Properties of High-Tc Single Crystals as Natural Interferometers in the THz Frequency Range

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Abstract

We consider oblique incidence of (p)TM-polarized wave on the anisotropic superconducting slab, immersed on a dielectric media, such that its uniaxial (c) axis is perpendicular to the surfaces. The below and above plasma frequency transmissivity patterns are studied and several of its properties determined, within the context of the Maxwell-London theory. Below, the regime is attenuated for any incident angle, and there is a transmissivity maximum, quite pronounced in case of a very high external dielectric constant. Above, a propagative regime exists where the superconductor is a natural optical resonator, and we show here that the minimum of the transmission peaks are modulated by an envelope function associated to the Brewster condition. We propose this set-up to obtain light with an extremely small transverse wavelength inside the superconductor.
I. INTRODUCTION

Conductors, well understood by the free-electron picture, undergo a transition from opacity to transparency at the plasma frequency. The effect, named after the frequency range where it occurs in metals, is known by ultraviolet transparency. Much below the plasma frequency the properties of conductors are dominated by dissipation resulting into an exponentially damped electromagnetic fields away from the surface, and so, nearly no light penetration into the bulk (skin effect). Similarly, light propagation in superconductors is also restricted to a strip near the surface, although by somewhat different reasons. Ideal superconductors do not dissipate, and the exclusion of electromagnetic fields from the bulk is essentially a consequence of the Meissner effect that forbids a magnetic field in its interior. Thus light propagation in superconductors is also confined to a strip below the surface, characterized by the London penetration depth. Indeed measurements involving the interaction of light with superconductors, below the pair-breaking threshold, are surface effects, like the surface impedance and the thin film plasmon. This is all in agreement with the view of no plasma frequency below the gap, thus condemning the superconductor to opacity throughout the relevant frequency range, that is, much below the frequency associated to the gap. Recently this view has completely changed due to the infrared reflectivity measurements of c-axis polarized light done in several high-Tc superconductors, such as La$_{2-x}$Sr$_x$CuO$_4$, YBa$_2$Cu$_3$O$_{7-x}$, and Bi$_2$Sr$_2$CaCu$_2$O$_{8-x}$. These measurements clearly indicate transparency starting at a critical plasma frequency, hereafter called $\omega_p$. The startling novelty is the evidence of a plasma mode below the energy gap, a collective charge oscillation between the CuO$_2$ planes, that was not considered in the early days arguments when such modes were proven forbidden to isotropic superconductors. Similarly to conductors, the reflectivity of layered superconductors changes dramatically depending whether the frequency range studied is below or above $\omega_p$. While below $\omega_p$ the behavior is the standard one, namely amplitudes decay exponentially from the surface, this is not the case above, where surprisingly, the anisotropic (ideal) superconductor behaves similarly to an anisotropic dispersive
dielectric medium. The recent theoretical literature\textsuperscript{18–20} has discussed several unexpected properties of the high-Tc ceramic single crystals, such as fiber-optic behavior, namely light is totally evanescent outside the slab, and in its interior, propagation can be pictured through ray optics with multiple internal reflection at the interfaces\textsuperscript{21}.

In this paper we consider a single crystal with the CuO$_2$ planes parallel to the external interface where a c-axis polarized THz source sheds light at an oblique angle of incidence. Single crystals are easily grown in this geometry that we claim to work as a natural resonator in the THz frequency region\textsuperscript{18,19}. Here we obtain several of its transmissivity characteristics below and above $\omega_p$ for this particular geometry. The present work applies at least for extremely low temperatures, where losses are negligible.

The study of the reflectivity at oblique incident on a superconducting slab has been previously considered by Artemenko and Kobel’kov\textsuperscript{22} in the context of kinetic equations for Green functions generalized to the case of layered superconductors with weak interlayer coupling. The present paper is a more detailed study of such system, where several new features are pointed out. In particular our study of light interaction with the anisotropic ideal superconductor is done in the context of the London-Maxwell theory, the simplest possible framework to understand the basic features of such systems near the plasma frequency. M. Tachiki et al.\textsuperscript{19}, have also obtained the light reflectivity and transmissivity through a film, however for a geometry distinct from ours. Notice that here the dielectric-superconductor interface has its normal along the c-axis, whereas in their case, it is orthogonal to the c-axis.

Recently a Fabry-Perot resonator was constructed with YBCO films\textsuperscript{23,24} and used to determine some of the superconductor properties, such as the complex conductivity, in the GHz region. When an integer number of half-wavelengths matches the distance between the two YBCO films, light is transmitted though this interferometer. This distance is an adjustable parameter that allows for the search of resonances at a given frequency. The simplest interferometer of all is the single slab, in our case a high-Tc single crystal immersed on a dielectric medium. The applications of the single slab interferometer are limited in comparison to the Fabry-Perot resonator because the thickness cannot be adjusted at will,
although varying the angle of incidence offers some degree of choice in this sense.

