Astrophysical Haloscopes

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We compute the fluxes of radio photons from conversion of axion-like particle dark matter in cosmic magnetic fields. We find that for axion-like particle masses around $10^{-6}$ eV and effective coupling constants to photons $g_{a\gamma} \gtrsim 10^{-13}$ GeV$^{-1}$ strongly magnetized nearby stellar winds may give detectable line-like radio photon signals, although predicted fluxes are highly uncertain due to the poorly known structure of the magnetic fields. Nevertheless, it may be worth while to conduct a dedicated search in the direction of such sources. When combined with a possible future laboratory detection of axion-like dark matter such observations may in turn provide information on the small scale magnetic field structure in such objects.

PACS numbers: 95.35.+d,14.80.Va,95.55.Jz
Axion-like particles (ALPs) have developed into an interesting alternative to the WIMP paradigm of cold dark matter. Originally axions were motivated by the strong CP problem which can be solved by promoting the CP-violating phase $\theta$, experimentally constrained to be smaller than $\sim 10^{-10}$, to a pseudo-scalar field $a$ via $\theta \rightarrow a/f_a$ with $f_a$ an energy scale known as the Peccei-Quinn scale. The field $a$ is then dynamically driven to zero in a suitable potential which would explain why the phase $\theta$ essentially vanishes. In such models the axion field couples to the gluon field strength tensor $G^\alpha_{\mu\nu}$ via a term of the form \[ \mathcal{L}_{aG} = \frac{\alpha_s}{8\pi f_a} a G^\alpha_{\mu\nu} \tilde{G}^{\alpha\mu\nu}, \] (1) where $\tilde{G}^{\alpha\mu\nu}$ is the dual to $G^\alpha_{\mu\nu}$ and $\alpha_s$ is the strong fine structure constant. In QCD axion models the axion mass $m_a$ is related to the Peccei Quinn scale by $m_a \approx 6 \times 10^{-6} \left( \frac{10^{12} \text{GeV}}{f_a} \right)$ eV. (2)

In generalizations of such scenarios to ALPs $f_a$ and $m_a$ are considered as independent parameters and there is a coupling term to photons of similar shape to the ALP-gluon coupling term. Using Lorentz-Heaviside units, $\epsilon_0 = \mu_0 = 1$ the parts of the Lagrangian depending on the ALP and photon fields can be written as \[ \mathcal{L}_{\alpha\gamma} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{\alpha_{\text{em}}}{8\pi} \frac{C_{\alpha\gamma}}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} - V_a(a), \] (3)

where $F_{\mu\nu}$ is the electromagnetic field strength tensor, $\tilde{F}_{\mu\nu}$ is its dual, $C_{\alpha\gamma}$ is a model dependent dimensionless number, and $V_a(a)$ is the effective ALP potential which can be expanded as $V_a(a) = \frac{1}{2} m_a^2 a^2 + \mathcal{O}(a^3)$ around $a = 0$.

ALPs can then contribute to cold dark matter as the phase $\theta$ freezes out at a random value of order one. Other contributions can result from cosmic strings that are formed when the $U(1)$ symmetry associated with the field $a$ is broken. For a review on computing ALP relic densities see Ref. 4. Obtaining the correct order of magnitude for the relic dark matter density requires $m_a \gtrsim 10^{-6}$ eV in the case of axions.

The ALP-photon coupling in Eq. (3) couples two photons (which can also be off-shell) to one ALP. This provides many possible experimental and observational tests for the existence of ALPs. In light shining through walls experiments a laser beam is partly converted to ALPs in a strong magnetic field in front of a wall which is then reconverted by a similar magnetic field within a high Qvalue optical cavity. For example, the Axion-Like Particle Search, alternatively called Any Light Particle Search (ALPS) [949] is operated at DESY and uses a 5T magnetic field and an optical cavity of 8.4m length [5, 6], and the OSQAR experiment at CERN has recently started [7]. Photons within stars can be converted to ALPs in the ambient magnetic fields. On the one hand, this leads to an additional energy loss mechanism that has been used to constrain $f_a$ and $m_a$, see Ref. 8 for a recent review, where for axions one obtains $m_a \lesssim 10^{-3}$ eV. In fact, there are recent hints for extra cooling in certain stellar objects, see Ref. 8 for a review. On the other hand, electronvolt scale ALPs emitted from the Sun in this way can be reconverted to X-ray photons in a strong magnetic field in a dark cavity which is used in so-called haloscopes such as CAST [10, 11] and its planned successor IAXO [12]. Further astrophysical tests include core collapse supernova explosions [13], cosmic microwave background distortions [14], a possible anomalous transparency of the Universe to $\gamma$-rays [15] and spectral distortions of astrophysical sources [16, 17].

