D-branes from M-branes

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ABSTRACT

The 2-brane and 4-brane solutions of ten dimensional IIA supergravity have a dual interpretation as Dirichlet-branes, or ‘D-branes’, of type IIA superstring theory and as ‘M-branes’ of an $S^1$-compactified eleven dimensional supermembrane theory, or M-theory. This eleven-dimensional connection is used to determine the ten-dimensional Lorentz covariant worldvolume action for the Dirichlet super 2-brane, and its coupling to background spacetime fields. It is further used to show that the 2-brane can carry the Ramond-Ramond charge of the Dirichlet 0-brane as a topological charge, and an interpretation of the 2-brane as a 0-brane condensate is suggested. Similar results are found for the Dirichlet 4-brane via its interpretation as a double-dimensional reduction of the eleven-dimensional fivebrane. It is suggested that the latter be interpreted as a D-brane of an open eleven-dimensional supermembrane.
1. Introduction

The importance of super p-branes for an understanding of the non-perturbative dynamics of type II superstring theories is no longer in doubt. For example, they are relevant to U-duality of toroidally-compactified type II superstrings [1,2], and symmetry enhancement at singular points in the moduli space of $K_3$ or Calabi-Yau compactified type II superstrings [3,4,5,6] as required by the type II/heterotic string-string duality [1,3,7]. Type II p-branes were first found as solutions of the effective D=10 supergravity theory [8,9,10,11]. Because their worldvolume actions involve worldvolume gauge fields [12,13], in addition to the scalars and spinors expected on the basis of spontaneously broken translation invariance and supersymmetry, they were not anticipated in the original classification of super p-branes [14]. For the same reason, the fully D=10 Lorentz covariant action for these type II super p-branes is not yet known. One purpose of this paper is to report progress on this front.

The type II p-branes are conveniently divided into those of Neveu/Schwarz-Neveu/Schwarz (NS-NS) type and those of Ramond-Ramond (RR) type according to the string theory origin of the $(p+1)$-form gauge potential for which they are a source. The supergravity super p-branes found in the NS-NS sector comprise a string and a fivebrane. The string has a naked timelike singularity and can be identified as the effective field theory realization of the fundamental string*. The fivebrane solution is non-singular and has a 5-volume tension $\sim \lambda^{-2}$ expected of a soliton, where $\lambda$ is the string coupling constant. Since a 5-brane is the magnetic dual of a string in D=10 [15], this solution is an analogue of the BPS magnetic monopole of D=4 super Yang-Mills (YM) theory.

In the RR sector the ten-dimensional (D=10) IIA supergravity has p-brane solutions for $p=0,2,4,6$, while the IIB theory has RR p-branes solutions for $p=1,3,5$

* Note that the existence of this solution is necessary for the consistency of any string theory with massless spin 2 excitations since a macroscopic string will then have long range fields which must solve the source free equations of the effective field theory.
With the exception of the 3-brane, which is self-dual, these p-branes come in \((p, \tilde{p})\) electric/magnetic pairs with \(\tilde{p} = 6 - p\). The RR p-brane solutions all have a p-volume tension \(\sim \lambda^{-1}\) so, although non-perturbative, they are not typically solitonic. Moreover, they are all singular, with the exception of the 3-brane, and even this exceptional case is not typical of solitons because the solution has an event horizon [19]. Thus, the RR p-branes are intermediate between the fundamental string and the solitonic fivebrane. It now appears [20] that they have their place in string theory as Dirichlet-branes, or D-branes [21,22].

It was shown in [23] how all the p-brane solutions of D=10 IIA supergravity (with \(p \leq 6\)) have an interpretation in D=11, extending previous results along these lines for the string and fourbrane [24,25,26]. In particular, the 0-branes were identified with the Kaluza-Klein (KK) states of D=11 supergravity and their 6-brane duals were shown to be D=11 analogues of the KK monopoles. The remaining p-brane solutions have their D=11 origin in either the membrane [25] or the fivebrane [27] solutions of D=11 supergravity. It was subsequently shown that D=11 supergravity is the effective field theory of the type IIA superstring at strong coupling [3] and then that various dualities in \(D < 10\) can be understood in terms of the electric/magnetic duality in D=11 of the membrane and fivebrane [28,29]. These results suggest the existence of a consistent quantum theory underlying D=11 supergravity. This may be a supermembrane theory as originally suggested [30], or it may be some other theory that incorporates it in some way. Whatever it is, it now goes by the name ‘M-theory’ [31,32].

