Topological defects and Goldstone excitations in domain walls between ferromagnetic quantum Hall effect liquids

Vladimir I. Fal’ko †, S.V. Iordanskii ‡

* School of Physics and Chemistry, Lancaster University, LA1 4YB, UK
† Departement de Physique, Université Joseph Fourier Grenoble 1, Grenoble, France
‡ Landau Institute for Theoretical Physics, ul. Kosygina 1, Moscow, Russia (today)

It is shown that the low-energy spectrum of a ferromagnetic quantum Hall effect liquid in a system with a multi-domain structure generated by an inhomogeneous bare Zeeman splitting $\varepsilon_Z$ is formed by excitations localized at the walls between domains. For a step-like $\varepsilon_Z(\mathbf{r})$, the domain wall spectrum includes a spin-wave with a linear dispersion and a small gap due to spin-orbit coupling, and a low-energy topological defects. The latter are charged and may dominate in the transport under conditions that the percolation through the network of domain walls is provided.

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Due to the electron-electron exchange interaction, two-dimensional (2D) electrons form a ferromagnetic liquid in the vicinity of odd-integer filling factors. Recently, an idea has been proposed [2,3] that the dissipation and thermodynamics of the liquid at the filling factor $\nu = 1$ may be dominated by a topological spin-texture (skyrmion) which provides a non-trivial degree of mapping of a 2D plane onto a sphere representing the local direction of spin-polarization $\hat{\tau}$, $n = 1$. As a particle-like excitation, the skyrmion carries electrical charge equal to its topological charge, and has a reduced contribution of the exchange to its energy, as compared to the dissociation energy of a spin-exciton into an electron-hole pair [4]. It has been suggested [5] that the conductivity $\sigma_{xx}$ at exactly $\nu = 1$ may be provided by thermal activation of skyrmion/anti-skyrmion pairs. However, the excess Zeeman energy of a skyrmion may compensate the gain in the exchange energy, due to a large number of improperly polarized electrons in it. To stabilize skyrmions, one should reduce the bare value of the single-electron Zeeman splitting, $\varepsilon_Z$ in the material accommodating the 2D electron gas. In GaAs/AlGaAS heterostructures and quantum wells, one may achieve this goal either by using the hydrostatic pressure which induces strains and progressively changes the conduction band Landé factor from negative to positive values, or by optically polarizing nuclear spins of Ga and As in the vicinity of the heterostructure, which affects $\varepsilon_Z$ via the hyperfine interaction [6]. Both of these methods have recently allowed one to obtain the 2D electron gas with an almost zero value of the single-electron Zeeman splitting [7,8], although it is not clear to which extend the induced variation of $\varepsilon_Z$ is spatially homogeneous over the plane of the 2D gas.

In this paper, we study the effect of an inhomogeneity of the single-electron spin splitting onto the ferromagnetic quantum Hall effect liquid at $\nu = 1$ under the extreme condition that the externally modified Zeeman energy approaches zero value and extend an analysis of skyrmion-type textures onto the latter case. For the sake of simplicity, we consider a strained quantum well in which $\langle \varepsilon_Z(\mathbf{r}) \rangle = 0$, but only as an average over the 2D plane, whereas locally it fluctuates between small positive and negative values, $\langle \varepsilon_Z^2(\mathbf{r}) \rangle = \varepsilon_Z^2$. Mesoscopic-size regions of a negative and positive $\varepsilon_Z$ generate a multi-domain structure of the ferromagnetic liquid. A soft disorder in the sign of the Zeeman energy is different from the traditionally studied potential disorder, since the former cannot be electrostatically screened. In a given sample, the lines where $\varepsilon_Z(\mathbf{r}) = 0$ define a contour of walls between oppositely polarized domains - their spatial configuration is determined by a distribution of stresses and by the interface roughness in a given quantum well device, or by inhomogeneity of the nuclear spin polarization [9]. Below, we analyze the spin-texture of a domain wall between two oppositely polarized $\nu = 1$ ferromagnetic liquids (DW), its excitations and a possible effect of topological defects in the DW on the transport properties of the 2D system.

