Catapulating towards massive and large spatial quantum superposition

Run Zhou, Ryan J. Marshman, Sougato Bose, and Anupam Mazumdar

Van Swinderen Institute, University of Groningen, 9747 AG Groningen, The Netherlands
Centre for Quantum Computation and Communication Technology, School of Mathematics and Physics, University of Queensland, Brisbane, Queensland 4072, Australia
Department of Physics and Astronomy, University College London, Gower Street, WC1E 6BT London, United Kingdom

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A large spatial quantum superposition of size $O(1-10) \mu m$ for mass $m \sim 10^{-17}-10^{-14}$ kg is required to probe the foundations of quantum mechanics and test the classical and quantum nature of gravity via entanglement in a laboratory. In this paper, we will show that it is possible to accelerate the two spin states of a macroscopic nanocrystal sourced by the inhomogeneous nonlinear magnetic field in a Stern-Gerlach-type setup. We will assume that the electronic spin can be embedded at the center of the nanocrystal, such as the nitrogen-vacancy (NV) center of diamond. Our analysis will be generic to any dopant or any material. We will show that we can create a desired superposition size within 1–2 s by catapulting the trajectories of the two spin states with a modest magnetic field gradient and then recombine the trajectories for a coherent interference. We will show the demanding nature of the precision required in the magnetic field to recover a 99% spin coherence confidence level at the moment of interference.

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I. INTRODUCTION

Gravity is one of the weakest interactions of nature, and it is very special because of its universality. Unlike any other known interactions, it is not yet clear whether gravity respects the rules of quantum mechanics at a microscopic level [1]. No experimental proof validates whether the gravitational interaction is indeed quantum. The spacetime we have witnessed so far in gravitational experiments is extremely classical without any hint of quantumness [2].

The conventional wisdom is that the gravitational effects will become important only when we approach the Planckian length or time scale, making it extremely challenging to test the quantum nature of gravity in a laboratory. Furthermore, neither tests from the cosmological perturbations in the cosmic microwave background radiation [3] nor the positive detection of primordial gravitational waves [4] confirms the quantum nature of gravity; as a matter of fact, neither do any other astrophysical tests [5]. They all have many astrophysical uncertainties, making it extremely challenging to conclude the true nature of gravity. Also, the feeble nature of the gravitational interaction makes it extremely hard to detect the graviton as an individual quantum [6].

Despite all these challenges, there is one hope for gravity. The gravitational interaction is a long-range interaction like in the case of quantum electrodynamics, hence it gives us a unique possibility to test its quantum properties in the infrared spectrum.

Recently, a tabletop experiment has been proposed to explore such a quantum origin of gravity with the help of quantum superposition and quantum entanglement [7,8]. The protocol known as the quantum-gravity-induced entanglement of masses (QGEM) is based on the quantum interaction of gravity with the quantum state of matter to generate the entanglement. The latter is purely a quantum observable and has no classical analog. If gravity is indeed quantum, it will entangle the two masses in quantum spatial superpositions [9,10]. In the canonical approach to quantum gravity, the gravitational interaction is being mediated by the hypothetical massless spin-2 graviton, whose quantum properties can be studied [10]; see also Refs. [7,11] for the path integral approach and Ref. [12] for the Arnowitt-Deser-Misner (ADM) approach. The critical point to note here is that a creation of a spatial superposition is governed by its own dynamical degree of freedom, which conserves the equations of motion governed by the electromagnetic properties of the material. To understand the theoretical aspects of the entanglement, we will always need to consider the dynamical aspects of two masses; see Ref. [10].

A large spatial superposition for a massive object tests the foundations of quantum mechanics [13,14], tests the equivalence principle of gravity [15,16], falsifies spontaneous collapse mechanisms [17,18], and places bounds on decoherence mechanisms [19,20]. Furthermore, as an application of a massive quantum interferometer, we can use them as a quantum sensor [21] and to probe very high frequency gravitational waves [22].

To realize some of these ambitious experiments, especially QGEM, we will require a large spatially localized state of

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supposition $\Delta Z \sim O(10–100) \mu m$ for large masses of order $m \sim 10^{-15}–10^{-14}$ kg [7,23]. We are assuming that the superposition is in the $z$ direction. These requirements are far beyond the scales achieved to date in any laboratory (e.g., macromolecules $m \sim 10^{-22}$ kg over $\Delta Z \sim 0.25 \mu m$, or atoms of mass $m \sim 10^{-25}$ kg over $\Delta Z \sim 0.5$ m [14,24–27]).

Despite numerous challenges, there are already physical schemes to obtain small $\Delta Z$ and $m$ [28–52], and there are arguments presented on how to achieve large superposition in a vacuum by using the Stern-Gerlach principle in the presence of a magnetic field gradient [45,50,53]. Based on these ideas, a feasibility experiment has been performed with the help of atoms, showing that such a Stern-Gerlach interferometer (SGI) for massive objects can indeed be realizable [48]. Of course, we will now need to increase the mass by nearly six to seven orders of magnitude, which will pose a serious technical, if not fundamental, challenge.

In this paper, we aim to improve the existing mechanism for creating a large spatial superposition [7,45,50,53]. In this paper we will consider the effect of a spatially dependent nonlinear magnetic field to create the superposition as opposed to the spatially dependent linear magnetic field in Ref. [53]. We will utilize the nonlinear magnetic field profile to further increase the spatial superposition size. We will concentrate on the one-dimensional interferometer, which avoids the issues related to the two-dimensional SGI [54]. We will be focusing on applying a much lower magnetic field gradient first described in the original paper about QGEM [7]. We are assuming that the electronic spin can be embedded at the center of the nanocrystal, such as in the case of a nitrogen-vacancy (NV) center in diamond. Our discussions will be generic to any dopant and material, but for illustration, we will use material properties similar to those of diamond. Also, we will avoid the low region of the magnetic field for the Majorana spin flip [55,56], discussed below.

Typically, we will require experimental configurations with a magnetic field which originates in a single current-carrying wire or a permanent magnet, where the magnetic field goes as $|B| \propto 1/z$, where $z$ is the distance from the current source. The magnetic field can then be expanded around a small region. Such configurations were considered in Ref. [53]. However, as we will see below, if we consider the nonlinear part of the magnetic field dependence, we can generate an even larger superposition size at a shorter time scale. Indeed, a detailed discussion of obtaining such a magnetic field profile will require separate consideration, such as a quadrupole field from coils in an anti-Helmholtz configuration.

