Frozen photons in Jaynes–Cummings arrays

Nikolaos Schetakis\textsuperscript{1,4}, Thomas Grujic\textsuperscript{2}, Stephen Clark\textsuperscript{2,3,4}, Dieter Jaksch\textsuperscript{2,3} and Dimitris Angelakis\textsuperscript{1,3,4}

\textsuperscript{1} Science Department, Technical University of Crete, Chania, Crete, 73100, Greece
\textsuperscript{2} Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, UK
\textsuperscript{3} Centre for Quantum Technologies, National University of Singapore, 2 Science Drive 3, Singapore 117542
\textsuperscript{4} Authors to whom any correspondence should be addressed.

Received 28 May 2013, in final form 10 September 2013
Published 7 November 2013
Online at stacks.iop.org/JPhysB/46/224025

Abstract

We study the origin of ‘frozen’ photonic states in coupled Jaynes–Cummings–Hubbard arrays. For the case of half the array initially populated with photons while the other half is left empty, we show the emergence of a self-localized photon or ‘frozen’ states for specific values of the local atom–photon coupling. We analyse the dynamics in the quantum regime and discover important additional features that do not appear to be captured by a semi-classical treatment, which we analyse for different array sizes and filling fractions. We trace the origin of this interaction-induced photon ‘freezing’ to the suppression of the excitation of propagating modes in the system at large interaction strengths. We discuss in detail the possibility of experimentally probing the relevant transition by analysing the emitted photon correlations both in the idealized lossless case and more realistic scenarios when reasonable losses are included. We find a strong signature of the effect in the emitted photons statistics.

1. Introduction

Since the first proposals to realize strongly correlated many-body states of light in resonator array architectures \cite{1–3} a rapid growth of interest in these structures has been seen along several lines of investigation. Perhaps most recently, significant attention has been paid to finding novel observable non-equilibrium dynamical effects in modest-sized arrays, both in the steady state of driven-dissipative systems \cite{4–9} and in explorations of coherent array dynamics \cite{10–14}. Resonator arrays are in many ways ideal platforms for the exploration of non-equilibrium quantum phenomena, allowing relative ease of access to dynamical observables via localized measurements of photon fields.

In this work, we investigate the time evolution of nonlinear arrays, going beyond the one- or two-photon limit into a strongly correlated many-body regime. We find evidence for an interaction-induced ‘freezing’ of domain walls of photons in initially half-filled one-dimensional array systems. The resonator nonlinearity must be sufficiently large for localization effects to set in, beyond which the photon population remains trapped in half of the system. We show that a semi-classical (\text{SC}) treatment similar to that first presented in \cite{11} for the limiting case of two coupled resonators predicts a sharp transition between localized domain formation and delocalized dynamics in which photons tunnel between both halves of the system.

Going beyond the \text{SC} approach, fully quantum calculations confirm the frozen photon dynamics for strongly nonlinear arrays while also revealing features not present in the \text{SC} calculation.

In some ways, our findings can be seen as a photonic analogue of the non-equilibrium dynamics of XXZ chains where it has been recently shown that strong nearest-neighbour interactions can lead to the formation of polarized domains which strongly influence the transport properties of spin chains \cite{15}. These ferromagnetic domains have been shown to be stable. They are spectrally separated from mobile states of the system which are capable of breaking them.
2. The system

The system we consider is a one-dimensional linear array of \( M \) coupled optical resonators. Each resonator features a relevant mode of frequency \( \omega_r \), and is coherently coupled to its nearest neighbours, as shown schematically in figure 1. A single two-level system (TLS) with the transition frequency \( \omega_a \) is coherently coupled via a Jaynes–Cummings interaction to each resonator, with coupling strength \( g \). In this work, we consider only the on-resonance case \( \Delta = \omega_r - \omega_a = 0 \). The governing Hamiltonian is then the well-known Jaynes–Cummings–Hubbard (JCH) Hamiltonian:

\[
\hat{\mathcal{H}} = \sum_j \left[ \left( \omega_a \hat{a}_j^\dagger \hat{a}_j + \omega_a \hat{\sigma}_j^+ \hat{\sigma}_j^- + g (\hat{a}_j^\dagger \hat{\sigma}_j^- + \hat{a}_j \hat{\sigma}_j^+) \right) \right] - J \sum_{\langle j, j' \rangle} \hat{a}_j^\dagger \hat{a}_{j'}.
\]

