On the Determination of the Gluon Density of the Proton from Heavy-Flavour Production at HERA

Stefano Frixione  
Dip. di Fisica, Università di Genova, and INFN, Sezione di Genova, Genoa, Italy

Michelangelo L. Mangano  
INFN, Scuola Normale Superiore and Dipartimento di Fisica, Pisa, Italy

Paolo Nason\footnote{On leave of absence from INFN, Sezione di Milano, Milan, Italy.} and Giovanni Ridolfi\footnote{On leave of absence from INFN, Sezione di Genova, Genoa, Italy.}  
CERN TH-Division, CH-1211 Geneva 23, Switzerland

Abstract

Using a recent next-to-leading-order calculation of the photoproduction double differential cross section for heavy quarks, we study the possibility of extracting the gluon density of the proton from heavy-quark photoproduction data. We discuss the theoretical uncertainties connected with this method, and we conclude that they are well under control in a wide $x$ domain.
The gluon density of the proton is usually extracted from fits to deep inelastic scattering data. There it affects the evolution of the quark density, according to the Altarelli-Parisi\cite{1} evolution equation. Its determination is indirect, and to some extent, it is influenced by the particular parametrization chosen for the fits. A direct determination is therefore highly desirable. In hadron-hadron collisions, processes such as direct photon\cite{2,3} and jet\cite{4} production are directly sensitive to the gluon distribution. Although these processes provide further constraints on the gluon density, they do not allow a precision comparable with the DIS experiments.

The $ep$ collider HERA offers new opportunities to measure the gluon density directly. Since the initial state is simpler than in hadron-hadron collisions, the gluon density enters in a simpler way. Furthermore, various methods can be used, thus allowing for consistency checks. Some of the methods are reviewed in ref.\cite{5}. The most promising one is based upon the measurement of the longitudinal structure function\cite{6}. It will probably allow a determination of the gluon density in the $x$ range between $10^{-3}$ and $5 \times 10^{-2}$ for $25 \text{ GeV}^2 < Q^2 < 150 \text{ GeV}^2$.

There are a number of methods that rely upon the study of hard production processes. At HERA, these processes arise both from the pure photon component and from the hadronic (sometimes called resolved) component of the photon. In jet production, the effect of the latter is strongly enhanced, because the typical elementary process $gg \rightarrow gg$ is about 20 times larger than the processes $\gamma g \rightarrow q\bar{q}$ and $\gamma q \rightarrow qg$. One possible strategy to suppress the hadronic component is to consider only off-shell photon jet production. This suppresses the rate, but eliminates the hadronic component. One should then disentangle the two pure photon processes $\gamma g \rightarrow q\bar{q}$ and $\gamma q \rightarrow qg$. The other possibility is to require heavy-quark production. In the hadronic component only the two subprocesses (at leading order) $gg \rightarrow Q\bar{Q}$ and $q\bar{q} \rightarrow Q\bar{Q}$ contribute, and they are two orders of magnitude smaller\cite{7} than the typical process $gg \rightarrow gg$. The hadronic component is not eliminated completely with this method, but there are large kinematical ranges in which it is negligible.

Along this line a popular method is to consider $J/\psi$ production\cite{8}. Although experimentally very convenient, this method suffers from several theoretical ambiguities, essentially due to difficulties in the computation of the $J/\psi$ production cross section. Open-charm production is a more promising method\cite{9}. It is less affected by theoretical ambiguities, and it is sensitive to the gluon density in the small-$x$ region. In ref.\cite{9} a study of the feasibility of measuring the gluon density using open charm
production was performed, using leading-order cross sections. It was concluded that
\( f^{(p)}_g(x) \) can be measured for \( x \) in the range between \( 10^{-1} \) and \( 10^{-3} \).

