Secure Degrees-of-Freedom of the MIMO X Channel with Delayed CSIT

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Abstract—In this paper, we study the secure degrees-of-freedom (SDoF) characterization for the multiple-input multiple-output (MIMO) X channel with confidential messages and delayed CSIT. In particular, we propose a transmission scheme, which can be regarded as a generalization of the state-of-the-art scheme without security and with delayed CSIT. The key of this generalization is performing the security analysis, by which we derive the optimal duration of the artificial noise transmission phase. As a result, we drive the sum-SDoF lower bound. Furthermore, we reveal that if the number of receive antennas, denoted by $N$, is fixed, the number of transmit antennas for the maximal sum-SDoF lower bound achieved by our scheme saturates at $\frac{1}{N^2 + \frac{1}{N}}$.

Index Terms—Delayed CSIT, information-theoretic security, lower bound, MIMO X channel, secure degrees-of-freedom.

I. INTRODUCTION

The degrees-of-freedom (DoF) characterization for the multiple-input multiple-output (MIMO) channel with delayed channel state information at the transmitters (CSIT) has attracted lots of interests [1–4]. In [1], a non-trivial sum-DoF lower bound was achieved by a novel transmission scheme for the single-input single-output (SISO) channel with delayed CSIT. Since each transmitter has messages for all receivers, the scheme in [1] for X channel is different from the schemes for broadcast channel [5] and interference channel [6]. This transmission scheme was shown to be linear sum-DoF optimal in [2]. Thereafter, in [3], the transmission scheme in [1] was generalized to the MIMO X channel with delayed CSIT. However, the study of [4] showed that the general transmission scheme in [3] was linear sum-DoF optimal, except one antenna configuration case. For this case, a linear sum-DoF optimal transmission scheme was proposed in [4] to fill the gap. Unlike the no delayed CSIT utilization for data transmission phase in [3], for this antenna configuration case, the scheme in [4] exploited the delayed CSIT in one transmitter when the other transmitter is sending data symbols.

The secure degrees-of-freedom (SDoF) region of MIMO interference channel with confidential messages (ICCM) was characterized in [7]. The sum-SDoF of SISO X channel with confidential messages (XCCM) was studied in [8], [9]. The research of SDoF with delayed CSIT was stemmed from [10], where the SDoF region for two-user MIMO broadcast channel with confidential messages (BCCM) was characterized. In [10], the key idea of the transmission scheme is to add an artificial noise (AN) transmission phase before the data transmission phase. For MIMO ICCM with delayed CSIT, a sum-SDoF lower bound was proposed in [11]. For MIMO XCCM with delayed CSIT and output feedback, the SDoF region was derived in [12]. With alternating no, delayed, and current CSIT, the SDoF region of two-user multiple-input multiple-output (MISO) BCCM was characterized in [13]. Under no eavesdropper’s CSIT, the sum-SDoF of one-hop wireless networks were obtained in [14]. However, there is no research explore the SDoF of the MIMO XCCM with delayed CIST, which is the focus of this paper.

The main contribution of this paper is that we obtain a non-trivial sum-SDoF lower bound by designing a transmission scheme. Our transmission scheme cannot be covered by those schemes in [10–11] and their extensions. Instead, the proposed transmission scheme can be regraded as a generalization of the scheme in [2] for symmetric antenna configurations, since the security issue is considered by us. To generalize the scheme in [4], we first add an AN transmission phase before data transmission phase. Next, the transmitted data symbols are masked with feedback received AN signals, where the arrangement of data transmission mimicks that in [4]. However, this raises a problem: What’s the optimal duration of the AN transmission phase? We answer this question by performing the security analysis. Similar to the security analysis in [10–12], we apply data processing inequality and Lemma 2 in [10] to transform the mutual information expression for information leakage into matrix rank expressions. Whereas, since the delayed CSIT setting for XCCM is not considered in [10–12], the deduced matrix expressions and their rank analysis are different from that in [10–12]. Thus, the derived optimal duration of AN transmission phase is new. Interestingly, our bound indicates that if the number of receive antennas is fixed, the number of transmit antennas for the maximal sum-SDoF lower bound achieved by our scheme has a saturation value.

