Imprints of ultralight axions on the gravitational wave signals of neutron star-black hole binary

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The axion or axion-like particle motivated from a natural solution of strong CP problem or string theory is a promising dark matter candidate. We study the observational effects of ultralight axion or axion-like particles in our Universe by the space-borne gravitational wave detector and the radio telescope. Taking the neutron star-black hole binary as an example, we demonstrate that the phase of gravitational waveform could be obviously modified by the slow depletion of the axion cloud around the black hole formed through the superradiance process. Other effects from dynamical friction with axion dark matter or dipole radiation are also discussed. Finally, we study the detectability of the ultralight axion particles at TianQin and LISA.

Introduction.— The discovery of gravitational waves (GWs) by LIGO and Virgo [1] has initiated new perspectives to explore the fundamental problem in particle physics and cosmology. The ultralight scalar boson, such as the axion [2] or axion-like particle [3] (In this letter, we use axion to represent axion and axion-like particle.), naturally correlates the particle physics, cosmology, and gravity in many important processes, like the ultralight dark matter (DM) [4, 5] formation and superradiance process [6–10] of Kerr black holes, which might be tested in various GW experiments including LIGO [11], TianQin [12, 13], LISA [14], and Taiji [15]. The well-known superradiance process can extract energy and angular momentum from fast-spinning black hole and amplify the amount of the ultralight bosons [6–10]. When the Compton wavelength of the axion particle is comparable to the size of the Kerr black hole, the number of bound-state axions could grow exponentially, until the superradiance condition is no longer satisfied as the angular momentum of the Kerr black hole decreases [16, 17]. The rotating black hole and its macroscopic axion cloud form a “gravitational atom”. The axion cloud grows at the expense of the black hole spin and finally could contain up to ∼10% of the black hole mass. The axion cloud could lead to various observable effects. One significant effect is the monochromatic GW signal from the annihilation process of axion particles [10, 16] in the axion cloud. The implications and constraints from this annihilation process have been well studied [17–20]. LIGO has put constraints on some mass range of axion particle [21–25]. However, these constraints might be relaxed as discussed in [26].

In this letter, we study how the existence of ultralight axion particle is imprinted in the GW signals emitted by the neutron star-black hole (NS-BH) binary. The ultralight axion could affect the binary system through three different mechanisms. One important mechanism is the mass depletion of axion cloud (DC). The axions in the cloud induced by the black hole superradiant instability could continuously annihilate into gravitons, gradually bringing down the mass of gravitational atom and changing the gravitational field between the binary. Another mechanism is that the surrounding axion DM could exert a dynamical friction (DF) force on the neutron star and drag it while it is orbiting the black hole. The third one is that when the neutron star is endowed with a scalar charge by axion particles, the NS-BH binary could emit scalar bosons through the dipole radiation (DR) during the inspiral. These three effects could take more orbital energy away and hence modify the binary evolution in the inspiral stage, which might be observable via GW detection and pulsar timing measurement. For GW detection, we also study the distinguishability of the GW waveforms with and without taking into account the effects from ultralight axion particles, mostly based on the phase shift of the GW signal at TianQin and LISA. In our analysis, we use the units in which $\hbar = G = c = 1$.

Binary with ultralight axion.— We pay attention to the inspiral binary system consisting of a neutron star and an intermediate mass Kerr black hole, whose mass $M_{\text{BH}}$ can range from $10^2$ to $10^6 \, M_\odot$. The black hole is much heavier than its neutron star companion. LIGO/Virgo has discovered black hole with mass larger than $10^2 \, M_\odot$. Such a black hole along with a stellar mass compact object can also emit low frequency GW sensitive to space-borne GW detector several decades before they are detected by ground-based GW detector.

