The Standard Model on the Quintic

Ralph Blumenhagen¹, Volker Braun¹,², Boris Körs³, and Dieter Lüst¹

¹ Humboldt-Universität zu Berlin, Institut für Physik, Invalidenstrasse 110, 10115 Berlin, Germany
e-mail: blumenha, braun, luest@physik.hu-berlin.de

² Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris, France
e-mail: volker.braun@lpt.ens.fr

³ Spinoza Institute, Utrecht University, Utrecht, The Netherlands
email: kors@phys.uu.nl

Abstract
We describe the general geometrical framework of brane world constructions in orientifolds of type IIA string theory with D6-branes wrapping 3-cycles in a Calabi-Yau 3-fold, and point out their immediate phenomenological relevance. These branes generically intersect in points, and the patterns of intersections govern the chiral fermion spectra and issues of gauge and supersymmetry breaking in the low energy effective gauge theory on their world volume. In particular, we provide an example of an intersecting brane world scenario on the quintic Calabi-Yau with the gauge group and the chiral spectrum of the Standard Model and discuss its properties in some detail. Additionally we explain related technical advancements in the construction of supersymmetric orientifold vacua with intersecting D-branes. Six-dimensional orientifolds of this type generalize the rather limited set of formerly known orbifolds of type I, and the presented techniques provide a short-cut to obtain their spectra. Finally, we comment on lifting configurations of intersecting D6-branes to M-theory on non-compact $G_2$ manifolds.

10/2002
1. Introduction

A central object of string phenomenology is to find a string vacuum whose low energy approximation is reproducing the known physics of the Standard Model or of its supersymmetric and grand unified extensions. As a first approach one may concentrate on finding models with the correct light degrees of freedom, the right gauge group and chiral fermion spectra, leaving the details of their dynamics aside for the moment. Intersecting brane worlds \[1\] have proven to be a candidate framework of model building which offers excellent opportunity to meet this requirement \[2-17\]. In these string compactifications, the fermion spectrum is determined by the intersection numbers of certain 3-cycles in the internal space, as opposed for instance to the older approaches involving heterotic strings, where the number of generations was given by the Euler characteristic in the simplest case. Beyond these topological data also some more geometrical issues have been addressed, which provide access to computing the scalar potential and determining the dynamics at least at the classical level \[10,18,19\]. Up to now, the construction has been limited to toroidal backgrounds and orbifolds thereof for the sake of simplicity. This has actually put severe constraints on the generality of the examples obtained and prevented any supersymmetric models, except for the one case of the \(T^6/\mathbb{Z}_2^2\) Calabi-Yau-orbifold \[20\], generalizing the former work on the six-dimensional \(T^4/\mathbb{Z}_2\) K3-orbifold \[7\].

In the present work, which covers only a fraction of the material presented first in \[21\], but also adds some extensions and slight improvements, we mostly describe the general framework of intersecting brane world constructions on any smooth background Calabi-Yau space\(^{1}\). This extends the range of accessible background spaces to include basically all geometrical string compactifications with supersymmetry in the bulk gravity sector. Therefore obstacles to finding supersymmetric Standard Model or GUT compactifications may possibly be overcome. For the time being, we give a new solution for a non-supersymmetric intersecting brane world on the quintic Calabi-Yau that not only realizes the chiral fermion spectrum of the Standard Model but can also be shown to have exactly the hypercharge as its only abelian gauge symmetry. It replaces the example given in \[21\]. The general philosophy of these models has been sketched in figure 1.

The internal Calabi-Yau space may split into various pieces which individually support 3-cycles wrapped by several D6-branes. On one of these regions, the Standard Model (SM)

\(^{1}\) See also the recent work \[22\] where non-compact local models of intersection brane configurations on Calabi-Yau spaces have been discussed.
fields are localized, while others may involve hidden sector (HS) gauge groups which couple only gravitationally to the visible sector (see figure 1). This kind of scenario offers at least two possible ways to address the issue of space-time supersymmetry breaking in intersecting brane worlds. In the first class of models the Standard Model brane configuration is already non-supersymmetric from the beginning (this is true for most of the models considered so far, including the CY example in [21]). This means that supersymmetry is broken at the string scale \( M_{\text{string}} \). In order to avoid the usual hierarchy problems \( M_{\text{string}} \) should be of the order of a few TeV, requiring that the volume of the internal CY space transverse to all Standard Model D-branes is large according to [23,24].

On the other hand it may happen that the Standard Model D-brane sector is itself supersymmetric, but the hidden sector preserves a different supersymmetry\(^2\) or is completely non-supersymmetric. Then the gravity mediated supersymmetry breaking appears naturally. In this case the following relation between the SUSY breaking scale in the Standard Model sector and the fundamental string scale is expected to hold:

\[
M_{3/2} \simeq \frac{M_{\text{string}}^2}{M_{\text{Planck}}}. \tag{1.1}
\]

With \( M_{3/2} \) of order TeV one obtains an intermediate string scale, \( M_{\text{string}} \simeq 10^{11}\text{GeV} \), a scenario which was already discussed in [28]. For D6-brane models the string scale, the string coupling constant \( g_{\text{string}} \), the typical length scale \( R_{\parallel} \) of the internal D6-brane volume

---

\(^2\) The relevant patterns of supersymmetry breaking in the effective low energy field theory have been discussed in [23,26,27].
Vol(D6) \sim R_{∥}^3 and the scale $R_{⊥}$ of the transversal internal volume are related to the gauge coupling $g_{YM}$ and the effective Planck mass in the following way:

\[ g_{YM}^2 = g_{\text{string}}(M_{\text{string}} R_{∥})^{-3}, \quad M_{\text{Planck}} = \frac{M_{\text{string}}^4}{g_{\text{string}}(R_{∥} R_{⊥})^{3/2}}. \quad (1.2) \]

Assuming that $M_{\text{string}} \simeq R_{∥}^{-1}$ this requires a moderately enlarged transversal space, namely $R_{⊥}^{-1} \simeq 10^9\text{GeV}$.

Finally there is another effect known to weaken the breaking effects in the effective four-dimensional theory on the Standard Model branes and to avoid the hierarchy problems. Namely, the backreaction of the geometry towards the presence of the branes, which is actually neglected in our approach, may involve a warped geometry, which may give rise to a scenario in the spirit of [29,30].

As an additional important motivation, we also hope to gain further insight into the dynamics of the models from the studies of Calabi-Yau geometry and mirror symmetry, in particular from the knowledge of scalar potentials [31-40]. For example, it was recently pointed out that type IIB vacua deformed by 3-form fluxes and D5-branes possess a rather particular structure reminiscent of the special geometry that governs the $\mathcal{N} = 2$ vacuum of the Calabi-Yau compactification [39,40].

An essential consistency condition is the cancellation of the Ramond-Ramond (RR) charge. There are two complementary methods in performing the actual computations of the cancellation as well as of the chiral spectra, the scalar potential and other relevant data. If the background is given by a string world sheet CFT, these can be extracted from certain string diagrams, notably the genus zero open string one-loop amplitude. On the other hand, if the background is smooth, one may go to the limit where the curvature is small everywhere and supergravity and classical geometry are valid. Since we are mostly interested in backgrounds defined in geometrical terms, we shall employ arguments taken from the effective supergravity action and from geometry. One of the main achievements of [21] actually was to show that for orbifold vacua, where the two regimes are connected by blowing up singularities, the geometrical point of view provides a much simpler formalism to compute the chiral spectra as compared to the CFT methods. We shall demonstrate this by studying orbifold limits of K3 in their blown-up version.
2. Intersecting brane worlds on Calabi-Yau 3-folds

In the brane world scenarios we are going to consider here, there are D-branes filling out the entire four-dimensional space-time providing the degrees of freedom for an effective gauge theory. The overall transverse six-dimensional space is compact, such that the internal excitations decouple from the effective theory below the string scale. The global consistency conditions in string models with D-branes that fill out the non-compact space-time involve the cancellation of the RR charges. Furthermore, supersymmetry requires the cancellation of the brane tensions and the corresponding Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles as well. If the latter is neglected, one can achieve the RR charge cancellation within type II vacua by including anti-branes, but these vacua usually suffer from run-away instabilities, if not even tachyons. The only setting in which objects with negative tension arise naturally in string theory are orientifolds \[41\], where the orientifold O-planes can balance the charge and tension of the D-branes. Therefore, orientifolds provide the framework where supersymmetric brane worlds may be found within string theory.

2.1. Definition

According to the above reasoning we will consider orientifold compactifications, where the ten-dimensional space-time \( \mathcal{X} \) is of the kind

\[
\mathcal{X} = \mathbb{R}^{3,1} \times \frac{\mathcal{M}^6}{\Omega \sigma}.
\]

Here \( \mathcal{M}^6 \) is a Calabi-Yau 3-fold with a symmetry under \( \sigma \), the complex conjugation

\[
\sigma : z_i \mapsto \bar{z}_i, \; i = 1, \ldots, 3,
\]

in local coordinates. It is combined with the world sheet parity \( \Omega \) to form the orientifold projection \( \Omega \sigma \). This operation is actually a symmetry of the type IIA string on \( \mathcal{M}^6 \). The construction has a T-dual or mirror symmetric description within type IIB, which is explained in some detail in \[21\] as well. Orientifold O6-planes are defined as the fixed locus \( \text{Fix}(\sigma) \) of \( \sigma \), which is easily seen to be a supersymmetric 3-cycle in \( \mathcal{M}^6 \). It is special Lagrangian (sLag) and calibrated with respect to the real part of the holomorphic 3-form \( \Omega_3 \). To see this define \( \Omega_3 \) and the Kähler form \( J \) in local coordinates

\[
\Omega_3 = dz_1 \wedge dz_2 \wedge dz_3, \quad J = i \sum_{i=1}^{3} dz_i \wedge d\bar{z}_i.
\]
From $\sigma(\Omega_3) = \Omega_3$ and $\sigma(J) = -J$ it then follows that

$$\Im(\Omega_3)|_{\text{Fix}(\sigma)} = 0, \quad J|_{\text{Fix}(\sigma)} = 0,$$

which implies

$$\Re(\Omega_3)|_{\text{Fix}(\sigma)} = d\text{vol}|_{\text{Fix}(\sigma)}.$$

It is also useful to define a rescaled 3-form

$$\tilde{\Omega}_3 = \frac{1}{\sqrt{\text{Vol}(\mathcal{M}^6)}} \Omega_3.$$

This orientifold projection truncates the gravitational bulk theory of closed strings down to a theory with 16 supercharges in ten dimensions, leading to 4 supercharges and $\mathcal{N} = 1$ in four dimensions, after compactifying on the Calabi-Yau. In order to cancel the RR charge of the O6-planes it is required to introduce D6-branes into the theory as well, which will provide the gauge sector of the theory. If we label the individual stacks of D6$_a$-branes with multiplicities $N_a$ by a label $a$, the gauge group of the effective theory will be given by

$$G = \prod_a U(N_a).$$

Here we exclude the possibility of branes which are invariant under the projection $\Omega \sigma$. They would give rise to $SO(N_a)$ or $Sp(N_a)$ factors. It is no conceptual problem to include them as well, but they are of little phenomenological interest.

