The Higgs Condensate as a Quantum Liquid

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Abstract
We model the Higgs condensate of the Standard Model as a relativistic quantum fluid analogous to superfluid helium. We find that the low-lying excitations of the Higgs condensate behave like two relativistic Higgs fields. The lighter Higgs boson has a mass of order $10^2$ GeV. We identify this light Higgs particle with the new LHC resonance at 125 GeV. The heavy Higgs boson has a mass around 750 GeV consistent with our recent phenomenological analysis of the preliminary LHC Run 2 data in the golden channel. We critical compare our theoretical scenario with two Higgs bosons to the available LHC Run 2 data.

Keywords Higgs boson · Large hadron collider

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1 Introduction
It is known since long time that in the Standard Model, within the non-perturbative description of spontaneous symmetry breaking [1–4], self-interacting scalar fields suffer the triviality problem [5], namely the renormalised self-coupling goes to zero when the ultraviolet cutoff is sent to infinity. Nevertheless, extensive numerical simulations showed that, even without self-interactions, the scalar bosons could trigger spontaneous symmetry breaking. Moreover, precise non-perturbative numerical simulations [6, 7] indicated that the excitation of the Bose-Einstein scalar condensate is a rather heavy scalar particle. In fact, our recent analysis of the preliminary LHC Run 2 data in the so-called golden channel [8, 9] (see also Ref. [10]), showed a rather convincing evidence of a broad scalar resonance with mass around 730 GeV, that seems to be consistent with a heavy Higgs boson.

Supposing that the full Run 2 data set will confirm the heavy Higgs boson proposal, we face with the problem of the existence of two Higgs bosons considering that the first runs of proton-proton collisions at the CERN Large Hadron Collider with center-of-mass energies $\sqrt{s} = 7$ and 8 TeV (Run 1) gave evidence for a spin-zero boson with mass 125 GeV [11, 12], and that it is now well established that this narrow resonance resembles closely the Higgs boson of the Standard Model [13, 14].
In the present paper we propose to look at the Higgs condensate as a quantum liquid analogous to the Bose-Einstein condensate in superfluid helium (helium II) \(^1\). We find that the low-lying excitations of the Higgs condensate resemble two Higgs bosons with masses of order 100 GeV and around 750 GeV, respectively. These condensate excitations parallel the phonons and rotons in superfluid helium.

The remainder of the paper is organised as follows. In Section 2 we briefly review the main properties of liquid helium in the superfluid phase. In Section 3 we discuss the Higgs mechanism taking into account the problem of triviality for self-interacting scalar fields in \((3+1)\)-dimensions. The presence of two Higgs bosons is addressed in Section 4. Section 5 is devoted to the phenomenological signatures of the two Higgs bosons and to a critical comparison with available experimental observations. Finally, in Section 6 we summarise the main results of the paper.

2 The Helium II

The condensation of a relativistic scalar field free asymptotically could appear paradoxical. Notwithstanding, in condensed matter physics it is known that an ideal non-relativistic Bose gas does display the Bose-Einstein condensation at sufficiently low temperatures. Indeed, in a non-interacting boson system at absolute zero temperature all particles will be in the state of zero momentum. An excitation of momentum \(\vec{p}\) will possess the free-particle energy \(\varepsilon = \vec{p}^2 / 2m\), \(m\) being the particle mass. However, note that, as we shall see later on, when the interactions between bosons are taken into account the quasi-particle excitation spectrum is drastically altered.

Soon after the remarkable discovery of superfluidity in liquid helium below the so-called \(\lambda\)-point \([15, 16]\), it was suggested that helium II should be considered as a degenerate ideal Bose gas that, indeed, manifests the Bose-Einstein condensation at a temperature close to the observed critical temperature \([17–20]\). However, Landau \([21, 22]\) pointed out that the suggestion by London and Tisza cannot account for the superfluidity of helium II below the \(\lambda\)-point. In fact, the remarkable properties of superfluid helium could be recovered if helium II were composed of an intimate mixture of two fluid, one fluid with zero viscosity and the other with normal viscosity. Landau \([21, 22]\) developed a peculiar two-fluid hydrodynamics model in which he explained the phenomenon of superfluidity as a consequence of an excitation spectrum of helium II derived empirically. Remarkably, the Landau two-fluid theory and the empirically derived excitation curve explain a great many of the superfluid properties of liquid helium. Actually, Landau assumed that every weakly excited state of helium II could be considered as an aggregate of elementary excitations. The potential motion of the quantum fluid was assumed to be due to sound waves and the corresponding elementary excitations were the phonons with a linear dispersion form:

\[
\varepsilon_{ph}(\vec{p}) = c_s |\vec{p}| ,
\]

where \(c_s\) is the sound velocity. The vortex motion of the fluid was ascribed to gapped elementary excitations, called rotons, with dispersion law:

\[
\varepsilon_{rot}(\vec{p}) = \Delta + \frac{(\vec{p}^2 - \vec{p}_0^2)}{2m^*} ,
\]

\(^1\) For a good account, see Refs. \([15, 16]\).
where $\Delta$ is a constant, $m^*$ some effective mass, and $\vec{p}_0$ is a momentum of order $|\vec{p}_0| \sim 1/d$ with $d$ the average distance between helium atoms, i.e. $d \simeq n^{-1/3}$, $n$ being the number density.

