Mathematical Modelling Of Extraction Of The Underground Fluids: Application To Peristaltic Transportation Through A Vertical Conduit Occupied With Porous Material

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Abstract. This paper gives a theoretical idea for extracting the underground fluids through peristalsis. Due to the similarity in aforesaid problem, peristaltic transportation of a viscous Newtonian fluid over a vertical porous conduit is studied by considering the impact of heat transfer. Presuming the long-wavelength approximation, explicit solutions are found as asymptotic expansions with reference to free convection and porosity parameters. Mathematical expressions for coefficient of heat transfer, temperature and mean flux are derived. It is experiential that for few precise values of dissimilar parameters under contemplation, the coefficient of heat transfer increases significantly as Grashof number and Eckert number increases. This narrates to optimization of heat transfer in some processes. Further, it has been observed that mean flow enhances with amplitude ratio, porosity in addition to pressure drop. This authorizes auxiliary research on the impacts of peristalsis on the flow features in vertical channels.

1. Introduction
Extracting oil and other liquids from stores profound underground is not as straightforward as simply penetrating and finishing a well. Any number of factors in the underground condition including the porosity of the stone and the thickness of the store can obstruct the free progression of product into the well. Earlier, it was normal to recoup as meagre as 10 percent of the reachable oil in a store, leaving the rest underground because that the innovative technology was not existed to bring the remaining to the earth surface. Today, the trend setting technology can bring about 60 percent of the total underground formation to the earth surface. The current enhanced recovery techniques are still leaving more than 40 percent of the reserve to the surface. In connection to some extent of this, an experimental work on rheology and lubrication of a drilling fluid is described by Beg et al. [1].

Peristalsis has all the earmarks of being a central mechanism for transport of fluids that propel either fluid or solid substance from one place to another through a sequential contraction of circular muscle. The fluid mechanical aspects of peristalsis and porosity have received lot of attention (Radhakrishnamacharya [2], Takabatake et al. [3], Vajravelu et al. [4], Radhakrishnamacharya and
Srinivasulu [5], Nadeem and Akbar [6], Ravi Kiran and Radhakrishnamacharya [7, 8], Ravi Kiran et al. [9, 10], Sivaiah and Reddy [11], Swamy Reddy et al. [12], Shamshuddin and Krishna [13], Radhakrishnamurthy and Sudam Sekhar [14], Ravi Kiran et al. [15, 16], Vaidya et al. [17]) in the past few years mostly because of their applications in diverse situations such as physiological flows, transport of fluids in oil and other industries and flow of water in trees.

The study of extraction of underground fluids through peristalsis is an application of the peristaltic motion of a fluid flow through upright channels. The fluid dynamical studies for both the problems would be the same. Keeping this in view, the problem of heat transfer in support of the transport of a viscous incompressible Newtonian fluid in a vertical channel included together with porous material, under the action of peristalsis is investigated. Momentum and energy equations have been solved, under the long wavelength approximation and closed form solutions for pressure drop, temperature, heat transfer and velocity have been calculated in terms of Grashof number \( G_m \) and porosity parameter \( \sigma^2 \), which are assumed to be very small. The effects of several parameters on temperature, mean flux and heat transfer have been studied. It is observed that for certain values of other variables, mean flux growths with \( G_m \).

2. Mathematical Model of the problem
Consider the 2-dimensional flow of a viscous, incompressible, Newtonian fluid through a vertical conduit, with flexible walls, filled with porous material. It is supposed that the progressive sinusoidal waves propagate along the walls of the channel. The wall is given by the equation.

\[
H(x, t) = a + b \sin \frac{2\pi}{\lambda}(x - ct)
\]

\[ \text{(1)} \]

Figure 1. Geometrical model of the Physical problem

In this equation, \( b \) be the amplitude, \( a \) be the half width of the original, undisturbed channel, \( \lambda \) be the wavelength, \( c \) be the speed of the wave propagating in the X-direction, \( t \) be the time, \( X \) and \( Y \) are Cartesian coordinates.