We choose a coordinate system (see fig.1) where the interfaces between the high-Tc single crystal and the dielectric media are at \( x = \frac{d}{2} \) and \( x = -\frac{d}{2} \). According to our choice of crystal geometry the c-axis is just the x-axis. The plane \( x - z \) corresponds to the so-called plane of incidence. Thus fields are plane waves given by \( \exp[-i(q_x x + q_z z - \omega t)] \), and \( \exp[-i(\tilde{q}_x x + \tilde{q}_z z - \omega t)] \), in the superconductor and in the dielectric medium, respectively. Below we list the remaining parameters used in the present model: the angle of incidence \( \theta \), the vacuum wave number, defined as \( k \equiv \frac{\omega}{c} \), the two London penetration lengths, transverse (\( \lambda_\perp \)), and longitudinal (\( \lambda_\parallel \)) to the surfaces; the dielectric constant of the non-conducting media exterior to the superconductor, \( \tilde{\varepsilon} \); the film thickness \( d \); and the distance between two consecutive CuO\(_2\) planes \( a \); the frequency independent dielectric constant \( \varepsilon_s \), the simplest phenomenological choice for the superconductor’s static dielectric constant along the c-axis. This constant takes into account the contribution from high-energy interband processes and the atomic core.\(^2\)

This paper is organized as follows. In section II, we introduce the London-Maxwell theory, the Fresnel equation and derive some noticeable properties below and above \( \omega_p \). In section III we analyze the transmission patterns expected for the two kinds of regimes characterized by \( q_x^2 \), the transverse wavenumber inside the superconducting slab: the attenuated (\( q_x^2 < 0 \)) and the propagative (\( q_x^2 > 0 \)). Based on the the present model we discuss on IV the optical properties of the high-Tc ceramic \( La_{2-x}Sr_xCuO_4 \), \( x = 0.16 \), at extremely low temperatures. Finally in section V we summarize our major results.

II. BASIC THEORY ABOVE AND BELOW \( \omega_p \)

The simplest possible theoretical framework able to describe light propagation on the anisotropic superconductor, below and above \( \omega_p \), is the London-Maxwell theory. The approach is limited to energies much lower than the pair breaking threshold. The dynamics of the electromagnetic fields is described by the Maxwell’s equations,
\[ \nabla \cdot \mathbf{E} = \frac{q}{\varepsilon_0} (n_s(x, t) - n_{0s}) \]  
(1)

\[ \nabla \cdot \mathbf{H} = 0 \]  
(2)

\[ \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \]  
(3)

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]  
(4)

In this paper we assume a low temperature regime, and so, ignore any normal current contribution, thus charge transport is just of the superfluid kind, described by the first London equation, adapted to the anisotropic problem.

\[ \mu_0 \lambda^2 \frac{\partial \mathbf{J}_\parallel}{\partial t} = \mathbf{E}_\parallel, \quad \mu_0 \lambda^2 \frac{\partial \mathbf{J}_\perp}{\partial t} = \mathbf{E}_\perp \]  
(5)

where \( \mathbf{E}_\parallel, \mathbf{J}_\parallel \) and \( \mathbf{E}_\perp, \mathbf{J}_\perp \) are the field and supercurrent components parallel and perpendicular to the interfaces, respectively.

The time dependent electromagnetic coupling of the charged superfluid induces density fluctuations, a new feature not present in the original formulation of London theory. Notice that in the above London equation the supercharge density is uniform and constant in space, \( n_{0s} \), since the London penetration depths are \( \lambda^2 = m/\varepsilon_0 q^2 \mu_0 \) and \( \lambda^2 = m/\varepsilon_0 q^2 \mu_0 \), however coupling through Maxwell’s equations forces the supercharge density, \( n_s(x, t) \), to fluctuate in time and space while only the neutralizing positive background remains uniform and constant: \( n_{0s} \). This is a linear theory that simply ignores second order position and time dependent corrections to the density. Such corrections are to a time independent process, which always has uniform and constant density, \( n_s(x, t) = n_{0s} \). For the case of interest here, namely wave propagation, this neutrality is not possible, and, according to Gauss’ law, the supercharge density is found no longer constant and uniform.

At this point we introduce the time dependence \( \exp(i \omega t) \) to all fields, assuming that the light frequency is much smaller than the frequency associated to the superconducting gap. Introducing the first London equation,

\[ i \omega \mu_0 \lambda^2 \mathbf{J}_\parallel = \mathbf{E}_\parallel, \quad i \omega \mu_0 \lambda^2 \mathbf{J}_\perp = \mathbf{E}_\perp, \]  
(6)
into Maxwell’s equations, gives that,

\[ \nabla \cdot D = 0 \]  \hspace{1cm} (7)

\[ \nabla \cdot H = 0 \]  \hspace{1cm} (8)

\[ \nabla \times E = -i\omega \mu_0 H \]  \hspace{1cm} (9)

\[ \nabla \times H = i\omega D \] where \( D = \varepsilon_s \varepsilon_0 E - iJ/\omega. \)  \hspace{1cm} (10)

where the continuity equation, \( \nabla \cdot J + i\omega q n_s = 0 \) has been used. The superconductor’s dielectric constant is tensorial, \( D = \varepsilon_s E - iJ/\omega = \varepsilon_0 \varepsilon \cdot E. \)

\[ \varepsilon = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} \] \hspace{1cm} \varepsilon_\perp = \varepsilon_s - \frac{1}{(k\lambda_\perp)^2} \hspace{0.5cm} \varepsilon_\parallel = \varepsilon_s' - \frac{1}{(k\lambda_\parallel)^2}, \]  \hspace{1cm} (11)

