Abstract

We present a stochastic theory for the nonequilibrium dynamics of charges moving in a quantum scalar field based on the worldline influence functional and the close-time-path (CTP or in-in) coarse-grained effective action method. We summarize (1) the steps leading to a derivation of a modified Abraham-Lorentz-Dirac equation whose solutions describe a causal semiclassical theory free of runaway solutions and without pre-acceleration pathologies, and (2) the transformation to a stochastic effective action which generates Abraham-Lorentz-Dirac-Langevin equations depicting the fluctuations of a particle’s worldline around its semiclassical trajectory. We point out the misconceptions in trying to directly relate radiation reaction to vacuum fluctuations, and discuss how, in the framework that we have developed, an array of phenomena, from classical radiation and radiation reaction to the Unruh effect, are interrelated to each other as manifestations at the classical, stochastic and quantum levels. Using this method we give a derivation of the Unruh effect for the spacetime worldline coordinates of an accelerating charge. Our stochastic particle-field model, which was inspired by earlier work in cosmological backreaction, can be used as an analog to the black hole backreaction problem describing the stochastic dynamics of a black hole event horizon.

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1 Introduction

We give in this paper a summary of our research program (commenced in [1]) on the quantum statistical dynamics of relativistic particles moving in a quantum field. The relativistic particles are not coupled to each other explicitly but only through their interactions with a quantum field. Because of this we need to take into consideration both the effect of each particle on the quantum field and the backreaction of the quantum field on each particle. A self-consistent treatment of the particle-field system dynamics is thus necessary. The backreaction from the field engenders nonlinear coupling amongst the particles, and non-Markovian dynamics. We present a new approach to this problem which highlights the stochastic effects of noise, decoherence, dissipation, fluctuations, and correlations, and their interconnections.

In [2] we presented the basic framework built on the concepts of quantum open systems [3], the model of quantum Brownian motion (QBM) [4], and the methodologies of the influence functional [5], closed-time-path (in-in) [6] coarse-grained effective action [7, 8], and world line quantization [9]. In [10] we applied this framework to spinless relativistic moving particles in a quantum scalar field, and derived the stochastic equations of motion, known as the Abraham-Lorentz-Dirac-Langevin (ALDL) equations, for their trajectories with self-consistent backreaction. The quantum fluctuations of the field are partially encoded as a...
stochastic noise in the ALDL equations. The mean trajectory obtained from taking the stochastic average is governed by Abraham-Lorentz-Dirac (ALD) equations with modified coefficients whose time-dependence enforce causality.

Here we use the approach developed in [10] to analyze a uniformly accelerated particle where the trajectory is self-consistently determined by its interaction with a quantum field (this includes the effects of radiation reaction and vacuum fluctuations). This important example illustrates the interconnections between radiation, radiation reaction, dissipation, quantum fluctuations, and the Unruh effect. Specifically, the uniform acceleration alters the quantum vacuum which in turn induces thermal-like fluctuations in the trajectory of a particle. This manifestation of the Unruh effect in terms of fluctuating trajectories is in contrast to the original formulation and more common derivations [11], where a particle with internal degrees of freedom (called a detector in the literature [11, 12]) moving on a uniformly accelerating prescribed trajectory feels hot at the Unruh temperature $T_U = \hbar a / 2\pi c k_B$.

Before delving into the technical details, in the remainder of this section we discuss the outstanding physical issues touched on in this work. In Sec. 2 we describe the relation of classical radiation, radiation reaction, vacuum fluctuations and quantum radiation, correcting the misconception of directly linking vacuum fluctuations with radiation reaction. In Sec. 3 we present the main structure of our approach based on the open system concept and the coarse-grained effective action techniques. We show the steps leading to a derivation of the ALD-Langevin equation for charges moving in a quantum field. In Sec. 4 we specialize to the case of a particle moving in a background field which imparts to it uniform acceleration on the average, and derive the Unruh effect. We discuss how this example can serve as an analog for studying the stochastic dynamics of black hole event horizons. In Sec. 5 we step back for a broader perspective and give some discussions of a) the special features of this new approach and its potential in overcoming the stumbling blocks present in existing approaches; and b) applications of these results and directions for further development.

1.1 Quantum, Stochastic, Semiclassical and Classical

In this program of investigation we take a microscopic approach and an open-systems perspective, using quantum field theory as the tool to provide a first-principles derivation of moving particles interacting with a quantum field. We begin with the closed system of quantum particles and fields. A closed system can be meaningfully partitioned into subsystems if there exists discrepancies in the scales describing each subsystem, or in accordance to the physical scales present in their interaction strengths (energy or time scales) in relation to the probing scale or resolution accuracy. If we are interested in the details of one such subsystem (e.g., particle dynamics) and decide to ignore certain details of the other subsystems (e.g., details in the quantum field configurations, such as correlations and phase information) comprising the environment, the distinguished subsystem is rendered an open system. The overall effect of the coarse-grained environment on the open-system can be captured by the influence functional technique of Feynman and Vernon [5], or the closely related closed-time-path (CTP) effective action method of Schwinger and Keldysh [6]. These are the initial value, or in-in, formulations suitable for following the time evolution of a system, in contradistinction to the usual in-out (Schwinger-DeWitt) effective action formulation useful for S-matrix calculations.

For the model of particle-field interactions under study, coarse-graining the quantum field yields a nonlocal coarse-grained effective action (CGEA) for the particle motion [7, 8]. An exact expression may be found in the special case that the particle and field are initially uncorrelated, and the field state is Gaussian. The CGEA may be used to treat the fully nonequilibrium quantum dynamics of interacting particles. However, when higher-order quantum corrections are suppressed by decoherence, the influence of the quantum fluctuations of the field may be encoded as stochastic noise, and the CGEA can be transcribed into a stochastic effective action [13, 14] describing the stochastic particle motion. Upon further coarse-graining, taking the stochastic average gives the semiclassical particle trajectory.

Let us analyze the physics of nonequilibrium processes for such particle-field systems at separate levels. **Classical** level: From the complete quantum microscopic description of the system (particle) and the environment (field), a classical description is reached when the system and environment are coarse-grained to an extent that both the particle and field histories are fully decohered [15, 16]. The fully coarse-grained
trajectory for the particle with backreaction from the fully coarse-grained field then obeys classical equations of motion, such as the ALD equation. The coarse-graining length scale necessary to achieve full classicality typically far exceeds the length scale for backreaction pathologies like preacceleration and runaway solutions.

**Semiclassical** level: This is often defined as a classical system (particles or detectors) interacting with a quantum environment (quantum field). It can be obtained from the complete quantum microscopic description by fully coarse-graining the environment (the quantum field) and finding the quantum average of the particle trajectory. In the special but important class when the field enters linearly in the particle-field interaction term, and without field self-interactions, coarse graining over the quantum field gives an exact CGEA. The CGEA yields the equations of motion for the system which in the leading order approximation is the semiclassical limit, followed by higher-order quantum corrections. In this particular situation, if decoherence can sufficiently suppress the nonlinear quantum corrections to the semiclassical equations of motion from the particle, then the semiclassical limit from the CGEA agrees with the classical limit.

**Stochastic** level: Going beyond a mean-trajectory (or fully coarse-grained) description of the particle, a stochastic component in the particle trajectory appears, as induced by the quantum field fluctuations manifesting as a classical stochastic noise counterbalancing the quantum dissipation in the system dynamics. The CGEA may be transcribed into a stochastic effective action, encoding much of the quantum statistical information of the field and the state of motion of the system in a noise correlator for the particle. The stochastic effective action yields classical stochastic equations of motion for the system which embodies a quantum dissipation effect (*over and above* the classical radiation reaction) that balances the quantum fluctuations via a fluctuation-dissipation relation (FDR).

1.2 Paradoxes and pathologies of backreaction

1.2.1 The ALD equations

The classical theory of moving charges interacting with a classical electromagnetic (EM) field, when the backreaction of the EM field is included, has many controversial difficulties [17]. The generally accepted classical equation of motion for a charged, spinless point particle including radiation-reaction is the Abraham-Lorentz-Dirac (ALD) equation [18]:

\[
\ddot{z} + \tau_0 (\dot{z} \dddot{z} + \dddot{z}) = \left(\frac{e}{mc}\right) \dot{z} F_{\text{ext}}(z).
\]

(See Sec. II for notations and definitions.) The timescale \( \tau_0 = \left(\frac{2e^2}{3mc^3}\right) \) determines the relative importance of the radiation-reaction term. For electrons, \( \tau_0 \sim 10^{-24} \text{ secs} \), which is the time it takes light to cross the electron classical radius \( r_0 \sim 10^{-15} \text{ m} \).

The ALD equation is pathological. Because it is a third order differential equation, it requires the specification of the initial acceleration in addition to the usual position and velocity required by second order differential equations. This leads to the existence of runaway solutions. Physical (e.g. non-runaway) solutions may be enforced by transforming Eq. (1) to a second order integral equation with boundary condition such that the final energy of the particle is finite and consistent with the total work done on it by external forces. But the removal of runaway solutions yields acausal solutions that pre-accelerate on timescales \( \tau_0 \). This is a source of lingering questions on whether the classical theory of point particles and fields is causal.

