Error Performance Analysis of Wireless Video Communication Systems Employing Multi-level MPSK Modulation and MIMO Technologies

Mussawir Ahmad Hosany

Abstract — Real-time communications of high definition video with the available limited channel bandwidth is a fundamental requirement for modern mobile and wireless networks such as Wi-Fi, WiMax, 3G, LTE and the emerging 5G. To fulfill this requirement, H.264/AVC and High Efficiency Video Coding (HEVC) or H.265 standards have been implemented to achieve high error performance over wireless channels and large compression efficiency thereby optimizing channel bandwidth. The problem of establishing and maintaining reliable communication paths among mobile users employing multiclass bitstreams to achieve high spatial diversity and coding gains is a big challenge for researchers. In practice, multimedia standards employ data partitioning where different levels of importance are assigned to the multimedia content.

The problem of transmission of coded HEVC data over wireless channels employing MIMO technologies to maximize high spatial diversity and coding gains has not been explicitly explored in existing research works. Moreover, to meet the demand of 5G networks in providing ultra-reliable low latency communications it is required to study fully the performance of multimedia data over wireless channels. To this end we propose, in this paper, to study and analyze the performance of coded MIMO systems employing a triple-class bitstreams source namely a high priority, medium priority and low priority class. The objective of our study is to promote Unequal Error Protection (UEP) as well as spatial diversity of the coded bitstreams without sacrificing the channel bandwidth and increasing the computational complexity. Space-Time Block Codes with Orthogonal properties and Hierarchical or Multi-level 8PSK modulation are considered in our analysis and the adopted approach will be shown, through numerical evaluation and simulations, that excellent UEP capabilities as well as high spatial diversity gains can be achieved.

New Bit Error Rate (BER) expressions in closed form are derived for independent and identically distributed Rayleigh channel in the presence of Additive White Gaussian Noise (AWGN). Single and multiple fading channel reception systems with Maximum Ratio Combining (MRC) are considered. The simulations of our proposed UEP transmission model are designed and implemented in Matlab Simulink® R2017. It is shown that the most important priority data gives a coding gain of around 11 dB over the less important one at a BER of $10^{-4}$ for an 8-diversity order Orthogonal Space-Time Block Code (OSTBC) employing Hierarchical 8PSK modulation. This work can be extended to evaluate the error probability of multiuser coded systems for 4G and 5G networks.

Keywords — 5G networks, Coded MIMO systems, H.265, Fading channels, Multi-level modulation, Wireless JPEG, Wireless Space-Time codes.

I. INTRODUCTION

The concept of uneven data protection against transmission errors was first described in [1]. In a practical UEP scheme various classes of multimedia traffic data, having unequal level of significance, are assigned. An increasing demand for wireless communications has renewed interest in accurate techniques for characterizing the error assessment of UEP communication systems [2]. Wireless JPEG and HEVC coders compress a multimedia content by using signal processing techniques such as Transform coding, motion compensation/estimation and entropy coding as well [3]. The degradation of multi-class bitstreams will have catastrophic effects in the quality of the multimedia content. Considering the HEVC standard, headers are most important than motion vectors and the DCT coefficients have the least importance in the bitstream. Therefore, to enhance the performance of multi-class multimedia data UEP techniques are employed in practice. This concept has motivated us to design a UEP scheme with multi-class data streams and has resulted in the work presented in this paper.

The transmission system consists of full diversity OSTBCs [4], [5] and Hierarchical 8PSK modulation [6]. We use the MGF approach as described in [7] to derive error probabilities in closed form that are simple to evaluate. Moreover, we established error bounds for our proposed UEP coded MIMO system. The proposed analysis does not involve increased in computational complexity and bandwidth expansion. It is shown that by varying the Signal to Noise Ratio (SNR) and a modulation vector, which controls the level of error protection, significant error performance can be achieved. The practical use of our proposed theoretic design is in the transmission of HEVC coded video data over wireless fading channels. The prioritized video bitstreams can be directly applied to our proposed UEP transmission model.

