Force and strength analysis of FSAE racing suspension based on spatial analytic geometry

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Abstract: Based on spatial analytic geometry, the paper established a mechanical of suspension system, which is aimed at the structure of double transverse arm suspension of FSAE racing car. Combined with the typical driving conditions of the car, we have worked it out with the MATLAB. Based on that, we have made the finite element static checking analysis with ANSYS, which is aimed at providing effective basis for the design and optimization of racing car.

Key words: Double wishbone suspension; space analytic geometry; force analysis; static checking.

1. Introduction
Suspension is one of the core parts of automobile driving system, and its performance determines the performance of the whole vehicle. The double wishbone suspension is widely used in racing cars because of its superior performance.[1] The connecting rod is an important guide and transmission element in the suspension system, so the analysis of the connecting rod is very important. Both the finite element analysis of the connecting rod, the study of the fracture and fatigue failure of the connecting rod and the tensile and crushing test of the connecting rod need to determine the force acting on each hinge point of the connecting rod as the boundary load of calculation and test. [2] If the plane mechanism is used as the layout of the suspension bars when calculating the forces on the connecting rod, and the kingpin heel Angle and anti-pitching Angle etc. [3-6] existed in the suspension geometry are ignored, the calculation results will deviate from the real value. Or using a simplified double wishbone suspension system model, the control arm and the body hinge is simplified into a ball hinge and an inline constraint, and the linear equations are established to calculate the force of the force of the suspension system. [7] The calculation results of this method show that it is more accurate to calculate the forces on the ball hinge connected with the steering knuckle of the control arm, but it is not able to accurately calculate the forces on the hinge of the control arm and the car body.

In this paper, the spatial analytical geometry is used as the basis for calculation research to ensure the accuracy of the spatial force model. In the process of racing, it is regarded as a state of dynamic equilibrium, which simplifies the direction of the space force, so that the magnitude and direction of the calculate force are closer to the real value.
2. Suspension mechanics model

The suspension of FSAE racing car mostly adopts the independent suspension structure with unequal length of double wishbone, and all of them are in the form of pushrod. Its 3d model is shown in figure 1:

![Double wishbone suspension model](image1)

**Fig. 1** double wishbone suspension model (front left wheel)

The suspension and wheel are analyzed as a whole, and the stress each point of the suspension system is solved by analytical method. The three-dimensional model of the suspension system is simplified, and its three-dimensional force analysis diagram is show in figure 2, and the following assumptions are made:

1. Except for springs, all components in the systems are rigid bodies without deformation.
2. Ignore the influence of gravity of each component of the suspension system and friction at the hinge points of each ball.
3. Under various driving conditions of the vehicle, the force exerted on the suspension system is a quasi-static process, that is, the inertial force and damping force under various working conditions are not considered.

![3d schematic diagram of front suspension](image2)

**Fig. 2** 3d schematic diagram of front suspension

In the figure, $F_1$ - the front point of upper transverse arm, $F_2$ - the rear point of upper transverse arm, $F_3$ - the front point of lower transverse arm, $F_4$ - the rear point of lower transverse arm, $F_5$ - the interior point of push rod. The magnitude and direction of these forces are unknown, so the direction is assumed to be outward along the rod. And following is set up, the coordinates of $uca_{front}$, $uca_{rear}$, $lea_{front}$, $lea_{rear}$, $prod_{inner}$, $lea_{outer}$, $uca_{outer}$, $prod_{outer}$, and $wheel_{center}$ are $(X_1, Y_1, Z_1)$, $(X_2, Y_2, Z_2)$, $(X_3, Y_3, Z_3)$, $(X_4, Y_4, Z_4)$, $(X_5, Y_5, Z_5)$, $(X_6, Y_6, Z_6)$, $(X_7, Y_7, Z_7)$, $(X_8, Y_8, Z_8)$, $(X_9, Y_9, Z_9)$. 

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*Note: The images are placeholders and should be replaced with actual figures.*
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b- horizontal distance from rear axle to center of mass, h- height of center of mass, L=a+b- wheelbase.

In the braking (acceleration) condition, the force diagram of the car is show in figure 3.