The point of the above summary is to show that the RR p-brane solutions of D=10 IIA supergravity theory currently have two quite different interpretations. On the one hand they are interpretable as D-branes of type IIA string theory.

† There is also a IIB 7-brane [16] and a IIA 8-brane [17] (see also [18]), but these do not come in electric/magnetic pairs and have rather different physical implications; for example, they do not contribute to the spectrum of particles in any \(D \geq 4\) compactification. Partly for this reason, only the \(p \leq 6\) cases will be discussed here.
On the other hand they are interpretable as solutions of $S^1$ compactified D=11 supergravity. In the $p = 2$ and $p = 4$ cases these D=11 solutions are also p-branes; since they are presumably also solutions of the underlying D=11 M-theory we shall call them ‘M-branes’. We shall first exploit the interpretation of the $p = 2$ super D-brane as a dimensionally reduced D=11 supermembrane to deduce its D=10 Lorentz covariant worldvolume action. The bosonic action has been found previously [22] by requiring one-loop conformal invariance of the open string with the string worldsheet boundary on the D-brane. One feature of the derivation via D=11 is that the coupling to background fields can also be found this way, and the resulting action has a straightforward generalization to general $p$. The coupling to the dilaton is such that the p-volume tension is $\sim \lambda^{-1}$, as expected for a D-brane [21]. The M-brane interpretation of the Dirichlet 4-brane is as the double-dimensional reduction of the eleven-dimensional fivebrane. We propose a bosonic action for the latter including a coupling to the bosonic fields of eleven-dimensional supergravity, and exploit it to deduce the coupling to background supergravity fields, including the dilaton, of the Dirichlet 4-brane. The result agrees with that deduced by generalization of the $p = 2$ case.

One intriguing feature of these results is that they suggest an interpretation of the eleven-dimensional fivebrane as a Dirichlet-brane of an open D=11 supermembrane, and we further suggest that the string-boundary dynamics is controlled by the conjectured [34], and intrinsically non-perturbative, six-dimensional self-dual string theory (which is possibly related to the self-dual string soliton [35], although this solution involves six-dimensional gravitational fields which are not, according to current wisdom, among the fivebrane’s worldvolume fields).

Finally, we show that a spherical D=10 2-brane can carry the same RR charge that is carried by the Dirichlet 0-branes; this charge is essentially the magnetic charge associated with the worldvolume vector potential. This suggests that the

\[ \uparrow \text{The action of [22] is not obviously equivalent to the bosonic sector of the one found here and the omission of a discussion of this point was a defect of an earlier version of this paper; fortunately, the equivalence has since been established by Schmidhuber [33].} \]
0-branes can be viewed as collapsed 2-branes. We point out that this is consistent with the $U(\infty)$ Supersymmetric Gauge Quantum Mechanics interpretation of the supermembrane worldvolume action [36,37], which further suggests an interpretation of the supermembrane as a condensate of 0-branes. Viewed from the D=11 perspective these results can be taken as further evidence that D=11 supergravity is the effective field theory of a supermembrane theory.

2. The D=10 2-brane as a D=11 M-brane

Consider first the D=10 2-brane. From its D-brane description we know that the worldvolume action is based on the D=10 Maxwell supermultiplet dimensionally reduced to three dimensions [38], i.e. the worldvolume field content is

$$\{X^a (a = 1, \ldots, 7), A_i (i = 0, 1, 2); \chi^I (I = 1, \ldots, 8)\} \tag{2.1}$$

