Fundamental parameters of RR Lyrae stars from multicolour photometry and Kurucz atmospheric models – I. Theory and practical implementation

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ABSTRACT

A photometric calibration of Kurucz static model atmospheres is used to obtain the following parameters of RR Lyrae stars: variation of stellar angular radius $\vartheta$, effective temperature $T_e$, gravity $g_e$ as a function of phase, interstellar reddening $E(B - V)$ towards the star and atmospheric metallicity $M$. Photometric and hydrodynamic conditions are given to find the phases of pulsation when the quasi-static atmospheric approximation (QSAA) can be applied. The QSAA is generalized to a non-uniformly moving spherical atmosphere, and the distance $d$, mass $M$ and atmospheric motion are derived from the laws of mass and momentum conservation. To demonstrate the efficiency of the method, the $UBV(RI)_C$ photometry of SU Dra was used to derive the following parameters: $[M] = -1.60 \pm 0.10$ dex, $E(B - V) = 0.015 \pm 0.010$, $d = 663 \pm 67$ pc, $M = (0.68 \pm 0.03)$ $M_\odot$, equilibrium luminosity $L_{eq} = 45.9 \pm 9.3$ $L_\odot$ and $T_{eq} = 6813 \pm 20$ K.

Key words: hydrodynamics – stars: atmospheres – stars: fundamental parameters – stars: variables: RR Lyrae.

1 INTRODUCTION

Although the determination of the fundamental parameters of RR Lyrae (RR) stars is interesting in itself, it is also important from a practical point of view because RR stars play a considerable role in establishing Galactic and extragalactic distance scales. The Preston index and spectroscopic observations are used to determine their atmospheric metallicity $[M]$ and interstellar reddening $E(B - V)$. Since confirmed RR-type components are not known in binary systems, the mass determination is based on both stellar evolution and pulsation theories. Due to the uncertainty of parallax data, the Baade–Wesselink (BW) method is mostly used to infer their distance $d$ (Smith 1995). In the BW analysis, and to determine $[M]$ and $E(B - V)$, the quasi-static atmosphere approximation (QSAA) is employed to interpret photometry and spectroscopy.

The QSAA was introduced by Ledoux & Whitney (1961): 'The simplest approach is to assume that at each phase, the atmosphere adjusts itself practically instantaneously to the radiative flux coming from the interior and to the effective gravity $g_e$,

$$g_e = GM/R^2 + \ddot{R}$$

(1)

where $R$, and $\ddot{R}$ are the instantaneous values of the radius and acceleration, which is supposed uniform throughout the atmosphere’, $G$ is the Newtonian gravitational constant, $M$ is the stellar mass and $t$ is a differentiation with respect to time $t$. One may then build a series of static model atmospheres’ and select one of them at each phase by spectroscopic or photometric observations. Its flux, colours, effective temperature $T_e$ and surface gravity $g_e$ are accepted as the atmospheric parameters of that phase providing a basis for the determination of other parameters such as angular radius, mass, distance, etc.

The subject of this paper is the QSAA and its generalization to a non-uniform atmosphere. We will investigate the QSAA from the point of view of atmospheric emergent flux and hydrodynamics. By comparing the observed colour indices with those of static model atmospheres (Castelli, Gratton & Kurucz 1997; Kurucz 1997), we select the phases when they coincide. Considering hydrodynamics, we do not construct a consistent dynamic model of an RR atmosphere. However, we find a better description of the pulsating atmosphere if we characterize it by pressure and density stratifications in addition to the two parameters $R$ and $\dot{R}$. We determine the fundamental parameters $M$ and $d$ from the hydrodynamic considerations without using the BW method or theories of stellar evolution and pulsation at all. Our method uses photometry as observational input; spectroscopy and radial velocity observations are not needed.

In Section 2, conditions of the QSAA are formulated for a spherically pulsating compressible stellar atmosphere with a velocity gradient. Practical methods are described to determine $[M]$, $E(B - V)$, $T_e(\psi)$ and log $g_e(\psi)$ from $UBV(RI)_C$ colours of Kurucz atmospheric models, $\psi$ being the pulsation phase. The laws of mass and momentum conservation are used to determine the mass and distance of the pulsating star. Section 3 presents the results obtained from the $UBV(RI)_C$ photometry of SU Dra. The discussion and conclusions are given in Sections 4 and 5, respectively.
2 THE QUASI-STATIC ATMOSPHERE APPROXIMATION

First, we describe a photometric method to select the phases in which QSAAS can be regarded as a good approximation from a point of view of fluxes. Next, we introduce the time-dependent pressure and density stratifications of the selected static models in the laws of mass and momentum conservation to find the phases in which QSAAS can be regarded as a more or less good approximation from a hydrodynamic point of view as well. Using the phases of valid QSAAS from photometric and hydrodynamic points of view, we determine mass, distance and infer the internal motions of the atmosphere with respect to the stellar radius \( R \). In this paper, we define \( R \) as where the optical depth is zero, since the emergent fluxes of a theoretical model are given for \( \tau = 0 \) in any photometric band. Of course, the zero boundary condition is a mathematical idealization (Ledoux & Whitney 1961). In the following, we tacitly assume that the density is not zero at the boundary, i.e. \( \rho(R) > 0 \) if \( 0 \leq \tau \ll 1 \).

