The Origin of Mass

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Abstract

The quark-lepton mass problem and the ideas of mass protection are reviewed. The hierarchy problem and suggestions for its resolution, including Little Higgs models, are discussed. The Multiple Point Principle is introduced and used within the Standard Model to predict the top quark and Higgs particle masses. Mass matrix ansätze are considered; in particular we discuss the lightest family mass generation model, in which all the quark mixing angles are successfully expressed in terms of simple expressions involving quark mass ratios. It is argued that an underlying chiral flavour symmetry is responsible for the hierarchical texture of the fermion mass matrices. The phenomenology of neutrino mass matrices is briefly discussed.

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1 Introduction

The origin of the quark and lepton masses, their mixing and three generation structure remains the major outstanding problem in particle physics. The charged fermion masses and mixing angles derive from Yukawa couplings, which are arbitrary parameters within the Standard Model (SM). Furthermore the non-vanishing neutrino masses and mixings are direct evidence for physics beyond the SM. So the experimental values of the fermion masses and mixings provide our best clue to this new physics. The main features requiring explanation are the following:

1. The large mass ratios between generations:
   \[ m_u \ll m_c \ll m_t; \quad m_d \ll m_s \ll m_b; \quad m_e \ll m_\mu \ll m_\tau. \]

2. The large mass splitting within the third (heaviest) generation:
   \[ m_\tau \sim m_b \ll m_t. \]

3. The smallness of the off-diagonal elements of the quark weak coupling matrix \( V_{CKM} \).

4. The tiny neutrino masses and the large off-diagonal elements of the lepton weak coupling matrix \( U_{MNS} \).

The charged fermion mass hierarchy ranges over five orders of magnitude, from \( \frac{1}{2} \) MeV for the electron to 174 GeV for the top quark. It is only the top quark which has a mass of order the electroweak scale \( \langle \phi_{WS} \rangle = 246 \) GeV and a Yukawa coupling of order unity. All of the other charged fermion masses are suppressed relative to the natural scale of the SM. The main problem in understanding the charged fermion spectrum is not why the top quark is so heavy but rather why the electron is so light. Indeed, as the top quark mass is the dominant term in the fermion mass matrix, it is likely that its value will be understood dynamically before those of the other fermions. In contrast there seems to be a relatively mild neutrino mass hierarchy and two of the leptonic mixing angles are close to maximal (\( \theta_{\text{atmospheric}} \simeq \pi/4 \) and \( \theta_{\text{solar}} \simeq \pi/6 \)). The absolute neutrino mass scale \( m_\nu < 1 \) eV is another puzzle, suggesting a new physics mass scale – the so-called see-saw scale \( \Lambda_{\text{seesaw}} \sim 10^{15} \) GeV.

We give an overview of the quark-lepton spectrum in section 2 and introduce the mechanism of mass protection by approximately conserved chiral charges in section 3. Various approaches to the gauge hierarchy problem are
considered in section 4, including supersymmetry, Little Higgs models and the Multiple Point Principle. In section 5 we discuss the connection between the top quark and Higgs masses and how they might be determined dynamically. Ansätze for the texture of fermion mass matrices and the resulting relationships between masses and mixing angles are considered in section 6. Mass protection is proposed as a natural explanation for the origin of fermion mass matrix texture in section 7. The neutrino mass problem is discussed in section 8 and finally, in section 9 we present a brief conclusion.

2 The Quark-Lepton Spectrum

The physical masses of the charged leptons can be directly measured and correspond to the poles in their propagators:

\[ M_e = 0.511 \text{ MeV} \quad M_\mu = 106 \text{ MeV} \quad M_\tau = 1.78 \text{ GeV} \quad (1) \]

However, due to confinement, the quark masses cannot be directly measured and have to be extracted from the properties of hadrons. Various techniques are used, such as chiral perturbation theory, QCD sum rules and lattice gauge theory. The quark mass parameters extracted from the data usually depend on a renormalisation scale \( \mu \) and the corresponding running masses \( m_q(\mu) \) are related to the propagator pole masses \( M_q \) by

\[ M_q = m_q(\mu = m_q) \left[ 1 + \frac{4}{3} \alpha_3(m_q) \right] \quad (2) \]

to leading order in QCD. The light \( u, d \) and \( s \) quark masses are usually normalised to the scale \( \mu = 1 \text{ GeV} \), corresponding to the non-perturbative scale of dynamical chiral symmetry breaking (or \( \mu = 2 \text{ GeV} \) for lattice measurements), and are typically given \([1]\) as follows:

\[
\begin{align*}
    m_u(1 \text{ GeV}) &= 4.5 \pm 1 \text{ MeV} \\
    m_d(1 \text{ GeV}) &= 8 \pm 2 \text{ MeV} \\
    m_s(1 \text{ GeV}) &= 150 \pm 50 \text{ MeV} 
\end{align*}
\]

However the renormalisation scale for the heavy quark masses is taken to be the quark mass itself which is in the perturbative regime:

\[
\begin{align*}
    m_c(m_c) &= 1.25 \pm 0.15 \text{ GeV} \\
    m_b(m_b) &= 4.25 \pm 0.15 \text{ GeV} \\
    m_t(m_t) &= 166 \pm 5 \text{ GeV} \quad (4)
\end{align*}
\]
Note that the top quark mass, \( M_t = 174 \pm 5 \text{ GeV} \), measured at FermiLab is interpreted as the pole mass.

The quark masses, of course, arise from the diagonalisation of the three generation mass matrices \( M_U \) and \( M_D \), by performing unitary transformations\(^1\) on the left-handed and right-handed quarks respectively:

\[
U_U M_U V_U^\dagger = \text{Diag}(m_u, m_c, m_t)
\]
\[
U_D M_D V_D^\dagger = \text{Diag}(m_d, m_s, m_b)
\]  

The quark mixing matrix

\[ V_{CKM} = U_U U_D^\dagger \]  

is a measure of the difference between the unitary transformations \( U_U \) and \( U_D \) acting on the left-handed up-type and down-type quarks and has also been measured:

\[
|V_{CKM}| = \begin{pmatrix}
0.9734 \pm 0.0008 & 0.2196 \pm 0.0020 & 0.0036 \pm 0.0007 \\
0.224 \pm 0.016 & 0.996 \pm 0.013 & 0.0412 \pm 0.002 \\
0.0077 \pm 0.0014 & 0.0397 \pm 0.0033 & 0.9992 \pm 0.0002
\end{pmatrix}
\]

Due to the arbitrariness in the phases of the quark fields, the mixing matrix \( V_{CKM} \) contains only one CP violating phase \(^2\), which is of order unity:

\[ \sin^2 \delta_{CP} \sim 1 \]

From solar and atmospheric neutrino oscillation data\(^3\), we know the neutrino mass squared differences:

\[ \Delta m^2_{21} \sim 5 \times 10^{-5} \quad \Delta m^2_{32} \sim 3 \times 10^{-3} \]

However we do not know the absolute neutrino masses although there is a similar upper limit of \( m_\nu \lesssim 1 \text{ eV} \) from tritium beta decay and from cosmology. Neutrino oscillation data also constrain the magnitudes of the lepton mixing matrix elements to lie in the following \( 3\sigma \) ranges\(^5\):

\[
|U_{MNS}| = \begin{pmatrix}
0.73 - 0.89 & 0.45 - 0.66 & < 0.24 \\
0.23 - 0.66 & 0.24 - 0.75 & 0.52 - 0.87 \\
0.06 - 0.57 & 0.40 - 0.82 & 0.48 - 0.85
\end{pmatrix}
\]

Due to the Majorana nature of the neutrino mass matrix, there are three unknown CP violating phases \( \delta, \alpha_1 \) and \( \alpha_2 \) in this case\(^4\). Note that, unlike the quark mixing matrix, \( U_{MNS} \) is not hierarchical; all its elements are of the same order of magnitude except for \( |U_{e3}| < 0.24 \).

