More on Meta-Stable Brane Configuration
by Quartic Superpotential for Fundamentals

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Abstract

For the case where the gauge theory superpotential has a quartic term as well as the mass term for quarks, the nonsupersymmetric meta-stable brane configuration was found recently. By adding the orientifold 6-planes and the extra fundamental flavors to this brane configuration, we describe the meta-stable nonsupersymmetric vacua of the gauge theory with antisymmetric flavor as well as fundamental flavors in type IIA string theory.
1 Introduction

It is known that the dynamical supersymmetry breaking in meta-stable vacua \cite{1, 2} occurs in the standard $\mathcal{N} = 1$ SQCD with massive fundamental flavors. The extra mass term for quarks in the superpotential implies that some of the F-term equations cannot be satisfied and then the supersymmetry is broken. The corresponding meta-stable brane realizations of type IIA string theory have been found in \cite{3, 4, 5}. Very recently Giveon and Kutasov \cite{6, 7} have found the type IIA nonsupersymmetric meta-stable brane configuration where an additional quartic term for quarks in the superpotential is present. Geometrically, this extra deformation corresponds to the rotation of D6-branes along the (45)-(89) directions while keeping the other branes described in \cite{3, 4, 5} unchanged. Classically there exist only supersymmetric ground states. By adding the orientifold 6-plane to this brane configuration \cite{6}, the meta-stable nonsupersymmetric vacua of the supersymmetric unitary gauge theory with symmetric flavor plus fundamental flavors is found \cite{8}.

Let us add an orientifold 6-plane and extra eight half D6-branes, located at the NS5$'$-brane, into the brane configuration of \cite{6} together with an extra NS5-brane and the mirrors for both D4-branes and rotated D6-branes. According to the observation of \cite{9, 10, 11}, this “fork” brane configuration contains the NS5$'$-brane embedded in an O6-plane at $x^7 = 0$. This NS5$'$-brane divides the O6-plane into two separated regions corresponding to positive $x^7$ and negative $x^7$. Then RR charge of the O6-plane jumps from $-4$ to $+4$. Furthermore, eight semi-infinite D6-branes are present in the positive $x^7$ region. This is necessary for the vanishing of the six dimensional anomaly. Then the type IIA brane configuration consists of two NS5-branes, one NS5$'$-brane, D4-branes, rotated D6-branes, an O6-plane and eight half D6-branes. We’ll see how the corresponding supersymmetric gauge theory, which is a standard $\mathcal{N} = 1$ SQCD with massive flavors together with the extra matters, occurs in the context of dynamical supersymmetry breaking in meta-stable vacua.

In this paper, we study $\mathcal{N} = 1$ $SU(N_c)$ gauge theory with an antisymmetric flavor $A$, a conjugate symmetric flavor $\tilde{S}$, $N_f$ fundamental flavors $Q$ and $\tilde{Q}$ and eight fundamental flavors $\hat{Q}$ in the context of dynamical supersymmetric breaking vacua. Now we deform this theory by adding both the mass term and the quartic term for quarks $Q, \tilde{Q}$ in the fundamental representation of the gauge group \cite{6}. Then we turn to the dual magnetic gauge theory \cite{12}. The dual magnetic theory giving rise to the meta-stable vacua is described by $\mathcal{N} = 1$ $SU(2N_f - N_c + 4)$ gauge theory with dual matter contents. The difference between the brane configuration of \cite{12} and the brane configuration of this paper is that the D6-branes are rotated in the (45)-(89) directions. By analyzing the magnetic superpotential, along the line
of [6] [7], we present the behaviors of gauge theory description and string theory description for the meta-stable vacua.

In section 2, the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c)$ gauge theory with above matter contents is given. In section 3, we construct the Seiberg dual magnetic theory which is $\mathcal{N} = 1$ $SU(2N_f - N_c + 4)$ gauge theory with corresponding dual matters. The rotation of D6-branes is encoded in the mass term for the meson field in the superpotential. In section 4, the nonsupersymmetric meta-stable minimum is found and the corresponding intersecting brane configuration of type IIA string theory is presented. In section 5, we comment on the future works.

