Analysis of optimal profile modification amount of spur gear under disturbance of backlash and center distance

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Abstract. Under high temperature conditions, backlash and center distance of gear pair deviate from the preset value, which makes original modification amount deviate from the optimal value. The Finite Element Method was used to verify the deviation of the optimal modification amount under the disturbance of backlash and center distance. Taking the backlash, center distance variation and modification amount as independent variables and the transmission error fluctuation as dependent variables, the mapping relationship between the four factors was established by the Kriging model. Then, the law of the influence of the three variables on the transmission error fluctuation was analyzed, and the optimal modification amount after the backlash and center distance disturbance was obtained. The results showed that when the resistance moment is constant, the increase of modification amount and backlash value leads to the increase of transmission error fluctuation, while the increase of center distance reduces the transmission error fluctuation. Among the three variables, the transmission error fluctuation is the most sensitive to backlash, followed by modification amount and the change of center distance. The modified modification amount reduces the transmission error fluctuation by 2.4% and the contact force fluctuation by 14.9%.

1. Introduction

In the working process of gear transmission system, affected by high temperature environment and various mating clearances, gear backlash and the center distance of gear will deviate from the preset value, which may greatly reduce the effectiveness of the optimal modification based on the original parameter environment in the new parameter environment, and further intensify the meshing vibration of gear [1].

In China, Baolin Wu [2] established the quantitative expression of the relationship between the geometric parameters and the maximum impact force of the meshing impact velocity in the process of gear transmission considering the deformation of the gear under load by applying the theory of mechanical dynamics. Huibo Zhang [3] studied and established the dynamic model of gear rotor considering the coupling of radial clearance and dynamic backlash, and analyzed the backlash of gear radial backlash. On the basis of considering the non-linear characteristics such as backlash, bearing clearance, time-varying meshing stiffness and tooth surface friction. Xiaoan Chen et al [4] put forward a non-linear dynamic analysis method of multi-clearance coupling gear transmission system based on finite element method for the first time. Siyu Chen [5] considered the meshing stiffness and static transmission error of actual gears, studied the dynamic behavior of gears with different modification amount and length. Overseas, Hamed Moradi [6] studied the influence of backlash nonlinearity on the non-linear vibration of spur gears by multi-scale method. Giorgio Bonori [7] estimated the static transfer error by using the nonlinear finite element method and optimized the modification parameters by using genetic algorithm. A Farshidianfar [8] used Melnikov analysis method to study the global homoclinic bifurcation and the transition to chaotic behavior of the nonlinear gear system. However, due to the complexity...
of the dynamic characteristics of gears under multi-gap coupling and the multi-factors affecting the meshing characteristics of gears, the research in the above literature lacks of the interaction between various factors.

Existing research mainly focuses on the influence of backlash on the dynamic characteristics of gear system under various working conditions, while few studies have been conducted on the influence of backlash and center distance on the optimal modification and the influence of the three on the transmission error fluctuation [9, 10, 11, 12]. In view of the above situation, the following two aspects were studied in this paper: (1) The effects of different backlash and center distance on the transmission error fluctuation and meshing vibration of gears and the offset of the optimal modification amount were compared and analyzed; (2) The influence of the backlash, center distance and the modification amount on the fluctuation of the transmission error was analyzed and the optimal amount of modification after the backlash and center distance disturbance was found.

2. Effect of gear backlash and center distance disturbance on optimal modification

In this paper, a pair of spur gears in a high-speed and light-load aero-gear system is taken as an object to study the influence of gear backlash and center distance disturbance on the optimal modification amount.

![Figure 1. Gear pair of aeronautical gear system](image)

Table 1. Basic parameters of gear pair

| Object  | Number of teeth | Modulus | Center distance [mm] | Pressure angle[°] | Tooth width [mm] | Rounding [mm] |
|---------|-----------------|---------|----------------------|------------------|-----------------|--------------|
| Gear 1  | 13              | 1.5     | 21.75                | 25               | 3               | 0.2          |
| Gear 2  | 16              |         |                      |                  |                 |              |

The parameters of the gear pair shown in Figure 1 are shown in Table 1. Gear 1 is the active gear. Speed is 14787r/min. Gear 2 is the driven wheel, and resistance moment is 0.636Nm. The average element size in the contact zone in the gear pair FE model is 0.32 mm. The element types of contact area are hexahedron element and triangular prism element. The contact type in LS-DYNA is “AUTOMATIC_SURFACE_TO_SURFACE”. The degrees of freedom in other directions of gears are limited except for the degree of freedom of axial rotation. Since the model does not take into account the influence of shaft and bearing, some simplifications are made in the gear pair FE model:

(1) The influence of elastic deformation of axle end support and bearing clearance is neglected.
(2) The static transmission error caused by manufacturing error and gear accuracy is neglected.
(3) Gear axial movement in meshing is ignored.

