Asymmetric metal-insulator transition in disordered ferromagnetic films

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We present experimental data and a theoretical interpretation on the conductance near the metal-insulator transition in thin ferromagnetic Gd films of thickness $b \approx 2 - 10\,\text{nm}$. A large phase relaxation rate caused by scattering of quasiparticles off spin wave excitations renders the dephasing length $L_\phi \lesssim b$ in the range of sheet resistances considered, so that the effective dimension is $d = 3$. The observed approximate fractional temperature power law of the conductivity is ascribed to the scaling regime near the transition. The conductivity data as a function of temperature and disorder strength collapse on to two scaling curves for the metallic and insulating regimes. The best fit is obtained for a dynamical exponent $z \approx 2.5$ and a correlation length critical exponent $\nu' \approx 1.4$ on the metallic side and a localization length exponent $\nu \approx 0.8$ on the insulating side.

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Since the scaling theory of Anderson localization has been proposed in 1979 \cite{1}, the metal-insulator transition in disordered conductors \cite{2} has been one of the most extensively studied cases of quantum phase transition, both experimentally and theoretically. In its simplest form it describes non-interacting electrons in a disordered potential, where the disorder can be controlled experimentally in a variety of ways, e.g. by systematic doping. One of the most dramatic predictions of the scaling theory is the absence of extended states, and therefore true metallic behavior, in systems in dimensions $d \leq 2$. This has been verified in numerous experiments \cite{3}. The other prediction is the existence of a critical point in $d > 2$ where the conductivity in the metallic phase goes to zero continuously with increasing disorder, in contrast to having a minimum metallic conductivity \cite{4}. Electron-electron interactions are known to modify the behavior near a metal-insulator transition in a significant way \cite{5}. For example, indications of a metallic state in two-dimensional systems have been found in experiment, and a number of theoretical scenarios explaining such a state have been developed \cite{6}.

Near the transition, the behavior is characterized by power laws with critical exponents. For example, the dc conductivity $\sigma(\lambda)$, with $\lambda$ being a measure of disorder, follows a power law $\sigma \sim t^s$, where $t = (1 - \lambda/\lambda_c)$ denotes the distance to the critical point at the critical disorder $\lambda_c$ and $s$ is the conductivity exponent. The dynamical conductivity at the critical point, on the other hand, is characterized by the dynamical exponent $z$ as $\sigma(\omega; \lambda_c) \sim \omega^{1/2}$. The correlation length on the metallic side ($\lambda < \lambda_c$) diverges at the critical point as $\xi \sim t^{-\nu'}$ and the localization length ($\lambda > \lambda_c$) diverges as $\xi \sim |t|^{-\nu}$. The critical exponents $\nu$ and $\nu'$ may be different. In $d = 3$ dimensions the relation $s = \nu'$ holds. The exponents in $d = 3$ have not been calculated in a reliable way up to now.

As for any quantum phase transition, the critical exponents cannot be measured experimentally at the true $T = 0$ critical point, but must be inferred from finite temperature $T$ measurements. Therefore, the emphasis has been to obtain the conductivity as a function of $T$, as close to $T = 0$ as possible. In spite of intense efforts over several decades \cite{8} it turned out to be rather difficult to access the critical regime in a reliable way. While so far all such experiments confirm the continuous nature of the transition, the values of the critical exponents remain controversial. Published experimental values of $s$ and $z$ vary from $s \approx 0.5$ \cite{9} to $s \approx 1$ \cite{10} to $s \approx 1.6$ and from $z \approx 2$ \cite{11} to $z \approx 2.94$ \cite{12}. For the Anderson transition (omitting interaction effects), numerical studies find the conductivity exponent $s \approx 1.6$ \cite{13} while theoretical prediction for the dynamical exponent is $z = 3$ \cite{14}.

