Atom holography

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We study the conditions under which atomic condensates can be used as a recording media and then suggest a reading scheme which allows to reconstruct an object with atomic reading beam. We show that good recording can be achieved for flat condensate profiles and for negative detunings between atomic Bohr frequency and optical field frequency. The resolution of recording dramatically depends on the relation between the healing length of the condensate and the spatial frequency contents of the optical fields involved.

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I. INTRODUCTION

In recent years, atomic optics has been rapidly emerging as a new and exciting subfield of atomic physics. The objective of atom optics is to manipulate atomic beams in a way similar to conventional optics by exploiting the wave properties of the atoms. Supported by advances in laser technology and microstructure fabrication a number of significant accomplishments have been realized in the laboratory with the demonstration of, e.g., mirrors, lenses, and diffraction gratings for atomic beams \[1\]. A natural way of extending these studies consists in exploring the possibilities of holographic imaging with atoms, the conventional optics analogue of which has been well-known for several decades \[2,3\].

Optical holography can be described as the three-dimensional reconstruction of the optical image of an arbitrarily shaped object. Typically, this is done in a two-step process where first the information about the object is stored in a hologram. This hologram is created by recording, e.g., with the help of a photographic film, the interference pattern between scattered light originating from the illuminated object and a (plane-wave) reference beam. The second step is the reconstruction, which is performed by shining a reading beam similar to the reference beam onto the hologram. The diffraction of the reading beam from the recorded pattern yields a virtual as well as a real optical image of the original object.

Drawing on this concept, the characteristic property of atomic holography is that at least the final reading step is performed with an atomic beam. In this way, an atom-optical image of the object is created which in certain situations can be thought of as some sort of material replica of the original. There are several reasons why the realization of atom holography is of interest: From a basic point of view, it significantly extends the already well-established range of analogies between light and matter waves. But more importantly perhaps, it may also have useful practical applications from atom lithography to the manufacturing of microstructures, or quantum microfabrication.

One of the prerequisites for an actual implementation of atomic holography is the availability of a reading beam of sufficient monochromaticity and coherence. Given the rapid advances in atom optics and especially in the realization of atom lasers, this requirement can be expected to be met in the near future. Another important question concerns the potentially detrimental influence of gravitational effects. One of the greatest challenges, however, is the manufacturing of the actual hologram where the information to be reconstructed is stored. Several schemes can be considered. One possibility is to diffract the atoms from a mechanical mask. The first successful realizations of such an approach have recently been reported in Ref. \[4\]. In these experiments the hologram was manufactured as a binary mask written onto a thin silicon nitride membrane. Such a hologram has the advantage of being permanent, however, as the mask only allows for complete or vanishing (binary) transmission of the beam at a given point one loses a significant amount of information about the optical image. Another interesting proposal was recently made in Ref. \[5\]. In this setup the atomic beam is diffracted from the inhomogeneous light field created by the superposition of object and reference beam. These beams thus directly form the hologram.

The purpose of the present paper is to investigate the perspectives of an alternative approach, namely the manufacturing of the hologram directly within a Bose-Einstein condensate (BEC). Atomic Bose condensates have recently been realized experimentally \[6,7\] and are now available almost routinely in several laboratories.

The motivation for the present study is twofold. First, the possibility of such an “all-atomic” scheme is interesting in itself and deserves further examination. Furthermore, it illustrates the wide potential applicability of condensates in atoms optics as a tool to influence the trajectories of atoms from external sources.

Our approach is based on two main ideas. The holographic information is encoded into the condensate in the form of density modulations by using writing and reference laser beams that form an optical potential for the condensate atoms. As we show later on, the density modulations follow closely the optical beam interference pattern if the condensate is of sufficiently high density, i.e., if a Thomas-Fermi description is applicable \[8\]. All-atomic reading is then accomplished in a way reminis-
cent of the Raman-Nath regime of diffraction between an atomic beam and a light field. Specifically, the reading beam atoms, that have a suitably chosen velocity, interact with the condensate atoms via $s$-wave scattering and acquire a spatially dependent phase shift reflecting the density modulations of the condensate. In the further spatial propagation of the atoms, this phase shift gives rise to the formation of the atom-optical image.

