Screening effect of nucleon electric dipole moments in atomic system

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We examine the screening effect of nucleon electric dipole moments (EDMs) and the nuclear EDM in an atom. The interactions of the nucleon EDMs with the electrons and protons induce the atomic EDM that survives the screening thanks to the nuclear finite-size effect. Since the leading-order contribution vanishes for spin $\frac{1}{2}$ nuclei, we evaluate the next-to-leading order contribution in $^{199}$Hg. The numerical result implies a lower sensitivity of the atomic EDM on the neutron EDM in comparison with a previous calculation based on a simple expansion of the Schiff moment operator.

I. SCHIFF MOMENT

The observation of a permanent electric dipole moment (EDM) of an atom indicates the existence of parity ($P$) and time-reversal ($T$) violating interactions between constituent particles. The interaction of the atomic EDM with an external electric field causes an energy shift, which has been sought in experimental research \[1, 2\]. Although EDMs of electrons, nucleons, and the nucleus are independently coupled to the electric field, those contributions to the energy shift are partially canceled out by higher-order effects. For example, the EDM of a point-like nucleus is completely screened by the atomic EDM induced by the interaction of the nuclear EDM with surrounding electrons.

$P, T$-odd nucleon-nucleon ($NN$) interactions allow a finite-size nucleus to have a nuclear Schiff moment as well as the nuclear EDM. Since the atomic EDM generated by the Schiff moment survives the screening effect, atomic EDMs particularly of diamagnetic atoms are sensitive to $P, T$-odd $NN$ interactions \[3–6\]. In this section we review the screening mechanism of the nuclear EDM induced by the $P, T$-odd meson-exchange $NN$ interaction.

The Hamiltonian of an atomic system that conserves $P$ and $T$ symmetries is written as

$$H_{\text{atom}} = H_{\text{nucl}} + H_e,$$

$$H_e = T_e + V^{(eN)} + V^{(eN)}_{\text{even-odd}},$$

where $H_{\text{nucl}}$ denotes $P, T$-even $NN$ interactions. The electron kinetic term $T_e$ and interactions between the electrons $V^{(eN)}$ are not relevant to the nuclear $P, T$ violation of interest. The electric interaction between the nucleus and the electrons is

$$V^{(eN)} = -e^2 \sum_{i=1}^{Z} \sum_{a=1}^{Z} \frac{1}{r_i' - r_a},$$

where $r_i'$ and $r_a$ are the coordinates of the electrons and protons, respectively. If $r_i' > r_a$, each term can be expanded as

$$\frac{1}{|r_i' - r_a|} = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \left( \frac{r_a \cdot \nabla}{r_i'} \right)^l \frac{1}{r_i'^l}.$$  

The atomic Hamiltonian excludes the odd-$l$ electron-nucleon ($eN$) interactions denoted by $V^{(eN)}_{\text{odd-odd}}$, which appear only if $P$ and $T$ symmetries are both violated in the nucleus. We treat $V^{(eN)}_{\text{odd-odd}}$ and $P, T$-odd interactions as perturbative interactions.

The nuclear ground state in the existence of the $P, T$-odd meson-exchange $NN$ interaction $\bar{V}_{eN,NN}$ can be approximated as

$$\left| \psi_{\text{g.s.}}^{(N)} \right> = \left| \psi_{\text{g.s.}}^{(N)} \right> + \sum_{n} \frac{1}{E_{g.s.}^{(N)} - E_{n}^{(N)}} \left| \psi_{n}^{(N)} \right> \langle \psi_{n}^{(N)} | \bar{V}_{eN,NN} | \psi_{\text{g.s.}}^{(N)} \rangle,$$

where $E_{g.s.}^{(N)}$ and $E_{n}^{(N)}$ denote the energies in the ground state $\left| \psi_{\text{g.s.}}^{(N)} \right>$ and excited states $\left| \psi_{n}^{(N)} \right>$ of the nuclear Hamiltonian $H_{\text{nucl}}$, respectively. $P, T$-odd $NN$ interactions induce the nuclear EDM defined by

$$d_{\text{nucl}} = \sum_{a=1}^{Z} e r_a,$$

where $e$ is the elementary charge. The interaction of the nuclear EDM with an external electric field represented in Fig. 1(a) causes the energy shift given by

$$\Delta E_{a}(\bar{V}_{eN,NN}, q_{N}) = \langle \psi_{\text{g.s.}}^{(N)} | -d_{\text{nucl}} \cdot E_{\text{ext}} | \psi_{\text{g.s.}}^{(N)} \rangle.$$  

This second-order effect of the $P, T$-odd meson-exchange $NN$ interaction is completely screened if the nucleus is a point-like particle \[3, 4\].

