O(\(\alpha_s\)) SPIN EFFECTS IN \(e^+e^- \rightarrow q\bar{q}\) (g)

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Abstract

We discuss O(\(\alpha_s\)) spin effects in \(e^+e^- \rightarrow q\bar{q}\) (g) for the polarization of single quarks and for spin-spin correlations of the final state quarks. Particular attention is paid to residual mass effects in the limit \(m_q \rightarrow 0\) which are described in terms of universal helicity flip and helicity non-flip contributions.

1 Introduction

In a series of papers we have investigated O(\(\alpha_s\)) final state spin phenomena in \(e^+e^- \rightarrow q\bar{q}\) (g). In [1–3] we have provided analytical results for the polarization of single quarks and in [4, 5] for longitudinal spin-spin correlations of the final state quarks including their dependence on beam-event correlations.\(^1\) By carefully taking the \(m_q \rightarrow 0\) limits of our analytical O(\(\alpha_s\)) results we have found that, at O(\(\alpha_s\)), the single-spin polarization \(P^{(l_1)}\) and the spin-spin polarization correlation \(P^{(l_1l_2)}\) do not agree with their \(m_q = 0\) counterparts. In fact, averaging over beam-event correlation effects, at O(\(\alpha_s\)) one has

\[
P^{(l_1)}_{m_q \rightarrow 0} = P^{(l_1)}_{m_q = 0} \left(1 - \frac{2}{3} \frac{\alpha_s}{\pi}\right), \quad P^{(l_1l_2)}_{m_q \rightarrow 0} = P^{(l_1l_2)}_{m_q = 0} \left(1 - \frac{4}{3} \frac{\alpha_s}{\pi}\right)
\]

(1)

where the residual \(m_q \rightarrow 0\) contributions are encased in square brackets. In Eq.(1) \(P^{(l_1)}\) denotes the longitudinal single-spin polarization of the final state quark and \(P^{(l_1l_2)}\) denotes the longitudinal spin-spin polarization correlation of the quark and antiquark. From Eq.(1) it is apparent that at O(\(\alpha_s\)) QCD(\(m_q = 0\)) \(\neq\) QCD(\(m_q \rightarrow 0\)) in polarization phenomena. We mention that the longitudinal polarization components are the only polarization components that survive in the high energy (or \(m_q = 0\)) limit. Here and in the following the residual mass contributions will be called anomalous contributions for the reason that the anomalous helicity flip contribution enters as absorptive input in the dispersive derivation of the value of the axial anomaly [7]. We shall see further on that \(P^{(l_1)}_{m_q = 0} = g_{14}/g_{11}\) where \(g_{14}\) and \(g_{11}\) are electroweak coupling coefficients and \(P^{(l_1l_2)}_{m_q = 0} = -1\) independent of the electroweak coupling coefficients. It is important to keep in mind that there are no residual mass effects at O(\(\alpha_s\)) in the unpolarized rate.

By an explicit calculation we have checked that the difference of the two results originates from the near-forward region (see [8]) which is very suggestive of an explanation in

\(^1\)Numerical O(\(\alpha_s\)) results on final state quark polarization effects in \(e^+e^- \rightarrow q\bar{q}\) (g) can be found in [6].
terms of universal near-forward contributions. That there is a universal helicity flip contribution in the splitting process \( q_\pm \rightarrow q_\mp + g \) has been noted some time ago in the context of QED [9] (see also [10, 11]). We have found that there also exists a universal helicity non-flip contribution whereas the anomalous contributions to the single-spin polarization \( P^{(\ell_1 \ell_2)} \) results to 100% from the universal helicity non-flip contribution whereas the anomalous contributions to the spin-spin correlation \( P^{(\ell_1 \ell_2)} \) is 50% helicity flip and 50% helicity non-flip.

