Radiative cooling of a horizontal layer of participating medium

S Prasanna and SP Venkateshan
HTTP Lab, IIT Madras, Chennai, India
E-mail: spv@iitm.ac.in

Abstract. The present paper studies the role of radiative cooling on natural convection in a square cavity. Radiative cooling of the air sets up convection. The interaction depends on multitude of parameters such as Planck number, optical length and the length scale of the enclosure. Both gray and non-gray gases have been considered for the study.

1. Introduction
During night time, the ground cools faster than the surrounding air. However under certain conditions the air layers very close to the ground have been observed to be cooler than the ground, for example Ramdas phenomenon (lifted temperature minimum)[1, 2, 3], convection in a radiative fog[4, 5]. Such temperature inversions are departure from the normal and understanding these phenomena are important for weather and meteorological studies.

Radiative cooling creates instability as the top layers of the medium are cooler than the ground, a condition akin to Rayleigh Benard convection. Gille and Goody[6] studied the effect of participating radiation on onset of convection in a fluid layer enclosed between two black walls maintained at constant temperature experimentally and observed that radiation has a tendency to suppress convection and hence delay the onset of convection. The study has been well supported by numerical and experimental studies[7, 8]. However, the nature of convection radiation interaction in the lower layers of an atmosphere is characteristically different from that between two parallel plates, as radiation is the sole cause of convection. Even though radiation has a tendency to damp out the perturbations and suppress convection, the radiative cooling of air layers would promote convection. The temperature inversion would occur when the radiative cooling rate of air layers is much higher than ground cooling rate. Studying the real dynamics of such an atmosphere is beyond the scope of the present work. Nevertheless, the present paper considers a simple geometry to deconstruct the role of radiative cooling on convective heat transfer. The motivation for the present study is to design suitable experiments to study the radiation convection interaction problem. A two dimensional square enclosure with top wall being a perfect window to radiation has been considered as the physical domain with background radiation accounting for the down-welling longwave radiative fluxes. Computations have been performed for steady state conditions for gray and non-gray fluid. The following sections would treat the mathematical formulation, numerical procedure and discussion of results.
2. Mathematical formulation

The physical domain of the present problem has been illustrated in figure 1. The bottom wall of the enclosure is maintained at a constant temperature of $T_{ref} = 300K$. The two side walls are insulated and diffuse. It has been assumed that the interaction of the enclosure with the surroundings is purely via radiation. This is unlike the real physical problem where the upper surface is a free surface continually influenced externally. However, the present study would enable understanding the influence of radiation on the state of affairs in a fluid layer. The mass and momentum equations for the fluid are the well known continuity and Navier Stokes equations. The fluid has been assumed to be incompressible and the Boussinesq approximation has been used to account for buoyancy. Since the temperature variation within the medium is not more than $10^\circ C$, such an assumption is justified. The presence of radiation does not have a direct effect on the mass and momentum equations. However radiation directly affects the energy equation. Even so, as radiation influences the temperature distribution and hence the flow, all the four equations have to be solved simultaneously. The non dimensionalized governing equations are

\begin{align}
\nabla \cdot \mathbf{v} &= 0 \\
\mathbf{v} \cdot \nabla \mathbf{v} &= -\nabla p + \nabla^2 \mathbf{v} - \frac{Ra^*}{Pr} (\theta - 1)\hat{y} \\
\mathbf{v} \cdot \nabla \theta &= \frac{1}{Pr} \nabla^2 \theta - \frac{1}{PrPr_l} \nabla \cdot q_r
\end{align}

where $Pr_l$ indicates relative strength of thermal diffusion and radiation and is equal to $\frac{k}{\sigma T_{ref}^4 \hbar}$. $\nabla \cdot q_r$, the divergence of the radiant flux, indicates the radiative heating or cooling in the fluid medium which in turn drives convection.

