Influence of Collective Nuclear Vibrations on Initial State Eccentricities in Pb + Pb Collisions

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Abstract—We study within the Monte Carlo Glauber model the influence of collective quantum effects in the Pb nucleus on the azimuthal anisotropy coefficients $\varepsilon_{2,3}$ in Pb + Pb collisions at the LHC energies. To account for the quantum effects, we modify the sampling of the nucleon positions by applying suitable filters that guarantee that the colliding nuclei have the mean squared quadrupole and octupole moments consistent with the ones extracted from the experimental quadrupole and octupole strength functions for the Pb nucleus with the help of the energy weighted sum rule. Our Monte Carlo Glauber model with the modified sampling of the nucleon positions leads to $\varepsilon_2(2)/\varepsilon_3(2) \approx 0.8$ at centrality $\leq 1\%$, which allows to resolve the $v_2$-to-$v_3$ puzzle.

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1. INTRODUCTION

It is now believed that hadron production in heavy ion collisions at the RHIC and LHC energies goes through quark-gluon plasma (QGP) stage. Hydrodynamic analyses of experimental data from RHIC and LHC data show that the QGP is formed at the proper time $\tau_0 \sim 0.5–1$ fm [1–3] after interaction of the Lorentz-contracted nuclei. The QGP fireball, formed between the nuclear disks receding from each other, inherits approximately the shape of the overlap region of the colliding nuclei. For non-central $AA$ collisions the overlap region has an almond shape. This can lead to a significant anisotropy in the transverse QGP expansion at later times, and eventually to azimuthal asymmetry of particle spectra [4]. In the presence of fluctuations of the initial QGP density, the azimuthal asymmetry can appear even for central collisions. The azimuthal dependence of hadron spectra is characterized by the flow coefficients $v_n$ in the Fourier expansion

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left[ 1 + \sum_{n=1}^{\infty} 2v_n \cos(n(\phi - \Psi_n)) \right],$$

where $N$ is the hadron multiplicity in a certain $p_T$ and rapidity region, $\Psi_n$ are the event reaction plane angles. In hydrodynamic models with smooth initial conditions, in the Fourier series (1) at midrapidity ($y = 0$) only the terms with $n = 2k$ survive (if the hadronization occurs without fluctuations). In this approximation, for $AA$ collisions with zero impact parameter the coefficients $v_{2k}$ should vanish due to the azimuthal symmetry. Hydrodynamic calculations show that for heavy ion collisions at small centralities in each event the flow coefficients $v_n$ with $n \leq 3$ are proportional, to good accuracy, to the anisotropy coefficients $\varepsilon_n$ for the initial entropy distribution [5–7]

$$v_n = k_n \varepsilon_n.$$

The coefficients $\varepsilon_n$ are defined as [8, 9]

$$\varepsilon_n = \frac{\int d\rho^b \rho^n \rho_s(\rho)}{\int d\rho \rho^n \rho_s(\rho)},$$

where $\rho_s(\rho)$ i.e. the initial fireball entropy density, and it is assumed that $\rho$ is calculated in the transverse c.m. frame, i.e., $\int d\rho \rho_s(\rho) = 0$.

Hydrodynamic calculations require the initial entropy density, which presently cannot be specified from first principles. There are currently several models in use for evaluation of the initial entropy distribution in heavy ion collisions. The most widely used and simple phenomenological method to generate the initial entropy distribution is the Monte Carlo (MC) wounded nucleon Glauber model [10, 11], in which the entropy density is expressed via a linear combination of the number of the participating nucleons and of the binary collisions. In the MC Glauber model event-by-event fluctuations of the entropy density is a combined effect of fluctuations of the nucleon positions in the colliding nuclei and fluctuations of the entropy production for a given geometry of the nucleon positions. The MC Glauber model has been quite successful in description within hydrodynamic

669
models of experimental data on the flow coefficients in AA collisions obtained at RHIC and the LHC. Hydrodynamic simulations with the Glauber model initial conditions demonstrated that the QGP produced at RHIC and the LHC has a very low shear viscosity to entropy density ratio, which is of the order of the lower quantum bound $1/4\pi$ [12, 13]. Another more recent phenomenological MC scheme for the entropy production in AA collisions, that was successfully used in hydrodynamic analyses, is the TRENTO model [14]. In the TRENTO model, similarly to the MC Glauber scheme, the entropy density fluctuations originate from fluctuations of the nucleon positions in the colliding nuclei and fluctuations of the entropy production for a given geometry of the positions of the participating nucleons. This differs from the QCD-inspired IP-Glasma [15] and MAGMA [16] models, in which the entropy density fluctuations come only from fluctuations of the nucleon positions.

Although hydrodynamic models can reproduce a vast body of data on heavy ion collisions from RHIC and the LHC, in recent years it was found that they have a tension with description of the ratio between $v_2$ and $v_3$ in ultra-central ($c \to 0$) Pb + Pb collisions at the LHC energies (so-called $v_2$-to-$v_3$ puzzle). Measurements of the flow coefficients in ultra-central 2.76 [17] and 5.02 [18] TeV Pb + Pb collisions show that $v_3$ and $v_3$ are close to each other. This is in disagreement with hydrodynamic calculations with the MC Glauber and TRENTO model initial conditions that give $v_2/v_3 \sim 1.25–1.4$ [19, 20, 23, 24]. This prediction is mainly due to the fact that for the elliptic flow the coefficient $k_2$, in the linear response relation (2), is bigger than the coefficient $k_3$ for the triangular flow (e.g. for ideal hydrodynamics the calculations performed in [19] give $k_2/k_3 = 1.35$ for 2.76 TeV Pb + Pb collisions in 0–2% centrality bin for 0.3 GeV $< p_T < 3$ GeV, and the ratio $k_2/k_3$ increases with the QGP shear viscosity [6, 19]). Simulations of the initial entropy distribution at zero impact parameter within the MC Glauber and TRENTO models give ellipticity $\epsilon_2(2)$ and triangularity $\epsilon_3(2)$ that are close to each other (here, as usual, $\epsilon_2(2) = \sqrt{\epsilon_2^2}$ is the root mean squared (RMS) eccentricity). Therefore, for $k_2/k_3 > 1$, the linear response relation (2) leads to $v_2/v_3 > 1$. The problem with reproducing the experimental ratio $v_2/v_3$ in ultra-central Pb + Pb collisions is clearly a serious challenge for the hydrodynamic paradigm of heavy ion collisions, because prediction that $k_2 > k_3$ seems to be quite reliable.

