Scalar and Spinor Two-Point Functions in Einstein Universe

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Abstract

Two-point functions for scalar and spinor fields are investigated in Einstein universe \((R \otimes S^{N-1})\). Equations for massive scalar and spinor two-point functions are solved and the explicit expressions for the two-point functions are given. The simpler expressions for massless cases are obtained both for the scalar and spinor cases.

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It is important to investigate quantum field theories in curved space-time in connection with phenomena under strong gravity as in the early universe. The fundamental object in dealing with quantum field theories in curved space-time is the two-point Green function which we shall study in the present communication. There are numerous previous studies which have dealt with the two-point functions in the maximally symmetric space-time $S^N$ and/or $H^N$. In the present paper we push forward the investigation and study the Einstein universe.

We consider the manifold $R \otimes S^{N-1}$ as an Euclidean analog of the $N$-dimensional Einstein universe. The manifold is defined by the metric
\begin{equation}
\begin{aligned}
ds^2 &= dr^2 + a^2(d\theta^2 + \sin^2 \theta d\Omega_{N-2}),
\end{aligned}
\end{equation}
where $d\Omega_{N-2}$ is the metric on a unit sphere $S^{N-2}$. The manifold is a constant curvature space with curvature
\begin{equation}
R = (N - 1)(N - 2) \frac{1}{a^2}.
\end{equation}

We start with the argument on scalar two-point functions. On the manifold $R \otimes S^{N-1}$ the scalar two-point function is defined by the equation
\begin{equation}
\begin{aligned}
((\partial_0)^2 + \Box_{N-1} - \xi R - m^2) G_F(y(0), y) &= - \frac{1}{\sqrt{g}} \delta^N(y(0), y),
\end{aligned}
\end{equation}
where $g$ is the determinant of the metric tensor $g_{\mu\nu}$, $\Box_{N-1}$ is the Laplacian on $S^{N-1}$, $\delta^N(y(0), y)$ the Dirac delta function in the manifold $R \otimes S^{N-1}$ and $\xi$ the coupling constant between the scalar field and the curvature. In the following discussions we fix $y(0)$ at the origin and write $G_F(y(0), y) = G_F(y)$. After performing the Fourier transformation in the time variable,
\begin{equation}
\begin{aligned}
G_F(y) &= \int \frac{d\omega}{2\pi} e^{-i\omega y} \tilde{G}_F(\omega, y), \\
\delta^N(y) &= \int \frac{d\omega}{2\pi} e^{-i\omega y} \delta^{N-1}(y),
\end{aligned}
\end{equation}
We obtain from Eq.(3),
\begin{equation}
\begin{aligned}
\left(\Box_{N-1} - \xi R - (m^2 + \omega^2)\right) \tilde{G}_F(\omega, y) &= - \frac{1}{\sqrt{g}} \delta^{N-1}(y),
\end{aligned}
\end{equation}
where \( y \) denotes the space component of \( y \). Equation (5) has the same form as the one for scalar Green functions on \( S^{N-1} \). For the maximally symmetric space \( S^{N-1} \) we can easily solve the equation for the Green function following the method developed by Allen and Jacobson.\(^3\) The Green function \( \tilde{G}_F(\omega, y) \) is obtained straightforwardly. Because of the symmetry on \( R \otimes S^{N-1} \), \( \tilde{G}_F(\omega, y) \) is represented as a function only of \( \sigma = a\theta \) which is the geodesic distance between \( y(0) \) and \( y \) on \( S^{N-1} \). The Laplacian acting on a function of \( \sigma \) is found to be

\[
\Box_{N-1} f(\sigma) = \frac{1}{a^2} (\sin \theta)^{2-N} \frac{d}{d\theta} (\sin \theta)^{N-2} \frac{d}{d\theta} f(\sigma) \\
= \left( \partial_\sigma^2 + \frac{N-2}{a} \cot \left( \frac{\sigma}{a} \right) \partial_\sigma \right) f(\sigma).
\]  

(6)

Using Eq.(6) we rewrite the Eq.(5) to obtain

\[
\left( \partial_\sigma^2 + \frac{N-2}{a} \cot \left( \frac{\sigma}{a} \right) \partial_\sigma - \xi R - (m^2 + \omega^2) \right) \tilde{G}_F = 0,
\]  

