The Non-Linear Higgs Legacy of the LHC Run I

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In the recent paper on \textit{The Higgs Legacy of the LHC Run I} we interpreted the LHC Higgs results in terms of an effective Lagrangian using the SFitter framework. For the on-shell Higgs analysis of rates and kinematic distributions we relied on a linear representation based on dimension-6 operators with a simplified fermion sector. In this addendum we describe how the extension of Higgs couplings modifications in a linear dimension-6 Lagrangian can be formally understood in terms of the non-linear effective field theory. It turns out that our previous results can be translated to the non-linear framework through a simple operator rotation. \footnote{This note will be included in the arXiv version of the original paper.}
In our recent analysis of The Higgs Legacy of the LHC Run I [1] we have searched for deviations of the observed Higgs boson from the Standard Model based on two different parametrizations. First, we studied shifted SM-like Higgs couplings [2],

\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \Delta_W g_{mW} W^\mu W_\mu + \Delta_Z \frac{g}{2c_W}m_Z H Z^\mu Z_\mu - \sum_{\tau,b,t} \Delta_f \frac{m_f}{v} H (\bar{f}_R f_L + \text{h.c.}) \]

+ \Delta_g F_{G} \frac{H}{v} G_{\mu\nu} G^{\mu\nu} + \Delta_{\gamma} F_{A} \frac{H}{v} A_{\mu\nu} A^{\mu\nu} .

While this $\Delta$-framework* does not represent a renormalizable field theory outside an effective field theory framework, it can be linked to a non-linear effective field theory of the Higgs sector [4].

In the same analysis [1], we expanded this $\Delta$-framework to a gauge-invariant linear effective Lagrangian, to be able to include kinematic distributions. Our 9-dimensional operator basis with the corresponding Wilson coefficients $f_j$ is

\[
\begin{align*}
\mathcal{O}_{GG} &= \phi^1 \phi^- G_{\mu\nu} G^{\mu\nu} & \mathcal{O}_{WW} &= \phi^1 \bar{W}_{\mu\nu} W^{\mu\nu} \phi & \mathcal{O}_{BB} &= \phi^1 \bar{B}_{\mu\nu} B^{\mu\nu} \phi \\
\mathcal{O}_W &= (D_{\mu})^\dagger \bar{W}_{\mu\nu} (D_{\nu}) \phi & \mathcal{O}_B &= (D_{\mu})^\dagger \bar{B}_{\mu\nu} (D_{\nu}) \phi & \mathcal{O}_{\phi,2} &= \frac{1}{2} \partial^\mu (\phi^i \phi) \partial_\mu (\phi^j \phi) \\
\mathcal{O}_t &= \phi^1 \phi (\mathcal{T}_3 \phi t_R) & \mathcal{O}_b &= \phi^1 \phi (\mathcal{T}_3 \phi b_R) , & \mathcal{O}_\tau &= \phi^1 \phi (\mathcal{T}_3 \phi \tau_R) .
\end{align*}
\]

To illustrate how this linear dimension-6 Lagrangian can be phenomenologically viewed as an expansion of the $\Delta$-framework we spell out the linear dimension-6 Lagrangian in terms of the physical Higgs field $H$

\[
\mathcal{L} = g_{Hgg} H G^{\mu\nu} G_{\mu\nu} + g_{H\gamma\gamma} H A_{\mu\nu} A^{\mu\nu} + \sum_{f=\tau,b,t} (g_{f_H f_L} f_R + \text{h.c.}) + g_{HZZ}^{(1)} A_{\mu\nu} Z^{\mu} Z^{\nu} H + g_{HZZ}^{(2)} A_{\mu\nu} Z^{\mu} Z^{\nu} H + g_{HZZ}^{(3)} A_{\mu\nu} Z^{\mu} Z^{\nu} H + g_{HWW}^{(1)} (W^\mu W^- H + \text{h.c.}) + g_{HWW}^{(2)} H W^\mu W^- H + g_{HWW}^{(3)} H W^\mu W^- H .
\]

Of these thirteen $g_{HXX}$ terms modified through the dimension-6 Lagrangian above, seven correspond to the $(1 + \Delta_x)$ defined in Eq.1, in the custodial limit $\Delta_W = \Delta_Z$, plus the addition of $\Delta_Z^\gamma$ if desired [1]. Four additional terms $g_{HVV'}^{(1,2)}$ add new structures to the $VVH$ couplings ($V = W, Z$), which can be tested experimentally.

