Symbolic relative entropy in quantifying nonlinear dynamics of equalities-involved heartbeats

Wenpo Yao
Nanjing Univ Posts & Telecom, Sch Telecomm & Informat Engn, Nanjing 210003, China

Wenli Yao
School of Mines, China University of Mining and Technology, Xuzhou 221116, China and
Department of Mining and Metallurgical Engineering,
Western Australian School Mines, Curtin University, Kalgoorlie, WA, Australia

Jun Wang
Nanjing Univ Posts & Telecom, Sch Geo & Bio Informat, Nanjing 210023, China

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Symbolic relative entropy, an efficient nonlinear complexity parameter measuring probabilistic divergences of symbolic sequences, is proposed in our nonlinear dynamics analysis of heart rates considering equal states. Equalities are not rare in discrete heartbeats because of the limits of resolution of signals collection, and more importantly equal states contain underlying important cardiac regulation information which is neglected by some chaotic deterministic parameters and temporal asymmetric measurements. The relative entropy of symbolization associated with equal states has satisfied nonlinear dynamics complexity detections in heartbeats and shows advantages to some nonlinear dynamics parameters without considering equalities. Researches on cardiac activities suggest the highest probabilistic divergence of the healthy young heart rates and highlight the facts that heart diseases and aging reduce the nonlinear dynamical complexity of heart rates.

I. INTRODUCTION

The cardiovascular modulation system is a complex system interacting with a collection of organs and influenced by many internal and external factors, and the derived heart rate is typical complex signal with nonlinearity and nonstationarity [1–4]. Besides classical time and frequency domain analysis, some nonlinear methods are applied to heartbeats analysis, such as chaotic deterministic parameters, entropy approaches, geometric methods, time irreversibility, and so on [5–14], among which symbolic dynamics analysis provides rigorous descriptions for complex system or chaotic phenomena by greatly reducing high demands on raw data and improving resilient to outliers or insensitive to noise [12–16].

Electrocardiogram is continuous electrical signals while heartbeats, mainly represented by RR interval, are discrete. It is reported that decreased heart rate variability (HRV), the tiny differences between successive heartbeats, is associated with increased mortality and is powerful predictor of arrhythmic complications [4,17], therefore under low signal acquisition precision we suppose the distributions of equal states in heartbeats may have important information about the cardiovascular regulation.

Accounting on the continuous distributions of real-world signals, equal states are not given deserved attention by some signal processings. Equal states are neglected in the original introduction of permutation entropy [18,19], and time irreversibility parameters like Porta [10,11], Guik [20] or Costa indexes [12,13] all do not take the equal states into consideration in measuring asymmetry of heart rates. In heart beat analysis, however, the ignorance is patently untrue and equal states should be given special attentions and treatments not only because of limits resolution of signal collection but also for some underlying structural information the equalities containing in practical contexts [21,22].

In our contributions, we map heart rates onto equality-involved symbolic ordinal patterns and apply relative entropy to measure the divergences among their probability distributions. Heartbeats of three groups of subjects, CHF (congestive heart failure) patients and healthy elderly and young volunteers, from PhysioNet database [23] are applied to testify the equality-associated symbolic relative entropy and verify our hypothesis that the equal states in heartbeats contain important underlying information about cardiac modulation.

II. SYMBOLIC TIME SERIES ANALYSIS

Symbolic time series analysis, or symbolic dynamics, is a coarse-graining method and deals with time series rigorously with finite precision. Symbolic dynamics analysis involves in symbolic transformation and statistical analysis for the symbolic sequence [24].

Symbolization simplifies time series by transforming raw data into symbolic sequence based on some given alphabet. Through symbolic transformation, some amount of detailed statistics are lost while some invariant dynamics, like periodicity, chaotic or symmetry, may be obtained. The transforms can be grouped into range-
partitioning static methods like Kurths-Wessel [21, 20] and base-scale symbolizations [8] and difference-based dynamic methods such as the classical permutation entropy and its modified version [18, 19, 21].

