Forward-backward correlations between multiplicities in windows separated in azimuth and rapidity.

V.V. Vechernin

St.-Petersburg State University
E-mail: vechernin@pobox.spbu.ru

Abstract
The forward-backward (FB) correlations between multiplicities in windows separated in rapidity and azimuth are analyzed in the framework of the model with strings as independent identical emitters. Both the short-range (SR) contribution, originating from the correlation between multiplicities produced by a single source, and the long-range (LR) contribution, originating from the fluctuation in the number of strings, are taken into account. The dependencies of the FB correlation coefficient ($b$) on rapidity and azimuthal acceptance of windows and on corresponding gaps between them are studied and compared with the experimental data. It is shown that the analysis of these dependencies enables to separate the contributions of two above mechanisms.

It is also demonstrated that the traditionally defined FB correlation coefficient $b$ has the strong nonlinear dependence on the acceptance of windows and suitable observables for the future FB correlation studies are proposed. The connection of the $b$ with the two-particle correlation function $C_2$ and the untriggered di-hadron correlation analysis is traced. It is shown by a model independent way that the measurements of the FB correlations between multiplicities in two small windows separated in rapidity and azimuth enable to find the two-particle correlation function $C_2$ even if the particle distribution in rapidity is not flat.

1 Introduction

For a long time, the considerable attention is devoted to the experimental [1]-[6] and theoretical [7]-[19] investigations of the so-called forward-backward (FB) correlation in high-energy pp and AA collisions - the correlation between multiplicities $n_F$ and $n_B$ of charged particles produced in two separated rapidity windows (“forward” and “backward”). The old problem in this correlation analysis is the separation of the so-called “volume” contribution, originating from the event-by-event fluctuation in the number of emitting sources [7].

In paper [17] it was suggested to use for this purpose the information on the event multiplicity in an additional third rapidity window, but as discussed in [18] it complicates the interpretation of obtained results. In present paper we argue that the investigation of the FB correlation between multiplicities in windows separated both in rapidity and azimuth can enable not only to separate the volume contribution, originating from the fluctuation in the number of sources, but also to obtain the important quantitative physical information on the magnitude of this fluctuation in the processes under consideration.

We also show that the traditional definition of the FB correlation coefficient leads to its strong dependence on the acceptance of the windows, with the correlation coefficient going to zero with the acceptance. As consequence the results obtained for the windows of different width can’t be compared directly. In this connection we propose suitable observables for the FB correlation studies, which have finite limit when the window acceptances go to zero.
To check our observations we use the simple two stage model \[9, 10, 20\], inspired by a string picture of hadronic interactions. In this model one suggests that at the initial stage of interaction some number of strings are formed, which then are considered as identical independent emitters of observed charge particles. In our note \[21\] we considered only the long-range (LR) part of the correlation, originating from the fluctuation in the number of sources (the strings or as suggested in \[7\] the cut pomerons). In the present paper we include into consideration in general form the short-range (SR) correlation between particles produced by a single string. This SR correlation can arise due to very different physical processes such as the formation and decay of clusters, resonances or minijets during the string fragmentation. Important that the presence of such SR correlation, along with the influence on the FB multiplicity correlation, inevitably turns a string into non-poissonian emitter.

We show that the investigation of the FB multiplicity correlation in the case of windows separated both in rapidity and azimuth enables to separate the LR and SR contributions. We also demonstrate by a model independent way that the measurements of the FB correlation coefficient between multiplicities in two small windows separated in rapidity and azimuth enable to find the two-particle correlation function $C_2$ even if the particle distribution in rapidity is not flat (as e.g. in the case of pA interactions) and the $C_2$ depends not only on the differences of rapidities.

The paper is organized as follows. In Sec.2 we discuss the different versions of the definition of the FB correlation coefficient and generalize this definition for the case of windows separated both in rapidity and azimuth. In Sec.3 the connection of the FB correlation coefficient with the two-particle correlation function $C_2$ is traced.

In Sec.4 we formulate the two stage model with strings as independent identical sources, introduce the pair correlation function of a single string and calculate the FB correlation coefficient including the LR and SR contributions in the framework of the model.

In Sec.5 we parameterize the pair correlation function of a single source, in accordance with a string decay picture, and fit the parameters using the data on the FB correlation strength between multiplicities in small azimuth and rapidity windows.

In Sec.6 in the framework of the model with all parameters fixed we calculate the values of the FB correlation coefficient for large $2\pi$-azimuth acceptance windows of different rapidity width and gap between them and compare the obtained results with the preliminary experimental data of ALICE \[30\].

In Sec.7 we discuss the introduction of suitable alternative observables for future FB multiplicity correlation studies.

Appendix A describes the calculation of integrals over rapidity and azimuth windows. In Appendix B we present the alternative derivation of the basic formulae and check that the calculated expression for the FB correlation coefficient in the case of the windows separated by a large rapidity gap turns into the expression for the LR correlation coefficient obtained earlier in \[21\]. In Appendix C we discuss the correspondence between the FB multiplicity correlations in windows separated in azimuth and rapidity and the so-called untriggered di-hadron correlations.

2 Definition of the FB correlation coefficient

Traditionally \[1, 2, 4, 5\] the FB correlation coefficient is defined as a coefficient $b$ in linear regression

\[
\langle n_B \rangle_{n_F} = a + b n_F .
\]  

(1)
In this case
\[ b = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{D_{n_F}}, \]  
\[ (2) \]
where \( D_{n_F} \) is the variance of the multiplicity in the forward window
\[ D_{n_F} = \langle n_F^2 \rangle - \langle n_F \rangle^2. \]
\[ (3) \]
Clear that the value of such defined correlation coefficient changes, if one will change independently the acceptances of the forward and/or backward windows. To avoid this trivial influence one can go from \( n_F \) and \( n_B \) to the relative or scaled observables \[ \nu_F = \frac{n_F}{\langle n_F \rangle} \] and \( \nu_B = \frac{n_B}{\langle n_B \rangle} \). In these observables
\[ \langle \nu_B \nu_F \rangle = a_{rel} + b_{rel} \nu_F \]
\[ (4) \]
In some papers \[ [3, 6] \] the following symmetrized form of \[ (2) \] is also used
\[ b_{sym} = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\sqrt{D_{n_F} D_{n_B}}}, \]
\[ (5) \]
for which one can prove that \( |b_{sym}| \leq 1 \). Note that in the case of symmetric windows, when \( \langle n_F \rangle = \langle n_B \rangle \) and \( D_{n_F} = D_{n_B} \), all these definitions lead to the same result
\[ b_{rel} = b_{sym} = b. \]
\[ (6) \]
In present paper we study the correlation between multiplicities \( n_F \) and \( n_B \) in windows separated both in rapidity and in azimuth. Denote by \( \delta \eta_F, \delta \phi_F \) and by \( \delta \eta_B, \delta \phi_B \) the width of the forward and backward windows in rapidity and in azimuth, and by \( \eta_F, \phi_F \) and \( \eta_B, \phi_B \) - the positions of the centers of the windows. We’ll also use the following short notation for the acceptance of forward and backward windows
\[ \delta_F \equiv \delta \eta_F \delta \phi_F / 2\pi, \quad \delta_B \equiv \delta \eta_B \delta \phi_B / 2\pi. \]
\[ (7) \]
By
\[ \eta_{FB} \equiv \eta_F - \eta_B, \quad \phi_{FB} \equiv \phi_F - \phi_B \]
\[ (8) \]
we denote the distance between the centers of the windows in rapidity and in azimuth.
These variables in an obvious way are connected with the so-called gaps \( \eta_{\text{gap}} \) and \( \phi_{\text{gap}} \) between window in rapidity and in azimuth:
\[ \eta_{FB} = \frac{\delta \eta_F}{2} + \eta_{\text{gap}} + \frac{\delta \eta_B}{2}, \quad \phi_{FB} = \frac{\delta \phi_F}{2} + \phi_{\text{gap}} + \frac{\delta \phi_B}{2}, \]
\[ (9) \]
or for symmetric windows, when \( \delta \eta_F = \delta \eta_B = \delta \eta \) and \( \delta \phi_F = \delta \phi_B = \delta \phi \):
\[ \eta_{FB} = \eta_{\text{gap}} + \delta \eta, \quad \phi_{FB} = \phi_{\text{gap}} + \delta \phi. \]
\[ (10) \]
3 Connection with two-particle correlation function

One can express the FB correlation coefficient through the two-particle correlation function \( C_2(\eta_1, \eta_2; \phi_1, \phi_2) \). For this purpose, we introduce the \( \rho_1(\eta, \phi) \) and \( \rho_2(\eta_1, \phi_1; \eta_2, \phi_2) \) – the one- and two-particle densities of charge particles:

\[
\rho_1(\eta, \phi) = \frac{d^2 N}{d\eta d\phi}, \quad \rho_2(\eta_1, \phi_1; \eta_2, \phi_2) = \frac{d^4 N}{d\eta_1 d\phi_1 d\eta_2 d\phi_2}.
\] (11)

If we integrate (11) over the forward acceptance interval, \( \eta \in \delta \eta_F, \phi \in \delta \phi_F \), we have [23]:

\[
\int_{\delta \eta_F} d\eta \int_{\delta \phi_F} d\phi \rho_1(\eta, \phi) = \langle n_F \rangle,
\] (12)

\[
\int_{\delta \eta_F} d\eta_1 \int_{\delta \eta_F} d\eta_2 \rho_2(\eta_1, \phi_1; \eta_2, \phi_2) = \langle n_F(n_F - 1) \rangle.
\] (13)

When we integrate over the forward, \( y_1 \in \delta \eta_F, \phi_1 \in \delta \phi_F \), and the backward, \( y_2 \in \delta \eta_B, \phi_2 \in \delta \phi_B \), acceptance intervals, we have

\[
\int_{\delta \eta_F} d\eta_1 \int_{\delta \eta_B} d\eta_2 \rho_2(\eta_1, \phi_1; \eta_2, \phi_2) = \langle n_F n_B \rangle.
\] (14)

The \( \langle n_F \rangle \) is an average multiplicity produced in the acceptance \( \delta \eta_F \delta \phi_F \).

