Closed-Time Path Integral Formalism and Medium Effects of Non-Equilibrium QCD Matter

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Abstract

We apply the closed-time path integral formalism to study the medium effects of non-equilibrium gluon matter. We derive the medium modified resummed gluon propagator to the one loop level in non-equilibrium in the covariant gauge. The gluon propagator we derive can be used to remove the infrared divergences in the secondary parton collisions to study thermalization of mini-jet parton plasma at RHIC and LHC.

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I. INTRODUCTION

Much effort has been given to detect a new form of matter known as quark-gluon plasma (QGP) for decades. The relativistic heavy-ion collider experiments at RHIC (Au-Au collisions at $\sqrt{s} = 200$ GeV) and LHC (Pb-Pb collisions at $\sqrt{s} = 5.5$ TeV) will provide the best opportunity to study such a state of matter in the laboratory. Perturbative QCD estimates that the energy density of produced jets and minijets might be larger than 50 and 1000 GeV/$fm^3$ at RHIC and LHC [1,2] which is much larger than the required energy density to produce quark-gluon plasma. As QGP lives for a very short time (several fermi) in a small volume ($\sim 100$ fm$^3$) a direct detection of this phase is not possible. Hence various indirect signatures are proposed for its detection. The prominent among them are: 1) $J/\Psi$ suppression [3], 2) strangeness enhancement [4], dilepton and direct photon production [5,6]. However, many uncertainties exist which make it difficult to claim the existence of the quark-gluon plasma. The main uncertainty lies in the lack of an accurate determination of the space-time evolution of the quarks and gluons produced just after the collision of two nuclei at RHIC and LHC. While the quark-gluon plasma during the equilibrium stage is described by Bjorken’s hydrodynamic evolution equations, it is much more difficult to determine the space-time evolution of partons in the pre-equilibrium stage. An accurate study of the pre-equilibrium stage will determine the equilibration time and initial conditions for hydrodynamic evolutions in the equilibrium stage. This study is also very important for the accurate determination of various signatures of the quark-gluon plasma.

To describe the space-time evolution of quark-gluon plasma at RHIC and LHC one needs to know how the partons are formed in these high energy nuclear collisions. The hard parton (jets and minijets) production can be calculated by using pQCD. The pre-equilibrium evolution of these hard partons can be studied by solving relativistic transport equations with secondary collisions among these partons taken into account [1,7–10]. However, soft parton production can not be computed within pQCD formalism. There are coherent effects for the soft partons and they may be described by formation of a classical chromofield [11–17]. For simplicity, we will only consider the evolution of minijet plasma which can be studied by solving relativistic transport equation:

$$p^\mu \partial_\mu f(x, p) = C(x, p),$$

with secondary collision among the partons taken into account. In the above equation

$$C(x, p) = \int \frac{d^3p_2}{(2\pi)^3 p_2^3} \int \frac{d^3p_3}{(2\pi)^3 p_3^3} \int \frac{d^3p_4}{(2\pi)^3 p_4^3} |M(pp_2 \rightarrow p_3 p_4)|^2 \delta^4(p + p_2 - p_3 - p_4)$$

$$[f(x, p_3)f(x, p_4)(1 + f(x, p))(1 + f(x, p_2)) - f(x, p)f(x, p_2)(1 + f(x, p_3))(1 + f(x, p_4))]$$

(2)

is the collision term for a partonic scattering process $pp_2 \rightarrow p_3 p_4$. In the above expression $p$, $p_2$, and $p_3$, $p_4$ are the four momentum of the partons before and after the collision. Different partonic scattering processes which can be considered are like $gg \rightarrow gg$, $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$ and $gq \rightarrow gq$ etc.. As the gluons are the dominant part of the minijet production we only consider $gg \rightarrow gg$ process in the following. The squared matrix element for this process is given by:

$$|M(\hat{s}, \hat{u}, \hat{t})|^2 = \frac{9\pi^2 \alpha_s^2}{8 \left[3 - \frac{\hat{u} \hat{t}}{\hat{s}^2} - \frac{\hat{u} \hat{s}}{\hat{t}^2} - \frac{\hat{s} \hat{t}}{\hat{u}^2}\right]},$$

(3)
where $\hat{s} = (p + p_2)^2 = (p_3 + p_4)^2$, $\hat{t} = (p - p_3)^2 = (p_2 - p_4)^2$, $\hat{u} = (p - p_4)^2 = (p_2 - p_3)^2$
are the Mandelstam variables. For massless gluon they are related by

$$\hat{t} = -\frac{\hat{s}}{2} [1 - \cos\theta], \quad \hat{u} = -\frac{\hat{s}}{2} [1 + \cos\theta],$$

where $\theta$ is the center of mass scattering angle which goes from $0 \rightarrow \frac{\pi}{2}$ for identical partons in the final state. When one puts $|M(\hat{s}, \hat{u}, \hat{t})|^2$ from Eq. (3) in Eq. (4) one encounters divergence in the collision term at small angle ($\theta \rightarrow 0$) or small momentum transfer $\hat{t} \rightarrow 0$. This divergence is inevitable as long as we use the free propagator in vacuum to evaluate the Feynman diagrams. However, this infrared divergence can be removed when one uses medium modified propagators instead of the vacuum propagator to evaluate the collision term in medium [18–20,8,21]. The medium modified propagators have been obtained in the thermal field theory for the case of an equilibrium plasma. However, these finite temperature calculations are valid only in equilibrium where there is a static temperature such as in the case of a heat bath and the system is isotropic. For realistic situations in the high energy heavy-ion collisions at RHIC and LHC the partons formed at the initial time are in non-equilibrium and finite temperature QCD is not applicable at this stage. One has to use closed-time formalism to compute various quantities in non-equilibrium [22–25]. In this paper, using closed-time path integral formalism, we derive the medium modified resummed gluon propagator in non-equilibrium to the one loop order in the covariant gauge which is necessary to obtain finite collision term to study equilibration of expanding minijet plasma at RHIC and LHC.

