Generalized Faddeev–Popov method for a deformed supersymmetric Yang–Mills theory

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ABSTRACT

In this paper, we will study the deformation of a three dimensional \( \mathcal{N} = 2 \) supersymmetry gauge theory. We will deform this theory by imposing non-anticommutativity. This will break the supersymmetry of the theory from \( \mathcal{N} = 2 \) supersymmetry to \( \mathcal{N} = 1 \) supersymmetry. We will address the problem that occurs in the Landau gauge due to the existence of multiple solutions to the gauge fixing condition. This problem will be addressed using a formalism that has been motivated by topological field theories. Finally, we will study the extended BRST symmetry that occurs in this theory.

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1. Introduction

The studies done on closed strings in the presence of a constant two form field, and the gravitational action induced by the bosonic string theory on a space-filling D-brane with a constant magnetic field, have motivated a noncommutative deformation of field theories [1–4]. The noncommutative field theory can be constructed by replacing all the products of fields in the action by Moyal products of those fields. This leads to the mixing of ultraviolet and infrared divergences [5]. These theories are also non-local, but the non-locality is introduced in a controlled manner [6]. The existence of such noncommutative deformations has motivated the study of a wider class of deformations for supersymmetric field theories. As the coordinate space of supersymmetric theories has Grassmann coordinates, it is possible to impose a noncommutative deformation between such coordinates and ordinary spacetime coordinates, along with introducing a non-anticommutative deformation between the Grassmann coordinates [7,8]. It is possible to deform the supersymmetric gauge theories using such deformations. [9,10]. It may be noted that the non-anticommutative deformation of field theories can be related to the existence of \( R - R \) backgrounds fields [11–14].

In this paper, we will analyze such a deformation for a three dimensional supersymmetric gauge theory. So, we will promote the Grassmann coordinates to non-anticommutating coordinates. This will break half the supersymmetry of the original theory. Thus, if we start from a four dimensional theory with \( \mathcal{N} = 1 \) supersymmetry, this deformation would break the supersymmetry down to \( \mathcal{N} = 1/2 \) supersymmetry [15,16]. This is because the four dimensional gauge theories have enough degrees of freedom to partially break the supersymmetry of the theory. However, if we tried to deform a three dimensional theory with \( \mathcal{N} = 1 \) supersymmetry, we would break all the supersymmetry of the theory. So, we will consider a theory with \( \mathcal{N} = 2 \) supersymmetry in three dimensions, and the non-anticommutativity will break half of this supersymmetry. Thus, the theory will have \( \mathcal{N} = 1 \) supersymmetry after this deformation. It may be noted that such deformation of supersymmetric field theories has been studied, and it has observed that the non-anticommutativity does break the supersymmetry from \( \mathcal{N} = 2 \) supersymmetry to \( \mathcal{N} = 1 \) supersymmetry [17,18].

As a gauge theory has a non-physical degrees of freedom, it cannot be quantized without fixing a gauge. The gauge fixing term can be incorporated at a quantum level by adding a ghost term and a gauge fixing term to the original classical action. This new action, which is obtained by the adding of the gauge fixing term and the ghost term to the original action is invariant under a symmetry called the BRST symmetry [19,20]. It is also invariant under another symmetry whose dual to the BRST symmetry is called the anti-BRST symmetry [21]. However, for non-perturbative gauge-fixing the Gribov ambiguity causes a problem [22,23]. This is because it is not possible to obtain a unique representative on gauge orbits once large scale field fluctuations in gauges like the Landau gauge take place. The Gribov ambiguity for supersymmetry theories has been recently studied [24,25].

