An efficient and high-precision algorithm on parameters estimation of LFM signal

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Abstract. At present, prevalent algorithms on parameters estimation of linear frequency modulation (LFM) signal include the fractional Fourier transform (FrFT) method, of which the crucial procedure is to estimate the optimal rotation angle. The estimated value of the optimal rotation angle has a great influence on the estimation accuracy of LFM parameters, and it also affects the computational complexity of the algorithm. This paper proposes an algorithm on parameters estimation of LFM signal with high efficiency and precision. Based on the mathematical relationship between the optimal and any two rotation angles of FrFT, the optimal rotation angle is calculated through three FrFT transforms and interpolation, and the LFM signal parameters are estimated with high efficiency and precision. Simulation results verify the effectiveness of the proposed algorithm.

1. Introduction

Linear frequency modulation (LFM) signals have good spread-spectrum characteristics and are widely used in radar, sonar, and communication fields. LFM signals are non-stationary, and time-frequency (TF) analysis methods are prevalently used to estimate their parameters1-2. Among the TF methods, the Wigner-Ville distribution (W-V) method has good time-frequency aggregation, but has the shortcoming of cross terms in practical applications. It is necessary to maintain good time-frequency resolution while suppressing cross terms3. The Fractional Fourier transform (FrFT) method has the characteristic of high energy accumulation and no cross term, which has been widely used in parameters estimation of LFM signal4-5. However, the important step of FrFT-based estimation is a two-dimensional search of the optimal rotation angle, which requires a large amount of calculation. Reference [6] and [7] proposed an efficient FrFT method for LFM signal parameters estimation with high efficiency. Using the geometric relationship between the rotation angle and the normalized projection length in the FrFT transform domain, the LFM signal can be detected and parameters can be estimated by performing three times FrFT. However, the projection length is sensitive to numerical discretization of the signal, which may bring in errors in estimating LFM parameters.

This paper presents an efficient and high-precision algorithm on parameters estimation of LFM signal. Based on the efficient FrFT method, interpolation is further performed to reduce the errors in the projection length, and hence a high-precision estimation of the optimal rotation angle can be obtained. This procedure is beneficial and realizes both high efficiency and precision in estimating the parameters. Compared with the traditional FrFT algorithm, the large amount of searching calculation for optimal rotation angle is avoided, so it has high computation efficiency; compared with the efficient FrFT algorithm, the accuracy is higher because of the interpolation procedure.
2. Algorithm principle

2.1. The FrFT-based algorithm

Supposing a LFM signal can be expressed as:

\[ s(t) = A \exp[j2\pi(f_c t + \frac{1}{2} \gamma t^2)] \]  

(1)

In which, \( A \) is amplitude, \( f_c \) is center frequency, \( \gamma \) is frequency modulation slope.

The fractional Fourier transform to signal \( x(t) \) can be defined as \(5\):

\[ X_\alpha(u) = \int_{-\infty}^{\infty} x(t) K_\alpha(u, t) dt \]

(2)

\[ K_\alpha(u, t) = \sqrt{1 - j \cot \alpha \exp\left(j\pi\left(t^2 + u^2\right) \cot \alpha - 2ut \csc \alpha\right)}, \]

\( \alpha \neq n\pi; \delta(u - t), \alpha = 2n\pi; \delta(u + t), \alpha = (2n \pm 1)\pi. \)  

(3)

Among them, \( n = 1, 2, \ldots; K_\alpha(u, t) \) is the kernel of FrFT, \( \alpha \) is rotation angle. When rotation angle \( \alpha = \pi / 2 \), FrFT changes into traditional Fourier transform.

![Figure 1. Principle of FrFT.](image)

![Figure 2. Sketch of the efficient FrFT algorithm.](image)

For FrFT, if the rotation angle \( \alpha \) is properly selected, the energy of the LFM signal will be highly concentrated in the FrFT domain. Thus, the FrFT method of LFM signal parameters estimation can be described as follows: calculate FrFT values of LFM signal at different rotation angles, search for the maximum absolute value after FrFT at all rotation angles, and the angle and frequency corresponding to the maximum absolute value are the results. The above process is a two-dimensional search. In the absence of a priori information, the initial search interval of rotation angle and the search step are extremely important and will directly determine the amount of computation.