There have been many efforts to understand charged particle radiation-reaction in the classical and quantum theory. To satisfy the self-consistency and causality requirements different measures are introduced to the underlying model. Examples include imposing a high-energy cutoff for the field, an extended charge distribution, special boundary conditions, particle spin, and the use of perturbation theory or order reduction techniques [19]. Previous work closely related to ours include those on quantum Langevin equations [20] [21], and the application of the Feynman-Vernon influence functional to non-relativistic particle coherence [22] and stochastic gravity [23]. A major distinction of our work is the focus on relativistic systems, non-equilibrium processes, and nonlinear particle-field interaction.
1.2.2 Coarse-graining and causality

Consider the situation where a localized particle at rest has some quantum fluctuations in its position just before an external force is applied at time $t_i$, and where, by chance, the fluctuation is one that seems to anticipate the force creating the appearance of preacceleration. Or, consider a situation where a quantum fluctuation produces the appearance of runaway acceleration for some period of time. While such specific fluctuations may be highly improbable to realize, they are among the set of fine-grained solutions in a sum-over-histories formulation of quantum mechanics. These examples illustrate the observational challenges to distinguishing cause and effect on a fine-grained quantum scale.

Why is there such a difficulty? The underlying microscopic theory may be consistent (though for field theory this may not be obvious) and causal, in the sense that the operator equations of motion are causal, but the quantum state (for particle or field) is inherently nonlocal. Observables (expectation values) of the particle motion involve both the operator equations of motion and the nonlocal quantum state, and thus the question of causality for trajectories requires more careful treatment. Operationally, we do not observe fine-grained histories; instead we observe coarse-grained histories where the coarse-graining scale is set by the measurement resolution in both time and space. With coarse graining, the quantum fluctuations giving highly nonclassical trajectories are suppressed, and we therefore expect that a “causal and consistent” quantum theory should yield, upon suitable coarse graining, a classical or semiclassical limit that is pathology free. This is an oversimplified sketch of a more elaborate theory [24] based on the decoherent history approach to quantum mechanics [16].

1.3 Uniform acceleration

1.3.1 Radiation and radiation reaction

A particularly interesting class of dynamics from the solutions to the ALD equation is that of a uniformly accelerated particle, defined by $\ddot{z}^\mu \dot{z}_\mu = a^2$. This class of dynamics exemplifies many of the important issues under discussion. First, it follows from the ALD equations that the classical radiation reaction force vanishes despite the existence of classical (Larmor) radiation registered as an energy flux at infinity. Hence, there is no direct balance between radiation and radiation reaction. There is one existing belief that since radiation is associated with radiation reaction, the extra work done on the particle against the radiation reaction force must directly provide the energy that goes into radiation, but this is a static viewpoint that neglects the full interplay of particle, near field (the so-called acceleration or Shott field), and far field (the radiation field) dynamics. Fields are dynamical entities with unusual properties (such as nonlocality and field correlations) much more complex than just radiation, which is a far-field definition, and energy can be attributed to a variety of sources other than radiation. For example, the acceleration field is known to contain energy and do work. One cannot simply equate work done against radiation reaction forces with the energy (“instantaneously”) radiated into infinity. It would require a ‘freezing out’ of the near-field’s ability to exchange or adjust the form of its energy consistently in time to be commensurate with the far field behavior known as radiation.

During periods of uniform acceleration, the energy “transfer” into radiation comes from acceleration fields, leaving zero radiation reaction. This is a special situation. During periods of nonuniform acceleration, there is radiation reaction, and one finds a mixture of field components; but even then, the energy apportioned to radiation cannot be instantaneously ascribed to work done against radiation reaction. On matters related to fields one should look carefully at the energy content locally and guard against making global statements.

1.3.2 Quantum radiation, vacuum fluctuations, and the Unruh effect

It has long been known that a uniformly accelerated detector (UAD) (a detector is defined as a particle with some internal degree of freedom) coupled to a quantum field following a prescribed trajectory registers its quantum field vacuum as a thermal state with temperature $T_U = \frac{ah}{2\pi k_B}$. This is the Unruh effect [11], often cited as an analog of Hawking effect [25]. But there are important differences: In the case of
a black hole, real radiation of thermal nature is emitted at the Hawking temperature. For a uniformly
accelerated detector (UAD) there is no emitted radiation associated with the Unruh effect (see, e.g., [26, 27]
and references therein) except for a transient period when the charge is coming into equilibrium with its
environment [28, 29]. For a uniformly accelerated charge (UAC) there is of course classical radiation, but it
is different from the Unruh radiation, which is a distinctly quantum effect.

We demonstrate below that for a uniformly accelerated charge interacting with a quantum field, the
vacuum fluctuations induce stochastic fluctuations in the particle trajectory with a thermal character. We
can refer to this as the Unruh effect for charges, the role of the “detector” is played by the particle motion
itself rather than an internal degree of freedom. In the semiclassical limit (neglecting quantum corrections
from the particle) the average emitted radiation is the usual classical radiation: there is no additional net
“quantum” radiation from the particle. What is new and interesting in our finding is that there could be
radiation associated with the stochastic component of the particle trajectory. For instance, these could
conceivably generate fluctuations in the emitted radiation, perhaps with a thermal character, or they could
produce other non-classical correlations in the radiation. This requires further investigation. It has been
shown that for a uniformly accelerated detector on a fixed trajectory the field correlation is altered around
the detector, but as these altered correlations fall off faster than radiation they can be considered as a
vacuum polarization effect [26, 30, 31]. Similar vacuum polarization effects are also expected for a moving
charge (contrast this prediction with [32]). Finally, the radiation emitted by an evaporating (shrinking) black
hole under non-equilibrium conditions is expected to have a non-thermal component [28] over and above the
usual thermal (Hawking) part. We are using the analog with non-uniformly accelerated detectors or charges
to investigate this issue [28].

2 Semiclassical limit

2.1 Coarse-grained effective action

We begin with the full microscopic theory of a spinless charged particle interacting with a scalar field. The
action for the closed system of particle plus field is

\[ S[z, \varphi, h] = S[z, h] + S[\varphi] + S[z, \varphi], \]  

(2)

where

\[ S[z, h] = \int d\tau \left( m_0 \sqrt{\dot{z}^\mu \dot{z}_\mu + h_\mu(\tau) \dot{z}^\mu(\tau)} \right) \]  

(3)

\[ S[\varphi] = \int d^4x \frac{1}{2} (\partial_\mu \varphi)^2 \]  

(4)

\[ S[z, \varphi] = \int d^4x j(x; z)(\varphi(x) + \varphi_{\text{ext}}(x)). \]  

(5)

The spacetime worldline coordinates of the particle are \( z^\mu(\tau), \varphi(x) \) is the scalar field, and \( \varphi_{\text{ext}}(x) \) is an
external field. The functions \( h_\mu(\tau) \) are auxiliary sources introduced to generate correlation functions, after
which they are set to zero. The field couples to the particle through the reparametrization invariant scalar
current

\[ j(x; z) = e \int d\tau \sqrt{\dot{z}^\mu \dot{z}_\mu} \delta(x - z(\tau)). \]  

(6)

The action \( S[z, \varphi, h] \) is manifestly invariant under reparametrizations \( \tau \to \tau' = \tau + \varepsilon(\tau) \). The particle
mass is \( m_0 \) and charge is \( e \). We use units where \( \hbar = c = 1 \), Greek indices \( \mu, \nu, ... = (0, 1, 2, 3) \), and
\( g_{\mu \nu} = (1, -1, -1, -1) \).

Consider an initial density matrix for the particle plus field at time \( t_i \) that has the factorized form

\[ \hat{\rho}_i = \hat{\rho}_z(t_i) \otimes \hat{\rho}_\varphi(t_i). \]  

(7)
where \( \hat{\rho}_z (t_i) \) is the initial density matrix for the particle subsystem \((z)\) and \( \hat{\rho}_\phi (t_i) \) is the initial density matrix for the field subsystem \((\phi)\). We note at the start that strictly speaking such factorized states are unphysical because they represent a completely decoupled particle and field. More general initial states could be considered, but treating a fully physical state remains an important challenge [33]. For \( \hat{\rho}_\phi (t_i) \) we assume that the initial field state is Gaussian (this includes thermal, squeezed, and coherent states, and therefore provides us with a fairly rich set of interesting and physical examples.) A relativistic covariant particle state \(|z\rangle \equiv \hat{\psi}^{(+)} (z) |0\rangle\), where \( \hat{\psi}^{(+)} (z) \) is the positive frequency operator for creating a field excitation centered at \( z \), has a localization size (in terms of the charge-density expectation value) of a Compton wavelength \( \lambda_c = h/\text{mc} \), imparting a natural minimum coarse-graining scale. Typical particle wavepackets have much larger (characteristic) de Broglie wavelengths, and thus require considerable further averaging of fine-grained trajectories to achieve a coarse-grained decoherent history. We assume that the initial particle state is a wavepacket localized around \( z_0 \) with width \( \Lambda \gg \lambda_c \). We will see that the degree of coarse graining does (weakly) effect the semiclassical trajectory via higher derivative terms that can generally be neglected. (Note that both \( \Lambda \) and \( \lambda_c \) are far larger than the classical radius of a charged particle \( r_0 \), the scale at which pathological backreaction behavior manifests).