It may be noted that the Gribov ambiguity is related to the existence of topological properties of field theories [26]. In fact, it
has also been demonstrated that gauge theories can be expressed in terms of topologically quantities [27–30]. This construction has motivated the study of two different kinds of topological field theories called the Schwarz type theories [27] and Witten type theories [29]. The Witten type theories can be related to the existence of Gribov ambiguity in field theory. This is because the Witten type theories are related to the cohomology, and can have a direct connection with the BRST symmetry in the gauge theory. In fact, it has been demonstrated that the Nicolai map can be constructed in Witten type theories [31]. The Nicolai map can be used to restrict the path integral to the moduli space of classical solutions [31]. So, the Nicolai map has also been used to address the Gribov ambiguity for usual gauge theories [32]. In this analysis, the BRST symmetry of the gauge theory was extended to include an extended BRST symmetry. We will apply this formalism to three dimensional non-anticommutative Yang–Mills theory.

The remaining of the paper is organized as follows. In Section 2, we will deform a three dimensional supersymmetric Yang–Mills theory in \( \mathcal{N} = 2 \) supersaturated formalism. This deformation will break the supersymmetry of the theory to \( \mathcal{N} = 1 \) supersymmetry. In Section 3, we will analyze the quantization of this theory. We will also address the problem that occurs in the Landau gauge due to the existence of multiple solutions to the gauge fixing condition. A formalism that has been motivated by the Nicolai map in topological field theories will be used to address this problem. In Section 4, we will analyze the extended BRST symmetry for this formalism. Finally, in Section 5, we will summarize our results and discuss some possible extensions of this work.

2. Non-anticommutativity

We will analyze a three dimensional gauge theory in \( \mathcal{N} = 2 \) superspace formalism. This space is parameterized by the coordinates, \((x^\mu, \theta_1, \theta_2)\), where \(\theta_1 = \theta_1^+, \theta_1^-\), and \(\theta_2 = \theta_2^+, \theta_2^-\). The generators of \( \mathcal{N} = 2 \) supersymmetry can be written as

\[
Q_{1a} = \bar{\theta}_{1a} - (y^\mu \theta_1)_a \partial_\mu,
Q_{2a} = \bar{\theta}_{2a} - (y^\mu \theta_2)_a \partial_\mu.
\]

The super-derivatives which commute with these generators of supersymmetry can be written as

\[
D_{1a} = \theta_1 + (y^\mu \theta_1)_a \partial_\mu,
D_{2a} = \theta_2 + (y^\mu \theta_2)_a \partial_\mu.
\]

It is possible to transform the \( \theta_1 \) and \( \theta_2 \) to another set of Grassmann coordinates

\[
\left( \begin{array}{c}
\theta_1 \\
\theta_2
\end{array} \right) = \left( \begin{array}{cc}
x_{11} & x_{12} \\
x_{21} & x_{22}
\end{array} \right) \left( \begin{array}{c}
\theta_1 \\
\theta_2
\end{array} \right),
\]

as long as the \( \det(x_{ij}) \neq 0 \). Now we choose \( x_{ij} \), such that \( \theta_1 = (\theta_{1a} + \theta_{2a})/2 \), \( \theta_2 = (\theta_{1a} - \theta_{2a})/2 \), and define another set of covariant derivatives as [33]

\[
D_a = \frac{1}{2}(D_{1a} + iD_{2a}), \quad \bar{D}_a = \frac{1}{2}(D_{1a} - iD_{2a}).
\]

These covariant derivatives satisfy

\[
[D_a, \bar{D}_b] = i(y^\mu \theta_1)_a \delta_b, \quad \{\bar{D}_a, \bar{D}_b\} = 0,
\]

\[
\{D_a, \bar{D}_b\} = 0.
\]

We also define \( D^2 = D^a D_a/2 \) and \( \bar{D}^2 = \bar{D}^a \bar{D}_a/2 \). This superspace is parameterized by the coordinates, \((x^\mu, \theta^a, \bar{\theta}^\alpha)\), where \(\mu = 0, 1, 2, 3\), and \(a, \alpha = 1, 2\). We will impose the non-anticommutative deformation between \( \theta^a \) as

\[
[\theta^a, \bar{\theta}^\alpha] = C^{a\beta}. \tag{6}
\]

