Probing TeV-scale gauge unification by hadronic collisions

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Abstract: Grand unified theories (GUTs) and extra dimensions are potential ingredients of the new physics that may resolve various outstanding problems of the Standard Model. If the inverse size of (one of) the extra dimension(s) is smaller than the GUT scale and standard gauge bosons are allowed to propagate in the bulk then, among other consequences, the evolution of the gauge couplings deviates from the usual logarithmic running somewhat below and between these two scales. In this work we show that if the compactification scale is the order of 10 TeV, then this modified running may be observable at the CERN Large Hadron Collider in the di-jet invariant mass distribution. We also demonstrate that dijets are highly sensitive to the renormalization effects of the extra dimensions, and are potential tools for determining the number of dimensions and the value of the compactification scale.

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1. Introduction

The goal of theoretical particle physics is to discover a unified theory of matter and interactions. String theory appears to be a candidate for such a theory, since various string modes can represent the matter, and their unique interaction gives rise to all the known forces, gauge and gravitational alike \[1\]. The consistent formulation of superstring theory requires ten space-time dimensions to achieve gauge (and gravitational) anomaly cancellations \[2\], which might be an indication that at the fundamental level space-time might have more than four dimensions.

Recently, revolutionary advances have been made in understanding the structure of string theory: the discovery of branes \[3\], dualities \[4, 5\], M-theory \[6\], and the AdS-CFT conjectures \[7\]. These developments inspired the study of the phenomenological aspects of effective field theory models with low fundamental scale(s) and additional space-time dimensions \[8, 9, 10, 11, 12\], in search for the solution of the gauge hierarchy and the cosmological constant problems. These models also present a fresh way to interpret other problems, such as the problem of symmetry breaking. These works suggest that it might be possible to formulate string theories with a fundamental scale close to the weak scale \[13\]. If these weak-scale strings are realized in Nature, then near-future particle accelerators may discover stringy phenomena like winding modes or Kaluza-Klein (KK) excitations.

In the field theory framework, gauge and Yukawa unification has proven to be an attractive assumption to economically explain the diverse features of matter and interactions, as for example the large number of parameters in the Minimal Supersymmetric Standard Model (MSSM). The advantages of unification and extra dimensions can be combined to solve problems of the Standard Model, the MSSM, and problems of grand unified theories (GUTs) formulated in four dimensions \[14, 15, 16\]. This picture is even more attractive considering that the unification and compactification scales can be lowered close to the weak scale, naturally avoiding the hierarchy between them.

In the weak-scale compactification scenario, when standard gauge bosons are allowed to propagate in the extra dimensions, precision electroweak measurements constrain the masses of the KK excitations of the gauge bosons. Global fits to electroweak observables provide lower bounds on the inverse compactification scale, \(1/R_C\), which are generally in the 2-5 TeV range \[17, 18\]. Within this model, the phenomenology of the virtual and real production of the KK excitations of the gauge bosons at various present and future colliders was recently examined by several authors \[19, 20, 21\]. The typical constraint on the compactification scale for the CERN Large Hadron Collider (LHC) with 100\(fb^{-1}\) of luminosity is \(M_C = 1/R_C \leq 6\) TeV \[20\]. Stronger constraints can be obtained in models with specific assumptions \[22\].

It is important to note that the earlier phenomenology work does not examine gauge unification together with the TeV-scale extra-dimensions. In this paper we
attempt to do so. The general idea is that, due to the effect of the extra dimensions, the evolution of the gauge couplings is modified by power law terms \[9, 10\]. This modified running can potentially be detected in processes which depend sensitively on one of the gauge couplings. We illustrate this using dijet production at the Fermilab Tevatron and at the LHC. This cross section, at the lowest order, is proportional to \(\alpha_s^2\). We calculate the cross section, including NLO QCD corrections, within the Standard Model, assuming that new space dimensions open up in the \(O(10 \text{ TeV})\) energy range and gluons can propagate in these new dimensions. Then we compare this to the result of standard QCD, evaluating the statistical significance of the deviation. In this scenario, we find that in the dijet channel the LHC discovers a common compactification scale up to \(M_C = 5-10 \text{ TeV}\) at 5\(\sigma\), depending on the treatment of KK thresholds as discussed below. The Tevatron run II should be able to discover a 1 TeV compactification scale in the optimistic case. We emphasize that these results can be generalized for other processes, different colliders, and other models with low scale gauge unification.