Bogoliubov [23] attempted to explain the phenomenon of superfluidity on the basis of the theory of Bose-Einstein condensation in a non-perfect gas. In fact, Bogoliubov considered a Bose gas with short-range repulsive interactions characterised by the $s$-wave scattering length $a_s$ in the dilute gas approximation, $a_s n^{1/3} \ll 1$. Using the methods of second quantisation and a new perturbative technique, Bogoliubov was able to show that the existence and the properties of the elementary excitations followed directly from the quantum-mechanical equations describing the Bose-Einstein condensation of the non-ideal gas. Moreover, Bogoliubov showed that the low excited states of the Bose gas can be described as a perfect Bose-Einstein gas of phonons. Finally, Feynman [24–26] showed that the excitation spectrum postulated by Landau could be derived within a first principle quantum-mechanical approach. Actually, Feynman convincingly showed that the only low-energy excitations in helium II were the Bogoliubov’s phonons. In addition, it turned out that from the microscopic point of view a roton may be considered like a small vortex ring. Therefore, rotons corresponds to high-energy excitations of the condensate localised on a region of order of the average distance between helium atoms.

To summarise, the remarkable superfluid behaviour of the helium II quantum liquid can be understood if the excitation spectrum is an almost ideal gas of elementary quasi-particles. The low-energy elementary excitations are the Bogoliubov’s phonons that are collective excitations that retain the needed quantum coherence for wavelengths $\lambda_{ph} \gg d$. The high-energy excitations are rotons, namely localised excitations of the Bose-Einstein condensate with wavelength of the order of the distance between helium atoms.

### 3 The Scalar Condensate

The Higgs mechanism in the Standard Model is implemented by the Bose-Einstein condensation of a relativistic scalar field. To illustrate in the simplest way the mechanism let us consider a real scalar field defined by the Lagrangian density:

$$
\mathcal{L}(x) = \frac{1}{2} \partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{1}{2} m^2 \phi^2(x) - \frac{\lambda}{4!} \phi^4(x) , \tag{3.1}
$$

with $\lambda > 0$. For $m^2 > 0$, the Lagrangian Eq. (3.1) describes a self-interacting scalar field with bare mass $m$. To implement the Bose-Einstein condensation we must assume $m^2 < 0$. In this case there is a macroscopic occupation of the zero mode of the scalar field. Accordingly, the vacuum expectation value of the quantum field $\hat{\phi}(x)$ is different from zero:

$$
< 0 | \hat{\phi} | 0 > = v . \tag{3.2}
$$

Therefore we are led to write:

$$
\phi(x) = h(x) + v , \tag{3.3}
$$

so that $< v | \hat{h} | v > = 0$. Rewriting the Lagrangian Eq. (3.1) in terms of the shifted field $h(x)$ we get:

$$
\mathcal{L}(x) = \frac{1}{2} \partial_\mu h(x) \partial^\mu h(x) - \frac{1}{2} m_h^2 h^2(x) - \frac{\lambda}{6} v^2 h^2(x) - \frac{\lambda}{4!} h^4(x) , \tag{3.4}
$$
with:

\[ m_{\hat{h}}^2 = \frac{1}{3} \lambda v^2 , \quad (3.5) \]

while the correct vacuum expectation value \( v \) corresponds to the vanishing of the tadpole. This perturbative implementation of the Bose-Einstein condensation of a relativistic scalar field is the generally accepted procedure in high energy physics. The standard perturbative scheme parallels closely the Bogoliubov’s perturbative approximation. The elementary excitations of the scalar condensate are given by a quantum scalar field \( \hat{h}(x) \) with mass given by Eq. (3.5) and cubic and quartic self-couplings. Thus, these elementary excitations are coherent long-range collective excitations of the scalar condensate that are analogous to the phonons in helium II. In the following we shall call these excitations the Bogoliubov’s branch of the condensate excitation spectrum. It is worthwhile to observe that the presence of the Bogoliubov’s branch is assured by the short-range repulsive interaction given by the positive quartic self-interaction term. Within the Bogoliubov’s perturbative approximation there is no way to recover the roton branch of the excitation spectrum. However, this perturbative scheme is doomed to failure since self-interacting scalar fields are subject to the triviality problem [5], i.e. the renormalised self-coupling \( \lambda \to 0 \) when the ultraviolet cutoff is sent to infinity. If this is the case, the Lagrangian Eq. (3.1) should reduce to the Lagrangian of a free scalar field. Naively one expects that the spontaneous symmetry breaking mechanism cannot be implemented without the scalar quartic self-coupling. However, one should keep in mind that a non-relativistic ideal Bose gas does develop the Bose-Einstein condensation. In the case of a trivial relativistic scalar field the onset of the condensation phase is given by the vanishing of the mass term \( m^2 = 0 \). Writing:

\[ \phi(x) = H(x) + v , \quad (3.6) \]

extensive numerical studies [6, 7] showed that the fluctuating field \( H(x) \) behaves as a free massive scalar field with mass finitely related to \( v \):

\[ m_H = \xi v . \quad (3.7) \]

Moreover, it turned out that in the continuum limit [6, 7] :

\[ \xi = 3.07 \pm 0.11 , \quad (3.8) \]

where the uncertainties include both the statistical and systematic errors. Assuming that \( v \) is the known weak scale of the Standard Model:

\[ v \simeq 246 \text{ GeV} , \quad (3.9) \]

from Eqs. (3.8) and (3.9) we get:

\[ m_H = 756 \pm 28 \text{ GeV} . \quad (3.10) \]

We see, then, that the excitations over the condensate are like the rotons in superfluid helium. The Bogoliubov’s branch of the excitation spectrum is absent since the triviality of the theory implies \( \lambda = 0 \).

### 4 Two Higgs Bosons

In the previous Section we have discussed the Bose-Einstein condensation for a relativistic real scalar field. Obviously, one could object that our discussion is not directly related to
the Standard Model since the relevant scalar sector is the $O(4)$-symmetric self-interacting scalar theory. However, the known Higgs mechanism eliminates three scalar fields (the Goldstone bosons) leaving as the physical Higgs field the radial excitation whose dynamics is described by the one-component (i.e. real) self-interacting scalar field theory.

We said that a real self-interacting scalar field is trivial, namely it is a free field asymptotically when the ultraviolet cutoff is sent to infinity. Even though a rigorous proof of triviality in (3+1)-dimensions is lacking, there are several convincing numerical studies that leave little doubt on the triviality of the scalar theories. *Rebus sic standibus*, the elementary excitations of the Higgs condensate should be a massive scalar field with a rather heavy mass given by Eq. (3.10). In our previous paper [9] we identified this elementary excitation as the true Higgs mode. As a matter of fact, we showed [9] that there is some evidence of this Higgs mode in the so-called golden channel. Nevertheless, it is widely accepted that the Higgs boson is the new narrow resonance at 125 GeV detected by both the ATLAS and CMS Collaborations [11, 12]. In the present Section we will show that there are two Higgs bosons. The true Higgs mode is the heavy resonance already discussed, while the resonance at 125 GeV is the light Higgs boson that correspond to the Bogoliubov’s branch of the excitation spectrum of the Higgs condensate. To see this we need a non-zero quartic self-interaction of the Higgs field.

Due to the triviality of self-interacting scalar fields the Higgs mode can interact only through the couplings to gauge and fermion fields. In fact, the interactions with vector bosons and fermion fields will induce an effective scalar self-coupling. If we define the effective quartic term in the Higgs potential:

$$ V^{(4)} = \frac{\lambda_{\text{eff}}}{6} \left[ \Phi^4(x) \Phi(x) \right] $$

then the renormalisation-group equation for the self-coupling $\lambda_{\text{eff}}$ can be easily obtained following Refs. [27, 28].

In the lowest approximation the relevant Feynman diagrams are displayed in Fig. 1. We have:

$$ \frac{d\lambda_{\text{eff}}(t)}{dt} \simeq \frac{1}{16\pi^2} \left\{ \frac{9}{4} \left[ 2g^4 + (g^2 + g'^2)^2 \right] - 9k^2\lambda_t^4 \right\}. $$

In Eq. (4.2) $g$ and $g'$ are the couplings to the weak $SU(2)$ and $U(1)$ respectively, which are related to the electric charge according to the well-known formula:

$$ g = \frac{e}{\sin \theta_W}, \quad g' = g \tan \theta_W = \frac{e}{\cos \theta_W}, $$

$\theta_W$ being the Weinberg’s angle. According to our previous paper [9], we are considering only the Yukawa coupling to the top quark, $\lambda_t = \frac{\sqrt{2} m_t}{v}$, where $m_t \simeq 173$ GeV is the top quark mass.

