Scalable Anytime Algorithms for Learning Formulas in Linear Temporal Logic

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Abstract
Linear temporal logic (LTL) is a specification language for finite sequences (called traces) widely used in program verification, motion planning in robotics, process mining, and many other areas. We consider the problem of learning LTL formulas for classifying traces; despite a growing interest of the research community existing solutions suffer from two limitations: they do not scale beyond small formulas, and they may exhaust computational resources without returning any result. We introduce a new algorithm addressing both issues: our algorithm is able to construct formulas an order of magnitude larger than previous methods, and it is anytime, meaning that it in most cases successfully outputs a formula, albeit possibly not of minimal size. We evaluate the performances of our algorithm using an open source implementation against publicly available benchmarks.

1 Introduction
Linear Temporal Logic (LTL) is a prominent logic for specifying temporal properties (Pnueli 1977), which has been recently used for finite traces (De Giacomo and Vardi 2013). It has become a de facto standard in many fields such as model checking, program analysis, and motion planning for robotics. Over the past five to ten years learning temporal logics (of which LTL is the core) has become an active research area and identified as an important goal in artificial intelligence: it formalises the difficult task of building explainable models from data. Indeed, as we will see in the examples below and as argued in the literature, e.g., by Camacho and McIlraith (2019) and Roy et al. (2020), LTL formulas are typically easy to interpret by human users and therefore useful as explanations.

The variable free syntax of LTL and its natural inductive semantics make LTL a natural target for building classifiers separating positive from negative traces.

The fundamental problem we study here is to build an explainable model in the form of an LTL formula from a set of positive and negative traces. More for-
mally (we refer to the next section for formal definitions), given a set \( u_1, \ldots, u_n \) of positive traces and a set \( v_1, \ldots, v_n \) of negative traces, the goal is to construct a formula \( \varphi \) of LTL which satisfies all \( u_i \)'s and none of the \( v_i \)'s. In that case we say that \( \varphi \) is a separating formula or—using machine learning terminology—a classifier.

To make things concrete let us introduce our running example, a classic motion planning problem in robotics and inspired by Grover et al. (2021). A robot collects wastebin contents in an office-like environment and empties them in a trash container. Let us assume that there is an office \( o \), a hallway \( h \), a container \( c \) and a wet area \( w \). The following are possible traces obtained in experimentation with the robot (for instance, through simulation):

\[
\begin{align*}
  u_1 &= h \cdot h \cdot h \cdot o \cdot h \cdot c \cdot h \\
  v_1 &= h \cdot h \cdot h \cdot c \cdot o \cdot h \cdot h
\end{align*}
\]

In LTL learning we start from these labelled data: given \( u_1 \) as positive and \( v_1 \) as negative, what is a possible classifier including \( u_1 \) but not \( v_1 \)? Informally, \( v_1 \) being negative implies that the order is fixed: \( o \) must be visited before \( c \). We look for classifiers in the form of separating formulas, for instance

\[
F(o \land Fc),
\]

where the \( F \)-operator stands for “\( e \)Eventually”. Note that this formula requires to visit the office first and only then visit the container.

Assume now that two more negative traces were added:

\[
\begin{align*}
  v_2 &= h \cdot h \cdot h \cdot o \cdot w \cdot c \cdot h \cdot h \\
  v_3 &= h \cdot h \cdot h \cdot w \cdot o \cdot w \cdot c \cdot w \cdot w
\end{align*}
\]

Then the previous separating formula is no longer correct, and a possible separating formula is

\[
F(o \land Fc) \land G(\neg w),
\]

which additionally requires the robot to never visit the wet area. Here the \( G \)-operator stands for “globally”.

Let us emphasise at this point that for the sake of presentation we consider only exact classifiers: a separating formula must satisfy all positive traces and none of the negative traces. However our algorithm naturally extends to the noisy data setting where the goal is to construct an approximate classifier, replacing ‘all’ and ‘none’ by ‘almost all’ and ‘almost none’.

**State of the art**

A number of different approaches for learning temporal logics have been proposed, leveraging SAT solvers (Neider and Gavran, 2018), automata (Camacho and McIlraith, 2019), and Bayesian inference (Kim et al., 2019), and extended to more expressive logics such as Property Specification Language (PSL) (Roy et al., 2020) and Computational Tree Logic (CTL) (Ehlers et al., 2020). Applications include program specification (Lemieux et al., 2015), anomaly and fault detection (Bombara et al., 2016), robotics (Chou et al., 2020), and many more: we refer to Camacho and McIlraith (2019), Section 7, for a list of practical applications.
Existing methods do not scale beyond formulas of size 10, making them hard to deploy for industrial cases. A second serious limitation is that they often exhaust computational resources without returning any result. Indeed theoretical studies [Fijalkow and Lagarde (2021)] have shown that constructing the minimal LTL formula is NP-hard already for very small fragments of LTL, explaining the difficulties found in practice.

