Privacy Threats Against Federated Matrix Factorization

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Abstract
Matrix Factorization has been very successful in practical recommendation applications and e-commerce. Due to data shortage and stringent regulations, it can be hard to collect sufficient data to build performant recommender systems for a single company. Federated learning provides the possibility to bridge the data silos and build machine learning models without compromising privacy and security. Participants sharing common users or items collaboratively build a model over data from all the participants. There have been some works exploring the application of federated learning to recommender systems and the privacy issues in collaborative filtering systems. However, the privacy threats in federated matrix factorization are not studied. In this paper, we categorize federated matrix factorization into three types based on the partition of feature space and analyze privacy threats against each type of federated matrix factorization model. We also discuss privacy-preserving approaches. As far as we are aware, this is the first study of privacy threats of the matrix factorization method in the federated learning framework.

1 Introduction
Recommender systems play a significant role in various applications, such as e-commerce and movie recommendation. Matrix Factorization (MF) \cite{Koren et al., 2009b}, as a typical Collaborative Filter (CF) method, has positioned itself as one of the effective means of generating recommendations and is widely adopted in real-world applications. Traditionally, for one company, it is essential to accumulate sufficient personal rating data to build a performant MF model. However, due to the sparse nature of user-item interactions, it can be hard for a single company to collect sufficient data to build an MF model. Moreover, recently enacted stringent laws and regulations such as General Data Protection Regulation (GDPR) \cite{Albrecht, 2016} and California Consumer Privacy Act (CCPA) \cite{Ghosh, 2018} stipulate rules on data sharing among companies and organizations, making collaboration between companies by sharing personal rating data illegal and impractical.

To tackle the challenge of protecting individual privacy and remitting the data shortage issue, federated learning (FL) \cite{Konečný et al., 2016, McMahan et al., 2017} provides a promising way that enables different parties collaboratively build a machine learning model without exposing private data in each party. It addresses data silos and privacy problems together. In FL, data can be partitioned horizontally (example-partitioned) or vertically (feature-partitioned) into different parties. When records are not aligned between parties and the feature spaces among parties are heterogeneous, federated transfer learning can be adopted. The use of FL in recommender systems has been studied over different data distributions. For example, \cite{Chai et al., 2019} considers horizontally partitioned rating data among clients, which hold ratings of the same user-item interaction matrix. Federated multi-view MF is studied where participants hold item interaction data, item features, or user features \cite{Planagan et al., 2020}. Each participant holds a part of model parameters, while some common parameters are shared among participants. Existing studies generally categorize horizontal and vertical federated recommender systems regarding on whether user alignment is required before FL \cite{Yang et al., 2019, Chai et al., 2019}. For example, participants sharing different users and the same set of items implies horizontal federated recommender systems. In our paper, we categorize different settings based on the partition of feature space, as shown in Fig. 1, which is consistent with other FL systems.

Although most existing studies of federated recommender systems adopt privacy preserving techniques, including homomorphic encryption (HE) \cite{Paillier, 1999}, secure multiparty computation (MPC) \cite{Paymann and Yupeng, 2017} and differential privacy (DP) \cite{Dwork, 2008} to protect data privacy. There is little exploration of how data privacy can be breached in federated MF. It is shown that the gradient present in the global model is the potential to breach data privacy in horizontal FL for deep learning \cite{Melis et al., 2019} or logistic regression \cite{Li et al., 2019}. However, a comprehensive study of the privacy threats against the plaintext federated matrix factorization in different data partitions is still required.

Inspired by this research gap, we investigate the potential privacy risks in federated matrix factorization. Specifically,
we classify the federated MF into horizontal, vertical federated MF as well as federated transfer MF based on the data partition approaches. We then demonstrate how private user preference data can be breached during FL training to honest and curious participants. Finally, we discuss how cryptographic techniques can be adopted to protect privacy. The contribution of this paper can be summarised as follows:

- We identify and formulate three types of federated MF problems based on the way of data partition.
- We demonstrate the privacy threats in each type of federated MF, by showing how the private preference data can be leaked to honest-but-curious participants.
- We investigate privacy-preserving approaches to protect privacy in federated MF.

