Modeling and analysis of the process of forming a vertical tether group of nano-satellites

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Abstract. The task of forming a tether group of nano-satellites in a circular orbit near the Earth is considered. The tether system consists of the basic spacecraft and several nanosatellites. Nominal programs for the deployment of the tether system in the vertical position according to the simplified equations of motion are constructed. The feasibility of the obtained programs for the development of the tether system for a more complex model of motion is confirmed. Example of forming a tether system consisting of the basic spacecraft and ten nano-satellites is provided.

1. Introduction
Prospectivity of the use of orbital tether technologies is not in doubt because of the possibility of lightweight long structures in space. In recent times, there are many projects of using tether groups of small spacecraft (SC) and nanosatellites (NS). Making agreed flight in orbit, these space systems can have many useful applications [1-5]. These are various measuring long systems (tens and even hundreds of kilometers) for monitoring of gravitational and magnetic fields, the upper layers of the atmosphere; systems of remote sensing of the Earth with a large base; near and far space observation systems; space debris collection systems covering a fairly large altitude range of orbits, etc.

Space tether system consisting of a main SC, which allows the release of tether, and several satellites connected by tethers in series to each other is considered. Two schemes for the formation of the orbital tether system (OTS) in vertical position are available. The first scheme - the scheme of "fast" deployment of TSS is characterized by a shorter time of system formation, but requires more complex control. In this case, it is necessary to ensure the separation of each NS in a given direction relative to the local vertical and at a given speed. The second scheme - the scheme of "slow" deployment of OTS is based on the separation of the process of the formation of OTS in separate stages. After each stage, the OTS takes the position close to vertical. In this case, the separation of each satellite occurs in a direction close to the local vertical.

The proposed nominal control programs transfer the system to a given vertical equilibrium position. These control programs are a generalization of similar programs [1, 6-8] to the case of vertical group of satellites.

Nominal program for the second scheme of the formation of OTS by the method of Lagrange mathematical model of motion of a chain of series-connected material points in the mobile orbital coordinate system is developed. The resulting model for any finite number of satellites can be used.
Implementation of the proposed programs of formation of OTS on a more complete model of the motion in the geocentric coordinate system is checked. This model takes into account the elasticity of the tethers, the functioning of the control mechanism, its inertia and the change in the orbit parameters of the center of mass of the system. In all used models of the motion of the tethers connecting the satellites, is considered weightless. The control mechanism operates in accordance with the principle of feedback, using the measurements of length and speed of tether.

2. The mathematical model and the program of "fast" deployment of OTS

Nominal program of "fast" deployment of OTS is built with the use of a mathematical model for an inelastic tether from the article [7] for the case when the mass of the main SC is a lot more mass of the rest of the OTS. This model is written for a system consisting of a main and a small spacecraft, taking into account the mass of the tether. To use this model proposes to build a nominal program to make the transition from a discrete chain of point masses of the NS to a continuous distribution of mass on the tether. In this case, the tether with increased linear density, equivalent to the total mass of all internal points of the chain, is obtained. The correctness of this approach is checked by mathematical modeling of the motion of OTS in the geocentric coordinate system using the discrete model of point-masses.

Let \( n \) and \( L_f \)–the number of space vehicles (NS plus main SC) and the total length of the tether. In this case, if \( m \) – the mass of one NS (with the exception of the lower NS), then the equivalent linear density of the tether is obviously equal to \( \rho = m(n-2)/L_f \). Then the mathematical model of the flat motion of the system and the nominal dynamic control program will have the form

\[
(m_n + \rho L)\ddot{L}_p = (m_n + \rho L_n / 2)\dot{L}_p F_{11} - T - \rho \dot{L}_p^2 \quad (1)
\]

\[
(m_n + \rho L / 3)\dot{L}_p^2 \ddot{\theta} = -2(m_n + \rho L / 2)\dot{L}_p F_{21} + (m_n + \rho L / 3)\dot{L}_p F_{22} \quad (2)
\]

\[
T = (m_n + \rho L / 2)\Omega^2 \left[ a(L_p - L_f) + bL_p / \Omega + 3L_f \right] \quad (3)
\]

