\( \pi N\sigma \) Term, \( \bar{s}s \) in Nucleon, and Scalar Form Factor — a Lattice Study

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Abstract

We report on a lattice QCD calculation of the \( \pi N\sigma \) term, the scalar form factor, and \( \langle N|\bar{s}s|N\rangle \). The disconnected insertion part of \( \sigma_{\pi N} \) is found to be \( 1.8 \pm 0.1 \) times larger than the connected insertion contribution. The \( q^2 \) dependence of \( \sigma_{\pi N}(q^2) \) is about the same as \( G_E(q^2) \) of the proton so that \( \sigma_{\pi N}(2m_\pi^2) - \sigma_{\pi N}(0) = 6.6 \pm 0.6 \) MeV. The ratio \( y = \langle N|\bar{s}s|N\rangle/\langle N|\bar{u}u + \bar{d}d|N\rangle = 0.36 \pm 0.03 \). Both results favor a \( \sigma_{\pi N} \sim 53 \) MeV, slightly larger than our direct calculation of \( \sigma_{\pi N} = 49.7 \pm 2.6 \) MeV. We also compute \( F_s \) and \( D_s \) and find that the agreement with those from the octect baryon mass splittings crucially depends on the inclusion of the large disconnected insertion. Finally, we give our result for the \( KN\sigma \) term.

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Like the pion mass in the meson sector, the $\pi N\sigma$ term is a measure of the explicit chiral symmetry breaking in the baryon sector. It is considered a fundamental quantity which pertains to a wide range of issues in the low-energy hadron physics, such as quark and baryon masses, strangeness content of the nucleon, pattern of SU(3) breaking, $\pi N$ and $KN$ scatterings, kaon condensate in dense matter, trace anomaly, and decoupling of heavy quarks. Defined as the double-commutator of the isovector axial charge with the Hamiltonian density taken between the nucleon states, i.e. $\sigma_{\pi N} = \frac{1}{3} \sum_{a=1,3} \langle N | [Q^5_a, [Q^5_a, \mathcal{H}(0)]] | N \rangle$ which appears in the off-shell $\pi N$ scattering amplitude $[1]$, it has in QCD the expression

$$\sigma_{\pi N} = \hat{m} \langle N | \bar{u}u + \bar{d}d | N \rangle,$$

where $\hat{m} = (m_u + m_d)/2$.

It is shown $[2]$ that at lowest order in $m_\pi$ (i.e. $m_\pi^2$), it is equal to the unphysical, but on-shell, isospin even $\pi N$ scattering amplitude at the Cheng-Dashen point, $\Sigma_{\pi N} = f_\pi^2 \hat{T}^+ (s = m_N^2, t = q^2 = 2m_\pi^2)$. Thus $\Sigma_{\pi N}$ can be extracted from $\pi N$ scattering experiment via fixed-$t$ dispersion relation for instance $[2]$. It is further shown $[3]$ that the next higher order term which is nonanalytic in quark mass (i.e. proportional to $\hat{m}^3/2$ or $m_\pi^3$) drops out if $\Sigma_{\pi N}$ is identified with $\sigma_{\pi N}(2m_\pi^2)$ $[3]$ which is only a function of $q^2$. This shows that the difference $\Delta_R$ in the relation $\Sigma_{\pi N} = \sigma_{\pi N}(2m_\pi^2) + \Delta_R$ is of the order $m_\pi^4/m_N^4$ and has been shown to be indeed negligible ($\sim 0.35$ MeV) in a chiral perturbation calculation $[3, 4]$.