\(^1\)Note that with an appropriate choice of the unitary matrices, it is possible to arrange that the mass eigenvalues \( m_i \) are real and positive for arbitrary mass matrices.
3 Fermion Mass and Mass Protection

A fermion mass term

\[ \mathcal{L}_{\text{mass}} = -m \bar{\psi}_L \psi_R + h.c. \]  

(11)
couples together a left-handed Weyl field \( \psi_L \) and a right-handed Weyl field \( \psi_R \), which then satisfy the Dirac equation

\[ i \gamma^\mu \partial_\mu \psi_L = m \psi_R \]  

(12)

If the two Weyl fields are not charge conjugates \( \psi_L \neq (\psi_R)^c \) we have a Dirac mass term and the two fields \( \psi_L \) and \( \psi_R \) together correspond to a Dirac spinor. However if the two Weyl fields are charge conjugates \( \psi_L = (\psi_R)^c \) we have a Majorana mass term and the corresponding four component Majorana spinor has only two degrees of freedom. Particles carrying an exactly conserved charge, like the electron carries electric charge, must be distinct from their anti-particles and can only have Dirac masses with \( \psi_L \) and \( \psi_R \) having equal charges. However a neutrino could be a massive Majorana particle.

If \( \psi_L \) and \( \psi_R \) have different quantum numbers, i.e. belong to inequivalent representations of a symmetry group \( G \) (\( G \) is then called a chiral symmetry), a Dirac mass term is forbidden in the limit of an exact \( G \) symmetry and they represent two massless Weyl particles. Thus the \( G \) symmetry “protects” the fermion from gaining a mass. Such a fermion can only gain a mass when \( G \) is spontaneously broken.

The left-handed and right-handed top quark, \( t_L \) and \( t_R \) carry unequal Standard Model \( SU(2) \times U(1) \) gauge charges \( \bar{Q} \):

\[ \bar{Q}_L \neq \bar{Q}_R \]  

(Chiral charges)

(13)

Hence electroweak gauge invariance protects the quarks and leptons from gaining a fundamental mass term (\( \bar{t}_L t_R \) is not gauge invariant). This mass protection mechanism is of course broken by the Higgs effect, when the vacuum expectation value of the Weinberg-Salam Higgs field

\[ <\phi_{WS}> = \sqrt{2} v = 246 \text{ GeV} \]  

(14)
breaks the gauge symmetry and the SM gauge invariant Yukawa couplings \( y_i \) generate the running quark masses \( m_i \), such as:

\[ m_t = y_t \frac{<\phi_{WS}>}{\sqrt{2}} = y_t v = y_t 174 \text{ GeV} \]  

(15)

\[ m_b = y_b \frac{<\phi_{WS}>}{\sqrt{2}} = y_b v = y_b 174 \text{ GeV} \]  

(16)
for the top and bottom quarks. In this way a top quark mass of the same order of magnitude as the SM Higgs field vacuum expectation value (vev) is naturally generated (with \( y_t \) unsuppressed). Thus the Higgs mechanism explains why the top quark mass is suppressed, relative to the fundamental (Planck, GUT...) mass scale of the physics beyond the SM, down to the scale of electroweak gauge symmetry breaking. However the further suppression of the other quark-lepton masses (\( y_b, y_c, y_s, y_u, y_d \ll 1 \)) remains a mystery, which it is natural to attribute to mass protection by another approximately conserved chiral (gauge) charge (or charges) beyond the SM, as discussed in section 7.

We remark here that, in the Minimal Supersymmetric Standard Model (MSSM), the SM Higgs field and its complex conjugate are replaced by two Higgs fields:

\[
\begin{align*}
\phi_{WS} & \rightarrow H_D & v_1 &= \frac{<H_D>}{\sqrt{2}} = v \cos \beta \\
\phi_{WS}^\dagger & \rightarrow H_U & v_2 &= \frac{<H_U>}{\sqrt{2}} = v \sin \beta
\end{align*}
\] (17)

So, in the MSSM, the top and bottom quark masses are expressed as follows in terms of their Yukawa coupling constants:

\[
\begin{align*}
m_t &= y_t \frac{<H_U>}{\sqrt{2}} = y_t v_2 = y_t 174 \sin \beta \text{ GeV} \tag{18} \\
m_b &= y_b \frac{<H_D>}{\sqrt{2}} = y_b v_1 = y_b 174 \cos \beta \text{ GeV} \tag{19}
\end{align*}
\]

Fermions which are vector-like under the SM gauge group (\( \vec{Q}_L = \vec{Q}_R \)) are not mass protected and are expected to have a large mass associated with new (grand unified, string...) physics. The Higgs particle, being a scalar, is not mass protected and \textit{a priori} would also be expected to have a large mass; this is the well-known gauge hierarchy problem.

\section{The Hierarchy Problem}

As just discussed, there is no symmetry within the SM model which protects the Higgs particle from acquiring a mass associated with physics beyond the SM. The Higgs boson mass squared gets corrections depending quadratically
on the SM ultra-violet (UV) cut-off $\Lambda$ from the one-loop diagrams of Fig. 1. However precision data from LEP indicate that the Higgs mass lies in the range $114 \text{ GeV} < M_h < 200 \text{ GeV}$. Therefore the sum of the bare mass squared term and the radiative corrections $\sim \Lambda^2$ from Fig. 1 must give a mass in this range. This leads to a fine-tuning problem for $\Lambda > 1 \text{ TeV}$ when, for example, the magnitude of the top loop contribution exceeds the physical Higgs mass. We now briefly discuss some proposed solutions to this problem.

### 4.1 Supersymmetry

The most popular approach to solving the hierarchy problem is based on supersymmetry (SUSY). Indeed the popularity of SUSY is largely based on its success in solving this problem and the consistency of the Minimal Supersymmetric Standard Model (MSSM) with the supersymmetric grand unification of the $SU(3) \times SU(2) \times U(1)$ running gauge coupling constants at a scale $\mu = \Lambda_{\text{GUT}} \sim 3 \times 10^{16} \text{ GeV}$, as illustrated in Fig. 2. In SUSY, essentially all the quadratically divergent loop diagrams in Fig. 1 have corresponding superpartner diagrams, involving stop, gaugino and higgsino loops. In the limit of exact supersymmetry, the diagrams cancel completely reflecting the opposite sign between fermion and boson loops. However in the MSSM, it is supposed that SUSY is softly broken at a scale $\mu = M_{\text{SUSY}} \sim 1 \text{ TeV}$. So, with
Figure 2: The running of the inverse fine structure constants in the MSSM.

typical superpartner masses of order $M_{SUSY}$, the cancellation is incomplete and the SM cut-off $\Lambda$ is replaced by $M_{SUSY}$.

It follows that SUSY solves the technical hierarchy problem, in the sense that once the ratio $\frac{M^2}{\Lambda^2}$ is set to be of order $10^{-28}$ in the MSSM at tree level, then it remains of order $10^{-28}$ to all orders of perturbation theory. Thus SUSY stabilizes the hierarchy between the electroweak and grand unified scales, but does not explain why the gauge hierarchy exists in the first place. This is reflected in the so-called doublet-triplet splitting problem of grand unified theories (GUTs). In the simplest GUT, the Weinberg-Salam Higgs doublet field combines with a coloured triplet Higgs field to form a 5-plet of SU(5). The Higgs doublet remains light of order 100 GeV, whereas the triplet Higgs contributes to proton decay and must therefore have a mass of order $10^{16}$ GeV. In the minimal SUSY SU(5) model, this requires a fine-tuning of parameters with an accuracy of 1 part in $10^{14}$.