Transmission error is the main cause of vibration of gear transmission system [13]. In this paper, the KISSsoft software is used to calculate influence of the parameters including backlash, center distance and modification on transmission error fluctuation of a gear pair transmission. Four different combinations of these three parameters are used for calculation as shown in Table 2 and the corresponding calculations are shown in Figure 2.

Table 2. Model parameters

| Number | Modification [μm] | Backlash [mm] | Center distance [mm] |
|--------|-------------------|---------------|----------------------|
| 1      | 20                | 0.2           | 21.75                |
| 2      | 20                | 0.12          | 21.75                |
| 3      | 20                | 0.12          | 21.78                |
| 4      | 15                | 0.2           | 21.75                |

Curves a, b, c and d in Figure 2 correspond to the first, second, third and fourth gear parameters in Table 2, respectively. As can be seen from Figure 2, the fluctuation of transmission error also changes with the change of backlash value and center distance. From the comparison of the transmission error fluctuations of group 1 and group 2, group 1 and group 4, we can see that the increase of backlash and modification amount leads to
the increase of transmission error fluctuations, while the comparison of transmission error fluctuations of group 2 and group 3 shows that the transmission error fluctuations decrease with the increase of center distance.

Figure 2. Transmission error fluctuation when the torque is constant

Contact force fluctuation can visually indicate the meshing vibration of gears. Four groups of models in Table 2 are analyzed by ANSYS/LS-DYNA and the contact force of the contact area is obtained as shown in Figure 3.

In Figure 3, Figures a, b, c and d correspond to the first, second, third and fourth set of gear parameters in Table 2, respectively. The curves in Figure 3 are the two wave periods selected after the contact force fluctuation is stable. Combining with Figure 2 and comparing with Figure b and Figure c in Figure 3, it can be seen that under the same amount of modification, when the backlash value and the center distance change respectively, the contact force fluctuation law and the fluctuation amount of the gear pair will be affected. The increase of the backlash value increases the transmission error fluctuation amount, resulting in the gear pair to increase the fluctuation of contact force by 44%; while the increase of the center distance reduces the fluctuation of transmission error, and the meshing vibration of the gear pair is relieved, but the effect is weaker than the influence of the backlash on the contact force fluctuation. The decline was only 5.6%. Comparing curve a, curve d in Figure 2 and Figure a Figure d in Figure 3, respectively, it can be seen that when the backlash is changed, Correcting the amount of modification can reduce the amount of transmission error fluctuation of the gear pair. Thereby causing the contact force fluctuation of the gear pair to return to the previous level. As shown in Figure 3d, the contact force fluctuation amount becomes 123N, which is only 1.6% different from in Figure 3b. The above analysis shows that the change of backlash and center distance actually makes the original modification amount deviate from the optimal modification amount, which leads to the intensification of
meshing vibration of gears. That is to say, the meshing vibration of gear pairs after the disturbance of backlash and center distance can be improved by modifying the modification amount.

3. Determination of the range of parameters
To get the law of influence of backlash, center distance and modification amount on transmission error fluctuation, and then provide direction for correcting amount of modification, an agent model based on Kriging is established.

3.1. Determination of backlash range
Gear backlash is divided into circumferential backlash $j_l$ and normal backlash $j_n$. Normal backlash is often used in the calculation of tooth backlash. The formula for calculating the minimum backlash [14] includes two parts, one is necessary for lubrication and the other is necessary due to thermal expansion. The formula for calculating the minimum backlash is as follows.

$$j_{nmin} = j_{na} + j_{nb}$$

In the formula,

$$k = \sqrt{f_{pb1}^2 + f_{pb2}^2 + 2(f_{\beta} \cos \alpha)^2 + (f_{x} \sin \alpha)^2 + (f_{y} \cos \alpha)^2}$$

where $E_{ss1}$ and $E_{ss2}$ are respectively the upper deviation of tooth thickness of two gears. $f_{pb1} = f_{pb2} = f_{pb}$ are the limit deviation of base section. $F_{pb}$ is tooth tolerance. $F_{\beta} = F_{x}, f_{\beta} = F_{y} / 2 f_{x}$. $\beta$ is helical angle. $\alpha$ is tooth angle (pressure angle). $f_{x}$ is limit deviation of center distance.

