Chiral Effective Field Theory for Nucleonic Matter

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Abstract. Predictions for the interaction part of the symmetry energy obtained from different microscopic approaches are reviewed and discussed in comparison with updated constraints recently obtained at GSI. The discussion is then extended to the neutron skin thickness in $^{208}$Pb and its relation to the density derivative of the symmetry energy. We highlight the importance of giving proper consideration to theoretical uncertainties of microscopic predictions in order to guide phenomenological analyses.

1. Introduction

The energy per particle as a function of density in infinite nuclear matter, known as the equation of state (EoS), contains rich information about the nature of the nuclear force in hadronic medium. Selected reaction observables and other nuclear properties have been found to be sensitive to the EoS, and therefore their measurements can provide useful constraints for the latter. The dependence of the EoS of isospin asymmetric matter on the neutron excess parameter brings up the symmetry energy, a quantity of fundamental importance for a variety of neutron-rich systems. Presently our knowledge of the symmetry energy is limited, particularly its density dependence around and above saturation density. From the experimental side, heavy-ion (HI) reactions are an established way to seek such constraints, based on the fact that the EoS is an important part of the input in transport models describing HI collisions.

Other nuclear properties have been found highly sensitive to specific aspects of the EoS of asymmetric matter. The neutron skin thickness, for instance, is sensitive to the slope of the symmetry energy, which determines to which extent neutrons are pushed outwards to form the skin. A strong correlation is also found between the pressure in the interior of a heavy nucleus and the radius of the average-mass neutron star. In summary, studies of nucleonic matter are especially timely and important, as they support on-going and future experimental effort, in both astrophysics and terrestrial laboratories.

The earlier FOPI-LAND data [1, 2], reanalysed in Ref. [3] and compared with transport model calculations, have suggested a softer-than-linear to linear term for the potential energy part of the symmetry energy, when the latter is parametrized as a power law. More recently, the directed and elliptic flows of neutrons and light charged particles in the reaction $^{197}$Au + $^{197}$Au at 400 MeV per nucleon were measured in the ASY-EOS experiment at GSI [4]. The updated findings confirm a moderately soft density dependence, but in Ref. [4] the authors report a more stringent constraint up to twice normal density.

Naturally, when updated constraints are made available, comparison with microscopic calculations becomes especially timely. Within that spirit, modern predictions of the symmetry energy based on chiral effective field theory (EFT) [5] are reviewed and discussed, following
closely Ref. [6]. For comparison, more “traditional” approaches are also considered, such as those based on meson-theoretic or phenomenological nucleon-nucleon (NN) potentials and three-nucleon forces (3NF). These approaches, which were particularly popular in the 1990’s and are still frequently used today, follow a very different philosophy. Thus, their inclusion in the comparison will provide a realistic measure of the spreading of contemporary theoretical predictions. We will explore whether the theoretical predictions are consistent with the recent constraints. An important point we wish to raise is the relevance of including microscopic predictions when correlating the density dependence of the symmetry energy to the neutron skin thickness in $^{208}$Pb.

Before moving to the next section, we recall that the parametrization used in the analysis of Ref. [4] is

$$e_{\text{sym}}(\rho) = 22 \text{ MeV}\left(\frac{\rho}{\rho_0}\right)^\gamma + 12 \text{ MeV}\left(\frac{\rho}{\rho_0}\right)^{2/3},$$

which fixes the symmetry energy at $\rho_0$ to be 34 MeV. The power law coefficient, $\gamma$, is reported as $0.72 \pm 0.19$. The same coefficient was found to be $0.9 \pm 0.4$ from the FOPI-LAND data [1, 2, 3]. Other useful quantities for the discussion which follows are the so-called “L” coefficient,

$$L = 3\rho_0\left(\frac{\partial e_{\text{sym}}}{\partial \rho}\right)_{\rho_0},$$

and the closely related symmetry pressure, $P_0 = \rho_0L/3$.

