Evanescent Airy beams

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In this Letter we propose the concept of the evanescent Airy beam. We analyze the structure of the ideal evanescent Airy beam, which initial profile has the Airy form, while the spectral decomposition consists of only evanescent partial waves. Also, we discuss the refraction of the Airy beam through the interface and investigate the field of the transmitted evanescent Airy beam. © 2009 Optical Society of America

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The propagating Airy beams are well defined by the paraxial wave equation [1, 2]

$$\frac{\partial u}{\partial z} + \frac{1}{2k} \frac{\partial^2 u}{\partial x^2} = 0. \quad (1)$$

The above equation provides the solution in the form of a non-diffracting Airy beam, which accelerates in $x$ direction (see Refs. [2–7]). For highly non-paraxial beams both non-diffractivity and acceleration are not the case [8]. Such beams possess comparable inputs of propagating and evanescent waves. The evanescent Airy beams are studied in this Letter.

At first we will define the field of the evanescent Airy beam. We start with the solution of Maxwell’s equations for the TE-polarized plane wave

$$E_y(x, z, k_x) = c(k_x)e^{i k_x x + ik_z z}, \quad (2)$$

where $E_y$ is the electric field orthogonal to the plane of wavevectors $(x, z)$, $c(k_x)$ is an amplitude, $k_x$ and $k_z = \sqrt{k_0^2 - k_x^2}$ are the transverse and longitudinal wavenumbers (their names are defined with respect to the propagation direction of the Airy beam), $k_0 = \omega/c$ is the wavenumber in vacuum, $\varepsilon$ and $\mu$ are the dielectric permittivity and magnetic permeability of the medium, respectively. The appropriate choice of the amplitude $c(k_x)$ yields the needed initial intensity distribution (beam’s profile). If

$$c(q) = \frac{1}{2\pi} e^{-aq^2} e^{i(q^3 - 3a^2q - ia^3)/3}, \quad (3)$$

the initial beam is the Airy beam $A(q/x_0) e^{i(aq/x_0)}$, where $a$ is a decay factor limiting the beam energy at $x < 0$, $x_0$ is an arbitrary transverse scale [3, 4]. Here and in what below the dimensionless parameters are used: transverse wavenumber $q = k_x x_0$, wavenumber in vacuum $\chi = k_0 x_0$, spatial ranges $\hat{x} = x/x_0$ and $\hat{z} = z/x_0$. Fourier transform results in the non-paraxial wave solution

$$E_y(x, z) = \int_{-\infty}^{\infty} c(q)e^{iq\hat{x} + i\sqrt{\chi^2 - q^2} \hat{z}} dq. \quad (4)$$

The spectrum of transverse wavenumbers can be divided into two domains. For $-n\chi < q < n\chi$ the partial plane waves are the propagating waves ($n = \sqrt{\varepsilon\mu}$ is the refractive index). In the rest $q$-region the waves are evanescent, because they are characterized by the imaginary longitudinal wavenumber. In paraxial approximation (\(\chi\) is great) the spectrum of propagating waves is wide and these waves dominate the evanescent ones. In the opposite case, when $\chi << 1$, the spectrum of propagating waves is narrow and we can neglect these waves. Finally, the solution for evanescent waves can be written with disregarding propagating waves as

$$E_y(x, z) = \frac{A}{2\pi} \int_{-\infty}^{\infty} e^{i\hat{x}q - |q|^2 e^{-aq^2} e^{i(q^3 - 3a^2q - ia^3)/3}} dq. \quad (5)$$

The above integral can be interpreted as the evanescent Airy beam because of two reasons. First, each of the partial plane waves in the Fourier integral [5] is evanescent. Second, the initial (at $\hat{z} = 0$) beam has the ideal limiting Airy profile $A(q/x_0) e^{i(aq/x_0)}$. If compared with an evanescent Bessel beam [9], the situation appears to be very similar. Bessel beam can be also presented as the superposition of the plane waves, the wavevectors forming a cone. That is why the longitudinal wavenumber of all the partial plane waves is the same. In the case of the evanescent Airy beam beam this wavenumber becomes a complex number. Since it is equal for each partial wave, it can be carried out the integral. For an Airy beam the longitudinal wavenumbers of the partial plane waves are different and we need to integrate over transverse wavenumber $q$.