![Fig. 1 Lowest-order contributions to the quartic self-coupling renormalisation](image)
mass. Moreover, our phenomenological analysis suggested that the coupling of the Higgs mode to the top quark were strongly suppressed such that:

$$\lambda_t^2 \rightarrow \kappa \lambda_t^2, \quad \kappa \approx 0.15 . \quad (4.4)$$

Finally, in Eq. (4.2) we set \( t = \ln(M/\mu) \), where \( \mu \ll v \).

By solving Eq. (4.2) one gets the effective self-coupling at the scale \( M > \mu \) once the couplings are fixed at the starting scale \( \mu \). We note that the triviality of the Higgs scalar field assures that \( \lambda_{eff}(\mu) \approx 0 \). Therefore, to the lowest-order approximation we obtain:

$$\lambda_{eff}(M) \approx \frac{1}{16\pi^2} \left\{ \frac{9}{4} \left[ 2g^4(\mu) + (g^2(\mu) + g'^2(\mu))^2 \right] - 9 \kappa^2 \lambda_t^4(\mu) \right\} \ln(M/\mu) . \quad (4.5)$$

It is useful to rewrite this last equation as:

$$\lambda_{eff}(M) \approx \left\{ \frac{9}{4} \alpha_{QED}(\mu) \left[ \frac{2}{\sin^2 \theta_W(\mu)} + \left( \frac{1}{\sin^2 \theta_W(\mu)} + \frac{1}{\cos^2 \theta_W(\mu)} \right)^2 \right] - \frac{9}{16\pi^2} \kappa^2 \lambda_t^4(\mu) \right\} \ln(M/\mu) . \quad (4.6)$$

Once we have an effective self-coupling, within the Bogoliubov’s approximation, we recover the phonon branch of the condensate excitation spectrum that behaves like a scalar field \( h(x) \). Since the Bose-Einstein condensation sets in at \( m^2 = 0 \), the mass of the scalar field \( h(x) \) is now:

$$m^2_h = \frac{1}{2} \lambda_{eff}(M) v^2 . \quad (4.7)$$

To avoid confusion or misunderstanding, it is necessary to pause and add some comments on our results. We are not saying that there are two different elementary Higgs fields. On the contrary, we have a unique quantum Higgs field. However, since the scalar condensate behaves like the helium II quantum liquid, when the Higgs field acts on the condensate it can give rise to two elementary excitations, namely the phonon-like and roton-like excitations corresponding to long-range collective and localised disturbances of the condensate, respectively. These elementary condensate excitations behave as weakly interacting scalar fields with vastly different mass. The main advance in our approach is that the Higgs boson masses are not free parameters, but these can be estimated from first principles.

To complete the mass calculations we must consider the effects due to the vector bosons and fermions. In the one-loop approximation we have the mass corrections displayed in Fig. 2.

Note that in Fig. 2 we are neglecting the one-loop term of order \( \lambda_{eff}^2 \), since it is easy to check that \( \lambda_{eff} \ll 1 \). In principle, the one-loop diagrams in Fig. 2 should modify the mass of the elementary excitations. Within the approximation of weakly interacting condensate excitations we may consider the phonon-like and roton-like excitations as independent scalar particles. Now, we have seen that the Bogoliubov’s branch is composed by

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**Fig. 2** Lowest-order contributions to the scalar field mass term
long-range collective excitations of the Higgs condensate that, therefore, retain the needed coherence to propagate for wavelengths up to the size of the roton-like excitations \( \sim 1/m_H \). So that, in evaluating the mass corrections we have that the integrals with loop momenta \( k \lesssim \Lambda_h \approx m_H \) are to be considered as finite mass corrections to the \( h(x) \) scalar field. On the other hand, for loop momenta \( k \gg \Lambda_h \) the resulting mass corrections (that contain quadratic and logarithmic divergences) must be incorporated into the mass term of the \( H(x) \) field. The only effect is that one must tune the bare mass term until the total mass is set to zero to ensure the onset of the Bose-Einstein condensation. As a consequence, we are left with:

\[
m_h^2 = \frac{1}{2} \lambda_{\text{eff}} v^2 + \delta m_h^2, \tag{4.8}
\]

where:

\[
\delta m_h^2 = \delta m_W^2 + \delta m_Z^2 + \delta m_{\lambda_{\text{eff}}}^2 + \delta m_t^2. \tag{4.9}
\]