2.1 QSAAS from the photometric point of view

The functions \( T_\epsilon[C_1, C_2, [M], E(B - V)] \) and \( g_\epsilon[C_1, C_2, [M], E(B - V)] \) of the Kurucz atmospheric models form a suitable grid for interpolation in the ranges \( 6000 < T_\epsilon < 8000 \) and \( 1.5 < \log g_\epsilon < 4.5 \) if the colour indices \( C_1 \) and \( C_2 \) are selected appropriately from \( UBV \) photometry. For fixed \([M]\) and \( E(B - V) \), the intersection of the two functions \( T_\epsilon(g_\epsilon, C_1, C_2, [M], E(B - V)) \) for a given pair of \( C_1 \), \( C_2 \). Although it is more practical to use \( U - 2B + V \) instead of \( U - B \) (Barcza & Benkő 2009), the other common colour indices, e.g. \( U - V \), etc., will also be kept.

Using \( UBV(R) \) \( C \) photometry, we can construct four independent colour indices \( C_{1,2,3,4} \), i.e. \( \lambda_2 \) = 6 combinations of colour indices results in six pairs of \( T_\epsilon, g_\epsilon \) for a given \( \phi \). Since the \( U \) band covers the Balmer jump, the indicator of \( g_\epsilon \), the combinations must contain at least one colour index containing \( U \) photometry. \( \lambda_1 \) = 56 such combinations can be constructed. The average of the 30 pairs of \( T_\epsilon, g_\epsilon \) will be accepted for a phase \( \phi \).

Condition I. If the scatter \( \Delta T_\epsilon(\phi), \Delta \log g_\epsilon(\phi) \) of \( T_\epsilon(\phi), \log g_\epsilon(\phi) \) are compatible with the expected scatter from the error of the colour indices, QSAAS provides a good approximation in this phase from the photometric point of view.

The bolometric corrections and \( UBV(R) \) \( C \) physical fluxes of a model with given \( T_\epsilon, g_\epsilon, [M], E(B - V) \) are available in a tabular form (Kurucz 1997), and they allow us to determine the angular radius of the star defined by

\[
\phi(\phi) = R(\phi)/d. \tag{2}
\]

Technical details of finding the appropriate model and determining \( \phi \) at a phase \( \phi \) are given in Barcza (2003) and Barcza & Benkő (2009).

On the scale provided by the Kurucz atmospheric models, the above procedure offers a possibility of determining reddening and atmospheric metallicity from photometry without using spectroscopy. In the shock-free phases, the QSAAS is expected to reproduce the colours well; therefore, \( \Delta T_\epsilon, \Delta \log g_\epsilon \) must be minimal as a function of the \( \phi \)-independent model parameters \( E(B - V), [M] \)

(Barcza & Benkő 2009). Of course, the method can also be applied for non-variable stars to determine \( E(B - V), [M], T_\epsilon, \log g_\epsilon, \theta \).

2.2 QSAAS from the hydrodynamic point of view

In the original form of the QSAAS (Ledoux & Whitney 1961), the atmospheric motion is described by the parameters \( R(\phi), R(\phi), \dot{R}(\phi) \) and the outward or inward accelerations are driven by the variable \( g_\epsilon(\phi) - g_\epsilon[R(\phi)] \) in (1), where \( g_\epsilon = G M/R^2 \). This is essentially a uniform atmosphere approximation (UAA). However, the atmosphere of an RR star is a dilute, compressible gaseous system with variable temperatures. Therefore, the introduction of additional parameters containing \( T_\epsilon(\phi) \) promises a better description in the frame of the QSAAS.

Considering the geometry of radial pulsation modes (Smith 1995) and neglecting rotation will result in all components of the vector field velocity \( v(r, t) \) being zero except for the radial component \( v_r(t) \). The viscosity is negligible at the velocities occurring in an RR atmosphere. Therefore, the momentum conservation is expressed by the Euler equation of hydrodynamics:

\[
\frac{\partial \dot{v}}{\partial t} + v \frac{\partial \dot{v}}{\partial r} + g_\epsilon(t) + \frac{1}{\rho} \frac{\partial \rho}{\partial r} + a^{(\text{ang})}(t, r) = 0, \tag{3}
\]

where \( t = P \phi \) and \( P \) is the pulsation period.

