Single-shot sub-Rayleigh imaging with pixel-limited detection

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For conventional imaging, the imaging resolution is determined by both the system’s Rayleigh limit and the detector’s pixel resolution. Even if the object’s transmitted intensity is detected at the imaging plane with single-pixel detectors, we report that imaging beyond the Rayleigh limit can be achieved by exploiting both the object’s sparsity in the representation basis and prior knowledge of the fixed optical system; this is supported by a numerical simulation and experiments. The image reconstruction algorithm is based on compressive sensing; factors affecting the reconstruction quality are also discussed. © 2014 The Japan Society of Applied Physics

Figure 1. Schematic of experimental setup for thermal light sub-Rayleigh imaging with pixel-limited detection.

For a standard conventional imaging system that directly records the object’s intensity distribution with a charge-coupled device (CCD) camera, both the imaging system’s Rayleigh limit and the camera’s pixel resolution restrict the optical system’s imaging resolution. For example, the imaging resolution is determined mainly by the optical system’s Rayleigh limit in remote sensing because the numerical aperture (NA) of the imaging system is usually small. However, for imaging in long-wavelength radiation bands, such as infrared and terahertz imaging, because cameras with large planar arrays are currently very hard to manufacture, the imaging resolution is limited mainly by the camera’s pixel resolution.

Many more methods have been invented to improve the imaging resolution by overcoming the imaging system’s Rayleigh limit than to do so by overcoming the camera’s pixel resolution. By exploiting the evanescent components in the object’s immediate proximity, sub-Rayleigh imaging can be achieved, but this method is applied only in the near-field range. Several microscopy techniques based on fluorescence have also been introduced to improve the imaging resolution. However, they require scanning or repetitive experiments, which limits real-time applications. By using additional a priori information on the optical system, an imaging resolution beyond the Rayleigh diffraction limit can be obtained. However, the degree of improvement is limited in practice because of the effect of detection noise. In addition, for an N-pixel image, these sub-Rayleigh imaging methods require at least N samples to reconstruct the image (this is called the Nyquist limit of the measurement).

The image’s sparsity has recently been taken as a quite general assumption because a natural object can be sparsely expressed in a proper representation basis (or under a suitable basis transform). A compressive sensing technique enables the reconstruction of an N-pixel image from much fewer than N measurements by using the image’s sparsity. This technique has already been successfully applied to super-resolution imaging, remote sensing, and compressive imaging. For conventional imaging, on the basis of the object’s sparsity, Yang et al. have demonstrated sub-Rayleigh imaging for a pixel-limited system, but the imaging system is not diffraction-limited. In addition, using prior knowledge of both the object’s sparsity and the system’s point spread function (PSF), Segev’s group has successively realized single-shot super-resolution imaging with coherent and incoherent illumination by sampling in the Fourier domain, but the imaging system is not pixel-limited but diffraction-limited. Therefore, it is natural to ask whether single-shot super-resolution imaging can be realized by direct sampling in the real-space domain for both pixel-limited and diffraction-limited conventional imaging systems. In this paper, we demonstrate that even if sparse-array single-pixel detectors (namely, a pixel-limited system) are used to record the object’s information at the imaging plane (namely, sampling below the Nyquist rate), single-shot sub-Rayleigh imaging is possible for diffraction-limited systems by exploiting both the assumption of the object’s sparsity in a representation basis and prior knowledge of the PSF. This will be very helpful for applications such as high-resolution imaging in wavelength regions without cameras and microscopy of sparse objects such as living cells and bacteria.

Figure 1 presents a schematic of the setup for experimental demonstration of thermal light single-shot sub-Rayleigh imaging with pixel-limited detection. The uniform light emitted from a halogen lamp is filtered by an optical filter (with a center wavelength of λ = 650 nm and a bandwidth of Δλ = 10 nm) and then collimated by a lens with a focal length of f0 = 200 mm. The object is illuminated by the collimated light, and its transmitted image is imaged onto the imaging plane D1 by a standard conventional imaging setup. In the experiment, a CCD camera was placed at the imaging plane D1 to detect the object’s transmitted intensity, and we sparsely chose the measurement data recorded by certain pixels of the CCD camera, which was equivalent to data from sparse-array single-pixel detectors.

