Uncertainties in Air Showers from Small-x pQCD Mini-Jets

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Received 31 October 2005

Abstract. To investigate uncertainties in air shower simulations caused by the small-$x$ regime, a model including leading-twist hard pQCD plus soft processes was built, which are separated by an energy dependent transverse momentum cut-off. We provide a fit of the cut-off to the total $pp$ cross section for different PDFs using the eikonal formalism and show that for modern PDF sets there is only a small uncertainty in the mini-jet cross section, and hence in the final-state multiplicity and the number of produced muons.

Keywords: UHECR, air showers, perturbative QCD, small $x$

PACS: 13.85.-t, 13.85.Tp, 96.50.sd

1. Introduction

Extensive air showers (EAS), are induced in the atmosphere due to collisions of a primary cosmic ray particle with air nuclei. The properties of cosmic rays, such as their origin, energy and composition are (partly) reconstructed from air shower observables. Thus, understanding the physics of the primary collision and of subsequent interactions in the atmosphere is important. The main uncertainties are dominated by hadronic interactions. For example, muon multiplicities at Auger, Kascade and CosmoLEP [1] might hint at a poor understanding of particle production at ultra-high cosmic ray energies. Given that these extend far beyond those of terrestrial accelerators, our knowledge of hadronic interactions has to be extrapolated to unknown regimes in any QCD inspired model attempting to describe the physics of EAS [2].

The QCD mini-jet cross section in $pp$ increases rapidly with energy and dominates the total cross section already for collider energies. It is sensitive to the small-$x$
regime of perturbative QCD (pQCD), and to the break-down of the leading-twist approximation at small transverse momentum, which requires us to introduce a cut-off \( p_{T_{\text{cut}}} \).

Another source of uncertainties is low-momentum “soft” particle production which dominates the very forward rapidity region and hence the momentum degradation of the cosmic ray in the atmosphere. For semi-central collisions with air nuclei an IR-safe approach to particle production is provided by gluon saturation effects, see [3] for an application to EAS. For peripheral \( pA \) and for \( pp \) collisions no intrinsic semi-hard scale exists and weak-coupling methods are therefore not available; here, we followed instead the Dual Parton Model [4], adjusting phenomenological parameters to (recent) accelerator data. However, our present focus is on uncertainties related to the perturbative QCD regime, in particular with respect to the multiplicity of secondaries (or \( d\sigma_{\text{jet}}/dy \)) which is related to the number of produced muons.

2. Mini-jet production

The hard jet cross section in pQCD can be written as [5]

\[
\frac{d\sigma_{\text{jet}}}{dy_3dy_4d^2p_Td^2b} = \sum_{i,j,l,k} x_1 f_i(x_1, Q^2, b) x_2 f_j(x_2, Q^2, b) \frac{1}{\pi} \frac{d\sigma}{dt}(i + j \rightarrow l + k),
\]

where the summation runs over all parton species, \( x_1 \) and \( x_2 \) are the light-cone momentum fractions carried by the scattering partons, \( t \) is the momentum transfer and \( b \) is the impact parameter. The kinematical variables \( x_1, x_2 \) and \( t \) are related in a simple way to the rapidities \( y_3, y_4 \) and \( p_T \) [5]. We do not employ a phenomenological K-factor since its energy dependence is not well known; this uncertainty does not affect our results significantly except for increasing the \( p_T \)-cutoff shown below slightly.

The functions \( f(x, Q^2, b) \) are the PDFs. The impact parameter dependence is assumed to factorize, \( f(x, Q^2, b) = T(b)f(x, Q^2) \), with \( T(b) \) the proton overlap function chosen according to [6]. HERA has measured the PDFs over a broad range of \( Q^2 = p_T^2 \), but only for \( x \gtrsim 10^{-4} \). Modern PDF packages extrapolate to lower \( x \) according to QCD evolution equations. We shall show results for both the (leading order) GRV98 and CTEQ6 parameterizations [7] which feature a BFKL-like growth of the gluon distribution at small \( x \). To illustrate the effect of small-\( x \) evolution, we also employ the old Duke-Owens (DO) parameterization, which exhibits a much flatter \( xg(x, Q^2) \) at small \( x \).