Our approach

To address both issues, we turn to approximation and anytime algorithms. Here approximation means that the algorithm does not ensure minimality of the constructed formula: it does ensure that the output formula separates positive from negative traces, but it may not be the smallest one. On the other hand, an algorithm solving an optimisation problem is called anytime if it can be interrupted anytime during its computation and yield some good albeit non-optimal solution. In other words, anytime algorithms work by refining solutions. As we will see in experiments, this implies that even if our algorithm timeouts it outputs some formula.

Hence at a high-level, our algorithm is an efficient and specialised enumeration algorithm of a fragment of LTL: boolean combinations of simple formulas. Our algorithm combines two ingredients:

- **Searching for simple formulas**: we develop succinct representations for normal forms of a fragment of LTL that we call simple LTL. The representation is exponentially more succinct than usual LTL formulas, yielding efficient manipulations of potentially large formulas. The key benefit is a space efficient dynamic programming algorithm for enumerating simple formulas.

- **Combining simple formulas**: we construct two algorithms for combining simple formulas. The first is an off-the-shelf decision tree algorithm, and the second is a new greedy algorithm called Boolean subset cover.

The two ingredients yield two subprocedures: the first one finds simple representation of formulas of increasing size, which are then fed to the second procedure in charge of combining them into a separating formula. This yields an anytime algorithm as both subprocedures can output separating formulas even with low computational budget and refine them over time.

Let us illustrate the two subprocedures in our running example. The first subprocedure enumerates so-called simple formulas in increasing size; we refer to the corresponding section for a formal definition. Intuitively, the simple formula $(\geq 0, o, \geq 0, c)$ represents the LTL formula $F(o \land F c)$, which requires to see $o$ at some point, then $c$ at any point later. The formulas $F(o \land F c)$ and $G(\neg w)$ are represented by two of the first generated simple formulas.

The second procedure constructs formulas as boolean combinations of the simple formulas. Without getting into the details of the algorithms, let us note that both $F(o \land F c)$ and $G(\neg w)$ satisfy $u_1$. The first does not satisfy $v_1$ and the
second does not satisfy \( v_2 \) and \( v_3 \). Hence their conjunction \( F(o \land F c) \land G(\neg w) \) is a solution, meaning it satisfies \( u_1 \) but none of \( v_1, v_2, v_3 \).

**Outline**

The mandatory definitions are given in the next section. We then describe the two phases of our algorithm in one section each. We conclude with an empirical evaluation.

2 Preliminaries

Let \( \mathbb{N} \) denote the set of natural numbers \( 0, 1, 2, \ldots \).

**Traces** Let \( \mathcal{P} \) be a finite set of atomic propositions. An *alphabet* is a finite non-empty set \( \Sigma = 2^\mathcal{P} \), whose elements are called *symbols*. A finite *trace* over \( \Sigma \) is a finite sequence \( t = a_1a_2 \ldots a_n \) such that for every \( 1 \leq i \leq n \), \( a_i \in \Sigma \). We say that \( t \) has length \( n \) and write \(|t| = n\). For example, let \( \mathcal{P} = \{p, q\} \), in the trace \( t = \{p, q\} \cdot \{p\} \cdot \{q\} \) both \( p \) and \( q \) hold at the first position, only \( p \) holds in the second position, and \( q \) in the third position. Note that throughout the paper we only consider finite traces.

A trace is a *word* if exactly one atomic proposition holds at each position: we used words in the introduction example for simplicity, writing \( h \cdot o \cdot c \) instead of \( \{h\} \cdot \{o\} \cdot \{c\} \).

Given a trace \( t = a_1a_2 \ldots a_n \) and \( 1 \leq i \leq j \leq n \), let \( t[i,j] = a_i \ldots a_j \) be the *infix* of \( t \) from position \( i \) up to and including position \( j \). Moreover, \( t[i] = a_i \) is the symbol at the \( i \)th position.

**Linear Temporal Logic** The syntax of Linear Temporal Logic (LTL, in short) is defined by the following grammar

\[
\varphi ::= p \in \mathcal{P} | \neg p | \varphi \lor \psi | \varphi \land \psi | X \varphi | F \varphi | G \varphi
\]

Formulas in LTL are evaluated over finite traces. To define the semantics of LTL we introduce the notation \( t, i \models \varphi \), which reads ‘the LTL formula \( \varphi \) holds over trace \( t \) from position \( i \)’. We say that \( t \) satisfies \( \varphi \) and we write \( t \models \varphi \) when \( t, 0 \models \varphi \). The definition of \( \models \) is inductive on the formula \( \varphi \):

- \( t, i \models p \in \mathcal{P} \) if \( p \in t[i] \).
- \( t, i \models X \varphi \) if \( i < |t| \) and \( t, i + 1 \models \varphi \). It is called the \( \text{neXt} \) operator.
- \( t, i \models F \varphi \) if \( t, i' \models \varphi \) for some \( i' \in [i, |t|] \). It is called the eventually operator (\( F \) comes from \( \text{Finally} \)).
- \( t, i \models G \varphi \) if \( t, i' \models \varphi \) for all \( i' \in [i, |t|] \). It is called the \( \text{Globally} \) operator.

We use the standard formulas: \( \text{true} = p \lor \neg p, \text{false} = p \land \neg p \) and \( \text{last} = \neg X \text{true} \), which denotes the last position of the trace. As a shorthand we use \( X^\ast \varphi \) for \( X \ldots X \varphi \) \( \text{n times} \).