In recent years, there have been several attempts to explain why in ultra-central Pb + Pb collisions $v_2/v_3 > 1$. In [19] the flow coefficients in ultra-central Pb + Pb collisions have been addressed using the MC Glauber and the MC-KLN [21, 22] initial conditions. It was found that both the MC Glauber and KLN models fail to reproduce the ratio $v_2/v_3$. The authors of [19] concluded that the observed ratio $v_2/v_3 \approx 1$ in ultra-central Pb + Pb collisions requires $\epsilon_3(2)/\epsilon_3(2) \sim 0.5–0.7$, which is inconsistent both with predictions of the MC Glauber and MC-KLN models. In [23] the effects of bulk viscosity on flow coefficients in ultra-central Pb + Pb collisions have been studied. It was shown that for the IP-Glasma initial conditions the inclusion of bulk viscosity can somewhat reduce $v_2/v_3$ ratio. Although the effect is not strong enough to reproduce the experimental $v_2/v_3$ well. In [24] the effect of the QCD equation of state on $v_2/v_3$ has been investigated for the TRENTO model initial conditions. The authors have found that in ultra-central Pb + Pb collisions $v_2/v_3 \approx 1.2$, and concluded that the variation of the equation of state does not allow to solve the $v_2$-to-$v_3$ puzzle.

In [19, 23, 24] it was assumed that the $^{208}$Pb nucleus is spherical. A scenario with an octupole (pear shape) deformation of the $^{208}$Pb nucleus has been addressed in [20] for the TRENTO initial conditions. This scenario seems to be appealing in the context of the $v_2$-to-$v_3$ puzzle, since for a given ratio $k_2/k_3$, $v_2/v_3 \propto \epsilon_3(2)/\epsilon_2(2)$. And one can expect that the pear deformation of the $^{208}$Pb nucleus should increase somewhat $\epsilon_3(2)$ (without a significant modification of $\epsilon_3(2)$), and consequently should reduce $v_2/v_3$. A pear shape of the $^{208}$Pb nucleus was supported by the results of [25] where, within the generator-coordinate extension of the Hartree-Fock–Bogoliubov method, the authors obtained the octupole deformation parameter $\beta_3 \sim 0.0375$ for the ground state. However, there the value $\beta_3 = 0$ has been found within the ordinary Hartree-Fock–Bogoliubov method. More recently, in [26] the value $\beta_3 = 0$ for the $^{208}$Pb nucleus ground state has also been obtained within the covariant density functional theory. The results of [20] show that, for reasonable values of $\beta_3$, the scenario with the octupole deformation of the $^{208}$Pb does not lead to a significant improvement in description of the ratio $v_2/v_3$ in ultra-central Pb + Pb collisions.

In the studies [19, 20, 23, 24], the initial conditions have been generated using the MC sampling of the nucleon positions with the Woods-Saxon (WS) nuclear distribution. In fact, presently this method is the standard approach for MC calculations of the initial conditions in heavy ion collisions. One of the apparent shortcomings of the MC WS sampling of the nucleon positions is that this approach completely ignores the collective dynamical effects for the long range fluctuations of the nucleon positions (which are especially important for calculations of the azimuthal anisotropy coefficients $\epsilon_n$). Indeed, it is well known that the long range nuclear density fluctuations have a
collective nature [27, 28]. The collective effects manifest themselves in the presence of giant resonances/vibrations, which correspond to coherent oscillations of the nucleons [27, 28] (for more recent reviews see [29, 30]). Since the long range collective effects are ignored in the MC WS sampling of the nucleon positions, there is no guarantee that this approach can mimic the true long range fluctuations of the nuclear density. One can expect that, in the context of the anisotropy coefficients $\varepsilon_2$ and $\varepsilon_3$ for ultracentral Pb + Pb collisions, the most crucial giant vibration modes are the quadrupole and octupole ones. In [31], we have investigated the possible effect of the isosinglet quadrupole giant vibration mode on the $\varepsilon_2(2)/\varepsilon_3(2)$ ratio. The analysis [31] was motivated by the fact, established in [32], that the MC WS sampling of the nucleon positions leads to a significant overprediction of the mean squared quadrupole moment of the 208Pb nucleus as compared to that obtained via the experimental parameters of the isosinglet giant quadrupole resonance (ISGQR). The quantum calculation with the help of the energy weighted sum rule (EWSR) for the quadrupole strength function (for a review, see [33]) gives the mean squared quadrupole moment that is smaller than the one calculated with the WS nuclear density by a factor of ~2.2 [31] (after correcting an error made in [32]). This means that prolate and oblate elliptic fluctuations of the 208Pb nucleus are significantly weaker than predicted by the MC WS sampling of the nucleon positions. For this reason, one can expect that the true many-body nuclear density should give a smaller ellipticity $\varepsilon_2$ than the MC simulation with the standard WS nuclear density. To study quantitatively this effect, in [31] we have performed the MC Glauber model calculations of the anisotropy coefficients $\varepsilon_{2,3}$ for central Pb + Pb collisions using a modified method of the MC sampling of the nucleon positions that guarantees that the averaged over all collisions squared quadrupole moments of the colliding nuclei coincide with the mean squared quadrupole moment of the 208Pb nucleus obtained using the EWSR. The results of [31] show that the modified MC sampling with filtering the nucleon positions by the value of the quadrupole moment leads to a noticeable reduction of the ellipticity $\varepsilon_2$. It was found that the quadrupole moment filtering practically does not change the prediction for the triangularity $\varepsilon_3$. We have obtained that the MC Glauber model with the quadrupole moment filtering of the nucleon positions gives $\varepsilon_3(2)/\varepsilon_3(2) \approx 0.8$ for 2.76 and 5.02 TeV central Pb + Pb collisions. Then, adopting the hydrodynamic linear response coefficients $k_{2,3}$ from [19, 20, 34, 35]), we obtained $\nu_3(2)/\nu_3(2) \approx 0.96$–1.12 which agrees reasonably with the data from ALICE [18].

One weakness of the analysis [31] is that the central Pb + Pb collisions are treated as collisions at zero impact parameter, i.e., calculations of [31] correspond to $b$-centrality, in the terminology of [36], defined in terms of the impact parameter $b$ ($c = \pi b^2/\sigma_{in}$ [37]). However, experimentally, the centrality of a collision is usually estimated through charged particle multiplicity $N_{ch}$ in a certain kinematic region. This $n$-centrality is defined as [36, 37]

$$c(N_{ch}) = \sum_{N=N_{ch}}^{\infty} P(N),$$

where $P(N)$ is the probability for observing the multiplicity $N$. Due to multiplicity fluctuations (at a given impact parameter), there is some mismatch between the $b$- and $c$-centrals [36, 37]. For this reason, one can reasonably worry about the effect of this mismatch on the results of [31] where the effect of $n$-centrality smearing at a given $b$-centrality has been ignored. Therefore, it is highly desirable to extend the calculations of [31] to the case of the $n$-centrality. This is our main purpose in the present paper. Also, we extend the analysis of [31] to the case of the octupole fluctuations. Investigation of the role of the filtering the nucleon positions by the octupole moment in the MC simulations of Pb + Pb collisions is interesting because the collective pear shape fluctuations may potentially affect the triangularity $\varepsilon_3$ of the fireball. From the available experimental data one can conclude that for the octupole shape fluctuations the mean squared octupole moment of the 208Pb nucleus may be somewhat larger than the one obtained from the MC WS calculations (see Appendix). The latter possibility seems to be very interesting in the context of the $\nu_2$-to-$\nu_3$ puzzle, because it should lead to an increase of $\varepsilon_3$ (similarly to the case with the pear shape deformation of the ground state [20]) and, consequently, to a smaller value of the ratio $\varepsilon_3(2)/\varepsilon_3(2)$. Note that contrary to the analysis of [31], in the present work we perform calculations for the whole range of centrality. As in [31], we use the MC Glauber model developed in [38, 39], which allows to account for the presence of the meson-baryon component in the nucleon light-cone wave function.