The paper is organized as follows. Section 2 outlines the related research studies to the proposed work and in Section III we explain our proposed transmission system and present briefly the concepts and constructions of wireless Space-Time Block Codes. Error rates and upper bounds for Single Rayleigh path reception of Hierarchical phase modulation signals or MPSK modulation are derived in Section IV. The error performances and upper bounds for OSTBCs employing Multi-level 8PSK modulation over multi-paths Rayleigh
channel are theoretically studied in Section V. In Section VI we give some numerical examples of our proposed work and show various theoretical results as well as simulated ones. The proposed simulated UEP system has been designed and implemented in Matlab Simulink R2017. Section VII concludes the paper.

II. RELATED WORKS

Recently, there has been intensive research focus on Multiple-Input Multiple-Output (MIMO) systems since these achieve spatial multiplexing with high diversity gains. MIMO systems have been successfully implemented in Long Term Evolution (LTE) 3GPP and WiMAX [8], [9]. A coded MIMO system typically employs an OSTBC and a linear modulator to achieve spatial diversity. In [6], the exact BER for a generalized Hierarchical MPSK modulation scheme is reported over AWGN channel for a single channel MRC receiver and a constellation parameter is used to control the relative message importance. The error performance of OSTBCs over various wireless channel models such as Rayleigh, Nakagami and Rician have been studied for uniform MPAM and MPSK signals in [10].

In [2], Kim et al. studied thoroughly the BER of a non-uniform MPSK modulation for a single transmit/multiple receive antenna system and a UEP transmission system was designed. They derived theoretical error probability bounds of such UEP system over the Rayleigh fading channel and further optimized the various traffic classes. Moreover, the UEP analysis was extended to the OSTBC system with multiple transmit/multiple receive antenna system. However, the adopted approach did not make use of the Moment Generating Function (MGF) to derive BER expressions resulting in an approximate evaluation of their UEP system. In [11], the calculation of error rates involves the approach of a Moment Generating Function (MGF) to derive BER closed-form expressions for single and multiple channel reception systems. These BER expressions have been shown to be exact and simple to evaluate as compared to those BER expressions established earlier in the literatures [12]-[14].

Recently, a UEP system was designed by Barmada et al. [15] through the use of turbo codes and multi-level modulation employing MIMO systems. Two priority levels were defined, and it was shown that this arrangement of unequal importance levels lead to enhanced performance as compared to an equal error protection system. Results showed that the proposed system outperformed other semi-unequal and equal error protection systems over a wide range of channel Signal-to-Noise Ratio (SNR). The UEP system developed has not been analysed theoretically in terms of error performance bounds and hierarchical modulation has not been employed to achieve UEP. The application of unequal error protection to Internet of Things (IoT) systems has been studied in [16]. The UEP system made use of polar codes as error-correcting codes to provide different levels of transmission reliabilities to support a wide variety of IoT devices. However, the use of such codes leads to an increase in computational complexity of the UEP system. Furthermore, communication systems employing UEP have been well investigated in the literatures [17]-[19]. In this paper, we address the shortcomings of prior works, and the specific contributions of our proposed work are given as follows:

1. We consider the design of a UEP system with OSTBCs employing Hierarchical 8PSK modulation over the Rayleigh fading channel. The objectives of our design are to achieve full spatial diversity and coding gains without increased in channel bandwidth and computational complexity.
2. We derive theoretical error performance bounds for our proposed UEP transmission system.
3. We designed and implemented our proposed systems using Matlab Simulink®.
4. We analyzed and compared the simulated and theoretical error performances of our proposed UEP system.

III. THE COMMUNICATION SYSTEM MODEL AND WIRELESS SPACE-TIME BLOCK CODES

A hierarchical 8PSK constellation, with non-equal space signal points, has 3 bits per symbol with each bit representing a sub-channel of unequal multimedia data priority. There are 3 sub-channels referred to as $i_1,i_2, i_3$ and the phase angles evolve in a hierarchical fashion. We define these phase angles in the form of a row vector as $\theta = [\theta_1 \theta_2]$. The modulated symbol is then encoded by an OSTBC encoder with $L$-branch diversity gain and the encoded output is sent over the MIMO communication system with $N_t$ transmit and $N_r$ receive antennas. Consider a MIMO system and assume that perfect CSI as well as the exact values of the noise variances in the MIMO system are available at the receiver. Furthermore, consider an OSTBC encoder that $N_t$ symbols from the modulator which are transmitted through the wireless channel [20].