where the $\chi^I$ are eight $Sl(2; \mathbb{R})$ spinors and $A_i$ is a worldvolume vector potential\(^\star\). As for every other value of $p$, only the bosonic part of the 10-dimensional Lorentz covariant action constructed from these fields is currently known [22]. However, the alternative interpretation of the 2-brane as an M-brane allows us to find the complete action. In this interpretation, the IIA 2-brane is the direct (as against double) dimensional reduction of the D=11 supermembrane. The worldvolume fields of the dimensionally reduced D=10 supermembrane are, before gauge-fixing, $\{X^m (m = 0, 1, \ldots, 9); \varphi; \theta\}$, where $\theta$ is a 32-component Majorana spinor of the D=10 Lorentz group and $X^m$ is a 10-vector. After gauge fixing the physical fields are

$$\{X^a (a = 1, \ldots, 7), \varphi; \chi^I (I = 1, \ldots, 8)\} \tag{2.2}$$

The difference between (2.1) and (2.2) is simply that the scalar $\varphi$ of (2.2) is replaced in (2.1) by its 3-dimensional dual, the gauge vector $A$. By performing this duality

\(^\star\) Throughout this paper we shall use the letter $A$ to denote worldvolume gauge fields, of whatever rank, and $B$ to denote spacetime gauge fields, of whatever rank.
transformation in the action prior to gauge fixing we can determine the fully D=10 Lorentz covariant Dirichlet supermembrane action.

The first step of this procedure is to isolate the dependence of the D=11 supermembrane action on $X^{11}$, which is here called $\varphi$. We shall first consider the case for which the D=11 spacetime is the product of $S^1$ with D=10 Minkowski spacetime, returning subsequently to consider the interaction with background fields. It is convenient to use the Howe-Tucker (HT) formulation of the action for which there is an auxiliary worldvolume metric $\gamma_{ij}$. It is also convenient to introduce the spacetime supersymmetric differentials

$$\Pi^m = dX^m - i\bar{\theta}\Gamma^m d\theta.$$  (2.3)

The action, given in [30], is

$$S = -\frac{1}{2} \int d^3\xi \sqrt{-\gamma} \left[ \gamma^{ij} \Pi_i^m \Pi_j^n \eta_{mn} + \gamma^{ij} (\partial_i \varphi - i\bar{\theta}\Gamma_{11} \partial_i \vartheta)(\partial_j \varphi - i\bar{\theta}\Gamma_{11} \partial_j \vartheta) - 1 \right]$$

$$- \frac{1}{6} \int d^3\xi \, \varepsilon^{ijk} [b_{ijk} + 3b_{ij}\partial_k \varphi],$$

(2.4)

where $\eta$ is the D=10 Minkowski metric, and

$$\varepsilon^{ijk} b_{ijk} = 3\varepsilon^{ijk} \left\{ i\bar{\theta}\Gamma_{mn}\partial_t [\Pi_i^m \Pi_j^n \eta_{mn} + i\Pi_i^m (\bar{\theta}\Gamma^m \partial_j \vartheta) - \frac{1}{3}(\bar{\theta}\Gamma^m \partial_i \vartheta)(\bar{\theta}\Gamma^m \partial_j \vartheta)]$$

$$+ (\bar{\theta}\Gamma_{11} \partial_i \vartheta)(\bar{\theta}\Gamma_{11} \partial_j \vartheta)(\partial_k X^m - \frac{2i}{3}\bar{\theta}\Gamma^m \partial_k \vartheta) \right\},$$

(2.5)

while

$$\varepsilon^{ijk} b_{ij} = -2\varepsilon^{ijk} i\bar{\theta}\Gamma_m \Gamma_{11} \partial_t \vartheta (\partial_j X^m - \frac{i}{2}\bar{\theta}\Gamma^m \partial_j \vartheta).$$

(2.6)

The second step, the replacement of the worldvolume scalar $\varphi$ by its dual vector field, can be achieved by promoting $d\varphi$ to the status of an independent worldvolume one-form $L$ while adding a Lagrange multiplier term $AdL$ to impose the constraint $dL = 0$. Eliminating $L$ by its algebraic equation of motion yields the
dual action in terms of the fields $X^m$ and the worldvolume field strength two-form $F = dA$. This action is

$$S = -\frac{1}{2} \int d^3 \xi \sqrt{-\gamma} \left[ \gamma^{ij} \Pi_i^m \Pi_j^n \eta_{mn} + \frac{1}{2} \gamma^{ik} \gamma^{jl} \hat{F}_{ij} \hat{F}_{kl} - 1 \right]$$

$$- \frac{1}{6} \int d^3 \xi \varepsilon^{ijk} \left[ b_{ijk} - 3i(\bar{\theta} \Gamma_{11} \partial_i \theta) \hat{F}_{jk} \right].$$

