Neutron halo and the impact-parameter dependence of the
\( \pi^+ - \pi^- \) asymmetry in heavy ion collisions

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Abstract

The theoretical nuclear physics predictions and experimental results on antiprotonic atoms lead to different proton and neutron spatial distributions. At CERN SPS energies production of positive and negative pions differs for \( pp, pn, np \) and \( nn \) scattering. These two facts lead to an impact parameter dependence of the \( \pi^+ \) to \( \pi^- \) ratio in \( {}^{208}Pb + {}^{208}Pb \) collisions. A recent experiment at CERN seems to confirm qualitatively these predictions. The results for two models considered are almost identical. It may open a new possibility for determination of neutron density distribution in nuclei.

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The whole traditional microscopic nuclear physics is based on the assumption that nuclei are built of protons and neutrons. The so-called y-scaling analysis seems to confirm that this is a good assumption [1].

Electron scattering off nuclei provides direct information about charge distribution (see for instance [2]), which is closely related to the spatial distribution of protons. The information about the neutron spatial distribution is not accessible directly.

Experimentally one usually determines the difference \( \Delta r_{np} \) between the neutron and proton mean square radii. However, this does not allow one to make a difference between the neutron skin or neutron halo model [3]. The radiochemical method applied to antiprotonic atoms [4, 5, 6] allows one to measure the neutron-to-proton ratio at the peripheries of nuclei. This measurement suggests a density profile of the halo type, where half-radii for neutron and proton densities are equal, and distributions differ only in their diffuseness.

Another method of investigating neutron distribution is based on the analysis of X-ray spectra of antiprotonic atoms [3]. It was also suggested in [7] that parity violating electron scattering could measure neutron density (for a newer review see [8]).

On the theoretical side the difference between the proton and neutron distributions can be obtained in the framework of Hartree-Fock (HF) method (see for example [9]) or Hartree-Fock-Bogoliubov method (see for example [10]). For heavy nuclei one usually uses zero-range nucleon-nucleon interaction of the Skyrme type. Reliable neutron densities can be also obtained from a global analysis of intermediate-energy elastic proton-nucleus scattering [11, 12].

Stable heavy nuclei exhibit an excess of neutrons over protons. A recent experiment at CERN [13] for charged pion production in the \( ^{208}Pb + ^{208}Pb \) collision has observed an interesting dependence of the ratio \( \frac{d\sigma^{\pi^+}}{dx_F} / \frac{d\sigma^{\pi^-}}{dx_F} \) on the Feynman variable \( x_F \). Surprisingly, the ratio for central and peripheral collisions differs significantly [13].

In Fig. 1 we present the ratio \( \rho_n(r)/\rho_p(r) \) as obtained from the HFB densities of \( ^{208}Pb \) [10]. The experimental point, taken from Ref. [3], is a result of interpolation of results obtained for other nuclei (the radiochemical method can be used only for a limited set of elements). The grey area shows a direct estimate of \( \rho_n(r)/\rho_p(r) \) obtained from X-ray spectra of antiprotonic Pb atoms. This is slightly model dependent and was obtained under assumption of Fermi-like density profiles of halo type. At large radii the HFB calculations are consistent with these experimental results.
In the following we assume that pions are produced in elementary nucleon-nucleon collisions. Without losing more generality we can write

\[
\frac{d\sigma^{A_1A_2 \to \pi^\pm}}{dx_F} (b, x_F; W) = \sum_{\alpha \beta = p,n} N_{\alpha \beta \to \pi^\pm} (b; W) \frac{d\sigma^{\alpha \beta \to \pi^\pm}}{dx_F} (x_F; W),
\]

where \( W \) is energy per binary nucleon-nucleon collision and \( N_{\alpha \beta \to \pi^\pm} \) are numbers of collisions of a given type. Only elementary cross sections for the \( pp \to \pi^\pm \) processes are known experimentally. Therefore in the following we shall use elementary cross sections \( \frac{d\sigma}{dx_F} (x_F; W) \) calculated in the HIJING model \([14]\). We shall leave the problem of the quality of the model for a separate detailed analysis.

In Fig.2 we show \( x_F \) dependence of the cross section for different types of collisions. We have used \( \sigma_{NN}^{NN} = 30 \text{ mb} \) to normalize all elementary cross sections. A clear difference between different cross sections can be seen. This asymmetry in elementary collisions in conjunction with the asymmetry of proton and neutron spatial distributions should lead to asymmetries in nuclear collisions which will be discussed in the present note. Only four elementary cross sections are independent. The cross sections for the \( \pi^- \) production (right panel) can be obtained from those for the \( \pi^+ \) production (left panel) thanks to isospin symmetry. In addition at \( x_F > 0 \) one has \( \sigma_{pp \to \pi^\pm} \approx \sigma_{pn \to \pi^\pm} \) and \( \sigma_{nn \to \pi^\pm} \approx \sigma_{np \to \pi^\pm} \).

