New pathway to bypass the $^{15}\text{O}$ waiting point

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We propose the sequential reaction process $^{15}\text{O}(p,\gamma)(\beta^+)^{16}\text{O}$ as a new pathway to bypass of the $^{15}\text{O}$ waiting point. This exotic reaction is found to have a surprisingly high cross section, approximately $10^{10}$ times higher than the $^{15}\text{O}(p,\beta^+)^{16}\text{O}$. These cross sections were calculated after precise measurements of energies and widths of the proton-unbound $^{16}\text{F}$ low lying states, obtained using the $^{15}\text{O}(p,p)\gamma^{15}\text{O}$ reaction. The large $(p,\gamma)(\beta^+)$ cross section can be understood to arise from the more efficient feeding of the low energy wing of the ground state resonance by the gamma decay. The implications of the new reaction in novae explosions and X-ray bursts are discussed.

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Unbound nuclei play a major role in astrophysics. The proton-unbound $^4\text{He}$ and the alpha-unbound $^3\text{Be}$ nuclei illustrate this fact. The former is involved in the $p(p,\beta^+)d$ reaction, first reaction of the $pp$ chain of reactions governing the energy generation in the sun. The latter, whose lifetime is about $10^{-16}$ seconds, is involved in the triple alpha reaction which is at the origin of the formation of all the heavier elements. The proton-unbound nuclei $^{15}\text{F}$ and $^{16}\text{F}$ play an important role in X-ray bursts. These astronomical events are known to happen in close binary systems, where accretion takes place from an extended companion star on the surface of a neutron star (type I X-ray burst). The accumulated material is compressed until it reaches sufficiently high pressure conditions to trigger a thermonuclear runaway. In these explosive events, the carbon and nitrogen elements are mainly transformed into $^{14}\text{O}$ and $^{15}\text{O}$ by successive proton captures. Then, the pathway for new proton captures is hindered by the proton-unbound nuclei $^{15}\text{F}$ and $^{16}\text{F}$. The reaction flux and the energy generation are then limited by the relatively slow $\beta^+$-decay of $^{14}\text{O}$ ($t_{1/2}=71$ s) and $^{15}\text{O}$ ($t_{1/2}=122$ s), which create waiting points. The sudden and intense release of energy observed in X-ray bursts requires to circumvent the limited energy generation in breakout reactions. The $^{15}\text{O}(p,\gamma)^{16}\text{Ne}$ reaction is considered to be one of the key reactions in this context. It makes the transition into the nucleosynthetic rp process (rapid proton capture) which is responsible for an increased rate of energy generation and the synthesis of heavier elements. In such explosive environments, $^{16}\text{F}$ is strongly populated in the ground state (g.s.) or in the first excited state, and leads to an equilibrium between formation and decay of this proton-unbound nucleus. From time to time before the proton is emitted, $^{16}\text{F}$ can capture another proton thus producing the $^{17}\text{Ne}$ particle stable isotope. This two-proton capture process was calculated to be significant for extreme densities (larger than $10^{11}$ g/cm$^3$). In this letter, $\beta^+$-decay of $^{16}\text{F}$ to $^{16}\text{O}$ is proposed as an alternative channel. Two reactions eventually proceed through the $\beta^+$-decay of the intermediate unbound $^{16}\text{F}$ g.s., which is fed directly by a proton capture or indirectly through a proton capture to the first excited state followed by a $\gamma$-emission. The $\gamma$-decay occurs noticeably to the low energy wing of the g.s. resonance. Subsequent proton emission is dramatically hindered due to the fact that the low energy proton has to tunnel through the Coulomb potential of the $^{15}\text{O}$ nucleus. These reaction channels have not been investigated so far and could speed-up the energy generation, competing with breakout reactions. The calculation of these reaction cross sections require the measurement of the energies, widths, spins and parities of the low lying states of $^{16}\text{F}$. These were obtained from the measurement of the $^{15}\text{O}(p,\gamma)^{15}\text{O}$ resonant elastic excitation function using low energy $^{15}\text{O}$ beam at the SPIRAL facility.

