Towards Task-Prioritized Policy Composition

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Abstract—Combining learned policies in a prioritized, ordered manner is desirable because it allows for modular design and facilitates data reuse through knowledge transfer. In control theory, prioritized composition is realized by null-space control, where low-priority control actions are projected into the null-space of high-priority control actions. Such a method is currently unavailable for Reinforcement Learning. We propose a novel, task-prioritized composition framework for Reinforcement Learning, which involves a novel concept: The “indifferent-space” of Reinforcement Learning policies. Our framework has the potential to facilitate knowledge transfer and modular design while greatly increasing data efficiency and data reuse for Reinforcement Learning agents. Further, our approach can ensure high-priority constraint satisfaction, which makes it promising for learning in safety-critical domains like robotics. Unlike null-space control, our approach allows learning globally optimal policies for the compound task by online learning in the indifference-space of higher-level policies after initial compound policy construction.

I. INTRODUCTION

Policy composition, i.e. the combination of already learned policies into a more capable compound policy, is desirable in Reinforcement Learning (RL). Composition also increases modularity and allows for knowledge transfer between tasks, thus enabling efficient use of data and learning time. State-of-the-art RL literature offers two perspectives on composition: Conjunctive (“AND”) composition [1, 2, 3], where tasks have to be solved concurrently, and disjunctive (“XOR”) composition [4], where tasks are switched over time. When the individual tasks are very different, “AND” composition often leads to sub-optimal policies and “XOR” composition always ignores some of the tasks. Classic control literature provides yet another perspective on composition: Hierarchical task composition, where a low-priority control action is projected into the null-space of a high-priority control action such that the high-priority task is still achieved by the combined action [5]. This method, implemented by null-space control, is a form of local, prioritized task composition that is re-computed at each step. However, this can result in “deadlocks” (local optima) where the agent fails to achieve low-priority tasks in the long term.

In this paper, we are interested in task-prioritized composition similar to null-space control but for RL. Consider the scenario in Fig. 1 where we first train two RL policies for two separate tasks and want to combine them for a new, more complex task. The first task (modeled by reward) is to avoid the U-shaped obstacle and the second task is to reach the goal. The arrows in Fig. 1 (left) show optimal policies for the two tasks. The two tasks are completely separate such that never has to reach the goal and never interacts with the obstacle during training. In the new task (right), we want to reach the goal while avoiding the obstacle and for this, we want to combine already learned policies and to a new policy.

Existing “AND” composition approaches model the new task as the sum [1], the mean [2], or a convex combination of the reward signals [3]: , for task- prioritized composition because they cannot implement a strict priority ordering of tasks. Even when we define the new task as , existing methods cannot guarantee that the compound policy avoids the obstacle. Depending on the scale of individual rewards and , the compound policy will drive straight into the obstacle. What we need is a compound policy that follows to avoid the obstacle (with high priority) while making progress towards the goal of (with low priority) whenever possible. For this, the contribution of the low-priority policy has to be adjusted such that it does not interfere with the high-priority policy, e.g. in safety-critical situations where the high-priority policy must be in full control (for example at the marker in Fig. 1).

While “AND” composition in RL is well-defined as obtaining a policy a sum of rewards, e.g. , for task-prioritized policy composition it is not clear what the new task’s reward is because the contribution of the low-propriety task depends on the policies of the high-priority task. Informally, we are interested in obtaining the policy for...
a new task $r_{1>2}$ that jointly optimizes a high-priority task $r_1$ and a low-priority task $r_2$ in such a way that the return for the high-priority task $r_1$ is not considerably lower under the composed policy $\pi_{1>2}$ compared to the original, optimal policy $\pi_1^*$ for the task $r_1$. “AND” composition frameworks are insufficient for this, because the optimal policy for the compound task $r_{1+2}$ will very likely obtain less reward under $r_1$ compared to the optimal task policy $\pi_1^*$, due to the influence of the low-priority reward signal $r_2$, a phenomenon referred to as “tragedy of the commons” [1].

More formally, we think of task-prioritized policy composition in RL as solving a new task $r_{1>2} = r_1 + w(s_t, a_t)r_2$, where $w(s_t, a_t) \in [0, 1]$ is some weight that is small whenever the contribution of the low-priority task $r_2$ would considerably worsen return for the high-priority task $r_1$ in the long run. This effectively prevents the low-priority task $r_2$ from contributing to the action-selection in such situations. This means that our compound task $r_{1>2}$ is only indirectly defined by the mentioned constraint on the return for $r_1$.

In the following, we first explain how policies with priorities are locally composed in null-space control, then we introduce the novel concept of “indifference space” of RL policies. After that, we explain how to use the “indifference space” to compose RL policies that observe task priorities and how our method can obtain globally optimal solutions to prioritized task compositions by learning online.