2. Theoretical Input

2.1. The chiral EoS

Chiral EFT is presently a popular approach which starts from a low-energy realization of quantum chromodynamics [5]. In chiral EFT, one retains the basic degrees of freedom typical of low-energy nuclear physics, pions and nucleons, while fitting unresolved nuclear dynamics at short distances to the properties of two- and few-nucleon systems. More specifically, EFT is a theory in which the properties governed by low-energy physics are specified by the choice of degrees of freedom and symmetries, and can be computed systematically. Short-range physics is included through the processes of regularization and renormalization. The microscopic EoS of symmetric nuclear matter and neutron matter based on chiral EFT employed here are calculated as described in Ref. [7]. The predictions at next-to-next-to-next-to-leading order (N$^3$LO) and at next-to-next-to-leading order (N$^2$LO) are based on high-precision chiral NN potentials at the respective orders [8] together with the leading 3NF, which is treated as in Ref. [9].

Since estimating the truncation error at N$^3$LO requires the predictions at N$^2$LO, those will be shown as well. (N$^4$LO predictions are not yet available.) Note that the N$^2$LO calculation is complete, in the sense that both the two-nucleon force (2NF) and the 3NF are consistently at next-to-next-to-leading order. This is not the case, though, for the N$^3$LO calculation, where the 3NF at N$^2$LO is employed. Efforts are in progress in our group to remove this inconsistency.

2.2. Relativistic meson-theoretic potentials and the Dirac-Brueckner-Hartree-Fock approach to the equation of state

The relativistic approach to nuclear matter, particularly the Dirac-Brueckner-Hartree-Fock (DBHF) approximation, started with the observation that the DBHF theory, unlike conventional Brueckner theory, has the inherent ability to predict realistic values for the saturation energy and density of nuclear matter. The characteristic feature of the DBHF approach is the fact that important 3NF are effectively taken into account through the density dependence of the nucleon spinors. This effective 3NF, which originates from virtual excitations of nucleon-antinucleon pairs and is, for that reason, known as “Z-diagrams”, provides a strong (repulsive and density-dependent) saturating effect and, thus, an important mechanism missing from conventional
Figure 1. Frame on the upper left: microscopic predictions of the interaction part of the symmetry energy at N2LO and N3LO of chiral perturbation theory, and corresponding power-law fits. See inset for the definition of the various curves. Upper right: The potential part of the symmetry energy as predicted with Bonn B and the DBHF calculation compared with a power-law fit. Lower panel: the same, but for the APR model [12].

2.3. Variational approaches
Alternatively, the energy per particle in nuclear matter can be obtained combining the 2NF with meson-theoretic or phenomenological 3NF to generate the additional repulsion essential to improve saturation. Nonrelativistic calculations of symmetric and neutron matter based on variational methods [11] and phenomenological 2NF and 3NF have been used extensively. To also represent this point of view, we will include predictions [12] based on popular phenomenological 2NF and 3NF from the 90’s, namely the Argonne v18 NN potential [13] together with the Urbana model IX [14] 3NF. These will be referred to as the “APR” model.
3. Predictions and Discussion

We calculate the symmetry energy for the theories and models mentioned in the previous section and subtract the kinetic contribution. The microscopic values of the potential energy part are then fitted with a power law for each of the models being considered, $\rho_0$ is the actual saturation density for that particular EoS.

On the left-hand side of Fig. 1, we display the potential energy part of the symmetry energy as obtained from microscopic calculations at N$^2$LO (solid green line) and at N$^3$LO (solid red line), together with approximations given by functions of the form

$$e_{sym}^{pot} = V_0 (\rho/\rho_0)^\gamma,$$

(3)
dashed green for N$^2$LO and dashed red for N$^3$LO. The fit is done by searching for the single parameter $\gamma$, setting $V_0$ equal to the microscopically predicted value at the appropriate saturation density. The density range considered in the fit covers approximately from 0.03 to 0.33 fm$^{-3}$, with all points carrying the same weight.

