Relativistic effects and emf localization in a unipolar generator

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Abstract. The paper deals with the issue of induced EMF localization in a Faraday's unipolar generator which is a revolving magnetic disk with its center and periphery connected by a conductor. Attention has also been given to the linear DC generator consisting of a long magnet and a conductor. Functioning as a closed loop, this conductor moves within the magnetic field of the magnet, the ends sliding on its surfaces. It is shown that in a linear unipolar generator the EMF is induced in a conductor irrespective of a reference frame chosen. Reasons are given in favour of the fact that when a magnetic disk is revolving, the EMF is induced in a stationary conductor of a unipolar generator.

1. Introduction
In theoretical physics the thesis of the rotation of a uniform magnetic field is currently not considered. In theoretical physics fields neither rotate nor move, but transform [1]. Therefore, it is considered, for example, that in a Faraday DC rotary unipolar generator, which is a magnetized rotating disk with its center and periphery connected by a conductor, the EMF is induced in the disk because the points of the disk move in their own non-rotating (stationary) magnetic field of the rotating disk.

In violation of the rules of theoretical physics in electrical engineering, particularly referring to the theory of electric machines, it is often the case that the concept of the rotating magnetic field is resorted to. And although this rotation refers to a non-uniform magnetic field and is viewed upon merely as a convenient technique, the existence of this concept promotes the fact that in a similar - electrotechnical - examination of a unipolar generator one often expresses an opinion that a disk-associated magnetic field rotates together with the disk as well. Herewith it is assumed that there is no effect on conductive electrons of the rotating disk of the unipolar generator, and the EMF is induced in the conductor it being crossed by the lines of force of the magnetic field rotating together with the magnet. In line with this point of view, if the conductor connecting the axle with the periphery of the disk is absent, then there should be no accumulated electric charge on the periphery of the rotating disk.

2. Rotary and linear DC generators
Consider a magnetic disk (or solid cylinder) 2 (fig. 1) mounted on the axle 1 (the vector of magnetic induction \( B \) of the magnetic disk is parallel to the disk axis) and a conductor 3, which via sliding contacts 4 connects the axle 1 with the current-collecting low-resistance rim 5 of the disk. In line with the main
version explaining EMF induction the rotation of the disk brings about EMF induction that at the finite resistivity of the disk material produces electric current in a closed circuit.

Fig. 1. Mounted on the axle 1, the magnetic disk 2 rotates with an angular velocity \( \omega \). The conductor 3 via the sliding contacts 4 connects the axle 1 to the current-collecting rim 5 of the disk. The magnetic induction vector \( B \) is directed perpendicular to the plane of the disk upwards (arrows).

In line with this explanation it is assumed that a magnetic field of a magnetic disk rotating at an angular velocity \( \omega \) remains stationary, and a Lorentz force \( F \) acting on free conduction electrons of the disk emerges in the disk moving in its own magnetic field and at a distance \( r \) from the center of the disk \( F = e\omega rB \), where \( e \) is the electron charge. This is due to the appearance of an electric field \( E = \omega rB \) producing EMF induction in a closed loop equal to \( \oint (vB)dl \) [2].

If the conductor 3 is absent, then after some time depending on the resistivity of the disk after the start of its rotation the movement of free electrons according to this explanation should lead to accumulation of electric charges in the disk material and on the conductive rim, creating an electric field in the magnetic disk that compensates for the electric field produced by the motion of the magnetic disk in its own magnetic field. If the resistivity of the disk material (magnetically solid ferrite or rubber ferrite) is high enough, the axle and the outer current-collecting rim form a cylindrical capacitor. In this case in the absence of the conductor 3 while the disk is rotating the cylindrical capacitor must be charged. In this case the presence of charge on the outer rim (in the presence of a compensating charge on the axle and within the magnet) is a prerequisite for the absence of electric field in a disk with finite conductivity. When the Lorentz electric field in the disk is equal to \( \omega rB \), the compensation of the electric field occurs if there is a volume charge \( \rho \) equal to \( 2B_0e_0\omega \) [3]. Upon abrupt termination of the disk rotation and disappearance of Lorentz forces, the charge on the rim and in the volume of the magnet for a certain period must preserve, depending on the charge relaxation time.

