Solving a Fuzzy Transportation Problem by Stepping Stone Method

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Abstract: In this paper a new algorithm namely fuzzy stepping stone method is proposed for finding a fuzzy optimal solution for a fuzzy transportation problem where the transportation costs are hexagonal fuzzy numbers. The initial solution of for the fuzzy transportation problem the fuzzy north west corner method was used and for determining the optimality of obtained solution fuzzy stepping stone method is used. The solution procedure is illustrated with a numerical example. It is observed that the proposed algorithm gives a better fuzzy optimal solution to the given fuzzy transportation problem.

Keywords: Hexagonal fuzzy numbers, centroid ranking, stepping stone Method.

I. INTRODUCTION

The transportation problem is an important problem which has been widely used in Operation research. It has been often used to simulate different real life problems. In this problem we determine the optimal shipping patterns between origins or sources and destinations. Several methods were introduced for ranking of fuzzy numbers. The idea of fuzzy set was first introduced by Zadeh in the year 1965[4].Chanas and Kuchta [1], further proposed a method for solving fuzzy transportation problem. Chanas et. al.,[3] developed a method for solving fuzzy transportation problems. Liu and Kao [4] proposed a new method for the solution of the fuzzy transportation problem by using the Zadeh’s extension principle. Using parametric approach, Nagoorgani and Abdul Razak [7] obtained a fuzzy solution for a two stage Fuzzy Transportation problem with trapezoidal fuzzy numbers. Omar et. al., [7] also proposed a parametric approach for solving transportation problem under fuzziness. Fegad et.al.,[8] Pandian and Natarajan [4] proposed a fuzzy zero point method to find the fuzzy optimal solution of fuzzy transportation problems. Narayana Murthy et.al., [3] also proposed Russell’s method for the solution of Fuzzy Transportation problem with trapezoidal fuzzy numbers.

In this paper we use the Centroid Ranking Method [5][6] where the idea is to transform a problem with fuzzy parameters into a crisp form. The objective of a fuzzy transportation problem is to determine the transportation schedule that minimizes the total fuzzy transportation cost while satisfying the availability and the requirement limits. In order to solve a transportation problem the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp value. But in real life applications supply, demand and unit transportation cost may be uncertain due to several factors. These imprecise data may be represented by fuzzy numbers.

II. PRELIMINARIES

A. Crisp Set

A crisp set is defined in such a way as to divide the individuals in some given universe into two groups.

1) Members that certainly belong to the set.
2) Members that certainly do not belong to the set.

B. Fuzzy Set

A fuzzy set is characterised by a membership function mapping elements of domain, space or universe of discourse X to the unit interval [0,1]. $\tilde{A} = \{ (x, \mu_\tilde{A}(x)) : x \in X \}$ Here $\mu_\tilde{A}: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_\tilde{A}(x)$ is called the membership value of $x \in X$ in the fuzzy set A.

C. Fuzzy Number

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_\tilde{A}: R \rightarrow [0,1]$ has the following properties

1) A must be a normal fuzzy set.
2) $\alpha A$ must be a closed interval for every $\alpha \in [0,1]$.
3) The support of A, must be bounded.
D. Hexagonal Fuzzy Numbers

A fuzzy number \( \bar{A} = (a, b, c, d, e, f) \) is said to be a hexogonal fuzzy number if its membership function is given by

\[
\mu_{\bar{A}}(x) = \begin{cases} 
\frac{1}{a} (x-a), & a \leq x \leq b \\
\frac{1}{b-a} \left( x - \frac{b-a}{2} \right), & b \leq x \leq c \\
1, & a \leq x \leq a \\
\frac{1}{c-b} \left( x - \frac{c-b}{2} \right), & c \leq x \leq d \\
\frac{1}{f-e} \left( f - x \right), & a \leq x \leq b \\
0, & \text{otherwise}
\end{cases}
\]

E. Proposed Ranking Method

An efficient approach for comparing the fuzzy numbers is by using of a ranking function \( R: F(R) \rightarrow R \). Where F(R) is a fuzzy number defined on set of real numbers, which maps each fuzzy number into a real number. Where natural order exists. Here a centroid based distance approach to rank fuzzy number is used. The ranking function is defined as

\[
R(\bar{A}_h) = \left( \frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18} \right) \left( \frac{W}{18} \right)
\]

