The study of double stratification and viscous dissipation effects on chemically radiative MHD free convective flow of micropolar fluid embedded in non-Darcian porous medium

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Abstract

Numerical expression for steady 2D natural convection magnetohydrodynamics flow of micropolar fluid past along a vertical surface embedded through non-Darcian porous medium in the presence of heat source, chemical reaction, viscous dissipation and thermal radiation is obtained. Variable wall temperature and concentration in a doubly stratified are taken into account. The initial governing boundary layer equations are transformed to a system of ODEs, which are then solved numerically by using the Runge–Kutta–Fehlberg fourth–fifth order method along shooting technique. A parametric study is conducted and so that numerical result are obtained for the velocity, micro-rotation, temperature and concentration as well as the local friction factor coefficient, local couple stress, local Nusselt and Sherwood numbers for different values of the governing parameters, namely, the magnetic parameter, material parameter, Prandtl number, radiation parameter, chemical reaction parameter and viscous dissipation and discussed in detail. Comparison in limiting case is also part of present study to validate the obtained results.

Keywords: Chemical reaction, viscous dissipation, MHD, thermal radiation, micropolar fluid, double stratification, Darcy number, porous medium.
Introduction

Micropolar fluids are a subset of the micromorphic fluid theory introduced in a pioneering paper by Eringen [1-2]. Micropolar fluids are those consisting of randomly oriented particles suspended in a viscous medium, which can undergo a rotation that can affect the hydrodynamics of the flow, making it a distinctly non-Newtonian fluid. They constitute an important branch of non-Newtonian fluid dynamics where microrotation effects as well as microinertia are exhibited. Eringen’s theory has provided a good model for studying a number of complicated fluids, such as colloidal fluids, polymeric fluids and blood. A good insight of this division is given by many researchers (see [3] – [9]).

Porous medium is a very important aspect in Science and Engineering which is described as a medium or material that contains pores or spaces between solid materials or solid matrix through which liquids or gases can pass. Rosali et al. [10] studied the micropolar fluid flow towards a stretching sheet and shrinking sheet in present of porous medium with suction. The effect of medium permeability on thermal convection in micropolar fluids is considered by Sharma and Gupta [11]. The micropolar fluid flow through porous media was studied by researchers [Kamel et al. [12], Hamdan and Rehkopf [13], Kim and Fedorov [14]].

Research in the area of magnetohydrodynamics (MHD) and thermal radiation has also been developed in many directions, and industry has exploited the use of magnetic fields in controlling a range of fluid flow and thermal processes. The influence of magnetism on electrically conducting flows has been reported with a plethora of other physical phenomena. The study of flow and heat transfer for an electrically conducting micropolar fluid past a porous plate under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems, such as MHD generators, plasma studies, nuclear reactors, oil exploration, and geothermal energy extraction. Thus, keeping in mind the specific industrial application related to polymer processing technology, numerous attempts have been made to analyze the effects of a transverse magnetic field on boundary layer flow characteristics. Das [15 ] prepared a report on effects of chemical reaction and thermal radiation on heat and mass transfer flow MHD micropolar fluid in a rotating frame of reference. The combined effects of viscous dissipation and Joule heating on MHD free convection flow of
micropolar fluid in presence of constant heat and mass fluxes is carried out by Haque et al. [16]. Ibrahim et al. [17] illustrated the impact of viscous dissipation on MHD mixed convection flow of micropolar fluids under the influence of thermal radiation. Radiation effect on convective heat and mass transfer flow over a porous plate embedded in a Darcy-Forchheimer porous medium was investigated by Mukhopadhyay et al. [18]. Pal and Mondal [19] explored the influence of thermal radiation on MHD Darcy-Forchheimer mixed convection flow past a stretching sheet embedded in porous medium. MHD free convection heat and mass transfer flow in a micropolar fluid over a vertical porous medium with radiation and chemical reaction was proposed by Abdou and EL-Kabeir [20]. Abo-Eldahab and EL-Gendy [21] explained convective heat transfer past a continuously moving plate embedded in a non-Darcian porous medium under the influence of magnetic field. Mabood and Ibrahim [22] have studied effects of Soret and non-uniform heat source on MHD non-Darcian convective flow over a stretching sheet in a micropolar fluid in presence of viscous dissipation and radiation.

