Local Franck-Condon factors in suspended carbon nanotube quantum dots

Fabio Cavaliere¹, Eros Mariani², Renaud Leturcq³,⁴, Christoph Stampfer³,⁵ and Maura Sassetti¹

¹ Dipartimento di Fisica, Università di Genova and CNR-SPIN, Via Dodecaneso 33, 16146 Genova, Italy
² Centre for Graphene Science, School of Physics, University of Exeter, Stocker Road, Exeter, EX4 4QL, United Kingdom
³ Laboratory for Solid State Physics, ETH Zurich, 8093 Zurich, Switzerland
⁴ IEMN CNRS-UMR 8520, ISEN, Avenue Poincaré, BP 60069, 59652 Villeneuve d’Ascq Cedex, France
⁵ JARA-FIT and II. Institute of Physics, RWTH Aachen University, 52074 Aachen, Germany
E-mail: cavalieri@fisica.unige.it

Abstract. We investigate the effects of the coupling between quantized vibrational modes (vibrons) and electronic degrees of freedom in suspended carbon nanotube quantum dots. The elastic deformations couple both to the total dot charge and to its spatial density fluctuations. The latter, neglected in the Anderson-Holstein model, have profound consequences if the size of the vibron and of the dot do not coincide, as is generically the case in experimental devices. When the vibron is smaller than the quantum dot, spatially varying local Franck-Condon factors emerge with striking effects on the transport properties of the system. The theoretical results are supported by our experimental observations.

1. Introduction
The intriguing transport properties of single-wall carbon nanotubes (CNTs) attracted considerable interest in recent years [1]. Impurities or geometrical defects along a CNT may behave as tunnel barriers leading to the formation of a CNT quantum dot connected to the external CNT parts acting as source and drain contacts [2, 3, 4]. Quantum dots formed in CNTs display a Coulomb diamond pattern showing signatures of the fourfold degeneracy of electronic states in graphene (out of which the CNT are made) [5]. On top of their peculiar electronic structures, suspended CNTs possess a rich variety of vibrational degrees of freedom, ranging from stretching and breathing modes to soft transverse bending ones [6]. Electrons tunnelling in suspended CNT quantum dots have been shown to excite stretching vibrons, leading to the appearance of vibrational sidebands in the nonlinear conductance spectra [7, 8, 9, 10].

Most commonly, the transport properties of suspended CNT dots are described in terms of the Anderson-Holstein (AH) model [11, 12], in which the vibrational motion couples to the total excess charge of the dot. The AH model leads to the Franck-Condon (FC) blockade recently observed in experiments [9], with position-independent FC factors [13, 14, 15, 16].

In general, however, vibrons couple both to the total charge and to the spatial modulations of the charge density in the dot [17, 18, 19]. The scenario obtained considering the latter can differ
Figure 1. (a) SEM image of the suspended CNT device (scale bar: 200 nm): red marks denote its position relative to source (S) and drain (D) contacts and the suspended top gate (TG). (b) Colour plot of the differential conductance $G$ (units $e^2/h$) as a function of top gate $V_g$ (units V) and bias $V$ (units mV) voltages. (c) Scheme of the CNT dot (orange) connected to CNT contacts (yellow) via tunneling barriers (red) at $y_{1,2}$. The vibron is shown as a spring (see text).

