Investigation on the wear of spur gears generated by modified cutter

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Abstract: Tooth surface wear damage is one of the main causes of gearing system failure. Excessive wear leads to tooth profile loss and an increase in transmission errors, as the worn gear surfaces are no longer conjugate. Thus, the enhancement of gear durability against wear is important for gear application. Recent works show that cutter modification can aid in reducing the tool wear in gear processing, while the wear performance of the gears produced by modified cutters is still unknown. Therefore, this study focuses on the wear performance of the gear generated by modified cutter. Numerical results show that the wear resistance can be enhanced through proper cutter modification.

Keywords: gear wear; cutter modification; gear modification

1 Introduction

Spur gears, which are the key components in many mechanical systems [1–3], are widely used in modern industries, such as aerospace, marine, agriculture, and construction [4–6]. As advanced driving systems enable high speed, wear damages have become the main cause of failure of gear components [7–10].

Thus, gear wear has been actively researched in recent times, and some of these studies are focused on wear mechanism and wear calculation methods.

For mild surface wear, Archard’s wear model [11] is widely used, although it is rather imperfect. As gear wear is usually regarded as a special case of surface wear, research on gear wear can be dated back to the doctoral thesis of Andersson [12]. Wu and Chen [13] then proposed a simple wear calculation method for spur gear and revealed that the worst wear occurred at the beginning of the gear mesh (tooth tips of the driven gear). Flodin and Andersson then published a series of works on spur and helix gear wear prediction [14, 15], in which a modified Archard’s wear model was established, a “single point observation principle” was used, and the tooth stiffness was considered through an empirical model developed by Simon [16]. Brauer and Andersson [17] investigated the wear in spur gears caused by interference using a mixed finite element and analytical approach. They validated that the interference was caused by deviation from the ideal gear geometry and unfavorable deformation during operation; Bajpai et al. [18] proposed a wear prediction methodology for spur and helical gears using a finite element-based gear contact mechanics model in conjunction with Archard’s wear formulation to predict the wear of contacting tooth surfaces. They also investigated the influence of tooth profile deviation through intentional tooth profile modification [19]; Tunaliğer and Tuğ [20] researched the wear in internal gears with the combination of theoretical calculation and experiment. They designed and manufactured a fatigue and wear test equipment that was similar to Forschungsstelle für Zahnradern