We start with an inspiral NS-BH binary without considering the effects from axions in the quasicircular approximation. Firstly, for NS-BH binary in vacuum, the energy loss of inspiral is equal to the power of GW radiation, i.e.,

$$- \frac{dE_0}{dt} = P_{\text{GW}},$$

where

$$P_{\text{GW}} = \frac{32}{5} \mu^2 r_4 \omega^6,$$
\[ \mu = \frac{M_{\text{BH}} m_{\text{NS}}}{M_{\text{BH}} + m_{\text{NS}}}, \]  

(3)

with \( P_{\text{GW}} \) being the GW emission power, \( \mu \) the reduced mass, \( \omega \) and \( r \) the angular velocity and separating distance of binary, and \( m_{\text{NS}} \) the neutron star mass. Note that

\[ \omega = \pi f = \sqrt{\frac{M_{\text{BH}} + m_{\text{NS}}}{r^3}} \]  

(4)

and \( f \) is GW frequency. Eq. (4) tells us the relation between orbiting radius and GW frequency of the NS-BH binary.

With the existence of ultralight axion DM, besides the energy radiated by GW, more energy could be radiated through several mechanisms. In the followings, we will discuss them one by one. The benchmark values for \( M_{\text{BH}}, m_{\text{NS}} \) and \( f \) are 100 M\( \odot \), 1.5 M\( \odot \) and 0.01 Hz.

**Depletion of Axion Cloud.**— The boundary condition at horizon of the Kerr black hole would lead to imaginary frequency of axion particles and then the exponential growth of the bound-state axions. In other words, the fast-spinning black hole can spontaneously transform energy and angular momentum to produce a macroscopic axion cloud and form a gravitational atom through superradiance process [6–10]. In the gravitational atom, the gravitational fine-structure constant is defined by \( \alpha = M_{\text{BH}} m_{a} \), where \( m_{a} \) is the axion mass. This parameter decides the efficiency of the superradiant instability and the time scale of axion cloud formation. Since a considerable cloud formation speed requires \( \alpha \sim \mathcal{O}(10^{-2} - 1) \), we are concerned with the axion mass ranging from \( 10^{-12} \) to \( 10^{-14} \) eV for \( M_{\text{BH}} \sim 100 \, M_{\odot} \). The superradiance condition [10] reads

\[ \alpha m < \frac{\chi}{2 \left( 1 + \sqrt{1 - \chi^2} \right)}, \]  

(5)

where \( \chi \) is the dimensionless spin of rotating black hole \((0 < \chi < 1)\) and \( m \) is the magnetic quantum number of axion bound-state level. When the exponential growth stops the axion cloud extracts part of angular momentum from black hole, and the number of axion can be evaluated by [17]

\[ N_{\text{axion}} \sim 10^{78} \left( \frac{\Delta \chi}{0.1} \right) \frac{1}{m} \left( \frac{M_{\text{BH}}}{100 \, M_{\odot}} \right)^2, \]  

(6)

where \( \Delta \chi \) is typically the difference between the initial and last black hole spin.

To clearly demonstrate the model-independent effects of the superradiance and the following DC process, we only consider the gravitational interaction of axion, which, unlike other couplings to the standard model, is universal. More specifically, the process of axions annihilation into gravitons is a more promising channel than level transitions within the gravitational atom [17], since the excited level needs much longer time to grow to a large occupation number in contrast with the first level.

Assuming \( \alpha = 0.2 \), for 2p level axion cloud, the superradiance rate depends on axion mass and black hole host spin [27, 28], i.e.,

\[ \Gamma_{\text{SR}} = \frac{1}{48} g_p \left( 1 - \frac{\omega_{211}}{\Omega_{H}} \right) \chi \alpha^8 m_a, \]  

(7)

where

\[ g_p = (1 - \chi^2) + \left( \frac{1 - \omega_{211}}{\Omega_{H}} \right)^2 \chi. \]  

(8)

Here, \( \omega_{211} \) is the frequency eigenvalues of the axion cloud and \( \Omega_{H} \) is the black hole angular velocity at the outer horizon, given by

\[ \Omega_{H} = \frac{\chi}{2 M_{\text{BH}} \left( 1 + \sqrt{1 - \chi^2} \right)}. \]  