The backlash value calculated from the above formula combined with the data in Table 1 is 0.28mm. The recommended backlash value of this type of gear in Chinese national standard is 0.08mm to 0.18mm. The recommended backlash value range for this drive system is 0.32mm to 0.4mm. The final determined backlash range is 0.1mm to 0.4mm.

3.2. Determination of the range of center distance variation
At 200 °C, the distance variation of central pore caused by thermal expansion is the main factor. ANSYS/LS-DYNA was used to analyze the thermal deformation of the box body shown in Figure. 4 at 200 °C. As shown in Table 3, the initial center distance of gear pair is 21.75mm. The center distance increased by 60μm after thermal expansion. Considering that the center distance may become smaller, it is finally determined that the center distance range is -20 to 60μm.

3.3. Determination of the range of modification amount
The profile modification can eliminate the interference of the gear pair due to the deformation of the gear teeth and smooth gear meshing process. Here, the gear pair is modified in the direction of tooth height. The modified area is shown in the shaded area of Figure 5. The modification parameters mainly include three parameters: the maximum modification amount $\Delta_{max}$, modification length $h_{max}$ and type of modification curve.
Figure 5. Profile modification area

The commonly used methods for estimating the maximum amount of modification are as follows: ideal estimation method, gear manufacturing precision selection method and empirical formula method. Among them, the deformation during the gear meshing process, the gear deformation caused by the temperature and the manufacturing error of the gear is comprehensively considered by ideal estimation method to calculate the maximum shape of the gear. Ideal estimation method is used to calculate the amount of modification here. The formula is as follows:

$$
\Delta_{\text{max}} = \delta + \delta_T + \delta_m
$$

In the formula,

- $\delta = \frac{F_t}{bC_{\gamma}}$;

- $\delta_T = \pi \cdot m \cdot \cos \alpha \cdot \Delta_{\gamma} \cdot \gamma \times 10^3$;  

- $\delta_T$ —— The amount of loading Deformation of Gear Tooth Surface;

- $\delta_m$ —— The amount of deformation caused by temperature changes;

- $\delta_m$ —— Gear manufacturing error;

- $C_{\gamma}$ —— Tooth surface comprehensive stiffness;

- $\Delta_{\text{max}}$ —— Temperature difference during gear transmission;

- $\gamma$ —— Gear material linear expansion coefficient;

Ignore the effects of manufacturing errors in this analysis, so $\Delta_{\text{max}} = \delta + \delta_T$. Thus, the theoretical tooth profile modification amount after rounding was 10μm. Since the manufacturing error and other factors are neglected, the above theoretical calculation value is on the low side. In order to get a more general rule in a wider range of modification, the modification range is set to 10 to 30μm.

The modified lengths of gear 2 and gear 1 are 0.4899mm and 0.4823mm, respectively.

The modification curve has straight line modification, Walker modification curve, arc modification curve and so on. Arc-modification curve was used here.

In summary, modification parameters are as follows. The amount of modification is 10 to 30μm. The modified lengths of gear 2 and gear 1 are 0.4899mm and 0.4823mm, respectively. The modification curve is a circular curve.

4. Agent model construction

4.1. Select sample points

The experimental design theory can help determine the sample points [15]. Common experimental design methods include uniform test design, Latin hypercube design and optimized Latin square design. Here, the optimized Latin square test design is used to select the sample points. In this experiment, 10 sets of samples were selected. The selected sample points are shown in Figure 6.
4.2. Kriging model

