Topological Defects: Fossils from the Early Universe

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ABSTRACT

In the context of current particle physics theories, it is quite likely that topological defects may be present in our universe. An observation of these fossils from the early universe would lead to invaluable insight into cosmology and particle physics, while their absence provides important constraints on particle-cosmology model building. I describe recent efforts to address cosmological issues in condensed matter systems such as $^3$He and a possible solution to the magnetic monopole problem due to defect interactions.

I. INTRODUCTION

The electroweak model, GUTs and almost all other particle physics models are based on “spontaneous symmetry breaking” i.e. phase transitions. If these descriptions of particle physics are correct, the inescapable implication is that the early universe must have seen phase transitions much like the freezing of water, and, the magnetization of iron. Then, the consequences of phase transitions that we observe in the laboratory can be expected to apply to the universe as well. In particular, relics of the high temperature phase of condensed matter systems called “topological defects” are routinely observed in the laboratory and similar relics of the early high temperature universe could exist in the present universe. In other words, these are possible fossils from the early universe. Their observations would be invaluable for gleaning information about very high energy particle physics and cosmology. It would also have implications for astrophysics and astronomy. An observed absence of topological defects too is very useful since it imposes severe constraints on particle physics model building. Indeed, the absence of magnetic monopoles inspired the inflationary revolution in cosmology and GUTs models are constrained to provide the requisite amount of inflation.

The hunt for cosmic topological defects depends crucially on their properties. The last two decades has seen extensive research on topological defects and their potential role in cosmology. Very recently, the lack of experimental input has been relieved by enterprising condensed matter physicists who have been performing experiments in the laboratory to answer questions of great interest to cosmologists.

In this talk I will first describe some recent experimental results obtained in $^3$He that have direct implications for cosmology. I will then describe a possible new solution to the cosmological monopole over-abundance problem highlighting the rich processes involved in the study of topological defects.

II. TOPOLOGICAL DEFECTS IN $^3$HE

Over the last several years, a number of condensed matter experiments of a cosmological flavour have been performed. These are:

- The experiment in nematic liquid crystals by Chuang et. al. [1990] where the authors studied the relaxation of a network of strings.

- The formation of defects in liquid crystals by Bowick et. al [1994].
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- The formation of vortices in $^4$He by Peter McClintock and his group in Lancaster [Hendry et. al., 1994].
- The “missing energy” experiments in $^3$He conducted in Grenoble and Lancaster [Bauerle et. al., 1996] to study the formation of vortices.
- The ingenious experiment in $^3$He in Helsinki [Ruutu et. al., 1996] to study the formation and distribution of strings.
- The $^3$He experiment confirming the analog of the baryon number anomaly important for baryogenesis [Bevan et. al., 1997].
- The $^3$He experiment studying the conversion of a baryon number analog into what would correspond to a cosmological magnetic field [Krusius, Vachaspati and Volovik, 1998].

Here I will summarize the Helsinki experiment studying the distribution of vortices formed at a phase transition and the violation of the $^3$He analog of baryon number.

A. Distribution of Strings

The first quantity one needs to determine in studying topological defects is the number density of defects formed during a phase transition. This was the subject of the McClintock experiment and the Grenoble, Lancaster and Helsinki experiments. But the Helsinki experiment went further and determined the distribution of string sizes formed during the phase transition, i.e. they obtained the “spectrum” of defects. Here is how they did it.

First they prepared a sample of superfluid $^3$He-B at typical temperatures of order 1 mK. Then they reheated a bubble in the sample by sending in neutrons that collided with the $^3$He nucleus and underwent the following nuclear reaction:

$$n + ^3He \rightarrow p + ^3H_1 + 764 \text{ keV}.$$  

The deposited 764 keV restores the symmetry in the bubble (of size about 20 microns) which then cools down and undergoes a phase transition. (During this phase transition, quasiparticles are produced and their energy can be detected in the Grenoble and Lancaster experiments. This energy is short of the injected 764 keV. The missing energy presumably goes into a network of vortices.) The $^3$He sample in the Helsinki experiment is rotating which means that there is a relative flow between the superfluid and normal components of $^3$He in the sample. This relative flow causes a force on the vortices inside the bubble, that stretches out some of the vortices (and collapses others). The stretched out vortices eventually become straight and settle to the axis of rotation where they can be counted using NMR. The spectrum of loop sizes is determined by the Helsinki group since only loops larger than a critical size can be stretched out by any given rotational velocity. By varying the rotational velocity, smaller loops can be counted and, in this way, the spectrum of loop sizes obtained.