significantly from that of the conventional AH model. Indeed, if the vibron length $L_v$ is smaller than the dot size $L_d$, position-dependent FC factors arise [19], with dramatic consequences on the transport properties. Indeed, if the vibron locates asymmetrically along the dot only the electrons tunneling across the barrier next to the vibron may excite the vibrational motion. As a result, a selective suppression of the family of conductance traces corresponding to tunneling through the barrier farther from the vibron occurs. Only when $L_v \gg L_d$ the effects of the density fluctuations are less relevant and one recovers the standard AH model. In this paper, the transport properties of a suspended CNT dot will be explored, with a particular emphasis on the regime $L_v < L_d$. The theory will be compared with the results of our transport experiment performed on a suspended CNT, which also exhibits the suppression of a family of conductance traces. The CNT is suspended between source and drain contacts, to which a bias voltage $V$ is applied, see Fig. 1a. Here, a quantum dot is formed below an electrically isolated suspended top gate [9], and its excess number of electrons may be tuned by means of the top-gate voltage $V_g$. Figure 1b shows a color map of the differential conductance $G$ as a function of $V_g$ and $V$. Besides the almost perfect fourfold symmetry of the Coulomb blockade diamonds, one observes a rich spectrum of excited states related to the excitation of stretching vibrons [9, 6], with a level spacing of about 0.8 meV corresponding to a vibron size $L_v \approx 60$ nm [9]. The dot length, estimated from the size of Coulomb diamonds, is $L_d \approx 240$ nm $> L_v$. Two main features are present: (i) a striking suppression of conductance traces with negative slope, and (ii) a regular alternating pattern of positive (PDC, red) and negative differential conductance (NDC, blue). By taking into account charge density fluctuations in the regime $L_v < L_d$, we show how the point (i) can be addressed as a result of asymmetric FC factors stemming from vibronic excitations near the barrier on the source side 1. We point out that the usual AH model with symmetric FC factors is not able to reproduce such a suppression of conductance traces. In parallel, we show that asymmetries between the tunneling rates on different barriers and orbital states in the dot yield a dynamical trapping of electronic states which can finally be responsible for the repeated PDC/NDC pattern of point (ii) [20, 21, 22].

1 According to the bias scheme of the device [9], conductance traces with negative slope correspond to tunneling events through the drain barrier.
2. Position-dependent electron-vibron coupling

We illustrate the mechanism giving rise to position-dependent FC factors studying the simplest model for the dot, namely a single spinless electronic state, coupled to the lowest vibrational stretching mode. The extension of this model to the case of a CNT-based quantum dot will be presented in Sec. 3. Tunnel barriers at $y_1 = 0$ and $y_2 = L_d$ define the dot region while the vibron occupies the CNT region $y_0 \leq y \leq y_0 + L_v$, see Fig. 1c. The dot is modeled as a Luttinger liquid (LL) with open boundary conditions (OBC) [23], with the electronic field operator $\psi(y)$ satisfying $\psi(y_1) = \psi(y_2) = 0$. The field operator is decomposed as [23] $\psi(y) \approx e^{i k_F y} \psi_R(y) + e^{-i k_F y} \psi_L(y)$ with $k_F$ the Fermi momentum and $\psi_R/L(y)$ the field operators for right (R) and left (L) movers. Due to OBC, $\psi_L(y) = - \psi_R(-y)$, allowing a description in terms of right movers, with

$$\psi_R(y) = \frac{1}{\sqrt{2\pi}} e^{i q/2} e^{i \pi y N/L_d} e^{i \phi(y)}$$ (1)

the bosonized form of their field operator. Here, $N$ is the dot excess charge zero mode, $\xi$ a short length cutoff and $[\theta, N] = i$. Furthermore,

$$\phi(y) = \sum_{q>0} \frac{1}{\sqrt{q}} e^{-\pi q y/2 L_d} \left[ e^{-i \pi q y/2 L_d} b_q^\dagger + \text{h.c.} \right]$$ (2)

is the plasmonic field with $b_q^\dagger$ creating the $q$-th plasmon mode with frequency $\omega_q = \pi v_c q/L_d$ and group velocity $v_c$. In the following, we will assume $v_c \approx v_F$ due to gate-induced screening [24]. The dot and vibron Hamiltonians ($H_d$, $H_v$) read ($\hbar = 1$)

$$H_d = E_c N^2 + \sum_q \omega_q b_q^\dagger b_q = H_d^{(N)} + H_d^{(pl)} \quad ; \quad H_v = \frac{p_0^2}{2 M} + \frac{M \omega_0^2}{2} x_0^2 ,$$ (3)

with $E_c$ the charging energy, $M$ the vibron mass, $\omega_0 = \pi v_s/L_v$ its frequency and $v_s$ the stretching mode velocity [24]. In a CNT, $v_s < v_c$ which in experiments implies $\omega_0 < \omega_1$ [9, 8]. The vibron coordinate $x_0$ represents the amplitude of the distortion field $u(y) = \sqrt{2} x_0 \sin[\pi(y - y_0)/L_v]$, while $p_0$ is the conjugate momentum. The electron-vibron coupling is described by