The product of superfields of \( \theta^a \) can be Weyl ordered. This is done by the ordinary product of superfields by a star product, which is a fermionic version of the Moyal product

\[
V(x, \theta, \bar{\theta}) \ast V'(x, \theta, \bar{\theta}) = V(x, \theta, \bar{\theta}) \exp \left( \frac{-C^{a\beta} \partial_\theta^a \partial_{\bar{\theta}}^\beta}{2} \right) \ast V'(x, \theta, \bar{\theta}), \tag{7}
\]

where \( V(x, \theta, \bar{\theta}) \) and \( V'(x, \theta, \bar{\theta}) \) are supervector fields. It may be noted that the Grassmann coordinate \( \bar{\theta}_a \) satisfies

\[
[\bar{\theta}_a, \bar{\theta}_b] = 0, \quad \{\bar{\theta}_a, \theta_b\} = 0,
\]

\[
[\bar{\theta}_a, x^\mu] = 0, \quad \theta_a = 0, \quad \{y^\mu, y^\nu\} = 0.
\]

and the bosonic coordinates satisfy

\[
[x^\mu, \theta^a] = \bar{\theta}^a C^\mu, \quad \{x^\mu, \theta^a\} = i C^a\delta^{\mu}_\beta \bar{\theta}^\beta.
\]

where \( C^\mu = \delta^a_\beta \epsilon_{\beta\gamma}^\alpha \). It is possible to write

\[
[\bar{\theta}_a, y^\mu] = 0, \quad \{\bar{\theta}_a, y^\nu\} = 0,
\]

\[
[y^\mu, y^\nu] = 0,
\]

where \( y^\mu = x^\mu + i \delta^a_\beta y^\nu \bar{\theta}_\beta \). The superfields can be expressed as functions of \((y^\mu, \theta^a, \bar{\theta}^\alpha)\) [78]. The supervector field \( V(y, \theta, \bar{\theta}) \) in the Wess–Zumino gauge is given by

\[
V(y, \theta, \bar{\theta}) = -\theta^a \bar{\theta}^\alpha A_\mu + i \theta^a \bar{\theta} \lambda^\mu,
\]

\[
\lambda^\mu = \frac{1}{4} \epsilon_{\beta\gamma}^\alpha C^\beta \epsilon_{\beta\gamma}^\alpha \bar{\theta}^\beta A_\mu,
\]

\[
\lambda^\mu = \frac{1}{2} \epsilon_{\beta\gamma}^\alpha C^\beta \epsilon_{\beta\gamma}^\alpha \bar{\theta}^\beta A_\mu,
\]

\[
\lambda^\mu = \frac{1}{2} \epsilon_{\beta\gamma}^\alpha C^\beta \epsilon_{\beta\gamma}^\alpha \bar{\theta}^\beta A_\mu,
\]

where \( V^a(y, \theta, \bar{\theta})T_a = V(y, \theta, \bar{\theta}) \). Here \( T_a \) are generators of SU(N) Lie algebra.

\[
[T_a, T_b] = i f_{abc} T_c.
\]

We can define the Chiral and anti-Chiral field strength for this theory as

\[
W_\alpha = -\frac{1}{4} \bar{D}De^\alpha \ast D_a e^\alpha,
\]

\[
\bar{W}_\alpha = \frac{1}{4} \bar{D}De^\alpha \ast \bar{D}_a e^\alpha.
\]

The action for \( \mathcal{N} = 1 \) gauge theory can be written as

\[
S_{\text{SYM}} = Tr \int d^3 x d^2 \theta W^\alpha \ast W_\alpha + Tr \int d^3 x d^2 \bar{\theta} \bar{W}^\alpha \ast \bar{W}_\alpha.
\]

