Intrinsic Gap of the $\nu = 5/2$ Fractional Quantum Hall State

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The fractional quantum Hall effect is observed at low magnetic field, in a regime where the cyclotron energy is smaller than the Coulomb interaction energy. The $\nu = 5/2$ excitation gap is measured to be $262 \pm 15 \text{ mK}$ at $\sim 2.6 \text{ T}$, in good agreement with previous measurements performed on samples with similar mobility, but with electronic density larger by a factor of two. The role of disorder on the $\nu = 5/2$ gap is examined. Comparison between experiment and theory indicates that a large discrepancy remains for the intrinsic gap extrapolated from the infinite mobility (zero disorder) limit. In contrast, no such large discrepancy is found for the $\nu = 5/2$ Laughlin state. The observation of the $5/2$ state in the low-field regime implies that inclusion of non-perturbative Landau level mixing may be necessary to better understand the energetics of half-filled fractional quantum Hall liquids.

PACS numbers: 73.43.-f,73.63.Hs,03.67.-a

Since the discovery of the fractional quantum Hall effect (FQHE), understanding the role played by electron-electron interactions has been the source of major breakthroughs in our understanding of strongly interacting two-dimensional electron gases (2DEGs). Chief among these is the composite fermion picture of the incompressible FQH liquid\textsuperscript{[1, 2]}, extremely successful at explaining both the complete series of observed FQH states in the first Landau level (FLL), and the absence of such a liquid at precisely half-filling. In the second Landau level (SLL), however, the situation is more complex where experiments have shown, unambiguously, exact quantization of the Hall resistance at filling factor $\nu = 5/2$, $3/2$, $1/2$ and $\nu = 7/2$. In 1991, Moore and Read\textsuperscript{[3]} proposed an elegant many-body wave function to explain this phenomenon that described the $5/2$ FQH state as a ‘condensation process’ of composite fermions. In recent years, this Moore-Read ‘Pfaffian’ state has received considerable interest owing to built-in quantum statistics that are now predicted to be non-abelian. The non-abelian composite particles that comprise the $\nu = 5/2$ FQH state underlie a paradigm for fault-tolerant topological quantum computation first proposed by Kitaev\textsuperscript{[4]} and recently exploited by Das Sarma, Freedman and Nayak\textsuperscript{[5]}. Yet, in spite of these many recent theoretical advances, an unequivocal experimental verification of the Moore-Read description is still missing. Furthermore, continued discrepancies between experiment and theory, such as the large difference between the measured and calculated activation energy gap, remain problematic.

In an effort to better understand electron-electron interaction at half filling, we present in this work a detailed analysis of the $\nu = 5/2$ state for a sample with, to our knowledge, the lowest electron density reported to date (by nearly a factor of two). This allows the study of the FQHE in a regime where the cyclotron energy is smaller than the Coulomb interaction energy. We compare the measured energy gap with neighbouring FQH states in the SLL, and discuss these results in the context of previous studies allowing us to deduce the intrinsic gap in the zero-disorder limit. Our analysis shows that large discrepancies remain between theory based on a Moore-Read Pfaffian state and experiment at $\nu = 5/2$ that cannot be attributed to disorder alone. In contrast, a similar analysis for the $\nu = 5/2$ Laughlin state shows much better agreement with current models.

The sample used in this study was a 40 nm wide, modulation-doped, GaAs/AlGaAs quantum well, with a measured density of $1.6(1) \times 10^{11} \text{ cm}^{-2}$ and mobility of $14(2) \times 10^{6} \text{ cm}^{-2}/\text{V.s}$. The sample was cooled in a dilution fridge enclosed inside a shielded room, with a base temperature of $\sim 16 \text{ mK}$ in continuous mode, and equipped with a 9 Tesla magnet. Treatment with a red LED was used during the cooldown.