We will further assume that we can achieve the required level of internal cooling of the nanocrystal and the external cooling for maintaining the coherence of the spin for 1–2 s; see the bounds on ambient temperatures in Refs. [7,23,57]. We will also assume that the crystal is ideal; in this respect, we are assuming that the impurities are very small, such that the spin coherence can be maintained. The internal cooling for the crystal will also suppress the phonon vibration sufficiently to maintain the spin coherence [7]. Given that all these effects are under control, we will ask, how large a superposition size can we achieve for objects of mass $10^{-17}$, $10^{-16}$, and $10^{-15}$ kg?

We will apply the inhomogeneous magnetic field profile and the bias magnetic field. We will consider the nonlinear dependence of the magnetic field in one direction, $z$, without loss of any generality. In this regard, we will create the superposition primarily in the $z$ direction. We will first create a velocity difference between the two paths of the spins by creating anharmonic oscillations and create a sufficiently large velocity difference between the two paths to catapult the trajectories as far as possible to create a large $\Delta Z$. Then we will bring the trajectories back to cause the two paths to interfere, and we will study the spin coherence [58,59].

While creating the spatial superposition, we will lose the spin coherence; therefore, to create interference, we will need to ensure that the spin coherence is restored at the moment of interference. We will demand that the spin coherence be 99%, which will place a severe constraint on the two paths and hence any fluctuations they incur in creating the superposition. We will see that our analysis following Refs. [58,59] will put a stringent constraint on the magnetic field fluctuations, which we can tolerate. We will also assume that the entire setup is performed in a free-fall experiment, such that the Earth’s gravitational acceleration can be negligible. The latter is necessary for avoiding any gravity-induced and relative acceleration noise; the details can be found in Ref. [21].

The paper is organized as follows. In Sec. II, we will discuss the foundations of the SGI setup and discusses the nonlinear magnetic field profile and the constraints on the magnetic field. In Sec. III, we will discuss various stages of the two trajectories for masses $10^{-17}$, $10^{-16}$, and $10^{-15}$ kg. In Sec. IV, we will discuss the scaling behavior of the superposition size. In Sec. V, we will discuss the constraint on the magnetic field fluctuation, which we can tolerate for the spin coherence, and in Sec. VI, we will conclude our paper.

II. STERN-GERLACH INTERFEROMETER

We can write the Hamiltonian of the spin embedded in the nanocrystal as [45,53,60]

$$
H = \frac{\hat{p}^2}{2m} + \hbar \hat{D} \hat{S}^2 - \frac{\chi_m m}{2\mu_0} \hat{B}^2 - \hat{\mu} \cdot \hat{B},
$$

(1)

where $m$ is the mass of the nanocrystal and $\hat{p}$ and $\hat{S}$ are momentum and spin operators, respectively. $D$ is the NV zero-field splitting. $\chi_m$ is the magnetic susceptibility. $\mu_0$ is the vacuum permeability. $\hat{\mu} = -g \mu_B \hat{S}$ is the spin magnetic moment, where $g \approx 2$ is the Landé $g$ factor, $\mu_B = e \hbar / 2m$ is the Bohr magneton, $e$ is the electron charge, and $m_n$ is the electron rest mass. $\hat{B}$ is the magnetic field. We will assume that the spin is embedded in the center of the nanocrystal. We will not consider the effects of external torque in this paper; we are assuming that we can engineer a situation so that the torque and the rotational effects of the mass are negligible or decoupled from the translation (for possible mechanisms to cool rotation, see Refs. [61–65]). At this point, we also consider an idealized system with no impurities. Of course, in reality, we will need to consider the impurities. However, for this toy model, we will not consider these effects here for the time being. Note that we are neglecting the gravitational potential here. We will be interested in experimenting with a free-fall setup to minimize gravity gradient noise and dephasing due to Earth’s gravitational potential; see the discussion in Ref. [21].
With these assumptions, the last two terms in Eq. (1) represent the potential energy
\[
\hat{U} = \frac{X_m m}{2\mu_0} \mathbf{B}^2 - \hat{\mu} \cdot \mathbf{B},
\]  
from which we can calculate the acceleration of the nanocrystal as
\[
\hat{a} = -\frac{1}{m} \nabla \hat{U} = \frac{X_m}{2\mu_0} \nabla \mathbf{B}^2 - \frac{g e h}{2m m_e} \nabla \hat{S} \cdot \mathbf{B}. \tag{3}
\]
Equation (3) shows that if the specific form of the magnetic field is determined, then the acceleration can be calculated to obtain the trajectory of the nanocrystal in the magnetic field.

We will assume that the magnetic field takes the following simple form:
\[
\mathbf{B} = (B_0 + \eta z^2 - \eta x^2) \hat{z} - 2\eta z \hat{x}, \tag{4}
\]
where $B_0$ is a fixed constant magnetic field. We will explain below why we need this bias magnetic field to align the NV spin in the $z$ direction. The $\eta$ is a coefficient with the dimension $T/m^2$, while $\eta z$ will determine the magnetic field gradient [66]. Here, we have assumed that at the initial moment the coordinate of the NV center along the $x$ axis is zero $(x = 0)$ and that the embedded spin is aligned in the $z$ direction $(S_z = 0)$.

Let us now calculate $\nabla \mathbf{B}^2 = 2(B_0 + \eta z^2)2\eta z \hat{z}$ and $\nabla \hat{S} \cdot \mathbf{B} = (2\eta z) \hat{x}$, respectively, and then combine the results to get the expression for the acceleration.
\[
a_z = \left(\frac{X_m}{\mu_0} (B_0 + \eta z^2)2\eta z - 2\eta z \hat{S}_z \right) \hat{z}. \tag{5}
\]
The initial state superposition is given by $(|\uparrow_z\rangle + |\downarrow_z\rangle)/\sqrt{2}$, the internal spin of the NV center, and the $z$ direction for the wave packet separation.

