Image description by harmonic Fourier moment

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Abstract. In this paper, Two kinks of orthogonal moments are constructed with Trigonometric functions, named Harmonic-Fourier moments (HFMs), which include Cosine-Fourier moments (CFMs) and Sine-Fourier moments (SFMs). These kinds of moments can be used in image and object analysis. The radial functions of HFMs have more zeros than Zernike polynomials of the same degree so that the property makes HFMs have stronger ability in image description. Furthermore, the formulas of HFMs are extremely simple. This property makes it possible that HFMs can be computed at a higher speed.

1. Introduction
Since Hu introduces moments invariants[1], image moments have been used as pattern features in many different fields, such as watermarking and data-hiding[2], visual pattern recognition [3, 4], object classification[5]. Among the different kinds of moments, geometric moments[1] have played important roles in image recognition. However, geometric moments have no orthogonality, which leads to geometric moments contains too much redundancy information[6]. In addition, it is difficult to reconstruct image with these moments.

To solve the problem associated with geometric moments, many kinds of orthogonal image moments are proposed based on various kinds of polynomials, such as orthogonal Fourier-Mellin moments(OFMs)[7], Chebyshev-Fourier moments[8] and Jacobi-Fourier moments(JFMs)[9], Bessel-Fourier moments(BFMs)[10].

The calculation of polynomials need cost a great deal of CPU time, so all above moments with polynomial radial functions are quite computationally expensive. In 2002, Ping et al. pointed out that orthogonal moments can be constructed with arbitrary orthogonal functions[8]. In 2003, Radial harmonic Fourier moments (RHFM) based on the trigonometric functions are proposed[11].

According to the orthogonal theory, the function family \( \cos(n\theta) \) \((n = 0,1,2,...)\) and \( \sin(m\theta) \) \((m = 1,2,3,...)\) are complete and orthogonal over the interval \((0,\pi)\), thus, we can directly use the cosine and sine functions as radial functions to construct orthogonal image moments respectively. In this paper, two different moments are proposed, namely, Cosine-Fourier moments (CFMs) and Sine-Fourier moments (SFMs), we group them under the name Harmonic-Fourier moments (HFMs). The kernel functions of HFMs are significantly simpler compared with those of ZMs, PZMs and other moments based on polynomials, thus, computational complexity of HFMs is far less than those of moments based on polynomials. Similar to other orthogonal moments with polynomials, it is easy to use them to conduct image reconstruction and recognition. However, they have better performance than ZMs, PZMs and other moments based on polynomials.
2. Harmonic-Fourier moments (HFM)

2.1 The radial function of the Cosine-Fourier moments

Kernel functions of orthogonal image moment based on cosine and sine function are as follows:

\[
T_n^C(r) = \begin{cases} 
\frac{1}{\sqrt{2\pi r}} & n = 0 \\
\frac{1}{\sqrt{\pi r}} \cos(n\pi r) & n \geq 1 
\end{cases}
\]

\(n = 0, 1, 2, 3, 4, \ldots\)

\(T_n^S(r) = \frac{1}{\sqrt{\pi r}} \sin(n\pi r)\)

\(n = 1, 2, 3, 4, \ldots\)

Obviously, the set \(T_n^C(r)\) and \(T_n^S(r)\) are orthogonal and complete over the interval \(0 \leq r \leq 1\). \(T_n^C(r)\) and \(T_n^S(r)\) are defined as the radial function of the Cosine-Fourier moments, which is orthogonal and complete over interior of the unit circle.

2.2 Harmonic Fourier moments

In the polar system, we can use \(T_n^C(r)\) as radial function to define Cosine-Fourier moments (CFMs) as:

\[
H_{nm}^C = \int_0^{2\pi} \int_0^1 f(r, \theta) \left[ T_n^C(r) \right]^* \exp(-jm\theta) r dr d\theta 
\]

Where \(f(r, \theta)\) is the image and \(n = 0, 1, 2, 3, 4, \ldots\), \(m = \pm 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots\), are the moment orders, \(\left[ T_n^C(r) \right]^*\) is the conjugate function of the \(T_n^C(r)\).

Similar to the form of \(H_{nm}^C\), We define Sine-Fourier moments (SFMs) with the radial function \(T_n^S(r)\) in the polar system as:

\[
H_{nm}^S = \int_0^{2\pi} \int_0^1 f(r, \theta) \left[ T_n^S(r) \right]^* \exp(-jm\theta) r dr d\theta 
\]

Where \(f(r, \theta)\) is the image and \(n = 1, 2, 3, 4, \ldots\), \(m = \pm 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots\), are the moment orders, \(\left[ T_n^S(r) \right]^*\) is the conjugate function of the \(T_n^S(r)\).

3. Property of Harmonic Fourier moments

The more zeros radial functions have, the stronger ability to catching images information the moments possess, and the positions of their zeros indicate the moments’ sampling positions in the image[12].