The incident light wavelength, \( 2\pi/k \), is assumed much larger than the London penetration depth along the surface, thus rendering an always negative dielectric constant component \( \varepsilon_\parallel. \)

\[ \varepsilon_\parallel \approx -\frac{1}{(k\lambda_\parallel)^2}, \hspace{0.5cm} k\lambda_\parallel \ll 1 \]  \hspace{1cm} (12)

A consequence of an extremely high anisotropy is that a similar property does not hold perpendicularly to the film. Consequently the incident light wavelength matches the transverse London penetration depth even much below the pair breaking threshold. Thus the present model’s description near \( \omega_p \) has physical content and we use it to describe phenomena at the frequency window where this dielectric component \( \varepsilon_\perp \) flips sign.

\[ \varepsilon_\perp(\omega = \omega_p) = 0 \hspace{1cm} \omega_p = \frac{c}{\sqrt{\varepsilon_s\lambda_\perp}} \]  \hspace{1cm} (13)

The interesting polarization is the so-called \( p \) or \( TM \) (Transverse Magnetic), the only one that probes properties of the dielectric component \( \varepsilon_\perp. \) We notice, for this polarization, a superficial charge density build-up at the interfaces whose oscillation yields interesting low frequency properties. Its field components \( (E_x, H_y, E_z) \) imply on a transverse current \( J_x \), not present in the external insulating dielectric media, that lead to this superficial
charge density. The s or TE (Transverse Electric) polarized wave, with its field components \( (H_x, E_y, H_z) \), only probes \( \varepsilon_\parallel \), and consequently fields are confined to a strip near the interfaces. for this reason it is not considered in this paper.

Long before the advent of Maxwell’s equations, Fresnel had developed a geometrical method to explain the properties of light propagation on anisotropic dielectric media characterized by more than one refraction index. It is straightforward to derive from the present Maxwell-London theory the so-called Fresnel’s equation of wave normals,

\[
\frac{q_z^2}{\varepsilon_\perp} + \frac{q_x^2}{\varepsilon_\parallel} = k^2
\]  

inside the superconductor \((-d/2 < x < d/2)\), and,

\[
\tilde{q}_z^2 + \tilde{q}_x^2 = \tilde{\varepsilon} k^2
\]  

in the dielectric media \((x < -d/2; x > d/2)\). According to Fig. we solve Eq.(15) for the incident wave in the dielectric, using the previously introduced parameters: \( \tilde{q}_x = -k \sqrt{\varepsilon} \cos \theta \), and \( \tilde{q}_z = k \sqrt{\varepsilon} \sin \theta \). Snell’s law states that the wavenumber parallel to the surface is the same for the incident, reflected and transmitted waves: \( q_z = \tilde{q}_z \). Finally we obtain an expression, central for our purposes here, that gives \( q_x \), the transverse wavenumber inside the superconductor, in the approximation described by Eq.(12).

\[
q_x^2 = \frac{1}{\lambda_\parallel^2} \left( \frac{\varepsilon}{\varepsilon_\perp(\omega)} \sin^2 \theta - 1 \right)
\]  

This equation is a valuable tool to understand many of the unusual properties of light interaction with the (ideal) anisotropic superconductor. For instance, the normal incidence \((\theta = 0)\) is always attenuated exponentially decaying over a distance \( \lambda_\parallel \). Next we derive further results, in special several frequency parameters useful to understand the properties of wave propagation inside the superconducting slab.

\textbf{A. Below } \omega_p

For \( \varepsilon_\perp < 0 \) the transverse wavenumber inside the superconductor, \( q_x \), given by Eq.(13), is imaginary, and so, the TM wave is attenuated in this frequency range. The attenuation
length shortens as the frequency increases from zero to \( \omega_p \). To see this, notice that deeply below \( \omega_p \), the transverse dielectric constant is approximately given by \( \varepsilon_\perp \approx -1/(k_\perp \lambda_\perp)^2 \), similarly to Eq.\((12)\), resulting that, \( q_x^2 \approx -[(k_\perp \lambda_\perp)^2 \sin^2 \theta + 1]/\lambda_{\parallel}^2 \). Thus the attenuation length, typically of order \( \lambda_{\parallel} \) for low frequencies, diminishes as the frequency increases, and eventually vanishes at \( \omega_p \).