Symbol-sequence analysis mainly targets on their probability distribution, and measures include classical statistics like visual histograms and information theory like Shannon or Renyi entropies.

A. Symbolization in original permutation entropy

Symbolic transformation in permutation entropy [18, 19, 27, 20], a local dynamical symbolization on the basis of comparison of neighboring values [24], is defined for arbitrary real-world time series with features of simplicity, fast calculation and invariance property. Multi-dimensional phase space is firstly reconstructed as \( X^m_x(i) = [x(i), x(i + \tau), \ldots, x(i + (m - 1)\tau)] \) for different dimension \( m \) and delay \( \tau \). And then reorganize each vector according to their values \( x_{mr}(j_1) \leq x_{mr}(j_2) \leq \cdots \leq x_{mr}(j_l) \) which will be mapped onto ordinal pattern \( \pi_j = \{j_1, j_2, \ldots, j_l\} \). The 6 order patterns of \( m=3 \) and symbols on the basis of a given alphabet \( \{0, 1, \ldots, 5\} \) are illustrated in Fig. 1.

![FIG. 1. Order patterns and their symbols (in parentheses) of embedding dimension 3 without considering equal states.](image)

B. Symbolization in modified permutation entropy

On accounting continuous distributions and very rare equal values of real-world time series, Bandt et al. and others rank equalities according to their order of emergence or add small random perturbations to numerically break equal states [18, 19, 27, 28, 30], while when negligible amounts of equalities are existent, Bian et al. method [21] could be an interesting alternative.

Bian and Ma take distributions of equal values into consideration and propose the modified permutative method by revising the indexes of equalities in ordinal patterns [21]. Equal values, \( x_{mr}(j_1) = x_{mr}(j_1+1) \) or \( x_{mr}(j_l) = x_{mr}(j_l+1) = x_{mr}(j_l+2) \), are in adjacent continuous orders in permutations, therefore, their neighboring indexes in the order pattern \( j_1, j_1+1, j_l, j_l+1, j_l+2 \) could be revised to be identical to the smallest \( j_1 \) of each group of equalities as \( j_l, j_1 \) and \( j_l, j_l, j_l \). Taking vector \( (1.2, 0.9, 1.8, 0.9, 1.8, 1.8) \) as an example, its ascending reorganization is \((0.9, 0.9, 1.2, 1.8, 1.8, 1.8)\), and the original order pattern could be \( '241356' \) while in modified approach the order pattern should be \( '221333' \).

![FIG. 2. Modified permutations and their symbols with two equalities when \( m \) is 3. The two gray elements are equalities, and the white ones represent the third values in the front, middle or back of and bigger (above) or smaller (below) than the equal pairs.](image)

C. Symbolic relative entropy

Permutation entropy and its modified version are both Shannon entropy for the probability distributions of all ordinal types [18, 27, 28], and there are other informational forms like Tsallis or Renyi entropies [31, 32].

In our nonlinear dynamics analysis, we apply relative entropy [33] to measure the divergences of ordinal patterns’ probability distributions in Eq. 1. As a reference, we also use the Kullback-Leibler divergence, as Eq. 2, in the original literature [34] for its merits of symmetry and non-negative behaviors.

\[
S\text{Re}D = \sum_{i=1}^{N-1} \sum_{j=1}^{N-i} p(\pi_i) \log \frac{p(\pi_i)}{q(\pi_{i+j})} \tag{1}
\]

\[
S\text{Re}J = \sum_{i=1}^{N-1} \sum_{j=1}^{N-i} (p(\pi_i) - q(\pi_{i+j})) \log \frac{p(\pi_i)}{q(\pi_{i+j})} \tag{2}
\]