In order to extract the gluon density from open-charm production to next-to-leading accuracy, a next-to-leading calculation of the heavy-quark photoproduction cross section is needed. Such calculations for single-inclusive cross sections (i.e. integrated over the momentum of one of the heavy quarks) have been available for some time\[^{[10,11]}\]. They are not, however, very useful for our purpose, since they do not allow for the full reconstruction of the \( x \) variable of the parton in the proton from the final-state observables.

We have recently completed a next-to-leading calculation of the double differential heavy-quark photoproduction cross section. This calculation, which will be described in more detail in a forthcoming publication\[^{[12]}\], is an extension of the results presented in refs. \[^{[13]}\] and \[^{[14]}\]. In the present work, we will apply this result to the problem of extracting the gluon density of the proton from open-charm production. The main goal of this work is to show that the theoretical uncertainties associated with this procedure are well under control, and to assess their magnitude. The possible sources of uncertainty in the problem are the following: radiative corrections of even higher order (i.e. \( \mathcal{O}(\alpha_{\text{em}}^2) \)), not included in our calculation, the poor knowledge of the value of the heavy-quark mass, uncertainties on the value of \( \Lambda_{\text{QCD}} \), and the presence of a hadronic component of the photon. Furthermore, one should not forget the large hadronization effects that are usually found in charm production. There are good reasons to believe that these effects are smaller in the photoproduction than in the hadroproduction case. We will also argue that there are several ways of getting rid of the major hadronization ambiguities.

We begin by writing the heavy-quark cross section at the leading order in the following form

\[
\frac{d\sigma^{(0)}}{dy_{\gamma\gamma} dM^2_{\gamma\gamma}} = x_{\gamma} \frac{d\sigma^{(0)}}{dx_{\gamma} dM^2_{\gamma\gamma}} = \frac{1}{E^2} f^{(e)}_{\gamma}(x_{\gamma}, \mu_0^2) f^{(p)}_g(x_{\gamma}, \mu_0^2) \sigma^{(0)}_{\gamma g}(M^2_{\gamma\gamma}),
\]

(1)

where \( M_{\gamma\gamma} \) is the invariant mass of the heavy-quark pair, and \( y_{\gamma\gamma} \) is the rapidity of the pair in the electron-proton centre-of-mass frame (we choose positive \( y \) in the direction of the incoming photon). \( E \) is the electron-proton centre-of-mass energy, and

\[
x_{\gamma} = \frac{M_{\gamma\gamma}}{E} \exp(y_{\gamma\gamma})
\]

(2)
\[ x_g = \frac{M_{Q\bar{Q}}}{E} \exp(-y_{Q\bar{Q}}). \]  

(3)

The function \( f^e_\gamma \) is the photon density in the electron in the Weizsäcker-Williams approximation,

\[ f^e_\gamma(x, \mu_0^2) = \frac{\alpha_{\text{em}}}{2\pi} \frac{1 + (1 - x)^2}{x} \log \frac{\mu_0^2(1 - x)}{m_e^2x^2} \]  

(4)

and \( f^g_\gamma \) is the density of gluons in the proton. The Born level partonic cross section \( \tilde{\sigma}^{(0)}_{\gamma g}(s) \) is given by

\[ \tilde{\sigma}^{(0)}_{\gamma g}(s) = \frac{e_\gamma^2 \alpha_{\text{em}} \alpha_s(\mu_R) \pi \beta \rho}{m^2} \left[ 3 - \beta^4 \log \frac{1 + \beta}{1 - \beta} - 4 + 2\beta^2 \right], \]  

(5)

where \( m \) is the heavy-quark mass, \( e_\gamma \) is its charge in electron charge units, and

\[ \rho = \frac{4m^2}{s}, \quad \beta = \sqrt{1 - \rho}. \]  

(6)

The renormalization scale is set to \( \mu_R = \mu_0 \), and the reference scale \( \mu_0 \) is defined by

\[ \mu_0 = \sqrt{(p_T^2 + \bar{p}_T^2)/2 + m^2}, \]  

(7)

where \( p_T \) and \( \bar{p}_T \) are the transverse momenta of the heavy quark and of the heavy antiquark. The factorization scale for the proton is set to \( 2\mu_0 \).

