Axion-Dilaton Destabilization and the Hubble Tension

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The discrepancy in measurements of the Hubble constant indicates new physics in dark energy, dark matter, or both. Drawing inspiration from string theory, where axions interact with the other moduli fields, including the dilaton, here we demonstrate that the dynamics of an interacting dilaton and axion naturally realizes the proposal of Early Dark Energy. In this setup, stabilization of the the dilaton is in part due to the axion, and in the early universe the dilaton contributes to dark energy. The combined axion-dilaton system is destabilized when the Hubble constant falls below the mass of the axion, triggering a phase of fast-roll evolution of the dilaton wherein its equation of state is \( w = 1 \), and the early dark energy redshifts away as \( a^{-4.4} \).

I. INTRODUCTION

With the refined measurement of the distance to the Large Magellanic Cloud \(^1\), the discrepancy between the observed and inferred values of the Hubble constant has raised in significance to 4.4\(\sigma \). The data supporting this discrepancy comes from a wide range of redshift, providing the greatest challenge yet to the ΛCDM model. This growing observational evidence motivates the search for theoretical explanations.

In contrast with past cosmological glitches, e.g. anomalies in the Cosmic Microwave Background, there are still relatively few working theoretical explanations \(^2\). However, some of the necessary ingredients of a solution are now known. As emphasized in \(^7\), to rectify the cosmological distance ladder with the inverse distance ladder and the CMB requires a modification of the sound horizon \( r_s \), and thus any solution must include new dynamics in the very early universe \(^7\).

The scenario of ‘Early Dark Energy’ \(^2\)\(^9\) implemented a modification of \( r_s \) via the decay of dark energy into an exotic fluid that redshifts faster than radiation, i.e. with equation of state \( w > 1/3 \). This exotic fluid can arise via a scalar field with potential that is a polynomial in the cosine of the field, \( V = \mu^4 (\cos \phi)^p \) \(^2\), with parameters tuned so that the decay happens shortly before recombination.

In an attempt to avoid such an unnatural potential, \(^3\) developed single-field models with polynomial potentials that can realize the early dark energy dynamics via non-standard field space trajectories. The best fit model was found to be \( V = e^{4(\phi/m_H)} \), corresponding to a massless \( \lambda \phi^4 \) theory with \( \lambda \approx 10^{-112} \) and \( \phi \) undergoing a Planckian field excursion.

In this letter we demonstrate that the requisite dynamics can be realized via the interacting dynamical system of a dilaton field, referred to as such due to an exponential potential \( V = V_0 e^{-\lambda \phi/m_H} \), and an axion field with cosine potential. At early stages the axion is held up its potential by Hubble friction, and due to an interaction, the dilaton is also held up its potential, and thus acts as dark energy. Eventually, when the Hubble constant falls below the mass of the axion, the axion rolls down its potential and begins to oscillate. At this point the dilaton is destabilized and also rolls down its potential. For \( \lambda > 1 \), this rolling can be fast-roll with the energy density in \( \phi \) predominantly in kinetic energy. The equation of state of \( \phi \) is \( w = 1 \) and the energy density redshifts as \( a(t)^{-6} \).

This realizes early dark energy, wherein the onset of the decay of dark energy is due to the small axion mass, which is itself in agreement with general intuition from the string theory axiverse, and the ‘exotic fluid’ is simply the kinetic energy of the early dark energy field.

As a final note before we proceed, we remark that the motivation for this setup comes from a promising approach to cosmology in string theory. The existence of moduli fields is one of the few model-independent predictions of string theory (or more generally, extra-dimensional theories). The prevailing approach is to stabilize and subsequently ignore all these fields, in order to focus on the minimal fields necessary to describe our universe \(^4\). However, the dynamics of these moduli fields can leave tell-tale signs on cosmological observables: In the context of inflation, this can lead to observable gravitational waves in small field inflation \(^13\). In the post-inflation universe, oscillations of the moduli fields can drive a period of matter domination \(^14\). The present letter suggests that string moduli may also solve the Hubble tension.

The structure of this letter is as follows: In Section II we review the dynamics of axions and dilatons. In

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\(^1\) See e.g. \(^10\)\(^12\) for early examples.
Section III we propose our mechanism for solving the Hubble tension, and in Section IV we show demonstrate quantitatively the validity of this solution. We close with a discussion in Section V.

II. THE COSMOLOGICAL DYNAMICS OF SCALAR FIELDS

Axions were first posited as a solution to the strong CP-problem of the standard model [15–17]. They were later realized to be excellent dark matter candidates [18–20], and later yet realized to be ubiquitous in theories of quantum gravity [21–23]. The basic premise is a scalar field with a continuous shift symmetry that is broken by non-perturbative effects to a discrete shift symmetry.