Double stratifications are a natural process that describes the layering of bodies of water based on their temperature. It occurs mainly because of temperature variations due to the presence of different fluids of different density. This natural process creates a transition zone of temperature gradient between cold and hot fluid zones. In a case of natural convection and boundary layer analysis, thermal stratification plays an important role in vertical temperature distribution. The concept of thermal stratification is based on the division of water bodies about a surface/plate into three layers known as epilimnion, metalimnion and hypolimnion. Considering the model of Double stratifications, many scientists have problems of engineering interests viz. Singh and Kumar [23], Srinivasacharya and RamReddy [24], Murthy et al. [25], Upendar and Srinivasacharya [26], Hayat et al. [27]. Srinivasacharya, and Surender [28].

To the best of our knowledge, there are no works reported on the double stratification and viscous dissipation on MHD natural convective micropolar fluid along a vertical surface embedded in non-Darcian porous medium with different levels of thermal radiation. The nonlinear ODEs are solved numerically by using the Runge–Kutta–Fehlberg fourth–fifth order method along shooting technique. Moreover graphical and tabular results are demonstrated and investigated. The present paper is the extension of the previous study on the effects of thermal
radiation on MHD free convective micropolar fluid flow along a vertical surface embedded in a non-Darican thermally stratified medium (Koriko et al. [29]).

Mathematical Formulation

We consider a steady 2D viscous incompressible electrically conducting micropolar fluid along a vertical surface embedded in a non-Darcian double stratified porous medium. Here keeping the origin fixed, the sheet is then stretched with a velocity \( u(x) \), varying linearly with the distance from the slit. The flow is assumed to flow in \( x \)-direction which is along vertical surface and \( y \)-axis is normal to it. Fluid suction/injection is imposed at the plate surface. The temperature of the surface \( T_w \) is held uniform at which is higher than the ambient temperature \( T_\infty \) (i.e \( T_w > T_\infty \)). The concentration of the surface \( C_w \) is held uniform at which is higher than the ambient concentration \( C_\infty \) (i.e \( C_w > C_\infty \)). In this investigation, the thermal and solutal stratification is properly accounted for by modifying all temperature(concentration) of the surface and ambient temperature(concentration). The uniform magnetic field of magnitude \( B_0 \) is applied normal to the plate. Also the magnetic Reynolds number is assumed to be small so the induced magnetic field is negligible in comparison to the applied magnetic field. Under the foregoing assumptions with the Boussinesq approximation, the governing equations of the MHD free convection flow are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \frac{\mu + k}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho} \frac{\partial N}{\partial y} - \frac{\sigma B^2(x)}{\rho} u - \frac{9}{k} u - \frac{b^2}{k} u^2 + g \beta_T (T - T_\infty) + g \beta_C (C - C_\infty), \tag{2}
\]

\[
u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \left( \frac{\mu + k}{\rho} \right) \frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho} \frac{2N + \partial u}{\partial y}, \tag{3}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_l}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_e}{\partial y} + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{k_l a}{\rho c_p} \left( A(T_w - T_0) e^{-\frac{\mu}{\sqrt{B}}} + B(T - T_\infty) \right) \tag{4}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - Kr (C - C_\infty) \tag{5}
\]
Corresponding boundary conditions are

\[ u = u_w(x) = ax, \quad v = v_w(x), \quad N = -n \frac{\partial u}{\partial y}, \quad T = T_w, \quad C = C_w, \quad \text{at } y = 0, \]

\[ u \to 0, \quad N \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as } y \to \infty \]  

(6)

