An Iterative Algorithm for Optimal Carrier Sensing Threshold in Random CSMA/CA Wireless Networks

Dong Min Kim and Seong-Lyun Kim

Abstract—We investigate the optimal carrier sensing threshold in random CSMA/CA networks considering the effect of binary exponential backoff. We propose an iterative algorithm for optimizing the carrier sensing threshold and hence maximizing the area spectral efficiency. We verify that simulations are consistent with our analytical results.

Index Terms—CSMA/CA, carrier sensing threshold, iterative algorithm, stochastic geometry

I. INTRODUCTION

To enhance wireless connectivity and capacity, efficient multiple access schemes for spatially randomly distributed nodes are necessary. The most widely used multiple access scheme is Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA). In this letter, we propose an iterative algorithm for finding the optimal carrier sensing threshold of spatially randomly distributed CSMA/CA wireless networks.

The fundamental processes in CSMA/CA are carrier sensing and random backoff. Carrier sensing provides the spatial resolution to concurrent transmitters and random backoff gives the temporal resolution to concurrent transmitters at the nearby place. The IEEE 802.11 Distributed Coordination Function (DCF) [1] utilizes physical carrier sensing (optionally virtual carrier sensing) and binary exponential backoff (BEB). In physical carrier sensing, such as the energy detection method, the node senses the medium to measure the aggregate interference, and transmission can begin only if the measured interference is below the carrier sensing threshold. In virtual carrier sensing, the node that intends to transmit performs proactive actions to prevent nodes in the vicinity from transmitting simultaneously with it. When the medium becomes idle, multiple transmitters would access simultaneously, causing collisions. By utilizing BEB, contention conflict is avoidable.

The carrier sensing threshold is a significant parameter to balance the tradeoff between the spatial reuse and the packet collision by controlling the aggregate interference. In [2], it is noted that optimizing carrier sensing is important to increase the throughput performance. The authors of [2] investigate the optimal carrier sensing range under the regular hexagonal topology, whereas we consider the spatially randomly distributed interferers to realistically capture the effect of interference using stochastic geometry [3].

Some researches conducted to determine the spatial distribution of transmitting nodes in the CSMA/CA network. In one such work [4], the authors applied the Matérn hard-core process (MHP) [3] to model the spatial transmitter pattern.

MHP is a dependent thinning process of the Poisson point process (PPP) used to create separation of the marked points by at least a certain minimum distance. In [5], the authors proposed a simple sequential inhibition (SSI) point process to model for the same purpose, which is not mathematically tractable. In [4] and [5], backoff scheme was not considered and the optimal carrier sensing threshold is not provided.

Later, the authors of [6] investigated the throughput performance of dense CSMA networks from the stochastic geometry point of view. However, the authors of [6] did not find the optimal carrier sensing threshold and did not consider the effect of random backoff either. In [7], the authors investigated the optimal carrier sensing threshold based on a lower bound for the outage probability. They considered the effect of one strong interference and simplified the backoff scheme, whereas we consider the effect of aggregate interference and the effect of BEB, such as, collisions in the contention period, increasing of the backoff interval and backoff freezing behavior.

II. PROBLEM DEFINITION

The area spectral efficiency $\eta$ (ASE), which is defined as the product of successfully transmitting node density and the data rate, provides a framework to quantify the capacity of the wireless network [8]. Our problem is to find the optimal carrier sensing threshold $I_s^*$ that maximizes ASE $\eta$ as follows:

$$I_s^* = \arg \max_{I_s} \lambda_s \log_2 (1 + \beta) p_s,$$

where $\lambda_s$ denotes the active transmitter density in the contention free period of CSMA/CA and $\beta$ means the target SIR. The transmission success probability is denoted by $p_s$.

We propose an iterative algorithm for finding $I_s^*$ as described in Algorithm 1. We will explain how the proposed algorithm is obtained and show the performance of the algorithm.