The size of a formula is the size of its underlying syntax tree.
The LTL Learning Problem  The LTL exact learning problem studied in this paper is the following: given a set $P$ of positive traces and a set $N$ of negative traces, construct a minimal LTL separating formula $\varphi$, meaning such that $t \models \varphi$ for all $t \in P$ and $t \not\models \varphi$ for all $t \in N$.

There are two relevant parameters for a sample: its size, which is the number of traces, and its length, which is the maximum length of all traces.

The problem is naturally extended to the LTL noisy learning problem where the goal is to construct an $\epsilon$-separating formula, meaning such that $\varphi$ satisfies all but an $\epsilon$ proportion of the traces in $P$ and none but an $\epsilon$ proportion of the traces in $N$. For the sake of simplicity we present an algorithm for solving the LTL exact learning problem, and later sketch how to extend it to the noisy setting.

3 Searching for simple formulas

The first insight of our approach is an exponentially succinct representation of a fragment of LTL that we call simple LTL.

Simple formulas  Let us introduce some notations.

- A partial symbol is a conjunction of positive or negative atomic propositions. We write $s = p_0 \land p_2 \land \neg p_1$ for the partial symbol specifying that $p_0$ and $p_2$ hold and $p_1$ does not. The definition of a symbol satisfying a partial symbol is natural: for instance the symbol $\{p_0, p_2, p_4\}$ satisfies $s$. The width of a partial symbol is the number of atomic propositions it uses.

- A constraint is either $\geq n$ or $= n$ for some $n \in \mathbb{N}$.

- A simple formula is a sequence alternating partial symbols and constraints and ending with a partial symbol.

Informally, a simple formula is specifying the existence of a subsequence in a word satisfying a number of constraints. Before giving a formal definition of the LTL formula represented by a simple formula, let us give a few examples.

The simple formula $\langle \geq 0, p \land q, \geq 2, \neg p \rangle$ represents the LTL formula

$$F((p \land q) \land F \cdot X^2 \neg p),$$

which reads: there exists a position satisfying $p \land q$, and at least two positions later there exists a position satisfying $\neg p$.

The simple formula $\langle p, \geq 10, \neg q, = 3, p \rangle$ represents the LTL formula

$$p \land F \cdot X^{10}(\neg q \land X^3 p),$$

which reads: the first position satisfies $p$ and at least ten positions later there exists a position satisfying $\neg q$ and exactly three positions later the position satisfies $p$.

We write $|\theta|$ for the formula represented by the simple formula $\theta$, it is defined by induction on the length of $\theta$. There are two cases:
If the simple formula $\theta$ starts with a partial symbol $s$, let us write $\theta = (s, \theta')$, then $\llbracket \theta \rrbracket = s \land \llbracket \theta' \rrbracket$.

If the simple formula $\theta$ starts with a constraint, let us write $\theta = (c, \theta')$. If $c$ is ‘$\geq n$’ then $\llbracket \theta \rrbracket = F^n \llbracket \theta' \rrbracket$, and if $c$ is ‘$= n$’ then $\llbracket \theta \rrbracket = X^n \llbracket \theta' \rrbracket$.

By representing constraints in binary we have an exponentially succinct representation: the simple formula $\langle = n, a \rangle$ uses $O(\log(n))$ bits and represents the LTL formula $X^n a$ of size $O(n)$.

We call simple LTL the fragment of LTL represented by simple formulas. Note that it only uses the next and eventually temporal operators as well as conjunctions and atomic propositions.

Generating simple formulas Let us consider the following problem: given the sample $S = P \cup N$, we want to generate all simple formulas together with the subset of $S$ they satisfy.

Our first technical contribution and key to the scalability of our approach is an efficient solution to this problem based on dynamic programming.

Let us define a natural order in which we want to generate simple formulas. They have two parameters: length, which is the number of partial symbols in the sequence, and width, which is the sum of the widths of the partial symbols in the sequence. We consider the order based on summing these two parameters: $(1, 1), (2, 1), (1, 2), (3, 1), (2, 2), (1, 3), \ldots$

(We note that in practice, slightly more complicated orders on pairs are useful since we want to increase the length more often than the width.) Our enumeration algorithm works by generating all simple formulas of a given pair of parameters, in a recursive fashion. Assuming that we already generated all simple formulas for the pair of parameters $(i, j)$, we define two procedures, one for generating the simple formulas for the parameters $(i + 1, j)$, and the other one for $(i, j + 1)$.

Let us define the dynamic programming table $\text{LastUsedPos}$ as follows, where $\varphi$ is a simple formula and $t$ a trace in $S$:

$$\text{LastUsedPos}(\varphi, t) = \{i \in [1, |t|] : t[i] = \varphi\}.$$ 

The goal is to compute $\text{LastUsedPos}$ for all simple formulas with small parameters. A useful idea is to change the representation of the set of traces $S$, by precomputing the lookup table $\text{Index}$ defined as follows, where $t$ is a trace in $S$, $s$ a symbol, and $i$ in $[1, |t|]$:

$$\text{Index}(t, s, i) = \{j \in [i + 1, |t|] : t[j] = s\}.$$ 

The table $\text{Index}$ can be precomputed in linear time from $S$, and makes the dynamic programming algorithm easier to formulate.

