A comparative analysis to study the characteristic properties of charged particle production in hadron-nucleus collisions at high energies, by utilising the approaches from different statistical models is performed. Predictions from different approaches using the Negative Binomial distribution, shifted Gompertz distribution, Weibull distribution and the Krasznovszky-Wagner distribution are utilised for a comprehensive study of the relative successes of these forms. These distributions are derived from a variety of functional forms based on either phenomenological parameterizations or some model of the underlying dynamics. Some of these distributions have been used to study the highest energy data at the LHC for both proton-proton and nucleus-nucleus collisions.

I. INTRODUCTION

Investigation of charged particle production in collisions of high energy particles has always been of interest. Gradual transition of small number of such particles produced in fixed target experiments by manifolds in collider experiments has not diminished this interest. The increase in numbers follows logarithmic rise in the average values with the increasing energy of collision in center of mass system (c.m.s.). Several theoretical and phenomenological studies have explored the dynamics of particle production. Numerous statistical models predicting the trends have been proposed, studied and used to successfully describe the particle production in hadron-hadron (pp and pp), lepton-lepton ($e^+e^-$), hadron-nucleus ($hA$) and nucleus-nucleus ($AA$) collisions over a wide range of c.m.s. energy. Some of the statistical distributions and their modified forms which have been widely used are Negative Binomial distribution (NB) \cite{1}, Gamma distribution \cite{2}, Tsallis distribution \cite{3, 4}, Weibull distribution (WB) \cite{5, 8} etc. In one of our previous papers, we introduced shifted Gompertz distribution (SG) for the description of particle production and have shown that it explains very well the multiplicity distributions in different kinds of collisions. The modified version of SG distribution described the data extremely well as shown in the references \cite{9, 10}. Another distribution which remarkably well described the data on hadron-hadron and hadron-nucleus interactions, from a large number of experiments was Krasznovszky-Wagner distribution (KW) \cite{11, 12}.

In this paper, the first study of multiplicity distributions in hadron-nucleus interactions from fixed target experiments in terms of four distributions namely, NB, SG, WB and KW is reported. The data used were collected almost four decades ago from different projectile-target combinations at different energies. In one of our earlier papers \cite{13} we analysed the data on hadron-nucleus interactions to study the Tsallis non-extensive entropy model. We reanalyse the data in the light of new concepts. An outline of the models used is described in the following section. The paper is organised as follows: After this brief introduction in section I, we describe in section II the essential steps of all four distributions mentioned above. Section III gives details of the data used and results obtained from the comparison of our analysis of these distributions. Section IV presents the conclusions.

II. PROBABILITY FUNCTIONS AND THE PARTICLE MULTIPLICITY

The probability of producing $n$ charged particles in an interaction can be understood in terms of a probability distribution function (PDF), expressed as $P(n|\mu)$, where $\mu$ represents a set of parameters which influence the shape and scale of the PDF. The number of particles produced are distributed according to some PDF which depending upon $\mu$ produces different types of distributions. We discuss four such PDFs in the following section;

A. The Negative Binomial (NB) distribution

The experimental results are understood in terms of distributions of negative binomial form both for total multiplicities and multiplicities in restricted pseudorapidity windows. A plausible explanation of the experimental findings leads to a concept of cluster formation and an insight in to the properties of clusters. The negative binomial distribution is characterised by two free parameters, which determine the mean $<n>$ and shape $k$ of the distribution. The PDF is given by:

$$P_{\text{NB}}(n|<n>,k) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \frac{(n > /k)^n}{(1+n > /k)^{n+k}}$$

the fit parameters of the distribution are $<n>$ as the mean value of $n$ and $k$ as the shape parameter.
B. Shifted Gompertz (SG) distribution

We introduced the shifted Gompertz distribution to study its validity for the multiplicity data from $pp$, $\bar{p}p$ and $e^+e^-$ collisions from various experiments at different energies [9,10,11]. The detailed studies have shown that SG distribution describes the data trends very well and the precision is better than NB in several cases. It is defined in terms of the probability density function (PDF) by two non-negative parameters, $b$, the scale parameter and $\eta$ the shape parameter. Following equations define the PDF and the mean of the distribution:

$$P_n(n|b,\eta) = be^{-bn}e^{-\eta e^{-bn}}[1 + \eta(1 - e^{-bn})] \quad \text{for } n > 0 \quad (2)$$

Mean of the distribution:

$$\left(-\frac{1}{\eta}\right)(E[ln(\zeta)] - ln(\eta)) \quad \text{where } \zeta = \eta e^{-bn} \quad (3)$$

and

$$E[ln(\zeta)] = \left[1 + \frac{1}{\eta}\right] \int_0^\infty e^{-\zeta}[ln(\zeta)]d\zeta - \frac{1}{\eta} \int_0^\infty \zeta e^{-\zeta}[ln(\zeta)]d\zeta \quad (4)$$

Although SG distribution has been studied recently, with very good results in hadronic and leptonic interactions, this is our first attempt to analyse the $p/\pi^-$—nucleus data from the fixed target experiments, in terms of shifted Gompertz distribution.

