Research on Transverse Vibration of Hoisting Vertical Rope

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Abstract. By analysing the source of vibration of the main rope, and deriving the excitation function of the head sheave and the cage guides, a physical model of the vibration of the main rope is established. The generalized Hamilton principle is used to establish the transverse and longitudinal coupled vibration equation of the main rope. Considering the influence of rigid cage guides and head sheaves excitation on the main rope vibration, the main rope vibration control equation is discretized and solved based on the Galerkin truncation method to obtain the main rope transverse vibration displacement curve. At the same time, the machine vision image matching method is used to realize the measurement of the transverse vibration of the vertical rope, and the pixel coordinates of the target are converted into actual physical coordinates through coordinate conversion, thereby calculating the transverse vibration displacement of the vertical rope. Finally, the results of comparison experiments show that the above matching algorithm has strong practicability for measuring the transverse vibration of the hoisting main rope.

1. Introduction
With the increase in mining depth and production capacity, the safety of coal mining has become increasingly prominent. The wire rope is directly connected to the hoisting conveyance, and is responsible for the transportation of coal, personnel and various equipment materials [1]. The transverse vibration characteristics of the wire rope directly affect the safety of the hoisting system. In severe cases, broken rope, conveyance jam or casualties might take place [2]. Traditional wire rope detection methods mostly use contact vibration acceleration sensors for monitoring. The data measured by this method is affected by the distance of the wire rope movement, and there are difficulties in installation and signal transmission. Therefore, the method of machine vision measurement is proposed. Machine vision has wider measurement range, and it’s non-contact and easy to operate [3]. A visual measurement method based on template matching is adopted to measure the transverse vibration of the moving vertical steel wire rope.

2. Mathematical model of the hoisting main rope
In the study of the hoisting wire rope, the longitudinal and transverse damping of the steel wire rope has little influence on the transverse and longitudinal vibration [4]. It is considered as a continuous elastic body without slender longitudinal and transverse damping, and the vibration mechanical model of the main hoisting rope is constructed by using the method of distributed parameters [5].

The vibration model of the hoisting system is shown in figure 1. It is mainly composed of hoisting main rope, head sheave, rigid cage guides and the hoisting conveyance. With the separation point of
main rope and head sheave as the origin O, establish the x-o-y coordinate system, set the direction upward as positive, and take the distance from the wire rope to the origin O is the position point P with the length of \(x(t)\), the position point has vibration in both transverse and longitudinal directions, where the Y direction is the transverse vibration \(u(x(t), t)\), and the X direction is the longitudinal vibration \(w(x(t), t)\), the longitudinal movement speed of the hoisting conveyance and the main hoisting rope is \(v(x(t), t) = L(t)\), and the longitudinal acceleration is \(a(t) = \dot{v}(x(t), t) = \ddot{L}(t)\).

![Figure 1. Model construction](image)

2.1. Derivation of vibration equation of hoisting main rope

According to the generalized Hamilton principle, the kinetic energy, elastic potential energy, gravitational potential energy and damping virtual work of the hoisting system are obtained [6].

The overall kinetic energy \(E_k(t)\) during the operation of the hoisting system can be expressed as equation (1):

\[
E_k(t) = \frac{1}{2} \rho \int_0^{l(t)} \left( v + \frac{Dw}{Dt} \right)^2 dx + \int_0^{l(t)} \left( \frac{Du}{Dt} \right)^2 dx + \frac{1}{2} M \left( v + \frac{Dw}{Dt} \right)^2 |x = l(t) + \frac{1}{2} M \left( \frac{Du}{Dt} \right)^2 |x = l(t) \quad (1)
\]

Elastic potential energy \(E_e(t)\) is

\[
E_e(t) = E_{e1}(t) + E_{e2}(t) = E_{e0} + \int_0^{l(t)} \left( T \varepsilon + \frac{1}{2} EA \varepsilon^2 \right) dx + \frac{1}{2} ku^2 |x = l(t) \quad (2)
\]

Gravitational potential energy \(E_g(t)\) is

\[
E_g(t) = E_{g0} - \int_0^{l(t)} \rho gw dx - Mgwx = l(t) \quad (3)
\]

The damping virtual work in system is

\[
W_\delta = (-c \frac{Du}{Dt}, \delta u + cs + k\delta)|x = l(t) \quad (4)
\]

Substitute the above Equation (1) to (4) into the generalized Hamilton principle formula.

\[
\int_{t_1}^{t_2} (\delta E_k - \delta E_e - \delta E_g + \delta \delta) dt = 0 \quad (5)
\]

Leibnitz's law and partial integral method are also needed for variational and integral operations. The differential control equation and boundary condition equation of the hoisting system without excitation are obtained through a series of operations of equation (5). The expressions are as equation (6).
\[ \rho(a + w_n + aw_x + 2vw_x + v^3w_x) - T_x(1 + w_x) - TW_{xx} - EA(w_{xx} + 3w_xw_{xx} + \frac{3}{2}w_x^3w_{xx} + u_xu_{xx} + u_xuu_{xx} + \frac{1}{2}u_x^3w_{xx}) - \rho g = 0 \]

\[ \rho(u_n + au_x + 2vu_{xx} + v_xu_{xx}) - Tu_x - Tu_{xx} - EA(u_xw_x + w_xu_{xx} + w_xw_{xx}u_x + \frac{1}{2}w_x^2u_{xx} + \frac{3}{2}u_x^2u_{xx}) = 0 \]

\[ M(a + aw_x + 2vw_x + v_xw_{xx}) + T(1 + w_x) + EA(w_x + \frac{3}{2}w_x^3 + \frac{1}{2}w_x^3) + \frac{1}{2}u_x^3w_x + M(a + aw_x + 2vw_x + v_xw_{xx}) - Tu_x + E(A(u_xw_x + w_xu_{xx} + \frac{1}{2}w_x^2u_{xx} + \frac{1}{2}u_x^3) + ku + c(u_x + vu_x) = 0 \]

(6)

2.2. Vibration equation under boundary excitation

During the operation, due to the external excitation caused by the rigid cage guides and the head sheave, the upper and lower ends of the wire rope are subject to interference excitation. Under the external excitation of \( x = l(t) \), the expression of the boundary condition of lateral vibration is expressed as equation (7).

\[
\begin{align*}
    u(l(t), t) &= e_1(t) \\
    u(l(t), t) &= y(l(t), t) + h(x, t) \\
    y(l(t), t) &= 0
\end{align*}
\]

(7)

Under the external excitation of \( x = 0 \), the expression of transverse vibration boundary condition is

\[
\begin{align*}
    u(0, t) &= e_2(t) \\
    u(0, t) &= y(0, t) + h(0, t) \\
    y(0, t) &= 0
\end{align*}
\]

(8)

In this case, the boundary condition is a nonhomogeneous boundary condition, which becomes the sum of the corresponding homogeneous boundary condition expression \( y(x, t) \) and the non-homogeneous boundary condition expression \( h(x, t) \). The transverse vibration displacement function \( u(x, t) \) is expressed as equation (1-9).

\[ u(x, t) = y(x, t) + h(x, t) \]

(9)

2.3. Solution of vibration equation

Substituting equation (9) into (6), the solution is obtained as equation (10).

\[
\begin{bmatrix}
    M_{xx} \\
    0
\end{bmatrix}
\begin{bmatrix}
    \dot{P} \\
    \dot{Q}
\end{bmatrix} + \begin{bmatrix}
    C_{xx} \\
    C_y
\end{bmatrix}
\begin{bmatrix}
    \dot{P} \\
    \dot{Q}
\end{bmatrix} + \begin{bmatrix}
    K_{xx} \\
    0
\end{bmatrix}
\begin{bmatrix}
    P \\
    \dot{Q}
\end{bmatrix} = \begin{bmatrix}
    CP_x \\
    CP_y
\end{bmatrix} + \begin{bmatrix}
    F_x \\
    F_y
\end{bmatrix}
\]

(12)
Among them, \( P = [p_1, p_2, ..., p_n]^T \) and \( Q = [q_1, q_2, ..., q_n]^T \) represent the generalized coordinate vector of the main rope longitudinal and transverse respectively. \( M_w(t) \) and \( M_y(t) \) represent the masses corresponding to the generalized coordinate vectors \( P \) and \( Q \), \( C_w(t) \) and \( C_y(t) \) represent the damping corresponding to the generalized coordinate vectors \( P \) and \( Q \), \( K_w(t) \) and \( K_y(t) \) are the stiffness corresponding to the generalized coordinate vectors \( P \) and \( Q \), \( F_w(t) \) and \( F_y(t) \) are the generalized force matrix corresponding to the generalized coordinate vectors \( P \) and \( Q \), \( CP_w(t) \) and \( CP_y(t) \) are relative standard coupling item. Through the partial differential equations, \( y(x, t) \) is obtained.

With known \( y(x, t) \) and \( h(x, t) \), the displacement function \( u(x, t) \) of lateral vibration under external excitation is solved.

3. Research on machine vision main rope measurement

Machine vision technology is an important branch that has been extended with the development of computer science. It has the advantages of convenient installation, strong adaptability, wide measurement range and non-contact, which can realize measurement, tracking, positioning and target recognition [8]. Aiming at the problem of visual measurement of steel wire rope vibration, it is mainly divided into four processing steps: video data input, data image pretreatment, matching positioning and result output.

3.1. Data image preprocessing

Before the matching and positioning of the measuring points, it is necessary to pre-process the collected images. It is known that the main hoisting rope is in the initial position before the hoisting system is not running. When it’s running, the main hoisting rope is in dynamic operation. The offset distance between the dynamic point position and its initial position in the static state is the vibration displacement. The vibration displacement measurement of a certain point in the main hoisting rope space is actually the movement of the point in the horizontal direction of the image.

Therefore, a virtual auxiliary line perpendicular can be added to the main hoisting rope and passing through the measuring point, as shown in figure 2. The intersection is regarded as the measuring point of the steel rope.

![Effect picture of main hoisting rope pretreatment](image)

**Figure 2. Effect picture of main hoisting rope pretreatment**

3.2. NCC matching algorithm optimization and result analysis

When using the NCC matching algorithm to calculate the location of the wire rope measurement point, the calculation of the similarity measurement formula of the NCC matching algorithm is too complicated, which causes the calculation of the wire rope vibration data result to be too large. To analyse the similarity measurement formula \( R(i, j) \), Let \( T_1 = |Q(m, n) - E(Q)| \), then

\[
R(i, j) = \frac{\sum_{m=1}^{n} \sum_{n=1}^{m} S_{ij}(m,n)R_1}{\sum_{m=1}^{n} \sum_{n=1}^{m} |S_{ij}(m,n) - E(S_{ij})|^2} \tag{13}
\]

where \( E(S_{ij}) \), \( E(Q) \) are the average grey values of the search sub-image and the template image respectively, and the numerator part is the convolution operation of the sub-image and the template, which can introduce the idea of difference summation for calculation [9]. The specific difference steps are as follows:

First, from the difference and summation theorem, \( \sum_{k=1}^{n} f_1(x)g_2(x) = \sum_{k=1}^{n} f_1(x)F_2(x) \). \( T_1 \) in the numerator of equation (13) are stored in descending order as a one-dimensional array, denoted as \( f_1(x) \).
\[ f_1(x - 1) \geq f_1(x), \quad x = 1, 2, \ldots m_n \]  
\[ F_1(x) = f_1(x - 1) - f_1(x) \]

where \( f_1(m_n) = 0 \).

When sorting in descending order, it is necessary to record the initial position of each gray value before sorting, so as to maintain the consistency of the corresponding point positions of the search sub-image and the template image during the operation. After that, the search subgraph \( S \) is saved as a one-dimensional array in the descending order recorded when sorting by the previous \( T_1 \), denoted as \( f_2(x) \), and cumulatively summed. \( F_2(x) \) is the array after the cumulative sum of \( f_2(x) \).

\[ F_2(x) = F_2(x - 1) + f_2(x) \]

In equation (16), \( F_2(0) = 0 \), a large number of 0 and 1 elements will be generated in the \( F_1 \) calculation result, which reduces the calculation amount of the similarity measurement formula \( R(i,j) \).

3.3. Result output

Calculate the vibration displacement \( d(t_i) \) of the measuring point at time \( t_i \) through the scale factor [10].

\[ d(t_i) = \begin{cases} \lambda \sqrt{(x_i - x_1)^2 + (y_i - y_1)^2}, & y_i \geq y_1 \\ -\lambda \sqrt{(x_i - x_1)^2 + (y_i - y_1)^2}, & y_i < y_1 \end{cases} \]

where \( \lambda \) is the scale factor, used to convert the pixel distance in the image coordinate system into the actual distance, and the unit is mm; \( t \) is the total number of frames or the camera's acquisition frequency, \( I = 1, 2, 3, \ldots \)

\[ \lambda = \frac{d}{D} \]

4. Experimental verification and data analysis

To verify the effectiveness and robustness of the vibration measurement based on template matching, comparison is made between theoretical analysis and experiments. The test platform is shown as figure 3.

Figure 3. Physical picture of single rope winding hoist test platform
(a) Winding hoist, (b) Hoisting conveyance

The machine vision monitoring system is shown as figure 4.

Figure 4. Overall layout of machine vision system
(a) The camera, (b) Photographed wire rope
4.1. Validation of vibration model of hoisting main rope
A set of 1.5 mm bulges are set at the side of the cage guide at 9.3 m (T=18 s). Influence of transverse vibration at the hoisting speed of 0.5 m/s is investigated. The final vibration displacement curve of the main rope at 2 m away from the head sheave is shown in figure 5.

![Figure 5. Vibration displacement of the main rope 2 m away from the head sheave](image)

(a) Theoretical curve, (b) Measured curve

Through comparison between the theoretical curve and the measured curve in figure 5, it can be seen that the measured amplitude in the steady state is 2.1 mm, and the amplitude at the bulge is 4.2 mm, while the corresponding theoretical values are 2 mm and 5.1 mm. The relative errors are 4.7% and 17.7%.

4.2. Influence of the vibration characteristics of the main rope under the excitation of different rigid cage guide positions
During the experiment, the industrial camera is installed on the wellhead platform with a horizontal distance of 1.6 m from the hoisting main rope, and the spatial position of the main hoisting rope at a distance of 2 m from the head sheave platform is monitored. The following three comparison methods are arranged for the failure excitation position of the rigid cage guide, that is, set a set of bumps of 1.5 mm in size at 9.3 m, two sets of bumps of 1.5 mm at 6 m and 9.3 m, and two sets of bumps of 1.5 mm at 2.5 m and 9.3 m. The theoretical and actual transverse vibration displacement after wavelet denoising [11] is shown in figure 6.

![Figure 6. Theoretical and measured curve at different cage guide excitation positions](image)

(a) Theoretical curve, (b) Measured curve

Comparing the theoretical and experimental result, it is found that the change law remains the same. The transverse vibration of the main rope generally increases from a small amount, then remains stable, and finally decreases gradually. Among them, if there is a rigid cage guide excitation, a curve mutation will occur. By changing the distance between the fault location of the rigid cage guide and the monitoring point, it can be seen that as the fault location gradually moves away from the monitoring...
point, the amplitude of the transverse vibration of the sudden change position of the main hoisting rope also decreases, but it is significantly greater than the transverse vibration in the steady state amplitude.

5. Conclusion
The experiment shows that there is a certain degree of difference between the experimental and theoretical results. The error may be caused by: the boundary excitation is obtained under the pre-set ideal conditions, which is different from the excitation received during the operation of the actual hoisting system, and besides, wind resistance, frictional damping might also lead to some errors.

It can be concluded that the change trend of the measured curve and the theoretical curve is consistent, and the overall error value is within a reasonable range, which demonstrates the feasibility of the image matching measurement method to monitor the transverse vibration of the hoisting main rope.

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