In this study, wall temperature (concentration) and free stream temperature (concentration) are defined as

\[ T_w = T_0 + m_1 x, \quad T_\infty = T_0 + m_2 x, \quad C_w = C_0 + m_3 x, \quad C_\infty = C_0 + m_4 x, \]  

(7)

with \( m_1, m_2, m_3 \) and \( m_4 \) as constants.

where \( u \) and \( v \) are components of velocity in \( x \) and \( y \) directions respectively, \( u_w(x) \) is the wall shrinking or stretching velocity, \((a>0)\) for stretching, \((a < 0)\) for shrinking and \((a = 0)\) for static wall, \( v_w(x) \) is the wall mass flux velocity, \( \rho \) is the fluid density, \( \mu(=\nu \rho) \) is the dynamic viscosity, \( \nu \) is the kinematic viscosity, \( \sigma \) is the electrical conductivity, \( j \) is the micro-inertial density, \( \tau \) is the vortex viscosity, \( \beta ' \) is the thermal expansion coefficient, \( \beta \) is the solutal expansion coefficient, \( Kr \) is the chemical reaction rate constant, \( \alpha = \frac{k}{\rho \alpha} \) is the thermal diffusivity, \( k_1 \) is the thermal conductivity, \( q_w \) is the wall heat flux, \( n \) is a constant such that \( 0 \leq n \leq 1 \). When \( n = \frac{1}{2} \) , we have the vanishing of anti-symmetric part of the stress tensor and denotes weak concentration of microelements, the case \( n = 1 \) is used for the modeling of turbulent boundary layer flows. This investigation reports that the case we consider is when \( n = 0 \) (called strong concentration) which represents cent rated particle flows in which the microelements close to the wall are unable to rotate, then, \( N = 0 \) near the wall and \( N \) is the micro-rotation or angular velocity whose direction of rotation is in the \( xy \)-plane. In this paper, we study, a case when \( n = 0 \) is considered. The micopolar parameter or material parameter is \( K = \frac{k}{\mu}, \quad K \neq 0 \) for micropolar fluid and \( K = 0 \) for classical Newtonian fluid. Any of these assumptions is invoked to allow the field of equations that predicts the correct behavior in the limiting case when the microstructure effects became negligible and the total spin \( N \) reduces to
the angular velocity Adhikari and Maiti [30]. By the Rosseland approximation the radiative heat flux can be reduced in the form

\[ q_r = -\frac{4\sigma_s}{3k^*} \frac{\partial T^4}{\partial y} \]  

(8)

where \( \sigma_s \) is the Stephen Boltzmann constant and \( k^* \) is the mean absorption coefficient.

It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences within the flow are sufficiently small, then equation (7) can be linearized by expanding \( T^4 \) into the Taylor series about \( T_\infty \), which after neglecting higher order terms takes the form

\[ T^4 \approx 4T^3_\infty T - 3T^4_\infty \]  

(9)

In view of the equations (8) and (9), equation (4) becomes;

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_1}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{16\sigma_s T^3_\infty}{3k^*} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{k_1 a}{\rho c_p} g \left( A(T_u - T_0) e^{-\frac{\eta}{\gamma_y}} + B(T - T_\infty) \right) \]  

(10)

The continuity equation (1) is satisfied by introducing a stream function \( \psi \) such that

\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \]  

(11)

The momentum, angular momentum, energy and concentration equations can be transformed into the corresponding ordinary differential equations by the following transformation

\[ \zeta = y \sqrt{\frac{a}{g}}, \quad \psi(x, y) = x \sqrt{a g}(\zeta), \quad N = ax \sqrt{a g}(\zeta), \quad \theta = \frac{T - T_\infty}{T_u - T_0}, \quad \phi = \frac{C - C_x}{C_w - C_0} \]  

