Critical Scaling of Compression-Driven Jamming of Frictionless Spheres

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Athermal isotropic compression-driven jamming:

work has focused on quasi-static compression and behavior in the jammed solid phase above $\phi_J$

We are interested in behavior in the liquid phase below $\phi_J$, particularly to probe for a diverging time scale as $\phi \to \phi_J$ from below.

Do stress-isotropic (compression-driven) and stress-anisotropic (shear-driven) jamming have the same critical universality?

Ikeda et al, PRL 124, 058001 (2020) computed the relaxation time $\tau$ of initial unjammed configurations below $\phi_J$. A common divergence of $\tau$ for both random isotropic configurations and for configurations in steady-state simple shear, suggests that compression-driven and shear-driven jamming have a common universality.

Nishikawa et al, J Stat Phys 182, 37 (2021) have questioned Ikeda et al’s results; find $\tau \sim \ln N$ when the system becomes sufficiently large, so $\tau$ has no proper thermodynamic limit.
Athermal isotropic compression at a finite compression rate:

We probe time scales below $\phi_J$ by compressing at a finite rate $\dot{\epsilon}$. The finite rate introduces a control time scale with which to probe the critical time scale.

We measure the bulk viscosity, which we find to diverge at $\phi_J$, and compare to the shear viscosity to look for a universality of stress-isotropic and stress-anisotropic jamming.

Model: size-bidisperse, soft core spheres, in non-Brownian suspension

$$N_s = N_b = N/2 \quad d_b/d_s = 1.4 \quad \phi = \frac{1}{L_D \sum_i V_i}$$

packing fraction

one-sided harmonic elastic contact interaction

$$f^\text{el}_{ij} = -\frac{d}{d\mathbf{r}_i} \left[ \frac{1}{2} k_e \left( 1 - \frac{r_{ij}}{d_{ij}} \right)^2 \right] \quad r_{ij} = |\mathbf{r}_i - \mathbf{r}_j| \quad d_{ij} = (d_i + d_j)/2$$

viscous dissipative drag due to host medium

$$f^\text{dis}_i = -k_d V_i \left[ \mathbf{v}_i - \mathbf{v}_{\text{host}} (\mathbf{r}) \right]$$

dynamics:

$$m_i \ddot{\mathbf{r}}_i = \sum_j f^\text{el}_{ij} + f^\text{dis}_i$$

dimensionless parameters:

$$\dot{\epsilon} t_0, \quad t_0 = (D/2) k_d V_s d_s^2 / k_e = 1$$

$$Q \lesssim 1 \Rightarrow \text{overdamped} \quad N = 16,384$$

$$Q = \sqrt{m_s k_e / k_d V_s d_s} \quad 32,768$$
Results: pressure $p$

2D

\[ \phi < \phi_J : \quad p \to 0 \text{ as } \dot{\epsilon} \to 0 \]

\[ \phi > \phi_J : \quad p \to \text{constant as } \dot{\epsilon} \to 0 \]

3D

\[ \phi < \phi_J : \quad \zeta \to \text{constant as } \dot{\epsilon} \to 0 \]

\[ \phi > \phi_J : \quad \zeta \to \infty \text{ as } \dot{\epsilon} \to 0 \]
Critical scaling ansatz:

\[ p = \dot{\varepsilon}^q f \left( \frac{\phi - \phi_J}{\dot{\varepsilon}^{1/z\nu}} \right) \]

\( \phi > \phi_J: \) since \( p \to \text{constant as } \dot{\varepsilon} \to 0, \)
then \( f(x \to +\infty) \to |x|^{qz\nu} \)

\( \phi < \phi_J: \) since \( \zeta \equiv p/\dot{\varepsilon} \to \text{constant as } \dot{\varepsilon} \to 0, \)
then \( f(x \to -\infty) \to |x|^{-(1-q)z\nu} \)

\[ \lim_{\dot{\varepsilon} \to 0} p \sim (\phi - \phi_J)^y \]
\[ y = qz\nu \]

\[ \lim_{\dot{\varepsilon} \to 0} \zeta \sim (\phi_J - \phi)^{-\beta} \]
\[ \beta = (1 - q)z\nu \]

plot \( p/\dot{\varepsilon}^q \) vs \( (\phi - \phi_J)/\dot{\varepsilon}^{1/z\nu} \) – data for different \( \dot{\varepsilon} \)
should collapse to a common curve.
\[ p = \dot{\varepsilon}^q f \left( \frac{\phi - \phi_J}{\dot{\varepsilon}^{1/z\nu}} \right) \]
\[ \zeta = p/\dot{\varepsilon} \]
\[ \lim_{\dot{\varepsilon} \to 0} p \sim (\phi - \phi_J)^y \]
\[ \lim_{\dot{\varepsilon} \to 0} \zeta \sim (\phi_J - \phi)^{-\beta} \]
\[ y = qz\nu \]
\[ \beta = (1 - q)z\nu \]

Test of sensitivity of fitted parameters to size of data window used in fit
Conclusions:

|       | compression | simple shearing (numerical results) | theory: marginal stability |
|-------|-------------|-----------------------------------|---------------------------|
| 2D    | $\beta = 2.63 \pm 0.09$ | $\beta = 2.77 \pm 0.20$ [1], $\beta = 2.58 \pm 0.10$ [2] | $\beta = 2.83$ [4] |
|       | $y = 1.12 \pm 0.04$ | $y = 1.08 \pm 0.03$ [1], $y = 1.09 \pm 0.01$ [2] |               |
| 3D    | $\beta = 3.07 \pm 0.15$ | $\beta = 2.56$ [3], $\beta = 2.8$ [4], $\beta = 2.94$ [5], $\beta = 3.8 \pm 0.1$ | $\beta = 2.83$ [4] |
|       | $y = 1.22 \pm 0.03$ | $y = 1.16 \pm 0.01$ [6] |               |

**compression:**

- **bulk viscosity:**
  \[ \phi < \phi_J \lim_{\dot{\epsilon} \to 0} p/\dot{\epsilon} \sim (\phi - \phi_J)^{-\beta} \]

- **pressure:**
  \[ \phi > \phi_J \lim_{\dot{\epsilon} \to 0} p \sim (\phi - \phi_J)^y \]

**shearing:**

- **pressure analog of shear viscosity:**
  \[ \phi < \phi_J \lim_{\dot{\gamma} \to 0} p/\dot{\gamma} \sim (\phi - \phi_J)^{-\beta} \]

- **pressure on yield stress line:**
  \[ \phi > \phi_J \lim_{\dot{\gamma} \to 0} p_\gamma \sim (\phi - \phi_J)^y \]

Our results are consistent with a common universality for stress-isotropic compression-driven jamming vs stress-anisotropic shear-driven jamming (though 3D is tentative)

[1] Olsson and Teitel, PRE 83, 020201(R) (2011)
[2] Olsson and Teitel, PRL 109, 108001 (2012)
[3] Kawasaki et al, PRE 91, 012203 (2015)
[4] DeGiuli et al, PRE 91, 062206 (2015)
[5] Lerner et al, PNAS 109, 4798 (2012)
[6] Olsson, PRL 122, 108003 (2019)