On the magnetic field topology in reconnection region

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Abstract. Magnetic reconnection is generally accompanied by large and quasi-turbulent spatio-temporal fluctuations from the MHD scales down to the kinetic ones. The study of these turbulent fluctuations is generally based on the investigation of spectral and scaling features. In a recent paper by Consolini et al. [1] it has been shown how the statistics of geometrical invariants of coarse-grained gradient tensor of plasma velocity is able to provide valuable information on the topology of the turbulent structures. Here, we present a preliminary study of the topology of magnetic field structures at kinetic scales in the X-line/dissipation region of a reconnection event observed by MMS constellation [2]. The analysis evidenced vortex sheet Ohmic-dissipation.

1. Introduction
Several space environments display complex dynamics characterised by multi-scale processes, such as fluid and magnetoydodynamic (MHD) turbulence. In the framework of heliospheric plasmas, turbulence is a phenomenon widely documented in several different regions: interplanetary medium, Earth’s magnetosheath and tail central plasma sheet, etc. Furthermore, the occurrence of turbulent spatio-temporal fluctuations has been observed also in the course of fast dynamical processes such as the magnetic reconnection and/or current disruption events.

The understanding and characterisation of spatio-temporal turbulent fluctuations are very important for modelling plasma heating, acceleration and transport across boundary regions. In this framework of a great importance is the study of the scaling features and intermittent nature of such fluctuations down to the kinetic domain, where plasma acceleration and heating is mainly taking place in the course of fast relaxation processes.

The traditional approach to space plasma turbulence is based on Fourier-spectral analysis and statistical methods assuming an Eulerian point-of-view. Although using such an approach much has been gathered on the general spectral and statistical features of these fluctuations, less is known about the topology of the relevant magnetic field and plasma structures responsible for them.

In this framework, information on the relevant topologies involved in the spatio-temporal turbulence observed in several space plasma regions can be obtained by investigating the statistical features of the geometrical invariants of the coarse-grained gradient tensor of velocity and magnetic field. Recent multi-satellite missions such as the ESA-Cluster and the NASA-MMS, due to their quasi-tetrahedral configuration, allow the computation of the magnetic and velocity field gradient tensor, $A_{ij} = \partial_i u_j$ and $X_{ij} = \partial_i b_j$, at the tetrahedron scale (the so called coarse-grained gradient tensor), providing more detailed information on the spatial features of the observed fluctuations and their evolution with the scale.
Recently, Consolini et al. [1] have shown the potentiality of the investigation of coarse-grained velocity gradient tensor and its geometrical invariants at small-scales for a case study of space plasma turbulence using the ESA-CLUSTER mission, following the same methodology used in the case of fluid turbulence [3, 4, 5]. In this work they have analysed the joint statistics $P(R, Q)$ of the second and third geometric invariants, $Q$ and $R$, of the coarse-grained velocity gradient tensor, showing the presence of a good similarity to what is found for the low end of the inertial range in fluid turbulence. Clearly, a full comprehension of the role that structure topology plays in magnetised turbulent plasmas would also require the investigation of magnetic field gradient tensor features [6].

This work presents a preliminary analysis of magnetic field topologies in the kinetic domain (i.e., below the ion-cyclotron frequency) in the X-line/dissipation region of a reconnection event observed by MMS constellation [2] by evaluating the statistics $P(R_X, Q_X)$ and $P(Q_J, Q_K)$ of the geometric invariants of the coarse-grained magnetic field gradient tensor.

2. Magnetic field gradient tensor and geometrical invariants: a brief introduction

In turbulent media the analysis of the structures involved in the dissipation and in the generation of intermittency is central, and can be achieved by studying the statistics of the geometrical invariants of velocity field gradient tensor, $A_{ij} = \partial_i u_j$, which provides an information of the role that nonlinear self-stretching plays in generating small-scale large fluctuations [7, 8, 9].

The same can be done for the magnetic field gradient tensor, $X_{ij} = \partial_i b_j$ ($X = \nabla b$), whose geometrical invariant quantities can be useful to characterize the local geometrical features of the magnetic field lines [6].

On the basis of the Cayley-Hamilton Theorem the characteristic polynomial of the magnetic field gradient tensor,

$$||X - \lambda_i I|| = \lambda_i^3 + Q_X \lambda_i + R_X = 0$$

is invariant under SO3 group, so that the quantities,

$$Q_X = -\frac{1}{2} X_{im} X_{mi}$$

and

$$R_X = -\frac{1}{3} X_{im} X_{mk} X_{ki},$$

are geometrical invariants under rotations and reflections. In particular, depending of the values of $Q_X$ and $R_X$ we get different topologies of the magnetic field lines. Thus, as in the fluid case, we can use two invariants, $Q_X$ and $R_X$ to classify the most relevant topological structures of magnetic field lines using the $(R_X, Q_X)$ plane and investigating the joint statistics $P(R_X, Q_X)$ of the invariant quantities. In detail, the $(R_X, Q_X)$ plane can be divided into four sectors using the $R_X = 0$ axis and the discriminant line $D = 0$, i.e.,

$$D = \frac{27}{4} R_X^2 + Q_X^3 = 0,$$

so that different topologies of the field lines can be associated to the different sectors: i) unstable vortex compressing [$D, R_X > 0$]; ii) stable vortex stretching [$D > 0$ and $R_X < 0$]; iii) stable tube-like structures [$D, R_X < 0$] iv) unstable sheet-like structures [$D < 0$ and $R_X > 0$].

