Extreme wave height analysis using Weibull and Rayleigh distribution in some coastal areas affected by tsunami disasters and earthquakes in September 2018 at Central Sulawesi

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Abstract. One of the most important measures is the availability of data and information through research related to the analysis of extreme wave heights at each location to be built. This study aims to provide methods and information in determining extreme wave heights that have been required to be known when designing coastal structures and public infrastructure on the coast. In this study, the extreme wave height was calculated based on a wave model simulation with an incident wave scenario that has an extreme wave height at each research location. The results showed that the distribution of extreme wave height in the waters of Palu Bay, the coast of Banawa District, was closer to the Rayleigh distribution, while the waters of Sirenja District, Donggala Regency were more in accordance with the Weibull distribution. This difference is caused by the attenuation factor of wave height due to differences in coastal morphology. The attenuation value of wave height at Banawa beach reaches 53 - 78%, while in Sirenja District it is only 35 - 46% compared to the extreme wave height in the deep sea.

1. Introduction
The distribution of wave height based on coastal conditions is very important in estimating the extreme wave heights that are likely to threaten the coast [1]. The form of the wave threat originates from the simultaneous effect of sea level and waves that occur simultaneously. The dominant sea level parameter is influenced by tidal conditions, while one of the important parameters of wave threat is the extreme wave height generated by various generating sources [2]. [3] stated that in fact, in certain situations, there are always waves that are higher than significant waves which are called extreme waves. The probability of an extreme wave occurring is relatively small so that the analysis of wave height using only field measurement methods or numerical methods will not be able to analyze the extreme wave height [4].

Extreme wave height should be the basis for the analysis to determine the height of coastal structures or embankments in coastal reclamation to prevent overtopping waves in residential areas and public facilities. This extreme wave height analysis requires a high degree of accuracy so that it has an impact on good and cost-effective designs [5]. This study combines the results of numerical hydrodynamic modeling with the modified Weibull and Rayleigh distributions to determine extreme wave heights at the coast. The results of the numerical model prior to statistical analysis were verified by field measurement data.
The numerical model used in this study is the SWAN model [6]. This model has often been used to determine frequency domain wave parameters in shallow waters. The distribution of the modeled wave height that has been verified with field data is then analyzed using statistical methods (Weibull and Rayleigh distribution) to determine a reliable distribution of wave height and extreme wave height.

In addition to using a numerical model to calculate wave height and period, this study calculates the extreme wave height by applying the modified Weibull and Rayleigh contributions. The use of the modified Rayleigh Distribution to determine extreme waves has been implemented on the east coast of the Brazilian Sea which is one of the beaches in the southern part of the Atlantic Ocean. The calculation results show that the ratio between the maximum wave height ($H_{\text{max}}$) and the significant wave height ($H_s$) is $\frac{H_{\text{max}}}{H_s} = \left[\ln(N)/2\right]^{1/2} = 2.2$ times. However, this value becomes divergent if the wave value becomes large and the empirical relationship for large waves changes to $\frac{H_{\text{max}}}{H_s} = \left[\ln(N^2)/2\right]^{1/4}$ [1].

This research will provide an accurate calculation of extreme wave height, especially in several coastal locations that were damaged by the Tsunami and Earthquake in Central Sulawesi on September 28, 2018. Some of the damaged coastal locations were roads, settlements had land subsidence (down lift) and other public infrastructure facilities. The results of this study will be very useful because they provide accurate data and information about extreme wave heights to produce a good and cost-effective repair design. The problems that are the focus of this research can be formulated as follows in two important parts, namely, how to model significant wave heights ($H_{1/3}$) based on the results of a verified numerical model with field data in each measurement location and how to determine wave height at the research site. Based on these problems, the aim of this study is to determine the height of extreme waves in coastal areas, especially those that experienced a high level of damage during the September 2018 earthquake in Central Sulawesi.

2. Materials and methods

2.1 Wave modeling

The basic assumptions based on mathematical logic about the probability density function of the wave height are based on the Rayleigh function which is stated as follows [3][4]:

$$f(h) = \frac{2h}{a^2} \cdot \exp \left( -\left( \frac{h}{a}\right)^2 \right)$$

(1)

Where a is the scale parameter ($a, h > 0$).