The contribution of the neglected tangential motions is the average \( a^{(\text{ang})} = \frac{4\pi}{\rho} \int_0^\infty \int_0^{2\pi} \int_0 \phi \frac{\partial \dot{v}}{\partial \phi} \frac{d\phi}{\rho} d\phi d\theta \frac{\rho \sin \theta}{2\phi} d\phi \) (Landau & Lifshitz 1980). In perfect spherical symmetry, \( a^{(\text{ang})} = 0 \). The actual pressure and density stratifications were divided into two parts: \( \dot{p}(r, t) = p_r(t) + p^{(\text{dyn})(r, t)} \) and \( \dot{g}(r, t) = g(r, t) + g^{(\text{dyn})(r, t)} \), where \( p_r(t) \) and \( g(r, t) \) denote the values of a static model atmosphere, while \( p^{(\text{dyn})(r, t)} \) and \( g^{(\text{dyn})(r, t)} \) are the dynamical corrections due to pulsation.

From the hydrodynamic point of view, the essence of the QSAAS is that at a given \( t \)

\[
- \frac{1}{\rho} \frac{\partial \rho}{\partial r} = g_\epsilon(t) \geq 0, \tag{4}
\]

i.e. the \( r \)-independent effective gravity of a static model atmosphere is introduced in (3) and \( p^{(\text{dyn})(r, t)} = 0, g^{(\text{dyn})(r, t)} = 0 \). In what follows, we drop this latter restriction and define a dynamical correction term of acceleration

\[
a^{(\text{dyn})} = \frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{1}{\rho} \frac{\partial \rho}{\partial r}, \tag{5}
\]

to account for the difference of a static and dynamical atmosphere. Because of the lack of dynamical model atmospheres, \( a^{(\text{dyn})} \) must be determined empirically from the quantities derived from photometry.

Equation (3) is transformed to definition (1) of QSAAS if \( \dot{v}/\partial r = 0 \) and \( a^{(\text{ang})} = 0 \), \( a^{(\text{dyn})} = 0 \), because these simplifications imply \( \dot{v}/\partial t = \dot{R} \) and \( g_\epsilon(R) = G M R^{-2} \), i.e. the UAA is valid. As a dynamical equation, this simplified equation of motion is the basis of mass determination in BW analyses (e.g. Cacciari, Clementini & Buser 1989; Liu & Janes 1990).

If the sound velocity is assumed as an upper limit for \( v_{\text{L}} \) and \( v_{\text{L}} \), \( a^{(\text{ang})} \) is some 0.1 ms\(^{-2}\). It is negligible in comparison with the other components of the acceleration at any \( r < R \). Typical values are \( g_\epsilon \approx 10^{-2} \text{ms}^{-2} \) during the pulsation cycle of an RR star while \( g_\epsilon \) can exceed 100 ms\(^{-2}\) when the atmosphere is in the state of maximal compression. Thus, \( a^{(\text{ang})} \) will be neglected and

\[
g_\epsilon(t) = g_\epsilon(t) - g_\epsilon(r), \tag{6}
\]
is a periodic function of \( t \) with zero-points.
If the r-dependence of the temperature T is neglected and a constant density upper boundary condition is assumed, an integration of (4) over the interval [r, R] with the equation of state of a perfect gas gives the approximate density stratification of the model atmosphere:

\[ g(r, t) = g(R) \exp[-h_0(R, t)[r - R(t)]] , \]

where \( h_0(R, t) = \mu g_0(t) / R T(R) \), R, \( \mu \) are the reciprocal barometric scaleheight at R, the universal gas constant and the average molecular mass, respectively. Essentially, we use the static model atmospheres to measure \(-\varrho^{-1}(\partial \varrho / \partial r)\). The atmospheric pulsation is driven by \( g(r, t) - a^{\text{dyn}} \) and the thermal processes are represented by the variable \( h_0 \).

**Condition II.** If \( \partial v / \partial t + \varrho \partial v / \partial r \approx g \), i.e. \( |a^{\text{dyn}}(r, t)| \lesssim 2 \Delta g \), QSAA provides a good approximation in this phase from the hydrodynamic point of view.

This condition formulates the fact that the dynamical excess of acceleration in the upper photosphere is smaller than the \( \Delta g \) error of \( g \) and the same error is assumed for \( \partial v / \partial t + \varrho \partial v / \partial r \). The combination of (5) and (3) allows us to estimate the constant and \( O(d) \) terms of \( a^{\text{dyn}} \). However, the satisfaction of Condition II can be checked afterwards when the terms with other powers of \( d \) were determined. The quotient \( g(r, t) / a^{\text{dyn}} / g \) characterizes the degree of excellence of QSAA. The QSAA is exact from the hydrodynamic point of view if \( a^{\text{dyn}} = 0 \).

