Freeze-out and thermalization in relativistic heavy ion collisions

Sourendu Gupta,1 Debasish Mallick,2 Dipak Kumar Mishra,3, Bedangadas Mohanty,2,∗, Nu Xu4,5

1Department of Theoretical Physics, Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400005, India
2School of Physical Sciences, National Institute of Science Education and Research, HBNI,
Jatni 752050, India
3Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai 400085, India,
4Institute of Modern Physics, Chinese Academy of Sciences,
509 Nanchang Road, Lanzhou 730000, China
5College of Physical Science and Technology, Central China Normal University,
152 Luyu Road, Wuhan 430079, China

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Abstract

High energy heavy-ion collisions in laboratory produce a form of matter that
can test Quantum Chromodynamics (QCD), the theory of strong interactions, at
high temperatures. One of the exciting possibilities is the existence of thermody-
amically distinct states of QCD, particularly a phase of de-confined quarks and
gluons. An important step in establishing this new state of QCD is to demonstrate
that the system has attained thermal equilibrium. We present a test of thermal
equilibrium by checking that the mean hadron yields produced in the small impact
parameter collisions as well as grand canonical fluctuations of conserved quantities
give consistent temperature and baryon chemical potential for the last scattering
surface. This consistency for moments up to third order of the net-baryon number,
charge, and strangeness is a key step in the proof that the QCD matter produced

∗Chinese Academy of Sciences President’s International Fellowship Initiative visiting scientist at In-
stitute of Modern Physics, Lanzhou 730000, China
in heavy-ion collision attains thermal equilibrium. It is a clear indication for the first time, using fluctuation observables, that a femto-scale system attains thermalization. The study also indicates that the relaxation time scales for the system are comparable to or smaller than the life time of the fireball.

Relativistic collisions of heavy-ion are being carried out at the Relativistic Heavy Ion Collider (RHIC) in BNL and the Large Hadron Collider (LHC) facilities in CERN. These experiments have demonstrated the existence of a deconfined state of quarks and gluons, which are the fundamental constituents of baryonic matter. The experiments observe the collective flow of a fireball made of such deconfined matter [1, 2, 3, 4, 5] (see also a review [6]). This state of matter replicates baryonic matter in the universe when it was a few microseconds old. It is also conjectured that the cores of astrophysical objects like neutron stars may contain a related form of baryonic matter, albeit at lower temperature but higher density [7].

By studying the flow of matter in the femto-scale fireballs produced at RHIC and LHC, it has been shown that the ratio of shear viscosity to entropy density, $\eta/s$, is the lowest among all known fluids [8, 9]. The values of this ratio extracted from experiment are close to the bounds expected for a strongly coupled quantum fluid [10]. Further, it has been recently shown this fluid has large vortical effect [11].

Theoretical studies of strongly interacting matter using lattice field theory shows a cross over from hadronic matter to a deconfined state of quarks and gluons [12]. Consistent with these expectations, no sign of a phase transition has been found at the top energies in the experiments at RHIC and LHC.

Lattice studies have revealed an experimentally reachable critical point [13, 14, 15] in an extremely rich phase diagram of strongly interacting matter [16, 17, 18, 19]. Currently experimental studies are underway to vary the collision energy in order to study this phase diagram [20, 21, 22]. Experimental data reveals tantalising hints of critical behaviour.
[23][24][25], but these interpretations of experiments have the underlying assumption that
the fireball produced in the collisions should have come to local thermal equilibrium during
its evolution. Experimental tests of thermalization are non-trivial for these femto-scale
system: not just because the systems are small, but also because they are expanding [20].
This is what we examine critically in this paper.

Systems which are significantly larger than the measuring instruments allow ensembles
of measurements on a single system, and one may exploit the hierarchy of scales between
the size of the full system, the measuring apparatus, and the microscopic length scales.
This allows us to create ensembles of measurements on identical systems by measuring
the system at multiple points. This philosophy is used routinely to measure fluctuations
in the early universe [26][27]. On the other hand it is problematic when the measurement
involves the whole system. This happens for low-multipole fluctuations in the universe,
and is called the cosmic variance problem. For the femto-scale system which we study,
the hierarchy of length scales collapses for all measurements, and one constructs the
ensemble by repeating the experiment. This requires additional checks to make sure that
the repeated trials are indeed comparable.