It is possible to expand this in component form as

\[
S_{\text{SYM}} = Tr \int d^3 x \left[ -4 i \bar{\theta}^a \sigma^\mu \partial_\mu \lambda - F^\mu \nu F_{\mu \nu} + 2D^2 \right] + \text{Tr} \int d^3 x \left[ -2i C^\mu \nu F_{\mu \nu} \bar{\lambda} \lambda + \frac{C^\mu \nu \mu \nu (\bar{\lambda} \lambda)^2}{2} \right].
\]
3. Quantization

In the previous section, we analyzed the non-anticommutative deformation of a gauge theory in $\mathcal{N} = 2$ superspace formalism. This deformation broke the supersymmetry of the gauge theory from $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$ supersymmetry. In this section, we will analyze the quantization of this $\mathcal{N} = 1$ supersymmetry. We will perform this analysis using a covariant formalism. We can also express this deformed three dimensional theory using the covariant derivative [34].

$$V_A = -i[D_{\alpha}, \tilde{D}_{\beta}]_*, D_{\alpha}, \tilde{D}_{\alpha},$$

$$\exp(V) \star \nabla_A \star \exp(-V) = \exp(-V) \star \tilde{D}_{\alpha} \star \exp(V),$$

(16)

where $D_{\alpha} = \exp(-V) \star D_{\alpha} \star \exp(V)$, and $\tilde{D}_{\alpha} = \exp(V) \star \tilde{D}_{\alpha} \star \exp(-V)$. We can express it using a covariant formalism as [34]

$$V_A = D_A - i\Gamma_A,$$

(17)

where

$$D_A = (\partial_{\alpha\beta}, D_{\alpha}, \tilde{D}_{\beta}),$$

$$\Gamma_A = (\Gamma_{\alpha\beta}, \Gamma_\alpha, \tilde{\Gamma}_\alpha).$$

(18)

It is possible to express the Bianchi identity as $\{V_A, H_{BC}\}_* = 0$, where $H_{AB} = \{V_A, V_B\}_* = \Gamma_{AB}^\gamma V_{\gamma} - \Gamma_{AB}V_{\gamma} - \Gamma_{\gamma AB}.$ The gauge transformation of this covariant derivative is given by $V_A \rightarrow \epsilon^{\alpha A} \star V_A \star e^{-i\lambda A}$, and $e^{\gamma A} \star \nabla_A \star e^{-i\gamma A} \rightarrow e^{\gamma A} \star \nabla_A \star e^{-i\gamma A}$. However, it is possible to construct a covariant derivative in a different representation, and the gauge transformations of this covariant derivative is given by

$$V_A \rightarrow u \star V_A \star u^{-1},$$

(19)

where $u$ is defined as $u = e^{iK}$, and the parameter $K = K^A T_A$ is a real superfield [34]. The transformation of the spinor fields can be written as

$$\Gamma_\alpha \rightarrow iu \star \nabla_\alpha \star u^{-1},$$

$$\Gamma_{\alpha\beta} \rightarrow iu \star \nabla_{\alpha\beta} \star u^{-1},$$

$$\Gamma_{\alpha\beta} \rightarrow iu \star \nabla_{\alpha\beta} \star u^{-1}.$$

(20)

As this theory has gauge symmetry, we will have to introduce a ghost term and a gauge fixing term to the original action. However, to calculate non-perturbative effects, we will have to deal with the Gribov ambiguity. So, we will apply the extended BRST symmetry to this theory [32]. Thus, for a gauge fixing condition $F[\Gamma] = 0$, and $F[\bar{\Gamma}] = 0$, where $g$ is an element of $SU(N)$. For the Landau gauge, we can write

$$F[\Gamma] = D^a \Gamma_a = 0, \quad \bar{F}[\bar{\Gamma}] = \bar{D}^a \bar{\Gamma}_a = 0.$$

(21)

