Entropic Dynamics and the Quantum Measurement Problem*

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Abstract

We explore the measurement problem in the entropic dynamics approach to quantum theory. The dual modes of quantum evolution—either continuous unitary evolution or abrupt wave function collapse during measurement—are unified by virtue of both being special instances of entropic updating of probabilities. In entropic dynamics particles have definite but unknown positions; their values are not created by the act of measurement. Other types of observables are introduced as a convenient way to describe more complex position measurements; they are not attributes of the particles but of the probability distributions; their values are effectively created by the act of measurement. We discuss the Born statistical rule for position, which is trivially built into the formalism, and also for generic observables.

1 Introduction

Quantum mechanics introduced several new elements into physical theory. One is indeterminism, another is the superposition principle embodied in both the linearity of the Hilbert space and the linearity of the Schrödinger equation. Between them they dealt a very severe blow to the classical conception of reality. The founders faced the double challenge of locating the source of indeterminism and of explaining why straightforward consequences of the superposition principle are not observed in the macroscopic world. Despite enormous progress the challenge does not appear to have been met yet—at least as evidenced by the number of questions that stubbornly refuse to go away.

The quantum measurement problem embodies most of these questions\textsuperscript{1}. One is the problem of macroscopic entanglement; another is the problem of definite

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\textsuperscript{1}A clear formulation of the problem is [1]; see also [2]. Modern reviews with references to the literature appear in [3] and [4].
outcomes. Since it is not possible to consistently assign objective values to physical properties, when and how do values become actualized? How does a measurement yield a definite outcome or how do events ever get to happen? Are the values of observables created during the act of measurement?

An early “solution” due to von Neumann [2] was to postulate a dual mode of wave function evolution, either continuous and deterministic according to the Schrödinger equation, or discontinuous and stochastic during the measurement process. It is in the latter process—the wave function collapse or projection postulate [3][6]—where probabilities are introduced. Other proposed solutions involved denying that collapse ever occurs which led to the many worlds, the many minds, and the modal interpretations. These issues and others (such as the preferred basis problem) can nowadays be tackled within the decoherence program [6] but with one strong caveat. Decoherence works but only at the observational level—it saves the appearances. In this view quantum mechanics is merely empirically adequate; it does not aim to provide an objective picture of reality. Is this acceptable?

Our goal here is to revisit the problem of measurement from the fresh perspective of Entropic Dynamics (ED) which introduces some new elements of its own. [7][8] In the standard view, which remains popular to this day, quantum theory is considered an extension of classical mechanics and therefore deviations from causality demand an explanation. In the entropic view, on the other hand, quantum mechanics is an example of entropic inference, a framework designed to handle insufficient information. [9] From the entropic perspective indeterminism requires no explanation. Uncertainty and probabilities are the norm; it is certainty and determinism that demand explanations. The general attitude is pragmatic [10]: physical theories are mere models for inference; they do not attempt to mirror reality and, therefore, the best one can expect is that they be empirically adequate, that is, good “for all practical purposes”. And this is not just the best one can do, it is the best one ever needs to do. Therefore in the entropic framework the program of decoherence is completely unobjectionable.

Once one accepts quantum theory as a theory of inference the dichotomy between two distinct modes of wave function evolution is erased. Continuous unitary evolution and discontinuous collapse correspond to two modes of processing information, namely entropic updating in infinitesimal steps and Bayesian updating in discrete finite steps. Indeed, as shown in [11] these two updating rules are not intrinsically different; they are special cases within a broader scheme of entropic inference. [9]

The other element that is significant for our present purpose is the privileged role ascribed to the position observable. In ED, unlike the standard interpretation of quantum mechanics, the positions of particles have definite values just as they would in classical physics. Therefore the problem of definite outcomes does not arise; the process of observation is essentially classical. No inconsistencies arise because in ED position is the only observable. More explicitly: other observables such as momentum, energy, angular momentum and so on are not
attributes of the particles but of the probability distributions. This opens the opportunity of explaining all other “observables” in purely informational terms.

