NEW CLASSIFICATION OF SOLAR FLARES BASED ON THE MAXIMUM FLUX IN SOFT X-RAYS AND ON DURATION OF FLARE

E.A. Bruevich
Lomonosov Moscow State University, Sternberg Astronomical Institute, Universitetsky pr., 13, Moscow 119992, Russia

e-mail: red-field@yandex.ru

Abstract. Solar flare activity is characterized by classification systems, both in optical and Soft X-ray ranges. The most generally accepted classifications of solar flares describe important parameters of flare such as the maximum of brightness of the flare in optical range in \( H_\alpha \) (flare class in \( H_\alpha \) is changed from F to B), area of the flare in \( H_\alpha \) which is changed from S (less than 2 square degrees) to 4 (more than 24.7 square degrees) and the maximum amplitude of the Soft X-ray flux in the band 0.1-0.8 nm (X-ray flare of classes from C to X). A new classification of solar flares which is proposed in this paper – the X-ray index of flare XI, based on measurements of radiation in the range 0.1-0.8 nm on the GOES satellites. The XI index has a clear physical interpretation associated with the total flare energy in the range 0.1-0.8 nm. XI is easily calculated for each flare with use of available GOES data. The XI index can be used to assess flares along with other important geoeffective parameters.

Key words. solar flares, solar flares classification, solar proton fluxes.

1 Introduction

Solar flares are a complex of physical phenomena in plasma combined into one interconnected process of energy accumulation and release. Flares manifest themselves in all ranges of the electromagnetic spectrum, which makes it possible to study the physical processes occurring in them.

The ultraviolet emission, X-rays, gamma-rays, as well as part of infrared and radio emission can be registered only from space. The flare activity of the Sun has become more accessible for analysis in recent years with the
development of space technology, and in particular through the regular X-ray observations on the GOES satellites and in the ultraviolet range on the SDO orbital observatory. Ultraviolet and X-ray flares dramatically increase the ionization in the upper atmosphere of the Earth, in the ionosphere. This can lead to violations of radio communication, malfunctions in the navigation devices of ships and aircrafts, radar systems, accidents on long power lines. Particles of high energies from the flares penetrate the upper atmosphere of the Earth with destroying the ozone layer. Shock waves and solar plasma emissions after large flares cause severe disturbances of the magnetosphere of the Earth – the magnetospheric storms.

When observing solar flares, all the parameters characterizing this phenomenon are important: the area of the flare, its average brightness, the shape of the light curves both in the optical range and in the ultraviolet and X-ray ranges. Both the maximum amplitude of the flare in different bands and lines of the emission spectrum and the total energy that came from the flare to the Earth are very important, [1].

Numerous studies in the field of solar-terrestrial physics classify solar flares as fundamental events affecting the Earth. The source of geoeffective disturbances are not only electromagnetic radiation from solar flares, but also increase of proton fluxes in the flare and huge emissions of substances from the Sun’s corona, the so-called coronal mass ejections (CME), which often accompany large flares. The largest flares are sometimes accompanied by the so-called ground level events – GLE (Ground Level Enhancement).

In the current cycle 24 there is a rather low flare activity in general, [2, 3]. A comparative analysis of several solar activity indices in cycles 22, 23 and 24 showed that the relative differences in the amplitudes of variations of activity indices from the minimum to the maximum of the cycle vary significantly during the transition from cycles 22 and 23 to cycle 24. But, the maximum amplitudes of variations of the flare index FI, of the relative number of sunspots SSN and of the UV flux in the hydrogen line 121.6 nm \( F_{\lambda \alpha} \) are significantly decreased (by 20 - 50%) already in cycle 23 compared to cycle 22.

Amplitudes of variations of the MgII index 280 nm (center/wings), of radio emission flux on the 10.7 cm \( F_{10.7} \), of the background flux in the Soft X-ray range and of the solar constant TSI decreased by 20-30%, and variations of SSN and of FI decreased by 30 – 50% in cycle 24 compared to cycle 23, [4].

Currently, two classification systems are used to determine the flare class:
(1) - the optical classification (flares class in $H\alpha$ changed from SF to 4B), supplemented by the flare index FI (considering the full duration of the flare in minutes) and (2) - X-ray classification based on the absolute maximum of the flare flux in the X-ray range 0.1-0.8 nm (changed from C1 to X27). In the X-ray classification, the flare duration and the shape of the X-ray luminosity curve are not considered.

In an active region the accumulation of energy in the form of magnetic energy of the current layer in the upper chromosphere and in the corona before the flare occurs. The current layer has a magneto-plasma structure, at least two-dimensional structure and usually two-scale structure. The first, who attract attention to the importance of the process of formation of the current layer in the corona near a special line of the magnetic field, was S. I. Syrovatsky, [5].

A huge energy is suddenly released at the beginning of a large flare at the top of the arch of the magnetic field according to the present ideas about the development of the flare process: in the soft X-ray region the flare radiation is orders of magnitude higher than the radiation of the entire Sun without of flares in this range. Around primary energy release, electrons (and sometimes protons) are accelerated to high energies and plasma is heated to the temperatures of 20-30 million K.

The problem of predicting the occurrence of large solar flares is actively studied in spectral-polarization observations of the Sun in the microwave range. On the RATAN-600 radio telescope is carried out the continuous monitoring of the flare-productive active regions on the Sun which can generate of powerful X-ray flare events of M and X classes.

