How the presence of a gas giant affects the formation of mean-motion resonances between two low-mass planets in a locally isothermal gaseous disc

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ABSTRACT
In this paper we investigate the possibility of a migration-induced resonance locking in systems containing three planets, namely an Earth analog (1\(M_\oplus\)), a super-Earth (4\(M_\oplus\)) and a gas giant (one Jupiter mass). The planets have been listed in order of increasing orbital periods. All three bodies are embedded in a locally isothermal gaseous disc and orbit around a solar mass star. We are interested in answering the following questions: Will the low-mass planets form the same resonant structures with each other in the vicinity of the gas giant as in the case when the gas giant is absent? More in general, how will the presence of the gas giant affect the evolution of the two low-mass planets? When there is no gas giant in the system, it has been already shown that if the two low-mass planets undergo a convergent differential migration, they will capture each other in a mean-motion resonance. For the choices of disc parameters and planet masses made in this paper, the formation of the 5:4 resonance in the absence of the Jupiter has been observed in a previous investigation and confirmed here. In this work we add a gas giant on the most external orbit of the system in such a way that its differential migration is convergent with the low-mass planets. We show that the result of this set-up is the speeding up of the migration of the super-Earth and, after that, all three planets become locked in a triple mean-motion resonance. However, this resonance is not maintained due to the low-mass planet eccentricity excitation, a fact that leads to close encounters between planets and eventually to the ejection from the internal orbits of one or both low-mass planets. We have observed that the ejected low-mass planets can leave the system, fall into a star or become the external planet relative to the gas giant. In our simulations the latter situation has been observed for the super-Earth. It follows from the results presented here that the presence of a Jupiter-like planet might have a strong influence on the architecture of planetary systems. Moreover, the planet ejections due to the gas giant action may lead to the formation of a population of low-mass freely floating planets.

Key words: methods: numerical - planets and satellites: formation

1 INTRODUCTION
The rapid progress in instrumentation, observational techniques and strategies has provided an enormous amount of data, which has been used for intensive planet searches. The rate of planetary system discoveries is impressive: After 20 years of observations, we know already over 1000 confirmed planets and a few thousand of planet candidates (Borucki et al., 2011; Batallia et al., 2012). An even more compelling motivation for the research presented here is the growing evidence that multi-planet systems are the rule rather than the exception. The number of such systems at the time of writing this paper has reached over 170. In some of them the planetary orbits remain in mean-motion commensurabilities (e.g. Lissauer et al., 2011b; Steffen et al., 2012; Fabrycky et al., 2012; Steffen et al., 2013). For example, the authors of the recent paper (Steffen et al., 2013) have reported that in all thirteen systems confirmed by their investigations there are planets near one of the first order mean-motion resonances. More precisely, they have found pairs of planets near the 2:1, 3:2, 4:3, 5:4 and 6:5 commensurabilities. There is a strong indication that in some of those systems there could be multiple pairs of planets near first order mean-motion resonances which might form triple resonances. This seems to be the case of...
Kepler-53 (4:2:1) or Kepler-60 (20:15:12). The importance of 3-body resonances has been recently discussed also by Migaszewski et al. (2012) in the context of the dynamics of Kepler-11. All the planets involved in the mean-motion commensurabilities mentioned above are super-Earths with masses in the range of 3 - 10 $M_{\oplus}$. This suggests that complex multiple resonant structures might form in systems with low-mass planets only. Systems of this kind are probably not rare, because preliminary results of the Kepler mission seem to confirm the expectations that low-mass planets are numerous (Ida & Lin, 2005). The case of two resonant low-mass planets migrating in a gaseous protoplanetary disc has been studied in details by Papaloizou & Szuszkiewicz (2005, 2010). Their main finding is that, once convergent migration takes place, it is very likely that the planets end up in one of the mean-motion commensurabilities, whose type is critically dependent on the migration rate. In the most commonly realized resonances the values of $p$ appearing in the period ratio $(p+1) : p$ are greater when the planet masses are disparate than when they are equal. This is a straightforward consequence of the difference in the migration rates. In fact, the differential migration of equal mass planets in the Earth mass range is slower than that of disparate mass planets. An analytic criterion for the occurrence and maintenance of resonances between low-mass planets has been provided in Papaloizou and Szuszkiewicz (2010).