III. EQUAL STATES IN HEART BEATS

Three groups of heart rates derived from ECGs from PhysioBank [22] are applied in our contributions. The 20 healthy young (aged 21-34, mean 25±4 yrs) and 20 elderly (aged 68-81, mean 74±4 yrs) participants of Fantasia database [35] (sampling frequency of 250Hz and 16
bit resolution) and 15 patients (aged 22-71, mean 56±11 yrs) with congestive heart failure (NYHA class 3-4) of chfhd [20] (sampling frequency of 250Hz and 12 bit resolution) have been used repeatedly in the nonlinear dynamics analysis of cardiac activities and present valuable information about aging and diseases [7, 8, 12, 21, 37].

Equalities are not rare in heart beats [see Fig. 3], order patterns with equal states, symbolized from 6 to 12, in CHF heartbeats have comparable distributions to those without equalities, and their proportions are higher than those of the two healthy groups of heartbeats. More comprehensive analysis to the equalities in the three kinds of heart rates is conducted, and the probability distributions of different equal states are shown in visual histograms Fig. 4 and statistical tests listed in Table I.

TABLE I. T tests for distributions of heartbeats’ equal states. 'e2t1' denotes two-equal values with τ = 1 and ‘CHF-Eld’ represents t test for the equalities in heart rates of CHF patients and the elderly subjects.

| p value | CHF-Eld | CHF-Yng | Eld-Yng |
|---------|---------|---------|---------|
| e2t1    | 5.9 * 10^{-6} | 6.0 * 10^{-6} | 3.1 * 10^{-5} |
| e2t2    | 6.3 * 10^{-6} | 1.5 * 10^{-6} | 9.6 * 10^{-5} |
| e2t3    | 3.0 * 10^{-6} | 8.8 * 10^{-9} | 2.2 * 10^{-4} |
| e3t1    | 7.9 * 10^{-5} | 9.0 * 10^{-6} | 1.5 * 10^{-2} |
| e3t2    | 1.9 * 10^{-5} | 1.3 * 10^{-5} | 7.0 * 10^{-3} |
| e3t3    | 6.0 * 10^{-5} | 8.3 * 10^{-7} | 2.0 * 10^{-3} |

Analyzing the probability distributions of two- and three-equal values in the three groups of heartbeats, equal states patently have high-distribution existence and sudden cardiac death could be clearly indicated by low heart rate variability which may be transformed into equalities in heartbeats under low precision of signal collection. The high proportion of equalities in heart rates, therefore, should not be explained simply from the limits of signal collection because they contain valuable physiological information about cardiac modulation.

IV. SYMBOLIC RELATIVE ENTROPY OF HEART RATES

In this section, we focus on the influence of equal states on three kinds of HRVs’ probabilistic divergence of order patterns.

A. Symbolic relative entropy (m=2) and Porta, Costa indexes

When m=2, only two neighboring values are compared, and the symbolic relative entropy shares similarity with temporal asymmetric parameters, Porta and Costa indexes, in measuring the asymmetric distributions of ups Δx+ and downs Δx−. Porta’s index [10, 11], in Eq. (3), measures the distributions of downs in time series. Costa et al. consider energy requirements of each transition in heart rate [12] and formulate a simplified temporal asymmetric index [13] as Eq. (4) where H is the Heaviside function.

\[
P% = \frac{N(\Delta x^-)}{N(\Delta x \neq 0)} \cdot 100 \tag{3}
\]

\[
A = \frac{\sum H(-\Delta x^-) - \sum H(\Delta x^+)}{N(\Delta x \neq 0)} \tag{4}
\]

Considering the continuous distributions of real-world signals whose equal values are very rare, Porta index is \( P = p(\Delta x^-) \times 100 \), the Costa index is then expressed as Eq. (3) and the two versions of symbolic relative entropy are rewritten as Eq. (5) and (2) where \( P_d = p(\Delta x^-)/p(\Delta x^+) = P/(1 - P) \).

