The thermal structure of sunspots from ring diagram analysis

Charles S. Baldner\textsuperscript{1}, Richard S. Bogart\textsuperscript{2}, and Sarbani Basu\textsuperscript{1}

\textsuperscript{1}Astronomy Department, Yale University, P.O.Box 208101, New Haven, CT, 06530-8101, USA
\textsuperscript{2}Hansen Experimental Physics Laboratory, Stanford University, Palo Alto, CA, 94305, USA
E-mail: charles.baldner@yale.edu

Abstract. We present a large sample of 264 active regions from solar cycle 23, analysed using ring diagrams. The frequencies of these rings are inverted to determine the thermal structure (sound speed and adiabatic index) of these regions as a function of radius. The large sample allows us to describe in a statistically significant way how the thermal properties of the outer layers of the Sun change in the presence of magnetic fields.

1. Introduction
Understanding the subsurface structure of sunspots is one of the current major avenues of research in solar physics. Helioseismology provides a unique opportunity to determine the structure of sunspots empirically, but this determination involves substantial difficulties. In this work, we present the analysis of a large number of ring diagrams of active regions taken from the Michelson Doppler Imager (MDI) instrument on board the Solar and Heliospheric Observatory (SOHO).

Ring diagrams are three-dimensional power spectra of localised areas on the solar surface [1]. The presence of strong surface magnetic fields in active regions is known to change the mode parameters of ring diagrams [2, 3, 4, 5]. Inversions of ring diagram frequencies for structure have been performed on small numbers of rings [6, 7] to determine the changes in sound speed and adiabatic index. In these works, sound speed and adiabatic index were found to be enhanced in the layers between approximately 0.975\,R\odot and 0.985\,R\odot, and depressed in the shallower layers between 0.99\,R\odot and 0.998\,R\odot.

Previous studies have used small numbers of active regions — typically less than 20. In this work, we present structure inversions of a much larger sample of active regions. The mode parameters of this sample were described in [5]. In section 2 we describe the sample and the inversion techniques, in section 3 we present the results, and in section 4 we compare the results to previous studies and discuss further work.

2. Method
In the present work, we have a sample of 264 active regions. This sample was previously described in [5]. The ring diagrams are constructed from Michelson Doppler Imager (MDI) full disk Doppler-grams. The region is a 16\degree\,\times\,16\degree patch tracked across the disk for 8192 minutes, projected to a square grid using Postel’s projection, and appodized to a uniform circular
Figure 1. Inversions for squared adiabatic sound speed $c^2$, shown for four representative regions.

aperture. The ring diagram itself is the three dimensional power spectrum of the tracked velocity data [8].

Active regions are selected from the NOAA active region catalog. Construction of ring diagrams is limited by the availability of suitable data, which is typically only guaranteed during MDI dynamics campaigns, which run two to three months of every year. We require data coverage of at least 85%. The quality of the rings suffers greatly for lower data coverage rates. For every active region in the sample, we also construct one or two comparison ring diagrams of nearby quiet sun regions. These regions are tracked at the same solar latitude to minimize systematic effects in the frequency differences due to geometry.

To characterize each ring we use a measure of the total unsigned line-of-sight magnetic flux. This measure is the Magnetic Activity Index (MAI), defined in [6]. In this study, we invert the differences of the mode frequencies between the active region and the quiet comparison region, thus each measurement is characterized by the difference in MAI, $\Delta$ MAI. The sense of the inversion and the $\Delta$ MAI is active minus quiet.

The power spectra are fit using a 13-parameter function defined in [8]. The characteristics of the mode parameters were described in [5]. The frequency differences can then be inverted for structural quantities. In this work, we use Subtractively Optimised Local Averages (SOLA) as described in [9]. The inversion minimizes the difference between the inversion averaging kernel and a target kernel. In our case, the form of the target kernel is fixed with a variable parameter controlling the width. We also remove a smoothly varying function of frequency from
the frequency differences. This function is commonly referred to as the surface term, which we construct using basis splines. In our inversions, then, we have four free parameters: the width of the target kernel, the error suppression parameter, the cross-term suppression parameter, and the number of knots in the surface term. The cross-term arises from the need for two thermodynamic quantities to fully describe the changes we invert for. In this work, both the sound speed and adiabatic index cross-terms are density. These are the standard four free parameters in SOLA inversions of helioseismic data (e.g. [6]).

The principal effort in performing an inversion for structure from helioseismic data is the difficulty in choosing the appropriate inversion parameters. A thorough exploration of the parameter space was performed on a small subset of the rings, including comparisons to rings published in earlier works. It was found that good inversions could be obtained for all the rings from a small set of combinations of parameter values. The differences in inverted quantities and averaging kernels between inversions with these different parameter sets are easily seen, and so the appropriate value for the inversion parameters could quickly be chosen for the remaining rings.