C. The two parameter Weibull (WB) distribution

The Weibull distribution was originally proposed as a model for studying the material breaking strength. It is commonly used in many fields such as engineering sciences, biology etc. to assess product reliability, analyze life data and model failure times.

Two different probability distribution functions (PDF) of Weibull exist; one having three free parameters and another having two free parameters. Weibull distribution can also be fitted to non-symmetrical data. The two parameter Weibull has been used during the last few years to describe the collision data from high energy experiments [12,13].

The PDF of a Weibull random variable is:

$$P_n(n|\lambda,K) = \left\{ \begin{array} {ll} \frac{k}{\lambda}\left(\frac{n}{\lambda}\right)^{(K-1)}exp^{-(\frac{n}{\lambda})^K} & n \geq 0 \\ 0 & n < 0 \end{array} \right. \quad (5)$$

The standard Weibull has a scale parameter $\lambda > 0$ and a shape parameter $K > 0$ for its two free parameters. The mean of the distribution function is given by:

$$\bar{n} = \lambda \Gamma\left(\frac{K + 1}{K}\right) \quad (6)$$

D. The Krasnovszky-Wagner (KW) distribution

This distribution based on the generalized geometrical-optical model in the impact parameter representation [11,12] has three free parameters; a mean $<n>$ and two others, $m$ and $A$:

$$P_n(n|<n>,m,A) = \frac{mF(A)^A}{\Gamma(A)} \quad (7)$$

where $z = n/ <n>$ and

$$F(A) = \frac{\Gamma^m(A + 1/m)}{\Gamma^m(A)} \quad (8)$$

The distribution have been used to study for $e^+e^-$, $pp$ and particle production in $\pi^-p$ interactions mainly in the context of KNO scaling [11,13].

III. MOMENTS OF MULTIPLICITY DISTRIBUTIONS

The shape of the charged particle multiplicity distribution is dependent on the mechanism of particle production and hence serves as a fundamental tool to study the production dynamics. It is well established that in the case of independent emission of single particles, the shape is Poissonian. Any deviations from this shape points to the presence of correlations [10]. To study the shape, we use normalised moments $C_q$, and normalised factorial moments $F_q$ defined as:

$$C_q = \frac{\langle n^q \rangle}{\langle n \rangle^q} \quad (9)$$

$$F_q = \frac{\langle n(n-1)\cdots(n-q+1) \rangle}{\langle n \rangle^q} \quad (10)$$

The multiplicity distribution shape is rather non-trivial in terms of its dependence on the interaction energy. A scaling behaviour was predicted by Koba–NielsenOlesen (KNO) [17] which means that probability distributions at all energies fall on to one curve when plotted as a function of a variable $z = n/ <n>$. Study of moments of distribution reveals that if the scaling hypothesis holds, the moments $C_q$, equation (9) are independent of energy. And if the moments depend upon energy, the KNO scaling breaks down. The moments $F_q$ equation (10) show correlations in the production of up to $q$ particles. For the particle distribution following Poissonian shape, all $F_q$ are equal to unity. The distribution becomes broader if the particles are correlated, and narrower if they are anti-correlated. In case of a positive(negative) correlation the $F_q$ are more(less) than unity.
IV. RESULTS

The following sections describe the data analysed and the distributions used.

A. The data used

The above mentioned distributions have been used to study the interaction dynamics by fitting the data from different varieties of collider data such as $e^+e^−$ at LEP, $pp$ and heavy ion data at the Large Hadron Collider (LHC). The study of nuclear interactions has been recently done by using different types of heavy ions such as $Au$, $Pb$ etc. at the LHC. In the present work we extend the analysis to check the validity of these distributions in hadron-nucleus interactions in fixed target experiments using nuclear emulsion as a target. Nuclear emulsion acts as a photographic film, capable of recording three-dimensional tracks of charged particles with submicron spatial resolution, which can be studied under a microscope. The emulsion consists of silver bromide (AgBr) crystals dispersed in a gelatin layer which contains carbon, nitrogen and oxygen atoms. The projectile can undergo interaction with any one of these nuclei. On the average the interaction of the projectile is estimated as an interaction of the projectile with a nucleus of atomic weight 72. The latest usage of emulsion technology has been in the OPERA experiment [18], which aims at a direct observation of tau neutrino appearance in a pure muon neutrino beam. The data used for the present study, are though old, and at relatively lower center of mass energies, carry valuable information suitable for testing new models. In addition, for comparison, we also use the conventional NB distribution which has been the most frequently tested for almost all types of interactions. We present analysis of the following inelastic interactions from the fixed target experiments using $p$ or $π^±$ as the projectile.