**B. Above \( \omega_p \)**

The regime is propagative for oblique incidence, no matter how small \( \theta \) is, provided that a frequency window is selected sufficiently close to the plasma frequency (\( \varepsilon_\perp \sim \varepsilon_s + 0^+ \)). Thus a frequency window exists, starting at \( \omega_p \) and ending at \( q_x = 0 \), this last condition associated to the frequency below, obtained from Eq.\(16\).

\[
\omega_0 = \frac{\omega_p}{\sqrt{1 - \frac{\varepsilon_s}{\tilde{\varepsilon}} \sin^2 \theta}}
\]  

The propagative frequency window, \( (\omega_p, \omega_0(\theta)) \) exists until a critical angle is reached, thus defined only for an angular window \( (0, \theta_c) \).

\[
\theta_c = \begin{cases} 
\arcsin \sqrt{\varepsilon_s/\tilde{\varepsilon}} & \tilde{\varepsilon} > \varepsilon_s \\
\pi/2 & \tilde{\varepsilon} < \varepsilon_s 
\end{cases}
\]  

Notice that for \( \tilde{\varepsilon} > \varepsilon_s \) there is another attenuated region above the critical angle, \( \theta_c < \theta \leq \pi/2 \).

A well known property of oblique TM wave, incident on the interface between two dielectric, is of no reflection at the so-called Brewster angle. We show here that for the interface dielectric-superconductor this property of no reflection occurs in the whole propagative angular region \( (0, \theta_c) \). This is a dispersive medium property, that for any angle of incidence allows for the choice of a refraction index by tuning the frequency, such that the Brewster condition is met. This Brewster condition of no reflection is given below, following standard arguments, obtained from the study of a plane wave incident on the interface between the two media.
Before proceeding any further, we briefly show that the above relation yields the Brewster angle in case of a single interface between two non-conducting dielectric media. For the sake of the argument ignore the frequency dependence of $\tilde{\varepsilon}$ and solve Eq. (19): $\cos \theta / \sqrt{\tilde{\varepsilon}} = \cos \tilde{\theta} / \sqrt{\tilde{\varepsilon}}$, where $\tilde{\theta}$ and $\theta$ are the incident and transmitted angles, respectively. Added of Snell’s law, $\sqrt{\tilde{\varepsilon}} \sin \theta = \sqrt{\varepsilon} \sin \tilde{\theta}$, one finally obtains the Brewster angle condition, $\theta + \tilde{\theta} = \pi / 2$. To obtain this result we have introduced into Eq. (19) that $q_x = -k \sqrt{\tilde{\varepsilon}} \cos \theta$ and $\tilde{q}_x = -k \sqrt{\varepsilon} \cos \tilde{\theta}$. Back to the superconductor-dielectric interface, square Eq. (19) and introduce both Eq. (16) and Eq. (12). The Brewster condition is determined by the transverse wavenumber and the frequency obtained at a given $\theta$.

$$q_{x,B}^2 = \left[ \sqrt{(\alpha - 1)^2 - 4 \alpha \beta} + (\alpha - 1) \right] / 2 \lambda_{||}^2$$

$$\omega_B^2 = \omega_p^2 \left[ \sqrt{(\alpha - 1)^2 + 4 \alpha \beta} - (\alpha - 1) \right] / (2 \beta)$$

$$\alpha = \frac{\varepsilon_s}{\tilde{\varepsilon}} \left( \frac{\lambda_{\perp}}{\lambda_{||}} \right)^2 \cos^2 \theta$$

$$\beta = 1 - \frac{\varepsilon_s}{\varepsilon_{s}} \sin^2 \theta$$

We find that properties of the Brewster condition depend whether the ratio $\tilde{\varepsilon} / \varepsilon_s$ is smaller or larger than one. Let us restrict the discussion, in both cases, to the highly anisotropic superconductors, defined through the condition $\sqrt{\varepsilon_s / \tilde{\varepsilon}} (\lambda_{\perp} / \lambda_{||}) \gg 1$. For $\tilde{\varepsilon} / \varepsilon_s < 1$ the Brewster frequency varies from $\omega_B \approx \omega_p$, for $\theta = 0^+$, to $\omega_B \approx \omega_0$, for $\theta = \pi / 2$. The transverse wavenumber associated to each of these extreme angles is $q_{x,B}(0) \approx \sqrt{\varepsilon_s / \tilde{\varepsilon}} (\lambda_{\perp} / \lambda_{||}^2)$, and $q_{x,B}(\pi / 2) \approx 0$, respectively. For $\tilde{\varepsilon} / \varepsilon_s > 1$ the propagative regime also begins at $\theta = 0^+$, like the previous case, and ends at $\theta_c$ with the Brewster frequency $\omega_B \approx \omega_p \sqrt{\alpha' / (\alpha' - 1)}$, $\alpha' \equiv (\varepsilon_s / \tilde{\varepsilon})(1 - \varepsilon_s / \tilde{\varepsilon})(\lambda_{\perp} / \lambda_{||})^2$. The corresponding transverse wavenumber is $q_{x,B}(\theta_c) \approx \sqrt{\alpha' / \lambda_{||}}$.