By (12) and (13) for windows of small acceptance in rapidity and azimuth we have

\[
\rho_1(\eta_F, \phi_F) = \frac{\langle n_F \rangle}{\delta \eta_F \delta \phi_F}, \quad \rho_2(\eta_F, \phi_F; \eta_B, \phi_B) = \frac{\langle n_F n_B \rangle}{\delta \eta_F \delta \phi_F \delta \eta_B \delta \phi_B},
\] (14)

\[
\rho_2(\eta_F, \phi_F; \eta_F, \phi_F) = \frac{\langle n_F(n_F - 1) \rangle}{(\delta \eta_F \delta \phi_F)^2}.
\] (15)

These formulae enable the experimental measurement of the one- and two-particle densities of charge particles \( \rho_1(\eta, \phi) \) and \( \rho_2(\eta_1, \phi_1; \eta_2, \phi_2) \), and hence the two-particle correlation function \( C_2 \), which is introduced in a standard way:

\[
C_2(\eta_1, \eta_2; \phi_1, \phi_2) = \frac{\rho_2(\eta_1, \eta_2; \phi_1, \phi_2)}{\rho_1(\eta_1, \phi_1) \rho_1(\eta_2, \phi_2)} - 1.
\] (16)

Substituting (14) in (16) we get for windows of small acceptance in rapidity and azimuth:

\[
C_2(\eta_F, \phi_F; \eta_B, \phi_B) = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\langle n_F \rangle \langle n_B \rangle} = \left( \frac{n_F}{\langle n_F \rangle} \frac{n_B}{\langle n_B \rangle} \right) - 1.
\] (17)

Important that by (17) the observation of multiplicity-multiplicity correlation with two small (in azimuth and rapidity) windows, enables to measure the two-particle correlation function \( C_2 \) in accordance with the standard definition (16) even in the case of nonhomogeneous distributions (e.g. in the absence of the translation invariance in rapidity) and without using an event mixing procedure, usually applied in a di-hadron correlation analysis (see the discussion in Appendix C).

Note also that if in formula (17) we’ll mean by \( n_F \) and \( n_B \) the multiplicities of particles with the certain transverse momenta belonging correspondingly to the intervals \( \delta p_T F \) and \( \delta p_T B \), than by (17) one can measure the two-particle correlation function \( C_2 \) between particles
and \( \delta p \)
\( \delta \phi \)
\( \delta \eta \)
\( \delta \rho \)
\( \delta p_{TF} \)
\( \delta p_{TB} \)
\( C_2(\mathbf{p}_F, \mathbf{p}_B) \)
\( \mathbf{p}_F \)
\( \mathbf{p}_B \)
\( \delta \eta_F \)
\( \delta \phi_F \)
\( \delta \eta_B \)
\( \delta \phi_B \)
\( \rho_1(\eta, \phi) = \rho_1(\eta)/2\pi \)
\( \rho_2(\eta_1, \phi_1; \eta_2, \phi_2) = \rho_2(\eta_1, \eta_2; \phi_1 - \phi_2)/(2\pi)^2 \) (18)
\( C_2(\eta_1, \eta_2; \phi_1 - \phi_2) = \frac{\rho_2(\eta_1, \eta_2; \phi_1 - \phi_2)}{\rho_1(\eta_1)\rho_1(\eta_2)} - 1 \) (19)
\( \langle n_F \rangle_{n_B} - \langle n_F \rangle \langle n_B \rangle = \langle n_F \rangle \langle n_B \rangle I_{FB} \) (20)
\( D_{n_F} = \langle n_F \rangle + \langle n_F \rangle^2 I_{FF} \) (21)
\( \langle n_F \rangle = \frac{\delta \phi_F}{2\pi} \int_{\delta \eta_F} d\eta \rho_1(\eta) \) (22)
\( I_{FB} = \frac{1}{(2\pi)^2 \langle n_F \rangle \langle n_B \rangle} \int_{\delta \eta_F} d\eta_1 d\phi_1 \int_{\delta \eta_B} d\eta_2 d\phi_2 \rho_1(\eta_1)\rho_1(\eta_2)C_2(\eta_1, \eta_2; \phi_1 - \phi_2) \) (23)
\( I_{FF} = \frac{1}{(2\pi)^2 \langle n_F \rangle^2} \int_{\delta \eta_F} d\eta_1 d\phi_1 \int_{\delta \eta_F} d\eta_2 d\phi_2 \rho_1(\eta_1)\rho_1(\eta_2)C_2(\eta_1, \eta_2; \phi_1 - \phi_2) \) (24)
\( b_{rel} = \frac{\langle n_F \rangle}{\langle n_B \rangle} b = \frac{\langle n_F \rangle I_{FB}}{1 + \langle n_F \rangle I_{FF}} \) (25)
Important that in the absence of correlation, when \( C_2 = 0 \), we have \( I_{FB} = I_{FF} = 0 \) and simultaneously by (21) \( D_{n_F} = \langle n_F \rangle \) for the multiplicity distribution in the forward window.
Further simplification of the integrals (23) and (24) is discussed in Appendix A. Note only that, for example, in the important case of FB windows separated only in rapidity (i.e. when \( \delta \phi_F = \delta \phi_B = 2\pi \)), by (24) we have
\( \langle n_F \rangle = \int_{\delta \eta_F} d\eta \rho_1(\eta) \) (26)
\( I_{FB} = \frac{1}{\langle n_F \rangle \langle n_B \rangle} \int_{\delta \eta_B} d\eta_1 \int_{\delta \eta_F} d\eta_2 \rho_1(\eta_1)\rho_1(\eta_2)C_2(\eta_1, \eta_2) \) (27)
\( I_{FF} = \frac{1}{\langle n_F \rangle^2} \int_{\delta \eta_F} d\eta_1 \int_{\delta \eta_F} d\eta_2 \rho_1(\eta_1)\rho_1(\eta_2)C_2(\eta_1, \eta_2) \) (28)
where
\( C_2(\eta_1, \eta_2) = \frac{1}{\pi} \int_0^\pi d\phi C_2(\eta_1, \eta_2; \phi) \) (29)
For windows, which are small both in rapidity and in azimuth (within which one can consider the \( C_2(\eta_1, \eta_2; \phi_1 - \phi_2) \) and \( \rho_1(\eta) \) to be constant) we have
\( \langle n_F \rangle = \rho_1(\eta_F)\delta_F \)
\( \langle n_B \rangle = \rho_1(\eta_B)\delta_B \) (30)
\[ I_{FB} = C_2(\eta_F, \eta_B; \phi_{FB}) , \]
\[ I_{FF} = C_2(\eta_F, \eta_F; 0) , \]
\[ D_{n_F} = \langle n_F \rangle [1 + \langle n_F \rangle C_2(\eta_F, \eta_F; 0)] , \]

and
\[ b_{rel} = \frac{\langle n_F \rangle}{\langle n_B \rangle} b = \frac{\langle n_F \rangle C_2(\eta_F, \eta_B; \phi_{FB})}{1 + \langle n_F \rangle C_2(\eta_F, \eta_F; 0)} . \]

Remember our short notations (7) and (8). We see that the correlation coefficient (4), even defined in scaled variables, still depends through \( \langle n_F \rangle \) on the acceptance \( \delta_F \) of the forward window, that was observed earlier [20, 21] in a framework of a simple model.

In the case when both small FB windows are situated in the central region, where one can suppose the translation invariance in rapidity:
\[ \rho_1(\eta) = \rho_0 , \quad C_2(\eta_1, \eta_2; \phi) = C_2(\eta_1 - y_2; \phi) \]
the formulae (30)–(34) admit further simplification:
\[ \langle n_F \rangle = \rho_0 \delta_F , \quad \langle n_B \rangle = \rho_0 \delta_B , \]
\[ D_{n_F} = \langle n_F \rangle [1 + \delta_F \rho_0 C_2(0, 0)] , \]
\[ b_{rel} = \frac{\delta_F}{\delta_B} b = \frac{\delta_F \rho_0 C_2(\eta_{FB}, \phi_{FB})}{1 + \delta_F \rho_0 C_2(0, 0)} . \]

At last for large windows situated in the central rapidity region along with (35) and (36) we must to use the formulae (21) and (25), with the following expressions for \( I_{FB} \) and \( I_{FF} \):
\[ I_{FB} = (\delta \eta_F \delta \phi_F \delta \eta_B \delta \phi_B)^{-1} \int d\eta_1 d\phi_1 \int d\eta_2 d\phi_2 C_2(\eta_1 - y_2; \phi_1 - \phi_2) , \]
\[ I_{FF} = (\delta \eta_F \delta \phi_F)^{-2} \int d\eta_1 d\phi_1 \int d\eta_2 d\phi_2 C_2(\eta_1 - y_2; \phi_1 - \phi_2) \]
(further simplification of the integrals see in Appendix A).