The beam of radioactive $^{15}\text{O}$ nuclei was produced at the SPIRAL-GANIL facility through the projectile fragmentation of a 95 A.MeV $^{16}\text{O}$ primary beam on a thick carbon target. Mean intensities of 10$^7$pps at an energy of 1.2 A.MeV were obtained after post acceleration by the CIME cyclotron. A beam contamination of less than 1% of $^{15}\text{N}$ was achieved using a vertical betatron oscillation.
selection device [9] and a suitable degrader in the analysis line of LISE spectrometer [9] where the measurements were made. Two stable beams, $^{14}$N and $^{15}$N, were also used in similar experimental conditions for calibrations. The excitation function for the elastic scattering at these low energies can be described by the Rutherford scattering, but shows "anomalies", i.e. various resonances that are related to individual states in the compound nucleus. The principle of the measurement is described in [10, 11] and references therein. A 31(1) μm thick polyethylene (CH$_2$)$_n$ target was used, thick enough to stop the beam inside. The scattered protons were detected by a silicon detector, placed at forward angles (180° in the center of mass frame) within an angular acceptance of 2°. Protons were identified using their energy and time-of-flight. The energy resolution was 3 keV in the center of mass frame. Fig. 1 shows the excitation function for the H($^{15}$O,$p$)$^{15}$O reaction measured from 0.450 MeV to 1.1 MeV. The measured cross section was reproduced by an R-matrix calculation using the code ANARKI [12] which is seen to be in a good agreement with the data. A value of $S_p = -534 \pm 5$ keV was obtained for the proton separation energy in agreement with the recommended value [14].

The R-matrix analysis was also used to extract the properties of the first three states in $^{16}$F, given in Table I. A significant difference was found between the present and the recommended value of the width for the first excited state [14]. This width is an important parameter used in the calculations in the next section.

The calculation of the $^{15}$O($p$,$\beta^+$)$^{16}$O cross section was made using the properties of $^{16}$F g.s. resonance measured in the present work and the Breit-Wigner formula for a single-level resonance [13]:

\[
\sigma(E_p) = \pi\lambda^2 \frac{2J_f + 1}{(2J_i + 1)(2J_f + 1)} \frac{\Gamma_{in} \Gamma_{out}}{(E_p - E_R)^2 + (\frac{\Gamma_{total}}{2})^2} 
\]

FIG. 1: Excitation function for the H($^{15}$O,$p$)$^{15}$O reaction at 180° in the center of mass frame. The line is a result of an R-matrix calculation using parameters from Table I.

where $\lambda$ is the de Broglie wavelength, $J$ are the spins, and $E_R, \Gamma_{total}, \Gamma_{in}, \Gamma_{out}$ are the resonance energy, total width, and partial widths of the incoming and outgoing channels. In the ($p$,β$^+$) case, $E_F = E_{g.s.}$ the energy of the g.s. resonance, $\Gamma_{in} = \Gamma_{p}^{g.s.}$ the proton width, and $\Gamma_{out}$ corresponds to the $\beta^+$-decay partial width. The energy dependence of the proton width $\Gamma_{p}^{g.s.}(E_p)$ for the incoming channel was taken into account by using the relation:

\[
\Gamma_{p}^{g.s.}(E_p) = \Gamma_{p}^{g.s.}(E_{g.s.}) \frac{P(E_p)}{P(E_{g.s.})} 
\]

where $\Gamma_{p}^{g.s.}(E_{g.s.})$ is the proton width at the resonance energy and $P(E_p)$ is the penetrability function under the Coulomb potential barrier. A partial lifetime for $^{16}$F($\beta^+$) of 1 second and a negligible branching ratio to the $^{15}$O($p$,β$^+$)$^{16}$C+α final decay channel were assumed. This assumption is supported by the $\beta^+$-decay properties measured in the mirror nucleus $^{16}$N [14]. The $\beta^+$-decay partial width was taken as a constant since the energy dependence of the Fermi function is small due to large $Q_{\beta^+}=15417(8)$ keV [14]. The calculated $^{15}$O($p$,$\beta^+$)$^{16}$O cross section is shown in Fig. 2 as a function of the center of mass energy. The maximum of the cross section is observed at the energy of 534 keV corresponding to the $^{16}$F g.s. resonance. At this energy the ($p$, $\beta^+$) cross section is very small, about $10^{-20}$ barns, since $^{16}$F mainly decays by proton emission, which is $\approx 10^{20}$ times stronger than the $\beta^+$-decay (since $\Gamma_{p}^{g.s.}(E_{g.s.}) = 25$ keV and $\Gamma_{out} = 0.66 \times 10^{-18}$ keV).