Theoretically, the scale dependent conductivity at finite $T$ is obtained from $\sigma(\omega)$ by replacing $\omega$ by $T$. The critical dynamical scaling is found even away from the critical point, at frequencies $\omega > \omega_c$ where $\omega_c = \frac{1}{\tau}(\xi/l)^{-z}$. Here $l$ and $\tau$ are the mean free path and the momentum relaxation time, respectively. More precisely, in the above scaling regime the dynamical conductivity obeys the scaling law,

$$\sigma(\omega; \lambda) = \xi^{-1}G(\pm 1, \xi\omega^{1/2}), \quad t \gtrless 0. \quad (1)$$

At the critical point, when $\xi \rightarrow \infty$, it follows that $G(\pm 1, \xi\omega^{1/2}) \sim \xi\omega^{1/2}$. Using the sheet resistance $R_0$ as the disorder parameter controlled in experiment, so that $\xi \propto |R_0 - \lambda_c|^{-\nu}$, and replacing $\omega$ by $T$, the conductivity should obey the scaling,

$$|R_0 - \lambda_c|^{-\nu}\sigma(T; R_0) = G(\pm 1, |R_0 - \lambda_c|^{-\nu}T^{1/z}) \quad (2)$$

with $(R_0 - \lambda_c) \gtrless 0$, where $\lambda_c$ is the critical resistance.
Below we will see that the exponent $\nu$ can be different on the two sides of the transition.

In this letter we report the study of the conductivity near the metal-insulator transition in a thin-film geometry where it is possible not only to increase the disorder directly by changing the atomic structure, e.g. by varying the film deposition parameters and by annealing, but also indirectly by varying the film thickness. Both factors determine the sheet resistance, which is the single important parameter controlling the distance to the critical point. We are able to measure the disorder as well as the temperature dependence of the conductivity quite reliably and reproducibly. In particular, we are able to determine the location of the critical point with unprecedented precision. As we will explain below, at the relatively high temperatures and large sheet resistances of the experiment, the films are in an effectively three-dimensional regime.

Two series of thin films of Gd (series 1 and series 2) were grown by r.f. magnetron sputtering through a shadow mask onto sapphire substrates held at a temperature of 130 K. The current and voltage leads of the deposited sample overlapped with predetermined palladium contacts, thus allowing reliable electrical connection with low contact resistance for in situ measurements of the electrical properties. The experiments were performed in a specialized apparatus in which the sample can be transferred without exposure to air from the high vacuum deposition chamber to an adjoining low-temperature cryostat and electrically reconnected for transport measurements. Immediately after deposition, the samples were transferred to the cryostat and held at a temperature of 77 K or below. At these temperatures the samples are stable and do not undergo any time-dependent changes in resistance. If however the temperature is temporarily raised back to the deposition temperature (130 K), annealing marked by a slow irreversible increase in resistance occurs.

To parameterize the amount of disorder in a given film[1], we use the sheet resistance $R_0 \equiv R_{xx}(T = 5 K)$ where $R_{xx}$ is the longitudinal resistance. In our experiments $R_0$ spans the range from 4 k$\Omega$ (35 Å thick) to 40 k$\Omega$ (< 20 Å thick). Controlled thermal annealing thus allows us to advantageously tune a single sample through successive stages of increased disorder. Our series 1 samples comprise 5 separate depositions with two of the samples undergoing 12 successive anneals thus giving a total of 17 measurements at different stages of disorder. Our series 2 samples comprise a single sample undergoing 15 successive anneals for a total of 16 measurements spanning the critical region where the metal-insulator transition occurs. The resistance was measured using four-terminal dc techniques.