Several features of a domain wall between quantum Hall effect ferromagnets can be assessed qualitatively. Due to the stiffness of a polarization field, it can be regarded as a smooth variation of $\hat{\tau}(\mathbf{r})$ created by conditions imposed by the crystal: from $n_z(x > 0) = 1$ to $n_z(x < 0) = -1$. The long-range character of the variation of the order parameter $\hat{\tau}(\mathbf{r})$ across the DW, at the length scale $L_w \gg \lambda = \sqrt{\hbar c / e B}$, is guaranteed by the smallness of the Zeeman energy as compared to the electron-electron exchange, $\varepsilon_Z \ll \varepsilon_0$. A spin-texture inside the domain wall can be described as a mapping of the 2D plane on to the unit sphere, such that $\hat{\tau}(\mathbf{r})$ retraces a path from the ‘south’ to the ‘north’ pole when a path in the real space traverses the border between domains. In the case of a flat border between $\varepsilon_Z > 0$ and $\varepsilon_Z < 0$ regions, the geodesics on a sphere naturally minimize the DW exchange energy.

Without spin-orbit coupling, the DW energy would be degenerate with respect to the choice of a geodesic,
i.e., with respect to the angle $\varphi$ between the planar component of the polarization vector $\vec{m} = (\sin \nu \cos \varphi, \sin \nu \sin \varphi, \cos \nu)$ and the locally determined normal direction to the line where $n_z = \cos \nu = 0$. This degeneracy results in a soft collective mode corresponding to the rotation of the spin coordinate system relative to the orbital one. Symmetry transformations responsible for this residual degeneracy belong to the group $\text{SO}_2$ of rotations of a spin-texture around the axis $\vec{T}_z$, the magnetic field orientation. The spin-orbit coupling of a form proposed in lifts the degeneracy of the DW ground state and orients the field $\vec{m}$ in the middle of the DW perpendicular to the border between domains. As a one-dimensional object, the DW has an action that resembles the action of a classical sine-Gordon model. Its low-lying excited states have the form of (a) spin-waves and (b) of topologically stable defects — kinks in the angle $\varphi$.

The ground state of a single straight domain wall is characterized by one properly chosen geodesic line on a sphere $|\vec{m}| = 1$. Spin-waves correspond to small rotations, $\Delta \varphi \ll 1$, of a geodesic. A kink can be described as a $2\pi$ rotation of the polarization vector $\vec{m}$ around the axis $\vec{T}_z$. Topological classes of kinks are given by the homotopic group $\pi_1(\text{SO}_2) = Z$, or, equivalently, by the degree of mapping of a line representing the middle of the DW, $n_z = 0$ onto the equator of a unit sphere. As a 2D object, a kink can be viewed as a mapping of a plane into a sphere, $R^2 \to S^2$, such that a set of geodesics collected upon moving along the domain wall covers the entire sphere $|\vec{m}| = 1$. Since the excess density of electrons, $\delta \rho$ carried by a polarization texture is equal to

$$\delta \rho = \left|\left(\partial_y \cos \nu \right) \left(\partial_z \varphi \right) - \left(\partial_z \cos \nu \right) \left(\partial_y \varphi \right)\right| / 4\pi,$$ (1)

the electrical charge of a kink is equal to the degree of mapping $R^2 \to S^2$ provided by the defect, which coincides with its topological charge classified by $\pi_1(\text{SO}_2)$. That makes kinks akin skyrmions, and the latter are classified using the homotopic group $\pi_2(\text{SU}_2/\text{U}_1) = Z$, and these two solitonic excitations are related to different boundary conditions at the infinities of a 2D plane. Stability of a kink with respect to the decay of its charge density into the bulk is provided by its relatively small energy, as compared to the bulk skyrmion, which is supported by the quantitative analysis. The kinks we discuss in this paper are also similar to skyrmionic textures on the edge of the quantum Hall effect liquid discussed in [4].

