Optimization and selection of forming depth and pressure for box shaped Superplastic forming using grey based fuzzy logic

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Abstract: Superplastic forming (SPF) is the first choice of designers for manufacturing parts with complexity as used in aircraft and automobile industries, where the strength to weight ratio is the main criterion. Superplastic forming of a sheet metal has been extensively used to produce the parts with greater complexities that are much stronger at the same time lighter than with other methods. Superplastic forming of sheets invariably results in thickness variation. Minimum thickness results at the portion where sheet comes in to contact with the die last. Pressure, forming depth and complexity of the part affect this thinning. The present investigation aims for simultaneous optimisation of forming depth and pressure of box shaped Superplastic forming using grey based fuzzy logic. In the present study Sn-Pb chosen; which is a model material for SPF to carryout experiments, the same results could be applicable for any other Superplastic material. Results revealed that depth at level1 (D1) and pressure at level 3 (P3) parameter settings minimize the time of forming, and maximize the thinning ratio, simultaneously.

1. Introduction
Ductility is the ability to undergo shape change without failing under the action of external mechanical stresses. Elongations in excess of 200\% indicate the superplastic behaviour in the materials. Superplastic forming is the forming process uses this high extensibility of these materials. Because of this property SPF is widely used in the manufacturing of parts with greater complexity at lower costs compared to conventional machining [1]. Certain conditions are necessary for materials to exhibit this phenomenon of super plasticity [2]. The Pb – 61.9\% and Sn – 38.1\% alloy is a model material to conduct experiments on superplastic forming. Most researchers used both symmetrical [3-7] and asymmetrical [7] rolling for the grain refinement and could achieve grain sizes below 10 microns. Superplastic forming of sheet invariably results in thickness variation. To meet the tight tolerance limits of the parts, it is very important to control this thickness variation. Processing of the material to obtain a high ‘m’ value, part/die design changes to minimize local stress concentrations, forming profiled sheet of varying thickness, and pressure application in a profiled and controlled manner to control the strain rate are some of the methods developed by researches to control this thickness variation. Most of the researchers focus on thinning during superplastic forming of hemi-spherical and conical shaped products [3-6].

Kalaichelvan, et.al [3] conducted experiments on Pb–Sn sheet and concluded that combination of variable pressure and preforming gives better results compared to applying only variable strain rate method. Babu and colleagues [4-6] presented thinning in hemi-spherical and conical shaped products. Their results showed that variation in pressure during forming reduced the thinning. This review concludes that very few researchers focused on thickness variation in particularly in the case of box shaped components [7, 8]. Hence, the present investigation focused to study the effect of pressure and depth on thickness variation and forming time of a box shaped component.
Deng [9] proposed Grey system that has capability to handle the data with uncertainty [10]. A grey relational grade converts the optimisation of multiple performance features or responses into a single grey relational grade (GRG) [11]. GRG can be obtained by the aggregation the grey relational coefficients of the individual features. Researchers have been using grey relational analysis for multiple objective optimisation in various fields of engineering [10–12]. Fuzzy logic theory deals vague data with uncertainty [13]. Application of these two techniques together improves the accuracy of solution for multi criteria optimisation problems [10]. Researchers have been using this hybrid method for process optimisation problems and concluded that this method improved the performance of the process significantly [10, 12, 14]. Present study focuses on multi-objective optimisation of Superplastic forming of box shaped component using Grey based fuzzy logic method.

2. Experimental Procedure

Material used for solders i.e., Lead – Tin alloy of composition Pb-40 % and Sn-60%, used to conduct experiments. Soldering rods are casted in a die of dimensions 120 mm x 120 mm x 10 mm to make billets. Cast billets now rolled to a thickness of ~ 2 mm sheet in six roll passes. Due to high strain imposed on the materials the average grain size reduces to ~3 microns. Box shaped forming tests were carried out using an experimental setup shown in Fig.1(a). Blanks of dimensions 40 mm x 40mm and thickness 2 mm formed in to box shape using Argon gas in a die of 20mm X 20mm with different depths of 10, 12.5 and 15mm under constant gas pressures 30, 34 and 37.5 bar. Experiments are conducted using orthogonal L9. For each experiment forming time was noted in hours. Thinning ratio defined as a ratio of thickness at the flat surface of the box shape to the initial thickness was also calculated. Thickness of flat surface of the formed part was obtained by a setup having a combination of a dial gauge and a surface gauge, details of measurement method was presented elsewhere [11,12]. Experimental results are shown in Table.1. Images of these formed parts shown in Fig. 1(b)