Various estimates of $\Sigma_{\pi N}$ have ranged from 22 to 110 MeV over the years, but eventually settled around 60 MeV $[4]$. On the other hand, a puzzle was raised by Cheng $[5]$. If one assumes that $\langle N | \bar{s}s | N \rangle = 0$, a reasonable assumption from the OZI rule, the $\sigma_{\pi N}^{(0)}$ obtained from the octet baryon masses gives only 32 MeV, almost a factor two smaller than $\Sigma_{\pi N}$ extracted from the $\pi N$ scattering. This puzzle was tackled from both ends. First, the scalar form factor was calculated $[1]$ in chiral perturbation theory ($\chi$PT) with the two correlated pions as the dominating intermediate state. As a result, the scalar form factor is found to be exceedingly soft which leads to a large change of $\sigma_{\pi N}(q^2)$ in a small range of $q^2$, i.e. $\Delta \sigma_{\pi N} = \sigma_{\pi N}(2m_\pi^2) - \sigma_{\pi N}(0) = 15.2 \pm 0.4$ MeV. Thus, this reduces $\sigma_{\pi N}$

$$\sigma_{\pi N} = \Sigma_{\pi N} - \Delta \sigma_{\pi N}$$

(2) to $\sim 45$ MeV. The remaining discrepancy between $\sigma_{\pi N}$ and $\sigma_{\pi N}^{(0)}$ is reconciled if one admits the possibility of a large $\bar{s}s$ content in the nucleon $[3, 6]$. From the pattern of SU(3) breaking in the octet baryon masses, one finds $[4, 6]$

$$\sigma_{\pi N} = \sigma_{\pi N}^{(0)}/(1 - y),$$

(3) where $y = 2 \langle N | \bar{s}s | N \rangle / \langle N | \bar{u}u + \bar{d}d | N \rangle$. Given $\sigma_{\pi N}^{(0)} = 32$ MeV from the octet baryon masses $[3]$, or $35(5)$ MeV from the one loop $\chi$PT calculation $[1]$ and $\sigma_{\pi N} = 45$ MeV, eq. (3) implies $y = 0.2 - 0.3$.

Hence, a consistent solution seems to have emerged which suggests that $\sigma_{\pi N} \sim 45$ MeV, $\Delta \sigma_{\pi N} \sim 15$ MeV, and $y \sim 0.2 - 0.3$. In this letter, we undertake a lattice
QCD calculation of the above quantities to scrutinize the viability of this resolution. It turns out that our study strongly points to a significantly different solution as we shall show.

The calculation of $\sigma_{\pi N}$ in lattice QCD has been attempted by several groups [7, 8] who employed the Feynman-Hellman theorem

$$\hat{m} \frac{\partial M_N}{\partial \hat{m}} = \hat{m} \langle N | \bar{u}u + \bar{d}d | N \rangle_{C.I.} + \hat{m} \langle N | \bar{u}u + \bar{d}d | N \rangle_{D.I.}$$

and obtained $\sigma_{\pi N}$ through the derivative of the nucleon mass. We note that in eq. (4) the connected insertion (C.I.) part comes from the differentiation with respect to the valence quark propagator; whereas the disconnected insertion (D.I.) comes from the derivative of the fermion determinant. Their contributions to the scalar current $\bar{\psi}\psi$ in the nucleon are shown schematically in Fig. 1. In the quenched approximation, it is found that $\sigma_{\pi N}$ obtained from the derivative of the nucleon mass is only about 15 — 25 MeV [7]. This is much smaller than the phenomenological value of $\sim 45$ MeV [4]. The smallness of $\sigma_{\pi N}$ in this case is traced to the fact that the nucleon mass in the quenched approximation is calculated with the determinant set to a constant, so that its derivative corresponds to the C.I. only (which is verifiable by comparing to the direct evaluation of the C.I. [9, 10]) and it does not involve D.I. which can be substantial. Indeed, when the derivative of $M_N$ is calculated with dynamical fermions included, it is found [8] that the l.h.s. of eq. (4) which now includes the D.I. becomes $\sim 2$ to 3 times larger than the C.I. contribution. This implies a large contribution of the D.I.. Since the error on $\partial M_N/\partial \hat{m}$ is quite large [10, 8], we decided to calculate the D.I. directly [10] with the help of the $Z_2$ noise [11]. Following our calculation of the flavor-singlet $g_0^A$ [12], we calculate the C.I. and D.I. of $\sigma_{\pi N}$ directly in the quenched approximation. In terms of the Feynman-Hellman theorem, it would correspond to calculating $\partial M_N/\partial \hat{m}$ by taking the derivative of the determinant first before setting it to a constant.

