CONSTRAINING THE FLARING REGION OF SAGITTARIUS A\(^*\) BY 1.3 mm VLBI MEASUREMENTS

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ABSTRACT

We use a model of an accretion flow coupled with an emergent flare to interpret the latest 1.3 mm very long baseline interferometry (VLBI) measurements for Sagittarius A\(^*\). The visibility data constrained the distances from the flare center to the black hole center as \(d_{\text{EW}}\lesssim 20 R_\odot\) and \(d_{\text{NS}}\lesssim 80 R_\odot\) in the east–west and north–south directions, respectively. If interpreted by the hot-spot model, the flare was preferred to pass in front of the black hole at a radius much larger than \(d_{\text{EW}}\). If interpreted by the episodic jet launched from a nearly edge-on hot accretion flow, the flare was preferred to be ejected with \(i \gtrsim 40^\circ\) off the black hole rotating axis. This method can be generalized to help us understand future submillimeter VLBI observations and study the millimeter/submillimeter variabilities in the vicinity of the Galactic center supermassive black hole.

Key words: Galaxy – center – submillimeter: general

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1. INTRODUCTION

Sagittarius A\(^*\) (Sgr A\(^*\)) is the best black hole candidate that can be studied by very long baseline interferometry (VLBI). Doeleman et al. (2008) reported 1.3 mm VLBI detections of Sgr A\(^*\) on two baselines, namely, ARO/SMT (Arizona)–CARMA (California) and ARO/SMT–JCMT (Hawaii). These data were fitted by a circular Gaussian component with an FWHM of \(\sim 43\mu\text{as}\), reaching the event-horizon scale of the Galactic center supermassive black hole. Several groups soon interpreted these data independently by an accretion flow around a rotating black hole with a high inclination angle (Broderick et al. 2009; Huang et al. 2009a, 2009b; Dexter et al. 2009).

Recently, Fish et al. (2011) reported new measurements on all the three baselines of the ARO/SMT–CARMA–JCMT array over three nights: 2009 April 5–7 (Days 95–97). The data on Day 95 and Day 96 show a high level of consistency with the data in 2007 April by Doeleman et al. (2008), fitted by a circular Gaussian component with FWHMs of \(\sim 41\mu\text{as}\) and \(\sim 44\mu\text{as}\), respectively. But the correlated flux density on all the baselines was preferred to be ejected with \(i \gtrsim 40^\circ\) off the black hole rotating axis. This method can be generalized to help us understand future submillimeter VLBI observations and study the millimeter/submillimeter variabilities in the vicinity of the Galactic center supermassive black hole.

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2. CONSTRAINING THE POSITION OF FLARING COMPONENT

Hereafter, for simplicity, we denote the three baselines from short to long, namely, ARO/SMT–CARMA, CARMA–JCMT, and ARO/SMT–JCMT by baselines I, II, and III, respectively. We apply all the data on Days 95–97 (Fish et al. 2011) and take averages if there is more than one data point measured on the same baseline coordinates \((u, v)\) on the UV plane. We interpret the measurements on Day 95 and Day 96 as contributions by an accretion flow in its quiescent state. The magnetorotational-instability-dominated relativistic accretion flow (Huang et al. 2009b), named the Q Model, with a black hole spin \(a = 0.5\), an inclination angle \(i = 60^\circ\), and a position angle \(\Theta = 100^\circ\), is adopted to account for the emission in the quiescent state. The black hole spin and orientation are typical in our earlier work, which are also in agreement with the estimates made by other groups (Dexter et al. 2010; Shcherbakov 2010; Broderick et al. 2011).

We simulated the image (shown in the left panel of Figure 1) of the accretion flow by ray-tracing with polarized relativistic radiative transfer (Shcherbakov & Huang 2011; Huang & Shcherbakov 2011). We further plot the visibility data of this theoretical image in the right panel of Figure 1, with interstellar scattering considered (Shen et al. 2005). The total flux density \(S_\text{Q0} = 2.1\text{ Jy}\). The amplitude of visibility, i.e., the correlated flux density \(S_C\) shown in black lines fit the data on Day 95 with reduced chi-squares \(\chi^2_\text{of} = 1.06\) and those on Day 96 with \(\chi^2_\text{of} = 1.07\); here the degrees of freedom (dof) = 12 on both days. The closure phases \(\phi_{\text{CL}}\) of baselines I, II, and III averaged over 10 minutes, as shown in plus signs, is in the range \((0^\circ, 25^\circ)\), which is also consistent with the observations at \(0^\circ \pm 40^\circ\) (Fish et al. 2011).

