Interaction of a Bose-Einstein condensate with a gravitational wave

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Partly motivated by recent proposals for the detection of gravitational waves, we study their interaction with Bose-Einstein condensates. For homogeneous condensates at rest, the gravitational wave does not directly create phonons (to lowest order), but merely affects the propagation of existing phonons or indirectly creates phonon pairs via quantum squeezing – an effect which has already been considered in the literature. For inhomogeneous condensate flows such as a vortex lattice, however, the impact of the gravitational wave can directly create phonons. This more direct interaction can be more efficient and could perhaps help bringing such a detection mechanism for gravitational waves a step closer towards experimental realizability – even though it is still a long way to go. Finally, we argue that super-fluid Helium might offer some advantages in this respect.

I. INTRODUCTION

A century after their prediction \(^{1,2}\), gravitational waves have been detected at LIGO \(^{3,4}\), which was one of the major breakthroughs in modern physics. To fully exploit this new window into our Universe, there have been many proposals for alternative gravitational wave detectors, some on larger scale (such as LISA \(^5\)) and others on smaller scale.

Pushing this idea to the extreme limit of very small systems, there has been a proposal \(^6\) based on the creation of phonons in Bose-Einstein condensates by the gravitational wave. Note that this scheme is somewhat different from an interferometric set-up as used in LIGO, which measures the deformation during the gravitational wave (e.g., with light or matter waves). Instead, the scheme proposed in \(^6\) envisions detecting the created phonons after the gravitational wave passed through – which is more similar to resonant mass antennas such as Weber bars.

On the one hand, the smallness of Bose-Einstein condensates made of atomic vapor seems to suggest that their interaction with gravitational waves is extremely tiny – but, on the other hand, one might hope that the specific properties of Bose-Einstein condensates such as their coherence could help detecting this tiny interaction. In the following, we try to adopt an unbiased point of view and address the general question of how Bose-Einstein condensates interact with gravitational waves. For homogeneous condensates at rest, the gravitational wave interacts with Bose-Einstein condensates interact with gravitational waves and whether this interaction could, at least in principle, be employed to detect them.

To this end, we start with a fully relativistic effective description of Bose-Einstein condensates in flat space-time, see also \(^5\), and derive the non-relativistic limit in Sec. \(\text{III}\). Then, after briefly reviewing the well-known derivation of the phonon modes in flat space-time (see Sec. \(\text{III}\)), we consider the impact of the gravitational wave in Sec. \(\text{IV}\). In Section \(\text{V}\), an analogy to an effective electric field based on an alternative scaling ansatz is introduced. Finally, we discuss the detectability of this effect in Sec. \(\text{VI}\).

II. BOSE-EINSTEIN CONDENSATE

We consider bosonic atoms with total spin zero and neglect their internal structure, treating them effectively as point particles. Since the total atom number is of course conserved, we describe them by a complex Klein-Fock-Gordon field \(\phi\) carrying a conserved current. In order to model the local interaction of the atoms (in the s-wave scattering approximation), we consider a \(\lambda \phi^4\) self-interaction. However, as we shall see below, other suitable interactions terms yield the same results (in the non-relativistic limit considered here). Assuming that a large number of atoms is condensed into the same single-particle quantum state, we treat \(\phi\) as a classical complex scalar field. Altogether, we start with the Klein-Fock-Gordon equation with self-interaction, see also \(^7\)

\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2 c^2}{\hbar^2} + \lambda |\phi|^2 \right) \phi = 0, \tag{1}
\]

where \(m\) denotes the mass of the atoms and \(\lambda\) the their coupling strength. Since the atoms are supposed to be ultra-cold, we may consider the non-relativistic limit and thus separate the fast temporal oscillations stemming from their rest energy \(mc^2\) from the remaining slow time-dependence via the ansatz

\[
\phi(t, r) = \psi(t, r) \exp \left\{ -i \frac{mc^2}{\hbar} t \right\}. \tag{2}
\]

Insertion of this ansatz into (1) yields

\[
i\hbar \ddot{\psi} + \frac{\hbar^2}{2m} \nabla^2 \psi - \frac{\lambda \hbar^2}{2m} |\psi|^2 \psi = \frac{\hbar^2}{2mc^2} \ddot{\psi}. \tag{3}
\]

If we neglect the tiny relativistic correction on the right-hand side, we recover the Gross-Pitaevskii equation

\[
i\hbar \ddot{\psi} = \left( -\frac{\hbar^2}{2m} \nabla^2 + g|\psi|^2 \right) \psi, \tag{4}
\]

after identifying the coupling strength \(g = \lambda \hbar^2/(2m)\).