The calculation of the $^{15}$O($p$,$\gamma$)$^{16}$O reaction was performed in a sequential manner, a schematic represen-

TABLE I: Measured properties for the low-lying states in $^{16}$F.

| $E_{CM}$ (keV) | $E_p$ (keV) | $E_x$ (keV) | $E_1$ (keV) | $J^{+}$ | $\Gamma_p$ (keV) | $\Gamma_x$ (keV) |
|---------------|------------|------------|------------|-------|----------------|-----------------|
| 534 ± 5      | 0          | 0          | 40 ± 20    | 25 ± 5
| 732 ± 10     | 193 ± 6   | 198 ± 10   | 40 < 40    | 70 ± 5
| 958 ± 2      | 424 ± 5   | 425 ± 2    | 21 ± 30    | 6 ± 3

$^a$Recommended values $^b$This work.

FIG. 2: $^{15}$O($p$,$\beta^+$)$^{16}$O and $^{15}$O($p$,$\gamma$)$^{16}$O reaction cross sections are shown as a function of the center of mass energy.
tation of this reaction is shown in Fig. 3. A proton capture reaction to the first excited state of $^{16}\text{F}$ is considered, followed by a gamma decay to the g.s. resonance, from which a $\beta^+$-decay branching ratio is taken into account. The cross section $\sigma_{p,\gamma}\beta(E_p)$ for the $(p,\gamma)(\beta^+)$ reaction at the energy $E_p$ is an integration of the differential cross section over all possible energies of the gamma transition (since the g.s. has a large width):

$$\sigma_{p,\gamma}\beta(E_p) = \int \sigma_{p,\gamma}(E_p, E_\gamma)P_\gamma(E_\gamma)P_\beta(E_p, E_\gamma)dE_\gamma \quad (3)$$

where $\sigma_{p,\gamma}(E_p, E_\gamma)$ is the cross section to capture the proton at the energy $E_p$ and to emit a gamma ray with an energy $E_\gamma$, $P_\gamma(E_\gamma)dE_\gamma$ is the strength function [2, 15], that is the probability for the gamma ray to have an energy between $E_\gamma$ and $E_\gamma + dE_\gamma$, and $P_\beta(E_p, E_\gamma)$ is the branching ratio function for the $^{16}\text{F}$ nucleus to decay by $\beta^+$-ray emission.

The first term $\sigma_{p,\gamma}(E_p, E_\gamma)$ is calculated using a Breit-Wigner formula with the following parameters $E_1$, $\Gamma_{Ttot}(E_p, E_\gamma)$, $\Gamma_p^1(E_\gamma)$, $\Gamma_p^1(E_\gamma)$ being the energy, total width, proton width and gamma width for the resonance corresponding to the first excited state of $^{16}\text{F}$. The gamma-ray is emitted from a $1^-$ state to the 0$^-$ g.s., which corresponds to a M1 transition, whose energy dependence of the gamma width $\Gamma_p^1(E_\gamma)$ is:

$$\Gamma_p^1(E_\gamma) = \Gamma_p^1(E_1 - E_{g.s.})\left\{\frac{E_\gamma}{E_1 - E_{g.s.}}\right\}^3 \quad (4)$$

A gamma lifetime of 1 ps was obtained from the mirror nucleus [14], which corresponds to the partial width $\Gamma_p^1(E_1 - E_{g.s.}) = 0.661 \times 10^{-3}$ eV. The strength function of the $^{16}\text{F}$ g.s. resonance was calculated assuming a Breit-Wigner parametrization:

$$P_\gamma(E_\gamma)dE_\gamma = \frac{1}{N} \frac{dE_\gamma}{(\Delta E)^2 + (\Gamma_p^\gamma(E_p - E_\gamma))^2} \quad (5)$$

and the normalization constant is:

$$N = \int \frac{1}{(\Delta E)^2 + (\Gamma_p^\gamma(E_p - E_\gamma))^2}dE_\gamma \quad (6)$$

with $\Delta E = E_p - E_{c} - E_{g.s.}$ and $\Gamma_p^\gamma(E_p - E_\gamma) = \Gamma_{out} + \Gamma_p(E_p - E_\gamma)$ is the total width of the g.s. resonance. The $\beta^+$ branching ratio is calculated using:

$$P_\beta(E_p, E_\gamma) = \frac{\Gamma_\beta}{\Gamma_\beta + \Gamma_p^\gamma(E_p - E_\gamma)} \quad (7)$$