II. TASK COMPOSITION IN NULL-SPACE CONTROL

In control-theory, prioritized task combination is achieved through null-space control [5,6]. Null-space control involves a hierarchy of tasks and attempts to solve a lower-priority task in the null-space of a higher-priority task, which is that part of space that contains all equally valid solutions to the higher-priority task. Null-space control is possible when control vectors can be added together, in which case it is implemented by projecting the low-priority control vector $\theta_2$ into the null-space of the high-priority controller. For this, we multiply $\theta_2$ with a filtering term $(I - J_{ee}^TJ_{ee}^-)$, where $I$ is the identity matrix, $J_{ee}^T$ is the transpose of the task Jacobian $J_{ee}$ that relates changes in control with changes in task space, with $J_{ee}^-)$ being its pseudo inverse. This allows for the prioritized combination of a high-priority control signal $\theta_1$ and a low-priority control signal $\theta_2$ such that the resulting control $\theta_{1>2}$ will only execute those components of $\theta_2$ that do no interfere with the desired task-space effect of $\theta_1$.

Importantly, null-space control does not guarantee that the control $\theta_{1>2}$ will produce globally optimal behavior, i.e. it is possible that only the high-priority control will be achieved (i.e. collision avoidance), while the null-space projection of the low-priority control results in a global sub-optimal outcome. In Fig. 1 the robot would drive in a straight line towards the goal (in the null-space of the high-priority controller) up to the marker, at which point the high-priority controller would prevent it from advancing further to prevent a collision with the obstacle. Thus, classic null-space control can be thought of as a local task-prioritized policy composition because the null-space projection is applied separately in every state, without considering the long-term consequences of this projection.

III. INDIFFERENCE-SPACES OF POLICIES

The general intuition of classical null-space control is also applicable to RL policies. We refer to this as the indifference-space of policies and their $Q$-functions. We define all actions $a \in A$ to be in the indifference-space of a policy $\pi$ and the $Q$-function $Q^\pi$ in state $s \in S$, if they have the same (or better) expected return as $\pi$. In our example, the indifference-space of $\pi_1^*$ contains all actions as long as the state is not next to the obstacle, where it only contains one action. In such a case, we relax this definition and allow some loss of return $\varepsilon$, such that all actions $a$ are part of the soft indifference-space if

$$Q^\pi(s, a) - Q^\pi(s, \pi(s)) \leq \varepsilon. \quad (1)$$

Intuitively, when the soft indifference-space of a high-priority policy contains multiple actions, we can allow the low-priority policy to select which of those near-optimal actions to execute. In this way, the low-priority task can be achieved in the long run with actions that are in the high-priority policy’s soft indifference space.

In some tasks, this soft indifference-space might be very large (i.e. sparse reward navigation tasks) while in tasks with more constrained optimal policies this space might be rather small. In either case and to the best of our knowledge, how to find the soft indifference-space or a similar concept for RL policies is an open research question, since existing RL composition approaches only consider “AND” [1,2,3,7] or “XOR” [4] task compositions. Furthermore, how to have a low-priority policy select actions from the high-priority policy’s soft indifference-space, what the resulting task-prioritized compound policy or its $Q$-function is, and what this means in terms of reward are also open research questions. In this paper, we propose a possible method for finding the soft indifference-space of a policy, for exploiting it for local task-prioritized RL policy composition, and for a constrained exploration based on this space.

IV. INDIFFERENCE-SPACE REINFORCEMENT LEARNING

For our task-prioritized composition framework, we consider a Markov Decision Process (MDP) with $d$-dimensional action space $A$ and state space $S$, transition dynamics $p(s_{t+1}|s_t, a_t)$, the marginal $\rho$, and a discount factor $\gamma$. Tasks are defined by reward functions $r \in \mathcal{R}$ for the same MDP and we are also allowed to access to optimal task policies $\pi_1^*$, $\pi_2^*$ and $Q$-functions $Q_1^*$, $Q_2^*$ for tasks $r_1$, $r_2$. There are multiple challenges to overcome: As mentioned before, we do not know the task $r_{1>2}$ in terms of reward, we cannot exclusively rely on the individual $Q$-functions for evaluating the compound task $\pi_{1>2}$, and we must ensure the priorities while allowing progress on the lower-priority task when possible.