III. SYSTEM MODEL

1) Topology and Channel Modeling: Consider a wireless network, in which all transmitters communicate with their receivers over a common wireless channel. Transmitters are located according to a homogeneous PPP with intensity $\lambda$. This kind of network topology is called the Poisson bipolar network [9]. Each transmitter $i$ has infinite backlogged data to transmit. The transmitter/receiver pairs vary over time, but we focus on a snapshot of the overall communication process. The channel gain from transmitter $i$ to receiver $j$ is modeled by $g_{ij}d_{ij}^{-\alpha}$, where $g_{ij}$ is an independently and identically distributed exponential random variable with unit mean, which reflects the effect of Rayleigh fading. The distance between nodes $i$ and $j$ is denoted by $d_{ij}$ with the path loss exponent $\alpha$. Using a common channel, different communication pairs can interfere with one another. Let $P$ be the transmit power and
an associated receiver \(j\) is at a distance of \(r_i\) from the typical transmitter \(i\). Assuming the network is interference-limited and the receiver noise is ignored, then the signal-to-interference-ratio (SIR) \(\gamma_j\) is given by:

\[
\gamma_j = \frac{g_i r_i^{-\alpha} P}{\sum_{u \in \mathcal{T}_i, j \neq i} g_u r_u^{-\alpha} P} = \frac{g_i r_i^{-\alpha} P}{I},
\]

where \(I\) denotes the aggregate interference and \(\mathcal{T}_i\) denotes the set of concurrently transmitting (interfering) nodes when node \(i\) transmits. For a given target SIR \(\beta\), a transmission succeeds if \(\gamma_j\) is greater than or equal to \(\beta\). The data rate of the typical transmitter \(i\) is a function of \(\beta\). We use Shannon’s formula

\[
\text{capacity} = \frac{h}{1 + \beta} \log_2 (1 + \beta)
\]

Algorithm 1 Proposed Algorithm.

1. Initialize \(I_{next}^s\) with a small value less than \(r_i^{-\alpha} P\)
2. \(I_{current}^s \leftarrow I_{next}^s + 1\)
3. while \(I_{next}^s \neq I_{current}^s\) do
4. \(I_{next}^s \leftarrow \tau_{current}^s\)
5. \(\tau_{current}^s \leftarrow 1, \tau_{next}^s \leftarrow 0\)
6. while \(\tau_{next}^s \neq \tau_{current}^s\) do
7. \(\tau_{current}^s \leftarrow \tau_{next}^s\)
8. \(\tau_{next}^s \leftarrow \tau_{current}^s - \frac{(\tau_{next}^s - \lambda t_{current})}{(1 - h' t_{current})} \quad \triangleright h\) is RHS of (5)
9. end while
10. \(\tau \leftarrow \tau_{next}^s\)
11. update \(\eta\) with \(\tau\) \quad \triangleright \eta\) is (10)
12. \(I_{next}^s \leftarrow I_{current}^s - \frac{\eta (t_{current})}{\eta (t_{current})}\)
13. end while
14. \(I_s \leftarrow I_{next}^s\) \quad \triangleright get the optimal carrier sensing threshold

### Table I: The medium access probability \(p_b\).

| \(I_s\) (dBm) | 0.0001 | 0.001 | 0.01 |
|---------------|--------|-------|------|
| \(\beta\) (dB) | -10    | -10   | -10  |
| \(r\) (simul.)| 0.045  | 0.05  | 0.05 |
| \(r\) (analy.)| 0.037  | 0.04  | 0.05 |

Due to space limitation, we omit the derivation of (5). Please see http://hertz.yonsei.ac.kr/tau.pdf.

A homogeneous Poisson point process with \(\lambda\) in infinite area becomes a uniform distribution of \(k\) nodes on the finite area of size \(A\), where \(k = \lambda A\).

A. B. 

1. Due to space limitation, we omit the derivation of (5). Please see http://hertz.yonsei.ac.kr/tau.pdf.

2. A homogeneous Poisson point process with \(\lambda\) in infinite area becomes a uniform distribution of \(k\) nodes on the finite area of size \(A\), where \(k = \lambda A\).
MHP [3], the active transmitter density $\lambda_t$ in the contention-free period can be modeled as follows:

$$\lambda_t = \frac{1 - \exp\left(-\lambda_t \pi R_s^2\right)}{\pi R_s^2},$$

(7)

where $R_s$ denotes the sensing range. The value $\tau$ is the probability that an arbitrary transmitter in the network completes the BEB process (i.e., backoff counter reaches 0) and accesses the channel. Therefore, $\lambda_t \tau$ accurately represents the node density of contending nodes. In this regard, we used a thinned node density $\lambda_t \tau$ instead of $\lambda$ to model the effect of BEB in (7).