The quantitative analysis of the DW energetics in this paper is based on the $\sigma$-model approach [4]. We started from the Grassman functional integral for interacting electrons where the kinetic energy, Zeeman splitting and spin-orbit coupling terms were present. Then, we split interactions by means of the Hubbard-Stratonovich transformation, and derived the 2D $\sigma$-model for the polarization $\vec{m} = (\sin \nu \cos \varphi, \sin \nu \sin \varphi, \cos \nu)$ using the gradient expansion [7]. Below, we parametrize the polarization by using Euler angles: $\nu$, $|\varphi| < \pi$, with respect to the normal to the plane and $\varphi$ with respect to the direction across the domain wall. The mean field $\vec{m}(r,t)$ enters into a unitary transformation, $U = \left[\left(1 + n_x\right)^{1/2} + i \left(1 + n_x\right)^{-1/2} \left(n_y \sigma_x - n_x \sigma_y\right)\right] / \sqrt{2}$, which locally determines the spinor part of the electron wave function. The gradient expansion with respect to the matrices $U(-i\lambda \nabla)U^\dagger$, $U(-i\partial_t)U^\dagger$ was based on the assumption that the chemical potential in the system lies in the gap, and we took into account the Landau level mixing both by interactions and the spin-orbit coupling. That brings us to the action in the form

$$\int \left[ F_\nu + (F_\nu + F_C) + F_Z + F_{\text{so}} \right] \, dr / (2\pi) - F_{\text{top}},$$ (2)

which is composed of the gradient terms ($F_\nu + F_C$), the Zeeman energy $F_Z$, the spin-orbit coupling term $F_{\text{so}}$, the topological term $F_{\text{top}} = (\mu - 3\lambda / 2) \int d\rho / \pi$, where $\mu$ is the single-electron chemical potential, and $F_\nu = -\left(i / 2 \lambda^2 \cos \nu \partial_r \varphi\right)$. The latter term should be kept to calculate the spin-wave spectrum of the DW [5].

The polarization field stiffness and the Zeeman energy are described by

$$F_\nu = 3_1 \sum \left(\nabla n_\nu\right)^2 / 8 = 3_1 \left(\sin^2 \nu \left(\nabla \varphi\right)^2 + \left(\nabla \nu\right)^2\right) / 8,$$

$$F_Z = (\varepsilon_Z / 2\lambda^2) \text{sign}(x) n_z = (\varepsilon_Z / 2\lambda^2) \text{sign}(x) \cos \nu. \quad (3)$$

The Zeeman energy in Eq. (3) corresponds to the model of a step-like $\varepsilon_Z(r) = \varepsilon_Z \text{sign}(x)$. The exchange interaction of electrons at the $n$-th Landau level with electrons from a completely filled Landau level $n=0$ is given by $3_3 = \int_0^\infty V(R) e^{-R^2 / L_n} R^2 dR$, where $R = r / \lambda$. For an unscreened Coulomb interaction, $V(r) = e^2 / r \chi$, $3_1 = 3_3 / 2$, and $3_3 = \sqrt{\pi / 2} \lambda^2 / \chi$. The ‘Coulomb term’ $F_C$ in Eq. (3) is the result of the higher order expansion in gradients [6]. It is mentioned only to be neglected later, since $F_C$ is enough to provide an ultraviolet cut-off in the DW defect energy calculation.

In the main approximation, that is, before the spin-orbit coupling is taken into account, the domain wall energy is minimized by any texture with $\varphi = \text{const}$ and with $\nu(x)$ obeying the saddle-point equation,

$$\lambda^2 \partial_x^2 \nu = (4\varepsilon_Z / 3_0) \text{sign}(x) \sin \nu. \quad (4)$$