The Faddeev–Popov operator can be written as

$$M_F[\Gamma] = \left(\frac{\delta F[\Gamma]}{\delta g}\right), \quad \bar{M}_F[\bar{\Gamma}] = \left(\frac{\delta \bar{F}[\bar{\Gamma}]}{\delta \bar{g}}\right).$$

(22)

In the standard Faddeev–Popov method, the following expression is used

$$1 = \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][F[\Gamma]] + \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][\bar{F}[\bar{\Gamma}]].$$

(23)

However, as we can have multiple solutions in the Landau gauge, we can generalize the standard Faddeev–Popov to multiple solutions as follows [32],

$$N_F[\Gamma, \bar{\Gamma}] = \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][F[\Gamma]] + \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][\bar{F}[\bar{\Gamma}]]$$

where $N_F[\Gamma, \bar{\Gamma}]$ denotes the number solutions corresponding to a gauge fixing condition.

It is known that for usual gauge theories, the fundamental modular region is a unique representation of every gauge orbit [35,36]. Even though the theory considered here is a supersymmetric Yang–Mills theory, it is a gauge theory and hence we expect such a unique representation of every supergauge orbit. Furthermore, it is possible to generalize the arguments used for obtaining these results for a supersymmetric theory. It may be noted that even though this theory has $\mathcal{N} = 1$ supersymmetry, the gauge symmetry for this theory is different from both the undeformed Yang–Mills theory with $\mathcal{N} = 1$ supersymmetry and the un-deformed Yang–Mills theory with $\mathcal{N} = 2$ supersymmetry. This is because the gauge transformations are defined in terms of the deformed superspace coordinates, and thus involve fermionic version of the Moyal product. However, as this theory has a gauge symmetry, we can in principle define unique representation of every supergauge orbit on deformed superspace. So, we will denote the fundamental modular region $\Lambda$, as the set of absolute minima of the functional

$$V_{\Gamma, \bar{\Gamma}} = \int d^3x d^2\theta (\bar{\delta} \Gamma)^2 + \int d^3x d^2\bar{\theta} (\bar{\delta} \bar{\Gamma})^2.$$  

(25)

The stationary points of $V_{\Gamma, \bar{\Gamma}}$ satisfy the Landau gauge condition, and the boundary of the fundamental modular region $\partial \Lambda$ is the set of degenerate absolute minimum of $V_{\Gamma, \bar{\Gamma}}$. The fundamental modular region $\Lambda$ lies within the Gribov region, and the operators $M_F[\Gamma]$ and $M_F[\bar{\Gamma}]$ obtain zero modes in the boundary of the Gribov region, which is called the Gribov horizon. The gauge orbits can be labeled using this fundamental modular region, and denoted $\Gamma(v)$, $\bar{\Gamma}(v)$ as configurations in the fundamental modular region. So, we have $N_F[\Gamma, \bar{\Gamma}]$, as every orbit crosses the fundamental modular region only once. Since the integrand is positive the minima of $V_{\Gamma, \bar{\Gamma}}$ are those $\Gamma_0, \bar{\Gamma}_0$ satisfying the Landau gauge condition $D^a \Gamma_a = 0$ and $D^2 \Gamma_a = 0$. The boundary $\partial \Lambda$ is the set of degenerate absolute minima of $V_{\Gamma, \bar{\Gamma}}$.

The expectation value of a gauge invariant operator $O_\epsilon$, defined on the deformed superspace, can be written as

$$\langle O[\Gamma, \bar{\Gamma}] \rangle = \int \frac{D\Omega[\Gamma, \bar{\Gamma}] e^{-S_{\text{SYM}}}}{D\exp e^{-S_{\text{SYM}}}},$$

(26)

where $D = D[\Gamma(v) \bar{\Gamma}(v)]$ is a suitably defined measure for the path integral. This is well defined if there are unique solutions to the gauge fixing condition. Since $N_F$ is finite, we obtain

$$\langle O[\Gamma, \bar{\Gamma}] \rangle = \left[ \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][F[\Gamma]] \right] \left[ \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][\bar{F}[\bar{\Gamma}]] \right],$$

$$\left[ \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][\bar{F}[\bar{\Gamma}]] \right] \left[ \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][F[\Gamma]] \right].$$