2. TeV-scale gauge unification

In this section, we outline the theoretical framework that we adopted. Motivated by the facts listed in the Introduction, we assume the existence of a higher (than 4) dimensional underlying theory in which all the fundamental scales are close to the weak scale and the standard gauge bosons propagate in the extra dimensions. In particular, gauge and Yukawa unifications happen somewhat above the TeV energy range at \(M_{\text{GUT}}\). In the meantime, the compactification scale is between the two scales: \(M_{Z^0} \lesssim M_C = 1/R_C \lesssim M_{\text{GUT}}\). A working example of this is the model outlined in Refs. \([9, 10]\). In this case, the underlying theory has \(3+\delta\) independent space directions and one time dimension. It is assumed that the \(\delta\) additional space dimensions compactify on circles with a common radius \(R_C = O(0.1 \text{ TeV}^{-1})\). The known fermions are confined to the observed 4 space-time dimensions, while the known gauge bosons (especially the gluons) and possibly existing Higgs bosons propagate in the full space (\(4+\delta\) dimensions). If low energy supersymmetry exists, then one can assume that some of the additional chiral families access the extra dimensions. In this work we assume that this does not happen\(^1\).

Following \([9]\), we assume that below the compactification scale (in the TeV energy range) the theory can be well approximated by a field theory formulated in a 4-dimensional space-time. In Refs. \([9, 10]\) this theory is assumed to be the MSSM, supplemented with the KK excitations of its non-chiral sector. It is shown that the presence of these KK excitations affects the renormalization evolution of the

\(^1\)In the notation of \([9, 10]\), this means that the number \(\eta\) of chiral fermions propagating in the bulk is set to zero.
gauge couplings. This effect is quantified by power-law type corrections to the usual logarithmic scale dependence. In the scenario of [9], at the lowest order, the scale dependence of the gauge couplings is given by

\[ \alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{b_i - \tilde{b}_i}{2\pi} \ln \frac{\mu}{\mu_0} - \frac{\tilde{b}_i}{4\pi} \int_{\mu_0}^{\mu} \frac{dt}{t} \left[ \theta_3 \left( \frac{it}{\pi R^2} \right) \right]^\delta. \] (2.1)

where \( i = 1, 2, 3 \) labels the gauge groups of the MSSM, and the coefficients of the usual one loop beta functions\(^2\)

\[ (b_1, b_2, b_3) = (33/5, 1, -3) \] (2.2)

are supplemented by new contributions from the properly supersymmetrized KK towers

\[ (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (3/5, -3, -6) + \eta (4, 4, 4) \] (2.3)

(where, for simplicity, we set \( \eta = 0 \)). In the last term of Eq. (2.1) \( \theta_3 \) denotes the elliptic Jacobi function and

\[ r = \pi (X_\delta)^{-2/\delta} \quad \text{with} \quad X_\delta = \frac{2\pi^{\delta/2}}{\delta \Gamma(\delta/2)}. \] (2.4)

We note that in Refs. [9, 10] an approximate expression is used to calculate the running of the couplings, but in our work we use the exact formula (2.1).

The power-law term in Eq. (2.1) accelerates the running of the gauge couplings and makes them meet earlier than the usual unification scale of \( 2 \times 10^{16} \) GeV. In particular, for \( \eta = 0 \), the strong coupling decreases faster than what the logarithmic running describes. This deviation from the standard evolution is highly enhanced for low compactification scales, as illustrated by Fig. 1 of Ref. [9]. We found that, using Eq. (2.1), the strong coupling decreases by 25% at 10 TeV for \( M_C = 10 \) TeV and \( \delta = 2 \).

