Numerical Simulation of Energy Distribution of Turbulent Vortices in Atmosphere Effects on Aircraft Wake Vortices

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Abstract. The effect of energy distribution of turbulent vortices in atmosphere on the evolution of wake vortices is studied using the numerical simulation approach. Solution-Based Dynamic adaptive mesh method is applied to compute the wake vortex evolution using FLUENT. Two series of cases are designed to study the role of larger and small-scale vortices in the decay of wake vortex, respectively. The results show that energy distribution of different scale vortices has a significant effect on the onset time of vortex linking and the decay rate of wake vortices at the turbulent diffusion phase and fast decay phase.

1. Introduction

A wake vortex system, as a reaction to the lift generated by an aircraft, is a pair of counter-rotating vortices. The wake vortex with high energy intensity puts the aircraft in danger during the approach phase. It became very important to investigate the evolution characteristics of wake vortices. In natural conditions, atmospheric turbulence is the key factor affecting the development of wake vortices. The evolution of wake vortices under different intensity levels of atmospheric turbulence has been studied using numerical simulation method in many researches[1-4]. Some relationships between the intensity of wake vortex and turbulence have been established. However, most previous studies focused on the effects of different atmospheric turbulence dissipation rates on the wake vortex, and less attention is paid to the energy distribution of vortices with different scales under the same dissipation rate. The energy distribution also is a significant parameter of atmospheric turbulence and its influence on wake vortices development should not be ignored. In this paper, different energy distributions of turbulent vortices in atmosphere effects on aircraft wake vortices are investigated and analyzed.

2. Numerical Method and Initial Conditions

The evolution of wake vortices is simulated using Fluent LES code and the Boussinesq-approximated Navier-Stokes equations are as follows:

\[
\frac{\partial u_i}{\partial x_i} = 0
\]  

(1)

\[
\frac{\partial u}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + (\nu + \nu_t) \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} \right)
\]  

(2)
where $\nu_t$ are sub-grid kinematic viscosity. The Lagrangian dynamic model is adopted to close the equations. The second-order upwind and the bounded second-order implicit scheme are used for spatial discretization and time integration, respectively. Solution-Based Dynamic mesh Adaption is carried out at every time step is carried out at each time step by local refinement and coarsening according to the refinement criteria based on the magnitude of vorticity. Figure 1 shows the initial mesh with refiner resolution around the vortex core.

![Figure 1. Mesh adaption and vorticity magnitude $|\omega|$ (1/s) of the initial vortex.](image)

The velocity field of the wake vortex is initialized by the L-O vortex, which given by

$$v_0(r) = \frac{\Gamma_0}{2\pi} \left[1 - \exp\left(-1.2526\left(\frac{r}{r_c}\right)^2\right)\right]$$

(3)

where $\Gamma_0$ is the initial circulation, $r_c$ is the vortex core radius and $r$ is the distance from the core. An approaching A340 aircraft is chosen for generating wake vortices. The initial circulation $\Gamma_0=458\,m^2/s$, the vortex space $b_0=47\,m$ and the vortex core radius $r_c=3\,m$. The initial descent velocity $w_0 = \frac{\Gamma_0}{2\pi b_0} = 1.55\,m/s$ and hence the reference time $t_0=b_0/w_0=30.3\,s$.

The computational domain size is $L_x \times L_y \times L_z = 8b_0 \times 5.4b_0 \times 5.4b_0$ with a uniform grid resolution of $\Delta=2\,m$, where $x$, $y$ and $z$ correspond to the axial, lateral and vertical directions, respectively. After refinement, the resolution is 0.5m in the region of the vortex core. All boundaries of the computational domain are periodic. The numerical time step is fixed at 0.01s.

The velocity field of wake vortices superimposed over a pre-generated turbulent flow field is the initial condition of the numerical simulation. The ambient atmosphere is regarded as homogeneous and isotropic and hence the Rogallo method[5] was used to generate the turbulent flow field. Our initial ambient turbulent field obeys a modified von karman spectrum[6]

$$E(k) = \frac{2/3K}{k_e} \left(\frac{k}{k_e}\right)^4 \left[1 + \left(\frac{k}{k_e}\right)^2\right]^{\nu/2} \exp\left[-2\left(\frac{k}{k_{kol}}\right)^2\right]$$

(4)

where $k_e$ is the peak wave number of the energy spectrum and $k_{kol} = (\varepsilon / \nu^3)^{1/4}$ is the Kolmogorov wave number, where $\varepsilon$ is the eddy dissipation rate (EDR) and $\nu$ is the viscosity. $k_e$ determines the range of wave number in the energy-containing range of the spectrum or the energy distribution of vortices with different sizes. To investigate the influence of vortex size on the wake vortex evolution, several cases (Cases A) with variable $k_e$ are set under moderate ambient turbulence and the parameters as displayed in Table 1.