We ascribe the increase of visibility on Day 97 relative to Days 95–96 to the emergence of a flaring component. For simplicity, we assume that the brightness distribution of this flare can be represented by a circular Gaussian function \(B_r(S_P_0, D_P, x_P, y_P)\) with four parameters, namely, \(S_P_0\) (total flux...
density), \(D_p\) (FWHM), \(x_p\) (shift of flare center in abscissa), and \(\gamma_p\) (shift in ordinate). The brightness distribution of the flaring state of the accretion flow, \(B_p\), is simply assumed to be the superposition of the brightness distribution of the quiescent state, \(B_Q\), and that of the flare, \(B_f\), i.e., \(B_p = B_Q + B_f\). The visibility distribution with interstellar scattering considered, \(V_p = \mathcal{F}[B_p, \text{scal}]\), is also the sum of \(V_Q = \mathcal{F}[B_Q, \text{scal}]\) and \(V_f = \mathcal{F}[B_f, \text{scal}]\), according to the property of linearity of the Fourier transform \(\mathcal{F}\). Thus, we obtain

\[ V_p = V_Q + V_{p0} \]

\[ V_{p0} = \mathcal{F}[B_{p, \text{scal}}(S_{p0}, D_p, 0, 0)] e^{-2\pi i(x_p u + y_p v)}, \]

where \(u\) and \(v\) are the baseline coordinates on the UV plane.

We search over wide ranges of the parameters, \(S_{p0}\), \(D_p\), \(x_p\), and \(\gamma_p\), and fit the data on Day 97 to calculate the reduced chi-square \(\chi^2_{\text{red}}\) with dof = \(N_{\text{data}} - N_{\text{param}} = 14 - 4 = 10\). We choose sets of parameters that yield \(\chi^2_{\text{red}} = 1 \pm 1\) by preference. Thus, we obtain \(S_{p0} = 0.95 \pm 0.55\) Jy and \(D_p = 6.2 \pm 6.2\ R_g\). We estimate the observational angular size of the flare as \(\theta_{\text{obs}} = 38^{+28}_{-17}\) mas with the formula \(\theta_{\text{obs}}^2 = (D_p \cdot \theta_{RG})^2 + \theta_{\text{scat}}^2\), where \(\theta_{RG} \approx 0.005\) mas is the angular size of gravitational radius and \(\theta_{\text{scat}} \approx 0.021\) mas is the extrapolated angular size of scattering screen in the EW direction (Shen et al. 2005).

The corresponding brightness temperature of the flare is \(T_B = 1.22 \times 10^{22}(S_{p0}/\text{Jy})(v/\text{GHz})^{-2}(\theta_{\text{obs}}/\text{mas})^{-2} \approx 1.5_{-0.7}^{+0.9} \times 10^{10}\) K. This value is consistent with the estimation for a plasmon adopted to interpret the variability of Sgr A* on 3 mm wavelength observed by ATCA (Li et al. 2009). We show the boundaries of the preferred positions in dashed white lines in the left panels of Figure 2, i.e., all the preferred pairs of \((x_p, \gamma_p)\) are located in the region surrounded by the boundaries. This region is narrow along the direction of \(\sim 65^\circ\) east-of-north, hereafter denoted as the EW direction, and elongated along the direction of \(\sim 25^\circ\) west-of-north, hereafter the NS direction. The region has an asymmetrical shape that slightly varies with the parameters of the Q Model. Generally speaking, the preferred distances from the flare center to the black hole center are \(d_{EW} \lesssim 20\ R_g\) and \(d_{NS} \lesssim 80\ R_g\), along the EW and NS directions, respectively.

The data on Day 97 impose a strict constraint on \(d_{EW}\) but a relatively relaxed one on \(d_{NS}\). This is mainly because the measurements on the tracks of the two long baselines II and III are roughly aligned in the EW direction. These baselines are longer than \(3G\lambda\), giving a high angular resolution of 85 \(\mu\)as, or 17 \(R_g\), in the EW direction. A flaring component emerging at \(d_{EW} \gtrsim 17\ R_g\) would cause a null/valley point in the visibility profile at a baseline length \(\lesssim 3G\lambda\), so that the amplitudes of visibility on baseline II might be lower than those on baseline III. However, the amplitudes actually have a monotonically decreasing profile from baseline II to baseline III, which implies there is no structural variability with scales larger than 17 \(R_g\) in the EW direction. Moreover, the track of baseline I can also detect the separation of the two components in the EW direction sensitively, although its precise direction is \(\sim 70^\circ\) west-of-north, which is \(\sim 45^\circ\) off those of baselines II and III. On the contrary, \(d_{NS}\) cannot be well constrained since there are no long enough projected baselines available in the NS direction.

