Abstract

Within the context of biomass production this paper proposes a method for logistics optimisation involving multiple tours to be executed simultaneously by groups of harvesting units (HUs) and support units (SUs) to first harvest biomass from multiple agricultural fields, before supplying the biomass to multiple biogas plants (BPs) via shuttling SUs. This problem is relevant on a larger scale in particular for contractors. This problem is complex since there are three interconnected optimisation levels: (i) the assignment of BPs to tours and the ordering of BPs assigned to each tour, (ii) the assignment of fields to BPs and the ordering of fields assigned to each BP, and (iii) determining the number of HUs and SUs assigned to each tour, whereby different HUs and SUs may in general have different working rates and loading capacities. The dimensions of the problem considered in this paper are inspired by a large real-world scenario with 7 tours, 20 biogas plants, 1200 corn fields, 7 HUs and a total of 42 SUs operated on an area of 80km × 80km. Both problem modeling and a solution method are discussed. For the latter a GPU-accelerated heuristic search algorithm is proposed. For the former an optimisation criterion minimising both total accumulated path length and the maximum completion time over all harvesters tours, an embedded local minimisation for the assignment of SUs to tours, and demand fulfilment constraints for BPs are discussed. In stochastic simulation experiments it is found that permitting unconstrained assignment of any available field that a contractor services to any available BP independent of their ownerships is crucial to attain maximum path length savings and also to best balance and minimise uniform completion times over all harvesters tours such that weather-dependent harvesting windows can be exploited optimally.

Keywords: Biomass Supply Chain; Logistics Optimisation; Assignment Problem; Routing Problem; GPU-acceleration.

1. Introduction and Problem Formulation

Biomass belongs to the trending class of renewable energy sources. Two main benefits are (i) its versatility in generating not only electricity but also heat and biofuels, and (ii) its storage capability to generate energy on-demand. The typical supply chain for agricultural biomass production involves planting, harvesting, road transport, storage and utilisation for energy exploitation. A critical characteristic is that typically only a very limited and weather-dependent optimal harvesting period is available. This demands optimised logistical operations. The largest fraction of cost in biomass energy generation originates from logistics operations (Rentizelas et al. (2009)).

For the architecture of a biomass supply chain simulation model and functional modeling see Pavlou et al. (2016a) and Pavlou et al. (2016b). A simulation model for agricultural biomass logistics systems in terms of demand fulfilment and storage management was presented by Ebadian et al. (2011). Sokhansanj et al. (2006) proposed a model to simulate the flow of biomass from a field to a biorefinery involving collection, storage, and transport operations. Marufuzzaman & Eksioglu (2017) discussed a dynamic multi-modal facility location model to alleviate the impacts of congestion on biomass supply chain performance due to seasonality of biomass.

Fig. 1. Problem sketch with exemplary $N_T = 2$ tours, $N_B = 3$ biogas plants (blue houses) and $N_F = 11$ fields (green dots). Paths traveled by harvesting units (HUs), empty support units (eSUs) and loaded support units (lSUs) are denoted by black, dashed-red and blue arrows, respectively. All fields (numbered for identification) provide an expected supply of biomass, whereas all BPs have a minimum demand. Subject to constraints the optimisation tasks are: (i) assignments of BPs to tours, (ii) their ordering for each assignment, (iii) assignment of fields to BPs, (iv) their ordering for each assignment, (v) assignment of $N_{HU}(g)$, $\forall g = 1, \ldots, N_T$, and (vi) assignment of $N_{SU}(g)$, $\forall g = 1, \ldots, N_T$. 

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They proposed a mixed integer nonlinear program to trade-off investment, transportation, and congestion management decisions. A linear approximation of the model was then solved using a hybrid Benders-based rolling horizon algorithm. Cundiff et al. (1997) presented a linear programming approach for a biomass delivery system. General opportunities and challenges in biomass and biofuel supply chain modeling were summarised by Yue & You (2016). A review on operations research approaches for the design of biomass feedstock supply chains was provided by Sun et al. (2018). Other reviews are by Malladi & Sowlati (2018), Atashbar et al. (2016), Castillo-Villar (2014), Yue et al. (2014) and Gold & Seuring (2011).

In Bochtis et al. (2013) the finding of a permutation schedule for a number of geographically dispersed fields where sequential biomass handling operations have to be carried out (baling, loading and transport) was formulated as a flow shop problem with sequence dependent setup times. They considered two machinery systems, a baling system (tractor-baler) and a loading system (fork-lift/transport truck), working in series on a total of 4 fields and starting from a central depot. A sensitivity analysis was also provided that demonstrated the importance of accurate and predictable parameters in the optimisation problem. Without predictable parameters it was found that solution quality can degrade quickly. Related to baling, Zamar et al. (2017) proposed for the bale collection problem a constrained K-means and nearest neighbour approach. This problem occurs after harvest and baling operations of a crop and consists of defining the sequence in which bales spread over a field are collected. A method for scheduling of machinery fleets in biomass multiple-field operations was proposed by Orfanou et al. (2013). For their case study they considered grass harvesting on a total of 5 fields with a maximum of 2 mowing, 2 raking and 2 baling machines. In Busato et al. (2019), a method to determine the optimal number of transport units in individual fields during harvest for silage production is proposed minimising operational costs subject to time constraints. For their field trials they considered 4 different locations with between 2 to 5 fields each. Gracia et al. (2014) discussed an application of the vehicle routing problem to the biomass collection problem. A hybrid approach based on genetic algorithms and local search methods was presented to solve a real case study, where they considered 1 central storage location, 12 vehicle routes and 146 orchards. Aguayo et al. (2017) discussed a corn-stover harvest scheduling problem that arises when a cellulosic ethanol plant contracts with farmers to harvest corn stover after the grain harvest has been completed. The plant contracts a fleet of harvesting crews, which must be assigned by the plant scheduler to harvest fields as they are called in by the farmers over the harvest season. The objective is to minimize the total cost incurred by the plant by determining the optimal number of balers required, the type of balers to harvest each field, the number of periods needed to complete the harvest of each field, and the routing of balers over the planning horizon. For their case study they considered 1 cellulose ethanol plant and 85 fields.