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Nomenclature

\( l_p(u,r) \)  \hspace{1cm} \text{Cutter tooth modification function}  \\
\( \xi \)  \hspace{1cm} \text{Tooth profile modification magnitude}  \\
\( \mu_0 \)  \hspace{1cm} \text{Tooth profile modification range}  \\
\( r_c(u,r) \)  \hspace{1cm} \text{Tooth surface of the cutter with parabolic modification}  \\
\( n_c(u,r) \)  \hspace{1cm} \text{Tooth normals of the cutter with parabolic modification}  \\
\( s_t \)  \hspace{1cm} \text{Translation distance of the rack in gear generating}  \\
\( \varphi_g \)  \hspace{1cm} \text{Rotating angle of gear in gear generating}  \\
\( r_g \)  \hspace{1cm} \text{Pitch radius of the gear}  \\
\( \omega_g \)  \hspace{1cm} \text{Velocity of generated gear}  \\
\( v_c \)  \hspace{1cm} \text{Velocity of rack}  \\
\( \varphi_{eg}(u,r,\varphi_g) \)  \hspace{1cm} \text{Relative velocity between the rack and the gear}  \\
\( n_{eg}(u,r,\varphi_g) \)  \hspace{1cm} \text{Tooth normals of the rack in the ground coordinate system}  \\
\( r_{eg}(u,r,\varphi_g) \)  \hspace{1cm} \text{Tooth surface of the modified spur gear}  \\
\( n_{eg}(u,r,\varphi_g) \)  \hspace{1cm} \text{Tooth normals of the modified spur gear}  \\
\( \varphi_i \)  \hspace{1cm} \text{Rotating angle of drive gear in tooth contact analysis (TCA)}  \\
\( \varphi_o \)  \hspace{1cm} \text{Rotating angle of driven gear in TCA}  \\
\( E_{wi} \)  \hspace{1cm} \text{Center distance of the gear pair}  \\
\( r_{ei}(u,r) \)  \hspace{1cm} \text{Tooth surface of drive gear in TCA}  \\
\( r_{eo}(u,r) \)  \hspace{1cm} \text{Tooth surface of driven gear in TCA}  \\
\( r_{ei}(u,r,\varphi_i) \)  \hspace{1cm} \text{Tooth surface of drive gear in ground coordinate system in TCA}  \\
\( r_{eo}(u,r,\varphi_o) \)  \hspace{1cm} \text{Tooth surface of driven gear in ground coordinate system in TCA}  \\
\( n_{ei}(u,r,\varphi_i) \)  \hspace{1cm} \text{Tooth normals of drive gear in ground coordinate system in TCA}  \\
\( n_{eo}(u,r,\varphi_o) \)  \hspace{1cm} \text{Tooth normals of driven gear in ground coordinate system in TCA}  \\
\( v_i(u,r,\varphi_i) \)  \hspace{1cm} \text{Velocity of contact points in drive gear in TCA}  \\
\( v_o(u,r,\varphi_o) \)  \hspace{1cm} \text{Velocity of contact points in driven gear in TCA}  \\
\( v_{wo}(u,r,\varphi_i,\varphi_o) \)  \hspace{1cm} \text{Relative velocity of the contact points in TCA}  \\
\( K_i \)  \hspace{1cm} \text{Curvature of drive gear}  \\
\( K_o \)  \hspace{1cm} \text{Curvature of driven gear}  \\
\( K_{wo} \)  \hspace{1cm} \text{Relative curvature between the drive and driven gears}  \\
\( n_{ej} \)  \hspace{1cm} \text{Number of instant contact points}  \\
\( r_j \)  \hspace{1cm} \text{Position vector in each contact point}  \\
\( F_j \)  \hspace{1cm} \text{Normal contact force in each contact point}  \\
\( C_j \)  \hspace{1cm} \text{Comprehensive stiffness matrix in contact}  \\
\( \delta_j \)  \hspace{1cm} \text{Total deformation of the gear teeth}  \\
\( n_i \)  \hspace{1cm} \text{Unit normal vector of the contact point}  \\
\( P_{HI} \)  \hspace{1cm} \text{Maximum pressure in each contact point}  \\
\( b \)  \hspace{1cm} \text{Contact width}  \\
\( l \)  \hspace{1cm} \text{Tooth width}  \\
\( a_j \)  \hspace{1cm} \text{Semi-Hertzian contact width}  \\
\( v_j \)  \hspace{1cm} \text{Poisson’s ratio for drive gear}  \\
\( \nu_o \)  \hspace{1cm} \text{Poisson’s ratio for driven gear}  \\
\( h \)  \hspace{1cm} \text{Wear depth}  \\
\( K_w \)  \hspace{1cm} \text{Wear coefficient}  \\
\( s_i \)  \hspace{1cm} \text{Slider distance in each micro segment}  \\

und Getreibbau [21] closed circuit power circulation system in working principle. Masjedi and Khonsari [22] developed a procedure to predict film thickness, traction coefficient, and wear rate with provision for the interactions between the surface asperities of the teeth of gear. Additionally, Henneberg et al. [23] proposed a quasi-stationary approach to achieve particle concentration and distribution in gear oil for performing the wear estimation.

Some other works have focused on enhancing the wear resistance to render the gear system more durable: Mao [24] adopted the micro-geometry modification method to investigate gear fatigue wear reduction and proposed an optimized micro-geometry gear tooth through advanced non-linear finite element analysis. İmrek and Düzçükoğlu [25] focused on the relationship between wear and tooth width modification in spur gears. They indicated that the wear depth of the modified gear along the meshing area was almost uniform and better than that of the unmodified gear; Karpat and Ekwaro-Osire [26] studied the influence of tip relief modification on spur gears with asymmetric teeth to reveal that an excessive increase in tip relief modification should be avoided, and the level of
excessive increase was based on tip relief configuration; PrabhuSekar and Sathishkumar [27, 28] proposed a method for enhancing the wear resistance of normal contact ratio spur gear pairs through non-standard gears. They showed that the use of non-standard pinion and gear of unequal tooth thickness could aid in reducing the wear damage. Zhou et al. [29] investigated the normal and tangential oil film stiffness of modified spur gears with non-Newtonian elastohydrodynamic lubrication. They revealed that the modification was valid for inhibiting the jump of oil film stiffness and improving the gear wear resistance. These studies together established that reasonable flank modification could aid in enhancing the gear wear resistance. However, on one hand, as these works are conducted based on an assumed gear tooth modified geometry, realizing their proposed methods in practice is difficult (gear modification is generally obtained by cutter modification) [30–32]. On the other hand, as the modified gears are not conjugate, transmission errors and change of dynamic performance may occur [33].

