The Inert Doublet Model (IDM) is one of the simplest extensions of the Standard Model (SM), providing a dark matter candidate. It is a two Higgs doublet model with a discrete $Z_2$ symmetry, that prevents the scalars of the second doublet (inert scalars) from coupling to the SM fermions and makes the lightest of them stable. We study a large number of IDM scenarios, which are consistent with current constraints on direct detection and relic density of dark matter, as well as with all collider and low-energy limits. We propose a set of benchmark points with different kinematic features, that promise detectable signals at future $e^+e^-$ colliders. Two inert scalar pair-production processes are considered, $e^+e^- \rightarrow A \ H$ and $e^+e^- \rightarrow H^+H^-$, followed by decays of $A$ and $H^\pm$ into final states which always include the lightest and stable neutral scalar dark matter candidate $H$. Significance of the expected observations is studied for different benchmark models and different running scenarios, for centre-of-mass energies from 250 GeV up to 3 TeV. For low mass scenarios, high significance can be obtained for the signal signatures with two muons or an electron and a muon in the final state. For high mass scenarios, which are only accessible at high energy stages of CLIC, the significance is too low for the leptonic signature and the semi-leptonic final state has to be used as the discovery channel. Results presented for this channel are based on the fast simulation of the CLIC detector response with the DELPHES package.
1. Inert Doublet Model

A number of astrophysical observations based on gravitational interactions point to the existence of dark matter (DM) in the Universe, which cannot be described with the Standard Model. One of the simplest extensions of the Standard Model which can provide a dark matter candidate is the Inert Doublet Model [1, 2, 3]. In this model, the scalar sector is extended by a so-called inert or dark doublet $\Phi_D$ (the only field odd under $Z_2$ symmetry) in addition to the SM Higgs doublet $\Phi_S$. This results in five physical states after electroweak symmetry breaking: the SM Higgs boson $h$ and four dark scalars: two neutral, $H$ and $A$, and two charged, $H^\pm$. A discrete $Z_2$ symmetry prohibits the inert scalars from interacting with the SM fermions through Yukawa-type interactions and makes the lightest neutral scalar, chosen to be $H$ in this work, a good dark matter candidate.

Two sets of benchmark points (BPs) in agreement with all theoretical and experimental constraints were proposed in [4], covering different possible signatures at $e^+ e^-$ colliders, with masses of IDM particles extending up to 1 TeV. Prospects for the discovery of IDM scalars at CLIC running at 380 GeV, 1.5 TeV and 3.5 TeV were then described in detail in [5] and summarized in [6], focusing on leptonic final states. In this contribution we update these results and extend them to ILC running at 250 GeV and 500 GeV. We also include new results based on the semi-leptonic channel analysis, for CLIC running at 1.5 TeV and 3 TeV, which supersede results presented in [7].

2. Benchmark scenarios

Distributions of the scalar masses for the IDM benchmark scenarios considered in [4] are shown in Fig. 1. For the considered benchmark scenarios $H$ is the lightest, stable neutral scalar, which can be much lighter than the other two, $A$ and $H^\pm$. On the other hand the mass splitting between $A$ and $H^\pm$ is limited by existing constraints to about 70 GeV.

The following tree-level production processes of inert scalars at $e^+ e^-$ collisions are considered: neutral scalar pair-production, $e^+ e^- \rightarrow A H$, and charged scalar pair-production, $e^+ e^- \rightarrow H^+ H^-$. The leading-order cross sections for these processes are presented in Fig. 2 for collision...
energies of 380 GeV and 1.5 TeV. As the couplings of inert scalars to SM bosons are determined by SM parameters, production cross sections are determined by the scalar masses and depend very weakly on additional model parameters. Far from the kinematic threshold, the production cross section, dominated by the $s$-channel $Z$-boson exchange, decreases as $1/s$ with the collision energy.

In the scenarios considered in this paper the produced dark scalar $A$ decays to a (real or virtual) $Z$ boson and the (lighter) neutral scalar $H$, $A \rightarrow Z^{(*)} H$, while the produced charged boson $H^{\pm}$ decays predominantly to a (real or virtual) $W^{\pm}$ boson and the neutral scalar $H$, $H^{\pm} \rightarrow W^{\pm(*)} H$, where the DM candidate $H$ escapes detection. The mono-$Z$ signature of the neutral scalar pair-production can be considered in the leptonic or hadronic $Z$-boson decay channel. For the charged scalar pair production, resulting in two $W$ bosons in the final state, leptonic, semi-leptonic and hadronic final states are possible.