4 The model

We now calculate the FB correlations in windows separated in rapidity and azimuth using the simple two stage model [9, 10, 20], inspired by a string picture of hadronic interactions. In this model we suggests that at the initial stage of interaction some number \( N \) of strings are formed, which fluctuates event-by-event with some variance \( D_N = \langle N^2 \rangle - \langle N \rangle^2 \) and scaled variance
\[ \omega_N = D_N / \langle N \rangle . \]

Note that the fluctuation in the number of strings in pp and especially in AA collisions is not poissonian [28] and hence \( \omega_N \neq 1 \). Its value depends on the collision energy. At next stage of the interaction one considers these strings as identical independent sources of observed charge particles.

In the present work, along with the so-called long-range (LR) part of the correlation [7, 10, 21], originating from the fluctuation in the number of strings, we take into account also the short-range (SR) contribution, originating from the correlation between particles produced by a single string.
4.1 Pair correlation function of a single string

To characterize the last property of a string we introduce, similarly to the consideration in the section [3], the two-particle correlation function for charged particles produced from a decay of a single string

\[
\Lambda(\eta_1, \eta_2; \phi_1 - \phi_2) = \frac{\lambda_2(\eta_1, \eta_2; \phi_1 - \phi_2)}{\lambda_1(\eta_1)\lambda_1(\eta_2)} - 1 ,
\]

where the \(\lambda_1(\eta)\) and \(\lambda_2(\eta_1, \eta_2; \phi_1 - \phi_2)\) are the one- and two-particle densities of charge particles produced by one string. We suppose that the particle emission from a string is isotropic in \(\phi\).

Similarly to (12) and (13) one can write

\[
\int_{\delta\eta, \delta\phi} d\eta d\phi \lambda_1(\eta, \phi) = \langle \mu_F \rangle ,
\]

\[
\int_{\delta\eta, \delta\phi} d\eta_1 d\phi_1 \int_{\delta\eta, \delta\phi} d\eta_2 d\phi_2 \lambda_2(\eta_1, \eta_2; \phi_1 - \phi_2) = \langle \mu_F (\mu_F - 1) \rangle ,
\]

\[
\int_{\delta\eta, \delta\phi} d\eta_1 d\phi_1 \int_{\delta\eta_B, \delta\phi_B} d\eta_2 d\phi_2 \lambda_2(\eta_1, \eta_2; \phi_1 - \phi_2) = \langle \mu_B \mu_F \rangle ,
\]

where now the \(\langle \mu_F \rangle\) and \(\langle \mu_B \rangle\) are the average multiplicities produced by one string in the forward \(\delta\eta F \delta\phi F\) and backward \(\delta\eta_B \delta\phi_B\) windows.

By (42)–(44) and we have

\[
\langle \mu_B \mu_F \rangle - \langle \mu_B \rangle \langle \mu_F \rangle = \langle \mu_B \rangle \langle \mu_F \rangle J_{FB} ,
\]

\[
D_{\mu_F} = \langle \mu_F \rangle + \langle \mu_F \rangle^2 J_{FF} ,
\]

where

\[
\langle \mu_F \rangle = \frac{\delta \phi_F}{2\pi} \int_{\delta\eta_F} d\eta \lambda_1(\eta) ,
\]

\[
J_{FB} = \frac{1}{(2\pi)^2 \langle \mu_F \rangle \langle \mu_B \rangle} \int_{\delta\eta, \delta\phi} d\eta_1 d\phi_1 \int_{\delta\eta_B, \delta\phi_B} d\eta_2 d\phi_2 \lambda_1(\eta_1)\lambda_1(\eta_2)\Lambda(\eta_1, \eta_2; \phi_1 - \phi_2) ,
\]

\[
J_{FF} = \frac{1}{(2\pi)^2 \langle \mu_F \rangle^2} \int_{\delta\eta, \delta\phi} d\eta_1 d\phi_1 \int_{\delta\eta, \delta\phi} d\eta_2 d\phi_2 \lambda_1(\eta_1)\lambda_1(\eta_2)\Lambda(\eta_1, \eta_2; \phi_1 - \phi_2) .
\]

By (46) we see that the presence of SR correlation turns the string into non-poissonian emitter.

For small windows in which one can consider \(\Lambda(\eta_1, \eta_2; \phi_1 - \phi_2)\) and \(\lambda_1(\eta)\) to be constant

\[
\langle \mu_F \rangle = \lambda_1(\eta_F)\delta_F , \quad \langle \mu_B \rangle = \lambda_1(\eta_B)\delta_B ,
\]

\[
J_{FB} = \Lambda(\eta_F, \eta_B; \phi_{FB}) ,
\]

\[
J_{FF} = \Lambda(\eta_F, \eta_F; 0) ,
\]

where we have used our short notations \(\delta_F\) and \(\delta_B\) \(\Box\) for the window acceptances.

If the both small windows are situated in the central rapidity region, where each string contributes to the particle production in the whole rapidity range, then due to the translation invariance in rapidity

\[
\lambda_1(\eta) = \mu_0 , \quad \Lambda(\eta_1, \eta_2; \phi) = \Lambda(\eta_1 - \eta_2; \phi)
\]

and the formulae (50)–(52) take the form

\[
\langle \mu_F \rangle = \mu_0\delta_F , \quad \langle \mu_B \rangle = \mu_0\delta_B ,
\]
\[ J_{FB} = \Lambda(\eta_{FB}, \phi_{FB}) , \]
\[ J_{FF} = \Lambda(0,0) . \]

Recall that \( \eta_{FB} \) and \( \phi_{FB} \) are the distances between the centers of forward and backward windows in rapidity and azimuth [8].

In the case of large windows situated in the central rapidity region, along with (54) one must use the formulae (45) and (46) with \( J_{FB} \) and \( J_{FF} \), which are given by the following expressions:

\[ J_{FB} = (\delta \eta_{f} \delta \phi_{f} \delta \eta_{b} \delta \phi_{b})^{-1} \int_{\delta \eta_{f} \delta \phi_{f}} d\eta_{1} d\phi_{1} \int_{\delta \eta_{b} \delta \phi_{b}} d\eta_{2} d\phi_{2} \Lambda(\eta_{1} - \eta_{2}, \phi_{1} - \phi_{2}) , \]
\[ J_{FF} = (\delta \eta_{f} \delta \phi_{f})^{-2} \int_{\delta \eta_{f} \delta \phi_{f}} d\eta_{1} d\phi_{1} \int_{\delta \eta_{f} \delta \phi_{f}} d\eta_{2} d\phi_{2} \Lambda(\eta_{1} - \eta_{2}, \phi_{1} - \phi_{2}) \]
(see Appendix A for further simplifications).

### 4.2 Resulting correlation strength

In general in the framework of the model one can write for \( N \) independent identical sources [23]:

\[ \rho_{1}^{N}(\eta) = N \lambda_{1}(\eta) , \]
\[ \rho_{2}^{N}(\eta_{1}, \eta_{2}; \phi) = N \lambda_{2}(\eta_{1}, \eta_{2}; \phi) + N(N - 1) \lambda_{1}(\eta_{1}) \lambda_{1}(\eta_{2}) . \]

Then the one- and two-particle densities of charge particles [11] are given by

\[ \rho_{1}(\eta) = \langle \rho_{1}^{N}(\eta) \rangle = \langle N \rangle \lambda_{1}(\eta_{1}) , \]
\[ \rho_{2}(\eta_{1}, \eta_{2}; \phi) = \langle \rho_{2}^{N}(\eta_{1}, \eta_{2}; \phi) \rangle = \langle N \rangle \left[ \lambda_{2}(\eta_{1}, \eta_{2}; \phi) - \lambda_{1}(\eta_{1}) \lambda_{1}(\eta_{2}) \right] + \langle N^{2} \rangle \lambda_{1}(\eta_{1}) \lambda_{1}(\eta_{2}) . \]

This leads to the following connection between correlators:

\[ \rho_{2}(\eta_{1}, \eta_{2}; \phi) - \rho_{1}(\eta_{1}) \rho_{1}(\eta_{2}) = \langle N \rangle \left[ \lambda_{2}(\eta_{1}, \eta_{2}; \phi) - \lambda_{1}(\eta_{1}) \lambda_{1}(\eta_{2}) \right] + D_{N} \lambda_{1}(\eta_{1}) \lambda_{1}(\eta_{2}) , \]
where \( D_{N} \) is the event-by-event variance \( D_{N} = \langle N^{2} \rangle - \langle N \rangle^{2} \) of the number of sources. As a result we have the following expression of the two-particle correlation function \( C_{2}(\eta_{F}, \eta_{B}; \phi_{FB}) \) [19] through the pair correlation function of a single string \( \Lambda(\eta_{F}, \eta_{B}; \phi_{FB}) \) [12]:

\[ C_{2}(\eta_{1}, \eta_{2}; \phi) = \frac{\omega_{N} + \Lambda(\eta_{1}, \eta_{2}; \phi)}{\langle N \rangle} , \]

where \( \omega_{N} \) is the event-by-event scaled variance [11] of the number of sources.

Important that by (64) we see that the value of the common constant (pedestal) in \( C_{2} \) is physically important. The height of the pedestal, \( \omega_{N} / \langle N \rangle = D_{N} / \langle N \rangle^{2} \), contains the important physical information on the magnitude of the fluctuation of the number of emitters \( N \) at different energies and centrality fixation [7].