Notations: The identity matrix of dimensions $m$ is denoted by $I_m$. The determinant of matrix $A$ is denoted by $\text{det}(A)$. The block-diagonal matrix with blocks $P$ and $Q$ is denoted by $\text{bd}\{P, Q\} = [P, 0; 0, Q]$. The $\log$ function is referred to $\log_2$.

II. SYSTEM MODEL AND MAIN RESULTS

A. $(M, M, N, N)$ MIMO XCCM with Delayed CSIT

We consider a $(M, M, N, N)$ MIMO XCCM has two transmitters with $M$ antennas and two receivers with $N$ antennas, i.e., transmitters 1, 2, and receivers 1, 2. The transmitter $i = 1, 2$ has a confidential message $W_{i,j}$ for receiver $j = 1, 2$. The complex input signal at transmitter $i = 1, 2$ and time slot (TS) $t$ is denoted by $x_i[t] \in \mathbb{C}^M$. The complex received
signal at receiver $j = 1, 2$ and TS $t$ is denoted by $y_{j,t} \in \mathbb{C}^N$. Mathematically, the input-output relationship is written as

$$y_{j,t} = H_{1,j}[t]x_1[t] + H_{2,j}[t]x_2[t] + z_{j,t}, \quad j = 1, 2,$$

where the CSI matrix from the transmitter $i = 1, 2$ to the receiver $j = 1, 2$ at TS $t$ is denoted by $H_{i,j}[t] \in \mathbb{C}^{N \times M}$, and the additive white Gaussian noise (AWGN) vector at the receiver $j$ and TS $t$ is denoted by $z_{j,t}$. We assume that $H_{i,j}[t] \forall t$ is non-static (time-varying) and linearly independent. We denote the collection of CSI matrices for TS $1$ to TS $t - 1$ as $H^{-1} = [H_{1,j}[1], \ldots, H_{j,j}[t - 1]], i, j = 1, 2$. At TS $t$, due to feedback delay, $H^{-1}$ is available at two transmitters.

**B. Sum-SDoF**

A $(2nR_{i1}(\text{SNR}), 2nR_{i2}(\text{SNR}), 2nR_{i2}(\text{SNR}), 2nR_{i2}(\text{SNR}), n)$ code with secure achievable rates $R_{i,j}(\text{SNR})$, $i, j = 1, 2$ is defined as follows: The communication process takes $n$ channel uses with confidential messages $W_{i,j} = [1, \ldots, 2nR_{i,j}(\text{SNR})], i, j = 1, 2$. A stochastic encoder $f_i(\cdot)$ at the transmitter $i = 1, 2$, encodes confidential message $W_{i1}, W_{i2}$, and $H^{-1}$, to a codeword $x_i^n = [x_i[1], \ldots, x_i[n]]$. At the TS $t$, the input signal is encoded by $x_i[t] = f_i(W_{i1}, W_{i2}, H^{-1}), i = 1, 2$. A decoder $g_{i,j}(\cdot)$ at the receiver $j = 1, 2$ decodes the output signal $y_{j,t}^i \triangleq [y_{j,1}[t], \ldots, y_{j,n}[t]]$ to an estimated message $\hat{W}_{i,j}$, which is given by $\hat{W}_{i,j} = g_{i,j}(H^n, y^n_{j,t}), j = 1, 2$, where two receivers are assumed to have perfect CSI. In addition, the secure code should satisfy the reliability criterion, i.e., $P_r[W_{i,j} \neq \hat{W}_{i,j}] \leq \epsilon_n, i, j = 1, 2$, and the secrecy criterion,

