Near-IR Surface Brightness Fluctuations and Optical Colours of Magellanic Star Clusters

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Accepted ?. Received ?; in original form ?

ABSTRACT

This work continues our efforts to calibrate model surface brightness fluctuation luminosities for the study of unresolved stellar populations, through the comparison with data of Magellanic Cloud star clusters. We present here the relation between absolute $K_s$-band fluctuation magnitude and $(V-I)$ integrated colour, using data from the 2MASS and DENIS surveys, and from the literature. We compare the star cluster sample with the sample of early-type galaxies and spiral bulges studied by Liu et al. (2002). We find that intermediate-age to old star clusters lie along a linear correlation with the same slope, within the errors, of that defined by the galaxies in the $\bar{M}_{K_s}$ vs. $(V-I)$ diagram. While the calibration by Liu et al. was determined in the colour range $1.05 < (V-I)_{0} < 1.25$, ours holds in the interval $-5 < \bar{M}_{K_s} < -9$, $0.3 < (V-I) < 1.25$. This implies, according to Bruzual & Charlot (2003) and Mouhcine & Lancerot (2003) models, that the star clusters and the latest star formation bursts in the galaxies and bulges constitute an age sequence. At the same time, a slight offset between the galaxies and the star clusters [the latter are $\sim 0.7$ mag fainter than the former at a given value of $(V-I)$], caused by the difference in metallicity of roughly a factor of two, confirms that the $\bar{M}_{K_s}$ vs. $(V-I)$ plane may contribute to break the age-metallicity degeneracy in intermediate-age and old stellar populations. The confrontation between models and galaxy data also suggests that galaxies with $K_s$ fluctuation magnitudes that are brighter than predicted, given their $(V-I)$ colour, might be explained in part by longer lifetimes of TP-AGB stars. A preliminary comparison between the $H$ 2MASS data of the Magellanic star clusters and the sample of 47 early-type galaxies and spiral bulges observed by Jensen et al. (2003) through the $F160W$ HST filter leads to the same basic conclusions: galaxies and star clusters lie along correlations with the same slope, and there is a slight offset between the star cluster sample and the galaxies, caused by their different metallicities. Magellanic star clusters are single populations, while galaxies are composite stellar systems; moreover, the objects analysed live in different environments. Therefore, our findings mean that the relationship between fluctuation magnitudes in the near-IR, and $(V-I)$ might be a fairly robust tool for the study of stellar population ages and metallicities, could provide additional constraints on star formation histories, and aid in the calibration of near-IR SBFs for cosmological distance measurements.

Key words: astronomical data bases: miscellaneous — galaxies: star clusters — galaxies: stellar content — infrared: galaxies — infrared: stars — Magellanic Clouds — stars: AGB and post–AGB
1 INTRODUCTION

Whereas surface brightness fluctuation (SBF) measurements are currently being pursued as a powerful distance indicator (e.g., Tonry et al. 1996, 1997; Liu & Graham 2001; Mei et al. 2001; Jensen et al. 2001, 2003), their potential for the study of unresolved stellar populations, although recognized long since (Tonry & Schneider 1983; Tompkins et al. 1990; Liu, Charlot, & Graham 2000; Blakeslee, Vazdekis, & Alhaidari 2001; Jensen et al. 2003), has so far remained mostly unfulfilled. We have recently (González, Liu, & Bruzual 2004, 2005; Mouhcine et al. 2004) started an effort to calibrate near-infrared (near-IR hereafter) SBFs, mostly for their use as probes of the characteristics of stellar populations. The idea is to compare SBFs derived from single stellar population (SSP) models with observed fluctuation luminosities of Magellanic Cloud (MC) star clusters. Our contribution has been to build “superclusters” with the data, in order to reduce the stochastic effects produced by the inadequate representation, in single star clusters, of the luminosity functions of stars evolving through short evolutionary phases (Santos & Frogel 1997; Bruzual 2002; Cervino et al. 2002; Cervino & Valls-Gabaud 2003; Cantiello et al. 2003; González, Liu, & Bruzual 2004); i.e., the asymptotic giant branch (AGB) and upper red giant branch (RGB). Superclusters, first introduced in González, Liu, & Bruzual (2004, henceforth Paper I), are built by coadding MC clusters, in the compilation van den Bergh (1981), that have the same SWB class (Searle, Wilkinson, & Bagnuolo 1981). The SWB classification is based on two reddening-free parameters, derived from integrated $ugv$ photometry of 61 rich star clusters in the Magellanic Clouds. Later, Elson & Fall (1983) assigned SWB classes to 147 more clusters using $UBV$ photometry.

The SWB ranking constitutes a smooth, one-dimensional sequence that Searle et al. interpreted as one of increasing age and decreasing metallicity; this interpretation is supported by stellar photometry and spectroscopy in individual clusters [e.g., references in Searle, Wilkinson, & Bagnuolo (1981); Frogel et al. (1990)]. The sequence discovered by Searle and collaborators was arbitrarily segmented by them to define a classification with seven types of clusters; however, facts like the apparently smooth progression in age and metallicity along the sequence, and the similarity between the handful of type VII MC clusters and the old and metal-poor halo globular clusters of the Milky Way have translated into the customary adoption of the working hypothesis that clusters within each SWB type have approximately the same stellar population. In our case, as we have already explained in Paper I, binning the data into superclusters is the compromise we have found most convenient to adopt in order to try to circumvent the problem of small-number statistics posed by individual star clusters. We have devised eight superclusters, one for each of the seven different SWB classes, plus one “Pre-SWB-class” supercluster with the youngest objects in the sample.

In the present paper, we focus on the relationship between $M_{K_s}$ and the $(V - I)$ colour. The merits of going to infrared wavelengths have been discussed at length [e.g., Liu et al. (2000); Paper I; Mouhcine et al. (2005)]. Summarising, the light of intermediate-age and old populations is dominated by bright and cool stars for which the spectral energy distributions peak in the near-IR, at the same time that dust extinction is reduced. On the other hand, there is a scarcity in the near-IR of both accurate empirical calibrations and self-consistent models, a fact that we hope we are contributing to remedy with our work.

