S1 Results from Independence Simulation Settings

The simulation results under the independence setting for stepwise procedures comparisons are shown in this section. Tables S1, S2 and Tables S3, S4 respectively provide the results of numerical comparisons of single-step procedures using Fisher and Binomial Exact Tests (as plotted in Figures 1 and 2).
Table S1: Simulated FWER comparisons for single-step procedures with independent \( p \)-values generated from Fisher’s Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

| \( m = 5 \) | \( \pi_0 = 0.2 \) | \( \pi_0 = 0.4 \) | \( \pi_0 = 0.6 \) | \( \pi_0 = 0.8 \) |
|-----------|----------------|----------------|----------------|----------------|
| \( N = 25 \) | MBonf 0.0045 | MBonf 0.0025 | MBonf 0.0005 | MBonf 0.0050 |
| Tarone 0.0030 | Tarone 0.0025 | Tarone 0.0015 | Tarone 0.0025 | Tarone 0.0015 |
| Bonf 0.0005 | Bonf 0.0005 | Bonf 0.0005 | Bonf 0.0005 | Bonf 0.0005 |
| Sidak 0.0005 | Sidak 0.0005 | Sidak 0.0005 | Sidak 0.0005 | Sidak 0.0005 |
| \( N = 50 \) | MBonf 0.0060 | MBonf 0.0030 | MBonf 0.0040 | MBonf 0.0050 |
| Tarone 0.0060 | Tarone 0.0030 | Tarone 0.0040 | Tarone 0.0050 | Tarone 0.0040 |
| Bonf 0.0003 | Bonf 0.0003 | Bonf 0.0003 | Bonf 0.0003 | Bonf 0.0003 |
| Sidak 0.0005 | Sidak 0.0005 | Sidak 0.0005 | Sidak 0.0005 | Sidak 0.0005 |
| \( N = 75 \) | MBonf 0.0075 | MBonf 0.0035 | MBonf 0.0045 | MBonf 0.0055 |
| Tarone 0.0075 | Tarone 0.0035 | Tarone 0.0045 | Tarone 0.0055 | Tarone 0.0045 |
| Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 |
| Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 |
| \( N = 100 \) | MBonf 0.0075 | MBonf 0.0035 | MBonf 0.0045 | MBonf 0.0055 |
| Tarone 0.0075 | Tarone 0.0035 | Tarone 0.0045 | Tarone 0.0055 | Tarone 0.0045 |
| Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 |
| Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 |
| \( N = 125 \) | MBonf 0.0075 | MBonf 0.0035 | MBonf 0.0045 | MBonf 0.0055 |
| Tarone 0.0075 | Tarone 0.0035 | Tarone 0.0045 | Tarone 0.0055 | Tarone 0.0045 |
| Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 |
| Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 |
| \( N = 150 \) | MBonf 0.0075 | MBonf 0.0035 | MBonf 0.0045 | MBonf 0.0055 |
| Tarone 0.0075 | Tarone 0.0035 | Tarone 0.0045 | Tarone 0.0055 | Tarone 0.0045 |
| Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 | Bonf 0.0015 |
| Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 | Sidak 0.0015 |
Table S2: Simulated minimal power comparisons for single-step procedures with independent $p$-values generated from Fisher’s exact test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

| $m = 5$ | $N = 25$ | $N = 50$ | $N = 75$ | $N = 100$ | $N = 125$ | $N = 150$ |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|
| $\pi_0 = 0.2$ | MBonf 0.2550 | 0.5060 | 0.6855 | 0.8195 | 0.9145 | 0.9505 |
|          | Tarone 0.1945 | 0.3900 | 0.5775 | 0.7680 | 0.8655 | 0.9275 |
|          | Bonf 0.1125 | 0.3825 | 0.5765 | 0.7680 | 0.8655 | 0.9275 |
|          | Sidak 0.1125 | 0.3825 | 0.5850 | 0.7680 | 0.8710 | 0.9340 |
| $m = 5$ | $\pi_0 = 0.4$ | MBonf 0.2110 | 0.4085 | 0.5785 | 0.7405 | 0.8375 | 0.9025 |
|          | Tarone 0.1605 | 0.3110 | 0.4715 | 0.6705 | 0.7695 | 0.8625 |
|          | Bonf 0.0880 | 0.3000 | 0.4700 | 0.6705 | 0.7695 | 0.8625 |
|          | Sidak 0.0880 | 0.3000 | 0.4770 | 0.6705 | 0.7765 | 0.8680 |
| $m = 5$ | $\pi_0 = 0.6$ | MBonf 0.1550 | 0.3130 | 0.4320 | 0.5835 | 0.7025 | 0.7845 |
|          | Tarone 0.1180 | 0.2365 | 0.3370 | 0.5145 | 0.6255 | 0.7245 |
|          | Bonf 0.0605 | 0.2190 | 0.3355 | 0.5145 | 0.6255 | 0.7245 |
|          | Sidak 0.0605 | 0.2190 | 0.3420 | 0.5145 | 0.6330 | 0.7345 |
| $m = 5$ | $\pi_0 = 0.8$ | MBonf 0.0945 | 0.1800 | 0.2570 | 0.3595 | 0.4660 | 0.5050 |
|          | Tarone 0.0740 | 0.1330 | 0.1920 | 0.2955 | 0.3950 | 0.4850 |
|          | Bonf 0.0330 | 0.1190 | 0.1920 | 0.2955 | 0.3950 | 0.4850 |
|          | Sidak 0.0330 | 0.1190 | 0.1955 | 0.2955 | 0.4025 | 0.5005 |
| $m = 10$ | $\pi_0 = 0.2$ | MBonf 0.3155 | 0.6130 | 0.8090 | 0.9110 | 0.9765 | 0.9930 |
|          | Tarone 0.2075 | 0.4695 | 0.7220 | 0.8550 | 0.9415 | 0.9820 |
|          | Bonf 0.1575 | 0.4660 | 0.7220 | 0.8550 | 0.9415 | 0.9820 |
|          | Sidak 0.1575 | 0.4660 | 0.7220 | 0.8595 | 0.9425 | 0.9830 |
| $m = 10$ | $\pi_0 = 0.4$ | MBonf 0.2700 | 0.5220 | 0.7180 | 0.8455 | 0.9440 | 0.9750 |
|          | Tarone 0.1770 | 0.3905 | 0.6065 | 0.7720 | 0.8905 | 0.9505 |
|          | Bonf 0.1235 | 0.3795 | 0.6065 | 0.7720 | 0.8905 | 0.9505 |
|          | Sidak 0.1235 | 0.3795 | 0.6065 | 0.7775 | 0.8920 | 0.9575 |
| $m = 10$ | $\pi_0 = 0.6$ | MBonf 0.2050 | 0.4030 | 0.5615 | 0.7300 | 0.8450 | 0.9035 |
|          | Tarone 0.1330 | 0.2990 | 0.4525 | 0.6315 | 0.7590 | 0.8525 |
|          | Bonf 0.0800 | 0.2825 | 0.4525 | 0.6315 | 0.7590 | 0.8525 |
|          | Sidak 0.0800 | 0.2825 | 0.4525 | 0.6375 | 0.7615 | 0.8585 |
| $m = 10$ | $\pi_0 = 0.8$ | MBonf 0.1115 | 0.2440 | 0.3500 | 0.4775 | 0.6140 | 0.6935 |
|          | Tarone 0.0760 | 0.1680 | 0.2645 | 0.3810 | 0.5165 | 0.6060 |
|          | Bonf 0.0390 | 0.1555 | 0.2645 | 0.3810 | 0.5165 | 0.6060 |
|          | Sidak 0.0390 | 0.1555 | 0.2645 | 0.3880 | 0.5170 | 0.6185 |
| $m = 15$ | $\pi_0 = 0.2$ | MBonf 0.3370 | 0.6715 | 0.8820 | 0.9495 | 0.9915 | 0.9965 |
|          | Tarone 0.2520 | 0.4995 | 0.7530 | 0.8910 | 0.9765 | 0.9895 |
|          | Bonf 0.1390 | 0.4870 | 0.7515 | 0.8910 | 0.9765 | 0.9895 |
|          | Sidak 0.1390 | 0.4870 | 0.7515 | 0.8960 | 0.9765 | 0.9895 |
| $m = 15$ | $\pi_0 = 0.4$ | MBonf 0.2880 | 0.5815 | 0.7910 | 0.9025 | 0.9635 | 0.9830 |
|          | Tarone 0.2110 | 0.4105 | 0.6475 | 0.8050 | 0.9335 | 0.9745 |
|          | Bonf 0.1030 | 0.3870 | 0.6460 | 0.8050 | 0.9335 | 0.9745 |
|          | Sidak 0.1030 | 0.3870 | 0.6460 | 0.8125 | 0.9335 | 0.9745 |
| $m = 15$ | $\pi_0 = 0.6$ | MBonf 0.2135 | 0.4485 | 0.6570 | 0.7925 | 0.8840 | 0.9500 |
|          | Tarone 0.1495 | 0.3070 | 0.5085 | 0.6730 | 0.8315 | 0.9140 |
|          | Bonf 0.0700 | 0.2760 | 0.5065 | 0.6730 | 0.8315 | 0.9140 |
|          | Sidak 0.0700 | 0.2760 | 0.5065 | 0.6790 | 0.8315 | 0.9140 |
| $m = 15$ | $\pi_0 = 0.8$ | MBonf 0.1205 | 0.2635 | 0.4270 | 0.5490 | 0.6710 | 0.7780 |
|          | Tarone 0.0830 | 0.1785 | 0.3050 | 0.4295 | 0.5890 | 0.7020 |
|          | Bonf 0.0335 | 0.1480 | 0.3040 | 0.4290 | 0.5890 | 0.7020 |
|          | Sidak 0.0335 | 0.1480 | 0.3040 | 0.4345 | 0.5895 | 0.7020 |
Table S3: Simulated FWER comparisons for single-step procedures with independent \( p \)-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