The results of the experiment seem to agree with the scale invariant prediction found by Alex Vilenkin and me in 1984 [Vachaspati and Vilenkin, 1984]. Using numerical simulations, we found that the number density, $dn$, of loops with size between $R$ and $R + dR$ is:

$$dn = \alpha \frac{dR}{R^4}.$$  

The coefficient $\alpha$ depends on the conditions of the phase transition (for example, the pressure in the $^3$He sample) but the exponent of $R$ (equal to $-4$) is the scale-invariant prediction that experiment seems to be confirming.
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An important prediction for the cosmology of cosmic strings from our 1984 paper is that the string network consists of infinite strings separate from the loop population. This has not been seen so far in the experiments. However, it is likely that the absence of infinite strings may be due to the fixed phase boundary conditions on the surface of the bubble. This topic needs further theoretical investigation. And then to find the infinite string population, will need still greater ingenuity on the part of experimentalists.

B. Baryon Number Violation

In the standard electroweak model, baryon number is violated by anomalous processes. The classical conservation of the baryonic current is modified to:

$$\partial_\mu j_B^\mu = \frac{N_F}{32\pi^2} [-g^2 W_\mu^a \tilde{W}^{a\mu} + g'^2 Y_\mu^a \tilde{Y}^{a\mu}] \tag{1}$$

in the usual notation. If we consider the case where only the $Z$ gauge field is non-vanishing and integrate over four volume, this reduces to:

$$\Delta Q_B = N_F \frac{\alpha^2}{32\pi^2} \cos(2\theta_w) \Delta \int d^3 x \vec{B} \cdot \vec{Z} \tag{2}$$

where, $Q_B$ is the baryonic charge and $\Delta (\cdot)$ represents the difference at two different times. The integral on the right-hand side of (2) is the helicity of the $Z$ magnetic field and so, changes in the helicity of electroweak strings (which are tubes of $Z$ flux for the present purpose) yield changes in baryon number. Now helicity has a very simple interpretation for $Z$ strings - it is simply the linking number of two or more loops of string, or, the twist of one string [Vachaspati and Field, 1994].

This argument made no direct reference to the fermions in the standard model. However, the final result can also be derived by considering fermion zero modes on two electroweak string loops that are linked together. The fermion zero modes on one of the string loops will feel the other loop due to an Aharanov-Bohm interaction. This interaction shifts the Dirac sea and leads to a non-trivial baryon number [Garriga and Vachaspati, 1995].

In $^3\text{He}$, the ingredients of the standard electroweak model are also present since there are fermionic quasiparticles that play the role of standard model fermions, and an order parameter that plays the role of the gauge and Higgs field. There is a direct analog of electroweak strings in $^3\text{He}$-A and quasiparticle zero modes too! Then the question is if we can observe the analog of baryon number violation on $^3\text{He}$ vortices.

It is not possible for me to give all the details in this short report, but the main points are as follows. In $^3\text{He}$, the motion of vortices relative to the normal fluid is effectively like an electric field along the vortex. The quasiparticle zero modes feel this electric field, lifting the entire Dirac sea along the zero mode. This creates quasiparticles from the vacuum in direct analogy with the creation of baryon number along electroweak strings that move across an ambient magnetic field.

The experiment, however, cannot measure the number of quasiparticles produced from the vacuum. Instead it measures the momentum gained by the quasiparticles which is given by the formula:

$$\partial_t \vec{P} = \frac{1}{2\pi^2} \int d^3 x (p_F \hat{\vec{l}}) \vec{E} \cdot \vec{B}$$

where, $p_F$ is the Fermi momentum and $\hat{\vec{l}}$ is the orientation of the Cooper pair angular momentum. The way the measurement is made is that the anomalous change in the quasiparticle momentum leads to an extra force on the moving vortices. This force is measured, leading
to the confirmation of “momentogenesis” along $^3$He vortices and baryogenesis in the standard model.

