Modeling oil and gas inflow to horizontal wells

Am At Khayrullin, A V Myasnik, Az Am Khayrullin

Industrial University of Tyumen, 38, Volodarskogo Str., Tyumen, 625000, Russia

E-mail: hajrullinaa@tyuiu.ru

Abstract. The paper presents the modeling of the flow of oil and gas to a horizontal well in an isotropic and uniformly anisotropic reservoir. As a result, analytical methods for calculating oil and gas flow rates in horizontal wells were obtained, which can be used for further development of hydrocarbon fields.

1. Introduction

Over the last few years, both in Russia and abroad, oil fields have been developed mainly by directional or horizontal wells. Their main purpose is to increase the surface area of contact with the producing reservoir, which, in turn, significantly increases their recovery capabilities. Drilling such wells is more promising as they, in comparison with conventional vertical wells, allow:

• developing reserves previously not covered by displacement by increasing the coverage factor by area;
• controlling coning in deposits with a close location of WOC and GOC zones;
• increasing productivity by increasing the surface area of filtration;
• increasing the recovery ratio of residual reserves when extracting high-viscosity oil or bitumen;
• developing deposits on the continental shelf.

The main goal of this work is to develop an objective methodology for predicting the productivity of oil and gas wells with a horizontal tailing-in, drilled in technologically-altered heterogeneous reservoirs, completely tapped by a horizontal wellbore.

2. Modelling of oil inflow to horizontal well

This problem was solved based on the method of A.P. Telkov [3]. The essence of this problem is that instead of several vertical wells, one horizontal well of length \( l \) is drilled. The formula considered in this article is quite simple and therefore quite clearly shows the influence of various parameters and factors on the total productivity of a horizontal well. In the development of this formula, A.P. Telkov took the ideas of Yu.P. Borisov and I.A. Charny [2] as a basis, in which the total filtration resistance of a section of an oil reservoir with a well mathematically described by special functions is divided into parts and is represented by a sequence of filtration resistances (called external and internal) mathematically described by simple elementary functions, therefore, clearly evident and easily analysed.

So, let us consider a horizontal well with a length that completely drains a homogeneously anisotropic reservoir with the same physical properties at all points, as well as with impermeable boundaries of the roof and the bottom. Its width \( 2\sigma \) and length \( m\sigma \) instead of a chain of \( m \) vertical wells may contain one horizontal well with a length of horizontal section \( l \) and a radius \( r_0 \) (Fig. 1).
Figure 1. Model of a horizontal well with the external boundary in the form of a circle

In its turn, the length of the horizontal section $l$ should be much less than the length of the producing reservoir $m2\sigma$.

According to Yu.P. Borisov, the equation of inflow to the chain of several pairs of vertical wells has the form:

$$ q_m = \frac{kh}{\mu} \left( \frac{P_r - P_b}{m \frac{L}{2\sigma} + \frac{1}{2\pi} \ln \left( \frac{2\sigma}{2\pi r_w} \right)} \right) $$

where $\frac{kh}{\mu}$ – flow capacity of an oil reservoir;

$P_r$ – reservoir pressure, MPa;

$P_b$ – bottomhole pressure, MPa;

$m$ – number of producing wells (in this case $m = 2$);

$L$ – distance from the well line to the reservoir pressure line, in this case $L = \sigma$, i.e. half the distance between adjacent rows of producing wells, m;

$2\sigma$ – distance between adjacent rows of producing wells, m;

$r_w$ – well radius, m.

If we use the A.P. Telkov method and replace a pair of vertical wells with one horizontal well, then the flow rate of this well can be calculated using the following formula:

$$ q_h = \frac{kh}{\mu} \frac{P_r - P_b}{J_e + J_i} $$

where $J_e$ – external filtration resistance (according to Yu.P. Borisov), equal to:

$$ J_e = \frac{1}{2} \frac{L}{m2\sigma - l} \ln \frac{m2\sigma}{l} $$

$J_i$ – internal filtration resistance (according to Yu.P. Borisov), equal to:

$$ J_i = \frac{h}{l} \frac{1}{2\pi} \ln \frac{h}{2\pi r_w} $$

Let us determine the dependence of the increase in the well flow rate as the length of the horizontal section increases. The radius of the well $r_c = 0.1$ m, and the parameter $2\sigma = 500$ m. Figure 2 shows the graph of this dependence.
This graph clearly shows the increase in the flow rate from 112.5 m$^3$/day in the case of a horizontal hole length equal to 100 meters to 204.6 m$^3$/day with a length of a horizontal section of 450 meters.

3. Reservoir heterogeneity impact on the filtration process

However, all the above-mentioned calculations are valid only for a completely homogeneous reservoir. In actual conditions, practically all productive layers are characterized by a rather high degree of heterogeneity, i.e. the difference in the permeability of the strata along the strike and along the bedding. The heterogeneity of the porous medium can greatly influence the filtration processes in the reservoir. For example, it is known that this influence both affects the quantitative characteristics of fluid filtration, and to some extent alters their flow geometry.

