MASER AND LASER ACTION WITH ONE ATOM

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We present a theory which can explain the micromaser as well as its optical counterpart, the microlaser, for appropriate values of dissipative parameters. We show that, in both the the cases, the cavity radiation fields can have sub-Poissonian photon statistics. We further analyse if it is possible to attain a Fock state of the radiation field. The microlaser is precluded for such analyses due to the damping of its lasing levels making transitions at optical frequencies. Hence, we focus our attention on the micromaser and our exact simulation of the dynamics shows that it is not possible to generate a Fock state of the cavity radiation field.

I. INTRODUCTION

The subject of one-atom maser and its optical counterpart, the one-atom laser, has generated extensive interest after the recent experimental demonstrations that they are capable of generating nonclassical states of the radiation fields [1-3]. In the one-atom maser experiment [1], two-level $^{85}\text{Rb}$ atoms in their upper Rydberg states of the transitions $63^2p_{3/2} \rightarrow 61^2d_{5/2} (21.5065 \text{ GHz})$ and $63^2p_{1/2} \rightarrow 61^2d_{5/2} (21.456 \text{ GHz})$ are pumped into a microwave cavity at such a rate that, at most, one atom is present there at any time. So, there can be one of the two situations in the cavity: only one atom is present in the cavity or the cavity is empty of any atom. The cavity is tuned to one of the above two transitions in individual experiments and the sub-Poissonian nature of the cavity radiation field having variance less than that for a coherent state field has been inferred. It may be noted here that a well stabilized conventional laser can generate radiation fields that can at best be close to a coherent state field only. Sub-Poissonian radiation fields have photon distribution functions narrower than that for a coherent state field which has a Poissonian photon statistics.

Whereas in the one-atom laser experiment [2], the pump rate of two-level $^{138}\text{Ba}$ atoms in their upper states into a resonant optical cavity gives a stream of an average number of atoms present there satisfying the condition $\langle N \rangle \leq 1.0$. However, in order to have a sub-Poissonian radiation field, an uniform atom-field interaction independent of cavity mode structre is essential. The technique adopted by An el al [3] to improve their earlier setup [2] provides such a situation. Single-atom events in this cavity would be suitable for generating sub-Poissonian field in the optical regime since such arrangements in the microwave regime produced nonclassical fields [1].

In the following, we describe a unified formalism which shows that the one-atom maser (micromaser) as well as the one-atom laser (microlaser) cavity fields can have sub-Poissonian photon statistics. Our unified theory also explains the conventional laser dynamics. Further, the hope is that such sub-Possionon radiation fields would further shrink to bosonic Fock states. Our analysis below shows that this unified theory is not suitable for looking into these possibilities. Hence, we examine the possibilities of generating Fock states of the micromaser cavity field by an exact numerical simulation of the dynamics. In the case of microlaser, the damping of the atomic levels precludes such possibilities.

II. THE FORMALISM

We assume that atoms arrive individually at the cavity with an average interval $\bar{t}_c = 1/R$ where $R$ is the flux rate of atoms. We have $t_c = t + t_{cav}$ where $t$ is the interaction time, fixed for every atom, and $t_{cav}$ is the random time lapse between one atom leaving and successive atom entering the cavity. $t_c$ is the average of $t_c$ taken over a Poissonian distribution in time of incoming atoms. The cavity field evolves by this repititive dynamics from near vacuum as thermal photons in the optical cavity are almost nonexistant. Thus, during $\tau$, we have to solve the equation of motion by taking into account the cavity dissipation as well as atomic damping, important at optical frequencies. Thus, we have

$$\dot{\rho} = -i[H, \rho] - \kappa(\sigma^a \rho - 2\sigma \rho^a + \rho \sigma^a) - \gamma(S^+ S^- \rho - 2S^- \rho S^+ + \rho S^+ S^-)$$

