Bilinear Time-frequency Analysis for Lamb Wave Signal Detected by Electromagnetic Acoustic Transducer

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Abstract. Accurate acquisition of the detection signal travel time plays a very important role in cross-hole tomography. The experimental platform of aluminum plate under the perpendicular magnetic field is established and the bilinear time-frequency analysis methods, Wigner-Ville Distribution (WVD) and the pseudo-Wigner-Ville distribution (PWVD), are applied to analyse the Lamb wave signals detected by electromagnetic acoustic transducer (EMAT). By extracting the same frequency component of the time-frequency spectrum as the excitation frequency, the travel time information can be obtained. In comparison with traditional linear time-frequency analysis method such as short-time Fourier transform (STFT), the bilinear time-frequency analysis method PWVD is more appropriate in extracting travel time and recognizing patterns of Lamb wave.

1. Introduction

Electromagnetic acoustic transducer (EMAT) can be effectively used for the non-contact nondestructive testing and evaluating (NDT&E) of metallic materials. The main advantage of the detection technique is that no coupling agent is required [1]. The ultrasonic Lamb wave (ULW) has been widely used in NDT & E of the plate-like material due to its fast detection speed. The cross-hole tomography technique based on ULW detected by EMAT has received considerable attention nowadays [2,3]. The method of cross-hole imaging is to divide the imaging area into a lot of grids, to iteratively calculate the slowness distribution based on the extracted travel time and the propagation paths of all Lamb waves and achieve image reconstruction. The travel time is the time when ULW signal propagates from the transmitting transducer to the receiving transducer. The accuracy of the travel time extraction determines the performance of image reconstruction. The previous work of extracting the travel time of the Lamb wave is based on the time-domain or the Hilbert transform methods. The signal detected by EMAT is a typical unsteady signal, which is difficult to be completely described the characteristics by using a single time-domain or frequency domain analysis method. Time-frequency analysis is the abbreviation of Joint Time-Frequency Analysis, which clearly offers the relationship between the frequency component of a given signal and a function of time. It describes the signal’s density and intensity at different times and frequencies.

Using time-frequency analysis method, the energy density of each frequency component of the ULW signal can be effectively obtained with time, which can help to achieve more accurate extraction of travel time. Short-time Fourier transform (STFT) is an early and widely used linear time-frequency analysis method, which has been used to recognize the overlapped Lamb wave signals detected in the time domain [4]. Wavelet transform (WT) is another linear time-frequency analysis method, and its
largest advantage is that it can automatically adjust the analysis resolution. K. S. Ho et al. used a wavelet transform method to analyze the propagation of multimode Lamb wave [5].

Compared with the linear time-frequency analysis methods, the bilinear time-frequency distribution has good time-frequency focusing and is very suitable to process non-stationary and multi-frequency signals. Some members of the bilinear time-frequency analysis methods are Wigner-Ville distribution (WVD), the pseudo-Wigner-Ville distribution (PWVD), windowed Wigner-Ville distribution (WWVD) and smooth-windowed Wigner-Ville distribution (SWWVD) [6]. In this paper, WVD and PWVD are chosen to analyze and obtain the travel time information from Lamb waveforms.

2. WVD and PWVD Theory
The common linear time-frequency representation is STFT and wavelet transform. The STFT realizes the localization in the time-domain of the signal. The wavelet transform has the characteristics of multi-resolution and greatly enriches the signal analysis and processing method. We will use WVD and PWVD to analyze the multimode Lamb waves. WVD is the most common bilinear time-frequency analysis method. It can provide an advantage which has high time-frequency resolution, energy focus and can almost meet all the expected features of time-frequency distribution. However, this advantage comes at the expense of some limitations when dealing with the multi-component signal [7,8]. The WVD algorithm is defined as:

$$W(t, f) = \int s^\prime(t - \frac{\tau}{2})s(t + \frac{\tau}{2})e^{-j2\pi f \tau} d\tau$$  \hspace{1cm} (1)$$