In this paper we are interested on the finite thickness superconducting slab immersed on a dielectric medium. A well-known optical property of a film is the resonant condition, that leads to total light transmission through the film, namely an integer number of transverse half-wavelengths perfectly matching the thickness, $N \lambda_x / 2 = d$. This condition on Eq. (16)
gives that,

\[ q_x = N \frac{\pi}{d} \quad \omega_N = \omega_p \sqrt{1 + \left( \frac{N \pi \lambda_{\|}}{d} \right)^2} \]

where \( N \) is the number of half-wavelengths. Notice that the propagative frequency window is swept backwards for increasing \( N \), i.e., the maximum propagative frequency of Eq. (17) is associated to the smallest integer, \( N = 0 \). Near the minimum propagative frequency, \( \omega_p \), \( N \) is large, but there is a maximum value associated to the limit of this theory’s validity, which cannot describe a transverse wavelength so small as the interlayer distance \( a \). Another model, able to describe individual layers, such as the Lawrence-Doniach model\(^2\), must be considered in this limit.

\[ N_{\text{max}} = 2d/a \quad q_{xN_{\text{max}}} = 2\pi/a \quad \omega_{N_{\text{max}}} = \omega_p \sqrt{1 + \left( \frac{2\pi \lambda_{\|}}{a} \right)^2} \left( 1 - \frac{\bar{\varepsilon}}{\varepsilon_s} \sin^2 \theta + \left( \frac{2\pi \lambda_{\|}}{a} \right)^2 \right) \]

In the usual cases this maximum frequency falls extremely close to the plasma frequency, \( \omega_{N_{\text{max}}} \sim \omega_p^+ \), a direct consequence of a longitudinal London penetration depth much larger than the inter-plane distance: \( \lambda_{\|} \gg a \).

It follows from Eq. (24) the enlargement of the distance between two consecutive resonances, \( \omega_{N-1} - \omega_N \), for an increasing propagative window \( (\omega_p, \omega_0) \). This is a valuable remark, considering that the choice on \( \omega_0 \) depends on other parameters such as an appropriate angle of incidence, \( \theta \) and (or) the external dielectric medium constant, \( \bar{\varepsilon} \). Let us check that indeed this is the case for two special cases of Eq. (24): (i) Near \( \omega_p \), one has that \( d/N_{\text{max}} \pi \lambda_{\|} \ll 1 \) and, in this limit, it holds that \( \omega_{N-1} - \omega_N \approx \omega_p \left[ 1 - (\omega_p/\omega_0)^2 \right] (d/\pi \lambda_{\|})^2 / N^3 \).

(i) Near \( \omega_0 \) we just provide the distance between the square of the last two maxima, \( \omega_0^2 - \omega_1^2 = \omega_0^2 (\pi \lambda_{\|}/d)^2 [1 - (\omega_p/\omega_0)^2] / [(\omega_p/\omega_0)^2 + (\pi \lambda_{\|}/d)^2] \). Both limits clearly indicate that for increasing \( \omega_0 \), and so small ratio \( \omega_p/\omega_0 \), the distance between consecutive frequency maxima also increases.
III. TRANSMISSION PATTERN ANALYSIS

Applying the standard arguments of continuity of the transverse fields, \( H_y \) and \( E_z \), one obtains the ratio between the bottom and the top normal components of the Poynting's vector, and so, the transmissivity coefficient, \( T \), that measures the transmission of energy through the film. For the case of identical top and bottom dielectric media, \( T \) is a sole function of the Fresnel reflection coefficient (\( \rho \)) of a single surface, and of the phase parameter (\( \eta \)).

\[
\rho = \frac{1 - F}{1 + F}, \quad \eta = \exp \left( i q_x d \right), \quad T = \frac{|\eta(1 - \rho^2)|^2}{1 - \eta^2 \rho^2}; \quad (26)
\]

where \( F \) has been previously defined (Eq.(19)).

A. The Attenuated Regions

There are two possible attenuated regions for a fixed \( \theta \), below the plasma frequency, \( 0 < \omega < \omega_p \), and above the propagative window, \( \omega > \omega_0 \). Both regions display the common feature that \( 1 - (\tilde{\varepsilon}/\varepsilon_{\perp}) \sin^2 \theta > 0 \). In these regions the transverse wavelength, \( q_x \), is imaginary thus being interpreted as a penetration depth. The ratio defined in Eq.(19) is imaginary, the phase parameter \( \eta \) real, and so, the transmissivity coefficient of Eq.(26) becomes,

\[
T = \frac{8 f^2}{(1 + f^2)^2 \cosh (2d/l_x) - (1 - 6 f^2 + f^4)}
\]

\[
f^2 = \left( \frac{\omega \lambda_{||}}{c} \right)^2 \frac{\tilde{\varepsilon}}{\cos^2 \theta} (1 - \frac{\tilde{\varepsilon}}{\varepsilon_{\perp}} \sin^2 \theta) \quad l_x = \frac{\lambda_{||}}{\sqrt{1 - \frac{\tilde{\varepsilon}}{\varepsilon_{\perp}} \sin^2 \theta}} \quad (27)
\]