(7)

where we restrict ourselves to the region \( \sigma \neq 0 \). To solve Eq.(7) we make a change of the variable \( z = \cos^2 \left( \frac{\sigma}{2a} \right) \) and find

\[
\left[ z(1-z) \partial_z^2 + \left( \frac{N-1}{2} - (N-1)z \right) \partial_z - (N-1)(N-2)\xi - \frac{(m^2 + \omega^2)a^2}{4} \right] \tilde{G}_F = 0.
\]  

(8)

Equation (8) is known as the hypergeometric differential equation. The solution of this equation is given by the linear combination of hypergeometric functions,\(^3\)

\[
\tilde{G}_F = qF \left( \frac{N-2}{2} + i\alpha, \frac{N-2}{2} - i\alpha, \frac{N-1}{2} ; z \right) + pF \left( \frac{N-2}{2} + i\alpha, \frac{N-2}{2} - i\alpha, \frac{N-1}{2} ; 1-z \right),
\]  

(9)

where we define \( \alpha \) by

\[
\alpha = \sqrt{\left( m^2 + \omega^2 \right)a^2 + (N-1)(N-2)\xi - \frac{(N-2)^2}{4}}.
\]  

(10)
As Green function $\tilde{G}_F$ is regular at the point $\sigma = a\pi$, we find that $p = 0$. To determine the constant $q$ we consider the singularity $\tilde{G}_F$ of $\tilde{G}_F$ in the limit $\sigma \to 0$

$$
\tilde{G}_F \to q \frac{\Gamma\left(\frac{N-1}{2}\right) \Gamma\left(\frac{N-3}{2}\right)}{\Gamma\left(\frac{N-2}{2} + i\alpha\right) \Gamma\left(\frac{N-2}{2} - i\alpha\right)} \left(\frac{\sigma}{2a}\right)^{3-N},
$$

and compare it with the singularity of the Green function in flat space-time. This procedure is justified because the singularity on a curved space-time background has the same structure as that in the flat space-time. For $\sigma \sim 0$ the Green function in the flat space-time behaves as

$$
\tilde{G}_F^{\text{flat}}(\sigma) \sim \frac{1}{4\pi^{(N-1)/2}} \Gamma\left(\frac{N-3}{2}\right) \sigma^{3-N}.
$$

Comparing Eq.(11) with Eq.(12) we obtain the constant $q$:

$$
q = \frac{a^{3-N}}{(4\pi)^{(N-1)/2}} \frac{\Gamma\left(\frac{N-2}{2} + i\alpha\right) \Gamma\left(\frac{N-2}{2} - i\alpha\right)}{\Gamma\left(\frac{N-1}{2}\right)}.
$$

From Eqs.(4),(9) and (13) we finally obtain the scalar two-point function $G_F$,

$$
G_F(y) = a^{3-N} \frac{(4\pi)^{(N-1)/2}}{\Gamma\left(\frac{N-2}{2} + i\alpha\right) \Gamma\left(\frac{N-2}{2} - i\alpha\right)} \frac{\Gamma\left(\frac{N-1}{2}\right)}{\Gamma\left(\frac{N-2}{2} + i\alpha\right) \Gamma\left(\frac{N-2}{2} - i\alpha\right)}
\times F\left(\frac{N-2}{2} + i\alpha, \frac{N-2}{2} - i\alpha, \frac{N-1}{2}; \cos^2 \left(\frac{\sigma}{2a}\right)\right).
$$

The two-point function (14) develops many singularities at $\sigma = 2\pi na$ where $n$ is an arbitrary integer. This property is a direct consequence of the boundedness of the space $S^{N-1}$. In other words the geodesic distance $\sigma$ is bounded in $[0, 2\pi a)$. Thus the two-point function (14) satisfies the periodic boundary condition $G_F(y^0, \sigma) = G_F(y^0, \sigma + 2\pi na)$. In the two dimensional limit $N \to 2$ the two-point function reduces to the well-known formula

$$
G_F(y) \to \frac{a}{2} \int \frac{d\omega}{2\pi} e^{-i\omega y^0} \frac{\cosh(\alpha (\pi - \sigma/a))}{\alpha \sinh(\alpha \pi)}
\times \sum_{n=\infty}^{\infty} \int \frac{d\omega}{2\pi} e^{-ipy} \frac{1}{p^2 + m^2},
$$