The paper is organized as follows. In section II we describe the formulation of closed-time path integral of SU(3) gauge theory. In section III we derive the resummed gluon propagator to one loop level. We summarize and conclude our main results in section IV.

**II. CLOSED-TIME PATH INTEGRAL FORMALISM IN GAUGE THEORY**

We consider SU(3) pure gauge theory which is QCD without quarks. In high energy heavy-ion collisions at RHIC and LHC most of the parton formed are gluons. Hence we concentrate on gluons only. However, extending close-time path integral formalism to quarks is straightforward. As the two nuclei travel almost at a speed of light at RHIC and LHC the system is dynamically evolving and many quantities have to be formulated in Boost invariant way [26]. For this expanding system of partons we work in the covariant gauge in this paper. The QCD action without quark is given by:

$$S = \int d^4x \left[-\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu}^a - \frac{1}{2\xi} (G^a)^2 - \mathcal{L}_{FP}\right],$$

where the gluon field tensor:

$$F^{a\mu\nu} = \partial^{\mu} A_{a\nu}^a - \partial^{\nu} A_{a\mu}^a + g f^{abc} A_{b\mu}^b A_{c\nu}^c,$$

the ghost Lagrangian density:

$$\mathcal{L}_{FP} = [\partial_{\mu} \bar{C}] D^\mu [A] C$$

3
and the gauge fixing term given by

\[ G^a = \partial_\mu A^{a\mu}. \]  

(8)

In Eq. (8) \( \xi \) is the gauge fixing parameter in covariant gauge. In the closed-time path integral formalism the gauge field \( A \), the ghost field \( C \) and the corresponding sources \( j, \chi \) are defined in both the time branches. To make the formulas simpler we denote the fields and the corresponding sources by following common notations:

\[ Q = (A, C, \bar{C}), \quad J = (j, \bar{\chi}, \chi). \]  

(9)

Denoting \( Q^+, Q^- \) and \( J^+, J^- \) the fields and the sources on the upper and lower branch of the time path, the in-in generating functional becomes:

\[ Z[J^+, J^-, \rho] = \int DQ^r < Q^+, t_0 | \rho | Q^-, t_0 > e^{i[S[Q^r] + J^r Q^r]}, \]  

(10)

where \( S[Q^r] = S[Q^+] - S^*[Q^-] \) with \( r = +, - \). More explicitly, the above equation can be written as:

\[ Z[J^+, J^-, \rho] = \int [dQ^+] [dQ^-] < Q^+, t_0 | \rho | Q^-, t_0 > e^{i[S_0[Q] + S_{int}[Q] + Tr J \cdot Q]}, \]  

(11)

where

\[ S_0[Q] = \sum_{r,s=+,-} \int \frac{d^4p}{(2\pi)^4} \left[ \frac{1}{2} A^{\mu}_{\mu} (p) [G^{-1}_{\mu\nu}(p)]^{rs} A^{\nu}_{\nu}(-p) + \bar{C}^r (p) [S^{-1}(p)]^{rs} C^s(-p) \right], \]  

(12)

and

\[ Tr J \cdot Q = \int \frac{d^4p}{(2\pi)^4} [j^+_\mu (p) A^{+\mu}_{\mu} (p) + \bar{C}^+ (-p) \chi^+(p) + \bar{\chi}^+ (-p) C^+ (p) + (+ \rightarrow -)]. \]  

(13)

In the Eq. (12) \( G_{\mu\nu}(p) \) and \( S(p) \) are gluon and ghost free propagators and in Eq. (11) \( S_{int}[Q] = S_{int}[Q^+] - S^*_{int}[Q^-] \).

In covariant gauge the further complications arise because of the presence of the unphysical ghost fields. In equilibrium one can define the ghost distribution function (BE) and hence can deal with ghost fields in the medium even in covariant gauge. However, in non-equilibrium there is no easy procedure to obtain a ghost distribution function in the QCD medium at RHIC and LHC. Quark and gluon distribution functions in non-equilibrium situations at RHIC and LHC can be obtained from minijets by using pQCD or via other methods \[ 4 \] [11][27][28]. For this reason we work in the Landshoff and Rehan scheme of frozen ghost formalism \[ 29][30], where the gauge theory in the covariant gauge is obtained by restricting the space of initial state \( |Q^r, t_0> \) to the physical one. This means gluons with spatially transverse polarization will contribute to the trace. Hence the non-local Kernel \( K \) appearing in the path integral of the generating function \[ 25 \] will couple only to the transverse component of the gauge field \( A \).

Now we consider a cylindrically symmetric expanding system of partons in 1+1 dimension. For this purpose we introduce the flow velocity of the medium
where $\eta = \frac{1}{2} \ln \frac{t+z}{t-z}$ is the space-time rapidity and $u_\mu u^\mu = 1$. We define the four symmetric tensors \[31,32,29\]:

\[
T_{\mu\nu}(p) = g_{\mu\nu} - \frac{(u \cdot p)(u_\mu p_\nu + u_\nu p_\mu) - p_\mu p_\nu - p^2 u_\mu u_\nu}{(u \cdot p)^2 - p^2},
\]

\[
L_{\mu\nu}(p) = \frac{-p^2}{(u \cdot p)^2 - p^2} \left( u_{\mu} \left( \frac{(u \cdot p)p_{\mu}}{p^2} \right) \left( u_{\nu} \left( \frac{(u \cdot p)p_{\nu}}{p^2} \right) \right) \right),
\]

\[
C_{\mu\nu}(p) = \frac{1}{\sqrt{2[(u \cdot p)^2 - p^2]}} \left[ \left( u_{\mu} \left( \frac{(u \cdot p)p_{\mu}}{p^2} \right) p_{\nu} + \left( u_{\nu} \left( \frac{(u \cdot p)p_{\nu}}{p^2} \right) \right) \right) \right] \text{ and}
\]

\[
D_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2},
\]

which are required for a cylindrically symmetric system in $1\oplus1$ expanding plasma at RHIC and LHC in the very early stage.