The transmissivity has a peak below the plasma frequency since at extremely small frequencies it is an increasing function, growing proportionally to \( \omega^2 \), and very near \( \omega_p \) it becomes a decreasing function that eventually vanishes at \( \omega_p \). We obtain here the asymptotic limit of Eq.(27), in this last regime that shows the exponential decay of the transmissivity below and extremely close to \( \omega_p \). For \( \theta \neq 0 \) the transmissivity is approximately described by \( T \approx 16 f^{-2} \exp \left( -2d/l_x \right) \) that leads to,
\[
T = 32\left(\frac{\varepsilon_s}{\varepsilon} \frac{\lambda}{\lambda_\|}\right)^2 \frac{1}{\tan^2 \theta} \left(\frac{\omega_p}{\omega} - 1\right) \exp\left(-\frac{d}{\lambda_\|} \sqrt{\frac{2\varepsilon}{\varepsilon_s} \sin \theta \frac{\omega_p}{\omega} - 1}\right)
\] (28)

**B. The Propagative Region**

In the propagative window, \((0, \theta_c)\) and \((\omega_p, \omega_0)\), one has that \((\varepsilon/\varepsilon_\perp) \sin^2 \theta - 1 > 0\) and the transmissivity becomes,

\[
T = \frac{8 F^2}{(1 + 6F^2 + F^4) - (1 - F^2)^2 \cos(2q_x d)}
\] (29)

where \(F^2 = (\omega \lambda_\|/c \cos^2 \theta)^2 [(\varepsilon/\varepsilon_\perp) \sin^2 \theta - 1]\), according to Eq.(19), and \(q_x\) is given by Eq.(16). The transmissivity maximum \((T = 1)\) corresponds to \(\eta = 1\) \((q_x = N\pi/d)\), the resonance condition previously described (Eq.(24)).

The Brewster condition is well defined in the context of the envelope curve that fits all the transmissivity minima. The maximum of this envelope function is a convenient definition of the Brewster condition. The reason for this is very simple, when the transmissivity minima and maxima are equal no wave is reflected and the Brewster condition is met. The transmissivity minima occur for \(\cos(2q_x d) = -1\). Hence the minimum transmissivity is given by \(T_{\min} = 4F_{\min}^2/(1 + F_{\min}^2)^2\) where \(F_{\min}\) is obtained from Eq.(19) under the minimum wavenumber condition, \(q_x = (N + 1/2)\pi/d\). These points correspond to an integer number of transverse half-wavelengths, added of an extra one forth of a wavelength, matching the thickness.

From its turn the envelope curve, defined by the continuous function \(T_{\text{env}} \equiv 4F^2/(1 + F^2)^2\) of parameter \(q_x\), has a maximum at \(F^2 = 1\), the Brewster condition previously discussed in Eq.(19). Obviously \(T_{\text{env}}\) fits all the transmissivity minima within the propagative window. Near the Brewster point interferometric properties of the finite superconducting slab are no longer useful.
IV. DISCUSSION

For our discussion we choose the high-Tc ceramic $La_{2-x}Sr_xCuO_4$, which has a single superconducting layer per unit cell, following Tamasaku, Nakamura, and Uchida data on high-quality single crystals of this superconductor. For the composition $x = 0.16$ the critical temperature is $T_c = 34K$. For $T = 8K$ these authors obtain from the reflectivity data, and Kramers-Kronig analysis, the transverse dielectric constant, $\varepsilon_\perp = \varepsilon_s - \omega'^2_\perp/\omega^2 - (\omega^2_\perp - \omega'^2)/[\omega(\omega + i\gamma)]$, fitted by the following parameters: $\varepsilon_s = 25, \omega'_\perp = 8.4 \times 10^{12} \text{rad/s}, \omega_\perp = 9.0 \times 10^{12} \text{rad/s}$ and $\gamma = 5.0 \times 10^{10} \text{rad/s}$. The two frequency dependent terms in the dielectric constant are, respectively, the contribution of the condensed carriers and the thermally excited quasi-particles. In this paper losses are being totally discarded, thus we extrapolate the above data to $\gamma = 0$, thus obtaining the transverse London penetration $\lambda_\perp = c/\omega_\perp = 33 \mu m$, and the plasma frequency $\omega_p = \omega_\perp/\sqrt{\varepsilon_s} = 1.8 \times 10^{12} \text{rad/s}$. For the zero-temperature London penetration length along the $CuO_2$ planes we take that $\lambda_\parallel = 0.2 \mu m$, thus yielding an effective anisotropy of $\lambda_\perp/\lambda_\parallel = 165$. The distance between two consecutive $CuO_2$ planes in this compound is $a = 0.7 \text{nm}$. For illustration purposes we choose the slab sufficiently thin in order to deal just with fairly small number of frequency resonances in the propagative window: $d = 0.1 \mu m$ and, consequently, $N_{max} = 285$, according to Eq.(25).

