Improved Collective Field Formalism for
an Antifield Scheme for
Extended BRST Symmetry

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Abstract

We present a new collective field formalism with two rather than one
collective field to derive the antifield formalism for extended BRST
invariant quantisation. This gives a direct and physical proof of the
scheme of Batalin, Lavrov and Tyutin, derived on algebraic grounds.
The importance of the collective field in the quantisation of open alge-
bras in both the BRST and extended BRST invariant way is stressed.

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1 Introduction

Today, the best Lagrangian quantisation scheme that is capable of quantising any gauge theory of all known types, is the antifield formalism or the Batalin-Vilkovisky quantisation scheme. Let us give a non-exhaustive list of assets of this scheme. The antifields that are introduced, are sources for the BRST transformations. Hence, the renormalisation of gauge theories as described by Zinn-Justin is naturally incorporated in the BV scheme. Secondly, the antifields allow you, at least formally, to fix a gauge, do the calculation you are interested in, and still be able to transform the result to other gauges. This holds especially for the quantum counterterms needed for maintaining the Ward identities. The antifields thus prevent you from accidentally finding a vanishing anomaly (see and references therein). A third, and major, asset of the BV scheme is that the fields and their associated antifields allow for an algebraic structure which is very similar to classical mechanics. There is an analog of the Poisson bracket, the so-called antibracket. The fields and antifields are canonically conjugated with respect to this bracket. Also, canonical transformations, i.e. transformations of the fields and the antifields which leave the antibracket invariant, play a key part. Gauge fixing can be understood as a canonical transformation.

Despite all these qualities, the scheme is less used than could be expected, probably because the meaning of the scheme is somewhat obscured by its usual algebraic derivation. Recently, Alfaro and Damgaard gave an explicit derivation of the BV scheme for closed algebras starting from ordinary BRST quantisation. This was generalised to open algebras in . The guiding principle is that whatever the original BRST algebra, it has to be extended to include the Schwinger-Dyson BRST symmetry (below denoted SD-symmetry). This is implemented using a collective field, which leads to extra shift symmetries. The antighosts of this shift symmetry are the BV antifields. This point of view does not only give more insight in the fact that the BV antifield formalism incorporates the Schwinger-Dyson equations, it also provides us with an explicit and intuitive road from ordinary BRST quantisation to the BV scheme.

The same basic idea of extending the BRST algebra with the SD-symmetry has also been implemented in the Hamiltonian quantisation formalism of Batalin, Fradkin and Vilkovisky in . This way, it is possible to prove the equivalence of the Hamiltonian and Lagrangian formalism. Let us note
in passing that, contrary to what is stated in [9], the proof is valid for open algebras.

Although BRST–anti-BRST (or extended BRST) quantisation [10] allows apparently for nothing more than ordinary BRST quantisation, there has been a growing interest in it. In [11], Batalin, Lavrov and Tyutin developed an antifield formalism for extended BRST invariant quantisation on algebraic grounds. This was partly motivated by the hope to construct a superfield formulation for the quantisation of general gauge theories [12]. In this antifield scheme, one has three rather than one antifield: a source for the BRST transformation, a source for the anti-BRST transformation and finally a sourceterm for mixed transformations. In [13], the collective field approach is used to derive this antifield formalism. The only difference is the way the antifields are removed after gauge fixing. In [11], this is done by fixing them to zero, while in [13] a Gaussian integral was used for that purpose. This latter fact however restricts the validity of [13] to closed algebras [6]. In this paper, we show how modifying the collective field scheme, by introducing two rather then one collective field, leads to the same gauge fixing procedure as the algebraic approach. We argue that then indeed open algebras can be quantised and point out the crucial importance of the collective fields for this.

The plan of this paper is the following. In section 2, we give a short review of the SD approach to the ordinary BV scheme, in order to make the analogies and differences with our treatment of the extended BRST case more clear. In section 3, we introduce our modified collective field technique and show how it leads to the Schwinger-Dyson equations as Ward identities. The next section contains the heart of the paper. There we derive the antifield formalism of [11] for closed algebras. Ward identities and the quantum master equation are discussed in section 5. A comment on open algebras is given in section 6. Finally, we draw our conclusions.