2.2. RR charges and brane tension

The charge cancellation conditions are often obtained by regarding divergences of one-loop open string amplitudes, but can also be determined from the consistency of the background in the supergravity equations of motion or Bianchi identities. The Chern-Simons action for D$p$-branes and O$p$-planes are given by\cite{12,16}

$$S_{\text{CS}}^{(Dp)} = \mu_p \int_{Dp} \text{ch}(\mathcal{F}) \wedge \sqrt{\hat{A}(\mathcal{R}_T)/\hat{A}(\mathcal{R}_N)} \wedge \sum_q C_q,$$

$$S_{\text{CS}}^{(Op)} = Q_p \mu_p \int_{Op} \sqrt{\hat{L}(\mathcal{R}_T/4)/\hat{L}(\mathcal{R}_N/4)} \wedge \sum_q C_q.$$ 

The relative charge of the orientifold planes is given by $Q_p = -2^{p-4}$ and $\text{ch}(\mathcal{F})$ denotes the Chern character, $\hat{A}(\mathcal{R})$ the Dirac genus of the tangent or normal bundle, and the $\hat{L}(\mathcal{R})$ the
Hirzebruch polynomial. The physical gauge fields and curvatures are related to the skew-hermitian ones in (2.8) by rescaling with $-4i\pi^2\alpha'$. These expressions simplify drastically for sLag 3-cycles, where $\text{ch}(\mathcal{F})|_{D_p} = \text{rk}(\mathcal{F})$, the other characteristic classes become trivial and finally the only contribution in the CS-term (2.8) then comes from $C_7$.

In the following we denote the homology class of $\text{Fix}(\sigma)$ by $\pi_{O6} = [\text{Fix}(\sigma)] \in H_3(\mathcal{M}^6)$ and the homology class of any given brane stack $D_{6a}$-brane by $\pi_a$. By our assumptions the $\pi_a$ are never invariant under $\sigma$ but mapped to image cycles $\pi'_a$. Therefore, a stack of D6-branes is wrapped on that cycle by symmetry, too. The RR charge cancellation can now easily be deduced by looking at the equation of motion of $C_7$

$$\frac{1}{\kappa^2} d \ast dC_7 = \mu_6 \sum_a N_a \delta(\pi_a) + \mu_6 \sum_a N_a \delta(\pi'_a) + \mu_6 Q_6 \delta(\pi_{O6}), \quad (2.9)$$

where $\delta(\pi_a)$ denotes the Poincaré dual form of $\pi_a$, $\mu_p = 2\pi(4\pi^2\alpha')^{-(p+1)/2}$, and $2\kappa^2 = \mu_7^{-1}$. Upon integrating over $\mathcal{M}^6$ the RR-tadpole cancellation condition becomes a relation in homology

$$\sum_a N_a (\pi_a + \pi'_a) - 4\pi_{O6} = 0. \quad (2.10)$$

In principle it involves as many linear relations as there are independent generators in $H_3(\mathcal{M}^6, \mathbb{R})$. But, of course, the action of $\sigma$ on $\mathcal{M}^6$ also induces an action $[\sigma]$ on the homology and cohomology. In particular, $[\sigma]$ swaps $H^{2,1}$ and $H^{1,2}$, and the number of conditions is halved.

Similarly one can determine the disc level tension by integrating the Dirac-Born-Infeld effective action. It is proportional to the volume of the D-branes and the O-plane, so that the disc level scalar potential reads

$$\mathcal{V} = T_6 \frac{e^{-\phi_4}}{\sqrt{\text{Vol}(\mathcal{M}^6)}} \left( \sum_a N_a (\text{Vol}(D_{6a}) + \text{Vol}(D_{6a}')) - 4\text{Vol}(O6) \right). \quad (2.11)$$

The potential is easily seen to be positive semidefinite and its vanishing imposes conditions on some of the moduli, freezing them to fixed values. Whenever the potential is non-vanishing, supersymmetry is broken and a classical vacuum energy generated by the net brane tension. It is easily demonstrated that the vanishing of (2.11) requires all the cycles wrapped by the D6-branes to be calibrated with respect to the same 3-form as are the O6-planes. In a first step, just to conserve supersymmetry on their individual world volume theory, the cycles have to be calibrated at all, which leads to

$$\mathcal{V} = T_6 e^{-\phi_4} \left( \sum_a N_a \int_{\pi_a} \hat{\Omega}_3 \bigg| + \sum_a N_a \int_{\pi'_a} \hat{\Omega}_3 \bigg| - 4 \int_{\pi_{O6}} \hat{\Omega}_3 \bigg) \right). \quad (2.12)$$
Since $\hat{\Omega}_3$ is closed, the integrals only depend on the homology class of the world volumes of the branes and planes and thus the tensions also become topological. If we further demand that any single D$_{6a}$-brane conserves the same supersymmetries as the orientifold plane the cycles must all be calibrated with respect to $\Re(\hat{\Omega}_3)$. We can then write

$$V = T_6 e^{-\phi_4} \int_\sum a N_a (\pi_a + \pi'_a) - 4\pi_{O6} \Re(\hat{\Omega}_3).$$

In this case, the RR charge and NSNS tension cancellation is equivalent, as expected in the supersymmetric situation.

2.3. Massless closed string modes

The action of $\Omega \sigma$ on the cohomology determines the spectrum of the closed string bulk modes as usual. One simply needs to consider all the massless fields of the $\mathcal{N} = 2$ type IIA theory after compactification on the Calabi-Yau 3-fold and project out those that are odd under $\Omega \sigma$ when truncating to $\mathcal{N} = 1$ supersymmetry. Before the projection there were $h^{(1,1)}$ abelian vector multiplets and $h^{(2,1)}$ hypermultiplets. The $h^{(1,1)}$ vector multiplets consist of one scalar coming from the dimensional reduction of the gravity field (the Kähler modulus), another scalar resulting from the reduction of the NSNS 2-form and a four-dimensional vector from the reduction of the RR 3-form along the 2-cycle. If the $(1, 1)$ form is invariant under $\Omega \sigma$ an $\mathcal{N} = 1$ chiral multiplet survives the projection and if it is odd we get an $\mathcal{N} = 1$ vector multiplet. Note, that the surviving chiral multiplets still contain the complexified Kähler moduli. On the other hand the four scalars of the $h^{(2,1)}$ hypermultiplets contain two scalars from the ten-dimensional gravity field (the complex structure moduli) equipped with two scalars arising from the dimensional reduction of the RR 3-form along the two associated 3-cycles referring to $H^{2,1}(\mathcal{M}^3)$ and $H^{1,2}(\mathcal{M}^3)$. Under the $\Omega \sigma$ projection one of the two components of the complex structure is divided out and moreover one linear combination of the RR scalars survives, so that the former quaternionic complex structure moduli space gets reduced to a complex moduli space of dimension $h^{(2,1)}$.

2.4. Massless open string modes

In this section we are going to present the most important input for constructing intersecting brane world models of particle physics, the formulae that determine the spectrum of the chiral fermions of the effective theory in terms of topological data of the brane
configuration and the Calabi-Yau manifold. Roughly speaking, at any intersection point of two stacks of D6-branes a single chiral fermion is localized [47], transforming in the bifundamental representation of the two respective gauge groups. One needs to take care that also non-chiral matter in sectors where intersection points of opposite orientation cancel will generically become massive, so that the massless spectrum is really determined by the intersection numbers. As was mentioned already, the search for a viable model close to the Standard Model particle content boils down to looking for Calabi-Yau spaces with an involution $\sigma$ and an intersection form for its 3-cycles that allows to realize the desired particle spectrum at the intersections.

Catching up with the above discussion of the brane tension and the induced scalar potential, we can say a bit more: If we want to construct a supersymmetric intersecting brane world we need the desired intersection pattern to be realized within a set of sLag cycles all calibrated by the same 3-form, which makes the task a lot harder. In the following we shall actually be presenting a model on the Calabi-Yau defined by the Fermat quintic polynomial in $\mathbb{CP}^4$ which is built out of calibrated cycles, but not all calibrated with respect to the same calibration form. In supersymmetric models, the total massless spectrum is easily found by adding superpartners to the fermions. Upon breaking supersymmetry, it is to be expected that all fields except gauge bosons and chiral fermions will get masses through interactions.

To obtain the chiral spectrum of a given set of D6-branes wrapped on cycles $\pi_a$ with their images on $\pi'_a$ and the O6-planes wrapped on $\pi_{O6}$ a few considerations are necessary. The only novelty is that in addition to the standard operation by $\Omega$ a permutation of the branes and intersection points by $\sigma$ occurs, formally encoded in acting by a permutation matrix on the Chan-Paton labels that determine the representation under the gauge group. First note that the net number of self-intersections of any stack vanish, due to the anti-symmetry of the intersection form (denoted by $\odot$). Whenever a brane intersects its own image, there are two cases to distinguish: The intersection can itself be invariant under $\sigma$, such that the Chan-Paton labels are anti-symmetrized by $\Omega$. Alternatively, it can also be mapped to a second intersection, such that no projection applies and the symmetric and antisymmetric parts are kept. Finally, if any two different stacks intersect, there are always bifundamental representations localized at the intersection. According to these rules, the spectrum of left-handed massless chiral fermions is shown in table 1.
The above classification can be obtained directly from string amplitudes when a CFT description is available, e.g. in the orbifold limit, while at large volume one can apply the Atiyah-Singer index theorem to infer the zero-modes of the Dirac operator. This is actually a tautology, since the number of chiral modes is given as an integral over the point-like common world volume of any pair of D6-branes with a trivial integrand,

\[ \int_{D6_a \cap D6_b} ch(F_a) \wedge ch(F_b^*) \wedge \delta(R) = \text{rk}(F_a) \text{rk}(F_b) \int_{\mathcal{M}^6} \delta(\pi_a) \wedge \delta(\pi_b), \] (2.14)

which only counts the intersection numbers again. In the mirror symmetric type IIB picture chirality is in fact induced exclusively by the non-triviality of the gauge and spin connection.

Due to the topological nature of the chiral spectrum table 1 should hold for every smooth Calabi-Yau manifolds and even the six-dimensional torus [48]. Little can be said about the fate of the D-brane setting away from the limit of classical geometry, when venturing into the interior of the Kähler moduli space, where potentials may be generated. Therefore, the configuration will in general not be stable, but the important point is, whenever the setting is describable purely in terms of D6-branes on sLag 3-cycles table 1 applies.

To make a first check of the consistency of the spectrum, the non-abelian gauge anomaly $SU(N_a)^3$

\[ A_{\text{non-abelian}} \sim \pi_a \circ \pi_a \] (2.15)

vanishes due to the antisymmetry of the intersection form.
2.5. The Quintic

Now that we have collected the machinery to construct intersecting brane worlds on general Calabi-Yau 3-folds, we proceed to discuss the example of the quintic. It is probably the most studied and best understood example of a hypersurface in a projective space. It even appears that there are no other examples known in the literature where a sufficiently large class of sLag cycles has been found and their intersection form classified.

