Enhancing squeezed light optomechanical interferometer using quantum optical restoring force

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It was known that application of frequency dependent squeezed vacuum improves the sensitivity beyond standard quantum limit by a factor of $e^{-r}$, where $r$ is the squeezing parameter. In this work, we show that application of squeezed light along with optical restoring force can enhance the sensitivity beyond standard quantum limit by a factor of $\sqrt{e^{-2\zeta/4\Delta}}$, where $0 < \zeta/\Delta < 1$, with $\Delta$ as the detuning between cavity eigenfrequency and driving field. The technique described in this letter is restricted to the frequencies much smaller than the resonance frequency of the optomechanical mirror.

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The quest to detect gravitational waves [1, 2] has revolutionized precision measurements using optical interferometer. The laser interferometer gravitational wave detector is based on the coupling of optical modes with mechanical modes, which is known as optomechanics [3–6]. With the miniaturization [7–9] of mechanical mirrors, optomechanics has emerged as one of the best physical systems to design ultraprecise sensors [10–13]. Such a sensor can be designed by embedding optomechanical cavities into the arms of an optical interferometer [14, 15].

Shot noise and radiation pressure noise (RPN) [16–18] are two major noises in optomechanics. Shot noise arises from the randomness in the photon counting while the RPN arises because of the randomness in the radiation pressure force exerted on the mechanical mirror. Shot noise can be decreased by increasing the laser power, however, this leads to an increase in RPN. This trade-off between shot noise and RPN imposes standard quantum limit (SQL) [19–21]. Several techniques [22–32] were developed to overcome SQL. One of the most popular methods is to use squeezed light [33–42]. A squeezed light [43–47] is a special quantum state in which frequency dependent fluctuations induced by radiation pressure force.

The mechanical mirror divides the total cavity into two sub-cavities each with length $l$ and eigenfrequency $\omega_c$. The annihilation operators for optical field inside the cavity are given by $a$ and $\hat{c}$ as shown in Fig. 1. There is no tunnelling between $\hat{a}$ and $\hat{c}$ and vice-versa as the mechanical mirror is perfectly reflective. A co-sinusoidal classical force $f \cos(\omega_r t)$, with $\omega_r$ as frequency and $t$ as time, changes the position $\hat{z}$ of the mechanical mirror. The total Hamiltonian $\hat{H}$ of the optomechanical cavity [54] is given as

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega_m^2 \hat{z}^2 + \hbar \omega_c \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \left( 1 - \frac{\hat{z}}{l} \right) + \hbar \omega_c \left( \hat{c}^\dagger \hat{c} + \frac{1}{2} \right) \left( 1 + \frac{\hat{z}}{l} \right) - f \cos(\omega_r t) \hat{z} + \hat{H}_r,$$

where $\hat{p}$, $\omega_m$ and $m$ are momentum, eigenfrequency and the mass of the mechanical mirror, respectively. $\hat{H}_r$ is the Hamiltonian for the environment and its coupling with the optomechanical cavity. $\hbar$ is the reduced Planck’s constant. The optical fields $\hat{a}$ and $\hat{c}$ are driven by external fields with annihilation operators $b$ and $d$, respectively. The operators $b$ and $d$ are normalized such that their optical powers are given by $\hbar \omega_d (\hat{d}^\dagger \hat{d})$ and $\hbar \omega_a (\hat{b}^\dagger \hat{b})$, respectively. The operators $\hat{b}$ and $\hat{d}$ are the an-

![FIG. 1. Interferometer with membrane in the middle of an optomechanical cavity. The optomechanical membrane is perfectly reflective, so there is no tunnelling between $\hat{a}$ and $\hat{c}$. The cavity fields $\hat{a}$ and $\hat{c}$ are synthesized such that optical restoring force counters the fluctuations induced by radiation pressure force.](image-url)

