Open quantum systems in noninertial frames

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Received 4 November 2010
Published 4 January 2011
Online at stacks.iop.org/JPhysA/44/045305

Abstract

We study the effects of decoherence on the entanglement generated by the Unruh effect in noninertial frames by using phase flip, phase damping and depolarizing channels. It is shown that decoherence strongly influences the initial state entanglement. Entanglement sudden death can occur irrespective of the acceleration of the noninertial frame under the action of phase flip and phase damping channels. It is investigated that an early sudden death occurs for large acceleration under the depolarizing environment. Moreover, entanglement increases for a highly decohered phase flip channel.

PACS numbers: 03.65.Ud, 03.65.Yz, 03.67.Mn, 04.70.Dy

1. Introduction

Entanglement is one of the potential sources of quantum theory. It is the key concept and a major resource for quantum communication and computation [1]. In the last few years, enormous efforts have been made to investigate various aspects of quantum entanglement and its benefits in a number of setups, such as teleportation of unknown states [2], quantum key distribution [3], quantum cryptography [4] and quantum computation [5, 6]. Recently, the study of quantum entanglement of various fields has been extended to the relativistic setup [7–12] and interesting results on the behavior of entanglement have been obtained. The study of entanglement in the relativistic framework is important not only from a quantum information perspective but also to understand deeply black hole thermodynamics [13, 14] and the black hole information paradox [15, 16].

Earlier investigations on quantum entanglement in the relativistic framework are mainly focused on isolated quantum systems. In fact, no quantum system can be completely isolated from its environment and may result in a non-unitary dynamics of the system. Therefore, it is important to study the effect of environment on the entanglement in an initial state of a quantum system during its evolution. The interaction between an environment and a quantum system leads to the phenomenon of decoherence and it gives rise to an irreversible transfer of information from the system to the environment [17–19].
In this paper we work out the effect of decoherence on the entanglement of the Dirac field in a noninertial system. Alsing et al [7] have shown that the entanglement between two modes of a free Dirac field is degraded by the Unruh effect and asymptotically reaches a nonvanishing minimum value in the infinite acceleration. We investigate how the loss of entanglement through the Unruh effect is influenced in the presence of decoherence by using a phase flip, phase damping and depolarizing channel in the Kraus operator formalism. The effect of the amplitude damping channel on the Dirac field in a noninertial system was recently studied by Wang and Jing [20]. We consider two observers, Alice and Rob that share a maximally entangled initial state of two qubits at a point in flat Minkowski spacetime. Then Rob moves with a uniform acceleration and Alice stays stationary. To achieve our goal, we consider two cases. In one case we allow only Rob’s qubit to interact with a noisy environment and in the second case both qubits of the two observers interact with a noisy environment. Let the two modes of Minkowski spacetime that correspond to Alice and Rob be, respectively, given by $|n\rangle_A$ and $|n\rangle_R$, respectively. Moreover, we assume that the observers are equipped with detectors that are sensitive only to their respective modes and share the following maximally entangled initial state:

$$|\psi\rangle_{A,R} = \frac{1}{\sqrt{2}}(|00\rangle_A|0\rangle_R + |11\rangle_A|1\rangle_R),$$  

(1)

where the first entry in each ket corresponds to Alice and the second entry corresponds to Rob. From the accelerated Rob’s frame, the Minkowski vacuum state is found to be a two-mode squeezed state [7]

$$|0\rangle_M = \cos r |0\rangle_I |0\rangle_{II} + \sin r |1\rangle_I |1\rangle_{II},$$  

(2)

where $\cos r = (e^{-2\pi \omega c/a} + 1)^{-1/2}$. The constants $\omega$, $c$ and $a$, in the exponential stand, respectively, for Dirac particle’s frequency, the speed of light in a vacuum and Rob’s acceleration. In equation (2) the subscripts $I$ and $II$ of the kets represent the Rindler modes in regions $I$ and $II$, respectively, in the Rindler spacetime diagram (see figure 1). The excited state in Minkowski spacetime is related to Rindler modes as follows [7]:

$$|1\rangle_M = |1\rangle_I |0\rangle_{II}.$$

(3)
Table 1. Single-qubit Kraus operators for the phase flip, phase damping and depolarizing channels.