$$H_{d-v} = c \int_{\text{max}[L_d y_0 + L_v]}^{\text{min}[L_d y_0 + L_v]} dy \rho(y) \partial_y u(y)$$ (4)

with $c$ the deformation potential coupling constant [24, 25, 26] and $\rho(y)$ the electron density of the dot. In the spirit of LL we approximate $\rho(y) \approx \rho_R(y) + \rho_R(-y)$ where $\rho_R(y) = (N/2 L_d) + (1/2 \pi) \partial_y \phi(y) = \rho_0/2 + \delta \rho(y)$ is the electron density of right movers [23] with $\rho_0 = N/L_d$ the average density and $\delta \rho(y) = (1/2 \pi) \partial_y \phi(y)$ the density fluctuations. This approximation neglects terms $e^{i k_F y_q} \psi_q^\dagger(y) \psi_L(y) + \text{h.c.}$ with spatial oscillations of short-wavelength $k_F^{-1}$ and is valid if $L_v \gg k_F^{-1}$, which implies $N \gg L_d/\pi L_v$, a condition satisfied in our experiment.

Introducing generalized plasmonic coordinates $x_q = (b_q + b_q^\dagger)/\sqrt{2 \omega_q}$ the coupling in Eq. (4) can be decomposed into a contribution $H_d^{(N)} = \sqrt{M} c_0 x_0 N$ due to the zero mode and $H_d^{(pl)} = x_0 \sqrt{M} \sum_{q=1}^\infty c_q x_q$ due to the plasmonic density fluctuations. Here we have introduced

$$c_0 = \frac{\omega_0^2}{\pi} \lambda_m \sin \left[ \pi \left( 1 + \frac{y_0}{L_d} \right) \right] \theta \left( \frac{y_0}{L_d} \right) \theta \left( 1 + \frac{y_0}{L_d} \right)$$ (5)

$$c_q = \frac{2 \omega_0^2}{\pi} \sqrt{\omega_1 \lambda_m q^2 \delta^2} \left\{ \sin \left[ \frac{y_0}{L_d} \right] \sin \left[ \pi q \delta \left( 1 + \frac{y_0}{L_d} \right) \right] \right\}$$ (6)
where \( \delta = L_v/L_d \) and \( \lambda_m = c/(\nu_s \sqrt{M \omega_0}) \).

Note that, for simplicity, we have quoted only the expressions for \( \delta \leq 1 \) and \( y_0 + L_v \leq L_d \). The total Hamiltonian \( H_{\text{dot-vib}} = H_A + H_v + H_{\text{d-v}} \) is quadratic in the vibron and plasmon generalized coordinates: introducing the two vectors \( X_\mu = (\sqrt{M} x_0, x_1, x_2, \ldots) \) and \( P_\mu = (p_0/\sqrt{M}, p_1, p_2, \ldots) \) it has the form

\[
\sum_{\mu=0}^\infty [P_\mu^2 + X_\mu^2]/2 + X_0 \sum_{\nu=1}^\infty c_\mu X_\mu + c_0 X_0 N \text{ and is thus easily diagonalized} \ [27, 16].
\]