| Table 1. Parameters of energy spectrum for Cases A. |
|--------------------------------------------------|
| EDR  | $k_e(m^{-1})$ | $k_e^* (k_e b_0/\pi)$ |
|-------|----------------|---------------------|
| $\varepsilon^*=0.05$ | $2\pi/75$ | 0.63 |
|       | $2\pi/90$ | 0.52 |
In order to further analyze the contribution of vortices of different scales to the decay of wake vortices, two cases (Cases B) with the same magnitude of kinetic energy and different $k_e$ are also compared and relevant parameters are listed in Table 2.

Table 2. Parameters of energy spectrum for Cases B.

| EDR   | $k_e (m^{-1})$ | $K (m^2/s^2)$ |
|-------|----------------|---------------|
| $\epsilon^* = 0.23$ | $\frac{2\pi}{90}$ | 0.065 |
| $\epsilon^* = 0.168$ | $\frac{2\pi}{220}$ |   |

In this study, the eddy dissipation rate is normalized according to $\epsilon^* = (\epsilon b_0)^{1/3} / w_0$. Figure 2 shows the energy spectrum of all cases.

Figure 2. Energy spectrum with different $k_e$ for Cases A (left) and Cases B (right).

3. Results and Discussion

In order to assess the strength of wake vortices, the mean circulation $\Gamma_{5\rightarrow15}$ is used in this paper, which is averaged over 11 circular planes with radii from 5 to 15 m. $\Gamma_{5\rightarrow15}$ is a non-dimensional quantity normalized by the initial circulation and it is defined by

$$\Gamma_{5\rightarrow15}^* = \frac{1}{11} \sum_{r=5}^{15} \left( \int_{r}^{15} \omega dA \right) / \Gamma_0$$

Figure 3 shows the circulation evolution with time for Cases A. It can be seen that all these cases appear two-phase decay process: the turbulent diffusion phase and fast decay phase. The difference of dissipation rate between the two phases is obvious, which is caused by the instability of the wake vortex at the fast decay phase. For different $k_e$, the evolution of circulation with time is quite different. As expected, with the decrease of $k_e$, the increase of the energy of large-scale vortices, the onset time $t_{onset}$ of fast decay of wake vortices is advanced, that is, the connection of vortex pairs occurs earlier. However, it should be noted that different $k_e$ has little effect on the decay rate of the two phases. The turbulence field generated by smaller $k_e$ has relatively more high-energy and large-scale turbulence vortices, while in turbulence field with the larger $k_e$, these vortices contain relatively less energy.
For Cases B, the total energy of the ambient turbulence filed in the two cases is the same, but the distribution of energy in different vortex scales is different. The energy spectrum in Figure 2 shows that, compared with \( e^*=0.23 \), \( e^*=0.168 \) has higher energy in large scale vortices and lower energy in small-scale vortices, but the total energy design is basically the same. The evolution of the wake circulation with the time of the two cases is also shown in Figure 3. It can be seen that although the turbulent kinetic energy of the two cases is the same, the details of the decay of wake vortices are quite different. First of all, the two-phase decay process of the two cases is obvious, but the dissipative acceleration is different. The decay rate of \( e^*=0.23 \) is faster than that of \( e^*=0.168 \) in the diffusion phase, while that of \( e^*=0.168 \) is faster in the fast decay stage. Compared with the two cases, \( e^*=0.123 \) has a larger decay rate of turbulent energy consumption and larger energy of small-scale vortex, which is just corresponding to the difference of decay rate in the diffusion stage and this can be considered as evidence that small-scale vortices control the energy dissipation of wake vortices. On the other hand, it is also observed from the circulation evolution with time that \( t_{\text{link}}^*=3.3 \) (\( t_{\text{link}}^*=t_{\text{link}}/t_0 \)) for \( e^*=0.168 \) and 4.3 for \( e^*=0.23 \). The vortex linking of the former is earlier, and the energy spectrum distribution shows that the energy of large-scale vortices in the former is higher. This relationship indicates that the large-scale vortices control the onset times of fast decay, which is caused by the long-wave instability. It is concluded that the large-scale vortex in the background turbulence will cause the deformation and irregularity of wake vortices, while the small-scale vortex will directly act on the wake vortex and promote the decay through mixing and energy exchange.

