Corrigan-Ramond Extension of QCD at Nonzero Baryon Density

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Abstract

We investigate the Corrigan-Ramond extension of one massless flavor Quantum Chromo Dynamics at nonzero quark chemical potential. Since the extension requires the fermions to transform in the two index antisymmetric representation of the gauge group, one finds that the number of possible channels is richer than in the ’t Hooft limit. We first discuss the diquark channels and show that for a number of colors larger than three a new diquark channel appears. We then study the infinite number of color limit and show that the Fermi surface is unstable to the formation of the Deryagin-Grigoriev-Rubakov chiral waves. We discover, differently from the ’t Hooft limit, the possibility of a colored chiral wave breaking the color symmetry as well as translation invariance.
I. INTRODUCTION

Different limits have been proposed in order to get a better understanding of the complex structure of strongly coupled gauge dynamics and more specifically of Quantum Chromo Dynamics (QCD). At zero temperature and baryon density, one of the best known limits is the large number of colors $N$ proposed by ‘t Hooft. Here, one increases the number of colors while the quarks transform in the fundamental representation of $SU(N)$. It is then possible to organize a diagrammatic expansion in the inverse of the number of colors. The hope is that the leading terms in the large $N$ expansion may provide a good description of the three color physical world. Indeed a number of QCD properties have a simple understanding in this limit. Recently, also modern lattice simulations have explored the ‘t Hooft limit. However, there are cases in which the leading terms in the ‘t Hooft expansion are not sufficient to capture basic properties of QCD. For example it has been recently shown, that the low energy meson-meson scattering amplitudes are not well represented by the ‘t Hooft large $N$ limit. Hence, it is interesting to investigate different limits which for certain values of the theory parameters also yield QCD.

Soon after the ‘t Hooft limit, Corrigan and Ramond (CR) proposed an alternative QCD extension. Here the quarks transform, with respect to the gauge group, according to the two index antisymmetric representation. To be more precise Corrigan and Ramond suggested a generalization of QCD in which some flavors were in a higher dimensional representation and others still in the fundamental representation. The focus there was on the baryonic and the $\eta'$ properties of QCD. It is easy to show that for three colors the CR limit is identical to QCD. However at large number of colors the CR and ‘t Hooft limits are very different. For instance quark loops are suppressed in the ‘t Hooft limit but not in the CR extension of QCD. Some of the properties at leading number of colors and the possible relevance for QCD were suggested already in the Corrigan and Ramond work and further investigated by Kiritsis and Papavassiliou.

Recently, Armoni, Shifman and Veneziano have also proposed an interesting relation between certain sectors of the two index antisymmetric (and symmetric) theories at large number of colors and sectors of super Yang-Mills (SYM). This has lead to a renewed interest in this limit. Using a supersymmetric inspired effective Lagrangian approach some of the $1/N$ corrections were investigated.
interesting consequences about the spectrum and the vacuum of QCD and also of non-QCD like theories. From this work one can start understanding the mechanism which makes the scalar companions of the lowest-lying pseudoscalars heavy with respect to their chiral partners. It is important to note that despite the enormous amount of theoretical information about supersymmetric gauge theories we still do not have knowledge of basic properties. For example, one cannot even answer the simple question of which hadronic states are the lightest ones in SYM. In [11] it was shown that the lightest states are constituted by the supermultiplet containing the gluinoball while the supermultiplet of glueballs is heavier. If supersymmetry will not be observed in experiments this might be the only way we can infer some experimental information on these theoretically relevant theories. Different sectors in the CR large N limit are, however, not mapped in SYM.

Besides these two limits a third one for massless one-flavor QCD, which is, somewhat, in between the ’t Hooft and the CR one, has been proposed very recently [12]. Here, one first splits the QCD Dirac fermion into the two elementary Weyl fermions and afterwards assigns one of them to transform according to a rank-two antisymmetric tensor while the other remains in the fundamental representation of the gauge group. For three colors one reproduces one-flavor QCD and for a generic number of colors the theory is chiral. Such a theory turns out to be a particular case of the generalized Georgi-Glashow (gGG) model.