3. Leptonic channel analysis

Isolated leptons (electrons and muons) can be identified and reconstructed with very high efficiency and purity, and the signatures based solely on lepton measurements are usually considered “golden channels”, if the expected statistics of signal events is high enough. For purely leptonic final state, the detector acceptance cuts can be applied on the generator level and other detector
Searching Inert Scalars at Future $e^+ e^-$ Colliders

Aleksander Filip Zarnecki

Figure 3: Signal Feynman diagrams for the considered production and decay process for: (left) neutral scalar production, $e^+ e^- \rightarrow HA \rightarrow HHll$, and (right) charged scalar production, $e^+ e^- \rightarrow H^+ H^- \rightarrow HHl'l'\nu\nu'$.

Figure 4: Distribution of the lepton pair invariant mass, $M_{\mu\mu}$, as a function of the lepton pair longitudinal momentum, $P^\mu_{Z\mu}$, for IDM signal (green points) and Standard Model background (red points). Signal events were simulated for BP1 scenario (left) and BP9 scenario (right), for centre-of-mass energy of 250 GeV. The blue box indicates the cut used to remove the dominant background from $e^+ e^- \rightarrow \mu^+ \mu^- (\gamma)$ process.

Effects are expected to have marginal impact on the outcome of the analysis. Therefore, in [5] we focused on leptonic decays of $Z$ and $W^\pm$, leading to a signature of leptons and missing transverse energy. We considered the $\mu^+ \mu^-$ final state as a signature of the neutral scalar pair-production, while the different flavour lepton pairs, $\mu^+ e^-$ and $e^+ \mu^-$, were considered as a signature for production of charged inert scalars, see Fig. 3.

Signal and background samples were generated with WHIZARD 2.2.8 [8, 9]. Generator level cuts reflecting detector acceptance for leptons and initial state radiation photons were applied. For the neutral inert scalar pair production, $e^+ e^- \rightarrow AH$, the invariant mass of the lepton pair from (virtual) $Z$ decay depends on the mass splitting between $A$ and $H$ and is relatively small, $M_{\mu\mu} \lesssim M_Z$. We apply pre-selection cuts on the invariant mass and the longitudinal boost of the lepton pair to suppress the dominant background process $e^+ e^- \rightarrow \mu^+ \mu^- (\gamma)$, see Fig. 4. Distributions of selected kinematic variables describing the leptonic final state in $AH$ analysis, after the pre-selection cuts, are presented in Fig. 5. Presented in Fig. 6 (left) is the lepton pair invariant mass distribution after pre-selection cuts and additional selection based on lepton pair energy, transverse momentum, production angle (polar angle of the $Z$ boson) and the difference of the lepton azimuthal angles. Already with this simplest, cut-based approach, the IDM signal would result in the visible excess in the invariant mass distribution for the number of benchmark scenarios. For the final selection...
Searching Inert Scalars at Future e+e− Colliders

Aleksander Filip Żarnecki

Figure 5: Distributions of the kinematic variables describing the leptonic final state in $AH$ analysis: lepton pair energy, $E_{\mu\mu}$, and total transverse momentum, $p_{T\mu\mu}$. Expected distributions for representative benchmarks BP1 (red histogram), BP2 (green) and BP7 (blue) are compared with expected background (black histogram) simulated for 1 ab$^{-1}$ collected at 250 GeV.

Figure 6: Left: distribution of the lepton pair invariant mass, $M_{\mu\mu}$, for BP1 (red histogram), BP2 (green) and BP7 (blue) signal scenarios, compared with the expected Standard Model background (black histogram), after event selection cuts (see text for details). Right: response distributions of the BDT classifiers used for the selection of $AH$ production events, for BP1. Samples are normalised to 1 ab$^{-1}$ collected at 250 GeV.

of signal-like events, a multivariate analysis is performed using a Boosted Decision Tree (BDT) classifier [10] with 8 input variables [5]. The standard approach in this type of analysis is to train BDT to separate the considered signal scenario from the background events. However, this approach, also used in our previous study [5], is only valid if we do have some initial estimates on the model parameters, scalar masses in particular. For the results presented here we modified our approach and we train BDTS using all accessible (at given energy) benchmark scenarios from given category (separately for virtual and real $Z$ in the final state) but for the one we look for. This procedure, which we consider a more general (“scenario-independent”) approach, results in the expected significances of observation lower by up to 20% compared to the “educated-selection” results.

