A possibility of CPT violation in the Standard Model

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Abstract – It is shown that there is a possibility of violation of CPT symmetry in the Standard Model which does not contradict the famous CPT theorem. To check this possibility experimentally it is necessary to increase the precision of measurements of the proton and antiproton mass difference by an order of magnitude.

Introduction. – Invariance under the combined CPT transformation is considered as one of the most fundamental symmetries of local quantum field theory. Here C is the operator of charge conjugation, P and T are space reflection and time reversal operators. The famous CPT theorem was proved in [1–6]. The content of the CPT theorem is approximately as follows: a Lagrangian of any local Lorentz-invariant quantum field theory with the usual reflection and time reversal operators. The famous CPT theorem was proved in [1–6]. The content of the CPT theorem is approximately as follows: a Lagrangian of any local Lorentz-invariant quantum field theory with the usual reflection and time reversal operators. The famous CPT theorem was proved in [1–6]. The content of the CPT theorem is approximately as follows: a Lagrangian of any local Lorentz-invariant quantum field theory with the usual reflection and time reversal operators.

CPT symmetry may explain the matter-antimatter asymmetry of the Universe [7].

In the present paper we demonstrate that there is a possibility to violate the CPT symmetry of the Lagrangian of the Standard Model which does not contradict the famous CPT theorem. To check this possibility experimentally it is necessary to increase the current precision of measurements of mass difference of a proton and an antiproton by an order of magnitude.

It should be mentioned that a discovery of violation of such a fundamental symmetry as the CPT symmetry would be by itself of essential value. Besides, violation of CPT symmetry may explain the matter-antimatter asymmetry of the Universe [7].

CPT theorem. – Let us remind the definitions of the operators of charge conjugation C, of space reflections P and time reversal T. For simplicity we will consider only fields with spins 0, 1/2 and 1. We will use (until the opposite case is underlined) the interaction representation for fields which in particular allows to define the C, P and T operators not depending on time even if the corresponding symmetries are violated.

The unitary operator of charge conjugation C acts on the scalar, vector and spinor fields in the following way.

A scalar field:

\[ C\phi(x)C^{-1} = \eta_C(\phi)\phi^+(x). \]  (1)

A vector field:

\[ CV_\mu(x)C^{-1} = \eta_C(V)V^+_{\mu}(x). \]  (2)

A spinor field:

\[ C\psi(x)C^{-1} = \eta_C(\psi)c\bar{\psi}_T(x), \]  (3)

where, as usual, \( \bar{\psi} = \psi^+\gamma_0 \) and the unitary matrix \( c \) is defined by the equations

\[ c\gamma_\mu^T c^{-1} = -\gamma_\mu, \quad c^+ = c^{-1}, \quad c^T = -c, \]  (4)

\( \gamma_\mu \) are Dirac matrices.

In eqs. (1)–(3) \( \eta_c \) are arbitrary phase factors. If one demands that the square of the operator \( C \) coincides with the identity, then in the case of Hermitian fields arbitrariness is restricted: \( \eta_c \) can have only values \( \pm 1 \).

The unitary operator of space reflection P is defined as follows.

A scalar field:

\[ P\phi(x)P^{-1} = \eta_P(\phi)\phi(x_0, -\mathbf{r}). \]  (5)

A vector field:

\[ PV_\mu(x)P^{-1} = \eta_P(V)\epsilon_\mu V^\mu(x_0, -\mathbf{r}), \]  (6)

\[ \epsilon_\mu = (1, -1, -1, -1). \]
A spinor field:

\[ P\psi(x)P^{-1} = \eta P(\psi)\gamma_0\psi(x_0, -\mathbf{x}). \]  

(7)

In eqs. (5)–(7) \( \eta P \) are again arbitrary phase factors. If one demands that the square of the operator \( P \) coincides with the identity then in the case of boson fields \( \eta P = \pm 1 \) and in the case of fermion fields the phases are restricted by the values \( \pm 1 \) or \( \pm i \).

The antiunitary operator of time reversal \( T \) is defined in the following way.