(12)

where \( \eta \) is the independent dimensionless similarity variable. Thus \( u \) and \( v \) are given by

\[ u = ax f'(\zeta), \quad v = -\sqrt{a g}(\zeta), \]  

substituting variables (12 ) into equations (2) (3) (11 ) and (5), we obtain the following ODEs

\[ (1 + K) f'' + f f' - f'^2 + Kg' - Mf' = -\frac{F_s}{Da} f'^2 - Psf' + Gr\theta + Gc\phi = 0 \]  

(13)
\begin{equation}
\left(1 + \frac{K}{2}\right)g'' + fg' - f'g - K(2g + f') = 0
\end{equation}
\begin{equation}
\left(1 + \frac{4}{3R}\right)\theta'' - \Pr \varepsilon_1 f' - \Pr \theta f' + \Pr f' \theta + \Pr E c f'' + \Pr (A e^{-\eta} + B \theta) = 0
\end{equation}
\begin{equation}
\phi'' + Sc (f \phi' - f' \phi) - Sc \varepsilon \phi - Sc \varepsilon \phi = 0
\end{equation}

The corresponding boundary conditions are
\[ f(0) = S, \quad f'(0) = 1.0, \quad g(0) = 0, \quad \theta(0) = 1 - \varepsilon_1, \quad \phi(0) = 1 - \varepsilon_2 \]
\[ f'(\infty) \to 0, \quad g(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0 \]

In the above equations, primes denote differentiation with respect to \( \zeta \). The dimensionless velocity, angular velocity, temperature and concentration are represented as \( f(\zeta), g(\zeta), \theta(\zeta) \) and \( \phi(\zeta) \) respectively,

\( Gr = \frac{g \beta_f (T_w - T_0)}{\alpha^2 x} \) is the Grashof number, \( Gc = \frac{g \beta_c (C_w - C_0)}{\alpha^2 x} \) is the modified Grashof number, \( M = \frac{\sigma B_0^2}{\rho a} \) is the magnetic parameter, \( S = -\frac{v_w(x)}{\sqrt{a \theta}} \) is the constant mass flux with \( s > 0 \) for suction and \( s < 0 \) for injection, \( F_s = \frac{b^*}{x} \) is local Forchheimer parameter, \( Da = \frac{k^*}{x^2} \) local Darcy parameter, \( P_s = \frac{\theta}{k^* a} \) Porosity parameter, \( Pr = \frac{\theta}{a} \) is the Prandtl number, \( Ec = \frac{\theta^2 x^2}{c_p (T_w - T_\infty)} \) is Eckert number, \( \varepsilon_1 = \frac{m_1}{m} \) is thermal stratification parameter, \( \varepsilon_2 = \frac{m_1}{m_3} \) is solutal stratification parameter, \( R = \frac{kk^*}{4 \sigma \gamma T^3} \) is the radiation parameter, \( Sc = \frac{\theta}{D_m} \) is the Schmidt number and \( \gamma = \frac{K}{\partial a^2} \) is the chemical reaction parameter.

The physical quantities of the interest are the skin friction coefficient \( C_f \), local Nusselt number \( Nu_s \) and local Sherwood number \( Sh_s \) are defined as;
\[ C_f = \tau_w/\rho U^2/2, \quad Nu_x = \frac{xq_w}{k_1(T_w - T_\infty)}, \quad Sh_x = \frac{xq_w}{k_m(C_w - C_\infty)} \]

where the wall shear stress \( \tau_w \), heat flux \( q_w \) and mass flux \( k_m \) are given by

\[ \tau_w = \left( (\mu + k) \frac{\partial u}{\partial y} + kN \right)_{y=0}, \quad q_w = -k_1 \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad q_m = -k_m \left( \frac{\partial C}{\partial y} \right)_{y=0} \]

Where \( k \) being the thermal conductivity, using the similarity variables \( (\cdot) \), we get

\[ \frac{1}{2} C_f \text{Re}_{\infty}^{1/2} = (1 + (1-n)K) f^*(0), \quad \text{Re}_{\infty}^{1/2} = g'(0) \quad \text{Nu}/\text{Re}_{\infty}^{1/2} = -\theta'(0), \quad \text{Sh}/\text{Re}_{\infty}^{1/2} = -\phi'(0) \]

where the local Reynolds number is given by \( \text{Re}_{\infty} = \frac{ux}{\nu} \).