However, it is licit to expect that most systems will contain not only low-mass planets, but also more massive planets like gas giants. For this reason, the aim of this paper is to investigate the behaviour of pairs of planets with masses in the Earth mass range, similar to those considered in Papaloizou and Szuszkiewicz (2005), when there is in the system an additional, more massive planet. In other words, we ask ourselves how the presence of a giant planet can influence the dynamics of a system containing two low-mass planetary companions. The continuous flow of observational information about the dynamics and structure of such systems offers a strong motivation for our investigations on the characteristics of the planetary configurations formed during the early stages of the evolution with particular attention given to resonant configurations.

The situation is more complex if we have more than two planets, as it was shown for example by Marzari et al. (2010) or Moeckel and Armitage (2012) for a three giant planet system. In the case of a system with three planets we can observe indeed a variety of dynamical behaviours, such as planet merging, orbital exchange or scattering of the planets. We study first the evolution of a system containing only two low-mass planets embedded in a gaseous protoplanetary disc. Then, we add a Jupiter-like gas giant on the external orbit relatively to both low-mass planets and compare the evolution of the low-mass planets before and after this addition. We choose to put a gas giant on the external orbit in order to create an environment favourable for the formation of the commensurabilities between the super-Earth and the gas giant. Podlewksa & Szuszkiewicz (2008). It is very difficult to get a resonance capture in the opposite configuration (Podlewksa & Szuszkiewicz, 2009, Podlewksa et al., 2012).

We expect that the evolution of the system might be strongly dependent on the migration rate of the giant planet, so we perform our simulations in discs with different viscosities, changing in this way the migration speed of the gas giant. As it is shown by the simulations, it turns out that also the migration of low-mass planets can be affected by the viscosity of the disc. A similar effect has been already observed by Masset et al. (2006). They have explained it by non-linearities of the flow around a low-mass planet that can cause an additional torque from the co-orbital region which might slow down its migration rate.

We find that in the locally isothermal discs the following sequence of events occurs during the evolution of a three-planet system consisting initially of two low-mass planets (an Earth analog and a super-Earth with the mass of four Earth masses) and a gas giant (with the mass of Jupiter): The system evolves first into an unstable triple resonance, followed by the ejection of one or both low-mass planets. Therefore, the two most frequent outcomes of the system evolution are that the gas giant remains the most internal planet or one of the low-mass planets stays in resonant configuration with the gas giant. If similar conditions are common among the observed young planetary systems, the planets ejected during the evolution of such systems might form a population of planets freely floating in the space.

The paper is organized as follows. In Section 2 we describe the methodology followed in the numerical simulations, while in Section 3 we present the behaviour of a system with two low-mass planets, showing how the viscosity affects their evolution. The results of the simulations with a gas giant with two low-mass planets are discussed in Section 4 and partly also in Section 5, where our conclusions are also drawn.

2 DESCRIPTION OF THE NUMERICAL SIMULATIONS

We have performed full 2D numerical simulations of planets embedded in a gaseous protoplanetary disc using the hydrodynamical code NIRVANA. For details on the numerical scheme employed in the code see Nelson et al. (2000). We use polar coordinates ($r, \phi$) with the origin located at the position of the central star.

