**CP Violation in the Decay $B \to X_d e^+e^-$**

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**ABSTRACT**

The decay $b \to d e^+e^-$ has an amplitude containing comparable contributions proportional to $V_{tb}V_{td}^*$, $V_{cb}V_{cd}^*$ and $V_{ub}V_{ud}^*$. These pieces involve different unitarity phases produced by $c\bar{c}$ and $u\bar{u}$ loops. The simultaneous presence of different CKM phases and different dynamical phases leads to a calculable asymmetry in the partial widths of $b \to d e^+e^-$ and $\bar{b} \to \bar{d} e^+e^-$. Using the effective Hamiltonian of the standard model, we calculate this asymmetry as a function of the $e^+e^-$ invariant mass. The effects of $\rho$, $\omega$ and $J/\psi$ resonances are taken into account in the vacuum polarization of the $u\bar{u}$ and $c\bar{c}$ currents. As a typical result, an asymmetry of $-5\%$ ($-2\%$) is predicted in the nonresonant domain $1 \text{ GeV} < m_{e^+e^-} < m_{J/\psi}$, assuming $\eta = 0.34$ and $\rho = 0.3 \ (-0.3)$. The branching ratio in this domain is $1.2 \times 10^{-7} \ (3.3 \times 10^{-7})$. Results are also obtained in the region of the $J/\psi$ resonance, where an asymmetry of $3 \times 10^{-3}$ is expected, subject to certain theoretical uncertainties in the $b \to dJ/\psi$ amplitude.

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I. INTRODUCTION

The decays $B \to X_{s,d}l^+l^-$ are important probes of the effective Hamiltonian governing the flavour-changing neutral current transition $b \to s(d)l^+l^-$ [1]. The matrix element contains a term describing the virtual effects of the top quark proportional to $V_{tb}V_{ts}^*$, $q = s,d,$ and in addition terms induced by $c\bar{c}$ and $u\bar{u}$ loops, proportional to $V_{cb}V_{cs}^*$ and $V_{ub}V_{us}^*$. In the case of the decay $b \to s l^+l^-$, the relevant CKM factors have the order of magnitude $V_{tb}V_{ts}^* \sim \lambda^3$, $V_{cb}V_{cs}^* \sim \lambda^3$, $V_{ub}V_{us}^* \sim \lambda^5$, where $\lambda = \sin \theta_C \simeq 0.221$. This has the consequence that the $u\bar{u}$ contribution is very small, and the unitarity relation for the CKM factors reduces approximately to $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* \approx 0$. Thus the effective Hamiltonian for $b \to s l^+l^-$ essentially involves only one independent CKM factor $V_{tb}V_{ts}^*$, so that $CP$ violation in this channel is strongly suppressed, within the standard model [2, 3].

The situation is quite different for the transition $b \to d l^+l^-$. The internal top-quark contribution is proportional to $V_{tb}V_{td}^*$, while the terms related to $c\bar{c}$ and $u\bar{u}$ loops are proportional to $V_{cb}V_{cd}^*$ and $V_{ub}V_{ud}^*$. All of these CKM factors are of order $\lambda^4$, and, a priori, can have quite different phases. In addition, the $c\bar{c}$ and $u\bar{u}$ loop contributions are accompanied by different unitarity phases corresponding to real intermediate states. We thus have a situation in which the amplitude contains pieces with different CKM phases as well as different dynamical (unitarity) phases. These are precisely the desiderata for observing $CP$-violating asymmetries in partial rates. The purpose of this paper is to derive quantitative predictions for the $CP$-violating partial width asymmetry between the channels $b \to d e^+e^-$ and $\bar{b} \to \bar{d} e^+e^-$.  

II. THE EFFECTIVE HAMILTONIAN FOR $b \to d l^+l^-$

The effective Hamiltonian for the decay $b \to d l^+l^-$ in the standard model can be written as

