Is BFKL factorization valid for Mueller-Tang jets?

Dimitri Colferai E-Mail: colferai@fi.infn.it

Department of Physics, University of Florence and INFN Florence

Presented at the Low-x Workshop, Elba Island, Italy, September 27–October 1 2021

The perturbative QCD description of high-energy hadroproduction of two hard jets separated by a large rapidity gap void of emission (also called Mueller-Tang jets) is based on a factorization formula of BFKL type that represents exchanges of colour-singlet objects among the external particles. This formula resums to all perturbative orders a certain class of Feynman diagrams that are supposed to dominate the cross-section in the Regge limit. However, the explicit calculations at next-to-leading logarithmic order questions the validity of such factorization when an IR safe jet algorithm is used to reconstruct jets. We show the origin of such violation of factorization, and quantify its impact for LHC phenomenology. In this connection, we estimate the impact of other contributions to the cross-section that are not included in the Mueller-Tang factorization formula — colour non-singlet exchanges — that, at low rapidity separation, compete with the singlet ones.

1 Introduction

Mueller-Tang (MT) jets [1] are important for studying perturbative high-energy QCD and the Pomeron at hadron colliders. They are characterized by final states with at least 2 jets with comparable hard transverse momenta \( k_{J1} \sim k_{J2} \gg \Lambda_{QCD} \), well separated in rapidity \( Y \equiv y_{J1} - y_{J2} \), and absence of emission in a given interval of pseudo-rapidity \( \Delta \eta \lesssim Y \) in the central region (the so-called gap). For this reason, they are also called "jet-gap-jet" events, and a typical final state is depicted in fig. 1a.

The presence of the gap suggests that these events mainly occur when the momentum exchange between the forward and backward systems is due to a colour-singlet virtual state: a non-singlet exchange would be characterized most of the times by final state radiation deposited in the central region.

A large rapidity interval is possible because at LHC the center-of-mass (CM) energy is much larger than the jet transverse energy. In this case, the coefficients of the perturbative series are enhanced by powers of \( Y \approx \log(s/k_J^2) \), and an all-order resummation of the leading terms \( \sim (\alpha_s Y)^n \) is needed for a proper determination of the amplitude.
1.1 Cross section in leading logarithmic approximation

At lowest perturbative order, a colour-singlet exchange in the $t$-channel is due to two gluons in
colour-singlet combination. At higher orders, as just mentioned, the partonic elastic amplitude
is affected by powers of $Y \simeq \log(s/k_1^2)$ due to gluon ladder-like diagrams. Such contributions can
be resummed into the so-called BFKL gluon Green function (GGF) \cite{BFKL}. It is interesting to observe
that such LL loop diagrams are both UV and IR finite.

By squaring the partonic amplitude, the LL partonic cross-section is then given by the product of 2 GGFs,
which embody the energy-dependence, and two impact factors (IFs), that couple
the gluons to the external particles. In the LL approximation the IFs are just a trivial product of
coupling constants and colour factors.

Finally, the cross section for MT jets in the LL approximation can be expressed by the factorization formula (see fig. 1b)

$$\frac{d\sigma^{(LL)}}{dJ_1 dJ_2} \simeq \int d(x_1, x_2, t_1, t_1', t_2, t_2') f_A(x_1) \Phi_A(x_1, t_1, t_2; J_1) G(x_1 x_2 s, t_1, t_2)$$

$$\times G(x_1 x_2 s, t_1', t_2') \Phi_B(x_2, t_1', t_2'; J_2) f_B(x_2).$$

(1.1)

Here $J = (y_J, k_J)$ represents the set of jet variables, the GGFs $G$ describe universal gluon dynamics,
the IFs $\Phi_i$ describe the coupling of the reggeized gluons or pomerons to the external particles,
and the PDFs $f_i$ describe the partonic content of hadrons.

1.2 First phenomenological analyses of jet-gap-jet events

The importance of considering such BFKL contributions to the cross section has been emphasized
since the first analysis by CMS. The plot in fig. 2 shows the number of events as a function of the
multiplicity of charged particles in the gap region. We see that both Herwig and Pythia are
able to describe the data if one or more particles are observed between the jets, but only Herwig
agrees in the first bin without observed particles, and this happens because Herwig includes the
contribution of colour-singlet exchange from BFKL at LL.

If one looks at differential distributions of JGJ events, like distributions in $p_\perp$ or in rapidity
distance $Y$, however, the situation is not so nice. Here LL predictions are unable to describe data
(see fig. 3a). Even if one improves the BFKL GGF by adding next-to-leading logarithmic (NLL)
contributions \cite{NLL}, none of the implementations is able to simultaneously describe all the features
of the measurements (see fig. 3b).
Figure 2: CMS measurements of multiplicity of charged particles in the gap region, and comparison with Pythia and Herwig predictions.

- Charged-particle multiplicity in the gap region between the tagged jets compared to PYTHIA and HERWIG predictions.
- HERWIG 6: include contributions from color singlet exchange (CSE), based on BFKL at LL.
- PYTHIA 6: inclusive dijets (tune Z2*), no-CSE.

Figure 3: Comparison of various differential measurements by D0 for jet-gap-jet events, and comparison with various theoretical models implementing the BFKL GGF in NLL approximation.

- Ratio $R = \frac{N_{\text{jet-gap-jet}}}{N_{\text{inclusive dijet}}}$ of jet-gap-jet events to inclusive dijet events as a function of $p_t$ and the rapidity gap $Y$.
- NLL $^*$ BFKL calculations different implementations of the soft rescutting processes (EEI models), describe many features of the data, but none of the implementations is able to simultaneously describe all the features of the measurement. Ekstedt, Enberg, Ingelman, [1703.10919]
2 Impact factor in next-to-leading logarithmic approximation

2.1 Structure of the calculation and final result

It appears thus compelling to provide a full NLL description of MT jets. The idea is to generalize the BFKL factorization formula for MT jets to the NLL approximation.

The NLL BKFL GGF is known in the non-forward case [3], but due to its complexity, only the forward version [4, 5], has been used in order to estimate the contribution of NL logarithmic terms to the cross section [6]. However, this is not relevant for our study of the impact factors.

The determination of the NL IF can be done with a NLO calculation, which is affected, of course, by IR (soft and collinear) divergencies. Actually, the very existence of NL IF is not a trivial statement. By summing virtual and real contributions at first perturbative order, one has to prove that

- the $\log(s)$ terms from virtual corrections reproduce the BFKL kernel (and this we already know);
- the const term of the virtual corrections, which are IR divergent and constitutes the virtual part of the IF, when combined with real emission terms, must provide a finite remainder, after subtraction of the collinear singularities (proportional to the Altarelli-Parisi splitting functions) to be absorbed in the PDF;

Such finite remainder defines the next-to-leading impact factor. A sketch of this decomposition is depicted in fig. 4

![Diagram of the decomposition of the (real + virtual) NLO calculation for the determination of the NL impact factors.](image)

**Figure 4**: Schematics of the decomposition of the (real + virtual) NLO calculation for the determination of the NL impact factors.

The calculation of NL IF for MT jets was performed [8, 9] using Lipatov’s effective action, and has been confirmed by our independent calculation [10]. This is the structure of the result in the case of incoming quark:

$$
\Phi(l_1, l_2, q) = \frac{\alpha_s^3}{2\pi(N_c^2 - 1)} \int_0^1 dz \int dk \, S_J(k, q, z) \, C_F \left(\frac{1 + (1 - z)^2}{z}\right) \\
\times \left\{ C_F^2 \frac{z^2 q^2}{k^2 (k - z q)^2} + C_F C_A f_1(l_1, k, q, z) + C_A^2 f_2(l_1, k, q) \right\}
$$

It is important to understand the kinematics of the process (see fig. 5): after the “upper” incoming quark interacts with the two gluons in colour-singlet, a quark and a gluon can be found in the forward hemisphere of the final state; the “lower” parton $p_2$ remains intact and is just slightly deflected in the backward hemisphere.
Let me denote with $k$ the outgoing gluon momentum, with $k$ its transverse momentum and with $z$ its longitudinal momentum fraction with respect to the parent quark; $q$ is the overall $t$-channel transferred momentum. $k$ and $z$ are integration variables. Virtual contributions are contained as delta-function contributions at $z = 0$ and $k = 0$.

We can see the quark-to-gluon splitting function $P_{gq}$ as overall factor, and then three terms with different colour structures. The integration in the phase space of the gluon and quark final state has to be restricted by an IR-safe jet algorithm $S_J$, such as the $k_\perp$-algorithm.

### 2.2 Violation of BFKL factorization

In the diffractive process we are considering, one quark moves in the backward direction and is identified with the backward jet. The other two partons, whose distance in azimuth-rapidity is denoted by $\Delta \Omega = \sqrt{\Delta \phi^2 + \Delta y^2}$, are emitted in the forward hemisphere, so as to produce at least one jet. There are 3 possibilities:

- $\Omega < R$ corresponding to a composite jet;
- $\Omega > R$ where the gluon is the jet and the quark is outside the jet cone;
- $\Omega > R$ where the quark is the jet and the gluon is outside the jet cone.

In the last configuration there is a problem due to the $dz/z$ integration of the $C_2^A f_2$ term. In fact, when the quark is the jet, the gluon can become soft and its phase-space integration is essentially unconstrained.

The limit $z \to 0$ at fixed $k$ corresponds to find the gluon in the central (and backward) region, where the emission probability of the gluon turns out to be flat in rapidity, and formally the $z$ (or $y_g$) integration diverges.

If we believe the above transition probability to be reliable at least in the forward hemisphere ($y_g > 0$), the longitudinal integration yields a $\log(s)$. But a $\log(s)$ in the IF is not acceptable, being against the spirit of BFKL factorization where all the energy-dependence is embodied in the GGFs.

All this looks strange, because one would have argued that gluon emission in the central region should be dynamical suppressed, due to the singlet exchange in the $t$-channel. We will solve this puzzle later on. For the moment, we discuss a proposal to cope with this fact in practice.

In order to avoid this problem, the authors of \cite{8,9} impose an upper limit $M_{X,\text{max}}$ on the invariant mass of the forward diffractive system. In that case, the $z$ variable is bounded from below and the $z$-integral is finite. However, a crucial question then arises: do we really need to impose a cut on the diffractive mass? Actually, \textit{can} we impose such a constraint?

If it were possible, then one could avoid $\log(s)$ terms in the IF, though at the price of introducing logarithms of the diffractive mass: $\log(M_{X,\text{max}}^2/k_J^2)$. However, from the experimental point of view, in order to impose such constraint one should be able to measure the spectator partons, i.e.,
the proton remnants (or the intact proton in case of diffractive events). Since this is not possible with the present experimental detectors at hadron colliders, other solutions must be found.

In order to find the origin of such logarithmic contributions in the IF, let us consider a pair of diagrams contributing to the $C_A^2 f_2$ term and drawn in fig. 6. It is clear that, if the two $t$-channel (vertical) gluons emitted by the lower quark are in a colour-singlet state, by colour conservation the (one or two) upper gluons cannot be in such a state, since a (coloured) gluon is emitted in the final state. Therefore we cannot claim that this diagram involves a colour-singlet exchange between the upper and lower system — the final state gluon being in the central region.

The option of defining MT jets by selecting those diagrams that involve only colour-singlet exchanges is not viable, in particular this would end up in a non-gauge-invariant procedure. We therefore claim that this problem cannot be avoided and that MT jets are not describable by the naive factorization formula originally proposed.

At this point, it remains to estimate the size of such violation and possibly to resum another set of diagrams, if the violation is sizeable. Given the fact that we cannot measure particles (partons, hadrons) below some energy threshold $E_{th} \sim 200$ MeV, we can at most require no activity above that threshold within the rapidity gap. This prescription is IR safe, because it is inclusive for gluon energies $E_g < E_{th}$ (our analysis here proceeds at partonic level). Since such soft gluons can have arbitrary values of rapidity between the two jets, we can easily estimate that the logarithmic contribution to the impact factor is of the order

$$\Phi_{\log} \sim C_A^2 \frac{E_{th}^2}{k_f^2} \log \frac{s}{k_f^2}$$

Note that this term is regular for $E_{th} \to 0$, actually it vanishes, at variance with the $O(C_F^2)$ term in the IF which diverges in the same limit. When evaluated with the values of energies and momenta of typical processes analysed at LHC, this term turns out to be small, of order 1% or less with respect to other terms [10].

Although not really needed from a quantitative point of view at the moment, one could envisage to resum such logs in the same BFKL spirit, i.e. by considering diagrams where an arbitrary number of soft (below threshold) gluons are emitted in the gap (without being detected). This enlarge considerably the number of diagrams to be taken into account. Some of them, like those in fig. 7a, could be incorporated in the IF, which however acquire a dependence on the jet rapidity as well as on the gap extension and threshold.

Other diagrams, like those of fig. 7b, contribute in a completely different way, outside the structure of the MT factorization formula. Actually, they are just diagrams of a Mueller-Navelet (two jet inclusive) process, with the restriction that the energy of all gluons emitted in the gap region is below threshold. For threshold energies $E_{th} \ll |k|$, this contribution can be easily estimated

---

**Figure 6**: Examples of diagrams that involve a non-singlet emission “above” the emitted gluon, thus producing a $\log(s)$ term in the impact factor. The pink rectangles denote colour-singlet projection.
Figure 7: Examples of diagrams resumming gluon emission from non-singlet gluon exchange. (a) Corrections to the IF; the pink rectangles denote colour-singlet projection. (b) Mueller-Navelet-like contributions outside the BFKL factorization formula (1.1).

In LL approximation, at least as far as the $E_{th}$-dependence is concerned: for soft emissions, virtual and real corrections cancel each other. Since virtual corrections are always fully taken into account, while imposing a void gap constrains only real emission, what remains is essentially equal to the virtual corrections with momenta above $E_{th}$. In the LL approximation, virtual corrections are provided by the exponentiation of the intercept of the reggeized gluon $\omega(k)$ with its internal momentum integrated below $E_{th}$, resulting in

$$\omega_{th}(k) \approx -\frac{\alpha_s N_c}{\pi} \log \frac{|k|}{E_{th}}$$

$$\frac{d\sigma_{\text{oct}}}{dt} \approx \frac{d\sigma_0}{dt} \exp \left( -\frac{\alpha_s N_c}{\pi} \log \frac{k^2}{E_{th}^2} Y \right)$$

to be compared with the MT asymptotic cross section

$$\frac{d\sigma_{\text{sing}}}{dt} \approx \frac{d\sigma_0}{dt} \left( \frac{\alpha_s C_F \pi}{2} \right)^2 \exp \left( \frac{\alpha_s N_c}{\pi} 8 \log 2 Y \right) \left[ \frac{1}{2} \alpha_s N_C \zeta(3) Y \right]^3,$$

d$\sigma_0/dt$ being the lowest order (one-gluon exchange) cross section.

Figure 8: Ratio of differential cross sections in rapidity $Y$ of non-singlet (octet) exchanges (fig. 7b) emitting gluon having energies below threshold and singlet-exchange (fig. 1b). Two values of $\alpha_s$ are considered.

Such comparison was already made by Mueller and Tang in their original paper [1], but for very large values of $Y \approx 12$ and other parameters not corresponding to LHC kinematics, where
the non-singlet exchanges is strongly suppressed with respect to the singlet ones. Repeating such comparison with realistic LHC parameters \( k = 30 \text{ GeV}, \ E_{th} = 0.2 \text{ GeV} \) we find (fig. 8) that for \( Y \sim 3 \) the two contributions are of the same order, and at \( Y \sim 4 \) the non-singlet one is still important, about 10% of the singlet one.

3 Conclusions

To summarize, we have demonstrated that, for jet-gap-jet observables, there is violation of the standard BFKL factorization at NLL level, since the IFs present logarithmically enhanced energy-dependent contributions. However such terms are rather small, below 1% for current measurements of Mueller-Tang jets at LHC, and their resummation looks not compelling.

Nevertheless, colour non-singlet contributions are expected to be non-negligible at LHC, in particular for small values of the rapidity distance \( Y \) between jets. Mueller-Navelet contrubution below threshold should in this case be included, unless NLL corrections to the latter provide a further suppression so as to render them irrelevant. But this requires further studies.

Comments: Presented at the Low-x Workshop, Elba Island, Italy, September 27–October 1 2021.

Acknowledgements

I thank Krzysztof Kutak and Leszek Motyka for useful discussions on this subject.

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 824093.

References

[1] A. H. Mueller and W. K. Tang, Phys. Lett. B 284 (1992), 123-126 doi:10.1016/0370-2693(92)91936-4

[2] L. N. Lipatov, Sov. J. Nucl. Phys. 23 (1976), 338-345;
   E. A. Kuraev, L. N. Lipatov and V. S. Fadin, [Zh. Eksp. Teor. Fiz. 72 (1977) 377] ;
   I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.

[3] V. S. Fadin and R. Fiore, Phys. Rev. D 72 (2005), 014018.

[4] V.S. Fadin and L.N. Lipatov, Phys. Lett. B 429 (1998) 127.

[5] G. Camici and M. Ciafaloni, Phys. Lett. B 386 (1996) 341; Phys. Lett. B 412 (1997) 396,
   [Erratum-ibid.Phys. Lett. B 417 (1997) 390]; Phys. Lett. B 430 (1998) 349.

[6] O. Kepka, C. Marquet and C. Royon, Phys. Rev. D 83 (2011), 034036 [arXiv:1012.3849 [hep-ph]].

[7] A. Ekstedt, R. Enberg and G. Ingelman, [arXiv:1703.10919 [hep-ph]].

[8] M. Hentschinski, J. D. Madrigal Martinez, B. Murdaca and A. Sabio Vera, Nucl. Phys. B 887 (2014),
   309-337 [arXiv:1406.5625 [hep-ph]].

[9] M. Hentschinski, J. D. M. Martinez, B. Murdaca and A. Sabio Vera, Nucl. Phys. B 889 (2014), 549-579
   [arXiv:1409.6704 [hep-ph]].

[10] D. Colferai, F. Deganutti and C. Royon, in preparation.