| Channel             | Operators                                                                 |
|---------------------|---------------------------------------------------------------------------|
| Phase flip channel  | $E_0 = \sqrt{1-p} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $E_1 = \sqrt{p} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ |
| Phase damping channel | $E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}$, $E_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{pmatrix}$ |
| Depolarizing channel | $E_0 = \sqrt{1-p} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $E_1 = \sqrt{p/3} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $E_2 = \sqrt{p/3} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $E_3 = \sqrt{p/3} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ |

In terms of Minkowski modes for Alice and Rindler modes for Rob, the maximally entangled initial state of equation (1) by using equations (2) and (3) becomes

$$|\psi\rangle_{A,I,II} = \frac{1}{\sqrt{2}} \left( |0\rangle_A |0\rangle_I |0\rangle_{II} + \sin r (|0\rangle_A |1\rangle_I |1\rangle_{II} + |1\rangle_A |1\rangle_I |0\rangle_{II}) \right).$$  \hfill (4)

Since Rob is causally disconnected from region $II$, we must take trace over all the modes in region $II$. This leaves the following mixed density matrix between Alice and Rob, that is,

$$\rho_{A,I} = \frac{1}{2} \left( \cos^2 r |00\rangle_{A,I} \langle 00| + \cos r \sin r (|01\rangle_{A,I} \langle 10| + |10\rangle_{A,I} \langle 01|) \right) \times \sin^2 r (|01\rangle_{A,I} \langle 01| + |11\rangle_{A,I} \langle 11|).$$  \hfill (5)

2. Single qubit in a noisy environment

In this section we consider that only Rob’s qubit is coupled to a noisy environment. The final density matrix of the system in the Kraus operator representation becomes

$$\rho_f = \sum_i (\sigma_0 \otimes E_i) \rho_{A,I} (\sigma_0 \otimes E_i^*) \left( \right),$$  \hfill (6)

where $\rho_{A,I}$ is the initial density matrix of the system given by equation (5), $\sigma_0$ is the single-qubit identity matrix and $E_i$ are the single-qubit Kraus operators of the channel under consideration. The Kraus operators of the channels we use are given in table 1. The spin–flip matrix of the final density matrix of equation (6) is defined as $\tilde{\rho}_f = (\sigma_2 \otimes \sigma_2) \rho_f (\sigma_2 \otimes \sigma_2)$, where $\sigma_2$ is the Pauli matrix. The degree of entanglement in the two-qubit mixed state in a noisy environment can be quantified conveniently by the concurrence $C$, which is given as [21, 22]

$$C = \max \{0, \sqrt{\lambda_1 - \sqrt{\lambda_2 - \sqrt{\lambda_3 - \sqrt{\lambda_4}}}} \} \quad \lambda_i \geq \lambda_{i+1} \geq 0,$$  \hfill (7)

where $\lambda_i$ are the eigenvalues of the matrix $\rho_f \tilde{\rho}_f$. The eigenvalues under the action of the phase flip channel become

$$\lambda_1^{PF} = (1 - 2p + p^2) \cos^2 r,$$
$$\lambda_2^{PF} = p^2 \cos^2 r,$$
$$\lambda_3^{PF} = \lambda_4^{PF} = 0,$$  \hfill (8)

where the superscript $PF$ corresponds to the phase flip channel. Similarly, the eigenvalues under the action of phase damping and depolarizing channels are, respectively, given by

$$\lambda_{1,2}^{PD} = \frac{1}{4} (2 - p \pm 2\sqrt{1 - p}) \cos^2 r,$$
$$\lambda_3^{PD} = \lambda_4^{PD} = 0,$$  \hfill (9)
\[ \lambda_{1}^{DP} = (-1 + p)^2 \cos^2 r, \]
\[ \lambda_{2}^{DP} = \lambda_{3}^{DP} = \lambda_{4}^{DP} = \frac{1}{9} p^2 \cos^2 r, \]

where the superscripts PD and DP stand for phase damping and depolarizing channels, respectively. In all these equations \( p \in [0, 1] \) is the decoherence parameter. The upper and lower values of \( p \) correspond to the undecohered and fully decohered cases of the channels, respectively. The concurrence under the action of every channel reduces to the result of [7] when the decoherence parameter \( p = 0 \).