The perfect spherical symmetry means that in a Euler picture (Pringle & King 2007), we have to use (3)–(7) with \( a^{\text{lin}}(r, t) = 0 \) and the mass conservation law

\[
\frac{\partial \varrho}{\partial t} + \varrho \frac{\partial v}{\partial r} + \frac{\partial}{\partial r} \left( \frac{\varrho v^2}{r} \right) = 0
\]

must be taken into account. With assumption (7), the analytic solution to (8) is

\[
v(r, t) = -a_1 r + a_0 + a_2 r^{-1} + a_3 r^{-2},
\]

where \( a_1 = Ch_0^{-1}(\partial h_0 / \partial t) \), \( a_0 = \bar{R} + C a_1 (R - 3 h_0^{-1}) \), \( a_2 = 2 C a_0 h_0^{-1} \), \( a_3 = C a_1 - h_0^{-1} \), \( C = 1 \). C was introduced to incorporate UAA by taking \( a^{\text{dyn}}(r, t) = 0 \) and \( C = 0 \). The main term in \( v \) is \( \bar{R} \) and the convergence of (9) is excellent because \( R h_0 \gg 1 \).

After differentiations of \( v, h_0 \), and \( \partial \), taking \( r = R \), the following form is convenient for a numerical solution:

\[
M(d, t_1) - M(d, t_2) = 0,
\]

where \( M(d, t) = (g_v(t) - (\partial a^{\text{lin}} / \partial t) - \varrho (\partial v / \partial r) - a^{\text{dyn}}(R, t)) R^2 / G \). It can be solved if there exist two or more \( r \) intervals satisfying Condition I and \( a^{\text{dyn}}(R, t) \) is identical for them. The assumed differences \( a^{\text{dyn}}(R, t_1) - a^{\text{dyn}}(R, t_2) \approx 0 \) and the satisfaction of Condition II must be verified afterwards. Equation (10) must be solved for different phase pairs \((\varphi_1, \varphi_2) \) and the values \( d \) and \( M \) must be omitted from the final averaging if \( a^{\text{dyn}}(R, \varphi_1) \neq a^{\text{dyn}}(R, \varphi_2) \).

To account for the difference of the local and effective temperatures, the boundary temperature \( T(R) = 2^{-0.25} T_e \) of a grey atmospheric model at \( \tau = 0 \) (Mihalas 1978) will be used in \( h_0(R, t) \). Hydrogen and helium are mainly neutral at the boundary temperature of RR stars. Consequently, assuming a typical chemical composition means that \( \mu = 1.3 \) is the appropriate choice in \( h_0(R, t) \).

### 3 RESULTS FOR SU DRA

Barcza (2002) derived good quality light curves of the RRA star SU Dra by homogenizing 45 yr of UBVR(I)c observations (see his table 7). The method described in the previous section will now be applied. Instead of smoothing the light curves, binned data will be used to keep the results as close as possible to observed quantities. The 30 colour index pairs containing at least one \( U \) were obtained from CI \(|n=10| = \{ U - 2 B + V, U - V, U - R, U - I, B - V, B - R, B - I, V - R, V - I, R - C, R - I \} \). Taking \( |M| = -1.60 \) and \( E(B - V) = 0.015 \) (Liu & Janes 1990), the basic quantities for (10) are summarized in Table 1 for different phases. Fig. 1 is a plot of \( T_e(\varphi), \vartheta(\varphi), \log g_v(\varphi), h_0(R, \varphi) \) for one whole pulsation.

**Table 1.** The basic quantities for (10) at various phases. The units are cm s\(^{-2}\), K, rad \( \times 10^{10} \) and cm\(^{-1}\) \( \times 10^{10} \).

| \( \varphi \) | \( \log g_v \) | \( T_e \) | \( \vartheta \) | \( h_0(R) \) |
|---|---|---|---|---|
| 0.1 | 3.08 ± 0.07 | 7418 ± 27 | 1.627 ± 0.004 | 30.0 | −, − |
| 0.15 | 2.59 ± 0.03 | 7021 ± 10 | 1.723 ± 0.003 | 10.4 | I, II |
| 0.2 | 2.16 ± 0.04 | 6735 ± 14 | 1.792 ± 0.004 | 3.99 | I, − |
| 0.25 | 2.03 ± 0.04 | 6596 ± 12 | 1.801 ± 0.004 | 3.00 | I, II |
| 0.3 | 2.14 ± 0.04 | 6519 ± 13 | 1.799 ± 0.004 | 3.96 | I, II |
| 0.35 | 2.39 ± 0.07 | 6495 ± 18 | 1.774 ± 0.004 | 7.10 | −, − |
| 0.4 | 2.67 ± 0.08 | 6524 ± 21 | 1.723 ± 0.004 | 13.4 | −, − |
| 0.45 | 2.70 ± 0.05 | 6519 ± 13 | 1.692 ± 0.004 | 14.3 | −, − |
| 0.5 | 2.57 ± 0.01 | 6418 ± 3 | 1.717 ± 0.003 | 10.7 | L, − |
| 0.55 | 2.38 ± 0.04 | 6328 ± 8 | 1.740 ± 0.003 | 7.11 | I, II |
| 0.6 | 2.29 ± 0.06 | 6292 ± 11 | 1.741 ± 0.004 | 5.83 | −, − |
| 0.65 | 2.30 ± 0.06 | 6374 ± 11 | 1.678 ± 0.004 | 5.77 | −, − |
| 0.7 | 2.25 ± 0.06 | 6448 ± 17 | 1.616 ± 0.006 | 5.18 | −, − |
| 0.93 | 3.95 ± 0.06 | 7055 ± 22 | 1.546 ± 0.006 | 235 | −, − |
| 0.94 | 4.05 ± 0.03 | 7300 ± 12 | 1.545 ± 0.003 | 289 | L, − |
| 0.94 | 4.17 ± 0.05 | 7502 ± 27 | 1.520 ± 0.006 | 371 | −, II |
| 0.95 | 4.01 ± 0.07 | 7582 ± 42 | 1.520 ± 0.008 | 164 | −, − |