3. Analysis of typical working conditions
In the running process of a racing car, its stress state is very complex. It is composed of various driving conditions (29 working conditions specified by General Motors), which can determine the load acting on the wheels, and then calculate the load acting on the ball hinge and bushing of the control arms.[3] Only the basic conditions of braking (acceleration) and uniform turning are analyzed.

3.1. The condition of braking (acceleration)
In the braking (acceleration) condition, the force diagram of the car is show in figure 3.

In the figure, F1- inertia force, G-vehicle weight, Fyf, Fyv- vertical load of front and rear wheels, Frf, Frv- friction of front and rear wheels, a- horizontal distance from front axle to center of mass, b- horizontal distance from rear axle to center of mass, h- height of center of mass, L=a+b- wheelbase.

![Fig. 3 Force analysis diagram of braking (acceleration) condition](image-url)
According to the FSAE racing rules, when the car is braking, the four wheels must lock together at the same time, then there is a backward acceleration, a forward inertia force $F_I$, where the attachment coefficient is 1.4 (The same is true for acceleration condition with opposite accelerations and inertial forces).

Plane equilibrium equation of braking condition:

\[
\begin{align*}
F_{Nf} &= \frac{G}{L} (b + \psi \cdot h) \\
F_{Nr} &= \frac{G}{L} (a - \psi \cdot h) \\
F_{Bf} &= F_{Nf} \cdot \psi \\
F_{Br} &= F_{Nr} \cdot \psi
\end{align*}
\] (5)

3.2. Uniform turning
When the racing car enters the corner at a constant speed, its front suspension force is shown in figure 5.

In the figure, $F_I$ -- inertia force, $G_1$ -- load distribution of vehicle weight in front axle, $B_1$ -- front wheelbase. $F_{NW}, F_{Nh}$ -- vertical load of outer and inner wheel, $F_{cw}, F_{cn}$ -- lateral force of outer and inner wheel, $K_{zf}, K_{zn}$ -- the camber stiffness of front and rear suspension.
For the moment balance equilibrium equation of the bearing point of the front right wheel, it can be obtained:

\[
\begin{align*}
F_{NW} &= \frac{G \cdot b}{2L} + \frac{G \cdot b \cdot w_f \cdot A_y}{g \cdot L \cdot B_1} + \frac{G \cdot h \cdot A_y \cdot K \cdot \psi F}{g \cdot B_1 (K \cdot \psi F + K \cdot \psi R)} \\
F_{NN} &= \frac{G \cdot b}{2L} - \frac{G \cdot b \cdot w_f \cdot A_y}{g \cdot L \cdot B_1} - \frac{G \cdot h \cdot A_y \cdot K \cdot \psi F}{g \cdot B_1 (K \cdot \psi F + K \cdot \psi R)}
\end{align*}
\]

(7)

\[\begin{align*}
F_C &= F_{NW} \cdot \psi \\
F_C &= F_{NN} \cdot \psi
\end{align*}\]

4. Example analysis of front suspension under typical driving conditions

The force formula of each hinge point of the suspension system has been derived above. The main parameters of the vehicle and suspension parameters are shown in table 1, 2:

| Table 1 Main parameters of the vehicle |
|---------------------------------------|
| The mass M (Kg) | 285 |
| Height of center of mass h (mm) | 270 |
| Wheel base L (mm) | 1550 |
| Wheelspan B1/B2 (mm) | 1200 1150 |
| The ratio of the axial load | 0.45 0.55 |
| Height of roll center h1/w (mm) | 20 40 |
| Maximum linear acceleration a (g) | 1 |
| Maximum roll acceleration aR (g) | 1.7 |
| Lateral stiffness (Nm/°) | 432.562 410.72 |
| Adhesion coefficient of braking | 1.4 |
| Lateral adhesion coefficient | 1.7 |

| Table 2 Coordinates of hard points (hinge points) |
|-----------------------------------------------|
| hard points (hinge points) | X | Y | Z |
| uca_front (point 1) | -111.643 | -265 | 279.599 |
| uca_rear (point 2) | 120.012 | -265 | 279.599 |
| lca_front (point 3) | -136.217 | -136.217 | 130.263 |
| lca_rear (point 4) | 138.189 | -220 | 146.467 |
| prod_inboard (point 5) | -4.184 | -286.119 | 595.401 |
| uca_outer (point 6) | 4.184 | -144.291 | 318.945 |
| lca_outer (point 7) | -4.184 | -549.858 | 148.759 |
| prod_outboard (point 8) | -4.184 | -509.878 | 177.477 |
| wheel_center (point 9) | 0 | -600 | 228.6 |