Assuming $T$ is the number of available time slots to transmit $X$ symbols then the Space-Time encoder matrix with orthogonal characteristics can be represented by (1). Each $x_j$ symbol is sent from the $j$th transmit antenna in the $j$th time slot for $i=1,2,..., N_r$ and $j=1,2,..., T$. The coderate $R$, of such OSTBC is defined by (2). It is shown in [4,5] that full rate OSTBC exists from orthogonal designs and other variable coders are also possible. These codes achieve full diversity and low error probabilities over fading channels which make them attractive to MIMO transmission systems.

\[
X = \begin{bmatrix} x_{11} & x_{21} & x_{31} \\
 x_{12} & x_{22} & x_{32} \\
 x_{1T} & x_{2T} & x_{3T} 
\end{bmatrix} \quad (1)
\]

\[
R = \frac{x}{T} \quad (2)
\]

The received signal, given by (3), is an $N_r \times T$ matrix and $H$ is the channel coefficient matrix with i.i.d samples which are modeled as circular complex Gaussian random variables and $X^T$ is the transpose of $X$ with size $N_r \times T$. $W_nT$ is an $N_r \times T$ Gaussian noise matrix modeled with zero mean and variance $N_0/2$.

\[
Y_{nT} = H X^T + W_{nT} \quad (3)
\]
IV. PERFORMANCE ANALYSIS FOR SINGLE RAYLEIGH CHANNEL OF MULTI-LEVEL 8PSK MODULATION

A Hierarchical 8PSK constellation consists of a BPSK embedded in a QPSK and in turn embedded in an 8PSK constellation. Every information bit, starting with the MSB and ending with the LSB, is mapped onto 3 levels of protection in a unique modulated symbol as shown in Fig. 1. The ratio $\frac{\theta_1}{M^2}$ controls the degree of protection and a higher ratio indicates a stronger UEP system. When $\theta_1=\pi/4$ and $\theta_2=\pi/8$ the modulation become uniform with a conventional 8PSK constellation [21].

![Multi-level 8PSK constellation with Gray code mapping.](image)

The phase offset angles for general non-uniform PSK constellation $\theta$ are simplified as follows [19], [21]:

$$\theta_i = \frac{\pi}{2} \beta_i, i = 1, 2, \ldots, m - 1$$  \hfill (4)

where $m = \log_2 M$ where $M$ is the alphabet size. The Hierarchical 8 PSK modulator outputs 8 signals given in Table I with different phase angle rotations and Table II gives the decision rules for demodulating the received symbol where this demodulator is a Maximum Likelihood (ML) hard decision detector.

| TABLE I: MODULATOR SYMBOL | Symbol | Phase Angle |
|----------------------------|--------|-------------|
| 000                        | $\theta_1 + \theta_2$ |
| 001                        | $\theta_1 - \theta_2$ |
| 010                        | $2\pi - (\theta_1 + \theta_2)$ |
| 011                        | $2\pi - (\theta_1 - \theta_2)$ |
| 100                        | $\pi - (\theta_1 + \theta_2)$ |
| 101                        | $\pi - (\theta_1 - \theta_2)$ |
| 110                        | $\pi + (\theta_1 + \theta_2)$ |
| 111                        | $\pi + (\theta_1 - \theta_2)$ |

| TABLE II: ML DEMODULATOR | Decision rules for Phase angle received | Demod. Symbol |
|---------------------------|----------------------------------------|---------------|
| $0 < \theta \leq \pi/2$  | 000                                    |
| $\pi/2 < \theta \leq \pi$ | 001                                    |
| $-\pi/2 < \theta \leq -\pi$ | 010                                    |
| $-\pi < \theta < 0$       | 011                                    |
| $\pi/2 < \theta < \pi$   | 100                                    |
| $-\pi < \theta < -\pi/2$ | 101                                    |
| $-\pi/2 < \theta < \pi$  | 110                                    |
| $\pi < \theta < 2\pi$   | 111                                    |