4.2 Composite Higgs and Pseudo-Goldstones

The idea of replacing the elementary Higgs scalar field by composite pseudo-Goldstone bosons is motivated by the success of QCD as a theory of strong interactions. QCD neatly avoids the hierarchy problem, by the very definition
of the scale of strong interactions $\Lambda_{QCD} \sim 200$ MeV as the scale at which
the asymptotically free QCD fine structure constant $\alpha_3(\mu)$ diverges. The SM
one loop renormalisation group equation for $\alpha_3(\mu)$

$$16\pi^2 \frac{dg_3}{d\ln \mu} = b_3 g_3^2 \quad b_3 = -7$$

(20)
generates a logarithmic dependence on the energy scale $\mu$. Consequently
with an input value at the Planck scale, $\Lambda_{Planck} \sim 10^{19}$ GeV, for the QCD
coupling constant $g_3$ of order unity, $g_3(\Lambda_{Planck}) = 0.7$ corresponding to
$\alpha_3(\Lambda_{Planck}) = g_3^2(\Lambda_{Planck})/4\pi \sim 0.04$, the QCD scale is naturally generated to
be $\Lambda_{QCD} = \Lambda_{Planck} \exp(-32\pi^2/7) \sim 200$ MeV, where $\alpha_3(\Lambda_{QCD}) \rightarrow \infty$. Thus
the hierarchy $\Lambda_{QCD}/\Lambda_{Planck} \sim 4 \times 10^{-20}$ is obtained without fine-tuning.

The confining QCD force is responsible for the formation of a $\bar{q}q$ quark con-
densate in the vacuum, which spontaneously breaks the approximate global
$G = SU(2)_L \times SU(2)_R$ chiral symmetry associated with the light $u$ and $d$
quarks ($m_u, m_d \ll \Lambda_{QCD}$) down to the diagonal isospin group $H = SU(2)_L + R$
of hadronic physics. Thus three composite pseudo-Goldstone bosons – the
pions – arise as a result of this spontaneous breakdown of $\dim G - \dim H = 3$
of the global symmetry generators; the pions are not exactly massless Gold-
stone bosons because the global symmetry is explicitly broken by the small
quark masses $m_u$ and $m_d$. The quark condensate also spontaneously breaks
the electroweak $SU(2) \times U(1)$ gauge symmetry. So, in the absence of the
Higgs field, the pions would become the longitudinally polarised weak gauge
boson states, generating a mass for say the $W$ boson of $\frac{1}{2}g_2 f_\pi \sim 30$ MeV,
where $f_\pi \sim \Lambda_{QCD}/2 \sim 95$ MeV is the pion decay constant. However, in
the SM, there is an elementary Higgs field and the pions remain as phys-
ical pseudoscalar mesons, whose low energy dynamics is well described by
chiral perturbation theory based on the $G/H$ non-linear sigma model. This
non-linear sigma model becomes strongly coupled at an UV cut-off scale
$\Lambda \sim 4\pi f_\pi \sim 1$ GeV, where non-Goldstone hadronic bound states and reso-
nances are formed.

In technicolor models [6], electroweak symmetry breaking is provided by
an asymptotically free $SU(N_{TC})$ gauge theory without elementary scalars,
alogous to QCD, with a confinement scale of order the electroweak gauge

$$\Lambda_{TC} \sim v = 174 \text{ GeV}$$

(21)
Massless technifermions, analogous to the up and down quarks, form vac-
uum condensates spontaneously breaking the electroweak gauge symmetry
and generating massless Goldstone bosons known as technipions. These technipions provide the longitudinal $W$ and $Z$ gauge boson states and their physical masses. The generated $W$ boson mass is given in this case by $M_W = \frac{1}{2}g_2 f_{TC} \sim 80$ GeV, where $f_{TC} = \sqrt{2}v = 246$ GeV is the technipion decay constant. So the hierarchy problem is naturally solved without fine-tuning. The UV cut-off for the non-linear sigma model description of the technipion interactions is given by $\Lambda \sim 4\pi f_{TC} \sim 2$ TeV, which is the scale where the new technihadron physics enters. However the simple technicolor mechanism does not communicate the electroweak symmetry breaking to the quarks and leptons, leaving them massless. In order to generate quark and lepton masses, it is necessary to complicate the model significantly, introducing phenomenological problems with flavour changing neutral currents and precision electroweak data.

There have also been attempts to identify the Higgs boson as a bound state of a $t$ and a $\bar{t}$ quark, as a consequence of a large Yukawa coupling of the top quark and the formation of a top quark condensate. Top quark condensation models lead to the infrared fixed point prediction for the top quark mass (see section 5.1).

The idea of identifying the Higgs boson itself as a pseudo-Goldstone boson has recently been revived in the so-called Little-Higgs models [7]. The new ingredient is to arrange that the UV cut-off $\Lambda \sim 4\pi f$ of the associated $G/H$ non-linear sigma model is postponed to 10 TeV without fine-tuning, corresponding to a pseudo-Goldstone decay constant $f$ of order 1 TeV. It is hoped that the higher (10 TeV) scale for the new physics, underlying the $G \rightarrow H$ global symmetry breaking, is sufficient to avoid it giving phenomenological problems associated with precision data and flavour changing neutral currents. The non-zero masses for the pseudo-Goldstone bosons are generated by the gauge, Yukawa and scalar couplings, which explicitly break the global symmetries. In order to obtain a realistic light Higgs scenario, it is necessary that the one loop quadratically divergent contributions to the Higgs mass in Fig. 1 are naturally cancelled by weakly coupled physics at the scale $f$. These cancellations involve one loop diagrams containing new particles, related to the top quark, the $W$ and $Z$ gauge bosons and the Higgs particle, having appropriate coupling constant values. Furthermore, in contrast to supersymmetry, they occur between particles with the same statistics. As a consequence of these cancellations, quadratically divergent corrections to the Higgs mass arise only at two and higher loop order, making the small Higgs mass natural and motivating the name “Little Higgs”. This Little Higgs
scenario requires that the gauge, Yukawa and scalar couplings are organised in such a way that each one separately preserves enough of the global symmetry to forbid the Higgs mass. Then two or more different couplings are required to act together, in two or higher loop diagrams, in order to generate quadratically divergent contributions to the Higgs mass.

As a concrete example, we outline the structure of the simplest model based on an $SU(5)/SO(5)$ non-linear sigma model and known as the Littlest Higgs model [8]. The $SU(5)$ global symmetry contains the extended gauge group $[SU(2) \times U(1)]^2$. At the same time as the global symmetry breakdown, $SU(5) \rightarrow SO(5)$, the gauge symmetry $[SU(2) \times U(1)]^2$ breaks down to its diagonal subgroup $SU(2) \times U(1)$ identified as the SM electroweak gauge group. The global symmetry breakdown results in 14 Goldstone bosons, which decompose under the electroweak gauge group $SU(2) \times U(1)$ as a real singlet $1_0$, a real triplet $3_0$, a complex doublet $2_{\pm1/2}$ and a complex triplet $3_{\pm1}$. The real singlet and triplet bosons become the longitudinal components of the extra gauge bosons $Z^0$ and $W'^{\pm0}$, giving them masses of order $f \sim 1$ TeV. The gauge and Yukawa couplings break the $SO(5)$ global symmetry and generate a Coleman-Weinberg type potential [9] for the remaining Goldstone bosons. This potential gives rise to a vacuum expectation value $v = 174$ GeV for the complex doublet. However the complex triplet is not protected by the global symmetry from one loop divergent corrections and hence obtains a heavy mass of order $f \sim 1$ TeV. Finally, in order to cancel the quadratic divergence in the top quark one loop correction to the Higgs mass, it is necessary to introduce a vector-like (i.e. non-chiral) weak isosinglet quark $T$ of charge $2/3$. As explained in section [8] vector-like fermions are not mass protected and so a bare mass term is allowed for the $T$ quark, which is chosen to be of order $f \sim 1$ TeV.

