Accurate Control for Gimbal System of Single Gimbal Control Moment Gyro Based on Nonlinear Cascade Extended State Observer

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Abstract. The nonlinear friction, unmodeled disturbance and coupling torque caused by the movement of satellite are the main factors limiting the angular speed accuracy in gimbal system of the single gimbal control moment gyro (SGCMG). Consequently, a high-precision servo control system based on the nonlinear cascade extended state observer (NCESO) is established in this paper in order to reduce the influence of those disturbances and improve the output angular velocity accuracy of the gimbal system. The simulation results confirmed the feasibility and effectiveness of the proposed method.

1. Introduction
Control moment gyro (CMG) is a type of inertial actuator applied to a long-running spacecraft and is used widely because of the advantages of large output torque, continuous adjustable torque, high precision, and long service life [1]. According to the degrees-of-freedom of the gimbal system, CMG can be divided into single gimbal CMG (SGCMG) and double gimbal CMG (DGCMG) [2]. SGCMG is applied more in spacecrafts due to its simple structure and easy to control.

The SGCMG output torque \( T \) is the vector cross product of high-speed rotor moment of inertia \( H \) and angular velocity \( w \) provided by the gimbal system, therefore the accuracy of \( w \) is directly affects the torque performance of the entire gyro system in the case of \( H \) unchanged [3].

There are three major disturbances that affect the speed accuracy of the gimbal system. The movement of satellite will cause coupling torque to the gimbal system, the influence of the gyro effect leads to the nonlinear friction, and the unmodeled disturbance also should be considered. The existent interferences of the above-mentioned make the gimbal system unable to output high-precision angular velocity.

In recent years, many scholars proposed various methods handling the problems in the gimbal system, In [4], a feedforward-feedback compound controller is proposed to weaken the influence of coupling torque and moving gimbal effect. While the effect of the coupling torque is attenuated to some degree, the complete elimination of coupling torque is not achieved, and the nonlinear friction and unmodeled dynamics have not been considered. To restrain the influence of nonlinear friction, some research efforts have been focused on the nonlinear friction model of gimbal system in CMG [5].
However, it is very difficult to calculate the parameters in the nonlinear friction model accurately because of the gyro effect and that is the reason why it’s less employed [6].

The extended state observer (ESO), which was proposed by Han in his works, can estimate the uncertainties along with the states of system without requiring much model information [7]. With this method, the effects of uncertainties which are not acquired precious are regarded as lumped disturbance and are extended as an incremental state of the original system [8]. Although ESO can estimate various uncertainties with little model information and has been widely used in many fields, if the order of the system is greater than 2, it’s hard to obtain the gains. Thus, a concept of cascade ESO (CESO) was proposed in [9], and it’s easier to configure the only two parameters. Moreover, the stability of CESO was verified in [10], and linear CESO (LCESO) presented in it aims to inhibit the disturbance. Although a certain effect is achieved, the LCESO has some deviations from the actual disturbance observations, which limits the disturbance suppression effect of the system. And on basis of [10], a nonlinear CESO (NCESO) is put forward in this paper to observe the interference more exactly to improve the angular speed accuracy of the gimbal system.

In this paper, the mathematical model of SGCMG is shown in Section II. The design of NCESO is presented in Section III. And the results of simulation about the method are discussed in Section IV. The conclusions are summarized in Section V.

2. Gimbal System Model of SGCMG

As can be seen from the system structure of SGCMG shown in Fig.1, the SGCMG consists of the gimbal system, high speed rotor system and the base. Besides, the resolver is used to measure angular position of the gimbal rotor and the current of the gimbal motor is obtained by the Hall current sensor.

![Fig.1 System structure of SGCMG](image)

The coordinates of SGCMG are defined in Fig. 2, where \( o \) is the geometry center of the stator in the gyro room, \( oxyz \) and \( oxyz_i \) are the coordinates of base and gimbal system respectively. \( oxyz_i \) rotate around axis \( oy \) relative to \( oxyz \), and the spinning angular of the gimbal is defined as \( \theta \), spinning angular speed is defined as \( \dot{\theta} \).
The mathematical model of the output torque of the gyro gimbal servo system can be expressed as
\[ T = k \cdot i = J \cdot \dot{\theta} + F_{ud} + F_{nf} + F_{f} \]  
(1)

where \( T \) is the output torque, \( k \) is the torque coefficient, \( s \) represents three-phase (u, v and w), \( i \) represents the current, \( J \) is the equivalent rotational inertia of gimbal system, \( F_{ud} \), \( F_{nf} \) and \( F_{f} \) represent the unmodeled disturbance, nonlinear friction and pedestal coupling disturbance, respectively.