In the particular case of the correlation between $M_{K_s}$ and $(V - I)$, Liu et al. (2002) confirmed a linear dependence of $M_{K_s}$ with $(V - I)$ in a sample of 26 ellipticals, S0s, and spiral bulges in the Local Group, Fornax, Virgo, Eridanus, and Leo. The relation had already been hinted at by the data set of Jensen et al. (1998), which covered only half the range in colour [$\sim 1.16 < (V - I) < \sim 1.24$]; it was more clearly shown by Mei et al. (2001), mainly through the inclusion of data for NGC 4489 at $(V - I) \lesssim 1.05$. Liu et al. filled significantly the gap in colour with observations of Fornax objects. Probing the extent (in age, metallicity and, ultimately, environment) to which this relation remains valid has obvious implications for an accurate calibration of near-IR SBFs as distance indicators, and hence for a precise determination of $H_0$ through these measurements [see Liu et al. (2002) for a detailed discussion].

There are also implications of this correlation from the point of view of stellar population studies. Since in general observed properties of stellar populations are luminosity-weighted, and given that after $\sim 12$ Myr the youngest stars are also the most luminous in the optical and near-IR wavelengths, the tendency uncovered by Liu et al. (2002) could be tracing the most recent burst of star formation in each of these systems. This is the conclusion reached in the cited work, after comparing their results with predictions by Liu, Charlot, & Graham (2000) [based on the Vazdekis et al. (1996) models]. Although there are disagreements among such predictions (especially in the derived metallicities obtained, respectively, from the Liu, Charlot, & Graham models and from those of Blakeslee, Vazdekis, & Alhaidari on one hand, and from those of González-Lópezlira et al. on the other), all of them point toward a large spread in age for the sample, from less than 5 Gyrs to more than 12 Gyrs. Furthermore, in all three sets of models confronted by Liu et al. (2002), the age and metallicity sequences are not parallel in at least some regions of the $M_{K_s}$ vs. $(V - I)$ plane, opening the possibility of using it as a diagnostic for breaking the age-metallicity degeneracy.

This study of the relationship between $M_{K_s}$ and $(V - I)$ in the MC star clusters is complementary in two advantageous ways to the investigation by Liu et al. (2002). Firstly, since the superclusters are approximately single age, single metallicity stellar populations, the star formation bursts that they represent are not masked by an underlying population, as it occurs in galaxies; hence, they likely constitute a better set for comparison with simple stellar population models. Secondly, the MC clusters span an even larger ex-
tent in age, i.e., from a few $10^6$ yr to $\sim 10^{10}$ yr, than that of the galaxies and bulges in the Liu et al. sample. Consequently, they cover a range roughly three times larger in $M_K$, and four times more extended in $(V-I)$.

For this investigation, we have made use of $K_s$-band data retrieved from the Two Micron All Sky Survey (2MASS; Skrutskie et al. 1997); $I_{Gunn}$ (I$_g$ or I henceforth) data from the Deep Near-Infrared Southern Sky Survey (DENIS; Epchtein et al. 1997); and $V$ data from different sources in the literature. This paper is organized as follows. In §2, we give a brief summary of the data acquisition and characteristics, as well as of our own treatment of such data. In §3, we present the ingredients of the stellar population synthesis models, and a comparison between the two sets of models used here. In §4, we compare the theoretical predictions to the observations. Finally, in §5, the results of the present work are discussed and summarised.

2 OBSERVATIONAL DATA OF MAGELLANIC CLOUD STAR CLUSTERS

2.1 The 2MASS data

$K_s$ data for 191 MC clusters from the compilation of van den Bergh (1988) and analysed by Elson & Fall (1985, 1988) were retrieved from the 2MASS archive. The data processing has been presented at length in Paper I. Succinctly, the fluctuation luminosity is the ratio of the second moment of the luminosity function ($\Sigma n_i L_i^2$) to its first moment (the integrated luminosity, $\Sigma n_i L_i$), as expressed by the equation:

$$\bar{L} = \frac{\Sigma n_i L_i^2}{\Sigma n_i L_i}. \quad (1)$$

Bright stars are the main contributors to the numerator, whereas faint stars contribute significantly to the denominator. The second moment of the luminosity function was derived by measuring the flux of resolved, bright stars in the MC clusters, while the integrated luminosity was computed from the total light detected in the images, after removing the sky background emission. In order to reduce the stochastic errors produced by the small numbers of luminous and cool RGB and AGB stars in single star clusters, eight superclusters were assembled with the 2MASS data, one for each of the seven different SWB classes, plus one “pre-SWB-class” supercluster; this was accomplished by stacking individual clusters with the same SWB type.2 The mosaics were used to measure the integrated light of the superclusters. To derive the second moment of the luminosity function, star lists for each one of the superclusters were integrated with entries from the 2MASS Point Source Catalog (PSC). The photometry of the point sources was performed by the 2MASS collaboration on individual cluster frames, previously to and independently from this work (i.e., we did not measure fluxes from point sources on the stacked supercluster frames), following the standard procedure of profile-fitting plus curve-of-growth aperture correction.3 Since all

2 Individual cluster images were multiplicatively scaled to a common photometric zero-point (determined by the 2MASS team) and dereddened; SMC clusters were geometrically magnified to place them at the distance of the LMC.

3 http://www.ipac.caltech.edu/2mass/releases/allsky/doc/sec1_4b.html

near-IR zero-points were obtained in a uniform fashion, it is unlikely that they are an important source of systematic error.

As explained in Paper I, the quoted fluctuation errors include stochastic variations produced by small-number statistics. These were calculated following a statistical approach introduced by Buzzoni (1989) and Cerviño et al. (2002), based on the assumption that the variables involved have a Poissonian nature. In this framework, stochastic errors scale as $M_{tot}^{-1/2}$, where $M_{tot}$ is the total mass of the stellar population. We refer the reader to the appendix for a short discussion on the subject of stochastic variations.