In situ powder filters and RC filters were used on the sample leads to ensure efficient cooling of the 2DEG. Temperatures were monitored with a RuO resistive thermometer, and a CMN magnetization thermometer, both calibrated with superconducting fixed point thermometers in the 2DEG. Transport measurements were performed using a standard lock-in technique at $\sim 6.5 \text{ Hz}$.

Fig. 1 shows the magnetoresistance ($R_{xx}$) and corresponding Hall resistance ($R_{xy}$), taken around $\nu = 5/2$ in the SLL at $\sim 20 \text{ mK}$. A vanishingly small magnetoresistance is observed at $\nu = 5/2$, which, together with a wide plateau in the corresponding Hall trace, indicates the $5/2$ state is exceptionally well formed. The unambiguous $5/2$ state observed here, occurring at $\sim 2.63 \text{ T}$, represents to our knowledge the lowest magnetic field observation of the $5/2$ to date\textsuperscript{[3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]}. Strong FQHE minima are also observed at...
ν = 14/5, 8/3, 7/3, and 11/5, each of which exhibit plateaus in \( R_{xy} \). A hint of an emerging \( R_{xx} \) minimum can also be seen at \( \nu = 12/5 \). The four reentrant phases observed in the Hall trace on either side of the \( 5/2 \) plateau (two peaks tending towards \( R_{xy} = h/3e^2 \) and two tending towards \( R_{xy} = h/2e^2 \)) together with the observation of the \( \nu = 14/5 \) state, and the emerging minimum at \( \nu = 16/7 \), are all signatures of an extremely high quality sample [15, 19, 20].

The deep \( R_{xx} \) minima appearing in the reentrant insulating phase at \( \sim 2.55 \) T (Fig. 1b) is similar to that observed elsewhere upon lowering the electronic temperature to a regime where the reentrant state is fully formed [15, 19, 20].

FIG. 1: (a) Hall resistance and (b) corresponding magnetoresistance in the second Landau level of our low density, high mobility 2DEG (T \( \sim 20 \) mK).

The temperature dependence of the FQHE minima are shown in Fig. 2b-e, with all data acquired at fixed magnetic field in order to avoid heating effects caused by varying fields. The corresponding energy gaps were determined by linear fits to the thermally activated transport region, where the resistance is given by the equation \( R_{xx} \propto e^{-\Delta/k_BT} \). The gap error quoted on each plot was estimated from the goodness of the linear fit. Examination of weakly formed FQHE states under single shot of the dilution fridge down to \( \sim 9 \) mK indicated the electrons continued to cool, suggesting that the low temperature tail-off observed in the data of Fig. 2 does not reflect a saturation in the electronic temperature. Instead, it may indicate a transition from activated conduction to hopping conduction [21], and/or could result from the energy dependent Landau level broadening due to disorder [22]. In the \( 7/5, 2/3 \), and \( 8/3 \) states (Figs. 2c-d), there is a deviation from activated behaviour at high temperature whose onset temperature scales with the corresponding gap value. Likely, this results from \( k_BT \) approaching the gap value. However, the same deviation is not observed in the \( 11/5 \) and \( 16/7 \) FQH states, which have lower measured gaps than the \( 7/3 \) and \( 12/5 \) states. Interestingly, recent work has suggested the \( 11/5 \) and \( 16/7 \) to be Laughlin states, while the \( 7/3 \) and \( 12/5 \) are proposed to be non-Laughlin [17, 24, 25].

We also observed a FQH state at \( \nu = 5/2 \) the electron-hole conjugate of \( \nu = 5/2 \), appearing at a magnetic field less than \( 2 \) T. However, due to a competition between the weakly formed \( 5/2 \) and rapidly emergent neighbouring FQH states, plotted in Coulomb energy units. Open circles are our data. Solid squares and circles, respectively, refer to the “high mobility” and “low mobility” samples in reference [4, 14, 15, 17]. The four reentrant phases observed in the Hall trace on either side of the \( 5/2 \) plateau (two peaks tending towards \( R_{xy} = h/3e^2 \) and two tending towards \( R_{xy} = h/2e^2 \)) together with the observation of the \( \nu = 14/5 \) state, and the emerging minimum at \( \nu = 16/7 \), are all signatures of an extremely high quality sample [15, 19, 20].