(15)
where $p^\mu$ and $y^\mu$ are given by
\[
\begin{align*}
  p^\mu &= \left(\omega, \frac{n}{a^2}\right), \\
  y^\mu &= (y^0, \theta) = \left(y^0, \frac{\sigma}{a}\right).
\end{align*}
\]
(16)

In deriving Eq.(15) we employ the following summation formula:
\[
\frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{\cos(nx)}{n^2 + a^2} = \frac{\cosh(a(\pi - x))}{\sinh(a\pi)}.
\]
(17)

Equation (15) is nothing but the ordinary two-point function with the periodic boundary condition for the spatial coordinate.

If $m = 0$ and $\xi = \frac{N-2}{4(N-1)}$, the field equation (3) is invariant under conformal transformation. In this case we can perform the Fourier integral in Eq.(14) at $y = y(0)$,
\[
G_{\text{conformal}}(y = y(0)) = \frac{2a^{2-N}}{(4\pi)^{(N+1)/2}} \Gamma \left(\frac{N-1}{2}\right) \Gamma \left(\frac{N}{2} - 1\right) \Gamma \left(1 - \frac{N}{2}\right).
\]
(18)

The two-point function at $y = y(0)$ obtained above is known to be useful in calculating the effective potential and studying the vacuum structure of the theory.

Next we consider the spinor two-point function on $R \otimes S^{N-1}$. The spinor two-point function $D$ is defined by the Dirac equation
\[
(\nabla + m)D(y) = -\frac{1}{\sqrt{g}}\delta^N(y).
\]
(19)

We introduce the bispinor function $G$ defined by
\[
(\nabla - m)G(y) = D(y).
\]
(20)

According to Eq.(19) $G(y)$ satisfies the following equation,
\[
(\nabla^\nabla - m^2)G(y) = -\frac{1}{\sqrt{g}}\delta^N(y).
\]
(21)

On $R \otimes S^{N-1}$ we rewrite Eq.(21) in the following form
\[
\left((\partial_0)^2 + \Box_{N-1} - \frac{R}{4} - m^2\right)G(y) = -\frac{1}{\sqrt{g}}\delta^N(y).
\]
(22)
where $\Box_{N-1}$ is the Laplacian on $S^{N-1}$. Performing the Fourier transformation

$$G(y) = \int \frac{d\omega}{2\pi} e^{-i\omega y^0} \tilde{G}(\omega, y),$$

(23)

we rewrite Eq.(22) in the form

$$\left( \Box_{N-1} - \frac{R}{4} - (m^2 + \omega^2) \right) \tilde{G}(\omega, y) = -\frac{1}{\sqrt{g}} \delta^{N-1}(y).$$

(24)

Equation (24) is of the same form as the one for the spinor Green function on $S^{N-1}$. According to the method developed by Camporesi, we can find the expression of $\tilde{G}(\omega, y)$. The general form of the Green function $\tilde{G}(\omega, y)$ is written as

$$\tilde{G}(\omega, y) = U(y)(g_{N-1}(\omega, \sigma) + g'_{N-1}(\omega, \sigma)n_i\gamma^i),$$

(25)

where $U$ is a matrix in the spinor indices, $g_{N-1}$ and $g'_{N-1}$ are scalar functions only of $\omega$ and $\sigma$, $n_i$ is a unit vector tangent to the geodesic $n_i = \nabla_i \sigma$, $\gamma^i$ is the Dirac gamma matrices and Roman index $i$ runs over the space components ($i = 1, 2, \cdots, N - 1$). Inserting Eqs.(25) into Eq.(24) we get

$$\left[ U \Box_{N-1}(g_{N-1} + g'_{N-1}n_i\gamma^i) + 2(\nabla_j U)\nabla^j (g_{N-1} + g'_{N-1}n_i\gamma^i) \right. + \left. (\Box_{N-1} U)(g_{N-1} + g'_{N-1}n_i\gamma^i) - \left( \frac{R}{4} + (m^2 + \omega^2) \right) U(g_{N-1} + g'_{N-1}n_i\gamma^i) \right] = 0,$$