\begin{equation}
\nabla \cdot q_r = \frac{1}{4\pi T_{ref}^4} \int_{\eta=0}^{\infty} \tau_{h,\eta}(4e_{b\eta} - g_{\eta}) d\eta
\end{equation}

where $e_{b\eta}$ and $g_{\eta}$ are respectively the spectral black body emissive power given by Planck function and spectral energy density.

\begin{equation}
g_{\eta} = \int_{4\pi} I_{\eta} d\Omega
\end{equation}

The equation of transfer for radiation intensity is given by

\begin{equation}
\mu \frac{dI_{\eta}}{dx} + \xi \frac{dI_{\eta}}{dy} = \tau_{h,\eta}(I_b - I_{\eta})
\end{equation}
The boundary conditions for the present problem are listed below:

(i) \( u = 0 \) and \( v = 0 \) at the boundaries of the enclosure
(ii) \( \theta = 1 \), at \( y = 0 \)
(iii) \( \frac{\partial \theta}{\partial y} = 0 \), at \( y = h \)
(iv) at the side walls, there is a balance between conduction and radiation, \( \frac{\partial \theta}{\partial y} - \frac{1}{\Pi T q_r} = 0 \)
(v) \( \varepsilon = 0, \rho = 0 \) at the top wall
(vi) at side walls, \( \varepsilon = 1 \) (unless specified otherwise) and diffuse radiation
(vii) at bottom wall, \( \varepsilon = 1 \) and diffuse radiation

The background radiation is at a lower temperature compared to \( T_{ref} \).

3. Solution Procedure

The governing equations with appropriate boundary conditions have been solved using the finite volume method. As the problem is coupled, all the variables, velocity, temperature and intensity have to be solved for simultaneously. This has been achieved by performing sequential iteration for each variable. SIMPLE algorithm has been used to couple pressure and velocity. Convection terms have been represented by first order upwinding. Staggered grid has been used to discretize the domain, with pressure, temperature and intensities at the geometric center of the control volume and the velocities at the nodes on the boundaries of each control volume. Cosine grid has been used to discretize the physical domain where mesh is fine close to the wall and coarse away from the wall.

Weighted Sum of Gray Gas method (WSGG) has been used to model non-gray radiation. The non gray gas is represented by a weighted sum of gray gases of different absorption coefficients such that the radiative heat flux of the non gray gas is represented by an equivalent mixture of gray gases.

\[
\frac{dI_j}{dx} + \xi \frac{dI_j}{dy} = \tau_{h,j}(a_j \frac{\sigma T^4}{\pi} - I_j)
\]

where \( I_j, a_j \) and \( \tau_{h,j} \) are the intensity, weightage and optical path of \( j^{th} \) gray gas. The total divergence of heat flux can be calculated from spectral radiative heat flux as

\[
\nabla \cdot q_r = \sum_{j}^{\text{no of gases}} \nabla \cdot q_{r,j} = \sum_{j}^{\text{no of gases}} \tau_{h,j} a_j \theta^4 - \frac{1}{4} \tau_{h,j} g_j^* = \frac{\tau_h \theta^4}{4} - \frac{1}{4} \sum_{j}^{\text{no of gases}} \tau_{h,j} g_j^*
\]

As the temperature variation within the medium is less than 10°C, the medium is assumed to be homogeneous, thereby simplifying the calculations. The coefficients in WSGG are calculated from the line by line spectrum of the constituent gases available in HITRAN database[9] using the \( k \)-distribution method [10, 11]. The basic idea of this approach is to group the spectral intervals having the same absorption coefficient i.e. transforming from spectral distribution to absorption coefficient distribution. From the line by line data, the absorption coefficients over the entire spectral range is to be calculated which for Lorentz profile is given by

\[
\kappa_{\eta} = \sum_{i}^{\text{no of lines}} \frac{S_i}{\pi (\eta - \eta_i)^2 + \gamma_i^2}
\]

where, \( S_i \) is the line strength, \( \gamma_i \) is the line half width and \( \eta_i \) is the location of line center. A cumulative distribution function of absorption coefficient weighted by the Planck function is calculated for the gas mixture. This is done by dividing absorption coefficient range into a large number of equal intervals. Due to large range of absorption coefficients, the division is
performed using a logarithmic scale. The spectral range is also divided into a large number of narrow bands such that Planck function can be assumed to be constant within a given band. The weights for each absorption coefficient is determined by summing up the weighted Planck function corresponding to the absorption coefficient interval.