|     | \( \pi_0 = 0.2 \) | \( \pi_0 = 0.4 \) | \( \pi_0 = 0.6 \) | \( \pi_0 = 0.8 \) |
|-----|------------------|------------------|------------------|------------------|
| \( m = 5 \) | \( \alpha = 0.05 \) | MBonf | 0.0020 | 0.0060 | 0.0075 | 0.0165 |
|     |                  | Tarone | 0.0010 | 0.0030 | 0.0055 | 0.0105 |
|     |                  | Bonf   | 0.0010 | 0.0020 | 0.0025 | 0.0030 |
|     |                  | Sidak  | 0.0010 | 0.0020 | 0.0025 | 0.0030 |
| \( m = 10 \) | \( \alpha = 0.05 \) | MBonf | 0.0010 | 0.0045 | 0.0130 | 0.0160 |
|     |                  | Tarone | 0.0000 | 0.0010 | 0.0050 | 0.0115 |
|     |                  | Bonf   | 0.0000 | 0.0005 | 0.0025 | 0.0025 |
|     |                  | Sidak  | 0.0000 | 0.0005 | 0.0025 | 0.0025 |
| \( m = 15 \) | \( \alpha = 0.05 \) | MBonf | 0.0010 | 0.0065 | 0.0045 | 0.0150 |
|     |                  | Tarone | 0.0000 | 0.0010 | 0.0020 | 0.0070 |
|     |                  | Bonf   | 0.0000 | 0.0005 | 0.0000 | 0.0000 |
|     |                  | Sidak  | 0.0000 | 0.0005 | 0.0000 | 0.0000 |
| \( m = 5 \) | \( \alpha = 0.1 \) | MBonf | 0.0070 | 0.0125 | 0.0200 | 0.0365 |
|     |                  | Tarone | 0.0020 | 0.0065 | 0.0110 | 0.0285 |
|     |                  | Bonf   | 0.0020 | 0.0055 | 0.0065 | 0.0130 |
|     |                  | Sidak  | 0.0020 | 0.0055 | 0.0065 | 0.0130 |
| \( m = 10 \) | \( \alpha = 0.1 \) | MBonf | 0.0040 | 0.0080 | 0.0275 | 0.0350 |
|     |                  | Tarone | 0.0000 | 0.0030 | 0.0165 | 0.0195 |
|     |                  | Bonf   | 0.0000 | 0.0015 | 0.0055 | 0.0060 |
|     |                  | Sidak  | 0.0000 | 0.0015 | 0.0055 | 0.0060 |
| \( m = 15 \) | \( \alpha = 0.1 \) | MBonf | 0.0060 | 0.0155 | 0.0185 | 0.0315 |
|     |                  | Tarone | 0.0005 | 0.0060 | 0.0045 | 0.0200 |
|     |                  | Bonf   | 0.0000 | 0.0010 | 0.0020 | 0.0025 |
|     |                  | Sidak  | 0.0000 | 0.0010 | 0.0020 | 0.0025 |
Table S4: Simulated minimal power comparisons for single-step procedures with independent $p$-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

| Procedure | $\pi_0 = 0.2$ | $\pi_0 = 0.4$ | $\pi_0 = 0.6$ | $\pi_0 = 0.8$ |
|-----------|----------------|----------------|----------------|----------------|
| $m = 5$   |                |                |                |                |
| MBonf     | 0.9205         | 0.8805         | 0.7845         | 0.5565         |
| Tarone    | 0.8815         | 0.8240         | 0.7395         | 0.5235         |
| Bonf      | 0.8735         | 0.8055         | 0.6610         | 0.4045         |
| Sidak     | 0.8735         | 0.8055         | 0.6610         | 0.4045         |
| $\alpha = 0.05$ |                |                |                |                |
| $m = 10$  |                |                |                |                |
| MBonf     | 0.9850         | 0.9635         | 0.9035         | 0.7390         |
| Tarone    | 0.9470         | 0.9240         | 0.8630         | 0.6855         |
| Bonf      | 0.9315         | 0.8635         | 0.7050         | 0.4775         |
| Sidak     | 0.9315         | 0.8635         | 0.7050         | 0.4775         |
| $\alpha = 0.05$ |                |                |                |                |
| $m = 15$  |                |                |                |                |
| MBonf     | 0.9925         | 0.9810         | 0.9555         | 0.8210         |
| Tarone    | 0.9825         | 0.9500         | 0.9095         | 0.7845         |
| Bonf      | 0.9820         | 0.9475         | 0.8560         | 0.6135         |
| Sidak     | 0.9820         | 0.9475         | 0.8560         | 0.6135         |

Tables S5 and S6 provide numerical results of step-down procedures comparisons using Fisher Exact Test, which are also plotted as Figures S1 and S2. Tables S7 and S8 provide numerical results of step-up procedures comparisons using Fisher Exact Test, which are plotted as Figures S3 and S4.
Table S5: Simulated FWER comparisons for step-down procedures with independent \( p \)-values generated from Fisher’s Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

|            | \( N = 25 \) | \( N = 50 \) | \( N = 75 \) | \( N = 100 \) | \( N = 125 \) | \( N = 150 \) |
|------------|--------------|--------------|--------------|--------------|--------------|--------------|
| \( m = 5 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.2 \) | MHolm 0.0030 | 0.0090 | 0.0065 | 0.0115 | 0.0150 | 0.0150 |
|            | TH 0.0015 | 0.0045 | 0.0030 | 0.0075 | 0.0090 | 0.0140 |
|            | Holm 0.0010 | 0.0045 | 0.0030 | 0.0075 | 0.0090 | 0.0140 |
| \( m = 5 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.4 \) | MHolm 0.0055 | 0.0155 | 0.0135 | 0.0230 | 0.0225 | 0.0225 |
|            | TH 0.0030 | 0.0080 | 0.0080 | 0.0185 | 0.0140 | 0.0180 |
|            | Holm 0.0020 | 0.0075 | 0.0080 | 0.0185 | 0.0140 | 0.0180 |
| \( m = 5 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.6 \) | MHolm 0.0100 | 0.0215 | 0.0215 | 0.0290 | 0.0305 | 0.0320 |
|            | TH 0.0065 | 0.0115 | 0.0115 | 0.0220 | 0.0185 | 0.0205 |
|            | Holm 0.0030 | 0.0110 | 0.0115 | 0.0220 | 0.0185 | 0.0205 |
| \( m = 5 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.8 \) | MHolm 0.0155 | 0.0285 | 0.0285 | 0.0360 | 0.0375 | 0.0440 |
|            | TH 0.0115 | 0.0145 | 0.0160 | 0.0260 | 0.0240 | 0.0270 |
|            | Holm 0.0050 | 0.0140 | 0.0160 | 0.0260 | 0.0240 | 0.0270 |

