Allocation of Multi-Robot Tasks with Task Variants

Zakk Giacometti and Yu Zhang

Abstract
Task allocation has been a well-studied problem. In most prior problem formulations, it is assumed that each task is associated with a unique set of resource requirements. In the scope of multi-robot task allocation problem, these requirements can be satisfied by a coalition of robots. In this paper, we introduce a more general formulation of multi-robot task allocation problem that allows more than one option for specifying the set of task requirements—satisfying any one of the options will satisfy the task. We referred to this new problem as the multi-robot task allocation problem with task variants. First, we theoretically show that this extension fortunately does not impact the complexity class, which is still NP-complete. For solution methods, we adapt two previous greedy methods for the task allocation problem without task variants to solve this new problem and analyze their effectiveness. In particular, we “flatten” the new problem to the problem without task variants, modify the previous methods to solve the flattened problem, and prove that the bounds still hold. Finally, we thoroughly evaluate these two methods along with a random baseline to demonstrate their efficacy for the new problem.

Introduction
Given a set of tasks and a set of resources, the problem of task allocation is to determine the allocation of resources (e.g., robots) to tasks so as to maximize the overall utility achieved. The task allocation problem has been well studied in the robotics community. (Gerkey and Matari 2004) classifies the problem according to three categories: Single-Task/Multi-Task (ST/MT) robot, Single-Robot/Multi-Robot (SR/MR) task, and Instantaneous/Time-Extended Assignment (IA/TA). (Korsah, Stentz, and Dias 2013) extends this taxonomy by considering interrelated utilities and constraints among the tasks. In this paper, we focus on the allocation of multi-robot tasks with single-task robots and instantaneous assignments. This problem has many applications in the real-world, such as for urban search and rescue, automated manufacturing and warehousing, etc. In this formulation, each task has a set of pre-specified resource requirements and each robot is associated with a set of resources (a.k.a. capabilities). A task can be satisfied if the set of robots assigned to it satisfy the resource requirements.

One assumption made in prior problem formulations is that each task is associated with a unique set of resource requirements. This however may not always be the case for real-world applications. Consider a monitoring task for an open area. It may be achieved by multiple mobile robots with cameras or a single UAV. The resources required for each way of achieving the task are very different. In this paper, we set out to address the multi-robot task allocation problem with task variants, which represent different ways to achieve a task. We first theoretically prove that this extension does not impact the complexity class, which is still NP-complete. To provide a solution, we adapt two previous greedy methods that are introduced for the multi-robot task allocation problem. We show that it is not difficult to compile the new problem into a “flattened” problem, for which the previous method would apply. With slight modifications to the proofs, we show that the solution bounds carry over to the new problem. Finally, we thoroughly evaluate these two methods along with a random baseline to demonstrate their efficacy for the new problem.

Related Work
The multi-robot task allocation problem is known to be NP-complete (Gerkey and Matari 2004), and is closely related to the coalition formation problem (Sandholm et al. 1999) in the multi-agent community. In fact, (Shehory and Kraus 1998) first looked at the task allocation problem via coalition formation and provided a greedy method based on the set covering problem. (Service and Adams 2011) studied the task allocation problem that maximizes utility rather than minimizing cost, and showed that this seemingly innocuous change resulted in very different solution bounds. A greedy heuristic was provided to solve this problem. (Zhang and Parker 2013b) further analyzed this problem and proposed a new heuristic that incorporates the influence of resource requirements between tasks when making assignments. Our work adapted the heuristics in these earlier works to solve the new problem with task variants.

Our work falls in line with many prior approaches that aimed at extending the applicability of the task allocation problem. (Vig and Adams 2006; Vig and Adams 2007) adapted prior task allocation methods to work in multi-robot systems with additional constraints and preferences (e.g., balanced workload) that are present in physical robotic sys-
A function Cost:

\[ \text{Cost}(c_j, \tau_{k,l}) \]

\[ \sum_h P_{k,l}[h] W[h] - \text{Cost}(c_j, \tau_{k,l}) \]

in which \( m_{jk,l} \) represents an assignment of a coalition \( c_j \) to a task configuration \( \tau_{k,l} \).

The problem is then to search for a set of assignments \( S \) that maximizes:

\[ \sum_{m_{jk,l} \in S} U(m_{jk,l}) \]

subject to the constraints that no assignments must have overlapping robots and any task must have at most one task configuration assigned in the solution. Next, we analyze the complexity of this new formulation.

**Theorem 1.** The decision problem of whether there exists an assignment of no less than a given utility value for the multi-robot task allocation problem with task variants is NP-complete.