$P, T$-odd $NN$ interactions also induce the odd-$l$ $eN$ interactions, which violate $P$ and $T$ symmetries both in the nucleus and in the electron system. Thus, the $P, T$-odd meson-exchange $NN$ interaction induces the atomic EDM defined by

$$d_{\text{atom}} = - \sum_{i=1}^{Z} e r_i',$$

where $r_i'$ is the coordinate of the ith electron. The expression for the atomic EDM can be written as

$$d_{\text{atom}} = \sum_{a=1}^{Z} e r_a - \sum_{i=1}^{Z} e r_i',$$

where $r_a$ is the coordinate of the $a$th proton. The expression for the atomic EDM can be written as

$$d_{\text{atom}} = \sum_{a=1}^{Z} e r_a - \sum_{i=1}^{Z} e r_i'.$$

The numerical result implies a lower sensitivity of the atomic EDM on the neutron EDM in comparison with a previous calculation based on a simple expansion of the Schiff moment operator.

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which has a non-zero value only if $P$ and $T$ symmetries are both violated in the electron system.

The atomic EDM contributes to the energy shift in the third order of perturbation represented in Fig. 1(b) as

$$
\Delta E_3(\gamma_{NN}, qN, q_c) = \sum_m \frac{1}{E_m - E_m'} \left[ \langle \bar{\psi}_{g.s.}(A) | \right] 
$$

$$
\times \left[ \bar{\psi}_{g.s.}(A) \right] - d_{\text{atom}} \cdot E_{\text{ext}} \left| \bar{\psi}_{m}(c) \right\rangle \left| V_{\text{odd-l}}(c) \right| \langle \bar{\psi}_{g.s.}(A) \rangle 
$$

+ c.c.

The eigenstates of the atomic system are expressed except for the Clebsch-Gordan coefficients as

$$
\left| \bar{\psi}_{g.s.}(A) \right\rangle = \left| \nucl \right\rangle \otimes \left| e_{\text{NSM}}(c) \right\rangle,
$$

(10)

$$
\left| \bar{\psi}_{m}(A) \right\rangle = \left| \nucl \right\rangle \otimes \left| e_{\text{NSM}}(c) \right\rangle,
$$

(11)

where $\left| \psi_{g.s.}(c) \right\rangle$ and $\left| \psi_{m}(c) \right\rangle$ denote the ground state and excited states of the electron system described by $H_e$ with the energies $E_{g.s.}^{(c)}$ and $E_{m}^{(c)}$, respectively.

The screening effect of the nuclear EDM is demonstrated by employing a Hermitian operator

$$
U_{\text{nucl}} = \frac{1}{Ze} d_{\text{nucl}} \sum_i Z_i \nabla_i^2,
$$

(12)

where

$$
\langle d_{\text{nucl}} \rangle = \left\langle \bar{\psi}_{g.s.}(A) \right| d_{\text{nucl}} \left| \bar{\psi}_{g.s.}(A) \right\rangle.
$$

(13)

The external interaction of the nuclear EDM is transformed as

$$
\left\langle \bar{\psi}_{g.s.}(A) \right| - d_{\text{nucl}} \cdot E_{\text{ext}} \left| \bar{\psi}_{g.s.}(A) \right\rangle 
$$

$$
= i\left\langle \bar{\psi}_{g.s.}(A) \right| U_{\text{nucl}} - d_{\text{atom}} \cdot E_{\text{ext}} \left| \bar{\psi}_{g.s.}(A) \right\rangle,
$$

(14)

and the interaction of the nuclear EDM with the electrons corresponding to the $l = 1$ component in Eq. (13) is also transformed as

$$
\left\langle \bar{\psi}_{m}(A) \right| V_{l=1} \left| \bar{\psi}_{g.s.}(A) \right\rangle
$$

$$
= i\left\langle \bar{\psi}_{m}(A) \right| U_{\text{nucl}} - d_{\text{atom}} \cdot E_{\text{ext}} \left| \bar{\psi}_{g.s.}(A) \right\rangle
$$

$$
- i\left\langle \bar{\psi}_{m}(A) \right| U_{\text{nucl}} - d_{\text{atom}} \cdot E_{\text{ext}} \left| \bar{\psi}_{g.s.}(A) \right\rangle.
$$

