On Kaon production in $e^+e^-$ and Semi-inclusive DIS reactions

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Abstract. We consider semi-inclusive unpolarized DIS for the production of charged kaons and the different possibilities to test the conventionally used assumptions $s - \bar{s} = 0$ and $D_s^{K^+ - K^-} = 0$. The considered tests have the advantage that they do not require any knowledge of the fragmentation functions. We also show that measurements of both charged and neutral kaons would allow the determination of the kaon FFs $D_q^{K^+ + K^-}$ solely from SIDIS measurements, and discuss the comparison of $(D_u - D_d)^{K^+ - K^-}$ obtained independently in SIDIS and $e^+e^-$ reactions. All analysis are performed in LO and NLO in QCD. The feasibility of the tests to HERMES SIDIS data is considered.

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1 Introduction

It is well known that neutral current inclusive deep inelastic scattering (DIS) yields information only about quark plus antiquark parton densities. When neutrino experiments are possible one can obtain separate knowledge about quark and antiquark densities, but for the case of polarized DIS this is impossible experimentally. For this case semi-inclusive DIS (SIDIS), where some final hadron is detected, plays an essential role, but requires a knowledge of the fragmentation function (FF) for a given parton to fragment into the relevant hadron. As pointed out in [1] and more recently in [2] a precise knowledge of the FFs is vital. In this paper we examine what we can learn about the kaon FFs from experimental data.

When the spin state of the detected hadron is not monitored, it is possible to learn about the FFs from both $e^+e^- \rightarrow hX$ and unpolarized SIDIS $l + N \rightarrow lhX$. In the case of pion production $SU(2)$ plays a very helpful role in reducing the number of independent FFs needed. For production of charged kaons, which is important for studying the strange quark densities, $SU(2)$ is less helpful, and even a combined analysis of $e^+e^-$ and SIDIS data on both protons and neutrons does not allow an unambiguous determination of the kaon FFs $K$.

It is thus conventional to make certain reasonable sounding assumptions about the strange quark densities and the kaon FFs. In the first part of this paper we discuss to what extend these assumptions can be justified and tested experimentally. We shall discuss tests based on both a leading order (LO) and a next-to-leading order (NLO) approach. For although it has been often assumed that an NLO treatment is essential, in our paper we have kept the LO treatment for two reasons – one always starts with LO and then follows the natural hierarchy LO $\rightarrow$ NLO and also because a recent study in [2] showed that a very acceptable description of the combined polarized DIS and SIDIS data can be achieved in a LO approximation as well and thus LO cannot be ruled out yet.

As mentioned above, SU(2) symmetry is of little help if only charged kaons are measured. However, it is well known that charged and neutral kaons are combined in SU(2) doublets. This relates the FFs of $K^0$ to those of $K^\pm$, which implies that no new FFs appear in $K^0$-production. In the second part of our paper we examine to what extent detecting neutral as well as charged kaons can help to determine the kaon fragmentation functions. We carry out the analysis in LO and NLO.

In Section 2 we recall the general formulae for inclusive $e^+e^-$ and SIDIS. In Section 3 we consider semi-inclusive $K^\pm$ production and possible tests whether, for the quark densities, $s(x) = \bar{s}(x)$, and whether, for the fragmentation functions, $D_s^{K^+}(z) = D_s^{K^-}(z)$. In Section 4 we discuss production of $K^\pm$, $K^0$; in Sections 5 and 6 we consider the combinations $K^+ + K^- - 2K^0$ and $K^+ + K^- + 2K^0$ respectively, both in LO and NLO. Possible tests for the reliability of the leading order treatment of the processes are discussed.

2 General formula for $e^+e^-$ and unpolarized SIDIS

For convenience we shall recall some general formulae for the cross sections and asymmetries in $e^+e^- \rightarrow hX$ and $e + N \rightarrow e + h + X$. 


2.1 $e^+e^- \to hX$

There are two distinct measurements of interest: the total cross section $d\sigma_T^h(z)$ and the forward backward (FB) asymmetry $A_{FB}^h$. If $d^2\sigma^h/(dz d\cos \theta)$ is the differential cross section for $e^+e^- \to hX$, these quantities are defined as:

$$d\sigma_T^h(z) = \int_{-1}^{+1} \left( \frac{d^2\sigma^h}{dz d\cos \theta} \right) d\cos \theta$$

$$A_{FB}^h(z) = \left[ \int_{-1}^{0} - \int_{0}^{+1} \right] \left( \frac{d^2\sigma^h}{dz d\cos \theta} \right) d\cos \theta,$$

where $\theta$ is the CM scattering angle and $z$ is, neglecting masses, the fraction of the momentum of the fragmenting parton transferred to the hadron $h$: $z = 2(P_{h,q})/q^2 = E_h/E$, where $E_h$ and $E$ are the CM energies of the final hadron $h$ and the initial lepton.