(i) A set of $p-Em$ interactions at $P_{Lab} = 27, 67, 200, 300, 400$ and $800$ GeV [19,23].

(ii) $p-Au$ interactions at $P_{Lab} = 200$ GeV [23].

(iii) A set of $π^-Em$ interactions at $P_{Lab} = 50, 200, 340$ and $525$ GeV [25,28].

The data for $50$ GeV and $340$ GeV $π^-Em$ interactions are our own data, collected using nuclear emulsion stacks. In one case, the emulsion stack was irradiated by a $50$ GeV $π^-$ beam under the effect of a strong pulsed magnetic field of intensity $180$ KOe and is of a unique kind. The magnetic field was used for identifying the charged particles from the curvatures of the tracks produced.

(iv) $π^±Ne$ interactions at $P_{Lab} = 30$ and $64$ GeV [29].

V. COMPARISON OF PDFS OF DIFFERENT DISTRIBUTIONS OF MULTICITY

The experimental data of hadron-nucleus inelastic collisions mentioned in the previous section, are studied in terms of various distributions:

The PDFs from the Negative Binomial, shifted Gompertz, two parameter Weibull and Krasznovszky-Wagner distributions, are calculated by using equations (1), (2), (5) and (7). Minimum $χ^2$ fits to the $p-Em/Au$ data are shown in figure 1. Table I gives the parameters of the fits for all the distributions and a comparison of corresponding $χ^2/n_{dof}$ and $p$-values is given in table II. The minimum $χ^2$ fits were done by using CERN data analysis framework ROOT 6.19.

For $p-Em/Au$ interactions, one finds that NB, SG and PW distributions reproduce the data very well at almost all the energies with SG giving the best result with lowest $χ^2/n_{dof}$. Out of the four, WB distribution gives reasonable fit results at the lower energies but fails completely at the highest energies with $p$-values corresponding to $CL < 0.01\%$ making the fits statistically excluded at these energies. The detailed comparison between the three functions, NB, SG and KW shows that the $χ^2/n_{dof}$ values are roughly comparable and fits have $p$-values corresponding to $CL > 0.1\%$ in each case.

Figure 2 shows the dependence of shape parameter on the projectile energy ($P_{Lab}$) for each of the distributions for $p-Em$ interactions. It is observed that the shape parameter increases slowly with $P_{Lab}$. The dependence is almost linear. However for the NB distribution, scale parameter measures the average multiplicity, as shown in equation (1) which then must coincide with the mean value of the distribution. This makes the shape parameter a bit less flexible for fitting procedure. The dependence is parameterised as shown in equation (11) and the fit parameters ($p_0$ and $p_1$) are given in table III.

\[
shape\text{-}parameter = p_0 + p_1 \ast (P_{Lab}) \quad (11)
\]

Figures 3 shows the data and fits to NB, SG, WB, KW distributions for $π^-Em$ and $π^±Ne$ interactions at various energies. Parameters of the fit and $χ^2/n_{dof}$ and $p$-values are given in the tables I and II. We find that the data for $π^-Em$ interactions at $340$ GeV and $π^±Ne$ at $30$ GeV are statistically excluded with $p$-values corresponding to $CL < 0.01\%$ for almost all distributions. However, while SG distribution fits most of the data well at all energies, KW produces the best $χ^2/n_{dof}$ value for most of the data. Figure 4 shows the dependence of shape parameter of each distribution on $P_{Lab}$. Again, it is observed that the dependence is almost linear. However, due to very few data points, the fits suffer from large uncertainties. Parameters of the linear fits ($p_0$ and $p_1$) are given in table III. Though the $χ^2/n_{dof}$ is very large on account of very low number of fit points, yet it shows the trends how the shape parameter changes with energy, in both $p-Em$ and $π^-Em$ interactions.
FIG. 1: Data (points) on charged particle multiplicity in inelastic \( p\text{-Em/Au} \) collisions in fixed target experiments, with different projectile energies, fitted with NB, SG, WB and KW distributions (solid lines).

**FIG. 2:** Comparison of the shape parameter of NB, SG, WB and KW distributions in inelastic \( p\text{-Em} \) collisions at different energies.