In the following parts of this paper, section 2 gives related works of privacy issues of MF method and federated MF. We give backgrounds of MF and security model in section 3. Then, section 4 gives privacy attacks to each type of federated MF. In section 5 we discuss privacy-preserving approaches. Finally, conclusions are drawn in section 6.

2 Related Work

Privacy risks in general recommender systems are studied in [Jeckmans et al., 2013] [Lam et al., 2006], which analysis the privacy concerns that happened in different phases and caused by different entities. However, the federated learning framework is not considered, which introduces parameter exchanges between participants and thus enlarges the attack surface. Though [Chai et al., 2019] investigates privacy risks in horizontal federated MF and adopts HE to harness the privacy risks, it assumes honest-and-curious participants, and the proposed approach only defends against an honest-but-curious server. We demonstrate that other curious clients can easily infer private training data.

Many works are exploring privacy-preserving techniques for federated recommender systems. [Qi et al., 2020] adopts local differential privacy to train a neural network in horizontal federated news recommendation. [Flanagan et al., 2020] studies federated multi-view MF over heterogeneous data held by different users. [Canny, 2002] proposes federated CF based on partial Singular Value Decomposition and adopts HE for model aggregation. Fully HE [Kim et al., 2016] as well as garbled circuit [Nikolaenko et al., 2013] is also investigated for privacy-preserving MF, where secure MF is conducted between the server and a crypt-service provider. [Gao et al., 2019] [Ju et al., 2020] are the first to investigate the feasibility of the FL framework to enable a distributed training of deep models from multiple heterogeneous datasets for brain-computer interface. Although these works propose various privacy-preserving approaches, they fail to investigate the potential privacy loss the intermediate transferred parameters can breach during FL training.

3 Background

In this section, we introduce the matrix factorization method based on stochastic gradient descent, as well as the security model considered in this paper.

3.1 Matrix Factorization

We consider $n$ users rate a subset of $m$ items. For $[n] := 1, \ldots, n$ the set of $n$ users, and $[m] := 1, \ldots, m$ the set of $m$ items, the user/item pairs that generate the ratings are denoted by $M = \mathbb{C}[n \times m]$. The total number of ratings is $M = |M|$. Finally, for $(i, j) \in M$, we denote by $r_{i,j} \in \mathbb{R}$ the rating generated by user $i$ for item $j$. Matrix factorization uses a $d \in \mathbb{N}$ dimensional vector to represent a user as $u$ and an item as $v$, referred as a profile, and models the relevance of an item to a user as the inner product of their profiles. MF computes the user profiles and item profiles $u_i, v_j \in \mathbb{R}^d$ following the regularized mean squared error as follows:

$$\min_{U, V} \frac{1}{M} \sum_{(i, j) \in M} (r_{i,j} - \langle u_i, v_j \rangle)^2 + \lambda_u \sum_{i \in [n]} \|u_i\|_2^2 + \lambda_v \sum_{j \in [m]} \|v_j\|_2^2$$

(1)

for positive constants $\lambda_u, \lambda_v$, the inner product $\langle u_i, v_j \rangle$ is the predicted unobserved ratings $r_{i,j}$.

Stochastic gradient descent (SGD) is widely applied to optimize the profiles $u_i$ and $v_j$ as follows:

$$u_i = u_i^{t-1} - \gamma \cdot (\nabla_{u_i} F(U^{t-1}, V^{t-1}) + 2\lambda_u u_i^{t-1})$$

(2)

$$v_j = v_j^{t-1} - \gamma \cdot (\nabla_{v_j} F(U^{t-1}, V^{t-1}) + 2\lambda_v v_j^{t-1})$$