We first proceed diagonalizing \( H_{\text{d-v}} \) and introduce the new normal modes \( \tilde{X}_\mu = (\tilde{x}_0, \tilde{x}_1, \tilde{x}_2, \ldots, \tilde{P}_\mu = (\tilde{p}_0, \tilde{p}_1, \tilde{p}_2, \ldots) \) with \( X_\mu = \sum_{\nu=0}^\infty k_{\mu \nu} \tilde{X}_\nu \) and \( P_\mu = \sum_{\nu=0}^\infty k_{\mu \nu} \tilde{P}_\nu \). The diagonalizing matrix elements \( k_{\mu \nu} = c_\mu (\Omega^2 - \omega_\mu^2)^{-1/2} \) for \( \mu \geq 1 \) depend on \( k_{0\nu} = \left[ 1 + \sum_{q=1}^\infty c_q^2 / \left( \Omega^2_q - \omega_\mu^2 \right) \right]^{-1/2} \), with the new eigenfrequencies \( \Omega_\nu \) given by the self-consistent equation \( \Omega^2_\nu = \omega_0^2 + \sum_{q=1}^\infty c_q^2 / \left( \Omega^2_q - \omega_\mu^2 \right) \). The new modes with \( \nu \geq 1 \) represent damped plasmons with energy \( \Omega_\nu > \omega_\nu \), while the term \( \nu = 0 \) with \( \Omega_0 < \omega_0 \) represents the vibron excitation dressed by a plasmonic cloud. In order to eliminate the remaining term \( H_{\text{d-v}}^{(N)} = c_0 N \sum_{\nu \geq 0} k_{0\nu} \tilde{x}_\nu \) we employ a Lang-Firsov transformation \ [23], generated by \( \mathcal{U} = \exp \left[-iN \sum_{\nu \geq 1} \eta_\nu \tilde{p}_\nu \right] \) with \( \eta_\nu = c_0 k_{0\nu} / \Omega^2_\nu \). Finally, we obtain \( H_{\text{dot-vib}} = E_c^N + \sum_{\nu \geq 0} \Omega_\nu \left( a^\dagger_\nu a_\nu + 1/2 \right) \), where \( a^\dagger_\nu \) creates the \( \nu \)-th eigenmode and \( E_c^N = E_c - \sum_{\nu \geq 0} \left( c_0 k_{0\nu} / \sqrt{\Omega^2_\nu} \right)^2 \) contains a polaronic shift. Dot-vibron states are labeled by \( |N, l \rangle \), with \( l \) the occupation number of the dressed-vibron states.

The above transformations affect the electronic field as well. Crucially, due to the mixing of vibrons and plasmons induced by \( H_{\text{d-v}} \), vibronic operators appear in the electronic field. Since \( \dot{\psi} \) contains a polaronic shift. Dot-vibron states are labeled by \( |N, l \rangle \), with \( l \) the occupation number of the dressed-vibron states.

The transformed field operator Eq. (7) removes an electron from the system, with the simultaneous excitation of the vibron mode.

In the sequential regime, tunneling rates can be evaluated via the golden rule. For the localized injection of an electron at \( y \) along the nanotube, with a scattering \( l \rightarrow l' \) of the vibron state, one obtains \( \Gamma(y) \propto X_{l,l'}(y) \), with \( X_{l,l'}(y) = 2\pi \xi \left| (N + 1, l'| \psi^\dagger_R(y)|N, l \rangle \right|^2 \). From Eq. (7),

\[
X_{l,l'}(y) = e^{-\lambda^2(y)} \left[ \lambda^2(y) \right]^{l_2-l_1} \left[ \frac{\Gamma^l_{l'} \Gamma_{l_1}}{l_2-l_1} \left[ \lambda^2(y) \right] \right]^2 ,
\]

where \( \lambda^2(y) = [\lambda_N + \lambda_-(y)]^2 + \lambda^2_+(y) \), \( l_2 = \min(l, l') \), \( l_1 = \max(l, l') \) and \( \Gamma^l_{l'}(x) \) are the generalized Laguerre polynomials.

This constitutes our main result: Eq. (9) represents a position-dependent Franck-Condon factor, due to the position-dependent effective electron-vibron coupling \( \lambda(y) \), in sharp contrast with the predictions of the AH model. The latter fails in cases when \( \max \{ \lambda_\pm(y) \} \gg \lambda_N \) so that the position dependence of the coupling can no longer be neglected. Two regimes can be identified according to the vibron being smaller than the dot \( (L_v < L_d) \) or larger \( (L_v > L_d) \).

