Inventory Model Considering Deterioration, Stock-Dependent and Ramp-Type Demand with Reserve Money and Carbon Emission

Dharmendra Yadav, S.R. Singh, Manisha Sarin

Abstract: Now a day, government is more concerned about the environment, so inventory model for deteriorating product for multi-product with partial backlogging is modeled here by considering carbon emission cost under the influence of inflation. It is also assumed buyer have sufficient amount of money to pay the vendor in the beginning of the business but still buyer focus to avail the offer of trade credit offered by vendor. As demand of many products such as fashionable products, cold drinks etc., get stabilized after the acceptance by the market and take the form of ramp-type. So, while developing the model, ramp-type initial stock-dependent demand is considered. As the life time of the product is finite so finite planning horizon is considered here. To obtain the optimal solution, search algorithm is provided. To illustrate and validate the model, numerical example is provided. Further, to study the effect of important parameters, sensitive analysis is also carried.

Keywords: Reserve Money, Multi-Product, Deterioration, Initial Stock-Dependent and Ramp-Type Demand, Partially Backlogging, Carbon Emission

I. INTRODUCTION

Every inventory practitioners while designing inventory control policies tries their best to optimize the total inventory cost which is blocked ideally in the form of inventory. To achieve this task, inventory practitioners try to boost the demand by analyzing the policy of trade credit. Due to this, the purchasing cost for the retailer reduces as well as the cost to hold the inventory of the supplier also reduces. Traditionally, retailer accepts this offer as she/he does not have sufficient money to pay at once to the vendor and finalize the account. Retailer can deposit the revenue in interest generating account which is received by selling the product during this period. On deposited amount retailer can earn interest. As the trade credit period is over, the vendor charges interest on the balance amount. Hence, trade credit period offered by supplier is a important promotional tool to attract the retailer. Retailer can take this as an alternative incentive policy. On acquainted with the benefits of trade credit option, two researchers Haley and Higgins (1973) introduced the inventory model taking permissible delay in payments. They considered that demand of the customer is fixed. Discounted cash flow (DCF) approach was adopted by Huang and Huang (2004) to investigate the EPQ model under the effect of time value of money and inflation by considering defective products. Mahata and Goswami (2006) developed and analyzed the fuzzy production lot-size inventory model. They incorporated the concept of permissible delay in payments. They considered that demand rate \(d\) and production rate \(k\) as triangular fuzzy numbers (TFN). An inventory model is analyzed by Singh et al. (2008). They assumed that received lot contains imperfect items. They also assumed that supplier offers trade credit to the retailer. Singh and Jain (2009) developed a deterministic inventory model for deteriorating items. Developed model is analyzed under the effect of inflation. Due to the short life time, they assumed that planning horizon is finite. They further considered that retailer had required amount to settle the vendor’s dues in the beginning of the planning period, but she/he/ wish to avail the offer of trade credit policy offered by vendor. Proposed inventory models have been solved by Singh and Jain with two different approaches. In first, they optimize the inventory cost and in other method they maximize the net profit function. Integrated economic order quantity model for retailer is investigated by Shastri et al. (2014) by considering two-levels of trade credit. Further, they assumed that items present in stock are perishable. An inventory policy is developed by Yadav et al. (2015) to obtain the payment time to settle the account by the retailer to the vendor under the effect of inflation. An inventory model is investigated by Cárdenas-Barrón et al. (2019) for buyer’s point of view. They assumed that buyer accept the offer of trade credit which is offered by vendor to boost the demand. From above, it is observed that most of the inventory researchers while modeling inventory models considered that buyer have no money to settle the dues of vendor at the beginning of the planning horizon. Retailers have to take loan to clear the account of the vendor. In the present study, a situation that the retailer has sufficient money to settle the dues of vendor but still retailer avail the offer of trade credit offered by vendor.

From the literature, it is observed that most of the inventory researcher considered that demand function is the function of continuous and differentiable function of time. From market analysis it is observed that demand of various goods such as canned foods, soft drinks, newly launched items (such as fashionable accessories, smart phone) etc., not remain identical throughout the planning horizon. It is also observed that demand is practically proportional to the displayed stock. But the assumption that demand cannot increase or decrease regularly over time seem to unrealistic. Due to the attraction of price or quality demand of product increases to the certain duration of the planning period. Demand of the product gets stabilized when the product is accepted by the user in the market. Then after, demand remains fixed for the remaining planning period.

