Soft Topology on Function Spaces

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Abstract

Molodtsov initiated the concept of soft sets in [16]. Maji et al. defined some operations on soft sets in [14]. The concept of soft topological space was introduced by some authors. In this paper, we introduce the concept of the pointwise topology of soft topological spaces and the properties of soft mappings spaces. Finally, we investigate the relationships between some soft mappings spaces.

Key Words and Phrases. soft set, soft point, soft topological space, soft continuous mapping, soft mappings spaces, soft pointwise topology.

1. INTRODUCTION

Many practical problems in economics, engineering, environment, social science, medical science etc. cannot be dealt with by classical methods, because classical methods have inherent difficulties. The reason for these difficulties may be due to the inadequacy of the theories of parameterization tools. Molodtsov [16] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Maji et al. [13], [14] research deal with operations over soft set. The algebraic structure of set theories dealing with uncertainties is an important problem. Many researchers have contributed towards the algebraic structure of soft set theory. Aktaş and Çağman [2] defined soft groups and derived their basic properties. U. Acar et al. [1] introduced initial concepts of soft rings. F. Feng et al. [8] defined soft semirings and several related notions to establish a connection between soft sets and semirings. M. Shabir et al. [20] studied soft ideals over a semigroup. Qiu Mei Sun et al. [23] defined soft modules and investigated their basic properties. C. Gunduz(Aras) and S. Bayramov [9], [10] introduced fuzzy soft modules and intuitionistic fuzzy soft modules and investigated some basic properties. T. Y. Ozturk and S. Bayramov defined chain complexes of soft modules and their soft homology modules. T. Y. Ozturk et al. introduced the concept of inverse and direct systems in the category of soft modules.

Recently, Shabir and Naz [21] initiated the study of soft topological spaces. Theoretical studies of soft topological spaces have also been by some authors in [22], [3], [25], [13], [6], [12]. In the study [5] were given different soft point concepts from the studies [22], [3], [25], [13], [6], [12]. In this study, soft point concepts in the study [5] is used.

In the present study, the pointwise topology is defined in soft continuous mappings space and the properties of soft mappings spaces is investigated. Subsequently, the relations is given between on some soft mappings spaces.
2. PRELIMINARY

In this section we will introduce necessary definitions and theorems for soft sets. Molodtsov [14] defined the soft set in the following way. Let \( X \) be an initial universe set and \( E \) be a set of parameters. Let \( P(X) \) denotes the power set of \( X \) and \( A \subseteq E \).

**Definition 1.** [14] A pair \((F,A)\) is called a soft set over \( X \), where \( F \) is a mapping given by \( F : A \rightarrow P(X) \).

In other words, the soft set is a parameterized family of subsets of the set \( X \). For \( e \in A \), \( F(e) \) may be considered as the set of \( \varepsilon \)-elements of the soft set \((F,A)\), or as the set of \( \varepsilon \)-approximate elements of the soft set.

**Definition 2.** [14] For two soft sets \((F,A)\) and \((G,B)\) over \( X \), \((F,A)\) is called soft subset of \((G,B)\) if

1. \( A \subseteq B \) and
2. \( \forall e \in A \), \( F(e) \) and \( G(e) \) are identical approximations.

This relationship is denoted by \((F,A) \subseteq (G,B)\). Similarly, \((F,A)\) is called a soft superset of \((G,B)\) if \((G,B)\) is a soft subset of \((F,A)\). This relationship is denoted by \((F,A) \supseteq (G,B)\). Two soft sets \((F,A)\) and \((G,B)\) over \( X \) are called soft equal if \((F,A)\) is a soft subset of \((G,B)\), and \((G,B)\) is a soft subset of \((F,A)\).

**Definition 3.** [14] The intersection of two soft sets \((F,A)\) and \((G,B)\) over \( X \) is the soft set \((H,C)\), where \( C = A \cap B \) and \( \forall e \in C \), \( H(e) = F(e) \cap G(e) \). This is denoted by \((F,A) \cap (G,B) = (H,C)\).

**Definition 4.** [14] The union of two soft sets \((F,A)\) and \((G,B)\) over \( X \) is the soft set, where \( C = A \cup B \) and \( \forall e \in C \),

\[
H(e) = \begin{cases} 
F(e), & \text{if } e \in A - B, \\
G(e), & \text{if } e \in B - A, \\
F(e) \cup G(e), & \text{if } e \in A \cup B.
\end{cases}
\]

This relationship is denoted by \((F,A) \cup (G,B) = (H,C)\).

**Definition 5.** [14] A soft set \((F,A)\) over \( X \) is said to be a NULL soft set denoted by \( \Phi \) if for all \( e \in A \), \( F(e) = \emptyset \) (null set).

**Definition 6.** [14] A soft set \((F,A)\) over \( X \) is said to be an absolute soft set denoted by \( \tilde{X} \) if for all \( e \in A \), \( F(e) = X \) (null set).

**Definition 7.** [21] The difference \((H,E)\) of two soft sets \((F,E)\) and \((G,E)\) over \( X \), denoted by \((F,E) \setminus (G,E)\), is defined as \( H(e) = F(e) \cap G(e) \) for all \( e \in E \).