The experiment is remarkable confirmation of the physics of anomalies on vortices which is an important ingredient in cosmological baryogenesis scenarios. (The electroweak sphaleron itself may be viewed as an electroweak string [Vachaspati, 1994; Hindmarsh and James, 1994].) The experiment, however, does not say anything about the creation of matter in the universe since that depends on the cosmological environment and other factors such as CP violation and departures from thermal equilibrium.

III. ON THE MONOPOLE PROBLEM

There are three known solutions to the GUT monopole over-abundance problem. The first is that the GUT phase transition that produces magnetic monopoles is followed by a period of inflation that dilutes the monopole density to acceptable levels [Guth, 1981]. The second is that the GUT model includes a period during which electromagnetism is broken. During this period, the magnetic monopoles will get connected by strings, leading to rapid annihilation and dilution [Langacker and Pi, 1980]. The third possibility is that the GUT phase transition never occurred and the universe was always in a state of broken GUT symmetry [Dvali, Melfo and Senjanovic, 1995].

All three solutions of the monopole over-abundance problem require fine tuning of parameters and/or model building solely for the purpose of eliminating magnetic monopoles. The solution I have recently conjectured in collaboration with Gia Dvali and Hong Liu [Dvali, Liu and Vachaspati, 1997] does not appear to suffer from fine tuning or excessive model building. The reason I say that the solution is “conjectured” is because it involves a knowledge of defect interactions that is just beginning to be investigated.

The basic scenario is in the following steps:

• At the GUT phase transition, magnetic monopoles are formed together with an infinite unstable domain wall that percolates throughout space.

• The domain wall moves through space, sweeping out the entire volume.

• When a monopole is hit by the wall, it gets captured. On the wall the monopole unwinds and its energy spreads and propagates along the wall. The collision of monopoles and anti-monopoles on the wall leads to their annihilation.

• At some epoch, the domain wall start collapsing as it is unstable. When the domain wall collapses and disappears, so does all the magnetic charge. In this way, neither domain wall nor magnetic monopole survive today.

At every step, there are constraints on the parameters in the model that need to be satisfied for the scenario to be successful. In the first step, we need the unstable domain wall to percolate in space. This imposes a mild constraint on the parameters. In the second step, we need to make sure that the domain wall sweeps up nearly all the monopoles in the universe. We know that stable ($Z_2$) domain walls will indeed sweep up the whole universe. So the constraint here is that the domain wall cannot be too unstable. The third step assumes that the monopoles do not pass through the wall, or, if there is a probability associated with passing through, this is very small. This problem has been studied in other solitonic systems and by the results available so far, in no case did the soliton pass through a domain wall [Kudryavtsev, Piette and Zakrzewski, 1997; Trebin and Kutka, 1995; Misirpashaev, 1991; Krusius, Thuneberg and Parts, 1994]. For the fourth step to be successful, the domain walls should not be too stable, otherwise they would start dominating the universe. This would impose a constraint on the model parameters.
In the paper with Dvali and Liu, we evaluated the constraints on the parameters and, from our results, it seems that there is no fine tuning involved. Further, there is no excessive model building involved since the scenario works in minimal GUT models such as that based on $SU(5)$. Therefore, the interaction of defects might resolve the monopole problem, relieving GUTs from the obligation of providing inflation.

**IV. OUTLOOK**

The discovery of cosmic topological defects would provide us with a direct window on the early universe. They would yield invaluable non-perturbative information about particle physics. If the defects are massive enough, they could be important astrophysically and may have played a vital role in large-scale structure formation.

In this connection, I should mention recent results from two groups finding that, in the simplest cosmological model, GUT scale (gauge) cosmic strings do not provide the appropriate anisotropy in the cosmic microwave background, nor enough power for large-scale structure formation [Allen et. al, 1997; Albrecht, Battye and Robinson, 1997]. Recently, other cosmological models have also been studied with improvements in the fit to observation [Avelino et. al., 1997; Battye, Robinson and Albrecht, 1997]. This is real progress. But these results are largely numerical and are hard to test and understand. A clearer picture is needed before one can definitively claim that GUT scale cosmic strings do not play an astrophysical role. Probably the cleanest way to rule out, or find, GUT scale cosmic strings is by a *direct* search for them via their gravitational lensing of background galaxies and quasars [de Laix, Krauss and Vachaspati, 1997]. Results from such searches are expected to be available within the next decade.

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