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{td}^* \left\{ \sum_{i=1}^{10} c_i(\mu)\mathcal{O}_i(\mu) \right. \\
- \lambda_u \left[ c_1(\mu)\left(\mathcal{O}_1^u(\mu) - \mathcal{O}_1(\mu)\right) + c_2(\mu)\left(\mathcal{O}_2^u(\mu) - \mathcal{O}_2(\mu)\right) \right] \right\}, \quad (2.1)$$
where we have used the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix
\[ V_{tb}V_{td}^* + V_{ub}V_{ud}^* = -V_{cb}V_{cd}^*, \]
and \( \lambda_u \equiv V_{ub}V_{ud}^*/V_{tb}V_{td}^* \). For the purpose of this paper it is convenient to use the Wolfenstein representation \([4]\) of the CKM matrix with four real parameters \( \lambda = \sin \theta_C \approx 0.221, A, \rho, \) and \( \eta \), where \( \eta \) is a measure of CP violation. In terms of these parameters
\[ \lambda_u = \rho(1 - \rho) - \eta^2 \left( \frac{1}{(1 - \rho)^2 + \eta^2} - i \frac{\eta}{(1 - \rho)^2 + \eta^2} \right) + \cdots, \]  
(2.2)
where the ellipsis denotes higher-order terms in \( \lambda \). Furthermore, we will make use of
\[ \frac{|V_{tb}V_{td}|^2}{|V_{cb}|^2} = \lambda^2 \left( (1 - \rho)^2 + \eta^2 \right) + O(\lambda^4). \]  
(2.3)
The operator basis \( \{O_i\} \) for \( H_{\text{eff}} \) is given in Refs. \([5,6]\) with the obvious replacement \( s \rightarrow d \), and the additional operators \( O_{1,2}^u \) read
\[ O_1^u = (\bar{d}_\alpha \gamma_\mu P_L u_\beta)(\bar{b}_\beta \gamma^\mu P_L b_\alpha), \quad O_2^u = (\bar{d}_\alpha \gamma_\mu P_L u_\alpha)(\bar{b}_\beta \gamma^\mu P_L b_\beta), \]  
(2.4)
with \( P_{L,R} = (1 \mp \gamma_5)/2 \). The evolution of the Wilson coefficients \( c_i(\mu) \) in Eq. (2.1) from the scale \( \mu = m_W \) down to \( \mu = m_b \) by means of the renormalization group equation has been discussed in several papers, and we refer the reader to the review article by Buchalla et al. \([7]\). The resulting QCD-corrected matrix element can be written as
\[ \mathcal{M} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{td}^* \frac{\alpha}{4\pi} \left\{ c_9^\text{eff} (\bar{d} \gamma_\mu P_L b) \bar{b} \gamma^\mu l + c_{10}^\text{eff} (\bar{d} \gamma_\mu P_L b) \bar{b} \gamma^\mu \gamma^5 l \right. \]
\[ \left. - 2 c_7^\text{eff} \bar{d} i \sigma_{\mu \nu} \frac{q^\nu}{q^2} (m_b P_R + m_d P_L) b \bar{b} \gamma^\mu l \right\}. \]  
(2.5)
Neglecting terms of \( O(m_q^2/m_W^2) \), \( q = u, d, c \), the analytic expressions for all Wilson coefficients, except \( c_9^\text{eff} \), are the same as in the \( b \rightarrow s \) analogue, and can be found in Refs. \([3,4]\). Using the parameters given in Appendix \[A\], we obtain in leading logarithmic approximation
\[ c_7^\text{eff} = -0.315, \quad c_{10} = -4.642, \]  
(2.6)
and in next-to-leading approximation

\[
\begin{align*}
    c_{9}^{\text{eff}} &= c_9 + 0.124 \omega(\hat{s}) + g(\hat{m}_c, \hat{s}) (3c_1 + c_2 + 3c_3 + c_4 + 3c_5 + c_6) \\
    &+ \lambda_u (g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s})) (3c_1 + c_2) - \frac{1}{2} g(\hat{m}_d, \hat{s}) (c_3 + 3c_4) \\
    &- \frac{1}{2} g(\hat{m}_b, \hat{s}) (4c_3 + 4c_4 + 3c_5 + c_6) + \frac{2}{9} (3c_3 + 4c_4 + 3c_5 + c_6) ,
\end{align*}
\]

with

\[
\begin{align*}
    c_1 &= -0.249, \quad c_2 = 1.108, \quad c_3 = 1.112 \times 10^{-2}, \quad c_4 = -2.569 \times 10^{-2} , \\
    c_5 &= 7.404 \times 10^{-3}, \quad c_6 = -3.144 \times 10^{-2}, \quad c_9 = 4.227 ,
\end{align*}
\]

and the notation \( \hat{s} = q^2/m_b^2 \), \( \hat{m}_q = m_q/m_b \). In the above formula \( \omega(\hat{s}) \) represents the one-gluon correction to the matrix element of the operator \( O_9 \) (see Appendix B), while the function \( g(\hat{m}_q, \hat{s}) \) arises from the one-loop contributions of the four-quark operators \( O_1-\hat{O}_6 \), i.e.

\[
g(\hat{m}_q, \hat{s}) = -\frac{8}{9} \ln(\hat{m}_q) + \frac{8}{27} + \frac{4}{9} y_q - \frac{2}{9} (2 + y_q) \sqrt{|1 - y_q|} \\
\times \left\{ \Theta(1 - y_q)(\ln \left( \frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}} \right) - i\pi) + \Theta(y_q - 1)2 \arctan \frac{1}{\sqrt{y_q - 1}} \right\} ,
\]

with \( y_q \equiv 4\hat{m}_q^2/\hat{s} \).

### III. LONG-DISTANCE EFFECTS: \( \rho, \omega \) AND THE \( J/\psi \) FAMILY

A more complete analysis of the above decay has to take into account long-distance contributions, which have their origin in real \( u\bar{u}, d\bar{d}, \) and \( c\bar{c} \) intermediate states, i.e. \( \rho, \omega, \) and \( J/\psi, \psi', \) etc., in addition to the short-distance interaction defined by Eqs. (2.5)–(2.8). In the case of the \( J/\psi \) family this is usually accomplished by introducing a Breit-Wigner distribution for the resonances through the replacement

\[ g(\hat{m}_c, \hat{s}) \rightarrow g(\hat{m}_c, \hat{s}) - \frac{3\pi}{\alpha^2} \sum_{V=J/\psi, \psi'} \frac{\hat{m}_V \text{Br} (V \rightarrow l^+l^-) \hat{\Gamma}_V}{\hat{s} - \hat{m}_V^2 + i\hat{m}_V \hat{\Gamma}_V} , \]

where the properties of the vector mesons are listed in Ref. [12].
We prefer to follow a different procedure, discussed in our previous paper [13], which uses the renormalized photon vacuum polarization $\Pi_\gamma^{\text{had}}(\hat{s})$, related to the measurable quantity $R_{\text{had}}(\hat{s}) \equiv \sigma_{\text{tot}}(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. This allows us to implement the long-distance contributions using experimental data.