$$\frac{1}{n}I(W_{1,1}, W_{2,1}; y^n_{2,t}) \leq \epsilon_n, \quad (2a)$$

$$\frac{1}{n}I(W_{1,2}, W_{2,2}; y^n_{1,t}) \leq \epsilon_n, \quad (2b)$$

where $\epsilon_n \to 0$ as $n \to \infty$. The secure sum-capacity is defined as the maximal achievable sum-rate, which is written as $C = \max \sum_{i=1}^{2} \sum_{j=1}^{2} R_{i,j}(\text{SNR})$. The sum-SDoF is a first-order approximation of the secure sum-capacity in the high SNR regime and defined as follows:

$$\frac{2}{\text{SNR}} \sum_{i=1}^{2} \sum_{j=1}^{2} d_{ij} \geq \lim_{\text{SNR} \to \infty} \frac{C}{\log \text{SNR}}. \quad (3)$$

**C. Main Results**

**Theorem 1:** Consider the $(M, M, N, N)$ MIMO XCCM with delayed CSIT. The sum-SDoF lower bound is given by

$$\sum_{i=1}^{2} \sum_{j=1}^{2} d_{ij} \geq \begin{cases} 0, & M \leq N, \\ \frac{3N(M-N)}{2M-N}, & N < M \leq \frac{7N}{8}, \\ \frac{6M-N}{6M-N}, & \frac{7N}{8} < M \leq 2N, \\ \frac{8M-N}{8M-N}, & 2N < M. \end{cases} \quad (4)$$

**Proof:** Please refer to Section-III.

**Remark:** Fig. 1 shows: 1) The derived sum-SDoF lower bound has a gain over the sum-SDoF lower bound of MIMO ICCM with delayed CSI [11], where the gain comes from the joint data transmission from two transmitters; 2) The derived sum-SDoF lower bound is less than that of the scenarios with better CSIT conditions, i.e., the sum-SDoF of MIMO ICCM with perfect CSIT [7], the sum-SDoF of SISO XCCM with perfect CSIT [9], and the sum-SDoF of MIMO XCCM with delayed CSIT and output feedback [12]; 3) There is a maximum in the derived sum-SDoF lower bound, which indicates that for a fixed $N$, the number of transmit antennas saturates at $\frac{4N}{7}$. This implies that we can switch off the extra antennas, if they do not increase the derived lower bound.

**III. PROOF OF THEOREM 1**

**A. $M \leq N$: Keep Two Transmitters Silent**

The intuition is given as follows: The transmitted AN symbols from one transmitter will be immediately decoded by the eavesdropper, which disables the security of data transmission superposed feedback received AN signals. Hence, the sum-SDoF lower bound is 0 by keeping two transmitters silent.

**B. $N < M \leq 2N$: The Proposed Transmission Scheme**

The following pre-assignated matrices: $\phi[k] \in \mathbb{C}^{M \times \tau_i N}, k = 1, \ldots, \tau_2, \omega[k] \in \mathbb{C}^{M \times \tau_i N}, k = 1, \ldots, \tau_2, \gamma[k] \in \mathbb{C}^{M \times \tau_i N}, k = 1, \ldots, \tau_4$, are linearly independent and full rank. Holistically, we denote $\Phi = [\phi[1]; \ldots; \phi[\tau_2]], \Omega = [\omega[1]; \ldots; \omega[\tau_2]], \Gamma = [\gamma[1]; \ldots; \gamma[\tau_2]],$ and $\Theta = [\theta[1]; \ldots; \theta[\tau_4]]$.

**Phase-I (AN Symbol Transmission for Receiver 1):** This phase spans $\tau_1$ TSs. At TS $t = 1, \ldots, \tau_1$, $M$ AN symbols are sent from transmitter 1, i.e., $x_1^t = u_1[t]$. Meanwhile, the transmitter 2 keeps silent. The holistic transmitted signal for Phase-I is written as

