Comprehensive model of a hermetic reciprocating compressor

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Abstract: A comprehensive simulation model is presented to predict the performance of a hermetic reciprocating compressor and to reveal the underlying mechanisms when the compressor is running. The presented model is composed of sub-models simulating the in-cylinder compression process, piston ring/journal bearing frictional power loss, single phase induction motor and the overall compressor energy balance among different compressor components. The valve model, leakage through piston ring model and in-cylinder heat transfer model are also incorporated into the in-cylinder compression process model. A numerical algorithm solving the model is introduced. The predicted results of the compressor mass flow rate and input power consumption are compared to the published compressor map values. Future work will focus on detailed experimental validation of the model and parametric studies investigating the effects of structural parameters, including the stroke-to-bore ratio, on the compressor performance.

1. Introduction
Many simulation models for reciprocating compressors are presented in the literature. Navarro et al. [1] proposed a phenomenological model for a hermetic reciprocating compressor to predict the overall isentropic efficiency and volumetric efficiency. The efficiencies were expressed as a function of ten parameters representing various kinds of the main losses in the compressor, including the refrigerant superheating by motor cooling, mechanical loss dissipation, and pressure losses at inlet and outlet valves. Perez-Segarra et al. [2] presented a detailed model for the thermodynamic efficiencies to characterize hermetic reciprocating compressors. The authors split the volumetric efficiency, isentropic efficiency and combined mechanical-electrical efficiency into several partial efficiencies so as to denote effects of different thermodynamic phenomena. Winandy et al. [3] presented a simplified steady-state model for a reciprocating compressor to identify the processes which have an influence on refrigerant mass flow rate, compressor power and discharge temperature. Estupinan and Santos [4] presented a dynamic model of a hermetic reciprocating compressor that considered the lubrication of the journal bearings, but the model
did not include the frictional loss between the piston rings and cylinder wall. Todescat et al. [5] developed a model for the thermodynamic energy analysis in a hermetic reciprocating compressor. However, their model did not include the motor model.

In this paper, a comprehensive model for a hermetic R410A reciprocating compressor is presented. This model includes sub-models for the kinematics, compression process, frictional power loss and overall energy balance. In addition, the in-cylinder heat transfer sub-model, leakage sub-model and valve sub-model are included in the compression process model. Both the frictional power losses at the piston ring and bearings are considered in the current model. Moreover, a single-phase induction motor model similar to that of Dutra and Deschamps [6] is included. The algorithm of the solution to the whole compressor model is also provided. This comprehensive model can be very helpful for the compressor designer by investigating the effect of varying different parameters, including the reed valve shape and stiffness, on compressor performance.

2. Geometry and Kinematics Model

A schematic of the geometry and kinematics of the reciprocating compressor is shown in Fig. 1. The bottom dead center (B.D.C.) is the starting point of the crank angle $\theta$. The piston displacement $x$, velocity $U$, and acceleration $a$, are expressed as a function of the crank angle as follows,

$$x = \frac{S}{2}[1 - \cos(\theta + \pi) + \frac{\lambda}{4}(1 - \cos 2(\theta + \pi))]$$

$$U = \frac{S}{2}\omega(\sin \theta + \frac{\lambda}{2}\sin 2\theta)$$

$$a = \frac{S}{2}\omega^2(\cos \theta + \lambda \cos 2\theta)$$

Where, $\lambda$ is the ratio of the crank radius to the connecting rod length; $\omega$ is the compressor rotational speed in rad/s; $S$ is the piston stroke.