The absorptive part of the vacuum polarization is given by

$$\text{Im } \Pi_\gamma^{\text{had}}(\hat{s}) = \frac{\alpha}{3} R_{\text{had}}(\hat{s}), \quad (3.2)$$

whereas the dispersive part may be obtained via a once-subtracted dispersion relation [14]

$$\text{Re } \Pi_\gamma^{\text{had}}(\hat{s}) = \frac{\alpha \hat{s}}{3\pi} P \int_{4\hat{m}_q^2}^{\infty} \frac{R_{\text{had}}(\hat{s}')}{\hat{s}'(\hat{s}' - \hat{s})} d\hat{s}', \quad (3.3)$$

where $P$ denotes the principal value.

To derive an expression that relates $g(\hat{m}_q, \hat{s})$ and $R_{\text{had}}(\hat{s})$, let us start with the electromagnetic current involving $u$, $d$ and $c$ quarks, which is relevant to the production of $\rho$, $\omega$ and $J/\psi$ resonances:

$$j_{\text{em}}^\mu = \frac{2}{3} \bar{u}\gamma^\mu u - \frac{1}{3} \bar{d}\gamma^\mu d + \frac{2}{3} \bar{c}\gamma^\mu c + \cdots. \quad (3.4)$$

Using Eq. (3.4), the vacuum polarization may then be written as

$$\Pi_\gamma^{\text{had}} = \frac{4}{9} \Pi^{cc} + \frac{4}{9} \Pi^{uu} + \frac{1}{9} \Pi^{dd} + \cdots \quad (3.5)$$

The vacuum polarization $\Pi^{q\bar{q}}$ associated with a $q\bar{q}$ loop is related to $g(\hat{m}_q, \hat{s})$ via

$$\Pi^{q\bar{q}} = \frac{9 \alpha}{4 \pi} \left( g(\hat{m}_q, \hat{s}) + \frac{4}{9} + \frac{8}{9} \ln \hat{m}_q \right). \quad (3.6)$$

Next we define currents corresponding to the quantum numbers of $\rho$, $\omega$, and $J/\psi$

$$j_\rho^\mu = \frac{1}{2}(j_\mu^u - j_\mu^d), \quad j_\omega^\mu = \frac{1}{6}(j_\mu^u + j_\mu^d), \quad j_{J/\psi}^\mu = \frac{2}{3} j_\mu^c, \quad (3.7)$$

with $j_\mu^q = \bar{q}\gamma_\mu q$, in terms of which the vacuum polarization, Eq. (3.3), can be rewritten as

$$\Pi_\gamma^{\text{had}} = \Pi^{J/\psi} + \Pi^\omega + \Pi^\rho + \cdots. \quad (3.8)$$
With the assumption $m_u = m_d$ it follows immediately that $\Pi^{u\bar{u}} = \Pi^{d\bar{d}}$, and we arrive at

$$\Pi^{c\bar{c}} = \frac{9}{4} \Pi^{J/\psi},$$  \hspace{1cm} (3.9)$$

$$\Pi^{u\bar{u}} = \Pi^{d\bar{d}} = \frac{9}{5} (\Pi^\omega + \Pi^\rho),$$ \hspace{1cm} (3.10)$$

so that

$$\text{Im} \ g(\hat{m}_c, \hat{s}) = \frac{\pi}{3} R^{J/\psi}_{\text{had}} (\hat{s}),$$ \hspace{1cm} (3.11)$$

$$\text{Im} \ g(\hat{m}_u, \hat{s}) = \text{Im} \ g(\hat{m}_d, \hat{s}) = \frac{4\pi}{15} (R^\rho_{\text{had}} (\hat{s}) + R^{\omega}_{\text{had}} (\hat{s})).$$ \hspace{1cm} (3.12)$$

For the real part of the one-loop function $g(\hat{m}_q, \hat{s})$ one finds

$$\text{Re} \ g(\hat{m}_c, \hat{s}) = -\frac{8}{9} \ln \hat{m}_c - \frac{4}{9} \hat{s} + \frac{\hat{s}}{3} P \int_{4\hat{m}_c^2}^{\infty} \frac{R^{J/\psi}_{\text{had}} (\hat{s}')} {\hat{s}' (\hat{s}' - \hat{s})} d\hat{s}',$$ \hspace{1cm} (3.13)$$

and

$$\text{Re} \ g(\hat{m}_q, \hat{s}) = -\frac{8}{9} \ln \hat{m}_q - \frac{4}{9} \hat{s} + \frac{4\hat{s}}{15} P \int_{4\hat{m}_q^2}^{\infty} \frac{R^\rho_{\text{had}} (\hat{s}')} {\hat{s}' (\hat{s}' - \hat{s})} d\hat{s}', \quad q = u, d. \hspace{1cm} (3.14)$$