For a small vibron \( (L_v < L_d) \), the spatial modulations of the electron density \( \delta \rho(y) \) can never be neglected. In particular, if the vibron is located within the dot \( (0 \leq y_0 \leq L_d - L_v) \) we
find $\lambda_N \propto c_0 \to 0$, see Eq. (5), and density fluctuations provide the only mechanism through which electrons and vibrons couple via $\lambda_{\pm}(y)$. When the vibron crosses either tunneling barrier ($y_0 < y_i < y_0 + L_v$ for $i = 1, 2$) on the other hand, both $\lambda_N$ and $\lambda_{\pm}(y)$ are of the same order. Fig. 2a shows $\lambda(y)$ for $\delta = 0.2$ and different vibron positions. When the latter sits entirely within the dot, the coupling is sizeable only in the proximity of the vibron. With the vibron across the dot ($y_0 < y_i < y_0 + L_v$), $\lambda(y)$ increases due to the contribution of $\lambda_N$ but is nonetheless peaked in the vibron region. The above results imply a localized suppression of the tunneling conductance which would allow to detect the vibron location and extension, in contrast with the AH model, which would predict a constant suppression in the whole dot region.

Figure 2b shows $\lambda(y)$ for a vibron sitting next to the left tunneling barrier case ($y_0 = y_1 = 0$) and different values of $\delta$: as $L_v$ increases the spatial dependence of $\lambda(y)$ becomes less dramatic. The values of the effective coupling at the two tunneling barriers $\lambda_{1,2} = \lambda(y_{1,2})$ are particularly important in order to describe lateral tunneling in CNT-dots embedded into longer CNTs, as in the experiment discussed here. In Fig. 2c, $\lambda_{1,2}$ are shown as a function of $\delta$ for $y_0 = y_1 = 0$: when $\delta \to 0$ the small value of $\lambda_2$ implies that the excitation of vibron modes is inhibited for tunneling on the right barrier. As discussed in the next section, this lies at the origin of the strong suppression of a family of conductance traces observed in our experiment.

For $\delta = 1$ and a symmetric vibron $y_0 = (L_d - L_v)/2$ one has $\lambda_1 = \lambda_2$. In this symmetric situation the lateral tunneling may be formally treated within an effective AH model with coupling constant $\lambda_{AH} = \lambda_{1,2}$. Notice that the physical mechanism of electron-vibron coupling is however different from the AH model, since here the coupling to the total charge vanishes, $\lambda_N = 0$.

In the case of a large vibron ($L_v > L_d$), with $\delta \gg 1$ and the dot embedded into the vibron, the term $\partial_y u(y)$ in Eq. (4) is essentially constant over the integration domain (the dot region). As a result, the contribution due to density fluctuations $\delta \rho(y)$ averages to zero yielding $\lambda_{\pm}(y) = 0$ and the only residual coupling due to the total charge $N$. As a result, one obtains $\lambda_N \approx \lambda_m \cos(\pi y_0/L_v)$, a position-independent coupling, and the usual AH model is recovered. The maximum value of the effective coupling is $0.24$, obtained for $\delta < 1$ depends strongly on the ratio $v_s/v_c$ and on $\lambda_m$. In all the calculations above, we have adopted usual parameters for graphene and CNTs. However, several effects may lead to a renormalization of the above quantities. As an example, the coupling to the radial breathing mode has been shown to reduce the charge velocity [17, 28], thus increasing $v_s/v_c$ and allowing to reach $\lambda_1 > 1$ while still $\lambda_2 \ll 1$. Also the value of the deformation potential coupling parameter $c$ has been experimentally found larger than expected [29], which would lead to a higher estimate for $\lambda_m$.
3. Transport properties of a vibrating CNT

