Transverse muon polarization in $K^{\pm} \rightarrow \pi^0 \mu^{\pm} \nu$ decay induced by the two-photon final-state interaction

V.P. Efrosinin$^a$, I.B. Khriplovich$^{b,c}$, G.G. Kirilin$^{b,c}$, Yu.G. Kudenko$^a$

$^a$Institute for Nuclear Research, 117312 Moscow, Russia
$^b$Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia
$^c$Novosibirsk University, 630090 Novosibirsk, Russia

Abstract

We calculate the transverse muon polarization in the decay $K^{\pm} \rightarrow \pi^0 \mu^{\pm} \nu$ induced by the electromagnetic two-photon final-state interaction. For the central part of the Dalitz plot the typical value of this polarization is about $4 \times 10^{-6}$.

1 Introduction

Measurement of the muon transverse polarization $P_T$ in the decay $K^{\pm} \rightarrow \pi^0 \mu^{\pm} \nu$ ($K_{\mu3}$) can provide important insight to new physics beyond the Standard Model (SM). In the case of $K_{\mu3}$ decay, $P_T$ is a T-odd observable $s_\mu \cdot (p_\pi \times p_\mu)$ determined by the momenta $p_\pi$ of the $\pi^0$, $p_\mu$ of the $\mu$ and the spin $s_\mu$ of the $\mu$. This observable is very small in the SM, but it is an interesting probe of non-SM CP-violation mechanisms [1] where $P_T$ can be as large as $10^{-3}$ in either $K_{\mu3}$ or $K^{\pm} \rightarrow \mu^{\pm}\nu\gamma$ ($K_{\mu2\gamma}$). The recent measurement of $P_T$ in the $K^+ \rightarrow \pi^0 \mu^+ \nu$ decay provided by KEK E246 [2] gives a value of $P_T$ consistent with zero with experimental error $\sigma \sim 5 \times 10^{-3}$ and will be further improved by a factor of 4. The first limit on $P_T$ in $K_{\mu2\gamma}$ will also be obtained in this experiment. Proposed experiments [3] could reach sensitivity to $P_T$ of $\leq 10^{-4}$.

In fact, whether CP or T is violated or not, a nonvanishing $P_T$ in $K_{\mu3}$ decays can be induced by electromagnetic final-state interactions (FSI). In the neutral kaon decays this correlation arises due to the imaginary part of the diagram in Fig. 1, and can be as large as $\sim 10^{-3}$ [4, 5]. In this sense, $K^{\pm} \rightarrow \pi^0 \mu^{\pm} \nu$ decays have a major advantage: here the final pion is neutral, and thus there is no elastic electromagnetic FSI. The single-photon contribution to $P_{Te}^{\pm}$ arises in these decays due to the imaginary part of the two-loop diagram shown in Fig. 2, and was estimated as $P_{Te}^{\pm} \lesssim 10^{-6}$ [6]. Such a strong suppression of this imaginary part is caused by the small value of the phase space of the three-particle intermediate state.

In the present work we demonstrate that the two-photon effects result in a larger contribution to the transverse muon polarization in the $K^{\pm} \rightarrow \pi^0 \mu^{\pm} \nu$ decays.

Let us note that the FSI contribution to the muon transverse polarization in the $K^+ \rightarrow \pi^0 \mu^+ \nu$ and $K^- \rightarrow \pi^0 \mu^- \bar{\nu}$ decays is the same: $P_{Te}^{m+} = P_{Te}^{m-}$, while the CP-odd contributions are of opposite signs: $P_{TcP+} = -P_{TcP-}$. To be definite, we present all intermediate formulae for the $K^- \rightarrow \pi^0 \mu^- \bar{\nu}$ decay.

For the $K_{\mu3}$ decay, the most general invariant amplitude is

$$M_{K_{\mu3}} = -\frac{G_F}{\sqrt{2}} V_{us} \frac{1}{\sqrt{2}} \left[(p_K + p_\pi)_{\mu} f_+(t) + (p_K - p_\pi)_{\mu} f_-(t)\right] \bar{u}(p_\mu) \gamma_\mu (1 + \gamma_5) v(p_\nu).$$