\[
f(\kappa_i) = \frac{1}{I_b} \sum_{j=1}^{n_{\text{bands}}} I_{b,j} W(\kappa_j - \delta \kappa, \kappa_j + \delta \kappa)
\]

\[
g(\kappa_i) = \sum_{j=1}^{i} f(\kappa_j)
\]

During nocturnal conditions, the down-welling radiation is due to emission and scattering of radiation from the atmosphere. As the atmospheric radiation is non-gray in nature, a gray approximation may not be suitable. STREAMER[12] has been used to calculate the spectral down-welling radiation. STREAMER is a radiative transfer model that can determine radiative fluxes, longwave and shortwave, for various atmospheric and surface conditions. STREAMER performs radiative transfer calculations for water vapour, carbon dioxide, ozone and selected aerosol profiles. The model calculates spectral radiative fluxes by dividing the longwave spectrum (20 – 2480 cm\(^{-1}\)) into 105 bands. Standard tropical profile provided by STREAMER, in absence of clouds and aerosols, has been used as the atmospheric profile for all the simulations. Figure 2 shows the spectral down-welling radiation obtained from STREAMER for the standard tropical atmospheric profile. The total down-welling radiative flux equals that of a black body at \(T_{\text{back}} \approx 0.97 T_{\text{ref}}\) and this has been used as the background radiation for gray calculations. Figure 3 shows the cumulative distribution functions for saturated air. The weights and corresponding absorption coefficients for WSGG are determined using the Gauss Chebyshev quadrature[13]. Radiative transfer of equation for each gray gas is solved using the Discrete Ordinate method

![Figure 2: Downwelling radiation from STREAMER.](image1)

![Figure 3: Cumulative distribution function for Planck distribution and background radiation.](image2)
used. The quadrature scheme satisfies the moment equations and also satisfies first moment over half range along the principal directions \((x, y)\). The intensity equation for a discrete direction \(i\) is

\[
I_{p,j}^i = \frac{\tau_{h,j} \Delta x \Delta y a_j I_h + |\mu| \Delta y I_{w,j}^i + |\xi| \Delta x I_{s,j}^i}{\tau_{h,j} \Delta x \Delta y + |\mu| \Delta y + |\xi| \Delta x}
\] (11)

where \(I_{w,j}^i\) and \(I_{s,j}^i\) are the intensities at the west and south (directions with respect to the ordinate direction) boundaries of the control volume. The intensity at cell center and cell boundary are related by interpolation given by

\[
I_p = f I_e + (1-f) I_w = f I_n + (1-f) I_s
\] (12)

where \(f\) can lie anywhere between 0 and 1. Step scheme \(f = 1\) has been used in the present numerical study. Combining equations 8 and 9, intensities are calculated iteratively by marching from one end of the boundary.

The coupling between temperature and radiation is through the divergence of radiative heat flux (equation 8). The fourth power of temperature means the coupled set of equations are nonlinear in nature. Newton Raphson method has been used to linearize the source term \((\tau_h \theta^4)\) in the energy equation.

\[
\tau_h \theta^4 = \tau_h (4\theta_{old}^3 \theta_{new} - 3\theta_{old}^4)
\] (13)

The coupled energy equation and radiative transfer equation are solved sequentially. Bi-conjugate gradient method has been used to solve the energy equation and Gauss-Siedel iterations have been performed for radiative transfer equation. The procedure has been implemented using FORTRAN 90.