One defines the Fermat quintic by the following hypersurface in

$$\mathbb{CP}^4$$

$$Q : \sum_{i=1}^{5} z_i^5 = 0 \subset \mathbb{CP}^4.$$  \hfill (2.16)

It has the obvious involution from the complex conjugation of the coordinates $$z_i \rightarrow \overline{z}_i$$ as a symmetry. The fixed points of $$\sigma$$ are the real quintic $$\sum_{i=1}^{5} x_i^5 = 0 \subset \mathbb{RP}^4$$, topologically a sLag $$\mathbb{RP}^3$$. As a further symmetry of the Fermat quintic a $$\mathbb{Z}_5$$ acts via

$$z_i \mapsto \omega^{k_i} z_i$$, \quad $$\omega = e^{2\pi i 5}$$, \quad $$k_i \in \mathbb{Z}_5.$$  \hfill (2.17)

We can use this symmetry to generate a whole class of sLag cycles from the one prototype above. Because the diagonal $$\mathbb{Z}_5$$ is trivial, this produces $$5^4 = 625$$ different minimal $$\mathbb{RP}^3$$, labeled by the integers $$k_i$$ and defined by

$$|k_2, k_3, k_4, k_5\rangle \overset{\text{def}}{=} \left\{ x_1 : \omega^{k_2} x_2 : \omega^{k_3} x_3 : \omega^{k_4} x_4 : \omega^{k_5} x_5 \big| x_i \in \mathbb{R}, \sum_{i=1}^{5} x_i^5 = 0 \right\}.$$  \hfill (2.18)

The only information further needed is their intersection form, determined in [31]. The intersection of any cycle $$|k_2, k_3, k_4, k_5\rangle$$ with the one $$|1, 1, 1, 1\rangle$$ is given by the coefficient of the monomial $$g_2^{k_2} g_3^{k_3} g_4^{k_4} g_5^{k_5}$$ in

$$I_{\mathbb{RP}^3} = \prod_{i=1}^{5} (g_i + g_i^2 - g_i^3 - g_i^4) \in \mathbb{Z}[g_1, g_2, g_3, g_4, g_5]/\langle g_5^5 = 1, \prod g_i = 1 \rangle.$$  \hfill (2.19)

All the other intersection numbers are then obtained by applying the $$\mathbb{Z}_5^4$$ symmetry. The ensuing intersection matrix $$M \in \text{Mat}(625, \mathbb{Z})$$ has rank $$204 = b_3$$, so the $$|k_2, k_3, k_4, k_5\rangle$$ generate the full $$H_3(Q; \mathbb{R})$$.

Of course, only one fifth of the minimal $$\mathbb{RP}^3$$ are $$\mathbb{R}(\Omega_3)$$ calibrated, while the others are calibrated with respect to $$\mathbb{R}(\omega^k \Omega_3)$$. To determine the $$\mathbb{R}(\Omega_3)$$ sLags we need to know how $$\mathbb{Z}_5^4$$ acts on the holomorphic volume form. From the residue formula

$$2\pi i \Omega_3 = \oint \frac{\epsilon^{i_1 \cdots i_5} z_{i_1} dz_{i_2} \wedge dz_{i_3} \wedge dz_{i_4} \wedge dz_{i_5}}{\sum_{i=1}^{5} z_i^5}.$$  \hfill (2.20)
it is evident that $\Omega_3$ transforms as $\Omega_3 \mapsto (\prod_i \omega^{k_i})\Omega_3$, such that the $|k_2, k_3, k_4, k_5\rangle$ are calibrated with respect to $\Re(\Omega_3)$ precisely if $\sum_{i=2}^{5} k_i = 0 \mod 5$. Using the intersection matrix for these 125 sLags one can check that they generate a 101-dimensional subspace of $H_3(Q)$. As was discussed at length, in order to construct any supersymmetric brane world model, it would be necessary to use D6-branes wrapping these 125 cycles only. Only in this case the scalar potential generated by the tension of the branes would be balanced by the negative tension of the O6-planes. Unfortunately, this turns out to be impossible with the present class of sLags, since it is found that the intersections among themselves all vanish. Therefore, a chiral spectrum cannot be reconciled with a supersymmetric groundstate. The validity of this statement is in fact rather limited. One cannot even conclude that a supersymmetric brane world model is not accessible on the Fermat quintic, because it may well happen to exist within another set of sLag cycles. Not to mention that the general quintic may have points in its moduli space where another involution can be used to define $\mathfrak{g}$ and completely different sets of sLags exist.

2.6. The Quintic Standard Model

We have seen that using only the 3-cycles in (2.18) we cannot obtain interesting brane configurations if we want to preserve supersymmetry, which would restrict us to only using the 3-cycles in (2.18). However, it is indeed possible to construct a model with the correct intersection numbers by dropping the requirement of supersymmetry, as we shall demonstrate in the following. The breaking of supersymmetry is still of a special and somehow weak nature, since the individual stacks still respect some supersymmetry generators, just not all the same. This can have interesting consequences for the dynamics of the effective gauge theory.

A further generalization of the set of sLags is needed since all intersection numbers of the 625 minimal $\mathbb{RP}^3$ are in the range $-2, \ldots, 2$, while we need $\pm 3$ for some cycles to reproduce the three generation structure of the Standard Model fermion spectrum. So we must use linear combinations. There are some subtleties with this step. First notice that by adding the sLag cycles in homology, their volumes also just add up in accordance with the topological nature of their tension. Furthermore, if the two cycles in question are calibrated with respect to the same calibration 3-form, their sum in homology, represented by the geometrical union of the two submanifolds, will as well be calibrated by this 3-form. But the union of two sLag submanifolds will usually not be a smooth and connected submanifold itself, and thus cannot simply be wrapped by any single stack of D-branes.
Let us illustrate this problem with a simple example of sLag submanifolds on a four-dimensional torus $T^4 = T^2_1 \times T^2_2$. Take the two $T^2_1$ to be given by square tori of volume 1. Now consider 2-cycles calibrated by $\Re(\Omega_2) = dx_1 \wedge dx_2 - dy_1 \wedge dy_2$, given by lines on any one of the two $T^2_1$ satisfying $\varphi_1 + \varphi_2 = 0$ for their relative angles $\varphi_I$ with respect to the $x_I$-axis on each $T^2_I$. Any such line is defined homologically by specifying the two 1-cycles it wraps on the $T^2_I$. So let us denote them by two times two integers $(n^1_a, m^1_a)$, $a$ labeling the stacks. The calibration condition is solved by $n^1_a = n^2_a$, $m^1_a = -m^2_a$ and the volume of any such sLag is given by $\text{Vol}_a = (n^1_a)^2 + (m^1_a)^2$. One may pick a basis in homology by using all 2-cycles with two entries 0 and 1 respectively. Now compare for instance the cycles $(1, 0; 1, 0) + (0, 1; 0, -1)$ and $(1, 1; 1, -1)$, both with total volume 2. The first is the sum of two sLag cycles of volume 1, each projecting onto either the $x_I$-axes or the $y_I$-axes of the $T^2_I$, while the second one is the product of the diagonals. This relation closely resembles what we are seeking in the Calabi-Yau case of the quintic. We add up two sLags of equal volume 1. The union of the two submanifolds first consists of two components intersecting in a point at the origin of the two $T^2$. But we also find a smooth sLag in the homology class $\langle 1, 1; 1, -1 \rangle$ which is the sum of the two individual cycles plus a component that is not calibrated by $\Re(\Omega_2)$. This demonstrates that a calibrated cycle cannot be decomposed into basis elements all calibrated by the same calibration form. Neither can one expect sums of calibrated basis cycles to be represented by smooth and connected sLag submanifolds.

But there is a physical argument for the existence of a smooth connected representative in exactly the class of the sum of two sLags, that may apply, whenever the two original cycles intersect. At the intersection point there will be localized scalar moduli in the bifundamental of the gauge groups on the two stacks. Turning on vacuum expectation values for these corresponds geometrically to deforming the singular geometry at the intersection into a smooth submanifold, where the two components have joined together. The product gauge group is actually broken to the diagonal and the decomposition of the bifundamental contains a neutral scalar which parameterizes the deformation. We therefore expect a smooth connected cycle of minimal volume to exist in the homology class of the sum of any two intersecting sLag representatives. As in the toroidal example above, it may not fall into the original class of sLag cycles, and there is no control over its calibration condition. On the other hand, cycles which do not intersect in homology may be disentangled geometrically by smooth deformations. D-branes wrapped on such classes have to be expected to decay into disjoint components, i.e. multiple stacks of D-branes.
We shall now use this criterion to improve the example for a Standard Model brane world given in [21].

In order to realize the three generation Standard Model spectrum on an intersecting brane world on the Fermat quintic we then employ a combination of the sLag $\mathbb{RP}^3$ cycles and linear combinations of such cycles which intersect each other. Any single stack is then still expected to maintain four linearly realized supersymmetry generators, but not all of them the same. Concretely, we have an O6-plane on the cycle $\pi_{O6} = |0, 0, 0, 0\rangle$ and can choose to wrap D6-branes on the following 3-cycles

$$
\begin{align*}
\pi_a &= \pi_c - \pi_d - |0, 2, 1, 4\rangle - |0, 3, 4, 1\rangle \\
\pi_b &= |0, 3, 1, 1\rangle \\
\pi_c &= |1, 4, 3, 4\rangle + |4, 4, 3, 2\rangle \\
\pi_d &= |0, 3, 0, 3\rangle - |2, 0, 3, 4\rangle
\end{align*}
$$

On the whole we have been able to find a fairly large number of hundreds of configurations with the same chiral fermion spectrum meeting the requirements. The one given above looks rather complicated because it is designed to meet the additional condition

$$
(\pi_a - \pi'_a) - (\pi_c - \pi'_c) + (\pi_d - \pi'_d) = 0
$$

for a massless hypercharge gauge boson, as to be explained in section 3.3. It is also straightforward to check that the homology classes are primitive, i.e. not a multiple of some other class in $H_3(Q, \mathbb{Z})$, by finding at least one other cycle such that the intersection number with this other one is $\pm 1$. This was an important consistency requirement for toroidal models. The intersection numbers of the given 3-cycles are shown in table 2. The matrix has the following symmetries

$$
\begin{align*}
\pi_i \circ \pi_j &= -\pi_j \circ \pi_i = \pi'_j \circ \pi'_i = -\pi'_i \circ \pi'_j, \\
\pi_i \circ \pi'_j &= \pi_j \circ \pi'_i = -\pi'_i \circ \pi_j = \pi_j \circ \pi'_i.
\end{align*}
$$

Table 2 just reproduces the “intersection numbers of the Standard Model” as proposed in [8]: If one wraps 3 branes on $\pi_a$, 2 branes on $\pi_b$, and a single brane on $\pi_c$ and $\pi_d$, the gauge group is $U(3) \times U(2) \times U(1)^2$ before performing any anomaly analysis.
The intersection matrix then produces the bifundamental fermions of the Standard Model and nothing else, as shown in table 3. Since $\pi_i \circ \pi'_i = \pi_i \circ \pi_{O6} = 0$ there are no chiral fermions in the symmetric or antisymmetric representations of the gauge groups.