(3)

where $\gamma > 0$ is the learning rate. $U$ and $V$ are the user profile matrix and item profile matrix with each row as a profile, and $\nabla_{u_i} F(U, V)$ and $\nabla_{v_j} F(U, V)$ can be computed as follows:

$$\nabla_{u_i} F(U, V) = -\frac{2}{M_{u_i}} \sum_{j: (i, j) \in M} v_j (r_{i,j} - \langle u_i, v_j \rangle)$$

(4)

$$\nabla_{v_j} F(U, V) = -\frac{2}{M_{v_j}} \sum_{i: (i, j) \in M} u_i (r_{i,j} - \langle u_i, v_j \rangle)$$

(5)

3.2 Security Model

We assume all participants as well as the server if there is any are honest-but-curious (a.k.a. semi-honest). An honest-but-curious participant follows the protocol honestly but tries to infer private information from the intermediate information it knows.

4 Federated MF and Privacy Threats

In this section, we discuss federated MF in three settings that differ in data partition. Then we investigate how privacy can be breached towards adversarial participants in each setting. For simplicity and without loss of generality, we consider FL systems consisting of two participants, $P^A$ and $P^B$. Fig. 1 compares the data partition in each setting of federated MF discussed in this paper. We adopt a sparse representation of partitioned data in Fig 2 to demonstrate the nature of horizontal, vertical, and transfer federated learning. In the horizontal FL setting Fig. 2(a) participants share the same feature space. In the vertical FL setting Fig 2(c) participants
hold heterogeneous feature space, and only \( P^A \) holds the ratings. Whereas, in federated transfer MF Fig.2(b) participants share partial models (e.g., item profiles) for knowledge transfer.

### 4.1 Horizontal Federated MF

In horizontal federated MF, \( P^A \) and \( P^B \) share the same user-item interaction matrix (i.e., the same user and item feature space), as shown in Fig.1(a) and Fig. 2(a). Therefore, each participant holds the profiles of all users and items, and can locally compute gradient of the whole MF model. Only model aggregation requires communication between A and B [McMahan et al., 2017]. In model aggregation, the global user profiles matrix is computed by \( \hat{U}_{\text{Global}} = \frac{1}{2}(U_{PA} + U_{PB}) \), and item profiles matrix \( V_{\text{Global}} = \frac{1}{2}(V_{PA} + V_{PB}) \). \( P^A \) can compute the gradient \( \nabla F_B(U,V) \) of \( P^B \) following

\[
\nabla F_B(U^{T-1}, V^{T-1}) = \frac{1}{\gamma} (2 \cdot U_{\text{Global}}^{T} - U_{PA}^{T} - U_{PB}^{T}) - 2\lambda_u U^{T-1}
\]

where \( T \) is the index of round. Since the user-item interaction matrix is sparse, that is, \( M = \Theta(n + m) \), which is much smaller than the number of potential ratings \( n \cdot m \) [Nikolaenko et al., 2013]. For one update in SGD, it is very likely to have one rating record for each item or user. Therefore, according to Equation 4, \( P^A \) can easily find the \((i,j)\) pair and the corresponding \( v_j \), by checking the gradient and comparing \( \nabla_{u_j} F_B(U,V) \) to each of \( \{u_j\}_{j=1}^m \) as well as comparing \( \nabla_{v_j} F_B(U,V) \) to each of \( \{v_j\}_{j=1}^m \). Then, \( P^A \) further infer the private rating score by

\[
\hat{r}_{ij} = -\frac{\nabla_{u_j} F_B(U^{T-1}, V^{T-1})}{2 \cdot v_j^{T-1}} + \langle u_i^{T-1}, v_j^{T-1} \rangle.
\]

This way, \( P^A \) may complete inference attack and extracts raw private user preference data \((i,j,\hat{r}_{ij})\) of \( P^B \) from the plaintext global model in horizontal federated MF.

