Influence of Particles Fall Velocity, Archimedean’s Force, and Depth of Debris Flow on a Relationship between Ultimate Density of Debris-Flow Mass and Channel Slope

Boris S. STEPANOV 1 and Roza K. YAFYAZOVA 1

1 National Hydrometeorological Service “Kazhydromet” (Abay St., 32, Almaty 050022, Kazakhstan)
E-mail: bs.stepanov@gmail.com

Based on theoretical studies, ambiguous dependence of ultimate density of debris-flow mass on channel slope (debris flow path) and possibility of discontinuous increase of debris-flow mass density, when critical value of channel slope is exceeded, have been established. This allows to develop methods for calculating changes in characteristics of debris flow as it moves in a mountain valley and on a fan, to assess the effects of confluence of debris flow with surface and (or) underground water feeders, to develop optimal measures to reduce damage caused by debris flows by active impact on the rheological characteristics of debris-flow mass, to assess the risk of economic activity in debris-flow dangerous areas, and to develop optimal measures to reduce damage caused by debris flows.

Key words: debris flow, debris-flow mass density, grain-size distribution, viscosity and plasticity

1. INTRODUCTION

Until the mid of 20th century it was believed that formation of the catastrophic debris flows in the Northern Tien Shan were due to heavy and long-continued rainfalls, the frequency of which is extremely rare (once every 10,000 years), and that debris-flow mass density does not exceed 1300-1500 kg/m$^3$. However, characteristics of the 1921 debris flow deposited in the area of the city of Almaty (slope angle of 1-3°) indicated that this debris flow sediments had a high density (2350-2400 kg/m$^3$ or more).

The results of studying conditions of debris flows formation in 1956, 1958, 1963, 1973, 1977, as well large-scale experiments at the Chemolgan field test site in 1972, 1973, 1975, 1976 and 1978 showed that catastrophic debris flows (with density of 2400 kg/m$^3$ or more) can be formed by interacting of a water flow (formed as a result of breaking of surface and underground water reservoirs or intense rainfalls) with soils containing particles with sizes differing by 8-10 orders (from silt and clay particles to boulders of 5-10 m or more) at channel slope exceeding 11° and water discharge over 6-30 m$^3$/s. The experimental data could not be explained by existing theories of sediment transportation.

Until now, a dominated view is that relationship between debris-flow mass density and minimum channel slope (debris flow path) at which debris-flow mass does not stop or partially disintegrate (large particles deposition) and relationship between a channel slope and maximum debris-flow mass density are unique: characteristics of debris flow (discharge, volume, debris-flow mass density), formed on steep slope, should be decreased when slope is decreased.

Until 1972 it was believed that in the actual conditions a water flow which is formed due to emptying of a glacial water reservoir or liquid precipitation cannot be transformed into a debris flow with the volume concentration of 0.82-0.88 (at the average density of solid component of 2650 kg/m$^3$). The experiments conducted by Kazakh Institute for Hydrometeorological Research on the artificial replication of debris flows at the Chemolgan test site in the period of 1972-1978 showed that when the length of a debris flow origination site of 700 m, its maximum slope of 17°, grain-size distribution of soils is polydisperse (particle size from $10^{-4}$ to 10° mm), the average density of rocks of 2650 kg/m$^3$, and water flow discharge of 28 m$^3$/s, a debris flow with discharge of 430 m$^3$/s and peak density of 2400 kg/m$^3$ (the volume concentration of 0.85) is formed [Khonin et al., 1977]. The results of study of actual debris flows formed in the period of 1970-2015 confirmed the results of experiments [Vinogradov, 1977 b; Kirenskaya et al., 1977; Popov et al., 1980; Khaydarov and Fominov, 1988; Khaydarov and Shevyrtalov, 1989; Yafyazova, 2011].
It is known, that debris flows with high density are not formed on the light slope \( (1 - 3^3) \). However debris flows with high density having sufficiently plasticity can move on the light slope \( (1 - 3^3) \).

The aim of research, which the results are given in this paper, was establishing the relationship between the volume concentration of solid component of debris-flow mass (for a different grain-size distribution and density of solid component of debris-flow mass) and a minimum channel slope on which the debris-flow mass can move without disintegration (sedimentation) or stop.

2. SEDIMENT ENTRAINMENT BY SURFACE WATER FLOW IN DEBRIS FLOW ORIGINATION SITE

2.1 Review of existing debris-flow models for sediment transport in suspension

In equations describing process of sediment transportation, a concept of power was first appeared in proceedings of Rubey [1933], and later, Knapp [1938], Zamarin et al. [1952], Velikanov [1954], Bagnold [1954; 1966], Mostkov [1957], Bogomolov and Mikhaylov [1972], Stepanov [1978], Stepanov and Stepanova [1991] where flowing fluid was considered as a transportation machine.

Zamarin et al. [1952] constructed a model of debris flow that illustrated as

\[
P = 0.022 \frac{v}{w} \sqrt{H \sin \alpha},
\]

where \( P \) — flow transportation capacity, i.e. amount of suspended sediments in kilograms contained in \( 1 \) m\(^3\) of water; \( v \) — flow velocity; \( w \) — fall velocity; \( H \) — flow depth; \( \sin \alpha \) — energy slope.

Bagnold [1966] constructed a model of debris flow as illustrated by

\[
P = \rho g H \sin \alpha \left( \frac{0.01v^4}{w} \right),
\]

where \( \rho \) — density of fluid; \( g \) — gravity acceleration; \( e = 0.01 \) — efficiency, the coefficient 0.01 against the second suspension term is the theoretical suspension efficiency 0.015 reduced by the factor 2/3 on account of the stream power already dissipated in bedload transport.