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reentrant states, the $R_{xx}$ minimum did not fall significantly with temperature near base. The “quasi-gap” was therefore determined by measuring the depth of the $\frac{5}{2}$ minima with respect to the average resistance of the two neighbouring peaks ($R_{\text{peak}}$) $\frac{5}{2}$ $\pm$ 0.018. The resulting Arrhenius plot, which clearly shows activated behaviour (Fig. 2f) gives an estimate for the $\frac{5}{2}$ gap value of $\sim 25$ mK.

In Fig. 2g, the gap values are plotted in Coulomb energy units, $e^2/\epsilon l_B$, where $l_B = \sqrt{\hbar/eB}$ is the magnetic length, and $\epsilon = 12.9$ is the dielectric constant. Results from recent gap measurements at the SLL by Choi et al. are also shown for comparison. The excellent agreement between our data set and that of the Choi et al. ‘low mobility’ sample ($\mu_B = 10.5 \times 10^5 \text{cm}^2/\text{V} \cdot \text{s}$) is surprising given the factor of two difference in electron densities between our sample (1.6 $\times 10^{11} \text{cm}^{-2}$) and theirs (2.8 $\times 10^{11} \text{cm}^{-2}$ and 3.2 $\times 10^{11} \text{cm}^{-2}$ for the “low mobility” and “high mobility” respectively). Simple dimensional considerations imply that the interaction energy, and hence the FQH gap, should scale as $\sqrt{B}$, which would predict a $\sim 40\%$ enhancement in the gap between the low density (ours) and the high density (Choi et al.) samples. Our finding that the gap is almost the same for the two samples with similar mobility (independent of density), while significantly enhanced in samples with higher mobility (Choi et al. “high mobility”) indicates that disorder more strongly affects the gap than the applied magnetic field. Furthermore, the similar gap value measured in a low magnetic field where the cyclotron energy is reduced compared to the Coulomb interaction suggests non-perturbative Landau level coupling may affect the $\nu = \frac{5}{2}$ FQH gap in a way not yet understood theoretically.

In Fig. 3a, we show a plot of all the $\frac{5}{2}$ gap values found in the literature versus the inverse transport lifetime, $\tau_{\text{tr}}^{-1}$, deduced from the reported mobilities $\mu_B$ $\times$ $10^5$ cm$^2$/V s. In spite of the large spread in the $\frac{5}{2}$ data, owing to wide ranging differences in sample parameters, i.e. dopant, well width, etc., a clearly discernible trend (indicated by the solid curve as a guide-to-the-eye) is observed pointing towards a disorder-free intrinsic gap value in the range of $\Delta_{i/2} \sim 0.005-0.010$ $e^2/\epsilon l_B$. This estimate is in good agreement with a similar extrapolation reported very recently by Pan et al. [20, 24]. Moreover, examination of the low field Shubnikov de Haas oscillations gave the level broadening, $\Gamma$, in our sample to be $\Gamma = 0.168 \pm 0.040$ K. This gives a direct experimental estimate for the intrinsic gap, $\Delta_i = \Delta_{i}^{\text{exp}} + \Gamma$, of $\sim 0.005$ $e^2/\epsilon l_B$, also in good agreement with the extrapolated intrinsic gap value in Fig. 3a. Importantly, the experimentally measured gap inferred from this data remains well below (by a factor of three to five) the theoretically estimated intrinsic gap value for a Moore-Read type Pfaffian wave function, $(\sim 0.025$ $e^2/\epsilon l_B)$ [27, 28].