In the left panels of Figure 2, we show three examples of the flaring state, each consisting of the accretion flow overlaid by a different flare. \(P_1\), shown in the top left panel, with \(S_{p10} = 0.8\) Jy, \(D_{p1} = 6.6\ R_g\), and \((x_{p1}, \gamma_{p1})/R_g = (-4.08, -2.04)\), is located within the preferred region. The visibility corresponding to \(Q + P_1\) is shown in the top right panel, fitting the data on Day 97 by \(\chi^2_{\text{red}} = 1.13\). The closure phase \(\phi_{\text{CL}}\) is within the range \((-20^\circ, 20^\circ)\), indicating high symmetry of the total image. \(P_2\), shown in the middle left panel, with \(S_{p20} = 0.8\) Jy, \(D_{p2} = 6.6\ R_g\), and \((x_{p2}, \gamma_{p2})/R_g = (-12.24, -6.12)\), is outside the preferred region. The visibility fits the data by \(\chi^2_{\text{red}} = 8.27\). As shown in the middle right panel, a flare slightly beyond the boundaries in the EW direction can decrease \(S_C\) a lot on baseline I and cause a valley structure on baseline II. \(\phi_{\text{CL}}\) is in the range \((-40^\circ, 40^\circ)\), deviating greatly from zero, and inferring that the symmetry of the total image in the EW direction is somehow broken. \(P_3\), shown in the bottom left panel, with \(S_{p30} = 0.95\) Jy, \(D_{p3} = 6.2\ R_g\), and \((x_{p3}, \gamma_{p3})/R_g = (-16.34, 34.08)\), is within the preferred region. This is the case with minimal \(\chi^2_{\text{red}} = 0.25\), i.e., the best fit. As shown in the bottom right panel, it even reproduces the \(\sim 10\%\) increase in \(S_C\) related to baseline length on baseline I. However, considering the uncertainties of the data and the simplicity of the static models adopted here, we think it may have overfitted the data, especially on baseline I.

3. DISCUSSION

We introduce a simple but useful method to understand the time-variable emission of Sgr A* detected by 1.3 mm VLBI (Fish et al. 2011). We interpret the data on Days 95–96 by an accretion flow in its quiescent state, and the increase of correlated flux density on all the three baselines on Day 97 by the emergence of an extra flare, modeled by a circular Gaussian spot. The visibility measurements impose a strict constraint on the distance from the flare center to the black hole center in the EW direction \(65^\circ\) east-of-north as \(d_{EW} \lesssim 20\ R_g\). They also
place another relatively relaxed constraint on the distance in the NS direction (25° west-of-north) as $d_{NS} \lesssim 80 R_g$. General discussions on various aspects follow in the rest of this section.

3.1. Resolution of Baselines

The longest baseline length on the tracks of baseline III (ARO/SMT–JCMT) is $\sim 3.55G\lambda$, corresponding to the highest resolution of $\theta_{beam} \sim 70 \mu as$. The apparent size of the flaring component assumed in this Letter is $\lesssim 66 \mu as$, slightly super-resolved by the current baseline. Here, we follow the criterion provided in Shen et al. (1997) to judge the degree of resolution.

$\theta_{LIM}$, the limit of source size that can be resolved, is defined as

$$\theta_{LIM} = [\theta_{LIM}^{\text{Statistical}}]^4 + \theta_{LIM}^{\text{Systematic}}]^4]^{1/4},$$

where

$$\theta_{LIM}^{\text{Statistical}} = \frac{0.53}{\sqrt{S/N}} \cdot \theta_{beam}$$

$$\theta_{LIM}^{\text{Systematic}} = \frac{0.53}{\sqrt{|F_v/\Delta F_v|}} \cdot \theta_{beam}. \quad (2)$$

With a mean signal-to-noise ratio (S/N) of 5.5 on the ARO/SMT–JCMT baseline (Doeleman et al. 2008) and $|\Delta F_v/F_v|$ being the sum of the uncertainties of the flux density measurements $\sim 30\%$, this limit is estimated to be $\theta_{LIM} \sim 20 \mu as$. Thus, we think the flare was resolved in an optimistic manner.

