Stripe State in the Lowest Landau Level

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The stripe state in the lowest Landau level is studied by the density matrix renormalization group (DMRG) method. The ground state energy and pair correlation functions are systematically calculated for various pseudopotentials in the lowest Landau level. We show that the stripe state in the lowest Landau level is realized only in a system whose width perpendicular to the two-dimensional electron layer is smaller than the order of magnetic length.

KEYWORDS: fractional quantum Hall effect, stripe, ground state, phase diagram, two dimension, density matrix, renormalization group

1. Introduction

The two-dimensional electron systems in a high magnetic field exhibit many interesting properties. The transport measurements on quantum Hall systems show the presence of zero-resistance states at various fractional fillings \( \nu = n/(2n \pm 1) \),\(^{1,2}\) which is due to the formation of incompressible liquid states,\(^{3}\) while finite conductance is observed at \( \nu = 1/2 = \lim_{n \to \infty} n/(2n \pm 1) \) even in the limit of low temperature,\(^{4}\) which suggests the formation of compressible liquid state predicted in the composite fermion theory.\(^{5}\) Transitions to insulating states have also been observed at low fillings around \( \nu \sim 1/5,6-8\) and they are thought to be caused by the formation of Wigner crystal, which is pinned by impurity potentials.

Analytical and numerical studies\(^{5,9-13}\), have confirmed the existence of these ground states in the lowest Landau level, but there still remain questions on the ground state between the fractional quantum Hall states at \( \nu = n/(2n \pm 1) \). In particular, recent DMRG calculations show the presence of a weak stripe state in the lowest Landau level around \( \nu = 0.42, 0.37 \) and below \( 0.32 \),\(^{14}\) whose correlation functions are different from those expected in the composite fermion theory,\(^{15}\) while recent experiments on ultra high-mobility wide quantum wells did not yield any evidence of the stripe formation in the lowest Landau level.\(^{16}\)

In the present paper, we show that the stripe state in the lowest Landau level is realized only in a system of narrow quantum wells whose width is smaller than the order of magnetic length. There is a phase transition to a liquid state as the width increases, which is caused by a decrease in short-range pseudopotentials. This may be a reason why the stripe state in the lowest Landau level has not been observed in wide quantum wells.

We calculate the ground-state wavefunction by employing the density-matrix renormalization-group (DMRG) method,\(^{17}\) which is applied to two-dimensional systems in a magnetic field.\(^{18-20}\) This method enables us to obtain the essentially exact ground state of large size systems extending the limitation of the exact diagonalization method with controlled accuracy.

2. Model and Method

We use the Hamiltonian of two-dimensional electrons in a high perpendicular magnetic field. The kinetic energy of the electrons is quenched by the magnetic field and we can omit this energy. Thus the Hamiltonian contains only the Coulomb interaction

\[
H = \sum_{i<j} e^{-q^2/2} V(q) e^{iq \cdot (R_i - R_j)}
\]

where \( R_i \) is the guiding center coordinate of the \( i \)th electron, which satisfies the commutation relation, \( [R_i^x, R_j^y] = i \ell^2 \delta_{jk} \). \( V(q) = 2\pi e^2/\varepsilon q \) is the Fourier transform of the Coulomb interaction. The magnetic length \( \ell \) is set to be 1 and we use \( e^2/\varepsilon \ell \) as units of energy. We assume that the magnetic field is strong enough to polarize the spins and suppress the Landau level mixing.

In the present DMRG calculations, we divide the system into unit cells \( L_x \times L_y \) with the periodic boundary conditions for both \( x \) and \( y \) directions. We calculate the ground-state energy and wavefunction for systems with up to 24 electrons in the unit cell with various aspect ratios \( L_x/L_y \), and obtain the pair correlation function \( g(r) \) defined by

\[
g(r) = \frac{L_xL_y}{N_e(N_e-1)} \langle \Psi | \sum_{i \neq j} \delta(r + R_i - R_j) | \Psi \rangle
\]

from the ground state wavefunction \( | \Psi \rangle \). The correlation functions in the stripe state are calculated in the unit cell which has the minimum energy with respect to \( L_x/L_y \). These correlation functions are expected to have the correct period of the stripes realized in the thermodynamic limit.

