Quark matter in light neutron stars

Márcio Ferreira,† Renan Câmara Pereira,† and Constança Providência‡

†CFisUC, Department of Physics, University of Coimbra, P-3004 - 516 Coimbra, Portugal

(Dated: August 31, 2020)

Higher-order repulsive interactions are included in the three-flavor NJL model in order to describe the quark phase of an hybrid star. The effect of 4-quark and 8-quark vector-isoscalar interactions in the stability of hybrid star configurations is analyzed. The presence of a 8-quark vector-isoscalar channel is seen to be crucial in generating large quark branches in the M(R) diagram. This is due to its stiffening effect on the quark matter equation of state which arises from the non-linear density dependence of the speed of sound. This additional interaction channel allows for the appearance of a quark core at moderately low NS masses, \( \sim 1M_\odot \), and provides the required repulsion to preserve the star stability up to \( \sim 2.1M_\odot \). Furthermore, we show that both the heaviest NS mass generated, \( M_{\text{max}} \), and its radii, \( R_{\text{max}} \), are quite sensitive to the strength of 8-quark vector-isoscalar channel, leading to a considerable decrease of \( R_{\text{max}} \) as the coupling increases. This behavior imprints a considerable deviation from the purely hadronic matter equation of state in the \( \Lambda(M) \) diagram, which might be a possible signature of the quark matter existence, even for moderately low NS masses, \( \sim 1.4M_\odot \). The resulting \( M(R) \) and \( \Lambda(R) \) relations are in accordance with the latest astrophysical constraints from NICER and Ligo/VIRGO observations, respectively.

I. INTRODUCTION

Neutron stars (NS) have been the focus of many experimental and theoretical studies in astrophysics, nuclear and particle physics. Their inner composition still remains an open question. The extreme densities reached in NS cores might originate some exotic matter, such as hyperons, Bose-Einstein condensates or quark matter [1].

The two solar mass pulsars PSR J1614-2230 \( (M = 1.908 \pm 0.016 M_\odot) \) and PSR J0348+0432 \( (M = 2.01 \pm 0.04 M_\odot) \) [2] and MSP J0740+6620 [3], \( (M = 2.14^{+0.10}_{-0.09}M_\odot) \) impose tight constraints on the nuclear matter equation of state (EoS). Multi-messenger astrophysics that combines astrophysical observations of different type, electromagnetic radiation, gravitational waves (GW) and different types of particles provide deeper insights on NS properties. The analysis by the LIGO/Virgo collaborations of the GW from the NS merger GW170817 gave us important information about the NS structure [4] [5], e.g., an upper limit of the tidal deformability of a NS star, that allows us to to set extra constraints on the high density EoS. Moreover, the detection of the gamma-ray burst (GRB) GRB170817A [6], and the electromagnetic transient AT2017gfo [7] that followed up the GW170817 event has further established constraints on the lower limit of the tidal deformability [8] [12]. The Neutron Star Interior Composition Explorer (NICER) experiment is presently another important source of observational data that may shed some light into the structure of NS. Recently, two different teams of NICER have estimated the mass and radius of the millisecond-pulsar PSR J0030+0451 [13].

While massive pulsars rule out soft EoS at high densities, a too stiff EoS, which gives rise to large radii, is incompatible with the tidal deformability from GW observations [14]. The high density region of the EoS is thus severely constrained, which may exclude exotic, i.e. non-nucleonic, degrees of freedom inside NS, such as quark matter. However, the existence of a first order phase transition from hadronic to quark matter, depending on its properties, may balance the two features mentioned above and still explain the observational data [13]. Detecting observational signatures that indicate the presence of exotic matter inside neutron stars is a major difficulty. For instance, it is hard to establish a clear physical distinction between a purely hadronic NS and one with a quark core solely from NS observables, such as the star mass, radius and tidal deformability. However, the presence of a first order phase transition between hadronic and quark matter can lead to observational signatures that could be exploited in more neutron star binary mergers observations, favoring the hypothesis of quark matter in the neutron star core [14] [16].

One way to study quark degrees of freedom in NS matter is through effective models, which incorporate the most important properties and symmetries of the strong interactions. The NJL model is an widely used effective model of QCD. Some of its applications are the study of the phase diagram of QCD, the behavior of mesons at finite temperature and density and also to study of the possible existence of quark matter inside neutron stars [17] [26]. The NJL model Lagrangian is built considering symmetry preserving interactions, specially chiral symmetry [27] [28].