(2.7)

where

$$\hat{F}_{ij} = F_{ij} - b_{ij}.$$  

(2.8)

Thus (2.7) is the fully D=10 Lorentz covariant worldvolume action for the D=10 IIA Dirichlet supermembrane. The bosonic action, obtained by setting the fermions to zero in (2.7), is equivalent to the Born-Infeld-type action found by Leigh [22]. The equivalence follows from the recent observation of Schmidhuber [33] that dualizing the vector to a scalar in the action of Leigh yields the action of a D=11 membrane, which was precisely the (bosonic) starting point of the construction presented here*. It is interesting to note that a sigma-model one-loop calculation in the string theory is reproduced by the classical supermembrane.

It can now be seen why it was advantageous to start from the HT form of the action; whereas the auxiliary metric is simply eliminated from (2.4), leading to the standard Dirac-Nambu-Goto (DNG) form of the action, its elimination from (2.7) is far from straightforward, although possible in principle. The point is that the $\gamma_{ij}$ equation is now the very non-linear, although still algebraic, equation

$$\gamma_{ij} = \left( 1 + \frac{1}{2} \gamma^{kp} \gamma^{lq} \hat{F}_{kl} \hat{F}_{pq} \right)^{-1} \left( g_{ij} + \gamma^{kl} \hat{F}_{ik} \hat{F}_{lj} \right)$$

(2.9)

where $g_{ij} = \Pi_i^m \Pi_j^n \eta_{mn}$. This equation can be solved as a series in $\hat{F}$ of the form

$$\gamma_{ij} = g_{ij} \left[ 1 - \frac{1}{2} g^{kp} g^{lq} \hat{F}_{kl} \hat{F}_{pq} \right] + g^{kl} \hat{F}_{ik} \hat{F}_{jl} + O(\hat{F}^4),$$

(2.10)

and the approximation $\gamma_{ij} = g_{ij}$ yields the quadratic part of the action in $\hat{F}$.

* The equivalence with Born-Infeld for $p=1$, i.e. the D-string, was shown in [39]
Invariance of the action (2.7) under supersymmetry requires \( \hat{F} \) to be invariant. To see how this comes about, we observe that the two-form \( b \) in \( \hat{F} \) is precisely the one that defines the WZ term in the Green-Schwarz superstring action; it has the property that the three-form \( h = db \) is superinvariant, which implies that \( \delta_\epsilon b = da \) for some one-form \( a(\epsilon) \), where \( \epsilon \) is the (constant) supersymmetry parameter. The modified two-form field strength \( \hat{F} \) is therefore superinvariant if we choose \( \delta_\epsilon A = a \). The \( \kappa \)-transformation of \( A \) is similarly determined by requiring \( \kappa \)-gauge invariance of the action, but it can also be deduced directly from those of the D=11 supermembrane given in [30]. The result is most simply expressed in terms of the variations of the supersymmetric forms \( \Pi^m \) and \( \hat{F} \), which are†

\[
\delta_\kappa \Pi^m = -2i(\delta_\kappa \bar{\theta}) \Gamma^m d\theta \\
\delta_\kappa \hat{F} = i(\delta_\kappa \bar{\theta}) \Gamma^m_\epsilon \Gamma_{11} d\theta \wedge \Pi^m \\
\delta_\kappa \theta = (1 + \Gamma)\kappa,
\]

where

\[
\Gamma = \frac{1}{6\sqrt{-\gamma}} \varepsilon^{ijk} \Pi^m_i \Pi^p_j \Pi^q_k \Gamma_{mnp} - \frac{1}{2} \gamma^{ik} \gamma^{jl} F_{kl} \Pi^m_i \Pi^p_j \Gamma_{mn} \Gamma_{11}
\] (2.12)

and \( \kappa(\xi) \) is the D=10 Majorana spinor parameter.