If the suspended solids are heterogeneous in size, the work rate is given by the arithmetic mean of the fall velocities [Bagnold, 1966]

\[
w = \frac{\sum w_i p_i}{\sum p_i},
\]

where \( p_i \) is the weight of any individual constituent grade whose fall velocity is \( w_i \).

Mostkov [1957] built a model of debris flow as illustrated by Eq. 4. This model takes into account energy consumption for saltation and suspension of particles, their rolling and impingement.

\[
C = \frac{1.21 \sin \alpha}{1 - 2 \sin \alpha},
\]

where \( C \) — volume concentration of solids in debris-flow mass expressed as a decimal fraction.

Based on the gravitation theory of Velikanov [1948; 1954]

\[
C = \frac{1 - \frac{1}{2} \psi}{\beta},
\]

where \( \psi = \frac{\alpha w}{\beta} \); \( \alpha = \frac{\rho_s}{\rho_t} \); \( \beta = \frac{aw}{\sin \alpha} \); \( w = 0.2 \); \( \rho_t \) — debris flow solid component density; \( \rho_s \) — water density.

If \( \alpha = 1.65 \), maximum volume concentration \( C_{max} \) is calculated as

\[
C_{max} = \frac{1 - \frac{1}{2} \psi}{\beta} = 0.38.
\]

According to Sanoyan [1980]

\[
P = 132 \frac{v \sin \alpha}{w} + 0.56 \left( \frac{v \sin \alpha}{w} \right)^2,
\]

where \( n_c = nd^{1/8} \sin \alpha^{1/6} \); roughness coefficient \( n = 0.14 \); \( d \) — particle size.

**Figure 1** presents results of calculation by Eqs. 2, 4, 5 and 7.

It was used in the calculation the water density \( \rho_t = 1000 \text{ kg/m}^3 \), the average density of solid component in debris-flow mass \( \rho_s = 2650 \text{ kg/m}^3 \), roughness coefficient \( n = 0.14 \), the average particle size \( d = 0.0003 \text{ m} \), fall velocity \( w = 0.08 \text{ m/s} \), flow depth \( H = 1 \text{ m} \), flow velocity \( v \) calculated by \( v = k \sqrt{H \sin \alpha} \), where Chezy’s coefficient \( k = 15 \).

**Figure 1** shows the relationships between maximum possible volume concentration of solids in...
debris-flow mass and channel slope is single-valued function.

Models for describing the effects of phase shift in large-scale debris flow given in Bagnold [1954], Takahashi [1981], Hotta and Miyamoto [2008], Nishiguchi et al. [2011], Hotta et al. [2013], Uchida et al. [2013].

2.2 Approach to sediment transport problem from general physics

Hyper-concentrated debris flows can be formed in the desert area if the solid component of debris-flow mass is fine particles, the length of channel is sufficient, and its slope (depending on the depth of water flow) exceeds a critical value. Under such conditions, the particles move in suspension.

In the mountain area where the particles size is heterogeneous (the maximum particles size is in the range 100-10,000 mm or more), the channel slope have to exceed 10-12° and the water flow discharge of 5-10 m³/s and more. Under such conditions large particles (size of 100 mm and more) can move under gravitation by rolling, and particles size of less than 100 mm move in suspension. If density of debris-flow mass increases, the large particles are in “gravity solution” (a mixture of smaller particles and water).

At high concentration of solids in debris-flow mass and the presence of clay particles, debris-flow mass can acquire plasticity. With increasing plasticity the solid component in debris-flow mass may be in quasi-suspension. If the debris flow depth is more than the size of large particles, they can move on slopes of less than 1° due to their potential energy [Stepanov and Yafyazova, 2014].

Data about effect of mineralogical composition and grain-size distribution of soils on rheological characteristics of debris-flow mass, Archimedes’ force values on different size particles, concentration of solids on fall velocity of particles allowed to develop a model describing dependence between concentration of solids in debris-flow mass and channel slope.

At present, it is theoretically impossible to determine the flow efficiency. However, the data about the large-scale experimental and actual debris flows under various conditions allow estimating the value of flow efficiency. For the conditions of Tien Shan, the Pamirs, the Dzungarsky Alatau, the Caucasus e =0.07 For more information about determining the flow efficiency to read the section 3.

In Eq. 8 the expression \( \sum_{i=1}^{n} (\rho_i - \rho_{-1}) w_i C_i (1 - C_i)^{\alpha} \) is the power \( (N) \) required to maintain solids in suspension. It is easy to be convinced that the product \( w_i C_i (1 - C_i)^{\alpha} \) increases with increasing volume concentration \( C \) of solids in debris-flow mass, it reaches a maximum and then it decreases. Formally, when \( C \rightarrow 1 \), the power required to maintain solids in suspension tends to zero \( (N \rightarrow 0) \).

In Eq. 8 the expression \( \sum_{i=1}^{n} \rho_i C_i \) is the power \( (N) \) which flow can use (spend) to maintain solids in suspension. Evidently, that this power (until the flow regime is turbulent) increases with increasing volume concentration of solids, it reaches a maximum and then it decreases because of the increase of viscosity and plasticity debris-flow mass resulting decrease of flow velocity.

As a result of changes of concentration of solids in debris-flow mass (Eq. 8) with a typical characteristics for mountains the mineralogical composition and grain-size distribution of loose rocks and flow depth the relationship between the concentration of solids in debris-flow mass and path slope of its movement can be ambiguous.