A possible approach to construct an hybrid EoS is the two model approach: one that describes the hadronic (confined) phase and a second model describing the quark (deconfined) phase. The matching of the two EoS may be carried out within different approaches, in particular considering local charge neutrality or global charge neutrality [29]. In the present approach we will consider a Maxwell construction to describe a first-order phase
transition from hadron matter to a quark phase. This approach is considered to be quite realistic if the surface tension between hadron and quark matter, a still unknown quantity, is large. This methodology has been widely used, where an hadronic model and an independent quark model were considered, see \cite{21,30,34}. Using the NJL model to describe the quark phase of a hybrid EoS, previous works have successfully predicted neutron stars with at least $2M_\odot$. \cite{22,35}. The presence of the vector-isoscalar interaction was shown to be very important in stiffening the EoS to sustain $2M_\odot$. The inclusion of 8-quark interactions in the scalar and in the vector-isoscalar channel within the two-flavor NJL model was explored in \cite{30,31} in the context of hybrid stars. In \cite{35}, local and nonlocal NJL models with vector interaction among were seen to typically give no hybrid stars (or just small quark branches).

It has been shown by several authors that the onset of the $\Delta$ may compete with the onset of hyperons, and due to its large isospin and the still lack of information to fix the coupling constants these particles may set in at densities below the onset of hyperons, just above saturation density \cite{36-39}. In particular, the onset of $\Delta$s may occur in low mass stars making compatible relativistic mean-field models with the constraint set by GW170917 on the tidal deformability. In the present work, we will show an alternative scenario and will show that the onset of quarks at densities below twice saturation density may also have a similar effect of pushing down the tidal deformability of stars with masses $\sim 1.4 M_\odot$ or below.

Using a constant-speed parametrization for the high-density EoS region \cite{40,41}, the authors concluded that for a strong first-order phase transition to quark matter to be compatible with $M_{\text{max}} > 2 M_\odot$ requires a large speed of sound in the quark phase, $v_s^2 \gtrsim 0.5$ for soft hadronic EoS and $v_s^2 \gtrsim 0.4$ for stiff hadronic EoS. Using the same formalism, the work \cite{42} points in the same direction: strong repulsive interactions in quark matter are required to support the NS masses $M \gtrsim 2.0 M_\odot$.

In \cite{43}, the authors studied the possibility of occurrence of stars with quark cores, imposing well known constraints, both observational and theoretical ab-initio calculations, to a large set of EoS built using metamodels parametrized by the speed of sound. They propose that $1.4 M_\odot$ stars are compatible with hadronic stars. Besides, they infer that massive stars with a mass $\approx 2 M_\odot$ and a speed of sound not far from the conformal limit will have large quark cores. We would like to understand whether it is possible to arrive to similar conclusions starting from a set of quark matter EoS that satisfy a given number of constraints set by properties of mesons in the vacuum which, also have been derived from a model with intrinsic chiral symmetry.

To attain this aim, we will work in the framework of the three-flavor NJL model, and we will analyze the effect of 4-quark and 8-quark vector-isoscalar interactions in hadron-quark hybrid EoS. NJL models typically give rather low values for the speed of sound in the quark matter phase ($v_s^2 \sim 0.2 - 0.3$) and have a small dependence on the density. Furthermore, the speed of sound is quite insensitive to the NJL model parameters $\Lambda$, $m_{u,d}$, $m_s$, $G_S$, $G_D$, i.e. the cutoff, current masses and couplings of the scalar and $t$'Hooft terms. We will investigate the impact of the vector interactions in the speed-of-sound and in the quark phase and thus on the stability of hybrid stars sequences. Moreover, exploring these additional interactions, we will analyze the possibility of having quark cores in light NS and, at the same time, fulfill all observational constraints.

This paper is organized as follows: in Section II the quark model is detailed. The results are presented in Section III followed by our conclusions, in Section IV.

### II. MODEL AND FORMALISM

The SU(3)$_f$ NJL Lagrangian density, including four and six scalar-pseudoscalar interactions and four and eight vector-isoscalar interactions is:

$$
\mathcal{L} = \bar{\psi}(i\partial - \hat{m} + \mu\gamma^0)\psi \\
+ G_S \sum_{a=0}^8 \left[ \left(\bar{\psi}\lambda^a\psi\right)^2 + \left(\bar{\psi}i\gamma^5\lambda^a\psi\right)^2 \right] \\
- G_D \left[ \det(\bar{\psi}(1 + \gamma_5)\psi) + \det(\bar{\psi}(1 - \gamma_5)\psi) \right] \\
- G_\omega \left[ \left(\bar{\psi}\gamma^\mu\lambda^0\psi\right)^2 + \left(\bar{\psi}\gamma^\mu\gamma_5\lambda^0\psi\right)^2 \right] \\
- G_{\omega^2} \left[ \left(\bar{\psi}\gamma^\mu\lambda^0\psi\right)^2 + \left(\bar{\psi}\gamma^\mu\gamma_5\lambda^0\psi\right)^2 \right] .
$$

(1)

The diagonal matrices $\hat{m} = \text{diag}(m_u, m_d, m_s)$ and $\mu = \text{diag}(\mu_u, \mu_d, \mu_s)$ are the quark current masses and chemical potential matrices, respectively. The matrices $\lambda^a$ with components $a = 1, 2, \ldots, 8$, are the Gell-Mann matrices of the SU(3) group while, the zero component, is a matrix proportional to the identity matrix, $\lambda^0 = \sqrt{\frac{2}{3}}I$. The quark field has $N_f$-components in flavor space.