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nihilation operators for the output field from the sub-cavity-a and sub-cavity-c, respectively. We adopt the following notation through out this article: for any operator $\hat{O}$, $\hat{O}$ represents its steady state while $\hat{\delta}_O$ represents its quantum fluctuation. The steady state solutions are

$$\tilde{a} = \sqrt{\frac{c}{\delta_0}} \frac{\sqrt{\frac{d}{2}\hat{\delta}_c}}{\Delta + i g \tilde{z} + \frac{c}{\delta_0}} \tilde{z} = \frac{\sqrt{\frac{d}{2}\hat{\delta}_c}}{\Delta + i g \tilde{z} + \frac{c}{\delta_0}} \frac{\delta_0}{\delta_0^2 + \gamma \omega},$$

where $g = \omega_0/\lambda$, same for both the sub-cavities [55, 56], $\zeta$ is the cavity decay rate, $\gamma$ is the decay rate of mechanical mirror, and $\Delta = \omega_0 - \omega_0^2$, with $\omega_0$ as the frequency of the external driving fields $b$ and $d$. The $\tilde{z}$ in $\tilde{a}$ and $\tilde{c}$ leads to optomechanical bi-stability. The bi-stability in Eq. (2) can be avoided by choosing $|\tilde{a}|^2 = |\tilde{c}|^2$. Then the average radiation pressure force on the mechanical mirror from both the sub-cavities is equal but opposite in direction. Hence, the $\tilde{z}$ becomes zero. We assume the beam-splitters in Fig. 1 are 50:50. The input fields $\tilde{b}$ and $\tilde{d}$ are phase adjusted such that, $\tilde{b} = (\hat{E} + i\hat{N}) e^{-i\phi}/\sqrt{2}$ and $\tilde{d} = (\hat{F} + i\hat{G}) e^{-i\phi}/\sqrt{2}$, where $\phi = \tan^{-1}(2\Delta/\zeta)$, $\hat{E}$ and $\hat{F}$ are the laser annihilation operators while $\hat{V}$ and $\hat{U}$ are the vacuum annihilation operators. Then the steady state cavity fields $\tilde{a} = \sqrt{\frac{c}{\delta_0}} \tilde{E} / \sqrt{2(\Delta^2 + \zeta^2/4)}$ and $\tilde{c} = \sqrt{\frac{c}{\delta_0}} \tilde{F} / \sqrt{2(\Delta^2 + \zeta^2/4)}$ can be set to be real by taking $\tilde{E}$ and $\tilde{F}$ as real. The steady-state output fields are given by the input-output formalism [57] as

$$\hat{B} = \hat{D} = -\frac{\hat{E}}{\sqrt{2}} e^{i\phi}.$$  

As there are no external losses in the sub-cavities, Eq. (3) implies that the average optical power of the output field is equal to the average optical power of the input field. As $\tilde{z} = 0$, there is no optomechanical contribution in Eq. (3). The $\phi$ is a consequence of detuning $\Delta$ between driving laser field and cavity eigenfrequency. The equations of motion for fluctuations are given as

$$\dot{\hat{M}} = \left(-i\Delta - \frac{\zeta}{2}\right)\hat{M} + i2g_0\hat{\delta}_c + \sqrt{\frac{c}{\delta_0}} \hat{\delta}_c,$$

$$m \left(\hat{\delta}_c + \gamma\hat{\delta}_c + \omega_m^2\hat{\delta}_c\right) = \hbar g (\hat{M} + \hat{M}^\dagger) + \hat{O} + f \cos (\omega_1 t),$$

where $\hat{M}_1 = \hat{\delta}_b - \hat{\delta}_d$, $M = \hat{\delta}_a - \hat{\delta}_b$, and $\hat{O}$ is the noise operator for the mechanical mirror. We have used the relation $\tilde{a} = \tilde{a}^* = \tilde{c}$ in writing Eq. (4) and Eq. (5). The $f \cos (\omega_1 t)$ is treated like classical fluctuation and included in Eq. (5). Position of the mechanical mirror can be inferred by measuring the phase of the output field at the detectors $D_1$ and $D_2$ or $D_1$ and $D_2$. However, as we are dealing with $\hat{M}$, which is a joint operator of $\hat{\delta}_c$ and $\hat{\delta}_c$, we measure the relative phase between $\hat{B}$ and $\hat{D}$. Hence the general homodyne measurement is slightly modified to measure $\hat{Q}$, which is given as