![Figure 1: Schematic of geometry and kinematics of a reciprocating compressor](image)

3. Compression process model

3.1 Governing equations

The refrigerant inside the cylinder is assumed to be at uniform state, which means the thermal properties of the refrigerant are uniform anywhere in the cylinder. The refrigerant thermal properties are evaluated using REFPROP [7]. According to the law of continuity, the mass change of refrigerant in the cylinder can be expressed by,
\[
\frac{dm_c}{dt} = \frac{dm_{suc}}{dt} + \frac{dm_{li}}{dt} - \frac{dm_{dis}}{dt} - \frac{dm_{ho}}{dt}
\]  
(4)

Where, \( m_c \) refers to the refrigerant mass in the cylinder. The conservation equation of energy can be derived by employing the first law of thermodynamics,

\[
\frac{dQ}{dt} + \frac{dW}{dt} = \frac{dm_{dis}}{dt} h_{dis} + \frac{dm_{ho}}{dt} h_{ho} + \frac{d(m_u c_v)}{dt} - \frac{dm_{suc}}{dt} h_{suc} - \frac{dm_c}{dt} h_i
\]  
(5)

Eq. (5) can be further transformed into the following equation which describes the refrigerant temperature in the cylinder as a function of crank angle,

\[
\frac{dT_c}{dt} = \frac{1}{m_c C_v} \left[ \frac{dQ}{dt} + \frac{dm_{suc}}{dt} (h_{suc} - h_c) + \frac{dm_{li}}{dt} (h_{li} - h_c) - T_c \left( \frac{\partial P_c}{\partial T_c} \right) \left( A_p U - v_c \frac{dm_c}{dt} \right) \right]
\]  
(6)

The instantaneous heat transfer rate between the refrigerant and the cylinder wall is determined using the heat transfer correlation proposed by Adair et al. [8],

\[
Nu = 0.053 Re^{0.8} Pr^{0.6}
\]  
(7)

Then, the heat transfer rate between refrigerant and cylinder wall is calculated,

\[
Q_{heat} = \frac{dQ}{dt} = h_{heat} A_{heat} (T_c - T_{wall})
\]  
(8)

The heat transfer coefficient \( h_{heat} \) is determined based on the Nusselt number calculated in Eq. (7). Additionally, Adair et al. [8] reported that the cylinder wall temperature changed little in a reciprocating compressor compared with its time averaged value in a cycle. Thus, the cylinder wall temperature is assumed to be constant, which is calculated by incorporating the overall energy balance model with the compression process model.

### 3.2 Valve model

The refrigerant mass flow rates through the suction and discharge valves are determined by assuming an isentropic, compressible flow,

\[
m_{valve} = \frac{dm_{valve}}{dt} = C_{valve} A_{valve} (2 \rho_{high} P_{high})^{\frac{1}{2}} \left\{ \kappa \frac{\rho_{low}}{\kappa - 1} \left( \frac{P_{low}}{P_{high}} \right)^{\frac{\kappa}{\kappa - 1}} \left( \frac{P_{low}}{P_{high}} \right)^{\frac{\kappa + 1}{2}} \right\}^{\frac{1}{2}}
\]  
(9)

The discharge coefficient \( C_{valve} \) of 0.58 is used for the suction valve and a coefficient of 0.6 is used for the discharge valve [9]. \( A_{valve} \) denotes the valve opening area for the suction and discharge valves. For a valve, there are two dominant regions, pressure dominant region and mass flux dominant region. In the pressure dominant region, the valve motion is dominated by the pressure difference across the valve. However, beyond the transition point the assumption that there is a significant pressure difference across the valve is no longer applicable and the valve motion is driven by the gas mass-flux. The transitional valve plate lift, below which the pressure force dominates, can be calculated by [10],
Where, \(D_{\text{port}}\) and \(D_{\text{valve}}\) are the diameter of valve port and valve plate, respectively. \(A_{\text{valve}}\) is the valve flow area. Both the suction valves and discharge valves are reed-type. The motion of the reed valve under the force due to the gas pressure difference across the valve can be approximated as a motion of a cantilever beam exerted by the same force. According to Soedel [11], the transverse displacement of a cantilever beam can be described by,

\[
EI \frac{\partial^4 u}{\partial x^4} + \rho A u = q
\]  

(11)

Where, \(E\) is the Young’s modulus; \(I\) is the moment of inertia of the beam cross section. \(\rho\) is the density of valve material; \(A\) is the cross-sectional area; \(q\) is the load per unit length; \(u\) is the instantaneous transverse displacement at position, \(x\).