The proof is straightforward as verifying the solution of this problem would only take polynomial time, so the problem is in NP. Furthermore, since the task allocation problem without task variants is clearly a special case of this new formulation, which is NP-complete, this new problem must also be NP-complete.

**Solution Methods**

Since the new problem is NP-complete, instead of looking for exact solutions, we propose to study approximate solutions.

**Random Task Configuration:** The first thought is to randomly pick from the set of task configurations for each task, which essentially turns the new problem into a task allocation problem without variants. We can then apply any of the state-of-the-art task allocation methods. This also becomes our baseline approach for comparison. This method clearly would perform poorly in situations where a very bad task configuration has been chosen, e.g., it renders all the remaining tasks achievable.

**Flattening Formulation:** A better idea is to try to convert the new problem into a problem without task variants such that prior task allocation solutions can be applied. An obvious solution we consider here is a flattening approach that treats every task configuration as an independent task. This “flattens” the extra dimension of task variant decompositions are not known prior to assigning robots. A recent work (Cano et al. 2018) that studied the task allocation problem with task variants, applying to the domain of process scheduling. However, the problem studied was single-robot tasks with multi-task robots (i.e., MTSR) while we are addressing the multi-robot task allocation problem (i.e., ST-MR).

On the aspect of task variants, the information invariant theory (Donald 1995) discussed different ways that a task may be achieved by different sensori-computational systems, which are considered equivalent for achieving the task. (Tang and Parker 2005, Zhang and Parker 2013a) applied this idea to the problem of coalition formation, resulting in greater flexibility in dynamic environments compared to traditional approaches. The task variants in our work can be considered as static ways of capturing information invariant for tasks.

**Problem Formulation**

Following prior work, we formulate our variation of the ST-MR-IA problem below, only redefining tasks as sets of task configurations (i.e., task variants). A multi-robot task allocation problem with task variants is a tuple \( (R, C, T, W, V, \text{Cost}, U) \):

- A set of robots \( R = \{r_1, r_2, \ldots\} \). Each robot \( r_i \) is associated with a vector \( B_i \) of H real non-negative capabilities, in which \( H \) is assumed to be a constant that specifies the maximum number of capabilities for a domain.
- A set of coalitions, \( C = \{c_1, c_2, \ldots\} \). Each coalition \( c_j \) satisfies \( c_j \subseteq R \).
- A set of tasks to be assigned \( T = \{t_1, t_2, \ldots\} \). Each task \( t_k \) is associated with a set of task configurations \( T_k = \{\tau_{k,1}, \tau_{k,2}, \ldots\} \), where each task configuration \( \tau_{k,l} \) requires a vector \( P_{k,l} \) of \( H \) real non-negative capabilities for achieving task \( t_k \) using configuration \( \tau_{k,l} \).
- A vector \( W \) of real non-negative costs for capabilities: the use of the capability indexed by \( h \) incurs \( W[h] \) cost per unit.
- A vector \( V \) of real positive rewards for tasks: achieving task \( t_k \) with any of its configurations receives \( V[k] \) reward.
- A function \( \text{Cost}: C \times T \rightarrow \mathbb{R}^0 \) that computes real non-negative communication and coordination costs for an assignment based on the coalition and task configuration pair, where \( \tau \) is used above to denote the union of task configurations for all the tasks.

- A utility function \( U \) for assignments, defined as:

\[ U_s(m_{jk,l}) = V[k] - \sum_h P_{k,l}[h] W[h] - \text{Cost}(c_j, \tau_{k,l}) \]

\[ U(m_{jk,l}) = \begin{cases} U_s(m_{jk,l}) & \forall h : \sum_r \in c_j B_i[h] \geq P_{k,l}[h] \\ 0 & \text{otherwise} \end{cases} \]

subject to the constraints that no assignments must have overlapping robots and any task must have at most one task configuration assigned in the solution. Next, we analyze the complexity of this new formulation.

**Theorem 1.** The decision problem of whether there exists an assignment of no less than a given utility value for the multi-robot task allocation problem with task variants is NP-complete.

The proof is straightforward as verifying the solution of this problem would only take polynomial time, so the problem is in NP. Furthermore, since the task allocation problem without task variants is clearly a special case of this new formulation, which is NP-complete, this new problem must also be NP-complete.

**Solution Methods**

Since the new problem is NP-complete, instead of looking for exact solutions, we propose to study approximate solutions.