Lattice calculations of three-point functions have been used to study the EM [13], axial (isovector) [14], pseudoscalar $\langle \pi NN \rangle$ [15] form factors, and the flavor-singlet $g_0^A$ [12]. For the scalar current, we use $S(x) = 2\kappa/8\kappa_c[\bar{u}u(x) + \bar{d}d(x)]$, where we have implemented the mean-field improvement factor $8\kappa_c$ to define the lattice operator [16].

The C.I. is calculated in the same way as the isovector axial coupling $g_3^A$ [14]. Numerical details are given in Ref. [14]. Like in Ref. [12], the lattice renormalized $g_{S,con}^L = \langle N | \bar{u}u + \bar{d}d | N \rangle_{con}^{L}$ has been calculated for $\kappa = 0.154, 0.152, \text{ and } 0.148$, corresponding to quark masses of about 120, 200, and 370 MeV respectively (the scale $a^{-1} = 1.74(10)$ GeV is set by the nucleon mass), and is plotted in Fig. 2(a). The calculations were done on a quenched $16^3 \times 24$ lattice at $\beta = 6.0$ with 24 gauge configurations as in the previous cases [14, 12]. Due to the fact that the quenched $\chi$PT calculation exhibits a leading non-analytic behavior of $m^{3/2}$ for the nucleon mass [17], we extrapolate $g_{S,con}^L$ to the chiral limit ($\kappa_c = 0.1568$) with the form $C + Dm^{1/2}$. This is so because $g_{S,con}^L = \partial M_N/\partial \hat{m}$ in the quenched approximation as we alluded to ear-
lier in eq. (1). As a result, we obtain \( g_{S,\text{con}}^L = 3.04(9) \) as shown in Fig. 2(a). The \( g_S \) in the continuum with the \( MS \) scheme is related to its lattice counterpart by the relation \( g_S = Z_S g_S^L \), where \( Z_S \) is the finite lattice renormalization constant. The one-loop calculation gives \( Z_S = 0.995 \) for \( \beta = 6.0 \) \cite{4}, from which we find \( g_{S,\text{con}} = 3.02 \pm 0.09 \).

We also computed isovector \( g_3^S = \langle N | \bar{u}u - \bar{d}d | N \rangle \) which does not involve the D.I. and find it to be 0.63(7).

Since \( \sigma_{\pi N} \) is renormalization group invariant, the C.I. contribution is \( \sigma_{\pi N,\text{con}} = \hat{m} g_{S,\text{con}}^L \) where \( \hat{m} \) is the lattice quark mass. From \( m_\pi^2 \) and \( M_N \), we find \( \hat{m} = 5.84(13) \) MeV. Thus, \( \sigma_{\pi N,\text{con}} = 17.8(9) \) MeV which agrees well with previous calculations \cite{4,44,24}. The C.I. part of the form factor is obtained by extrapolating \( g_{S,\text{con}}^L \) at different \( \kappa \) to the chiral limit. It is plotted in Fig. 2(b) together with \( g_A^S \), the isovector axial form factor. We see that they are almost identical within errors. In so far as the concept of meson dominance goes, this reflects in part the fact that the isovector scalar meson and \( A_1 \) are essentially degenerate in the lattice calculation.

Like in the case of the axial coupling constants \cite{12}, we also find that the ratio \( R_S = g_3^S / g_{S,\text{con}}^L \) dips below the SU(6) result of 1/3 as the quark mass becomes lighter. This is interpreted as due to the cloud quark/anti-quark effect and is responsible for the \( \bar{u} - \bar{d} \) parton difference reflected in the Gottfried sum rule \cite{21}. Only when the cloud degree of freedom is eliminated in the valence approximation \cite{21} where the Fock space is limited to the valence do we recover the SU(6) limit. This indirectly shows the effect of the cloud quarks in the C.I.