In our simulations the mass unit is the mass of the central star. The initial semi-major axis of the super-Earth defines the unit of length. The unit of time is the orbital period of the initial orbit of the super-Earth divided by $2\pi$. In these units, the gravitational constant $G = 1$. With the above settings, our computational domain extends from 0.33 to 5 and it is divided uniformly into 489 and 512 grid cells in the radial and azimuthal directions respectively. The radial resolution of the grid is therefore $\delta r = 9.6 \times 10^{-3} = 0.192 H/r$, where $H$ is the semi-thickness of the disc. This means that, before the low-mass planet motion becomes dominated by the presence of the gas giant in the disc, i.e. when $r \geq 0.78$, $H$ is resolved by no less than 4 grid cells at the planet locations. Instead, the width of the horseshoe region is covered just by one grid cell, which indicates that the corotation torque is poorly resolved. However, this should not affect our results, because we expect the corotation torque to saturate under the conditions adopted in this study (see Section 5). The resolution of our grid is a compromise between making the size of the disc appropriate for modelling our three-planet system and being able to follow the evolution of planets for a sufficiently long period of time. In
order to strengthen the validity of our results, we have verified that the final outcome of our simulations does not change if we double the resolution in each direction (978 and 1024 grid cells in the radial and azimuthal directions respectively). We choose open radial boundary conditions, so that the material in the disc can outflow through the boundaries of the computational domain according to the viscous evolution of the disc. The disc has constant aspect ratio \( h = H/r = 0.05 \) and uniformly distributed surface density \( \Sigma_0 = 6 \times 10^{-4} \). This value of \( \Sigma_0 \) corresponds to the minimum mass solar nebula (MMSN) around 5.2 AU if one takes the length unit equal to 5.2 AU and the mass unit equal to the solar mass. We use the locally isothermal equation of state and do not take into account the disc self-gravity. Those assumptions have been made in the previous paper (Papaloizou & Szuszkiewicz, 2003), which is the starting point for the present study. This is why we have not employed here a more realistic equation of state and the disc self-gravity has been neglected. The gravitational potential is softened with softening parameter \( \varepsilon = 0.8H \) and we do not exclude any material within the Hill sphere. Crida et al. (2009) argue that the material inside the Hill sphere should be very carefully taken into account in numerical simulations when the self-gravity of the gas is neglected. How this should be realized in a proper way is still a subject of intensive research. However, to take fully into account self-gravity is not essential for the purpose of this study. We have rather concentrated on the influence of the relative migration rate which has been tuned by the choice of different values of the viscosity. Finally, to close the list of assumptions, the planets do not accrete matter from the disc. In all simulations in which there is a gas giant in the system, all planets are kept on fixed orbits until the giant planet opens a deep gap in the disc and only after that time they are allowed to migrate.

In the simulations presented here the central star has a Solar mass, so the masses of the planets are respectively \( m_E = 1 M_\oplus \), \( m_{SE} = 4 M_\oplus \) and \( M_J = 1 M_\oplus \), where \( M_\oplus \) is the Earth mass and \( M_J \) is the Jupiter mass. The planet with mass \( m_E = 1 M_\oplus \) (called hereafter “the Earth analog” or “the Earth-like planet”) is located initially on the innermost circular orbit at the distance \( r_E = 0.83 \). Another planet with the mass \( m_{SE} = 4 M_\oplus \) (which will be called “the super-Earth”) is located initially at \( r_{SE} = 1 \), also on the circular orbit. This configuration of the low-mass planets is similar to the one described in Papaloizou & Szuszkiewicz (2003). With respect to Papaloizou & Szuszkiewicz (2003), in the present work the kinematic viscosity \( \nu \) is introduced and the effects of the addition of a gas giant with the mass of Jupiter in the system are investigated. We have initially placed the gas giant at \( r_J = 1.62 \), which enables the formation of any of the first order mean-motion resonances. Then, we have moved its location to \( r_J = 1.35 \) and \( r_J = 1.28 \). The kinematic viscosity is changed from simulation to simulation and can take the values \( \nu = 0.2 \cdot 10^{-6}, 6 \cdot 10^{-6} \) and \( 10^{-5} \) expressed in our units. The motion of the planets is calculated using a standard leapfrog integrator.

3 THE EVOLUTION OF A SYSTEM WITH TWO LOW-MASS PLANETS

First of all, we investigate the behaviour of a system containing two low-mass planets embedded in a gaseous protoplanetary disc. This will be a useful reference for comparison with the full three-planet system that is the main goal of this study. The initial set up for our simulations is illustrated in Fig. 1. Before adding a gas giant to the system, we check the dependence of the planet migration rate on the value of the kinematic viscosity of the disc. The values of \( \nu \) range from 0 to \( 10^{-5} \), so that we can control the effects of the viscosity on the planet migration. This will allow us to perform in the next Section a series of simulations with different migration speeds of the gas giant while retaining full control of possible changes caused by the viscosity in the low-mass planet migration. The migration rates of the two low-mass planets in discs with four different values of viscosity are shown in Fig. 2 separately for the Earth analog and the super-Earth before the two planets start to interact strongly with each other.