The sensing range $R_s$ is a function of the physical carrier sensing threshold $I_s$. In [14], the authors used the mean value of the sensing range, which is given as follows:

$$R_s = D_s \int_{0}^{D_0} f_1 (r) \, dr + \sum_{i=1}^{5} D_{s,i} \int_{D_{s,i}}^{D_0} f_2 (r) \, dr + D_0 \int_{D_0}^{\infty} f_3 (r) \, dr,$$

(8)

where $f_1 (r) = r^2 \exp(-\lambda r^2)$, $D_1 = \sqrt{(i-1)P/I_s}$ for $i = 0, \ldots, 5$. The variable $D_i$ means the minimum distance from an arbitrary node to the interferers. It is clear that $R_s$ is in inverse proportion to $I_s$. However, the dynamics between them is affected by the node density. In the sparse node density case, it is most probable that there is only one interferer nearby the sensing node. As node density grows, at most six strong interferers can exist at the same distance (refer to [14] for detail). In this regard, $R_s$ can be approximated as $R_s \approx D_0$ and $R_s \approx D_5$ for sparse and dense cases, respectively. In next section, we will find the optimal carrier sensing threshold $I_s^*$.

IV. OPTIMAL CARRIER SENSING THRESHOLD

When $I_s$ is high, most transmitters are simultaneously transmitting, making the success probability low. For lower $I_s$, more transmitters are silent, and the aggregate interference is less, leading to a higher success probability. Thus, there exists an optimal carrier sensing threshold that maximizes the ASE of (1). Fig. 1 shows the ASE of the CSMA/CA scheme as a function of $I_s$. The simulation and our analytical results are congruent. We observe that an optimal $I_s$ exists, which is obtained by solving (1). To this end, the transmission success probability $p_s$ in (1) is derived in the next subsection.

A. Transmission Success Probability of CSMA/CA

With the carrier sensing range $R_s$, the other transmitters within $R_s$ should be silenced. The transmission of a typical transmitter is successful if $\gamma_j \geq \beta$ is satisfied. Assuming path loss exponent $\alpha = 4$, which is validated for urban area, the transmission success probability $p_s$ can be approximated in closed-form as follows:

$$p_s \approx \exp\left(-\pi \lambda_s \sqrt{\beta r_s^2} \arctan\left(\frac{\sqrt{\beta r_s^2}}{R_s^2}\right)\right).$$

(9)

Details of the derivation are contained in Appendix.

B. Proposed Algorithm

We now explain our main result for the optimal carrier sensing threshold. Using (9), the ASE of (1) is as follows:

$$\eta = \lambda_t \log_2 (1 + \beta) \exp\left(-\pi \lambda_t \sqrt{\beta r_s^2} \arctan\left(\frac{\sqrt{\beta r_s^2}}{R_s^2}\right)\right).$$

(10)

Equation (10) is a function of $R_s$, and $R_s$ is a function of $I_s$ as shown in (8). The value $\lambda_t$ is obtained using (7). Unfortunately, the closed-form solution of (1) is hard to find. One way to deal with the problem is making an algorithm where the transmitters update their sensing thresholds iteratively and distributively. Our proposed algorithm is given as follows:

1. First, initialize the carrier sensing threshold $I_s^{(0)}$ with a small value less than $\eta^{-1} P$.
2. Next, find the value $\tau$ using Newton’s method (6).
3. Update $I_s^{(n+1)}$ using the following Newton’s method:

$$I_s^{(n+1)} = I_s^{(n)} - \frac{\eta' (I_s^{(n)})}{\eta'' (I_s^{(n)})},$$

(11)

where $\eta'$ and $\eta''$ denote the first and second derivatives of $\eta$ of (10) with respect to $I_s$.
4. Repeat procedures 2) and 3) until the solution is found.

The pseudo code of proposed algorithm is described in Algorithm 1. The proposed algorithm converges to an optimal value within a few iterations. The transmission distance $r_i$ can be estimated by the received signal strength (RSS) and/or Global Positioning System (GPS) information. We conducted simulation based on the RSS method [15]. The RSS measurements are relatively inexpensive and simple to implement in hardware. To estimate the node density, a node collects the received power samples from its nearest neighbors and performs the maximum likelihood estimation. According to [16], the estimation results are highly accurate and the procedure is uncomplicated.

