Development of a simulation model for the supply of spare parts for heating equipment

I Ya Lvovich¹, A P Preobrazhenskiy², O N Chaporov³

¹Information systems and technologies department, Voronezh institute of high technologies, 73a, Lenin st, Voronezh 394043, Russia
²Information security department, Voronezh state technical university, 14, Moscow dist., Voronezh 394026, Russia

E-mail: komkovvivt@yandex.ru

Abstract. The problems associated with the supply of spare parts for heating equipment are considered. The main stages of development of a simulation model that allows you to simulate the incoming flow of applications for different types of parts for heating equipment are presented. The analysis of the time series of the supply of parts for heating equipment. Then there are opportunities for modeling autocorrelated flows by ordering for spare parts that have a certain relationship between each other according to the items of the item. Cross-correlation and autocorrelation functions of time series were considered, and it was revealed that these functions are not similar to each other.

1. Introduction

Today, the movement of spare parts for heating equipment in warehouse networks is possible only when ordering original spare parts. An original is considered such a spare part that is manufactured by a company of heating equipment or produced by another company that has the legal right to manufacture modules, elements, parts under the brand name of a manufacturer of heating equipment.

The disadvantages of such a supply system are:

1. quite a long time for delivery of an order for spare parts from a central warehouse (from 2 to 12 weeks);
2. the manufacture of spare parts in the artisanal way. Non-original spare parts have a lower cost, and as a result, lower quality;
3. far from cheap original products (the manufacturer reserves the right to non-disclosure of pricing)

In order to maximize the details of the supply of spare parts for heating equipment, it is necessary to develop a system in which there will be a scientifically sound management of stocks of spare parts at the service station.

2. Development of a simulation model with unplanned needs for spare parts for heating equipment

The main problem in developing a simulation model of this type is the modelling the incoming flow of applications for different types of parts for heating equipment, however, the results obtained should be
similar to the results of a previous statistical analysis on the functioning of the heating equipment maintenance departments, taking into account seasonal variations and the dynamics of receipt orders.

In order to simulate the flows of supplies of spare parts with the presence of mutual correlation, in this work it was decided to use autoregressive models, since they have the properties necessary for solving this problem, namely: autoregressive models have the property of interdependence of values from each other, in the area of one row [1, 2].

We consider the expression (1), where a linear combination with a random value and the presence of previous values is presented, it is in this form that the value of the autoregressive model is represented:

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \ldots + \alpha_p y_{t-p} + \epsilon_t.$$  

We consider the simplest model of the autoregressive process (2), which is called the autoregressive model of the first row [3]. Note that the current level of the autoregressive model depends solely on the number of previous members of the series, so autoregressive models are determined by the order of the series and denote autoregressive processes

$$y_t = \alpha y_{t-1} + \epsilon_t.$$  

Where \( \alpha \) is a constant that can only be less than unity in magnitude |\( \alpha \)|<1, \( \epsilon \) - a sequence of independent random variables. In this case, the following properties are fulfilled for the error \( \epsilon_t \): 

$$M(\epsilon_t), D(\epsilon_t) = \sigma^2 = \text{Const}, \text{i} \neq 0 \Rightarrow \gamma = M(\epsilon_t, \epsilon_{t+i}) = 0.$$  

Next, we consider Figure 1, which presents a simulation of first-order autoregression processes, where the trajectories have a constant and unchanging character, with the absence of non-stationary components.

![Figure 1. First-order autoregressive model with a constant equal to \( \alpha = 0.9 \).](image)

The model (2) presented above is also a Markov process, in these cases the autoregressive function has the following form: 

$$p_t = p(y_t, y_{t+1}) = \alpha^t.$$

Next, we consider a second-order autoregressive model, which is referred to as the “Yule process”. This model is given by (3):

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \epsilon_t,$$

where also represents a sequence of independent random variables. The first coefficients from this expression in order to implement the autocorrelation process [4, 5] must be expressed as follows:
Note that in order for the stationary conditions of the process to be satisfied, the presented values must be satisfied by the following relation (5):

\[-1 < p_1 < 1, -1 < p_2 < 1, p_2 < 2p_2^2 < 1.\]  

It is important to know that with private autocorrelation values for logs that are more than two, they are equal to zero. The autoregressive model of the second order, in contrast to the same model of the first order, is less regular in character with a selective trajectory. Next, we consider Figure 2, which presents a simulation of second-order autoregression processes. The latter model is the most used, not counting the processes of the so-called "white noise".

