Activated Sphalerons and Large Extra Dimensions

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Abstract

We present a new scenario for the baryon number violation that may take place in models with large extra dimensions. Our idea is interesting because leptogenesis with a low reheating temperature requires an alternative source of the $B + L$ violation to convert the produced leptons into baryons. If one considers a model with an intermediate compactification radius, the reheating temperature may be allowed to become higher than the critical temperature of the conventional electroweak phase transition. Our mechanism is still important in this case, because it provides the phase boundary, which is an essential ingredient of the electroweak baryogenesis.

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1 Introduction

In spite of the great success in the quantum field theory, there is still no consistent scenario in which the quantum gravity is included. The most promising scenario in this direction would be the string theory, where the consistency is ensured by the requirement of additional dimensions. Initially the sizes of extra dimensions had been assumed to be as small as $M_p^{-1}$, however it has been observed later that there is no reason to believe such a tiny compactification radius\[1\]. The large extra dimension may solve the hierarchy problem. Denoting the volume of the $n$-dimensional compact space by $V_n$, the observed Planck mass is obtained by the relation $M_p^2 = M^*_{n+2} V_n$, where $M^*$ denotes the fundamental scale of gravity. If one assumes more than two extra dimensions, $M^*$ may be close to the electroweak scale without conflicting any observable bounds. Although such a low fundamental scale considerably improves the situations of the traditional hierarchy problem, the scenario requires some degrees of fine-tuning. The largeness of the quantity $V_n$ is perhaps the most obvious example of such fine-tuning.

In theories with large extra dimensions, measured fermion masses represent a window into ultraviolet physics. One important avenue would be to find models in which the hierarchical patterns of the fermion masses can be produced within natural couplings of order unity in the underlying theory. There is an interesting approach to this problem, which utilizes the locality rather than the symmetry to generate the hierarchy in the Yukawas\[2\].

There is an another problem related to the fine-tuning of the coupling constant. The problem of the $\mu$-term is important when one considers supersymmetric extension.

In this paper we show that some of the mechanisms, which had been put forward to solve the above problems, can trigger the electroweak symmetry restoration at a low temperature to activate sphaleron interactions in a false-vacuum domain. We stress that the false-vacuum domain is a natural product of the required system. Our mechanism is important, because it provides a source of the baryon number violation at a low temperature. The observed baryon number asymmetry of the Universe requires baryon number violating interactions to have been effective but non-equilibrium at the early stages of the Universe. The production of the net baryon asymmetry requires baryon number violating
interactions, C and CP violation and a departure from thermal equilibrium\[3\]. When the fundamental mass scale is sufficiently high, the first two of these ingredients are naturally contained in conventional GUTs or other string-motivated scenarios, and the third can be realized in an expanding universe\[4\]. On the other hand, recent observations of neutrino mixing and the measured values for the differences in mass-squareds make it more plausible for us to include heavy Majorana neutrinos to the Standard Model. These additional neutrinos can naturally be heavy since they are singlets of the Standard Model gauge groups and their masses are not determined by the electroweak scale. If these heavy Majorana neutrinos had existed in the early Universe and had effective CP violation in their decay modes, they can be natural candidates for producing lepton asymmetry via out-of-equilibrium decays at later period. The leptonic asymmetry produced by the decay of these heavy Majorana neutrinos is expected to be converted into the baryon asymmetry of the Universe by sphaleron interactions\[3\]. In models with supersymmetry, lepton number may be produced by the Affleck-Dine mechanism\[3\].

In models with large extra dimensions, however, the situation is rather involved because of the requirement for the low reheating temperature\[7, 11\]. In this respect, it is very important to propose ideas for realizing the baryon number violation at low temperature. In this paper we consider supersymmetric models with large or intermediate extra dimensions and present a mechanism where the required baryon number violation appears in the domain of the false vacuum where sphalerons are activated at the temperature below $T_{EW}$\[11\]. Our idea is very simple and naturally contained in the corresponding models for the Yukawa hierarchy\[2\] or the $\mu$-problem\[14\].