Finally, if ALPs contribute significantly to the cold dark matter, they can be converted to photons in a strong magnetic field within a dark cavity. These are called haloscopes examples of which are ADMX [18] which scans the mass range between $1.9\mu eV$ and $3.7\mu eV$, and planned future experiments such as MADMAX [19] and BRASS which use layered dielectrica in a strong magnetic field. For $10^{-9}$ eV $\lesssim m_a \lesssim 10$ eV current constraints can be roughly summarised by $g_{a\gamma} \lesssim 10^{-10}$ GeV$^{-1}$.

The haloscope effect can also occur in astrophysical magnetic fields where it can lead to radio emission from strongly magnetized astrophysical objects. For the case of resonant conversions in essentially homogeneous magnetic fields around neutron stars this has been considered in Ref. 20, and for non-resonant transitions from around the Galactic center in Ref. 21. In the present paper we estimate the radio fluxes from non-resonant conversions and discuss their prospects for detection more systematically.

In the next section we derive general expressions for diffuse fluxes and fluxes from discrete sources. In section III we apply these expressions to concrete astrophysical cases and in section IV we compare the predicted fluxes with the sensitivities of present and future radio telescopes. We conclude in section V.
II. CONVERSION OF AXION-LIKE PARTICLES INTO PHOTONS IN AMBIENT MAGNETIC FIELDS

The ALP-photon coupling term in Eq. (3) can also be written as

$$\frac{\alpha_{em} C_{a\gamma}}{8 \pi f_a} a \tilde{F}_{\mu\nu} = \frac{e^2}{32 \pi^2} \frac{C_{a\gamma}}{f_a} a \tilde{F}_{\mu\nu} = \frac{\alpha_{em} C_{a\gamma}}{8 \pi} a \tilde{F}_{\mu\nu} = \frac{g_{a\gamma}}{4} a \tilde{F}_{\mu\nu},$$

where $\alpha_{em} = e^2/(4\pi\epsilon_0)$ and

$$g_{a\gamma} \equiv \frac{\alpha_{em} C_{a\gamma}}{2\pi f_a}.$$  \hspace{1cm} (5)

Note that whereas $e^2 F_{\mu\nu} \tilde{F}^{\mu\nu}$ is independent of the electromagnetic units, $\alpha_{em} F_{\mu\nu} \tilde{F}^{\mu\nu}$ and thus $g_{a\gamma}$ is not. We will furthermore generally use units in which $k_B = c_0 = h = 1.$

We now consider the Primakoff effect, the conversion of an ALP of energy-momentum $(E_a, k_a)$ and mass $m_a$ into a photon of energy-momentum $(\omega, \lambda)$ in an external magnetic field whose Fourier transform is defined by

$$B(\omega, \lambda) = \frac{1}{(2\pi)^2} \int dt \int d^3r \, B(t, r) e^{i(\omega t - \lambda \cdot r)}.$$  \hspace{1cm} (6)

Energy-momentum conservation requires

$$E_a = (m_a^2 + k_a^2)^{1/2} = \omega - \omega_\lambda - k_a = k_\lambda - k,$$

where for an electron density $n_e$ the plasma frequency is given by

$$\omega_{pl} = \left( \frac{e^2 n_e}{\epsilon_0 m_e} \right)^{1/2} \sim 1.3 \times 10^3 \left( \frac{n_e}{\text{cm}^{-3}} \right)^{1/2} \text{rad s}^{-1}.$$  \hspace{1cm} (8)

One can then show that for non-relativistic ALPs, $|k_a| \ll m_a, k_\lambda$, the conversion rate can be written as

$$R_{a\rightarrow\gamma} = \frac{\pi \epsilon_0 g_2^2 a^2}{2 \alpha_{em} n_a} \int \frac{d\omega}{T} d^3k_\lambda \delta(\omega + E_a - \omega_\lambda) \sum_\lambda |B(\omega, k_\lambda - k_a) \cdot \epsilon_\lambda(k_\lambda)|^2$$

$$= \frac{\pi \epsilon_0 g_2^2 a^2 n_a}{2 \alpha_{em} T} \int d^3k_\lambda \sum_\lambda |B(\omega - E_a, k_\lambda - k_a) \cdot \epsilon_\lambda(k_\lambda)|^2,$$