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Ritchie (1980) was the inventory practitioner who considered this factor while modeling the model. Ritchie was the first researcher who coined the term ‘ramp-type’ to illustrate such type of demand. Wu (2001) developed an inventory model considering ramp-type demand with Weibull distribution deterioration and partially backlogging. Inventory policy for the seasonal products was developed by Panda et al. (2008). Demand of the seasonal product is represented by three times period classified ramp-type demand function. Lot size inventory model for deteriorating items is proposed by the researcher Yang et al. (2010). Proposed model is analyzed under the effect of stock-dependent demand with partially backlogging. Yang et al. assumed that replenishment cycle and shortages are time-varying. Economic production model for single product was developed by Pal et al. (2014) for deteriorating items with ramp type demand rate. Pal et al. assumed that production rate and demand rate are related to each other. Rate of deterioration is considered as function of two parameter Weibull's distribution. Retailer’s optimal strategy was examined by Cardenas Barron (2019) for nonlinear holding and stock dependent demand when retailer received offer of trade credit from the seller.

Most of the inventory practitioners assumed that due to evaporation, decay and spoilage causes deterioration of the products i.e., there is deviation in its worth from the original value (Zhang et al. 2015). Chen et al. (2017) found in literature that around 45% of North America retail industries and around 30% of supermarket compromises deteriorating products. Dye and Yang (2016) reviewed that due to deterioration, United States’ grocery stores suffered a loss of $30 billion every year. Chen et al. (2017) pointed out that most of the super-markets have shifted their pricing policy to increase the demand and to reduce the spoilage. An inventory model is investigated by Tashakkor et al. (2018) with the objective to maximize the profit of the system so that dynamic prices and replenishment cycle are optimal.

A. The Research Gap and Our Contribution

From literature survey, it is observed that although many inventory models have been developed but as far as our knowledge is concerned no researcher have raised the following issues while developing the inventory model:
1. As government is more concerned about the environment, carbon emission cost is considered here.
2. It is assumed buyer have money to settle the dues of vendor in the beginning but still buyer offer of trade credit which is offered by vendor.
3. Today, no business concerned for single product. So, multi-product inventory model is proposed here.
4. As product (fashionable products, cold drinks etc.) accepted by the market, demand of products get stabilized and take the form of ramp-type. So, here ramp-type stock-dependent demand is considered.
5. As we know that life of the product is finite, so we consider that the planning horizon is finite.
6. Effect of inflation and deterioration is also considered to increase the utility of the model.
7. Partial backlogging is considered here.

The rest of the study of the work is organized as follows: section-II contains the notations and assumptions which are used for the development of the proposed model. Section-III represents the systematic development and mathematical solution of proposed inventory model. Section IV contains solution algorithm. Section-V demonstrates numerical example and their solutions which are used to validate the proposed model. Section-VI contains the sensitivity analysis of the results obtained from the examples. Section-VII represents the concluding remarks and the scope for future research of the proposed work.

II. ASSUMPTIONS AND NOTATIONS

With the help of following assumption, an inventory model for multi-item has been developed in this paper:
1. There are ‘n’ products in inventory.
2. Lead time is taken negligible as the replenishment rate is very high.
3. Due to the finite life cycle of many products, time horizon of the inventory system is assumed to be finite.
4. Due to the presence of the products in stock, deterioration process of the product take place.
5. Items deteriorated during the planning horizon are not repairable.
6. Study has been carried out under the influence of inflation and time-value.
7. Shortages are allowed for each products and unmet demand partially backlogged which is the function of waiting time ‘t’.
8. Trade credit option is offered by the vendor to the buyer to enhance the demand.
9. Shape of the demand function of the i th products is consider as ramp type and it is the function of initial stock level of the products.
10. Carbon emission is considered due to the result of keeping the deteriorating items and warehousing the products.
11. Length of replenishment cycle for each product is same.