(9)

Besides, the saturation time scale of axion cloud growth is \( \tau_{\text{SR}} = 1 \text{ day} \left( \frac{M_{\text{BH}}}{100 \, M_{\odot}} \right)^{9 \frac{0.2}{\alpha}} \left( \frac{0.9}{\chi} \right) \) [29]. Also the annihilation rate for one pair of axions is approximately

\[ \Gamma_a \approx \frac{1}{320} \alpha^{12} m_a^3, \]  

(10)

and this result has been improved through numerical calculations [30]. Note that in this letter, we use the numerical result [29, 31], which is smaller than the analytical result in Eq. (10). The timescale of axion cloud formation is about 6 orders of magnitude shorter than the annihilation timescale. Thus the depletion of the axion caused by the annihilation effect is considered under assumptions that the process of axion cloud growth had stopped and the number of axion has already reached its peak.

Despite of the orbital energy loss of NS-BH binary via the GW radiated by the inspiral binary, there is another part of the orbital energy taken away by the DC effect. The DC effect is produced mainly due to the axions annihilation in the cloud, which cause the mass depletion of the gravitational atom. The DC effect brings a phase shift to the inspiral GW emitted by the neutron star and gravitational atom binary. In this case, the mass depletion rate of gravitational atom’s mass \( \mathcal{P} \) depends on the annihilation rate and the number of axions occupying the level, and is up to

\[ \frac{\mathcal{P}}{M_{\text{BH}}} \approx 10^{-6} \, \text{yr}^{-1} \left( \frac{m_a}{10^{-12} \, \text{eV}} \right) \]  

(for \( \alpha = 0.2 \)).  

(11)

It is also necessary to discuss the DC effect duration. Generally, the timescale of this process can be evaluated by \( \tau_{\text{DC}} \approx (\Gamma_a N_{\text{axion}})^{-1} \sim 10^4 \text{ yrs} \) [17]. This timescale is much longer than the orbital period of the binary and
the observation time of space-borne GW detector. This helps to ensure a sufficient GW signal duration.

Dynamical Friction.— If ultralight axion DM exists, the binary system could be surrounded by diffused DM gas [32, 33]. The DF of such DM gas with density $\rho_{\text{DM}}$ modifies the GW waveform of the binary [34]. To clearly show the effects from DF, we parameterize the axion DM density around the neutron star as

$$\rho_{\text{DM}} = 10^n \rho_0 ,$$

where factor $n$ demonstrates how dense DM is and it could be as great as $\sim 14$ for extreme case [32, 33]. Here, $\rho_0 = 0.3 \text{ GeV/cm}^3$. The exact value of $n$ depends on the exact location of the neutron star and the DM distribution function. Following Refs. [34, 35], we assume the black hole is approximately static with respect to the DM, and the drag force exerted on the neutron star is opposite to its orbiting velocity $v$ and decelerates it. The friction is given by

$$F_{\text{DF}} = \frac{4\pi m_{\text{NS}}^2 I(r, v) \rho_{\text{DM}}}{v^2} ,$$

where $I(r, v) \sim \mathcal{O}(1)$ is the Coulomb logarithm [36]. We assume a constant value of $I(r, v)$ throughout the binary inspiral as in Refs. [35, 37]. Having the friction force, the energy loss of the binary and the phase shift of its GW induced by DF can be evaluated.