In line with the ‘electrotechnical’ version of the explanation of the EMF induction mechanism in a unipolar generator, nothing occurs in a disk rotating together with its magnetic field, and the EMF is induced in the conductor 3. If the conductor 3 is absent, then the cylindrical capacitor formed by the axle and the external current collector rim must remain uncharged during rotation.

The reason for the existence of essentially contradicting explanations of the mechanism of unipolar generation lies in the fact that in a unipolar generator the law according to which the circulation of the
electric field strength vector (EMF) $\mathbf{E}$ in a closed loop is equal to the velocity $-\mathbf{v}$ of the flow.

At first sight the question of the location of EMF induction in a loop can be solved experimentally and in an elementary way. It would seem that if, for example, the electrical resistance of a magnetic disk between the axle and the current collector rim is much greater than that of the conductor (or vice versa), then the voltage between the axle and the current collector rim should depend on the location of EMF induction.

Fig. 2. The equivalent circuits of the generator with different EMF localization. The left generator is a magnetic disk with an EMF source $1$ having internal resistance $R_m$. The right generator is an EMF inducing conductor connecting the axle to the current collector rim.

As seen from the equivalent circuits (fig. 2) of the unipolar generator for the cases of EMF induction in different sections of the circuit, the output voltage must be different. It must be equal to zero if the current source (marked with dashed lines) is a disk (fig. 2a) with internal resistance $R_m$, and the voltage drops on the internal resistance of the disk. If the voltage source is a conductor with practically zero internal resistance (fig. 2b), then the voltage on an EMF-generating conductor connected to a high-resistance load $R_m$ of the disk should be equal to the EMF. However, in practice the EMF localization by measuring the voltage of the generator using, for example, a voltmeter is difficult enough. The reason for this is the fact that if EMF is induced in the conductor 3, it is induced in any auxiliary conductor connected in parallel to the conductor 3. The EMF does not depend on the shape and arrangement of the conductors and is the same for all parallel conductors. This is understandable if one takes into account that any two parallel conductors form a closed loop (see fig. 3, where the contour is formed by the conductor 3 and the conductor 6 with the load 7), which has no sliding contacts. Since there is no change in magnetic flux in any such closed and having no sliding contacts contour located near the rotating disk, then the circulation of the electric field strength vector in this contour is zero. But if the circulation in such a contour is zero, then when EMF is induced in one of the conductors, exactly the same oppositely directed EMF when traveling around the circuit must emerge in the other conductor. For this reason, if a voltmeter via conductors 6 is connected to the axle and the current collector rim as the load 7, then if the EMF is induced in the conductor 3, an additional source of EMF will appear in the voltmeter circuit, exactly the same as in the conductor 3.
Fig. 3. A unipolar generator consisting of the axle 1, the magnetic disk 2, and the conductor 3 with the sliding contacts 4 touching both the axle and the conductive rim 5 contains an additional conductor 6 with the load 7. The conductor 6 has an arbitrary shape. The circuit formed by the conductor 3 and the conductor 6 with the load 7 is closed.

Fig. 4. The equivalent circuit of the generator. The EMF is induced both in the conductor and in the voltmeter circuit. 1 – EMF sources, 2 – voltmeter, 3 – conductor, $R_m$ – disk resistance, $R_v$ – internal resistance of voltmeter.

When an EMF source is present in the voltmeter circuit, an equivalent arrangement in the case of EMF induction in external circuits by a rotating magnetic field is the circuit given in fig. 4.
Using Kirchhoff's circuits law, it is easy to show that in the case given in fig. 4 the voltage indicated by the voltmeter as in the case represented by the equivalent circuit in fig. 2a, will be zero. That is, the measurement of the conductor voltage will not produce the expected result to localize EMF induction.

