OBSERVATIONAL EVIDENCE FOR STOCHASTIC BIASING

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ABSTRACT

We show that the galaxy density in the Las Campanas Redshift Survey cannot be perfectly correlated with the underlying mass distribution since various galaxy subpopulations are not perfectly correlated with each other, even taking shot noise into account. This rules out the hypothesis of simple linear biasing and suggests that the recently proposed stochastic biasing framework is necessary for modeling actual data.

Subject headings: galaxies: statistics — large-scale structure of universe

1. INTRODUCTION

Measurements of clustering in upcoming galaxy redshift surveys hold the potential for measuring cosmological parameters with great accuracy (Tegmark 1997; Goldberg & Strauss 1998), especially when complemented by measurements of the cosmic microwave background (Eisenstein, Hu, & Tegmark 1999; Hu, Eisenstein, & Tegmark 1998; Hu et al. 1999). Since such measurements are only as accurate as our understanding of biasing, there has been a recent burst of work on the relation between the distribution of galaxies and the underlying mass.

Dekel & Lahav (1998; Dekel 1997, § 5.5; Lahav 1996, § 3.1) have proposed a robust framework termed “stochastic biasing,” which drops the assumption that the galaxy density $\rho_g(r)$ is uniquely determined by the matter density $\rho$. Writing the corresponding density fluctuations as $g = \rho_g / \rho - 1$ and $\delta = \rho_g / \rho - 1$, $g$ is modeled as a function of $\delta$ plus a random term. It has been known since the outset (see, e.g., Dressler 1980) that any deterministic biasing relation $g = f(\delta)$ must be complicated, depending on galaxy type. Even allowing this, however, deterministic biasing still implies that the peaks and troughs of the two fields must coincide spatially, which need not be the case. Restricting attention to second moments, all the information about the stochastic and nonlinear nature of $f(\delta)$ can be contained in a single new function $r$ (Pen 1998; Tegmark & Peebles 1998, hereafter TP98). Grouping the densities into a two-dimensional vector

$$\mathbf{x} \equiv \begin{pmatrix} \delta \\ g \end{pmatrix}$$

and assuming nothing except translational invariance, its Fourier transform $\hat{\mathbf{x}}(k) \equiv \int e^{-ikr} \mathbf{x}(r) dr$ obeys

$$\hat{\mathbf{P}}(k) \hat{\mathbf{P}}(k') = (2\pi)^3 \delta^{(3)}(k - k') \begin{pmatrix} P(k) & P_g(k) \\ P_g(k) & P_c(k) \end{pmatrix}$$

for some $2 \times 2$ power spectrum matrix that we will denote $\mathbf{P}(k)$. Here $P$ is the conventional power spectrum of the mass distribution, $P_g$ is the power spectrum of the galaxies, and $P_c$ is the cross spectrum. It is convenient to rewrite this covariance matrix as

$$P(k) = P(k) \begin{pmatrix} 1 & b(k)r(k) \\ b(k)r(k) & b(k)^2 \end{pmatrix}$$

where $b \equiv (P/P_g)^{1/2}$ is the bias factor (the ratio of luminous and total fluctuations) and the new function $r \equiv P_g/(P/P_g)^{1/2}$ is the dimensionless correlation coefficient between galaxies and matter. The special case $r = 1$ gives the simple deterministic biasing relation $g = bb$. Both $b$ and $r$ may generally depend on scale.

Since the function $r(k)$ is a cosmologically important quantity, it has received much recent attention. Pen (1998) has shown how it can be measured using redshift-space distortions and nonlinear effects, Scherrer & Weinberg (1998) have computed $r$ for a number of theoretical models, and Blanton et al. (1998, hereafter B98) have estimated $b$ and $r$ from hydrodynamic simulations. TP98 have computed the time evolution of bias in the linear regime, while Taruya, Koyama, & Soda (1999) and Taruya & Soda (1999) have generalized this result to the perturbatively nonlinear case (see also Mo & White 1996; Mo, Jing, & White 1997; Matarrese et al. 1997; Bagla 1998; Catelan et al. 1998a; Catelan, Matarrese, & Porciani 1998b; Colin et al. 1998; Sheth & Lemson 1998; Moscardini et al. 1998; Porciani et al. 1998; Wechsler et al. 1998). Some of these numerical and theoretical predictions have been borne out in observational data that indicate very high bias values ($b \sim 4-6$) at $z \sim 3$, decreasing rapidly with time (Giavalisco et al. 1998).