2 Collective fields for BV

In this section we give a very short summary of the collective field approach for the construction of the BV antifield formalism [3]. We start from a classical action $S_{0}[\phi^i]$, depending on a set of fields $\phi^i$. Suppose that this classical action has gauge invariances which are irreducible and form a closed algebra. Then one can construct a nilpotent BRST operator acting on an extended set of fields $\phi^A$. The $\phi^A$ include the original gauge fields $\phi^i$, the ghosts $c^a$ and the pairs of trivial systems needed for the construction of the gauge fermion and for the gauge fixing. We summarise all their BRST transformation rules by $\delta\phi^A = R^A[\phi^A]$. We enlarge the set of fields by replacing the field $\phi^A$ wherever it occurs, by $\phi^A - \varphi^A$. $\varphi^A$ is called the collective field. This leads to a new symmetry, the shift symmetry, for which we introduce a ghost field $c^A$, and a trivial pair consisting of an antighost
field $\phi^*_A$ and an auxiliary field $B_A$. The BRST transformation rules are given by:

\[
\begin{align*}
\delta \phi^A &= c^A \\
\delta \varphi^A &= c^A - \mathcal{R}^A[\phi - \varphi] \\
\delta c^A &= 0 \\
\delta \phi^*_A &= B_A \\
\delta B_A &= 0.
\end{align*}
\]  

(1)

Now there are two gauge symmetries to fix. We do this as follows:

\[
\begin{align*}
S_{gf} &= S_0[\phi^i - \varphi^i] - \delta[\phi^*_A \phi^A] + \delta \Psi[\phi^A] \\
&= S_0[\phi^i - \varphi^i] + \phi^*_A \mathcal{R}^A[\phi - \varphi] - \phi^*_A c^A + \frac{\delta \Psi}{\delta \phi^A} c^A - \varphi^A B_A. \\
&= S_{BV}(\phi - \varphi, \phi^*) - \phi^*_A c^A + \frac{\delta \Psi}{\delta \phi^A} c^A - \varphi^A B_A. \\
\end{align*}
\]  

(2)

This gives the following form of the partition function, which is typical for the BV scheme:

\[
Z = \int [d\phi^A][d\phi^*_A] \delta \left( \phi^*_A - \frac{\delta \Psi[\phi]}{\delta \phi^A} \right) e^{\frac{i}{\hbar}S_{BV}(\phi, \phi^*)}. \\
\]  

(3)

The fact that the gauge fixed action is still BRST invariant, leads to the classical master equation for $S_{BV}$, using that $\delta \varphi^A = c^A - \frac{\delta S_{BV}(\phi - \varphi)}{\delta \phi^*_A}$:

\[
\frac{\delta S_{BV}(\phi, \phi^*)}{\delta \phi^A} \frac{\delta S_{BV}(\phi, \phi^*)}{\delta \phi^*_A} = 0. \\
\]  

(4)

Using a BRST invariant action as weight in the partition function, we have Ward’s identities $\langle \delta X \rangle = 0$, for any $X$. Remember that quantum counterterms may be needed in order to guarantee the validity of the Ward identities. Imposing that the Schwinger-Dyson equations should be derivable as Ward identities, restricts their form to $\hbar \mathcal{M}(\phi - \varphi, \phi^*)$. Hence, we replace $S_{BV}(\phi - \varphi, \phi^*)$ as weight in the path integral by $W(\phi - \varphi, \phi^*) = S_{BV}(\phi - \varphi, \phi^*) + \hbar \mathcal{M}(\phi - \varphi, \phi^*)$. Considering quantities $X(\phi, \phi^*)$, and integrating out all fields of the collective field formalism, except these, this gives the Ward identity

\[
0 = \int [d\phi][d\phi^*] [(X, W) - i\hbar \Delta X] e^{\frac{i}{\hbar}W(\phi, \phi^*)} \delta (\phi^*_A - \Psi_A), \\
\]  

(5)

with the antibracket defined by

\[
(F, G) = \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi^*_A} - \frac{\delta F}{\delta \phi^*_A} \frac{\delta G}{\delta \phi^A}; \\
\]  

(6)
and with the delta-operator
\[ \Delta X = (-1)^{\epsilon_A + 1} \frac{\delta}{\delta \phi^*_A} \frac{\delta}{\delta \phi^A} X = (-1)^{\epsilon_A} (-1)^{\epsilon_A} \frac{\delta}{\delta \phi^*_A} \frac{\delta}{\delta \phi^A} X. \] (7)

We also denoted \( \frac{\delta \Psi}{\delta \phi^A} = \Psi_A \).

