Consistency and Phenomenology of Four-Dimensional Strings

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In this talk we discuss string consistency requirements on four dimensional string models, namely the cancellation of target space duality anomalies. The analysis is explicitly performed for (hypothetical) orbifold models assuming the massless spectrum of the supersymmetric standard model. In addition, some phenomenological properties of four-dimensional strings, like the unification of the standard model gauge coupling constants and soft supersymmetry breaking parameters, are investigated.
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ABSTRACT

In this talk we discuss string consistency requirements on four dimensional string models, namely the cancellation of target space duality anomalies. The analysis is explicitly performed for (hypothetical) orbifold models assuming the massless spectrum of the supersymmetric standard model. In addition, some phenomenological properties of four-dimensional strings, like the unification of the standard model gauge coupling constants and soft supersymmetry breaking parameters, are investigated.

1. Introduction

Four-dimensional string theories are regarded as excellent candidates for unification of all interactions. However it is still an enormous challenge for string theories to make contact with low energy physics, i.e. with the phenomena around the weak scale. One difficulty which emerged after the discovery of the ten-dimensional heterotic string\(^1\) is the vast proliferation of consistent models in four dimensions. Some of the four-dimensional string models possess in fact very attractive phenomenological features as the standard model gauge group (plus some hidden gauge symmetry), three families of quarks and leptons (plus some extra vector-like states), computable, semi-realistic Yukawa couplings, etc. But unfortunately, there is so far not a single completely realistic model; in particular there is no model with the standard model gauge group, three families and no extra gauge non-singlet states.

The interactions of the massless string degrees of freedom are described by an effective field theory where one expands the relevant terms up to a certain power in the external momenta. The effective low-energy lagrangian has to obey many of the symmetry properties one knows in point particle field theory. In particular, the requirement of the absence of gauge and gravitational anomalies puts severe constraints on the form of the massless fermionic spectrum of the theory. In string theory, however, one expects that there exist much more symmetry than in point particle field theory due to the finite extension of the string. One known example of enlarged “stringy” symmetries are the target-space duality symmetries\(^2\) which describe the invariance of the string theory under the inversion of certain length parameters. The duality symmetries are potentially anomalous in the low-energy
field theory. These “stringy” duality anomalies must be cancelled since one knows that duality symmetries are preserved in any order of string perturbation theory. The requirement of the absence of target-space duality anomalies provides new constraints on the massless string spectrum not present in any point particle field theory.

In this talk we want to discuss whether classes of string compactifications, where each class contains a large number of different four-dimensional string models, can be free of target-space duality anomalies when assuming a certain massless string spectrum which looks consistent from the particle point of view. We will focus on the spectrum of the minimal supersymmetric standard model (MSSM). This is what we call minimal superstring compactification (MSSC). Clearly, this investigation can be performed for any favored, hypothetical massless string spectrum. Furthermore we will put MSSC’s under the phenomenological test of proper coupling constant unification. Finally we will discuss some phenomenological consequences of MSSC’s for the soft supersymmetry (SUSY) breaking parameters in model independent way. The material presented in this talk is largely based on ref. where a more complete list of references can be found.

2. The effective lagrangian of MSSC

Let us define more precisely what we mean by MSSC. (Recall that so far a MSSC was not explicitly constructed.) It is a hypothetical four-dimensional string model with the following properties: (i) It has as gauge group \( G = SU(3) \times SU(2) \times U(1) \times G_{\text{hidden}} \) up to the Planck mass \( M_P \). (This does not exclude the unification of this gauge group at \( M_P \).) (ii) It has \( N = 1 \) SUSY down to the weak scale. (iii) The massless \( SU(3) \times SU(2) \times U(1) \) non-singlet spectrum is that of the MSSM. These observable chiral \( N = 1 \) fields are denoted by \( \Phi_i = (\phi_i, \psi_i) \) with the flavor index \( i = Q, U, D, L, E, H, \overline{H} \). In addition, each string compactification has some gauge singlet states, including the dilaton chiral superfield \( S = (s, \psi_s) \) and several moduli fields \( T_\alpha = (t_\alpha, \psi_\alpha) \). The massive states are at \( M_P \).

