\text{eff}} + 3H\rho_{\text{eff}}(1 + w_{\text{tot}}) = 0, \]
(9)

where

\[ w_{\text{tot}} = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} \]
(10)

is the total equation of state (EoS) parameter. In the absence of any matter field, ‘\( F(R) \) dominated universe’, the EoS parameter has only the gravitational contribution as

\[ w_R = -1 - \frac{\dot{H} f'(R) + 3(3H\ddot{H} - 4H^2\dot{H} + 4\dot{H}^2 + \ddot{H}) f''(R) + 18(\ddot{H} + 4H\dot{H})^2 f'''(R)}{f(R) - 6(\dot{H} + H^2) f'(R) + 36(4H^2\dot{H} + H\ddot{H}) f''(R)} \].
(11)

In this case the first modified Friedmann equation (4) yields

\[ 3H^2 = \rho_R, \]
(12)

where \( \rho_R \) is the gravitational contribution of energy density (6) for \( \rho_m = 0 \).

Taking time derivative of Eq. (12) and using continuity equation (9), one can find a new relation for EoS parameter as

\[ w_R = -1 - \frac{2\dot{H}}{3H^2}. \]
(13)

For \( \dot{H} > 0 \), we have \( w_R < -1 \), indicating the phantom DE dominated universe. In the case of \( \dot{H} < 0 \), we have \( w_R > -1 \), representing the quintessence DE dominated universe. The deceleration parameter in the \( F(R) \) dominated universe is obtained as

\[ q_R = -\frac{\ddot{a}}{a^2} = -1 - \frac{\dot{H}}{H^2} = -\frac{3w_R + 1}{2}. \]
(14)
In the case of $w_R < -1/3$, we have $q < 0$ indicating the accelerated expansion of the universe and for $w_R > -1/3$, we have $q > 0$, representing the decelerated expansion phase.

In the theories of modified gravity there are two classes of scale factor that have been usually used in the literature \[16\]. The first one has been given by

$$ a(t) = a_0(t_s - t)^{-h}, \quad t \leq t_s, \quad h > 0. $$

(15)

Using Eqs. (8) and (15), one can obtain

$$ H = \frac{h}{t_s - t} = \left[ \frac{hR}{12h + 6} \right]^{1/2}, \quad \dot{H} = H^2/h, \quad \ddot{H} = \frac{2H^3}{h^2}. $$

(16)

Here a big rip singularity will be happened at $t = t_s$ \[17\]. It is easy to see that for the first class of scale factor given by (15) we have $\dot{H} > 0$. Hence from (13), one can conclude that for this class of scale factor $w_R < -1$, indicating the phantom DE dominated universe. This scale factor is usually so-called phantom scale factor.

The second class of scale factor has been defined as

$$ a(t) = a_0 t^h, \quad h > 0, $$

(17)

The Hubble parameter, in this case, is obtained as

$$ H = \frac{h}{t} = \left[ \frac{hR}{12h - 6} \right]^{1/2}, \quad \dot{H} = -H^2/h, \quad \ddot{H} = \frac{2H^3}{h^2}. $$

(18)

From (13) one can see that $\dot{H} = -H^2/h < 0$ represents the quintessence dominated universe. Hence, the model (17) is so-called the quintessence scale factor.

In sections (3) and (4), by using the above classes of scale factor, we reconstruct the different $f(R)$-gravities according to the ghost and modified ghost DE models.

### III. F(R) RECONSTRUCTION FROM GHOST DE

The energy density of ghost DE is given by

$$ \rho_{\Lambda} = \alpha H $$

(19)

where $\alpha$ is a constant of model.
A. phantom scale factor

For the first class of scale factor (15) and using (16), the energy density of ghost DE in (19) is written as

$$\rho = \alpha \sqrt{hR_{12}}$$

Equating (20) and (6) by using (16) gives the following solution

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \alpha \gamma_p R^{\frac{1}{2}}$$

where

$$m_{\pm} = \frac{h + 3 \pm \sqrt{h^2 - 10h + 1}}{4}$$

and

$$\gamma_p = \frac{2}{9} \sqrt{12h + 6}$$

Here $\lambda_{\pm}$ are the constants of integration and can be obtained from the initial conditions. In order to generating accelerated expansion at the present time, one can assume that $f(R)$ is a small constant at the present time as follows