Little Higgs models have characteristic physics signals at the TeV scale which may be seen at the LHC. The presence of a vector-like $T$ quark of charge $2/3$ is generically required to cancel the divergence from the top quark loop. It could be pair produced or singly together with a jet at hadron colliders, decaying to $Wb$, $th$ and $tZ$. We also expect new gauge bosons which cancel the $W$ and $Z$ gauge boson loops. Their quantum numbers, decay modes and production mechanisms are model dependent. Finally one expects extra scalars some of which could have a TeV scale mass, like the triplet in the Littlest Higgs model, and some (typically second Higgs doublets) could be protected from one loop divergent masses becoming as light as the SM Higgs. These new states generate contributions to precision electroweak observables.
and flavour changing neutral currents, which provide important constraints on Little Higgs models.

### 4.3 Multiple Point Principle

In this approach \[10\] to the hierarchy problem, the idea is not to introduce any new physics beyond the SM, but rather a new fundamental principle: the Multiple Point Principle (MPP). According to this MPP, nature chooses coupling constant values such that a number of vacuum states have the same energy density. This MPP assumption, of course, explicitly introduces a fine-tuning mechanism, namely the degeneracy of the vacua. We shall use MPP in section \[5.2\] to predict the top quark and SM Higgs masses, by postulating the existence of a vacuum state at the fundamental (Planck) scale \(<\phi_{WS}>\sim \Lambda_{Planck} \sim 10^{19}\) GeV degenerate with the usual SM vacuum having \(<\phi_{WS}> = 246\) GeV. This application of the MPP assumes the existence of the hierarchy \(v/\Lambda_{Planck} \sim 10^{-17}\). In order to actually derive this hierarchy, it is necessary to have another vacuum at the electroweak scale degenerate with the above two. This third vacuum is postulated to correspond to a phase in which an effective 6\(t\) and 6\(T\) S-wave bound state scalar field develops a non-zero vev. The vacuum degeneracy conditions then predict values for the top quark Yukawa coupling at both the fundamental scale \(g_t(\mu_{fundamental})\) and the electroweak scale \(g_t(v)\). Now having MPP predicted values for \(g_t(\mu)\) at two different scales, we can calculate the ratio of these scales from the SM renormalisation group equations (the SM gauge coupling constants contributing to the running are considered as given). In this way we can derive the gauge hierarchy ratio \(v/\mu_{fundamental} \sim 10^{-17}\) and show that \(\mu_{fundamental} \sim \Lambda_{Planck} \sim 10^{19}\) GeV. This scenario is discussed in more detail in Holger Nielsen’s lectures \[10\].

### 5 Top Quark and Higgs Masses

As the top quark mass is the dominant term in the SM fermion mass matrix, it is likely that its value will be understood dynamically before those of the other fermions. It has been known for some time \[11, 12\] that the self-consistency of the pure SM up to some physical cut-off scale \(\Lambda\) imposes constraints on the top quark and Higgs boson masses. The first constraint is the so-called triviality bound: the running Higgs coupling constant \(\lambda(\mu)\)
should not develop a Landau pole for $\mu < \Lambda$. The second is the vacuum stability bound: the running Higgs coupling constant $\lambda(\mu)$ should not become negative leading to the instability of the usual SM vacuum. These bounds are illustrated \[13\] in Fig. 3 where the combined triviality and vacuum stability bounds for the SM are shown for different values of the high energy cut-off $\Lambda$. The allowed region is the area around the origin bounded by the co-ordinate axes and the solid curve labelled by the appropriate value of $\Lambda$. In the following we shall be interested in large cut-off scales $\Lambda \simeq 10^{15} - 10^{19}$ GeV, corresponding to the grand unified (GUT) or Planck scale. The upper part of each curve corresponds to the triviality bound. The lower part of each curve coincides with the vacuum stability bound and the point in the top right hand corner, where it meets the triviality bound curve, is the quasi-fixed infra-red fixed point for that value of $\Lambda$. The vacuum stability curve will be important for the discussion of the MPP prediction of the top quark and SM Higgs boson masses in section 5.2. Before this however, we will discuss their quasi-fixed point values in the SM and the MSSM.

5.1 Infra-red Fixed Point

The idea that the top quark mass might be determined dynamically as an infrared fixed point of the renormalisation group equations is quite old \[14\]. In practice one finds an effective infrared stable quasifixed point at the scale $\mu = m_t$, where the QCD gauge coupling constant $g_3(\mu)$ is slowly varying \[15\]. The quasifixed point prediction of the top quark mass is based on two assumptions: (a) the perturbative renormalisation group equations are valid up to some high (e.g. GUT or Planck) energy scale $\Lambda \simeq 10^{16} - 10^{19}$ GeV, and (b) the top Yukawa coupling constant is large at the high scale $g_t(\Lambda) \gtrsim 1$.

Neglecting the lighter quark masses and mixings, which is a good approximation, the SM one loop renormalisation group equation for the top quark Yukawa coupling $g_t(\mu) = \sqrt{2} m_t(\mu)/\langle \phi_{WS} \rangle$ is:

$$16\pi^2 \frac{dg_t}{d \ln \mu} = g_t \left( \frac{9}{2} g_t^2 - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \right)$$

(22)

The gauge coupling constants $g_i(\mu)$ satisfy the equations:

$$16\pi^2 \frac{dg_i}{d \ln \mu} = b_i g_i^3 \quad \text{where} \quad b_1 = \frac{41}{6}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7 \quad \text{in the SM}.$$ 

(23)
Figure 3: SM bounds in the ($M_t, m_H$) plane for various values of $\Lambda$, the scale at which new physics enters.
The nonlinearity of the renormalisation group equations then strongly focuses $g_t(\mu)$ at the electroweak scale to its quasifixed point value. We note that while there is a rapid convergence to the top Yukawa coupling fixed point value from above, the approach from below is much more gradual. The renormalisation group equation for the Higgs self-coupling

$$16\pi^2 \frac{d\lambda}{d\ln \mu} = 12\lambda^2 + 3 \left(4g_t^2 - 3g_2^2 - g_1^2 \right) \lambda + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4$$

similarly focuses $\lambda(\mu)$ towards a quasifixed point value, leading to the SM fixed point predictions [15] for the running top quark and Higgs masses:

$$m_t \simeq 225 \text{ GeV} \quad m_H \simeq 250 \text{ GeV} \quad (25)$$

Unfortunately these predictions are inconsistent with the Fermilab results, which require a running top mass $m_t \simeq 166 \pm 5 \text{ GeV}$.

The fixed point top Yukawa coupling is reduced by about 15% in the MSSM to a value of $g_t(m_t) \simeq 1.1$ as shown in Fig. [14][16], due to the contribution of the superpartners to the renormalisation group beta functions. Also the top quark couples to just one of the MSSM Higgs doublets $H_U$, leading to the fixed point prediction

$$m_t(m_t) \simeq (190 \text{ GeV}) \sin \beta \quad (26)$$

for $\tan \beta \lesssim 15$. Including the stop gluino one-loop SUSY correction the experimental running top quark mass becomes $m_t = 160 \pm 5 \text{ GeV}$. Thus

Figure 4: The rapid convergence from above of the top Yukawa coupling constant $\lambda_t \equiv g_t$ to the MSSM infra-red quasi-fixed point value as $\mu \to m_t$. 