In order to develop the multi-item inventory model in this paper following notations have been used for the ith products where i=1, 2,……., n:

\[ Q_i(t) \] : Inventory of ith product at any instant of time ‘t’
\[ \theta_i \] : Deterioration rate for the ith product
\[ D_i(t) = \left( \frac{a_i e^{b_i t} + a_i Q_0}{\left( \frac{a_i e^{b_i t} + a_i Q_0}{\mu_i} \right) \mu_i t} \right) \] Demand rate for the ith product
\[ r_i \] : It is constant parameter which represents the difference between inflation rate and the discount rate and inflation rate
\[ \phi_i(t) = \left[ \frac{1}{1 + \delta_i (t - t)} \right] \] Backlogging rate for the ith product which is the function waiting time ‘t’
\[ K_i \] : Ordering cost per order for the ith product
\[ c_i \] : Purchasing cost per unit for the ith product
\[ s_i \] : Selling price per unit for the ith product
\[ c_{ih} \] : Inventory holding cost per unit for the ith product
\[ c_{ie} \] : Average of carbon emission cost for holding items in stock for the ith product
\[ c_i \] : Shortage cost per unit backordered per unit time for the ith product
\[ c_l \] : Penalty cost per unit for lost sale for the ith product
\[ c_{id} \] : Deterioration cost per unit for the ith product
\[ c_{de} \] : Average of carbon emission cost from
III. FORMULATION OF MATHEMATICAL FORM OF INVENTORY MODEL CONSIDERING CARBON EMISSION WITH RAMP-TYPE DEMAND

Two phases are considered during the replenishment cycle. During the first phase of replenishment cycle, inventory level is positive while in other phase it is negative. Initially, retailers ordered $Q_i (= Q_0^i + B_i)$ quantity of $i^{th}$ product out of which $B_i$ is utilized to fulfill the backlogged quantities. Rest of $Q_0^i$ units of stock decreases due to the demand of the customers and due to the deterioration until stock level reaches to zero at time $t = t_1^i$. At $t = t_1^i$ shortages occurred. This shortages backlogged partially at the rate $\frac{1}{1+\delta(T-t)}$. At the end of cycle, the stock level of $i^{th}$ product reaches a maximum shortage level $"B_i"$. Order placed in the subsequent cycle is such that ordered quantity clears the backlog. Also the ordered quantity raises the stock level upto $Q_0^i$. Fig.1 represents the inventory level for the whole planning period for the $i^{th}$ product. Planning period is divided into $N$ cycles. $T = \frac{H}{N}$ is the length of each cycle. Order are placed at $nT$, $n = 0, 1, 2 \ldots \ldots (N - 1)$.

Hence, the stock level $Q_i(t)$ for the $i^{th}$ product is represented by the following differential equation:

$$\frac{dQ_i(t)}{dt} = -(a_i e^{b_i t} + a_i Q_0^i) - \theta_i Q_i(t), \quad 0 \leq t \leq \mu_i^i$$

with equation of continuity is $Q_i'(\mu_i^-) = Q_i'(\mu_i^+) \ldots \ldots (1)$

$$\frac{dQ_i(t)}{dt} = -(a_i e^{b_i t} + a_i Q_0^i) - \theta_i Q_i(t), \quad \mu_i^i \leq t \leq t_1^i \ldots \ldots (2)$$

and $Q_i(t_1^i) = 0$

Further, at $t = t_1^i$ shortages occur. During the shortage period, demand of the product at any time $t$ partially backlogged at a rate $\frac{1}{1+\delta(T-t)}$. Thus, stock level for the $i^{th}$ item during $t_1^i \leq t \leq T$ is illustrated with the help of following differential equation:

$$\frac{dQ_i(t)}{dt} = -(a_i e^{b_i t} + a_i Q_0^i) \quad t_1^i \leq t \leq T \ldots \ldots (3)$$

with the boundary condition $Q_i(t_1^i) = 0$.