From CP invariance it follows that

$$d\sigma_T^h(z) = d\sigma_T^h(-z), \quad A_{FB}^h(z) = -A_{FB}^h(-z),$$

where $h$ is the C-conjugate of the hadron $h$. Eq. (3) implies that the total cross section $d\sigma_T^h$ actually provides information only about $D_{h^+}^q = D_q^h + D_{\bar{q}}^h$, while measurement of $A_{FB}^h$ determines the non-singlet (NS) combinations $D_{h^-}^q = D_q^h - D_{\bar{q}}^h$, and this is true in all orders of QCD.

In LO the formula are especially simple:

$$d\sigma_T^h(z) = 3\sigma_0 \sum_q \hat{c}^2_q D_{q^h}^h, \quad \sigma_0 = \frac{4\pi\alpha_s^3}{3s}$$

$$A_{FB}^h(z) = 3\sigma_0 \sum_q \frac{3}{2} \hat{a}_q D_{q^h}^h.$$

Assuming both photon and Z-boson exchange we have:

$$\hat{c}^2_q(s) = c^2_q - 2c_q v_q v_{qZ} \Re h_Z + \left( v^2_q + a^2_q \right) \left( v_{qZ}^2 + a_{qZ}^2 \right) |h_Z|^2$$

$$\hat{a}_q = 2 a_c a_q \left( -c_{qZ} \Re h_Z + 2 v_{qZ} |h_Z|^2 \right),$$

where $h_Z = [s/(s-m_{Z}^2 + imZ)]/\sin^22\theta_W$. In (6) $v_q$ is the charge of the quark $q$ in units of the proton charge, and, as usual,

$$v_e = -1/2 + 2 \sin^2\theta_W, \quad a_e = -1/2,$$

$$v_{q} = I_{3}^{u} - 2 e_q \sin^2 \theta_W, \quad a_q = I_{3}^{u}, \quad I_{3}^{u} = 1/2, \quad I_{3}^{d} = -1/2.$$

2.2 Unpolarized SIDIS $e + N \to e + h + X$

In semi-inclusive deep inelastic scattering, we consider the non-singlet difference of cross-sections $\sigma_{N}^{h-h}$, where the measurable quantity is the ratio:

$$R_{N}^{h-h} = \frac{\sigma_{N}^{h-h}}{\sigma_{N}^{DIS}}, \quad \sigma_{N}^{h-h} = \sigma_{N}^{h} - \sigma_{N}^{\bar{h}}.$$
3 Production of charged kaons

As seen from (12), in \( R_{K^+K^-} \) both \( s - \bar{s} \) and \( D_{d}^{K^+K^-} \) appear. They are expected to be small, and the usual assumption is that they are equal to zero. Here we examine to what extent one can test these assumptions experimentally in SIDIS.

It was shown in [3], that even if we combine data on the forward-backward asymmetry \( A_{FB}^{K^+K^-} \) in \( e^+e^- \) annihilation with measurements of \( K^+ \) and \( K^- \) production in SIDIS, we cannot determine the fragmentation functions without assumptions. The reason is that we have 3 measurements for the 4 unknown quantities \( D_{u,d,s}^{K^+K^-} \) and \( (s - \bar{s}) \). Thus, one needs an assumption: either \( s - \bar{s} = 0 \) or \( D_{d}^{K^+K^-} = 0 \). In fact, up to now, all analyses of experimental data have been performed assuming both \( s - \bar{s} = 0 \) and \( D_{d}^{K^+K^-} = 0 \).

Note, that from the quark content of \( K^\pm \), the assumption \( D_{d}^{K^+K^-} = 0 \) seems very reasonable if the \( K^\pm \) are directly produced. However, if they are partly produced via resonance decay this argument is less persuasive. Of course \( e^+e^- \rightarrow \Lambda^\pm \Lambda^\mp \) sheds no light on this issue.