Another dimension of the problem is whether the system evolves with time or not.
Again when the time scale of evolution, of measurement, and the microscopic dynamics are
well separated, simplicities emerge. In a static large system, repeated measurements can
be made at effectively the same time. This can probe thermodynamic stability such as the
convexity of free energies, or the lack of entropy production. However, in quasi-statically
evolving systems the entropy may change simply because the temperature changes with
time, and not because the system is driven towards or out of equilibrium. When there
is no clear hierarchy between the rate of evolution and microscopic relaxation time, then
one has to qualify the notion of thermal equilibrium.
Distribution functions are driven out of equilibrium, and different parts may evolve at different rates. For a small decrement of temperature, $\Delta T$, the equilibration of mean energies will require small changes. In a Boltzmann gas, for example, the average energy changes by the factor $3\Delta T/2$. This change is easily made by small exchanges of momentum among those particles which lie near the peak of the thermal distribution, and can proceed quickly. The tail of the distribution, on the other hand, has to change by a large factor. There are two ways for this to happen. One is that particles carrying these rare large momenta collide with others having equally large momenta and both emerge with smaller momentum in one scattering. Another way is for the large momentum particle to collide many times with particles of much smaller momenta and decrease the momentum in small steps. Either route gives longer relaxation times. Similarly, local changes in the chemical potential, $\Delta \mu$, distort the local number distribution, different part of which relax at different rates. Due to such differential relaxation of different parts of the distribution function different moments of the distributions may not correspond to the same temperature and chemical potential. One also knows that approach to a critical point brings with it correlated fluctuations of the whole system in both space and time, thereby increasing relaxation times dramatically. Studying the moments of the number distributions to find evidences for both equilibrium and large departures scenarios are therefore crucial tests for the physics of these femto-systems.

In the relativistic femto-systems created by heavy-ion collisions, the conserved quantities are the net-baryon number ($B$), strangeness ($S$) and electric charge ($Q$). Since these are additive quantum numbers, every anti-particle has a charge opposite to that of the particle, and the net quantum number is the algebraic sum over particles and anti-particles. Conserved quantities measured for the whole system will always be equal to that in the initial state. However, using the limited acceptance of the detector, only a
part of the thermalized system is accessed experimentally and hence can be treated within
the framework of grand canonical ensemble. This allows us to measure fluctuations of the
conserved quantities.

If the fireballs were static and in thermal equilibrium, they could be described by a
temperature, $T$, and three chemical potentials $\mu_B$, $\mu_S$ and $\mu_Q$. Since the system is not
static, these four quantities are functions of space and time. One can only observe parti-
cles which come to the detector after the last scattering. This region of last scattering is
characterized by a specific value of $T$ and chemical potentials which are generally called
freezeout parameters. It is common to quote as the freezeout parameters the values of
these thermodynamic variables which give the best match between particle yields (i.e
means of number distributions) in a statistical model of an ideal gas of hadrons and reso-
nances (HRG)\cite{28, 29, 30, 31, 32}. Characterizing the yield at fixed $\sqrt{s_{NN}}$ and acceptance
window by a single set of freezeout parameters was seen to work well for charged hadrons
\cite{28, 29}. Characterizing strange and non-strange hadrons by different freezeout condi-
tions works slightly better in some cases \cite{33}. Other variants of the HRG have also been
shown to work \cite{34, 35, 36, 37, 38, 39}. However, these HRG models also seem to describe
yield data in $e^+e^-$ and pp collisions, where one does not expect thermalized matter to
be formed \cite{40, 41, 42}. Similarly, they can describe yields in highly peripheral collisions.
These observations introduce a severe uncertainty in the interpretation of the freezeout
parameters derived using yields in terms of thermal conditions. Here we propose that a
way out of this impasse is to ask for common thermal descriptions of more than just the
mean particle number.

