Feynman–Smoluchowski engine at high temperatures and the role of constraints

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Abstract. Feynman’s ratchet and pawl is a paradigmatic model for energy conversion using thermal fluctuations in the mesoscopic regime. Here, we optimize the power output of the ratchet as a heat engine in the high temperatures limit, and derive the universality of efficiency at maximum power up to second order, using a non-linear approximation. On the other hand, the linear model may be optimized by constraining the internal energy scales in different ways. It is shown that simple constraints lead to well-known expressions of thermal efficiency in finite-time thermodynamics. Thereby, the constrained ratchet, in the linear regime, has been mapped to an effective finite-time thermodynamic model.

Keywords: Brownian motion, exact results, fluctuation phenomena, heat conduction
1. Introduction

Feynman–Smoluchowski (FS) ratchet [1, 2] has motivated the modeling of Brownian or molecular motors [3–11] and sharpened the understanding of thought experiments like Maxwell’s demon [12, 13]. Subsequently, various analogs and generalizations [14–24], have been studied in literature. The ratchet is designed to rectify thermal fluctuations across a mechanical link whose two asymmetric ends experience different fluctuations due to their being embedded in different (hot and cold) baths. The processes of heat and work transfer are assumed to occur at finite rates, thus generating a finite output power. Feynman’s analysis [1] concluded that the device could operate with reversible efficiency in the quasi-static limit which implies a vanishing output power. Based on this analysis, we shall also assume a strong coupling between the fluxes, i.e. there is no heat leakage between the heat baths (see [24–26] for contrasting views).

In the finite-power regime, one may extract maximum power by tuning the system’s internal energy scales to appropriate values [18, 19, 22]. A related quantity of interest, that also places the FS system in a broader thermodynamic context, is the efficiency at maximum power (EMP). It shares a universal property with many other finite-time models [27–29], viz. for small differences in bath temperatures, EMP behaves as \( \eta_c/2 + \eta_c^2/8 + O(\eta_c^3) \), where \( \eta_c = 1 - T_2/T_1 \) is the Carnot bound, with \( T_2(T_1) \) as the cold (hot) bath temperatures. The first-order term can be explained using the strong-coupling assumption within linear irreversible thermodynamics [30], while the second-order term is beyond linear response, and has been related to a certain symmetry property in the model [31, 32].

In this paper, we focus on the performance of FS ratchet at maximum power in the regime where thermal energy of a bath is much higher than the internal energy scale excited by the bath. We highlight new features of the device in this regime, not discussed earlier in literature. We note that it is not possible to optimize power—simultaneously over both internal scales—within the linear regime. However, a two-parameter optimization is possible if one extends the operational domain to non-linear approximation. Interestingly, one is able to then recover EMP that retains the same
universality up to second order as for the EMP of the original problem, equation (8) below. We then impose some simple constraints over the internal energy scales, such that optimization of power over a single parameter can be performed using the linear model. These constrained optimization problems yield some well-known forms of EMP found in other finite-time models. Moreover, under each of these constraints, it is possible to give an effective finite-time thermodynamic model for the FS engine.

The plan of the paper is as follows. In section 2, we briefly describe the model of FS engine and discuss its optimal performance. In section 3, two-parameter optimization of ratchet engine in high temperatures limit is discussed. Section 4 is devoted to optimization of the ratchet in linear regime, subject to constraints. In section 5, FS engine is mapped to effective thermodynamic models depending on the constraints used in the previous section. Section 6 is devoted to a discussion of the results, with concluding remarks.

2. Feynman’s ratchet and pawl model

Feynman’s model [1] consists of a vane, immersed in a hot reservoir at temperature $T_1$, and connected through an axle with a ratchet in contact with a cold reservoir at $T_2$. In the center of the axle, there is a wheel from which a weight $Z$ is suspended. Because of the collisions of gas molecules, the vane is subjected to Brownian fluctuations. But the ratchet is restricted to rotate in one direction only due to a pawl which in turn is connected to a spring. Let $\epsilon_2$ be the amount of energy to overcome the elastic energy of the spring. Let in each step, the wheel rotate an angle $\phi$ and the torque induced by the weight be $Z\phi$. Then the system requires a minimum of $\epsilon_1 = \epsilon_2 + Z\phi$ energy to lift the weight hanging from the axle. Hence the rate of forward jumps of the ratchet is given as $R_F = r_0 e^{-\epsilon_1/k_B T_1}$, where $r_0$ is a rate constant and $k_B$ is Boltzmann’s constant, which we set equal to unity. In other words, temperature has the dimensions of energy. A part of the energy $\epsilon_1$ is converted into work $Z\phi$, and other is transferred as heat $\epsilon_2$ to the cold thermal bath through the interaction between the ratchet and the pawl. Similarly, the rate of the backward jumps is $R_B = r_0 e^{-\epsilon_2/T_2}$. One may regard $Z\phi$ and $-Z\phi$ as the work done by and on the system, respectively. If $R_F > R_B$, this system works as two-reservoir heat engine. Then, the rates of heat related to the hot and the cold reservoirs, are given as