The above equation (5) shows that the wave packet only separates in the $z$ direction and that in the $x$ coordinate of the NV center the acceleration is always zero. The spin state in the $x$ and $y$ basis will experience a rapid Larmor precession; therefore averaging the spin yields no net force along the $x$ axis in our case. This means that as long as $B_0$ in Eq. (4) is not zero, we can ensure that the spin direction is approximately aligned along the $z$ axis and avoid the Majorana spin flips; see Ref. [53]. However, this is an ideal situation. In the actual experiment, the spin would have a Larmor precession around the $z$ axis. The minimum allowable value of $B_0$ in Eq. (4) can be determined by both the Larmor precession frequency and the adiabatic condition of the frequency which forbids the particle motion along the $x$ axis; see, for details, Ref. [53]. The Larmor precession frequency is given by
\[
\omega_L = \frac{g e \mathbf{B}(x, z)}{2m_e}, \tag{6}
\]
which will be required to satisfy the adiabatic condition $\omega_L \ll \omega_z^2$ [53]. Combining Eqs. (4) and (6), and the adiabaticity condition, we are able to obtain the minimum magnetic field, labeled $B_{\text{min}}$. The minimum Larmor precession frequency corresponding to the minimum magnetic field that satisfies the adiabatic condition is
\[
\omega_L^{\text{min}} = \frac{g e}{2m_e} B_{\text{min}}. \tag{7}
\]
In this paper, we will set $B_0 \geq B_{\text{min}}$, and $B_0$ is the minimum magnetic field coordinates experienced by the wave packet; so the adiabatic condition is always satisfied during the evolution of the wave packet. We will take $B_0 \approx 5.7 \times 10^{-4}$ T, which ensures that the adiabatic condition is always satisfied in this paper.

III. CATAPULTING TRAJECTORIES

Note that the difference between the two wave packet trajectories is mainly caused by the difference in the spin eigenvalues of the second term on the right-hand side in Eq. (5). We would expect to get a large superposition size by increasing the value of $\eta$. However, increasing the value of $\eta$ will only increase the motional frequency of the wave packet and will not directly increase the superposition size $\Delta Z$. Furthermore, it maintains the superposition size for a longer period. We clearly see this result in Fig. 1. When we fix the mass of the nanocrystal and increase the value of $\eta$, we find that the motional frequency of the wave packet increases with $\eta$ while the maximum superposition size remains almost unchanged, such that it can reach $\Delta Z \sim 40$ $\mu$m within $\tau \sim 1.2$ s for the case $m = 10^{-17}$ kg. Similar results were found for the other two masses considered.

Although the superposition size does not increase with $\eta$, we can increase the velocity difference between the two wave packets in a short time by increasing the value of $\eta$ (as shown in Fig. 2). When there is a large velocity difference between the two wave packets and the spatial position coincides, we can adjust the magnetic field so that the two wave packets are located at the lowest point of the potential energy (which is $z = 0$). By doing so, we can catapult the two wave packets around in the magnetic field and achieve a large superposition size.

We will now discuss the trajectories of the wave packets. Let us first consider the case where the nanocrystal has a mass of $10^{-17}$ kg. A similar analysis will arise for all the other masses under consideration. We will discuss the implementation in three stages. The purpose of the first stage is to obtain a large velocity difference between the two trajectories in a short time (around, say, 0.2 s) by applying $(\eta = 1 \times 10^8 \ T/m^2)$ [67]. The purpose of the second stage is to generate, and then close, a large spatial superposition of the two trajectories. To do this, we decrease the acceleration of the wave packets by decreasing the value of $\eta$. By adjusting the value of $\eta$ to an appropriate value, we can get a large superposition size in a relatively short coherent time scale ($\sim 1$ s) [69,70]. When the spatial positions of the two wave packets coincide again, we begin the third stage. We then adjust the magnetic field gradient and the position of the particle in the potential energy. Doing so carefully will close the interferometer, causing the spatial and momentum differences to become zero in both arms (Fig. 3). Figure 4 shows these steps graphically. The behavior of the nanocrystals in the magnetic field for masses of $10^{-16}$ and $10^{-15}$ kg is very similar to that for a mass of $10^{-17}$ kg, and we have included the numerical results for both those cases ($10^{-16}$ and $10^{-15}$ kg) in Appendix C. We can also consider these stages in more detail.
FIG. 1. Trajectories of the two wave packets under different magnetic field gradients and the corresponding superposition size. \( \eta \) is a parameter associated with the magnetic field profile. With the increase in the value of \( \eta \) [(a) and (d) \( 1.4 \times 10^6 \), (b) and (e) \( 2.4 \times 10^6 \), and (c) and (f) \( 3.4 \times 10^6 \) \( \text{T/m}^2 \)], the motional frequency of the wave packet increases accordingly, but the maximum superposition size that can be achieved over the same period of time remains almost unchanged (the maximum superposition size is about 40 \( \mu \text{m} \) within 1.2 s). The mass here is \( 10^{-17} \text{kg} \).

**Initialization**, \( t = 0 \). The wave packet enters the inhomogeneous magnetic field region. The initial velocity along the \( z \) direction is \( \dot{z}(0) = 0 \), and the initial position \( z_0 = 0 \). The parameter \( \eta = 1 \times 10^8 \text{T/m}^2 \).

**Stage I**, \( 0 < t < T_1 \). The two wave packets oscillate rapidly in the magnetic field, and the velocity difference between them grows larger and larger; see Figs. 4(a) and 4(g). The magnetic field gradient is \( \eta_z \approx -1 \times 10^4 \text{T/m} \) at \( t = T_1 \), where \( \eta = 1 \times 10^8 \text{T/m}^2 \), \( z \approx -1 \times 10^{-4} \text{m} \).

At \( t = T_1 \). Since the difference in the change in the spatial position [Fig. 4(j)] between the two wave packets is opposite to that of the difference in the velocity, it is possible to find a moment when the velocity difference is large enough and the superposition size is 0. This moment is marked \( T_1 \). At time

FIG. 2. The velocity curve for the two wave packets under different \( \eta \) and the corresponding velocity difference. By increasing the value of \( \eta \) [(a) and (d) \( 1.4 \times 10^6 \), (b) and (e) \( 2.4 \times 10^6 \), and (c) and (f) \( 3.4 \times 10^6 \) \( \text{T/m}^2 \)], the maximum velocity difference that can be achieved over the same period of time also increases (around 400, 700, and 1100 \( \mu \text{m/s} \) in less than 1.2 s, respectively). The mass here is \( 10^{-17} \text{kg} \).
$T_1$, if we adjust the magnetic field such that the coordinate of the two wave packets in the magnetic field is $z = 0$ [see Fig. 4(b)], then we find the value of $\eta \sim 1 \times 10^7 \text{T/m}^2$, and $\partial B_z = \eta z = 0$ at $z = 0$.