$$x_1^t = u_1. \quad (5)$$

The holistic received signals for Phase-I are written as

$$y_j^i = H_{1,j}[t]u_1 + z_j^i, \quad j = 1, 2, \quad (6)$$

where the AWGN signal at receiver $j$ is denoted by $z_j^i$, $u_1 = [u_1[1]; \ldots; u_1[\tau_1]] \in \mathbb{C}^{\tau_1 M}$, and $H_{1,j}[t] = \text{bd}([H_{1,j}[1], \ldots; H_{1,j}[\tau_1]]) \in \mathbb{C}^{\tau_1 N \times \tau_1 M}, j = 1, 2$. **Phase-II (AN Symbol Transmission for Receiver 2):** This phase is same as Phase-I, except the role of the transmitters 1 and 2 is swapped. Hence, this phase spans $\tau_1$ TSs as
The holistic received signals for Phase-II are written as

$$y_{j}^{H} = H_{2,j}^{H} u_{2} + z_{j}^{H}, \quad j = 1, 2,$$  

where the AWGN signal at receiver $j$ is denoted by $z_{j}^{H}$, $u_{2} = [u_{2}[1]; \cdots ; u_{2}[\tau_{1}]] \in \mathbb{C}^{T \times M}$, and $H_{2,j}^{H} = bd\{H_{2,j}[\tau_{1} + 1], \cdots ; H_{2,j}[2\tau_{1}]\} \in \mathbb{C}^{2N \times 2M}, j = 1, 2.$

Phase-III (Data Symbol Transmission for Receiver 1 from Two Transmitters): This phase spans $\tau_{2}$ TSSs. With the CSI matrices of Phase-I and Phase-II, transmitters 1 and 2 re-construct $y_{1}^{H}$ and $y_{2}^{H}$, respectively, when the AWGN is ignored. At $T s = 2\tau_{1} + 1, \cdots ; 2\tau_{1} + 2\tau_{3}$. $M$ data symbols (for receiver 1) superposed received AN signals are sent from transmitter 1, i.e., $x_{1}^{H}[t] = a_{1}^{H}[t - 2\tau_{1} - \tau_{3}] + \phi[t - 2\tau_{1} - \tau_{3}] y_{1}^{H}$. Meanwhile, $M$ data symbols (for receiver 2) superposed received AN signals are sent from transmitter 2, i.e., $x_{2}^{H}[t] = b_{2}^{H}[t - 2\tau_{1} - \tau_{3}] + \phi[t - 2\tau_{1} - \tau_{3}] y_{2}^{H}$. The holistic transmitted signals for Phase-III are written as

$$x_{1}^{H} = b_{1} + \Phi y_{1}^{H}, \quad (13a)$$

$$x_{2}^{H} = b_{2}^{H} + \Phi y_{2}^{H}, \quad (13b)$$

Phase-IV (Data Symbol Transmission for Receiver 1 from Transmitter 1): This phase spans $\tau_{3}$ TSSs. With the CSI matrices of Phase-III, the transmitter 2 re-constructs $H_{2,2}^{H} x_{2}$. At $T s = 2\tau_{1} + 2\tau_{3} + 1, \cdots ; 2\tau_{1} + 2\tau_{2} + 2\tau_{3}$. $M$ data symbols (for receiver 1) superposed received AN signals are sent from transmitter 1, i.e., $x_{1}^{H}[t] = a_{1}^{H}[t - 2\tau_{1} - \tau_{2}] + \phi[t - 2\tau_{1} - \tau_{2}] y_{1}^{H}$. Meanwhile, the transmitter 2 sends $x_{2}^{H}[t] = y_{2}^{H}$. The holistic transmitted signals for Phase-IV are written as

$$x_{1}^{H} = \Gamma H_{1,1}^{H} y_{1}^{H}, \quad (15a)$$

$$x_{2}^{H} = b_{2}^{H} + \Omega y_{2}^{H}, \quad (15b)$$

The holistic received signals for Phase-IV are written as

$$y_{j}^{H} = H_{1,j}^{H} x_{1}^{H} + H_{2,j}^{H} x_{2}^{H} + z_{j}^{H}, \quad j = 1, 2,$$  

where the AWGN signal at receiver $j$ is denoted by $z_{j}^{H}$, $a_{1}^{H} = [a_{1}^{H}[1]; \cdots ; a_{1}^{H}[\tau_{3}]] \in \mathbb{C}^{T \times M}$, and $H_{1,j}^{H} = bd\{H_{1,j}[\tau_{1} + 1], \cdots ; H_{1,j}[2\tau_{1} + \tau_{2} + 1], \cdots ; H_{1,j}[2\tau_{1} + 2\tau_{2} + \tau_{3}]\} \in \mathbb{C}^{2N \times 2M}, j = 1, 2.$