where \( a, b \) – parameters of program, \( \Omega \) – the angular velocity of the motion of the center of mass of the system in the orbit, \( L_p \) – nominal (program) length of tether, \( \theta \) – the deviation angle of tether from vertical, \( T \) – the nominal force of the tether tension, \( L_f \) – final length of tether, \( F_{11} = \dot{\theta}^2 + 2\Omega \ddot{\theta} + 3\Omega^2 \cos^2 \theta \), \( F_{21} = \dot{\theta} + \Omega \), \( F_{22} = -1.5\Omega^2 \sin 2\theta \), \( m_n \) – mass of lower NS, \( \dot{L}_p = dL_p / dt \), \( \ddot{L}_p = d^2L_p / dt^2 \) etc. It is assumed that the mass of the lower NS can differ from that of other NS's. This is due to the fact that too low mass the NS can lead respectively to small tether tension immediately after separation of the satellite from the main SC, which will lead to increased demands for the control mechanism. At construction of mathematical model (1), (2) it is assumed that the parameters of the orbit of the center of mass of the system during its deployment does not change and the length of the tether is much smaller than the distance of the centre of mass of system from the attracting centre.

Using the technique of [7], it can be shown that if in the program the parameters \( a > 3 \) and \( b > 0 \), then in accordance with the model (1), (2) the vertical equilibrium positions \( \theta = 0 \) and \( \theta = \pi \) (\( \dot{\theta} = \ddot{L}_p = \dot{\theta} = \ddot{L}_p = 0 \), \( L_p = L_f \neq 0 \)) are asymptotically stable. In addition, if \( b > b_* \), where \( b_* \)–a certain critical value, inequalities \( L_p < L_f \), \( \dot{L}_p > 0 \) are met.

3. Mathematical model of motion of OTS in geocentric coordinate system

The mathematical model of motion of OTS in geocentric coordinate system is a set of material points connected among themselves by elastic one-sided mechanical links

\[
m_i \ddot{r}_i = G_i + T_i - T_{i+1}, \quad i = 1, 2, ..., n
\]
where $\mathbf{r}_i$ and $m_i$ – radius-vector and mass of the material point with number $i$; $T_i$ – the tensioning force acting between the material points, $m_i$ – mass of the main spacecraft. The gravitational forces $\mathbf{G}_i$, here correspond to the Central Newtonian field. Aerodynamic forces and dissipative forces inside the tether are not taken into account. When writing the equations of motion (4), it is assumed that the main plane of the right geocentric coordinate system $OXYZ$ coincides with the plane of the orbit of the center of mass of the OTS, which is stationary. The axis $OX$ is directed along the line of the orbit nodes (for the equatorial orbit to the point of the spring equinox).

The tension forces are determined by Hooke’s law taking into account the one-sidedness of mechanical bonds

$$T_i = \begin{cases} c(\gamma_i - 1)\Delta L_i / \Delta L_{0i}, & \text{if } \gamma_i \geq 1 \\ 0, & \text{if } \gamma_i < 0 \end{cases}$$

where $c$ – the stiffness coefficient of the tether, $\gamma_i = \Delta L_i / \Delta L_{0i}$, $\Delta L_i = |\mathbf{r}_{i+1} - \mathbf{r}_i|$ and $\Delta L_{0i}$ – deformed and undeformed lengths of elementary parts of the tether, $\Delta L_i = \mathbf{r}_{i+1} - \mathbf{r}_i$.

The dynamics of the control mechanism is described by the following approximate equations [9]

$$\mu \dot{L}_i = T_i - F_c$$

where parameter $\mu$ takes into account the inertia of the control mechanism, $L$ – the unstretched length of the tether, $F_c$ – friction force in the brake mechanism, $T_i$ – the tension force on the first part of the tether, counting from the main spacecraft. In modeling the movement of the parameter $\mu$ is not changed.

The control force is calculated according to the feedback principle

$$F_c = T + p_L (L - \dot{L}_p) + p_V (\dot{L} - \dot{L}_p)$$

where $T$ – nominal tension force (3), $p_L$ and $p_V$ – the feedback coefficients, $L$ and $\dot{L}_p$ – nominal values calculated from the model (1), (2); $L$ and $\dot{L}$ – the length and speed of the tether defined when integrating the system (4) together with the equation (6). The control force $F_{min}$ is limited to a certain value, which characterizes the brake mechanism.