When calculating the ratio of \( \pi^+ \) to \( \pi^- \) cross sections it is sufficient to know the fractions \( f_{\alpha \beta} = \frac{N_{\alpha \beta \to \pi^\pm}}{\sum_{\alpha \beta} N_{\alpha \beta \to \pi^\pm}} \). It is far from obvious how to calculate the fractions \( f_{\alpha \beta} \). In the most naive approach

\[
f_{\alpha \beta} = \frac{N_{\alpha/1N_{\beta/2}}}{\sum_{\alpha \beta = p,n} N_{\alpha/1N_{\beta/2}}},
\]

where \( N_{\alpha/1}, N_{\beta/2} \) are fractions of protons/neutrons in the nucleus \( A_1 \) and \( A_2 \), respectively. In this simple model there is neither impact parameter nor energy dependence of \( f_{\alpha \beta} \), i.e. as a consequence of Eq.(1) the ratio

\[
R_{+/−} (x_F, b; W) \equiv \frac{d\sigma^{A_1A_2 \to \pi^+(x_F, b; W)/dx_F}}{d\sigma^{A_1A_2 \to \pi^-(x_F, b; W)/dx_F}}
\]

depends only on the Feynman \( x_F \) and center-of-mass energy. Recent preliminary experimental results of the NA49 collaboration \([13]\) seem to contradict this simple approach.

It is often assumed that the dynamics of nuclear collisions is governed by the number of binary collisions \([15]\). The number of binary nucleon-nucleon collisions at a given impact parameter is proportional to the nucleus-nucleus thickness \([19]\)

\[
T_{A_1A_2}(\vec{b}) = \int d^2s_1 T_{A_1}(\vec{s}_1) T_{A_2}(\vec{s}_1 - \vec{b}) = \int d^2s_2 T_{A_1}(\vec{s}_2 - \vec{b}) T_{A_2}(\vec{s}_2).
\]

3
In Eq. (4) we introduced \( T_{A_i}(\vec{b}) = \int dz_i \rho_{A_i}(\vec{b}, z_i) \), where \( \rho_{A_i} \) is the density function of the nucleus \( A_i \) normalized to the number of nucleons. Analogously the number of binary collisions of a given type in Eq. (1) can be written as

\[
N_{\alpha\beta \rightarrow \pi \pm}(b; W) = T_{A_1A_2}^{\alpha\beta}(\vec{b}) \cdot \sigma_{\text{inc}}^{NN}(W) ,
\]

where

\[
T_{A_1A_2}^{\alpha\beta}(\vec{b}) = \int d^2s_1 T_{A_1}^\alpha(\vec{s}_1) T_{A_2}^\beta(\vec{b} - \vec{s}_1) = \int d^2s_2 T_{A_1}^\alpha(\vec{b} - \vec{s}_2) T_{A_2}^\beta(\vec{s}_2) ,
\]

where now \( T_{A_i}^p \) and \( T_{A_i}^n \) are nucleus thicknesses of protons and neutrons, respectively. We have assumed one universal inelastic cross section \( \sigma_{\text{inc}}^{\alpha\beta}(W) = \sigma_{\text{inc}}^{NN}(W) \). When properly normalized, Eq. (6) describes the probability of the binary \( \alpha\beta \) collisions and is a simple generalization of Eq. (4). Of course

\[
T_{A_1A_2}(\vec{b}) = \sum_{\alpha\beta} T_{A_1A_2}^{\alpha\beta}(\vec{b}) .
\]

As a second limiting case we shall consider the wounded nucleon model [16]. We shall assume that the production of particles is proportional to the number of wounded nucleons rather than to the number of binary collisions. The simplest version of the model must be generalized to include different distributions of protons and neutrons and differences of elementary cross sections. Somewhat schematically the cross section for the nuclear collision can be written as

\[
\frac{d\sigma_{A_1A_2 \rightarrow \pi \pm}}{dx_F}(b, x_F; W) \propto \sum_{\alpha, \beta = p, n} \left[ N_{wou}^{\alpha/A_1}(b, W) \ w_2^\beta(b) \ \frac{d\sigma_{\alpha\rightarrow \pi \pm}}{dx_F}(x_F; W) \right.
\]

\[
+ \ N_{wou}^{\beta/A_2}(b, W) \ w_1^\alpha(b) \ \frac{d\sigma_{\beta\rightarrow \pi \pm}}{dx_F}(x_F; W) \right] .
\]