$$f(R_0) = -2R_0, \quad \text{and} \quad f'(R_0) \sim 0,$$

where $R_0 \sim (10^{33} \text{ev})^2$ is the current curvature $[18]$. Using the boundary condition (24), the constants $\lambda_{\pm}$ in (21) are obtained as

$$\lambda_+ = \frac{4\sqrt{R_0 m_+} + \alpha \gamma_p (1 - 2m_+)}{2R_0^{m_+ - \frac{1}{2}} (m_+ - m_-)}$$

and

$$\lambda_- = \frac{-4\sqrt{R_0 m_+} + \alpha \gamma_p (1 - 2m_-)}{2R_0^{m_- - \frac{1}{2}} (m_+ - m_-)}$$

Inserting (21) into (11) and using (16) obtains the EoS parameter of ghost $f(r)$ gravity as

$$w_R = -1 - \frac{2}{3h},$$

which describe the phantom phase of accelerated universe. It must be mention that, from Eq(32), the reconstruction can be performed provided that

$$0 < h \leq (5 - 2\sqrt{6}) \quad \text{or} \quad h \geq (5 + 2\sqrt{6})$$
The EoS parameter (27), shows that the reconstructed \( f(R) \) gravity from the ghost DE can cross the phantom divide, while in the scenario of DE, the ghost model can cross the phantom divide if the interaction between dark matter and DE is included [19].

Using (27) and (14), the deceleration parameter is constant and calculated as

\[
q = -1 - \frac{1}{h}.
\]  

(29)

It shows that, in this case, for all values of \( h \) in the range (28), our universe is expanded under an accelerating phase \( q > 0 \).

B. Quintessence scale factor

Now we reconstruct ghost \( f(R) \) gravity for the second class scale factor in (17). By using (18), the energy density of ghost DE in (19) can be written as follows

\[
\rho_\Lambda = \alpha \sqrt{\frac{hR}{12h - 6}}
\]  

(30)

Equating (30) and (6) by using (18) gives the following solution

\[
f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \alpha \gamma_q \frac{R^{\frac{1}{2}}}{h}
\]  

(31)

where

\[
m_\pm = \frac{3 - h \pm \sqrt{h^2 + 10h + 1}}{4}
\]  

(32)

and

\[
\gamma_q = \frac{2}{9} \sqrt{\frac{12h - 6}{h}}
\]  

(33)

Using the boundary condition (24), the constants \( \lambda_\pm \) in (31) can be obtained as

\[
\lambda_+ = \frac{4 \sqrt{R_0} m_- + \alpha \gamma_q (1 - 2m_-)}{2R_0^{m_- - \frac{1}{2}} (m_+ - m_-)}
\]  

(34)

and

\[
\lambda_- = -\frac{4 \sqrt{R_0} m_+ + \alpha \gamma_q (1 - 2m_+)}{2R_0^{m_+ - \frac{1}{2}} (m_+ - m_-)}
\]  

(35)

Inserting (31) into (11) and using (18) results the EoS parameter of ghost \( f(r) \) gravity as

\[
w_R = -1 + \frac{2}{3h}, \quad h > 1,
\]  

(36)
which corresponds to the quintessence regime, i.e., \(-1 < w_R < -1/3\). In this case, the deceleration parameter is constant and obtained as

\[
q = -1 + \frac{1}{h}. \tag{37}
\]

In this manner we find that in quintessence regime with \(h > 1\), the parameter \(q < 0\), while for \(1/2 < h \leq 1\), we have \(q \geq 0\), which reveal a deceleration expansion at early time. Therefore by this reconstruction, both deceleration and acceleration have been permitted, separately for various the parameter \(h\).

**IV. F(R) RECONSTRUCTION FROM GENERALIZED- Ghost DE**

Here we reconstruct the \(f(R)\) gravity for generalized-ghost DE. The energy density of phenomenological model is given by

\[
\rho_\Lambda = \alpha H + \beta H^2 \tag{38}
\]

where \(\alpha\) and \(\beta\) are the constants of the model. This model was first proposed in [20] to get an accelerating universe. For other motivation to consider this form see [21]. When \(\beta \to 0\), we recovered the model discussed in Sec. III and for \(\alpha \to 0\), this model give the holographic DE model with Hubble scale as IR-cutoff. Also the subleading term \(H^2\) in the ghost DE model might play a crucial role in the early evolution of the universe [10]. In fact in this model we have a ‘XghostDE’ model where ‘X’ denotes the second term in (38).