**Stage II**, $T_1 < t < T_2$. In this stage, the two wave packets have different initial velocities at the new initial potential position, which is equivalent to ejecting the two wave packets away from each other. Moreover, due to the reduction in the magnetic field gradient, the spatial position difference between the two wave packets can be significantly increased; see Fig. 4(k).

At $t = T_2$. The two wave packets meet again after a half time period of motion, and we mark the time of their meeting as $T_2$; see Fig. 4(b). At time $T_2$, we need to adjust the coordinates of the wave packet in the magnetic field and select an appropriate value of $\eta$, so that the trajectory of the two wave packets can be closed in a relatively short time ($\sim 1$ s).

At time $T_2$, the magnetic field of the wave packet is adjusted to $z_{T_2} = -102.8 \mu \text{m}$. Parameter $\eta$ is adjusted to $3.4 \times 10^7 \text{T/m}^2$.

**Stage III**, $T_2 < t < T_3$. The two wave packets still oscillate rapidly in the magnetic field, but their spatial position difference and velocity difference will be smaller and smaller; see Figs. 4(i) and 4(l).

At $t = T_3$. The trajectories of the two wave packets are closed. The superposition size is zero, and the velocity difference is zero; see Fig. 3(a). We will analyze this case separately in the context of spin coherence.

In this experimental setup, the magnetic field was changed twice, but we did not consider the effect of the change in the magnetic field here. Note that the magnetic field changes in a short time, so it will only cause a small disturbance to the wave packet trajectory, but we are here taking this effect to be negligible. In fact, we can introduce switching functions of the magnetic field, for example, as discussed in Ref. [53], and our main results will not be adversely affected. It is worth noting that the parameter $\eta$ used in stage I is as high as $10^8 \text{T/m}^2$, and the corresponding maximum magnetic field gradient is $\partial B_z \sim 10^8 \text{T/m}$ (which can be achieved in the laboratory [71]), since the maximum magnetic field coordinates experienced by the wave packet is only $100 \mu \text{m}$ [see Fig. 4(a) and Appendix C, Figs. 15(a) and 16(a), for different masses].

Since we initialize the coordinates of the NV center in the magnetic field in stage II and stage III, the coordinates of the NV center in the magnetic field are inconsistent with their spatial coordinates in the latter two stages. To avoid confusion, unless otherwise specified, the coordinates mentioned in this paper refer to the coordinates of the NV center in the magnetic field.

We have set the value of $\eta$ at the first stage to be $\eta \sim 1 \times 10^7 \text{T/m}^2$, which limits the time of the first stage to about 0.2 s, and we set the limit of the time of the second stage to about 0.5 s and require the trajectories of the two wave packets to be closed within 1.5 s; then we can get the motion of the wave packets and the superposition size with different masses as shown in Fig. 4 and Appendix C, Figs. 15 and 16.

**IV. SCALING BEHAVIOR**

We can use Eq. (5) to roughly analyze the motion of the wave packet for any masses. By substituting the values of each physical quantity into Eq. (5), it can be shown that when the mass $m \geq 10^{-17} \text{kg}$, the motion of the wave packet is mainly dominated by $\eta z^2$ in the first term on the right-hand side of the equation. This term has nothing to do with mass; that is, as long as the values of $\eta$ and the initial position are determined, the maximum velocity of the wave packet can be determined. This result can be seen in Fig. 4(d) and Appendix C, Figs. 15(b) and 16(b). The velocity difference between the two wave packets is caused by the second term on the right-hand side of Eq. (5). The value of this term is inversely proportional to the mass. Since the size of the velocity difference determines the superposition size, the maximum superposition size we can achieve should also be inversely proportional to the mass, i.e., $\Delta Z \sim 1/m$. This result is also borne out from our numerical results, as shown in Fig. 4(k) and Appendix C, Figs. 15(k) and 16(k).

It should be noted that in order to compare the behavior of the wave packets with different masses, the values of $\eta$ and the initial position are chosen properly at different stages of the experiment, but this does not mean that for the wave packets with different masses we can only take these parameter values. For example, if we do not limit the time of the second stage of the experiment to about 0.5 s, we can choose a smaller value of $\eta$ and get a larger superposition size.

We first perform a linear fitting of the velocity difference in stage I. The fitting formula is

$$\Delta V_{\text{fit}} = \left( \frac{5.4 \times 10^{-13} \text{kg}}{m} \right) \left( \frac{T_i}{1 \text{s}} \right) 10^{-6} \text{m/s},$$  \(8\)
FIG. 4. Dynamical aspects for the mass $m = 10^{-17}$ kg during the three experimental stages: (a)–(c) the magnetic field coordinates (potential coordinates) experienced, (d)–(f) the velocities, (g)–(i) the velocity differences, and (j)–(l) the superposition size. We set different values of $\eta$ and the initial position of the wave packet in the magnetic field at different stages. Stage I, $\eta = 1 \times 10^8$ T/m$^2$, with an initial coordinate $z = 100 \mu$m. Stage II, $\eta = 1 \times 10^5$ T/m$^2$, with an initial coordinate $z = 0 \mu$m. Stage III, $\eta = 3.4 \times 10^7$ T/m$^2$, with an initial coordinate $z = -102.8 \mu$m. The initial coordinates here refer to the initialization coordinates of the NV center in the magnetic field at different experimental stages. Times $T_1$ and $T_2$ are determined by constraining the moment when the superposition size is zero (with an accuracy of $10^{-6}$ $\mu$m). Time $T_3$ is the moment when both the velocity difference between the two wave packets and the superposition size are zero.

where $\Delta V_{\text{fit}}$ is the maximum velocity difference reached in stage I. $T_1$ is a variable here, representing the end time of stage I. The values of mass $m$ are $10^{-17}$, $10^{-16}$, and $10^{-15}$ kg, respectively. The linear fitting results are shown in Fig. 5.