These observations show the existence of a rather long preceding phase in the pre-flare emission of the active regions. This phase, which precedes to the flares of large power, is characterized by the rise of a new magnetic flux and by multiple inversions of the sign of circular polarization in the wavelength range from 2 to 5 cm, [6-8].

Such unique opportunities in flares forecasting relate to improvement of parameters of the multi-octave solar spectral-polarization complex of high resolution (SPCHR) on RATAN-600 and with successful implementation of the Program of Regular Observations and Data Processing using modern technologies, [9].

Thus, the beginning and further development of the flare in different spectral ranges (and different spectral lines of the short-wave part of the spectrum formed both in the chromosphere and in the corona) occurs in different ways,
so for a more complete description of the observed flare parameters, both optical and X-ray classifications are used.

An important point for determining the flare class in the optical classification is the identification of letters and numbers with the real parameters of the X-ray classification based on the magnitude of the fluxes at the flare maximum [10].

In [11], the value determining the flare index in the optical range \( Q = i \cdot t \) which is proportional to the total energy emitted by the flare was introduced for the first time. In this equation \( i \) represents the class of \( H_\alpha \)-flares in a special scale, \( t \) defines the duration of the \( H_\alpha \)-flare in minutes. The value of \( i \) changes from 0.5 for SF, SN and SB \( H_\alpha \)-flare, to 4.0 for 4B \( H_\alpha \)-flare. Now for \( Q \) received the FI (Flare Index) designation, [10,11].

The FI value is calculated as the averages of the day and adjusted for the total observation time during the day. The archived data of FI from 1976 to 2014 are available on the website of the National Geophysical Data Center – NOAA [12].

The system of evaluation of solar flares power by X-ray radiation (classes C, M and X) adopted all over the world currently relies on measurements of radiation flux in SXR (Soft X-ray Emission) on 0.1-0.8 nm region, see, for example, [13-15]. The most powerful flares in this classification – flares of X class corresponds to the absolute flux of more than \( 10^{-4} \) W/m\(^2\) in the band of 0.1-0.8 nm, X-ray flares of M1–M9 classes corresponds to the flux from \( 10^{-5} \) W/m\(^2\) to \( 10^{-4} \) W/m\(^2\), X-ray flares of C1–C9 classes corresponds to the flux from the \( 10^{-6} \) W/m\(^2\) to \( 10^{-5} \) W/m\(^2\).

2 Comparison of flares in the parameters of the optical and X-ray classifications

Figure 1 demonstrates, for 36 flares of cycle 24 (from the Table 1, see lower) the relationships between the two most common flare’s classification systems – the optical classification (expressed in the FI index value) and X-ray classification, based on the GOES data in the SXR range.

On Figure 1 the linear regression (continuous line - 1 which is curved due to the logarithmic scale along the X and Y axes) and quadratic regression (dotted line - 2) are marked. It can be seen that the amplitude of the flare at the maximum in SXR-range and the FI optical index, which is the energy
analogue of the flare in $H_\alpha$, do not show a noticeable relationship although on average the greater value of the maximum in SXR-range corresponds to the larger value of FI. It can be noted that for flares of X-ray class X1 – X9, the FI optical index is in the range of 80-700, while for M1 – M9 classes FI varies from 10 to 300.

As can be seen on Figure 2 the relationship between the FI optical flare index and the total energy $E_{0.1-0.8}$ in the Soft X-ray range, calculated in this work for the flares from Table 1 by the formula (1), closer than between the flare indices in Figure 1.

Both of flare indices which are shown in Figures 1, 2 are a measure of the energy emitted in the optical and X-ray bands. Note that the duration of the flare in the SXR-range is determined quite accurately, as the data of the GOES measurements are presented with an interval of time in 2.5 seconds.

In the optical range, the duration of the flare and its optical score can vary between observations at different observatories (up to tens of minutes.
Table 1: Parameters of 36 flares of 24-th cycle