2.2. The efficient FrFT-based algorithm

The efficient FrFT algorithm is based on the geometric relationship between FrFT and W-V distribution. Through the mathematical relationship between the optimal and any two rotation angles, the optimal rotation angle can be estimated by using two FrFT transforms \(7\). The process is shown in figure 2. Among them, \( \theta \) is time-frequency angle of W-V transform, and \( \alpha \) is rotation angle of FrFT; \( L_\theta \) is normalized time-frequency length in the W-V distribution; \( L_\alpha \) is normalized FrFT length under the rotation angle \( \alpha \). Normally, \( L_\alpha \) is the width of 0.5 times of the normalized maximum amplitude.
According to reference [7], we assume that two rotation angles $\alpha_1$ and $\alpha_2$ are known, and the corresponding normalized FrFT lengths are $L_{\alpha_1}$ and $L_{\alpha_2}$, respectively. Then, we have the following equations

$$L_{\alpha_1} = |L_{\alpha_1} \sin(\phi - \alpha_1)|$$  \hspace{1cm} (4)

$$L_{\alpha_2} = |L_{\alpha_2} \sin(\phi - \alpha_2)|$$  \hspace{1cm} (5)

The optimal rotation angle can be expressed as

$$\tilde{\alpha} = \phi = \arccot \frac{L_{\alpha_2} \cos \alpha_2 + L_{\alpha_1} \cos \alpha_1}{L_{\alpha_2} \sin \alpha_2 + L_{\alpha_1} \sin \alpha_1}$$  \hspace{1cm} (6)

Considering the dimensional normalization in discrete FrFT numerical calculation, the real frequency modulation slope and the initial frequency can be calculated as

$$\gamma = -\cot \tilde{\alpha} \cdot f_s / T = -\frac{L_{\alpha_2} \cos \alpha_2 + L_{\alpha_1} \cos \alpha_1}{L_{\alpha_2} \sin \alpha_2 + L_{\alpha_1} \sin \alpha_1} \cdot f_s / T$$  \hspace{1cm} (7)

Among them, $T$ is sampling duration while $f_s$ is sampling frequency.

Performing FrFT to the LFM signal with a rotation angle of $\tilde{\alpha}$, and the initial frequency can be calculated as

$$\begin{align*}
\bar{u} &= \arg \max \left| S_\phi(u) \right| \\
\tilde{f}_c &= \bar{u} \csc \tilde{\alpha} / T \end{align*}$$  \hspace{1cm} (8)

The optimal rotation angle can be calculated according to the mathematical relationship between rotation angles and the corresponding projection length of the LFM signal on the time-frequency distribution plane. This method only need three times of FrFT to realize parameter estimation of a LFM signal, and the computational complexity is greatly reduced.

In practice, the FrFT is a numerical calculation, and the signal should be discretely sampled and quantized. Because of the discreteness, the estimated value of the normalized FrFT length includes errors, which will ultimately affect the estimation accuracy of the optimal rotation angle. As shown in Figure 3, the normalized amplitudes of the two adjacent points at 462 and 463 are 0.5252 and 0.4614, respectively. This means we may not be able to have a normalized FrFT length without error. In order to further reduce the error introduced by the discrete effect, interpolation can be used to ensure the normalized amplitude is exactly half of the maximum, and then we can obtain the normalized FrFT length with much less error. When linear interpolation is utilized, the procedure can be expressed as

$$\begin{align*}
x_i &= \frac{0.5 - y_i}{y_{i+1} - y_i} (x_{i+1} - x_i) + x_i \hspace{1cm} (9) \\
x_k &= \frac{0.5 - y_k}{y_{k+1} - y_k} (x_{k+1} - x_k) + x_k \hspace{1cm} (10)
\end{align*}$$

$$\Delta x = |x_j - x_l|$$  \hspace{1cm} (11)

Among them, assuming that the coordinates of the two adjacent points near 0.5 times of the normalized maximum amplitude on the rising edge are $(x_i, y_i)$ and $(x_{i+1}, y_{i+1})$, while the two on the falling edge are $(x_k, y_k)$ and $(x_{k+1}, y_{k+1})$. $x_i$ and $x_k$ are the X-axis coordinates where their normalized amplitude are exact 0.5 times both on the rising and falling edges. 