We calculate the D.I. in Fig. 1(b) the same way we did for the D.I. part of \( g_A^0 \) \cite{12} by summing over \( t \). For \( t_f >> a \), this sum becomes

\[
\sum_t \frac{G_{PS}^\alpha(t_f, \vec{p}, t, \vec{q})G_{PP}^\alpha(t, \vec{p})}{G_{PP}(t_f, \vec{p})G_{PP}(t, \vec{q})} \rightarrow t_f >> a \quad \text{const} + t_f g_{S,\text{dis}}^L(q^2) \tag{5}
\]

Thus, we calculate the sum as a function of \( t_f \) and take the slope to obtain the D.I. part of \( g_S^L \). Since the D.I. involves quark loops which entail the calculation of traces of the inverse quark matrices, we use the proven efficient algorithm to estimate these traces stochastically with the \( Z_2 \) noise \cite{11} which was applied to the study of \( g_A^0 \) \cite{12}.

The results of eq. (5) with 300 complex \( Z_2 \) noise and 50 gauge configurations for \( \kappa = 0.148, 0.152 \) and 0.154 are presented in Fig. 3. The corresponding \( g_{S,\text{dis}}^L = \langle N | \bar{u}u + \bar{d}d | N \rangle_{\text{dis}}^L \) are obtained from fitting the slopes in the region \( t_f \geq 8 \) where the nucleon is isolated from its excited states with the correlation among the time slices taken into account \cite{12}. The resultant fits covering the ranges of \( t_f \) with the minimum \( \chi^2 \) are plotted in Fig. 3. Finally, the errors on the fit, also shown in the figure, are obtained by jackknifing the procedure.

Plotted in Fig. 4(a) are the results of \( g_{S,\text{dis}}^L \) with the same sea-quark mass as those of the valence- (and cloud-) quarks in the nucleon. They suggest a non-linear behavior in the quark-mass. This is enhanced by our finding of a very soft form factor (Fig. 4(b)) which is consistent with the expectations of \( \chi PT \) \cite{8} where the pion loop leads to a non-analytic behavior in \( m_q^3/2 \). Furthermore, this non-linear behavior is
seen prominently in hadron masses when dynamical fermions are included [22]. For these reasons, we fit \( \langle N|\bar{u}u + \bar{d}d|N\rangle_{\text{dis}} \) with the linear plus \( m^{1/2} \) form as for the C.I. and get a small \( \chi^2 \) (see Fig. 4(a)). The extrapolation to the chiral limit is carried out in the same way as in the case of \( g_A^0 \) [12]. To calculate \( \langle N|\bar{s}s|N\rangle \), we fix the sea-quark mass at 0.154 and extrapolate the valence-quark mass to the chiral limit with the form \( C + D\sqrt{m + m_s} \) to reflect the \( m_k^3 \) dependence of the nucleon mass from the kaon loop in \( \chi PT \). These results are also plotted in Fig. 4(a).