**Results and Discussion**

Eqs. (13)-(16) constitute a highly non-linear coupled boundary value problem of third and second-order. So we develop most effective numerical shooting technique with sixth-order Runge-Kutta integration algorithm. To select \( \zeta_{\infty} \) we begin with some initial guess value and solve the problem with some particular set of parameters to obtain \( f^*(0), g(0), \theta'(0) \) and \( \phi'(0) \).

The solution process is repeated with another larger value of \( \zeta_{\infty} \) until two successive values of \( f^*(0), g(0), \theta'(0) \) and \( \phi'(0) \) differ only after desired digit signifying the limit of the boundary along \( \zeta \).

To analyze the results, numerical computations are carried out for different in the governing parameters such as Grashof number \( Gr \), modified Grashof number \( Gc \), magnetic factor \( M \), Forchheimer parameter \( Fs \), Darcy number \( Da \), suction/injection parameter \( S \), permeability parameter \( Ps \), thermal Stratification parameter \( \varepsilon_1 \), Solutal Stratification parameter \( \varepsilon_2 \), Prandtl number \( Pr \), thermal radiation parameter \( Nr \), temperature dependent heat source \( A/B \), Eckert number \( Ec \), Schmidt number \( Sc \) and chemical reaction factor \( \gamma \) on dimensionless velocity, angular velocity, temperature and concentration. In the current study, the following default parameter values are adopted for computations: \( Gr = 2.0, Gc = 2.0, K = 1.0, M = 2.0, S = 0.5, Fs = 0.5, Da = 10, Ps = 0.5, \varepsilon_1 = 0.3, \varepsilon_2 = 0.3, R = 2.0, Pr = 0.71, Ec = 0.05, A = 0.5, B = 0.3, Sc = 0.6 \) and \( \gamma = 0.5 \).
These values are conserved as common unless specifically pointed out in the appropriate graphs and tables.

Figs. 1-2 display the relationship between thermal Grashof number $Gr$ and velocity profiles. It is noticed that velocity rises with $Gr$. This is due to the fact that the rise in thermal buoyancy force enhances the velocity. From Fig. 2, it is noted that that velocity rises with $Ge$. This is due to the fact that the increase in species buoyancy force increases the velocity.

The effect of the magnetic field factor $M$ on the dimensionless velocity distributions is shown in Fig. 3. It is identified that velocity decreases with the magnetic parameter $M$. This is due to the Lorentz force created by the magnetic field.

Fig. 4 shows the dimensionless velocity profiles for various values of Darcy parameter $Da$. It can be observed that increase in $Da$ raises the velocity. The depreciation of velocity with porosity parameter $Ps$ is given in Fig. 5. This is due to the increase of resistive force introduced by the porosity of the medium.

The influence of the Forchheimer parameter $Fs$ on the velocity field is presented in Fig. 6. In the absence of $Fs$, the present study implies to the MHD free convective heat and mass transfer flow in a micropolar fluid along a vertical surface embedded in a porous medium under the influence of radiation and double stratification. The increase in $Fs$ indicates that the porous medium is offering more resistance to the fluid flow. This results in depreciation of the velocity profile.

The effects of the micopolar parameter $K$ on non-dimensional velocity profiles is displayed in Fig. 7. As $K$ increases, the microconcentration which alter the flow field increases and as a result boundary layer thickness also increases.

