Vacua of $N = 10$ three-dimensional gauged supergravity

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Abstract

We study scalar potentials and the corresponding vacua of $N = 10$ three-dimensional gauged supergravity. The theory contains 32 scalar fields parametrizing the exceptional coset space $E_6(-14)/SO(10) \times U(1)$. The admissible gauge groups considered in this work involve both compact and non-compact gauge groups which are maximal subgroups of $SO(10) \times U(1)$ and $E_6(-14)$, respectively. These gauge groups are given by $SO(p) \times SO(10-p) \times U(1)$ for $p = 6, \ldots, 10$, $SO(5) \times SO(5)$, $SU(4, 2) \times SU(2)$, $G_2(-14) \times SU(2, 1)$ and $F_4(-20)$. We find many AdS$_3$ critical points with various unbroken gauge symmetries. The relevant background isometries associated with the maximally supersymmetric critical points at which all scalars vanish are also given. These correspond to the superconformal symmetries of the dual conformal field theories in two dimensions.

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1. Introduction

Gauged supergravities play an important role in many aspects of string theory. Some of them arise as effective theories of string compactifications in the presence of fluxes of various p-form fields, see, for example, [1] for a recent review. Furthermore, they are very useful in the AdS/CFT correspondence [2]. This is due to the fact that in gauged supergravity theories, supersymmetry allows scalar potentials which admit some critical points with negative cosmological constants, AdS critical points. These critical points are of particular interest in the context of the AdS/CFT correspondence because they correspond to conformal field theories on the boundary of AdS space.
In the original AdS$_5$/CFT$_4$ correspondence, critical points of $N = 8$ five-dimensional gauged supergravity found in [3] describe various phases of $N = 4$ SYM. The correspondence is now extended to other dimensions as well. These include AdS$_4$/CFT$_3$ and AdS$_3$/CFT$_2$ correspondences. The former is of interest in the sense that it might give some insight to condensed matter systems, for example, superconductors. Gauged supergravities in four dimensions are useful to this study in much the same way as five-dimensional gauged supergravities in AdS$_5$/CFT$_4$. Vacua of $N = 8$ four-dimensional gauged supergravity have been classified in [4, 5] soon after its construction [6], and recently, some new vacua of this theory have been identified in [7, 8]. Although, a lot of works have been done in finding critical points of this theory, it is expected that many critical points remain to be found. On the other hand, AdS$_3$/CFT$_2$ correspondence is a good place to test and study many aspects of the AdS/CFT correspondence. This is because there are many known two-dimensional conformal field theories, and things are more controllable in two dimensions. So we hope to understand AdS$_3$/CFT$_2$ in much more detail than the higher-dimensional analogues. In this case, three-dimensional gauged supergravities are, of course, the natural framework. In comparison with the higher-dimensional counterparts, AdS$_3$/CFT$_2$ is important not only for understanding the AdS/CFT correspondence but also for the study of black hole entropy, see [9] for a review and references therein.

Three-dimensional Chern–Simons gauged supergravity, see, for example, [10–13] and [14] for the construction, has a much richer structure than the analogous theories in higher dimensions due to the duality between vectors and scalars in three dimensions. The admissible gauge groups include compact, non-compact, non-semisimple and complex ones. Supersymmetry determines unique scalar target spaces for theories with $N > 8$ [15]. Some works have been done in studying critical points or vacua of gauged supergravities in three dimensions [16–21]. The theories considered in these works have $N = 4, 8, 9, 16$ supersymmetry, respectively. In this paper, we study $N = 10$ theory whose 32 scalar fields parametrize the coset $\frac{E_{6(-14)}}{SO(10) \times U(1)}$. The admissible gauge groups are subgroups of $E_{6(-14)}$. Some of the compact and non-compact admissible gauge groups have been classified in [14]. These are gauge groups that we will study in this work. The compact gauge groups are $SO(p) \times SO(10 - p) \times U(1)$ for $p = 6, \ldots, 10$ and $SO(5) \times SO(5)$. The non-compact gauge groups are $G_{2(-14)} \times SU(2, 1), SU(4, 2) \times SU(2)$ and $F_{4(-20)}$. All of these gauge groups are maximal subgroups of $SO(10) \times U(1)$ and $E_{6(-14)}$, respectively.

We will study some critical points of the scalar potentials in all of the gaugings mentioned above by using the technique introduced in [4]. In this ‘subgroup method’, we start by choosing a particular subgroup of the gauge group and study the potential on the restricted scalar manifold which is invariant under this subgroup. As a consequence of Schur’s lemma, the critical points found on this invariant manifold are critical points of the potential on the whole scalar manifold, 32-dimensional $\frac{E_{6(-14)}}{SO(10) \times U(1)}$ manifold in this work. This method has been used to study critical points of scalar potentials of $N = 16$ gauged supergravity in [20] and in other dimensions as well.

The paper is organized as follows. In section 2, we review some useful ingredients to construct $N = 10$ gauged supergravity theory. We use the parametrization of the scalar coset manifold $\frac{E_{6(-14)}}{SO(10) \times U(1)}$ in much the same way as the $\frac{SU(20)}{SO(10)}$ coset in $N = 9$ theory. All details of the gauge group generators and other needed information can be found in appendix A. Various vacua are given in section 3 including the background isometries of the maximally supersymmetric critical points at which all scalars vanish. The computations are carried out with the help of the computer program Mathematica [22]. We finally summarize our results and give some conclusions in section 4.
2. \( N = 10 \) three-dimensional gauged supergravity

In this section, we construct \( N = 10 \) three-dimensional gauged supergravity using the formulation given in [14]. The procedure is essentially the same as that given in [18], so we will only give the needed ingredients and refer the reader to [14] for the full detail of the construction.

We start by giving a description of symmetric spaces. In three-dimensional gauged supergravity with \( N > 8 \), scalar fields parametrize a unique coset space of the form \( G/H \).

The group \( G \) given by some non-compact real form of an exceptional group is the global symmetry of the theory with the maximal compact subgroup \( H \). The subgroup \( H \) is further decomposed into \( SO(N) \times H' \) in which \( SO(N) \) is the R-symmetry. Note that the additional factor \( H' \) does not appear when \( SO(N) \) is the maximal compact subgroup of \( G \). This is the case for \( N = 9 \) and \( N = 16 \) theories in which \( G \) is given by \( E_{6(-26)} \) and \( E_{8(8)} \), respectively.

The \( G \) generators \( t^M \) decompose into \( \{X^{ij}, X^\alpha \} \) which are generators of \( \{SO(N), H'\} \) and non-compact generators \( V^\alpha \).

In general, the ungauged Lagrangian of the three-dimensional supergravity coupled to a nonlinear sigma model is not invariant under diffeomorphisms of the sigma model target space. In the formulation of [14], the invariance of the Lagrangian is constructed from some isometries of the target space including appropriate field-dependent \( SO(N) \) transformations.

The \( G \) algebra, \( g \), is then formed by the isometries of the target space that can be extended to an invariance of the Lagrangian. As shown in [14], under the map

\[
\mathcal{V} : g \rightarrow a, \quad \mathcal{V}_M t^A = \frac{1}{2} \mathcal{V}_M t^{ij} + \mathcal{V}_M t^a + \mathcal{V}_M t^A, 
\]

the algebra \( g \) is mapped to an associative subalgebra of \( a = \{t^{ij}, t^a, t^A\} \). The algebra \( a \) is an extension of \( SO(N) \times H' \) algebra, \( \mathfrak{so}(N) \times \mathfrak{h}' \), with the commutation relations given by

\[
\begin{align*}
[t^{ij}, t^{KL}] &= -4\delta^{[i[K} f^{j]L]}, \quad [t^{ij}, t^A] = -\frac{1}{2} f^{ij,AB} t_B, \quad [t^a, t^b] = f^{ab} t^c, \\
[t^A, t^B] &= \frac{1}{2} f^{AB} t^C + \frac{1}{8} C_{\alpha} h^{\alpha} t^A, \quad [t^a, t^A] = h^a t^b.
\end{align*}
\]

where \( C_{\alpha} \) and \( h^a \) are an \( H' \) invariant tensor and anti-symmetric tensors defined in [14]. \( f_{ij} \) tensors are constructed from \( N - 1 \) almost complex structures \( f^P, p = 2, \ldots, N \). For symmetric target spaces, all the \( \mathcal{V}' \)s are given by the expansion

\[
L^{-1} V_M L = \frac{1}{2} \mathcal{V}_M X^{ij} + \mathcal{V}_M X^a + \mathcal{V}_M Y^A,
\]

and the map \( \mathcal{V} \) is now an isomorphism, see [14] for further detail. We have introduced ‘flat’ indices \( A, B, \ldots \) for the scalar manifold. The target space metric \( g_{ij}, i, j = 1, 2, \ldots, d = \text{dim } G/H \), is given by

\[
g_{ij} = e_i^A e_j^B \delta_{AB}
\]

where the vielbein \( e_i^A \) is encoded in the expansion

\[
L^{-1} \partial_i L = \frac{1}{2} Q_{ij} X^{ij} + Q_i^a X^a + e_i^A Y^A.
\]

\( Q_{ij} \) and \( Q_i^a \) are composite connections for \( SO(N) \) and \( H' \), respectively. R-symmetry indices \( I, J, \ldots = 1, \ldots, N \) and \( \alpha, \beta, \ldots = 1, \ldots, \text{dim } H' \). Finally, the coset representative \( L \) transforms under \( G \) and \( H' \) by multiplications from the left and right, respectively.

The scalar manifold of the \( N = 10 \) theory is a 32-dimensional symmetric space \( E_{6(−14)}/SO(10) \times SO(7) \). We will use the \( E_6 \) generators constructed in [24]. Note that there is an additional factor \( H' = U(1) \) in this theory in contrast to \( N = 9 \) and \( N = 16 \) theories studied in [18] and [20]. The 78 generators of \( E_6 \) are given in [23] for the first 52 generators and in [24] for the remaining 26. We can construct the non-compact form \( E_{6(−14)} \) by making 32 generators...
non-compact using ‘Weyl unitarity’. These transform as a spinor representation of \( SO(10) \) and are given by

\[
Y^A = \begin{cases} 
  i c_{A+21} & \text{for } A = 1, \ldots, 8 \\
  i c_{A+28} & \text{for } A = 9, \ldots, 16 \\
  i c_{A+37} & \text{for } A = 17, \ldots, 32 
\end{cases}
\]  

(6)

The 46 compact generators are the generators of \( SO(10) \times U(1) \) and are given in appendix A.

We now come to various gaugings described by the gauge invariant embedding tensor \( \Theta_{MN} \). This tensor acts as a projector on the symmetry group \( G \) to the gauge group \( G_0 \). The gauge generators are given by

\[
J_M = \Theta_{MN} T^N.
\]

(7)

The dimension of the gauge group is given by the rank of \( \Theta_{MN} \). The requirement that these generators form an algebra gives

\[
\left[ J_M, J_N \right] = \hat{f}^{MP}_{MN} J_P
\]

(8)

where \( \hat{f}^{MP}_{MN} \) are structure constants of the gauge group. Using the \( G \) algebra \( \left[ t^M, t^N \right] = f^{MN}_{R} T^R \), we can write (8) as

\[
\Theta_{MP} \Theta_{NQ} f^{PQ}_{R} = \hat{f}^{MN}_{MP} \Theta_{PR}.
\]

(9)

Together with the gauge invariant condition \( \hat{f}^{MN}_{MP} \Theta_{Q} + \hat{f}^{MP}_{QP} \Theta_{QN} = 0 \), this implies the so-called quadratic constraint

\[
\Theta_{PL} f^{KL}_{MN} (\Theta_{NP} T^N) = 0
\]

(10)

Supersymmetry imposes a projection constraint on \( T_{IJ,KL} \)

\[
\Theta_{MN} T^N = 0
\]

(11)

where \( \Theta_{MN} \) denotes the representation \( \Theta \) of \( SO(N) \). For symmetric target spaces, it has been shown in [14] that the embedding tensor of the admissible gauge group must satisfy

\[
\Theta_{MN} = 0.
\]

(12)

The embedding tensors for the compact gaugings with gauge groups \( SO(p) \times SO(10 - p) \times U(1) \), \( p = 6, \ldots, 10 \), and \( SO(5) \times SO(5) \) are given by [14]

\[
\Theta_{IJ,KL} = \delta_{IJ}^{KL} + \delta_{IJ}^{KL} \Theta_{U(1)} + \frac{1}{2} (5 - p) \Theta_{U(1)}
\]

(13)
where
\[ \Xi_{IJ} = \begin{cases} 2(1 - \frac{p}{10})\delta_{IJ} & \text{for } I \leq p, \\ -\frac{2}{5}\delta_{IJ} & \text{for } I > p, \end{cases} \quad \theta = \frac{p - 5}{5}. \] (16)

For \( p = 5 \), the gauge group is \( SO(5) \times SO(5) \) which lies entirely in \( SO(10) \). This is the case in which the \( U(1) \) is not gauged. The generators for these gauge groups can be obtained by choosing appropriate generators of \( SO(10) \), and the \( U(1) \) generator is simply given by \( 2\epsilon_{70} \).

We refer the reader to appendix A for further details.

Non-compact gaugings considered in this work are those given in [14]. The gauge groups are \( SU(4, 2) \times SU(2) \), \( G_{2(-14)} \times SU(2, 1) \) and \( F_{4(-20)} \). We find the following embedding tensors:

\[ \Theta_{G_{2(-14)} \times SU(2, 1)}: \Theta_{MN} = \frac{1}{3} \eta_{G_{2(-14)}} - \frac{2}{3} \eta_{SU(2, 1)} \] (17)

\[ \Theta_{SU(4, 2) \times SU(2)}: \Theta_{MN} = \eta_{SU(4, 2)} - \eta_{SU(2)} \] (18)

\[ \Theta_{F_{4(-20)}}: \Theta_{MN} = \eta_{F_{4(-20)}} \] (19)

where \( \eta_{G_0} \) is the Cartan Killing form of the gauge group \( G_0 \). The gauge generators of these three gaugings are given in appendix A.

We finally repeat the stationarity condition for the critical points of the scalar potential [14]
\[ 3A_i^{IK} A_j^{JK} + N g_{ij} A_3^{IK} A_3^{JK} = 0 \] (20)
where \( A_3^{ij} \) is defined by
\[ A_3^{ij} = \frac{1}{N^2} \left[ -2D_i D_j A_1^{ij} + g_{ij} A_1^{ij} + A_1^{[ij} f_1^{j]} + 2T_{ij} \epsilon^{ij} - 4D_i T_{lij} - 2T_{klj} f^{[ij]} \right]. \] (21)

For supersymmetric critical points, the unbroken supersymmetries are encoded in the condition
\[ A_1^{IK} A_1^{IJ} \epsilon^i = -\frac{V_0}{4g} \epsilon^i = \frac{1}{N} \left( A_1^{ij} A_1^{ij} - \frac{1}{2} N g_{ij} A_3^{ij} A_3^{ij} \right) \epsilon^i. \] (22)

The notations and all definitions are the same as those in [14]. In the next section, we will give the scalar potential for each gauging along with the corresponding critical points.

### 3. Vacua of \( N = 10 \) gauged supergravity

In this section, we give some vacua of the \( N = 10 \) gauged theory with the gaugings described in the previous section. We will also discuss the isometry groups of the background with maximal supersymmetry at \( L = 1 \). This is a supersymmetric extension of the \( SO(2, 2) \sim SO(1, 2) \times SO(1, 2) \) isometry group of AdS3. A similar study has been done in [20] and [18] for \( N = 16 \) and \( N = 9 \) theories, respectively. For the full list of superconformal groups in two dimensions, we refer the reader to [25]. As a general strategy, we give the trivial critical point in which all scalars are zero, \( L = 1 \), as the first critical point. It is also useful to compare the cosmological constants of other critical points with the trivial one. According to the AdS/CFT correspondence, the cosmological constant \( V_0 \) is related to the central charge in the dual CFT as \( c \sim \frac{1}{V_0} \), so we will give the ratio of the central charges for each nontrivial critical point with respect to the trivial critical point at \( L = 1 \). We first start with compact gaugings.
3.1. Vacua of compact gaugings

The compact gauging includes gauge groups $SO(p) \times SO(10-p) \times U(1)$ for $p = 6, \ldots, 10$ and $SO(5) \times SO(5)$. We give the scalar potential in $SO(p) \times SO(10-p) \times U(1)$ for $p = 7, \ldots, 10$ gaugings in the $G_2$ invariant scalar sector. For $SO(6) \times SO(4) \times U(1)$ gauging, we study the potential in $SO(4)_{\text{diag}}$ and $SU(3)$ sectors. Finally, for $SO(5) \times SO(5)$ gauging, we study the potential in $SO(5)_{\text{diag}}$, $SO(4)_{\text{diag}}$ and $SO(3)_{\text{diag}}$ sectors. All notations are the same as in [16] and [18].

3.1.1. $SO(10) \times U(1)$ gauging. We will study the potential in the $G_2$ invariant scalar manifold. From 32 scalars, there are four singlets under $G_2 \subset SO(p), p = 7, \ldots, 10$. These four scalars correspond to non-compact directions of $SU(2,1)$. We use the same parametrization as in [20], namely using three compact generators of the $SU(2)$ subgroup and one non-compact generator. With this parametrization, the coset representative takes the form

$$L = e^{a_1 c_{18}} e^{a_2 c_{19}} e^{a_3 c_{18}} e^{b_1 (Y_1 + Y_0)} e^{-a_1 c_{19}} e^{-a_2 c_{18}} e^{-a_3 c_{18}}.$$ (23)

This choice of $L$ will also be used in the next three gauge groups. In this $SO(10) \times U(1)$ gauging, the potential is given by

$$V = \frac{1}{2} g^2 [1 - 101 - 28 \cosh(2\sqrt{2}b_1) + \cosh(4\sqrt{2}b_1)].$$ (24)

The potential does not depend on $a_1, a_2$ and $a_3$.

The first critical point is the trivial one in which all scalars are zero. We find

$$V_0 = -64g^2, \quad A_1 = -4I_{10}.$$ (25)

We use the notation $I_n$ for an $n \times n$ identity matrix from now on. This is the critical point with $(10,0)$ supersymmetry according to our convention. The corresponding background isometry is $Osp(10|2, \mathbb{R}) \times SO(2,1)$.

The second critical point is at $b_1 = \frac{\cosh^{-1} 2}{\sqrt{2}}$ with the cosmological constant $V_0 = -100g^2$. This is a non-supersymmetric point. The ratio of the central charges between this point and the maximally supersymmetric point is

$$\frac{c_{(0)}}{c_{(1)}} = \frac{V_0^{(1)}}{V_0^{(0)}} = \frac{5}{4}.$$ (26)

Here and from now on, the notations $c_{(0)}$ and $c_{(i)}$ mean the central charges of the trivial and $i$th nontrivial critical points, respectively.

For $a_1 = a_3 = 0$, the coset representative (23) has a larger symmetry $SO(7)$. This $SO(7)$ is embedded in $SO(8)$ in such a way that it stabilizes one component of the $SO(8)$ spinor. In [20], this $SO(7)$ has been called $SO(7)^\perp$ according to a component of $8$, or $\overline{8}$, is stabilized. Our critical point is parametrized only by $b_1$, so has $SO(7)$ symmetry. Note that this point is very similar to the non-supersymmetric $SO(7) \times SO(7)$ critical point of the $SO(8) \times SO(8)$ gauged $N = 16$ theory given in [20] and the $SO(7)$ point in $SO(9)$ gauged $N = 9$ theory studied in [18]. The similarity mentioned here and in the following means that the location and the value of the cosmological constant relative to the trivial point are similar for these points. We do not know whether this is only an accident or there is a precise relation (to be specified if exists) between these critical points.

3.1.2. $SO(9) \times U(1)$ gauging. The potential in this gauging is much more complicated than the previous gauge group and depends on all four scalars. So we use the local $H = SO(10) \times U(1)$ symmetry to remove the $e^{-a_1 c_{18}} e^{-a_2 c_{19}} e^{-a_3 c_{18}}$ factor in (23) to simplify the
computation and reduce the calculation time. The potential is given in appendix C. Although we do not have a systematic way of finding critical points of this complicated potential, we find some critical points numerically.

The first critical point is the maximally supersymmetric (9,1) point
\[ a_1 = a_2 = a_3 = b_1 = 0, \quad V_0 = -64g^2, \]
\[ A_1 = \text{diag}(-4, -4, -4, -4, -4, -4, -4, -4, -4) . \]  
(27)
The background isometry is given by $Osp(9|2, \mathbb{R}) \times Osp(1|2, \mathbb{R})$.

The second critical point is given by
\[ b_1 = \frac{1}{\sqrt{2}} \cosh^{-1} \frac{7}{3}, \quad a_1 = \pi, \quad a_2 = \frac{3\pi}{2}, \quad a_3 = \frac{\pi}{2}, \quad V_0 = -\frac{1024}{9}g^2, \]
\[ A_1 = \text{diag}(-8, -8, -8, -8, -8, -8, -8, -8, 16\sqrt{\frac{2}{3}}, -16\sqrt{\frac{2}{3}}) . \]  
(28)
This $G_2$ critical point has (2,1) supersymmetry with
\[ \frac{c(0)}{c(1)} = \frac{4}{3} . \]  
(29)
This critical point should be compared with the (1,1) $G_2 \times G_2$ point in the $SO(8) \times SO(8)$ gauged $N = 16$ theory. The two points have similar locations and values of the cosmological constant relative to the trivial point.

The third critical point in this gauging is given by
\[ b_1 = \frac{1}{\sqrt{2}} \cosh^{-1} 2, \quad a_1 = a_3 = \frac{\pi}{2}, \quad a_2 = \text{arbitrary}, \quad V_0 = -100g^2, \]
\[ A_1 = \text{diag}(-7, -7, -7, -7, -7, -7, -7, -7, -5) . \]  
(30)
This is a (1,0) point with $G_2$ symmetry and
\[ \frac{c(0)}{c(2)} = \frac{5}{4} . \]  
(31)

3.1.3. $SO(8) \times SO(2) \times U(1)$ gauging. The potential in the $G_2$ sector is given by
\[ V = \frac{1}{4096} e^{-4\sqrt{2}b_1} g^2 \left[ 3(-1 + e^{4\sqrt{2}b_1})^2 \cos(4a_1) + 4(-1 + e^{4\sqrt{2}b_1})^6 \cos(2a_1) \right] \]
\[ + 170 e^{4\sqrt{2}b_1} + 27 e^{4\sqrt{2}b_1} + 4(e^{4\sqrt{2}b_1} - 1)^2 \cos^2 a_1 \cos(2a_3) \]
\[ + 8(e^{4\sqrt{2}b_1} - 1)^6 \cos^2 a_1 [2(13 + 86 e^{4\sqrt{2}b_1} + 13 e^{4\sqrt{2}b_1}) \cos(2a_3) \]
\[ + (e^{4\sqrt{2}b_1} - 1)^2 \cos^2 a_1 \cos(4a_3)] - 2 e^{4\sqrt{2}b_1}[88 549 + 21 112 \cosh(\sqrt{2}b_1) \]
\[ + 22 148 \cosh(2\sqrt{2}b_1) - 56 \cosh(3\sqrt{2}b_1) - 681 \cosh(4\sqrt{2}b_1)] \].  
(32)
The potential does not depend on $a_2$. We find the following critical points.

First of all, when $a_1 = a_2 = a_3 = b_1 = 0$, we find the maximally supersymmetric critical points. At this point, we find
\[ V_0 = -64g^2, \quad A_1 = \text{diag}(-4, -4, -4, -4, -4, -4, -4, -4, -4) . \]  
(33)
This point has (8,2) supersymmetry and $Osp(8|2, \mathbb{R}) \times Osp(2|2, \mathbb{R})$ as the background isometry group.

The next point is given by
\[ b_1 = \cosh^{-1} 2, \quad a_1 = a_3 = 0, \quad V_0 = -100g^2 . \]  
(34)
This is an $SO(7)$ non-supersymmetric point with
\[ \frac{c_0}{c_1} = \frac{5}{4}. \] (35)
This point is very similar to the non-supersymmetric $SO(7) \times SO(7)$ point of the $SO(8) \times SO(8)$ gauged $N = 16$ theory studied in [20].

The third critical point is given by
\[ b_1 = \frac{1}{\sqrt{2}} \cosh^{-1} \frac{7}{3}, \quad a_1 = 0, \quad a_3 = \frac{\pi}{2}, \quad V_0 = -\frac{1024}{9} g^2, \]
\[ A_1 = \begin{pmatrix} -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{16}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1 x_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2 x_3 \end{pmatrix} \] (36)
where
\[ x_1 = -\frac{7}{2} [5 + \cos(2a_2)], \quad x_2 = \frac{7}{3} \sin(2a_2), \quad x_3 = \frac{7}{3} [5 + \cos(2a_2)]. \] (37)
We find that this is the $(1,1)$ point with $G_2$ symmetry, and the diagonalized $A_1$ tensor is given by
\[ A_1 = \text{diag} \left( -8, -8, -8, -8, -8, -8, -8, -8, -8, \frac{16}{3}, \frac{16}{3} \right). \] (38)
The ratio of the central charges is
\[ \frac{c_0}{c_2} = \frac{4}{3}. \] (39)
This point is similar to the $G_2 \times G_2$ point with $(1,1)$ supersymmetry in $SO(8) \times SO(8)$ gauged $N = 16$ theory.

3.1.4. $SO(7) \times SO(3) \times U(1)$ gauging. In this gauging, we still work with the $G_2$ invariant scalar sector. The potential is given by
\[ V = -\frac{1}{52} g^2 [1301 + 448 \cosh(\sqrt{2} b_1) + 308 \cosh(2\sqrt{2} b_1) - 9 \cosh(4\sqrt{2} b_1)]. \] (40)
This case is very similar to the $SO(10) \times U(1)$ gauging in the sense that the potential does not depend on $a_1$, $a_2$ and $a_3$ and admits two critical points.

The first critical point is as usual at $L = \mathbf{1}$. This point is a $(7,3)$ point with
\[ V_0 = -64 g^2, \quad A_1 = \text{diag} \left( -4, -4, -4, -4, -4, -4, -4, -4, -4, 4, 4, 4 \right). \] (41)
The background isometry is $Osp(7|2, \mathbb{R}) \times Osp(3|2, \mathbb{R})$.

The second critical point is given by
\[ b_1 = \frac{1}{\sqrt{2}} \cosh^{-1} \frac{7}{5}, \quad V_0 = -\frac{1024}{3} g^2. \] (42)
The $A_1$ tensor is very complicated, so we give its explicit form in appendix B, equation (B.1). Remarkably, the complicated matrix $M_3^{(1)}$ can be diagonalized to $\text{diag} \left( 8, 8, \frac{16}{3} \right)$. This gives
\[ A_1 = \text{diag} \left( -8, -8, -8, -8, -8, -8, -8, 8, 8, \frac{16}{3} \right). \] (43)
So this critical point has \( (0,1) \) supersymmetry with
\[
\frac{c(0)}{c(1)} = \frac{4}{3},
\]
(44)
Note that this point has \( G_2 \) symmetry although it is characterized only by \( b_1 \). This is because the \( SO(7) \) in the gauge group is not the same as \( SO(7) \) \( \bar{\pm} \), and \( b_1 \) is not invariant under this \( SO(7) \). The \( SO(7) \) in the gauge group is embedded in \( SO(8) \) as \( 8_v \rightarrow 7 + 1 \). This point is similar to the \((1,1) \times \bar{G}_2 \times G_2\) point in \([20]\).

3.1.5. \( SO(6) \times SO(4) \times U(1) \) gauging. \ We first study the potential in the \( SO(4)_{\text{diag}} \) scalar sector. There are four singlets in this sector corresponding the non-compact directions of \( SO(2,2) \sim SO(2,1) \times SO(2,1) \). We parametrize the coset representative by
\[
L = e^{a_i[V_i,V_j]} e^{b_i[V_i,V_j]} e^{-a_i[V_i,V_j]} e^{b_i[V_i,V_j]} e^{-a_i[V_i,V_j]},
\]
(45)
where
\[
V_1 = j_1 + j_2,
V_2 = j_3 - j_4,
V_3 = j_3 + j_4,
V_4 = j_1 - j_2,
\]
(46)
and
\[
j_1 = Y_1 + Y_5 - Y_9 + Y_{13} - Y_{17} - Y_{21} + Y_{30} + Y_{32},
j_2 = Y_2 + Y_{10} - Y_{11} + Y_{18} + Y_{19} - Y_{28} + Y_{31} + Y_3,
j_3 = Y_4 + Y_7 + Y_{12} - Y_{15} + Y_{20} + Y_{23} + Y_{26} - Y_{27},
j_4 = Y_6 - Y_8 + Y_{14} + Y_{16} - Y_{22} + Y_{24} + Y_{25} - Y_{29}.
\]
(47)
We find the potential
\[
V = -2 e^{-4\sqrt{2}[b_1 + b_2]} [1 + 4 e^{4\sqrt{2}b_1} + e^{8\sqrt{2}b_1} + 4 e^{4\sqrt{2}b_2} + e^{8\sqrt{2}b_2} + 12 e^{4\sqrt{2}[b_1 + b_2]} + e^{8\sqrt{2}[b_1 + b_2]} + 4 e^{4\sqrt{2}[b_1 + 2b_2]} + 4 e^{4\sqrt{2}[b_1 + 2b_1]_b}] g^2.
\]
(48)
There is no non-trivial critical point in this potential. So there is no critical point with \( SO(4)_{\text{diag}} \) symmetry.

Next, we will consider the \( SU(3) \) invariant sector. The \( SU(3) \) is a subgroup of \( SO(6) \sim SU(4) \). There are eight singlets in this sector. The coset representative is parametrized by
\[
L = e^{a_i[V_i,V_j]} e^{b_i[V_i,V_j]} e^{-a_i[V_i,V_j]} e^{b_i[V_i,V_j]} e^{-a_i[V_i,V_j]},
\]
(49)
in which the eight scalars correspond to non-compact directions of \( SU(2,2) \). As usual, we have used the local \( H \) symmetry to simplify the parametrization of \( L \). The potential is given in a separate Mathematica file. We find two critical points.

The trivial \((6,4)\) critical point at \( L = 1 \) is given by
\[
V_0 = -64g^2,
A_1 = \text{diag}(-4, -4, -4, -4, -4, 4, 4, 4, 4, 4).
\]
(50)
The background isometry is \( Osp(6|2, \mathbb{R}) \times Osp(4|2, \mathbb{R}) \).

The nontrivial critical point is given by
\[
a_i = \frac{\pi}{2}, \quad i = 1, \ldots, 6,
b_1 = b_2 = \cosh^{-1} \sqrt{3}, \quad V_0 = -144g^2,
A_1 = \text{diag}(-10, -10, -10, -10, -10, -10, 6, 6, 10, 10).
\]
(51)
This point preserves (0,2) supersymmetry and SU(3) symmetry. The ratio of the central charges is
\[ \frac{c(0)}{c(1)} = \frac{3}{2}, \]  
(52)

3.1.6. $SO(5) \times SO(5)$ gauging. We start with the potential in the $SO(5)_{\text{diag}}$ scalar sector. There are two singlets in this sector corresponding to the non-compact directions of $SL(2)$. We parametrize the coset representative by
\[ L = e^{a_1 V} e^{b_1 U} e^{-a_1 V} \]  
(53)
where the compact and non-compact generators of $SL(2)$ are given by
\[ V = \frac{1}{\sqrt{2}} \left( c_{11} - c_{17} + c_{32} - c_{48} + c_{75} + \sqrt{3} \tilde{c}_{70} \right), \]  
(54)
\[ U = Y_3 - Y_5 - Y_{12} + Y_{16} + Y_{17} - Y_{18} + Y_{27} + Y_{29}. \]  
(55)
The potential is given by
\[ V = -8g^2 (5 + 3 \cosh(4b_1)) \]  
(56)
which does not have any non-trivial critical points.

We then move to smaller unbroken gauge symmetry namely $SO(4)_{\text{diag}}$. The parametrization of $L$ is the same as in (45). The potential turns out to be the same as that of $SO(6) \times SO(4) \times U(1)$ gauging and, of course, does not have any non-trivial critical points.

To proceed further, we need to reduce the residual symmetry to a smaller group. The next sector we will consider is $SO(3)_{\text{diag}}$. There are eight singlets in this sector. These are non-compact directions of $SO(4, 2)$ $\sim$ $SU(2, 2)$. We parametrize the coset representative in this sector by
\[ L = e^{a_1 V} e^{a_2 c_{14}} e^{a_3 c_{15}} e^{a_4 c_{19}} e^{a_5 c_{20}} e^{a_6 c_{21}} e^{b_1 Z_1} e^{b_2 Z_2} \]  
(57)
where
\[ Z_1 = Y_1 + Y_{11} - Y_{20} - Y_{29}, \quad Z_2 = Y_2 + Y_{13} - Y_{24} + Y_{27}. \]  
(58)
The potential depends on all eight scalars. Its explicit form is given in a separate Mathematica file.

The trivial (5,5) critical point at $L = I$ is characterized by
\[ V_0 = -64g^2, \quad A_1 = \text{diag} (-4, -4, -4, -4, 4, 4, 4, 4). \]  
(59)
The corresponding background isometry group is $Osp(5|2, \mathbb{R}) \times Osp(5|2, \mathbb{R})$.

We find a nontrivial critical point given by
\[ a_i = \frac{\pi}{2}, \quad i = 1, \ldots, 6, \quad b_2 = 0, \]  
\[ b_1 = \frac{\cosh^{-1} 5}{2}, \quad V_0 = -256g^2. \]  
(60)
\[ A_1 = \text{diag} (-8, -8, -8, 16, 16, -16, 16, 16, 16, 16, 16). \]

This critical point has (3,0) supersymmetry with the ratio of the central charges
\[ \frac{c(0)}{c(1)} = 2. \]  
(61)
3.2. Vacua of non-compact gaugings

We now consider non-compact gaugings with gauge groups $SU(4, 2) \times SU(2)$, $G_{2(-14)} \times SU(2, 1)$ and $F_{4(-20)}$. At $L = \mathbf{1}$, the gauge group is broken down to its maximal compact subgroup, and the bosonic part of the background isometry is formed by this subgroup and $SO(2, 2)$. These three gauge groups contain $SU(3)$ subgroup, so we study the potential in the $SU(3)$ scalar sector in all non-compact gaugings. For $G_{2(-14)} \subset SU(2, 1)$ and $F_{4(-20)}$ gaugings, the $SU(3) \subset G_2$ sector consists of eight scalars which is twice the number of scalars in the $G_2$ sector. The $SU(3)$ is embedded in $G_2$ as $7 \rightarrow 3 + \tilde{3} + \mathbf{1}$. The eight scalars correspond to non-compact directions of the $SO(4, 2) \sim SU(2, 2) \subset E_{6(-14)}$. For $SU(4, 2) \times SU(2)$ gauging, the $SU(3)$ is embedded in $SU(4) \subset SU(4, 2)$ as $4 \rightarrow 3 + \mathbf{1}$. Similarly, the eight scalars are described by non-compact directions of $SU(2, 2)$. This sector is essentially the same as that used in $SO(6) \times SO(4) \times U(1)$ gauging.

Fortunately, we do not need to deal with all eight scalars. In these three gaugings, four of the eight $SU(3)$ singlets lie along the gauge group, so only four directions orthogonal to the gauge group are relevant. This is because the singlets which are parts of the gauge group will drop out from the potential and correspond to flat directions of the potential. The relevant four singlets are contained in the $SU(2, 1)$ subgroup of $SU(2, 2)$. We also study the potentials in other sectors specific to each gauging. The details of these sectors will be explained below.

3.2.1. $G_{2(-14)} \times SU(2, 1)$ gauging. If we study the potential in the $G_2$ sector in this gauging, we will find the constant potential. This is because all scalars in the $G_2$ sector are parts of the gauge group and will drop out from the potential. We then start with the $SU(3) \subset G_2$ sector. As discussed above, this sector contains four relevant scalars parametrized by

$$L = e^{bi_1\phi_1}e^{bi_2\phi_2}e^{bi_3\phi_3}e^{bi_4\phi_4}e^{-ai_1\phi_1}e^{-ai_2\phi_2}e^{-ai_3\phi_3}e^{-ai_4\phi_4}.$$  

(62)

The potential is given by

$$V = \frac{1}{16\pi^2} g^2 \{-101 - 28 \cosh(2\sqrt{2}b_1) + \cosh(4\sqrt{2}b_1)\}.$$  

(63)

There are two critical points. The first one is the trivial critical point given by $L = \mathbf{1}$ and

$$V_0 = -\frac{64}{9} g^2, \quad A_1 = \text{diag}( -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}, \frac{2}{3}).$$  

(64)

We find that this point has $(7,3)$ supersymmetry. The symmetry of this point is given by the maximal compact subgroup $G_2 \times SU(2) \times U(1)$ of $G_{2(-14)} \times SU(2, 1)$. The left-handed supercharges transform as 7 under $G_2$ while the right-handed supercharges transform as 3 under the $SU(2) \sim SO(3)$. So the background isometry is given by $G(3) \times Osp(3|2, \mathbb{R})$.

The second critical point is characterized by

$$b_1 = \frac{\cosh^{-1} \frac{12}{\sqrt{2}}}{\sqrt{2}}, \quad V_0 = -\frac{100}{9} g^2.$$  

(65)

$$A_1 = \begin{pmatrix}
-\frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{7}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{11}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{7}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{7}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & y_1 & y_4 & y_5 \\
0 & 0 & 0 & 0 & 0 & 0 & y_4 & y_2 & y_6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & y_5 & y_6 & y_3 \\
\end{pmatrix}.$$
where
\begin{align*}
y_1 &= \frac{1}{6}[13 - \cos(2a_1) - 2 \cos^2 a_1 \cos(2a_2)], \\
y_2 &= \frac{1}{6}[13 + \cos(2a_1) - 2 \cos(2a_2) \sin^2 a_1], \\
y_3 &= \frac{1}{3}(6 + \cos(2a_2)), \\
y_4 &= \frac{1}{3} \cos^2 a_2 \sin(2a_1), \\
y_5 &= -\frac{1}{3} \cos a_1 \sin(2a_2), \\
y_6 &= \frac{1}{3} \sin a_1 \sin(2a_2).
\end{align*}

We can diagonalize \( A_1 \) to
\[ A_1 = \text{diag}(-\frac{11}{3}, -\frac{7}{3}, -\frac{2}{3}, -\frac{7}{3}, -\frac{2}{3}, \frac{7}{3}, \frac{5}{3}) \]
from which we find that this is a \((0,1)\) supersymmetric critical point. The ratio of the central charges relative to the \( L = 1 \) point is
\[ \frac{c_{(0)}}{c_{(1)}} = \frac{5}{4}. \]

This \( SU(3) \) point is closely related to the \((0,1)\) \( SU(3) \) point in the \( G_{2(-14)} \times SL(2) \) gauged \( N = 9 \) theory in [18].

We now study the potential in a different sector, \( SU(2) \text{diag} \) sector. From the \( SU(3) \) sector discussed above, the next symmetry to be considered could be the \( SU(2) \subset SU(3) \). In general, we expect more scalars than those appearing in the \( SU(3) \) sector. This will make the calculation take much longer time. We then consider the \( SU(2) \text{diag} \) sector in which \( SU(2) \text{diag} \subset SU(2) \times SU(2) \). The first and second \( SU(2) \)'s are subgroups of \( SU(3) \subset G_{2(-14)} \) and \( SU(2, 1) \), respectively. There are four singlets in this sector corresponding to the non-compact directions of \( SO(4, 1) \sim Sp(1, 1) \). We choose to parametrize the coset representative by applying three \( SO(3) \subset SO(4) \sim SO(3) \times SO(3) \) rotations as follows:
\[ L = e^{a_1 c_{(2)}} e^{a_2 c_{(2)}} e^{a_3 c_{(2)}} e^{b_1 (Y_{12} - Y_{13} + Y_{23})} e^{-a_3 c_{(2)}} e^{-a_2 c_{(2)}} e^{-a_1 c_{(2)}}. \]

The potential is
\[ V = \frac{1}{12} g^2 [-269 - 192 \cosh(2b_1) - 52 \cosh(4b_1) + \cosh(8b_1)]. \]

There is one non-trivial critical point given by
\[ b_1 = \cosh^{-1} \sqrt{2}, \quad V_0 = -16g^2. \]

This is a supersymmetric point with the associated \( A_1 \) tensor given in appendix B, equation (B.3). After diagonalization, we find
\[ A_1 = \text{diag}(-4, -4, -4, -4, -\frac{10}{3}, -2, -2, 2, 2, 2) \]
which gives \((2,3)\) supersymmetry. The ratio of the central charges is
\[ \frac{c_{(0)}}{c_{(2)}} = \frac{3}{2}. \]

This critical point has \( SU(2) \text{diag} \times U(1) \) symmetry.

3.2.2. \( F_{4(-20)} \) gauging. In this gauging with a simple gauge group, we study the potential in the \( G_2 \) and \( SU(3) \) scalar sectors. We start with the \( G_2 \) sector. Two of the four scalars are parts of the gauge group, so we only need to parametrize the coset representative with the other two scalars. These two scalars correspond to the non-compact directions of \( SL(2) \). The \( L \) is then parametrized by
\[ L = e^{a_1 c_{(2)}} e^{b_1 (Y_{23} + Y_{32})} e^{-a_1 c_{(2)}}. \]
The potential is
\[ V = \frac{g^2}{8} \left[ -101 - 28 \cosh(2\sqrt{2}b_1) + \cosh(4\sqrt{2}b_1) \right]. \]  
(75)

There are two critical points. The first one is trivial and given by
\[ L = I, \quad V_0 = -16g^2, \quad A_1 = \text{diag} (-2, -2, -2, -2, -2, -2, -2, -2). \]  
(76)

This is the maximally supersymmetric point with (9,1) supersymmetry. The gauge symmetry is broken down to its maximal compact subgroup \( SO(9) \), and the background isometry is \( OSP(9|2,\mathbb{R}) \times OSP(1|2,\mathbb{R}) \).

The second critical point is given by
\[ b_1 = \frac{\cosh^{-1} 2}{\sqrt{2}}, \quad V_0 = -25g^2, \quad A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{7}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{2} & \end{pmatrix} \]  
(77)

where
\[ w_1 = -3 - \frac{1}{2} \cos(2a_1), \quad w_2 = \frac{1}{2} \left[ -6 + \cos(4a_1) \right], \quad w_3 = \cos a_1 \sin a_1. \]  
(78)

The \( A_1 \) tensor can be diagonalized to
\[ A_1 = \text{diag} \left( \frac{11}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2} \right). \]  
(79)

This critical point is a (1,0) point with
\[ \frac{c_{(0)}}{c_{(1)}} = \frac{5}{4} \]  
(80)

and preserves \( SO(7) \subset SO(9) \subset F_{4(-20)} \) symmetry.

In the \( SU(3) \) sector, there are eight singlets, but four of them are parts of the \( F_{4(-20)} \). So there are four singlets orthogonal to the gauge group. These are non-compact directions of \( SU(2,1) \), and \( L \) can be parametrized by
\[ L = e^{a_1 e_i c_{a_1}} e^{a_2 e_i c_{a_2}} e^{a_3 e_i c_{a_3}} e^{b_1 e_i e_j c_{a_4}} e^{-a_1 e_i c_{a_1}} e^{-a_2 e_i c_{a_2}} e^{-a_3 e_i c_{a_3}}. \]  
(81)

The potential is given by
\[ V = \frac{g^2}{8} \left[ -101 - 28 \cosh(2\sqrt{2}b_1) + \cosh(4\sqrt{2}b_1) \right] \]  
(82)

which is the same as the potential in the \( G_2 \) sector. The non-trivial critical point is at the same position and cosmological constant, \( b_1 = \cosh^{-1} 2, V_0 = -25g^2 \). The residual symmetry is \( SO(7) \) as in the previous critical point. Although the \( A_1 \) tensor in this case is more complicated, it is the same as (79) after diagonalization. The explicit form of \( A_1 \) is given in appendix B, equation (B.5).
3.2.3. $SU(4, 2) \times SU(2)$ gauging. This gauging is the most difficult one to find a suitable scalar sector in order to reveal nontrivial critical points and still have a manageable number of scalars. We start with the $SO(4)_{\text{diag}}$ scalar sector. The $SO(4)_{\text{diag}}$ is formed by taking the subgroup $SU(2) \times SU(2) \times SU(2) \times SU(2)$ of $SU(4, 2) \times SU(2)$. The first two $SU(2)$’s are subgroups of $SU(4) \subset SU(4, 2)$, the third $SU(2)$ is the $SU(2) \subset SU(4, 2)$. Our $SO(4)_{\text{diag}}$ is the diagonal subgroup of $(SU(2) \times SU(2)) \times (SU(2) \times SU(2)) \sim SO(4) \times SO(4)$. There are two singlets in this sector. These are non-compact directions of $SL(2)$, and $L$ can be parametrized by

$$L = e^{b_1 c_{13} + b_2 c_{15}} e^{b_3 Y} e^{-a_1 c_{13}},$$

which, unfortunately, gives a constant potential $V = -16g^2$. So we move to a smaller residual symmetry to obtain a non-trivial structure of the potential.

We now study the potential in the scalar sector parametrizing the $SU(3)$ invariant manifold. This $SU(3)$ is a subgroup of $SU(4) \subset SU(4, 2)$. The eight singlet scalars in this sector are the non-compact directions of $SO(4, 2) \sim SU(2, 2)$. The four directions which are orthogonal to the gauge group are non-compact directions of $SU(2, 1) \subset SU(2, 2)$. The coset representative is given by

$$L = e^{a_1 (c_{13} + c_{15})} e^{a_2 (c_{15} + c_{17})} e^{a_3 (c_{17} - c_{15})} e^{b_3 Y} e^{-a_1 (c_{23} - c_{25})} e^{-a_2 (c_{19} + c_{13})} e^{-a_3 (c_{15} + c_{17})}.$$ (84)

We find the potential

$$V = -2g^2(5 + 3 \cosh(2b_1))$$ (85)

which, again, does not admit any non-trivial critical points.

The next sector we will study is $SU(2)_{\text{diag}}$. This symmetry is a diagonal subgroup of $SU(2) \times SU(2)$ in which the first $SU(2)$ is a subgroup of $SU(4) \subset SU(4, 2)$, and the second $SU(2)$ is the $SU(2)$ factor in the gauge group. There are four scalars in this sector. These scalars are non-compact directions of $SU(2, 1)$, and $L$ can be parametrized by

$$L = e^{b_1 c_{13} + b_2 c_{15}} e^{b_3 Y} e^{-a_1 c_{13}} e^{2a_2 c_{17}} e^{-a_3 c_{15}}.$$ (86)

where

$$Y = Y_1 - Y_6 - Y_{12} - Y_{16} + Y_{17} + Y_{18} + Y_{30} + Y_{31}.$$ (87)

The corresponding potential is

$$V = \frac{g^2}{8} [-101 - 28 \cosh(4\sqrt{2}b_1) + \cosh(8\sqrt{2}b_1)].$$ (88)

We now discuss its trivial critical point at $L = 1$. This point is characterized by

$$V_0 = -16g^2, \quad A_1 = \text{diag}(-2, -2, -2, -2, -2, 2, 2, 2, 2).$$ (89)

The critical point has $(6, 4)$ supersymmetry. The gauge group is broken down to its maximal compact subgroup $SU(4) \times SU(2) \times U(1)$ under $SU(4) \sim SO(6)$ while the right-handed supercharges transform as 4 under $SU(2) \sim SU(2) \sim SO(4)$. So the background isometry is given by $Osp(6|2, \mathbb{R}) \times Osp(4|2, \mathbb{R})$. The nontrivial critical point with $SU(2)_{\text{diag}} \times SU(2) \times SU(2) \times U(1)$ symmetry is given by

$$b_1 = \frac{1}{\sqrt{2}} \cosh^{-1} \sqrt{\frac{3}{2}}, \quad V_0 = \frac{5g^2}{2}.$$ (90)
G 11

Minkowski and dS vacua have been found. All critical points found in this paper are listed in

In this paper, we have studied critical points of

So this is a (4,0) point with

The associated A 1 tensor is given in appendix B, equation (B.7), which can be diagonalized to

Hence, the critical point is (4,0) and the unbroken SUSY gauge symmetry is

Correspondingly, the critical points

Some critical points of


g = SO(10) × U(1) along with their scalar potentials and the

g = SO(10) × U(1) along with their scalar potentials and the

It is interesting to study gaugings with other gauge groups which are not

The gauge groups considered in this work are only maximal subgroups of SO(10) × U(1)

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Furthermore, studies of RG flows between

Table 1. Some critical points of N = 10 gauged supergravity in three dimensions.

| Critical point | Gauge group | V₀ | Unbroken SUSY | Unbroken gauge symmetry |
|---------------|-------------|----|--------------|-------------------------|
| 1             | SO(10) × U(1) | -64g² | (10, 0) | SO(10) × U(1) |
| 2             | SO(10) × U(1) | -100g² | – | SO(7) |
| 3             | SO(9) × U(1) | -64g² | (9, 1) | SO(9) × U(1) |
| 4             | SO(9) × U(1) | -\frac{1004}{4}g² | (2, 1) | G₂ |
| 5             | SO(9) × U(1) | -100g² | (1, 0) | G₂ |
| 6             | SO(8) × SO(2) × U(1) | -64g² | (8, 2) | SO(8) × SO(2) × U(1) |
| 7             | SO(8) × SO(2) × U(1) | -100g² | – | SO(7) |
| 8             | SO(8) × SO(2) × U(1) | -\frac{1004}{4}g² | (1, 1) | G₂ |
| 9             | SO(7) × SO(3) × U(1) | -64g² | (7, 3) | SO(7) × SO(3) × U(1) |
| 10            | SO(7) × SO(3) × U(1) | -\frac{1004}{4}g² | (0, 1) | G₂ |
| 11            | SO(6) × SO(4) × U(1) | -64g² | (6, 4) | SO(6) × SO(4) × U(1) |
| 12            | SO(6) × SO(4) × U(1) | -144g² | (0, 2) | SU(3) |
| 13            | SO(5) × SO(5) | -64g² | (5, 5) | SO(5) × SO(5) |
| 14            | SO(5) × SO(5) | -256g² | (3, 0) | SO(3) diag |
| 15            | G_{2(-14)} × SU(2, 1) | -\frac{64}{3}g² | (7, 3) | G_{2(-14)} × SU(2) × U(1) |
| 16            | G_{2(-14)} × SU(2, 1) | -\frac{100}{9}g² | (0, 1) | SU(3) |
| 17            | G_{2(-14)} × SU(2, 1) | -16g² | (2, 3) | SU(2) diag × U(1) |
| 18            | F_{4(-20)} | -16g² | (9, 1) | SO(9) |
| 19            | F_{4(-20)} | -25g² | (1, 0) | SO(7) |
| 20            | SU(4, 2) × SU(2) | -16g² | (6, 4) | SU(4) × SU(2) × SU(2) × U(1) |
| 21            | SU(4, 2) × SU(2) | -25g² | (4, 0) | SU(2) diag × SU(2) × SU(2) × U(1) |

The associated A 1 tensor is given in appendix B, equation (B.7), which can be diagonalized to

Hence, the critical point is (4,0) and the unbroken SUSY gauge symmetry is

4. Conclusions

In this paper, we have studied critical points of N = 10 three-dimensional gauged supergravity

with both compact and non-compact gauge groups. Remarkably, all critical points found in

this paper are AdS critical points. This is in contrast to the results of [20] in which some

Minkowski and dS vacua have been found. All critical points found in this paper are listed in

table 1.

The gauge groups considered in this work are only maximal subgroups of SO(10) × U(1)

and E_{6(-14)}. It is interesting to study gaugings with other gauge groups which are not

maximal subgroups of SO(10) × U(1) and E_{6(-14)} along with their scalar potentials and the

corresponding critical points. In particular, non-semisimple gaugings are very interesting in the

sense that they are related to semisimple Yang–Mills gaugings which arise from dimensional

reductions of higher-dimensional theories [26]. Furthermore, studies of RG flows between
critical points identified in this work are of particular interest in studying deformations of the dual two-dimensional CFTs. We hope to give further results on these issues in future works.

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Appendix A. Essential formulae

In this appendix, we give all necessary formulae in order to obtain the scalar potential. We use the 52 generators of the \( F_4 \) subgroup of \( E_6 \) from [23]. The remaining 26 generators are given in [24]. The generators are normalized by

\[
\text{Tr}(c_i c_j) = -6\delta_{ij}.
\]

(A.1)

With this normalization, we find that

\[
\mathcal{V}^{IJ} = -\frac{1}{6} \text{Tr}(L^{-1} T^2_0 LX^{IJ})
\]

(A.2)

\[
\mathcal{V}^A = \frac{1}{6} \text{Tr}(L^{-1} T^2_0 LX^A)
\]

(A.3)

\[
\mathcal{V}_{IJ} = -\frac{1}{6} \text{Tr}(L^{-1} X L X^{IJ})
\]

(A.4)

\[
\mathcal{V}^A_{IJ} = \frac{1}{6} \text{Tr}(L^{-1} X L X^A)
\]

(A.5)

where we have introduced the symbol \( T^2_0 \) for gauge group generators as in [18]. \( T^2_0 \) will be replaced by some appropriate generators of the gauge group being considered in each gauging.

The following mapping provides the relation between \( c_i \) and \( X^{ij} \), generators of \( SO(10) \),

\[
X^{12} = c_1, \quad X^{13} = -c_2, \quad X^{23} = c_3, \quad X^{34} = c_6, \quad X^{14} = c_4, \quad X^{24} = -c_5, \\
X^{15} = c_7, \quad X^{25} = -c_8, \quad X^{35} = c_9, \quad X^{45} = -c_{10}, \quad X^{56} = -c_{15}, \quad X^{16} = c_{11}, \\
X^{26} = -c_{12}, \quad X^{46} = -c_{14}, \quad X^{56} = c_{13}, \quad X^{17} = c_{16}, \quad X^{27} = -c_{17}, \quad X^{47} = -c_{19}, \\
X^{57} = c_{18}, \quad X^{67} = -c_{21}, \quad X^{37} = -c_{20}, \quad X^{18} = -c_{36}, \quad X^{28} = c_{30}, \quad X^{38} = -c_{31}, \\
X^{48} = -c_{33}, \quad X^{58} = c_{32}, \quad X^{68} = -c_{35}, \quad X^{39} = -c_{34}, \quad X^{29} = -c_{46}, \quad X^{19} = c_{45}, \\
X^{49} = -c_{48}, \quad X^{59} = c_{47}, \quad X^{69} = -c_{50}, \quad X^{39} = -c_{49}, \quad X^{29} = -c_{52}, \quad X^{19} = -c_{51}, \\
X^{1,10} = -c_{71}, \quad X^{2,10} = c_{72}, \quad X^{3,10} = -c_{73}, \quad X^{4,10} = c_{74}, \quad X^{5,10} = c_{75}, \\
X^{6,10} = c_{76}, \quad X^{7,10} = c_{77}, \quad X^{8,10} = c_{78}, \quad X^{9,10} = \tilde{c}_{53}.
\]

(A.6)

The \( \tilde{c}_{53} \) and \( \tilde{c}_{70} \) are defined by [24]

\[
\tilde{c}_{53} = \frac{1}{2} c_{53} + \frac{\sqrt{3}}{2} c_{70} \quad \text{and} \quad \tilde{c}_{70} = -\frac{\sqrt{3}}{2} c_{53} + \frac{1}{2} c_{70}.
\]

(A.7)

All the \( \beta^j \)'s components can be obtained from the structure constants of the \( [X^{IJ}, Y^A] \) given in [23] and [24].

Generators of the \( SO(p) \times SO(10 - p) \) compact gauge group are given by

\[
T^{IJ}_1 = X^{IJ}, \quad I, J = 1, \ldots, p, \quad T^{IJ}_2 = X^{IJ}, \quad I, J = p + 1, \ldots, 10.
\]

(A.8)

The \( U(1) \) subgroup is generated by \( X = 2\tilde{c}_{70} \).
In the non-compact $G_{2(-14)} \times SU(2, 1)$ gauging, the generators of $G_{2(-14)}$ can be obtained from combinations of $SO(7)$ generators [27]

\[
T_1 = \frac{1}{\sqrt{2}}(X^{36} + X^{41}), \quad T_2 = \frac{1}{\sqrt{2}}(X^{31} - X^{46}),
\]

\[
T_3 = \frac{1}{\sqrt{2}}(X^{43} - X^{16}), \quad T_4 = \frac{1}{\sqrt{2}}(X^{73} - X^{24}),
\]

\[
T_5 = -\frac{1}{\sqrt{2}}(X^{21} + X^{47}), \quad T_6 = -\frac{1}{\sqrt{2}}(X^{26} + X^{71}),
\]

\[
T_7 = \frac{1}{\sqrt{2}}(X^{76} - X^{21}), \quad T_8 = \frac{1}{\sqrt{6}}(X^{16} + X^{43} - 2X^{72}),
\]

\[
T_9 = -\frac{1}{\sqrt{6}}(X^{41} - X^{36} + 2X^{25}), \quad T_{10} = -\frac{1}{\sqrt{6}}(X^{31} + X^{46} - 2X^{57}),
\]

\[
T_{11} = \frac{1}{\sqrt{6}}(X^{73} + X^{24} + 2X^{15}), \quad T_{12} = -\frac{1}{\sqrt{6}}(X^{74} - X^{23} + 2X^{65}),
\]

\[
T_{13} = \frac{1}{\sqrt{6}}(X^{26} - X^{71} + 2X^{35}), \quad T_{14} = \frac{1}{\sqrt{6}}(X^{21} + X^{76} - 2X^{45}).
\]

These generators are essentially the same as those used in [18], but we repeat them here for convenience. The $SU(2, 1)$ generators are given by

\[
J_1 = -c_{52}, \quad J_2 = -\tilde{c}_{53}, \quad J_3 = -c_{78}, \quad J_4 = \tilde{c}_{70},
\]

\[
J_5 = \frac{1}{\sqrt{2}}(Y_1 + Y_6), \quad J_6 = \frac{1}{\sqrt{2}}(Y_9 + Y_{14}),
\]

\[
J_7 = \frac{1}{\sqrt{2}}(Y_{21} + Y_{24}), \quad J_8 = \frac{1}{\sqrt{2}}(Y_{25} + Y_{30}).
\]

We have normalized these generators according to the embedding tensor given in section 2.

In $SU(4, 2) \times SU(2)$ gauging, the relevant generators are given by

- $SU(4, 2)$:

\[
Q_i = c_i, \quad i = 1, \ldots, 15,
\]

\[
Q_{16} = \frac{1}{\sqrt{2}}(c_{53} + c_{77}), \quad Q_{17} = \frac{1}{\sqrt{2}}(c_{51} - c_{78}), \quad Q_{18} = \frac{1}{\sqrt{2}}(\tilde{c}_{53} - c_{36}),
\]

\[
Q_{19} = \tilde{c}_{70}, \quad Q_{20} = \frac{1}{\sqrt{2}}(Y_1 + Y_{23}), \quad Q_{21} = \frac{1}{\sqrt{2}}(Y_2 - Y_{22}),
\]

\[
Q_{22} = \frac{1}{\sqrt{2}}(Y_3 + Y_{24}), \quad Q_{23} = \frac{1}{\sqrt{2}}(Y_4 - Y_{21}), \quad Q_{24} = \frac{1}{\sqrt{2}}(Y_5 + Y_{20}),
\]

\[
Q_{25} = \frac{1}{\sqrt{2}}(Y_6 + Y_{18}), \quad Q_{26} = \frac{1}{\sqrt{2}}(Y_7 - Y_{17}), \quad Q_{27} = \frac{1}{\sqrt{2}}(Y_8 - Y_{19}),
\]

\[
Q_{28} = \frac{1}{\sqrt{2}}(Y_9 + Y_{27}), \quad Q_{29} = \frac{1}{\sqrt{2}}(Y_{10} - Y_{29}), \quad Q_{30} = \frac{1}{\sqrt{2}}(Y_{14} - Y_{25}),
\]

\[
Q_{31} = \frac{1}{\sqrt{2}}(Y_{12} + Y_{30}), \quad Q_{32} = \frac{1}{\sqrt{2}}(Y_{13} + Y_{26}), \quad Q_{33} = \frac{1}{\sqrt{2}}(Y_{14} - Y_{28}),
\]

\[
Q_{34} = \frac{1}{\sqrt{2}}(Y_{15} - Y_{32}), \quad Q_{35} = \frac{1}{\sqrt{2}}(Y_{16} + Y_{31}).
\]

- $SU(2)$:

\[
K_1 = \frac{1}{2}(c_{51} + c_{78}), \quad K_2 = -\frac{1}{2}(c_{52} - c_{77}), \quad K_3 = \frac{1}{2}(c_{36} + \tilde{c}_{53}).
\]
To find the above generators, we first look at the generators of the compact subgroup $SU(4) \times SU(2) \times U(1)$ of the $SU(4,2)$. Using the fact that $SU(4) \sim SO(6)$ and $SU(2) \times SU(2) \sim SO(4)$, we can identify $SU(4) \times SU(2) \times SU(2)$ with $SO(6) \times SO(4) \subset SO(10)$. The $U(1)$ generator is simply $c_{70}$.

The final non-compact gauge group is $F_{4(-20)}$. Its generators can be easily identified by $c_1, \ldots, c_{52}$ in the construction of the $E_6$ given in [24].

We can now compute the $T$-tensors using

$$T^{IJ, KL} = \gamma^{I,J, KL, \beta} \delta^{SO(p)}_{\alpha \beta} - \gamma^{I,J, A, \delta} \delta^{SU(10-p)}_{\alpha \beta} + \frac{1}{2} (5 - p) \gamma^{I,J} \gamma^{KL},$$  \hspace{1cm} (A.13)

$$T^{IJ, A} = \gamma^{I,J, A, \delta} \delta^{SO(p)}_{\alpha \beta} - \gamma^{I,J, A, \delta} \delta^{SU(10-p)}_{\alpha \beta} + \frac{1}{2} (5 - p) \gamma^{I,J} \gamma^{A},$$  \hspace{1cm} (A.14)

for compact gaugings and

$$T^{IJ, KL} = \gamma^{I,J, KL, \beta} \eta_{\alpha \beta} - K \gamma^{I,J, KL} \gamma^{KL, \beta} \eta_{\alpha \beta},$$  \hspace{1cm} (A.15)

$$T^{IJ, A} = \gamma^{I,J, A, \delta} \eta_{\alpha \beta} - K \gamma^{I,J, A} \gamma^{A, \delta} \eta_{\alpha \beta},$$  \hspace{1cm} (A.16)

for non-compact gaugings with $K$ being $\frac{2}{7}$ and 6 for $G_1 \times G_2$ being $G_{2(-14)} \times SU(2,1)$ and $SU(4,2) \times SU(2)$, respectively. As in [18], we use summation convention over gauge indices $\alpha, \beta$ with the notation $\delta^{G_0}$ and $\eta^{G_0}$ meaning that the summation is restricted to the $G_0$ generators. For $F_{4(-20)}$ gauging, we have the simpler expressions for the $T$-tensors namely

$$T^{IJ, KL} = \gamma^{I,J, KL, \beta} \eta_{\alpha \beta}, \quad T^{IJ, A} = \gamma^{I,J, A, \delta} \eta_{\alpha \beta}.$$  \hspace{1cm} (A.17)

### Appendix B. Explicit forms of the $A_1$ tensors

In this appendix, we give the explicit forms of the $A_1$ tensors mentioned in the main text. We collect them here due to their lengthy and complicated forms.

- $SO(7) \times SO(3) \times U(1)$ gauging

  $G_2$ sector:

  $$A_1 = \begin{pmatrix} -8I_7 & 0 \\ 0 & M_3^{(1)} \end{pmatrix}, \quad M_3^{(1)} = \begin{pmatrix} m_1 & m_4 & m_5 \\ m_4 & m_2 & m_6 \\ m_5 & m_6 & m_3 \end{pmatrix}. \hspace{1cm} (B.1)$$

  The elements of the matrix $M^{(1)}$ are given by

  $$m_1 = \frac{1}{4} \left[ 21 - \cos(2a_2) - \cos(2a_1)(1 + 3 \cos(2a_3)) + 4 \cos(2a_2) \sin^2(a_1) \sin^2 a_3 + 4 \sin(2a_1) \sin a_2 \sin(2a_3) \right]$$

  $$m_2 = -\frac{1}{3} \left[ -1 - \cos(2a_2) - 2 \cos^2 a_2 \cos(2a_3) \right]$$

  $$m_3 = \frac{1}{3} \left[ 21 - \cos(2a_1) + \cos(2a_1) \cos(2a_3) \right] + 4 \cos^2 a_1 \cos(2a_2) \sin^2 a_3 - 4 \sin(2a_1) \sin a_2 \sin(2a_3)$$

  $$m_4 = \frac{8}{3} \cos a_2 \sin a_3 \cos a_1 \cos a_3 - \sin a_1 \sin a_2 \sin a_3$$

  $$m_5 = \frac{1}{4} \left[ -2 \cos^2 a_2 + (-3 + \cos(2a_2)) \cos(2a_2) \sin(2a_1) - 4 \cos(2a_2) \sin a_2 \sin(2a_3) \right]$$

  $$m_6 = \frac{8}{3} \cos a_2 \sin a_3 \sin a_1 \cos a_1 \sin a_2 \sin a_3. \hspace{1cm} (B.2)$$
\bullet G_{2(1|4)} \times SU(2, 1) gauging

\[ SU(2)_{\text{diag}} \] sector:

\[
A_1 = \begin{pmatrix}
-4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m_{22} & 0 & 0 & m_{52} & 0 & m_{72} & 0 & 0 \\
0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 \\
0 & m_{52} & 0 & 0 & m_{55} & 0 & m_{75} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 \\
0 & m_{72} & 0 & 0 & m_{75} & 0 & m_{77} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2
\end{pmatrix}
\] (B.3)

where

\[
m_{22} = \frac{1}{12} [-30 + \cos[2(a_1 - a_2)] + \cos[2(a_1 + a_2)] - 2 \cos(2a_3) \\
+ \cos(2a_1)(2 + 6 \cos(2a_3)) + \cos(2a_2)(2 - 4 \cos^2 a_1 \cos(2a_3)) \\
+ 8 \sin(2a_1) \sin a_2 \sin(2a_3)]
\]

\[
m_{52} = \frac{1}{6} \{(-2 \cos^2 a_1 + (-3 + \cos(2a_2)) \cos(2a_3)) \sin(2a_1) \\
+ 4 \cos(2a_1) \sin a_2 \sin(2a_3)\}
\]

\[
m_{72} = \frac{4}{3} \cos a_2 \sin a_3 \cos a_1 \sin a_1 - \cos a_1 \sin a_2 \sin a_3
\]

\[
m_{55} = \frac{1}{12} [-\cos[2(a_1 - a_2)] - \cos[2(a_1 + a_2)] - 2 \cos(2a_1)(1 + 3 \cos(2a_3)) \\
+ \cos(2a_2)(2 - 4 \cos(2a_3) \sin^2 a_1) - 2(15 + \cos(2a_3)) \\
+ 4 \sin(2a_1) \sin a_2 \sin(2a_3)]
\]

\[
m_{75} = \frac{4}{3} \cos a_2 \sin a_3 \cos a_1 \cos a_3 + \sin a_1 \sin a_2 \sin a_3
\]

\[
m_{77} = \frac{1}{3} [-7 - \cos(2a_2) + 2 \cos^2 a_2 \cos(2a_1)].
\] (B.4)

\bullet F_{4(20)} gauging

\[ SU(3) \] sector:

\[
A_1 = \begin{pmatrix}
\frac{7}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{7}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{7}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{7}{4} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_{55} & 0 & 0 & a_{55} & 0 \\
0 & 0 & 0 & 0 & -\frac{7}{4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{7}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_{55} & 0 & 0 & a_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\] (B.5)

where

\[
a_{55} = \frac{1}{16} [-50 - 2 \cos(2a_1) + 3 \cos[2(a_1 - a_3)] - 8 \cos^2 a_1 \cos(2a_2) \cos^2 a_3 \\
- 2 \cos(2a_3) + 3 \cos[2(a_1 + a_3)] + 8 \sin(2a_1) \sin a_2 \sin(2a_3)]
\]

\[
a_{85} = \frac{1}{4} [2 \cos^2 a_2 + (-3 + \cos(2a_2)) \cos(2a_1)] \sin(2a_1) \\
+ 4 \cos(2a_1) \sin a_2 \sin(2a_3)]
\]
Appendix C. Scalar potential for $SO(9) \times U(1)$ gauging in the $G_2$ sector

$$V = -\frac{1}{327680} \rho^2 e^{-4\sqrt{2}h} \left[ -2(4(-1 + e^{\sqrt{2}h})^3)(1 + e^{\sqrt{2}h}) \cos[2a_1](1 + 3 \cos[2a_3]) \\
+4(1 + e^{\sqrt{2}h})^2(9 + 6 e^{\sqrt{2}h} + 29 e^{\sqrt{2}h} - (1 + e^{\sqrt{2}h})^2 \cos[2a_1] + 4(1 + e^{\sqrt{2}h})^2 \cos[2a_1] \sin[a_1]^2 - \sin[2a_1] \sin[a_2] \sin[2a_3])^2 \\
+20((-1 + e^{\sqrt{2}h})^4(4 \cos[a_2]^2 \cos[2a_1] + 2 \cos[2a_1](-2 \cos[a_2]^2 \\
+(-3 + 2\cos[2a_2]) \cos[2a_1] + 8 \sin[2a_1] \sin[a_2] \sin[2a_3])^2 \\
- 2621 440 e^{4\sqrt{2}h} \cos[a_1]^2 \cos[a_2]^2 (\cos[a_1] \sin[a_1] + \cos[a_1] \sin[a_2])^2 \right]$$
\[
\times \sinh \left[ \frac{b_1}{\sqrt{2}} \right]^6 - 384 e^{\sqrt{2}b_1} (-1 + e^{\sqrt{2}b_1})^6 (4 \cos[2a_1] \sin[2a_1] \sin[a_2] \\
+ (3 \cos[2a_1] - 2 \cos[a_1]^2 \cos[2a_2] - 1) \sin[2a_1])^2 - 96 (-1 + e^{2\sqrt{2}b_1})^2 \\
\times (2 (4 (3 + 2 e^{\sqrt{2}b_1} + 3 e^{2\sqrt{2}b_1}) - 4 (1 + e^{\sqrt{2}b_1})^2 \cos[a_1]^2 \cos[a_2]^2 \\
+ (-1 + e^{\sqrt{2}b_1})^2 ((3 + \cos[2a_2] - 2 \cos[2a_1] \sin[a_2]^2) \sin[a_1]^2 - 2 \sin[2a_1] \\
\times \sin[a_2] \sin[2a_1]))^2 - 4 (-1 + e^{\sqrt{2}b_1})^2 (2 (29 - 2 e^{\sqrt{2}b_1} (-3 + \cos[2a_2])) \\
+ \cos[2a_1] + e^{\sqrt{2}b_1} (29 + \cos[2a_2]) + (e^{2\sqrt{2}b_1} - 1)^2 \cos[2a_1] (2 \cos[a_2]^2) \\
- \cos[2a_2] - 3 \cos[2a_3]) - 2 (e^{2\sqrt{2}b_1} - 1)^2 (\cos[a_2]^2 \cos[2a_3] \\
+ 2 \sin[2a_1] \sin[a_2] \sin[2a_3]))^2 - 16 e^{\sqrt{2}b_1} (-1 + e^{\sqrt{2}b_1})^6 (12 \cos[2a_1] \sin[2a_1] \\
+ 16 \cos[2a_1] \sin[2a_1] \sin[a_2] - 4 (1 + 2 \cos[a_1]^2 \cos[2a_2])) \sin[2a_1])^2 \\
- 4 (-1 + e^{\sqrt{2}b_1})^3 (1 + e^{2\sqrt{2}b_1}) \cos[2a_1] (1 + 3 \cos[2a_3]) \\
\times 29 + 6 e^{2\sqrt{2}b_1} + 29 e^{\sqrt{2}b_1} - (-1 + e^{\sqrt{2}b_1})^2 \cos[2a_1] \\
\times 4 (-1 + e^{\sqrt{2}b_1})^2 (\cos[a_1]^2 \cos[2a_2] \sin[a_1]^2 - \sin[2a_1] \sin[a_2] \sin[2a_3]))^2 \\
\right) 
\]

(C.1)

References

[1] Samtleben H 2008 Lectures on gauged supergravity and flux compactifications Class. Quantum Grav. 25 214002 (arXiv:0808.4076)

[2] Maldacena J M 1998 The large N limit of superconformal field theories and supergravity Adv. Theor. Math. Phys. 2 231–52 (arXiv:hep-th/9711200)

[3] Khavanin A, Pilch K and Warner N P 2000 New vacua of gauged N = 8 supergravity Phys. Lett. B 487 14–21 (arXiv:hep-th/9812035)

[4] Warner N P 1983 Some new extrema of the scalar potential of gauged N = 8 supergravity Phys. Lett. B 128 169

[5] Warner N P 1984 Some properties of the scalar potential in gauged supergravity theories Nucl. Phys. B 231 250

[6] de Wit B and Nicolai H 1982 N = 8 supergravity Nucl. Phys. B 208 323

[7] Fischbacher T 2010 Fourteen new stationary points in the scalar potential of SO(8)-gauged N = 8, D = 4 supergravity J. High Energy Phys. JHEP09(2010)068 (arXiv:0912.1636)

[8] Fischbacher T, Pilch K and Warner N P 2010 New supersymmetric and stable, non-supersymmetric phases in supergravity and holographic field theory arXiv:1010.4910

[9] Kraus P 2008 Lectures on black holes and the AdS3/CFT2 correspondence Lect. Notes Phys. 755 193–247 (arXiv:hep-th/0609074)

[10] Nicolai H and Samtleben H 2001 Maximal gauged supergravity in three dimensions Phys. Rev. Lett. 86 1686–9 (arXiv:hep-th/0010076)

[11] Nicolai H and Samtleben H 2001 Compact and noncompact gauged maximal supergravities in three dimensions J. High Energy Phys. JHEP04(2001)022 (arXiv:hep-th/0103032)

[12] Fischbacher T, Nicolai H and Samtleben H 2004 Non-semisimple and complex gaugings of N = 16 supergravity Commun. Math. Phys. 249 475–556 (arXiv:hep-th/0306276)

[13] Nicolai H and Samtleben H 2001 N = 8 matter coupled AdS3 supergravities Phys. Lett. B 514 165–72 (arXiv:hep-th/0106153)

[14] de Wit B, Herger I and Samtleben H 2003 Gauged locally supersymmetric D = 3 nonlinear sigma models Nucl. Phys. B 671 175–216 (arXiv:hep-th/0307006)

[15] de Wit B, Tollsten A K and Nicolai H 1993 Locally supersymmetric D = 3 nonlinear sigma models Nucl. Phys. B 392 5–38 (arXiv:hep-th/9208074)

[16] Gava E, Karndumri P and Narain K S 2010 Ads, vacua and RG flows in three dimensional gauged supergravities J. High Energy Phys. JHEP04(2010)117 (arXiv:1002.3760)

[17] Berg M and Samtleben H 2002 An exact holographic RG flow between 2D conformal field theories J. High Energy Phys. JHEP05(2002)006 (arXiv:hep-th/0112154)
[18] Chatrabhuti A and Karndumri P 2010 Vacua and RG flows in $N = 9$ three-dimensional gauged supergravity J. High Energy Phys. JHEP10(2010)098 (arXiv:1007.5438)

[19] Fischbacher T 2002 Some stationary points of gauged $N = 16$, $D = 3$ supergravity Nucl. Phys. B 638 207–19 (arXiv:hep-th/0201030)

[20] Fischbacher T, Nicolai H and Samtleben H 2002 Vacua of maximal gauged $D = 3$ supergravities Class. Quantum Grav. 19 5297–334 (arXiv:hep-th/0207206)

[21] Fischbacher T 2009 The many vacua of gauged extended supergravities Gen. Rel. Grav. 41 315 (arXiv:0811.1915)

[22] Wolfram S 2003 The Mathematica Book 5th edn (Champaign, IL: Wolfram Media)

[23] Bernardoni F, Cacciatori S L, Cerchiai B L and Scotti A 2008 Mapping the geometry of the $F_4$ group Adv. Theor. Math. Phys. 12 889–994 (arXiv:07053978)

[24] Bernardoni F, Cacciatori S L, Cerchiai B L and Scotti A 2008 Mapping the geometry of the $E_6$ group J. Math. Phys. 49 012107 (arXiv:0710.0356)

[25] Fradkin E S and Linetsky Y Ya 1992 Results of the classification of superconformal algebras in two dimensions Phys. Lett. B 282 352–6 (arXiv:hep-th/9203045)

[26] Nicolai H and Samtleben H 2002 Chern–Simons vs. Yang–Mills gaugings in three dimensions Nucl. Phys. B 638 207–19 (arXiv:hep-th/0303213)

[27] Günyaydin M and Ketov S V 1996 Seven-sphere and the exceptional $N = 7$ and $N = 8$ superconformal algebras Nucl. Phys. B 467 215–46 (arXiv:hep-th/9601072)