The solution of Eq. (11) can be expressed as the combination of infinite modal series,

\[
u(x,t) = \sum_{k=1}^{\infty} \eta_k(t) U_k(x)
\]

(12)

Where, \(U_k(x)\) is the \(k\)-th natural vibration mode; \(\eta_k(t)\) is the time varying weight factor corresponding to \(k\)th natural vibration mode. The \(k\)-th natural vibration mode is expressed as,

\[
U_k(x) = \cosh \left( \frac{\lambda_k x}{L} \right) - \cos \left( \frac{\lambda_k x}{L} \right) - \cosh \lambda_k + \frac{\cosh \lambda_k}{\sinh \lambda_k} \frac{\sinh \left( \frac{\lambda_k x}{L} \right)}{\sinh \lambda_k} - \cosh \lambda_k + \sin \frac{\lambda_k x}{L} \right]
\]

(13)

Where, \(L\) is the length of the beam. For the reed valve presented in this paper, only the first mode shape is considered, i.e., \(k=1, \lambda_i=1.875\). Substituting Eq. (12) into Eq. (11) and utilizing orthogonality condition,

\[
\ddot{\eta}_k + 2 \zeta \omega_k \dot{\eta}_k + \omega_k^2 \eta_k = F_k
\]

(14)

Where,

\[
\omega_k = \frac{\lambda_k^2}{L} \sqrt{\frac{EI}{\rho A}}, \quad \zeta = \frac{\lambda_k}{2 \rho \omega_k}, \quad N_k = \int U_k^2(x) dx, \quad F_k = \frac{F_{\text{valve}}}{\rho AN_k}, F_{\text{valve}} \text{ is the flow force}
\]

(15)

The solution to the Eq. (14) can be obtained by

\[
\eta_k = e^{-\zeta \omega_k t} \left[ A \cos(\gamma_k t) + B \sin(\gamma_k t) \right] + \frac{F_k}{\omega_k^2}
\]

(16)

Where,

\[
\gamma_k = \omega_k \sqrt{1 - \zeta_k^2}
\]

(17)

Differentiate the equation with respect to time once and twice respectively,

\[
\ddot{\eta}_k = -\zeta \omega_k e^{-\zeta \omega_k t} \left[ A \cos(\gamma_k t) + B \sin(\gamma_k t) + e^{-\zeta \omega_k t} [-A \gamma_k \sin(\gamma_k t) + B \gamma_k \cos(\gamma_k t)] \right]
\]

(18)
\[ \ddot{\eta}_k = e^{-\zeta_k \omega_k t} \left[ (\zeta_k \omega_k M - N \gamma_k) \cos(\gamma_k t) + (\zeta_k \omega_k N - M \gamma_k) \sin(\gamma_k t) \right] \]  

Where,

\[ A = \eta_k(0) - \frac{F_k}{\omega_k}, \quad B = \frac{\ddot{\eta}_k(0) \omega_k + \eta_k(0) \zeta_k \omega_k^2 - F_k \zeta_k}{\gamma_k \omega_k}, \quad M = B \gamma_k - A \zeta_k \omega_k, \quad N = A \gamma_k + B \zeta_k \omega_k \]

Using Eq. (15), Eq. (18), Eq. (20) and Eq. (21), the instantaneous transverse displacement, velocity and acceleration of a cantilever beam at position \( x \) can be determined.

### 3.3 Leakage sub-model

The gas leaking through the piston ring gap is modeled as an isentropic, compressible fluid flowing through an orifice. The mass flow rate of the gas leakage is calculated by,

\[ m_{\text{gap}} = \frac{dm_{\text{gap}}}{dt} = C_{\text{gap}} A_{\text{gap}} P_u \left( \frac{2}{ZRT_u} \right)^{\frac{1}{\kappa}} \left( \frac{P_{\text{d}}}{P_u} \right)^{\frac{2}{\kappa}} - \left( \frac{P_{\text{d}}}{P_u} \right)^{\frac{\kappa+1}{\kappa}} \left( \frac{P_{\text{d}}}{P_u} \right)^{\frac{1}{\kappa+1}} \right), \quad P_{\text{d}} > 0.54 \]

\[ m_{\text{gap}} = \frac{dm_{\text{gap}}}{dt} = C_{\text{gap}} A_{\text{gap}} P_u \left( \frac{1}{ZRT_u} \right)^{\frac{1}{\kappa+1}} \left( \frac{2}{\kappa} \right)^{\frac{1}{\kappa+1}} \left( \frac{P_{\text{d}}}{P_u} \right)^{\frac{1}{\kappa+1}} \left( \frac{P_{\text{d}}}{P_u} \right) \leq 0.54, \text{choked} \]