Some studies focused on the tool wear that happens during the cutting process. Bouzakis et al. [34, 35] established a sophisticated numerical model to predict tool wear in gear hobbing cutting process and revealed that coated hob teeth performed better than the uncoated ones, and tooth profile geometry influenced tool wear. Claudin and Rech [36] developed a new rapid characterization method to enhance the wear resistance of hob in gear manufacturing and established that cutter edge geometry also influenced tool wear; Karpuschewski et al. [37] investigated the geometrical influence of tool profile on wear behavior in gear hobbing. This study suggested that 30\% of the total piece cost for the hobbing process can be reduced with the application of optimized tool profiles and corresponding cutting parameters. These studies established that tool profile modification could aid in enhancing the wear resistance of the tool in the cutting process. However, the wear performance of the gear cut by modified cutter remained unknown.

Based on all the works discussed so far, this study aims to investigate the wear performance of spur gears with tooth profiles generated by modified hobbing cutters. Firstly, the tooth geometry of spur gears generated by modified cutters is obtained, including the tooth modification of basic rack generator and tooth surface of the modified gear. Then the tooth wear calculation is discussed in detail, including the tooth contact analysis, contact force calculation, and wear depth calculation. Finally, the wear performance under different cutter modifications is discussed by considering the planet gear and sun gear of an epicyclical transmission used in the wheel hub of an electric automobile as an example, establishing that wear resistance can be enhanced with appropriate cutter modification.

2 Tooth geometry of spur gears with cutter modification

Face-hobbing cutter is widely used in the manufacture of external spur gears [38]. As the basic geometry of the cutter includes a rack, the cutting process can be regarded as the meshing of a rack and a gear [39]. The modified gear tooth geometry should thus be deduced based on the modified cutter and the kinematic relations generated.

2.1 Tooth modification for the rack generator

As is shown in Fig. 1, the basic unmodified tooth profile appears to be a straight line with a pressure angle \( \alpha \) relative to the pitch line. The coordinate system \( S_o(O_y - x_y y_y z_y) \) is attached at the middle of the rack with its \( x_y \)-axis along the pitch line of the rack. To modify the tooth profile, coordinate systems \( S_i(O_i - x_i y_i z_i) \) and \( S_i(O_r - x_i y_i z_i) \) are established with their corresponding \( x_i \)-axis and \( x_r \)-axis along the basic
unmodified tooth profile. In coordinate systems \( S_i \) and \( S_r \), the tooth profile is defined by parameter \( u \). Two parameters \( u_{0a} \) and \( u_{0b} \) are used to define the starting and ending points of modification, and function \( l_{pf}(u) \) is used to present the offset of the modification. In this study, drawn from the research in Ref. [40], a parabolic function is used to modify the tooth profile, which can be defined in coordinate systems \( S_i \) and \( S_r \) as follows:

\[
l_{pf}(u, r) = \begin{cases} 
\xi(u - u_0)^2, & u > u_0 \\
0, & -u_0 \leq u \leq u_0 \\
-\xi(u - u_0)^2, & u < -u_0 
\end{cases}
\]

where \( u_0 = \mu_0 h_i(r) / \cos(\alpha) \), \( \xi \) is used to define the tooth profile modification magnitude, and \( \mu_0 \) is used to define the tooth profile modification range.

The origins of coordinate systems \( S_i \) and \( S_r \), respectively \( O_i \) and \( O_r \), are the intersection points of the pitch line and unmodified tooth profile. Their distance from \( O_i \) denotes the tooth thickness modification of the gear \( \Delta s_m \).

Thus, the tooth surface of the cutter with parabolic modification can be obtained as follows:

\[
r_e(u, r) = M_{st,r} \left[ \pm l_{pf}(u) \ u \ 0 \ 1 \right]^T
\]

where \( M_{st,r} = \begin{bmatrix} \cos(\alpha) & \pm \sin(\alpha) & 0 & \pm s_m(r) \\ \pm \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \).