For the closed system, the density matrix at a later time \( t_f \) is

\[
\hat{\rho} (t_f) = \hat{U} (t_f, t_i) \hat{\rho}_z (t_i) \hat{U}^\dagger (t_f, t_i),
\]

where \( \hat{U} \) is the unitary evolution operator for the fully interacting particle plus field system. The matrix elements of \( \hat{U} \) can be given a sum over histories representation:

\[
K_h (\varphi_f, z_f, t_f; \varphi_i, z_i, t_i) \equiv \langle \varphi_f, z_f | \hat{U}_h (t_f, t_i) | \varphi_i, z_i \rangle = \int_0^{\varphi_f} \int_{z_i}^{z_f} D\varphi Dz \exp \{i S [z, \varphi, h]\},
\]

where the functional measures are

\[
\int_0^{\varphi_f} D\varphi = \int \prod_x d\varphi (x),
\]

\[
\int_{z_i}^{z_f} Dz = \int \prod_{\mu, \tau} dz^\mu (\tau),
\]

subject to appropriate boundary conditions that are discussed in [10]. The in-in or closed-time-path (CTP) generating functional is given by

\[
Z_{\text{in-in}} [h, h'] = \text{Tr}_{\varphi, z} \left[ \hat{U}_h (t_f, t_i) \hat{\rho}_z \hat{U}^\dagger_h (t_f, t_i) \right] = \int d\varphi d\varphi' d\varphi'' d\varphi''' d^3z d^3z' \times K_h (\varphi_f, z_f, t_f; \varphi_i, z_i, t_i) K^*_h (\varphi_f, z_f, t_f; \varphi_i', z_i', t_i) \times \rho (\varphi_i, z_i; \varphi_i', z_i'; t_i).
\]

This generating functional is a tool for computing the average particle trajectory and its fluctuations (i.e. the correlation functions for the particle worldline coordinates) including the effects of backreaction from the coarse-grained field self-consistently.

Using the assumed initially factorized state and the sum-over-histories representation, the generating functional may be written as

\[
Z_{\text{in-in}} [h, h'] = \int Dz Dz' e^{i \{ S[z, h] - S[z', h'] \}} F [z, z'],
\]

where \( F [z, z'] \) is a functional of the particle worldline coordinates. We will see that the degree of coarse graining does (weakly) effect the semiclassical trajectory via higher derivative terms that can generally be neglected. The in-in or closed-time-path (CTP) generating functional is given by

\[
Z_{\text{in-in}} [h, h'] = \text{Tr}_{\varphi, z} \left[ \hat{U}_h (t_f, t_i) \hat{\rho}_z \hat{U}^\dagger_h (t_f, t_i) \right] = \int d\varphi d\varphi' d\varphi'' d\varphi''' d^3z d^3z' \times K_h (\varphi_f, z_f, t_f; \varphi_i, z_i, t_i) K^*_h (\varphi_f, z_f, t_f; \varphi_i', z_i', t_i) \times \rho (\varphi_i, z_i; \varphi_i', z_i'; t_i).
\]
where
\[ F[z, z'] = \int D\varphi D\varphi' e^{i\left[ S[\varphi]+S[z, \varphi]-S[\varphi'-S[z', \varphi']\right]} \rho_i(\varphi_i, \varphi'_i, t_i) \]
\[ \equiv \exp \left\{ iS_{IF}[z, z'] \right\} \]
(14)
is the Feynman-Vernon influence functional, and \( S_{IF}[z, z'] \) is the influence action. From the definition of the in-in generating functional it follows that
\[ \left( \frac{1}{i} \right)^n \frac{\delta^n Z_{in-in}[h, h']}{\delta h_\mu(\tau_n) \ldots \delta h_\mu(\tau_1)} \bigg|_{h, h'} = \text{Tr}_{\varphi, z}(\hat{z}_\mu(\tau_n) \ldots \hat{z}_\nu(\tau_1) \hat{\rho}_i). \]
(15)
Hence, we see that \( Z_{in-in} \) is indeed a generating function for the expectation values of the worldline coordinates at an arbitrary proper time \( \tau_i \).

Evaluating the full generating functional is virtually impossible. The interaction term \( S[z, \varphi] \), while linear in \( \varphi \), is highly non-linear in the worldline coordinate \( z \). However, if the initial state of the field \( \rho_z(t_i) \) is Gaussian, the influence functional involves purely Gaussian integrands in the variables \( \varphi \), and thus can be evaluated exactly, given the well-known result (see e.g., [28])
\[ S_{IF} = \int d^4x d^4x' \left[ j^{(-)}(x) G_R(x, x') j^{(+)}(x') + \frac{i}{4} j^{(-)}(x) G_H(x, x') j^{(-)}(x') \right], \]
(16)
where
\[ G_R(x, x') = \text{Tr}_{\varphi} \left( i \left[ \hat{\varphi}(x), \hat{\varphi}(x') \right] \hat{\rho}_A(t_i) \right) \theta(t, t'), \]
(17)
\[ G_H(x, x') = \text{Tr}_{\varphi} \left( \left[ \hat{\varphi}(x), \hat{\varphi}(x') \right] \hat{\rho}_\phi(t_i) \right) \]
(18)
are the retarded and Hadamard Green’s functions, respectively. The brackets \( [\cdot, \cdot] \) and \( \{\cdot, \cdot\} \) denote commutators and anticommutators. The sum and difference currents \( j^{(\pm)} \) are defined by
\[ j^{(-)}(x; z, z') = (j(x) - j'(x)) \]
(19)
\[ j^{(+)}(x; z, z') = (j(x) + j'(x))/2. \]
(20)
We also define
\[ z^{(-)}(\tau) = (z(\tau) - z'(\tau)) \]
(21)
\[ z^{(+)}(\tau) = (z(\tau) + z'(\tau))/2, \]
(22)
and
\[ h^{(-)}(\tau) = (h(\tau) - h'(\tau)) \]
(23)
\[ h^{(+)}(\tau) = (h(\tau) + h'(\tau))/2. \]
(24)
The in-in generating functional can be compactly expressed as
\[ Z_{in-in}[h^{(\pm)}] = \int Dz^{(+)} Dz^{(-)} e^{iS_{CGEA}[z^{(\pm)}, h^{(\pm)}]}, \]
(25)
where
\[ S_{CGEA}[z^{(\pm)}, h^{(\pm)}] = S_z[z, h] - S_z[z', h'] + S_{IF}[z^{(\pm)}] \]
(26)
is the in-in or CTP coarse-grained effective action (CTPCGEA). We emphasize that \( S_{CGEA} \) encapsulates the full effects of the coarse-grained quantum field, and \( Z_{in-in}[h^{(\pm)}] \) generates, in principle, the full information about the quantum dynamics of the particle. The semiclassical description of the particle dynamics is
given by the particle trajectory’s expectation value $\bar{z}^\mu = \langle \hat{z}^\mu (\tau) \rangle$, neglecting particle quantum corrections, obtained from the equations of motion

$$
\frac{\delta S_{CGEA}}{\delta \bar{z}^{-1}_\mu (\tau)} \bigg|_{z(\tau)=0,z(\tau)=\bar{z}} = 0.
$$

Upon setting $z(\tau) = 0$ the imaginary part of $S_{CGEA}$, which involves the Hadamard function, gives vanishing contribution. Consequently, the semiclassical equations of motion depends only on the retarded green’s function and are therefore real and causal.

### 2.2 Modified ALD equation with time-dependent coefficients

The semiclassical trajectory is a solution of the equation derived from Eq. (27)

$$
m_0 \ddot{z}_\mu = \frac{e^2}{c^3} \int d\tau' \bar{w}^\mu G^R_\tau (z(\tau), z(\tau')) + e\bar{w}_\mu \varphi_{ext}(z),
$$

where

$$
\bar{w}^\mu \equiv \dot{z}^\nu \bar{z}[\mu \bar{\nabla}] - \dot{\bar{z}}^\mu.
$$

The external potential $\varphi_{ext}$ provides an external force $F^\mu_{\text{ext}} = e\bar{w}^\mu \varphi_{ext}$ with two main components: a gradient force $e\dot{z}^\nu \bar{z}[\mu \bar{\nabla}] \varphi_{ext} = e(\bar{g}^\mu_{\nu} - \dot{\bar{z}}^\nu) \partial^\nu \varphi_{ext}$, and an effective contribution to the particle mass, $e\varphi_{ext} \dot{\bar{z}}^\mu$. Note that $\dot{\bar{z}}^\mu \bar{w}_\mu = \dot{\bar{z}}^\mu \bar{z}_\mu = 0$ identically, and therefore $\dot{\bar{z}}^\mu F^\mu_{\text{ext}} = 0$, as required of a relativistic force that preserves the mass-shell constraint $\dot{\bar{z}}^2 = 1$. The scalar field force $F^\mu_{\text{ext}}$ is analogous to the electromagnetic force $F^\mu_{EM} = \vec{\nabla} F_\mu$, which satisfies $\dot{\bar{z}}^\mu F^\mu_{EM} = 0$.