As shown in Fig. 2a, the results of the iterative solution and exhaustive search are coherent. The optimal carrier sensing threshold varies with the target SIR $\beta$. If $\beta$ increases, $I_s$ should be decreased to lower the active transmitter density. For the comparison, we also plot the optimal carrier sensing threshold obtained by ignoring the BEB (dashed line in Fig 2). In this case, the optimal carrier sensing range can be approximated as $1.1278 \sqrt{\beta r_s}$ (derivation in Appendix). By ignoring the BEB, the active transmitter density is overestimated, where the corresponding optimal carrier sensing threshold is lower, causing performance degradation as shown in Fig. 2b. Fig. 2 shows...
practical backoff random variable and taking expectation of
in the design and optimization of high performing CSMA
by NS-3 simulations. Our analytical results could be employed
the proposed algorithm shows acceptable performance.

V. CONCLUDING REMARKS

We proposed a tractable approach for the optimal carrier
sensing threshold of the random CSMA/CA networks. Most
previous works using stochastic geometry overlooked the
effect of the random backoff. We considered the effect of the
practical backoff scheme and verified accuracy of our analysis
by NS-3 simulations. Our analytical results could be employed
in the design and optimization of high performing CSMA/CA
networks. The spectrum sensing based cognitive radio network
(CRN) is one of the viable applications of our work. If
the spectrum is sensed as available, the multiple secondary
transmitters would access concurrently, causing collisions. To
avoid this situation, the CSMA/CA-based MAC protocol for
the CRN is desirable. Our results can be used to find an
optimal spectrum sensing level.

APPENDIX

1) Derivation of (9): We denote \( I_{R_s} \) as the aggregate
interference from the outside of region with the radius \( R_s \).
Using the fact that the channel gain \( g_{i,j} \) is an exponential
random variable and taking expectation of \( I_{R_s} \), then \( p_s \) is:

\[
p_s = \Pr \left[ g_{i,j} \frac{P}{I_{R_s}} \geq \beta \right] = \mathcal{E}[I_{R_s}] \exp \left(-\frac{\beta P}{n(I_{R_s})} \right).
\]

(12)

By substituting \( s = \beta P / n \), (12) becomes the Laplace transform
of shot-noise process \( I_{R_s} \). Using the result of [9], (12) is:

\[
p_s = \exp \left(-2 \pi \lambda \int_{R_s}^{\infty} \left(1 - \mathcal{E}[e^{-\beta P g}]\right) dv \right),
\]

(13)

where \( v \) is a dummy variable representing the distance to a
random interferer. Using the moment generating function of
the exponential random variable, the probability \( p_s \) is:

\[
p_s \approx \exp \left(-2 \pi \lambda \int_{R_s}^{\infty} \left(1 - \frac{\beta P}{I_{R_s}} \right) dv \right).
\]

(14)

Assuming \( \alpha=4 \), closed-form is obtained as follows:

\[
p_s \approx \exp \left(-\pi \lambda \sqrt{\frac{f^R_s}{R_s}} \arctan \left(\frac{f^R_s}{R_s}\right)\right).
\]

(15)

2) Derivation of Optimal Sensing Threshold ignoring BEB:

Assuming high node density and \( \alpha=4 \), the value \( \lambda_s \) approxi-
mates \( \lambda \approx 1 / \pi R_s^2 \). The objective function of (1) becomes:

\[
\eta = \frac{\log_2 (1 + \beta)}{\pi R_s^2} \exp \left(-\sqrt{\frac{\beta^R_s}{R_s^2}} \arctan \left(\frac{\beta^R_s}{R_s^2}\right)\right).
\]

(16)

Differentiating (16) with \( R_s \) and simplifying exponential term:

\[
\frac{d\eta}{dR_s} \approx \frac{2 \log_2 (1 + \beta)}{\pi R_s^3} \exp \left(-\frac{\beta^R_s}{R_s^3} \left(\frac{\beta^R_s}{R_s^3} \right) (\frac{\beta^R_s}{R_s^3} + \frac{\beta^R_s}{R_s^3} - 1) = 0.
\]

(17)

By solving (17), \( R_s^* \) is obtained:

\[
R_s^* = \left(0.5 \left(1 + \frac{1}{\sqrt{2}} \beta^R_s \right) \right)^{1/4} \approx 1.1278 \beta^R_s R_s.
\]

(18)

REFERENCES

[1] IEEE 802.11-2007: Wireless LAN Medium Access Control (MAC) and
Physical Layer (PHY) Specifications, IEEE, June 2007.