The coupling of the action (2.7) to background fields can also be deduced from its D=11 origin. We shall consider here only the bosonic membrane coupled to bosonic background fields. Consider first the NS-NS fields. In the D=10 membrane action obtained by direct dimensional reduction from D=11, the NS-NS two-form potential \( B \) couples to the topological current \( \varepsilon^{ijk} \partial_k \varphi \). In the dual action this coupling corresponds to the replacement of \( F \) by \( F - B \). The coupling to the D=10 spacetime metric is obvious so this leaves the dilaton; to determine its coupling we

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† As explained in detail in [40], it is not necessary to specify the transformation of the metric \( \gamma_{ij} \) if use is made of the ‘1.5 order’ formalism.
recall (see e.g. [26]) that the D=11 metric is

$$ds_{11}^2 = e^{-\frac{2}{3}\phi} ds^2 + e^{\frac{4}{3}\phi} d\phi^2 ,$$  \(2.13\)

where $ds^2$ is the string-frame D=10 metric and $\phi$ is the dilaton. A repetition of the steps described above, but now for the purely bosonic theory and carrying along the dependence on the NS-NS-spacetime fields, leads (after a redefinition of the auxiliary metric to the action

$$S = -\frac{1}{2} \int d^3 \xi \ e^{-\phi} \sqrt{-\gamma} \left[ \gamma^{ij} g_{ij} + \frac{1}{2} \gamma^{ik} \gamma^{jl} (F_{ij} - B_{ij}) (F_{kl} - B_{kl}) - 1 \right] ,$$  \(2.14\)

where now $g_{ij} = \partial_i X^m \partial_j X^n g_{mn}$. The appearance of $F$ through the modified field strength $F - B$ could also have been deduced simply by the replacement of the flat superspace two-form potential $b$ in $\hat{F}$ by its curved superspace counterpart, since setting the fermions to zero then yields precisely $F - B$. As for the RR fields, the coupling to the 3-form potential is of course the standard Lorentz coupling while the coupling to the 1-form potential has interesting implications which will be discussed at the conclusion of this article.

The above result, and the known form of the bosonic p-brane action in the absence of worldvolume gauge fields, suggests that the corresponding bosonic part of the worldvolume action of the Dirichlet super p-brane is

$$S = -\frac{1}{2} \int d^{(p+1)} \xi \ e^{-\phi} \sqrt{-\gamma} \left[ \gamma^{ij} g_{ij} + \frac{1}{2} \gamma^{ik} \gamma^{jl} (F_{ij} - B_{ij}) (F_{kl} - B_{kl}) -(p-1) \right] .$$  \(2.15\)

Since the vacuum expectation value of $e^\phi$ is the string coupling constant $\lambda$, it follows from this result that the p-brane tension is $\sim \lambda^{-1}$, as expected for D-branes. Of course, the steps leading to this result were particular to $p = 2$ but we shall shortly arrive at the same result for $p = 4$ via a different route. Although the action (2.15) is only guaranteed to be correct to quadratic order in $F$ for $p \neq 2$, this will prove sufficient for present purposes.
3. The D=11 5-brane as a supermembrane D-brane

Consider now the Dirichlet 4-brane. In this case its M-brane interpretation is as a double-dimensional reduction of the D=11 5-brane. The (partially) gauge-fixed field content of the latter consists [41,42] of the fields of the N=4 six-dimensional antisymmetric tensor multiplet, i.e.

\[
\{X^a (a = 1, \ldots, 5), A^+_{ij} (i, j = 0, 1, \ldots, 5); \chi^I (I = 1, \ldots, 4)\}
\]

(3.1)

where \(\chi^I\) are chiral symplectic-Majorana spinors in the 4 of \(USp(4) \cong Spin(5)\), and \(A^+\) is the two-form potential for a self-dual 3-form field strength \(F = dA^+\). Because of the self-duality of \(F\) we cannot expect to find a worldvolume action (at least, not one quadratic in \(F\)). We might try to find an action that leads to all equations except the self-duality constraint which we can then just impose by hand, as advocated elsewhere in another context [43]. We shall adopt this strategy here, but it is important to appreciate an inherent difficulty in its present application. The problem is that the self-duality condition involves a metric and it is not clear which metric should be used, e.g. the induced metric or the auxiliary metric; the possibilities differ by higher order terms in \(F\). Because of this ambiguity we should consider the action as determining only the lowest order, quadratic, terms in \(F\). With this proviso, an obvious conjecture for the D=11 5-brane action is

\[
S = -\frac{1}{2} \int d^6 \xi \sqrt{-\gamma} \left[ \gamma^{ij} \partial_i X^M \partial_j X^N \eta_{MN} + \frac{1}{2} \gamma^{il} \gamma^{jm} \gamma^{kn} F_{ijk} F_{lmn} - 4 \right],
\]