| Date of flare | X-ray class/ $H_\alpha$-score/ $H_\alpha$-duration, min | X-ray index | Index | Energy $E_{0.1-0.8}$, emitted by flare, ($J/m^2$) |
|---------------|-------------------------------------------------|-------------|-------|-----------------------------------------------|
| 13.02.2011    | M6.6/1N/72                                       | XI9.56E-2   | 72    | 0.072                                         |
| 15.02.2011    | X2.3/2N/60                                       | XI3.24E-1   | 120   | 0.263                                         |
| 18.02.2011    | M6.6/1F/20                                       | XI2.77E-2   | 20    | 0.053                                         |
| 24.02.2011    | M3.5/1F/19                                       | XI4.62E-2   | 19    | 0.042                                         |
| 07.03.2011    | M3.7/1F/90                                       | XI2.59E-1   | 90    | 0.179                                         |
| 08.03.2011    | M5.3/1F/20                                       | XI1.30E-1   | 20    | 0.0806                                        |
| 08.03.2011    | M4.4/1F/23                                       | XI1.32E-1   | 23    | 0.081                                         |
| 09.03.2011    | X1.5/2B/63                                       | XI1.3E-1    | 157.5 | 0.107                                         |
| 14.03.2011    | M4.2/1N/45                                       | XI1.51E-2   | 45    | 0.028                                         |
| 03.08.2011    | M5.3/2B/60                                       | XI1.92E-1   | 150   | 0.148                                         |
| 04.08.2011    | M9.3/2B/84                                       | XI1.18E-1   | 210   | 0.112                                         |
| 09.08.2011    | X6.9/2B/72                                       | XI2.96E-1   | 190   | 0.2574                                        |
| 06.09.2011    | X2.1/2B/55                                       | XI1.67E-1   | 137.5 | 0.118                                         |
| 07.09.2011    | X1.8/3B/74                                       | XI1.30E-1   | 259   | 0.101                                         |
| 08.09.2011    | M6.7/1N/60                                       | XI6.65E-2   | 60    | 0.059                                         |
| 22.09.2011    | X1.5/2N/118                                      | XI9.27E-1   | 236   | 0.75                                          |
| 24.09.2011    | X1.9/2B/49                                       | XI1.87E-1   | 122.1 | 0.143                                         |
| 25.09.2011    | M7.4/2N/70                                       | XI1.69E-1   | 140   | 0.094                                         |
| 03.11.2011    | X1.9/2B/84                                       | XI1.96E-1   | 210   | 0.168                                         |
| 26.12.2011    | M2.3/SF/37                                       | XI4.00E-2   | 18.5  | 0.045                                         |
| 31.12.2011    | M2.4/1F/19                                       | XI1.73E-2   | 19    | 0.025                                         |
| 23.01.2012    | M8.7/2B/135                                      | XI4.43E-1   | 337   | 0.389                                         |
| 12.07.2012    | X1.4/2B/250                                      | XI9.1E-1    | 625   | 0.792                                         |
| 20.10.2012    | M9.1/SF/30                                       | XI6.35E-2   | 15    | 0.064                                         |
| 23.10.2012    | X1.9/1F/90                                       | XI1.04E-1   | 90    | 0.119                                         |
| 06.09.2017    | X9.3/2B/40                                       | XI1.12E0    | 100   | 0.343                                         |
| 10.09.2017    | X8.2/3B/120                                      | XI2.73E0    | 360   | 2.62                                          |
Figure 2: The connection between the optical flare index (FI) and the total flare energy of $E_{0.1-0.8}$ in the range of 0.1-0.8 nm for 36 flares of 24-cycle. A quadratic regression line is shown.

for long-duration flares) which creates an additional error in the definition of FI. Traditionally, the FI index was calculated daily by several observatories. Data on daily and monthly averages of FI from 1976 to 2014 are available on NASA’s website [12].

To determine the FI values related to individual flares, we used information from the Catalogs, published in [16, 17], where unfortunately, the moments of the end of flares were not always determined with sufficient accuracy.

As for the X-ray classification, the example for flares 9.03.11 and 12.07.12 (Figure 3 and Figure 4) illustrates its wrongness although this X-ray classification is used most frequently. These flares are about of the same X-ray class (equal to X1.6 and X1.4) but their total energy emitted in this range varies greatly: $E_{0.1-0.8} = 0.15 J/m^2$ for the flare 9.03.11 of X1.6 class and $E_{0.1-0.8} = 0.75 J/m^2$ for the flare 12.07.12 of X1.4 class, respectively.
Note that the total energy $E_{0.1-0.8}$ emitted by the flare is calculated in this paper using the time integration of the flux $F_{0.1-0.8}(t)$ given the values of the background radiation $F_{\text{background}}$ from the beginning to the end of the flare:

$$E_{0.1-0.8} = \int (F_{0.1-0.8}(t) - F_{\text{background}}) dt \quad (1)$$

Such a significant difference in the total energy of 9.03.11 and 12.07.12 flares is explained by the difference in the shape of the curves of brightness and in durations of flares. Along with the X-ray class, its optical class (from SF to 4B) is used to describe the flare. To determine the optical class of the flare, you need to have access to observations of both the flare’s area and its brightness in the $H_\alpha$ line.

The FI flare activity index complements the information about the area and brightness of the flare with information about its duration in the optical range. The difficulty in calculating FI is that the flare duration in the optical range may differ for observations in different observatories. In this sense, the GOES series of observations available in real time and with common absolute calibration (which allows comparison of flare events since 1978) have a huge advantage over the classification in optical range: no wonder that the X-ray classification based only on knowledge of the amplitude in the flare maximum is currently most popular.

The most powerful flares of 24-th cycle that occurred at the decline phase of cycle during the September 2017 confirm that the classification based only on the maximum amplitude value does not carry a complete information.

According to this classification, the flare of September 6, 2017 of class X9.3 is considered to be more powerful than the X8.2 flare of September 10, 2017. These flares were generated from the same active region. In fact, the flare X8.2 was more strong and more geoeffective than the flare X9.3, the curve of dependence of flare power versus time was kind of more gently sloping and so the total energy of X8.2 flare $E_{0.1-0.8}$ was much greater than for X9.3 flare ($2.52 J/m^2$ vs $0.35 J/m^2$ respectively).

Related to this is the fact which is important for the effect of flares on the magnetosphere and the ionosphere of the Earth: a stream of protons ($I_{pr}$) in the channel with $E > 10$ MeV caused by flare X8.2, has had more power than after flare X9.3. For the more hard-energy protons with $E > 100$ MeV the flare X 9.3 has shown almost no increase in the flux above the background level, but in flare X8.2 the strengthening of the proton flux reached a record
3 The XI – new X-ray solar flares index, determined from observations on the satellites of the Goes series

The most clear option for a physically justified classification of flares is the addition of the information about flare’s duration to the X-ray classification of flares in terms of the maximum SXR-flux (in the range of 0.1-0.8 nm).