2 The $\mathcal{N} = 1$ supersymmetric electric brane configuration

The type IIA supersymmetric electric brane configuration [9, 10, 11, 12] corresponding to $\mathcal{N} = 1$ $SU(N_c)$ gauge theory with an antisymmetric flavor $A$, a conjugate symmetric flavor $\tilde{S}$, eight fundamental flavors $\hat{Q}$ and $N_f$ fundamental flavors $Q, \tilde{Q}$ [13] can be described as follows: one middle NS5'-brane(012389), two NS5-branes(012345) denoted by NS5L-brane and NS5R-brane respectively, $N_c$ D4-branes(01236) between them, $2N_f$ D6-branes(0123789), an orientifold 6 plane(0123789) of positive RR charge, an orientifold 6 plane(0123789) of negative RR charge and eight half D6-branes. The transverse coordinates $(x^4, x^5, x^6)$ transform as $(-x^4, -x^5, -x^6)$ under the orientifold 6-plane(O6-plane) action. Let us introduce two complex coordinates [14]

$$v \equiv x^4 + ix^5, \quad w \equiv x^8 + ix^9.$$  

Then the origin of the coordinates $(x^6, v, w)$ is located at the intersection of $x^6$ coordinate and O6-plane. The left NS5L-brane is located at the left hand side of O6-plane while the right NS5R-brane is located at the right hand side of O6-plane. The $N_c$ color D4-branes are suspended between NS5L-brane and NS5R-brane. Moreover the $N_f$ D6-branes are located between the NS5L-brane and the middle NS5'-brane and its mirrors $N_f$ D6-branes are located between the middle NS5'-brane and the NS5R-brane. The antisymmetric and conjugate symmetric flavors $A$ and $\tilde{S}$ are 4-4 strings stretching between D4-branes located at the left hand side of O6-plane and those at the right hand side of O6-plane, $N_f$ fundamental flavors $Q$ and $\tilde{Q}$ are strings stretching between $N_f$ D6-branes and $N_c$ color D4-branes and eight fundamental flavors $\hat{Q}$ are strings stretching between eight half D6-branes which are on top of O6−-plane and $N_c$ color D4-branes.
Let us deform this theory which has vanishing superpotential by adding both the mass term and the quartic term for $N_f$ fundamental quarks. The former can be achieved by “translating” the D6-branes along $\pm v$ direction leading to their coordinates $v = \pm v_{D6}$ \[^1\] while the latter can be obtained by “rotating” the D6-branes \[^2\] by an angle $\theta$ in $(w, v)$-plane. We denote them by $D6_{\theta}$-branes which are at angle $\theta$ with undeformed unrotated D6-branes(0123789). Then their mirrors $N_f$ D6-branes are rotated by an angle $-\theta$ in $(w, v)$-plane according to O6-plane action and we denote them also by $D6_{-\theta}$-branes \[^3\]. Then, in the electric gauge theory, the deformed superpotential is given by

$$W_{\text{elec}} = \frac{\alpha}{2} \text{tr}(Q\tilde{Q})^2 - m \text{tr} Q\tilde{Q} - \frac{1}{2\mu} \left[ (A\tilde{S})^2 + QS\tilde{A}\tilde{Q} + (Q\tilde{Q})^2 \right] + \hat{Q}\tilde{S}\hat{Q},$$

where $\alpha = \tan \frac{\theta}{\Lambda}$, $m = \frac{v_{D6}}{2\pi \ell_s^2}$.

Note that the convention for $D6_{\theta}$-branes in \[^1\] was such that the angle between unrotated D6-branes and $D6_{\theta}$-branes was not $\theta$ but $(\frac{\pi}{2} - \theta)$.

\[^3\]
is, the $k$ D4-branes among $N_f$ D4-branes (stretched between $NS5_R$-brane and $D6_{-\theta}$-branes) are reconnecting with those number of D4-branes stretched between the middle $NS5'$-brane and $NS5_R$-brane. Then those resulting $k$ D4-branes are moving to $\pm v$ direction and the remaining $(N_c - k)$ D4-branes are stretching between the middle $NS5'$-brane and $NS5_R$-brane and $(N_f - k)$ D4-branes are stretched between the $NS5_R$-brane and $D6_{-\theta}$-branes(and their mirrors).

Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration with deformed superpotential (2.1) for the $SU(N_c)$ gauge theory with an antisymmetric flavor $A$, a conjugate symmetric flavor $\tilde{S}$, eight fundamentals $\hat{Q}$, and $N_f$ fundamental massive flavors $Q, \tilde{Q}$. The origin of the coordinates $(x^6, v, w)$ is located at the intersection of $x^6$ and O6-plane. It is evident that the two deformations are characterized by both translation and rotation for D6-branes. As in [12], a combination of a middle $NS5'$-brane, $O6^+$-plane, $O6^-$-plane and eight half D6-branes is represented by $NS5'/O6/D6$.