Naively, one might have expected to obtain a small cross section for the $(p,\gamma)(\beta^+)$ reaction, similar to the $(p,\beta^+)$ one, since $\gamma$- and $\beta$-widths are much smaller than proton-widths. Contrary to naive expectations, the $(p,\gamma)(\beta^+)$ cross section is about $10^{10}$ times larger than the $(p,\beta^+)$ cross section, as shown in Fig. 2. The large ratio can be explained in the following way. As it has been shown previously, there is only one $(p,\beta^+)$ reaction for $10^{20} (p,p)$ reactions. In the $(p,\gamma)(\beta^+)$ case, one gamma-ray is emitted for $10^6$ incident protons (from the ratio of the widths) and about one gamma-transition over $10^5$ populates the low energy wing of the g.s. resonance (less than 50 keV above the proton emission threshold) where it is almost always followed by a $\beta^+$-decay ($P_\beta \approx 1$). This implies that one incident proton over $10^{10}$ induces a $(p,\gamma)(\beta^+)$ reaction, that is a factor $10^{10}$ times larger than in the $(p,\beta^+)$ reaction.

In the following, uncertainties in the calculations and their evaluated effects on the results are discussed. The position and width of the low lying $^{16}\text{F}$ states were measured with a high precision (see Table 1). The effect of the uncertainties in these measured parameters results in a change by less than a factor two in the calculated cross sections. The calculated $(p,\gamma)(\beta^+)$ cross section is insensitive to the $^{16}\text{F}$ $\beta^+$-decay lifetime, as a variation by a factor of 100 causes the cross section to change by only a factor of 2. The lifetime of the $\gamma$-transition is a sensitive parameter since the $(p,\gamma)(\beta^+)$ cross section is almost directly proportional to this parameter. A value measured in the mirror nucleus was used, but this assumption works only to within a factor of 10 [16]. The other excited states in $^{16}\text{F}$ were also studied and found to be negligible. The $^{15}\text{O}(p,\gamma)(p,\gamma)^{17}\text{Ne}$ double indirect proton capture reaction was not taken into account, neither the cross sections calculated, since it requires an appropriate 3-body calculation. Moreover, cross sections may change by several effects, which remain to be evaluated as: non-resonant direct capture contributions, quantum interferences, continuum couplings [17].

At a given temperature $T$ of the gas inside the star, protons exhibit a Maxwellian distribution and the reaction rates $N_A < \sigma v >$ are calculated by integrating numerically the Maxwellian-averaged cross sections $\sigma(E_p)$ over all possible proton energies. The obtained reactions rates are shown in Fig. 4 (a) as a function of the temperature. The rate of the $(p,\beta^+)$ reaction is negligible compared to that of the reaction $(p,\gamma)(\beta^+)$.
FIG. 4: (a) $^{15}$O($p,\beta^+$)$^{16}$O and $^{15}$O($p,\gamma$)$^{15}$O reactions rates are shown as a function of the temperature. The $^{15}$O($\alpha,\gamma$)$^{19}$Ne reaction rate is also shown for comparison. (b) Density versus temperature conditions where the $^{15}$O($p,\gamma$)$^{15}$O reaction represents 10 to 50 % of the total reaction flux initiated by the $^{15}$O nucleus.

for all temperatures. To evaluate the impact of this latter reaction, it has to be compared with the competing $\beta^+$-decay of $^{15}$O and the $^{15}$O($\alpha,\gamma$)$^{19}$Ne alpha capture reaction. Fig. 4 (b) shows the temperature and density conditions where the $^{15}$O($p,\gamma$)$^{15}$O reaction represents 10 to 50 % of the total reaction flux initiated by the $^{15}$O nucleus. Boxes delimit conditions where novae and X-ray bursts can happen. For the lowest temperatures ($< 10^8$ K), the ($p,\gamma$)$^{15}$O reaction requires extreme densities ($> 10^{10}$ g cm$^{-3}$) to compete with the $^{15}$O($\beta^+$)-decay. For the highest temperatures ($> 1.1 \times 10^9$ K), the ($\alpha,\gamma$) always dominates. In novae explosions, $^{15}$O nuclei mainly decay by $\beta^+$-ray emission, the ($p,\gamma$)$^{15}$O reaction representing less than 1 % of the flux from $^{15}$O. In X-ray bursts, the ($p,\gamma$)$^{15}$O reaction can represent up to 30 % of the total flux. Within the uncertainties of the calculations, the ($p,\gamma$)$^{15}$O reaction could be faster than the ($\alpha,\gamma$) reaction for temperatures up to $10^9$ K. A more precise evaluation depends on the ($\alpha,\gamma$) reaction rate (not well known) and on the relative abundances in hydrogen and helium, since one reaction consumes protons and the other alpha particles. In these extreme conditions, a new cycle of reactions is operating: $^{15}$O($p,\gamma$)$^{16}$O($p,\gamma$)$^{17}$F($p,\gamma$)$^{18}$Ne$^{18}$F($p,\alpha$)$^{15}$O. This new cycle could speed-up the CNO cycle and occur complementary to breakout reactions. The role of this new proposed cycle of reactions remains to be studied more carefully under various X-ray bursts conditions.