3.1 LO approximation, \( K^\pm \)

In LO we have:

\[
\hat{\sigma}_p^{K^+K^-} = \frac{1}{9} [4 u_V D_u^{K^+K^-} + d_V D_d^{K^+K^-} + (s - \bar{s}) D_{\bar{s}}^{K^+K^-}], \\
\hat{\sigma}_n^{K^+K^-} = \frac{1}{9} [4 d_V D_u^{K^+K^-} + u_V D_d^{K^+K^-} + (s - \bar{s}) D_{s}^{K^+K^-}].
\]

From a theoretical point of view it is more useful to consider the following combinations of cross-sections, which, despite involving differences of cross-sections, are likely to be large:

\[
(\hat{\sigma}_p - \hat{\sigma}_n)^{K^+K^-} = \frac{1}{9} [(u_V - d_V) (4 D_u - D_d)^{K^+K^-} + 2 (s - \bar{s}) D_{s}^{K^+K^-}], \\
(\hat{\sigma}_p + \hat{\sigma}_n)^{K^+K^-} = \frac{1}{9} [(u_V + d_V) (4 D_u + D_d)^{K^+K^-} + 2 (s - \bar{s}) D_{s}^{K^+K^-}] .
\]

We define:

\[
R_+(x, z) \equiv \frac{(\hat{\sigma}_p + \hat{\sigma}_n)^{K^+K^-}}{u_V + d_V}, \\
R_-(x, z) \equiv \frac{(\hat{\sigma}_p - \hat{\sigma}_n)^{K^+K^-}}{u_V - d_V} .
\]

From a study of the \( x \) and \( z \) dependence of these we can deduce the following:

1) if \( R_-(x, z) \) is a function of \( z \) only, then, since \( D_{s}^{K^+K^-} \) is a favoured transition, we can conclude that \( (s - \bar{s}) = 0 \).

2) if \( R_+(x, z) \) is also a function of \( z \) only, then, since \( D_{d}^{K^+K^-} \) is a favoured transition, we can conclude that \( (s - \bar{s}) = 0 \).

3) if \( R_+(x, z) \) and \( R_-(x, z) \) are both functions of \( z \) only, and if in addition, \( R_+(x, z) = R_-(x, z) \), then both \( s - \bar{s} = 0 \) and \( D_{d}^{K^+K^-} = 0 \).

4) if \( R_+(x, z) \) and \( R_-(x, z) \) are both functions of \( z \) only, but they are not equal, \( R_+(x, z) \neq R_-(x, z) \), we conclude that \( s - \bar{s} = 0 \) but \( D_{d}^{K^+K^-} \neq 0 \).

5) if \( R_-(x, z) \) is not a function of \( z \) only, then NLO corrections are needed, which we consider below.

The above tests for \( s - \bar{s} = 0 \) and \( D_{d}^{K^+K^-} = 0 \) can be spoilt either by \( s - \bar{s} \neq 0 \) and/or \( D_{d}^{K^+K^-} \neq 0 \), or by NLO corrections, which are both complementary in size.

That's why it is important to formulate tests sensitive to both \( s - \bar{s} = 0 \) and/or \( D_{d}^{K^+K^-} = 0 \) solely, i.e. to consider NLO.

3.2 NLO approximation, \( K^\pm \)

In an NLO treatment it is still possible to reach some conclusions, though less detailed than in the LO case. We now have:

\[
(\hat{\sigma}_p - \hat{\sigma}_n)^{K^+K^-} = \frac{1}{9} [(u_V - d_V) \otimes (1 + \alpha_s C_{qq}) \otimes (4 D_u - D_d)^{K^+K^-}], \\
(\hat{\sigma}_p + \hat{\sigma}_n)^{K^+K^-} = \frac{1}{9} [(u_V + d_V) \otimes (4 D_u + D_d)^{K^+K^-} + 2 (s - \bar{s}) \otimes D_{s}^{K^+K^-}] \otimes (1 + \alpha_s C_{qq}).
\]

Here \( C_{ij} \) are

\[
C_{ij}(y) = C_{ij}^M + [1 + 4\gamma(y)] C_{ij}^L , \\
\gamma(y) = \frac{1 - y}{1 + (1 - y)^2}.
\]

\( C_{ij}^M, L \) being the corresponding Wilson coefficients [4]. Suppose we try to fit both [18] and [19] with one and the same fragmentation function \( D(z) \):

\[
(\hat{\sigma}_p - \hat{\sigma}_n)^{K^+K^-} \approx \frac{4}{9} [(u_V - d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D(z)], \\
(\hat{\sigma}_p + \hat{\sigma}_n)^{K^+K^-} \approx \frac{4}{9} [(u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D(z)] .
\]

If this gives an acceptable fit for the \( x \) and \( z \)-dependence of both \( p - n \) and \( p + n \) data, we can conclude that both \( s - \bar{s} \approx 0 \) and \( D_{d}^{K^+K^-} \approx 0 \) and that \( D(z) = D_{s}^{K^+K^-} \).