**A. Phantom scale factor**

Same as previous section, first we choose the phantom scale factor (15). Therefore the energy density of generalized-ghost DE, by using (16), is rewritten as

\[
\rho_\Lambda = \alpha \sqrt{\frac{hR}{12h + 6}} + \beta \frac{hR}{12h + 6} \tag{39}
\]

Equating (39) and (6) by using (16) results the following solution for \(f(R)\)

\[
f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \alpha \gamma_p R^4 - \frac{\beta}{3} R \tag{40}
\]
where parameters $m_\pm$ and $\gamma_p$ are the same as Eqs. (22) and (23), respectively. Applying the boundary conditions in (24), the constants $\lambda_\pm$ in (40) are determined as

$$\lambda_+ = \frac{2\beta \sqrt{R_0(1 - m_\pm)} + 3\alpha \gamma_p (1 - 2m_\pm) + 12\sqrt{R_0 m_\pm}}{6R_0^{m_+ - \frac{1}{2}}(m_+ - m_-)} \quad (41)$$

$$\lambda_- = \frac{-2\beta \sqrt{R_0(1 - m_\pm)} + 3\alpha \gamma_p (1 - 2m_\pm) + 12\sqrt{R_0 m_\pm}}{6R_0^{m_- - \frac{1}{2}}(m_+ - m_-)} \quad (42)$$

Substituting (40) into (11) and using (16) yields the EoS parameter of generalized-ghost $f(R)$ gravity as

$$w_R = -1 - \frac{2}{3h} \left(1 + \frac{R\beta}{12\alpha H + 6\alpha H h^{-1} + \beta R^2} \right). \quad (43)$$

The above EoS parameter is time-dependent. Here in contrast with constant EoS parameter in (27), The dynamical behavior of EoS parameter is given and we see that at infinity it merge to a constant value (27) of Sec. III.

In this case, the time varying deceleration parameter is obtained as

$$q = -1 - \frac{1}{h} - \frac{R\beta}{12\alpha H + 6\alpha H + \beta R}. \quad (44)$$

Same as the previous section, this give us the accelerating expanding universe for all values of $h$ in the range (28).

**B. Quintessence scale factor**

We now choose the quintessence scale factor (47). In this case the energy density of generalized-ghost DE, by using (18), is rewritten as follows

$$\rho_\Lambda = \alpha \sqrt{\frac{hR}{12h - 6}} + \frac{\beta}{12h - 6} \quad (45)$$

Equating (45) and (6) by using (18) results the following solution for $f(R)$

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \alpha \gamma_p R^{\frac{1}{2}} - \frac{\beta}{3} R \quad (46)$$

where parameters $m_\pm$ and $\gamma_p$ are the same as Eqs. (32) and (33), respectively. Applying the boundary conditions in (24), the constants $\lambda_\pm$ in (40) are determined as

$$\lambda_+ = \frac{2\beta \sqrt{R_0(1 - m_-)} + 3\alpha \gamma_p (1 - 2m_-) + 12\sqrt{R_0 m_-}}{6R_0^{m_+ - \frac{1}{2}}(m_+ - m_-)} \quad (47)$$
FIG. 1: The evolution of deceleration parameter, \( q \), versus \( a \) in generalized-ghost DE model for parameters: \( (h = 1.3, \, \alpha = 2, \, \beta = 0.3) \).

\[
\lambda_- = -\frac{2\beta\sqrt{R_0(1 - m_+)} + 3\alpha\gamma_q(1 - 2m_+) + 12\sqrt{R_0m_+}}{6R_0^{m_+ - \frac{1}{2}}(m_+ - m_-)} \tag{48}
\]

Substituting (46) into (11) and using (18) obtains the EoS parameter of generalized-ghost \( f(R) \) gravity as

\[
w_R = -1 + \frac{2}{3h} \left( 1 + \frac{R\beta}{12\alpha h H - 6\alpha H h^{-1} + \beta R} \right), \tag{49}
\]

which can represent the quintessence regime of accelerated universe. Here we see that the EoS parameter is dynamical, in contrast with constant EoS parameter in (36). The dynamical behavior of EoS parameter is achieved and we see that at infinity it merge to a constant value (36) of Sec. III.