### 4.2 Vertical Federated MF

In vertical federated MF as shown in Fig.1(b) and Fig. 2(c), \( P^A \) holds the user-item interaction matrix, and \( P^B \) holds some auxiliary data of users (or items). We adopt the model presented in [Koren et al., 2009a] to leverage auxiliary data provided by \( P^B \) in vertical federated MF. For each user \( i \), \( P^B \) hold distinct factor vectors \( y_a \in \mathbb{R}^d \) corresponds to each attribute. The user \( i \) can thus be described through the set of user-associated attributes \( A(u) \) as \( \sum_{a \in A(u)} y_a \). For vertical federated MF model, Equation 1 can be modified as follows:

\[
\begin{align*}
\min_{U,V} & \frac{1}{M} \sum_{(i,j) \in M} (r_{ij} - \langle (u_i + k_i), v_j \rangle)^2 + \\
& \lambda_u \sum_{i \in [n]} ||u_i||^2_2 + \lambda_v \sum_{j \in [m]} ||v_j||^2_2,
\end{align*}
\]

where \( k_i = |N(u)|^{-0.5} \sum_{l \in N(u)} x_l \sum_{a \in A(u)} y_a \) is the auxiliary information of user \( i \). \( x \) is the implicitly preferred item set, \( y \) is \( i \)'s attributes (e.g., demographic info).

To conduct federated vertical MF, \( P^B \) locally computes and sends \( k_i \) to \( P^A \), and \( P^A \) sends nothing to \( P^B \). Therefore, \( P^A \) has no privacy leakage to \( P^B \), while \( P^B \) leaks \( k_i \) to \( P^A \). In such a setting, user ID leakage during the user alignment stage causes a major privacy threat.

### 4.3 Federated Transfer MF

Without loss of generality, in federated transfer MF, we assume \( P^A \) and \( P^B \) holds ratings given by different users on the same set of items, and \( P^A \) tries to infer private data of
Federated Transfer MF (Common item set)

In horizontal federated recommender systems, users as participants sharing the same set of items and a different set of participants sharing the same set of users and a different set of ratings from the same set of users and items. Therefore, each participant only shares the same set of users (or items), each participant locally holds its user (or item) profiles sub-matrix and the global item (or user) profiles matrix for local SGD. For federated transfer MF, one party holds rating data; the other holds auxiliary data, each party holds partial parameters with shared parameters such as user profiles matrix. For both horizontal federated MF and Federated transfer MF, clients can locally conduct SGD optimization without the need for communication. Participants only need to exchange parameters during the model aggregation process. For vertical federated MF, two participants need to collaboratively compute the estimated rating for each update, which dramatically increases the communication cost. The resilience of each setting against the inference attack is also shown. Horizontal federated MF breaches most private information, including user ID and user preference data. For vertical federated MF, recommender $P^A$ leaks no information to data provider $P^B$, and data provider sends the intermediate data to the recommender. The user ID is breached for both participants. For federated transfer MF, only private rating data and user profiles are leaked, and no user ID is breached.

Tab. 1 demonstrates the comparison of three settings based on the way to update the model, the partition of model parameters, and the resilience of the FL system against inference attack. In horizontal federated MF, all participants share ratings from the same set of users and items. Therefore, each participant locally holds the whole user profiles matrix and item profiles matrix for local SGD. For federated transfer MF, participants only share the same set of users (or items), each participant locally holds its user (or item) profiles sub-matrix and the global item (or user) profiles matrix for local SGD. For vertical federated MF, one party holds rating data; the other holds auxiliary data, each party holds partial parameters with shared parameters such as user profiles matrix. For both horizontal federated MF and Federated transfer MF, clients can locally conduct SGD optimization without the need for communication. Participants only need to exchange parameters during the model aggregation process. For vertical federated MF, two participants need to collaboratively compute the estimated rating for each update, which dramatically increases the communication cost. The resilience of each setting against the inference attack is also shown. Horizontal federated MF breaches most private information, including user ID and user preference data. For vertical federated MF, recommender $P^A$ leaks no information to data provider $P^B$, and data provider sends the intermediate data to the recommender. The user ID is breached for both participants. For federated transfer MF, only private rating data and user profiles are leaked, and no user ID is breached.
| Problem setting | Parameter partition in each party | Gradient computation | Resilience against inference attack |
|-----------------|----------------------------------|----------------------|-----------------------------------|
| Horizontal FedMF | The whole model params | Locally | Weak |
| Vertical FedMF | Partial params without shared params | Collaboratively | P⁺ strong, P⁻ weak |
| Federated Transfer MF | Partial params with shared params | Locally | Medium |

Tab. 1: Comparison of different problem settings including horizontal and vertical federated MF as well as federated transfer MF. FedMF denotes federated MF.