The interactions of the above degrees of freedom are described by an \( N = 1 \) supergravity lagrangian. The kinetic energies of the matter fields and of the moduli follow from the Kähler potential which can be expanded around \( \phi_i = 0 \):

\[
K = K_0(t_\alpha, \overline{t}_\alpha) + K_{ij}(t_\alpha, \overline{t}_\alpha)\phi_i\overline{\phi}_j + \ldots
\]

The target space duality group \( \Gamma \) is given by those discrete reparametrizations on the moduli,

\[
\Gamma : \quad t_\alpha \to \overline{t}_\alpha(t_\alpha),
\]

which leave the underlying string theory, and therefore also the effective Lagrangian invariant. Due to the moduli dependence of the low-energy couplings,
Γ acts non-trivially on the Kähler potential and on the Kähler metrics. First, Γ acts on $K$ as a $U(1)$ Kähler transformation like

$$K_0 \rightarrow K_0 + g(t_{\alpha}) + \overline{g(t_{\alpha})}. \quad (2.3)$$

Second, Γ induces a change of the matter Kähler metric of the form

$$K_{ij} \rightarrow h_{il}(t_{\alpha})^{-1} h_{jk}(t_{\alpha})^{-1} K_{lk}. \quad (2.4)$$

It follows that the matter fields possess a non-trivial transformation behavior under Γ-transformations in order to obtain duality-invariant kinetic-energy terms for the matter fields:

$$\phi_i \rightarrow h_{ij}(t_{\alpha}) \phi_j. \quad (2.5)$$

Due to the non-trivial action, eq.(2.3), of Γ on the Kähler potential, the matter fermions transform with an additional phase as

$$\lambda_a \rightarrow e^{-\frac{1}{4}(g-\overline{g})} \lambda_a, \quad \psi_i \rightarrow e^{\frac{1}{4}(g-\overline{g})} h_{ij} \psi_j. \quad (2.6)$$

($\lambda_a$ are the gaugino fields.)

In the following we will restrict the discussion to symmetric $\mathbb{Z}_M$ and $\mathbb{Z}_M \times \mathbb{Z}_N$ orbifolds, since for these type of models the effective Langrangian can be constructed in a rather explicit way. Our formulas will be valid for a large class of (0,2) models with non-standard gauge embeddings and/or with the presence of Wilson lines. These compactifications include some examples that are of phenomenological interest since the gauge group can be different from $E_6 \times E_8$. In fact, there exist models with standard model gauge group $G = SU(3) \times SU(2) \times U(1)$ and three generations plus additional vector-like matter fields. Every orbifold of this type has three complex planes, and each orbifold twist $\tilde{\theta} = (\theta_1, \theta_2, \theta_3)$ acts either simultaneously on two or all three planes. For simplicity, we will consider the dependence of the effective action on the three untwisted moduli fields $t_{\alpha}$ ($\alpha = 1, 2, 3$) which describe the sizes of the three complex planes. For a more general discussion see. The moduli space for the $t_{\alpha}$-fields is locally given by the non-compact coset space $[SU(1,1)/U(1)]^3$. Since target-space duality transformations are discrete reparametrizations, Γ must be a discrete subgroup of $[SL(2,\mathbb{R})]^3$:

$$\Gamma: \quad t_{\alpha} \rightarrow \frac{a_{\alpha} t_{\alpha} - i b_{\alpha}}{i c_{\alpha} t_{\alpha} + d_{\alpha}}, \quad a_{\alpha} d_{\alpha} - b_{\alpha} c_{\alpha} = 1. \quad (2.7)$$

The parameters $a_{\alpha}, b_{\alpha}, c_{\alpha}, d_{\alpha}$ are in general a discrete set of real numbers. For the overall modulus $t = t_1 = t_2 = t_3$, the duality group is often given by the modular group $SL(2,\mathbb{Z})$ with integer parameters. Specifically, for (2,2) or (0,2) compactifications with possibly non-standard embedding of the twist into the gauge group $E_8 \times E_8$, but without discrete Wilson lines, the duality group is given by
In this case, the effective lagrangian must be modular invariant and is determined by automorphic functions of $SL(2, \mathbb{Z})$. However, for models with discrete background parameters some parts of the modular symmetries could be broken. Then the parameters $a_\alpha, b_\alpha, c_\alpha, d_\alpha$ in eq.(2.7) form a restricted set of integers, and the effective lagrangian will contain automorphic functions of the relevant modular subgroup.