Figs. 8-10 depict the effect of thermal stratification parameter, solutal stratification parameter and $v$ suction/injection parameter on non-dimensional velocity profile. The thermal stratification depreciates the effective convective potential between the heated plate and ambient fluid in the medium as a result velocity decreases with the increase of thermal stratification parameter $\varepsilon_1$. It is shown in Fig. 8. Similar effect is observed with the solutal stratification parameter $\varepsilon_2$ which is shown in Fig. 9. From Fig. 10, as suction parameter increases, the momentum boundary layer thickness declines, while the reverse phenomenon is observed for the case of injection.

The results included in Fig. 11 clarify that increasing values of micropolar parameter $K$ increasing the angular velocity fields. Figs. 12 and 13 provide the effect thermal stratification
parameter $\varepsilon_1$ and solutal stratification parameter $\varepsilon_2$ on angular velocity. It is observed that angular velocity increases rapidly up to $\zeta < 1$ and then declines gradually in the flow domain $1 < \zeta < 5$.

In case of higher Prandtl values the diffusion of heat away from the heated surface is very slow when compared to the smaller Prandtl values. Hence temperature decreases with the increase Prandtl number $Pr$ as shown in the Fig. 14. The effect of radiation parameter $R$ on temperature is displayed in Fig. 15. It is noticed that the temperature depreciates with the increase of $Nr$ due to the absorption of temperature from the fluid.

Fig. 16 reveals the impact of heat source parameters $A$ and $B$ on temperature. It is see that temperature increases with $A$ and $B$. The appearance of exponential term in the space dependent heat source is to yield additional heat energy across the flow region leads to increase in temperature. Fig. 17 is plotted to explain the effect of viscous dissipation $Ec$ on temperature. From these plots, it can be seen that temperature increases with $Ec$. The thermal stratification parameter $\varepsilon_1$ reduces the effective temperature difference between the plate and the ambient fluid and as a result temperature decreases with $\varepsilon_1$. This is given in Fig. 18.

Figs. 19 – 20 show the dimensionless concentration profiles for different values of Schmidt number $Sc$ and chemical reaction parameter $\gamma$. As the $Sc$ and $\gamma$ increases, concentration profile decreases. It is observed that, higher the molecular diffusivity, larger the concentration in the flow region. Fig. 21 describes the effect of solutal stratification parameter $\varepsilon_2$ on concentration profiles. It is noticed that the concentration of the fluid decreases with the increase of $\varepsilon_2$.

A comparative study in the special situation is made with the work done by Koriko et al. [29]. Here good agreement is noted which is represented in Table 1.

The variation in local skin-friction coefficient, local couple stress, local Nusselt number and local Sherwood number for various parameters are investigated through tables 2 - 3. The behavior of theses physical parameters is self evident from tables 2 and 3 and hence they are not discussed any further to keep brevity.
Fig. 1. The effect of Gr on the velocity profile.

Fig. 2. The effect of Gc on the velocity profile.
Fig. 3. The effect of $M$ on the velocity profile.

Fig. 4. The effect of $Da$ on the velocity profile.
Fig. 5. The effect of $Ps$ on the velocity profile.

Fig. 6. The effect of $Fs$ on the velocity profile.
Fig. 7. The effect of $K$ on the velocity profile.

Fig. 8. The effect of $\varepsilon_1$ on the velocity profile.
Fig.9. The effect of $\varepsilon_2$ on the velocity profile.

Fig.10. The effect of $S$ on the velocity profile.
Fig. 11. The effect of $K$ on the angular velocity profile.

Fig. 12. The effect of $\varepsilon_i$ on the angular velocity profile.
Fig. 13. The effect of $\varepsilon_2$ on the angular velocity profile.

Fig. 14. The effect of $Pr$ on the temperature profile.
Fig. 15. The effect of $Nr$ on the temperature profile.

Fig. 16. The effect of $A$ and $B$ on the temperature profile.
Fig. 17. The effect of $Ec$ on the temperature profile.

Fig. 18. The effect of $\epsilon_1$ on the temperature profile.
Fig. 19. The effect of $Sc$ on the concentration profile.