For a uniform MPSK modulation it can be shown that the $n^{th}$ signal in the constellation is given by $\exp(2\pi i(n-1)/2^k)$ where $n=1,2,\ldots,k=\log_2 M$. The phase vectors are represented as $\varphi = [\varphi_0 \varphi_1 \ldots \varphi_{M-1}]$ and given by (4). The coefficients $P_i$ and $Q_i$ are found from (5) and (6) respectively.

$\varphi_j = P_j \pi + \sum_{i=1}^{\log_2 M} \omega_i$, $\omega_j = 0, \ldots, M-1$  \hfill (4)

By replacing (5) and (6) in (4) a general expression is obtained in (7). As a result the hierarchical modulated symbol for the $f^{th}$ signal can be represented by $\exp(\varphi f \sqrt{-1})$.

$$P_j = \left[ \frac{4^j}{M} - \frac{2^j}{M} \right] \quad j = 0, 1, 2, \ldots, M - 1$$ \hfill (5)

$$Q_{i,j} = \left[ \frac{4^j}{2^{\log_2 M-i-1}} + 1 - \frac{2^j}{M} \right] \quad i = 0, 1, \ldots, \log_2 M - 1 \quad j = 0, 1, \ldots, M - 1$$ \hfill (6)

$$\varphi_j = \pi \left[ \frac{4^j}{M} - \frac{2^j}{M} \right] + \sum_{i=0}^{\log_2 M-1} \omega_i \left[ \frac{4^j}{2^{\log_2 M-i-1}} + 1 - \frac{2^j}{M} \right]$$  \hfill (7)

For example, the Hierarchical 4PSK modulation phase angles and modulator signals can be found by using (5),(6),(7) and given as follows: $\psi_{4-2/4PSK} = [\theta_1, \pi - \theta_1, \pi + \theta_1, 2\pi - \theta_1]$ and $\exp(i\theta_1), \exp(i(\pi - \theta_1)), \exp(i(\pi + \theta_1)), \exp(i(2\pi - \theta_1))$.

A. Error Rates Analysis

The average error probability, given by (8), can be found by employing (9) which has been provided by Craig [23]. Replacing (9) in (8) gives (10) which be easily evaluated by using a Moment Generating Function (MGF) that can be expressed by (11). Hence the average error probability of (10) reduces to (12) when (11) is substituted in (10) where $a$ is a constant that depends on the type of modulation used.

$$P = \int_0^\infty Q(a\sqrt{\gamma})p_\gamma(\gamma) \, d\gamma$$ \hfill (8)

$$Q(a\sqrt{\gamma}) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left( -\frac{a^2 \gamma}{2 \sin^2 \theta} \right) \, d\theta$$ \hfill (9)

$$P = \int_0^\infty \frac{1}{\pi} \int_0^{\pi/2} \exp\left( -\frac{a^2 \gamma}{2 \sin^2 \theta} \right) \, d\theta \, p_\gamma(\gamma) \, d\gamma = \frac{1}{\pi} \int_0^{\pi/2} \int_0^\infty \exp\left( -\frac{a^2 \gamma}{2 \sin^2 \theta} \right) \, p_\gamma(\gamma) \, d\gamma \, d\theta$$ \hfill (10)

$$M_\gamma(s) = \int_0^\infty p_\gamma(\gamma) \, e^{sy} \, d\gamma$$ \hfill (11)

$$P = \frac{1}{\pi} \int_0^{\pi/2} M_\gamma \left( -\frac{a^2}{2 \sin^2 \theta} \right) \, d\theta$$ \hfill (12)