where $n_a$ is the ALP number density, assumed to be spatially homogeneous, the sum over $\lambda$ is over the two photon polarization states represented by the vector $\epsilon_\lambda(k)$, and $T$ is the time scale over which the integration in Eq. (6) is performed. Note that due to the Wiener-Chintschin theorem $|B(\omega, k)| \propto T^{1/2}V^{1/2}$ with $V$ the volume over which is integrated in Eq. (3). Therefore, $R_{a\rightarrow\gamma}$ is independent of $T$ and proportional to $V$, as it should. Similar expressions have been discussed in Ref. [2]. Note that for $\omega_{pl} = m_a$ there is a contribution from $\omega_\lambda = E_a, k_\lambda = k_a$ which leads to a resonance in as static and homogeneous magnetic field. In the following we will, however, assume dilute plasmas in which the plasma mass can be neglected, $\omega_\lambda = |k_\lambda| = k_\lambda = 2\pi x$. According to Eq. (5), when considering photons with $\omega_\lambda \geq 10 \text{ MHz}$ this is justified for $n_e \lesssim 2.3 \times 10^3 (\nu/10 \text{ MHz})^2 \text{ cm}^{-3}$.

If the magnetic field contains magnetohydrodynamic waves with dispersion relations $\omega \simeq v_m k$ with $v_m$ a characteristic velocity scale, Eq. (9) shows that the photon energies are concentrated around $k_\gamma \simeq m_a$ with a characteristic relative width

$$\Delta \equiv \frac{\Delta k_\lambda}{k_\lambda} \simeq v_a^2/2 + v_m + \Delta v \sim 10^{-3},$$

where $v_a = k_a/m_a \sim 10^{-3}$ is the characteristic ALP velocity in the Galaxy and $\Delta v$ is the velocity dispersion within the object considered which is $\Delta v \sim 10^{-3}$ for the Galactic objects we will consider. In a turbulent magnetized medium $v_m$ is of the order of the Alfvén velocity which is itself of the order of $\Delta v$. The last term in Eq. (10) results from the Doppler effect due to relative motion between the dark matter, magnetic field and observer. Since $v_m \lesssim \Delta v$ the observational signature is thus a line-like photon spectrum with relative width $\Delta \sim 10^{-3}$.

For order of magnitude estimates we will usually consider the static limit in which

$$|B(\omega, k)|^2 \rightarrow \frac{1}{2 \pi} 2\pi \delta(\omega) |B(k)|^2,$$
with the static Fourier transform defined by

$$B(k) = \frac{1}{(2\pi)^{3/2}} \int d^3r B(r) e^{-ik\cdot r}. \quad (11)$$

In this limit Eq. (9) turns into

$$R_{a\rightarrow \gamma} = \frac{\pi \epsilon_0}{2} g_{a\gamma}^2 n_a \int d^3k \delta(k - E_a) \sum_{\lambda} |B(k - k_a) \cdot \epsilon_{\lambda}(k)|^2. \quad (12)$$

This result has been first derived in Ref. [22] in a slightly different notation. Let us now approximate

$$\sum_{\lambda} |B(k - k_a) \cdot \epsilon_{\lambda}(k)|^2 \approx B^2(k - k_a)$$

and express the latter in terms of the magnetic field power spectrum. Assuming homogeneity and isotropy one can write the magnetic field energy density as

$$\rho_m = \frac{1}{2\mu_0 V} \int d^3r |B(r)|^2 = \frac{1}{2\mu_0 V} \int d^3k |B(k)|^2 = \int d\ln k \rho_m(k), \quad (13)$$

thus

$$\rho_m(k) = \frac{2\pi}{\mu_0 V} k^3 |B(k)|^2. \quad (14)$$

This allows to rewrite Eq. (12) as

$$R_{a\rightarrow \gamma} \approx \pi g_{a\gamma}^2 M_a \rho_m(m_a), \quad (15)$$

where we have taken the non-relativistic approximation $|k - k_a| \approx k_a \approx m_a$ and expressed $n_a$ in terms of the total ALP mass within volume $V$, $M_a \approx n_a m_a V$.

We will now apply this formula to the case of diffuse emission and to discrete sources emitting into a small angular range. For diffuse emission Eq. (15) gives the specific intensity

$$I \approx \frac{\pi}{4} \frac{g_{a\gamma}^2 M_a}{m_a^2} \rho_m(m_a), \quad (16)$$

where the integral is along the line of sight and we have taken into account that both the ALP mass density $\rho_a = n_a m_a$ and the magnetic field power density may depend on the position along the line of sight and $\Delta$ is the relative photon line width estimated in Eq. (10).