The retarded Green’s function is singular when $\tau = \tau'$, and so requires regularization. We choose

$$
G^R_{\Omega} (\sigma) = \frac{\Omega J_1 (\Omega \sigma)}{4\pi \sigma},
$$

where $J_1$ is a first order Bessel function and $\sigma^2 = (z - z')^\mu (z - z')_\mu$. In the limit $\Omega \rightarrow \infty : G^R_{\Omega} \rightarrow \delta (\sigma^2)$. Consistency requires that the Hadamard Green’s function is regulated with the same effective cutoff: $G_{\Omega} \rightarrow G^R_{\Omega}$. We assume that the high-frequency mode cutoff $\Omega$ comes from the limit of the measurement resolution or the preparation scale for the initial particle state, so that $\Omega \sim 1/\Lambda$. It is reasonable to assume (though this argument needs to be made rigorous) that on finer-grained scales the correlations of the particle and field are undisturbed by the motion of the particle. This suggests viewing the initial factorized state as representing a particle that, through the initial state preparation process at time $t_i$, is uncorrelated only with the longer wavelength modes of field, while it remains fully dressed by the (undisturbed) short wavelength modes. The regulated Green’s function is therefore a model for the response of the long wavelength field modes to the particle motion, rather than a short wavelength (high energy) modification of the underlying quantum field theory.

Using our explicit choice for $G^R_{\Omega} (\sigma)$, the resulting equations of motion are

$$
m(r) \ddot{z}_\mu (\tau) = e\bar{w}^\mu \varphi_{ext} + \frac{\tau_0}{2} J_1^{(0)} (r) \left( \bar{z}^2 \dot{z}^\mu \bar{z}^\mu - \dot{\bar{z}}^\mu \dot{\bar{z}}^\mu \right) + \frac{\tau_0}{2} \sum_{n=1}^{\infty} \frac{J_1^{(n)} (r)}{\Omega^n} \mu^{(n)}_\mu ,
$$

where $r = \tau - \tau_i$ is the elapsed proper-time from the initial time $\tau_i = \tau_i (t_i)$ [Recall that $\tau_0 = (2e^2/3mc^3)$]. The time-dependent radiation reaction term proportional to $\Omega \bar{z}_\mu$ has been absorbed into a time-dependent mass $m(r)$. At $r = 0$, corresponding to the initial time when the field and particle are uncorrelated, $m(0) = m_0$ ($m_0$ is the bare mass which includes the renormalization from the short wavelength field modes above the cutoff $\Omega$). The field dresses the particle, changing its effective mass, which reaches an asymptotic value $m \equiv m(\infty) = m_0 - e^2 \Omega / 8\pi c^3$ on the cutoff timescale $1/\Omega$. It is interesting to note that this shift is $-1/2$
Radiation reaction is given by the last two terms on the right side (RHS) of Eq. (31). The second term has the usual third derivative form for ALD radiation reaction, except for the time-dependent coefficient \( f_{\Omega}^{(0)}(r) \). This function depends on the exact choice of the regulated retarded Green’s function, but it has the general property that \( f_{\Omega}^{(0)}(0) = 0 \) and \( f_{\Omega}^{(0)}(\infty) = 1 \), with the timescale for the transition from 0 to 1 being \( 1/\Omega \). The last term on the RHS of Eq. (31) represent a set of higher derivative contributions to radiation reaction. These are suppressed at low energies by factors of \( 1/\Omega^n \). The functions \( f_{\Omega}^{(n)}(r) \) vanish when \( r = 0 \), and go to constant values on a timescale \( 1/\Omega \). At the initial time \( r = 0 \) the radiation reaction terms, including the higher derivative contributions, vanish, and this makes the solutions to Eq. (31) unique (requiring only the usual position and velocity initial data), causal (no preacceleration), and runaway free.

The functions \( u_{\mu}^{(n)} \) involve fourth and higher derivative contributions:

\[
u_{\mu}^{(n)} = u_{\mu}^{(n)} \left( \frac{d^{n+3}z}{dr^{n+3}}, \ldots, \frac{dz}{d\tau} \right).
\]  

(32)

The dependence of the equations of motion in Eq. (31) on \( \Omega \) and the higher derivative terms reflects the degree to which the particle subsystem (trajectory) is distinguished from the full system (particle and field) by coarse graining. The larger \( \Omega \) the more point-like the particle. For sufficiently point-like particles (finer grained trajectories) and longer time scales \( (r \gg 1/\Omega) \), the higher derivative terms are strongly suppressed, and the dominant contribution to the equations of motion are independent of \( \Omega \), giving

\[
m \ddot{z}_{\mu} (\tau) = e \bar{\nu}_{\mu} \varphi_{\text{ext}} (z) + \frac{\tau_0}{2} \left( \dot{z}_{\mu} \dot{z}^2 + \dot{\varphi}_{\mu} \right).
\]  

(33)

Thus, at low energies and long times, the radiation reaction force is of the usual ALD form. (Note that the coefficient \( \tau_0/2 \) is 1/2 of the value for radiation reaction in the EM field. Again viewing the EM field as equivalent to two scalar fields, this result is natural.) Since in this limit \( \tau_0 \) is a very small parameter, in practice we can treat Eq. (33) perturbatively. The lowest order solution is found by neglecting the radiation reaction term so that the trajectory is determined by the background field \( \varphi_{\text{ext}} \), giving

\[
\ddot{z}_{\mu} = e \left( m + e \varphi_{\text{ext}} (z) \right)^{-1} \dot{z}^\nu \dot{z}_{[\nu} \partial_{\mu]} \varphi_{\text{ext}} (z).
\]  

(34)

Taking one proper-time derivative gives

\[
\ddot{z}_{\mu} = \frac{e}{m} \frac{d}{d\tau} \left( m + e \varphi_{\text{ext}} (z) \right)^{-1} \dot{z}^\nu \dot{z}_{[\nu} \partial_{\mu]} \varphi_{\text{ext}} (z).
\]  

(35)

These two expression for \( \ddot{z} \) and \( \dot{z} \) can be substituted into the radiation reaction term in Eq. (33), yielding an order \( e^4 \) equation for the radiation reaction force containing only first derivatives \( \dot{z} \). This procedure can be iterated to produce a runaway-free and causal perturbative approximation to any order in \( e \), but clearly this expansion which contains only first derivatives \( \dot{z} \) can not converge to the third-derivative ALD equation. To fully understand causality the non-Markovian short-time behavior must be examined, with Eq. (31) being an example of how non-Markovian effects can enforce causality.

We have analyzed the special case when the particle and field are initially uncorrelated below some coarse grained preparation scale \( \Omega \). The nonequilibrium time-dependence of the radiation reaction force reflects the fact that in any nonequilibrium quantum setting it takes time for the particle’s self-field to adjust to changes in its motion. This lag preserves causality unless one takes the cutoff to infinity (finest grained histories): but this would imply that the high-energy structure of the fundamental theory is critical to the equations of motion even at low energy. This, while in principle possible, is in contradiction with our experience that realistic physical conditions are described by low-energy effective theories. Our analysis is a first step
in demonstrating the full causality of the coarse-grained semiclassical limit of particle motion in quantum field theory. Next steps include considering more general initial conditions, rigorously deriving a coarse-grained (regulated) Green’s function from a physical coarse-graining mechanism, and analyzing the effects of higher-order quantum corrections.

3 Stochastic limit

3.1 Stochastic effective action

We now go beyond the average trajectory and include the trajectory fluctuations induced by the quantum fluctuations of the field. To do this, we return to the full quantum generating function, Eq. (25). In practice, to make use of the in-in generating functional the non-linear $z^{(\pm)}$ path integrals must be evaluated using some approximation. To find quantum corrections to the semiclassical solution, the loop expansion is often invoked. We describe here a different approach that transforms $S_{CGEA}$ into a stochastic effective action.

Noting that $G^\Omega_H (x, x')$ is real and symmetric, and therefore has positive eigenvalues, it follows that

$$\text{Im} \left( S_{IF} \left[ z^{(\pm)} \right] \right) = \frac{1}{4} \int dx dx' j^{(-)} (x) G^\Omega_H (x, x') j^{(-)} (x') \geq 0,$$

and

$$\left| F \left[ z^{(\pm)} \right] \right| = |e^{iS_{CGEA}}| = e^{-\text{Im} (S_{IF} [z^{(\pm)}])} < 1. \quad (37)$$

The influence functional is suppressed for large $j^{(-)}$, which reflects the decoherence associated with the radiation produced by the particle current. Consistency with the cutoff $\Omega$ requires that the current $j^{(-)}$ does not produce radiation with frequency greater than $\Omega$. The contribution to $S_{CGEA}$ from the imaginary part of the influence functional may be re-written as

$$|e^{iS_{CGEA}}| = \int D\tilde{\phi} P [\tilde{\phi}] e^{i \int j^{(-)}(x) \tilde{\phi}(x) dx}, \quad (38)$$

where

$$P [\tilde{\phi}] = N e^{-\int dx dx' \tilde{\phi}(x) G^\Omega_H (x, x')^{-1} \tilde{\phi}(x') \quad (39)$$

is a normalizable probability distribution for a stochastic field $\tilde{\phi} (x)$. The two-point function for the stochastic field is

$$\langle \{ \tilde{\phi} (x), \tilde{\phi} (x') \} \rangle_{\text{stoch}} = G^\Omega_H (x, x') = \langle \{ \tilde{\phi} (x), \tilde{\phi} (x') \} \rangle_{\text{quantum}}, \quad (40)$$

showing that $\tilde{\phi}$ encodes the same information as the quantum field anti-correlation function, with noise (or correlations) above the cutoff assumed to be coarse grained and averaged out.