3 The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration

The magnetic theory is obtained by interchanging the $D6_{\pm \theta}$-branes and $NS5_{L,R}$-branes while the linking number is preserved. After one moves the left $D6_{\theta}$-branes to the right all the way(and their mirrors, right $D6_{-\theta}$-branes to the left) past the middle $NS5'$-brane and the right $NS5_R$-brane, the linking number counting [12] implies that one should add $N_f$ D4-branes, corresponding to the meson $M(\equiv Q\tilde{Q})$, to the left side of all the right $N_f$ $D6_{\theta}$-branes(and their mirrors). Note that when a D6-brane crosses the middle $NS5'$-brane, due to
the parallelness of these, there was no creation of D4-branes. That is, when the $D6_{\pm \theta}$-branes approach the middle NS5$'$-brane, one should take $\theta = 0$ limit (making D6-branes to be parallel to the middle NS5$'$-brane) and then after they cross the middle NS5$'$-brane, they return to the original positions given by $D6_{\pm \theta}$-branes as follows:

$$D6_{\pm \theta} \rightarrow D6 \rightarrow D6_{\pm \theta}$$

(3.1)

Next, let us move the left NS5$_L$-brane to the right all the way past O6-plane (and its mirror, right NS5$_R$-brane to the left), and then the linking number counting \[12\] leads to the fact that the dual number of colors was $(2N_f - N_c + 4)$. There was a creation of D4-branes when the NS5-brane crosses an O6-plane because they are not parallel to each other. From this, the constant term 4 in the dual color above arises, compared with the case of \[15\]. Now we draw the magnetic brane configuration in Figure 2 where some of the flavor D4-branes are recombined with those of $(2N_f - N_c + 4)$ color D4-branes and those combined resulting D4-branes are moved into $\pm v$ direction. One takes $k$ D4-branes from $N_f$ flavor D4-branes and reconnect them to those from $(2N_f - N_c + 4)$ color D4-branes in Figure 2 such that the resulting branes are connecting from the $D6_{\theta}$-branes to the NS5$_L$-brane directly. Their coordinates between $D6_{\theta}$-branes and $k$ D4-branes will be $v = -v_{D6}$ in order to minimize the energy. This Figure 2 also can be obtained from the magnetic brane configuration of \[6\] by adding O6-planes and eight half D6-branes with the right presence of mirrors under the O6-plane action.

Then the low energy dynamics is described by the dual magnetic theory with gauge group $SU(2N_f - N_c + 4)$ and this theory is higgsed down to $SU(2N_f - N_c + 4 - k)$ in the $k$-th vacuum by nonzero vacuum expectation value for dual quarks which is determined later. The matters are $N_f$ flavors of fundamentals $q, \bar{q}$, an antisymmetric flavor $a$, a conjugate symmetric flavor $\tilde{s}$, eight fundamentals $\hat{q}$, gauge singlet $M$ which is magnetic dual of the electric meson field $Q\bar{Q}$ and other gauge singlet $\hat{M}$ that is $\hat{Q}\hat{Q}$. Then the superpotential including the interaction between the meson field $M$ and dual matters with $\mu \rightarrow \infty (\omega \rightarrow 0)$ limit is described by

$$W_{mag} = \frac{1}{\Lambda} M q \bar{s} a \bar{q} + \frac{\alpha}{2} \text{tr} M^2 - m \text{tr} M + q \bar{s} \hat{q} + \hat{M} \bar{q} \bar{q}, \quad M \equiv Q\bar{Q}, \quad \hat{M} \equiv \hat{Q}\hat{Q}$$

(3.2)

where the second and third terms originate from the two deformations \[2.1\], and the fourth term also comes from electric theory. The $\theta$-dependent coefficient function, $\alpha$, in front of quadratic term of the meson field also occurs in the geometric brane interpretation for the different supersymmetric gauge theory \[16\]. The $\alpha = 0$ limit reduces to the theory given by \[12\]. The parameters $\alpha$ and $m$ are the same as before in \[2.1\].

\[2\] For arbitrary rotation angles of $D6_{\pm \theta}$-branes and NS5$_{\pm \omega}$-branes, there exist also other meson fields
Figure 2: The $N = 1$ supersymmetric magnetic brane configuration for the $SU(2N_f - N_c + 4 - k)$ gauge theory with an antisymmetric flavor $a$, a conjugate symmetric flavor $\tilde{s}$, $N_f$ fundamental flavors $q, \tilde{q}$ and eight fundamentals $\hat{q}$. The $N_f$ flavor D4-branes connecting between $NS5_L$-brane and $D6_\theta$-branes are related to the dual gauge singlet $M$ and are splitting into $(N_f - k)$ and $k$ D4-branes. The location of intersection between $D6_\theta$-branes and $(N_f - k)$ D4-branes is given by $(v, w) = (0, v_{D6} \cot \theta)$ while the one between $D6_\theta$-branes and $k$ D4-branes is given by $(v, w) = (-v_{D6}, 0)$.

