**I. INTRODUCTION**

One of the main mysteries of the standard cosmological paradigm is the explanation of the excess of matter over antimatter, which is verified by Cosmic Microwave Background observations [1], as well as from Big Bang nucleosynthesis predictions [2]. Gravitational baryogenesis is one of the mechanisms that have been proposed for the generation of such baryon-anti-baryon asymmetry [3-9]. Moreover, the gravitational baryogenesis has some crucial effects on singular inflation (see for example [10]). This mechanism for baryon asymmetry incorporates one of Sakharov’s criteria [11], and the baryon-anti-baryon asymmetry is obtained by the presence of a CP-violating interaction term of the form

\[
\frac{1}{M^2} \int d^4x \sqrt{-g} (\partial_{\mu} R) J^\mu.
\]

Such a term could be acquired from higher-order interactions in the fundamental gravitational theory [3]. In particular, \(M_\star\) is the parameter that denotes the cutoff scale of the underlying effective theory, \(J^\mu\) is the baryonic matter current, and \(g\) and \(R\) are respectively the metric determinant and the Ricci scalar. If one applies the above in the case of a flat Friedmann-Robertson-Walker (FRW) geometry, then the baryon-to-entropy ratio \(\eta_B/s\) is proportional to \(R\), and especially in the case where the matter fluid corresponds to relativistic matter with equation-of-state parameter \(w = 1/3\) then the net baryon asymmetry generated by the term (1) is zero.

In the present work we are interested in investigating the gravitational baryogenesis mechanism in the framework of \(f(T)\) gravity, which is a gravitational modification based on the torsional (teleparallel) formulation of gravity. In particular, in the Teleparallel Equivalent of General Relativity (TEGR) \([12, 15]\) the gravitational Lagrangian is the torsion scalar \(T\), and hence one can construct torsional modified gravity by extending it to \(f(T)\) \([16, 17]\) (see \([18]\) for a review). The interesting point is that although TTEGR is completely equivalent with general relativity at the equation level, \(f(T)\) gravity corresponds to different gravitational modification than \(f(R)\) one, and therefore its cosmological implications bring novel features \([20, 22]\).

Particularly, we shall examine in detail the effects of various gravitational baryogenesis terms which are proportional to \(\partial_{\mu} T\) or \(\partial_{\mu} f(T)\). As we will show, for the simplest choice of \(f(T)\) gravity, the resulting baryon-to-entropy ratio can be compatible to observations, only if some parameters are chosen to be abnormally large. Furthermore, we will constrain the functional form of more general \(f(T)\) grammes which can realize a radiation dominated Universe, and finally we shall discuss how more general cosmologies can be contribute successfully to the gravitational baryogenesis scenario.

This paper is organized as follows: In section III we briefly review the fundamental properties of \(f(T)\) gravity. In section IV we discuss various gravitational baryogenesis scenarios in the context of \(f(T)\) gravity, and we examine the qualitative implications on the baryon-to-entropy ratio which we calculate in detail for each case under study. Finally, the conclusions follow in the end of the paper.

**II. TELEPARALLEL AND \(f(T)\) GRAVITY**

In teleparallel gravity the dynamical variables are the tetrads \(e^\mu_A\), which form an orthonormal base for the tangent space at each spacetime point. The metric is then expressed as

\[
g_{\mu\nu} = \eta_{AB} e^\mu_A e^\nu_B,
\]

where Greek and Latin indices indices span the coordinate space and tangent space respectively. Moreover,
we obtain the modified Friedmann equations

\[ T^\rho_{\mu\nu} \equiv e^\lambda_A (\partial_\mu e^A_{\nu} - \partial_\nu e^A_{\mu}) . \]  

(3)

The Lagrangian of the theory is the torsion scalar \( T \), constructed by contractions of the torsion tensor as

\[ T = \frac{1}{4} T^{\rho\mu\nu} T_{\mu\nu\rho} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\rho} - T^{\rho}_{\mu\nu} T^{\mu\nu}_{\rho} . \]  

(4)

Similarly to \( f(R) \) gravity where one extends the Einstein-Hilbert Lagrangian, namely the Ricci scalar \( R \), to an arbitrary function \( f(R) \), one can generalize \( T \) to \( f(T) \) gravity by variation \( f \) of the total action

\[ S = \int d^4x e \left[ \frac{T + f(T)}{2\kappa^2} \right] , \]  

(5)

where \( e = \det (e^A_{\mu}) = \sqrt{-g} \) and \( \kappa^2 = 8\pi G = M_p^{-1} \) is the gravitational constant, with \( M_p \) the Planck mass.