The flow efficiency is a combined function. It depends on the density of particles, their shape and size, concentration in debris-flow mass, mineralogical composition and grain-size distribution, flow velocity, its depth. Mostkov [1959] believed that the flow efficiency is about 20%. According to Goncharov [1962] the flow efficiency is less than 10%, and Greenwald [1974], the flow efficiency can vary between 2 – 19%.

At present, it is theoretically impossible to determine the flow efficiency. However, the data about the large-scale experimental and actual debris flows under various conditions allow estimating the value of flow efficiency. For the conditions of Tien Shan, the Pamirs, the Dzungarsky Alatau, the Caucasus e =0.07 For more information about determining the flow efficiency to read the section 3.

In Eq. 8 the values of \( C_i, \rho_{-1}, w_i, \) and \( v \) are determined by Eqs. 9, 10, 11–15 and 22.

\[
C = \frac{CP}{1 - C + C \sum_{i=1}^{n} P_i} \tag{9}
\]

where \( C \) – volume concentration of solid component in debris-flow mass, \( P_i \) – part of solid component, \( \sum_{i=1}^{n} P_i =1 \).

The mixture density is determined as

\[
\rho_{-1} = \rho_w = 1000 \text{kg/m}^3, \\
\rho_i = \rho_w C_i + \rho_{-1}(1 - C_i), \\
\rho_e = \rho_w C_e + \rho_{-1}(1 - C_e),
\]

In Eq. 8 the expression \( \sum_{i=1}^{n} (\rho_i - \rho_{-1}) w_i C_i (1 - C_i)^{\alpha} \) is the power \( (N) \) required to maintain solids in suspension. It is easy to be convinced that the product \( w_i C_i (1 - C_i)^{\alpha} \) increases with increasing volume concentration \( C \) of solids in debris-flow mass, it reaches a maximum and then it decreases. Formally, when \( C \rightarrow 1 \), the power required to maintain solids in suspension tends to zero \( (N \rightarrow 0) \).

In Eq. 8 the expression \( \sum_{i=1}^{n} \rho_i C_i \) is the power \( (N) \) which flow can use (spend) to maintain solids in suspension. Evidently, that this power (until the flow regime is turbulent) increases with increasing volume concentration of solids, it reaches a maximum and then it decreases because of the increase of viscosity and plasticity debris-flow mass resulting decrease of flow velocity.

As a result of changes of concentration of solids in debris-flow mass (Eq. 8) with a typical characteristics for mountains the mineralogical composition and grain-size distribution of loose rocks and flow depth the relationship between the concentration of solids in debris-flow mass and path slope of its movement can be ambiguous.

The flow efficiency is a combined function. It depends on the density of particles, their shape and size, concentration in debris-flow mass, mineralogical composition and grain-size distribution, flow velocity, its depth. Mostkov [1959] believed that the flow efficiency is about 20%. According to Goncharov [1962] the flow efficiency is less than 10%, and Greenwald [1974], the flow efficiency can vary between 2 – 19%.

At present, it is theoretically impossible to determine the flow efficiency. However, the data about the large-scale experimental and actual debris flows under various conditions allow estimating the value of flow efficiency. For the conditions of Tien Shan, the Pamirs, the Dzungarsky Alatau, the Caucasus e =0.07 For more information about determining the flow efficiency to read the section 3.

In Eq. 8 the values of \( C_i, \rho_{-1}, w_i, \) and \( v \) are determined by Eqs. 9, 10, 11–15 and 22.

\[
C = \frac{CP}{1 - C + C \sum_{i=1}^{n} P_i} \tag{9}
\]

where \( C \) – volume concentration of solid component in debris-flow mass, \( P_i \) – part of solid component, \( \sum_{i=1}^{n} P_i =1 \).

The mixture density is determined as

\[
\rho_{-1} = \rho_w = 1000 \text{kg/m}^3, \\
\rho_i = \rho_w C_i + \rho_{-1}(1 - C_i), \\
\rho_e = \rho_w C_e + \rho_{-1}(1 - C_e),
\]

In Eq. 8 the expression \( \sum_{i=1}^{n} (\rho_i - \rho_{-1}) w_i C_i (1 - C_i)^{\alpha} \) is the power \( (N) \) required to maintain solids in suspension. It is easy to be convinced that the product \( w_i C_i (1 - C_i)^{\alpha} \) increases with increasing volume concentration \( C \) of solids in debris-flow mass, it reaches a maximum and then it decreases. Formally, when \( C \rightarrow 1 \), the power required to maintain solids in suspension tends to zero \( (N \rightarrow 0) \).

In Eq. 8 the expression \( \sum_{i=1}^{n} \rho_i C_i \) is the power \( (N) \) which flow can use (spend) to maintain solids in suspension. Evidently, that this power (until the flow regime is turbulent) increases with increasing volume concentration of solids, it reaches a maximum and then it decreases because of the increase of viscosity and plasticity debris-flow mass resulting decrease of flow velocity.

As a result of changes of concentration of solids in debris-flow mass (Eq. 8) with a typical characteristics for mountains the mineralogical composition and grain-size distribution of loose rocks and flow depth the relationship between the concentration of solids in debris-flow mass and path slope of its movement can be ambiguous.

The flow efficiency is a combined function. It depends on the density of particles, their shape and size, concentration in debris-flow mass, mineralogical composition and grain-size distribution, flow velocity, its depth. Mostkov [1959] believed that the flow efficiency is about 20%. According to Goncharov [1962] the flow efficiency is less than 10%, and Greenwald [1974], the flow efficiency can vary between 2 – 19%.

At present, it is theoretically impossible to determine the flow efficiency. However, the data about the large-scale experimental and actual debris flows under various conditions allow estimating the value of flow efficiency. For the conditions of Tien Shan, the Pamirs, the Dzungarsky Alatau, the Caucasus e =0.07 For more information about determining the flow efficiency to read the section 3.