The proposed method is hence fundamentally different from a recent suggestion to arbitrarily shape the center-of-mass wave function of an atom (“wave-front engineering”). Instead of pursuing an holographic approach, this latter method makes use of a sequence of suitably shaped laser pulses to obtain the desired wave front. In fact, our proposal is more closely related to Ref. [9], which also suggests using of Bose-Einstein condensates to control particle deflection. Finally, we note that the present work is also related to the discussion of the analogy between matter-wave mixing phenomena in ultracold atomic samples and conventional nonlinear optics, including in particular matter-wave phase conjugation [11–13] and four-wave mixing [14–16].

The paper is organized as follows. After briefly recollecting the principles of optical holography Sec. II gives a general discussion of our approach to atomic holography introduced above. As an illustration in Sec. III the atom-optical imaging of a simple object is worked out in detail. Summary and conclusions are given in Sec. IV.

II. HOLOGRAPHIC IMAGING WITH ATOMIC BEAMS

A. Principles of optical holography

The principles of optical holography in their most basic form are shown in Fig. 1a; detailed expositions can be found, e.g., in Ref. [17]. An object is illuminated with a laser wave and the resulting field $E_o(r)$ is brought to interference with the reference beam $E_r(r)$. The ensuing superposition field is recorded on a suitable medium, e.g., a photographic plate, in such a way that the optical transmission of the medium becomes proportional to the total field intensity

$$I = |E_r + E_o|^2 = |E_o|^2 + |E_r|^2 + E_o^*E_r + E_r^*E_o.$$  (1)

The wave front of a reading beam $E_{rd}(r)$ impinging upon this hologram is thus proportional to $I(r)E_{rd}(r)$ after it has penetrated the medium. In this expression, the terms of interest are $E_oE_r^*E_{rd} + E_o^*E_rE_{rd}$. They contain the original object wavefront and its conjugate, and can be used to construct a virtual and a real image of the object. In optics several techniques have been developed which allow to separately view each of these terms, such as side-band Fresnel holography, Fraunhofer holography, Fourier transform holography, etc. [17]. For the present discussion of atom-optical holography we make use of ideas from side-band Fresnel holography, which does not rely on lenses and thus allows for a simple extension to matter waves.

FIG. 1. Set-ups for Fresnel side-mode holography: (a) all-optical realization; (b) optical/matter-wave realization. (Note the $-z$-axis in the last sketch.)
B. Atomic Bose-Einstein condensates as recording media

The idea of storing information onto atomic condensates is based on the observation that the density distribution of a condensate in the Thomas-Fermi limit closely reflects the behavior of the confining potential. This yields the possibility of accurate external control.

The Gross-Pitaevskii equation which governs the evolution of the macroscopic wave function \( \Phi(\mathbf{r}, t) \) describing the state of an atomic condensate with \( N \) atoms is given by

\[
i\hbar \frac{\partial \Phi}{\partial t} = \frac{\mathbf{p}^2}{2M} \Phi + V(\mathbf{r})\Phi + g|\Phi|^2\Phi,
\]

where \( \mathbf{p} \) denotes the atomic center-of-mass momentum, \( M \) the atomic mass, and \( V(\mathbf{r}) \) the external potential. The strength of atomic two-body interactions is determined by \( g = 4\pi\hbar^2a/M \) with \( a \) being the \( s \)-wave scattering length. The normalization condition for the condensate wave function reads

\[
\int d^3 \mathbf{r} |\Phi(\mathbf{r})|^2 = N.
\]

The steady state of a condensate can thus be described with a time-independent wave function \( \phi(\mathbf{r}) \) which is defined by

\[
\Phi(\mathbf{r}, t) = e^{-i\mu t/\hbar} \phi(\mathbf{r}),
\]

where \( \mu \) is the chemical potential. In the Thomas-Fermi limit, where the effect of kinetic energy is much weaker than the mean-field potential, the contribution of the term \( \mathbf{p}^2/2M \) can be neglected and the condensate density becomes

\[
|\phi(\mathbf{r})|^2 = |\mu - V(\mathbf{r})|/g,
\]

where \( \mu \) is determined by the normalization condition Eq. (3). From this expression we see immediately that the form of the external potential is replicated in the density profile of the atomic condensate.