The distances z1, z2 and the focal length of the lens f obey the Gaussian thin-lens equation:

\[ \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}. \]

On the basis of the Rayleigh criterion, the resolution limit is determined by the wavelength λ and NA of the lens f:

\[ \Delta x_s = \frac{\lambda}{2 \sin \theta}. \]
\[
\Delta s_0 = 0.61 \frac{1}{\text{NA}} \approx 1.22 \frac{d_2}{L},
\]

where \( L \) is the effective transmission aperture of the imaging lens \( f \), and NA is approximately \( L/(2d_2) \).

When the light is fully spatially incoherent and uniform, the intensity at the imaging plane \( D_t \) is the convolution of the intensity at the object plane with the absolute value of the incoherent PSF: \( I_t(x, y) \), which denotes the convolution symbol, \( h(x, y) \) is the optical system’s PSF, \( I_{obj}(x, y) \) is the object intensity, and \( I_t(x, y) \) is the image intensity. For the optical system shown in Fig. 1, the system’s PSF is

\[
h(x, y) \propto \text{somb}^2 \left[ \frac{\pi L}{2d_2} \sqrt{x^2 + y^2} \right],
\]

where \( \text{somb} \) is the spherical Bessel function of the first kind.

From Eqs. (2) and (3), compared with the original object, the image is blurred and has low spatial resolution at the imaging plane \( D_t \) when the transmission aperture of the imaging lens \( f \) is small. To restore a high-resolution image, the relation shown in Eq. (2) usually yields \( F[I_t(x, y)] = F[I_{obj}(x, y)]H \) in the spatial frequency domain, where \( F \) denotes the Fourier transform, and \( H \) is the optical transfer function. For the optical system shown in Fig. 1, when the image is sampled at or above the Nyquist rate at the imaging plane \( D_s \), many iterative methods can be used to restore the object using prior knowledge of the optical system. However, when the detection system is characterized as sparse-array single-pixel detectors, the problem is that we wish to reconstruct the object from a blurred image smeared by a low-pass filter with samples below the Nyquist rate, which becomes much more difficult by traditional iterative methods.

Using a different method from that described above, we try to directly restore an object represented by the optical system shown in Fig. 1, exploiting both the assumption of the object’s sparsity in the representation basis and prior knowledge of the fixed optical system. Mathematically speaking, any image can be expanded by an orthonormal basis (such as a Fourier basis and a wavelet basis). However, only a small number of the expansion coefficients are nonzero, and the largest coefficients can express the image’s main features. Thus, the image is considered to be sparse or compressible in an appropriate representation basis, for example, a transmission double slit in real space. The smaller the number of nonzero coefficients in the representation basis is, the sparser the image is. In the framework of compressive sensing, there are an infinite number of images, which—after being convoluted by the PSF — will produce the image recorded by the sparse-array single-pixel detectors for the setup shown in Fig. 1; our goal is to find the sparsest one. It has been mathematically and experimentally demonstrated that when the compressive sensing technique is used, if the object is sparse enough, then any sparsity-based reconstruction method is bound to find the sparsest solution with measurements below the Nyquist rate. Here, we employed the gradient projection for the sparse reconstruction algorithm.

The object \( T \) can be reconstructed by solving the following convex optimization program:

\[
\begin{align*}
T & = \arg \min_{\|t\|_1} \frac{1}{2} \left\| I'_t(x, y) - h(x, y) \otimes |t'(x, y)|^2 \right\|_2^2 + \tau \left\| \Psi |t'(x, y)|^2 \right\|_1, \forall s = 1, \ldots, M,
\end{align*}
\]

where \( \tau \) is a nonnegative parameter, \( |t'(x, y)|^2 \) is the intensity recorded by the \( s \)th single-pixel detector, and \( M \) is the total number of single-pixel detectors at the imaging plane \( D_t \) (namely, the total measurement number). \( \Psi \) denotes the transform operator to the sparse basis. \( \|P\|_2 \) and \( \|P\|_1 \) represent the Euclidean norm and the \( L_1 \)-norm of \( \Psi \), respectively.