The Kähler potential has a particularly simple dependence on the moduli fields $t_\alpha$:

$$K = -\sum_{\alpha=1}^{3} \log(t_\alpha + \bar{t_\alpha}) + \delta_{ij} \prod_{\alpha=1}^{3} (t_\alpha + \bar{t_\alpha})^{n_i^\alpha} \phi_i \overline{\phi_i} \cdots$$ (2.8)

Thus each matter field is characterized by a number $n_i^\alpha$. Using eq.(2.6) it follows that the normalized matter scalars and fermions transform under $\Gamma$ transformations (2.7) as

$$\phi_i \to \prod_{\alpha=1}^{3} \left( \frac{-ic_\alpha t_\alpha + d_\alpha}{ic_\alpha t_\alpha + d_\alpha} \right)^{-\frac{1}{2} n_i^\alpha} \phi_i, \quad \psi_i \to \prod_{\alpha=1}^{3} \left( \frac{-ic_\alpha \bar{t_\alpha} + d_\alpha}{ic_\alpha \bar{t_\alpha} + d_\alpha} \right)^{-\frac{1}{2} - \frac{1}{4} n_i^\alpha} \psi_i.$$ (2.9)

Therefore, the numbers $n_i^\alpha$ are called duality charges or modular weights of the matter fields.

In principal, the modular weights $n_i^\alpha$ are undetermined parameters in the effective lagrangian. However it is very important to realize that the allowed range of possible $n_i^\alpha$'s can be computed in string theory for any standard model field and for any class of orbifold compactification. Without presenting any detail, let us just state the main result of this investigation for the overall modular weights $n_i = \sum_{\alpha=1}^{3} n_i^\alpha$:

$$n = -1 \quad (\phi \text{ untwisted})$$
$$n = -2 - p + q \quad (\phi \text{ twisted with } \bar{\theta} \text{ acting on all planes})$$
$$n = -1 - p + q \quad (\phi \text{ twisted with } \bar{\theta} \text{ acting on two planes})$$ (2.10)

where $p$ ($q$) is the number of twisted oscillators with positive (negative) chirality in the vertex operator of $\phi_i$. Moreover, examining the modular transformation properties of the vertex operator of the space-time supersymmetry charge, one exactly recovers the additional Kähler phase for the fermions in eqs.(2.6),(2.9).

The maximal number of oscillators is limited by the requirement that the conformal dimension $h$ of the vertex operators must be one. Therefore $p_{\text{max}}$ and $q_{\text{max}}$ depend on contribution $h_{\text{KM}}$ of the Kac-Moody part, associated to the gauge group $SU(3) \times SU(2) \times U(1)$, to the vertex operator of each field. $h_{\text{KM}}$ is finally determined by the level of the $SU(3) \times SU(2) \times U(1)$ Kac-Moody algebra. The strongest constraints on $p_{\text{max}}$ and $q_{\text{max}}$ arise for the lowest Kac-Moody levels $3/5k_1 = k_2 = k_3 = 1$. Then, for the case of $\mathbb{Z}_3$, the standard model fields must not have any oscillators, and the modular weights can only be -1 and -2. For the other orbifolds the allowed ranges of $n_i$ can be bigger; the complete list of cases can be found in ref.3.
3. Consistency of MSSC – Target Space Duality Anomalies

Consider the supersymmetric non-linear $\sigma$-model of the moduli $T_\alpha$ coupled to gauge and matter fields as described in the last section. At the one-loop level one encounters a triangle diagram with two gauge bosons of the gauge group $G = \prod G_a^\star$ and one modulus field as external legs and massless gauginos and charged (fermionic) matter fields circulating inside the loop. This anomalous diagram leads, together with the tree-level part which is given by the dilaton/axion field $S$, to the following (non-local) one-loop effective supersymmetric lagrangian\textsuperscript{12,13,10}:

$$\mathcal{L}_{nl} = \sum_a \int d^2 \theta \frac{1}{4} W^a W^a \left\{ k_a S - \frac{1}{16\pi^2} \frac{1}{16} \Box^{-1} \nabla \nabla D \nabla D \sum_{\alpha=1}^{3} b^\alpha_a \log(T_\alpha + \mathcal{T}_\alpha) \right\} + \text{h.c.}$$

(3.1)

Here $W^a$ are the Yang–Mills superfields and $k_a$ are the levels of the $G_a$ Kac-Moody algebras. The anomaly coefficients $b^\alpha_a$ look like\textsuperscript{13,12}

$$b^\alpha_a = -C(G_a) + \sum_{\mathcal{R}_a} T(\mathcal{R}_a)(1 + 2n_{\mathcal{R}_a}^\alpha).$$

(3.2)

Writing the expression (3.1) in components it leads to a non-local contribution to the $CP$ odd term $F_{\mu\nu} \tilde{F}_{\mu\nu}$ and to a local contribution to the gauge coupling constant.