Consider the following method of modeling the flow of orders for spare parts of heating equipment: a moving average model.

In this model, by the method of determining the value of a series, a linear combination of errors is made [6], as the difference between the actual values before the simulation and the values after the simulation, the difference is determined using the following expression (6):

\[y_t = \varepsilon_t - \beta_1 \varepsilon_{t-1} - \beta_2 \varepsilon_{t-2} - \ldots - \beta_p \varepsilon_{t-p},\]  

where \(\varepsilon_t\) is a sequence of non-dependent random variables.

An assumption is made that over the past periods model errors contain information about the behavior of the process. In such a situation, the process is set based on a weighted sum of past and present values in the form of a linear model. The transition from the MA model (5) to the AR model (1) is performed on the basis of sequential substitution instead of \(\varepsilon_{t-1}, \varepsilon_{t-2}\ldots\) their calculated values:

\[y_t = \varepsilon_t - \sum_{i=1}^{\infty} \beta_i y_{t+i},\]  

Figure 2. Second-order autoregressive model with a constant equal to \(\alpha = 0.9\).
In MA models, to satisfy the stationary condition, there are no restrictions on the parameters \( \beta_1, \beta_2, ..., \beta_q \), but for a possible conversion to the AR model, the condition for the convergence of a number of coefficients \( \beta_1, \beta_2, ..., \beta_q \). The autocorrelation function of the MA model is expressed in terms of the coefficients and is equal to:

\[
p_{\tau} = \begin{cases} 
-\beta_\tau + \beta_1 \beta_{\tau+1} + \beta_2 \beta_{\tau+2} + ... + \beta_q \beta_{\tau+q} & \text{when } \tau = 1, 2, ..., q \\
1 + \beta_1^2 + ... + \beta_q^2 & \text{when } \tau > q
\end{cases}
\]

(8)

Thus, after the lag, the autocorrelation function is equal to zero, which is a characteristic property of the MA model.

From the graph in Figure 3 you can see a smoother character of the sample trajectory, which indicates the interconnection of orders or requirements for spare parts for short time intervals. In the general case, it is possible to combine the processes defined by the AR model and the MA model, which is a combined model (ARMA model). This model has p lags in the AR process and q lags in the MA process and is defined by the relation:

\[
y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + ... + \alpha_p y_{t-p} + \varepsilon_t - \beta_1 \varepsilon_{t-1} - ... - \beta_q \varepsilon_{t-q}.
\]

(9)

Figure 3. Smoothed time series for expenses for the main heat exchanger (Z1), electronic board (Z2) and heat-insulating panel (Z3).

The stationarity of the ARMA process will be determined only by the AR component of the overall process. If the ARMA process is stationary, it certainly has an infinitely large AR representation or an infinitely large MA representation. Therefore, the ARMA (p, q) process is very convenient to represent and has all (p + q) parameters, which is the main advantage of this ARMA (p, q) model.

The mathematical expectation of a stationary ARMA (p, q) process will be zero. The simplest ARMA (1,1) is considered - the process:

\[
y_t = \alpha y_{t-1} + \varepsilon_t - \beta \varepsilon_{t-1}.
\]

(10)
The dispersion of the ARMA (1,1) process is determined as there is a relationship between the coefficients given below:

\[
D(y_t) = \gamma_0 = \frac{1 + \beta^2 - 2\alpha\beta}{1 - \alpha^2} \sigma^2, \quad \gamma_1 = \frac{(1 - \alpha\beta)(\alpha - \beta)}{1 - \alpha^2} \sigma^2.
\]  

(11)

The higher-order autocovariance coefficients \( \gamma_\tau = \alpha^{\tau-1} \), are related by the ratio and the first ARMA (1,1) process autocorrelation coefficient is defined as \( p_1 = \frac{\gamma_1}{\gamma_0} = \frac{(1 - \alpha\beta)(\alpha - \beta)}{1 + \beta^2 2\alpha\beta} \).

