Distributed Frequency Regulation in Power Grids with Low and Time-Varying Inertia

Manasa Muralidharan, Jan Kleissl and Patricia Hidalgo-Gonzalez

Abstract—This paper presents a distributed frequency control method for power grids with high penetration of inverter-connected resources under low and time-varying inertia due to renewable energy (RE). We provide a distributed virtual inertia (VI) allocation method using the distributed subgradient algorithm. We implement our distributed control strategy under full and sparse communication architectures. The distributed full and sparse communication controllers achieve comparable performance to a centralized controller and stabilize the test system within 6 s. We study the sensitivity of the controller performance to varying objective function weights on phase angle versus frequency deviation, gradient step sizes, and allowed rate of change of inertia (RoCol) coefficients. We observe that the setting time of the states and the controller performance and effort are susceptible to changes in the gradient step size and objective weights on the frequency and angle deviation. While a higher objective weight on angle versus frequency deviations positively affects their settling times, it negatively impacts controller performance and control effort. The impact of this work is to propose distributed control schemes as new mechanisms that act before or in alignment with primary and secondary control to safely regulate the frequency in future power grids.

I. INTRODUCTION

A. Motivation

The frequency stability of the power grid depends on the balance between power supply and demand at all times [1]. Conventional grids are dominated by synchronous generators (SGs) that have kinetic energy stored in their rotating masses (rotational inertia). SGs help maintain grid frequency stability by extracting this stored kinetic energy in response to an instantaneous local mismatch in power supply and demand. The initial grid frequency response after a contingency is a function of the weighted average rotational inertia of all the SGs, which is assumed to be constant in a SG-dominated grid [2].

Unlike SGs, renewables interface with the grid via power electronic converters that inherently lack rotational inertia due to the absence of mechanical rotating parts [2]. Low aggregated rotational inertia correlated to renewable penetration [3] and fast frequency dynamics [2], [4] are, therefore, characteristic of a renewable-dominated grid. Rotational inertia is also temporally and spatially-varying as a result of high penetration of variable renewable energy [2], [4], [5]. Low and variable rotational inertia poses a challenge to grid frequency stability because a large disturbance from the nominal frequency can trigger frequency-dependent load shedding and other protection schemes. The triggering of protection schemes ultimately causes cascading failures that could result in a system-wide blackout [2], [4], [6].

B. Literature Review

Virtual inertia (VI) emulation, also referred to as synthetic inertia provision, is a proposed solution to maintain grid frequency stability in low inertia grids. VI emulation relies on inverter-based generation (IBG) mimicking the behavior of a SG. The work in [7]–[9] summarizes different VI emulation strategies for IBG that vary in how closely they approximate the behavior of SGs. These works model the complexity of the frequency dynamics to different degrees, with swing equation-based models being the prevalent choice for system-level frequency stability analysis. Several works study the optimal allocation of VI and damping, with the approaches varying in the metrics used to evaluate post-contingency system stability [10]. The work in [11], [12] characterizes the post-contingency system stability in terms of conventional metrics like the rate of change of frequency and frequency nadir, while [13] attempts to minimize frequency overshoot and damping ratio via pole placement. The authors of [14] treat the post-contingency response as a disturbance signal and evaluate the energy expended in minimizing the disturbance via the $H_2$ norm. The work in [15] further presents a comparison of VI and damping allocation using system-norm ($H_2$, $H_{\infty}$) approaches versus pole placement and concludes that the optimal performance metric cannot be generalized and may be system-specific. [11]–[15] all approach VI and damping allocation as a planning problem rather than a real-time control problem.

Only a few papers have considered the temporal variation in rotational inertia in designing frequency control techniques for VI and damping allocation. The work in [16] models, for the first time, the time-varying frequency dynamics as a switched affine hybrid system, with the system switching through different modes representing different levels of inertia. Using this framework, the authors solve a receding-horizon model predictive control problem for dynamic VI placement. The work in [17] models the temporal variation in inertia as a stochastic disturbance and minimizes the $H_2$ norm of the disturbance. Whereas [18] designs a fuzzy logic-based adaptive control to regulate frequency under time-varying inertia.