|            | \( N = 25 \) | \( N = 50 \) | \( N = 75 \) | \( N = 100 \) | \( N = 125 \) | \( N = 150 \) |
|------------|--------------|--------------|--------------|--------------|--------------|--------------|
| \( m = 10 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.2 \) | MHolm 0.0020 | 0.0070 | 0.0125 | 0.0130 | 0.0160 | 0.0185 |
|            | TH 0.0005 | 0.0040 | 0.0070 | 0.0080 | 0.0115 | 0.0125 |
|            | Holm 0.0005 | 0.0040 | 0.0070 | 0.0080 | 0.0115 | 0.0125 |
| \( m = 10 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.4 \) | MHolm 0.0050 | 0.0155 | 0.0200 | 0.0215 | 0.0280 | 0.0265 |
|            | TH 0.0025 | 0.0090 | 0.0125 | 0.0125 | 0.0200 | 0.0175 |
|            | Holm 0.0025 | 0.0090 | 0.0125 | 0.0125 | 0.0200 | 0.0175 |
| \( m = 10 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.6 \) | MHolm 0.0095 | 0.0250 | 0.0285 | 0.0290 | 0.0360 | 0.0350 |
|            | TH 0.0060 | 0.0150 | 0.0185 | 0.0155 | 0.0220 | 0.0215 |
|            | Holm 0.0045 | 0.0140 | 0.0185 | 0.0155 | 0.0220 | 0.0215 |
| \( m = 10 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.8 \) | MHolm 0.0175 | 0.0340 | 0.0360 | 0.0380 | 0.0420 | 0.0405 |
|            | TH 0.0090 | 0.0215 | 0.0235 | 0.0195 | 0.0255 | 0.0230 |
|            | Holm 0.0055 | 0.0190 | 0.0225 | 0.0195 | 0.0255 | 0.0230 |

|            | \( N = 25 \) | \( N = 50 \) | \( N = 75 \) | \( N = 100 \) | \( N = 125 \) | \( N = 150 \) |
|------------|--------------|--------------|--------------|--------------|--------------|--------------|
| \( m = 15 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.2 \) | MHolm 0.0045 | 0.0070 | 0.0070 | 0.0140 | 0.0125 | 0.0120 |
|            | TH 0.0025 | 0.0035 | 0.0030 | 0.0090 | 0.0060 | 0.0085 |
|            | Holm 0.0005 | 0.0030 | 0.0030 | 0.0090 | 0.0060 | 0.0085 |
| \( m = 15 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.4 \) | MHolm 0.0095 | 0.0165 | 0.0145 | 0.0255 | 0.0255 | 0.0285 |
|            | TH 0.0060 | 0.0080 | 0.0075 | 0.0160 | 0.0175 | 0.0165 |
|            | Holm 0.0010 | 0.0070 | 0.0075 | 0.0160 | 0.0175 | 0.0165 |
| \( m = 15 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.6 \) | MHolm 0.0165 | 0.0260 | 0.0215 | 0.0345 | 0.0350 | 0.0345 |
|            | TH 0.0090 | 0.0130 | 0.0105 | 0.0190 | 0.0215 | 0.0195 |
|            | Holm 0.0020 | 0.0105 | 0.0100 | 0.0190 | 0.0215 | 0.0195 |
| \( m = 15 \) |              |              |              |              |              |              |
| \( \pi_0 = 0.8 \) | MHolm 0.0215 | 0.0350 | 0.0315 | 0.0415 | 0.0465 | 0.0390 |
|            | TH 0.0120 | 0.0170 | 0.0165 | 0.0225 | 0.0290 | 0.0260 |
|            | Holm 0.0020 | 0.0135 | 0.0165 | 0.0225 | 0.0290 | 0.0260 |
Table S6: Simulated minimal power comparisons for step-down procedures with independent $p$-values generated from Fisher's Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

|        | $N = 25$ | $N = 50$ | $N = 75$ | $N = 100$ | $N = 125$ | $N = 150$ |
|--------|----------|----------|----------|-----------|-----------|-----------|
| $m = 5$ |          |          |          |           |           |           |
| $\pi_0 = 0.2$ | MHolm | 0.2555  | 0.5070  | 0.6855  | 0.8200  | 0.9145  | 0.9505  |
|          | TH      | 0.1945  | 0.3905  | 0.5780  | 0.7680  | 0.8660  | 0.9280  |
|          | Holm    | 0.1130  | 0.3830  | 0.5770  | 0.7680  | 0.8660  | 0.9280  |
| $m = 5$ |          |          |          |           |           |           |
| $\pi_0 = 0.4$ | MHolm | 0.2120  | 0.4090  | 0.5790  | 0.7405  | 0.8375  | 0.9030  |
|          | TH      | 0.1605  | 0.3115  | 0.4725  | 0.6705  | 0.7695  | 0.8630  |
|          | Holm    | 0.0880  | 0.3005  | 0.4710  | 0.6705  | 0.7695  | 0.8630  |
| $m = 5$ |          |          |          |           |           |           |
| $\pi_0 = 0.6$ | MHolm | 0.1555  | 0.3150  | 0.4330  | 0.5855  | 0.7035  | 0.7855  |
|          | TH      | 0.1185  | 0.2365  | 0.3375  | 0.5160  | 0.6265  | 0.7260  |
|          | Holm    | 0.0605  | 0.2190  | 0.3360  | 0.5160  | 0.6265  | 0.7260  |
| $m = 5$ |          |          |          |           |           |           |
| $\pi_0 = 0.8$ | MHolm | 0.0950  | 0.1815  | 0.2585  | 0.3615  | 0.4690  | 0.5530  |
|          | TH      | 0.0745  | 0.1330  | 0.1920  | 0.2965  | 0.3960  | 0.4860  |
|          | Holm    | 0.0330  | 0.1190  | 0.1920  | 0.2965  | 0.3960  | 0.4860  |
| $m = 10$|          |          |          |           |           |           |
| $\pi_0 = 0.2$ | MHolm | 0.3160  | 0.6130  | 0.8095  | 0.9120  | 0.9765  | 0.9930  |
|          | TH      | 0.2075  | 0.4700  | 0.7220  | 0.8550  | 0.9415  | 0.9820  |
|          | Holm    | 0.1575  | 0.4660  | 0.7220  | 0.8550  | 0.9415  | 0.9820  |
| $m = 10$|          |          |          |           |           |           |
| $\pi_0 = 0.4$ | MHolm | 0.2705  | 0.5220  | 0.7185  | 0.8455  | 0.9445  | 0.9750  |
|          | TH      | 0.1770  | 0.3905  | 0.6065  | 0.7720  | 0.8905  | 0.9505  |
|          | Holm    | 0.1235  | 0.3795  | 0.6065  | 0.7720  | 0.8905  | 0.9505  |
| $m = 10$|          |          |          |           |           |           |
| $\pi_0 = 0.6$ | MHolm | 0.2010  | 0.4035  | 0.5615  | 0.7300  | 0.8450  | 0.9035  |
|          | TH      | 0.1330  | 0.2990  | 0.4525  | 0.6315  | 0.7590  | 0.8525  |
|          | Holm    | 0.0800  | 0.2825  | 0.4525  | 0.6315  | 0.7590  | 0.8525  |
| $m = 10$|          |          |          |           |           |           |
| $\pi_0 = 0.8$ | MHolm | 0.1115  | 0.2440  | 0.3500  | 0.4780  | 0.6145  | 0.6935  |
|          | TH      | 0.0760  | 0.1680  | 0.2645  | 0.3810  | 0.5175  | 0.6065  |
|          | Holm    | 0.0390  | 0.1555  | 0.2645  | 0.3810  | 0.5175  | 0.6065  |
| $m = 15$|          |          |          |           |           |           |
| $\pi_0 = 0.2$ | MHolm | 0.3375  | 0.6715  | 0.8820  | 0.9495  | 0.9915  | 0.9965  |
|          | TH      | 0.2520  | 0.4995  | 0.7530  | 0.8910  | 0.9765  | 0.9895  |
|          | Holm    | 0.1390  | 0.4870  | 0.7515  | 0.8910  | 0.9765  | 0.9895  |
| $m = 15$|          |          |          |           |           |           |
| $\pi_0 = 0.4$ | MHolm | 0.2885  | 0.5825  | 0.7915  | 0.9025  | 0.9635  | 0.9830  |
|          | TH      | 0.2110  | 0.4105  | 0.6475  | 0.8055  | 0.9335  | 0.9745  |
|          | Holm    | 0.1030  | 0.3870  | 0.6460  | 0.8055  | 0.9335  | 0.9745  |
| $m = 15$|          |          |          |           |           |           |
| $\pi_0 = 0.6$ | MHolm | 0.2135  | 0.4495  | 0.6575  | 0.7930  | 0.8840  | 0.9500  |
|          | TH      | 0.1495  | 0.3070  | 0.5085  | 0.6730  | 0.8315  | 0.9140  |
|          | Holm    | 0.0700  | 0.2760  | 0.5065  | 0.6730  | 0.8315  | 0.9140  |
| $m = 15$|          |          |          |           |           |           |
| $\pi_0 = 0.8$ | MHolm | 0.1205  | 0.2645  | 0.4280  | 0.5495  | 0.6730  | 0.7780  |
|          | TH      | 0.0835  | 0.1785  | 0.3055  | 0.4295  | 0.5890  | 0.7030  |
|          | Holm    | 0.0335  | 0.1480  | 0.3045  | 0.4290  | 0.5890  | 0.7030  |
Figure S1: Simulated FWER comparisons for different step-down procedures based on FET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).
Figure S2: Simulated minimal power comparisons for different step-down procedures based on FET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).
Table S7: Simulated FWER comparisons for step-up procedures with independent $p$-values generated from Fisher’s Exact Test statistics, including Procedure 3.3 ($\text{MHoch}$), the Roth procedure ($\text{Roth}$), and the conventional Hochberg procedure ($\text{Hoch}$).