$$\dot{Q}_1 = r_0 \epsilon_1 \left( e^{-\epsilon_1/T_1} - e^{-\epsilon_2/T_2} \right) > 0,$$

$$\dot{Q}_2 = r_0 \epsilon_2 \left( e^{-\epsilon_1/T_1} - e^{-\epsilon_2/T_2} \right) > 0.$$  

According to the model, $\epsilon_1 > \epsilon_2$, and so positivity of the fluxes implies: $\epsilon_2/T_2 > \epsilon_1/T_1$. The power output, $P = \dot{Q}_1 - \dot{Q}_2$, is given by:

$$P = r_0 (\epsilon_1 - \epsilon_2) \left( e^{-\epsilon_1/T_1} - e^{-\epsilon_2/T_2} \right).$$

The efficiency of the engine, $\eta = P/\dot{Q}_1$ is given by

$$\eta = 1 - \frac{\epsilon_2}{\epsilon_1} \leq \eta_c.$$
For given bath temperatures, it is natural to optimize the power output with respect to the internal energy scales $\epsilon_1$ and $\epsilon_2$, which yields the following solution [19]

$$\epsilon_1^* = T_1 \left[1 - (\eta_c^{-1} - 1) \log(1 - \eta_c)\right],$$

$$\epsilon_2^* = T_1 (\eta_c^{-1} - 1) (\eta_c - \log(1 - \eta_c)),$$

with the expressions for the optimal power and EMP [19] as given by

$$P^* = r_0 e^{-\epsilon_1 / T_1} (e^{-\epsilon_2 / T_2}) \eta_c^2 (1 - \eta_c)^{(\eta_c^{-1} - 1)},$$

$$\eta^* = \eta_c \left[1 - (\eta_c^{-1} - 1) \log(1 - \eta_c)\right]^{-1}.$$  \hfill (8)

Notably, $\eta^*$ depends only on the ratio of the reservoir temperatures. Further, the above expression of efficiency also holds for EMPs of a two-level atomic system [33] and a simple model of classical particle transport [34]. Equation (8) has the following expansion for small values of $\eta_c$:

$$\eta^* = \frac{\eta_c}{2} + \frac{\eta_c^2}{8} + \frac{7 \eta_c^3}{96} + O(\eta_c^4).$$  \hfill (9)

The above series displays the universality up to second order mentioned in the Introduction.

3. Ratchet in high temperatures regime

In the following, we are interested in the regime, where the energies associated with forward and backward jumps are very small compared to the temperatures of reservoirs. Therefore, we can expand $e^{-\epsilon_1 / T_1} (e^{-\epsilon_2 / T_2})$ as Taylor series, say, up to first or second order. First, we look for a possible two-parameter power optimization in this regime. Keeping terms up to the first order, we have the approximate expression for power as

$$P = r_0 (\epsilon_1 - \epsilon_2) \left(\frac{\epsilon_2}{T_2} - \frac{\epsilon_1}{T_1}\right).$$  \hfill (10)

We address the above approximation as the linear model [18]. Similarities between the above model and a thermoelectric generator were recently discussed in [24].

Now, a two-parameter optimization of the above expression, over $\epsilon_1$ and $\epsilon_2$, yields the condition $T_1 = T_2$, which is clearly not a meaningful result. This suggests that the individual scales $\epsilon_1$ and $\epsilon_2$ may not be varied independently within the linear model. We consider this idea in further detail in section 4. However, if we retain terms up to second order in the exponentials, then power is given by

$$P = r_0 (\epsilon_1 - \epsilon_2) \left(\frac{\epsilon_2}{T_2} - \frac{\epsilon_1}{T_1} + \frac{\epsilon_1^2}{2T_1^2} - \frac{\epsilon_2^2}{2T_2^2}\right).$$  \hfill (11)