Consider then replacing the photographic plate in Fig.1b by a pancake-shaped BEC as the recording medium. In case the writing of the information into the condensate is achieved by optical beams, which are assumed to be far detuned from atomic resonance, they create an optical potential proportional to \( I/\delta \) where \( I \) is given by Eq. (1) and \( \delta = \omega - \omega_L \) is the detuning between the atomic resonance \( \omega \) and the laser frequency \( \omega_L \). The total potential \( V(\mathbf{r}) \) acting on the condensate is then the sum of the trap potential, taken to be slowly varying, and this optical potential. From Eq. (4), it then follows in full analogy with optical holography that all terms in Eq. (4) are stored in the density distribution of the condensate ground state. However, this atomic-condensate recording is in some ways more akin to “real-time” holography, since the optical fields should be continuously present in order to maintain the density modulations in the condensate.

C. Reading from atomic condensates

As already mentioned in the introduction, we consider an all-atomic reading scheme, which has the fundamental advantage of allowing one to reconstruct a material “replica” of the stored object. Specifically, the reading beam is a monoenergetic atomic beam of velocity \( v_{rd} \) impinging at some angle onto the condensate. We assume that the internal state of these incoming atoms is such that they are only weakly perturbed by the writing and trap potentials, so that their dominant interaction is scattering by the atoms in the condensate. It is important at this point to emphasize that the atoms in the reading beam need not be of the same species as the condensate atoms. In principle they could be of just about any element or even molecule.

We consider specifically reading beam velocities such that the interaction between the incoming atoms and the condensate can be described in terms of \( s \)-wave scattering. This condition is fulfilled provided that

\[
a_{rc}m_{rc}v_{rd}/\hbar \ll 1,
\]

where \( a_{rc} \) denotes the \( s \)-wave scattering length for collisions between reading and condensate atoms and \( m_{rc} \) is their relative mass. For simplicity, we further assume that the density of the reading beam is low enough that collisions between atoms in that beam can be neglected. Under these conditions the time evolution of the reading atoms’ wave function \( \varphi(\mathbf{r}, t) \) in the mean field of the condensate is determined by the equation

\[
i\hbar \dot{\varphi}(\mathbf{r}, t) = \left[ \frac{\mathbf{p}^2}{2M_{rd}} + g_{rd}|\phi(\mathbf{r})|^2 \right] \varphi(\mathbf{r}, t)
\]

where \( M_{rd} \) is the mass of the atoms in the reading beam, \( g_{rd} = 2\pi\hbar^2a_{rc}/m_{rc} \). This equation assumes that to a good degree of approximation, the condensate stays in its ground state during the whole reading process.

Over the course of time, the condensate gradually loses atoms due to scattering by the incoming atoms and other processes, but it is assumed that its density distribution remains given by Eq. (4) with \( \mu \) slowly varying due to the change in the number of atoms \( N \), so that \(|\phi(\mathbf{r})|^2\) has to be changed adiabatically in Eq. (6). Under these circumstances the shape of the holographic image will gradually

\[\text{[Note that the use of optical fields is not essential to the present discussion: other interactions susceptible of imposing a spatially dependent potential } V(\mathbf{r}) \text{ on the condensate can also be considered.}}\]
change and eventually distort when $|\phi(r)|^2$ deviates too much from the Thomas-Fermi expression. However, the time scale for this process, the lifetime of the condensate, can be long in comparison to the time necessary to form the image, the flight time of the reading atoms.

The reading and reconstruction of the condensate information into a material “replica” is easily achieved if the condensate is sufficiently thin that its density distribution can be regarded as effectively two-dimensional, and its interaction with the reading atoms is short enough that the Raman-Nath (or thin hologram) approximation can be invoked. The condensate then acts as a phase grating for the reading beam, whose waveform after penetrating the condensate is given by

$$\varphi(r, z_c+, \tau) = \exp[-ig_{rd} |\phi(r)|^2 \tau / \hbar] \varphi(r, z_c-, 0). \quad (7)$$

Here $\tau$ denotes the time it takes the probe atoms to pass through the condensate of length $l_z$, $z_c-$ and $z_c+$ are the $z$-coordinates just before and past the condensate respectively. The situation described by Eq. 7 is reminiscent of phase holography in optics. In case

$$g_{rd} \max[|\phi'(r)|^2] / \hbar \ll 1 \quad (8)$$

we obtain

$$\varphi(r, z_c+, \tau) \approx (1 - ig_{rd} |\phi(r)|^2 \tau / \hbar) \varphi(r, z_c-, 0),$$

i.e., the holographic information stored in the condensate is indeed transferred to the reading beam. Subsequent free space propagation allows to separate the different terms contained in $|\phi(r)|^2$ and to reconstruct the atom-optical replica of the stored object.