Instead of the \(\lambda \phi^4\)-interaction considered above, one might start with a more general ansatz for the interaction term \(j^\mu(x) W_{\mu\nu}(x-x') j^\nu(x')\) where \(j_\mu \propto \text{Im}(\phi^* \partial_\mu \phi)\)
is the Noether current and $W_{\mu\nu}(x - x')$ some interaction kernel. Assuming that the range of this interaction is much shorter than the relevant length scales of the condensate, we may approximate it by a local term $W_{\mu\nu}(x - x') \approx W_{\mu\nu}^0 \delta^4(x - x')$, which is analogous to the s-wave scattering approximation. Then, after inserting the ansatz (2), we see that the rest mass density

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

and the Hamilton-Jacobi (eikonal) equation

$$\dot{S} + g \rho + \frac{(\nabla S)^2}{2m} = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}},$$

with the so-called quantum pressure term on the right-hand side. Neglecting this term, we obtain the Bernoulli equation for the condensate.

In order to study phonons, we linearize $\rho = \rho_0 + \delta \rho$ and $S = S_0 + \delta S$ these expressions around a given background solution $\rho_0$ and $S_0$. Linearizing the equation of continuity (1) yields

$$(\partial_t + \nabla \cdot \mathbf{v}_0) \delta \rho + \nabla \cdot \left( \frac{\rho_0}{m} \nabla \delta S \right) = 0,$$

and similarly for the eikonal equation (7)

$$(\partial_t + \mathbf{v}_0 \cdot \nabla) \delta S + g \delta \rho = \frac{\hbar^2}{4m} \frac{\rho \nabla^2 (\delta \rho/\sqrt{\rho_0}) - \delta \rho \nabla^2 \sqrt{\rho_0}}{\sqrt{\rho_0}^3}.$$

If we again neglect the quantum pressure term on the right-hand side, we find the wave equation for sound

$$(\partial_t + \mathbf{v}_0 \cdot \nabla) (\partial_t + \mathbf{v}_0 \cdot \nabla) \delta S = \nabla \cdot \left( \frac{\rho_0}{m} \nabla \delta S \right)$$

with the convective (co-moving) derivative and the speed of sound $c_s^2 = g \rho_0/m$.

### III. PHONONS

Now, before considering gravitational waves, let us briefly recapitulate the standard derivation of the phonon wave equation from (1). To this end, it is convenient to employ the eikonal (WKB) ansatz

$$\psi(t, \mathbf{r}) = \sqrt{\rho(t, \mathbf{r})} \exp \left\{ i S(t, \mathbf{r})/\hbar \right\},$$

which expresses $\psi$ in terms of density $\rho$ and phase $S$. Inserting this ansatz into (1), we obtain the equation of continuity with the velocity $\mathbf{v} = \nabla S/m$

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

and the Hamilton-Jacobi (eikonal) equation

$$\dot{S} + g \rho + \frac{(\nabla S)^2}{2m} = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}.$$

### IV. GRAVITATIONAL WAVE

Now we are in the position to investigate how the above derivations change in the presence of a gravitational wave. We employ the usual transverse trace-less (TT) gauge and consider a wave with a fixed (+) polarization, propagating in z-direction

$$ds^2 = c^2 dt^2 - [1 - h] dx^2 - [1 + h] dy^2 - dz^2,$$

where the strain field $h(t, z) = h(t - z/c)$ describes the gravitational wave and is very small, e.g., $h = \mathcal{O}(10^{-21})$. The metric determinant simply reads $|g| = 1 + \mathcal{O}(h^2)$, and we shall neglect all terms of second and higher order in $h$ in the following. As a result, the only change induced by the gravitational wave will be the modification of the Laplacian $\nabla^2 \rightarrow [1 + h] \partial_z^2 + [1 - h] \partial_x^2 + \partial_y^2$ in (10) and accordingly in the subsequent equations.