(27)

So, we have $N_F[\Gamma(v), \bar{\Gamma}(v)] = N_F[\Gamma(v), \bar{\Gamma}(v)] = N_F[\Gamma, \bar{\Gamma}]$. So, we can write

$$\langle O[\Gamma, \bar{\Gamma}] \rangle = \left[ \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][\bar{F}[\bar{\Gamma}]] \right] \left[ \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][F[\Gamma]] \right],$$

$$\left[ \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][\bar{F}[\bar{\Gamma}]] \right] \left[ \int D\bar{\delta} \bar{\Delta}_F[\bar{\delta}][F[\Gamma]] \right].$$

(28)
It may be noted that this expression involves the fermionic version of the Moyal product. This expression can be written in terms of the power series involving the deformation matrix $C_{αβ}$. The first term in the series will correspond to the usual un-deformed supersymmetric Yang–Mills theory. Thus, the expectation value of a gauge invariant operator also receives corrections from the deformation of the Yang–Mills theory. Furthermore, in absence of supersymmetry, this expression reduces to the expression for the usual Yang–Mills case. This can be seen by setting by making all the fermionic fields in the expression to vanish. However, this will be different from breaking all the supersymmetry by imposing two non-anticommutative deformations. The expectation value of gauge invariant operators in this case, will be different from the expectation value of gauge invariant operators of the usual Yang–Mills theory.

4. Symmetries

In the previous section, we analyzed the quantization of a three dimensional non-anticommutative gauge theory. We generalized the standard Faddeev–Popov method to incorporate the existence of multiple solutions to the gauge fixing condition. This was done to address the existence of non-perturbative effects. In this section, we will analyze the BRST symmetry for this generalized Faddeev–Popov method. The partition function for this generalized Faddeev–Popov method, can be written as

$$ Z_{gf} = \left[ \int \mathcal{D}\delta(F[\Gamma]) |\det M_{F}[\Gamma]| \delta(\bar{F}[\Gamma]) \right] \left[ \det \tilde{M}_{F}[\Gamma] \right] e^{-S_{YM}(F,\bar{F})}. $$

(29)

This is valid for non-perturbative field theory, as it takes the modulus of the determinant into account. Thus, we can write [32]

$$ |\det M_{F}[\Gamma]|_* = [\text{sgn}(\det M_{F}[\Gamma]) |\det M_{F}[\Gamma]|_* ,
|\det \tilde{M}_{F}[\Gamma]|_* = [\text{sgn}(\det \tilde{M}_{F}[\Gamma]) |\det \tilde{M}_{F}[\Gamma]|_* .$$

(30)

We can express the action corresponding to $\det M_{F}[\Gamma]$ and $\det \tilde{M}_{F}[\Gamma]$ as follows,

$$ S_{det} = \int d^3 x d^2 \theta \left[ -b^{a} * D^{a} \Gamma_{\alpha}^{a} + \frac{\xi}{2} b^{a} * b^{a} + \tilde{c}^{a} * M_{F}^{a} * c^{a} \right] $$

$$ + \int d^3 x d^2 \tilde{\theta} \left[ -\tilde{b}^{a} * D^{a} \tilde{\Gamma}_{\alpha}^{a} + \frac{\xi}{2} \tilde{b}^{a} * \tilde{b}^{a} + \tilde{c}^{a} * \tilde{M}_{F}^{a} * c^{a} \right] .$$

(31)