\[
A = p(\Delta x^-) - p(\Delta x^+) = P - (1 - P) = 2P - 1 \tag{5}
\]

\[
SReD = p(\Delta x^-)log\frac{p(\Delta x^-)}{p(\Delta x^+)} = PlogP_d \tag{6}
\]

\[
SReJ = (p(\Delta x^-) - p(\Delta x^+))log\frac{p(\Delta x^-)}{p(\Delta x^+)} = AlogP_d \tag{7}
\]

Porta, Costa parameters and the symbolic relative entropy are different in dealing with equalities, however, in case of no or very few equal values they are mathematically equivalent in measuring the divergence of ups and downs that equal states imply directional symmetry considering only two values. In this subsection, we make comparisons of these methods considering equal states.

From Table I, Porta, Costa indexes and two relative entropic parameters do not have consistent outcomes under the existence of non-negligible amount of equalities in heartbeats. Costa index shares the conventional knowledge of complexity-loss theory [7, 8, 12, 16, 21, 37–39] about the nonlinear relationships of the three groups.
FIG. 3. Heart rates and their ordinal patterns’ probability distributions. a) heartbeats b) Histogram of symbolic original permutations c) Histogram of symbolic modified permutations. In modified ordinal patterns, the probabilities of 7 symbols containing equal values of CHF heartbeats are between 0.06 and 0.08 and those of healthy elderly people are around 0.02 and 0.03, and slightly lower than 0.01 to the healthy young.

FIG. 4. The probability distributions of equal values of heartbeats (mean±std). a) Two-equal values, $x_i = x_{i+\tau}$. b) Three-equal values, $x_i = x_{i+\tau} = x_{i+2\tau}$.

of heartbeats (CHF<Eld<Yng) while fails in statistical analysis (CHF-Eld p value is 0.420, CHF-Yng is 0.202 and Eld-Yng is 0.533). Porta index and two relative entropies are showing absolutely reverse relationships of temporal asymmetry, CHF>Eld>Yng. About the inconsistencies, Costa gives explanations that some single-scale based algorithms fail to account for the inherent multiscale information [13, 39, 40] (more detailed information are in discussions), while from our perspective, the equal states may also play important roles that these inconsistent results may be related to the ignorance of underlying information contained by the equal states.

Two-equal values in heartbeats have high proportions (even to 20% for CHF heart rate), therefore, it is not enough to just measure the asymmetric probability distributions of ups and downs. We take ups, downs together with equalities into account, and measure their probability distributions’ divergence by the modified permutation relative entropy with delay factors from 1 to 5.

As time irreversibility shown in Fig. 5 and statistical analysis listed in Table III both the modified permutive relative entropic parameters have consistent outcomes with relevant literatures about broken asymmetry in heart diseases and aging and the distinctions among the three groups of heartbeats are significant statistically (p values are all smaller than $1.0 \times 10^{-6}$), which shows the necessity of considering equal states in quantifying distributions asymmetry. What’s more, the equality-involved permutation relative entropy verify the facts from heartbeats’ directional asymmetry that heart diseases CHF leads to reduction of time irreversibility of heartbeats and aging causes lower nonlinear dynamical complexity than the healthy young heart rates.

|                | CHF     | Elderly | Young  |
|----------------|---------|---------|--------|
| $P_{50}$       | 7.615±2.693 | 4.294±2.859 | 2.756±2.371 |
| Costa          | 0.027±0.033 | 0.036±0.027 | 0.042±0.029 |
| ReIrD          | 0.259±0.103 | 0.139±0.101 | 0.087±0.078 |
| ReIrJ          | 0.075±0.048 | 0.033±0.030 | 0.017±0.017 |

TABLE II. Porta, Costa indexes and original permutation relative entropy of three kinds of heartbeats (mean±std). We choose $P_{50} = |P - 50|$ as an alternative to Eq. (1) because it measures asymmetry by determining its distance from 50.
TABLE III. Independent sample t tests for modified permutative relative entropy (m=2) of the three groups of heartbeats.