Eq. (4) is the result of the variational principle applied to the energy $F_\nu + F_Z$. Its solution should be antisymmetric, so that in the half-plane $x > 0$ it satisfies the boundary conditions $\cos \nu(0) = 0$ and $\cos \nu(\infty) = 1$. The optimal $\nu_0(x)$ can be found in the form of

$$\cos \nu_0 = \left[1 - 2 \cosh^{-2} \left(\left|w\right| + \ln(\sqrt{2} + 1)\right)\right] \text{sign}(w),$$ (5)
where \(w = (2x/\lambda) \sqrt{\varepsilon_z/\mathcal{Z}_0}\), which confirms that the domain wall width, \(L_w = 2\lambda \sqrt{\mathcal{Z}_0/\varepsilon_z}\), is large, \(L_w \gg \lambda\). The result of Eq. (3) allows us to calculate the ground state DW energy per magnetic length: \(E_w^0 = \sqrt{\varepsilon_z/\mathcal{Z}_0}(1 - \sqrt{1/2}) / \pi\) [19].

At this scale of energies, the structure of the wall is degenerate with respect to choice of the geodesic on the unit sphere along which the polarization \(\overrightarrow{\tau}\) rotates, that is, with respect to the angle \(\varphi\). At a finer scale of energies, such a degeneracy is lifted by the spin-orbit coupling. It is natural to assume that \(\hbar \omega_c \gg \mathcal{Z}_0 = \frac{\varepsilon^2}{2\lambda} \sqrt{\varepsilon_z} \gg \varepsilon_z\), which confines the ground state electrons to the lowest Landau level. Since the spin-orbit coupling \([13,20]\), such as \(\nu_{so}[\overrightarrow{\tau} \times \overrightarrow{\sigma}]\), is not diagonal in the Landau level basis, it appears only in the form of \(\varepsilon_{so}\mathcal{Z}_0/\hbar \omega_c\), \(\varepsilon_{so} = \hbar \nu_{so}/\lambda\), which approves its perturbative treatment. To be more specific, we shall limit the spin-orbit coupling energy by the constraint \((\varepsilon_{so}/\hbar \omega_c)^2 < \varepsilon_z/\mathcal{Z}_0\), which enables us to exclude a spin-wave instability of the ground state of a homogeneous liquid in the bulk [17]. The spin-orbit coupling term in Eq. (4) can be found in the form of

\[
F_{so} = (\varepsilon_{so}\mathcal{Z}_0/\hbar \omega_c) \, n_z(x) \partial_y n_z(x)
\]

\[
= \frac{\varepsilon_{so}\mathcal{Z}_0}{2\hbar \omega_c \lambda} \left( \cos \varphi \partial_x + \sin \varphi \partial_y \right) \left( \nu - \frac{\sin 2\nu}{2} \right),
\]

which has a minimum at \(\varphi = 0\) for the ground state of the DW with \(\nu(x) = \nu_0(x)\) and determines the energies of the low-lying excitations of this object.

First of all, we analyze the energetics of a topological defect. A kink corresponds to the texture with the planar component of the polarization (followed along the middle of the DW) making a full circle when the coordinate changes from \(y = -\infty\) to \(y = \infty\). If the DW texture varies along the wall much slower than across it - an assumption which can be verified after the solution is found, the problem may be formulated as a one-dimensional one. The free energy of the wall (regarded as a 1D system) related to the lifted residual \(SO_2\) degeneracy can be obtained from Eqs. (3) [1] by integrating out the transverse form-factor of the DW, \(\cos \nu_0(x)\):

\[
\mathcal{E} = \mathcal{Z}_0 \int \frac{dy}{8\lambda} \left[ \frac{\mathcal{Z}_0}{\varepsilon_z} (\alpha \lambda \partial_y \varphi)^2 + \varepsilon_{so} (1 - \cos \varphi) \right]
\]