Figures 5 shows the dependence of average charged particle multiplicity \( < n > \), calculated from different distributions fitted to the data, on projectile energy \( P_{\text{Lab}} \) for \( p\text{-Em} \) and \( \pi^-\text{-Em} \) interactions. The dependence is parameterised as a quadratic equation in ln(\( P_{\text{Lab}} \)) as shown in equation (12). The fit parameters are given in table IV. We compare in table IV the values of \( < n > \), expected from different distribution fits with the experimentally measured values [19–29]. It is observed that in most of the cases, the two values are in agreement to each other, within the limits of error. This shows that the distributions represent the data very well.

\[
< n > = a + b \times (\ln P_{\text{Lab}}) + c \times (\ln P_{\text{Lab}})^2
\]  

(12)

Figures 6 and 7 show the dependence of normalised moments \( C_q \) on \( P_{\text{Lab}} \) in \( p\text{-Em} \) and \( \pi^-\text{-Em} \) interactions, at different energies. Comparison between data and fit values for different distributions is shown for each fitted distribution. The values of the \( C_q \) moments are given in tables VI and VII. It may be observed that the moments are nearly independent of energy. This confirms that the KNO scaling is valid up to this energy range. The center-of-mass energy of the \( p\text{-Em} \) interaction for \( P_{\text{Lab}} = 800 \text{ GeV} \) is approximately 335 GeV, assuming the average atomic weight of emulsion as 72. Even for the \( \pi^-\text{-Ne} \), it was shown in [29] that the KNO scaling is approximately satisfied.

Figures 8 and 9 show the dependence of normalised factorial moments \( F_q \) on \( P_{\text{Lab}} \) in \( p\text{-Em} \) and \( \pi^-\text{-Em} \) interactions, at different energies. Comparison between data and fit values for different distributions is shown for each fitted distribution. The values of the moments are given in tables VI and VII. It may be observed that the factorial moments are all greater than unity. This indicates that...
FIG. 3: Data on charged particle multiplicity distributions in inelastic $\pi^\pm -E_{m}$ and $\pi^\pm -N_{c}$ collisions at different energies with fits from NB, SG, WB and KW distributions.

FIG. 4: Comparison of the shape parameter of NBD, SGD, WB and KW (top to bottom) distributions in inelastic $\pi^\pm -E_{m}$ interactions at different energies.

FIG. 5: Comparison of experimental average multiplicity (data points) and the values from best fit (solid lines) of NB, SG, WB and KW distributions in inelastic $p - E_{m}$ (top) and $\pi^\pm - E_{m}$ (bottom) interactions at different energies.
FIG. 6: Normalised moments calculated from the data and from NB, SG, WB and KW fit distributions in inelastic $p$-$Em$.

FIG. 7: Normalised moments calculated from the data and of NB, SG, WB and KW fit distributions in inelastic $\pi^-$-$Em$. 
FIG. 8: Normalised factorial moments calculated from the data and of NB, SG, WB and KW fit distributions in inelastic $p$-Em.

FIG. 9: Normalised factorial moments calculated from the data and of NB, SG, WB and KW fit distributions in inelastic $\pi^-$-Em.
the particles produced are correlated and the interaction dynamics is influenced by these correlations. No distribution has $F_q$ smaller than unity, prohibiting the negative correlations.

VI. CONCLUSION

A detailed analysis to compare the multiplicity spectra of charged particles produced in $p/\pi$-nucleus interactions in the fixed target experiments, with projectile energies ranging from 27 GeV to 800 GeV, has been done in the framework of the Negative Binomial, the shifted-Gompertz, the Weibull distributions and the Krasznovszky-Wagner distribution. The relevance of the comparison is on account of the similar nature of these distributions, whereby each of these distributions has two free parameters. It is interesting to revisit the old data collected with the visual detectors which offers an opportunity to test and study the particle production with new concepts. During the times when these data were collected, the particle production at different energies was studied in terms of KNO scaling \[17\,\text{[29]}\]. The understanding in terms of NB distribution came into picture much later, and was used mostly for the data from colliders. Often, owing to the success of NB, it became a benchmark with respect to which the performance of other distributions were studied. Our analysis of the data in terms of Negative Binomial, shifted Gompertz, the Weibull and the Krasznovszky-Wagner distribution shows that all four distributions produce acceptable fits in terms of $p$-values and NB is not superior in its performance. However, none of these is an obvious choice to be used for different projectiles and data at all energies. The analysis of normalised moments and the normalised factorial moments shows that KNO scaling is valid up to these energies, the particles produced have positive correlations which leads to the interaction dynamics being influenced by these correlations.