In light of all this activity, it would be timely to measure $r(k)$ observationally. This is the topic of the present Letter. Unfortunately, measuring $r$ directly requires knowledge of the true matter distribution $\delta$. Although this information, in principle, may be obtained from, e.g., the POTENT reconstruction from peculiar velocity measurements (Dekel, Bertschinger, & Faber 1990), the quasi-linear redshift-space method of Pen (1998), or gravitational lensing (Van Waerbeke 1998), it will likely require better data than is presently available since systematic errors in the estimate of $\delta$ masquerade as $r < 1$. We therefore adopt an indirect approach, based on the following simple idea. If two different types of galaxies are both perfectly correlated with the matter, then they must also be perfectly correlated with each other. If we can demonstrate imperfect correlations between two galaxy subsamples, then we will have shown that $r < 1$ for at least one of them. We show that $r < 1$ at high statistical significance and place quantitative limits on $r$ in § 3.
2. RULING OUT SIMPLE BIASING

We base our analysis on galaxies in the Las Campanas Redshift Survey (LCRS; Shectman et al. 1996) that are grouped into six types or "clans" according to the spectral classification scheme of Bromley et al. (1998a, 1998b), ranging from very early type galaxies (clan 1) to late-type objects (clan 6). To reduce shot noise, we define our clan 4 as the combination of the original clans 4, 5, and 6. The clustering properties of these clans (shown in Fig. 1) vary in a systematic way, revealing a progression in the relative bias factors 

\[ r \text{ depends strongly on the formation epoch of a galaxy population, and so one might suspect that some of the LCRS clans have } r < 1. \]

The LCRS consists of \( n_z = 327 \) rectangular fields in the sky, and we further subdivide the volume into \( n_z = 10 \) radial shells in the range \( 10,000 \text{ km s}^{-1} < cz < 45,000 \text{ km s}^{-1} \). Discarding galaxies outside of this redshift range (as in Lin et al. 1996) leaves 519, 8282, 5152, and 5669 galaxies in the four clans. For each of our \( n = n_z \) spatial volumes \( V_\alpha (\alpha = 1, \ldots, n) \) and each clan \( i = 1, \ldots, n_z \), we count the number of observed galaxies \( N^{(i)}_\alpha \) and compute the expected number of galaxies \( N^{(i)}_\alpha \) using the selection function of Lin et al. (1996) with the clan-dependent Schechter parameters of Bromley et al. (1998a). We write the map of observed density fluctuations for the \( i \)th clan as an \( n \)-dimensional vector \( g^{(i)} \), defined by \( g^{(i)}_i = N^{(i)}_\alpha / \tilde{N}^{(i)}_\alpha - 1 \). In a simple \( r = 1 \) linear biasing model, we would have

\[ g^{(i)}_i = b_i \delta^{(i)}_\alpha + \epsilon^{(i)}_\alpha, \]

where \( b_i \) is the bias of the \( i \)th clan, \( \delta^{(i)}_\alpha \) is the matter density fluctuation in \( V_\alpha \), and the shot noise contributions \( \epsilon^{(i)}_\alpha \) have zero mean and a diagonal covariance matrix

\[ N^{\epsilon^{(i)}}_{\alpha \beta} = \langle \epsilon^{(i)}_\alpha \epsilon^{(i)}_\beta \rangle = \delta^{(i)}_{\alpha \beta} \tilde{N}^{(i)}_\alpha. \]

Can we rule this out? For a pair of clans \( i \) and \( j \), consider the difference map

\[ \Delta g = g^{(i)} - \tilde{g}^{(i)} \]

for different values of the factor \( f \). If \( f = b_i / b_j \), then equation (4) shows that the (unknown) matter density fluctuations \( \delta^{(i)}_\alpha \) will cancel out, and \( \Delta g \) will consist of mere shot noise whose covariance matrix is \( N = \langle \Delta g \Delta g^\ast \rangle = N^{\epsilon^{(i)}} + f^2 N^{\epsilon^{(j)}} \).