**Definition 8.** [21] Let \( Y \) be a non-empty subset of \( X \), then \( \tilde{Y} \) denotes the soft set \((Y,E)\) over \( X \) for which \( Y(e) = Y \) for all \( e \in E \).

In particular, \((X,E)\) will be denoted by \( \tilde{X} \).

**Definition 9.** [21] Let \((F,E)\) be a soft set over \( X \) and \( Y \) be a non-empty subset of \( X \). Then the sub soft set of \((F,E)\) over \( Y \) denoted by \((Y F,E)\), is defined as follows \( Y F(e) = Y \cap F(e) \), for all \( e \in E \). In other words \((Y F,E) = \tilde{Y} \cap (F,E)\).
Definition 10. Let \((F, A)\) and \((G, B)\) be two soft sets over \(X_1\) and \(X_2\), respectively. The cartesian product \((F, A) \times (G, B)\) is defined by \((F \times G)_{(A \times B)} (e, k) = (F, A)(e) \times (G, B)(k), \forall (e, k) \in A \times B\).

According to this definition, the soft set \((F, A) \times (G, B)\) is soft set over \(X_1 \times X_2\) and its parameter universe is \(E_1 \times E_2\).

Definition 11. Let \((F_1, E_1)\) and \((F_2, E_2)\) be two soft sets over \(X_1\) and \(X_2\), respectively and \(p_i : X_1 \times X_2 \to X_i, q_i : E_1 \times E_2 \to E_i\) be projection mappings in classical meaning. Then the soft mappings \((p_i, q_i), i \in \{1, 2\},\) is called soft projection mapping from \(X_1 \times X_2\) to \(X_i\) and defined by

\[
(p_i, q_i)((F_1, E_1) \times (F_2, E_2)) = (p_i, q_i)((F_1 \times F_2), (E_1 \times E_2)) = p_i(F_1 \times F_2), q_i(E_1 \times E_2) = (F, E)_i.
\]

Definition 12. Let \(\tau\) be the collection of soft set over \(X\), then \(\tau\) is said to be a soft topology on \(X\) if

1) \(\Phi, X\) belongs to \(\tau\);
2) the union of any number of soft sets in \(\tau\) belongs to \(\tau\);
3) the intersection of any two soft sets in \(\tau\) belongs to \(\tau\).

The triplet \((X, \tau, E)\) is called a soft topological space over \(X\).

Definition 13. Let \((X, \tau, E)\) be a soft topological space over \(X\), then members of \(\tau\) are said to be soft open sets in \(X\).

Definition 14. Let \((X, \tau, E)\) be a soft topological space over \(X\). A soft set \((F, E)\) over \(X\) is said to be a soft closed in \(X\), if its relative complement \((F, E)'\) belongs to \(\tau\).

Proposition 1. Let \((X, \tau, E)\) be a soft topological space over \(X\). Then the collection \(\tau_e = \{F(e) : (F, E) \in \tau\}\) for each \(e \in E\), defines a topology on \(X\).

Definition 15. Let \((X, \tau, E)\) be a soft topological space over \(X\) and \((F, E)\) be a soft set over \(X\). Then the soft closure of \((F, E)\), denoted by \(\overline{(F, E)}\), is the intersection of all soft closed super sets of \((F, E)\). Clearly \(\overline{(F, E)}\) is the smallest soft closed set over \(X\) which contains \((F, E)\).

Definition 16. Let \(x \in X\), then \((x, E)\) denotes the soft set over \(X\) for which \(x(e) = \{x\}\) for all \(e \in E\).

Definition 17. Let \((F, E)\) be a soft set over \(X\). The soft set \((F, E)\) is called a soft point, denoted by \(x_e, E\), if for the element \(e \in E\), \(F(e) = \{x\}\) and \(F(e') = \emptyset\) for all \(e' \in E - \{e\}\).

Definition 18. Two soft points \((x_e, E)\) and \((y_{e'}, E)\) over a common universe \(X\), we say that the points are different points if \(x \neq y\) or \(e \neq e'\).

Definition 19. Let \((X, \tau, E)\) be a soft topological space over \(X\). A soft set \((F, E)\) in \((X, \tau, E)\) is called a soft neighborhood of the soft point \((x_e, E) \in (F, E)\) if there exists a soft open set \((G, E)\) such that \((x_e, E) \in (G, E) \subset (F, E)\).
Definition 20. Let \((X, \tau, E)\) and \((Y, \tau', E)\) be two soft topological spaces, \(f : (X, \tau, E) \to (Y, \tau', E)\) be a mapping. For each soft neighbourhood \((H, E)\) of \(f(x), E\), if there exists a soft neighbourhood \((F, E)\) of \((x, E)\) such that \(f((F, E)) \subset (H, E)\), then \(f\) is said to be soft continuous mapping at \((x, E)\).

If \(f\) is soft continuous mapping for all \((x, E)\), then \(f\) is called soft continuous mapping.

Definition 21. Let \((X, \tau, E)\) be a soft topological space over \(X\). A subcollection \(\beta\) of \(\tau\) is said to be a base for \(\tau\) if every member of \(\tau\) can be expressed as a union of members of \(\beta\).