5 Privacy Preservation in Federated MF

According to the privacy threats investigated in section 4, we give some advises for privacy preservation in federated MF. For horizontal federated MF, the global user and item profile matrices computed by aggregation should be protected against each participant. For vertical federated MF, the auxiliary data provider should keep its computed feature sent to the recommender secret. For federated transfer MF, the shared user or item profile matrix should be kept secret to any honest-but-curious participant throughout the FL training, as the rating score and private profile can be potentially implied.

To keep intermediate parameters private, there are mainly three types of approaches cryptography-based, obfuscation-based and hardware-based approaches. Cryptography-based approaches generally use HE and MPC to keep intermediate transactions private. Obfuscation-based approaches such as DP obfuscate private data by randomization, generalization or represssion. Hardware-based approaches rely on trusted execution environment (TEE) to conduct FL learning in a trusted enclave. By using cryptography-based approaches, fully HE can be introduced to prevent decryption during training [Kim et al., 2016]. Secret sharing schemes can also be introduced following a two-server architecture [Damgard I, 2012]. Since the user-item interaction matrix is sparse, applying DP may introduce too much noise and make the model unavailable. TEE can also be applied by encrypting private data and conducting private training inside TEE [Chen et al., 2020b].

6 Conclusion

We identify and formulate three types of federated MF problems based on the partition of feature space. Then, we demonstrate the privacy threats against each type of federated MF. We show how the private user preference data, private user/item profiles matrix, and user ID can be potentially leaked to honest-but-curious participants. Finally, We discuss privacy-preserving approaches to protect privacy in federated MF. For future work, we will experimentally study the power of the proposed privacy attacks by measuring the portion and accuracy of the inferred private data. Privacy threats against alternating least squares-based MF and other recommender systems also require further comprehensive study.

References

[Albrecht, 2016] Jan Philipp Albrecht. How the gdpr will change the world. *European Data Protection Law Review*, 2:287–289, 2016.
[Canny, 2002] John Canny. Collaborative filtering with privacy. In *Proceedings 2002 IEEE Symposium on Security and Privacy*, pages 45–57. IEEE, 2002.
[Chai et al., 2019] Di Chai, Leye Wang, Kai Chen, and Qiang Yang. Secure federated matrix factorization. *arXiv preprint arXiv:1906.05108*, 2019.
[Chen et al., 2020a] Chaochao Chen, Bingzhe Wu, Wenzin Fang, Jun Zhou, Li Wang, Yuan Qi, and Xiaolim Zheng. Practical privacy preserving poi recommendation. *arXiv preprint arXiv:2003.02834*, 2020.
[Chen et al., 2020b] Yu Chen, Fang Luo, Tong Li, Tao Xiang, Zheli Liu, and Jin Li. A training-integrity privacy-preserving federated learning scheme with trusted execution environment. *Information Sciences*, 522:69 – 79, 2020.
[Damgard I, 2012] Smart N P et al. Damgard I, Pastor V. Multiparty computation from somewhat homomorphic encryption. In *In Proc. of Advances in Cryptology (CRYPTO’12)*, 2012.
[Dwork, 2008] Cynthia Dwork. Differential privacy: A survey of results. In *TAMC*, pages 1–19, 2008.
[Flanagan et al., 2020] Adrian Flanagan, Were Oyomno, Alexander Grigorievskiy, Kuan Eeik Tan, Suleiman A. Khan, and Muhammad Ammad-Ud-Din. Federated multi-view matrix factorization for personalized recommendations, 2020.
[Gao et al., 2019] Dashan Gao, Ce Ju, Xiguang Wei, Yang Liu, Tianjian Chen, and Qiang Yang. Hhhfl: Hierarchical heterogeneous horizontal federated learning for electroencephalography. *NeurIPS Workshop on Federated Learning for Data Privacy and Confidentiality*, 2019.
[Ghosh, 2018] Dipayan Ghosh. What you need to know about california’s new data privacy law. *Harvard Business Review*, 2018.
[Jeckmans et al., 2013] Arjan J. P. Jeckmans, Michael Beye, Zekeriyaa Erkin, Pieter H. Hartel, Reginald L. Lagendijk, and Qiang Tang. Privacy in recommender systems. In *Social Media Retrieval*, 2013.
[Ju et al., 2020] Ce Ju, Dashan Gao, Ravikiran Mane, Ben Tan, Yang Liu, and Cuntai Guan. Federated transfer learning for eeg signal classification. *IEEE Engineering in Medicine and Biology Society*, 2020.
[Kim et al., 2016] Sungwook Kim, Jinsu Kim, Dongyoung Koo, Yuna Kim, Hyunsoo Yoon, and Junbum Shin. Efficient privacy-preserving matrix factorization via fully homomorphic encryption. In *Proceedings of the 11th ACM on Asia Conference on Computer and Communications Security*, pages 617–628, 2016.
[Konečný et al., 2016] Jakub Konečný, H Brendan McMahan, Daniel Ramage, and Peter Richtárik. Federated op-
timization: Distributed machine learning for on-device intelligence. *arXiv preprint arXiv:1610.02527*, 2016.