## 2 Definition of polarization observables

In the limit \( m_q \rightarrow 0 \) the relevant spin degrees of freedom are the longitudinal polarization components \( s^\ell_1 = 2\lambda_q \) and \( s^\ell_2 = 2\bar{\lambda}_q \) of the quark and the antiquark. One defines an unpolarized structure function \( \bar{H} \) and single–spin and spin–spin polarized structure functions \( H^{(\ell_1)}, H^{(\ell_2)} \) and \( H^{(\ell_1 \ell_2)} \), resp., according to

\[
H(s^\ell_1 s^\ell_2) = \frac{1}{4} (H + H^{(\ell_1)} s^\ell_1 + H^{(\ell_2)} s^\ell_2 + H^{(\ell_1 \ell_2)} s^\ell_1 s^\ell_2).
\]

Eq. (2) can be inverted to give

\[
\begin{align*}
H &= \left[H(\uparrow\uparrow)\right] + H(\uparrow\downarrow) + H(\downarrow\uparrow) + \left[H(\downarrow\downarrow)\right], \\
H^{(\ell_1)} &= \left[H(\uparrow\uparrow)\right] + H(\uparrow\downarrow) - H(\downarrow\uparrow) - \left[H(\downarrow\downarrow)\right], \\
H^{(\ell_2)} &= \left[H(\uparrow\uparrow)\right] - H(\uparrow\downarrow) + H(\downarrow\uparrow) - \left[H(\downarrow\downarrow)\right], \\
H^{(\ell_1 \ell_2)} &= \left[H(\uparrow\uparrow)\right] - H(\uparrow\downarrow) - H(\downarrow\uparrow) + \left[H(\downarrow\downarrow)\right].
\end{align*}
\]

We have indicated in (3) that the spin configurations \( H(\uparrow\uparrow) \) and \( H(\downarrow\downarrow) \) can only be populated by anomalous contributions in the \( m_q \rightarrow 0 \) limit. The normalized single-spin polarization \( P^{(\ell_1)} \) and the spin–spin correlation \( P^{(\ell_1 \ell_2)} \) are then given by

\[
P^{(\ell_1)} = \frac{g_{14}}{g_{11}} \frac{H^{(\ell_1)}}{H}, \quad P^{(\ell_1 \ell_2)} = \frac{H^{(\ell_1 \ell_2)}}{H},
\]

where \( g_{14} \) and \( g_{11} \) are \( q^2 \)-dependent electroweak coupling coefficients (see e.g. [3]). For the ratio of electroweak coupling coefficients one finds \( g_{14}/g_{11} = -0.67 \) and -0.94 for up-type and down-type quarks, respectively, on the \( Z_0 \) resonance, and \( g_{14}/g_{11} = -0.086 \) and -0.248 for \( q^2 \rightarrow \infty \).

Considering the fact that one has \( H(\uparrow\uparrow) = H(\downarrow\downarrow) = 0 \) for \( m_q = 0 \), \( H^{pc}(\uparrow\downarrow) = H^{pc}(\downarrow\uparrow) \) for the p.c. structure functions \( (H, H^{(\ell_1 \ell_2)}) \), and \( H^{pv}(\uparrow\downarrow) = -H^{pv}(\downarrow\uparrow) \) for the p.v. structure function \( H^{(\ell_1)} \) with \( H^{pv}(\uparrow\downarrow) = H^{pc}(\downarrow\uparrow) \), it is not difficult to see from Eq. (3) that \( H^{(\ell_1)}/H = +1 \) and \( H^{(\ell_1 \ell_2)}/H = -1 \) for \( m_{q=0} \) to all orders in perturbation theory.
3 Polarization results for $m_q \to 0$ and $m_q = 0$

The results of taking the $m_q \to 0$ limit of our analytical finite mass results in [1–5] can be concisely written as

$$H^{pc}(s_1^\ell, s_2^\ell) = \frac{1}{4} \left( H^{pc} + H^{pc(t_1 \ell_2)} s_1^\ell s_2^\ell \right) = N_c q^2 \left( 1 - s_1^\ell s_2^\ell \right) \left( 1 + \frac{\alpha_s}{\pi} \right) + \left[ \frac{4}{3} \times \frac{\alpha_s}{\pi} s_1^\ell s_2^\ell \right]$$,

$$H^{pv}(s_1^\ell, s_2^\ell) = \frac{1}{4} \left( H^{pv(t_1)} s_1^\ell + H^{pv(t_2)} s_2^\ell \right) = N_c q^2 (s_1^\ell - s_2^\ell) \left( 1 + \frac{\alpha_s}{\pi} - \left[ \frac{2}{3} \times \frac{\alpha_s}{\pi} \right] \right). \quad (5)$$

We have again indicated the parity nature of the structure functions ((pc): parity conserving; (pv): parity violating). The $m_q = 0$ results are obtained from Eq. (5) by dropping the anomalous square bracket contributions (see [3, 5]).