The theoretical curves appear reasonably described by the simple ansatz up to their respective saturation densities and somewhat above it, whereas the constraint should be applicable up to about 2$\rho_0$, which amounts to approximately 0.3 fm$^{-3}$ by the definition of Ref. [4]. Furthermore, an analytical approximation which appears reasonable may be less than satisfactory with respect to derivatives. This point will be addressed again later in the paper.

In the middle frame of Fig. 1, one can see to which degree the interaction part of the symmetry energy can be approximated by a single-term power law for the Bonn B meson-exchange potential [15] used in a DBHF calculation, while a similar comparison is shown on the right-hand side of the same figure for the potential symmetry energy obtained from the EoS of Ref. [12].

Of course the N$^3$LO predictions must be seen in the context of EFT theoretical uncertainties. Therefore, in Fig. 2 the N$^3$LO predictions are shown with their estimated truncation error. In Fig. 2, the shaded area represents the empirical constraint. The predictions fall within the empirical constraints at the lower densities but are otherwise softer.

![Figure 2. Microscopic predictions of the interaction part of the symmetry energy at N$^3$LO with EFT truncation error. The shaded area shows the empirical constraint from Ref. [4].](image)

Obviously, the parametrization given in Eq. 1 was found to be consistent with the reaction observables measured in the GSI experiment. However, here one learns that, although a moderately soft (less than linear) dependence is preferred by microscopic models, Eq. 1 is overall not a satisfactory representation of these theoretical predictions. It would be interesting to move beyond the power-law parametrization when analyzing elliptic-flow ratios.
Table 1. Neutron skin ($S$) of $^{208}$Pb with varying power law, $\gamma$, in the interaction symmetry energy within the range determined by the ASY-EOS analysis. The third column displays the slope of the symmetry energy at about 2/3 of saturation density, followed by the $L$ parameter and the symmetry pressure. For further details, see discussion in the text.

| $\gamma$ (fm) | $S$ (fm) | $\frac{\partial e_{\text{sym}}}{\partial \rho}$ (MeV fm$^3$) | $L$ (MeV) | $P_0$ (MeV/fm$^3$) |
|---------------|----------|-------------------------------------------------|-----------|-------------------|
| 0.53          | 0.14     | 154                                             | 60.1      | 3.14              |
| 0.72          | 0.18     | 177                                             | 72.6      | 3.80              |
| 0.91          | 0.21     | 195                                             | 85.2      | 4.45              |

Next we extend the discussion to the neutron skin of $^{208}$Pb in relation to density derivatives and pressure. Due to lack of space, the reader is referred to Ref. [6] and references therein for details on the neutron skin calculations.

In Table 1, for values of $\gamma$ spanning the uncertainty of the ASY-EOS constraint, we show (second, fourth, and fifth columns, respectively), the skin thickness of $^{208}$Pb, the $L$ parameter, defined in Eq. (2), and the symmetry pressure $P_0$. This confirms that the skin is sensitive to the pressure in the neutron-enriched core of $^{208}$Pb which pushes excess neutrons towards the low-density edges of the nucleus.

On the other hand, the average density in nuclei is less than saturation density and, therefore, typical nuclear observables (such as, for instance, those used to construct phenomenological forces) actually probe densities somewhat lower than saturation density. Therefore, Table 1 also shows how the slope of the symmetry energy at $\rho=0.1$ fm$^{-3}$ varies in relation to the neutron skin (third column).

The observations collected above can be summarized as

$$S_{\text{empirical}} \approx (0.18^{+0.03}_{-0.04}) \text{ fm}; \quad \left(\frac{\partial e_{\text{sym}}}{\partial \rho}\right)_{\frac{2}{3}\rho_0} \approx (177^{+18}_{-23}) \text{ MeV fm}^3; \quad L \approx (72.6 \pm 13) \text{ MeV}. \quad (4)$$