Note that in many cases the evaluation of the dispersion integral may be carried out analytically (see e.g. Ref. [13]). The cross-section ratios appearing in Eqs. (3.11)–(3.14) may be written as

$$R^{J/\psi}_{\text{had}} (\hat{s}) = R^{c\bar{c}}_{\text{cont}} (\hat{s}) + R^{J/\psi}_{\text{res}} (\hat{s}),$$ \hspace{1cm} (3.15)$$

$$R^\rho_{\text{had}} (\hat{s}) + R^{\omega}_{\text{had}} (\hat{s}) = R^{u\bar{u} + d\bar{d}}_{\text{cont}} (\hat{s}) + R^\rho_{\text{res}} (\hat{s}) + R^{\omega}_{\text{res}} (\hat{s}),$$ \hspace{1cm} (3.16)$$

where the subscripts “cont” and “res” refer to the contributions from the continuum and the resonances respectively. The $J/\psi$ resonances and $\omega$ are well described through a relativistic Breit-Wigner form, i.e.

$$R^{J/\psi}_{\text{res}} (\hat{s}) = \sum_{V=J/\psi, \psi, ...} \frac{9\hat{s}}{\alpha^2} \frac{\text{Br} (V \rightarrow l^+ l^-) \hat{\Gamma}^V_{\text{total}} \hat{\Gamma}^V_{\text{had}}}{(\hat{s} - \hat{m}_V^2)^2 + \hat{m}_V^2 \hat{\Gamma}^V_{\text{total}}^2},$$ \hspace{1cm} (3.17)$$
and
\[ R_{\text{res}}^\omega (\hat{s}) = \frac{9\hat{s}}{\alpha^2} \frac{\text{Br} (\omega \rightarrow l^+l^-) \hat{\Gamma}_{\text{total}}^{\omega^2}}{(\hat{s} - \hat{m}_\omega^2)^2 + \hat{m}_\omega^2 \hat{\Gamma}_{\text{total}}^{\omega^2}}, \] (3.18)
with a $\hat{s}$-independent total width, which is quite adequate for our purposes. The $\rho$ resonance may be introduced through
\[ R_{\text{res}}^\rho (\hat{s}) = \frac{1}{4} \left( 1 - 4\frac{\hat{m}_\rho^2}{\hat{s}} \right)^{3/2} |F_\pi(\hat{s})|^2, \] (3.19)
$F_\pi(\hat{s})$ being the pion form factor, which is represented by a modified Gounaris-Sakurai formula \[16\]. The continuum contributions can be parametrized using the experimental data from Ref. \[17\], and are given in Appendix A.

### IV. BRANCHING RATIO AND CP-VIOLATING ASYMMETRY

The differential branching ratio for $b \rightarrow d l^+l^-$ in the variable $\sqrt{\hat{s}}$ including next-to-leading order QCD corrections is given by
\[ \frac{d \text{Br}}{d \sqrt{\hat{s}}} = \frac{\alpha^2}{2\pi^2} \frac{|V_{tb}V_{td}^*|^2 \text{Br} (B \rightarrow X c \bar{\nu}_e)}{|V_{cb}|^2 f(m_c) \kappa(m_c)} \lambda^{1/2}(1, \hat{s}, \hat{m}_d^2) \sqrt{\hat{s} - 4\hat{m}_l^2 \Sigma}, \] (4.1)
where we have neglected nonperturbative corrections of $O(1/m_b^2)$ \[18\]. The various factors appearing in Eq. (4.1) are defined by
\[ \lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ac), \] (4.2)
\[ \Sigma = \left\{ \frac{12 \text{Re}(\epsilon_c^\text{eff} \epsilon_9^\text{eff}) F_1(\hat{s}, \hat{m}_d^2) + 4\hat{s} |\epsilon_7^\text{eff}|^2 F_2(\hat{s}, \hat{m}_d^2)}{\hat{s}} \right\} \left( 1 + 2\frac{\hat{m}_l^2}{\hat{s}} \right) \]
\[ + \left( |\epsilon_9^\text{eff}|^2 + |c_{10}|^2 \right) F_3(\hat{s}, \hat{m}_d^2, \hat{m}_l^2) + 6\hat{m}_l^2 \left( |\epsilon_9^\text{eff}|^2 - |c_{10}|^2 \right) F_4(\hat{s}, \hat{m}_d^2) \}, \] (4.3)
with
\[ F_1(\hat{s}, \hat{m}_d^2) = (1 - \hat{m}_d^2)^2 - \hat{s}(1 + \hat{m}_d^2), \]
\[ F_2(\hat{s}, \hat{m}_d^2) = 2(1 + \hat{m}_d^2)(1 - \hat{m}_d^2)^2 - \hat{s}(1 + 14\hat{m}_d^2 + \hat{m}_d^4) - \hat{s}^2(1 + \hat{m}_d^2), \]
\[ F_3(\hat{s}, \hat{m}_d^2, \hat{m}_l^2) = (1 - \hat{m}_d^2)^2 + \hat{s}(1 + \hat{m}_d^2) - 2\hat{s}^2 + \lambda(1, \hat{s}, \hat{m}_d^2) \frac{2\hat{m}_l^2}{\hat{s}}, \]
\[ F_4(\hat{s}, \hat{m}_d^2) = 1 - \hat{s} + \hat{m}_d^2, \] (4.4)
while the ratio of CKM matrix elements in terms of the Wolfenstein parameters $\rho$
TABLE I: Branching ratio $\text{Br}(B \to X_d l^+ l^-)$, where $l = e, \mu$ or $\tau$, for different values of $(\rho, \eta)$ excluding the region ($\pm 20$ MeV) around the J/$\psi$ and $\psi'$ resonances.