We require that $\pi_1^*$ and $\pi_2^*$ are stochastic, maximum entropy policies, meaning our method is based on the
maximum-entropy framework [8, 9], where policies not only maximize reward, but also the entropy $H$:

$$\pi_{ME}^* = \underset{\pi}{\arg \max} \frac{1}{N} \sum_{t} \mathbb{E}_{(s_t, a_t) \sim \rho^s} \left[ r(s_t, a_t) + \alpha H(\pi(\cdot|s_t)) \right].$$

(2)

The maximum entropy framework is particularly adequate for policy composition, as described by Haarnoja et al. [2], while also offering desirable mathematical properties. As we will see, the policy definition

$$\pi(a_t | s_t) \propto \exp(-\frac{1}{\alpha} Q_{\text{Soft}}(s_t, a_t)),$$

(3)
together with Amortized Stein Variational Gradient Descent (ASVGD) and the Soft Q-Learning [9] update mechanism conveniently allow us to learn a sampling network for the intractable policy density reflecting dynamically weighted Q-function mixture resulting from our method.

Additionally, for prioritized composition, we must define an ordering of tasks $1 \geq 2$, such that task priority can be determined. Given such an ordering, we are now interested in obtaining the policy $\pi_{1 \geq 2}$ for task $r_{1 \geq 2}$, which maximizes both $r_1$ and $r_2$, while guaranteeing that the return for the high-priority task $r_1$ is not considerably worse under $\pi_{1 \geq 2}$ compared to its optimal policy $\pi_1^*$.

### A. Indifference-weighting for local composition

To obtain the task-prioritized compound policy $\pi_{1 \geq 2}$, we define its $Q$-function as

$$Q_{1 \geq 2}(s_t, a_t) = Q_1^*(s_t, a_t) + w_{1 \geq 2}(s_t, a_t)Q_2^*(s_t, a_t)$$

(4)

and let $Q_{1 \geq 2}$ induce $\pi_{1 \geq 2}$ via Eq. (5). The question is how to choose $w_{1 \geq 2}$ for it to implement the desired task-prioritized composition based on the high-priority soft indifference-space. Informally and considering our definition of the soft indifference-space of a policy, $w_{1 \geq 2}(s_t, a_t)$ must tend to zero for all actions that are far from local maxima in the high-priority $Q$-function $Q_1^*$, such that the low-priority $Q$-function $Q_2^*$ can only contribute to $Q_{1 \geq 2}$ at points that have similar high return under the high-priority task $r_1$.

We can calculate such a factor by considering the first- and second-order information of the high-priority policy. The Jacobian $J Q_1^*$ of the high-priority policy’s $Q$-function (evaluated at point $s, a$) is small around critical points, while the corresponding Hessian $H Q_1^*$ is negative around local maxima. Thus, when $Q_1^*$ is continuous and differentiable w.r.t $a_t$, we can perform a point-wise second-order derivative test to find approximate local maxima of the high-priority policy $\pi_1$, construct the soft indifference-space around those local maxima, and define $w_{1 \geq 2}$ as a measure of local optimality:

$$w_{1 \geq 2}(s_t, a_t) \equiv \kappa(\|J Q_1^*\|, 0) \cdot F(\|H Q_1^*\|).$$

(5)

Here, $\|\cdot\|$ denotes an arbitrary matrix norm, $\kappa$ denotes the squared-exponential (SE) kernel $\kappa(x, x') = \sigma^2 \exp(-(x-x')^2/2\ell^2)$ that will be large when $x$ and $x'$ are similar, while $F$ denotes the Fermi-Dirac distribution $F(\varepsilon) = \frac{1}{\exp(\varepsilon - \mu/k_B T) + 1}$, with the Boltzmann constant $k_B$ and the parameters for the point of symmetry $\mu$ and absolute temperature $T$. To provide further intuition for Eq. (5), consider Fig. 2, which shows the two component functions of $w_{1 \geq 2}$. Eq. (5) simply ensures that $w_{1 \geq 2}$ tends to 1 when the Jacobian of $Q_1$ is close to zero and the Hessian of $Q_1$ is negative, while tending to zero otherwise, which is the desired factor. However, $\kappa$ and $F$ have critical parameters that control how much deviation from the optimal action $w_{1 \geq 2}$ allows, which require a task-dependent tuning-process that might be brittle if performed manually. It might be possible to automatically tune these parameters such that Eq. (1) holds for a desired $\varepsilon$, but how to do this concretely is ongoing research.

Given this definition for $w_{1 \geq 2}$, we can now obtain the task-prioritized compound $Q$-function $Q_{1 \geq 2}$ in Eq. (4) from two constituent $Q$-functions and a given priority ordering. $w_{1 \geq 2}$ ensures that the low-priority $Q$-function can only select actions that are in the soft indifference-space of the high-priority $Q$-function. Next, we show one way for obtaining

Fig. 2: Desired weights as result of Fermi-Dirac and SE kernel parametrization in restrictive and permissive scenarios.
the policy that corresponds to the zero-shot, task-prioritized composition $Q$-function $Q_{1 \geq 2}$ and note that this approach is just one of potentially many ways for implementing task-prioritized composition in RL.