The fermion spectrum leaves two of the abelian factors free of anomalies. One is the Standard Model hypercharge and given by

$$U(1)_Y = \frac{1}{3} U(1)_a - U(1)_c + U(1)_d.$$  \hspace{1cm} (2.24)
the four abelian factors are anomalous and decouple through a generalized Green-Schwarz mechanism, leaving the Standard Model gauge group with one extra $U(1)$. In order to ensure that the second gauge boson does get a mass, while the hypercharge gauge boson really remains massless, one additionally has to analyze the couplings to the various axions that descend from the dimensional reduction of the RR 5-form potential. We shall come to this point later and show that the extra restriction can be satisfied.

An important point to notice is that the model so far does have a non-vanishing RR tadpole. The sum of the homology classes

$$3(\pi_a + \pi'_a) + 2(\pi_b + \pi'_b) + (\pi_c + \pi'_c) + (\pi_d + \pi'_d) - 4\pi_{O6}$$

(2.25)

does not vanish. To cancel the RR charge without changing the spectrum, one can easily introduce a hidden brane sector that carries the right charge to cancel the tadpole but does not intersect the Standard Model branes, so there is no chiral matter charged under both the visible and the hidden gauge group.

**2.7. Large transverse volume**

In order to reconcile the string scale supersymmetry breaking which occurs in the model just presented with the hierarchy problem one may refer to a large extra dimension scenario with a fundamental string scale of the order of a TeV $[23,24]$. The four-dimensional gauge couplings $g_a$ and Planck mass $M_{pl}$ are obtained by dimensional reduction from the fundamental string scale $M_s$ and string coupling $g_s$ according to

$$\frac{1}{g_a^2} \sim g_s \frac{\text{Vol} (\text{D6}_a)}{l_s^3}, \quad M_{pl}^2 \sim \frac{M_s^2 \text{Vol} (\mathcal{M}^6)}{g_s^2 l_s^6}. \quad (2.26)$$

The conditions for a high effective Planck scale with a TeV string scale can then be phrased

$$\frac{\text{Vol} (\mathcal{M}^6)}{l_s^6} \gg \frac{\text{Vol} (\text{D6}_a)^2}{l_s^6}. \quad (2.27)$$

Numerical values for plausible assumptions have been discussed in the introduction already.
3. Scalar potential, anomalies and gauge boson masses

Non-supersymmetric brane configurations are in general unstable. On the one hand, depending on the intersection angles there can be tachyons localized at the intersection points. Phenomenologically it was suggested that these tachyons might be interpreted as Standard Model Higgs fields [49,1], where in particular in [50] it was demonstrated that the gauge symmetry breaking is consistent with this point of view. In general we expect that tachyons can well be avoided in a large subset of the parameter space, as was due in the simpler setting of toroidal compactifications. On the other hand, even if tachyons are absent one generally faces uncanceled NSNS tadpoles, which might destabilize the configuration [19,10,51,52]. In [19] it was shown that for appropriate choices of the D-branes the complex structure moduli can be stabilized by the induced tree level potential. The stabilization of the dilaton remains a major challenge as in all non-supersymmetric string models.

For supersymmetric intersecting brane worlds we can expect much better stability properties. First tachyons are absent in these models due to the Bose-Fermi degeneracy. However, since for orientifolds on Calabi-Yau spaces the configuration only preserves $\mathcal{N} = 1$ supersymmetry, in general non-trivial F-term and D-term potentials can be generated.

3.1. F-term superpotential and D-terms

There are strong restrictions known for the contributions that can give rise to corrections to the effective $\mathcal{N} = 1$ superpotential of a type II compactification on a Calabi-Yau 3-fold with D6-branes and O6-planes on supersymmetric 3-cycles. The standard arguments about the non-renormalization of the superpotential by string loops and world sheet $\alpha'$ corrections apply. The only effects then left are non-perturbative world sheet corrections, open and closed world-sheet instantons. In general, these are related to non-trivial $\mathbb{C}P^1$ and $\mathbb{R}P^2$ with boundary on the O6-plane in the Calabi-Yau manifold for the closed strings and discs with boundary on the D6-branes for open strings. In fact, only the latter contribute to the superpotential. The typical form for the superpotential thus generated is known, but explicit calculations are only available for non-compact models. Usually, they make use of open string mirror symmetry arguments. In many cases, there is an indication that the non-perturbative contributions to the superpotentials tend to destabilize the vacuum, and it would be a tempting task to determine a class of stable $\mathcal{N} = 1$ supersymmetric intersecting brane models.
The tension of the D6-branes and O6-planes in addition introduces a vacuum energy which is described in terms of D-terms in the language of $\mathcal{N} = 1$ supersymmetric field theory. These depend only on the complex structure moduli and do not affect the Kähler parameter of the background. The most general form for such a potential is given by

$$V_{D-term} = \sum_a \frac{1}{2g_a^2} \left( \sum_i q_a^i |\phi_i|^2 + \xi_a \right)^2,$$

with $g_a$ the gauge coupling of a $U(1)_a$, $\xi_a$ the FI parameter, and the scalar fields $\phi_i$ are the superpartners of some bifundamental fermions at the intersections. They become massive or tachyonic for non-vanishing $\xi_a$, depending on their charges $q_a^i$. Due to the positive definiteness of the D-term, $\mathcal{N} = 1$ supersymmetry will only be unbroken in the vacuum, if the potential vanishes.

This requirement can be compared to the calibration condition for the 3-cycles wrapped by the branes. If they are just individually sLag, one can use (2.13) to write

$$V = 2 T_6 e^{-\phi_4} \sum_a N_a \left( \left| \int_{\pi_a} \hat{\Omega}_3 \right| - \int_{\pi_a} \Re(\hat{\Omega}_3) \right).$$

To apply (3.1) we have to use the properly normalized gauge coupling

$$\frac{1}{g^2_U(1)_a} = \frac{N_a}{g_a^2} = \frac{N_a M_s^3}{(2\pi)^4} e^{-\phi_4} \left| \int_{\pi_a} \hat{\Omega}_3 \right|. \quad (3.3)$$

Hence, the FI-parameter $\xi_a$ can be identified as

$$\xi_a^2 = \frac{M_s^4}{2\pi^2} \frac{\left| \int_{\pi_a} \hat{\Omega}_3 \right| - \int_{\pi_a} \Re(\hat{\Omega}_3)}{\left| \int_{\pi_a} \hat{\Omega}_3 \right|}, \quad (3.4)$$

which vanishes precisely if the overall tension of the branes and planes cancels out, i.e. if all are calibrated with respect to the same 3-form. Since the FI-term is not a holomorphic quantity one expects higher loop corrections to the classical result (3.4).

3.2. Anomalies

While the non-abelian anomalies of the chiral fermion spectrum given in table 3 vanish in any case, the mixed and abelian anomalies do not. Their cancellation requires various axions to participate in a generalized Green-Schwarz mechanism to render the theory
consistent. One can actually check in detail that the relevant couplings match with the anomalous contribution
\[ A_{ab} = \frac{N_a}{2} (-\pi_a + \pi'_a) \circ \pi_b \]  
(3.5)
from the chiral matter spectrum. It is in fact sufficient to consider the case of the $U(1)_a - SU(N_b)^2$ diagrams for $a \neq b$.

To do so, it is useful to define an integral basis for $H_3(M^6, \mathbb{Z})$, given by 3-cycles $\alpha^I$, $\beta_J$, $I, J = 0, \ldots, h^{(2,1)}$ with the property $\alpha^I \circ \alpha^J = \beta_I \circ \beta_J = 0$ and $\alpha^I \circ \beta_J = \delta^I_J$. We then expand the $\pi_a$ in terms of the basis cycles $\alpha^I$ and $\beta_J$,
\[ \pi_a = e_a^I \alpha^I + m_a^J \beta_J, \]  
(3.6)
with integers $e_a^I$, $m_a^J$, and similarly for $\pi'_a$. The general Chern-Simons couplings reduce to
\[ \int_{\mathbb{R}^{1,3} \times (\pi_a + \pi'_a)} C_3 \wedge \text{Tr} \left( F_a \wedge F_a \right), \quad \int_{\mathbb{R}^{1,3} \times (\pi_a + \pi'_a)} C_5 \wedge \text{Tr} \left( F_a \right). \]  
(3.7)
The four-dimensional axions $\Phi_I$ and the dual 2-form $B^I$, $I = 0, \ldots, h^{(2,1)}$ are
\[ \Phi_I = \int_{\alpha^I} C_3, \quad \Phi_{I+h^{(2,1)}+1} = \int_{\beta_I} C_3, \]  
\[ B^I = \int_{\beta_I} C_5, \quad B_{I+h^{(2,1)}+1} = \int_{\alpha^I} C_5. \]  
(3.8)
More precisely, $(d\Phi_I, dB^I)$ and $(d\Phi_{I+h^{(2,1)}+1}, dB_{I+h^{(2,1)}+1})$ are Hodge dual to each other in four dimensions. The general couplings (3.4) can now be expanded
\[ \int_{\mathbb{R}^{1,3} \times (\pi_a + \pi'_a)} C_3 \wedge \text{Tr} \left( F_a \wedge F_a \right) = \sum_I \left( e_a^I + (e_a^I)' \right) \int_{\mathbb{R}^{1,3}} \Phi_I \wedge \text{Tr} \left( F_a \wedge F_a \right) \]  
\[ + \sum_I \left( m_a^I + (m_a^I)' \right) \int_{\mathbb{R}^{1,3}} \Phi_{I+h^{(2,1)}+1} \wedge \text{Tr} \left( F_a \wedge F_a \right), \]  
\[ \int_{\mathbb{R}^{1,3} \times (\pi_a + \pi'_a)} C_5 \wedge \text{Tr} \left( F_a \right) = N_a \sum_I \left( m_a^I - (m_a^I)' \right) \int_{\mathbb{R}^{1,3}} B^I \wedge F_a \]  
\[ + N_a \sum_I \left( e_a^I - (e_a^I)' \right) \int_{\mathbb{R}^{1,3}} B_{I+h^{(2,1)}+1} \wedge F_a. \]  
(3.9)
Adding up all terms for the $U(1)_a - SU(N_b)^2$ anomaly, one finds
\[ A^{(2)}_{ab} \sim N_a \sum_I \left( (e_a^I + (e_a^I)') \left( m_b^I - (m_b^I)' \right) + (m_a^I + (m_a^I)') \left( e_b^I - (e_b^I)' \right) \right) \sim \]  
\[ \sim 2N_a \left( \pi_a - \pi'_a \right) \circ \pi_b, \]  
(3.10)
which has just the right form to cancel the anomalous contribution (3.3) of the chiral fermions.
### 3.3. Gauge boson masses

The starting gauge group of our model contained four abelian factors, of which two anomalous ones get massive by the Green-Schwarz mechanism. A very important input to decouple some of the superfluous abelian factors from the unbroken and anomaly-free gauge group is the occurrence of Stückelberg mass terms from axionic couplings even without an anomaly \[8\]. These couplings arise from the reduction of the RR 5-form in (3.9). In principle, $C_5$ can be reduced along any 3-cycle to produce $2h^{(2,1)} + 2$ 2-forms, as stated above. Again, only two of them are effectively involved in the Green-Schwarz mechanism to give masses to the gauge bosons of the two anomalous $U(1)$. At first sight, it then looks unlikely that any of the other two gauge bosons could evade getting a mass, being outnumbered by axions. One can rewrite the coupling term for the 2-forms $B^I$ in the Lagrangian like

$$M^a_I B^I \wedge F_a = \sum_I B^I \wedge \sum_a N_a (m^a_I - (m^a_I)'), F_a$$

with a $4 \times (h^{(2,1)} + 1)$ matrix $M^a_I$, and similarly a second such term for the $B_{I+h^{(2,1)}+1}$. The requirement to have a massless hypercharge gauge boson now translates into

$$M^a_I (1, 0, -3, 3)_a = (0, \ldots, 0)^I.$$

This implies that the $\beta_I$-components of the cycles $N_a (\pi_a - \pi'_a)$ are required to be linearly dependent. A similar relation has to hold for the couplings to the 2-forms $B_{I+h^{(2,1)}+1}$ and the $\alpha^I$-components, as well. This is very suggestive: Every independent cycle introduces one axionic coupling and the axion is eaten by one of the $U(1)$ gauge bosons. In order to have a surviving gauge boson, a linear relation needs to hold between the cycles. If the second $U(1)$ is meant to decouple (3.12) should be the only linear relation among the $N_a (\pi_a - \pi'_a)$. For the toroidal case one can check that the conditions derived in \[8\] in the dual type IIB picture precisely reproduce (3.12). The concrete model defined in (2.21) does in fact satisfy (3.12) or equivalently (2.22) and in addition $M^a_I$ has rank equal to 3. This means that the only unbroken abelian gauge symmetry is the hypercharge $U(1)_Y$.