In the formula above \( N_{wou}^{\alpha/A_i} \) is the number of wounded \( \alpha \) (p or n) in nucleus \( A_1 \) or \( A_2 \) and \( w_i^\beta \) is the probability that the wounded \( \alpha \) interacted with \( \beta \) (p or n) from nucleus \( A_2 \) or \( A_1 \), respectively. Eq. (8) is equivalent to Eq. (1) with

\[
N_{\alpha\beta \rightarrow \pi \pm} = N_{\alpha/A_1}^{wou}(b, W) \ w_2^\beta(b) + N_{\beta/A_2}^{wou}(b, W) \ w_1^\alpha(b) .
\]

By construction, in our approach the numbers of wounded protons and neutrons reproduce the well known formula from [16] for the number of wounded nucleons

\[
N_{N/A_i}^{wou}(b) = N_{p/A_i}^{wou}(b) + N_{n/A_i}^{wou}(b) .
\]
Numbers of wounded protons and neutrons were calculated in the zero-range nucleon-nucleon approximation. In calculating the number of wounded nucleons we have taken $\sigma_{\text{inel}}^{NN} = 30$ mb, relevant for CERN SPS energies. Our construction requires also

$$w_i^p(b) + w_i^n(b) = 1 .$$  \hspace{1cm} (11)

In this communication the fractions $w_i^\alpha$ were estimated as

$$w_i^\alpha(b) = \frac{N_{\text{wou}}^{\alpha/A_i}(b)}{N_{\text{wou}}^{p/A_i}(b) + N_{\text{wou}}^{n/A_i}(b)} ,$$  \hspace{1cm} (12)

which by construction fulfils (11). With this prescription Eq.(9) reduces to:

$$N_{\alpha\beta\rightarrow\pi\pm}(b) = \left( N_{N/A_1}^{\text{wou}}(b, W) + N_{N/A_2}^{\text{wou}}(b, W) \right) w_i^\alpha(b) w_j^\beta(b) .$$  \hspace{1cm} (13)

In the present note we use proton and neutron densities calculated in the Hartee-Fock-Bogoliubov method [10] with Skyrme interaction SLy4 (see Fig.1).

Above we have only sketched the model. More details will be presented and discussed in detail elsewhere [17].

In Fig.3 we present the ratio $R_{+/-}$ (see Eq.(3)) as a function of $x_F$ for different values of the impact parameter $b$ in the two models considered. For comparison we show preliminary experimental data for ”central collisions” (solid circles) and ”peripheral collisions” (open circles) presented in [13]. The notion of central and peripheral collisions was not specified in [13]. Therefore the data can be used only as an indication of the effect. At present no quantitative comparison of model results to experimental data is possible. Our approach explains the experimental data provided they are extremely peripheral (this requires their more detailed analysis). Our results are reliable up to $x_F \approx 0.2$. It is known that the HIJING code does not describe the $\pi^+ - \pi^-$ asymmetry in elementary collisions in the region of large $x_F$.

For completeness in Fig.4 we present the ratio $R_{+/-}$ as a function of the impact parameter $b$ for different values of $x_F$. Surprisingly the binary collision picture gives very similar results to the predictions of the wounded nucleon model.

The predictions of the two models are almost identical. This suggests that a detailed comparison of model results with the well defined $(x_F, b)$ experimental data could open a new possibility to study the neutron density profile. We expect that the NA49 collaboration at CERN will be able to gather the corresponding experimental data in the near future.
Can it provide a method competitive to that offered by proton-nucleus elastic scattering, antiprotonic atoms or parity violating electron scattering? Of course results of these methods must finally converge. Therefore one may hope that together they will provide more reliable information on neutron distribution in nuclei.

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[19] In the present analysis we take zero-range NN interaction to avoid introduction of new not well known parameters. The effect of finite-range interaction will be discussed elsewhere.

FIG. 1: Neutron-to-proton ratio \( \frac{Z n(r)}{p n(r)} \), \( \frac{Z n(r)}{p p(r)} \) for \( ^{208}Pb \). The experimental point is taken from [3] while the experimental grey band from [18]. The result of the HFB calculation [10] is shown by the thin solid line.
FIG. 2: $\frac{d\sigma}{dx_F}$ as a function of Feynman $x_F$ for $\alpha\beta \rightarrow \pi^+$ (left panel) and $\alpha\beta \rightarrow \pi^-$ (right panel) for different subcollisions: $pp$ (black solid), $nn$ (grey solid), $pn$ (black dashed) and $np$ (grey dashed).

FIG. 3: The ratio $R_{+/−}$ as a function of the Feynman $x_F$ for the model with binary collision (left panel) and for the model with the scaling with number of wounded nucleons (right panel). Also shown are the preliminary experimental data from Ref.[13].
FIG. 4: The ratio $R_{+/−}$ as a function of impact parameter for the model with binary collision (left panel) and for the model with the scaling with number of wounded nucleons (right panel).