A scalar field:

\[ T\phi(x)T^{-1} = \eta T(\phi)\phi(-x_0, \mathbf{x}). \]

(8)

A vector field:

\[ TV_\mu(x)T^{-1} = \eta T(V)(-\epsilon_\mu)V_\mu(-x_0, \mathbf{x}), \]

\[ \epsilon_\mu = (1, -1, -1, -1). \]

(9)

A spinor field:

\[ T\psi(x)T^{-1} = \eta T(\psi)t\psi(-x_0, \mathbf{x}), \]

(10)

where, in eqs. (8)–(10) \( \eta_\tau \) are arbitrary phase factors and the unitary matrix \( t \) is defined by the condition

\[ t\gamma_\mu t^{-1} = \gamma_\mu^T. \]

(11)

The operator \( T \) can be defined only as an antiunitary operator. An antiunitary operator has the specific property

\[ \Theta\lambda\Theta^{-1} = \lambda^*, \]

(12)

where \( \lambda \) is an arbitrary c-number.

Now one can define the CPT operator which we denote for shortness as \( \Theta \):

\[ \Theta \equiv \text{CPT}. \]

(13)

Transformations for scalar, vector and spinor fields are

\[ \Theta\phi(x)\Theta^{-1} = \eta_\phi(\phi)^+(-x), \]

\[ \Theta V_\mu(x)\Theta^{-1} = \eta_\phi(V)\nu_\mu^+(x), \]

\[ \Theta\psi(x)\Theta^{-1} = \eta_\psi(\psi)\gamma_5\psi_\tau^T(x), \]

(14)

(15)

(16)

where \( \eta_\phi \) are some phases and \( \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 \). For definiteness one can assume, e.g., the standard representation of \( \gamma \)-matrices.

It is well known that the operator \( \Theta \) can be defined only as an antiunitary operator because the time reversal operator \( T \) is antiunitary.

The CPT theorem in the Lagrangian formalism is as follows. If quantum field theory satisfies the following six postulates:

1) field equations are local;

2) the Lagrangian is invariant with respect to the proper Lorentz group;

3) one has usual connection between spin and statistics;

4) boson fields commute with all other fields, kinematically independent fermion fields anticommute;

5) any product of field operators is symmetrized, in cases of boson fields, and antisymmetrized, in cases of fermion fields (normal ordering of operators possesses this property);

6) the Lagrangian is Hermitian;

then the Lagrangian of any such theory of interacting fields with spins 0, 1/2 and 1 is invariant with respect to the following antiunitary operator \( \Theta \) in the Hilbert space

\[ \Theta\phi(x)\Theta^{-1} = \phi^+(-x), \]

\[ \Theta V_\mu(x)\Theta^{-1} = -V_\mu^+(-x), \]

\[ \Theta\psi(x)\Theta^{-1} = -i\gamma_5\psi_\tau^T(-x). \]

(17)

One can see that this operator \( \Theta \) up to phase factors coincides with the product of the three operators \( C, P \) and \( T \) defined in eqs. (14)–(16).

Let us now consider the Jost CPT theorem [6] in terms of Wightman functions (in terms of vacuum expectations of fields) [8].

The postulates of this formalism are:

a) invariance of the theory with respect to the proper Lorentz group;

b) positivity of energy, the existence of vacuum;

c) weak causality,

\[ \langle 0|\Phi_1(x_1)\Phi_2(x_2)\ldots\Phi_n(x_n)|0\rangle = (-1)^n \langle 0|\Phi_n(x_n)\ldots\Phi_2(x_2)\Phi_1(x_1)|0\rangle, \]

(18)

for all \( (x_1, x_2, \ldots, x_n) \) for which \( \sum \lambda_i(x_i - x_{i+1}) \) is always a spacelike vector if \( \lambda_i \geq 0 \) and \( \sum_i \lambda_i = 1 \), where \( \sigma \) is the number of permutations of fermionic fields. Weak causality is valid if usual causality conditions of postulates 3) and 4) of the CPT theorem in the Lagrangian formalism are valid, but it is essentially weaker of these postulates.