Substituting now (64) into (63) we get the following expression for the FB correlation coefficient [1] in the case of FB windows which are small both in rapidity and in azimuth:

\[ b_{rel} = \frac{\langle n_{F} \rangle}{\langle n_{B} \rangle} b = \frac{\langle n_{F} \rangle [\omega_{N} + \Lambda(\eta_{F}, \eta_{B}; \phi_{FB})] / \langle N \rangle}{1 + \langle n_{F} \rangle [\omega_{N} + \Lambda(\eta_{F}, \eta_{F}; 0)] / \langle N \rangle} . \]
If these small FB windows are situated in the central rapidity region, where the translation invariance in rapidity takes place, then the \( \Lambda(\eta_F, \eta_B; \phi_{FB}) \) will depend only on the difference \( \eta_{FB} = \eta_F - \eta_B \) of rapidities and the formula (65) takes the following form

\[
b_{rel} = \frac{\delta_F}{\delta_B} b = \frac{\delta_F \mu_0[\omega_N + \Lambda(\eta_{FB}, \phi_{FB})]}{1 + \delta_F \mu_0[\omega_N + \Lambda(0, 0)]},
\]

where \( \mu_0 \) is the average rapidity density of the charged particles produced by one string. Note that in this case the basic formula (64) can be obtained also by an alternative way (see Appendix B).

One can present the result for the FB correlation coefficient (66) as the sum of two terms

\[
b_{rel} = b_{rel}^{LR} + b_{rel}^{SR},
\]

where:

\[
b_{rel}^{LR} = \frac{\delta_F \mu_0 \omega_N}{1 + \delta_F \mu_0[\omega_N + \Lambda(0, 0)]},
\]

and

\[
b_{rel}^{SR} = \frac{\delta_F \mu_0 \Lambda(\eta_{FB}, \phi_{FB})}{1 + \delta_F \mu_0[\omega_N + \Lambda(0, 0)]}.
\]

The first term (67) depends on the acceptance \( \delta_F \) of the forward window, but doesn’t depend on the distance between the centers of forward and backward windows in rapidity \( \eta_{FB} \) and in azimuth \( \phi_{FB} \), what justifies the name of this contribution as the long range (LR) one. This contribution reveals itself as the common pedestal when one plots the value of the FB correlation coefficient \( b \) as a function of \( \eta_{FB} \) and \( \phi_{FB} \), the height of the pedestal is determined by the event-by-event fluctuation of the number of the strings (sources) \( N \) and can be used for the evaluation of the extent of this fluctuation. Note that at any fixed number of sources there will be no such contribution, as \( \omega_N \equiv D_N/N = 0 \).

The second term (68) is proportional to the pair correlation function \( \Lambda(\eta_{FB}, \phi_{FB}) \) of a single string with some common factor depending on the acceptance \( \delta_F \) of the forward window (7). In plots of the FB correlation coefficient \( b \) as a function of \( \eta_{FB} \) and \( \phi_{FB} \) this contribution manifests itself as some structures above the level of the common LR pedestal (see Figs.1 and 2 below), which justifies the name of this contribution as the short range (SR) one.

We would like to emphasize that if the pair correlation function of a single string is equal to zero, \( \Lambda(\eta_{FB}, \phi_{FB}) = 0 \), and there are no SR correlations, \( b_{\Lambda=0}^{SR} = 0 \), we still have nonzero FB correlations due to the LR contribution:

\[
b_{rel}^{\Lambda=0} = b_{\Lambda=0}^{LR} = \frac{\delta_F \mu_0 \omega_N}{1 + \delta_F \mu_0 \omega_N},
\]

which originates from the event-by-event fluctuation in the number of strings \( N \). We note also, that at \( \Lambda = 0 \) by (46) and (49) one has for the multiplicity distribution from a string \( D_{\mu_F} = \langle \mu_F \rangle \) and the expression for the correlation coefficient (69) is in accordance with the earlier results obtained in [10, 20, 21] for this case.

In general case of windows of an arbitrary width in rapidity and azimuth, within which one can’t consider \( \Lambda(\eta_1, \eta_2; \phi) \) to be constant, the formula (65) by (20)–(25) and (64) takes the form

\[
b_{rel} = \frac{\langle n_F \rangle}{\langle n_B \rangle} b = \frac{\langle \mu_F \rangle[\omega_N + J_{FB}]}{1 + \langle \mu_F \rangle[\omega_N + J_{FF}]},
\]

where the integrals \( J_{FB}, J_{FF} \) are given by the formulae (48), (49) and the \( \langle \mu_F \rangle \) is the mean multiplicity produced in the forward window by a single string (47). If both of these windows are located in a central region and the translation invariance in rapidity takes place, then one can use for the integrals \( J_{FB} \) and \( J_{FF} \) more simple expressions (57) and (58).
Note that as it’s shown in the Appendix A, in the important case of symmetric FB windows (\(\delta \eta_F = \delta \eta_F = \delta \eta\)), separated only in rapidity (i.e. of the full acceptance in azimuth, \(\delta \phi_F = \delta \phi_B = 2\pi\)), the expressions (57) and (58) for \(J_{FB}\) and \(J_{FF}\) transform to

\[
J_{FB} = \frac{1}{\delta \eta^2} \int_{-\delta \eta}^{\delta \eta} d\eta \Lambda(\eta + \eta_{FB}) t_{\delta \eta}(\eta) ,
\]

\[
J_{FF} = \frac{2}{\delta \eta^2} \int_{0}^{\delta \eta} d\eta \Lambda(\eta)(\delta \eta - \eta) = \frac{2}{\delta \eta} \int_{0}^{\delta \eta} \Lambda(\eta) d\eta - \frac{2}{\delta \eta^2} \int_{0}^{\delta \eta} \Lambda(\eta) \eta d\eta ,
\]

where

\[
\Lambda(\eta) = \frac{1}{\pi} \int_{0}^{\pi} d\phi \Lambda(\eta, \phi)
\]

and \(t_{\delta \eta}(\eta)\) is the "triangular" weight function, arising at the integration over \((\eta_1 + \eta_2)/2\) in (57) and (58) and reflecting the corresponding phase space (see the formulae (91) and Fig.6 in the Appendix A).

5 Rapidity-azimuth dependence of the FB multiplicity correlation

5.1 Parametrization of the pair correlation function of a single string

To calculate the FB multiplicity correlation strength we need to know the pair correlation function of string \(\Lambda(\eta, \phi)\), which as it was shown above reflects the contribution of the short-range correlations.

In accordance with the standard picture of string decay one can use the following parametrization for the pair correlation function \(\Lambda(\eta, \phi)\) of a single string:

\[
\Lambda(\eta, \phi) = \Lambda_1 e^{-\frac{\eta}{\eta_1}} + \Lambda_2 \left( e^{-\frac{|\eta - \eta_0|}{\eta_2}} + e^{-\frac{|\eta + \eta_0|}{\eta_2}} \right) e^{-\frac{(\phi - \pi)^2}{\phi_2^2}} .
\]

The first term in the formula corresponds to the near-side correlation peak at \(\phi = 0\), originating from the hadronization of some string segment. The amplitude and the width of this peak in azimuth and in rapidity are characterized by the parameters \(\Lambda_1, \eta_1\) and \(\phi_1\). The second term in the formula corresponds to the away-side ridge-like structure at \(\phi = \pi\), arising due to the overlap of two lower and wider symmetric humps (with parameters \(\Lambda_2, \eta_2\) and \(\phi_2\)), shifted by \(\pm \eta_0\) in rapidity correspondingly the near-side peak. They originates from the hadronization of two string segments adjoining to the first one (\(\eta_0\) is the mean rapidity length of a string decay segment). We imply that in formula (74)

\[
|\phi| \leq \pi .
\]

At \(|\phi| > \pi\) one must to use the periodic extension \(\phi \rightarrow \phi + 2\pi k\) of the \(\Lambda(\eta, \phi)\). With such completion the \(\Lambda(\eta, \phi)\) meets the following requirements

\[
\Lambda(-\eta, \phi) = \Lambda(\eta, \phi) , \quad \Lambda(\eta, -\phi) = \Lambda(\eta, \phi) , \quad \Lambda(\eta, \phi + 2\pi k) = \Lambda(\eta, \phi) .
\]
Figure 1: The forward-backward (FB) correlation coefficient \(b\) in pp collisions at 0.9 TeV for symmetric windows of small acceptance \((\delta \eta_F = \delta \eta_B = \delta \eta = 0.2\) in rapidity and \(\delta \phi_F = \delta \phi_B = \delta \phi = \pi/4\) in azimuth), calculated with taking into account the long-range (LR) \((67)\) and short-range (SR) \((68)\) contributions by the formulae \((70)\), \((95)\) and \((96)\), at different values of distance between the centers of windows in azimuth \(\phi_{FB} = \phi_{sep}\) \((0^\circ, 90^\circ, 180^\circ\) in the left panel and \(0^\circ, 45^\circ, 135^\circ\) in the right panel) as a function of distance between the centers of windows in rapidity \(\eta_{FB} = \eta_{sep}\). Corresponding experimental data points are taken from \([29]\).

5.2 Fitting the model parameters

Using the parametrization \((74)\) for the pair correlation function \(\Lambda(\eta, \phi)\) of a string, one can calculate the FB multiplicity correlation coefficient for windows of an arbitrary width in rapidity and azimuth by the formula \((70)\) with the \(J_{FB}\) and \(J_{FF}\) given by the formulae \((57)\) and \((58)\) valid in the mid-rapidity region. The technical details of the calculation and the resulting formula \((106)\) for the correlation coefficient \(b_{corr}\) in this particular case are presented in the Appendix A.