Note. Satisfaction of Conditions I and/or II is indicated in the last column by I and/or II, respectively.

**Figure 1.** \( T_e, \vartheta, \log g_v, h_0(R) \) of SU Dra as a function of phase. Filled circles indicate the phases when Condition I is satisfied.

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points lying below or close to the dotted lines. At phases lying above the dotted lines, the monochromatic flux of static models and SU Dra differs significantly on a level which has noticeable effect on the broad-band colours $UBV(RI)_C$. The assumed $[M] = -1.6, E(B-V) = 0.015$ were verified by a variation procedure (Barcz & Benkő 2009). In the shock-free phases $\psi = 0.15, 0.5, 0.55$, minimization of $\Delta T_{\psi} (\psi)$, $\Delta \log g_{\psi} (\psi)$ resulted in $[M] = -1.60 \pm 0.10, E(B-V) = 0.015 \pm 0.01$.

To demonstrate the difference between good and poor QSSA, the histograms of the 30 log $g_{\psi} \Delta T_{\psi}$ values are plotted in the upper panels of Fig. 2 for $\psi = 0.5, 0.98$ of SU Dra and BD $M$ 2010. The Author. Journal compilation RAS, MNRAS $i = \pm 0.15, 0.5, 0.55$, minimization of $i = \pm 0.93$. $i = -\phi$ $1, [\bar{\Delta}] = \pm \phi$, $\theta = \phi^{\psi}$, $T_{\psi} (T_{\psi})$, $C I = \pm \phi$.

$\phi = 0.98$, dashed: BD $67^\circ$ 708. Lower panels: standard errors $\Delta \log g_{\psi}$ and $\Delta T_{\psi}$ from 30 possible combinations of CI, as a function of phase. At phases below the dotted lines, the scatter can be attributed to observational errors of the colour indices.

$\phi = 0.015$ i.e. $\phi = 0.93$ was found. Thus, $\phi = 0.35$ Condition I is moderately violated, but Condition II is satisfied and $a^{\phi 0}(R, t) / g_{\psi} (R, t) \approx -0.07$; therefore, this phase was included to obtain $d, M$ of $[\star]$. Condition I is satisfied at $\phi = 0.2, 0.5$; however, these phases had to be excluded from the mass and distance determination because of the large $a^{\phi 0}(R, \phi = 0.2, 0.5) = 5.8, -3.3 \text{ m s}^{-2}$ and $q = -0.52, 38$, respectively.

Using our best solution for $M$ and $d$, the radius variation, velocities and the components of acceleration were computed in physical units and are plotted in Figs 3(b)–(d). Velocities and accelerations are plotted only for the phases of more or less good QSSA ($\phi = 0.15-0.3, 0.5, 0.55$), including the slightly shocked phases $\phi = 0.35-0.45$.

At the phases $\phi = 0.15, 0.25, 0.3, 0.35, 0.55 a^{\phi 0}(R, \phi = -79, 8, -16, 65, 93 \text{ cm s}^{-2}$ and Condition I is satisfied, $a^{\phi 0}(R, t) / g_{\psi} (R, t) < 0.13$, i.e. $a^{\phi 0}(R, \phi = 0.39, 0.02, 0.06, -0.10, -0.13$ were found. Thus, $\phi = 0.35-0.45$.

Table 2. Distance and mass of SU Dra from (10).

| $d$ (pc) | $M$ ($M_\odot$) | Used phases | Remark |
|----------|-----------------|-------------|--------|
| 663 ± 67 | 0.68 ± 0.03     | 0.15, 0.25–0.35, 0.55 | C = 1, [*] |
| 658 ± 104| 0.65 ± 0.03     | 0.15, 0.25, 0.30, 0.55 | C = 1     |
| 618 ± 200| 0.54 ± 0.10     | 0.20, 0.30, 0.50, 0.55 | C = 0, UAA |

Note. [*] indicates the values which we accept as the best ones.