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agreement with the fixed point prediction would require \( \tan \beta = 1.5 \pm 0.3 \), which is ruled out in the MSSM at the 3\( \sigma \) level by LEP data \cite{1}. However we note that the infrared fixed point scenario for the top quark mass is consistent in the next to minimal supersymmetric standard model (NMSSM) \cite{17}.

For large \( \tan \beta \) it is possible to have a bottom quark Yukawa coupling satisfying \( g_b(\Lambda) \gtrsim 1 \) which then approaches an infrared quasifixed point and is no longer negligible in the renormalisation group equation for \( g_t(\mu) \). Indeed with \( \tan \beta \approx 60 \), we can trade the mystery of the top to bottom quark mass ratio for that of a hierarchy of vacuum expectation values, \( v_2/v_1 \approx 60 \), and have all the third generation Yukawa coupling constants large:

\[
g_t(\Lambda) \gtrsim 1 \quad g_b(\Lambda) \gtrsim 1 \quad g_\tau(\Lambda) \gtrsim 1\tag{27}
\]

Then \( m_t, m_b \) and \( m_\tau \) all approach infrared quasifixed point values\(^2\) compatible with experiment \cite{18,19}. This large \( \tan \beta \) scenario is consistent with the idea of Yukawa unification \cite{20,21}:

\[
g_t(\Lambda) = g_b(\Lambda) = g_\tau(\Lambda) = g_G\tag{28}
\]

as occurs in the SO(10) SUSY-GUT model with the two MSSM Higgs doublets in a single 10 irreducible representation and \( g_G \gtrsim 1 \) ensures fixed point behaviour. However it should be noted that the equality in eq. (28) is not necessary to obtain the fixed point values. Also the lightest Higgs particle mass is predicted to be \( m_{h^0} \approx 120 \text{ GeV} \).

### 5.2 Multiple Point Principle

The application of the MPP to the pure Standard Model \cite{22}, with a cut-off close to \( \Lambda_{\text{Planck}} \), implies that the SM parameters should be adjusted, such that there exists another vacuum state degenerate in energy density with the vacuum in which we live. This means that the effective SM Higgs potential \( V_{\text{eff}}(\phi_{WS}) \) should have a second minimum degenerate with the well-known first minimum at the electroweak scale \( \langle \phi_{WS} \rangle = \phi_{\text{vac}1} = 246 \text{ GeV} \). Thus we predict that our vacuum is barely stable and we just lie on the vacuum stability curve in the top quark, Higgs particle (pole) mass \((M_t, M_H)\) plane, shown \cite{23} in Fig. 5 for a cut-off \( \Lambda = 10^{19} \text{ GeV} \). Furthermore we expect the

\(^2\)However we note, in the large \( \tan \beta \) scenario, SUSY radiative corrections to \( m_b \) are generically large: the bottom quark mass gets a contribution proportional to \( v_2 \) from some one-loop diagrams, whereas its tree level mass is proportional to \( v_1 = v_2/\tan \beta \).
second minimum to be within an order of magnitude or so of the fundamental scale, i.e. $< \phi_{WS} > = \phi_{vac 2} \simeq \Lambda_{Planck}$. In this way, we essentially select a particular point on the SM vacuum stability curve and hence the MPP condition predicts precise values for $M_t$ and $M_H$.

For the purposes of our discussion it is sufficient to consider the renormalisation group improved tree level effective potential $V_{eff}(\phi_{WS})$. We are interested in values of the Higgs field of the order $\phi_{vac 2} \simeq \Lambda_{Planck}$, which is very large compared to the electroweak scale, and for which the quartic term strongly dominates the $|\phi_{WS}|^2$ term; so to a very good approximation we have:

$$V_{eff}(\phi_{WS}) \simeq \frac{1}{8} \lambda(\mu = |\phi_{WS}|) |\phi_{WS}|^4$$

(29)

The running Higgs self-coupling constant $\lambda(\mu)$ and the top quark running Yukawa coupling constant $g_t(\mu)$ are readily computed by means of the renormalisation group equations, which are in practice solved numerically, using the second order expressions for the beta functions.

The vacuum degeneracy condition is imposed by requiring:

$$V_{eff}(\phi_{vac 1}) = V_{eff}(\phi_{vac 2})$$

(30)

Now the energy density in vacuum 1 is exceedingly small compared to $\phi_{vac 2}^4 \simeq \Lambda_{Planck}^4$. So we basically get the degeneracy condition, eq. (30), to mean that
the coefficient $\lambda(\phi_{\text{vac}})$ of $|\phi_{\text{vac}}|^4$ must be zero with high accuracy. At the same $\phi$-value the derivative of the effective potential $V_{\text{eff}}(\phi_{WS})$ should be zero, because it has a minimum there. Thus at the second minimum of the effective potential the beta function $\beta_\lambda$ also vanishes:

$$\beta_\lambda(\mu = \phi_{\text{vac}}) = \lambda(\phi_{\text{vac}}) = 0$$

which gives to leading order the relationship:

$$\frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 = 0$$

between the top quark Yukawa coupling and the electroweak gauge coupling constants $g_1(\mu)$ and $g_2(\mu)$ at the scale $\mu = \phi_{\text{vac}} \simeq \Lambda_{\text{Planck}}$. We use the renormalisation group equations to relate the couplings at the Planck scale to their values at the electroweak scale. Figure 6 shows the running coupling constants $\lambda(\phi)$ and $g_t(\phi)$ as functions of $\log(\phi)$. Their values at the electroweak scale give our predicted combination of pole masses [22]:

$$M_t = 173 \pm 5 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV}$$

6 Mass Matrix Ansätze and Texture

The motivation for considering mass matrix ansätze is to obtain testable relationships between fermion masses and mixing angles, which might reduce the number of parameters in the Yukawa sector and provide a hint to the physics beyond the SM. We shall focus on the charged fermion sector here, particularly the quarks, and postpone discussion of neutrino masses and mixings to section 8. The hierarchical structure of the mass spectrum should be reflected in the fermion mass matrices. So it is natural to speculate that the smaller matrix elements may contribute so weakly to the physical masses and mixing angles that they can effectively be neglected and replaced by zero—the so-called texture zeros. The most famous ansatz incorporating such a texture zero is the Fritzsch hermitian ansatz [24] for the two generation quark mass matrices:

$$M_U = \begin{pmatrix} 0 & B \\ B^* & A \end{pmatrix} \quad M_D = \begin{pmatrix} 0 & B' \\ B'^* & A' \end{pmatrix}$$

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Figure 6: Plots of $\lambda$ and $g_t$ as functions of the scale of the Higgs field $\phi$ for degenerate vacua with the second Higgs VEV at the Planck scale $\phi_{\text{vac}2} = 10^{19} \text{ GeV}$. We formally apply the second order SM renormalisation group equations up to a scale of $10^{25} \text{ GeV}$.