For a discrete source at distance $d$ we get for the total flux density

$$S \approx \frac{\pi}{4d^2} \frac{g_{a\gamma}^2}{m_a^2} \frac{1}{\Delta} \int d^3r \rho_a(r) \rho_m(m_a, r) \approx \frac{\pi}{4d^2} \frac{g_{a\gamma}^2}{m_a^2} \frac{1}{\Delta} M_a \rho_m(m_a), \quad (17)$$

where in the last step we have assumed the ALP and magnetic field densities to be roughly constant with $M_a$ the total ALP mass within the object. If the discrete source covers a solid angle $\Omega_s \approx \pi (r_s/d)^2$ with $r_s$ the radius of the source, one can also express the flux density as a specific intensity $I = S/\Omega_s$,

$$I \approx \frac{1}{4r_s^2} \frac{g_{a\gamma}^2}{m_a^2} \frac{1}{\Delta} M_a \rho_m(m_a), \quad (18)$$

which does not depend on the distance to the source.

III. APPLICATION TO ASTROPHYSICAL SOURCES

The relations between photon frequency, wavenumber and ALP mass are given by

$$\nu = \omega_\gamma / (2\pi) = 242 \left( \frac{m_a}{\mu eV} \right) \text{MHz}, \quad \frac{1}{k} = 20 \left( \frac{m_a}{\mu eV} \right)^{-1} \text{cm}. \quad (19)$$
We are interested in the ALP mass range $10^{-6}$ eV $\lesssim m_a \lesssim 10^{-3}$ eV because this corresponds to the frequency range in which radio telescopes are sensitive and for axions this is the characteristic window in which they are good candidates for cold dark matter. For the magnetic field power spectrum we make a power law ansatz

$$\rho_m(k) = \frac{B^2}{2\mu_0} f(k), \quad (20)$$

where $B$ is the characteristic total r.m.s. field strength and $f(k)$ is the fraction of the magnetic field energy within one decade around wavenumber $k$. For turbulent magnetic fields this can be approximated by

$$f(k) \simeq (kl_c)^n, \quad (21)$$

where $l_c$ is the magnetic field coherence length and $n$ is a spectral index which for Kolmogorov turbulence would be $n = -2/3$. Turbulence is expected to extend between the scale $l_c$ and the resistive scale which is thought to be fractions of centimeters and thus smaller than the scale Eq. (19). Since in the astrophysical context $kl_c$ is typically very large, the factor $f(k)$ will be an important limiting factor with a large uncertainty.

Let us also estimate the conversion rate for a single ALP. It is obtained by dividing Eq. (15) by the total number of ALPs within volume $V$, $N_a = M_a/m_a$ and using the ansatz Eq. (20),

$$\frac{1}{\tau_a} \simeq \pi g_{a\gamma} \rho_m(m_a) \simeq 9.7 \times 10^{-29} \left( g_{a\gamma} 10^{14} \text{GeV} \right)^2 \left( \frac{m_a}{\mu\text{eV}} \right)^{-1} \left( \frac{B}{G} \right)^2 \left( \frac{f(m_a)}{s^{-1}} \right), \quad (22)$$

This shows that ALPs will not significantly convert within the age of the Universe unless $g_{a\gamma}$ is close to its current experimental upper limit, $g_{a\gamma} \lesssim 10^{-10} \text{GeV}^{-1}$, and/or the magnetic fields at scale $m_a$ are much stronger than Gauss. It is nevertheless interesting to see what happens if the rate Eq. (22) becomes faster than ALPs can be replaced. In an object of linear size $r_s$ this happens if $r_s/\tau_a \gtrsim v_a$, in numbers

$$B \gtrsim 5.6 \times 10^{14} \left( g_{a\gamma} 10^{14} \text{GeV} \right)^{-1} \left( \frac{m_a}{\mu\text{eV}} \right)^{1/2} \left( \frac{10^6 \text{cm}}{r_s} \right)^{1/2} \left( \frac{1}{f(m_a)} \right) \text{G}. \quad (23)$$

In this case the total flux density cannot exceed the limit

$$S_{\text{max}} \simeq \rho_a \frac{v_a}{\Delta} \left( \frac{r_s}{d} \right)^2 \simeq 10^{-10} \left( \frac{m_a}{\mu\text{eV}} \right)^{-1} \left( \frac{r_s}{10^6 \text{cm}} \right)^2 \left( \frac{d}{\text{kpc}} \right)^{-2} \text{Jy}, \quad (24)$$

where $d$ is again the distance to the object and $r_s$ is the maximal length scale over which Eq. (22) applies and the numbers apply as long as the ALP density is not significantly enhanced in such objects. Eq. (24) is in particular relevant for strongly magnetized neutron stars which have been considered in Ref. [20] and for which $r_s \sim 10 \text{km}$. Flux densities from such small objects are therefore unlikely to be detectable even with next generation telescopes.