Furthermore, by decomposing the magnetic field gradient tensor into its symmetric and antisymmetric parts other invariants can be introduced:

$$Q_K = -\frac{1}{2} Tr(K^2),$$
\[ R_K = -\frac{1}{3} \text{Tr}(K^3) \]  

and

\[ Q_j = -\frac{1}{2} \text{Tr}(J^2) = \frac{1}{4} j^2. \]

Here, the first two are related to the symmetric strain rate tensor \( K \), while the last one is associated to the current density \( j \).

Among these last invariant quantities the two scalars \( Q_j \) and \( Q_K \) can be used to evaluate the relative importance of rotational part of the energy and the energy associated to the strain. In this framework, the joint probability distribution \( P(Q_j, -Q_K) \), defined on the plane \((Q_j, -Q_K)\), allows to identify the topologies that are mainly involved in the Ohmic dissipation \( [6] \).

Here, we investigate the two joint probabilities \( P(R_X, Q_X) \) and \( P(Q_j, -Q_K) \) to get an information of the most relevant topologies involved in the spatio-temporal turbulent fluctuations observed in the magnetic reconnection dissipation region.

### 3. Data and Methods

To study the topological features of the magnetic field spatio-temporal fluctuations at the non-MHD scales in the dissipation region of a magnetic reconnection event we consider the NASA-MMS encounter of the dissipation region already studied by Burch et al. \( [2] \) on October, 16, 2015 from 13:06:45 UT to 13:07:15 UT. During the selected time interval the MMS constellation crosses the reconnection X line/dissipation region, where jet reversals, intense currents and strong plasma heating is observed [see Burch et al. \( [2] \) for a more detailed description]. Furthermore, the MMS satellite configuration resembles a quasi-perfect tetrahedral shape (pseudo-sphere), being the tetrahedral shape characterised by a planarity coefficient \( P = 0.19 \), an elongation factor \( E < 0.01 \) and a characteristic scale (radius of the circumscribed sphere) \( L \sim 10 \) km \( [10] \). This characteristic scale \( L \) is intermediate between the electron inertial length \( \eta_e \sim 2 \) km and the proton inertial length \( \eta_p \sim 70 \) km, so that MMS configuration allows to investigate the magnetic field topologies at non-MHD (kinetic) scales.

To evaluate the coarse-grained gradient tensor the dataset used in our analysis consists in the magnetic field measurements from FGMs and SCM experiments and satellite attitude data. Data come from NASA-CDAWeb (https://cdaweb.sci.gsfc.nasa.gov/index.html). FGMs and SCM data have been combined so to get a resolution of 1024 samples per second following a general scheme reported in Ref. \( [11] \).

Figure 1 shows a sample of the obtained power spectral densities (PSD) of the three magnetic field FGMs-SSC combined components relative to satellite MMS1. No anomalous effects of the data merging are visible at the Nyquist frequency \( (f_N = 64 \text{ Hz}) \) corresponding to FGM sampling rate. Furthermore, the observed PSDs show a common power-law behaviour, \( S(f) \sim f^{-\beta} \), with \( \beta \sim -8/3 \) over nearly two orders of magnitude above the proton cyclotron frequency \( (f_\Omega \sim 0.4 \text{ Hz}) \) computed using the magnetosheath local mean magnetic field.

To compute the coarse-grained gradient tensor of the magnetic field at the tetrahedron scale we make use of the TETRAD method introduced by Chertkov et al. \( [3] \). Indicating with the \( \tilde{X}_{ij} \) the coarse-grained gradient tensor \( X_{ij} = \partial_i b_j \) evaluated on the volume \( \Gamma \), i.e.

\[ \tilde{X}_{ij} = \frac{1}{\Gamma} \int_{\Gamma} \partial_i b_j d^3 x, \]

it results,

\[ \tilde{X}_{ij} = \left( \tilde{\rho}^{-1} \right)_{ik} \delta_{kj} - \frac{\delta_{ij}}{3} \text{Tr} \left( \tilde{\rho}^{-1} \hat{b} \right), \]
where \( \hat{\rho} \) and \( \hat{b} \) are the distance matrix tensor and the magnetic field one, computed as described in Refs. [3, 1] using the four measurements at the vertexes of the tetrahedron. In Eq. (9) the second term on the right hand side is the constraint of a divergence-free field, i.e., \( \nabla \cdot \hat{b} = 0 \). The quantity, \( \tilde{X}_{ij} \), represents the average gradient tensor of the magnetic field at the scale of the characteristic size \( L \) of the tetrahedron, and is a coarse-grained approximation of the local magnetic field gradient tensor.