$M(d, \psi)$ for the phases $t = \psi P$, $\psi = 0.15-0.35, 0.5, 0.55$ with $a^{\phi 0}(R, t) = 0$. The average and standard error of $M, d$ are given in Table 2 from the pairs ($\psi_1 = 0.25$ and $\psi_2 = 0.15, 0.3, 0.35, 0.55$) and ($\psi_1 = 0.35$ and $\psi_2 = 0.55$) in (10) as our best values denoted by $[\star]$. At $\psi = 0.35$ Condition I is moderately violated, but Condition II is satisfied and $a^{\phi 0}(R, t)/g_{\psi} (R, t) \approx -0.07$; therefore, this phase was included to obtain $d, M$ of $[\star]$. Condition I is satisfied at $\psi = 0.2, 0.5$; however, these phases had to be excluded from the mass and distance determination because of the large $a^{\phi 0}(R, \phi = 0.2, 0.5) = 5.8, -3.3 \text{ m s}^{-2}$ and $q = -0.52, 38$, respectively.

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4 DISCUSSION

From the point of view of radiative transfer, the plane-parallel approximation of the model atmospheres (Kurucz 1997) could be applied beyond doubt, because \( h_0^{-1} \leq 0.03 \) \( R \) holds for the whole pulsation cycle of an RR star. In favour of applying QSAA, semi-quantitative arguments were that the temperature changes are very slow even in a high amplitude RR star such as SU Dra: 5 K/2500 s < \(|\partial T_e/\partial t| < 1500 \) K/5000 s, while the characteristic time-scale of the radiative processes is below 200 s (Oke, Giver & Searle 1962; Buonaura et al. 1985). The effect of the variable effective gravity can be characterized by our Condition II, which provides information on whether QSAA may be assumed. The limits are 4.5 cm s\(^{-2}\)/2500 s < \(|\dot{g}_e/\dot{t}| < 1400 \) cm s\(^{-2}\)/2500 s. The upper limit comes from the rising branch of the light curve, when the main shock hits the atmosphere.

In general, the photometric input of the present method is identical with that of the BW method. In order to obtain the fundamental parameters, radial velocity data and their problematic conversion to pulsation velocities are not necessary. On the other hand, \( \vartheta \) and \( h_0 \) must be differentiated numerically; differential quotients are sensitive to the non-validity of QSAA.

Our photometric and hydrodynamic considerations revealed empirical quantitative conditions to find the phase intervals of the pulsation when the static model atmospheres of Kurucz (1997) are satisfactory to derive the variable and non-variable physical parameters of the pulsating atmosphere. Outside these intervals, dynamical model atmospheres are necessary to refine the parameters from QSAA, which is beyond the scope of this paper. The fundamental parameters \( d, M, [M] \), \( E(B - V) \) were determined using photometric quantities only in phases when the QSAA was a good approximation (i.e. both Conditions I and II were satisfied). The values obtained from averaging over the entire pulsation cycle can be considered as a first approximation only, because QSAA was assumed in all phases regardless of it being a good or poor approximation. The large error of \( L_\odot \) and \( (M_V) \) originates from \( \Delta d/\dot{d} \approx 0.1 \) of our best value [\( \star \)] in Table 2.

To give a good impression of the accuracy of inverting the \( UBV(RI) \)c photometry to physical parameters, the comparison star BD +67°708 was used, because its colours are similar to those of SU Dra (table 3 in Barcza 2002). The results are \( [M] = -0.77 \pm 0.03 \), \( E(B - V) = 0.000 \), \( \vartheta = (1.913 \pm 0.002) \times 10^{-10} \) rad, \( \log g = 3.59 \pm 0.01 \) and \( T_e = 7505 \pm 5 \) K. The errors are roughly in the same order of magnitude as those of SU Dra when Conditions I and II are satisfied.

4.1 Kinematics of the atmosphere

Figs 3(c) and (d) demonstrate that significant corrections must be added to \( \dot{R} \). If the true pulsation velocity and acceleration are required at \( 0 \leq \tau < 1 \). Moreover, the triangles show another correction: \(-4.6 \) km s\(^{-1}\) ≤ \( \dot{v} \) (\( R, \vartheta \)) = \( h_0^{-1} \vartheta \partial \vartheta/\partial t < 8.3 \) km s\(^{-1}\) if \( 0 < \vartheta < 0.6 \), i.e. \( v(r, \vartheta) = v(r) - \vartheta (r, \vartheta) h_0(R, \vartheta) \vartheta \partial \vartheta/\partial t \) is the velocity profile. In other words, even in the shock-free phases, considerable phase-dependent velocity and acceleration gradients exist in the layers \( R - h_0^{-1} < r < R \) of the line formation. The problem of \( \dot{v} \neq 0 \) is present in all phases, i.e. even in the shock-free intervals, which are used in modern BW analyses (e.g. Liu & Janes 1990).