The assumed hierarchical structure gives the following conditions:

$$|A| \gg |B|, \quad |A'| \gg |B'|$$

among the parameters. It follows that the two generation Cabibbo mixing is given by the well-known Fritzsch formula

$$|V_{us}| \approx \left| \sqrt{\frac{m_d}{m_u}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right|$$

where $\phi = \arg B' - \arg B$. This relationship fits the experimental value well, provided that the phase $\phi$ is close to $\frac{\pi}{2}$. The generalisation of the Fritzsch ansatz to three generations:

$$M_U = \begin{pmatrix} 0 & C & 0 \\ C^* & 0 & B \\ 0 & B^* & A \end{pmatrix} \quad M_D = \begin{pmatrix} 0 & C' & 0 \\ C'^* & 0 & B' \\ 0 & B'^* & A' \end{pmatrix}$$

with the assumed hierarchy of parameters:

$$|A| \gg |B| \gg |C|, \quad |A'| \gg |B'| \gg |C'|$$
however leads to an additional relationship

$$|V_{cb}| \simeq \sqrt{\frac{m_s}{m_b} - e^{-i\phi_2}} \sqrt{\frac{m_c}{m_t}}$$

(39)

which is excluded by the data for any value of the phase $\phi_2$. Consistency with experiment can, for example, be restored by introducing a non-zero $(M_U)_{22}$ mass matrix element [25].

Several ansätze have been proposed—for example, see [26] for a systematic analysis of symmetric quark mass matrices with texture zeros at the SUSY-GUT scale. Here I will concentrate on the lightest family mass generation model [27], which successfully expresses all the quark mixing angles in terms of simple, compact formulae involving quark mass ratios. According to this model the flavour mixing for quarks is basically determined by the mechanism responsible for generating the physical masses of the up and down quarks, $m_u$ and $m_d$ respectively. So, in the chiral symmetry limit, when $m_u$ and $m_d$ vanish, all the quark mixing angles vanish. Therefore we are led to consider an ansatz in which the diagonal mass matrix elements for the second and third generations are practically the same in the gauge (unrotated) and physical bases.

The mass matrix for the down quarks ($D = d, s, b$) is taken to be hermitian with three texture zeros of the following form:

$$M_D = \begin{pmatrix} 0 & a_D & 0 \\ a_D^* & A_D & b_D \\ 0 & b_D^* & B_D \end{pmatrix}$$

(40)

where

$$B_D = m_b + \delta_D \quad A_D = m_s + \delta'_D \quad |\delta_D| \ll m_s \quad |\delta'_D| \ll m_d$$

(41)

It is, of course, necessary to assume some hierarchy between the elements, which we take to be: $B_D \gg A_D \sim |b_D| \gg |a_D|$. The zero in the $(M_D)_{11}$ element corresponds to the commonly accepted conjecture that the lightest family masses appear as a direct result of flavour mixings. The zero in $(M_D)_{13}$ means that only minimal “nearest neighbour” interactions occur, giving a tridiagonal matrix structure. Since the trace and determinant of the hermitian matrix $M_D$ gives the sum and product of its eigenvalues, it follows that

$$\delta_D \simeq -m_d$$

(42)
while $\delta'_D$ is vanishingly small and can be neglected in further considerations.

It may easily be shown that equations (40 - 42) are entirely equivalent to the condition that the diagonal elements ($A_D$, $B_D$) of $M_D$ are proportional to the modulus square of the off-diagonal elements ($a_D$, $b_D$):

$$\frac{A_D}{B_D} = \left| \frac{a_D}{b_D} \right|^2$$

(43)

Using the conservation of the trace, determinant and sum of principal minors of the hermitian matrix $M_D$ under unitary transformations, we are led to a complete determination of the moduli of all its elements, which can be expressed to high accuracy as follows:

$$|M_D| = \begin{pmatrix}
0 & \sqrt{m_d m_s} & 0 \\
\sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\
0 & \sqrt{m_d m_b} & m_b - m_d
\end{pmatrix}$$

(44)

The mass matrix for the up quarks is taken to be of the following hermitian form:

$$M_U = \begin{pmatrix}
0 & 0 & c_U \\
0 & A_U & 0 \\
c_U^* & 0 & B_U
\end{pmatrix}$$

(45)

The moduli of all the elements of $M_U$ can also be readily determined in terms of the physical masses as follows:

$$|M_U| = \begin{pmatrix}
0 & 0 & \sqrt{m_u m_t} \\
0 & m_c & 0 \\
\sqrt{m_u m_t} & 0 & m_t - m_u
\end{pmatrix}$$

(46)

The CKM quark mixing matrix elements can now be readily calculated by diagonalising the mass matrices $M_D$ and $M_U$. They are given in terms of quark mass ratios as follows:

$$|V_{us}| = \sqrt{\frac{m_d}{m_s}} = 0.222 \pm 0.004 \quad |V_{us}|_{exp} = 0.221 \pm 0.003$$

(47)

$$|V_{cb}| = \sqrt{\frac{m_d}{m_b}} = 0.038 \pm 0.004 \quad |V_{cb}|_{exp} = 0.039 \pm 0.003$$

(48)

$$|V_{ub}| = \sqrt{\frac{m_u}{m_t}} = 0.0036 \pm 0.0006 \quad |V_{ub}|_{exp} = 0.0036 \pm 0.0006$$

(49)
As can be seen, they are in impressive agreement with the experimental values.

The proportionality condition \( (43) \) is not so easy to generate from an underlying symmetry beyond the Standard Model, but it is possible to realise it in a local chiral \( SU(3) \) family symmetry model \( [28] \).

It is common to make ansätze in the context of SUSY-GUT models, which of course give relationships between the fermion mass parameters at the grand unified scale \( \Lambda_{GUT} \). In grand unified theories, the SM gauge group \( SMG \equiv SU(3) \times SU(2) \times U(1) \) is embedded in a simple Lie group \( G \) and the SM interactions become unified for scales \( \mu > \Lambda_{GUT} \). Also each generation of quark and lepton SM irreducible representations are combined into larger irreducible representations of \( G \), introducing symmetries between quarks and leptons. In the minimal \( SU(5) \) model the two MSSM Higgs doublets are promoted into \( 5 \) and \( \overline{5} \) representations, giving the well-known GUT relation

\[
m_b(\Lambda_{GUT}) = m_\tau(\Lambda_{GUT}),
\]

which is consistent with experiment. However the model also predicts the equality of the full down quark and charged lepton mass matrices \( M_D = M_E \) and, in particular:

\[
m_d/m_s = m_e/m_\mu
\]

which fails phenomenologically by an order of magnitude. So it is necessary to introduce a more complicated group theoretical structure.

Georgi and Jarlskog \( [29] \) introduced a Higgs field in the the 45 dimensional representation of \( SU(5) \) and postulated that the Weinberg Salam Higgs field \( H_D \) resides in a linear combination of the \( 5 \) and \( \overline{45} \) representations. Furthermore they assumed the following ansatz, in which the \( \overline{45} \) only contributes to the \((M_D)_{22}\) and \((M_E)_{22}\) matrix elements and dominates them:

\[
M_D = \begin{pmatrix} 0 & F & 0 \\ F & E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad M_E = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}
\]

Then, with the assumed hierarchical parameters \( |D| \gg |E| \gg |F| \), we are led to the mass relations

\[
m_b(\Lambda_{GUT}) = m_\tau(\Lambda_{GUT}), \quad m_s(\Lambda_{GUT}) = m_\mu(\Lambda_{GUT})/3
\]

and

\[
m_d(\Lambda_{GUT}) = 3m_e(\Lambda_{GUT}),
\]

which are consistent with experiment. This ansatz is easily extended \( [31] \) to \( SO(10) \), in which right-handed neutrinos are introduced for each generation. There are then 16 fermion states for each generation, which fit into the single \( 16 \) representation of \( SO(10) \) and
are such that the sum of the \((B - L)\) charges for the 16 fermions vanishes. So the \((B - L)\) charge can be gauged and \(SU(5) \times U(1)_{B-L}\) is a subgroup of \(SO(10)\). However, in order to also generate realistic up quark masses and CKM matrix, it is necessary to further complicate the group theoretical ansatz, see for example \[32, 33, 34\].