Next, we take the velocity difference obtained in stage I as the initial velocity of the wave packet to study the trajectory of the wave packet in stage II. In Sec. V and Appendix A we discuss the fitting of wave packet trajectories. Now we only need to move the simple harmonic motion [Eq. (26)] by the $-\pi/2$ phase to fit the trajectory of the wave packet with the initial velocity $V_{\text{fit}}$ and initial position $z = 0$. The fitting formula of the wave packet trajectory in stage II is

$$z(t) = \Delta Z_0 \cos \left( \sqrt{A} t - \frac{\pi}{2} \right),$$

(9)

where $\Delta Z_0$ and $\sqrt{A}$ are the amplitude and frequency of the wave packet motion, respectively. The specific expression of $A$ is found later, in Eq. (27). Combining Eqs. (8) and (9) gives
respectively. We have set the gradient, so we need to add a parameter to consider the initial velocity of the wave packet in addition to only one parameter in the equation. In our case, we need to determine the magnetic field gradient, so there is only one parameter in the equation. In Ref. [53], we only considered the magnetic field gradient, so there is only one parameter in the equation. In our case, we need to determine the magnetic field gradient, so there is only one parameter in the equation.

The SGI splits the two wave packets in the superposition state. First, they lose their spin coherence and then recombine to recover the spin coherence. We first use the definition of the spin coherence given in Ref. [59] to calculate the expression of the spin coherence in the case of our magnetic field profile and then study what experimental conditions are needed to recover the spin coherence. Heisenberg’s equation of motion is given by

$$i\hbar \frac{d\hat{A}_H(t)}{dt} = [\hat{A}_H(t), \hat{H}_H(t)],$$

where $\hat{A}_H(t)$ and $\hat{H}_H(t)$ are the Hermitian and the Hamiltonian operator in the Heisenberg picture, respectively. Using

$$\Delta Z_0 = \frac{\Delta V_{\text{fit}}}{\sqrt{A}} = \left( \frac{5.4 \times 10^{-13} \text{ kg}}{m} \right) \left( \frac{1 \text{ Hz}}{\sqrt{A}} \right) \left( \frac{T_1}{1 \text{ s}} \right) 10^{-6} \text{ m}.$$  

(10)

For the sake of discussion, let $\sqrt{A} = (\pi/0.4) \text{ Hz}$. That is, the half period of wave packet motion is 0.4 s, which is consistent with the time set in stage II of the experiment in this paper. It can be seen from Eq. (10) that the maximum superposition size of the ejection trajectory is inversely proportional to the mass of the nanocrystal. The scaling behavior of the superimposed size is shown in Fig. 6.

This formula is similar to Eq. (11) of Ref. [53]. In Ref. [53], we only considered the magnetic field gradient, so there is only one parameter in the equation. In our case, we need to consider the initial velocity of the wave packet in addition to the gradient, so we need to add a parameter $T_1$, which represents the initial velocity of the wave packet. If we fix the value of $T_1$, that is, the magnitude of the initial velocity, then our expression is the same as Eq. (11) of Ref. [53]. However, since we have two parameters to play with, we obtain a larger size of superposition compared with Ref. [53]. For instance, for $m = 10^{-15} \text{ kg}$, we can obtain $\Delta Z = 16 \mu m$ with our current proposal in a total time of flight of roughly 1.4 s. In Ref. [53], we had obtained $\Delta Z = 0.11 \mu m$ for the same time period.

When discussing the scaling behavior of the superposition size by numerically fitting the velocity difference in stage I and the trajectory of the wave packet in stage II, we should make the following points.

(i) The maximum velocity difference achieved in stage I is determined by the gradient parameter $\eta$ and the initial position of the nanocrystal in stage I. The greater the value of $\eta$ and the initial position, the greater the maximum velocity difference. The fitting formula for the velocity difference in Eq. (8) only holds for $\eta = 1 \times 10^8 \text{ T/m}^2$, initial position = $100 \mu m$.

(ii) Time $T_1$ in Eq. (8) is a variable, with an upper bound determined by both the maximum velocity difference and the mass of the nanocrystal. In this paper, the maximum velocity difference that can be achieved in stage I is about $1.4 \times 10^5 \mu m/s$. Therefore the upper bound on $T_1$ is about 2.6, 26, and 260 s for masses equal to $10^{-17}$, $10^{-16}$, and $10^{-15} \text{ kg}$, respectively.

(iii) As can be seen from Eq. (10), by decreasing the value of $\sqrt{A}$, we can get a larger superposition size, but correspondingly, we also need a longer time to close the wave packet trajectory.

(iv) In Eq. (8), the reason for using the velocity difference obtained in stage I to calculate the amplitude directly is that the velocity of the two wave packets corresponding to the linear fitting velocity difference in stage I is almost the same with the direction being opposite. The amplitude calculated from the velocity difference is equal to the sum of the amplitudes of the two wave packets and gives the maximum superposition size.

V. RECOVERING SPIN COHERENCE

When discussing the scaling behavior of the superposition size by numerically fitting the velocity difference in stage I and the trajectory of the wave packet in stage II, we should make the following points.

(i) The maximum velocity difference achieved in stage I is determined by the gradient parameter $\eta$ and the initial position of the nanocrystal in stage I. The greater the value of $\eta$ and the initial position, the greater the maximum velocity difference. The fitting formula for the velocity difference in Eq. (8) only holds for $\eta = 1 \times 10^8 \text{ T/m}^2$, initial position = $100 \mu m$.

(ii) Time $T_1$ in Eq. (8) is a variable, with an upper bound determined by both the maximum velocity difference and the mass of the nanocrystal. In this paper, the maximum velocity difference that can be achieved in stage I is about $1.4 \times 10^5 \mu m/s$. Therefore the upper bound on $T_1$ is about 2.6, 26, and 260 s for masses equal to $10^{-17}$, $10^{-16}$, and $10^{-15} \text{ kg}$, respectively.

(iii) As can be seen from Eq. (10), by decreasing the value of $\sqrt{A}$, we can get a larger superposition size, but correspondingly, we also need a longer time to close the wave packet trajectory.