**Random Task Configuration:** The first thought is to randomly pick from the set of task configurations for each task, which essentially turns the new problem into a task allocation problem without variants. We can then apply any of the state-of-the-art task allocation methods. This also becomes our baseline approach for comparison. This method clearly would perform poorly in situations where a very bad task configuration has been chosen, e.g., it renders all the remaining tasks achievable.

**Flattening Formulation:** A better idea is to try to convert the new problem into a problem without task variants such that prior task allocation solutions can be applied. An obvious solution we consider here is a flattening approach that treats every task configuration as an independent task. This “flattens” the extra dimension of task variants and allows us to consider all possible task configurations at once while allowing prior methods to be directly applied. A remaining problem, of course, is that this formulation can potentially lead to invalid solutions, since the same task may be assigned multiple times as different task configurations. This seems to imply that we cannot consider different task configurations at the same time. The dilemma here, hence, is to incorporate the influences among the task variants when
making assignments while preserving the validity of solutions. We will show soon that this in fact is not difficult at all.

![Figure 1: An illustration of how our methods maintain valid solutions after flattening. Circles on the left hand side represent task configurations and squares represent coalitions. Circles on the right represent assignments. Arrows indicate a feasible assignment and dotted lines represent conflicts, e.g., c1 and c2 above conflict (due to overlapping coalitions). The top on the left is a problem without flattening while the bottom with flattening. To maintain valid solutions, the intuition is to maintain the conflicts in the new problem formulation by updating the definition of M.](image)

Maximum Utility with Flattening (FlatMaxUtil)

First, we introduce a natural greedy heuristic that selects an assignment that maximizes the utility among those remaining at every greedy step, similar to those in (Service and Adams 2011). Since we consider all task variants as separate tasks, we need to ensure the validity of our solution. A simple way to achieve this is to eliminate assignments to all variants of an assigned task at each greedy iteration. At each step, the following metric is to be maximized by the chosen assignment:

\[ m^\lambda = \max_{m_{xy} \in M'(\lambda)} U(m_{xy}) \]  \hspace{1cm} (4)

where \( m_{xy} \) refers to the assignment of coalition \( x \) to task \( y \). Note that given the flattened formulation, we no longer need to consider the task configuration. A special note on the definition of \( M'(\lambda) \) above, where \( \lambda \) refers to the greedy step, \( M'(\lambda) \) represents the remaining valid assignments to be considered, and \( m^\lambda \) refers to the assignment chosen at the greedy step. In (Service and Adams 2011), there is a definition of \( M(\lambda) \), where assignments that have coalitions overlapping with the chosen assignment \( m^\lambda \) or for the same task will be removed for the next iteration. In our formulation, in order to maintain the validity of the solution, we change \( M(\lambda) \) to \( M'(\lambda) \), which additionally removes assignments that represent different task configurations for the chosen task. In this way, we have preserved all the task configurations to be considered at any greedy step while ensuring that no invalid solution will be produced. Fig. 1 provides an illustration of this intuition.

**Theorem 2.** Applying FlatMaxUtil to the ST-MR-IA problem with task variants without restricting the maximum coalition size yields a worst case ratio \( \theta = |R + T| \), while restricting the maximum coalition size to be \( k \) yields a worst case ratio of \( \theta = k + 2 \).

**Proof.** Given a task allocation problem with task variants, first, for each task \( t_k \), we change the problem by adding a robot \( r^k \) that is shared among all the assignments for all the task configurations for \( t_k \). Furthermore, we modify the problem such that each \( r^k \) has only a unique capability that is not used by any task. This essentially allows at most one of the assignments for a task being made, which is exactly how we ensure a valid solution. As a result, the bounds in (Service and Adams 2011) are directly applicable to the flattened problem after this modification (which are \( |R| \) and \( k + 1 \) above, respectively, without task variants). Since we add a total of \( T \) robots and the maximum coalition size is increased by 1, we have the bounds holds.

Resource Centric with Flattening (FlatRC)

The FlatMaxUtil method is expected to perform poorly in many scenarios, as it only considers the utility of assignment for each greedy choice and does not consider the influences of assignments on each other. This effect is first observed in (Zhang and Parker 2013b).