These parameters are substituted into equations (1) to (10), and the following data are obtained by the calculation of MATLAB:

| Table 3 Braking (acceleration) force calculation results |
|---------------------------------------------------------|
| Front single wheel load (N) | 1108.641 |
| Rear single wheel load (N) | 287.859 |
| Front wheel braking force (N) | 1552.097 |
| Rear wheel braking force (N) | 403.002 |
| Force on the front point of upper arm (N) | -2527.486 |
| Force on the rear point of upper arm (N) | 540.730 |
| Force on the front point of lower arm (N) | 2304.321 |
| Force on the rear point of lower arm (N) | 147.345 |
| Force on the pushrod (N) | -1065.571 |
Table 4 Calculation results of turning forces

| Force on the front point of lower arm (N) | -2260.253 |
| Force on the rear point of upper arm (N) | 475.976 |
| Force on the front point of upper arm (N) | 2079.700 |
| Force on the rear point of lower arm (N) | 76.190 |
| Lateral force on the outer side of front wheel (N) | 768.075 |
| Load on the inside of rear wheel (N) | 131.360 |
| Force on the rear point of upper arm (N) | 2060.455 |
| Lateral force on the inside of rear wheel (N) | 1401.857 |
| Lateral force on the inside of front wheel (N) | 44.818 |
| Lateral force on the outside of front wheel (N) | 134.293 |
| Force on the front point of upper arm (N) | 2097.970 |
| Load on the inside of front wheel (N) | 5. Finite element analysis of suspension system

The CAD model was established for double wishbone of suspension by CATIA, which was then converse into STP format and imported into ANSYS workbench. Boundary conditions, such as constraints and loads, were imposed on geometry and meshed, and then rigid body static simulation calculation was carried out.

The results of braking (acceleration) conditions are shown in FIG 6:

Fig. 6 Calculation cloud chart for braking (acceleration) condition

The results of turning conditions are shown in figure 7:
Analysis of simulation result:

From the distribution of the overall cloud chart, it can be clearly concluded that under the braking (acceleration) condition, the large stress and strain areas of the upper transverse arm and the lower transverse arm are mainly distributed in the sector at the outer point. Considering that the joint bearing installed at the sector will bear the force with changing direction, it is necessary to re-check and optimize the design of the sector and prepare spare parts. Under the turning condition, the larger stress and strain areas are distributed on the inner point side. Because in the FSAE competition, there are multiple combined turning that the racing driver will repeatedly enter the turning, so the welding of the bearing sleeve at the inner point bar end must be tested repeatedly to ensure its reliability.

By analyzing the maximum stress and strain, it can be concluded that under the basic working condition of the example, the maximum stress of the upper transverse arm and the pushrod is 61.773Mpa (the pushrod is 32.233Mpa), which is far lower than the yield strength of the 45 steel 355Mpa. Therefore, the design margin of the upper transverse arm and the pushrod is too large. The rod with smaller pipe diameter or wall thickness can be considered to satisfy the lightweight design. As for the lower transverse arm, it shows obvious stress concentration. The maximum stress reached 527.55Mpa, which was greater than the yield strength of 45 steel 355Mpa. Consequently, the design of sector and pushrod support could not meet the requirements of working conditions, so a new design was required.

6. Conclusion

(1) In this paper, a mechanical model of the FSAE racing suspension system was built based on the spatial analytic geometry for the FSAE racing double wishbone suspension, and the calculation and solution were completed based on MATLAB for the vehicle driving conditions.

(2) Finite element analysis was conducted on the control arms and pushrod by ANSYS Workbench. The analysis results clearly showed the distribution of stress and strain, and the conclusion was drawn
that both the upper transverse arm design margin was too large and the lower transverse arm design was insufficient, and the design should be reconsidered.

(3) In this paper, the calculation method for the force analysis of double wishbone suspension is established based on the basic mechanical model. Within the allowable range of error and model simplification, the calculation results are closer to the real value, and the modeling solution method is also applicable to other forms of suspension.

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