Where the discharge coefficient \( C_{\text{gap}} \) is assumed to be 0.86 [12] and the compressible factor \( Z \) is determined by using REFPROP.

### 4. Dynamics and Frictional Power Loss Model

The force analysis on the crank-connecting rod driving mechanism during the up-stroke and down-stroke are given in Fig. 2 and Fig. 3 respectively. Employing the force balance on the system, all the force loads are determined which can be used to calculate the frictional power loss on the main and crank pin journal bearings.

![Figure 2: Schematic of force and moment analysis when the piston is in up-stroke](image1.png)

![Figure 3: Schematic of force and moment analysis when the piston is in down-stroke](image2.png)
4.1 Piston Ring Lubrication Model

The main frictional power loss in the reciprocating compressor can be located at the piston ring frictional power loss and journal bearing frictional power loss. There is a piston ring located between the piston and cylinder wall to seal the in-cylinder refrigerant. Considering the effect of the surface roughness on the lubrication performance, the model proposed by Yang and Zhao[13] is used to get the frictional power loss at the piston ring-cylinder wall interface.

4.2 Bearing Frictional Power Loss Model

As mentioned above, with the determined force loads from the force balance analysis, the frictional power losses at the main and crank pin journal bearings can be predicted using the regression method proposed by Stachowiak and Batchelor [14],

$$H_{friction} = 3.9307 e^{0.706 \nu_1, oil} \cdot 1.577 \nu_2, oil\cdot D_j^{2.240} R_j^{0.477} c^{2.240} T_s^{0.204} \left(1 + \ln W^* \right)^{1.324}$$

(22)

Where, \( \nu_1, oil \) and \( \nu_2, oil \) are the kinematic viscosities of the oil at 37.8°C and 93.3°C. \( T_s \) is the oil temperature in °F. The dimensionless load capacity \( W^* \) and clearance \( c \) are calculated by,

$$W^* = \frac{F^2 + F_j^2 + F_j^2 R_j^2}{\mu \omega U L R_j^2}$$

(23)

$$c = R_{bush} - R_{journal}$$

(24)

5. Single-Phase Motor Model

A single-phase induction electric motor is used in this conventional reciprocating compressor. As pointed out by Dutra and Deschamps [6], the single-phase induction electric motor can be analyzed using an equivalent circuit. Fig. 4 shows a typical equivalent circuit for a single-phase induction electric motor.

![Equivalent circuit of the single phase induction motor](image)

Figure 4: Equivalent circuit of the single phase induction motor [6]

\( Z_{stator}, Z_{rotor}, Z_m \) are stator impedance, rotor impedance and magnetized impedance. \( R \) and \( X \) are resistance and reactance respectively. The positive and negative sign refer to the forward loop and backward loop. Given the input voltage, resistance, reactance and motor slip ratio, the motor shaft angular speed, motor efficiency and power consumption can be determined.
6. Overall Energy Balance Model

The energy flow in the conventional reciprocating refrigeration compressor is shown in Fig. 5. It is assumed that there is no temperature gradient in the individual compressor components, i.e., the lumped temperature analysis method is used.

Fig. 6 shows the thermal resistance network for the overall energy flow in the compressor. The heat transfer rate between two compressor components is calculated as the ratio of the temperature difference to the thermal resistance. In order to find the thermal resistance, empirically correlated averaged Nusselt numbers are determined [10].