3.2. Challenges to the Flare Models

The origin of flares in Sgr A* is still controversial. Various models can be put into two general classes, namely, the whole change and the transient structure. The whole change means enhancement in flux density in the whole emission region, caused by changes in physical quantities of the accretion flow, e.g., an increase in heating coefficient or the creation of power-law electrons (e.g., Yuan et al. 2003; Liu et al. 2004). We can fit the Day 97 data by the Q Model itself with $\chi^2_{dof} = 1.07$, $dof = N_{data} = 14$, if the total flux density is scaled up to 3 Jy. This implies that those models of whole change are plausible for the Day 97 data, i.e., no extra component is required for data interpretation. This is consistent with the result of $d_{EW} \lesssim 20 R_g$, indicating that the flare happened very close to or even inside the emission region of accretion flow.

We are also interested in challenging models of the transient structure, which means an extra component emerged to
contribute to structural variability. Generally speaking, the Day 97 data do not preclude any flare model if the transient structure is located within the preferred flaring region.

A popular model used to interpret short-time variability of Sgr A* in millimeter and near-infrared bands is a hot spot orbiting in Keplerian angular velocity (e.g., Broderick & Loeb 2005; Do včiak et al. 2008). Such a hot spot varying in an hourly timescale is predicted to be detectable by submillimeter VLBI (Doeleman et al. 2009). We assume that the flare on Day 97 was caused by a hot spot orbiting at \( r_\parallel \), the projective radius from the black hole center. The normal direction of the orbital plane coincides with the projective rotating axis of the black hole. According to earlier work, the position angle of Sgr A* is preferred to be in the range \((-90^\circ, 180^\circ)\) or \((0^\circ, 90^\circ)\), rather than in the range \((-180^\circ, -90^\circ)\). We then obtain \( r_\parallel < 40R_g \) as calculated from the constraints on the flaring region shown in the above section. The period of the Keplerian orbit is constrained as \( T_{\text{Kep}}(r=r_\parallel) < 8 \text{ hr} \), which is insensitive to black hole spin. The observational duration \( T_{\text{obs}} \) was \(\sim 2\) hr on Day 97, i.e., \( T_{\text{obs}} > 0.25T_{\text{Kep}}(r=r_\parallel) \), implying that an apparent light curve should be detected with a nearly edge-on disk assumed. However, this is inconsistent with the observed sustaining high flux density. Therefore, we would either exclude this model or explain this inconsistency by two arguments. One is that the measurements of the total flux density had rather large errors. The other is that the radius was shortened by projection effect, i.e., the hot spot might be passing in front of the black hole at a radius \( r_\perp > 40R_g \) with a component aligned with the line of sight.

We further generalize this model into a hot spot moving with sub-Keplerian angular velocity and radial velocity comparable to the light speed, the same as the velocity of the background transonic accretion flow. In this case, the hot spot would quickly fall into the event horizon of the black hole and disappear. We integrate the curve of radial velocity shown in Huang et al. (2009b) to calculate the typical timescale of falling at the specific starting radii \( r_\parallel < 40R_g \). The lifetime of the hot spot is constrained as \( T_{\text{fall}}(r=r_\parallel) < 1 \text{ hr} \), i.e., \( T_{\text{obs}} > 2T_{\text{fall}} \). Similar to the Keplerian-rotating model, we would either exclude this model or interpret it as a special case in which the hot spot started at a larger radius in front of the black hole with \( r_\perp > 60R_g \) and plunged into the event horizon.

Furthermore, we consider a different model of episodic jets proposed by Yuan et al. (2009), in which the flare is contributed by an episodic ejection of plasmoid from a hot accretion flow. As they calculated, a plasmoid with initial location of \(\sim 10R_g \) to Sgr A* can accelerate from rest to \(\sim 0.8c \) in 35 minutes. This predicts that during the 2 hr observation, the plasmoid can travel \(\sim 250R_g \) from the black hole center. To take into account both the constraint on \( d_{\text{NS}} \) and the projection effect by a disk with \( i \gtrsim 60^\circ \), the plasmoid must be ejected in a direction with \( \theta_j \gtrsim 40^\circ \) off the rotating axis of the black hole.