The above model can be easily extended to the case of a CNT with two valley \( \eta = \pm 1 \) and two spin channels \( \sigma = \pm 1 \) with \( N_{\eta \sigma} \) excess electrons per channel. It is convenient to introduce total (\( + \)) and relative (\( - \)) charge (\( c \)) and spin (\( s \)) modes in analogy to Eq. (7). Furthermore, \( \Phi \) appear in \( \psi \) branches model. Due to the coupling of the vibron to the electrochemical potentials whose difference is proportional to the bias voltage \( V \) modeled as LLs [4] with four branches and OBC at the tunneling barriers the charging energy renormalized by the polaronic shift. The term \( H \psi \eta,\sigma \) describes an energy shift \( V \) is the right movers field operator of the four-branches model. Due to the coupling of the vibron to the \( c+ \) mode dressed vibron operators appear in \( \psi_{R,\eta,\sigma}(y) \), in which we further neglect the high-energy dynamics of the collective charge and spin modes in analogy to Eq. (7). Furthermore, \( \Phi^{(\alpha)}_{R,\eta,\sigma}(y) \) is the field operator of right movers in lead \( \alpha \) and \( \tau_{\alpha,\eta} \) the tunneling amplitude for the level \( \eta \) with spin \( \sigma \). Leads, modeled as LLs [4] with four branches and OBC at the tunneling barriers \( y = y_\alpha \), are kept at electrochemical potentials whose difference is proportional to the bias voltage \( V \).

To describe the steady-state occupation probability of the dot states in the sequential regime, we employ a rate equation [13, 32]. Tunneling rates into the branch \( \eta \) on barrier \( \alpha \) read

\[
\Gamma^{(\text{in})}_{\alpha,\eta} = \Gamma_0 \left| \frac{\tau_{\alpha,\eta}}{\tau_{2,+}} \right|^2 X_{ll'}(y_\alpha) f \left( \Delta E + (-1)^{\alpha+1} \frac{\nu V}{2} \right),
\]

where \( \Gamma_0 = 2\pi \nu |\tau_{2,+}|^2 / \xi^2 \), \( \nu \) is the density of states assumed equal on both leads and \( f(E) \) is the Fermi function at temperature \( T \) with \( \Delta E \) the energy difference between initial and final dot states. They contain the barrier-dependent FC factors discussed in the previous section. It is useful to introduce the parameter \( A = |\tau_{1,\eta}|^2 / |\tau_{2,\eta}|^2 \), describing the asymmetry between source and drain tunnel barriers, and \( \gamma = |\tau_{\alpha,-1}|^2 / |\tau_{\alpha,+1}|^2 \).

Sequential tunneling is justified for \( k_B T \gg \Gamma_0 \). Additionally, the above treatment neglects coherences among vibron states, which is appropriate if \( \Omega_0 \gg \Gamma_0 \). In our experiment both conditions are satisfied, as we can estimate \( k_B T \approx 90 \mu eV \) (for \( T \approx 1 \) K), \( \Gamma_0 \approx 1 \mu eV \) (from current-voltage characteristics) and \( \Omega_0 \approx 0.8 \mu eV \) (from the separation of vibron subbands). We also extract \( E_c \approx 4.5 \) meV from the size of the Coulomb diamonds [9].

Restricting for simplicity to \( N_g \approx N_{g*} = 1/2 + 4K + 3\epsilon/2E_c \) where the resonance between \( N_c = 4K \) with \( N_{\eta,\sigma} = K \pm \eta, \sigma \) and \( N_c = 4K + 1 \) with \( K \) a positive integer occurs, the rate equation is numerically solved and the differential conductance \( G = \partial I / \partial V \) is obtained.

As already discussed in the introduction, the nonlinear transport spectra exhibit two main features: (i) a striking suppression of conductance traces with negative slope and (ii) a regular pattern of alternating PDC and NDC. Concerning (i), the suppression of traces with negative slope can only be reproduced by our model if asymmetric FC factors are employed in the

\[\text{For tunneling out, a similar expression holds.}\]
Figure 3. Colour maps of the numerical differential conductance $G$ (units $0.01 \, e^2/h$) as a function of $\bar{N}_g = N_g - N_g^*$ (see text) and $V$ (units meV). (a) Plot for $A = 1/40$, $\gamma = 40$, $\lambda_1^2 = 2$, $\lambda_2^2 = 0.2$ and $\Delta \varepsilon = 0.48$ meV; (b) as in (a) but $\lambda_1^2 = \lambda_2^2 = 2$ (AH model); (c) as in (a) but for $A = \gamma = 1$; (d) as in (a) but for $\gamma = 1$; (e) as in (a) but for $\Delta \varepsilon = 0$; (f) as in (a) but for $A = 1$. In all panels, $\Omega_0 = 0.8$ meV, $k_B T = 50 \, \mu$eV, $E_c = 4.5$ meV and $\Gamma_0 = 0.8 \, \mu$eV.

calculation. A simple asymmetry of the tunneling barriers $A \neq 1$ is not able to reproduce such a feature, regardless the asymmetry strength. Concerning $(ii)$, the occurrence of NDC requires $\gamma > 1$, $\Delta \varepsilon \approx \Omega_0/2$ and $A < 1$.