[Koren et al., 2009a] Y. Koren, R. Bell, and C. Volinsky. Matrix factorization techniques for recommender systems. *Computer*, 42(8):30–37, 2009.

[Koren et al., 2009b] Yehuda Koren, Robert Bell, and Chris Volinsky. Matrix factorization techniques for recommender systems. *Computer*, 42(8):30–37, 2009.

[Lam et al., 2006] Shyong K. Lam, Dan Frankowski, and John Riedl. Do you trust your recommendations? an exploration of security and privacy issues in recommender systems. In *ETRICS*, 2006.

[Li et al., 2019] Zhaorui Li, Zhicong Huang, Chaohao Chen, and Cheng Hong. Quantification of the leakage in federated learning, 2019.

[McMahan et al., 2017] H Brendan McMahan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Agüera y Arcas. Communication-efficient learning of deep networks from decentralized data. In *Proceedings of the 20th International Conference on Artificial Intelligence and Statistics*, pages 1273–1282, 2017.

[Melis et al., 2019] Luca Melis, Congzheng Song, Emiliano De Cristofaro, and Vitaly Shmatikov. Exploiting unintended feature leakage in collaborative learning. *2019 IEEE Symposium on Security and Privacy (SP)*, May 2019.

[Nikolaenko et al., 2013] Valeria Nikolaenko, Stratis Ioannidis, Udi Weinsberg, Marc Joye, Nina Taft, and Dan Boneh. Privacy-preserving matrix factorization. In *Proceedings of the 2013 ACM SIGSAC conference on Computer & communications security*, pages 801–812, 2013.

[Paillier, 1999] Pascal Paillier. Public-key cryptosystems based on composite degree residuosity classes. In *EUROCRYPT*, pages 223–238, 1999.

[Payman and Yupeng, 2017] Mohassel Payman and Zhang Yupeng. SecureML: A system for scalable privacy-preserving machine learning. *IACR Cryptology ePrint Archive*, page 396, 2017.

[Qi et al., 2020] Tao Qi, Fangzhao Wu, Chuhan Wu, Yongfeng Huang, and Xing Xie. Fedrec: Privacy-preserving news recommendation with federated learning. *arXiv*, pages arXiv–2003, 2020.

[Yang et al., 2019] Q. Yang, Y. Liu, Y. Cheng, Y. Kang, T. Chen, and H. Yu. *Federated Learning*. 2019.