We now turn to the question of the dynamics underlying the hierarchical texture of the above ansätze.

### 7 Origin of Texture

As pointed out in section 3, a natural explanation of the charged fermion mass hierarchy would be mass protection, due to the existence of some approximately conserved chiral charges beyond the SM. These chiral charges, which we should like to identify with gauge quantum numbers in the fundamental theory beyond the SM, provide selection rules forbidding the transitions between the various left-handed and right-handed quark-lepton states, except for the top quark. In order to generate mass terms for the other fermion states, we have to introduce new Higgs fields, which break the symmetry group \(G\) of the fundamental theory down to the SM group. We also need suitable intermediate fermion states to mediate the forbidden transitions, which we take to be vector-like Dirac fermions with a mass of order the fundamental scale \(M_F\) of the theory. In this way effective SM Yukawa coupling constants are generated \[35\], which are suppressed by the appropriate product of Higgs field vacuum expectation values measured in units of \(M_F\). We assume that all the couplings in the fundamental theory are unsuppressed, i.e. they are all naturally of order unity.

Consider, for example, the model obtained by extending the Standard Model gauge group \(SMG = SU(3) \times SU(2) \times U(1)\) with a gauged chiral abelian flavour group \(U(1)_f\). This \(SMG \times U(1)_f\) gauge group is assumed to be broken to \(SMG\) by the vev of a scalar field \(\phi_S\) where \(\langle \phi_S \rangle < M_F\) and \(\phi_S\) carries \(U(1)_f\) charge \(Q_f(\phi_S) = 1\). Suppose further that the \(U(1)_f\) charges of the Weinberg Salam Higgs field and the left- and right-handed bottom quark fields are:

\[
Q_f(\phi_{WS}) = 0 \quad Q_f(b_L) = 0 \quad Q_f(b_R) = 2
\] (53)
Then it is natural to expect the generation of a mass for the $b$ quark of order:

$$m_b \simeq \left( \frac{\langle \phi_S \rangle}{M_F} \right)^2 \langle \phi_{WS} \rangle$$

(54)

via the tree level diagram shown in Fig. 7 involving the exchange of two $\langle \phi_S \rangle$ tadpoles, in addition to the usual $\langle \phi_{WS} \rangle$ tadpole, with two appropriately charged vector-like fermion intermediate states of mass $M_F$. We identify $\epsilon_f = \langle \phi_S \rangle/M_F$ as the $U(1)_f$ flavour symmetry breaking parameter. In general we expect mass matrix elements of the form:

$$M(i, j) = \gamma_{ij} \epsilon_f^{n_{ij}} \langle \phi_{WS} \rangle$$

(55)

between the $i$th left-handed and $j$th right-handed fermion components, where

$$\gamma_{ij} = \mathcal{O}(1), \quad n_{ij} = | Q_f(\psi_{Li}) - Q_f(\psi_{Rj}) |$$

(56)

So the effective SM Yukawa couplings of the quarks and leptons to the

$$\begin{array}{cccc}
Q_f = 0 & Q_f = 1 & Q_f = 1 & Q_f = 2 \\
b_L & M_F & M_F & b_R \\
\langle \phi_S \rangle & \otimes & \langle \phi_{WS} \rangle & \langle \phi_S \rangle
\end{array}$$

Figure 7: Feynman diagram which generates the $b$ quark mass via superheavy intermediate states.

Weinberg-Salam Higgs field $y_{ij} = \gamma_{ij} \epsilon_f^{n_{ij}}$ can consequently be small even though all fundamental Yukawa couplings of the “true” underlying theory are of $\mathcal{O}(1)$. We are implicitly assuming here that there exists a superheavy spectrum of states which can mediate all of the symmetry breaking transitions; in particular we do not postulate the absence of appropriate superheavy states in order to obtain exact texture zeros in the mass matrices. The mass matrix elements, eq. (55), are only predicted in order of magnitude, unless one can specify the spectrum of superheavy fermions and their Yukawa couplings.
It does not seem possible to explain the SM fermion mass spectrum with an anomaly free set of flavour charges in an \( SMG \times U(1)_f \) model with a single Higgs field \( \phi_S \) breaking the \( U(1)_f \) gauge symmetry. One can introduce a second gauged abelian\(^3\) flavour group \( U(1)_{f'} \) and obtain a realistic charged fermion spectrum in an \( SMG \times U(1)_f \times U(1)_{f'} \) model\(^4\). In supersymmetric models one can allow an abelian flavour gauge symmetric extension of the MSSM to be anomalous \([40],[41]\), this anomaly being cancelled by the Green-Schwartz mechanism \([42]\). Models with this property clearly point towards a string theory origin.

The texture of some of the fermion mass matrix ansätze in SUSY-GUT models, such as \([32],[33]\), can be generated from global chiral abelian flavour models possibly combined with some discrete symmetries. However the required spectrum of Higgs fields and superheavy matter fields tends to be rather complicated. Texture zeros appear as a consequence of the absence of the superheavy states needed to mediate the transition between the corresponding SM Weyl states in diagrams similar to Fig. 7. For example an explicit \( SO(10) \) SUSY-GUT model has been constructed by Albright and Barr \([43]\), in which the \( SO(10) \) and mass protecting flavour quantum numbers of all the Higgs and matter fields are specified. It is based on a global \( U(1) \times Z_2 \times Z_2 \) flavour symmetry that stabilizes a solution to the doublet-triplet splitting problem, mentioned in section 4.1, and determines the structure of the Higgs and Yukawa superpotentials from which the fermion mass matrices are constructed. The required \( SO(10) \) representations of Higgs fields are: one \( 45 \), two \( 16 \oplus \bar{16} \) pairs, six \( 10 \) and five \( 1 \). In addition to the three \( 16 \) representations for the the quarks and leptons, the superheavy matter fields comprise the following \( SO(10) \) representations: two \( 16 \oplus \bar{16} \) pairs, two \( 10 \) and six \( 1 \). Finally to avoid too rapid proton decay, there is also a discrete \( Z_2 \) matter symmetry. The resulting mass matrices for the down quarks and leptons are

\[
M_D = \begin{pmatrix}
\eta & \delta & \delta' e^{i\phi} \\
\delta & 0 & \sigma + \epsilon/3 \\
\delta' e^{i\phi} & -\epsilon/3 & 1
\end{pmatrix} m_b^0
\]

\[
M_E = \begin{pmatrix}
\eta & \delta & \delta' e^{i\phi} \\
\delta & 0 & -\epsilon \\
\delta' e^{i\phi} & \sigma + \epsilon & 1
\end{pmatrix} m_b^0
\]

\( (57) \)

\(^3\)Non-abelian \( SU(2) \) and \( SU(3) \) flavour symmetry groups have also been considered, see for example \([28],[36],[57]\) for some recent examples.

\(^4\)This \( SMG \times U(1)^2 \) model has exactly the same charged fermion spectrum as the so-called anti-grand unification model \([58]\).
and the up quark and Dirac neutrino\(^5\) mass matrices are:
\[
M_U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} m_\nu^0 \\
M_N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & -\epsilon & 1 \end{pmatrix} m_\nu^0
\]
where there is a natural hierarchy of parameters \(\sigma \sim 1 \gg \epsilon \gg \delta, \delta' \gg \eta\). Since the spectrum of heavy states has been fully specified, there are no missing \(\mathcal{O}(1)\) factors in eqs. 57 and 58. The nine SM charged fermion masses, the three CKM mixing angles and CP violating phase are well-fitted with the 8 parameters in the above matrices, after renormalisation group evolution from the GUT scale.