The deceleration parameter, in this case, is time varying and obtained as

\[
q = -1 + \frac{1}{h} + \frac{R\beta}{12\alpha h H + 6\alpha H + \beta h R}. \tag{50}
\]

Here in contrast with (37) in previous section, the deceleration parameter evolves with time. In Fig. I for \( h > 1 \), an early time transition from the deceleration to acceleration phases is demonstrated.
V. CONCLUSION

The reconstruction of $f(R)$ modified gravity for different ghost and generalized-ghost DE models in FRW flat universe has been investigated. The reconstruction was performed by considering two classes of scale factors, containing i) phantom scale factor, $a = a_0(t_s - t)^h$ and ii) quintessence scale factor, $a = a_0 t^h$. The equation of state and deceleration parameter of reconstructed $f(R)$-ghost/generalized-ghost, have been calculated. We showed that the corresponding $f(R)$ gravity of ghost/generalized ghost DE model can behave like phantom or quintessence. In $f(R)$-generalized ghost case, the EoS parameter can vary with time. Also, the transition between deceleration to acceleration regime is happened at early time. These behaviors are in contrast with Pure $f(R)$-ghost and $f(R)$-holographic DE \cite{12, 13}. Therefore the generalized-ghost DE model give us a stronger view of the universe in $f(R)$ reconstruction point of view.

\begin{thebibliography}{99}
\bibitem{1} S. W. Allen, et al., Mon. Not. R. Astron. Soc. \textbf{353}, 457 (2004); C. L. Bennett et al., Astrophys. J. \textbf{148}, 1 (2003); P. Astier et al., Astron. Astrophys. \textbf{447}, 31 (2006); K. Abazajian et al., Astron. J. \textbf{128}, 502 (2004); K. Abazajian et al., Astron. J. \textbf{129}, 1755 (2005); S. Perlmutter et al., Astrophys. J. \textbf{517}, 565 (1999); D. N. Seprgel et al., Astrophys. J. Suppl. Ser. \textbf{148}, 175 (2003); M. Tegmark et al., Phys. Rev. D \textbf{69}, 103501 (2004).
\bibitem{2} E. Witten, Nucl. Phys. B \textbf{156}, 269 (1979); G. Veneziano, Nucl. Phys. B \textbf{159}, 213 (1979); C. Rosenzweig, j. Schechter and C. G. Trahern, Phys. Rev. D \textbf{21}, 3388 (1980); P. Nath and R. L. Arnowitt, Phys. Rev. D \textbf{23}, 473 (1981); K. Kawarabayashi and N. Ohta, Nucl. Phys. B \textbf{175}, 477 (1980); Prog. Theor. Phys. \textbf{66}, 1789 (1981); N. Ohta, Prog. Theor. Phys. \textbf{66}, 1408 (1981).
\bibitem{3} A. R. Zhitnitsky, Phys. Rev. D \textbf{84}, 124008 (2011); A. R. Zhitnitsky, Phys. Rev. D\textbf{82}, 103520 (2010); B. Holdom, Phys. Lett. B\textbf{697}, 351 (2011); E. Thomas, A. R. Zhitnitsky, \texttt{(arXiv:1109.2608)} [hep-th].
\bibitem{4} F. R. Urban and A. R. Zhitnitsky, Phys. Lett. B \textbf{688}, 9 (2010); Phys. Rev. D \textbf{80}, 063001 (2009); JCAP \textbf{0909}, 018 (2009); Nucl. Phys. B \textbf{835}, 135 (2010).
\bibitem{5} N. Ohta, Phys. Lett. B \textbf{695}, 41 (2011).
\end{thebibliography}
[6] J. Bjorken, SLAC-PUB-9063 (2001) [arXiv:hep-th/0111196]; R. Schutzhold, Phys. Rev. Lett. 89, 081302 (2002); J. D. Bjorken, SLAC-PUB-10676 (2004) [arXiv:astro-ph/0404233]; F. R. Klinkhamer and G. E. Volovik, Phys. Rev. D 77, 085015 (2008); F. R. Klinkhamer and G. E. Volovik, Phys. Rev. D 78, 063528 (2008); F. R. Klinkhamer and G. E. Volovik, Phys. Rev. D 79, 063527 (2009); Y. B. Zeldovich, JETP Lett. 6, 316 (1967) [Pisma Zh. Eksp. Teor. Fiz. 6, 883 (1967)]; W. Zimdahl, H. A. Borges, S. Carneiro, J. C. Fabris and W. S. Hipolito-Ricaldi, JCAP 04, 028 (2011); S. Nojiri and S. D. Odintsov, Phys. Rev. D 72, 023003 (2005); S. Capozziello, V. F. Cardone, E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Rev. D 73, 043512 (2006).