Here $T^{\mu\nu}$ is transverse with respect to the flow-velocity but $L^{\mu\nu}$ and $D^{\mu\nu}$ are mixtures of space-like and time-like components. These tensors satisfy the following transversality properties with respect to $p^\mu$:

\[
p_\mu T^{\mu\nu}(p) = p_\mu L^{\mu\nu}(p) = 0, \quad p_\mu p_\nu C^{\mu\nu}(p) = 0.
\]

In addition to this, the above tensors satisfy the following properties:

\[
T \cdot L = T \cdot C = T \cdot D = 0, \quad T + L + D = 1,
\]

\[
T \cdot T = T, \quad L \cdot L = L, \quad C \cdot C = \frac{1}{2}(L + D),
\]

\[
TrC \cdot L = TrC \cdot D = 0.
\]

Any symmetric tensor $S^{\mu\nu}$ can be written in terms of the above four tensors:

\[
S^{\mu\nu} = a T^{\mu\nu} + b L^{\mu\nu} + c C^{\mu\nu} + d D^{\mu\nu}
\]

with

\[
a = \frac{1}{2} Tr T \cdot S, \quad b = Tr L \cdot S, \quad c = -Tr C \cdot S, \quad d = Tr D \cdot S.
\]

In terms of this tensor basis the gluon propagator in the covariant gauge is given by:

\[
G_{\mu\nu}(p)_{ij} = -iT_{\mu\nu}(p) \left( [G(p)]_{ij}^{vac} + [G(p)]_{ij}^{med} \right) - i(L_{\mu\nu}(p) + \xi D_{\mu\nu}(p)) [G(p)]_{ij}^{vac}
\]

\[
= -i \left( g_{\mu\nu} + (\xi - 1)D_{\mu\nu}(p) \right) [G(p)]_{ij}^{vac} - iT_{\mu\nu}(p) [G(p)]_{ij}^{med},
\]

where $i, j = +, -$. The forms of $G(p)_{ij}$ are

\[
[G(p)]_{ij}^{vacuum} = \left( \begin{array}{cc} \frac{1}{p^2 + i\epsilon} & 0 \\ 0 & \frac{-1}{p^2 - i\epsilon} \end{array} \right) - 2\pi i\delta(p^2) \left( \begin{array}{cc} 0 & \theta(-p_0) \\ \theta(p_0) & 0 \end{array} \right).
\]

and
\[ [G(p)]_{ij}^{\text{medium}} = -2\pi i \delta(p^2) \tilde{f}(\vec{p}) \left( \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right), \quad \tilde{f}(\vec{p}) = f(\vec{p}) \cdot \theta(p_0) + f(-\vec{p}) \cdot \theta(-p_0). \]  

(22)

Here \( f(\vec{p}) \) is the distribution function. Note that in this Landshoff-Rehban scheme the transverse component of the gauge propagator proportional to \( T^{\mu\nu} \) contains the medium effect which is gauge-parameter independent. As the initial density matrix contains only sum over transverse polarizations of gluons the ghost propagator is simply given by:

\[ S(p)_{ij} = -i [G(p)]_{ij}^{\text{vac}}, \]

(23)

because ghost fields do not couple to the kernel \( K \) in the generating function.

III. RESUMMED GLUON PROPAGATOR AT ONE LOOP LEVEL IN NON-EQUILIBRIUM

In the last section we derived the gauge field propagator in the medium for the free part of the action \( S_0 \). In this section we will consider the interaction term of the action \( S_{\text{int}} \). For such a situation one will have to consider the full propagator instead of the free propagator in the medium. This implies one has to solve the full Schwinger-Dyson equation which is practically impossible. Usually one has to truncate the series at one or two loop level of the self-energy. Such a truncation demands that the QCD coupling constant should not be too large. We consider here the evolution of the gluon-minijet plasma at RHIC and LHC where the average transverse momentum of the partons are found to be large at the early stage [3]. Therefore the coupling constant corresponding to such average transverse momentum is found to be small. In this situation we truncate the Schwinger-Dyson equation at one loop level of the self energy and consider the corresponding resummed two-point Green’s function \( \tilde{G} \).

The resummed two-point Green’s function \( \tilde{G} \) of gluon can be decomposed as:

\[ \tilde{G}_{\mu\nu}(p) = -i T_{\mu\nu}(p) \tilde{G}^T(p) - i L_{\mu\nu}(p) \tilde{G}^L(p) - i \xi D_{\mu\nu}(p) \tilde{G}^D(p), \]

(24)

where \( \tilde{G}^T, \tilde{G}^L, \tilde{G}^D \) correspond to \( T, L \) and \( D \) components respectively. The last part \( \tilde{G}^D(p) \) is identical to the vacuum part [3] and hence we do not consider it any more. It can be mentioned that there are separate Dyson-Schwinger equation for different components which does not couple with each other.