Now, it is easy to recognize that $\mathcal{L}_{nl}$, eq.(3.1), is not invariant under the discrete target space duality transformation (2.7). It follows that the duality anomalies must be cancelled by adding new terms to the effective action. Specifically, there are two ways of cancelling these anomalies. In the first one\textsuperscript{12,10} the $S$ field may transform non-trivially under duality transformations, $S \to S - \frac{1}{8\pi} \sum_{\alpha=1}^{3} \delta_{GS}^\alpha \log(i\alpha T_\alpha + d_\alpha)$, and cancels in this way some part or all of the duality non-invariance of eq.(3.1). This non-trivial transformation behavior of the $S$–field is completely analogous to the Green–Schwarz mechanism for the case of an anomalous $U(1)$ gauge group and leads to a mixing between the moduli and the $S$–field in the one-loop Kähler potential.

Second, the target space duality anomaly can be possibly cancelled by a local contribution to $\mathcal{L}_{nl}$, which is related to the one-loop threshold contributions to the gauge coupling constants due to the massive string states. The threshold contributions are given in terms of automorphic functions of the target space duality group, which have the required transformation behavior under the discrete duality transformations. This topic will be discussed in section 4.

It is clear that the part of the duality anomaly which is removed by the Green–Schwarz mechanism is universal, i.e. gauge group independent. Thus for cases where

* Analogously, there is also a mixed gravitational, $\sigma$-model anomaly with two gravitons and one modulus field as external legs\textsuperscript{10,3,11}.
there are no moduli dependent threshold contributions from the massive states the anomaly coefficients have to coincide for each gauge group factor $G_a$:

$$\frac{b'^{\alpha}}{k_a} = \frac{b'^{\alpha}}{k_b} = \frac{b'^{\alpha}}{k_c} = ... \quad (3.3)$$

This particularly interesting constraint arises for orbifold compactifications where the moduli dependent threshold contributions are absent because of an enlarged $N = 4$ supersymmetry in the massive spectrum. In more technical terms, eq.(3.3) applies if all orbifold twists $\vec{\theta}$, which define a particular $\mathbb{Z}_M$ or $\mathbb{Z}_M \times \mathbb{Z}_N$ orbifold, act non-trivially on the corresponding $\alpha$th complex plane of the underlying six-torus.

Let us investigate the cancellation of duality anomalies for the phenomenologically most interesting case of MSSC. Then the anomaly coefficients of the standard model gauge groups take the following form:

$$b'^1_3 = 3 + \sum_{g=1}^{3} (2n^\alpha_{Q_g} + n^\alpha_{U_g} + n^\alpha_{D_g}), \quad b'^2_2 = 5 + \sum_{g=1}^{3} (3n^\alpha_{Q_g} + n^\alpha_{L_g}) + n^\alpha_{H} + n^\alpha_{\overline{H}},$$

$$b'^1_1 = 11 + \sum_{g=1}^{3} (\frac{1}{3}n^\alpha_{Q_g} + \frac{8}{3}n^\alpha_{U_g} + \frac{2}{3}n^\alpha_{D_g} + n^\alpha_{L_g} + 2n^\alpha_{E_g}) + n^\alpha_{H} + n^\alpha_{\overline{H}} \quad (3.4)$$

Whether eqs.(3.3) have any solutions crucially depends on the distribution of the allowed modular weights of the standard model fields. Of course, similar constraints may be obtained for other extended gauge groups and particle contents.