In connection with all of the above-mentioned, when designing and further developing the fields, it is necessary to take into account with full accuracy the inhomogeneity of the producing reservoir. Therefore, the expressions presented above should be generalized taking into account the anisotropy of the reservoir, which can be considered in the internal filtration resistance $J_i$. To do this, we generalize our oil-saturated thickness $h$ taking into account the heterogeneity in the vertical and horizontal permeability of the reservoir:

$$\chi = \sqrt{k_h/k_v}$$

where $k_h$ and $k_v$ – horizontal and vertical permeability, respectively.

Then, the generalized oil-saturated thickness, taking into account the anisotropy of the reservoir, will take the form:

$$h' = h \cdot \chi$$

The value of the reservoir thickness also appears in the formula of the internal filtration resistance $J_i$, and, consequently, it will undergo the following transformation taking into account the generalization of the oil-saturated thickness:

$$J'_i = \frac{h'}{l} \frac{1}{2\pi} \ln \frac{h'}{2\pi r_w}.$$
\[ S = \left( \frac{k}{k_s} - 1 \right) \ln \frac{r_s}{r_w} \]

where

\( k \) – reservoir permeability, m\(^2\)
\( k_s \) – permeability in the skin area, m\(^2\)
\( r_s \) – skin area radius, m
\( r_w \) – wellbore radius, m

But for horizontal wells there is no exact definition of the concept of skin factor in scientific works. Everything is related to the fact that the inflow to horizontal wells differs significantly from the inflow to vertical wells and, in general, is not radial. A degraded zone around a horizontal well does not have a strictly defined geometry. Therefore, when taking into account the skin factor in calculating the productivity of a real horizontal well, generalized methods based on Hawkins’ formula are used.

In this calculation, we will use the modified Hawkins formula used for a homogeneous reservoir with a cylindrical wellbore zone [5], corrected for the geometry and anisotropy of the reservoir:

\[ S = \frac{\chi \cdot h \cdot k}{l} \left( \frac{k}{k_s} - 1 \right) \ln \frac{r_s}{r_w} \]

where

\( \chi \) – reservoir anisotropy
\( h \) – reservoir thickness, m
\( l \) – length of the horizontal section, m

Thus, the initial formula for calculating the productivity of a horizontal well in an infinite uniformly anisotropic reservoir becomes:

\[ q_h = \frac{kh}{\mu} \frac{P_r - P_b}{J_e + J_i^l + S} \]

The flow rate of the horizontal oil well under consideration, taking into account the positive skin factor, will fall significantly (Fig. 3). In this case, if the length of the horizontal section varies from 100 to 450 meters, the flow rate of the well will be 48.6 m\(^3\)/day and 77.9 m\(^3\)/day, respectively.

**Figure 3.** Dependence of the increase in the flow rate as the length of the horizontal section (HS) of the producing well increases and taking into account the effect of the skin factor

**4. Modelling of gas inflow to horizontal well**

This solution about inflow modeling is valid only for a horizontal oil well. Modeling the inflow of gas to horizontal wellbores is a much more complex process, requiring consideration of a much larger number of concomitant factors. Probably, that is why the number of studies aimed at studying the performance of oil wells far exceeds the available number of studies to determine the productivity of horizontal gas wells.

Further in this article, a model of horizontal gas well productivity will be considered according to a method based on the works of Z.S. Aliiev [1].

In their work, Z.S. Aliiev and V.V. Sheremet propose presenting the entire working area of filtration in the form of two fictitious zones with different flow geometry. During gas filtration, there is a disruption of the linear relationship between the filtration rate and the pressure gradient.
An approximate expression of productivity for a gas horizontal well that tapped the entire strip-like formation with the thickness $h$ for a two-term filtering law of the form $\frac{\Delta P}{\Delta x} = \alpha u + \beta u^2$ will take the form:

$$P_r^2 - P_b^2 = AQ + BQ^2,$$

where $A$ – coefficient of filtration resistance, taking into account the friction pressure loss; $B$ – coefficient for the inertial component; $\alpha, \beta$ – constant coefficients; $u$ – fluid filtration rate, m/s;

In this case, the formula for the flow rate $Q$ will be:

$$Q = \frac{-A + \sqrt{A^2 + 4B(\rho_r^2 - \rho_w^2)}}{2B},$$

The coefficients of filtration resistance $A$ and $B$ are determined by the following formulas:

$$A = \frac{A^*}{2L} \left[ \frac{2}{z - r_w} \left( \frac{h}{2} - r_w + r_w \ln \frac{r_w}{r_w + \frac{h}{2} - r_w} \right) + \frac{R_e - r_w - \frac{h}{2}}{R_e + r_w + \frac{h}{2}} \right],$$

$$B = \frac{B^*}{8L} \left[ \ln \frac{r_w + \frac{h}{2} - r_w}{r_w + \frac{h}{2} - r_w} - \frac{h}{r_w + \frac{h}{2} - r_w} \right] + \frac{R_e - r_w - \frac{h}{2}}{(R_e + r_w + \frac{h}{2})^2}.$$

