Bearings stiffness parameter estimation by sensitivity method

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Abstract. The sensitivity method is applied for bearings stiffness parameter estimation of an experimental rotor system. For model updating, critical shaft speeds are used. The reliability of the sensitivity method is tested on a simple example. The results show that this method can be successfully applied to adjust the finite element model to the experimental data.

1. Introduction

In complex engineering problems, the agreement between analytical and experimental results is not always good. The discrepancy can be explained by the assumptions used in the development of the mathematical model. Often, more detailed modelling of a physical system does not yield the desired agreement of results and also is limited by the computing capacity of the hardware used. Because of this, methods that allow changing the model parameters to achieve better agreement between analytical and measured data are developed.

Description of the procedures for updating the model parameters and their application for simple test examples can be found in \cite{1, 2}, where mainly direct and iterative methods are presented. Direct methods allow obtaining the values of the updated parameters without iterative calculations. However, these values can go beyond the physically possible ranges after updating. So, contrary to iterative methods, direct methods are not widely used in practice.

Among iterative methods, the sensitivity method is actively used. It allows using different parameters and experimental data for adjusting the finite element model (FEM) to the test results. One can get acquainted with the application examples of the sensitivity method, as well as its description in \cite{3, 4}.

Other methods of updating the FEM parameters have been developing lately. The method of reducing the parameter identification problem to an optimization problem has gained huge popularity. The method allows imposing different constraints on the updated parameters, whereas many other methods cannot do it because of the method of solution they are based on. In \cite{5}, approaches to updating FEM using an optimization method were considered. The successful application of these algorithms is presented in \cite{6-9}.

Over the last decades, the neural network-based approach has been actively developing \cite{10, 11}.

For estimating dynamic characteristics of rotor system bearings, most of the above-mentioned methods have to be modified, but special identification methods are developed sometimes. For example, a method of estimation of non-linear stiffness parameters of rolling element bearings from
random response of rotor-bearing systems was developed in [12]. Paper [13] presented an algorithm of identification of bearings parameters for multi-degree-of-freedom rotor-bearing systems where the shaft was considered flexible, and the parameters of supports depended on the rotation speed. Many approaches to identification of bearing parameters are presented in [14].

In this study, the sensitivity method was used to estimate the bearings stiffness parameter of an experimental rotor system. For model updating, critical shaft speeds are used. The method can be applied to adjust a rotor model based on the test results of a more complicated dynamic system. Calculations were done in MATLAB.

2. Theory
Let us introduce a vector of updated model parameters \( \mathbf{p} \), a vector of analytical \( \mathbf{z} \) and experimental \( \mathbf{z}_m \) critical speeds

\[
\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix}, \quad \mathbf{z}_m = \begin{bmatrix} \omega_{m_1} \\ \omega_{m_2} \\ \vdots \\ \omega_{m_n} \end{bmatrix}.
\]

(1)

The updating equations for this method are based on a linear approximation of the experimental data using Taylor’s series expansion

\[
\mathbf{z}_m = \mathbf{z}_j + \mathbf{S}_j (\mathbf{p} - \mathbf{p}_j),
\]

(2)

where \( j \) is the iteration number, \( \mathbf{S}_j \) is the sensitivity matrix (rectangular in a general case). Its components are the first derivatives of the critical speeds with respect to the updated parameters.

From (2), the following equation for updating model parameters can be derived:

\[
\mathbf{p}_{j+1} = \mathbf{p}_j + \left[ \mathbf{S}_j^T \mathbf{S}_j \right]^{-1} \mathbf{S}_j^T (\mathbf{z}_m - \mathbf{z}_j).
\]

(3)

After the initial parameter values are set, the calculations are conducted according to equation (3) until the change in the parameters between successful iterations becomes small.

3. Simple example
Consider an example of rotor supports stiffness updating using the sensitivity method. This example demonstrates the reliability of the applied parameter estimation approach.

The rotor model is comprised of a shaft, a disk, and supports on the left and right shaft ends. The shaft model consists of ten beam elements with a length of 50 mm and a circular solid cross section with a diameter of 10 mm. The elasticity modulus and the density of the material are \( 2 \cdot 10^5 \) MPa and 7800 kg · m\(^{-3} \), accordingly.

The rotor supports are modelled as isotropic constant stiffness springs. The stiffness of both springs is 100000 N · m\(^{-1} \). The disk is located at the shaft center. It is modelled as a lumped inertia element having the mass of 1 kg, the polar moment of inertia of \( 5 \cdot 10^4 \) kg · m\(^2\) and the diameter moments of inertia of \( 2.5 \cdot 10^{-4} \) kg · m\(^2\).

