Design Secondary Optical System Applied in White-LED General Illumination

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Abstract: As a new generation of light source, LED has many advantages that other light sources do not have. However, due to the nonuniform lighting of LED, secondary LED optical system design is particularly important. Freeform surface tailoring method, an important method of lighting design, establishes a light intensity change model after smooth surface refraction (reflection) of the light and simplifies the solution process for more complex issues of solution using the free surface tailoring method. Based on this method, secondary LED optical system is designed, and the light intensity distribution is simulated after LED light passes through the secondary optical system. The results show that the method has not only simplified the calculation process of the free surface tailoring method, but also the designed LED secondary optical system has achieved the purpose of uniform lighting to a certain degree.

Keywords: Freeform surface tailoring method, LED lighting, Geometrical optics, Secondary optical system

Introduction
As a new solid cold light source, LED, unmatched by traditional light sources, has many advantages such as energy saving, resistance to impact, resistance to moisture, longer service life and so on. In recent years, with the continuous advancement of semiconductor lighting technology, luminous efficiency of LED has already reached 100 lm/W, so that LED lighting lamps have gradually replaced traditional light sources and been applied to general lighting field.

Luminescent properties of LED lamp are similar to Lambert, which results in its nonuniform illumination light field. When it is applied to lighting, its light intensity must be converted using a secondary optical system. If LED is regarded as a point light source, its secondary optical system can be designed using the free-form surface tailoring method. Free surface tailoring method is a design method that partial differential equations are solved using numerical values so as to construct the surface figure⁠[¹] of free-form surface in lighting optics, it is very widely applied in the field of lighting design. One of the most important steps in designing tailored free-form surface is to calculate⁠[²] the light intensity after light passes through the optical surface with light wavefront as the carrier. As shape of the light wavefront has changed after passing through the optical surface, therefore, changes⁠[³] of the light wavefront curvature tensor and optical surface tensor must also be considered, so that complexity of the differential equation that is required to be ultimately solved is increased. In
this paper, a light intensity change model that does not take the light wavefront as the light intensity carrier is put forward, which reduces the complexity of optical surface to be solved and designs the secondary optical system for LED lamps using an improved method. The simulation results have proved the efficiency of this method.

1 Improved theoretical model

It is hypothesized that both the optical surface \( S_1 \): \( \vec{r}_1 = \vec{r}_1(u, v, r) \in \mathbb{R}^3 \), \((u, v, r) \in D_1 \subset \mathbb{R}^2 \) and target surface for illumination \( S_2 \): \( \vec{r}_2 = \vec{r}_2(u, v, r) \in \mathbb{R}^3 \), \((u, v, r) \in D_2 \subset \mathbb{R}^2 \) are regular surfaces of \( \mathbb{R}^3 \), that is, there are definite unit normal vectors at various points on \( S_1 \) and \( S_2 \), which are recorded as \( \vec{N}_1(u, v) \) and \( \vec{N}_2(u, v) \) respectively. Point light source \( T : \vec{r} = \vec{r}_0 \) is outside \( S_1 \), any incident ray \( \vec{TP}_1, (P_1 \in S_1) \) is reflected via \( S_1 \) to obtain its reflected ray \( \vec{P}_1\vec{P}_2 (P_2 \in S_2) \). Due to the smoothness and regularity of curved surface, a continuous mapping \( \sigma : \Delta S_1 \rightarrow \Delta S_2 \) can be defined between sufficiently small neighborhoods \( \Delta S_1 \subset S_1 \) of the reflection point \( P_1 \) and \( \Delta S_2 \subset S_2 \) of the receiver point \( P_2 \), and this mapping can be known as one-to-one correspondence by the free surface tailoring method. \( P_1 \) is solely determined by \( \vec{r}_1(u, v) \), and \( P_2 \) is solely determined by \( \vec{r}_2(u, v) \) and \( \sigma : \Delta S_1 \rightarrow \Delta S_2 \), therefore, the mapping can be recorded as

\[
\sigma : \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} f(u, v, r) \\ g(u, v, r) \end{bmatrix}
\]