Assuming that we identify the left-hand side of eq. (1) with the data, we can invert the equation, and get a first determination of \( f^g_\gamma \):

\[ f^{(0)}_g(x_g, \mu_0^2) = x_g \frac{d\sigma_{\text{data}}}{dxdg\,dM^2_{Q\bar{Q}}} \frac{E^2}{f^e_\gamma(x_\gamma, \mu_0^2)\tilde{\sigma}^{(0)}_{\gamma g}(M^2_{Q\bar{Q}})}. \]  

(8)

The inclusion of radiative corrections does not pose any problem. We write the full cross section as

\[ x_g \frac{d\sigma}{dxdg\,dM^2_{Q\bar{Q}}} = \frac{1}{E^2} f^e_\gamma(x_\gamma, \mu_0^2) f^g_\gamma(x_g, \mu_2^2) \tilde{\sigma}^{(0)}_{\gamma g}(M^2_{Q\bar{Q}}) + \Delta(f^g_\gamma, x_g, M^2_{Q\bar{Q}}), \]  

(9)

where \( \Delta \) represents all the radiative effects. In \( \Delta \) we have also indicated explicitly the functional dependence upon the gluon density in the proton. The light quarks, which enter at the next-to-leading order via the \( \gamma q \to qQ\bar{Q} \) process, give a small contribution (less than 5% for all values of \( x_g \) and \( M_{Q\bar{Q}} \) considered here). The \( \Delta \) term is given by

\[ \Delta(f^g_\gamma, x_g, M^2_{Q\bar{Q}}) = \sum_j \int dx\,dx_\gamma f^e_\gamma(x_\gamma, \mu_0^2) f^g_j(x_g, \mu_2^2) \alpha_{\text{em}} \alpha_s^2(\mu_R) \times \frac{d\hat{\sigma}^{(1)}_{ij}}{dM^2_{Q\bar{Q}}dy_{Q\bar{Q}}}(x_\gamma x E^2, M^2_{Q\bar{Q}}, \hat{y}_{Q\bar{Q}}, \mu_F, \mu_R, \mu_\gamma), \]  

(10)
where
\[ \hat{y}_{Q\gamma} = \frac{1}{2} \log \frac{M_{Q\gamma}^2}{x_g x_{Q\gamma} E^2} \]  
(11)
is the heavy-quark pair rapidity in the partonic centre of mass frame. The factorization scale of the photon \( \mu_\gamma \) is set to 1 GeV (we verified that the results are rather insensitive to the choice of \( \mu_\gamma \)). For a complete discussion of all the scale choices we refer the reader to a forthcoming publication [12].

We now write the full \( f_g^{(p)} \) as
\[ f_g^{(p)}(x, \mu^2) = f_g^{(0)}(x, \mu^2) + f_g^{(1)}(x, \mu^2), \]  
(12)
where the second term is the next-to-leading correction, and plug it back into eq. (9). We get
\[ f_g^{(1)}(x_g, \mu^2) = -\frac{E^2 \Delta(f_g^{(0)}, x_g, M_{Q\gamma}^2)}{f_\gamma^{(e)}(x_\gamma, \mu^2_g) \hat{\sigma}^{(0)}_{Qg}(M_{Q\gamma}^2)}. \]  
(13)

We have neglected the \( f_g^{(1)} \) piece contained in the \( \Delta \) term, the corresponding contribution being of order \( \alpha_{em} \alpha_S^3 \). It could also be easily incorporated by iterating eq. (13), using the full structure function in the right-hand side. In the simple illustration we have given, we have integrated the cross section over all variables but \( x_g \) and \( M_{Q\gamma} \). In general, in realistic experimental configurations, other cuts will be applied to the data. The procedure for the extraction of \( f_g^{(p)} \) outlined above can be carried out also in this case, provided the same cuts as applied to the hadronic final states in the data are also applied to the partonic final state in the calculation. One interesting possibility is to apply this procedure to the invariant mass of the two-jet system, instead of the invariant mass of the heavy-quark pair. This makes no difference at the Born level, but next-to-leading corrections will turn out to be different, and one should compute them with an appropriate jet definition (which should match the jet definition used in the analysis of the data) in order to get a meaningful answer (see refs. [15] for a discussion of this point). Furthermore, with this procedure there would be no need to identify both charmed mesons, and the result would be less sensitive to fragmentation effects.