Within this context the motivation and contribution of this paper is to propose a method for logistics optimisation for biomass production involving multiple tours to be executed simultaneously by groups of harvesting units (HUs) and support units (SUs) to first harvest biomass from multiple agricultural fields, before supplying the biomass to multiple biogas plants (BPs) via shuttling SUs. The problem formulation is visualised in Fig. 1. This logistics problem is relevant on a larger scale in particular for contractors and in practice often solved manually by human schedulers. Part of this contribution is to quantify optimisation-based gains with respect to the working practice. This is achieved by comparing and initialising the optimisation procedure with a solution that is replicating the in practice common decision process. This logistics problem is not the least complex since there are three interconnected optimisation levels: (i) the assignment of BPs to tours and the ordering of BPs assigned to each tour, (ii) the assignment of fields to BPs and the
ordering of fields assigned to each BPs, and (iii) determining of the number of HUs and SUs, \( N_{HU}(g) \) and \( N_{SU}(g) \), assigned to each tour \( g = 1, \ldots, N_T \), whereby different HUs and SUs may have different working rates and loading capacities. The dimensions of the problem considered in this paper are significantly larger than those of case studies reported in the literature. Likewise, to the author’s knowledge, proposing a solution based on GPU computation has not yet been proposed within the agricultural biomass supply chain context.

The remaining paper is organised as follows: problem modeling including the discussion of constraints and cost function, the solution method including the discussion of the GPU-implementation, numerical results and the conclusion are described in Sections 2.3.

2. Problem Modeling

Extending Fig. 1, modeling assumptions, constraints and the cost function are discussed.

2.1. Constraints and Parameters

First, there are multiple groups of HUs and SUs dispatched to multiple BPs and fields. Since typically \( N_H > N_T \), multiple BPs must be served along each tour. The number of active tours is always bounded by the number of total available HUs, i.e., \( N_T \leq N_{\text{total}}^{\text{HU}} \). However, there must not necessarily be \( N_{\text{total}}^{\text{HU}} \) active tours since multiple HUs can in general operate along the same tours (e.g., to increase working rates). HUs never drive to BPs. Instead, they travel from field to field. Thus, it is assumed that they are maintained and refilled with fuel directly on the fields without returning to HQ. Similarly, also SUs return to HQ only after harvest completion on all assigned fields.

Second, it holds that

\[
N_{\text{total}}^{\text{HU}} = \sum_{g=1}^{N_T} N_{HU}(g), \quad N_{\text{total}}^{\text{SU}} = \sum_{g=1}^{N_T} N_{SU}(g),
\]

(1)

and, in general, \( N_{HU}(g) = \sum_{i=1}^{N_{\text{classes}}} N_{i}^{\text{HU}}(g) \) and \( N_{SU}(g) = \sum_{i=1}^{N_{\text{classes}}} N_{i}^{\text{SU}}(g), \forall g = 1, \ldots, N_T \), where \( N_{\text{classes}}^{\text{HU}} \) and \( N_{\text{classes}}^{\text{SU}} \) different classes of HUs and SUs may differ by working rates and loading capacities, respectively.

Third, all BPs must be served a minimum biomass supply,

\[
\sum_{f \in F} \hat{s}_f \geq d_b^{\text{min}}, \forall b \in B,
\]

(2)

where \( B, \mathcal{F}_b \subset \mathcal{F}, d_b^{\text{min}}, \) and \( \hat{s}_f \) denote the set of BPs, the set of fields assigned to a specific BP \( b \in B \), the minimum demand of biomass, and the expected supply from each field \( f \in \mathcal{F} \), respectively. It is assumed that there is no upper bound on the maximum demand for each BP. Thus, good storage capabilities at BPs are assumed (Rentizelas et al., 2009).

Fourth, each field supplies a certain expected supply that must be assumed at the time of logistics planning. For simplicity, a linear relationship is assumed for the mapping from field size to biomass, \( \hat{s}_f = \hat{\epsilon}_f h_f, \forall f \in \mathcal{F} \), where field size and a lumped parameter are denoted by \( h_f \) and \( \hat{\epsilon}_f \), respectively. For simulations, a constant, \( \epsilon_{\text{conv}} = \hat{\epsilon}_f, \forall f \in \mathcal{F} \), is assumed and thus,

\[
\hat{s}_f = \epsilon_{\text{conv}} h_f, \forall f \in \mathcal{F}.
\]

(3)

Fifth, SUs are assumed to have a loading capacity limit denoted by \( \rho_{\text{load}}^{SU} \). For any given field \( f \in \mathcal{F}_b \) the number of shuttling drives to its assigned BP is thus

\[
N_{\text{shuttle}}^{SU,f} = \frac{\hat{s}_f}{\rho_{\text{load}}^{SU}}, \quad l = 1, \ldots, n_{\text{classes}}.
\]

(4)

Sixth, a comment about the path network is made. In case locations are not directly adjacent, shortest paths are implicitly computed along the path network to connect two locations. Seventh, remaining parameters relevant to the algorithm proposed in the next section are stated. These are (i) average travel velocities along the path network for HUs and SUs, \( v_{\text{edge}} \) and \( v_{\text{SU}} \), which are assumed to be uniform over all mobile machinery, (ii) the in-field area working rates (ha/h) of HUs, \( v_{\text{area}}^{HU} \), \( l = 1, \ldots, n_{\text{classes}} \), and (iii) fill times required to load up SUs, \( \tau_{\text{load}}^{SU} \), \( l = 1, \ldots, n_{\text{classes}} \), which are assumed to be proportional to loading capacities \( \rho_{\text{load}}^{SU} \). These parameters are assumed to be uniform among the path network and all fields.

2.2. Path length-based Cost Function

The final cost metric used for the evaluation of different solution candidates is selected as path length-based. Thus, cost is measured as the total distance accumulated by all HUs and SUs involved for mission completion. An extending alternative monetary units-based cost function is discussed further below. Cost is abbreviated by symbol \( C \).

A path length-based cost metric has several advantages. There are comparatively few parameters that need to be identified from data, non-negativity of path lengths implies existence of a global optimal solution, and all optimisation tasks according to Fig. 1 are addressed since accumulated total distance is clearly affected by (i) assignments of BPs to tours, (ii) their ordering within each tour, (iii) assignment of fields to BPs, (iv) their ordering for each assignment, (v) assignment of \( \rho_{\text{load}}^{HU} \), \( \forall g = 1, \ldots, N_T \), and (vi) assignment of \( N_{\text{load}}^{SU}(g), \forall g = 1, \ldots, N_T \).

The cost function is evaluated tracing traveled distances according to Fig. 1. Two aspects are emphasised. First, the assigned number of HUs and SUs, \( N_{HU}(g) \) and \( N_{SU}(g) \), multiply distances for each tour. Second, the number of shuttling drives according to \( 3 \), \( N_{\text{shuttle}}^{SU,f} \), is multiplied by 2 in order to account for back-and-forth distances for SUs shuttling between fields and their assigned BPs.