We now argue why the films are effectively three dimensional. As discussed in Ref. [14], the phase relaxation rate in ferromagnetic films is dominated by the scattering off spin wave excitations, yielding $\tau_\perp^{-1} \propto T$, in $d = 2, 3$ dimensions. The effective dimensionality of the system is $d = 3$ if the temperature dependent correlation length $L_c(T) = \sqrt{\sigma_T/N_0} \ll b$, where $N_0$ is the 3d density of states. The inequality is satisfied for temperatures $T \gg T_c = T_0(bk_F)^{-3}g(\beta^4/h^2)\tau_{\perp,0}/h$ , where $\epsilon_F, k_F$ are Fermi energy and wave number, respectively, and $1/\tau_{\perp,0} \sim 1 K$ is the phase relaxation rate at $T = T_0 = 1 K$ and $\beta$ is a coefficient of order unity. For not too thin samples, $bk_F > 10$, the 3d condition is seen to be met in our experiments.

To gain a first impression of the critical behavior exhibited by the data we use a fit to the expression

$$\sigma(T; R_0)/L_0 = BT^p + A.$$  (3)

Here $L_0 = e^2/h$ is the conductance quantum. The functional form of the conductivity given by Eq. (3) provides a good description of all our data for fourteen samples in the range 15 k$\Omega$ < $R_0$ < 31 k$\Omega$. We fit the data to Eq. (3) allowing the parameters $p$, $B$ and $A$ to vary with disorder. Figure 1 shows the fit for the sample with a value of $R_0 = 22.67 k\Omega$ closest to critical disorder $R_c$ where $A = 0$. We note that a positive (negative) value of $A$ indicates the delocalized (localized) regime. It follows from the scaling relation Eq. (1) that $A = 0$ at the critical point. Fortunately, the sample depicted in Fig. 1 with $R_0 = 22.67 k\Omega$ and $A = -0.01(1)$ is close enough to criticality so that we can confidently make the identification $R_c = 22.67 k\Omega$. We note that $R_c$ is of order unity in units of $h/e^2$. Using the same fitting procedures for the thirteen samples away from criticality (i.e., $A \neq 0$) we allowed all three parameters, $A, B, p$ to vary with $R_0$. It turns out that $B$ and $p$ do vary with disorder, if only slightly. The scaling property Eq. (1) requires the parameters $B, p$ to be independent of disorder. At criticality shown in Fig. 1 we determine the parameter values (see inset) $p = 0.39(3)$ and $B = 0.622(9)K^{-p}$. We infer from the value of $p$ a critical dynamical exponent $z = 1/p \approx 2.5$.

Next we analyse the data in a completely unbiased way by using a scaling plot of the scaling function $G$ defined in Eq. (2). For practical reasons this requires picking a value of the critical disorder, $R_c$, which we take from the above analysis as $R_c = 22.67 k\Omega$. We may now take different values of the critical exponents $\nu'$ and $\nu$ to see how well the data points collapse onto single curves on both sides of the transition. The best data collapse is found for $\nu' = 1.38$ on the metallic side and $\nu = 0.77$ on the insulating side. The two panels of Fig. 2 show how well the temperature-dependent data for each of the samples indicated in the legends collapse onto linear scaling curves for the two best fit values of $\nu'$ and $\nu$. For convenience, we have normalized the axes of each panel to unity using the highest temperature value of the conductivity of the samples $R_0 = 21.54 k\Omega$ ($R_0 = 23.77 k\Omega$) closest to criticality on the metallic (insulating) side of the transition.
the metallic regime and reminiscent of Eq. (3) with, however, fixed coefficients on the metallic side can be represented by an expression the sensitivity of and hence further away from criticality. The minima of the scaling analysis of Fig. 2 were determined by comparing the intercepts of the linear fits in Fig. 2.

In the insets to the two panels of Fig. 2, we show the dependence of Chi-square ($\chi^2$) on $\nu'$ in the metallic regime and $\nu$ in the insulating regime. The $\chi^2$ calculation is performed using statistical weights proportional to the inverse of the ordinate values, thereby increasing the sensitivity of $\chi^2$ to the data sets close to the origin and hence further away from criticality. The minima of $\chi^2$ are clearly defined for $p = 0.39$ at $\nu' = 1.38(11)$ in the metallic regime and $\nu = 0.77(11)$ in the insulating regime. Accordingly, the conductivity exponent $s$ in 3d has been experimentally determined to have the value $s = \nu' = 1.38(11)$.