Dyson-Schwinger equations for different components can be written as

\[ \left[ \tilde{G}^{T,L}_{\mu\nu}(p) \right]_{ij} = \left[ G^{T,L}_{\mu\nu}(p) \right]_{ij} + \sum_{l,k} \left[ G^{T,L}_{\mu\nu}(p) \right]_{il} \cdot \left[ \Pi^{T,L}_{\mu\nu}(p) \right]_{lk} \cdot \left[ \tilde{G}^{T,L}_{\mu\nu}(p) \right]_{kj}. \]

(25)

Please remember that \( i,j,k,l = +, - \) and suppression of Lorentz and color indices in the above equation is understood.

Instead of using the matrix form of this equation, we prefer to use the retarded, advanced and symmetric Green’s functions. The retarded and advanced resummed Green’s function are found to be:

\[ \tilde{G}^{T,L}_{R,A}(p) = G^{T,L}_{R,A}(p) + G^{T,L}_{R,A}(p) \cdot \Pi^{T,L}_{R,A}(p) \cdot \tilde{G}^{T,L}_{R,A}(p). \]

(26)
The straightforward solution of the above equation gives:

\[
\tilde{G}^{T,L}_{R,A}(p) = \frac{G^{T,L}_{R,A}(p)}{1 - G^{T,L}_{R,A}(p) \cdot \Pi^{T,L}_{R,A}(p)} = \frac{1}{p^2 - \Pi^{T,L}_{R,A}(p) \pm \text{sgn}(p_0)\epsilon},
\]

(27)

where the self-energy contains the medium effects. Similar but more complicated equation is obtained for the resummed symmetric Green’s function:

\[
\tilde{G}^{T,L}_S(p) = G^{T,L}_S(p) + G^{T,L}_R(p) \cdot \Pi^{T,L}_R(p) \cdot \tilde{G}^{T,L}_S(p)
+ G^{T,L}_S(p) \cdot \Pi^{T,L}_A(p) \cdot \tilde{G}^{T,L}_A(p) + G^{T,L}_R(p) \cdot \Pi^{T,L}_S(p) \cdot \tilde{G}^{T,L}_A(p).
\]

(28)

After some algebra it can be shown that

\[
\tilde{G}^{T,L}_S(p) = [1 + 2f(p)\text{sgn}(p_0)](\tilde{G}^{T,L}_R(p) - \tilde{G}^{T,L}_A(p))
+ (\Pi^{T,L}_S(p) - (1 + 2f(p)\text{sgn}(p_0))\Pi^{T,L}_R(p) - \Pi^{T,L}_A(p)]) \times \tilde{G}^{T,L}_R(p) \times \tilde{G}^{T,L}_A(p).
\]

(29)

Let us evaluate the various components of the self-energy. For short hand notation we denote \(Gl\) for gluon loop, \(Ta\) for tadpole diagram (see Fig. 1) and \(FP\) represents ghost loop diagram. Explicitly the expressions for various self energies are given by:

\[
\left[\Pi_{Gl;\mu\nu}^{ab}(p)\right]_{kl} = (k \times l)\frac{g^2}{2t} \delta_{ab} N_c \cdot \int \frac{d^4q}{(2\pi)^4} [G^{\alpha\gamma}(q)]_{kl} [G^{\beta\delta}(p - q)]_{kl} \cdot [g_{\mu\alpha}(-p - q)_\beta + g_{\alpha\beta}(2q - p)_\mu + g_{\beta\mu}(2p - q)_\alpha]
+ [g_{\nu\gamma}(p + q)_\delta + g_{\gamma\delta}(p - 2q)_\nu + g_{\nu\delta}(q - 2p)_\gamma].
\]

\[
\left[\Pi_{Ta;\mu\nu}^{ab}(p)\right]_{kl} = -g^2 N_c \delta_{ab} \delta_{kl} \int \frac{d^4q}{(2\pi)^4} (g_{\mu\lambda}g_{\nu\sigma} - g_{\sigma\lambda}g_{\mu\nu}) [G^{\alpha\lambda}(q)]_{kl}.
\]

\[
\left[\Pi_{FP;\mu\nu}^{ab}(p)\right]_{kl} = (k \times l)ig^2 \delta_{ab} N_c \cdot \int \frac{d^4q}{(2\pi)^4} [G^{vac}(q)]_{kl} [G^{vac}(p - q)]_{kl} \cdot q_\mu(p - q)_\nu.
\]

(30)

It has to be remembered that \(k, l = (-, +)\) are not contracted in the right hand side of the above equations. So it is obvious that \(\Pi_{Ta;\mu\nu}^{ab}(p)_{kl} = 0\), when \(k \neq l\). In the following calculation we neglect the vacuum part and concentrate on the medium part of the self energy. Note that the distribution function is contained only in the medium part of the self energy. The divergence of the vacuum part of the self energy is absorbed in the redefinition of the bare quantities which is well known. Like the Green’s function the self energy can be written as \([31,32]\):

\[
\Pi_{\mu\nu}(p) = T_{\mu\nu}(p)\Pi^T(p) + L_{\mu\nu}(p)\Pi^L(p).
\]

(31)

Using Eq. (15) \(\Pi\)'s are decomposed as:

\[
\Pi^{aT}(p) = \Pi^a_{\mu\nu}(p) \cdot \frac{T_{\mu\nu}(p)}{2}, \quad a = R, A, S,
\quad \text{and}
\]

\[
\Pi^{aL}(p) = \Pi^a_{\mu\nu}(p) \cdot L_{\mu\nu}(p), \quad a = R, A, S.
\]

(32)

