Field Gradient Measurement using a Stern-Gerlach Atomic Interferometer with Butterfly Geometry

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Atomic interferometers have been studied as a promising device for precise sensing of external fields. Among various configurations, a particular configuration with butterfly-shaped geometry has been designed to sensitively probe the field gradient. We introduce a Stern-Gerlach (SG) butterfly interferometer by incorporating magnetic field in the conventional butterfly-shaped configuration. The atomic trajectories can be flexibly adjusted by varying the magnetic field to increase the sensitivity of the interferometer, while the conventional butterfly interferometer using Raman transitions can be understood as a special case. We also show that the SG interferometer can keep high contrast against a misalignment in position and momentum caused by the field gradient.

I. INTRODUCTION

Atomic interferometry is a state-of-the-art instrument that manipulates a coherent superposition of atomic states for precise field sensing tasks [1]. Among various types of atomic interferometers, an atomic fountain configuration using stimulated Raman transitions [2,3] has attracted much attention due to the high sensitivity and has been employed for a precise measurement of the acceleration of gravity [4–8], the Newtonian gravitational constant $G$ [9–12], gravity gradient [13–15], rotations [16–18], gravitational redshift [19,20], and fine structure constant [21]. The high sensitivity of the atomic interferometer has opened a promising direction to test fundamental nature of gravity beyond the Newtonian limit, such as Einstein’s equivalence principle [22–27], the inverse law of gravitation [28,29], gravitational wave detection [30,31], and dark energy [32,33]. Moreover, a promising scheme to test a non-classical nature of gravity has recently been proposed [34,35].

The basic procedure of an atomic fountain is to prepare a superposition of the internal energy states of atoms and to split their paths depending on the internal states so that the atom in each trajectory acquires a different phase [2,3]. The phase difference induced by the external field can be detected by measuring the population of phase $\pi/2$ trajectories. The phase difference with respect to the zeroth-order in the field gradient strength. Applying the Raman pulses inducing the Raman transitions. The SG interferometer enables the atomic ensemble to have the phase difference with $T^3$ scaling where $T$ is the total interferometer time. This provides a higher sensitivity to the acceleration induced by the external field than the conventional atomic fountain with $T^2$ scaling. Along with this proposal, it has been shown that external fields instead of laser-pulses can be exploited to enhance the precision of matter-wave interferometers [36,37].

Atomic interferometer can also be utilized for probing the external field having non-vanishing gradient, i.e. when the acceleration varies by position. The precise measurement of the field gradient is particularly important for testing fundamental physics such as the variation of the gravitational constant $G$ and the violation of $1/r^2$ law [28,29,31], and it has various technical applications including detecting subsurface mass anomalies [32,33], inertial navigation system [34], underground structure detection [35], and mineral exploration [36,37].

The effect of the field gradient appears in the conventional Mach-Zehnder type atomic interferometers as it causes a systematic misalignment of position and momentum, which eventually diminishes visibility [12,38]. The obstacle can be overcome if a precise estimate of the gravity gradient is given [38,59]. An interesting way to employ an atomic interferometer for measuring the field gradient is to construct a butterfly configuration [14,17,50–57]. The butterfly interferometer enables us to analyze the second-order effect precisely as the areas in space-time diagram between two wave packets cancel out, which contribute the first order effect [58].

In this paper, we introduce a SG interferometer in a butterfly configuration to estimate the field gradient. By applying a magnetic field to split the atoms’ trajectories depending on the magnetic moments of the internal atomic states, the interferometer can be closed up to the zeroth-order in the field gradient strength. Applying the magnetic field provides a more flexible control of the interferometer paths than laser-pulses, resulting in a higher sensitivity in the phases difference. We compare the performance of the proposed interferometer with that of the conventional one. From this, we show that the phase difference can be efficiently increased by inducing a larger momentum splitting in the interferometer and using larger...
magnetic moments of the internal atomic states. We also verify that the interferometer maintains a high visibility even though the misalignment of the interferometer can occur in the higher orders of the field gradient strength.