All of the previous very encouraging results suggest that it is relevant to explore, within the CR extension of QCD, other regimes. At nonzero temperature, for example, the confinement/deconfinement phase transition problem was studied in [13]. In this paper it was shown that an alternating pattern, as a function of number of colors, with respect to the symmetries of the center group appears. The confinement/deconfinement properties in the CR limit are expected to be different than the SYM ones [13]. It has also been very instructive to investigate the relation between confinement and chiral symmetry in the CR extension of QCD and then confront them with the ’t Hooft one [13].

In the present work we provide the first investigation of the CR extension of QCD when turning on a nonzero baryon density. Much recent work has been devoted to try to unveil possible phases of QCD at nonzero baryon density. It has been established, using various techniques, that the ground state of QCD with two or three flavors at large quark chemical potential displays a color superconductive phase [14, 15, 16, 17]. One can also determine the pattern of chiral symmetry breaking at asymptotically high density and show, for example,
that for three degenerate and light flavors the Color-Flavor-Locked phase (CFL) [18] emerges.

In the hope to gain some further insight, the large \( N \) ‘t Hooft limit at nonzero baryon density has also been investigated, first in [19] and more recently in [20]. One discovers that color superconductivity is suppressed in the ‘t Hooft large \( N \) limit and that the Fermi surfaces are unstable to chiral waves. These waves correspond to chiral condensates that vary as a function of the space coordinates and therefore break translational invariance.

Since the CR extension requires the fermions to transform according to the two index antisymmetric representation of the gauge group we find a richer group theoretical structure than in the ‘t Hooft limit. We first, briefly, discuss the diquark channels and show that when \( N \) is larger than three a new diquark channel appears. Another interesting feature is that for an even number of colors the center group symmetry breaks to a \( Z_2 \) and hence we expect these theories to display also color confinement [13]. We then study the infinite number of color limit and show that, as for the ‘t Hooft case, color superconductivity is not favored with respect to the formation of the Deryagin-Grigoriev-Rubakov (DGR) chiral waves. We discover, differently from the ‘t Hooft limit, the possibility of the presence of a colored chiral wave breaking the color symmetry as well as translation invariance. The presence of the DGR instability in the CR limit of QCD shows that, even at an infinite number of colors the baryonic sector of the theory does not decouple at nonzero matter density.

II. NOTATION AND CONVENTIONS

It is useful to formulate in detail the theory for quarks in the antisymmetric representation of the gauge group. The Lagrangian reads:

\[
\mathcal{L} = -\frac{1}{2} \text{Tr} [F^{\mu\nu} F_{\mu\nu}] + i \bar{q} \gamma^{\mu} D_\mu q + \mu q \dagger q,
\]

with the Dirac fermion \( q \):

\[
q^{[ij]} = q^{\tilde{\alpha}} (t^{\tilde{\alpha}})^{ij}, \quad \tilde{\alpha} = 1, \ldots, \frac{N(N-1)}{2},
\]

and \( i, j = 1, \ldots, N \). The generators, in the fundamental representation of the \( SU(N) \) gauge group are the matrices \( t^a \) with \( a = 1, \ldots, N^2 - 1 \) normalized according to \( \text{Tr} [t^a t^b] = \delta^{ab}/2 \) and \( t^{\tilde{\alpha}} \) are the subset of these matrices which are antisymmetric. The gauge field and the field strength are defined as:

\[
A_\mu = A_\mu^a t^a, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu].
\]
The covariant derivative acting on the fermionic fields is:

\[ D_\mu \tilde{q}^\alpha = \partial_\mu \tilde{q}^\alpha + igA_\mu^a(T^a)_{\tilde{\alpha}}^\alpha \tilde{q}^\beta, \]  

with the generator \( T^a \) written explicitly for the two index antisymmetric representation via the generators in the fundamental representation:

\[ (T^a)_{\tilde{\alpha}}^\alpha = 4 \text{Tr} [t^\alpha t^a t^\beta], \quad \text{with} \quad \text{Tr} [T^a T^b] = \frac{N-2}{2} \delta^{ab}. \]  