Thus, by analogy with the value of the optical flare index FI (proportional to the total energy radiated in $H_\alpha$), X-ray flare activity index XI based on GOES data (considering the duration of the flare in the X-ray range and the shape of the flare light curve) is introduced in this paper. The new flare index XI is also an analogue of the total energy $E_{0.1-0.8}$, radiated by the flare in the SXR-band.

To determine XI, we use the value of quarter of the maximum flux (FWQM - full width quarter-maximum), see [14]. In Figure 3 and Figure 4 the value of $a$ corresponds to the time in minutes that has elapsed from the level of the flux in a quarter of the maximum to the maximum in the rise phase of a flare, the value of $b$ corresponds to the time in minutes from the maximum to the level of the flux in a quarter of the maximum in the decline phase of a flare. We determine the value of the flare index XI as the composition of the X-ray flux in 0.1-0.8 nm at the flare maximum $F_{0.1-0.8}^{max}$ with the flare duration at the FWQM level which is equal to $(a + b)$. Interval of time $(a + b)$ is expressed in seconds:

$$XI = F_{0.1-0.8}^{max} \cdot (a + b)$$  \hspace{1cm} (2)

The XI has the dimension $J/m^2$ and is approximately equal to the total energy $E_{0.1-0.8}$ which is calculated as the integral under the light curve after subtraction of the background by the formula (1).

The data of daily observations of the flux values $F_{0.1-0.8}(t)$ with a temporal interval of 2.5 seconds are available on the GOES website from 2001 to the present in a real time practically, see https://satdat.ngdc.noaa.gov/sem/goes/data/new_full/

To determine the $t_a$ and $t_b$ moments for the flare, according to the GOES data, we have to find the moment of maximum of the flare – $t_m$. Then we
Figure 3: Flare 09.03.2011. The radiation flux in the range 0.1-0.8 nm \( F_{0.1-0.8} \) according to the observations of GOES-15. The moments of the maximum of flare flux \( t_m \) and moments of a quarter of the maximum flux value (FWQM) - \( t_a \) and \( t_b \) are showed.

determine the level FWQM and find the moments \( t_a \) and \( t_b \) using a magnitude of the flux at a maximum \( F_{0.1-0.8}^{\text{max}} \). In Figure 3 \( t_a = 23^h20^m, t_m = 23^h23^m \) and \( t_b = 23^h34^m \). Accordingly, the time interval \( (a + b) = t_b - t_a = 14 \) minutes (840 seconds).

Thus, from the calculations using formula (2) for the flare of 09.03.2011, the X-ray flare index \( XI = 1.6E^{-4} \cdot 840 = 0.1344(J/m^2) \). That is, the flare of 09.03.2011 has an X-ray index XI1.344E-1. For comparison, the total energy \( E_{0.1-0.8} \) which is equal to the area under the flare light curve with allowance for the background level, calculated by the formula (1), \( E_{0.1-0.8} = 0.107J/m^2 \).

For 12.07.2012 flare X-ray index XI is equal to \( XI = 1.4E^{-4} \cdot 6480 = 0.907(J/m^2) \) (XI1.907E0) and the energy of this flare is equal to \( E_{0.1-0.8} = 0.792J/m^2 \). Thus, the value of XI can be successfully used as a preliminary estimate of the total energy \( E_{0.1-0.8} \), which in turn is the most important
Figure 4: Flare 12.07.2012. The radiation flux in the range 0.1-0.8 nm $F_{0.1-0.8}$ according to the observations of GOES-15. The moments of the maximum of flare flux $t_m$ and moments of a quarter of the maximum flux value (FWQM) - $t_a$ and $t_b$ are showed.

A geoeffective characteristic of the flare [1,18].

For a comfortable representation of the characteristics of flares, all information about the flare can be represented by analogy with the X-ray classification: the 09.03.2011 flare (Fig.3) of class X1.6 is characterized by the X-ray flare index XI1.3E-1, the 12.07.2012 flare of class X1.4 is characterized by X-ray flare index XI9.1E-1.

A representation of XI value as 1.3E-1 and 0.907E0 is widely accepted for use in many computer applications (EXCELL, ORIGH PRO, etc.).

If we mean a flare of classes M and C, then it turns out that the XI values will be two or three orders of magnitude lower than for flares of the class X, because of weaker flares for the of M and C classes the fluxes in the maximum of flares is 1–2 orders of magnitude lower than for flares of X classes, and because of weaker flares have usually shorter durations.

That is, for flares of classes M1 – M9, the X-ray index value may be of the order of XI1E-4 – XI1E-2 depending on the flare duration, for the flares
Figure 5: Dependence of the total flare energy 0.1-0.8 on the new X-ray flare index XI for 36 flares of cycle 24. Flares with subsequent proton events are represented by filled asterisks, flares not accompanied by proton events are represented by hollow asterisks. The quadratic regression line and the standard deviation are shown. The largest flares of cycle 24 of 06.09.2017 and 10.09.2017 are marked.

of classes C1 – C4 the XI value varies in the range of XII1E-5 – XII1E-3.

The Table 1 presents data on 36 major flares in 2011-2014. For comparison, data are given on the two largest flares of the 24-th cycle that occurred in September 2017.

The information on the X-ray class of flares and their duration at the FWQM level for a calculation of the X-ray flare index XI from the formula (2) is obtained from the archival data available on the GOES website. Information on the parameters of flares in the $H_\alpha$ line is available on [10, 11].

So, the energy $E_{0.1-0.8}$ of the flares, and the X-ray flare indices XI are calculated in this paper using equations (1) and (2) and using the data of the GOES website.