In this work it is proposed to use the Non Line-Of-Sight (NLOS) flat Rayleigh fading channel. The Rayleigh distribution SNR per bit is given by (13) where $\gamma$ refers to the average SNR per bit and its Laplace Transform is represented by (14). Substituting (14) in (12) and using Tables of Integrals from [24, p. 177, eq. (2.562.1)] gives (15), which is a simple expression to find the average error probability given any $\gamma$.

$$P_\gamma(\gamma) = \frac{1}{\gamma} \exp\left( -\frac{\gamma}{\gamma} \right)$$ \hfill (13)

$$M_\gamma(-s) = \frac{1}{(1+s\gamma)}$$ \hfill (14)
\[
P \equiv P(\alpha, \gamma) = \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{a^2 \gamma}{2 \sin^2 \theta} \right)^{-1} d\theta = \frac{1}{2} \left(1 - \frac{a^2 \gamma/2}{1 + a^2 \gamma/2} \right)
\]

(15)

The Average Symbol Error Probability (SEP) of any uniform MPSK modulation scheme over the Rayleigh channel is found by using (16) which results in the closed-form expression given by (17) [7, eq. 5A.15]. The evaluation of the integral can be easily worked out by using Tables of Integrals [24, p. 177, eq. (2.562.1)] as shown below and the average Bit Error Probability, \( P_b \), can be found by using (24).

\[
P_s = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M \gamma \left( -\frac{\sin^2 \frac{\pi}{M}}{\sin^2 \theta} \right) d\theta = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \frac{\sin^2 \theta}{\sin^2 \theta + c} d\theta
\]

(16)

where

\[
\mu = \left[ \frac{\gamma \sin^2 \frac{\pi}{M}}{1 + \gamma \sin^2 \frac{\pi}{M}} \right]^{\frac{1}{2}}
\]

Having presented some known results to evaluate the average error probability for a uniform MPSK modulation scheme we now turn our attention to derive error rates for the Hierarchical SPSK modulation over the Rayleigh fading channel where \( P_s \) is given by (18) and \( \psi_{ij} \) is the \( j^{th} \) phase error between the transmitted and received phases for the \( i^{th} \) bit in the Hierarchical MPSK symbol. \( i=1,2,.., \log_2 M \) represents the bitstream class or sub-channel number.

\[
P_b \equiv P_b(M, \psi_{ij}, \gamma) = \frac{1}{2\pi} \int_{-\pi - |\psi_{ij}|}^{\pi - |\psi_{ij}|} \left( -\frac{\sin^2 \psi_{ij}}{\sin^2 \theta} \right) d\theta
\]

(18)

Using the same principle of derivation of (16) together with the Table of integrals [24, p. 177, eq. (2.562.1)] we performed the integration and show that (18) reduces to a new BER closed-form expression given by (19).

\[
P_b \equiv P_b(M, \psi_{ij}, \gamma) = \frac{\text{sgn}(\psi_{ij})}{2\pi} \int_{-\pi}^{\pi} \left| \psi_{ij} \right| \sin^2 \theta \sin^2 \theta + c d\theta
\]

\[
c = \gamma \sin^2 \psi_{ij} \quad P_b = \frac{\text{sgn}(\psi_{ij})}{2\pi} \left( \pi - |\psi_{ij}| - \mu \left( \frac{\pi}{2} + \tan^{-1}(\mu \cot |\psi_{ij}|) \right) \right)
\]

(19)

To illustrate the usefulness of our derived BER expression of (19) consider an example of finding the BER of Hierarchical 2/4 PSK modulation over a Rayleigh channel. In this case \( M=4, \) \( i=2 \) and with \( j=2 \) the MSB phase angles are \( \psi_{1j}=\theta_1-\pi/2 \) and \( \psi_{2j}=\theta_1+\pi/2; \) for the LSB, \( \psi_{2j}=-\theta_1 \) and \( \psi_{2j}=\pi+\theta_1. \) Replacing these values in (19) we obtained the BER expressions given by (20) and (21). For a higher order modulation scheme such as Hierarchical 2/4/8 PSK the same principle described above can be followed to derive other BER expressions.