III. MATHEMATICAL FORMULATION OF FUZZY TRANSPORTATION PROBLEM

In general, let there be m sources of supply, \( s_i \) \((i=1,2,\ldots,m)\) having \( a_i \) \((i=1,2,\ldots,m)\) units of supply to be transported among ‘n’ destinations \( D_j \)(j = 1, 2, ..., n) with \( b_j \)(j = 1, 2, ..., n) units of demand. Also let, \( C_{ij} = \) cost of transporting per unit of commodity from source \( i \) to destination \( j \), and \( x_{ij} = \) quantity transported from source \( i \) to destination \( j \). Then the problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying supply and demand constraints.

Mathematically, the problem may be stated as a linear programming problem as follows:

Minimize \( Z = \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ij} x_{ij} \)

|   | 1  | 2  | 3  | ... | j  | ... | n  |
|---|----|----|----|-----|----|-----|----|
| 1 | \( \hat{c}_{11} \) | \( \hat{c}_{12} \) | \( \hat{c}_{13} \) | ... | \( \hat{c}_{1j} \) | ... | \( \hat{c}_{1n} \) |
| 2 | \( \hat{c}_{21} \) | \( \hat{c}_{22} \) | \( \hat{c}_{23} \) | ... | \( \hat{c}_{2j} \) | ... | \( \hat{c}_{2n} \) |
| ... | ... | ... | ... | ... | ... | ... | ... |
| 1 | \( \hat{c}_{m1} \) | \( \hat{c}_{m2} \) | \( \hat{c}_{m3} \) | ... | \( \hat{c}_{mj} \) | ... | \( \hat{c}_{mn} \) |

A. New Algorithm To Find The Optimal Solution To A Fuzzy Transportation Problem

1) Step 1: determine an initial basic feasible solution.
2) Step 2: The number of occupied cells is exactly equal to \( m + n + 1 \). Where \( m \) is number of rows and \( n \) is number of columns.
3) Step 3: Evaluate the co-efficient of shipping goods via transportation routes not currently in solution. This testing of each unoccupied cell is conducted by the following five steps as follows:
   a) Select an unoccupied cell, where a shipment should be made.
   b) Beginning at this cell, trace a closed path using the most direct route through at least three only horizontal and vertical moves. Further, since only the cell at the turning point are occupied cells used in the solution and then back to the original occupied cell and moving with considered to be on the closed path, both unoccupied and occupied boxes may be skipped over. The cells at the turning points are called stepping stone on the path.
   c) Assigning plus(+) and minus(-) signs alternatively on each corner cell of the closed path just traced, starting with plus sign at the unoccupied cell to be evaluated.
d) Compute the 'net change in the cost' along the closed path by adding together the unit cost in each square containing the minus sign.
e) Repeat the sub-step(a) through sub-step(b) until 'net change' in cost has been calculated for all unoccupied cells of the transportation table.

4) **Step 4:** Check the sign of each of the changes. If all net changes computed are greater than are equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution and decreases total shipping costs.

5) **Step 5:** Select the unoccupied cells having the highest negative net cost change and determine the maximum number of units that can be assigned to a cell mark with a minus sign on the closed path corresponding to this cell. Add this number to the unoccupied cell and to all other cells on the path marked with a plus sign, subtract this number from cells on the closed path marked with a minus signs.

6) **Step 6:** Go to step (2) and repeat the procedure until we get an optimal solution.