The accuracy of the results is controlled by the density matrix used in the calculation. The truncation error in the norm of the wavefunction obtained in the present calculation is typically \( 10^{-4} \) with keeping 200 eigenstates of the density matrix.

3. Stripe Ground State at \( \nu = 3/8 \)

The stripe state in the lowest Landau level has already been obtained in the previous DMRG study,\(^{14}\) which shows the enhancement of the stripe correlation between the incompressible liquid states around \( \nu \sim 0.42, 0.37 \) and between \( \nu \sim 0.32 \) and 0.15. Here, we present de-
Figure 1 shows the ground-state energy and the pair correlation functions at $\nu = 3/8$ obtained for the systems of $N_e = 12, 15, 18$ and 24, where $N_e$ is the number of electrons in the unit cell. We find that the stripe state has the lowest energy independent of the number of electrons. There is a liquid ground state in the system of 12 electrons at the aspect ratio larger than 1.8, but the energy is higher than that of the stripe state around $L_x/L_y \sim 1.3$ and we think this liquid state is not realized in large systems.

The energy of the stripe state in Fig. 1 clearly depends on the aspect ratio. This clear dependence shows the existence of an optimum stripe structure because the change in the aspect ratio modifies the period of the stripes. Indeed, the pair correlation functions calculated at the energy minimum have almost the same stripes structure as shown in Figs. 1 (e) – 1 (h). Since the size dependence is small, we expect the existence of the stripes even in the thermodynamic limit. In order to examine the size dependence in more detail, we plot the correlation functions along the $x$-axis for various sizes of systems. The results for $N_e = 12, 15, 18$ and 24 presented in Fig. 2 (a) show that the size dependence on the amplitude and the period of the stripes is very small, which strongly suggests that almost the same correlation function is realized in the thermodynamic limit. We note that the period of the stripes is about $4\ell$, which is much smaller than $10\ell$ expected in the composite fermion theory.

For comparison with the stripe state in higher Landau levels, we present in Fig. 2 (b) the correlation functions in higher Landau levels. We find that the amplitude of the stripes in the lowest Landau level is relatively small and the short range correlations around $r \sim 2\ell$ are different. Furthermore, the stripe state in the lowest Landau level has clear oscillations along the stripes as shown in Fig. 2 (c). Since the correlation functions are qualitatively different, we call the stripe state in the lowest Landau level, type-II stripe state. All the stripe states found in the low-
Fig. 3. The energies of the stripe state and the liquid state for various Haldane’s short-range pseudopotentials $V_1$ and $V_3$. $\delta V_n$ is the difference from the value of pure Coulomb interaction in the ideal two-dimensional system. $\delta V_3 = -0.008$ in (a)–(c), and it is $-0.024$ in (d)–(f). $N_e = 12$. $\delta V_n = 0$ for $n \geq 5$.

4. Transition to a Liquid State

Although the stripe state is expected in the numerical calculations, clear experimental evidence has not been obtained. In particular, transport properties of recent high-mobility wide quantum wells suggest the liquid ground state around $\nu = 1/4$,\(^\text{16}\) while clear stripe correlations are obtained in the DMRG calculations. Since the previous DMRG study assumed the ideal two-dimensional system, we here consider the effect of finite width perpendicular to the two-dimensional layer.

When the wavefunction is enlarged in the perpendicular direction, the short-range effective repulsion is reduced. This reduction is represented by a decrease in Haldane’s pseudopotentials $V_n$,\(^\text{21}\) where $n$ is the relative angular-momentum between the two electrons. In the present study, we decrease the short-range components, $V_1$ and $V_3$, and see how the ground state is modified. In general, a decrease in $V_n$ for $n \geq 5$ is present, but the amount of the change rapidly decreases with the increase in $n$. We therefore neglect the effects of $V_n$ for $n \geq 5$ in order to simplify the problem.