For MSB, \( P_b \equiv P_b(4, \psi_{1j}, \gamma) = P_b(4, \psi_{11}, \gamma) - P_b(4, \psi_{12}, \gamma) \)

(20)

For LSB, \( P_b \equiv P_b(4, \psi_{2j}, \gamma) = P_b(4, \psi_{21}, \gamma) - P_b(4, \psi_{22}, \gamma) \)

(21)

B. Error Bounds Analysis

The conditional upper bound error probability of a uniform MPSK signal can be obtained by setting \( \theta = \pi/2 \) in (16) resulting in (22). For our case setting \( \theta = \pi/2 \) in (18) gives (23) which is an upper bound error probability of every bit in the Multi-Level MPSK symbol over the Rayleigh fading channel.

\[
P_s \leq \frac{M-1}{M} \left( 1 + \gamma \sin^2 \frac{\pi}{M} \right)^{-1}
\]

(22)

\[
P_b(M, \psi_{ij}, \gamma) \leq \frac{\pi - |\psi_{ij}|}{\pi} \left( 1 + \gamma \sin^2 \psi_{ij} \right)^{-1}
\]

(23)

V. PERFORMANCE ANALYSIS FOR CODED MIMO SYSTEMS OVER RAYLEIGH FADING MULTICHANNEL RECEPTION OF MULTI-LEVEL MPSK SIGNALS

Consider a Coded MIMO system employing a \( L \)-branch diversity MRC receiver which is optimal and independent of the fading characteristics since it is a Maximum Likelihood Receiver. The combined SNR for every received symbol, \( \gamma \), is found by using (24).

\[
\gamma_l = \sum_{l=1}^{L} \gamma_l
\]

(24)

A. Error Rates Analysis

Using the MGF approach as described in Section 4, \( P_0(E) \) can be found by using (25). The MGF of the SNR per symbol, \( \gamma \), for our proposed work is a Rayleigh Multipath fading channel defined by \( M_r(-s) = (1 + s \gamma)^{-1} \). Assuming fading characteristics is i.i.d for all \( L \) branches then (25) can be represented by (26) and for a uniform MPSK modulation scheme with coherent demodulation employing MRC receiver it can be shown that \( P_0(E) \) is given by (27).

\[
P_0(E) = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{\sin^2 \pi/M}{\sin^2 \theta} \right) d\theta
\]

(25)

\[
P_0(E) = \frac{1}{\pi} \int_0^{\pi/2} \left[ M_r \left( -\frac{\sin^2 \pi/M}{\sin^2 \theta} \right) \right]^L d\theta
\]

(26)
\[ P_b(E) = \frac{1}{\pi \log_2 M} \int_0^{(M-1)M} \left[ M^\gamma \left( \frac{\sin^2 \theta}{\sin^2 \theta + c} \right)^L \right] d\theta \] (27)

For the Rayleigh multipath fading channel (27) is simplified to an integral that has been evaluated in [7 eq. 5A.17] and is presented in (28). For a single Rayleigh fading channel with L=1, it can be shown that (28) reduces to (17) hence showing the exactness of the analysis.

\[ P_S = \frac{1}{\pi} \int_0^{(M-1)M} \left( \frac{\sin^2 \theta}{\sin^2 \theta + c} \right)^L d\theta \]

\[ = \frac{(M-1)}{M} - \frac{\mu}{\pi} \left( \frac{\pi}{2} + \tan^{-1} \alpha \right) \left( \frac{L-1}{2} \left( \frac{2k}{k} \right)^{\frac{1}{4(1+c)^k}} + \sin (\tan^{-1} \alpha) \left( \frac{L-1}{2} \left( \frac{2k}{k} \right)^{\frac{1}{4(1+c)^k}} \right) \right) \]

(28)

where

\[ c = \gamma \sin^2 \frac{\pi}{M} \quad \mu = \frac{\pi}{\sqrt{1+c}} \]

\[ \alpha = \mu \cot \frac{\pi}{M} \]

\[ T_{lk} = \frac{\left( \frac{2k}{k} \right)^{\frac{1}{4(1+c)^k}}}{\left( \frac{2(k-i)}{(k-i)} \right)^{\frac{1}{4(1+c)^k}}} \]