The strange behavior of the closed contour of the unipolar generator, which leads to the appearance of a current in the absence of change in the magnetic flux in the contour, is mostly attributed to the presence of a sliding contact in the contour.

This explanation can hardly be considered satisfactory because sliding contacts occur in generators based on a different operating principle, e.g. in DC generators, but there the law of induction is ‘at work’. A more profound explanation of the phenomenon as to the disparity between the circulation of the electric field strength and the rate of change in the magnetic flux is given by A. Kholmetskii [4], who relates this law to a violation of the continuity of the dependence of velocity functions of the contour elements and the vector potential on the radius and time at the point of a sliding contact. However, even this explanation being purely mathematical does not remove the question of EMF localization and does not contradict the assumption of EMF localization in a conductor. One can get convinced in the absence of such a contradiction by passing on from the consideration of the rotating magnetic disk to the rectilinear motion of the magnetized bar.

![Diagram](image)

**Fig. 5.** The rectangular high-resistance magnet 1 on its sides covered with low-resistance current collector plates 2 rests in the reference system K. The conductor 3 moves with a velocity v in a magnetic field, contacting the plates 2 with sliding contacts 4. The stationary voltmeter 5 is connected to the plates 2 of the stationary magnet. The magnetic induction vector is directed upwards (arrows).

Consider a device that will be called a Faraday DC linear generator. The generator (fig. 5) is a high-resistance magnetic bar 1 resting in the reference frame K, the lateral low-resistance current collector plates 2 of which are connected to each other by the conductor 3 via sliding contacts 4, as shown in the figure. The magnetic induction vector of the magnetic bar is directed perpendicular to the upper and lower planes of the bar. A voltmeter 5, also resting in the reference frame K, is connected to plates 2.
Let us set the conductor 3 in motion. The EMF is induced in a conductor moving in a magnetic field, and a current appears in the closed loop formed by the conductor 3 and the magnet 1, the strength of which depends on the EMF and the resistance of the magnet between the plates 2. If the resistance of the magnet by far exceeds that of the conductor 3, then free charges appear on the plates of the capacitor, whose role is played by the current collector plates 2 separated by high-resistance magnet material. If the voltage induced by the conductor 3 moving in a magnetic field is measured on the current collector plates of a stationary magnetic bar, then the voltmeter will show the voltage \( vBl \) Volt when the distance \( l \) between the plates is small compared to other linear dimensions of the magnet. In principle, it is also possible to measure free charges on the plates.

![Diagram](image)

**Fig. 6.** The rectangular high-resistance magnet 1 on its sides covered with low-resistance current collector plates 2 rests in the reference frame K. The conductor 3 moves with a velocity \( v \) in a magnetic field, contacting the plates 2 via the sliding contacts 4. The voltmeter 5 moving together with the conductor 3 is connected to the sliding contacts 4 via conductors 6. The magnetic induction vector is directed upwards (arrows).

We have based our previous reasoning on the unconditional and generally accepted fact of EMF induction in a conductor moving in a magnetic field. We now turn to the reference frame K' where the conductor is at rest while the magnetic bar fitted with a voltmeter is in motion. The unconditional fact is that the voltmeter reading equal to \( vBl \) with the change of a reference frame remains the same. But the presence of a final voltage on the current collector plates unequivocally indicates that in the reference frame K' EMF is generated in a conductor and not in a magnetic bar. If EMF were generated in a moving bar, then because the generator was short-circuited by the conductor at rest in this system, the voltmeter reading would be zero. When an induced EMF is localized in the bar, the reading must be zero for another reason, namely, because of the compensating EMF induction in the voltmeter circuit moving together with the magnetic bar in its magnetic field.
The fact of EMF generation in a conductor and not in the magnet of a linear generator indicates the possibility of such EMF localization under the discontinuity of the velocity of the contour elements and of the vector potential as noted by A. Kholmetzkii, which also occurs in a linear generator.