We now begin with a study of the charm differential distribution in the variable \( x_g \) (as defined in eq. (3)) at HERA, for various cuts in \( M_{Q\gamma} \). These are the distributions which are relevant to the extraction of the gluon density at HERA. All our results are obtained for \( \sqrt{s} = 314 \) GeV. In fig. [4] we plot the leading-order and next-to-leading-order pure photon cross section (throughout this work, we use the proton
structure function set HMRS B of ref. [16]). From the figure we first notice that radiative corrections are moderate, but not negligible. This is an indication that the perturbative expansion is reliable in this case. We also observe that the effect of the radiative corrections cannot be described by a simple $K$-factor. From fig. 1, neglecting for the moment the contribution of the hadronic component of the photon, we can infer that the distribution is sensitive to the gluon density of the proton down to $x_g$ of the order of a few times $10^{-4}$.

We now examine the effect of the hadronic component of the photon. This contributes another term to the cross section, given by

$$d\sigma_H = \sum_{ij} \int dx \int dx_G \left[ \int_1^{x_G} f_{i}^{(e)}(z, \mu_0^2) f_i^{(\gamma)} \left( \frac{x_G}{z}, \mu_G^2 \right) \frac{dz}{z} \right] f_j^{(p)}(x, \mu_F^2) \, d\hat{\sigma}_{ij}. \quad (14)$$

This is formally of the same order as eq. (1), since the photon parton densities $f_i^{(\gamma)}$ are of order $\alpha_{em}/\alpha_S$. The contribution of the hadronic component, including next-to-leading orders (computed using the program of ref. [13]) is displayed in fig. 2 for two different parametrizations of the photon structure function. We have chosen the set LAC 1 (ref. [17]) and the set ACFGP HO-mc (ref. [18]). These two sets have extreme gluon distributions (as can be easily seen from fig. 10 of ref. [19]), and they may therefore give an idea of the magnitude of the hadronic component contribution (experimental results from HERA will help to better specify the value of this component). We see from fig. 2 that the hadronic component gives a large contribution for the smallest invariant-mass cut. Even in this case, however, there is a region of small $x_g$ in which the hadronic component is negligible with respect to the pure photon term. When we increase the invariant mass cut, we notice that the hadronic component decreases faster than the pure photon contribution. This is due to the fact that in the hadronic component, when we increase the invariant mass, the production process is suppressed by the small value of the gluon density of the photon at large $x$. We therefore see that a large region in $x_g$ is accessible by this method. By pushing the invariant-mass cut to 20 GeV we can reach the region of $x_g = 10^{-1}$. Observe that statistics will not be a problem even at these large invariant-mass cuts. With 100 pb$^{-1}$ of integrated luminosity, there would be about $10^5$ events per bin in the case of a 20 GeV cut (before accounting for experimental efficiencies). We therefore find that the conclusions of ref. [4] are not spoiled by theoretical ambiguities due to higher-order effects or to the hadronic component of the photon. We then conclude that the theoretical uncertainties in the heavy-flavour cross section, in the range of
$10^{-3} < x_g < 10^{-1}$, are small enough to allow for a determination of the gluon density in the proton.