The field equations for \( f(T) \) gravity arise by variation of the total action \( S + S_M \), where \( S_M \) is the matter action, in terms of the tetrads, namely

\[ e^{-1} \partial_\mu (e^\alpha_A S^\rho_{\mu\nu}) (1 + f_T) - (1 + f_T) e^\lambda_A T^{\rho\mu\lambda} S^\nu_{\rho\mu} + \frac{e^\rho_A S^\mu_{\rho\nu}}{2} \partial_\mu (T + f(T)) = \frac{k^2}{4} e^\rho_A T^{(M)}_{\rho \nu} . \]  

(6)

with \( f_T = df(T)/dT \) and \( f_T = df(T)/dT^2 \), and where the “super-potential” tensor \( S^\rho_{\mu\nu} = \frac{1}{2} (K^{\rho}_{\mu \nu} + \delta_\rho^{\mu} T^{\alpha\nu}_{\alpha} - \delta_\nu^{\mu} T^{\alpha\rho}_{\alpha}) \) is defined in terms of the co-torsion tensor \( K^{\mu}_{\nu \rho} = \frac{1}{2} (T^{\mu\rho\nu} - T^{\nu\rho\mu} - T^{\rho\mu\nu}) \). Additionally, \( T^{(M)}_{\rho \nu} \) denotes the energy-momentum tensor corresponding to \( S_M \). Note that when \( f(T) = T \) one obtains the teleparallel equivalent of general relativity, in which case equations (3) coincide with the field equations of the latter.

Let us now apply \( f(T) \) gravity in a cosmological framework. We consider a flat FRW background geometry with metric

\[ ds^2 = dt^2 - a(t)^2 (\delta_{ij} dx^i dx^j) , \]  

(7)

with \( a(t) \) the scale factor, which arises from the diagonal vierbein

\[ e^\lambda_A = \text{diag}(1, a, a, a) . \]  

(8)

Inserting the vierbein choice (8) into the field equations (6) we obtain the modified Friedmann equations

\[ H^2 = \frac{8\pi G}{3} \rho - \frac{f}{6} + \frac{T f_T}{3} , \]  

\[ \dot{H} = -\frac{4\pi G(\rho + P)}{1 + f_T + 2T f_{TT}} , \]  

(9) (10)

with \( H \equiv \dot{a}/a \) is the Hubble parameter, and where a “dot” denotes the derivative with respect to \( t \). In the above equations \( \rho \) and \( P \) correspond to the effective energy density and the pressure of the matter content of the Universe. Finally, note that we have used the relation

\[ T = -6H^2 , \]  

(11)

which according to (4) holds for an FRW Universe.

### III. Baryogenesis in \( f(T) \) Gravity

Let us now work in the torsional formulation of gravity, and consider a \( CP \)-violating interaction term of the form

\[ \frac{1}{M_p^2} \int d^4x \sqrt{-g} (\partial_\mu (-T)) J^\mu . \]  

(12)

In the analysis to follow we assume that thermal equilibrium exists, and in all cases which we study we assume that the Universe evolves slowly from an equilibrium state to an equilibrium state, with the energy density being related to the temperature \( T \) of each state as

\[ \rho = \frac{\pi^2}{30} g_* T^4 , \]  

(13)

where \( g_* \) denotes the number of the degrees of freedom of the effectively massless particles \( \mathcal{E} \). Hence, for the \( CP \) violating interaction of Eq. (12), the induced chemical potential straightforwardly reads \( 3 \mu \sim \pm T/M_p^2 \), and thus the corresponding baryon-to-entropy ratio becomes

\[ \frac{n_B}{s} \sim \frac{15g_b T}{4\pi^2 g_* M_p^2 T} \bigg|_{TD} , \]  

(14)

where \( TD \) is the temperature in which the baryon current violation decouples. Now depending on the specific torsional gravity that controls the evolution, certain differences may occur, which we discuss in the following two subsections.