**Motivating Example:** As a motivating example, consider a task allocation problem with 3 tasks, 2 variants per task: \( T = \{t_1, t_2, t_3\} \), \( T_1 = \{t_{11}, t_{12}\} \), \( T_2 = \{t_{21}, t_{22}\} \) and \( T_3 = \{t_{31}, t_{32}\} \), with capability requirements: \( P_{1,1} = (2, 0, 0, 0), P_{1,2} = (1, 1, 0, 1), P_{2,1} = (1, 1, 1, 0), P_{2,2} = (1, 1, 0, 1) \). Suppose we only have two robots with the first capability, but sufficient robots with the other three capabilities. Also assume that robots have at most one unit of each capability, all tasks have equal rewards, all capabilities have equal costs, and \( Cost \) always returns 0 for all assignments. Maximizing solely on utility will cause \( t_{11} \) to be chosen, preventing assignment of either variant of \( t_2 \), reducing the utility of the final solution.

Similar to the ResourceCentric heuristic in (Zhang and Parker 2013b), we use a similar heuristic that maximizes the following metric after flattening:

\[ \rho_{xy} = U(m_{xy}) - \sum_{m_{jl} \in M'_{jl}(\lambda)} \frac{1}{|M'_{jl}(\lambda)|} \cdot U(m_{jl}) \]  \hspace{1cm} (5)

where \( M'_{jl}(\lambda) \) represents the set of assignments conflicting with \( m_{jl} \) (assignment of \( c_j \) to task \( t_l \) after flattening), with conflicts defined similarly to how we remove conflicting assignments in \( M' \) in Eq. (4), differing from \( M_{jl} \) in (Zhang and Parker 2013b). It follows that the approximation bounds remain similar to those in (Zhang and Parker 2013b).

**Corollary 1.** Applying FlatRC to the ST-MR-IA problem with task variants while restricting the maximum coalition size to be \( k \) yields a worst case ratio of \( \theta = \min(2k + 4, \max_{m_{jl} \in S^*}(|M'_{jl}(1)|)) \), in which \( S^* \) the optimal solution.

The proof proceeds nearly identically to that shown for FlaxMaxUtil given the bound in (Zhang and Parker 2013b) (which is \( \min(2k + 2, \max_{m_{jl} \in S^*}(|M'_{jl}(1)|)) \)).
Complexity Analysis: The algorithm for FlatRC follows almost identically to ResourceCentric in Zhang and Parker (2013b). As we now have multiple configurations per task, the worst case complexity is increased, but only linearly. For clarity, let $|T_{max}| = max_{t_k \in T} |T_k|$, the size of the largest task configuration set. Then, the complexity is bounded by $O(|T||C||T_{max}||M|)$, where $M$ is the set of assignments. Each greedy step is bounded by $O(|M|^2)$. As there can be at most $min(|R|, |T||T_{max}|)$ assignments, the overall complexity is bounded by $O(min(|R|, |T||T_{max}|) \cdot |T|^2|T_{max}|^2|C|^2)$.

Approximated FlatRC (FlatRCA)

To improve the computational performance, we also adapt the ResourceCentricApprox heuristic in Zhang and Parker (2013b) to our problem, after flattening. Following a similar reasoning, we wish to reduce the complexity of our algorithm as $|C|$ grows exponentially with $|R|$. To this end, we compute $\beta_{il} = \frac{|M'_{il}|}{|M|}$, which measures how much task $t_i$ is flattened) depends on robot $r_l$. Then we compute the average expected loss for each task $t_i$ due to the assignment of $r_l$, $\varphi_{il}$. Finally, we compute the greedy criteria $\hat{\beta}_{il}$ from this value and the utility of each remaining assignment:

$$\hat{\beta}_{il} = \frac{|M'_{il}|}{|M_i|}$$  \hspace{1cm} (6)

$$\varphi_{il} = \hat{\beta}_{il} \cdot U(m_{jl}, M_i \setminus \{\lambda\})$$  \hspace{1cm} (7)

$$\rho_{xy} = U(m_{xy}) - \sum_{r_i \in C \setminus l} \varphi_{il}$$  \hspace{1cm} (8)

Simulation Results

In this section, we provide simulation results for the task variant problem. We focus mainly on randomly generated allocation scenarios, varying key parameters. In all cases when evaluating performance ratios we compare against the upper bound of the optimal solution as (Shehory and Kraus 1998): the sum of the feasible assignments with the maximum utility for each task without checking for conflicts. The costs of each capability (i.e. $W$) are randomly generated from $[0.0, 1.0]$. Each task or robot has a 50% chance to need/provide any capability. Capability values, unless specified otherwise, are generated from $[0, 8]$. The number of capabilities $H$ is fixed at 7. The maximum size of coalitions is fixed at 5 ($k = 5$). Task rewards (i.e. $V$) are generated randomly from $[100, 200]$. $Cost$ is defined as a linear function of coalition size, $4n$. Measurements are made over 1000 runs.