Response distributions of the BDT classifier used for the selection of $AH$ production events for the benchmark scenario BP1 at 250 GeV are presented in Fig. 6 (right). Expected significance
Searching Inert Scalars at Future $e^+e^-$ Colliders

Aleksander Filip Zarnecki

Figure 7: Significance of the deviations from the Standard Model predictions, expected for 1 ab$^{-1}$ of data collected at centre-of-mass energy of 250 GeV, 380 GeV and 500 GeV, for events with two muons in the final state, for all considered low mass benchmark scenarios. Only significance above 5$\sigma$ is shown.

of the deviations from the Standard Model predictions, assuming 1 ab$^{-1}$ of data collected at centre-of-mass energy of 250 GeV, 380 GeV and 500 GeV, are shown in Fig. 7. Only scenarios resulting in significances above 5$\sigma$ are shown.

The selection of $H^+H^-$ production events is more challenging as the two leptons in the final state no longer originate from a single (on- or off-shell) intermediate state. No pre-selection cuts are applied (except for the detector acceptance cuts on the generator level), but we focus on electron-muon pairs in the final state, avoiding large SM background from the direct lepton pair production.

In Fig. 8 (left) the distribution of the lepton pair invariant mass, $M_{\ell\ell}$, for three benchmark scenarios (BP1, BP3 and BP6) is compared with Standard Model expectations for centre-of-mass energy of 380 GeV. The expected background cross section for the considered final state is over two orders of magnitude higher than for the considered benchmark points. However, kinematic distributions are very different, as two massive scalars are produced in the signal case, reducing the kinematic space available for lepton pair production, allowing for efficient selection of signal-enhanced sample of events using the multivariate analysis. The same procedure and the same set of input variables is used as for the $AH$ analysis.

Response distributions of the BDT classifier used for the selection of $H^+H^-$ production events for the benchmark scenario BP1 at 380 GeV are presented in Fig. 8 (right). In Fig. 9 we show the expected significance of the deviations from the Standard Model predictions for 1 ab$^{-1}$ of data collected at 250 GeV, 380 GeV and 500 GeV, for scenarios resulting in the significances above 5$\sigma$.

We found that for scenarios accessible at a certain energy, up to 500 GeV, high significance can be expected for leptonic signature at future $e^+e^-$ colliders. The significance is mainly related to the inert scalar production cross sections. We display the dependence of the expected significance on the inert scalar masses, for events with two muons and for events with and electron-muon pair in the final state, in Fig. 10. With 1 ab$^{-1}$ of integrated luminosity collected at 250 GeV, 380 GeV and 500 GeV, the expected discovery reach of $e^+e^-$ colliders extends up to neutral scalar mass sum of 220 GeV, 300 GeV and 330 GeV, respectively, and for charged scalar pair-production up to charged scalar masses of 110 GeV, 160 GeV and 200 GeV.
Searching Inert Scalars at Future $e^+e^-$ Colliders

Aleksander Filip Żarnecki

Figure 8: Left: distribution of the lepton pair invariant mass, $M_{\mu\mu}$, for BP1 (red histogram), BP3 (green) and BP6 (blue) signal scenarios, compared with the expected Standard Model background (black histogram). Right: response distributions of the BDT classifiers used for the selection of $H^+H^-$ production events, for BP1. Samples are normalised to $1\text{ ab}^{-1}$ collected at $380\text{ GeV}$.

Figure 9: Significance of the deviations from the Standard Model predictions, expected for $1\text{ ab}^{-1}$ of data collected at centre-of-mass energy of $250\text{ GeV}$, $380\text{ GeV}$ and $500\text{ GeV}$, for events with an electron and a muon in the final state, for all considered low mass benchmark scenarios. Only significance above $5\sigma$ is shown.

For collision energies much above the threshold, the inert scalar pair-production cross section decreases fast with the collision energy (see Fig. 2). Still, significant increase in discovery reach is expected for $2.5\text{ ab}^{-1}$ of data collected at $1.5\text{ TeV}$ CLIC, see Fig. 11. The neutral scalar pair-production can be discovered in the leptonic channel for $m_A + m_H < 450\text{ GeV}$ and the charged scalar production for $m_{H^\pm} < 500\text{ GeV}$. No further improvement is expected when running at $3\text{ TeV}$.