From Fig. 4(a), we find that \( \langle N|\bar{u}u + \bar{d}d|N\rangle_{\text{dis}} \) is significantly improved from the valence quark model which predicts \( \Delta \approx 0.06(9) \text{ fm} \) can be interpreted as the size of the two-\( \pi \) intermediate state in the \( \chi PT \) calculation [4]. This possibility can be seen in Fig. 1(b) with two \( \pi \) dominance. Indeed, if we assume that the D.I. part completely saturates \( \sigma_{\pi N} \) with \( g_S = 8.43(24) \), it would give \( \Delta \sigma_{\pi N} = 11.5(2.1) \text{ MeV} \) similar to that of the \( \chi PT \) calculation [4]. However, there is also the C.I. part (fig. 2(b)) which is much harder than the D.I. When combined, it yields a scalar form factor \( g_S(q^2) \) which is softer than \( g_A^0(q^2) \) and becomes close to \( G_E(q^2) \) of the proton. They are plotted in Fig. 5 for comparison. Fitting the \( g_S(q^2) \) to a dipole form gives a dipole mass \( m_D = 0.80(4) \text{ GeV} \). This predicts \( \Delta \sigma_{\pi N} = 6.6(6) \text{ MeV} \), much smaller than the 15.2(4) MeV obtained solely based on the two-\( \pi \) dominance. We conclude from this that the \( \chi PT \) calculation [4] is relevant to the D.I. but missed the C.I. which maybe dominated by the scalar meson. On the other hand, the \( \langle N|\bar{s}s|N\rangle(q^2) \) comes only from the D.I., hence is very soft. Its r.m.s. radius \( \langle r^2 \rangle^{1/2}_{\bar{s}s} = 1.06(9) \text{ fm} \) can be interpreted as the size of the \( K\bar{K} \) meson cloud in the scalar channel (see Fig. 1(b)).
For the parameter $y$ in eq. (3), we find it to be 0.36(3). Both $\Delta \sigma_{\pi N}$ and $y$ differ significantly from the phenomenological solution based on $\chi PT$ as mentioned earlier which did not take into account the $C. I.$ with a possible scalar dominance. Our results on $\Delta \sigma_{\pi N}$ and $y$ strongly suggest a higher $\sigma_{\pi N} = \Sigma_{\pi N} - \Delta \sigma_{\pi N} \sim 53$ MeV, assuming $\Sigma_{\pi N} \sim 60$ MeV and $\sigma(0) \sim 32 - 35$ MeV. Now, we turn to our result of $\sigma_{\pi N}$. Our direct calculation gives $\langle N|\bar{u}u + \bar{d}d|N \rangle = 8.43(24) \text{ and } \sigma_{\pi N} = 49.7(2.6)$ MeV. This is about one and half $\sigma$ smaller than 53 MeV inferred from $\Delta \sigma_{\pi N}$ and $y$. Since the direct computation of $\sigma_{\pi N}$ involves the determination of the quark mass which is more susceptible to systematic errors (such as the extrapolation in the quark mass and the infinite volume limit) than the $q^2$ dependence of the form factor and the ratio $y$, we believe that our result on $\sigma_{\pi N}$ is less reliable than $\Delta \sigma_{\pi N}$ and $y$. To examine the sensitivity of these three quantities as far as the chiral limit extrapolation is concerned, we fit them to a linear function in $m$ instead of $m^{1/2}$ and find that $\Delta \sigma_{\pi N} = 4.7(8)$ MeV, $y = 0.42(3)$, and $\sigma_{\pi N} = 39.0(2.0)$ MeV. Again, we see that both $\Delta \sigma_{\pi N}$ and $y$ favor a higher $\sigma_{\pi N} \sim 55$ MeV which is very close to the above estimate of 53 MeV with the $m^{1/2}$ extrapolation. Yet, the directly calculated $\sigma_{\pi N}$ falls short of this expectation and is also much smaller than that of the $m^{1/2}$ extrapolation.

Table 1: Scalar contents, $\Delta \sigma_{\pi N}$, $y$ and $\sigma_{\pi N}$ in comparison with phenomenology

| Lattice | Phenomenology |
|---------|--------------|
| $\langle p|\bar{u}u|p \rangle$ | 4.55(16) |
| $\langle p|\bar{d}d|p \rangle$ | 3.92(16) |
| $\langle N|\bar{s}s|N \rangle$ | 1.53(7) |
| $F_S$ | 1.51(12) |
| $D_S$ | -0.88(28) |
| $\langle r^2 \rangle_{S/2}^{1/2}(ud)$ | 0.85(4) fm |
| $\langle r^2 \rangle_{S/2}^{1/2}(s)$ | 1.06(9) fm |
| $\Delta \sigma_{\pi N}$ | 6.61(59) MeV |
| $\sigma_{\pi N}$ | 49.7(2.6) MeV |
| $\sigma_{KN}$ | 362(13) MeV |

Clearly, calculations on larger lattices and smaller quark masses will be needed to bring the systematic errors under control and obtain a completely consistent solution on $\Delta \sigma_{\pi N}$, $y$, and $\sigma_{\pi N}$. Eventually dynamical fermions need to be included to complete the picture. Nevertheless, based on what we have on a qualitative and semi-quantitative level, we find that a consistent solution might be close to $\Delta \sigma_{\pi N} = 6.6(6)$ MeV, $y = 0.36(3)$, and $\sigma_{\pi N} \sim 53$ MeV which are significantly different from the present phenomenological values. We should stress that our results on $F_S$ and $D_S$, like their counterparts in the axial couplings, agree well with those deduced from the SU(3) breaking pattern of the octet baryon masses and that the $D.I.$ is the important ingredient for this agreement. In addition, we report the $KN\sigma$ term $\sigma_{KN} = (\hat{m} + m_s)\langle N|\bar{u}u + \bar{d}d + 2\bar{s}s|N \rangle/4$ in Table 1. If we assume that they
are similarly depressed as in $\sigma_{\pi N}$, we would then predict the final $\sigma_{KN}$ at 389(14) MeV. It agrees with $\sigma_{KN} = 395$ MeV from a recent chiral analysis of $K\pi$ scattering [24]. Finally, we note that $m_s \langle N|\bar{s}s|N \rangle = 183(8)$ MeV. Together with the kinetic and potential energy contribution of $-90$ MeV [25], the strange quark contributes about 90 MeV to the nucleon mass.