Total self energy of the medium part is the sum of gluon loop and tadpole contributions (see Fig. 1) because the ghost loop contribution is present in the vacuum sector in the frozen ghost formalism. The real and imaginary part of the total self energy can be written as:

\[
\text{Re}\Pi^{T,L}(p)_a = \text{Re}\Pi^{T,L}(p)_{Gl;a} + \Pi^{T,L}(p)_{Ta;a}, \quad a = R, A, S,
\]
\[ \text{Im} \Pi_{T,L}^T(p) = \text{Im} \Pi_{T,L}^T(p)_{Gl:a}, \quad a = R, A, S. \]

One can check that the retarded and advanced self energy are related by:

\[
\text{Re} \Pi_{Gl,R}^{T,L}(p) = \text{Re} \Pi_{Gl,A}^{T,L}(p); \quad \text{Im} \Pi_{Gl,R}^{T,L}(p) = -\text{Im} \Pi_{Gl,A}^{T,L}(p),
\]

\[
\text{Re} \Pi_{Gl,R}^{T,L}(p) = \text{Re} \Pi_{Gl,A}^{T,L}(p); \quad \text{Im} \Pi_{Gl,R}^{T,L}(p) = -\text{Im} \Pi_{Gl,A}^{T,L}(p)
\]

\[ \Pi_{Ta;R}^{T,L}(p) = \Pi_{Ta;A}^{T,L}(p). \quad (33) \]

Furthermore we have \( \Pi_{Ta;S}^{T,L}(p) = 0 \). Therefore we only need calculate \( \text{Re} \Pi_{Gl,R}^{T,L}, \text{Im} \Pi_{Gl,R}^{T,L}, \Pi_{Gl;S}^{T,L} \) and \( \Pi_{Ta;R}^{T,L} \) which we compute in this paper for any non-equilibrium gluon distribution function \( f(p) \). Simplifying Eq. (30) and using Eq. (32) the explicit expression of retarded self energy of the gluon loop diagram is found to be:

\[
\Pi_{Gl,R}^{T,L}(p) = \frac{g^2}{2} \delta_{ab} N_c \int \frac{d^4q}{(2\pi)^4} \left[ \frac{\tilde{f}(\bar{q})\delta(q^2)H_{T,L}^{T,L}(q, p)}{(p - q)^2 + \text{isgn}(p_0 - q_0)\epsilon} + \frac{\tilde{f}(\bar{p} - \bar{q})\delta((p - q)^2)H_{T,L}^{T,L}(p - q, p)}{q^2 + \text{isgn}(q_0)\epsilon} \right], \quad (34)
\]

where

\[
H_T(q, p) = 8 \frac{(u \cdot q)(q \cdot p)(u \cdot p)}{(u \cdot p)^2 - p^2} - 4 \frac{(q \cdot p)^2}{(u \cdot p)^2 - p^2} - 4 \frac{p^2(q \cdot u)^2}{(u \cdot p)^2 - p^2}
\]

\[
- \left[ (p + q)^2 \right] \left[ 1 - \frac{(q \cdot u)(u \cdot p)}{(u \cdot q)((u \cdot p)^2 - p^2)} \right] + \frac{(q \cdot p)^2}{2((u \cdot p)^2 - p^2)} + \frac{p^2}{2((u \cdot p)^2 - p^2)} \right]
\]

\[
- 4p^2 + 8 \frac{(q \cdot p)(u \cdot p)}{(u \cdot q)} - 4 \frac{(q \cdot p)^2}{(u \cdot q)^2}
\]

\[
+ \frac{(\xi - 1)(p^4(-q^2 + 2p \cdot u)(p \cdot u)(q \cdot p) + (q \cdot u)^2(p^2 - 2(p \cdot u)^2)))}{2(q \cdot u)^2((q \cdot p)^2 - p^2)(p^2 - (p \cdot u)^2)} \quad (35)
\]
Similar but more complicated calculation for symmetric self energy yields:

\[ H^L(q, p) = \frac{8p^2}{(u \cdot p)^2 - p^2} \left[ (u \cdot q) - \frac{(u \cdot p)(q \cdot p)}{p^2} \right] - [(p + q)^2] \left[ \frac{2(q \cdot p)(p \cdot u)^3}{(q \cdot u)p^2((p \cdot u)^2 - p^2)} - \frac{(q \cdot p)^2(p \cdot u)^2}{(q \cdot u)^2p^2((p \cdot u)^2 - p^2)} - \frac{(p \cdot u)^2}{((p \cdot u)^2 - p^2)} \right] - 4p^2 + 8 \frac{(q \cdot p)(u \cdot p)}{(u \cdot q)} - 4 \frac{(q \cdot p)^2}{(u \cdot q)^2} + (\xi - 1) \frac{(p \cdot u)^3 - 2(q \cdot u)(p \cdot u)(q \cdot p) + (q \cdot u)^2p^2}{(q \cdot u)^2(p^2 - (p \cdot u)^2)}. \] (36)

Using the \( \delta \) function we get:

\[ \Pi_{GL; R}^{T,L}(p) = \frac{g^2}{2} \delta_{ab}N_c \int \frac{d^3q}{(2\pi)^3} \frac{1}{2|q|} \left[ f(q) \cdot H^{T,L}(q, p)|_{q_0=|q|} \cdot \frac{f(-q) \cdot H^{T,L}(q, p)|_{q_0=-|q|}}{2|q|} + \frac{f(q) \cdot H^{T,L}(q, p)|_{q_0=|q|}}{2|q|} \cdot f(-q) \cdot \tilde{f}(\vec{p} - q) \right] \]

\[ + \delta \left( (|q| - p_0)^2 - |\vec{q} - \vec{p}|^2 \right) \cdot \frac{K^{T,L}(q, p)|_{q_0=|q|}}{2|q|} \cdot f(-q) \cdot \tilde{f}(\vec{p} - q), \] (37)