The plan of the paper is as follows. In Section 2 we discuss the theoretical framework. In Section 3 we present our numerical results. We give conclusions in Section 4. In Appendix we discuss calculations of the mean squared quadrupole and octupole moments of the 208Pb nucleus using the EWSR.

2. THEORETICAL FRAMEWORK

In the present analysis, to generate the initial entropy density we use the MC Glauber approach developed in [38, 39]. This MC Glauber model allows to perform calculations of the entropy production in the standard way, when each nucleon is treated as a one-body state, and accounting for the presence of the
meson cloud in the nucleon, when the physical nucleon light-cone function includes the bare nucleon and meson-baryon Fock states. The results of our previous analyses [39, 40] show that, for both the versions, the predictions of this model for centrality dependence of the midrapidity charged multiplicity density are in very good agreement with experimental data for 0.2 TeV Au+Au collisions at RHIC, 2.76 and 5.02 TeV Pb + Pb, and 5.44 TeV Xe+Xe collisions at the LHC.

2.1. Outline of the MC Glauber Scheme

In this subsection we briefly outline the algorithm used in our MC Glauber model for the version without the meson-baryon component (in this case, our scheme is similar to the MC Glauber generator GLISSANDO [11]). The entropy generation occurs through the wounded nucleons (WNs) and through the hard binary collisions (BCs). We assume that for each pair of colliding nucleons the cross section of the hard binary collision is suppressed by a factor $\alpha$ [41]. The total entropy density in the transverse plane is written as (we consider the central rapidity region)

$$\rho_s(\rho) = \frac{N_{wn}}{2} s_{wn}(\rho - \rho_s) + \frac{N_{bc}}{2} s_{bc}(\rho - \rho_s),$$  \hspace{1cm} (5)

where the $s_{wn}$ terms corresponds to the WN sources and $s_{bc}$ terms to the BC sources. $N_{wn}$ and $N_{bc}$ are the numbers of the WNs and BCs, respectively. We write $s_{wn}$ and $s_{bc}$ as

$$s_{wn}(\rho) = \frac{(1 - \alpha)}{2} s(\rho), \quad s_{bc}(\rho) = s(\rho),$$ \hspace{1cm} (6)

where $s(\rho)$ is the source entropy distribution. We use for $s(\rho)$ a Gaussian form

$$s(\rho) = s_0 \exp(-\rho^2 / \sigma^2) / \pi \sigma^2$$ \hspace{1cm} (7)

with $s_0$ the total entropy of the source, and $\sigma$ width of the source. We assume that the center of each WN entropy source coincides with the WN position, and for each BC the center of the entropy source is located in the middle between the colliding nucleons.

For each entropy source, we treat $s_0$ as a random variable. We assume that the QGP expansion is isentropic. In this approximation we can treat each entropy source as a source of the charged multiplicity $n = a s_0$ in the unit pseudorapidity interval $|\eta| < 0.5$ with $a \approx 7.67$ [42]. We describe the fluctuations of $n$ by the Gamma distribution

$$\Gamma(n, \langle n \rangle) = \left( \frac{n}{\langle n \rangle} \right)^{\kappa-1} e^{-n \kappa / \langle n \rangle} \langle n \rangle^{\kappa} \Gamma(\kappa)$$ \hspace{1cm} (8)

with the parameters $\langle n \rangle$ and $\kappa$ adjusted to fit the experimental mean charged multiplicity and its variance in the unit pseudorapidity window $|\eta| < 0.5$ for $pp$ collisions.

As in the analyses [39, 40], in the version with the meson-baryon component of the nucleon, for the total weight of the $MB$ states in the physical nucleon we take 40% that allows one to describe the DIS data on the violation of the Gottfried sum rule [43]. In the sense of the entropy sources, calculation of the initial entropy density in this version is similar to that for the version without $MB$ component. However, in this case, the entropy sources can be produced in $BB$, $MB$, and $MM$ collisions. The results of [39, 40] show that both the versions give similar predictions for the midrapidity charged multiplicity density $dn_{ch}/d\eta$. However, the version with the $MB$ component requires somewhat smaller value of the parameter $\alpha$ to fit the measured midrapidity $dn_{ch}/d\eta$. In the present analysis we use the values $\alpha = 0.14 (0.09)$ for the versions without (with) the meson-baryon component of the nucleon. These values allow to reproduce very well the data on the centrality dependence of $dn_{ch}/d\eta$ at $\eta = 0$ for 2.76 and 5.02 TeV Pb + Pb collisions. For more details on our MC Glauber scheme we refer the reader to [39, 40].

2.2. Sampling of Nucleon Positions

The MC Glauber model gives the algorithm for calculation of the entropy distribution in each $AA$ collision for given nucleon positions in the colliding nuclei. It should be supplemented by a prescription for the MC sampling of the nucleon positions. Usually, in event-by-event simulations of heavy ion collisions, the nucleon positions are generated using the uncorrelated WS distribution (or the WS distribution with a restriction on the minimum distance between two nucleons [11, 44] to model the NN hard core). However, this procedure completely ignores the collective nature of the long range fluctuations of the nuclear density, and can lead to an incorrect description of the 3D fluctuations of the many-body density of the colliding nuclei. This can translate to incorrect predictions for fluctuations of the initial entropy distribution in $AA$ collisions. As already mentioned in the introduction, from the point of view of heavy ion collisions, the most important collective fluctuations are related to the quadrupole and octupole vibration modes. Their magnitude can be characterized by the squared $L$-multipole moment (we denote it as $Q^2_L$) for $L = 2$ and 3 defined via the spherical harmonics (see Appendix). In [31] we have suggested a simple systematic method for calculations of the mean squared multipole moments, $\langle Q^2_L \rangle$, for arbitrary $L$ from experimental strength functions using the EWSR (for completeness, in Appendix we outline it). For the $L = 2$ mode this method gives the mean squared quadrupole moment of the $^{208}$Pb nucleus that is smaller than predicted by the MC simulation with the WS nuclear density by the factor $r_2 \approx 2.25$ (see Appendix). One can expect that the overprediction of the $L = 2$ fluctuations of the
\( ^{208}\text{Pb} \) nuclear density may translate to an overprediction of the ellipticity \( \epsilon_2 \) in the MC simulation of ultra-central \( \text{Pb} + \text{Pb} \) collisions. In [31] we have proposed a simple method for curing this problem by performing the MC sampling of the nucleon positions with a suitable \( Q_2 \)-filter, which should guarantee the true value of \( \langle Q_2 \rangle \) for the final sample of the nucleon positions. In [31] we have performed calculations using two different \( Q_2 \)-filters with smooth and sharp filtering. In the smooth version we used a \( Q_2 \)-filter that generated a set of the nucleon positions with distribution in \( Q_2^2 \) which was equal to the rescaled by the factor \( r_2 Q_2^2 \) distribution for the WS nuclear distribution. In the second method we merely selected only the nucleon configurations with \( Q_2^2 < Q_{2\text{max}}^2 \) with \( Q_{2\text{max}}^2 \) adjusted to provide in the MC sampling the correct EWSR \( \langle Q_2 \rangle \). It was found that these two very different filters give practically identical results for \( \epsilon_{2,3} \).