Wherein, \((u, v, r) \in \Delta D_1 \subset D_1 \), \((u, v, r) \in \Delta D_2 \subset D_2 \), this mapping is only related to the light source \( T : \vec{r} = \vec{r}_0 \) and curved surfaces \( S_1 \) and \( S_2 \). When the reflection point changes in a tiny neighborhood \( \Delta S_1 \) of \( P_1 \) on \( S_1 \), a tiny neighborhood \( \Delta S_2 \) of \( P_2 \) on \( S_2 \) is obtained according to the continuity of \( \sigma \) and under the action of mapping. Thereupon, lights shined into \( \Delta S_1 \) can all be projected onto \( \Delta S_2 \). If illumination intensity at \( P_1 \) is \( E_1 \), light intensity at \( P_2 \) is \( E_2 \), area of \( \Delta S \) is recorded as \( S(\Delta S) \), and diameter of \( \Delta S \) is recorded as \( d(\Delta S) \), then there must be

\[
\lim_{d(\Delta S) \to 0} E_1 \cdot S(\Delta S) = \lim_{d(\Delta S) \to 0} E_2 \cdot S(\Delta S)
\]
energy, that is, the following formula can be obtained:

\[
\frac{E_2}{E_1} = \left( \lim_{d(\Delta S_1) \to 0} S(\Delta S_1) \right) / \left( \lim_{d(\Delta S_2) \to 0} S(\Delta S_2) \right)
\] (2)

Let \( I_1 / I_2 = \lambda \), then

\[
\lambda = \lim_{d(\Delta S_2) \to 0} \frac{S(\Delta S_2)}{S(\Delta S_1)} = \frac{\left| r_{2w_1} \, du \times r_{2v_1} \, dv \right|}{\left| r_{1w_1} \, du \times r_{1v_1} \, dv \right|} = \left[ \left( r_{2u_1} \cdot f_{u_1} + r_{2v_1} \cdot g_{u_1} \right) du \times \left( r_{2u_1} \cdot f_{v_1} + r_{2v_1} \cdot g_{v_1} \right) dv \right] / \left| r_{1u_1} \, du \times r_{1v_1} \, dv \right| \] (3)

Unit direction vector of the incident ray \( \overrightarrow{TP_1} \) is recorded as \( \overrightarrow{\rho} \), if \( \overrightarrow{n_1} \) is the unit normal vector of \( S_1 \) at \( P_1 \), then direction vector of the emergent ray can be obtained as

\[
\overrightarrow{A} = \frac{n_1}{n_2} \overrightarrow{\rho} + \left( -\left( \overrightarrow{\rho} \cdot \overrightarrow{N_1} \right) \cdot \frac{n_1}{n_2} + \sqrt{1 - \left( \frac{n_1}{n_2} \right)^2} \right) \overrightarrow{N_1} \] (4)

according to the Snell’s law. In addition, direction of the emergent ray can also be expressed as \( \overrightarrow{P_2P} \), which is recorded as \( \overrightarrow{b} \), that is, \( \overrightarrow{b} = \overrightarrow{P_2P} = \overrightarrow{r_2} - \overrightarrow{r_1} \). Then \( \overrightarrow{b} = \overrightarrow{r_2} (f(u_1,v_1),g(u_1,v_1)) - \overrightarrow{r_1}(u_1,v_1) \) is obtained by the definition of the mapping \( \sigma \).

Both \( \overrightarrow{A} \) and \( \overrightarrow{b} \) represent the direction of emergent ray, therefore, \( \overrightarrow{A} // \overrightarrow{b} \), that is

\[
\overrightarrow{A} \times \overrightarrow{b} = 0
\] (4)

Formula (4) is the implicit definition of the mapping \( \sigma \). Both sides of formula (4) are derived by \( u_1 \) to obtain

\[
\overrightarrow{A} \times [r_{2u_1} \cdot f_{u_1} + r_{2v_1} \cdot g_{u_1}] = \overrightarrow{A} \times \frac{r_{1u_1}}{A_{u_1}} \times \overrightarrow{b}
\] (5)

Likewise, there is

\[
\overrightarrow{A} \times [r_{2u_1} \cdot f_{v_1} + r_{2v_1} \cdot g_{v_1}] = \overrightarrow{A} \times \frac{r_{1v_1}}{A_{v_1}} \times \overrightarrow{b}
\] (6)

Formula (7) can be obtained by mutual cross product of both sides of formulae (5) and (6) as follows:
\[
(A \times r_{2v_2}) \times (A \times r_{2v_2}) \cdot \frac{\partial (f, g)}{\partial (u_1, v_1)} = (A \times \hat{b} - A \times r_{1u}) \times (A \times \hat{b} - A \times r_{1v}) \tag{7}
\]