The left panel presents the migration rate of the inner planet with mass of \( 1 M_\oplus \). We can see that for lower viscosity the migration is faster. In the right panel of Fig. 2 we plot the migration rates of the outer planet with the mass of \( 4 M_\oplus \). For both planets the migration rate changes with the viscosity in a similar quantitative way, but the migration of the super-Earth is affected by the viscosity much stronger than that of the less massive planet. The lowest black line appearing in the left panel and that in the right panel denote the migration rates of the planets with masses \( 1 M_\oplus \) and \( 4 M_\oplus \) respectively calculated according to well known 3D prescription of the type I migration given by equation (70) in Tanaka et al (2002). However, recent developments in the calculations of the torque acting on low-mass planets (Paardekooper & Mellema 2006; Baruteau & Masset 2008; Kley & Crida 2008; Paardekooper et al. 2010, 2011) have revealed that the speed and even the direction of the migration can be changed if we take into account a more refined treatment of the disc structure. According to Paardekooper et al. (2010) for the locally isothermal approximation which we use here, we expect to have a slower migration rate than those described in Tanaka et al. (2002).

This is illustrated in Fig. 2 where the upper black lines denote the migration rates obtained for locally isothermal
The migration rates of planets with the masses $1M_\oplus$ (left panel) and $4M_\oplus$ (right panel) in the discs with four different kinematic viscosities. The grey curves denote the results for $\nu = 10^{-5}, 6\cdot 10^{-6}, 2\cdot 10^{-6}$ and 0 from top to bottom respectively. The upper and lower black lines denote the migration rates calculated according to Paardekooper et al. (2010) (equation (49) in their paper) and Tanaka et al. (2002) (equation (70) in their paper) respectively.

The left panel present the semi-major axis ratios of the planets embedded in a disc with the kinematic viscosities 0 (gray colour) and $2\cdot 10^{-6}$ (black colour). The right panel shows the semi-major axis ratios of the planets embedded in the disc with the viscosities $6\cdot 10^{-6}$ (gray colour) and $\nu = 10^{-5}$ (black colour) where the differential migration is so slow that planets attain the 5:4 resonance after a longer evolution time.

Our simulations fits better to the upper black lines obtained after Paardekooper et al. (2010) than to the lower ones by Tanaka et al. (2002). Despite the differences in the migration rates due to the disc viscosity, in all our simulations the planets are captured eventually in the 5:4 resonance. To illustrate this, in Fig. 3 we have plotted the evolution of the semi-major axis ratios of the two low-mass planets in discs with four different values of the viscosity. The results of our simulations are presented in two separate panels, because the resonant capture occurs at different timescales. The resonance capture in the discs with the two lower values of the viscosity ($\nu = 0$ and $\nu = 2\cdot 10^{-6}$) took place after 6000 and 11000 time units respectively, see left panel. In the discs with the higher values of the viscosity ($\nu = 6\cdot 10^{-6}$ and $\nu = 10^{-5}$) it occurred after 40000 and 50000 time units respectively, see right panel. Note that the same resonance (5:4) obtained here has been reported by Papaloizou & Szuszkiewicz (2003) for planets embedded in an inviscid disc with the same surface density as in our simulations. In Fig. 3 (right panel) one can observe another interesting phenomenon, namely the passage through one of the higher order resonances (most likely 9:7) in the case of viscosity $\nu = 10^{-5}$ (black curve) at around $2\cdot 10^4$ time units.