Let's note an important circumstance. We have considered the case in which the voltmeter is at rest relative to the magnet. It is this case that makes it possible to reveal the localization of EMF generation. If the ends of a conductor are connected to a voltmeter, which while at rest relative to the conductor moves along with the latter at a speed $v$ relative to the magnetic bar (fig. 6), then the voltage measured on it will be zero. In this case the equality of the voltage to zero is due to the fact that in the circuit of the voltmeter that is under the same conditions as the conductor exactly the same EMF is induced as in the conductor. The contrary direction of the EMF in the conductor and in the circuit of the voltmeter when passing round the closed circuit \('\text{conductor 3 - conductor 6 fitted with voltmeter 5}'\) causes zero reading of the voltmeter as is also shown by the equivalent circuit in fig. 4. The situation with the use of a voltmeter connected in this way to localize EMF as shown in fig. 6 is just as hopeless as with the case of a rotating magnetic disk. The example with a linear generator shows that in order to localize EMF induction in a unipolar generator with a rotating disk it is necessary to directly or indirectly place the voltmeter circuit on the rotating disk.

Now consider a hypothetical generator given in fig. 7, which is a rotating high-resistance magnetic ring 1 with an arbitrarily large radius $R$. The ring is fitted with an external 2 and internal 3 low-resistance current collector rings connected to each other by a conductor 4 via sliding contacts 5.

![Fig. 7. Rotating with an angular velocity $\omega$ a high-resistance magnetic ring 1 fitted with outer 2 and inner 3 low-resistance current collector rings. The conductor 4 is connected to the conductive rings via sliding contacts 5. The magnetic induction vector is directed upwards (arrows).](image)

This arrangement is not fundamentally different from the disk considered above (it differs only in the size of the inner ring), therefore a current must appear in the closed conductor-magnet circuit. Not different from the disk, this large-diameter arrangement is fundamentally little different from a linear generator in the sense that the short sections of the ring move practically rectilinearly at each instant of time. If EMF is induced in the magnet, then in the absence of a conductor 4 that closes the circuit a free charge must appear on the outer and inner rings, the electric field of which compensates for the electric
field that is induced in the magnet due to the motion of the magnetic ring. If EMF is induced in the conductor 4, then in the absence of the latter there must be no free charge on the rings.

When considering a generator given in fig. 7, once again the questions arise whether EMF is induced in a magnetic ring and whether free charges appear on the current-collecting rings in the absence of a conductor closing the outer and inner conductor rings. What will the stationary voltmeter connected via sliding contacts to the inner and outer ring of the rotating magnetic ring show, and what will the voltmeter connected to the inner and outer ring and located on the rotating ring show?

We will try to answer these questions by considering the mechanism of EMF induction in the conductor of a linear generator from the reference frame where the magnet is in motion.

Fig. 8. In the reference frame K₀ the belt 1 on the rotating pulleys 2 moves with the speed \( u_0 \) relative to the plates A and B. The surface charge density of the plates A and B is minus \( \sigma_0 \). The surface charge density of the moving belt is \( \sigma_0 \). The electric field between the far section \( a \) and the near section \( b \) of the belt is absent. The arrows indicate the magnetic induction vector. In the reference frame K, the structure moves in the direction of the X axis (in the figure shown by an arrow on the left) with a velocity \( v \) (also shown by an arrow). The conductor 3 will remain at rest in the reference frame K.

3. The mechanism of EMF induction in the conductor of a linear generator

The total effect of electrons creating a magnetic field of permanent magnets is ‘equivalent to the current circulating over the surface of the magnet’ [3]. Noting this fact, R. Feynman writes: ‘Therefore, it is no accident that a magnetic rod is equivalent to a solenoid’ [5]. We note that in this case we are not talking about the conduction current, which has also been mentioned by A.N. Matveyev [2].

