Cylinder diameter measurement with displacement and rotation error correction for non-telecentric optics

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Abstract. The paper presents the implementation of a method for measuring the diameter of cylinders in traffic along a transport line using machine vision systems with correction of the error of displacement and rotation of the product for the case of using non-telecentric optics. Diameters were measured for a set of displacements and rotations of the cylinder, and estimates of the achievable errors of this implementation of the layout of the measuring installation were made.

1. Introduction
Measuring the geometric parameters of cylindrical products is an important scientific and technical task. There are many industries where it is required to measure the cylinder diameter with high accuracy. This is necessary in the production of high-frequency coaxial cables, precision bearings, cartridge casings, uranium tablets for the production of fuel cells for nuclear power plants and other tasks. Non-compliance with dimensional tolerances in the production of cylindrical products is fraught with the inability to operate the final device or may even lead to emergency situations with serious consequences.

So as usual in the production of manufactured hundreds, or even thousands of items per minute, it is necessary to provide continuous measurements and discarding of products directly on the production line. And this imposes a limitation on the choice of measurement method. Thus, when the cylinder moves along the conveyor in the measuring zone, it may shake (Fig. 1) or rotate the product, which increases the measurement error. Shadow and diffractive optical direct scattering measurement methods provide the best measurement accuracy. Such methods allow achieving a diameter measurement error of the order of 0.01% of the measurement range [1], [2]. However, it is not always possible to provide measurements on direct light scattering and they require the use of telecentric optics for both the lighting and receiving parts.

For measurements of the geometric dimensions of objects using backscattering, the measurement accuracy is already about 0.1% of the measurement range [3], [4]. The use of backscattering vision systems or laser profilometers to control the geometric parameters of products is possible for those applications where this error is considered satisfactory. Laser profilometers measure the distance to each point of the object along the laser line on the object, and thereby it is possible to calculate the diameter of the cylinder from the value of the height difference between the object and the substrate. However, in order to provide an error of about 0.1%, it is necessary to ensure high uniformity of the substrate in height, which is rarely feasible in practice. The disadvantage of this method is that it has a high error when measuring cylindrical surfaces. Such a disadvantage is caused by a distortion of the measured geometric position of the points on the cylindrical surface due to different tilt angles of the
light-scattering cylindrical surface. Using high-resolution digital vision systems, there are additional requirements for the stability of the measurement facility and lighting conditions. The use of inexpensive lenses with high spatial aberration introduces an additional error in the measurement of dimensions, since they do not allow you to get a true image of the controlled product [5].

The aim of the work is to develop methods of measuring the diameter of cylindrical objects using non-cylindrical optics, which provide reliable and accurate measurements in technological tasks.

2. Task definition
The problem of cylinder diameter measurement is reduced to a triangulation method of measurement using illumination in the form of a light section and a matrix photodetector. In this case, when using non-telecentric optics, it is necessary to compensate for distortions associated with the arbitrary positioning of the object in the measuring volume.

![Figure 1. Changing the size of the visible image when object displaced along the optical axis occurs.](image)

F is the focal length of the lens, D is the diameter of the object, D' and D'' – apparent diameter, dX is the displacement along the axis.

In order to compensate for the error associated with the movement of the object along the optical axis (Fig. 1), a method of measuring the diameter of the cylindrical object was proposed and tested, based on the triangulation method of light section adapted for measuring a cylindrical surface[6]. The essence of this method is to measure two parameters: the visible diameter along the laser line on the Y object and the scaling factor M. The scaling factor is calculated as a result of the analysis of the image of the laser line curved by a cylindrical surface.

\[ D = M \cdot Y \]  

(1)

This method application has acceptable (~ 0.1%) error in measuring the cylinder diameter with a minimum of equipment costs in the displacements range along the optical axis by ± 1 mm.

However, using this method does not solve the problem of object rotation in the measurement zone (Fig. 2). When measuring the visible diameter Y along the laser line, due to the rotation of the object, the true diameter D is always less than the visible one:

\[ D = Y \cdot \cos(\alpha) \]  

(2)

where \( \alpha \) is the angle of rotation of the cylinder. To solve this problem, a modification of this method with correction of the rotation error for non-telecentric optics is proposed.