The NJL model is nonrenormalizable in four dimensional space-time. Hence some regularization procedure must be employed in order to regularize the integrals. Alongside, the Matsubara formalism to derive the thermodynamical potential we are going to regularize the integrals. The multi-quark interactions considered are all chiral symmetry preserving. The four scalar and pseudoscalar quark interaction is present in the original formulation of the NJL model and is essential to incorporate in the model spontaneous chiral symmetry breaking. The $t$'Hooft determinant for three quark flavours corresponds to a six quark interaction which incorporates the explicit $U_A(1)$ symmetry breaking in the model. Incorporating vector interaction in the model has been found to be necessary to model the medium to high density behaviour of the EoS and predict $2M_\odot$ neutron stars. The inclusion of all possible chiral-symmetric set of eight quark vector interactions was performed in \cite{44} in order to study the masses of the lowest spin-0 and spin-1 meson states. Following previous works, the vector-isoscalar quark interactions
have been shown to be essential to build $2M_\odot$ neutron stars.

In the present work, we will restrict our analysis to four and eight vector-isoscalar quark interactions and study their influence on the EoS of hybrid neutron stars. These vector interactions have free coupling constants, $G_\omega$ and $G_{\omega\omega}$ respectively. In general, both of these couplings can be fixed in the vacuum by fitting the omega meson mass. However, as discussed in the literature [28, 45], the vector-isoscalar terms are proportional to density degrees of freedom and their couplings might be density dependent. Hence, to take into account the possible in-medium dependence of the vector couplings $G_\omega$ and $G_{\omega\omega}$, we will not fix their magnitudes in the vacuum and leave them as free parameters. As in our previous works [33], we will study different models defined by different values for the ratios $\xi_\omega = G_\omega/G_S$ and $\xi_{\omega\omega} = G_{\omega\omega}/G_S^2$.

The thermodynamical potential of the NJL model is calculated in the mean field approximation (MF), where the product between quark bilinear operators are linearized around their mean field values, and a linear Lagrangian density can be obtained (for more details on the linear product between $N$ operators see [46]). The quark fields can then be integrated out.

Using the Matsubara formalism and the linearized Lagrangian density of the NJL model, $\Omega$, is derived from the lagrangian written in Equation (1). For finite temperature and chemical potential it can be written as:

$$\Omega - \Omega_0 = 2G_S(\sigma_u^2 + 2\sigma_d^2 + \sigma_s^2) - 4G_D\sigma_u\sigma_d - \frac{2}{3}G_\omega(p_u + p_d + p_s)^2 - \frac{4}{3}G_{\omega\omega}(p_u + p_d + p_s)^4$$

$$- 2TN_c \sum_{i=u,d,s} \int_0^\Lambda d^3p \frac{1}{(2\pi)^3} \ln \left(1 + e^{-\frac{(E_i + \mu_i)}{T}}\right)$$

$$- 2TN_c \sum_{i=u,d,s} \int_0^\Lambda d^3p \frac{1}{(2\pi)^3} \ln \left(1 + e^{-\frac{(E_i - \mu_i)}{T}}\right)$$

$$- 2N_c \sum_{i=u,d,s} \int_0^\Lambda d^3p \frac{1}{(2\pi)^3} E_i.$$  (2)

The constant $\Omega_0$ is calculated in such a way that the potential vanishes in the vacuum. Also, $E_i = \sqrt{p^2 + M_i^2}$ and $\sigma_i$ and $\rho_i$ are the condensate and density of the quarks with flavor $i$, respectively.

For $i \neq j \neq k \in \{u,d,s\}$, the effective mass, $M_i$, and effective chemical potentials, $\tilde{\mu}_i$, are found to be:

$$M_i = m_i - 4G_S\sigma_i + 2G_D\sigma_j\sigma_k,$$  (3)

$$\tilde{\mu}_i = \mu_i - \frac{4}{3}G_\omega(p_i + p_j + p_k) - \frac{16}{9}G_{\omega\omega}(p_i + p_j + p_k)^3.$$  (4)

In the MF approximation the thermodynamical potential must be stationary with respect to the effective mass, $M_i$, and effective chemical potentials $\tilde{\mu}_i$, i.e.,

$$\frac{\partial \Omega}{\partial M} = \frac{\partial \Omega}{\partial \tilde{\mu}_i} = 0.$$  (5)

Applying these stationary conditions to the thermodynamical potential yields a closed expression for the quark condensate, $\sigma_i$, and density, $\rho_i$. For the explicit expressions see [37].