To fit the model parameters we calculate at first the FB multiplicity correlation strength in the most simple case, when the observation windows are small both in azimuth and rapidity. Using the experimental data on the FB multiplicity correlation strength with the small windows situated at different rapidity \(\eta_{FB}\) and azimuth \(\phi_{FB}\) distances from each other, one can fix all parameters entering the SR \((68)\) and the LR \((67)\) contributions. Further we will use everywhere only the FB correlation coefficient \(b_{corr}\) defined with symmetrical FB windows. As we have seen above \((6)\), in this case \(b_{corr} \equiv b_{rel} = b\).

We fit the parameters of the pair correlation function \(\Lambda(\eta, \phi)\) \((74)\) of a single string and the parameter \(\omega_N\) \((11)\), characterizing the event-by-event fluctuation of the number of strings,
using the experimental data on FB correlation coefficient, \( b_{\text{corr}} \), for charged particles with transverse momenta \( 0.3 < p_T < 1.5 \) GeV/c, obtained in [29] for symmetric windows of \( \delta \eta_F = \delta \eta = 0.2 \) width in rapidity and \( \delta \phi_F = \delta \phi = \pi/4 \) in azimuth.

The results of the fitting in the case of pp collisions at 0.9 and 7 TeV are presented in Figs. 1 and 2 correspondingly. The values of the model parameters used at fitting are presented in Table 1. Note that only the products \( \mu_0 \omega_N \), \( \mu_0 \Lambda_1 \) and \( \mu_0 \Lambda_2 \) enter the resulting formula (106) for \( b_{\text{corr}} \), where the \( \mu_0 \) is the average rapidity density of the charged particles produced by one string (53) and \( \omega_N \) is the event-by-event scaled variance of the number of strings (11).

In Figs. 1 and 2 we see that the general behavior of the correlation coefficient \( b_{\text{corr}} \) in the case of small FB windows, calculated with \( \Lambda(\eta, \phi) \) (74) expected from the mechanism of string break up, are characterized by the following features. There are the large narrow near-side peak at \( \eta_{FB} = 0 \), \( \phi_{FB} = 0 \), and the away-side structure at \( \phi_{FB} = \pi \), which is smaller in amplitude and wider in rapidity. There is also the common pedestal corresponding to the LR correlation (67), originating from the event-by-event fluctuation in the number of emitting sources.

In Table 1 we see that with the increase of the energy from 0.9 to 7 TeV the value of \( \mu_0 \omega_N \) increases three times. At that the values of the parameters, characterizing the pair correlation function \( \Lambda(\eta, \phi) \) (74) of a single string, do not change considerably. The near-side peak with the increase of the energy becomes a little bit narrower and higher, with the values of the away-side ridge-like structure parameters remaining the same.
| $\sqrt{s}$, TeV | 0.9 | 7.0 |
|---------------|-----|-----|
| LRC | $\mu_0\omega_N$ | 0.7 | 2.1 |
| SRC | $\mu_0\Lambda_1$ | 1.5 | 2.3 |
| | $\eta_1$ | 0.75 | 0.75 |
| | $\phi_1$ | 1.2 | 1.1 |
| SRC | $\mu_0\Lambda_2$ | 0.4 | 0.4 |
| | $\eta_2$ | 2.0 | 2.0 |
| | $\phi_2$ | 1.7 | 1.7 |
| SRC | $\eta_0$ | 0.9 | 0.9 |

Table 1: The parameters of the model with strings as independent identical emitters (see formulae (41), (70) and (74)) used at the comparison with the experimental data [29, 30].

6 Comparison with the experimental data

In previous section we have fixed all model parameters by the data on the FB correlation coefficient with small acceptance windows (see the Table 1). So now we can calculate by formulae (57), (58) and (70) without any additional free parameters the values of the FB correlation coefficient $b_{corr}$ for large acceptance windows, within which one cannot consider the $\Lambda(\eta, \phi)$ to be constant.

As an example, we calculate the values of the FB correlation coefficient $b_{corr}$ in the practically important case of symmetric windows separated only in rapidity (i.e. with the $2\pi$ azimuthal acceptance) and study the dependencies of $b_{corr}$ on the width of the windows and on the size of the rapidity gap between them. In this case in the formula (70) for $b_{corr}$ one can use the values of $J_{FB}$ and $J_{FF}$ given by the formulae (71)–(73). (For technical details see the formulae (100)–(106) in the Appendix A.)

The results of the calculations with the FB windows width from 0.2 to 0.8 rapidity units in the case of pp collisions at 0.9 and 7 TeV are presented in Figs.3-5. In Fig.3 the results are presented as a function of the rapidity gap between windows ($\eta_{gap} \equiv \eta_{FB} - \delta\eta$) (10) and in Figs.4 and 5 as a function of windows width ($\delta\eta_F = \delta\eta_F = \delta\eta$) at zero rapidity gap between them ($\eta_{gap} = 0$). In the same plots the corresponding preliminary experimental results of ALICE [30] on the $b_{corr}$ for charged particles with transverse momenta $0.3 < p_T < 1.5$ GeV/c are also shown.

In Figs.3-5 we see the nice agreement of the results obtained by the formulae (70)–(73) in the framework of the model with strings as independent identical emitters (SM) with the experimental data on the FB correlations in large $2\pi$ azimuthal windows [30]. Note that the agreement was achieved without using any additional adjusting parameters, all model parameters were already fixed by the experimental data [29] on the FB correlations with small windows separated in azimuth and rapidity (see subsection 5.2 and the Table 1).

In Fig.4 we see also that the relative contribution of the long-range correlation (LRC) (67) considerably increases with the energy growth from 0.9 to 7 TeV, whereas the contribution of the short-range correlation (SRC) (68), characterizing the properties of a single source, remains practically the same. This reflects a significant increase with energy of the event-by-event fluctuations in the number of emitting sources (strings), characterizing by the parameter $\omega_N$ (11) (see Table 1).
Figure 3: The FB correlation coefficient $b$ for symmetric windows of full $2\pi$-acceptance in azimuth and $\delta \eta = 0.2$, 0.4, 0.6 and 0.8 width in rapidity for pp collisions at 0.9 and 7 TeV, as a function of rapidity gap $\eta_{\text{gap}} = \eta_{FB} - \delta \eta$ between windows. The curves are the results of calculations in the model with strings as independent identical emitters (SM) by the formulae (57), (58) and (70) with the values of parameters shown in Table I. The points are the corresponding preliminary experimental data of ALICE [30].

7 Alternative observables

In the conclusion on the base of above analysis we would like to discuss the introduction of more suitable observables for the future FB correlation studies.

From the equations (30)–(34) we see that if the acceptance of small symmetric FB windows goes to zero, $\delta F = \delta B \to 0$, then all traditionally defined FB correlation coefficients $b$, $b_{rel}$ and $b_{sym}$ also go to zero. Recall that in this case $b = b_{rel} = b_{sym}$. This unpleasant dependence of the correlation coefficients on the width of the windows arises due to behavior of the variance $D_n$ in the denominator of (2). Really by (20) and (30)–(33) we see model-independent way that in this limit $\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle \sim \delta F \delta B$ and $D_n \sim \delta F$. We can rid of this drawback if in (2) we normalize the correlator $\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle$ by the product $\langle n_F \rangle \langle n_B \rangle$ instead of the $D_n$ and introduce the observable

$$\beta_{\text{mod}} \equiv \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\langle n_F \rangle \langle n_B \rangle} = \left( \frac{n_F}{\langle n_F \rangle} \frac{n_B}{\langle n_B \rangle} \right) - 1 . \tag{77}$$

Then for windows, which are small both in rapidity and in azimuth, by (31) we have

$$\beta_{\text{mod}} = C_2(\eta_F, \eta_B; \phi_{FB}) \tag{78}$$
Figure 4: The same as in Fig. 3, but as a function of windows width $\delta \eta_F = \delta \eta_B = \delta \eta$ at zero rapidity gap between them ($\eta_{\text{gap}} = 0$). The relative contributions of the long- (LRC) (67) and short-range correlations (SRC) (68) calculated in the framework of the model with strings as independent identical emitters (SM) (70) are shown.

In the case of the FB windows, which are small only in rapidity and large ($\delta \phi_F = \delta \phi_B = 2\pi$) in azimuth

$$\beta_{\text{mod}} = C_2(\eta_F, \eta_B) = \frac{1}{\pi} \int_0^{\pi} d\phi C_2(\eta_F, \eta_B; \phi), \quad (79)$$

where we have take into account (26)–(29). We see that at small acceptances of windows the $\beta_{\text{mod}}$ (77) has the two-particle correlation function as a finite limit at $\delta_F = \delta_B \to 0$ in contrast to $b$, $b_{\text{rel}}$ and $b_{\text{sym}}$.

Note that the traditionally defined (2) correlation coefficient $b$, used above, is also proportional to the two-particle correlation function as a finite limit at $\delta_F = \delta_B \to 0$ in contrast to $b$, $b_{\text{rel}}$ and $b_{\text{sym}}$.