Note that for all above tests, both in LO and NLO approximation, we don’t require a knowledge of \( D_{u,d}^{K^+K^-} \).
This is especially important since the $e^+e^-$ total cross section data determine only the $D^{K^+K^-}_{u,d,s}$, and these are relatively well known, while $D^{K^+K^-}_{u,d}$ can be determined solely from $A_{FB}$ in $e^+e^-$ or from SIDIS.

The results of the above tests would indicate what assumptions are reliable in trying to extract the fragmentation functions $D^{K^+K^-}_{u,d,s}$ from the same data.

4 Production of charged and neutral kaons

The description of SIDIS and $e^+e^-$ reactions, in which one monitors neutral $K^0 = (K^0 + K^0)/\sqrt{2}$ as well as charged $K^\pm$ does not require any further FFs. This is due to SU(2) invariance which relates the neutral to the charged kaon FFs:

$$D^{K^+K^-}_{u,d} = D^{K^0K^0}_{d}, \quad D^{K^+K^-}_{d,d} = D^{K^0K^0}_{u,u}, \quad D^{K^+K^-}_{s} = D^{K^0K^0}_{s}. \quad (23)$$

In principle this helps to determine the kaon FFs $D^{K^+K^-}_{u,d,s}$ solely from SIDIS measurements, without the problem of combining $e^+e^-$ data and SIDIS data at widely different value of $Q^2$.

Two possible measurements can be performed: with $(K^+ + K^- - 2K^0)$ and with $(K^+ + K^- + 2K^0)$.

5 The combination $K^+ + K^- - 2K^0$

In NLO we have for $e^+e^-$:

$$\frac{d\sigma_T^{K^+K^- - 2K^0}}{dQ^2}(x,z) = 3\sigma_0(\hat{e}_u^2 - \hat{e}_d^2)\alpha_s \left[ 1 + \frac{\alpha_s}{2\pi} C_F (c_{\lambda}^2 + c_{\lambda,L}^2) \right]$$

$$\times (D_u - D_d)^{K^+K^-}, \quad (24)$$

$$\frac{d\sigma_T^{K^+K^- - 2K^0}}{dQ^2} = d\sigma_T^{K^+K^-} + d\sigma_T^{K^+K^-} - 2d\sigma_T^{K^0},$$

where $c_{\lambda}^2$, $c_{\lambda,L}^2$ are the Wilson coefficients for the contribution of the transverse (T) and longitudinal (L) virtual boson.

and for SIDIS:

$$\frac{d\sigma_T^{K^+K^- - 2K^0}}{dQ^2}(x,z) =$$

$$= \left\{ \frac{1}{9} [4(u + \bar{u}) - (d + \bar{d})] (1 + \frac{\alpha_s}{2\pi} C_{Q\gamma}) \right\} (D_u - D_d)^{K^+K^-}, \quad (25)$$

$$\frac{d\sigma_T^{K^+K^- - 2K^0}}{dQ^2}(x,z) =$$

$$= \left\{ \frac{1}{9} [4(d + \bar{d}) - (u + \bar{u})] (1 + \frac{\alpha_s}{2\pi} C_{Q\gamma}) \right\} (D_u - D_d)^{K^+K^-}, \quad (26)$$

Thus, due to SU(2)-invariance, in all orders of QCD all three processes always measure the same NS combination of fragmentation functions $(D_u - D_d)^{K^+K^-}$, whose evolution does not involve the very poorly known gluon fragmentation functions.

The difference of cross sections $K^+ + K^- - 2K^0$, involving neutral kaons, is essential in order to eliminate, due to SU(2) invariance, the $s + \bar{s}$-quark parton densities and the gluon FF.

Note that the combinations of quark densities in the above do have a singlet component and thus depend on $g(x)$, but that is not a problem.