3. Method description
An optical design (Fig.3) for measuring the diameter of the cylinder with compensation for displacement and rotation of the object has two perpendicular laser light sections of minimum possible thickness in the measuring zone. The sections are arranged so as to “cut” the cylinder at the angle of +45 and −45 degrees to the axis of the cylinder (Fig. 4). The axis of the section planes is located at an angle to the optical axis of the camera. Due to this angle, when moving the product along the optical axis, the cross-section image on the resulting image moves and it is possible to measure the cylinder.
diameter using any laser section using the method described in [6]. Angle of cylinder inclination is determined by pair of upper or lower extreme points on the cylinder surface from light section. To calculate the true diameter, the measured diameter is corrected by the cosine of the inclination angle.

**Figure 2.** Error in measuring diameter when rotating an object in the measurement zone.

Let the points $A$, $B$, $C$, and $D$ in the received image (Fig. 4) have the coordinates $(X_a, Y_a)$, $(X_b, Y_b)$, $(X_c, Y_c)$, and $(X_d, Y_d)$, respectively. Then the diameter of the cylinder $D$ will depend linearly on the vertical size of the imaginary vertical line:

$$D = M \cdot Y = M \cdot \left( \frac{Y_b + Y_d - Y_a - Y_c}{2} \right) \cdot \cos(\alpha),$$

where $M$ is the conversion factor of the object diameter. The parameter $M$ has a quadratic dependence on the distance to the measured object $L$:

$$M(L) = k \cdot L^2 + m \cdot L + n$$

where $k$, $m$ and $n$ are coefficients calculated as a result of calibration. The distance $L$ is calculated using the calibration function $L(X)$, which can be approximated by a second-degree polynomial:

$$L(X) = L\left( \frac{X_a + X_b + X_c + X_d}{2} \right) = a \cdot X^2 + b \cdot X + c$$

**Figure 3.** Optical design of a cylinder diameter measuring system with correction of displacement and rotation of an object.

The cos ($\alpha$) value can be measured from the visible inclination of a cylindrical object, which can be estimated from the positions of points $A$, $C$, and $D$, $B$ from the trigonometric ratios:
To measure the diameter of the cylinder, it is necessary to determine its angle of inclination and build two calibration functions: the change in the visible vertical size from the distance to the object $M(L)$ and the dependence of the distance to the object on the horizontal position of the light cross-section line $L(X)$.

$$\tan(\alpha) = \frac{Y_A - Y_C}{X_A - X_C} + \frac{Y_D - Y_B}{X_D - X_B}$$

(6)

4. Laboratory layout and calibration

To test this method for measuring diameter, a laboratory setup model was assembled (Fig. 5). A semiconductor laser emitter (1) (650 nm, 5 mW), projecting a cross-section in the measuring zone, was used as a light source. Short-focus lens (2) is installed in measurement area to reduce thickness of light planes. Light planes intersect measured object (3) located on a manual coordinate table. The analyzed image arrived on the photodetector (4) (5 Mpx the DMK 23UP031 video camera and lens for cameras 5 Mpx with variable focal distance of 15-8 mm) located at an angle 45 degrees to optical axis of the emitter. All layout elements were located on a hard base to eliminate the effects of vibrations and temperature drifts. The maximum sharpness of the obtained image was achieved at a distance of 138 mm from the receiving lens along the optical axis of the photodetector.

In order to calibrate the distance to the object from the apparent displacement of the light lines on the received image and the scale factor along the vertical axis, depending on the distance to the object, the cylinder was replaced with a reference plane with a calibration template (Fig. 6). The template size is 5x5 mm with a manufacturing error of no more than 0.015 mm. The plane of the template was moved along the optical axis of the photodetector in a range of distances from 132 to 145 mm with a pitch of 0.5 mm with a positioning error of the order of ± 0.1 mm. In this range of movements, the optical properties of the system made it possible to distinguish confidently the received image.
As a result of calibration, the X coordinates of the center of intersection of lines on the image in pixels are obtained. Figure 7 shows the resulting graph of the dependence of the distance L to the object from the X coordinate.