II. CONVENTIONAL BUTTERFLY INTERFEROMETER

Let us consider a quantum system exposed to an external field having a quadratic potential \( V(z) = -F_0 z - \frac{1}{2}Kz^2 \) in 1-D with constants \( F_0 \) and \( K \). The Hamiltonian of the system is then described as

\[
\hat{H} = \frac{\hat{p}^2}{2m} - F_0 \hat{z} - \frac{1}{2}K \hat{z}^2.
\]  

(1)

Here, \( \hat{z} \) and \( \hat{p} \) are quantized position and momentum operators in the z-axis, satisfying the canonical commutation relation \( [\hat{z}, \hat{p}] = i\hbar \). When \( K < 0 \), the last term is a harmonic potential that provides a restoration force to a stable point determined by \( F_0 \) and \( K \). When \( K > 0 \), a force from the last term pushes the atoms outward. For example, the Hamiltonian can describe the effect of gravity by setting \( F_0 = -mg \) and \( K = m\Gamma \) with the gravitational acceleration \( g \) and gravity gradient \( \Gamma \).

We consider a problem of measuring the field gradient, given by the parameter \( K \). The conventional butterfly interferometer [67] measures the constant \( K \) as follows: First, a \( \pi/2 \) pulse is applied to the atomic cloud prepared in the ground state \( |1\rangle \) of the internal state to generate an equal superposition between the ground state \( |1\rangle \) and the excited state \( |2\rangle \). By applying the pulse, the atom in the internal state \( |2\rangle \) gains a momentum \( \delta p = \hbar k \) proportional to the wave number \( k \) of the laser pulse. Second, after waiting \( T_{\text{dis}} \) for displacement of position, a \( \pi \) pulse is applied to swap the momentum states of two different trajectories. Third, the wave packets freely evolve for a free evolution time \( T_f \). Finally, we collect the wave packets by redoing the first step in the reverse way and measure the internal state on the basis \( \{|\pm\rangle = (|1\rangle \pm |2\rangle)/\sqrt{2} \} \).

The procedure is illustrated in Fig. 1. Since the trajectory of each wave packet is different, the phase accumulation is also different for each trajectory. One can find that the phase difference before the measurement is written as [68]

\[
\delta \Phi_c = -\frac{KT^2k}{32m} \left( \frac{p_0}{m} T + \frac{\hbar k}{2m} T + \frac{F_0}{2m} T^2 \right),
\]  

(2)

where \( T = 2T_{\text{dis}} + T_f \) is the total interferometer time, and \( p_0 \) is the initial momentum of the wave packet. Since the probability of obtaining outcomes \( \{|\pm\rangle \} \) is \( P_{\pm} = (1 \pm C \cos \delta \Phi_c)/2 \), the parameter \( K \) can be inferred by the population of each state. Here, \( C \) represents the visibility that quantifies the interference of the atomic interferometer. We discuss the effect of the visibility in the following section.

One can easily verify that the interferometer is closed in the zeroth order of \( K \) when the free evolution time satisfies

\[
T_f = \frac{2m\delta z}{\delta p},
\]

(3)

where \( \delta z \) is the difference in position right before the free evolution. In the case of the conventional butterfly configuration, the preparation time \( T_{\text{dis}} \) for displacement has to obey \( T_{\text{dis}} = T_f/2 \) to meet the condition Eq. (3) by noting that \( \delta z = \delta p T_{\text{dis}}/m \). Thus, the total time duration of the interferometer is given by \( T = 2T_f \). When we consider the gravitational acceleration and gravity gradient, we recover the phase difference from the gravity gradient [15, 67].

More specifically, Eq. (2) presents that the phase difference is proportional to (i) the velocity difference \( \delta \dot{z}/m \) of the two wave packets and (ii) the difference between the initial and final positions of the wave packet. Thus, the sensitivity of measuring \( K \) can be improved by increasing the difference of the velocity of the wave packets or that of the initial and final position of the wave packet. As recently pointed out [50], the laser-pulse based atomic interferometer has a limitation of increasing the momentum difference of the wave packets while employing external fields to give a momentum difference may resolve this drawback. As a remark, since the difference between the initial and final positions is required to be large, it is beneficial to have the direction of an initial velocity to be the same as that of the linear force. For example, if the final position is the same as the initial one, which may arise in a fountain configuration, the phase difference the atoms have will vanish.
moments factor, and the magnetic quantum number along the y-axis, respectively. Including the magnetic field contribution, the force exerted on the atoms becomes time-dependent,

$$\hat{H} = \frac{\hbar^2}{2m} - F(t)\hat{z} - \frac{1}{2}K\hat{z}^2,$$

where $F(t)$ consists of a given linear force $F_0$ and the time-dependent magnetic force that we apply;

$$F(t) = F_0 + \mu_1 \frac{\partial B_y(t)}{\partial z} \langle 1 | \hat{z} | 1 \rangle + \mu_2 \frac{\partial B_y(t)}{\partial z} \langle 2 | \hat{z} | 2 \rangle,$$

where $|1\rangle$ and $|2\rangle$ are the internal quantum states having magnetic moments $\mu_1$ and $\mu_2$, respectively. Because of the difference between the magnetic moments, the wave packets experience different forces from the magnetic field.