|                  | $N = 25$ | $N = 50$ | $N = 75$ | $N = 100$ | $N = 125$ | $N = 150$ |
|------------------|----------|----------|----------|-----------|-----------|-----------|
| $m = 5$          |          |          |          |           |           |           |
| $\pi_0 = 0.2$    | MHoch    | 0.0030   | 0.0090   | 0.0070    | 0.0115    | 0.0150    | 0.0155    |
|                  | Roth     | 0.0020   | 0.0045   | 0.0040    | 0.0085    | 0.0115    | 0.0155    |
|                  | Hoch     | 0.0015   | 0.0045   | 0.0040    | 0.0085    | 0.0115    | 0.0155    |
| $\pi_0 = 0.4$    | MHoch    | 0.0060   | 0.0155   | 0.0140    | 0.0235    | 0.0230    | 0.0245    |
|                  | Roth     | 0.0035   | 0.0080   | 0.0085    | 0.0185    | 0.0160    | 0.0200    |
|                  | Hoch     | 0.0025   | 0.0075   | 0.0085    | 0.0185    | 0.0160    | 0.0200    |
| $\pi_0 = 0.6$    | MHoch    | 0.0105   | 0.0215   | 0.0215    | 0.0290    | 0.0305    | 0.0325    |
|                  | Roth     | 0.0065   | 0.0115   | 0.0115    | 0.0220    | 0.0195    | 0.0215    |
|                  | Hoch     | 0.0030   | 0.0110   | 0.0115    | 0.0220    | 0.0195    | 0.0215    |
| $\pi_0 = 0.8$    | MHoch    | 0.0160   | 0.0285   | 0.0285    | 0.0360    | 0.0380    | 0.0445    |
|                  | Roth     | 0.0115   | 0.0145   | 0.0160    | 0.0265    | 0.0245    | 0.0280    |
|                  | Hoch     | 0.0050   | 0.0140   | 0.0160    | 0.0265    | 0.0245    | 0.0280    |
| $m = 10$         |          |          |          |           |           |           |
| $\pi_0 = 0.2$    | MHoch    | 0.0025   | 0.0070   | 0.0125    | 0.0140    | 0.0170    | 0.0200    |
|                  | Roth     | 0.0005   | 0.0040   | 0.0070    | 0.0080    | 0.0120    | 0.0135    |
|                  | Hoch     | 0.0005   | 0.0040   | 0.0070    | 0.0080    | 0.0120    | 0.0135    |
| $\pi_0 = 0.4$    | MHoch    | 0.0055   | 0.0155   | 0.0200    | 0.0225    | 0.0290    | 0.0275    |
|                  | Roth     | 0.0025   | 0.0090   | 0.0125    | 0.0125    | 0.0200    | 0.0185    |
|                  | Hoch     | 0.0025   | 0.0090   | 0.0125    | 0.0125    | 0.0200    | 0.0185    |
| $\pi_0 = 0.6$    | MHoch    | 0.0095   | 0.0250   | 0.0285    | 0.0290    | 0.0360    | 0.0350    |
|                  | Roth     | 0.0060   | 0.0150   | 0.0185    | 0.0155    | 0.0220    | 0.0215    |
|                  | Hoch     | 0.0045   | 0.0140   | 0.0185    | 0.0155    | 0.0220    | 0.0215    |
| $\pi_0 = 0.8$    | MHoch    | 0.0180   | 0.0340   | 0.0360    | 0.0380    | 0.0420    | 0.0405    |
|                  | Roth     | 0.0095   | 0.0210   | 0.0235    | 0.0195    | 0.0255    | 0.0235    |
|                  | Hoch     | 0.0055   | 0.0190   | 0.0225    | 0.0195    | 0.0255    | 0.0235    |
| $m = 15$         |          |          |          |           |           |           |
| $\pi_0 = 0.2$    | MHoch    | 0.0045   | 0.0070   | 0.0070    | 0.0140    | 0.0125    | 0.0130    |
|                  | Roth     | 0.0020   | 0.0035   | 0.0030    | 0.0090    | 0.0060    | 0.0095    |
|                  | Hoch     | 0.0005   | 0.0030   | 0.0030    | 0.0090    | 0.0060    | 0.0095    |
| $\pi_0 = 0.4$    | MHoch    | 0.0100   | 0.0165   | 0.0145    | 0.0255    | 0.0255    | 0.0290    |
|                  | Roth     | 0.0060   | 0.0080   | 0.0075    | 0.0160    | 0.0175    | 0.0165    |
|                  | Hoch     | 0.0010   | 0.0070   | 0.0075    | 0.0160    | 0.0175    | 0.0165    |
| $\pi_0 = 0.6$    | MHoch    | 0.0175   | 0.0260   | 0.0215    | 0.0345    | 0.0350    | 0.0345    |
|                  | Roth     | 0.0090   | 0.0130   | 0.0105    | 0.0190    | 0.0220    | 0.0195    |
|                  | Hoch     | 0.0020   | 0.0105   | 0.0100    | 0.0190    | 0.0220    | 0.0195    |
| $\pi_0 = 0.8$    | MHoch    | 0.0215   | 0.0350   | 0.0315    | 0.0415    | 0.0465    | 0.0390    |
|                  | Roth     | 0.0120   | 0.0170   | 0.0165    | 0.0225    | 0.0290    | 0.0265    |
|                  | Hoch     | 0.0020   | 0.0135   | 0.0165    | 0.0225    | 0.0290    | 0.0265    |
Table S8: Simulated minimal power comparisons for step-up procedures with independent $p$-values generated from Fisher’s Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

| $m$ | $\pi_0$ | MHoch | Roth | Hoch |
|-----|---------|-------|------|------|
| 5   | 0.2     | 0.2600| 0.1975| 0.1170|
|     |         | 0.5075| 0.3915| 0.3845|
|     |         | 0.6885| 0.5820| 0.5810|
|     |         | 0.8240| 0.7685| 0.7685|
|     |         | 0.9170| 0.8695| 0.8695|
|     |         | 0.9525| 0.9300| 0.9300|
| 5   | 0.4     | 0.2155| 0.1630| 0.0885|
|     |         | 0.4105| 0.3115| 0.3010|
|     |         | 0.5810| 0.4755| 0.4740|
|     |         | 0.7410| 0.6705| 0.6705|
|     |         | 0.8400| 0.7715| 0.7715|
|     |         | 0.9055| 0.8660| 0.8660|
| 5   | 0.6     | 0.1580| 0.1200| 0.0605|
|     |         | 0.3155| 0.2365| 0.2190|
|     |         | 0.4340| 0.3380| 0.3365|
|     |         | 0.5860| 0.5165| 0.5165|
|     |         | 0.7045| 0.6280| 0.6280|
|     |         | 0.7875| 0.7275| 0.7275|
| 5   | 0.8     | 0.0955| 0.0745| 0.0330|
|     |         | 0.1815| 0.1330| 0.1190|
|     |         | 0.2585| 0.1920| 0.1920|
|     |         | 0.3615| 0.2970| 0.2970|
|     |         | 0.4695| 0.3965| 0.3965|
|     |         | 0.5535| 0.4870| 0.4870|
| 10  | 0.2     | 0.3215| 0.2080| 0.1580|
|     |         | 0.6155| 0.4685| 0.4660|
|     |         | 0.8110| 0.7225| 0.7225|
|     |         | 0.9130| 0.8555| 0.8555|
|     |         | 0.9765| 0.9420| 0.9420|
|     |         | 0.9930| 0.9820| 0.9820|
| 10  | 0.4     | 0.2735| 0.1770| 0.1240|
|     |         | 0.5245| 0.3840| 0.3795|
|     |         | 0.7200| 0.6070| 0.6065|
|     |         | 0.8465| 0.7720| 0.7720|
|     |         | 0.9450| 0.8920| 0.8920|
|     |         | 0.9755| 0.9510| 0.9510|
| 10  | 0.6     | 0.2030| 0.1335| 0.0800|
|     |         | 0.4045| 0.2910| 0.2825|
|     |         | 0.5615| 0.4525| 0.4525|
|     |         | 0.7310| 0.6315| 0.6315|
|     |         | 0.8450| 0.7600| 0.7600|
|     |         | 0.9045| 0.8530| 0.8530|
| 10  | 0.8     | 0.1135| 0.0765| 0.0390|
|     |         | 0.2440| 0.1625| 0.1555|
|     |         | 0.3500| 0.2645| 0.2645|
|     |         | 0.4780| 0.3810| 0.3810|
|     |         | 0.6150| 0.5175| 0.5175|
|     |         | 0.6935| 0.6075| 0.6075|
| 15  | 0.2     | 0.3405| 0.2520| 0.1390|
|     |         | 0.6720| 0.5010| 0.4875|
|     |         | 0.8830| 0.7545| 0.7535|
|     |         | 0.9505| 0.8910| 0.8910|
|     |         | 0.9915| 0.9765| 0.9765|
|     |         | 0.9965| 0.9900| 0.9900|
| 15  | 0.4     | 0.2895| 0.2110| 0.1030|
|     |         | 0.5830| 0.4115| 0.3870|
|     |         | 0.7925| 0.6485| 0.6470|
|     |         | 0.9025| 0.8060| 0.8060|
|     |         | 0.9635| 0.9335| 0.9335|
|     |         | 0.9830| 0.9745| 0.9745|
| 15  | 0.6     | 0.2150| 0.1495| 0.0700|
|     |         | 0.4500| 0.3080| 0.2760|
|     |         | 0.6595| 0.5095| 0.5075|
|     |         | 0.7935| 0.6730| 0.6730|
|     |         | 0.8845| 0.8320| 0.8320|
|     |         | 0.9505| 0.9150| 0.9150|
| 15  | 0.8     | 0.1210| 0.0835| 0.0335|
|     |         | 0.2645| 0.1785| 0.1480|
|     |         | 0.4285| 0.3055| 0.3045|
|     |         | 0.5500| 0.4295| 0.4290|
|     |         | 0.6730| 0.5895| 0.5895|
|     |         | 0.7780| 0.7035| 0.7035|
Figure S3: Simulated FWER comparisons for different step-up procedures based on FET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).
Figure S4: Simulated minimal power comparisons for different step-up procedures based on FET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).

