The Stellar Orbital Structure in Axisymmetric Galaxy Models with Supermassive Black Hole Binaries

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Received 2018 February 10; revised 2018 October 16; accepted 2018 October 17; published 2018 December 6

Abstract

It has been well established that particular centrophilic orbital families in non-spherical galaxies can, in principle, drive a black hole binary to shrink its orbit through three-body scattering until the black holes are close enough to strongly emit gravitational waves. Most of these studies rely on the orbital analysis of a static supermassive black hole (SMBH)-embedded galaxy potential to support this view; it is not clear, however, how these orbits transform as the second SMBH enters the center. So our understanding of which orbits actually interact with an SMBH binary is not ironclad. Here, we analyze two flattened galaxy models, one with a single SMBH and one with a binary, to determine which orbits actually do interact with the SMBH binary and how they compare with the set predicted in single SMBH-embedded models. We find close correspondence between the centrophilic orbits predicted to interact with the binary and those that are actually scattered by the binary, in terms of energy and \( L_z \) distribution, where \( L_z \) is the \( z \) component of a stellar particle’s angular momentum. Of minor note: because of the larger mass, the binary SMBH has a larger radius of influence than in the single SMBH model, which allows the binary to draw from a larger reservoir of orbits to scatter. Of the prediction particles and scattered particles, nearly half have chaotic orbits, 40% have \( f_x f_y = 1:1 \) orbits and 10% have other resonant orbits.

Key words: black hole physics – galaxies: elliptical and lenticular, cD – galaxies: kinematics and dynamics – galaxies: nuclei – galaxies: structure

1. Introduction

Supermassive black holes (SMBHS), with masses in the range of \( 10^6 M_\odot \)–\( 10^9 M_\odot \), are a part of nearly every galaxy center (Kormendy & Richstone 1995; Kormendy & Ho 2013). As galaxies grow through merging, the SMBHSs will sink to the center of the remnant, eventually forming an SMBH binary (SMBHBs; Begelman et al. 1980) that will coalesce—though the timescale from a galaxy merger to black hole merger could be more than a Hubble time. The process that governs SMBHB coalescence can be broken into three stages. First, dynamical friction from the background stars and gas drives the two black holes closer until they become bound as a binary. As the orbit shrinks and becomes a hard binary, few-body scattering of nearby stars dominates. These stars have low angular momentum and are located in a volume of phase space called the loss cone (Quinlan 1996; Yu 2002; Milosavljević & Merritt 2003). If three-body scattering is efficient, the binary orbit shrinks by a factor of a few hundred, resulting in a binary which will start to emit gravitational waves and merge into one. SMBHB mergers are thought to be the most powerful gravitational wave sources in the universe (Hughes 2003). Of the three stages, the three-body scattering stage is widely thought to be the longest for binaries having SMBHs of comparable masses. For black holes with mass ratios of \( q \ll 0.1 \), dynamical friction may not be efficient enough and black holes do not form a binary (Tremmel et al. 2015; Dosopoulou & Antonini 2017).

Although lingering questions remain for the dynamics of SMBHBs, the consensus is that there are a number of ways to drive the SMBHB through the three-body scattering stage relatively quickly. One way is shape: triaxial and flat galaxy models keep the binary supplied with stars on centrophilic orbits to help the black hole binary merge (Berczik et al. 2006; Holley-Bockelmann & Sigurdsson 2006; Khan et al. 2011, 2013; Pretor et al. 2011; Vasiliev et al. 2014). Rotation also helps shrink the binary orbit (Holley-Bockelmann & Khan 2015; Mirza et al. 2017), an effect often excluded from simulations despite the near-ubiquity of bulk rotation in classical and pseudobulges (Bender et al. 1992; Kormendy & Kennicutt 2004; Khochfar & Silk 2009; Bois et al. 2011; Gadotti 2012; Tsatsi et al. 2015). A third intruder black hole from a subsequent galaxy merger can accelerate the coalescence of the SMBHB through the combined action of Kozai–Lidov resonances and gravitational wave emission (Hoffman & Loeb 2007; Bonetti et al. 2016; Ryu et al. 2018). Finally, the viscous drag from a gaseous disk around the SMBHB may increase the binary’s orbital decay rate (Haiman et al. 2009; Lodato et al. 2009).