5.1 LO approximation, $K^+ + K^- - 2K^0$

The LO expressions are particularly simple and obtained from (23) with $\alpha_s = 0$. They imply that SIDIS determines $(D_u - D_d)^{K^+K^-}$ given $(u + \bar{u})$ and $(d + \bar{d})$ are known.

The difference $\tilde{\sigma}_p - \tilde{\sigma}_n$ is:

$$\tilde{\sigma}_p - \tilde{\sigma}_n = (\sigma_p - \sigma_n) (x,z) =$$

$$= 5 \frac{1}{9} [(u + \bar{u}) - (d + \bar{d})] (D_u - D_d)^{K^+K^-}, \quad (27)$$

which is a non-singlet in both the PDs and the FFs. This implies that in its $Q^2$-evolution and in all orders in QCD it will always contain the same NS combinations, convoluted with the corresponding Wilson coefficients when higher orders are considered.

The fact that $e^+e^-$ and SIDIS measure the same combination $(D_u - D_d)^{K^+K^-}$ allows to combine $e^+e^-$ data at $Q^2 \approx m_Z^2$, where $Z^0$-exchange is the dominant contribution, with SIDIS experiments at $Q^2 << m_Z^2$ where $\gamma$-exchange dominates. For example one could test the relation

$$\frac{9 d\sigma_T^{K^+K^- - 2K^0}}{dQ^2}(x,z) = 3 \frac{d\sigma_T^{K^+K^- - 2K^0}}{dQ^2}(x,z) =$$

$$= \frac{[4(u + \bar{u}) - (d + \bar{d})](x,Q^2)}{3 \sigma_0(\hat{e}_u^2 - \hat{e}_d^2) m_Z^4}. \quad (28)$$

Here $d\sigma_T^{K^+K^- - 2K^0}(x,z)$ denotes that the data is measured at $m_Z^2$ and then evolved to $Q^2$ according to the DGLAP equations. This would be a test of LO, but also a test of the factorization of SIDIS into parton densities times FFs.

Tests for whether LO is a reasonable approximation for the SIDIS reactions can be made as follows. In LO one should have:

1) for proton targets

$$\frac{d\sigma_T^{K^+K^- - 2K^0}}{dQ^2}(x,z) = \text{function of } z \text{ only} =$$

$$\equiv f_p(z) = (D_u - D_d)^{K^+K^-}(z) \quad (29)$$
2) for neutron targets
\[
\frac{\sigma^{K^+ + K^− - 2K^0}_n(x, z)}{4(d + d) - (u + \bar{u})} = \text{function of } z \text{ only } = \\
\equiv f_{n}(z) = (D_{u} - D_{d})^{K^+ + K^−}(z), \tag{30}
\]
where the PD's are determined in LO, see for example [6].

3) and if measurements for both proton and neutron targets are available, then also
\[
f_{p}(z) = f_{n}(z) \tag{31}
\]
should hold, as expected from (29) and (30).

The above LO-tests do not require knowledge of the FFs. Concerning the measurement of FFs, an attempt was made in [11] to combine data on $e^+e^-$ and SIDIS. The evolution involved there required an estimate of the gluon FF which induced quite large errors. In the present case, we study the NS combination $(D_{u} - D_{d})^{K^+ + K^−}$, which can be measured both in $e^+e^-$ and SIDIS, (24) - (26), and whose evolution in $Q^2$ is straightforward since it does not involve the gluon FFs.

5.2 NLO approximation, $K^+ + K^− - 2K^0$

In higher orders of QCD the cross sections on $p$ and $n$ with $K^+ + K^− - 2K^0$ depend on the gluon PD – eqs. (25) and (26). The difference of the cross sections on proton and neutron eliminates $g(x)$:
\[
(\bar{\sigma}_p - \bar{\sigma}_n)^{K^+ + K^− - 2K^0}_s(x, y, z) = \\
= \frac{5}{9}[(u + \bar{u}) - (d + \bar{d})]\left(1 + \frac{\alpha_s}{2\pi} \otimes C_{qq}\otimes\right) \\
\times(D_{u} - D_{d})^{K^+ + K^−}., \tag{32}
\]
and (32) determines $(D_{u} - D_{d})^{K^+ + K^−}$ without the influence of even the gluon quarks or any other FF. Note that $(u + \bar{u}) - (d + \bar{d})$ is a NS and thus $g(x)$ will not creep back through the $Q^2$-evolution.