The mathematical model of the gimbal motor can be expressed as
\[ u_s = R_s \cdot i + L_s \cdot \frac{di}{dt} + k_e \cdot \dot{\theta} \]  
(2)

where \( u_s \) is control voltage of the gimbal motor, \( R_s \) represents the stator phase resistance, \( R_s = R_v = R_w = R \) and \( k_e \) is the back electromotive force coefficient, \( L_s \) is the phase inductance, \( L_u = L_v = L_w = L \).

Define state variables as \( x = [\theta, \dot{\theta}, i]^T \), and the unmodeled disturbances, nonlinear friction, and other disturbances are treated as lumped disturbances, so the state-space equation of gimbal servo system can be expressed as
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= k / J \cdot x_3 + d \\
\dot{x}_3 &= -k_e / L \cdot x_2 - R / L \cdot x_1 + 1 / L \cdot u
\end{align*}
\]  
(3)

where \( d = -1 / J \cdot (F_{ud} + F_{nf} + F_{f}) \) is the lumped disturbance.

It can be seen that the lumped disturbance and the control inputs are not in the same channels, resulting in the so-called mismatched condition.
3. Cascade Extended State Observer

3.1. Design of NCESO

According to (3), it can be seen that it does not meet the structural requirements of the traditional CESO's integral series, thus (3) is performed coordinate transformation to get a new spatial state variable as $\bar{x} = [\bar{x}_1, \bar{x}_2, \bar{x}_3]^T$, and the relationship between the old and new state variables is shown as (4)

$$\begin{align*}
\bar{x}_1 &= x_1 \\
\bar{x}_2 &= x_2 \\
\bar{x}_3 &= k/J \cdot x_3 + d
\end{align*}$$

The new state space equation of the gimbal servo system can be expressed as (5)

$$\begin{align*}
\dot{\bar{x}}_1 &= \bar{x}_2 \\
\dot{\bar{x}}_2 &= \bar{x}_3 \\
\dot{\bar{x}}_3 &= f + b \cdot u
\end{align*}$$

where $f = -k \cdot k_c / (J \cdot L) \cdot \bar{x}_2 - R / L \cdot \bar{x}_3 + R / L \cdot d + \bar{d}$ is the lumped disturbance that enters the system through $\bar{x}_3$ channel, and $b = k / (J \cdot L)$.

In order to make full use of the mature second-order ESO and simplify the parameter configuration process, the CESO proposed in [9] is applied to the disturbance rejection control of the SGCMG. NCESO estimated variables are defined as $\bar{z} = [z_1, z_2, z_3, z_4, z_5, z_6]^T$ where $z_1$ estimates $\bar{x}_1$, $z_2$ estimates $\bar{x}_2$, $z_3$ estimates $\bar{x}_3$, $z_4$ estimates $\bar{x}_4$, $z_5$ estimates $f$, $z_6$ and $z_7$ are NCESO intermediate variables. NCESO estimated errors are defined as $\bar{e}_i (i=1,\ldots,5)$ expressed as (6)

$$\begin{align*}
\bar{e}_1 &= z_1 - \bar{x}_1 \\
\bar{e}_2 &= z_2 - \bar{x}_2 \\
\bar{e}_3 &= z_3 - \bar{x}_3 \\
\bar{e}_4 &= z_4 - \bar{x}_4 \\
\bar{e}_5 &= \bar{e}_6 - f
\end{align*}$$

And the state equation of NCESO is defined as (7).

$$\begin{align*}
\frac{\dot{z}_1}{\dot{z}_2} \frac{\dot{z}_3}{\dot{z}_4} \frac{\dot{z}_5}{\dot{z}_6} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix} - \begin{bmatrix} \bar{\beta}_1 \cdot E_1(\bar{e}) \\ \bar{\beta}_2 \cdot E_2(\bar{e}) \\ \bar{\beta}_3 \cdot E_3(\bar{e}) \\ \bar{\beta}_4 \cdot E_4(\bar{e}) \\ \bar{\beta}_5 \cdot E_5(\bar{e}) \\ \bar{\beta}_6 \cdot E_6(\bar{e}) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -b \end{bmatrix} \cdot u
\end{align*}$$

where $\bar{\beta}_i$ and $\bar{\beta}_j$ are observer parameters of NCESO, and $E_i(*)$ and $E_2(*)$ are error functions defined in (8)

$$\begin{align*}
E_1(\bar{e}) &= \bar{e}_1 \\
E_2(\bar{e}) &= \begin{cases} \frac{|\bar{e}| \cdot \text{sign}(\bar{e})}{|\bar{e}| > b} & i = 1, 3, 5 \\
\frac{\bar{e}}{b^*} & |\bar{e}| \leq b \end{cases}
\end{align*}$$

NCESO errors equation satisfy the following function
Besides, observer parameters are configured as [10], where $\delta = 2\omega_o$, $\bar{\delta} = \omega_o^2$, and $\omega_o$ is the observer bandwidth.

The structure of NCESO is shown as Fig. 3.