The pseudocode for both enumeration tasks is given in [1]. For the width increase algorithm, we say that two simple formulas are compatible if they share the set of constraints. The actual implementation of SCARLET refines the algorithms in some places. For instance: 

- If the simple formula $\theta$ starts with a partial symbol $s$, let us write $\theta = (s, \theta')$, then $\llbracket \theta \rrbracket = s \land \llbracket \theta' \rrbracket$.
- If the simple formula $\theta$ starts with a constraint, let us write $\theta = (c, \theta')$. If $c$ is ‘$\geq n$’ then $\llbracket \theta \rrbracket = F^n \llbracket \theta' \rrbracket$, and if $c$ is ‘$= n$’ then $\llbracket \theta \rrbracket = X^n \llbracket \theta' \rrbracket$.

By representing constraints in binary we have an exponentially succinct representation: the simple formula $\langle = n, a \rangle$ uses $O(\log(n))$ bits and represents the LTL formula $X^n a$ of size $O(n)$.

We call simple LTL the fragment of LTL represented by simple formulas. Note that it only uses the next and eventually temporal operators as well as conjunctions and atomic propositions.
• Line 2: two simple formulas of the same length and width can induce formulas of LTL of different size if the constants involved are very different; we enumerate in increasing sizes of the resulting LTL formulas.

• Line 3: instead of considering all partial symbols, we restrict to those appearing in at least one positive trace.

• Line 7: some computations for \( \geq j' \) can be made redundant if we step over the next position in LastUsedPos, a finer data structure factorises the computations.

• Line 24/25: using a refined data structure we only enumerate compatible simple formulas.

Algorithm 1: Recursive enumeration of simple formulas for the set of traces \( S \)

1: procedure Search simple formulas – length increase \((\ell, w)\)
2: for all simple formulas \( \varphi \) of length \( \ell \) and width \( w \) do
3:   for all partial symbols \( s \) of width at most \( w \) do
4:     for all \( t \in S \) do
5:       \( I = \text{LastUsedPos}(\varphi, t) \)
6:       for all \( i \in I \) do
7:         \( J = \text{Index}(t, s, i) \)
8:         for all \( j \in J \) do
9:           \( \varphi_{=j} \leftarrow (\varphi_{=j}, s) \)
10:          add \( j \) to \( \text{LastUsedPos}(\varphi_{=j}, t) \)
11:       end for
12:     for all \( j' \leq \max(J) \) do
13:       \( \varphi_{\geq j'} \leftarrow (\varphi_{\geq j'}, s) \)
14:       add \( J \cap [j', |t|] \) to \( \text{LastUsedPos}(\varphi_{\geq j'}, t) \)
15:     end for
16:   end for
17: end for
18: end procedure

21: procedure Search simple formulas – width increase \((\ell, w)\)
22: for all simple formulas \( \varphi \) of length \( \ell \) and width \( w \) do
23:   for all simple formulas \( \varphi' \) of length \( \ell \) and width 1 do
24:     if \( \varphi \) and \( \varphi' \) are compatible then
25:       \( \varphi'' \leftarrow \varphi \land \varphi' \)
26:     for all \( t \in S \) do
27:       \( \text{LastUsedPos}(\varphi'', t) \leftarrow \text{LastUsedPos}(\varphi, t) \cap \text{LastUsedPos}(\varphi', t) \)
28:     end for
29:   end if
30: end for
31: end procedure
The Boolean subset cover problem. The formulas $\varphi_1$, $\varphi_2$, and $\varphi_3$ satisfy the words encircled in the corresponding area. In this instance $(\varphi_1 \land \varphi_2) \lor \varphi_3$ is a separating formula.

The dual point of view We use the same algorithm but swapping positive and negative traces to produce formulas in a dual fragment to simple LTL, which uses the next and globally temporal operators, the last predicate, as well as disjunctions and atomic propositions. Indeed from a simple formula its negation is obtained as follows:

\[
\neg X \varphi = \text{last} \lor X \neg \varphi \quad ; \quad \neg F \varphi = G \neg \varphi \quad ; \quad \neg (\varphi_1 \land \varphi_2) = \neg \varphi_1 \lor \neg \varphi_2.
\]

4 Combining simple formulas

As explained in the previous section, we can efficiently generate simple formulas and dual simple formulas together with the sets of traces they satisfy. Now we need to combine these formulas in order to construct separating formulas, as illustrated in the introduction.

Boolean combination of formulas Let us consider the following subproblem: given a set of formulas and the sets of positive and negative traces they satisfy, does there exist a Boolean combination of some of the formulas which is a separating formula? We call this problem the Boolean subset cover, it is illustrated in Figure 1. In this example we have three formulas $\varphi_1$, $\varphi_2$, and $\varphi_3$, each satisfying subsets of $u_1, u_2, u_3, v_1, v_2, v_3$ as represented in the drawing. Inspecting the three subsets reveals that $(\varphi_1 \land \varphi_2) \lor \varphi_3$ is a separating formula.