$$\hat{Q} = \left(B^\dagger \hat{R}_1 + \hat{R}_1^\dagger B\right) - \left(D^\dagger \hat{r}_1 + \hat{r}_1^\dagger D\right),$$

where $\hat{R}_1$ and $\hat{r}_1$ are the reference fields at the output of the optical cavities in arm-a and arm-b, respectively. These optical cavities are on resonance with the incoming fields, and have rigidly fixed mirrors with the upper mirrors being perfectly reflective while the lower mirrors have the decay rate $\zeta$. The reference fields can be written in terms of input fields as

$$\hat{R}_1(\omega) = \frac{H_i \hat{E}(\omega) + \hat{V}(\omega)}{\sqrt{2}}, \quad \hat{r}_1(\omega) = \frac{H_i \hat{F}(\omega) + \hat{U}(\omega)}{\sqrt{2}},$$

where $H = (i\omega + \zeta/2)/(i\omega - \zeta/2)$, with $\omega$ as Fourier frequency. The quantum fluctuation in the output fields is given as

$$\hat{Y}_b(\omega) - \hat{Y}_d(\omega) = \hat{G}_1 \left(\hat{\delta}_b^\dagger(\omega) - \hat{\delta}_d^\dagger(\omega)\right) + \hat{G}_2 \left(\hat{\delta}_b(\omega) - \hat{\delta}_d(\omega)\right) + \hat{G}_3 \hat{O}(\omega),$$

where $\hat{Y}_D(\omega) = i \left[\hat{\delta}_D^\dagger(\omega) - \hat{\delta}_D(\omega)\right], \text{with} \ O = B, D\ and\ G_1 = i + \frac{i\zeta - i(\alpha - \Delta\zeta)}{\Delta + i\zeta}, \ G_3 = \sqrt{\frac{3}{\pi}} \frac{\alpha \Delta}{\zeta^2}, \ G_2 = -i + \frac{i\zeta - i(\alpha - \Delta\zeta)}{\Delta + i\zeta},$

with $\alpha = 4\hbar g^2/\delta_0^2$. Substituting Eq. (8) in the quantum fluctuation $\hat{\delta}_Q$ part of Eq. (6) gives

$$\hat{\delta}_Q(\omega) = \frac{E}{\sqrt{2}} \left[\hat{Y}_b(\omega) - \hat{Y}_d(\omega)\right] + B^\dagger \left[\hat{\delta}_R_1(\omega) - \hat{\delta}_R_1(\omega)\right] + B \left[\hat{\delta}_R_1^\dagger(\omega) - \hat{\delta}_R_1^\dagger(\omega)\right].$$

We have used the relation $\hat{B} = \hat{D}$ in writing Eq. (9). The fluctuations $\hat{\delta}_R_1$ and $\hat{\delta}_R_1$ in the reference fields are given as $\hat{\delta}_R_1(\omega) = H (i\hat{\delta}_E(\omega) + \hat{\delta}_d(\omega))/\sqrt{2}$, $\hat{\delta}_R_1(\omega) = H (i\hat{\delta}_E(\omega) + \hat{\delta}_d(\omega))/\sqrt{2}$. The cavities in arm-a and arm-b have rigidly fixed mirrors and they do not have any external losses. Hence the steady state reference fields are given as $\hat{R}_1 = \hat{r}_1 = i\hat{E}/\sqrt{2}$ (because $\hat{E} = \hat{F}$). The action of $f \cos (\omega_1 t)$ changes the equilibrium position of the mechanical mirror leading to signal $\hat{Q}$ as

$$\hat{Q} = \frac{\sqrt{2}}{2} \left[\frac{G_3(-\omega_f) e^{i\omega_f t} + G_3(\omega_f) e^{-i\omega_f t}}{\Delta + \zeta^2/4}\right].$$