Given the alternative hypothesis that there is a residual signal with some covariance matrix \( S \), so that \( \langle \Delta g \Delta g^\ast \rangle = N + S \), the most powerful "null-buster" test for ruling out the null hypothesis \( \langle \Delta g \Delta g^\ast \rangle = N \) is using the generalized \( \chi^2 \) statistic (Tegmark 1998)

\[ \nu = \frac{\Delta g \cdot N^{-1} S N^{-1} \Delta g - \operatorname{Tr} (N^{-1} S)}{2 \operatorname{Tr} (N^{-1} S N^{-1} S)^{1/2}}, \]

which can be interpreted as the number of "sigmas" at which the noise-only null hypothesis is ruled out. We choose \( S_{ab} = \xi (r_a - r_b) \), where \( r_a \) is the center of volume \( V_a \) and \( \xi \) is the correlation function measured by the LCRS (Tucker et al. 1997). We plot the results in Figure 2 for three pairs of clans \( i \) and \( j \), after each corresponding valley-shaped curve tells us a number of things. The fact that \( \nu \gg 1 \) on the left-hand side (as \( f \rightarrow 0 \), with all the weight on clan \( i \)) means that there is a strong detection of cosmological fluctuations above the shot noise level (\( \nu \sim 1 \)). Likewise, \( \nu \gg 1 \) on the right-hand side (as \( f \rightarrow \infty \)), which demonstrates a cosmological signal in clan \( j \). The fact that the curve dips for intermediate \( f \)-values tells us that the two density maps are correlated (\( \nu > 0 \)) and have common signal. The minimum is attained at the value \( f \) which gives the best-fit relative bias \( b_i / b_j \) for this common signal. However, the fact that \( \nu \gg 1 \) even at the minimum proves that even though some of the signal is shared in common, not all of it is: there are no values of \( b_i \) for which equation (4) can hold for any pair of clans \( i \) and \( j \).

3. MEASURING r(k)

Having ruled out nonstochastic linear biasing, i.e., having demonstrated that \( r < 1 \) for some clans, let us now study in
more detail what constraints can be placed on \( r \). Upper limits on \( r \) may be set as follows. Let us lengthen the two-dimensional vector \( x = (\delta, g) \) from equation (1) to an \((1 + n_i)\)-dimensional vector including all \( n_i \) galaxy clans \( (n_i = 4) \): \( x_0 = \delta \) and \( x_i = g^{(i)}, i = 1, \ldots, n_i \), i.e., \( x = (\delta, g^{(1)}, g^{(2)}, g^{(3)}, g^{(4)}) \). Next we factor the \((1 + n_i) \times (1 + n_i)\) covariance matrix \( P = (x x^T) \) as \( P_y = \sigma \sigma^T R \), where \( \sigma \cdot \sigma = P_{ii}^{1/2} \) is the rms fluctuation of the \( i \)th component and \( R \equiv P_{ij}/\sigma \) is the dimensionless correlation matrix. The components \( R_{ii} \) clearly correspond to the correlations we discussed earlier, except that there is now one \( r \) for each clan, a vector of correlations \( r \). These are the numbers we are interested in, although all we can measure directly are the remaining degrees of freedom in \( R \), the galaxy-galaxy correlation matrix \( Q \) (the \( n \times n \) submatrix in the lower right corner). What constraints can we place? Consider first the simpler case involving only \( n_i = 2 \) clans. Then

\[
R = \begin{pmatrix} 1 & r \\ r & Q \end{pmatrix} = \begin{pmatrix} 1 & r_1 & r_2 \\ r_1 & 1 & \rho \\ r_2 & \rho & 1 \end{pmatrix},
\]

(8)

where \( \rho \) denotes the one number that we can measure: the correlation between clans 1 and 2. Since a correlation matrix cannot have negative eigenvalues, det \( R \geq 0 \). After some algebra, this gives the constraint

\[
|r_1 - \rho r_2| \leq [(1 - \rho^2)(1 - r_2^2)]^{1/2}.
\]

(9)

Thus, the larger we make \( r_2 \), the more tightly constrained \( r_1 \) becomes and vice versa. If \( r_2 = 1 \), then the right-hand side of equation (9) vanishes, forcing \( r_1 = \rho \). This gives an upper bound on the smaller of \( r_1 \) and \( r_2 \). The most weakly constrained case is clearly \( r_1 = r_2 \), and so we obtain

\[
\min (r_1, r_2) \leq \left( \frac{1 + \rho}{2} \right)^{1/2}.
\]

(10)

For the case with \( n_i = 4 \) clans, we get analogous inequalities for each of the \( n_i(n_i + 1)/2 = 10 \) pairs of clans. Additional, more complicated constraints follow from requiring a non-negative determinant for all \( 4 \times 4 \) submatrices and for the whole \( 5 \times 5 \) matrix, although these will not be considered here.