S2 Results from Dependence Simulation Settings

In this section, we provide the details for simulating the block dependent binomial exact test (BET) statistics and the simulation results for the stepwise procedures comparisons. The following steps illustrate how to generate the dependent BET statistics and corresponding $p$-values.

**Step 1. Generate dependent Poisson observed counts for each group**

In order to generate $m$ dependent BET statistics $T_i$, we use the following algorithm to generate $m$ dependent Poisson random variables within each group, noting that the Poisson random variables between two groups are independent.

1. Let $\lambda_{i1} = 2$ for $i = 1, \ldots, m$, generate $m$ independent Poisson random variable
\[ Y_{1i} \sim \text{Poi}((1 - \rho)\lambda_{i1}) \text{ and one } Y_{01} \sim \text{Poi}(2\rho). \]

2. Let \( X_{i1} = Y_{i1} + Y_{01} \) for \( i = 1, \ldots, m \), then \( X_{i1} \sim \text{Poi}(2) \) and the correlation between \( X_{i1} \) and \( X_{j1} \) is \( \frac{\text{Cov}(X_{i1}, X_{j1})}{\sqrt{\text{Var}(X_{i1})}\sqrt{\text{Var}(X_{j1})}} = \frac{\text{Var}(Y_{01})}{\sqrt{2}\sqrt{2}} = \frac{2\rho}{2} = \rho \) for \( i, j = 1, \ldots, m \) and \( i \neq j \).

3. Let \( \lambda_{i2} = 2 \) for \( i = 1, \ldots, m_0 \) and \( \lambda_{i2} = 10 \) for \( i = m_0 + 1, \ldots, m \), generate \( m \) independent Poisson random variable \( Y_{i2} \sim \text{Poi}((1 - \rho)\lambda_{i2}) \) for \( i = 1, \ldots, m \), one \( Y_{02} \sim \text{Poi}(2\rho) \), and one \( Y_{02}' \sim \text{Poi}(10\rho) \).

4. Let \( X_{i2} = Y_{i2} + Y_{02} \) for \( i = 1, \ldots, m_0 \) and \( X_{i2} = Y_{i2} + Y_{02}' \) for \( i = m_0 + 1, \ldots, m \), then \( X_{i2} \sim \text{Poi}(2) \) for \( i = 1, \ldots, m_0 \) and \( X_{i2} \sim \text{Poi}(10) \) for \( i = m_0 + 1, \ldots, m \). For \( i, j = 1, \ldots, m_0 \) and \( i \neq j \), the correlation between \( X_{i2} \) and \( X_{j2} \) is \( \frac{\text{Cov}(X_{i2}, X_{j2})}{\sqrt{\text{Var}(X_{i2})}\sqrt{\text{Var}(X_{j2})}} = \frac{\text{Var}(Y_{02})}{\sqrt{2}\sqrt{2}} = \frac{2\rho}{2} = \rho \). Similarly, for \( i, j = m_0 + 1, \ldots, m \) and \( i \neq j \), the correlation between \( X_{i2} \) and \( X_{j2} \) is also equal to \( \rho \); for \( i = 1, \ldots, m_0 \) and \( j = m_0 + 1, \ldots, m \), the correlation between \( X_{i2} \) and \( X_{j2} \) is equal to zero.

**Step 2. Obtain the conditional test statistics**

Since the generated Poisson random variables between two groups are independent, we can directly conduct BET for each hypothesis. After generating Poisson observed counts \( x_{i1} \) and \( x_{i2} \), let \( c_i = x_{i1} + x_{i2} \) be the total observed count for two groups. Then the test statistics \( T_i \) is conditional test statistics \( X_{i1} \) given \( X_{i1} + X_{i2} = c_i \) and the critical value is the observed count \( x_{i1} \) for Group 1.

**Step 3. Conditional distribution of the test statistics**

Based on the conditional inference in Lehmann and Romano [1], which is the BET in our paper, the conditional distribution of \( X_{i1} \) given \( X_{i1} + X_{i2} = c_i \) is Binomial, \( \text{Bin}(c_i, p_i) \), where \( p_i = \frac{\lambda_{i1}}{\lambda_{i1} + \lambda_{i2}} \).

**Step 4. Calculate available \( p \)-value \( P_t \) and attainable \( p \)-values**

When \( H_i \) is true, i.e., \( \lambda_{i1} = \lambda_{i2}, p_i = 0.5 \). Thus, \( X_{i1}|X_{i1} + X_{i2} = c_i \sim \text{Bin}(c_i, 0.5) \) under \( H_i \). Therefore, the available conditional \( p \)-value for \( H_i \) can be calculated by

\[
P_t = \Pr_{H_i}\{X_{i1} \geq x_{i1}|X_{i1} + X_{i2} = c_i\} = \sum_{j=x_{i1}}^{c_i} \binom{c_i}{j} 0.5^j (1 - 0.5)^{c_i - j} = \sum_{j=x_{i1}}^{c_i} \binom{c_i}{j} 0.5^{c_i}.
\]

The corresponding attainable \( p \)-values can be calculated by

\[
\Pr_{H_i}\{X_{i1} \geq x|X_{i1} + X_{i2} = c_i\} = \sum_{j=x}^{c_i} \binom{c_i}{j} 0.5^{c_i} \text{ for } x = 0, 1, \ldots, c_i.
\]
The simulation results under the above simulation setting for stepwise procedures comparisons are shown in Tables S9 - S14 and Figures S5 - S8. It is easy to see that in such block dependence simulation setting, the $p$-values calculated based on the Poisson outcomes satisfies the PRDS Assumption 2.2, since $\rho \geq 0$ and the tests are one-sided.

**R-package for MHTdiscrete:** R-package MHTdiscrete [3] contains R code to implement our proposed methods and several existing FWER controlling procedures for discrete data, which are described in this paper. The package can be downloaded from [https://cran.r-project.org/web/packages/MHTdiscrete](https://cran.r-project.org/web/packages/MHTdiscrete).

**Web Application for MHTdiscrete:** A web application containing the proposed procedures and several comparable procedures can be accessed at [https://allen.shinyapps.io/MTPs](https://allen.shinyapps.io/MTPs).

**References**

[1] Lehmann, E. L. and Romano, J. P. (2005). *Testing Statistical Hypotheses, 3rd Edition*. Springer.

[2] R Development Core Team (2018). *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing.