It is remarkable and somewhat surprising that the scaling functions are very well represented by straight lines in the regime considered. Thus the conductivity on the metallic side can be represented by an expression reminiscent of Eq. (3) with, however, fixed coefficients $B, p = 1/z, \sigma(T; R_0)/L_0 = BT^{1/z} + a |R_0 - R_c|^{\nu'}$, and a disorder independent coefficient $a$ determined from the intercepts of the linear fits in Fig. 2.

As pointed out in Ref. 4, reasons for the earlier experiments not agreeing with one another have been traced to difficulties in having a system allowing sufficient access into the critical region and possessing a well defined critical point. In contrast, the current work is done on a system where the critical point can be clearly identified, and the critical region is experimentally accessible. Note that the number of data sets (14 total) around $R_c$ to be kept in the scaling analysis of Fig. 2 were determined by comparing the $\chi^2$ fits for different number of data points kept. A minimum in $\chi^2$ was obtained when the data points were restricted from about $15k\Omega - 31k\Omega$. We can also theoretically estimate the width of the critical region from the boundary frequency, $\omega_c = \frac{1}{4}(\xi/l)^{-2} \sim \frac{1}{4}(|R_0 - R_c|/R_c)^2$, where $\xi$ is equal to $\nu'z$ on the metallic and $\nu z$ on the insulating side. At $T \sim 20K$ and using $1/\tau \sim 10^3K$, this gives $|R_0 - R_c|/R_c \sim 0.4$ on the metallic side and $\sim 0.2$ on the insulating side, corresponding to a critical region extending from about $15k\Omega - 28k\Omega$, as shown in Fig. 3. This is a sufficiently large experimentally accessible region that allows us to obtain the critical exponents quite reliably.

Although the annealing treatment of a sample in the critical regime allows one to move the system through the transition point in small steps, it is desirable to have
in addition a continuous control parameter. We have observed that an applied magnetic field $H$ shifts the parameter $A$ smoothly. A continuous variation of $H$ therefore should allow to access the critical behavior of $A$ in the range one order of magnitude smaller than the present one. Work in this direction is in progress.

Our finding of the two distinctly different values for the critical exponent $\nu$ in the metallic and insulating phase, respectively, is unexpected. The quality of the data and of the fit to the scaling function appears to be so good that the difference of $\nu$ and $\nu'$ cannot be explained by uncertainties in the measurement or deviations from the scaling form. From the point of view of theory, different critical exponents of the correlation length on both sides of the transition may indicate a different structure of critical modes. We can only speculate that this behavior may be related to the system considered here to be ferromagnetic. In ferromagnets we expect spin transport by spin wave propagation, which should render it different from charge transport. To our knowledge a theoretical treatment of this problem is not available at present.

To summarize, we have studied the metal-insulator transition in thin ferromagnetic disordered films in an effectively three-dimensional regime experimentally and theoretically. We concentrate on the critical regime, for which the dynamical scaling behavior at the critical point is known to be $\sigma(\omega) \propto \omega^{1/2}$. At finite temperatures the relevant frequencies $\omega$ are given by the temperature $T$. A first fit of the $\sigma(T)$ data to a temperature power law plus a $T$-independent constant $A(R_0)$ showed an exponent $p \approx 0.4$ and serves to determine the critical resistance $R_c$ from the zero of $A(R_0)$. A full scaling analysis of the data from a reasonably accessible regime around the critical point ($\pm 30\%$) allows to determine the critical exponent of the correlation length as $\nu' = 1.38(11)$ on the metallic side and the localization length exponent $\nu = 0.77(11)$ on the insulating side and the dynamical critical exponent as $z \approx 2.5$.

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