Of course one can also define the generators of the two index antisymmetric representation directly as tensor product of the generators in the fundamental representation. We have also introduced directly the chemical potential for the baryon number of the theory and limit ourself to the one flavor case. It is also easy to check that for three colors the previous Lagrangian describes one massless flavor QCD.

III. CR AND THE DGR INSTABILITY

We start our preliminary study of the CR extension of QCD at nonzero quark chemical potential. We consider the limit in which the baryon density is large enough that perturbative techniques are applicable. Throughout the work we assume the fermions to be massless and identify the Fermi energy and momentum (e.g. \( p_F = \mu \)). Some of the key properties of the large chemical potential limit can be explored using renormalization group techniques. At small temperatures the presence of a large baryon chemical potential induces the formation of a Fermi surface of quarks. Renormalization group techniques allows us to concentrate on the relevant scattering processes. The large number of colors limit together with a nonzero baryon chemical potential in the ’t Hooft limit has been investigated in [19, 20], here we will extend this analysis to the CR limit.

Cooper pair formation is favored by any attractive interaction near the Fermi surface. This fact leads to the phenomenon of superconductivity. For QCD at high chemical potential the Cooper pairs carry color charges and hence generically color symmetry breaks spontaneously.

The formation of a Cooper pair is often described as due to the presence of an instability near the Fermi surface, i.e. the BCS instability. In general there will be more than one attractive interaction in the system. Each different attractive interaction tends to form
Cooper pairs with different quantum numbers. Renormalization Group (RG) analysis near the Fermi surface can be used to determine which attraction dominates.

A practical procedure to uncover the presence of an instability in a given channel is to determine first the low energy effective coupling constant of the associated 4-fermion interaction near the Fermi surface. This effective coupling constant contains color, flavor and spin dependence. If the effective coupling constant develops a Landau pole, this indicates an instability in the associated channel. It is expected that the channel in which the Landau pole is reached first is the one which occurs. A review of the RG analysis for BCS theories can be found in [21] and was introduced for investigating color superconductivity in [22].

The situation at asymptotically large chemical potentials where perturbation theory is valid is more involved. As Son [23] has demonstrated magnetic gluons are not screened at large quark chemical potential which means that the one gluon exchange does not reduce exactly to a contact term interaction.

Given the above we start our analysis of the CR limit by considering quark-quark scattering at the Fermi surface. Since we are only interested in the large N limit behavior we will not attempt a full analysis of the ground state at a low number of colors. For our purposes it will be sufficient to investigate the overall color structure.

Since our quarks transform according to the 2-index antisymmetric representation of the gauge group, for a generic number of colors \( N > 3 \), the scattering leads, in color space, to the three channels depicted in Fig. 1 via Young-Tableaux and labelled by (a), (b), and (c) respectively.

For \( N = 3 \) the channel (a) on the RHS of Fig. 1 disappears and the two channels ((b) and (c)) correspond to the antitriplet and the sextet of QCD. The former leads to the instability responsible for color superconductivity in QCD.

Note that when extending the theory to a number of colors larger than 3 while keeping the fermions in the fundamental representation, as in the 't Hooft limit, only two channels appear. They are the straightforward generalization of the ones in QCD. However in the CR generalization of QCD also the channel (a) appears and it turns out to be the most attractive one. This is so since it is antisymmetric with respect to all of the color indices. Also channel (b) is attractive but is surely dominant only for \( N = 3 \), i.e. for QCD. We note that for four colors the most attractive channel is a color singlet.