[3] Zhu, Y. and Guo, W. (2017). *MHTdiscrete: Multiple Hypotheses Testing for Discrete Data*. R package version 1.0.0.
Table S9: Simulated FWER comparisons for single-step procedures with dependent \( p \)-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

| \( \rho \) | 0  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|------------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( m = 5 \) | MBonf | 0.0045 | 0.0050 | 0.0035 | 0.0045 | 0.0025 | 0.0055 | 0.0025 | 0.0080 | 0.0045 | 0.0040 |
|           | Tarone | 0.0015 | 0.0010 | 0.0005 | 0.0020 | 0.0010 | 0.0030 | 0.0005 | 0.0040 | 0.0010 | 0.0005 |
|           | Bonf | 0.0015 | 0.0010 | 0.0005 | 0.0015 | 0.0010 | 0.0025 | 0.0005 | 0.0035 | 0.0000 | 0.0005 |
|           | Sidak | 0.0015 | 0.0010 | 0.0005 | 0.0015 | 0.0010 | 0.0025 | 0.0005 | 0.0035 | 0.0000 | 0.0005 |
| \( \pi_0 = 0.4 \) | | | | | | | | | | |
| \( m = 5 \) | MBonf | 0.0125 | 0.0145 | 0.0155 | 0.0160 | 0.0115 | 0.0135 | 0.0110 | 0.0060 | 0.0105 | 0.0045 |
|           | Tarone | 0.0060 | 0.0025 | 0.0060 | 0.0050 | 0.0050 | 0.0045 | 0.0045 | 0.0055 | 0.0040 | 0.0025 |
|           | Bonf | 0.0025 | 0.0015 | 0.0020 | 0.0035 | 0.0020 | 0.0010 | 0.0010 | 0.0000 | 0.0020 | 0.0010 |
|           | Sidak | 0.0025 | 0.0015 | 0.0020 | 0.0035 | 0.0020 | 0.0010 | 0.0010 | 0.0000 | 0.0020 | 0.0010 |
| \( \pi_0 = 0.6 \) | | | | | | | | | | |
| \( m = 5 \) | MBonf | 0.0035 | 0.0005 | 0.0020 | 0.0015 | 0.0025 | 0.0025 | 0.0025 | 0.0010 | 0.0025 | 0.0000 |
|           | Tarone | 0.0005 | 0.0005 | 0.0010 | 0.0005 | 0.0010 | 0.0000 | 0.0005 | 0.0005 | 0.0005 | 0.0005 |
|           | Bonf | 0.0005 | 0.0005 | 0.0010 | 0.0005 | 0.0010 | 0.0000 | 0.0005 | 0.0005 | 0.0005 | 0.0005 |
|           | Sidak | 0.0005 | 0.0005 | 0.0010 | 0.0005 | 0.0010 | 0.0000 | 0.0005 | 0.0005 | 0.0005 | 0.0005 |
| \( \pi_0 = 0.8 \) | | | | | | | | | | |
| \( m = 5 \) | MBonf | 0.0080 | 0.0065 | 0.0095 | 0.0095 | 0.0065 | 0.0035 | 0.0025 | 0.0055 | 0.0070 | 0.0030 |
|           | Tarone | 0.0020 | 0.0030 | 0.0060 | 0.0050 | 0.0020 | 0.0020 | 0.0010 | 0.0030 | 0.0030 | 0.0025 |
|           | Bonf | 0.0000 | 0.0015 | 0.0020 | 0.0015 | 0.0000 | 0.0010 | 0.0005 | 0.0010 | 0.0010 | 0.0005 |
|           | Sidak | 0.0000 | 0.0015 | 0.0020 | 0.0015 | 0.0000 | 0.0010 | 0.0005 | 0.0010 | 0.0010 | 0.0005 |
| \( m = 10 \) | MBonf | 0.0185 | 0.0105 | 0.0115 | 0.0135 | 0.0135 | 0.0150 | 0.0100 | 0.0090 | 0.0050 | 0.0085 |
|           | Tarone | 0.0120 | 0.0080 | 0.0075 | 0.0095 | 0.0075 | 0.0100 | 0.0065 | 0.0030 | 0.0040 | 0.0045 |
|           | Bonf | 0.0005 | 0.0005 | 0.0005 | 0.0010 | 0.0010 | 0.0000 | 0.0020 | 0.0005 | 0.0010 | 0.0010 |
|           | Sidak | 0.0005 | 0.0005 | 0.0005 | 0.0010 | 0.0010 | 0.0000 | 0.0020 | 0.0005 | 0.0010 | 0.0010 |
Table S10: Simulated minimal power comparisons for single-step procedures with dependent \( p \)-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Bonferroni (Bonf) and Sidak (Sidak) procedures.

| \( \rho \) | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|------------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( m = 5 \) | \( \pi_0 = 0.4 \) | MBonf | 0.9095 | 0.8880 | 0.8580 | 0.8455 | 0.8135 | 0.7745 | 0.7545 | 0.7295 | 0.6840 | 0.6305 |
| | | Tarone | 0.8500 | 0.8350 | 0.8000 | 0.7945 | 0.7585 | 0.7070 | 0.6815 | 0.6630 | 0.6130 | 0.5605 |
| | | Bonf | 0.7530 | 0.7465 | 0.7010 | 0.6920 | 0.6750 | 0.6205 | 0.5930 | 0.5675 | 0.5260 | 0.4685 |
| | | Sidak | 0.7530 | 0.7465 | 0.7010 | 0.6920 | 0.6750 | 0.6205 | 0.5930 | 0.5675 | 0.5260 | 0.4685 |
| \( m = 5 \) | \( \pi_0 = 0.6 \) | MBonf | 0.8150 | 0.8020 | 0.7755 | 0.7740 | 0.7655 | 0.7195 | 0.7255 | 0.6790 | 0.6750 | 0.6260 |
| | | Tarone | 0.7635 | 0.7435 | 0.7210 | 0.7075 | 0.7185 | 0.6775 | 0.6815 | 0.6425 | 0.6255 | 0.5865 |
| | | Bonf | 0.6135 | 0.5985 | 0.5740 | 0.5615 | 0.5725 | 0.5410 | 0.5270 | 0.5070 | 0.4715 | 0.4365 |
| | | Sidak | 0.6135 | 0.5985 | 0.5740 | 0.5615 | 0.5725 | 0.5410 | 0.5270 | 0.5070 | 0.4715 | 0.4365 |
| \( m = 5 \) | \( \pi_0 = 0.8 \) | MBonf | 0.5965 | 0.6055 | 0.5955 | 0.5925 | 0.6075 | 0.6095 | 0.5960 | 0.5960 | 0.6075 | 0.6120 |
| | | Tarone | 0.5635 | 0.5730 | 0.5675 | 0.5600 | 0.5755 | 0.5880 | 0.5730 | 0.5730 | 0.5935 | 0.5985 |
| | | Bonf | 0.3825 | 0.3845 | 0.3805 | 0.3760 | 0.3875 | 0.4000 | 0.3960 | 0.3735 | 0.3820 | 0.3810 |
| | | Sidak | 0.3825 | 0.3845 | 0.3805 | 0.3760 | 0.3875 | 0.4000 | 0.3960 | 0.3735 | 0.3820 | 0.3810 |
| \( m = 10 \) | \( \pi_0 = 0.4 \) | MBonf | 0.9760 | 0.9460 | 0.9175 | 0.8925 | 0.8570 | 0.8250 | 0.7885 | 0.7270 | 0.6895 | 0.6090 |
| | | Tarone | 0.9470 | 0.8940 | 0.8535 | 0.8295 | 0.7875 | 0.7585 | 0.7120 | 0.6525 | 0.6040 | 0.5260 |
| | | Bonf | 0.8805 | 0.8260 | 0.7625 | 0.7500 | 0.6845 | 0.6695 | 0.6075 | 0.5550 | 0.5045 | 0.4410 |
| | | Sidak | 0.8805 | 0.8260 | 0.7625 | 0.7500 | 0.6845 | 0.6695 | 0.6075 | 0.5550 | 0.5045 | 0.4410 |
| \( m = 10 \) | \( \pi_0 = 0.6 \) | MBonf | 0.9250 | 0.9125 | 0.8845 | 0.8425 | 0.8300 | 0.7920 | 0.7470 | 0.7160 | 0.6590 | 0.6180 |
| | | Tarone | 0.8820 | 0.8630 | 0.8370 | 0.7705 | 0.7680 | 0.7285 | 0.6925 | 0.6645 | 0.6090 | 0.5745 |
| | | Bonf | 0.7420 | 0.7260 | 0.7030 | 0.6220 | 0.6285 | 0.5710 | 0.5425 | 0.4995 | 0.4440 | 0.4155 |
| | | Sidak | 0.7420 | 0.7260 | 0.7030 | 0.6220 | 0.6285 | 0.5710 | 0.5425 | 0.4995 | 0.4440 | 0.4155 |
| \( m = 10 \) | \( \pi_0 = 0.8 \) | MBonf | 0.7675 | 0.7595 | 0.7390 | 0.7330 | 0.7320 | 0.6975 | 0.6865 | 0.6665 | 0.6340 | 0.6160 |
| | | Tarone | 0.7145 | 0.7055 | 0.6910 | 0.6860 | 0.6885 | 0.6540 | 0.6445 | 0.6310 | 0.5925 | 0.5875 |
| | | Bonf | 0.4935 | 0.4880 | 0.4655 | 0.4710 | 0.4630 | 0.4235 | 0.4145 | 0.3975 | 0.3725 | 0.3495 |
| | | Sidak | 0.4935 | 0.4880 | 0.4655 | 0.4710 | 0.4630 | 0.4235 | 0.4145 | 0.3975 | 0.3725 | 0.3495 |
Table S11: Simulated FWER comparisons for step-down procedures with dependent $p$-values generated from Binomial Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