### III. EXAMPLE: ATOM-OPTICAL IMAGING OF A SMALL APERTURE

In this section we illustrate the principle of atom holography in the case of imaging of a simple object. This example allows one to investigate more closely under which conditions and to which degree the general scheme of Section II can be realized in practice.

#### A. Optical potentials

The geometry we are considering is shown in Fig. 1b. The aperture and the condensate are parallel to each other, their centers being located at the points $(0, 0, 0)$ and $(0, 0, z_c)$, respectively. The aperture is illuminated by a plane optical wave $E_{\text{inc}}$ of amplitude $E_0$ and wave vector $k_L$ propagating along the $z-$direction. The emerging electric field is the well-known Kirchhoff’s solution to the associated diffraction problem \[\text{[17]}\]

$$E_o(r, z_c) = -\frac{1}{2\pi} \int_{\text{obj}} d^2r_0 E_{\text{inc}}(r_0, 0) \left( ik_L - \frac{1}{R} \right) \times \cos \theta \frac{\exp(ik_L R)}{R}, \quad (9)$$

where $r_0$ is a two-dimensional vector in the object plane and $R = \sqrt{r^2 + z_c^2}$ the distance from an object point $(r_0, z) = 0$ to a point $(r, z_c)$ on the thin condensate acting as a recording medium. Finally, $\cos \theta = z / R$, and the integration is performed over the object boundaries.

The diffracted electric field acquires a simpler form in the Fresnel regime (i.e., the paraxial or parabolic approximation) where $z_c > \lambda_L / 2$ so that $|r| \sim 1$ and $R \sim z_c + |r - r_0|^2 / z_c$. The object field at the location of the condensate can then be approximated as

$$E_o(r, z_c) \propto e^{ik_L z_c} e^{i\pi r^2 / \lambda L z_c} \frac{\mathcal{F}_2[O(r_0) e^{|r|^2 / \lambda L z_c}]}{\rho = |r| / \lambda L z_c}, \quad (10)$$

where $O(r_0)$ defines the shape of the object and $\mathcal{F}_2[\rho] r$ denotes the two-dimensional Fourier transform with respect to $\rho$.

In order to construct the optical potential $V(r)$ that imprints the information onto the condensate, the object field $E_o(r, z_c)$ is interfered as in conventional holography with a plane wave reference beam of amplitude $E_r$ and wave vector $k_r = k_L (\sin \beta, 0, \cos \beta) = (k_L, 0, k_z)$, see Fig. 1b. The intensity of the superposition at the location of the condensate is thus

$$I(r, z_c) = |E_o(r, z_c)|^2 + |E_r|^2$$

$$+ \left| \mathcal{F}_2[\ldots] E^* r \int_{\text{obj}} |E_o(r_0, 0) e^{-ik_L z_c} - ik_L z_c e^{-i\pi r^2 / \lambda L z_c} + c.c.] \right|^2 \quad (11)$$

To make things clear, let us backtrack for a moment and imagine that instead of a matter-wave hologram, we create an optical hologram from the intensity distribution \[\text{[1]}\]. When illuminating that hologram with the reading beam $E_{rd} = E_o e^{ik_L z_c} e^{i\pi r^2 / \lambda L z_c}$, we see the emergence of three wavefronts: the background wave $E_{rd}(E_0^2 + |E_r|^2)$ travelling along the direction $(-k_L, 0, k_z)$; the object wave $E_{rd}^* E_r$; and the conjugate beam $E_{rd}^* E_r$, which constitutes a converging wavefront travelling in the $z-$direction. Upon propagating a distance $z_c$ after the plane of the hologram, the quadratic phase in the conjugate beam is undone and a real image is created. Indeed, by applying again Kirchhoff’s solution in the Fresnel approximation of Eq. (10) one obtains

$$E_{\text{im}}(r, 2z_c) \rightarrow E_{rd}^* E_r \propto e^{ik_L z_c} e^{i\pi r^2 / \lambda L z_c}$$

$$
\times \frac{\mathcal{F}_2[O(r_0) e^{|r|^2 / \lambda L z_c}]}{\rho = |r| / \lambda L z_c}, \quad (12)$$

which is precisely proportional to $O(r)$. The object wavefront, on the other hand, corresponds to a virtual image. We now study the conditions under which this same procedure can be applied to atom holography.
B. The writing process

We assume for concreteness that the condensate consists of sodium atoms, so that laser fields with wavelengths of about \( \lambda_J \sim 10^{-6}\) m can be used to create the optical potentials \([19]\). It is assumed to be trapped in a square well potential, as this provides a homogeneous density of a condensate and thus avoids distortions of the holographic image. However, one could also work with the approximately constant density distribution near the center of a harmonic trap that is very wide in the transverse directions.