Removing all derivatives from \( X \), the Ward identity can be used to derive the quantum master equation, as it should hold for any \( X \). We get
\[
i \hbar \int [d\phi][d\phi^*] X(\phi, \phi^*_A + \Psi_A) \Delta \exp \left[ \frac{i}{\hbar} W(\phi, \phi^*_A + \Psi_A) \right] \delta(\phi^*) = 0. \quad (8)
\]
and thus the quantum master equation
\[
\Delta \exp \left[ \frac{i}{\hbar} W(\phi, \phi^*_A + \Psi_A) \right] = 0. \quad (9)
\]

Let us end this far too short overview with a comment on open algebras. In [16], De Wit and van Holten gave a recipe to construct a BRST invariant action for gauge symmetries with an open algebra. It consists in modifying the BRST transformation rules and the action itself by adding an expansion to both in powers of the derivatives of the gauge fermion with respect to the gauge fields. For the case of the gauge fermion in the collective field formalism, \( F = \phi^*_A \phi^A + \Psi[\phi] \), we thus have to expand in powers of \( \phi^*_A \) and \( \Psi_A \).

A solution which is linear in the latter can be found, and only an expansion in the antifield remains. This way, the form of \( S_{BV} \) for open algebras, that is, an extended action with terms that are of quadratic or higher order in the antifields, is recovered. For more details, see [1].

A posteriori, the collective field formalism can be seen as a justification of the procedure of De Wit and van Holten. We will develop this point of view here in some detail, as it will be our starting point for the treatment of open algebras in the extended BRST antifield formalism. When quantising a gauge theory, one always has to choose a set of functions \( F^\alpha \), defining a gauge. This is at least so for every known scheme today. The quantisation should at least satisfy the following three requirements. (i) The partition function and expectation values should be well-defined, owing to a careful choice of the functions \( F^\alpha \). This is the admissibility condition for the gauge fixing. (ii) Although defined using specific \( F^\alpha \), the partition function should be invariant under (infinitesimal) deformations of the functions \( F^\alpha \), i.e. gauge independent. (iii) When putting the \( F^\alpha \) to zero in the expressions for the partition function and expectation values, they should reduce to the ill-defined expressions one started from.

The introduction of collective fields allows us to construct the BRST transformation rules such that \( \delta^2 \phi^A = 0 \) as we can shift the off-shell nilpotency problem of open algebras to the transformation rules of the collective field.

\(^1\)This point of view was stressed by P.H. Damgaard.
The originally present gauge symmetry can thus be fixed in a manifestly BRST invariant way by adding $\delta \Psi(\phi)$. So, the gauge fixed action can be decomposed as $S_{gf} = S_{inv} + \delta \Psi$. The second requirement for a good quantisation scheme can be satisfied by taking $S_{inv}$ to be BRST invariant, as Ward identities then imply gauge independence.

Another restriction on $S_{inv}$ is that when used as weight in the partition function (i.e. when putting $\Psi$ to zero), the original, ill-defined path integrals are recovered. It is clear that

$$S_{inv} = S_{BV}(\phi - \varphi, \phi^*) - \phi^*_A e^A - \varphi^A B_A$$  \hspace{1cm} (10)

with the boundary condition that $S_{BV}(\phi, \phi^* = 0) = S_0[\phi]$ satisfy this requirement. Moreover, imposing that the original, ill-defined SD equations are recovered as Ward identities restrict us to this form \[4\]. We are now free to include in $S_{BV}$ whatever expansion in the antifields $\phi^*$ we want, as they are fixed to zero anyway when $\Psi = 0$. The only condition is that $(S_{BV}, S_{BV}) = 0$, as this leads indeed to a BRST-invariant $S_{inv}$. The question whether open algebras can be quantised in BV amounts then to proving that the classical master equation can be solved for open algebras \[14\].