Definition 22. Let \((X, \tau, E)\) be a soft topological space over \(X\). A subcollection \(\delta\) of \(\tau\) is said to be a subbase for \(\tau\) if the family of all finite intersections members of \(\delta\) forms a base for \(\tau\).

Definition 23. \(\{(\varphi_i, \psi_i) : (X, \tau, E) \to (Y_i, \tau_i, E_i)\}_{i \in \Delta}\) be a family of soft mappings and \(\{(Y_i, \tau_i, E_i)\}_{i \in \Delta}\) is a family of soft topological spaces. Then, the topology \(\tau\) generated from the subbase \(\delta = \{(\varphi_i, \psi_i)^{-1}_{i \in \Delta}(F, E) : (F, E) \in \tau_i, i \in \Delta\}\) is called the soft topology (or initial soft topology) induced by the family of soft mappings \(\{(\varphi_i, \psi_i)\}_{i \in \Delta}\).

Definition 24. \(\{(X_i, \tau_i, E_i)\}_{i \in \Delta}\) be a family of soft topological spaces. Then, the initial soft topology on \(X = \prod_{i \in \Delta} X_i\) generated by the family \(\{(p_i, q_i)\}_{i \in \Delta}\) is called product soft topology on \(X\). (Here, \((p_i, q_i)\) is the soft projection mapping from \(X\) to \(X_i\), \(i \in \Delta\).)

The product soft topology is denoted by \(\prod_{i \in \Delta} \tau_i\).

3. Soft Topology on Function Spaces

Let \(\{(X_s, \tau_s, E)\}_{s \in S}\) be a family of soft topological spaces over the same parameters set \(E\). We define a family of soft sets \(\prod_{s \in S} X_s, \tau, E\) as follows;

If \(F_s : E \to P(X_s)\) is a soft set over \(X_s\) for each \(s \in S\), then \(\prod_{s \in S} F_s : E \to P(\prod_{s \in S} X_s)\) is defined by \(\left(\prod_{s \in S} F_s\right)(e) = \prod_{s \in S} F_s(e)\). Let’s consider the topological product \(\prod_{s \in S} X_s, \prod_{s \in S} \tau_s, \prod_{s \in S} E_s\) of a family of soft topological spaces \(\{(X_s, \tau_s, E)\}_{s \in S}\).

We take the contraction to the diagonal \(\Delta \subset \prod_{s \in S} E_s\) of each soft set \(F : \prod_{s \in S} E_s \to P(\prod_{s \in S} X_s)\). Since there exist a bijection mapping between the diagonal \(\Delta\) and the parameters set \(E\), then the contractions of soft sets are soft sets over \(E\).

Now, let us define the topology on \(\prod_{s \in S} X_s, E\). Let \((p_{s_0}, 1_E) : \prod_{s \in S} X_s, \tau, E \to (X_{s_0}, \tau_{s_0}, E)\) be a projection mapping and the soft set \((p_{s_0}, 1_E)^{-1}(F_{s_0}, E)\) be for each \((F_{s_0}, E) \in \tau_{s_0}\). Then
\[ (p_{s_0}, 1_E)^{-1} (F_{s_0}, E) = (p_{s_0}^{-1}(F_{s_0}), E) = \left( F_{s_0} \times \prod_{s \neq s_0} \tilde{X}_s, E \right). \]

The topology generated from \[ \left\{ \left( F_{s_0} \times \prod_{s \neq s_0} \tilde{X}_s, E \right) : s_0 \in S, (F_{s_0}, E) \in \tau_{s_0} \right\} \]
a soft subbase and the soft topology is denoted by \( \tau = \prod_{s \in S} \tau_s. \)

**Definition 25.** The soft topological space \( \left( \prod_{s \in S} X_s, \tau, E \right) \) is called the product of family of the soft topological spaces \( \{(X_s, \tau_s, E)\}_{s \in S}. \)

It is clear that each projection mapping \((p_s, 1_E) : \left( \prod_{s \in S} X_s, \tau, E \right) \to (X_s, \tau_s, E)\)
is soft continuous. Additionally, the soft base of the soft topology \( \tau \) is formed by the soft sets

\[
\left\{ \left( F_{s_1} \times \prod_{s \neq s_1} \tilde{X}_s, E \right) \cap \ldots \cap \left( F_{s_n} \times \prod_{s \neq s_n} \tilde{X}_s, E \right) \right\} = \left( F_{s_1} \times \ldots \times F_{s_n} \times \prod_{s \neq s_1 \ldots s_n} \tilde{X}_s, E \right).
\]

Let \((X, \tau, E)\) be a soft topological space, \(\{(Y_s, \tau_s, E)\}_{s \in S}\) be a family of soft topological spaces and \(\{(f_s, 1_E) : (X, \tau, E) \to (Y_s, \tau_s, E)\}_{s \in S}\) be a family of soft mappings. For each soft point \(x_e \in (X, \tau, E)\), we define the soft map \(f = \triangle f_s : (X, \tau, E) \to \left( \prod_{s \in S} Y_s, \tau, E \right)\)
by \(f(x_e) = \{f_s(x_e)\}_{s \in S} = \{(f_s(x))_e\}_{s \in S}\). If \(f : (X, \tau, E) \to \left( \prod_{s \in S} Y_s, \tau, E \right)\) is any soft mapping, then \(f = \triangle f_s\) is satisfied for the family of soft mappings \(\{f_s = p_s \circ f : (X, \tau, E) \to (Y_s, \tau_s, E)\}_{s \in S}\).