![Figure 5: Energy balance of the conventional reciprocating piston compressor](image1)

![Figure 6: Thermal resistance network of the overall energy balance](image2)

7. Numerical Algorithm

The flow chart of the algorithm used to solve the presented comprehensive compressor model is given in Fig. 7. Given the initial guess values for the temperatures of different compressor components, in-cylinder refrigerant mass/temperature and the suction valve state, the determination of the instantaneous in-cylinder refrigerant pressure and temperature becomes an initial value problem, which can be solved by a variable step size method RKF45. However, the end states of both the in-cylinder refrigerant and suction valve after one cycle need to match the initial state at the very beginning of the cycle. Moreover, the temperatures of the compressor components are also required to meet the overall energy balance. Therefore, Broyden’s method is employed to solve the nonlinear system of equations.

8. Results and Discussion

The hermetic reciprocating compressor investigated in this paper uses R410A as its working fluid. There are two cylinders located in the shell. Since the compression process of the refrigerant in each cylinder is the same, except for the compression process phase difference, the in-cylinder process in one of the cylinders is investigated. The bottom dead center (B.D.C.) is chosen as the starting point of the piston. Fig. 8 shows the P-V indicated diagram at different pressure ratios for one of the cylinders. The suction pressure stays constant and there is a 11°C superheat at the compressor inlet. As it is shown, the duration of the re-expansion process for a higher discharge pressure case is longer than that for a lower discharge pressure case. This is one of the reasons accounting for the decrease of the volumetric efficiency with an increment
in pressure ratio, since less volume is available for the suction gas through the suction valve if a higher discharge pressure is applied.

**Figure 7:** The flow chart of algorithm for the conventional reciprocating piston compressor model

**Figure 8:** P-V indicated diagram for one cylinder

Fig. 9 shows the variation of the discharge valve displacement with respect to the compressor crank shaft rotation angle for different discharge pressures. It is seen that the discharge valve is still open when the re-expansion process starts. Back flow occurs at the discharge valve due to the late closing of the valve plate, which can be seen in Fig. 10.

**Figure 9:** Discharge valve plate displacement v.s. crank shaft rotation angle

**Figure 10:** Discharge valve mass flow rate v.s. crank shaft rotation angle

The comparison between the predicted mass flow rate/input power consumption and the online performance table values are given in Table 1. The relative error of the mass flow rate and input power are around 5% and 10%, respectively.

| Tevap [°C] | Tcond [°C] | Ts [°C] | mass flow rate [kg/s] | input power [W] |
|-----------|-----------|---------|-----------------------|-----------------|
|           |           |         | map value | predicted | error (%) | map value | predicted | error (%) |
| 10        | 25        | 11      | 0.078     | 0.081     | 3.3       | 1442      | 1415      | -1.9      |
| 10        | 35        | 11      | 0.074     | 0.077     | 5.1       | 2025      | 2269      | 12.0      |
| 5         | 25        | 11      | 0.065     | 0.068     | 5.2       | 1593      | 1739      | 9.2       |
| 5         | 30        | 11      | 0.063     | 0.067     | 6.1       | 1830      | 2035      | 11.2      |
9. Conclusions

A comprehensive model for a hermetic R410A reciprocating compressor is presented. This model consists of kinematics model, in-cylinder compression process model, frictional power loss model, single phase induction motor model and overall energy balance model. The valve, leakage and heat transfer model are included in the compression process model. A numerical algorithm used to solve the whole compressor model is introduced.

The P-V diagram is given for one cylinder. The discharge valve is found to be still open even when the re-expansion process starts which brings the back flow of refrigerant from the discharge plenum to the in-cylinder control volume. The predicted mass flow rate and input compressor power consumption have a relative error of 5% and 10% respectively compared to published table values.