Once the magnetic field coarse-grained gradient tensor, \( \tilde{X}_{ij} \), is computed, all the main four geometrical invariants, \( \{Q_X, R_X, Q_K, Q_j\} \), can be evaluated using the expressions of Eqs. (2, 3, 5) and (7). The estimation of the associated joint probabilities \( P(R_X, Q_X) \) and \( P(Q_j, -Q_K) \) allows the investigation of the most relevant magnetic field line topologies at the observational scale \( L \).

The computation of the joint probabilities \( P(R_X, Q_X) \) and \( P(Q_j, -Q_K) \) is done by applying a kernel-based method as described in Kaiser & Schreiber [12]. The used kernel is a Gaussian one, whose width is twice the resolution scale adopted to compute the joint PDFs.

4. Results

Following the procedure described in the previous Section 3 we have computed point-by-point the four geometrical invariants, \( \{Q_X, R_X, Q_K, Q_j\} \) and successively estimated the associated joint probabilities \( P(R_X, Q_X) \) and \( P(Q_j, -Q_K) \) for the X-line/dissipation region crossing.

Figure 2 shows the computed joint probability \( P(R_X, Q_X) \) of the first two geometrical invariants, \( Q_X \) and \( R_X \). The joint PDF \( P(R_X, Q_X) \) shows a cigar-like shape and is mainly concentrated in upper part of the \( (R_X, Q_X) \) plane, i.e., above the discriminant zero-line \( D = 0 \). This suggests that the most relevant topologies of the magnetic field line structures are typically vortical structures. In particular we observe both stable vortex stretching and unstable vortex compressing, while there is a less evidence for the formation of sheet-like and tube-like structures, which are associated to the lower part of the \( [R_X, Q_X] \) plane below the discriminant zero-line. Furthermore, the distribution core is quite well in agreement with the one observed in MHD decaying turbulence simulations by Dallas & Alexakis [6].

Apart from the distribution core we can observe some spotty increments of the probability in the upper-left \( [R_X, Q_X] \) plane. These probability spots are associated with the occurrence of extreme events that corresponds with those points where strong electron heating is observed (data not-shown: please refer to Burch et al. [2]).

Figure 3 reports the joint probability \( P(Q_j, -Q_K) \) of the other two invariants. As already said in Section 2 the distribution \( P(Q_j, -Q_K) \) on the \( (Q_j, -Q_K) \) plane characterises the dissipative structures, i.e., the structures where the Ohmic dissipation is mainly concentrated [6]. In our case we can find that the PDF is mainly aligned along the bisector of the \( (Q_j, -Q_K) \) plane.

\[ S(f) \sim f^{-8/3}, \text{ with } \beta \sim -8/3, \text{ while the vertical dashed line indicates the average proton cyclotron frequency, } f_\Omega \sim 0.4 \text{ Hz.} \]
The joint probability \( P(R_X, Q_X) \) of the first two geometrical invariants. The dashed red line is the discriminant zero-line. The two invariants are normalized using the mean-squares value of the current density, \( j \), i.e., \( j_{RMS}^2 = \langle |j| ^2 \rangle \).

This result, which is in agreement with previous numerical study by Dallas & Alexakis [6] in a MHD decaying turbulence regime, suggests that the Ohmic dissipation mainly occurs in current layers located in vortex sheets. The same it occurs also for those extreme events identified in the \((R_X, Q_X)\) plane.

The joint probability \( P(Q_j, -Q_K) \) of the other two geometrical invariants. The white line is the bisector line. The two invariants are normalised using the mean-squares value of the current density, \( j \).

5. Summary and Conclusions
In this work we have presented a preliminary study of the geometrical/topological features of the magnetic field turbulent fluctuations at non-MHD/kinetic scales in a reconnection dissipation region performed by the analysis of the coarse-grained gradient tensor topological scalars. The principal aim of this work is to stress the potentiality of this multi-point method in unveiling the most relevant topologies that play a fundamental role in the observed turbulent spatio-temporal fluctuations.

Our analysis of the most relevant topologies has essentially shown that:
• the magnetic field streamlines are mainly elliptic, i.e. characterised by swirling streamlines associated with stable vortex stretching and unstable vortex compressing;
• the Ohmic dissipation concentrates in current layers in vortex sheets;
• there is a good correlation between electron heating and Ohmic dissipation in vortex sheets.

The emerging scenario from our analysis on the topologies in the dissipation region of a reconnection site is that of a complex domain, characterised by turbulent multi-scale structures. We believe that this complex and turbulent topology can strongly affect the reconnection phenomenon and plasma heating as already discussed in some previous works [see e.g. Refs. [13, 14, 15, 16] and references therein].

Clearly, more observational and theoretical studies, capable of describing the evolution of coarse-grained gradient tensor of the magnetic field from MHD scales down to kinetic ones, are necessary to explain the interrelationship between the observed turbulent structures and the occurrence of strong plasma acceleration events at kinetic scales.

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