| $(\rho, \eta)$ | $\text{Br}(B \to X_d e^+ e^-)$ | $\text{Br}(B \to X_d \mu^+ \mu^-)$ | $\text{Br}(B \to X_d \tau^+ \tau^-)$ |
|----------------|-----------------|-----------------|-----------------|
| (0.3, 0.34)    | $2.7 \times 10^{-7}$ | $1.8 \times 10^{-7}$ | $0.7 \times 10^{-8}$ |
| (-0.07, 0.34)  | $5.5 \times 10^{-7}$ | $3.8 \times 10^{-7}$ | $1.6 \times 10^{-8}$ |
| (-0.3, 0.34)   | $7.9 \times 10^{-7}$ | $5.4 \times 10^{-7}$ | $2.3 \times 10^{-8}$ |

and $\eta$ has already been given in Eq. (2.3). In order to remove the uncertainties in Eq. (4.1) due to an overall factor of $m_b^5$, we have introduced the inclusive semileptonic branching ratio via the relation

$$\Gamma(B \to X_c e \bar{\nu}_e) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 f(\hat{m}_c) \kappa(\hat{m}_c), \quad (4.5)$$

where $f(\hat{m}_c)$ and $\kappa(\hat{m}_c)$ represent the phase space and the one-loop QCD corrections [19] to the semileptonic decay respectively, and are given in Appendix B. Integrating the distribution in Eq. (4.1) for $l = e, \mu$, and $\tau$ over $\sqrt{s}$, we obtain the branching ratio $\text{Br}(B \to X_d l^+ l^-)$, depending on the specific choice of $\rho$ and $\eta$. The results are shown in Table I, for typical values of $(\rho, \eta)$ in the experimentally allowed domain [1]. Note that the branching ratio is quite sensitive to the Wolfenstein parameter $\rho$. For instance, the branching ratio for $B \to X_d e^+ e^-$ varies from $2.7$ to $7.9 \times 10^{-7}$, when $\rho$ is varied from $+0.3$ to $-0.3$.

Let us now turn to the $CP$-violating rate asymmetry, which is defined as follows:

$$A_{CP}(\sqrt{s}) = \frac{d\Gamma/d\sqrt{s} - d\Gamma/d\sqrt{s}}{d\Gamma/d\sqrt{s} + d\Gamma/d\sqrt{s}}, \quad (4.6)$$

where

$$\frac{d\Gamma}{d\sqrt{s}} \equiv \frac{d\Gamma(b \to d l^+ l^-)}{d\sqrt{s}}, \quad \frac{d\Gamma}{d\sqrt{s}} \equiv \frac{d\Gamma(\bar{b} \to \bar{d} l^+ l^-)}{d\sqrt{s}}. \quad (4.7)$$

The physical origin of a $CP$-violating asymmetry in the reaction can be understood by considering the term proportional to $c_9^{\text{eff}}$ in the matrix element, which can be

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1The branching ratio for different regions of $\sqrt{s}$ will be discussed below.
written symbolically as
\[ \mathcal{M} \sim A + \lambda_u B \, . \] (4.8)

The corresponding matrix element for \( \bar{b} \to \bar{d} l^+ l^- \) is
\[ \mathcal{M} \sim A + \lambda_u^* B \, , \] (4.9)
giving an asymmetry
\[ A_{CP} = -2 \text{Im} \lambda_u \text{Im} (A^* B) \frac{\Delta}{|A|^2 + |\lambda_u B|^2 + 2 \text{Re} \lambda_u \text{Re} (A^* B)} \, , \quad (4.10) \]
which provides a measure for \( CP \) violation. The asymmetry results from the presence of \( CP \) violation in the CKM matrix (\( \text{Im} \lambda_u \neq 0 \)) and unequal unitarity phases in the amplitudes \( A \) and \( B \) (\( \text{Im} (A^* B) \neq 0 \)).

The complete result contains an additional term due to the interference of \( c_{eff}^7 \) with \( c_{eff}^9 \), and the asymmetry takes the final form
\[ A_{CP}(\sqrt{s}) = -2 \text{Im} \lambda_u \Delta \frac{\Delta}{\Sigma + 2 \text{Im} \lambda_u \Delta} \approx -2 \text{Im} \lambda_u \Delta \frac{2 \eta}{(1 - \rho)^2 + \eta^2} \frac{\Delta}{\Sigma} \, , \quad (4.11) \]
with \( \Sigma \) defined in Eq. (4.3), and
\[ c_{eff}^9 \equiv \xi_1 + \lambda_u \xi_2 \, , \]
\[ \Delta = \text{Im} (\xi_1^* \xi_2) f_1(\hat{s}) + \text{Im} (c_{eff}^7 \xi_2) f_1(\hat{s}) \, , \]
\[ f_1(\hat{s}) = 6 F_1(\hat{s}, \hat{m}_d^2) \left( 1 + \frac{2 \hat{m}_l^2}{\hat{s}} \right) \, , \]
\[ f_+ (\hat{s}) = F_3(\hat{s}, \hat{m}_d^2, \hat{m}_l^2) + 6 \hat{m}_l^2 F_4(\hat{s}, \hat{m}_d^2) \, , \quad (4.12) \]

where the phase-space functions \( F_1 \) and \( F_{3,4} \) are given in Eq. (4.4). Notice that \( A_{CP} \) vanishes as \( m_u \to m_c \), since in that limit \( \xi_2 \to 0 \) (see Eq. (2.7))