The Kriging form is as follows:

\[ y(x) = \sum_{i=1}^{m} \beta_i f_i(x) + z(x) \]  \tag{4}

\( f_i(x) \) is the basis function. A polynomial function is generally used. \( z(x) \) is the associated stochastic process function. The mean is 0. The process variance \( \sigma^2 \) and spatial covariance functions are defined as follows:

\[ \text{cov}(Z(x_i), Z(x_j)) = \sigma^2 R(x_i, x_j) \]  \tag{5}

\[ \sigma^2 = \frac{1}{P} (y - X\beta)^T R^{-1} (y - X\beta) \]  \tag{6}

In the formula, \( R[x_i, x_j] \) is the correlation function of \( x_i \) and \( x_j \). The correlation function has different forms, and the Gaussian correlation function is used here:

\[ R(x_i, x_j) = \exp(-\sum_{k=1}^{m} \theta_k [x_i^k - x_j^k]^2) \]  \tag{7}

When the correlation function is determined, the response at any point is estimated to be:

\[ \hat{y} = \hat{\beta} + r^T(x)R^{-1}(y - f \hat{\beta}) \]  \tag{8}

\( y \) is a column vector of length \( n_x \), which is the response value of the sample data. When \( f(x) \) is a constant, \( f \) is a unit column vector of length \( n_x \). \( r^T(x) \) is the correlation vector between test point \( x \) and sample point \( \{x^1, x^2, ..., x^5\} \) which length is \( n_x \).

5. Results analysis

5.1. Model accuracy check

To know the accuracy of the model, 5 sets of data were randomly generated within the sample space. The corresponding values were obtained using the fitted model, and then compared with the calculated values of KISSsoft. As shown in Table 4, A2 indicates the amount of modification. B2 indicates the backlash. C2 indicates the amount of variation in the center distance. \( y_1 \) indicates the response value of the fitted model, and \( y_2 \) indicates the calculated value of KISSsoft.

Table 4. Comparison between Kriging model response value and KISSsoft calculation result

| Samples | Number | A2 [μm] | B2 [mm] | C2 [μm] | \( y_1 \) [μm] | \( y_2 \) [μm] |
|---------|--------|---------|---------|---------|----------------|----------------|
| 1       | 20     | 0.1     |         | 20      | 1.0514         | 1.0454         |
| 2       | 25     | 0.25    | 60      |         | 1.0909         | 1.0975         |
| 3       | 30     | 0.325   | 0       |         | 1.1343         | 1.1417         |
| 4       | 15     | 0.4     | 20      |         | 1.1402         | 1.1467         |
| 5       | 10     | 0.175   | 40      |         | 1.0426         | 1.0415         |

Figure 7 shows the values of \( y_1 \) and \( y_2 \) in a graph form. It can be seen that the relative error is below 1\%, which indicates that the Kriging model has high precision and can meet the analysis requirements.
Figure 7. Relative error between Kriging and KISSsoft

5.2. Influence of backlash, modification amount and center distance variation on the fluctuation of transmission error

When the center distance fluctuation, backlash and shape modification are fixed values, the variation of the transmission error fluctuation with the other two variables is shown in Figure 8.

Figure 8. Variation of transmission error fluctuation with variables

It can be seen from Figure 8. that the influence of the amount of modification, backlash and center distance variation on the fluctuation of the transmission error is monotonous when the resistance moment is constant. The fluctuation of the transmission error increases with the amount of modification, increases with the increase of backlash value and decreases with the increase of center distance. In the actual situation, the increase of the center distance will lead to an increase in the backlash. Therefore, the influence of the increase of the center distance on the fluctuation of the transmission error and the meshing vibration has two aspects. (1) The influence of the increase of center distance itself on the fluctuation of transmission error. (2) The influence of the increase of center distance on the fluctuation of transmission error by increasing the backlash value. The sensitivity of transmission error fluctuation to various factors can be obtained by deriving the variables in the fitting model, as shown in Figure 9.

Figure 9. Sensitivity of transmission error fluctuation to variables

It can be obtained from Figure 9. that among the three variables, the transmission error fluctuation is the most sensitive to the backlash, followed by the modification, and the lowest to the center distance variation. From the influence of the variation of the center distance on the fluctuation of the transmission error, the influence of center distance variation on transmission error fluctuation is smaller than that of backlash variation. So in the actual situation, the center distance increases. The positive effect of center distance increase on transmission error fluctuation will be covered by the negative effect of backlash value increase caused by
center distance increase. Therefore, the two effects caused by the increase of the center distance are negative after superposition.