A detailed protocol for measuring the field gradient $K$ is given as follows: We prepare a quantum state in superposition between two different internal states by applying a $\pi/2$ pulse

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}} (|1\rangle|\psi_1(0)\rangle + |2\rangle|\psi_2(0)\rangle),$$

where the first and second kets represent the internal state and the spatial wave function of the atom, respectively. The spatial wave function of the wave packet obtains a momentum $\hbar K$ from the pulse.

We then apply magnetic field $\vec{B}(t)$ for $T_{\text{dis}}$. As the simplest configuration, we assume a sequence of the magnetic field,

$$\vec{B}(t) = \begin{cases} \vec{B}_0 & 0 \leq t < t_1 \\ 0 & t_1 \leq t < t_1 + t_2 \\ -\vec{B}_0 & t_1 + t_2 \leq t \leq T_{\text{dis}} \end{cases}$$

where we denote $t_1, t_2, t_3$ as the time duration for each step composing the total time duration for displacement $T_{\text{dis}} = t_1 + t_2 + t_3$, as illustrated in Fig. 2. Thus, $\vec{B}(t)$ is constant in each step. Due to the magnetic field, the dynamics of the wave packets depend on their magnetic moments such that

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left( |1\rangle \hat{\mathcal{U}}_1(t,0)|\psi_1(0)\rangle + |2\rangle \hat{\mathcal{U}}_2(t,0)|\psi_2(0)\rangle \right),$$

where the unitary operators applied to the motion are written as [68]

$$\hat{\mathcal{U}}_i(t,0) = e^{i\Phi_i(t)} \hat{D}[z_i(t), p_i(t)] \hat{V}(t,0).$$

Here, $\Phi_i(t)$ is the phase accumulation on each trajectory, and the dynamics of the spatial wave function can be divided into the time-evolution operator $\hat{V}(t) = \exp \left[ -\frac{i}{\hbar} \left( \frac{\hbar^2}{2m} - \frac{1}{2}K\hat{z}^2 \right) \right]$ without linear force and the displacement operator $\hat{D}[z_i(t), p_i(t)] = e^{i[p_i(t)\hat{z} - z_i(t)\hat{p}]/\hbar}$ due to the force exerted on the atoms. The displacement parameters $z_i$ and $p_i$ follow the dynamics of classical trajectories of the atoms. Explicitly, for constant magnetic field in time, the dynamics is described by

FIG. 2: SG butterfly interferometer in a free falling frame assuming $\hbar c \rightarrow 0$. (a) The trajectories describe the mean positions of the atomic states $|1\rangle$ and $|2\rangle$ having magnetic moments $\mu_1$ and $\mu_2$. (b) The momentum difference between the two wave packets, $\Delta p(t) = p_2(t) - p_1(t)$, the magnetic gradient $\partial B_y(t)/\partial z$ that is applied to the atoms, and $\vec{B}_0 = (0, B_{0y}, 0)$. Note that the two wave packets do not need to have the same direction of momentum.

III. SG BUTTERFLY INTERFEROMETER

A. Implementation of SG butterfly interferometer

We propose a SG butterfly interferometer that exploits an external magnetic field, which is a variant of the conventional butterfly interferometer. The configuration of the interferometer is illustrated in Fig. 2. Such a configuration can be experimentally implemented by applying a magnetic field $\vec{B}(t) = (0, B_y(t), 0)$ to give momentum kicks to wave packets differently based on their magnetic moments $\mu_1$ and $\mu_2$ (Fig. 2) with $\mu_i = \mu_B g_F m_F$. Here, $\mu_B$, $g_F$, and $m_F$ denote the Bohr magneton, the Landé factor, and the magnetic quantum number along the y axis, respectively. Including the magnetic field contribution, the force exerted on the atoms becomes time-dependent.

$$\hat{H} = \frac{\hbar^2}{2m} - F(t)\hat{z} - \frac{1}{2}K\hat{z}^2,$$

where $F(t)$ consists of a given linear force $F_0$ and the time-dependent magnetic force that we apply;
where \( t_0 \) is the initial time, and we define \( \omega \equiv \sqrt{K/m} \), and \( F_i(t) = F_0 + \mu_i \partial B_y(t)/\partial z \) is the force that each wave packet experiences. The phase accumulation of each wave packet is written as

\[
\Phi_i(t) = \frac{1}{2\hbar} \int_0^t d\tau z_i(\tau) F_i(\tau) + \phi_i,
\]

where \( \phi_i \) denotes the phase accumulation from \( \pi/2 \) pulses.