### 4. Intersecting brane worlds in six dimensions

The methods developed above for constructing four-dimensional intersecting brane world models on smooth Calabi-Yau backgrounds can also be applied to orbifolds. In this
case one first needs to resolve the singular geometry in order to be able to compare to the classical data encoded in the intersection numbers. In this section we demonstrate the elegance and technical simplicity of the construction for six-dimensional K3-orbifolds, which is slightly simpler than Calabi-Yau-orbifolds, and via the six-dimensional constraints on anomaly cancellation offers an excellent check on the consistency of the results. Of course, some modifications need to be applied to the four-dimensional prescriptions in order to adapt to the K3. We are not going to explain everything in detail, but refer the reader to [21] for more instructions and proper definitions. Further extensions to F-theory and M-theory vacua were also discussed in this reference.

4.1. K3 compactification

In general, the compactification of type IIB on a K3 leaves $\mathcal{N} = (0, 1)$ supersymmetry in six dimensions. The closed string fields usually do not cancel the gravitational anomaly of the graviton multiplet by their own. The well known condition

$$n_H - n_V + 29 n_T = 273 \quad (4.1)$$

determines whether the irreducible $R^4$ coefficient cancels out, when taking the open string states into account as well.

The involution $\sigma$ now leaves fixed sLag 2-cycles in the K3, which are wrapped by O7-planes, whose charge is then canceled by D7-branes, according to the cancellation condition

$$\sum_a N_a (\pi_a + \pi_a') - 8 \pi_{O7} = 0 \quad (4.2)$$

The gauge group supported by the various stacks is again given by (2.7), while the chiral spectrum can be determined in complete analogy to the four-dimensional case. It is summarized in table 4, the subscripts denoting the representation under the little group $SO(4) \simeq SU(2) \times SU(2)$, which is to be flipped for a negative intersection number.

| Representation | Multiplicity |
|----------------|--------------|
| $[\text{Adj}]_{(1,2)}$ | $\pi_a \circ \pi_a$ |
| $[A_a + A_a']_{(1,2)}$ | $\frac{1}{2} (\pi_a \circ \pi_a' + \pi_a \circ \pi_{O7})$ |
| $[S_a + S_a']_{(1,2)}$ | $\frac{1}{2} (\pi_a \circ \pi_a' - \pi_a \circ \pi_{O7})$ |
| $[(N_a, N_b) + (\tilde{N}_a, \tilde{N}_b)]_{(1,2)}$ | $\pi_a \circ \pi_b$ |
| $[(N_a, \tilde{N}_b) + (\tilde{N}_a, N_b)]_{(1,2)}$ | $\pi_a \circ \pi_b'$ |

Table 4: Chiral spectrum in $d = 6$
Because there are no other contributions to the irreducible $\text{Tr}(F_a^4)$ anomaly coefficient these cancel automatically by the tadpole cancellation (2.10). The gravitational $R^4$ anomaly comes out to be

$$A_{\text{op}} = 14 \pi_{O7} \circ \pi_{O7},$$

in terms of $\mathcal{N} = (0, 1)$ supermultiplets. In fact, it had to be expected that the net contribution is independent of the concrete set of charged matter and only depends on its total self-intersection, since (4.3) must cancel the contribution $A_{\text{cl}} = 273 - n_H - 29 n_T = 28(9 - n_T)$ to the anomaly from the gravity multiplet. It now follows that

$$\pi_{O7} \circ \pi_{O7} = 2(9 - n_T) = \frac{1}{32} \sum_{a,b} N_a N_b (\pi_a \circ \pi_b + \pi_a \circ \pi_b'),$$

(4.4)

a strong consistency requirement that relates the topology of the $O7$-plane and the number of tensor multiplets in the effective theory. In [21] we have indeed given a purely mathematical proof of (4.4) that only rested on the sLag nature of $\text{Fix}(\sigma)$.

One can demonstrate that the spectrum of table 4 reproduces essentially all known orbifold models of type IIB orientifolds on $K3$ [53-60], although their results are usually obtained after lengthy CFT computations and tedious Chan-Paton algebra. In this sense, the concept of intersecting branes also offers a technical short-cut to produce such supersymmetric orientifold spectra. In the following we shall now discuss just one example of a $K3$-orbifold.

4.2. The $T^4/\mathbb{Z}_2$ $K3$-orbifold

A general $K3$-orbifold group $\mathbb{Z}_N = \{\Theta, \Theta^2, ..., 1\}$ acts crystallographically on $T^4$ which we may assume to be a direct product of two-dimensional tori $T^2_I$, $I = 1, 2$. The factorization implies a diagonal period matrix $\tau^{IJ}$,

$$dz_I = dx_I + \sum_{J=1}^2 \tau^{IJ} dy_J = dx_I + \tau^I dy_I, \quad z_I \equiv z_I + 1, \quad z_I \equiv z_I + \tau^I,$$

(4.5)

with diagonal orbifold action

$$\Theta z_I = e^{2\pi i v_I} z_I, \quad v_1 + v_2 = 0,$$

(4.6)

and $\overline{\sigma}$ reflecting $\Im(z_I)$. The entire orientifold group is generated by $\Omega \overline{\sigma}$ and $\Theta$. The calibration condition is now rephrased in terms of the relative angles $\varphi_I$ of any D7-brane with respect to the O7-plane,

$$\varphi_1 \pm \varphi_2 = \theta,$$

(4.7)
with some fixed angle \( \theta \) and an arbitrary sign. To preserve the same supersymmetry as does the O7-plane, one needs to demand

\[ \varphi_1 + \varphi_2 = 0. \] (4.8)

For the sake of brevity we now specialize to give just a single example of a K3-orbifold, the \( \mathbb{Z}_2 \) orbifold limit of K3. The action of \( \mathbb{Z}_2 \) on the \( z_1, z_2 \) is defined by \( v_I = (1/2, -1/2) \) as in (4.6). The homology includes some 2-cycles \( \pi_a \) on the K3 which are inherited from the torus cycles \( \pi_a \), corresponding to massless modes in the untwisted sector of the CFT in the singular limit. They are organized in orbits under \( \mathbb{Z}_2 \)

\[ \pi_a = \pi_a + \Theta \pi_a. \] (4.9)

The intersection form of these cycles is given by

\[ \pi_a \circ \pi_b = \frac{1}{2} \left( \sum_{i=0}^{1} \Theta^i \pi_a \right) \circ \left( \sum_{j=0}^{1} \Theta^j \pi_b \right). \] (4.10)

In the case at hand, there are six elements \( \pi_{ij} \) in \( H_2(T^4, \mathbb{Z}) \), which we denote as

\[ \{\pi_{13}, \pi_{24}, \pi_{14}, \pi_{23}, \pi_{12}, \pi_{34}\}. \] (4.11)

The indices \( (1, 2, 3, 4) \) are referring to the coordinates \( (x_1, y_1, x_2, y_2) \) along the four 1-cycles of the \( T^2 \). Their intersection form reads

\[ I_{T^4} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \text{diag}(1, -1, -1). \] (4.12)

Each of these six 2-cycles \( \pi_{ij} \) on \( T^4 \) gives rise to a 2-cycle on the orbifold, \( \pi_{ij} \). A subtlety arises in their proper normalization as generators of \( H_2(T^4/\mathbb{Z}_2, \mathbb{Z}) \), which is provided by \( \pi_{ij} = 2\pi_{ij} \). The intersection matrix

\[ I_{T^4/\mathbb{Z}_2}^{\text{Torus}} = 2 I_{T^4} \] (4.13)

follows for the cycles on the orbifold.

In addition the resolution of the fixed points of \( \Theta \) give rise to exceptional 2-cycles, massless fields in the twisted sectors of the orbifold CFT. For the \( \mathbb{Z}_2 \) orbifold there are
16 2-cycles blown-up to \( \mathbb{CP}^1 \) at the 16 fixed points \( P_{ij} \). The exceptional divisors are then denoted \( e_{ij} \). Their intersections read

\[
e_{ij} \circ e_{kl} = -2 \delta_{ik} \delta_{jl},
\]

the Cartan matrix of \( A_{16}^{1} \). As can be deduced from comparing to the CFT limit \([59,61-63]\), the O7-planes only wrap 2-cycles \( \pi_a \) inherited from the torus and no exceptional divisors. Its homology class is then given by

\[
\pi_{O7} = 2(\pi_{13} + \pi_{24}).
\]

To determine the action of \( \Omega \sigma \) on the cohomology of K3 one needs to take the intrinsic reflection of all twisted fields by \( \Omega \) into account. We then write \([\sigma] = (-1) \otimes p\), with some permutation \( p \) of twisted sectors,

\[
e_i \mapsto -e_{p(i)}.
\]

The actual operation \( p \) depends crucially on the complex structure of the background torus, for which two inequivalent choices are compatible with the required symmetries. In the particular case we shall be discussing, this distinction is unimportant, however. Referring to the \( A \) type choice, the complex structure \( \tau_I \) of any single \( T_2 \) is defined by selecting the two lattice vectors \( 1, \tau_I = i \Im(\tau_I) \) as a basis, where \( \sigma \) reflects along the real line. With this choice of complex structure all the fixed points \( P_{ij} \) are geometrically invariant. Through the intrinsic reflection of the blow-up mode, \([\sigma] \) then reflects all \( e_{ij} \). On \( T^4 \) only the cycles \( \pi_{13} \) and \( \pi_{24} \) are invariant and the action of \([\sigma] \) is summarized together by

\[
[\sigma]_{AA} = \text{diag}(1_2, -1_{20}),
\]

with \( 1_n \) denoting unit matrices of rank \( n \). From this the number of tensor multiplets follows as the number of eigenvalues +1 minus 1 to be \( n_T = 1 \). Computing the self-intersection number of the orientifold plane \((4.15)\) we find \( \pi_{O7} \circ \pi_{O7} = 16 \), consistent with \((4.4)\).