**Model:** The hadron resonance gas (HRG) model that we utilize in this work is briefly
discussed below and details can be found at Ref.\cite{31}. In a grand canonical ensemble with
a gas of hadrons and resonances, the thermodynamic pressure ($P$) can be obtained from
the logarithm of partition function in the limit of large volume as

\[
P(T, \mu_B, \mu_Q, \mu_S, V) = \frac{T}{V} \sum_i \ln Z_i = \sum_i \pm \frac{T g_i}{2\pi^2} \int d^3 k \ln \{1 \pm \exp [(\mu_i - E)/T]\}.
\]  

(1)

where \(k\) is the momentum, \(g_i\) is the degeneracy factor of the \(i^{th}\) species of hadron or resonance, and the plus signs are for Fermions whereas the minus signs are for Bosons. The sum is carried out over all particles in thermal equilibrium with masses up to 2.5 GeV which are listed in Particle Data Group (PDG) booklet.

The freezeout conditions for the fireball are characterized by the temperature \(T\), freezeout volume \(V\), and the three chemical potentials \(\mu_B\), \(\mu_Q\), and \(\mu_S\). If the system is in chemical equilibrium then the chemical potential for the \(i^{th}\) species is given by \(\mu_i = B_i \mu_B + Q_i \mu_Q + S_i \mu_S\), where \(B_i\), \(Q_i\) and \(S_i\) are the baryon number, electric charge and strangeness of the particle. In the analysis that we present in this paper, the detector’s acceptance and resonance decay effects are taken into account, for details see Ref. [31].

The effect of radial flow (collective expansion of the system) seen in heavy-ion collisions do not affect the moments as they are obtained integrated over the measured \(p_T\) range. HRG model calculations with and without flow shows a small difference for the observable studied [31] and those are used as systematic uncertainties associated with the model calculations in the paper.

The grand canonical approach is justifiable as experimental data used in the current work analyses a portion of the fireball produced in heavy-ion collisions. It is akin to an open system, unlike for the data for full 4\(\pi\) coverage which would have required a canonical treatment with exact conservation of charges. In addition, the criteria for applicability of grand canonical ensemble, \(VT^3 > 1\) holds true for the bulk of the produced particles in heavy-ion collisions. Such an approach has been widely used to understand the yields
of produced hadrons [20, 43, 44] and fluctuations [45, 46, 47] in the field of high-energy nuclear collisions. Further, as discussed in Ref. [48], the effect of global conservation of charges depends on the fraction of charges accepted in the detector, which is found to be small and within the experimental uncertainties for the data up to third order moment used in the current work.

**Observable:** In this paper, we study the cumulants of different order such as: $C_1^X = \langle X \rangle$, $C_2^X = \langle (X - \langle X \rangle)^2 \rangle$, $C_3^X = \langle (X - \langle X \rangle)^3 \rangle$, $C_4^X = \langle (X - \langle X \rangle)^4 \rangle - 3(C_2^X)^2$ and so on, where $X$ can be one of $B$, $Q$ and $S$. Since the observed cumulants in thermal equilibrium are related to the susceptibilities:

$$C_n^X = (V/T)T^n\chi^{(n)}_X(T,\mu) ,$$

there is a possible way to check thermalization and the predictions of lattice QCD [49, 45, 50, 51, 52]. The $n$th order generalised susceptibilities ($\chi^{(n)}_X$), where $X$ represents baryon, strangeness or electric charge indices, can be expressed as [13],

$$\chi^{(n)}_X(T,\mu_B,\mu_Q,\mu_S) = \frac{d^n P(T,\mu_B,\mu_Q,\mu_S)}{d\mu_X^n} .$$