Now, a vital point is the choice of the background solution. As one option, one could expand around a background solution $\rho_0(t, \mathbf{r})$ and $S_0(t, \mathbf{r})$ in the presence of a gravitational wave, which may be time-dependent in general. In this way, one would obtain a homogeneous wave equation for sound whose coefficients are altered a bit due to the presence of the gravitational wave. This route has been taken in [3] for a general metric, showing that the phonon propagation can be understood in terms of an effective acoustic metric, see also [9, 10]. This results of [3] were used in [6] in order to propose a detection mechanism for gravitational waves.

However, the change of the condensate itself $\rho_0(t, \mathbf{r})$ and $S_0(t, \mathbf{r})$ due to the gravitational wave is not captured in this homogeneous wave equation for sound. But this change can be interpreted as a direct creation of phonons – instead of merely affecting their propagation – which could be a more direct signature of the gravitational wave. Hence, we compare the two scenarios with and without the gravitational wave and thus linearize around a stationary background solution $\rho_0(\mathbf{r})$ and $\mathbf{v}_0(\mathbf{r})$ in flat space-time (i.e., without a gravitational wave), e.g., the ground state. Any departure from this stationary solution can then be interpreted as the creation of a quasi-particle (e.g., phonon) and thereby a signature of the gravitational wave.

Following this strategy, Eq. (10) now reads

$$(\partial_t + \mathbf{v}_0 \cdot \nabla) (\partial_t + \mathbf{v}_0 \cdot \nabla) \delta S = \nabla \cdot \left( \frac{\rho_0}{m} \nabla \delta S \right)$$

$$= h \partial_x (\mathbf{v}_0 \mathbf{v}_0^\prime) - h \partial_y (\mathbf{v}_0^\prime \mathbf{v}_0^\prime),$$

where the scalar product is taken with respect to the Minkowski metric. Similarly Eq. (9) acquires a source term and becomes, again after neglecting the quantum pressure term

$$(\partial_t + \mathbf{v}_0 \cdot \nabla) \delta S + g \delta \rho = mh \frac{(\mathbf{v}_0^\prime)^2 - (\mathbf{v}_0^\prime)^2}{2}.$$
The source terms stemming from the quantum pressure contribution are a bit more lengthy, but can be derived in complete analogy.

For a homogeneous ($q_0 = \text{const}$) condensate at rest ($v_0 = 0$), all the source terms vanish and thus no phonons are directly created by the gravitational wave. Note that the modification of the propagation of phonons corresponds to terms of higher order $\mathcal{O}(\hbar \delta S)$ which are neglected in our first-order treatment. These higher-order terms can induce the indirect creation of phonon pairs via quantum squeezing, see, e.g., [6].

In a non-trivial velocity field $v_0(r)$ such as a vortex lattice, however, these source terms are non-vanishing and thus the gravitational wave could directly generate phonons. Note that a vortex lattice is also advantageous from another point of view: Since gravitational waves are shear waves, a medium or set-up with resistance or response to shear is usually considered to be favorable for gravitational wave detection, see, e.g., [11]. But a fluid has per definition no resistance to shear (on large length and time scales) by itself. However, a vortex lattice does induce a resistance to shear and thus can also support shear waves, which are referred to as Tkachenko waves, see, e.g., [12].