Now, optimizing the above expression over $\epsilon_1$ and $\epsilon_2$, we get the following solution
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\[ \epsilon^*_{\text{hot}} = T_1 \frac{(4 - 3\eta_c)}{3(2 - \eta_c)}, \quad \epsilon^*_{2\text{hot}} = T_1 \frac{(1 - \eta_c)(4 - \eta_c)}{3(2 - \eta_c)}. \] (12)

It is clear that the expression for efficiency remains as in equation (4). Thus, we obtain the expressions for optimal power and EMP in the high temperatures regime

\[ P^*_{\text{hot}} = \frac{2r_o T_1 \eta_c^2}{27(2 - \eta_c)}, \] (13)

\[ \eta^*_{\text{hot}} = \frac{2 - \eta_c}{4 - 3\eta_c}. \] (14)

If we expand \(\eta^*_{\text{hot}}\) in Taylor series near equilibrium, we obtain

\[ \eta^*_{\text{hot}} = \frac{\eta_c}{2} + \frac{\eta_c^2}{8} + \frac{9\eta_c^3}{96} + O(\eta_c^4). \] (15)

The above series shows that the universality of EMP up to second order \([19, 31]\) survives in the high temperatures limit, using a non-linear approximation in the power output. The above form of efficiency is compared with equation (8) in figure 1, where we also compare the optimal power, equation (7), with the optimal power in high temperatures non-linear regime, equation (13). It is to be noted that whereas the latter approximation overestimates EMP, the power output is underestimated as compared to optimal power.

4. Linear regime with constraints

In this section, we impose simple constraints on the energy scales of the ratchet system in the linear regime. This allows us to define a single-parameter optimization problem for power output, equation (10). These constraints may be interpreted as a form of control on the design of the device. We are interested in the form of EMP under the following constraints \([35]\).

(a) \(\epsilon_1 = k_1 > 0\). Then optimizing power (equation (10)) with respect to \(\epsilon_2\), we get

\[ \eta_1 = \frac{\eta_c}{2}, \] (16)

a universal expression independent of the chosen \(k_1\) value.

(b) On the other hand, consider setting \(\epsilon_2 = k_2 > 0\). On optimization of power, we obtain \(\epsilon_1 = k_2(2 - \eta_c)/(2 - 2\eta_c)\), and EMP as

\[ \eta_2 = \frac{\eta_c}{2 - \eta_c}, \] (17)
which is again a universal formula depending only on the ratio of bath temperatures, but independent of the chosen constant \( k_2 \). Of course, the expressions for optimal power do depend on the chosen constant.

(c) A more general constraint \( \gamma \epsilon_1 + (1 - \gamma) \epsilon_2 = k_3 \) where \( 0 \leq \gamma \leq 1 \). Here, the constraint involves two fixed parameters. Optimization of power subject to this constraint, leads to the following optimal values:

\[
\epsilon_1^* = \frac{k_3(2 - (1 - \gamma)\epsilon_c)}{2(1 - (1 - \gamma)\epsilon_c)}, \quad \epsilon_2^* = \frac{k_3(2 - (2 - \gamma)\epsilon_c)}{2(1 - (1 - \gamma)\epsilon_c)}
\]

and the EMP is Schmiedl–Seifert (SS) efficiency [36]

\[
\eta_{SS} = \frac{\epsilon_c}{2 - (1 - \gamma)\epsilon_c}.
\]

Clearly, (a) and (b) are special cases, with \( \gamma = 1 \) and \( \gamma = 0 \), respectively. Here, EMP is independent of \( k_3 \), but depends on \( \gamma \). The above form has been obtained in [27, 32, 36–39], where the parameter \( \gamma \) may be defined, for example, in terms of the ratio of the dissipation constants or thermal conductivities of the thermal contacts [27, 37].

(d) If the constraint \( \epsilon_1 \epsilon_2 = k_4 \) is imposed, the optimal power is obtained at Curzon–Ahlborn (CA) efficiency [28]:

\[
\eta_{CA} = 1 - \sqrt{1 - \eta_c},
\]

at optimal values of \( \epsilon_1 \) and \( \epsilon_2 \):

\[
\epsilon_1^* = \sqrt{k_4(1 - \eta_c)^{1/4}}, \quad \epsilon_2^* = \frac{\sqrt{k_4}}{(1 - \eta_c)^{1/4}}.
\]
5. Mapping to effective thermodynamic model

The expressions for EMP, obtained in the above, are also encountered in many thermodynamic models based on different assumptions [27, 36–39]. They are obtained in finite-time as well as quasi-static models [32, 40] of heat engines. Thus, it is natural to enquire about the thermodynamic underpinning of the constrained FS model. In this section, we show that FS engine in the linear regime can be mapped to a specific endoreversible model, under the constraints described above. In the endoreversible approximation [28, 41, 42], the work-extracting part of the engine operates in a reversible way, and any irreversibility in the cycle is attributed solely to thermal contacts with the reservoirs due to finite conductance of the heat exchangers.