To see how the concurrence and hence the entanglement is influenced by the decoherence parameter \( p \) in the presence of the Unruh effect, we plot the concurrence for each channel against \( p \) for various values of \( r \). In figure 2, the concurrence under the action of phase flip channel is plotted against \( p \). The figure shows that for smaller values of \( p \), the entanglement is strongly acceleration dependent, such that for large values of Rob’s acceleration (the value of \( r \)) it gets weakened. However, as \( p \) increases the dependence of entanglement on acceleration decreases and the increasing value of \( p \) causes a rapid loss of entanglement. The entanglement sudden death occurs irrespective of the acceleration of Rob’s frame for a 50% decoherence. Figure 3 shows the effect of decoherence on the concurrence under the action of the phase damping channel. In this case, the degradation of entanglement due to decoherence is smaller compared to the degradation in the case of phase flip. The entanglement vanishes for all values of acceleration only when the channel is fully decohered. The concurrence under the action of the depolarizing channel is exactly equal to the one for the phase flip channel. Hence it influences the entanglement in a way exactly similar to the phase flip channel as shown in figure 2.
3. Both qubits in a noisy environment

In this section we consider that both Alice’s and Rob’s qubits are influenced simultaneously by a noisy environment. The final density matrix in this case can be written in the Kraus operators formalism as follows:

\[
\rho_f = \sum_k E_k \rho_{A,I} E_k^\dagger,
\]

where \( \rho_{A,I} \) is given by equation (5) and \( E_k \) are the Kraus operators for a two-qubit system, satisfying the completeness relation \( \sum_k E_k E_k^\dagger = I \) and are constructed from the single-qubit Kraus operators of a channel by taking the tensor product of all the possible combinations in the following way:

\[
E_k = \sum_{i,j} E_i \otimes E_j,
\]

where \( E_{i,j} \) are the single-qubit Kraus operators of a channel given in table 1. We consider that both Alice’s and Rob’s qubits are influenced by the same environment, that is, the decoherence parameter \( p \) for both qubits is same. Proceeding in a similar way like the case of single qubit coupled to the environment, the eigenvalues of the matrix \( \rho_f \tilde{\rho}_f \) under the action of the phase flip channel become

\[
\lambda_{PF}^1 = (1 + 2(-1 + p)p^2 \cos^2 r, \\
\lambda_{PF}^2 = 4(-1 + p)^2 p^2 \cos^2 r, \\
\lambda_{PF}^3 = \lambda_{PF}^4 = 0.
\]
Likewise the eigenvalues for phase damping and depolarizing channels, respectively, become
\[
\begin{align*}
\lambda_{1}^{PD} &= \frac{1}{4}(-2 + p)^2 \cos^2 r, \\
\lambda_{2}^{PD} &= \frac{1}{4} p^2 \cos^2 r, \\
\lambda_{3}^{PD} &= \lambda_{4}^{PD} = 0,
\end{align*}
\]
(14)
\[
\begin{align*}
\lambda_{1,3}^{DP} &= \frac{1}{1296} [324 + p(-3 + 2p)(387 + 152p(-3 + 2p)) \\
&\quad + 4(3 - 4p)^2(9 + 5p(-3 + 2p)) \cos 2r \\
&\quad + (3 - 4p)^2 p(-3 + 2p) \cos 4r \pm 4(3 - 4p)^2 \cos r \\
&\quad \times (3(54 + p(-3 + 2p)(33 + 8p(-3 + 2p))) \\
&\quad + (3 - 4p)^2(2(9 - 6p + 4p^2) \cos 2r + p(-3 + 2p) \cos 4r)^{1/2}], \\
\lambda_{2}^{DP} &= \lambda_{4}^{DP} = \frac{1}{1296} p(-3 + 2p)(-9 + 4p) \\
&\quad + (-3 + 4p) \cos 2r)(3 + 4p + (-3 + 4p) \cos 2r). \quad (15)
\end{align*}
\]

The ‘±’ sign in equation (15) corresponds to the eigenvalues \(\lambda_{1}\), and \(\lambda_{3}\) respectively. It is necessary to point out here that the concurrence under the action of each channel reduces to the result of [7] when we set the decoherence parameter \(p = 0\).