where \( K^{T,L}(q, p) \) are given by:

\[ K^T(q, p) = \frac{p^2(p^2 + 4(q \cdot u)((q \cdot u) - (p \cdot u)))}{4(q \cdot u)^2(q \cdot u - p \cdot u)^2(p^2 - (p \cdot u)^2)} \left[ 8(q \cdot u)^4 - 16(p \cdot u)(q \cdot u)^3 - 12(p^2 - 2(p \cdot u)^2)(q \cdot u)^2 + 4(3p^2(p \cdot u) - 4(p \cdot u)^3)(q \cdot u) + 8(p \cdot u)^4 + p^4 - 8p^2(p \cdot u)^2 \right] \] (39)

and

\[ K^L(q, p) = \frac{p^2[(p \cdot u) - 2(q \cdot u)]^2}{4(q \cdot u)^2(q \cdot u - p \cdot u)^2(p^2 - (p \cdot u)^2)} \left[ 8(q \cdot u)^4 - 16(p \cdot u)(q \cdot u)^3 + 4(2p \cdot u)^2 + p^2)q \cdot u)^2 - 4p^2(p \cdot u)(q \cdot u) + p^4. \right] \] (40)

The expressions for the transverse and longitudinal part of the self energy coming from the Tadpole part (Eq. (30) and (32)) are found to be:

\[ \Pi_{T;r}^{T,L}(p) = g^2 \delta_{ab}N_c \int \frac{d^3q}{(2\pi)^3} \left[ \frac{f(q)}{2|q|} \left[ 1 + \frac{(u \cdot p)(q \cdot p)}{2(q \cdot u)((q \cdot u)^2 - p^2)} - \frac{(q \cdot p)^2}{2(q \cdot u)^2((q \cdot u)^2 - p^2)} \right] \right] \]

\[ - \frac{p^2}{2((u \cdot p)^2 - p^2)} \left| q_0=|q| \right| + \frac{f(-q)}{2|q|} \left[ 1 + \frac{(u \cdot p)(q \cdot p)}{2(q \cdot u)((q \cdot u)^2 - p^2)} - \frac{(q \cdot p)^2}{2(q \cdot u)^2((q \cdot u)^2 - p^2)} \right] \]

\[ - \frac{p^2}{2((u \cdot p)^2 - p^2)} \left| q_0=-|q| \right|. \] (41)
and

\[
\Pi_{T;A,R}(p) = g^2 \delta_{ab} N_c \int \frac{d^3q}{(2\pi)^3} \frac{f(q)}{2|q|} [3 - 2 \frac{(u \cdot p)(q \cdot p)}{p^2(u - q)} + \frac{p^2}{p^2(u - q)} (1 - \frac{(u \cdot p)(q \cdot p)}{p^2(u - q)})^2]|_{q_0 = -|q|} \\
+ \frac{f(-q)}{2|q|} [3 - 2 \frac{(u \cdot p)(q \cdot p)}{p^2(u - q)} + \frac{p^2}{p^2(u - q)} (1 - \frac{(u \cdot p)(q \cdot p)}{p^2(u - q)})^2]|_{q_0 = |q|} 
\]

(42)

respectively. To summarize Eqs. (37), (38), (41) and (42) contain all the expressions for different parts of the self energy in non-equilibrium which will be used in the medium modified propagator to obtain a finite collision term. In this paper we make hard momentum loop approximation in non-equilibrium which is similar to hard thermal loop approximation in equilibrium [33]. In the hard momentum loop approximation the self energies are found to be gauge parameter independent (see Appendix).

To obtain the collision term for the process \(gg \rightarrow gg\) in the medium we have to use the resummed Feynman propagator \(\tilde{G}(p)_{++}\) which can be obtained from the resummed advanced, retarded and symmetric propagators via the relation:

\[
[\tilde{G}(p)]_{++} = \frac{1}{2} \left[ \tilde{G}_S(p) + \tilde{G}_A(p) + \tilde{G}_R(p) \right], 
\]

(43)

where

\[
\tilde{G}_{R,A}(p) = \frac{1}{p^2 - \Pi_{R,A}(p) \pm isgn(p_0)\epsilon},
\]

(44)

and

\[
\tilde{G}_S(p) = [1 + 2f(\tilde{p})] sgn(p_0)[\tilde{G}_R(p) - \tilde{G}_A(p)] \\
+ (\Pi_S(p) - (1 + 2f(\tilde{p}) sgn(p_0)[\Pi_R(p) - \Pi_A(p)]) \times \tilde{G}_R(p) \times \tilde{G}_A(p). 
\]

(45)

Different parts of the self energy appearing in the above equations are given in Eqs. (37), (38), (41) and (42). Finally, using the relation between various parts of the self energy (see Eq. (38)) we find from the above equations:

\[
\tilde{G}_{++}(p) = \frac{p^2 - Re\Pi_R(p) + \frac{1}{2}\Pi_S(p)}{(p^2 - Re\Pi_R(p))^2 + (Im\Pi_R(p))^2} 
\]

(46)

which is the required expression for the medium modified resummed Feynman gluon propagator in non-equilibrium at one loop level of the self energy.