Proceeding in a similar way, the spreading of the predictions from the “family” of models in Table 2 can be summarized as

$$S_{\text{Theories}} \approx (0.18 \pm 0.02) \text{ fm}; \quad \left(\frac{\partial e_{\text{sym}}}{\partial \rho}\right)_{\frac{2}{3}\rho_0} \approx (154^{+22}_{-19}) \text{ MeV fm}^3; \quad L \approx (58^{+14}_{-11}) \text{ MeV}. \quad (5)$$

Concentrating on the chiral predictions, the difference between the predictions at N$^2$LO and N$^3$LO is a (pessimistic) estimate of the truncation error at N$^3$LO. Proceeding as detailed in Ref. [6], the chiral predictions at N$^3$LO is estimated to be

$$S_{N^3LO} \approx (0.17 \pm 0.02) \text{ fm}; \quad \left(\frac{\partial e_{\text{sym}}}{\partial \rho}\right)_{\frac{2}{3}\rho_0} \approx (148 \pm 14) \text{ MeV fm}^3; \quad L \approx (48 \pm 15) \text{ MeV}. \quad (6)$$

These observations may be more easily captured in a visual way, as presented in Figure 3. The area shaded in blue is obtained from the empirical constraint for $L$ and the corresponding constraint for the neutron skin. Including the predictions from Table 2 generate the green area. Finally, Eq. (6) produces the pink region. Clearly, the uncertainty in the density derivative is much larger than could be inferred from the blue area, that is, from the correlation based on the empirical information.

In Ref. [4], the authors do mention that the sharp value of $e_{\text{sym}}^{\text{pot}}(\rho_0)$ is the result of choosing a power law as in Eq. (1) and that using lower values of $e_{\text{sym}}^{\text{pot}}(\rho_0)$ leads to lower values of $L$. 


Figure 3. Relation between the $L$ parameter and the neutron skin in $^{208}$Pb. Blue shaded area: empirical constraint for $L$ and corresponding constraint for the neutron skin. Green area: Predictions from Table 2. Equation (6) produces the pink region.

Table 2. As in Table 1, but for each of the theoretical approaches under consideration.

| Theor. Approach | $S$ (fm) | $\frac{\partial e_{sym}}{\partial \rho}$ (MeV fm$^3$) | $L$ (MeV) | $P_0$ (MeV/fm$^3$) |
|-----------------|----------|---------------------------------|------------|-----------------|
| DBHF            | 0.16     | 135                             | 46.8       | 2.45            |
| $N^3$LO         | 0.17     | 148                             | 47.5       | 2.50            |
| APR             | 0.18     | 158                             | 65.0       | 3.43            |
| $N^2$LO         | 0.20     | 176                             | 72.0       | 3.79            |

still within acceptable error margins. However, the results from Ref. [16] (which are based on Skyrme phenomenology), are no longer met with the alternative parametrization. The results which have been reviewed here show that adhering to Eq. (1) is not recommendable from the theoretical standpoint.

4. Conclusions
Theoretical calculations of the symmetry energy, in particular from modern ab initio predictions, are timely and important as they complement on-going and future experimental efforts. In this paper, existing predictions for the interaction part of the symmetry energy have been explored in the light of new and more stringent constraints recently obtained for this quantity. We considered a few but fundamentally different approaches to obtain a realistic idea of the spreading in microscopic predictions.

Within the simplest assumption of a single-term power law, it is found that various predictions based on chiral EFT, relativistic meson-theory, or phenomenological forces and the variational method, can only approximately be described by a power law consistent with the constraint. The $N^3$LO prediction is generally softer than the constraint.

The focus then moves to the neutron skin thickness of $^{208}$Pb. We stress the importance of
experiments keeping in close touch with ab initio predictions. Caution must be exercised with regard to the possibility of constraining the density slope from the knowledge of the skin based only on correlations obtained with families of simple phenomenological interactions, as the latter procedure may underestimate the uncertainties.

Work in progress includes generating systematic order-by-order predictions of neutron star masses and radii. Our emphasis is on the role of the pressure around normal density in the formation of both the neutron skin and the radius of a typical neutron star.

Acknowledgments
This work was supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, under Award Number DE-FG02-03ER41270.

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