(1)
Here $G_F$ is the Fermi constant; $V_{us}$ is the element of the Cabibbo-Kobayashi-Maskawa matrix; $f_+(t)$ and $f_-(t)$ are form factors, $p_K$, $p_\pi$, $p_\mu$, and $p_\nu$ are the momenta of the kaon, pion, muon, and antineutrino, respectively; $t = (p_K - p_\pi)^2$ is the momentum transfer squared to the lepton pair. Our convention for $\gamma_5$ is
\[
\gamma_5 = \left( \begin{array}{cc} 0 & -I \\ -I & 0 \end{array} \right).
\tag{2}
\]

Expression (1) can be conveniently rewritten as
\[
M = -\frac{G_F}{\sqrt{2}} V_{us} \sqrt{2} f_+(t) \bar{u}(p_\mu)(\hat{p}_K + \chi \hat{p}_\mu)(1 + \gamma_5)v(p_\nu),
\tag{3}
\]
where
\[
\chi = \frac{1}{2}(\xi - 1), \quad \xi = \frac{f_-}{f_+} = -0.35 \pm 0.15 \ [7].
\tag{4}
\]
The standard parameterization is
\[
f_+(t) = f_+(0) \left( 1 + \lambda \frac{t}{m_\pi^2} \right), \quad \lambda = 0.0286 \pm 0.0022 \ [7].
\tag{5}
\]
Experimental data are compatible with a constant, i.e., $t$-independent, $f_-$. The covariant, 4-dimensional form of the transverse polarization vector is
\[
P_{T\alpha} = -2\text{Im} \chi m_\mu \varepsilon_{\alpha\beta\gamma\delta} p_\mu p_\nu p_\gamma p_\delta / \Phi,
\tag{6}
\]
\[
\varepsilon_{0123} = 1, \quad \Phi = 2(p_\mu p_K)(p_\nu p_K) - m_K^2(p_\mu p_\nu) + 2\chi m_\mu^2(p_\nu p_K) + |\chi|^2 m_\mu^2(p_\mu p_\nu).
\]
The covariant expression for the degree of transverse polarization is

\[ P_T = 2 \text{Im} \chi m_\mu \sqrt{D}/\Phi, \]  

where

\[ D = m_\mu^2 m_\pi^2 m_K^2 + 2(p_\mu p_\pi)(p_\mu p_K)(p_\pi p_K) - m_\mu^2 (p_\pi p_K)^2 - m_\pi^2 (p_\mu p_K)^2 - m_K^2 (p_\mu p_\pi)^2. \]  

Figure 2: The contribution of the two-loop diagram to \( P_T^{\text{em}} \) in \( K^- \to \pi^0 \mu^- \bar{\nu} \) decay. Crosses mark on-mass-shell particles.

### 2 Some details of the calculation

We are interested in the transverse muon polarization due to \( \text{Im} \chi \) induced by the FSI with two photons. Here the transverse muon polarization is proportional to the imaginary parts of diagrams shown in Fig. 2. In diagrams a, c, e, g, h both photons are real and the intermediate muon is off-mass-shell. In diagrams b, d, f the intermediate muon and one photon are on-mass-shell, and the second photon is virtual. In the diagrams, the on-mass-shell intermediate particles are marked by crosses. Among the diagrams, those with both photons attached to the kaon, are absent. Obviously, there is no \( P \)-even effective \( KK\pi \) vertex.

The common vertex for the diagrams \( \mathfrak{b} \), \( \mathfrak{a} \) is the \( \pi^0 \to \gamma\gamma \) amplitude:

\[ M(\pi^0 \to \gamma\gamma) = -\sqrt{2} \frac{\alpha}{\pi f_\pi} \varepsilon_{\alpha\beta\gamma\delta} e_{1\alpha} e_{2\beta} k_{1\gamma} k_{2\delta}, \]

where \( e_{1\alpha}, e_{2\beta} \) are the polarization vectors of the photons, \( k_{1\gamma}, k_{2\delta} \) are their momenta and \( f_\pi = 0.13 \) GeV. We use here the theoretical expression for the coupling constant; the \( \pi^0 \) life time calculated with it reproduces the experimental value within the accuracy of the latter.
Figure 3: The contribution of two-photon final-state interactions to $P_T^{em}$ in $K^- \rightarrow \pi^0 \mu^- \bar{\nu}$ decay. Crosses mark on-mass-shell particles.
The $K^- \rightarrow \mu^- \nu$ vertex in the Inner Bremsstrahlung (IB) diagrams $3a$–d, is

$$M(K_{\mu 2}) = -\frac{G_F}{\sqrt{2}} V_{us} f_K p_{K \mu} \bar{u}(p_\mu) \gamma_\mu (1 + \gamma_5) v(p_\nu),$$  \hspace{1cm} (10)$$

where $f_K = 0.16$ GeV.