Besides, the normals of the cutter are defined as follows:

\[
n_e(u, r) = \frac{dr_e(u, r)}{du} \times \frac{dr_e(u, r)}{dr}
\]

2.1 Tooth surface of the modified gear

As shown in Fig. 2, \( O_0 \) is the center of the gear, and \( O_1 \) is the contact point between the pitch line of the rack and pitch circle of the gear. Four coordinate systems are established to present the generated kinematics. The coordinate system \( S_i(O_0 - x_{0i}y_{0i}z_{0i}) \) is fixed on the ground with \( x_{0i} \)-axis along \( O_0O_1 \), while the coordinate system \( S_r(O_1 - x_{ri}y_{ri}z_{ri}) \) is fixed on the ground with \( x_{ri} \)-axis along the direction of \( x_{0i} \)-axis.

The coordinate system \( S_g(O_g - x_{gi}y_{gi}z_{gi}) \) is attached on the gear with \( z_{gi} \)-axis along the gear axis, and its angle relative to \( S_0 \) is the rotating angle of the gear. The coordinate system \( S_g(O_r - x_{gri}y_{gri}z_{gri}) \) is attached on the rack with \( y_{gri} \)-axis along the translational direction of the rack, and its distance from \( S_i \) denotes the translation distance of the rack \( s_r \).

Based on pure rotation relation, the angle of gear and the translation of the rack must fulfill the following criteria [41]:

\[
s_r = \varphi \left( r_g \right) \quad \text{(4)}
\]

where \( r_g \) is the pitch radius of the gear.

With the coordinate systems established, the relationship between the rack and gear can be obtained as follows:

\[
M_{gr} (\varphi) = M_{s0} (\varphi) M_{0i} M_{1u} (s_r) \quad \text{(5)}
\]

where

\[
M_{0i} (\varphi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \varphi r_g \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad M_{si} = \begin{bmatrix} 1 & 0 & 0 & r_g \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

and

\[
M_{g0} (\varphi) = \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]
Thus, the envelope surface of the rack is defined as follows:
\[
r_g(u, r, \varphi_g) = M_g \varphi_g r_e(u, r)
\] (6)

The meshing equation can be obtained as follows [42]:
\[
n_{wr}(u, r) \cdot v_{eg}(u, r, \varphi_g) = 0
\] (7)

Suppose the velocity of the generated gear \( \omega_g = 1 \), the velocity of the generator must be \( v_c = r_f \varphi_c \), the normals of the generator and the relative velocity in the coordinate system \( S_0 \) can be calculated as follows:
\[
\begin{bmatrix}
v_{eg}(u, r, \varphi_g)
v_{c}
n_{eg}(u, r, \varphi_g)
\end{bmatrix}
= M_{u1} M_{v1} (\varphi_g) r_e(u, r) - M_{u1} M_{v1} (\varphi_g) v_c
\] (8)

Equation (8) is solved using a numerical method. When the parameter \( \varphi_g \) is obtained as \( \varphi_g = \varphi_g(u, r) \) and substituted into Eq. (6), the final tooth surface of the modified spur gear can be obtained as follows:
\[
r_e(u, r) = r_g [\mu, r, \varphi_g(u, r)]
\] (9)

Therefore, the tooth normal is obtained as follows:
\[
n_g(u, r) = M_{g} \varphi_g(u, r) n_c(u, r)
\] (10)

### 3 Calculation of wear

#### 3.1 Tooth contact analysis

To calculate the wear for all the points on the tooth profile, it is important that for any point on the tooth profile of the drive gear, there is a corresponding contact point on the driven gear. Though several methods have been proposed earlier for gear tooth contact analysis [43, 44], this study employs an analytical method.