As in [31], in the present analysis we perform calculations using smooth and sharp \( Q_2 \)-filterings of the nucleon positions. In the first case, we use in the MC sampling of the nucleon positions a smooth \( Q_2 \)-filter which generates the nucleon positions with the \( Q_2^2 \) distribution given by

\[
P(Q_2^2) = C \exp(-Q_2^2/a_2^3) P_{WS}(Q_2^2),
\]

where \( P_{WS} \) is the \( Q_2^2 \) distribution for the ordinary unfiltered MC WS sampling of the nucleon positions, \( C \) is the normalization constant, and \( a_2 \) is the parameter adjusted to have \( \langle Q_2^2 \rangle = \langle Q_{2,WS}^2 \rangle / r_2 \). From the point of view of numerical computations, the ansatz (9) with the Gaussian suppression factor \( \exp(-Q_2^2/a_2^3) \) is simpler than the method of [31], with rescaling the original WS distribution \( P_{WS}(Q_2^2) \). In the second method, as in [31], we use a sharp filter with a cutoff \( Q_2^2 < Q_{2\text{max}}^2 \) with \( Q_{2\text{max}}^2 \) adjusted to have \( \langle Q_2^2 \rangle = \langle Q_{2,WS}^2 \rangle / r_3 \). As in [31], we have found that predictions for \( \epsilon_{2,3} \) obtained for the smooth and sharp \( Q_2 \)-filters are practically indistinguishable.

In the present analysis, in addition to the effect of the quadrupole vibrations addressed in [31], we also study the influence on the anisotropy coefficients \( \epsilon_{2,3} \) of the octupole \((L = 3)\) vibrations of the \( ^{208}\text{Pb} \) nucleus. Similarly to the case of the quadrupole fluctuations of the 3D nuclear density, an inappropriate description of the octupole 3D nuclear density fluctuations in the MC WS sampling of the nucleon positions can lead to incorrect predictions for the 2D initial entropy fluctuations in \( \text{Pb} + \text{Pb} \) collisions. It is reasonable to expect that for ultra-central \( \text{Pb} + \text{Pb} \) collisions the changes in the octupole 3D fluctuations of the nuclear density will mostly affect the triangularity \( \epsilon_3 \).

Unfortunately, there are rather large uncertainties in the experimental data on the octupole strength function of the \( ^{208}\text{Pb} \) nucleus (see appendix), which translate to considerable uncertainties in the value of the mean squared octupole moment obtained using the EWSR. Calculations using the EWSR and the available data on the octupole strength function of the \( ^{208}\text{Pb} \) nucleus, show that the ratio of the mean squared octupole moment predicted by the WS \( ^{208}\text{Pb} \) nuclear density to the true one should most likely lie within the range \( 0.7 < r_3 < 0.84 \) (see Appendix). Thus, contrary to the situation with the quadrupole mode, it is possible that the WS nuclear density somewhat underpredicts the 3D octupole fluctuations of the \( ^{208}\text{Pb} \) nucleus. To model the effect of possible enhancement of the octupole fluctuations for the \( ^{208}\text{Pb} \) nucleus on the initial entropy distribution, we use, similarly to the case of the quadrupole mode, two types of filters in the sampling of the nucleon positions. In this first method, we use a smooth \( Q_2 \)-filter, which generates the nucleon positions with the distribution in \( Q_2^2 \) given by

\[
P(Q_2^2) = C \{1 - \exp(-(Q_2^2/a_3^2))\} P_{WS}(Q_2^2).
\]

In the second method, we use a sharp filter that selects only the configurations with \( Q_2^2 > Q_{3\text{min}}^2 \). The values of \( a_3 \) and \( Q_{3\text{max}}^2 \) are adjusted to have \( \langle Q_2^2 \rangle = \langle Q_{2,WS}^2 \rangle / r_3 \). Both these prescriptions push the \( \langle Q_2^2 \rangle \) to higher values. As for the \( L = 2 \) mode, we have found that predictions for \( \epsilon_{2,3} \) obtained for the smooth and sharp \( Q_2 \)-filters are practically identical. It is worth noting that although our \( Q_2 \)-filters give significant changes in the distributions in \( Q_2^2 \) for the generated set of the nucleon positions, they give almost zero effect on the one-nucleon density distribution (i.e., after the \( Q_2 \)-filtering we have the same WS density distribution).

In Fig. 1a we plot the distributions in the squared \( L = 2 \) multiple moment obtained for the MC sampling of the nucleon positions for uncorrelated WS density of the \( ^{208}\text{Pb} \) nucleus without and with \( Q_2 \)-filtering (for the smooth \( Q_2 \)-filter that corresponds to \( r_2 = 2.25 \)). In Fig. 1b we show similar results for the \( L = 3 \) mode. For this mode we show the results for two filtered distributions for \( r_3 = 0.84 \) and 0.7. We use in Fig. 1 the dimensionless variables \( q_L = Q_2^2/AR_L^{2L} \), where \( R_L \) is the nucleus radius in the WS parametrization of the \( ^{208}\text{Pb} \) nuclear density \((A.1)\).
It is worth noting that our numerical calculations show that the \(Q_2^2(Q_1^1)\)-filtering practically does not affect the \(Q_2^2(Q_1^1)\) distribution. This occurs because to very good accuracy the original two dimensional distribution in \(Q_{2,3}\), for the MC WS sampling of the nucleon positions, can be written in a factorized form

\[
P_{\text{WS}}(Q_2^2, Q_3^3) = P_{\text{WS}}(Q_2^2) P_{\text{WS}}(Q_3^3). \tag{11}
\]

Note that, similarly to the cases when the \(Q_2^2\)- and \(Q_3^3\)-filters are applied separately, our numerical calculations show that for simultaneous use of the \(Q_2^2\)- and \(Q_3^3\)-filters predictions for \(\epsilon_{2,3}\) turn out to be practically identical for the smooth and sharp filters.