From the magnetic superpotential (3.2), the supersymmetric vacua are obtained and the F-term equations are given as follows:

$$
\tilde{s}a\tilde{q}M = 0, \quad a\tilde{q}Mq + \tilde{q}\tilde{q} = 0,
$$

$$
\tilde{q}M\tilde{s}\tilde{q} = 0, \quad M\tilde{q}\tilde{s}a + \tilde{M}\tilde{q} = 0,
$$

$$
\tilde{s}\tilde{q} + \tilde{q}\tilde{M} = 0, \quad \tilde{q}\tilde{q} = 0,
$$

$$
\frac{1}{\Lambda} \tilde{q}s\tilde{a}q = m - \alpha M. \tag{3.3}
$$

By multiplying $M$ into the last equation with $\tilde{q} = 0 = \tilde{M}$, the matrix equation is satisfied $mM = \alpha M^2$. Because the eigenvalues are either 0 or $\frac{m}{\alpha}$, one can take $N_f \times N_f$ matrix with containing $A$ or $\tilde{S}$: $M_1 \equiv Q\tilde{S}AQ, P \equiv Q\tilde{S}Q$ and $\tilde{P} \equiv \tilde{Q}A\tilde{Q}$. They couple to the dual quarks and other flavors in the superpotential via $M_1\tilde{q}\tilde{s}q + \tilde{P}\tilde{q}\tilde{a}\tilde{q}q$. As emphasized in [12], the geometric constraint (3.1) at the intersection between D6-branes and the middle NS5'-brane removes the presence of these gauge singlets, $M_1, P$ and $\tilde{P}$. That is, when $D6_\theta$-branes are crossing the middle NS5'-brane, they do not produce any D4-branes because they are parallel at the origin $x^6 = 0$. This implies there is no $M_1$ term in the magnetic superpotential. In general, $P$ and $\tilde{P}$-terms arise when $D6_\theta$-branes intersect with its mirrors $D6_{-\theta}$-branes around the origin $x^6 = 0$. But they are also parallel to each other due to (3.1). This leads to the fact that there are no $P$ or $\tilde{P}$-terms in the magnetic superpotential. Therefore, we are left with (3.2).
$k$’s eigenvalues 0 and $(N_f - k)$’s eigenvalues $\frac{m}{\alpha}$:

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{m}{\alpha} \mathbf{1}_{N_f-k} \end{pmatrix}$$

where $k = 1, 2, \cdots, N_f$ and $\mathbf{1}_{N_f-k}$ is the $(N_f-k) \times (N_f-k)$ identity matrix [7]. The expectation value of $M$ is represented by the fundamental string between the flavor brane displaced by the $w$ direction and the color brane from Figure 2. The $k$ of the $N_f$ flavor D4-branes are connected with $k$ of $(2N_f - N_c + 4)$ color D4-branes and the resulting D4-branes stretch from the $D6_\theta$-branes to the $NS5_L$-brane and the coordinate of an intersection point between the $k$ D4-branes and the $NS5_L$-brane is given by $(v, w) = (-v_{D6}, 0)$. This corresponds to exactly the $k$’s eigenvalues 0 of $M$ above. Now the remaining $(N_f - k)$ flavor D4-branes between the $D6_\theta$-branes and the $NS5'$-brane are related to the corresponding eigenvalues of $M$ given by $\frac{m}{\alpha} \mathbf{1}_{N_f-k}$. The coordinate of an intersection point between the $(N_f - k)$ D4-branes and the $NS5'$-brane is given by $(v, w) = (0, v_{D6} \cot \theta)$. Note that using the expressions for $\alpha$ and $m$ from (2.1), one obtains $\frac{m}{\alpha} = \Lambda \frac{v_{D6} \cot \theta}{2 \pi \ell_s^2}$ which must be $\Lambda \frac{w}{2 \pi \ell_s^2}$. Then $w = v_{D6} \cot \theta$.

Substituting (3.4) into the last equation of (3.3) leads to

$$\tilde{q} \tilde{s} \tilde{a} \tilde{q} = \begin{pmatrix} m \Lambda \mathbf{1}_k & 0 \\ 0 & 0 \end{pmatrix}.$$ (3.5)

Since the rank of the left hand side is at most $2N_f - N_c + 4$, one must have more stringent bound $k \leq (2N_f - N_c + 4)$. In the $k$-th vacuum the gauge symmetry is broken to $SU(2N_f - N_c + 4 - k)$ and the supersymmetric vacuum drawn in Figure 2 with $k = 0$ has $q\tilde{s} = a\tilde{q} = 0$ and the gauge group $SU(2N_f - N_c + 4)$ is unbroken. The expectation value of $M$ in this case is given by $M = \frac{m}{\alpha} \mathbf{1}_{N_f} = m \Lambda \cot \theta \mathbf{1}_{N_f}$ explicitly. By moving the D6-branes into the place between the middle NS5$'$-brane and the $NS5_L,R$-branes, one obtains other brane configuration. There exist $(N_f - k)$ flavor D4-branes connecting $D6_{\pm \theta}$-branes and the $NS5_L,R$-branes after this movement.