(3.2)

where the fields \(X^M, M = (0, 1, \ldots, 10)\), are maps from the worldvolume to the D=11 Minkowski spacetime. This action has an obvious coupling to the bosonic fields \((g_{MN}, B_{MNP})\) of D=11 supergravity*. The coupled action is

\[
S = -\frac{1}{2} \int d^6 \xi \sqrt{-\gamma} \left[ \gamma^{ij} g^{(11)}_{ij} + \frac{1}{2} \gamma^{il} \gamma^{jm} \gamma^{kn} (F_{ijk} - B_{ijk}) (F_{lmn} - B_{lmn}) - 4 \right]
\]

(3.3)

where \(g^{(11)}_{ij}\) is the pullback of the 11-metric \(g_{MN}\) and \(B_{ijk}\) is the pullback of the 3-form potential \(B_{MNP}\). Up to quartic terms in \(F_{ij}\), and setting to zero the RR

* although consistency with the self-duality condition is now problematic.
spacetime fields, the double dimensional reduction of (3.3) to D=10 reproduces the action (2.15) with p=4, as required for the M-brane interpretation of the Dirichlet 4-brane. In particular, the dilaton dependence is exactly as given in (2.15).

The worldvolume vector of the D=10 Dirichlet p-branes allows not only a coupling to the 2-form potential of string theory but also to the endpoints of an open string via a boundary action [21,22,44]. Let \( X^m(\sigma, \tau) \) be the locus in spacetime of the string’s worldsheet, with boundary at \( \tau = 0 \). If this boundary lies in the worldvolume of a p-brane, then

\[
X^m(\sigma, \tau) \big|_{\tau=0} = X^m(\xi(\sigma)) ,
\]

where \( X^m(\xi) \) is the locus in spacetime of the p-brane’s worldvolume. It is also convenient to introduce the conjugate momenta to the worldsheet scalar fields at the worldsheet boundary, \( \pi_m \), defined by

\[
\pi_m(\sigma) = \sqrt{-g} g_{mn}(X(\sigma, \tau)) \left. \frac{dX^n(\sigma, \tau)}{d\tau} \right|_{\tau=0} .
\]

The D=10 Lorentz covariant boundary action can then be written as

\[
S_b(string) = \oint d\sigma \left[ A_i(\xi(\sigma)) \frac{d\xi^i(\sigma)}{d\sigma} + X^m(\xi(\sigma))\pi_m(\sigma) \right] .
\]

Similarly, the worldvolume antisymmetric tensor \( A^+ \) of the D=11 5-brane allows not only a coupling to the 3-form potential of D=11 supergravity but also to the boundary of an open membrane. Let \( X^M(\sigma, \rho, \tau) \) be the locus in the D=11 spacetime of the membrane’s worldvolume, with boundary at \( \tau = 0 \). If this boundary lies in the worldvolume of a fivebrane, with coordinates \( \xi^i \), then

\[
X^M(\sigma, \rho, \tau) \big|_{\tau=0} = X^M(\xi(\sigma, \rho)) ,
\]

where \( X^M(\xi) \) is the locus in spacetime of the fivebrane’s worldvolume. Defining, as before, the conjugate momenta \( \pi_M \) to the membrane scalar fields at the membrane’s
boundary, we can write down the following natural generalization of (3.6):

$$S_b(membrane) = \oint d\sigma d\rho \left[ A_{ij}^b(\xi) \frac{d\xi^i}{d\sigma} \frac{d\xi^j}{d\rho} + X^m(\xi)\pi_M \right]. \quad (3.8)$$