As soon as the capture in the mean-motion resonance...
 occurs, the eccentricities of the planets increase. Assuming that the eccentricity of the super-Earth $e_{SE}$ is negligible, which is true for our simulations ($e_{SE} \approx 0.007$), Papaloizou & Szuszkiewicz (2001) gave an analytic prescription for the final value of the eccentricity of the inner planet $e_E$ in the inviscid disc

$$e_E^2 = \left( \frac{m_{SE}}{m_E} \left( \frac{a_{SE}}{a_E} \right)^{1/2} - 1 \right) \times \frac{m_E}{0.578(p+1)(m_E + m_{SE})} \left( \frac{H}{r} \right)^2,$$

where $p$ is an integer. According to this estimate, the eccentricity should reach 0.048 in the 5:4 resonance ($p=4$), which is in very good agreement with the eccentricity of the Earth-like planet in an inviscid disc shown in black colour in Fig. 4. As mentioned previously, if the viscosity in the disc is higher, the migration of low-mass planets is slower. Thus, also the eccentricity should reach lower values in discs with higher viscosity, as it is indeed observed in our simulations. For example, in Fig. 4 it is shown (gray colour line) the evolution of the eccentricity of the Earth-like planet in the disc with viscosity $2 \cdot 10^{-6}$. A similar correlation between the migration rate and the eccentricity evolution was observed in Crida et al. (2008) for systems of two giant planets.

### 4 THE EVOLUTION OF TWO LOW-MASS PLANETS IN THE PRESENCE OF A GAS GIANT

In the previous Section we have presented the most likely resonant configuration for a two low-mass planet system: an Earth analog and a super-Earth embedded in a locally isothermal gaseous disc with the properties described in Section 2. Here, we show how the presence of a gas giant can influence the evolution of such a system. The giant planet is located initially on the external orbit (see Fig. 5 outside any first order mean-motion resonance and migrates with speed determined by the viscosity of the disc. In our simulations we consider two values of $\nu$, namely $2 \cdot 10^{-6}$ and $10^{-5}$. Fig. 4 shows how the gas giant modifies the migration rate of the super-Earth. The left panel of that Figure displays how the semi-major axes of the planets' orbits evolve in a disc with viscosity $2 \cdot 10^{-6}$. The same information, but for a disc with $\nu = 10^{-5}$, is presented in the right panel.

The initial evolution of the semi-major axes of the two low-mass planets in the presence of the gas giant (gray colour) is compared with the situation in which there is no massive planet in the system (black colour). The presence of the gas giant in the system makes the migration of the 4$M_E$ planet faster than before. This results in turn in a faster differential migration of the two low-mass companions. The relatively slow migration of the gas giant, which occurs in the disc with $\nu = 2 \cdot 10^{-6}$, causes the low-mass planets to pass through the 5:4 mean-motion commensurability without being captured in it. They proceed instead to another first order commensurability, namely the 6:5. In the disc with higher viscosity ($\nu = 10^{-5}$), the Jupiter migrates very fast and this leads to the 1:1 resonant locking between the super-Earth and the Earth analog. In the left and right panel of Fig. 5 we plot the evolution of the semi-major axis ratios of the planets in the discs with the kinematic viscosities $2 \cdot 10^{-6}$ and $\nu = 10^{-5}$ respectively. The aim of these plots is to show what will be the final outcome of the evolution of the two systems described above: Will the 6:5 resonance actually take place and will the 1:1 resonance configuration survive during further migration?