The effective Lagrangian based on a non-linear realization of the electroweak symmetry breaking and including a light scalar $H$ has been studied in detail in Refs. [1-6]. In the non-linear realization the Higgs is not embedded in a doublet. We can nevertheless link the non-linear and linear operator sets in terms of canonical dimensions. This connection is usually established through the ratio of scales $(v/f)^2$, where $f$ can be related to the scale of strong dynamics. A detailed analysis of the non-linear model reveals a double ordering: first, there is the chiral expansion, and in addition there is the classification in powers of $(v/f)^2$. Following Ref. [6], a subset of the non-linear Lagrangian can be linked to the $\Delta$-framework of Eq.1, with the additional assumption $\Delta_W = \Delta_Z$ [7]. As for the linear representation, an extended set of non-linear operators provides a natural extension to include kinematic distributions at the LHC.

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* Our $\Delta$-framework [2] is essentially identical to the experimental $\kappa$ framework [3].
For our study, based on [5], we start with the reduced bosonic CP-even operator set of order \((v/f)^2\),

\[
\begin{align*}
\mathcal{P}_C &= -\frac{g^2}{4} \text{Tr}(V^a V^a) F_C(H) \\
\mathcal{P}_T &= -\frac{g^2}{4} \text{Tr}(TV^a) \text{Tr}(TV^a) F_T(H) \\
\mathcal{P}_H &= \frac{1}{2} (\partial_\mu H)(\partial^\mu H) F_H(H) \\
\mathcal{P}_W &= -\frac{g^2}{4} W^a_{\mu\nu} W^{a\mu\nu} F_W(H) \\
\mathcal{P}_G &= -\frac{g^2}{4} G^a_{\mu\nu} G^{a\mu\nu} F_G(H) \\
\mathcal{P}_B &= -\frac{g^2}{4} B_{\mu\nu} B^{\mu\nu} F_B(H) \\
\mathcal{P}_{\square H} &= \frac{1}{v^2} (\partial_\mu \partial^\mu H)^2 F_{\square H}(H) \\
\mathcal{P}_1 &= \frac{g g'}{2} B_{\mu\nu} \text{Tr}(TW^{\mu\nu}) F_1(H) \\
\mathcal{P}_2 &= ig B_{\mu\nu} \text{Tr}(T[V^\mu, V^\nu]) F_2(H)
\end{align*}
\]

where \(V^a \equiv (D_\mu U)^a \) and \(T \equiv U \sigma_3 U^\dagger \) are the vector and scalar chiral fields transforming in the adjoint of \(SU(2)_L\). The dimensionless unitary matrix \(U \) contains the Goldstone modes, \(U = e^{i(\sigma_3 \pi/v)} \), and transforms as a bi-doublet \(U \to LUR^\dagger \) under the global \(SU(2)_{L,R}\) transformations. The covariant derivatives are

\[
\begin{align*}
D_\mu U &= \partial_\mu U + \frac{i}{2} g W^a_{\mu \sigma_3} U - \frac{ig'}{2} B_{\mu} U \sigma_3 \\
D_\mu V^\nu &= \partial_\mu V^\nu + ig \left[ W^a_{\mu} \sigma_3, V^\nu \right]
\end{align*}
\]

where in the second line \(D_\mu \) is the covariant derivative for the adjoint representation of \(SU(2)_L \) [5]. The model-dependent functions \(F_i(H)\) introduce the anomalous Higgs couplings.