The strongest constraints arise for the $\mathbb{Z}_3$ and $\mathbb{Z}_7$ orbifolds where eq.(3.3) must be satisfied with respect to all three complex planes, and where the choice of possible values for the modular weights is very limited. Let us investigate the most common case of level one Kac–Moody algebras, $k_1 = 5/3$, $k_2 = k_3 = 1$. (For $\mathbb{Z}_3$ orbifolds, our results are also true for arbitrary $k_1$.) With the help of a computer program one can now check that the equations (3.3), together with eq.(3.4), have no simultaneous solutions at all. In this way we have ruled out the MSSC $\mathbb{Z}_3$ and $\mathbb{Z}_7$ compactifications with lowest Kac-Moody level by general consistency arguments. The requirement of target space anomaly freedom forces us to introduce additional fields. For $\mathbb{Z}_3$ we have analyzed the anomaly conditions further. It turns out that one needs 12 more $SU(2)$ doublets that $SU(3)$ triplets in all models. This excludes in particular the minimal Higgs content.

For all other classes of orbifolds the duality anomaly matching conditions are unfortunately not strong enough to rule out the MSSC scenario. The reasons are the enlarged ranges of possible modular weights. In addition, all other models have at least one complex plane which is left unrotated by one of the orbifolds twists. Then the anomaly cancellation condition eq.(3.3) cannot be applied with respect to this particular complex plane.
4. Phenomenology of MSSC

4.1. Threshold Effects and Gauge Coupling Unification

In the last section we have demonstrated that string consistency requirements like the cancellation of target space duality anomalies put severe constraints on the massless spectrum of four-dimensional strings. Now we use the phenomenological constraint of proper unification of the running gauge coupling constants to get further information on the massless particle spectrum.

The one-loop running gauge coupling constants in four-dimensional strings take the following form:

\[
\frac{1}{g_a^2(\mu)} = \frac{k_a}{g_{\text{string}}^2} + \frac{b_a}{16\pi^2} \log \frac{M_{\text{string}}^2}{\mu^2} - \frac{1}{16\pi^2} \sum_{\alpha=1}^{3} (b'_{\alpha} - k_a \delta_{GS}^\alpha) \log[(t_\alpha + \bar{t}_\alpha)|\Delta(t_\alpha)|^2].
\]

(4.1)

\(b_a\) is the \(N = 1\) \(\beta\)-function coefficient, and \(M_{\text{string}}\) is the typical string scale, which is of the order of the Planck mass. Its precise value, using the \(\overline{MS}\) scheme, is determined by the universal string coupling constant \(g_{\text{string}}\) as \(M_{\text{string}} = 0.5 \times g_{\text{string}} \times 10^{18}\text{GeV} \simeq 3.5 \times 10^{17}\text{GeV}\). The moduli dependent term in eq.(4.1) describes the one-loop threshold contribution to the gauge coupling constant from the massive momentum and winding modes. Here we have assumed that the field independent contributions to the threshold corrections are small compared to the moduli dependent pieces. This was shown to be true for the \((2,2)\) \(Z_3\)-orbifold compactification in ref.\(^{15}\). \(\Delta(t_\alpha)\) is an automorphic function of the duality group \(\Gamma\). Duality invariance of \(g_a^2(\mu)\) requires that the function \(\Delta(t_\alpha)\) must transform (up to a phase) under (2.7) as \(\Delta(t_\alpha) \rightarrow \Delta(t_\alpha)(ic_t a_\alpha + d_\alpha)\). For \(\Gamma_\alpha = SL(2,\mathbb{Z})\), \(\Delta(t_\alpha)\) is given by the Dedekind function, \(\Delta(t_\alpha) = \eta(t_\alpha)^2\), where \(\eta(t) = e^{-\pi t/12} \prod_{n=1}^{\infty} (1 - e^{2\pi nt})\).

Note that the threshold contribution is vanishing if \(t_\alpha\) corresponds to a completely rotated complex plane, since in this case one has \(b'_{\alpha} = k_a \delta_{GS}^\alpha\). Thus for the \(Z_3\) and \(Z_7\) orbifolds there are no moduli dependent threshold corrections.