The equations governing the flow in a porous channel simplify, under long wavelength approximation to,

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

\[ \text{(2)} \]
Equation of momentum: 
\[-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 U}{\partial Y^2} + \rho g \beta (T - T_0) - \frac{\mu}{k_0} U = 0\]  
(3)

Equation of energy: 
\[K \frac{\partial^2 T}{\partial Y^2} + \mu \left( \frac{\partial U}{\partial Y} \right)^2 + \frac{\mu}{k_0} U^2 = 0\]  
(4)

where U and V represent velocity profiles in X and Y directions, µ is the viscosity coefficient, k_0 is the pore permeability, ρ is the density of the fluid, g is the acceleration due to gravity of the fluid, β is the coefficient of expansion, K is the thermal conductivity and T is the temperature of the fluid.

The no-slip boundary condition for velocity gives \( U = 0 \) at \( Y = \pm H(x,t) \)  
(5)

Further, the channel walls are continued at constant temperature \( T = T_0 \) at \( Y = \pm H(x,t) \)  
(6)

Using the following transformation from stationary coordinate system \((x, y)\) translating with wave velocity \(c\) in the positive X-direction
\[x = X - ct, \quad y = Y \]
\[u = U - c, \quad v = V \]  
(7)

and then using the subsequent dimensionless quantities
\[y' = \frac{y}{a}, \quad x' = \frac{x}{\lambda}, \quad u' = \frac{u}{c}, \quad \theta = \frac{T - T_0}{T_0}, \quad v' = \frac{\lambda}{ac} v, \quad p' = \frac{p}{\mu c \lambda a^2}\]  
(8)

the equations (2) to (6) reduce to the form (after leaving the primes),
\[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\]  
(9)

\[-\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \sigma^2 (u + 1) + G_m \theta = 0\]  
(10)

\[\frac{\partial^2 \theta}{\partial y^2} + E_m \left( \frac{\partial u}{\partial y} \right)^2 + \sigma^2 E_m (u + 1)^2 = 0\]  
(11)

\[u = -1 \] on \( y = \pm \eta(x) \)  
(12)

\[\theta = 0 \]  
(13)

where \(\eta(x) = 1 + \varepsilon \sin(2\pi x)\)

\[\sigma^2 = \frac{a^2}{k_0} \] (porosity parameter), \( G_m = \frac{g \beta T_0 a^3}{v^2} \) (Grashof number)

\[E_m = \frac{c^2}{KT_0} \] (Eckert number), \( \varepsilon = \frac{b}{a} \) (amplitude ratio)

3. Analysis

The equations (10) and (11) are nonlinear coupled equations in \(u\) and \(\theta\). It is not feasible to obtain exact solutions for random values of the entire parameters. Hence, we take perturbation solution with respect to the small parameters \(G_m\) and \(\sigma^2\) as given below:
\[F = (F_{00} + G_m F_{01} + \ldots) + \sigma^2 (F_{10} + G_m F_{11} + \ldots) + \ldots\]  
(14)

where \(F\) is a flow parameter.
Applying equation (14) in equations (10) and (11) and neglecting all higher order terms, we attain the subsequent sets of equations collectively with the respective boundary conditions,

\[
\begin{align*}
- \frac{\partial p_{00}}{\partial x} + \frac{\partial^2 u_{00}}{\partial y^2} &= 0 \\
- \frac{\partial^2 \theta_{00}}{\partial y^2} + E_m \left( \frac{\partial u_{00}}{\partial y} \right)^2 &= 0 \\
\text{Boundary conditions} \\
&\ u_{00} = -1, \theta_{00} = 0 \text{ at } y = \pm \eta
\end{align*}
\]

(15)

\[
\begin{align*}
- \frac{\partial p_{01}}{\partial x} + \frac{\partial^2 u_{01}}{\partial y^2} + \theta_{00} &= 0 \\
- \frac{\partial^2 \theta_{01}}{\partial y^2} + 2E_m \left( \frac{\partial u_{00}}{\partial y} - \frac{\partial u_{01}}{\partial y} \right) &= 0 \\
\text{Boundary conditions} \\
&\ u_{00} = -1, \theta_{00} = 0 \text{ at } y = \pm \eta
\end{align*}
\]

(16)

\[
\begin{align*}
- \frac{\partial p_{10}}{\partial x} + \frac{\partial^2 u_{10}}{\partial y^2} - \left( u_{00} + 1 \right) &= 0 \\
- \frac{\partial^2 \theta_{10}}{\partial y^2} + 2E_m \left( \frac{\partial u_{00}}{\partial y} - \frac{\partial u_{10}}{\partial y} + E_m \left( u_{00} + 1 \right)^2 \right) &= 0 \\
\text{Boundary conditions} \\
&\ u_{00} = 0, \theta_{00} = 0 \text{ at } y = \pm \eta
\end{align*}
\]