The number of operators given in Eq. (4) can be further reduced when considering on-shell Higgs measurements [5]. First, in analogy to our linear ansatz we neglect \(P_1\) and \(P_2\) because of their tree-level contribution to the \(S\) and \(T\) parameters respectively. Next, operators containing the combination \(D_\mu V^a\) are irrelevant for on-shell gauge bosons or when the fermion masses in the process are neglected. This removes \(P_9\) and \(P_{10}\) from our Higgs analysis. While \(P_2\) and \(P_3\), together with \(P_4\) and \(P_5\), are responsible for one of the interesting de-correlations that may allow us to distinguish linear from non-linear electroweak symmetry breaking [5], they do not affect three-point Higgs couplings. For the same reason we also omit \(P_6\) and \(P_8\). Finally, for on-shell Higgs amplitudes \(P_7\) and \(P_{\square H}\) lead to coupling shifts [8], i.e. their effect can be accounted for by a re-definition of the remaining non-linear operator coefficients.

Again in analogy to the linear ansatz of Eq. (5) we also add three Yukawa-like non-linear operators of the type

\[
\mathcal{P}_i = \frac{m_f}{\sqrt{2}} \overline{Q}_L U F_i(H) t_R + \text{h.c.}
\]

where a factor \(m_f/v\) has been introduced with respect to [5] for a better comparison with [1]. With this simplification of the fermion sector we can absorb a combination of \(P_C\) and \(P_H\) in the equations of motion and arrive at the 9-dimensional non-linear operator set

\[
\{ \mathcal{P}_G, \mathcal{P}_B, \mathcal{P}_W, \mathcal{P}_H, \mathcal{P}_4, \mathcal{P}_5, \mathcal{P}_7, \mathcal{P}_6, \mathcal{P}_i \}
\]

with the appropriate coefficients \(c_i\), where the \((v/f)^2\) factors are implicitly absorbed. Assuming a truncated polynomial for the Higgs functions of all operators, \(F_i = 1 + 2a_i H/v + \ldots\), and working in the fermion mass basis we define the non-linear extension to the SM Lagrangian in analogy to Eq. [5]. The combined coefficients we denote as
Figure 1: 68% and 95% CL error bars on the coefficients defined for the non-linear operator analysis. The underlying SFitter analysis is identical to Ref.[1].

$$a_j = c_j \tilde{a}_j$$ for the gauge operators and $$a_j = c_j (2\tilde{a}_j - 1)$$ for the fermion operators. This way we can link the non-linear Lagrangian and the linear Lagrangian relevant for our Higgs analysis,

$$\frac{v^2 f_{BB}}{2 \Lambda^2} = a_B,$$
$$\frac{v^2 f_{BW}}{8 \Lambda^2} = a_4,$$
$$\frac{v^2 f_t}{\Lambda^2} = a_t,$$
$$\frac{v^2 f_{WW}}{2 \Lambda^2} = a_W,$$
$$\frac{v^2 f_{GG}}{(4\pi)^2 \Lambda^2} = a_G,$$
$$\frac{v^2 f_{BB}}{8 \Lambda^2} = a_4,$$
$$\frac{v^2 f_{WW}}{4 \Lambda^2} = a_5,$$
$$\frac{v^2 f_t}{\Lambda^2} = a_b,$$
$$\frac{v^2 f_{GG}}{\Lambda^2} = a_H,$$
$$\frac{v^2 f_{BB}}{8 \Lambda^2} = a_4,$$
$$\frac{v^2 f_{WW}}{4 \Lambda^2} = a_5,$$
$$\frac{v^2 f_t}{\Lambda^2} = a_b,$$

We emphasize that these relations are valid only when we study the effects of the operators restricted to trilinear Higgs interactions.

Based on these relations we can express the SFitter Higgs results from Ref. [1] in terms of this non-linear subset of operators. In Figure 1 we show the 68% and 95% CL allowed regions based on all on-shell Higgs event rates and including kinematic distributions.

Summarizing, in this addendum we have presented the results of the SFitter Run I Higgs analysis [1] in terms of non-linear effective operators. While the $\Delta$-framework can be linked to a subset of operators in a non-linear Lagrangian, additional non-linear operators allows us to also describe kinematic distributions. This is the same logic as the extension of the $\Delta$-framework to a linear dimension-6 Lagrangian. If we restrict our analysis to on-shell Higgs measurements, we find a one-to-one correspondence between the linear and a non-linear operator set. The LHC results in the two approaches can be translated into each other through a simple operator rotation.

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