10. Nomenclature

\begin{itemize}
\item \(A\) area
\item \(C\) coefficient, specific heat capacity
\item \(D\) diameter
\item \(E\) Young’s modulus
\item \(H\) Power
\item \(I\) moment of inertia of the beam cross section area
\item \(L\) Length
\item \(P\) Pressure
\item \(Q\) heat transfer rate
\item \(R\) gas constant, radius
\item \(S\) piston stroke
\item \(T\) temperature
\item \(U\) velocity
\item \(V\) volume
\item \(W\) Load
\item \(Z\) compact factor, impedance
\item \(H\) unit enthalpy, or valve lift
\item \(M\) mass
\item \(Q\) load per unit length of beam
\item \(U\) transverse displacement of the beam
\item \(N\) specific volume
\item \(X\) piston displacement
\item \(\theta\) crank angle
\item \(\mu\) dynamic viscosity
\item \(\nu\) kinematic viscosity
\item \(\eta_k\) weight factor of k-th vibration mode
\item \(\kappa\) ratio of specific heat capacity
\item \(\lambda\) ratio of crank radius to connecting rod length
\item \(\rho\) density
\item \(\omega\) compressor angular speed
\item \(c\) refrigerant in control volume
\item \(cond\) condensing
\item \(cyl\) cylinder
\item \(d\) downstream
\item \(dis\) discharge
\item \(evap\) evaporating
\item \(high\) high pressure side
\item \(li, lo\) leak in and out
\item \(low\) low pressure side
\item \(mean\) mean velocity of piston
\item \(oil\) lubricating oil
\item \(p\) piston
\item \(sp\) superheat
\item \(suc\) suction
\item \(u\) upstream
\item \(valve\) valve
\end{itemize}
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12. References

[1] Navarro, E., Granryd, E., Urchueguia, J.F., Corberan, J.M., 2007. A phenomenological model for analyzing reciprocating compressors. Int. J. Refrigeration 30, 1254-1265.

[2] Perez-Segarra, C.D., Rigola, J., Soria, M., Oliva, A., 2005. Detailed thermodynamic characterization of hermetic reciprocating compressors. Int. J. Refrigeration 28, 579-593.

[3] Winandy, E., Saavedra, O.C., Lebrun, J., 2002. Simplified modeling of an open-type reciprocating compressor. Int. J. Therm. Sci. 41, 183-192.

[4] Estupinan, E.A., Santos, I.F., 2007. Dynamic modeling of hermetic reciprocating compressors, combining multibody dynamics, finite elements method and fluid film lubrication. Int. J. Mechanics 1 (4), 36-43.

[5] Todescat, M. L., Fagotti, F., Prata, A. T., Ferreira, R. T. S. 1992. Thermal energy analysis in reciprocating hermetic compressors. Proc. of the Int. Comp. Eng. Conf. Purdue University, West Lafayette, IN USA, pp. 1419-1428.

[6] Dutra, T., Deschamps, C. J., 2015. A simulation approach for hermetic reciprocating compressors including electrical motor modeling. Int. J. Refrigeration 59, 168-181.

[7] NIST. 2010. Reference fluid thermodynamic and transport properties (REFPROP) Version 9.0.

[8] Adair, R. P., Qvale, E. R., Pearson, J. T., 1972. Instantaneous heat transfer to the cylinder wall in reciprocating compressors. In: Proceedings of the International Compressor Engineering Conference. Purdue University, West Lafayette, IN USA, pp. 521-526.

[9] Lin, M., Sun, S.Y., 2006. Theory of the piston compressor. Xi’an Jiaotong University Press. Xi’an, China.

[10] Kim, J-H, Groll, E.A., 2007. Feasibility study of a bowtie compressor with novel capacity modulation. Int. J. Refrigeration 30, 1427-1438.

[11] Soedel, W., Sound and vibrations of positive displacement compressors, 2007. CRC Press, 162-168.

[12] Furuhama, S., Tada, T., 1961. On the flow of the gas through the piston-rings (1st report, the discharge coefficient and temperature of leakage gas). Bulletin of JSME 4 (16), 684-690.

[13] Yang, B., Zhao, Y., 2011. Piston ring-cylinder liner lubrication analysis in a CO2 refrigeration reciprocating compressor. Inst. of Mech. Eng., Part C: J. Mech. Eng. Sci., 225(11), pp. 2638-2648.

[14] Stachowiak, G.W., Batchelor, A.W., 2001. Engineering Tribology, 2nd ed. Butterworth Heinemann, Boston.