Phase-V (Interference Recurrence): This phase spans $\tau_{4}$ TSSs, which is used to re-transmit the combination of previous interference signals. This re-transmission will not incur new interference, but create useful equations for decoding. With the CSI matrices of Phase-III to Phase-VI, the transmitter 1 re-constructs $H_{2,1}^{H} x_{2} - H_{1,1}^{H} x_{1}^{H}$ and the transmitter 2 re-constructs $H_{2,j}^{H} x_{2} - H_{1,j}^{H} x_{1}^{H}$. At $T s = 2\tau_{1} + 2\tau_{2} + 3\tau_{3} + 1, \cdots ; 2\tau_{1} + 2\tau_{2} + 2\tau_{3} + \tau_{4}$. The transmitter 1 sends $x_{1}^{H}[t] = \theta[t - 2\tau_{1} - 2\tau_{2} - 2\tau_{3}] y_{1}^{H}$. The transmitter 2 sends $x_{2}^{H}[t] = \theta[t - 2\tau_{1} - 2\tau_{2} - 2\tau_{3}] y_{2}^{H}$. The holistic transmitted signals for Phase-VII are written as

$$x_{1}^{VII} = \Theta H_{2,1}^{H} H_{1,1}^{H} x_{1}^{H}, \quad (17a)$$

$$x_{2}^{VII} = \Theta b_{2}^{H} + \Theta H_{1,2}^{H} x_{1}^{H} - H_{2,2}^{H} x_{2}^{H}, \quad (17b)$$

The holistic received signals for Phase-VII are written as

$$y_{j}^{VII} = H_{1,j}^{VII} x_{1}^{VII} + H_{2,j}^{VII} x_{2}^{VII} + z_{j}^{VII}, \quad j = 1, 2,$$  

where the AWGN signal at receiver $j$ is denoted by $z_{j}^{VII}$. $a_{1}^{H} = H_{2,1}^{H} x_{2} - H_{1,1}^{H} x_{1}^{H}$ and $H_{2,j}^{H} = bd\{H_{1,j}[2\tau_{1} + 2\tau_{2} + \tau_{4}]\} \in \mathbb{C}^{2N \times 2M}, j = 1, 2.$
The optimal equation at receiver $i$ for security, due to the symmetry, we only need to perform analysis at one receiver. The final decoding equation at receiver 1 is given in (19), where the AWGN signal is denoted by $z_i$. The decoding of data symbols is only related to $H_1$, since the impact of $y_1^\text{VIII}$ and $y_1^\text{IV}$ can be removed. The rank of $H_1$ in (19) is $\min\{N(\tau_2 + \tau_3 + \min\{\tau_2, \tau_3\}), M(2\tau_2 + \tau_3)\}$, whose reason is given in Appendix A. Since the impact of $y_1^\text{VIII}$ and $y_1^\text{IV}$ is removed for decoding, the optimal $\tau_2^*, \tau_3^*$ can be found in (2), which is given by $(\tau_2^*, \tau_3^*) = (2N - M, 2M - \tau_2^*, \tau_3^*)$. It can be verified that the rank of $H_1$ is equal to the number of data symbols for receiver 1, i.e., $\min\{N(\tau_2^* + \tau_3^* + \min\{\tau_2, \tau_3\}), M(2\tau_2^* + \tau_3^*)\} = M(2\tau_2^* + \tau_3^*)$.