Eq. (5) describes the ideal evanescent Airy beam. In real situations, the beam contains also the propagating waves in the narrow $q$-domain, exactly $-n\chi < q < n\chi$. One more approximation is the replacing of the expression $\sqrt{\chi^2 - q^2}$ by $|q|$, because we assume small parameter $\chi$. This leads to the independence of the electric field [5] on the refractive index of the medium. In Fig. 1 we compare the Airy beams calculated using Eq. (5) and using the exact formula (4). One observes that the lobes
Fig. 1. Comparison of (a) an ideal evanescent beam and (b) realistic one. Intensity \( |E_y/A_1|^2 \) is calculated. The decay parameter \( a = 0.05 \). Parameters for figure (b): \( n = 1, \chi = 0.1 \).

Fig. 2. Evanescent Airy beams at decay parameters (a) \( a = 0.1 \) and (b) \( a = 0.5 \).

of the beam are formed by the evanescent waves. The propagating waves added in Fig. 1(b) only strengthen the beam, but do not influence on its structure. The intensity of propagating waves is distributed homogeneously over the transverse coordinate \( \tilde{x} \). Therefore, all the lobes are intensified approximately equally. The main lobe propagates farther than that for purely evanescent waves due to the propagating waves. If parameter \( \chi \) diminishes, the contribution of the propagating waves becomes smaller and both figures (a) and (b) become more and more similar. Condition \( \chi << 1 \) implies large wavelength compared with the transverse scale, i.e. \( \lambda >> 2\pi x_0 \). With such condition the evanescent profile shown in Fig. 1 can be experimentally realized.

Evanescent Airy beams have the only parameter in Eq. (5), it is the decay parameter \( a \). From Fig. 2 we conclude that small decays provide the side lobes of the evanescent Airy beam. Until \( a = 0.1 \) the evanescent beams are very similar, what can be seen from comparison of Fig. 1(a) and Fig. 2(a). The beam structure stays approximately the same. This can be analyzed using the spectrum (6). At small \( a \) the main input takes the cubic term \( e^{i\chi^3/3} \), which is responsible for the fringes. However, great decay parameters \( a \) significantly change the beam structure. The side lobes almost disappear at \( a = 0.5 \) (see Fig. 2(b)). It should be noted that the main lobe is approximately the same as in Fig. 2(a), i.e. its properties are not determined by the parameter \( a \). The side lobes disappear in the full agreement with the results of Ref. [4], the authors of which showed that the limited Airy beam at large \( a \) lose the fringes. Addressing again to Eq. (5) we conclude that at great \( a \) the cubic term does not play an important part in comparison with the other terms. This assertion is valid for both propagating and evanescent Airy beams.

To illustrate the concept of the evanescent Airy beam we consider transmission of the incident propagating Airy beam through the interface between two non-magnetic media, the first medium with refractive index \( n \), the second one with refractive index 1. Taking only propagating plane waves in the spectrum of the incident beam we describe it as follows

\[
E_y(x, z) = \int_{-n\chi}^{n\chi} c(q)e^{i\tilde{x}q + i\sqrt{n^2 - q^2}z}dq.
\]

Each plane wave in this spectrum transmits through the interface \( \tilde{z} = \tilde{z}_0 \) according to the Fresnel formula

\[
\tau(q) = \frac{2\zeta_1(q)}{\zeta_1(q) + \zeta_2(q)} e^{i(\zeta_1(q) - \zeta_2(q))\tilde{z}_0},
\]

where \( \zeta_1 = \sqrt{n^2 - q^2} \) and \( \zeta_2 = \sqrt{\chi^2 - q^2} \) are the normalized longitudinal wavenumbers.

After transmission, some of the waves (with \( -\chi < q < \chi \)) stay propagating waves. The others are subjected to the total internal reflection, therefore, they become evanescent waves. So, the electric field of the transmitted Airy beam can be written as

\[
E_y(x, z) = \int_{-n\chi}^{n\chi} \tau(q)c(q)e^{i\tilde{x}q + i\sqrt{n^2 - q^2}z}dq.
\]

The numerical simulation of the refraction of the Airy beam is demonstrated in Fig. 3. The “free” beam, which is not undergone to the interaction, has some bright side lobes (see Fig. 3(a)). We place the interface at \( \tilde{z}_0 = 2 \) and observe the transmitted Airy beam. Reflected beam is not the case for our investigation.

