Thermal diffusivity effect in opto-thermal skin measurements

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Abstract. We present our latest study on the thermal diffusivity effect in opto-thermal skin measurements. We discuss how thermal diffusivity affects the shape of opto-thermal signal, and how to measure thermal diffusivity in opto-thermal measurements of arbitrary sample surfaces. We also present a mathematical model for a thermally gradient material, and its corresponding opto-thermal signal. Finally, we show some of our latest experimental results of this thermal diffusivity effect study.

1. Introduction
Opto-thermal transient emission radiometry (or OTTER) \cite{1,2} is an infrared remote sensing technique that has been used in skin hydration \cite{3-5}, skin pigments \cite{6}, and trans-dermal drug delivery \cite{7} measurements. It uses a pulsed laser (Er: YAG, 2.94 $\mu$m, 100ns pulse duration, 1mm spot size, <4mJ/pulse) as heat source to heat up the sample surface, and fast infrared detector (Mercury Cadmium Tellurium, 13.1 $\mu$m) to pick up the consequent blackbody radiant changes. The shape of measurement signal is dependent on sample’s optical properties, thermal properties, and its layer structure. To date, all our studies are focused on the optical properties of the samples, which are assumed thermally homogeneous. However, for samples like in-vivo human skin, we know it is not thermally homogeneous, and we know that its thermal properties will change when the water content in skin changes\cite{8}. Also, the thermal properties at skin surface are different from that at deep inside. In this paper, we will investigate the thermal diffusivity effect in opto-thermal skin measurements. We will discuss how thermal diffusivity will affect the shape of opto-thermal signal, and how to measure thermal diffusivity in opto-thermal measurements.

2. Theory
For semi-infinite homogeneous materials, after the laser pulse, the temperature field within the sample can be described by following partial differential equation, where $\alpha$ is the optical absorption coefficient for the excitation light, $D(z)$ is sample’s thermal diffusivity, and $\theta(z,t)$ is the temperature field at depth $z$ and time $t$ within the sample, $C$ is sample’s specific heat, $\rho$ is sample’s density, and $E_\alpha$ is the energy density absorbed from the excitation laser pulse.
When $D(z)$ is a constant, the above equation can be easily solved and the opto-thermal signals can then be calculated by [1,2],

$$S(t) = \zeta \int_0^\tau \beta e^{-\beta z} \theta(z,t) dz$$

$$= A e^{\zeta t} \text{erfc}(\sqrt{\pi/\tau}) \quad (\text{when } \alpha \gg \beta)$$

where $A = \frac{E_0 \xi \beta}{\rho C}$ is the amplitude of the signal, $\beta$ is the optical absorption coefficient for the emission light, and $\tau = \frac{1}{\beta' D}$ is the decay lifetime of the signal. Parameter $\zeta = \zeta(\lambda_{\text{em}})$ includes factors that depend on the black body emission curve, detector sensitivity, focusing and alignment, but is independent of the properties of the sample per se.

2.1. Optical Limit and Thermal limit of the Opto-thermal Signal

When $t \to 0$, opto-thermal signal Eq.(2) can be expanded as a power of series

$$\lim_{t \to 0} S(t) = A \left\{ 1 - \frac{2}{\sqrt{\pi}} \left( \sqrt{\frac{1}{\tau}} + \frac{2}{3} \left( \sqrt{\frac{1}{\tau}} \right)^3 + \frac{4}{15} \left( \sqrt{\frac{1}{\tau}} \right)^5 + \ldots \right) \right\}$$

Eq(3) shows that there more optical information in the early part of the signal, and the initial amplitude, $S(0)$, only depends upon sample’s optical properties and volumetric specific heat. This is called optical limit. This is a unique feature of impulse excitation. When $t \to \infty$, Eq. (2) can be expressed as following power of series

$$\lim_{t \to \infty} S(t) = \frac{E_0 \xi}{\rho C \sqrt{\pi D t}} \left\{ 1 - \frac{\tau}{2t} + \frac{3\tau^2}{2t^2} - \frac{15\tau^3}{8t^3} + \ldots \right\}$$

Eq(4) shows that the thermal properties will dominate the opto-thermal signal at long times, until the limiting $\frac{1}{\sqrt{D t}}$ law of uni-dimensional diffusion is obtained. In this limit, the signal does not depend on the optical properties of the material, but is inversely proportional to its thermal diffusivity, therefore it is called thermal limit of the impulse response curve. Optical limit and thermal limit provide the fundamental principles for our data analysis, as illustrated in Figure 1A. The results show that, at longer time (>10s), the opto-thermal signals are clearly dominated by thermal properties.