(15)

Although the same transformations are realized even if one adopts

$$
U_{\nucl}^i = \frac{1}{Ze} d_{\text{nucl}} \sum_i Z_i \nabla_i^2
$$

(16)

instead of $U_{\text{nucl}}$, the resulting nuclear moment is a more complicated two-body operator than the Schiff moment.

Using these transformations, one obtains

$$
\Delta E_3(\gamma_{NN}, qN, q_c) = -\Delta E_2(\gamma_{NN}, qN)
$$

$$
+ \sum_m \frac{1}{E_m - E_m'} \left[ \left\langle \bar{\psi}_{g.s.}(A) \right| - d_{\text{atom}} \cdot E_{\text{ext}} \left| \bar{\psi}_{m}(c) \right\rangle \left\langle \bar{\psi}_{m}(c) \right| V_{\text{NSM}} \left| \psi_{m}(c) \right\rangle 
$$

(17)

$$
+ c.c. \right].
$$

The first term cancels out the second-order effect. Considering $l \leq 3$, the remaining $e^3$ interaction violating $P$ and $T$ symmetries in the electron system is

$$
\left\langle \bar{\psi}_{m}(c) \right| V_{\text{NSM}} \left| \psi_{m}(c) \right\rangle
$$

$$
= \left\langle \bar{\psi}_{m}(c) \right| V_{l=3} \left| \psi_{m}(c) \right\rangle
$$

$$
- i\left\langle \bar{\psi}_{m}(c) \right| U_{\text{nucl}} \left| V_{l=2} \right| \left| \psi_{m}(c) \right\rangle
$$

$$
= \left\langle \bar{\psi}_{m}(c) \right| - 4\pi e \sum_i Z_i \nabla_i^2 \delta(r_i) \left| \psi_{m}(c) \right\rangle.
$$

(17)

Here, nuclear rank 3 operators are omitted because those interactions do not arise in nuclei with a spin of $\frac{1}{2}$ such as $^{129}$Xe, $^{199}$Hg, and $^{225}$Ra. The rank 1 tensor $S_{\text{short}}$ is the expectation value of the nuclear Schiff moment, which is calculated by

$$
S_{\text{short}} = \sum_n \frac{1}{E_{g.s.} - E_{g.s.}'(N)} \left\langle \bar{\psi}_{g.s.}(N) \right| \frac{1}{E_{g.s.} - E_{g.s.}'(N)} \times \left| \psi_{g.s.}(N) \right\rangle \left| \psi_{g.s.}(N) \right\rangle.
$$

(19)

The Schiff moments of actinide nuclei would be enhanced thanks to octupole correlations and parity doubling of the ground states. It is expected from recent nuclear many-body calculations that the Schiff moment of $^{225}$Ra is greater than that of $^{199}$Hg by orders of magnitude, although the uncertainty is still large.
The explicit form of the Schiff moment operator is

\[ S_k = \frac{1}{10} e \sum_{a=1}^{Z} \left[ r_a^2 a_{k, a} + \frac{5}{3} r_{a, k} \langle r^2 \rangle_{\text{ch}} - \frac{4}{3} r_{a, j} \langle Q_{jk} \rangle_{\text{ch}} \right], \]

(20)

where the charge mean values are defined by

\[ \langle r^2 \rangle_{\text{ch}} = \frac{1}{Z} \sum_{n=1}^{Z} \langle \psi^{(N)}_{\text{g.s.}} | r_a^2 | \psi^{(N)}_{\text{g.s.}} \rangle \]

(21)

\[ \langle Q_{jk} \rangle_{\text{ch}} = \frac{1}{Z} \sum_{n=1}^{Z} \langle \psi^{(N)}_{\text{g.s.}} | Q_{a, jk} | \psi^{(N)}_{\text{g.s.}} \rangle, \]

(22)

and

\[ Q_a^{(2)} = \sqrt{\frac{3}{2}} [r_a \otimes r_a]^{(2)} \]

(23)

is the quadrupole moment of proton. Since the \( P, T \)-odd meson-exchange \( NN \) interaction is scalar, only the \( z \)-component \( S_z \) can have non-zero values. The third term of the Schiff moment operator (20) must vanish in spin \( \frac{1}{2} \) nuclei.