After this magnetic field sequence, assuming \( \omega T \ll 1 \), the displacement of the position and momentum before the free evolution, i.e., at \( t = T_{\text{dis}} \), can be easily found by using Eqs. (10) and (11), and one can determine the free evolution time \( T_f \) by the condition given by Eq. (8). After the free evolution time \( T_f \), we apply the magnetic field in the reverse sequence to Eq. (7), as illustrated in Fig. 2. Finally, the wave packets are collected for measurements that detect the population in the basis \( \{\pm\} \). The phase difference between two wave packets is written as

\[
\delta \Psi = \Phi_2(T) - \Phi_1(T)
+ \phi_2 - \phi_1 + \frac{z_1(T)p_2(T) - z_2(T)p_1(T)}{2\hbar}.
\]

The term \( \Phi_2(T) - \Phi_1(T) + \phi_2 - \phi_1 \) is the desired phase difference acquired by the different trajectories. On the other hand, the last term in Eq. (13) arises from the misalignment of the wave packets caused by non-zero \( K \). As the misalignment not only leads to an additional phase but also causes a degradation of visibility of the interferometer, it is demanded to minimize it in practice. We first focus on the phase difference \( \delta \Psi \) and then discuss the effect of diminished visibility in Sec. III. C. Note that in general, the phase difference depends on the initial position and momentum of the wave packets.

B. Magnetic field gradient pulse regime

Let us focus on an important limit that a magnetic field gradient pulse is applied, i.e., we take a limit \( t_1, t_3 \ll T_f \) with fixing the momentum transfer of magnetic fields as

\[
(\mu_2 - \mu_1) \frac{\partial B_y}{\partial z} t_1 = \delta p - \hbar k
\]

and

\[
(\mu_2 - \mu_1) \frac{\partial B_y}{\partial z} t_3 = -2\delta p.
\]

Here, \( t_2 \) is determined by the condition (3) as \( t_2 = T_f/2 \). Under these conditions, the phase difference is given by

\[
\delta \Phi = \left( \frac{\delta p}{\hbar k} \right) \delta \Phi_c + \left( \frac{\mu_1 + \mu_2}{\mu_2 - \mu_1} \right) \left( \frac{K T^3}{32 m^2 \hbar} \right) \frac{\delta p}{3} + \frac{\hbar k}{2}.
\]

where we omitted the \( K \)-independent phase difference in \( \delta \Phi_c \). In this pulse regime, the role of the magnetic field gradient pulse is similar to the \( \pi/2 \) and \( \pi \) pulses in the conventional butterfly configuration. Especially by assuming \( \delta p = \hbar k \) and \( \mu_1 = -\mu_2 \), the phase difference \( \delta \Phi \) is exactly the same as that of the conventional butterfly interferometer, \( \delta \Phi_c \). This case, no magnetic field is applied during \( 0 \leq t \leq t_1 \) as we have \( t_1 = 0 \) from Eq. (14), and the magnetic field \( t_2 \leq t \leq t_3 \) has the same role as the \( \pi \) pulse in the conventional butterfly setup.

We highlight that the magnetic field gradient pulse can provide an additional control in the momentum transfer \( \delta p \) to increase the sensitivity of the interferometer compared to using the conventional laser pulses [50]. One can observe that the first term of Eq. (16) has the same form as in the conventional butterfly interferometer but scales linearly by increasing the momentum difference \( \delta p \). Thus, when the magnetic field induces a larger momentum transfer \( \delta p \) than that from the Raman transition \( \hbar k \), the accumulated phase difference can be increased with the factor of \( \delta p/(\hbar k) \).