Besides the problem of small-number statistics, assessing crowding is crucial for accurate SBF measurements of star clusters. Through the blending of sources, crowding can in principle make the numerator of equation II larger and hence the SBF magnitude brighter. Another source of systematic error is the sky level, which impacts the denominator of equation II. Both crowding and sky determination have been lengthily addressed in Paper I, and revisited in Mouhcine et al. (2005) to consider the corrected fluctuation measurements of superclusters type I and II (González, Liu, & Bruzual 2005). In regard to crowding, we did not follow there the usual procedure of analysing the effects of the addition of artificial stars on the measured fluctuations. Instead, we compared the fluctuations derived from annular regions of the superclusters (i.e., regions with diverse crowding properties), and checked whether the different regions would be deemed crowded by the criterion, developed by Ajhar & Tonry (1994), that the two brightest magnitudes of stars cover more than $2\%$ of the area. We also tested our hypothesis that the PSC quality flags would be very helpful to eliminate blended sources by obtaining, again in annuli, fluctuation values with and without stars with faulty photometry. While crowding affects preferentially the centers of clusters, errors in the sky subtraction will impact more the fainter, outer regions. Our analysis allowed us to determine that the circular regions within $1^\prime$ of the center of the superclusters provide the better balance of uncertainties owing to crowding and sky subtraction, at the same time that they are less vulnerable to stochastic effects than smaller annular regions. Consequently, for the SBF measurements we use only integrated light and point sources within $1^\prime$ (at the distance of the LMC) from the centers of the superclusters. Just point sources with good photometry, according to the PSC quality flags,4 were included.

We adopted distance moduli of $(m-M)_0 = 18.50 \pm 0.13$ to the LMC and $(m-M)_0 = 18.99 \pm 0.05$ for the SMC (Ferrarese et al. 2001).

2.2 The DENIS data

The observations for DENIS were carried out between 1995 and 2001, and in particular the MC data were taken between 1995 and 1998. Cioni et al. (2000) give a good summary of the DENIS instrument, and data acquisition and characteristics. The instrument was mounted at the Cassegrain focus of the 1-m ESO telescope at La Silla, Chile, and could obtain simultaneously images at $I_g$, $J$, and $K_s$ with 3 cam-
erars. These had, respectively, a Tektronix CCD with 1024×1024 pixels (each 1′′×1′′), and two NICMOS infrared detectors with 256×256 pixels (each 3.′×3.′), but the J and Ks exposures were dithered to a 1′′ pseudo-resolution. DENIS scanned the Southern sky in strips of 30′′ in declination and 12′ in right ascension. Each strip comprises 180 12′×12′ images, with an overlap of 2′ between every pair. The integration time at I is 9 s; at J and Ks, the integration time is 1 s, but each released image is made up of 9 individual microscanned exposures, for a total integration time also of 9 s.

The nominal 5σ limiting magnitude at I is 18 mag, and the typical size of a detected point source is smaller than 2′′ FWHM. However, for this particular project we are concerned with the photometric accuracy at I achieved for extended sources. From those clusters for which more than one calibrated exposure is available, we find the photometric error to be ∼0.1 mag.

Flattened and bias-subtracted I images that contain the Magellanic clusters in our sample were retrieved from the DENIS archive. The sample is the same one from which the $M_{K_s}$ measurements were obtained, with the exception of seven clusters, presented in Table 1; there are no calibrated DENIS data for six of them, while the other one (NGC 1777) has two bright foreground stars and we do not know whether the V measurement has been duly corrected. Instrumental zero-points were obtained from the DENIS archive as well. Next, the I-band flux was derived for each cluster, in a diaphragm with the same size as the one used for the corresponding V-band measurement. V magnitudes and diaphragms were taken from van den Bergh (1981), except for those clusters listed in Table 2. It is worth noticing that, since we do not know the coordinates of the centres of the V observations, the derived (V−I) colour could be slightly biased to the red. The sky emission for each cluster was determined from an annulus separated from the photometric diaphragm by a buffer area; the best sizes of both buffer and sky annuli were chosen after visually inspecting the images, with the purpose of excluding from the sky measurements bright foreground stars and, most importantly, residual cluster light.

The individual cluster V and I-band flux values were then corrected for extinction as in Paper I, Table 1, and averaged to obtain the V and I fluxes for the superclusters, and their (V−I) colours. As before, when clusters do not have individually measured reddening, we have assumed $E(B−V) = 0.075$ for the LMC and $E(B−V) = 0.037$ for the SMC (Schlegel et al. 1998). Once again, given that the extinction corrections of all the data (V, I, and Ks) were done consistently by us, this is probably not an important source of systematic error. Absolute zero points were taken for V from Bessell (1979), and for I from Fouqué et al. (2004), who determined it especially for DENIS. The average (V−I) colours of the superclusters [and, for completeness, also the $M_{K_s}$ values derived in Paper I and in González, Liu, & Bruzual (2003)] are presented in Table 3.

3 STELLAR POPULATION MODEL PREDICTIONS

Central to this paper is the comparison between, on the one hand, the observed optical photometry and $K_s$-band fluctuation magnitudes of MC superclusters, and, on the other, the properties of synthetic single age, single metallicity stellar populations as predicted by, respectively, Bruzual & Charlot (2003, BC03) and Mouchine & Lançon (2003, ML03) stellar population synthesis models. BC03 models range from 0.1 Myr to 17 Gyr in age, while those of ML03 go from 1.2 Myr to 16 Gyr. Both sets of models span initial stellar metallicities of $Z/Z_\odot = 1/50$ to $Z/Z_\odot = 2.5$. Here, we describe briefly the main ingredients of the stellar population synthesis models, referring to the quoted papers for more details. Note that other sets of theoretical predictions of galaxy magnitudes. We also remark that the dispersion of (V−I) colours among the individual constituents of each supercluster is typically ∼0.2−0.3 mag. Although some fraction of this spread could be attributed to the fact that superclusters are not really single stellar populations, it is nevertheless consistent with the scatter among different low-mass realizations of a true SSP. For example, using Monte Carlo simulations, Bruzual (2002) finds 3σ fluctuations in (V−K) of almost 2 mag for a $1\times10^4M_\odot$ cluster at a given age. Moreover, the distribution of (V−I) colours of individual clusters within each supercluster looks reasonably Gaussian for all except supercluster SWB VII. There are only 12 star clusters type VII, and while the mode of the distribution (3 clusters) lies around $(V−I) = 1$, there is a tail (2 clusters) with $(V−I) > 1.4$. This tail could be produced by real population differences or, again, by small-number statistics.