(26)

where we restrict ourselves to the region $\sigma \neq 0$. To evaluate Eq.(26) we have to calculate the covariant derivative of $U$ and $n_i$. $U$ is the operator which makes parallel transport of the spinor at point $y^{(0)}$ along the geodesic to point $y$. Thus the operator $U$ must satisfy the following parallel transport equations:

$$\left\{ \begin{array}{l}
n^i \nabla_i U = 0, \\
U(y^{(0)}) = 1,
\end{array} \right.$$

(27)

Here we set

$$\nabla_i U \equiv V_i U.$$

(28)
From the integrability condition on $V_i$,

$$\nabla_i V_j - \nabla_j V_i - [V_i, V_j] = \frac{1}{a^2} \sigma_{ij},$$

(29)

and the parallel transport equation (27) we easily find that

$$V_i = -\frac{1}{a} \tan \left( \frac{\sigma}{2a} \right) \sigma_{ij} n^j,$$

(30)

where $\sigma_{\mu\nu}$ are the antisymmetric tensors constructed by the Dirac gamma matrices, $\sigma_{\mu\nu} = \frac{1}{4} [\gamma_{\mu}, \gamma_{\nu}]$. To find $V_i$ we have used the fact that the maximally symmetric bitensors on $S^{N-1}$ is represented as a sum of products of $n_i$ and $g_{ij}$ with coefficients which are functions only of $\sigma$. After some calculations we get the Laplacian acting on $U$

$$\Box_{N-1} U = -\frac{N - 2}{4a^2} \tan^2 \left( \frac{\sigma}{2a} \right) U.$$

(31)

The first and second derivatives of $n_i$ are also the maximally symmetric bitensors and found to be

$$\nabla_i n_j = \frac{1}{a} \cot \left( \frac{\sigma}{a} \right) (g_{ij} - n_i n_j),$$

$$\Box_{N-1} n_i = -\frac{1}{a^2} \cot^2 \left( \frac{\sigma}{a} \right) (N - 2) n_i.$$

Therefore Eq.(26) reads

$$\left( \partial_\sigma^2 + \frac{N - 2}{a} \cot \left( \frac{\sigma}{a} \right) \partial_\sigma - \frac{N - 2}{4a^2} \tan^2 \left( \frac{\sigma}{2a} \right) - \frac{R}{4} - (m^2 + \omega^2) \right) g_{N-1}$$

$$+ n_i \gamma^i \left( \partial_\sigma^2 + \frac{N - 2}{a} \cot \left( \frac{\sigma}{a} \right) \partial_\sigma - \frac{N - 2}{4a^2} \cot^2 \left( \frac{\sigma}{2a} \right) - \frac{R}{4} - (m^2 + \omega^2) \right) g'_{N-1} = 0.$$  

(33)

Since the two terms in Eq.(33) are independent, each term in the left-hand side of Eq.(33) has to vanish. We define the functions $h_{N-1}(\omega, \sigma)$ and $h'_{N-1}(\omega, \sigma)$ by $g_{N-1}(\omega, \sigma) = \cos \left( \frac{\sigma}{2a} \right) h_{N-1}(\omega, \sigma)$ and $g'_{N-1}(\omega, \sigma) = \sin \left( \frac{\sigma}{2a} \right) h'_{N-1}(\omega, \sigma)$ respectively and make a change of variable by $z = \cos^2 \left( \frac{\sigma}{2a} \right)$. We then find that Eq.(33) is rewritten in the form of two hypergeometric differential equations:

$$\left[ z(1 - z) \partial_z^2 + \left( \frac{N + 1}{2} - N z \right) \partial_z - \frac{(N - 1)^2}{4} - (m^2 + \omega^2) a^2 \right] h_{N-1}(\omega, z) = 0.$$  

(34)
\[
\left[ z(1 - z)\partial_z^2 + \left( \frac{N - 1}{2} - Nz \right) \partial_z - \frac{(N - 1)^2}{4} - (m^2 + \omega^2)a^2 \right] h'_{N-1}(\omega, z) = 0. \tag{35}
\]