**Theorem 1.** \(f : (X, \tau, E) \to \left( \prod_{s \in S} Y_s, \tau, E \right)\) is soft continuous if and only if \(f_s = p_s \circ f : (X, \tau, E) \to (Y_s, \tau_s, E)\) is soft continuous for each \(s \in S\).

**Proof.** \(\implies\) The proof is obvious.

\[ \iff \text{Let} \left( F_{s_1} \times \ldots \times F_{s_n} \times \prod_{s \neq s_1 \ldots s_n} \tilde{Y}_s, E \right) \text{be any soft base of product topology.} \]

\[
f^{-1} \left( F_{s_1} \times \ldots \times F_{s_n} \times \prod_{s \neq s_1 \ldots s_n} \tilde{Y}_s, E \right) = f^{-1} (p_{s_1}^{-1}(F_{s_1}) \cap \ldots \cap p_{s_n}^{-1}(F_{s_n}), E) = (f^{-1} p_{s_1}^{-1}(F_{s_1}) \cap \ldots \cap f^{-1} p_{s_n}^{-1}(F_{s_n}), E)\]
Since the soft mappings \( p_{s_1} \circ f, \ldots, p_{s_n} \circ f \) are soft continuous, the soft set
\[
(f^{-1}p_{s_1}^{-1}(F_{s_1}) \cap \ldots \cap f^{-1}p_{s_n}^{-1}(F_{s_n}), E)
\]
is soft open. Thus, \( f : (X, \tau, E) \to \left( \prod_{s \in S} Y_s, \tau, E \right) \) is soft continuous. \( \square \)

If \( \{f_s : (X_s, \tau_s, E) \to (Y_s, \tau'_s, E)\}_{s \in S} \) is a family of soft continuous mappings, then the soft mapping \( \prod_{s \in S} f_s : \left( \prod_{s \in S} X_s, \tau, E \right) \to \left( \prod_{s \in S} Y_s, \tau'_s, E \right) \) is soft continuous.

Now, let the family of soft topological spaces \( \{(X_s, \tau_s, E)\}_{s \in S} \) be disjoint, i.e., \( X_{s_1} \cap X_{s_2} = \emptyset \) for each \( s_1 \neq s_2 \). For the soft set \( F : E \to \bigcup_{s \in S} X_s \) over the set \( E \), define the soft set \( F|_{X_s} : E \to X_s \) by
\[
F|_{X_s}(e) = F(e) \cap X_s, \forall e \in E
\]
and the soft topology \( \tau \) define by
\[
(F, E) \in \tau \iff (F|_{X_s}, E) \in \tau_s.
\]

It is clear that \( \tau \) is a soft topology.

**Definition 26.** A soft topological space \( \left( \bigcup_{s \in S} X_s, \tau, E \right) \) is called soft topological sum of the family of soft topological spaces \( \{(X_s, \tau_s, E)\}_{s \in S} \) and denoted by \( \oplus \{ (X_s, \tau_s, E) \} \).

Let \( (i_s, 1_E) : (X, \tau, E) \to \oplus \{ (X_s, \tau_s, E) \} \) be an inclusion mapping for each \( s \in S \). Since
\[
(i_s, 1_E)^{-1} (F, E) = (F|_{X_s}, E) \in \tau_s, \text{ for } (F, E) \in \tau,
\]
\( (i_s, 1_E) \) is soft continuous.

Let \( \{(X_s, \tau_s, E)\}_{s \in S} \) be a family of soft topological spaces, \( (Y, \tau', E) \) be a soft topological space and \( \{f_s : (X_s, \tau_s, E) \to (Y, \tau', E)\}_{s \in S} \) be a family of soft mappings. We define the function \( f = \nabla_{s \in S} f_s : \oplus \{ (X_s, \tau_s, E) \} \to (Y, \tau', E) \) by \( f(x_e) = f_s(x_e) = (f_s(x))_e \), where each soft point \( x_e \in \oplus \{ (X_s, \tau_s, E) \} \) can belong to a unique soft topological space \( (X_{s_0}, \tau_{s_0}, E) \). If \( f : \oplus \{ (X_s, \tau_s, E) \} \to (Y, \tau', E) \) is any soft mappings, then \( \nabla_{s \in S} f_s = f \) is satisfied for the family of soft mappings
\[
\{f_s = f \circ i_s : (X_s, \tau_s, E) \to (Y, \tau', E)\}_{s \in S}.
\]

**Theorem 2.** The soft mapping \( f : \oplus_{s \in S} (X_s, \tau_s, E) \to (Y, \tau', E) \) is a soft continuous if and only if \( f_s = f \circ i_s : (X_s, \tau_s, E) \to (Y, \tau', E) \) are soft continuous for each \( s \in S \).