In Fig. 3, we plot the ground state energy for various $\delta V_1 = V_1 - (V_1)_{2D}$ and $\delta V_3 = V_3 - (V_3)_{2D}$ where $(V_n)_{2D}$ are the values for the pure Coulomb interaction in the ideal two-dimensional system. In this figure, we can see that $-\delta V_1$ increases the energy of both the stripe and liquid states. On the other hand, $-\delta V_3$ decreases the energy of the liquid state compared with the energy of the stripe state; the stripe state at $\delta V_3 = -0.008$ has lower energy compared with the liquid state around $L_x/L_y \sim 1.8$, while the liquid state apparently has lower energy for $\delta V_3 = -0.024$. Furthermore, the aspect ratio dependence on the energy of the stripe state at $\delta V_3 = -0.024$ is very small. These results indicate that the stripe structure at $\delta V_3 = -0.024$ is not stable against changes in the period and the ground state becomes a liquid state which has lower energy. Thus we can expect a transition to a liquid state between $\delta V_3 \sim -0.01$ and $-0.02$. Since each state has different total momentum, this transition is expected to be of the first order.

The existence of the transition is also found in larger systems. Since the $V_1$ dependence on the transition is small as shown in Fig. 3, we consider the case of $\delta V_1 = \delta V_3$. The results for $N_e = 15$ and 18 are shown in Fig. 4, which reveals that the liquid state at $\delta V_3 = -0.032$ has lower energy and the aspect ratio dependence on the energy of the stripe state is very small, while the liquid state for $\delta V_3 = -0.008$ has higher energy compared with the energy of the stripe state. These features are the same as those of the system of $N_e = 12$. Since the size dependence on the transition is small, we expect the transition to a liquid state around $\delta V_3 \sim -0.018$ even in the thermodynamic limit.

In order to see the effects on the ground-state wavefunction, we calculate the ground-state pair correlation functions for $\delta V_1 = \delta V_3 = -0.008$, and $-0.032$. The results are shown in Fig. 5, where we can see that the stripe correlations shown in (a) of $\delta V_3 = -0.008$ and (b) of $\delta V_3 = -0.032$ are almost the same, although the stripe state at $\delta V_3 = -0.032$ has higher energy compared with the energy of the liquid state as shown in Fig. 4 (d). Since the correlation functions in the liquid state are completely different from those in the stripe state as shown in Figs. 5 (b) and 5 (c), the first-order transition between the stripe state and the liquid state is confirmed.
5. Ground State Phase Diagram around $\nu = 3/8$

In order to investigate the phase boundary away from $\nu = 3/8$, we have calculated the ground-state energy and correlation functions at various fillings between $\nu = 2/5$ and $\nu = 1/3$ with various $V_3$. The obtained results are summarized in the phase diagram shown in Fig. 6. We find that the stripe state is stable around $\nu = 3/8$ for small $|\delta V_3|$. The phase boundary to the liquid state moves toward smaller $|\delta V_3|$ as $\nu$ approaches the incompressible liquid states at $\nu = 1/3$ and 2/5. The transition to the liquid state occurs even at $\delta V_3 = 0$ at $\nu \sim 0.35$ and 0.385. These phase boundaries at $\delta V_3 = 0$ are consistent with the previous DMRG calculations.14

Since the stripe state is not realized for large $-\delta V_1$ and $-\delta V_3$, the ground state of wide quantum wells is expected to be a liquid state. When we assume an infinitely deep quantum well and neglect the effects from $\delta V_n$ for $n \geq 5$, the critical width of about 6$\ell$ is obtained. In the case of realistic quantum wells, the critical width may be smaller than this value because the wavefunction penetrates through the potential barrier. The line of the phase boundary in Fig. 6 is just a guide for the eyes. If we precisely study the excitation spectrum around the boundary, some structures may be found. This remains for future investigation.

6. Summary

In the present work we have studied the stripe state in the lowest Landau level using the DMRG method. The ground-state energy and the pair-correlation functions are systematically calculated at various fillings and pseudopotentials around $\nu = 3/8$. We have shown that the stripe state in the lowest Landau level is realized in a system whose width perpendicular to the two-dimensional layer is sufficiently small.

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