Lattice supersymmetric Ward identities

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SUSY Ward identities for the N=1 SU(2) SUSY Yang-Mills theory are studied on the lattice in a non-perturbative numerical approach. As a result a determination of the subtracted gluino mass is obtained.

1. Introduction

The formulation of SUSY gauge theories on the lattice is problematic since the discretization breaks the Poincaré invariance, a sector of the superalgebra. In the Wilson approach the suppression of unphysical states in the fermionic sector is obtained by the introduction of an extra-term (Wilson term) which explicitly breaks SUSY. The restoration of SUSY in the continuum limit can be verified by considering the related lattice Ward identities (SUSY WIs) \cite{1}.

We focus on the N=1 SU(2) SUSY Yang-Mills theory (SYM) (see also \cite{2} and references therein). This is the SUSY version of quantum gluodynamics where gluons are accompanied by fermionic partners (gluinos) in the same (adjoint) representation of the color group. As a consequence of the explicit breaking of the symmetry, the SUSY WIs assume in the lattice theory a peculiar form. We restrict the analysis to the on-shell regime \cite{3}. A subtracted gluino mass $m_S$ appears; in addition, the SUSY current $S_\mu (x)$ gets a multiplicative factor $Z_S$ and a new mixing term $Z_T \partial_\mu T_\mu (x)$ is added to the nominal WIs of the continuum.

In this contribution we present the non-perturbative determination of the quantities $m_S Z_S^{-1}$ and $Z_T Z_S^{-1}$ from the numerical analysis of the SUSY WIs. Preliminary results were presented in \cite{4}. More details, including related theoretical issues, will be presented in a forthcoming publication. This study is also complemented by a perturbative computation \cite{5}.

The numerical computations were performed on the CRAY-T3E computers at John von Neumann Institute for Computing (NIC), Jülich. We thank NIC and the staff at ZAM for their kind support.

2. Method

We consider the zero momentum lattice SUSY WI with insertion $\mathcal{O}(y)$

\begin{equation}
Z_S \sum_x \langle (\nabla_0 S^l_0(x)) \mathcal{O}(y) \rangle + Z_T \sum_x \langle (\nabla_0 T^l_0(x)) \mathcal{O}(y) \rangle = m_S \sum_x \langle \chi^l(x) \mathcal{O}(y) \rangle + O(a) .
\end{equation}

This WI is valid in the on-shell regime where $x \neq y$ and for gauge-invariant operators $\mathcal{O}(x)$.
latter contains temporal links, for which a multi-hit procedure is more appropriate than smearing. Such a procedure is however computationally too expensive in our setup with dynamical fermions.

For a given insertion $O(x)$ the WI results in two independent equations when composing the spins of sink and insertion operators. The solution of the $2 \times 2$ linear system allows the non-perturbative determination of $m_S Z_{S}^{-1}$ and $Z_{T} Z_{S}^{-1}$ for each time-separation $t = x_0 - y_0$. See Fig. 3 for an example. Alternatively we solve the overdetermined linear system for several time-separations ($t_{min}, \cdots, L_t/2$); the values of $m_S Z_{S}^{-1}$ and $Z_{T} Z_{S}^{-1}$ are obtained in this way through a least-square fit.

3. Results

Configurations were generated on a $12^3 \times 24$ lattice at $\beta = 2.3$ by means of the two-step multibosonic algorithm (TSMB). See [2] and references therein for more details on the algorithm. See also [6] for an application to QCD with three dynamical quark flavors. The configurations at $\kappa = 0.1925$ were produced in [2]. Results concerning $\kappa = 0.1925$ and 0.194 were already presented in [4]. We add here more statistics at $\kappa = 0.194$ and a new simulation point, $\kappa = 0.1955$. The algorithmic setup was optimized in order to reduce autocorrelations for light gluinos.

In Table 4 we report the complete results for the global fit over a range of time-separations. The smallest time-separation included in the fit $t_{min}$ was chosen such that contact terms in the correlations are absent; this means $t_{min} = 3$ for insertion $\chi^{(sp)}(x)$ and $t_{min} = 4$ for $T_0^{(loc)}(x)$. Discretization effects can be evaluated by comparing determinations from different insertions, see data for $\kappa = 0.1925$ and $\kappa = 0.194$. For $\kappa = 0.1925$ we also report results for different definitions of $\chi^{(sp)}(x)$, namely for the simple-plaquette definition of the field tensor and for different smearing parameters. The deviation ranges between 20% and 40% for $m_S Z_{S}^{-1}$. Data from $T_0^{(loc)}(x)$ are however subject to large statistical fluctuations and thus $O(a)$ effects cannot be reliably estimated.

In Fig. 3 we report the determination of $m_S Z_{S}^{-1}$
Table 1
Summary of results.

| $\kappa$ | operator | $mSZ^{-1}_S$ | $ZTZ^{-1}_S$ | $mSZ^{-1}_T$ | $ZTZ^{-1}_S$ |
|---------|----------|-------------|-------------|-------------|-------------|
| 0.1925  | $\chi^{(sp)}$ | 0.176(5)    | -0.015(19)  | 0.166(6)    | 0.183(14)   |
| 0.1925  | $\chi^{(sp)}$ (*) | 0.182(6)    | -0.044(16)  | 0.173(6)    | 0.176(14)   |
| 0.1925  | $\chi^{(sp)}$ (**) | 0.1969(47)  | -0.058(14)  | 0.1821(47)  | 0.146(11)   |
| 0.1925  | $T_0^{(loc)}$ | 0.132(16)   | 0.11(7)     | 0.144(18)   | 0.29(6)     |
| 0.194   | $\chi^{(sp)}$ | 0.148(6)    | -0.038(19)  | 0.124(6)    | 0.202(15)   |
| 0.194   | $T_0^{(loc)}$ | 0.095(27)   | 0.11(13)    | 0.076(30)   | 0.27(9)     |
| 0.1955  | $\chi^{(sp)}$ | 0.0839(4)   | -0.051(13)  | 0.0532(40)  | 0.179(10)   |

* With plaquette field tensor.
** With plaquette field tensor and different smearing.

Figure 2. $mSZ^{-1}_S$ as a function of $\kappa^{-1}$ with insertion $\chi^{(sp)}(x)$ (diamonds) and $T_0^{(loc)}(x)$ (triangles) and point-split currents. The filled circle is the result of the extrapolation, the filled triangle is the determination of $\kappa_c$ of [7].

as a function of the inverse hopping parameter. The expectation is that $mSZ^{-1}_S$ vanishes linearly when $\kappa \to \kappa_c$. We see a clear decrease when $\kappa$ is increased towards $\kappa_c$. An extrapolation using data from insertion $\chi^{(sp)}(x)$ gives as a result: $\kappa_c = 0.19750(38)$ for the point-split currents and $\kappa_c = 0.19647(27)$ for the local ones. The result can be compared with the estimate $\kappa_c = 0.1955(5)$ from the study of the first order phase transition [7]. An analogous analysis for the quantity $ZTZ^{-1}_S$ (fitting to a constant, in this case) gives $ZTZ^{-1}_S = -0.039(7)$ for the point-split currents and $ZTZ^{-1}_S = 0.185(7)$ for the local ones.

Our results demonstrate the feasibility of implementing lattice SUSY WIs in order to verify supersymmetry restoration in a non-perturbative framework.

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