### 3 Schwinger-Dyson Equations from two collective fields

In this section, we will present the formalism with two collective fields and derive the SD equation from them as a Ward identity without the complication of gauge symmetries. In the derivation of the Ward identity, we will already meet one peculiarity which will also be crucial in the next section. We start from an action $S_0[\phi]$, depending on one bosonic degree of freedom $\phi$. It has to be such that when exponentiated and put under a path integral, it leads to a well-defined partition function and perturbation series. We introduce two copies of the original field, the two so-called collective fields, $\varphi_1$ and $\varphi_2$ and consider the action $S_0[\phi - \varphi_1 - \varphi_2]$. This leads to two gauge symmetries for which we introduce two ghostfields $\pi_1$ and $\phi_2^*$ and two antighost fields $\phi_1^*$ and $\pi_2$. The BRST–anti-BRST transformation rules are

\begin{align*}
\delta_1 \phi &= \pi_1 \\
\delta_1 \varphi_1 &= \pi_1 - \phi_2^* \\
\delta_1 \varphi_2 &= \phi_2^* \\
\delta_1 \pi_1 &= 0 \\
\delta_1 \phi_2^* &= 0 \\
\delta_2 \phi &= \pi_2 \\
\delta_2 \varphi_1 &= -\phi_1^* \\
\delta_2 \varphi_2 &= \pi_2 + \phi_1^* \\
\delta_2 \pi_2 &= 0 \\
\delta_2 \phi_1^* &= 0.
\end{align*}  \hspace{1cm} (11)

Imposing $(\delta_2 \delta_1 + \delta_1 \delta_2) \phi = 0$ gives the extra condition $\delta_2 \pi_1 + \delta_1 \pi_2 = 0$, while analogously $(\delta_2 \delta_1 + \delta_1 \delta_2) \varphi_1 = 0$ gives $\delta_1 \phi_1^* + \delta_2 \phi_2^* = \delta_2 \pi_1$. $(\delta_2 \delta_1 + \delta_1 \delta_2) \varphi_2 = 0$ leads to no new condition. We introduce two extra bosonic fields $B$ and $\lambda$.
and the BRST transformation rules:
\[
\begin{align*}
\delta_1 \pi_2 &= B \\
\delta_1 B &= 0 \\
\delta_1 \phi^*_1 &= \lambda - \frac{B}{2} \\
\delta_1 \lambda &= 0 \\
\delta_2 \pi_1 &= -B \\
\delta_2 B &= 0 \\
\delta_2 \phi^*_2 &= -\lambda - \frac{B}{2} \\
\delta_2 \lambda &= 0.
\end{align*}
\] (12)

All these rules together guarantee that \( \delta_1^2 = \delta_2^2 = \delta_1 \delta_2 + \delta_2 \delta_1 = 0. \)

With all these BRST transformation rules at hand, we can construct a gauge fixed action that is invariant under extended BRST symmetry. We will fix both the collective fields to be zero. To that end, we add
\[
S_{col} = \frac{1}{2} \delta_1 \delta_2 [\phi^*_1 - \phi^*_2]
\]
\[
= - (\phi_1 + \phi_2) \lambda + \frac{B}{2} (\phi_1 - \phi_2) + (-1)^a \phi^*_a \pi_a.
\] (13)

In the last term, there is a summation over \( a = 1, 2. \) Denoting \( \phi_{\pm} = \phi_1 \pm \phi_2, \) we have the gauge fixed action
\[
S_{gf} = S_0[\phi - \phi_+] - \phi_+ \lambda + \frac{B}{2} \phi_- + (-1)^a \phi^*_a \pi_a.
\] (14)

The Schwinger-Dyson equations can be derived as Ward identities in the following way.
\[
0 = \langle \delta_1 [\phi^*_1 F(\phi)] \rangle
\]
\[
= \int d\mu \left[ \phi^*_1 \frac{\delta F}{\delta \phi} \pi_1 + (\lambda - \frac{B}{2}) F(\phi) \right] e^{i \tilde{S}_{gf}}.
\] (15)

We denoted the integration measure over all fields by \( d\mu. \) The term \( \langle BF(\phi) \rangle \) is zero. This can be seen by noticing that \( B = \delta_1 \delta_2 \phi_+. \) The Ward identities themselves allow to integrate by parts to get
\[
\langle BF(\phi) \rangle = -\langle \phi_+ \delta_2 \delta_1 F(\phi) \rangle,
\] (16)

which drops out as \( \phi_+ \) is fixed to zero. The same trick will be useful in deriving the Ward identities of the extended BRST symmetry in the antifield scheme.

The SD equation then results as in [7, 5] by integrating out \( \pi_a, \phi^*_a, \lambda, B, \phi_+ \) and \( \phi_- \). Of course, the SD equations can also be derived as Ward identities of the anti-BRST transformation.