The specific intensity is often expressed in terms of the brightness temperature,

$$T_b(\nu) \equiv \frac{c_0^2 I}{2\nu^2}, \quad (25)$$

which in our case for $\nu \simeq m_a/(2\pi)$ gives

$$T_b(m_a) \equiv 2\pi^2 c_0^2 I/m_a^2 = 0.56 \left( \frac{I}{\text{Jy}/\text{sr}} \right) \left( \frac{m_a}{\mu\text{eV}} \right)^{-2} \text{mK}. \quad (26)$$

Let us now compute numerical estimates for these quantities. For the Galactic diffuse emission we obtain from Eq. (15) for the specific intensity

$$I \simeq 1.8 \left( g_{a\gamma} 10^{14} \text{GeV} \right)^2 \left( \frac{m_a}{\mu\text{eV}} \right)^{-2} \left( \frac{10^{-3}}{\Delta} \right) \left( \frac{\rho_a}{0.3 \text{GeVcm}^{-3}} \right) \left( \frac{L}{8 \text{kpc}} \right) \times \left( \frac{B}{5 \mu\text{G}} \right)^2 \left( \frac{f(m_a)}{\text{mJy}/\text{sr}} \right), \quad (27)$$

where $L$ is the characteristic linear size of the Milky Way and for $\rho_a$ we have substituted the canonical local dark matter density. Inserting this into Eq. (20) gives brightness temperatures in the micro Kelvin range for the fudge factors, times $f(m_a)$. However, this latter factor is likely to be very small: The typical coherence length of Galactic
magnetic fields is $l_c \sim \text{pc}$ so that for a Kolmogorov power spectrum $f(m_a) \approx (m_a l_c)^n \lesssim 10^{-13}$. Even for dark matter and magnetic field profiles that are enhanced toward the Galactic center the total flux is not enhanced by much more than an order of magnitude. It is thus unlikely that this diffuse emission is detectable in the foreseeable future.

Let us now turn to relatively compact objects which could be small enough for the factor $f(m_a)$ not to be too small. For a discrete source from Eq. (17) we obtain for the total flux density

$$S \approx 2.8 \times 10^{-11} \left( g_{\gamma} 10^{14} \text{ GeV} \right)^2 \left( \frac{m_a}{\mu \text{eV}} \right)^{-2} \left( \frac{10^{-3}}{\Delta} \right) \left( \frac{M_a}{10^{-10} M_\odot} \right) \left( \frac{d}{2 \text{kpc}} \right)^{-2} \left( \frac{B}{10^{-3} \text{ G}} \right)^2 f(m_a) \text{ Jy}. \quad (28)$$

Inserting the corresponding specific intensity $I = S/\Omega_s$ into Eq. (29) then gives for the distance independent brightness temperature

$$T_b \approx 5 \left( g_{\gamma} 10^{14} \text{ GeV} \right)^2 \left( \frac{m_a}{\mu \text{eV}} \right)^{-4} \left( \frac{10^{-3}}{\Delta} \right) \left( \frac{M_a}{10^{-10} M_\odot} \right) \left( \frac{r_s}{\text{pc}} \right)^{-2} \left( \frac{B}{10^{-3} \text{ G}} \right)^2 f(m_a) \text{ nK}. \quad (29)$$

The Crab nebula at a distance $d \approx 2 \text{kpc}$ has a radius $r_s \approx 2 \text{pc}$ and a field $B \approx 10^{-3} \text{ G}$. This implies $M_a \approx 0.3 M_\odot$. Inserting this into the above formulae yields

$$S \approx 2.1 \times 10^{-8} \left( g_{\gamma} 10^{14} \text{ GeV} \right)^2 \left( \frac{m_a}{\mu \text{eV}} \right)^{-2} \left( \frac{10^{-3}}{\Delta} \right) \left( \frac{d}{2 \text{kpc}} \right)^{-2} \left( \frac{B}{10^{-3} \text{ G}} \right)^2 f(m_a) \text{ Jy},$$

$$T_b \approx 3.8 \left( g_{\gamma} 10^{14} \text{ GeV} \right)^2 \left( \frac{m_a}{\mu \text{eV}} \right)^{-4} \left( \frac{10^{-3}}{\Delta} \right) \left( \frac{B}{10^{-3} \text{ G}} \right)^2 f(m_a) \text{ \mu K}. \quad (30)$$