**Proof.** \( \implies \) The proof is obvious.

\( \iff \) Let \( (F, E) \in \tau' \) be a soft open set. The soft set \( f^{-1}(F, E) \) belongs to the soft topology \( \oplus \{ \tau_s \} \) if and only if the soft set \( \left( f^{-1}(F)|_{X_s}, E \right) \) belongs to \( \tau_s \). Since
Let \( \{ f_s : (X_s, \tau_s, E) \rightarrow (Y_s, \tau'_s, E) \}_{s \in S} \) be a family of soft continuous mappings. We define the mapping \( f = \bigoplus f_s : \bigoplus (X_s, \tau_s, E) \rightarrow \bigoplus (Y_s, \tau'_s, E) \) by \( f(x_e) = f_s(x_e) \) where each soft point \( x_e \in \bigoplus (X_s, \tau_s, E) \) belongs to \( (X_s, \tau_s, E) \). It is clear that if each \( f_s \) is soft continuous then \( f \) is also soft continuous.

**Theorem 3.** Let \( \{(X_s, \tau_s, E)\}_{s \in S} \) be a family of soft topological spaces. Then

\[
\left( \prod_{s \in S} X_s, \tau_e \right) = \prod_{s \in S} (X_s, \tau_{s_e}) \quad \text{and} \quad \left( \bigoplus_{s \in S} X_s, \tau_e \right) = \bigoplus_{s \in S} (X_s, \tau_{s_e})
\]

are satisfied for each \( e \in E \).

**Proof.** We should show that \( \tau_e = \prod_{s \in S} (\tau_s)_e \). Let us take any set \( U \) from \( \tau_e \). From the definition of the topology \( \tau_e \), there exist a soft open set

\[
(F_{s_1}, E) \times \cdots \times (F_{s_n}, E) \times \prod_{s \neq s_1 \ldots s_n} X_s
\]

such that the set \( U = (F_{s_1}(e) \times \cdots \times F_{s_n}(e) \times \prod_{s \neq s_1 \ldots s_n} X_s) \) belongs to the topology \( \prod_{s \in S} (\tau_s)_e \).

Conversely, let \( (U_{s_1} \times \cdots \times U_{s_n} \times \prod_{s \neq s_1 \ldots s_n} X_s) \in \prod_{s \in S} (\tau_s)_e \). Then from the definition of the topology \((\tau_s)_e\), there exist soft open sets \((F_{s_1}, E), \ldots, (F_{s_n}, E)\) such that \( F_{s_1}(e) = U_{s_1}, \ldots, F_{s_n}(e) = U_{s_n} \). Then

\[
U_{s_1} \times \cdots \times U_{s_n} \times \prod_{s \neq s_1 \ldots s_n} X_s = F_{s_1}(e) \times \cdots \times F_{s_n}(e) \times \prod_{s \neq s_1 \ldots s_n} X_s \in \tau_e.
\]

The topological sum can be proven in the same way. \( \square \)

Let \( (X, \tau, E) \) and \( (Y, \tau', E) \) be two soft topological spaces. \( Y^X \) is denoted the all soft continuous mappings from the soft topological space \((X, \tau, E)\) to the soft topological space \((Y, \tau', E)\), i. e.,

\[
Y^X = \left\{ (f, 1_E) : (X, \tau, E) \rightarrow (Y, \tau', E) \mid (f, 1_E) \text{ is a soft continuous map} \right\}.
\]

If \( (F, E) \) and \( (G, E) \) are two soft set over \( X \) and \( Y \), respectively then we define the soft set \( (G^F, E) \) over \( Y^X \) as follows;
\[ G^F(e) = \left\{ (f, 1_E) : (X, \tau, E) \to \left( Y, \tau', E \right) \mid f(F(e)) \subset G(e) \right\} \text{ for each } e \in E. \]

Now, let \( x_\alpha \in (X, \tau, E) \) be an any soft point. We define the soft mapping \( e_{x_\alpha} : (X^X, \tau_P, E) \to \left( Y, \tau', E \right) \) by \( e_{x_\alpha}(f) = f(x_\alpha) = (f(x))_\alpha \). This mapping is called an evaluation map. For the soft set \( (G, E) \) over \( Y \), \( e_{x_\alpha}^{-1}(G, E) = (G^x_\alpha, E) \) is satisfied. The soft topology that is generated from the soft sets \( \left\{ (G^x_\alpha, E) \mid (G, E) \in \tau' \right\} \) as a subbase is called pointwise soft topology and denoted by \( \tau_P \).

**Definition 27.** \((X^X, \tau_P, E) \) is called a pointwise soft function space (briefly PISFS).

**Remark 1.** The evaluation mapping \( e_{x_\alpha} : (X^X, \tau_P, E) \to \left( Y, \tau', E \right) \) is a soft continuous mapping for each soft point \( e_{x_\alpha} \in (X, \tau, E) \).

**Proposition 2.** A soft map \( g : (Z, \eta, E) \to (X^X, \tau_P, E) \), where \( (Z, \eta, E) \) is a soft topological space, is a soft continuous mapping if and only if the soft mapping \( e_{x_\alpha} \circ g : (Z, \eta, E) \to \left( Y, \tau', E \right) \) is a soft continuous mapping for each \( x_\alpha \in (X, \tau, E) \).