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TABLE I: Scale and shape parameters for NB, SG, WB and KW distributions.

| Energy (GeV) | NB | SG | WB | KW |
|--------------|----|----|----|----|
| p-Em         |    |    |    |    |
|              | $\chi^2/n_{dof}$ | p-value | $\chi^2/n_{dof}$ | p-value | $\chi^2/n_{dof}$ | p-value | $\chi^2/n_{dof}$ | p-value |
| 27           | 1.00 (19.01/9) | 0.4561 | 0.95 (18.10/9) | 0.5156 | 1.38 (26.24/9) | 0.1238 | 1.49 (28.27/9) | 0.0785 |
| 67           | 1.12 (25.67/23) | 0.3169 | 1.09 (24.96/23) | 0.3521 | 1.19 (27.47/23) | 0.2405 | 1.22 (28.08/23) | 0.2128 |
| 200          | 0.90 (28.80/32) | 0.6295 | 0.83 (26.50/32) | 0.7412 | 1.18 (37.83/32) | 0.2203 | 1.34 (42.94/32) | 0.0938 |
| 300          | 1.23 (49.30/40) | 0.1488 | 1.26 (50.51/40) | 0.1232 | 1.78 (71.30/40) | 0.0017 | 2.11 (84.53/40) | 0.0001 |
| 400          | 1.28 (52.41/41) | 0.1091 | 1.25 (51.30/41) | 0.1300 | 2.38 (97.61/41) | < 0.0001 | 2.56 (105.16/41) | < 0.0001 |
| 800          | 1.60 (75.02/47) | 0.0058 | 1.70 (80.06/47) | 0.0019 | 2.37 (111.35/47) | < 0.0001 | 2.41 (113.35/47) | < 0.0001 |
| p-AU         | 0.67 (4.67/7) | 0.6998 | 0.73 (5.14/7) | 0.6432 | 0.77 (5.36/7) | 0.6159 | 1.03 (7.18/7) | 0.4109 |
| p-Em         | 1.59 (34.96/22) | 0.0391 | 1.99 (43.73/22) | 0.0038 | 1.65 (36.29/22) | 0.0283 | 1.66 (36.51/22) | 0.0268 |
| 67           | 1.06 (32.82/31) | 0.3778 | 0.97 (30.02/31) | 0.5163 | 1.25 (38.85/31) | 0.1572 | 1.30 (40.27/31) | 0.1231 |
| 340          | 2.55 (71.48/28) | < 0.0001 | 2.42 (67.65/28) | < 0.0001 | 3.07 (86.03/28) | < 0.0001 | 2.98 (83.37/28) | < 0.0001 |
| 525          | 1.18 (54.10/46) | 0.1927 | 0.99 (45.75/46) | 0.4825 | 1.93 (88.88/46) | 0.0002 | 1.83 (83.97/46) | 0.0005 |
| p-AU         | 3.57 (89.06/16) | < 0.0001 | 7.42 (118.79/16) | < 0.0001 | 5.98 (95.74/16) | < 0.0001 | 5.84 (93.46/16) | < 0.0001 |
| p-Em         | 1.74 (31.35/18) | 0.0262 | 1.94 (34.96/18) | 0.0097 | 2.13 (38.35/18) | 0.0035 | 2.06 (37.05/18) | 0.0052 |
| p-AU         | 2.72 (46.31/17) | 0.0902 | 3.62 (61.57/17) | < 0.0001 | 3.78 (64.20/17) | < 0.0001 | 4.09 (69.45/17) | < 0.0001 |
| p-Em         | 1.13 (19.24/17) | 0.3148 | 1.31 (22.32/17) | 0.1728 | 1.38 (23.45/17) | 0.1350 | 1.37 (23.37/17) | 0.1377 |

TABLE II: Comparison of $\chi^2/n_{dof}$ and p-values for NB, SG, WB and KW distributions.

| Energy (GeV) | NB | SG | WB | KW |
|--------------|----|----|----|----|
| p-Em         |    |    |    |    |
|              | $\chi^2/n_{dof}$ | p-value | $\chi^2/n_{dof}$ | p-value | $\chi^2/n_{dof}$ | p-value | $\chi^2/n_{dof}$ | p-value |
|              | 3.5021 | 0.0001 | 6.17 (24.67/4) | < 0.0001 | 5.0414 | -0.0008 | 5.74 (11.48/2) | < 0.0001 |
|              | 3.0744 | 0.0009 | 3.38 (13.51/4) | < 0.0001 | 3.8972 | 0.0012 | 4.19 (8.39/2) | < 0.0001 |
|              | 1.7781 | 0.0002 | 3.53 (14.11/4) | < 0.0001 | 1.9626 | 0.0002 | 4.02 (8.05/2) | < 0.0001 |
|              | 0.8412 | 0.0002 | 2.97 (11.86/4) | < 0.0001 | 0.9599 | 0.0003 | 3.78 (7.56/2) | < 0.0001 |