The potential for an axion field is given by

\[ V(\chi) = m_\chi^2 f^2 \cos \frac{\chi}{f}, \quad (1) \]

and the equation of motion is,

\[ \ddot{\chi} + 3H \dot{\chi} + V_\chi = 0. \quad (2) \]

The canonical axion dynamics are as follows. Consider the interacting axion-dilaton system,

\[ V(\phi, \chi) = V_0 \lambda^2/(H^2 m_{\text{pl}}^2). \quad (3) \]

The parameter \( \lambda \) is referred to as the “slow-roll” parameter [24], \( \lambda = m_{\text{pl}}/(\partial_\phi V/V) \). The phase space dynamics of this model are well understood, and exhibits fixed points wherein the dilaton ‘tracks’ the background. We will be interested in dynamics away from these tracking solutions.

To understand the dynamics, let’s again look for a slow-roll solution in a background where \( \phi \) is subdominant and \( H \) is a constant. If we assume the second derivative \( \ddot{\phi} \) is initially subdominant, then the solution is given by

\[ \phi(t) = \frac{m_{\text{pl}}}{\lambda} \log \left[ 1 + \frac{\lambda^2 t V_0}{3Hm_{\text{pl}}^2} \right]. \quad (4) \]

To assess slow-roll, we can evaluate the sub-dominance of the second-derivative. At \( t = 0 \):

\[ \frac{\dddot{\phi}}{H^2 \phi} \bigg|_{t=0} = \frac{V_0 \lambda^2}{3H^2 m_{\text{pl}}^2}. \quad (5) \]

Thus when \( V(\phi) \) is a subdominant contribution to \( H \), i.e. \( V_0 \ll H^2 m_{\text{pl}}^2 \), the field will slow roll if \( \lambda \ll 1 \).

However, if \( \lambda \gg 1 \), and with initial conditions away from a tracker fixed point, the solution is fast-roll. During this fast-roll phase, the potential is subdominant to the other terms in the equation of motion and the solution is simply

\[ \dot{\phi} \propto a(t)^{-3}. \quad (6) \]

Thus the kinetic energy redshifts as \( a^{-6} \) and the energy initially stored in \( V(\phi) \) is rapidly redshifted away.

The fast-roll phase can not last forever, since \( V,\phi \phi \propto m_\chi^2 \) is decreasing:

\[ \frac{V,\phi \phi}{H^2} = \frac{V_0 \lambda^2}{m_{\text{pl}}^2 H^2} e^{-\lambda \phi/m_{\text{pl}}}. \quad (7) \]

Thus, in a reversal of the dynamics of the axion, fast-roll will terminate when \( V,\phi \phi \propto H^2 \), i.e. when Hubble becomes greater than the mass of the field. We must check then that the period of fast-roll can be sufficiently long-lived so as to dilute the early dark energy.

Setting equation (7) equal to 1, this occurs when \( \phi \) is given by:

\[ \phi_{\text{end-FR}} = \frac{m_{\text{pl}}}{\lambda} \log \left[ \frac{V_0 \lambda^2}{H^2 m_{\text{pl}}^2} \right]. \quad (8) \]

To compute the amount of redshifting of energy, we can solve for \( \phi \) in a radiation dominated universe, \( a(t) = (t/t_0)^{1/2} \), with the initial condition that \( \phi(t_0) = 0 \),

\[ \phi(a) = \phi(t_0)t_0 \left( 1 - \frac{1}{a} \right). \quad (9) \]

Thus \( \phi \) asymptotes to a value \( \phi(t_*)t_* \), where \( t_* \) can be written in terms of \( H_* \) via,

\[ H_* = \frac{1}{2t_*} \quad (10) \]

We can then solve for the value of \( a \) when \( \phi = \phi_{\text{end-FR}} \),

\[ a_{\text{end-FR}} = \frac{\dot{\phi}_0}{\dot{\phi}_0 - H_0 \phi_{\text{end-FR}}}. \quad (11) \]

This is large if the initial velocity \( \dot{\phi}_0 \) is tuned to \( \dot{\phi}_0 \approx H_0 \phi_{\text{end-FR}} \).

The total redshifting of the energy density is \( a^{-6} \), and thus all of the energy density can be redshifted away by an adequate choice of initial condition for the fast-roll phase. In the context of the coupled axion-dilaton system, this initial condition will be chosen dynamically, and the fast-roll phase continues long after the axion is destabilized.

III. THE MECHANISM: EARLY DARK ENERGY FROM AXION-DILATON DYNAMICS

Consider the interacting axion-dilaton system,

\[ V(\phi, \chi) = m_\chi^2 f^2 e^{\beta \phi/m_{\text{pl}}}(1 + \cos \frac{\chi}{f}) + V_0 e^{-\lambda \phi/m_{\text{pl}}}. \quad (12) \]
One could also include a coupling of the dilaton to the kinetic energy of the axion, but this effect is degenerate with the other two terms and so we ignore it. These couplings naturally arise between the axion and Kahler moduli of string theory, and the dilaton, related to the string coupling $g_s = e^\phi$ [23]. Axion interactions in dark energy models have also been studied in a different context in [23, 26].