The calibration dependence of the distance to the object on the coordinate of the line of the laser light section in the camera image is described by a polynomial of the second degree:

\[ L(X) = 0.000003 \cdot X^2 - 0.03 \cdot X + 167.948, \]  

(7)

where X is the coordinate in pixels, L is the coordinate in millimeters. The interpolation error at the nodal points was checked. The maximum interpolation deviation from these reference points is 0.1 mm, which is a satisfactory result that is well consistent with the positioning error. This error can be improved with more thorough calibration. For the scaling factor depending on the distance, calibration dependence was also obtained (Fig. 8).

The dependence is quadratic (4) and is described by the following expression:

\[ M(L) = -0.002 \cdot L^2 + 0.33 \cdot L - 39.129, \]  

(8)

where L is the coordinate in millimeters, and M is the conversion factor of the object height in microns/pixel. The deviation of the measured data from the linear function does not exceed 0.2% of the values at the nodal points, which is consistent with the positioning error and the manufacturing error of the scale.

![Figure 8. The dependence of the scale factor M on the distance to the object.](image)

For this layout, the expected diameter measurement errors were calculated. The pixel size of the selected camera is 2.2 x 2.2 microns. A cylinder with a diameter of approximately 8.2 mm at a distance of 138 mm was visible in the image as 960 pixels or 2.112 mm. Thus, the size reduction ratio was 3.8 times. The coordinates of points A, B, C and D can be measured at best with an error of 1 pixel (sub-pixel interpolation was not used). But due to the reduction coefficient, the measurement error, the determination error will be equal to 8.4 microns. Therefore, the relative error of the diameter measurement (as the difference in the coordinates \( Y_A \) and \( Y_B \)) can be about 16.8 microns.

It is possible to reduce this error by using a lens with a different focal length so that the image of the cylinder occupies the entire area of the array of the video camera. This will reduce the coefficient M to 2. As a consequence, this will reduce the diameter error by about 2 times, to 8.8 μm. Further improvement in accuracy is possible through subpixel interpolation.

5. Experimental results

To test the operability of the object rotation compensation method, an experiment was performed to measure the cylinder diameter when it was moved in the range L from 133 to 143 mm in 1 mm increments. A matte metal cylinder with a diameter of 8,196 mm was used as the object of measurement. At each step in the displacement range, ten measurements of the cylinder diameter were
made at various arbitrary angles of rotation in the range of ±15 degrees. Table 1 presents the results of the arithmetic mean $D_{\text{mean}}$ of the cylinder diameter at each point along the axis of displacements, the deviation from the true value of $\Delta D$ and the standard deviation $SD$ in each series.

| $L$, mm | $D_{\text{mean}}$, mm | $\Delta D$, μm | SD, μm |
|-------|------------------|----------------|--------|
| 133   | 8.201            | -5             | 15     |
| 134   | 8.194            | 2              | 9      |
| 135   | 8.192            | 4              | 12     |
| 136   | 8.201            | -5             | 14     |
| 137   | 8.202            | -6             | 21     |
| 138   | 8.189            | 7              | 12     |
| 139   | 8.196            | 0              | 7      |
| 140   | 8.191            | 5              | 19     |
| 141   | 8.188            | 8              | 14     |
| 142   | 8.208            | -12            | 18     |
| 143   | 8.203            | -7             | 17     |

The maximum measurement error of the mean diameter was 12 μm. The standard deviation for all cylinders ranged from 7 to 21 microns. This is in good agreement with the calculated errors.

**Conclusions**
The use of the rotation error correction method for non-telecentric optics made it possible to reduce the error in determining the diameter of cylindrical products at their displacements along the optical axis in the working area. On the model of the laboratory setup, it was possible to achieve an error in determining the diameter of the cylinders of not more than 12 microns with an offset of ± 5 mm.

This machine vision system provides measurement accuracy acceptable for many tasks with low equipment costs.

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