Unfortunately, the Crab nebula is very bright at radio frequencies, of the order of $10^3 \text{ Jy}$, corresponding to a brightness temperature of $\approx 10^5 \text{ K}$. This would make it very difficult to extract this small line signal from this large astrophysical foreground, unless $g_{\gamma \gamma} \gtrsim 10^{-9} \text{ GeV}^{-1}$ even if $f(m_a)$ is not much smaller than one. On the other hand, Wolf-Rayet stars can produce stellar winds with parameters similar to the Crab nebula, and are also thought to accelerate high energy cosmic rays [23, 24]. In more detail, in an expanding and rotating stellar wind conservation of angular momentum can produce stellar winds with parameters similar to the Crab nebula, and are also thought to accelerate high energy cosmic rays [23, 24]. In more detail, in an expanding and rotating stellar wind conservation of angular momentum leads to magnetic breaking of the plasma which in turn can cause the coherent magnetic field $B_e$ to obtain the topology of a Parker spiral for which $B_e(r) \propto \text{const.}$ If the turbulent magnetic field component is comparable in strength, then $B(r) \propto B(r_s)$ for $r \approx r_s$. For a constant ALP density so that $M_a(r_s) = 4\pi \rho_a r_s^2/3$. Here, $r_s$ can be identified with the radius of the wind termination shock and can be estimated by equating the mass swept up from the interstellar medium with the ejected mass $M_e$ which gives

$$r_s \approx \left( \frac{3 M_e}{4 \pi m_N n_0} \right)^{1/3} \approx 2.1 \left( \frac{M_e}{M_\odot} \right)^{1/3} \left( \frac{1 \text{ cm}^{-3}}{n_0} \right)^{1/3} \text{pc}, \quad (31)$$

where $n_0$ is the baryon number density of the interstellar medium and $m_N$ the nucleon mass. With $B(r) \lesssim 10^{16} \text{ G cm}$ this gives $B(r_s) \lesssim 2 \times 10^{-3} \text{ G}$, and thus similar to the Crab nebula case. Another way to estimate the magnetic field is by assuming rough equipartition between the kinetic wind and the magnetic field energies which gives

$$B(r_s) \approx \left( \frac{3 \mu_0 M_e}{4 \pi r_s^2} \right)^{1/2} v_w \approx (\mu_0 m_N n_0)^{1/2} v_w \approx 1.9 \times 10^{-3} \left( \frac{n_0}{1 \text{ cm}^{-3}} \right)^{1/2} \left( \frac{v_w}{10^{-2} \text{ G}} \right), \quad (32)$$

where $v_w$ is the wind velocity and we have used Eq. (31) in the second step. This gives thus values very similar to the numbers above. Stellar winds from Wolf-Rayet stars have been observed with flux densities below 0.1 Jy at $d \approx 1 \text{kpc}$, corresponding to brightness temperatures of order a few degrees, and thus have much lower electron acceleration efficiency than Crab type supernova remnants [24]. This would considerably simplify the search for a line signal.

We also note that galaxy clusters predict numbers similar to Eq. (30) but since coherence scales are likely larger than parsecs, the suppression factor $f(m_a)$ is likely to be dramatic again.

Let us now try to estimate the suppression factor $f(m_a)$ for such compact objects in a bit more detail. The magnetic field coherence length in such objects is likely much smaller than for the Galactic magnetic fields. The Weibel instability may produce magnetic fields on length scales of the Debye length. If one component of the medium is given by accelerated relativistic electrons and the other by free electrons of density $n_e$ at temperature $T_e$ it is given by

$$\lambda_D = \frac{\bar{v}}{\omega_{pl}} \approx \left( \frac{e \bar{v}_e}{e^2 n_e} \right)^{1/2} \approx 6.9 \times 10^3 \left( \frac{T_e}{10^6 \text{ K}} \right)^{1/2} \left( \frac{\text{cm}^{-3}}{n_e} \right)^{1/2} \text{ cm}, \quad (33)$$
with \( \bar{v} \) the thermal velocity. The Bell instability can amplify magnetic fields on the scale of the gyro radius of cosmic rays of momentum \( p \) and charge \( Z \) which is given by

\[
rg \sim 3 \times 10^9 \left( \frac{10^{-3} G}{B} \right) \left( \frac{p/Z}{\text{GeV}} \right) \text{ cm},
\]

(34)

In any case, if cosmic rays are accelerated in such objects significant magnetic power on such scales is required to enable diffusive acceleration. The scales in Eqs. (33) and (34) are larger than the scale Eq. (19) only by a few orders of magnitude so that the suppression factor \( f(m_a) \) may be moderate.

We also note that for \( \nu \gtrsim 1 \text{ MHz} \) free-free absorption is generally negligible within the Galaxy, as can be seen from the absorption rate which in the Rayleigh-Jeans regime \( \nu \lesssim k_B T_e / h = 2.1 \times 10^{13} (T_e / 10^3 \text{ K}) \text{ Hz} \) is given by

\[
a_{\nu}^{\text{ff}} \simeq 9 \times 10^{-4} \left( \frac{n_e}{0.1 \text{ cm}^{-3}} \right)^2 \left( \frac{10^3 \text{ K}}{T_e} \right)^{3/2} \left( \frac{\text{MHz}}{\nu} \right)^2 \text{ pc}^{-1},
\]

(35)

where \( T_e \gtrsim 10^3 \text{ K} \) is the temperature of the medium.