The quoted errors in (V−I) were derived as follows: the dispersion of I flux measurements was calculated for those clusters for which multiple images were available; for clusters with single images, the average dispersion was adopted; the error in I for the superclusters was found by adding the individual dispersions in quadrature, and then dividing by $(N−1)^{1/2}$, with N the number of individual clusters in each supercluster; finally, the error in (V−I) was computed by assigning to the V value the same uncertainty as the one for I, and assuming a correlation coefficient of 0.5 between V and I (see, for example, Cervino et al. (2003); Paper I). We note, however, that while we find an average dispersion at I of ∼0.1 mag for individual clusters, there might be systematics, like flattening defects, that would increase this error somewhat. Indeed, Paturel et al. (2003) give an average uncertainty of ∼0.2 mag at I for their measurements of galaxy magnitudes. We also remark that the dispersion of (V−I) colours among the individual constituents of each supercluster is typically ∼0.2 − 0.3 mag. Although some fraction of this spread could be attributed to the fact that superclusters are not really single stellar populations, it is nevertheless consistent with the scatter among different low-mass realizations of a true SSP. For example, using Monte Carlo simulations, Bruzual (2002) finds 3σ fluctuations in (V−K) of almost 2 mag for a $1\times10^4M_\odot$ cluster at a given age. Moreover, the distribution of (V−I) colours of individual clusters within each supercluster looks reasonably Gaussian for all except supercluster SWB VII. There are only 12 star clusters type VII, and while the mode of the distribution (3 clusters) lies around $(V−I) = 1$, there is a tail (2 clusters) with $(V−I) > 1.4$. This tail could be produced by real population differences or, again, by small-number statistics.

7 The compilation by van den Bergh (1981) does not quote errors for the V photometry, but marks as uncertain (U−B) or (B−V) values for which any two observations differ by more than 0.1 mag. Among van den Bergh’s sources, only van den Bergh & Hagen (1968) and Alcaino (1978) list errors for V. Hence, the clusters in our work with published errors go from 18% for SWB class VI to 75% for SWB type VII. For supercluster type VII, we have calculated the uncertainty in $(V−I)$ using the errors in these papers; it differs only by 0.01 mag from the one derived with the procedure described above.
3.1 Single Stellar Population Models

ML03 stellar population synthesis models were designed to reproduce the near-IR properties of both resolved and unresolved stellar populations, with an emphasis on intermediate age stellar populations. The library of evolutionary tracks used by ML03 is based on the models of Bressan et al. (1993) and Fagotto et al. (1994a,b,c). We will refer to these sets as the Padova tracks hereafter. The sets of tracks cover major stellar evolutionary phases: from the main sequence to the end of the early-AGB phase for low- and intermediate-mass stars, and to the central carbon ignition for massive stars. The Padova tracks do not extend to the end of the Thermally Pulsing Asymptotic Giant Branch phase (TP-AGB hereafter). The high luminosities and low effective temperatures of stars evolving through this phase make them among the main contributors to the integrated near-IR light of stellar systems within the age interval when these stars are alive (e.g., Frogel et al. 1990; Mouchine & Lançon 2002). The extension of these tracks to cover the TP-AGB phase is then needed. Until this happens, the evolution of low- and intermediate-mass stars through the TP-AGB evolutionary phase is followed using the so-called synthetic evolution modelling (e.g., Iben & Truran 1978; Renzini & Voli 1981; see also Marigo et al. 2003 for another attempt to include the TP-AGB in stellar population models). The complex interplay between different processes affecting stellar evolution through the TP-AGB phase are taken into account in the synthetic evolution models used by ML03. The properties of TP-AGB stars are allowed to evolve according to semi-analytical prescriptions. These prescriptions take into account the effect of metallicity on the instantaneous properties of TP-AGB stars. The evolution of TP-AGB stars is stopped at the end of the AGB phase, since their contribution to the optical/near-IR light is almost negligible once stopped at the end of the AGB phase, since their contribution to the optical/near-IR light is almost negligible once they have evolved beyond this phase. The models predict the effective temperature, bolometric luminosities, and lifetimes of TP-AGB stars. The models predict a qualitatively similar evolution over the age range comprised by them; i.e., the $K_s$-band surface brightness fluctuation magnitudes get fainter as the $(V-I)$ colour gets redder. For both sets of stellar population model predictions, the $(V-I)$ colour increases gradually to redder values as the stellar populations age. On the other hand, it is predicted by both BC03 and ML03 models that at a fixed age, single metallicity stellar populations show redder $(V-I)$ colours at higher metallicity. This is because, at fixed initial stellar mass, lowering metallicity causes stars to evolve at higher effective temperatures. In view of the monotonic and smooth evolution of the $(V-I)$ colour, and given its weaker sensitivity (compared to the near-IR wavelength range) to the presence of very cool and luminous stars, this colour index can be regarded as a primary age indicator at fixed metallicity.

For the stellar populations dominated by red supergiant stars, i.e., younger than a few $\times$ 10 Myr, the $K_s$-band SBF magnitude gets drastically fainter with age; on the other hand, the $(V-I)$ changes by a modest factor around $\sim$ 0.3 mag. This trend is produced by the combination of two facts:
(1) the masses, and hence the luminosities, of the red supergiant stars that drive the fluctuation signal change significantly, while (2) the combined mass of main sequence stars, that determine the integrated optical properties, stays almost constant.

When red supergiant stars disappear from a stellar population, AGB stars drive the evolution of the near-IR properties up to 1.5–2 Gyr. For populations dominated by short-lived massive AGB stars, i.e., younger than \( \sim 200 \) Myr, the predicted evolution in the \( M_{K_s} \) vs. \( (V-I) \) diagram is complex. When the first (massive) AGB stars emerge in the stellar population, a brightening of the \( K_s \)-band surface brightness fluctuation magnitude is predicted, at almost fixed \( (V-I) \) colour. This is because of the overluminosity produced by the envelope burning that affects AGB stars with large initial stellar masses, i.e., \( M_{\text{init}} \gtrsim 3.5–4 M_\odot \) (see e.g., Mouchine & Lançon 2002 for more details on the effects of envelope burning on intermediate-age stellar population properties). The observed counterparts of single age, single metallicity stellar populations within this age range are expected to cluster at the same location in the \( M_{K_s} \) vs. \( (V-I) \) diagram, i.e., in the region around \( (V-I) \sim 0.4 \) and \( M_{K_s} \sim -7.5 \). During this age interval, the models have no ability for stellar population age-dating, and/or metallicity estimate.