While it is true that centrophilic orbits in non-spherical and/or rotating systems are thought to be key in shrinking the SMBHB orbit, an analysis of the orbits that are scattered by the binary has not been undertaken. Instead, most studies analyze the orbits within a black hole-embedded primary galaxy before the second SMBH enters, keeping track of those with the potential of interacting with an SMBHB, if one were there (Holley-Bockelmann et al. 2001, 2002; Jesseit et al. 2005; Hoffman et al. 2010; Bryan et al. 2012; Valluri et al. 2012; Röttgers et al. 2014). However, the perturbation from a second black hole may be significant enough to obliterate stable centrophilic orbital families as well as to generate new regions of stability in phase space. It is important to understand the
orbits that really do interact with the binary, as these orbits can influence the eccentricity and plane of the binary itself.

Here we examine the orbits of particles scattered by an SMBHB in a flat galaxy model during the hard binary evolution regime in a direct $N$-body simulation and compare these to the number and type of orbits predicted to interact with the binary within the initial conditions of a single SMBH-embedded galaxy model.

This paper is organized as follows. In Section 2 we describe the technique used to identify the orbits that are predicted to interact with the binary versus the orbits that actually do interact. Our results are featured in Section 3, and the implications are covered in Section 4.

2. Method

Our base galaxy model, with one million particles, is in equilibrium, non-rotating, and flattened, having an axis ratio of $c/a = 0.75$ and a single SMBH. It follows the Dehnen density profile (Dehnen 1993) having a central logarithmic slope equal to 1. In system units, the black hole has a mass of 0.005, and each stellar particle has a mass of $1 \times 10^{-6}$. We introduce a second equal-mass SMBH in orbit within the base galaxy and perform direct $N$-body simulations using $\phi$-GPU (Berczik et al. 2011, 2013). Separation between SMBHs shrinks and a hard Keplerian binary forms. We record the output in 53 snapshots—52 for details see Section 2.1

2.1. Particles with Promise: Identifying Centrophilic Orbits in the Base Potential

As in Li et al. (2015), for $t = 0$, 13, 26, 39, we freeze the potential of the base model at each snapshot and run each particle to $t = 52$. For each orbit, we record its minimum separation from the SMBH, $r_{\text{min}}$. If the orbit of a particle has $r_{\text{min}}$ less than the hardening radius of the SMBHB (if it were present in the base model), we define it as one of the particles with promise. We select all of those particles at the four snapshots. The hardening radius is expressed as

$$a_h = G\mu/4\sigma^2$$

(Quinlan 1996). We select as model units the gravitational constant, $G = 1$, the reduced mass of the SMBHB, $\mu = 0.0025$ (since each black hole has a mass of 0.005), and the stellar velocity dispersion within the radius of influence of the SMBH, $\sigma = 0.4$. Therefore, here $a_h = 0.004$.

2.2. Scattered Particles: Tracking Particles that Interact with the SMBHB

Before the SMBHB hardens, a star may gain or lose energy after interacting with the black holes. Since we are interested in interactions in general, we track all particles with a significant change in energy, and we define significant as greater than a 10% change.