We summarize here the color factors entering the RG analysis for the quark-quark scat-
tering three channels stemming from the one gluon exchange interaction of Fig. 1:

\[ C_a = 2 + \frac{2}{N}, \quad C_b = \frac{2}{N}, \quad C_c = -1 + \frac{2}{N}. \]  

The coupling of the relevant four fermion interaction will be proportional, in each channel,

\[ \begin{array}{ccc} \otimes & \otimes & \otimes \\ \oplus & \oplus & \oplus \\ \end{array} \]

FIG. 1: Decomposition of the tensor product of two quarks in the 2-index antisymmetric representation. In the text we refer to the three channels that appear in the RHS as (a), (b) and (c) respectively.

As an aside we note that differently from the case with fermions in the fundamental representation, for an even number of colors the underlying theory still preserves a \( Z_2 \) center group symmetry. This fact allows one to define the Polyakov loop as a good order parameter for confinement. This is interesting since for an even number of colors we predict the presence of a well defined deconfining phase transition \cite{13}. It would then be interesting to investigate the relation between the confinement and superconductive phase transition. We expect this case to be similar to the one discussed in \cite{24, 25}.

The large number of colors is also interesting in the CR limit due to the possibility that, at least in vacuum, certain sectors of the theory are expected to be mapped in super Yang Mills. Confinement properties, however, cannot be matched at zero and nonzero temperature \cite{13}. Since super Yang Mills does not possess any baryon number it is clear that little information can be deduced using the correspondence when exploring the presence of a nonzero chemical potential. However the CR limit is still well defined and using the large chemical potential limit one can derive results which are under perturbative control. One could, however, still hope that at large number of colors the baryon number decouples and the knowledge of the supersymmetric theory be of help. A possible naive argument is that at large number
of colors nonplanar diagrams are suppressed and hence diquark condensation is suppressed and one expects a quark-antiquark condensate.

As we shall shortly see superconductivity is indeed suppressed at large \( N \) in the CR limit but now the Fermi surface is unstable with respect to the development of chiral waves with \( 2\mu \) wave number:

\[
\langle \bar{q}(x)q(y) \rangle = e^{iP \cdot (x+y)} \int d^4q e^{-iq \cdot (x-y)} f(q),
\]

where \( \vec{P} \) has modulus \( P = \mu \) and has arbitrary direction. We will discuss the color structure of the previous condensate in the large CR limit later in the text.

The DGR instability has been first discovered at infinite number of colors in the ’t Hooft limit of QCD [19]. Due to the new group structure, the CR large N limit leads to different possibilities in the color composition of the pair with respect to the ’t Hooft case.

There is an important difference with respect to the ordinary constant chiral condensate. In vacuum the pairing happens between a particle and an antiparticle moving in opposite directions. In the present case one pairs a particle and a hole near the Fermi surface moving in the same direction and with a momentum near the Fermi one. The scattering is nearly in the forward direction and the scattering amplitude becomes singular favoring the formation of a pair.

To determine the existence of the DGR instability in the large N CR limit we use again the renormalization group approach and consider an infinite number of colors [19, 20]. So we start with the study of \( \bar{q}q \) scattering. In this case we have a quark in the 2-index antisymmetric representation and a hole in the anti-2-index representation. In Fig. 2 we show the decomposition of the tensor product of the two representations. In the RHS of Fig. 2 we have respectively the totally antisymmetric channel (singlet), the adjoint and a third one. The first two channels correspond exactly to the ones in the ’t Hooft limit while the third one is a specific feature of the CR extension of QCD. Note, however, that since the scattering states are in the two index antisymmetric representation this also affects the overall color factor for the scattering process under consideration so that one cannot simply use the results of the large N a la ’t Hooft for the singlet and the adjoint channel. As we mentioned at the beginning of the section one might imagine the possibility of the formation of a homogeneous condensate \( \langle \bar{q}q \rangle \). In this case the condensate is associated to pairing a quark and an antiquark at large quark matter densities. However this is not an option.
FIG. 2: Decomposition of the tensor product of a quark in the 2-index antisymmetric representation and an antiquark in the anti-2-index antisymmetric one. The three channels in the RHS are the singlet, the adjoint and an extra channel not present in the ’t Hooft limit.