Another possibility, which follows from (33), is to use for the normalization in (5) the differences $D_{n_F} - \langle n_F \rangle$ and $D_{n_B} - \langle n_B \rangle$ instead of $D_{n_F}$ and $D_{n_B}$ and to introduce

$$\beta_{\text{rob}} = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\sqrt{D_{n_F} - \langle n_F \rangle} \sqrt{D_{n_B} - \langle n_B \rangle}}. \quad (80)$$

Then again for windows, which are small both in rapidity and in azimuth, by (31) and (33) we have

$$\beta_{\text{rob}} = \frac{C_2(\eta_F, \eta_B; \phi_{FB})}{\sqrt{C_2(\eta_F, \eta_B; 0) C_2(\eta_B, \eta_B; 0)}}. \quad (81)$$
\begin{align*}
\beta_{\text{rob}} &= \frac{C_2(\eta_{FB}, \phi_{FB})}{C_2(0, 0)} \quad (82) \\
\beta_{\text{rob}} &= \frac{C_2(\eta_F, \eta_B)}{\sqrt{C_2(\eta_F, \eta_F)C_2(\eta_B, \eta_B)}} \quad (83),
\end{align*}

for small windows in mid-rapidity region. In the case when the FB windows are \( \delta \phi_F = \delta \phi_B = 2\pi \) in azimuth and small in rapidity, we’ll have

\begin{align*}
\beta_{\text{rob}} &= \frac{C_2(\eta_{FB})}{C_2(0)} \quad (84)
\end{align*}

in the central rapidity region. We see that the \( \beta_{\text{rob}} \) as the \( \beta_{\text{mod}} \) has a finite limit at small acceptances of windows in contrast to traditionally used \( b \), \( b_{rel} \) and \( b_{sym} \).

Note that the second definition (80) is closely connected with so-called robust variance [23, 24, 25]:

\begin{align*}
R_n &= \frac{D_n - \langle n \rangle}{\langle n \rangle^2} \quad (85)
\end{align*}
By (77) and (80) we have

$$\beta_{rob} = \frac{\beta_{mod}}{\sqrt{R_{nf} R_{nF}}}.$$  \hspace{1cm} (86)

In the framework of the model described in Section 4 in the case of small FB observation windows we find for the alternative observables $\beta_{mod}$ (77) and $\beta_{rob}$ (80):

$$\beta_{mod} = C_2(\eta_F, \eta_B; \phi_{FB}) = \frac{\omega_N + \Lambda(\eta_F, \eta_B; \phi_{FB})}{\langle N \rangle},$$  \hspace{1cm} (87)

$$\beta_{rob} = \frac{\omega_N + \Lambda(\eta_F, \eta_B; \phi_{FB})}{\omega_N + \Lambda(\eta_F, \eta_F; 0)}.$$  \hspace{1cm} (88)

We see that they have finite limits at small acceptances $\delta_F, \delta_B$ and are simply connected with the pair correlation function of a single string $\Lambda(\eta_F, \eta_B; \phi_{FB})$, the mean number $\langle N \rangle$ and the scaled variance $\omega_N = D_N/\langle N \rangle$ (41) of the number of strings.

Note that by (87) the $\beta_{mod}$ decrease with the $\langle N \rangle$, so if the mean number of sources is large (as e.g. in AA collisions), it can be suitable to use as a correlation measure instead of $\beta_{mod}$ the product:

$$\rho_1(0) \beta_{mod} = \frac{dN_{ch}}{d\eta} \Bigg|_{\eta=0} \cdot C_2(\eta_F, \eta_B; \phi_{FB}) = \mu_0 \omega_N + \mu_0 \Lambda(\eta_F, \eta_B; \phi_{FB}).$$  \hspace{1cm} (89)

We have used that $\rho_1(0) = \mu_0 \langle N \rangle$, where the $\mu_0$ is the rapidity density of the charged particles produced by one string (53).

So comparing the different definitions of the multiplicity correlation coefficient, we see that the traditional definitions (2), (4) and (5) of the FB correlation coefficient lead to its strong dependence on the acceptance of windows, with the correlation coefficient going to zero with the window width. Hence, the results obtained for windows of different width can’t be compared directly. In this connection it can be suitable to use in the FB correlation studies the proposed observables (77) and (80), which values have nonzero limits when the acceptance of windows goes to zero.

### 8 Conclusions

We are extending the FB multiplicity correlation analysis to the case of the windows separated both in rapidity and azimuth. We are showing that this enables to separate the contributions, arising due to the event-by-event fluctuation in the number of sources and originating from the pair correlation function of a single source.

In mid-rapidity region the first contribution doesn’t depend on the distance between the windows in rapidity and azimuth, leading to the long range (LR) correlation. This LR contribution is proportional to the scaled event-by-event variance of the number of sources $\omega_N$. So the analysis of this contribution enables to obtain the important quantitative physical information on the magnitude of this fluctuation in the processes under consideration.

The second contribution, originating from the correlation between multiplicities produced by a single source, is an effect of the very different physical processes such as the details of string break up, the formation and decay of clusters, resonances or minijets during the string fragmentation. Its value (68) depends on the distances between the centers of the backward and forward windows in rapidity $\eta_{FB}$ and in azimuth $\phi_{FB}$. This contribution is proportional to the pair correlation function of a single source $\Lambda(\eta_{FB}, \phi_{FB})$ and decreases at large separation.
between windows, leading only to the sort range (SR) correlation. The dependence of the SR contribution on $\eta_{FB}$ and $\phi_{FB}$ can be retrieved from the experimental data on the multiplicity correlation with the FB windows, which are small both in rapidity and in azimuth (see Figs. 1 and 2 and Table 1).

We also see (46) that the presence of the SR correlation along with the influence on the FB multiplicity correlation inevitably turns a string into a non-poissonian emitter.

We have compared the different definitions of the multiplicity correlation coefficient and see that the traditional definitions (2), (4) and (5) of the FB correlation coefficient lead to its strong dependence on the acceptance of the windows, with the correlation coefficient going to zero with the acceptance. Hence, the results obtained for the windows of different width can’t be compared directly. In this connection we propose suitable observables (77) and (80) for the future FB correlation studies, which values have nonzero limit when the acceptance go to zero. At that the strong non-linear dependence of the traditionally defined FB correlation coefficient on the width of the windows and on the value of gap between them, is well described in the framework of the model with strings as independent identical sources (see Figs. 3–5).

We show by a model independent way that the measurements of the FB correlation coefficient between multiplicities in two small windows separated in rapidity and azimuth enable to find the two-particle correlation function $C_2$ even if the particle distribution in rapidity is not flat (as e.g. in the case of pA interactions) and the $C_2(\eta_1, \eta_2; \Delta \phi)$ depends not only on the difference ($\Delta \eta = \eta_1 - \eta_2$) of rapidities but also on $\eta_1$ and $\eta_2$. Important that this approach also does not need to use an event mixing procedure, usually applied in the so-called di-hadron correlation analysis, in which one assumes from the very beginning the dependence of two-particle correlation function only on the differences $\Delta \eta$ and $\Delta \phi$.

Even in a mid-rapidity region, where the application of the di-hadron correlation approach is justified, the results obtained by this method depend on the details of ”track and/or event mixing” used in the approach for the imitation of the ”uncorrelated” particle production, what leads to the uncertainty in determination of the common ”pedestal” in $C_2$, the height of which contains the important physical information on the scaled event-by-event variance of the number of sources $\omega_N$. (see the discussion in Appendix C).

Acknowledgements

The author thanks M.A. Braun, G.A. Feofilov and I. Altsybeev for useful discussions and Ewen Gillies for the help in preparation of the manuscript. The work was supported by the RFBR grant 12-02-00356-a and the Saint-Petersburg State University grant 11.38.197.2014.

Appendix A. Calculation of the integrals over rapidity and azimuth windows.

For symmetric rapidity windows $\delta \eta_B = \delta \eta_F = \delta \eta$ with the distance $\eta_{FB} = \eta_F - \eta_B$ between their centers one has

$$\int_{-\delta \eta}^{\delta \eta} d\eta \int_{-\delta \eta}^{\delta \eta} d\eta_2 f(|\eta_1 - \eta_2|) = \int_{-\delta \eta}^{\delta \eta} d\eta f(|\eta_{FB} + \eta|) \, t_\delta(y)(\eta) , \tag{90}$$

where $t_\delta(y)$ is the ”triangular” weight function (see Fig.6):

$$t_\delta(y) = [\theta(-y)(\delta y + y) + \theta(y)(\delta y - y)] \theta(\delta y - |y|) . \tag{91}$$
The formula \( \text{(90)} \) is valid for any distance between the centers of windows, in particular for coinciding windows. In the last case \( \eta_{FB} = 0 \) and we have

\[
\int_{-\delta \eta}^{\delta \eta} d\eta f(|\eta_1 - \eta_2|) = \int_{-\delta \eta}^{\delta \eta} d\eta f(|\eta|) t_{\delta \eta}(\eta) = 2 \int_{0}^{\delta \eta} d\eta f(|\eta|)(\delta \eta - \eta) \tag{92}
\]

The same formula

\[
\int_{-\delta \phi}^{\delta \phi} d\phi f(|\phi_1 - \phi_2|) = \int_{-\delta \phi}^{\delta \phi} d\phi f(|\phi_{FB} + \phi|) t_{\delta \phi}(\phi) \tag{93}
\]

is valid for the integration over azimuthal windows, but in this case one has also to take into account the periodicity: \( f(|\phi|) = f(|\phi + 2\pi k|) \). The last leads to significant simplification of the formula \( \text{(93)} \) in the case of full, \( 2\pi \), azimuth acceptance windows:

\[
\int_{-2\pi}^{2\pi} d\phi f(|\phi_{FB} + \phi|) t_{2\pi}(\phi) = 4\pi \int_{0}^{\pi} d\phi f(|\phi|) . \tag{94}
\]

So for large symmetric windows in the central rapidity region by \( \text{(90)} - \text{(92)} \) the formulae \( \text{(57)} \) and \( \text{(58)} \) in general case can be written in the following form

\[
J_{FB} = (\delta \eta \delta \phi)^{-2} \int_{-\delta \eta}^{\delta \eta} d\eta \int_{-\delta \phi}^{\delta \phi} d\phi \Lambda(\eta_{FB} + \eta; \phi_{FB} + \phi) t_{\delta \eta}(\eta) t_{\delta \phi}(\phi) , \tag{95}
\]

\[
J_{FF} = 4(\delta \eta \delta \phi)^{-2} \int_{0}^{\delta \eta} d\eta \int_{0}^{\delta \phi} d\phi \Lambda(\eta; \phi) (\delta \eta - y) (\delta \phi - \phi) . \tag{96}
\]

The \( \delta \eta \) and \( \delta \phi \) are the width of the observation windows in rapidity and in azimuth, and the \( \eta_{FB} \) and \( \phi_{FB} \) are the corresponding distances between their centers. We imply that \( \Lambda(\eta; \phi) \) satisfies the conditions \( \text{(76)} \). The similar formulae take place for the integrals \( I_{FB} \) \( \text{(39)} \) and \( I_{FF} \) \( \text{(40)} \) in section \( \text{B} \).