TABLE III: Fit parameters, equation [11] giving dependence of shape parameters on $P_{lab}$ for different distributions.
TABLE IV: Comparison of average charged multiplicity $<n>$ derived from different distributions with the experimentally measured values.

| Energy (GeV) | $<n>_{exp}$ | $<n>_{NB}$ | $<n>_{SG}$ | $<n>_{WB}$ | $<n>_{KW}$ |
|-------------|-------------|-------------|-------------|-------------|-------------|
| p-Em        |             |             |             |             |             |
| 27          | 6.23 ± 0.20 | 7.01 ± 0.12 | 7.33 ± 0.01 | 6.97 ± 0.11 | 6.98 ± 0.12 |
| 67          | 9.73 ± 0.23 | 10.52 ± 0.28 | 10.42 ± 0.03 | 10.30 ± 0.25 | 10.24 ± 0.27 |
| 200         | 13.2 ± 0.2  | 13.51 ± 0.36 | 12.91 ± 0.04 | 13.24 ± 0.31 | 13.05 ± 0.33 |
| 300         | 15.1 ± 0.2  | 15.35 ± 0.23 | 14.89 ± 0.05 | 15.28 ± 0.21 | 15.24 ± 0.23 |
| 400         | 16.8 ± 0.4  | 16.82 ± 0.20 | 16.41 ± 0.05 | 16.58 ± 0.18 | 16.53 ± 0.19 |
| 800         | 19.37 ± 0.37| 19.75 ± 0.35 | 19.06 ± 0.13 | 19.59 ± 0.32 | 19.50 ± 0.33 |
| p-AU        |             |             |             |             |             |
| 200         | 21.6 ± 1.20 | 16.47 ± 1.18 | 15.51 ± 0.24 | 16.27 ± 0.88 | 15.86 ± 1.00 |

TABLE V: Fit parameters, equation (12) giving dependence of average multiplicity on $P_{Lab}$ for different distributions.

| Distribution | a     | b      | c      | $\chi^2/n_{dof}$ | a     | b      | c      | $\chi^2/n_{dof}$ |
|--------------|-------|--------|--------|------------------|-------|--------|--------|------------------|
| NB           | -1.2529 | 1.9289 | 0.1786 | 2.08 (6.23/3)    | 17.1065 | -5.4421 | 0.8280 | 2.69 (2.69/1)    |
| SG           | 0.7924 | 1.1187 | 0.2257 | 2.71 (8.13/3)    | 16.5492 | -5.7465 | 0.8703 | 18.23 (18.23/1)  |
| WB           | -0.3299 | 1.5108 | 0.2172 | 2.24 (6.73/3)    | 16.8092 | -5.3383 | 0.8116 | 1.99 (1.99/1)    |
| KW           | 0.1983 | 1.2767 | 0.2396 | 2.26 (6.78/3)    | 17.6899 | -5.6651 | 0.8438 | 1.61 (1.61/1)    |

TABLE VI: Normalised moments $C_n$ and normalised factorial moments $F_n$ of experimental multiplicity distributions at different energies with different projectiles.
### TABLE VII: Normalised moments $C_n$ and normalised factorial moments $F_n$ of NB fit distribution for data at different energies with different projectiles.