To be specific, we investigate the imaging of a rectangular aperture of width \( D = 10 \lambda_L \) located at a distance \( z \sim 1000 \lambda_L \) from the condensate. The emerging diffraction pattern has an angular width of \( \theta_d = \lambda_L / D \sim 0.1 \), so that the condensate must have an extension of at least \( l_x \sim 100 \lambda_L \sim 10^{-4} \) m in the \( x \)-direction. Its extension \( l_y \) along the \( y \)-axis, as well as the width of the aperture, are both assumed large enough that diffraction effects are negligible in that direction, reducing the problem to an effective two-dimensional geometry. This allows us to express the condensate wave function as \( \phi(r) = \psi(x) / \sqrt{l_y l_z} \) where \( l_z \) is the condensate thickness, which is assumed to be very small as we recall from sec. II C. Indeed, an upper limit to \( l_z \) is provided by the condition that the density distribution has to be effectively two-dimensional. The periodicity of the intensity distribution (11) along the \( z \)-direction is determined by the angle between reference and writing beams; quantitatively, one obtains the requirement

\[
l_z \ll 2 \pi / k_L (1 - \cos \beta).
\]

In our numerical example we choose \( \beta = 30^\circ \) and \( l_z = 10^{-6} \) m so that this condition is well satisfied. From the normalization condition for a condensate in a square well potential one immediately finds that the chemical potential is given by

\[
\mu = N g / l_x l_y l_z
\]

and thus from Eq. (11)

\[
|\psi(x)|^2 = N / l_x
\]

The strength of the optical fields is determined from the requirement that the recorded density profile of the condensate is mainly determined by the light field intensity, and not by the trap ground state profile (pedestal). This means that the field intensity must yield modulations of the optical potential deeper than the trapping potential, i.e.

\[
\max |\hbar \Omega^2(r, z_c) / \delta| \gg \mu,
\]

where the Rabi frequency \( \Omega^2(r, z_c) \propto I(r, z_c) \). In addition, since the atomic density of Eq. (4),

\[
|\psi(r, z_c)|^2 = \frac{1}{g} \left[ \mu - V_{\text{trap}}(r) - \frac{\hbar \Omega^2(r, z_c)}{\delta} \right]
\]

where \( V_{\text{trap}} \) is the trap potential, is non-negative, one should choose a negative detuning \( \delta \) so that \( |\psi(r, z_c)|^2 \) reaches the approximate value

\[
|\psi(r, z_c)|^2 \simeq \left| \frac{\hbar \Omega^2(r, z_c)}{g \delta} \right|
\]

For the Thomas-Fermi approximation and thus Eq. (4) to be valid, the condensate healing length \( \xi \) needs to be much smaller than the characteristic length scale \( \lambda \) over which the external potential varies, i.e.,

\[
\xi = (8 \pi n)^{-1/2} \ll \lambda
\]

where \( n \) is the condensate density. In our case, \( \lambda \) is of course of the order of the optical writing beam wavelength, \( \lambda \sim \lambda_L \). The healing length determines the length scale of a density variation whose quantum pressure (kinetic energy contribution) is of the order of the interaction energy \( \frac{\hbar \Omega^2}{\sqrt{g}} \). This leads to the requirement

\[
8 \pi N a l_x / l_y l_z \gg (l_x / \lambda_x)^2
\]

which, with all other values previously fixed, translates into \( N / l_y \gg 2 \times 10^9 \). In our numerical example we work with the value \( N / l_y = 2 \times 10^{11} \) which correspond to \( N = 2 \times 10^7 \) for \( l_y = 10^{-4} \) m.