We have also investigated the effect of the modification of the distribution in the isovector dipole moment. For the isovector dipole fluctuations, the MC sampling of the nuclear configurations with the WS nuclear density leads to the mean squared dipole moment that is larger by a factor of \(\sim 5-6\) than that obtained from the parameters of the isovector dipole resonance [32, 45]. The isovector giant dipole resonance corresponds to the collective oscillations of the protons and neutrons in opposite directions [27, 28]. This mode can lead to a prolate form of the nucleon distribution (i.e. it generates some quadrupole moment), and in principle inadequate description of this mode could affect the geometry of the entropy distribution in Pb + Pb collisions. However, we have found the effect of the modification of the MC sampling of the nucleon positions for the isovector dipole mode (in the same way as we do it for the isosinglet quadrupole mode) on the results for \(\epsilon_{2,3}\) turns out to be practically negligible. Physically, this is due to a very small statistical weight (among the quadrupole fluctuations) of the fluctuations with the collective displacement of all the protons and all the neutrons in opposite directions. Therefore, a modification of the distribution in the isovector dipole moment in the MC sampling of the nucleon positions gives almost zero effect on \(\epsilon_{2,3}\).

It is worth noting that the sampling of the nucleon positions for the WS nuclear density leads to some overprediction of the pure radial fluctuations, corresponding to the monopole \((L = 0)\) vibration mode, as compared to prediction from the EWSR for the experimental monopole strength function (see Appendix). However, intuitively one could expect that the effect of the radial fluctuations should be immaterial for the eccentricities \(\epsilon_{2,3}\) (especially at small centralities), and the disagreement between the \(L = 0\) moments for the WS sampling of the nucleon positions and that obtained from the EWSR should not be important. Our calculations confirm this, we have found that adding the filtering for the \(L = 0\) mode practically does not affect the azimuthal coefficients \(\epsilon_{2,3}\), hence we have not used a filter for the \(L = 0\) mode.

Finally, we would like to emphasize that the fact that all our predictions for \(\epsilon_{2,3}[2]\) for the smooth and
sharp filters are practically identical is very encouraging from the point of view of the validity of the strategy to mimic collective effects by simple $Q^2_{2,3}$-filterings the nucleon positions. Indeed, our smooth and sharp filters lead to radically different distributions in $Q^2$ and $Q^3$. It is clear that the many-body densities for these filters are also radically different. Nevertheless, we obtain practically identical $\varepsilon_{2,3}^2$, if both the versions lead to the same values of $\langle Q^2_{2,3} \rangle$ and $\langle Q^3 \rangle$, and the difference in their other characteristics (say, the difference in the values of $\langle Q^2_{2,3} \rangle^2$) has a negligible effect on $\varepsilon_{2,3}(2)$. This feature of the Glauber model predictions for $\varepsilon_{2,3}^2$ allows us to expect that our results for $\varepsilon_{2,3}^2$ should be close to those for the true many-body density, provided that we use the $Q^2_{2,3}$-filters guaranteeing the correct values of $\langle Q^2 \rangle$ and $\langle Q^3 \rangle$.

3. NUMERICAL RESULTS FOR $\varepsilon_{2}^2$ AND $\varepsilon_{3}^2$

In this section we present our numerical results for the rms ellipticity $\varepsilon_2^2$ and triaxiality $\varepsilon_3^2$ for 5.02 TeV Pb + Pb collisions. The results for 2.76 TeV Pb + Pb collisions are very close to those for 5.02 TeV, and hence we do not show them. For the versions with the $Q^2_{2,3}$-filtering, we present the results obtained with the smooth filters (as we already said, the results for the versions with the smooth and sharp $Q^2_{2,3}$-filters are practically indistinguishable). The results have been obtained by generating $\sim 6 \times 10^6$ Pb + Pb collisions, i.e. we have about 6 $\times 10^4$ events in the region $c \leq 1\%$, which is most interesting in the context of the $v_2$-to-$v_3$ puzzle. We have performed calculations for the Glauber schemes with and without the meson-baryon component of the nucleon. We present the results obtained for the entropy sources with the Gaussian width parameter [3] $0.4$ fm. In the small centrality region ($\leq 5-10\%$), that is interesting in the context of the $v_2$-to-$v_3$ puzzle, the predictions for $\varepsilon_{2,3}^2$ have very low sensitivity to the value of $\sigma$. We have checked this by performing calculations for the value $\sigma = 0.7$ fm. In this case, $\varepsilon_{2,3}^2$ become somewhat smaller at large centralities (by $\sim 5-7\%$ at $c \sim 50\%$), but at centrality $\leq 5-10\%$ the results are very close to those for $\sigma = 0.4$ fm.

In Figs. 2a and 2b we show the results for centrality dependence of $\varepsilon_{2,3}^2$ for the ordinary MC WS sampling of the nucleon positions (i.e., without applying any $Q^2_{2,3}$-filters). From Figs. 2a and 2b one can see that the results for $\varepsilon_{2,3}^2$ in the versions without and with the meson-baryon component are very similar. For the curves shown in Fig. 2, we have on average $\varepsilon_{2}^2/\varepsilon_{3}^2 = 0.94-0.95$ at $c \leq 1\%$. In Figs. 3a and 3b we show $\varepsilon_{2,3}^2$ obtained with the MC sampling of the nucleon positions with applying the smooth $Q^2$-filter, which gives for the colliding nuclei $\langle Q^2 \rangle = \langle Q^2 \rangle_{WS}/r_2$ with $r_2 = 2.25$, i.e., the mean squared quadrupole moment consistent with the EWSR prediction. From comparison of the results shown in Figs. 2 and 3, one case see that the presence of the $Q^2$-filter noticeably reduces $\varepsilon_2^2$, but almost does not affect $\varepsilon_3^2$. For Fig. 3, at $c \leq 1\%$ we have on average $\varepsilon_2^2/\varepsilon_3^2 = 0.82-0.84$. Note that the value of the ratio $\varepsilon_2^2/\varepsilon_3^2$ at $c \leq 0.1\%$ for the curves shown in Fig. 3, is just by $\sim 2\%$ larger than that obtained in [31] in similar calculations for zero impact parameter.