In summary, it is shown that unbound nuclei can be involved in specific reactions that could play a role in astrophysics. Sequential ($p,\gamma$)$^{15}$O($\beta^+$) reaction, proceeding through an intermediate proton-unbound nucleus, was studied for the first time. The calculated $^{15}$O($p,\gamma$)$^{15}$O($\beta^+$) reaction cross section is found to be almost $10^{10}$ times larger than the direct $^{15}$O($p,\beta^+$)$^{16}$O reaction cross section. The large increase is mainly due to a strong feeding of the low energy wing of the $^{16}$F g.s. resonance, where the subsequent $\beta^+$-decay is favored. The ($p,\gamma$)$^{15}$O($\beta^+$) could act in X-ray bursts, and would provide a steady burning scenario with a continuous depletion of $^{15}$O. It is of great importance to study carefully the effects of this new reaction under various X-ray bursts conditions, and to demonstrate experimentally the existence of ($p,\gamma$)$^{15}$O($\beta^+$) reactions. The cross section of the $^{15}$O($p,\gamma$)$^{15}$O($\beta^+$) reaction is calculated to be in the nanobarns range and can be measured using next generation intense RIB. More generally, several other unbound nuclei as $^{19}$Na or $^{15}$F could also be involved in this type of reaction and remain to be studied. We thank the GANIL crew for delivering the $^{15}$O beam, M. Ploszajczak and A. Navin for stimulating discussions. This work has been supported by the IN2P3-IFIN-HH Program.

[1] H.A. Bethe, C.L. Critchfield, Phys. Rev. 54, 248 (1938).
[2] C. S. Rolfs and W. S. Rodray, Cauldrons in the Cosmos, The University of Chicago Press, Chicago 60637 (1988).
[3] K. Langanke et al., Z. Phys. A 324, 147 (1986).
[4] R.K. Wallace and S.E. Woosley, Astrophys. J. Suppl. Ser. 45, 389 (1981).
[5] M. Wiescher, H. Schatz, and A.E. Champagne, Phil. Trans. Roy. Soc. London A 356, 2105 (1998).
[6] H. Schatz et al., Phys. Rev. Lett. 86, 3471 (2001).
[7] J. Görres et al., Phys. Rev. C 51, 392 (1995).
[8] P. Bertrand et al., 17th International Conf. on Cyclotrons and their applications, Tokyo (2004).
[9] R. Anne et al., Nucl. Instr. and Meth. A257, 215 (1987).
[10] V.Z. Golberg et al., Phys. At. Nucl. 60, 1061 (1997).
[11] F. de Oliveira et al., Eur. Phys. J. A 24, 237 (2005).
[12] A.M. Lane and R.G. Thomas, Rev. Mod. Phys. 30, 257 (1958).
[13] E. Berthoumieux et al., Nucl. Instr. and Meth. B 138, 55 (1998).
[14] G.Audi et al., Nucl. Phys. A 729, 337 (2003); D.R. Tilley et al., Nucl. Phys. A564, 1 (1993).
[15] A. Messiah, Quantum Mechanics, North-Holland, Amsterdam, Vol. 2 (1962).
[16] F. de Oliveira et al., Phys. Rev. C 55, 3149 (1997).
[17] J. Rotureau et al., Phys. Rev. Lett. 95, 042503 (2005); R. Chatterjee et al., Nucl. Phys. A 764, 528 (2006).