Let us also represent a magnet in the form of a structure (fig. 8) that includes dielectric plates: the far plate \( A \) and the near plate \( B \) both carrying a negative electric charge, and an endless dielectric belt 1 of \( l_0 \) width on the pulleys 2. The belt carries a positive electric charge and, with the pulleys rotating, moves...
relative to the plates $A$ and $B$ with the speed $u_0$. The structure contains moving elements, but as a whole it is at rest in the reference frame $K_0$.

The surface charge density of the plates is minus $\sigma_0$, the intrinsic surface charge density of the belt $\sigma'_{\text{prop}}$ is such that in the reference frame $K_0$ when the belt moves (relative to the plates $A$ and $B$) with the speed $u_0$ there is no electric field of strength $E_0$ between the flat sections $a$ and $b$ of the belt.

The motion of the belt carrying a charge whose surface density in the reference frame $K_0$ is $\sigma_0$ creates in this reference frame an electric current of strength

$$I_0 = u_0 \sigma_0' I_0.$$  \hfill (1)

At a sufficiently large width $l_0$ of the belt the current within the structure creates a magnetic field with an induction $B$ equal to $\mu_0 I_0/l_0$, or, taking into account (1) $B = \mu_0 u_0 \alpha_0$.

It is clear that a condition for the absence of an electric field between the sections $a$ and $b$ of the belt is an equality of the surface density $\sigma_0$ of the moving belt charge to an absolute value of the negative charge density of the plates. Taking into account the Lorentz contraction of the belt, this condition can be expressed by the formula

$$\sigma'_{\text{prop}} = \sigma_0 \sqrt{1 - u_0^2 / c^2}. \hfill (2)$$

Consider the given structure in the reference frame $K$, in which the reference frame $K_0$ with the considered structure contained within moves in the direction of the $X$ axis with the speed $v$. If the directions of the movement of the far belt $a$ and of the structure coincide, the speed $v_\alpha$ of the far section of the belt in the reference frame $K$ according to the relativistic velocity-addition formula can be represented as

$$v_\alpha = \frac{v - u_0}{1 - vu_0/c^2}. \hfill (3)$$

The surface charge density of the far section $a$ of the moving belt in the reference frame $K$ is related to the speed $v_\alpha$ of the far section of the belt by the formula

$$\sigma = \sigma'_{\text{prop}} / \sqrt{1 - v_\alpha^2 / c^2}. \hfill (4)$$

or taking into account (3)

$$\sigma = \sigma'_{\text{prop}} / \sqrt{1 - (v - u_0)^2 / (1 - vu_0/c^2)^2 c^2}. \hfill (5)$$

After elementary transformations (see Appendix 1, p. 13), the formula (5) takes the form:

$$\sigma = \frac{1 - vu_0/c^2}{\sqrt{1 - v^2/c^2} \sqrt{1 - u_0^2 / c^2}} \sigma'_{\text{prop}}. \hfill (6)$$
Since by the condition (2) \( \sigma'_{\text{prop}} = \sigma_0 \sqrt{1 - u_0^2 / c^2} \), from the formula (6) it follows

\[
\sigma = \frac{1 - vu_0 / c^2}{\sqrt{1 - v^2 / c^2}} \sigma_0.
\] (7)

Taking into account the fact that the surface density of the negative charge of the plate \( A \) when moving in the reference frame \( K \) acquires due to Lorentz contraction a value equal to minus \( \sigma_0 \sqrt{1 - v^2 / c^2} \), the total value \( \sigma_{A,\text{sum}} = \sigma - \sigma_0 \sqrt{1 - v^2 / c^2} \) of the surface density of the far section of the belt \( a \) and of the plate \( A \) becomes equal to

\[
\sigma_{A,\text{sum}} = -\frac{vu_0}{c^2 \sqrt{1 - v^2 / c^2}} \sigma_0
\] (8)

Similarly, (see Appendix 2, p. 13), using the velocity addition formula \( v_a = \frac{v + u_0}{1 + vu_0 / c^2} \) and considering in the reference frame \( K \), the surface charge density in the near section of the belt \( b \) and the surface charge density on the plate \( B \), it is easy to find that the total charge density \( \sigma_{Bb,\text{sum}} \) on this section of the belt \( b \) and on the plate \( B \) is

\[
\sigma_{Bb,\text{sum}} = \frac{vu_0}{c^2 \sqrt{1 - v^2 / c^2}} \sigma_0
\] (9)

and differs from the total charge density at the top only by the sign.

The electric field strength \( E \) between the plates of the moving flat capacitor, the absolute value of the charge density on the plates of which is \( \sigma_{\text{sum}} \), is set by the equality \( E = \frac{\sigma_{\text{sum}}}{\varepsilon_0} \) or taking into account (9)

\[
E = \frac{vu_0 \sigma_0}{c^2 \varepsilon_0 \sqrt{1 - v^2 / c^2}}.
\] (10)

The non-zero surface charge of the moving capacitor as well as the accompanying electric field of strength \( E \) as set by the formula (10) is a purely relativistic value, and appears only in the reference frame in which the ‘capacitor’ moves. And the charge is localized not on the conductive plates, but in the layers of the moving magnet, which are adjacent to the plates.

An electric field of strength \( E \) displaces free electric charges in the conductor 3 at rest in the reference frame \( K \). The displacement of free charges and their accumulation on the plates \( A \) and \( B \) or at the ends of the conductor 3 (for example, in the absence of plates \( A \) and \( B \)) is an absolute effect that does not depend on a reference frame. Similarly, in a reference frame \( K_0 \) the free charges become also displaced, which leads to the formation of EMF in the conductor 3. The EMF source, which is the conductor 3, being connected to the body of the magnet at rest in the reference frame, creates an electric current in a closed circuit.

The mechanism of EMF induction in a conductor of a unipolar generator given in fig. 7 is the same. Representing the unipolar generator given in fig. 1 in the form of thin rings similar to the generator in
fig. 7, the emergence of a current in a closed circuit can be explained by EMF induction in a conductor, and the very appearance of EMF on small sections of the conductor by the relativistic polarization of a section of a magnet moving relative to a section of the conductor.

Taking into account the equality \( c = \sqrt{\frac{e_0 \mu_0}{\epsilon_0 \mu_0}} \), the formula (10) can be written as:

\[
E = \frac{\mu_0 v u_0 \sigma_0}{\sqrt{1 - v^2 / c^2}}. \tag{11}
\]

Using (1), the formula (11) can be transformed into

\[
E = \frac{\mu_0 v I_0}{l_0 \sqrt{1 - v^2 / c^2}}. \tag{12}
\]

The expression \( \mu_0 I_0 / l_0 \) gives the value of magnetic induction \( B_0 \) within the moving belt in the reference frame \( K_0 \). Thus,

\[
E = \frac{v B_0}{\sqrt{1 - v^2 / c^2}}. \tag{13}
\]

Since the induction \( B \) of the magnetic field between the plates in the reference frame \( K \) is related to the induction \( B_0 \) of the magnetic field in the reference frame \( K_0 \) by the formula of Lorentz transformations of the fields \( B = B_0 / \sqrt{1 - v^2 / c^2} \), then the electric field strength represented by the formula (13) can be written as

\[
E = vB. \tag{14}
\]

The formulas (10) and (14) show that in the zone of the conductor 3 at rest in the reference frame \( K \) given in fig. 8 the electric field can be considered both as a consequence of the relativistic polarization of the moving magnet, which produces the surface density of the bound charge \( \frac{v u_0 \sigma_0}{c^2 \sqrt{1 - v^2 / c^2}} \), and purely conditionally, as a consequence of the magnetic field motion by the induction \( B \) with a speed \( v \).

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