Our numerical results for the asymmetry together with the differential branching ratio, Eq. (4.1), are shown in Figs. 1–3 for different values of \( \rho \) and \( \eta \). It is interesting
\footnote{We have also calculated the asymmetry in the \( b \to s \) transition, which is roughly one order of magnitude smaller than in \( b \to d \). Our results for the asymmetry differ somewhat from those given in Ref. [3], which uses an incorrect sign for the absorptive part of the one-loop function \( g(\hat{m}_q, \hat{s}) \). The correct sign is given in Refs. [8] and [10].}
TABLE II: Branching ratio $\text{Br}(B \to X_d e^+e^-)$ and average asymmetry $\langle A_{\text{CP}} \rangle$ for different regions of $\sqrt{s}$, below the $J/\psi$ resonance ($\varepsilon = 20 \text{ MeV}$).

| $(\rho, \eta)$  | $2m_e < \sqrt{s} < 1 \text{ GeV}$ | $1 \text{ GeV} < \sqrt{s} < (m_{J/\psi} - \varepsilon)$ |
|----------------|-----------------------------------|----------------------------------|
| $\text{Br}$    |                                   |                                  |
| $(0.3, 0.34)$  | $1.1 \times 10^{-7}$              | $1.2 \times 10^{-7}$             |
| $(-0.07, 0.34)$| $2.4 \times 10^{-7}$              | $2.3 \times 10^{-7}$             |
| $(-0.3, 0.34)$ | $3.4 \times 10^{-7}$              | $3.3 \times 10^{-7}$             |
| $\langle A_{\text{CP}} \rangle$ |                                   |                                  |
| $(0.3, 0.34)$  | $-8.4 \times 10^{-3}$             | $-5.3 \times 10^{-2}$            |
| $(-0.07, 0.34)$| $-4.0 \times 10^{-3}$             | $-2.7 \times 10^{-2}$            |
| $(-0.3, 0.34)$ | $-2.9 \times 10^{-3}$             | $-1.9 \times 10^{-2}$            |

TABLE III: Branching ratio $\text{Br}(B \to X_d e^+e^-)$ and average asymmetry $\langle A_{\text{CP}} \rangle$ for the large $\sqrt{s}$ region, excluding the $J/\psi$ and $\psi'$ resonances ($\varepsilon = 20 \text{ MeV}$).

| $(\rho, \eta)$  | $(m_{J/\psi} + \varepsilon) < \sqrt{s} < (m_{\psi'} - \varepsilon)$ | $(m_{\psi'} + \varepsilon) < \sqrt{s} < \sqrt{s}_{\text{max}}$ |
|----------------|---------------------------------------------------------------|---------------------------------------------------------------|
| $\text{Br}$    |                                                               |                                                               |
| $(0.3, 0.34)$  | $0.3 \times 10^{-7}$                                         | $1.6 \times 10^{-8}$                                         |
| $(-0.07, 0.34)$| $0.5 \times 10^{-7}$                                         | $3.4 \times 10^{-8}$                                         |
| $(-0.3, 0.34)$ | $0.8 \times 10^{-7}$                                         | $4.9 \times 10^{-8}$                                         |
| $\langle A_{\text{CP}} \rangle$ |                                                              |                                                               |
| $(0.3, 0.34)$  | $-5.1 \times 10^{-2}$                                        | $5.2 \times 10^{-3}$                                        |
| $(-0.07, 0.34)$| $-2.5 \times 10^{-2}$                                        | $2.1 \times 10^{-3}$                                        |
| $(-0.3, 0.34)$ | $-1.8 \times 10^{-2}$                                        | $1.5 \times 10^{-3}$                                        |

to note that the $\rho$ resonance is barely visible in the invariant mass spectrum, but has a strong influence on the asymmetry in the region up to 1 GeV. We have evaluated the branching ratio and average asymmetry $\langle A_{\text{CP}} \rangle$ for different regions of $\sqrt{s}$ using Eq. (4.6), and our results are displayed in Tables II–IV.

A variation of $m_t$ in the interval $176 \pm 10 \text{ GeV}$ changes these numbers by $\lesssim 10\%$. \footnote{A variation of $m_t$ in the interval $176 \pm 10 \text{ GeV}$ changes these numbers by $\lesssim 10\%$.}
TABLE IV: Branching ratio $\text{Br}(B \to X_d e^+ e^-)$ and average asymmetry $\langle A_{\text{CP}} \rangle$ near the $J/\psi$ and $\psi'$ resonances ($\varepsilon = 20$ MeV).

| $(m_{J/\psi} - \varepsilon) < \sqrt{s} < (m_{J/\psi} + \varepsilon)$ | $(m_{\psi'} - \varepsilon) < \sqrt{s} < (m_{\psi'} + \varepsilon)$ |
|-----------------|-----------------|
| $\text{Br}$ | $3.7 \times 10^{-6}$ | $1.8 \times 10^{-7}$ |
| $\langle A_{\text{CP}} \rangle$ | $0.6 \times 10^{-3}$ | $4.4 \times 10^{-3}$ |
| $\langle A_{\text{CP}} \rangle^a$ | $2.9 \times 10^{-3}$ | $6.7 \times 10^{-3}$ |

$^a$ Including OZI correction, induced by one-photon exchange as specified in Eq. (5.1).