In Figs. 4a and 4b we plot $\varepsilon_{2,3}^2$ obtained with the MC sampling of the nucleon positions with applying simultaneously the smooth $Q^2$- and $Q^3$-filters, that give for the colliding nuclei $\langle Q^2_{2,3} \rangle = \langle Q^2_{2,3} \rangle_{WS}/r_{2,3}$ with $r_2 = 2.25$ and $r_3 = 0.84$. The addition of the $Q^3$-filtering for $r_3 = 0.84$ increases $\varepsilon_3^2$ by $\sim 2\%$ at $c \leq 1\%$, and in this centrality region we have now on average $\varepsilon_3^2/\varepsilon_3^2 = 0.8-0.82$. In Fig. 5 we show the results similar to that plotted in Fig. 4, but for $r_3 = 0.7$. In this version, at $c \leq 1\%$ we have on average $\varepsilon_2^2/\varepsilon_3^2 = 0.78-0.81$. From comparison of the results shown in Fig. 3 with that shown in Figs. 4 and 5, one can see that the $Q^3$-filtering increases slightly $\varepsilon_2^2$, without a noticeable effect on the value of $\varepsilon_3^2$. The results shown in Figs. 3–5 demonstrate that the effect of the $Q^3_{2,3}$-filters becomes seen only at $c \leq 10\%$. From Figs. 3–5, one can see that, in the most interesting (in the context of the $v_2$-to-$v_3$ puzzle) region of small centralities $c \leq 1\%$, modification of the MC sampling of the nucleon positions with the $Q^2_{2,3}$- and $Q^3$-filters increases the difference $\varepsilon_2^2 - \varepsilon_3^2$ by a factor of $\sim 3$. Note that our values for the ratio $\varepsilon_2^2/\varepsilon_3^2$ at $c \leq 1\%$, for the versions with the $Q^3$-filtering, are smaller than those obtained within the MC-KLN model in [19] by $15–20\%$, and by $\sim 10–15\%$ than obtained within the TRENTO scheme in [20] (for the octupole deformation parameter $\varepsilon_3 = 0–0.0375$). As compared

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1 The reason for this property of $\varepsilon_{2,3}^2$ is unclear. It may be connected with the fact that in the Glauber bombed nucleon model the variance of $\varepsilon_n$ (as $\langle Q^2_n \rangle^2$) depends only on the two-nucleon correlators for the colliding nuclei. While $\langle Q^2_{2,3} \rangle^2$ depend on the four-nucleon correlators as well, which are not important for the variance of $\varepsilon_n$ at all.

2 Note that our calculations show that the $Q^2$- and $Q^3$-filterings give almost zero effect on the higher harmonics $\varepsilon_4$ and $\varepsilon_5$, and hence we do not show them.
to calculations of [46] within the MAGMA model our values of the ratio $\epsilon_2(2)/\epsilon_3(2)$ are smaller by a factor of ~1.65.

From the point of view the $v_2$-to-$v_3$ puzzle, it is interesting to know the ratio $k_2/\epsilon_2(2)/k_3/\epsilon_3(2)$. Hydrodynamic simulations of Pb + Pb collisions at the LHC energies give $k_2/\epsilon_2(2)/k_3/\epsilon_3(2) \approx 1.2-1.4$ [19, 20, 34, 35] for small centralities ($c \leq 2\%$). Our results shown in Figs. 4 and 5 with the MC sampling of the nucleon positions with the simultaneous $Q_2^2$-filtering (with $r_2 = 2.25$) and $Q_3^2$-filtering for centrality $\sim 0.1-0.2\%$ give $\epsilon_2(2)/\epsilon_3(2) \approx 0.8(0.78)$ at $r_3 = 0.84(0.7)$. These values of $\epsilon_2(2)/\epsilon_3(2)$ lead to $0.96(0.94) < k_2/\epsilon_2(2)/k_3/\epsilon_3(2) < 1.12(1.1)$ for $r_3 = 0.84(0.7)$ and $1.2 < k_2/\epsilon_2(2)/k_3/\epsilon_3(2) < 1.4$. This agrees reasonably with the ALICE measurements [18] for 2.76 and 5.02 TeV Pb + Pb collisions that give at $c \rightarrow 0 \epsilon_2(2)/\epsilon_3(2) \approx 1.08 \pm 0.05$.

The above results have been obtained for the uncorrelated WS nuclear density. We also performed calculations replacing it by the WS nuclear density with the hard $NN$ repulsion for the expulsion radius $r_c = 0.9$ [44] and 0.6 [47] fm. We have found that the $NN$ hard core changes slightly the values of $\epsilon_2, \epsilon_3$ for MC simulations without the $Q_{1,3}^2$-filtering. However, for the version with the simultaneous $Q_{1,3}^2$-filtering predictions for $\epsilon_{2,3}(2)$ are very close to those for the
uncorrelated WS nuclear density. This fact shows that predictions for $\langle Q^2 \rangle$ depend mostly on the large-scale ($L \sim R_a$) properties of the many-body nuclear distribution, and its properties on the small-scale distances ($L \sim r_c \ll R_a$) are of minor importance. This may be viewed as another argument in favor of our basic idea to model the collective effects in the $^{208}$Pb nucleus by applying suitable $Q^2$-filterings of the nucleon positions in the MC simulations, which guarantee that the selected set of the nucleon positions reproduces the EWSR predictions for $\langle Q^2 \rangle$.

In connection with modeling the effect of the $NN$ hard core in MC simulations of $AA$ collisions, it is worth noting that it is not evident that models with the excluded volume are physically better justified than simulations with the uncorrelated WS nuclear density. The point is that it is possible that in reality the “excluded volume” is not empty. Indeed, the short range $NN$ interaction can be successfully described in the dibaryon picture (for reviews, see [48, 49]), in which the expulsion region is not empty, but occupied by a 6$q$-cluster. In this case, similarly to $hD$-scattering [50], the 6$q$-clusters should participate in the $t$-channel gluon exchanges between the colliding nuclei and contribute to the entropy production in $AA$ collisions. It is clear that in this scenario the use of the uncorrelated WS nuclear density is more adequate for simulation of the initial conditions in heavy ion collisions.

**Fig. 4.** The rms azimuthal coefficients $\epsilon_n^2$ (solid) and $\epsilon_n^3$ (dashed) vs. centrality for 5.02 TeV Pb + Pb collisions obtained within the MC Glauber model without (a) and with (b) the meson-baryon component of the nucleon using the MC WS sampling of the nucleon positions with the smooth $Q^2$- and $Q^3$-filters that give $\langle Q^2 \rangle = \langle Q^2 \rangle_{WS}/r_2$ with $r_2 = 2.25$ and $\langle Q^3 \rangle = \langle Q^3 \rangle_{WS}/r_3$ with $r_3 = 0.84$ (see text for explanations).