V. SCALING ANSATZ

Since the gravitational wave acts as a periodic stretching and compressing of the $x$ and $y$ coordinates such that the total area/volume stays constant, it might be illuminating to consider an appropriate ansatz for the wave function $\psi(t, x, y, z)$ which is adapted to this change. To this end, we exploit the fact that the condensate mainly feels time-dependence of this end, we exploit the fact that the condensate mainly

\[
\psi(t, x, y, z) \rightarrow \psi(t, |1 - h/2|x, |1 + h/2|y, z). \tag{15}
\]

The scale factors $1 \mp h/2$ are chosen such that the internal derivatives cancel the scale factors $1 \pm h$ in front of $\partial_x^2$ and $\partial_y^2$ to linear order in $h$. As a result, $h$ disappears from the spatial derivatives, but re-enters via the time-derivative

\[
\dot{\psi}(t, x, y, z) \rightarrow \dot{\psi} - \frac{\hbar}{2} \left( x \partial_x \psi - y \partial_y \psi \right). \tag{16}
\]

Accordingly, the Gross-Pitaevskii equation now reads

\[
\frac{\hbar}{2m} \partial_t \dot{\psi} + (\partial_t^2 \dot{\psi} + g|\psi|^2 \psi - \frac{\hbar^2}{2m} \nabla^2 \psi + g|\psi|^2 \psi, \tag{17}
\]

and thus has the same form as in the presence of an effective vector potential $A_{\text{eff}} \propto h(x e_x - y e_y)$ corresponding to a quadrupolar electric field $E_{\text{eff}} \propto \hat{h}(x e_x - y e_y)$.

In terms of this scaling ansatz (analogous to a different choice of coordinates), the source terms for the equation of continuity and the eikonal equation (again neglecting the quantum pressure contribution) are more symmetric

\[
\left( \partial_t + \nabla \cdot v_0 \right) \delta \psi + \nabla \cdot \left( \frac{\hbar g q_{00}}{m} \nabla \delta S \right) = \frac{\hbar}{2} \left( x \partial_x - y \partial_y \right) \varrho_0,
\]

\[
\left( \partial_t + v_0 \cdot \nabla \right) \delta S + g \delta \varrho = \frac{\hbar}{2} \left( x \partial_x - y \partial_y \right) S_0. \tag{18}
\]

Again we find source terms for the wave equation of sound which vanish for homogeneous condensates at rest.

VI. DETECTABILITY

Now, what remains is the important question of whether this effect is actually detectable. After all, the strength $\hbar$ is extremely tiny, suppressed by twenty orders of magnitude or more.

Let us first briefly review how this problem is solved at LIGO. Of course, LIGO is a highly complex machine with a very clever design, but we shall greatly simplify our consideration by just counting how many orders of magnitude are gained by which main mechanisms. First, LIGO exploits a huge ratio of different length scales. The arm length of the interferometer (4 km) in comparison to the wavelength of light (around 1 $\mu$m) gives nine orders of magnitude. The second source for large numbers is the ratio of time scales. During one half-period of the gravitational wave (in the 10 ms range), the light bounces back and forth between the mirrors several hundred times, which also increases the accuracy. Finally, the huge number of photons within the interferometer (in the $\mathcal{O}(10^{19})$ regime), together with our ability to detect light down to the single-photon limit, renders it possible to measure position changes of a tiny fraction of the photon wavelength, which accounts for the remaining orders of magnitude.

After this brief reminder, let us discuss if one could possibly bridge this gap of more than twenty orders of magnitude with a Bose-Einstein condensate. As discussed above, the lowest-order interaction Hamiltonian describing the coupling to a gravitational wave $h$ reads

\[
\hat{H}_{\text{int}} = \hbar^2 \int d^3 r \frac{(\partial_x \hat{\Psi})(\partial_x \hat{\Psi}^\dagger) - (\partial_y \hat{\Psi})(\partial_y \hat{\Psi}^\dagger)}{2m} h. \tag{19}
\]