On solving equation (1), (2) and (3) by using the boundary condition, we get

$$Q_i(t) = e^{-\theta_i t} \left[ \int_{t_1^i}^{t} (a_i e^{b_i y} + a_i Q_0^i) e^{\theta_i y} dy + a_i e^{b_i t} + a_i Q_0^i \right]$$

$$Q_i(t) = e^{-\theta_i t} \left[ \int_{t_1^i}^{t} (a_i e^{b_i y} + a_i Q_0^i) e^{\theta_i y} dy + \int_{t_1^i}^{t} (a_i e^{b_i y} + a_i Q_0^i) e^{\theta_i y} dy \right]$$

$$Q_i(t) = - \left( a_i e^{b_i t} + a_i Q_0^i \right) \frac{1}{(1+\delta(T-t))} \int_{t_1^i}^{t} e^{\theta_i y} dy$$

Now, the maximum positive inventory level for the $i^{th}$ product per cycle is

$$Q_i^0 = \frac{1}{1+\theta_i t_1^i} \left[ \int_{t_1^i}^{T} (a_i e^{b_i y} + a_i Q_0^i) e^{\theta_i y} dy + a_i e^{b_i t} + a_i Q_0^i \right]$$

Maximum quantity of demand which can be backlogged for the $i^{th}$ product in each cycle is

$$B_i = -Q_i(T) \ldots \ldots (8)$$

Hence, the total ordering quantity for the $i^{th}$ product in each cycle is

$$Q_i = Q_0^i + B_i$$

$$Q_i = \frac{1}{1+\alpha_i t_1^i} \left[ \int_{t_1^i}^{T} (a_i e^{b_i y} + a_i Q_0^i) e^{\theta_i y} dy \right]$$

Now, we proceed to calculate the different cost associated with inventory step by step as follows:

Present Value of Ordering Cost ($OC_i$) for $i^{th}$ Product:

Order is placed at the beginning of each cycle. Thus, for every cycle, ordering cost for the $i^{th}$ product is as follows:

$$OC_i = K_i$$

Present Value of Purchasing Cost ($PC_i$) for $i^{th}$ Product:

We purchased the product at the beginning of each cycle. But retailer paid the account to the vendor at the end of the trade credit period offered by vendor. Thus, purchasing cost for the $i^{th}$ product is as follows:

$$PC_i = c_i Q_i e^{-rt_i M_i}$$

Present Value of Holding Cost ($HC_i$) for $i^{th}$ Product:

During $[0, t_1^i]$, inventory level of the product is positive. For the $i^{th}$ product, holding cost is the sum of the traditional inventory holding cost ($c_{i h}$) and the carbon emission cost ($c_{i e}^i$). Thus, holding cost is

$$HC_i = \left( c_{i h} + c_{i e}^i \right) \int_{t_1^i}^{T} Q_i(t) e^{-rt_i} dt + \chi_i + c_i e^{\theta_i t_i Q_i(t)}$$

$$HC_i = \left( c_{i h} + c_{i e}^i \right) \int_{t_1^i}^{T} e^{-(r+\theta_i)t} \left( \int_{t_1^i}^{T} (a_i e^{b_i y} + a_i Q_0^i) e^{\theta_i y} dy \right) dt$$
(c_{ih} + c_{he}) \int_{\tau_{ij}}^{t_{i+1}} e^{-(t_{i+1}-\tau_{ij})} \left( \int_0^{t_{i+1}} a_i e^{b_i y} + a_i' Q_0) d\tau \right) dt \\

Present Value of Deterioration Cost ($DC_i$) for $i^{th}$ Product:

Deterioration takes place due to the physical presence of the product in the inventory. So, deterioration occurs during $[0,t_i]$. Hence, deterioration cost for the $i^{th}$ product is the sum of the total deterioration cost ($c_{dh}$) and carbon emission cost ($c_{ue}$). Thus, deterioration cost is:

$$DC_i = (c_{ih} + c_{he}) \int_{\tau_{ij}}^{t_{i+1}} \left( \int_0^{t_{i+1}} a_i e^{b_i y} + a_i' Q_0) d\tau \right) e^{-\tau_{ij}} dt$$

$$= \int_{\tau_{ij}}^{t_{i+1}} \left( c_{ih} + c_{he} \right) \left( \int_0^{t_{i+1}} a_i e^{b_i y} + a_i' Q_0) d\tau \right) e^{-\tau_{ij}} dt$$

Present Value of Shortage Cost ($SC_i$) for $i^{th}$ Product:

Shortages occur in the inventory system during the period $[t_i, T]$. Shortage level is maximum at $t=T$. The total value of shortage cost during this time for $i^{th}$ product can be calculated as follows:

$$SC_i = c_{is} \int_{t_i}^{T} \left( \int_0^{t_i} a_i e^{b_i y} + a_i' Q_0) d\tau \right) e^{-\tau_i} dt$$

$$\int_{t_i}^{T} \left( c_{is} \left( \int_0^{t_i} a_i e^{b_i y} + a_i' Q_0) d\tau \right) e^{-\tau_i} dt$$

Present Value of Lost-sale Cost ($LS_i$) for $i^{th}$ Product:

Shortages occurred during the stock-out period $[t_i, T]$. It is also observed that all customers are not ready to wait for the next shipment to fulfill their demand. Hence, some loss of sale occurs. Due to which there is loss in profit as well as goodwill of the retailer in the market. The present value of this lost-sale cost for the $i^{th}$ product can be calculated as follows:

$$LS_i = c_{iL} \int_{t_i}^{T} \left( \int_0^{t_i} a_i e^{b_i y} + a_i' Q_0) d\tau \right) \left( 1 - \frac{1}{1 + e^{b_i y}(T - \tau_i)} \right) dt$$

Present Value of Interest Earned ($IE_i$) for $i^{th}$ Product:

According to the trade credit period, following two different cases are arises to evaluate the interest earned for the $i^{th}$ product:

(i) $M_i \leq t_i$

(ii) $M_i \geq t_i$

Case-1: $M_i \leq t_i$

As retailer have sufficient money in the starting of each cycle to purchase the $Q_i$ quantity of items. Retailer prefers to opt trade credit policy to earn the money. Revenue generated due to sale can be deposited by the retailer in the interest bearing account and settle all the dues of supplier when trade credit period ends up. During the trade credit period, retailer can deposited generated revenue in the same account till the end of the cycle.

$$IE_i = c_{iQ} \int_{t_i}^{M_i} a_i e^{b_i y} dt + s(t_i - \mu) + aQ0ie=rttd + sileit1I(11-t)iaiebijui+aQ0ie=rttd + sileit1I(t11-t)iaiebijui+aQ0ie=rttd$$

Present Value of Total Cost for the $i^{th}$ Product:

The present value of the total cost for the $i^{th}$ product = 'ordering cost' + 'purchasing cost' + 'holding cost' + 'deterioration cost' + 'shortage cost' + 'lost-sale cost' - 'interest earned'

$$TC_i = K_i + c_{iQ} e^{-rT} M_i +$$
Total Inventory Cost of the System:

Here, it is considered that there are ‘n’ products in the inventory system. Now, we evaluate the expression of total inventory cost of the system in two different cases.

Case-1: $M^i \leq t_1^i$

There are ‘N’ equal cycles in the planning horizon ‘H’. $T=H/N$ is the cycle length. $k_i$ is the point of replenishment point for the planning horizon for where $n=0,1,2,\ldots, (N-1)$. Here, in the beginning inventory level is ‘Q’ units and at the end there are shortages. In the subsequent cycle, ordered quantity is such that it clears up the backlogged quantity and the inventory level builds up to $Q_0$. Hence, for the whole planning horizon, the total inventory cost is

$$TC_1 = TC_1^i \left( 1 + e^{-rT} + e^{-2rT} + \ldots + e^{-(N-1)rT} \right)$$

Total inventory cost for the inventory system containing ‘n’ product is

$$TC_1 = \sum_{i=1}^{n} \left( e^{-(i-1)rT} \right) TC_1^i$$

To determine the inventory system in two different cases.

Case-2: $M^i \geq t_1^i$

Total inventory cost for the inventory system containing ‘n’ product is

$$TC_2 = \sum_{i=1}^{n} \left( e^{-(i-1)rT} \right) TC_2^i$$

Here, it is considered that there are “n” products in the inventory system. Now, we

$$= \left( 1 + e^{-rT} + e^{-2rT} + \ldots + e^{-(N-1)rT} \right)$$

end there are shortages. In the subsequent cycle, ordered quantity is such that it clears up the backlogged quantity and the inventory level builds up to $Q_0$. Hence, for the whole planning horizon, the total inventory cost is