The $K^- \rightarrow \mu^- \bar{\nu} \gamma$ vertex in the structure dependent (SD) radiation diagrams in Fig. $3a$–g is described by two independent amplitudes:

$$M(SD_V) = -\frac{G}{\sqrt{2}} V_{us} \sqrt{4\pi \alpha} F_V \varepsilon_{\alpha\beta\gamma\delta} e_{1\alpha} \bar{u}(p_\mu) \gamma_\beta (1 + \gamma_5) v(p_\nu) k_{1\gamma} (p_K - k_1)_\delta;$$ \hspace{1cm} (11)$$

$$M(SD_A) = -\frac{G}{\sqrt{2}} V_{us} \sqrt{4\pi \alpha} (-i) F_A [\delta_{\alpha\beta} (p_K k_1) - (p_K - k_1)_\alpha k_{1\beta}] e_{1\alpha} \bar{u}(p_\mu) \gamma_\beta (1 + \gamma_5) v(p_\nu).$$ \hspace{1cm} (12)$$

Let us note here that when going over from the $K^- \rightarrow \mu^- \bar{\nu} \gamma$ decay to $K^+ \rightarrow \mu^+ \nu \gamma$, not only the lepton current in both amplitudes changes to $\bar{u}(p_\nu) \gamma_\beta (1 + \gamma_5) v(p_\mu)$, but in the second amplitude, $M(SD_A)$, $(-i)$ changes to $i$. Theoretically, the vector form factor $F_V$ is directly related to the $\pi^0 \rightarrow \gamma \gamma$ amplitude [3] (see also [4, 5]). In the case of $\pi \rightarrow e \nu \gamma$ decay one should delete $4\pi \alpha$ from the constant $\sqrt{2\alpha/\pi f_\pi}$ in [8], and then divide by $\sqrt{2}$. In our case of the $K \rightarrow \mu \nu \gamma$ decay one should also change $f_\pi \rightarrow f_K$. Thus we obtain

$$F_V = \frac{1}{4\pi^2 f_K} = \frac{0.079}{m_K}. \hspace{1cm} (13)$$

The value of the axial form factor $F_A$, as predicted by chiral perturbation theory, is [1]

$$F_A = \frac{0.043}{m_K}. \hspace{1cm} (14)$$

The experimental study of the decay $K^+ \rightarrow \mu^+ \nu \gamma$ [2] results in the following information on the corresponding form factors in this decay:

$$-0.04/m_K < F_V - F_A < 0.24/m_K, \hspace{0.5cm} |F_V + F_A| = (0.165 \pm 0.007 \pm 0.011)/m_K.$$  

As to the diagram $3h$ with a double SD radiation, qualitative arguments will be presented below which allow one to neglect it.

A common feature of all diagrams where both intermediate photons are real ($3a$, c, e, g, h), is that their contributions are proportional to $m_\pi^2$. Indeed, in the limit $m_\pi \rightarrow 0$ the 4-momenta of the photons in the $\pi^0 \rightarrow \gamma \gamma$ decay become collinear, and the matrix element (9) vanishes. On the scale of the $K_{\mu 3}$ problem, $m_\pi^2$ is relatively small, and correspondingly, the contributions of diagrams $3a$, c, e, g, h are rather small as well.

On the other hand, there is a certain similarity between contributions of the intermediate states with two real photons and real photon and muon in diagrams of the same structure, i.e., between imaginary parts corresponding to diagrams of Fig. $3$ a and b, c and d, e and f, respectively. Therefore, it is expedient to calculate the respective pairs of imaginary parts in parallel. Strong cancellations occur between the terms proportional to $m_\pi^2$ in diagrams a and b, c and d, e and f. In particular, the most formidable integrals, which arise in diagrams a and b, cancel completely.