However, in general, the particles gaining energy are more than that losing energy, therefore the net energy of all of the interacting particles gain is positive, which should be equal to the energy the SMBHB loses. The total energy of the whole system is conserved to $\sim 10^{-4}$. For the snapshot at $t = 0$, among the one million particles, $\sim 640,000$ particles gain energy and $\sim 360,000$ particles lose energy. However, for most particles the energy change is just due to the background galaxy potential evolving with time (see Figure 1 in Section 3) and two-body encounters with other stellar particles. Only particles with significant energy change are the ones that really interact with the SMBHB. Therefore we order the stellar particles according to the absolute value of their energy change percentage, $|\Delta E/E|$, from high to low. The criterion we use here to find the scattered particles with the SMBHB is that of the total energy that the first $N_s$ most-energy-changing particles get equal to that of the SMBHB loses. With this criterion, we get $N_s = 14,226$ for the snapshot at $t = 0$. We note that among these 14,226 particles only 200 particles lose energy, all of the other particles gain energy. In order not to miss some particles with high $|\Delta E|$ but low $|\Delta E/E|$, we also order the particles by $|\Delta E|$ from high to low and make use of the same criterion obtaining $N_s = 11,839$. The union of these two groups of particles is considered as the scattered particles. The total number is $\sim 14,900$, mainly made of the ones obtained using $|\Delta E|$. For all of the scattered particles, the $\Delta E/E$ is significant, which is between 10% and 1000%, with minimum value 10% and maximum value 41,200%. We use the same method to obtain the scattered particles for other snapshots.
After identifying the particles with promise and scattered particles at the four snapshots, we run the eight groups of particles for 100 dynamical times to get dominant frequencies along the principle axes for each particle. For the first 50 dynamical times we obtain $f_{x1}$, $f_{y1}$, $f_{z1}$, for the second 50 dynamical times we obtain $f_{x2}$, $f_{y2}$, $f_{z2}$. We use Laskar’s frequency mapping technique to classify orbits according to the dominant frequency ratios along the principle axes (Holley-Bockelmann et al. 2001, 2002; Holley-Bockelmann & Sigurdsson 2006; Valluri et al. 2010, 2012; Vasiliev et al. 2014, 2015; Holley-Bockelmann & Khan 2015) and determine the orbital families present by integrating all of the particles to complete the phase coverage of the orbit. To identify chaotic orbits, we use the criterion that any two of $f_{x}$, $f_{y}$, $f_{z}$ must satisfy both $|f_{1} - f_{2}| > 2f_{bin}$, and $|f_{1} - f_{2}|f_{bin} > 10^{-1.22}$ (Valluri et al. 2010). As shown in Figure 7, the main resonant orbital type is $f_{x}f_{z} = 1:1$. The way we identify this tube orbit is excluding all of the chaotic particles; if a particle satisfies $f_{x}/f_{y} > 0.95$ and $f_{x}/f_{y} < 1.05$, it is considered as a $f_{x}f_{z} = 1:1$ orbit. The remaining orbits are resonant orbits.

### 3. Results

Table 1 lists the energy components and energy change of the stellar particles and the SMBHB in the two-SMBH model at $t = 0$ and $t = 52$. The first four columns show the stellar particles’ potential relative to the stellar background, $E_{pot, in}$.

| energy Table |
|-----------------|-----------------|-----------------|-----------------|
| Stellar Particles, $E_{pot, in}$ | Stellar Particles, $E_{pot, ext}$ | Stellar Particles, $E_{k}$ | Stellar Particles, $E_{tot}$ | BHB, $E_{pot}$ | BHB, $E_{k}$ | BHB, $E_{tot}$ |
| $t = 0$ | $-0.305$ | $-0.016$ | $0.157$ | $-0.164$ | $0.000$ | $0.001$ | $0.000$ |
| $t = 52$ | $-0.290$ | $-0.017$ | $0.159$ | $-0.146$ | $-0.034$ | $0.017$ | $-0.017$ |
| $\Delta E$ | $0.016$ | $-0.001$ | $0.002$ | $0.018$ | $-0.034$ | $0.017$ | $-0.018$ |