After a brief review of background material on ED (section 2) we discuss the measurement of observables other than position and derive the corresponding Born rule (section 3). The issue of amplification is addressed in section 4 and we summarize our conclusions in section 5. A more detailed treatment of the quantum measurement problem is given in [12].

2 Entropic Quantum Dynamics

To set the context for the rest of the paper we briefly review the three main ideas that form the foundation of entropic dynamics. Several important topics and most technical details are not discussed here. For a detailed account of, for example, how time is introduced into an essentially atemporal inference scheme, or the entropic nature of the phase of the wave function, or the introduction of constants such as $\hbar$ or $m$, see [8]. For simplicity here we discuss a single particle. The first idea is about the subject matter: the goal is to predict the position $x$ of a particle on the basis of some limited information. We assume that in addition to the particle the world contains other variables—we call them $y$. Not much needs to be known about the $y$ except that they are described by a probability distribution $p(y|x)$ that depends on the particle position. The entropy of the $y$ variables is given by

$$S[p, q] = - \int dy p(y|x) \log \frac{p(y|x)}{q(y)} = S(x).$$  \hspace{1cm} (1)$$

Neither the underlying measure $q(y)$ nor the distribution $p(y|x)$ need to be specified further. Note that $x$ enters as a parameter in $p(y|x)$ and therefore its entropy is a function of $x$: $S[p, q] = S(x)$.

The second idea concerns the method of inference: we use the method of maximum entropy subject to appropriate constraints to calculate the probability $P(x'|x)$ that the particle takes a short step from $x$ to a nearby point $x'$. The constraints reflect the relation between $x$ and $y$ given by $p(y|x)$, and the fact that motion happens gradually—a large step is the result of many infinitesimally short steps. Thus entropic dynamics does not assume any underlying sub-quantum mechanics whether it be classical or not. The successive accumulation of many such short steps results in a probability distribution $\rho(x,t)$ that satisfies the Fokker-Planck equation

$$\frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \vec{v})$$  \hspace{1cm} (2)$$

\hspace{1cm} \footnote{The case of momentum is discussed in [13].}

\hspace{1cm} \footnote{And this is why the $y$ variables are not hidden variables. The technical term ‘hidden variables’ refers to variables introduced to explain the emergent quantum behavior as a reflection of an essentially classical dynamics—whether stochastic or not—operating at a deeper level. The $y$ variables do not play this role because in ED there is no underlying classical dynamics.}
where the current velocity \( \vec{v} \) is
\[
\vec{v} = \frac{\hbar}{m} \vec{\nabla} \phi \quad \text{with} \quad \phi(x, t) = S(x, t) - \log \rho^{1/2}(x, t).
\] (3)

These equations show how the entropy \( S(x, t) \) guides the evolution of \( \rho(x, t) \).

The third idea is an energy constraint: the time evolution of \( S(x, t) \) is determined by imposing that a certain “energy” be conserved\(^4\). Thus, we require the diffusion to be non-dissipative. To this end introduce an energy functional,
\[
E[\rho, S] = \int d^3x \rho(x, t) \left[ \frac{\hbar^2}{2m} (\vec{\nabla} \phi)^2 + \frac{\hbar^2}{8m} (\vec{\nabla} \log \rho)^2 + V \right].
\] (4)

Note that this energy is a statistical concept; it is not assigned to the particle but to \( \rho \) and \( S \). Imposing that the energy be conserved for arbitrary initial choices of \( \rho \) and \( S \) leads to the quantum Hamilton-Jacobi equation,
\[
\hbar \dot{\phi} + \frac{\hbar^2}{2m} (\vec{\nabla} \phi)^2 + V - \frac{\hbar^2}{2m} \frac{\nabla^2 \rho^{1/2}}{\rho^{1/2}} = 0.
\] (5)

This equation shows how the distribution \( \rho(x, t) \) affects the evolution of the entropy \( S(x, t) \).