**Theorem 4.** If the soft topological space \( \left( Y, \tau', E \right) \) is a soft \( T_1 \)-space for each \( i = 0, 1, 2 \) then the soft space \( (X^X, \tau_P, E) \) is also a soft \( T_1 \)-space.

**Proof.** The soft points of the soft topological space \( (Y^X, \tau_P, E) \) denoted by \((f, E)\) i.e., if \( \beta \neq \alpha \) then \( f_\alpha(\beta) = \varnothing \) and if \( \beta = \alpha \) then \( f_\alpha(\beta) = f \). Now, let \( f_\alpha \neq g_\beta \) be two soft points. Then it should be \( f \neq g \) or \( \alpha \neq \beta \). If \( f = g \), then \( (f(x))_\alpha \neq (g(x))_\beta \) is satisfied. In both cases, \( (f(x_0))_\alpha \neq (g(x_0))_\beta \) is satisfied for at least one \( x_0 \in X \). Since \( \left( Y, \tau', E \right) \) is a soft \( T_1 \)-space, there exists soft open sets \( (F_1, E) \), \( (F_2, E) \) in \( \tau' \), where the condition of the soft \( T_1 \)-space is satisfied. Then the soft open sets \( (F_1^{x_\alpha}, E) = e_{x_\alpha}^{-1}(F_1, E) \) and \( (F_2^{x_\beta}, E) = e_{x_\beta}^{-1}(F_2, E) \) are neighbours of soft points \( f_\alpha \) and \( g_\beta \), respectively where the conditions of soft \( T_1 \)-space are satisfied for these neighbours. \( \square \)

Now, we construct relationships between some function spaces. Let \( \{ (X_\alpha, \tau_\alpha, E) \}_{\alpha \in S} \) be a family of pairwise disjoint soft topological spaces, \( \left( Y, \tau', E \right) \) be a soft topological spaces and \( \prod_{\alpha \in S} (X_\alpha, \tau_\alpha, E), \oplus (X_\alpha, \tau_\alpha, E) \) be a product and sum of soft topological spaces, respectively. Define

\[
\nabla : \prod_{\alpha \in S} (X_\alpha, \tau_\alpha, E) \to \left( \bigoplus_{\alpha \in S} X_\alpha, \left( \tau \bigoplus_{\alpha \in S} \right)_P, E \right)
\]

such that \( \forall \{(f_\alpha, 1_E)\} \in \prod_{\alpha \in S} X_\alpha^X, \forall x_\alpha \in \oplus (X_\alpha, \tau_\alpha, E), \nabla \{(f_\alpha)\} (x_\alpha) = f_{x_0} (x_\alpha) = (f_{x_0} (x))_\alpha \), where \( x_\alpha \) belongs to unique \( (X_\alpha, \tau_\alpha, E) \). We define the mapping
$$\nabla^{-1} : \left( Y_{s \in S} \otimes X_s, \left( \otimes_{\tau \in S} \right) \right) \rightarrow \prod_{s \in S} \left( Y^{X_s}, \tau_{s_p}, E \right)$$

by $\nabla^{-1}(f) = \{ f \circ i_s = f \mid_{X_s} : X_s \rightarrow Y \} \in \prod_{s \in S} \left( Y^{X_s}, \tau_{s_p}, E \right)$ for each $f : \oplus_{s \in S} X_s \rightarrow Y$. It is clear that the mapping $\nabla^{-1}$ is an inverse of the mapping $\nabla$.

**Theorem 5.** The mapping

$$\nabla : \prod_{s \in S} \left( Y^{X_s}, \tau_{s_p}, E \right) \rightarrow \left( Y_{s \in S} \otimes X_s, \left( \otimes_{\tau \in S} \right) \right), E$$

is a soft homeomorphism in the pointwise soft topology.

**Proof.** To prove the theorem, it is sufficient to show that the mappings $\nabla$ and $\nabla^{-1}$ are soft continuous. For this, we need to show that the soft set $\nabla^{-1} \left( e_{x_n}^{-1}(F, E) \right)$ is a soft open set, where each $e_{x_n}^{-1}(F, E)$ belongs to a soft subbase of the soft space

$$\left( Y_{s \in S} \otimes X_s, \left( \otimes_{\tau \in S} \right) \right), E$$.

$$e_{x_n}^{-1}(F, E) = \left\{ f : \oplus_{s \in S} X_s \rightarrow Y \mid f(x_n) \in (F, E) \right\} = \left\{ f_{s_0} : X_{s_0} \rightarrow Y \mid f_{s_0}(x_n) \in (F, E) \right\}.$$

Since

$$\nabla^{-1} \left( e_{x_n}^{-1}(F, E) \right) = \nabla^{-1} \left( \left\{ f_{s_0} : X_{s_0} \rightarrow Y \mid f_{s_0}(x_n) \in (F, E) \right\} \right)$$

$$= \left\{ f_{s_0} : X_{s_0} \rightarrow Y \mid f_{s_0}(x_n) \in (F, E) \right\} \times \prod_{s \neq s_0} \left( Y^{X_s}, \tau_{s_p}, E \right)$$

is the last soft set, $\nabla^{-1} \left( e_{x_n}^{-1}(F, E) \right)$ is a soft open set on the product space $\prod_{s \in S} \left( Y^{X_s}, \tau_{s_p}, E \right)$.