Since the laws of quantum mechanics imply that one can only distinguish orthogonal quantum states with certainty, an unambiguous detection of a gravitational wave is only possible if the quantum state $|\psi\rangle$ without a gravitational wave is orthogonal to the state $\hat{U}_{\text{int}} |\psi\rangle$ after the interaction with the gravitational wave. In other words, the fidelity

\[
|\langle \psi | \hat{U}_{\text{int}} |\psi\rangle| = |\langle \psi | \mathcal{T} \exp \left\{ -i \hbar \int dt \hat{H}_{\text{int}}(t) \right\} |\psi\rangle|, \tag{20}
\]

should be zero or at least well below unity (for a reasonable detection probability). This is only possible if the smallness of $\hbar = \mathcal{O}(10^{-21})$ in (19) is compensated by some large number(s).
As a major advantage of a Bose-Einstein condensate, the field operator $\hat{\Psi}$, after acting on the coherent condensate state $|\psi\rangle$, does indeed generate a large number

$$\hat{\Psi}(r)|\psi\rangle \approx \psi_{\text{cond}}(r)|\psi\rangle = \mathcal{O}(\sqrt{N}),$$

(21)

where $\psi_{\text{cond}}(r)$ is the condensate wave function, which scales with the square root of the number of condensed atoms $N$. As a result, the interaction Hamiltonian scales with $\mathcal{O}(Nh)$. This enhancement mechanism can be understood via the following simple picture: Neglecting the interactions between the atoms, we may approximate the $N$-particle wave function of the condensate by the product ansatz $\psi_N(r_1, \ldots, r_N) = \psi_1(r_1) \ldots \psi_1(r_N)$, where $\psi_1(r)$ is the single-atom wave function, related to the condensate wave function via $\psi_{\text{cond}}(r) = \sqrt{N}\psi_1(r)$. Now, if the fidelity (measuring the response to a gravitational wave) for a single atom is given by $1 - \varepsilon$, where $\varepsilon \ll 1$ is a small number, the fidelity for the whole condensate would be $(1 - \varepsilon)^N \approx 1 - N\varepsilon$. This shows the advantage of the coherent state (see also [14]) of the condensate in comparison to $N$ incoherent atoms, for example, where one would have to add probabilities $\propto \varepsilon^2$ instead of amplitudes $\propto \varepsilon$, which gives the usual $N\varepsilon^2$ versus $N\varepsilon$ scaling.

Note that the scaling $\mathcal{O}(Nh)$ is due to the fact that the gravitational wave interacts with the whole condensate (as considered in the previous sections) and creates phonons, for example. If we have a homogeneous condensate at rest $\nabla \psi_{\text{cond}} = 0$ plus a few phonons, the gravitational wave would only act on these phonons and thus the scaling would be reduced to $\mathcal{O}(nh)$ where $n$ is the number of phonons, which is much smaller $n \ll N$. This can be understood by inserting the usual mean-field ansatz $\hat{\Psi}(r) \approx \psi_{\text{cond}}(r) + \chi(r)$ where $\chi(r)$ are the Bogoliubov-de Gennes (or phonon) modes. Their action on $|\psi\rangle$ scales with $\sqrt{n}$ instead of $\sqrt{N}$, i.e., $\chi|\psi\rangle = \mathcal{O}(\sqrt{n})$.

Unfortunately, Bose-Einstein condensates of ultra-cold atoms do typically not contain enough atoms to compensate twenty orders of magnitude. Since the characteristic length and time scales of such condensates are usually in the $\mu$m and ms regime, it is also not easy to generate further large numbers by ratios of length or time scales. One option (also discussed in [6]) could be based on resonance effects, which would require a sufficiently long life-time of the condensate with a high enough Q factor for the relevant modes as well as a gravitational wave with precisely the right frequency (which must also be stable over that time). In view of these obstacles, overcoming the twenty orders of magnitude would require tremendous experimental progress and new ideas.

In addition, achieving a sufficiently small fidelity [20] is not the end of the story. This just implies that the laws of quantum mechanics do not forbid the detection of gravitational waves via this mechanism. To actually measure the difference between the states $|\psi\rangle$ and $\hat{U}_{\text{int}}|\psi\rangle$, e.g., to detect single phonons is a highly non-trivial task (see, e.g., [15]). This shows another advantage of LIGO, because our experimental capabilities to detect light (in the optical or near-optical regime) down to the single-photon level is well developed.