The plan of our paper is the following. In section 2 we show the basic idea\[13\]. In section 3 we consider the mechanism in ref.\[2\] where the observed fermion masses

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Some examples are already discussed in ref.\[12\] for models with warped extra dimension. Aspects of baryogenesis with large extra dimensions are already discussed by many authors. For example, in ref.\[1\], it is argued that the Affleck-Dine mechanism can generate adequate baryogenesis. In ref.\[8\], the global-charge non-conservation due to quantum fluctuations of the brane surface is discussed. The baryogenesis by the decay of heavy X particle is discussed in ref.\[9, 13\]. In ref.\[10\], it is argued that a dimension-6 proton decay operator, suppressed today by the mechanism of quark-lepton separation in extra dimensions can generate baryon number if one assumes that this operator was unsuppressed in the early Universe due to a time-dependent quark-lepton separation.
and mixings are generated by localizing the three generations of matter and the two Higgs fields at different locations in a compact extra dimension. The required domain configuration is a natural ingredient of the model. We show that the Yukawa hierarchy is destabilized and the top Yukawa is exponentially suppressed in the domain of the degenerated false vacuum. Then the electroweak symmetry restoration will take place because the radiative correction from the top is suppressed in the domain. In section 4 we consider another example where the $\mu$-problem is solved by the localization of wavefunctions in the extra dimension. In this case, the hierarchical tiny coupling is not maintained in the false vacuum and large $\mu$-term appears. Then the Higgs potential is stabilized and the electroweak symmetry restoration takes place in the false-vacuum domain.

2 Localized wavefunctions and the false vacuum

Our idea is based on ref. [13]. In ref. [13], we argued that the localized wavefunctions in extra dimensions are displaced in the false vacuum in the four-dimensional spacetime so that the exponentially suppressed interactions are enhanced in the domain of the false vacuum. Localization of a superfield is already discussed in ref. [2, 16].

2.1 Localized wavefunctions along the fat domain wall

To show the elements of our idea, here we limit ourselves to the non-supersymmetric construction with fermions localized within only one extra dimension [17]. To localize fields in an extra dimension, it is necessary to break higher dimensional translation invariance, which is accomplished by a spatially varying expectation value of the five-dimensional scalar field $\phi_A$ that forms a thick wall along the extra dimension. If the scalar field $\phi_A$ couples to a five-dimensional fermionic field $\psi_i$ through the five-dimensional Yukawa interaction $g_{\phi_A} \bar{\psi_i} \psi_i$, it is possible to show that the fermionic field localizes at the place where the total mass in the five-dimensional theory vanishes. For definiteness, we consider

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See also ref. [15]
the Lagrangian

\[ L = \bar{\psi}_i (i \gamma_5 + g_i \phi_A(y) + m_{5,i}) \psi_i + \frac{1}{2} \partial_\nu \phi_A \partial^\nu \phi_A - V(\phi_A), \] (2.1)

where \( y \) is the fifth coordinate of the extra dimension. For the special choice \( \phi_A(y) = 2 \mu^2 y \), which corresponds to approximating the kink with a straight line interpolating two vacua, the wave function in the fifth coordinate becomes gaussian centered around the zeros of \( g_i \phi_A(y) + m_{5,i} \). It is also shown\(^[1]\) that a chiral fermionic field in the four-dimensional representation can result from the localization mechanism. When two fermions \( \psi_p \) and \( \psi_q \) have five-dimensional masses \( m_{5,p} \) and \( m_{5,q} \), the corresponding localizations are at \( y_p = -\frac{m_{5,p}}{2g_\mu^2} \) and \( y_q = -\frac{m_{5,q}}{2g_\mu^2} \), respectively. The shapes of the fermion wave functions along the fifth direction are

\[ \Psi_p(y) = \frac{\mu^{1/2}}{(\pi/2)^{1/4}} e^{\mu^2(y - y_p)^2} \quad \Psi_q(y) = \frac{\mu^{1/2}}{(\pi/2)^{1/4}} e^{\mu^2(y - y_q)^2}. \] (2.2)

The operator that contains both \( \psi_p \) and \( \psi_q \) is exponentially suppressed in the effective four-dimensional theory. For example, one may expect the following operator in the five-dimensional theory,

\[ \mathcal{O}_5 \sim \int d^5x [\phi_B \psi_p^5 \psi_q^5] \] (2.3)

where \( \psi_p^5 \) and \( \psi_q^5 \) are five-dimensional representations of the fermionic fields. The corresponding four-dimensional operator is obtained by simply replacing the five-dimensional fields by the zero-mode fields and calculating the wave function overlaps along the fifth dimension \( y \). The result is

\[ \mathcal{O}_4 \sim \epsilon \times \int d^4x \phi_B \psi_p^4 \psi_q^4, \] (2.4)

where \( p \) and \( q \) denote the four-dimensional representations of the chiral fermionic fields. The small overlap of the fermionic wavefunctions along the fifth dimension suppresses the effective coupling, i.e., \( \epsilon \sim e^{-\mu^2 |y_p - y_q|^2} \).