Usually ratios of cumulant or products of moments are calculated in order to cancel the dependence on system volume. Experiments use the notation $M = C_1$ for the mean, $\sigma^2 = C_2$ for the variance, $S = C_3/C_2^{3/2}$ for the skewness, and $\kappa = C_4/C_2^2$ for the kurtosis. In terms of these variables, the ratios of cumulants are $\sigma^2/M = C_2/C_1$, $S\sigma = C_3/C_2$ and $\kappa\sigma^2 = C_4/C_2$ [23, 53, 54]. Though all the three ratios of higher cumulants ($\sigma^2/M$, $S\sigma$, $\kappa\sigma^2$) and second order off-diagonal cumulants ($\sigma^2_{XY}$) reported by the STAR experiment are expected to carry the fluctuation signals, only observables up to third order moments ($M$, $\sigma^2_{XY}$, $\sigma^2_{XX}$, $S\sigma$) are used in this work. The observable, $\kappa\sigma^2$, will be used elsewhere to test departures from thermal equilibrium. The observables calculated in the HRG model are the same as measured in the experiment.
Experimental Data: The STAR experiment has published measurements on cumulants up to 4th order and their ratios for the net-proton number (NP) \[^{23}\], the net-kaon number (NK) \[^{53}\] and the net-charge (NQ) \[^{54}\]. The experiment has also published second order off-diagonal cumulants \[^{55}\]. The effect of choosing NP as a proxy of net-baryon and NK as a proxy of net-strangeness has been studied in detail in \[^{56, 57}\]. Event-by-event net-charge distributions are obtained using the Time Projection Chamber (TPC) and Time of Flight (TOF) detectors at STAR in the momentum range \(0.2 < p_T(\text{GeV}/c) < 2.0\) and a pseudo-rapidity range of \(|\eta| < 0.5\) \[^{54}\]. Net-proton distribution is measured \[^{23}\] using TPC detector only in a momentum range of \(0.4 < p_T(\text{GeV}/c) < 0.8\) and net-kaon \[^{53}\] using both TPC and TOF detector within a momentum range of \(0.2 < p_T(\text{GeV}/c) < 1.6\). The second order off-diagonal cumulants \(\sigma_{Qp}^2\), \(\sigma_{QK}^2\) and \(\sigma_{pK}^2\) representing correlations between electric charge-baryon number, electric charge-strangeness number and baryon-strangeness number are obtained by STAR by detecting particles within of acceptance of \(|\eta| < 0.5\) and \(0.4 < p_T(\text{GeV}/c) < 1.6\) \[^{55}\]. All the above measurements are done for small impact parameter (called central 0-5%) Au on Au collisions at a range \(\sqrt{s_{NN}} = 7.7\)–200 GeV. The cumulants of distributions are corrected for detector inefficiencies and other analysis artefacts as discussed in \[^{23, 53, 54}\]. So the experimental data available for this work are the observables, \(M_{\pi^+, \pi^-}, M_{K^+, K^-}, M_p, M_{\bar{p}}, \sigma^2(NP), \sigma^2(NK), \sigma^2(NQ), \sigma_{Qp}^2, \sigma_{QK}^2\) and \(\sigma_{pK}^2\), \(S\sigma(NP), S\sigma(NK)\) and \(S\sigma(NQ)\). It has been previously shown by several authors \[^{58, 59, 61}\] that the \(\sigma^2/M\) for net-charge and net-protons are not properly explained by several variants of HRG models. The reason has been understood to be due to the acceptance limitation in the measurements — resonance decays can throw part of the charge outside the detector acceptance, thereby increasing the value of \(\sigma^2/M\) over the intrinsic thermal distribution for the fireball. Interestingly the decay distortion is absent from the observables \(\sigma^2/M(NK)\) and all three \(S\sigma\). In order to deal with the
stochasticity due to decays, we introduce the three volume independent observable using the second order cumulants for the current study: \( \sigma_{QK}^2/\sigma^2(NK) \), \( \sigma_{pK}^2/\sigma^2(NK) \), and \( \sigma_{Qp}^2/\sigma^2(NP) \) [60].

**Methodology:** The freezeout parameters, namely the \( T \), \( \mu_B \), \( \mu_S \) and \( \mu_Q \), are extracted at each \( \sqrt{s_{NN}} \), from the experimental data by minimizing

\[
\chi^2 = \sum_{i=1}^{N} \left( \frac{\Delta_i}{E_i} \right)^2 \quad \text{where} \quad \Delta_i = R_i^{\text{exp}} - R_i^{\text{HRG}},
\]

where \( N \) is the number of observables used in this calculation. \( R_i^{\text{exp}} \) and \( R_i^{\text{HRG}} \) are experimental measurements and HRG model calculations respectively for the \( i \)th observable, and \( E_i \) is the statistical uncertainty in the experiment.