Let us start from the case in which the two low-mass planets are approaching the 6:5 resonance, i.e. when they are embedded in a disc with viscosity $\nu = 2 \cdot 10^{-6}$. At the beginning of the simulations, the gas giant is placed very close to the 2:1 resonance with the super-Earth. For the first several hundred time units of its evolution, the differential migration of these two planets is convergent. There is a sign of the excitation of the super-Earth eccentricity, which is clearly seen in Fig. 5 (left panel) and which is due to the vicinity of the 2:1 resonance. However, the capture into the 2:1 commensurability did not take place at this early stage, because, after the short period of the convergent migration, the planets have migrated divergently for several hundreds of time units. Only after that time the process of the formation of the 2:1 resonance started and we have to wait for its completion for almost 40000 time units. The low-mass planets have reached a position close to the resonance after approximately 6000 time units. This is revealed by the libration of the resonance angle $\phi = 6 \lambda_{SE} - 5 \lambda_E - \omega_E$ around a fixed value. Here $\lambda_{SE}$ and $\lambda_E$ denote the mean longitudes of the super-Earth (SE) and the Earth analog (E) respectively, while $\omega_E$ is the longitude of the pericentre of the Earth analog. Initially the libration amplitude is large (170 degrees), but by the time of 20000 time units it decreases to 50 degrees and after 35000 time units is around 25 degrees, as it can be seen from Fig. 5. The appearance of the resonant angle defined above is not associated with a significant increase in the eccentricities of the planets' orbits. The signature of the 6:5 resonance is present also in the behaviour of the angle between the apsidal lines, shown in Fig. 5 (left panel). In this way, the system has attained the configuration in which the Jupiter and the super-Earth are locked in the 2:1 commensurability and, in addition, the super-Earth and the Earth analog are locked in the 6:5 commensurability. This means that the resonant configuration is established also between the Jupiter and the Earth-like planet. Their period ratio equals 12:5. Thus, the three planets in the system are locked in the triple 12:6:5 mean-motion
Figure 6. The comparison of the evolution of semi-major axes of low-mass planets in the presence of the gas giant (gray colour) and for the system without the Jupiter (black colour). The left and right panels show the evolution of low-mass planet in the discs with the kinematic viscosities $\nu = 2 \cdot 10^{-6}$ and $10^{-5}$ respectively. The location of the Jupiter-like planet is not shown in these figures. The upper pair of lines refers to the semi-major axis of the super-Earth, the lower one to that of the Earth-like planet.

Figure 7. The evolution of the semi-major axis ratios of the planets embedded in the discs with the kinematic viscosities $\nu = 2 \cdot 10^{-6}$ (left panel) and $\nu = 10^{-5}$ (right panel). The horizontal lines denote the exact position of the resonance.

resonance. The illustration of this statement is presented in Fig. 10. However, such resonant configuration is not maintained during the further evolution. Soon after the capture in the triple resonance, the orbits of both low-mass planets become chaotic (see Fig. 7, left panel, the uppermost and lowest lines almost at the end of the displayed period of the evolution) and the super-Earth is shifted on the most external orbit. On the other hand, in the disc with viscosity $10^{-5}$ the gas giant first captures the super-Earth in the 2:1 commensurability and then the super-Earth captures the Earth-like planet into the 1:1 commensurability. As a consequence, the three planets form the triple 2:1:1 resonant configuration which, however, does not survive during the further evolution. The lightest planet is ejected from the system after roughly 9000 time units, see Fig. 4 right panel, lower line. Despite the scattering occurring in the system, the gas giant and the super-Earth remain locked in the commensurability till the end of the simulation (see Fig. 11). The capture in the 1:1 resonance has already been observed in a variety of numerical studies, see e.g. Thommes (2005), Beauge et al. (2007), Crida (2009), Cresswell & Nelson (2008, 2009), Podlewska-Gaca & Szuszkiewicz (2011). Here we would like to take a closer look at the type of motion of the two low-mass planets in the co-orbital configuration induced by the vicinity of a migrating gas giant and affected by interaction of the 2:1 resonance. In Fig. 11 we present the evolution of the orbits of the two co-orbital planets and the resonance angle, which for the 1:1 commensurability is just equal to the difference between the mean longitudes of the planets. The amplitude of libration of the resonance angle is very large, which makes difficult to determine the type of the planet motion. During the first part of the co-orbital evolution, after a very short transient event (close encounter), which took place at about 2300 time units, the planets gradually settled down on mutual horseshoe orbits. The resonant angle librates roughly around 180 degrees and the relative variation of the semi-major axis of the Earth-like planet is large. At around 5000 time units the situation has changed. The amplitude of the semi-major axis of the Earth-like planet has decreased significantly and a value around which the resonant angle librates decreases slowly in time, reaching towards the end of our simulations (between 7000 and 7800 time units) the value about 60 degrees. This could be an indication that at this stage of the evolution the planets move on the tadpole orbits. Finally, at around 9000 time units,
the Earth-like planet is ejected from the system, falling into the star.