Meanwhile, the second term in Eq. (16), which is magnetic moment dependent, is a distinct feature of the SG butterfly interferometer. In contrast to the first term or \( \delta \Phi_c \), this term does not depend on the initial and fi-
nal position of the atoms in the interferometer, thus can yield a non-vanishing phase difference even when the final position of the wave packet is the same as the initial position which leads to $\delta \Phi = 0$. We also note that $\alpha \equiv (\mu_1 + \mu_2)/(\mu_2 - \mu_1)$ determines the prefactor of the magnetic-moment-dependent phase difference from the butterfly configuration of the SG interferometer. In order to increase the factor $\alpha$, the magnitude of the magnetic moments $\mu_1$ and $\mu_2$ should be as large as possible, while keeping their difference $|\mu_2 - \mu_1|$ small. Note that $\alpha = 0$ when $\mu_1 = -\mu_2$, in which case the interferometer has a symmetric geometry. For example, a recent experiment on the atomic interferometer [18] has implemented the $5^2S_{1/2}$ manifold of $^{87}\text{Rb}$ atom to generate superposition between $|F = 2, m_F = 2\rangle$ and $|F = 2, m_F = 1\rangle$. In this case, the prefactor is $\alpha = 3$. This prefactor can be increased by employing an atom in superposition between higher magnetic moments. Rydberg atoms can be a promising option to realize this as they can have high quantum numbers of $n$, $l$, and $m$. Superposition between circular Rydberg states $|55c\rangle = |n = 55, l = 54, m = 54\rangle$ and $|56c\rangle = |n = 56, l = 55, m = 55\rangle$ recently realized in experiment [69] can boost the factor to $\alpha = 109$, which is significantly higher than using the $5^2S_{1/2}$ manifold of $^{87}\text{Rb}$.

In order to focus on the effect of the magnetic field, let us consider the case $\hbar k \ll \delta p_0 F_0 T$ so that the initial and final $\pi/2$ pulses have a negligible role in the momentum transfer and the phase difference. We simply denote such regime as $\hbar k \to 0$. A relevant physical situation would be when the atoms’ internal states $|1\rangle$ and $|2\rangle$ undergo the Raman transition at radio frequencies [19]. In this case, the phase is written as

$$\delta \Phi = -\frac{KT^2}{32\hbar m} \left[ \frac{p_0}{m} + \frac{F_0}{2m} T^2 - \frac{\delta p}{m} \frac{\mu_1 + \mu_2}{3(\mu_2 - \mu_1)} T \right].$$

As an illustrative example, we have plotted in Fig. 3 the phase differences in the SG butterfly interferometer induced by the gravitational field. As physical parameters, we take with experimentally feasible values of $m = 1.42 \times 10^{-25}\text{kg}$, $g = 9.8\text{m/s}^2$, $\Gamma = mK = 3 \times 10^{-6}\text{s}^{-2}$, $\delta v = \delta p/m = 0.02\text{m/s}$ based on Ref. [18]. First of all, the figure shows that when the initial and final positions are the same, only the phase difference from the magnetic moment is non-vanishing. Also, when Rydberg atoms are employed with $\alpha = 109$, the phase difference from the magnetic moment can be large enough to increase the sensitivity.

In realistic experiments, the magnetic field in the displacement stage has to be large enough to attain the phase $\phi_0$ of the above limit. Taking into account practical situations, we analyze the phase accumulation with a finite time duration of displacement instead of the pulse regime. In this case, from the condition of Eq. [2], the free evolution time is determined by $T_f = t_1 + 2t_2$. We show the phase accumulation from finite time of displacement in Fig. 4. As shown in the figure, the finite time duration for displacement still leads to a similar phase difference to that we obtained in the limit $t_1 \to 0$.

C. Visibility

Finally, let us discuss the degradation of the visibility due to the misalignment. The misalignment changes the probability of detecting $|\pm\rangle$ as $P_{\pm} = (1 \pm C \cos \delta \Phi)/2$, where

$$C = |\langle \psi_0(T) | \hat{V}^\dagger(T) \hat{D}(\Delta z(T), \Delta p(T)) \hat{V}(T) | \psi_0(T) \rangle|$$

represents the visibility. Here, $\Delta z(T) = z_2(T) - z_1(T)$ and $\Delta p(T) = p_2(T) - p_1(T)$. Unless $\Delta z(T) = \Delta p(T) = 0$, the visibility of the signal is strictly less than 1. In the butterfly interferometer case with a pulse regime, up to order of $K$, we have

$$\Delta z(T) \approx \frac{KT^3}{32m^2} \delta p, \quad \Delta p(T) \approx 0.$$ (19)