Still, the calculation remains quite tedious, and only its final results will be presented.
3 Internal Bremsstrahlung

We start with the contribution of the IB diagrams 3a-d:

\[
2 \text{Im} \chi(\text{IB}) = \frac{\alpha^2 f_K}{4\pi f_\pi} \frac{1}{f_+(t)} \left\{ 4 + 3\chi \right. \\
+ \frac{1}{I^2_{\mu\pi}} \left[ (p_\mu p_\pi) m_{\pi}^2 + \chi m_{\mu}^2 m_{\pi}^2 - \frac{(p_\mu p_\pi) m_{\mu}^2 \chi(p_\pi P)}{p^2} \right] - \frac{2m_{\mu}^2}{p^2} \\
+ \frac{m_{\mu}^2 L_2}{2} \left[ 1 - \frac{m_{\mu}^2}{I^2_{\mu\pi}} ((p_\mu p_\pi) + (1 + \chi)m_{\mu}^2) \right] + \frac{2(1 + \chi)m_{\mu}^2}{m_{\pi}^2 + 2(p_\mu p_\pi)} \\
+ \frac{2(p_\mu p_\pi)(p_k p_\pi) - m_{\pi}^2(p_k p_\mu)}{2D} \left[ L_2 ((p_k p_\pi)(p_\mu p_\pi) - m_{\pi}^2(p_k p_\mu) + \chi I_{\mu\pi}^2 \\
- (1 + \chi)((p_\mu p_\pi)(p_k p_\pi) - m_{\mu}^2(p_k p_\mu)) \right] + L_3 \left( \chi((p_k p_\mu)(m_{\mu}^2 - (p_\mu p_\pi)(p_k p_\pi)) \\
- I_{\mu\pi}^2 - (1 + \chi)((p_k p_\mu)(p_k p_\pi) - m_{\mu}^2(p_k p_\mu)) \right) \\
+ L_4 \left( I^2 + \chi((p_k p_\pi) - m_{\mu}^2)(p_\mu P) - (p_k p_\mu)((p_\pi P) - (p_k P)) \right) \\
\left. \right\} \right].
\]

Here \( \chi = \chi(t) \) is defined in fact by the same relation (13) and does not include \( \text{Im} \chi \) which is being calculated, \( D \) is given in (8), \( P = p_\mu + p_\pi \),

\[
L_2 = \frac{1}{I_{\mu\pi}} \ln \left( \frac{p_\pi P}{(p_\pi P) - I_{\mu\pi}} \right), \quad L_3 = \frac{1}{I_{\pi\pi}} \ln \left( \frac{(p_k p_\pi) + I_{\pi\pi}}{(p_k p_\pi) - I_{\pi\pi}} \right), \quad L_4 = \frac{1}{I} \ln \left( \frac{(p_k P) + I}{(p_k P) - I} \right),
\]

\[
I_{\mu\pi} = \sqrt{(p_\mu p_\pi)^2 - m_{\mu}^2 m_{\pi}^2}, \quad I_{\pi\pi} = \sqrt{(p_k p_\pi)^2 - m_{\pi}^2 m_{\pi}^2}, \quad I = \sqrt{(p_k P)^2 - P^2 m_{\pi}^2},
\]

\[
C = \frac{1}{2} \left\{ L_2 [(p_\mu p_\pi)(p_k p_\mu)m_{\pi}^2 - 2(p_\mu p_\pi)^2(p_k p_\pi) + (p_k p_\pi)m_{\mu}^2 m_{\mu}^2] \\
- (L_3 - L_4) [(p_k p_\pi)(p_\mu p_\pi)m_{\mu}^2 - 2(p_\mu p_\pi)(p_k p_\pi)^2 + (p_\mu p_\pi)m_{\mu}^2 m_{\mu}^2] \\
+ L_4 [(p_\mu p_\pi)^2 m_{k}^2 - m_{\mu}^2(p_k p_\pi)^2 - D] \right\}.
\]

The first line \( 4 + 3\chi \) in braces in (13) is dominating, it constitutes about 70% at the center of the Dalitz plot. The typical value of the IB contribution in this region is

\[
2 \text{Im} \chi(\text{IB}) = 1.4 \times 10^{-5}.
\]
4 Structure-dependent radiation

The vector SD amplitude (11) is operative in all three diagrams 3e–g. Its contribution equals