| $\rho$ | 0    | 0.1  | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  |
|--------|------|------|------|------|------|------|------|------|------|------|
| $m = 5$ |      |      |      |      |      |      |      |      |      |      |
| $\pi_0 = 0.4$ | MHolm | 0.0085 | 0.0090 | 0.0070 | 0.0090 | 0.0105 | 0.0120 | 0.0095 | 0.0150 | 0.0115 | 0.0055 |
|         | TH   | 0.0055 | 0.0060 | 0.0040 | 0.0050 | 0.0065 | 0.0100 | 0.0040 | 0.0130 | 0.0095 | 0.0045 |
|         | Holm | 0.0025 | 0.0040 | 0.0035 | 0.0035 | 0.0055 | 0.0015 | 0.0075 | 0.0040 | 0.0035 |      |
| $m = 5$ |      |      |      |      |      |      |      |      |      |      |
| $\pi_0 = 0.6$ | MHolm | 0.0170 | 0.0095 | 0.0125 | 0.0140 | 0.0125 | 0.0095 | 0.0160 | 0.0150 | 0.0090 | 0.0065 |
|         | TH   | 0.0110 | 0.0065 | 0.0095 | 0.0095 | 0.0090 | 0.0070 | 0.0115 | 0.0125 | 0.0075 | 0.0055 |
|         | Holm | 0.0030 | 0.0020 | 0.0035 | 0.0055 | 0.0040 | 0.0030 | 0.0045 | 0.0025 | 0.0040 | 0.0030 |
| $m = 5$ |      |      |      |      |      |      |      |      |      |      |
| $\pi_0 = 0.8$ | MHolm | 0.0160 | 0.0205 | 0.0200 | 0.0190 | 0.0170 | 0.0175 | 0.0135 | 0.0075 | 0.0120 | 0.0070 |
|         | TH   | 0.0110 | 0.0145 | 0.0120 | 0.0150 | 0.0120 | 0.0120 | 0.0115 | 0.0065 | 0.0100 | 0.0060 |
|         | Holm | 0.0035 | 0.0015 | 0.0020 | 0.0015 | 0.0025 | 0.0025 | 0.0025 | 0.0010 | 0.0025 | 0.0000 |
| $m = 10$ |      |      |      |      |      |      |      |      |      |      |
| $\pi_0 = 0.4$ | MHolm | 0.0130 | 0.0150 | 0.0115 | 0.0095 | 0.0090 | 0.0100 | 0.0115 | 0.0130 | 0.0095 | 0.0090 |
|         | TH   | 0.0060 | 0.0030 | 0.0075 | 0.0040 | 0.0040 | 0.0075 | 0.0080 | 0.0105 | 0.0070 | 0.0075 |
|         | Holm | 0.0005 | 0.0005 | 0.0015 | 0.0005 | 0.0015 | 0.0015 | 0.0010 | 0.0030 | 0.0010 | 0.0010 |
| $m = 10$ |      |      |      |      |      |      |      |      |      |      |
| $\pi_0 = 0.6$ | MHolm | 0.0160 | 0.0150 | 0.0185 | 0.0165 | 0.0175 | 0.0105 | 0.0125 | 0.0130 | 0.0140 | 0.0055 |
|         | TH   | 0.0055 | 0.0085 | 0.0115 | 0.0100 | 0.0125 | 0.0075 | 0.0065 | 0.0100 | 0.0125 | 0.0045 |
|         | Holm | 0.0000 | 0.0020 | 0.0025 | 0.0000 | 0.0015 | 0.0005 | 0.0015 | 0.0025 | 0.0005 |      |
| $m = 10$ |      |      |      |      |      |      |      |      |      |      |
| $\pi_0 = 0.8$ | MHolm | 0.0230 | 0.0160 | 0.0195 | 0.0195 | 0.0200 | 0.0215 | 0.0145 | 0.0120 | 0.0090 | 0.0130 |
|         | TH   | 0.0160 | 0.0130 | 0.0145 | 0.0130 | 0.0145 | 0.0165 | 0.0130 | 0.0100 | 0.0075 | 0.0120 |
|         | Holm | 0.0005 | 0.0005 | 0.0005 | 0.0010 | 0.0010 | 0.0000 | 0.0020 | 0.0005 | 0.0010 | 0.0010 |
Table S12: Simulated minimal power comparisons for step-down procedures with dependent \( p \)-values generated from Binomial Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

| \( \rho \) | 0   | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( \pi_0 = 0.4 \) | MHolm | 0.9095 | 0.8880 | 0.8580 | 0.8455 | 0.8135 | 0.7745 | 0.7545 | 0.7300 | 0.6840 | 0.6305 |
| m = 5 | TH | 0.8500 | 0.8350 | 0.8000 | 0.7945 | 0.7585 | 0.7070 | 0.6815 | 0.6630 | 0.6130 | 0.5605 |
| Holm | 0.7535 | 0.7465 | 0.7010 | 0.6920 | 0.6750 | 0.6205 | 0.5930 | 0.5675 | 0.5260 | 0.4690 |
| \( \pi_0 = 0.6 \) | MHolm | 0.8155 | 0.8020 | 0.7755 | 0.7740 | 0.7655 | 0.7195 | 0.7255 | 0.6795 | 0.6750 | 0.6260 |
| m = 5 | TH | 0.7640 | 0.7440 | 0.7210 | 0.7080 | 0.7185 | 0.6775 | 0.6815 | 0.6425 | 0.6255 | 0.5865 |
| Holm | 0.6135 | 0.5985 | 0.5750 | 0.5615 | 0.5725 | 0.5415 | 0.5270 | 0.5070 | 0.4715 | 0.4365 |
| \( \pi_0 = 0.8 \) | MHolm | 0.5980 | 0.6060 | 0.5975 | 0.5930 | 0.6080 | 0.6095 | 0.5965 | 0.5960 | 0.6080 | 0.6125 |
| m = 5 | TH | 0.5640 | 0.5740 | 0.5685 | 0.5600 | 0.5755 | 0.5880 | 0.5735 | 0.5730 | 0.5940 | 0.5990 |
| Holm | 0.3830 | 0.3845 | 0.3805 | 0.3760 | 0.3875 | 0.4000 | 0.3690 | 0.3735 | 0.3820 | 0.3810 |
| \( \pi_0 = 0.4 \) | MHolm | 0.9760 | 0.9460 | 0.9175 | 0.8925 | 0.8570 | 0.8250 | 0.7885 | 0.7270 | 0.6895 | 0.6090 |
| m = 10 | TH | 0.9470 | 0.8940 | 0.8535 | 0.8295 | 0.7875 | 0.7585 | 0.7120 | 0.6525 | 0.6040 | 0.5260 |
| Holm | 0.8805 | 0.8260 | 0.7625 | 0.7500 | 0.6845 | 0.6695 | 0.6075 | 0.5550 | 0.5045 | 0.4410 |
| \( \pi_0 = 0.6 \) | MHolm | 0.9265 | 0.9125 | 0.8845 | 0.8425 | 0.8300 | 0.7920 | 0.7470 | 0.7160 | 0.6590 | 0.6180 |
| m = 10 | TH | 0.8820 | 0.8630 | 0.8380 | 0.7705 | 0.7680 | 0.7285 | 0.6925 | 0.6645 | 0.6090 | 0.5745 |
| Holm | 0.7420 | 0.7260 | 0.7030 | 0.6220 | 0.6285 | 0.5710 | 0.5425 | 0.4995 | 0.4440 | 0.4155 |
| \( \pi_0 = 0.8 \) | MHolm | 0.7680 | 0.7600 | 0.7390 | 0.7330 | 0.7325 | 0.6975 | 0.6870 | 0.6665 | 0.6340 | 0.6160 |
| m = 10 | TH | 0.7165 | 0.7060 | 0.6925 | 0.6865 | 0.6895 | 0.6540 | 0.6450 | 0.6310 | 0.5925 | 0.5875 |
| Holm | 0.4935 | 0.4880 | 0.4655 | 0.4710 | 0.4630 | 0.4235 | 0.4145 | 0.3975 | 0.3725 | 0.3495 |
Table S13: Simulated FWER comparisons for step-up procedures with dependent \( p \)-values generated from Binomial Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