![Experimental Setup](image1.png)

**Figure 1.** (a)Experimental Setup, (b-d) Samples formed to different depths (b) 10 mm (c) 12.5 mm (d) 15 mm at various pressures of 30, 34, 37.5 bar
Table 1. Experimental results

| Experiment No | Depth [mm] | Pressure [bar] | Forming time [min] | Thinning ratio |
|---------------|-----------|----------------|--------------------|---------------|
| 1             | 10        | 30             | 37                 | 0.775         |
| 2             | 10        | 34             | 25                 | 0.760         |
| 3             | 10        | 37.5           | 21                 | 0.714         |
| 4             | 12.5      | 30             | 56                 | 0.671         |
| 5             | 12.5      | 34             | 50                 | 0.642         |
| 6             | 12.5      | 37.5           | 40                 | 0.617         |
| 7             | 15        | 30             | 63                 | 0.329         |
| 8             | 15        | 34             | 55                 | 0.317         |
| 9             | 15        | 37.5           | 47                 | 0.305         |

3. Optimization using grey based fuzzy logic.

This section presents the use of grey relational analysis with fuzzy logics for optimisation and selection of forming parameters, considering the optimisation of both forming time and thickness ratio simultaneously. It consists of the following steps:

1. Normalisation of experimental values. The normalisation can be done for three different approaches based on the requirement of performance characteristics: Eq. (1) for larger- the -better, Eq. (2) for smaller- the- better, Eq. (3) for targeted value better.

\[ X_i(k) = \frac{x_i^0(k) - \min x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \]

\[ X_i(k) = \frac{\max x_i^0(k) - x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \]

\[ X_i = 1 - \frac{|x_i^0(k) - x^0|}{\max x_i^0(k) - \min x_i^0} \]

In the present study thickness ratio which is the ratio of final thickness of the formed part to the initial thickness of the sample is considered for optimisation. Higher thickness ratio indicates less thinning during the forming process. Ideal forming is with thickness ratio as 1. But this condition cannot be achieved as thinning is inevitable in superplastic forming process. Hence higher the better characteristics are used for thickness ratio. For forming time smaller the better characteristics is used.

2. Calculation of grey relational coefficient, which can be calculated by Eq. (4).

\[ \xi(k) = \frac{\Delta_{\min} + \zeta \Delta_{\max}}{\Delta_{\max} + \zeta \Delta_{\min}} \]

Here \( \Delta_{\min}(k) \) = the deviation sequence, \( X_0(k) \) = reference sequence, and \( X_i(k) \) = comparability sequence. Where \( \Delta_{\min}(k) = \|X_0(k) - X_i(k)\| \), \( \Delta_{\max} = \max \max \|X_0(k) - X_i(k)\| \), and \( \Delta_{\min} = \min \min \|X_0(k) - X_i(k)\| \)

\( \zeta \) is identification coefficient \( \zeta \in [0, 1] \). \( \zeta = 0.5 \) is generally used.

3. Calculation of grey relational grade

The grey relational grade can be determined by averaging the grey relational coefficients corresponding to each performance characteristic as given in Eq. (5).
4. Fuzzification of grey relational coefficients and grey relations grades.
This is done by mapping the input grey relational coefficients of thickness ratio (TR) and time (T) and output (GFRG) by mapping the membership functions between 0 & 1

\[ \gamma_i = \frac{1}{n} \sum_{k=1}^{n} \zeta_{i}(k) \]  

(5)

5. Development of fuzzy rules
Total 9 fuzzy rules are developed for Time, Thickness ratio and GRG using Eq. (6).

First rule: if TR is A1, T is B1; then G is C1, else:
Second rule: if TR is A2, T is B2; then G is C2; else;

\[ \text{nth rule}: \text{if TR is } A_n, \text{ T is } B_n; \text{ then G is } C_n \]  

(6)

These rules are developed by using the results obtained from the experiments for inference.