For security, due to the symmetry, we only need to perform analysis at one receiver. Given the notations $y_1^\text{VIII} = [y_1^\text{VIII}, \ldots, y_1^\text{IV}]$ and $u = [u_1; u_2]$, the information leakage $I(b_2^a, b_2^b; b_1; y_1^a; a_1^a, a_1^b, a_2)$ is calculated in (20), where the reason of each step is given as follows:

(a) $I(b_2^a, b_2^b; b_1; y_1^a; a_1^a, a_1^b, a_2) = I(b_2^a, b_2^b; b_1; y_1^a; a_1^a, a_1^b, a_2) + I(u; y_1^a; b_2^b; b_1; a_1^a, a_1^b, a_2)$, and applying the data processing inequality for the Markov chain $(b_2^a, b_2^b, b_1, u) \rightarrow (H_{1,1}^\text{VII}; u_1, H_{2,2}^\text{IV}; u_2, H_{1,1}^\text{VII}; (b_1 + \Phi H_{1,2}^\text{IV}) u_1)$ + $H_{2,2}^\text{IV}; (b_2^b + \Phi H_{2,2}^\text{IV}) u_2) \rightarrow (y_1^a; a_1^a, a_1^b, a_2)$.

(b) $\log \text{SNR} \rightarrow \infty$

(c) $N(2\tau_2 + \tau_3) \log \text{SNR} - \min\{N(2\tau_1 + \min\{\tau_1, \tau_2\} + \min\{\tau_2, \tau_3\}), 2M\tau_1\} \log \text{SNR}$. (20)
To maximize the sum-SDoF lower bound achieved by our scheme, $\tau_1$ should be as small as possible. This is because, Phase-I and Phase-II do not contain any fresh data symbols. Consequently, the optimal $\tau_1$ is given by

$$\tau_1^* = \begin{cases} \frac{N(M-N)}{2(M-N)}, & N < M \leq \frac{M+N}{2}, \\ \frac{N}{2M-N}, & \frac{M+N}{2} < N \leq 2N. \end{cases}$$

(23)

Our scheme has delivered $2M(2\tau_2 + \tau_3)$ data symbols over $2\tau_1 + 2\tau_2 + 2\tau_3 + \tau_4$ TSSs. With the above ($\tau_1^*, \tau_2^*, \tau_3^*, \tau_4^*$), it can be seen that the sum-SDoF lower bound in (4) for $N < M \leq 2N$ is achieved.

C. $2N < M$: Adopt the Transmission Scheme in [7]

The intuition is given as follows: Since the number of useful equations at the two receivers is at most $2N$ per TS, the data symbols cannot be decoded by interference recurrence if we send more than $2N$ data symbols per TS. This motivates us to send $2N$ data symbols from one transmitter for one receiver at one TS, which is the same as the transmission scheme in [7] for MIMO ICCM with delayed CSIT does. According to [7], the sum-SDoF lower bound is $4N/5$.

IV. CONCLUSIONS

We have obtained a sum-SDoF lower bound of the MIMO XCCM with delayed CSIT by proposing a transmission scheme. This transmission scheme can be deemed as a generalized version of the scheme in [4] for symmetric antenna configurations. We have derived the optimal phase duration for AN transmission based on security analysis. In the future, the research can be devoted to: 1) Finding a linear sum-SDoF upper bound, to examine the optimality of our design; and 2) Extending the proposed transmission scheme to the one for arbitrary antenna configurations with the absence of symmetry.

APPENDIX

A. Rank Analysis for Matrix $H_1$

The rank of $H_1$ is equal to the sum of the rank of

$$L = \begin{bmatrix} H_{11}^{III} & 0 & H_{12}^{III} \\ 0 & H_{21}^{IV} & H_{22}^{IV} \end{bmatrix},$$

and the rank of

$$U = \begin{bmatrix} H_{11}^{VIII} & H_{12}^{VI} & H_{12}^{IV} & H_{11}^{VII} & 0 \end{bmatrix}.$$