### B. Learning the task-prioritized compound policy

Given $Q_{1 \geq 2}$ from above, we must still learn a sampling model for the corresponding, potentially high-dimensional and intractable density of the task-prioritized compound maximum entropy policy $\pi_{1 \geq 2}$. We rely on the ASVGD \cite{10}, as in original Soft Q-learning \cite{9}, to train a DNN $f$ parameterized by $\phi$ to map random noise inputs $\zeta$ to action samples $a_t$ from the maximum entropy target policy $\pi_{1 \geq 2}$ such that $a_t = f(\phi, s_t)$. The Stein variational gradient that minimizes the Kullback-Leibler divergence between the policy $\pi^\phi$ (induced by $f(\phi)$) and the desired, intractable policy density $\pi_{1 \geq 2}$ corresponding to the soft $Q$-function $Q_{1 \geq 2}$ is given by

$$
\nabla f(\phi, s_t) = \mathbb{E}_{a_t \sim \pi^\phi} \left[ \kappa(a_t, f(\phi, s_t)) \nabla Q_{1 \geq 2}(s_t, a_t) \right. \\
\left. + \alpha \nabla k(f(\phi, s_t)) \right],
$$

where $\kappa$ is an arbitrary kernel. This gradient can directly be used to update the sampling network (for details see \cite{9, 10}). The reason this works is the proportion in Eq. (3), which implies that the soft $Q$-function $Q_{1 \geq 2}$ corresponds to the un-normalized density of $\pi_{1 \geq 2}$, hence making the ASVGD method applicable. Thus, as long as we can calculate $Q_{1 \geq 2}(s_t, a_t)$ for a batch of states and actions, we can train a sampling network for the corresponding intractable policy $\pi_{1 \geq 2}$. In this fashion, we can directly use the soft Q-learning ASVGD update procedure to learn sampling networks for high-dimensional, task-prioritized composition policy densities induced by indifference-weighted Q-functions.

The procedure outlined so far is a semi-direct adaptation of null-space control for RL policies and Q-functions, meaning it shares the limitations mentioned in Sec. \cite{4} namely that it only yields a locally optimal policy for the task $r_{1 \geq 2}$ that is optimal for $r_1$ but not necessarily for $r_2$, which can lead to the locally optimal behavior sketched in Fig. \cite{1} With a classical null-space controller, this would be the resulting, locally optimal behavior, with no way of improving it, except by manually programming a low-priority controller that optimally solves the low-priority task in the null-space of the high-priority controller. However, a globally optimal solution clearly exists, namely driving around the obstacle as seen in Fig. \cite{1} on the right. Our indifference-space composition approach for RL can learn this globally optimal policy for the new compound task $r_{1 \geq 2}$, as we explain in the next section.

### C. Online learning with indifference-space exploration

Given our procedure for finding the soft indifference-space of a $Q$-function, while keeping the high-priority $Q$-function $Q^*_1$ fixed, we can continue to train the low-priority $Q$-function $Q^*_2$. By sampling exploratory actions in the soft indifference-space of the high-priority policy, the low-priority policy can learn to adapt to the constraints imposed by the high-priority policy. The reason we can do this while still guaranteeing constrain-satisfaction for the high-priority task is that the indifference weight $w_{1 \geq 2}$ will filter out the contribution from the low-priority $Q$-function $Q_2$ at all points that are far from local maxima in $Q^*_1$, such that even an adversarial low-priority $Q$-function that maximizes the task $-r_1$ opposite to $r_1$ would not result in constrain violation for the high-priority task $r_1$. Thus, while keeping the parameters of $Q^*_1$ frozen, we can take exploratory actions in the soft indifference-space of $Q^*_1$ to learn a new state-action value function $Q^*_2$ (possibly with initial parameters from $Q_2$) that adapts to the constraints imposed by $Q^*_1$. Thus, while Eq. \cite{4} can be thought of as the equivalent to locally optimal null-space control in RL, with this indifference-space exploration strategy we can obtain the globally optimal policy $\pi_{1 \geq 2}$ induced by

$$
Q^*_1(s_t, a_t) = Q^*_1(s_t, a_t) + w_{1 \geq 2}(s_t, a_t)Q^*_2(s_t, a_t), \tag{6}
$$
as described in the previous section. $\pi_{1 \geq 2}$ will be globally optimal for the task $r_{1 \geq 2}$, meaning both tasks will be maximized jointly, in the best possible way under the task priority ordering. In Fig. \cite{1} this policy will produce the desired, globally optimal behavior in which the agent navigates around the U-obstacle instead of down the middle.

We aim to present initial experimental results of our method at the 2nd RL-CONFORM workshop at IROS 2022.

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