We find:

\[
\delta m_W^2 \simeq \frac{3}{16\pi} \frac{\alpha_{\text{QED}}(\Lambda_h)}{\sin^2 \theta_W} \left[ \Lambda_h^2 - M_W^2 \ln \left( \frac{\Lambda_h^2 + M_W^2}{M_W^2} \right) \right] + \frac{3}{8\pi} \frac{\alpha_{\text{QED}}(\Lambda_h)}{\sin^2 \theta_W \cos^2 \theta_W} M_W^2 \left[ 1 - \ln \left( \frac{\Lambda_h^2 + M_W^2}{M_W^2} \right) - \frac{M_Z^2}{\Lambda_h^2 + M_H^2} \right], \tag{4.10}
\]

\[
\delta m_Z^2 \simeq \frac{3}{32\pi} \frac{\alpha_{\text{QED}}(\Lambda_h)}{\sin^2 \theta_W \cos^2 \theta_W} \left[ \Lambda_h^2 - M_Z^2 \ln \left( \frac{\Lambda_h^2 + M_Z^2}{M_Z^2} \right) \right] + \frac{3}{8\pi} \frac{\alpha_{\text{QED}}(\Lambda_h)}{\sin^2 \theta_W \cos^2 \theta_W} \left[ 1 - \ln \left( \frac{\Lambda_h^2 + M_Z^2}{M_Z^2} \right) - \frac{M_Z^2}{\Lambda_h^2 + M_Z^2} \right], \tag{4.11}
\]

\[
\delta m_{\lambda_{\text{eff}}}^2 \simeq \frac{\lambda_{\text{eff}}}{64\pi^2} \left[ \Lambda_h^2 - m_H^2 \ln \left( \frac{\Lambda_h^2 + m_H^2}{m_H^2} \right) \right], \tag{4.12}
\]

\[
\delta m_t^2 \simeq \frac{\kappa \lambda_t^2}{32\pi^2} \left[ -\Lambda_h^2 + 3 m_t^2 \ln \left( \frac{\Lambda_h^2 + m_t^2}{m_t^2} \right) + \frac{3 m_t^4}{\Lambda_h^2 + m_t^2} - 3 m_t^2 \right]. \tag{4.13}
\]

To evaluate \( \lambda_{\text{eff}} \) and \( \delta m_h \) we set:

\[
\Lambda_h \simeq m_H \simeq 730 \text{ GeV}, \quad \sin^2 \theta_W \simeq 0.223. \tag{4.14}
\]

Concerning the mass scale \( \mu \), we assumed \( 1 \text{ GeV} \lesssim \mu \lesssim 100 \text{ GeV} \) and obtained:

\[
m_h \simeq 50 - 60 \text{ GeV}. \tag{4.15}
\]

We are led, thus, to the remarkable prediction that there are two kind of elementary excitations of the Higgs condensate that resemble closely a heavy Higgs boson with mass around 750 GeV and a light Higgs boson with mass of order 100 GeV. Obviously, the light Higgs boson is naturally identified with the new narrow resonance at 125 GeV. Note, however, that according to Eq. (4.15) we have:

\[
m_h^{\exp} \simeq 2.5 m_h. \tag{4.16}
\]
We believe that the difference between the theoretical estimate Eq. (4.15) and the observed mass is due to the fact that our approximations completely neglect quantum correlation effects. Indeed, in condensed matter it is well known that correlations lead to elementary quasi-particle with an effective mass different from the “free” mass. Equation (4.16) suggests that the scalar condensate behaves as a quantum liquid with non-negligible correlations. These correlations are expected to affect appreciably the long-range phonon-like excitations. On the other hand, we do not expect sizeable correlation effects on the roton-like excitations since these arise from localised disturbance of the scalar condensate.

We would like to end this Section by attempting at least a qualitative estimate of the size of the correlation effects. We push further the analogy with liquid helium by assuming that the role of the average distance between helium atoms is naturally played by $d \sim 1/m_H$. We have seen that the interactions of the roton-like scalar condensate excitations with mass $m_H$ with the vector bosons and fermions induce an effective positive quartic self-coupling. This repulsive short-range interaction will distort the condensate over a distance $D \sim \frac{1}{\sqrt{\lambda_{eff} v}}$.

The Bogoliubov’s dilute gas approximation corresponds here to $D \gg d$ or, equivalently, $\lambda_{eff} \ll 1$. Indeed, one can easily check that $D \gtrsim 10 d$. The distortion of the condensate by quantum fluctuations will, in turn, increase the inertia of the long-range phonon-like excitations. In fact, observing that $\langle v|\nabla \hat{h}|v \rangle \sim \frac{\hbar^2}{D^2}$, we see that the mass of long-range condensate excitations increases by $\sim 1/D \sim \sqrt{\lambda_{eff} v}$. This should push the mass of the light Higgs boson closer to the experimental value.