![Figure 3. Circulation evolution with time for Cases A (left) and Cases B (right).](image)

Therefore, the decay rate of different \( k_e \) in the diffusion stage is basically the same, because the energy contained in the small-scale vortices in Cases A is the same (see the energy spectrum), and the initial decay of the wake vortex is consistent. The effect of \( k_e \) on the onset time of fast decay is realized by the effect of large-scale turbulent vortices. The larger \( k_e \) is, the more energy the large-scale vortices contain, and the stronger the Crow instability of the wake is, and the faster the formation of vortex ring (the structure is shown in Figure 4). This is different from the decay mechanism of wake vortices in the cases with different \( e \). In most previous studies, the change of ambient turbulence intensity is achieved by adjusting the energy of all scale vortices[7, 8]. The difference of small-scale turbulent energy consumption affects the dissipation behavior of wake vortices at the beginning of the diffusion stage, so different eddy dissipation rate leads to the difference of the circulation decay in the diffusion phase.
Figure 4. Structure of the wake vortex for $k_e^* = 0.44$ at $t^* = 5.3$ (left), 6.1 (middle) and 6.9 (right).

Figure 5 shows the axial mode energy for $k_e^* = 0.23$ ($k_e = 2\pi/200 \text{ m}^{-1}$) and $k_e^* = 0.52$ ($k_e = 2\pi/90 \text{ m}^{-1}$) in Cases A. The curves of large and small wavelengths respectively show the development of long and short wave instability energy of wake vortices. It can be seen that the energy changes of different wavelengths in the two cases have similar processes. Firstly, in the early stage of the development of vortices, all wave numbers have an obvious energy growth process. The wake vortex starts to trigger the instability of different wavelengths under the disturbance of different scale vortices from the ambient turbulence. It is also noted that the larger the wavelength, the faster the energy growth. In the two cases, $\lambda = 8b_0$ has the maximum energy growth rate in the early stage. On the contrary, the energy growth rate of $\lambda = 0.04b_0$ is the slowest, which indicates that the deformation of wake vortices is dominated by long-wave instability under the atmospheric conditions of moderate turbulence intensity ($\varepsilon^* = 0.05$), which is consistent with the previous analysis of circulation evolution. After a relatively rapid increase, the energy of the small wavelength begins to stabilize, which can be considered as reaching its maximum value. Finally, the energy of large and small wavelengths begins to decrease, and the latter occurs earlier.

Moreover, some differences in detail can be also seen in Figure 5. Obviously, the energy of the wavelength $\lambda = 8b_0$ with $k_e^* = 0.23$ is larger than that of $k_e^* = 0.52$ in the initial stage, which is reasonable because the kinetic energy of the former in the energetic region is greater, and the large-scale vortex drives the development of the long-wave instability of the wake vortex, which makes the deform of the wake vortex earlier. The time to reach peak energy is also different in the two cases. The energy of $\lambda = 8b_0$ for $k_e^* = 0.23$ reaches the peak value at $t^* = 6$, while the time of $k_e^* = 0.52$ is about $t^* = 8$, which is obviously later than the onset time of fast decay stage ($k_e^* = 0.23$: $t_{\text{link}}^* = 4.75$; $k_e^* = 0.52$: $t_{\text{link}}^* = 6.4$). The former is 26% later than $t_{\text{link}}^*$ and the latter is 25%, which are mostly consistent. According to the previous analysis, after wake vortex linking, the vortex ring begins to form, and the circulation decreases rapidly, while the axial mode energy continues to increase after $t_{\text{link}}$. This phenomenon seems to be contrary because, in this study, the circulation is calculated by integrating the vorticity in the flight direction, which is convenient for the risk assessment of wake encounter in the parallel approach phase. When the wake vortex deforms in curvature, the calculated circulation value is less than the integral along the vortex axis direction. Therefore, the strength of the wake vortex itself should be greater. If the circulation obtained by the integration in the direction of the vortex axis is investigated, the circulation decay rate of the wake in the fast decay phase will be significantly reduced. This is because the wake vortex still maintains high energy, and the deformation of the vortex ring formed in the flight direction is continuous, which makes the axial fluctuating energy continue to increase.
4. Conclusion

Through adjusting the peak wave number $k_e$ of von karman spectrum, the turbulence field with energy distribution is generated. Large eddy simulation of the evolution of wake vortices is carried out under these different turbulent conditions. The main conclusions are as follows:

- The large-scale vortex in ambient turbulence will cause the deformation and irregularity of wake vortices, while the small-scale vortex will directly act on the wake vortex and promote the decay through mixing and energy exchange.
- The large vortex controls the onset time of vortex linking and fast decay.
- The time that wake vortex energy reaches its peak value is obviously later than the onset time of the fast decay phase.

Acknowledgments

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