Noting that the Green function is regular at the point \( \sigma = a\pi \) we write the solutions of Eqs. (34) and (35) by only one kind of the hypergeometric function

\[
h_{N-1}(\omega, z) = c_{N-1} F\left( \frac{N - 1}{2} + i\beta, \frac{N - 1}{2} - i\beta, \frac{N + 1}{2}; z \right), \tag{36}
\]

\[
h'_{N-1}(\omega, z) = c'_{N-1} F\left( \frac{N - 1}{2} + i\beta, \frac{N - 1}{2} - i\beta, \frac{N - 1}{2}; z \right), \tag{37}
\]

where we define \( \beta \) by

\[
\beta = a\sqrt{m^2 + \omega^2}. \tag{38}
\]

As we remained in the region where \( \sigma \neq 0 \) the normalization constants \( c_{N-1} \) and \( c'_{N-1} \) are yet undetermined. To obtain \( c_{N-1} \) and \( c'_{N-1} \) we consider the singularity of \( \tilde{G} \) in the limit \( \sigma \to 0 \),

\[
\tilde{G} \rightarrow c_{N-1} \frac{\Gamma \left( \frac{N + 1}{2} \right) \Gamma \left( \frac{N - 3}{2} \right)}{\Gamma \left( \frac{N - 1}{2} + i\beta \right) \Gamma \left( \frac{N - 1}{2} - i\beta \right)} \left( \frac{\sigma}{2a} \right)^{3-N} \frac{\left( \Gamma \left( \frac{N - 1}{2} \right) \right)^2}{\Gamma \left( \frac{N - 1}{2} + i\beta \right) \Gamma \left( \frac{N - 1}{2} - i\beta \right)} \left( \frac{\sigma}{2a} \right)^{3-N}, \tag{39}
\]

Comparing Eq. (39) with Eq. (12), the over-all factors \( c_{N-1} \) and \( c'_{N-1} \) are obtained :

\[
c_{N-1} = \frac{a^{3-N}}{(4\pi)^{(N-1)/2}} \frac{\Gamma \left( \frac{N - 1}{2} + i\beta \right) \Gamma \left( \frac{N - 1}{2} - i\beta \right)}{\Gamma \left( \frac{N - 1}{2} \right)} \tag{40}
\]

\[
c'_{N-1} = 0. \tag{41}
\]

Therefore \( g'_{N-1}(\omega, \sigma) \) disappears in the final expression of the Green function.
Eqs. (36) and (40) we find the expression of $\tilde{G}(\omega, y)$,

$$
\tilde{G}(\omega, y) = U(y)g_{N-1}(\omega, \sigma) = U(y)\frac{a^{3-N}}{(4\pi)^{(N-1)/2}} \frac{\Gamma\left(\frac{N-1}{2} + i\beta\right)\Gamma\left(\frac{N-1}{2} - i\beta\right)}{\Gamma\left(\frac{N+1}{2}\right)} \times \cos\left(\frac{\sigma}{2a}\right) F\left(\frac{N-1}{2} + i\beta, \frac{N-1}{2} - i\beta, \frac{N+1}{2}; \cos^2\left(\frac{\sigma}{2a}\right)\right).
$$

Thus the Green function $G(y)$ on $R \otimes S^{N-1}$ is obtained.

The spinor two-point function $D(y)$ is derived from the Green function $G(y)$. Inserting Eqs. (23) and (25) into Eq. (20) we get

$$
D(y) = \int \frac{d\omega}{2\pi} e^{-i\omega y^0} (-i\omega \gamma^0 + \gamma^i \nabla_i - m) U g_{N-1}
= \int \frac{d\omega}{2\pi} e^{-i\omega y^0} \left[ \gamma_i n^i U \left( \partial_\tau - \frac{N-2}{2a} \tan\left(\frac{\sigma}{2a}\right) \right) g_{N-1} - (i\omega \gamma^0 + m) U g_{N-1} \right].
$$