Further, being a NS it would not be a problem to compare the two independent measurements: in $e^+e^-$ annihilation at $Q^2 \simeq m_Z^2$, eq. (21), and in SIDIS at $Q^2 << m_Z^2$, eq. (32). They should give the same result, when evolved to the same $Q^2$ according to the DGLAP equations, and thus present a test of the hypothesis that SIDIS is a product of the quark-production and quark-fragmentation processes. This test would be independent of the gluon and strange PDs or any other FFs and hold in any order in QCD.

Having thus determined $(D_{u} - D_{d})^{K^+ + K^−}$ one may proceed to determine the gluon PD, without the uncertainties of $s + \bar{s}$, measuring the sum of the same cross sections on $p$ and $n$:
\[
(\bar{\sigma}_p + \bar{\sigma}_n)^{K^+ + K^− - 2K^0}_s(x, y, z) = \\
= \frac{1}{3}\left[[u + \bar{u}) + (d + \bar{d})]\left(1 + \frac{\alpha_s}{2\pi} \otimes C_{qq}\otimes\right) + \\
+2\frac{\alpha_s}{2\pi} g \otimes C_{qq}\otimes\right)(D_{u} - D_{d})^{K^+ + K^−}, \tag{33}
\]

6 The combination $K^+ + K^− + 2K^0$

The general expressions in NLO are rather lengthy, so we begin by discussing the LO case which already exhibits the main properties. For brevity we use the notation $(K) = K^+ + K^− + 2K^0$.

6.1 LO approximation, $K^+ + K^− + 2K^0$

In LO we have for $e^+e^-$:
\[
\begin{align*}
\sigma_T^{(K)}(z) &= 3\sigma_0 \left[ (\hat{e}_u^2 + \hat{e}_d^2) m^2_z (D_{u} + D_{d})^{K^+ + K^−} + \\
&+ 2\hat{e}_d^2 D_{s}^{K^+ + K^−}\right], \tag{34}
\end{align*}
\]
and for SIDIS:
\[
\begin{align*}
\hat{\sigma}_T^{(K)}(x, z, Q^2) &= \frac{1}{9} [(4(u + \bar{u}) + \\
&+(d + \bar{d}))(D_{u} + D_{d})^{K^+ + K^−} + 2(s + \bar{s})D_{s}^{K^+ + K^−}] \tag{35}
\end{align*}
\]
\[
\begin{align*}
\hat{\sigma}_n^{(K)}(x, z, Q^2) &= \frac{1}{9} [(4(d + \bar{d}) + \\
&+(u + \bar{u}))(D_{u} + D_{d})^{K^+ + K^−} + 2(s + \bar{s})D_{s}^{K^+ + K^−}] \tag{36}
\end{align*}
\]

Eqs. (34) - (36) imply that due to SU(2) invariance, the three cross sections $\sigma_T^{(K)}$, $\hat{\sigma}_p^{(K)}$, and $\hat{\sigma}_n^{(K)}$ always measure only two combinations of FFs: $(D_{u} + D_{d})^{K^+ + K^−}$ and $D_{s}^{K^+ + K^−}$. Note that, as this is a property of SU(2)-symmetry, it will hold in all orders of QCD, only the gluon FF will enter in addition in higher orders.

From (34) - (36) it follows that in LO we have three measurements for two unknown quantities $(D_{u} + D_{d})^{K^+ + K^−}$ and $D_{s}^{K^+ + K^−}$. This implies in particular that measurements of $K^+ + K^− - 2K^0$ and $K^+ + K^− + 2K^0$ in SIDIS – eqs. (24), (25), and (26), are already enough to determine $(D_{u} + D_{d})^{K^+ + K^−}$ and $D_{s}^{K^+ + K^−}$ and it is not necessary to use data from $e^+e^-$ performed at very different $Q^2$.

The difference $\hat{\sigma}_p^{(K)} - \hat{\sigma}_n^{(K)}$ determines $(D_{u} + D_{d})^{K^+ + K^−}$ only through the NS combination $(u + \bar{u}) - (d + \bar{d})$:
\[
\hat{\sigma}_p^{(K)} - \hat{\sigma}_n^{(K)} = \frac{1}{3}[(u + \bar{u}) - (d + \bar{d})](D_{u} + D_{d})^{K^+ + K^−} \tag{37}
\]

Once we have thus determined $(D_{u} + D_{d})^{K^+ + K^−}$, we can use $\hat{\sigma}_{p,n}^{(K)}$ (or equivalently their sum $\hat{\sigma}_p^{(K)} + \hat{\sigma}_n^{(K)}$) to obtain $D_{s}^{K^+ + K^−}$.