The Boolean subset cover problem is a generalization of the well known and extensively studied subset cover problem, where we are given $S_1, \ldots, S_m$ subsets of $[1, n]$, and the goal is to find a subset $I$ of $[1, m]$ such that $\bigcup_{i \in I} S_i$ covers all of $[1, n]$ – such a set $I$ is called a cover. Indeed, it corresponds to the case where all formulas satisfy none of the negative traces: in that case conjunctions are not useful and we can ignore the negative traces.

The subset cover problem is known to be NP-complete. However there exists a polynomial time $\log(n)$-approximation algorithm called the greedy algorithm: it is guaranteed to construct a cover which is at most $\log(n)$ times larger than the minimal cover. This approximation ratio is optimal in the following sense: [Dinur and Steurer, 2013]: there is no polynomial time $(1 - o(1)) \log(n)$-approximation algorithm for subset cover unless P = NP. Informally, the greedy algorithm for the subset cover problem does the following: it iteratively constructs a cover $I$
by sequentially adding the most ‘promising subset’ to $I$, which is the subset $S_i$ maximising how many more elements of $[1,n]$ are covered by adding $i$ to $I$.

We introduce an extension of the greedy algorithm to the Boolean subset cover problem. The first ingredient is a scoring function, which takes into account both how close is the formula to being separating, and how large it is. We use the following formula:

$$Score(\varphi) = \frac{\text{Card} \{ i \in [1,n] : u_i \models \varphi \} + \text{Card} \{ i \in [1,n] : v_i \not\models \varphi \}}{\sqrt{|\varphi|} + 1},$$

where $|\varphi|$ is the size of $\varphi$. The use of $\sqrt{\cdot}$ is empirical, it is used to mitigate the importance of size over being separating.

The algorithm maintains a set of formulas $F$ which is initially the set of formulas given as input, and add new formulas to $F$ until finding a separating formula. Let us fix a constant $K$, which in the implementation is set to 5. At each point in time the algorithm chooses the $K$ formulas $\varphi_1, \ldots, \varphi_K$ in $F$ and constructs all disjunctions and conjunctions of $\varphi_i$ with formulas in $F$. For each $i$, we keep the disjunction or conjunction with maximal score, and add this formula to $F$ if it has higher score than $\varphi_i$. We repeat this procedure until either finding a separating formula, or no formula is added to $F$.

Another natural approach to the Boolean subset cover problem is to use decision trees: we use one variable for each trace and one atomic proposition for each formula to denote whether the trace satisfies the formula. We then construct a decision tree classifying all traces.

We experimented with both approaches and found that the greedy algorithm is both faster and yields smaller formulas. We do not report on these experiments because the formulas output using the decision tree approach are prohibitively larger and therefore not useful for explanations. Let us however remark that using decision trees we get a theoretical guarantee that if there exists a separating formula as a Boolean combination of the formulas, then the algorithm will find it.

5 The full algorithm

The algorithm combines the two ingredients defined above: we generate simple formulas of increasing length and width, and run the induced Boolean subset cover algorithm to find separating formulas.

For memory related issues we empirically bound the number of formulas to be considered for each generation. An important optimisation is to keep an upper bound on the size of a separating formula, which we use to cut off computations that cannot lead to smaller formulas in the Boolean subset cover algorithm. Another implementation gain is to store scores of already considered formulas and to perform efficient computations of newly formed ones.

Extension to the noisy setting The algorithm is seamlessly extended to the noisy setting by updating the Boolean set cover algorithm: instead of outputting only separating formulas we also output $\varepsilon$-separating formulas.
6 Experimental evaluation

In this section, we answer the following research questions to assess the performance of our LTL learning algorithm.

**RQ1:** How effective are we in learning concise LTL formulas from samples?

**RQ2:** How much scalability do we achieve through our algorithm?

**RQ3:** What do we gain from the anytime property of our algorithm?

**Experimental Setup** To answer the questions above, we have implemented a prototype of our algorithm in Python 3 in a tool named SCARLET\(^1\) (SCalable Anytime algoRithm for LEarning lTl), which we will make publicly available. We compare the performance of SCARLET against two state-of-the-art tools for learning logic formulas from examples:

1. **FLIE\(^2\)** developed by Neider and Gavran (2018), infers minimal LTL formulas using a learning algorithm that is based on constraint solving (SAT solving).

2. **SYSLITE\(^3\)** developed by Arif et al. (2020), originally infers minimal past-time LTL formulas using an enumerative algorithm implemented in a tool called CVC4SY Reynolds et al. (2019). For our comparisons, we use a version of SYSLITE that we modified (which we refer to as SYSLITE) to infer LTL formulas rather than past-time LTL formulas. Our modifications include changes to the syntactic constraints generated by SYSLITE as well as changing the semantics from past-time LTL to ordinary LTL.

In both tools, we disabled the \(U\)-operator since we only consider formulas from the fragment of LTL without this operator.

All the experiments are conducted on a single core of a Debian machine with Intel Xeon E7-8857 CPU (at 3 GHz) using up to 6 GB of RAM. The timeout was set to be 900 s for all experiments.