5 Phenomenology of the Two Higgs Bosons

We have seen that the perturbations of the scalar condensate due to the quantum Higgs field behave as two independent massive scalar fields in the dilute gas approximation that is the relevant regime for the LHC physics. To see what are the experimental signatures of our proposal it is necessary to examine the interactions of the condensate elementary excitations. The most evident consequence of our approach is the prevision of two Higgs bosons. The light Higgs boson is a natural candidate for the new LHC scalar resonance at 125 GeV. Therefore we shall indicate our light Higgs boson with $h(125)$. On the other hand, our previous phenomenological analysis of the preliminary LHC Run 2 data in the golden channel [9] suggested the presence of a broad scalar resonance with central mass at 730 GeV. Accordingly, we shall denote the heavy Higgs boson with $H(730)$. Note that this mass value is consistent with the lattice determination Eq. (3.10). Obviously, these two Higgs bosons will interact with the gauge vector bosons. We already pointed out [8, 9] that the couplings of the Higgs condensate elementary excitations to the gauge vector bosons are fixed by the gauge symmetries. As a consequence, both the Higgs bosons $h(125)$ and $H(730)$ will be coupled to gauge bosons as in the usual perturbative approximation of the Standard Model. As concern the coupling to fermion fields, if we admit the presence of the Yukawa terms in the Lagrangian, then, after taking into account Eq. (4.1), we get:

$$\hat{L}(x) = \frac{\lambda_f}{\sqrt{2}} v \hat{\psi}_f(x) \hat{\psi}_f(x) + \frac{\hat{\psi}_f(x) \hat{\psi}_f(x) \hat{H}(x) }{\sqrt{2}} ,$$

(5.1)

where $\hat{\psi}_f(x)$ is a generic fermion quantum field. The first term in Eq. (5.1) gives the interaction of the massless fermion field with the condensate, while the second term is the interaction of the quantum Higgs field with fermions. As is well known, the repeated
scatterings of the massless fermions with the (almost) uniform Higgs condensate generate a fermion mass given by:

\[ m_f = \frac{\lambda_f}{\sqrt{2}} v. \]  

(5.2)

In perturbation theory \( \hat{H}(x) \) is an elementary quantum field. So that the coupling of the elementary Higgs field to fermions is related to the fermion mass by:

\[ \lambda_f = \frac{\sqrt{2} m_f}{v}. \]  

(5.3)

However, in our approach the scalar quantum field \( \hat{H}(x) \) can create two different quasiparticles. In the dilute gas approximation these quasiparticles can be described by two weakly-interacting elementary quantum fields, \( \hat{h}(125)(x) \) and \( \hat{H}(730)(x) \), except that particle creation and destruction operators must be replaced by the quasiparticle creation and destruction operators. Therefore, instead of Eq. (5.1) we have:

\[
\hat{\mathcal{L}}(x) = \frac{\lambda_f}{\sqrt{2}} v \hat{\psi}_f(x) \hat{\psi}_f(x) + \sqrt{Z_{\psi h}^f} \frac{\lambda_f}{\sqrt{2}} \hat{\psi}_f(x) \hat{\psi}_f(x) \hat{h}(125)(x)
\]

\[ + \sqrt{Z_{\psi H}^f} \frac{\lambda_f}{\sqrt{2}} \hat{\psi}_f(x) \hat{\psi}_f(x) \hat{H}(730)(x) \]  

(5.4)

where \( Z_{\psi h}^f \) and \( Z_{\psi H}^f \) are wavefunction renormalisation constant [29, 30] that, roughly, take care of the eventual mismatch in the overlap between the fermion and quasiparticle wavefunctions. Note that the gauge symmetries assure that there are not renormalisations in the coupling of the quasiparticles to the gauge fields. This corresponds in condensed matter to the well-known fact that a quasielectron has exactly the same electric charge of a free electron.

A direct calculation of the wavefunction renormalisation constants is not easy. Nevertheless, we can fix these constants from a comparison with the experimental observations. After the end of the Run 2 at the Large Hadron Collider it resulted that the narrow scalar resonance at 125 GeV were consistent with the perturbative Higgs boson of the Standard Model [13, 14]. In particular, the Yukawa couplings of the resonance at 125 GeV with the top and bottom quarks and with the \( \tau \) lepton are consistent with the theoretical predictions from perturbation theory. As a consequence, we are led to assume that:

\[ Z_{\psi h}^f \simeq 1. \]  

(5.5)

A remarkable consequence of Eq. (5.5) is that our light Higgs boson \( h(125) \) is indistinguishable from the perturbative Higgs boson. The unique difference derives from the Higgs self-coupling. In the perturbative approach the self-coupling is a free parameter related to the Higgs boson mass by Eq. (3.5):

\[ \lambda_{SM} = \frac{3 m_h^2}{v^2}. \]  