We proceed by first determining numerically the ground state of the condensate subject to the writing optical potentials, with the goal of justifying the approximate density profile (17). This is achieved by solving the Hartree-Fock wave function evolution governed by the Gross-Pitaevskii equation in imaginary time. For the parameter values listed, we find a very good agreement between the shape of the density modulation of the condensate ground state, see Fig. 3, and the optical intensity distribution, see Fig. 3. Further numerical simulations show that this density profile can actually be prepared by adiabatically turning on the writing and reference beams: Starting from the condensate in the trap ground state, the optical fields are switched on slowly enough that no density oscillations become significantly excited. In our specific example, we consider first the square-well ground state \( \psi(x) = \sqrt{N / l_x} \) and turn on the optical potential Eq. (11), with a switch-on function \( [1 - \exp(-t / t_c)]^2 \), where \( t_c \approx 0.01 \) s is short in comparison to typical condensate lifetimes. The resulting condensate density profile is given in Fig. 3. This illustrates that in this way the condensate wave function is transferred without difficulty from the trap ground state to the new ground state in presence of the optical potential, see Fig. 3.
FIG. 2. Condensate ground state density [m$^{-1}$] in a square-well potential with the optical fields on. The object size is $10\lambda_L$, and the distance from the object to the condensate is $z_c = 1000 \cdot \lambda_L$. The sodium condensate parameters are $N = 2 \cdot 10^7$, $l_x = 3 \cdot 10^{-4}$m, $l_y = 10^{-4}$m, $l_z = 10^{-6}$m, $V_{\text{trap}}(|x| > l_x) = \mu$. The optical field parameters are $|\bar{\hbar}\Omega^2(r, z_c)/\delta| = 100 \cdot \mu$, $E_r = 0.1E_o$, $\beta = 30^\circ$.

FIG. 3. Optical hologram for the same optical fields as used for writing on the atomic condensate in Fig. 2.

C. The reading process

The reading and construction of a replica of the store object is achieved by an off-axis atomic beam impinging upon the condensate from the side opposite to the writing optical beams. The velocity of this beam needs to be carefully selected, as it must be confined between a lower and an upper bound resulting from the thin hologram and $s$-wave scattering approximations, respectively.

As we have seen, a lower bound in atomic velocities $v_{rd}$ in that beam is determined by the condition of validity of Raman-Nath, or thin hologram, approximation. The physical meaning of this condition is that the longitudinal contribution to the kinetic energy of the probe atoms must be large compared to their interaction energy with the condensate atoms

$$\frac{p_{rd}^2}{2M_{rd}} \gg g_{rd}\max|\phi(r)|^2$$

and the typical transverse deflection inside the condensate must remain small compared to the length scale of the density fluctuations,

$$\frac{p_{rd}\tau}{M_{rd}} \ll \lambda_L.$$  \hfill (20)

Under these circumstances, the kinetic energy term in Eq. (11) can be dropped. Eq. (19), together with the requirement of small phase variations in the reading beam upon propagation through the condensate (see Eq. (8)), gives the lower bound for $v_{rd}$.

In addition, Eq. (20), together with the requirement that the $s$-wave scattering approximation is valid, see Eq. (5), determines an upper bound for $v_{rd}$.

Our numerical simulations are for $v_{rd} \sim 10^{-1}$ m/sec, which is a reasonable value for the experiments with ultra cold atomic beams and lies within these lower and upper bounds.

Similarly to the optical case, the atomic wave function acquires a quadratic phase upon free propagation, a result of the fact that in the paraxial approximation, the dispersion relations of optical and matter waves are both quadratic and essentially the same. Indeed,

$$\varphi(x, z_c + \Delta z, \tau + \Delta \tau) =$$

$$\int d\xi e^{ix\xi} e^{-\frac{\hbar^2}{2M_{rd}} \Delta \tau} \int dx' e^{-ix\xi} \varphi(x', z_c + \tau)$$

$$= e^{i\frac{M_{rd} \hbar^2}{2M_{rd}} \xi^2} \int dx' \varphi(x', z_c + \tau) e^{i\frac{M_{rd} \hbar^2}{2M_{rd}} \xi'^2} e^{-i\frac{M_{rd} \hbar^2}{2M_{rd}} \xi'x'$$
\[ e^{i \frac{M_M k_M}{\hbar} x^2} \mathcal{F}_2[\phi(x', z_c + \tau)] e^{i \frac{M_M k_M}{\hbar} x'^2}] \bigg|_{x = \frac{M_M k_M}{\hbar} x} \] (21)

Here \( \phi(x', z_c + \tau) \) is defined according with Eq. \( (3) \) and \( \Delta z = v_r \Delta t \). The free propagation time \( \Delta t \) is determined from the same requirement as in the optical case, namely that the quadratic phase \( \exp(-i \pi x^2/\lambda_L z_c) \) be exactly compensated. This gives

\[ \Delta t = \frac{M_M v_r}{2\hbar} \left( \frac{\lambda_L z_c}{\pi} \right). \] (22)