\[ 2 \text{Im} \chi (SD_V) = \frac{\alpha^2 F_V}{4 \pi f_\pi f_+(t)} \left\{ \frac{1}{2} \left[ \left( 1 - \frac{m_\mu^2}{P^2} \right)^2 - \frac{m_\pi^2}{I_{\mu\pi}^2} \left( \frac{(p_\mu p_\pi) m_\pi^2 L_2}{2} \right) - (p_\mu p_\pi) \right. \right. \]
\[ + \frac{m_\mu^2 (p_\pi P)}{P^2} \] \left. \left. \right\} (P^2 + \chi (P p_\mu) + \chi L_2 m_\pi^4 \right) \]
\[ + \frac{\chi}{2} \left( 1 - \frac{m_\mu^2}{P^2} \right) \left[ m_\pi^2 + \left( 1 - \frac{m_\mu^2}{P^2} \right) \left( \frac{(P p_\pi)}{2} \right) + \frac{1}{3} \left( 1 - \frac{m_\mu^2}{P^2} \right)^2 \right. \]
\[ \times \left( 2 m_k^2 + P^2 \right) \right] + A \left[ (P p_k) + \chi (p_k p_\mu) \right] + \frac{m_\pi^2}{I_{k\pi}^2} \left( p_\mu p_\pi \right) - \frac{m_k^2 m_\pi^2}{2} L_3 \]
\[ \times \left[ \chi \left( (p_k p_\pi) - m_\pi^2 \right) + (1 + \chi) \left( (p_k p_\mu) - m_k^2 \right) \right] \right\} . \] (17)

Numerically, this contribution at the center of the Dalitz plot is

\[ 2 \text{Im} \chi (SD_V) = -0.9 \times 10^{-6}. \] (18)

Let us consider now the contribution of the axial SD amplitude (12). In this case diagram 3h is not operative, and diagrams 3e–f give

\[ 2 \text{Im} \chi (SD_A) = \frac{\alpha^2 F_V}{4 \pi f_\pi f_+(t)} \left\{ \frac{1}{2} \left[ \left( 1 - \frac{m_\mu^2}{P^2} \right)^2 - \frac{m_\pi^2}{I_{\mu\pi}^2} \left( \frac{(p_\mu p_\pi) m_\pi^2 L_2}{2} \right) - (p_\mu p_\pi) \right. \right. \]
\[ + \frac{m_\mu^2 (m_\pi^2 + (p_\mu p_\pi))}{P^2} \] \left. \left. \right\} (P^2 + \chi (P p_\mu) + \chi L_2 m_\pi^4 \right) \]
\[ + \frac{\chi}{2} \left( 1 - \frac{m_\mu^2}{P^2} \right) \left[ m_\pi^2 + \left( 1 - \frac{m_\mu^2}{P^2} \right) \left( \frac{(P p_\pi)}{2} \right) + \frac{1}{3} \left( 1 - \frac{m_\mu^2}{P^2} \right)^2 \left( 2 m_k^2 + P^2 \right) \right. \]
\[ \times \left( 2 m_k^2 + P^2 \right) \right] \right] + A \left( (P p_k) - m_k^2 - \chi ((p_k p_\mu) - (P p_\mu)) \right) \right\} . \] (19)

Here

\[ A = \frac{1}{I_{\mu\pi}^2} \left\{ 2 (p_\mu p_\pi)^2 - m_\mu^2 (P p_\pi) \left( 1 - \frac{m_\mu^2}{P^2} \right) \right\} - \frac{m_\mu^2 m_\pi^4 L_2}{2 I_{\mu\pi}^2} \] (20)

If one assumes for \( F_A \) its theoretical value (14), then the contribution of (19) constitutes numerically at the center of the Dalitz plot

\[ 2 \text{Im} \chi (SD_A) \simeq 0.3 \times 10^{-6}. \] (21)

And at last, the double structure-dependent (DSD) radiation (see diagram 3h). The small magnitude even of the single SD effects, gives us serious reasons to assume that the contribution of the DSD diagram 3h to \( 2 \text{Im} \chi \) can be safely neglected, so much the more that it is proportional to \( m_\pi^2 \).
5 Conclusion

Among the effects of the FSI, it is the two-photon ones which provide the main contribution to the transverse muon polarization in $K^\pm \rightarrow \pi^0 \mu^\pm \nu$ decay. The typical value of $2 \text{Im} \chi$ in the central part of the Dalitz plot is close to

$$2 \text{Im} \chi = 2 \text{Im} \chi(I_B) + 2 \text{Im} \chi(SD_V) + 2 \text{Im} \chi(SD_A) \simeq 1.3 \times 10^{-5}. \quad (22)$$

The deviation from this value over the Dalitz plot does not exceed 15%.

![Dalitz plot for $P_{em}^m$ in $K^\pm \rightarrow \pi^0 \mu^\pm \nu$ decay. Contours correspond to constant $P_{em}^m$.](image)

Figure 4: Dalitz plot for $P_{em}^m$ in $K^\pm \rightarrow \pi^0 \mu^\pm \nu$ decay. Contours correspond to constant $P_{em}^m$.