In this example, the simulated data is obtained from a similar model with other stiffness of springs equal to 150000 N · m\(^{-1} \). For updating the parameter, the first critical shaft speed is used. Tables 1 and 2 show the results of applying the sensitivity method.
Table 1. Simulated and analytical data.

| Critical speed, rpm | Error, % | Initial FEM | Updated FEM | Simulated | Initial FEM | Updated FEM |
|---------------------|----------|-------------|-------------|-----------|-------------|-------------|
| 1608                |          | 1658        | 1658        | 3         | 0           |             |

Table 2. Updated parameter.

| Parameter                  | Value                     | Error, % | Initial FEM | Updated FEM | Simulated |          |
|----------------------------|---------------------------|----------|-------------|-------------|-----------|----------|
| Springs stiffness, N·m⁻¹   | 100000                    | 150000   | 150000      | 150000      | 0         |          |

Figure 1 shows the plot of the parameter value vs. the iteration number.

![Figure 1](Image)

**Figure 1.** Convergence of the parameter for simple example.

It is seen that stiffness of the springs can be determined quite accurately. The method works correctly, and it can be applied for the estimation bearings stiffness parameter of an experimental rotor system.

4. Experimental example

This section presents the application of the sensitivity method to the problem of estimation bearings stiffness parameter of an experimental rotor system *Bently Nevada RK4 Rotor Kit*, figure 2. The kit comprises a frame, a shaft, a disk, supports, an electric motor with a coupling, proximity probes, a motor control unit, and proximity probes’ data acquisition unit. In each rotor support, a bearing with a rubber ring seal is installed, figure 3.

![Figure 2](Image)

**Figure 2.** Experimental setup.

![Figure 3](Image)

**Figure 3.** Bearing.
Table 3 shows the parameters of the shaft and disk. The disk was located closer to the left support. The distance between them was 170 mm.

Table 3. Parameters of the shaft and disk.

| Parameter               | Value  | Unit  |
|-------------------------|--------|-------|
| Shaft length            | 552    | mm    |
| Shaft diameter          | 10     | mm    |
| Disk mass               | 0.812  | kg    |
| Disk diameter           | 75     | mm    |
| Disk thickness          | 25     | mm    |
| Shaft material density  | 7796   | kg·m⁻³|
| Shaft material Young modulus | 2.1·10⁵ | MPa |

On the experimental system, the relationships of vibration amplitude vs. shaft speed for run-up and run-down were obtained, figure 4. Based on the plots, critical rotor speeds were obtained. The data was acquired near the support located closer to the electric motor. It is known [15] that the maximum displacement amplitude during the run-up and run-down is observed at different speeds, so an average between the run-up and run-down critical speed was taken (1874 rpm).

Figure 4. Vibration amplitude: (a) run-up; (b) run-down.

Figure 5 shows the rotor model.

Figure 5. Rotor model.

The shaft is modelled with 11 beam elements. The disk is modelled as a lumped inertial element with a mass, a polar moment of inertia and the equal diameter moments of inertia. The rotor supports were modelled as isotropic constant stiffness springs.

Shaft model parameters correspond to the data given in Table 3. Table 4 gives the parameters of the inertia element. The distance between the inertia element and the supports correspond to the disk.
location in the experimental system. The value of springs stiffness is not known, and its initial value was set to 50000 N m⁻¹.

### Table 4. Inertia element parameters.

| Parameter                  | Value       | Unit       |
|----------------------------|-------------|------------|
| Mass                       | 0.812       | kg         |
| Polar moment of inertia    | 5.75 · 10⁻⁴ | kg m²⁻¹    |
| Diameter moment of inertia | 3.30 · 10⁻⁴ | kg m²⁻¹    |

The analytical first critical rotor shaft speed was 1668 rpm. The error between analytical and experimental critical speed was 11%. The inertia rotor characteristics were modelled accurately enough. As the influence of damping on the critical speeds is small, springs stiffness (bearings stiffness parameter) is selected to update. Its initial value corresponds to the data presented in the model description. Tables 5 and 6 show the results of applying the sensitivity method.

### Table 5. Analytical and experimental data.

| Critical speed, rpm | Initial FEM | Updated FEM | Experimental | Initial FEM | Updated FEM | Error, % |
|---------------------|-------------|-------------|--------------|-------------|-------------|----------|
|                     | 1668        | 1874        | 1874         | 11          | 0           |

### Table 6. Updated parameter.

| Parameter                  | Value       |
|----------------------------|-------------|
| Springs stiffness, N m⁻¹   | 50000       |
|                            | 119731      |

Figure 6 shows the plot of the parameter value vs. the iteration number.

![Stiffness vs. Iterations](image)

**Figure 6.** Convergence of the parameter for experimental example.

It is seen from figure 6 that the iterative process of bearings stiffness parameter estimation converged successfully. After updating, the error between the analytical and experimental critical speed is small.
5. Conclusion

In this study, the sensitivity method was applied to updating the rotor dynamic model. The reliability of the method was tested on a simple example. The bearings stiffness parameter of an experimental rotor system was estimated using the sensitivity method. The considered method allows adjusting the rotor model to replicate the experimentally observed critical speed.

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