(iv) In Eq. (8), the reason for using the velocity difference obtained in stage I to calculate the amplitude directly is that the velocity of the two wave packets corresponding to the linear fitting velocity difference in stage I is almost the same with the direction being opposite. The amplitude calculated from the velocity difference is equal to the sum of the amplitudes of the two wave packets and gives the maximum superposition size.
Eq. (11), we can get the equation of motion for the position,

\[
\frac{d\hat{r}(t)}{dt} = \frac{1}{i\hbar} [\hat{r}(t), \hat{H}_t(t)] = \frac{\hat{p}(t)}{m},
\]

and the equation of motion for the momentum,

\[
\frac{d\hat{p}(t)}{dt} = \frac{1}{i\hbar} [\hat{p}(t), \hat{H}_t(t)]
= \frac{\hbar m}{2\mu_0} \nabla (\mathbf{B}[\mathbf{r}(t)])^2 + \nabla (\mathbf{\mu}(t) \cdot \mathbf{B}[\mathbf{r}(t)]).
\]

By integrating Eqs. (12) and (13), the formal solution of the evolution of the position and the momentum with time can be written as [59]

\[
p(t) = p_0 + \int_0^t \left( \frac{\hbar m}{2\mu_0} \nabla (\mathbf{B}[\mathbf{r}(t')]^2) - \nabla (\mathbf{\mu} \mathbf{\sigma} \cdot \mathbf{B}[\mathbf{r}(t')]) \hat{a}(t') \hat{a}(t')^\dagger \right) dt',
\]

\[
r(t) = r_0 + \frac{t}{m} p_0 + \frac{1}{m} \int_0^t (t - t') \left( \frac{\hbar m}{2\mu_0} \nabla (\mathbf{B}[\mathbf{r}(t')]^2) - \nabla (\mathbf{\mu} \mathbf{\sigma} \cdot \mathbf{B}[\mathbf{r}(t')]) \hat{a}(t') \hat{a}(t')^\dagger \right) dt',
\]

where \(r_0\) and \(p_0\) are the initial position and the momentum, respectively, \(\mathbf{\sigma}\) is the spin operator, and \(\hat{a}(t)\) and \(\hat{a}(t')\) are creation and annihilation operators with \(S(t) = \mathbf{\sigma} \hat{a}(t) \hat{a}(t')\). Since the beams’ trajectories are split by an inhomogeneous magnetic field along the \(z\) direction, the wave packet motion along the \(z\) direction is studied next. With a bit of rearrangement, Eqs. (14) and (15) become

\[
p_z(t) - p_z = \Delta p_z,
\]

\[
z(t) - z_0 - \frac{t}{m} p_z = \Delta z,
\]

where

\[
\Delta p_z = \int_0^t \left( \frac{\hbar m}{2\mu_0} \frac{\partial}{\partial z} B_z(t')^2 - \mathbf{\mu} \mathbf{\sigma}_z \frac{\partial}{\partial z} B_z(t') \right) dt',
\]

\[
\Delta z = \frac{1}{m} \int_0^t (t - t') \left( \frac{\hbar m}{2\mu_0} \frac{\partial}{\partial z} B_z(t')^2 - \mathbf{\mu} \mathbf{\sigma}_z \frac{\partial}{\partial z} B_z(t') \right) dt'
\]

are the variations in the \(z\) component of the position and the momentum. \(p_z\) and \(p_z\) are the initial \(z\) component of the position and the momentum, respectively. We take into account that the wave function of a massive particle localized in the position space at \(t = 0\) is a Gaussian wave packet

\[
\psi(z,0) = \left( \frac{1}{2\pi \delta z^2} \right)^{1/4} e^{-\frac{z^2}{2\delta z^2}},
\]

with a minimum uncertainty \(\delta z \delta p_z = \hbar/2\). In Appendix B, we study the evolution of the wave packet in the presence of the nonlinear magnetic field. We show that the expected value of the position of each arm of the interferometer coincides with the classical trajectories in Eq. (5).
the first-order term gives
\[ \langle \delta z(t) \rangle \lesssim 1 - \frac{1}{2} \left( \left| \frac{\delta \Phi(t)}{\delta z} \right|^2 + \left| \frac{\Delta z(t)}{\delta \zeta} \right|^2 \right) \lesssim 1 - \epsilon^2. \] (24)

Here, we assumed \( \epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon \) for simplicity. When \( \epsilon = 0.1 \), the confidence level for spin coherence is 99%. Next, we need to make an approximation for Eq. (5), so that we can analytically solve the equation of motion, which is convenient for estimating the trajectory deviation caused by the imprecision of magnetic field control.

We model Eq. (5) with the following acceleration expression:
\[ \frac{d^2z(t)}{dt^2} = a_z = \left( C_{\text{corr}} \frac{\kappa m}{\mu_0} B_0 - S_z \frac{g \hbar}{m m_e} \right) \eta, \] (25)

where \( C_{\text{corr}} \) is a dimensionless correction factor. The value of this correction factor is related to \( \eta \). When \( \eta \) is taking different values, we need to adjust the correction factor to make the approximate trajectory as close to the exact trajectory as possible.

In this paper, we have considered the masses \( m \sim 10^{-17}, 10^{-16}, \) and \( 10^{-15} \) kg; substituting the values of other physical quantities into Eq. (25), we can find that the coefficient \( C_{\text{corr}} \frac{\kappa m}{\mu_0} B_0 - S_z \frac{g \hbar}{m m_e} < 0 \), which gives rise to the following harmonic oscillator equation with a solution of Eq. (25):
\[ z(t) = z_0 \cos(\sqrt{A}t), \] (26)

where \( A \) is the square of frequency
\[ A = \left( C_{\text{corr}} \frac{\kappa m}{\mu_0} B_0 - S_z \frac{g \hbar}{m m_e} \right) \eta > 0 \] (27)

and \( z_0 \) is the amplitude. Here, we select the value of \( \eta \) corresponding to the second stage of the experiment to calculate the minimum accuracy required to control the magnetic field. This is because the fluctuation in the magnetic field is inversely proportional to \( A \). The greater the value of \( A \), the higher the accuracy requirements for the magnetic field control. Also, the value of \( A \) is proportional to \( \eta \) [Eq. (27)]. The larger

| Mass (kg) | \( S_z \) | \( \frac{\Delta x}{x} \) | \( \frac{\Delta z}{z} \) |
|----------|----------|----------------|----------------|
| \( 10^{-17} \) | 1        | \( \lesssim 3.7 \times 10^{-4} \) | \( \lesssim 6.9 \times 10^{-8} \) |
|          | -1       | \( \lesssim 3.8 \times 10^{-4} \) | \( \lesssim 7.1 \times 10^{-8} \) |
| \( 10^{-16} \) | 1        | \( \lesssim 2.1 \times 10^{-4} \) | \( \lesssim 2.2 \times 10^{-8} \) |
|          | -1       | \( \lesssim 2.1 \times 10^{-4} \) | \( \lesssim 2.2 \times 10^{-8} \) |
| \( 10^{-15} \) | 1        | \( \lesssim 1.2 \times 10^{-4} \) | \( \lesssim 6.9 \times 10^{-9} \) |
|          | -1       | \( \lesssim 1.2 \times 10^{-4} \) | \( \lesssim 7.0 \times 10^{-9} \) |

FIG. 8. The deviation between the approximate trajectory and the exact trajectory. Here, \( m = 10^{-17} \) kg, \( \eta = 1 \times 10^6 \) T/m, and \( C_{\text{corr}} = 27.3467 \).