**Sample generation** To provide a comparison among the learning tools, we follow the literature Neider and Gavran (2018); Roy et al. (2020) and use synthetic benchmarks generated from real-world LTL formulas. For benchmark generation, earlier works rely on a fairly naive generation method. In this method, starting from a formula \(\varphi\), a sample is generated by randomly drawing traces and categorizing them into positive and negative examples depending on the satisfaction with respect to \(\varphi\). This method, however, often results in samples that can be separated by formulas much smaller than \(\varphi\). Moreover, it often requires a prohibitively large amount of time to generate samples (e.g., if almost all traces satisfy or violate a formula, as happens for instance for \(G\,p\)) and, hence, often does not terminate in a reasonable time.

To alleviate the issues in the existing method, we have designed a novel generation method for the quick generation of large samples. In our method, we first convert the starting formula into an equivalent DFA and then extract

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1. \url{https://github.com/rajarshi008/Scarlet}
2. \url{https://github.com/cryhot/samples2LTL}
3. \url{https://github.com/CLC-UlOwa/SySLite}
accepted and rejected words to obtain a sample of the desired size. We provide more details on this new generation method used in the appendix.

### 6.1 RQ1: Performance Comparison

To address our first research question, we have compared all three tools on a synthetic benchmark suite generated from eight LTL formulas. These formulas originate from a study by Dwyer et al. (1999), who have collected a comprehensive set of LTL formulas arising in real-world applications (see Table 1 for an excerpt). The selected LTL formulas have, in fact, also been used by FLIE for generating its benchmarks. While FLIE also considered formulas with $U$, we did not consider them for generating our benchmarks.

| Absence: | $G(\neg p)$, $G(q \rightarrow G(\neg p))$ |
| Existence: | $F(p)$, $G(\neg p) \lor F(p \land F(q))$ |
| Universality: | $G(p)$, $G(q \rightarrow G(p))$ |
| Disjunction of patterns: | $G(\neg p) \lor F(p \land F(q)) \lor G(\neg s) \lor F(r \land F(s)), F(r) \lor F(p) \lor F(q)$ |

Table 1: Common LTL formulas used in practice
Our benchmark suite consists of a total of 256 samples (32 for each of the eight LTL formulas), generated using our generation method. The number of traces in the samples range from 50 to 2,000, while the length of traces range from 8 to 15.

Figure 2a presents the runtime comparison of FLIE, SYSLITE and SCARLET on all 256 samples. From the scatter plots, we observe that SCARLET ran faster than FLIE on all samples. Likewise, SCARLET was faster than SYSLITE on all but eight (out of 256) samples. SCARLET timed out on only 13 samples, while FLIE and SYSLITE timed out on 85 and 36, respectively (see Figure 2b).

The good performance of SCARLET can be attributed to its efficient formula enumeration technique. In particular, SCARLET only considers formulas that have a high potential of being a separating formula since it extracts simple LTL formulas from the sample itself. FLIE and SYSLITE, on the other hand, search through arbitrary formulas (in order of increasing size), each time checking if the current one separates the sample.

Figure 3 presents the comparison of the size of the formulas inferred by each tool. On 170 out of the 256 samples, all tools terminated and returned an LTL formula with size at most 7. In 150 out of this 170 samples, SCARLET, FLIE, and SYSLITE inferred a formulas of equal size, while the formula inferred by SCARLET was larger on the remaining 20 samples. The latter observation indicates that SCARLET misses certain small, consistent formulas because they do not fall into the supported fragment of LTL.

However, it is important to highlight that the formulas learned by SCARLET are in most cases not significantly larger than those learned by FLIE and SYSLITE. This can be seen from the fact that the average size of formulas inferred by SCARLET (on benchmarks in which none of the tools timed out) is 3.21, while the average size of formulas inferred by FLIE and SYSLITE is 3.07.

Overall, SCARLET displayed significant speed-up over both FLIE and SYSLITE while learning a formula similar in size to that by FLIE and SYSLITE. This answers question RQ1 in the positive.

6.2 RQ2: Scalability

To address the second research question, we investigate the scalability of SCARLET in two dimensions: the size of the sample and the size of the formula from which the samples are generated.

Scalability with respect to the size of the samples

For demonstrating the scalability with respect to the size of the samples, we consider two formulas \( \varphi_{cov} = F(a_1) \land F(a_2) \land F(a_3) \) and \( \varphi_{seq} = F(a_1) \land F(a_2) \land F(a_3) \), both of which appear commonly in robotic motion planning [Fainekos et al. (2005)]. While the formula \( \varphi_{cov} \) describes the property that a robot eventually visits (or covers) three regions \( a_1, a_2, \) and \( a_3 \) in arbitrary order, the formula \( \varphi_{seq} \) describes that the robot has to visit the regions in the specific order \( a_1, a_2, a_3 \).

We have generated two sets of benchmarks for both formulas for which we varied the number of traces and their length, respectively. More precisely, the first benchmark set contains 90 samples of an increasing number of traces (5 samples for each number), ranging from 200 to 100,000, each consisting of traces of fixed length 10. On the other hand, the second benchmark set contains 90
samples of 200 traces, containing traces from length 10 to length 50. As the results on both benchmark sets are similar, we here discuss the results on the first set and refer the reader to the appendix for the second set.