Only in LO are SIDIS measurements enough to determine $D_{s}^{K^+ + K^−}$. It is thus important to have reliable tests of LO approximation. It’s an advantage that using the same expressions (35) - (36) one can form possible tests of the LO in these processes.
1) In LO we have:

\[
3(\hat{\sigma}_p - \hat{\sigma}_n)_{K^+K^- + 2K^0}(x, z) = \text{function of } z \text{ only} = (D_u + D_d)_{K^+K^- + 2K^0}(z).
\]

2) If \(K^0\) are not measured, LO would be a good approximation:

\[
9(\hat{\sigma}_p - \hat{\sigma}_n)_{K^+K^-}(x, z) = \text{function of } z \text{ only} = (4D_u - D_d)_{K^+K^-}(z).
\]

\(\hat{\sigma}_p, \hat{\sigma}_n\) measure different combinations of the three unknown FFs:

\[
(D_u + D_d)_{K^+K^-}, \quad (D_s + K^-) \quad \text{and} \quad (D_g + K^-) \quad \text{(40)}
\]

(The general expressions for NLO are rather lengthy, so we present below only those relevant for our discussion.)

Combined with measurements of \(K^+ + K^- + 2K^0\), we have enough measurements to determine all kaon FFs: \((D_u + D_d)_{K^+K^- + 2K^0}, (D_s + K^-) \quad \text{and} \quad (D_g + K^-)\).

Solely from SIDIS, and without the influence of the strange and gluon PDs, in NLO one can determine \(D_{u+d}K^+K^-\) and \(D_gK^+K^-\). The difference of \((\hat{\sigma}_p - \hat{\sigma}_n)_{K^+K^-}\) determines a combination of \((D_u + D_d)_{K^+K^-} + D_gK^+K^-\) where the PDs enter only as a common factor in the combination \((u + \bar{u}) - (d + \bar{d})\):

\[
(\hat{\sigma}_p - \hat{\sigma}_n)_{K^+K^- + 2K^0}(x, y, z) = 3\alpha_s + \frac{\alpha_s}{2\pi} C_{qq} \times (D_u + D_d)_{K^+K^- + 2K^0}.
\]

As these FFs are not NS and thus have a different \(Q^2\)-evolution, the above equation would give information on both \((D_u + D_d)_{K^+K^-} \quad \text{and} \quad D_gK^+K^-\). Further, combined with measurements on \((D_u - D_d)_{K^+K^-}\), one can determine \((D_u \pm D_d)_{K^+K^-} \quad \text{and} \quad D_gK^+K^-\) in NLO solely in SIDIS and they will depend on the parton densities only through the combination \((u + \bar{u}) - (d + \bar{d})\).

Further one can combine the measurements of \(D_{K^+K^-}\) and \(D_gK^+K^-\) with measurements of \(e^+e^-\) annihilation or \(p + n\) SIDIS cross section to determine \(D_{K^+K^-}\).

In summary, if in addition to charged \(K^\pm\) also neutral \(K^0\) are measured, we showed that in LO all FFs \(D_{u+d}K^+K^-\) can be determined solely from SIDIS, i.e. it is not necessary to use data from \(e^+e^-\) performed at very different \(Q^2\). In NLO \(e^+e^-\) data should be included, as well, and then all FFs can be determined without the influence of the strange and gluon PDs. The non-singlet \((D_u - D_d)_{K^+K^-}\) can be singled out in both \(e^+e^-\) and SIDIS. Since comparing the two measurements at different \(Q^2\) is straightforward, one can test the factorization of the SIDIS cross section into parton densities and fragmentation functions both in LO and NLO.

\[\text{7 Conclusions}\]

The paper considers the possibilities to obtain the kaon FFs in \(e^+e^-\) annihilation and SIDIS. It consists of two parts. In the first part we have considered possible tests for \(s = \bar{s} = 0 \quad \text{and} \quad D_{K^+K^-} = 0\) in unpolarized SIDIS with final charged \(K^\pm\), both in LO and NLO of QCD.