Fig.(1) provides a pictorial intuitive view of a p-polarized wave incident on a superconducting slab immersed on a dielectric media of dielectric constant $\tilde{\varepsilon}$. The anisotropic superconductor has its c-axis perpendicular to the interfaces, and is characterized by two-dielectric constants, the longitudinal ($\varepsilon_\parallel$), monotonous and always negative in any frequency window, and the transverse ($\varepsilon_\perp$), that flips sign, creating the below and above plasma frequency regimes.

Fig.(2) displays the below plasma transmissivity for $\theta = 20^\circ, 50^\circ, \text{and } 80^\circ$. In this frequency range the transmissivity has a peak, whose height is quite pronounced due to the choice of a nonconducting bounding medium of very high dieletric constant, namely $SrTiO_3, \tilde{\varepsilon} \approx 2.0 \times 10^4$. In case the slab is immersed in air ($\tilde{\varepsilon} = 1$), the transmissivity
maxima are $2.1 \times 10^{-4}$, $4.5 \times 10^{-4}$, and $6.0 \times 10^{-3}$, for the above angles of incidence, respectively, all occurring at frequencies extremely close to the plasma frequency ($\omega \approx 0.99 \omega_p$). Therefore the peak still exists in case of a small dielectric constant, although very small.

Fig.(3) and Fig.(4) show the above $\omega_p$ transmissivity for $\theta = 20^\circ$, and $80^\circ$, respectively. The many spikes, observed in these transmissivity patterns, are the resonances labeled by the number of transverse half-wavelengths that fit transversally into the slab. In order to best visualize the spikes we have chosen the frequency window to $(\omega_p, \omega(10))$. Indeed the spikes are less dense near $\omega_0$ than near $\omega_p$ as shown in these two Figures. Properties of the spikes, such as the density, are studied through the functions,

$$Q_1(N) \equiv \frac{\omega(N)}{\omega(N - 1/2) - \omega(N + 1/2)} \quad (30)$$

$$Q_2(N) \equiv \frac{2}{T(N - 1/2) + T(N + 1/2)} \quad (31)$$

The first function provides information on the spike density. In fact near $\omega_p$, where this density is high $d/\pi\lambda_|| N \ll 1$, one obtains that $Q_1(N) \approx N^3(\pi\lambda_||/d)^2/[1 - (\omega_p/\omega_0)^2]$. Near $\omega_0$ consecutive peaks are well separated and $Q_1(N)$ is small. The second function is just the ratio of the transmissivity maximum and the average transmissivity taken between the two neighbor transmissivity minima. In case the minima are not pronounced, namely are approximately one, $Q_2(N)$ becomes approximately equal to one. For interferometric purposes the best resolution for a spike is sought and, for this purpose, both $Q_1(N)$ and $Q_2(N)$ must be much larger than one in the frequency range under investigation. Fig.(5) and Fig.(6) display the above functions versus $\omega - \omega_p$, for $\theta = 20^\circ$, and $80^\circ$. The frequency range displayed ranges from $\omega(N_{max} - 1)$, very near $\omega_p$, to $\omega(1)$, the last spike before the end of the propagative regime at $\omega(0)$. Fig.(5) shows that $Q_1$ drops over many orders of magnitude for increasing frequency, confirming that the density of spikes is maximum near $\omega_p$ and minimum near $\omega_0$. The minimum of $Q_2$, shown in Fig.(6) for both angles, has in its minimum the Brewster condition, since the transmissivity maximum and minimum are equal to one. Notice that the frequency associated to the minimum of $Q_2$ is just the maximum of the envelope function shown in Fig.(3) and Fig.(4). This frequency, $\omega_B$, shows,
in a explicit way, that the Brewster condition of no $TM$ reflected polarized light is more than just an angle, being a set of parameters, including $\omega_B$ and its associated angle of incidence.

Lastly Fig.(7) displays $\omega_B - \omega_p$ vs. $\theta$. For very small angle of incidence the Brewster frequency is close to $\omega_p$ and for a grazing angle moves to an intermediate position in the propagative window. This figure also shows the behavior of the lower $(\omega(N_{max}) - \omega_p)$ and upper $(\omega_0 - \omega_p)$ limits of the propagative window vs. $\theta$.

V. CONCLUSION

In this paper we have studied transmissivity of the TM-polarized light through a superconducting slab immersed on a dielectric media. The superconductor is anisotropic and the surface, where light is obliquely incident, is orthogonal to the c-axis. The present study is done in the context of the London-Maxwell theory, which gives a good account of the physical situation for scales larger than the interplane separation. The anisotropic superconductor has a tensorial dielectric constant. Along the surfaces the (longitudinal) component is negative because the incident beam wavelength is always much larger than the longitudinal London penetration depth. However the situation is quite distinct along the transverse direction. The incident wavelength can be comparable to the transverse London penetration depth implying that the physical range where this dielectric constant component vanishes is an interesting one even though we are only interested in phenomena taking place much below the pair breaking threshold.