(5.6)

On the contrary, as discussed in Section 4, in our approach the Higgs self-coupling can be estimate. In fact, we found in the lowest-order approximation that \( \lambda_{eff} \ll 1 \). More precisely, we have:

\[ \frac{\lambda_{eff}}{\lambda_{SM}} \lesssim 0.1. \]  

(5.7)
In principle Eq. (5.7) can be contrasted with the experimental observations. Indeed, the Higgs self-coupling gives rise to triple and quartic Higgs vertices with well-defined experimental signatures. The test of the quartic Higgs vertex probably is not possible even at the high luminosity LHC. However, the triple Higgs vertex can be constrained experimentally from searches for double Higgs boson production. In fact, recently the ATLAS Collaboration [31] was able to set limits on the Higgs boson self-coupling by combining the single Higgs boson analyses with the double Higgs boson analyses in several different decay channels using data at $\sqrt{s} = 13$ TeV with an integrated luminosity up to 79.8 fb$^{-1}$ for the single Higgs boson and up to 36.1 fb$^{-1}$ for the double Higgs boson. By assuming that new physics affects only the triple self-coupling $\lambda_{HHH}$, they reported:

$$-2.3 < \frac{\lambda_{HHH}}{\lambda_{SM}} < 10.3 \quad \text{ATLAS} \quad (5.8)$$

at the 95% confidence level. Likewise, in Ref. [14] the CMS Collaboration set limits on the Higgs self-coupling by combining measurements of the production and decay rates of the Higgs boson using the data set recorded at $\sqrt{s} = 13$ TeV corresponding to an integrated luminosity of up to 137 fb$^{-1}$, depending on the decay channel:

$$-3.5 < \frac{\lambda_{HHH}}{\lambda_{SM}} < 14.5 \quad \text{CMS} \quad (5.9)$$

at the 95% confidence level. It is evident that to distinguish the perturbative Higgs boson from our proposal it is necessary to increase considerably the integrated luminosity.

Fortunately, our proposal can be more easily contrasted to observations by looking at the heavy Higgs boson $H(730)$. Again, the phenomenological signatures of the heavy Higgs boson $H(730)$ depend on the couplings to gauge bosons and fermions. Since the couplings to the gauge vector bosons are fixed by the gauge symmetries, it follows that the main decay modes of the heavy Higgs boson $H(730)$ are given by the decays into $W^+W^-$ and $Z^0Z^0$ with [9]:

$$\text{Br}(H(730) \rightarrow W^+W^-) \simeq 2 \text{Br}(H(730) \rightarrow Z^0Z^0) \quad (5.10)$$

Moreover, for a heavy Higgs boson the relevant fermion coupling is the Yukawa coupling to the top quark. Actually, in our previous phenomenological analysis on the heavy Higgs boson proposal [8, 9] we suggested that the top Yukawa coupling could be strongly suppressed according to Eq. (4.4). We were led to this suppression from the results of a search for heavy neutral resonances produced by gluon-gluon fusion and decaying into two massive vector bosons reported by both ATLAS and CMS Collaborations using the preliminary LHC data at $\sqrt{s} = 13$ TeV. Interestingly enough, comparing Eq. (4.4) with Eq. (5.4) we infer that:

$$\kappa = Z_{wf}^H \quad (5.11)$$

On general grounds, it is known that $0 < Z_{wf}^H \leq 1$. Indeed, from the comparison with the experimental observations we have concluded that $Z_{wf}^H \simeq 1$ and $Z_{wf}^H = \kappa \ll 1$. Even though we cannot evaluate the wavefunction renormalisation constant, we would like to present some arguments that make plausible the small value of $Z_{wf}^H$. Indeed, we said that the wavefunction renormalisation is determined by the mismatch between the overlap of the fermion and the quasiparticle wavefunctions. Now, the light and heavy Higgs bosons are collective excitations corresponding to disturbances of the scalar condensate over a region with size of order $D \sim 1/m_h$ and $d \sim 1/m_H$ respectively. Therefore, we expect
that the heavy quasiparticle will suffer a more severe mismatch with respect to the light quasiparticle:

\[
\frac{Z_{WF}^H}{Z_{WF}^b} \approx \frac{d}{D} \approx \frac{m_h}{m_H}.
\]  

(5.12)

Since \(Z_{WF}^h \approx 1\), we get:

\[
Z_{WF}^H = \kappa \approx \frac{m_h}{m_H} \approx 0.17.
\]  

(5.13)

It should be stressed that our estimate Eq. (5.13) is, at best, a phenomenological educated guess. Nevertheless, it is reassuring to see that Eq. (5.13) is close to the phenomenological parameter \(\kappa\) used in Ref. [9].