Figure 4a shows the average runtime results of SCARLET, FLIE, and SYSLITE on the first benchmark set. We observe that SCARLET substantially outperformed the other two tools on all samples. However, this result is expected since both \(\varphi_{cov}\) and \(\varphi_{seq}\) are of size eight, and FLIE and SYSLITE are generally not powerful enough to learn formulas of size greater or equal to eight from large samples.

From Figure 4a, we further observe a significant difference between the run times of SCARLET on samples generated from formula \(\varphi_{cov}\) and from formula \(\varphi_{seq}\). This is evident from the fact that SCARLET failed to learn formulas for samples of \(\varphi_{seq}\) starting at a size of 6000, while it could learn formulas for samples of \(\varphi_{cov}\) up to a size of 50000. Such a result is due to the order of enumeration used by SCARLET: while \(\varphi_{cov}\) is a boolean combination of simple formulas of length 1 and width 1, \(\varphi_{seq}\) is a simple formulas of length 3 and width 1.

### Scalability with respect to the size of the formula

To demonstrate the scalability with respect to the size of the formula used to generate samples, we have extended \(\varphi_{cov}\) and \(\varphi_{seq}\) to families of formulas \((\varphi_{cov}^n)_{n \in \mathbb{N} \setminus \{0\}}\) with \(\varphi_{cov}^n = F(a_1) \land F(a_2) \land \ldots \land F(a_n)\) and \((\varphi_{seq}^n)_{n \in \mathbb{N} \setminus \{0\}}\) with \(\varphi_{seq}^n = F(a_1 \land F(a_2 \land F(\ldots \land F(a_n))))\), respectively. These family of formulas describe properties similar to that of \(\varphi_{cov}\) and \(\varphi_{seq}\), but the number of regions is parameterized by \(n \in \mathbb{N} \setminus \{0\}\). We consider formulas from the two families by varying \(n\) from 2 to 5 to generate a benchmark suite consisting of samples (5 samples for each formula) having 200 traces of length 10.

Figure 4b shows the average run time comparison of the tools for samples from increasing formula sizes. We observe a trend similar to Figure 4a: SCARLET performs better than the other two tools and learns formulas of family \(\varphi_{cov}^n\) faster than that of \(\varphi_{seq}^n\). However, contrary to the near linear increase of the runtime with the number of traces, we notice an almost exponential increase of the runtime with the formula size.

Overall, our experiments show better scalability with respect to both sample size and formula size when compared against the other two tools. This answers RQ2 in the positive.

### 6.3 RQ3: Anytime Property

To show the advantage of the anytime property of our algorithm, we use the benchmark suite used to show formula size scalability in RQ2. In particular, we consider the samples generated from the formula \(\varphi_{cov}^5\).

On each of the five samples generated from the formula, we observe that SCARLET timed out as can be seen from Figure 4b. However, due to the anytime property of our algorithm, SCARLET was able to output the exact formula \(\varphi_{cov}^5\) and that too within one second (0.7 seconds in average) in all five samples. The reason SCARLET was not able to terminate is because it was not able to validate if the formula it found is in fact, the minimal formula that it can find from the fragment of LTL that we consider.
Our observations from the experiments demonstrate that in certain cases, especially where all the tools including SCARLET timeout, we obtain a minimal separating formulas using the anytime property. This answers RQ3 in the positive.

![Graph comparing SCARLET, SYSLITE, and FLIE](image)

Figure 4: Comparison of SCARLET, FLIE and SYSLITE on synthetic benchmarks. In Figure 4(a), all times are in seconds and ‘TO’ indicates timeouts.

7 Conclusion

We have proposed a new approach for learning temporal properties from examples, fleshing it out in an approximation anytime algorithm. We have shown in experiments that our algorithm outperforms existing tools in two ways: it scales to larger formulas and input samples, and even when it timeouts it often outputs a separating formula.

Our algorithm targets a strict fragment of LTL, restricting its expressivity in two aspects: it does not include the U (Until) operator, and we cannot nest the eventually and globally operators. We leave for future work to extend our algorithm to full LTL.

An important open question concerns the theoretical guarantees offered by the greedy algorithm for the Boolean subset cover problem. It extends a well known algorithm for the classic subset cover problem and this restriction has
been proved to yield an optimal log(n)-approximation. Do we have similar guarantees in our more general setting?

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R. Roy, D. Fisman, and D. Neider. Learning interpretable models in the property specification language. In International Joint Conference on Artificial Intelligence, IJCAI, pages 2213–2219, 2020. doi: 10.24963/ijcai.2020/306.
A Sample generation method

To evaluate the performance of the tools FLIE, SYSLITE, and SCARLET effectively, we rely on our novel sample generation algorithm to generate benchmarks from LTL formulas. The outline of the generation algorithm is presented in Algorithm 2. The crux of the algorithm is to convert the LTL formula \( \varphi \) into its equivalent DFA \( A_\varphi \) and then extract random traces from the DFA to obtain a sample of desired length and size.