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References

[1] S. Weinberg, Phys. Rev. Lett. 17, 616 (1966).
[2] T. P. Cheng and R. Dashen, Phys. Rev. Lett. 26, 594 (1971).
[3] J. Gasser, M.E. Sainio, and A. Švarc, Nucl. Phys. B307, 779 (1988); L.S. Brown, W.J. Pardee, and R.D. Pecci, Phys. Rev. D4, 2801 (1971).
[4] J. Gasser, H. Leutwyler, and M.E. Sanio, Phys. Lett. B253, 252, 260 (1991).
[5] T.P. Cheng, Phy. Rev. D13, 2161 (1976); *ibid* D38, 2869 (1988).
[6] J. Gasser and H. Leutwyler, Phys. Rep. 87, 77 (1982); J. F. Donoghue and C. R. Nappi, Phys. Lett. B168, 105 (1986).
[7] S. Güsken et al., Phys. Lett. B212, 216 (1988); P. Bacilieri et al., *ibid*. B214, 115 (1988); S. Cabasino et al., *ibid*. B258, 195 (1991).
[8] R. Gupta et al., Phys. Rev. D44, 272 (1991); C. Bernard et al., *ibid*. D48, 4419 (1993).
[9] L. Maiani et al., Nucl. Phys. 293, 420 (1987).
[10] S. J. Dong and K. F. Liu, Nucl. Phys. B (Proc. Suppl.) 26, 353 (1992).
[11] S.J. Dong and K.F. Liu, Phys. Lett. B328, 130 (1994).
[12] S.J. Dong, J.-F. Lagaë, and K.F. Liu, Phys. Rev. Lett. 75, 2096 (1995).
[13] T. Draper, R.M. Woloshyn and K.F. Liu, Phys. Lett. 234B, 121 (1990); W. Wilcox, T. Draper, and K.F. Liu, Phys. Rev. D 46, 1109 (1992).
[14] K.F. Liu, S.J. Dong, T. Draper, J.M. Wu, and W. Wilcox, Phys. Rev. D 49, 4755 (1994).
[15] K.F. Liu, S.J. Dong, T. Draper, and W. Wilcox, Phys. Rev. Lett. 74, 2172 (1994).
Figure Captions

Fig. 1 (a) Connected insertion. (b) Disconnected insertion.

Fig. 2 (a) The lattice $g_{L,con}^L$ for the C. I. as a function of the quark mass $m_a$. The chiral limit result is indicated by $\bullet$. (b) The form factor $g_{L,con}^L(q^2)$ at the chiral limit.

Fig. 3 The ratios of eq.(5) for the scalar current are plotted for the 3 $\kappa$ cases. ME is the fitted slope.

Fig. 4 (a) The D.I. of $\langle N|\bar{u}u + \bar{d}d|N \rangle$ and $\langle N|\bar{s}s|N \rangle$ as a function of $m_a$. The chiral limit result is indicated by $\bullet$. (b) The corresponding form factors.

Fig. 5 The normalized form factor $g_{S}(q^2)/g_{S}(0)$ compared with $G_E(q^2)$ and $g^{A}_{3}(q^2)$ and their respective experimental results.
Fig. 4

Disconnected Insertion

Form factors of disconnected insertion

(a)

(b)

g_S(q^2), G_E(q^2) & g_A(q^2)

Fig. 5
Fig. 1

Fig. 2

Fig. 3