Even with being able to detect single phonons, there is still the task of distinguishing the phonons created by a gravitational wave from other noise effects. In this regard, it is important to remember a crucial difference between LIGO and the scheme discussed here: While LIGO measures the position changes of the mirrors while the gravitational wave passes through, one would detect phonons in the condensate after the interaction with the gravitational wave. From this point of view, the condensate is more analogous to resonant mass antennas such as Weber bars (as mentioned in the Introduction). As a result, the matching to gravitational wave form templates as used in LIGO cannot be applied in the same way here, which makes it necessary to employ other mechanisms to filter out the noise.

To end this Section with some speculations, one might consider using super-fluid Helium instead of ultra-cold atomic vapor, see also [12]. As a drawback, super-fluid Helium is a strongly interacting system which is harder to model theoretically and only a small fraction of the atoms (a bit below 10%) are actually condensed. As an advantage, however, the number of atoms and thus also the phase space for length and time scales can be much larger. Of course, the issues related to detecting single or a small number of phonons or Tkachenko quanta and filtering out noise are analogous.

VII. CONCLUSIONS & OUTLOOK

Starting with a fully relativistic effective description [1] of a Bose-Einstein condensate, we study its interaction with a gravitational wave [14]. We find that a vital point is the choice of the background solution around which the phonon modes are linearized. In order to calculate how many phonons (or other quasi-particle excitations) are directly created by the gravitational wave, we choose a background solution $v_0$ and $\theta_0$ in flat spacetime, i.e., without a gravitational wave. Then we obtain an inhomogeneous wave equation [14]

$$\Box v_{0, \theta_0} \delta S = \mathcal{D}_{v_{0, \theta_0}} h,$$

(22)

for the phonons $\delta S$ where the gravitational wave $h$ generates a source term – unless we have a homogeneous condensate at rest. In contrast, if we choose a background solution $v_g$ and $\theta_g$ in the presence of a gravitational wave (or, equivalently, if we have a homogeneous condensate at rest), we obtain a homogeneous wave equation

$$\Box^h v_{g, \theta_g} \delta S = 0,$$

(23)

where the gravitational wave only modifies the coefficients a little bit, see, e.g., [6, 8]. This modification is of higher order $\mathcal{O}(h\delta S)$ than the effect in [22] which is linear in both, $h$ and $\delta S$.

The difference between the wave equations [22] and [23] does also affect the strength of the interaction [19].
In the first case, it scales with the number \( N \) of atoms in the condensate \( \mathcal{O}(Nh) \), while, in the second case, it merely scales with the number \( n \) of phonons \( \mathcal{O}(nh) \), which is typically much smaller. Nevertheless, since \( Nh \) is still a tiny number in typical Bose-Einstein condensates made of atomic vapor, further large numbers (such as ratios in length and time scales, as in LIGO) would be required to reach the regime necessary for gravitational wave detection. In view of the characteristic length and time scales of typical Bose-Einstein condensates made of atomic vapor, this seems to be an extremely challenging task. As a speculation, this task might be a bit less challenging for super-fluid Helium, where the number \( N \) and also the spatial dimensions can be much larger. In addition, super-fluid Helium is the real ground state of the system instead of the meta-stable state of Bose-Einstein condensates made of atomic vapor (which entails problems with three-body losses etc.).

It is also important to note that we did not include a potential \( V \) (such as the trapping potential) in our considerations. Its interaction Hamiltonian reads

\[
\hat{H}_\text{int}^V = \int d^3r \hat{\Psi}^\dagger \hat{\Psi} \frac{\partial V}{\partial h} \hbar.
\]

Ideally, one should also start with a fully relativistic description of the potential (e.g., generated by laser beams) and then derive its change \( \partial V/\partial h \) due to the gravitational wave. However, all the arguments above would still apply (at least qualitatively) since the above Hamiltonian would only act as an additional source for phonons (or other excitations), it would require tremendous fine-tuning to have the phonons created by \( \hat{H}_\text{int}^V \) cancel the other phonons created in the condensate.

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