since the amount of energy needed in order to make such a pair is twice the quark chemical potential. Following Deryagin, Grigoriev, and Rubakov [19] and later Shuster and Son [20] we consider the possibility to form Cooper pairs among a quark and a hole. This pairing is energetically favored with respect to the quark-antiquark one.

Near the Fermi surface we consider a quark with momentum $P + q$, and the hole with momentum $P - q$. The total momentum of the pair is $2P$ and is approximately equal to twice the Fermi momentum. $q$ is a small fluctuation of the momentum above the Fermi surface.

Shuster and Son introduced an elegant way to derive the DGR results in the 't Hooft limit. The method consists in reducing the problem to a 1+1 dimensional one and it was invented in the past in a different context [26]. Since the derivation uses only kinematical considerations which can be applied directly to the CR limit we will not repeat them here. We just recall that one can decompose the small momentum $q$ in a component parallel to $P$ and the orthogonal one. One can show that at sufficiently large chemical potential and within a well defined kinematical window one can neglect the orthogonal component.

Remarkably the relevant effective theory in 1+1 dimensions is the following non-abelian Thirring model but with a momentum dependent coupling constant [20]

$$L_{\text{eff}} = i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{g^2}{4\pi} \ln \frac{\Delta}{q_\parallel} (\bar{\Psi} \gamma^\mu \frac{T^a}{2} \Psi)^2.$$  \hspace{1cm} (7)

$\Delta$ denotes the scale for the perpendicular momentum $q_\perp$ to $P$ and $q_\parallel$ is the longitudinal component of $q$. The $\gamma_\mu$ are two dimensional gamma matrices. If we choose $P$ to be along
the $z$ axis, $\Psi$ is the following field in terms of the original four dimensional Dirac quark:

$$
\Psi = \begin{pmatrix}
    e^{-i\mu z} q_{L2} \\
    e^{i\mu z} q_{R2} \\
    e^{-i\mu z} q_{R1} \\
    e^{i\mu z} q_{L1}
\end{pmatrix},
$$

(8)

where $q_{L2}$, $q_{R2}$, $q_{R1}$, and $q_{L1}$ are identified as the four components of the usual Dirac spinor $q^T = (q_{L1} q_{L2} q_{R1} q_{R2})$ in the Weyl basis. We have suppressed the color indices. This particular decomposition becomes more clear when we recognize that we are only considering the positive energy particle states (i.e. $q_{L2}(L_1), q_{R1}(R_2)$) when the particle’s momentum is near $\vec{P}(-\vec{P})$. The exponential factors are introduced to eliminate the fast spatial variations of the fields.

Using the above identification, it is easy to see that a constant condensate of the form $\langle \bar{\Psi} \Psi \rangle$ corresponds to space-dependent condensates of $\langle \bar{q} q \rangle$, because $\langle \bar{\Psi} \Psi \rangle = \cos(2\mu z)\langle \bar{q} q \rangle - i \sin(2\mu z)\langle \bar{q} \gamma^0 \gamma^3 q \rangle$. In the RHS of the last expression we are using the four dimensional gamma matrices.

The running of the coupling of the effective theory is governed again by an equation of the type:

$$
\frac{\partial \lambda(s)}{\partial s} = 2\frac{C}{\pi} \lambda^2(s).
$$

(9)

The effective coupling constant depends on $q_{||}$ and consequently $\lambda$ should be a function of the RG parameter $s$ and $q_{||}$.