2.3. Extension: Alternative Cost Metric

Above cost function is motivated (i) from the perspective of a contractor with full-time hired employees reimbursed at a fixed monthly rate, and (ii) with the objective of having to deal with only few and easy-to-quantify parameters. In a different setting with hourly reimbursements and more parameters an alternative monetary units-based cost function may be derived. By first translating distance to “time” as the dependent variable through

\[
\begin{align*}
N_{\text{shuttle}}^{SU,f} = & \frac{\hat{s}_f}{\rho_{\text{load}}^{SU}}, \quad l = 1, \ldots, n_{\text{classes}}; \\
\end{align*}
\]

(4)
knowledge of average travel speeds along all edges of the path network and in-field working rates of HUs, the two main cost positions can then be identified as: (i) hourly driver rates for all HUs and SUs, and (ii) hourly fuel consumption rates times Diesel prices for HUs and SUs. Here, complexity arises since fuel consumption varies for in-field work, along rural, gravel or paved roads, and dependent on whether SUs are loaded or empty during their shuttling services. Likewise, fuel prices may vary substantially. These variations and uncertainties in parameters make monetary units-based cost function modeling more complex and less reliable with respect to practical employment.

Finally, note that all cost functions derived from translating distance to time as the dependent variable before multiplications by different hourly cost rates can always be reformulated as a nonlinearly weighted version of the distance-based cost function from Section 2.2. Constraints and likewise the solutions by different distance to time as the dependent variable before multiplication procedure a human scheduler may implicitly conduct. Step 1 involves (i) assigning an approximately equal number of HUs to each tour, e.g., \( N_{HU}(g) = \frac{N_H}{N_T} \), \( g = 1, \ldots, N_T - 1 \), with \( N_{HU}(N_T) \) capturing the integer residual, and (ii) a sequential nearest neighbour search, i.e., from HQ to a closest first BP, from the first BP to a closest second BP, and so forth until all BPs are assigned to all tours and ordered.

Step 2 involves the assignment of fields to BPs. Guiding notion is that every BP has a minimum demand for biomass, \( a^\text{min}_b \), \( \forall b \in B \). Therefore, fields are picked and assigned to their spatially closest BP that is not yet supplied up to its minimum demand. Once all BPs are supplied with their minimum demand, all remaining (not yet assigned) fields are assigned to their spatially closest BP. This is enabled by assumption according to problem modeling of Section 2 about the absence of any upper bound on supplied biomass and the assumption about good storage capabilities at BPs.

Step 2 was not yet concerned about the ordering of fields for each BP-assignment. This is conducted as part of Step 3 in form of a sequential nearest neighbour search, i.e., from HQ to the first field of its assigned BP, from that field to a closest second field assigned to the same BP, and so forth until fields assigned to each BP are ordered.

In Step 4, the solution of Steps 1-3 is summarised by generic vector \( x^* \) before its cost is computed. The initialisation solution is, in general, suboptimal because of its sequential derivation. Furthermore, it is fully deterministic if fields in Step 2 are selected according to a deterministic rule such as, e.g., their field identification numbers. In Step 2 stochasticity may be inserted by randomly drawing fields. In Section 4 about numerical examples the initialisation solution will be considered as the baseline reference in order to quantify any iterative solution improvement from conducting Steps 5-16 of Algorithm 1.

In Step 6, \( x \) is updated according to

\[
x = \begin{cases} 
  x^*, & \text{if } u < \epsilon, \\
  x_{\text{old}}, & \text{otherwise}, 
\end{cases}
\]

where \( u \in [0,1] \) is drawn as a uniform random variable and \( \epsilon \in [0,1] \) is a hyperparameter. This update rule can be considered as trading off exploitation and exploration for the evolution of new \( x \) candidates. This update rule is motivated by the presence of multiple optimisation layers including both the assignment of BPs to tours and fields to BPs. For example, suppose a BP is newly assigned to an alternative tour. Then, this assignment may initially be suboptimal because fields assigned to this BP are not yet optimally ordered to fit the new tour scenario. Thus, initially there is no progress with respect to \( x^* \). However, throughout the course of iterations and better orderings of fields a new optimal \( x^* \) may eventually be found.

Step 8 involves a simple remove- and reinsert-operation for vertices, which can represent either a field or a BP. The insertion heuristic is discussed. First, in general any field can be removed from any BP-assignment and arbitrarily reassigned to any BP. Second and in contrast, in case the removed vertex is a BP it can be reassigned to any arbitrary tour or to a different ordering index within the existing tour. Importantly, in case a BP is reassigned, also all of the fields assigned to that BP are

| Algorithm 1: Logistics for Biomass Production |
|---------------------------------------------|
| 1. Assign BPs to tours and order their visit within each tour. |
| 2. Assign fields to BPs. |
| 3. Order the visit of fields assigned to each BP. |
| 4. Summarise the solution of Steps 1-3 as \( x^* \), compute the resulting cost \( C^* \), and set \( x_{\text{old}} \leftarrow x^* \). |
| 5. For \( N_{\text{iters}} \) do |
| 6. Update \( x \) based on \( x_{\text{old}} \), \( x^* \) and the exploration heuristic of (5). |
| 7. If CPU-version then |
| 8. Randomly remove a vertex (field or BP) from \( x \) and randomly reinsert that vertex into \( x \) following insertion heuristics. Check constraints, compute cost if feasible, and update \( x^* \) and \( C^* \) if improving. |
| 9. Set \( x_{\text{old}} \leftarrow x \). |
| 10. Else if GPU-version then |
| 11. Broadcast \( x \) to GPU-workers \( i = 1, \ldots, n \), and set \( x_i, x^*_i \leftarrow x \). |
| 12. On GPU: Conduct Step 8 for each GPU-worker, and compute cost \( C_i \). |
| 13. Return \( C_i \) of each worker \( i = 1, \ldots, n \) to the host. |
| 14. Compute \( i^* = \arg \min_i \{ C_i \} \). Reconstruct \( x_{i^*} \) on the host. |
| 15. If \( C_{i^*} < C^* \), update \( x^* \leftarrow x_{i^*} \) and \( C^* \leftarrow C_{i^*} \). |
| 16. Set \( x_{\text{old}} \leftarrow x_{i^*} \). |

3. Problem Solution

3.1. Main Algorithm

Algorithm 1 is proposed to solve the problem formulated in the previous section.