V. CONCLUSIONS

The principal results of our analysis are as follows:

1. In the region excluding the $J/\psi$ resonances, we find a sizeable $CP$-violating asymmetry between the decays $b \to d e^+ e^-$ and $\bar{b} \to \bar{d} e^+ e^-$. This asymmetry amounts to $-5.3\% (-1.9\%)$ for the invariant mass region $1 \text{ GeV} < \sqrt{s_{e^+e^-}} < m_{J/\psi} - 20 \text{ MeV}$, assuming $\eta = 0.34$ and $\rho = 0.3 (-0.3)$. The corresponding branching ratio is $1.2 \times 10^{-7} (3.3 \times 10^{-7})$. The asymmetry scales approximately as $\eta ((1 - \rho)^2 + \eta^2)^{-1}$, while the branching ratio scales as $(1 - \rho)^2 + \eta^2$. For a nominal asymmetry of $5\%$ and a branching ratio of $10^{-7}$, a measurement at $3\sigma$ level requires $4 \times 10^{10} B$ mesons. In view of the clear dilepton signal, such a measurement might be feasible at future hadron colliders. It should be noted, however, that identification of the reaction $b \to d e^+ e^-$ in the presence of the much stronger reaction $b \to s e^+ e^-$ would require a study of the decay vertex, in order to select final states such as $\pi^+, \pi^+ \pi^- \pi^+$ etc. (accompanied by any numbers of neutrals). In the inclusive analysis of $e^+ e^-$ pairs, only those with invariant mass in the range $(M_B - M_K) < \sqrt{s} < (M_B - M_\pi)$ can be unambiguously ascribed to $b \to d e^+ e^-$.  

2. In the neighbourhood of the $J/\psi$ resonance ($m_{J/\psi} - 20$ MeV < $\sqrt{s} < m_{J/\psi} +$
20 MeV), the branching ratio is substantial (Br = 3.7 × 10^{-6}), but the asymmetry is very small (⟨A_{CP}^{J/ψ}⟩ = 0.6 × 10^{-3}). This smallness in asymmetry is the inevitable result of a very large c ¯c amplitude near the J/ψ, interfering with a small nonresonant background.

3. It is pertinent to ask if some refinement of the effective Hamiltonian underlying our calculation might lead to a higher asymmetry in the J/ψ region. In this connection, the following comments are in order.

(i) Our prescription for incorporating resonances into the effective Hamiltonian via the vacuum polarization function Π_{lad}^γ(§) implicitly assumes that the transition b → dJ/ψ is adequately described by the leading term (3c_1 + c_2 + 3c_3 + c_4 + 3c_5 + c_6) appearing in the Wilson coefficient c_{9}^{eff}, Eq. (2.7). With the values of c_i (i = 1, . . . , 6) given in Eq. (2.8), the theoretical branching ratio for the related reaction b → sJ/ψ is known to be ∼ 5 times smaller than measured [1, 3, 20]. It could be argued that for the purposes of calculating the b → dJ/ψ amplitude the coefficient (3c_1 + c_2 + · · ·) should accordingly be corrected to κ_V(3c_1 + c_2 + · · ·), with κ_V ∼ √5. While such a procedure enhances the branching ratio of b → dJ/ψ by a factor κ_V^2, it reduces the asymmetry by a factor κ_V. Outside the J/ψ and ψ′ regions, the branching ratio is essentially independent of κ_V. The asymmetry for √s < m_{J/ψ} is likewise unaffected, while that between J/ψ and ψ′ is reduced by ∼ κ_V. In the region √s > m_{ψ′} the asymmetry is quite sensitive to κ_V and can even be enhanced by an order of magnitude. This corner of phase space accounts, however, for only about 6% of the decay rate.

(ii) The asymmetry may be slightly enhanced if one takes into account mixing of the c ¯c current with the u ¯u and d ¯d currents. Such a mixing can give rise to an OZI-rule violating transition u ¯u → J/ψ, mediated by a one-photon (or 3-gluon) intermediate state [21, 22]. The QED effect can be incorporated into our calculation of the asymmetry near the J/ψ resonance by
the replacement

\[ \lambda_u g(\hat{m}_c, \hat{s}) \longrightarrow \lambda_u (1 + i \frac{4}{9} \alpha) g(\hat{m}_c, \hat{s}) \]  

(5.1)

in the coefficient \( c_9^{\text{eff}} \). The resulting asymmetry increases from \( 0.6 \times 10^{-3} \) to \( 2.9 \times 10^{-3} \) (see Table IV).

(iii) Finally, it is possible to contemplate gluonic corrections to the effective Hamiltonian, that allow the transition \( b \rightarrow dJ/\psi \) to take place not only through a colour-singlet \((c\bar{c})_1\) intermediate state (i.e. \( b \rightarrow d(c\bar{c})_1 \rightarrow dJ/\psi \)) but also through a colour-octet intermediate configuration \((b \rightarrow d(c\bar{c})_8 \rightarrow dJ/\psi)\). An illustrative calculation by Soares [22] yields an asymmetry of about 1% from such a mechanism.