We make two comparisons: varying the number of robots and tasks. We also compare time when varying robots. Varying the number of task variants showed similar trends to varying robots and is not shown. Our time analysis we only consider the time required to assign coalitions to tasks. Note that in most of our results, FlatRC and FlatRCA overlap significantly. We show a clear improvement in applying FlatRC and FlatRCA over the simple greedy heuristic for varying numbers of robots and tasks.

Conclusion and Future Work

First, we introduced a new formulation of the ST-MR-IA problem that allows for more realistic and flexible scenarios of achieving tasks in the form of task configuration variants. A simple but effective method of solving this problem is to “flatten” it into a task allocation problem without the variants. With slight modifications, this allows the application of existing greedy heuristics that provide good approximation bounds. However, this method effectively discards some finer information about the interaction between task variants. It is clear that improved methods that utilize this information may be devised, but the increased complexity of the problem do not make it trivial to do so. In future work, we plan to investigate such a method if it does exist and compare its performance with those discussed in this work.
References

[Cano et al. 2018] Cano, J.; White, D.; Bordallo, A.; McCreesh, C.; Michala, A.; Singer, J.; and Nagarajan, V. 2018. Solving the task variant allocation problem in distributed robotics. *Autonomous Robots* 42(7):1477–1495.

[Donald 1995] Donald, B. R. 1995. On information invariants in robotics. *Artificial Intelligence* 72(1-2):217–304.

[Gerkey and Matari 2004] Gerkey, B. P., and Matari, M. J. 2004. A formal analysis and taxonomy of task allocation in multi-robot systems. *The International Journal of Robotics Research* 23(9):939–954.

[Korsah, Stentz, and Dias 2013] Korsah, G. A.; Stentz, A.; and Dias, M. B. 2013. A comprehensive taxonomy for multi-robot task allocation. *The International Journal of Robotics Research* 32(12):1495–1512.

[Liemhetcharat and Veloso 2014] Liemhetcharat, S., and Veloso, M. 2014. Weighted synergy graphs for effective team formation with heterogeneous ad hoc agents. *Artificial Intelligence* 208:41–65.

[Luo, Chakraborty, and Sycara 2011] Luo, L.; Chakraborty, N.; and Sycara, K. 2011. Multi-robot assignment algorithm for tasks with set precedence constraints. In *2011 IEEE International Conference on Robotics and Automation*, 2526–2533.

[Sandholm et al. 1999] Sandholm, T.; Larson, K.; Anderson, M.; Shehory, O.; and Tohm, F. 1999. Coalition structure generation with worst case guarantees. *Artificial Intelligence* 111(1):209–238.

[Service and Adams 2011] Service, T., and Adams, J. 2011. Coalition formation for task allocation: theory and algorithms. *Autonomous Agents and Multi-Agent Systems* 22(2):225–248.

[Shehory and Kraus 1998] Shehory, O., and Kraus, S. 1998. Methods for task allocation via agent coalition formation. *Artificial Intelligence* 101(1):165–200.

[Tang and Parker 2005] Tang, F., and Parker, L. E. 2005. ASyMTRe: Automated synthesis of multi-robot task solutions through software reconfiguration. In *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*, 1501–1508.

[Vig and Adams 2006] Vig, L., and Adams, J. 2006. Multi-robot coalition formation. *IEEE Transactions on Robotics* 22(4):637–649.

[Vig and Adams 2007] Vig, L., and Adams, J. 2007. Coalition Formation: From Software Agents to Robots. *Journal of Intelligent and Robotic Systems* 50(1):85–118.

[Walsh and Wellman 1998] Walsh, W. E., and Wellman, M. P. 1998. A market protocol for decentralized task allocation. In *Proceedings International Conference on Multi Agent Systems (Cat. No.98EX160)*, 325–332.

[Zhang and Parker 2012] Zhang, Y., and Parker, L. E. 2012. Task allocation with executable coalitions in multirobot tasks. In *2012 IEEE International Conference on Robotics and Automation*, 3307–3314.

[Zhang and Parker 2013a] Zhang, Y., and Parker, L. E. 2013a. Iq-asymtre: Forming executable coalitions for tightly coupled multirobot tasks. *IEEE Transactions on Robotics* 29(2):400–416.

[Zhang and Parker 2013b] Zhang, Y., and Parker, L. 2013b. Considering inter-task resource constraints in task allocation. *Autonomous Agents and Multi-Agent Systems* 26(3):389–419.