FIG. 9. Comparison of approximate and exact trajectories of two wave packets. Here, \( m = 10^{-17} \) kg, \( \eta = 1 \times 10^6 \) T/m, \( C_{\text{corr}} = 2526.82 \), and \( p_z = 0 \).
the deviation of the trajectory can be expressed as due to an inaccuracy in the control of the magnetic field. Then expressions (37) and (38) below.

For \( m = 10^{-17} \) kg (see Fig. 4), the value of \( \eta = 1 \times 10^6 \) T/m² at stage II, and the corresponding correction factor \( C_{\text{correct}} = 27.3467 \). The corrected approximate trajectory is compared with the exact trajectory as shown in Fig. 7 [72].

We can compare the approximated trajectory with the exact trajectory (see Fig. 7 for \( \eta = 1 \times 10^6 \) T/m² at stage II, and the corresponding correction factor \( C_{\text{correct}} = 27.3467 \). The corrected approximate trajectory is compared with the exact trajectory as shown in Fig. 7 [72].

With the abovementioned approximations, we can now address the question of spin coherence and any fluctuations within 4 s is less than \( 10^6 \) T/m².

Eqs. (23) and (27)–(29), we can get

\[
\left( \frac{\Delta p_z}{\eta} \right)_z \leq \frac{2 \sqrt{\epsilon} \delta t}{\Delta z_{\text{corr}} \delta z},
\]

where the subscript \( z \) represents the accuracy required to obtain from the positional uncertainty.

Similarly, by using the last inequality about the momentum in Eq. (23), we can also obtain a requirement for the accuracy of the magnetic field. The momentum can be obtained by taking the derivative of Eq. (26):

\[
p_z(t) = m \frac{dz}{dt} = -mz_0 \sqrt{A} \sin(\sqrt{A}t).
\]

Since the initial momentum is zero, we can directly express the deviation of momentum as

\[
\Delta p_z(t) = -mz_0 \sqrt{A} \Delta z \sin(\sqrt{A}t) - m \left[ \Delta \frac{\Delta z}{\Delta A} \right] \sin(\sqrt{A}t).
\]

\[
\leq \frac{mz_0 \sqrt{A} \Delta z}{2 \sqrt{A}} \sin \left( \frac{\Delta \sqrt{A}}{2 \sqrt{A}} t \right) \leq \frac{1}{4} \frac{mz_0 t}{A} (\Delta A + A)^2 - \frac{1}{4} mz_0 A.
\]

\[
\frac{mz_0 t}{A} (\Delta A + A)^2 - \frac{1}{4} mz_0 A.
\]
The two trajectories coincide. Here, the center’s position and the classical trajectory of the NV center.

In Table I we have used \( \varepsilon = 0.1 \), which corresponds to recovering 99% spin coherence; \( z_0 = 1 \times 10^{-4} \) m, which is the initial center position of the wave packet; \( t = \frac{2\pi}{\Delta F} \approx 0.7 \) s, which is the duration of experimental stage; and \( \eta = 1 \times 10^6 \) T/m², which is the gradient parameter.

**VI. CONCLUSION**

In this paper, we have provided a simple mechanism for creating a large spatial superposition with heavy masses and with embedded spin. We have shown that it is possible to achieve \( \Delta z \sim O(10^3) \) \( \) \( \mu m \) for \( m = 10^{-17} \) kg, \( \Delta z \sim O(10^2) \) \( \mu m \) for \( m = 10^{-16} \) kg, and \( \Delta z \sim O(10^1) \) \( \mu m \) for \( m = 10^{-15} \) kg within \( \sim 1.4 \) s. There is indeed an order-of-magnitude gain in the splitting of the wave function compared with our earlier proposal [53], where we had taken only the gradient term in the magnetic field and could not achieve such a large spatial superposition in a short time scale (within 1–1.5 s). In this regard, catapulting the trajectory of the two wave packets has yielded a better result with a magnetic field gradient of order \( O(10^0–10^1) \) T/m.

We highlighted that there are primarily three stages of the trajectory. First, we create a large velocity difference between the two wave packets, which experience differential spin-dependent forces. The anharmonic oscillations gradually increase the amplitude, and when the two trajectories meet at \( z = 0 \), their velocity difference is large, and the trajectories catapult to achieve a large spatial splitting. We employ three different values of the \( \eta \) parameter which controls the magnetic field gradient; see Fig. 4 and Appendix C, Figs. 15 and 16. We have ensured that the interference is completed within \( O(1–1.5) \) s, where the wave function overlap is such that the position and the momentum match to interfere with the two paths. We have also analyzed the conditions required to maintain the spin coherence. To achieve 99% coherence, we have obtained the stringent bound on the magnetic field fluctuations. The most stringent condition on the fluctuation in the magnetic field arises from stage I (see Appendix A, Table II), and similarly for stage II (see Table I). We have also analyzed the spreading of the wave function and showed that the wave packets evolve and do not satisfy the minimum uncertainty principle throughout the trajectory at every moment but that the largest \( \Delta z \delta \rho \lesssim 4\hbar \) (this restriction only holds when the initial conditions are \( \Delta z \sim 5 \times 10^{-3} \) \( \mu m \) and the initial position \( z \sim 5 \times 10^{-2} \) \( \mu m \)) and it oscillates with a period of roughly 0.5 s where it satisfies the minimum uncertainty principle for \( m = 10^{-17} \) kg and for \( \eta = 10^6 \) T/m². However, the wave function’s classical and quantum trajectories match extremely well; see Appendix B, Fig. 14.

Indeed, in all of our analysis, the time duration of the spin coherence is an important factor for the experiment, but the spin coherence times are perpetually rising (approaching 1 s [69,70], even 30 s [74,75]); adapting these to nanocrystals remains an open problem, but there are no fundamental constraints [76]. The spatial coherence times can be made 100 s; see Refs. [7,21,23]. There are indeed other challenges, but the requisite pressures, temperatures, distances from other sources, and fluctuations are achievable [23]. For example, a decoherence rate below 0.1 Hz is achievable for diamond.
spheres of masses $10^{-14}$ kg. This is expected [7,23] for internal temperatures of 0.15 K, an environmental temperature of 1 K, and the environmental gas number density of $10^{-8}$ m$^{-3}$. In addition to these, we will need to take into account the effect of the rotation of the diamond [65] and the excitation of phonons [77] on the spin coherence. However, we will study these effects separately.