Finally, we point out that in Ref. [9] the matching of the asymptotic regions of the evolution below and above of the KK mass thresholds is approximate. In [9] it is suggested that above the compactification scale Eq. (2.1) is used, while below it the same with vanishing \( \tilde{b}_i \). This approximation completely neglects the width of the KK states. On the other hand, in our case this width is not negligible. For KK excitations of gluons the width is given by

\[ \Gamma_n = 2\alpha_s(Q)m_n, \] (2.5)

\(^2\)The difference of the gauge evolutions between the Standard Model and the MSSM is negligible compared to the effect of an O(TeV) size extra dimension in the \( \mu = 1-10 \) TeV range. Keeping in accord with Refs. [9, 10], we use the MSSM beta functions.
where \( m_n = n/R_C \) is the mass of the resonance. Even with a reduced value of \( \alpha_S \), a 10 TeV resonance has a width of about 2 TeV, which is comparable to the mass. Thus, a step-function style matching at the threshold is pessimistic, because it underestimates the deviation of the running below the resonance thresholds.

The matching at the KK thresholds is discussed in detail in Ref. [23], and is beyond the scope of this work. To demonstrate our point, and for simplicity, we use the pessimistic matching prescription of Ref. [9]. Meanwhile we define an optimistic prescription, which somewhat overestimates the KK width effect, by equating \( \mu_0 \) with the \( Z^0 \) mass in Eq. (2.1). Neither of these prescriptions is correct, but they can be viewed as extrema of the exact treatment.

3. Effect on the hadronic dijet production

In this section we examine the sensitivity of the near future hadronic accelerators to the effect of the extra dimensions on the running of the strong coupling. In order to detect the possible deviation from the standard evolution of the gauge couplings, we have to select processes which are highly sensitive to one of the couplings and can be precisely measured at the near future colliders. Dijet production is such a process since, at leading order, it depends on the square of the strong coupling constant \( \alpha_S \), and is independent from the other couplings\(^3\). Moreover, the dijet final state can be fully reconstructed experimentally and can be used to determine the energy \( \hat{s} \) that entered into the hard partonic subprocess. This energy is the virtuality of the particle exchange in the s–channel, and also the scale at which the coupling constant involved in the subprocess should be evaluated. Finally, the expected rate of dijet production is relatively high both at the Tevatron run II and at the LHC. At the latter, for example, the production cross section even at 4 TeV jet-pair invariant mass \( (M_{jj}) \) is about 0.3 fb [24]. We will show that this event rate allows for a good statistical discrimination.

To quantify the difference between the standard expectation and the one with the modified running we define the statistical significance

\[
S = \frac{|N_{SM} - N_{XD}|}{\sqrt{N_{SM}}} \tag{3.1}
\]

where

\[
N_{SM} = \mathcal{L} \int_{M_{\text{min}}}^{M_{\text{max}}} dM_{jj} \frac{d\sigma_{SM}}{dM_{jj}} \tag{3.2}
\]

is the number of dijet events produced with invariant mass higher than \( M_{\text{min}} \) assuming the standard running of gauge couplings, \( M_{\text{max}} \) is the maximal dijet energy at the given collider, and \( \mathcal{L} \) is the integrated luminosity of the experiment at hand.\(^4\)

\(^3\)This is important, since the accelerated running of the gauge couplings can partially cancel if the process equally depends on more than one of them.

\(^4\)In our numerical calculations we use 100 fb\(^{-1}\) for the LHC and 10 fb\(^{-1}\) for the Tevatron.
The number of dijet events is calculated using the modified running, \( N_{XD} \), defined similarly to \( N_{SM} \) with the standard cross section \( d\sigma_{SM}/dM_{jj} \) replaced by the extra-dimensional one taking into account the modified running of \( \alpha_S \).

The bulk of the dijets at the Tevatron and the LHC is produced by the standard model \( q\bar{q}, gg \), the \( qg, \bar{q}g \rightarrow qg, \bar{q}g \) and the \( gg \rightarrow q\bar{q}, gg \) processes. In the context of TeV-scale extra dimensional models with gauge bosons propagating in the bulk, dijet production at the LHC was examined in [19] and [21]. These works study the effect of the KK excitations on the rate without modifying the running of the strong coupling. In these studies it was shown that KK excitations of the gluon contribute a significant portion and increase the rate while also changing the shape of the dijet cross section. But in a model where \( \alpha_S \) decreases significantly at scales around 10 TeV due to the effect of the extra dimensions, the change of the coupling partially counter-balances the effect of the KK excitations leading to a less conclusive signal for the higher compactification scales.