Fig1 shows the plots of the radial function \(T_n^C(r)\) with \(n = 0, 1, 2, 3, 4, 5\), Fig2 shows the plots of the radial function \(T_n^S(r)\) with \(n = 0, 1, 2, 3, 4, 5, 6\), Fig3 shows the Zernike polynomials \(R_n^{(m)}(r)\) with \(m = 10\) and \(n = 10, 12, 14, \ldots, 28\), Fig4 shows the RHFMs polynomials \(Q_n(r)\) with \(n = 0, 1, 2, 3, 4, 5\). As illustrated in Fig1, the radial polynomial \(T_n^C(r)\) has \(n\) zeros in the interior of the interval \(0 \leq r \leq 1\), and the Fig2 shows that the radial polynomial \(T_n^S(r)\) has \(n+1\) zeros, while there are only \((n-m)/2\) zeros in the Zernike polynomial \(R_n^{(m)}(r)\) of the same degree \(n\). The occurrences of zeros in \(T_n^C(r)\) and \(T_n^S(r)\) are similar to that in \(Q_n(r)\). Therefore, the radial functions need much lower degree \(n\) to represent an image.

Moreover, comparing the positions of the zeros of \(T_n(\theta)\) and \(R_n^{(m)}(r)\), It can be seen that the zeros
of $T_n(r)$ are almost uniformly distributed over the interval $0 \leq r \leq 1$, which shows that margin and center of the image have the same contribution for HFM. However, the zeros of $R_n^H(r)$ are stand far from the origin, its uneven distribution of zeros leads to unnecessary emphasis on marginal part of the image and negligence of the center.

4. Experiments and analysis
In this section, several experimental results provided prove the theoretical framework proposed in the above sections.

Fig. 1 the radial function $T_n^C(r)$ of CFMs with $n = 0, 1, 2, 3, 4, 5$;

Fig. 2 the radial function $T_n^S(r)$ of SFMs with $n = 1, 2, 3, 4, 5, 6$;

Fig. 3 Zernike radial polynomial $R_n^{10}(r)$ with $m=10$ and $n = 10, 12, ..., 28$;
4.1 Image reconstruction from Harmonic Fourier moments (HFM)

In this subsection, we discuss the image reconstruction capability of HFM. In this experiment, the mean square reconstruction error (MSRE) is used to measure the performance of the reconstruction. The formula of image reconstruction from HFM is as follows:

$$\hat{f}(r, \theta) = \sum_{n=-m_{\text{max}}}^{m_{\text{max}}} \sum_{m=-n_{\text{max}}}^{m_{\text{max}}} H_{n m}(r) \exp(j m \theta)$$

Where $m_{\text{max}}$, $n_{\text{max}}$ are the highest orders of the HFM used in image reconstruction.

To reconstruct image with moments, the letter “R” with size of 64x64, as shown in Fig. 5, is used as test image, and the experimental results are compared with those of PZMs and ZMs. Fig. 6 shows the comparison between the reconstructed images. As more moments are added to the reconstruction image, the images reconstructed are getting closer to Fig. 6. As shown in these figures, in early orders, the images reconstructed from HFM more resemble the original image than those from ZMs and PZMs. This result can be explained by Fig 1-3. Zernike polynomials have less zeros than Radial functions of the Harmonic Fourier moments, and the their zeros are less than Radial functions of the Harmonic Fourier moments, which makes it less effective for Zernike moments to describe an image. As for PZM, the reconstructed images are not only fuzzier than those from Sine-Fourier moments, but also getting blurred rapidly as the number of moments is increasing, which is resulted from the numerical instability breakdown of PZMs.

Fig. 7 shows the reconstruction error plots for the different numbers of moments. As shown in the figures, the MSRE of PZMs climbs suddenly as the number of moments increasing to a certain level, which implies that the computation of PZMs breaks down. The numerical stability issue also leads to the fact that the MSRE of CFMs begins to go up as the number of moments increasing to a certain value. SFMs and ZMs also have numerical instability issue, they break down later in the curve (not shown in the figure). The experimental results show that SFM has better image representation capability than the rest.

4.2 Computational complexities

In this section, the computational complexity of HFM is discussed by computing certain number of image moments.

Fig. 5 Original image with size 64x64, the range of pixel values is [0, 255]
Fig. 6 reconstructed images of the letter ‘R’ of 64x64. \( K = 1, 2, 3\ldots 33 \).

Fig. 7 Comparative study of reconstruction error of SFMs, CFMs, ZMs and PZMs of the letter “R”, the size of image is 64x64.

Since the radial functions of HFMs are Trigonometric functions, their formulas are very simple. Their computational complexity is much less than those of the moments based on polynomials. An image of size 100x100 was used as testing object. Fig. 8 gives a comparison of computation time between the ZMs, PZMs, HFMs and RHFMs. As shown in the figure, the time consumption of Harmonic Fourier moments is only approximate half of ZMs and much less than Pseudo-Zernike moments, which is equal to RHFMs.

Fig. 8 The time consumption for computing the Harmonic moments, Zernike moments, Pseudo-Zernike and radial-Harmonic-Fourier moments

5. Conclusion
In this paper, two new orthogonal moments, named Cosine-Fourier moments (CFMs) and Sine-Fourier moments are constructed. We group them under the name Harmonic-Fourier moments(HFMs), and they
have similar properties to RHFMs, but their formulas are simpler. Orthogonal Harmonic functions have much more zeros than polynomials of the same degree. This property makes HFMs have stronger ability in describing image than ZMs, but the radial functions of Cosine-Fourier moments have a singularity at the origin, this property results in numerical instability issue. Furthermore, the computational complexities of the Harmonic-Fourier moments are much less than other moments with polynomials and approximately equal to RHFMs. The study shows that in image analysis the new moments can be used as feature descriptor.

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