In the second part we show that, if in addition to \(K^\pm\) also the neutral \(K^0\) are measured 1) in LO the kaon FFs can be obtained solely from SIDIS, and 2) in NLO the combined data of the total cross section in \(e^+e^-\) annihilation in addition to SIDIS is also needed; then the FFs can be determined without the uncertainties of the strange and gluon PDs. Different possibilities to test the LO approximation in unpolarized SIDIS are discussed and in all proposed tests no knowledge of the fragmentation functions is necessary. We show that, in all orders of QCD, the non-singlet combination \((D_u - D_d)_{K^+K^-}\) can be measured directly both in \(e^+e^-\) and in SIDIS without any influence of the strange and gluon parton densities or any other FFs. Comparing the measurements in \(e^+e^-\) and SIDIS allows tests of the factorization of SIDIS into parton densities and fragmentation functions in any order in QCD.

In our approach we consider the sum and difference of cross sections for hadron \(h\) and its C-conjugate \(\bar{h}\). The cross section differences, \(h - \bar{h}\), are NS and, both their \(Q^2\)-evolution and NLO corrections in QCD are straightforward, since they don’t mix with other PDs or FFs. But
they involve poorly known quantities such as the non-singlets $s-\bar{s}$ and $D_d^{K^+K^-}$, and we suggest some tests for these quantities. Quite the opposite is true when the sum of cross sections $h+\bar{h}$ is considered. In this case the $Q^2$-evolution and NLO corrections involve the poorly known gluon FFs, but the cross sections contain the best known combinations of PDFs $q+\bar{q}$, measured in DIS, and $D_{h+\bar{h}}$ measured in $e^+e^-$. We have tried to exploit some of the advantages of both types of combinations of data. Note that though we often consider difference asymmetries, the quantities that they determine are not small and thus, we hope, measurable.

We want to add few remarks on the measurability of the discussed asymmetries. In general, these are difference asymmetries and high precision measurements are required. In addition, the data should be presented in bins in both $x$ and $z$. Quite recently such binning was done in [7] for the very precise data of the HERMES collaboration in DESY on $K^\pm$-production in semi-inclusive DIS on Proton and Deuterium targets. These results show that for $0.350 \leq z \leq 0.450$ and for $0.450 \leq z \leq 0.600$ in the $x$-interval $0.023 \leq x \leq 0.300$ the accuracy of the data allows to form the differences $(\sigma_p+\sigma_n)^{K^+K^-}$ and $(\sigma_p-\sigma_n)^{K^+K^-}$ with errors not bigger than 7-13% and 10-15% respectively. Having these cross sections, given that $uV$ and $dV$ are well known, one can form the ratios $R_+$ and $R_-$ with these precisions. Then, if we do not obtain an acceptable fit to $R_+(x,z_0)$ which is independent of $x$, then $s-\bar{s}=0$ is not a good approximation. This conclusion assumes the success of the LO-test involving $R_-(x,z_0)$, and is independent of our knowledge of the FFs.

If however, an acceptable x-independent fit to $R_+(x,z_0)$ is obtained, then the precision of this fit will put limits on $(s-\bar{s})D_d^{K^+K^-}$. Using these limits in the expression for $R_+-R_-$, and comparing it with experiment at the same values $z_0$ will then put limits on $D_d^{K^+K^-}$.

If we work in NLO and we do not succeed to obtain an acceptable fit for (21) and (22) with the same $D(z)$, then $s-\bar{s} \simeq 0$ and $D_d^{K^+K^-} \simeq 0$ cannot hold simultaneously, at least one of these assumptions fails.

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**References**

1. S. Kretzer, E. Leader, E. Christova, Eur.Phys.J. C22 (2001) 269-276
2. D. de Florian, G.A.Navarro and R. Sassot, Phys. Rev. D71 (2005) 094018
3. E. Christova, E. Leader, Proceedings of XI-th workshop on high energy physics, Dubna-SPIN-2005 (Russia), pp. 17-21 (Preprint: hep-ph/0512075)
4. D. de Florian, M. Stratmann and W. Vogelsang, Phys. Rev. D57 (1998) 5811
5. S. Kretzer, Phys. Rev. D62 (2000) 054001
6. A.D Martin, R.G. Roberts, W.J.Stirling and R.S.Thorne, Phys. Lett. B531 (2002) 216
7. Achim Hillenbrand, Measurement and Simulation of the Fragmentation Process at HERMES, Ph.D. theses, DESY-2005