The fact that triple resonances are not maintained might be a consequence of the eccentricity excitation of the low-mass planet orbits. Indeed, after the capture in the 2:1 resonance, the eccentricities of both low-mass planets increase significantly. For the disc with viscosity $2 \cdot 10^{-6}$, the eccentricity of the Earth-like planet reaches 0.12 and that of the super-Earth approaches 0.45. In the case of the disc with higher viscosity ($\nu = 10^{-5}$), the eccentricities increase to even higher values, namely 0.55 and 0.8 for the Earth analog and the super-Earth respectively. The motion on such highly eccentric orbits may lead to the chaotic behaviour in the system which is observed in our simulations and may be related to the disappearance of the triple resonances.

Thus, we argue that, under the mutual conditions discussed in this Section, in systems containing a gas giant and two low-mass planetary companions, the capture in the triple resonance due to convergent migration can be very common at the early stages of the evolution. However, the final configuration of such systems consists of a gas giant and only one low-mass planet on the internal orbit. The second low-mass planet is moved from the internal orbit onto the external one or is entirely ejected from the system due to the eccentricity excitation of the planetary orbits.

5 DISCUSSION AND CONCLUSIONS

In Section 4 we have considered the situation in which the gas giant is located initially outside any first order mean-motion resonance, namely at the distance $r_J = 1.62$. Now, we discuss the possible final architectures for a few other initial locations of the Jupiter-like planet, assuming that such configurations were formed in the previous stages of the evolution of the planetary systems.

First, we place the gas giant at the distance $r_J = 1.35$. If the viscosity of the disc is low ($2 \cdot 10^{-6}$), the most inner planet moves on a chaotic orbit since the beginning of the simulation. Meanwhile, the Jupiter and the super-Earth approach the 3:2 commensurability. The eccentricity of the super-Earth increases only to about 0.1, which is lower than what was observed for this resonance in our previous simulations (Podlewski & Szuszkiewicz, 2008). Moreover, the angle between the apsidal lines oscillates in the whole range from 0 to $2\pi$. Finally, after 16000 time units both low-mass planets are scattered from the disc. If instead the viscosity of the disc is high ($\nu = 10^{-5}$), the convergent migration leads to the temporary capture in the triple 4:3:2 mean-motion resonance. During the evolution of such configuration, the eccentricities of the low-mass planets increase and, as a result, the Earth analog is ejected from the system. This event has a big effect on the system, because the commensurability between the Jupiter and the super-Earth is destroyed. Soon after this happens, also the second low-mass planet is scattered from the disc. The final result in both cases, regardless of the value of the viscosity, is just a single gas giant orbiting the star.

In the next simulation, the gas giant is placed initially even closer than before, i.e. at $r_J = 1.28$. In this case, we find that in the disc with low viscosity ($\nu = 2 \cdot 10^{-6}$), the Jupiter captures the super-Earth into the 3:2 resonance. Then, the Earth-like planet is ejected from the system and the resonant structure between the gas giant and the super-Earth is lost. However, during the further evolution of the system, the convergent migration brings the super-Earth and the gas giant back into that commensurability.
Figure 10. Left panel: The evolution of the angles between apsidal lines of the Earth-like planet and the super-Earth (top panel), the Earth-like planet and the gas giant (middle panel), and the super-Earth and the gas giant (bottom panel) in the disc with the viscosity $2 \times 10^{-6}$. Right panel: The same as in left panel but for the disc with the viscosity $10^{-5}$.

In the disc with viscosity $10^{-5}$, the super-Earth is scattered into the most external orbit at a distance close to $r = 2$ and slowly migrates towards the Jupiter. We expect that the super-Earth will stop to migrate just outside the 2:1 external resonance with the gas giant as it has been shown in Podlewska et al. (2012). The super-Earth still did not reach the predicted location at the time of the end of simulations. The Jupiter, instead, captures the Earth analog in the 2:1 resonance and this commensurability survives till the end of the simulation. Again, the final configuration consists of
the gas giant and only one low-mass planet on the internal orbit.