| \( \rho \) | \( \pi_0 = 0.4 \) | \( m = 5 \) | MHoch | Roth | Hoch |
|---|---|---|---|---|---|
| 0 | 0.0085 | 0.0090 | 0.0070 | 0.0090 | 0.0105 | 0.0120 | 0.0110 | 0.0175 | 0.0130 | 0.0130 | 0.0080 |
| 0.1 | 0.0060 | 0.0070 | 0.0050 | 0.0065 | 0.0075 | 0.0095 | 0.0060 | 0.0140 | 0.0105 | 0.0070 |
| 0.2 | 0.0025 | 0.0040 | 0.0035 | 0.0035 | 0.0035 | 0.0060 | 0.0020 | 0.0090 | 0.0055 | 0.0060 |
| 0.3 | 0.0170 | 0.0100 | 0.0125 | 0.0160 | 0.0130 | 0.0105 | 0.0165 | 0.0170 | 0.0120 | 0.0120 |
| 0.4 | 0.0120 | 0.0075 | 0.0095 | 0.0110 | 0.0085 | 0.0070 | 0.0125 | 0.0140 | 0.0095 | 0.0105 |
| 0.5 | 0.0030 | 0.0020 | 0.0035 | 0.0055 | 0.0040 | 0.0030 | 0.0050 | 0.0050 | 0.0050 | 0.0075 |
| 0.6 | 0.0165 | 0.0210 | 0.0200 | 0.0200 | 0.0170 | 0.0180 | 0.0135 | 0.0095 | 0.0140 | 0.0115 |
| 0.7 | 0.0110 | 0.0145 | 0.0130 | 0.0150 | 0.0115 | 0.0130 | 0.0100 | 0.0075 | 0.0120 | 0.0085 |
| 0.8 | 0.0035 | 0.0015 | 0.0020 | 0.0015 | 0.0025 | 0.0025 | 0.0030 | 0.0010 | 0.0030 | 0.0035 |
| 0.9 | 0.0145 | 0.0165 | 0.0115 | 0.0100 | 0.0105 | 0.0105 | 0.0135 | 0.0155 | 0.0110 | 0.0125 |

| \( \rho \) | \( \pi_0 = 0.6 \) | \( m = 5 \) | MHoch | Roth | Hoch |
|---|---|---|---|---|---|
| 0 | 0.0145 | 0.0165 | 0.0115 | 0.0100 | 0.0105 | 0.0105 | 0.0135 | 0.0155 | 0.0110 | 0.0125 |
| 0.1 | 0.0075 | 0.0045 | 0.0080 | 0.0035 | 0.0045 | 0.0075 | 0.0095 | 0.0120 | 0.0080 | 0.0115 |
| 0.2 | 0.0005 | 0.0005 | 0.0015 | 0.0010 | 0.0015 | 0.0015 | 0.0015 | 0.0015 | 0.0025 | 0.0060 |
| 0.3 | 0.0165 | 0.0150 | 0.0185 | 0.0165 | 0.0185 | 0.0115 | 0.0150 | 0.0155 | 0.0190 | 0.0085 |
| 0.4 | 0.0060 | 0.0070 | 0.0110 | 0.0100 | 0.0110 | 0.0070 | 0.0070 | 0.0105 | 0.0135 | 0.0070 |
| 0.5 | 0.0000 | 0.0020 | 0.0020 | 0.0025 | 0.0005 | 0.0015 | 0.0010 | 0.0020 | 0.0045 | 0.0015 |
| 0.6 | 0.0235 | 0.0170 | 0.0205 | 0.0200 | 0.0210 | 0.0225 | 0.0165 | 0.0140 | 0.0105 | 0.0170 |
| 0.7 | 0.0145 | 0.0120 | 0.0125 | 0.0130 | 0.0155 | 0.0155 | 0.0130 | 0.0075 | 0.0080 | 0.0110 |
| 0.8 | 0.0005 | 0.0005 | 0.0005 | 0.0010 | 0.0010 | 0.0000 | 0.0020 | 0.0005 | 0.0100 | 0.0035 |
Table S14: Simulated minimal power comparisons for step-up procedures with dependent \( p \)-values generated from Binomial Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

| \( \rho \) | \( m = 5 \) | \( m = 10 \) | \( \pi_0 \) | \( m = 5 \) | \( m = 10 \) | \( \pi_0 \) | \( m = 5 \) | \( m = 10 \) | \( \pi_0 \) |
|---|---|---|---|---|---|---|---|---|---|
| \( \pi_0 = 0.4 \) | | | | | | | | | |
| MHoch | 0.9110 | 0.8905 | 0.8595 | 0.8490 | 0.8155 | 0.7780 | 0.7595 | 0.7370 | 0.6985 | 0.6525 |
| Roth | 0.8565 | 0.8430 | 0.8075 | 0.8020 | 0.7640 | 0.7200 | 0.6945 | 0.6755 | 0.6300 | 0.5955 |
| Hoch | 0.7640 | 0.7555 | 0.7105 | 0.7025 | 0.6835 | 0.6310 | 0.6050 | 0.5860 | 0.5435 | 0.4945 |

\( m = 5 \) \( \pi_0 = 0.6 \)

| MHoch | 0.8165 | 0.8065 | 0.7785 | 0.7775 | 0.7690 | 0.7255 | 0.7290 | 0.6880 | 0.6835 | 0.6425 |
| Roth | 0.7670 | 0.7515 | 0.7245 | 0.7115 | 0.7250 | 0.6845 | 0.6840 | 0.6505 | 0.6325 | 0.5990 |
| Hoch | 0.6210 | 0.6045 | 0.5780 | 0.5665 | 0.5800 | 0.5460 | 0.5345 | 0.5140 | 0.4845 | 0.4530 |

\( m = 5 \) \( \pi_0 = 0.8 \)

| MHoch | 0.8165 | 0.8065 | 0.7785 | 0.7775 | 0.7690 | 0.7255 | 0.7290 | 0.6880 | 0.6835 | 0.6425 |
| Roth | 0.7670 | 0.7515 | 0.7245 | 0.7115 | 0.7250 | 0.6845 | 0.6840 | 0.6505 | 0.6325 | 0.5990 |
| Hoch | 0.6210 | 0.6045 | 0.5780 | 0.5665 | 0.5800 | 0.5460 | 0.5345 | 0.5140 | 0.4845 | 0.4530 |

\( m = 10 \) \( \pi_0 = 0.4 \)

| MHoch | 0.9770 | 0.9465 | 0.9205 | 0.8945 | 0.8630 | 0.8300 | 0.7925 | 0.7380 | 0.7055 | 0.6305 |
| Roth | 0.9495 | 0.8980 | 0.8595 | 0.8325 | 0.7910 | 0.7610 | 0.7190 | 0.6605 | 0.6190 | 0.5625 |
| Hoch | 0.8815 | 0.8260 | 0.7630 | 0.7520 | 0.6850 | 0.6705 | 0.6095 | 0.5565 | 0.5075 | 0.4485 |

\( m = 10 \) \( \pi_0 = 0.6 \)

| MHoch | 0.9285 | 0.9155 | 0.8855 | 0.8475 | 0.8340 | 0.7985 | 0.7540 | 0.7235 | 0.6735 | 0.6405 |
| Roth | 0.8870 | 0.8650 | 0.8425 | 0.7760 | 0.7735 | 0.7335 | 0.7000 | 0.6700 | 0.6145 | 0.5875 |
| Hoch | 0.7425 | 0.7260 | 0.7030 | 0.6225 | 0.6290 | 0.5710 | 0.5430 | 0.5000 | 0.4465 | 0.4205 |

\( m = 10 \) \( \pi_0 = 0.8 \)

| MHoch | 0.7725 | 0.7625 | 0.7400 | 0.7380 | 0.7350 | 0.6995 | 0.6890 | 0.6710 | 0.6400 | 0.6285 |
| Roth | 0.7200 | 0.7055 | 0.6945 | 0.6895 | 0.6905 | 0.6515 | 0.6460 | 0.6365 | 0.5970 | 0.5965 |
| Hoch | 0.4935 | 0.4880 | 0.4655 | 0.4710 | 0.4630 | 0.4235 | 0.4145 | 0.3975 | 0.3725 | 0.3495 |
Figure S5: Simulated FWER comparisons for different step-down procedures based on the blocking dependent BET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).
Figure S6: Simulated minimal power comparisons for different step-down procedures based on the blocking dependent BET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).
Figure S7: Simulated FWER comparisons for different step-up procedures based on the blocking dependent BET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).
Figure S8: Simulated minimal power comparisons for different step-up procedures based on the blocking dependent BET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).