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FIG. 15. Dynamical aspects for the mass $m = 10^{-16}$ kg during the three experimental stages: (a)–(c) the magnetic field coordinates (potential coordinates) experienced, (d)–(f) the velocities, (g)–(i) the velocity differences, and (j)–(l) the superposition size. We set different values of $\eta$ and the initial position of the wave packet in the magnetic field at different stages. Stage I, $\eta = 1 \times 10^8$ T/m$^2$, with initial coordinate $z = 100 \mu$m. Stage II, $\eta = 9 \times 10^5$ T/m$^2$, with initial coordinate $z = 0 \mu$m. Stage III, $\eta = 3.445 \times 10^7$ T/m$^2$, with initial coordinate $z = -103.7 \mu$m. Times $T_1$ and $T_2$ are determined by constraining the moment when the superposition size is zero (with an accuracy of $z = 10^{-6} \mu$m). Time $T_3$ is the moment when both the velocity difference between the two wave packets and the superposition size are zero.
FIG. 16. Dynamics for the mass \( m = 10^{-15} \) kg during the three experimental stages: (a)–(c) the magnetic field coordinates (potential coordinates) experienced, (d)–(f) the velocities, (g)–(i) the velocity differences, and (j)–(l) the superposition size. We set different values of \( \eta \) and the initial position of the wave packet in the magnetic field at different stages. Stage I, \( \eta = 1 \times 10^8 \) T/m², corresponding to the coordinate \( z = 100 \) µm. Stage II, \( \eta = 6 \times 10^6 \) T/m², corresponding to the initial coordinate \( z = 0 \) µm. Stage III, \( \eta = 1 \times 10^8 \) T/m², with an initial coordinate \( z = -30 \) µm. Times \( T_1 \) and \( T_2 \) are determined by constraining the moment when the superposition size is zero (with an accuracy of \( z = 10^{-6} \) µm). Time \( T_3 \) is the moment when both the velocity difference between the two wave packets and the superposition size are zero.
corresponding values of \( \eta \) and the correction factor. Now we set \( \eta = 1 \times 10^8 \) T/m\(^2\) and the corresponding correction factor \( C_{\text{correct}} = 252.682 \). Using Eqs. (5) and (26), we can compare the approximate and the exact trajectories of the wave packets as shown in Fig. 9.

Figure 10 shows the deviation of the approximate trajectory from the exact trajectory. We can see that the maximum deviation between trajectories within 1 s is less than 9 \( \mu \)m.

Using Eqs. (37) and (38), we can get \( \delta \eta/\eta \) in the magnetic field fluctuation for different masses, as shown in Table II.

In Table II we have used \( \varepsilon = 0.1 \), which corresponds to maintaining 99\% spin coherence; \( z_0 = 1 \times 10^{-4} \) m, which is the initial center position of the wave packet; \( t = \frac{z_0}{c} \approx 0.2 \) s, which is the duration of the experimental stage; and \( \eta = 1 \times 10^8 \) T/m\(^2\), which is the gradient parameter.

**APPENDIX B: SPREADING OF THE WAVE PACKET**

In Sec. V, we assumed that the wave packet is always kept to a minimum uncertainty when calculating the accuracy required of the magnetic field control to restore spin coherence. In this Appendix, we study the evolution of the wave packet width in the quartic potentials, providing a theoretical basis for our hypothesis in Sec. V.

The Schrödinger equation is

\[
\frac{i\hbar}{d t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle. \tag{B1}
\]

By substituting the specific forms of the Hamiltonian [Eq. (1)] and magnetic field [Eq. (4)] into the Schrödinger equation and making the initial state the Gaussian wave packet shown in Eq. (20), we can use the Trotter expansion method to numerically calculate the evolution of the wave packet [78] as shown in Fig. 11.

It can be seen from Fig. 11 that the spreading of the wave packet in the quartic potential exhibits a periodic oscillation behavior. For the purpose of illustration, we will take a single value of \( \eta = 10^6 \) T/m\(^2\) for mass \( m = 10^{-17} \) kg. A similar analysis can be performed for different values of \( \eta \), but the physical properties will not alter much.

**APPENDIX C: TRAJECTORIES FOR MASSES 10\(^{-16}\) AND 10\(^{-15}\) kg**

The time evolution of the nanocrystals for different masses in a nonlinear magnetic field shows a very similar pattern to that of \( m = 10^{-17} \) kg. The main difference between these evolutions is that the maximum velocity difference between the wave packets with the opposite spin orientations is inversely proportional to the mass for the same magnetic field gradient parameter \( \eta \) (Fig. 6). The difference in velocity between the wave packets then determines the superposition size that we can obtain in the same time span. The magnetic field used to control the motion of the wave packets in the second and third stages will change accordingly for different masses and the difference in velocity between the wave packets at the end of the first stage. We have shown the numerical results of specific parameters and the evolution of the nanocrystals in Figs. 15 and 16. An important point to note is that for the heaviest mass \( m = 10^{-15} \) kg we can obtain the spatial superposition size of 15 \( \mu \)m, which is the required value we require for testing the quantum nature of gravity in a laboratory by including the Casimir screening; see Ref. [23]. The simple scaling of the superposition size \( \Delta Z_0 \) is given by Eq. (10).

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[67] The actual magnetic field gradient is small, as $\partial B/\partial z \approx \eta z$ and $z \ll 1$ m. For $z \sim 100 \mu m$, the maximum magnetic field gradient will then be $\partial B/\partial z \approx \eta z \approx 10^4 T/m$. These values of the magnetic field gradient can be achievable in a laboratory [47,68].

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[73] See Appendix B, where we analyze the spread in the wave packet: Figure 11 shows the evolution of the probability density of the wave packet for one of the spins for $m = 10^{-17}$ kg for $\eta = 10^6 T/m^2$. The numerical values will not alter much for different values of $\eta$. Note that the expectation value of the wave packet position with the classical trajectory is shown in Fig. 14. We can see from Fig. 13 that the minimum uncertainty is not always followed throughout the trajectory but it is satisfied at certain times. If we could manage to close the interference at those moments, we would be able to recover the spin coherence as desired in the text.

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