where \(\alpha = \sqrt{(2 - \sqrt{2}/2)/3\pi} \approx 0.37\). The energy minimum of a kink is provided by the texture with \(\cos \varphi(y) = 1 - \frac{2}{\cosh^2(u)}\) and \(\sin \varphi(y) = \frac{2\sinh(u)}{\cosh^2(u)}\), \(u = \frac{\varphi}{\alpha \lambda} \left( \varepsilon_{so} \mathcal{Z}_0/\hbar \omega_c \varepsilon_z \right)^{1/2} \). \(\Theta = \pm 1\) is the sign of a topological charge of the defect. The charge density distribution in a single kink calculated using Eqs. (1) is shown in Fig. 1. The activation energy of the kink/anti-kink pair (two times the spin-deformation energy of one kink) is equal to \(\mathcal{E}_a = \alpha \mathcal{Z}_0 (\mathcal{Z}_0/\varepsilon_z)^{1/4} (\varepsilon_{so}/\hbar \omega_c)^{1/2}\).

Since the pair of defects with an opposite sign has total topological number \(N = 0\), the term \(F_{so}\) plays no role in determining \(\mathcal{E}_a\). Note that the activation energy in Eq. (4) is smaller than the activation energy of a skyrmion in the 2D bulk, \(\mathcal{E}_a < \mathcal{Z}_0\) (since we assume that \((\varepsilon_{so}/\hbar \omega_c)^2 < \varepsilon_z/\mathcal{Z}_0\)). This prevents the kink from decaying into the bulk excitations.

The spectrum of spin-waves propagating along the DW can be found by expanding the Lagrangian in the vicinity of the homogeneous ground state, \(\nu = \nu_0(w)\) and \(\varphi = 0\) - up to the second order in small variations, \(\varphi = e^{i\delta w/\lambda - i\omega t}/h \varphi(w)\) and \(\nu = \nu_0(w) + e^{i\delta w/\lambda - i\omega t}/h \delta \nu(w)\) (the coordinate across the wall is normalized by its width \(L_w, w = (2x/\lambda) \sqrt{\varepsilon_z/\mathcal{Z}_0}\). To select excitations localized near the boundary between domains, one should study only those solutions for which \(\delta \nu(\pm \infty) = 0\). In contrast, the boundary conditions for \(\varphi\) are free, since the fluctuations of \(\varphi\) have no sense in the regions where \(\sin \nu = 0\). Therefore, it is easier to formulate the eigenvalue problem for a different function, \(\phi(w) = \sin \nu_0(w)\varphi(w)\), which has more suitable boundary conditions \(\phi(\pm \infty) = 0\).

After applying the variational principle to the result of such an expansion, and in the limit of \(q \ll \sqrt{\varepsilon_z/\mathcal{Z}_0}\) (the wavelength along the wall is much longer than the DW width), we arrive at the eigenvalue equations,

\[
i \omega \phi \approx \varepsilon_z \left[ -\partial_w^2 + \text{sign}(w) \cos \nu_0 \right] \delta \nu,
\]

\[
i \omega \delta \nu \approx \varepsilon_z \frac{\mathcal{Z}_0}{\mathcal{Z}_0} \Pi \phi + \left[ \frac{q^2}{4} - \frac{2 \varepsilon_{so}}{\hbar \omega_c \varepsilon_z} \right] \sqrt{\varepsilon_z/\mathcal{Z}_0} \left( \partial_w \nu_0 \right) \phi.
\]

The operator on the right hand side of equation Eq. (8) has a spectrum of eigenvalues with gap \(u_0 \sim 1\). Eigenvalues of the operator \(\Pi\) in Eq. (9),

\[
\Pi = -\partial_w^2 + (\partial_w^2 \sin \nu_0)/\sin \nu_0,
\]

start from zero - for the function \(\phi_0 = \text{Const} \times \sin \nu_0(w)\) - and are of the order of unity for other ‘excited states’ \(\phi_n\) [17]. The lowest eigenvalue corresponds to the excitation with \(\varphi\) constant across the wall, but varying along it. This is the Goldstone mode. Other excited states are spin-waves in the bulk of the domain affected by the presence of a wall. Their spectrum starts above a gap of the order of Zeeman energy, \(\varepsilon_z\). If we treat Eqs. (9) perturbatively and approximate the lowest eigenstate by \(\phi_0\), taking into account the expression on its left-hand side in the diagonal approximation, we find that the soft excitation has the dispersion