(1)

where $H$ is the Jaynes-Cummings Hamiltonian [6] and $\kappa$ and $\gamma$ are the cavity-mode and atomic decay constants respectively. $\sigma$ is the photon annihilation operator and $S^+$ and $S^-$ are the Pauli pseudo-spin operators for the two-level system. During $t_{cav}$, the cavity field evolves under its own dynamics and is represented by

$$\dot{\rho} = -\kappa(1 + \bar{n}_{ih})(\sigma^a \rho - 2\sigma \rho^a + \rho \sigma^a)$$

$$-\kappa \bar{n}_{ih}(\sigma^a \rho - 2\sigma \rho^a + \rho \sigma^a)$$

(2)
The method for obtaining a coarse-grained time derivative for the photon number distribution \( P_n = \langle n| \rho |n \rangle \) is given in detail in Ref. [4]. The steady-state photon statistics is then

\[
P_n = P_0 \prod_{m=1}^{n} v_m
\]

and \( P_0 \) is obtained from the normalisation \( \sum_{n=0}^{\infty} P_n = 1 \).

The \( v_n \) is given by a continued fractions involving the system parameters \( N = R/2\kappa \), the number of atoms passing through the cavity in a photon lifetime, \( \kappa/g \) and \( \gamma/g \) where \( g \) is the atom-field coupling constant. Once \( P_n \) is obtained, we can get the laser intensity, proportional to \( \langle n \rangle \), and its variance. This represents micromaser or microlaser photon statistics depending upon the relative values of the dissipative parameters and the coupling constant. We introduce another parameter, the pump parameter \( D = \sqrt{Ng\tau} \) useful for the description of laser characteristics. It is interesting to note that we can recover the conventional laser photon statistics from Eq. (2) by an appropriate choice of the above parameters.

### III. ONE-ATOM MASER DYNAMICS

This involves the Rydberg levels of the active atoms having spontaneous lifetime \( t_s = 1/2\gamma \) in the order of a fraction of a second. Whereas their flight time \( \tau \) through the cavity is of the order of microseconds. Hence we can safely put \( \gamma = 0 \) in the Eq. (1). We find from our formalism that the normalized variance defined by

\[
v = \sqrt{\langle n^2 \rangle - \langle n \rangle^2}/\langle n \rangle
\]

is 0.5522 which is very close to the experimental finding \( v = 0.5477 \) in Ref. 1a near \( N = 30 \) and values of the other parameters reported there. Thus we find that our formalism describes the micromaser dynamics quite accurately. It may be noted here that \( v = 1 \) for coherent state radiation fields. Thus, this novel device does generate radiation fields with sub-Poissonian photon statistics.

### IV. ONE-ATOM LASER DYNAMICS

The lasing frequency here is in optical regime and, hence, the atomic lifetimes in these cases are short. Hence, we have to take \( \gamma \neq 0 \) for the microlaser dynamics. For the typical parameters in the experiments in Ref. 3, we find that \( \gamma/g = 0.1 \). From our photon statistics in Eq. (2), we obtain the laser intensity proportional to \( \langle n \rangle \) as a function of the pump parameter \( D \). The structures in \( \langle n \rangle \) as \( D \) is varied for fixed \( N \) reflect the characteristics of the Jaynes-Cummings interaction[6]. Soon after the threshold is attained at about \( D = 1.0 \), the photon number rises sharply. The reason is as follows. The field is almost in vacuum before the very first atom enters the cavity. Thus the atoms initially in their upper states contribute varying fractions of their energies to the cavity and at \( n = N \) and \( D = \pi/2 \) the atoms get completely inverted. Thus \( \langle n \rangle \) is peaked at about \( D = 1.6 \) depending on \( \kappa \), \( \gamma \) and \( N \). For higher \( \kappa \) and \( \gamma \), the peak moves slightly towards higher \( D \) due to threshold being attained at higher \( D \). We further find that \( n > 0 \) around \( D = 31.4, 62.8, 94.2, ... \) giving \( g\tau = \pi, 2\pi, 3\pi, ... \) respectively. At such values of \( g\tau \), the atom absorbs the photon it has emitted before leaving the cavity.