Concluding, in Section 3 we have investigated systems of two planets with masses $1M_\oplus$ and $4M_\oplus$ embedded into a locally isothermal disc. It has already been demonstrated that for such systems, evolving in the inviscid disc, the capture into a first order mean-motion resonance is very likely if the relative migration of the planets is convergent (Papaloizou & Szuszkiewicz 2002, 2010). We confirm here this result also for discs with viscosities in the range between $0$ and $10^{-5}$. In all our simulations the planets attain finally the 5:4 resonance, even if the path through which it occurs can be very different from one case to another.

The main goal of this study was to investigate how the architecture of a system changes if it contains not only a pair of low-mass planets, but also a Jupiter mass gas giant located on the most external orbit. Shall we expect to assist to the formation of a resonant configuration in such systems? Apart from those cases in which both or one low-mass planets are ejected from the system at the very early stages of the simulations, the most common configuration is a triple commensurability. However, in all our simulations this is just a transitional configuration. It never survives for a long time. Soon after the resonant locking, the eccentricity of both low-mass planets increases and close encounters between planets occur. Finally, the ejection of one or both low-mass planets from the internal orbits takes place. The most likely final scenarios of the evolution of the systems described in this paper, for given planetary masses and initial planet locations, are that i) only the giant planet is left in the system or ii) a resonant pair of planets remains that can consist in one of the following: The gas giant and the super-Earth on the internal orbit or the gas giant and an Earth analog on the external orbit.

Within our simulations we have observed also cases in which a low-mass planet moves from the internal to the external orbit due to the action of the Jupiter-like planet. Interestingly enough, the low-mass planet is located close to the 2:1 resonance but not exactly in resonance. This might be a consequence of the interaction of the low-mass planet with the density waves excited by the gas giant described in details in Podlew ska et al. (2012). To sum up, it is important to stress here that the variety of outcomes described above is mainly due to the action of the gas giant. Because of the presence of the Jupiter in the disc, in fact, the initial migration rates of the low-mass planets undergo very quickly an alteration. After that, the motion of the low-mass planets is dominated by the interaction with the gas giant and the waves excited by it in the disc.

Our conclusions are concerning a system of three planets embedded in a locally isothermal gaseous disc. One needs to be aware of all limitations these assumptions can bring. Let us present a straightforward argument, which will help to justify the use of the locally isothermal equation of state in our simulations. To this purpose, we have first evaluated the opacity of the disc at the initial location of the super-Earth, namely at $r = 5.2$ AU. Adopting the values of surface density, aspect ratio, and distance considered in our paper, we have found that the disc is optically thick at that position. Next, we have evaluated the migration rate from equations (50-53) in Paardekooper et al. (2011), which give the total torque acting on a low-mass planet consisting of the Lindblad torque plus the corotation torque. Finally, we have compared this migration rate with that obtained in the case of the locally isothermal limit using equation (49) in Paardekooper et al. (2010), getting an agreement between the two values within a few percents. This upshot confirms that we have studied here a particular case of the migration rates of the planets where the corotation torque can be neglected, namely the planets, which move in an optically thick disc well modelled by the locally isothermal approximation. Other disc conditions may lead to other migration rates (in particular for the terrestrial planets), see Bitsch et al. (2013).

The most obvious extension of this work will be to move into a more realistic equation of state or ideally to use a self-consistent radiative model of the protoplanetary discs. This will allow us to understand how the low-mass planets migrate in the given physical conditions occurring in the observed discs. Much work has already been done towards this aim (see for instance Paardekooper & Mellema 2003; Paardekooper & Papaloizou 2008; Kley & Crida 2008; Baruteau & Masset 2008; Bitsch et al. 2010, 2011).

So far, we know examples of systems in which low-
mass planets are close to the 5:4 commensurability (Kepler-11, Lissauer et al., 2011a) or form triple resonances or even multiple commensurabilities involving a 1:1 resonance (Lissauer et al., 2011b; Rivera et al., 2010). All these structures are seen also in our numerical simulations at some stage of the evolution. On the other side, neither a super-Earth nor an Earth analog orbiting on the internal orbit relative to a gas giant and locked with it in a mean-motion resonance have been detected yet. However, our simulations indicate that these configurations are quite likely to form at the early stages of the planetary system evolution. The increasing number of data (from the numerous planet searches) should shed more light on the existence of such configurations soon.

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