#### A. Simple teleparallel gravity

In simple TEG, if the Universe is filled with a perfect fluid with constant equation of state parameter \( \omega = P/\rho \), the torsion scalar is given by (11), which using (9) becomes

\[ T = -16\pi G \rho . \]  

(15)

Interestingly enough, according to (15) it can be seen that the resulting baryon-to-entropy ratio is not zero, independently of the equation-of-state parameter of the Universe. This is a radical contrast with general relativity, where in the case of a radiation dominated era the resulting baryon-to-entropy ratio is zero (9), since the Einstein-Hilbert equations are of the form

\[ R = -8\pi G(1 - 3\omega) \rho . \]  

(16)
Let us see what the simple form of equation (15) implies for the baryon-to-entropy ratio. Firstly, by assuming a radiation dominated Universe, the energy density is of the form $\rho = \rho_0 a^{-4}$, with $a(t)$ the scale factor. Consequently, the differential equation (15) can be analytically solved to yield the scale factor

$$a(t) = \left(8\sqrt{\pi G \rho_0 t + 1}\right),$$  

(17)

where we have assumed that $a(0) = 1$. We mention that mathematically there is an additional solution to Eq. (15) for a radiation dominated Universe, but since it is unphysical we omit it. By using (15), (17) and (13), we can obtain the decoupling time $t_D$ as a function of the decoupling temperature $T_D$, namely

$$t_D = \frac{3}{4\pi G} \sqrt{\frac{5}{\rho_0 g_* T_D^3} - 1}.$$  

(18)

Note that the relation (18) is an exact relation and not a leading order approximation, and this situation is unique in the cases we shall consider in this work, since in more complicated $f(T)$ theories leading order approximations are going to be used. Then, inserting everything in (14), the resulting baryon-to-entropy ratio $n_B/s$ reads

$$\frac{n_B}{s} \simeq -\frac{576\pi^5/2 G^{3/2} g_\ast T_D^3 a^4}{g_\ast M_\ast^2 \rho_0^{5/2} \left[\pi T_D^2 (8\sqrt{\pi G \rho_0} - 1) - 6 \sqrt{\frac{5}{g_\ast}}\right]^3}.$$  

(19)

We proceed by investigating how the free parameters of the theory affect the resulting baryon-to-entropy ratio in this simple torsional theory. As a specific example we assume that the cutoff scale $M_\ast$ is equal to $M_\ast = 10^{12}$GeV, the decoupling temperature is $T_D = M_\ast = 2 \times 10^{16}$GeV, with $M_\ast$ the upper bound for tensor-mode fluctuations constraints on the inflationary scale, and also that $g_\ast \simeq O(1)$, $\rho_0 \simeq 10^{-6}$GeV and $g_\ast \simeq 106$, which is the total number of the effectively massless particle in the Universe [3]. Then, by transforming to Planck units for simplicity, the resulting baryon-to-entropy ratio (19) is found to be $n_B/s \simeq -1.02 \times 10^{-42}$, which is extremely small, compared to the observed value $n_B/s \simeq 9.2 \times 10^{-11}$ and more importantly it is negative, which means that this theory predicts an excess of anti-matter over matter, which is unphysical. The analysis shows that the baryon-to-entropy ratio is robust against the changes of the initial energy density $\rho_0$. The resulting picture reveals that in order to acquire physically consistent predictions, the simple TEGR seems not to produce a viable baryon-to-entropy ratio, and thus a more complicated torsional term is required.

**B. General $f(T)$ theories**

In this subsection we extend the above discussion in the case of generalized $f(T)$ theories. As we will see, we can use the baryon-to-entropy ratio in order to constrain the functional form of $f(T)$ gravity. We start by considering the power-law cosmic evolution with scale factor

$$a(t) = A t^{\gamma},$$  

(20)

with $A, \gamma$ being a positive constants.

Let us now consider the three viable $f(T)$ cases according to observations [24].