In the case of Mach-Zehnder type of atomic interferometer, the misalignment is given by

$$\Delta z(T) \approx \frac{KT^3}{8m^2} \delta p, \quad \Delta p(T) \approx \frac{KT^2}{4m} \delta p.$$ (20)

In order to analyze the effect of the misalignment, let us assume the initial wave packet to be Gaussian, with the uncertainty of the position and momentum along the $z$-axis as $\sigma_z$ and $\sigma_{p_z}$. We assume that the correlation of the position and momentum is zero for simplicity. In this case, the visibility $C$ for a pure state can be simplified as

$$C = |\langle \psi(0) | \hat{D}(\Delta z', \Delta p_{z}') | \psi(0) \rangle| = \exp \left[ \frac{-\Delta z'^2}{8\sigma_z^2} - \frac{\Delta p_{z}'^2}{8\sigma_{p_z}^2} \right].$$ (21)
FIG. 5: Visibility of atomic interferometers using $^{87}$Rb atoms with the position uncertainty $\sigma_z = 200 \mu$m and the velocity uncertainty $\sigma_p/m = 0.44 \text{mm/s}$, which is achieved with a temperature $T_{\text{eff}} = 2 \text{nK}$. The solid curve represents the visibility of the butterfly configuration and the dashed curve that of the Mach-Zehnder configuration.

and for a mixed initial state $\hat{\rho}(0)$,

$$C = |\text{Tr}\left[\hat{D}(\Delta z', \Delta p_z')\hat{\rho}_0\right]| = \exp\left[-\frac{\sigma_z^2 \Delta p_z^2 + \sigma_p^2 \Delta z^2}{2\hbar}\right],$$

(22)

where $\Delta z' = \Delta z \cosh \omega T - \frac{\Delta p_z}{m\omega} \sinh \omega T$ and $\Delta p_z' = \Delta p_z \cosh \omega T + m\omega \Delta z \sinh \omega T$. Fig. 5 shows the visibility of an example with different interferometer time $T$. It shows that the visibility is approximately 1 for 5 seconds and 3 seconds for the butterfly configuration and the Mach-Zehnder type interferometer, respectively. Thus, the butterfly configuration is more robust to the misalignment.

In the case when visibility is degraded by the misalignment, one may overcome this obstacle by adjusting experimental parameters more elaborately. For example, for an atomic interferometer based on laser pulses, by changing the frequency of the laser depending on the gravity gradient one can align the trajectories of the atomic to reduce the amount of the misalignment. The same technique can be applied by changing the magnetic field gradient accordingly.

IV. CONCLUSION

We have introduced a variant of a butterfly interferometer that enables a precise measurement of the field gradient. In our SG butterfly configuration, the atoms are prepared in superposition between the internal states having different magnetic moments, so that the external magnetic field can split the trajectories in the time-dependent manner.

The proposed interferometer is flexible in controlling the momentum transfer of the atoms by using the external magnetic field, while it implements the conventional butterfly configuration as a special case. We have shown that the phase difference between the different atomic trajectories arising from the field gradient can be decomposed into magnetic moment independent and dependent parts. The magnetic moment independent phase can be compared to the phase difference in the conventional butterfly interferometer, but can linearly increase by the momentum difference of the interferometer $\delta p$ induced by the external magnetic field. The magnetic moment dependent phase provides a distinct feature of the SG butterfly interferometer scaling in $T^3$ and can be increased by enlarging the magnetic quantum number of the internal atomic state.

We have also investigated the degradation of the visibility when the interferometer is not perfectly closed due to the field gradient and have demonstrated that the robustness of the interferometer against the misalignment is comparable or better than the Mach-Zehnder type of atomic interferometer.

Acknowledgments

CO and LJ acknowledge support from the ARL-CDQI (W911NF-15-2-0067), ARO (W911NF-18-1-0020, W911NF-18-1-0212), ARO MURI (W911NF-16-1-0349), AFOSR MURI (FA9550-15-1-0015, FA9550-19-1-0399), DOE (DE-SC0019406), NSF (EFMA-1640959, OMA-1936118), and the Packard Foundation (2013-39273). HK’s work is supported by the QuantERA ERA-NET cofund in Quantum Technologies implemented within the European Union’s Horizon 2020 Programme and EPSRC (EP/R044082/1).

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