Figure 5 demonstrates that for a given set of 24-th cycle flares, the X-ray
index XI is closely related to the total flare energy $E_{0.1-0.8}$, where the value of XI which is calculated by formula (2) absolutely coincides with $E_{0.1-0.8}$ in dimension $(J/m^2)$ and practically coincides in magnitude.

The relationship between $E_{0.1-0.8}$ and XI is described by the equation:

$$E_{0.1-0.8} = 0.0198 + 0.658 \cdot XI + 0.106 \cdot XI^2 \quad (3)$$

Figure 5 also shows the two largest flares that occurred in September 2017. It can be seen that the September 6, 2017 X9.3 flare (the fifth largest since the introduction of the X-ray classification of flares) significantly loses to the September 10, 2017 flare of X8.2 class as by the value of the total energy $E_{0.1-0.8}$, and by the value of the X-ray flare index XI.

4 XI – X-RAY SOLAR FLARE INDEX AND SOLAR PROTON EVENTS (SPE)

Patrolling observations of the Sun in the soft X-ray range is the basis of the modern classification of Solar Proton Events (SPE). It should be understood that each flare is individual and not all flare events are manifested in the same way. X-ray observations on the spacecrafts YOHKOH and RHESSI showed the appearance of centres of radiation arising from the flare [6, 7].

In X-ray photons of limbic flares, three radiation sources are observed, two of which are located in the photosphere at the foot of the flare loops. They are associated with electron beams falling along the field lines, accelerated in longitudinal currents in accordance with the prediction of the electrodynamic model. The third source is located above the flare loop in the corona, where, according to the electrodynamic model, the radiation source that appears due to the heating of the plasma after reconnecting in the current layer should be located, [7].

In [2,19] it is shown that most of the favourably located (on the Western part of the solar disk) flares of > M5 class in the soft X-ray range 0.1-0.8 nm are accompanied by SPE, and almost all SPE can be identified with a particular solar flare. The duration and intensity of the injection of protons for the observer in the Ecliptic plane varies from one proton event to other proton event by several orders of magnitude.

The physical basis for the communication of SPE with soft x-ray radiation is the fact that the source of heating of the flare plasma can be accel-
Figure 6: Dependence of the X-ray index XI of the parent flare versus proton flux with energies $E > 10$ MeV in cycles 23 and 24. The Ground Level Events (GLE) are marked.

erated electrons, which are accelerated simultaneously with protons. At the same time, there is no theory that connects quantitatively different types of electromagnetic and corpuscular radiation of solar flares [20]. According to statistical analysis [20, 21] which was made for several largest proton events of the 23rd and 24th cycle, the time difference from the maximum of the flare of 0.1-0.8 nm to the maximum of SPE is time interval from 35 minutes to 2 hours depending on the path of proton propagation.

In this case, from the graphic data GOES (www.n3kl.org/sun/noaa_archive/) it can be seen that the phase of a sharp increase in the proton flux coincides with the phase of the flare flux decline. Most often (see graphic data GOES), the proton flux ($I_{pr}$) quickly reaches a maximum and for some time (depending on the flare power - up to a couple of days) is kept at a constant level and then slowly decreases.

An important property of the X-ray index XI, as an energy characteristic
Figure 7: Dependence of the X-ray index $\xi$ of the parent flare versus proton flux with energies $E > 100$ MeV in cycles 23 and 24. The Ground Level Events (GLE) are marked.

of the flare, is its obvious connection with proton flares (proton events). Usually, when discussing the relationship between thermal and non-thermal electromagnetic radiation of solar flares, we refer to the similarity of time profiles of the intensity of non-thermal radiation and the derivative on time of soft X-ray flux (Neupert effect). This similarity corresponds to a single-loop evaporation model, but it is not observed in more than 50% of long-term events and this is due to the long and multiple acceleration of electrons with a variable spectrum in the system of flare loops in different physical conditions [20].

In the study of the energy release of solar flares, it is necessary to have an idea of the fine structure of the flare region, since many energy release channels depend on the geometric parameters of the magnetic loops along which the energy transfer occurs. The fine structure can affect the density of the accelerated electrons in the beam and their propagation in the plasma.

In addition, the density of the electric current flowing along the magnetic lines also depends on the transverse dimensions of the current tubes. However, in describing the flare process is commonly used model in the flare
region excluding the thin spatial structure [22].

All these features of different flares are manifested in the ambiguity of the relationship between X-ray fluxes and proton fluxes in flares.

Traditionally, a statistical analysis of flares with subsequent SPE is carried out to identify patterns of interaction between X-ray flares and geoeffective proton events. Since 1970 SREs, in which protons with energy $E > 10$ MeV and fluxes $I_{pr} \geq 1$ pfu ($1\,\text{pfu} = 1\,\text{cm}^{-2}\cdot\text{s}^{-1}\cdot\text{sterad}^{-1}$) were observed, has been collected in Catalogs edited by Yu. I. Logachev, available at www.wdcb.ru/stp/online_data.ru.html. These Catalogs are characterized by homogeneous and long time series.

The most large flares which are accompanied by so-called ground-based increases (GLE) are among the most powerful SPE and proton fluxes with $E > 100$ MeV for such flares are enclosed in the range of $100 - 1000$ pfu.