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The non-uniform motion of the outermost layers introduces uncertainty when pulsation velocities are determined, because the centre-of-mass velocity $v_\gamma$ must be subtracted from the observed radial velocities. A recent exposition of the problem for Cepheid stars is given in Nardetto et al. (2009). By definition $\theta, \vartheta, \hat{\vartheta}$ refer to $0 \leq \gamma \ll 0.1$, while the variable component of the radial velocity is an average of velocities (e.g. 9) over the layers $R - h_0^{-1} \lesssim r < R$, i.e. $0 \leq \vartheta \leq 0.3$. The effect on $v_\gamma$ and $d$ has not yet been studied at all. Nevertheless, the importance is obvious, since an error of $1$ km s$^{-1}$ in $v_\gamma$ results in an error $\Delta d/d \approx 0.1$ (Gautschy 1987). There is a considerable uncertainty of $v_\gamma$ in the literature, suggesting an error of $d$ as large as a factor of 2. It suffices to mention that for SU Dra, $v_\gamma = -161$ and $-166.9$ km s$^{-1}$ are given by Oke et al. (1962) and Liu & Janes (1990) from high dispersion spectra and spectral masking method CORAVEL-technique, respectively.

A wavy fine structure in the variation of $R$ is clearly seen in Figs 1 and 3(b). The outward motion starts at $\varphi \approx 0.45$, 0.74, 0.94–0.98 while the outward motion is reversed at $\varphi \approx 0.25$, 0.55, 0.78, i.e. the atmospheric velocity is $v(r, \varphi) \approx 0$ at these phases. The wavy fine structure is without doubt real, because the beginnings of the outward motion are connected with significant maxima in $T_e(\varphi)$, $\log g_e(\varphi)$. The well-known bump and hump at $\varphi \approx 0.74, 0.94$–0.98 (Smith 1995) in the light curve are caused by the precursor and main shocks, respectively. At $\varphi \approx 0.45$, a small change is observable in the slope of the light curve (Barcza & Benkő 2009). Here, the existence of a shock is a new finding. It is a pre-precursor shock; we propose the designation jump for it. By integrating radial velocities, authors tend to smooth out the fine structure of motions mentioned [e.g. Liu & Janes (1990) in the case of SW And]. This practice seems to be unjustified.

The velocity of the atmosphere is $v(r, \varphi) \approx 0$ in the interval $\varphi \approx 0.82$–0.90, the atmosphere is roughly at a standstill, the brightness starts rising, a small depression of $\vartheta(\varphi)$ and $\log g_e(\varphi)$ is visible at $\varphi \approx 0.84$–0.86, and $T_e(\varphi)$ is monotonic. It is not clear whether the small undulation of $\vartheta(\varphi)$ around $(1.571-1.579) \times 10^{-10}$ is real or not, because the maximal $\Delta \log g_e = 0.116$ was found just at $\varphi = 0.88$.

### 4.2 $T_e$ scale, mass, distance, absolute brightness

Surprisingly, in comparison with previous studies, considerable differences were found only in $(T_e, T_{\text{eq}})$. This is due to the fact that the interpolated $T_e(\varphi)$ depends on $\log g_e(\varphi)$ and that the information from a five-colour photometry was used in a more complex manner: an averaged value from 30 colour index pairs was utilized. The photometry covers the whole spectrum between 350 and 1000 nm. The use of only one colour index, e.g. $V-K$ solely ‘because of its apparent merits’ (Liu & Janes 1990), can result in a systematic error of $T_e$. It can explain the $\approx 350$ K difference, because in that previous work QSSA was assumed in all phases despite violating both Conditions I and II. An inspection of the functions $\{T_0(\log g_e, C_1, C_2, [M], E(B-V))\}_{1:2}$ derived from the tables (Kurucz 1997) shows that if merely one colour index is to be used, the optimal choice for determining $T_e$ would be $R_C - I_C$. This is because $R_C - I_C$ is almost independent of $\log g_e$ for the actual values of $[M]$ and $E(B-V)$ of SU Dra. However, in phases violating Condition I, the sole use of $R_C - I_C$ would also introduce a systematic error, similar to the use of $V-K$.

There is a remarkable decrease of $\Delta d = 200 \rightarrow 67$ pc, $\Delta M = 0.10 \rightarrow 0.03 M_\odot$ if our compressible QSSA is substituted for UAA, i.e. more physical input is used in the frame of a one-dimensional model in space. Since (10) was derived from the dynamical equation (3), the mass determination is more accurate from this while the distance is more uncertain. This is reflected in the shape of the curves in Fig. 3(a). The large formal error of $d$ originates from the non-separability of errors in $\theta, \vartheta, \hat{\vartheta}, h_0, \Delta h_0/\Delta t, \Delta^2 h_0/\Delta t^2$ and $a_{\text{dyn}}(r, t)$.