For stellar populations older than \( \sim 300 \) Myr, in which AGB stars are not affected by envelope burning, both sets of models predict a monotonic dimming of the \( K_s \)-band surface brightness fluctuation magnitudes as the \( (V-I) \) colour gets redder. This evolutionary pattern continues for older stellar populations, i.e., older than 2–3 Gyr, when RGB stars drive their near-IR properties. This is because of the evolution of late-type giant star content. As a stellar population ages, the mass of the stars fueling the evolution of near-IR properties, i.e., AGB stars for ages younger than \( \sim 1.5 \) Gyr, and red giant stars for older ages, decreases. Consequently, at a fixed metallicity, the average luminosity of these stars decreases with stellar population age. Conversely, at a fixed age, \( M_{K_s} \) magnitudes get brighter and the \( (V-I) \) colour gets redder as the stellar population metallicity increases. Thus, both the BC03 and ML03 sets of models predict that, as the metallicity increases, populations with the same age move to the upper right in the \( M_{K_s} \) vs. \( (V-I) \) diagram.

Despite the qualitative agreement between the BC03 and ML03 sets of theoretical predictions, differences between the two are apparent. At a given stellar metallicity, the evolutionary track in the \( M_{K_s} \) vs. \( (V-I) \) diagram predicted by ML03 models is systematically redder than the one predicted by BC03 models. For stellar populations with \( Z=0.0004, \) 0.008, and 0.02, and between ages 300 Myr and 1.5 Gyr, the models based on ML03 isochrones predict, at similar \( (V-I) \) colours, brighter \( K_s \)-band SBF magnitudes [see also Mouchine et al. (2003)]. Observationally, this means that for a given \( (V-I) \) colour, models based on BC03 isochrones will attribute a higher metallicity to a star cluster with a certain \( M_{K_s} \) magnitude. The \( K_s \)-band SBF magnitude in the models based on ML03 isochrones is more sensitive to the presence of AGB stars. This is due to the fact that the AGB lifetimes used in the ML03 stellar population synthesis models are larger, thus increasing the contribution of these stars to the \( K_s \)-band light budget.

4 MODELS VERSUS OBSERVATIONAL DATA

Figures 2 and 3 show the comparison of \( M_{K_s} \) versus \( (V-I) \) colour of MC superclusters with both sets of models. BC03 models are presented in the left panels, while ML03 ones are displayed on the right. Fig. 4 presents models with \( Z=0.0004, \) 0.004, and 0.02. Even though the superclusters all have \( Z \lesssim 0.01 \), for completeness Fig. 5 illustrates the evolution of models with solar metallicity and \( Z=0.05 \); the models with \( Z=0.008 \) are also shown in Fig. 4 for comparison purposes. The solid dots represent the MC supercluster data, and the rectangle marks the general locus of the Liu et al. (2002) galaxy sample. The observed \( (V-I) \) galaxy colours reported by Liu et al. were transformed to \( (V-I) \), assuming that the mean stellar metallicities of the sample galaxies cover the range \( 0.008 < Z < 0.05 \), by the following equation:

\[
(V-I) = (1.050 \pm 0.006)(V-I) - (0.005 \pm 0.002)
\]

Focusing first on the data and on Fig. 2 we notice that the youngest Pre-SWB and SWB 1 superclusters, respectively at \( (V-I)=0.34 \) and \( (V-I)=0.61 \), are observed with significantly redder optical colours than predicted by the models with \( Z=0.008 \), the ones closest to their metallicity of \( Z=0.01 \) (Cohen 1982). Given their young ages, though, this is not surprising; Charlot & Fall (2000) offer the prescription that populations younger than \( \sim 10^7 \) years suffer from about three times more reddening than later in their lifetimes. Concordantly, for example, Grebel & Chu (2000) have measured a total colour excess (including foreground extinction) \( E(B-V) = 0.28 \pm 0.05 \) for Hodge 301, a relatively old cluster in 30 Dor for which these authors also derive an age of 20–25 Myr. This is the mean age of the star clusters that compose the supercluster type I (Paper I), if one adopts the age calibration by Elmegreen & Fall (1987). Interestingly, if one dereddens the SWB I supercluster by an extra 0.20 mag in \( E(B-V) \), or the difference between the measurement by Grebel & Chu (2000) and the average 0.08 mag that was adopted for the LMC (Schlegel et al. 1998), the data point falls exactly on the models with \( Z=0.008 \). We follow a similar procedure for the Pre-SWB supercluster. Both Parker (1993) and Dickens et al. (1994) have measured an average reddening of \( E(B-V) \approx 0.43 \) towards the whole of the 30 Dor region, while Selman et al. (1999) have determined a radially dependent reddening in the direction of R136 that reaches \( E(B-V) \gtrsim 0.5 \) at the center of the cluster and declines to \( E(B-V) \sim 0.3 \) at \( r=1' \). Dereddening the youngest, Pre-SWB supercluster by \( E(B-V) = 0.35 \) above the average of the LMC places it right where the BC03 models predict a population younger than 5 Myr should be in Figures 2 and 3. It is possible that the fluctuation magnitudes derived for the Pre-SWB and type I superclusters suffer from a certain degree of crowding (Mouchine et al. 2003). If this is the case, the correct values would be fainter than the reported ones, but would still lie along the same model sequence, at ages, respectively, slightly younger and slightly older. For the rest of the MC supercluster data points we attempt no further correction.

8 Exactly the same relation between age and reddening was found observationally by van den Bergh & Hagen (1968) for the MC clusters.
Figure 1. Comparison of model $M_{K_s}$ versus $(V - I_g)$ colour for $Z = 0.0004$ (top left), $Z = 0.004$ (top middle), $Z = 0.008$ (top right), $Z = 0.02$ (bottom left), and $Z = 0.05$ (bottom middle). Solid lines: Bruzual & Charlot (2003, BC03); dotted lines: Mouhcine & Lançon (2003, ML03). For $Z = 0.0004$ and $Z = 0.004$, evolution of models is shown only between $\sim 300$ Myr and $\sim 16$ Gyr. In the remaining panels, BC03 and ML03 models start, respectively, at 2.4 Myr and 12 Myr. From left to right, tickmarks indicate 5 and 10 Myr (only for BC03, $Z \gtrsim 0.008$ models), 100 and 200 Myr (for BC03 and ML03 models with $Z \gtrsim 0.008$), 0.4, 1, 4, and 12 Gyr (all models).