The in-in generating functional may be re-expressed as

$$Z_{\text{in-in}} \left[ h^{(\pm)} \right] = \int Dz^{(+)} Dz^{(-)} \int D\tilde{\phi} P [\tilde{\phi}] e^{iS_{\text{stoch}} [z^{(\pm)}; \tilde{\phi}]}, \quad (41)$$

where

$$S_{\text{stoch}} \left[ z^{(\pm)}; \tilde{\phi} \right] = \text{Re} \left\{ S_{CGEA} \left[ z^{(\pm)} \right] \right\} + \int j^{(-)} (x) \tilde{\phi}(x) dx \quad (42)$$

$$= S_z [z, h] - S_z [z', h'] + \int dx j^{(-)} (x) \left( \tilde{\phi}(x) + \int dx' G^\Omega_H (x, x') j^{(+)} (x') \right)$$

is the stochastic effective action.
The generating functional now involves both a sum over trajectories and a sum over stochastic field configurations. If we neglect higher order quantum corrections and take \( S_{\text{stoch}}[\bar{z}^{(\pm)}; \hat{\varphi}] \simeq S_{\text{stoch}}[z^{(\pm)}; \hat{\varphi}] \), where \( z^{(\pm)} \) are the classical (extremal solutions), then we obtain a stochastic generating functional:

\[
Z_{\text{stoch}}^{\text{in-in}}[h^{(\pm)}] = \int D\hat{\varphi} P[\hat{\varphi}] e^{iS_{\text{stoch}}[z^{(\pm)}; \hat{\varphi}]}.
\]  

(43)

This describes the stochastic regime, where higher order quantum effects are washed out by decoherence but some measure of quantum field induced stochasticity remains. We emphasize that this is not a stochastic theory of scalar field QED where stochasticity is introduced in an ad hoc manner. Rather, we have started with a microscopic quantum theory and shown the conditions (namely strong decoherence that suppresses higher order corrections) whereby its behavior can be captured by its stochastic effects.

### 3.2 ALD-Langevin (ALDL) equations

The stochastic effective action generates nonlinear stochastic equations of motion, defined by

\[
\frac{\delta S_{\text{stoch}}}{\delta \bar{z}^{(-)}(\tau)}|_{z^{(-)}=0,z^{(+)}=\bar{z}} = 0,
\]  

(44)

where \( z \) is now taken to be a stochastic variable. Using Eqs. \( 34 \) and \( 35 \), we find that

\[
m_0 \ddot{z}_\mu = e\bar{w}_\mu \left( \varphi_{\text{ext}}(z) + \frac{c}{2} \int d\tau' G_\Omega^0(z(\tau),z(\tau')) \right).
\]  

(45)

Or, using the regulated Green’s function and dropping higher derivative terms, the nonlinear stochastic equations of motion take the form (in the late-time limit \( \tau \gg 1/\Omega \))

\[
m \ddot{z}_\mu = e\bar{w}_\mu [\varphi_{\text{ext}}(z) + \dot{\varphi}(z)] + \frac{\tau_0}{2} \left( \dot{z}_\mu \dot{z}_\mu + \dddot{z}_\mu \right).
\]  

(46)

The perturbative procedure illustrated in Eqs. \( 34 \) and \( 35 \) can be similarly be applied to Eq. \( 46 \) to obtain first-derivative equations of motion.

The stochastic field provides a stochastic force \( \eta_\mu(\tau) = e\bar{w}_\mu \dot{\varphi}(z) \) which, just like the force from \( \varphi_{\text{ext}} \), satisfies \( \dddot{z}_\mu \eta_\mu = 0 \), and hence also preserves the mass-shell constraint. Note that the stochastic field generates an effective stochastic mass term, \( e\dot{\varphi}(z) \dddot{z}_\mu \), in parallel with the effective mass term generated by \( \varphi_{\text{ext}} \). In the short-time limit these equations are (like the semiclassical equations) modified by nonequilibrium (time-dependent) effects.

The noise \( \eta_\mu(\tau) \) enters the Langevin equation in a highly nonlinear way, and its correlator is conditional on the particle history, making the equation very complicated. Since the notion of a semiclassical trajectory already requires substantial coarse-graining, which averages out short time and higher order quantum effects, we make the analog of the background field separation by expanding the equations around the semiclassical trajectory \( \bar{z}_\mu(\tau) \) to obtain linearized Langevin equations for the trajectory fluctuations \( y^\mu(\tau) \equiv z^\mu(\tau) - \bar{z}_\mu(\tau) \). This gives

\[
m \ddot{y}_\mu = f^\mu_{\text{ext}}(y) + \int d\tau' D_{\mu\nu}(\tau,\tau') y^\nu(\tau') + \eta_\mu.
\]  

(47)

The linearized noise \( \eta_\mu(\tau) \equiv e\bar{w}_\mu \dot{\varphi}(\bar{z}) \) has vanishing mean, \( \langle \eta_\mu(\tau) \rangle = 0 \), and a two-point correlator

\[
\langle \{ \eta_\mu(\tau), \eta_\nu(\tau') \} \rangle \equiv e^2 \bar{w}_{\mu\nu} \bar{z}(\tau) \bar{z}(\tau') G_\Omega^0(\bar{z}(\tau),\bar{z}(\tau'))
\]  

(48)

that is independent of the particle fluctuations \( y \), but is dependent upon the self-consistently determined semiclassical trajectory \( \bar{z}_\mu(\tau) \). The operator \( \bar{w}_\mu \) is evaluated in terms of the solution \( \bar{z} \). The linear force from the external field is given by

\[
f^\mu_{\text{ext}}(y) \equiv \int d\tau' \frac{\delta}{\delta z^\nu(\tau')} \{ e\bar{w}_\mu \varphi_{\text{ext}}(\bar{z}) \} y^\nu(\tau').
\]  

(49)
The second term on the RHS of Eq. (47) represents the radiation reaction force, where

\[
D_{\mu\nu}(\tau, \tau') \equiv \frac{\gamma_0}{2} \frac{\delta}{\delta z^\nu(\tau')} \left\{ \ddot{z}_\mu \dot{z}^2 + \dot{z}_\mu \dot{z}^\nu \right\}
\]  

(50)
is the dissipation kernel for linearized fluctuations of the trajectory around its average (a first-derivative perturbative form of \(D_{\mu\nu}\) can be obtained using Eqs. (44) and (46) to remove the \(\ddot{z}\) and \(\dot{z}\) factors). The kernel \(D_{\mu\nu}\) represents what we have described as quantum (or stochastic) dissipation: the backreaction in response to the quantum field induced fluctuations. While there is no direct link between the classical (ALD) radiation reaction force and quantum fluctuations, there is a relationship between the quantum dissipation and quantum fluctuations (27, 29).

4 Uniformly accelerated charges

In a weak electric field, the vacuum of an uniformly accelerated charge (UAC) assumes thermal attributes (the Unruh effect) and additional fluctuations are induced in the particle motion. In a stronger field, charged particle pair-creation becomes important. These are two distinct types of particle production processes. In the semiclassical-stochastic limit (in which we have been working) one does not have charged-particle creation effects. In the analogous situation of semiclassical gravity and Hawking radiation this entails the neglect of quantum gravitational effects. For sufficiently high intensity fields, charged particle pair creation can become a significant process, and the creation of both photons and charges must be treated as a fully quantum problem. The gravitation analogy would entail calculations based on some theory of quantum gravity. There, the emergence of classical spacetime from a theory of quantum gravity is an issue of fundamental importance. Prior investigation has led to some preliminary understanding (35). One can use this simpler theory of particle-field interaction to gain some insight into the stochastic limit of quantum cosmology and quantum gravity. This was one of the original motivations for this line of research undertaken currently. An even more ambitious goal is to follow in detail the transition from stochastic to quantum, including quantum particle aspects. We can use the more “tractable” problem here of particle-field interactions to gain insight into the transition from stochastic and quantum regimes of gravity in the so-called ‘bottom-up’ approach to quantum gravity (96). On this issue some interesting results have been obtained (87, 88) based on quantum Brownian motion (QBM) models. The significant nonlinearities of locally interacting particles and fields in the present problem makes for a better model to study this issue.