\[
\hbar \omega = (\alpha \varepsilon_z \mathcal{Z}_0/A)^{1/2} \sqrt{\varepsilon_{so}/\hbar \omega_c} \sqrt{\varepsilon_z/\mathcal{Z}_0} + (\alpha q^2),
\]

where \(A = \sum_w \left[ \int dw f_k(w) \sin \nu_0(w) \right]^2 / u_k \sim 1\) is determined by the normalized eigenstates, \(f_k(w)\) and eigenvalues, \(u_k\) of an equation \([-\partial_w^2 + |\cos \nu_0| - u_k] f_k(w) =\)
0. So, the spectrum of a soft mode has the gap \( \epsilon_0 (\varepsilon_{so}/\hbar \omega_c)^{3/4} \) and a linear dispersion \( q \sqrt{\varepsilon_{so}} \) at \( (\varepsilon_{so}/\hbar \omega_c)^{1/2} < q < \sqrt{\varepsilon_{so}}/\hbar \omega_c \).

Note that, as a one-dimensional object, DW between oppositely polarized QHE liquids resembles a 1D antiferromagnet. This analogy can be traced throughout the properties of kinks and linear-dispersion spin-waves, and also can be extended onto identification of regimes when classically obtained solutions are sound. Classical treatment of spin-waves is provided by their stability against decay into kink-antikink pairs, since \( \mathcal{E}_u \gg \hbar \omega_c \).

The latter may be compared to the known condition for the border between the classical solution for the 1D Sin-Gordon model and the Fermi-gas of kinks \( \varepsilon_0/2 \pi \approx (\varepsilon_{so}/\hbar \omega_c)^{1/2} (\varepsilon_{so}/\hbar \omega_c) < 1 \). It has been also shown previously \( \varepsilon_0 \approx \hbar \omega_c \) that the topological defect energy in an antiferromagnet is lowered by quantum corrections arising from scattering of spin-waves on it. One should, therefore, analyze similar corrections to the DW-kink energy, \( \mathcal{E}_u \). However, our estimation \( \varepsilon_0 \approx \hbar \omega_c \) indicates that those corrections do not alter the relation \( \mathcal{E}_u \gg \hbar \omega_c \).

As we have shown above, the domain wall (DW) has an excitation spectrum with energies lower than the energies of excitations in the bulk. Therefore, the low-gap spin-waves in it may be responsible for the low-frequency absorption of a system in the regime when the multi-domain structure is formed. Moreover, the fact that topological defect in the DW is charged and has a low activation energy (since they are formed on the top of a texture defect in the DW), may lead to the dominating role of kink/anti-kink pairs in the dissipative conductivity, \( \sigma_{xx} \), when the network of DW's forms a percolation cluster through the entire 2D plane. Since DW's separate regions of a positive and negative value of \( \varepsilon_Z \), the critical regime is realized when the sample areas under the \( \varepsilon_Z > 0 \) and \( \varepsilon_Z < 0 \) domains are equal \( \varepsilon_0/\hbar \omega_c \), that is, when \( \langle \varepsilon_Z (r) \rangle = 0 \). This scenario suggests that in the proximity of external conditions providing \( \varepsilon_Z \rightarrow 0 \), dissipative transport at \( \nu = 1 \), \( \sigma_{xx}(T) \) may be determined by percolation of thermally activated kinks through an infinite cluster of DW's. That may be the reason why, in the high-pressure experiment \( \varepsilon_0 \), the vicinity of the pressure value where the conduction band \( g \)-factor in GaAs should nominally change its sign, the \( \nu = 1 \) activation energy falls sharply below the value expected for the skyrmion/anti-skyrmion pairs in a homogeneous ferromagnetic liquid.

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FIG. 1. 2D charge-density distribution around a kink. The scratch shows the middle of the DW, where \( g(r) = 0 \). The coordinates across and along the DW, \( w \) and \( u \), are normalized by \( L_w \) and \( L_k \), respectively.