For the simple case of a \( \mathbb{Z}_2 \) orbifold group, one can directly compare the results of this procedure to the standard \( \mathbb{Z}_2 \) orientifold of type IIB string theory \([64,53]\), since the projection by \( \Omega \sigma \) is equivalent to the standard projection by \( \Omega \) upon performing T-dualities along the two circles parameterized by \( \Im(z_i) \). One particular solution of the tadpole constraints will then be found recovering the spectrum and gauge group first discovered by Bianchi and Sagnotti and described in terms of D-branes by Gimon and
Polchinski. In order to do so, we introduce just two stacks of fractional D7-branes with multiplicities $N_1 = N_2 = 16$, supporting $U(16)^2$. The cycles are
\[
\begin{align*}
\pi_1 &= \frac{1}{2} (\pi_{13}) + \frac{1}{2} (e_{11} + e_{12} + e_{21} + e_{22}), \\
\pi_2 &= \frac{1}{2} (\pi_{24}) + \frac{1}{2} (e_{11} + e_{13} + e_{31} + e_{33}),
\end{align*}
\]
plus their images under $\Omega \bar{\sigma}$. Their intersections follow
\[
\begin{align*}
\pi_1 \circ \pi_1 &= -2, & \pi_2 \circ \pi_2 &= -2, & \pi_1 \circ \pi'_1 &= 2, & \pi_2 \circ \pi'_2 &= 2, \\
\pi_1 \circ \pi_{O7} &= 2, & \pi_2 \circ \pi_{O7} &= 2, & \pi_1 \circ \pi_2 &= 0, & \pi_1 \circ \pi'_2 &= 1
\end{align*}
\]
(4.19)
to produce the chiral massless spectrum shown in table 5.

| Representation | Multiplicity |
|----------------|-------------|
| $[(\text{Adj}, 1) + (1, \text{Adj})]_{(2,1)}$ | 2 |
| $[(\text{A}, 1) + (1, \text{A}) + c.c.]_{(1,2)}$ | 2 |
| $[(16, 16) + c.c.]_{(1,2)}$ | 1 |

Table 5: BS/GP model

which is identical to the results of $\cite{64,53}$. Note, that in the orbifold limit chirality was induced by a non-trivial projection on the Chan-Paton indices, while the intersections of the brane were vanishing. However, in the smooth case the gauginos are localized at the self-intersections of the D7-branes. At the orbifold point the model is supersymmetric, as here the D7-branes simply lie on top of the orientifold plane. According to the discussion above, this does not change as long as all D7-branes wrap cycles calibrated by $\Re(\Omega_2)$.

In the view of the very general prescriptions to construct intersecting brane models, it is apparent that the solution found by the CFT methods is not the only supersymmetric vacuum of the $T^4/\mathbb{Z}_2$ K3-orbifold. To demonstrate this explicitly, let us consider an example that has a gauge group of reduced rank. It involves just a single stack of $N = 16$ branes, wrapped on
\[
\pi = \frac{1}{2} (\pi_{13} + \pi_{24} + \pi_{14} + \pi_{23}) + \frac{1}{2} (e_{11} + e_{44} + e_{14} + e_{41})
\]
(4.20)together with its image stack, with gauge group $U(16)$. The relevant intersection numbers are
\[
\begin{align*}
\pi \circ \pi &= -2, & \pi \circ \pi' &= 4, & \pi \circ \pi_{O7} &= 4,
\end{align*}
\]
(4.21)
giving rise to the chiral massless spectrum

| Representation | Multiplicity |
|----------------|-------------|
| $\text{Adj}_{(2,1)}$ | 2 |
| $[A + \overline{A}]_{(1,2)}$ | 4 |

Table 6: Chiral spectrum

Now the tadpoles are not canceled locally even at the singular orbifold point before blowing-up. Therefore, supersymmetry is not preserved in a trivial manner and the vanishing of the scalar D-term potential imposes constraints on the complex structure moduli of the torus. The scalar potential (2.11) at the orbifold point reduces to

$$
V = T_7 e^{-\phi_6} \left[ \prod_{I=1}^{2} \sqrt{3(\tau^I) + \frac{1}{3(\tau^I)}} - \left( \sqrt{3(\tau^1) 3(\tau^2)} + \frac{1}{\sqrt{3(\tau^1) 3(\tau^2)}} \right) \right]
$$

which vanishes precisely if $3(\tau^1) = 3(\tau^2)$. Moving away from this supersymmetric locus, the intersection angles do no longer satisfy (4.8), supersymmetry is broken spontaneously, and an open string tachyon appears. The corresponding FI term depends on $\xi \sim 3(\tau^1) - 3(\tau^2)$. In six dimensions the D-term potential has the general form

$$
V_{\text{D-term}} \sim \left( \sum_i q_i |\phi_i|^2 - \sum_i q_i |\tilde{\phi}_i|^2 - \xi \right)^2,
$$

where now $\phi_i$ and $\tilde{\phi}_i$ denote two complex scalars with charge $q_i$. Independent of the sign of $\xi$, one always gets a tachyonic mode if $\xi \neq 0$, which is in accord with the string theory picture.

5. M-theory lift of $\mathcal{N} = 1$ intersecting brane worlds via $G_2$ manifolds

As it is well known, 11-dimensional M-theory is supposed to describe the strong coupling regime of the type IIA superstring; hence we expect that by lifting intersecting brane world models with non-abelian gauge groups and chiral fermions interesting non-perturbative informations about the gauge dynamics can be obtained. Furthermore, as we will describe, an $\mathcal{N} = 1$ supersymmetric intersecting brane world scenario with only D6-branes, and possibly O6-planes, can be nicely described in purely geometrical terms.
via M-theory compactification on a seven-dimensional Ricci-flat manifold $X_7$ with reduced $G_2$ holonomy group (for some papers on M-theory compactifications on $G_2$ spaces see [53-83]). This can be easily seen by looking at the fields which couple to the 6-brane background. The D6-branes are the magnetically charged monopoles of the KK vector, and thus only couple to components of the eleven-dimensional metric, the RR 1-form $C_1$ and the dilaton of type IIA. The geometric lift of isolated D6-branes in flat ten-dimensional space-time is then a non-trivial $U(1)$ fibration over an $S^2 \subset \mathbb{R}^3$, a Taub-Nut space. Similarly, the O6-planes lift to an Atiyah-Hitchin space.

More precisely, the relation between the geometrical M-theory picture and the type IIA brane picture is as follows: If $X_7$ has a suitable $U(1)$ isometry, one obtains a type IIA superstring interpretation upon dimensional reduction to ten dimensions. This circle is usually non-trivially fibered over a six-dimensional base $B_6$ which serves as the geometric background of the corresponding IIA superstring theory, i.e. $B_6 = X_7/U(1)$. The space $B_6$ is in general non Ricci-flat, whereas the curvature of $B_6$ reflects the gravitational back reaction of the 6-branes on the type IIA metric. The specific form of the IIA brane configuration depends very much on the choice of the $U(1)$ action. In order to obtain a configuration that contains D6-branes, one has to ensure that the $U(1)$ action has a codimension 4 fixed point set $L$ which describes the world volume locus of the 6-branes. In M-theory language non-abelian gauge bosons arise, if $X_7$ has an A-D-E singularity of codimension four. The non-abelian gauge bosons correspond to massless M2-branes wrapped around collapsing 2-cycles. Product gauge groups with chiral bi-fundamental matter representations are provided by colliding singularities, i.e. by two or more sets of fixed points $L = L_1 \cup L_2 \ldots \cup L_i$, which intersect at a point on $X_7$. In the IIA brane picture this is described by the intersection of 6-branes. Hence massless fermions are supported by isolated (conical) singularities of codimension 7 of $X_7$, whose metrics are given in terms of a radial cone on a six-dimensional base space $Y_6$:

$$ds^2_{X_7} = dr^2 + r^2 d\Omega^2_{Y_6}. \tag{5.1}$$

In order for $X_7$ to have $G_2$ holonomy, it is known that $Y_6$ has to have weak $SU(3)$ holonomy. Resolving the point like singularity at $r = 0$ means that the corresponding product gauge group gets spontaneously broken to a diagonal subgroup by the vev of a bifundamental scalar field and that the associated fermions become massive.
So far, metrics for compact $G_2$ spaces have not yet been constructed. However a few examples of non-compact $G_2$ metrics are explicitly known. One can view these non-compact $G_2$ spaces as describing the local neighborhood of a compact $G_2$ space around some local singularity, e.g. around the locus of (intersecting) D6-branes. Being on a non-compact $G_2$ manifold, the gravitational degrees of freedom decouple, and one is left only with the local gauge degrees of freedom. For the corresponding IIA superstring theory this means that the global RR tadpole conditions do not need to be satisfied, since part of RR fluxes can escape to infinity on a non-compact direction.

Basically the known examples of non-compact $G_2$ spaces group together into two classes [84,85]: one is topologically a $\mathbb{R}^4$ bundle over $S^3$ and the other a $\mathbb{R}^3$ bundle over a quaternionic base space $Q$. The first class can be e.g. generalized by an $\mathbb{R}^4/\mathbb{Z}_N$ bundle over $S^3$, see [84]. This situation corresponds to $N$ wrapped, but non-intersecting D6-branes around the $S^3$ of the deformed conifold; the associated gauge theory is $\mathcal{N} = 1$ $SU(N)$ super Yang-Mills without chiral matter fields.

In the second class one indeed obtains examples with intersecting D6-branes. Specifically, examples with known metrics are given by the quaternionic spaces $Q = S^4$ and $Q = \mathbb{C}P^2$. Consider briefly the first example with $Q = S^4$ where the metric of $X_7$ can be written as a cone on $Y_6 = \mathbb{C}P^3$ [70]. The associated fixed point set of the $U(1)$ action is given by $L = \mathbb{R}^3 \cup \mathbb{R}^3$, meeting at the origin in $\mathbb{R}^6$. This corresponds to two intersecting D6-branes at special angles such that supersymmetry is preserved. The related field theory is given by an $\mathcal{N} = 1$, abelian $U(1) \times U(1)$ gauge theory with one charged chiral matter field.

Next let us discuss in some more detail the case of three intersecting D6-branes which belongs to $Q = \mathbb{C}P^2$ [70]. The associated metric describes a cone on $Y_6 = SU(3)/(U(1) \times U(1))$. Here the fixed point set $L = \mathbb{R}^3 \cup \mathbb{R}^3 \cup \mathbb{R}^3$ corresponds to three D6-branes which intersect in one point at supersymmetric angles. Resolving the point like singularity at the origin $r = 0$ such that the cone is deformed to a smooth $G_2$ manifold, the fix point set becomes $L = S^2 \times \mathbb{R} \cup \mathbb{R}^3$ with zero intersection of the two branches. This corresponds to two disjoint D6-branes which do not intersect anymore. Hence there are no more massless fermions on the resolved singularity. As explained in [70] this has the following nice field theory interpretation. In the singular case the gauge group of the three intersecting D6-branes is $U(1)^3$ with three chiral matter fields $\Phi_1 = (1,-1,0)$, $\Phi_2 = (-1,0,1)$ and $\Phi_3 = (0,1,-1)$. In addition there is also a world sheet instanton generated superpotential of the form $W \sim \Phi_1 \Phi_2 \Phi_3$. The flat directions of this superpotential always allow one
charged scalar field, say \( \phi_3 \), to take an arbitrary vev. So in the generic case the gauge group is Higgsed to \( U(1)^2 \) and all fermions are massive. This Higgsing precisely corresponds to resolving the cone singularity. Two D6-branes recombine into a single D6-brane under the Higgsing, which does not anymore intersect the third D6-brane.