Several explanatory comments are made. Steps 1-4 represent the solution initialisation and replicate a sequential solution procedure a human scheduler may implicitly conduct. Step 2 involves the assignment of fields to BPs. Guiding notion is that every BP has a minimum demand for biomass, \( a^\text{min}_b \), \( \forall b \in B \). Therefore, fields are picked and assigned to their spatially closest BP that is not yet supplied up to its minimum demand. Once all BPs are supplied with their minimum demand, all remaining (not yet assigned) fields are assigned to their spatially closest BP. This is enabled by assumption according to problem modeling of Section 2 about the absence of any upper bound on supplied biomass and the assumption about good storage capabilities at BPs.
always also completely reassigned to the new tour or to the new
ordering position, such that the attachment of fields to that BP
is maintained. Notice that since in practice $N_f \gg N_b$, it is more
likely to draw fields instead of BPs. Variations can be thought of.
For example, in Section 4.1 an in practice important variation is
discussed where the assignment of fields to BPs is fixed (e.g.,
due to different ownerships or contractual agreements) and only
the sequence in which the fields are harvested can be optimised.
Furthermore, the reassignment of BPs may ultimately cause a
tour to become inactive, i.e., if the last and only BP of a tour is
reassigned more efficiently to an alternative tour. This is done
on purpose since it is a priori not obvious what the most efficient
number of active tours is.

Throughout Algorithm 1 checking constraints imply to ver-
ify if for a given $x$ candidate the minimum biomass supply con-
straints in (2) are satisfied for all BPs. If this is not the case the $x$
candidate is discarded (and its cost not evaluated).

If all constraints are satisfied the cost for a candidate solution
$x$ is evaluated. This involves 3 steps as discussed next.

First, the assignment of HUs to tours is discussed. To ob-
tain an intuition it is here focused on the case of uniform HUs
for $N_{HU}(g)$, $\forall g = 1, \ldots, N_f$. The assignment method for
the case with non-uniform HUs with different working rates is men-
tioned later. The general strategy for the assignment of HUs is to
achieve approximately uniform completion times, $T_{HU_{\text{compl}}}(g)$,
over all harvesters tours in order to simultaneously explo-
tion time over all harvesters tours to rank GPU-candidates solu-
tions. Given road travel velocities and in-field working rates as
well as all distances and field sizes the completion times for all
harvesters tours can be calculated. Initially, the average number
of HUs is assigned to each group. Then, in an iterative pro-
cedure tours with minimum and maximum completion times, $g_{\text{min}}$ and $g_{\text{max}}$, are identified, before their assigned number of HUs,
$N_{HU}(g_{\text{min}})$ and $N_{HU}(g_{\text{max}})$, is decreased and increased by one
HU, respectively. This is repeated until the maximum comple-
tion time over all tours cannot be further reduced.

Second, a logic must be devised for the assignment of SUs
to all groups is $N_{SU}$ different SU classes. Suppose further that these classes only
differ by their in-field fill times, $t^{\text{load}}_{SU}$, $\forall l \in \mathcal{L}$, but by their
traveling speed along roads (edges) between any pair of fields
and BPs (vertices). Then, two different waiting times for HUs
can be defined as,

$$
t^{\text{wait}_{\text{compl}}}_{HU,f_{SU},b_{SU}}(g) = \frac{t^{\text{edge}}_{f_{SU},b_{SU}} - (N_{SU}^{\text{has}}_g) \sum_{l \in \mathcal{L}_{(l_{SU})}} N_{l_{SU}}^{\text{has}}_g t^{\text{load}}_{SU} l_{SU} + 1}{\sum_{l \in \mathcal{L}_{(l_{SU})}} N_{l_{SU}}^{\text{has}}_g} 
$$

for the two cases $I_{SU}^{\text{has}} \in \{I_{SU}^{\text{largest}}, I_{SU}^{\text{mallest}}\}$, and where $t^{\text{edge}}_{f_{SU},b_{SU}}$ denotes the total travel time from field $f_{SU} \in \mathcal{F}_{b_{SU}} \subset \mathcal{F}$ to its
assigned BP $b_{SU} \in \mathcal{B}_{SU} \subset \mathcal{B}$ and back (which can be com-
puted based on path network distances and travel velocities),

and where $l_{SU}^{\text{largest}} \in \mathcal{L}$ and $l_{SU}^{\text{mallest}} \in \mathcal{L}$ denote the SU
classes with largest and smallest fill times, and where $g \in \{1, \ldots, N_f\}$. Negative $t^{\text{wait}_{\text{compl}}}_{HU,f_{SU},b_{SU}}(g) < 0$ (and likewise $t^{\text{wait}_{\text{compl}}}_{HU,f_{SU},b_{SU}}(g) > 0$) imply that there are sufficiently many SUs assigned to the tours such
that HUs can operate continuously without any interruption or waiting time. An interruption would occur if at a given time
during harvest there is no SU to which a HU can transfer the
harvested biomass. This is the case if $t^{\text{wait}_{\text{compl}}}_{HU,f_{SU},b_{SU}}(g) > 0$. Comparing $t^{\text{wait}_{\text{compl}}}_{HU,f_{SU},b_{SU}}(g)$ and $t^{\text{wait}_{\text{compl}}}_{HU,b_{BU},b_{BU}}(g)$ it is found after simplifications
that $t^{\text{wait}_{\text{compl}}}_{SU_{bg}} < t^{\text{wait}_{\text{compl}}}_{SU_{bg}}$. This implies that $t^{\text{wait}_{\text{compl}}}_{HU,f_{SU},b_{SU}}(g)$ is more
and an update rule must be devised based on that equa-
tion and SUs must be assigned based on ordering according to
their fill times. Algorithm 2 is therefore proposed for the
assignment of SUs to tours.

**Algorithm 2: Assignment of SUs to Different Tours**

1. Compute average waiting times for each tour,

$$
T^{\text{wait}}_{HU}(g) = \frac{1}{|B_g|} \sum_{b \in B_g} \frac{1}{|T_{b_{SU}}|} \sum_{l \in \mathcal{L}_{(l_{SU})}} t^{\text{wait}_{\text{compl}}}_{HU,f_{SU},b_{SU}}(g),
$$

for all $g = 1, \ldots, N_f$.

2. Compute indices,

$$
g^{\text{max}} \leftarrow \arg \max_{g \in \{1, \ldots, N_f\}} T^{\text{wait}}_{HU}(g),
$$

$$
g^{\text{max},2nd} \leftarrow \arg \max_{g \in \{1, \ldots, N_f\}} T^{\text{wait}}_{HU}(g),
$$

$$
g^{\text{min}} \leftarrow \arg \min_{g \in \{1, \ldots, N_f\}} T^{\text{wait}}_{HU}(g).
$$

3. Determine $L(g^{\text{min}})$ as the set of available SU classes for
tour $g^{\text{min}}$ and sort it according to rising fill times .