Table 1. Clusters in González et al. (2004) not included in this paper

| Supercluster | Name | Reason |
|--------------|------|--------|
| I . . . . . . | L 51 | No available calibrated DENIS data |
| SL 477 | No available calibrated DENIS data |
| NGC 1951 | No available calibrated DENIS data |
| NGC 1986 | No available calibrated DENIS data |
| III . . . . . | NGC 1953 | No available calibrated DENIS data |
| V . . . . . . | NGC 1777 | Two bright foreground stars |
| NGC 2193 | No available calibrated DENIS data |
The evolution predicted by the models of the $K_s$-band SBF magnitudes as a function of the $(V - I)$ colour agrees remarkably well with the observed sequence defined by the MC superclusters. Superclusters of SWB types II, III, and IV have ages, respectively, of 35, 105, and 320 Myr, using the SWB class-age transformation of Elson & Fall (1985). Their observed loci concur nicely with the predicted location of stellar populations with ages between a few $\times \sim 10$ Myr and $\sim 300$ Myr, in the region at $(V - I) \sim 0.4$ mag and $M_{K_s} \sim -7.5$ mag. SWB types V, VI, and VII correspond to ages of $\sim 1$ Gyr, 3 Gyr, and 9 Gyr, respectively. For older stellar populations, the predicted monotonic fading of the $K_s$-band SBF magnitudes as the $(V - I)$ colour gets redder agrees with the observed properties of intermediate-age and old MC superclusters.

However, one cannot fail to notice that the MC superclusters older than $\sim 3$ Gyr (i.e., with SWB types VI and VII) are not very well matched by the models. Given their respective ages and metallicities, the type VI superclusters mostly seems brighter in $M_{K_s}$, while type VII mainly appears redder in $(V - I)$ than the model predictions. We remind the reader here that the measured $(V - I)$ colours could be slightly biased to the red as a consequence of our ignorance of the exact telescope pointings used for the acquisition of the $V$ data (see §2.2). We have discussed in the same section that the colour distribution of the individual SWB VII clusters shows evidence of slight population differences, although we could also be seeing the effects of small-number statistics (12 clusters).

As advanced in §1 it is readily apparent in Figures 2 and 3 that the MC superclusters have greatly increased the range in $M_{K_s}$ versus $(V - I)$ that is now accessible for exploration. We have plotted in both panels of Fig. 3 as a solid straight line, the linear fit found by Liu et al. (2002) to their early-type galaxy and spiral bulge data set. After transforming $(V - I_0)$ to $(V - I)$ with eq. 2 this relation becomes

$$M_{K_s} = (-5.84 \pm 0.04) + (3.4 \pm 0.8)((V - I)_0 - 1.20).$$

It is remarkable that, with the exception of the extremely young supercluster Pre-SWB, and the old, metal-

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Table 2. $V$ photometry sources different from \textit{van den Bergh} (1981)

| Supercluster | Name      | Source                  |
|--------------|-----------|-------------------------|
| Pre-SWB . .  | NGC 1727  | Bica et al. (1996)      |
|              | NGC 1936  | Bica et al. (1996)      |
|              | NGC 2014  | Bica et al. (1996)      |
|              | NGC 2074  | Bica et al. (1996)      |
|              | NGC 2091  | Bica et al. (1996)      |
| I . . . . . . | NGC 299   | Alcaino (1978)          |
| IV . . . . . .| SL 663     | Mackey & Gilmore (2003)² |
| V . . . . . . | SL 363     | Bernard & Bigay (1974); Bernard (1975); Bica et al. (1996) |
|              | SL 556     | Bica et al. (1996)      |
|              | SL 855     | Mackey & Gilmore (2003)² |
| VI . . . . . .| SL 842     | Bica et al. (1996)      |
| VII . . . .  | ESO 121-SC03 | Mateo, Hodge, & Schommer (1986) |
|              | NGC 1786   | \textit{van den Bergh} (1981) and Bica et al. (1996) b |

² \textit{F555W} magnitudes were first obtained by integrating over the radial luminosity profiles published by Mackey & Gilmore (2003), then transformed to $V$ with the relations of Dolphin (2000) (without charge transfer terms), assuming ($B - V$) colours from Elson & Fall (1985) appropriate for the s types Elson & Fall (1988) of the clusters.

b \textit{van den Bergh} (1981) lists $V = 10.88$ and states that only observations corrected for the contribution of a foreground star have been considered, but neglects giving a diaphragm. Bica et al. (1996) register both a $V$ mag of 10.88 and a diaphragm of 60".

Table 3. $M_{K_s}$ and $(V - I)$ values

| Supercluster | $M_{K_s}$ | $(V - I)$ |
|--------------|-----------|-----------|
| pre . . . . . | -7.70±0.40 | 0.34 ± 0.14 |
| I . . . . . . | -8.85±0.12 | 0.61 ± 0.08 |
| II . . . . .  | -7.84±0.28 | 0.48 ± 0.10 |
| III . . . .  | -7.45±0.24 | 0.47 ± 0.06 |
| IV . . . . . | -7.51±0.18 | 0.54 ± 0.05 |
| V . . . . . . | -6.69±0.20 | 0.78 ± 0.11 |
| VI . . . . . | -6.21±0.24 | 1.02 ± 0.07 |
| VII . . . . .| -4.92±0.38 | 1.06 ± 0.13 |

\footnote{For these SWB types, 0.002 $\lesssim Z \lesssim 0.0008$ \textit{Frogel et al. 1991}}
Figure 2. Comparison of $M_{K_s}$ versus $(V - I_g)$ colour of Magellanic Cloud superclusters with low-$Z$ models by BC03, left panel, and by ML03 right panel. Dotted line: model with $Z = 0.0004$; dashed line: $Z = 0.004$; solid line: $Z = 0.008$. Tickmarks as in Fig. 1. The thick arrows deredden the Pre-SWB and SWB I superclusters by $E(B - V) = 0.2$ above the average of the LMC. The thin arrow dereddens the Pre-SWB supercluster by an additional $E(B - V) = 0.15$ (see text). The rectangle marks locus of the Liu et al. (2002) galaxy sample.

Figure 3. Comparison of $M_{K_s}$ versus $(V - I_g)$ colour of Magellanic Cloud superclusters with high-$Z$ models by BC03 (left panel), and by ML03 (right panel). Solid line: model with $Z = 0.008$; dotted line: $Z = 0.02$; dashed line: $Z = 0.05$. Evolution before $\sim 300$ Myr is now shown for all models. Symbols as in Figures 1 and 2, except that BC03 model with $Z = 0.02$ is graphed without tickmarks, since it runs so closely to the $Z = 0.008$ model. The straight solid line is best fit to Liu et al. (2002) sample of early-type galaxies and spiral bulges. The straight dotted-dashed line is our fit to superclusters I through VI (see text).