[7] Rong-Gen Cai, Z. L. Tuo and H. B. Zhang, Phy. Rev. D84, 123501 (2011).

[8] A. R. Zhitnitsky, [arXiv:1112.3365][hep-ph].

[9] A. Rozas-Fernandez, Phys. lett. B 709, 313 (2012); A. Sheykhi, M. Sadegh Movahed and E. Ebrahimi, Astrophys. space Sci. 339, 93 (2012); A. Sheikhi and A. Bagheri, Europhys. Lett. 95, 39001 (2011).

[10] Rong-Gen Cai et al. [arXiv:1201.2494][astro-ph.CO]; Chao-Jun Feng, Xin-Zhou Li, Xian-Yong Shen, [arXiv:1202.0058][astro-ph.CO].

[11] S. Nojiri and S. D. Odintsov, Phys. Rev. D 74, 086005 (2006a); S. Nojiri and S. D. Odintsov, J. Phys. Conf. Ser. 66, 012005 (2007); J. Phys. A 40, 6725 (2007); S. Capozziello, S. Nojiri, S. D. Odintsov and A. Troisi, Phys. Lett. B 639, 135 (2006); E. Elizalde and D. Saez-Gomez, [arXiv:0903.2732]; A. de la Cruz-Dombriz and A. Dobado, Phys. Rev. D 74, 087501 (2006); J. L. Cortes and J. Indurain, Astropart. Phys. 31, 177 (2009); I. H. Brevik, Gen. Rel. Grav. 38, 1317 (2006); L. N. Granda, The Problems of Modern Cosmology, Tomsk State Pedagogical University Press (2009); X. Wu and Z. H. Zhu, Phys. Lett. B 660, 293 (2008); K. Bamba, C. Q. Geng, S. Nojiri and S. D. Odintsov, Phys. Rev. D 79, 083014 (2009); K. Bamba, S. Nojiri and S. D. Odintsov, JCAP 0810, 045 (2008); K. Bamba and C. Q. Geng, Phys. lett. B679, 282 (2009); A. Khodam - mohammadi, P. majari, M. Malekjani, Astrophys. space Sci. 331, 673 (2011); M. R. Setare, Int. J. Mod. Phys. D17, 2219 (2008).

[12] K. Karami and M.S. Khaledian, JHEP 1103, 086 (2011).

[13] M. R. Setare, Astrophys. Space Sci. 326, 27 (2010).

[14] S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007a).

[15] S. Nojiri and S. D. Odintsov, [arXiv:0910.1464].

12
[16] H. M. Sadjadi, Phys. Rev. D 73, 063525 (2006).
[17] S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. 38, 1285 (2006b).
[18] S. Nojiri and S. D. Odintsov, Phys. Lett. B 657, 238 (2007b); S. Nojiri, S. D. Odintsov, phys. Rev. D 77, 026007 (2008).
[19] A. Sheykhi and M. Sadegh Movahed, Gen. Relativ. Gravit. 44, 449 (2012).
[20] J. Grande, A. Pelinson and J. Sola, Phys. Rev. D 79, 043006 (2009); J. Sola, J. Phys. A 41, 164066 (2008); I. L. Shapiro, J. Sola and H. Stefancic, JCAP 0501, 012 (2005).
[21] K. Freese, F. C. Adams, J. A. Frieman and E. Mottola, Nucl. Phys. B 287, 797 (1987); O. Bertolami, Nuovo Cim. B 93, 36 (1986); J. M. Overduin and F. I. Cooperstock, Phys. Rev. D 58, 043506 (1998); V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000); K. Freese, New Astron. Rev. 49, 103 (2005); Y. Z. Ma, Nucl. Phys. B 804, 262 (2008).