### 4 Extended antifield formalism for closed, irreducible algebras

The starting point is the same as in [13]. Given any classical action \( S_0[\phi^i] \) with a closed and irreducible gauge algebra, the configuration space is enlarged...
by introducing the necessary ghosts, antighosts and auxiliary fields, as is described e.g. in [13]. The complete set of fields is denoted by $\phi_A$ and their BRST-anti-BRST transformation rules are all summarised by $\delta_a \phi_A = R_{Aa}(\phi)$. For $a = 1$, we have the BRST transformation rules, for $a = 2$ the anti-BRST transformation. Since the algebra is closed, we have that

$$\frac{\delta R_{Aa}(\phi)}{\delta \phi_B} R_{Ba}(\phi) = 0$$

and that

$$\frac{\delta R_{A1}(\phi)}{\delta \phi_B} R_{B2}(\phi) + \frac{\delta R_{A2}(\phi)}{\delta \phi_B} R_{B1}(\phi) = 0. \quad (18)$$

In the first formula, there is no summation over $a$.

Instead of constructing a gauge fixed action that is invariant under the extended BRST symmetry, we will introduce collective fields and associated extra shift symmetries. Contrary to the previous collective field approach to extended BRST invariant quantisation of [13], we introduce two collective fields $\phi^{A1}$ and $\phi^{A2}$, commonly denoted by $\phi_{Aa}$, and replace everywhere $\phi_A$ by $\phi_A - \phi^{A1} - \phi^{A2}$. We now have two shift symmetries for which we introduce the ghosts $\pi^{A1}$ and $\phi^{*A2}$ with ghostnumber $gh(\pi^{A1}) = gh(\phi^{*A2}) = gh(\phi_A) + 1$ and the antighosts $\phi^{*A1}$ and $\pi^{A2}$ with ghostnumber $gh(\pi^{A2}) = gh(\phi^{*A1}) = gh(\phi_A) - 1$. Again, we will use $\pi_{Aa}$ and $\phi^{*Aa}$ as compact notation. Of course, one has to keep in mind that for $a = 1$, $\pi_{Aa}$ is a ghost, while for $a = 2$, $\pi_{Aa}$ is an antighost and vice versa for $\phi^{*Aa}$.

We construct the BRST-anti-BRST transformations as follows:

$$\delta_a \phi_A = \pi_{Aa}$$
$$\delta_a \phi_{Ab} = \delta_{ab} \left[ \pi_{Aa} - \epsilon_{ac} \phi^{*A_c} - R_{Aa} \left( \phi - \phi^{A1} - \phi^{A2} \right) \right] + (1 - \delta_{ab}) \epsilon_{ac} \phi^{*A_c}, \quad (19)$$

with no summation over $^2 a$ in the second line. These are chosen such that

$$\delta_a (\phi_A - \phi^{A1} - \phi^{A2}) = R_{Aa} (\phi - \phi^{A1} - \phi^{A2}). \quad (20)$$

The two collective fields lead to even more freedom than the one in the collective field formalism for BV to shift the $R_{Aa}$ in the transformation rules. However, we will see that the choice above leads to what was already known as the antifield formalism for extended BRST symmetry [11]. Furthermore, the discussion of open algebras in section 2 also indicates that it is useful to construct the rules such that $\delta^2_a \phi_A = 0$ and $(\delta_1 \delta_2 + \delta_2 \delta_1) \phi_A = 0$, independently of the closure of the algebra. Imposing that $\delta^2_a \phi_A = 0$ ($a = 1, 2$) and that $\delta_1 \delta_2 + \delta_2 \delta_1 = 0$ when acting on any field, we are led to the introduction of two extra fields $B_A$ and $\lambda_A$ and the new transformation rules:

$$\delta_a \pi_{Ab} = \epsilon_{ab} B_A$$

$^2$Our convention: $\epsilon_{12} = 1, \epsilon^{12} = -1.$
\[ \delta_a B_A = 0 \]
\[ \delta_a \phi_A^a = -\delta_a^2 \left[ (-1)^a \lambda_A + \frac{1}{2} \left( B_A + \frac{\delta R_{A1}(\phi - \varphi_1 - \varphi_2)}{\delta \phi_B} R_{B2}(\phi - \varphi_1 - \varphi_2) \right) \right] \]
\[ \delta_a \lambda_A = 0. \]