In Eq. 8 the values of \( C_i, \rho_{-1}, w_i, \) and \( v \) are determined by Eqs. 9, 10, 11–15 and 22.

\[
C = \frac{CP}{1 - C + C \sum_{i=1}^{n} P_i} \tag{9}
\]

where \( C \) – volume concentration of solid component in debris-flow mass, \( P_i \) – part of solid component, \( \sum_{i=1}^{n} P_i =1 \).

The mixture density is determined as

\[
\rho_{-1} = \rho_w = 1000 \text{kg/m}^3, \\
\rho_i = \rho_w C_i + \rho_{-1}(1 - C_i), \\
\rho_e = \rho_w C_e + \rho_{-1}(1 - C_e),
\]
Algorithm for determining the fall velocity $w_i$ is as follow:

1. According to Goncharov [1962], the fall velocity (laminar flow) is calculated as

$$w_i = \frac{d_i^4 (\rho_f - \rho_{water})}{24 \mu_{i-1}}.$$  \hspace{1cm} (11)

If the debris-flow mass has plasticity ($\tau_o > 0$), the fall velocity is determined by Stepanov

$$w_i = \frac{d_i^4 (\rho_f - \rho_{water}) - 6 \tau_o}{24 \mu_{i-1}}.$$  \hspace{1cm} (12)

2. The fall velocity (turbulent flow around a body) is calculated as [Goncharov, 1962]

$$w = 3.6 \sqrt{\frac{(\rho_f - \rho_{water}) d_i}{\mu_{i-1}}}.$$  \hspace{1cm} (13)

If $w$ determined by Eqs. 11 or 12 is more than $w$ determined by Eq. 13, $w$ is calculated by Eq. 13.

If determined by Eqs. 11 or 12 is less than $w$ determined by Eq. 13, $w$ is calculated by Eqs. 11 or 12.

Velikanov [1948] considered that fall velocity of monodispersed particles depend on their volume concentration

$$w_{i(c)} = w_i (1 - C_i),$$  \hspace{1cm} (14)

where $w_{i(c)}$ – fall velocity of particles taking into account their concentration in the debris-flow mass.

Kizevalter [1979] referring to Happel and Brenner [1976] proposed to take into account the influence of concentration of solids on fall velocity as follows

$$w_{i(c)} = w_i (1 - C_i)^4.$$  \hspace{1cm} (15)

Values of $k$ for different Reynolds numbers obtained experimentally. These values $k$ were given by Kizevalter [1979] as follow

$$k = 4.65 \text{ for } \text{Re} < 0.2$$

$$k = \left(4.45 + 18 \frac{d}{D}\right) \text{Re}^{-0.1} \text{ for } 1 \text{ < Re < 200},$$

$$k = 4.45 \text{Re}^{-0.1} \text{ for } 200 < \text{Re} < 500$$

$$k = 2.39 \text{ for } 500 < \text{Re},$$

where Reynolds number is determined based on the values of $(\rho_f - \rho_{water}), \mu_{i-1}, d, w_i$. 

Perhaps the most basic work in the rheology of suspensions was due to Einstein, who derived a formula for the relative viscosity of dilute suspensions of uniform-sized spherical particles [Farris, 1968].

Chong et al. [1971] obtained the empirical dependence between viscosity and concentration of the solid component for a high-concentration suspension.

Farris [1968] developed a method for calculating the viscosity of suspension when the solid component is polydisperse.

In [Stepanov and Stepanova, 1991] to calculate the viscosity of debris-flow mass, when the solid component is polydisperse, it is recommended to use the empirical equation (19)

$$\mu_{w} = K^{*+1} \mu_{o} \prod_{i=1}^{n} \mu_{i},$$  \hspace{1cm} (19)

where $n$ – a number of decimal intervals of grain-size distribution (grain-size distribution is divided by intervals in such a way that minimum and maximum sizes of particles of each interval differ from each other ten times); $K$ – coefficient considering mutual influence of particles of complementary intervals; $K = 1.34^c$.

If $n = 1$, $\mu_{i} = \mu_{o}$, where $\mu_{o}$ – water viscosity [Ns/m²].

When $C_i < 0.5$ [Stepanov and Stepanova, 1991],

$$\mu_i = \mu_{o} (1 + 2.5 C_i) (10 C_i^2 + \exp 11 C_i^2).$$  \hspace{1cm} (20)

When $C_i > 0.5$ [Stepanov and Stepanova, 1991],

$$\mu_i = \mu_{o} (1 + 2.5 C_i) (10 C_i^2 + \exp 11 C_i^2) \times \exp (20 C_i - 10).$$  \hspace{1cm} (21)

Rheological model for debris flow that is based on the relationship between stress and rate of its deformation is given in [Vinogradov, 1977a].

Vinogradov believed that the deformation rate of debris-flow mass is determined both the debris flow resistance caused by molecular and turbulent viscosity and the “internal friction” by Amontona-Coulomb’s
Further investigation [Stepanov and Stepanova, 1991] showed that the main role in the debris flow resistance under debris-flow mass density of 2350-2450 kg/m\(^3\) belongs to shearing resistance due to the presence of colloidal particles (under the stipulation that the flow depth exceeds the size of the largest particles of solid component).