- **The power-law model of Bengochea and Ferraro [16],** namely

  $$f(T) = B (-T)^n,$$  

(21)

with $B$ a constant and $n > 1$. Then from Eq. (9) we obtain that the energy density at leading order is

$$\rho \simeq C T^{-2n},$$  

(22)

with the parameter $C$ being equal to $C = B 6^{n-1}(1-18n)\gamma^{2n}$. Then, the decoupling time $t_D$ as a function of the decoupling temperature $T_D$ is found to be

$$t_D = \left(\frac{\pi^2 g_\ast}{30 C}\right)^{1/2} T_D^{-\frac{3}{2}},$$  

(23)

and by using the expression for the torsion scalar (11), the resulting baryon-to-entropy ratio $n_B/s$ becomes

$$\frac{n_B}{s} \simeq -\frac{45 g_\ast (30 C)^{3/2}}{g_\ast M_\ast (\pi^2 g_\ast)^{-\frac{3}{2}} T_D^{-\frac{3}{2}} + 1}.$$  

(24)

Thus, by choosing $M_\ast = 10^{12}$GeV, $T_D = M_\ast = 2 \times 10^{16}$GeV, $\gamma = 0.6$, $n = 5.5$ and $B = -10^{-6}$, the baryon-to-entropy ratio becomes $n_B/s \simeq 7.53 \times 10^{-11}$, which is in very good agreement with observations. In Fig. 1 we present the $n$-dependence of the baryon-to-entropy ratio in a specific example. As it can be seen, both the parameters $B$ and $n$ affect the baryon-to-entropy ratio in a crucial way, and thus the baryon-to-entropy ratio may be used to constrain the functional form of the $f(T)$ gravity.

- **The Linder model [17]  

  $$f(T) = B (1 - e^{-p\sqrt{|T|}}),$$  

(25)

where $B$ and $p$ are the model parameters. In this case Eq. (9) at leading order gives

$$\rho \simeq \frac{B}{6} + \frac{\gamma^2}{t^2},$$  

(26)

The decoupling time $t_D$ as a function of the decoupling temperature $T_D$ becomes

$$t_D \simeq \frac{\sqrt{30y}}{\sqrt{\pi g_\ast T_D^3 - 5 B}},$$  

(27)
and by using the expression for the torsion scalar \(\ddot{T}f\), the resulting baryon-to-entropy ratio \(n_B/s\) reads

\[
\frac{n_B}{s} \approx \frac{5 \sqrt{2} g_W \left( \frac{g \pi^2 T_D^2}{90 B} \right)^{3/2}}{2 g_* M_* p^{3/2} \pi T_D^2}.
\]

Hence, by choosing \(T_D\) and \(M_*\) as previously and also \(p = 10^{-10}, B = 1,\) and \(\gamma = 10^{-2}\) the resulting baryon-to-entropy ratio is \(\frac{n_B}{s} \approx 2.62 \times 10^{-11}\), which is in very good agreement with observations. Finally, note that there is a wide range of values for the parameters \(\gamma\) and \(p\), for which we can achieve compatibility of the baryon-to-entropy ratio with observations.

C. Generalized baryogenesis term

In this section we consider a more general baryogenesis interaction term than \(12\), namely we extend it to

\[
\frac{1}{M_*^2} \int d^4x \sqrt{-g} \left[ \partial_\mu (-T + f(-T)) \right] J^\mu.
\]

For calculation convenience in the following we consider only the \(-T\) part, i.e.

\[
\frac{1}{M_*^2} \int d^4x \sqrt{-g} \left[ \partial_\mu (-T) \right] J^\mu,
\]

since the \(-T\) term was considered in \(12\) and was analyzed in the previous subsection. One can always find the results of the full \(-T + f(-T)\) consideration by adding the \(\frac{1}{M_*^2}\) results of the separate \(-T\) and \(f(-T)\) investigations.

In the case \(54\), the resulting baryon-to-entropy ratio becomes

\[
\frac{n_B}{s} \approx -\frac{15 g_W g_\pi T_F(T)}{4 \pi^2 g_* M_*^2 T_D} |\tau_D|.
\]

Let us apply these into a power-law cosmological evolution of the form

\[
a(t) = A t^\gamma,
\]

with \(A, \gamma > 0\). Similarly to the previous subsection, we consider the three viable \(f(T)\) cases according to observations \(24\).

