Rotator for electromagnetic emission based on conformal mapping

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Abstract. Using conformal mapping it is proposed a device, which demonstrates the illusion effect. Emission from the current sheet source placed on the X-axis centered at the origin is almost fully rotated at an angle of 90 degrees. As a result, the observer would detect that the current sheet source placed on Y-axis.

1. Introduction
For centuries materials have led the progress of humanity: from stone to copper and glass. In the 21st century, it can be created materials with artificial predefined properties. Using these materials it can be achieved “magical” effects, for instance, different kinds of illusions including invisibility. One convenient mathematical tool to calculate the refractive index of material with interesting properties is transformation optics and conformal mapping. For example, in paper [1] it was proposed a simple method of illusion optics based on conformal mapping. Authors made an object look like another with a significantly different shape. Another illusion was achieved in the article [2]. The resulting emission of two point sources placed symmetrically to the centre of the axis is the same as from a single point source placed in the centre. One more kind of optical illusion was proposed by Vozianova A. and colleagues [3]. Using the cylindrical polar angle-dependent coordinate transformation researchers achieved a creation the mirage of a point source radiation at any predetermined angular direction. In the paper [4] authors showed an illusion with a different source: proposed lenses can make one active source appear omnidirectionally as two in-phase sources. In this paper, we will use a slightly modified conformal mapping which will allows obtaining more feasible refractive index with almost the same functionality.

2. Theory and results
By the following conformal mapping authors [4] designed a new class of gradient index lenses:

\[ w^N = z^N - 1 \] (1)

Such lens could make one active source appear omnidirectionally as two (when \( N = 2 \)) in-phase sources. The proposed lens had the main disadvantage, if we talk in terms of implementation, refractive index in some area is less than 1.

In the theory of conformal mappings, there are two complex planes (or spaces): physical \( z = x + iy \) (P1) and virtual \( w = u + iv \) (P2). To calculate a refractive index profile corresponding to any conformal mapping \( w(z) \), it should be taken a derivative [5]
\[ n = \left| \frac{dw}{dz} \right| n' \]  

where \( n' \) is the refractive index in virtual space which is taken as 1 if the virtual space is empty. In this paper we use a conformal mapping

\[ w(z) = k \left( (z^2 - 1)^{1/2} + Arc \right) \]  

where \( k = h/\pi \) – is arbitrary constant. According to the equation (2) a refractive index profile is taken as:

\[ n(x, y) = \left| \frac{z}{(z^2 - 1)^{1/2}} + \frac{1}{(z^2 - 1)^{1/2}} \right| \]  

where we used \( k = 1 \) and \( n' = 1 \).

In point \( z = 0 \) refractive index \( n \) is equal to 1 and at \( z \rightarrow \infty \) it turned to 1. Further, there are two points \( z = \pm 1 \) with extremal refractive index. In these points refractive index turns to infinity. The proposed refractive index profile is shown in figure 1. A material with such refractive index is easier to fabricate than the one in [4].

**Figure 1.** The refractive index profile based on conformal mapping (3)

As it was predicted from the equation (4) refractive index is not less than 1 in the whole plane \( XY \).

To simulate a proposed effect current sheet source (placed from \( z = -1 \) to \(+1\)) radiated in the medium with the demonstrated above refractive index (see figure 1).
Figure 2 (a,b). The electric field distribution (a) and the radiation pattern diagram (b) for one current source placed on the X-axis between points $z = \pm 1$ in the proposed medium.

It can be noticed that the emission from the original source is rotated and propagates at an angle of 90 degrees. Thus, the observer would see a rotated source radiation. To compare the source radiation in the proposed material and air we consider the same current source placed on Y-axis between points $z = \pm i$ in air.

Figure 3 (a,b). The electric field distribution (a) and the radiation pattern diagram (b) for one current source placed on the X-axis between points $z = \pm i$ in the air.

It can be seen (figure 2 (b), 3 (b)) that the emission from the source in the studied medium is more collimated than the one in the air. Moreover, it can be shown another kind of emission rotation. The well-known Eaton lens [7] is a perfect retroreflector. All incident light is reflected back to the direction it came from. With the proposed refractive index, the same result can be achieved.
Figure 4 (a,b). The electric field distribution (a) of symmetrically placed source and the ray trajectories (b) from one-half source from (a)

The whole incident light from the current sheet source rotates around the point $z = -1$ and then propagates back. Thus, with the proposed conformal mapping can be achieved one-directional Eaton lens effect.

3. Conclusion
In conclusion, we have proposed a device to rotate emission from the specifically placed source in the medium based on conformal mapping (3). The refractive index of this device is calculated according to (4). As a result, the original source would be perceived as rotated one by a detector or the observer. And the emission would be highly collimated. This effect was described previously [4] but with another conformal mapping. The advantage of our device is the refractive index profile does not become less 1. This feature makes proposed structure more feasible. In addition, it should be noted that this effect can be realized not only for electromagnetic emission but also for any kind of waves (sound, plasmons) [6].

References
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