It is not difficult to see that one has $H^{pc} = -H^{pc(t_1 \ell_2)} = H^{pv(t_1)} = -H^{pv(t_2)}$ for $m_q = 0$. By commuting $\gamma_5$ through the relevant $m_q = 0$ diagrams, as Eq. (5) shows these relations no longer hold true for the anomalous contributions showing again that the anomalous contributions originate from residual mass effects which obstruct the simple $\gamma_5$-commutation structure of the $m_q = 0$ contributions.

4 Near-forward gluon emission

In Table 1 we list the helicity amplitudes $h_{\lambda_1, \lambda_2, \lambda_3}$ for the splitting process $q(p_1) \rightarrow q(p_2) + g(p_3)$ and the $\cos \theta$ dependence of the helicity amplitudes in the near-forward region. We also list the difference of the initial helicity and the final helicities $\Delta \lambda = \lambda_1 - \lambda_2 - \lambda_3$.

| $h_{\lambda_1, \lambda_2, \lambda_3}$ | $\Delta \lambda$ | $\cos \theta$ dependence |
|----------------------------------|-----------------|--------------------------|
| $h_{1/2 1/2 +1}$                | −1              | $\sim \sqrt{1 - \cos \theta}$ |
| $h_{1/2 1/2 -1}$                | +1              | $\sim \sqrt{1 - \cos \theta}$ |
| $h_{1/2 -1/2 +1}$               | 0               | $\sim m_q/E$             |
| $h_{1/2 -1/2 -1}$               | +2              | 0                        |

Table 1. Helicity amplitudes and their near-forward behaviour.

Column 2 shows helicity balance $\Delta \lambda = \lambda_1 - \lambda_2 - \lambda_3$.

In the forward direction $\cos \theta = 1$ the only surviving helicity amplitude is $h_{1/2 -1/2 +1}$ for which the helicities satisfy the collinear angular momentum conservation rule $\Delta \lambda = 0$.

Squaring the helicity flip amplitude $h_{1/2 -1/2 +1}$ and adding in the propagator denominator factor $2p_2p_3 = 2E^2 x (1 - x) (1 - \sqrt{1 - m^2/E^2})$ one obtains $(x = E_3/E; E$ is the energy of the incoming quark and $s = 4E^2$)

$$\frac{d\sigma_{[h,f]}}{dx \, d\cos \theta} = \sigma_{Born}(s) C_F \frac{\alpha_s}{4\pi} x \frac{m_q^2}{E^2} \frac{1}{\left( \cos \theta \sqrt{1 - m_q^2/E^2} \right)^2} \quad (6)$$

Note that the helicity flip contribution $h_{1/2 -1/2 +1}$ is not seen in a $m_q = 0$ calculation. The helicity flip splitting function $d\sigma_{[h,f]}/d\cos \theta$ is strongly peaked in the forward direction.
Using the small \((m_q^2/E^2)\)-approximation one finds that \(\sigma_{[hf]}\) has fallen to 50% of its forward peak value at \(\cos \theta = 1 - (\sqrt{2} - 1)m_q^2/(2E^2)\).

Integrating Eq. (6) over \(\cos \theta\), we obtain
\[
\frac{d\sigma_{[hf]}}{dx} = \sigma_{\text{Born}}(s)C_F \frac{\alpha_s}{2\pi} x \equiv \sigma_{\text{Born}}(s)D_{[hf]}(x) \tag{7}
\]
where \(D_{[hf]} = C_F(\alpha_s/2\pi)x\) is called the helicity flip splitting function \([11]\). The integrated helicity flip contribution survives in the \(m_q \rightarrow 0\) limit since the integral of the last factor in Eq.(6) is proportional to \(1/m_q^2\) which cancels the overall \(m_q^2\) factor in Eq.(6). It is for this reason that the survival of the helicity flip contribution is sometimes called an \(m/m\)-effect.

For the helicity non-flip contribution one finds
\[
\frac{d\sigma_{nf}}{dx d\cos \theta} = \sigma_{\text{Born}}(s)C_F \frac{\alpha_s}{\pi} \frac{1 + (1 - x)^2}{x} \left(1 - \cos \theta \sqrt{1 - m_q^2/E^2}\right)^2 \tag{8}
\]

The helicity non-flip splitting function \(d\sigma_{nf}/d\cos \theta\) vanishes in the forward direction but is strongly peaked in the near-forward direction. Using again the small \((m_q^2/E^2)\) approximation \(\sigma_{nf}\) can be seen to peak at \(\cos \theta = 1 - m_q^2/(2E^2)\).