| Energy (GeV) | Normalised moments (NB) | Normalised factorial moments (NB) |
|--------------|-------------------------|----------------------------------|
| p-Au         |                         |                                  |
| 200          | 1.430 ± 0.049           | 2.628 ± 0.073                   |
|              | 5.541 ± 0.542           | 12.803 ± 2.198                  |
|              | 1.368 ± 0.040           | 2.359 ± 0.065                   |
|              | 4.589 ± 0.446           | 9.595 ± 1.647                   |
|              |                         |                                  |
| π⁺-Em        | 50                      | 1.112 ± 0.022                   |
|              | 1.692 ± 0.004           | 2.922 ± 0.082                   |
|              | 5.562 ± 0.313           | 1.009 ± 0.020                   |
|              | 1.324 ± 0.004           | 1.888 ± 0.061                   |
|              | 2.856 ± 0.182           |                                  |
| 200          | 1.555 ± 0.026           | 1.988 ± 0.006                   |
|              | 3.626 ± 0.096           | 7.369 ± 0.393                   |
|              | 1.173 ± 0.025           | 1.679 ± 0.002                   |
|              | 2.702 ± 0.076           | 4.733 ± 0.267                   |
| 340          | 1.094 ± 0.026           | 1.599 ± 0.011                   |
|              | 2.612 ± 0.035           | 4.633 ± 0.165                   |
|              | 1.027 ± 0.025           | 1.356 ± 0.010                   |
|              | 1.943 ± 0.027           | 2.932 ± 0.107                   |
| 525          | 1.225 ± 0.020           | 1.853 ± 0.006                   |
|              | 3.212 ± 0.042           | 6.207 ± 0.193                   |
|              | 1.163 ± 0.019           | 1.624 ± 0.005                   |
|              | 2.551 ± 0.035           | 4.392 ± 0.142                   |
| π⁺-Ne        | 30                      | 1.185 ± 0.021                   |
|              | 1.705 ± 0.016           | 2.790 ± 0.005                   |
|              | 5.075 ± 0.075           | 1.032 ± 0.019                   |
|              | 1.190 ± 0.010           | 1.473 ± 0.008                   |
|              | 1.930 ± 0.042           |                                  |
| 64           | 1.204 ± 0.030           | 1.749 ± 0.020                   |
|              | 2.880 ± 0.016           | 5.238 ± 0.128                   |
|              | 1.077 ± 0.028           | 1.312 ± 0.014                   |
|              | 1.727 ± 0.015           | 2.402 ± 0.073                   |
| π⁻-Ne        | 30                      | 1.234 ± 0.019                   |
|              | 1.844 ± 0.015           | 3.174 ± 0.005                   |
|              | 6.120 ± 0.075           | 1.078 ± 0.017                   |
|              | 1.306 ± 0.009           | 1.736 ± 0.008                   |
|              | 2.476 ± 0.044           |                                  |
| 64           | 1.223 ± 0.033           | 1.789 ± 0.022                   |
|              | 2.954 ± 0.014           | 5.583 ± 0.126                   |
|              | 1.098 ± 0.030           | 1.355 ± 0.016                   |
|              | 1.797 ± 0.013           | 2.493 ± 0.070                   |

### TABLE VIII: Normalised moments $C_n$ and normalised factorial moments $F_n$ of SG fit distribution for data at different energies with different projectiles.

| Energy (GeV) | Normalised moments (SG) | Normalised factorial moments (SG) |
|--------------|-------------------------|----------------------------------|
| p-Au         |                         |                                  |
| 200          | 1.420 ± 0.091           | 2.591 ± 0.103                   |
|              | 5.427 ± 0.117           | 12.467 ± 0.131                  |
|              | 1.358 ± 0.084           | 2.325 ± 0.082                   |
|              | 4.487 ± 0.068           | 9.326 ± 0.026                   |
|              |                         |                                  |
| π⁺-Em        | 50                      | 1.115 ± 0.025                   |
|              | 1.749 ± 0.012           | 3.153 ± 0.025                   |
|              | 6.307 ± 0.138           | 1.014 ± 0.021                   |
|              | 1.382 ± 0.005           | 2.086 ± 0.030                   |
|              | 3.373 ± 0.102           |                                  |
| 200          | 1.270 ± 0.032           | 2.042 ± 0.025                   |
|              | 3.808 ± 0.001           | 7.931 ± 0.077                   |
|              | 1.188 ± 0.029           | 1.731 ± 0.016                   |
|              | 2.861 ± 0.013           | 5.164 ± 0.082                   |
| 340          | 1.102 ± 0.033           | 1.622 ± 0.037                   |
|              | 2.685 ± 0.044           | 4.842 ± 0.057                   |
|              | 1.034 ± 0.030           | 1.377 ± 0.028                   |
|              | 2.004 ± 0.025           | 3.084 ± 0.022                   |
| 525          | 1.240 ± 0.024           | 1.908 ± 0.023                   |
|              | 3.403 ± 0.014           | 6.823 ± 0.016                   |
|              | 1.178 ± 0.022           | 1.677 ± 0.017                   |
|              | 2.723 ± 0.004           | 4.898 ± 0.030                   |
| π⁻-Ne        | 30                      | 1.190 ± 0.021                   |
|              | 1.775 ± 0.020           | 3.079 ± 0.014                   |
|              | 6.037 ± 0.011           | 1.037 ± 0.017                   |
|              | 1.254 ± 0.011           | 1.690 ± 0.001                   |
|              | 2.478 ± 0.022           |                                  |
| 64           | 1.213 ± 0.032           | 1.805 ± 0.032                   |
|              | 3.084 ± 0.027           | 5.863 ± 0.005                   |
|              | 1.086 ± 0.028           | 1.365 ± 0.020                   |
|              | 1.897 ± 0.005           | 2.797 ± 0.020                   |
| π⁻-Ne        | 30                      | 1.243 ± 0.018                   |
|              | 1.909 ± 0.018           | 3.438 ± 0.012                   |
|              | 7.027 ± 0.012           | 1.086 ± 0.015                   |
|              | 1.364 ± 0.009           | 1.933 ± 0.002                   |
|              | 3.004 ± 0.023           |                                  |
| 64           | 1.229 ± 0.036           | 1.829 ± 0.038                   |
|              | 3.096 ± 0.039           | 5.775 ± 0.034                   |
|              | 1.105 ± 0.031           | 1.393 ± 0.025                   |
|              | 2.759 ± 0.004           |                                  |
| Energy (GeV) | Normalised moments (WB) | Normalised factorial moments (WB) |
|-------------|-------------------------|----------------------------------|
| p-Em        |                         |                                  |
| 27          | 1.185 ± 0.030 1.757 ± 0.020 2.954 ± 0.017 5.478 ± 0.139 | 1.055 ± 0.027 1.304 ± 0.014 1.735 ± 0.016 2.432 ± 0.081 |
| 67          | 1.214 ± 0.042 1.818 ± 0.023 3.053 ± 0.040 5.583 ± 0.233 | 1.122 ± 0.039 1.483 ± 0.019 2.117 ± 0.032 3.170 ± 0.146 |
| 200         | 1.232 ± 0.027 1.949 ± 0.002 3.481 ± 0.100 6.811 ± 0.104 | 1.160 ± 0.026 1.673 ± 0.002 2.652 ± 0.079 4.492 ± 0.279 |
| 300         | 1.270 ± 0.020 2.003 ± 0.008 3.574 ± 0.034 6.986 ± 0.172 | 1.206 ± 0.019 1.759 ± 0.008 2.835 ± 0.027 4.901 ± 0.122 |
| 400         | 1.237 ± 0.015 1.877 ± 0.006 3.196 ± 0.023 5.931 ± 0.113 | 1.179 ± 0.015 1.658 ± 0.006 2.560 ± 0.019 4.216 ± 0.081 |
| 800         | 1.185 ± 0.022 1.759 ± 0.013 2.914 ± 0.017 5.230 ± 0.108 | 1.137 ± 0.021 1.581 ± 0.013 2.402 ± 0.013 3.880 ± 0.078 |