The significance is mainly driven by the signal production cross section and is approximately proportional to the square-root of the integrated luminosity. Shown in Fig. 12 are the significance results scaled to the integrated luminosity of $1\text{ ab}^{-1}$, presented as a function of the signal production cross section. For the $AH$ channel, which leads to $\mu^+\mu^-$ final state, a universal linear dependence on the signal cross section is observed which does not seem to depend on the running energy. Significant (above $5\sigma$) observation is possible for cross sections roughly larger than $0.5\text{ fb}$. For the
Searching Inert Scalars at Future $e^+e^-$ Colliders

Aleksander Filip ˙Zarnecki

4. Semi-leptonic channel

For charged scalar pair-production, significant improvement of the discovery reach for scenarios with high scalar masses can be achieved using the semi-leptonic final state, see Fig. 13. As the signal cross section increases by an order of magnitude and a similar scaling is expected for the background processes (dominated by the $W^+W^-$ production), the significance of the observation in the semi-leptonic channel should increase by a factor of about 3. Additional improvement is possible due to kinematic constraints which can be imposed on the hadronic final state (corresponding to one of the produced $W$ bosons). However, detector response has to be taken into account in more details.

Results presented in the following are based on the signal and background event samples were generated with WHIZARD 2.7.0, taking into account the beam energy profile expected for CLIC
Searching Inert Scalars at Future $e^+e^-$ Colliders

Aleksander Filip Żarnecki

Figure 12: Significance of the deviations from the Standard Model predictions expected at different CLIC running stages, assuming the same integrated luminosity of 1 ab$^{-1}$, as a function of the signal cross section in the considered channel, for: (left) events with two muons in the final state ($\mu^+\mu^-$) and (right) events with an electron and a muon in the final state ($e^+\mu^-$ or $e^-\mu^+$).

Figure 13: Signal Feynman diagram for the charged scalar pair-production in semi-leptonic decay channel: $e^+e^- \rightarrow H^+H^- \rightarrow HHjj\nu$.

Running at 1.5 TeV and 3 TeV. We assume running with -80% electron beam polarisation and the corresponding integrated luminosity of 2 ab$^{-1}$ and 4 ab$^{-1}$ respectively. For realistic simulation of the CLIC detector response fast simulation framework DELPHES [11] was used, with control cards prepared for the new detector model CLICdet [12].

Selected for the analysis are events with exactly one isolated lepton (electron or muon) and two exclusive jets reconstructed with the VLC algorithm$^1$ [13]. Also rejected are events with an isolated photon with energy above 10 GeV or with the energy sum of the energy-flow objects outside the two reconstructed jets higher than 20 GeV. In Fig. 14, distributions of the jet pair invariant mass, $M_{jj}$, and the sum of jet energies, $E_{j_1} + E_{j_2}$, for the two signal scenarios, are compared with the expected SM background for CLIC running at 3 TeV.

The analysis procedure is similar to the one used for the leptonic channel. Huge background coming mainly from $W^+W^-$ and $ZZ$ pair-production is first suppressed by the pre-selection cuts based on lepton and jet kinematics. Then a multivariate analysis is performed using the BDT classifier with 11 input variables: total energy in an event, missing transverse momentum, missing (recoil) mass; energy, transverse momentum and scattering angle of the isolated lepton; energy,

---

$^1$The VLC algorithm is run with parameter $R = 1$ at 1.5 TeV and $R = 1.2$ at 3 TeV, and with $\beta = \gamma = 1$. 

8
Searching Inert Scalars at Future $e^+ e^-$ Colliders

Aleksander Filip Żarnecki

Figure 14: Distributions of the kinematic variables describing the semi-leptonic final state in $H^+ H^-$ analysis: jet pair invariant mass, $M_{jj}$, and the sum of jet energies, $E_{j1} + E_{j2}$. Expected distributions for benchmark scenarios BP23 (blue histogram) and HP15 (red) are compared with expected background (black histogram) simulated for 4 ab$^{-1}$ of data collected at 3 TeV width -80% electron beam polarisation.