Inspired by [13, 23], we will gauge fix both the collective fields to zero in a BRST–anti-BRST invariant way. For that purpose, we need a matrix \( M^{AB} \), with constant c-number entries and which is invertible. Moreover, it has to have the symmetry property \( M^{AB} = (-1)^{a \epsilon_{AB}} M^{BA} \) and all the entries of \( M \) between Grassmann odd and Grassmann even sectors have to vanish. It should be such that \( \phi_A M^{AB} \phi_B \) has over all ghostnumber zero and has even Grassmann parity. Except for the constraints above, the precise form of \( M \) is of no concern. It will drop out in the end completely [13]. The collective fields are then gauge fixed to zero by adding the term

\[ S_{col} = -\frac{1}{4} \epsilon^{ab} \delta_a \delta_b \left[ \varphi_{A1} M^{AB} \varphi_{B1} - \varphi_{A2} M^{AB} \varphi_{B2} \right] \]
\[ = - (\varphi_{A1} + \varphi_{A2}) M^{AB} \lambda_B + \frac{1}{2} (\varphi_{A1} - \varphi_{A2}) M^{AB} B_B \]
\[ + (-1)^{a \epsilon_{AB}} \phi_{A1}^a M^{AB} \pi_{B1} + (-1)^{a \epsilon_{AB}} \phi_{A2}^a M^{AB} \pi_{B2} \]
\[ + (-1)^{a \epsilon_{AB}} M^{AB} R_{B1}(\phi - \varphi_1 - \varphi_2) + (-1)^{a \epsilon_{AB}} \phi_{A2}^a M^{AB} R_{B2}(\phi - \varphi_1 - \varphi_2) \]
\[ + \frac{1}{2} (\varphi_{A1} - \varphi_{A2}) M^{AB} \frac{\delta R_{B1}(\phi - \varphi_1 - \varphi_2)}{\delta \phi_C} R_{C2}(\phi - \varphi_1 - \varphi_2). \]

The relative sign between the two contributions of the gauge fixing is needed to make two terms containing the product \( \phi_{A1}^a M^{AB} \phi_{B2}^b \), cancel. Redefine now \( \varphi_{A\pm} = \varphi_{A1} \pm \varphi_{A2} \), which allows us to rewrite the gauge fixing terms in a more compact and suggestive form:

\[ S_{col} = -\varphi_{A+} M^{AB} \lambda_B + \frac{1}{2} \varphi_{A-} M^{AB} B_B + (-1)^a (-1)^{a b} \phi_{A1}^a M^{AB} \pi_{Ba} \]
\[ + \frac{1}{2} \varphi_{A-} M^{AB} \frac{\delta R_{B1}(\phi - \varphi_+)}{\delta \phi_C} R_{C2}(\phi - \varphi_+) \]
\[ + (-1)^{a+1} (-1)^{a b} \phi_{A1}^a M^{AB} R_{B+}(\phi - \varphi_+). \]

Notice that this time a summation over \( a \) is understood in the third and fifth term. The \( \phi_{A1}^a \) have indeed become source terms for the BRST and anti-BRST transformation rules, while the difference of the two collective fields \( \varphi_{A-} \) acts as a source for mixed transformations. The sum of the two collective fields is just fixed to zero.

The original gauge symmetry can be fixed in an extended BRST invariant way by adding the variation of a gauge boson \( \Psi(\phi) \), of ghostnumber zero. We take it to be only a function of the original fields \( \phi_A \). This gives the extra terms

\[ S_{\Psi} = \frac{1}{2} \epsilon^{ab} \delta_a \delta_b \Psi(\phi) \]
\[
\begin{align*}
\delta \Psi & = \frac{\delta \Psi}{\delta \phi_A} B_A + \frac{1}{2} \epsilon^{ab}(-1)^{\epsilon_B+1} \left[ \frac{\delta}{\delta \phi_A} \frac{\delta}{\delta \phi_B} \Psi \right] \pi_A \pi_B.
\end{align*}
\] (24)

In order to show that we now have the antifield formalism which was derived on algebraic grounds in [11], we first have to make the following (re)definitions. We incorporate the matrix \(M^{AB}\) introduced above in the antifields:

\[
\begin{align*}
\phi^{*Aa} & = (-1)^{\epsilon_A} \phi_B^{*a} M^{BA} (-1)^{a+1} \quad a = 1, 2 \\
\bar{\phi}^A & = \frac{1}{2} \bar{\phi}_B M^{BA}.
\end{align*}
\] (25)