Dipole Radiation.— We also discuss the DR of the NS-BH binary. The binary could radiate axion particles under some conditions during the evolution at the cost of orbital energy. The interacting distance of axion mediated force depends on the angular frequency of axion $\omega_a$. In other words, the NS-BH binary radiates axion only when the orbital frequency $\omega$ becomes comparable to, or larger than the angular frequency of the axion we discussed. Following Refs. [38], the axion radiation power of the binary is given by

$$P_{\text{DR}} \sim \frac{Q^2 r^2 \omega^4}{24\pi} \left(1 - \frac{\omega_a^2}{\omega^2}\right)^{\frac{3}{2}} \Theta(\omega^2 - \omega_a^2) ,$$

where $Q$ is the neutron star’s scalar charge and the Heaviside function indicates that the axion radiation turns off when the orbital frequency is much less than the axion’s angular frequency. The power of the axion radiation, produced by a time-dependent dipole of scalar charge, has a slightly weaker frequency dependence ($\propto f^\frac{2}{3}$ when $f \gg \omega_a/\pi$) than the GW radiation power ($\propto f^{\frac{5}{3}}$). Moreover, using method in Refs. [38], the scalar charge of neutron star can be calculated by $Q = \pm 4\pi^2 f_a R_{\text{NS}}$, where $f_a$ is the axion decay constant and $R_{\text{NS}}$ is the neutron star radius. One can see that larger neutron star might contain a larger amount of scalar charge, which leads to more stronger DR power.

Comparison of three axion effects.— In order to extract the influence of axion existence and resulting effects (DC, DF and DR), we investigate and compare their contributions on binary orbit evolution. We denote $M$ as the total mass of black hole and axion cloud. Hereafter, we adopt the default values, $\alpha = 0.2, M = 100 M_\odot, m_{\text{NS}}$ as $1.5 M_\odot$, and $f$ as $0.01 \text{ Hz}$, in general discussion, and simultaneously these require $m_a$ to be $10^{-12.5} \text{ eV}$. We focus on the $2p$ level axion cloud ([211] state) and omit the influence of other levels. Therefore, to sum up what we discuss above, with the existence of ultralight axions, the DC, DF and DR effects increase the reduction rate of orbital radius. For the evolution of binary orbital radius with axions, we obtain

$$\frac{dr}{dt} = \left(-\frac{M m_{\text{NS}}}{2r^2}\right)^{-1} (P_{\text{GW}} + P_{\text{DC}} + P_{\text{DF}} + P_{\text{DR}})$$

with

$$P_{\text{DC}} = \frac{m_{\text{NS}}}{2r} \frac{dI}{dt} ,$$

$$P_{\text{DF}} = \frac{4\pi m_{\text{NS}}^2 I(r, v) \rho_{\text{DM}}(r)}{v r} .$$

Figure 1. The normalized power of radiated energy for different axion effects. The power for each effect is normalized to the power of GW radiation $P_{\text{GW}}$ in vacuum. The purple line represents the normalized power of radiated energy from the depletion of axion cloud (DC). The green, blue and orange lines depict the dynamical friction (DF) effect with different axion DM density. In this case, the power of dipole radiation (DR) vanishes. These results are shown with different GW frequency $f$ or orbiting radius $r$. In the light blue region, the merger time of NS-BH binary is less than 5 years. When GW frequency is low, the power of orbital energy loss induced by the DC effect is comparable to the power of GW radiation.
\( P_{\text{DR}} \) has been given in Eq. (14). Here \( \dot{\mathcal{P}} \) is the time derivative of gravitational atom mass as in Eq. (11). It should be noted that for the NS-BH binary we discussed, orbital frequency at which the binary begin to generate axion radiation is approximately \( O(10^2) \) rad/s, so DR effect would not change the GW waveform of the inspiral binary less than this frequency.