Using (18) the average bit error probability for an OSTBC with L-branched diversity can be represented by (29). Assuming i.i.d fading statistics as well as same SNR/bit, \( \gamma \) for all the L branches and using the MGF of Rayleigh defined by \( M_{\gamma}(s) = (1 + \gamma s)^{-1} \) it can be shown that (29) reduces to (30).

\[ P_b \triangleq \frac{P_b(M, \psi_{ij}, \gamma)}{2\pi} = -\frac{\text{sgn}(\psi_{ij})}{\pi} \int_0^{\pi} \left[ M_{\gamma} \left( -\frac{\sin^2 \theta}{\sin^2 \theta + c} \right)^L \right] d\theta \] (29)

\[ P_b \triangleq \frac{P_b(M, \psi_{ij}, \gamma, L)}{2\pi} = -\frac{\text{sgn}(\psi_{ij})}{\pi} \left( \frac{\pi}{2} - \mu \left\{ \left( \frac{\pi}{2} + \tan^{-1} \left( \frac{\pi}{2} \right) \right) \left( \frac{L-1}{2} \left( \frac{2k}{k} \right)^{\frac{1}{4(1+c)^k}} + \sin (\tan^{-1} \alpha) \left( \frac{L-1}{2} \left( \frac{2k}{k} \right)^{\frac{1}{4(1+c)^k}} \right) \right) \right\} \right) \]

\[ \text{where} \quad c = \gamma \sin^2 \psi_{ij} \quad \mu = \frac{e}{\sqrt{1+c}} \] (30)

The integrand of (30) is similar to that of (28) for the uniform MPSK case and therefore it can be easily integrated with only a change in the upper limit. After integration we ended up with a new BER closed form expression given by (31). To test the validity of our expression we set \( L=1 \) (single Rayleigh channel link) in (32) and (33) reduces the upper bounds to those derived in (22) and (23).

\[ P_b \leq \frac{(M-1)}{M} \left( 1 + \gamma \sin^2 \frac{\pi}{M} \right)^{-L} \]

(32)

\[ P_b(M, \psi_{ij}, \gamma, L) \leq \frac{\pi |\psi_{ij}|}{\pi} \left( 1 + \gamma \sin^2 \psi_{ij} \right)^{-L} \]

(33)

VI. NUMERICAL RESULTS AND DISCUSSIONS

In this section, numerical results as well as simulated ones are presented to illustrate and verify all our derived error probability expressions obtained in the previous sections. The SNR per bit as well as the constellation phase angle vector \( \theta \) are varied and the error probabilities for various modulation schemes are calculated and plotted. The simulations of our proposed UEP transmission model are designed and implemented in Matlab Simulink® R2017. We used the in-built Multipath Rayleigh fading, AWGN and the OSTBC Encoder/Combiner Simulink blocks in all our designs. Since the number of different cases covered by our analysis is quite broad, we present only some examples that demonstrate the usefulness of our proposed UEP transmission system. The derived closed-form expressions have been verified extensively by Monte Carlo simulations.

In order to analyze the transmission of video data over wireless networks an experiment is set up by using the H.264/AVC Annex B (JM14.1) reference source code [3] for encoding the CIF (352x288 pixels) Bus video test sequence. The video sequence was encoded at 30 frames/second for a GOP of 12 frames (IDR B P B P B P B P B I ...). With IDR rate of 24 frames. The following error resiliency features were enabled: data partitioning, FMO dispersed mode and Constrained Intra Prediction. The NAL size of 200 bytes was considered. A robust decoder with error concealment schemes is used.

Fig. 2 and 3 show the error performances for uniform QPSK and 8PSK modulation schemes respectively. The derived expressions of (17) and (22) were used and it can be
clearly seen that with increasing diversity gain, low error rates or coding gains are achieved, and all theoretical results are in accordance with their respective upper bound ones. The error performance results of Hierarchical QPSK and 8PSK modulation over a single Rayleigh fading channel link are illustrated in Fig. 4 and 5 respectively. We employ (19) in our numerical analysis with the vector $\theta$ set to $\theta_1 = \frac{\pi}{5}$ for Hierarchical QPSK modulation and $[\theta_1, \theta_2] = [\frac{\pi}{4}, \frac{\pi}{8}]$ for the 8PSK modulation case.