Figure 15: Significance of the deviations from the Standard Model predictions in the leptonic channel (open circles) and the semi-leptonic channel (filled squares) as a function of twice the charged scalar mass for expected CLIC running scenario: 1 ab$^{-1}$ of data collected at 380 GeV, 2.5 ab$^{-1}$ at 1.5 TeV and 5 ab$^{-1}$ at 3 TeV.

invariant mass and emission angle of the jet pair; reconstructed angles of the hadronic $W$ decay. As before, the BDT is trained separately for scenarios with virtual $W^\pm$ production (when the difference of $H^\pm$ and $H$ masses is smaller than the mass of $W^\pm$) and with real $W^\pm$ production (larger mass differences).

Shown in Fig. 15 is the significance for observing deviations from the Standard Model predictions. Results based on the semi-leptonic channel analysis for CLIC running at 1.5 TeV and 3 TeV are compared with the leptonic channel sensitivity presented in Sec. 3. Huge increase of the signal significance is observed, up to a factor of 6, and the discovery reach for charged scalar pair-production is extended up to $m_{H^\pm} \sim 1$ TeV.
5. Conclusions

The Inert Doublet Model is one of the simplest SM extensions providing natural candidate for dark matter. Light IDM scenarios, with scalar masses in \(O(100\,\text{GeV})\) range are still not excluded by the current experimental and theoretical constraints. Low mass IDM scenarios can be observed with high significance in the di-lepton channels already at a \(e^+e^-\) collider with 250 GeV center-of-mass energy. Discovery reach increases for higher \(\sqrt{s}\) and significant improvement in the discovery reach is observed when considering the semi-leptonic final state. Full simulation study of the charge scalar pair-production in the semi-leptonic decay channel is ongoing.

Acknowledgements

This contribution was supported by the National Science Centre, Poland, the OPUS project under contract UMO-2017/25/B/ST2/00496 (2018-2021) and the HARMONIA project under contract UMO-2015/18/M/ST2/00518 (2016-2019), and by the European Regional Development Fund - the Competitiveness and Cohesion Operational Programme (KK.01.1.1.06 - RBI TWIN SIN).

References

[1] N. G. Deshpande and E. Ma, Pattern of Symmetry Breaking with Two Higgs Doublets, Phys. Rev. D 18 (1978) 2574.

[2] Q. H. Cao, E. Ma and G. Rajasekaran, Observing the Dark Scalar Doublet and its Impact on the Standard-Model Higgs Boson at Colliders, Phys. Rev. D 76 (2007) 095011 [arXiv:0708.2939].

[3] R. Barbieri, L. J. Hall and V. S. Rychkov, Improved naturalness with a heavy Higgs: An Alternative road to LHC physics, Phys. Rev. D 74 (2006) 015007 [hep-ph/0603188].

[4] J. Kalinowski, W. Kotlarski, T. Robens, D. Sokolowska and A. F. Zarnecki, Benchmarking the Inert Doublet Model for \(e^+e^-\) colliders, JHEP 1812 (2018) 081 [arXiv:1809.07712].

[5] J. Kalinowski, W. Kotlarski, T. Robens, D. Sokolowska and A. F. Zarnecki, Exploring Inert Scalars at CLIC, JHEP 1907 (2019) 053 [arXiv:1811.06952].

[6] J. de Blas et al., The CLIC Potential for New Physics, CERN Yellow Rep. Monogr. Vol. 3 (2018) [arXiv:1812.02093].

[7] D. Sokolowska et al., Inert Doublet Model signatures at future \(e^+e^-\) colliders, PoS(EPS-HEP2019)570 [arXiv:1911.06254].

[8] W. Kilian, T. Ohl and J. Reuter, WHIZARD: Simulating Multi-Particle Processes at LHC and ILC, Eur. Phys. J. C 71 (2011) 1742 [arXiv:0708.4233].

[9] T. Ohl, O’mega: An Optimizing matrix element generator, AIP Conf. Proc. 583 (2002) no.1, 173 [hep-ph/0011243].

[10] A. Hocker et al., TMVA - Toolkit for Multivariate Data Analysis, physics/0703039.

[11] J. de Favereau et al. [DELPHES 3 Collaboration], DELPHES 3, A modular framework for fast simulation of a generic collider experiment, JHEP 1402 (2014) 057 [arXiv:1307.6346].

[12] E. Leogrande, P. Roloff, U. Schnoor and M. Weber, A DELPHES card for the CLIC detector, arXiv:1909.12728.

[13] M. Boronat, J. Fuster, I. Garcia, P. Roloff, R. Simoniello and M. Vos, Jet reconstruction at high-energy electron-positron colliders, Eur. Phys. J. C 78 (2018) no.2, 144 [arXiv:1607.05039].