To verify the concept, Figs. 2 and 3 show simulated and experimental demonstrations of imaging a double slit using the optical system depicted in Fig. 1. The slit width of the double slit (80 \( \times \) 80 pixels, with a pixel size of 6.45 \( \mu \)m \( \times \) 6.45 \( \mu \)m), as shown in Figs. 2(a1) and 3(a1), is \( a = 30 \mu \)m, and its center-to-center separation is \( d = 60 \mu \)m. The double slit includes about 370 nonzero pixel values in the real-space domain. The parameters listed in Fig. 1 are set as follows: the transverse size of the hole \( D = 20.0 \) mm, \( z_1 = z_2 = 800 \) mm, the focal length of the lens \( f = 400 \) mm, and its effective transmission aperture \( L = 7.9 \) mm. In addition, the pixel size of the single-pixel detector is 6.45 \( \mu \)m \( \times \) 6.45 \( \mu \)m, and the single-shot exposure time is set to 1 ms. According to Eq. (1), the imaging system’s resolution limit is \( \Delta s_0 \approx 80 \mu \)m, and the cross section of the system’s PSF between the object plane and the detection plane \( D_s \) is shown in Figs. 2(a3) and 3(a3). In this case, the object’s image at the imaging plane \( D_s \) as shown in Figs. 2(b1) and 3(b1), cannot be resolved for conventional imaging. Figures 2(b1)–2(b5) and 3(b1)–3(b5) present the intensity distributions recorded by the detectors when the number of single-pixel detectors at the imaging plane \( D_t \) is \( M = 6400, 1600, 400, 100, \) and 64. For a double slit expressed in a real-space basis, the corresponding simulated and experimental sparse reconstruction results, given by Eq. (4), are depicted in Figs. 2(c1)–2(c5) and 3(c1)–3(c5), respectively. Both the simulated and experimental results
Fig. 3. Experimental demonstration of sub-Rayleigh imaging with pixel-limited detection by imaging a double slit, corresponding to Fig. 2.

Fig. 4. Effect of PSF measurement error on reconstruction quality of the same double slit, using the measurement data recorded by $M=400$ single-pixel detectors displayed in Figs. 2(b) and 3(b). From left to right, the transmission aperture of the lens $f$ is $L \approx 7.4$, 7.6, 7.9, 8.1, and 8.3 mm, respectively. (a)–(a3) are the simulated sparse reconstruction results; (b)–(b3) are the corresponding experimental reconstruction results.

illustrated in Figs. 2 and 3, respectively, clearly demonstrate that sub-Rayleigh imaging with pixel-limited detection can be achieved when both the assumption of the object’s sparsity in a representation basis and prior knowledge of the optical system’s PSF are used. In addition, if the center-to-center distance between two single-pixel detectors is increased such that the measurement number is less than the lower value required for image recovery by compressive sensing,13,14 the restored image will be distorted [Figs. 2(c3) and 3(c3)]. However, as shown in Figs. 2(c4) and 3(c4), only $M=100$ measurements are used to reconstruct the object, which is far fewer than the number suggested by the Nyquist rate. In conventional imaging, the optical system’s PSF is often space-variant, and the system’s PSF measured in practical applications also deviates from the true distribution. In Fig. 4, we show the effect of the PSF measurement error on the reconstruction quality of the images. By changing the transmission aperture $L$ of the lens $f$, we can obtain different PSFs. For transmission apertures of $L \approx 7.4$, 7.6, 7.9, 8.1, and 8.3 mm, Figs. 4(a1)–4(a3) and 4(b1)–4(b3) show the simulated and experimental results, respectively, of imaging the same double slit shown in Figs. 2 and 3 using the measurement data recorded by $M=400$ single-pixel detectors displayed in Figs. 2(b1) and 3(b1). As shown in Fig. 4, the images can be stably restored if the PSF measurement error is less than 2.5%.