In Fig. 1, we show the normalized power of radiated energy for different axion effects. The power for each effect is normalized to the power of GW radiation \( P_{\text{GW}} \) in vacuum and we take binary GW frequency \( f \) from \( 10^{-4} \) to \( 10^{-1} \) Hz. The purple line represents the normalized power of radiated energy from the DC effect. The green, blue and orange lines mark the DF effect for different DM densities with \( n = 2, 5 \) and \( 10 \), in which we adopt \( I(r,v) \approx 3 \) [35]. Since the binary orbital frequency here is much smaller comparing to the angular frequency of axion, the DR effect is negligible. In the light blue region, the NS-BH binary merger time is less than 5 years, which is the typical operation time of space-borne GW detector. We find that the DC effect is at least several orders of magnitude larger than the DF effect numerically. Even if we assume the axion DM density with \( n = 10 \), the DF contribution is much less than the DC contribution throughout the whole frequency band we are interested in. Moreover, when the GW frequency is low (e.g., \( f = 10^{-3} \) Hz), the power of orbital energy loss induced by the DC effect is comparable with the GW power for the NS-BH binary in vacuum, and as the binary getting closer, the DC effect becomes relatively weaker. In other words, the DC effect could play an important role when binary GW frequency is in the low frequency band, while in the high frequency band, the influence of DC effect is small.

**Effects on gravitational wave detector and pulsar timing measurements.**—The existence of ultralight axion can change both the phase and the amplitude of GW emitted by the NS-BH binary. To determine the impact of DC on the GW signals in the space-borne and ground-based detector, here we calculate the GW phase modification induced by it. When the effects of DF and DR is relatively insignificant, only axion annihilation power \( \dot{\mathcal{P}} \) brings a shift into GW frequency evolution. From Eq. (15) as well as Eq. (4), we get

\[
\frac{df}{dt} = \frac{3}{2} \dot{\mathcal{P}} f + \frac{96}{5} \pi^{8/3} M_c^{5/3} f^{11/3} ,
\]

where \( M_c \) is chirp mass of this binary. As a comparison, for the binary in vacuum, Eq. (18) becomes

\[
\frac{df_{\text{vac}}}{dt} = \frac{96}{5} \pi^{8/3} M_c^{5/3} f_{\text{vac}}^{11/3} ,
\]

where we use \( f_{\text{vac}} \) to denote GW frequency without axion.

Thus, the time-domain GW phase can be evaluated by integrating \( f(t) \),

\[
\phi = \int_0^T 2\pi f(t) \, dr , \tag{20}
\]

where \( T \) is the integrating time. In practice, \( T \) can be considered as the observation time of GW detector. Equivalently, the frequency-domain phase is

\[
\phi(f) = \int \frac{2\pi}{r'(t)} \, dr . \tag{21}
\]

Here, \( r' \) denotes the time derivative of \( r \). The total phase shift is the phase difference between considering the binary with ultralight axion DM and in a vacuum.

As for GWs from NS-BH binary, we can calculate the phase shift triggered by DC. To begin with, space-borne observatory are expected to detect the binary GW when they are in the inspiral stage. The GW signal has a duration of several years and carries phase information about the binary system. In this way, it can provide us the information of axion DM. We consider the benchmark binary in low frequency window. The DC effect brings a phase shift to the inspiral waveform. Using the result of \( \dot{\mathcal{P}} \) and solving the equations, the expected phase shift is

\[
\Delta \phi \sim 15\pi \left( \frac{m_a}{10^{-12} \text{eV}} \right) \left( \frac{f_T}{10^{-2} \text{Hz}} \right) \left( \frac{T}{5 \text{ yrs}} \right)^2 . \tag{22}
\]

for space-borne detector, where \( f_T \) is GW frequency when it was detected. We adopt that the observation time of space-borne GW detector \( T \) is approximately 5 years in this estimation [12, 14]. We see that, when the mass of ultralight axion is \( 10^{-12} \) eV and detecting GW frequency is \( 10^{-2} \) Hz, the phase shift in 5-year observation of space-borne detector is larger than \( 2\pi \). That probably indicates it is necessary to consider the effect in waveform modeling. On the other hand, if the black hole is dressed by a newly formed axion cloud and has a neutron star companion, it can be interpreted in the binary GWs about the characteristic of the axion field.