Here, \( \hat{a}_j \) is the photon destruction operator for the resonator \( j \) and \( \hat{a}_j^\dagger \) are the raising/lowering operators for the TLS coupled to the resonator \( j \). The set of nearest-neighbour resonators is denoted by \( \langle j, j' \rangle \). The Hamiltonian \( \hat{\mathcal{H}} \) commutes with the total excitation number operator

\[
\hat{N} = \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{\sigma}_j^+ \hat{\sigma}_j^-).
\]

We remove the free evolution of the resonators coupled to the TLS by transforming to a frame rotating at \( \omega_r = \omega_a \), such that only the competing terms governing the atom–resonator interaction and resonator tunnelling remain. We note that for the form of the Jaynes–Cummings coupling to be valid in equation (1), we must operate in the regime \( g \ll \omega_r = \omega_a \). Throughout this work, we consider initial states with one half of the system contiguously populated with \( N_0 \) photons per resonator, while the other half remains empty. The TLS in each resonator is initialized in its ground state:

\[
|\Psi(0)\rangle = \prod_{j=1}^{M/2} |g, N_0\rangle_j \otimes \prod_{j=M/2+1}^M |g, 0\rangle_j,
\]

where \( |g, n\rangle \) denotes a photonic Fock state of \( n \) photons in resonator \( j \).

Generalizing [11] to the case of an extended system, of central interest in the following analysis will be the photon imbalance \( Z(t) \) between the left (L) and right (R) halves of the system, as defined by

\[
Z(t) = \frac{\sum_{j=1}^{M/2} \langle 0 | \hat{a}_j^\dagger \hat{a}_j \rangle(t) - \sum_{j=M/2+1}^M \langle 0 | \hat{a}_j^\dagger \hat{a}_j \rangle(t)}{\sum_{j=1}^M \langle 0 | \hat{a}_j^\dagger \hat{a}_j \rangle(t)}.
\]

In particular, we find that time-averaging \( Z(t) \) neatly encapsulates details of the photon dynamics. We denote such time averages in the following by \( \bar{Z} \). Throughout this work, \( \bar{Z} \) is defined as the average of \( Z(t) \) over the interval \( t \in [0, 20] \).

We have empirically found that this interval is sufficiently long to capture the nature of the long-time dynamics and short enough that the array is in a non-trivial state at the end of the interval once losses are included. Values of \( \bar{Z} \) close to zero imply the delocalization of photons across the two halves of the system, either oscillating back and forth in some manner or reaching an approximately even distribution. Meanwhile, \( \bar{Z} \approx 1 \) implies a photon population trapped in one side of the system for a substantial period of time.

3. Semi-classical treatment

We begin our analysis of the dynamics of the system equation (1) at the SC level, thereby making contact with the related previous work by the authors of [11]. We first use the Heisenberg equation of motion \( \frac{d}{dt} \hat{O} = i[\hat{\mathcal{H}}, \hat{O}] \) to generate evolution equations for the photonic and TLS operator expectation values. As the SC approximation entails factorizing the expectation values of the operator products into the products of expectation values (e.g. \( \langle \hat{a}_j^\dagger \hat{\sigma}_j^- \rangle = \langle \hat{a}_j^\dagger \rangle \langle \hat{\sigma}_j^- \rangle \)), we only need to generate equations for the three operators \( \hat{a}_j, \hat{\sigma}_j^+, \hat{\sigma}_j^- \).

Defining \( (\alpha_j, m_j, z_j) \equiv (\langle \hat{a}_j \rangle, \langle \hat{\sigma}_j^+ \rangle, \langle \hat{\sigma}_j^- \rangle) \), we obtain the set of coupled differential equations for their evolution:

\[
\frac{d}{dt} \alpha_j = -i \omega_a \alpha_j - igm_j + i/2 \left( (1 - \delta_{j,1}) \alpha_{j-1} + (1 - \delta_{j,M}) \alpha_{j+1} \right)
\]

\[
\frac{d}{dt} m_j = -2i \omega_a m_j + ig \alpha_j z_j,
\]

\[
\frac{d}{dt} z_j = -2i g (\alpha_j m_j - m_j \alpha_j),
\]

where the delta functions take into account the open boundary conditions.

An SC description is not capable of properly describing the quantum mechanical Fock states of equation (3). We therefore take an array pumped with coherent states with the same photon number \( \alpha_j \alpha_j^\dagger = N_0 \) as initial conditions of equations (5):

\[
\alpha_j = \begin{cases} \sqrt{N_0} & : j \leq M/2, \\ 0 & : j > M/2, \end{cases}
\]

\[
z_j = -1,
\]

\[
m_j = 0.
\]
We note that it has been shown that for the case of $M = 2$ resonators, a qualitative change in the population imbalance dynamics occurs sharply at a critical coupling $g_c \approx 2.8\sqrt{N_0J}$ [11]. Specifically, for $g < g_c$, photons move between the resonators with a characteristic tunnelling time. Around $g = g_c$, this period diverges, leading to a ‘self-trapped’ regime for $g > g_c$, in which the photon population remains localized in one resonator.