Now we proceed to measure the galaxy-galaxy correlation matrix \( Q \) itself. \( Q_{ii} = G_{ii}/(G_{kk})^{1/2} \), where \( G \) is the \( n \times n \) galaxy-galaxy covariance matrix (the bottom right submatrix of \( P \)). Based on the analysis of Tegmark et al. (1998), we compute an estimate \( G \) of the form \( G_{ii} \equiv g^{(i)} E g^{(i)} \) minus shot noise, removed, as in Appendix B of Tegmark et al. (1998), by simply omitting self-pairs. The pair weighting is given by a matrix \( E \equiv N^{-1} F^{-1} N = (N^{-1} S_i i) \) as a compromise. We make the choice \( F = I \), which corresponds to measuring the rms fluctuations in the cells. Computing the window function \( W \), defined by \( \langle G_{ii} \rangle = \int P(k) W(k) d^3 k \), we find that \( G_{ii} \) mainly probes scales around \( \lambda \equiv 2 \pi / k \sim 10^{-4} h^{-1} \) Mpc. When reading Tables 1 and 2, the reader should thus bear in mind that the error bars reflect shot noise only.

Converting \( G \) back to \( Q \), we calculate error bars for its elements and all quantities derived below by generating 104 Monte Carlo realizations of Poisson shot noise for \( N_{ij} \), adding this to the observed values and processing the result in exactly the same manner as the real data. Since our primary goal is to rule out \( r = 1 \), we clearly do not care about sample variance in \( Q \): if biasing were simple, one would measure \( r = 1 \) in all samples.

The results are shown in Table 1. The second column confirms that the clustering strength drops with clan number, as shown by Bromley et al. (1998a). All off-diagonal elements in the correlation matrix are seen to be significantly below unity, ranging from the 88% correlation between clans 2 and 3 to a mere 42% correlation between the earliest and latest galaxy types. Correlations generally decrease with separation in clan number, as expected. Applying equation (9) or equation (10) to Table 1 gives a plethora of constraints on \( r \). For example, equation (10) tells us that either \( r_1 \) or \( r_4 \) must be below \( [(1 + 0.42)/2]^{1/2} \approx 84\% \).

Finally, we return to the correlation matrix \( G \) and diagonalize it, so that \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \) are the eigenvalues of \( G \), sorted from largest to smallest, with \( e_i \) the corresponding unit eigenvectors \( (Ge_i = \lambda_i e_i) \). It is instructive to decompose the fluctuation vector \( x \) into its uncorrelated principal components \( y_i \equiv e_i \cdot x \):

\[
x = \sum_{i=1}^{n} e_i y_i,
\]

(11)

where \( (\langle x_i y_i \rangle) = \delta \lambda_i \). If biasing were simple and linear, we would have \( x = b \delta \) for some vector \( b \) of bias values \( b_i \). Equation (11) would therefore have only one term, with \( e_i \propto b \) and \( y_i \propto \delta \). Additional terms in equation (11) would correspond to random physical processes, uncorrelated with \( \delta \), that are pushing \( r \) below unity. If we want a model with a minimal amount of stochasticity, we should therefore interpret the first (largest) principal component as tracing the underlying matter distribution; i.e., we should assume that \( \delta = a y_1 = a e_1 \) for some constant \( a \). Since \( \langle x e_i \rangle = G \), this assumption gives \( \langle x \delta \rangle = a \langle x e_i \rangle = a \lambda_i e_i \), \( \langle \delta \rangle^2 = a^2 \langle x^\prime e_i \rangle e_i = a^2 \lambda_i ^2 e_i \) and hence the correlation coefficients

\[
r_j = \frac{\langle x_j \delta \rangle}{\langle (x_j \delta)^2 \rangle^{1/2}} = \frac{\lambda_j}{\sqrt{G_{jj}}},
\]

(12)

These coefficients are shown in the last row of Table 2. The fraction of the variance of clan \( j \) that is caused by matter fluctuations is \( (\langle x_j \delta \rangle)^2/\langle \delta \rangle^2 = (e_j^T \lambda_j G_{jj} e_j) = r_j^2 \), which is simply the square of the correlation coefficient, and so this is a useful way to interpret \( r \).