Given the bias setup [9], traces with negative slope correspond to transitions on the right tunnel barrier: therefore we assume $\lambda_2 \ll \lambda_1$ with $\lambda_2 \approx 0$, corresponding to a vibron smaller than the dot, located next to the left tunneling barrier. Figure 3a shows $G$ as calculated with parameters chosen with all the above prescriptions. The results of our model are in excellent agreement with the experimental findings, see Fig. 1b. In particular, we note an almost perfect suppression of the conductance traces with negative slope due to the smallness of $\lambda_2$. In sharp contrast, results obtained within the standard AH model ($\lambda_1 = \lambda_2$) but same other parameters, shown in Fig. 3b, display no suppression of the conductance traces with negative slope, despite the strong asymmetry of the tunneling barriers.

Figures 3c-f illustrate the role of the other parameters of the system in creating NDC, for fixed $\lambda_1 \neq \lambda_2$ as in Fig. 3a. For a symmetrical setup $A = \gamma = 1$, no NDC can be achieved, see Fig 3c. Choosing $\gamma > 1$ ($\gamma < 1$) results in a bottleneck for the tunneling events involving the state $\eta = +1$ ($\eta = -1$). This leads to a dynamical trapping of one of the orbital states, which acquires a high occupation probability with a corresponding decrease in the current [20, 21, 22].

No qualitative difference is present between $\gamma < 1$ and $\gamma > 1$. Tuning $\gamma = 1$ instead suppresses NDC, as shown in Fig. 3d, since no dynamical trapping can be achieved.

A regular pattern of repeating NDC and PDC traces is obtained if $\Delta \varepsilon \approx \Omega_0/2$. In this case, the vibrational subbands for $\eta = +1$ are interleaved with those of $\eta = -1$, resulting in an alternating dynamical trapping and un-trapping of the dot states. With completely degenerate states $\Delta \varepsilon = 0$, as shown in Fig. 3e, trapping and un-trapping compete and the net result is the disappearance of NDC. We note that the smallness of the $\Delta \varepsilon$ required for observing NDC is not sufficient to spoil appreciably the fourfold symmetry of the Coulomb blockade diamonds observed in Fig. 1b. Tuning the left/right tunnel barriers to $A < 1$ yields NDC lines with the
same positive slope at any voltage. With $A = 1$ one has NDC with positive slope for $V < 0$ and suppressed NDC with negative slope for $V > 0$ (see Fig. 3f) in contrast with the experiment.

4. Conclusions
A microscopic model describing the coupling between electrons and vibrons in a suspended CNT-quantum dot has been presented. In contrast with the usual AH model, vibrons couple both to the total dot charge and to the spatial fluctuations of the electronic density. As a result, a position-dependent effective coupling $\lambda(y)$ and related position-dependent FC factors arise when the vibron is smaller than the dot, while the conventional results of the AH model are recovered for vibrons larger than the dot itself. Small vibrons located asymmetrically with respect to the dot center result in a large effective coupling on the tunnel barrier closer to the vibron. This implies a strong suppression of one family of conductance traces in the transport characteristics of the system. Such a suppression is indeed observed in our experiment and cannot be explained with the usual AH model. The alternating NDC/PDC traces observed in the experiment have been addressed in terms of asymmetric tunnel couplings on the tunneling barriers and between different orbital state of the CNT dot.

Acknowledgments
Financial support by INFM-CNR via Seed Project PLASE001 and “Progetto giovani”, and by the EU-FP7 via ITN-2008-234970 NANOCTM are gratefully acknowledged.

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