(17)

Solving the above sets of equations, the non-dimensional velocity and temperature can be obtained as

\[
\begin{align*}
&\ u = (u_{00} + G_u u_{01}) + \sigma^2 u_{10} \\
&\ \theta = (\theta_{00} + G_u \theta_{01}) + \sigma^2 \theta_{10}
\end{align*}
\]

(18) (19)

where

\[
\begin{align*}
u_{00} &= -1 + \frac{1}{2} \frac{\partial p_{00}}{\partial x} \left( y^2 - \eta^2 \right) \\
\theta_{00} &= -E_m \left( \frac{\partial p_{00}}{\partial x} \right)^2 \frac{1}{12} \left( y^4 - \eta^4 \right) \\
u_{01} &= -1 \frac{\partial p_{01}}{\partial x} \left( y^2 - \eta^2 \right) + E_m \frac{1}{12} \left( \frac{\partial p_{01}}{\partial x} \right)^2 \left[ \frac{y^6 - \eta^6}{30} - \eta^4 \left( y^2 - \eta^2 \right) \right] \\
\theta_{01} &= -2E_m \left[ - \frac{\partial p_{01}}{\partial x} - \frac{\partial p_{01}}{\partial x} \frac{1}{12} \left( y^4 - \eta^4 \right) \right] + E_m \left( \frac{\partial p_{01}}{\partial x} \right)^2 \left[ \frac{y^8 - \eta^8}{3380} - \eta^4 \left( y^4 - \eta^4 \right) \right] \\
u_{10} &= -1 \frac{\partial p_{10}}{\partial x} \left( y^2 - \eta^2 \right) - \eta^2 \left( y^2 - \eta^2 \right) + \frac{1}{2} \frac{\partial p_{10}}{\partial x} \left( y^2 - \eta^2 \right)
\end{align*}
\]

(20) (21) (22) (23) (24)
The dimensionless pressure drop above single wavelength is given as

\[
\Delta p = (\Delta p_{00} + G_m \Delta p_{01}) + \sigma^2 \Delta p_{10} + O(G_m, \sigma^2)
\]

where

\[
\Delta p_{00} = \frac{9\varepsilon^2 - 3\vec{Q}_{00}(2 + \varepsilon^2)}{2(1 - \varepsilon^2)^2}
\]

\[
\Delta p_{01} = -3\vec{Q}_{01} \left[ \frac{2 + \varepsilon^2}{5} + \frac{24}{35}E_m \left\{ 1 + \frac{(\vec{Q}_{00} - 1)^2}{(1 - \varepsilon^2)\varepsilon^2} + 2(\vec{Q}_{00} - 1) \right\} \right]
\]

\[
\Delta p_{10} = -3\vec{Q}_{10} \left[ \frac{2 + \varepsilon^2}{5} - \frac{8}{5} \left[ 1 + \frac{(\vec{Q}_{00} - 1)^2}{(1 - \varepsilon^2)\varepsilon^2} \right] \right]
\]

The dimensionless mean flux is given by

\[
\bar{Q} = (\bar{Q}_{00} + G_m \bar{Q}_{01}) + \sigma^2 \bar{Q}_{10} + O(G_m, \sigma^2)
\]

Heat transfer coefficient \( H \) on the wall is defined as

\[
H = (H_{00} + G_m H_{01}) + \sigma^2 H_{10}
\]

where

\[
H_{00} = \frac{\partial\eta}{\partial x} + \frac{\partial\theta_{00}}{\partial y}
\]

\[
H_{01} = \frac{\partial\eta}{\partial x} + \frac{\partial\theta_{01}}{\partial y}
\]

\[
H_{10} = \frac{\partial\eta}{\partial x} + \frac{\partial\theta_{10}}{\partial y}
\]

### 4. Numerical Results and Discussion

The closed form expressions for temperature, flux and coefficient of heat transfer are given by the equations (19), (30) and (31) respectively and they depend upon the porosity parameter \( \sigma^2 \), Grashof number \( G_m \), amplitude ratio \( \varepsilon \).