The quark sector of the cold hybrid EoS can be easily calculated from Equation (2) in the $T = 0$ limit. The pressure and energy density are given by:

$$P = -\Omega,$$  (6)

$$\epsilon = -P + \sum_{i} \mu_i \rho_i.$$  (7)

Aside from the free vector couplings, $G_\omega$ and $G_{\omega\omega}$, the remaining parameters of the model are fixed in order to reproduce the values of some meson masses and decay constants. The used parameter set can be found in Table I. In Table II we present the values of some meson masses and lepton decay constants within the parameter set in Table I and the respective experimental values.

$$\begin{array}{cccccccc}
\hline
\Lambda & m_{u,d} & m_s & G_S \Lambda^2 & G_D \Lambda^2 & M_{u,d} & M_s & \\
[MeV] & [MeV] & [MeV] & [MeV] & [MeV] & [MeV] & [MeV] \\
623.58 & 5.70 & 136.60 & 1.67 & 13.67 & 332.2 & 510.7 & \\
\hline
\end{array}$$

Table I. Parameters of the NJL model used in the present work: $\Lambda$ is the model cutoff, $m_{u,d}$ and $m_s$ are quark current masses, $G_S$ and $G_D$ are coupling constants. $M_{u,d}$ and $M_s$ are the resulting constituent quark masses in the vacuum. This parameter set yields, in the vacuum, a light quark condensate of $\langle \bar{q} q \rangle^{1/3} = -243.9$ MeV and strange quark condensate of $\langle \bar{s} s \rangle^{1/3} = -262.9$ MeV.

$$\begin{array}{cccc}
\hline
NJL & SU(3) & Experimental & [18] \\
$\langle \bar{q} q \rangle$ & [MeV] & [MeV] & \\
$[\bar{q} q]$ & 139.6 & 139.6 & \\
$\langle \bar{s} s \rangle$ & 92.0 & 92.2 & \\
$\langle \bar{u} u \rangle$ & 493.7 & 493.7 & \\
$\langle \bar{d} d \rangle$ & 96.4 & 110.4 & \\
$\langle \bar{c} c \rangle$ & 515.6 & 547.9 & \\
$\langle \bar{b} b \rangle$ & 957.8 & 957.8 & \\
\hline
\end{array}$$

Table II. The masses and decay constants of several mesons within the model and the respective experimental values.

The NJL model pressure and energy density are defined up to a constant $B$, analogous to the MIT bag constant [21]. It is essential in building hybrid EoS that sustain two-solar mass neutrons stars. In [21, 33], $B$ was fixed by requiring that the deconfinement occurs at the same baryonic chemical potential as the chiral phase transition.
More recently in [22], an effective bag constant was also used to control the density at which the phase transition from hadron to quark matter happened. In the presence of a finite bag constant, the quark EoS is modified by \( P \to P + B \) and \( \epsilon \to \epsilon - B \). Hence the NJL quark EoS will be defined by three parameters: the model vector coupling ratios, \( \xi_\omega = G_\omega/G_S \) and \( \xi_{\omega\omega} = G_{\omega\omega}/G_S^2 \) and the bag constant \( B \).

For the hadronic part of the hybrid stars we use the DDME2 model [49]. This is a relativistic mean-field model with density dependent couplings that describes two solar mass stars and satisfies a well established set of nuclear matter and finite nuclei constraints [50, 51], including the constraints set by the ab-initio calculations for neutron matter using a chiral effective field theoretical approach [52]. This has been the low density constraint set in [53].

III. RESULTS

Herein, we analyze the effect of the vector-isoscalar couplings \( \xi_\omega = G_\omega/G_S \) and \( \xi_{\omega\omega} = G_{\omega\omega}/G_S^2 \) on the hybrid EoS and respective NS properties. The effect of the bag constant \( B \) was already studied in [18, 21–26, 33, 47, 54–60], where it was found that the onset of quark matter in the hybrid EoS happens at lower densities as \( B \) increases. Although we have explored several values for \( B \), we have decided to keep it fixed in the following analysis to \( B = 10 \) MeV/fm\(^3\). As free parameters, we consider \( \{ \xi_\omega, \xi_{\omega\omega} \} \) which give a considerable flexibility to span a wide range of EoS with the required properties. In the following, charge-neutral neutron star matter in \( \beta \)-equilibrium, with a first-order phase transition (via a Maxwell construction) from hadronic matter to quark matter happens, is studied.

The main effect of the 4-quark vector term is to stiffen the quark EoS and shift the onset of quark matter to larger densities as discussed in [26, 33]. Moreover, the larger the coupling constant, \( \xi_\omega \) the smaller the quark core. This behavior has been described considering a constant speed of sound model for the quark phase [57].