|        | CHF-Eld | CHF-Yng | Eld-Yng |
|--------|---------|---------|---------|
| m=2    |         |         |         |
| t1     | $1.0 \times 10^{-6}$ | $4.6 \times 10^{-13}$ | $1.0 \times 10^{-6}$ |
| t2     | $8.7 \times 10^{-8}$ | $3.7 \times 10^{-14}$ | $9.8 \times 10^{-8}$ |
| t3     | $1.1 \times 10^{-7}$ | $9.6 \times 10^{-14}$ | $1.6 \times 10^{-7}$ |
| t4     | $5.3 \times 10^{-9}$ | $1.5 \times 10^{-14}$ | $5.4 \times 10^{-9}$ |
| t5     | $8.1 \times 10^{-10}$ | $5.4 \times 10^{-16}$ | $9.0 \times 10^{-15}$ |

TABLE IV. Statistical analysis of original permutation relative entropy (SReD) of the heartbeats when $\tau = 1$. '0.000' in the table is 0.00033.

| m     | CHF-Eld | CHF-Yng | Eld-Yng |
|-------|---------|---------|---------|
| 3     | 0.579   | 0.101   | 0.280   |
| 4     | 0.130   | 0.002   | 0.190   |
| 5     | 0.058   | 0.000   | 0.131   |

With the adjustments of embedding parameters, it is important to note that relationships among the three kinds of heart signals change accordingly, and even when the distinctions are consistent with conventional wisdoms there is no acceptable statistical result. The original permutative methods without considering equal states do not have reliable outcomes and no conclusive conclusion can be reached.

Now we test the modified permutative relative entropy for the three types of heart beats, and the results and t tests are shown in Fig. 7 and Table V.

From Fig. 7 as the embedding dimension and delay factor increase, the size relationships of the permutation relative entropy of the CHF patients, healthy elderly and young volunteers are consistent and consistent with the complexity-lose wisdom (CHF $\prec$ Eld $\prec$ Yng). The distinctions among the heart signals are significantly different when m are 3 and 4 while the modified permutative relative entropy does not effectively separate the two groups of healthy subjects when m=5 and $\tau$>2 in statistics shown by the bold values in Tab 5.

From the above comparisons, the modified permutation considering equal states greatly improve the original ordinal methods in the relative entropic asymmetry analysis and have effective detections of heart diseases and aging in different heart beats. Equalities may be lead by the limitations of low resolution of signals collection or the accuracy of data extraction, however, they contain valuable underlying information about cardiac regulation system. In heartbeats analysis, therefore, equal states deserve more attentions and treatments to avoid potential misunderstandings and misleading conclusions.

The equality-involved permutative relative entropy of heart beats further suggests that aging and heart diseases like CHF leads to loss of nonlinear dynamics complexity of heart activities.
FIG. 6. Original permutation relative entropy (SReD) of the three groups of heartbeats with m=3, 4 and 5 and τ=1, 2, 3, 4 and 5. Normal complexity-loss relationship when t=1 (left of the dotted line), no clear regular discriminations when t are 2, 3 and 4 (between the two dotted lines), opposite to the complexity-loss relationship when t=5 (right of the dotted line).

FIG. 7. Modified permutative relative entropy of the three groups of heart electrical signals with m=3, 4, 5 and τ=1, 2, 3, 4 and 5.

For calculative and practical convenience, embedding dimension is recommended chosen no bigger than 5 that the amounts of ordinal patterns will increase sharply and the comparisons between each pair of distributions will be very large, which may contain too much redundant information. Whether the parameters settings in our heart rates analysis are suitable in other physiological signals or applications is need to be verified.