A first attempt to derive $d$ and $M$ of an RR star from the oversimplified version (1) of (3) was described by Barcza (2003). The present results for the distance and mass of SU Dra differ only slightly from the previous $d = (647 \pm 16)$ pc and $M = (0.66 \pm 0.03) M_\odot$. The most probable reason of the very good coincidence is the existence of the phase island with the good QSSA at $\varphi \approx 0.93$, just when the atmosphere is at a standstill; consequently, (1) is a good approximation because of $\varphi \approx 0$.

The values $d = 640$ pc, $M = 0.47 M_\odot$ given by Liu & Janes (1990) are very close to the present ones. However, some caution is appropriate because of the underestimated uncertainties in their derivation. Their $M$ originates from using (1), i.e. $C = 0$, the UAA corresponding to line 3 in Table 2. Furthermore, because of the uncertain value of $v_\gamma$, $\Delta d/d \lesssim 0.6$ is well possible.

In the BW method, the propagation of the error $\Delta v_\gamma = 1$ km s$^{-1}$ can be estimated from (2) as follows. Typical radius changes of an RR star are $R_{\text{max}} - R_{\text{min}} \approx 5 \times 10^5$ km within $P/2 \approx$ half-day. The error of the radius change is $\Delta R \approx \Delta v_\gamma$. $P/2 \approx 5 \times 10^4$ km, i.e. $\Delta R/(R_{\text{max}} - R_{\text{min}}) \approx 0.1$. By $d = (R_{\text{max}} - R_{\text{min}} \pm \Delta R)/(\vartheta_{\text{max}} - \vartheta_{\text{min}})$, the final error will be $\Delta d/d \approx 0.1$. This considerably exceeds the error originating from the projection factor (e.g. Liu & Janes 1990) converting the observed radial velocity to pulsation velocity. Furthermore, by (2) and (1), the error $\Delta d/d \approx 0.1$ propagates to an error $\Delta M/M \approx 0.2$.

The lower limit of the magnitude-averaged visual absolute brightness is $M_V > +0.48$ mag if $d < 724$ pc, while $M_V = +0.26$ and $+0.01$ mag belong to the improbable values $d = 800$ and 900 pc, respectively.

### 5 Conclusions

Observed colours and magnitudes of a spherically pulsating star have been compared with those of static Kurucz model atmospheres to determine fundamental parameters of the star in the frame of QSSA. Photometric and hydrodynamic conditions have been formulated for the validity of the QSSA in spherically pulsating stars.

1. The QSSA has been generalized for a non-uniform, compressible atmosphere with a radial velocity gradient. This is a step forwards because the hitherto available UAA described the motion of the atmosphere by effective gravity and differentiating the sole parameter radius $R$ of the zero optical depth with respect to time.

2. Our combined photometric and hydrodynamic method uses the variation of effective gravity, angular radius, velocity and acceleration in the Euler equation. The complete input of the Euler equation has been derived from photometry and theoretical model atmospheres. Spectroscopic and radial velocity observations were used neither explicitly nor implicitly. This is a definite advantage in comparison with the BW method, because less observational efforts are needed to get the fundamental parameters and the problematic conversion of observed radial velocities to pulsation velocities is not necessary.

3. Concerning the interpretation of multicolour photometry, the inputs are identical with those of the BW method. The present refinements are the quantitative conditions whether photometry can or cannot be interpreted by static model atmospheres.

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First, the phases have been selected by the photometric conditions when the QSAA is valid. Secondly, in a number of these phases, the laws of mass and momentum conservation have been applied in the Euler formalism of hydrodynamics to determine the mass and distance of the star from the motion of the atmospheric layers in the neighbourhood of zero optical depth. Afterwards, it was checked whether the hydrodynamic condition of the QSAA was satisfied in the used phases; i.e. phases that satisfied the photometric condition but violated the hydrodynamic condition had to be excluded. Atmospheric dynamical mass seems to be an appropriate term to indicate that the mass has been derived by a method which is completely different from a mass derived by pulsation or evolution theories (Smith 1995).

As a by-product, a variation procedure has been given for estimating atmospheric metallicity and interstellar reddening towards a star from photometry. This method was successfully applied for the non-variable comparison star BD+67°708, giving $[M] = -1.60 \pm 0.10$, $E(B - V) = 0.015 \pm 0.010$, $d = (663 \pm 67)$ pc, $R_{\text{min}} = 4.46 R_\odot$, $R_{\text{max}} = 5.29 R_\odot$, $M = (0.68 \pm 0.03) M_\odot$, $L_{\text{eq}} = (45.9 \pm 9.3) L_\odot$, $T_{\text{eq}} = (6813 \pm 20)$ K.

$L_{\text{eq}}$ and $T_{\text{eq}}$ are approximate values, since they originate from averaging over the whole pulsation cycle containing phases in which the QSAA is merely a first approximation.

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