Let us now analyze how \( \xi_{\omega\omega} \) affects the quark matter EoS. Figure 1 shows the pressure (right) and the speed of sound squared (left) as a function of baryonic density for \( \xi_\omega = 0 \) (herein, we use \( c = 1 \)). The speed of sound, \( v_s^2 = dp/d\epsilon \), characterizes how stiff the EoS is. It is clear from both panels that the 8-quark term, characterized by the coupling \( \xi_{\omega\omega} \), allows the quark EoS to become stiffer so that a larger quark core will be sustained in the hybrid NS: this term gives rise to a density dependent speed of sound that increases non-linearly with density. The main role of \( \xi_{\omega\omega} \) is played at large densities: it affects in a much smaller extension the onset of quark matter than the \( \xi_\omega \) coupling. This is clearly seen in Figure 2 where the onset density of quark matter, for each hybrid EoS, is shown by a color degrade in terms of the parameters \( \xi_{\omega\omega} \) and \( \xi_\omega \). The change of color is only slightly dependent on \( \xi_{\omega\omega} \).

The sudden decrease of the speed of sound \( v_s^2 \) at \( n \approx 0.5 \) fm\(^{-3}\) is due to the onset of strangeness. Note, however, that the appearance of the strange quark occurs via a crossover and thus in a continuous way. Since the vector terms introduced are flavor invariant [28], the onset of strangeness does not depend of the vector terms and is completely defined by the properties of the model shown in Table I. The amount of strangeness inside the star, will, therefore, be determined by the central density that depends on both vector terms.

We plot in Fig. 3 our set of EoS on a pressure vs energy density graph for \( \xi_\omega = 0 \), and include in the background the acceptable region of EoS defined in [53]. We conclude that our set of EoS covers a quite large fraction of the proposed region. The red color indicates a region with a speed of sound \( v_s^2 \ll 0.3 \) as shown in Fig. 1. Our most massive stars (purple color) lie close to the border of the region and are associated with central speed of sound well above the conformal limit, which can be as large as 0.9c. Some interesting conclusions are: a) our set of EoS also defines a change of slope. This could be due to the fact that we work with a model with chiral symmetry incorporated. This kind of knee is also present in other studies [58]; b) we get low mass stars with a quark core below the knee; c) our heaviest stars with a large quark core have a speed of sound far from the conformal limit;
d) the red dots identify EoS with a speed of sound close to the conformal limit and lie in the center of the region as obtained in [53]; e) the vector interactions considered in this work do not span the whole region of the Fig. 3. Including extra four and eight quark vector interactions, for instance in the scalar and vector-isovector channels, may increase this region. This is left as future work.

![Figure 3](image1)

Figure 3. The EoS used in the present study in pressure vs energy density. The color scale refers to the parameter $\xi_{\omega\omega}$. At low densities the DDME2 EoS is represented followed by the hadron-quark phase transition at constant pressure (Maxwell construction). All EoS shown are causal. On the background the contours of the region defined in [52] for the acceptable EoS that interpolate between the neutron matter EoS determined for a chiral effective field theory approach in [52] and the pQCD EoS calculated in [59]. The black dots identify the maximum mass stars.

In order to study the NS properties we have integrated the Tolmann-Oppeheimer-Volkoff (TOV) equations [61] and the tidal deformabilities $\Lambda$ are calculated as in [62]. Fig. 3 shows the $M(R)$ diagram for each hybrid EoS, parametrized by $(\xi_{\omega}, \xi_{\omega\omega})$. For the sake of clarity, we have fixed $\xi_{\omega}$ in each panel: $\xi_{\omega} = 0.0$ (left), $\xi_{\omega} = 0.1$ (center), and $\xi_{\omega} = 0.2$ (right). The color scale encodes the value of $\xi_{\omega\omega}$. The effect of $\xi_{\omega}$ is clear: as its value increases, quarks appear at larger masses, shorter quark star branches are obtained, which reach higher $M_{\text{max}}$. As expected, given that both represent repulsive interactions, $\xi_{\omega\omega}$ shows the same trend as $\xi_{\omega}$. Higher values of $\xi_{\omega\omega}$ originate longer quark branches capable of reproducing more massive NS. The most interesting cases occur for smaller values of $\xi_{\omega}$ and for considerable values of $\xi_{\omega\omega}$, see left and center panels. Under these conditions, quarks are already present inside light NS, $M > 0.9M_{\odot}$, and it is still possible to attain quite massive and compact NS, $M \approx 2.2M_{\odot}$ and $R \approx 11$ km. For $\xi_{\omega\omega} > 10$, hybrid NS with $M > 1.9M_{\odot}$ that predict already some quark content for $M \approx 1.0M_{\odot}$ NS are possible.

We have represented two shaded regions in Figure 3 that indicate the $(M, R)$ constraints obtained by two independent analysis using the NICER x-ray data from the millisecond pulsar PSR J0030+0451 [13, 63]. The set of hybrid EoS in the present work are in good agreement with both constraints.

