Cavity-enhanced light scattering in optical lattices to probe atomic quantum statistics

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Studies of ultracold atoms in optical lattices link various disciplines. Fundamental quantum many-body theories, formulated initially for condensed matter, can be tested in better controllable atomic systems [1], e.g., strongly correlated phases, quantum simulators. Such studies influence different areas [1]: quantum information processing, ultracold collisions, exotic molecules, etc.

While mean-field approaches describe only the average atomic density, the main goal is to study quantum properties of these gases. They are most prominent in lattices, where one has phase transitions between states of similar density but radically different quantum fluctuations.

Standard methods to measure quantum properties are based on matter-wave interference of atoms released from a trap [2] destroying the system. “Bragg spectroscopy” using stimulated matter-wave scattering by laser pulses proved successful [3, 4] but destructive. Alternative less destructive methods observing scattered light were proposed mainly for homogeneous Bose-Einstein condensates (BEC) [5, 6, 7, 8], but not yet implemented.

Our model is based on nonresonant collective light scattering into a cavity. Angle resolved measurements of photon number and variance give information about atom-number fluctuations and pair correlations without single-site access. Observation at angles of diffraction minima provides information on quantum fluctuations insensitive to classical noise. For transverse probing, no photon is scattered into a cavity from a Mott insulator phase, while the photon number is proportional to the atom number for a superfluid.

Different quantum states of atoms in optical lattices can be nondestructively monitored by off-resonant collective light scattering into a cavity. Angle resolved measurements of photon number and variance give information about atom-number fluctuations and pair correlations without single-site access. Observation at angles of diffraction minima provides information on quantum fluctuations insensitive to classical noise. For transverse probing, no photon is scattered into a cavity from a Mott insulator phase, while the photon number is proportional to the atom number for a superfluid.

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FIG. 1: (Color online) Setup. A lattice is illuminated by a probe at the angle θ₀ which is scattered into a cavity at θ₁.

lattice with M sites. A region of K ≤ M sites is illuminated by probe light which is scattered into another mode (cf. Fig. 1). Although, each mode could be a freely propagating field, we will consider cavity modes whose geometries (i.e. axis directions or wavelengths) can be varied. A related manybody Hamiltonian is given by

\[ H = \sum_{l=0,1} \hbar \omega_l a^\dagger_l a_l + \int d^3r \Psi^\dagger(r) H_{a1} \Psi(r), \quad (1) \]

\[ H_{a1} = \frac{p^2}{2m_a} + V_{cl}(r) + \hbar g_0^2 \sum_{l,m=0,1} \frac{u_l^*(r) u_m(r) a^\dagger_l a_m}{\Delta_{ma}}, \]

where \( a_0 \) (\( a_1 \)) are the annihilation operators of the probe (scattered) light with the frequencies \( \omega_{0,1} \), wave vectors \( k_{0,1} \), and mode functions \( u_{0,1}(r) \); \( \Psi(r) \) is the atom-field operator. In the effective single-atom Hamiltonian \( H_{a1} \), \( p \) and \( r \) are the momentum and position operators of an atom of mass \( m_a \) trapped in a classical potential \( V_{cl}(r) \); \( g_0 \) is the atom–light coupling constant. We consider the field-atom detunings \( \Delta_{ma} = \omega_l - \omega_a \) larger than the spontaneous emission rate and Rabi frequencies. Thus, in \( H_{a1} \) the adiabatic elimination of the upper state was used.

Assuming weak fields \( a_{0,1} \), we expand \( \Psi(r) \) in Eq. (1) using localized Wannier functions corresponding to \( V_{cl}(r) \) and keep only the lowest vibrational state at each site: \( \Psi(r) = \sum_{i=1}^M b_i w(r - r_i) \), where \( b_i \) is the atom annihi-
lational operator at site with coordinate $r_i$. Substituting this in Eq. (1), one can get a generalized Bose-Hubbard Hamiltonian including light scattering. However, in contrast to our previous work [12] and “Bragg spectroscopy” [4], we do not consider lattice excitations here and focus on scattering from atoms in a prescribed state.