As the classical force $f \cos (\omega_1 t)$ drives the mechanical mirror at frequency $\omega_f$, the $\hat{Q}$ is also oscillating at the same frequency. For $\omega_f \ll \omega_m$, substituting $G_3$ from Eq. (8) into Eq. (10) gives

$$\hat{Q} = \frac{\sqrt{\frac{c}{\delta_0}} f \hat{\delta}_c}{\delta_0^2} \cos (-\omega_f t + \phi),$$

where $\phi = \phi + \tan^{-1}(4\epsilon\Delta^2/\zeta^2)$. Eq. (11) is derived by assuming $\omega_m \gg \omega_f$, $\omega_m \gg \gamma$, and $1 > \zeta^2/\Delta^2 > \epsilon$. As the signal in Eq. (11) is oscillating at $\omega_f$, we only need to know the magnitude of noise at $\omega_f$.\n

The analytical expressions for the noise $N_o$ and the signal $S_o$ at $\omega = \omega_f$, when $\Delta = 0$, can be obtained from Eq. (8) and Eq. (10) as

$$N_o = \sqrt{2|E|^2 + \frac{1024h^2g^4|E|^6}{m^2\omega_m^4\gamma^2} + \frac{64\hbar g^2|E|^4\omega_f\gamma}{m\omega_m^6\gamma^2}}, \quad (12a)$$

$$S_o = \frac{8g|E|^2}{m\omega_m^2}\cos(\omega_f t). \quad (12b)$$

The first term on the RHS of Eq. (12a) gives the shot noise while the second and third terms give the RPN and thermal noise, respectively. Temperature is assumed to be zero Kelvin in Eq. (12a). The force sensitivity $F_o$ is given as

$$F_o = \frac{m\omega_m^2\xi}{4g} \sqrt{\frac{1}{2|E|^2} + \frac{256\hbar^2g^4|E|^2}{m^2\omega_m^4\gamma^2} + \frac{16\hbar g^2\omega_f\gamma}{m\omega_m^2\gamma}}. \quad (13)$$

The contribution from shot noise and RPN compete in Eq. (13) leading to SQL at $|E|^2 = I_{opt}$. $I_{opt}$ is the intensity at which both the shot noise and RPN are equal. Using Eq. (13), the $I_{opt}$ can be estimated as

$$I_{opt} = \frac{m\omega_m^2\xi^2}{16\sqrt{2}\hbar g^2}. \quad (14)$$

Substituting Eq. (14) into Eq. (13) gives the force sensitivity at $\Delta = 0$ as $F_1 = \sqrt{2}F_{sql}$, where $F_{sql} = \hbar m\omega_m^2$, since $\gamma \ll \omega_m$ and $\omega_f \ll \omega_m$. A prominent property of Eq. (13) is its dependence on $|E|^2$. For $|E|^2 > I_{opt}$ the shot noise contribution decreases but the RPN increases, similarly for $|E|^2 < I_{opt}$ the RPN decreases but the shot noise increases. Hence in Eq. (13), for best sensitivity, we must set $|E|^2 = I_{opt}$ which enforces SQL.

Equation (13) establishes the presence of shot noise, RPN, and thermal noise. The objective of this letter is not only to go beyond the SQL but also to break the squeezed light limit. As a first step optical restoring force is used to suppress RPN and hence surpassing SQL. But also to break the squeezed light limit. As a first step optical restoring force is used to suppress RPN and hence surpassing SQL. However, the strength of the RPN can be significantly reduced by setting $\Delta - \mathcal{R}(\alpha) = 0$ for $\omega \ll \omega_m$ and $\gamma \ll \omega_m$, where $\mathcal{R}$ stands for the real part. Setting $\Delta - \mathcal{R}(\alpha) = 0$ eliminates the RPN contribution from $\mathcal{R}(\alpha)$. The residual RPN from the imaginary part of $\alpha$ is significantly less than the shot noise in the low frequency regime as $\varepsilon \ll 1$. As a result, the method described in this letter is strictly limited to frequencies much smaller than the resonance frequency of the mechanical mirror. At these lower frequencies, the force sensitivity is less because of large RPN. Hence suppressing RPN in the low frequencies is quite important. Note that $\bar{Q}$ is oscillating at $\omega_f$, so we only need to bother about noise at $\omega_f$.