On the other hand, in [21] it was also found that the high mass gluonic KK excitations tend to decay into dijets with very high transverse momenta, while the standard model background has lower jet \( p_T \). We use this fact to disentangle the competing effects of the KK-excitations and the running of the strong coupling. Fig. 3 of Ref. [21] shows that for compactification scales up to 10 TeV the KK contribution to the cross section is below 1% if \( p_T \lesssim 3 \) TeV, and at most a few percent if \( p_T = 5 \) TeV. That is, for \( 1/R_C = 10 \) TeV, the KK contribution is negligible up to \( M \sim 2p_T \lesssim 6 \) TeV, and it is a few percent up to \( M = 10 \) TeV. This conclusion is further strengthened by Fig. 4 of Ref. [21], which shows that if events are selected such that the minimal jet \( p_T \) is about 0.5 TeV, then the KK contribution is less than 1% for all the compactification scales relevant in this work. For this reason, in the rest of this work we confine ourselves to the study of jets with \( 560 \) GeV < \( p_T < 5 \) TeV. With this cut in place, the effect of the extra dimensions on the strong coupling can be observed unbiased by the KK excitations.

In Fig. 1 we show the significance (3.1) as the function of the minimal dijet mass \( M_{\text{min}} \) at the LHC for the optimistic matching prescription. The qualitative shape of the curves is easy to understand. For low \( M_{\text{min}} \) the small deviation from the standard running results in a low significance. For high \( M_{\text{min}} \) the statistical error of the sample increases, which diminishes the significance. There is an optimal \( M_{\text{min}} \) around 5 TeV, where the significance is maximal.

From Fig. 1 we observe that, based on the modified running of \( \alpha_S \), in dijet production the LHC can discover compactification scales up to 10 TeV at 5\( \sigma \) almost independently from the number of extra dimensions. For the pessimistic matching the discovery reach is reduced to 5 TeV at 5\( \sigma \). For the Tevatron run II, based on the projected numbers in Ref. [25], we obtain a 1 TeV discovery reach at 5\( \sigma \) for the optimistic matching scenario. Fig. 1 also shows that the peak position of the statistical significance is sensitive to the number of extra dimensions \( \delta \), which may
Figure 1: The statistical significance (in units of \( \sigma \)'s) of the deviation from the Standard Model, as the function of the minimal dijet mass \( M_{\text{min}} \) at the LHC, for different numbers of dimensions and for a compactification scale of 10 TeV.

serve as dimensiometer if \( S \) is measured precisely enough as the function of \( M_{\text{min}} \).

Fig. 2 shows that the value of the compactification scale is well correlated with the maximum achievable significance at the LHC. This is shown for the optimistic matching scenario, but the pessimistic one is qualitatively the same. From the two figures it is clear that the overall shape of the significance is sensitive to the compactification scale and the number of extra-dimensions, thus a global fit to this variable can determine both of these quantities simultaneously.

Our results also imply that calculations that are performed in a similar framework to ours, with compactification scale close to the weak scale and (some of) the gauge bosons propagating in the bulk, have to account for the modified running of the gauge couplings. In particular, couplings of KK-excitations of gauge bosons to matter are modified significantly at \( O(10 \text{ TeV}) \), which changes earlier conclusions on their discovery limits. Also, the modified running of the strong coupling might effect the evolution of the partonic distribution functions of the proton which are utilized in processes with hadronic initial state. These effects are outside of the scope of this work and should be investigated elsewhere.

4. Conclusions

In this work, we examined a scenario in which extra dimensions open up close above
Figure 2: The statistical significance (in units of $\sigma$'s) of the deviation from the Standard Model, as the function of the minimal dijet mass $M_{\text{min}}$ at the LHC, for different compactification scales and for 4 extra dimensions.

the weak scale, not much below a possible unification scale, standard gauge bosons propagating in them. In this case, the renormalization evolution of the gauge couplings deviates from the standard running already somewhat above the weak scale. We showed that this deviation is measurable at the LHC at $5\sigma$ in the dijet channel, provided that the compactification scale is $5-10$ TeV$^{-1}$, exact values depending on the details of the treatment of the KK thresholds. The Tevatron run II can observe the extra dimensions up to a maximal scale of 1 TeV. We also demonstrated that dijets are highly sensitive to the renormalization effects of the extra dimensions, and are potential tools for the extraction of the number of dimensions and the value of the compactification scale.

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