Neglecting atomic tunneling, the Hamiltonian reads:

$$H = \sum_{i=0,1} \hbar \omega_i a_i^\dagger a_i + \hbar \omega_0 \sum_{i \neq 0,1} \frac{g^2}{\Delta a_i} \sum_{K} J_{i}^{Km} \sum_{m=0,1} \left( \sum_{i=1}^{K} J_{i}^{Km} \hat{n}_i \right),$$

where $\hat{n}_i = b_i^\dagger b_i$. We define the operator of the atom number at illuminated sites as $\hat{N}_K = \sum_{i=1}^{K} \hat{n}_i$. For a deep lattice the coefficients $J_{i}^{Km}$ can be studied even by classical scattering [14] neglecting atom spreading, which can be studied even by classical scattering [14].

The Heisenberg equation for the scattered light in the frame rotating with $\omega_0$ ($\Delta_{01} = \omega_0 - \omega_1$) thus reads:

$$\dot{a}_i = \left[ \Delta_{01} - \frac{g^2}{\Delta a_i} \sum_{i=1}^{K} |u_1(r_i)|^2 \hat{n}_i \right] a_i - i \frac{g^2 a_0}{\Delta a_0} \sum_{i=1}^{K} u_0^* (r_i) u_0 (r_i) \hat{n}_i - \kappa a_i,$$

where $\kappa$ is the cavity decay rate and $a_0$ will be assumed a classical field given by a c-number constant.

**Light properties.** Though the dispersion shift of a cavity mode is sensitive to atom statistics through $\hat{n}_i$, we assume it is much smaller than $\kappa$ or $\Delta_{01}$. A stationary solution of Eq. (2) for $a_1$ and photon number then reads

$$a_1 = C \hat{D}, \quad n_{ph} = a_1^\dagger a_1 = |C|^2 \hat{D} \hat{D}, \quad \hat{D} = \sum_{i=1}^{K} A_i \hat{n}_i,$$

with $C \equiv i \frac{g a_0}{\Delta a_0 (i \Delta_{01} - \kappa)}$ and the coefficients $A_i(\theta_0, \theta_1) \equiv u_1^* (r_i) u_0 (r_i)$. This expression of the light operators through the atomic ones is a central result here.

For a 1D lattice of period $d$ and atoms trapped at $x_m = md$ ($m = 1, 2, \ldots, M$) the mode functions are $u_{01}(r_m) = \exp (i m k_0 d)$ for standing waves with $k_0 = \frac{1}{d}$ and $u_{01}(r_m) = \cos (m k_0 d)$ for standing waves with $k_0 d = \frac{1}{d}$ (cf. Fig. 1). For the atomic quantum state we use the assumptions: (i) the mean atom number at all sites is $\langle \hat{n}_i \rangle = n = N/M$ ($\langle \hat{N}_K \rangle = N_K \equiv n M$) and (ii) the pair correlations $\langle \hat{n}_i \hat{n}_j \rangle$ are identical for any $i \neq j$, which is valid for a deep lattice, and will be denoted as $\langle \hat{n}_i \hat{n}_j \rangle$ (with $a \neq b$).

Thus, $\langle a_1 \rangle \sim \langle \hat{D} \rangle = \sum_{i=1}^{K} A_i \langle \hat{n}_i \rangle = n A$ showing that the field amplitude only depends on the mean density and exhibits the angular distribution of classical diffraction $A(\theta_0, \theta_1) \equiv \sum_{i=1}^{K} A_i(\theta_0, \theta_1)$ with diffraction maxima and minima. The central point now is that the photon number [3] is not just the amplitude squared, but we get

$$\langle \hat{n}_a \hat{n}_b \rangle = n^2 (1 - 1/N) + n, \quad \langle \hat{D}_a \hat{D}_b \rangle = n (1 - 1/M) + 0, \quad R(\theta_0, \theta_1) \equiv \langle \hat{D}_a \hat{D}_b \rangle - \langle \hat{D}_a \rangle \langle \hat{D}_b \rangle = 0.$$
a photon number proportional to the atom number $N$.

A coherent state approximates a SF but without pair
correlations. In the limit $N, M \to \infty$, it well describes
scattering from a small region ($K \ll M$) of a partially
illuminated superfluid ($SF_K$). However, we proved that
even in this limit it fails to describe scattering at angles
of Bragg maxima from a large lattice region ($K \sim M$).