The variance of the cavity field \( \sigma \) increases sharply at about \( D = 1.6 \) where \( \langle n \rangle \) is also peaked. We find that near this value of \( D \), the \( P_n \) is doubly peaked at \( n = 0 \) and \( n \approx N \) and this increases the variance in photon number. However, for slightly higher value of \( D \), the cavity field is highly sub-Poissonian in nature. It also appears for further higher values of \( D \). But, \( \langle n \rangle \) is very small for such values of \( D \) due to increase in the interaction time \( \tau \) which increases the influence of atomic as well as cavity reservoirs on the atom-field interaction. It may be noted here that these characteristics are clearly different from that of conventional lasers.

### V. CAN THERE BE A FOCK STATE OF THE RADIATION FIELD?

We see that the one-atom dynamics is capable of narrowing the photon distribution function of the cavity radiation field compared to that for a coherent state field. This raises the question whether the distribution can be further reduced to a number state of the radiation field. The reason why such possibilities may arise is due to the fact that the reservoir influences are minimal in these dynamics. In fact, the analysis in Ref. [4] indicates this possibility in the micromaser dynamics when the condition

\[
f(n) = -2n\bar{n}_{th} - 2N\sin^2(\sqrt{n}g\tau) \times \exp[-\gamma - (2n - 1)\kappa] = 0
\]

is satisfied. Clearly \( f(n) = 0 \) only when \( \bar{n}_{th} = 0 \) with \( g\tau = \pi/2 \). The later condition can be met by an adjustment of the atomic velocity with the help of the velocity selector in the micromaser setup [1]. But the other condition that the cavity temperature \( T = 0 \) obviously cannot be satisfied. In the micromaser experiment [1b], the quality factor of the cavity is \( Q = 3.4 \times 10^9 \) and
its temperature is maintained at $T = 0.3$ K which gives thermal photons $\bar{n}_{th} = 0.033$. The Rydberg level involved in the atomic transitions have lifetimes of the order of a second which is much longer compared to its flight time of about $\tau = 40 \mu s$ through the cavity. These values of the parameters governing the dissipation dynamics make their influences very small. In the case of one-atom laser, the lifetimes of atomic levels at optical frequencies are short which makes the influence of atomic reservoir non-negligible. Hence, even though the thermal photons $\bar{n}_{th} = 0$ in the optical cavity, the one-atom laser [2,3] would not be a right choice for this study. Thus, the micromaser turns out to be a suitable system to look into the possibilities of generating a number state of the cavity radiation field.

We then have to take into account the exact influences of the reservoirs to ascertain if a Fock state of the cavity field can actually be generated. Hence, instead of Eq. (1), we have to deal with

$$\dot{\rho} = -i[H, \rho] - \kappa(1 + \bar{n}_{th})(a^\dagger \rho a - 2a \rho a^\dagger + \rho a^\dagger a)$$

whenever a atom is present in the cavity. As mentioned earlier, a atom takes a time $\tau$ to pass through the cavity. These atomic events are separated by random durations, $t_{cav}$, during which the cavity evolves under its own dynamics. Hence we set $H = 0$ during $t_{cav}$. Processes like these atomic events separated by random intervals are known as Poisson processes in literature encountered in various branches of physics, for example, radioactive materials emitting alpha particles. A sequence of durations of such processes can be obtained from uniform deviates, also called random numbers, $x$ generated using a computer such that $0 < x < 1$, and then by using the relation [7]

$$t_R = -\mu \ln(x)$$

where $t_R = t_{cav} + \tau$, $\mu = 1/R$ where $R$ is the flux rate of atoms.