Fig. 4 shows the existence of UEP between the MSB and LSB sub-channels. At a BER of $10^{-2}$ a theoretical gain of around 3 dB is observed between the high priority MSB and low priority LSB sub-channels. For the Hierarchical 8PSK modulation case, Fig. 5 illustrates similar error performances ($P_{b1}$ and $P_{b2}$) for the first 2 sub-channels. This is because the embedded QPSK signal consists of 2 BPSK signals at $\theta_1 = \pi/4$ which has also been confirmed in [6, Fig. 6].

Fig. 6 compares the UEP capabilities among three Hierarchical 8PSK sub-channels when the vector $\theta$ is set to $[\pi/5, \pi/15]$. At a BER of $10^{-2}$, a theoretical gain of around 3 dB is observed between the MSB and middle bit sub-channels while around 7 dB is obtained between the middle bit and LSB sub-channels. Hence, it can be deduced that by only setting the phase vector to the above values, a total of around 10 dB is achievable between the highest and lowest priority sub-channels with our proposed system.
of the Hierarchical 8PSK modulation scheme. The illustrated theoretical and upper bound results were obtained from our derived BER expressions of (31) and (33). It can be clearly seen that as the diversity and phase angle vector are increased higher coding gains are achieved among the UEP sub-channels. At a BER of $10^{-6}$ and when $\theta_1$ is set to $\pi/5$, a theoretical gain of around 3 dB is observed between the MSB and LSB sub-channels for a 8-branch diversity OSTBC.

Theoretical and upper bound results for 8-diversity OSTBCs employing Hierarchical 8PSK modulation scheme with phase vector $\theta = \begin{bmatrix} \pi/5 & \pi/15 \end{bmatrix}$ are illustrated in Fig. 8. It can be clearly observed that significant gains are achieved in our proposed UEP system hence fulfilling entirely the objective of our research work. The highest priority bitstream gives a gain of approximately 11 dB over the lowest one at a BER of $10^{-6}$.

**Fig. 8.** Error performances for 8-branch diversity OSTBCs employing Hierarchical 8PSK modulation.

Fig. 11 compares the computational times of our derived BER expression given by (31) for two error protection schemes. By setting the phase angle vector $\theta = \begin{bmatrix} \pi/4 & \pi/8 \end{bmatrix}$ we obtain an Equal Error Protection (EEP) scheme and with this vector we varied the diversity order of the OSTBC and calculated the computation times of (31) for the high and low priority sub-channels. Similarly, we set $\theta = \begin{bmatrix} \pi/5 & \pi/15 \end{bmatrix}$ to obtain a UEP scheme and calculated the corresponding computation times which are then plotted against the diversity order of the OSTBCs. It can be seen from the curves that nearly similar computation times are obtained for the EEP and UEP schemes. Hence, we can deduce that the performance analysis of our UEP scheme does not involve higher computational complexity than the EEP counterpart.

**Fig. 11.** Comparison of computation times between EEP and UEP schemes.

**VII. CONCLUSION**

In this paper it has been shown that a possible solution to communicate ultra-reliable multiclass bitstreams over wireless channels exists in the form of an unequal error protection coded MIMO system that achieves high spatial diversity and coding gains without bandwidth expansion and computational complexity. New BER closed-form expressions have been established for the proposed UEP transmission system and these are verified through Monte Carlo simulations over the Rayleigh fading channel. We have observed that by simply varying the modulation vector a range of high performance gains are achievable. It has been shown that around 11 dB coding gain is obtained between the
highest priority bitstream and its lowest counterpart when the coded MIMO system employs an 8-branch diversity order OSTBC at a BER of $10^{-6}$. The proposed communication system model can find direct applications to the transmission of wireless JPEG and HEVC data as well as in digital video broadcasting systems.

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