To construct a required false-vacuum configuration, we extend the above model to include an another scalar field \( \phi_B \) that determines the five-dimensional mass \( m_{5,i}(\phi_B) \).
The field $\phi_B$ determines the position of the center of the fermionic wavefunction along the fifth dimension. We assume that $\phi_B$ does not make a kink configuration along the fifth dimension, but does make a defect configuration in the four-dimensional spacetime. For definiteness, we consider the Lagrangian

$$\mathcal{L} = \bar{\psi}_i \left( i \partial_5 + g_i \phi_A(y) + m(\phi_B)_{5,i} \right) \psi_i + \frac{1}{2} \partial_\nu \phi_k \partial^\nu \phi_k - V(\phi_k),$$

(2.5)

where indices represent $i = p, q$ and $k = A, B$. Here $\phi_A$ makes the kink configuration along the fifth dimension while $\phi_B$ develops defect configuration in the four-dimensional spacetime. For example, $m_{5,i}$ are written as $m(\phi_B)_{5,i} = k_i \phi_B$ and the potential for $\phi_B$ takes the form:

$$V_B = -m_B \phi_B^2 + \lambda_B \phi_B^4.$$  

(2.6)

In this case, because of the effective $Z_2$ symmetry of the potential, the resultant defect is the cosmological domain wall that interpolates between two degenerated vacua. The center of the fermionic wavefunction in the fifth dimension is shifted by the defect configuration of $\phi_B$ in the four-dimensional spacetime.

### 2.2 Localized wavefunctions and the orbifold

If the extra dimension is an orbifold, one can localize the wavefunction at the fixed point [2, 18]. In this case one can find the required two degenerated vacua without adding new field. One is the positive configuration for $0 < y < L$, and the other is the negative one. If the sign is positive, the zero-mode is concentrated at $y = 0$. If it is negative, the zero mode is concentrated at $y = L$. In general, two degenerated vacua generate the domain configuration. If the hierarchical couplings are induced by the above-mentioned mechanism of the orbifold, the hierarchy is destabilized in the quasi-degenerated false vacuum.
3 Destabilized fermion mass hierarchy

In this section we consider a model for fermion masses, where the Yukawa coupling hierarchy is generated due to the localization of fields in extra spatial dimensions. For our purpose, we consider the model in ref. [2], which differs from the Arkani-Hamed/Schmaltz model [17]. In this model, small Yukawa couplings are due to the position of each generation relative to the localized Higgs fields and not due to the splittings of left and right handed fermions. Assuming that the Higgs vacuum expectation value is confined to one of the orbifold fixed points, and that the fermions are localized with O(1) width along the fifth dimension, one can obtain the hierarchical Yukawa couplings by localizing only the third generation quarks at the Higgs boundary. Other quarks are localized at the opposite side so that their Yukawa couplings are suppressed.

For definiteness, we consider a Lagrangian for localizing fermionic field,

\[ \mathcal{L} = \bar{\psi}_i \left( i \partial_5 + g_i \phi(y) \right) \psi_i + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{\lambda}{4} \left( \phi^2 - v^2 \right)^2 , \]  

(3.1)

where the couplings \( g_i \) and \( \lambda \) are real. Applying the simplest set of boundary conditions in ref. [18] and compactifying on a \( Z_2 \) orbifold, one can obtain the localized fermions at the fixed points at \( y = 0 \) or \( y = L \). If \( g_i \phi \) is positive, the zero-mode of \( \psi_i \) is concentrated at \( y = 0 \). If it is negative, the zero mode is concentrated at \( y = L \). In this case one can obtain two degenerated solutions. Two (quasi) degenerated vacua generates a

\[ \text{We notice that our idea is applicable to other models for the Yukawa hierarchy, where the orbifold boundary condition is not responsible for the localization [16,17]. If the Yukawa hierarchy is explained by splittings of the left and the right handed fermions along the fat domain wall in the extra dimension, the large Yukawa coupling may be exponentially suppressed in the false vacuum because the top quark wavefunction along the extra dimension can be displaced by a defect configuration of } \phi_B \text{ in the four-dimensional spacetime. The same idea is already discussed in ref. [13] to solve the difficulties in baryogenesis with large extra dimensions.} \]

\[ \text{The degeneracy is broken if there is an another scalar field } \phi' \text{ that satisfies the same boundary condition. Although the } Z_2^{sim} \text{ symmetry that corresponds to the simultaneous flips of } < \phi > \text{ and } < \phi' > \text{ will remain, the } Z_2 \text{ symmetries of their independent flips are explicitly broken if there exists a cross term in the effective potential. In this case } \phi' \text{ may be a constant which is called “odd mass” in ref. [2]. Even if the degeneracy is not destabilized, the domain wall collapses when there is a bias when they are produced [11].} \]
domain that interpolates between them. What we want to consider in this section is the cosmological implications of the domain structure.