We have carried out numerical simulation of the dynamics with the data taken from the experimental arrangements [1] in which $g = 39 \text{ kHz}$ and the $\tau = 40 \mu s$ was one of the atom-field interaction times. In this case $\gamma \tau = 1.56$ which produces Fock states of $n$ photons where $n$ satisfies $\sin \gamma \sqrt{n + 1} = 0$ in an ideal cavity ($Q = \infty$). Since the experimental arrangements are close to ideal situation, it was hoped that such Fock states could be attained experimentally. Indeed, such results have been reported in Ref. 1b. However, our numerical simulations does not confirm these conclusions. Instead, it gives photon fields with very narrow distribution functions (sub-Poissonian) centred about $n$. Figs. 1 and 2 display distribution function $P(n)$ narrowly centred about $n = 14$.

The reason for these results is simple. The cavity dissipation, although very small, effects the coherent atom-field interaction and moreover the randomness in $t_{cav}$ makes the photon distribution function fluctuate all the time centred about $n$ in addition to making it broader. In this experiment [1], the atoms coming out of the cavity are subjected to measurements from which state of the cavity field is inferred. The atoms enter the cavity in the upper $|a\rangle$ of the two states $|a\rangle$ and $|b\rangle$. The exiting atom is, in general, in a state

$$\langle \psi | = a|a\rangle + b|b\rangle$$

with $p_a = |a|^2$ and $p_b = |b|^2$ are the probabilities of the atom being in the states $|a\rangle$ and $|b\rangle$ respectively. According to the Copenhagen interpretation of quantum mechanics [8], this wave function collapses (or is projected) to either $|a\rangle$ or $|b\rangle$ the moment a measurement is made on it. Due to this inherent nature of quantum mechanics, a noise is associated with the measurement which is know as quantum projection noise [9]. We define the projection operator $J = |a\rangle\langle a|$. The variance in its measurement is given by

![FIG. 1. Cavity photon distribution function at the exit of the 7000th atom.](image1)

![FIG. 2. $P(n)$ vs $n$ at the moment of the 9000th atom leaving the cavity.](image2)
\[
(\Delta J)^2 = \langle J^2 \rangle - \langle J \rangle^2 = p_a (1 - p_a)
\] (9)

We find that \((\Delta J)^2 = 0\) only when \(p_a = 1\) or \(0\). For the generation of a Fock state, it is necessary that the atom should leave the cavity unchanged in its upper state [4]. Hence, for such a situation we must have \(p_a = 1\) in which case \((\Delta J)^2\) should be 0. We find from our numerical simulations that that \(p_a = P(a)\) is mostly about 0.8 [Fig. 3] and, hence, \((\Delta J)^2 \neq 0\) always. This obviously indicates that the cavity field is in a linear superposition of Fock states giving a photon distribution function with the variance, defined in Eq. (3), \(v > 0\) (For a Fock state \(v = 0\)). Indeed, we find that the \(v\) is about 0.5 in our calculations, presented in Fig. 4, indicating a sub-Poissonian nature of the cavity field. By itself, it carries a signature of quantum mechanics.

We further notice in Fig. 4 that there are small fluctuations in \(v\) due to the fluctuations in \(P(n)\) [Figs 1 and 2]. Also, \(v\) is nowhere near 0 in Fig. 4.

![FIG. 3. Population of the upper state of the individual atoms at the exit from the cavity.](image)

![FIG. 4. Fluctuations in \(v\) at the exit of successive atoms from the cavity.](image)

The steady-state photon statistics in Eq. (2) gives \(v = 0.597\) which lies within the range of fluctuations in Fig. 4. This is due to the fact that the coarse-graining process involved in obtaining steady-state results neglects these small fluctuations (A detailed discussion can be found in Ref. [5]). Our exact numerical simulations, however, shows that they are crucial in the generation of a Fock state. We have carried out simulation until about 10000 atoms passed through the cavity and carrying out the simulations any further would only be a repetition of the above fluctuations.

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