2.2. Opto-thermal Signal of Thermal Gradient Material

When $D(z)$ is not constant, say $D(z) = D + w_D z$, a linear thermal gradient, Eq(1) can no longer be solved analytically. However, we can numerically solve the $\theta(z,t)$ from Eq(1) using Finite Element Methods, and then use Eq(2) to calculate the corresponding opto-thermal signals. Figure 1B shows the opto-thermal signals of thermal gradient material at different thermal gradient $w_D$ and $D$. Apparently, thermal gradient also affect the shapes of opto-thermal signals.
Figure 1 Optical limit and thermal limit of the opto-thermal signal (A) and opto-thermal signals of thermal gradient material (B) at different thermal gradient \( D \) and \( \text{Dw} \). The calculation parameters used are \( \beta = 2.9 \times 10^3 \text{m}^{-1} \) and \( D = 10^{-7} \text{m}^2/\text{s} \).

In principle, we can measure sample’s optical properties using signal’s initial amplitude, and measure sample’s thermal diffusivity and possibly its thermal gradient using longer time signal. However, in practice, accurate initial amplitude measurements and longer time OTTER measurements (>1s) are very difficult due to hardware limitations. To get around this problem and get thermal information, we have developed a new method. First, we analyze the opto-thermal signals by least squares fitting using Eq.(2) to yield best fit \( \tau \) and \( \alpha \); then, from value \( \alpha \), by using pure water signal as a reference and assuming that parameters \( (E_a, \zeta) \) do not change during a set of measurements, we can get a relative value of \( \beta \), denoted \( \beta* \), where \( \beta* \propto \beta \). Finally, we can get a relative value of thermal diffusivity \( D \) from value \( \tau \), denoted \( D* = 1/\beta*^2 \tau \), apparently \( D* \propto D \).

3. Experimental Results and Discussions

3.1. Thermal Diffusivity of Nail at Different Water Concentration

The thermal diffusivity measurements of nail at different water concentration are performed by soaking the left index finger nail in water for 10 minutes. The finger nail is carefully pat dried using a soft tissue after soaking. Opto-thermal measurements are performed before the hydration, and subsequently after, over a period of 25 minutes. Figure 2A shows the thermal diffusivity \( D* \) and the optical absorption coefficient for the emission light \( \beta* \) of left index finger nail before and after the 10 minutes immersive hydration. Both \( D* \) and \( \beta* \) increase significantly even after just 10 minutes of soaking, then as the nail gradually lost it excess water to ambient air, \( D* \) and \( \beta* \) gradually recover back to their normal levels, the recovery time is about 25 minutes.

3.2. Thermal Diffusivity of Finger Skin at Different Water Concentration

The thermal diffusivity measurements of finger skin at different water concentration are performed on the left index finger, which is soaked in water for 30 minutes until wrinkled. Again, the finger skin is carefully pat dried using a soft tissue after soaking. Opto-thermal measurements are performed before and after the hydration. Figure 2B shows the thermal diffusivity \( D* \) and the optical absorption coefficient for the emission light \( \beta* \) of left index finger nail before and after the 10 minutes immersive hydration. Both \( D* \) and \( \beta* \) increase significantly even after just 10 minutes of soaking, then as the finger gradually recover itself under ambient conditions, \( D* \) and \( \beta* \) gradually recover back to their normal levels, the recovery time is about 70 minutes.

3.3. Discussions

When using 13.1 \( \mu \text{m} \) wavelength detection interference filter, \( \beta \) is proportional to skin hydration (\%, v/v). Using the first order linear approximation, we have [4,5]

\[
\beta = \beta_\omega \cdot H + \beta_\delta \cdot (100 - H)
\]  

(5)
where $\beta_w$ is the optical absorption coefficient of pure water, and $\beta_d$ is the optical absorption coefficient of dry skin. Using the value of $\beta_w = 2.9 \times 10^3 \, \text{m}^{-1}$, $\beta_d = 1 \times 10^4 \, \text{m}^{-1}$ [4,5], we can then work out the skin hydration from $\beta$. By plotting thermal diffusivity $D^*$ against hydration $H$, we can also work out how thermal diffusivity changes against skin hydration. Figure 2C shows the hydration dependent thermal diffusivity of left index finger skin and its finger nail. Although the result is based on crude approximations, it is the first attempt, and it does show how different sample’s thermal diffusivities depend differently on hydration levels.

**Figure 2** $D^*$ and $\beta^*$ values of left index finger nail after 10 minutes immersive hydration (A), $D^*$ and $\beta^*$ values of left index finger after 30min immersive hydration (B), thermal diffusivity of left index finger skin and left index finger nail at different hydration levels (C).

### 4. Conclusions and Future Work

We have investigated the thermal diffusivity effect in opto-thermal skin measurements, and showed how the thermal diffusivity and thermal gradient can affect the signal shape. We have also developed a method for getting thermal information in opto-thermal measurements. The next step is to develop new hardware in order to make it possible for more accurate initial amplitude measurements and longer time opto-thermal measurements.

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