Our main purpose is to remove infrared divergence appearing in the small angle partonic scatterings which plays an important role in the production and equilibration of minijet plasma. For this purpose it is necessary to study the infrared behaviour of the self energies obtained in this paper for non-equilibrium situations. We note that in the static limit \((p_0 = 0, |\vec{p}| \rightarrow 0)\) one obtains \(\Pi^T(p_0 = 0, |\vec{p}| \rightarrow 0) = m_D^2\) (the Debye screening mass) and \(\Pi^T(p_0 = 0, |\vec{p}| \rightarrow 0) = m_g^2\) (the magnetic screening mass). It is widely believed that while Debye screening mass is non-zero the magnetic screening mass is zero at one loop level of the self energy. For this reason one expects that while the electric field is screened, the magnetic field is not screened and one still has infrared divergence in the magnetic sector at one loop level. However, this is true for a system where the gluon distribution function is isotropic.
(in momentum space) or in equilibrium. This is not true for non-isotropic gluon distribution function which is the case in the early stage of the heavy-ion collisions at RHIC and LHC. The static limit result of the transverse part of the self energy derived in this paper is not zero for non-isotropic (in momentum space) gluon distribution function. The typical values of the magnetic screening masses at one loop level is found to be 257 MeV at RHIC and 330 MeV at LHC by using non-equilibrium gluon-minijet distribution function \[27\] at the initial time. Hence the medium modified gluon propagator derived in this paper is safe from infrared divergences both in electric and magnetic sector in non-equilibrium situations at RHIC and LHC. Therefore these propagators can be used to obtain finite collision terms to study equilibration of minijet plasma at RHIC and LHC. Note that the above values are obtained by using the minijet distribution function at the initial time. The time evolution of these screening masses have to be determined by solving relativistic transport equations with Bjorken’s boost invariance picture taken into account. As these involves extensive numerical work (see [1]) we will report the self-consistent space-time evolution study elsewhere.

\section*{IV. CONCLUSIONS}

In this paper we have applied the closed-time path integral formalism to study medium effects of the minijet plasma in non-equilibrium. In particular we have derived the medium modified resummed gluon propagator to the one loop order of self energy which is necessary to obtain a finite collision term to study equilibration of parton plasma at RHIC and LHC. These medium modified propagators have been studied in more detail in finite temperature QCD formulations. However, at RHIC and LHC the parton momentum distributions at early stage are anisotropic and finite temperature QCD formulations can not be applied to these non-equilibrium situations. This is because a parton inside the nucleus (which travels almost at the speed of light at RHIC and LHC) carries mostly longitudinal momentum before an ultra relativistic nuclear collisions. After jets and minijets are formed and suffer secondary collisions the isotropy between longitudinal and transverse momentum may be achieved. According to Bjorken’s proposal [26] many quantities are expected to be expressed in terms of boost invariant parameters. For this reason and to have a covariant formulation we have worked in the covariant gauge which is a suitable gauge for expanding plasma. We give the result of the resummed gluon propagator upto one loop level of self energy in non-equilibrium. Furthermore it is shown that these propagators are infrared divergence free, both in electric and magnetic sector.

The medium modified resummed gluon propagator we derived in this paper will be used to obtain finite collision term for the $gg \rightarrow gg$ scattering process in non-equilibrium to solve the relativistic kinetic equation (Eq. (1)) to study equilibration of minijet plasma at RHIC and LHC. In this way one does not have to put ad-hoc values for the momentum transfer cut-off which crucially changes all the properties and hence determination of all the signatures of the quark-gluon plasma. In future we hope to use our results of the resummed gluon propagators in the collision term to study production and equilibration of the minijet plasma at RHIC and LHC. As the solution of the relativistic transport equation involves extensive numerical work (see [1]) we will report it elsewhere.
ACKNOWLEDGMENTS

We thank Fred Cooper for useful discussions. We thank Joerg Ruppert for his help in using Feynman package. C-W. K. and G. C. N. acknowledge the financial support from Alexander von Humboldt Foundation.
Appendix A: The explicit forms of self energies in Hard Loop Momentum Approximation:

We use hard momentum loop (HML) approximation in non-equilibrium which is equivalent to hard thermal loop (HTL) approximation in equilibrium. In the hard momentum loop approximation the loop momentum $q$ is harder than external momentum $p$. In HML approximation we find that the whole expressions are independent of gauge-fix parameters $\xi$. In the hard momentum loop approximation we find:

\[
Re\Pi_{GLR}^T(p) \sim \frac{g^2}{2} \delta_{ab} N_c \frac{1}{(u \cdot p)^2 - p^2} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2|q|} \left[ \frac{f(q)}{(|q| - p_0)^2 - |q - \vec{p}|^2} \right] \\
\cdot \left[ 8(u \cdot q)(q \cdot p)(u \cdot p) - 4(q \cdot p)^2 - 4p^2(q \cdot u)^2 \right] \\
- 2(p \cdot q)(u \cdot p)^2 + p^2(p \cdot q) + 2 \frac{(p \cdot q)^2(u \cdot p)}{u \cdot q} - 2 \left( \frac{q \cdot p}{u \cdot q} \right)^3 \bigg]_{q_0 = |q|} \\
+ \frac{f(-\vec{q})}{(|q| + p_0)^2 - |q - \vec{p}|^2} \left[ 8(u \cdot q)(q \cdot p)(u \cdot p) - 4(q \cdot p)^2 - 4p^2(q \cdot u)^2 \right] \\
- 2(p \cdot q)(u \cdot p)^2 + p^2(p \cdot q) + 2 \frac{(p \cdot q)^2(u \cdot p)}{u \cdot q} - 2 \left( \frac{q \cdot p}{u \cdot q} \right)^3 \bigg]_{q_0 = -|q|} \\
+ (q \to p - q),
\]