In equation (1), \(t\) represents time and \(f\) represents frequency. \(s^\prime\) is the complex conjugate. The integrand \(\int s^\prime(t - \frac{\tau}{2})s(t + \frac{\tau}{2})\) is called the instantaneous autocorrection function of WVD (autocorrection for short) and the result is calculated from \(-\infty\) to \(+\infty\). When there are more than one frequency components contained in \(s(t)\), there is a cross-term in WVD and that will affect the signal judgment and analysis. To eliminate the interference of the cross distribution, one common method is to use a time limited function \(h(\tau)\) to add window processing to \(s^\prime(t - \frac{\tau}{2})s(t + \frac{\tau}{2})\) in equation (1). The PWVD algorithm is defined as

$$W_p(t, f) = \int_{-\infty}^{\infty} h(\tau)s^\prime(t - \frac{\tau}{2})s(t + \frac{\tau}{2})e^{-j2\pi f \tau} d\tau$$  \hspace{1cm} (2)$$

Where \(h(\tau)\) is a time-domain window function, which is equivalent to perform low-pass filtering on the signal in the frequency-domain. Thus only the distribution within the limited time-frequency range is considered. Therefore, using a window function can reduce the interference.

3. Experiment Setup
In this paper, we used the two-layer spiral coil [9] on the 3-mm thick aluminum plate (1000mm × 1000mm) and used the perpendicular coupling magnetic field to generate and detect A0 and S0 mode Lamb wave at the driving frequency 240kHz. The experimental system for ultrasonic Lamb waves is shown in Fig.1. The Ritec 5000 SNAP ultrasonic experiment platform is used for testing, which is controlled by the computer to generate tone-burst signal for the spiral coil EMAT transmitter excitation, and connecting the received signals to the oscilloscope for display and output. The spiral coil of the transmitter and receiver is manufactured by the flexible printed circuit (FPC). The spatial separation \(Ax\) between the transmitter and receiver line is 600mm.

The frequency-thickness of the experiment was 0.72MHz·mm. According to the dispersion curve of aluminum plate, there are two modes of the signal under the above experimental conditions: A0 and S0 modes. When the driving frequency is 240kHz, the theoretical group velocities of the A0 and S0 mode are 3052m/s and 5267m/s, respectively.
4. Travel Time Extraction

Fig 2(a) shows the time domain signal detected by the EMAT receiver transducer when the transmitter and receiver are 600mm apart from each other in the experimental system in Fig.1. As can be seen, there are two ultrasonic signals in it. The corresponding arrival times are around 120us and 200us, respectively. The corresponding frequency-domain signal is shown in Fig 2(b). According to Lamb wave group velocity curve, the S0 and A0 signal theoretical travel time should be 600/5267=113.9(us), and 600/3052=196.6(us). So the two ultrasonic signals (shown in Fig. 2(a)) with 120us and 200us arrival time are S0 and A0 modes, respectively.

Fig.1. The schematic diagram of the experimental system for ultrasonic Lamb waves generated and detected with spiral-coil EMATs.

Fig.2. Lamb wave signals at $f = 240$ kHz; (a) time-domain signal, (b) frequency-domain signal

In order to distinguish the ultrasonic Lamb wave mode shown in Fig. 2(a), we used bilinear time-frequency methods WVD and PWVD to analyze and extract the travel time in this paper. To make the
results more accurate and eliminate interference as far as possible, the time-frequency analysis method used in this paper all didn’t take the straight-through signal into account.