The typical value of the transverse muon polarization in the central part of the Dalitz plot is about

$$P_{em}^T \simeq 4 \times 10^{-6}. \quad (23)$$

The distribution of this polarization over the Dalitz plot is shown in Fig. 4. At low pion energy it can reach the level of $6 \times 10^{-6}$. 

This work was supported in part by the Russian Foundation for Basic Research through Grants No. 98-02-17797 and 99-02-17814, I.B.K. and G.G.K. acknowledge also the support by the Ministry of Education through Grant No. 3N-224-98, and by the Federal Program Integration-1998 through Project No. 274.

Appendix

Here we present for convenience some useful integrals.

1. Two-photon intermediate states

\[ d\rho_{(\gamma\gamma)} = \frac{1}{(2\pi)^2} \frac{\delta(p_\pi - k_1 - k_2)}{2\omega_1 2\omega_2} d^3k_1 d^3k_2, \quad k_1^2 = 0, \quad k_2^2 = 0. \]

\[ \int d\rho_{(\gamma\gamma)} = \frac{1}{8\pi}. \]

\[ \int k_{1\alpha} d\rho_{(\gamma\gamma)} = \frac{p_{\pi\alpha}}{16\pi}. \]

\[ \int \frac{d\rho_{(\gamma\gamma)}}{(p_\mu k_2)} = \frac{1}{8\pi} L_1. \]

\[ \int \frac{k_{2\alpha}}{(p_\mu k_2)} d\rho_{(\gamma\gamma)} = \frac{p_{\mu\alpha}}{8\pi I_{\mu\pi}^2} \left[ \frac{m_{\pi}^2(p_\mu p_\pi)}{2} L_1 - m_{\pi}^2 \right] + \frac{p_{\pi\alpha}}{8\pi I_{\mu\pi}^2} \left[ (p_\mu p_\pi) - \frac{m_{\mu}^2(p_\mu p_\pi)}{2} L_1 \right]. \]

\[ \int \frac{d\rho_{(\gamma\gamma)}}{(p_\mu k_1)} = \frac{1}{8\pi} L_3. \]

\[ \int \frac{k_{1\alpha}}{(p_\mu k_1)} d\rho_{(\gamma\gamma)} = \frac{p_{\kappa\alpha}}{8\pi I_{\kappa\pi}^2} \left[ \frac{m_{\pi}^2(p_\kappa p_\pi)}{2} L_3 - m_{\pi}^2 \right] + \frac{p_{\pi\alpha}}{8\pi I_{\kappa\pi}^2} \left[ (p_\kappa p_\pi) - \frac{m_{\kappa}^2(p_\kappa p_\pi)}{2} L_3 \right]. \]

Here

\[ L_1 = \frac{1}{I_{\mu\pi}} \ln \frac{(p_\mu p_\pi) + I_{\mu\pi}}{(p_\mu p_\pi) - I_{\mu\pi}}. \]

\[ I_{\mu\pi}, \ I_{\kappa\pi}, \ \text{and} \ L_3 \ \text{are defined in the text.} \]

2. Muon-photon intermediate states

\[ d\rho_{(\mu\gamma)} = \frac{1}{(2\pi)^2} \frac{\delta(p_\mu + p_\pi - k_1 - p)}{2\omega_1 2p_0} d^3k_1 d^3p, \quad p^2 = (p_\mu + k_2)^2 = m_{\mu}^2, \quad k_1^2 = 0. \]

\[ \int d\rho_{(\mu\gamma)} = \frac{1}{8\pi} \left( 1 - \frac{m_{\mu}^2}{P^2} \right). \]

\[ \int k_{1\alpha} d\rho_{(\mu\gamma)} = \frac{1}{16\pi} \left( 1 - \frac{m_{\mu}^2}{P^2} \right)^2 P_\alpha. \]

\[ \int k_{1\alpha} k_{1\beta} d\rho_{(\mu\gamma)} = \frac{1}{96\pi} \left( 1 - \frac{m_{\mu}^2}{P^2} \right)^3 \left( g_{\alpha\beta} P^2 - 4P_\alpha P_\beta \right). \]
\[\int \frac{d\rho(\mu\gamma)}{(p_\pi - k_1)^2} = -\frac{1}{16\pi} (L_1 + L_2).\]