Here the ghosts and anti-ghosts are denoted by $c^{a}, \tilde{c}^{a}$ and $\tilde{c}^{a}, \tilde{\tilde{c}}^{a}$, respectively. The Nakanishi–Lautrup auxiliary fields are denoted by $b^{a}, \tilde{b}^{a}$. So, we obtain

$$ \lim_{\xi \to 0} \frac{1}{\xi} \int D\xi^{-S_{det}} = [\delta(F[\Gamma]) \det M_{F}[\Gamma] \delta(\bar{F}[\Gamma]) \det \tilde{M}_{F}[\Gamma] *,$n

(32)

where the measure $D\xi$ includes an integral over the ghosts, anti-ghosts and auxiliary fields. We can also write the action corresponding to $\text{sgn}(\det M_{F}[\Gamma])$ and $\text{sgn}(\det \tilde{M}_{F}[\Gamma])$ as

$$ S_{sgn} = \int d^3 x d^2 \theta \left[ b^{a} * M_{F}^{a} * b^{a} - \tilde{b}^{a} * \tilde{M}_{F}^{a} * \tilde{b}^{a} + \frac{1}{2} b^{a} * b^{a} \right] $$

$$ + \int d^3 x d^2 \tilde{\theta} \left[ \tilde{b}^{a} * \tilde{M}_{F}^{a} * \tilde{b}^{a} - \tilde{\tilde{b}}^{a} * \tilde{\tilde{M}}_{F}^{a} * \tilde{\tilde{b}}^{a} + \frac{1}{2} \tilde{b}^{a} * \tilde{b}^{a} \right] .$$

(33)

Here $d^a, d^a, \tilde{d}^a, \tilde{d}^a$ are new Grassmann odd superfields and $d^a, \tilde{d}^a, b^a, \tilde{b}^a$ are new auxiliary Grassmann even superfields. Completing the square, the $B$ field can be integrated out, and so we can define the effective action as

$$ S_{eff} = \int d^3 x d^2 \theta \left[ \frac{1}{2} d^a * (M_F) \tilde{c} + \frac{1}{2} \tilde{d}^a * M_{\tilde{F}} \tilde{c} - \tilde{c}^2 * M_{F} * d^a \right] $$

$$ + \int d^3 x d^2 \tilde{\theta} \left[ \frac{1}{2} \tilde{d}^a * (M_{\tilde{F}}) \tilde{c} + \frac{1}{2} \tilde{d}^a * M_{\tilde{F}} \tilde{c} - \tilde{c}^2 * \tilde{M}_{F} * \tilde{d}^a \right] .$$

(34)

Thus, the partition function can be written as

$$ Z_{gf} = \int \mathcal{D}N_{F}[\Gamma, \bar{\Gamma}]^{-1} e^{-S_{YM} - S_{det} - S_{sgn}}.$$
where $\mathcal{U}_c = \text{diag}(\tilde{c}^2 \ast F^2, \tilde{d}^2 \ast (iM^\text{ph} + \tilde{B}^2/2))$ and $\tilde{\mathcal{U}}_c = \text{diag}(\tilde{c}^2 \ast \tilde{F}, \tilde{d}^2 \ast (iM^\text{ph} + \tilde{B}^2/2))$. The anti-BRST transformation can be written as $S\tilde{T}_c^\alpha = (\gamma^{\text{ph}} \ast \tilde{c}^\alpha)_i$, $S\tilde{c}_c^\alpha = (\gamma^{\text{ph}} \ast \tilde{c}^\alpha)_i$, $\tilde{S}\tilde{c}_c^\alpha = \tilde{c}^\beta \ast \tilde{F} \ast \tilde{c}^\alpha$, $\tilde{S}\tilde{c}_c^\alpha = -\tilde{F} \ast \tilde{c}^\alpha$, $\tilde{S}\tilde{F}^\alpha = 0$, $\tilde{S}\tilde{B}^\alpha_i = 0$. We can write $\Sigma_{\text{det}} = + \text{Tr} \int d^3x d^2\theta \tilde{S} \tilde{S} \tilde{V} \ast \Sigma_{\text{det}} = + \text{Tr} \int d^3x d^2\theta \tilde{S} \tilde{S} \tilde{V} \ast$.