Figure 3 shows the coordinate systems for the calculation of contact points. Points \( O_1 \) and \( O_2 \) are respectively the drive and driven gear center. The coordinate system \( S_0 \) is fixed on the ground with \( x_f \)-axis along the center distance vector \( O_1 O_2 \). The coordinate system \( S_f \) is attached on the driven gear with its angle relative to \( S_f \) being the rotating angle of drive gear \( \varphi_f \). The coordinate system \( S_o \) is attached on the drive gear with its angle relative to \( S_f \) being the rotating angle of drive gear \( \varphi_o \). The coordinate transformation from the drive and driven gears to the fixed coordinate \( S_f \) can be obtained as follows:
\[
M_{f1}(\varphi_f) = \begin{bmatrix}
\cos(\varphi_f) & -\sin(\varphi_f) & 0 & 0 \\
\sin(\varphi_f) & \cos(\varphi_f) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
(11)
\[
M_{fo}(\varphi_o) = \begin{bmatrix}
\cos(\varphi_o) & -\sin(\varphi_o) & 0 & 0 \\
\sin(\varphi_o) & \cos(\varphi_o) & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where \( E_{io} \) is the center distance of the gear pair.

The tooth profile of the drive and driven gears are redefined as \( r_{g1}(u, r) \) and \( r_{g2}(u, r) \), respectively, and the tooth surface in \( S_f \) can be obtained as follows:
\[
\begin{bmatrix}
r_{f1}(u, r, \varphi_f)
r_{f2}(u, r, \varphi_o)
\end{bmatrix}
= M_{f1}(\varphi_f) r_{g1}(u, r) - M_{fo}(\varphi_o) r_{g2}(u, r)
\] (12)

Similarly, the tooth normals in \( S_f \) are obtained as follows:
\[
\begin{bmatrix}
n_{f1}(u, r, \varphi_f)
n_{f2}(u, r, \varphi_o)
\end{bmatrix}
= M_{f1}(\varphi_f) n_{g1}(u, r) - M_{fo}(\varphi_o) n_{g2}(u, r)
\] (13)

In addition, the velocity of contact points in the coordinate system \( S_f \) relative to the drive and driven gears are respectively defined as follows:
Further, the relative velocity of the contact points between the drive and driven gears is obtained as follows:

\[
\begin{aligned}
\mathbf{v}_d(u, r, \phi) &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \times \mathbf{r}_d(u, r, \phi) \\
\mathbf{v}_o(u, r, \phi) &= \begin{bmatrix} 0 \\ 0 \\ \phi \end{bmatrix} \times \mathbf{r}_o(u, r, \phi)
\end{aligned}
\]  
(14)

Thus, for any conjugate point, including parameter \(u, r\) in the drive gear and \(u, r\) in the driven gear, the following conditions must be satisfied:

\[
\begin{aligned}
\mathbf{n}_d(u, r, \phi) &= \mathbf{n}_o(u, r, \phi) \\
\mathbf{r}_d(u, r, \phi) &= \mathbf{r}_o(u, r, \phi) \\
\mathbf{n}_d(u, r, \phi) \times \mathbf{n}_o(u, r, r, \phi) &= 0
\end{aligned}
\]  
(16)

This implies that for any point \(P_i(u, r, r)\) on the tooth of the drive gear, the angular position of the drive \(\phi_i\), angular position of the driven gear \(\phi_o\), and the contact point \(P_o(u, r, r)\) on the tooth of the driven gear can be obtained.

For spur gears, the tooth surface can be simplified as a planar curve. Therefore, the curvature of the drive gear \(K_i\) and that of the driven gear \(K_o\) can be obtained based on the planar curvature equation. The relative curvature is defined as follows:

\[
K = K_i + K_o
\]  
(17)

### 3.2 Contact force

As shown in Fig. 4, for any moment during the driving process defined by subscript \(j\), suppose the instant contact point number is \(n_j\), then each of the points is defined as \(P_j(j = 1, 2, ..., n_j)\). The total torque applied to the driven gear is shared by these contact points.

\[
T_o = \sum_{j=1}^{n} T_j \times r_j
\]  
(18)

where \(T_o\) is the load applied to the driven gear, and \(r_j\) is the position vector of the contact point. \(T_j\) is the normal contact force for each point, which is calculated as follows:

\[
T_j = C \delta_j n_j
\]  
(19)

where \(C\) is the comprehensive stiffness matrix, \(\delta_j\) is the total deformation of the gear teeth, and \(n_j\) is the unit normal vector of the contact point.

Due to the complicated tooth geometry of spur gears, no accurate analytical algorithm has been proposed for force calculation so far. Therefore, Simon proposed an empirical method based on finite element method [16]. However, this study was established to be inaccurate and applicable only to standard involute gears. Thus, our current work applies the finite element method directly for force investigation.