For autocorrelation coefficients of higher orders, the relation holds \( p_\tau = \alpha p_{\tau-1}, \tau \geq 2 \). Moreover, starting from the second, the autocorrelation coefficients of the ARMA (1,1) process decrease exponentially. In the framework of solving the problem of comparative analysis of forecast accuracy, an algorithm for constructing sample functions of interrelated random time series (hereinafter, as an example of a three-dimensional series) is proposed [7].

Initially, a dispersion matrix \( D \) of vector random variables is determined that determines the value of the multidimensional random process at a given point in time. Using the given matrix \( D \), one searches for the matrix \( A \), which implements a linear transformation of an array of independent random vectors with given mathematical expectations and unit variance [8, 9]. This matrix \( A \) is a solution to a system of nonlinear equations, which in vector form can be represented as \( A \cdot A^T = D \).

\[
D = \begin{pmatrix} 1 & 0.6 & 0.5 \\ 0.6 & 0.999 & 0.3 \\ 0.5 & 0.3 & 1 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 0.341 & 0.888 & 0.309 \\ 0.823 & 0.171 & 0.541 \\ -0.292 & 0.368 & 0.883 \end{pmatrix}
\]  

(12)

As a result of such a transformation, three time series are obtained, correlations for which take place only at the same moment in time. Next, based on the moving average procedure, we obtain a set of interconnected and rounded values of time series (Figure 4), where \( Z_1, Z_2, Z_3 \) are the time series of application flows for the heat exchanger tube, three-way valve, and fastening clips of the heat exchanger, respectively.

![Figure 4. A lot of interconnected and autocorrelated time series of parts supplies.](image_url)
Thus, we determined both cross-correlation and autocorrelation functions of time series, and it was revealed that these functions are not similar to each other.

3. Conclusion
In the paper, the problem of developing an algorithm was solved with the aim of modeling interrelated time series describing the movement of spare parts of heating equipment. To solve this problem, it was decided to use autoregressive models, where the average has a sliding function, and also combined time series models. Based on the results, a scheme was developed with the simulation of autocorrelated flows of orders for spare parts, which have some kind of relationship with each other according to the nomenclature items. Thus, this algorithm was developed in which cross-correlation is not 0.

Based on the models of factor analysis and scalar time series forecasting procedures, a procedure has been developed for predicting an interconnected set of order volumes for various parts, which allows taking into account the revealed statistical patterns. The order flows of interconnected parts, such as the main heat exchanger, electronic circuit board and thermal insulating panel, were modeled.

References
[1] Orlova D E 2018 Stability of solutions in ensuring the functioning of organizational and technical systems Modeling, Optimization and Information Technologies 6(1) 325–336
[2] Sorokin S O 2018 Optimization modeling of the functioning of the system of homogeneous objects in a multidimensional digital environment Modeling, optimization and information technologies 6(3) 153–164
[3] Talluri S, Kim M K, Schoenherr T 2013 The relationship between operating efficiency and service quality: are they compatible? Int. J Prod Res 51 548–2567
[4] Parmezan A R S, Souza V M A, Batista G E 2019 Evaluation of statistical and machine learning models for time series prediction: Identifying the state-of-the-art and the best conditions for the use of each model Information Sciences 484 302–337
[5] Zhang J, Lee G M and Wang J 2016 A comparative analysis of univariate time series methods for estimating and forecasting daily spam in United States Twenty-second Americas Conference on Information Systems (San Diego, USA) pp 1–10
[6] Golden RM, Henley SS, White H, et al 2016 Generalized information matrix tests for detecting model misspecification Econometrics 4(4) 46
[7] Shah A and Ghahramani Z 2015 Parallel predictive entropy search for batch global optimization of expensive objective functions Advances in Neural Information Processing Systems pp 3330–3338
[8] Groefsema H, van Beest N R T P 2015 Design-time compliance of service compositions in dynamic service environments Int. Conf. on Service Oriented Computing & Applications pp 108–115
[9] Yao Y and Chen J 2010 Global optimization of a central air-conditioning system using decomposition-coordination method, Energy and Buildings 5 570–583