The flares accompanied by GLE are of great interest both for the determination of the mechanisms of acceleration and propagation of charged particles in the Sun and in the interplanetary medium, and for the determination of radiation danger in the near-Earth space. According to the Catalogs [16,17] in 24th cycle only three events with GLE were recorded (17.05.2012, 06.01.2014 and 10.09.2017), while in 23rd cycle 15 events with GLE were recorded.

From the Catalogs [16,17] it also follows that in 2018, when we are already at the minimum between 24th and 25th cycles, the number of events with energy of protons $E > 10$ MeV throughout 24th cycle happened approximately 30% less than in cycle 23 (60 versus 89, respectively). Comparison of flare parameters from Table 1 is shown in Figure 5. Blackened asterisks indicate flares which accompanied by SPE with different levels of flux but exceeding the value of 10 pfu for protons with $E > 10$ MeV. Figure 5 demonstrates that flare with subsequent proton event is most likely characterized by X-ray index greater than $10\,\text{MeV}$.

Table 2 shows all 24-cycle flares accompanied by SPE and characterized by $I_{pr} > 2$ pfu (13 weakest flares with $I_{pr} = 1$ divided by 74 flares with SPE are not included), as well as the most significant 23rd cycle flares (21 flares with $I_{pr}$ having from 100 to 10000 pfu for SPE with $E > 10$ MeV). Some of these flares are accompanied by SPE with $E > 100$ MeV.

On Figure 6 and on Figure 7 the dependences of the X-ray index of the X-ray index $X_{I}$ vs values of proton fluxes with $E > 10$ MeV and $E > 100$ MeV for 84 parent flares from Table 2 are presented.
The linear regression is marked by line curved because of the logarithmic scale for the Y-axis. The standard deviation ($\sigma$) of the regression coefficients is relatively large (15 – 25% of the values of the coefficients) to assert of existence of close connections between X-ray index XI and $I_{pr}$.

Data on proton fluxes in the ranges with $E > 10$ MeV and $E > 100$ MeV are taken from the Catalogs [16, 17] and refined directly from the GOES archive data, available at (www.n3kl.org/sun/noaa_archive/).

Figure 6 and Figure 7 show that the spread in values of proton fluxes is sufficiently large, which confirms the complexity and variety of flares and associated SPEs. Note that the calculation of the X-ray index XI for a particular flare is very simple: we have to determine from the GOES archive data the level of FWQM which is equal to one quarter of the flux at the maximum, then to determine the moments $t_a$ and $t_b$ and to use formulae (2). Observations of GOES are presented with an interval of 2.5 seconds, which entails an error in determining of the X-ray index XI only in the third-fourth significant digit (less than 0.1%). A calculation the value of energy $E_{0.1−0.8}$ (as the flare characteristics which is compared to XI) requires much more efforts: to integrate the flare light curve, you need to convert the archived GOES data, linking them to the real time-scales and correctly select the background level. Finally, the error in the definition of $E_{0.1−0.8}$ can already reach several percent. In particular, due to the fact that the background level before and after the flash may vary, and this introduces uncertainty in determining the flash end time.

5 Relationship between rising and declining phases of solar flares

To calculate the index XI, it is important to determine the times $t_a$ and $t_b$ (intervals a and b) most precisely. For GOES data, this is not difficult, except when flares go on overlapping one another before they reach the level of the background flow during the phase of decline. In this case, the following comparative analysis of the data on the relationship between the phase of rise and phase of decline of flares can help.

In this paper, we investigate the relationship between the parameters a and b for a sample of 36 significantly more powerful flares of classes M3 - X9 from the Table 1.
Table 2: Parameters of 84 flares of 23-rd and 24-th cycles and subsequent Solar Proton Events affecting the earth environment

| Date of flare | X-ray class/ $H_{\alpha}$-score | X-ray index | Proton flux $I_{pr}$ E>10MeV, pfu | Proton flux $I_{pr}$ E>100MeV, pfu |
|---------------|---------------------------------|-------------|-----------------------------------|-----------------------------------|
| 24.08.1998    | X1.0/3B/180                     | 0.55        | 220                               | 4                                 |
| 30.09.1998    | M2.8/2N/150                     | 0.43        | 800                               | 3.5                               |
| 14.07.2000    | X5.7/3B/105/GLE                 | 4.11        | 8000                              | 400                               |
| 08.11.2000    | M7.4/3F/80/GLE                  | 0.66        | 10000                             | 300                               |
| 24.11.2000    | X2.3/2B/22                      | 0.255       | 94                                | 1.2                               |
| 02.04.2001    | X18.4/1B/70                     | 2.85        | 1000                              | 5.5                               |
| 09.04.2001    | M7.9/1B/70                      | 0.251       | 5                                 | 0.45                              |
| 10.04.2001    | X2.3/3N/176                     | 0.538       | 100                               | 0.4                               |
| 15.04.2001    | X10.4/3B/130/GLE                | 1.87        | 951                               | 100.3                             |
| 24.09.2001    | X2.6/2B/155                     | 1.178       | 1050                              | 10                                |
| 01.10.2001    | M4.2/2B/160                     | 0.76        | 600                               | 0.2                               |
| 04.11.2001    | X1.0/3B/450/GLE                 | 0.85        | 3000                              | 50                                |
| 22.11.2001    | X1.0/2B/160/GLE                 | 1.1         | 3500                              | 4                                 |
| 26.12.2001    | M7.1/1B/230/GLE                 | 1.28        | 779                               | 50                                |
| 21.04.2002    | X1.5/1F/125                     | 2.16        | 2000                              | 20                                |
| 26.10.2003    | X1.2/1N/170                     | 1.65        | 466                               | 1.0                               |
| 28.10.2003    | X10.7/4B/280/GLE                | 5.8         | 7500                              | 150                               |
| 29.10.2003    | X10.0/2B/150/GLE                | 3.2         | 2500                              | 100                               |
| 02.11.2003    | X8.3/2B/170                     | 1.86        | 1060                              | 40                                |
| 03.11.2003    | X3.9/2F/50                      | 0.875       | 1000                              | 3                                 |
| 04.11.2003    | X17/3B/80                       | 1.98        | 600                               | 1.1                               |
| 28.01.2011    | M1.3/1F/30                      | 0.01        | 3                                 | -                                 |
| 15.02.2011    | X2.2/2F/25                      | 0.02        | 3                                 | -                                 |
| 07.03.2011    | M3.7/1F/90                      | 0.259       | 50                                | 0.15                              |
| 07.06.2011    | M2.5/2N/100                     | 0.1         | 50                                | 4                                 |
| 14.06.2011    | M1.3/SF/40                      | 0.30        | 8                                 | -                                 |
| 04.08.2011    | M9.3/2B/84                      | 0.118       | 70                                | 1.5                               |
| 08.08.2011    | M3.5/1B/55                      | 0.054       | 4                                 | -                                 |
| 09.08.2011    | X6.9/2B/72                      | 0.296       | 25                                | 2.5                               |
| 06.09.2011    | X2.1/2B/55                      | 0.167       | 15                                | 0.1                               |
| 07.09.2011    | X1.8/3B/74                      | 0.13        | 8                                 | 0.3                               |
Table 3: Continuation of Table 2