We finally observe that in order for the image to be formed on-axis, the reading beam needs to propagate off-axis at an angle \( \beta_A \) such that it compensates the angle of an optical reference beam, i.e.,

\[ \sin \beta_A = k_L \sin \beta/k_A. \] (23)

This is a very small angle since \( k_A \equiv v_r M_M/\hbar \gg k_L \). Fig. \( 3 \) shows the numerically computed atomic density profile in the plane \( z_c + \Delta z \) for this choice of parameters. This demonstrates explicitly that the original rectangular aperture is indeed reconstructed on axis. In contrast, the virtual image, for which the quadratic phase stored in the condensate is not compensated, propagates off-axis, and so does the background contribution.

FIG. 5. Reconstructing a replica of the original object from the atomic hologram. The reading beam consists of a monochromatic beam of sodium atoms moving at an angle \( \beta_A \) from the \( z \)-axis at a velocity \( v_r = 0.1 \) m/sec. Shown is the atomic density profile at a distance \( \Delta z \) from the condensate such that the quadratic phase shift of the conjugate image is precisely canceled. The insert compares the reconstructed and original objects. The off-axis feature for positive \( x \) corresponds to the real object, for which the quadratic phase is still present. The large off-axis feature at negative \( x \) is background.

IV. SUMMARY AND CONCLUSIONS

In this paper, we have theoretically discussed an atom holography scheme where the hologram is stored in a Bose-Einstein condensate. An important feature of the proposed scheme is that the reading beam needs not consist of the same element as the condensate atoms. It merely needs to be a slow monochromatic atomic or molecular beam that interacts with the condensate atoms via \( s \)-wave scattering, or any other interaction leading to a cubic nonlinearity in the nonlinear Schrödinger equation. Matter-wave holography, once experimentally realized, is certain to open up the way to numerous potential applications, in particular in microfabrication. One should note that the slow atoms discussed in the present paper have large de Broglie wavelengths, which severely limit the spatial resolution of the material replica that can be reconstructed. However, the wavelength of matter waves can easily be shortened, for example by gravitation. It is readily conceivable that this can be used to achieve miniaturized structures with nanometer-scale features. This, and other aspects of atom holography, will be the subject of future studies.

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[1] C. S. Adams, M. Sigel, and J. Mlynek, Physics Reports 240, 143 (1994).
[2] D. Gabor, Nature 161, 777 (1948).
[3] D. Gabor, Proc. R. Soc. A 197, 454 (1949).
[4] M. Moringa, M. Yaeuda, T. Kishimoto, and F. Shimizu, Phys. Rev. Lett. 77, 802 (1996).
[5] A. Soroko, J. Phys. B 30, 5621 (1997).
[6] M. Anderson et al., Science 269, 198 (1995).
[7] K. B. Davis et al., Phys. Rev. Lett. 75, 5820 (1995).
[8] P. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. in press (1998).
[9] M. Olshanii, N. Dekker, C. Herzog, and M. Prentiss, quant-ph/9811021 (1998).
[10] S. A. Chin and H. A. Forbert, cond-mat/9810269 (1998).
[11] E. V. Goldstein, K. Plättner, and P. Meystre, Quant. Semiclas. Optics 7, 743 (1995).
[12] E. V. Goldstein, K. Plättner, and P. Meystre, Jrnl. Res. Nat. Inst. Stand. Technol. 101, 583 (1996).
[13] E. V. Goldstein and P. Meystre, Phys. Rev. A 59, 1509 (1999).
[14] C. K. Law, H. Pu, and N. P. Bigelow, Phys. Rev. Lett. 81, 5257 (1998).
[15] M. Trippenbach, Y. B. Band, and P. S. Julienne, Optics Express 3, (1998).
[16] L. Deng et al., Nature 398, 218 (1999).
[17] J. B. DeVelis and G. O. Reynolds, Theory and applications of holography (Addison-Wesley, London, 1967).
[18] E. M. Lifshitz and L. P. Pitaevskii, Statistical Physics, Part 2 (Pergamon Press, New York, 1980).
[19] D. M. Stamper-Kurn et al., Phys. Rev. Lett. 80, 2027 (1998).