- For the power-law model of Bengoechea and Ferraro \(21\), and assuming that \(n = 2m + 1,\) with \(m\) a positive integer, it can be shown that the resulting baryon-to-entropy ratio at leading order is

\[
\frac{n_B}{s} \approx -\frac{15 g_W 2^{n+1} 3^n B n g_*^\gamma}{4 \pi^2 g_* M_*^2} \left( \frac{30 C_1}{T_D^2 (2n+1)^{n+1}} \right),
\]

with \(C_1\) being equal to, 

\[
C_1 = 2^{-1-n} 3^{-1+n} B(1 - 18n).
\]

Choosing the values \(M_* = 10^{12}\)GeV, \(T_D = M_I = 2 \times 10^{16}\)GeV, \(n = 15,\) \(\gamma = 1.03 \times 10^{-3.8}\) and \(B = 10^{-10},\) in which case we find that \(n_B/s \approx 9.24 \times 10^{-11}\), which is very close to the observationally accepted value. Alternatively, one can use \(M_* = 10^{12}\)GeV, \(T_D = M_I = 2 \times 10^{16}\)GeV, \(n = 3,\) \(\gamma = 1.23 \times 10^{-23}\) and \(B = 10^{-60},\) and the resulting baryon-to-entropy ratio is \(n_B/s \approx 2.01 \times 10^{-11},\) but in this case, both \(\gamma\) and \(B\) have significantly small values.

- For the Linder model \(23\), the resulting baryon-to-entropy ratio is at leading order,

\[
\frac{n_B}{s} \approx \sqrt{\frac{B}{g_* M_*^2}} \left( \frac{\gamma}{\sqrt{g_*}} \frac{-5B + g_* \pi^2 T_D^2}{\pi^2 T_D^2} \right) \times e^{-2p(-5B + g_* \pi^2 T_D^2)} / \sqrt{s}.
\]
which is significantly small due to the presence of the exponential, regardless of the values we choose (unless of course we choose $p$ to be enormously small, but that is not so appealing).

- For the exponential model [20], the baryon-to-entropy ratio at leading order is,

$$\frac{n_B}{s} \simeq -\frac{10\sqrt{6}Be^{\gamma^2}W\left(\frac{a_\gamma T^4}{m_*}\right)g_0\gamma^2W\left(\frac{2\pi^2T^4}{m_*}\right)^{3/2}}{g_*M_*\sqrt{\pi^2T_D}},$$

(39)

where $W(x)$ is again the Lambert function. Obviously this result is unphysical, since the resulting baryon-to-entropy ratio is negative which means that there is an excess of anti-matter over matter. Therefore, not all $f(T)$ gravities yield similar results.

IV. CONCLUSIONS

In this paper we studied the gravitational baryogenesis scenario, generated by an $f(T)$ theory of gravity. In the context of $f(T)$ baryogenesis, the baryon-to-entropy ratio depends on $T$, and we discussed two cases of $f(T)$ theories of gravity, the case $f(T) = T$ and also $f(T) \sim (-T)^n$. In the first case, the resulting picture is not so appealing since in order for the predicted baryon-to-entropy ratio to be compatible to the observational value, some of the parameters must give abnormally small values. The case $f(T) \sim (-T)^n$ is more interesting and we investigated which values should the parameter $n$ take in order to have compatibility with the data. As we showed, the variable $n$ plays a crucial role in the calculation of the baryon-to-entropy ratio. In both cases the interesting new feature is that the baryon-to-entropy ratio is nonzero for a radiation dominated Universe, in contrast to the Einstein-Hilbert gravitational baryogenesis scenario. Finally, we investigated how more general cosmologies affect the baryon to entropy ratio, and when the gravitational baryogenesis term is of the form $\partial_\alpha T$, inconsistencies may occur in the theory. As we showed, the remedy to this issue is to modify the gravitational baryogenesis term, so that the baryon current is coupled to $\partial_\alpha f(T)$. In this way more general cosmological evolutions can be considered and the resulting baryon-to-entropy ratio is compatible to the observational data.

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