With these results in mind, we now look at the time-averaged photon imbalance $\bar{Z}$ for larger arrays as calculated by the time evolution of the set of equations (5), as shown in figure 2. We see that SC theory still predicts a sharp localized/delocalised transition at the critical coupling $g = g_c$, regardless of the system size $M$.

4. The fully quantum regime

Going now beyond SC theory, which can only be valid in the limit of a large number of excitations, we investigate whether an analogue of the localization predicted by the SC equations persists in the fully quantum regime of a few ($N_0 \leq 4$) excitations. Explicitly constructing a matrix representation of the Hamiltonian of equation (1) and time evolving by applying the unitary operator $U(t) = \exp(-i\hat{H}t)$ to the initial state, $|\Psi(t)\rangle$ becomes numerically challenging beyond even $M = 2$ resonators. The two-species nature of the JCH Hamiltonian, coupled with the necessity of retaining a sufficient number of photons per resonator in calculations so as to avoid the truncation error, leading to a large Hilbert space dimension. Some progress is possible by projecting the dynamics into fixed particle number subspaces; however, we turn instead to a compact matrix product state (MPS) representation of the wavefunction [16]. This representation is ideally suited for representing the state of one-dimensional systems with at most nearest-neighbour couplings as in our case. Efficient and accurate Hamiltonian evolution of the MPS is achieved via the time-evolving block decimation (TEBD) algorithm [17].
Figure 4. An exploration of the time-averaged photon imbalance $\bar{Z}$. (a) As a function of the system size $M$, holding the initial excitation $N_0$ fixed. (b) $\bar{Z}$ as a function of the initial excitation $N_0$, holding instead the system size fixed.

Figure 5. The projection of the initial state $|\Psi(0)\rangle$ into each of the eigenstates spanning the subspace consistent with the total number of excitations in the system, $N_0 M/2$, for the simplest case of a dimer of $M = 2$ resonators. (a) Overlaps for an initial excitation of just $N_0 = 1$ photon in the first resonator. (b) The corresponding projections for an initial state of $N_0 = 4$ photons. Both plots show the evolution of the different projections as the Jaynes–Cummings nonlinearity $g$ is ramped up. Eigenmodes marked ‘P’ (for ‘propagating’) have a nonzero photon correlation function $C$ across the two halves of the system. Modes marked ‘N’ have vanishing $C$ as $g$ increases. Note that the larger dimension of the subspace for $N_0 = 4$ results in more eigenstates in (b).

Figure 6. Charting the localization–delocalization transition for a minimal $M = 2$ resonator array through both the time-averaged photon imbalance $\bar{Z}$ (top row) and the time-averaged photon correlator $\bar{g}^{(2)}$ (bottom row) for increasing initial photon number $N_0$. $\bar{Z}$ and $\bar{g}^{(2)}$ are calculated for systems with no losses (left column) and for small finite loss rates $\kappa = 0.05J$.

5. Probing the frozen dynamics in an experiment

Finally, we present calculations showing experimentally relevant photonic observables which give signatures of the transition between the localized and delocalized dynamics. While the photon number imbalance $\bar{Z}$ between the two halves of the system may be measurable via quantum non-demolition measurements on frequency shifts of the TLS, we focus here on purely photonic observables that can be extracted from the emitted photons from the structure. In particular, we find that the measurement of the local second-order photon correlations $g_L^{(2)} = \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle / \langle \hat{a}^\dagger \hat{a} \rangle^2$ yields signatures of the transition. Figure 6 shows that the freezing of population in one-half of the system (where the photon imbalance is $\bar{Z} \approx 1$) is accompanied by a qualitative change in the on-site correlations from $g^{(2)} > 1$ to $g^{(2)} \approx 1$. Figures 6(a) and (c) show that both the photon imbalance and the time-averaged correlator $\bar{g}^{(2)}$ approach a limiting behaviour as the initial number of photons grows large, with a sharp transition in observables in the vicinity of the critical point predicted by SC theory $g = g_c$.