If the hierarchy of fermion masses is due to the above-mentioned localization mechanism of the orbifold, the hierarchy may be destabilized in the false vacuum. If the wavefunction of the top is displaced toward the opposite side of the extra dimension, its Yukawa coupling is exponentially suppressed in the false vacuum. The small top Yukawa prevents the radiative breakdown of the electroweak symmetry in supersymmetric theories. According to ref. [23], the false-vacuum domain may safely survive until \( T = \left( \frac{\sigma^2}{M_p^2} \right)^{1/4} \), where \( \sigma \) is the tension of the corresponding domain wall. This implies that the electroweak symmetry restoration may take place at the temperature lower than \( T_{EW} \) if the tension is smaller than \((10^7 GeV)^3\). If the potential for the field \( \phi \) is a flat potential, the tension of the domain wall is \( \sigma \sim m_{3/2} v^2 \).

## 4 Destabilized \( \mu \)-term

To solve the \( \mu \)-problem within the setup of the extra dimensions, one may use the shining mechanism [14, 24]. Here we consider a model in ref. [14]. The MSSM matter and Higgs fields are assumed to live on a (3+1)-dimensional brane embedded in one extra dimension. Following ref. [24], we employ four-dimensional \( \mathcal{N}=1 \) superspace notation and treating the fifth coordinate \( y \) as a label. The action for a massive five-dimensional hypermultiplet \((\Phi, \Phi^c)\) is

\[
\int d^4x \int d^2\theta \left( \int d^4\Phi^\dagger \Phi + \Phi^c \Phi^c \right) + \int d^2\theta \Phi^c (m + \partial_y) \Phi
\]

\[
+ \int d^4x \int d^2\theta \left( -\delta(y) \sqrt{M_s} J \Phi^c - \delta(y - L) \sqrt{M_s} J^c \Phi + \delta(y) \frac{K}{\sqrt{M_s}} \Phi^c H_u H_d \right)
\]

(4.1)

Although it seems rather difficult to produce these defects merely by the thermal effect after inflation, the nonthermal effect may create such defects during the reheating period of inflation. The nonthermal creation of matter and defect has raised a remarkable interest. In particular, the efficient production of such products during the period of coherent oscillations of inflaton has been studied by many authors [20]. There is an another possibility that the defects are generated after the first brane inflation, while the reheating temperature after the second thermal brane inflation is kept much lower than the electroweak scale [21]. The cosmological constraint on the domain wall that is produced before thermal inflation is already discussed in ref. [22].

\footnote{See ref. [2] for more detail.}
The vacuum equations for the scalar fields are then

$$\Phi_F = -\delta(y - L) \sqrt{M_* J^c} + (m - \partial_y) \phi^c = 0$$

$$\Phi^c_F = -\delta(y) \sqrt{M_* J} + (m - \partial_y) \phi = 0$$

(4.2)

The source $J^c$ at $y = 0$ shines an expectation value for $\phi^c$, which generates the exponentially suppressed $\mu$-term on the matter brane. We are interested in the case where the centers of the localized matter fields in the extra dimension are displaced by a cosmological defect in the four-dimensional spacetime. If the center of the source $J$, or Higgs fields in the extra dimension are displaced in the false-vacuum domain of the four-dimensional spacetime, the hierarchical suppression for the $\mu$-term will be destabilized. The resulting large $\mu$-term stabilizes the Higgs potential in the effective four-dimensional theory. Then the standard scenario of the electroweak symmetry breaking fails and the symmetry restoration takes place in the local domain.

5 Conclusions and Discussions

Leptogenesis with large extra dimensions suffers a serious problem because of the low reheat temperature that makes it impossible to convert the produced leptons into baryons by the $B + L$ violating sphalerons. In this paper we have presented a new scenario for sphaleron activation at the temperature below $T_{EW}$.

If one considers a model with an intermediate fundamental mass, the reheating temperature may be allowed to become higher than the critical temperature of the conventional electroweak phase transition. Our mechanism is still important in this case, because it provides the phase boundary, which is the essential ingredient of the electroweak baryogenesis.

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