The authors of ref. [20] reach a conclusion that contrasts with ours, that is to say that the ambiguities related to the hadronic component of the photon spoil the predictivity of the method. This conclusion is based upon a single inclusive calculation. They find that in particular the set LAC 3 of the photon structure functions of ref. [17] gives a hadronic contribution that competes with the pure photon contribution for all rapidities. We find that even with the LAC 3 set, when looking at the double differential quantities we have considered, it is still possible to perform the extraction of the gluon density in the proton, although in a more restricted $x_g$ range. However, we have chosen not to include this structure function set in our analysis for the following reasons. The LAC 3 set has an unphysically hard gluon structure function. For $Q^2 = 5 \text{ GeV}^2$, it peaks at $x = 0.9$ and carries 70% of the total momentum of the hadronic component of the photon. Because of this very pronounced peak, it is roughly as hard as the pure photon. We have examined the origin of this behaviour. As can be seen from ref. [17], the set LAC 3, unlike the sets LAC 1 and LAC 2, is obtained by fitting data for $F_2^\gamma(x, Q^2)$ at $Q^2$ values down to 1.31 GeV$^2$ (for the other sets the fits start at $Q^2 = 4.3 \text{ GeV}^2$). From $Q^2 = 1.3 \text{ GeV}^2$ to $Q^2 = 4.3 \text{ GeV}^2$, $F_2^\gamma(x, Q^2)$ grows rapidly in the region $x > 0.2$. In order to reproduce this growth using the QCD evolution, one is forced to assume a very hard gluon component, which under evolution will feed down a large quark component at moderate values of $x$. On the other hand, at this low $Q^2$ values, there may be other reasons, having nothing to do with the QCD evolution, that can cause the observed growth. For example, new thresholds for single resonances or resonance-pair production can be opened. After all, at $Q^2 = 1.3 \text{ GeV}^2$ and $x = 0.3$ the mass of the produced hadronic system is only 1.74 GeV. We therefore believe that these data are more consistent with a change in regime for $F_2^\gamma$, from a low-energy VMD behaviour to a high-energy point-like behaviour, than with perturbative evolution; one should thus not attempt to fit it with QCD evolution alone. Indications that one should use $Q^2$ values larger than 4 GeV$^2$ were also given in ref. [21]. Furthermore, the authors of ref. [17] also express some scepticism with respect to their set 3. Finally, experimental results on jet production in photon-photon collisions [22] and photon-hadron collisions [23] clearly disfavour the set LAC 3.

We will now turn to a discussion of the other theoretical uncertainties involved in our procedure, in order to estimate the precision of the method. One source of
uncertainty is due to the truncation of the perturbative expansion. We estimate this uncertainty by varying the renormalization and factorization scales by a factor of two above and below their reference value. The result is plotted in fig. 3. We see that the effect of the variation of the factorization scale is moderate, while the renormalization scale uncertainty amounts to an uncertainty of ±10 to 20% on the result.

In fig. 4 we show the dependence of our result upon the heavy quark mass. Even in this case the variation of the cross section is of the order of 10 to 20%. We should remind, however, that the mass of the heavy quark (unlike the renormalization scale) is a physical parameter that enters the cross section. It therefore makes sense to reduce this uncertainty by using values of the mass that give a good fit of the data. There are of course also uncertainties associated with the error on the present determination of $\Lambda_{QCD}$. In practice, it will be more convenient to measure $\alpha_s f_g(p_T(x))$, which is much less sensitive to the value of $\Lambda_{QCD}$.

We observe that perturbation theory alone makes a prediction for the ratio of our differential distributions for different mass cuts in the $x_g$ region in which the hadronic component is small. In leading order and up to scaling violation, the gluon density in the proton cancels out in this ratio. We present a plot of these ratios in fig. 5. As can be seen from the figure, the mass and scale dependence of the ratios for the pure photon contribution is negligible, while significant changes can be observed when the hadronic component is included. This in principle might allow for an independent check of the hadronic structure of the photon.