The Jost theorem is: for any quantum field theory satisfying postulates a)–c), vacuum expectations are invariant with respect to the operator \( \Theta \) defined in (17), that is, for any set \( (x_1, x_2, \ldots, x_n) \) one has

\[ \langle 0|\Phi_1(x_1)\Phi_2(x_2)\ldots\Phi_n(x_n)|0\rangle = \langle 0|\Theta^{-1}\Phi_1(x_1)\Theta^{-1}\Phi_2(x_2)\Theta^{-1}\Phi_n(x_n)\Theta^{-1}|0\rangle, \]

\[ \ldots \]

\[ \langle 0|\Theta^{-1}\Phi_1(x_1)\Theta^{-1}\Phi_2(x_2)\Theta^{-1}\Theta^{-1}\Phi_n(x_n)\Theta^{-1}|0\rangle. \]

(19)

For example, for scalar fields it means

\[ \langle 0|\phi_1(x_1)\phi_2(x_2)\ldots\phi_n(x_n)|0\rangle = \langle 0|\phi_1^{+}(-x_1)\phi_2^{+}(-x_2)\ldots\phi_n^{+}(-x_n)|0\rangle. \]

(20)

Let us stress that, in the general theory of interacting fields formulated in terms of Wightman functions, one takes full field operators in the Heisenberg representation.
The check of the equality of masses of particles and antiparticles is one of fundamental tests of CPT invariance. Let us consider the case of a proton which, as a stable particle, is most appropriate for precise direct mass measurements. The results of experiments are [9]

\[ m_p = 938.272081 \pm 0.000006 \text{ MeV}, \]

\[ |m_p - m_{\overline{p}}|/m_p < 7 \times 10^{-10} \quad \text{at CL } 90\%. \]  \hspace{1cm} (21)

There are also measurements comparing the charge-to-mass ratio of the proton and of the antiproton [9]

\[ \frac{q_p}{m_p} / \frac{q_{\overline{p}}}{m_{\overline{p}}} = 1.00000000000 \pm 0.00000000007. \]  \hspace{1cm} (22)

Assuming that the charge of proton and antiproton is the same, one gets an accuracy one order of magnitude better than in eq. (21).

The most impressive test of CPT symmetry comes [9] from the limit on the mass difference between neutral kaons \( K^0 \) and \( \overline{K}^0 \):

\[ |m_{K^0} - m_{\overline{K}^0}|/m_{K^0} \leq 0.8 \times 10^{-18} \quad \text{at CL } 90\%. \]  \hspace{1cm} (23)

One should mention that this restriction is based on the quantum mechanical picture of neutral kaons as the two-level system whose evolution is taken in the Wigner-Weisskopf approximation.

But our further considerations will not concern this special case of neutral kaon system (and will not contradict this restriction).

A possibility of CPT violation. – In spite of the theoretical perfectness of the CPT theorem one can still assume that the operator \( \Theta \) defined by this theorem in eq. (17) is unphysical, i.e., it does not transform physical states into physical ones.

One can assume that the physical CPT operator \( \Theta_{\text{ph}} \) differs from the theoretical operator \( \Theta \) by another choice of the CPT phases \( \eta \) in eq. (17). And it turns out that it is possible to violate CPT invariance of the Lagrangian of the Standard Model with a non-standard choice of phases \( \eta \) for quark fields.

The Standard Model Lagrangian density is

\[ L_{\text{SM}}(x) = \frac{g}{2\sqrt{2}} W^\mu_\nu (x) \bar{\psi} \gamma^\mu (1 - \gamma^5) (d(x) \cos \theta_c + s(x) \sin \theta_c) + h.c. + \ldots, \]  \hspace{1cm} (24)

where we have written explicitly only terms of interactions of light u, d and s quarks with the \( W \) bosons interesting for us.