As a first step in the analysis we tested whether a fit can distinguish between a thermal and a non-thermal system. This was done by taking simulated data obtained from the non-thermal model called UrQMD [62], and trying to extract freezeout conditions from it by the process outlined in Eq. 4. Approximately one million UrQMD events were analyzed for 0-5% Au+Au collisions at each collision energy. In this case we found that the value of \( \chi^2/\text{ndf} \sim O(1000 - 10000) \) for all observable rules out a reasonable fit, and the best possible freezeout parameters obtained from the yields, \( \sigma^2/M(NP,NK,NQ) \), and \( S\sigma(NP,NK,NQ) \) are quite different. Even carrying out the studies without \( \sigma^2/M \) for net-charge and net-protons in UrQMD yields a \( \chi^2/\text{ndf} \sim O(100 - 10000) \). Since UrQMD model is known to reproduce many other aspects of the final state obtained in heavy-ion collisions like multiplicity and mean \( p_T \) [63], this shows that accidental agreement of the data with the HRG model is unlikely. We have additionally repeated the analysis using a different non-thermal model that includes parton degrees of freedom, A Multi Phase Transport (AMPT) model [64], for Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV and found that the thermal fit failed.
Top 5% Au + Au Collisions at RHIC

(a) $\sqrt{s_{NN}} = 27$ GeV

(b) $\sqrt{s_{NN}} = 200$ GeV

Figure 1: Detailed comparison of the best fit predictions of the HRG model at (a) $\sqrt{s_{NN}} = 27$ GeV and (b) $\sqrt{s_{NN}} = 200$ GeV, which minimizes $\chi^2$ given in Eq. (4) with the experimental measurements. This representative case at two collision energies shows the difference between data and HRG model ($\Delta$) divided by the statistical uncertainty along the ordinate for each of the observables mentioned in the abscissa. The $\sigma^2/M(NQ)$ and $\sigma^2/M(NP)$ values shown as open circles are scaled down by a factor 100. The figure also shows a comparison of the magnitudes of systematic and statistical uncertainties, since the former are not included in the definition of $\chi^2$ to improve the discriminatory power of the fits.
Results: In Figure 1 we show the quality of a fit to fifteen pieces of data (shown in x-axis) at $\sqrt{s_{NN}} = 27$ GeV and $\sqrt{s_{NN}} = 200$ GeV obtained by varying only $T$, $\mu_B$, $\mu_S$ and $\mu_S$. The $\sigma^2/M$ for net-charge and net-protons behaves quite differently from the others due to the effect of resonance decay as discussed above. However, the ratios of second order cumulants ($\sigma_{QK}^2/\sigma^2(NK)$, $\sigma_{pK}^2/\sigma^2(NK)$, and $\sigma_{Qp}^2/\sigma^2(NP)$) are well explained by the model.

![Figure 2](image.png)

Figure 2: The best fit parameters of the HRG model at different $\sqrt{s_{NN}}$ obtained by fitting data for central Au+Au collisions. Shown as red open circles (with points slightly displaced for clarity of presentation) are the comparison of the freezeout parameters to those obtained using only the mean yields of various produced hadrons by STAR experiment [20]. Also shown as open blue squares are the comparison of the freeze-out parameters at $\sqrt{s_{NN}} = 200$ GeV obtained by fitting to the mean yields of only $\pi^\pm$, $K^\pm$ and $p(\bar{p})$.

At every $\sqrt{s_{NN}}$, we fitted the parameters of the HRG model to the thirteen observables ($M_{\pi^+}$, $M_{\pi^-}$, $M_{K^+}$, $M_{K^-}$, $M_p$, $M_{\bar{p}}$, $\sigma^2/M(NK)$, $\sigma_{QK}^2/\sigma^2(NK)$, $\sigma_{pK}^2/\sigma^2(NK)$, $\sigma_{Qp}^2/\sigma^2(NP)$, $S\sigma(NP), S\sigma(NK)$ and $S\sigma(NQ)$) shown in Figure 1 as solid circles. In Fig. 2 we show
the best fit values of \(T\), \(\mu_B\), \(\mu_S\) and \(\mu_Q\) as a function of \(\sqrt{s_{NN}}\). The \(\chi^2/\text{ndf}\) are of the \(O(1)\), except for \(\sqrt{s_{NN}} = 7.7\) GeV, where the value is 11.9. The results of the fits, which now include higher second and third order moments of particle multiplicity distributions are in good agreement with those obtained only using the mean yields of produced particles (shown as red open circles) by STAR experiment \cite{20}. The slight difference in \(T\) values arises because of inclusion of multi-strange hadrons in the fits to the yields. As shown in the figure, for \(\sqrt{s_{NN}} = 200\) GeV (as blue open square), using similar observables as in the current study, i.e the yields of \(\pi^\pm\), \(K^\pm\) and \(p(\bar{p})\), has a better agreement with our results. Including systematic uncertainties in the experimental data by adding in quadrature improves the \(\chi^2/\text{ndf}\) values to 1.0 and 4.6 for \(\sqrt{s_{NN}} = 200\) and 7.7 GeV, respectively. We have verified that the measured \(p_T\) distributions of pion, kaon, proton and the anti-particles for 0-5\% Au+Au collisions at \(\sqrt{s_{NN}} = 200\) and 19.6 GeV are reproduced using a thermal model with the extracted thermal parameters and average radial flow velocity of 0.55\(c\) and 0.46\(c\), respectively.