As $\omega_f \ll \omega_m$, we are interested in finding noise where $\varepsilon \ll 1$ is already satisfied. Hence setting $\Delta = \mathcal{R}(\alpha)$ should suppress RPN in our system. Assuming $\omega_f \ll \omega_m$, the magnitude of symmetrized [59] noise spectral density $N$ at $\omega_f$ is evaluated as

$$N = 2|E|^2 + \frac{16g^2|E|^4\hbar m\omega_f\gamma\coth(\hbar \omega_f/2k_BT)}{m^2\omega_m^2(\Delta^2 + \zeta^2/4)}. \quad (16)$$

There is no RPN in Eq. (16) as it is suppressed. We use the thermal correlation [60] $(\bar{\Theta}(\omega)\bar{\Theta}(\omega')) = \hbar m\omega\gamma(1 + \coth(\hbar \omega/2k_BT))\delta(\omega - \omega')$ with temperature $T$, and $k_B$ as Boltzmann constant. We simplify Eq. (16) by assuming that $1 > \zeta/\Delta > \zeta^2/\Delta^2 \gg \varepsilon$. The force sensitivity $F_{\delta}$ is given as

$$F_{\delta} = \frac{m\omega_m^2\sqrt{\Delta^2 + \zeta^2}/4}{2g} \sqrt{1 + \frac{4g^2\omega_f\gamma\coth(\hbar \omega_f/2k_BT)}{m^2\omega_m^2(\Delta^2 + \zeta^2/4)}}. \quad (17)$$

The condition that $\zeta/\Delta$ should lie between 1 and $\varepsilon$ is not necessary for RPN suppression but required for improving $F_{\delta}$ beyond SQL. The first term of the RHS of Eq. (17) gives shot noise contribution while the second term gives the thermal noise contribution. The shot noise in Eq. (17) can be decreased by increasing the intensity, however, the input intensity is constrained by the condition $\Delta = \mathcal{R}(\alpha)$ as

$$\Delta = \frac{4\hbar g^2|\bar{a}|^2}{m\omega_m^2} \Rightarrow 2|E|^2 = \frac{m\omega_m^2(\Delta^2 + \zeta^2/4)}{\hbar g^2\zeta}. \quad (18)$$

Substituting Eq. (18) into Eq. (17) gives the best force sensitivity achievable as

$$F_{\delta} = F_{sql}\sqrt{\frac{\zeta}{4\Delta} + \frac{\gamma\omega_f}{\omega_m^2}}. \quad (19)$$

We assumed $T = 0K$ in Eq. (19). Note that $\gamma\omega_f/\omega_m^2 \ll \zeta/\Delta$. Hence $F_{\delta}$ is better than $F_{sql}$ by a factor of $\sqrt{\zeta/4\Delta}$. The intensity in Eq. (18) is larger than $I_{opt}$ by a factor of $(\Delta^2/\zeta^2 + 1/2\sqrt{2}\Delta/\zeta)$. With suppression of RPN, we are able to increase the intensity beyond $I_{opt}$. However the signal in Eq. (11) is reduced by a factor of $1/\sqrt{4\Delta^2/\zeta^2 + 1}$. Combining these two factors, as net, we see an improvement by a factor of $\sqrt{\zeta/4\Delta}$ beyond $F_{sql}$. Eq. (13) is a well-known equation in optomechanics for establishing the SQL. On the other hand, Eq. (17) is derived after suppressing the RPN. We used approximation given in Eq. (15) and $1 > \zeta/\Delta > \zeta^2/\Delta^2 \gg \varepsilon$, in combination with the strategy of setting $\Delta = \mathcal{R}(\alpha) = 0$ in order to realize Eq. (17). In Fig. 2 we plot the force sensitivity directly using Eq. (9) and Eq. (10).

The simulation parameters for the Fig. 2 are: $m = 10^{-7}$Kg, $\omega_m = 10^5$Hz, $\omega_f = 100$Hz, $\Delta = 100\zeta$, $\zeta = 10^8$Hz, $\gamma = 1$Hz.
The ratio between $\xi$ to $\bar{\xi}$ in Fig. 2 is given by $\bar{\xi}$ of optical power (10$^{-15}$ W) at optical power (1.885 W) corresponds to $|\hat{E}|^2 (4.741 \times 10^{15}$ Hz).