Here we take a modest step in this direction by applying our methods to a uniformly accelerated charge in a scalar field. We choose a background field \(\varphi_{\text{ext}}\) such that \(\ddot{z}_\mu \ddot{z}_\mu = a^2 = \text{constant}\) is a solution to Eq. (41) if we ignore the radiation reaction terms. It can be immediately verified that uniformly accelerating solutions satisfy \((\ddot{z}_\mu \ddot{z}^\nu + \ddot{z}_\nu \dot{z}^\mu) |_{a^2=\text{constant}} = 0\). Hence, neglecting the suppressed higher derivative terms and quantum corrections, the radiation reaction force identically vanishes, and the uniformly accelerating trajectory is a solution to Eq. (41) including radiation reaction. (This results also holds if we use the perturbative first derivative approximation to the semiclassical equations of motion.)

We choose a coordinate frame where the average trajectory is in the \(\ddot{z}^2 = \ddot{z}^3 = 0\) plane, so that

\[
\ddot{z}^0(\tau) = a^{-1} \sinh(\alpha \tau), \tag{51}
\]

\[
\ddot{z}^1(\tau) = a^{-1} \cosh(\alpha \tau), \tag{52}
\]

\[
\ddot{z}^2(\tau) = \ddot{z}^3(\tau) = 0. \tag{53}
\]

Defining right-moving coordinates \(u(\tau) = (\ddot{z}^1 - \ddot{z}^0)/2\) and left-moving coordinates \(v(\tau) = (\ddot{z}^1 + \ddot{z}^0)/2\), the uniformly accelerated trajectory is \(u(\tau) = (1/2a) e^{-\alpha \tau}\) and \(v(\tau) = (1/2a) e^{+\alpha \tau}\). The future particle horizon, shown in Fig. 1 (a), is given by \(u = 0\).

Let us now examine the linearized stochastic fluctuations of the particle, induced by the stochastic force \(\eta_\mu = c \ddot{w}_\mu \ddot{z} \), around the semiclassical trajectory. The structure of the field fluctuations are characterized...
by the anticooperator (the Hadamard function) along the accelerated trajectory:

$$G_H (\vec{z} (\tau), \vec{z} (\tau')) = \langle \{ \hat{\phi} (\vec{z} (\tau)), \hat{\phi} (\vec{z} (\tau')) \} \rangle.$$

For a massless scalar field in the vacuum state, evaluated at two different times $\tau$ and $\tau'$, the Hadamard function gives

$$G_H (\vec{z} (\tau), \vec{z} (\tau')) = \text{Im} \left( \frac{i}{4\pi^2} \int_0^\infty dk \frac{\sin k |\vec{z}^1 (\tau) - \vec{z}^1 (\tau')|}{|\vec{z}^1 (\tau) - \vec{z}^1 (\tau')|} \exp \left( k \vec{z}^0 (\tau) - \vec{z}^0 (\tau') \right) \right),$$

where

$$r (\tau, \tau') = |\vec{z}^1 (\tau) - \vec{z}^1 (\tau')|,$$

and

$$t (\tau, \tau') = (\vec{z}^0 (\tau) - \vec{z}^0 (\tau')).$$

Substituting the uniformly accelerated solution, it becomes (re-inserting factors of $c$)

$$G^U_H (\tau, \tau') = \int_0^\infty \frac{dk}{k} g (k, a) \left\{ \coth \frac{c\pi k}{a} \right\} \cos k (t - \tau'),$$

where the general form for $g (k, a)$ for a massive scalar field in $D + 1$ spacetime dimensions is given in [39] as

$$g (k, a) = \frac{c^2 k \sinh (c\pi k / a)}{2D-3a\pi(D+3)/2} \int_0^\infty r^{D-2} K_{ik/\alpha} (c^2 \sqrt{m^2 + r^2 / a})^2 dr.$$

$K_{ik/\alpha}$ is a modified Bessel function of imaginary order. The function $g (k, a)$ is the effective spectral density of the scalar field environment seen by the particle. In general it depends on the particle's acceleration $a$ and
mass \( m \). When \( m = 0 \), the integral can be evaluated in closed form. For \( D = 3 \) (the case of \( 3 + 1 \) spacetime which is considered here), the effective spectral density is independent of the particle state of motion:

\[
g(k, a) = \frac{k^2}{2\pi c^2}. \tag{59}
\]

The form of the Hadamard function in Eq. (57) is precisely the same as that for a bath of harmonic oscillators with temperature \( T = \hbar a/(2\pi ck_B) \), and spectral density given by \( k^2/2\pi c^2 \). Thus, the scalar field vacuum along the accelerated particle trajectory looks like a thermal state, with temperature proportional to the acceleration.

The correlator of the linearized stochastic force on the particle, given by

\[
\langle \{ \eta^\mu (\tau), \eta^{\nu} (\tau') \} \rangle \equiv e^2 \bar{w}^{\mu} (\bar{z} (\tau)) \bar{w}^{\nu} (\bar{z} (\tau')) G_{\Omega H} (\bar{z} (\tau), \bar{z} (\tau')), \tag{60}
\]

governs the response of the particle to the quantum field fluctuations. Note that \( \bar{w}_2 (\bar{z} (\tau)) = \bar{w}_3 (\bar{z} (\tau)) = 0 \), and thus the induced fluctuations act only in the \((z^0, z^1)\) plane: \( \eta_{\bar{z} 0} (\tau) = \eta_{\bar{z} 1} (\tau) = 0 \), in the linearized response approximation. (This result does not carry over to the nonlinear Langevin equation [Eq. (46)] when higher order effects are included). Because the stochastic force is given by \( \eta_{\bar{z}} = e\bar{w}_{\bar{z}} \hat{\Phi} (\bar{z}) \), the response of the particle depends on its state of motion through \( \bar{w}_{\bar{z}} (\bar{z}) \). While the underlying quantum field looks thermal along the uniformly accelerated particle’s average trajectory, the fluctuations that are induced in the motion are complicated by non-isotropic and relativistic effects. In fact these effects make this model more interesting. First, we note that the Langevin equation describes the stochastic behavior of a particle’s worldline, depicting fluctuations in both the space and time coordinates. This makes this system a simple analog for quantum cosmology with the stochasticity in time induced by quantum fluctuations.

Second, the particle motion is an interesting analog for stochastic black hole horizon dynamics. If we view the particle trajectory in the far future as effectively determining the horizon, the Langevin equations for \( \tau \to \infty \) then describes a stochastic distribution of horizon positions [see Fig. 1(b)]. We note that most of the stochastic spread of the particle trajectory (and hence the horizon) occurs for \( \tau < c/a \). When the particle approaches the velocity of light, Lorentz time dilation suppresses (i.e. slows) fluctuations from the perspective of external observers. The analogy with black hole horizon formation suggests that most of the stochastic spread in a black hole’s horizon occurs in the early stages of its formation. We are using stochastic relativistic particle dynamics as a guide to the investigation of the horizon fluctuations in black hole dynamics.

### 5 Discussions, Applications and Further Development

The open system concept coupled with the coarse-grained effective action and the influence functional techniques have been applied to quantum Brownian motion [4], interacting quantum fields [7, 10, 13], and stochastic semiclassical gravity [23]. The new task we have undertaken in recent years is the adoption of the worldline quantization method to this existing framework for the treatment of relativistic charged particles moving in a quantum field [11, 2]. In this alternative formulation of QED the particle worldline structure is highly efficient for describing the particle-like degrees of freedom (such as spacetime position), while the field structure most easily describes processes like radiation and pair creation. Here in the Discussions, we summarize the special features and merits of this approach and then mention its applications and extensions.

### 5.1 Special Features of World-line Influence Functional Approach

#### 5.1.1 Particle versus field formulation, fixed versus dynamic background

The contrasting paradigms of particles versus fields give very different representations and descriptions for the same physical processes. The success of quantum field theory for the past century has catapulted the field concept to the forefront, with the particle interpreted as excitations of field degrees of freedom.
In recent years, the use of the quantum-mechanical path integral in string theory has inspired a renewed effort towards particle-centric quantum formalisms. The so-called worldline quantization method—where the particle’s spacetime coordinate $x^\mu(\tau)$ is quantized—is especially useful for calculating higher-loop processes in non-dynamical classical background fields. Background field treatment is a useful and well-defined method in conditions where the quantum fluctuations of the fields are small. However, many problems involving the dynamical degrees of freedom of the field require the inclusion of field dynamics beyond the fixed-background approximation. A simple example is radiation reaction. More examples are found in semiclassical gravity \cite{41}, where spacetime playing the role of ‘particle’ is treated classically while interacting with the quantum fields.

The proper treatment of radiation-reaction requires, at the least, consistency between the particle’s (quantum) average trajectory and the mean-field equations of motion. When one treats the particle as quantum mechanical, but coupled only to the mean-field background (and not the quantum field fluctuations) one is working within the semiclassical regime \cite{41}. (When quantum fluctuations of the particle motion are sufficiently small, one recovers the classical regime \cite{41}). In our program, we go beyond the semiclassical approximation to include the influence of quantum field fluctuations on the moving particle, thus reaching over to the stochastic regime. To do this properly, we must begin with the full quantum theory for both particle and field, since a particle can never fully decouple from the field. Also, since most field degrees of freedom remain unobserved, we treat the particle as a quantum open system with the field acting as an environment. This is how nonequilibrium and stochastic mechanics ideas enter, and why the self-consistent quantum particle evolution is aptly described by stochastic quantum dynamics.