We wish to comment on a model of tidal disruption of asteroid explored by Zubovas et al. (2011), in which the flare is related to asteroids rather than properties of hot accretion flow. They predicted a small size for a flaring region \(\lesssim 1 \text{ AU} \) or \(\lesssim 25R_g \), which is roughly included in our constraints with boundaries of \( d_{\text{EW}} \) and \( d_{\text{NS}} \). However, the timescale of an asteroid in a
parabolic orbit around Sgr A* is estimated to be $\lesssim 1.5 \text{hr}$, which is disfavored by the Day 97 flare.

3.3. Visibility Prediction for an Array of Five Stations

The flaring region of Sgr A* can be constrained better if suitable baselines in the NS direction are available. Theoretically, the separation of $\Delta S \approx d_{\text{NS}} \lesssim 80 R_g$ between two components, the quiescent accretion flow and the flare, will cause first a null/valley point in the visibility profile at $L_0 \gtrsim 0.25 G\lambda$ in the NS direction. We include two additional stations, APEX and ARO in Chile and LMT in Mexico, which are promising for contributing to submillimeter VLBI observations in the near future. The tracks of seven additional baselines, from short to long, ARO–SMT–LMT, CARMA–LMT, APEX–LMT, JCMT–LMT, APEX–ARO, SMT, APEX–CARMA, and APEX–JCMT, cover from 0.5G$\lambda$ to 7G$\lambda$ in length close to the NS direction. In an optimistic view, the separation of $\Delta S \approx d_{\text{NS}} \lesssim 40 R_g$ could be detected if all the baselines work. In the left panels of Figure 3, we choose two examples of flares, P4 and P5, both with $S_p = 0.95 \text{Jy}$ and $D_p = 6.2 R_g$, but with $(x_{p4}, y_{p4})/R_g = (-2.04, 4.9)$ and $(x_{p5}, y_{p5})/R_g = (-16.32, 40.8)$, respectively. The corresponding visibilities are shown in the right panels. The image of the Q Model and P4, with $\Delta S \approx 5 R_g$, causes the first valley point at $\sim 4G\lambda$ on the baseline APEX–LMT. The image of the Q Model and P5, with $\Delta S \approx 40 R_g$, causes the first valley point at $\sim 0.5G\lambda$ on the baseline ARO–SMT–LMT, which appears as a drop in amplitude between baselines ARO–SMT–CARMA and ARO–SMT–LMT. At the same time, closure phases by a station group, including APEX and LMT, also imply a change to symmetry caused by the flare. For example, $|\phi_{\text{CL}}|$ by the group of APEX–ARO/SMT–LMT are shown in blue plus signs in the right panels of Figure 3.

4. SUMMARY

The detection of time-variable emission of Sagittarius A* by 1.3 mm VLBI indicates instability on event-horizon scales. Modeling the flare by a circular Gaussian spot, the data constrained the spot size as $D_0 \sim 6.2 \pm 6.2 R_g$ and deviation to the black hole center in the east–west direction ($65^\circ$ east-of-north) as $d_{\text{EW}} \lesssim 20 R_g$, both being comparable to the size of the black hole shadow ($\sim 10 R_g$). Alternatively, we can interpret this flaring activity as an enhancement in the whole emission region, and we would prefer such an interpretation to models of transient structures that require an extra component. This is mainly due to the sustaining high flux density in a long observational duration, $T_{\text{obs}} \sim 2 \text{hr}$. The duration is more than one-fourth of the orbiting period if modeled by a hot spot in Keplerian rotation at a preferable orbiting radius $\lesssim 40 R_g$. It is also more than twice the falling timescale if modeled by a hot spot with sub-Keplerian angular velocity and high radial velocity, and with a preferable initial radius $\lesssim 40 R_g$. A hot spot located at $d_{\text{EW}}$ cannot maintain high flux density during the observational duration, unless it passed in front of the black hole at a radius much larger than $d_{\text{EW}}$ so that the observed distance is shortened by projection effect. If interpreted by an episodic jet, the ejecting plasmoid cannot be confined within the preferred flaring region with an assumed edge-on accretion flow, unless the angle between the ejecting direction and the black hole rotating axis is greater than $40^\circ$. This method of visibility analysis can be generalized for future submillimeter VLBI measurements. We would have a better understanding of the nature of variability in the black hole vicinity of the Galactic center if $d_{\text{NS}}$, the deviation of the flare to the black hole center in the north–south direction ($25^\circ$ west-of-north), could be constrained more precisely with new stations included.

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