The effects of \( E_m, G_m, \sigma^2, \varepsilon \) on temperature have been displayed in figure2 to figure5. It is examined that for unchanging values of other flow variables, temperature primarily rises downwards the channel and then diminishes i.e., oscillatory behaviour is observed which may be caused by peristalsis. It is seen from figure2 and figure 3 that, the temperature enhances as Eckert number or Grashof number raises. Additionally, it is inspected that the temperature profile increases as \( \sigma^2, \varepsilon \) increases (figure4 and figure 5). Physically, this gives an understanding of variation in temperature due to peristalsis. It is also clear that peristalsis controls greater improvement in temperature and there by regulates the flow without interruption.
Deviation of mean flux $\langle Q \rangle$ with amplitude ratio $\varepsilon$ is represented in figure 6 – figure 9. It is noticed that for unchanging values of remaining variables, mean flux amplifies with $\varepsilon$ and also with pressure change $\Delta p$. figure 6 – figure 9 show that $\langle Q \rangle$ increases as $E_m$ or $G_m$ increases while it decreases as $\sigma^2$ increases. The physical significance of enhancement in Grashof number leads to improvement in buoyancy forces which speeds up the flow in the axial direction. This in turn improves the mean flux.
Further, figure 6 – 9 depict a decline of flux with the change in porous parameter. Physically, this result can be understood as greater permeability reduces the mean flux in vertical channels. This may be due to the gravitational force. This evidences the importance of porous material for the peristaltic flow in vertical channels.
Figure 6. Deviation in mean flux with $\varepsilon$ ($E_m=2$, $G_m=1$, $\sigma^2=1$)

Figure 7. Deviation in mean flux with $\varepsilon$ ($E_m=4$, $G_m=1$, $\sigma^2=1$)
The coefficient of heat transfer $H$ on the wall of the conduit is calculated mathematically and the outcomes are shown in table 1 to table 4. It can be observed that $H$ first increases down the channel and then decreases. This variation is same as in the case of temperature, which may be again because of peristalsis. Also, from table 1 and table 2, it is perceived that, for fixed values of all other parameters, heat transfer increases as Eckert number or Grashof number enhances. table 3 and table 4 illustrate that $H$ growths as $\sigma^2$ or $\varepsilon$ increases.
It has been experiential that the mean flux boost by about 18% to 20% as free convection parameter ($G_m$) rises from 1 to 2 for specified values each other flow quantities. This is vital outcome of this research. The proposed method enhances the mean flux to remarkable possibility over the existing prospects.

| Table 1: Variation of $H$ with $E_m$ ($G_m=3, \varepsilon = 0.1$) |
|---------------------------------------------------------------|
| $X$   | $E_m=1$ | $E_m=5$ |
|-------|--------|--------|
| 0.0   | 3.054  | 8.709  |
| 0.4   | 4.758  | 12.963 |
| 0.8   | 2.134  | 5.145  |

| Table 2: Variation of $H$ with $G_m$ ($E_m=3, \varepsilon = 0.1$) |
|---------------------------------------------------------------|
| $X$   | $G_m=1$ | $G_m=5$ |
|-------|--------|--------|
| 0.0   | -2.559 | 2.696  |
| 0.4   | 7.224  | 8.799  |
| 0.8   | -6.173 | 1.889  |

| Table 3: Variation of $H$ with $\sigma^2$ ($G_m=3, E_m=3, \varepsilon = 0.1$) |
|---------------------------------------------------------------|
| $X$   | $\sigma^2=1$ | $\sigma^2=2$ |
|-------|-------------|-------------|
| 0.0   | 4.827       | 10.722      |
| 0.4   | 10.909      | 14.307      |
| 0.8   | -1.844      | 3.829       |

| Table 4: Variation of $H$ with $\varepsilon$ ($G_m=3, E_m=3$) |
|---------------------------------------------------------------|
| $X$   | $\varepsilon =0.1$ | $\varepsilon =0.2$ |
|-------|-----------------|-----------------|
| 0.0   | 1.088           | 2.138           |
| 0.4   | 7.512           | 11.138          |
| 0.8   | 2.858           | 6.830           |

5. Conclusion: This research paper gives a clear idea on variations of pertinent flow characteristics in peristaltic transport of underground fluids. It was experienced that the flux increases with peristalsis. Therefore, as it was predicted earlier, peristalsis brings up good amount of underground fluids without any lubrication. Hence, this study gives a remarkable idea which minimizes a huge expenditure that was incurred for the previous methods of extraction.

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