![Figure 4](image2)

Figure 4. $M(R)$ diagrams for $\xi_{\omega} = 0$ (left), $\xi_{\omega} = 0.1$ (center), and $\xi_{\omega} = 0.2$ (right). The color scale indicates the $\xi_{\omega\omega}$ value and the black line represents the purely hadronic sequence. The bag constant is fixed at $B = 10$ MeV/fm$^3$. The colored regions indicate the $(M, R)$ constraints obtained by two independent analysis using the NICER x-ray data from the millisecond pulsar PSR J0030+0451 [13, 63].

The $\Lambda(M)$ diagrams are shown in Figure 5. Like in Figure 4 we show three panels: $\xi_{\omega} = 0.0$ (left), $\xi_{\omega} = 0.1$ (center), and $\xi_{\omega} = 0.2$ (right). The red dashed line represents the constraint $70 < \Lambda_{1.4M_{\odot}} < 580$ (90% level) obtained from the GW170817 event [64]. We see that, with the combination of low $\xi_{\omega}$ and high $\xi_{\omega\omega}$, it is possible to generate an hybrid EoS that softens the hadronic EoS (solid black line) at low baryonic densities, and satisfies the GW170817 $\Lambda_{1.4M_{\odot}}$ constraint. Another interesting result is that the radius of the heaviest stable NS, $R_{\text{max}}$, is quite sensitive to the $\xi_{\omega\omega}$ value, and it is possible to predict sequences in the $\Lambda(M)$ diagram that clearly deviate from the purely hadronic EoS one. Small values of $\Lambda$ for a low/intermediate mass star could be an important signature indicating the presence of quark matter in NS, which would be accessible through observational results on $(M_i, R_i, \Lambda_i)$.

![Figure 5](image3)

Figure 5. $\Lambda(M)$ diagrams for $\xi_{\omega} = 0$ (left), $\xi_{\omega} = 0.1$ (center), and $\xi_{\omega} = 0.2$ (right). The color scale indicates the $\xi_{\omega\omega}$ value and the black line represents the purely hadronic sequence. The bag constant is fixed at $B = 10$ MeV/fm$^3$. The dashed red line indicates the constraint $70 < \Lambda_{1.4M_{\odot}} < 580$ (90% level) from the GW170817 event [64].

In Figure 6 we show how the central density, $n_{\text{max}}$
at the maximum NS, $M_{\text{max}}$, depends on $(\xi_{\omega}, \xi_{\omega})$. The overall effect of $\xi_{\omega}$ is to decrease the central density of $M_{\text{max}}$, while $\xi_{\omega}$ shows a clear non-monotonic impact on $n_{\text{max}}$. The maximum value of $n_{\text{max}}$ is reached for $\xi_{\omega} = 0$ and $\xi_{\omega} \approx 11$. This is already seen in Figure 4 (left panel), where the $R_{\text{max}}$ shows a non-monotonic behavior: it increases up to $\xi_{\omega} = 10$ and then starts to decrease for higher $\xi_{\omega}$ values. Since the onset of the s-quark occurs at $\approx 0.5 \text{ fm}^{-3}$ independently of the vector interaction, as we have seen before, we conclude that all stars have some fraction of s-quarks. However, if $\xi_{\omega} > 0.1$ the amount of strangeness is quite small. This behavior has also been found in hadronic matter with hyperons: if the coupling to the vector mesons is strong the strangeness content of the star is small [65,66]. It is interesting, however, to realize that the 8-quark term stiffens the EoS but still allows very large central baryonic densities, and, as a consequence, a large strangeness content.

Figure 6. Central density at the maximum NS mass, $n_{\text{max}}$ [in units of saturation density, $n_0 = 0.155 \text{ fm}^{-3}$], as a function of both $\xi_{\omega}$ and $\xi_{\omega}$. The dashed lines represent the value of the maximum NS mass [in $M_\odot$] reached by each hybrid EoS, defined by $(\xi_{\omega}, \xi_{\omega})$.

In Figure 7, we display the speed of sound squared, $v_s^2$, attained at the central density of the heavier NS ($M_{\text{max}}$) for each hybrid EoS, i.e., $v_s^2(n_{\text{max}})$, which is a function of $(\xi_{\omega}, \xi_{\omega})$. $v_s^2$ is very sensitive to $\xi_{\omega}$ and is only slightly affected by $\xi_{\omega}$.