**Fig. 5.** Same as in Fig. 4 for $r_3 = 0.7$. 
4. CONCLUSIONS

The present study is an extension of our previous analysis [31] of the influence of the collective quantum effects in the nuclear many-body distribution on the anisotropy coefficients $\varepsilon_{2,3}$ in Pb + Pb collisions at the LHC energies, motivated by the $v_1$-to-$v_3$ puzzle in ultra-central Pb + Pb collisions. Contrary to our previous calculations [31], where only collisions at a zero impact parameter have been studied, we perform calculations for the $n$-centrality and in the whole centrality range. We model the collective effects in the colliding Pb nuclei by modifying the MC sampling of the nucleon positions by using suitable filters that guarantee that the mean squared quadrupole and octupole moments coincide with the ones obtained using the EWSR from the data on the quadrupole and octupole strength functions of the $^{208}$Pb nucleus. We have found that the EWSR and experimental data on the ISGQR of the $^{208}$Pb nucleus lead to the mean squared quadrupole moment that is smaller than the one for the uncorrelated WS nuclear density by the factor $r_2 \approx 2.25$. For the octupole mode, the available experimental data on the octupole strength function support that the ratio between the mean squared octupole moment for the uncorrelated WS nuclear density and the one obtained with the EWSR should be $0.7$–$0.84$.

We have performed the MC Glauber model calculations with applying the smooth and sharp $Q_{2,3}^2$-filters to generate the sample of the nucleon positions. We find that the results for $\varepsilon_{2,3}(2)$ obtained with the smooth and sharp $Q_{2,3}^2$-filterings are practically identical. Our numerical results show that the effect of the $Q_{2,3}^2$-filtering of the nucleon positions on the values of $\varepsilon_{2,3}(2)$ becomes seen at $c \leq 10\%$. At centralities $c \sim 0.1$–$1\%$ our MC Glauber model with the modified sampling of the nucleon positions gives to $\varepsilon_{2,3}(2)/\varepsilon_{1}(2) \sim 0.8$, which is by a factor of $\sim 1.2$ smaller than that for the ordinary MC sampling of the nucleon positions for the uncorrelated WS nuclear density. Such a value of the ratio $\varepsilon_{2,3}(2)/\varepsilon_{1}(2)$ allows to reach a reasonable agreement with the ratio $v_2(2)/v_1(2) \approx 1.08 \pm 0.05$ at $c \to 0$ obtained for 2.76 and 5.02 TeV Pb + Pb collisions by ALICE [18] for the ratio $k_2/k_3 \approx 1.35$, which is consistent with the window $1.2 < k_2/k_3 < 1.4$ supported by the hydrodynamic simulations of [19, 20, 34, 35].

Although, our analysis demonstrates the importance of the collective effects for the geometry of the initial QGP fireball for a spherical nucleus, one can expect that the collective effects may be important for collisions of the non-spherical nuclei as well (e.g. for $^{197}$Au + $^{197}$Au and $^{238}$U + $^{238}$U collisions). The collective effects may be important for nuclear shape investigation [51] and for interpretation of the results of the event shape engineering [52–54] in AA collisions at the RHIC and LHC energies, and at the NICA energy region, where the critical point effects may affect the medium expansion, and the account of suppression of the quadrupole fluctuations for the Au nucleus is especially important.

APPENDIX

Calculation of the Mean Squared Multipole Moments of the $^{208}$Pb Nucleus

For completeness, in this Appendix we briefly review the method of [31] for calculation of the mean squared multipole moments of the $^{208}$Pb nucleus with the help of the EWSR [27, 33], and give the ratios between the mean squared multipole moments obtained using the ordinary MC WS sampling of the nucleon positions and those calculated using the EWSR.

We assume that in the ground state the $^{208}$Pb nucleus is spherical. We write the nuclear density in the WS form

$$\rho_{n}(r) = \frac{\rho_{0}}{1 + \exp[(r + R_{s})/d]}$$  \hspace{0.5cm} (A.1)

with $R_{s} = 6.62$ fm, and $d = 0.546$ fm [55, 56]. We define the quadrupole and octupole moments in terms of the spherical harmonics, $Y_{l,m}$, with $L = 2$ and 3. The needed isosinglet $L$-multipole operator reads (see, e.g. [27, 28, 30])

$$F_{L} = \sum_{l=1}^{4} r_{l}^{L} Y_{l,m}(n_{l})$$  \hspace{0.5cm} (A.2)

with $n_{l} = r_{l}/|r_{l}|$. The mean squared $L$-multipole moment, $\langle Q_{L}^{2} \rangle$, of a nucleus in the ground state can be defined quantum mechanically as

$$\langle Q_{L}^{2} \rangle = \langle \langle F_{L}^{+} F_{L} \rangle(0), 0 \rangle.$$  \hspace{0.5cm} (A.3)

Classical calculation of $\langle Q_{L}^{2} \rangle$ for the uncorrelated WS nuclear density gives

$$\langle Q_{L}^{2} \rangle_{WS} = \langle F_{L}^{+} F_{L} \rangle_{WS} = \frac{A(2L + 1)}{4\pi} \langle r^{2L} \rangle.$$  \hspace{0.5cm} (A.4)

Of course, this formula becomes invalid if one includes the effect of the short range hard core $NN$ correlations. But their effect is not very strong (see below). To perform quantum calculation of $\langle Q_{L}^{2} \rangle$ of the $^{208}$Pb nucleus we use the EWSR (for a review, see [33]) for strength function, $S(\omega)$ of the operator $F_{L}$. It is defined as

$$S(\omega) = \sum_{n} |\langle n | F_{L} | 0 \rangle|^{2} \delta(\omega - \omega_{n}),$$  \hspace{0.5cm} (A.5)

$^{3}$We ignore in this Appendix a very small effect of the c.m. nucleon correlations. However, in our numerical simulations they have been treated properly.
where \( \omega_n = E_n - E_0 \) and \( E_n \) are the nucleus state energies. In terms of moments of the strength function, given by

\[
m_k = \int_0^\infty d\omega \omega^k S(\omega),
\]

we can write \( \langle 0| F_L^* F_L |0 \rangle = m_0 \). It is convenient to rewrite it as

\[
\langle 0| F_L^* F_L |0 \rangle = \frac{m_1}{E_c},
\]

where

\[
E_c = m_1/m_0
\]

is the so called the centroid energy \( E_c \), which can be viewed as the typical excitation energy for the operator \( F_L \) acting on the ground state. The representation (A.7) is more convenient than the one via \( m_0 \), because the experimental errors in the normalization of the strength function are not important for the ratio \( m_1/m_0 \), and the moment \( m_1 \) can be exactly calculated with the help of the EWSR, which for \( L \geq 2 \) [27, 30, 33] gives

\[
m_1 = \frac{A(L(L+1)/8\pi m_N^2)}{\langle Q_L^2 \rangle_{EWSR}},
\]

where \( m_N \) is the nucleon mass. Thus, we have

\[
\langle Q_L^2 \rangle_{EWSR} = \frac{A(L(L+1)/8\pi m_N^2)}{\langle Q_L^2 \rangle_{EWSR}}.
\]