After identifying the particles with promise and scattered particles at the four snapshots, we run the eight groups of particles for 100 dynamical times to get dominant frequencies along the principle axes for each particle. For the first 50 dynamical times we obtain $f_{x1}, f_{y1}, f_{z1}$, for the second 50 dynamical times we obtain $f_{x2}, f_{y2}, f_{z2}$. We use Laskar’s frequency mapping technique to classify orbits according to the dominant frequency ratios along the principle axes (Holley-Bockelmann et al. 2001, 2002; Holley-Bockelmann & Sigurdsson 2006; Valluri et al. 2010, 2012; Vasiliev et al. 2014, 2015; Holley-Bockelmann & Khan 2015) and determine the orbital families present by integrating all of the particles to complete the phase coverage of the orbit. To identify chaotic orbits, we use the criterion that any two of $f_{x}$, $f_{y}$, $f_{z}$ must satisfy both $|f_{1} - f_{2}| > 2f_{bin}$, and $|f_{1} - f_{2}|f_{bin} > 10^{-1.22}$ (Valluri et al. 2010). As shown in Figure 7, the main resonant orbital type is $f_{x}f_{z} = 1:1$. The way we identify this tube orbit is excluding all of the chaotic particles; if a particle satisfies $f_{x}/f_{y} > 0.95$ and $f_{x}/f_{y} < 1.05$, it is considered as a $f_{x}f_{z} = 1:1$ orbit. The remaining orbits are resonant orbits.

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From this table, we can notice that: the energy that the SMBHB loses is equal to what the stellar particles gain, as expected; the
stellar particles’ $E_{\text{pot, in}}$ changes a lot, while their $E_{\text{pot, ext}}$ and $E_K$ do not change too much; the energy in the system mainly is given from $E_{\text{pot}}$ (the SMBHB’s potential relative to each other), half obtained by their kinetic energy $E_K$, and half obtained by the stellar particles’ $E_{\text{pot, in}}$. This indicates that the stellar particles’ main energy change between $t = 0$ and $t = 52$ is due to the SMBHB changing the stellar structure by interacting with them. Therefore we plot Figure 1.

Figure 1 shows the stellar density in snapshots at $t = 6, 8, 10, 13, 26, 39, 52$, respectively. The origin is set up to the center of mass of the SMBHB. When the binary becomes hard, it ejects the stars passing by, making the density decrease gradually. The radius of influence is 0.05 in system units. We noticed that the interaction of the SMBHB and surrounding stars influences the stellar density within 0.2 system units (four times the radius of influence of a single SMBH). That means that the region that the SMBHB can impact is at least several (about three to four) times larger than the influence radius of a binary in agreement with Khan et al. (2012).

Figure 2 shows the number of scattered particles, particles with promise, and common particles of the two groups at each snapshot at $t = 0, 13, 26, 39$. The trend is that the number of particles with promise at any snapshot are more than that of the scattered particles at the same snapshot, indicating that some particles with the possibility to be scattered are no longer able to be scattered after the orbit shrinks. It is natural that at an earlier time the SMBHB can and needs to scatter more. For example, from $t = 0$ to $t = 52$ the binary can scatter particles 1.5 times its mass, while from $t = 39$ to $t = 52$ it can only scatter half its mass. The common particles of the two groups are rare, usually less than 30% of the particles with promise at the same snapshot. It is inferred that both the particles with promise and the scattered particles may be drawn from a larger collection in phase space, because even if they do not have many common particles, they still have the same characteristic quantities such as energy, $L_z$, etc., as shown below.

Figure 3 shows that the kinetic energy of the two SMBHs increases with time. After $t = 6$, the two SMBHs have nearly the same kinetic energy. Since they have equal masses, they therefore have equal velocity after $t = 6$.

The left and middle panels of Figure 4 show the energy histogram of the particles with promise and the scattered particles in the one-SMBH model in snapshots at $t = 0, 13, 26, 39$, in number and fraction, respectively. The trend is for any energy bin in any snapshot, except $t = 0$ where the number of particles of promise is bigger than that of the scattered particles. The remarkable thing is in the middle panel all of the lines overlap with each other perfectly except for the ones at $t = 0$, which shows that in all snapshots, except for $t = 0$, the particles from the two groups come from the same energy slice. The right panel of Figure 4 shows the energy distribution of the two groups and all of the one million particles, in fraction, for the $t = 0$ snapshot. We notice that for all the particles the energy peaks at $E = -0.1$, which is much bigger than the peak position of the particles with promise at $E = -1.3$ and the peak position of the scattered particles at $E = -1.6$. This indicates that the particles with promise and scattered particles both belong to a lower energy slice. Though not showing the energy of all particles in other snapshots, particles with promise and scattered particles at other snapshots are also from a lower energy slice.