In conclusion of this section, the \( P, T \)-odd meson-exchange \( NN \) interaction induces the atomic EDM as well as the nuclear EDM. The energy shifts due to the interactions of the nuclear EDM and atomic EDM with an external electric field are partially canceled out each other. The remaining contribution is given by

\[ \Delta E_2(\tilde{T}, N, q_N) = \sum_{n=1}^{A} \langle \psi^{(A)}_{\text{g.s.}} | -d_{\text{atom}} \cdot E_{\text{ext}} | \psi^{(A)}_{\text{g.s.}} \rangle \]

(24)

where the interaction of the Schiff moment with the electrons denoted by \( V_{\text{NSM}} \) induces the atomic EDM that survives the screening. The third-order process is illustrated in Fig. 2.

II. NUCLEON EDM

There are several attempts to identify the leading-order contribution of the nucleon EDM to the atomic EDM. In particular, the Schiff moment of \(^{199}\text{Hg}\) related to the nucleon EDM was computed in the random phase approximation (RPA). Using their result, an upper bound on the neutron EDM was evaluated from the experimental limit on the atomic EDM as \( d_n < 1.6 \times 10^{-26} \text{e} \cdot \text{cm} \). This constraint is competitive with the result of a recent direct measurement \( d_n < 1.8 \times 10^{-26} \text{e} \cdot \text{cm} \).

However, the RPA calculation is based on a simple expansion of the Schiff moment operator. That approach fails to explain the relation between the Schiff moment and nucleon EDM interactions. In this section we examine the screening effect of nucleon EDMs in an atom. The nuclear EDM interactions with the electrons induce the atomic EDM that causes the screening.

Figure 3(a) shows the nucleon EDM interaction with an external electric field, which contribute to the energy shift as

\[ \Delta E_1(d_N) = \sum_{a=1}^{A} \langle \psi^{(A)}_{\text{g.s.}} | -d_a \cdot E_{\text{ext}} | \psi^{(A)}_{\text{g.s.}} \rangle, \]

(25)

where \( d_a \) denotes the nucleon EDMs. For a point-like nucleus, this first-order contribution is completely screened even if the nucleons are relativistic.

The second-order effect represented in Fig. 3(b) is given by

\[ \Delta E_2(d_N, q_e) = \sum_{m} \frac{1}{E^{(e)}_{\text{g.s.}} - E^{(e)}_{\text{m}}} \times \langle \psi^{(A)}_{\text{g.s.}} | -d_{\text{atom}} \cdot E_{\text{ext}} | \psi^{(A)}_{\text{m}} \rangle | V_{\text{NSM}} | \psi^{(A)}_{\text{g.s.}} \rangle + c.c., \]

(26)

where the atomic EDM is induced by the nucleon EDM interaction with the electrons:

\[ \tilde{V}(e \vec{r}) = -e \sum_{i=1}^{Z} \sum_{a=1}^{A} \frac{d_a \cdot (r_i - r_a)}{|r_i - r_a|^3}. \]

(27)

The multipole expansion of each term is given by

\[ \frac{d_a \cdot (r_i - r_a)}{|r_i - r_a|^3} = \sum_{l=0}^{\infty} (-1)^l \frac{l!}{r_i^{2l+3}} \langle r_a \cdot \nabla_i \rangle L^{(l)} d_a \cdot r_i^l. \]

(28)
The ground state and excited states of the atomic Hamiltonian $H_{\text{atom}}$ without $P$, $T$-odd interactions are expressed as
\begin{align}
|\psi^{(A)}_m\rangle &= |\psi^{(N)}_{\text{g.s.}}\rangle \otimes |\psi^{(e)}_{m \text{g.s.}}\rangle, \quad (29) \\
|\psi^{(A)}_m\rangle &= |\psi^{(N)}_{\text{g.s.}}\rangle \otimes |\psi^{(e)}_{m \text{g.s.}}\rangle, \quad (30)
\end{align}
respectively.