## IV. NUMERICAL EXAMPLE

A company has 3 operational procedures weaving, processing and packing with capacity to produce 3 different types of cloths suitings, shirtings and woollens. The information about the cost of transformation is imprecise and here hexagonal fuzzy numbers are used to represent the cost, whereas the supply and demand are crisp numbers.

| From/To     | Weaving       | Processing    | Packing       | Supply |
|-------------|---------------|---------------|---------------|--------|
| Suiting     | 1, 2, 4, 6, 8, 9 | 4, 6, 9, 10, 12, 13 | 11, 12, 14, 16, 18, 20 | 56     |
| Shirting    | 9, 10, 11, 13, 16, 18 | 7, 9, 10, 14, 16, 18 | 3, 5, 7, 9, 10, 12 | 56     |
| Woollen     | 10, 12, 14, 16, 18, 20 | 8, 10, 12, 14, 16, 18 | 5, 7, 9, 10, 12, 14 | 55     |
| Demand      | 42            | 56            | 69            | 167    |

1) **Solution**

   a) **Step: 1** The above problem is a balanced fuzzy transportation problem. (i.e) \( Z = \sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} \) Using the centroid ranking method to obtain the crisp values of fuzzy transportation problem.

   | From/To | Weaving | Processing | Packing | Supply |
   |---------|---------|------------|---------|--------|
   | Suiting | 1       | 3          | 4       | 56     |
   | Shirting| 4       | 3          | 2       | 56     |
   | Woollen | 4       | 4          | 2       | 55     |
   | Demand  | 42      | 56         | 69      | 167    |

   b) **Step: 2** North west corner rule is used to obtain the initial basic feasible solution

   ![North west corner rule solution]

   The initial solution has 3+3-1=5 occupied cells and the transportation cost

   \[ \text{Min } Z = (42 \times 1) + (14 \times 3) + (42 \times 3) + (14 \times 2) + (55 \times 2) = 167 \]
c) **Step: 3** Select the unoccupied cell, and a closed path using the most direct route through at least three horizontal and vertical moves.

| From/To     | weaving | Processing | Packing | Supply |
|-------------|---------|------------|---------|--------|
| Suitting    | 42      | 1          | 3       | 56     |
| Shirting    | 4       | 42         | 3       | 56     |
| Woolen      | 4       | 4          | 55      | 2      |
| Demand      | 42      | 56         | 69      | 167    |

**d) Step: 4** Assigning plus(+) and minus(-) signs alternatively on each corner cell of the closed path. Compute the 'net change in the cost' along the closed path by adding together the unit cost in each square containing the minus sign.

| From/To     | weaving | Processing | Packing | Supply |
|-------------|---------|------------|---------|--------|
| Suitting    | 42      | 1          | 3       | 56     |
| Shirting    | 4       | 42         | 3       | 56     |
| Woolen      | 4       | 4          | 55      | 2      |
| Demand      | 42      | 56         | 69      | 167    |

d) **Step: 5** To determine next cost change, let us list down the changes as shown below.

| Unoccupied | Closed path                                                                 | Net cost change | Remark   |
|------------|-------------------------------------------------------------------------------|-----------------|----------|
| (1,3)      | (1,3) → (2,3) → (2,2) → (1,2)                                                | 4-2+3-3=2       | Cost increase |
| (2,1)      | (2,1) → (1,1) → (1,2) → (2,2)                                                | 4-1+3-3=3       | Cost increase |
| (3,1)      | (3,1) → (1,1) → (1,2) → (2,2) → (2,3) → (3,3)                               | 4-13-3+2-2=3    | Cost increase |
| (3,2)      | (3,2) → (2,2) → (2,3) → (3,3)                                                | 4-3+2-2=1       | Cost increase |

Total transportation cost = \(42 \times 1\) + \(14 \times 3\) + \(42 \times 3\) + \(14 \times 2\) + \(55 \times 2\)

\[= 42 + 42 + 126 + 28 + 110 = 348\]

The total transportation cost of the optimal solution is Rs.348.

A. **Comparison Table**

| S.NO | Method             | Iteration | Optimal Solution |
|------|--------------------|-----------|------------------|
| 1    | Stepping stone Method | 4         | 348              |
| 2    | Zero Point Method   | 10        | 348              |
| 3    | Blocking Zero Point Method | more than 10 | 348          |

Fig: 1.1 Graphical Representation
V. CONCLUSION

In this paper a balanced fuzzy transportation problem is considered. Different methods as Stepping stone method, zero point method and blocking zero point method has been applied to find the optimality of the fuzzy feasible solution. The optimum transportation cost is the same for all the three methods, Whereas the iterations are less in Stepping Stone method when compared to the other two methods. So we can conclude that stepping stone method is better than the other two.

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