To see how the entanglement behaves when both qubits are coupled to the noisy environment, we plot the concurrence against the decoherence parameter \(p\) for different values of \(r\) under the action of each channel separately. Figure 4 shows the dependence of concurrence on the decoherence parameter \(p\) under the action of the phase flip channel. The dependence of entanglement on acceleration of Rob’s frame is obvious in the region of lower values of \(p\). However, this dependence diminishes as \(p\) increases and a rapid decrease in the degree of entanglement develops. At a 50% decoherence level, entanglement sudden death occurs irrespective of Rob’s acceleration. It is interesting to see that beyond this point, the entanglement regrows as \(p\) increases. The dependence of entanglement on acceleration
Figure 5. The concurrence $C$ under the action of phase damping channel is plotted against the decoherence parameter $p$ for the case when both qubits are coupled to a noisy environment.

Figure 6. The concurrence $C$ under the action of depolarizing channel is plotted against the decoherence parameter $p$ for the case when both qubits are coupled to a noisy environment.

of Rob’s frame reemerges and the entanglement reaches the corresponding undecohered maximum value for a fully decohered case. The concurrence varies apparently like a parabolic function of the decoherence parameter $p$ with its vertex at $p = 0.5$. The dependence of entanglement on $p$ under the action of the phase damping channel is shown in figure 5. In this
case the entanglement decreases linearly as \( p \) increases and the dependence on acceleration diminishes. Whatever the acceleration of Rob’s frame may be, entanglement sudden death occurs when the channel is fully decohered. The influence of the depolarizing channel on the entanglement is shown in figure 6. Unlike the other two channels, the depolarizing channel does not diminish the effect of acceleration on the entanglement as the \( p \) increases. However a rapid decrease in entanglement appears which leads to entanglement sudden death at different values of the decoherence parameter for different acceleration of Rob’s frame. The larger the acceleration, the earlier entanglement sudden death occurs.

If we compare the single-qubit and the two-qubit decohering situations, it becomes obvious that the entanglement loss is rapid when both qubits are coupled to the noisy environment. For example, in the case of the phase flip channel, the concurrence behaves as a linear function of \( p \) for the single-qubit decohering case, whereas in the case of the two-qubit decohering case it varies like a parabolic function. Nevertheless, sudden death occurs at the same value of \( p \) \( (p = 0.5) \), irrespective of the acceleration, for both cases under the action of the phase flip channel. Whereas for the phase damping channel, sudden death occurs in both cases for all values of \( r \) at \( p = 1 \). For the depolarizing channel, however, this is not true.

**4. Conclusion**

In conclusion, we have investigated that the entanglement in Dirac fields is strongly dependent on coupling with a noisy environment. This result is contrary to the case of an isolated system in which the entanglement of Dirac fields survives even in the limit of infinite acceleration of Rob’s frame. In the presence of decoherence, the entanglement rapidly decreases and entanglement sudden death occurs even for zero acceleration. Under the action of the phase flip channel, the entanglement can regrow when both qubits are coupled to a noisy environment in the limit of large values of the decoherence parameter. The entanglement disappears, irrespective of the acceleration, under the action of the phase damping channel only when the channel is fully decohered both for single-qubit and two-qubit decohering cases. However, under the action of the depolarizing channel, an early sudden death occurs for larger acceleration when both qubits are coupled to the environment. In summary, the entanglement generated by the Unruh effect in a noninertial frame is strongly influenced by decoherence.

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