Due to different physical-morphological conditions, the Longuet-Higgins basic postulate may not always be suitable for ocean wave conditions. There is a need for a model that can fulfill two objectives, namely (a) accommodating the Rayleigh distribution under suitable conditions and (b) adjusting the data under more general conditions. The conditions can be met if we apply the Weibull probability density function to certain conditions [3]:

$$f(h) = \frac{b}{a} \left( \frac{h}{a} \right)^{b-1} \cdot \exp \left( -\left( \frac{h}{a}\right)^b \right)$$

(2)

Where a is the scale parameter and b the shape parameter ($a, b, h > 0$).

The Rayleigh model is a special feature of the Weibull distribution for its shape parameter $b = 2$. Another reason for considering the Weibull distribution for simulating extreme ocean wave heights is offered in terms of the intensity function [3]. In any case, it is important to note that the concept of Weibull's law does not contradict the Longuet-Higgins assumption by simply adding to it by expanding to more general conditions.
2.2 Determining of Significant Wave Height

If \( h_1, h_2, h_3, \ldots, h_n \) are wave sequence form based on its wave height, then \( h_p \) is the mean value calculated from the \( pN \) value where \( 0 \leq p \leq 1 \). The formula for calculating the \( h_p \) value is derived so that the significant wave height and the average wave height can be determined [3].

\[
1 - F(h) = \exp \left( \frac{h}{\alpha} + \alpha \right)^b \tag{3}
\]

Because this is a theoretical equation of \( p \), \( h = a \left( (\alpha^b - \ln(p))^\frac{1}{b} - \alpha \right) \)

The average \( h_p \) value of \( H \) values greater than \( h \) is:

\[
h_p = E \left( \frac{h}{H > h} \right) = E \left( \frac{h-h}{H > h} \right) + h \tag{4}
\]

Where \( E \) is the expectations operator, thus obtained:

\[
h_p = a \left( (\alpha^b - \ln(p))^\frac{1}{b} - \alpha \right) + p^{-1}a^{-1} \exp(\alpha^b). I_a^{\alpha^b-\ln(p)}(b^{-1}) \tag{5}
\]

where:

\[
I_t(p) = \int_t^{\infty} \exp(-x) . x^{p-1} dx
\]

is an incomplete Gamma function.

Significant wave height is expressed by \( p = 1/3 \) and the average wave height is \( 1/10 \) of the highest wave height with \( p = 1/10 \). The mean value of all waves is given when \( p = 1 \) [3].

2.3 Determine the Average Maximum Wave Height \( \bar{H}_{\text{max}} \)

The probability of the modified Weibull model density function for the distribution of the maximum wave height is expressed by [3]:

\[
f_1(h) = \frac{a \sigma(h)}{dh}
\]

Then the mean extreme wave height for the Weibull distribution model is:

\[
\bar{H}_{\text{max}} = a . b^{-1} . \Gamma \left( \frac{1}{b} \right) \cdot \left[ n \left( 1 - \alpha^b \right) - \frac{n-1}{2} \right] . \Gamma \left( \frac{1}{b} \right) \cdot \left( 1 - \alpha^b \right) + \cdots + \frac{(n-1)^{(r+1)}}{n!} \cdot \left( 1 - n . \alpha^b \right) \tag{6}
\]

where \( \Gamma(p) = \int_0^{\infty} \exp(-h) . h^{b-1} . dh \) are Gamma Functions and! is a factorial symbol.

2.4 Calculating Frequent Extreme Wave Height

Determining the height of the extreme wave height \( (H_{mfm}) \) is the mode of the \( G(h) \) distribution. This is the solution of the equation \( [df_1(h)/dh] = 0 \) which is represented by [3]:

\[
H_{mfm} = a \left[ \frac{A^b \sqrt{A^2 - 4(A - b \alpha^b + a \alpha^b + 1) a b(b-1)}}{2(A - b \alpha^b + a \alpha^b + 1)} \right] - \alpha \tag{7}
\]

where \( A = nb \alpha^b + b \alpha^b + nb - \alpha^b - 1 \)

If \( H_1, H_2, \ldots, H_N \) are the maximum daily wave height of a number of \( N \) sample waves then the probability of the extreme wave height not exceeding \( h_L \) is:
\[ G(H_L) = \left(1 - \exp\left(a^b - \left(\frac{h_L}{a} + \alpha\right)^b\right)\right)^N \]  

(8) 

