The light scalar nonets are studied using the QCD sum rules for the tetraquark operators. The operator product expansion for the correlators is calculated up to dimension 12 and this enables us to perform analyses retaining sufficient pole-dominance. To classify the light scalar nonets, we investigate the dependence on current quark mass and flavor dynamics. Especially, to examine the latter, we study separately SU(3) singlet and octet states, and show that the number of annihilation diagrams is largely responsible for their differences, which is also the case even after the inclusion of the finite quark mass. Our results support the tetraquark picture for isosinglets, while that for octets is not conclusive yet.

Keywords: Light scalar nonets; Tetraquark; Pseudo-peak artifacts.

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1. Light scalar nonets as tetraquarks

The structure of scalar mesons is a long-standing problem in hadron physics. In contrast to the other hadrons, two flavor nonets appear around 1 GeV in the scalar meson spectra. Especially, the lighter scalar nonets \( \sigma(600), \kappa(800), a_0(980), f_0(980) \) are candidates for tetraquark states since the \( q\bar{q}q\bar{q} \) assignments for these mesons naturally explain their mass ordering and decay patterns in contrast to the \( q\bar{q} \) assignments. These explanations are qualitatively favorable, but there still remain several questions to be answered by the quantitative studies: i) Can \( q\bar{q}q\bar{q} \) configurations provide light masses below 1 GeV despite of large number of quarks? If possible, which effects are responsible for the mass reduction? ii) Do all states in nonets have large tetraquark components? iii) How large are current quark mass effects on the mass splitting? We attempt to answer these questions with nonperturbative treatments of correlators for interpolating fields of light scalar nonets.

Especially, i) and ii) are deeply related to possible intermediate states depending...
on flavor structure of the nonets rather than current quark mass effects. To see this, it is convenient to consider the SU(3) singlet ($S$) and octet ($O_i$ ($i=1\sim8$)) states in the SU(3) chiral limit. Using diquark bases, $U=(\bar{d}s)$, $D=(\bar{s}u)$, $S=(\bar{u}d)$, these states are described as $S = (UU+DD+SS)/\sqrt{3}$, $O_1 = UD$, $O_2 = (UU+DD-2SS)/\sqrt{6}$, ...

The isodoublet $\kappa$ and isovector $a_0$ belong to purely the octet because of the nonzero isospin, while the isoscalar $\sigma$ and $f_0$ can be composed of the mixture of the singlet and octet states in the real world where the flavor SU(3) symmetry is broken by the quark masses. Thus the ideal mixing is expected to be realized as, $\sigma \sim \bar{S}S = \sqrt{1/3}S - \sqrt{2/3}O_2$, $f_0 \sim \sqrt{1/2}[\bar{D}D + \bar{U}U] = \sqrt{2/3}S + \sqrt{1/3}O_2$.

Here let us see the difference between singlet and octet states. The difference emerges from the number of the $q\bar{q}$ annihilation diagrams in which some quark lines disconnected between the space-time points $x$ and $0$ like Fig.1. For example, the correlator included in singlet case, $[\bar{U}U(x)\bar{U}U(0)] = [d\bar{s}\bar{s}(x)][d\bar{s}ds(0)]$, has larger number of annihilation diagrams than that in octet case, $[\bar{U}D(x)][\bar{U}D(0)] = [d\bar{s}\bar{s}(x)][d\bar{s}su(0)]$. We can verify that the ratio of number of annihilation diagrams is $O : \sigma : f_0 : S = 1 : 2 : 3 : 4$.

We note that the annihilation diagrams do not always represent the 2q mesonic propagations. As shown in Fig.1, annihilation diagrams can be interpreted as either s-channel and t-channel processes. Especially, we can interpret the latter as diquark-antidiquark correlation which was conjectured to largely reduce masses of the tetraquark states. We will see that the annihilation diagrams play key roles in the Borel analyses.

In later analyses, we will use the current $J(\theta) = \cos \theta J_P + \sin \theta J_S$ with $J_P = \epsilon_{abc}\epsilon_{def}[u_a^T C d_b][\bar{u}_d C \bar{d}_e^T]$ and $J_S = \epsilon_{abc}\epsilon_{def}[u_a^T C \gamma_5 d_b][\bar{u}_d \gamma_5 C \bar{d}_e^T]$. $\theta$ will be chosen to achieve the pole-dominance and the small threshold dependence with better degree.