5. Conclusion

In this paper, we analyzed a three dimensional supersymmetric Yang–Mills theory on deformed superspace. We deformed the superspace by imposing non-anticommutativity. The original theory has $\mathcal{N} = 2$ supersymmetry. The deformation of this theory broke half of the supersymmetry of the original theory. Thus, the final theory had only $\mathcal{N} = 1$ supersymmetry. We analyzed the quantization of this theory, and addressed the problem that occurs due to the Gribov ambiguity. This was done by generalizing the standard Faddeev–Popov method. This generalized Faddeev–Popov method was used for analyzing the existence of multiple solutions to the gauge fixing condition. Thus, non-perturbative effects could be addressed using this generalized Faddeev–Popov method. We derived an expression for calculating the expectation values of gauge invariant operators. A partition function for this deformed theory was also constructed. We analyzed the BRST and the anti-BRST symmetries this partition function. It was demonstrated that apart from being invariant under the usual BRST and the usual anti-BRST transformations, this partition function also invariant under double BRST and double anti-BRST transformations. We were able to combine the usual BRST and anti-BRST transformations, with these new BRST and anti-BRST transformations. These new BRST and anti-BRST transformations, take into account the existence of multiple solutions to the gauge fixing conditions, and so the results of this paper can be used for analyzing different aspects of non-perturbative phenomena.

It may be noted that a similar analysis can be done for theories with higher amount of supersymmetry. It would be interesting to apply this formalism for studying supersymmetric theories with boundaries. This is because it is possible to construct a three dimensional theory with $\mathcal{N} = 1/2$ supersymmetry by combining the boundary effects with non-anticommutativity [18]. However, the quantization of this theory has not been studied. It would be interesting to analyze the effect of boundaries on the results obtained in this paper. It may be noted that the existence of Gribov ambiguity is related to the gauge fixing procedure for quantizing Yang–Mills theories, and this problem is usually addressed in the Gribov–Zwanziger formalism [37–43]. It is possible to analyze effects coming from the existence of the Gribov copies in a local way using this formalism. This formalism has been used to analyze the infrared behavior of the gluon and ghost propagator. In fact, the Gribov–Zwanziger formalism has been used for studying the zero momentum value of the gluon propagator. These propagators have been used for analyzing the spectrum of gauge theories [44,45]. The supersymmetric generalization of the Gribov–Zwanziger formalism has also been performed [46]. This has been used for analyzing existence of the condensate and vanishing of the vacuum energy. The renormalization of supersymmetric Yang–Mills theory with $\mathcal{N} = 1$ supersymmetry was analyzed using the Gribov–Zwanziger formalism [47]. This was done by using the Landau duality. The proof of renormalizability of this theory to all orders was studied using an algebraic renormalization procedure. It was demonstrated that only three renormalization constants are needed for this theory. In fact, the non-renormalization theorem in the Landau gauge was analyzed using the Gribov–Zwanziger formalism. The renormalization factors for a non-linear realization of the supersymmetry were also studied in this formalism. It will be interesting to analyze the effect of non-anticommutative deformation on these results. This can be done by analyzing a deformed supersymmetric Yang–Mills theory in Gribov–Zwanziger formalism. Thus, we can study a non-anticommutative four dimensions supersymmetric Yang–Mills theory in $\mathcal{N} = 1$ superspace formalism. The non-anticommutativity will break the supersymmetry of the theory from $\mathcal{N} = 1$ supersymmetry to $\mathcal{N} = 1/2$ supersymmetry. It will also be interesting to perform a similar analysis for a deformed three dimensional theory in $\mathcal{N} = 2$ superspace formalism. Here the non-anticommutativity will break the supersymmetry of the theory from $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$ supersymmetry. It will be interesting to analyze the effect of this supersymmetry breaking on the existence of the condensate and vanishing of the vacuum energy.

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