Poor supercluster SWB class VII, the remaining MC superclusters lie roughly along the correlation found by Liu et al. (2002). In fact, a fit to superclusters I (corrected for extra extinction as explained in §4) through VI yields:

$$M_{K_s} = (-5.1 \pm 0.4) + (4.0 \pm 0.6)[(V - I_g)_0 - 1.20].$$

This is also shown in Fig. 3 as a straight dotted-dashed line. Within the errors, the relations for, respectively, MC superclusters and galaxies have the same slope,\(^\text{10}\) at the same

\(^{10}\) If in fact the fluctuation magnitude of supercluster type I is affected by crowding and its true value is a few tenths of a mag dimmer, the match between the two slopes would be even better.
time that the MC superclusters have a systematically lower $M_{K_s}$ magnitude, at a fixed $(V - I)$ colour, than what is predicted for early-type galaxies. The observed offset can be explained, according to both the BC03 and ML03 sets of models, as a metallicity effect. The metallicity of the MC superclusters is roughly (bar only SWB class VI) half that of the sample of early-type galaxies and spiral bulges. For a given $(V - I)$ colour, the metal-poor simple stellar populations are on average older than the metal-rich ones, having then fainter $M_{K_s}$ as a consequence of both lower metallicities and older ages.

We refer the reader to Paper I for a detailed discussion of the reasons for the monotonic decline of the $K_s$-band SBF brightness with age displayed by intermediate-age and old populations. When compared to both sets of models, the result obtained here for the MC superclusters backs the interpretation offered in Liu et al. (2002), that their galaxy sample constitutes a sequence. Namely, that objects with brighter SBFs have a more recent latest star formation burst (or a more extended star formation history), and that the most recent bursts in every object have all occurred at roughly constant metallicity. This is so because their sample lies on a straight line, parallel to the age sequence of their single metallicity models. Simultaneously, the fact that there exists a discernable offset between the galaxies and the MC superclusters confirms that the $M_{K_s}$ vs. $(V - I)$ plane may contribute to the decoupling of age and metallicity effects in intermediate-age and old stellar systems. Particularly worthy of notice is the observation that the two samples offer consistent results, considering that galaxies are composite stellar systems, while the superclusters are approximately single stellar populations. The most likely explanation is that, being mostly probes of the brightest stars in a population at a given wavelength, SBFs are relatively insensitive to an underlying older population in composite systems.

It is also interesting to remark here that, while the BC03 models attribute a higher metallicity to NGC 1419 and NGC 1389, the two Fornax objects left out of their fit (through sigma-clipping) by Liu et al. (2002), the ML03 model with solar metallicity (Fig. 4 right panel, dotted line) shows an excursion to brighter SBF magnitudes at exactly the right place to ascribe to them the same metallicity as to other galaxies that do lie on the linear fit. This is a consequence of the longer lifetimes of TP-AGB stars in ML03 models. Hence, longer TP-AGB lifetimes could partly explain the observations of galaxies with $M_{K_s}$ brighter than predicted, given their $(V - I)$ colour.

Last but not least, we want to mention here that the largest sample to date of galaxies put together for SBF studies has been presented by Jensen et al. (2003). The 65 objects in this sample, however, were observed with the Near-IR Camera and Multi-Object Spectrometer (NICMOS) camera 2 (NIC2) on board the Hubble Space Telescope (HST), using the F160W filter. The photometric transformations between the NIC2 and the commonly used near-IR ground-based filters is particularly difficult, for several reasons: the very deep molecular absorption bands in the star themselves; the significant differences between the HST and the ground-based filters; and the fact that telluric absorption features are very deep in ground-based observations but completely absent in data obtained from space (Stephens et al. 2003). Dealing with fluctuation magnitudes has an added degree of complexity, and conflicting statements can be found in the literature with regard to what colour to use to determine the correct transformation coefficients [i.e., the fluctuation colour of the population (Buzzoni 1993; Blakeslee, Vazdekis, & Ajhar 2001), or the mean of the integrated and the fluctuation colours (Tonry et al. 1997)]. With all these caveats, we have transformed the relation between $M_{F160W}$ and $(V - I)$, derived by Jensen et al. (2003) from the 47 galaxies in their sample that show no signs of dust in the NIC2 field of view, into one between $M_H$ and $(V - I)$. To this end, we have relied on eq. 2 and on the transformation between $M_H$ and $M_{F160W}$ obtained in eq. 2 of Paper I.\footnote{11} For $(J - K_s)$, we have substituted $0.92 \pm 0.06$, which is the average integrated colour of the mentioned 47 galaxies, as obtained from the 2MASS Extended Source Catalog (XSC), via the GATOR catalog query page. This is the result:

\begin{equation}
M_H = (-5.12 \pm 0.09) + (4.9 \pm 0.5) [ (V - I)_0 - 1.21 ].
\end{equation}

For the MC clusters, using the values for $M_H$ derived in Paper I and in González, Liu, & Bruzual (2003), with the fluctuation magnitude for the supercluster type I once again corrected for extra extinction as in $\parallel$, the relation reads:

\begin{equation}
M_H = (-4.0 \pm 0.5) + (4.8 \pm 0.7) [ (V - I)_0 - 1.21 ].
\end{equation}

Even if tentative, since the near-IR observations of galaxies and star clusters have not been performed through the same filter, the conclusion is the same as the one drawn from the $K_s$ fluctuations: the slope for early-type galaxies and MC superclusters is the same within the errors; the MC clusters have fainter fluctuation magnitudes at a given $(V - I)$ colour, as a result of their lower metallicity. It would certainly be worthwhile to obtain the data needed to perform a fairer comparison of galaxies and MC star clusters in the $H$ observing window in the future. Table 4 lists the coefficients of our fits to the MC supercluster data; the reduced chi-square ($\chi^2$) and the rms of the points (in magnitudes) after the fits are also included.