6. Calculation of multi response output \( \mu_{c_0}(G) \)
Fuzzy interface engine performs max–min interface operation formula, Eq.(7) to obtain multi response output.

\[ \mu_{c_0}(G) = (\mu_{A_1}(TR) \land \mu_{B_1}(T) \land \mu_{C_1}(G)) \lor \ldots \ldots \mu_{A_n}(TR) \land \mu_{B_n}(T) \land \mu_{C_n}(G) \]  

(7)

7. Calculation of Grey-fuzzy reasoning grade \( G_0 \)
\( G_0 \) can be obtained by using centroid de-fuzzification formula, Eq. (8)

\[ G_0 = \frac{\sum g \mu_{c_0}(G)}{\sum \mu_{c_0}(G)} \]  

(8)

8. Selection of optimum parameters: This can be done by selecting the parameter combination having higher GFRG.

4. Results and discussion
Experimental results were normalised between 0 and 1 using Eq.’s (1), (2). For thickness ratio (TR) Eq. (1), and for time (T) Eq.(2) respectively were used as explained in section 3. Table.2. shows the normalised values, grey relational coefficients and GRG for experimental results shown in Table 1. The grey-fuzzy reasoning grade (GFRG) was obtained from fuzzy logic tool box of MATLAB (R2014b). Grey relational coefficients for Thickness ratio (TR), Time (T) are inputs and GFRG is output. Triangular shaped membership functions were used for fuzzy modeling. To represent the grey relational coefficients (GRC) of inputs TR, and T the linguistic membership functions used were Lowest (LT), Low (L), Medium (M), High (H) and Highest (HT). For output grey fuzzy reasoning grade (GFRG) membership functions used were Lowest (LT), Very Low (VL), Medium Low (ML), Low (L), High (H), Medium High (MH), Higher (HR), Medium Higher (MHR) and Highest (HT). These membership functions are shown in Figs. 2(a) and 2(b).GFRG is shown in Fig.2(c) as indicated in rule viewer in Fuzzy tool box. Rows represent fuzzy rules and the first two columns show grey relational coefficients for TR, T. Last column gives the defuzzified GFRG. These GFRG values obtained for all the nine experiments along with the ranking sequence based on higher GFRG are shown in Table 2. From the Table 2, can be noted that the experiment number 3 shows the highest grey-fuzzy reasoning grade indicating that best multiple performance characteristics for Superplastic forming of the part.
Table 2. Data processing, grey relational coefficients and grey relational grade

| Experiment No. | Normalised values | Grey relational coefficients | Grey relational grade (GRG) | GFRG | Rank |
|----------------|-------------------|------------------------------|----------------------------|------|------|
|                | Time | Thickness ratio | Time | Thickness ratio |               |               |               |      |      |
| 1              | 0.6190 | 1.0000 | 0.5676 | 1.0000 | 0.78 | 0.846 | 3      |
| 2              | 0.9048 | 0.9681 | 0.8400 | 0.9400 | 0.89 | 0.917 | 2      |
| 3              | 1.0000 | 0.8702 | 1.0000 | 0.7939 | 0.90 | 0.926 | 1      |
| 4              | 0.1667 | 0.7787 | 0.3750 | 0.6932 | 0.53 | 0.506 | 6      |
| 5              | 0.3095 | 0.7170 | 0.4200 | 0.6386 | 0.53 | 0.536 | 5      |
| 6              | 0.5476 | 0.6638 | 0.5250 | 0.5980 | 0.56 | 0.551 | 4      |
| 7              | 0.0000 | 0.0511 | 0.3333 | 0.3451 | 0.34 | 0.381 | 9      |
| 8              | 0.1905 | 0.0255 | 0.3818 | 0.3391 | 0.36 | 0.400 | 8      |
| 9              | 0.3810 | 0.0000 | 0.4468 | 0.3333 | 0.39 | 0.416 | 7      |

Figure 2. (a) Membership functions for time and thickness ratio, (b) Membership functions for grey-fuzzy reasoning grade, (c) Fuzzy logic rule viewer (Experiment No.1)

5. Conclusions
This paper presents an application of grey-fuzzy logic for multiple performance characteristic optimisation and selection of process parameters for box shaped Superplastic forming process of Pb-Sn alloy. The conclusions of this present study are,
The present study shows highest GFRG with depth at level 1 (D1) and pressure at level 3 (P3). These parameter settings minimize the time of forming, and maximize the thinning ratio, simultaneously.

The method proposed simplifies the optimization of multi-criteria responses into a single GFRG and can be used to improve the process of box shaped Superplastic forming.

Future studies can be focused on use of ANOVA to determine most influencing parameter on GFRG.

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