Grand Unification of Cosmology: Common Origin of Inflation,
Dark Energy, Dark Matter and Baryon Asymmetry

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Abstract: I suggest a grand unified model of the inflation, reheating, dark energy, dark matter
and baryon asymmetry, which can account for the common origin of these things. The model
can coherently and completely characterize the universe evolution from the early inflation and
reheating to the late little inflation and hot expansion, moreover, it establishes the relationships
among these processes and particle physics. I give and solve the complete system of equations of
the dynamical evolution of each stage. The numerical results clearly show the slow-roll feature
of the primordial inflation, the inflation essence is a process of the dark matter growing from
the dark energy, the decay of the early superheavy dark matter into the SM particles leads to
both the universe reheating and the matter-antimatter asymmetry, the present-day dark energy
and light cold dark matter are derived from the late little inflation. This model with eight input
parameters not only perfectly reproduces the measured inflationary data and the current energy
density budget, but also gives some predictions, for instance, $r_{0.002} \approx 0.00153$, $m_{DM} \approx 0.391$
MeV, $\eta_B \approx 6.08 \times 10^{-10}$, $h \approx 0.674$, and so on. Finally, it is very promising to test the model
in the near future.

Keywords: cosmological model; inflation; dark energy; dark matter; baryon asymmetry
I. Introduction

The standard model of particle physics (SM) and the ΛCDM model of cosmology together have successfully accounted for all kinds of the cosmic observations from the BBN era to the present day [1], but they can not address the origin of the hot big bang of the universe [2], namely what happened before the standard hot expansion, and also can not answer the origins of the current dark energy, cold dark matter (CDM) and baryon asymmetry [3]. At present the theoretical and experimental investigations have clearly indicated that the very early universe surely underwent the inflation phase and the followed reheating one [4]. These two processes not only provide the initial conditions of the hot expansion, but also are closely related to the origin of the universe matter [5]. To solve the above problems, therefore, seeking an underlying theory beyond the SM and ΛCDM has become a focus of the researches of high energy physics and cosmology, this aspect is currently attracting more and more attentions of the theoretical and experimental physicists [6].

There have been numerous theories about the inflation, dark energy, dark matter, and baryon asymmetry, which include all kinds of paradigms of the inflation and the reheating [7], some special models of the dark energy [8], even some models based on the non-standard gravity [9], a variety of the CDM candidates [10], the modified Newtonian dynamics [11], and many mechanisms of leptogenesis or baryogenesis [12]. However, all of the proposals have a common shortcoming, namely, they are only aiming at one or two specific aspects of the universe rather than considering it as a whole, in other words, the above-mentioned things are dealt in isolation regardless of their connections, this is obviously unnatural and inadvisable because the uniqueness of the universe origin destines there are certainly some connections among them. In fact, the vast majority of these models have now been ruled out by the recent measured data and analyses [13].

By use of the analyses for the power spectra of the anisotropic and polarized temperature of the cosmic microwave background (CMB) [14], we have obtained the following inflationary data, the tensor-to-scalar ratio, the scalar spectral index, the running of the spectral index, and the scalar power spectra. On the other hand, from the global analyses of cosmology which includes CMB, BBN, structure formation, etc. [15], we have extracted the following universe data, the dark energy density, the dark matter density, the ratio of the baryon number density to the photon one, and the neutrino mass sum. The present optimum values of these data are given as [1],

\[
\begin{align*}
& r_{0.002} < 0.058, \quad n_s \approx 0.965, \quad \frac{dn_s}{d\ln k} \approx -0.004, \quad \ln(10^{10} \Delta^2_R) \approx 3.04, \\
& \Omega_{DE} \approx 0.685, \quad \Omega_{CDM} \approx 0.265, \quad \eta_B \approx 6.1 \times 10^{-10}, \quad \sum_i m_{\nu_i} \sim 0.1 \text{ eV}. 
\end{align*}
\]

(1)

All of the measured data undoubtedly contain the information of the origin of the universe, they severely constrain the models beyond the ΛCDM [16], so any one successful theory of cosmology has to confront them unavoidably.

I find a new approach to solve the above puzzles of the universe. Based on the universe concordance and the nature unification, I attempt to build a grand unified model of cosmology on the basis of the standard theory of gravity as well as the fundamental principle of cosmology. The model not only elegantly relates the above-mentioned ingredients to each other, but also completely solve the dynamical evolution of each period of the universe from the inflation era to
Figure 1: The sketch of the origin and evolution of the universe described by the cosmological unified model. The dark energy indicated by the grey background is step by step released and reduced through it converting into the dark matter, the whole evolution is just like a cascade of hydropower stations, so the “cosmological constant” problem is naturally removed.

The idea framework of the model can be described by the sketch shown as Fig. 1. The universe sequentially went through the earlier inflation, the followed reheating, the later little inflation and the standard hot expansion. The model contains a sole neutral scalar field $\varphi$, which can arises from the symmetry breaking of some GUT. At the superhigh scale of the very early universe, $\varphi$ is both the inflationary field and the primordial dark energy $\varphi_{DE}$. The inflation process is in essence that the superheavy dark matter $\varphi_{DM}$ is gradually grown from $\varphi_{DE}$, in other words, $\varphi_{DE}$ is slowly converted into $\varphi_{DM}$. The nature of $\varphi_{DE}$ and $\varphi_{DM}$ are regarded as the two energy forms of the $\varphi$ field, $\varphi_{DE}$ is an inert condensed state with a negative pressure, which has no couplings to other fields, whereas $\varphi_{DM}$ is an excited massive particle state with vanishing pressure, which can interact with the SM particles. The relationship between $\varphi_{DE}$ and $\varphi_{DM}$ is inappropriately analogous to ice and vapour, which are only the two morphologies of the same material. In short, I give the above explanations about the unknown nature of the dark energy and the dark matter, this is also based on Occam’s Razor. When the inflation is completed, the decay of $\varphi_{DM}$ into the SM Higgs and lepton is responsible for the reheating and the leptogenesis. After reheating the tiny residual dark energy whose density has greatly reduced is the source of the little inflation (namely the second inflation), by which a light and stable dark matter is newly grown, and eventually it becomes the current CDM. The current dark energy is exactly the final leftover one. Although there is a different about 120 orders of magnitude between the primordial dark energy and the current one, the dark energy is step by step released and reduced through it converting into the dark matter, the whole evolution is just like a cascade of hydropower stations, so the “cosmological constant” problem is naturally removed.
The remainder of this paper is organized as follows. I give a complete solution of the inflationary evolution in Section II. The reheating evolution is solved in Section III. I discuss the evolution of the little inflation and the hot expansion in IV Section. In Section V, I deal with the generation of the neutrino mass and the baryon asymmetry by the particle model calculation. Section VI is a summary of the numerical results of the model. Section VII is devoted to conclusions.

II. Inflation

According to the standard paradigm [17], the inflation field is denoted by $\varphi$, it is considered as spatially uniform distribution, but there are very small fluctuations, which will become sources of the structure formation. $\varphi$ has a super-high energy density in the earlier inflation and reheating periods. The dynamics of the inflation process is described as what follows. Under the flat FLRW Metric, the $\varphi$ Lagrangian is given as

$$\mathcal{L}_\varphi = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi), \quad g_{\mu\nu} = diag(-1, a^2(t), a^2(t), a^2(t)), \quad (2)$$

where $V(\varphi(t))$ is the inflationary potential and $a(t)$ is the scale factor of the universe expansion. The energy density and pressure of $\varphi$ are obtained by its energy-momentum tensor, namely

$$T^\mu_\nu(\varphi) = g^\mu\beta \partial_\beta \varphi \partial_\nu \varphi + \delta^\mu_\nu \mathcal{L}_\varphi \implies - T^0_0 = \rho_\varphi(t) = \frac{\dot{\varphi}^2(t)}{2} + V(\varphi), \quad \frac{1}{3} \delta^j_i T^i_j = P_\varphi(t) = \frac{\dot{\varphi}^2(t)}{2} - V(\varphi) = w_\varphi(t) \rho_\varphi(t), \quad (3)$$

where $\frac{\dot{\varphi}^2(t)}{2}$ is the $\varphi$ kinetic energy and $w_\varphi$ is the parameter-of-state relating the pressure to the energy density. It can be seen that the potential energy and the kinetic energy together determine $\rho_\varphi$ and $w_\varphi$, and vice versa.

I now introduce the dark energy $\varphi_{DE}$ and the dark matter $\varphi_{DM}$, they are only two energy components of the same $\varphi$ field, so they are defined by the following relations,

$$P_{\varphi_{DE}}(t) = -\rho_{\varphi_{DE}}(t) \quad (w_{\varphi_{DE}} \equiv -1), \quad P_{\varphi_{DM}}(t) = 0 \quad (w_{\varphi_{DM}} \equiv 0),$$

$$\rho_\varphi(t) = \rho_{\varphi_{DE}}(t) + \rho_{\varphi_{DM}}(t), \quad P_\varphi(t) = P_{\varphi_{DE}}(t) + P_{\varphi_{DM}}(t) = -\rho_{\varphi_{DE}}(t),$$

$$\implies \rho_{\varphi_{DE}} = -w_\varphi \rho_\varphi = \frac{-2 w_\varphi}{1 - w_\varphi} V(\varphi), \quad \rho_{\varphi_{DM}} = (1 + w_\varphi) \rho_\varphi = \frac{-3}{2} + \frac{1 + w_\varphi}{1 - w_\varphi} V(\varphi) = \varphi^2, \quad (4)$$

where I employ Eqs. (3). Eqs. (4) clearly show the relationships among all kinds of the energy forms of the $\varphi$ field. Note that $\rho_{\varphi_{DE}}$ only contains a part of $V(\varphi)$ without the kinetic energy since $\varphi_{DE}$ is always an inert condensed state with negative pressure, whereas $\rho_{\varphi_{DM}}$ carries the kinetic energy and the rest of $V(\varphi)$, but both are equal amount, since $\varphi_{DM}$ is an excited massive particle state with vanishing pressure. The physical implications of $\varphi_{DE}$ and $\varphi_{DM}$ will be further clear in the following context.

Friedmann equation, the continuity equation, and the growth equation of $\varphi_{DM}$ collectively control the dynamics of the inflationary evolution, namely

$$\rho_\varphi = \rho_{\varphi_{DE}} + \rho_{\varphi_{DM}} = 3M_p^2 H^2,$$

$$\dot{\rho}_\varphi + 3H \rho_\varphi (1 + w_\varphi) = \dot{\rho}_{\varphi_{DE}} + \dot{\rho}_{\varphi_{DM}} + 3H \rho_{\varphi_{DM}} = 0,$$

$$\dot{\varphi}_{\varphi_{DM}} = -2\eta H \rho_{\varphi_{DM}}, \quad (5)$$
where $\tilde{M}_p = \frac{1}{\sqrt{8\pi G}} \approx 2.43 \times 10^{18}$ GeV is the reduced Plank mass, $H(t) = \frac{\dot{a}(t)}{a(t)}$ is the universe expansion rate. $\eta$ is always negative, in fact, it is a slow-roll parameter which controls the $\varphi_{DM}$ growth. Once $\eta$ is specified, Eqs. (5) are a closed system of equations, from which one can derive all of the inflationary evolutions. It is very clear that the $\rho_{\varphi_{DM}}$ growth is entirely from the $\rho_{DE}$ reduction in the comoving volume, so the essence of the inflation is a process of $\varphi_{DE}$ converting into $\varphi_{DM}$. From Eqs. (4) and (5), we can easily derive

$$\epsilon(t) = \frac{\dot{H}}{H^2} = \frac{3(1 + w_\varphi)}{2}, \quad \eta(t) = -\frac{\ddot{\varphi}}{H \dot{\varphi}}, \quad (6)$$

$\epsilon$ and $\eta$ are namely two slow-roll parameters defined as usual, they can generally vary with the time in the inflation period, or else the inflation will continue on without termination. In general, the range of $-1 \leq w_\varphi \leq \frac{1}{3}$ is physically accepted, there is thus $0 \leq \epsilon \leq 2$ and $H \leq 0$, this means that both the expansion rate and the total energy density are always decreased in the inflation process. Put Eqs. (4) and Eqs. (6) together, all kinds of the energy forms can be expressed by $\rho_\varphi$ and $\epsilon$,

$$\rho_{\varphi_{DE}} = (1 - \frac{2\epsilon}{3})\rho_\varphi, \quad \rho_{\varphi_{DM}} = \frac{2\epsilon}{3}\rho_\varphi, \quad \frac{\dot{\varphi}^2}{2} = \frac{\epsilon}{3}\rho_\varphi, \quad V(\varphi) = (1 - \frac{\epsilon}{3})\rho_\varphi. \quad (7)$$

Eqs. (5)-(7) make up the fundamental equations of the inflation dynamics, in principle, if one can provide the evolution of any one of the nine inflationary quantities, $H$, $\rho_\varphi$, $\rho_{\varphi_{DE}}$, $\rho_{\varphi_{DM}}$, $\dot{\varphi}$, $V(\varphi)$, $\epsilon$, $\eta$, $w_\varphi$, then the evolutions of the rest of the inflationary quantities are completely determined by the system of equations. In what follows, we will find their solutions them by some special techniques.

One of the inflationary features is that the universe size expands about $10^{25}$ times in extremely short duration, therefore we use the e-fold number to characterize a time span of the inflation instead of the scale factor, it is defined as follows,

$$N(t) = \ln \frac{a(t_{inf})}{a(t)} = \int_{t}^{t_{inf}} H(t') dt' \implies \dot{N}(t) = -H(t), \quad (8)$$

where the “inf” subscript specially indicates the time of inflation finish, and the starting point of the inflation is set as $a(0) = 0$ and $N(0) = +\infty$. Eq. (8) now acts as the role of $\frac{\dot{a}}{a} = H$ since $N(t)$ replaces $a(t)$ as the time scale, it will frequently be employed in the following formula derivations.

By use of Eqs. (4)-(8), we can order by order give the slow-roll parameters by the total energy $\rho_\varphi$ as follows,

$$\epsilon = \frac{d \ln \rho_\varphi}{2dN}, \quad \eta = \frac{d \ln \rho_{\varphi_{DM}}}{2dN} = \frac{d \ln \rho_{\varphi_{DE}}}{2dN}, \quad \theta = \frac{d \ln (-\rho''_\varphi)}{2dN} = \frac{d \ln (-\rho_{\varphi_{DM}})}{2dN}, \quad \delta = \frac{d \ln \rho''_\varphi}{2dN}, \quad ... \quad (9)$$

$$\rho'_\varphi = \frac{d \rho_\varphi}{dN} = 3\rho_{\varphi_{DM}}, \quad \rho''_\varphi = \frac{d^2 \rho_\varphi}{dN^2} = 3\rho''_{\varphi_{DM}}, \quad ... \quad (10)$$

$$\implies \frac{d \ln \epsilon}{2dN} = \eta - \epsilon, \quad \frac{d \ln (-\eta)}{2dN} = \theta - \eta, \quad \frac{d \ln (\theta)}{2dN} = \delta - \theta, \quad ... \quad (11)$$
Hereinafter the “” superscript denotes a derivative with regard to $N$. These slow-roll parameters are closely related to the inflationary observable quantities by the following Eqs. (20), for example, $\epsilon$ and $\eta$ are relevant to the tensor-to-scalar ratio and the scalar spectral index, $\theta$ is involved in the running of the spectral index, therefore, a key of solving inflation problem is correctly finding the solutions of these slow-roll parameters.

If the inflationary potential $V(\varphi)$ is provided, then the slow-roll parameters are conventionally given by $V(\varphi)$ as follows,

$$
\epsilon_V = \frac{\dot{M}_p^2}{2} \left( \frac{dV}{d\varphi^2} \right)^2 = \left( \frac{3 - \eta}{3 - \epsilon} \right)^2 \epsilon, \\
\eta_V = \dot{M}_p^2 \frac{\dot{V}}{d\varphi^2} = \frac{(\epsilon + \eta)(3 - \eta) - 2\eta'}{3 - \epsilon}, \\
\xi_V = \frac{\dot{M}_p^4}{V \ddot{d}\varphi} \left( \frac{3 - \eta}{3 - \epsilon} \right)^2 \left[ 4\epsilon \eta + \eta' \left( 3 - 3\epsilon + 2\eta - 2\theta \right) - 2\eta\theta' \right],
$$

where $\eta' = \frac{d\eta}{dN} = 2\eta(\theta - \eta)$ and $\theta' = \frac{d\theta}{dN} = 2\theta(\delta - \theta)$, we have employed the foregoing equations. Eqs. (12) give the relations between these two sets of slow-roll parameters. However, it should be stressed that these approximations are held only in the case of $\epsilon, \eta, \theta \ll 1$, which is in the early and middle stages of the inflation. When the inflation is close to its end, some slow-roll parameters actually become $\sim 1$, thus the approximations are invalid.

Finally, we can derive the following useful relations from the previous fundamental equations,

$$
\dot{\varphi}^2 = 2\epsilon \dot{M}_p^2 H^2 \implies \dot{\varphi} = \frac{d\varphi}{dN} = M_p \sqrt{2\epsilon}, \\
\frac{\dot{\varphi}}{\varphi} + 3H + \frac{dV(\varphi)}{d\varphi} = 0 \implies \frac{\ddot{\varphi}'}{3 - \epsilon} - \frac{\varphi'}{3 - \epsilon} + \dot{M}_p^2 \frac{d\ln|V|}{d\varphi} = 0, \\
\frac{\ddot{a}}{a} = (1 - \epsilon) H^2 = -\frac{1 + 3w_\varphi}{2} H^2,
$$

where $\varphi < 0$ (namely $\varphi' > 0$) is default since the $\varphi$ value is gradually reduced. Eq. (14) is the equation of motion of $\varphi$, for the case of $\epsilon \ll 1$, it can be solved by provided $V(\varphi)$. Eq. (15) characterizes the expansion acceleration, the accelerating or decelerating expansion only depends on the value of $\epsilon$, namely, there is $\ddot{a} > 0$ if $\epsilon \ll 1$ and there is $\ddot{a} < 0$ if $\epsilon > 1$. In the inflation period, the universe is always accelerating expansion, when the inflation is completed, the universe immediately transforms from the accelerated expansion to the decelerated one, thus we obtain the following limits

$$
\ddot{a}(t) > \ddot{a}(t_{inf}) = 0, \quad 0 \leq \epsilon(t) < \epsilon(t_{inf}) = 1, \quad -1 < w_\varphi(t) < w_\varphi(t_{inf}) = -\frac{1}{3}.
$$

This is also the boundary condition for the inflation equations.

A traditional technique of solving the inflation problem is as the following procedure. Firstly, one has to design or guess a function form of $V(\varphi)$. Secondly, one puts $V(\varphi)$ into Eq. (14), and omits $\epsilon$ in the denominator since there is $\epsilon \ll 1$ in the most of the inflation duration, thus one can solve the $\varphi$ differential equation to obtain the function of $\varphi(N)$. Thirdly, one puts $\varphi(N)$ into Eq. (13) to work out $\epsilon(N)$, and further one can figure out $\eta(N)$ and $\theta(N)$ by Eqs. (11).
Lastly, one can calculate $\rho_\psi$, $\rho_{\psi\psi}$, $\rho_{\phi\phi}$ and $\dot{\phi}^2/2$ by $V(\phi(N))$ and $\epsilon(N)$ from Eqs. (7). Until now, the inflationary evolutions are completely solved out. Nevertheless, this procedure has two shortcomings, i) $\epsilon$ is actually $\sim 1$ rather than $\ll 1$ in the later period of the inflation, thus $\epsilon$ being neglected in Eq. (14) will lead to the incomplete inflationary solutions, in particular, this has great effect on the inflation termination and the subsequent reheating. ii) it is very difficult to completely fit all of the inflationary data in Eqs. (1) by means of this technique, a desirable function form of $V(\phi)$ can not be found at all after the countless endeavours have been made. Therefore, to reliably and completely solve the inflation problem in the standard gravity framework, we have to find a new approach.

In the system of the inflationary equations, since all of the unknown inflationary quantities have an equal status at least in mathematical sense, I can choose the slow-roll parameter $\eta$ as the starting point to solve the inflation problem instead of the potential $V(\phi)$. In principle, I can employ the following procedure, firstly, I can design or guess an evolution function of $\eta$, this amounts to specifying the law of the $\varphi_{DM}$ growth in Eqs. (5). Secondly, I can solve $\rho_\phi$ (and also obtain $H$) from the fundamental Eqs. (5). Thirdly, I can obtain $\epsilon$ and $w_\phi$ by Eqs. (6). Lastly, I can calculate $\rho_{\psi\psi}$, $\rho_{\phi\phi}$, $\dot{\phi}^2/2$ and $V(\phi)$ by Eqs. (7). Thus, all of the inflationary evolutions are completely solved out. By this reversal technique, the potential $V(\phi)$ is figured out rather than provided. Whatever technical means is employed, the only criterion is that it is able to fit all of the inflationary data correctly and completely.

After a careful analysis, I propose a function form of $\eta(N)$ such as

$$
\eta(N) = \eta(0)e^{-\frac{\alpha N^4}{N^2}} = -\frac{e^{\alpha(1-\frac{N^4}{N^2})}}{N^4 + 5},
$$

where $N_s$ and $\alpha$ are two fix parameters, $\eta(0)$ is also parameterized by them, in addition, $\theta$ is obtained by Eqs. (11). In fact, $N_s$ is corresponding to the inflationary e-fold number when the pivot scale of $k_\ast = 0.05$ Mpc$^{-1}$ exited from the horizon, its value will be calculated by the following Eq. (21) as $N_s \approx 52.7$. $\alpha$ is one of two input parameters in the inflation sector of the unified model, the other one is $H_{inf}$, by fitting the inflationary data we can determine $\alpha \approx 1.51$, see the summarized Tab. 1 in VI section. Evidently, Eqs. (17) imply there are $\eta(0) = \theta(0) = \frac{e^{\alpha}}{N_s + 5}$ and $\eta'(0) = 0$ at the end of the inflation.

Based on Eqs. (17), we can solve the inflationary evolutions as follows, firstly we put $\eta(N)$ into the second equality in Eqs. (9) so as to directly solve $\rho_{\phi\phi}(N)$, then we can work out $\rho_\phi(N)$ from the first equation in Eqs. (10), further we can obtain $\epsilon(N)$ by the second equality in Eqs. (7), the derived results are

$$
\frac{\rho_{\phi\phi}(N)}{\rho_{\phi}(0)} = \frac{2}{3} e^{2\int_0^N \eta(N')dN'}, \quad \frac{\rho_\phi(N)}{\rho_{\phi}(0)} = 1 + 3 \int_0^N \frac{\rho_{\phi\phi}(N')}{\rho_{\phi}(0)}dN', \quad \epsilon(N) = \frac{3}{2} \frac{\rho_{\phi\phi}(N)}{\rho_\phi(N)},
$$

where I use $\frac{\rho_{\phi\phi}(0)}{\rho_\phi(0)} = \frac{2}{3}$ due to $\epsilon(0) = 1$. Lastly, we can calculate the evolutions of $\rho_{\phi\phi}, \dot{\phi}^2/2$ and $V(\phi)$ by use of the relations in Eqs. (7), and also obtain $w_\phi$ in Eqs. (6) and $\eta_\phi$ in Eqs. (12). Note that all kinds of the energy densities are normalized to $\rho_\phi(0)$, which is namely $\rho_\phi(t_{inf})$.

Fig. 2 numerically shows the inflationary evolutions of all kinds of the energy forms with $N$ as time scale. In the early and middle phases of the inflation proceeding, the three curves of
Figure 2: The inflationary evolutions of the relevant energy forms with the e-fold number as
time scale, $N_* \approx 52.7$ is corresponding to the time of $k_*=0.05$ Mpc$^{-1}$ exiting horizon.

$\rho_{\varphi}, \rho_{\varphi_{DE}}, V_\varphi$ almost coincide with each other, moreover, they nearly keep a constant value, the
reason for this is that the growths of both $\varphi_{DM}$ and $T_\varphi$ are very slow at this stage, namely so-
called slow-roll. In the later phase of the inflation proceeding, the growths of $\rho_{\varphi_{DM}}$ and $T_\varphi$ have
been accumulated to a certain amount, so the curves of $\rho_{\varphi}, \rho_{\varphi_{DE}}, V_\varphi$ are gradually separated each
other. Eventually, $\rho_{\varphi_{DM}}$ fully exceeds $\rho_{\varphi_{DE}}$, the $\varphi_{DE}$-dominated universe is transformed into
the $\varphi_{DM}$-dominated one, the accelerating expansion is naturally terminated, thus the inflation
is over, this characteristic will more clearly be seen in the following Fig. 4. At the time of the
inflation finish, there are $\rho_{\varphi_{DE}} = \frac{V_\varphi}{2} = \frac{\rho_\varphi}{3}$ and $\rho_{\varphi_{DM}} = 2T_\varphi = \frac{2\rho_\varphi}{3}$, which are the case of $\epsilon = 1$ in
Eqs. (7). Note that the green curve predicts $\rho_{\varphi}(\infty)/\rho_{\varphi}(0) \approx 13.83$, this implies the $\rho_{\varphi}$ amount
only changes about a dozen times from the inflation beginning to its end. In short, Fig. 2 clearly
shows the inflation evolutions and its slow-roll feature.

Fig. 3 numerically shows the inflationary evolutions of the slow-roll parameters and the
parameter-of-state with $N$ as time scale. One can see the three remarkable features. i) In the
most of the inflation duration, these slow-roll parameters are essentially $\ll 1$ and the parameter-
of-state is $w_\varphi \approx -1$, so the variation of each of the curves is very insignificant. Only when
$N(t) \rightarrow N(t_{inf}) = 0$, the three parameters of $\epsilon, \eta_\varphi, w_\varphi$ sharply rise, this thus brings about the
inflation termination. ii) In the early and middle phases of the inflation, $\eta$ is coinciding with $\eta_\varphi$
due to $\eta \approx \eta_\varphi$, whereas in the last phase of the inflation, $\eta$ is coinciding with $\theta$ due to $\eta \approx \theta$.
iii) $\epsilon$ is always positive while $\eta, \theta$ are always negative, but the sign of $\eta_\varphi$ can change from early
negative to late positive. In the light of the following Eq. (23), $M_\varphi^2$ is proportional to $\eta_\varphi$, this
means that $M_\varphi$ is gradually grown from nothing as $\eta_\varphi$ evolving, of course, this is closely related
to the $\varphi_{DM}$ growing, therefore $M_\varphi$ essentially arises from the dark energy converting into the
dark matter. This mechanism of the mass generation of the inflation field is very different from
one of the usual particle mass generation, which results from the vacuum spontaneous breaking.
When the inflation process is completed, the universe transforms from the accelerating expansion
to the decelerating one. In short, these numerical results of Fig. 2 and Fig. 3 excellently explain
the inflation phase of the very early universe in Fig. 1.
Figure 3: The inflationary evolutions of the slow-roll parameters and the parameter-of-state with the e-fold number as time scale, \( N^* \approx 52.7 \) is corresponding to the time of \( k^*_\text{in} = 0.05 \text{ Mpc}^{-1} \) exiting horizon.

Now we set about addressing the inflationary observable data. In Fig. 3, at \( N^* \approx 52.7 \) the slow-roll parameters and the parameter-of-state are evaluated as follows,

\[
\begin{align*}
 \epsilon^*_V & \approx \epsilon_* \approx 0.0001047, \\
 \eta_* & = \frac{-1}{N_* + 5} \approx -0.01734, \\
 \theta_* & = \eta_* - \frac{2\alpha}{N_*} \approx -0.07469, \\
 \eta^*_V & \approx \epsilon_* + \frac{\epsilon^*_V}{3} \approx -0.0180, \\
 w^*_\varphi & = \frac{2\epsilon_*}{3} - 1 \approx -0.99993,
\end{align*}
\]

where I take \( \alpha = 1.51 \), hereinafter the “*” subscript specially indicates the \( N^* \) time. From the cosmological perturbation theory of the structure formation, we know that the above slow-roll parameters are directly related to the following inflationary observable quantities [18],

\[
\Delta^2_R(k) = \frac{H_*^2}{8\pi^2 M_p^2 \epsilon_*},
\]

\[
\frac{d\ln \Delta^2_R(k)}{d\ln k} = \frac{d\ln H^2 - d\ln \epsilon}{(\epsilon - 1)dN} = -4\epsilon_* + 2\eta_* \approx -6(\epsilon^*_V - \frac{\eta'_*}{9}) + 2\eta^*_V ,
\]

\[
\frac{dn_s}{d\ln k} \bigg|_{k_*} = \frac{dn_s}{(\epsilon_* - 1)dN} \bigg|_{k_*} = 4\epsilon'_* - 2\eta'_* = 8\epsilon_*(\eta_* - \epsilon_*) - 4\eta_*(\theta_* - \eta_*) \\
\approx -24\epsilon^*_V(\epsilon^*_V - \frac{2\eta'_*}{9}) + 16\epsilon^*_V\eta^*_V - 2\xi^2_{V^*} ,
\]

where \( k_* = 0.05 \) or \( 0.002 \text{ Mpc}^{-1} \) is the pivot scale exiting horizon at \( N^* \). \( H_{\text{in}f} \) is the expansion rate at the time of the inflation finish, which is the second input parameter in the inflation sector, I take \( H_{\text{in}f} \approx 2.72 \times 10^{12} \text{ GeV} \) to correctly fit the inflationary data. \( \frac{\rho_{\varphi}(0)}{\rho_{\varphi}(\infty)} \approx 13.83 \) is calculated out by Eqs. (18). Note that the terms with \( \frac{\eta'_*}{9} \) should not be omitted since they are the same order of magnitude as \( \epsilon^*_V \). Now one can use either one of the two sets of slow-roll parameters to calculate the inflationary quantities, put those values of Eqs. (19) into Eqs. (20),
we can perfectly reproduce all of the inflationary data in Eqs. (1), the detailed results are seen by the summarized Tab. 1 in VI Section.

The definition of $k_s$ and its calculation are given as follows,

$$\frac{\dot{a}}{a} = \frac{a_s H_s}{c} = \frac{H_0 H_{inf} H_s}{c} a_{inf} \frac{a_{inf}}{a_{ref}} \frac{a_{ref}}{a_0} = \left( \frac{3}{2} \right)^{\frac{1}{2}} \left( \Omega_\gamma(T_0) h^2 \right)^{\frac{1}{2}} \left( \frac{H_0}{c h} \right)^{\frac{1}{2}} \left( \frac{H_{inf}}{H_0} \right)^{\frac{1}{2}} \left( \frac{T_{req}}{T_0} \right)^{\frac{1}{2}} \left( \frac{\rho_{\varphi inf}}{\rho_{\varphi ref}} \right)^{\frac{1}{2}} e^{-N_*}, \quad (21)$$

where $c$ is the speed of light and $h$ is the scaling factor for Hubble expansion rate. A detailed derivation of Eq. (21) is seen by the Appendix. At the present-day, the universe expansion rate is $\frac{H_0}{c h} = \frac{100}{3 \times 10^5}$ Mpc$^{-1}$ or $H_0/h \approx 2.13 \times 10^{-42}$ GeV, the CMB temperature is $T_0 \approx 2.7255$ K or $T_0 \approx 2.35 \times 10^{-4}$ eV $[1]$. The photon energy density parameter is calculated as $\Omega_\gamma(T_0) h^2 \approx 2.5 \times 10^{-5}$ in IV section. $T_{req}$ is the reheating temperature at the time of the $\varphi_{DM}$-radiation equality, which will be solved out in III section. Note that Eq. (21) is independent of a detail value of $h$. Only input $k_s$ and $H_{inf}$ in Eq. (21), we can inversely work out $N_*$, the detailed results are seen in Tab. 1. However, it is worth emphasizing that Eq. (21) relates the above key quantities of the inflation, the reheating and the present-day together, this is rightly a characteristic of the unified model.

From Eq. (13), we derive the following relations,

$$\ddot{\varphi}(t) = \frac{\varphi(t) - \varphi(t_{inf})}{M_p}, \quad \frac{d\dot{\varphi}}{dN} = \sqrt{2\epsilon} \implies \dot{\varphi}(N) = \int_0^N \sqrt{2\epsilon} dN', \quad (22)$$

where I introduce the dimensionless reduced field $\varphi(t)$ for simplicity, it is monotonously reducing from the initial $\varphi(0)$ to the final $\varphi(t_{inf}) = 0$, thus the inflationary potential is characterized by $V(\varphi)$. The Taylor expansion of $V(\varphi)$ around $\varphi = 0$ leads to the following results,

$$V(\varphi) = V(0) + \frac{dV(\varphi)}{d\varphi} |_{\varphi=0} \varphi + \frac{d^2V(\varphi)}{2 d\varphi^2} |_{\varphi=0} \varphi^2 + \cdots \implies$$

$$M_\varphi^2 = \frac{d^2V(\varphi)}{M_p^2 d\varphi^2} = (3 - \epsilon)\eta_V H^2 \implies M_\varphi(t_{inf}) = \sqrt{2\eta_V(t_{inf}) H_{inf}}, \quad (23)$$

$$V_{min} = V(0) = \frac{1}{2} M_\varphi^2(t_{inf}) \varphi^2(t_{inf}) = \frac{2}{3} \rho_\varphi(t_{inf}) \implies \frac{\varphi(t_{inf})}{M_p} = \frac{2 H_{inf}}{M_\varphi(t_{inf})} = \sqrt{\frac{2}{\eta_V(t_{inf})}}, \quad (24)$$

where I use Eqs. (7) and Eqs. (12), $\eta_V(t_{inf}) \approx 1.4184$ is seen in Fig. 3. $M_\varphi$ is identified as the mass meanings of the inflationary field only when $\eta_V$ becomes positive, Fig. 3 shows how $M_\varphi$ is gradually grown from nothing as $\eta_V$ evolving from negative to positive, as stated earlier, this mechanism of generating $M_\varphi$ is in fact a characteristic of the inflation. Obviously, $M_\varphi(t_{inf})$ is the same size as $H_{inf}$, in addition, Eq. (24) indicates the value of $\varphi(t_{inf})$ is about $M_p$.

Finally, in order to find out the inflationary potential function, we need use $\varphi(t)$ to replace $N(t)$ as the time scale. Substitute the previous obtained $\epsilon(N)$ into Eq. (22) and make a numerical integration, then we can obtain a numerical function of $\dot{\varphi}(N)$, by which we can further translate all kinds of the evolutions with $N$ as variable in Fig. 2 into the new evolutions with $\varphi$ as variable, this is easily accomplished by a computer, the calculated results are shown in Fig. 4. $\varphi_* \approx 6.61$ is exactly corresponding to $N_* \approx 52.7$. Apparently, these evolutions in Fig. 4 are consistent
where 

\[ V(\hat{\phi}) = \frac{1}{2} \dot{\phi}^2 + \frac{1}{4} M^2 \phi^4 \]

with the ones in Fig. 2, nevertheless, they are more smooth and steady in the late phase of the inflation in comparison with the ones in Fig. 2, in addition, the amount of \( \hat{\phi}(t) \) varying in the inflation duration is much smaller than the amount of \( N(t) \) varying. In short, Fig. 4 more clearly shows all kinds of the features of the inflationary evolution, in particular, it really gives the numerical solution of \( V(\hat{\phi}) \) though we can not write out its analytical form, however, this surely provides us some insights into the inflationary potential.

### III. Reheating

At the end of the inflation, a substantial amount of the superheavy \( \varphi_{DM} \) have been generated and they have dominated the universe, thus the universe comes into the era of decelerating expansion, and it starts the reheating evolution. Because \( \varphi_{DM} \) is an unstable particle state, it can soon decay into the SM or beyond particles through some interactions with them. By contrast, the remaining \( \varphi_{DE} \) always keep the inert condensed state with negative pressure, so it has no any couplings to the SM or beyond particles, the only channel of \( \varphi_{DE} \) decay is still through it converting into \( \varphi_{DM} \). In fact, the \( \varphi_{DM} \) decay not only directly produces the hot-bath consisting of the SM particles, namely the universe is reheated, but also simultaneously generates the matter-antimatter asymmetry by a new leptogenesis mechanism, the relevant particle model will in detail be discussed in V section, here we are only concerned with the cosmological implications of the reheating process.

As the radiation energy is gradually grown from the \( \varphi_{DM} \) decay, now the total energy of the universe adds the new component \( \rho_R \). Friedmann equation, the continuity equation, and the
decay equation of $\varphi_{DM}$ collectively control the dynamics of the reheating evolution, namely

$$\rho_{\varphi_{DE}} + \rho_{\varphi_{DM}} + \rho_R = 3M_p^2 H^2, \quad \dot{\rho}_{\varphi_{DE}} + \dot{\rho}_{\varphi_{DM}} + 3H\rho_{\varphi_{DM}} = -\Gamma_{\varphi_{DM}}\rho_{\varphi_{DM}},$$

$$\dot{\rho}_R + 4H\rho_R = \Gamma_{\varphi_{DM}}\rho_{\varphi_{DM}}, \quad \frac{d(\rho_{\varphi_{DM}})}{dt} = -\beta\Gamma_{\varphi_{DM}}(\frac{\rho_{\varphi_{DM}}}{\rho_\varphi}), \quad \rho_\varphi = \rho_{\varphi_{DE}} + \rho_{\varphi_{DM}}, \quad (25)$$

$$\Gamma_{\varphi_{DM}} = \frac{M_{\varphi_{DM}}y_{eff}^2}{16\pi}, \quad (26)$$

where $\Gamma_{\varphi_{DM}}$ is the decay width of $\varphi_{DM}$, $M_{\varphi_{DM}} \approx 4.58 \times 10^{12}$ GeV is from Eq. (23), $y_{eff} \approx 0.001$ is an effective coupling coefficient provided by the particle model, we will derive Eq. (26) in V section. Eqs. (25) are a closed system of equations, their physical implications are very explicit, among others, we specify the decay equation of $\varphi_{DM}$ by the ratio of $\frac{\rho_{\varphi_{DM}}}{\rho_\varphi}$, $\beta \approx 0.03$ is the only input parameter in the reheating sector, which is determined by the global fits of the unified model, see Tab. 1.

In order to solve Eqs. (25), we can define the dimensionless energy densities and the dimensionless time variable as follows,

$$\tilde{\rho}(\tilde{t}) = \frac{\rho_1}{3M_p^2 \Gamma_{\varphi_{DM}}^2}, \quad \tilde{t} = \Gamma_{\varphi_{DM}}(t - t_{inf}) = \frac{t - t_{inf}}{\tau_{\varphi_{DM}}} \quad (0 < \tilde{t} \leq \tilde{t}_{ref} = \frac{t_{ref} - t_{inf}}{\tau_{\varphi_{DM}}}), \quad (27)$$

$$\Rightarrow \tilde{\rho}_R(0) = 0, \quad \tilde{\rho}_\varphi(0) = (\frac{H_{inf}}{\Gamma_{\varphi_{DM}}})^2, \quad \frac{\dot{\rho}_{\varphi_{DM}}(0)}{\dot{\rho}_\varphi(0)} = \frac{2}{3}, \quad \frac{\dot{\rho}_{\varphi_{DE}}(0)}{\dot{\rho}_\varphi(0)} = \frac{1}{3} = -\frac{w_\varphi(0)}{3}, \quad (28)$$

where $\tau_{\varphi_{DM}}$ denotes the $\varphi_{DM}$ lifetime, and the “ref” subscript specially indicates the time of the reheating finish, $\tilde{t}_{ref}$ will be given in the following Eq. (31). Eqs. (28) explicitly give the initial conditions of the reheating evolution. By use of Eqs. (27) and (28), we can recast Eqs. (25) as follows,

$$\frac{\dot{\rho}_\varphi + \dot{\rho}_R}{\dot{\rho}_\varphi} = \left(\frac{\Gamma_{\varphi_{DM}}}{H}\right)^2, \quad \frac{d(\dot{\rho}_\varphi)}{dt} + [3(\frac{\rho}{\Gamma_{\varphi_{DM}}}) + 1]\dot{\rho}_{\varphi_{DM}} = 0, \quad \frac{d\tilde{\rho}_R}{dt} + 4(\frac{H}{\Gamma_{\varphi_{DM}}})\tilde{\rho}_R = 0,$$

$$\frac{\dot{\rho}_{\varphi_{DM}}}{\dot{\rho}_\varphi} = \frac{2}{3} e^{-\beta \tilde{t}}, \quad \frac{\dot{\rho}_{\varphi_{DE}}}{\dot{\rho}_\varphi} = 1 - \frac{2}{3} e^{-\beta \tilde{t}} = -\frac{w_\varphi(\tilde{t})}{3}. \quad (29)$$

In Eqs. (29), $\frac{H}{\Gamma_{\varphi_{DM}}}$ is regarded as an unknown quantity, then we can numerically solve the closed system of equations, thus we obtain the reheating evolutions of $\dot{\rho}_R$, $\dot{\rho}_\varphi$, $\dot{\rho}_{\varphi_{DM}}$, $\dot{\rho}_{\varphi_{DE}}$, $w_\varphi$. Of course, the solutions must meet $H_{inf} > H \geq H_{ref}$ and $\ddot{a}(t) < 0$.

In fact, we are much more interested in the energy density parameters and the total parameter-of-state, which are defined by

$$\Omega_4(\tilde{t}) = \frac{\tilde{\rho}_1}{\dot{\rho}_\varphi + \dot{\rho}_R}, \quad w_T(\tilde{t}) = \frac{P_\varphi + P_R}{\rho_\varphi + \rho_R} = \frac{-\rho_{\varphi_{DE}} + \frac{1}{3}\rho_R}{\rho_\varphi + \rho_R} = -\frac{\Omega_{\varphi_{DE}}}{3} + \frac{\Omega_R}{3}. \quad (30)$$

Fig. 5 numerically shows the relating evolutions of the relevant energy density parameters with $\tilde{t}$ as time scale. As $\varphi_{DM}$ decay proceeding, its density is continuously decreasing from the initial $\Omega_{\varphi_{DM}} = \frac{1}{2}$ to the final $\Omega_{\varphi_{DM}} \approx 0$, namely $\varphi_{DM}$ is eventually exhausted, meanwhile, the radiation density is gradually increasing from the initial $\Omega_R = 0$ to the final $\Omega_R \approx 1$, as a result, universe is transformed from the initial $\varphi_{DM}$-dominated to the final R-dominated. Nevertheless, the most interesting thing is the curve of the $\Omega_{\varphi_{DE}}$ variation. In the early period of $0 < \tilde{t} \lesssim 100$, it
$\tilde{t} \approx 1.955$ is the time of the $\varphi_{DM} - R$ equality and $\tilde{t} \approx 400$ is the time of the reheating finish. Eventually, the radiation dominates the universe, $\varphi_{DM}$ is exhausted, and $\varphi_{DE}$ is left a tiny amount of remnants, which will become the source of the followed little inflation.

$\Omega_{\varphi_{DE}}$ is gradually reduced from the initial $\Omega_{\varphi_{DE}} \approx \frac{1}{3}$ to the minimal $\Omega_{\varphi_{DE}} \approx 10^{-7}$, but it however turn to rise in the late period of $100 \lesssim t \lesssim \tilde{t}_{\text{ref}} \approx 400$, the reason for this is that there is $\tilde{\rho}_{\varphi_{DE}} \approx \tilde{\rho}_{\varphi} \rightarrow \text{const}$ as $\tilde{\rho}_{\varphi_{DM}} \rightarrow 0$ in the last phase of the reheating, this is easily seen from the second equality in Eqs. (29), on the other hand, $\rho_{R}$ is decreasing duo to the hot expansion, so this leads to $\Omega_{\varphi_{DE}}$ rising. Therefore, till the reheating finish $\varphi_{DE}$ is not yet exhausted but rather left a tiny amount of remnants, $\Omega_{\varphi_{DE}}(\tilde{t}_{\text{ref}}) \approx 3.18 \times 10^{-6}$, which will become the source of the followed little inflation.

In Fig. 5, the time of the $\varphi_{DM} - R$ equality and the time of the reheating finish are determined by the following relations,

$$\Omega_{\varphi_{DM}}(\tilde{t}_{\text{req}}) = \Omega_{R}(\tilde{t}_{\text{req}}) \implies \tilde{t}_{\text{req}} = \frac{t_{\text{req}} - t_{\text{inf}}}{\tau_{\varphi_{DM}}} \approx 1.955,$$

$$\tilde{t} \geq \tilde{t}_{\text{ref}} \approx 400 \quad \Rightarrow \quad \tilde{\rho}_{\varphi_{DM}}(\tilde{t}) \approx 0, \quad \tilde{\rho}_{\varphi_{DE}}(\tilde{t}) \approx \tilde{\rho}_{\varphi}(\tilde{t}) \approx \text{const} , \quad (31)$$

where $\tilde{t}_{\text{req}}$ or $t_{\text{req}}$ specially indicates the time of the $\varphi_{DM} - R$ equality, which is an important time point in the reheating process. The universe energy is $\varphi_{DM}$-dominated in $t_{\text{inf}} < t \leq \tilde{t}_{\text{req}}$, but the universe enters into the radiation-dominated era once $t > \tilde{t}_{\text{req}}$. By contrast, the time point of $t_{\text{ref}}$ actually marks the reheating termination, the universe will start a new evolution of the next phase.

Fig. 6 numerically shows the reheating evolutions of the relevant parameters-of-state. In the early stage, the universe is $\varphi_{DM}$-dominated, $\Omega_{R}$ is very small, so there is $w_T \approx w_{\varphi} \approx -\frac{1}{3}$. In the late stage, the radiation gradually dominates the universe as $\varphi_{DM}$ is decaying, eventually $\Omega_{R}$ rises to $\Omega_{R} \approx 1$, only a tiny amount of $\Omega_{\varphi_{DM}}$ is left, so the final $w_T$ approaches to $w_T(\tilde{t}_{\text{ref}}) \approx \frac{1}{3}$. Obviously, the universe is indeed decelerating expansion in the reheating duration since there is always $w_T > -\frac{1}{3}$. As previously mentioned, at the end of the reheating there are $\rho_{\varphi_{DM}} \approx 0$ and
\[ t = \tilde{t} \rightarrow 400 \]

reheating finish

\[ \tilde{t} \approx 1.955 \]

\( \varphi_{DM} \sim \) R equality

\[ \rho_{\varphi} (\tilde{t}) \]

\( \rho_{\varphi} (\tilde{t}_{ref}) \)

\[ \tilde{t} = 1.955 \]

\( \varphi_{DM} \sim \) R equality

\[ \tilde{t} \approx 400 \]

reheating finish

Eventually, the radiation dominates the universe, \( \varphi_{DM} \) is exhausted, and \( \varphi_{DE} \) is left a tiny amount of remnants, which will become the source of the followed little inflation.

\[ \rho_{\varphi_{DE}} \approx \text{const}, \] so \( w_{\varphi} \) eventually approaches to \( w_{\varphi} (\tilde{t}_{ref}) \approx -1 \), which will become the starting point of the followed little inflation.

Finally, at these two time points of \( t_{req} \) and \( t_{ref} \) we can calculate the radiation temperature and the expansion rate as follows,

\[ \rho_{\varphi_{DM}} \approx \text{const} \]

\[ T_{ref} = \sqrt{\frac{\pi^2}{90}} \left( 90 \tilde{M}_p \tilde{t}_{ref} \right)^{\frac{1}{2}} \]

\( \rho_{\varphi} (\tilde{t}) \)

\( \rho_{\varphi} (\tilde{t}_{ref}) \)

\[ \tilde{t}_{req} = \sqrt{\frac{\pi^2}{90}} \left( 90 \tilde{M}_p \tilde{t}_{req} \right)^{\frac{1}{2}} \]

\[ \tilde{t}_{ref} = \sqrt{\frac{\pi^2}{90}} \left( 90 \tilde{M}_p \tilde{t}_{ref} \right)^{\frac{1}{2}} \]

where \( g_*(T_{req}) = g_*(T_{ref}) = 106.75 \) is the effective number of relativistic degrees of freedom, which includes all of the SM particles. Their calculated results are all listed in Tab. 1. Note that \( T_{req} \) is essentially the highest temperature in the reheating process. The data in Tab. 1 shows \( M_{\varphi_{DM}} > T_{req} > T_{ref} \), this means that \( \varphi_{DM} \) can not be reproduced from the hot-bath after the radiation dominates the universe, this also manifests the model self-consistency. However, it should be stressed that Eqs. (32) and (33) relate the relevant quantities of the inflation, reheating and particle physics together, this is another one characteristic of the unified model.

IV. Little Inflation and Hot Expansion

After the reheating is completed, the hot-bath of the SM particles has been formed, at the same time, the matter-antimatter asymmetry has also been generated by the leptogenesis (see V section), thus the thermal radiation in the SM sector will proceed with the evolution of the
standard hot expansion, namely the paradigm of hot big bang. By contrast, the remaining dark energy in the dark sector can newly start a new inflation, I call it as the little inflation. To some extent, the little inflation is similar to the primordial inflation discussed in II section, namely it is in fact a process of the dark matter growing from the dark energy, or a process of the dark energy converting into the dark matter, but now the universe is globally radiation-dominated instead of dark energy-dominated, therefore the little inflation is very different from the earlier inflation, its characteristic evolution will eventually lead to the dark energy and the CDM in the present-day universe.

From now on, in the little inflation we use these new subscripts of “DE” and “DM” to respectively indicate a dark energy quantity and a dark matter quantity, this is in order to distinguish from the earlier $\varphi_{DE}$ and $\varphi_{DM}$. Because the radiation energy density is now far larger than the dark energy one, the new dark matter grown from the dark energy is actually a very light and stable particle state, moreover, it is a relativistic particle in the early hot-bath. Note that the dark energy is still an inert condensed state with a negative pressure. The new relations of the energy density and the pressure in the little inflation are as follows,

$$
\rho_\varphi = \rho_{DE} + \rho_{DM}, \quad P_\varphi = P_{DE} + P_{DM} = -\rho_{DE} + \frac{\gamma_{DM}}{3} \rho_{DM} = w_\varphi \rho_\varphi, \\
\Rightarrow \frac{\rho_{DE}}{\rho_\varphi} = \frac{-3w_\varphi + \gamma_{DM}}{3 + \gamma_{DM}}, \quad \frac{\rho_{DM}}{\rho_\varphi} = \frac{3(1 + w_\varphi)}{3 + \gamma_{DM}}, \\
\gamma_{DM} = \left(\frac{P_{DM}}{E_{DM}}\right)^2 = 1 - \left(\frac{m_{DM}}{E_{DM}}\right)^2,
$$

where $m_{DM}$ is the light dark matter mass, and $\gamma_{DM}$ is a squared ratio of the dark matter momentum to its energy, which characterizes the relativistic degree of the dark matter particle. In the early phase of the little inflation, the dark matter is a relativistic particle owing to $E_{DM} \gg m_{DM}$, thus there is $\gamma_{DM} \approx 1$ and $P_{DM} = \frac{\rho_{DM}}{3}$, in this phase the dark matter is actually a dark radiation state (DR) with some pressure. As the universe expansion, the dark matter momentum is continuously decreased by the red-shift, in the late phase of the little inflation, its energy eventually reduces to $E_{DM} \approx m_{DM}$, thus there is $\gamma_{DM} \approx 0$ and $P_{DM} = 0$, so the dark matter becomes a non-relativistic particle without pressure, it is namely the CDM in the present-day universe. $m_{DM}$ is an input parameter in the little inflation sector, we will determine $m_{DM} \approx 0.391$ MeV by fitting the current energy density budget, see Tab. 1.

The little inflation in the dark sector and the hot expansion in the SM sector are simultaneously proceeding, they are both independent and interrelated. The closed system of equations of their evolutions are as follows,

$$
\rho_{DE} + \rho_{DM} + \rho_{AB} + \rho_R = 3\bar{M}_p^2 H^2, \\
\frac{d\ln \rho_{DE}}{dt} = -3(1 + w_\varphi)H \iff \dot{\rho}_{DE} + \rho_{DM} + (3 + \gamma_{DM})H\rho_{DM} = 0, \\
\frac{d\ln \rho_{DE}}{dt} = -[\kappa + 3(1 + w_\varphi)]\gamma_{DM}H, \\
\frac{d\ln \rho_{AB}}{dt} = -(3 + \gamma_B)H, \\
\frac{d\ln \rho_R}{dt} = \frac{d\ln \rho_R - d\ln (sa^3)^2}{dt} = -4H - \frac{1}{3} \frac{d\ln g_*(T)}{dt} + \frac{d\ln g'_*(T)}{dt},
$$

where $\rho_{AB}$ is the new energy component carried by the asymmetric baryon (AB), and $\gamma_B = 1 - \left(\frac{m_B}{E_B}\right)^2$ characterizes the relativistic degree of the baryon particle, its effect is similar to $\gamma_{DM}$.  

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Note that the symmetric baryon are always dealt as a part of \( \rho_R \) since they will eventually annihilate into the photon. Eq. (35) is Friedmann equation, which links the two sectors together and controls the expansion rate. Eq. (36) is the continuity equation of \( \rho_g \). Eq. (37) is the loss equation of \( \rho_{DE} \), one can obtain the growth equation of \( \rho_{DM} \) when Eq. (37) is substituted into Eq. (36), \( \kappa \) is the second input parameter in the little inflation, the global fits of the model will determine \( \kappa \approx 1.808 \), see Tab. 1. Eq. (38) and Eq. (39) are respectively the continuity equations of \( \rho_{AB} \) and \( \rho_R \), in particular, Eq. (39) is in charge of the hot expansion. In the derivation of Eq. (39), I use these relations of \( \rho_R \propto g_\ast(T)T^4 \) and \( sa^3 \propto g_\ast(T)T^3a^3 = \text{const} \) (entropy conservation), however \( g_\ast(T) \) and \( g_\ast(T) \) are depending on the temperature, so they have some effects on the radiation evolution. Note that there is \( g_\ast(T) = g_\ast(T) \) for a general case, but the two of them are not equal after the electron-positron annihilation, this will be seen in the following neutrino evolution.

To solve the above system of equations, similarly we can introduce the e-fold number of the little inflation as follows,

\[
N(t) = \ln \frac{a(t)}{a(t_{ref})} = \int_{t_{ref}}^t H(t')dt' \implies \dot{N}(t) = H(t),
\]

where the “0” subscript specially indicates the present-day time point, note that here \( \dot{N}(t) \) is positive, namely \( N \) is increasing with the time, this is different from the negative \( \ddot{N}(t) \) defined in Eq. (8), one should not confuse them. Make use of the entropy conservation, then \( N \) is given by the radiation temperature and \( g_\ast(T) \) as follows,

\[
N(T) = \ln \frac{T_{ref}}{T} + \frac{1}{3} \ln \frac{g_\ast(T_{ref})}{g_\ast(T)} \approx \frac{106.75}{2},
\]

where \( T_0 \approx 2.7255 \) K or \( T_0 \approx 2.35 \times 10^{-4} \) eV, \( g_\ast(T_{ref}) = 106.75 \), and \( g_\ast(T_0) = 2 \) (only the photon is still relativistic at the present day). We can figure out \( N_0 \approx 53.33 \) by use of the result of Eq. (33), see tab. 1, it is purely a coincidence that \( N_\ast \) and \( N_0 \) have the similar size.

Now we use \( N \) as the new time scale, the initial conditions of these energy evolutions are provided by the corresponding values at the end of the reheating, namely

\[
\begin{align*}
\rho_R(0) &= \rho_R(T_{ref}), \\
\frac{\rho_{AB}(0)}{\rho_R(0)} &= \frac{(n_B - \bar{n}_B) \bar{E}_Bs}{s} = \left( \frac{\eta_Bn_\gamma}{s} \right)_{T_0} \left( \frac{\rho_Bs}{n_B\rho_R} \right)_{T_{ref}} \approx \frac{7\eta_B}{6}, \\
\rho_{DM}(0) &= 0, \\
\frac{\rho_{DE}(0)}{\rho_R(0)} &= \frac{\rho_{DE}(T_{ref})}{\rho_R(T_{ref})} = \frac{\Omega_{DE}(T_{ref})}{\Omega_R(T_{ref})} \approx \Omega_{DE}(T_{ref}), \quad w_\phi(0) = -1,
\end{align*}
\]

where I use \( \bar{E}_B(T_{ref}) = \frac{\bar{E}_B}{n_B} \). The baryon asymmetry \( \eta_B \) is actually an input parameter provided by the particle physic, it will be calculated by the leptogenesis in V section, the global fits of the unified model will determine \( \eta_B \approx 6.08 \times 10^{-10} \). In Fig. 5 and Tab. 1, we have calculated out \( \Omega_{DE}(T_{ref}) \approx 3.18 \times 10^{-6} \).

By use of Eq. (40), then Eqs. (36) and (37) in the dark sector are recast as

\[
\frac{d\ln \rho_\phi}{dN} = -(3 + \gamma_{DM}) \frac{\rho_{DM}}{\rho_\phi}, \quad \frac{d\ln \rho_{DE}}{dN} = -\kappa \gamma_{DM} + 3(1 + w_\phi)(1 - \gamma_{DM}).
\]

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These two equations can analytically be solved for the two case of $\gamma_{DM} \rightarrow 1$ (namely the relativistic DR) and $\gamma_{DM} \rightarrow 0$ (namely the non-relativistic CDM). The dark matter transforms from the DR to the CDM when the universe temperature falls to $m_{DM}$, thus we can take $0.3m_{DM}$ as a reasonable critical temperature, from Eq. (41) the corresponding time point is then given by

$$N_{CDM} = \ln \frac{T_{ref}}{0.3m_{DM}} + \frac{1}{3} \ln \frac{106.75}{3.91},$$

(44)

where $m_{DM} \approx 0.391$ MeV will be determined by fitting the current energy density budget in Tab. 1, obviously, this critical temperature is below the electron-positron annihilation, so there is $g_*(T) = \frac{43}{p} \approx 3.91$. We can figure out $N_{CDM} \approx 33.08$ by use of $T_{ref}$ and $m_{DM}$ in Tab. 1.

When $T_{ref} > T \geq 0.3m_{DM}$, namely $0 < N \leq N_{CDM}$, the dark matter is consisting in the DR, thus there is $\gamma_{DM} \rightarrow 1$, for this case the solutions of Eqs. (43) are given by

$$\frac{\rho_{DE}(N)}{\rho_{\varphi}(N)} = e^{-\kappa N}, \quad \frac{\rho_{DM}(N)}{\rho_{\varphi}(N)} = 1 - e^{-\kappa N}, \quad \ln \frac{\rho_{\varphi}(N)}{\rho_{\varphi}(0)} = -4(N + \frac{e^{-\kappa N} - 1}{\kappa}) \implies$$

$$\ln \frac{\rho_{DE}(N)}{\rho_R(0)} = \ln \frac{\rho_{DE}(N)}{\rho_{\varphi}(N)} \frac{\rho_{\varphi}(N)}{\rho_{\varphi}(0)} \frac{\rho_{\varphi}(0)}{\rho_R(0)} = -\kappa N - 4(N + \frac{e^{-\kappa N} - 1}{\kappa}) + \ln \Omega_{DE}(T_{ref}),$$

$$\ln \frac{\rho_{DM}(N)}{\rho_R(0)} = \ln \frac{\rho_{DM}(N)}{\rho_{\varphi}(N)} \frac{\rho_{\varphi}(N)}{\rho_{\varphi}(0)} \frac{\rho_{\varphi}(0)}{\rho_R(0)} = \ln(1 - e^{-\kappa N}) - 4(N + \frac{e^{-\kappa N} - 1}{\kappa}) + \ln \Omega_{DE}(T_{ref}),$$

(45)

(46)

Here and below all kinds of the energy densities are normalized to $\rho_R(0)$. On the other hand, when $T < 0.3m_{DM}$, namely $N > N_{CDM}$, the dark matter has become the CDM, thus there is $\gamma_{DM} \rightarrow 0$, for this case the solutions of Eqs. (43) are

$$\frac{\rho_{DE}(N)}{\rho_{DM}(N_{CDM})} = 1, \quad \ln \frac{\rho_{DM}(N)}{\rho_{DM}(N_{CDM})} = -3(N - N_{CDM}) \implies$$

$$\ln \frac{\rho_{DE}(N)}{\rho_R(0)} = \ln \frac{\rho_{DE}(N_{CDM})}{\rho_R(0)} = -(\kappa + 4)N_{CDM} + \frac{4}{\kappa} + \ln \Omega_{DE}(T_{ref}),$$

$$\ln \frac{\rho_{DM}(N)}{\rho_R(0)} = \ln \frac{\rho_{DM}(N_{CDM})}{\rho_R(0)} = -3N - N_{CDM} + \frac{4}{\kappa} + \ln \Omega_{DE}(T_{ref}),$$

(47)

(48)

where $I$ omit $e^{-\kappa N_{CDM}} \ll 1$. It can be seen from Eqs. (47) and (48) that once the DR is cooled to the CDM, the conversion of the dark energy into the dark matter is immediately terminated, accordingly, the growth of the dark matter in the comoving volume is over, thus $\rho_{DE}$ will no longer change and keep a constant value in Eq. (47). From Eqs. (36) and (37), one can directly see when $\gamma_{DM} \rightarrow 0$, there are $\dot{\rho}_{DE} = 0$ and $\dot{\rho}_{DM} + 3H\rho_{DM} = 0$. Therefore, $N_{CDM}$ is exactly the time of the little inflation finish. The evolutions of the above $\rho_{DE}$ and $\rho_{DM}$ are shown in the following Fig. 7.

In a similar way, we can recast Eqs. (38) and (39) in the visible sector as

$$\frac{dln\rho_{AB}}{dN} = -(3 + \gamma_B), \quad \frac{dln\rho_R}{dN} = -4 - \frac{1}{3} \frac{dln\rho_\nu(T)}{dN} + \frac{dln\rho_\gamma}{dN},$$

(49)

they respectively characterize the hot expansion evolutions of $\rho_{AB}$ and $\rho_R$. As the universe is cooling, the massive baryon can transform from the relativistic state (namely radiation state)
to the non-relativistic one (namely matter state), the transition time point is given by

\[ N_B = \ln \frac{T_{\text{ref}}}{0.38m_B} + \frac{1}{3} \ln \frac{106.75}{61.75}, \tag{50} \]

where 0.38\(m_B\) is the baryon critical temperature, which is determined from the following Eqs. (63), it is very close to the phase-transition temperature of QCD, of course this is not accidental. We can figure out \(N_B \approx 24.14\), then the solution of \(\rho_{AB}\) is easily given as follows,

\[
\begin{align*}
\text{when } T_{\text{ref}} > T &> 0.38m_B, \ 0 < N \leq N_B, \ \gamma_B \to 1, \\
\Rightarrow \ln \frac{\rho_{AB}(N)}{\rho_R(0)} &= \ln \frac{\rho_{AB}(N) \rho_{AB}(0)}{\rho_{AB}(0) \rho_R(0)} = -4N + \ln \frac{7\eta_B}{6}, \\
\text{when } T < 0.38m_B, \ N > N_B, \ \gamma_B &\to 0, \\
\Rightarrow \ln \frac{\rho_{AB}(N)}{\rho_R(0)} &= \ln \frac{\rho_{AB}(N) \rho_{AB}(N_B)}{\rho_{AB}(N_B) \rho_R(0)} = -3(N - N_B) - 4N_B + \ln \frac{7\eta_B}{6}. \tag{52}
\end{align*}
\]

The evolution of \(\rho_{AB}\) is seen in the following Fig. 7.

The solution of \(\rho_R\) is slightly complex. Since the neutrino has been verified to have a sub-eV mass, eventually it can also transform from the relativistic state to the non-relativistic one, namely it will separate from the radiation. How the tiny mass of the neutrino is generated will be discussed in V section. In the radiation evolution, the electron-positron annihilation and the massive neutrino turning into non-relativistic state are two important time points, which are given by

\[ N_e = \ln \frac{T_{\text{ref}}}{m_e} + 1 \cdot 3 \ln \frac{106.75}{10.75}, \quad N_\nu = \ln \frac{T_{\text{ref}}}{0.29\Sigma m_\nu} + 1 \cdot 3 \ln \frac{106.75}{3.91}, \tag{53} \]

where 0.29\(\Sigma m_\nu\) is the neutrino critical temperature, which is determined by the following Eqs. (63). We can easily figure out \(N_e \approx 31.3\) and \(N_\nu \approx 48.8\) by use of the numerical values in Tab. 1. The solutions of \(\rho_R\) and \(\rho_\nu\) are given as follows,

\[
\begin{align*}
\text{when } T_{\text{ref}} > T &> m_e, \ 0 < N \leq N_e, \\
\Rightarrow \ln \frac{\rho_{\nu}(N)}{\rho_R(0)} &= -4N - \frac{1}{3} \ln g_\nu(T) + \frac{g_\nu(T)}{106.75}, \\
\text{when } m_e > T > 0.29\Sigma m_\nu, \ N_e < N \leq N_\nu, \\
\Rightarrow \ln \frac{\rho_{\nu}(N)}{\rho_R(0)} &= -4N - \frac{1}{3} \ln \frac{3.91}{106.75} + \ln \frac{3.36}{3.91}, \\
\text{when } T < 0.29\Sigma m_\nu, \ N > N_\nu, \\
\Rightarrow \ln \frac{\rho_{\nu}(N)}{\rho_R(0)} &= -4N - \frac{1}{3} \ln \frac{2}{106.75}, \\
\ln \frac{\rho_{\nu}(N)}{\rho_{\nu}(N_\nu)} &= \ln \frac{\rho_{\nu}(N) \rho_{\nu}(N_\nu) \rho_R(N_\nu)}{\rho_{\nu}(N_\nu) \rho_R(0)} = -3N - N_\nu - \frac{1}{3} \ln \frac{3.91}{106.75} + \ln \frac{1.36}{3.91}, \tag{57}
\end{align*}
\]

where I use \(\frac{\rho_{\nu}(N)}{\rho_{\nu}(N_\nu)} \approx \frac{1.36}{3.91}\) in Eq. (57). When \(N > N_\nu\), the neutrino temperature is different from the photon one, thus there is the distinction of \(g_\nu(T) = 3.36\) and \(g_\nu(T) = 3.91\) in Eq. (55). When \(N > N_\nu\), the neutrino becomes a non-relativistic particle, so it separates from the radiation, eventually the radiation is only left with the photon in Eq. (56).
Based on the above analytical solutions, Fig. 7. numerically shows the evolutions of the relevant energy densities from the reheating finish to the present day. In the SM sector, $\rho_{AB}$ (the blue curve), $\rho_R$ (the green curve) and $\rho_\nu$ (the black dotted curve) carry out the hot expansion evolutions. At $N_B \approx 24.14$ the asymmetric baryon turns into the non-relativistic state, at $N_\nu \approx 48.8$ the neutrino turns into the non-relativistic particle so that it separates from the radiation, eventually the radiation is only left with the photon since all of the symmetric matter have annihilated into the photon. By contrast, the dark sector undergoes a more complex evolution. Because the dark energy drives the little inflation, in the initial period of $0 < N \lesssim 1$, $\rho_{DM}$ (the pink dashed curve) sharply grows from nothing to the same size as $\rho_{DE}$ (the brown curve), after that $\rho_{DE}$ and $\rho_{DM}$ are continually diluted as the hot expanding of the R-dominated universe, in the meantime the conversion of $\rho_{DE}$ into $\rho_{DM}$ is not creased. At $N_{CDM} \approx 33.08$, the generated DR is turned into the CDM due to $\gamma_{DM} \to 0$, thus the little inflation is over, after then $\rho_{DE}$ no longer changes and keeps a constant. Therefore, the effect of $\gamma_{DM} \to 0$ amounts to a natural brake which can urgently stop the little inflation. At $N_{eq} \approx 44.3$ whose corresponding temperature is $\approx 1.5$ eV, the total matter ($\rho_{DM} + \rho_{AB}$) exceeds the radiation $\rho_R$, thereby the universe comes into the M-dominated era. Nevertheless, it is not far before the present $N_0 \approx 53.33$, at which the dark energy eventually exceeds the total matter, thus it newly dominates the universe. As we have observed today, the universe is currently accelerating expansion.

Finally, we can calculate the present density parameters of the relevant energy components
as follows,

\[ \Omega_i(N_0) = \frac{\rho_i(N_0)}{\rho_c(N_0)} \]

\[ \ln \Omega_i(N_0) h^2 = \ln \frac{\rho_i(N_0)}{\rho_R(0)} + 2 \ln \frac{H_{\text{ref}}}{H_0/h}, \]

\[ \sum_i \Omega_i(N_0) h^2 = h^2, \tag{58} \]

where \( \rho_c(N_0) = 3\tilde{M}_p^2 H_0^2 \) is the present-day critical energy density. All of \( \ln \frac{\rho_i(N_0)}{\rho_R(0)} \) can be calculated out by use of Eqs. (47)-(48), Eq. (52), and Eqs. (56)-(57), and \( H_{\text{ref}} \) has been worked out in Eqs. (33), therefore all of \( \Omega_i(N_0) h^2 \) are easily obtained, by which we can further predict \( h \), note that \( h \) is not an input parameter but an output result in the unified model. All kinds of the calculated results are listed in Tab. 1.

It is also very interesting to give some ratio relations among these density parameters, they are

\[ \ln \frac{\Omega_\gamma(N_0)}{\Omega_\nu(N_0)} = N_0 - N_\nu + \frac{1}{3} \ln \frac{2}{3.91} + \ln \frac{1.36}{3.91} = \ln \frac{0.29\Sigma m_\nu}{T_0} + \ln \frac{1.36}{3.91}, \tag{59} \]

\[ \ln \frac{\Omega_{AB}(N_0)}{\Omega_\gamma(N_0)} = N_0 - N_B + \frac{1}{3} \ln \frac{2}{106.75} + \ln \frac{7\eta_B}{6} = \ln \frac{0.38m_B}{T_0} + \frac{1}{3} \ln \frac{61.75}{106.75} + \ln \frac{7\eta_B}{6}, \tag{60} \]

\[ \ln \frac{\Omega_{DM}(N_0)}{\Omega_{AB}(N_0)} = N_B - N_{CDM} + \frac{4}{\kappa} + \ln \Omega_{DE}(T_{\text{ref}}) - \ln \frac{7\eta_B}{6} = \ln \frac{0.3m_{DM}}{0.38m_B} + \frac{1}{3} \ln \frac{3.91}{61.75} + \frac{4}{\kappa} + \ln \Omega_{DE}(T_{\text{ref}}) - \ln \frac{7\eta_B}{6}, \tag{61} \]

\[ \ln \frac{\Omega_{DE}(N_0)}{\Omega_{DM}(N_0)} = 3(N_0 - N_{CDM}) - \kappa N_{CDM}. \tag{62} \]

These equations clearly show that the present-day energy density budget is actually determined by these fundamental quantities of \( T_0, \Sigma m_\nu, m_B, \eta_B, m_{DM}, \kappa, \Omega_{DE}(T_{\text{ref}}) \) and \( T_{\text{ref}} \); among them, only \( \Sigma m_\nu, \eta_B, m_{DM} \) and \( \kappa \) are the input parameters of the unified model. As a self-consistency, one can check that from Eq. (58), Eq. (59) and Eq. (60), one can respectively derive the following well-known relations,

\[ \Omega_\gamma(T_0) h^2 = \frac{\pi^2 T_0^4}{45 \tilde{M}_p^2 (\frac{\rho_c}{\rho})^2}. \]

\[ \left( \frac{\rho_c}{\rho_\gamma} \right)_{T_0} = \Sigma m_\nu \left( \frac{3m_\gamma}{11\rho_\gamma} \right)_{T_0} = \frac{108\Sigma m_\nu}{11\pi^4 T_0}, \]

\[ \left( \frac{\rho_{AB}}{\rho_\gamma} \right)_{T_0} = m_B \left( \frac{\eta_B m_\gamma}{\rho_\gamma} \right)_{T_0} = \frac{36\eta_B m_B}{\pi^4 T_0}. \tag{63} \]

This thus confirms that our previous choices for the neutrino critical temperature and the baryon one are reasonable and correct. In brief, these quantities of \( T_0, \Sigma m_\nu, m_B \) and \( \eta_B \) are in charge of the current \( \Omega_\gamma, \Omega_\nu, \Omega_{AB} \) in the SM sector, whereas these quantities of \( m_{DM}, \kappa, \Omega_{DE}(T_{\text{ref}}) \) and \( T_{\text{ref}} \) are responsible for the current \( \Omega_{DE} \) and \( \Omega_{DM} \) in the dark sector. Eqs. (59)-(62) can successfully account for the current energy density budget, see Tab. 1.

V. Neutrino Mass and Baryon Asymmetry
The tiny neutrino mass and the baryon asymmetry, namely $\sum m_\nu$ and $\eta_B$, are two key cosmological parameters, as are seen in the last section. Now we address their generation mechanism by the following particle model. We can assume that after the GUT is broken, there are two species of neutral singlet fields besides the SM particles, one real scalar field $\varphi$ and one right-handed fermion $N_R$. $\varphi$ is namely the inflation field mentioned early, which has a special nature, whereas $N_R$ is a superheavy Majorana fermion, its mass is directly from the GUT breaking. The complete gauge interactions among these fields are characterized by the following Lagrangian,

$$
\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_\varphi + i N_R^\mu \partial^\mu N_R - \frac{1}{2} N_R^T C M_N N_R + h.c.
+ (\bar{N}_R Y D C N_R^T \varphi_{DM} + h.c.) + \mu H^\dagger H \varphi_{DM},
$$

(64)

where $\mathcal{L}_{SM}$ is the SM Lagrangian and $\mathcal{L}_\varphi$ has been given in Eq. (2). $C$ is the charge conjugate matrix. $M_N$ is a diagonalized mass matrix with the three eigenvalues of $M_N \sim 10^{12} < M_N^1 < M_N^3 \sim 10^{15}$ (GeV as unit). $l = (\nu_L, e_L)^T$ and $H = (H^0, H^-)^T$ are respectively the lepton and Higgs doublets of the SM. $\varphi_{DM}$ is the superheavy dark matter particle which arises from the primordial inflation, its mass is $M_{\varphi_{DM}} \approx 4.58 \times 10^{12}$ GeV, see Tab. 1. $Y_N$ and $Y_D$ are two 3 $ \times $ 3 coupling matrices, their size are generally $\lesssim 1$, note that $Y_D$ is a symmetric matrix. $\mu$ is a mass-dimensional coupling parameter, its value is set as $\mu \sim 10^8$ GeV.

From Eq. (64), the tiny neutrino mass is naturally generated by the type-I seesaw mechanism as follows [19],

$$
\mathcal{L}_\nu^{eff} = \frac{1}{2} H (Y_N M_N^{-1} Y_N^T) H^T C^T \Rightarrow \\
M_\nu = -\nu^2 (Y_N M_N^{-1} Y_N^T), \quad \sum_i m_\nu_i = \text{Tr} M_\nu \approx -\frac{\nu^2}{2 M_{N_1}} (Y_N^T Y_N)_{11},
$$

(65)

where $\nu = \sqrt{2} \times 174$ GeV is the vacuum expectation value of the Higgs field. Moderately provided $M_{N_1} \sim 10^{12}$ GeV and $(Y_N^T Y_N)_{11} \sim 0.01$, then we can naturally obtain $\sum m_\nu \sim 0.1$ eV. Based on both the neutrino oscillation experiments and the astrophysics investigations [1], we take $\sum m_\nu \sim 0.06$ eV as a suitable input value in the unified model, see Tab. 1.

From Eq. (64), which decay mode of $\varphi_{DM}$ can lead to the universe reheating and the successful leptogenesis? Because $M_{N_1}$ and $M_{\varphi_{DM}}$ are of the same order of magnitude, we can assume the reasonable limit of $M_{N_1} < M_{\varphi_{DM}} < 2 M_{N_1}$, then the direct decay of $\varphi_{DM} \rightarrow N_{1R} + N_{1R}$ is prohibited by this mass limit. On the basis of Eq. (64), the $\varphi_{DM}$ available decay channels only include, i) the two-body decay of $\varphi_{DM} \rightarrow H + H$, ii) the three-body decays of $\varphi_{DM} \rightarrow N_{1R} + l + H$ and $\varphi_{DM} \rightarrow N_{1R} + l^c + H$. Their partial widths and the total width are
Figure 8: The tree and one-loop diagrams of $\varphi_{DM} \rightarrow N_{1R} + l + H^c$. This decay process not only produces the earliest hot radiation, namely the universe reheating, but also generates the matter-antimatter asymmetry.

Calculated as follows,

$$
\Gamma(\varphi_{DM} \rightarrow H + H^c) = \frac{M_{\varphi_{DM}}}{16\pi} \left(\frac{\mu}{M_{\varphi_{DM}}}\right)^2,
$$

$$
\Gamma(\varphi_{DM} \rightarrow N_{1R} + l + H^c) = \frac{M_{\varphi_{DM}}}{32\pi} \left[ \frac{|(Y_N^1 Y_N)|^2 f(x)}{16\pi^2} \right] = \frac{M_{\varphi_{DM}} y_{eff}^2}{32\pi},
$$

$$
f(x) = f\left(\frac{M_{\varphi_{DM}}^2}{M_{N_1}^2}\right) = \frac{1}{4x^2} + \left(\frac{11}{3} + 2\ln x\right)x^{-1} - 3(1 - \ln x) - x + \frac{x^2}{12},
$$

$$\Rightarrow \Gamma_{\varphi_{DM}} = \Gamma(\varphi_{DM} \rightarrow H + H^c) + \Gamma(\varphi_{DM} \rightarrow N_{1R} + l + H^c) + \Gamma(\varphi_{DM} \rightarrow N_{1R}^c + l^c + H)
\approx \frac{M_{\varphi_{DM}} y_{eff}^2}{16\pi}, \quad (66)
$$

where the fraction in square bracket is defined as the squared effective coupling coefficient $y_{eff}^2$. Moderately provided $(Y_N^1 Y_N)|_{11} \sim 0.01, Y_{D11} \sim 1$, and $x = 1.6^2 \Rightarrow f(x) \approx 0.011$, then we figure out $y_{eff} \sim 0.001$, so it is taken as an input parameter of the unified model. Because there is $(\frac{\mu}{M_{\varphi_{DM}}})^2 \sim 10^{-8} \ll y_{eff}^2 \sim 10^{-6}$, the three-body decays absolutely dominate the total width of $\varphi_{DM}$, Eq. (66) correctly gives the decay rate of $\varphi_{DM}$. In short, all of the assumed coupling values are very moderate and self-consistent, these decays in Eq. (66) can successfully implement the universe reheating.

Fig. 8 shows the tree and one-loop diagrams of $\varphi_{DM} \rightarrow N_{1R} + l + H^c$. This decay process has the three following features. i) The lepton number is explicitly violated by two units, namely $\Delta L = 2$. ii) The $CP$ asymmetry of the decay rate is non-vanishing due to the interference between the tree amplitude and one-loop one, it is calculated as follows,

$$
A_{CP} = \frac{\Gamma(\varphi_{DM} \rightarrow N_{1R} + l + H^c) - \Gamma(\varphi_{DM} \rightarrow N_{1R}^c + l^c + H)}{\Gamma_{\varphi_{DM}}}
= \frac{\mu}{M_{\varphi_{DM}}} \frac{\sum_{i=2, 3} \text{Im}[|(Y_N^i Y_N)|_{11} Y_{D1i} Y_{D11}|^2 g(x)]}{4\pi (Y_N^1 Y_N)|_{11} Y_{D11}|^2 f(x)}
\approx \frac{M_{\varphi_{DM}}}{M_{N_1}^2} \frac{1}{3x^2} + \left(\frac{1}{2} + \ln x\right)x^{-1} - 1 + \frac{x}{6}, \quad (67)
$$

\text{where } g(x) = g\left(\frac{M_{\varphi_{DM}}}{M_{N_1}^2}\right) = \frac{1}{3x^2} + \left(\frac{1}{2} + \ln x\right)x^{-1} - 1 + \frac{x}{6}. \quad (67)
One can see from Eq. (67) that the CP-violating sources are rightly the complex phases in $Y_N$ and/or $Y_D$, the former is directly relevant to the leptonic CP violation in the low-energy experiments [20], whereas the latter purely arises from the inflation, however neither one can be ruled out as yet. In addition, the magnitude of $A_{CP}$ is actually dominated by the factor of $(\frac{\mu}{M_{\varphi_{DM}}})^2 \sim 10^{-8}$ because the numerator with $g(x)$ and the denominator with $f(x)$ are the same order of magnitude. iii) In the period of $t_{inf} < t < t_{req}$, the $\varphi_{DM}$ decay is certainly out-of-equilibrium since $\Gamma_{\varphi_{DM}} \sim H_{req} \ll H_{inf}$. After $t > t_{req}$, the radiation begins to dominate the universe and the thermal equilibrium is established, but $\varphi_{DM}$ can not be reproduced from the hot-bath due to $T_{req} < M_{\varphi_{DM}}$, see those discussions in III section and Tab. 1. In a word, the decay process of Fig. 8 fulfils the Sakharov’s three conditions [21], as a result, an asymmetry of the lepton number can indeed been generated when the reheating is completed.

In the followed hot expansion, the lepton asymmetry is partly converted into the baryon asymmetry through the SM sphaleron transition, which conserves the $B-L$ number [22]. The detailed relations are as follows,

$$Y_{B-L}(T_{req}) = \frac{n_{B-L} - \bar{n}_{B-L}}{S}|_{T_{req}} = \frac{(-2)A_{CP}}{g_*(T_{req})} \implies$$

$$\eta_B = \frac{n_B - \bar{n}_B}{n_\gamma}|_{T_0} = \frac{Y_{BS}}{n_\gamma}|_{T_0} = c_s Y_{B-L}(T_{req}) (\frac{s}{n_\gamma})_{T_0} \approx -0.024 A_{CP},$$

(68)

where $g_*(T_{req}) = 106.75$, $c_s = \frac{28}{73}$ and $(\frac{s}{n_\gamma})_{T_0} \approx 3.61$. Thus we can naturally obtain $\eta_B \sim 10^{-10}$ since $A_{CP} \sim 10^{-8}$. However, the accurate value of $\eta_B$ is determined by fitting the current $\frac{\Omega_{AB}}{\Omega_{CMB}}$ by use of Eq. (60).

Finally, we should point out that the usual thermal leptogenesis through $N_{1R} \rightarrow l + H^c$ can not occur in the model. In the reheating process, $N_{1R}$ is produced by the out-of-equilibrium decay of $\varphi_{DM}$, subsequently $N_{1R} \rightarrow l + H^c$ is only an equilibrium decay due to $\Gamma_{N_1} > \Gamma_{\varphi_{DM}} \sim H_{ req}$. After the radiation dominates the universe, $N_{1R}$ can not also be reproduced due to $M_{N_1} > T_{req}$. In brief, the particle model of Eq. (64) can simultaneously account for the tiny neutrino mass, the universe reheating and the baryon asymmetry very well.

VI. Summary for Numerical Results

Finally, I summarize all kinds of the numerical results calculated by the unified model so as to compare them with the experimental data. The used physical constants only include

$$M_p = 2.43 \times 10^{18} \text{ GeV}, \quad m_B = 0.9383 \text{ GeV},$$

$$T_0 = 2.7255 \text{ K} = 2.35 \times 10^{-4} \text{ eV}, \quad \frac{H_0}{h} = 2.13 \times 10^{-42} \text{ GeV}.$$

(69)

Tab. 1 in detail lists the input parameters of the model and its output predictions for the relevant cosmological quantities.

The unified model has in total eight input parameters. $H_{inf}$ and $\alpha$ are two inflationary parameters, their input values can excellently fit all of the inflationary data, see the inflation output results. $\beta$ is the only reheating parameter, while $\kappa$ and $m_{DM}$ are in the little inflation sector. We take two sets of typical values of $\beta, \kappa, m_{DM}$, each set of them can correctly fit the current energy density budget, see the last panel in Tab. 1, but there are small modifications in the reheating output results for the second set of these input values. The stability of the CDM
is no doubt since $m_{DM} < m_e$. The last three input parameters are from the particle model. By use of Eq. (60), we can determine $\eta_B \approx 6.08 \times 10^{-10}$ to fit the current $\Omega_{DE}$ data. These input values of $y_{eff}$ and $\sum m_\nu$ have been explained in the previous section. For $k_s = 0.05$ and $k_s = 0.002$ Mpc$^{-1}$, correspondingly there are two sets of the inflationary output results, $r_{0.002}$ is predicted to be one order of magnitude smaller than its current upper bound. Although there are no observation data of the universe heating as yet, the model gives many predictions (see the third panel), in particular, at the end of the reheating a tiny amount of residual dark energy become the source of the followed little inflation. All of the output density parameters are very well in agreement with the current data, furthermore, $h \approx 0.674$ is finely predicted. In the model, the primordial inflation occurs after the GUT phase transition, so all of the energy scales in Tab.1 are naturally below the GUT scale, of course the magnetic monopole problem is also eliminated.

Clearly, all of the numerical results are consistent and reasonable and without any fine-tuning, these output results are very well in agreement with the current measured data in Eqs. (1), in particular, we only use a small number of parameters to fit a great deal of the cosmological data, therefore this fully demonstrates the model success. Finally, we expect that the future experiments test this model.

VII. Conclusions

I suggest a grand unified framework of cosmology based on the standard gravity theory and the fundamental cosmology principle. The model gives an unified and coherent interpretation for the universe evolution from the early inflation and reheating to the late little inflation and hot expansion, moreover, it covers the origins of the neutrino mass and baryon asymmetry. In the model, the three things of inflationary field, dark energy and dark matter are unified by a
sole real scalar field with some special nature, its dynamical evolution step by step leads to the primordial inflation, the reheating, the little inflation, and eventually the current dark energy and CDM. In fact, both the primordial inflation and the little inflation are a conversion of the dark energy into the dark matter, namely a process of the dark matter growing from the dark energy. I give and solve the complete system of equations of the dynamical evolution of each stage, and also establish the inherent relationships between these processes and particle physics. All kinds of the numerical results are shown by the figures and the table.

The numerical solutions clearly show how the slow-roll inflation is successfully implemented. After the primordial inflation the generated superheavy dark matter decay into the SM particles through the Fig. 8 mode, this not only brings about the universe reheating, but also generates the matter-antimatter asymmetry. The reheating is over as the superheavy dark matter is exhausted, the R-dominated universe begins the standard hot expansion, at the same time, a tiny amount of the residual dark energy restarts a new inflation, namely the little inflation, from which the present-day dark energy and CDM are eventually derived. The whole evolution is just like a cascade of hydropower stations, the superhigh dark energy in the very early universe is step by step released so that it eventually reduced to the extreme-low dark energy in the present-day universe, thus the “cosmological constant” problem is naturally removed.

This unified model has only eight input parameters, but it can completely account for a great deal of the cosmological observations. The model not only perfectly reproduces the measured inflationary data and the current energy density budget, but also explicitly gives many interesting predictions, for instance, the dark matter and its mass are gradually growing from the dark energy as the inflation proceeding, the matter-antimatter asymmetry results from the superheavy dark matter decay in the reheating process, there is the little inflation in the dark sector besides the hot expansion in the SM sector, in addition, there are $r_{0.002} \approx 0.00153$, $m_{DM} \approx 0.391$ MeV, $\eta_B \approx 6.08 \times 10^{-10}$, $h \approx 0.674$, and so on. In short, the model is very successful and believable, so we expect that it is tested in the near future.

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Appendix
The derivation of Eq. (21) is as follows,

\[ k^* = \frac{a_s H_s}{c} = \frac{H_0 H_{\text{inf}}}{c} a_s \rho_{\text{inf}} a_{\text{req}} a_{\text{ref}} \]

\[ = \frac{H_0 H_{\text{inf}}}{c} \left[ \frac{\rho_{\varphi*}}{\rho_{\varphi\text{inf}}} \right]^{\frac{1}{2}} e^{-N_{\text{r}}} \left[ \frac{\rho_{\varphi\text{DM}}(t_{\text{req}})}{\rho_{\varphi\text{DM}}(t_{\text{inf}})} \right] \left[ \frac{\rho_R(t_{\text{req}})}{\rho_R(t_{\text{inf}})} \right] \]

\[ = \frac{H_0 H_{\text{inf}}}{c} \left[ \frac{\rho_{\varphi*}}{\rho_{\varphi\text{inf}}} \right]^{\frac{1}{2}} e^{-N_{\text{r}}} \left[ \frac{T_{\text{ref}}}{T_{\text{req}}} \right] \left[ \frac{g^* (T_{\text{req}}) T_0}{g^* (T_{\text{ref}}) T_0} \right] \]

\[ = \left[ \frac{3}{2} \frac{\Omega_0(T_0) h^2}{(H_0/c)} \right]^{\frac{1}{2}} \left[ \frac{H_{\text{inf}}}{H_0/h} \right]^{\frac{1}{2}} \left[ \frac{T_{\text{req}}}{T_0} \right]^{\frac{1}{2}} \left[ \frac{\rho_{\varphi*}}{\rho_{\varphi\text{inf}}} \right]^{\frac{1}{2}} e^{-N_{\text{r}}} \]

where I use \( a \propto \rho^{\frac{1}{4}} \) for the \( \varphi_{\text{DM}} \)-dominated stage in \( t_{\text{inf}} < t < t_{\text{req}} \) and \( a \propto \rho^{\frac{1}{4}} \) for the \( R \)-dominated stage in \( t_{\text{req}} < t < t_{\text{ref}} \); in addition, \( \rho_{\varphi\text{DM}}(t_{\text{inf}}) = \frac{2}{3} \rho_{\varphi}(t_{\text{inf}}) \), \( \rho_{\varphi\text{DM}}(t_{\text{req}}) = \rho_R(t_{\text{req}}) \), \( g^*(T_{\text{req}}) = g^*(T_{\text{ref}}) \) are employed.

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