The first and second panels of Figure 5 show the $r_{\text{min}}$ histogram of the particles with promise and the scattered particles in the one-SMBH model in snapshots at $t = 0, 13, 26, 39$, in number and fraction, respectively. In the first panel, the particles with promise have more particles in every $r_{\text{min}}$ bin. In the second panel all of the lines of the particles with promise are perfectly overlapping with each other, peaking at $r_{\text{min}} = 10^{-1.9}$, also all of the lines of the scattered particles are nearly overlapping with each other peaking at $r_{\text{min}} = 10^{-2.0}$, except for the one at $t = 0$, which indicates that scattered particles can have bigger $r_{\text{min}}$. The third panel shows the $r_{\text{max}}$ histogram of the particles with promise, scattered particles, and all particles at $t = 0$ in the one-SMBH model. It is seen that the two groups have much smaller $r_{\text{max}}$ than those of the particles with promise is that the one-SMBH model cannot mimic the real situation as well as the two-SMBH model. It is seen that the two-SMBH static model, the two groups of particles peak nearly the same position, i.e., the two-SMBH static model gives a better prediction of $r_{\text{min}}$. The complete comparison of the two models is in our following paper.

$L_z$ is conserved in axisymmetric galaxies. The left and middle panel of Figure 6 show the $L_z$ histogram of the particles with promise and scattered particles in the one-SMBH model for snapshots at $t = 0, 13, 26, 39$, in number and fraction, respectively. We can see from both panels that, for both groups, more than 90% of particles have very small $L_z$ around $10^{-7}$. The right panel shows the $L_z$ histogram of the particles with promise, the scattered particles, and all particles at $t = 0$ in the
one-SMBH model, illustrating that both groups have much lower $L_z$ than the group of all the particles does, which has $L_z$ peaking at around 1.

Figure 7 shows the frequency map of the eight groups of particles. The upper panel shows the frequency maps of the particles with promise at $t = 0$, the scattered particles at $t = 0$, the particles with promise at $t = 13$, and the scattered particles at $t = 13$, respectively. The lower panel shows the frequency maps of the particles with promise at $t = 26$, the scattered particles at $t = 26$, the particles with promise at $t = 39$, and the scattered particles at $t = 39$, respectively.

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the particles with promise at \( t = 13 \), and the scattered particles at \( t = 13 \), respectively. The lower panel shows the frequency maps of the particles with promise at \( t = 26 \), the scattered particles at \( t = 26 \), the particles with promise at \( t = 39 \), and the scattered particles at \( t = 39 \), respectively. It is seen that the main resonant orbital type is \( f_1f_2 = 1:1 \). We can also notice this in Figure 8, where the left panel shows the number of particles in each type for the eight groups, and the right panel shows the fraction of types in each group. Except for the scattered particles at \( t = 0 \), all of the other groups have chaotic orbits for the most part, then \( f_1f_2 = 1:1 \) orbits, then other resonant orbits. It is also no surprising that for each type, the number of particles decreases from \( t = 13 \) to \( t = 39 \). While also, except at \( t = 0 \), all of the other groups have the same fraction for the three orbital types—all have chaotic orbits around 50%, \( f_1f_2 = 1:1 \) resonant orbits around 40%, and other resonant orbits around 10%. This again verifies that the prediction groups are the same groups of particles as the scattered particles, except the groups at \( t = 0 \). Meanwhile, it shows that chaotic orbits dominate in number both in prediction and scattered groups, as is expected in the vicinity of an SMBHB.