In such situation, the flow velocity is calculated by Eq. 22 [Stepanov and Stepanova, 1991]

\[
v = \frac{\mu_{dm} C_f H}{2 \rho_{dm} \tau_s} \left( \frac{1 + \frac{2 H \sin \alpha}{l}}{\left( \frac{4 \rho_{dm} I^1}{\rho_{dm} I^2} \right)^{1.3}} \right)
\]

where \(\mu_{dm}\) – debris-flow mass viscosity [Ns/m\(^2\)]; \(C_f\) – form factor; \(\rho_{dm}\) – debris-flow mass density [kg/m\(^3\)]; \(l\) – empirical value of “mixing length” calculated according to [Stepanov and Stepanova, 1991]

\[
l = \frac{\sqrt{(C d_{75})^2 + (xH)^2}}{3.2 \left( \frac{H}{\Delta} \right)^{\frac{1}{2}} - \left( \frac{\Delta}{H} \right)^{\frac{1}{2}} + 0.3}
\]

where \(d_{75}\) – particle size of solid component, corresponding to 75% of complete grain-size distribution curve; \(x\) – empirical coefficient calculated according to [Stepanov and Stepanova, 1991]

\[
x \approx 0.4 \left( \frac{1 + C \left( \frac{\rho_f}{\rho_{dm}} - 1 \right)}{1 + 2.5 C} \right);
\]

\(\Delta\) – empirical coefficient of roughness calculated by \(\Delta = d_{75} + 0.2 H\); \(\tau_s\) – yield stress [N/m\(^2\)] by calculated in accordance with [Gavrishina, 1985] for monodisperse particles \(\tau_s\) is calculated from the empirical equation

\[
\frac{\tau_s}{\tau_{dm}} = \exp \left( 0.4 C^{0.3} \right) + 54.87 \left( \frac{C^{1.15}}{d^{0.31}} \right) \left( \frac{d}{\Delta R} \right)^{0.31}
\]

where \(\tau_{dm}\) – yield stress for dispersion medium; \(C\) – weight concentration of solids in debris-flow mass; \(d\) – particle size of solid component; \(\Delta R\) – size of working section of device for determining of yield stress for dispersion medium, [m].

For polydisperse particles \(\tau_s\) is calculated from cumbersome empirical equation, which is described in [Gavrishina, 1985].

In situations of practical interest, dependence between concentration of solids in debris-flow mass and channel slope (Z-function) calculated by Eq. 8.

Eq. 8 was developed by B. S. Stepanov, but W. W. Rubey, R. T. Knapf, E. A. Zamarin, K. V. Popov and V. V. Fandeev, M. A. Velikanov, R. A. Bagnold, M. A. Mostkov, A. I. Bogomolov and K. A. Mikhailov, B. S. Stepanov and T. S. Stepanova et al. may be considered as the authors of concept that to maintain solids in suspension it is necessary that a flow spent some of its potential energy.

On the curve, shown in Fig. 2, there is a segment (branch 3) with negative slope. The branches 1 and 2 are attractors which are sets of stable equilibrium points of debris-flow mass and coincide with \(\Xi\)-function (as shown in Fig. 3). Branch 3 represents an unstable state of debris-flow mass.

Thus, Z-function divides the plane \((\rho, \alpha)\) into area of disintegration of debris-flow mass (M) and area of increase of concentration of solids in debris-flow mass.
(N), which allows to predict the evolution of debris flow.

The presence of negative slope (branch 3) of Z-function allowed to theoretically justify a previously unknown natural phenomenon—a phenomenon of discontinuous increase of the debris-flow mass density when a channel slope exceeds a critical value for given values of channel and flow characteristics, as well mineralogical composition and grain-size distribution of solid component of debris-flow mass [Stepanov, 1992].

Figure 3 shows Ξ-function that reflects dependence of maximum possible value of density of debris-flow mass (for given values of flow depth and rheological characteristics of debris-flow mass) on channel slope. This curve is obtained from data shown in Fig. 2. It follows from Fig. 3, if debris flow path slope (channel slope) exceeds (infinitesimally) a value corresponding to the slope on which Z-function acquires a negative slope (branch 3) (see, Fig. 2), density of debris-flow mass increases by a finite value (the function is discontinuous).

From a physical point of view, this means that at an infinitesimal (relatively to critical value of αc angle in Fig. 2) an increase of channel slope debris flow becomes energetically possible (in a finite time, on finite length of channel) to increase debris-flow mass density by a finite amount. Fig. 3 shows that if at angle of αc debris-flow mass density exceeds ρc, then with an increase of channel slope by an infinitesimal amount debris-flow mass density reaches (after some finite time, with presence of loose rocks in a bed after moving of debris flow at some length) maximum possible value (for grain-size distribution used in the calculation).

3. COMPARISON OF THEORETICAL RESULTS WITH REAL OBSERVABLE CHARACTERISTICS OF DEBRIS FLOWS

The results of calculations by Eq. 8 are in good agreement with observations of debris flows in different physiographic conditions and allow making a quantitative interpretation of the features of debris flow processes, which even had not a qualitative explanation within general notions of debris flows.

The presence of negative slope (branch 3) of Z-function (see Fig. 2) in natural conditions it is reflected in the fact that a debris-flow mass density can be increased on a channel slope of 2-5° and even less, when debris-flow mass density of more than 2000 kg/m³. Deepening of channel and/or change of mineralogical composition and grain-size distribution of solid component of debris-flow mass are indications of increase of density.

The 1921 debris flow on the Malaya Almatinka River and the 1982 debris flow on the Sarkand River (Dzungarsky Alatau Mountain Ridge) may be as an example of increasing the debris-flow mass density on a small channel slope. Mineralogical composition and grain-size distribution of solid component of these debris flows are almost identical.

During the 1921 debris flow the channel was deepened by 5-10 m on the section with slope of less than 5°[Dyurnbaum, 1949]. The 1982 debris flow formed as a result of the interaction between a water flow and granitoid loose rock on the channel section with slope of about 10°. A further increase of the debris-flow mass density occurred on the channel section with slope of less than 2° and about 20 km in length in sedimentary rock.