The maximum pressure at each contact point can be calculated based on the Hertzian contact model [45, 46] as follows:

\[
P_{ij} = \frac{2F_j}{\pi b}
\]  
(20)

where \(b\) is the contact width, which is equal to the tooth width \(l\) in case of spur gears; \(a_j\) is the semi-Hertzian contact width, calculated as follows:

\[
a_j = \left( \frac{4F_j}{E_i E_o} \right)^{1/2} \frac{1 - \nu_i^2 + 1 - \nu_o^2}{K_w}
\]  
(21)

where \(\nu_i\) and \(\nu_o\) are the Poisson’s ratios for the drive and driven gears, respectively, and \(E_i\) and \(E_o\) are the Young’s modulus for the drive and driven gears, respectively.
3.3 Wear depth calculation

Based on the generalized Archard’s wear equation [7], the wear depth can be calculated as follows:

\[ h = \int_0^s K_w P \, ds \]  \hspace{1cm} (22)

where \( K_w \) is the wear coefficient, which is set to \( 5 \times 10^{-16} \text{ m}^2/\text{N} \) [15]. This equation can also be rewritten differently as follows:

\[ h_i = K_w P_i s_i + h_{i-1} \]  \hspace{1cm} (23)

where \( s_i \) is the slider distance in each micro segment, which can be obtained based on the point contact features of the gear surface.

For drive gear:

\[ s_i = d_i \frac{\left| v_w(u_i, r_i, u_i, r_i, \varphi_i, \varphi_i) \right|}{\left| v_f(u_i, r_i, \varphi_i) \right|} \]  \hspace{1cm} (24)

For driven gear:

\[ s_i = d_i \frac{\left| v_w(u_i, r_i, u_i, r_i, \varphi_i, \varphi_i) \right|}{\left| v_p(u_i, r_i, \varphi_i) \right|} \]  \hspace{1cm} (25)

4 Example and discussion

4.1 Finite element analysis

As discussed earlier, the force applied to the gear contact point is obtained through a finite element analysis in the third commercial software ANSYS, and the wear depth can be calculated based on the value of force.

In this research, the planet gear and sun gear of an epicyclical transmission used in the wheel hub of an electric automobile is considered as an example. The basic parameters of the gear pair are shown in Table 1.

As illustrated in Fig. 5, four tooth pairs are used to maintain the boundary adequately far for the tooth-loaded areas. The finite model is meshed with SOLID187 elements, which are high-order 3-D 10-node elements with quadratic displacement behavior and well suited for large strain deflections. The material is steel with a Young’s modulus of \( E = 2.1 \times 10^5 \text{ MPa} \) and Poisson’s ratio of 0.3. The contact type, without considering the frictional force, is set to be frictionless.

The pinion tooth flanks are set as master surfaces (CONTACT174 elements), and the wheel tooth flanks are set as slave surfaces (TARGE170 elements). Pure penalty formulation is used for calculating the contact.

To investigate the contact force at each point of time during a meshing cycle, the analysis type is set as transient structure. During the analysis, the revolution joints are set on the inner bore surface of the pinion and the gear. The velocity load is set to 6.28 on the pinion revolution, and a torque is set on the gear revolution. The analysis time is set to 0.08 s, and the sub step is set to 30 to enable the tooth surface pair contact from the root to the tip.

A convergence study is conducted with a fine mesh of elements at the contact region to assess the approximate number of elements required. The contact pressure at the initial position, obtained through the convergence study, is shown in Fig. 6. It is observed

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**Table 1** Basic design parameters for gear pair.

| Parameter                | Value |
|--------------------------|-------|
| Module (mm)              | 1.25  |
| Tooth number of pinion   | 19    |
| Tooth number of gear     | 66    |
| Addendum coefficient     | 1     |
| Dedendum coefficient     | 1.25  |
| Face width               | 20    |

**Table 2** Setting for the finite element analysis.

| Parameter                  | Value       |
|----------------------------|-------------|
| Young’s modulus            | 2.1×10^5 MPa|
| Poisson’s ratio            | 0.3         |
| Contact type               | Frictionless|
| Contact formulation        | Pure penalty|
| Master surfaces            | Pinion surface|
| Slave surfaces             | Gear surface|
| Torque applied the gear    | 1,000 N-mm  |
| Velocity of the pinion     | 6.28 rad/s  |
| Analysis time              | 0.08 s      |
| Analysis steps             | 30          |

**Fig. 5** Finite element model for spur gear.
that the contact pressure level converges to a constant level, when the number of elements is 45,000 and over. Thus, the model is discretized by a very fine mesh with 60,000 elements (60,000 > convergence level).