Fig. 20. The effect of $\gamma$ on the concentration profile.
Table 1. Comparison of the results of present and HAM on $f''(0), h'(0)$ and $\theta'(0)$ for the various values of $\varepsilon_1$ when $M = 1.0$, $K = 1.0$, $Gr = 1.0$, $A = 0.4$, $B = 0.2$, $Ps = 0.4$, $Pr = 0.71$, $Fs = 0.5$, $Da = 0.5$, $R = 0.7$, $S = 0.3$, $Gc = 0.0$, $Sc = 0.0$ and $Ec = 0.0$.

| $\varepsilon_1$ | $f''(0)$ | $h'(0)$ | $\theta'(0)$ | $f''(0)$ | $h'(0)$ | $\theta'(0)$ |
|-----------------|----------|----------|--------------|----------|----------|--------------|
| 0.4  | -1.07323092325528 | 0.29572619181767834 | -0.1817220296067319 | -1.0728488 | 0.2953457 | -0.18000991 |
| 0.6  | -1.14391400312143 | 0.3135060136745343 | -0.1402800127453434 | -1.1436765 | 0.3131998 | -0.13974543 |
| 0.8  | -1.2149010051245 | 0.33164300216441233 | -0.0961960011275332 | -1.2140489 | 0.3315309 | -0.09604299 |

Table 2 Variations in the local friction factor coefficient, local couple stress, local Nusselt and Sherwood numbers at different non-dimensional governing parameters

| $Gr$ | $Gc$ | $M$ | $Fs$ | $Da$ | $Ps$ | $f''(0)$ | $h'(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|------|------|-----|------|------|------|----------|----------|-------------|-------------|
| 2.0  | 2.0  | 1.0 | 0.5  | 0.5  | 0.5  | 1.34120  | 0.330142  | 0.357895    | 0.3847842   |
Table 3 Variations in the local friction factor coefficient, local couple stress, local Nusselt and Sherwood numbers at different non-dimensional governing parameters

| $K$  | $\varepsilon_1$ | $\varepsilon_2$ | $S$ | $A/B$ | $Nr$ | $Ec$ | $Sc$ | $\gamma$ | $f''(0)$ | $h'(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|------|-----------------|-----------------|-----|-------|------|------|------|---------|----------|--------|------------|------------|
| 3.0  | 0.5             | 0.5             | 0.5 | 0.5/0.3 | 0.5  | 0.03 | 0.6  | 0.5     | 1.44214  | 0.430545| 0.458885  | 0.484775  |
| 4.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 3.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 4.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 2.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 3.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 0.1  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 0.3  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 1.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 3.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 1.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 2.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 3.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 1.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 2.0  |                 |                 |     |        |      |      |      |         |          |        |            |            |
| 0.7/0.2 |               |                 |     |        |      |      |      |         |          |        |            |            |
| 1.0/0.5 |               |                 |     |        |      |      |      |         |          |        |            |            |
Conclusions

The present work deals with the numerical analysis of Double stratification on MHD free convective flow of a micropolar fluid along a vertical surface embedded in non-Darcian porous medium with radiation, chemical reaction, viscous dissipation and non-uniform heat source. From this study we observe that

- An increase in $\text{Sc}$ and $\gamma$ reduces the concentration boundary layer thickness.
- An increase in $\varepsilon_1$, depreciates velocity and temperature. It has negligible effect on concentration profiles. Angular velocity rises up to certain value $\zeta$ and then decreases.
- Rise in $\varepsilon_2$, causes depreciation velocity and concentration and increase in temperature. Angular velocity rises up to certain value $\zeta$ and then decreases.
- Temperature depreciates with $R$.
- The magnitudes of velocity and angular velocity enhance with $K$.
- Heat and mass transfer coefficients decline with $Fs$.
- Heat source parameters $A$ and $B$ cause a remarked decease in $f''(0)$, $g'(0)$ and $-\theta'(0)$ and increase in $-\phi'(0)$.
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