(47)

and

\[
Im\Pi_{GLR}^T(p) \sim \frac{g^2}{2} \delta_{ab} N_c \frac{1}{(u \cdot p)^2 - p^2} \int \frac{d^3q}{(2\pi)^3} \text{sgn}(p_0 - |q|) \delta \left( (p_0 - |q|)^2 - |q - \vec{p}|^2 \right) \left( \frac{f(q)}{2|q|} \right) \\
\left[ 8(u \cdot q)(q \cdot p)(u \cdot p) - 4(q \cdot p)^2 - 4p^2(q \cdot u)^2 \right] \\
- 2(p \cdot q)(u \cdot p)^2 + p^2(p \cdot q) + 2 \frac{(p \cdot q)^2(u \cdot p)}{u \cdot q} - 2 \left( \frac{q \cdot p}{u \cdot q} \right)^3 \bigg]_{q_0 = |q|} \\
+ \text{sgn}(p_0 + |q|) \delta \left( (|q| + p_0)^2 - |q - \vec{p}|^2 \right) \left( \frac{f(-\vec{q})}{2|\vec{q}|} \right) \\
\left[ 8(u \cdot q)(q \cdot p)(u \cdot p) - 4(q \cdot p)^2 - 4p^2(q \cdot u)^2 \right] \\
- 2(p \cdot q)(u \cdot p)^2 + p^2(p \cdot q) + 2 \frac{(p \cdot q)^2(u \cdot p)}{u \cdot q} - 2 \left( \frac{q \cdot p}{u \cdot q} \right)^3 \bigg]_{q_0 = -|q|} \\
+ (q \to p - q),
\]

(48)

Similarly for longitudinal components we obtain:

\[
Re\Pi_{GLR}^L(p) \sim \frac{g^2}{2} \delta_{ab} N_c \frac{1}{(u \cdot p)^2 - p^2} \int \frac{d^3q}{(2\pi)^3} \frac{f(q)}{2|q|} \left[ \frac{1}{(|q| - p_0)^2 - |q - \vec{p}|^2} \right] \left[ 8p^2((u \cdot q) - \frac{(u \cdot p)(q \cdot p)}{p^2})^2 \right] \\
- 4 \frac{(p \cdot q)^2(u \cdot p)^2}{(u \cdot q)p^2} + 2 \frac{(p \cdot q)^3(u \cdot p)}{(u \cdot q)^2p^2} + 2(u \cdot p)^2(q \cdot p) \bigg]_{q_0 = |q|} \\
+ \frac{1}{(2\pi)^3} \frac{1}{2|q|} \left[ \frac{f(-\vec{q})}{(|q| + p_0)^2 - |q - \vec{p}|^2} \left[ 8p^2((u \cdot q) - \frac{(u \cdot p)(q \cdot p)}{p^2})^2 \right] \\
- 4 \frac{(p \cdot q)^2(u \cdot p)^2}{(u \cdot q)p^2} + 2 \frac{(p \cdot q)^3(u \cdot p)}{(u \cdot q)^2p^2} + 2(u \cdot p)^2(q \cdot p) \bigg]_{q_0 = -|q|} \\
+ (q \to p - q),
\]

(49)
and

\[
Im \Pi_{Gl;R}^L(p) \sim \frac{g^2}{2} \delta_{ab} N_c \frac{8p^2}{(u \cdot p)^2 - p^2} \int \frac{d^3q}{(2\pi)^2} f(\bar{q}) \frac{1}{2|\bar{q}|} sgn(p_0 - |\bar{q}|)\delta \left( (|\bar{q}| - p_0)^2 - |\bar{q} - \bar{p}|^2 \right) \left[ 8p^2 \left( (u \cdot q) - \frac{(u \cdot p)(q \cdot p)}{p^2} \right)^2 \right] \\
- 4 \frac{(p \cdot q)^2(u \cdot p)^3}{(u \cdot q)p^2} + 2 \frac{(q \cdot p)^3(u \cdot p)^2}{(u \cdot q)^2p^2} + 2(u \cdot p)^2(q \cdot p) \right]_{p_0=|\bar{q}|} \\
+ \frac{f(-\bar{q})}{2|\bar{q}|} sgn(p_0 + |\bar{q}|)\delta \left( (|\bar{q}| + p_0)^2 - |\bar{q} - \bar{p}|^2 \right) \left[ 8p^2 \left( (u \cdot q) - \frac{(u \cdot p)(q \cdot p)}{p^2} \right)^2 \right] \\
- 4 \frac{(p \cdot q)^2(u \cdot p)^3}{(u \cdot q)p^2} + 2 \frac{(q \cdot p)^3(u \cdot p)^2}{(u \cdot q)^2p^2} + 2(u \cdot p)^2(q \cdot p) \right]_{p_0=-|\bar{q}|} \\
+ (q \to p - q) 
\]

(50)

For the symmetric part of the self energy we find:

\[
\Pi_{Gl;S}^{(T)}(p) = \frac{g^2}{2i} \delta_{ab} N_c \frac{4p^2}{(u \cdot p)^2 - p^2} \int \frac{d^3q}{(2\pi)^2} \delta \left( (|\bar{q}| - p_0)^2 - |\bar{q} - \bar{p}|^2 \right) \cdot |\bar{q}|(u_0 - \bar{u} \cdot \hat{q})^2 f(\bar{q}) \cdot \tilde{f}(\bar{p} - \bar{q}) \\
+ \delta \left( (|\bar{q}| + p_0)^2 - |\bar{q} - \bar{p}|^2 \right) \cdot |\bar{q}|(-u_0 - \bar{u} \cdot \hat{q})^2 f(-\bar{q}) \cdot \tilde{f}(\bar{p} - \bar{q}). 
\]

(51)

It is interesting to see that in the Hard Loop Momentum Approximation we have \( \Pi_{Gl;S}^{(T)}(p) \sim \Pi_{Gl;S}^{(L)}(p) \).
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