For the simplification of the further numerical calculations it’s important to take into account the factorization of the dependencies on \( \eta \) and \( \phi \) in near-side and away-side contributions, which takes place in the pair correlation function \( \Lambda(\eta, \phi) \), given by the formulae \( \text{(74)} \):

\[
\Lambda(\eta, \phi) = \sum_{i=1}^{2} \Lambda_i F_i(\eta)f_i(\phi) , \tag{97}
\]

Figure 6: The "triangular" weight function arising due to phase space at integration over non-periodical FB windows (see Appendix A for details).
where \( i = 1 \) corresponds to the near-side and \( i = 2 \) to the away-side contributions:

\[
F_1(\eta) = e^{-\frac{|\eta|}{\eta_1}}, \quad F_2(\eta) = e^{-\frac{|\eta-\eta_1|}{\eta_2}} + e^{-\frac{|\eta-\eta_2|}{\eta_1}},
\]

\[
f_1(\phi) = e^{-\frac{\phi^2}{\sigma_1^2}}, \quad f_2(\phi) = e^{-\frac{(\phi-\eta)^2}{\sigma_2^2}}.
\]

We must also remember the conditions (75) and (76). In this case the integrals (95), (96) and hence the resulting FB correlation coefficient \( b_{\text{corr}} \equiv b_{\text{rel}} = b \) (70) can be expressed through the simple one-fold integrals:

\[
J_{FB}(\eta_{\text{sep}}, \phi_{\text{sep}}) = \sum_{i=1}^{2} \Lambda_i H_i(\eta_{\text{sep}}) h_i(\phi_{\text{sep}}), \quad J_{FF} = \sum_{i=1}^{2} \Lambda_i H_i(0) h_i(0),
\]

where by (95):

\[
H_i(\eta_{\text{sep}}) = (\delta\eta)^{-2} \int_{-\delta\eta}^{\delta\eta} d\eta F_i(\eta + \eta_{FB}) t_{\delta\eta}(\eta),
\]

\[
h_i(\phi_{\text{sep}}) = (\delta\phi)^{-2} \int_{-\delta\phi}^{\delta\phi} d\phi f_i(\phi + \phi_{FB}) t_{\delta\phi}(\phi),
\]

and by (96):

\[
H_i(0) = 2(\delta\eta)^{-2} \int_{0}^{\delta\eta} d\eta F_i(\eta)(\delta\eta - \eta),
\]

\[
h_i(0) = 2(\delta\phi)^{-2} \int_{0}^{\delta\phi} d\phi f_i(\phi)(\delta\phi - \phi).
\]

Note that in the case of the \( 2\pi \)-azimuth windows (\( \delta\phi = 2\pi \)) due to parity and azimuthal periodicity (76) the formulae (102) and (104) admit further simplification by (94):

\[
h_i(\phi_{\text{sep}}) = h_i(0) = \frac{1}{\pi} \int_{0}^{\pi} d\phi f_i(\phi).
\]

Substituting (100) into (70), we get the resulting expression for the FB correlation coefficient in the case when the string pair correlation function \( \Lambda(\eta, \phi) \) is taken in the factorized form (97):

\[
b_{\text{corr}} = \frac{[\omega_N + \sum_{i=1}^{2} \Lambda_i H_i(\eta_{\text{sep}}) h_i(\phi_{\text{sep}})]\mu_0\delta_F}{1 + [\omega_N + \sum_{i=1}^{2} \Lambda_i H_i(0) h_i(0)]\mu_0\delta_F}.
\]

**Appendix B. Connection of the correlator and variance with the ones of a single source**

In the framework of the two stage model [9, 10, 20], under assumption that each string contributes to the particle production in both FB windows, one can express the observable correlator \( \langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle \) and the variance \( D_{n_F} \) through the correlator \( \langle \mu_F \mu_B \rangle - \langle \mu_F \rangle \langle \mu_B \rangle \) and the variance \( D_{\mu_F} \) for one source:

\[
\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle = \langle N \rangle (\langle \mu_F \mu_B \rangle - \langle \mu_F \rangle \langle \mu_B \rangle) + D_N \langle \mu_F \rangle \langle \mu_B \rangle,
\]

\[
D_{n_F} = \langle N \rangle D_{\mu_F} + D_N \langle \mu_F \rangle^2,
\]

where \( D_N = \langle N^2 \rangle - \langle N \rangle^2 \) and \( \langle N \rangle \) are the event-by-event variance and the mean number of sources (see [21] for derivation).
In the case of FB windows which are small both in rapidity and azimuth we have seen \((17)\) that
\[
C_2(\eta_F, \eta_B; \phi_{FB}) = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\langle n_F \rangle \langle n_B \rangle}.
\] (109)

One can also obtain this relation by \((20)\) and \((31)\). Similarly by \((45)\) and \((55)\) we have the same for the pair correlation function \(\Lambda(\eta_B, \eta_F; \phi_{FB})\) of a single string:
\[
\Lambda(\eta_B, \eta_F; \phi_{FB}) = \frac{\langle \mu_F \mu_B \rangle - \langle \mu_F \rangle \langle \mu_B \rangle}{\langle \mu_F \rangle \langle \mu_B \rangle}.
\] (110)

Then combining the formulae \((107)\)–\((110)\) and taking into account that
\[
\langle n_F \rangle = \langle N \rangle \langle \mu_F \rangle, \quad \langle n_B \rangle = \langle N \rangle \langle \mu_B \rangle,
\] (111)

we get again the formula \((64)\) of the text:
\[
C_2(\eta_F, \eta_B; \phi_{FB}) = \frac{\omega_N + \Lambda(\eta_F, \eta_B; \phi)}{\langle N \rangle}.
\]

Note that in the case of LR correlation, when the FB observation windows are separated by large enough rapidity gap at which one can neglect the correlations produced from the same source, we have by \((107)\)
\[
\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle = D_N \langle \mu_F \rangle \langle \mu_B \rangle.
\] (112)

The \((108)\) and \((112)\) lead to the following expression for the LR correlation coefficient \((4)\):
\[
b_{LR}^{rel} = \frac{a}{1 + a}, \quad a = \frac{\omega_N}{\omega_{\mu_F}} \langle \mu_F \rangle,
\] (113)

where \(\omega_N\) and \(\omega_{\mu_F}\) are the corresponding normalized variances:
\[
\omega_N = \frac{D_N}{\langle N \rangle}, \quad \omega_{\mu_F} = \frac{D_{\mu_F}}{\langle \mu_F \rangle}.
\] (114)

The expression \((113)\) coincides with the expression for the LR correlation coefficient obtained in [21].

Substituting the expression \((46)\) for \(D_{\mu_F} = \langle \mu_F \rangle + \langle \mu_F \rangle^2 J_{FF}\) into \((113)\) one obtains
\[
b_{LR}^{rel} = \frac{\langle \mu_F \rangle \omega_N}{1 + \langle \mu_F \rangle \omega_{N} + J_{FF}},
\] (115)

which coincides with the LR part of the FB correlation coefficient \((70)\) of the text. Recall that for FB windows which are small in rapidity and azimuth by \((52)\) one has \(J_{FF} = \Lambda(\eta_F, \eta_F; 0)\).

**Appendix C. Connection with the untriggered di-hadron correlation approach**

In practice, in di-hadron correlation analysis, the following alternative definition of the two-particle correlation function \(C\) is in use [26, 27]:
\[
C = S/B - 1,
\] (116)

where
\[
S = \frac{d^2 N}{d\Delta \eta \ d\Delta \phi}.
\] (117)
Here $\Delta \eta = \eta_1 - \eta_2$ and $\Delta \phi = \phi_1 - \phi_2$ are the distances between two particles in rapidity and in azimuth, and one takes into account all possible pair combinations of particles produced in given event in some one large rapidity interval $\Delta \eta \in (Y_1, Y_2)$. The $B$ is the same, but in the case of uncorrelated particle production, obtained by the event mixing procedure.

In this definition in contrast with (19) one implies from the very beginning that the translation invariance in rapidity takes place and the result depends only on $\Delta \eta = \eta_1 - \eta_2$ for any $\eta_1, \eta_2 \in (Y_1, Y_2)$. (All the pairs with the same value of difference $\eta_1 - \eta_2$ contribute to the same bin of the multiplicity distribution, irrespective of the value of $(\eta_1 + \eta_2)/2$, see also the discussion in [18].) This assumption is reasonable only in the central rapidity region at high energies. It means that we suppose that in the interval $(Y_1, Y_2)$:

$$\rho_1(\eta) = \rho_0, \quad \rho_2(\eta_1, \eta_2; \phi) = \rho_2(\eta_1 - \eta_2; \phi)$$ (118)

(see formula (35)).