Owing to the properties of the matrix \(M^{AB}\) above, the ghostnumber assignments after the redefinition are given by

\[
\begin{align*}
\text{gh} \left( \phi^{*Aa} \right) & = (-1)^{a} - \text{gh} \left( \phi_A \right) \\
\text{gh} \left( \bar{\phi}^A \right) & = -\text{gh} \left( \phi_A \right),
\end{align*}
\] (26)

while the Grassmann parities are of course

\[
\varepsilon_{\phi^{*Aa}} = \varepsilon_{\phi_A} + 1 \quad ; \quad \varepsilon_{\bar{\phi}^A} = \varepsilon_{\phi_A}.
\] (27)

We denote the so-called extended action of Batalin, Lavrov and Tyutin [11] by \(S_{BLT}\). Using the new variables and dropping the primes, it is defined by

\[
S_{BLT}(\phi_A, \phi^{*Aa}, \bar{\phi}^A) = S_0[\phi_A] + \phi^{*Aa} R_{Aa}(\phi) + \bar{\phi}^A \frac{\delta R_{A1}(\phi)}{\delta \phi_B} R_{B2}(\phi).
\] (28)

The remaining terms of \(S_{col}\), we denote by \(S_\delta\), hence

\[
S_\delta = -\varphi_A + M^{AB} \lambda_B + \bar{\phi}^A B_A - \phi^{*Aa} \pi_A a.
\] (29)

The notation stems from the fact that integrating over \(\pi_A a\), \(B_A\) and \(\lambda_B\) leads to a set of \(\delta\)-functions removing all the terms originating in the collective field formalism. The situation is then analogous to the BV scheme. Before the gauge fixing term \(S_\Psi\) is added, all antifields are fixed to zero.

With all these definitions at hand, we have that

\[
\begin{align*}
S_{gf} & = S_0[\phi - \varphi_+] + S_{col} + S_\Psi \\
& = S_{BLT}[\phi - \varphi_+, \phi^{*a}, \bar{\phi}] + S_\delta + S_\Psi,
\end{align*}
\] (30)

which gives the gauge fixed partition function

\[
Z = \int [d\phi][d\phi^{*a}][d\bar{\phi}][d\pi_A][dB] e^{S_{BLT}[\phi, \phi^{*a}, \bar{\phi}] + S_\Psi} e^{S_\delta}.
\] (31)
We already integrated out $\lambda$ and $\varphi_+$. Hence, $\hat{S}_\delta$ is $S_\delta$ with the $-\varphi_A M^{AB} \lambda_B$ omitted. The gauge fixing term $e^{\chi \hat{S}_\delta}$ can be obtained by acting with an operator $\hat{V}$ on $e^{\chi \hat{S}_\delta}$, i.e.

$$e^{\chi \hat{S}_\delta} e^{\chi \hat{S}_\delta} = \hat{V} e^{\chi \hat{S}_\delta}.$$  (32)

From the explicit form of $\hat{S}_\delta$ and $S_\Psi$, and using that $e^{a(y) \frac{\chi}{\hbar}} f(x) = f(x + a(y))$, we see that $\hat{V}(\Psi) = e^{-T_1(\Psi) - T_2(\Psi)}$ with

$$T_1(\Psi) = \frac{\bar{\delta} \Psi(\phi)}{\delta \phi_A} \cdot \frac{\bar{\delta}}{\delta \phi^a}$$

$$T_2(\Psi) = \frac{i \hbar}{2} \epsilon_{ab} \frac{\bar{\delta}}{\delta \phi^B} \frac{\bar{\delta}}{\delta \phi_A} \frac{\bar{\delta}}{\delta \phi_B} \Psi \frac{\bar{\delta}}{\delta \phi^{*Aa}}.$$  (33)

The convention is that derivatives act on everything standing on the right of them. The operator $\hat{V}$ can be integrated by parts, such that

$$\mathcal{Z} = \int [d\phi] [d\phi^a] [d\phi] \left[ \hat{U}(\Psi) e^{\chi S_{BLT}} \right] \delta(\phi^{*A1}) \delta(\phi^{*A2}) \delta(\bar{\phi}^A),$$  (34)

with the operator $\hat{U} = e^{+T_1 - T_2}$. This form of the path integral agrees with [14]. The quantisation prescription is then to construct $S_{BLT}$, function of fields and antifields. Then the gauge fixing is done by acting with the operator $\hat{U}(\Psi)$. Then the antifields $\phi^{*Aa}$ and $\bar{\phi}^A$ are removed by the $\phi$-functions which fix them to zero. This is the most important difference with the collective field formalism for extended BRST symmetry in [13]. There the antifields $\phi^{*Aa}$ were removed by a Gaussian integral, and it is precisely this procedure which prevented the generalisation of the results of [13] to open algebras [1].