5.1.2 Backreaction and Self-consistently Determined Trajectories

One major improvement of our approach to the problem of moving charges in a quantum field is the consideration of self-consistent backreaction of the quantum field on the particle in the determination of its trajectory. We also find that conceptual issues are easier to resolve if we deal with such problems at four distinct levels: quantum, stochastic, semiclassical and classical, as explained earlier. Confusion will arise when one mixes physical processes of one level with the other without knowing their interconnections, such as drawing the equivalence between radiation reaction with vacuum fluctuations. Before summarizing our thoughts for processes under nonequilibrium conditions, which cover most cases save a few special yet important situations, such as uniform acceleration, let us remark that these well-known cases are what we would call ‘test field’ or prescribed (trajectory) approximations. They are not self-consistently determined with backreaction considerations. These cases are easier to study because they possess some special symmetry, such as is present for the uniform acceleration case (Rindler spacetime), inertial case (Minkowski), or the eternal black hole spacetime. They are so legitimatized—meaning, physically relevant—only if the backreaction of the field on the particle permits such solutions (which may not be difficult to attain if the field strength is weak). It is under these special conditions that a detector will detect thermality. These very special conditions are the presuppositions of the Unruh and Hawking effects. In more general situations the noise kernel is nonlocal, which means that the noise in the detector is colored and temperature is no longer a strictly viable concept.

5.1.3 Quantum Origin of the ALD Equation

Quantum field fluctuations, while responsible for the decoherence in the quantum particle (system) leading to the emergent semiclassical and classical behaviors, also impart an effective classical stochastic noise in the equations of motion for the particle trajectory \cite{41}. These stochastic differential equations feature colored noise that encodes the influence of quantum fluctuations in the field. When there is sufficient decoherence, these equations give excellent approximations to the particle motion. Even in the case of weak decoherence they can provide a reasonable approximation to the particle correlation functions. More accurate results can be obtained from the n-particle-irreducible (nPI) closed-time-path effective action under a large N approximation, which is a nice way to systemize the hierarchy of correlation functions from the CGEA \cite{42}. This scheme has been applied to stochastic gravity \cite{43}. 

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5.2 Applications and further developments

Finally we want to connect our work with existing work in other areas, mention applications of the present results to related problems, and show directions where this new approach can stimulate further developments.

In terms of theory, the world-line quantization formalism has been applied to QED, QCD and string theory [44, 45]. Our interest in this present problem and its particular formulation stemmed partly from stochastic gravity [23], which is the stochastic generalization of semiclassical gravity. The relation of semiclassical and stochastic regimes was made clear in the investigation of quantum Brownian motion [4]. More interesting is the relation between the quantum and the stochastic regimes. Some work has appeared addressing this issue [37, 38] but more insight is needed. This is of particular interest in the ‘bottom-up’ approach to quantum gravity [46]. What features of quantum gravity can one detect from the lower energy (mesoscopic) phenomena as manifested in stochastic gravity? We don’t have a theory of quantum gravity but we can learn from a known and well-proven one such as QED. The investigations in this program offers a concrete example of how a full quantum theory can yield stochastic and semiclassical features. This could partially illuminate the common pathways from stochastic semiclassical gravity to quantum gravity.

In terms of applications our results are useful for a range of problems, from the consideration of the quantum aspects of beam physics (see, e.g., [47]) to the consideration of possible schemes for the detection of Unruh radiation, e.g., in linear accelerators or storage rings (note however our critique for certain proposals of the former [27] and misconceptions in the so-called ‘circular Unruh effect’ [29]). The stochastic trajectory from the backreaction of quantum fields studied in the last section is an interesting analog model to quantify black hole event horizon fluctuations.

The new approach as illustrated in this report can also serve as a launching pad for the exploration of nonequilibrium dynamics of charges undergoing arbitrary (self-consistently determined, not prescribed) motions, strongly correlated quantum systems, relativistic quantum kinetic and stochastic theories. It is currently being generalized to curved spacetime (see also [48]) and applied to gravitational radiation reaction problems. Recently a field-theoretical derivation of the MSTQW equation was given and a new stochastic radiation reaction equation proposed [49, 50]. This approach is also being applied to the study of quantum entanglement and teleportation of relativistic detectors [51] (see also [20, 28]) and environmental decoherence and entanglement [52, 53, 54] issues.

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References

[1] P. R. Johnson, Ph.D. thesis, University of Maryland, College Park, 1999 (unpublished).

[2] P.R. Johnson and B.L. Hu, “Stochastic Theory of Relativistic Particles Moving in a Quantum Field: Influence Functional and Langevin Equation” quant-ph/0012137.

[3] E. B. Davies, The Quantum Theory of Open Systems (Academic, London, 1976); U. Weiss, Quantum Dissipative Systems (World Scientific, Singapore, 1993).

[4] A. O. Caldeira and A. J. Leggett, Physica A 121, 587 (1983); Ann. Phys. (N.Y.) 149, 374 (1983); A. J. Leggett et al., Rev. Mod. Phys. 59, 1 (1987); V. Hakim and V. Ambegaokar, Phys. Rev. A 32, 423 (1985); F. Haake and R. Reibold, Phys. Rev. A 32, 2462 (1985); H. Grabert, P. Schramm, and G. L. Ingold, Phys. Rep. 168, 115 (1988); B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D 45, 2843 (1992); D 47, 1576 (1993).

[5] R. Feynman and F. Vernon, Ann. Phys. (N.Y.) 24, 118 (1963); R. Feynman and A. Hibbs, Quantum Mechanics and Path Integrals (McGraw-Hill, New York, 1965).
[6] J. Schwinger, J. Math. Phys. 2, 407 (1961); L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964) [Sov. Phys. JETP 20, 1018 (1965)]; G. Zhou, Z.B. Su, B. Hao, and L. Yu, Phys. Rep. 118, 1 (1985); Z.B. Su, L. Y. Chen, X.T. Yu, and K.C. Chou, Phys. Rev. B 37, 9810 (1988); B. S. DeWitt, in Quantum Concepts in Space and Time, edited by R. Penrose and C. J. Isham (Claredon, Oxford, 1986); R. D. Jordan, Phys. Rev. D 33, 444 (1986); E. Calzetta and B. L. Hu, Phys. Rev. D 35, 495 (1987); 37, 2878 (1988); 49, 6636 (1994); A. Campos and E. Verdaguer, Phys. Rev. D 49, 1861 (1994).

[7] B. L. Hu and Y. Zhang, “Coarse-Graining, Scaling, and Inflation” Univ. Maryland Preprint 90-186 (1990); B. L. Hu, in Relativity and Gravitation: Classical and Quantum, Proc. SILARG VII, Cocoyoc, Mexico 1990. eds. J. C. D’Olivo et al (World Scientific, Singapore 1991); Sukanya Sinha and B. L. Hu, Phys. Rev. D 44, 1028 (1991); F. Lombardo and F. D. Mazzitelli, Phys. Rev. D 53, 2001 (1996).

[8] Esteban A. Calzetta, Bei-Lok Hu, Francisco D. Mazzitelli, “Coarse-Grained Effective Action and Renormalization Group Theory in Semiclassical Gravity and Cosmology” Physics Report. 352 (2001) 459-520 [hep-th/0102199].

[9] A.O. Barut and I.H. Duru, Phys. Rev. 172, 1 (1989); M.J. Strassler, Nucl. Phys. B 385, 145 (1992); D.G.C. McKeon, Ann. Phys. (N.Y.) 224, 139 (1993); M.J. Schmidt and C. Schubert, Phys. Rev. Lett. 331, 69 (1994); Phys. Lett. B 318, 438 (1993); H. Kleinert, Path Integrals in Quantum Mechanics, Statistical and Polymer Physics (World Scientific, London, 1995); D. Fliegner, M. G. Schmidt, and C. Schubert, Nucl. Phys. B 51C, 174 (1996); J. Jalilian-Marian et al., Phys. Rev. D 62, 045020 (2000); R. Venugopalan and J. Wirstam, Phys. Rev. D 63, 125022 (2001).

[10] P.R. Johnson and B.L. Hu, Phys. Rev. D 65, 065015 (2002).

[11] W. G. Unruh, Phys. Rev. D 14, 3251 (1976).

[12] B. S. DeWitt, Phys. Rep. 19C, 297 (1975).

[13] B. L. Hu, in Proceedings of the Third International Workshop on Thermal Fields and its Applications, CNRS Summer Institute, Banff, August 1993, edited by R. Kobes and G. Kunstatter (World Scientific, Singapore, 1994) [gr-qc/9403061]; Yuhong Zhang, Ph.D. Thesis, University of Maryland 1990 (unpublished).

[14] S. A. Ramsey and B. L. Hu, Phys. Rev. D 56, 678 (1997); S. A. Ramsey, B. L. Hu and A. M. Stylianopoulos, Phys. Rev. D 57, 6003 (1998).

[15] M. Gell-Mann and J. B. Hartle, Phys. Rev. D 47, 3345 (1993).