In this case we have for the enumerator in (116):

$$S(\Delta \eta, \Delta \phi) = \int_{Y_1}^{Y_2} d\eta_1 d\eta_2 \rho_2(\eta_1 - \eta_2; \Delta \phi) \delta(\eta_1 - \eta_2 - \Delta \eta)$$ (119)

or in the case of commonly used symmetric interval $(-Y/2, Y/2)$:

$$S(\Delta \eta, \Delta \phi) = \rho_2(\Delta \eta; \Delta \phi) t_Y(\Delta \eta)$$ (120)

where the $t_Y(\Delta \eta)$ is the "triangular" weight function (91), defined in Appendix A (see Fig.6).

In the denominator of (116) for mixed events we should replace the $\rho_2(\eta_1, \eta_2; \Delta \phi)$ by the product $\rho_1(\eta_1)\rho_1(\eta_2)$, which due to the translation invariance in rapidity reduces simply to $\rho_0^2$. Then

$$B(\Delta \eta, \Delta \phi) = \rho_0^2 t_Y(\Delta \eta).$$ (121)

Substituting into (116) we get

$$C(\Delta \eta, \Delta \phi) = \frac{\rho_2(\Delta \eta; \Delta \phi)}{\rho_0^2} - 1 = C_2(\Delta \eta, \Delta \phi),$$ (122)

where we have taken into account (19) and (35). We see that if the translation invariance in rapidity takes place within the interval $(Y_1, Y_2)$, then the definition (116) is equivalent to the standard one (19) (see meanwhile the remark in the end of the appendix).

The drawback of this approach is that it supposes from the very beginning the translation invariance in rapidity and hence can’t be applied for an experimental determination of the two-particle correlation function $C_2$ for asymmetrical processes (such as e.g. the pA-interactions) and at large rapidity distances, when the translation invariance (35) is not valid. At that by (34), (78) and (81) we see that the approaches based on the analysis of the standard (2) or modified (77), (80) FB correlation coefficients with two remote windows of small acceptance in rapidity and azimuth enable in any case to measure the correlation function $C_2(\eta_1, \eta_2; \phi_1 - \phi_2)$ without using of any event mixing procedure.

Note that using of an event mixing procedure, applied in di-hadron correlation approach (116) for the determination of the $B$, can lead to an uncertainty in the experimental determination of the constant part of two-particle correlation function $C(\Delta \eta, \Delta \phi)$ even in central region where the translation invariance in rapidity takes place and the definitions of the correlation functions $C(\Delta \eta, \Delta \phi)$ (116) and $C_2(\Delta \eta, \Delta \phi)$ (19) are equivalent to each other (122).
Recall that the importance of the experimental determination of the constant part of two-particle correlation function $C_2$, which corresponds to the contribution of the long-range (LR) correlations, was discussed in subsection 4.2 (see the paragraph after the formula (68)).

One can illustrate the origin of the uncertainty in the constant term of $C_2(\Delta \eta, \Delta \phi)$ using the model with strings as independent identical emitters, described in the section 4. In the framework of the model by (120) and (62) we have for the enumerator and the denominator in (116):

\[ S(\Delta \eta, \Delta \phi) = \langle \rho_2^N(\Delta \eta; \Delta \phi) \rangle t_{\gamma}(\Delta \eta) = [\langle N \rangle \Lambda(\Delta \eta, \Delta \phi) + \langle N^2 \rangle] \mu_0^2 t_{\gamma}(\Delta \eta) , \quad (123) \]

\[ B(\Delta \eta, \Delta \phi) = \int_{-Y/2}^{Y/2} d\eta_1 d\eta_2 \langle \rho_1^N(\eta_1) \rho_1^N(\eta_2) \rangle \delta(\eta_1 - \eta_2 - \Delta \eta) = \langle N^2 \rangle \mu_0^2 t_{\gamma}(\Delta \eta) . \quad (124) \]

Then by (116) we get again that $C_2(\Delta \eta, \Delta \phi)$ is equal to $C_2(\Delta \eta, \Delta \phi)$, which is given by the equation (64).

But if one applies for the imitation of uncorrelated particle production another event mixing procedure, admitting, for example, the mixing only between events with the same multiplicity (what corresponds approximately to the same $N$), then instead of (124) we’ll have

\[ B(\Delta \eta, \Delta \phi) = \int_{-Y/2}^{Y/2} d\eta_1 d\eta_2 \langle \rho_1^N(\eta_1) \rho_1^N(\eta_2) \rangle \delta(\eta_1 - \eta_2 - \Delta \eta) = \langle N^2 \rangle \mu_0^2 t_{\gamma}(\Delta \eta) , \quad (125) \]

what by (116) and (123) leads to

\[ C(\Delta \eta, \Delta \phi) = \frac{\langle N \rangle}{\langle N^2 \rangle} \Lambda(\Delta \eta, \Delta \phi) \quad (126) \]

which does not correspond to the expression (64), based on the standard definition (29) of the two-particle correlation function $C_2$.

Comparing (126) and (64) we see that the resulting $C(\Delta \eta, \Delta \phi)$ (126) does not have the additional constant term $\omega_N/\langle N \rangle$ (the common pedestal in $C_2$), reflecting the contribution of the event-by-event fluctuation of the number of sources. The resulting expression (126) in this case occurs proportional to the pair correlation function of a single string $\Lambda(\Delta \eta, \Delta \phi)$ and, therefore, is equal to zero in the absence of the pair correlation from one string.

So we see that the two-particle correlation function $C(\Delta \eta, \Delta \phi)$, obtained by the di-hadron correlation approach (116), depends through $B$ on the details of the event mixing procedure, used for the imitation of uncorrelated particle production. Due to the uncertainties in the normalization factor $B$ one cannot measure correctly a value of the common pedestal, i.e. the long-range component of $C_2$. The same effect takes place also if one uses some arbitrary unjustified normalization procedures at the experimental determination of $B$ and/or $S$ in formula (116) (e.g. the normalization of $B(\Delta \eta, \Delta \phi)$ by $B(0,0)$).

Note that this long-range component of $C_2$ can be measured unambiguously in our approach (17), based on the studies of the FB correlations between multiplicities in windows separated in azimuth and rapidity, which does not use any event mixture procedure, leading to the uncertainties.

References

[1] S. Uhlig et al., Nucl. Phys. B 132, 15 (1978).
[2] K. Alpgard et al. (UA5 Collaboration), Phys. Lett. B 123, 361 (1983).
[3] R.E. Ansorge et al. (UA5 Collaboration), Z. Phys. C 37, 191 (1988).
[4] T. Alexopulos et al. (E735 Collaboration), Phys. Lett. B 353, 155 (1995).
[5] B.I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. 103, 172301 (2009).
[6] G. Aad et al. (ATLAS Collaboration), JHEP no.07, 019 (2012).
[7] A. Capella and A. Krzywicki, Phys. Rev. D 18, 4120 (1978).
[8] N.S. Amelin et al., Phys. Rev. Lett. 73, 2813 (1994).
[9] M.A. Braun, C. Pajares, V.V. Vechernin, Phys. Lett. B 493, 54 (2000).
[10] M.A. Braun, R.S. Kolevatov, C. Pajares, V.V. Vechernin, Eur. Phys. J. C 32, 535 (2004).
[11] P. Brogueira, J. Dias de Deus, and J.G. Milhano, Phys. Rev. C 76, 064901 (2007).
[12] N. Armesto, M.A. Braun, C. Pajares, Phys. Rev. C 75, 054902 (2007).
[13] N. Armesto, L. McLerran, C. Pajares, Nucl. Phys. A 781, 201 (2007).
[14] V.V. Vechernin, R.S. Kolevatov, Phys. Atom. Nucl. 70, 1797 (2007).
[15] M.A. Braun, Nucl. Phys. A 806, 230 (2008).
[16] V.P. Konchakovski et al., Phys. Rev. C 79, 034910 (2009).
[17] T. Lappi, L. McLerran, Nucl. Phys. A 832, 330 (2010).
[18] A. Bzdak, Phys. Rev. C 85, 051901 (2012).
[19] A. Olszewski, W. Broniowski, arXiv:1303.5280 [nucl-th] (2013).
[20] V.V. Vechernin, R.S. Kolevatov, arXiv:hep-ph/0304295 (2003); Vestnik SPbU, ser.4, no.2, 12 (2004).
[21] V.V. Vechernin, arXiv:1012.0214, 2010; Proceedings of the Baldin ISHEPP XX vol.2, JINR, Dubna (2011) 10.
[22] ALICE collaboration, J. Phys. G 32, 1295 (2006) [Section: 6.5.15 - Long-range correlations, p.1749].
[23] C. Pruneau, S. Gavin, and S. Voloshin, Phys. Rev. C 66, 044904 (2002).
[24] J. Whitmore, Phys. Rep. 27, 187 (1976).
[25] L. Foa, Phys. Rep. 22, 1 (1975).
[26] B.I. Abelev et al. (STAR Collaboration), Phys. Rev. C 80, 064912 (2009).
[27] CMS Collaboration, JHEP no.09, 091 (2010).
[28] V.V. Vechernin and H.S. Nguyen, Phys. Rev. C 84, 054909 (2011).
[29] I. G. Altsybeev, *Rapidity and Azimuth Topology of the Correlations between Charge Particle Yields in pp and Pb-Pb Collisions in ALICE Experiment at LHC*, PhD Thesis, Saint-Petersburg State University, Saint-Petersburg, 2013.

[30] G. A. Feofilov, et al. (for ALICE collaboration), *PoS* (Baldin ISHEPP XXI), 075 (2012).