Due to the power-law divergence of (1) as \( p_T \to 0 \) we need to introduce a low momentum cut-off. The (phenomenological) soft cross section \( \sigma_{\text{soft}} = 57 \) mb is assumed to be constant at high energies. It’s value is chosen to reproduce the total \( pp \) cross section at the lower end of relevant energies, \( E_{\text{cm}} \sim 100 \) GeV.

In order to avoid partial wave unitarity violation we use the eikonal formalism [8] to define the total cross section. In this formalism the total and inelastic cross
sections are related to $\sigma_{jet} + \sigma_{soft}$ by

$$\sigma_{tot} = 2 \int d^2 b [1 - e^{-T(b)(\sigma_{jet} + \sigma_{soft})}], \quad \sigma_{el} = \int d^2 b [1 - e^{-T(b)(\sigma_{jet} + \sigma_{soft})}]^2,$$

and $\sigma_{inel} = \sigma_{tot} - \sigma_{el}$.

3. Results

According to S-Matrix and Regge theory the total $pp$ cross section can be parameterized in a variety of ways at high energies. A parameterization fitted to the available experimental data is presented in [9]. Using expression (2) we fitted $pT_{cut}$ to that parameterization of $\sigma_{tot}$ (for $pp$) for different PDFs. As shown in fig. 1a, for modern PDFs the cut-off increases very rapidly at high energies, which is related to the small-$x$ growth of the gluon distribution. The fits for CTEQ6 and GRV98 return very similar values for $pT_{cut}$, with small discrepancies only for $E_{cm} \approx 100$ TeV. On the other hand, the DO set requires a constant cut-off within the relevant energy regime, which is due to the too flat small-$x$ gluon distribution for this old parameterization.

Using the above $pT_{cut}$ for a given energy and PDF, we can now integrate (1) over $y$ and $p_T$ to obtain $d\sigma_{jet}/dy$. In fig. 1b we show $d\sigma_{jet}/dy$ for different PDFs at $E_{CM} = 150$ TeV, a typical primary high-energy cosmic ray. Again we observe very good agreement among the modern PDFs, both of which predict a much higher multiplicity at central rapidity than the old DO set. Also, the latter gives a much wider rapidity plateau, implying a reduced momentum degradation in the atmosphere from inelastic interactions.

![Fig. 1](image-url)

**Fig. 1.** Fig a) Transverse momentum cut-off for leading-twist LO pQCD processes in $pp$, for various sets of PDFs. Fig b) Rapidity distribution of the (mini-) jet cross section in inelastic $pp$ collisions for those PDFs.
4. Conclusion

We have studied possible uncertainties in hard process extrapolated to high energies, which are sensitive to the small-$x$ regime of the PDFs. We fixed $p_T^{cut}(E_{cm})$ by fitting the total $pp$ cross section in the eikonal formalism. While modern sets of PDFs with a steep increase of the gluon distribution at small $x$ agree rather well, old parameterizations predict a much flatter $p_T^{cut}$.

We have also shown $d\sigma_{jet}/dy$ for different PDFs. There is little difference between CTEQ6 and GRV98 both at central and forward rapidity. Since mini-jets dominate particle production if the soft cross section is energy independent, we do not expect a large uncertainty in the number of produced muons (approximately proportional to the multiplicity of secondary hadrons) arising from the high-energy part of an EAS. For DO, which exhibits a much slower growth of the small $x$ gluon density one obtains a significantly wider rapidity plateau which would affect EAS. In the future, we plan to extend our model to $pA$ collisions and implement it into an EAS simulation package to study directly the effects of small-$x$ QCD interactions on air shower observables.

Acknowledgment(s)

It is a pleasure to thank the organizers of ”Quark matter 2005” for the opportunity to present this work and H.J. Drescher, A. Dumitru, Y. Nara and T. Kodama for their help with this project. Support by CAPES and CNPQ is acknowledged.

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