We introduce a Hermitian operator
\begin{equation}
U_N = i \frac{1}{Z e} \sum_{i=1}^{Z} \sum_{a=1}^{A} \langle d_a \cdot \nabla_i \rangle',
\end{equation}
where in contrast to $\langle d_{\text{nuc}} \rangle$ in Eq. (12),
\begin{equation}
\langle d_{a} \rangle = \langle \psi^{(N)}_{\text{g.s.}} \rangle |d_{a}\rangle |\psi^{(N)}_{\text{g.s.}}\rangle
\end{equation}
is the expectation value in the ground state of $H_{\text{nuc}}$. The nucleon EDM interactions are transformed as
\begin{equation}
\sum_{a=1}^{A} \langle \psi^{(A)}_{\text{g.s.}} \rangle | -d_{a} \cdot E_{\text{ext}} \rangle |\psi^{(A)}_{\text{g.s.}}\rangle = i \langle \psi^{(A)}_{\text{g.s.}} \rangle \left[ U_{N}, -d_{\text{atom}} \cdot E_{\text{ext}} \right] |\psi^{(A)}_{\text{g.s.}}\rangle,
\end{equation}
and
\begin{align}
\langle \psi^{(A)}_{m} | V_{\text{LO}}^{(N)} | \psi^{(A)}_{\text{g.s.}}\rangle &= i \langle \psi^{(A)}_{m} | \left[ U_{N}, V_{\text{LO}}^{(eN)} \right] | \psi^{(A)}_{\text{g.s.}}\rangle \\
&= i \langle \psi^{(A)}_{m} | \left[ U_{N}, H_{e} \right] | \psi^{(A)}_{\text{g.s.}}\rangle \\
&= i \langle \psi^{(A)}_{m} | \left[ U_{N}, V_{\text{LO}}^{(eN)} \right] | \psi^{(A)}_{\text{g.s.}}\rangle \\
&- i \langle \psi^{(A)}_{m} | \left[ U_{N}, V_{\text{LO}}^{(eN)} \right] | \psi^{(A)}_{\text{g.s.}}\rangle \\
&- i \langle \psi^{(A)}_{m} | \left[ U_{N}, V_{\text{LO}}^{(eN)} \right] | \psi^{(A)}_{\text{g.s.}}\rangle \
\end{align}
If the nucleus is a point-like particle, where the $eN$ interactions with $l \geq 2$ are absent, one obtains
\begin{equation}
\Delta E_{2}(d_{N}, q_{e}) = -\Delta E_{1}(d_{N}),
\end{equation}
which indicates the complete screening of the nucleon EDMs.

For a finite-size nucleus, the $eN$ interaction
\begin{equation}
\tilde{V}^{(eN)}_{\text{LO}} = \tilde{V}^{(eN)}_{l=2} - i \left[ U_{N}, V_{l=2}^{(eN)} \right]
\end{equation}
can induce the atomic EDM that survives the screening. The remaining contribution is given by
\begin{equation}
\Delta E_{1}(d_{N}) + \Delta E_{2}(d_{N}, q_{e}) = \sum_{m} \frac{1}{E_{m} - E_{m}^{(e)}} \times \langle \psi^{(A)}_{\text{g.s.}} | -d_{\text{atom}} \cdot E_{\text{ext}} | \psi^{(A)}_{\text{g.s.}}\rangle + \text{c.c.}
\end{equation}
The leading-order $eN$ interaction defined by Eq. (36) is explicitly written as
\begin{equation}
\tilde{V}^{(eN)}_{\text{LO}} = -\sqrt{15} e \sum_{i=1}^{A} \sum_{a=1}^{A} \left[ Q_{n}^{(2)} \otimes d_{a}^{(3)} \right] \cdot Q_{a}^{(3)}
\end{equation}
and
\begin{equation}
Q_{a}^{(3)} = \sqrt{\frac{5}{2}} \left[ r_{i}^{a} \otimes r_{i}^{a} \right]^{(2)} \cdot r_{i}^{a}
\end{equation}
is the octupole moment of electron. Since this interaction vanishes for the $^{199}$Hg nucleus with a spin of $\frac{1}{2}$, we should consider up to the third-order effect.