Let us now analyze how the quark core size depends on $(\xi_{\omega}, \xi_{\omega})$. Figure 8 displays both the mass of the quark core, $M_{\text{QC}}$ (right panel), and the radii, $R_{\text{QC}}$ (left panel), as a function of $(\xi_{\omega}, \xi_{\omega})$. We further indicate the maximum mass reached by each hybrid stars through contour lines as before (black dashed lines). For a fixed $\xi_{\omega}$ value, $M_{\text{QC}}$ increases with $\xi_{\omega}$, reaching a heavier quark core for low $\xi_{\omega}$ and high $\xi_{\omega}$. This is precisely when the central density is the largest. On the other hand, for a fixed $\xi_{\omega}$ value, $M_{\text{QC}}$ decreases as the value of $\xi_{\omega}$ gets bigger. Therefore, the extremes of $M_{\text{QC}}(\xi_{\omega}, \xi_{\omega})$ lie in opposite regions: the lighter quark core, $M \approx 0.8 M_\odot$, is found for $(\xi_{\omega} = 0.2, \xi_{\omega} = 0)$ while the heavier, $M \approx 1.8 M_\odot$, is generated for $(\xi_{\omega} = 0, \xi_{\omega} = 20)$. Actually, a quark core of $M \approx 1.8 M_\odot$ is generated in a region where $M_{\text{max}} \approx 2.1 M_\odot$, showing that 85% of the star has quark degrees of freedom. Even though $R_{\text{QC}}$ displays a similar trend as $M_{\text{QC}}$, there is a greater sensitivity to $\xi_{\omega}$ than $\xi_{\omega}$. Even for low $\xi_{\omega}$ values, the quark core radii can reach values as high as 9 km, although two solar mass stars are not attained for these values. The contour lines representing $M_{\text{max}}$ reflect a much stronger dependence on $\xi_{\omega}$ than on $\xi_{\omega}$.

Figure 7. Speed of sound at the central density of the most massive stable NS, $v_s^2(n_{\text{max}})$, as a function of both $\xi_{\omega}$ and $\xi_{\omega}$. The dashed lines represent the value of the maximum NS mass [in $M_\odot$] reached by each hybrid EoS, defined by $(\xi_{\omega}, \xi_{\omega})$.

Figure 8. The quark core mass $M_{\text{QC}}$ [in $M_\odot$] (left) and radii $R_{\text{QC}}$ [in km] (right) as a function of both $\xi_{\omega}$ and $\xi_{\omega}$. The dashed lines represent the value of the maximum NS mass [in $M_\odot$] reached by each hybrid EoS, defined by $(\xi_{\omega}, \xi_{\omega})$.

IV. CONCLUSIONS

In this work we have analyzed the effect of 4-quark and 8-quark vector-isoscalar interactions in hadron-quark hybrid EoS within the three flavor NJL model. Each hybrid EoS consists of charge-neutral matter in $\beta$–equilibrium, in which a first-order phase transition from hadronic to quark matter is present. We have analyzed how the stability of hybrid stars sequences and their properties
depending on the four and eight vector-isoscalar couplings, $\xi_\omega = G_\omega/G_S$ and $\xi_{\omega\omega} = G_{\omega\omega}/G_S^2$.

From the density dependence of the speed of sound of quark matter, one clearly recognizes the stiffening effect of both interactions. This behavior imprints interesting features in the sequences of stable star in the $M(R)$ diagram. We show that the size of the quark star branch is quite sensitive to both couplings, particularly to the $\xi_{\omega\omega}$ coupling. With a small value for $\xi_\omega$, there is a range of $\xi_{\omega\omega}$ values that predict quark matter in light NS, $\sim 1M_\odot$, and, at the same time, are able to sustain a quark core in quite massive NS, i.e., $\sim 2.1M_\odot$. Furthermore, the radii of the heaviest stable NS, $R_{\text{max}}$, is highly dependent on the strength of $\xi_{\omega\omega}$, leading to a considerable decrease of $R_{\text{max}}$ as the coupling increases. As a consequence, for a hybrid EoS a considerable deviation from the purely hadronic matter EoS prediction for the tidal deformability $\Lambda(M)$ is obtained. This occurs even for moderate NS masses, $\sim 1.4M_\odot$, in accordance with the astrophysical constraints from NICER and LIGO/Virgo observations.

We have also discussed how the size of the quark core depends on $\xi_\omega$ and $\xi_{\omega\omega}$. We have concluded that, for a fixed $\xi_\omega$ value, $M_{\text{QC}}$ increases with $\xi_{\omega\omega}$. While lighter quark cores, $\sim 0.8M_\odot$, are predicted for ($\xi_\omega = 0.2, \xi_{\omega\omega} = 0$), the heaviest cores, $\sim 1.8M_\odot$, are generated in the opposite regime, i.e., ($\xi_\omega = 0, \xi_{\omega\omega} = 20$). Quite massive quark cores, $\sim 1.8M_\odot$, are predicted for hybrid EoS in each $M_{\text{max}} \approx 2.1M_\odot$, showing that there are quark degrees of freedom in 85% of the star.

Concerning the conclusions drawn in [53], we obtain some similar results, in particular, we are able to describe two solar mass stars with a central speed of sound squared below 0.4, but more massive stars require larger central values for the speed of sound. However, some other aspects in our study differ from the ones discussed in [53]. We have obtained low mass stars with a quark core, and we can describe very massive stars with large quark cores and a speed of sound far from the conformal limit. This is also in divergence with the conclusions drawn in [14] because we were able of getting large quark cores even with a high central speed of sound, and the reason is that the model used to perform our study allows for a density dependent speed of sound, with a non-linear density dependence.