At the exit from the mountains, in the upper part of its fan (on slope of 2°) the channel was deepened with debris flow (Fig. 4). Large particles deposited in the lower part of the fan (on slope of less than 1°) due to decrease of the flow depth (Fig. 5) [Khaydarov and Fominov, 1988].

A verification of theoretical propositions about nature of the relationships between solid component concentration in debris-flow mass (Z-function) and channel slope (debris flow path) was done based on example of the debris flow in the Zhamankum Desert (Fig. 6) and debris flows occurred on the northern slope of the Zaliysky Alatau Mountain Ridge [Yafyazova, 2007].

Appropriateness of attracting characteristics of debris flow formed in the Zhamankum Desert, for verification of a model for debris flow state is due to relative uniformity of particles size: the average content of less than 0.1 mm particles—11.3%, and more than 0.5 mm—2.3%, and to the fact that during debris flow process, caused by emptying of wastewater storage reservoir with a volume of about 36 million m³, 36.5 million m³ of sand were moved. And the debris flow moved (on a slope with angle close to 0.5°) to tens of kilometers. Content of clay particles in the sand of the Zhamankum is so small that when concentrations of the solid component in debris-flow mass close to that at yield stress, debris-flow mass practically has no a property of plasticity. Consequently, sand particles moved at light slopes in suspension state due to mixing. To explain this phenomenon from the perspective of the theory of sediment transportation by water flow is not possible.

As a result of emptying of the natural wastewater storage reservoir in the Zhamankum Desert a canyon formed with the flat bottom, which width of 50-80 meters and a length of about 6 km. Lack of a channel on the flat bottom of the canyon testified that the width of the debris flow was equal to the width of the...
After emptying of wastewater storage reservoir the debris-flow process continued due to landslides, which were formed on the slopes of this storage reservoir. Landslides density was close to a maximum value, which allowed a formed debris flow to move over 6 km (even in the absence of plasticity) on a slope close to 0.5°. Otherwise, as a result of disintegration (sedimentation), solid component of the debris flow would be deposited increasing a slope of canyon bottom at the section adjacent to storage reservoir, and generated water flow would form a bed on the flat bottom of the canyon.

Consequently, a debris flow density was close to the value, which corresponds to the point C (Fig. 7). According to Eq. 8, such density (at the flow depth of 1 m) corresponds to the efficiency close to 0.14. Available data on the conditions for formation of debris flow in the Zhamankum desert it is insufficient to determine the coordinates of the point B (see. Fig. 7).

Figure 8 shows a curve of debris-flow mass state, calculated using grain-size distribution of glacial deposits in the Malaya Almatinka River basin. Debris flow depth is 1 m. While calculating the curve it was assumed that size fractions of 100-1000 and 1000-10,000 mm moved independently by rolling under gravity force down to the value of debris-flow mass density equal to 2000 kg/m³.

The coordinates of the point B (Fig. 8) make it possible to determine data on the 1982 debris flow in the Sarkand River basin. The channel slope where the 1982 debris flow was formed of about 10°. It is the value of the slope on which the large particles (particle size is more than 200 mm) by themselves can move by
rolling. Therefore, for the formation of debris flow it is necessary that only fractions which size is less than 100–200 mm moving in a turbulent flow in suspension. Application of Eq. 8 showed that the flow efficiency (at the flow depth of 1 m) is close to 0.07.

Determine the coordinates of the point C (see. Fig. 8) using data about the 1982 debris flow in the Sarkand River impossible, because when the volume concentration is close to 0.8–0.9, the main role in maintaining the solids in quasi-suspension belongs the plasticity of debris flow mass and Archimedian’s Force.

As shown by study of the conditions for the formation and movement of the 1988 debris flow in the Zhamankum Desert, the debris flows deposits on the fans of the Sarkand and Malaya Almatinka Rivers the angle of slope of 0.5–2° was sufficient for movement of particles which size was in the range from a fraction of a micron to 5–6 m [Khaydarov and Shevyrtalov, 1989; Khaydarov and Fominov, 1988; Vinogradov, 1977 b].

It follows from the above that even at a very large difference in the ranges of particles sizes in the grain-size distributions of solids the efficiency changes only two times. It is explained by that at the same concentration of solids in the debris-flow mass of the 1973, 1982, and 1988 debris flows the plasticity differed by many thousands of times (the yield stress of the 1988 debris flow was close to zero; the plasticity was not measured due to the lack of an instrument for measuring small quantity of plasticity). Eq. 8 can be used when the grain-size distribution of solids is close to the grain-size distribution of sand in the Zhamankum Desert or debris flows on the Malaya Almatinka or Sarkand Rivers.

In calculating a curve of debris-flow mass state (based on example of the 1988 debris flow in the Zhamankum Desert) grain-size distribution of solid component of debris-flow mass consisted of two size fractions: 0-0.1 mm and 0.1-0.5 mm. The flow depth was assumed to be 1 m, the flow efficiency, as a suspensions moving mechanism, was assumed to be equal to 0.14. Grain-size distribution of sand in the Zhamankum Desert is shown in Table 1. Curve of debris-flow mass state for the 1988 debris flow in the Zhamankum Desert is shown in Fig. 7.