Experimentally, the photon statistics encoded in $g^{(2)}$ are mapped on to the statistics of photons leaking from resonators with finite line widths as characterized by a loss rate $\kappa$. To assess whether the correlator $g^{(2)}$ can serve as a probe of the transition in realistic settings with a finite resonator loss rate,
we include Markovian photon loss processes at rate $\gamma$ and TLS de-excitation at rate $\kappa$ via a quantum master equation formalism, time evolving the system density matrix $\rho$ from the initial state of equation (3) under the evolution:

$$
\dot{\rho}(t) = -i[H, \rho] + \sum_{i=L,R} (\gamma L[\sigma_i^+\rho - \rho\sigma_i^-] + \kappa L[\sigma_i^-\rho - \rho\sigma_i^+]).
$$

(7)

where the action of the dissipator $L$ is defined as $L = (2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O)/2$. Figures 6(b) and (d) show that the introduction of a finite loss rate acts to smear the transition, pushing localization to larger nonlinearities $g$. However, for sufficiently large initial photon pumping (around $N_0 \approx 7$), the statistics of the emitted photons can be used to infer a change from delocalized physics (characterized by $g^{(2)} > 1$) to the localized case ($g^{(2)} \approx 1$). The value of gamma we use leads to a maximum ratio $g/\gamma \approx 200$, within reach of near future experiments in circuit QED architectures [18].

6. Conclusions

We have demonstrated the existence of a novel strongly correlated regime of ‘frozen’ photons in optical resonator arrays with large Jaynes–Cummings-type nonlinearities. For a sufficiently large initial excitation of part of the resonator array, the photon dynamics are dramatically suppressed due to a very small overlap with propagating modes of the system. As little as two pumped photons per resonator are sufficient to observe signatures of ‘frozen’ domains of photons, allowing access to the truly quantum few-excitation regime.

References

[1] Angelakis D, Santos M and Bose S 2007 Photon-blockade-induced Mott transitions and XY spin models in coupled cavity arrays Phys. Rev. A 76 31805
[2] Hartmann J 2006 Strongly interacting polaritons in coupled arrays of cavities Nature Phys. 2 849–55
[3] GreenTree A, Tahan C, Cole J and Hollenberg L 2006 Quantum phase transitions of light Nature Phys. 2 856–61
[4] Tomadin A, Giovannetti V, Fazio R, Gerace D, Carusotto I, Türeci H and Imamoglu A 2010 Signatures of the superfluid-insulator phase transition in laser-driven dissipative nonlinear cavity arrays Phys. Rev. A 81 061801
[5] Bamba M, Imamoglu A, Carusotto I and Ciuti C 2011 Origin of strong photon antibunching in weakly nonlinear photonic molecules Phys. Rev. A 83 021802
[6] Leib M and Hartmann M 2010 Bose–Hubbard dynamics of polaritons in a chain of circuit quantum electrodynamics cavities New J. Phys. 12 093031
[7] Carusotto I, Gerace D, Tureci H, Liberato S De, Ciuti C and Imamoglu A 2009 Fermionized photons in an array of driven dissipative nonlinear cavities Phys. Rev. Lett. 103 33601
[8] Gerace D, Türeci H, Imamoğlu A, Giovannetti V and Fazio R 2009 The quantum-optical Josephson interferometer Nature Phys. 5 281–4
[9] Grujic T, Clark S R, Jaksh D and Angelakis D G 2013 Repulsively induced photon superbunching in driven resonator arrays Phys. Rev. A 87 053846
[10] Makin M, Cole J, Hill C, GreenTree A and Hollenberg L 2009 Time evolution of the one-dimensional Jaynes–Cummings–Hubbard Hamiltonian Phys. Rev. A 80 43842
[11] Schmidt S, Gerace D, Houck A, Blatter G and Türeci H 2010 Nonequilibrium delocalization–localization transition of photons in circuit quantum electrodynamics Phys. Rev. B 82 100507
[12] Longo P, Schmitteckert P and Busch K 2011 Few-photon transport in low-dimensional systems Phys. Rev. A 83 063828
[13] Wong M and Law C 2011 Two-polariton bound states in the Jaynes–Cummings–Hubbard model Phys. Rev. A 83 055802
[14] Hartmann M and Plenio M 2008 Migration of bosonic particles across a Mott insulator to a superfluid phase interface Phys. Rev. Lett. 100 70602
[15] Mendoza-Arenas J, Mott insulator to a superfluid phase interface Phys. Rev. Lett. 100 70602
[16] Perez-Garcia D, Verstraete F, Wolf M and Cirac J 2007 Matrix product state representations Quantum Inf. Comput. 7 401–30
[17] Vidal G 2003 Efficient classical simulation of slightly entangled quantum computations Phys. Rev. Lett. 91 147902
[18] Houck A, Türeci H and Koch J 2012 On-chip quantum simulation with superconducting circuits Nature Phys. 8 292–9