4. for $l \in L(g^{\text{min}})$ do

5. Remove one SU candidate of class $l \in L(g^{\text{min}})$.

6. Add that SU to the group of $L(g^{\text{max}})$.

7. Recompute $T^{\text{wait}}_{HU}(g^{\text{max}})$.

8. if $T^{\text{wait}}_{HU}(g^{\text{max}}) < T^{\text{wait}}_{HU}(g^{\text{max},2nd})$ or $l$ represents

9. Accept that SU-assignment.

10. Break.

11. Repeat Steps 1-10 until $T^{\text{wait}}_{HU}(g^{\text{max}})$ cannot be further reduced.

In case HUs are also non-uniform, an equivalent method is
employed for the minimisation of the maximum completion time
over all harvesters tours, where the set of available HUs
classes is now sorted according to rising in-field working rates.
Finally, the third step for the evaluation of $x$ after the op-
timised assignment of different HUs and SUs to all groups is
discussed. The distance-based cost function is evaluated ac-
cording to Section 2.2. Importantly, the best solution of all
GPU workers is selected according to the criterion of minimis-

5
ing both accumulated total path length and maximum completion time over all $N_F$ harvesters tours such that approximately uniform completion times, $\mathcal{C}^{tot}(g)$, result for all tours. Note that minimising $\mathcal{C}^{tot}(g), \forall g = 1, \ldots, N_F$, is instead treated as a local embedded optimisation for the assignment of SUs. This is done (i) to avoid more constraining the GPU-candidates selection criterion, and (ii) since typically there are (many) more SUs in comparison to HUs such that waiting times can be controlled sufficiently well via a local embedded optimisation.

Based on an argument of symmetry it was also considered to repeat Step 8 of Algorithm 1 a second time. Suppose a scenario in which just one field is removed without its replacement by another field from a tour where all BPs are supplied the exact minimum demand of biomass. Then, it would become infeasible to sufficiently supply one of the BPs for that tour. Thus, no feasible improvement solution could ever be found eventhough one may exist and realised by a simple field exchange. Strictly, a repetition of Step 8 would guarantee feasibility only in expectation due to the random sampling involved and also only under the assumption that each field produces the same amount of biomass. Otherwise, for example, two smaller fields may be required to replace one larger field and so forth. In simulation experiments the solution with repeated Step 8 was tested. However, it was found that for the given large number of fields there were no symmetry issues and only solve times were prolonged.

The update of $x$ in Step 6 involving the exploration heuristic is implemented outside the GPU on the host. This is done to avoid having to send both $x_{old}$ and $x^*$ to the GPU.

Employment of a GPU permits to conduct Steps 8 in parallel on multiple workers. An implementation detail is discussed. By identical random process control on both GPU and host it is possible to broadcast only the current random seed and $x$ and recover only $C_t$ from all workers $i = 1, \ldots, n$. Thus, it is not necessary to return all $x_i, \forall i = 1, \ldots, n$. The optimal $x^*$ can then be reconstructed directly on the host.

Algorithm 1 is terminated after a maximum number of iterations, $N_{iter}$. An alternative early termination based on the length of non-decreasing plateaus in the cost function is avoided since it is difficult to tune and would add at least one additional hyperparameter.

In fact, an important perceived benefit of Algorithm 1 is its absence of many hyperparameters. The number of workers $n$ is prescribed by the GPU. Thus, the only two hyperparameters for Algorithm 1 are $\epsilon \in [0, 1]$ and $N_{iter} > 0$. The latter can be selected according to some desired maximum solve time (wall clock time). The former can easily be gridded due to its bounded range.

Algorithm 1 is scalable in that it is inherently parallelisable. Furthermore, there are no steps that are memory-intensive. In addition, there are no steps that involve any special linear algebra tools or other libraries.

Regarding convergence analysis, Algorithm 1 applies iterative local search leveraging random sampling. When leveraging parallel GPU-computations the solution can be interpreted as stochastic global search. Global search is not conducted explicitly on every GPU-worker. However, since GPU-workers are stochastically working together in parallel global search is implicitly conducted. Furthermore, by non-negativity of path lengths in the cost function and deterministic constraints a global optimal solution exists. All optimisation tasks according to Fig. 1 are addressed since accumulated total distance is clearly affected by (i) assignments of BPs to tours, (ii) their ordering within each tour, (iii) assignment of fields to BPs, (iv) their ordering for each assignment, (v) assignment of $N_{HU}(g), \forall g = 1, \ldots, N_F$, and (vi) assignment of $N_{SU}(g), \forall g = 1, \ldots, N_F$. Thus, in expectation Algorithm 1 is capable of finding the global optimal solution.

Finally as a remark, note that the problem of coupling crop assignment and routing for harvest planning discussed in Plessen (2019) can be reformulated as a reduced special case of the problem discussed here. Thus, the proposed solution methodology can in general be transferred.

### 3.2 Variations and Important System Parameters

Different problem variations are discussed. First, consider the important practical scenario in which the assignment of fields to BPs is fixed. This occurs if operators of BPs want to draw their supply from very specific fields that they potentially own or have leasing contracts with. Then, the optimisation potential is only with the order in which fields for that BP are harvested to minimise accumulated inter-fields distances. In such a scenario, any field removed from $x$ as part of Step 8 in Algorithm 1 must be reinserted into the same group of fields assigned to that given BP. This is the only change. The general structure of Algorithm 1 is otherwise not altered.

Second, analogously to the previous scenario one may consider use cases in which the assignment of sets of BPs to a specific tour is fixed, teams of HUs and SUs are fixed, and so forth.
Table 1. System parameters for problem modeling. All remaining location-dependent parameters are: (i) $\theta_f$, $\forall f \in F$, (ii) $d_{\min b}$, $\forall b \in B$, (iii) $h_f$, $\forall f \in F$, and (iv) one transition matrix indicating the distance costs for a path network connecting HQ, all BPs and all fields. Their modeling for 10 simulation experiments is discussed in Section 4.1.

| Symbol | Unit | Value |
|--------|------|-------|
| $N_{\text{total}}$ SU | - | 7 |
| $N_{\text{small}}$ SU | - | 14 |
| $N_{\text{large}}$ SU | - | 28 |
| $\nu_{\text{load,small}}$ SU | t | 12.5 |
| $\nu_{\text{load,large}}$ SU | t | 16.5 |
| $\nu_{\text{area}}$ SU | min | 6 |
| $\nu_{\text{load,small}}$ SU | min | 8 |
| $\nu_{\text{edge}}$ SU | km/h | 40 |
| $\nu_{\text{edge}}$ SU | km/h | 40 |
| $\nu_{\text{area}}$ SU | ha/h | 2.5 |
| $\epsilon_{\text{conv}}$ HU | t/ha | 40 |

Third, another variation involves the scenario in which in-field operating SUs transfer their load to out-of-field shuttling larger SUs such as trucks that wait along the field boundary. This scenario is mainly relevant if distances between fields and BPs are larger. In the most general case, i.e., if the number of such trucks exceeds the number of tours a new optimisation layer needs to be added for the assignment of these trucks. The proposed optimisation criterion would again be based on a waiting time similar to (6) whereby filling times would become a function of the number of the in-field operating SUs and, in general, also a function of the field shape and the in-field path plans driven by HUs and SUs. This scenario is out of the scope of this paper and also not considered for the experimental setup in the next section.