Here the weak coupling constant \( g \) is connected with the Fermi constant \( G_F \) in the usual way,

\[ \frac{g^2}{8\pi M_W^2} = \frac{G_F}{\sqrt{2}}, \quad G_F \approx 10^{-5}. \]  \hspace{1cm} (25)

The main point is that one can assume that, e.g., \( d \) and \( s \) quarks have opposite \( \eta \) phases with respect to the physical operator \( CPT_{\text{physical}} \equiv \Theta_{\text{ph}} \):

\[ \Theta_{\text{ph}} d(x) \Theta_{\text{ph}}^{-1} = \eta_0 (d) \gamma_5 d^+ (-x), \]
\[ \Theta_{\text{ph}} s(x) \Theta_{\text{ph}}^{-1} = \eta_0 (s) \gamma_5 s^+ (-x), \]
\[ \eta_0 (s) = -\eta_0 (d), \]  \hspace{1cm} (26)

where \( \eta_0 (d) = -i, \) as in eq. (17). It can be arranged, e.g., in models of composite quarks.

To see this let us suppose that fields corresponding to \( d \) and \( s \) quarks are composed of a constituent spinor field \( \psi \) in a model of composite quarks in the following way:

\[ d = (\overline{\psi} \gamma_0 \psi), \]
\[ s = (\overline{\psi} \gamma_5 \psi), \]  \hspace{1cm} (27)

where the normal ordering of operators is assumed. Now one can take into account that

\[ CPT \overline{\psi} O \psi (CPT)^{-1} = \eta (\psi) \overline{\psi} \gamma_0 \gamma_5 O \gamma_0 \gamma_5 \psi, \]  \hspace{1cm} (28)

where \( O \) is some combination of \( \gamma \)-matrices and \( \eta (\psi) \) is the CPT phase of the constituent field \( \psi \). Hence

\[ CPT \overline{\psi} \gamma_0 \psi (CPT)^{-1} = -\eta (\psi) \overline{\psi} \gamma_0 \psi, \]
\[ CPT \overline{\psi} \gamma_5 \psi (CPT)^{-1} = -\eta (\psi) \overline{\psi} \gamma_5 \psi. \]  \hspace{1cm} (29)

Using eqs. (29) one gets the following result for CPT transformations of \( d \) and \( s \) quarks:

\[ CPT d(x) (CPT)^{-1} = -\eta (\psi)^2 \gamma_5 d^+ (-x), \]
\[ CPT s(x) (CPT)^{-1} = -\eta (\psi)^2 \gamma_5 s^+ (-x). \]  \hspace{1cm} (30)

Thus \( d \) and \( s \) quarks obtain opposite CPT phases in such a model of composite quarks.

In this case the Standard Model Lagrangian will not be any more invariant under the physical CPT operator \( \Theta_{\text{ph}} \) but will consist of CPT even and CPT odd parts.

At first glance this extra minus in eq. (26) contradicts the powerful Jost theorem which does not allow extra minuses in eqs. (19), (17). But one should remember that the fields there are the full operators in the Heisenberg representation. The operator \( \Theta_{\text{ph}} \) does not commute with the Hamiltonian, hence it depends on time and is not restricted by the Jost theorem.

We would like to underline that the operator \( \Theta_{\text{ph}} \) does not commute with the Lagrangian both in the interaction representation and in the Heisenberg representation. We refer here to the Heisenberg representation just to demonstrate that our choice of opposite CPT phases for \( d \) and \( s \) quarks does not contradict the Jost theorem.

We underline once more that the considered type of CPT violation appears because we can choose the opposite CPT phases for \( d \) and \( s \) quarks. Then the terms of the weak Lagrangian, which are linear in \( d \) and \( s \) fields, obtain different signs after applying the CPT operator. This type of CPT violation happens in a theory with an interaction Lagrangian containing at least two terms with odd number of fields of different CPT phases.
Let us consider the influence of this \( CPT \) violation on the difference of proton and antiproton masses. For this purpose we will use the simplified quantum mechanical picture of a proton as a superposition
\[
|p\rangle = |uud\rangle + \xi|uus\rangle,
\]
where \(|uud\rangle\) and \(|uus\rangle\) are the eigenvectors of the Hamiltonian of strong interactions consisting of the corresponding light quarks \( u, d \) and \( s \).

The amplitude \( \xi \) appears due to opposite \( CPT \) phases of corresponding light quarks \( u \) and \( d \) and \( s \).