Table 1 Grain-size distribution of sand in the Zhamankum Desert [%]

| Sampling depth, m | Size fraction [mm] |
|-------------------|--------------------|
|                   | 0-0.5   | 0.5-0.25 | 0.25-0.1 | <0.1   |
| 0-3               | 3.4     | 50.3     | 37.5     | 8.8    |
| 3-6               | 1.2     | 32.0     | 56.1     | 10.7   |
| 6-9               | 2.2     | 36.3     | 55.2     | 6.3    |
| 9-12              | 1.1     | 22.1     | 59.9     | 16.9   |
| 12-20             | 4.2     | 34.3     | 49.6     | 11.9   |
| 20-50             | 1.8     | 30.0     | 54.9     | 13.3   |
| The average of six depths | 2.3 | 34.2 | 52.2 | 11.3 |

July 15, 1973 as a result of emptying moraine lake in the Malaya Almatinka River basin (the Zailiyskiy Alatau) catastrophic debris flow was formed, maximum discharge reached 10,000 m3/s, a volume was of 3.8 million m3 and density of the debris-flow mass was close to 2390 kg/m3 (with the average density of solid component of debris flow of 2650 kg/m3, the volume concentration is 0.85). Formation of the debris flow was occurred in the debris flow origination site with a length of 8 km long, the slope varied in a range of 8-17°. The bottom of the debris flow origination site deepened to 12-15 m (in some places up to 40 m). The debris flow deposited in the debris flow storage reservoir formed by a dam in the Medeo tract [Vinogradov, 1977 b; Yafyazova, 2011].

Debris-flow mass density was determined in three ways:

- using the volume and density of the solid component of debris-flow mass;
- using the suspension density and grain-size composition of the solid component;
- magnetometric method was used only during the Chemolgan experiments [Stepanov, 1982].

This debris flow was formed as a result of interaction of water flow with loose rocks represented by glacial deposits, similar to what was observed during the experiments in large-scale debris flow simulation at the Chemolgan field test site in 1972, 1973, 1975, 1976 and 1978.

The aim of the experiments was the following:

- identification of the transformation mechanism of water flow into a debris flow;
- determination of the critical discharge of water flow and channel slope at which the water flow can be transformed into a debris flow (when the solid component is glacial deposits);
- determination of regularities of change of velocity of debris flow depending on the debris-flow mass density, flow depth, and channel slope, etc.

Figures 7 and 8 show that the curves of state acquire a negative slope even when volume concentration of solids of debris-flow mass is less than 0.3, i.e., long before the transformation of turbulent debris flow into quasi-laminar.

Comparison of the curves of state given in Figs. 7 and 8 show that in case of excess of critical value of debris-flow mass density energy consumption for keeping solid component in suspension not only does not increase, but even decrease. Channel (debris flow path) slopes at which these events occur are different
and determined by the mineralogical composition and grain-size distribution of solid component of debris flow mass and flow depth.

Keeping particles suspended is promoted by plasticity of debris-flow mass through which a part (and at a high density even 100%) of particles can be in quasi-suspension that reduce (may even eliminate) energy consumption required to keep particles in quasi-suspension.

The results of experiments at the Chemolgan field test site described in detail in Zems et al., 1976; Khonin et al., 1977; Stepanova et al., 1978; Kirenskaya et al., 1980; Rickenmann et al., 2003. Some parameters of debris-flow experiments given in Table 2.

It has been found that turbulent mixing together with stirring caused by transverse flow circulations is maintained up to values of solid component concentration in debris-flow mass that are much higher than in pre-existing views. This radically changed understanding of the mechanisms for formation of a high density debris-flow mass. Grain-size distribution of glacial deposits (Table 3) is significantly different from soils of the Zhamankum Desert. Presence of silt and clay fractions in ancient glacial deposits is enough so that when debris-flow mass density is about 2400 kg/m$^3$ it acquires plasticity which is sufficient to keep almost all size fractions of solid component of debris-flow mass in quasi-suspension.

4. CONCLUSIONS

Confirmation in practice of theoretical conclusions about ambiguous dependence of ultimate density of debris-flow mass on channel slope (debris flow path) as well as a possibility of a discontinuous increase (or decrease) debris-flow mass density when bed slope excesses (or decreases) a critical value is of great practical interest. This allows to develop methods for calculating changes in characteristics of debris flow as it moves in a mountain valley and on a fan, to assess the risk of economic activity in debris flow hazardous areas, and to develop optimal measures to reduce damage caused by debris flows.

ACKNOWLEDGMENT: We thank Dr. Norifumi Hotta and anonymous reviewers for their valuable comments.

REFERENCES

Bagnold, R. A. (1954): Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear, Proceedings of the Royal Society London, Series A 225, pp. 49-63.

Bagnold, R. A. (1966): An approach to the sediment transport problem from general physics. Geological Survey Professional Paper 422-1.

Bogomolov, A. I. and Mikhaylov, K. A. (1972): Hydraulics, Moscow, Publishing house Stroyizdat. (in Russian)

Chong, J. S., Christiansen, E. B. and Baer, A. D. (1971): Rheology of concentrated suspensions, J. Appl. Polym. Sci., Vol. 15, pp. 2007-2021.

Dyurnbaum, N. S. (1949): Defence against debris flows of populated areas, Moscow-Leningrad: Publishing house Ministry for community facilities. (in Russian)

Farris, R. J. (1968): Prediction of the viscosity of multimodal suspensions from unimodal viscosity data, Trans. Soc. Rheol., Vol. 12, pp. 281-301.

Gavrishina, L. N. (1985): Model study of static shear stress of viscoplastic debris-flow mass, Ph. D. thesis, Almaty: Kazakh state university. (in Russian with English abstract)

Goncharov, V. N. (1962): Stream dynamics, Leningrad: Publishing house Gidrometeoizdat. (in Russian)

Hotta, N., Kaneko, T., Iwata T. and Nishimoto, H. (2013): Influence of fine sediment on the fluidity of debris flows, Journal of Mountain Science, Vol. 10, No. 2, pp. 233-238.