We assume that the initial condition for the axion is that it is displaced from the minimum of its potential, and is frozen by Hubble friction, or rather, it is slowly-rolling. The $e^\phi$ dependence of the axion potential then stabilizes the dilaton at the minimum of the combined potential [12],

$$\phi_0 = \frac{m_{pl}}{\beta + \lambda} \log \left[ \frac{V_0 \lambda}{\beta f^2 m_\chi^2 (\cos \chi/f)^2} \right].$$

For large $\lambda$, $\phi_0$ is very close to $0$. Thus the potential in the early universe, with the dilaton frozen at $\phi_0 \approx 0$, is given by

$$V_{early} = m_\chi^2 f^2 \left( 1 + \frac{\chi}{f} \right) + V_0.$$  \hspace{1cm} (14)

The constant $V_0$ is an additional contribution to the cosmological constant in the early universe.

Once the Hubble parameter becomes less than the mass of the axion, $H \lesssim m_\chi$, the axion rapidly rolls down its potential and begins to oscillate. The solution for the axion is given by,

$$\chi(t) \approx \frac{\chi_0}{a(t)^{3/2}} \cos m_\chi t.$$  \hspace{1cm} (15)

In order for this to occur shortly before recombination, we require $m_\chi \sim 10^{-27}$ eV. Thus $\chi$ is an ultralight axion. It is constrained to be a sub-dominant component of the dark matter [27].

When $\chi$ drops down its potential, $\phi$ is destabilized and begins to roll. As this occurs, the effective axion mass $m_{\chi eff}^2 = m_\chi^2 e^{\phi/m_{pl}}$ increases and $\chi$ undergoes extremely rapid damped oscillations. In this phase $\chi$ is well approximated by its time-averaged value $\langle \chi \rangle = \pi/f$. The dynamics of $\phi$ are then well described simply by the dilaton potential,

$$V_\phi = V_0 e^{-\lambda \phi/m_{pl}}.$$  \hspace{1cm} (16)

As shown in section [1] for $\lambda \gg 1$ the dynamics of $\phi$ are kinetic energy dominated and hence the energy evolves density as

$$\rho_\phi \approx \frac{V_0}{a^6},$$  \hspace{1cm} (17)

and thus quickly redshifts away.

![FIG. 1. Evolution of the fields in the coupled axion-dilaton system.](image1)

To demonstrate these dynamics, we numerically solve this system in a FRW universe for a fiducial set of parameters. We define $H(t) = H_*/(1 + H_* t)$ in units with $H_*$ normalized to 10, and initialize the system at $t = 0$ with $m_\chi/H_* = 10^{-6}$ and $f = 0.1$. We fix the exponents as $\beta = 1$ and $\lambda = 10$, which control the damping of the oscillations and the speed of the fast-roll evolution respectively. This result shown in Figure [1] where we observe the onset of destabilization, and a period of fast roll evolution. We plot the equation of state in Figure [2] which clearly shows a transition from $w = -1$ to $w = +1$, and thus of the dynamics of $\phi$ from dark energy to kinetic energy.

![FIG. 2. Evolution of the dilaton equation of state in the coupled axion-dilaton system.](image2)

### IV. Resolving the Hubble Tension

We now turn to the main goal of this letter. To solve the Hubble tension, we require that the early dark energy $V_0 e^{-\lambda \phi_0/m_{pl}} \approx eV^4$ be rapidly dissipated around recombination, $z_{rec} \sim 10^3$. The latter requirement sets the axion mass $m_\chi \lesssim H_{rec}$, which is given by $H_{rec} \approx eV/m_{pl} \sim 10^{-27} eV$. [27]
The former requirement then relates the normalization $V_0$, the decay constant $f$, and the exponents $\lambda$, $\beta$. We assume for simplicity that $\lambda \gg \beta$, in which case the condition $V_0 e^{-\lambda \phi_0/m_p} \simeq eV^4$, using the explicit expression for $\phi_0$, reads

$$f = \sqrt{\frac{\lambda eV^2}{\beta m_X}} \quad (18)$$

with $V_0$ left as a free parameter. For $m_X \sim 10^{-27}\text{eV}$, $f$ is of order the Planck scale.

V. DISCUSSION

In this letter we have proposed that the dynamics of a coupled axion-dilaton system can resolve the Hubble tension. In this scenario the canonical evolution of the axion leads to a destabilization of the dilaton, inducing a phase of fast-roll in the latter, and the conversion of potential energy into kinetic energy, which subsequently redshifts away as $a^{-6}$. For an axion of mass $10^{-27}\text{eV}$, this conversion occurs in a narrow redshift window around recombination, similar to the dynamics of [5].

Our work proposes a solution to the Hubble tension based in standard particle cosmology. In an upcoming work we will provide a detailed numerical computation of cosmological evolution and perturbation analysis as well as a Markov-Chain Monte-Carlo analysis of the model. We also plan to study of the genericity of these parameters in the moduli space of string theory.

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