When separating each NS, the speed of the main SC is corrected on the basis of the law of conservation of momentum of the system and the speed of the NS in the geocentric coordinate system is calculated

$$V_i = V_i - \frac{m_i}{m} V_{ri}, \quad V_i = V_i + V_{ri}$$

where $V_i$ and $V_i$ – speed main SC before and after the separation of the NS, $m_i$ – the mass of the main SC after separation of the NS, $V_{ri}$ – the relative speed of separation NS with number $i$.

When separating the first NS, the relative speed $V_{r1}$ ($i = n - 1$) is selected based on the provision of the necessary initial tension of the tether and is directed downwards along the local vertical line. For other satellites (to reduce disturbances at their separation), vectors of the relative velocities should correspond to the selected program for the deployment of the OTS.

When separating, the center of mass of the NS with the number $i$ is located at some distance from the center of mass of the main SC. The scheme of the separation, starting from the second satellite, is shown in figure 1.

According to figure 1, the position of the NS with number $i$ is defined as follows: $\Delta \mathbf{r}_i = l \mathbf{r}_{i+1}$, where $\mathbf{r}_{i+1} = \Delta \mathbf{r}_{i+1} / |\Delta \mathbf{r}_{i+1}|$, $\Delta \mathbf{r}_{i+1} = \mathbf{r}_{i+1} - \mathbf{r}_i$.

The required relative speed $V_{qi}$ is determined by the relative speed NS with number $i + 1$ from the corresponding proportion drawn up relative to the center of mass of the main SC under the assumption that in the nominal case, all vectors depicted in figure 1 are in the same plane. Then
The considered algorithm corresponds to the nominal conditions of motion and does not take into account the possible disturbances in the separation of the NS. In addition, this algorithm requires the rotation of the device from which the NS come out, which naturally complicates its design.

4. Mathematical model and program of "slow" deployment of OTS

The second scheme of deployment of the OTS consists of separate stages, at each of which the system is brought into a vertical position. Therefore, when building a mathematical model of movement, a case is considered when the system already includes \( n-k \) NS, where \( k \) is the number of NS closest to the main SC (figure 2). In this case, the control program should ensure that the OTS is brought into a vertical position when the following NS is added to the system.

Construction of a mathematical model is carried out in the orbital mobile right geocentric coordinate system \( O_{x_0}y_0z_0 \), where axis \( O_{x_0} \) is directed along the radius-vector main SC, axis \( O_{y_0} \) in the direction of orbital motion of the main SC.

Construction of the model is carried out taking into account the following assumptions: 1) the mass of the main spacecraft is much larger than the mass of the other elements of the OTS; 2) the center of mass of the main SC coincides with the center of mass of the OTS and moves along an unchanged unperturbed circular orbit; 3) satellites when deployment of the system are arranged on the same straight line; 4) the tethers connecting the individual elements of the OTS (material points) are inextensible and always stretched. The validity of the above assumptions is tested in the modeling of motion of the OTS in the geocentric coordinate system in accordance with equations (4), taking into account the extensibility of the tether and the operation of the stabilization system (7).

![Figure 1. Scheme of separation main SC and NS.](image1)

![Figure 2. Position of the satellites relative to the orbital coordinate system \( O_{x_0}y_0z_0 \).](image2)

For construction of the equations of plane motion of OTS Lagrange’s equations are used

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} = -\frac{\partial P}{\partial \dot{q}_j} + Q_j, \quad q_1 = s, \quad q_2 = \theta
\]  

(10)

where \( T \) and \( P \) – kinetic and potential energy of the system, \( q_j \) and \( \dot{q}_j \) \((j = 1,2)\) – generalized coordinates and velocities, \( \theta \) –the angle of deviation of the chain of material points from the vertical, \( Q_j \) – generalized forces for non-potential forces. The position of the orbital coordinate system \( O_{x_0}y_0z_0 \) relative to the fixed coordinate system \( OXYZ \) is determined by the angle \( u \).