Moreover,
\[
(A \times r_{2v_2}) \times (A \times r_{2v_2}) = r_{2v_2} \cdot (A \cdot (A \times r_{2v_2})) - A \cdot (r_{2v_2} \cdot (A \times r_{2v_2})) = 0
\]
\[
= \hat{A} \cdot (A \cdot (A \times r_{2v_2})) = \hat{A} \cdot (A \cdot (r_{2v_2} \times r_{2v_2}))
\tag{8}
\]

Formula (8) is substituted into formula (7) to obtain
\[
\hat{A} \cdot (A \cdot n_2) \cdot r_{2v_2} \times r_{2v_2} \cdot \frac{\partial (f, g)}{\partial (u_1, v_1)} = (A \times \hat{b} - A \times r_{1u}) \times (A \times \hat{b} - A \times r_{1v})
\tag{9}
\]

Numerator of \( \lambda \) is present in the left side of formula (9), which can be converted into
\[
\lambda = \frac{(\hat{b} \cdot n_2)}{(\hat{b} \cdot n_2)} \cdot (4K_1 \cdot |\hat{p}|^2 + \frac{|\hat{p}|^2}{|\hat{A}|} + 2 \cdot |\hat{p}|^2 + 1)
\]
\[
2 \cdot |\hat{p}|^2 \left[ 1 + \frac{|\hat{p}|}{|\hat{A}|} \right] \cdot (r_{1u} \cdot A, r_{1v} \cdot A) \left( -N_1 \right) \cdot \left( M_1 \right) \left( -L_1 \right) \left( M_1 \right) \left( -L_1 \right) \left( \frac{r_{1u}}{r_{1v}} \cdot \hat{A} \right) - 4H_1 \cdot \frac{|\hat{p}|^2}{(\hat{b} \cdot n_2)}
\tag{10}
\]

Wherein, \( E_1, F_1 \) and \( G_1 \) are coefficients of the first basic formula of the curved surface \( S_1 \), and \( M_1, N_1 \) and \( L_1 \) are coefficients of the second basic formula of the curved surface \( S_1 \).  

2 Improved free surface tailoring method

It is hypothesized that light source is located at point \( s \), direction vector of the incident ray at point \( \hat{p} \) on the optical surface is \( \hat{In} \), direction vector of the reflected (refracted) ray is \( \hat{Out} \), unit normal vector is \( \hat{N} \), and refractive indexes at both sides of the optical surface are \( n_1 \) and \( n_2 \), formula (11) can be obtained according to the Snell's law as follows:

\[
[1 + \frac{n_1^2}{n_2^2} - 2 \frac{n_1}{n_2} (\hat{Out} \cdot \hat{In})]^{1/2} \hat{N} = \hat{Out} - \hat{In}
\tag{11}
\]

The incident ray reaches the point \( \hat{i} \) on target surface after optical surface reflection (refraction), illumination intensity at point \( t \) can be obtained according to the derivation in Section 1 as follows:
It is hypothesized that the second-order equation is integrated and normal of the optical surface is determined, then necessary and sufficient condition of existence of a curved surface that is perpendicular to the vector field $N$ everywhere is that the vector field satisfies the following conditions [5]:

$$\overrightarrow{N} \cdot \text{curl}(\overrightarrow{N}) = 0$$  \hspace{1cm} (13)

Surface figure of the curved surface can be solved according to the above 3 equations.

### 3 Secondary LED optical system design

Luminescent properties of LED are similar to Lambert, when it is applied to lighting, its light intensity must be converted using a secondary optical system. LED is regarded as a point light source, and then its secondary optical system can be designed using the free-form surface tailoring method. During solution, a spherical coordinate system and an equation of the optical surface and target surface are established to solve the surface figure data of the optical surface by computer programming using a numerical method. Here, only the lens face in case of rotational symmetry is calculated to simplify the solution process. Figure (1) is the finally obtained lens. Figure (2) is the light intensity distribution on target surface.

It can be seen from Figure (2) that the purpose of uniform lighting is basically achieved on target surface, but light intensity at some points still fluctuates, causes of which are analyzed as follows: (1) Insufficient calculation precision; (2) Insufficiently precise LED simulation. But it still shows the efficiency of this method.
4 Conclusion
Free surface tailoring method, an important method of illumination optical design, is improved in this paper to simplify the calculation process of solving the optical surface figure. Secondary LED optical system is designed using an improved tailoring method, which has proved the efficiency of this method.

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