At this point it is necessary to check if our theoretical proposal of a heavy Higgs boson in consistent with the available LHC data. Firstly, we have redone the analysis presented in Ref. [9]. In Fig. 3, top panel, we display the (unofficial) combination, presented in Ref. [9], of the ATLAS and CMS data in the golden channel. The data are compared with the theoretical distribution, obtained following Ref. [9], with the parameter \(\kappa\) given by Eq. (5.13).

Looking at Fig. 3 we see that our theoretical estimate is still in reasonable agreement with the data. On the other hand, according to Eq. (5.10), the most stringent constraints come from the experimental searches for a heavy Higgs boson decaying into two W gauge bosons. In fact, both the ATLAS [32] and CMS [33] Collaborations presented the results on searches for a neutral heavy scalar resonance decaying into a pair of W boson using data at \(\sqrt{s} = 13\) TeV and corresponding to an integrated luminosity of 36.1 fb\(^{-1}\) and 35.9 fb\(^{-1}\), respectively. Since the resulting limits set by the two LHC Collaborations are compatible, we merely present the comparison with the ATLAS data. In Fig. 3, bottom panels, we display the observed limits at 95% confidence level on the heavy Higgs boson production cross section times the branching fraction \(Br(H \rightarrow WW)\) for the gluon-gluon fusion (left panel) and vector-boson fusion (right panel) production mechanisms in the narrow width approximation as reported in Ref. [32]. It should be remarked that, in the search for a heavy neutral resonance decaying into a WW boson pair, no significant excess of events beyond the Standard Model expected background were found in the explored mass range. This could lead to stringent constraints on our proposal. To this end, following Ref. [9], in Fig. 3 we report our estimate for the product of the gluon-gluon fusion production cross section and vector-boson fusion cross section times the branching ratio for the decay of the heavy Higgs boson into two W vector bosons. For the gluon-gluon fusion production mechanism we see that, in the relevant mass range, our theoretical cross section lies below the observed limits. This means that, at the moment, there is not enough sensitivity to detect the signal in this channel. On the other hand, the theoretical vector-boson fusion cross section falls within the \(\pm 2\sigma\) ranges around the expected limit for the Standard Model background only hypothesis. Even though the absence of a signal in this channel could seem problematic, our theoretical proposal is still viable. In fact, the suppression of the top Yukawa coupling implies that the expected signal for the gluon-gluon fusion production mechanism is well below the uncertainties of the expected background. Moreover, in the vector-boson fusion production mechanism the integrated luminosity is too low to safely disentangle the expected signal out of the background. Nevertheless, we expect that with the full LHC Run 2 data set there should be a signal at least for the vector-boson fusion production mechanism.
Fig. 3 (upper panel) Comparison to the LHC data, (green) full points, of the distribution of the invariant mass \( m_{ZZ} \) for the process \( H \rightarrow ZZ \rightarrow \ell\ell\ell\ell (\ell = e, \mu) \) in the high-mass region \( m_{ZZ} \gtrsim 600 \text{ GeV} \) with the expected signal histograms obtained assuming \( \kappa \simeq \frac{m_h}{m_H} \). The data have been taken from Ref. [9].

(lower panels) Limits on the production cross section times the branching fraction for the processes \( pp \rightarrow H \rightarrow WW \). The data have been taken from Fig. 5 of Ref. [31]. The dashed (green) lines demarcate the 95% confidence level region of the expected Standard Model background. The thick continuous (green) lines are the observed signal. The thin continuous black lines are our theoretical estimate for the gluon-gluon fusion (left panel) and vector-boson fusion (right panel) production cross section times the branching ratio \( \text{Br}(H \rightarrow WW) \).

6 Summary and Conclusions

In the present paper we proposed to picture the Higgs condensate of the Standard Model as a quantum liquid analogous to the superfluid helium. Our approach allowed us to uncover the spectrum of the elementary excitations. We found that there are two different kind of condensate excitations that are similar to phonon and rotons in helium II. We found that, in the weak interaction approximation, the Higgs condensate excitations behave as two Higgs bosons with mass around 100 GeV and 750 GeV respectively. The light Higgs boson was identified the the LHC narrow resonance at 125 GeV. The heavy Higgs boson found preliminary evidence in our previous phenomenological analysis in the golden channel of the preliminary LHC Run 2 data from ATLAS and CMS Collaborations. We have critically contrasted our theoretical proposal to the available LHC data. We concluded that up to now
the experimental observations are not yet in contradiction with the scenario of two Higgs bosons. We are confident that the full data set of the LHC Run 2 will corroborate our theoretical proposal.

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