4. Conclusions and Discussion

Earlier studies have shown that in realistic situations of a galaxy merger, where a remnant is non-spherical, an SMBHB evolves efficiently in the three-body scattering phase until the point where the strong emission of gravitational waves sets in. In this study we performed an orbital analysis for stellar orbits that interact with SMBHBs during the three-body scattering phase. We performed direct N-body simulations of an equal-mass pair of SMBHBs in a flat galaxy model that can accommodate an SMBH binary merger in few billion years, as was shown in Khan et al. (2013). We want to know if the particles, which in a one-SMBH-embedded-in galaxy can get very close to the center (i.e., \( r_{\text{min}} < a_{\text{th}} \)), still can do that in the same galaxy with two-SMBH-embedded-in making the SMBHB merge. As the merger progresses, the SMBHB loses energy, and the stellar particles gain energy. We define the particles with the most energy changed as the scattered particles by the SMBHB. At the same time we fix this model’s potential in the snapshots at \( t = 0, 13, 26, 39 \), making only one SMBH at the center of the galaxy and predict the particles that can potentially interact with the SMBHB by checking the particles’ \( r_{\text{min}} \) before \( t = 52 \) as in Li et al. (2015). We define those particles as the particles with promise. Then we rerun the two groups of particles for 100 dynamical times to obtain each particle’s frequency and the \( z \) component of angular momentum, \( L_z \). We use Laskar’s frequency mapping technique to classify orbits according to the dominant frequency ratios along the principle axes.

To summarize, we find that, after the SMBHB hardens, the particles with promise and the scattered particles are drawn from the same collection in phase space, although they do not have too many particles in common. Between \( t = 0 \) to \( t = 52 \) (the SMBHB hardens at \( t = 8 \)), for most of the particles, energy change comes from the variation of the central density of the galaxy and self interactions between them and not from the interaction with the SMBHB. Some main conclusions are as follows.

1. The number of scattered particles in all of the snapshots except the one at \( t = 0 \) is less than that of the particles with promise, which may be caused by \( a_{\text{th}} \) decreasing after the SMBHB’s hardening, while in the prediction we apply a constant of \( a_{\text{th}} \).
2. The energy and \( L_z \) distribution of the particles with promise and the scattered particles except at \( t = 0 \) are nearly the same, showing that the two groups of particles are drawn from the same collection in phase space.
3. An SMBHB scour(s) central cusp up to a few (~4) influence radii of one SMBH via the ejection of stars, which means SMBHBs can draw from a larger reservoir of orbits to scatter during the three-body scattering phase.
4. The \( r_{\text{min}} \) of the scattered particles is bigger than that of the particles with promise in the one-SMBH model, while in the two-SMBH static model the two groups of particles have the same peak for \( r_{\text{min}} \) i.e., the two-SMBH static model has a better prediction of \( r_{\text{min}} \). We will compare the two models in our following paper.
5. Except at \( t = 0 \), for both the particles with promise and the scattered particles, nearly 50% of them have chaotic orbits, 40% have \( f_1f_2 = 1:1 \) orbits, and the remaining 10% have other resonant orbits. This further verifies that the two groups of particles are drawn from the same collection in phase space. Generally, it is thought that the orbits in galactic nuclei are significantly made of chaotic orbits (Poon & Merritt 2001), while there is scare
literature on the orbital type of the particles that interact with the SMBHB. We will investigate if it is still the case in the two-SMBH static model in our following paper.

B.L. acknowledges Sarah Bird for help editing the paper. B.L. also acknowledges support from the International Postdoctoral Exchange Fellowship provided by the Office of China Postdoctoral Council. The simulations were performed on the facilities of the Center for High Performance Computing at Shanghai Astronomical Observatory. F.K. acknowledges the dedicated GPU cluster ACCRE at the Advanced Computing Center for Research and Education at Vanderbilt University, Nashville, TN, USA, on which direct N-body simulations are performed.

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