Here, $A^*$ and $B^*$ - dimensional coefficients that depend on fluid properties and thermobaric conditions in the reservoir determined by the following formulas:

$$A^* = \frac{1}{k \cdot T_{st}} \cdot \frac{\rho_{st} \cdot T_{st} \cdot z}{l_{rn} \cdot T_{st}};$$

$$B^* = \frac{1}{k \cdot T_{st}} \cdot \frac{\rho_{st} \cdot T_{st} \cdot z}{l_{rn} \cdot T_{st}};$$

where $l_{rn}$ – coefficient of macro-roughness of the rock, m. This coefficient is closely related to the coefficient $\beta^*$.

$$\beta^* = 2 \cdot 10^{11} \cdot e^{-0.021 \cdot k \cdot 10^{15}}$$

$z$ – coefficient of gas supercompressibility;

$P_{st}$ and $T_{st}$ – gas pressure and temperature in standard conditions;

$R_e$ – external radius of the well, m;

$r_w$ – wellbore radius, m.

The derivation of these formulas was carried out using the example of an ideal homogeneously isotropic reservoir. In practice, horizontal permeability $k_h$ is almost never equal to its vertical component $k_v$.

Figure 4 shows the layout diagram of a horizontal gas well in a homogeneously anisotropic reservoir with a thickness $H$ with different permeability components - vertical $k_v$ and horizontal $k_h$.

![Figure 4. Cross-sectional diagram of a homogeneously anisotropic formation with a plane passing through the axis of a horizontal gas well](image)

Changing the length of the horizontal section from 100 to 450 meters, we get the initial flow rate of $2.4 \cdot 10^5$ and $1.1 \cdot 10^6$ m$^3$, respectively.
However, to correctly predict the flow rate of a horizontal gas well directly during the development of a gas or oil and gas field, it is recommended to take into account the difference in bottomhole pressure over the entire length of the horizontal section of the well. This can be done by converting the formula of the gas HS flow rate to a system of two ordinary differential equations (DE) of the form:

\[
\begin{align*}
\frac{dp(L)}{dL} &= \frac{\alpha \cdot Q(L) \cdot p(L)}{2 \cdot \alpha \cdot Q(L)^2 - p(L)^2} \left( \frac{Q(L)}{D} + 2 \frac{dQ(L)}{dL} \right), \\
\frac{dQ(L)}{dL} &= -A + \sqrt{A^2 + 4A(p_r - p(L))^2} \\
&\quad \frac{2B}{2B}
\end{align*}
\]

where \( \alpha = \frac{8 \rho_s r p_s Z T_r}{\pi D^2 T_p} \);

\( \lambda \) – coefficient of hydraulic friction resistance.

This system of DE should be solved under the following initial conditions: \( L=0, p(0)=p_w, Q(L=0)=0 \) by dividing the entire horizontal wellbore by the number of elementary sections of \( n \), i.e. \( dL=L/n \). The greater the number of sections, the more accurate the values of bottomhole pressure will be.

The system of DE is a system of non-linear differential equations and is solved by numerical methods, for example, by the fourth-order Runge-Kutta method, and using almost any software package for mathematical calculations, such as Mathcad.

5. Conclusion

Therefore, the study resulted in obtaining equations of inflow of both oil and gas to a horizontal well in a homogeneously anisotropic reservoir. In the given dependences, the possible filtration resistances arising in the process of filtration of the fluid to the horizontal wellbore were taken into account, which significantly approximates the theoretical results obtained to their actual counterparts.

6. Acknowledgments

The authors of this work express their gratitude to the management of Tyumen Industrial University and in particular to the staff of the oil and gas development and exploration department for assistance in writing and financing this paper.

References

[1] Aliev Z S and Sheremet V V 1995 Determination of the productivity of horizontal wells that have tapped gas and gas-oil reservoirs (Moscow: Nedra)
[2] Borisov Yu P, Pilatovskiy V P and Tabakov V P 1964 Development of oil fields by horizontal and multilateral wells (Moscow: Nedra)
[3] Brekhuntzov A M and Telkov A P 2004 Development of the theory of fluid and gas filtration to horizontal wellbores (Tyumen: JSC SibNATS)
[4] Hawkins M F 1956 A Note on the Skin Effect (Trans: AIME)
[5] Renard G and Dupuy J M 1991 Formation damage effects on horizontal-well flow efficiency SPE 19414