Note that as the neutron star is orbiting the gravitational atom, the metric perturbation from this compact companion may trigger the reabsorption of the existed axion cloud [26]. For the cloud on the [211] state, the reabsorption effect happens after the binary leaves the low frequency window. In other words, for space-borne detector, it might still be reasonable to model the GW waveform without taking into consideration the termination of axion cloud induced by neutron star companion.

For ground-based GW observatory, we consider the phase shift in GW waveform. Generally, when the GWs of NS-BH binary are detected at ground-based detector, the binary are much closer to merger. In this case, the neutron star plays an important role on the evolution of axion cloud because it dives deeply into the cloud region. For the extreme case, if the neutron star com-
pletely destroys the axion bound state because of its tidal force, most of the axions might be absorbed by the black hole [26]. We return to the case in which only diffused DM gas exerting DF on the neutron star affects the binary inspiral. We had figured out in this circumstance the DF effect from diffused DM is negligible. In the opposite case, if the neutron star does not significantly change the stage of the bound-state axion, it enters the cloud and crosses over the cloud layer. In this case, the dominant axion effect for the considered NS-BH binary is stronger DF from the axion cloud [39]. However, we find that, when the GW frequency is sensitive to ground-based detector, the additional energy loss induced by this DF effect is at least 3 orders of magnitude smaller than the GW radiation. Given that the LIGO/Virgo GW event duration is a few seconds, the expected phase shift of this effect is less than the detecting precision of phase at current ground-based GW detector. Therefore, we could conclude that our analysis about ultralight axion effect is consistent with the existed GW observation.

The imprints of axions on the NS-BH binary period are also potentially observable through pulsar timing measurements, due to the fact that one of the inspiral objects is a rotating neutron star. Considering the DC effect, for the NS-BH binary we discuss above, at the moment when its orbiting period is $P = 100$ s, the rate of period change is deviated from the prediction without axion:

$$\Delta \dot{P} = \left| \dot{P} - \dot{P}_{\text{vac}} \right| \approx 10^{-12} \text{ s/s},$$

where $\dot{P}$ and $\dot{P}_{\text{vac}}$ represent period change rate with and without axions, respectively. The future measurement precision of $\dot{P}$ is approximately $\sim 10^{-15}$ s/s and therefore pulsar timing measurement could potentially detect the axion imprints [40].

**Detectability at TianQin and LISA.** We already point out the GW phase shift caused by the axion effect is too small to be detected by current ground-based detector, while the phase shift in low frequency GWs might be detectable by space-borne detector and comes up to $\sim \mathcal{O}(10)$. Furthermore, we quantitatively discuss the detectability and evaluate whether TianQin and LISA could unravel this axion effect from GW signals.

Firstly, in GW data analysis, one can define the noise-weighted inner product

$$\langle h_1 | h_2 \rangle = 2 \int_0^\infty \mathrm{d}f \tilde{h}_1(f) \tilde{h}_2^*(f) \frac{\tilde{h}_1^*(f) \tilde{h}_2(f)}{S_n(f)}, \tag{24}$$

where $h_i$ ($i = 1, 2$) represents time-domain signal, $\tilde{h}_i$ is the Fourier transformation of $h_i$, and $S_n(f)$ is the one-sided power spectral density for the instrument noise of the detector [41, 42]. The explicit formula of $S_n(f)$ for TianQin and LISA can be found in Ref. [43] and Ref. [44], respectively. The matched filtering method is used in search of GW signals, and the detector signal is correlated with template waveform. The signal-to-noise ratio (SNR) is defined by

$$\rho \equiv \frac{\langle h | h' \rangle}{\sqrt{\langle h' | h' \rangle}}, \tag{25}$$

where $h$ is the observed signal from the detector and $h'$ is the template. Notice that only if $h = h'$, $\rho$ is maximized.