Now it would be an interesting question how to generalize this scenario to the case of several intersecting D6-branes with associated non-abelian gauge structure. In [70] it was proposed to consider an \( \mathbb{R}^3 \) bundle over \( \mathbb{W} \mathcal{C} \mathbb{P}^2_{N_1, N_2, N_3} \) with at least two of the indices \( N_i \) are equal, say \( N_2 = N_3 \). This should then correspond to the intersection of 3 stacks of D6-branes. Just like in the abelian case, the corresponding gauge group \( U(N_1) \times U(N_2) \times U(N_2) \) will then be Higgsed to \( U(N_1) \times U(N_2) \) by the vev of the bifundamental scalar field when resolving the singular cone. In the following we will describe the work of [83], where it was tried to provide an explicit \( G_2 \) metric for this set-up by replacing the homogeneous, quaternionic spaces \( Q \), considered so far, by a non-homogeneous quaternionic space with only two isometries (see also [22] for related work). Specifically the quaternionic spaces used in [83] are based on the four-dimensional Minkowskian spaces with anti-self dual Weyl tensor introduced by Demiansky and Plebanski [86]. The corresponding Euclidean metric reads

\[
ds_4^2 = \frac{p^2 - q^2}{P} dp^2 + \frac{p^2 - q^2}{Q} dq^2 + \frac{P}{p^2 - q^2} \left( d\tau + q^2 d\sigma \right)^2 + \frac{Q}{p^2 - q^2} \left( d\tau + p^2 d\sigma \right)^2 ,
\]

with the forth order polynomials in the two coordinates \( p \) and \( q \):

\[
P = -\kappa (p - r_1)(p - r_2)(p - r_3)(p - r_4) ,
\]
\[
Q = \kappa (q - r_1)(q - r_2)(q - r_3)(q - r_4) ,
\]
\[
0 = r_1 + r_2 + r_3 + r_4 .
\]

Via the above constraint the quaternionic space depends on three parameters \( r_1, r_2 \) and \( r_3 \). The associated 7-dimensional metric with \( G_2 \) holonomy is given by

\[
ds^2 = \frac{1}{\sqrt{2\kappa|u|^2 + u_0}} \left( du^i + \epsilon^{ijk} A^j u^k \right)^2 + \sqrt{2\kappa|u|^2 + u_0} ds_4^2 ,
\]

which is topologically a \( \mathbb{R}^3 \) bundle (related to the coordinates \( u^i \)) over the quaternionic base space, given by the metric \( ds_4^2 \) with the \( SU(2) \) connection \( A^i \). For \( u_0 \neq 0 \) this space is smooth; however setting \( u_0 = 0 \) it develops a point like singularity, i.e. it will become a cone on a six-dimensional base \( Y_6 \).
In order to reduce to the ten-dimensional type IIA string with intersecting D6-branes, one has to choose an appropriate $U(1)$ Killing vector, which corresponds to the 11th M-theory directions. It will be a specific linear combination

$$k = \beta_1 \partial_\tau - \beta_2 \partial_\sigma .$$

(5.5)

Then the brane locations will depend on the fixed point set of $k$. Consider the following Killing vector

$$k = r_3^2 \partial_\tau - \partial_\sigma .$$

(5.6)

As discussed in [83] there are now two sets of 6-branes located at

$$D6_1 : p = r_3, \ u_1 = u_3 = 0 ,$$

$$D6_2 : q = r_3, \ u_1 = u_3 = 0$$

(5.7)

But by keeping generic values of the roots, there will be further codimension 6 fixed points at $q = r_2, \ p = r_4, \ u_1 = u_2 = 0$ and at $p = r_2, \ q = r_4, \ u_1 = u_2 = 0$. In order to avoid these fixed points we will set $r_1 = r_2$, which essentially moves these fixed points to infinity since the metric develops an infinite throat at $p \to r_2 = r_1$. In addition the parameters have to obey the constraint $0 = 2r_2 + r_3 + r_4$, such that the metric depends on two parameters, say $r_3$ and $r_4$. The number of D6-branes at the two fixed point sets is related to the surface gravity $|\nabla k|$ of the corresponding fixed point set, namely $N_i \sim \frac{1}{|\nabla k_i|}$. Calculating the surface gravity for the fixed point set given in eq.(5.7) gives

$$|\nabla k|_1 = |\nabla k|_2 = \frac{k}{4} (3r_3 + r_4)^2 (r_4 - r_3) .$$

(5.8)

(That both numbers coincide, is a consequence of the symmetry $p \leftrightarrow q$ of the metric.) Since this solution is characterized by two parameters $r_3$ and $r_4$, it is very tempting to identify this space as the one related to the weighted projective space $\mathbb{WCP}^2_{N_1,N_2,N_2}$. In our case the number of 6-branes in two stacks agree and we expect a gauge group $SU(N_1)^3$, where in the deformed case the Higgsing should be done in a way that the product of two equal gauge groups survives, because the two components of the fixed point set are related to the same number of 6-branes. At the moment, these conclusions are more speculative and further investigations are necessary.
Acknowledgments

We would like to thank the organizers of SUSY02, The 10th International Conference on Supersymmetry and Unification of Fundamental Interactions at DESY Hamburg, of The First International Conference on String Phenomenology at Oxford, of Strings 2002 at Cambridge, and finally of the 35th International Symposium Ahrenshoop on the Theory of Elementary Particles, Recent Developments in String/M-Theory and Field Theory at Berlin. The material presented in this article is an extended and combined version of the talks given at these conferences. D.L. like to thank Klaus Behrndt, Gianguido Dall’Agata and Swanpa Mahaptra for the pleasant collaboration on the material presented in the last chapter. The work is supported in part by the EC under the RTN project HPRN-CT-2000-00131.
References

[1] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, Noncommutative Compactifications of Type I Strings on Tori with Magnetic Background Flux, JHEP 0010 (2000) 006, hep-th/0007024.

[2] C. Angelantonj, I. Antoniadis, E. Dudas, A. Sagnotti, Type I Strings on Magnetized Orbifolds and Brane Transmutation, Phys. Lett. B 489 (2000) 223, hep-th/0007090.

[3] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, Magnetic Flux in Toroidal Type I Compactification, Fortsch. Phys. 49 (2001) 591, hep-th/0010198.

[4] C. Angelantonj, A. Sagnotti, Type I Vacua and Brane Transmutation, hep-th/0010279.

[5] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadas, A. M. Uranga, $D = 4$ Chiral String Compactifications from Intersecting Branes, J. Math. Phys. 42 (2001) 3103, hep-th/0011073.

[6] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadas, A. M. Uranga, Intersecting Brane Worlds, JHEP 0102 (2001) 047, hep-ph/0011132.

[7] R. Blumenhagen, B. Körs and D. Lüst, Type I Strings with $F$ and $B$-Flux, JHEP 0102 (2001) 030, hep-th/0012156.

[8] L. E. Ibanez, F. Marchesano, R. Rabadas, Getting just the Standard Model at Intersecting Branes, JHEP 0111 (2001) 002, hep-th/0105155.

[9] S. Förste, G. Honecker and R. Schreyer, Orientifolds with Branes at Angles, JHEP 0106 (2001) 004, hep-th/0105208.

[10] R. Blumenhagen, B. Körs, D. Lüst and T. Ott, The Standard Model from Stable Intersecting Brane World Orbifolds, Nucl. Phys. B 616 (2001) 3, hep-th/0107138.

[11] M. Cvetic, G. Shiu and A. M. Uranga, Chiral Four-Dimensional $N=1$ Supersymmetric Type IIA Orientifolds from Intersecting D6-Branes, Nucl. Phys. B 615 (2001) 3, hep-th/0107166.

[12] D. Bailin, G. V. Kraniotis and A. Love, Standard-like Models from Intersecting D4-branes, Phys. Lett. B 530 (2002) 202, hep-th/0108131.

[13] G. Honecker, Intersecting Brane World Models from D8-branes on $(T^2 \times T^4 / \mathbb{Z}_3)/\Omega R_1$ Type IIA Orientifolds, JHEP 0201 (2002) 025, hep-th/0201037.

[14] J. R. Ellis, P. Kanti and D. V. Nanopoulos, Intersecting branes flip SU(5), hep-th/0206087.

[15] C. Kokorelis, GUT Model Hierarchies from Intersecting Branes, JHEP 0208 (2002) 018, hep-th/0203187.

[16] D. Cremades, L. E. Ibanez and F. Marchesano, Standard Model at Intersecting D5-branes: Lowering the String Scale, hep-th/0205074.