### IV. DETECTABILITY

The effective solid angle of a single Gaussian beam is given by

\[
\Omega_b \simeq \theta^2 \simeq \frac{1}{(l \nu)^2} = \frac{1}{A \nu^2},
\]

(36)

where \( \theta \) is the angular radius of the beam, \( l \) is the effective length scale of the interferometer and \( A = l^2 \) its effective area. If a discrete source extends over several beams, the sensitivity in brightness temperature is increased by a factor \( N_{1/2} b = (\Omega_s / \Omega_b)^{1/2} \) relative to a single beam so that the minimal detectable brightness temperature is given by

\[
T_{b,\text{min}} \simeq \frac{T_{b,\text{min}0}}{N_{1/2} b} = T_{b,\text{min}0} \left( \frac{\Omega_b}{\Omega_s} \right)^{1/2},
\]

(37)

where \( T_{b,\text{min}0} \) is the sensitivity for a single beam. In general one has

\[
T_{b,\text{min}0} \simeq \frac{T_{\text{noise}}}{(B \nu)^{1/2}},
\]

(38)

where \( T_{\text{noise}} \) is the effective noise temperature, resulting from system and sky temperature added in quadrature, \( B \) is the bandwidth and \( \nu \) is the observing time. One also often uses the antenna temperature induced by a total flux density \( S \) defined by

\[
T_a = \frac{AS}{2} = 0.36 \left( \frac{A}{10^3 \text{ m}^2} \right) \left( \frac{S}{\text{Jy}} \right) \text{ K}.
\]

(39)

Combining this with Eqs. (25) and (30) and the relation \( S = I \Omega_s \) this shows that

\[
\frac{T_a}{T_b} = N_b = \frac{\Omega_s}{\Omega_b}.
\]

(40)

If the noise in one beam is again characterized by the temperature \( T_{b,\text{min}0} \), the noise in \( N_b \) beams corresponds to \( N_{1/2} b T_{b,\text{min}0} \). Comparing this with the total signal temperature \( T_a \) again gives a brightness temperature sensitivity improvement by a factor \( N_{1/2} b \). Equivalently, the smallest detectable total source flux density can be expressed as

\[
S_{\text{min}} = N_{1/2} b S_b,
\]

(41)

where \( S_b \) is the minimal detectable flux density per beam. Since \( S_{\text{min}} \) and \( S_b \) are proportional to \( T_{b,\text{min}0} \) which according to Eq. (38) is proportional to \( 1/\nu^{1/2} \), one often denotes the minimal detectable source flux density in units of Jy hr\(^{-1/2} \).
Let us now apply these estimates to various relevant experiments. LOFAR HBA \cite{25} has a beam size of $\sim 5$ arcsec which thus covers a solid angle $\Omega_b \simeq 2 \times 10^{-9}$ sr, at $\nu = 140$ MHz, corresponding to $m_a = 0.58$ $\mu$eV, see Eq. (19). For the stellar nebulae of radial extent $r_s \simeq 2$ pc at a distance $d$ discussed above this could increase the sensitivity by a factor $N_b^{1/2} \simeq 41 (2\text{kpc}/d)$. For a sensitivity of $S_b \sim 10^{-4}$ Jy per beam Eqs. (20) and (21) then predict a sensitivity of $T_b \simeq 2[d/(2\text{kpc})]$ K. Expressed in terms of total source flux density this corresponds to $S_{\text{min}} \simeq 4 \times 10^{-3} (2\text{kpc}/d)$ Jy according to Eq. (11). Comparing this with the prediction Eq. (30) suggests that couplings $g_{\alpha\gamma} \gtrsim 2.5 \times 10^{-12} [m_a/(0.58 \mu\text{eV})][d/(2\text{kpc})]^{1/2} \text{GeV}^{-1} / f(m_a)$ may be testable for $m_a \simeq \mu$eV.