The figure 2 shows the generalized coordinates \( s, \theta \) and accepted numbering of material points. Then the coordinates of the NS in the coordinate system \( OXYZ \) are determined by the expressions

\[
x_j = x_{ai} \cos u - y_{ai} \sin u, \quad y_j = x_{ai} \sin u + y_{ai} \cos u
\]  

(11)

where \( x_{ai} = r_i - \left[ s + (i-k)\Delta L \right] \cos \theta, \quad y_{ai} = -\left[ s + (i-k)\Delta L \right] \sin \theta, \quad \Delta L = L_j / (n-1), \quad i = k, k+1,...n \).
The kinetic and potential energies included in the equations (10) are determined by summing the corresponding expressions for the individual material points of the system

\[ T = \frac{1}{2} \sum_{i=k}^{n} (x_i^2 + y_i^2), \quad P = \sum_{i=k}^{n} P_i \]

where \( P_i = -Km_i / r_i, \quad r_i = \left[ x_i^2 + \left[ s + (i-k)\Delta L \right]^2 - 2n\left[ s + (i-k)\Delta L \right]\cos\theta \right]^{1/2} \)

Differentiating \( P_i \) by the generalized coordinates, we obtain

\[ \frac{\partial P_i}{\partial s} = K\left[ s + (i-k)\Delta L - r_i \cos\theta \right] / r_i^3, \quad \frac{\partial P_i}{\partial \theta} = Kr_i \sin\theta \left[ s + (i-k)\Delta L \right] / r_i^3 \]

Assuming that the distance \( r_i \) is much larger than the length of tether, we present the function \( 1 / r_i^3 \) as (given the first two terms of the series)

\[ \frac{1}{r_i^3} = \frac{1}{r_i^3} \left[ 1 + 3s + \left( i-k \right)\Delta L \cos\theta + \ldots \right] \]

For this task, only the non-potential force \( T \) generated by the brake mechanism is taken into account. This force is directed along the tether. Therefore \( Q_1 = -T, \quad Q_2 = 0 \). Further, substituting the expression (11-14) in the Lagrange equation (10) and conducting differentiation of the, we obtain

\[ \ddot{s} = m_i(s)\left( \Omega + \dot{\theta} \right)^2 + m_s(s)\Omega^2 \left( 3\cos^2\theta - 1 \right) - T / M_k \]

\[ \ddot{\theta} = -2m_i(s)\dot{s}(\Omega + \dot{\theta}) / J_k(s) - 1.5\Omega^2 \sin 2\theta \]

where \( M_k = m(n-k) + m, \quad m_i(s) = m_s\left[ s + (n-k)\Delta L \right] + m \sum_{i=k}^{n-1} \left[ s + (i-k)\Delta L \right] \)

\[ J_k(s) = m_s\left[ s + (n-k)\Delta L \right]^2 + m \sum_{i=k}^{n-1} \left[ s + (i-k)\Delta L \right]^2 \]

In equations (15), (16), it is assumed that all NS (except for the satellite with mass \( m_n \)) have the same mass, that is \( m_i = m, \quad i = k, \ldots, n-1 \). The parameter \( J_s \) represents the total moment of inertia of all NS with respect to the main SC. Equations (15), (16) include a set of models, each of which describes the movement of the OTS in the case when from the base spacecraft separated \( n - k + 1 \) NS, including the first satellite with mass \( m_n \).

Using the method of building the program (3) [7], it is easy to obtain a similar program for deploying system in a vertical position corresponding to the model (15), (16), and with a length of tether \( \Delta L \) at one stage of the deployment of system. Then

\[ T = m_i(\Delta L)\Omega^2 \left[ a(s - \Delta L) + b\dot{s} / \Omega + 3\Delta L \right] \]

where \( m_i(\Delta L) = m_n(n+1-k) + m \sum_{i=k}^{n-1}(i+1-k) \)

The program (17) has the same properties as the program (3): 1) equilibrium position \( s = \Delta L, \dot{s} = \theta = \dot{\theta} = 0 \) (or \( \theta = \pi \)) asymptotically stable, if \( a > 3, b > 0, \) such that all eigenvalues of the linearized system with respect to this equilibrium position are real; 2) with the increase of the value \( b \), the transition time increases. The second property leads to the fact that when approaching to the position of equilibrium inequalities \( s < \Delta L, \dot{s} > 0 \) are performed. In this case, the process of deployment of the OTS can be implemented using a control mechanism that works only on the braking of the tether.
5. The results of numerical simulation

To simulate the deployment of OTS, the following data is considered:

- Length of tether \( L_f = 10 \text{ km} \)
- Mass of each NS \( m = 2 \text{ kg} \)
- Mass of main SC \( m_i = 6000 \text{ kg} \)
- Stiffness of the tether \( c = 7000 \text{ N} \)
- Initial height of the circular orbit \( 300 \text{ km} \)
- Speed of separation of the first NS \( 2 \text{ m/s} \)
- Mass of tether \( \mu = 0.2 \text{ kg} \)

For the implementation of the first scheme of deployment of the OTS, two variants were considered: 1) the ideal separation of each NS with the necessary speed in a given direction in accordance with the algorithm from section 3; 2) the separation of the NS in the vertical direction, when the necessary direction of the relative speed is not implemented. The parameters of the nominal control program \( a = 4, b = 3.9 \). The simulation was carried out using a model in a geocentric coordinate system. The nominal values were calculated using the simplified model. If the ideal separation of each NS without perturbations is realized, then the final errors are minimal: errors in length and speed do not exceed, respectively, 0.1 m and 0.01 m/s. Maximum control errors occur during the separation of the first NS, but decrease rapidly after the transition process. During the deployment, satellites are located approximately on one straight line, the tethers between the NS are stretched, and the tension forces change quite smoothly. The position changes if all satellites are separated vertically, that is, the necessary direction of separation is not realized. In this case, the large perturbations of all the characteristics of the movement of OTS take place at the moment of separation of each NS. The maximum error of regulation arise in the separation of the seventh NS and they are equal respectively 7.7 m and 0.7 m. At the end of the deployment of OTS 0.1 m/s and 0.4 m. In this case, when the NS is separated, there are large jumps in the tension force and a short-term weakening of the tether. Maximum values of the tension force is about 13.5 N, although the nominal values do not exceed 0.7 N. In addition, the chain of NS is no longer on a straight line, but has a more complex shape. Figure 3 shows the position of the NS relative to the main SC (origin) at the time of separation of the next satellite, starting from the third, and in the final position of the system.

At realization in the second scheme of deployment, each phase of deployment (except the first) consists of fast speed-up and braking (figure 4(a)). After deployment of the system with following satellite, the system is placed in a position close to the vertical position. Further, the tether tension is loosened and the next stage of deployment is implemented (figure 4). After each stage of deployment, the mass characteristics of the OTS change and the new equilibrium position becomes asymptotically stable (in accordance with the nominal movement). However, if the parameter is \( \Delta L = L_f / (n - 1) \) in the deployment program, this significantly increases the deployment time at each stage. This is due to the fact that when the system (15), (16) approaches the asymptotically stable equilibrium position, the motion of the OTS slows down, since the system (15), (16) reaches the equilibrium position.
position only when $t \to \infty$. To avoid this effect, you can, for example, either end the deployment of each stage at a shorter length of tether than $\Delta L$, or set in the program (17) to a value $\Delta L + \varepsilon$, where $\varepsilon > 0$ - some small value. In the latter case, each stage ends when the length of the tether reaches $\Delta L$.

The analysis positions of the tether in the second scheme of the deployment of system shows that in the process of the formation of system all satellites at all stages of deployment are located practically on one line and are moving along similar trajectories. For the second scheme of deployment of the OTS, the same data was used as for the first scheme with the addition of program parameters (17): $a = 4$, $b = 10$, $\varepsilon = 50$ m. In the last stage, the parameter $b = 20$ to provide the restriction $\dot{s} > 0$ when the system approaches to the equilibrium position. Maximum control errors $-0.14$ m/s and $0.2$ m, at the end of the deployment - $0.05$ m/s and $0.06$ m.

![Figure 4](image)

**Figure 4.** The speed of the tether ($s$) and the tension force ($T_i$) near the SC (second stage of deployment), where $\tau$ - the number of turns in orbit.

When implement the second deployment scheme, the time to bring the system to the specified state is increased by approximately 4 times. The peculiarity of the second scheme of formation of the OS and the program (17) is that at the first stage of deployment immediately after the separation tension force of the tether are small and, accordingly, small values of the control force are required. In the example above $F_{\text{min}} = 0.001$ N, which leads to increased requirements for the control mechanism. Apparently, the only way to avoid this is to increase the mass of the first satellite, which is separated from the main SC. For example, if to increase the mass of the first satellite three times $m_s = 12$ kg, then $F_{\text{min}} = 0.008$ N, and it will be enough to ensure the stability of transients in the separation of objects.

6. References

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