| Date of flare    | X-ray class/ $H_{\alpha}$-score/ $H_{\alpha}$-duration, min | X-ray index XI | Proton flux $I_{pr}$ E>10MeV, pfu | Proton flux $I_{pr}$ E>100MeV, pfu |
|------------------|---------------------------------------------------------------|----------------|-----------------------------------|------------------------------------|
| 22.10.2011       | M1.3/1N/130                                                  | 0.26           | 5                                 | -                                  |
| 03.11.2011       | X1.9/2B/82                                                   | 0.4            | 4                                 | -                                  |
| 25.12.2011       | M4.0/1N/60                                                   | 0.054          | 3.0                               | -                                  |
| 23.01.2012       | M8.7/2B/335                                                  | 1.443          | 2500                              | 2.2                                |
| 27.01.2012       | X1.7/2F/96                                                   | 0.428          | 700                               | 11                                 |
| 05.03.2012       | X1.0/2B/105                                                  | 0.1            | 3                                 | -                                  |
| 07.03.2012       | X5.4/3B/220                                                  | 1.40           | 1600                              | 60                                 |
| 09.03.2012       | M6.3/SF/160                                                  | 0.329          | 500                               | 8                                  |
| 13.03.2012       | M7.9/1B/185                                                  | 0.469          | 150                               | 2                                  |
| 17.05.2012       | M5.1/1F/93/GLE                                               | 0.126          | 280                               | 20                                 |
| 14.06.2012       | M2.1/2B/150                                                  | 0.12           | 14                                | -                                  |
| 06.07.2012       | M6.2/1B/40                                                   | 0.1            | 25                                | -                                  |
| 08.07.2012       | M6.9/1N/23                                                   | 0.2            | 19                                | -                                  |
| 12.07.2012       | X1.4/2B/250                                                  | 0.873          | 80                                | 0.25                               |
| 17.07.2012       | M1.7/1F/250                                                  | 0.26           | 130                               | -                                  |
| 19.07.2012       | M7.7/SF/155                                                  | 0.667          | 70                                | 0.8                                |
| 08.11.2012       | M1.7/1F/40                                                   | 0.05           | 3                                 | -                                  |
| 14.11.2012       | M1.1/1F/10                                                   | 0.08           | 9                                 | -                                  |
| 15.03.2013       | M1.1/1N/180                                                  | 0.18           | 16                                | -                                  |
| 11.04.2013       | M6.5/3B/125                                                  | 0.133          | 100                               | 2                                  |
| 13.05.2013       | X1.7/1N/39                                                   | 0.21           | 41                                | -                                  |
| 15.05.2013       | X1.2/2N/66                                                   | 0.248          | 20                                | 0.1                                |
| 22.05.2013       | M5.0/3N/180                                                  | 0.273          | 600                               | 3                                  |
| 21.06.2013       | M2.9/1F/87                                                   | 0.11           | 6                                 | -                                  |
| 23.06.2013       | M2.9/1N/12                                                   | 0.09           | 14                                | -                                  |
| 28.10.2013       | X1.0/2N/50                                                   | 0.07           | 5                                 | -                                  |
| 29.10.2013       | X2.3/1N/20                                                   | 0.055          | 5                                 | -                                  |
| 01.11.2013       | M6.3/1B/70                                                   | 0.06           | 3                                 | -                                  |
| 06.11.2013       | M1.8/1F/40                                                   | 0.08           | 7                                 | -                                  |
| 19.11.2013       | X1.0/SF/80                                                   | 0.035          | 4                                 | -                                  |
Table 4: Continuation of Table 2