In order to quantify the difference of two given waveforms, the usual approach is to calculate their match,

$$M \equiv \frac{\langle h | h' \rangle}{\sqrt{\langle h' | h' \rangle \langle h | h \rangle}}, \tag{26}$$

with the observed waveform $h$ and putative template $h'$. Note that the template waveform $h'$ is calculated with absence of ultralight axion effect, and the match is equal to the SNR degradation in Ref. [45]. By contrast, the observed waveform $h$ is modified when considering the existence of axion cloud. With larger modification in realistic waveform, the match $M$ becomes smaller, and then it is more easier to distinguish $h$ from $h'$. In this letter, we employ the method derived from Ref. [46], and then for GW signal with SNR $\rho$, one takes the following criterion to distinguish them

$$\rho > \sqrt{\frac{D/2}{1 - M}} \tag{27}$$

where $D$ is the number of estimated parameters. In this way, it is straightforward to calculate the critical value of SNR for detecting the phase shift induced by ultralight axion effect with space-borne GW observatory. Eq. (27) also indicates that the detectability depends on both the difference between two waveforms and the intensity of GW signal.

For TianQin and LISA, typically $D = 10$. Therefore, we evaluate the critical SNR for TianQin and LISA to detect phase shift from GW signals. In other words, in order to detect the axion effect, the necessary condition is that the signal SNR should be larger than the critical value ($\sqrt{D/2(1 - M)}$). Besides this, for convinience, we assume that the SNR threshold to claim a detection of GW signals for both TianQin and LISA are $\sim 8$ for a NS-BH-like source. Notice that, when the critical value of SNR to detect the axion effect is smaller than the SNR threshold value to detect the GW signals, it can be interpreted for these conditions the axion effect is significant and the phase shift is observable as long as the GW signals can be detected by the space-borne detector. In Fig. 2, we plot the SNR range in which TianQin and LISA can distinguish the GW waveform with phase shift introduced by the DC effect from the regular waveform. Assuming black hole mass $100 \ M_\odot$, neutron star mass $1.5 \ M_\odot$ and the integration time 5 years, the results are shown with different initial frequency $f_{\text{start}}$. The red solid line shows the critical value of SNR to detect the
We assume that the initial frequency when GW is observed is $0.01 \text{ Hz}$ and the integration time is 5 years. The red solid line shows the critical value of SNR to detect the phase modification between the GW waveform with surrounding axions and that in vacuum. The dashed line marks SNR threshold to detect a NS-BH binary source at TianQin and LISA. Assuming black hole mass $100 M_{\odot}$, the integration time is 5 years. The results are shown with different initial frequency $f_{\text{start}}$. Note that for the NS-BH binary in the light orange region, the DC effect is detectable by GW detector. For the source in the light orange region, the DC effect is detectable. The dashed line marks SNR threshold to detect a NS-BH binary source at TianQin and LISA. For the source in the light orange region, the DC effect is detectable. The dashed line represents SNR threshold to detect the NS-BH binary at TianQin and LISA. For the NS-BH binary in the light orange region, the DC effect is detectable. We find that for axion mass ranging from $10^{-13.5} - 10^{-11.5} \text{ eV}$, the phase shift induced by the axion effect might be detectable if the space-borne detector can detect this type of NS-BH binary. However, the mass of black hole decides the strain of the binary GW, so for NS-BH binary with lower black hole mass, the detection of GW signals itself becomes harder. In general, the DC effect is potentially observable for the axion mass range $[0.2, 40] \times 10^{-13} \text{ eV}$.

**Conclusion.**—We have studied the ultralight axion particle’s impact on the gravitational wave signal of the NS-BH binary. Besides the gravitational wave radiation predicted by general relativity, axion cloud depletion, dynamical friction and scalar dipole radiation could also affect the orbital evolution. Among them, the axion cloud depletion effect dominates for the low-frequency gravitational wave detection. We found that TianQin and LISA could be used to explore the ultralight axion particles around the NS-BH binary. We expect the future multiband and multi-messenger observations would help to pin down the properties of the ultralight axion particle.