While some parameters such as the number of HUs and SUs or their loading capacities are easy to quantify there are other parameters that are affected to a larger extent by stochasticity such as seasonal crop yields or soil moisture. These parameters typically are difficult to predict at the time of harvest planning. Due to their importance and for an extending sensitivity analysis in practice they are here mentioned explicitly: $\nu_{\text{area}}^\text{HU}$, $\nu_{\text{load}}^\text{SU}$ and $\epsilon_{\text{conv}}^\text{biom}$.

4. Numerical Results

4.1. Experimental Setup

The results of 10 simulation experiments are summarised. The task of a contractor about planning logistics and implementing harvest of $N_F = 1200$ corn fields for the supply of biomass to $N_B = 20$ biogas plants is simulated. All parameters stem from real-world operations in Northern Germany and are summarised in Table 1. There is one class of HUs and two classes of SUs, which are for ease of notation in the following abbreviated by $N_{\text{SU}}^\text{small}(g)$ and $N_{\text{SU}}^\text{large}(g)$ for tours $g = 1, \ldots, N_T$. Field sizes are simulated with $h_f = 3 + 4u_f$.

Table 2. Summary of detailed results for one of the 10 experiments. The solutions of Cases 1 to 4 are compared.

| CASE | Unit | 1 | 2 | 3 | 4 |
|------|------|---|---|---|---|
| $\epsilon^*$ | - | - | 0 | 0.5 | 0.5 |
| $N_T$ | - | 7 | 7 | 7 | 7 |
| $N_{\text{small}}$ SU | - | 2 | 2 | 3 | 1 |
| $N_{\text{large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 3 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{edge}}$ SU | - | 2 | 2 | 2 | 3 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 3 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,small}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{load,large}}$ SU | - | 2 | 2 | 2 | 2 |
| $\nu_{\text{area}}$ SU | - | 2 | 2 | 2 | 2 |
Fig. 3. Convergence results for Experiment 1 and Cases 2 to 4. Top subplots illustrate the optimal cost $C^*_\epsilon$ as a function of optimisation iterations, $i_{iter}$, for 5 different $\epsilon$-levels. The legend which is uniformly valid for all cases is plotted for clarity only on the right. Bottom subplots illustrate indicator $I_{\epsilon \star}$ as a function of optimisation iterations for the cost-minimising best $\epsilon$-level. $I_{\epsilon \star}(i_{iter}) = 1$ and $I_{\epsilon \star}(i_{iter}) = 2$ imply that $C^*_\epsilon(i_{iter})$ could be decreased by improved reassignment of a field or a biogas plant, respectively. $I_{\epsilon \star}(i_{iter}) = 0$ implies that the optimal cost could not be further reduced at iteration $i_{iter}$.

Fig. 4. Routing results for Experiment 1 and Case 4. For each plot, the solid black line indicates the HU-tour and $\mathcal{B}(g)$ indicates the BPs to be served and their ordering for each tour. Dashed lines indicate the last transition of SUs back to HQ. Results are obtained minimising both accumulated total path length and maximum completion time over all 7 tours such that approximately uniform $T_{compl}^{HU}(g)$ result, see Table 2. The initial location data set is visualised in Fig. 2.
measured in hectares (ha) and for uniform random variables $u_f \in [0, 1]$, $\forall f \in \mathcal{F}$. Employing a local practical rule of thumb of one BP requiring a supply of about 250ha of corn fields, the minimum demand of each BP is in simulations also measured in ha and set as $d_{f}^{\text{min}} = 250$ha, $\forall f \in \mathcal{F}$. Then, the required “supply” of each field can here conveniently be also expressed as its field size, i.e., $s_f = h_f$, $\forall f \in \mathcal{F}$. In addition, a coverage area of 80km×80km is assumed in which all fields and biogas plants are uniformly distributed. See Fig. 2 for an example. The headquarter from which all mobile machinery initially starts is simulated to be at the origin. In order to evaluate proposed methods stochastically over 10 different simulation experiments and also for simplicity of generating these scenarios, the edge cost for the connection of any two vertices of the transition graph is calculated as the path length of straight line segments connecting any two vertices. In real-world employment this simplification is to be replaced by nonlinear paths representing the actual road network and involving shortest path computations over the path network in order to compute all edge costs.

4.2. Comparison of Four Solution Setups: Case 1 to 4

Four different solution methods (Case 1 to 4) are compared:

1. The initialisation method only; see Steps 1-4 of Algorithm 1. It produces a specific assignment of fields to BPs and enforces a fixed $N_T = N_{\text{HU}}^{\text{total}}$ such that $N_{\text{HU}}(g) = 1$, $\forall g = 1, \ldots, N_T$, and a fixed $N_{\text{SU}}^{\text{small}}(g) = 2$ and $N_{\text{SU}}^{\text{large}}(g) = 4$, $\forall g = 1, \ldots, N_T$. This method represents the initialisation step of Algorithm 1 and can be interpreted as a concatenation of nearest neighbour methods. This case serves as the baseline reference.

2. The assignment of fields to BPs generated from Case 1 is kept fixed. Likewise, $N_{\text{HU}}(g) = N_{\text{SU}}^{\text{small}}(g) = 2$ and $N_{\text{SU}}^{\text{large}}(g) = 4$, $\forall g = 1, \ldots, N_T$ remain fixed. Optimisation iterations are permitted to improve (i) the assignment of BPs (and all of their assigned fields) to different tours, and (ii) the ordering of fields (i.e., the harvest sequence) for each BP-assignment.