In Figure 3 we show the \(\Delta T\) and \(\Delta \mu_B\), the differences between the best fit value of \(T\) and \(\mu_B\) obtained from yields of \(\pi^\pm\), \(K^\pm\) and \(p(\bar{p})\), second order cumulants \(\sigma^2/M(NK)\), \(\sigma_{QK}^2/\sigma^2(NK)\), \(\sigma_{pK}^2/\sigma^2(NK)\), \(\sigma_{qK}^2/\sigma^2(NP)\) and those obtained by fitting the three \(S\sigma(NP)\), \(S\sigma(NK)\) and \(S\sigma(NQ)\) for various \(\sqrt{s_{NN}}\). Results for both central (left panel) and peripheral (right panel) collisions are shown. Since the minimum number of observables in a set is three, we have kept \(T\) and \(\mu_B\) parameters of HRG model free, fixed \(\mu_S\) values to those from STAR experiment \cite{20} and \(\mu_Q\) values to zero, for this test only. We note that in central collisions for \(\sqrt{s_{NN}} = 19.6\) GeV and higher collision energies, the best fit freezeout parameters obtained from different observables agree with each other within uncertainties. We further note that in peripheral collisions the different observables give very different freezeout conditions. A similar study with UrQMD central Au+Au collision
Figure 3: Detailed comparison of the best fit parameters of the HRG model at different $\sqrt{s_{NN}}$ obtained by fitting to different subsets of the data. The difference in temperature ($\Delta T$) and baryon chemical potential ($\Delta \mu_B$) from the third order to second order, third order to first order and second order to the first order moments are shown as filled-circles, open-squares and open-triangles, respectively. The figures show that consistent values of the freezeout parameters are obtained from different subsets of the data in central collisions at all $\sqrt{s_{NN}}$ except, possibly, the two smallest values. In peripheral collisions fits to different subsets of data give significantly different results, implying that thermalization is not seen.

data yields $\Delta T$ and $\Delta \mu_B$ values normalized to the respective uncertainties that varies in the range $-12$ to $18$, indicating no agreement between HRG thermal parameter values from the UrQMD data. The fact that results from HRG model are in excellent agreement with experimental data on various orders of fluctuation measure and with similar values of extracted thermal parameters $T$ and $\mu_B$ within the uncertainties, demonstrates that the
QCD matter produced in central Au on Au collisions at RHIC has attained thermalization at least for $\sqrt{s_{NN}} = 19.6$ GeV and above.

Figure 4: $T$ and $\mu_B$ values at freeze-out for Au on Au collision (0–5% centrality) at $\sqrt{s_{NN}} = 7.7–200$ GeV. The circles represent the best fit $T$ and $\mu_B$ values (black lines the uncertainties) obtained from the comparison of experimental yields of pion, kaon and proton, $\sigma^2/M$ of net-kaon, $\sigma_Q/K^2(NK)$, $\sigma_K/K^2(NK)$, $\sigma_Q/p^2(NP)$ and $S\sigma$ of net-proton, net-kaon and net-charge to corresponding results from HRG model with all four parameters free. Square markers represent $T$ and $\mu_B$ values (blue band the uncertainties) obtained from the simultaneous comparison of 2nd-order and 3rd-order moments to corresponding results from the HRG model. The stars are the results corresponding to the mean values of the yields of pion, kaon and protons. Coloured bands show uncertainties on freeze-out $T$ and $\mu_B$ values.