Example. Let us now show the most striking predic-
tions of this model at the basic example of a probe trans-
verse to the lattice ($\theta_1 = 0$). The scattered light is
collected in a cavity along the lattice ($\theta_1 = \pi/2$) with
atoms trapped at the antinodes ($d = \lambda/2$) [12, 13].

The operator $\hat{D} = \sum_{k=1}^{K} (-1)^{k+1} \hat{n}_k$ [14] here gives almost
zero average field amplitude independently on the
atomic state. This reflects the opposite phase of light
scattered from atoms separated by $\lambda/2$ (diffraction minim-
imum). However, the cavity photon-number is propor-
tional to $\langle \hat{D}^\dagger \hat{D} \rangle = (\langle \hat{n}^2 \rangle - \langle \hat{n}_a \hat{n}_b \rangle) K$ [cf. Eq. (13a)], which is determined by statistics of a particular state. Thus,
atoms in a MI state scatter no photons, while a SF scat-
ters number of photons proportional to the atom number:

$$\langle a_1 \rangle_{\text{MI}} = \langle a_1 \rangle_{\text{SF}} = 0, \quad \text{but} \quad \langle a_1 a_1 \rangle_{\text{MI}} = 0, \quad \langle a_1 a_1 \rangle_{\text{SF}} = |C|^2 N_K.$$  

Moreover, the photon number fluctuations $(\Delta n_{ph})^2$
are also different for various atomic states. In the MI
state, the variance $(\Delta \hat{a}^\dagger \hat{a})^2_{\text{MI}} = |D|^2_{\text{MI}} = 0$, whereas in SF, there is a strong noise $(\Delta \hat{a}^\dagger \hat{a})^2_{\text{SF}} \approx 2 N_K^2$.

Coupled light-matter dynamics in a cavity can lead to a
new self-organized phase [15] with atoms trapped at
every second site ($d = \lambda$), which gives $\hat{D} = \sum_{k=1}^{K} \hat{n}_k = \hat{N}_K$ [3]. If this state is a MI with $d = \lambda$, the number of photons $\langle a_1^\dagger a_1 \rangle_{\text{Self-arg}} = |C|^2 N_K^2$ is proportional to the
atom number squared and has a superradiant character.

Angular distributions. We will quantitatively dis-
cuss angular intensity distributions for scattering be-
tween two traveling waves, where Eq. (11) reduces to

$$R = \frac{\langle \delta \hat{n}_a \delta \hat{n}_b \rangle \sin^2 (K \alpha_{-}/2)}{\sin^2 (\alpha_{-}/2)} + (\langle \hat{n}^2 \rangle - \langle \delta \hat{n}_a \delta \hat{n}_b \rangle) K.$$  

While $|A|^2$ in the first term reproduces classical diffraction
with $\alpha_{-} = k_0 a \sin \theta_0 - k_1 a \sin \theta_1$, the second term in
Eq. (11) is simply isotropic. Thus, the noise quantity
is zero for MI, $R_{\text{MI}} = 0$, nonzero but isotropic for the coherent state, $R_{\text{Coh}} = N_K$, and angle dependent for a
SF. In a SF, even small pair correlations $\langle \delta \hat{n}_a \delta \hat{n}_b \rangle = -N/M^2$ give a large contribution near diffraction max-
ima $\alpha_{-} = 2\pi l, l = 0, 1, \ldots$, where the geometric factor is
$K^2$, invalidating the coherent-state approximation.

Figure 2 displays those angular distributions. Classical
diffraction $|D|^2$ with the only possible zero-order maxima at $\theta_1 = 0, \pi$ ($d = \lambda_0, l = 0 \pi$, $\theta_0 = 0$) is shown in
Fig. 2(a). $R$ for the coherent and SF$_K$ states are plotted
in Figs. 2(b), (c). For MI, $R = 0$. In SF, there is a noise suppression at maxima, which is total for all sites illuminated, $K = M$, and partial for $K = M/2$.

In a maximum, $D$, is reduced to $N_K$. Thus, the field amplitude is determined by $N_K = nK$, the intensity depends on $|D|^2$, while $R = (\Delta N_K)^2$ gives the atom-number variance at $K$ sites, which reflects the total and partial noise suppression in Figs. 2(b) and 2(c), since $|N_K|$ fluctuates for $K < M$. In diffraction minima, the field is zero, but the intensity is proportional to $\langle \hat{n}^2 \rangle - \langle \hat{n}_a \hat{n}_b \rangle$. Under scattering of spatially incoherent light, the intensity is isotropic and proportional to $\langle \hat{n}^2 \rangle$.