The planned SKA-low is sensitive in the frequency range between 50 and 350 MHz. It has a beam size of order of a square degree, $\Omega_b \simeq 3 \times 10^{-4}$ sr, which is typically larger than the angular size of the sources we have discussed here so that $N_b = 1$. The source flux density sensitivities are of order $10 \mu$Jy hr$^{-1/2}$. According to Eq. (20) in terms of brightness temperature this corresponds to $\simeq 10 \mu\text{K}$ hr$^{-1/2}$, see, e.g., Ref. \cite{26}. This can also be seen from Eq. (28) for $T_{\text{noise}} \simeq 10 \text{K}$ and $B \simeq 300$ MHz. Comparing this with the prediction Eq. (30) for $S$ this translates to possible sensitivities down to $g_{\alpha\gamma} \gtrsim 2 \times 10^{-13} [m_a/\mu\text{eV}] [d/(2\text{kpc})]^{1/2} \text{GeV}^{-1} / f(m_a)$ within about an hour of observing time. The planned SKA-mid is sensitive in the frequency range between 0.35 and 14 GHz and the sensitivity in terms of flux densities is about a factor 5 lower. On the other hand the predicted $S \propto (g_{\alpha\gamma}/m_a)^2$ so that constraints on $g_{\alpha\gamma}$ degrade by factors of a few. Note that according to Eqs. (2) and (5) for the QCD axion one has $g_{\alpha\gamma} \simeq 5 \times 10^{-15} (m_a/\mu\text{eV}) \text{GeV}^{-1}$.

Up to now we have quoted sensitivities for continuum emission. However, the predicted emission is line-like with a relative width of $\Delta \sim 10^{-3}$, see Eq. (10). This implies that the optimal effective bandwidth has to be set to

$$B \simeq \Delta \nu = 242 \left( \frac{m_a}{\mu\text{eV}} \right) \left( \frac{\Delta}{10^{-3}} \right) \text{kHz}. \quad (42)$$

This leads to sensitivities that are about a factor $\simeq 30(10^{-3}/\Delta)^{1/2}$ worse than the numbers quoted above and thus to limits on $g_{\alpha\gamma}$ that are degraded by a factor $\simeq 6(10^{-3}/\Delta)^{1/4}$, to $g_{\alpha\gamma} \gtrsim 10^{-12} [m_a/\mu\text{eV}] [d/(2\text{kpc})]^{1/2} \text{GeV}^{-1} / f(m_a)$ for SKA. On the other hand, since the signal at a given frequency or ALP mass scales as $1/\Delta$, the signal to noise ratio scales as $1/\Delta^{1/2}$. Overall the sensitivity to $g_{\alpha\gamma}$ scales as $1/\Delta^{1/4}$ and increases with observing time as $t^{1/4}$. Longer observing times and optimised broadband searches may thus increase the sensitivity.

V. OUTLOOK AND CONCLUSIONS

Under optimistic assumptions for the magnetic field power spectrum at meter scales we have shown that strongly magnetized stellar winds may probe ALP-photon coupling parameters below current upper limits $g_{\alpha\gamma} \lesssim 10^{-10}$ $\text{GeV}^{-1}$ in the ALP mass range around $10^{-6}$ eV through radio emissions that may be detectable with existing radio telescopes such as LOFAR HBA and with future experiments such as SKA. These experiments together cover frequencies between $\simeq 10$ MHz and $\simeq 15$ GHz, corresponding to ALP mass $0.1 \mu\text{eV} \lesssim m_a \lesssim 100 \mu\text{eV}$. Around $m_a \sim \mu\text{eV}$ observations of such discrete astrophysical objects should be sensitive to ALP-photon couplings $g_{\alpha\gamma} \gtrsim 10^{-12} [m_a/\mu\text{eV}] [d/(2\text{kpc})]^{1/2} \text{GeV}^{-1} / f(m_a)$ where $f(m_a) < 1$ describes the fraction of the magnetic field power on scales $k \simeq m_a$ which for turbulent spectra can be approximated by $f(m_a) \simeq (m_a l_c)^n$ with $n < 0$ and $l_c$ the coherence scale. The strongest constraints thus tend to be given by the most nearby sources. Furthermore, longer observation times and dedicated analysis methods may increase the sensitivity so that sensitivities down to $g_{\alpha\gamma} \simeq 10^{-13}$ $\text{GeV}^{-1}$ may be reachable. If laboratory haloscopes would find indications for the existence of ALP dark matter and fix its mass $m_a$ and coupling scale $g_{\alpha\gamma}$ the observation of a photon line in an astrophysical haloscope would in turn allow to derive characteristics of the magnetic field in the corresponding astrophysical object, in particular the power $\rho_{m}(m_a)$ around wavenumber $m_a$, or the combination $B^2 f(m_a)$.

ACKNOWLEDGMENTS

This work has been supported by the Deutsche Forschungsgemeinschaft through the Collaborative Research Center SFB 676 “Particles, Strings and the Early Univers”, and by the Helmholtz Alliance for Astroparticle Physics (HAP) funded by the Initiative and Networking Fund of the Helmholtz Association. We are grateful to Marcus Brüggen for
useful comments on the manuscript.

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