### 4.2 Results and discussion

In this study, for investigation of the influence of cutter modification on wear, a standard gear without cutter modification is denoted as Case 0, while the eight modified gears with different cutter modification parameters are denoted as Case 1 to Case 8 as shown in Table 3. Among them, for Case 1, Case 3, Case 5, and Case 7, four magnitude ranges are set to investigate the influence of the modification range. With any modification range of Case 1, Case 3, Case 5, and Case 7, a doubled modification magnitude is considered, as shown correspondingly in Case 2, Case 4, Case 6, and Case 8.

### Table 3 Modification parameters and tooth geometry in each case.

| Modification magnitude $\xi$ | Modification range $\mu$ | Tooth geometry |
|------------------------------|--------------------------|----------------|
| Case 0                       | 0                        | 0              |
| Case 1                       | 0.2                      | 0.75           |
| Case 2                       | 0.4                      | 0.75           |
| Case 3                       | 0.13                     | 0.5            |
| Case 4                       | 0.26                     | 0.5            |
| Case 5                       | 0.08                     | 0.25           |
| Case 6                       | 0.16                     | 0.25           |
| Case 7                       | 0.055                    | 0              |
| Case 8                       | 0.11                     | 0              |
Case 8, to investigate the influence of modification magnitude. It is worth noting that the influence of the modification magnitude is different with different modification ranges. Thus, the modification magnitude should not be considered the same under different modification ranges.

Varied tooth geometry against each case is also shown in the table. The influence of the modification parameters on the tooth shape can be deduced from the tooth geometry. The modification range determines the starting point of modification, and along with its decrease, the modified tooth shows a higher deviation from the standard tooth in Case 0. The modification magnitude influences the deviation magnitude, and along with its increase, the tooth tip and tooth root decline.

Figure 7 shows the normal contact force at different points in the gear. It is observed that the contact force is not symmetrical without cutter modification (Case 0), and it becomes relatively symmetrical with cutter modification (Case 1–Case 8). In addition, the maximum contact force is basically similar for all cases, implying that the maximum contact force is not influenced by cutter modification. However, the total contact force is increases with an increase in either tooth modification magnitude or modification range.

Figure 8 shows the semi contact width at different points in the gear. It is observed that the curve shape becomes symmetrical with cutter modification. In addition, for Case 1–Case 8, the value of contact width is almost the same at the middle tooth height point. However, at other points, it is increased with an increase in either modification magnitude or modification range. This can be explained by the change of contact force and relative curvature.

Figure 9 shows the sliding distance at different points in the gear. It is observed that the sliding distance decreases with an increase in the modification magnitude.
magnitude and modification range. This can be explained by the fact that the distance between the contact point and the corresponding instant center decreases with an increase in modification.

Figure 10 shows the wear depth at different points in the gear. For the standard case (Case 0), the wear depth is much higher near the tooth tip and lower near the tooth root. For the gear with cutter modification, the wear near the tooth tip reduces, while that near the tooth root increases. Case 4 is observed to be the best modification, in which the wear near the tooth tip and that near the tooth root are almost the same, and the maximum wear depth is lower than that of the standard gear.

Therefore, with proper cutter modification (Case 4), the surface wear becomes even, and the maximum wear depth is lower, implying that reshaping tooth geometry through cutter modification can enhance the wear resistance for spur gears.

5 Conclusions

This study establishes a mathematic model for modified gears generated by modified rack cutters and investigates wear performance under the different modifications of the cutter through a mixed finite element model and analytical approach. Based on the results and discussions, several conclusions are drawn as follows:

(1) With the proposed cutter modification, the gear flank wear reduces near the tooth tip, while it increases near the tooth root.

(2) With proper cutter modification, the gear flank wear is uniform throughout the flank, as the maximum wear depth is decreased, which enhances the wear resistance for spur gears (as shown in Case 4).

Fig. 9 Sliding distance at different points in the gear.

Fig. 10 Wear depth at different points in the gear.
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