[17] C. Kokorelis, New Standard Model Vacua from Intersecting Branes, JHEP 0209 (2002) 029, hep-th/0205147.
[18] R. Blumenhagen, B. Körs, D. Lüst and T. Ott, *Intersecting Brane Worlds on Tori and Orbifolds*, Fortsch. Phys. **50** (2002) 843, hep-th/0112015.
[19] R. Blumenhagen, B. Körs and D. Lüst, *Moduli Stabilization for Intersecting Brane Worlds in Type 0′ String Theory*, Phys. Lett. B **532** (2002) 141, hep-th/0202024.
[20] M. Cvetic, G. Shiu and A. M. Uranga, *Three-Family Supersymmetric Standard-like Models from Intersecting Brane Worlds*, Phys. Rev. Lett. **87** (2001) 201801, hep-th/0107143.
[21] R. Blumenhagen, V. Braun, B. Körs and D. Lüst, *Orientifolds of K3 and Calabi-Yau Manifolds with Intersecting D-branes*, JHEP **0207** (2002) 026, hep-th/0206038.
[22] A. M. Uranga, *Local Models for Intersecting Brane Worlds*, hep-th/0208014.
[23] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *The Hierarchy Problem and New Dimensions at a Millimeter*, Phys. Lett. B **429** (1998) 263, hep-ph/9803315.
[24] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *New Dimensions at a Millimeter to a Fermi and Superstrings at a TeV*, Phys. Lett. B **436** (1998) 257, hep-ph/9804398.
[25] D. Cremades, L. E. Ibanez and F. Marchesano, *SUSY Quivers, Intersecting Branes and the Modest Hierarchy Problem*, JHEP **0207** (2002) 009, hep-th/0201203.
[26] M. Klein, *Couplings in Pseudo-Supersymmetry*, hep-th/0205300.
[27] M. Klein, *Loop-Effects in Pseudo-Supersymmetry*, hep-th/0209206.
[28] C. P. Burgess, L. E. Ibanez and F. Quevedo, *Strings at the intermediate scale or is the Fermi scale dual to the Planck scale?*, Phys. Lett. B **447** (1999) 257, hep-ph/9810535.
[29] L. Randall and R. Sundrum, *A Large Mass Hierarchy from a Small Extra Dimension*, Phys. Rev. Lett. **83** (1999) 3370, hep-ph/9906064.
[30] L. Randall and R. Sundrum, *An Alternative to Compactification*, Phys. Rev. Lett. **83** (1999) 4690, hep-th/9906064.
[31] I. Brunner, M. R. Douglas, A. Lawrence and C. Römelsberger, *D-branes on the Quintic*, JHEP **0008** (2000) 015, hep-th/9906200.
[32] P. Kaste, W. Lerche, C. A. Lutken and J. Walcher, *D-branes on K3-fibrations*, Nucl. Phys. B **582** (2000) 203, hep-th/9912147.
[33] S. Kachru, S. Katz, A. Lawrence and J. McGreevy, *Open String Instantons and Superpotentials*, Phys. Rev. D **62** (2000) 026001, hep-th/9912151.
[34] K. Hori, A. Iqbal and C. Vafa, *D-branes and the Mass of the Quark*, Nucl. Phys. B **582** (2000) 203, hep-th/9912147.
[35] S. Kachru, S. Katz, A. Lawrence and J. McGreevy, *Open String Instantons and Superpotentials*, Phys. Rev. D **62** (2000) 026001, hep-th/9912151.
[36] K. Hori, A. Iqbal and C. Vafa, *D-branes and the Mass of the Quark*, Nucl. Phys. B **582** (2000) 203, hep-th/9912147.
[37] A. Hanany and A. Iqbal, *Quiver Theories from D6-branes via Mirror Symmetry*, JHEP **0204** (2002) 009, hep-th/0108137.
[38] P. Mayr, *N=1 Mirror Symmetry and Open/Closed String Duality*, hep-th/0108229.
[39] B. Feng, A. Hanany, Y. H. He and A. M. Uranga, *Toric Duality as Seiberg Duality and Brane Diamonds*, JHEP **0112** (2001) 035, hep-th/0109063.
[40] W. Lerche and P. Mayr, *On N = 1 Mirror Symmetry for Open Type II Strings*, hep-th/0111113.
[39] W. Lerche, P. Mayr and N. Warner, *Holomorphic N = 1 Special Geometry of Open-closed Type II Strings*, hep-th/0207259.
[40] W. Lerche, P. Mayr and N. Warner, *N=1 Special Geometry, Mixed Hodge Variations and Toric Geometry*, hep-th/0208039.
[41] C. Angelantonj and A. Sagnotti, *Open Strings*, hep-th/0204089.
[42] M. R. Douglas, *Branes within Branes*, hep-th/9512077.
[43] M. Green, J. A. Harvey, G. Moore, *I-Brane Inflow and Anomalous Couplings on D-Branes*, Class. Quant. Grav. 14 (1997) 47, hep-th/9605033.
[44] J. F. Morales, C. A. Scrucca and M. Serone, *Anomalous Couplings for D-branes and O-planes*, Nucl. Phys. B 552 (1999) 291, hep-th/9812071.
[45] C. A. Scrucca and M. Serone, *Anomalies and Inflow on D-branes and O-planes*, Nucl. Phys. B 556 (1999) 197, hep-th/9903145.
[46] B. Stefański, jr, *Gravitational Couplings of D-branes and O-planes*, Nucl. Phys. B 548 (1999) 275, hep-th/9812088.
[47] M. Berkooz, M. R. Douglas and R. G. Leigh, *Branes Intersecting at Angles*, Nucl. Phys. B 480 (1996) 265, hep-th/9606139.
[48] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Asymmetric Orbifolds, Noncommutative Geometry and Type I Vacua*, Nucl. Phys. B 582 (2000) 44, hep-th/0003024.
[49] C. Bachas, *A Way to Break Supersymmetry*, hep-th/9503030.
[50] D. Cremades, L. E. Ibáñez and F. Marchesano, *Intersecting Brane Models of Particle Physics and the Higgs Mechanism*, JHEP 0207 (2002) 022, hep-th/0203160.
[51] R. Blumenhagen, B. Körs, D. Lüst and T. Ott, *Hybrid Inflation in Intersecting Brane Worlds*, Nucl. Phys. B 641 (2002) 235, hep-th/0202124.
[52] J. García-Bellido and R. Rabaud, *Complex Structure Moduli Stability in Toroidal Compactifications*, JHEP 0205 (2002) 042, hep-th/0203247.
[53] E. G. Gimon and J. Polchinski, *Consistency Conditions for Orientifolds and D-Manifolds*, Phys. Rev. D 54 (1996) 1667, hep-th/9601038.
[54] E. G. Gimon and C. V. Johnson, *K3 Orientifolds*, Nucl. Phys. B 477 (1996) 715, hep-th/9604129.
[55] J. D. Blum and A. Zaffaroni, *An Orientifold from F Theory*, Phys. Lett. B 387 (1996) 71, hep-th/9607019.
[56] A. Dabholkar and J. Park, *A Note on Orientifolds and F-theory*, Phys. Lett. B 394 (1997) 302, hep-th/9607041.
[57] J. D. Blum, *F Theory Orientifolds, M Theory Orientifolds and Twisted Strings*, Nucl. Phys. B 486 (1997) 34, hep-th/9608053.
[58] A. Dabholkar and J. Park, *An Orientifold of Type IIB theory on K3*, Nucl. Phys. B 472 (1996) 207, hep-th/9602030; *Strings on Orientifolds*, Nucl. Phys. B 477 (1996) 701, hep-th/9604178.
[59] R. Blumenhagen, L. Görlich and B. Körs, *Supersymmetric Orientifolds in 6D with D-Branes at Angles*, Nucl. Phys. B **569** (2000) 209, [hep-th/9908130](http://www.arxiv.org/abs/hep-th/9908130).

[60] G. Pradisi, *Type I Vacua from Diagonal $\mathbb{Z}_3$-Orbifolds*, Nucl. Phys. B **575** (2000) 134, [hep-th/9912218](http://www.arxiv.org/abs/hep-th/9912218).

[61] R. Blumenhagen, L. Görlich and B. Körs, *Supersymmetric 4D Orientifolds of Type IIA with D6-branes at Angles*, JHEP **0001** (2000) 040, [hep-th/9912204](http://www.arxiv.org/abs/hep-th/9912204).

[62] S. Förste, G. Honecker and R. Schreyer, *Supersymmetric $\mathbb{Z}_N \times \mathbb{Z}_M$ Orientifolds in 4D with D-Branes at Angles*, Nucl. Phys. B **593** (2001) 127, [hep-th/0008250](http://www.arxiv.org/abs/hep-th/0008250).

[63] M. Bianchi and A. Sagnotti, *On the Systematics of Open String Theories*, Phys. Lett. B **247** (1990) 517.

[64] G. Papadopoulos and P. K. Townsend, *Compactification of D = 11 Supergravity on Spaces of Exceptional Holonomy*, Phys. Lett. B **357** (1995) 300, [hep-th/9506150](http://www.arxiv.org/abs/hep-th/9506150).

[65] B. S. Acharya, *On Realising N = 1 Super Yang-Mills in M theory*, [hep-th/0011089](http://www.arxiv.org/abs/hep-th/0011089).

[66] M. Atiyah, J. M. Maldacena and C. Vafa, *An M-theory Flop as a Large N Duality*, J. Math. Phys. **42** (2001) 3209, [hep-th/0011250](http://www.arxiv.org/abs/hep-th/0011250).

[67] M. Cvetic, G. W. Gibbons, H. Lu and C. N. Pope, *M3-branes, G(2) Manifolds and Pseudo-Supersymmetry*, Nucl. Phys. B **620** (2002) 3, [hep-th/0106026](http://www.arxiv.org/abs/hep-th/0106026).

[68] A. Brandhuber, J. Gomis, S. S. Gubser and S. Gukov, *Gauge Theory at Large N and New G2 Holonomy Metrics*, Nucl. Phys. B **611** (2001) 179, [hep-th/0106034](http://www.arxiv.org/abs/hep-th/0106034).

[69] B. Acharya and E. Witten, *M-theory Dynamics on a Manifold of G2 Holonomy*, [hep-th/0107177](http://www.arxiv.org/abs/hep-th/0107177).

[70] E. Witten, *Anomaly Cancellation on Manifolds of G2 Holonomy*, [hep-th/0108163](http://www.arxiv.org/abs/hep-th/0108163).

[71] M. Cvetic, G. W. Gibbons, H. Lu and C. N. Pope, *Cohomogeneity One Manifolds of Spin(7) and G2 Holonomy*, Phys. Rev. D **65** (2002) 106004, [hep-th/0108243](http://www.arxiv.org/abs/hep-th/0108243).

[72] T. Eguchi and Y. Sugawara, *String Theory on G2 Manifolds based on Gepner Construction*, Nucl. Phys. B **630** (2002) 132, [hep-th/0111012](http://www.arxiv.org/abs/hep-th/0111012).

[73] R. Blumenhagen and V. Braun, *Superconformal Field Theories for Compact G2 Manifolds*, JHEP **0112** (2001) 006, [hep-th/0110232](http://www.arxiv.org/abs/hep-th/0110232).

[74] R. Roiban and J. Walcher, *Rational Conformal Field Theories with G2 Holonomy*, JHEP **0112** (2001) 008, [hep-th/0110302](http://www.arxiv.org/abs/hep-th/0110302).

[75] T. Curio, B. Körs and D. Lüst, *Fluxes and Branes in Type II Vacua and M-theory Geometry with G2 and Spin(7) Holonomy*, Nucl. Phys. B **636** (2002) 197, [hep-th/0111163](http://www.arxiv.org/abs/hep-th/0111163).

[76] T. Friedmann, *On the Quantum Moduli Space of M Theory Compactifications*, Nucl. Phys. B **635** (2002) 384, [hep-th/0203250](http://www.arxiv.org/abs/hep-th/0203250).
[79] A. Brandhuber, $G_2$ Holonomy Spaces from Invariant Three-forms, Nucl. Phys. B 629 (2002) 393, hep-th/0112113.
[80] P. Berglund and A. Brandhuber, Matter from $G_2$ Manifolds, Nucl. Phys. B 641 (2002) 351, hep-th/0205184.
[81] K. Behrndt, Singular 7-manifolds with $G(2)$ holonomy and intersecting 6-branes, Nucl. Phys. B 635 (2002) 158, hep-th/0204061.
[82] L. Anguelova and C. I. Lazaroiu, M-theory on 'Toric' $G(2)$ Cones and its Type II Reduction, hep-th/0205074.
[83] K. Behrndt, G. Dall’Agata, D. Lüst and S. Mahapatra, Intersecting 6-branes from New 7-manifolds with $G(2)$ Holonomy, JHEP 0208 (2002) 027, hep-th/0207117.
[84] R. L. Bryant and S. Salamon, On the Construction of some Complete Metrics with Exceptional Holonomy, Duke Math. J. 58 (1989) 829.
[85] G. W. Gibbons, D. N. Page and C. N. Pope, Einstein Metrics On $S^3$, $R^3$ And $R^4$ Bundles, Commun. Math. Phys. 127 (1990) 529.
[86] J. F. Plebanski and M. Demianski, Rotating, Charged, and Uniformly Accelerating Mass in General Relativity, Annals Phys. 98 (1976) 98.