Finally, by combining the quantities \( \rho \) and \( S \) into a single complex function, \( \Psi = \rho^{1/2} e^{i\phi} \), the equations, (2) and (5), can be rewritten into the Schrödinger equation,
\[
i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi.
\] (6)

The fact that the Schrödinger equation turned out to be linear and unitary makes the language of Hilbert spaces and Dirac’s bra-ket notation particularly convenient—so from now we write \( \Psi(x) = \langle x|\Psi \rangle \).

To conclude this brief review we emphasize that the Fokker-Planck equation \( (2) \), the expression \( (3) \) for the current velocity as a gradient, and the relation between phase \( \phi \) and entropy \( S \) are derived and not postulated.

### 3 Measurement in ED

In practice the measurement of position can be technically challenging because it requires the amplification of microscopic details to a macroscopically observable scale. However, no intrinsically quantum effects need be involved: the position of a particle has a definite, albeit unknown, value \( x \) and its probability distribution is, by construction, given by the Born rule, \( \rho(x) = |\Psi(x)|^2 \). We can therefore assume that suitable position detectors are available; in ED the measurement of position can be considered as a primitive notion. This is not in any way different from the way information in the form of data is handled in any other\(^4\) There is a close parallel to statistical mechanics which also requires a clear specification of the subject matter (the microstates), the inference method (MaxEnt), and the constraints.
Bayesian inference problem. The goal there is to make an inference on the basis of given data; the issue of how the data was collected or itself inferred is not under discussion. If we want, of course, we can address the issue of where the data came from but this is a separate inference problem that requires an independent analysis. In the next section we offer some additional remarks of the amplification problem.

Our main concern here is observables other than position: how are they defined, how are they measured? See [12] and [14]. For simplicity, we will initially consider a measurement that leads to a discrete set of possible position outcomes. In this case, the continuous position probabilities become discrete,

$$\rho(x) \, dx = |\langle x | \Psi \rangle|^2 \, dx \rightarrow p_i = |\langle x_i | \Psi \rangle|^2 .$$  \hspace{1cm} (7)

Since position is the only objectively real quantity there is no reason to define other observables except that they may turn out to be convenient when considering more complex experiments in which before the particles reach the position detectors they are subjected to additional appropriately chosen interactions, say magnetic fields or diffraction gratings. Suppose the interactions within the complex measurement device \( A \) are described by the Schrödinger eq.(6), that is, by a particular unitary evolution \( \hat{U}_A \). The particle will be detected at position \( |x_i\rangle \) with certainty provided it was initially in state \( |a_i\rangle \) such that

$$\hat{U}_A |a_i\rangle = |x_i\rangle .$$  \hspace{1cm} (8)

Since the set \( \{|x_i\rangle\} \) is orthonormal and complete, the corresponding set \( \{|a_i\rangle\} \) is also orthonormal and complete,

$$\langle a_i | a_j \rangle = \delta_{ij} \quad \text{and} \quad \sum_i |a_i\rangle \langle a_i| = \hat{1} .$$  \hspace{1cm} (9)

Now consider the effect of this complex detector \( A \) on some arbitrary initial state vector \( |\Psi\rangle \) which can always be expanded as

$$|\Psi\rangle = \sum_i c_i |a_i\rangle ,$$  \hspace{1cm} (10)

where \( c_i = \langle a_i | \Psi \rangle \) are complex coefficients. The state \( |\Psi\rangle \) will evolve according to \( \hat{U}_A \) so that as it approaches the position detectors the new state is

$$\hat{U}_A |\Psi\rangle = \sum_i c_i \hat{U}_A |a_i\rangle = \sum_i c_i |x_i\rangle .$$  \hspace{1cm} (11)

which, invoking the Born rule for position measurements, implies that the probability of finding the particle at the position \( x_i \) is

$$p_i = |c_i|^2 .$$  \hspace{1cm} (12)

