Continuous media theory for MR fluids in non-shearing flows

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Abstract. The enhanced mechanical response of magnetorheological fluids under slow compression has been investigated by means of experiments, theory and particle-level simulations. A wide range of magnetic field strengths (0–354 kA/m), dispersing medium viscosities (20–500 mPa·s) and particle concentrations (5–30 vol%) were investigated. Plastic media theory in compressive flow was in good agreement with experimental data. Slight deviations from the theory were associated to the so-called strengthening effect as the yield shear stress could increase during compression. Particle-level simulations were in good agreement with both experiments and simulations.

1. Introduction
In general, available devices using magnetorheological (MR) fluids can be classified according to their flow mode as direct-shear flow mode, pressure-driven flow mode and squeeze-film flow mode. Among the three modes, it is well known that the squeeze flow mode provides the largest yield stress under the same field [1]. The rheological properties of MR fluids under shearing flows have been extensively investigated in the literature. However, the understanding of the behavior of MR fluids under non-shearing elongational flows, and particularly in squeeze flow mode is still far to be complete mainly because of the lack of both a thorough understanding of the basic MR mechanisms and reliable experimental data [2, 3].

First reports on squeeze flow magnetorheology were devoted to investigate the enhancement of MR performance by the so-called compression-assisted aggregation process [4-6]. Later, in a study, a series of low-strain tests on MR fluids where the behaviors under constant velocity squeezing flow and shear flow were compared [7]. A field dependence of $H^{0.91}$ was found for compression in contrast to the $H^{1.4}$ dependence observed under shearing.

In this work, we follow previous papers where we experimentally demonstrated the appearance of normal forces under no-slip compression in the filtration dominated regime [3, 8]. In the present study the long-standing question of whether a single theory for model plastic fluids is suitable to deal with the unidirectional compression problem in MR fluids was addressed. In this work, an extensive experimental and particle-level simulation investigation of the performance of MR fluids in slow-compression, no-slip, constant-volume squeeze mode under different magnetic field strengths (0–354 kA/m), dispersing medium viscosities (20–500 mPa·s) and particle concentrations (5–30 vol%) were investigated.

2. Theory
Plastic media theory in squeeze flow mode was first developed by Covey and Stanmore [9]. The solution of the motion equation for a Bingham fluid depends on the plasticity number, defined as:
where $\eta_p$ is the plastic viscosity, $v$ is the compression velocity, $R$ is the sample radius, $h$ is the plate-plate gap and $\tau_y$ is the shear yield stress. For low plasticity numbers, in the so-called filtration dominated regime, both Bingham and biviscous theories are applied that reduce to the following analytical expression in the case of constant volume tests [8]:

$$F = \frac{2\tau_y v^{3/2}}{3\sqrt{\pi h^5/2}} (1 - \varepsilon)^{5/2}$$  

where $V$ represents the total volume of the sample ($V = \pi R^2 h$) and the elongational strain $\varepsilon$ is defined here as the ratio of the moving distance of the upper plate to the initial distance between the plates $\varepsilon = (h_0 - h)/h_0$.

3. Simulations
A three-dimensional particle-level simulation method to understand the effect of particle concentration was also employed. Magnetic forces were approximated by the point-dipole limit and high-order magnetic multipoles were neglected. A free draining approximation was used for including hydrodynamic interactions. Short-range repulsive interactions between particles, and particles and walls were taken as an exponential interaction [3, 8].

Finally, the normal force $F^*$ acting on the plate is calculated by simply differentiating the total magnetic energy $U^*$ according to:

$$F^* = \frac{dU^*}{dh^*},$$

4. Results and discussion
Uniaxial monotonic compression tests were carried out in the presence of a uniaxial magnetic field at low compression rates under constant volume conditions. In all cases, the resulting normal force shows an initial ‘plateau’ in the low-strain region related to the compressive yield stress. This initial plateau is in agreement with equation 2 [3].

![Figure 1. Low-strain compressive stresses as a function of magnetic field strength H (squares), dispersing medium viscosity $\eta$ (circles) and particle volume fraction $\phi$ (triangles).](image)
The compressive yield stress was obtained from experimental data by dividing the initial normal force plateau value by the MR fluid area. For a comparative analysis, in Fig. 1, experimentally measured compressive yield stresses $\tau_{c,E} = A/\pi r_0^2$ (solid symbols) as well as calculations $\tau_{c,C}$ using $\tau_{c,C} = \frac{2\pi\gamma^3/2}{3\pi^{3/2}r_0^{5/2}\gamma^3} = \frac{2\pi\gamma_0^{3/2}}{3h_0}$ (open symbols) were shown. As demonstrated, calculated compressive yield stresses compare reasonably well with experimental measurements especially taking into account that there are not free parameters in these calculations. The yield compressive stress does increase with increasing the magnetic field strength, medium viscosity and particle content. A power law dependence is found in the three cases. Particle-level simulations show a similar dependence of the compressive yield stress with the particle concentration as shown in Fig. 2.

![Figure 2. Simulation results for the low strain plateau value $A$ as a function of the particle concentration $\phi$. Red line is the best fit and reveals a quadratic dependence (1.94 ± 0.22).](image)

In order to compare theory, experiments and particle-level simulations, a normalization of the normal force with the low-strain plateau value was carried out. According to equation 2, this normalized normal force must only depend on the elongational strain. As shown in Fig. 3, experimental data reasonably collapse in a master curve and are in good agreement with both theoretical plastic models and simulations [3, 8].

![Figure 3. Dimensionless normal force as a function of compressive strain $\varepsilon$ for different magnetic field strength, dispersing medium viscosity and particle volume fraction. Solid line corresponds to the prediction by the continuous media theory for plastic materials equation 2. Stars correspond to particle level dynamic simulations.](image)
Deviations from the theory can be associated to the so-called strengthening effect [10]. Under slow compression, particles may restructure in shorter, thicker and eventually stronger structures. As a consequence, the shear yield stress may enhance during compression. To further investigate the structural changes during squeeze, compression tests by superposition of a Small-amplitude oscillatory shear flow were carried out where the upper plate was made to undergo rotary oscillations about a mean position $5 \times 10^{-6}$ rad at a given frequency $f = 10$ Hz in the viscoelastic linear region. This kind of test would allow us to monitor viscoelasticity changes at the same time the sample is being compressed [3].

As shown in Fig. 4, viscoelastic moduli increased under compression following an exponential law. Moreover, viscoelastic moduli scale with a Mason-like number in a master curve. Strengthening effect is also suggested by rheomicroscopic structural observations and simulation results where the structure appears to become thicker under compression [3].

5. Conclusions
A unified description of the behavior of MR fluids in terms of a continuous media theory for plastic materials were proposed. This allowed us to obtain collapsed compression curves for a wide range of magnetic field strengths, medium viscosity and particle concentration. Deviations from the theory were explained in terms of the squeeze strengthening effect. On the one hand, a quadratic dependence with the magnetic field strength ($2.0 \pm 0.1$) and particle concentration ($2.0 \pm 0.2$) was found. On the other hand, a negligible dependence with the continuous phase viscosity was found. Experiments reported here validated another procedure to determine static yield shear stresses when slowly compressing the MR fluids. Particle-level dynamic simulations at large $1 - \varepsilon$ suggest a quadratic volume fraction dependence ($1.94 \pm 0.22$) in good agreement with experimental data.

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