3. The assignment of fields to BPs generated from Case 1 is kept fixed. However, optimisation iterations according to Steps 5-16 of Algorithm 1 are conducted to improve (i) $N_{\text{HU}}(g)$, $N_{\text{SU}}^{\text{small}}(g)$ and $N_{\text{SU}}^{\text{large}}(g)$, $\forall g = 1, \ldots, N_T$, (ii) the assignment of BPs (and all of their assigned fields) to different tours, and (iii) the ordering of fields for each BP-assignment.

4. The assignment of fields to BPs is now freely optimised as part of Steps 5-16 of Algorithm 1. This adds to the optimisation of (i) $N_{\text{HU}}(g)$, $N_{\text{SU}}^{\text{small}}(g)$ and $N_{\text{SU}}^{\text{large}}(g)$, $\forall g = 1, \ldots, N_T$, (ii) the assignment of BPs (and all of their assigned fields) to different tours, and (iii) the ordering of fields for each BP-assignment.

4.3. Results of 10 Simulation Experiments

The results for one of the 10 simulation experiments are illustrated in detail in Table 2, Fig. 3 and 4. The results over all 10 simulation experiments are quantitatively summarised in Table 3. Note that Algorithm 1 involves only two hyperparameters. For all experiments and solution setups it is set $N_{\text{Figs}} = 5000$ and it is uniformly gridded for $\epsilon \in [0.0, 25, \ldots, 1]$ to analyse the effect of the exploration heuristic, before the lowest cost solution and corresponding $*^*$ is returned. All simulation experiments were conducted on an Intel i7-7700K CPU@4.2GHz×8 and 1 TitanV-GPU.

A preliminary comment is made. The scope of this section is not to benchmark different solution methods and their solver speeds (e.g., different hybrid genetic algorithms, ant colony optimisation, and so forth). Rather, the scope of this section is to stress structural observations made which are most relevant in practice for contractors who manage logistics with service of many fields and BPs owned by multiple different parties.

The following observations are made. First, $N_T = 7$ in Table 2 implies $N_{\text{HU}}(g) = 1$, $\forall g = 1, \ldots, N_T$, and thus $N_T = N_{\text{HU}}^{\text{total}}$. While this was expected for Case 1 and 2 by enforcement, interestingly it also resulted for Case 3 and 4 throughout all 10 simulation experiments, even though Algorithm 1 is explicitly designed to also permit $N_T < N_{\text{HU}}^{\text{total}}$. It implies that every available HU forms exactly one harvester tour and there are never more than one HUs operating together on any tour. The reason for this result is explained by the optimisation criterion designed to minimise not only accumulated path length but also the maximum completion time, $T_{\text{HU}}^{\text{compl}}(g)$, over all tours $g = 1, \ldots, N_T$. When dropping the latter criterion and just optimising accumulated path length, indeed $N_T < N_{\text{HU}}^{\text{total}}$ could be observed. However, then in practice unacceptably unbalanced completion times, $T_{\text{HU}}^{\text{compl}}$, resulted such as, e.g., 800h for some groups and 150h for others. The takeaways message here is that for the given parameters setup the distribution of one HU per tour contributed importantly to achieve approximately uniform completion times over all harvesters tours with particularly balanced results for Case 4.

Second and in contrast to HUs, subtle differences in the assignment of both small and large SUs to the different groups could be observed for Case 3 and 4 throughout all experiments. See the listings $N_{\text{SU}}^{\text{small}}$ and $N_{\text{SU}}^{\text{large}}$ in Table 2. Of course, by construction this does not hold for Case 1 and 2 where the assignments are fixed with $N_{\text{HU}}(g) = 1$, $N_{\text{SU}}^{\text{small}}(g) = 2$ and $N_{\text{SU}}^{\text{large}}(g) = 4$, $\forall g = 1, \ldots, N_T$.

Third, typical convergence results are demonstrated in Fig. 3. For all Cases 2 to 4 the major reduction in cost $C^*$ is observed for initial iterations $i_{\text{iter}}$, before costs typically quickly plateaued. Differences in final cost levels dependent on hyperparameter $\epsilon$ result in particular for Case 4. Interestingly, most of the cost reductions are obtained by field exchanges, i.e., for $I_{\epsilon}(i_{\text{iter}}) = 1$. One explanation is that $N_T \gg N_{\text{Figs}}$, which implies that fields are drawn much more frequently as candidates for a reassignment in comparison to BPs. However, these results also imply that the initialisation method discussed in Section 3.1 for the initial assignment of BPs to groups is quite good. Nevertheless, for all Cases 2 to 4 also at least 1 BP-exchange resulted. Fig. 3 clearly visualises the significant improvement potential of Case 4. The attainable cost level is significantly lowered in comparison to Case 2 and 3. This is enabled by permitting an
The average quantifiers, $\bar{\tau}$, for Cases 1 to 4. In order to account for all 10 experiments, the completion times $\bar{T}_{\text{compl}}$ for Case 1 since the method involves no optimisation iterations with the current best solution at that time (exploitation step).

Ninth, it is observed that the cost reduction for Case 2 and 3 in comparison to the benchmark Case 1 is comparatively small with $-3.4\%$ and $-3.1\%$, respectively. Note further that the first sight surprising result of an on average slightly lower cost for Case 2 rather than for Case 3 is explained by the fact that Case 3 optimises both accumulated total path length and maximum completion time over all 7 tours. Consequently, the worst-case time, $\bar{T}_{\text{compl}}$, is much lower for Case 3 as Table 4 shows.

Tenth, one might consider adding hard constraints such that any GPU solution candidates are only permitted if waiting times over all tours are negative such that there is never an HU waiting. This was tested. It was found that in some of the 10 experiments no feasible solution could then be found. Thus, for the given location distribution of 20 BPs and 1200 fields, the given number of HUs and SUss according to Table 1 and in particular for a large coverage area of 80km×80km, in some scenarios there did not exist a solution without any waiting times. Therefore, instead of hard constraints, HUs-waiting time was added as a soft constraint in form of an embedded local minimisation.

Eleventh and to sum up, Case 4 clearly outperforms the other methods. While also offering the best results for $\bar{T}_{\text{HU,avg}}$ and $\bar{T}_{\text{wait, HU,avg}}$, the accumulated path length savings are uncomparably large with -66368.7km or -14.4% with respect to the baseline reference. While outperformance was in general expected since Case 4 implies the least constrained setup, the degree of improvement was unexpected. In practice this result is relevant for contractors and importantly also for environmental considerations. It implies that cleanest biomass production from a logistics point of view can only be achieved if all fields a contractor services can be freely assigned to any biogas plant independent of their ownerships. Note that all improvements are obtained without compromising the operation of BPs since supply to all BPs is still guaranteed. Thus, the same service is provided cleaner. Path length savings directly translate to fuel savings (Diesel) and thus directly to a reduction of exhaust gas and CO2 emissions.