The effects of a plasma frequency along the c-axis on the transmissivity patterns is studied here. Below the plasma frequency the regime is attenuated and an interesting feature arises in case the slab thickness is smaller than the longitudinal London penetration depth. The interaction between the two surfaces renders a transmissivity maximum that can be quite pronounced in case the external dielectric constant is very large. In particular we have shown here that the transmissivity vanishes exponentially below and very near to the plasma frequency. Above the plasma frequency there is a frequency and angular window where
the regime is propagative inside the superconductor independently on how thick slab is. Within this window, the superconducting slab works as a natural optical resonator displaying transmissivity resonances labeled by the number of transverse half-wavelength that exactly match the slab. A remarkable property of this regime is the transverse wavelength inside the superconductor, which can be as small as desired up the order of the inter-plane separation.

TM-polarized light, obliquely incident on a surface between two dielectric media, has the property of no reflection at the so-called Brewster angle. Similarly for the dielectric-superconductor interface a Brewster condition exists. We find here that this Brewster condition of total transmission exists for any angle of incidence within the propagative range, once the incident frequency is chosen suitably. In summary we find that the transmissivity minima are fitted by an envelope curve, whose maximum gives the Brewster condition.

The present work does not take into account losses thus being restricted to extremely low superconductors where thermally activated normal carriers can be ignored. In this context we have used parameters of the high-Tc ceramic $La_{2-x}Sr_xCuO_4$ to show the transmissivity features discussed here.

In summary we have determined in this paper several of the transmissivity properties of an anisotropic superconducting slab immersed on a dielectric medium which has the startling property of behaving, above the c-axis plasma frequency, as an anisotropic dielectric for incident TM polarized light.
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FIGURES

FIG. 1. A pictorial view of a TM polarized wave incident on a superconducting slab immersed on a nonconducting dielectric medium. The multiple reflections occurring at the interfaces suggest that an above $\omega_p$ situation is being represented here.

FIG. 2. The below $\omega_p$ transmissivity of the $La_{2-x}Sr_xCuO_4$ superconductor as described in the text, is shown here for three angles of incidence. The quite pronounced transmissivity peaks should be credited to an exterior dielectric medium of elevated constant, $SrTiO_3$: $\tilde{\varepsilon} \approx 2.0 \times 10^4 \text{ rad/s}$.

FIG. 3. The transmissivity is shown here in the window ($\omega_p, \omega(10)$) for $\theta = 20^\circ$. The remaining parameters are those of the $La_{2-x}Sr_xCuO_4$ superconductor, as described in the text. Notice that each spike is a resonance, labeled by the number of transverse half-wavelengths that exactly fit into the slab.

FIG. 4. The transmissivity is shown here in the window ($\omega_p, \omega(10)$) for $\theta = 80^\circ$. The remaining parameters are those of the $La_{2-x}Sr_xCuO_4$ superconductor, as described in the text. Notice that each spike is a resonance, labeled by the number of transverse half-wavelengths that exactly fit into the slab.

FIG. 5. The ratio between the resonance frequency and the distance between the two neighbor minima is shown here for two angles of incidence in the frequency window ($\omega(N_{max} - 1), \omega(1)$). The remaining parameters are those of the $La_{2-x}Sr_xCuO_4$ superconductor, as described in the text. The frequencies $\omega_1, \omega_B$ and $\omega_{N_{max} - 1}$ are indicated for the $\theta = 20^\circ$ curve.

FIG. 6. The ratio between the maximum transmissivity (one) and the average transmissivity of the two neighbor minima is shown here for two angles of incidence in the frequency window ($\omega(N_{max} - 1), \omega(1)$). The remaining parameters are those of the $La_{2-x}Sr_xCuO_4$ superconductor, as described in the text. The frequencies $\omega_1, \omega_B$ and $\omega_{N_{max} - 1}$ are indicated for the $\theta = 20^\circ$ curve.
FIG. 7. This figure shows the lower \((\omega(N_{max}) - \omega_p)\) and the upper \((\omega_0 - \omega_p)\) limits of the propagative window, as well as the Brewster frequency \((\omega_B - \omega_p)\), as a function of the angle of incidence \((\theta)\).
Superconducting film

Dielectric medium

E // polarization plane

Dielectric medium

Superconducting film

Dielectric medium
\[ \omega - \omega_p \left( 10^6 \text{ rad/s} \right) \]
$\theta = 80^\text{o}$
\[ \Delta \omega = \omega - \omega_p \]

\[ \Delta \omega_{N_{\text{max}}-1} \]

\[ \Delta \omega_B \]

\[ \omega_N/(\omega_{N-1/2} - \omega_{N+1/2}) \]

\[ \theta \]

\[ \Delta \omega_1 \]

\[ 20^\circ \]

\[ 80^\circ \]
\[ \Delta \omega = \omega - \omega_p \]

\[ \Delta \omega_{N_{\text{max}}}^{-1} \]

\[ \Delta \omega_B \]

\[ \theta \]

\[ 20^\circ \]

\[ 80^\circ \]

\[ \Delta \omega_1 \]
