The authors consider subadditive functions bounded above on a shift-compact set.

Let $X$ be an abelian topological group. A set $A \subseteq X$ is called shift-compact if for every sequence $(x_n)_{n \in \mathbb{N}}$ tending to 0 in $X$ there exists $x \in X$ such that the set $\{n \in \mathbb{N} : x + x_n \in A\}$ is infinite.

The main result is the following theorem.

**Theorem.** Let $X$ be an abelian metric group and $f : X \to \mathbb{R}$ a subadditive function. If $f$ is bounded above on a set $T \subset X$ whose $k$-fold sum $\sum_{i=1}^{k} T$ is shift-compact for some $k \in \mathbb{N}$, then $f$ is locally bounded at each point of $X$.

A consequence of the above theorem is that if $f : X \to \mathbb{R}$ is a subadditive function locally bounded at some point, then $f$ is locally bounded at each point of $X$.

The third section is devoted to some connections among WNT-property, boundedness on a shift-compact set, and local boundedness at a point.

Let us describe the WNT-property. Let $f : X \to \mathbb{R}$ be defined on an abelian metric group and $H^k = f^{-1}(-k, k)$ for $k \in \mathbb{N}$. Function $f$ is called a WNT-function if for every convergent sequence $(u_n)_{n \in \mathbb{N}}$ in $X$ there exist $k \in \mathbb{N}$, an infinite set $M \subset \mathbb{N}$ and $t \in X$ such that $\{t + u_m : m \in M\} \subset H^k$. For a function $f : X \to \mathbb{R}$ defined on an abelian metric group the following implications hold:

(i) if $f$ is locally bounded at a point, then $f$ is bounded on a shift-compact set in $X$;
(ii) if $f$ is bounded on a shift-compact set in $X$, then $f$ is WNT.

Furthermore, if $f : X \to \mathbb{R}$ defined on an abelian metric group is subadditive then the following conditions are equivalent:

(i) $f$ is locally bounded at some point;
(ii) $f$ is WNT;
(iii) $f$ is bounded above on a shift-compact set;
(iv) $f$ is bounded on a shift-compact set.

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**MSC:**

- 39B62 Functional inequalities, including subadditivity, convexity, etc.
- 28C10 Set functions and measures on topological groups or semigroups, Haar measures, invariant measures
- 54E52 Baire category, Baire spaces

**Keywords:**

shift-compact set; null-finite set; Haar-null set; Haar-meagre set; subadditive function; local boundedness

**Full Text:** DOI

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