Now, we prove that the mapping $\nabla^{-1} : \left( Y_{s \in S} \otimes X_s, \left( \otimes_{\tau \in S} \right) \right), E \rightarrow \prod_{s \in S} \left( Y^{X_s}, \tau_{s_p}, E \right)$ is soft continuous. Indeed, for each the soft set $\left( e_{x_n}^{-1}(F, E) \right)_{s_0} \times \prod_{s \neq s_0} \left( Y^{X_s}, \tau_{s_p}, E \right)$ belongs to the subbase of the product space $\prod_{s \in S} \left( Y^{X_s}, \tau_{s_p}, E \right)$,

$$\left( e_{x_n}^{-1} \right)_{s_0}(F, E) \times \prod_{s \neq s_0} \left( Y^{X_s}, \tau_{s_p}, E \right) = \left\{ \{ f_s \} \in \prod_{s \in S} Y^{X_s} \mid f_{s_0}(x_n) \in (F, E) \right\}$$

is satisfied.

Since the set
\[(\nabla^{-1})^{-1}\left((e_{x_{\alpha}})^{-1}_{s_0} (F, E) \times \coprod_{s \neq s_0} (Y^{X_s}, \tau_{s_p}, E)\right) = \nabla \left((e_{x_{\alpha}})^{-1}_{s_0} (F, E) \times \coprod_{s \neq s_0} (Y^{X_s}, \tau_{s_p}, E)\right) = \left\{ \nabla f_s : f_{s_0}(x_{\alpha}) \in (F, E) \right\}\]

belongs to sub-base of the soft topological space \(\left(\bigoplus_{s \in S} X_s, \left(\tau \bigoplus_{s \in S}\right)_p, E\right)\), the mapping \(\nabla^{-1}\) is soft continuous. Thus, the mapping

\[\nabla : \coprod_{s \in S} (Y^{X_s}, \tau_{s_p}, E) \to \left(\bigoplus_{s \in S} X_s, \left(\tau \bigoplus_{s \in S}\right)_p, E\right)\]

is a soft homeomorphism. \(\square\)

Now, \((Y_s, \tau'_s, E)\) be the family of soft topological spaces, \((X, \tau, E)\) be a soft topological space. We define mapping

\[\Delta : \coprod_{s \in S} (Y^{X_s}, \tau_{s_p}, E) \to \left(\bigoplus_{s \in S} X_s, \left(\tau \bigoplus_{s \in S}\right)_p, E\right)\]

by the rule \(\forall \{f_s : X \to Y_s\} \in \coprod_{s \in S} (Y^{X_s}, \tau_{s_p}, E), \Delta \{f_s\} = \Delta f_s.\)

Let the mapping \(\Delta^{-1} = \left(\prod_{s \in S} X_s, \left(\tau \prod_{s \in S}\right)_p, E\right) \to \prod_{s \in S} (Y^{X_s}, \tau_{s_p}, E)\) be

\[\Delta^{-1}(f) = \{p_s \circ f = f_s : X_s \to Y\}\]

for each \(f \in \left(\prod_{s \in S} X_s, \left(\tau \prod_{s \in S}\right)_p, E\right).\) It is clear that the mapping \(\Delta^{-1}\) is an inverse of the mapping \(\Delta.\)

**Theorem 6.** The mapping

\[\Delta : \coprod_{s \in S} (Y^{X_s}, \tau_{s_p}, E) \to \left(\bigoplus_{s \in S} X_s, \left(\tau \bigoplus_{s \in S}\right)_p, E\right)\]

is a soft homeomorphism in the pointwise soft topology.

**Proof.** Since \(\Delta\) is bijective mapping, to prove the theorem it is sufficient to show that the mappings \(\Delta\) and \(\Delta^{-1}\) are soft open. First, we show that the mapping \(\Delta\) is soft open. Let us take an arbitrary soft set
We define the following mapping \( e_{x_{a_1}}^{-1} \) \( (F_{s_1}, E) \times ... \times (e_{x_{a_k}}^{-1})_{s_k} (F_{s_k}, E) \times \left( \prod_{s \neq s_1 ... s_k} Y_s \right)^X \)

belongs to the base of the product space \( \prod_{s \in S} (Y^{X_s}, \tau'_s, E) \). Since the soft set

\[
\Delta \left( e_{x_{a_1}}^{-1} (F_{s_1}, E) \times ... \times (e_{x_{a_k}}^{-1})_{s_k} (F_{s_k}, E) \times \left( \prod_{s \neq s_1 ... s_k} Y_s \right)^X \right)
\]

\[
= \{ \{ f_s \} \mid f_s (x_{a_1}^1) \in (F_{s_1}, E), ..., f_{s_k} (x_{a_k}^k) \in (F_{s_k}, E) \}
\]

\[
= (F_{s_1}^{x_{a_1}}, E) \times ... \times (F_{s_k}^{x_{a_k}}, E) \times \left( \prod_{s \neq s_1 ... s_k} Y_s \right)^X
\]