The relevant kinematical range is for $\Delta^2/\mu < q_{||} < \Delta$. The renormalization group equation needs to be modified [20] since the four fermion coupling constant has also a direct dependence on the renormalization scale $s$ via the direct dependence on $q_{||}$. We can link $s$ and $q_{||}$ recalling that the typical momentum scale of the internal lines in the loop is of the order of $\Delta e^{-s}$ which sets also the vertex energy scale. One can show that the coupling constant hits a Landau pole, and hence signals an instability, when $s_L = \pi/h2$, where $h = 2g^2C/4\pi^2$.

This translates via the $\Delta e^{-s}$ to the following Landau energy scale

$$
E_L = \Delta e^{-\frac{\pi}{h}} = \Delta e^{-\frac{\pi^2}{2g^2C}}.
$$

(10)

Before discussing the results for the CR limit we review the 't Hooft limit ones [19, 20]. Here one has only two different channels, the totally antisymmetric (singlet) and the one
that corresponds to the octet in QCD. However the factor $C$ is proportional to $N$ only for the singlet case while it is a constant for the adjoint channel. From (10) it is clear that the larger the value of $C$ the earlier we meet the Landau pole in the RG flow. This is an indication in favor of the chiral wave instability in the singlet channel \cite{19,20}.

For the CR extension of QCD at large number of colors we find the following values of the one loop relevant coefficient $C$:

$$C_{\text{singlet}} = N, \quad C_{\text{Adjoint}} = \frac{N}{2}, \quad C_{\text{extra}} = -\frac{2}{N}.$$  \hspace{1cm} (11)

The subscript $\text{extra}$ refers to the third channel in the CR extension of QCD which is not present in the 't Hooft limit. The extra channel does not lead to an instability and is subleading in $N$ with respect to the other two channels. Our results show that both the singlet and the adjoint channel in Fig.2 are leading in $N$. This result is somewhat unexpected. We recall that in the 't Hooft limit the adjoint channel, although attractive, is subleading with respect to the singlet one. The difference in the large $N$ behavior here is due to the fact that we have many more fermions in the CR limit than in the 't Hooft one.

It is reasonable to expect that for $g^2N \ll 1$, see (10), the singlet channel dominates, since there is still a small numerical suppression due to the extra $1/2$ factor in the adjoint channel. A situation similar to the 't Hooft limit would then emerge, at least for $N \to \infty$. However such a small suppression factor does not preclude the possibility, especially when increasing the value of $g^2N$, that either both channels occur simultaneously or that the adjoint channel is the relevant one. In both cases color symmetries break spontaneously due to the presence of colored chiral waves.

Another remark is that one has shown that the theory is sensitive to the presence of the baryon number even at infinite number of colors. Besides, the condensate which forms has an explicit dependence on space (i.e. is a chiral wave). In this way one has also explicitly tested the baryon number dependence of the CR extension of QCD and shown that the baryon number, at nonzero chemical potential, does not decouple even in the planar limit \cite{9}. 

\hspace{1cm}
IV. CONCLUSIONS

We have investigated the CR extension of one massless flavor QCD at nonzero quark chemical potential. We have shown that this extension, both for a small as well as an infinite number of colors leads to a richer structure than in the ’t Hooft case.

We have first discussed the diquark channels and have seen that for a number of colors larger than three a new diquark channel appears. We have also considered the infinite number of color limit. Here, color superconductivity is not favored with respect to the Deryagin-Grigoriev-Rubakov chiral wave phase. Differently from the ’t Hooft limit there is also the possibility of a colored chiral wave which breaks both color symmetry and translation invariance.

One can now extend the work to a finite number of colors \[20\] or to a larger number of flavors. There are a number of possibilities, one can add new flavors in the fundamental (following CR) or in the two index antisymmetric representation. It is also very interesting to investigate in some detail the interplay between confinement and chiral symmetry in this limit at nonzero temperature and quark chemical potential. Our results show that even in the infinite number of colors limit in the CR extension of QCD one is still sensitive to the baryon symmetry.

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