Therefore the probability that the wave height is greater than the \(h_L\) value is expressed by:

\[ 1 - G(h_L) \]

Based on the observations that have been made on the daily extreme wave height, if it is obtained that the \(k\)-\(k\) data that exceeds the \(h_L\) wave height, the Geometric Law becomes:

\[(1 - G).G^{k-1}, k = 1, 2, 3, ...\]  

(9)

The mean values of \(k\) are:

\[ E(k) = (1 - G)^{-1} \]

The mean number of observations included between two adjacent wave heights in excess of \(h\) is \(E(k)\). The number of observed waves \(E(k)\) will equal the number of days that lie between the appearance of the maximum wave height exceeding \(h_L\).

\[ h_L = a \left[ \frac{\alpha^b - \ln\left(1 - \left(1 - \frac{1}{R_p}\right)^{\frac{1}{N}}\right)^\frac{1}{b}}{\alpha} \right] \]  

(10)

So the value of \(E(k)\) represents the period \(R_p\) where the extreme wave heights are being observed.

**Figure 1.** Bathymetry and measurement station for Makassar strait waters, Tompe district, Donggala regency

**Figure 2.** Bathymetry and measurement station of Palu bay waters, Banawa district, Donggala regency
2.5 Research sites
The research location (Figure (1)) is a location close to the epicenter of the September 28 2018 earthquake in Central Sulawesi. To determine the differences in wave characteristics, this study was applied to several stations as shown in Figure (1), representing the west side of Palu Bay and Figure (2), representing the eastern side of Palu Bay. The Research Flowchart can be seen in Figure 3.

![Research Flowchart](image)

Figure 3. Research flowchart.

3. Results and Discussion
3.1 Stationary wave height
The measurement of wave height in Makassar strait waters using the SWAN model consists of 5 (five) stations grouped based on wave height data (x coordinates) and frequency of occurrence (y-axis). These waves are generally a type of swell wave where the generating force originates away from the measuring station. The wave characteristics measured in Makassar strait waters tend to be the same, where the frequency of occurrence of waves with a wave height above 50 cm is very dominant. Figure 4 is one of the stationary wave characteristics at station 1 where the wave height reaching 70 cm occurs up to 8% of the total measured wave height.

The above data is then grouped by height to determine the scale parameter (a) and the shape parameter (b). The distribution of wave height at station 1 based on the frequency of occurrence of wave height is as follows:
These characteristics tend to be the same for all stations (station 1 to station 5) in the region. The impact of the September 28, 2018, earthquake and tsunami caused damage to road infrastructure and settlements along Palu Bay. One of the locations that suffered damage to both facilities and infrastructure was Banawa District, a coastal area of the capital city of Donggala Regency. Analysis of coastal conditions for the sake of rehabilitation of facilities and infrastructure was carried out at these locations as in Figure 2 above. It can be seen in the figure that the west coast bathymetry tends to be sloping compared to the eastern coast. One of the stationary wave height measurement stations in the waters of Palu Bay, Banawa District (Station 8), can be seen in Figure 6 and the results of wave height grouping to determine the stationary wave height distribution pattern at station 8 are shown in Figure 7.
Wave height parameters

Based on the wave data obtained at each measuring station, the Weibull distribution constant and the Rayleigh distribution can be calculated. The results of the calculation of scale parameters (a) and form parameters (b) for each distribution are shown in Table 1.

| Station | Weibull | Rayleigh |
|---------|---------|----------|
|         | Scale Parameters (a) | Parameter form (b) | Scale Parameters (a*) |
| 1       | 0.92    | 1.21     | 0.96      |
| 2       | 0.91    | 1.14     | 1.88      |
| 3       | 0.92    | 1.35     | 0.95      |
| 4       | 0.97    | 1.51     | 0.99      |
| 5       | 0.78    | 1.53     | 0.79      |
| 6       | 2.9     | 2.03     | 2.9       |
| 7       | 2.11    | 2.06     | 2.11      |
| 8       | 2.11    | 1.96     | 2.12      |
| 9       | 2.19    | 1.67     | 2.21      |
| 10      | 2.23    | 1.73     | 2.24      |

From Table 1, it can be seen that the difference in the Scale Parameter value between the Weibull and Rayleigh Methods. This difference occurs at Station 1 to Station 5 where the station has a high frequency wave.