V. DISCUSSIONS

There is arguably no definite equality between each two pairs of RR intervals if the resolution is sufficiently high, however, under the acceptable precision of signal collection, we can consider two intervals to be equal if we cannot distinguish them. The equal states, therefore, represent very low heart rate variability and contain information about cardiac autonomic modulation.

About the inconsistent results in Porta, Costa and symbolic relative entropy (m=2), except for the differ-
TABLE V. Independent sample t tests for three kinds of heartbeats’ equal states for modified permutative relative entropy (SReD).

| Scale Factor | CHF-Eld | CHF-Yng | Eld-Yng |
|--------------|---------|---------|---------|
| m3t1         | 9.8 * 10^{-7} | 2.2 * 10^{-15} | 1.7 * 10^{-7} |
| m3t2         | 9.7 * 10^{-9} | 2.5 * 10^{-15} | 1.0 * 10^{-6} |
| m3t3         | 5.8 * 10^{-9} | 5.0 * 10^{-16} | 6.2 * 10^{-5} |
| m3t4         | 2.7 * 10^{-9} | 6.0 * 10^{-15} | 2.0 * 10^{-5} |
| m3t5         | 7.6 * 10^{-9} | 3.2 * 10^{-16} | 9.0 * 10^{-6} |
| m4t1         | 4.7 * 10^{-7} | 2.5 * 10^{-15} | 1.7 * 10^{-7} |
| m4t2         | 7.6 * 10^{-9} | 9.2 * 10^{-19} | 9.0 * 10^{-6} |
| m4t3         | 4.7 * 10^{-10} | 7.5 * 10^{-19} | 1.4 * 10^{-4} |
| m4t4         | 3.0 * 10^{-9} | 9.8 * 10^{-19} | 1.9 * 10^{-4} |
| m4t5         | 9.8 * 10^{-10} | 5.1 * 10^{-17} | 1.5 * 10^{-4} |
| m5t1         | 5.8 * 10^{-5} | 2.4 * 10^{-10} | 7.0 * 10^{-5} |
| m5t2         | 1.0 * 10^{-6} | 9.3 * 10^{-9} | 0.049 |
| m5t3         | 6.3 * 10^{-11} | 8.0 * 10^{-13} | 0.129 |
| m5t4         | 3.1 * 10^{-8} | 3.5 * 10^{-9} | 0.116 |
| m5t5         | 2.5 * 10^{-8} | 2.7 * 10^{-8} | 0.081 |

different treatments for equal states, theoretical explanations could be phase space and multiple scale. For phase space, with adjustment of embedding dimension and delay factor, dynamical information from different dimensional spaces may lead to different outcomes. In this discussion, we focus on the multi-scale concept \[13, 29, 30, 39–41]\, \[m_{ij} = 1/\tau \sum_{t=-\infty}^{\infty} x_{i+t}, 1 \leq j \leq \frac{N}{\tau}, \text{where } \tau \text{ is scale factor.} \]

The coarse-grain procedure in the heartbeats analysis greatly reduces the equalities showing in Fig. 8. According to our research on equalities in heartbeats, the multiscale process obviously has impact on the further researches on nonlinear dynamics in heartbeats. The multi-scale theory, therefore we suggest, should be further validated by more representative numbers of heartbeats in more detailed experimental and theoretical analysis.

VI. CONCLUSIONS

Heart diseases and aging reduce heart beats variability and which bring in equal values under low precision of signal collection, and the high proportions of equalities contain important underlying regulation information about cardiac system. Negligence or insufficient treatment for these great amounts of equal values may result in incorrect outcomes and misunderstanding conclusions.

The symbolic relative entropy, probabilistic divergence of ordinal patterns, is a promising nonlinear parameter for the quantitative assessment of nonlinear dynamics complexity. Making full use of the equal states, the modified permutation relative entropy show great advantages to the original permutative method and some other temporal asymmetry parameters. In our contributions, we verify the the loss of nonlinear dynamics of heartbeats in aging and diseases which is consistent with the complexity loss theory.

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