Both incident and transmitted beams are shown in Fig. 3(b). We have calculated the amplitude (the square root of intensity) in this figure, because the amplitude of the beam profile is more vivid than intensity. The side lobes of the transmitted Airy beam rapidly decay. The main lobe spreads, so that it swallows up the nearest side lobes. Transmitted beam loses its accelerating property and shifts a bit to positive coordinates \( \tilde{x} \) after refraction.

So, what are the reasons of losing accelerating properties? At first, it is confined spectrum of the incident beam, which takes into account only propagating waves. Therefore, transmitted beam has no all evanescent components, and the intensity pattern of the evanescent waves (and of the whole beam) is distorted. Second, we consider great refractive index for the incident medium, so that much plane waves of the spectrum undergo the total internal reflection and become evanescent. Acceleration is the property of the propagating waves (in usual paraxial approximation all partial plane waves are propagating). Evanescent waves, as it has been discussed

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the energy from the interface. As well-known, it is not symmetric. The propagating waves carry energy from the right. The picture of the intensity distribution becomes almost symmetric. The propagating waves carry the energy from the interface. As well-known, it is not symmetric. The propagating waves carry the energy from the right. The picture of the intensity distribution becomes almost symmetric. The propagating waves carry the energy from the interface.

Fig. 3. Transmission of the Airy beam through the interface between dielectric media with $n = 3.5$ ($0 < \tilde{z} < 2$) and $n = 1$ ($\tilde{z} > 2$). (a) Intensity $I = |E_{\parallel}/A_1|^2$ of the incident Airy beam. (b) Transmission of the incident Airy beam through the interface. (c) Transmitted propagating waves. (d) Transmitted evanescent waves. For the figures (b)–(d), the quantity $\sqrt{T}$ is plotted. Parameters: $a = 0.1$, $\chi = 1$.

before, does not provide the acceleration of the beam. Third, the side lobes, which are essential for the accelerating Airy beams, are very weak, therefore, the beam spreads and is not more similar to the Airy beam.

The spreading of the propagating waves (Fig. 3(c)) is why the whole transmitted beam spreads. The side lobes of propagating waves are very weak. Except the side lobes on the left from the main lobe, the symmetric lobes due to the confined spectrum of the beam appear on the right. The picture of the intensity distribution becomes almost symmetric. The propagating waves carry the energy from the interface. As well-known, it is not the case for the evanescent waves. The intensity pattern of the evanescent waves has the fan-like form (see Fig. 4(d)). It is mostly symmetric and correspond to the rapidly decaying beam. However, the pattern does not look like in Fig. 2(a). It can be understood because of the following reasons. First, the beam has large parameter $\chi$, which does not allow us to exclude the propagating waves from the plane wave spectrum. Second, the spectrum of the evanescent waves is very restricted by the non-evanescent properties of the incident beam. Third, the interaction of the incident beam with the interface changes its ideal form. These items are responsible for the fan-like form of the transmitted evanescent beam.

If refractive index $n$ increases, the transmission provides qualitatively the same results as those in Fig. 3. However, the side lobes of the total transmitted and evanescent beams become more pronounced. If the incident Airy beam is paraxial and formed with both propagating and evanescent waves, it keeps the Airy structure after refraction as shown in Fig. 4. It should be noted that the transmitted beam accelerates faster than the incident one, because it propagates in the air.

In conclusion, we have demonstrated the ideal evanescent Airy beam, which consists of only evanescent partial plane waves and has the Airy profile at the initial plane $z = 0$. We have discussed the limitations of such a beam and concluded that it can be realized only in specific situation (it should be small parameter $\chi$ and all the wavenumbers $q$, from $-\infty$ to $\infty$, should be taken into account). We have considered the refraction of the incident propagating Airy beam through the interface and analyzed the transmitted evanescent beam and its deviation from the ideal one. We believe that the obtained theoretical results can be useful for the aims of microscopy.

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