Another deformation arises when we rotate the $NS5_{L,R}$-branes by an angle $\pm \theta'$ in the $(v, w)$-plane. This rotation provides an adjoint field of $SU(2N_f - N_c + 4)$ and couples to the magnetic dual matters. Integrating this adjoint field out leads to the fact that there exists a further contribution to the quartic superpotential for $q$ and $\tilde{q}$. In particular, when the rotated $NS5_{\pm \theta}$-branes are parallel to rotated $D6_{\pm \theta}$-branes, the coupling in front of $M^2$ in the magnetic superpotential vanishes.
4 Nonsupersymmetric meta-stable brane configuration

The theory has many nonsupersymmetric meta-stable ground states besides the supersymmetric ones we discussed in previous section. For the IR free region [12], the magnetic theory is the effective low energy description of the asymptotically free electric gauge theory. When we rescale the meson field as \( M = h\Lambda\Phi \), then the Kahler potential for \( \Phi \) is canonical and the magnetic quarks are canonical near the origin of field space. The higher order corrections of Kahler potential are negligible when the expectation values of the fields \( q, \tilde{q}, a, \tilde{s} \) and \( \Phi \) are smaller than the scale of magnetic theory. Then the magnetic superpotential can be written in terms of \( \Phi \) (or \( M \))

\[
W_{mag} = h\Phi q\tilde{s}a\tilde{q} + \frac{h^2 \mu_\phi}{2} \text{tr} \Phi^2 - h\mu^2 \text{tr} \Phi + \hat{q}\tilde{s}\hat{q} + \tilde{M}\hat{q}\hat{q}.
\]

From this, one can read off the following quantities

\[
\mu^2 = m\Lambda, \quad \mu_\phi = \alpha\Lambda^2, \quad M = h\Lambda\Phi.
\]

The classical supersymmetric vacua given by (3.4) and (3.5) can be described similarly and one decomposes the \((N_f - k) \times (N_f - k)\) block at the lower right corner of \( h\Phi \) and \( q\tilde{s}a\tilde{q} \) into blocks of size \( n \) and \((N_f - k - n)\) as follows:

\[
h\Phi = \begin{pmatrix}
0 & 0 & 0 \\
0 & h\Phi_n & 0 \\
0 & 0 & \frac{\mu^2}{\mu_\phi} 1_{N_f - k - n}
\end{pmatrix}, \quad q\tilde{s}a\tilde{q} = \begin{pmatrix}
\mu^2 1_k & 0 & 0 \\
0 & \varphi\tilde{\beta}\gamma\tilde{\varphi} & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]

Here \( \varphi \) and \( \tilde{\varphi} \) are \( n \times (2N_f - N_c + 4 - k) \) dimensional matrices and correspond to \( n \) flavors of fundamentals of the gauge group \( SU(2N_f - N_c + 4 - k) \) which is unbroken by the nonzero expectation value of \( q\tilde{s} \) and \( a\tilde{q} \). In Figure 3, they correspond to fundamental strings connecting the \( n \) flavor D4-branes and \((2N_f - N_c + 4 - k)\) color D4-branes. This Figure 3 can be also obtained from the meta-stable brane configuration of [13] by adding O6-planes and eight half D6-branes with appropriate mirrors under the O6-plane action. The antisymmetric and conjugate symmetric flavors \( \gamma \) and \( \tilde{\beta} \) are 4-4 strings stretching between \((2N_f - N_c + 4 - k)\) D4-branes located at the left hand side of O6-plane and those at the right hand side of O6-plane in Figure 3. Both \( \Phi_n \) and \( \varphi\tilde{\beta}\beta\tilde{\varphi} \) are \( n \times n \) matrices. The supersymmetric ground state corresponds to the vacuum expectation values given by \( h\Phi_n = \frac{\mu^2}{\mu_\phi} 1_n, \varphi\tilde{\beta} = 0 = \gamma\tilde{\varphi} \).