A low mass NS with a quark core would be confirmed if together with the BNS tidal deformability and mass, also the dominant post-merger GW frequency f peak would be measured. In [67] it was shown that this frequency would identify a first-order phase transition. In the presence of a first order phase transition the f peak comes at a much larger frequency: the larger the baryonic density gap at the phase transition the larger the frequency.

V. ACKNOWLEDGMENTS

This work was partially supported by national funds from FCT (Fundação para a Ciência e a Tecnologia, I.P., Portugal) under the IDPASC Ph.D. program (International Doctorate Network in Particle Physics, Astrophysics and Cosmology), with the Grant No. PD/-BD/128234/2016 (R.C.P.), under the Projects No. UID/FIS/04564/2019, No. UID/04564/2020, and No. POCI-01-0145-FEDER-029912 with financial support from Science, Technology and Innovation, in its FEDER component, and by the FCT/MCTES budget through national funds (OE).

[1] N. Glendenning, Compact Stars: Nuclear Physics, Particle Physics and General Relativity, Astronomy and Astrophysics Library (Springer New York, 2012).

[2] J. Antoniadis et al., Science 340, 6131 (2013) arXiv:1304.6875 [astro-ph.HE].

[3] H. T. Cromartie, E. Fonseca, S. M. Ransom, P. B. Demorest, Z. Arzoumanian, H. Blumer, P. R. Brook, M. E. DeCesar, T. Dolch, J. A. Ellis, R. D. Ferdman, E. C. Ferrara, N. Garver-Daniels, P. A. Gentile, M. L. Jones, M. T. Lam, D. R. Lorimer, R. S. Lynch, M. A. McLaughlin, C. Ng, D. J. Nice, T. T. Pennucci, R. Spiewak, I. H. Stairs, K. Stovall, J. K. Swiggum, and W. W. Zhu, Nature Astronomy 4(39) (2019) arXiv:1904.06759 [astro-ph.HE].

[4] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 119, 161101 (2017) arXiv:1710.05832 [gr-qc].

[5] B. A. et al. (The LIGO Scientific Collaboration and the Virgo Collaboration) (The LIGO Scientific Collaboration and the Virgo Collaboration), Phys. Rev. Lett. 121, 161101 (2018).

[6] B. P. Abbott et al. (LIGO Scientific, Virgo, Fermi-GBM, INTEGRAL, IceCube, AstroSat Cadmium Zinc Telluride Imager Team, IPN, Insight-Hxmt, ANTARES, Swift, AGILE Team, 1M2H Team, Dark Energy Camera GE-WM, DES, DLT40, GRAWITA, Fermi-LAT, ATCA, ASKAP, Las Cumbres Observatory Group, OzGrav, DWF (Deeper Wider Faster Program), ACT3, CAASTRO, VINROUGE, MASTER, J-GEM, GROWTH, JAGWAR, CaltechNRAO, TTU-NRAO, NuSTAR, Pan-STARRS, MAXI Team, TAC Consortium, KU, Nordic Optical Telescope, ePESSTO, GROND, Texas Tech University, SALT Group, TOROS, BOOTES, MWA, CALET, IKE-GW Follow-up, H.E.S.S., LOFAR, LWA, HAWC, Pierre Auger, ALMA, Euro VLBI Team, Pi of Sky, Chandra Team at McGill University, DFN, ATLAS Telescopes, High Time Resolution Universe Survey, RIMAS, RATIR, SKA South Africa/MeerKAT), Astrophys. J. 848, L13 (2017) arXiv:1710.05833 [astro-ph.HE].

[7] D. Radice, S. Bernuzzi, W. Del Pozzo, L. F. Roberts, and C. D. Ott, Astrophys. J. 842, L10 (2017) arXiv:1612.06429 [astro-ph.HE].

[8] D. Radice, A. Perego, F. Zappa, and S. Bernuzzi, Astrophys. J. 852, L29 (2018) arXiv:1711.03647 [astro-ph.HE].
[64] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. 121, 161101 (2018), arXiv:1805.11581 [gr-qc].

[65] S. Weissenborn, D. Chatterjee, and J. Schaffner-Bielich, Proceedings, 11th International Conference on Hypernuclear and Strangeness Physics (HYP 2012): Barcelona, Spain, October 1-5, 2012, Nucl. Phys. A914, 421 (2013).

[66] M. Oertel, C. Providência, F. Gulminelli, and A. R. Raduta, J. Phys. G42, 075202 (2015).

[67] A. Bauswein, N.-U. F. Bastian, D. B. Blaschke, K. Chatziioannou, J. A. Clark, T. Fischer, and M. Oertel, Phys. Rev. Lett. 122, 061102 (2019), arXiv:1809.01116 [astro-ph.HE].