Now we want to investigate the question whether the \(Z_M\) or \(Z_M \times Z_N\) orbifold compactifications can lead to the correct unification of the three coupling constants of the standard model gauge group \(SU(3) \times SU(2) \times U(1)_Y\), taking into account the threshold correction of the massive string excitations\(^{16,3}\). Our analysis will be based on the experimentally measured values of the strong coupling constant and the weak mixing angle: \(\alpha_3^{\exp} = 0.115 \pm 0.007\), \(\sin^2 \theta_W^{\exp} = 0.233 \pm 0.0008\). Considering the effect of the spectrum of the minimal supersymmetric standard model on the one-loop renormalization group equations\(^{17}\) without any threshold corrections with a SUSY threshold close to the weak scale, one finds\(^{18}\) that the quoted results for \(\alpha_3\) and \(\sin^2 \theta_W\) are in very good agreement with a unification mass \(M_X \simeq 10^{16}\text{GeV}\). Comparing this value with the string scale \(M_{\text{string}} \simeq 3.5 \times 10^{17}\text{GeV}\) one finds a
discrepancy. Thus the natural question is whether the correct unification of the three gauge coupling constants can be achieved by taking into account threshold contributions to $g_2^2(\mu)$. We will focus on the MSSC scheme. Making use of eq.(4.1) one gets for the value of the electroweak angle $\theta_W$ and for the strong coupling constant $\alpha_3$ ($k_3 = k_2 = 3/5k_1 = 1$):

$$\sin^2 \theta_W(\mu) = \frac{3}{8} - \frac{3\alpha_e(\mu)}{32\pi} \left( A \log \left( \frac{M_{\text{string}}^2}{\mu^2} \right) + \sum_{\alpha=1}^{3} A'^{\alpha} \log \left( |t_\alpha + \overline{t}_\alpha| \Delta(t_\alpha) |^2 \right) \right),$$

$$\frac{1}{\alpha_3(\mu)} = \frac{3}{8} \left( \frac{1}{\alpha_e(\mu)} - \frac{1}{4\pi} B \log \left( \frac{M_{\text{string}}^2}{\mu^2} \right) + \frac{1}{4\pi} \sum_{\alpha=1}^{3} B'^{\alpha} \log \left( |t_\alpha + \overline{t}_\alpha| \Delta(t_\alpha) |^2 \right) \right),$$

(4.2)

where $A = \frac{k_2}{k_1} b_1 - b_2$ and $B = b_1 + b_2 - \frac{k_1 + k_2}{k_3} b_3$. $A'^{\alpha}$ and $B'^{\alpha}$ have the same expressions after replacing $b_\alpha \rightarrow b'^{\alpha}_\alpha$. For the MSSC one has $A = 28/5$ and $B = 20$. Now we assume without loss of generality that the threshold contributions dominantly come from one modulus, i.e. from $t_1$, which belongs to a unrotated plane. Then one can eliminate the explicit $t_1$ dependence from eqs.(4.2), and with the experimental values for $\alpha_3^{\exp}$ and $\sin^2 \theta_W^{\exp}$ we obtain the following condition on the modular weights of the standard model particles: $2.7 \leq B'^1/A'^1 \leq 3.7$. Additional information about the allowed values of $A'^1$ and $B'^1$ can be extracted from the explicit $t_1$ dependence of $\Delta(t_1)$. Specifically, if $\Delta(t_1) = \eta(t_1)^{2}$ one knows that $\log[(t_1 + \overline{t}_1) \eta(t_1)^4] < 0$ for all possible values of $t_1$. For a general threshold correction $\Delta(t_1)$ this inequality is expected to hold, at least for sufficiently large values of $\text{Re} t_1$, which is anyway required for these corrections to be large. This originates from the expected behavior of $\Delta(t_1) \rightarrow e^{-t_1}$ for large $\text{Re} t_1$, i.e. the Kaluza-Klein limit of orbifold compactifications. Then the correct values low energy parameters are obtained provided $A'^1 < 0$, $B'^1 < 0$. Checking these conditions on the modular weights and taking into account also the duality anomaly conditions for completely rotated planes we obtain the following result: The correct unification of the three gauge coupling constants is possible for the $\mathbb{Z}_6$, $\mathbb{Z}_8'$ and all $\mathbb{Z}_M \times \mathbb{Z}_N$ orbifolds. Large enough threshold correction require for these cases that the radius of the relevant complex plane is relatively large: $\text{Re} t_1 \sim 10 - 20$.