Now the full one loop potential for \( \Phi_n, \varphi \equiv \varphi\tilde{\beta}, \tilde{\varphi} \equiv \gamma\tilde{\varphi} \) takes the form

\[
V = |\Phi_n\varphi|^2 + |\Phi_n\tilde{\varphi}|^2 + |\varphi\tilde{\varphi} - \mu^2 1_n + h\mu_\phi \Phi_n|^2 + b|h\mu|^2 \text{tr} \Phi_n^i \Phi_n^i.
\]
Figure 3: The nonsupersymmetric minimal energy brane configuration for the $SU(2N_f - N_c + 4 - k)$ gauge theory with an antisymmetric flavor $a$, a conjugate symmetric flavor $\tilde{a}$, $N_f$ fundamental flavors $q, \tilde{q}$ and eight fundamentals $\hat{q}$. This brane configuration is obtained by moving $n$ flavor D4-branes, from $(N_f - k)$ flavor D4-branes stretched between the NS5'-brane and the $D_6\theta$-branes in Figure 2 (and their mirrors).

where $b$ is given by $b = \frac{(\ln 4 - 1)}{8\pi^2}(2N_f - N_c + 4)$. Differentiating this potential with respect to $\Phi_n$ and putting $\hat{\varphi} = 0 = \tilde{\varphi}$, one obtains

$$h\Phi_n = \frac{\mu^2 \mu^*_\varphi}{|\mu^*_\varphi|^2 + b|\mu|^2} 1_n \simeq \frac{\mu^2 \mu^*_\varphi}{b|\mu|^2} 1_n \quad \text{or} \quad M_n \simeq \frac{\alpha\Lambda^3}{(2N_f - N_c + 4)} 1_n \quad (4.1)$$

for real $\mu$ and we assume here that $\mu^*_\varphi << \mu << \Lambda_m$. The vacuum energy $V$ is given by $V \simeq n|\mu^2|^2$. Expanding around this solution, one obtains the eigenvalues for mass matrix for $\hat{\varphi}$ and $\tilde{\varphi}$ will be

$$m^2_\pm = \frac{|\mu|^4}{(|\mu^*_\varphi|^2 + b|\mu|^2)^2} \left[ |\mu^*_\varphi|^2 \pm b|h|^2 \left(|\mu^*_\varphi|^2 + b|\mu|^2\right) \right] \simeq \frac{1}{b^2} \left(|\mu^*_\varphi|^2 \pm |bh\mu|^2\right).$$

Then for $|\mu^*_\varphi|^2 > \frac{|bh|^2}{1 - |bh|/\mu^2} \simeq |bh|^2$ in order to avoid tachyons the vacuum (4.1) is locally stable.

One can move $n$ D4-branes, from $(N_f - k)$ D4-branes stretched between the NS5'-brane and the $D_6\theta$-branes at $w = v_{D6} \cot \theta$, to the local minimum of the potential and the end points of these $n$ D4-branes are at a nonzero $w$ as in Figure 3. The remaining $(N_f - k - n)$ flavor D4-branes between the $D_6\theta$-branes and the NS5'-brane are related to the corresponding eigenvalues of $h\Phi$, i.e., $\frac{\mu^2}{\mu^*_\varphi} 1_{N_f - k - n}$.

The coordinate of an intersection point between the $(N_f - k - n)$ D4-branes and the NS5'-brane is given by $(v, w) = (0, v_{D6} \cot \theta)$. Finally, the remnant $n$ “curved” flavor D4-branes between the $D_6\theta$-branes and the NS5'-brane are related to the corresponding eigenvalues of $h\Phi_n$ by (4.1). Note that since $\frac{\mu^2}{b|\mu|^2} << \frac{\mu^2}{\mu^*_\varphi}$, the $n$ D4-branes are nearer to the $w = 0$ located at the NS5$_L$-brane.
As explained in [6], this local stable vacuum decays to the supersymmetric ground states. The end points of \(n\) “curved” flavor D4-branes on the NS5’-brane approach those of the \((2N_f - N_c + 4 - k)\) color D4-branes and two types of branes reconnect each other. For \(n \leq (2N_f - N_c + 4 - k)\), the final brane configuration is nothing but the supersymmetric vacuum of Figure 2 with the replacement \(k \rightarrow (k + n)\). When \(n > (2N_f - N_c + 4 - k)\), then the remnant \([n - (2N_f - N_c + 4 - k)]\) flavor D4-branes remain. On the other hand, the \(n\) D4-branes can move to larger \(w\) and return to the Figure 2. Also some of the D4-branes approach the intersection point between \(D6\)-branes and the NS5’-brane while the remaining D4-branes move to the one between \(D6\)-branes and the \(NS5_L\)-brane.