(a) In the linear regime, the heat flux entering from the hot reservoir \( \dot{Q}_1 \), equation (1), is given by

\[
\dot{Q}_1 = r_0 \epsilon_1 \left( \frac{\epsilon_2}{T_2} - \frac{\epsilon_1}{T_1} \right). \tag{22}
\]

\[
\equiv r_0 \epsilon_1^2 \left( \frac{1 - \eta}{T_2} - \frac{1}{T_1} \right). \tag{23}
\]

Here, we identify \( T'_1 = T_2 / (1 - \eta) \) as an effective temperature, satisfying \( T_2 < T'_1 < T_1 \). Therefore, when we impose \( \epsilon_1 = \text{constant} \), the heat flux satisfies \( \dot{Q}_1 \propto (1/T'_1 - 1/T_1) \), i.e. the flux is proportional to thermodynamic force as in linear irreversible thermodynamics. Then, it is assumed that the power is extracted between the temperatures \( T'_1 \) and \( T_2 \), with reversible efficiency given by \( \eta = 1 - T_2 / T'_1 \). Therefore,

\[
P = \eta \dot{Q}_1 = r_0 \eta \epsilon_1^2 \left( \frac{1 - \eta}{T_2} - \frac{1}{T_1} \right). \tag{24}
\]

Optimizing power with respect to \( \eta \) (\( \partial P / \partial \eta = 0 \)), we can obtain EMP as in equation (16).

(b) Similarly, in terms of \( \epsilon_2 \), the heat flux into the cold bath can be written as

\[
\dot{Q}_2 = r_0 \epsilon_2^2 \left( \frac{1}{T_2} - \frac{1}{T_1(1 - \eta)} \right)
\equiv r_0 \epsilon_2^2 \left( \frac{1}{T_2} - \frac{1}{T'_2} \right),
\]

where \( T'_2 = T_1 (1 - \eta) \) is an effective temperature, lying between values \( T_1 \) and \( T_2 \). Thus, for a fixed value of \( \epsilon_2 \), the heat flux \( \dot{Q}_2 \) is proportional to \( (1/T_2 - 1/T'_2) \), which plays the role of thermodynamic force. In this case, power is extracted at Carnot efficiency between \( T_1 \) and \( T'_2 \): \( \eta = 1 - T'_2 / T_1 \). Therefore,
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\[ P = \frac{\eta}{1 - \eta} Q_2 = r_0 e_2^2 \frac{\eta}{1 - \eta} \left( \frac{1}{T_2} - \frac{1}{T_1(1 - \eta)} \right). \]  (25)

Optimizing the above equation with respect to \( \eta \), we obtain equation (17).

(c) For the linear constraint \( \gamma \epsilon_1 + (1 - \gamma) \epsilon_2 = k_3 \), the effective thermodynamic model is more interesting. In terms of \( \eta \), this constraint equation can be written as

\[ \epsilon_1 = \frac{k_3}{\gamma + (1 - \gamma)(1 - \eta)} = \frac{k_3}{A}. \]  (26)

Then the expression for power becomes

\[ P = r_0 k_3^2 \frac{\eta}{A^2} \left( \frac{1 - \eta}{T_2} - \frac{1}{T_1} \right), \]  (27)

which can be rewritten as follows:

\[ P = r_0 k_3^2 \frac{\eta}{A} \left( \frac{1}{T_2} - \frac{1}{T_1} \right), \]  (28)

where the effective temperatures are defined as

\[ \tilde{T}_1 = T_1 A, \quad \tilde{T}_2 = \frac{T_2 A}{1 - \eta}. \]  (29)