### III. THIRD-ORDER EFFECT OF NUCLEON EDM

The nuclear EDM is also induced by the nucleon EDM interaction with the protons
\begin{equation}
\tilde{V}^{(N\bar{N})} = e d_{p} \sum_{a \neq b}^{Z} \sum_{b=1}^{Z} \sum_{a=1}^{A} | \sigma_{a} \cdot (r_{b} - r_{a}) |^{3},
\end{equation}
The energy shift due to the external interaction of the nuclear EDM, which is represented in Fig. 4(a), is given by
\begin{equation}
\Delta E_{2}(d_{N}, q_{N}) = \langle \tilde{V}^{(N\bar{N})}_{\text{g.s.}} \rangle - d_{\text{nuc}} \cdot E_{\text{ext}} \langle \tilde{V}^{(N\bar{N})}_{\text{g.s.}} \rangle,
\end{equation}
where
\begin{equation}
| \tilde{V}^{(N\bar{N})}_{\text{g.s.}} \rangle = | \psi^{(N\bar{N})}_{\text{g.s.}} \rangle
\end{equation}
It was claimed in Ref. [17] that this contribution is completely screened by the third-order effect of the nucleon EDM interaction, which is represented in Fig. 4(b). However, they considered the $eN$ interactions only up to $l = 1$ in the multipole expansions. In order to closely examine the screening effect of the EDM of a finite-size nucleus, we consider the $eN$ interactions with $l \leq 3$. The third-order effect is given by

$$
\Delta E_3(d_N, q_N, q_e) = \sum_m \frac{1}{E^{(e)}_{g.s.} - E^{(e)}_m}
\times \langle \psi^{(A)}_{g.s.} | - \boldsymbol{d}_{\text{atom}} \cdot \boldsymbol{E}_{\text{ext}} | \psi^{(A)}_m \rangle \langle \psi^{(N)}_m \rangle \langle \tilde{\psi}^{(e)}_{g.s.} | \tilde{\psi}^{(e)}_m \rangle + c.c. ,
$$

where $|\tilde{\psi}^{(A)}_{g.s.}\rangle = |\tilde{\psi}^{(N)}_{g.s.}\rangle \otimes |\tilde{\psi}^{(e)}_{g.s.}\rangle$.

\[ \text{FIG. 4. (a) The second-order and (b) third-order effects of the nucleon EDM interaction with the protons.} \]

The screening mechanism of $\Delta E_2(d_N, q_N)$ is similar to the case of $\Delta E_2(\gamma_{NN}, q_N)$, which is caused by the $P, T$-odd meson-exchange $NN$ interaction. As discussed in Sec. I, the screening effect of the nuclear EDM is incomplete because the atomic EDM induced by the Schiff moment survives the screening. The same discussion can be applied to the nuclear EDM induced by the nucleon EDM interaction. Thus, the total Schiff moment is given by

$$
S = S_{\text{short}} + S_{\text{long}}
$$

where

$$
S_{\text{long}} = \sum_n \frac{1}{E^{(N)}_{g.s.} - E^{(N)}_n}
\times \langle \psi^{(N)}_{g.s.} | S | \psi^{(N)}_n \rangle \langle \psi^{(N)}_n \rangle \langle \tilde{\psi}^{(N)}_{g.s.} | \tilde{\psi}^{(N)}_n \rangle + c.c. ,
$$

The Schiff moment related to the neutron EDM was calculated as $S_2 = 1.9d_n[\text{fm}^2]$ in a previous study [13]. Our result implies that the $^{199}$Hg atomic EDM is less sensitive to the neutron EDM. It should be noted that $S_{\text{long}}$ is a different quantity from $S_2$, which was derived from a simple expansion of the Schiff moment operator.

**IV. SUMMARY**

We have discussed the screening effect of electric dipole moments (EDMs) of nucleons and the atomic nucleus in an atom. In order to derive the leading-order contributions that survive the screening, we have incorporated the nucleon EDM interactions with an external electric field, the electrons, and the protons in the theory as perturbative interactions. This approach leads to a natural extension of the nuclear Schiff moment to include the effect of the nucleon EDM. The Schiff moment of $^{199}$Hg due to the neutron EDM has been evaluated in the independent particle model. Our result shows a lower sensitivity of the $^{199}$Hg atomic EDM on the neutron EDM than the previous RPA calculation [13]. On the other hand, it implies that the atomic EDM is relatively more sensitive to the $P, T$-odd meson-exchange $NN$ interaction.

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