5.3. Determination of the optimal modification

When the center distance and the backlash of the gear increase due to thermal expansion, it is known from the influence law of the amount of modification on the fluctuation of the transmission error that the amount of modification should be corrected in the direction of reducing the amount of modification. Take the gear pair in this article as an example. In actual working conditions, due to thermal expansion, the center distance of the gear pair increases by 60μm to 21.81 mm. At the same time, the backlash value increases by 0.04 mm to 0.36 mm. The original modification amount is 10μm. On the premise that gears do not interfere with meshing and meshing in non-involute area (except modification area), the amount of modification is reduced to 6μm in order to reduce the amount of transmission error fluctuation. On the basis of the above conditions, the calculation of the four sets of models in Table 5 is performed to check whether the corrected amount of the modification is valid. The amount of transmission error fluctuation and the contact force fluctuation are shown in Figure 10 and Figure 11, respectively.

Table 5. Simulation model

| Number | Backlash [mm] | Modification [μm] | Center distance [mm] |
|--------|---------------|-------------------|----------------------|
| 1      | 0.32          | 10                | 21.75                |
| 2      | 0.36          | 10                | 21.81                |
| 3      | 0.36          | 6                 | 21.81                |
| 4      | 0.36          | 0                 | 21.81                |

Figure 10. Fluctuation of transmission error

The first, second, third, and fourth sets of models in Table 5 correspond to the curves a, b, c, and d in Fig 10 and Figures a, b, c and d in Figure 11 respectively. Referring to Table 5 and Figure 10 and Figure 11, the comparison of the first and second sets of models shows that when the backlash and the center distance increase, the transmission error fluctuation and the contact force fluctuation of the gear pair increase. The increase of transmission error fluctuation and contact force fluctuation is 0.7% and 15.2% respectively. When the modification amount is reduced to 6μm, it can be seen that the transmission error fluctuation and the contact force fluctuation amount return to previous levels. The transmission error fluctuation was reduced by 2.4% and the contact force fluctuation was reduced by 14.9%. Continue to reduce the amount of modification to 0. That is the fourth group of models. The transmission error fluctuation amount is further reduced, but the contact force fluctuation amount of the gear pair increases and the contact force fluctuation amount increased up to 22.7% compared with the result of the modification of 6μm. The reason for that is that the too small amount of modification causes the gear pair to interfere or engage in non-involute zone engagement. When the amount of modification can ensure that the gear pair does not interfere and the gear pair does not engage in non-involute zone (except for the modification zone), the amount of modification is the optimal amount of modification. After the thermal expansion causes the backlash and the center distance to increase, the transmission error fluctuation increases, which also causes the optimal modification amount to shift to a smaller value. Therefore, the amount of modification should be reduced so that the amount of modification is optimum in a new parameter environment.

In summary, in the previous theoretical calculation method of shape modification, the influence of backlash and center distance on the amount of shape modification is often neglected. It can be seen from the above analysis that the disturbance of backlash and center distance will make the original modification amount Deviate from the optimal value. And by correcting the amount of modification, the meshing vibration of the gears pair can be reduced to some extent after backlash and center distance disturbance. It can be further
understood that there are different amounts of optimum modification under different backlashes and center distances.

![Figure 11. Contact force-time curve](image)

6. Conclusion
(1) The disturbance of backlash and center distance increase the fluctuation of the transmission error, which in turn exacerbates the meshing vibration and increases the fluctuation of contact force.
(2) It was verified that the original modification amount is deviated from the optimal value when the backlash and center distance disturbance.
(3) The Kriging-based agent model was established and its accuracy was verified. Based on that, the influence of the modification amount, backlash and center distance variation on the fluctuation of the transmission error was analyzed. The results showed that without considering the fluctuation of torque, the influence of the amount of modification, backlash and center distance variation on the fluctuation of the transmission error was monotonous. The increase of the amount of modification and the value of the backlash will increase the fluctuation of the transmission error, and increasing the center distance helps to reduce the fluctuation of transmission error. In the three variables, the transmission error fluctuation is the most sensitive to the backlash. The modification amount is the second and the sensitivity to the center distance variation is the lowest. It provides a reference for understanding the influence of modification, backlash and center distance variation on transmission error fluctuation and meshing vibration.
(4) The optimal modification amount of the gear after the backlash and center distance disturbance was found. The results showed that the modified amount of modification reduced the transmission error fluctuation by 2.4% and the contact force fluctuation by 14.9%. It provides an idea for correcting the amount of modification after backlash and center distance disturbance.

7. References
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