Now we show that FS system in the linear regime, and under the general constraint, is equivalent to a system of two coupled Carnot engines in which heat flux leaving the first engine (\( \dot{q}_1 \)), serves as input heat flux for the second engine, through a finite heat conductance (see figure 2(c)). Thus consider the power output from the first engine:

\[ P_1 = \frac{\eta_1}{1 - \eta_1} \dot{q}_1, \]  (30)

where \( \eta_1 \) is the reversible efficiency of engine 1 working between \( T_1 \) and \( \tilde{T}_1 \):

\[ \eta_1 = 1 - \frac{\tilde{T}_1}{T_1} = 1 - A, \]  (31)

and

\[ \dot{q}_1 = r_0 k_3^2 \left( \frac{1}{T_2} - \frac{1}{T_1} \right), \]  (32)
is the heat flux leaving engine 1. Thus $r_0k_3^2 \equiv \delta$ is the heat transfer coefficient of the heat exchanger connecting engines 1 and 2. So, we can rewrite equation (30) as

$$P_1 = r_0k_3^2 \frac{1 - A}{A} \left( \frac{1}{\bar{T}_2} - \frac{1}{\bar{T}_1} \right).$$

(33)

Now, engine 2 operates at Carnot efficiency $\eta_2$ between temperatures $\bar{T}_2$ and $T_2$:

$$\eta_2 = 1 - \frac{T_2}{\bar{T}_2} = 1 - \frac{1 - \eta}{A},$$

(34)

with the input heat flux as $\dot{q}_1$. Hence, the power of engine 2, $P_2 = \eta_2\dot{q}_1$ can be written as

$$P_2 = r_0k_3^2 \left( 1 - \frac{1 - \eta}{A} \right) \left( \frac{1}{T_2} - \frac{1}{\bar{T}_1} \right).$$

(35)

Adding equations (33) and (35), we get

$$P_1 + P_2 = P,$$

(36)

which is the total power, equation (28). Alternately, we can write $P_1 = (1 - \gamma)P$ and $P_2 = \gamma P$. Optimizing $P$ with respect to $\eta$, we obtain equation (19). It is clear that the maximum of $P_1$ and $P_2$ is also reached at the same value of $\eta$ as of $P$. Thus optimality of $P$ for the overall engine implies optimal power output of the sub-engines.
Now, the values $\gamma = 0$ and $\gamma = 1$ correspond to the special cases (a) and (b), respectively. Using $A = \gamma + (1 - \gamma)(1 - \eta)$, we can write

$$\eta_1 = (1 - \gamma)\eta, \quad \eta_2 = \frac{\gamma\eta}{1 - \eta + \eta\gamma}. \quad (37)$$

Also, the manner in which the two sub-engines are coupled, implies that the efficiencies of the sub-engines are related to the overall efficiency as: $\eta = 1 - (1 - \eta_1)(1 - \eta_2)$.

Finally, using equation (19), the EMPs for engine 1 and 2 are given by

$$\eta_1^* = \frac{(1 - \gamma)\eta_c}{2 - (1 - \gamma)\eta_c}, \quad \eta_2^* = \frac{\gamma\eta_c}{2 - 2(1 - \gamma)\eta_c}. \quad (38)$$

(d) For the constraint $\epsilon_1\epsilon_2 = k_4$, equation (10) for power becomes simplified as

$$P = r_0k_4\eta \left(\frac{1}{T_2} - \frac{1}{T_1(1 - \eta)}\right), \quad (39)$$

$$\equiv r_0k_4\frac{\eta}{\sqrt{1 - \eta}} \left(\frac{1}{T_2} - \frac{1}{T_1}\right), \quad (40)$$

where we have defined

$$\bar{T}_1 = T_1\sqrt{1 - \eta}, \quad \bar{T}_2 = \frac{T_2}{\sqrt{1 - \eta}}, \quad (41)$$

as the effective temperatures. Further, it is useful to decompose $\eta/\sqrt{1 - \eta}$ as follows:

$$\frac{\eta}{\sqrt{1 - \eta}} = \frac{1 - (1 - \eta)}{\sqrt{1 - \eta}} = \frac{1}{\sqrt{1 - \eta}} - \sqrt{1 - \eta}. \quad (42)$$

Thus, we can express equation (40) in the following form

$$P = r_0k_4 \left(\frac{1}{\sqrt{1 - \eta}} - 1\right) \left(\frac{1}{T_2} - \frac{1}{T_1}\right) + r_0k_4(1 - \sqrt{1 - \eta}) \left(\frac{1}{T_2} - \frac{1}{T_1}\right), \quad (43)$$

which can be rewritten as

$$P = \eta'_1 \frac{\eta'_1}{1 - \eta'_1} q'_1 + \eta'_2 \frac{\eta'_2}{1 - \eta'_2} q'_1 \equiv P_1 + P_2, \quad (44)$$