Table 2 is indeed consistent with the hypothesis that the first principal component traces the matter: since all of its four components (in boldface) have the same sign, we can interpret them as the (relative) bias factors that the clans would have if there were no randomness \( (\lambda_2 = \lambda_3 = \lambda_4 = 0) \). Since \( (e_j) \propto G_{jj}^{-1/2} \propto b_j r_j \), we can also interpret them as the best-fit slopes in linear regressions of \( x_j \) against \( \delta \), the quantity that Dekel & Lahav (1998) call simply “b.” The sharp decrease of

| \( h/b_h \) | Galaxy Correlation Matrix \( Q \) |
|---|---|
| 2.97 ± 0.04 | 0.63 ± 0.02 0.44 ± 0.02 0.42 ± 0.03 |
| 1.28 ± 0.01 | 0.63 ± 0.02 1 0.88 ± 0.02 0.81 ± 0.02 |
| 1.21 ± 0.01 | 0.44 ± 0.02 0.88 ± 0.02 1 0.76 ± 0.03 |
| 1.00 ± 0.01 | 0.42 ± 0.03 0.81 ± 0.02 0.76 ± 0.03 1 |
(e_i) toward later clan types (their ratios are 5.1:1:7:1:3:1) is thus caused by the joint decrease of both the measured relative bias b_i (the second column of Table 1) and the correlation r_i. Note that Figure 2 does not directly tell us which clans are most correlated, since low minima can signal either strong correlations, low fluctuations, or high shot noise—the latter being the case for the (rare) first clan.

4. CONCLUSIONS

Our basic conclusion is that bias is complicated. Rather than being describable by a single constant b_i, for the i-th galaxy type, evidence is mounting that bias is (1) stochastic and/or nonlinear, requiring a second quantity r_i to characterize second moments; (2) scale dependent (b_i and r_i depend on k_i); and (3) time dependent (b_i and r_i depend on redshift). Complication 1 was predicted by Dekel & Lahav (1998), and we have observationally confirmed it here. Complication 2 has long been observed on small scales (see, e.g., Mann, Peacock, & Heavens 1998, B98, and references therein), while complication 3 was predicted by Fry (1996) and TP98 and is gathering support from both simulations (Katz, Hernquist, & Weinberg 1998; B98) and observations (Giavalisco et al. 1998).

However, this is not cause for despair. The scale dependence is predicted to abate on large scales (Scherrer & Weinberg 1998), and the time evolution due to gravity is calculable (TP98). As long as we limit ourselves to second moments (power spectra, etc.), stochasticity merely augments P(k) and b_i(k) with an additional function r_i(k). Furthermore, our constraints on r_i using galaxy data alone—without knowledge of the underlying mass distribution—suggest strong regularities: the more similar two morphological types are, the stronger they are correlated.

More strikingly, our tentative identification of the first principal component of the galaxy covariance matrix as the underlying matter distribution is in excellent agreement with the recent simulations of B98: galaxies with high bias are almost perfectly correlated with matter (we find r_i ~ 98%), whereas the less biased populations have weaker correlations (we find r_i ~ 60%–80%). Thus, matter clustering explains only r_i^2 ~ 40%–60% of the variance of these galaxies.

This analysis is merely a first step toward observationally measuring the parameters of stochastic biasing. Exploring the scale dependence and time evolution of r_i as well as probing higher order moments of the joint distribution of galaxy types (see, e.g., Lahav & Saslaw 1992) remain to be done. However, our success in constraining r_i in the absence of difficult mass measurements is an encouraging indication for the analyses of upcoming galaxy surveys: With the techniques presented here, along with theoretical and numerical modeling, it may be possible to understand bias well enough to realize the full potential of these surveys for measuring fundamental cosmological parameters.

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TABLE 2

| \( \lambda \)  | Components of Eigenvector \( e_i \) |
|----------------|----------------------------------|
| 0.564 ± 0.013 | 0.91 ± 0.01                       |
| 0.122 ± 0.004 | -0.41 ± 0.01                      |
| 0.016 ± 0.002 | 0.01 ± 0.02                       |
| 0.008 ± 0.001 | -0.08 ± 0.01                      |

Clan correlation \( r_i \): 0.98 ± 0.02, 0.77 ± 0.02, 0.60 ± 0.02, 0.57 ± 0.03