Our final results are shown in Figure 4. This shows the results of the extraction of the freezeout parameters simultaneously using the 13 observables discussed earlier at each
\(\sqrt{s_{NN}}\) GeV. Also shown are the freeze-out \(T\) and \(\mu_B\) extracted from data and the HRG model using the combination of \(M_\pi\), \(M_K\) and \(M_p\), and combined higher order moments of \(\sigma^2/M\) of net-kaon, \(\sigma_{QK}^2/\sigma^2(NK)\), \(\sigma_{pK}^2/\sigma^2(NK)\), \(\sigma_{Qp}^2/\sigma^2(NP)\), \(S\sigma(NP,NK,NQ)\) observable for Au on Au collisions at \(\sqrt{s_{NN}} = 7.7\text{–}200\) GeV. The uncertainties are systematic associated with the implementation of various assumptions in the model and statistical, both added in quadrature. The model assumptions varied includes, radial flow effect, interactions through excluded volume effect, fixing some of the model parameters like \(\mu_Q\) and \(\mu_S\) from other measurements, resonance decay and considering width of resonances. The extracted \(T\) and \(\mu_B\) values are in agreement among all the three observables at each \(\sqrt{s_{NN}}\) within the uncertainties upto \(\sqrt{s_{NN}} = 19.6\) GeV, below that there are clear indications of deviations. The agreement between freezeout parameters extracted from different pieces of data shows that at freezeout the system is thermalized at large \(\sqrt{s_{NN}}\).

**Conclusions:**

We have reported a study of thermalization at the last stages of the evolution of the fireball created in heavy-ion collisions. The analysis of yields of hadrons (i.e., the means of the hadron number distributions) is known to be explained in thermal models for central and peripheral collisions of nuclei, in pp and \(e^+e^-\) collisions, and for synthetic data produced through event generators like UrQMD. This raises doubts about whether the analysis of yields is sufficient to tell us about thermalization of the fireball.

We found that a sensitive test of thermalization is the simultaneous description of yields as well as the event-to-event fluctuation measures \(\sigma^2/M\) (for net-kaons), \(\sigma^2/M\), \(\sigma_{QK}^2/\sigma^2(NK)\), \(\sigma_{pK}^2/\sigma^2(NK)\), \(\sigma_{Qp}^2/\sigma^2(NP)\) and \(S\sigma\) (for net-charge, net-protons and net-kaons). The variance measure \(\sigma^2/M\) for net-charge and net-protons has been found to be sensitive to acceptance, due to resonance decays. As a result, we cannot use these measures along with the rest. Instead we use the observables, \(\sigma_{QK}^2/\sigma^2(NK)\), \(\sigma_{pK}^2/\sigma^2(NK)\),
\[ \sigma_{\Delta p}^2 / \sigma^2(NP) \]. Agreement of all these observables for central Au+Au collisions with thermal gas predictions under common freezeout conditions is a strong indication that not only the region near the peak of the distribution, but also some part of the tail begins to approach a thermal distribution. This indicates that for the central Au+Au collisions at large \( \sqrt{s_{NN}} \) the medium densities are high enough, resulting in sufficient interactions among the constituents, to lead to thermalization. The relaxation time scales for the system are comparable to or smaller than the life time of the fireball.

Our studies shows a clear indication that an expanding femto-scale QCD system formed in central collisions of high energy heavy-ions [20] attains thermalization. Furthermore, this test shows that neither synthetic data, nor peripheral collisions can be considered as thermal or behaves like a bulk matter in thermodynamics. Interestingly the test also indicates that central collisions at the two lowest \( \sqrt{s_{NN}} \) may not be thermal. This lack of thermalization can be due to the fireballs being too dilute to ever come to equilibrium, or for there to be long relaxation time at these energies. Further studies are needed to establish which is the case, and thereby explore the part of the phase boundary which may be probed by the low-energy experiments.

For a thermal distribution, all orders of the cumulants should correspond to the same set of thermal parameters. We have not included fourth and higher order cumulants in this study. The higher order cumulants are, in principle, more sensitive to correlation lengths, and therefore are more sensitive probes of physics related to critical point and the nature of phase transition.

**Data availability.** The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Code availability.** The hadron resonance model code is available from the corresponding author upon reasonable request.
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