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where $\eta_1'(\eta_2')$ is Carnot efficiency of engine 1(2) operating between the temperatures $T_1(T_2)$ and $\tilde{T}_1(\tilde{T}_2)$, defined as

\begin{align}
\eta_1' &= 1 - \frac{\tilde{T}_1}{T_1} = 1 - \sqrt{1 - \eta}, \\
\eta_2' &= 1 - \frac{\tilde{T}_2}{T_2} = 1 - \sqrt{1 - \eta},
\end{align}

and

\begin{equation}
\dot{q}_1' = r_0 k_4 \left( \frac{1}{T_2} - \frac{1}{\tilde{T}_1} \right)
\end{equation}
is the heat flux leaving engine 1 and entering engine 2 (see figure 3(a)). Here $r_0 k_4$ is heat transfer coefficient of the heat exchanger connecting engines 1 and 2. We note that engine 1 and engine 2 deliver power at same efficiency. Then, the expressions for $\eta'_1$ and $\eta'_2$, at optimal power, are given by:

$$\eta'_1^* = \eta'_2^* = 1 - (1 - \eta_c)^{1/4}.$$  \hspace{1cm} (48)

6. Discussion and summary

Our choice of constraints is motivated by the fact that both $\epsilon_1$ and $\epsilon_2$ are the control parameters of the ratchet system. It is possible to tune either of them to obtain a desired performance of the engine. In other words, energy constraints can be imposed by setting a design goal. On the other hand, it is not straightforward to appreciate the nature of control with the general linear constraint $(c)$, though one can consider the equivalent thermodynamic model with effective temperatures as in equation (29). For a given value of $\gamma$, we can tune these temperatures and thus the efficiencies of engines 1 and 2. For $\eta = 0$, we have $\tilde{T}_1 = T_1$ and $\tilde{T}_2 = T_2$. In the reversible limit, when $\eta = \eta_c$, we have $\tilde{T}_1 = \tilde{T}_2 = \gamma T_1 + (1 - \gamma) T_2$, see figure 4. From equations (33) and (35), it is also clear that the power vanishes as $\tilde{T}_1 \to \tilde{T}_2$. Similar considerations can be made regarding the control of the effective temperatures in the case of constraint $(d)$.

However, note that the proposed thermodynamic model for the constrained FS system may not be unique. This may be shown by considering the case $(d)$. We have mapped this model to two coupled reversible engines connected by a heat flow with an inverse-temperature law. It has been shown that the EMP in this model is CA-efficiency. Usually, CA-value is associated with EMP for endoreversible models with Newtonian heat flows, i.e. heat flux is proportional to the difference of temperatures between which the heat flow takes place $[28, 42]$. In fact, it is possible to imagine an alternate model as follows (see figure 3(b)). By rewriting the power output, we get

$$P = \frac{r_0 k_4}{\tilde{T}_1 \tilde{T}_2} \frac{\eta}{1 - \eta} \left( T_1 (1 - \eta) - T_2 \right)$$

$$\equiv k' \frac{\eta}{1 - \eta} (\bar{T}'_2 - T_2),$$  \hspace{1cm} (49)

where we define $\bar{T}'_2 = T_1 (1 - \eta)$ as the effective temperature and $k' = r_0 k_4 / T_1 T_2$ as the coefficient of the exiting heat flux $\dot{Q}_2 = k'(\bar{T}'_2 - T_2)$, between temperatures $\bar{T}'_2$ and $T_2$.

Concluding, we have considered the optimization of output power in FS ratchet in the high temperatures regime, when the internal energy scales are much smaller in comparison to bath temperatures. A two-parameter optimization is possible if one includes the quadratic terms in the expansion of the exponentials. For the linear model, we have considered simple constraints on the internal scales, and obtained some well-known forms of EMP, such as SS-efficiency and CA-efficiency. The reason for these similarities is appreciated by showing that the constrained FS system can be mapped to a finite-time endoreversible model with appropriately defined heat flows, using effective temperatures. Note that the aforesaid mapping becomes possible since we are able to define an effective (intermediate) temperature as well as identify a thermodynamic force to simulate the dissipative flow of heat.
heat. Thus the linearity of the model, or the high temperatures limit plays an important role in the mapping. It is an interesting problem to explore whether the non-linear model, for instance based on equation (11), can be mapped to finite-time thermodynamic models. A possible mapping beyond the assumptions of linear irreversible thermodynamics (linear flux-force formalism) might be attempted. Finally, due to a formal analogy between FS system in the linear regime and thermoelectric models [24], and also specific types of quantum heat engines in the hot temperatures regime [35], the present analysis can provide a useful perspective on a broader class of energy conversion systems.

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