| Date of flare | X-ray class/ Hα-score/ Hα-duration, min | X-ray index X-ray index | Proton flux $I_{pr}$ | Proton flux $I_{pr}$ |
|---------------|----------------------------------------|-------------------------|----------------------|----------------------|
| 07.01.2014    | X1.2/2N/76/GLE                        | 0.481                   | 900                  | 4                    |
| 20.02.2014    | M3.0/SN/60                            | 0.16                    | 22                   | 0.8                  |
| 25.02.2014    | X4.9/2B/90                            | 0.777                   | 20                   | 0.8                  |
| 29.03.2014    | X1.0/2B/40                            | 0.0792                  | 3                    | 0.5                  |
| 18.04.2014    | M7.3/1N/50                            | 0.43                    | 58                   | 0.7                  |
| 10.09.2014    | X1.6/2B/240                           | 0.899                   | 30                   | 0.9                  |
| 13.12.2014    | M1.5/1F/15                            | 0.05                    | 3                    | -                    |
| 20.12.2014    | X1.8/3B/90                            | 0.038                   | 3                    | -                    |
| 15.03.2015    | M1.2/1F/40                            | 0.09                    | 8                    | -                    |
| 18.06.2015    | M1.2/1N/80                            | 0.11                    | 16                   | -                    |
| 21.06.2015    | M2.0/1N/120                           | 0.32                    | 100                  | 0.2                  |
| 21.06.2015    | M6.5/2B/170                           | 0.6                     | 500                  | 0.1                  |
| 25.06.2015    | M7.9/3B/63                            | 0.35                    | 22                   | -                    |
| 20.09.2015    | M2.1/2N/120                           | 0.05                    | 3                    | -                    |
| 09.11.2015    | M3.9/2N/65                            | 0.095                   | 4                    | -                    |
| 28.12.2015    | M1.8/1F/120                           | 0.02                    | 3                    | -                    |
| 01.01.2016    | M2.3/1N/105                           | 0.2                     | 21                   | -                    |
| 14.07.2017    | M2.4/1N/180                           | 0.23                    | 22                   | -                    |
| 04.09.2017    | M7.0/2N/90                            | 0.54                    | 800                  | 0.8                  |
| 06.09.2017    | X9.3/2B/40                            | 1.12                    | 40                   | 0.7                  |
| 07.09.2017    | X1.4/2B/120                           | 0.366                   | 400                  | 0.8                  |
| 10.09.2017    | X8.2/3B/120/GLE                       | 2.73                    | 1490                 | 60                   |
Figure 8: The dependency of $a$ from $b$ for 36 flares of cycle 24 studied in this work. The quadratic regression line and the standard deviation are shown.

The relationship between the parameters $a$ and $b$, expressed in seconds, is shown on Figure 6. The relationship between the values of $a$ and $b$ is described by the quadratic regression equation:

$$b = 356 + 1.32 \cdot a + 0.0018 \cdot a^2 \quad (4)$$

Thus, taking into account the dependence (4), it is possible to estimate the time $t_b$ in complex cases, when flares follow one after another and one flares is applied to subsequent flare. For some flares, there is a discrepancy in the determination of the time of begin and end of the flare in $H_\alpha$ in different observatories [15]. Sometimes observers indicated simply that the flare lasts more than a certain time. This introduces uncertainty in the calculation of the FI flare index.

To correctly determine the duration of $H_\alpha$ flares, as in [15], for 36 flares from Table 1, the connection between the duration from the beginning of the flare to its maximum ($t_{\text{rise}}$) and the duration from the flare maximum to its end ($t_{\text{decay}}$) is studied. $t_{\text{rise}}$ and $t_{\text{decay}}$ correspond to a full interval of time in the rise phase and in the decline phase of the flare, taking into account the excess of radiation from flares above the level of background in $H_\alpha$. 

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Figure 9: The dependency of $t_{\text{rise}}$ from $t_{\text{decay}}$ for 36 flares of cycle 24 studied in this work. The quadratic regression line and the standard deviation are shown.

The relationship between $t_{\text{rise}}$ and $t_{\text{decay}}$ is described by a quadratic regression equation with a relatively small second-order term:

$$t_{\text{decay}} = 17.68 + 2.006 \cdot t_{\text{rise}} + 0.0145 \cdot t_{\text{rise}}^2 \quad (5)$$

Using dependencies (4) and (5), you can determine the time intervals $b$ and $t_{\text{decay}}$ which are necessary to calculate the X-ray index XI and the optical index FI in cases where it is impossible to determine the parameters $b$ and $t_{\text{decay}}$ directly from observations.

### 6 Conclusions

The method proposed in this paper for determining the X-ray flare index XI by analogy with the optical flare index FI has the following advantages:

1. The X-ray index XI is easily calculated by formula (2). Data on the values of $a$, $b$, $F_{\text{max}}$, $F_{\text{background}}$ are available on the GOES web-site from
2001 to the present. Accordingly, for each flare, the X-ray index XI can be calculated, starting from 2001.

2. X-ray index XI is an analog of the total energy of flare $E_{0.1-0.8}$, calculated by formula (1). The relationship between XI and $E_{0.1-0.8}$ is described by equation (3), resulting if you know the index XI you can rapidly evaluate of the most important geoeffective flare parameter $E_{0.1-0.8}$.

3. By the value of the index XI, as well as by the value $E_{0.1-0.8}$, it is possible to determine flares with subsequent proton events (under the condition of localization of the flare region in the western part of the Sun’s disk suitable for propagation of protons towards the Earth).

4. X-ray index XI, as well as $E_{0.1-0.8}$, is the most important geoeffective parameter of the flare, associated with variations of solar cosmic rays and subsequent perturbation of the Earth’s magnetosphere and ionosphere.

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