3
In order to verify the effectiveness of the proposed algorithm, simulation experiments are performed here. The simulation parameters are as follows: The signal-to-noise ratio is 20dB, the initial frequency of the LFM signal is 15MHz, and the frequency modulation slope is $1.5 \times 10^6$ MHz/s, the pulse width 10 μs, and sampling frequency 51.1MHz. The simulation computer is configured as i5 CPU 2.53GHz, 4 GB memory.

Figure 4 shows the distribution of the LFM signal (using W-V transform) at the time-frequency domain. At first, the traditional FrFT method is used for parameter estimation. On this occasion, the initial search interval for the optimal rotation angle is $[0, \pi)$, and the search step is $0.005\pi$. Figure 5 shows the two-dimensional distribution of the LFM signal in the FrFT domain. The optimal rotation angle corresponding to the peak point is 106.2°, and the coordinate of the peak point on frequency axis is 144. Then we can calculate that the frequency modulation slope is $1.4846 \times 10^6$ MHz/s, and the starting frequency is 15.1MHz. It can be seen that the estimated value is close to the real one, but there are still some errors. At the same time, the calculation time based on the traditional FrFT method is 0.6027s. As the search step decreases, the estimation error becomes smaller, and the amount of computation also increases. Simulation results under different search steps of the rotation angle $\alpha$ are shown in the following table.

| Search step of rotation angle $\alpha$ (π/2) | RMS of chirp rate $\dot{\gamma}$ (MHz/s) | RMS of initial frequency $\hat{f}_i$ (kHz) | Time(s) |
|-------------------------------------------|----------------------------------------|----------------------------------------|---------|
| 0.01                                      | $1.5408 \times 10^4$                   | 4.5874                                 | 0.5052  |
| 0.001                                     | $6.6995 \times 10^3$                   | 2.2775                                 | 5.2694  |
| 0.0001                                    | $4.4418 \times 10^3$                   | 0.901                                 | 51.0341 |

Figure 6(a) and (b) show the results of efficient FrFT method. The values of rotation angle are $0.55\pi$ and $0.65\pi$, respectively. In figure 6(a) and (b), the normalized FrFT lengths before interpolation are 67 and 96, respectively. The calculated optimal rotation angle is 106.3868°. Therefore, the frequency modulation slope is $1.5026 \times 10^6$ MHz/s, and the initial frequency is 15.01MHz. The calculation time is 0.0529s, which is less than the traditional FrFT method. Simulation results indicate that the efficient FrFT method has higher computation efficiency.

According to the previous analysis, due to the discontinuity of numerical calculation, there are still considerable errors for the efficient FrFT method. By using linear interpolation, the new normalized FrFT lengths are 67.0388 and 96.7638, and the optimal rotation angle is 106.3537°. Hence, the estimated frequency modulation slope is $14.995 \times 10^6$ MHz/s, and the initial frequency is 15.007MHz. What is more, the calculation time is 0.065s, which is much less than the traditional FrFT method.
Simulation results indicate that the proposed method in this paper has both the advantages of high efficiency and accuracy.

Figure 4. Time-frequency distribution of the LFM signal (using W-V transform).

Figure 5. FrFT domain distribution of the LFM signal.

Figure 6. Discontinuity of the efficient FrFT algorithm.
4. Conclusion

Based on the efficient FrFT method, this paper proposes an improved algorithm on parameters estimation of LFM signal with high efficiency and accuracy. Compared with the traditional FrFT method, the proposed algorithm can obtain a high-precision normalized FrFT length by interpolation, and estimate the optimal rotation angle by performing only two FrFT transforms. The research in this paper could provide a practical solution for detecting and estimating LFM signal.

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