| Grid size | Angular quadrature | \(\frac{\partial \theta}{\partial y}\) |
|-----------|-------------------|------------------|
| 41 \times 41 | SN6 | 0.179 |
| 61 \times 61 | SN6 | **0.181** |
| 121 \times 121 | SN6 | 0.181 |
| 61 \times 61 | SN4 | 0.178 |
| 61 \times 61 | SN8 | 0.182 |

4. Results and Discussion

The code has been extensively validated with benchmark numerical results on convection, radiation and combined convection and radiation. Table 1 shows the comparison of numerical results from the present work and Yucel et al.[15]. A differential heated cavity with a participating medium was considered and the results of the present study compare well with the quoted results. A grid and angle sensitivity study has also been performed and is presented in Table 2. 61 \times 61 grids with 6 discrete directions in each octant (SN6) are chosen for the present study.

4.1. Gray gas

Numerical experiments have been conducted for various Planck numbers and optical lengths. The length scale of the phenomenon of interest is of the order of 10 cm and hence simulations have been performed for length scale = 10 cm. Accordingly the Ra* for a layer of air would be \(3 \times 10^7\). As both optical thickness and Planck numbers are dependent on length scale of the enclosure, length scale is an important parameter in the interaction phenomenon.

The interaction of radiation on natural convection has been illustrated in figures 4 and 5.
interesting to observe that radiation has contrasting roles to play in the interaction phenomena. The amount of radiative heat transfer is proportional to the optical thickness. Hence it would be reasonable to conclude that the temperature variation within the participating medium would increase with increase in optical thickness. This clearly means the Rayleigh number of the system would also increase. Therefore it would be logical to conclude convection to be stronger for a participating medium of high optical thickness. However, on the contrary it is observed that the convection is weaker for optical thickness 10 than 1. The convective strength follows a downward trend with optical thickness. However, this observation is not surprising at all, as it reaffirms the already established observation that radiation suppresses convection.

For the sake of understanding the observation, let us consider a participating medium of very high optical thickness $\tau_h \rightarrow \infty$. For such a situation, bulk of the cooling of the medium takes place close to the top wall or in the limit, from the wall itself. Accordingly, radiation can be modeled by diffusion approximation and hence the effective thermal conductivity of the system increases.

$$k_{eff} = k + k_{rad}$$  \hspace{1cm} (14)

where $k_{rad}$ is the “radiative conductivity” given by $\frac{16\sigma T^4_{ref}}{3k_{eff}}$. As a result, the effective Rayleigh number of the system reduces and would cause weak convection. Therefore, Rayleigh number defined in the conventional sense is not a good representative for a convective radiative system.

Figure 6 shows the center line vertical temperature distribution in the enclosure for $Pl = 0.01$. The temperature at the top wall clearly decreases with optical thickness. However, the temperature gradients at the bottom wall increase with decreasing optical thickness. The
possible cause of strong convection for a small optical thickness medium would be because of the strong temperature gradient close to the bottom wall. This explains the strong cell formation very close to the bottom wall. The large magnitudes of temperature gradients close to the wall is because of low values of Planck numbers. $Pl$ signifies the relative importance of radiation and thermal diffusion. As $Pl \to 0$ the system tends towards radiative equilibrium and other modes of heat transfer would become insignificant. In absence of thermal diffusivity, there would be a temperature jump observed at the bottom wall. The temperature jump would increase with decrease in optical thickness. However, thermal diffusivity smears out the temperature jump resulting in a steep temperature gradient. As $Pl$ increases, the thermal diffusion increases and hence the temperature variation within the medium also reduces considerably. This means the convection intensity also decreases considerably.