3.2. State Feedback Controller

According to the state feedback control theory and the Gyro gimbal system, a composite controller combing NCESO and state feedback control can be designed as (10)

$$u = \frac{[k_1 \cdot (r_p - \bar{x}_1) + k_2 \cdot (r_e - \bar{x}_1) - k_3 \cdot \bar{x}_1 - \bar{x}_6]}{b}$$

where $r_p$ and $r_e$ are the referenced angular position and speed of the gimbal system, respectively, $k_1$, $k_2$, and $k_3$ are the controller parameters. Besides, $\bar{x}_1$ is added to the controller to compensate the influence of the lumped disturbance.

4. Simulation analysis

In order to verify the angular speed tracking performance of the nonlinear CESO, the MATLAB/simulink software is used to simulate the gyro gimbal servo system.

As can be seen from (6) and (10), there are seven parameters to be designed in the system, which are $\bar{\delta}$, $\bar{\delta}$, $a$, $b$ of NCESO, and $k_1$, $k_2$, $k_3$ of the controller. The controller parameters $k_1$, $k_2$, $k_3$ are related to the controller bandwidth $\omega_o$, and configured as pole assignment method. And the controller bandwidth $\omega_o$ is related to the bandwidth of gimbal angular speed. The main simulation parameters are shown in Table 1.
Table 1. Simulation main parameters

| Num. | Parameter | Value | Parameter | Value |
|------|-----------|-------|-----------|-------|
| 1    | J(kg\cdot m^2) | 0.141 | k (Nm/A) | 1.73 |
| 2    | L(mH) | 7.5 | k_e(Nm/A) | 1.2 |
| 3    | R(Ω) | 5 | k_1 | 31000 |
| 4    | k_2 | 2960 | k_3 | 94 |
| 5    | a | 0.5 | b | 0.01 |
| 6    | $\alpha$ | 350 | $\beta$ | 2300 |

In the simulation, the step angular velocity reference signal of the gimbal is set to 5°/s at $t=1$s. And the disturbance torque caused by the movement of satellite is added at $t=2$s. Comparative simulation effects of step angular speed tracking and disturbance torque suppression of LCESO and NCESO are shown in Fig.4.

![Fig.4 Effect of step speed tracking and disturbance rejection](image-url)
It can be seen from Fig. 4 that when the disturbance torque is added at $t=2s$, the speed fluctuation of the gimbal used LCESO is $0.4^\circ/s$, while the speed fluctuation used NCESO is $0.07^\circ/s$. The effect of suppressing the disturbance torque is better for NCESO, and its steady-state error is $0.002^\circ/s$, which is also superior than the LCESO that is $0.004^\circ/s$. The gimbal speed stable accuracy of NCESO can reach 0.4‰.

In the design of the controller (10), the estimation of the lumped disturbance is added to compensate the effect of the disturbance. To further study the effect of residual disturbance to the gimbal system speed after disturbance compensation, a sinusoidal speed signal with the amplitude of $5^\circ/s$ and the frequency of 3Hz is given to the gimbal. The comparative disturbance estimation simulation results with LCESO and NCESO are shown in Fig. 5.

![Fig. 5 Comparative simulations of lumped disturbance $f$ and its estimation under (a) LCESO (b) NCESO](image)

It can be observed from Fig. 5 that when the gimbal system is given a sinusoidal reference speed, the gimbal is subjected to the influence of sinusoidal periodic disturbances. In spite of this, $z_6$ can perfectly estimate the lumped disturbance $f$ with the steady amplitude error of no more than 2% with both method. And the estimation error of the NCESO is obviously smaller than LCESO. The above analysis prove that the NCESO has better disturbance estimation performance than LCESO.

Comparative sinusoidal angular speed tracking simulation results with LCESO and NCESO are shown in Fig. 6.
Fig. 6 Effect of sinusoidal speed tracking and disturbance rejection

From Fig. 6, we can see that after compensating the influence of disturbance in the controller, the residual disturbance acted on the gimbal output is rather small. Compared with the angular speed tracking error of 1.05°/s and the lagged phase angular of 6° with the method of LCESO, the angular speed tracking error of the gimbal system is reduced to less than 0.16°/s, and the lagged phase angular is reduced to 1.1° with proposed NCESO method. The influence of disturbance on gimbal system output is significantly restrained. The above analysis verified the disturbance observation and disturbance rejection performance of the gimbal system.

5. Conclusion
In order to depress the unmodeled disturbance, nonlinear friction and unmodeled disturbances in the gyro gimbal system, and to achieve a high-precision angular speed tracking effect of the gimbal, a NCESO method is raised combined with the existing research results. Through simulation analysis, it is clear that the proposed method can effectively suppress various interferences and track the speed of the gimbal, so that the gimbal system can output high-precision angular speed. And it sufficiently guarantee that the entire gyro system can stably output the gyro moment.

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