So, in optical experiments, varying the geometry, the global statistics of $K$ sites, local single-site statistics, and pair correlations can be obtained even without a single-site access. Thus, light scattering gives a way to distin-
guish between atomic states. As shown by Eq. (11) and

\[ \frac{\langle \delta \hat{n}_a \delta \hat{n}_b \rangle \sin^2 (K \alpha_{-}/2)}{\sin^2 (\alpha_{-}/2)} + (\langle \hat{n}^2 \rangle - \langle \delta \hat{n}_a \delta \hat{n}_b \rangle) K. \]
Fig. 2 MI and SF$_M$ states are different in diffraction minima and in incoherent light. They are indistinguishable in maxima. SF$_M$ and coherent states differ in maxima only. The MI and coherent state are different at any angles.

The noise quantity or photon statistics are different in orders of $N_K$ for various states. Nevertheless, for large $N_K$, there could be practical problems to subtract large values in a maximum. In Refs. [6], this even led to a conclusion about the state indistinguishability by intensity measurements. In contrast to homogeneous BECs, in lattices, this problem has a natural solution: measurements outside maxima are free of the strong classical-like part of scattering and thus directly reflect fluctuations.

A classical analogy of different light scattering consists in different density fluctuations. A quantum treatment gives a deeper insight. Superfluid state is a superposition of all possible multisite Fock states giving distributions of $N$ atoms at $M$ sites. Various Fock states become entangled to scattered light of different phases and amplitudes. In contrast to a classical case (and MI with the only multisite Fock state), light fields entangled to various distributions do not interfere with each other due to the orthogonality of the Fock states. This reflects the which-way information and explains the zero amplitude but nonzero photon number in a diffraction minimum.

If at least one of the modes is a standing wave, the angle dependences become much richer. Besides new classical maxima given by $\alpha_\pm = k_{0,1}d\sin\theta_0 \pm k_{1,2}d\sin\theta_1$, the second, “noise,” term in Eqs. (4a) and (4b) is also not isotropic. It includes a sum of the geometric coefficients squared, which is equivalent to effective doubling of the lattice period (or light frequency doubling) leading to new features at angles, where classical diffraction predicts zero. In Fig. 3 a case of two standing waves is shown. Due to the effective period doubling (given by $k_{0,1}$, stronger than the isotropic component ($k_{1,2}$), new features appear at angles of, e.g., effective first-order maxima appear, though classically only the zero-order maxima are still possible.

The angle dependence of the photon number variance $(\Delta n_{ph})^2$ determined by $(\Delta |D|^2)^2$ shows anisotropic features due to “period doubling” even for two traveling waves. For the coherent state, the light at a maximum displays strong noise $(\Delta |D|^2)^2 = 4N_K^2 + 6N_K^3 + N_K$ because $(\Delta |D|^4) = N_K^4 + 6N_K^3 + 7N_K^2 + N_K$ and $(\Delta |D|^2) = N_K^2 + N_K$, stronger than the isotropic component ($N_K^2$ in highest order of $N_K$) and new features at $\theta_1 = \pm \pi/2$ (for $\theta_0 = 0$, 2$N_K^2$ in highest order of $N_K$). In SF$_M$, the noise at maxima can be suppressed, while at other angles it is nearly equal to that of the coherent state. In MI, $(\Delta |D|^2)^2 = 0$. Distinguishing between atomic states by light statistics is similar to that by the intensity.

In summary, we have shown that atomic quantum states can be nondestructively monitored by measuring scattered light. In contrast to homogeneous BECs, scattering from lattices exhibits advantageous properties: suppression of the classical scattering in Bragg minima, access to local and nonlocal correlations, angular distributions richer than classical diffraction. Also, other optical phenomena and quantities depending nonlinearly on the atom number operators will reflect quantum atom statistics [16, 17, 18], e.g., the dispersion of a medium will provide a spectral method of quantum state characterization [19]. Exploiting quantum properties of light should be applicable to other Bragg spectroscopy setups.

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