Comparing (A.10) with (A.4), we see that the ratio between the mean squared multiple moments for the ordinary MC sampling of the nucleon positions and that for quantum calculation with the help of the EWSR reads

\[
r_L = \frac{\langle Q_L^2 \rangle_{EWSR}}{\langle Q_L^2 \rangle_{EWSR}} = \frac{2m_N E_c\langle r^{2L} \rangle}{I(L(L+1)/r^{2L-2})}.
\]

We calculate the centroid energy using the Breit-Wigner parametrization of the strength function. Since the strength function is proportional to the imaginary part of the polarisability (susceptibility) \( \alpha \) (which, as usual, should satisfy the relation \( \alpha(-\omega^*) = \alpha^*(\omega) \) [57]) for the operator \( F_L \), for each resonance a double Breit-Wigner parametrization with the poles at \( \pm \omega_{R_i} - i\Gamma_{R_i}/2 \) (with the same residues) should be used (see Eq. (20) of [32]). For \( N \) resonances this gives

\[
E_c = \left[ \sum_{i=1}^N \frac{2f_i}{\pi \omega_i} \arctan \frac{2\omega_i}{\Gamma_i} \right]^{-1}
\]

with \( f_i \) the fraction of the \( i \)th resonance contribution to the EWSR.

For the isoscalar \( F_2 \) operator, the EWSR for the \( ^{208}\text{Pb} \) nucleus is practically exhausted by the isoscalar giant quadrupole resonance with \( \omega = 10.89 \text{ MeV} \) and \( \Gamma = 3 \text{ MeV} \) [58]. Formula (A.12) with these parameters gives \( E_c \approx 11.9 \text{ MeV} \), then from (A.11) one can obtain \( r_2 \approx 2.25 \). Thus, we see that probabilistic treatment of the \( ^{208}\text{Pb} \) nucleus with the WS nuclear density overestimates the 3D quadrupole fluctuations. It is clear that this can lead to incorrect predictions for the 2D fluctuations of the initial QGP fireball in \( AA \) collisions as well. As in [31], our strategy to cure this problem is to modify the MC sampling of the nucleon positions by applying a suitable filter that generates the nuclear configurations with the mean squared quadrupole moment consistent with the EWSR.

To calculate \( r_3 \) we need the strength function for \( F_3 \). For the \( ^{208}\text{Pb} \) nucleus, the function \( S(\omega) \) for the operator \( F_3 \) is distributed in a broad range of \( \omega \). There are several very narrow peaks in the low-energy region \( \omega \approx 7 \text{ MeV} \) [59–61], in which the low lying \( 3^- \) state with \( \omega \approx 2.615 \text{ MeV} \) exhausts \( \approx 20–25\% \) of the EWSR [59–61] and several more states in the region \( 4.7 \text{ MeV} \leq \omega \approx 7 \text{ MeV} \) (so called the low-energy octupole resonance (LEOR) region) that exhaust about \( 8–13\% \) of the EWSR [59, 60]. In the high-energy region there is a broad resonance at \( \omega \approx 16–20 \text{ MeV} \) with \( \Gamma \approx 5–8 \text{ MeV} \) [58, 61–65]. The measured EWSR fraction of the high-energy octupole resonance (HEOR) varies from \( \approx 20–50\% \) [63, 64] to \( \approx 60–90\% \) [58, 61, 62, 65]. Using the data from [60], that give \( 21\% \) for the EWSR fraction of the 2.615 MeV \( 3^- \) state, and \( 8.3\% \) for the EWSR fraction of the LEOR region, together with parameters of the HEOR from [58] (\( \omega \approx 19.6 \pm 0.5 \text{ MeV} \), \( \omega \approx 7.4 \pm 0.6 \text{ MeV} \) with the EWSR fraction \( 70 \pm 14\% \)) we obtain \( r_3 \approx 0.84 \). However, if we take \( 25\% \) for the EWSR fraction of the 2.615 MeV state as obtained in [61], and the parameters of the HEOR obtained in [63] (\( \omega \approx 16 \text{ MeV} \), \( \Gamma \approx 6 \text{ MeV} \)), then we obtain \( r_3 \approx 0.7 \). Thus, we see that the experimental data on the octupole strength function of the \( ^{208}\text{Pb} \) nucleus support that \( r_3 \leq 1 \). But due to the experimental uncertainties for the octupole strength function, we have uncertainties in the value of \( r_3 \) about \( 15–20\% \). In the present analysis we perform calculations for two values \( r_3 = 0.84 \) and 0.7.

The above values of the coefficients \( r_2 \) and \( r_3 \) correspond to the MC sampling of the nucleon positions with the uncorrelated WS nuclear density. Calculations using the WS distribution with restrictions on the minimum nucleon–nucleon distances, to mimic the \( NN \) hard core, give somewhat different values of \( r_{2,3} \). However, the effect of the \( NN \) hard core on \( r_{2,3} \) is relatively small: we obtained the reduction of \( r_2 \) by the factor 0.78 (0.926), and the reduction of \( r_3 \) by the factor 0.81 (0.928) for the core radius \( r_c = 0.9 \) (0.6) fm.

In the present analysis we modify the MC sampling of the nucleon positions only with the filters for the isoscalar \( L = 2 \) and 3 moments, which correspond to the nuclear shape fluctuations. We do not use a filter.
for the $L = 0$ mode, that corresponds to the pure radial fluctuations. The radial fluctuations may be characterized by the squared moment for the monopole isoscalar operator $F_{0} = \sum_{i=1}^{A} (r_{i}^{2} - \langle r^{2} \rangle)$. The EWSR for this operator gives $m_{1} = 2\langle r^{2} \rangle/m_{c}$ [66]. Using this formula, for the uncorrelated WS nuclear density, we obtain for the analogue of (A.11) in the case of the $L = 0$ mode

$$r_{0} = \frac{m_{c}E_{c}}{2} \left[ \langle r^{2} \rangle - \langle r^{2} \rangle \right].$$

(A.13)

For the isoscalar $L = 0$ mode the EWSR is practically exhausted by the isoscalar giant monopole resonance with $\omega \approx 13.6 - 13.9$ MeV and $\Gamma \approx 3$ MeV [58, 67]. These parameters give $E_{c} \approx 15$ MeV, and calculation using (A.13) for the WS distribution (A.1) gives $r_{0} \sim 1.6$. This means that for the MC sampling of the nuclear configurations with the uncorrelated WS nuclear density the magnitude of the pure radial fluctuations is somewhat overpredicted as compared to that extracted from the experimental monopole strength function. However, we have found that adding the filtering for the $L = 0$ mode, that decreases the mean squared $L = 0$ moment to its EWSR value, practically does not affect the azimuthal coefficients $\epsilon_{23}$. Therefore we do not use filtering for the $L = 0$ fluctuations.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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