Thus, the probability that the particle in state \( \hat{U}_A |\Psi\rangle \) is found at position \( x_i \) is \( |c_i|^2 \). But we can describe the same outcome from the point of view of the more complex detector. The particle is detected in state \( |x_i\rangle \) as if it had earlier been in the state \( |a_i\rangle \). We adopt a new language and say, perhaps
inappropriately, that the particle has effectively been “detected” in the state \( |a_i\rangle \), and therefore, the probability that the particle in state \( |\Psi\rangle \) is “detected” in state \( |a_i\rangle \) is \( |c_i|^2 = |\langle a_i | \Psi \rangle|^2 \)—which reproduces Born’s rule for a generic measurement device. The shift in language is not particularly fundamental—it is merely a matter of convenience but we can pursue it further and assert that this complex detector “measures” all operators of the form \( \hat{A} = \sum \lambda_i |a_i\rangle \langle a_i| \) where the eigenvalues \( \lambda_i \) are arbitrary scalars. Born’s rule is a postulate in the standard interpretation of quantum mechanics; within ED we see that it is derived as the natural consequence of unitary time evolution.

Note that it is not necessary that the operator \( \hat{A} \) have real eigenvalues, but it is necessary that its eigenvectors \( |a_i\rangle \) be orthogonal. This means that the Hermitian and anti-Hermitian parts of \( \hat{A} \) will be simultaneously diagonalizable. Thus, while \( \hat{A} \) does not have to be Hermitian (\( \hat{A} = \hat{A}^\dagger \)) it must certainly be normal, that is \( \hat{A} \hat{A}^\dagger = \hat{A}^\dagger \hat{A} \).

Note also that if a sentence such as “a particle has momentum \( \vec{p} \)” is used only as a linguistic shortcut that conveys information about the wave function before the particle enters the complex detector then, strictly speaking, there is no such thing as the momentum of the particle: the momentum is not an attribute of the particle but rather it is a statistical attribute of the probability distribution \( \rho(x) \) and entropy \( S(x) \), a point that is more fully explored in \[13\].

The generalization to a continuous spectrum is straightforward. Let \( \hat{A}|a\rangle = a|a\rangle \). For simplicity we consider a discrete one-dimensional lattice \( a_i \) and \( x_i \) and take the limit as the lattice spacing \( \Delta a = a_{i+1} - a_i \to 0 \). The discrete completeness relation, eq. (9),

\[
\sum_i \Delta a \frac{|a_i\rangle}{(\Delta a)^{1/2}} \frac{\langle a_i|}{(\Delta a)^{1/2}} = \hat{I} \quad \text{becomes} \quad \int da \ |a\rangle \langle a| = \hat{I} ,
\]

where we defined

\[
\frac{|a_i\rangle}{(\Delta a)^{1/2}} \to |a\rangle . \tag{14}
\]

We again consider a measurement device that evolves eigenstates \( |a\rangle \) of \( \hat{A} \) into unique position eigenstates \( |x\rangle \), \( \hat{U}_A(a) = |x\rangle \). The mapping from \( x \) to \( a \) can be represented by an appropriately smooth function \( a = y(x) \). In the limit \( \Delta x \to 0 \), the orthogonality of position states is expressed by a Dirac delta distribution,

\[
\frac{\langle x_i|}{\Delta x^{1/2}} \frac{|x_i\rangle}{\Delta x^{1/2}} = \frac{\delta_{ij}}{\Delta x} \quad \to \quad \langle x|x'\rangle = \delta(x - x') . \tag{15}
\]

An arbitrary wave function can be expanded as

\[
|\Psi\rangle = \sum_i \Delta a \frac{|a_i\rangle}{\Delta a^{1/2}} \frac{\langle a_i|}{\Delta a^{1/2}} \quad \text{or} \quad |\Psi\rangle = \int da \ |a\rangle \langle a|\Psi\rangle . \tag{16}
\]
The unitary evolution $\hat{U}_A$ of the wave function leads to