is soft open, \( \Delta \) is a soft open mapping. Similarly, it can be proven that \( \Delta^{-1} \) is soft open mapping. Indeed, for each soft open set \( e_{x_{a}}^{-1} \left( (F_{s_1}, E) \times ... \times (F_{s_k}, E) \times \prod_{s \neq s_1 ... s_k} Y_s \right) \) \( \in \left( \prod_{s \in S} Y_s \right)^X \left( \prod_{s \in S} \tau'_s, E \right) \),

\[
\Delta^{-1} \left( e_{x_{a}}^{-1} \left( (F_{s_1}, E) \times ... \times (F_{s_k}, E) \times \prod_{s \neq s_1 ... s_k} Y_s \right) \right)
\]

\[
= \Delta^{-1} \left( \left\{ f : X \to \prod_{s \in S} Y_s \mid f(x_{a}) \in (F_{s_1}, E) \times ... \times (F_{s_k}, E) \times \prod_{s \neq s_1 ... s_k} Y_s \right\} \right)
\]

\[
= \left\{ p_s \circ f \mid f(x_{a}) \in (F_{s_1}, E) \times ... \times (F_{s_k}, E) \times \prod_{s \neq s_1 ... s_k} Y_s \right\}
\]

\[
= \{ p_s \circ f \mid p_s \circ f(x_{a}) \in (F_{s_1}, E), ..., p_s \circ f(x_{a}) \in (F_{s_k}, E) \}.
\]

Hence this set is soft open and the theorem is proved. \( \square \)

Now, let \((X, \tau, E), (Y, \tau', E)\) and \((Z, \tau'', E)\) be soft topological spaces and \(f : (Z, \tau'', E) \times (X, \tau, E) \to (Y, \tau', E)\) be a soft mapping. Then the induced map \(\hat{f} : X \to Y^Z\) is defined by \(\hat{f}(x_{a})(z_{\beta}) = f(x_{a}, z_{\beta})\) for soft points \(x_{a} \in (X, \tau, E)\) and \(z_{\beta} \in (Z, \tau'', E)\). We define exponential law

\[
E : Y^Z \times X \to (Y^Z)^X
\]

by using induced maps \(E(f) = \hat{f}\) i.e., \(E(f)(x_{a})(z_{\beta}) = f(z_{\beta}, x_{a}) = \hat{f}(x_{a})(z_{\beta})\).

We define the following mapping

\[
E^{-1} : (Y^Z)^X \to Y^Z \times X
\]

which is an inverse mapping \(E\) as follows
\[ E^{-1}(\hat{f}) = f, \quad E^{-1}(\hat{f})(z_\beta, x_\alpha) = E^{-1}(\hat{f}(x_\alpha))(z_\beta) = f(z_\beta, x_\alpha). \]

Generally, in the pointwise topology for each soft continuous map \( g \), the mapping \( E^{-1}(g) \) need not to be soft continuous. Let us give the solution of this problem under the some conditions.

**Theorem 7.** Let \((X, \tau, E)\), \((Y, \tau', E)\) and \((Z, \tau'', E)\) be soft topological spaces and the mapping \( e : Y^Z \times X \rightarrow Z, e(f, z) = f(z) \) be soft continuous. Function space \( Y^X \) with pointwise soft topology and for each soft continuous mapping \( \hat{g} : X \rightarrow Y^Z \),

\[ E^{-1}(\hat{g}) : Z \times X \rightarrow Y \]

is also soft continuous.

**Proof.** By using the mapping

\[ 1_Z \times \hat{g} : Z \times Y \rightarrow Z \times Y^Z, \]

we take

\[ Z \times X \xrightarrow{1_Z \times \hat{g}} Z \times Y^Z \xrightarrow{t} Y^Z \times Z \xrightarrow{e} Y. \]

Hence \( e \circ t \circ (1_Z \times \hat{g}) \in Y^{Z \times X} \), where \( t \) denotes switching mapping. Let us apply exponential law \( E \) to \( e \circ t \circ (1_Z \times \hat{g}) \). For each soft point \( x_\alpha \in (X, \tau, E) \) and \( z_\beta \in (Z, \tau'', E) \),

\[
\begin{align*}
\left\{E\left(e \circ t \circ (1_Z \times \hat{g})\right)(x_\alpha)\right\}(z_\beta) &= \left(e \circ t \circ (1_Z \times \hat{g})\right)(z_\beta, x_\alpha) \\
&= e \circ t (z_\beta, \hat{g}(x_\alpha)) \\
&= e \left(\hat{g}(x_\alpha), z_\beta\right) \\
&= (\hat{g}(x_\alpha))(z_\beta).
\end{align*}
\]

Since \( E\left(e \circ t \circ (1_Z \times \hat{g})\right) = \hat{g} \), \( E^{-1}(\hat{g}) = e \circ t \circ (1_Z \times \hat{g}) \). Hence evaluation maps \( e \) and \( t \) are soft continuous, \( E^{-1}(\hat{g}) \) is soft continuous. \( \square \)

### 4. CONCLUSION

We hope that, the results of this study may help in the investigation of soft normed spaces and in many researches.

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