Additionally, the increased communication and information cost associated with centralized (C) control motivates the need for methods that consider the aggregation and coordination of different VI emulating DERs with existing SGs in maintaining grid frequency stability [7]. The authors of [19] present applications of distributed control methods from the multi-agent literature to classical power system
problems such as distributed power flow, frequency, and voltage control. However, distributed VI and damping allocation and its effect on grid frequency stability remain unexplored. Given that we can reformulate VI and damping allocation as an optimal resource allocation problem, distributed control methods developed for multi-agent networks are a suitable fit for solving this problem [10].

To the best of the authors’ knowledge, no existing work in literature provides a VI allocation method that is real-time, not centralized, and considers time-varying frequency dynamics.

C. Statement of Contributions

In this paper, we propose and implement a distributed optimization for VI allocation to empirically maintain the frequency stability of the grid. The contributions of this paper are a distributed control method for VI provision that is real-time, able to restore grid frequency even with sparse communication between the VI emulating resources, and considers time-varying power dynamics.

D. Organization of the Paper

The remainder of the paper is organized as follows. In Section II we introduce the general form of the optimal VI allocation problem and expand on the methodologies for both the centralized and distributed control approaches. Then in Section III we present parameters of the test system, and present our results for both the base case in Section IV.

A. Centralized VI Allocation

Assuming global knowledge of the system states, a solution to (1) exists if the objective function satisfies the first-order necessary condition for existence of an unconstrained minimizer [20], i.e., if \( \bar{m}(t) \) is an unconstrained minimizer of \( F(t) \) at time \( t \), the gradient of \( F(t) \) with respect to \( \bar{m}(t) \) is the zero vector.

**Assumption 1.** The local deviation in angle and frequency is only a function of the VI allocated locally i.e., \( \partial f_x / \partial m_i = 0 \) \( \forall i \neq j \). In other words, changes in local frequency and angles are only a result of the local inertia, local disturbances and power flow between the transmission lines. Frequency and angle deviations are not affected by changes in inertia from other nodes. Inter-area oscillations are assumed to be outside the scope of this study.

Under Assumption 1, the component of the gradient associated to node \( i \) at time \( t \), \( g_i(t) \), can be written as

\[
g_i(t) = \partial f_x(t) / \partial m_i(t) = \frac{\partial f_x(t)}{\partial \Delta \theta_i(t)} \frac{\partial \Delta \theta_i(t)}{\partial t} \frac{\partial t}{\partial m_i(t)}
\]

\[
= \left[ \gamma_0 \Delta \theta_i(t) + \gamma_\omega \Delta \omega_i(t) \frac{\partial \Delta \omega_i(t)}{\partial t} \frac{\partial t}{\partial \Delta \theta_i(t)} \frac{1}{1} \right].
\]

where \( \partial m_i(t)/\partial t = 2 \partial \theta_i(t) \), \( \partial \theta_i(t) \) is the rate of change of inertia (RoCol) at node \( i \), and is a function of the synchronous frequency, \( v_{sync} \) (Hz), the inertia coefficient at time \( t \), \( h_i(t) \) (s), and the rated power generation at node \( i \), \( S_i \) (MVA). The RoCol coefficient, \( \partial h_i(t)/\partial t \) dictates how much the inertia coefficient, \( h_i \), at a node \( i \) changes between consecutive time-steps.

**Assumption 2.** Let the RoCol coefficient, \( \partial h_i(t)/\partial t \) be equal to \( \delta h_i \), \( \forall t \), where \( \delta h_i \) is a non-zero scalar constant.

Given that the initial inertia coefficient at node \( i \) represents a predominantly renewable \( (h_i = 2 \text{ s}) \) or thermal \( (h_i = 4 - 10 \text{ s}) \) resource mix, a RoCol coefficient of 1 s/s implies that the resource mix at node \( i \) is still predominantly renewable/thermal at the next time step. This behavior occurs during time periods of peak renewable production or peak demand non-coincident with peak renewable production. Whereas, a 3 s/s RoCol coefficient implies that the resource mix can shift more rapidly in either direction from predominantly renewable to predominantly thermal or vice versa.

From Assumption 2, the RoCol, \( \partial m_i(t)/\partial t = 2 \delta h_i \), \( \delta h_i \) is also a scalar constant (denoted as \( \delta m_i \)). Further, from small signal approximation of the swing equation at node \( i \), assuming per unit voltage magnitudes, purely inductive lines [14] and no perturbation in power generation, we have:

\[
\frac{\partial \Delta \theta_i(t)}{\partial t} = \Delta \omega_i(t)
\]

\[
\frac{\partial \Delta \omega_i(t)}{\partial t} = -m_i(t)^{-1} \sum_{j=1}^n b_{ij} \Delta \phi_{ij}(t) + d_i \Delta \omega_i(t)
\]

where \( \Delta \phi_{ij} = \Delta \theta_i - \Delta \theta_j \) is the angle difference between...
nodes $i$ and $j$, and $d_i$ is the frequency-dependent load damping at node $i$. Substituting (3) in (2) and discretizing, we obtain the discrete-time gradient, \(\hat{g}(k) = [g_1(k), \ldots, g_n(k)]^T\), where each local gradient, $g_i(k)$, is written as

$$g_i(k) = \Delta\omega_i(k) \left[ \gamma_0 \Delta \theta_i(k) - \gamma_i m_i(k)^{-1} \Delta \phi_i(k) \right]$$

where $\Delta \phi_i(k) = \sum_{j=1}^n b_{ij} \Delta \phi_{ij}(k) + d_i \Delta \omega_i(k)$. Given an initial perturbation of the states, $x(0) = [\theta(0), \Omega(0)]^T$ and assuming that all nodes are controllable, we compute the allocated VI for all the nodes at discrete times $k = 1, \ldots, T$ by setting $\frac{\Delta \phi_i(k)}{\Delta \theta_i(k)}$ to zero and solving for $m_i(k)$ for all $i \in N$, i.e.,

$$m_i(k) = \frac{\gamma_i}{\gamma_0} \left( \sum_{j=1}^n b_{ij} \Delta \phi_{ij}(k) + d_i \Delta \omega_i(k) \right)$$

The system states are iteratively updated at every discrete time step as shown in (4) using a discretized swing equation.

The discretization that we use is the Forward Euler method with a step size of $h$.

$$\begin{bmatrix} \theta(k+1) \\ \omega(k+1) \end{bmatrix} = \begin{bmatrix} 0 & hI \\ -hM(k)^{-1}L - hM(k)^{-1}D \end{bmatrix} \begin{bmatrix} \theta(k) \\ \omega(k) \end{bmatrix}$$

where $M(k) \in \mathbb{R}^{n \times n}$ is a diagonal matrix whose diagonal elements correspond to the allocated VI obtained from (5). $D \in \mathbb{R}^{n \times n}$ is the diagonal damping matrix, and $L \in \mathbb{C}^{n \times n}$ is the network laplacian with off-diagonal elements $l_{ij} = -b_{ij}$, and diagonal elements $l_{ii} = \sum_{j=1,j \neq i}^n b_{ij}$. The computational cost of the centralized approach scales with the number of nodes in the network, $n$, as it requires full information from all the nodes.

**B. Distributed VI Allocation**

We propose a distributed method for VI allocation where each distributed controller (agent) optimally allocates VI based on limited information from its neighbors. We define the neighbors of agent $i$ as all agents that have an undirected edge with node $i$ [21]. In contrast to centralized VI allocation, each agent locally solves the following distributed optimization problem:

$$\min_{\tilde{m}^i(t) \in \mathbb{R}^n} F(t) = \sum_{i \in N} \frac{1}{2} \left( \gamma_0 \Delta \theta_i(t)^2 + \gamma_i \Delta \omega_i(t)^2 \right)$$

where the objective function is the same as in (1), but $\tilde{m}^i(t) = [m_1^i, \ldots, m_n^i]^T$ is now agent $i$’s estimate of allocated VI at all $n$ nodes (i.e., $m_1^i$ is agent $i$’s estimate of allocated VI at node $i$). Given each agent’s initial estimate of allocated inertia at all $n$ nodes, $\tilde{m}^i(0)$, agent $i$’s estimate of allocated inertia at discrete times $k = \{1, 2, \ldots, T\}$ can be computed using the distributed subgradient algorithm [22] as

$$\tilde{m}^i(k+1) = \sum_{j=1}^n a_{ij} \tilde{m}^j(k) - \alpha \hat{g}^i(k); a_{ij} > 0, \alpha > 0$$

where $a_{ij}$ is the $(i,j)_{th}$ element of the doubly stochastic weighted communication matrix, $A \in \mathbb{R}^{n \times n}$, $\alpha$ is the gradient step size, and $\hat{g}^i(k) \in \mathbb{R}^n$ is the subgradient of the local objective function, $f_x(k)$, with respect to the agent $i$’s allocated inertia vector, $\tilde{m}^i(k)$. Since each agent only has information about its own states, $x_i$, the components of the gradient corresponding to other nodes are zero i.e., $g_j^i = 0, \forall j \neq i$. Under Assumptions 1 and 2, agent $i$’s estimate of the gradient at time $k$ can be written as

$$\hat{g}^i(k) = [0_{1 \times (i-1)}, \kappa_i(k), 0_{1 \times (n-i)}]^T$$

$$\kappa_i(k) = \left[ \gamma_0 \Delta \theta_i(k) - \gamma_i m_i(k)^{-1} \Delta \omega_i(k) \right]$$

$$r_i(k) = \sum_{j=1}^n b_{ij} \Delta \phi_{ij}(k) + d_i \Delta \omega_i(k)$$

where $m_i(k)$ is agent $i$’s estimate of allocated VI at node $i$. The system states are iteratively updated using (6) similar to the centralized approach with the exception that the inertia matrix, $M(k)$ is a diagonal matrix whose diagonal elements correspond to agent $i$’s local estimate of allocated VI, i.e., $M(k) = \text{diag}(m_1^i(k), \ldots, m_n^i(k))$, where each $m_1^i(k)$ is obtained from (8).

**III. RESULTS**

**A. Case Study**

The 230 kV/300 MVA 12-bus 3-region network [13] shown in Fig. 1 is the chosen test system for this study. We allocate an initial inertia coefficient between 2-10 s [1]

![IEEE 12-bus 3-region test system](image)

at all the nodes via random sampling from a standard normal distribution, $N(0, 1)$. A node’s lower inertia coefficient at a given time-step indicates that its share of online generators is predominantly renewables. Conversely, a higher inertia coefficient indicates a SG-dominated node. The simulation uses a discrete step size, $h$, of 0.01 s. One node from each region of the 3-region test system, namely Nodes 3, 7, and 11, are simultaneously subject to an initial perturbation in frequency, $\Delta \omega(0)$, of 0.5 Hz. This setup allows the centralized and distributed controllers to allocate VI over a 10 s time horizon to stabilize the system, i.e., reduce the frequency and angle deviations to zero. Since the swing dynamics in (3) no longer describe a zero-inertia power system, we restrict the controllers to allocate VI greater than 0.1 s or less than −0.1 s. The objective function for the base case (BC) penalizes the angle deviation over the frequency deviation by assigning a higher weight to the
angle deviation ($\gamma_\theta = 70$) compared to that assigned to the frequency deviation ($\gamma_\omega = 0.1$). We choose a gradient step size, $\alpha$, of 0.6 and RoCol coefficient, $\delta_h$, of 3 s/s for the BC simulation. We study the sensitivity of the controllers to • RoCol coefficients, $\delta_h$, with 3 s/s and 1 s/s as possible values at all nodes $i$,
• a range of gradient step sizes, $\alpha \in \{0.1, 23, 45\}$, and
• varying objective weights, $\gamma_\omega \in \{0.01, 1.6\}$ and $\gamma_\theta \in \{0.01, 70, 2000\}$

We test two types of distributed communication architectures - (i) full communication with a uniformly-weighted communication matrix, i.e., $a_{ij} = 1/n$, where the number of nodes, $n$, is 11 in the test system, and (ii) sparse communication with Metropolis-based weights [23] defined as

$$a_{ij} = \begin{cases} \frac{1}{\max(N_i, N_j)} & i \to j \\ 0 & \text{otherwise} \end{cases}; \quad a_{ii} = 1 - \sum_{j \neq i} a_{ij} \quad (10)$$

where $N_i$ and $N_j$ are respectively the number of edges connected at nodes $i$ and $j$, i.e., the degree of the node. To ensure strong converge [22], the weighted communication matrices for the full and sparse communication architectures are doubly stochastic, as required by the distributed subgradient method described in Section II-B.

**B. Performance Metrics**

We quantify controller performance in terms of (1) the time that the frequency and angle deviations take to reach zero, i.e., settling time, (2) the cumulative deviations in frequency and angle over time evaluated using the $\ell_1$ norm, i.e., $||\Delta \omega||_1$ and $||\Delta \theta||_1$, and (3) the overshoot in angle deviation evaluated using the $\ell_\infty$ norm i.e., $||\Delta \theta||_\infty$. A shorter settling time in frequency and angle deviations, smaller cumulative deviations in frequency and angle over time ($||\Delta \omega||_1$ and $||\Delta \theta||_1$), and a smaller overshoot in angle deviation ($||\Delta \theta||_\infty$) are desirable to avoid critical states in the system.

We quantify the control effort required to achieve the desired controller performance using the $\ell_1$ and $\ell_\infty$ norms of allocated VI, i.e., $||h||_1$ and $||h||_\infty$, which respectively denote the cumulative and the maximum required resource (VI) capacity. A smaller $||h||_1$ and $||h||_\infty$ equates to less control effort.

**C. Simulation Results**

1) Base Case: Fig. 2 shows a snapshot of the first 6 s of the simulation results for the BC comparing the state trajectory of the distributed full communication (DFC) and distributed sparse communication (DSC) controllers in minimizing the frequency and angle deviation at the perturbed nodes. First, we note that all three controllers minimize the frequency and angle deviation and stabilize the system within 6 s. This response is within the 10 s time-horizon typical for inertial response [7]. Further, the settling times of both the DFC and DSC controllers are comparable to the centralized controller. Comparing the state trajectory of the distributed controllers, we observe a shorter settling time with the DFC than the DSC controller at Node 7 and vice versa at Nodes 3 and 11. Fig. 3 shows the corresponding VI allocated at the perturbed nodes over the first 6 s of simulation. The centralized controller quickly reaches the optimal value of allocated VI, provides a fixed amount of VI as the frequency and angle deviations decrease, and allocates almost no VI (equivalent to 0.1 s) once the system stabilizes. We expect this trend as the allocated VI in the centralized case is a function of the frequency and angle deviation, cf. (5). The DFC and DSC controllers initially allocate more VI than the centralized controller. Further, the DSC controller either over-allocates (cf. Node 7 in Fig. 3) or under-allocates (cf. Nodes 3 and 11 in Fig. 3) VI compared to the DFC controller in the initial 0.25 s as a result of sparse information. However, both distributed controllers converge to the same steady-state allocated VI as the frequency and angle deviations approach zero. We expect this response because as the frequency and angle deviations tend to zero, so does the local gradient of each agent $i$, $\bar{q}_i$. The consensus term in the distributed subgradient method, $\sum_{j=1}^{n} a_{ij} \bar{n}_j^t$, is all that remains in each agent $i$’s calculation of allocated VI, $\bar{n}_i^t$, cf. (6). As a result, there is no incentive to reduce the allocated VI once the agents achieve consensus.

Table II summarizes the cumulative deviation in states and overshoot in angle deviation for the DFC and DSC controllers (cf. Section III-B) normalized with respect to the centralized controller at 3 s/s and 1 s/s RoCol coefficients.
The DSC controller achieves better performance than the DFC controller at Nodes 3 and 11, as seen from the smaller $||\Delta\omega||_1$, $||\Delta\theta||_1$ and $||\Delta\theta||_\infty$ at both 3 s/s and 1 s/s RoCoI coefficients. At Node 7, the DFC controller has a smaller $||\Delta\omega||_1$ and $||\Delta\theta||_\infty$ at 3 s/s and 1 s/s RoCoI coefficients. However, the cumulative deviation in angle, $||\Delta\theta||_1$ at Node 7 is smaller with the DFC controller at 3 s/s RoCoI coefficient and larger, at 1 s/s RoCoI coefficient. Furthermore, comparing $||\Delta\omega||_1$, $||\Delta\theta||_1$ and $||\Delta\theta||_\infty$ at RoCoI coefficients of 3 s/s versus 1 s/s, we observe better controller performance at a lower RoCoI. This is expected because a lower RoCoI coefficient allows for more granular changes in the system inertia, which results in an easier system stabilization.

Comparing the control effort between the DFC and DSC at Nodes 7 and 11, we observe a trade-off between cumulative VI allocated ($||h||_1$) to minimize the frequency and angle deviations, and the required maximum VI ($||h||_\infty$). At Node 7, the DSC controller uses less cumulative VI to minimize the state deviations (smaller $||h||_1$) but allocates a higher maximum VI (larger $||h||_\infty$). At Node 11, the DFC controller uses the same (at 3 s/s RoCoI coefficient) or less (at 1 s/s RoCoI coefficient) cumulative VI but allocates a higher maximum VI. At Node 3, the DSC controller respectively uses less and more cumulative VI at 3 s/s versus 1 s/s RoCoI coefficients with the same maximum allocated VI. Although the control effort is higher in the DFC and DSC controllers compared to the centralized controllers in general, both the DFC and DSC controllers achieve optimal controller performance with the same maximum allocated VI ($||h||_1$ and $||h||_\infty$) as the centralized case at Node 3. While the cumulative VI allocated reduces with the RoCoI coefficient, the RoCoI coefficient has a negligible effect on the maximum allocated VI, as seen respectively from $||h||_1$ and $||h||_\infty$ at all the nodes.

2) Sensitivity Analysis: The performance of the distributed controllers is highly sensitive to changes in the gradient step size, and the objective weights on frequency and angle deviation. Fig. 4 shows the trajectory of frequency and angle deviations under a range of gradient step sizes, $\alpha = \{0.1, 23, 45\}$. We observe a shorter settling time in frequency and angle deviations with a larger gradient step size. We observe that the controller performance is either similar or better (by a factor of up to 81%) at a larger gradient step size, $\alpha$ of 45 compared to the BC controller performance. The cumulative and maximum allocated VI are also less (by a factor of up to 65%) or similar to that of the BC with a large gradient step size.

Fig. 5 and Fig. 6 respectively show the variation in the frequency objective weight at a BC angle objective weight of 0.1, BC gradient step size of 0.6, and 3 s/s RoCoI coefficient.
and vice versa, both at BC gradient step size and RoCoI coefficient. We observe a shorter settling time in frequency and angle deviations with a smaller $\gamma_\omega$ and larger $\gamma_\theta$. In other words, the settling time of the states is proportional to $\gamma_\omega/\gamma_\omega$. Although prioritizing the angle deviation in the objective function results in a shorter settling time in states, the cumulative deviations in frequency and angle over time as well as the overshoot in angle deviation are considerably higher than their BC values. The cumulative deviations in frequency and angle is higher by at least 503% at a smaller than the BC $\gamma_\omega$ of 0.01, and by at least 313% at a larger $\gamma_\theta$ than the BC of 2000. The overshoot in angle deviation is also higher by at least 460% at a small $\gamma_\omega$, and by at least 31% at a large $\gamma_\theta$. Comparing the control effort, the cumulative allocated VI is also similar or higher than their BC values by at least 320% at a small $\gamma_\omega$ of 0.01, and by at least 41% at a large $\gamma_\theta$ of 2000. Similarly, the maximum allocated VI is either similar or higher than the BC by at least 3% at a small $\gamma_\omega$ of 0.01. While the maximum allocated VI at a large $\gamma_\theta$ of 2000 is generally similar or higher by 3%, it is slightly less with the DSC controller by at least 3%. However, the high value of the corresponding cumulative VI overshadows any potential benefit in the form of lower maximum allocated VI at a larger $\gamma_\theta$ than the BC. These results suggest that there exist an upper bound on the value of $\gamma_\theta$ and a lower bound on the value of $\gamma_\omega$ that allows for shorter settling time in states without negatively impacting the performance and cost of control.

### IV. Conclusions

This paper presents, for the first time, a real-time distributed subgradient-based VI allocation control algorithm for coordinating aggregations of converter-based resources to regulate the grid frequency considering time-varying frequency dynamics. We implement the distributed VI allocation algorithms on a modified 12-bus 3-region test system under full and sparse communication architectures using a discrete time-step of 0.01 s. We observe a distributed controller performance with minimal control effort similar to the results of the centralized VI allocation. We study the sensitivity of the DFC and DSC controllers to operational parameters like the RoCoI coefficient and optimization parameters such as the objective weights on the frequency and angle deviation and the gradient step size.

Maintaining grid stability under low and variable inertia is an ongoing challenge in the ERCOT, the Australian National Electricity Market, and Central European grids [10]. This paper highlights the potential for implementing scalable, real-time fast frequency response that ensures reliable and secure operation of the power grid before primary and secondary controllers kick in. Open questions to be explored in future work include (1) mathematical bounds on the simulation parameters that maintain a comparable performance without increasing the computational cost, (2) the extent of sparsity with which the DSC would still be able to stabilize the system, and (3) the impact of the topology of the network on the controller performance and control effort.

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