Substituting Eq. (42) in Eq. (43) the spinor two-point function $D(y)$ is obtained

$$
D(y) = -\frac{a^{3-N}}{(4\pi)^{(N-1)/2}} \int \frac{d\omega}{2\pi} e^{-i\omega y^0} \frac{\Gamma\left(\frac{N-1}{2} + i\beta\right)\Gamma\left(\frac{N-1}{2} - i\beta\right)}{\Gamma\left(\frac{N+1}{2}\right)} \times \left[ (i\omega \gamma^0 + m) U \cos\left(\frac{\sigma}{2a}\right) F\left(\frac{N-1}{2} + i\beta, \frac{N-1}{2} - i\beta, \frac{N+1}{2}; \cos^2\left(\frac{\sigma}{2a}\right)\right) \right]
+ \gamma_i n^i U \frac{N-1}{2a} \sin\left(\frac{\sigma}{2a}\right) F\left(\frac{N-1}{2} + i\beta, \frac{N-1}{2} - i\beta, \frac{N+1}{2}; \cos^2\left(\frac{\sigma}{2a}\right)\right).\tag{44}
$$

According to the anticommutation relation of spinor fields the two-point function (44) satisfies the antiperiodic boundary condition $D(y^0, \sigma) = -D(y^0, \sigma + 2\pi na)$ where $n$ is an arbitrary integer. In the two dimensional limit $N \to 2$ the two-point function (44) simplifies:

$$
D(y) \to -\frac{a}{2} \int \frac{d\omega}{2\pi} e^{-i\omega y^0} \left[ (i\omega \gamma^0 + m) \sinh(\beta(\pi - \sigma/a)) \right] + \frac{a^3}{\cosh(\beta\pi)} \frac{\cosh(\beta(\pi - \sigma/a))}{\sinh(\beta\pi)}
= \frac{1}{2\pi a} \sum_{n=-\infty}^{\infty} \int \frac{d\omega}{2\pi} e^{-ipy} \frac{1}{i(py - m)}.\tag{45}
$$
where \( p^\mu \) and \( y^\mu \) are defined by

\[
\begin{align*}
p^\mu &= \left( \omega, \frac{2n-1}{2a^2} \right), \\
y^\mu &= \left( y^0, \frac{\sigma}{a} \right).
\end{align*}
\]  

To obtain Eq.\( (45) \) the following formulae were employed,

\[
\begin{align*}
\frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{\cos((2n-1)x)}{(2n-1)^2 + (2a)^2} &= \frac{\sinh(a(\pi - 2x))}{4a \cosh(a\pi)}, \\
\frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{(2n-1) \sin((2n-1)x)}{(2n-1)^2 + (2a)^2} &= \frac{\cosh(a(\pi - 2x))}{2 \cosh(a\pi)}.
\end{align*}
\]  

We easily see that the Eq.\( (45) \) is identical to the well-known two-point function of spinor with antiperiodic boundary condition for the spatial coordinate.

It should be noted that \( \text{tr} D(y = y(0)) \) is required in evaluating the effective potential:

\[
\text{tr} D(y = y(0)) = -\frac{\text{tr} \mathbf{1} a^{3-N}}{(4\pi)^{(N-1)/2}} \Gamma \left( \frac{3-N}{2} \right) \int \frac{d\omega}{2\pi} \frac{\Gamma \left( \frac{N-1}{2} + i\beta \right) \Gamma \left( \frac{N-1}{2} - i\beta \right)}{\Gamma (1 + i\beta) \Gamma (1 - i\beta)}.
\]  

For the massless case we can perform the Fourier integral in Eq.\( (48) \) and get

\[
\frac{\text{tr} D(y = y(0), m = 0)}{m} \longrightarrow -\frac{2\text{tr} \mathbf{1} a^{2-N}}{(4\pi)^{(N+1)/2}} \Gamma \left( \frac{N}{2} \right) \Gamma \left( \frac{N-1}{2} \right) \Gamma \left( 1 - \frac{N}{2} \right).
\]  

In the present paper we calculated scalar and spinor two-point functions on \( R \otimes S^{N-1} \). In the final expression for the scalar and spinor two-point functions, Eqs.\((14)\) and \((44)\), the Fourier integral is remained. For some cases we can perform the Fourier integral in Eqs.\((14)\) and \((44)\) at \( y = y(0) \). The resulting explicit expressions may be useful to investigate the vacuum structure of the Einstein universe.

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