4.2. Soft SUSY-breaking Parameters

We will now turn to the phenomenology of the soft SUSY-breaking parameters in four-dimensional strings (see also ref.19). The presence of these soft terms reflects itself into the SUSY-particle mass spectrum at low energies. If the idea of low-energy supersymmetry is correct, the latter should be amenable to experimental tests in future accelerators. In principle there are as many different soft terms as independent particles and/or couplings present. Imposing some symmetries at the GUT/Planck scale reduces the number of independent soft terms. In particular, it
would thus be very important to find constraints on the pattern of SUSY-breaking soft terms in effective low-energy lagrangians from strings. We will show that even without knowing the details of the supersymmetry breaking process one can obtain some characteristic features of the soft terms in a model independent way. We will only assume that the “seed” for SUSY-breaking is provided by the auxiliary fields of the dilaton $s$ and the moduli $t_{\alpha}$; this assumption is true in most supersymmetry breaking scenarios discussed up to now.

Let us consider first the gaugino masses for canonically normalized gauginos at the weak scale: $M_{a}(M_{W}) = 2\pi\alpha_{a}(M_{W})\tilde{M}_{a}$. The value of $\tilde{M}_{a}$ is given in a general $N = 1$ supergravity lagrangian by

$$\tilde{M}_{a} = m_{3/2} \sum_{a=s,t} f_{a}^{\alpha} K_{\alpha\beta}^{-1} G_{\beta}. \quad (4.3)$$

$m_{3/2}$ is the gravitino mass, $f_{a}^{\alpha}$ is the derivative of the gauge kinetic function $f_{a}$ with respect to the fields $s$ and $t_{\alpha}$, $K_{\alpha\beta}^{-1}$ is the inverse Kähler metric and $G_{\alpha}$ is the auxiliary field of $s$ and $t_{\alpha}$. The gauge kinetic function has the general form

$$f_{a}(s, t_{\alpha}) = k_{a}s + \frac{c}{16\pi^{2}} \log \Delta(t_{\alpha})^{2}. \quad (4.4)$$

Here, $\log \Delta(t_{\alpha})^{2}$ is the one-loop threshold contribution from the massive string excitations in terms of automorphic functions of the corresponding duality group $\Gamma$. The constant $c$ is generically of order one. As it stands, eq.(4.3), using eq.(4.4), only takes into account the one-loop contribution of the massive fields, however not the one-loop contribution of the massless fields in the Yang-Mills lagrangian which is described by the non-local effective langrangian (3.1). Considering also this non-local interaction simply amounts to replace $f_{a}^{t_{\alpha}}$ by the derivative of the one-loop gauge coupling constant with respect to $t_{\alpha}$. This means that one effectively has to replace in eq.(4.4) $\Delta(t_{\alpha})$ by a non-holomorphic function $\tilde{\Delta}(t_{\alpha}, \tilde{t}_{\alpha})$ which contains also the anomalous piece of the massless fields. Note that this contribution is also required by the duality invariance of the gaugino mass. Then $\tilde{M}_{a}$ takes the form

$$\tilde{M}_{a} = m_{3/2} \left( k_{a}(K_{s}^{-1}G_{s} + K_{s}^{-1}\tilde{G}_{s}) + \frac{1}{8\pi^{2}/\Delta} \frac{\partial \Delta}{\partial t_{\alpha}}(K_{t_{\alpha}}^{-1}G_{t_{\alpha}} + K_{t_{\alpha}}^{-1}\tilde{G}_{t_{\alpha}}) \right). \quad (4.5)$$

Here we have allowed for a mixing between the $s$-field and the moduli in the kinetic energy which occurs beyond the tree-level. Eq.(4.5) shows some interesting model-independent features. The existence of the threshold corrections implies in general that the gaugino masses $\tilde{M}_{a}$ are non-universal, i.e. gauge group dependent. In particular, if supersymmetry breaking occurs mainly in the moduli sector, i.e. $\tilde{G}_{t_{\alpha}} >> G_{s}$, which is often true for supersymmetry breaking by gaugino condensation in the hidden gauge sector, the gaugino masses dominantly originate from string loop effects. This means that $M_{a}$ is small compared to $m_{3/2}$.
For orbifolds one can explicitly parametrize the non-universality of the gaugino masses by the duality anomaly coefficients $b'_a^\alpha$. With eq.(4.1) one obtains

$$\tilde{M}_a = M^0(s, t)k_a + \sum_{\alpha=1}^{3} b'_a^\alpha M'^\alpha(s, t).$$

(4.6)