Our general conclusion is that a measurement of the branching ratio and partial width asymmetry in the channel \( b \rightarrow d e^+ e^- \) in the nonresonant continuum, would provide a theoretically clean and fundamental test of the idea that \( CP \) violation originates in the CKM matrix. The predicted asymmetry in the region \( 1 \text{ GeV} < m_{e^+e^-} < m_{J/\psi} \) is approximately

\[ -5.3\% \left( \frac{\eta}{0.34} \right) \left( \frac{1.2 \times 10^{-7}}{\text{Br}} \right), \]  

(5.2)

where \( \text{Br} \) denotes the branching ratio in the above interval. Measurements near the \( J/\psi \) resonance are predicted to show a very small asymmetry \((\sim 3 \times 10^{-3})\) that depends somewhat on the manner in which QCD modulates the effective interaction for \( b \rightarrow dJ/\psi \).

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APPENDIX A: INPUT PARAMETERS

\[ m_b = 4.8 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}, \quad m_u = m_d = m_\pi = 0.139 \text{ GeV}, \]
\[ m_t = 176 \text{ GeV}, \quad m_e = 0.511 \text{ MeV}, \quad m_\mu = 0.106 \text{ GeV}, \quad m_\tau = 1.777 \text{ GeV}, \]
\[ \mu = m_b, \quad \text{Br} (B \to X_c e\bar{\nu}_e) = 10.4\%, \quad \lambda = 0.2205, \quad \Lambda_{QCD} = 225 \text{ MeV}, \]
\[ \alpha = 1/129, \quad \sin^2 \theta_W = 0.23, \quad M_W = 80.2 \text{ GeV}. \] (A1)

\[
R_{\text{cont}}^{u\bar{u}+d\bar{d}}(\hat{s}) = \begin{cases} 
0 & \text{for } 0 \leq \hat{s} \leq 4.8 \times 10^{-2}, \\
1.67 & \text{for } 4.8 \times 10^{-2} \leq \hat{s} \leq 1.
\end{cases} \] (A2)

\[
R_{\text{cont}}^{e\bar{e}}(\hat{s}) = \begin{cases} 
0 & \text{for } 0 \leq \hat{s} \leq 0.60, \\
-6.80 + 11.33\hat{s} & \text{for } 0.60 \leq \hat{s} \leq 0.69, \\
1.02 & \text{for } 0.69 \leq \hat{s} \leq 1.
\end{cases} \] (A3)

APPENDIX B: USEFUL FUNCTIONS

As noted by Misiak [10], the function \( \omega(\hat{s}) \) can be inferred from [23] and is defined by

\[
\omega(\hat{s}) = -\frac{2}{9}\pi^2 - \frac{4}{3}\text{Li}_2(\hat{s}) - \frac{2}{3}\ln\hat{s}\ln(1 - \hat{s}) - \frac{5 + 4\hat{s}}{3(1 + 2\hat{s})}\ln(1 - \hat{s}) - \frac{2\hat{s}(1 + \hat{s})(1 - 2\hat{s})}{3(1 - \hat{s})^2(1 + 2\hat{s})}\ln\hat{s} + \frac{5 + 9\hat{s} - 6\hat{s}^2}{6(1 - \hat{s})(1 + 2\hat{s})}. \] (B1)

\[ f(\hat{m}_c) = 1 - 8\hat{m}_c^2 + 8\hat{m}_c^6 - 5\hat{m}_c^8 - 24\hat{m}_c^4\ln\hat{m}_c. \] (B2)

\[ \kappa(\hat{m}_c) = 1 - \frac{2\alpha_s(m_b)}{3\pi}\left(\pi^2 - \frac{31}{4}\right)(1 - \hat{m}_c)^2 + \frac{3}{2}. \] (B3)
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FIGURE CAPTIONS

**Figure 1** Branching ratio \( \text{Br}(B \to X_d e^+ e^-) \) (a) and \( CP \)-violating asymmetry \( A_{CP} \) (b) including next-to-leading order QCD corrections as well as long-distance contributions (solid line), i.e. \( \rho, \omega \), and the \( J/\psi \) family, as a function of \( \sqrt{\hat{s}} \), \( \hat{s} \equiv q^2/m_b^2 \). The dashed line in (a) corresponds to the nonresonant invariant mass spectrum. The Wolfenstein parameters are chosen to be \( (\rho, \eta) = (0.3, 0.34) \).

**Figure 2** Branching ratio \( \text{Br}(B \to X_d e^+ e^-) \) (a) and \( CP \)-violating asymmetry \( A_{CP} \) (b) for \( (\rho, \eta) = (-0.07, 0.34) \). The dashed line in (a) represents the nonresonant spectrum.

**Figure 3** Branching ratio \( \text{Br}(B \to X_d e^+ e^-) \) (a) and \( CP \)-violating asymmetry \( A_{CP} \) (b) for \( (\rho, \eta) = (-0.3, 0.34) \). The dashed line in (a) represents the nonresonant spectrum.
FIG. 1:

(a) $\frac{d\text{Br}}{d\sqrt{s}}$

(b) $A_{\text{CP}}$

$(\rho, \eta) = (0.3, 0.34)$
FIG. 2:
FIG. 3: