Bursts of cooperation triggered by threats in violent and other uncertain situations

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Abstract

During intergroup confrontations, agitating stimuli such as opponents’ provocations can trigger collective violence, even without the usual mechanisms of cooperation such as norms with sanctions. We examine video recordings of street fights between groups of young men. Collective violence in these attacks sometimes breaks out in a burst and at other times in a fizzle wherein only few group members participate. An adapted Ising spinglass model demonstrates that these two different outcomes can be predicted by a critical level of defectors, which is strongly supported by our data.

1 Introduction

When one observes how collective violence (by non-professionals) unfolds, one will notice one of two patterns, and rarely something else. Before punches are thrown, individuals in the focal group and their opponents insult and threaten, and thereby agitate, one another. When due to this turmoil, focal group members become more agitated, there is a bifurcation of subsequent collective actions. The first possibility is that at a critical level of turmoil, violence breaks out in a burst (or a series of short bursts) by a majority of individuals in the focal group (or a subgroup from a larger and more dispersed group) with face-to-face contact in close proximity [1, 2, 3]. Their targets are often stumbling, outnumbered, or otherwise vulnerable opponents [1, 4]. The second possibility is that, no matter how much turmoil, violence does not break out in a burst but in a fizzle wherein fewer individuals start fighting asynchronously. Understanding the unfolding of collective violence is complicated by other people present on the scene who may form an audience or try to deescalate [5, 6, 7], which happens often [8]. Given a common prelude of turmoil, our question is how to explain these two different outcomes. To
this end we use an Ising spinglass model \cite{9, 10, 11, 12, 13, 14, 15}, and test it on video recorded street fights between groups of (mostly) young men.

Collective violence involves a dilemma of collective action \cite{16, 17, 18, 19, 20}, and before predicting bursts and fizzes, we tell how the Ising model explains overcoming it. The face-to-face contacts of the focal group are the social ties of their network. We already know from public goods experiments that more than half of the participants are conditional cooperators willing to contribute if others do \cite{21}, thus conforming to their (weighted) average neighbor in the network \cite{22, 23}. Our empirical study is about relatively inexperienced individuals, not about seasoned fighters, hence our subjects experience uncertainty when they come under threat, which increases their conformity \cite{24, 25}. This makes sense from an evolutionary perspective when payoffs are hard to predict \cite{26}. Therefore subjects’ motivation not only depends on their expected payoff (as a share from the public good) but also on a preference to align with their group members.

Now think of two individuals, each with the behavioral options to defect or cooperate. (Behavioral options correspond to magnetic spins in the original, but that does not matter here.) The possibilities are: (1) both individuals defect, which avoids exploitation but does not yield any public good; (2) one individual cooperates while being exploited by the freeriding other, producing only half of the public good; or (3) both cooperate at a cost, which maximises the public good. Fig. 1 plots these possibilities (dashed line), with the normalised number of cooperators ($N_C/n$) from left to right. At the beginning, both defect, at the local minimum on the left. The vertical axis ($H$, or energy in the original) tells that moving uphill is unlikely, i.e., that one of the two will start contributing to the public good (the hill top). When moving to lower levels of $H$ if possible, it will certainly happen; thus, if one individual cooperates, the other conditional cooperator will join in, toward the global minimum on the right. The hill is the graphical representation of the dilemma of collective action under uncertainty; it is also drawn for a group of five. It shows that without further ado, it is very unlikely that a group of non-fighting individuals ($n \geq 2$) will overcome the hill and fight collectively, and it is more unlikely in larger groups.

Nevertheless, individuals may start fighting, and win over others to join. When agitating stimuli, such as opponents’ threats, increase, one or few individuals may accidentally cooperate, called “trembling hands” in game theory \cite{27}, which influences proximate others in their alignment, and may result in a cascade of cooperation. For the model, we define turmoil ($T$, or temperature in the original) as opponents’ behaviour that agitates members of the focal group. If it increases beyond a critical point ($T_c$), fighting can break out in a burst. Our contribution is to show that the outbreak of collective
violence depends on a second critical point, namely the critical proportion of steady defectors in the focal group \((p_c)\). In violent situations, people may steadily defect for several reasons. They may be too scared to fight [1], have empathy with their opponents, disagree with violence, i.e., value the public good differently, try to deescalate, are stopped in their tracks by deescalators, or they may have fought but got wounded, were wrestled to the ground, or got exhausted at some point and remained passive. If, for whatever cause or reason, their proportion stays below the critical level \((p \leq p_c)\), turmoil \((T \geq T_c)\) will be followed by a burst of violence, but above the critical level \((p > p_c)\), there is neither a critical level of turmoil nor a burst, and violence occurs in a fizzle. The Ising model allows us to identify \(p_c\), which we use as a prediction that we test empirically. Note that \(p\) not only incorporates deescalators in the focal group, but also focal group members who have been prevented to fight by deescalators in the focal group or by others.

In a recent special issue on intergroup conflict, a conflict participation function was introduced [28], which is based on the expected payoff, expected gain in reputation, care for one’s group, care for opponents, and opponents’ threat. After elaborating our model, we will explicate how it relates to this participation function, and thereby to a large body of conflict studies on humans and animals [28]. Across studies, the driving stimuli are different, but at an abstract level, the same Ising model can be used for various agitating stimuli, situations, and species.

Most models in the literature revolve around individual rewards and punishments, called selective incentives [19], on top of a share of the public good. For humans these incentives require norms about (in)appropriate behavior in a given situation [29], as well as monitoring of group members [30], and transmission of information (i.e., gossip) through the group’s network, which leads to reputations [31] that feed back through selective incentives, with or without leaders. This package of mechanisms is crucial for ongoing cooperation in the longer run but needs time to develop, which may not be available when threats are imminent. The more time people have, the better they can prepare themselves, which is especially important for high-risk situations. Examples are police, soldiers, firefighters, and combat medics who receive professional training that enables them to cooperate effectively and respond to situational stimuli in predetermined manners rather than spontaneously. Ordinary citizens who face violent opponents are less prepared or not prepared at all. For them, the uncertainties of outcomes, benefits, and costs are higher.

There are earlier models of cooperation without the usual package of cooperation mechanisms, namely models of thresholds [32], cascades [33], and critical mass [34]. These models draw on initiative takers or leaders to
initiate cooperation, and on strong rationality assumptions, such as perfect information on the numbers of defectors and cooperators. The spinglass model is more parsimonious because cooperation can be started by accidental cooperators rather than exceptionally zealous ones, as was already known for the prisoners’ dilemma [27]. If there are initiative takers or leaders [35], however, they can be accommodated, as well as exit as a third behavioral option. The spinglass model is also more parsimonious because it has no assumptions of strong rationality, which is bounded in violent and other uncertain situations by incomplete information about opponents and their threat, group members beyond face-to-face contact (versus [32, 34]), and payoffs.

2 Model

Members of a (fledgling) focal group can defect, $D$, or contribute, $C$, to a public good, with $0 < D < C$. In our empirical study, the public good is defense against or attack of opponents. Behavioral variable $S_i$ of conditional cooperators can take the value $S_i = C$ or $S_i = -D$, whereas steady defectors stay put at $S_j = -D$. Before a collective action, everyone in a focal group defects. Network ties among focal group members, $A_{ij} > 0$, mean that $i$ is in close proximity of and pays attention to group member $j$, else $A_{ij} = 0$. Because people tend to respond to proportions of their social environment rather than absolute numbers [33, 37], ties are row-normalised [with $w_{ij} = A_{ij} / \sum_{j=1}^{n} A_{ij}$ such that $\sum_{j=1}^{n} w_{ij} = 1$]. We assume that attention, hence ties, are reciprocal at least to some degree but not necessarily symmetric. Opponents’ turmoil, represented by $T$, has an effect on the focal group at the aggregate level, thus it equally affects everyone. Locally varying turmoil is discussed elsewhere [36]. The Ising model is the following Hamiltonian equation [38, 39],

$$H = -\sum_{i \neq j}^{n} w_{ij} S_i S_j.$$  \hspace{1cm} (1)

We do not assume that individuals know their payoffs in advance, but they will heuristically—and perhaps wrongly—distinguish between valuable ($C > D$) and nonvaluable ($C < D$) public goods. Note that payoffs are not used in the model’s calculations but are defined to provide a meaningful interpretation. When $i$ chooses between $C$ and $D$ amid $N_C$ cooperators, payoffs for cooperation, $P_C = \theta(N_C + 1)/n - 1$, and defection, $P_D = \theta N_C/n + Q$, with a synergy or enhancement factor $\theta \geq 1$, are the same as in evolutionary game theory [20] except for $Q$. This additional fac-
Figure 1: The dilemma of cooperation presented as a hill in between full defection (left) and full cooperation (right), with normalised cooperation ($N_C/n$) on the horizontal axis. The vertical axis ($H/n$) could be intuited as a negative likelihood, and is drawn for a dyad and a clique of five individuals. The larger the group is, the more rounded the hill becomes.

The function $Q$ assures that if $D$ approximates $C$, which means that the outcomes of defection and cooperation become equally valuable, $P_D$ approximates $P_C$:

$$Q = (\theta/n - 1)(1 - R); R = (C - D)/(C + D); \theta = \theta_0 + R,$$

with a base rate $\theta_0 \geq 1$.

For our empirical study, we have to choose values for $C$ and $D$ to predict the critical level of steady defectors, $p_c$. The most obvious choice is $C = 1$, as in game theory and the original Ising model. For $D$ we want to avoid two trivial values: if $D = C = 1$, there is no point in cooperating, as said above, and if $D = 0$, there is no dilemma (only a downward slope to the right in Fig. 1). Choosing $D$ halfway in between 0 and 1 seems to be a reasonable first approximation, hence we set $S_j = \{1, -1/2\}$ for all conditional cooperators, also in our examples. Because these two values were published earlier in two nonempirical papers [41, 36], one could say they were preregistered. For steady defectors we set $S_j = -1/2$, irrespective of their reasons. Note that all earlier Ising models had either the values $\{1, -1\}$ [9, 10, 11, 12, 13, 14, 15] or $\{1, 0\}$ [10]; the latter correctly predicts the size distribution of the number of pigtailed macaques participating in violent conflicts. Of all these models, only one represents a public goods game, be it for two persons [42, 43] whereas
our model is applicable to groups of any size.

Beyond our empirical study, the payoffs in the asymmetric Ising model can be generalised by relating $C$ and $D$ to the symmetric model through a mapping $\{C, -D\} \rightarrow \{S_0 + \Delta, S_0 - \Delta\}$, with a bias $S_0 = (C - D)/2$ with respect to 0, and the two behavioral options symmetrical at each side of $S_0$ at an offset $\Delta = \pm (C + D)/2$. (One could further generalise to variation across individuals, at the cost of many degrees of freedom, though.) It can be shown that the asymmetry in $S$ is equivalent to the symmetric model with an external field $2S_0$. The bias and offset are in the payoffs through $R = S_0/\Delta$. If $\Delta$ is set to a fixed value (0.75 in our examples), decreasing $S_0$ makes cooperation less valuable and is equivalent to an increasing threshold of cooperating network neighbors to win actor over to cooperate, also in other binary decision models. Cooperation can be made more valuable by increasing $S_0$, which corresponds to a decreasing threshold or proportion of cooperating network neighbors.

Solving the Ising model boils down to minimizing $H$. In the Appendix it is solved analytically by a mean field approach, but here it is solved computationally. Increasing $T$ increases the chance that randomly drawn conditional cooperator $i$ “flips” $S_i = -1/2$ to $S_i = 1$ or vice versa. In other words, increasing $T$ increases the amount of randomness in $i$’s decision, which then affects others in $i$’s neighborhood when it is their turn to decide. An individual decision happens in a Monte Carlo step in the Metropolis algorithm. It loops through great many Monte Carlo steps (counted by $t$; Fig. 2) to allow individuals’ interdependent behavior to settle down, and repeats this procedure over a range of $T$ values, in our study with increments of 0.01; $c_r$ at the bottom of Fig. 2 is a random number, $0 \leq c_r \leq 1$.

The continuous black line in Fig. 3 shows how cooperation in a group of conditional cooperators ($n = 5$) varies over turmoil. At low turmoil, collective action does not start, but at a critical level $T_c$ and $p = 0$, (almost) everybody bursts into cooperation, with a maximum at or near $T_c$ (where $N_C/n \approx 1$). A burst means that if $T$ increases to $T_c$, there is a phase transition: few accidental cooperators win over most others to join the collective action. If $p \leq p_c$, the effect of turmoil is nonmonotonic and the level of cooperation decreases if $T$ keeps increasing beyond $T_c$, which means that very strong turmoil becomes more confusing than agitating. Fighting ends when exhaustion sets in, a winner stands out, or others intervene.

We do not assume that there are leaders or initiative taken, but in many cases there are, usually with higher payoffs ($S_{0,i}$ values) than the rest. Simulations point out that leaders start cooperating at lower $T$ and thereby reduce $T_c$ for the entire group. Locally stronger turmoil has the same effect. Anger, ideology, and concerns for reputation may
Set all $S_i = -D; t = 0$

$t \leftarrow t + 1$

Randomly pick individual $i$

Calculate $H$ with $S_i$ and $H'$ with $S_i$ flipped

Implement the flip if $H' < H$ or $c_r < \exp(-(H'_i - H_i)/T)$

Figure 2: Metropolis algorithm, where index $i$ runs over the conditional cooperators, not the steady defectors.

Figure 3: Level of cooperation ($N_C/n$) with turmoil ($T$) in a clique of five individuals. The black line depicts the group without steady defectors ($p = 0$), the red line with one steady defector ($p < p_c$), and the dashed blue line with two steady defectors ($p > p_c$).

push individuals’ $S_{0,i}$ upward, whereas an intimidating majority of opponents will pull it downward.

If there are steady defectors (red line), $T_c$ increases, which is hardly visible
in Fig. 3 but more pronounced in larger networks, and maximum cooperation decreases because the number of conditional cooperators is lower. If the proportion of steady defectors reaches a critical level, $p_c$, there is a fizzle (dashed blue line in Fig. 3) with a lower maximum cooperation at a higher level of turmoil, further beyond $T_c$. A fizzle means that there is no phase transition, hence no critical level of turmoil, just a gradual increase of cooperators with increasing $T$. In a mean field analysis, there is no (burst of) cooperation if $p_c > S_0/\Delta$, independent of network size and density. Based on the mean field result we predict for our empirical study (given $S_0 = 0.15; \Delta = 0.75$) that $p_c = 1/3$. Simulations come very close to this value. If steady defectors are clustered together, however, they are less in the way of collective action (higher $p_c$) than if they are evenly spread out across the network (mean field).

The critical threshold of turmoil ($T_c$) increases with network size at a decreasing rate [36], but it also increases with the proportion of steady defectors. At $T_c$, simulations point out that cooperation starts in the smallest clusters of conditional cooperators. Empirically this is puzzling because larger groups have a better chance to win at lower individual costs. Yet if in a small (sub)group, someone starts fighting, he accounts for a relatively large proportion of his neighbors’ social contacts, and more readily wins them over to fight than in a large group where he would be a small minority. This bottom up mounting of cooperation is similar to bottom up synchronisation in the Kuramoto model [48]. Because we selected the videos for violence, we cannot be certain that turmoil is its cause, but since turmoil implies that violence starts in small subgroups, we can use this implication in an attempt to falsify the turmoil conjecture. Accordingly, we predict that smaller (sub)groups start cooperating earlier, i.e., at lower $T$. Our main prediction is that $p_c = 1/3$.

3 Data and Methods

To study violence, lab experiments lack the turmoil, agitation, and emotional intensity of violent confrontations due to ethical restrictions. Field studies, in contrast, cannot be based on a random sample of participants or groups, yet they are invaluable for realism. We obtained 42 videos from websites such as YouTube, LiveLeak, and WorldStarHipHop using search terms with the English keywords “brawl,” “street fight,” and “assault.” This sample is random with respect to temporal unfolding and (sub)group size. Of these clips, 36 are from English-speaking countries (mainly the US and the UK, with one from Canada and one from India); five of the remaining clips are from the Netherlands, and one is from Colombia. We did not observe dif-
ferences in relevant behavior related to the location of the recording. To keep distracting factors away from our analysis, we excluded clips with professional fighters, long range weapons, protective clothing, a referee, ambush attacks, or youths in a school yard. Most of our videos are phone recorded by bystanders and are left-truncated. In all likelihood, there had already been some turmoil that motivated bystanders to start filming. The shortest lasted 30 sec. and the longest was nearly 5 minutes (mean 101 sec.; s.d. 59 sec.). Out of a potential 2 x 42 groups, where the opponents in one analysis become the focal group in the next, 25 groups attacked a single individual rather than a group, who could not act collectively alone, which leaves 59 groups to examine. Most groups were small, \(2 \leq n < 10\) (mean 3.6), but one had 14 members. The smaller ones were simulated as cliques wherein everyone could see one another unless there were obstacles or deescalators obstructing visual contact. Obstruction was simulated by randomly removing \(m\) ties.

The videos were coded using Noldus Observer XT 14 software. Clips were played at half speed many times over, and one of us discussed the coding of each with one or two assistants. The assistants were unaware of the theoretical expectations. Each of 406 individuals was coded for belonging to a focal, opponent, or third-party group. Their behavior was interpreted and represented on the timeline.

We coded violence when force was used against another’s body (punching, slapping, kicking, hitting, stomping) and/or when another person’s body was forcibly moved (by pushing, shoving, dragging, wrestling, holding, etc.). Collective violence implies at least two fighting focal group members. For a burst of violence, we required that at least half of a group participated (Fig. 3), or both individuals did in a dyad, and they started fighting less than 2 seconds after the first, with a 5% margin. In the videos, it was not possible to distinguish leaders from initiative takers, but we noticed individuals who started violence on their own.

We subsumed the following behaviors of members of the opponent group under turmoil for the focal group: aggressing, including fighting gestures; pulling off clothing (jackets or vests); pulling up pants; pointing toward opponents; provocative gesturing with fingers or hands (as an invitation to engage); bending forward toward an opponent; encroaching (invading opponents’ personal space through using or damaging objects belonging to them); teasing, such as lightly hitting or ridiculing; and violence. We also included stumbling and falling because vulnerability of opponents tends to agitate focal group members as well as approaching the focal group in the

\[1\] The requirement that \(N_C \geq 0.5n\) is based on simulations of small networks just above \(p_c\), but in large networks with \(p \geq p_c\), \(N_C < 0.5n\).
context of confrontational tension, which under normal circumstances would not provoke. We calculated the total level of turmoil from the beginning of the clip until a focal group’s maximum participation in violence by the duration of each instance of turmoil and multiplying it by the number of opponents involved.

Steady defectors were coded as follows. First, we counted the focal group members who took de-escalatory action toward focal group members or opponents. Second, we noted how many focal group members were effectively stopped by others from using violence for at least five seconds. Third, we noted how many focal group members remained passive or were physically unable or spatially obstructed (by cars or people) to participate in violence for at least five seconds. For the proportion of steady defectors, \( p \), all these reasons and causes were taken into account before and during the first instance of violence by the focal group.

A plausible alternative explanation of the onset of collective action is synchrony of motion \([1, 50]\), which yields a feeling of oneness among group members \([51]\) and a stronger willingness to fight and take risks for group mates \([52]\). To measure the degree of synchronisation, we counted the number of synchronous pairs in a focal group with respect to simultaneous aggressing or moving toward or away from opponents, and divided the score by the maximum possible number of pairs.

4 Results

Of the 59 groups considered, there were 23 groups where violence started in a burst, 15 groups where violence was collective without a burst, and 21 cases of violence by a single group member. Turmoil preceded all collective violence with one exception, where two individuals suddenly assaulted a passive victim. The critical level of turmoil \( T_c \) for bursts is case-specific and depends on group size, both in absolute number and relative to the size of the opponent group, and on the proportion of steady defectors. Additionally, the use of weapons has an effect.

We confirm the finding of an earlier study \([1]\) that fighting tends to start in small groups or in small subgroups of larger groups. As predicted, we found that small (sub)groups burst into action at lower turmoil than larger groups \([36]\), and lower levels are of course reached earlier. In our data, bursts developed in 13 (37%) of the 35 smallest groups (dyads and triads, i.e., fully connected triples) and in 10 (42%) of the 24 larger groups. In bursts, the correlation between group size and turmoil is 0.53. The size-turmoil relation is slightly disturbed by dyads more often facing a larger opponent group.
and are therefore less likely to fight collectively than triads, which in turn are more robust in general [53]. As a matter of fact, larger groups always defeated smaller opponent groups (made them flee or worked them to the ground) with only one exception.

The proportions of steady defectors in groups with bursts (mean = 0.19; s.d. = 0.21) and groups without (mean = 0.49; s.d. = 0.27) are box-plotted in Fig. 4 (Welch test $t = 4.796; p = 6.411 \times 10^{-6}; df = 54.76$). Despite the simplicity of the model, the predicted critical threshold (vertical line in the figure) neatly separates the two boxes containing bursts and nonbursts. The threshold yields no perfect partitioning, though, and Fig. 4 also shows groups on the “wrong” side of the vertical line, as well as groups exactly on the line: 5 groups with a proportion $N_C : N_D = 2 : 1$ had a burst, as expected, but 3 did not. As expected (Fig. 3), there was slightly more turmoil preceding collective violence in the nonburst cases (16.6 versus 14.5 for bursts on average).

![Figure 4: The proportion of steady defectors and the predicted critical level (vertical line), which for our data separates the boxplots of bursts (1) and fizzles (0).](image)

We now focus on the 21 groups where violence was committed by a single member; 13 of these groups were dyads. In 11 of these 21 groups, one or few deescalators in the focal group successfully prevented participants from using violence. In 2 groups, members of the opponent group were able to avoid
collective violence by hampering focal group members from joining the fight. In 2 dyads, the participants were separated by their opponents and could not adjust their actions to each other any longer, leaving them to either fight on their own or flee. All these outcomes are consistent with the Ising model once interrupted ties and/or steady defectors are simulated. In 4 of the 6 remaining groups, opponents carried knives, a machete, a bat, or looked too intimidating even when unarmed, which apparently lowered $S_0$ in the focal groups. In the last 2 groups, members took turns attacking a fallen victim instead of using violence simultaneously, perhaps confident that they were in control (low uncertainty; high $S_0$). Because we took the same $S_0$ values for all conditional cooperators, variations of these values are below the resolution of our simple model, though.

An interesting alternative explanation is synchrony of motion \[1\] \[51\] \[52\]. Although in 21 out of 23 bursts, some degree of synchronisation (10.8 on average) preceded collective violence, there were 18 cases in which synchronisation (9.6 on average) was not followed by collective violence. In some of the latter cases, synchrony turned out to be a deceptive performance composed of blustering and aggrandising \[54\] without commitment to fighting. This does not imply that synchronisation is unimportant, just that it does not predict collective violence.

5 Discussion

The simple Ising model is now a century old \[55\] and has been applied to a wide range of problems \[56\] \[57\], to which we add the dilemma of collective action. It explains cooperation parsimoniously, based on stimuli without recourse to strong rationality, initiative takers, reputations, norms, feedback through selective incentives, or reliable information passing through the network. The Ising model provides insights into the temporal pattern of cooperation under uncertainty by predicting a critical threshold of steady defectors that distinguishes a burst of collective action from aizzle and is supported by the data. The model also explains why violent groups are often small or are small subgroups of larger groups despite greater risk. Small (sub)groups have a lower critical threshold of turmoil, and in a confrontation with opponents, lower levels are reached first. Whether the magnitude of turmoil predicts the severity of violence remains a question for future studies. We also investigated whether synchronous action precedes violence, but we found that synchronisation precedes both collective and solitary violence, and cannot predict either of these outcomes. However, synchronisation may still be important to increase solidarity \[58\].

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This study has several limitations. Because we selected no videos without violence, we cannot be certain that violence is caused by turmoil. When investigating the videos, however, we observed time and again that people reacted violently to turmoil, hence it still seems quite plausible that it triggers violence. Moreover, turmoil implies that small groups start cooperating first, which we were unable to falsify. Our measurements underestimated turmoil because the videos are left-truncated and depend on camera angle and vision width, thus we erred on the safe side. In the future, better domain specific scales of turmoil should be developed, which will make it possible to investigate the effect of turmoil in lab experiments. Another limitation is in the Ising model. Despite predicting the threshold of steady defectors well in general, it misclassifies some of the groups, hence also other effects beyond the model are at play. In future studies, it is important to expand the number and diversity of cases. Furthermore, when individuals find themselves more often in similar situations, they will learn, which is easier in smaller groups where they have a larger influence on their payoffs [59]. Some will change their strategy, and turn into unconditional (i.e., steady) defectors [60] who try to exploit other group members and maximise their individual payoff instead of maximising the group’s payoff. For the model, this would require decision rule updates during subsequent Monte Carlo steps.

A recent special issue of this journal [28] allows us to point out how our model relates to a broader literature. Therein, a general conflict participation function $q_i$ was introduced, $q_i = f(E(P), E(r), \alpha_I, \alpha_O, T)$, where $E(P)$ is $i$’s expected payoff, $E(r)$ is $i$’s expected gain in reputation, which we ignored because of data limitations, $\alpha_I$ is $i$’s care for his (and rarely her) ingroup, which increases with $C$ and decreases with $D$, $\alpha_O$ is $i$’s care for the outgroup, with opposite relations with $C$ and $D$, and $T$ stands for (the perception of) threat, turmoil, tension (e.g., due to economic volatility [61]), or temptation (e.g., to steal [62]). These variables, or subsets thereof, are important in a great many studies of conflicts of various kinds, including conflicts of interest without violence where the collective action is symbolic (e.g., signalling threat), and suggest applications of the Ising model to other species varying from quorum sensing bacteria [63] to herd bulls defending their group against approaching lions [64]. On top of the variables in the participation function, the Ising model also incorporates the effect of network topology, which makes a difference. Here we applied the Ising model to violence, but it can also be applied to other uncertain situations. Its dynamics are entirely consistent with temporal patterns of protests [65], which break out more often if (rumors say that) a government or its police are weakened [66, 67], analogous to vulnerable individuals in street fights. The model also seems applicable to spontaneous lynchings [68], and helping victims under uncer-
In this first empirical application, we showed that it can explain the unfolding of collective street violence, and in all likelihood, more discoveries lay ahead. Extensions to norms (as external fields) and noise in actors’ information about others’ behavior have been explored in simulations [69].

In a broader evolutionary perspective, random noise can solve coordination problems, such as collective responses to opponents’ threats, by shaking a group loose from its suboptimal (e.g., non-cooperative) state [70], but it can also mess up an optimal state. In the Ising model both happen depending on $T$: when in a group, turmoil approaches $T_c$ from below, random noise in decision making facilitates cooperation, but at high levels, too much noise entails misunderstandings and confusion.

### Appendix: Mean field analysis

In line with the social sciences, cooperation is defined as $N_C/n$ in the main text. Here we stay close to the Ising model and define cooperation in terms of the order parameter, $M = 1/n \sum_{i=1}^{n} S_i$. Consequently, $N_C/n = (M + D)/(C + D)$. Now the mean field assumption can be stated simply as $S_i = \bar{S}_i = M$.

We start out with the Hamiltonian, $H = -\sum_{i,j} w_{ij} S_i S_j$. We use the mapping $\{C, -D\} \rightarrow \{S_0 + \Delta, S_0 - \Delta\}$ with bias $S_0 = (C - D)/2$ and the offset $\Delta = (C + D)/2$ to rewrite the Hamiltonian as

$$H = -\sum_{i,j} w_{ij} (S_0 + \hat{S}_i)(S_0 + \hat{S}_j),$$

with $\hat{S}_i$ and $\hat{S}_j \in \{-\Delta, \Delta\}$.

### Mean field without and with unconditional defectors

To calculate the Boltzmann probabilities of a single spin (or an individual’s probabilities to cooperate or defect), we define the pertaining Hamiltonian $H_i$, taking into account the row normalization of the adjacency matrix ($\sum_j w_{ij} = 1$).
\[ H_i = -\sum_j w_{ij} (S_0 + \hat{S}_i)(S_0 + \hat{S}_j) \]  
(3)

\[ H_i = -\sum_j w_{ij} (S_0 + \hat{S}_i)M \]
\[ = -(S_0 + \hat{S}_i)M \]
\[ H^\pm_i = -S_0 M \mp \Delta M. \]  
(4)

In the subsequent derivation, \( \beta = 1/T \), without the Boltzmann constant. The average value of a spin, \( \bar{S}_i \), according to the Boltzmann distribution, with \( P(S^-_i) \) standing for the probability that \( S_i \) is negative and \( P(S^+_i) \) that it is positive, is

\[ \bar{S}_i = S^- P(S^-_i) + S^+ P(S^+_i) \]
\[ = S^- e^{-\beta H^-_i} + S^+ e^{-\beta H^+_i} \]
\[ = \frac{S^- e^{-\beta(-S_0 M + \Delta M)} + S^+ e^{-\beta(-S_0 M - \Delta M)}}{e^{-\beta(-S_0 M + \Delta M)} + e^{-\beta(-S_0 M - \Delta M)}} \]
\[ = \frac{S^- e^{-\beta \Delta M} + S^+ e^\beta \Delta M}{e^{-\beta \Delta M} + e^\beta \Delta M} \]
\[ = (S_0 - \Delta) e^{-\beta \Delta M} + (S_0 + \Delta) e^\beta \Delta M \]
\[ = S_0 - \Delta \tanh(\beta \Delta M). \]
(5)

So far, we only dealt with conditional cooperators, but there are also unconditional defectors in proportion \( p \). Accordingly, we define \( M_{cc} \) as the average spin value of the conditional cooperators and \( M_{ud} \) as the average spin value of the unconditional defectors. Note that \( M_{ud} = S^- \). We assume that the unconditional defectors are homogeneously distributed across the network. Consequently, the mean field equation becomes

\[ S_i = M = p M_{ud} + (1 - p) M_{cc} \]
\[ = p S^- + (1 - p) M_{cc}. \]  
(7)

The Hamiltonian for a single conditional cooperator becomes

\[ H^\pm_i = -(S_0 \pm \Delta)(p S^- + (1 - p) M_{cc}) \]
\[ = -S_0 p S^- \mp \Delta p S^- - S_0 (1 - p) M_{cc} \mp \Delta (1 - p) M_{cc}. \]  
(9)
In the derivation of Eq. 6, all terms that did not contain $\mp \Delta$ canceled each other out. For clarity, we remove these terms from Eq. 10, which results in

$$H_i^\pm = \mp \Delta pS^- \mp \Delta (1 - p)M_{cc}$$

The mean field analysis for conditional cooperator $i$ is

$$\bar{S}_i = S^- P(S_i^-) + S^+ P(S_i^+) = \frac{S^- e^{-\beta \Delta (pS^- + (1-p)M_{cc})} + S^+ e^{\beta \Delta (pS^- + (1-p)M_{cc})}}{e^{-\beta \Delta (pS^- + (1-p)M_{cc})} + e^{\beta \Delta (pS^- + (1-p)M_{cc})}}$$

$$= S_0 + \Delta \frac{e^{-\beta \Delta (pS^- + (1-p)M_{cc})} + e^{\beta \Delta (pS^- + (1-p)M_{cc})}}{e^{-\beta \Delta (pS^- + (1-p)M_{cc})} + e^{\beta \Delta (pS^- + (1-p)M_{cc})}}$$

$$= S_0 + \Delta \tanh (\beta \Delta (pS^- + (1-p)M_{cc})) = M_{cc}. \quad (14)$$

Using Eq. 8, we can express the self-consistency equation of $M_{cc}$ in $M$,

$$M = pS^- + (1 - p)(S_0 + \Delta \tanh (\beta \Delta (pS^- + (1-p)M_{cc})))$$

$$= pS^- + (1 - p)(S_0 + \Delta \tanh (\beta \Delta M))$$

$$= S_0 + \Delta p + (1 - p)\Delta \tanh (\beta \Delta M). \quad (16)$$

**Critical proportion of unconditional defectors**

Depending on $\beta$, the self-consistency equation has two stable and one unstable ferromagnetic solutions, or one stable paramagnetic solution. At a critical $\beta$ (or $T$), the system transitions between these two states. When the system is paramagnetic,

$$\frac{\partial}{\partial M} (S_0 - p\Delta + \Delta \tanh (\beta \Delta M)) < 1$$

at the solution of $M$. When the system is ferromagnetic,

$$\frac{\partial}{\partial M} (S_0 - p\Delta + \Delta \tanh (\beta \Delta M)) > 1$$

at the unstable solution of $M$. We can identify a critical $\beta$ when

$$\frac{\partial}{\partial M} (S_0 - p\Delta + \Delta \tanh (\beta \Delta M)) = 1$$

$$\frac{1}{\cosh^2 (\beta \Delta M)} \beta \Delta^2 (1 - p) = 1. \quad (18)$$
This equation can be rewritten as

\[
\cosh (\beta \Delta M) = \sqrt{\beta \Delta^2 (1 - p)} \tag{19}
\]

\[
\beta \Delta M = \pm \text{arcosh} (\sqrt{\beta \Delta^2 (1 - p)})
\]

\[
M = \frac{\pm \text{arcosh} (\sqrt{\beta \Delta^2 (1 - p)})}{\beta \Delta^2}
\]

\[
= \frac{\pm \text{arcosh} (\gamma \sqrt{(1 - p)})}{\gamma^2}, \tag{20}
\]

with \( \gamma = \sqrt{\beta \Delta^2} \). We can substitute this expression (20) in the self-consistency equation (16)

\[
M = S_0 - p \Delta + (1 - p) \Delta \tanh (\beta \Delta M)
\]

\[
\frac{\Delta M}{\Delta} = S_0 - p \Delta + (1 - p) \Delta \tanh (\gamma^2 \frac{M}{\Delta})
\]

\[
\Delta \pm \text{arcosh} (\frac{\gamma \sqrt{(1 - p)}}{\gamma^2}) = S_0 - p \Delta + (1 - p) \Delta \tanh (\pm \text{arcosh} (\gamma \sqrt{(1 - p)}))
\]

\[
\pm \frac{\text{arcosh} (\gamma \sqrt{(1 - p)})}{\gamma^2} = S_0 \frac{\Delta}{\Delta} - p \pm (1 - p) \tanh (\text{arcosh} (\gamma \sqrt{(1 - p)}))
\]

\[
\pm \frac{\text{arcosh} (\gamma \sqrt{(1 - p)})}{\gamma^2} = S_0 \frac{\Delta}{\Delta} - p \pm (1 - p) \frac{\gamma^2 (1 - p) - 1}{\gamma^2 (1 - p)}. \tag{21}
\]

The last substitution uses hyperbolic identities. Changing back to the original variables yields

\[
\pm \frac{\text{arcosh} (\sqrt{\beta \Delta^2 (1 - p)})}{\beta \Delta^2} = (1 - p) \sqrt{1 - \frac{1}{\beta \Delta^2 (1 - p)}} = S_0 \frac{\Delta}{\Delta} - p. \tag{22}
\]

We discard the equation that has no real numerical solutions and keep

\[
- \frac{\text{arcosh} (\sqrt{\beta \Delta^2 (1 - p)})}{\beta \Delta^2} = (1 - p) \sqrt{1 - \frac{1}{\beta \Delta^2 (1 - p)}} = S_0 \frac{\Delta}{\Delta} - p. \tag{23}
\]

Solutions of this equation become complex if \( p > \frac{S_0}{\Delta} \), hence \( p \leq \frac{S_0}{\Delta} \). The choice of \( C = 1 \) and \( D = 1/2 \) implies that there is no (burst of) cooperation if \( p > 1/3 \).
Data and code accessibility

All coded video data, the R script used to plot the data, and a Fortran script for simulations of the Ising model are available at https://osf.io/f25nq/

Authors’ contributions

JB made the asymmetric Ising model, and wrote the software and the paper. DW collected, interpreted, and analyzed the data. BM performed the mean field analysis.

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