To illustrate the nature of interaction, comparisons have been made for cases with and without convection(diffusion only). Figure 7 shows the temperature variation for various $Pl$ for $\tau_h = 0.1$. It is evident that there is little to choose between profiles of convection and pure conduction for $Pl = 0.01$. The difference between the two modes become significant as $Pl$ increases. This is true, even though $Ra$ is much higher for small $Pl$. This suggests that, although convection is very strong, the effective heat transfer and hence temperature profile are almost independent of convection. As $Pl$ increases, the heat transfer through conduction from the surface becomes important and hence the temperature is affected by both convection and radiation. The typical $Pl$ for an atmospheric layer of 10 cm and 1 m thick would be 0.01 and 0.001 respectively. This implies the presence of convection in such a scale of atmosphere would have little effect on the temperature. However, the real atmosphere is a non gray medium and it would be desirable to see the dynamics of such a medium.

4.2. Non gray medium

Water vapour is the main greenhouse gas contributing to radiative heat transfer. Let us consider a fully saturated layer of air ($RH = 100\%$). Figure 8 shows the temperature along the center of the enclosure for different number of gray gases used in the WSGG model. In order to capture the phenomena correctly, 100 gray gases have been used to represent the non gray phenomena. The absorption coefficients and weights are calculated according to the method discussed earlier. The $Pl$ number for such a configuration would be of the order of 0.01. Figure 9 shows the streamline and isotherm contour plots for a air layer saturated with water
vapour. The pattern is remarkably different from that for a gray medium. The side walls being perfectly black, interact with the participating medium. There would be heat loss in the window region of the participating medium, and hence the radiative losses from the side walls are much higher. As there is balance between conduction and radiation at the walls, strong temperature
Gradients are developed close to the vertical walls akin to a differentially heated cavity setting up convection. Simulations have been conducted by assuming the side walls to be re-radiating i.e. the side walls are perfectly reflecting (diffuse). This means there would be no radiation heat transfer from the side walls and thus eliminating the role of side walls. Figure 10 shows the temperature variation along the side walls for black and re-radiating side wall boundary conditions which shows excessive cooling of the participating medium in the presence of black side walls. It can be concluded that radiative modeling of the boundaries are very important for accurately capturing the physical phenomena as small perturbations can change the nature of flow conditions as seen in the case above.

Figure 11 shows the temperature distribution along the center line of the enclosure for different relative humidities. Relative humidity affects the nature of convective and radiative transfer from the cell. The temperature minimum occurs at the interior of the enclosure for $RH = 50\%$ and 100\%, where as for $RH = 10\%$ the lowest temperature occurs at the top. It has to be noted that the spectral radiation of humid air and the background radiation are similar. This could be a possible explanation for the maximum radiative cooling taking place from the center of the layer. There is a balance between the amount of down-welling radiation absorbed and radiation emitted. As $RH$ increases, the absorption coefficient increases and hence the amount of absorption from the top layers of the enclosure is also higher. As a result the top layers of the enclosure are tad hotter than the central regions. However, for lower $RH$ values, the emission from the layer would offset the role of absorption. Also, it is seen that convection alters the temperature profile to some extent as $RH$ increases. Figure 12 shows the streamline and isotherm contour plots for saturated air. Simulations have also been performed, for both gray and non gray gases, for height of the enclosure = 1 m. The solution for such cases did not converge to steady state suggesting the flow may be unstable.

5. Conclusion
Radiative cooling influences the temperature and convective heat flux to a large extent. When $Pl$ is small, the temperature field is mainly influenced by radiative flux. It can be said that convection acts like a small perturbation to the radiation dominated phenomena. The strength of convection decreases with increase in optical thickness. The thermal characteristic of a non-gray medium is remarkably different from a gray medium. The main differences occur in the application of the radiative boundary conditions. The length scale of a Ramdas layer would be up to a meter, which from the study indicates presence of strong advection within the layer. Another observation that can be made is that the humid air can cool to a temperature below the dew point, leading to condensation and radiative fog (not considered in the present study).
static environment, humid air can cause substantial cooling leading to secondary effects which would be of interest. However it has to be noted that in the presence of strong winds, radiative cooling can become less significant. The present study has considered a simple case without the complications involved in a regular atmospheric layer. However, the study has outlined the nature of interaction. The real flow and temperature field would have transient fluctuations which would be the emphasis of future works. It is also intended to design experiments to study the interaction problem.