Here $M'^\alpha$ is due to loop effects and is small compared to $m_3/2$. The size of the gauge group independent part $M^0$ depends on the details of the supersymmetry breaking mechanism. Generically it is of order $m_3/2$, however for $G_{t_{\alpha}} \gg G_s$ it is of one loop order and therefore comparable to $M'^\alpha$. It is instructive to eliminate the first term in the rhs. of (4.6) by taking certain linear combinations of gaugino masses. This leads to the following sum rule:

$$\tilde{M}_1(1 - \gamma_M k_2/k_1) + \tilde{M}_2(1 + \gamma_M) - k_1 + k_2 \tilde{M}_3 = 0.$$  

(4.7)

Here $\gamma_M = (\sum_{\alpha=1}^{3} B'^\alpha M'^\alpha)/(\sum_{\alpha=1}^{3} A'^\alpha M'^\alpha)$ and $A'^\alpha$ and $B'^\alpha$ are the linear combinations introduced in the previous section. Further simplification emerges (i) if for the particular orbifold considered only the first plane is left unrotated by some twist. Then only $M'^1$ will be non-vanishing. (ii) Or if the supersymmetry-breaking dynamics are such that the modulus of a particular complex plane plays a leading role (i.e. $|G_{t_{1}}| \gg |G_{t_{2}}|, |G_{t_{3}}|$), again only $M'^1$ will be relevant. In both cases the moduli dependence cancels from $\gamma_M$ yielding $\gamma_M = B'/A'$. In the MSSC scheme we need to have $2.7 \leq \gamma \leq 3.7$ for the correct unification of the gauge coupling constants. Then eq.(4.7) provides a rather strong constraint on the gaugino masses.

Finally let us briefly display the general form of the soft SUSY-breaking scalar masses. The scalar potential in the effective low-energy supergravity action has the form

$$V = e^{G}\left\{ \sum_{i,j} G_{\phi_i} G_{\phi_j} K_{\phi_i \phi_j}^{-1} \right\} - 3,$$

(4.8)

where $K$ is the total Kähler potential and sum runs over all charged massless chiral fields $\phi_i$. With the Kähler potential (2.8) one gets a general expression for the soft scalar masses of the form

$$m_i^2 = m_0^2(s, t, \tilde{t}) + \sum_{\alpha=1}^{3} n_i^\alpha m^2(s, t, \tilde{t}).$$  

(4.9)

The first term in the right-hand side is universal, i.e. does not depend on the particular matter field $\phi_i$ considered. The second term in eq.(4.9) depends on the modular weights of the matter fields and is in general not universal. Interesting phenomenological constraints on the modular weights may arise from absence of flavor-changing neutral currents demanding for example that $m_U^2$ and $m_C^2$ must be almost degenerate for the SUSY-GIM mechanism to work. This would suggest that both fields have similar modular weights. However for the case of gaugino condensation, the considerations in ref.22 indicate, that $m_U^2 \ll m_C^2$ such that the constraints on the modular weights might not be very tight.
5. Conclusions

In this talk we have considered some string consistency as well as phenomenological constraints on four-dimensional strings. One of the merits of our argumentation is that it allows us to discard large classes of models without needing to work on a (hopeless) model-by-model basis. Specifically we have shown that the requirement of the absence of duality anomalies rules out the existence of $\mathbb{Z}_3$ and $\mathbb{Z}_7$ orbifolds (at the lowest Kac-Moody level) with the massless spectrum of the supersymmetric standard model. We believe that absence of duality anomalies is intimately related to the world-sheet modular invariance of four-dimensional strings, in complete analogy to the relation between gauge/gravitational anomalies and world-sheet modular invariance.

Demanding for the additional, phenomenological requirement of correct gauge coupling unification all $\mathbb{Z}_M$ orbifolds except $\mathbb{Z}_6$ and $\mathbb{Z}_8$ are ruled out when one assumes the particle content of the minimal supersymmetric standard model. On the other hand, a consistent MSSC is not excluded (but not yet found) for $\mathbb{Z}_M \times \mathbb{Z}_N$. Finally we have shown that in generic string models the soft scalar masses are not universal but depend on the modular weight, or in more general terms, on the kinetic energy of the matter fields. Similarly, one sees that the soft gaugino masses are gauge group dependent. The departure from universality of gaugino masses may be related in specific models to the gauge coupling constant threshold effects. In some cases specific mass relationships are found.

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