Fig. 3(a) and (b) shows the analysis of multimode Lamb wave based on the WVD method. The corresponding Wigner-Ville distribution (WVD) spectrogram is shown in Fig. 3(a). The A0 and S0 wave theory time-frequency curve in Fig. 3(a) is calculated by the Lamb wave group velocity curve and the electromagnetic acoustic transceiver transducer distance, which is used to identify various Lamb wave modes. Between the distribution S0 mode and A0 mode, there is another signal distribution, which accounts for a large range of frequency direction. And its center is far away from the A0 and S0 theory time-frequency curve, so it’s not the distribution of Lamb wave signals. It is known from WVD theory that the distribution is a cross-term distribution. Therefore, in Fig. 3(a), the energy distribution density in the range of 100~150us is generated by the signal S0 in figure 2(a), and the energy density in the range of 180~210us is generated by the signal A0. The EMAT detecting S0 and A0 mode wave signal energy density distribution in the direction of the frequency, only in a narrow frequency band near 240kHz. This is because the driving frequency of the experiment is 240kHz, so the analysis of the experimental results in Fig. 3(a) are consistent with the objective fact. Fig 3(b) displays the component at the driving frequency \( f = 240 \text{ kHz} \). The signal in the middle is a cross-distributed signal and the corresponding S0 and A0 wave modes are marked respectively. The time of their peaks represents the travel time of the time-domain signal.

Fig. 3. (a) WVD spectrogram, (b) slice of WVD spectrogram at \( f = 240 \text{ kHz} \)

Fig. 4(a) and (b) shows the analysis of multimode Lamb wave based on PWVD method, where the cross distribution is effectively suppressed, which is beneficial to the extraction of more accurate travel time. The PWVD spectrogram is given in Fig. 4(a), where the straight-through signal is also not considered. Figure 4(b) shows the component at the driving frequency \( f = 240 \text{ kHz} \) and the corresponding S0 and A0 wave modes are marked respectively. The time of their peaks represents the travel time of the time-domain signal.
Comparing Fig 3(b) and Fig 4(b), we can see: In Fig 3(b), the appearance of the cross-distribution signal affects the shape of A0 wave and S0 wave signal and reduces the accuracy of the extraction. In Fig 4(b), the cross distribution is effectively suppressed, so the A0 wave and S0 wave signal is not affected, which makes the information extraction more accurate.

As comparison, the STFT method is also used to extract the travel time from the same waveform in Fig 2(a). Choosing the Hamming window and the window function’s width is 127. The results of the STFT spectrogram are given in figure 5(a) and it is clear that the signal contains both A0 and S0 modes. The corresponding travel time is extracted as shown in Fig. 5(b).

Comparing Fig 3(b), Fig 4(b) and Fig 5(b), we can see: The travel time of the S0 and A0 wave mode obtained by using WVD and PWVD are: 128us and 203us. The S0 and A0 mode wave travel times are obtained by using STFT are: 129us and 203.5us. Thus the practical group velocity of the two
modes can be obtained by the above three methods and their comparison with the theoretical group velocity is given in table 1.

**Table 1.** The comparison of S0 and A0 wave’s theoretical velocity and practical velocity obtained by three methods.

| Wave mode | Theoretical velocity(m/s) | STFT(m/s) | WVD(m/s) | PWVD(m/s) |
|-----------|---------------------------|-----------|----------|-----------|
| S0        | 5267                      | 4651      | 4688     | 4688      |
| A0        | 3052                      | 2948      | 2957     | 2957      |

Table 1 shows that the group velocities of the two modes obtained by WVD and PWVD are the same, which are more closer to the theoretical velocity compared with the group velocity obtained by STFT. The relative errors of the group velocity of S0 and A0 mode calculated by the above three methods are about 11% and 3%. Among them, the relative error of the group velocity of S0 mode is slightly larger, which may be caused by the straight-through signal or the experimental noise.

5. Conclusion
The accuracy of travel time extraction directly determines the accuracy of image reconstruction quality. In this paper, we used the Wigner-Ville Distribution (WVD) and the pseudo Wigner-Ville Distribution (PWVD) to analyze a multiple Lamb wave modes signal detected by EMAT, where the magnetic field is perpendicular coupling. Research results showed that, comparing with the traditional linear time-frequency analysis method STFT transform, the bilinear time-frequency analysis method PWVD transform has the capability to extract more accurate travel time. This paper provides a good way to extract multiple Lamb wave travel time.

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