The scale parameters of the Weibull Method and the Rayleigh Method have the same value when analyzing the wave height in the waters of Palu Bay, especially in Banawa District. At Station 6 and Station 7 the scale parameter (a) is the same and the shape parameter (b) of the Weibull method is close to number 2. Of course, this condition shows that the two wave data generated at Station 6, Station 7 and Station 8 follow the Rayleigh distribution. This condition does not occur at station 9 and station 10 where the scale parameter (a) of both methods is the same, but the shape parameter (b) does not approach number 2.

Significant wave height

To get the value of the extreme wave height of a water location, it begins with the calculation of its significant wave height. Based on the results of numerical modeling, the significant wave heights at several stations are as follows (Table 2):

The assumption in the numerical wave model used is that the wave height originates at 240° from the North as the highest fetch direction at that location. The swell wave height is 1.5 meters entering the...
beach in the model area and it turns out that the significant wave height at the beach is 0.8 meters to 0.98 meters. The highest significant wave occurred at Station 1 of 0.98 meters as in Figure 8:

Table 2. Significant wave heights of Sirenja District in Makassar Strait Waters

| Station | Significant Wave Height (meters) |
|---------|----------------------------------|
| 1       | 0.98                             |
| 2       | 0.8                              |
| 3       | 0.97                             |
| 4       | 0.94                             |
| 5       | 0.94                             |

Figure 8. The highest significant wave height in Sirenja District, Makassar strait waters

This significant variation in wave height is caused by the morphology of the coast and also the bathymetry of the waters. The significant wave height at the station on the coast of Palu Bay, Banawa is relatively smaller, around 0.4 meters to 0.7 meters. The model assumption used is the same as the previous assumption, namely that the swell wave height used at the open boundary is 1.5 meters. As a result of attenuation by bathymetry and other coastal morphology, the wave height that reaches the Banawa beach (station 6 to station 10) is as in Table 3:

Table 3. Significant wave heights of Banawa beach in Palu Bay waters

| Station | Significant Wave Height (meters) |
|---------|----------------------------------|
| 6       | 0.68                             |
| 7       | 0.42                             |
| 8       | 0.71                             |
| 9       | 0.6                              |
| 10      | 0.45                             |

The highest significant wave in the waters of the research location occurred at Station 8, which was 0.71 meter.

Figure 9. Significant wave height at Station 8, Banawa coast, Palu Bay
3.4 Extreme wave height

The extreme wave height is determined based on the value of the significant wave height obtained from the simulation model using SWAN. The equation used to calculate the $H_{max}$ value is as follows:

$$H_{max}/H_s = \left[\ln(N)/2\right]^{1/2}$$

If the value of $N = 150$ waves corresponds to the number of stationary waves that are measured successively, then $H_{max}/H_s = 1,583$. The results of the calculation of the data equivalence of Significant Wave Height to the maximum Wave Height are as shown in Table 4.

| Station | Significant Wave Height (m) | Extreme Wave Height (m) |
|---------|-----------------------------|-------------------------|
| 1       | 0.98                        | 1.55                    |
| 2       | 0.8                         | 1.27                    |
| 3       | 0.97                        | 1.54                    |
| 4       | 0.94                        | 1.49                    |
| 5       | 0.94                        | 1.49                    |
| 6       | 0.68                        | 1.08                    |
| 7       | 0.42                        | 0.66                    |
| 8       | 0.71                        | 1.12                    |
| 9       | 0.6                         | 0.95                    |
| 10      | 0.45                        | 0.71                    |

With this extreme wave height value, it can be used as information when rehabilitating damaged facilities and infrastructure during the September 2018 earthquake and tsunami.

4. Conclusion

Based on the results of statistical analysis, the height of extreme waves in the waters of Palu Bay, Banawa District is lower than the wave height in the waters of Sirenja District, Donggala Regency. The wave height in Banawa coastal waters experiences greater attenuation than the waters of Sirenja District. The 1.5-meter extreme wave height used as the incident wave height in this simulation has an attenuation of 53 - 78% for the waters of Palu Bay, the coast of Banawa District. The attenuation of extreme wave height in the waters of the Makassar Strait, the coast of Tompe District, is only 35 - 46%. Significant wave height at each station for the simulation conditions in the extreme wave height scenario, the value is about 1 meter for the waters in Sirenja District and 0.7 meters for the maximum value for waters on the Banawa coast.

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