3. Results
Inversions for the difference in squared sound speed $c^2$ and adiabatic index $\Gamma_1$ were performed for all regions in the sample. Figure 1 shows example sound speed inversions for four different rings with a range of active region strengths as a function of depth. Inversion parameters were chosen
Figure 3. Averages of inversions for $\varepsilon^2$ over two depth ranges are shown, plotted as a function of $\Delta$ MAI. Blue points are averages of inverted sound speed between 0.975$R_\odot$ and 0.985$R_\odot$; red crosses are averages of inverted sound speed between 0.99$R_\odot$ and 0.998$R_\odot$.

Figure 4. Averages of inversions for $\Gamma_1$ over two depth ranges are shown, plotted as a function of $\Delta$ MAI. Blue points are averages of inverted adiabatic index between 0.975$R_\odot$ and 0.985$R_\odot$; red crosses are averages of inverted adiabatic index between 0.99$R_\odot$ and 0.998$R_\odot$. 
to keep the averaging kernels as symmetric as possible and keep the cross-term contamination as small as possible. Figure 2 shows inversions for adiabatic index for the same regions.

Figure 3 shows averages of the inverted sound speed for all regions in our sample for different depth ranges. For sound speed averaged between \(0.975R_\odot\) and \(0.985R_\odot\), sound speeds are generally enhanced in the presence of magnetic fields, while in the region from \(0.99R_\odot\) to \(0.998R_\odot\), sound speeds decrease. In both regions, the magnitude of the change tends to increase with magnetic field strength, although the relationship seems to be more of an envelope than a linear relation. Further, there seems to be some saturation of the effect at very high magnetic field strengths. The inversion results for the adiabatic index are shown over the same range of depths. The behaviour of \(\Gamma_1\) is largely the same as sound speed.

In Figure 5, we plot the boundary between the positive and negative perturbations as a function of \(\Delta MAI\). There is no obvious dependence of the boundary point on magnetic activity.

4. Conclusions and further work
Earlier works [6, 7] have studied the structure of sunspots using ring diagrams, but have had much smaller sample sizes. We confirm the results of these earlier works. We find that both sound speed and adiabatic index are enhanced in the layers between approximately \(0.975R_\odot\) and \(0.985R_\odot\), and are depressed in the shallower layers between \(0.99R_\odot\) to \(0.998R_\odot\). We do not extend our inversions beyond these regions, as the helioseismic data contains less information outside these layers, and getting reasonable inversions requires great care.

[7] found a linear correlation between magnitude of the sound speed change and strength of the active region. We find a similar relation, but with substantial scatter. Further, we find that the correlation appears to saturate at high field strengths. We do not find that the depths of the enhancements or suppression of sound speed and adiabatic index depend on the strength of the active region in any way.
The scatter we see is to a certain extent due to instabilities in the inversions themselves. Although we have expended significant effort on selecting appropriate inversion parameters, it is not possible to explore the parameter space as thoroughly as earlier authors could on smaller data sets. We are continuing work to improve the quality of the inversions. Nevertheless, we believe that a substantial amount of the scatter is in the data itself, rather than due simply to the inversions. We are working on quantifying the intrinsic scatter.

It should be noted that what we have interpreted as a sound speed change is more precisely a wave speed change. Since the magnetic fields in the active regions change the speed of wave propagation as well as the structure, not all of the change in inverted wave speed is necessarily due to a thermal perturbation. The difference between the inverted sound speed and the actual sound speed, given a number of assumptions about the magnetic fields, was shown by [10].

Work is continuing in an effort to improve the quality of the inversions, to verify them against complimentary inversion techniques, and to more precisely determine the relationship between the structure and the strength of the active region.

Acknowledgments

CB is supported by a NASA Earth & Space Sciences Fellowship NNX08AY41H. SB is supported by NASA grant NNG06GD139 and NSF Career grant ATM-0348837. The Solar Oscillations Investigation / Michelson Doppler Imager project on SOHO is supported by NASA grant NNX09AI90G to Stanford University. SOHO is a project of international cooperation between ESA and NASA.

References

[1] Hill, F 1988, ApJ 333, 996
[2] Hindman, B, Haber, D, Toomre, J and Bogart, R 2000, Sol. Phys. 192, 363
[3] Rajaguru, S. P., Basu, S and Antia, H M 2001, ApJ 563, 410
[4] Rabello-Soares, M C, Bogart, R S and Basu, S 2008, J. Phys.: Conf. Series 118, 012084
[5] Baldner, C S, Bogart, R S, and Basu, S, 2010, HELAS IV in press
[6] Basu, S, Antia, H M and Bogart, R S 2004, ApJ 610, 1157
[7] Bogart, R S, Basu, S, Rabello-Soares, M C and Antia, H M 2008, Sol. Phys. 251, 439
[8] Basu, S and Antia, H M 1999, ApJ 525, 517
[9] Pijpers, F P and Thompson, M J 1994, A&A 281, 231
[10] Lin, C-H, Basu, S and Li, L 2009, Sol. Phys. 257, 37