The flavour symmetry group should of course be a natural feature of the fundamental theory beyond the SM. In the case of SUSY-GUT models, it is hoped that the group arises from string theory. An attractive possibility is that the flavour symmetry group is simply a sub-group of the gauge symmetry group of the fundamental theory. This is the case in the so-called anti-grand unification theory (AGUT) or family replicated gauge group model \(^{38}\)\(^{44}\). The AGUT model is based on a non-supersymmetric non-simple extension of the SM with three copies of the SM gauge group—one for each family or generation. With the inclusion of three right-handed neutrinos, the AGUT gauge group becomes \(G = (SMG \times U(1)_{B-L})^3\), where the three copies of the SM gauge group are supplemented by an abelian \((B - L)\) (= baryon number minus lepton number) gauge group for each family\(^6\). The AGUT gauge group \(G\) is the largest anomaly free group, transforming the known 45 Weyl fermions plus the three right-handed neutrinos into each other unitarily, which does not unify the irreducible representations under the SM gauge group. It is supposed to be effective at energies near to the Planck scale, where the \(i\)’th proto-family couples to just the \(i\)’th \(SMG \times U(1)_{B-L}\) factor. This gauge group is broken down by four Higgs fields \(W, T, \rho\) and \(\omega\), having vevs about one order of magnitude lower than the Planck scale, to its diagonal subgroup:
\[
(SMG \times U(1)_{B-L})^3 \rightarrow SMG \times U(1)_{B-L}
\]
(59)
The diagonal \(U(1)_{B-L}\) is broken down at the see-saw scale by another Higgs field \(\phi_{SS}\) and the diagonal \(SMG\) is broken down to \(SU(3) \times U(1)_{em}\) by the

\(^5\)\(M_N\) is inserted in the well-known see-saw formula for the light neutrino mass matrix given in section\(^{38}\)

\(^6\)The family replicated gauge groups \((SO(10))^3\) and \((E_6)^3\) have recently been considered by Ling and Ramond \(^{44}\).
Weinberg-Salam Higgs field. We note that the \((SMG \times U(1)_{B-L})^3\) gauge quantum numbers of the quarks and leptons are uniquely determined by the structure of the model and they include 6 chiral abelian charges—the weak hypercharge and \((B-L)\) quantum number for each of the three generations. With an appropriate choice of the abelian charges of the Higgs fields, it is possible to generate a good order of magnitude fit to the SM fermion masses, as discussed in Holger’s lectures [10]. In this fit, we do not attempt to guess the spectrum of superheavy fermions at the Planck scale, but simply assume a sufficiently rich spectrum to mediate all of the symmetry breaking transitions in the various mass matrix elements.

8 Neutrino Mass and Mixing

The recent impressive experimental developments in neutrino physics have been described here by John Simpson [3] and the formalism of neutrino oscillations has been described by Boris Kayser [4]. The phenomenology of neutrino oscillations [5] is summarised in the neutrino mass squared differences of eq. (9) and mixing matrix elements of eq. (10).

There are no right-handed neutrinos in the SM. Neutrinos can have a Majorana mass, but they are mass protected by electroweak gauge invariance and the SM Higgs mechanism does not generate a Majorana mass; a weak isotriplet Higgs boson would be required. However physics beyond the SM could easily generate an effective non-renormalisable interaction of the form

\[
\frac{1}{\Lambda} \phi_{WS} \phi_{WS} LL
\]  

(60)

where \(L\) is a SM lepton doublet and \(\mu = \Lambda\) is the energy scale where the SM breaks down. This interaction would generate a neutrino Majorana mass term

\[
m_\nu = \frac{2v^2}{\Lambda}
\]  

(61)

However we should note that, if there are some mass protecting chiral charges beyond the SM, the neutrino mass could be suppressed further and eq. (61) really provides an order of magnitude upper limit to the neutrino mass. If we take \(\Lambda = \Lambda_{GUT} \sim 3 \times 10^{16}\), this upper limit on the Majorana neutrino mass is \(m_\nu \sim 0.002\) eV. This is at least an order of magnitude less than the neutrino mass required to explain atmospheric neutrino oscillations, see eq. (3). So
there may well be new physics below the SUSY-GUT scale responsible for neutrino masses and, more generally, for the effective light neutrino Majorana mass matrix:

$$\overline{\nu_{Li}}(M_\nu)_{ij}\nu_{Lj}^c$$

(62)

Fermi-Dirac statistics means that $M_\nu$ must be symmetric.

If we assume that, as in many models, the charged lepton mass matrix is quasi-diagonal then the dominant contributions to the neutrino mixing matrix $U_{MNS}$ come from the Majorana neutrino mass matrix $M_\nu$. In this case there are two leading order forms [46, 47] for $M_\nu$, according to whether neutrino masses have a hierarchical or an inverse hierarchical spectrum\(^7\).

Hierarchical models, with neutrino masses satisfying $|m_3| \gg |m_2|, |m_1|$, have a leading order mass matrix of the type:

$$M_\nu \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{m}{2}$$

(63)

Inverted hierarchical models, with $|m_1| \approx |m_2| \gg |m_3|$, have a leading order mass matrix of the form:

$$M_\nu \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \frac{m}{\sqrt{2}}$$

(64)

where $m \simeq \sqrt{\Delta m^2_{23}} \simeq 0.05$ eV.

The best-known scenario for generating the effective light neutrino mass matrix from beyond the SM physics is the see-saw mechanism [48]. Here one introduces three right-handed neutrinos, which are not mass protected by the electroweak interactions. Hence they are expected to have a symmetric Majorana mass matrix $M_R$, with right-handed neutrino mass eigenvalues characteristic of some new physics energy scale—the see-saw scale. In addition one expects a normal electroweak scale Dirac neutrino mass matrix $M_N$, connecting the left-handed and right-handed neutrinos, as in eq. (58) for the $SO(10)$ model considered in the previous section. An effective light neutrino Majorana mass matrix is then naturally generated and given by the see-saw formula:

$$M_\nu = M_N M_R^{-1} M_N^T$$

(65)

\(^7\)We do not consider here the fine-tuned possibility of quasi-degenerate neutrino masses.
In the above $SO(10)$ model, the see-saw formula generates realistic neutrino masses with a right-handed neutrino see-saw scale $\Lambda_R = 3 \times 10^{14} \text{ GeV}$ and a large solar neutrino mixing angle \cite{49}. However the large atmospheric neutrino mixing angle is generated from the charged lepton mass matrix $M_E$ of eq. (57), which is not quasi-diagonal. It is also possible to fit the neutrino masses and mixing angles in the AGUT model \cite{44}, in which $M_E$ is quasi-diagonal.

9 Conclusion

We have reviewed some approaches to the quark-lepton mass problem. Of course we did not have time to cover the whole field and, for example, did not discuss the interesting ideas from models with extra dimensions \cite{50} for understanding fermion masses. The hierarchical structure of the spectrum was emphasized and interpreted as due to the existence of a mass protection mechanism, controlled by approximately conserved chiral flavour quantum numbers beyond the SM. Many flavour symmetry models have been constructed; however it must be admitted that none of them is totally convincing. The existence of two large neutrino mixing angles was not predicted and provides a challenge for model builders. So I leave you with the message that there is still much exciting research to be done—both theoretical and experimental—in the search for the physics of flavour underlying the Standard Model.

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