Finally, it is pointed out that results in Table 2 and Table 4 and in particular for Case 4, are benchmarked with respect to an optimised initialisation technique. It is stressed that in practice the fixed assignment of fields to BPs may be a lot worse, for example, due to contractual agreements between BP operators, field owners, leasing parties and so forth. Thus, path length savings and the environmental benefit are expected to be in practice (much) higher.

5. Conclusions

This paper contributed to the task of cleaner biomass production via logistics optimisation by (i) proposing methods for problem modeling and (ii) presenting a corresponding parallelisable solution algorithm suitable for GPU-acceleration. Multiple harvesters tours are to be executed simultaneously by unconstrained assignment of any field to any BP.

Fourth, typical routing results are demonstrated in Fig. 4. Interestingly, for the given data clustering-like solutions with closely located BPs result for tours $g=1, \ldots, 5$, whereas residuals are partitioned as shown in the plots for $B(6)$ and $B(7)$. As the related Table 2 shows corresponding path lengths for $C(6)$ and $C(7)$ are among the longest. However and crucially, completion times $\bar{T}_{\text{HU,comp}}(g)$, $\forall g=1, \ldots, N_T$, are still approximately uniform over all tours.

Fifth, it is further elaborated on different $\bar{T}_{\text{HU,comp}}$ observed for Cases 1 to 4. In order to account for all 10 experiments the average quantifiers, $\bar{T}_{\text{HU,avg}} = \frac{1}{\sum_{T=1}^{T}} \frac{1}{\sum_{g=1}^{G}} $ $\bar{T}_{\text{HU,comp}}(g)$ and $\bar{T}_{\text{HU,worst}} = \frac{1}{\sum_{T=1}^{T}} \max_{g(1, \ldots, G)} \bar{T}_{\text{HU,comp}}(g)$, are defined. As Table 3 demonstrates, the worst results are obtained for Case 2. This was expected since here the assignment of HUs and SUss is fixed, and the optimisation criterion is thus just about accumulated total path length but not accounting for balanced completion times. The best results are clearly obtained for Case 4 with $\bar{T}_{\text{HU,worst}} = 371.2$h. In addition, Case 4 also has the lowest $\bar{T}_{\text{HU,avg}} = 358.1$h.

Sixth, different $\bar{T}_{\text{wait}}$, for Case 1 to 4 are compared. Similarly to before, $\bar{T}_{\text{HU,avg}}$ and $\bar{T}_{\text{wait}}$, are defined. Table 4 summarises results. Again, Case 4 clearly outperforms the other methods. Interestingly, it achieves $\bar{T}_{\text{wait, HU,avg}} = 0.01$h. This implies that on average even for Case 4 there is a tiny delay for HUs operations. This implies that a simulated coverage area of 80km×80km as illustrated in Fig. 4 would in practice be too large or at least critical for the given machinery and parameters setup according to Table 1 in order to operate fully delay-free. For the 10 simulation experiments a large coverage area of 80km×80km was selected to emphasise this aspect.

Seventh, the average solve time, $\bar{T}_{\text{solve}}$, is obviously lowest for Case 1 since the method involves no optimisation iterations and no GPU-interaction. $\bar{T}_{\text{solve}}$ is comparable for Case 3 and 4, and $\approx 42\%$ smaller for Case 2. This is explained by the fact that Case 2 involves in contrast to Case 3 and 4 no embedded optimisation step with respect to the assignment of HUs and SUss to different groups.

| CASE  | Unit | 1 | 2 | 3 | 4 |
|-------|------|---|---|---|---|
| $\bar{\tau}$ | - | - | 0.0 | 0.4 | 0.4 |
| $N_T$ | - | 7 | 7 | 7 | 7 |
| $\bar{T}_{\text{HU,avg}}$ | h | 360.9 | 359.8 | 360.2 | 358.1 |
| $\bar{T}_{\text{HU,worst}}$ | h | 411.3 | 577.5 | 406.2 | 371.2 |
| $\bar{T}_{\text{wait, HU,avg}}$ | h | 0.12 | 0.08 | 0.09 | 0.01 |
| $\bar{T}_{\text{wait, HU,worst}}$ | h | 0.42 | 0.37 | 0.14 | 0.06 |
| $\bar{T}_{\text{solve}}$ | s | 0.0050 | 47.77 | 82.33 | 81.89 |
| $C^*$ | km | 461048.6 | 445043.6 | 446551.3 | 394679.9 |
| $\Delta C^*$ | km | -16005.0 | -14973.7 | -66368.7 | -66368.7 |
| $\Delta \bar{\tau}^*$ | % | -3.4 | -3.1 | 14.4 | 14.4 |

Table 3. Summary of results averaged over all 10 simulations experiments. The solutions of Cases 1 to 4 are compared.
groups of harvesting units (HUs) and support units (SUs) to first harvest biomass from multiple agricultural fields, before supplying the biomass to multiple biogas plants (BPs) via shuttling SUs. There are three interconnected optimisation levels: (i) the assignment of BPs to tours and the ordering of BPs assigned to each tour, (ii) the assignment of fields to BPs and the ordering of fields assigned to each BP, and (iii) determining the number of HUs and SUs assigned to each tour, whereby the HUs and SUs may in general have different working rates and loading capacities. In this paper, the optimisation criterion is not only accumulated total path length but also minimisation of the maximum completion time over all harvesters tours, and further, in an embedded optimisation loop, minimisation of waiting times for HUs by non-uniform assignment of SUs to each harvester tour. Large-scale experiments were conducted with 7 HUs, 14 small SUs, 28 large SUs, 20 BPs and 1200 corn fields operated over an area of 80km×80km. In 10 stochastic simulation experiments four different setups differing in their constraints setup were compared. The main finding was that for one setup (Case 4) significant path length savings can be achieved when permitting free and unconstrained assignment of any of the given fields a contractor services to any of the given biogas plants independent of their ownerships, while simultaneously obtaining approximately uniform completion times over all harvesters tours and minimising waiting times by suitable assignment of SUs to different harvesters tours, whereby a specific logic was proposed for the case that employed SUs have different loading capacities. The presented modeling approach is generic enough such that different extending setups can easily be tailored.

In order to evaluate proposed methods stochastically over 10 different simulation experiments and also for simplicity of generating these scenarios, all location vertices (the central depot, biogas plants and fields) were assumed to be connected by straight line segments. In real-world employment this simplification is to be replaced by nonlinear paths representing the actual road network.

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