When \(D6_{\pm\theta}\)-branes are moved to the place between the NS5’-brane and the \(NS5_{L,R}\)-branes, one gets the Figure 4 where the previous \((N_f - k)\) D6-branes in Figure 3 that were not connected to the \(NS5_{L,R}\)-branes, through the flavor D4-branes, are now connecting to the \(NS5_{L,R}\)-branes by the same number of D4-branes while the previous \(k\) D6-branes in Figure 3 that were connected to the \(NS5_{L,R}\)-branes, through the flavor D4-branes, are now not connecting to the \(NS5_{L,R}\)-branes. The former corresponds to a creation of D4-branes and the latter corresponds to an annihilation of D4-branes due to the Hanany-Witten transition [17, 14].

Figure 4: The nonsupersymmetric minimal energy brane configuration for the \(SU(2N_f - N_c + 4 - k)\) gauge theory with an antisymmetric flavor \(a\), a conjugate symmetric flavor \(\tilde{a}\), \(N_f\) fundamental flavors \(q, \tilde{q}\) and eight fundamentals \(\hat{q}\). This brane configuration can be obtained by moving \(N_f\) \(D6_{\pm\theta}\)-branes into the place between the \(NS5_{L,R}\)-branes and the middle NS5’-brane in Figure 3(and their mirrors). Note that there exists a creation of \((N_f - k)\) D4-branes connecting the \(D6\)-branes and the \(NS5_L\)-brane(and their mirrors).

In the remaining part, we focus on the gravitational potential of the \(NS5_L\)-brane. Let us
remind that the branes are placed as follows:

\begin{align*}
D6_\theta - \text{branes}(01237vw) : & \quad v = -v_{D6} + w \tan \theta, \\
NS5' - \text{brane}(012389) : & \quad v = 0, \\
NS5_L - \text{brane}(012345) : & \quad w = 0
\end{align*}

where we assume that $D6_\theta$-branes are located at the nonzero $y \equiv x^6$ and the $NS5'$-brane is located at $y = 0$ (the origin). The dependence of the distance between the $D6_\theta$-branes and $NS5'$ brane along the $NS5_L$-brane on $w$ can be represented by

$$\Delta x = | - v_{D6} + w \tan \theta |. \quad (4.2)$$

Then the partial differentiation of $\Delta x$ with respect to $w$ leads to an extra contribution for the computation of $\partial_w \theta_{i,w}$ where we define $\theta_{i,w}$ as

$$\begin{align*}
\theta_{1,w} & \equiv \cos^{-1} \left( \frac{y_m}{|w|} \right), \\
\theta_{2,w} & \equiv \cos^{-1} \left( \frac{y_m}{\sqrt{y^2 + |w|^2}} \right)
\end{align*} \quad (4.3)$$

with $y_m$ that is smallest value of $y$ along the D4-brane. Then the energy density of the D4-brane was computed in [18, 19] and is given by

$$E(w) = \frac{\tau_4}{2\ell_s} \sqrt{1 + \frac{\ell_s^2}{y_m^2} \left[ |w|^2 \sin^2 2\theta_{1,w} + (y^2 + |w|^2) \sin 2\theta_{2,w} \right]} \quad (4.4)$$

corresponding to (3.7) of [6] where $\tau_4$ is a tension of D4-brane in flat spacetime.

It is straightforward to compute the differentiation of $\left( \frac{\ell_s E(w)}{\tau_4} \right)^2$ with respect to $w$ and it leads to

$$\begin{align*}
\partial_w \left( \frac{\ell_s E(w)}{\tau_4} \right)^2 = & \ell_s^2 \bar{w} \left( \sin^2 \theta_{1,w} + \sin^2 \theta_{2,w} + \left[ \frac{\sqrt{y^2 + |w|^2}}{|w|} + \frac{|w|}{\sqrt{y^2 + |w|^2}} \right] \sin \theta_{1,w} \sin \theta_{2,w} \right) \\
+ & \left[ |w|^2 \sin 2\theta_{1,w} + (y^2 + |w|^2) \sin 2\theta_{2,w} \right] \times \left[ \left( \frac{\ell_s^2}{y_m^2} + |w|^2 \cos 2\theta_{1,w} \right) \partial_w \theta_{1,w} \\
+ & \left( \ell_s^2 + (y^2 + |w|^2) \cos 2\theta_{2,w} \right) \partial_w \theta_{2,w} + \frac{\bar{w}}{2} \left( \sin \theta_{1,w} + \sin \theta_{2,w} \right) \right]. \quad (4.5)
\end{align*}$$