Integrating Eq.(8) one obtains
\[
\frac{d\sigma_{nf}}{dx} = \sigma_{\text{Born}}(s)C_F \frac{\alpha_s}{\pi} \frac{1 + (1 - x)^2}{x} \ln \frac{E}{m_q} \equiv \sigma_{\text{Born}}(s)D_{nf}(x) \tag{9}
\]
where we have retained only the leading-log contribution. \(D_{nf}(x)\) can be seen to be the usual non-flip splitting function.

We now turn to the anomalous helicity non-flip contribution. Let us rewrite the non-flip contribution in the form
\[
\sigma_{nf} = \sigma_{\text{nf}} + \sigma_{[hf]} - \sigma_{[hf]} \tag{10}
\]

By explicit calculation we have seen that the total unpolarized rate \(\sigma_{\text{total}}\) has no anomalous contribution. The conclusion is that there is an anomalous contribution also to the non-flip transition with the strength \(-D_{[hf]} = -C_F(\alpha_s/2\pi)x\). We conjecture that the same pattern holds true for other processes, i.e. that there are no anomalous contributions to unpolarized rates but that there are anomalous contributions to both helicity flip and non-flip contributions in polarized rates.

Taking into account the anomalous helicity flip \(\sigma_{[hf]}\) and non-flip \(\sigma_{[nf]}\) contributions the pattern of the anomalous helicity contributions to the various spin configurations can then be obtained in terms of the Born term contributions and the universal flip and non-flip contributions \(\pm \int_0^1 D_{[hf]}(x)dx = \pm C_F \alpha_s/(4\pi) = \pm \alpha_s/(3\pi)\). Using a rather suggestive notation for the anomalous contributions one has
\[
\begin{align*}
[\uparrow \uparrow] & = (\uparrow \downarrow)_{[hf]} + (\downarrow \uparrow)_{[hf]} = (\uparrow \downarrow)_{\text{Born}} + (\downarrow \uparrow)_{\text{Born}} \left[ C_F \frac{\alpha_s}{4\pi} \right], \\
[\uparrow \downarrow] & = (\uparrow \downarrow)_{[nf]} + (\downarrow \uparrow)_{[nf]} = -2(\uparrow \downarrow)_{\text{Born}} \left[ C_F \frac{\alpha_s}{4\pi} \right], \\
[\downarrow \uparrow] & = (\downarrow \uparrow)_{[nf]} + (\uparrow \downarrow)_{[nf]} = -2(\downarrow \uparrow)_{\text{Born}} \left[ C_F \frac{\alpha_s}{4\pi} \right], \\
[\downarrow \downarrow] & = (\downarrow \uparrow)_{[hf]} + (\uparrow \downarrow)_{[hf]} = (\uparrow \downarrow)_{\text{Born}} + (\downarrow \uparrow)_{\text{Born}} \left[ C_F \frac{\alpha_s}{4\pi} \right].
\end{align*} \tag{11}
\]
Since one has \((\uparrow \downarrow)^{pc}_{\text{Born}} = (\downarrow \uparrow)^{pc}_{\text{Born}} = 2N_c q^2\) and \((\uparrow \downarrow)^{pv}_{\text{Born}} = -(\downarrow \uparrow)^{pv}_{\text{Born}} = 2N_c q^2\) one obtains the anomalous contributions in Eq.(5) using the factorizing relations (11). In particular one sees that anomalous contribution to the single-spin polarization \(P(\ell_1)\) results to 100% from the universal helicity non-flip contribution whereas the anomalous contributions to the spin-spin correlation function \(P(\ell_1\ell_2)\) is 50% helicity flip and 50% helicity non-flip. We conclude that the anomalous non-flip contribution are unavoidable in the \(O(\alpha_s)\) description of \(m_q \rightarrow 0\) spin phenomena in \(e^+e^- \rightarrow q\bar{q} (g)\). The normal contributions in Eq.(5) require an explicit calculation. The result \(H = 4N_c q^2(1 + \alpha_s/\pi)\) is, of course, well known since many years.

In this talk we did not discuss physics aspects of the anomalous helicity flip contribution. We refer the interested reader to the papers [10–13] which contain a discussion of various aspects of the physics of the anomalous helicity flip contributions in QED and QCD.

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