To convert \( \varphi \) into its equivalent DFA \( A_\varphi \) (Line 3), we rely on a python tool LTLf2DFA\footnote{https://github.com/whitemech/ltlf2DFA}. Essentially, this tool converts \( \varphi \) into its equivalent formula in First-order Logic and then obtains a minimal DFA from the formula using a tool named MONA \cite{Henriksen1995}.

For extracting random traces from the DFA (Line 5 and 9), we use a procedure suggested by \cite{Bernardi2010}. The procedure involves generating words by choosing letters that have a higher probability of leading to an accepting state. This requires assigning appropriate probabilities to the transitions of the DFA. In this step, we add our modifications to the procedure. The main idea is that we adjust the probabilities of the transitions appropriately to ensure that we obtain distinct words in each iteration.

Algorithm 2 Sample generation algorithm

```
Input: Formula \( \varphi \), length \( l \), number of positive traces \( n_P \), number of negative traces \( n_N \).
1: \( P \leftarrow \{\} \), \( N \leftarrow \{\} \)
2: \( A_\varphi \leftarrow \text{convert2DFA}(\varphi) \)
3: Loop \( n_P \) times
4: \( w \leftarrow \text{random accepted word of length } l \text{ from } A_\varphi. \)
5: \( P \leftarrow P \cup \{w\} \)
6: \text{end}
7: Loop \( n_P \) times
8: \( w \leftarrow \text{random accepted word of length } l \text{ from } A_\varphi^c. \)
9: \( N \leftarrow N \cup \{w\} \)
10: \text{end}
11: \text{return } S = (P, N)
```

Unlike existing sample generation methods, our method does not create random traces and try to classify them as positive or negative. This results in a much faster generation of large and better quality samples.
B  List of all formulas used for generating benchmarks

Table 2: Common LTL formulas used in practice Dwyer et al. (1999); Fainekos et al. (2005)

| Absence: | $G(\neg p_0), G(p_1 \rightarrow G(\neg p_0))$ |
|--------|-----------------------------------------------|
| Existence: | $F(p_0), G(\neg p_0) \lor F(p_0 \land F(p_1))$ |
| Universality: | $G(p_0), G(p_1 \rightarrow G(p_0))$ |
| Disjunction of patterns: | $G(\neg p) \lor F(p \land F(q)) \lor G(\neg s) \lor F(r \land F(s))$, $F(r) \lor F(p) \lor F(q)$ |
| Coverage family: | $F(a_1) \land F(a_2) \land \ldots \land F(a_n)$ |
| Sequence family: | $F(a_1 \land F(a_2 \land F(\ldots \land F(a_n))))$ |

C  Comparison of tools on existing benchmarks

To address our first research question RQ1 in the ‘Experimental evaluation’ section, we compared the performance of three tools on an existing benchmark suite Gaglione et al. (2021). The benchmark suite has been generated using a fairly naive generation method from the LTL formulas listed as Absence, Existence, Universality and Disjunction of patterns listed in Table 2.

Figure 5 represents the runtime comparison of FLIE, SYSLITE and SCARLET on 98 samples. From the scatter plots, we observe that SCARLET runs much faster than FLIE on all samples and than SYSLITE on all but two samples. Also, SCARLET timed out only on 3 samples while SYSLITE timed out on 6 samples and FLIE timed out on 15 samples.

Figure 6 presents the comparison of formula size inferred by each tool. On 84 out of 98 samples, where none of the tools timed out, we observe that on 65 samples, SCARLET inferred formula size equal to the one inferred by SYSLITE and FLIE. Further, in the samples where SCARLET learns larger formulas than other tools, the size gap is not significant. This is evident from the fact that the average formula size learned by SCARLET is 4.13 which is slightly higher than that by FLIE and SYSLITE, 3.84.

https://github.com/cryhot/samples2LTL
To address our second research question RQ2 in ‘Experimental evaluation’, we evaluated the scalability of our algorithm on two sets of benchmarks generated from formulas $\varphi_{cov}$ and $\varphi_{seq}$. While the first benchmark set contains 90 samples with increasing sizes but a fixed length, the second benchmark set contains 90 samples with 200 traces having lengths ranging from 10 to 50.

We provide the results for the second benchmark set here. Figure 7a depicts the results we obtained by running all three tools on it. The trends we observe here are similar to the ones we observe in the first benchmark set. **SCARLET** performs better on the samples from $\varphi_{cov}$ than it does on samples from $\varphi_{seq}$. The reason remains similar: it is easier to find a formula which is a boolean combination of length 1, width 1 simple LTL, than a simple LTL of length 3 and width 1.

Contrary to the results on the first benchmark set, we observe that the increase of runtime with the length of the sample is quadratic. This explains why on samples from $\varphi_{seq}$ on large lengths such as 50, **SCARLET** faces timeout. However, for samples from $\varphi_{cov}$, **SCARLET** displays the ability to scale way beyond length 50.
Figure 7: Comparison of SCARLET, FLIE and SYSLITE on synthetic benchmarks. In Figure 7a, all times are in seconds and ‘TO’ indicates timeouts.