To verify the applicability of sub-Rayleigh imaging with pixel-limited detection to more general images, we imaged a transmission plate of a “zhong” ring ($80 \times 80$ pixels), which consists of 1510 nonzero pixel values in the real-space domain, using the same experimental parameters as in Fig. 3. As in Figs. 2 and 3, the intensity distributions recorded by the detectors are displayed in Figs. 5(b1)–5(b3), and their corresponding sparse reconstruction results when the transmission plate is expressed in the real-space basis are shown in Figs. 5(c1)–5(c3). Compared with the result shown in Fig. 3(c4), the transmission plate’s image cannot be restored with $M=100$ measurements because the restoration quality is related to the object’s sparsity, and the plate (“zhong” ring) is much more complex than the double slit.13–15 The reproduction results when the transmission plate is expressed in the two-dimensional-discrete cosine transform (2D-DCT) basis are shown in Figs. 5(d1)–5(d3). From Figs. 5(c1)–5(c3) and 5(d1)–5(d3), the reconstruction quality of sub-Rayleigh imaging with pixel-limited detection is also related to the object’s sparse representation basis.24

In addition, as shown in Fig. 6, we also used simulation experiments to validate that sub-Rayleigh imaging with pixel-limited detection is effective for objects with multiple gray levels. In contrast to the object displayed in Fig. 2(a1), the gray double slit shown in Fig. 6(a) consists of four single slits, and the gray intensity ratio is 0.2:1:0.8:0.4. Using the same simulation parameters as in Fig. 2, the intensity distributions recorded by the detectors and their corresponding sparse reconstruction results are displayed in Figs. 6(b1)–6(b3) and 6(c1)–6(c3), respectively. Figure 6 clearly shows that the double slit with multiple gray levels can also be stably restored, and the relationship between the reconstructed image’s quality and $M$ is similar to that demonstrated in Fig. 2.

Two points regarding sub-Rayleigh imaging with pixel-limited detection should be emphasized. On the one hand, similar to the detection system on the test path in Ref. 25, each single-pixel detector at the imaging plane $D_i$ depicted in Fig. 1 can receive only local information from the object; thus, the center-to-center distance between two single-pixel detectors cannot be larger than the resolution limit, and the
image restoration is related to the image’s sparsity and that the reconstruction quality depends on the object’s sparse representation basis. This technique is very useful for applications such as imaging in long-wavelength radiation bands without a CCD camera, microscopy of living cells or bacteria, and imaging of atoms captured by ion traps, where the images are sufficiently sparse.

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Fig. 6. Simulated reconstruction results of imaging a gray double slit. (a) Original object; (b1)–(b5) and (c1)–(c5) intensity distributions recorded by the detectors at the imaging plane $D_i$ and their corresponding sparse reconstruction results, respectively, as in Fig. 2.

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single-pixel detectors at the imaging plane $D_i$ cannot be distributed at random, which differs from the method of global measurement described in Refs. 22–24. On the other hand, a high-resolution image can reportedly be stably reconstructed using the compressive sensing technique when the measurement matrix and representation basis satisfy the incoherence and restricted isometric property criteria.13,14 However, the measurement matrix and representation basis are not incoherent for the schematic shown in Fig. 1, which limits the degree of improvement in the imaging resolution. In addition, because the phase of the light field emitted from the halogen lamp is random, and the source transverse coherence length on the object plane is far smaller than the dimensions of the imaging object, the illumination can be considered to be incoherent by a certain exposure time in the experiment, and this method can also be generalized to conventional imaging schemes with partially incoherent and coherent illumination.

In conclusion, we realized single-shot sub-Rayleigh imaging with pixel-limited detection by exploiting both the imaging object’s sparsity in the representation basis and prior knowledge of the fixed optical system. Both simulated and experimental results demonstrated that sub-Rayleigh imaging with pixel-limited detection can overcome the limitations on the imaging resolution imposed by both the optical system’s Rayleigh limit and the camera’s pixel resolution. We also showed that the number of single-pixel detectors required for

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