| Energy (GeV) | Normalised moments (KW) | Normalised factorial moments (KW) |
|-------------|-------------------------|----------------------------------|
| p-Au        |                         |                                  |
| 200         | 1.420 ± 0.055 2.586 ± 0.045 5.390 ± 0.435 12.302 ± 1.809 | 1.358 ± 0.054 2.320 ± 0.039 4.454 ± 0.361 9.185 ± 1.364 |

**TABLE IX:** Normalised moments $C_n$ and normalised factorial moments $F_n$ of WB distribution for data at different energies with different projectiles.

| Energy (GeV) | Normalised moments (KW) | Normalised factorial moments (KW) |
|-------------|-------------------------|----------------------------------|
| p-Em        |                         |                                  |
| 27          | 1.176 ± 0.032 1.728 ± 0.026 2.866 ± 0.001 5.232 ± 0.082 | 1.047 ± 0.029 1.278 ± 0.019 1.671 ± 0.003 2.289 ± 0.046 |
| 67          | 1.209 ± 0.044 1.799 ± 0.030 2.995 ± 0.018 5.425 ± 0.166 | 1.117 ± 0.041 1.465 ± 0.025 2.069 ± 0.015 3.060 ± 0.101 |
| 200         | 1.216 ± 0.027 1.891 ± 0.003 3.301 ± 0.074 6.283 ± 0.303 | 1.144 ± 0.026 1.618 ± 0.005 2.496 ± 0.056 4.085 ± 0.200 |
| 300         | 1.252 ± 0.020 1.943 ± 0.011 3.394 ± 0.021 6.476 ± 0.123 | 1.188 ± 0.019 1.702 ± 0.011 2.680 ± 0.015 4.507 ± 0.083 |
| 400         | 1.224 ± 0.015 1.836 ± 0.008 3.084 ± 0.016 5.643 ± 0.087 | 1.166 ± 0.015 1.619 ± 0.008 2.462 ± 0.012 3.991 ± 0.060 |
| 800         | 1.175 ± 0.022 1.727 ± 0.015 2.830 ± 0.009 5.025 ± 0.082 | 1.127 ± 0.021 1.550 ± 0.015 2.327 ± 0.005 3.714 ± 0.056 |

**TABLE X:** Normalised moments $C_n$ and normalised factorial moments $F_n$ of KW distribution for data at different energies with different projectiles.