For comparison, a similar plot for the measured gap values of the $\frac{1}{3}$ Laughlin state is shown in Fig. 3b, [21, 23, 29, 30, 31, 32]. In contrast to the $\nu = \frac{5}{2}$ FQH state, the intrinsic gap determined at $\nu = \frac{1}{3}$ with our procedure ($\Delta_{1/3} \sim 0.045$ $e^2/\epsilon l_B$) is in good agreement with theory ($\sim 0.055$ $e^2/\epsilon l_B$) [25, 31]. Morf et al. proposed that since the disorder-induced Landau level broadening is expected to be roughly equal for FQH states corresponding to particle-hole conjugate pairs, then plotting the correspondind gap values as a function of Coulomb energy directly gives a measure for the intrinsic gap (slope of a fitted line to this data) $\Delta_{1/3}$. Fig. 3b shows the $\frac{1}{3}$ and $\frac{5}{2}$ gap values obtained in our low electron density sample (open squares) together with those from Ref. [14] (open triangles). The dashed line shows the predicted trend for a disorder free gap. The slope extracted from a linear fit gives the intrinsic gap for our sample to be $\sim 0.018$ $e^2/\epsilon l_B$, which is in remarkable agreement with the data from ref. [14] $(\sim 0.014$) and the theoretical value $(\sim 0.016$) corrected for the sample parameters specified in ref. [14]. This however disagrees with the intrinsic gap estimated both from our sample, and from the extrapolation towards the infinite mobility limit. Furthermore, the Landau Level broadening deduced from Fig. [14]...
implies a rather large value (∼1.25 K) that is an order of magnitude larger than the value determined experimentally from the SDH oscillations (∼0.17 K).

It is instructive to consider the energetics of our low-density \( \frac{5}{2} \) FQH state. At the observed field of 2.62 T the cyclotron energy is 52 K, the interaction energy is 81 K, and the Zeeman energy (assuming the GaAs band g-factor) is 0.75 K. The level broadening in our sample was measured to be ∼0.17 K and the mobility broadening ∼0.006 K [31]. Also important is the suppression of the ideal two-dimensional FQH excitation gap due to the finite width \( d = 40 \text{ nm} \) of our quasi-2D square well sample. For our \( \frac{5}{2} \) FQH gap, this is only about 15% (using \( d/l_B = 2.5 \) in [28], where \( l_B = 16 \text{ nm} \) at ∼2.6 T). Taking all of these energies into account we conclude: i) our measured gap value of 0.262 K is at least a factor of 5 lower than the ideal 2D theoretical \( \frac{5}{2} \) excitation gap (∼2 K at 2.6 T), even if the theoretical gap is corrected for finite width and level broadening suppression (∼1.5 K); ii) the cyclotron gap, i.e. the Landau level separation, is smaller than the interaction energy in our system, suggesting considerable non-perturbative inter-Landau level mixing (as well as disorder), not considered in the theoretical literature, may play an important role in the understanding of the enigmatic \( \frac{5}{2} \) FQH state.

In conclusion, the \( \frac{5}{2} \) energy gap was measured for a sample with an electron density nearly twice smaller than previously observed, and was found to be comparable to samples with higher densities, and similar mobilities. Extrapolating the experimentally measured energy gap values to zero disorder yields an estimate for the intrinsic gap which remains well below the theoretical value. By comparing our data for the \( \frac{5}{2} \) FQH state, our experiment rather points towards a spin-polarized state at \( \nu = \frac{5}{2} \) even in the zero-field limit, consistent with a Moore-Read Pfaffian wavefunction, the leading candidate for the \( \frac{5}{2} \) FQHE.

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\[
e^2/\epsilon_B (K) \quad \Delta (K)
\]

![Figure 4: Intrinsic gap energy at \( \nu = \frac{5}{2} \) from particle-hole paired states](image-url)

This work has been supported by the Natural Sciences and Engineering Research Council of Canada (NSERC), the Canada Fund for Innovation (CFI), the Canadian Institute for Advanced Research (CIFAR), the Canada Research Chairs program, MITACS, QuantumWorks and FQRNT (Québec). Two of the authors (G.G. and P.H.) acknowledge receipt of support from the Alfred P. Sloan Foundation through their fellowship program.

Special Note: Both authors marked with a star (*) contributed evenly to this work.

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