Moreover, the double-dimensional reduction of this membrane boundary action reproduces the string boundary action (3.6). This suggests that we interpret the D=11 5-brane as a Dirichlet-brane of an underlying open supermembrane. It seems possible that the dynamics of the membrane boundary in the fivebrane’s world-volume might be describable by a six-dimensional superstring theory, which one would expect to have N=2 (i.e. minimal) six-dimensional supersymmetry (e.g. on the grounds that it is a ‘brane within a brane’ [45]). However, since the 3-form field strength to which this boundary string couples is self-dual, this superstring theory would be, like the supermembrane itself, intrinsically non-perturbative. The existence of such a new superstring theory was conjectured previously [34] in a rather different context.

4. 0-branes from 2-branes and 2-branes from 0-branes.

One of the properties expected of the D=11 supermembrane theory or M-theory is that it have D=11 supergravity as its effective field theory. Various arguments for and against this have been given previously ([23] contains a recent brief review). A further argument in favour of this idea is suggested by the recent results of Witten concerning the effective action of n coincident Dirichlet p-branes [38]. He has shown that the (partially gauge-fixed) effective action in this case is the reduction from D=10 to (p + 1) dimensions of the $U(n)$ D=10 super Yang-Mills (YM) theory. Consider the 0-brane case for which the super YM theory is one-dimensional i.e. a model of supersymmetric gauge quantum mechanics (SGQM). If the 0-branes condense at some point then the effective action will be the $n \to \infty$ limit of a $U(n)$ SGQM. But this is just another description of the supermembrane! It is amusing to note that the continuity of the spectrum of the quantum supermembrane [46], in
the zero-width approximation appropriate to its D-brane description, might now be understood as a consequence of the zero-force condition between an infinite number of constituent 0-branes. However, it is known that quantum string effects cause the D-brane to acquire a finite size core [47], consistent with its M-brane interpretation as a solution with an event horizon [26], and it was argued in [23] that this fact should cause the spectrum to be discrete.

Actually, the supermembrane was usually stated as being equivalent to an $SU(\infty)$ SGQM model [36,37], but the additional $U(1)$ is needed to describe the dynamics of the centre of mass motion. Note that a $U(1)$ SGQM is precisely the action for a Dirichlet 0-brane. This suggests that there might exist some classical closed membrane configuration for which the ground state, on quantization, could be identified with the 0-brane. For this to be possible it would be necessary for the closed membrane to carry the RR charge associated with the 0-branes. We now explain how this can occur.

From the D=11 point of view the RR 0-brane charge is just the KK charge, i.e. the electric charge that couples to the KK vector field, which we shall here call $B_m$. The coupling of $B_m$ to the D=10 membrane can be found by dimensional reduction from D=11. To leading order this coupling has the standard Noether form $B_m \mathcal{J}^m$, where
\[ \mathcal{J}^m(x) = \int d^3\xi \sqrt{-\gamma} \gamma^{ij} \partial_i X^m \partial_j \varphi \delta^{10}(x - X(\xi)) . \] (4.1)
is the KK current density. After dualization of the scalar field this becomes
\[ \mathcal{J}^m(x) = \int d^3\xi \varepsilon^{ijk} \partial_i X^m F_{jk} \delta^{10}(x - X(\xi)) . \] (4.2)
The total KK charge is $Q \equiv \int d^9 x \mathcal{J}^0$. Choosing the $X^0 = \xi^0$ gauge one readily sees that
\[ Q = \oint F , \] (4.3)
i.e. the integral of the worldvolume 2-form field strength $F$ over the closed membrane.
Thus, a closed membrane can carry the 0-brane RR charge as a type of magnetic charge associated with its worldvolume vector field, and its centre of mass motion is described by the 0-brane $U(1)$ SQGM. This can be interpreted as further evidence that the 0-brane is included in the (non-perturbative) supermembrane spectrum. However, from the D=11 point of view the 0-brane is just a massless quantum of D=11 supergravity and supersymmetry implies the existence of all massless quanta given any one of them. Thus, we have found a new argument that the spectrum of the D=11 supermembrane (or, perhaps, M-theory) should include the massless states of D=11 supergravity.

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