In order to simplify this, one uses the partial differentiation of $\Delta x$ (4.2) with respect to $w$ which is equal to $\frac{1}{2} \tan \theta_{D6} \frac{\partial \Delta x}{\partial w}$. On the other hand, $\Delta x$ was known in [18] and it is

$$\Delta x = \frac{1}{2\ell_s} \left[ |w|^2 \sin 2\theta_{1,w} + (y^2 + |w|^2) \sin 2\theta_{2,w} \right] + \ell_s \left( \theta_{1,w} + \theta_{2,w} \right). \quad (4.6)$$
After differentiating this (4.6) with respect to \( w \) and equating it to the previous expression obtained from (4.2), one arrives at
\[
\left( \ell_s^2 + |w|^2 \cos 2\theta_{1,w} \right) \partial_w \theta_{1,w} + \left[ \ell_s^2 + (y^2 + |w|^2) \cos 2\theta_{2,w} \right] \partial_w \theta_{2,w} \\
+ \frac{\bar{w}}{2} (\sin 2\theta_{1,w} + \sin 2\theta_{2,w}) = \ell_s \frac{1}{2} \tan \theta \frac{\bar{w}\tan \theta - v_{D6}}{|w\tan \theta - v_{D6}|}
\]
(4.7)
corresponding to (3.21) of [6]. Now by putting this relation (4.7) into the second and third lines of (4.5), one gets
\[
\partial_w \left( \ell_s E(w) \right)^2 = \ell_s^2 \bar{w} \left( \sin^2 \theta_{1,w} + \sin^2 \theta_{2,w} + \left[ \frac{\sqrt{y^2 + |w|^2}}{|w|} + \frac{|w|}{\sqrt{y^2 + |w|^2}} \right] \sin \theta_{1,w} \sin \theta_{2,w} \right) \\
+ \left[ |w|^2 \sin 2\theta_{1,w} + (y^2 + |w|^2) \sin 2\theta_{2,w} \right] \frac{1}{2} \ell_s \tan \theta \frac{\bar{w}\tan \theta - v_{D6}}{|w\tan \theta - v_{D6}|}.
\]
(4.8)

It is easy to see that at \( w = v_{D6} \cot \theta \) which is an intersection point between \( D6_\theta \)-branes and \( NS5' \)-brane, \( \Delta x \) vanishes through (4.2) and this also implies that \( \theta_{i,w} \) vanishes from (4.6). Furthermore, the energy \( E(w) \) is zero from (4.4). This corresponds to the global minimal energy. For the parallel \( D6 \)-branes and \( NS5' \)-brane (i.e., \( \tan \theta = 0 \)), then the only stationary point is \( w = 0 \) [12]. If \( w \neq 0 \), then \( \sin \theta_{1,w} = \sin \theta_{2,w} \) from the first term of (4.8) but these are not physical solutions.

For real and positive parameters \( v_{D6}, w \) and \( \tan \theta \), we are looking for the solution with \( v_{D6} > w \tan \theta \) and setting the right hand side of (4.8) to zero, finally one gets with (4.3)
\[
\sin^2 \theta_{1,w} + \sin^2 \theta_{2,w} + \left( \frac{\sqrt{y^2 + w^2}}{w} + \frac{w}{\sqrt{y^2 + w^2}} \right) \sin \theta_{1,w} \sin \theta_{2,w} \\
= \frac{\tan \theta}{2\ell_s w}.
\]
(4.9)

Therefore, the brane configuration of Figure 4 has a local minimum where the end of \( D4 \)-brane are located at \( w \) given by (4.9). When the \( y \) goes to zero(\( \theta_{1,w} = \theta_{2,w} \equiv \theta_w \)), one can approximate (4.9) and one gets
\[
\tan \theta_w \sim \frac{w \tan \theta}{2\ell_s}.
\]
The gauge theory result is valid only when \( \theta \) and \( \frac{v_{D6}}{\ell_s} \) are much smaller than \( g_s \) while the classical brane construction with (4.9) is valid for any angle and the length parameters are of order \( \ell_s \) or larger.

5 Conclusions and outlook

In this paper, by adding the orientifold 6-planes and the extra fundamental flavors to the brane configuration [6], we have described the meta-stable nonsupersymmetric vacua of the
gauge theory with antisymmetric flavor as well as fundamental flavors, through the Figures 3 and 4, from type IIA string theory.

It would be interesting to deform the theories given in [20, 19, 21, 22, 23] where one of the gauge group factor has the same matter contents as the one of the present paper and see how the meta-stable ground states appear. Along the lines of [18, 19, 21, 22, 23], when the $D6_\theta$-branes are replaced by a single $NS5_\theta$-brane, it would be interesting to see how the present deformation arises in these theories. It is also possible to deform the symplectic or orthogonal gauge group theory with massive flavors [25] by adding an orientifold 4-plane. Similar application to the product gauge group case [26, 27, 28] is also possible to study. It is an open problem to see how the type IIB description [29] is related to the present work. To construct a direct gauge mediation [30, 31] for the present work is also possible open problem.

Acknowledgments

We would like to thank D. Kutasov for discussions. This work was supported by grant No. R01-2006-000-10965-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

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