Khaydarov, A. Kh. and Fominov, S. V. (1988): Some features of formation of debris flows in the Sarkand River basin, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeoro logical Research, No. 10, pp. 155-161. (in Russian)
Khaydarov, A. K. h. and Shevyrtalov, Ye. P. (1989) : Mudflows in the Zhamankum Desert 28-29 January 1988, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeorological Research, No. 11, pp. 49-59. (in Russian)

Konin, R. V., Keremkulov, V. A. and Mochalov, V. P. (1977) : The third experiment on the artificial replication of a debris flow, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeorological Research, No. 2, pp. 57-63. (in Russian)

Kirenskaya, T. L., Stepanov, B. S. and Konin, R. V. (1977) : The debris flow of 19 August 1975 in the Bolshaya Almatinka River basin, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeorological Research, No. 2, pp. 115-119. (in Russian)

Kirenskaya, T. L., Stepanova, T. S. and Balabayev, F. G. (1980) : Chemolgan-78, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeorological Research, No. 5, pp. 64-71. (in Russian)

Kizelvalter, B. V. (1979) : Theory of gravitational processes of enrichment, Moscow : Publishing house Nedra. (in Russian)

Knap, R. T. (1938) : Energy balance in stream flows carrying suspended load, A. Geophys. Union Trans., pp. 501-505.

Mostkov, M. A. (1957) : Hydraulic regularities in torrents, In Debris flows and their control measures, Moscow : Publishing house the USSR Academy of Sciences, pp. 18-54. (in Russian)

Mostkov, M. A. (1959) : Outlines of streamflow history, Moscow : Publishing house the USSR Academy of Sciences. (in Russian)

Nishiguchi, Y., Uchida, T., Tamura, K. and Satofuka, Y. (2011) : Prediction of run-out processes for a debris flow triggered by a deep rapid landslide, Italian Journal of Engineering Geology and Environment-Book, pp. 477-485.

Popov, V. I., Stepanov, B. S., Mochalov, V. P., Konin, R. V., Markov, I. N., Golubovich, V. A. and Bekarevich, V. Ye. (1980) : The Debris-flow events of 3-31 August 1977 in the Bolshaya Almatinka River basin, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeorological Research, No. 4, pp. 57-63. (in Russian)

Rickenmann, D., Weber, D. and Stepanov, B. (2003). Erosion by debris flows in field and laboratory experiments, Proceedings of the Third International Conference on Debris-Flow Hazards Mitigation : Mechanics, Prediction, and Assessment, September 10-12, Davos, Switzerland, pp. 883-894.

Rubey, W. W. (1933) : Equilibrium conditions in debris-laden streams, A. M. Geophys. Union Trans., 14 th Ann. Mtg., pp.497-505.

Sanoyan, V. G. (1980) : Carrying capacity in the general case of flow, In the book Erosion and debris flow processes and their control measures, No. 7, pp. 104-118. (in Russian)

Stepanov, B. S. (1978) : On a mechanism of erosion-shift process, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeorological Research, No. 3, pp. 130-133. (in Russian)

Stepanov, B. S. (1982) : The main characteristics of debris flows and debris-flow mass, Measurement methods, Proceeding of Kazakh Institute for Hydrometeorological Research., No. 79, pp. 3-137. (in Russian)

Stepanov, B. S. (1992) : A phenomena of discontinuous change of debris-flow mass density, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeorological Research, No. 12, pp. 141-172. (in Russian)

Stepanova, T. S., Khonin, R. V., Krzhechkovskaya, N. I. and Khaydarov, A. K. (1978) : Results of an experiment on the artificial replication of a debris flow in the Chemolgan River basin in 1976, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeorological Research, No. 3, pp. 86-92. (in Russian)

Takahashi T. (1981) : Debris flow, Annual Review of Fluid Mech., No. 13, pp. 57-77.

Uchida, T., Nishiguchi, Y., Nakatani, K., Satofuka, Y., Yamakoshi, T., Okamoto, A. and Mizuyama, T. (2013) : New Numerical Simulation procedure for large-scale debris flows (Kanako-LS), International Journal of Erosion Control Engineering, Vol. 6, No. 2, pp. 58-67.

Velikanov, M. A. (1948) : Land hydrology, Leningrad : Publishing house Gidrometeoizdat. (in Russian)

Velikanov, M. A. (1954) : Justification of gravitational theory for suspended sediments, In Proceedings of the USSR Academy of Sciences. Geographical Series, No. 4, pp. 349-359. (in Russian)

Vinogradov, Yu. B. (1977 a) : On the structure and dynamics of the debris-flow mass, In Debris flows, Collected Papers, Kazakh Institute for Hydrometeorological Research, No. 2, pp. 3-26. (in Russian)

Vinogradov, Yu. B. (1977 b) : Glacial outburst floods and debris flows, Leningrad : Gidrometeoizdat. (in Russian)

Yafyazova, R. K. (2007) : Nature of debris flows in the Zailiysky Ala-tau Mountains. Problems of adaptation, Almaty. (in Russian with English abstract)

Yafyazova, R. K. (2007) : Glacial outburst floods and debris flows, Leningrad : Gidrometeoizdat. (in Russian)

Zemin, A. E., Popov, K. V. and Fandeev, V. V. (1952) : Hydraulic structures, Moscow : Publishing house Agricultural literature. (in Russian)

Zemin, A. E., Popov, K. V. and Fandeev, V. V. (1952) : Hydraulic structures, Moscow : Publishing house Agricultural literature. (in Russian)