2. QCD Sum Rules and Borel analyses

We analyze tetraquark correlators using the QCD sum rules (QSR) which relate the nonperturbative aspects of QCD to the hadronic properties through the dispersion relation for the correlator $\Pi$. The Borel transformed QSRs with using the quark-
hadron duality above \( s_{th} (= E_{th}^2) \) are described as follows:

\[
\int_0^{s_{th}} ds \ e^{-s/M^2} \frac{1}{\pi} \text{Im}\Pi^<(s) = L_M \Pi^{\text{ope}}(-Q^2) - \int_{s_{th}}^\infty ds \ e^{-s/M^2} \frac{1}{\pi} \text{Im}\Pi^{\text{ope}}(s),
\]

where RHS includes the correlator \( \Pi^{\text{ope}} \) calculated by operator product expansion (OPE) and the hadronic parameters in LHS are evaluated as outputs.

Using Eq. (1), the effective mass and residue are evaluated by

\[
m^2_{\text{eff}}(M^2) = \int_0^{s_{th}} ds e^{-s/M^2} \frac{2}{\pi^2} \text{Im}\Pi^<(s), \quad \lambda^2_{\text{eff}}(M^2) = e^{-s/M^2} \int_0^{s_{th}} ds e^{-s/M^2} \text{Im}\Pi^<(s),
\]

where “effective” mass (residue) means that \( m_{\text{eff}} \) is averaged mass (residue) in the energy region from 0 to \( s_{th} \) including the width and background effects. If integral is well-saturated by single peak with mass \( m \), \( m_{\text{eff}} \) approaches to \( m \) with weak dependence on the value of \( M \), while if no large and sharp peak exists, \( m_{\text{eff}} \) shows large dependence on \( M \). The same argument is valid for the residue. To estimate these quantities of low energy excitations with good accuracy and small ambiguities, we need to treat sum rules in the appropriate \( M^2 \) region where low energy contributions below \( s_{th} \) are large enough compared to the contaminations from high energy components which have no relations with properties of low-lying resonances. Especially, without the sufficient pole contribution, we are stuck with the pseudo-peak artifacts which yield artificial stability of the masses against \( M^2 \) variation, which happens often in the QSR for multiquark-hadrons.

For these reasons, we calculate the OPE up to dimension 12 to set the reasonable \( M^2 \) window where we achieve both sufficient pole-dominance (pole contribution is larger than 50% of the total) and OPE convergence (highest dimension terms are less than 10% of whole OPE). We will use QSR within this \( M^2 \) window.

First we show in Fig. 2 the case of singlet and octet states (\( \theta = 7\pi/8 \)) in the SU(3) chiral limit to see the roles of the annihilation diagrams. The effective mass for the singlet case is found 700 ~ 850 MeV in consideration of the possible width effect. For the octet case the effective mass is estimated by 600 ~ 750 MeV, although the effective mass in the octet channel depends on \( M \) fairly indicating that the signal of the octet resonance is weak and considerably affected by low energy scattering.

![Fig. 2.](http://example.com/image.png)
Fig. 3. The effective masses for the $\sigma$, $f_0$, $a_0$ states with $\theta = 7\pi/8$, $6\pi/8$, $6\pi/8$, respectively. The downward and upward arrows indicate the lower and upper bounds of the $M^2$ window, respectively.

It should be noted that the residue of the octet state is smaller than that of the singlet state by factor $\sim 3$. This means that annihilation diagrams provide more sufficient strength for the singlet case than the octet case. The small strength in the octet case would make the effective mass fairly depend on $M^2$ value.

Including the finite strange quark mass 0.12 GeV, we show in Fig.3 are the effective mass plots for $\sigma(600)$, $f_0(980)$, and $a_0(980)$ with $\theta = 7\pi/8$, $6\pi/8$, $6\pi/8$, respectively. $\sigma$ and $f_0$ include the singlet component and show the sufficient pole strength yielding the moderate stability in the effective mass plots around 600 $\sim$ 800 MeV, 750 $\sim$ 900 MeV, respectively. On the other hand, $a_0$ includes only the octet state and show the rather large $M$ dependence. The other octet state, $\kappa$, also shows the same behavior as $a_0$ reflecting its octet nature rather than the quark mass effects.

In conclusion, we perform the QSR analyses for light scalar nonets retaining the sufficient pole dominance. Our tetraquark correlator analyses show the sufficient spectral strengths in the region below 1 GeV, in sharp contrast to the results from two quark meson correlators yielding typical mass around $\sim$ 1 GeV. Therefore our results support the tetraquark picture for isosinglets, while that for octets is still not conclusive because of rather large $M$ dependence, which is probably emerged from low energy scattering states. The origin of these differences are the annihilation diagrams, and it will be important to deduce more qualitative pictures from this fact.

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