One can assume that the physical \( CPT \) operator \( \Theta_{ph} \) transforms the proton state into the physical antiproton state
\[
\Theta_{ph}|p\rangle = \eta_p(|\bar{u}\bar{d}\bar{d}\rangle - \xi^*|\bar{u}\bar{d}\bar{s}\rangle) = \eta_p|\bar{p}\rangle,
\]
where \( \eta_p \) is the proton \( CPT \) phase factor, the minus sign appears due to opposite \( CPT \) phases of \( d \) and \( s \) quarks and \(|\bar{p}\rangle\) is the physical antiproton state.

Correspondingly, the action of \( \Theta_{ph} \) on the antiproton state will give the proton state
\[
\Theta_{ph}|\bar{p}\rangle = \eta_p|p\rangle.
\]

This case is similar to the case of the standard charge conjugation operator \( C \). The \( C \) operator is assumed to transform physical particles to physical antiparticles although the Lagrangian of weak interactions is not invariant with respect to \( C \).

On the other hand, the unphysical operator \( \Theta \) will act on the proton state as follows:
\[
\Theta|p\rangle = \eta_p(|\bar{u}\bar{d}\bar{d}\rangle + \xi^*|\bar{u}\bar{d}\bar{s}\rangle),
\]
producing an unphysical state which differs from the physical antiproton state (33) only by the phase in front of \( \xi^* \).

Let us remind that the minus sign in front of \( \xi^* \) for the physical antiproton in eq. (33) was obtained due to the opposite \( CPT \) phases of \( d \) and \( s \) quarks which can be arranged, \( e.g., \) in models of composite quarks, see eq. (30).

The full Hamiltonian is the sum of the \( CPT \) even and \( CPT \) odd parts:
\[
H = H_+ + H_-.
\]

The masses of proton and antiproton are
\[
m_p = \langle p|H_+|p\rangle + \langle p|H_-|p\rangle,
\]
\[
m_{\bar{p}} = \langle \bar{p}|H_+|\bar{p}\rangle + \langle \bar{p}|H_-|\bar{p}\rangle.
\]

Applying the \( CPT \) operator \( \Theta_{ph} \) we get
\[
m_p - m_{\bar{p}} = \langle p|H_+|p\rangle - \langle \bar{p}|H_-|\bar{p}\rangle = 2\langle uud + \xi uus|H_-|uud + \xi uus\rangle = 4\xi\langle uud|H_-|uus\rangle,
\]
where \( m_p - m_{\bar{p}} \approx 4\xi \sin \theta_c G_F \approx 6 \times 10^{-11} \),

which should be compared with present experiments, see eq. (21).

Thus to check the possibility of \( CPT \) violation it is necessary to improve the current experimental precision for \( m_p - m_{\bar{p}} \approx 4\xi \sin \theta_c G_F \approx 6 \times 10^{-11} \).

If one considers the charge-to-mass and assumes that the charge of proton and antiproton is the same then the precision is already one order of magnitude better (7 \( \times 10^{-11} \), see eq. (22). In this case it is desirable to improve the precision of experiments approximately twice.

There are of course other well-known consequences of \( CPT \) invariance which can be checked experimentally. For example cross-sections of scattering processes of some particles should coincide with cross-sections of reversed reactions of corresponding antiparticles with opposite spins. More precisely the transition probabilities for the following two processes should coincide: 1) particles with momenta \( p_i \) and spins \( \sigma_i \) react to produce particles with momenta \( p'_i \) and spins \( \sigma'_i \); 2) the corresponding antiparticles with \( p'_i, \sigma'_i \) react to produce antiparticles with \( p_i \) and \( -\sigma_i \).

Also there are interesting checks of \( CPT \) invariance in the system of neutral kaons, see, \( e.g., \) reviews in [9]. But the simplest tests of \( CPT \) invariance are the tests of equality of masses and life times of particles and their antiparticles. The measurement of the proton and antiproton mass difference seems to be the most proper experiment to check the possibility of violation of \( CPT \) symmetry considered in the present paper.

Conclusions. – We have shown that there is still the possibility of violation of \( CPT \) symmetry in the Standard Model. It can be achieved by a proper choice of the \( CPT \) phases for quarks. To check this possibility experimentally it is necessary to increase the accuracy of measurements of the proton and antiproton mass difference by an order of magnitude.

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