$$TC_1 = TC_1^i \left( 1 + e^{-rT} + e^{-2rT} + \ldots + e^{-(N-1)rT} \right)$$

Subject to $M^i \geq t_1^i$

$$TC_2 = \sum_{i=1}^{n} \left( e^{-(i-1)rT} \right) TC_2^i$$

$$= \left( 1 + e^{-rT} + e^{-2rT} + \ldots + e^{-(N-1)rT} \right)$$

there are shortages. In the subsequent cycle, ordered quantity is such that it clears up the backlogged quantity and the inventory level builds up to $Q_0$. Hence, for the whole planning horizon, the total inventory cost is

$$TC_1 = TC_1^i \left( 1 + e^{-rT} + e^{-2rT} + \ldots + e^{-(N-1)rT} \right)$$

$$= \left( 1 + e^{-rT} + e^{-2rT} + \ldots + e^{-(N-1)rT} \right)$$

... (12)

... (13)

... (14)

... (15)

IV. SOLUTION PROCEDURE

The objective of the problem is to find the optimal value of $t_1^i$ where $i=1,2,\ldots,n$ and $N$. After getting the values of $t_1^i$ and $N$, optimal inventory cost is obtained by substituting these values in the expression of total cost.
It is also obvious that if $TC_1'$ or $TC_2'$ is optimal than $TC_1$ or $TC_2$ is also optimal. To derive the optimal solutions of $TC_1'$ or $TC_2'$, the following classical optimization technique is used.

**Step-1:** Take the partial derivative of $TC_j'$ where $j=1,2$ with respect to $t_i^*$ and equating the results to zero. The necessary conditions for optimality are

$$\frac{\partial TC_j'}{\partial t_i^*} = 0$$

**Step-2:** Now, we solve the above equation to obtain the value of $t_i^* = t_i^{*'}$.

**Step-3:** With $t_i^{*'}$, we check the sufficient condition for optimality.

$$\left(\frac{\partial^2 TC_j'}{\partial (t_i^*)^2}\right) > c_i^*'0$$

**Step-4:** With $t_i^{*'}$ found in step-2, determine $TC_j'(t_i^{*'})$.

**Step-5:** Now, we derive the optimal value of $N$ such that it must satisfy the following condition:

$$TC_j^{*'}(N-1) \geq TC_j^{*'}(N) \leq TC_j^{*'}(N+1)$$

V. Numerical Illustration

In order to analyze the proposed model (case-1 of section 4), following input data in appropriate units has been taken.

**Example-1:** With the help of following data taken in appropriate unit, problem-1 is illustrated in the case when $M_1^*=t_i^*$

| No. of product | $c_1^*$ | $c_2^*$ | $c_{da}$ | $c_d^*$ | $c_i^*$ | $s^*$ | $a^*$ |
|---------------|-------|-------|---------|-------|-------|------|------|
| 2.4           | 0.28  | 0.0   | 0.0     | 0.0   | 0.6   | 6    | 1.4  |
| 2.5           | 0.30  | 0.0   | 0.0     | 0.0   | 0.6   | 6    | 1.5  |
| i=2           | $b_1^*$ | $a_1^*$ | $b_1^*$ | $Q_1^*$ | $I_1^*$ | $K_1^*$ | $H_1^*$ |
| 4.2           | 0.02  | 0.5   | 0.8     | 0.4   | 0.01  | 0.04 | 0.95 |
| 4.5           | 0.02  | 0.5   | 0.7     | 0.4   | 0.01  | 0.04 | 1.00 |

Table-1: Optimal Solution for Different Value of $M_i^*$:

| Trade Credit Period | N | T | $t_i^*$, i=1,2 | Total Inventory Cost |
|---------------------|---|---|----------------|----------------------|
| $M_1^*=0.18$        | 1 | 1 | $t_1^* = 0.58$, $t_2^* = 0.55$ | 4925.26 |
|                     | 2 | 0.5 | $t_1^* = 0.29$, $t_2^* = 0.27$ | 4738.34 |
|                     | 3 | 0.3 | $t_1^* = 0.18$, $t_2^* = 0.17$ | 4869.89 |
| $M_2^*=0.11$        | 1 | 1 | $t_1^* = 0.58$, $t_2^* = 0.57$ | 4989.97 |
|                     | 2 | 0.5 | $t_1^* = 0.29$, $t_2^* = 0.28$ | 4869.72 |
|                     | 3 | 0.3 | $t_1^* = 0.19$, $t_2^* = 0.18$ | 4978.06 |
| $M_3^*=0.10$        | 1 | 1 | $t_1^* = 0.59$, $t_2^* = 0.57$ | 5139.09 |
|                     | 2 | 0.5 | $t_1^* = 0.30$, $t_2^* = 0.29$ | 4989.98 |
|                     | 3 | 0.3 | $t_1^* = 0.20$, $t_2^* = 0.18$ | 5189.37 |

Table-1 shows the optimal solution for the different value of trade credit period. Table reflects that trade credit period and total inventory costs are negatively correlated. By decreasing the trade credit period, order quantity decreases. Therefore, ordering cost increases and hence inventory cost of the system increases.

VI. SENSITIVITY ANALYSIS

Sensitive analysis is performed in this section with respect to crucial parameter. We change the value parameter by -40%, -20%, 20% and 40% one by one keeping other parameters at their original values and observed its effect on optimal policy.

A. Sensitivity w.r.t ‘shortage cost’

Fig.2 reflects that shortage cost, time of positive inventory and total inventory cost are positively related. This shows that as the shortage cost increases, ordered quantity increases. Therefore, total inventory cost also increases.

B. Sensitivity w.r.t ‘Lost sale cost’

Fig.3 reflects that as the lost sale cost increase, time of positive inventory and total inventory cost also increases.

C. Sensitivity w.r.t ‘Backlogging Rate’

Fig.4 reflects that as the backlogging rate increases, total inventory cost also increases.
Fig. 4 reflects that time of non-negative inventory and rate of backlogging are positively correlated. Total inventory cost of the system increases due to increase in backlogging rate. Demand of the product during the shortage period decreases due to increase in rate of backlogging. Therefore, there is decline in the lost sale cost.

D. Sensitivity w.r.t ‘Rate of Deterioration’

On the analysis of Fig. 5, we can say that increase in rate of deterioration, time of positive inventory level decreases whereas inclination is observed in the total inventory cost. Deterioration cost increases due to increase in the rate of deterioration and hence inclination in the total inventory cost is observed.

E. Sensitivity w.r.t ‘Carbon Emission Cost of Holding Products’

It is observed from fig. 6, increase in the carbon emission cost of holding the products in inventory, inclination in total inventory cost is observed while no effect on the time of positive inventory is observed.

F. Sensitivity w.r.t ‘Carbon Emission Cost of Deteriorating Products’

Fig. 6 reflects that as the carbon emission cost of deteriorating products increases total inventory cost also increases but there is no effect on the time of positive inventory.

VII. CONCLUDING REMARKS AND FUTURE RESEARCH

In the proposed work, multi-product inventory model is analyzed by considering deterioration, time value of money, and partial backlogging. Present work is focused on the issue that the retailer has sufficient money to pay off the supplier but retailer adopted the trade credit policy offered by seller. In this paper, inventory model has been developed raising the practical issues such as carbon emission and ramp-type stock-dependent demand. Industries of fashionable products, electronic products etc., are examples where the proposed model is applicable. Proposed inventory model is validated by using numerical examples. At last, sensitive is workout with respect to important parameters of the inventory system.

From the sensitivity, it is observed that total inventory cost is highly sensitive with respect to shortage cost but least influenced by the carbon emission cost of holding products. So, retailer must pay special attention on shortages while making optimal policy. Inventory policy is also influenced by lost sale cost. Thus, while developing the inventory policy, retailer should choose a policy in such a way that lost sale cost and shortage cost can be reduced. It is also found that change in rate of backlogging have positive impact on the objective function of the system.

From the above analysis, it is observed that decision-maker, while developing the inventory policy should pay special attention on these parameters. By doing so, inventory cost will be reduced and retailer can manage to free the blocked amount for additional purposes. There are several promising areas to extend the proposed model to make it more fruitful. Proposed model can be reworked by taking different cost of inventory as imprecise parameters. Considering deterioration which follows Weibull distribution is also a very interesting extension.

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