\[\int \frac{k_{2\pi}}{(p_\pi - k_1)^2} d\rho(\mu\gamma) = \frac{1}{16\pi I_{\mu\pi}^2} \left\{ p_\mu \left[ \left( 1 - \frac{m_\mu^2}{P^2} \right) (p_\pi P) - \frac{(p_\mu p_\pi) m_\pi^2}{2} (L_1 + L_2) \right] \right. \]

\[\left. - p_\pi \left[ \left( 1 - \frac{m_\mu^2}{P^2} \right) (p_\mu P) - \frac{m_\mu^2 m_\pi^2}{2} (L_1 + L_2) \right] \right\}.\]

\[\int \frac{d\rho(\mu\gamma)}{(p_k k_1)} = \frac{1}{8\pi} L_4.\]

\[\int \frac{k_{1\alpha}}{(p_k k_1)} d\rho(\mu\gamma) = \frac{1}{8\pi I_{\mu\pi}^2} \left\{ \frac{1}{2} p_\kappa P^2 \left( \frac{1}{2} (p_k P) L_4 - 1 \right) \right. \]

\[\left. + P_\alpha \left[ (p_k P) - \frac{1}{2} m_k^2 P^2 L_4 \right] \right\}.\]

3. Integrals generating constants \(C\) and \(A\)

\[\int \frac{\varepsilon_{\alpha\beta\gamma\delta} p_\kappa p_\mu p_\beta p_\pi k_{1\delta} k_{1\rho} d\rho(\gamma\gamma) - \varepsilon_{\alpha\beta\gamma\delta} p_\kappa p_\mu p_\beta p_\pi k_{1\delta} k_{1\rho} d\rho(\mu\gamma)}{(p_k k_1)(p_k k_2)} = -\frac{C}{8\pi D} \varepsilon_{\alpha\beta\gamma\rho} p_\kappa p_\mu p_\beta p_\pi \gamma.\]

\[\int \frac{\varepsilon_{\alpha\beta\gamma\delta} p_\kappa p_\mu p_\beta p_\pi k_{1\delta} k_{1\rho} d\rho(\gamma\gamma) - \varepsilon_{\alpha\beta\gamma\delta} p_\kappa p_\mu p_\beta p_\pi k_{1\delta} k_{1\rho} d\rho(\mu\gamma)}{(p_k k_2)} = \frac{A}{32\pi} \varepsilon_{\alpha\beta\gamma\rho} p_\kappa p_\mu p_\beta p_\pi \gamma.\]

Explicit expressions for \(C\), \(D\), and \(A\) are given in the text.

**References**

[1] G. Garisto and G.Kane, Phys. Rev. D44 (1991) 2038; G. Bélanger and C.Q. Geng, Phys. Rev. D44 (1991) 2789; M. Kobayashi, T.-T. Lin, and Y. Okada, Prog. Theor. Phys. 95 (1995) 361; M. Fabbrichesi and F. Vissani, Phys. Rev. D55 (1997) 5334; G.-H. Wu and J.N. Ng, Phys. Lett. B392 (1997) 93.

[2] M. Abe et al., (KEK-E246 Collaboration) Phys. Rev. Lett. 83 (1999) 4253.

[3] M. Diwan et al., AGS Proposal E923, 1996; Yu.G. Kudenko and A.N. Khotjantsev, Yad. Fiz. 63 (2000) 1.

[4] N. Byers, S.W. MacDowell, C.N. Yang, High Energy Physics and Elementary Particles, IAEA, Vienna, 953 (1965).

[5] L.B. Okun, I.B. Khriplovich, Yad. Fiz. 6 (1967) 821.

[6] A.R. Zhitnitskii, Yad. Fiz. 31 (1980) 1014.

[7] Particle Data Group (C. Caso et al), Eur.Phys.J. C3 (1998) 1.
[8] V.G. Vaks, B.L. Ioffe, Nuovo Cim. 10 (1958) 342; Zh. Eksp. Teor. Fiz. 35 (1958) 221 [Sov. Phys. JETP 8 (1958)].

[9] V.F. Müller, Z. Phys. 173 (1963) 438.

[10] P. De Baenst, J. Pestieau, Nuovo Cim. 53 (1968) 137.

[11] J. Bijnens, G. Ecker, J. Gasser, Nucl. Phys. B396 (1993) 81.

[12] E787 Collaboration, S. Adler et al., hep-ex/0003019.