[16] R. B. Griffiths, J. Stat. Phys. 36, 219 (1984); R. Omnés, J. Stat Phys. 53, 893, 933, 957 (1988); Ann. Phys. (NY) 201, 354 (1990); Rev. Mod. Phys. 64, 339 (1992); The interpretation of quantum mechanics, (Princeton University Press, Princeton, 1994); M. Gell-Mann and J. B. Hartle, in Complexity, Entropy and the Physics of Information, ed. by W. H. Zurek (Addison-Wesley, Reading, 1990); J. B. Hartle, “Quantum Mechanics of Closed Systems” in Directions in General Relativity Vol. 1, edited by B. L. Hu, M. P. Ryan and C. V. Vishveswara (Cambridge Univ., Cambridge, 1993).

[17] P.W. Milonni, The Quantum Vacuum (Academic, San Diego, 1994); F. Rohrlich, Classical Charged Particles (Addison-Wesley, Reading, MA, 1965).

[18] M. Abraham and R. Becker, Electricity and Magnetism (Blackie, London, 1937); H.A. Lorentz, The Theory of Electrons (Dover, New York, 1952), pp. 49 and 253; P.A.M. Dirac, Proc. R. Soc. London A 167, 148 (1938).

[19] N.V. Kampen, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. 26 (1951); E. Moniz and D. Sharp, Phys. Rev. D 10, 1133 (1974); H. Levine, E. Moniz, and D. Sharp, Am. J. Phys. 45, 75 (1977); J. Wheeler and R. Feynman, Rev. Mod. Phys. 17, 157 (1945); F. E. Low, Annals Phys. 266, 274 (1998); V. Krivitskii and V. Tsytovich, Sov. Phys. Usp. 34, 250 (1991).
[20] H. Callen and T. Welton, Phys. Rev. 83, 34 (1951); H. Mori, Prog. Theor. Phys. 33, 423 (1965); G. W. Ford, M. Kac, and P. Masur, J. Math. Phys. 6, 504 (1965); H. Haken, Rev. Mod. Phys. 80, 67 (1975); H. Dekker, Phys. Rep. 80, 1 (1981).

[21] G. W. Ford, J. T. Lewis, and R. F. O’Connell, Phys. Rev. Lett. 55, 2273 (1985); Phys. Rev. A 37, 4419 (1988); G. W. Ford and R. F. O’Connell, Phys. Lett. A 157, 217 (1991); Phys. Lett. A 174, 182 (1993); G. W. Ford and R. F. O’Connell, Phys. Rev. A 57, 3112 (1998).

[22] L. Diósi, Found. Phys. 20, 63 (1990); P.M.V.B. Barone and A.O. Caldeira, Phys. Rev. A 43, 57 (1991); L. H. Ford, Phys. Rev. D 47, 5571 (1993); Phys. Rev. A 56, 1812 (1997); H-P. Breuer and F. Petruccione, in Relativistic Quantum Measurement and Decoherence, edited by H-P. Breuer and F. Petruccione (Springer-Verlag, Berlin, Germany, 2000).

[23] For reviews see, e.g., B. L. Hu and E. Verdaguer, “Stochastic gravity: A primer with applications”, Class. Quant. Grav. 20 (2003) R1-R42 [gr-qc/0211090]; “Stochastic gravity: Theory and Applications”, in Living Reviews in Relativity 7 (2004) 3; article number lrr-2004-3 [gr-qc/0307032].

[24] B. L. Hu, P. R. Johnson and A. Roura, “Some conceptual issues in self-consistent backreaction problems”, (in preparation).

[25] S. W. Hawking, Nature 248, 30 (1974); Com. Math. Phys. 43, 199 (1975).

[26] A. Raval, B. L. Hu and J. Anglin, Phys. Rev. D 53, 7003 (1996).

[27] B. L. Hu and A. Raval, “Is there Radiation in Unruh Effect?” in Quantum Aspects of Beam Physics, 18th Advanced IFCA Deam Dynamics Workshop, edited by Pisin Chen, World Scientific, Singapore (2001) quant-ph/0012135.

[28] A. Raval, B. L. Hu and D. Koks, Phys. Rev. D 55, 4795 (1997).

[29] B. L. Hu and P. R. Johnson, “Beyond Unruh effect: Nonequilibrium quantum dynamics of moving charges”, in Quantum Aspects of Beam Physics, 18th Advanced IFCA Deam Dynamics Workshop, edited by Pisin Chen, World Scientific, Singapore (2001).

[30] W.G. Unruh, Phys. Rev. D 46, 3271 (1992).

[31] S. Massar and R. Parentani, “From vacuum fluctuations to radiation. I. Accelerated detectors”, Phys. Rev. D 54, 7426-7443 (1996).

[32] Cl. Gabriel, Ph. Spindel, S. Massar, and R. Parentani, “Interacting charged particles in an electric field and the Unruh effect”, Phys. Rev. D 57, 6496-6510 (1998).

[33] L. D. Romero and J. P. Paz, Phys. Rev. A 55, 4070 (1997).

[34] S-Y. Lin pointed out (private communication) that the factors involving $\ddot{z}^{\mu}\varphi_{ext}$ were missing in Ref [10]. These factors didn’t play a role in the specific application treated in [10], but are necessary for the full consistency of the equations of motion. We thank him for this observation.

[35] J.P. Paz and S. Sinha, Phys. Rev. D 45, 2823 (1992).

[36] B. L. Hu, “General Relativity as Geometro-Hydrodynamics”, Invited talk at the Second Sakharov Conference, Moscow, May, 1996 gr-qc/9607070.

[37] C. Anastopoulos, “Quantum correlation functions and the classical limit”, Phys. Rev. D 63, 125024 (2001).

[38] E. Calzetta, A. Roura and E. Verdaguer, “Stochastic Description of Quantum Open Systems” Physica A 319, 188-212 (2003) quant-ph/0011097.
[39] J. R. Anglin, Phys. Rev. D 47, 4525 (1993).
[40] Fred Cooper, Salman Habib, Yuval Kluger, Emil Mottola, and Juan Pablo Paz, Phys. Rev. D 50, 2848–2869 (1994).
[41] Ya. Zel’dovich and A. Starobinsky, Zh. Eksp. Teor. Fiz 61, 2161 (1971) [Sov. Phys.- JETP 34, 1159 (1971)]; B. L. Hu and L. Parker, Phys. Rev. D 17, 933 (1978); F. V. Fischetti, J. B. Hartle and B. L. Hu, Phys. Rev. D 20, 1757 (1979); J. B. Hartle and B. L. Hu, Phys. Rev. D 20, 1772 (1979); P. A. Anderson, Phys. Rev. D 28, 271 (1983); 29, 615 (1984); E. Calzetta and B. L. Hu, Phys. Rev. D 35, 495 (1987); A. Campos and E. Verdaguer, Phys. Rev. D 49, 1861 (1994).
[42] E. Calzetta and B. L. Hu, “Correlations, Decoherence, Dissipation and Noise in Quantum Field Theory”, in Heat Kernel Techniques and Quantum Gravity, edited by S. Fulling (Texas A& M Press, College Station 1995) [hep-th/9501040]; E. Calzetta and B.L. Hu, Phys. Rev. D 61, 025012 (2000).
[43] A. Roura and E. Verdaguer, in preparation (2005).
[44] C. Schubert, “Perturbative Quantum Field Theory in the String-Inspired Formalism” Physics Reports 355 (2001) 73-234; Z. Bern and D. A. Kosower, Phys. Rev. Lett. 66, 1669 (1991).
[45] M. B. Green, J. H. Schwarz and E. Witten, Superstring Theory (Cambridge University, Cambridge, 1990); E. Witten, Physics Today 49, 24 (1996); J. Polchinsky, Superstring Theory (Cambridge University Press, Cambridge 1998).
[46] B. L. Hu, “A Kinetic Theory Approach to Quantum Gravity”, Int. J. Theor. Phys. 41, 2111-2138 (2002) [gr-qc/0204069].
[47] Pisin Chen editor, Quantum Aspects of Beam Physics, Proceedings of the Capri Workshop, Oct. 2000 (World-Scientific, Singapore, 2001).
[48] F. Bastianelli and A. Zirotti, Nucl. Phys. B 642, 372 (2002).
[49] Chad Galley and B. L. Hu, “Stochastic Self-Force from Radiation Reaction in Curved Spacetime”, (in preparation).
[50] Chad Galley, B. L. Hu and S.Y. Lin, “Electromagnetic and Gravitational Radiation Reaction Equations from Stochastic Field Theory”, in preparation.
[51] P. M. Alsing and G. J. Milburn, “Teleportation with a Uniformly Accelerated Partner”, Phys. Rev. Lett. 91, 180404 (2003).
[52] T. Yu and J. H. Eberly, “Finite-Time Disentanglement Via Spontaneous Emission”, Phys. Rev. Lett. 93, 140404 (2004).
[53] S. Shresta, C. Anastopoulos, A. Dragulescu, and B. L. Hu, “Non-Markovian qubit dynamics in a thermal field bath: Relaxation, decoherence and entanglement”, Phys. Rev. A [quant-ph/0408084].
[54] C. H. Chou and B. L. Hu, “Two qubits Disentanglement in an Effective Spin-Boson Model”, in preparation.