[2] X. Yang and N. Vaidya, “On physical carrier sensing in wireless ad hoc
networks,” in Proc. IEEE INFOCOM, USA, 2005.

[3] D. Stoyan, W. Kendall, and J. Mecke, Stochastic Geometry and its
Applications, 2nd ed. Wiley, 1995.

[4] H. Q. Nguyen, F. Baccelli, and D. Kofman, “A stochastic geometry
analysis of dense IEEE 802.11 networks,” in Proc. IEEE INFOCOM,
USA, 2007.

[5] A. Busson and G. Chelius, “Point processes for interference modeling
in CSMA/CA ad hoc networks,” in Proc. ACM MSWiM, Spain, 2009.

[6] G. Alfano, M. Garetto, and E. Leonard, “New insights into the
stochastic geometry analysis of dense CSMA networks,” in Proc. IEEE
INFOCOM, China, 2011.

[7] M. Kaynia, N. Jindal, and G. E. Oien, “Improving the performance of
wireless ad hoc networks through MAC layer design,” IEEE Trans.
Wireless Commun., vol. 10, no. 1, pp. 240–252, Jan. 2011.

[8] S. Weber, J. G. Andrews, and N. Jindal, “An overview of the trans-
mission capacity of wireless networks,” IEEE Trans. Commun., vol. 58,
no. 12, pp. 3593–3604, Dec. 2010.

[9] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, “Stochastic analysis of
spatial and opportunistic Aloha,” IEEE J. Sel. Areas Commun., vol. 27,
no. 7, pp. 1105–1119, Sept. 2009.

[10] G. Bianchi, “Performance analysis of the IEEE 802.11 distributed
coordination function,” IEEE J. Sel. Areas Commun., vol. 18, no. 3,
p. 535–547, Mar. 2000.

[11] F. Calli, M. Conti, and E. Gregori, “Dynamic tuning of the IEEE 802.11
protocol to achieve a theoretical throughput limit,” IEEE/ACM Trans.
Netw., vol. 8, no. 6, pp. 785–799, Dec. 2000.

[12] M. Souryal, B. Vojic, and R. Pickholtz, “Ad hoc, multihop CDMA
networks with route diversity in a Rayleigh fading channel,” in Proc.
IEEE MILCOM, USA, 2001.

[13] E. Ziooua and T. Antonakopoulos, “CSMA/CA performance under high
traffic conditions: throughput and delay analysis,” Comput. Commun.,
vol. 25, no. 3, pp. 313–321, Feb. 2002.

[14] J. Hwang and S.-L. Kim, “Cross-layer optimization and network coding
in CSMA/CA-based wireless multihop networks,” IEEE/ACM Trans.
Netw., vol. 19, no. 4, pp. 1028–1042, Aug. 2011.

[15] S. D. Chitte, S. Dasgupta, and Z. Ding, “Distance estimation from
received signal strength under log-normal shadowing: bias and variance,”
IEEE Signal Process. Lett., vol. 16, no. 3, pp. 216–218, Mar. 2009.

[16] E. Onur, Y. Durmush, and I. Niemegeers, “Cooperative density estimation
in random wireless ad hoc networks,” IEEE Commun. Lett., vol. 16,
no. 3, pp. 331–333, Mar. 2012.

Fig. 2: Optimal carrier sensing threshold and maximum area
spectral efficiency as a function of the target SIR (\( \lambda = 0.2, W_o=16, m=32, r_s=50 \text{ m, } P=30 \text{ dBm} \)).

the impact of estimation errors on \( r_s \) and \( \lambda \). The \( r_s=50 \text{ m} \)
is estimated by 54.8547m and the \( \lambda=0.2 \) is estimated by 0.1911.
Even though we adopted rather primitive estimation methods,
the proposed algorithm shows acceptable performance.