\section{Summary & Conclusions}

In this paper we have presented the relation between absolute $K_s$-band SBF magnitude and $(V - I)$ integrated colour for more than 180 MC star clusters. The newly reported results extend to fluctuation magnitudes $M_{K_s} \sim -9$ and optical colour $(V - I) \sim -0.4$ the linear relation observed already for early-type galaxies and spiral galaxy bulges in a more limited range of both fluctuation magnitudes and colour $[-5 \leq M_{K_s} \leq -7$, $1.05 \leq (V - I) \leq 1.25]$. This empirical relation has been compared to the predicted evolution of single age, single metallicity stellar population properties in the $M_{K_s}$ vs. $(V - I)$ diagram, based on both Bruzual & Charlot (2003) and Mouhcine & Lakan (2003) isochrones. The predicted evolution and the observed sequence agree quite well over the ranges of $M_{K_s}$ magnitude and $(V - I)$ colour covered by the data $[-5 \leq M_{K_s} \leq -9$, -0.4
\( \lesssim (V-I) \lesssim 1.25 \). With the exception of the extremely young supercluster Pre-SWB, and possibly the old and metal poor supercluster class VII, the remaining superclusters lie on the linear correlation already found by Liu et al. (2002). Such correlation is observed in the ranges \(-5 \lesssim M_K \lesssim -9\), 0.3 \( \lesssim (V-I) \lesssim 1.25 \). The existence of a linear correlation implies that star clusters, early-type galaxies and spiral bulges represent an age sequence, where younger stellar populations display brighter \( K \)-band SBF magnitudes and bluer \((V-I)\) colour. At the same time, the discernable offset between galaxies and MC star clusters confirms that the \( M_K \) vs. \((V-I)\) plane may contribute to distinguish the effects of age and metallicity in intermediate-age and old stellar systems. One other suggestive result that emerges from the comparison between the galaxy sample and the models is that longer lifetimes of TP-AGB stars might partly explain galaxies with near-IR SBFs that are brighter than anticipated, given their \((V-I)\) colour. A preliminary comparison between the 2MASS data of the MC star clusters and the sample of 47 early-type galaxies and spiral bulges observed by Jensen et al. (2003) through the \( F160W \) filter leads to the same basic conclusions: galaxies and star clusters lie along correlations with the same slope, and there is a slight offset between the star cluster sample and the galaxies, caused by their different metallicities.

We have found that results from star clusters in the MC (i.e., single stellar populations in Local Group irregular and consequently relatively metal-poor galaxies) agree with those of spiral bulges and early-type galaxies (i.e., composite systems with higher metallicities than the MC), located not only in the Local Group, but also in mildly dense clusters of galaxies like Fornax and Virgo. The implication is that the relationship between \( M_K \) and \((V-I)\) might be a fairly robust tool, rather insensitive to environment, at least in the local universe, for the study of ages and metallicities of unresolved stellar populations; could provide additional constraints on star formation histories; and aid in the calibration of the \( K \)-band SBFs for cosmological distance measurements. In this regard –the determination of cosmic distances–, the sensitivity of near-IR SBFs to stellar ages and metallicities, and perhaps to the details of AGB evolution in intermediate-age populations, is a potential caveat to bear in mind, albeit not necessarily in the local universe or in view of today’s observing capabilities.

For example, Ferreras et al. (1999) have analysed early-type galaxies in Coma and 17 clusters at \( 0.3 \lesssim z \lesssim 0.9 \); their results imply that only galaxies smaller than \( 0.5 \) \( L_* \), at redshifts \( z \geq 0.5 \), can be expected to harbour populations with metallicities below solar.

### ACKNOWLEDGMENTS

We thank the whole DENIS Team, especially G. Simon and its PI, N. Epchtein, for making available de DENIS data. The DENIS project is supported, in France by the Institut National des Sciences de l’Univers, the Education Ministry and the Centre National de la Recherche Scientifique, in Germany by the State of Baden Württemberg, in Spain by the DGICYT, in Italy by the Consiglio Nazionale delle Ricerche, in Austria by the Fonds zur Förderung der wissenschaftlichen Forschung and the Bundesministerium für Wissenschaft und Forschung. R.A.G. and M.A. acknowledge L. Carigi and A. Bressan for their very useful comments on the manuscript. We thank the referee, Joseph B. Jensen, for his careful reading of the paper; we are grateful for his suggestions.

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### Table 4. Fluctuation absolute magnitude vs. colour

| \( M \) | \( a \) | \( b \) | \( N \) | rms | \( \chi^2 \) |
|---|---|---|---|---|---|
| \( M_K \) | -5.1 ± 0.4 | 4.0 ± 0.6 | 6 | 0.85 | 1.02 |
| \( M_H \) | -4.0 ± 0.5 | 4.8 ± 0.7 | 6 | 1.04 | 1.28 |

Fits of the form \( \tilde{M} = a + b \left( (V - I)_0 \right) \) – reference colour]; the number of objects used for the fit is tabulated as \( N \). The resulting rms of the points (in magnitudes) after the fit and the reduced chi-square \((\chi^2)\) are also listed.
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It is beyond the scope of the present paper to investigate in depth the role of stochastic effects in the integrated properties of stellar populations. However, for illustration purposes, we show in Fig. 4 the colour-magnitude diagrams (CMDs) of superclusters type IV (left panel) and VI (right panel). Fig. 5 is a copy of the right panel of Fig. 4, this time indicating the locations of individual clusters types IV (open triangles) and VI (open circles) in the $M_K$ vs. $(V-I)$ plane. There are several facts worth noticing. Firstly, the colour magnitude diagrams, narrow at the top and wide at the bottom, resemble more those of SSPs with “normal” photometric errors than CMDs of composite populations (Bressan 2005, private communication). This notwithstanding, and even when the relative offset between the general loci of the two SWB classes is clearly discernible, single clusters of each SWB type show a large scatter, especially in fluctuation magnitude diagrams, narrow at the top and wide at the bottom, and the very large scatter of individual clusters within each SWB class. If this is the case, the construction of superclusters is an appropriate strategy to simulate more massive SSPs.

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Figure 4. Colour–magnitude diagrams of MC superclusters types SWB IV (left panel) and SWB VI right panel. Stars within 60 arcsec from the center (at the distance of the LMC). Average photometric errors are 0.04 mag in brightness and 0.02 mag in color for sources with $K_s \leq 13$; 0.06 and 0.03 mag for stars with $13 < K_s \leq 14$; and 0.13 and 0.07 mag (about the size of the dots) for sources with $14 < K_s \leq 15$.

Figure 5. $M_{K_s}$ versus $(V - I_g)$ colour of individual clusters classes SWB IV (open triangles) and SWB VI (open circles); all other symbols as in right panel of Fig. 2.