$$\hat{U}_A |\Psi\rangle = \sum_i \Delta a \frac{|x_i\rangle \langle a_i|\Psi\rangle}{\Delta a^{1/2}} = \sum_i \Delta x \frac{|x_i\rangle \langle a_i|\Psi\rangle}{\Delta x^{1/2}} \left( \frac{\Delta a}{\Delta x} \right)^{1/2} \Delta a^{1/2} \langle a_i | \Psi \rangle \Delta a^{1/2}$$

$$\rightarrow \int dx \ |x\rangle \langle a|\Psi\rangle \frac{da}{dx} \frac{dx}{|x\rangle \langle a|\Psi\rangle} \triangleq \frac{dx}{|x\rangle \langle a|\Psi\rangle} = \rho_A(a) da \quad (17)$$

so that

$$p_i = |\langle x_i | \hat{U}_A |\Psi\rangle|^2 = |\langle a_i |\Psi\rangle|^2 \quad \rightarrow \quad \rho(x)dx = |\langle a|\Psi\rangle|^2 \frac{da}{dx} \frac{dx}{|a\rangle \langle a|\Psi\rangle} = \rho_A(a) da . \quad (18)$$

Thus, “the probability that the particle in state $\hat{U}_A |\Psi\rangle$ is found within the range $dx$ is $\rho(x)dx$” can be rephrased as “the probability that the particle in state $|\Psi\rangle$ is found within the range $da$ is $\rho_A(a)da$” where

$$\rho_A(a) da = |\langle a|\Psi\rangle|^2 da , \quad (19)$$

which is the continuum version of the Born rule for an arbitrary observable $\hat{A}$.

4 Amplification

The technical problem of amplifying microscopic details so they can become macroscopically observable is usually handled with a detection device set up in an initial unstable equilibrium. The particle of interest activates the amplifying system by inducing a cascade reaction that leaves the amplifier in a definite macroscopic final state described by some pointer variable $\alpha$.

An eigenstate $|a_i\rangle$ evolves to a position $x_i$ and the goal of the amplification process is to infer the value $x_i$ from the observed value $\alpha_r$ of the pointer variable. The design of the device is deemed successful when $x_i$ and $\alpha_r$ are suitably correlated and this information is conveyed through a likelihood function $P(\alpha_r|x_i)$—an ideal amplification device would be described by $P(\alpha_r|x_i) = \delta_{ri}$. Inferences about $x_i$ follow from a standard application of Bayes rule,

$$P(x_i|\alpha_r) = P(x_i) \frac{P(\alpha_r|x_i)}{P(\alpha_r)} . \quad (20)$$

The point of these considerations is to emphasize that there is nothing intrinsically quantum mechanical about the amplification process. The issue is one of appropriate selection of the information (in this case $\alpha_r$) that happens to be relevant to a certain inference (in this case $x_i$). This is, of course, a matter of design: a skilled experimentalist will design the device so that no spurious correlations—whether quantum or otherwise—nor any other kind of interfering noise will stand in the way of inferring $x_i$.

It may seem that we are simply redrawing von Neumann’s line between the classical and the quantum with our treatment of the amplifying system. In some sense, we are doing just that. However, the line here is not between a classical
“reality” and a quantum “reality”—it is between the microscopic particle with a definite but unknown position and an amplifying system skillfully designed so its own microscopic degrees of freedom turn out to be of no interest. In fact, in [12] we showed that such an amplifier can be treated as a fully quantum system but it makes no difference to the inference.

5 Conclusions

The solution of the problem of measurement within the entropic dynamics framework hinges on two points: first, entropic quantum dynamics is a theory of inference not a law of nature. This erases the dichotomy of dual modes of evolution—continuous unitary evolution versus discrete wave function collapse. The two modes of evolution turn out to correspond to two modes of updating—continuous entropic and discrete Bayesian—which, within the entropic inference framework, are unified into a single updating rule.

The second point is the privileged role of position—particles have definite positions and therefore their values are not created but merely ascertained during the act of measurement. All other “observables” are introduced as a matter of linguistic convenience to describe more complex experiments. These observables turn out to be attributes of the probability distributions and not of the particles; their values are indeed “created” during the act of measurement.

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