$g = 10^{18}$ Hz/m. The optical power corresponding to the lowest point in Fig. 2 is given by $\hbar m_{\omega_f}|\hat{E}|^2$, where $|\hat{E}|^2$ is given by Eq. (18). The ratio between $F_s$ value from the plot and $F_{sql}$ is equal to $\sqrt{\xi/4\Delta}$. This is in complete agreement with Eq. (17).

The RPN arises because of the competitive evolution between canonically conjugate variables [61]. On the other hand, squeezed light is in a non-classical state [62, 63] which shrinks uncertainty in one conjugate variable while increasing uncertainty in the other. We eliminated RPN from Eq. (9) by suppressing the canonical quadrature of $\hat{Y}_b - \hat{Y}_d$. Now we can further enhance the force sensitivity by using squeezed light to suppress the noise from the remaining quadrature. The most interesting aspect, as shown below, is that the overall force sensitivity is better than what squeezed light alone can achieve.

The force sensitivity in Eq. (19) is derived by assuming that the input fields are vacuum and laser fields. Now lets squeeze the vacuum field [64–67] entering through the empty port of the interferometer so that

$$[U]_\xi = e^{\xi \hat{U}_\theta \hat{U}^\dagger} [0], \quad [V]_\xi = e^{\xi \hat{V}_\phi \hat{V}^\dagger} [0], \quad (20)$$

where $\xi = re^{-i\theta}$ with $r$ as the squeezing parameter and $\theta$ as the squeezing angle. Using Eq. (20) and Eq. (9), the symmetrized noise power spectral density $N_\xi$ of shot noise and RPN is given as

$$N_\xi = 2|\bar{E}|^2 [\cosh(2r) - \sinh(2r)\cos(\theta - 2\phi)] = 2|\bar{E}|^2 e^{-2r}. \quad (21)$$

The final result in Eq. (21) is obtained by considering frequency-dependent squeezing such that the squeezing angle $\theta = 2\phi$. As the squeezing is implemented only on the vacuum field, the signal remains same as in Eq. (7). Hence with the squeezed vacuum, the force sensitivity $F_\xi$ is given as

$$F_\xi = m_{\omega_m}^2 \sqrt{2\Delta^2 + \xi^2} \frac{\sqrt{e^{-2r} + 4g^2|\bar{E}|^4 h\gamma_{\omega_f}}}{8g} \frac{\sqrt{2|\bar{E}|^2 + m_{\omega_m}^2 (\Delta^2 + \xi^2/4)}}{\omega_m}. \quad (22)$$

Substituting Eq. (18) into Eq. (22) gives

$$F_\xi = F_{sql} \sqrt{\frac{\xi}{4\Delta} e^{-2r} + \frac{\gamma_{\omega_f}}{\omega_m^2}}. \quad (23)$$

The RHS of Eq. (23) shows that the sensitivity is improved by a factor of $\sqrt{e^{-2r}\xi/4\Delta}$ beyond $F_{sql}$. In Eq. (23), the squeezed light leads to $e^{-r}$ improvement while the optical restoring force leads to $\sqrt{\xi/4\Delta}$ improvement. Hence using squeezed light in combination with optical restoring force can enhance the interferometer performance beyond the squeezed light limit by a factor of $\sqrt{\xi/4\Delta}$. The method described in this letter is strictly limited to frequencies much smaller than the resonance frequency of the mechanical mirror, and the force sensitivity is improved to $\sqrt{e^{-2r}\xi/4\Delta}$.

**Simulation parameters :** For simulation, we use the following parameters: $m = 10^{-7}$ Kg, $\omega_m = 10^5$ Hz, $\omega_f = 100$ Hz, $\Delta = 100\zeta$, $\zeta = 10^6$, $\gamma = 1$ Hz, $g = 10^{18}$ Hz/m. For these parameters, the optical power is 1.88500 W and force sensitivity is $1.62 \times 10^{-17}$ N/$\sqrt{\text{Hz}}$ which is $4.2 \times 10^{-2}$ times smaller than the force sensitivity at $\Delta = 0$.

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