As suggested in ref. [24], an additional help to separate the pure from the hadronic photon contributions comes from tagging the photon energy by measuring the energy of the recoiling electron in the small-angle luminosity monitors. This approach might have, in general, problems at next-to-leading order: processes such as $\gamma q \rightarrow q\bar{Q}Q$, the light quark being emitted preferentially in the direction of the photon, will look and will be reconstructed as hadronic photon events. As was mentioned previously, however, the contribution of these processes is numerically negligible. The photon-tagging technique could then help to further constrain the gluon density of the photon.

We conclude by giving in table 1 the total cross sections for $b$ and $c$ production at HERA. Observe that the sensitivity to the variation of the scale and mass parameters is much smaller for $b$ than for $c$ production. This suggests that by using $b$ production a better precision may be achieved in the determination of the gluon density. On the other hand the $x_g$ region covered is smaller. We see in fact that using the structure
function set MSRD– (a set with a gluon density more singular at small $x$) the charm
cross section grows by 80%, while the $b$ cross section grows by less than 10% with
respect to our central value.

Observe the large difference in the hadronic component when using the two differ-
ent sets of photon structure functions. When going from the ACFGp to the LAC 1
set, the hadronic component of the charm cross section grows by a factor of 9, while
for bottom it grows only by a factor of 3. This difference is mainly due to the small
$x$ growth of the gluon density in the photon. It is likely that the charm cross section
and distributions at HERA will also help to constrain the gluon density in the photon
at small $x$. 
Table 1: Charm and bottom pair-production cross sections at HERA. The pure photon component and the hadronic component are shown separately. The central values for the scales are $\mu_R = m_c$ for charm and $m_b$ for bottom, $\mu_F = 2m_c$ for charm and $m_b$ for bottom. The central mass value for charm used here is 1.5 GeV, the low value is 1.2 GeV and the high value is 1.8 GeV. For bottom we set $m_b = 4.75$ GeV, the low value is 4.5 GeV and the high value is 5 GeV. The structure function set for the central value is HMRSB, with $\Lambda_4 = 190$ MeV. When studying the sensitivity to $\Lambda$ we use two HMRSB fits obtained with the values of $\Lambda$ indicated in the table. We also show the values obtained using the MRSD– structure functions, which have stronger enhancement at small $x$. For the hadronic component only the central values are used, since the uncertainty due to the choice of the photon structure function is by far the largest.
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**Figure Captions**

Fig. 1: Differential cross sections for charm production, histogrammed in the logarithm of $x_g = \exp(-y_0 q^\perp)M_0 q^\perp/E$, for $m = 1.5$ GeV, HMRS B structure functions for the proton, and default values for the scale choices. Only the pure photon component is included.

Fig. 2: Contribution of the hadronic component of the photon, compared with the pure photon component, for two extreme choices of the photon structure functions.

Fig. 3: Scale dependence of the cross section (pure photon only). The solid line corresponds to the choice $\mu_R = \mu_0$ and $\mu_F = 2\mu_0$, the dotted line to $\mu_R = \mu_0/2$ and $\mu_F = 2\mu_0$, the dashed line to $\mu_R = 2\mu_0$ and $\mu_F = 2\mu_0$, and the dot-dashed line to $\mu_R = \mu_0$ and $\mu_F = \mu_0/2$. This last curve is not shown for the smallest mass cut, because the scale goes outside the range of validity of the structure function parametrization.
Fig. 4: Charm mass dependence of the cross section (pure photon only). The solid line corresponds to the choice $m_c = 1.5$ GeV, the dotted line to $m_c = 1.8$ GeV and the dashed line to $m_c = 1.2$ GeV.

Fig. 5: Ratios of cross sections for different invariant-mass cuts. (a) Comparison between the pure photon case and the full (pure + hadronic) one, for two different photon parton distributions. (b) Same as (a), for a different invariant-mass cut. (c) Mass dependence of the ratio in the pure photon case, patterns as in fig. 4. (d) Scale dependence of the ratio in the pure photon case, patterns as in fig. 3.