### Nomenclature

| Symbol | Description |
|--------|-------------|
| $a_j$  | weightage of $j$th gas |
| $e_{bq}$ | spectral blackbody emissive power |
| $f$    | k-distribution function |
| $g$    | cumulative k-distribution function |
| $g_0$  | spectral energy density |
| $h$    | height of the square cavity, $m$ |
| $I$    | non dimensional intensity |
| $k$    | thermal conductivity, $W/mK$ |
| $p$    | dimensionless pressure |
| $P_l$  | Planck number, $h k T_{ref}^3 / 4\pi \sigma T_{ref}^4$ |
| $Pr$   | Prandtl number |
| $q_r$  | non dimensional radiative heat flux |
| $Ra^*$ | Rayleigh number, $(g \beta T_{ref} h^3 k \alpha) P_r$ |
| $RH$   | relative humidity |
| $S_i$  | line strength, $cm^{-2}$ |
| $T_{back}$ | background temperature, $K$ |
| $T_{ref}$ | reference temperature, 300 $K$ |
| $v$    | velocity vector |
| $x, y$ | dimensionless coordinates |

### Greek Symbols

| Symbol | Description |
|--------|-------------|
| $\kappa$ | absorption coefficient, $cm^{-1}$ |
| $\gamma$ | line half width, $cm^{-1}$ |
| $\varepsilon$ | hemispherical total emissivity |
| $\rho$ | hemispherical total reflectivity |
| $\eta$ | wave number, $cm^{-1}$ |
| $\theta$ | non dimensional temperature, $T/T_{ref}$ |
| $\tau_h$ | optical thickness, $\kappa h$ |
| $\nu$ | momentum diffusivity, $m^2/s$ |
| $\kappa$ | absorption index |
| $\mu, \xi$ | direction cosines |
| $\sigma$ | Stefan-Boltzmann constant, $5.67 \times 10^{-8} W/m^2 K^4$ |

### Subscripts

| Symbol | Description |
|--------|-------------|
| $b$    | blackbody |
| $i, j$ | indices |
| $\eta$ | spectral |

### References

1. Ramdas L and Atmanathan S 1932 *Beit. Geophys* **37** 116–117
2. Narasimha R 1994 *Current Science* (Bangalore) **66** 16–23
3. Mukund V, Ponnulakshmi V, Singh D, Subramanian G and Sreenivas K 2010 *Physica Scripta* **2010** 014041
4. Nishikawa T, Maruyama S and Sakai S 2004 *Boundary-Layer Meteorology* **113** 273–286
5. Price J 2011 *Boundary-Layer Meteorology* 1–25
6. Gille J and Goody R 1964 *J. Fluid Mech* **20** 12
7. Arpaci V, Go D et al. 1973 *Physics of Fluids* **16** 581
8. Béleoin F and Soufiani A 1997 *Physics of Fluids* **9** 3858
9. Rothman L, Gordon I, Barbe A, Benner D, Bernath P, Birks M, Boudon V, Brown L, Campargue A, Champion J et al. 2009 *Journal of Quantitative Spectroscopy and Radiative Transfer* **110** 533–572
10. Denison M and Webb B 1993 *ASME Journal of Heat Transfer* **115**
11. Modest M 2002 *Journal of heat transfer* **124** 30
12. Key J 2001 *Cooperative Institute for Meteorological Satellite Studies, University of Wisconsin* 96
13. Boyd J 2001 *Chebyshev and Fourier spectral methods* (Dover Pubns)
14. Carlson B and Lathrop K 1968 *Computing methods in reactor physics* 171–266
15. Yücel A, Acharya S and Williams M 1989 *Numerical Heat Transfer* **15** 261–278