Notice however that instead of acting with $\hat{U}$ on $e^{\chi S_{BLT}}$, it is a lot easier to take as realisation of the gauge fixing $S_\Psi + \hat{S}_\delta$, especially when $S_{BLT}$ becomes non-linear in the antifields.

Let us finally derive the classical master equations which are satisfied by $S_{BLT}$. They follow from the fact that $S_{gf}$ [10] is invariant under both the BRST and the anti-BRST transformation. Furthermore, one has to use the fact that the matrix $M^{AB}$ only has non-zero entries for $\varepsilon_A = \varepsilon_B$, and hence that $M^{AB} = (-1)^{e_A} M^{BA} = (-1)^{e_B} M^{BA}$. Also, in the collective field BRST transformation rules, we may replace $R_{Aa}(\phi - \varphi_+)$ by $\frac{\delta S_{BLT}}{\delta \phi^{*Aa}}$. Using that $\delta_a S_\Psi = 0$ on itself, we have that

$$0 = \delta_a S_{gf}$$

$$= \delta_a S_{BLT} + \delta_a S_\delta$$

$$= \frac{\bar{\delta} S_{BLT}}{\delta \phi_A} \cdot \frac{\bar{\delta} S_{BLT}}{\delta \phi^{*Aa}} + \epsilon_{ab} \phi^{*Aa} \frac{\delta S_{BLT}}{\delta \phi^A}.$$  (35)

We introduce two antibrackets, one for every $\phi^{*Aa}$, defined by

$$(F, G)_a = \frac{\bar{\delta} F}{\delta \phi_A} \cdot \frac{\bar{\delta} G}{\delta \phi^{*Aa}} - \frac{\bar{\delta} F}{\delta \phi^{*Aa}} \cdot \frac{\bar{\delta} G}{\delta \phi_A}.$$  (36)
Of course, they have the same properties as the antibrackets from the usual BV scheme, so that we finally can write the classical master equations as

$$\frac{1}{2}(S_{BLT}, S_{BLT})_a + \varepsilon_{ab} \phi^{*Aa} \frac{\delta S_{BLT}}{\delta \phi^A} = 0. \quad (37)$$

For closed, irreducible algebras, we know that the solution is of the form $[\mathcal{P}]$.

5 Ward’s Identities and Quantum Master Equation

In this section, we first derive the Ward identities for the extended BRST symmetry and then we take these identities as a starting point to derive the quantum master equation.

5.1 Ward’s Identities

As the gauge fixed action we constructed $[\mathcal{B}]$ is invariant under both the BRST and anti-BRST transformation rules, the standard procedure based on Shakespeare’s theorem $[\mathcal{C}]$ allows the derivation of 2 types of Ward identities. For any $X$, we have that

$$\langle \delta_1 X \rangle = 0 \quad \langle \delta_2 X \rangle = 0, \quad (38)$$

where $\langle A \rangle$ means the quantum expectation value using the gauge fixed action $[\mathcal{B}]$ of an operator $A$. As we are only interested in the theory after having integrated out $\varphi_+$, we will restrict ourselves to quantities $X(\phi_A, \phi^{*Aa}, \tilde{\phi})$. Furthermore, we assume that the quantum corrections - the counterterms - which may be needed to cancel the non-invariance of the measure and hence to guarantee the validity of the Ward identities, do not spoil the gauge fixing procedure described above. Like in the case of BV in section 2, if they would, the derivation of the SD equations as Ward identities would be invalidated. Hence, they are restricted to be of the form $M(\phi - \varphi_+, \phi^{*Aa}, \tilde{\phi})$ and

$$W_{BLT}(\phi, \phi^{*Aa}, \tilde{\phi}) = S_{BLT}(\phi, \phi^{*Aa}, \tilde{\phi}) + \hbar M(\phi, \phi^{*Aa}, \tilde{\phi}). \quad (39)$$

The Ward identities then become

$$0 = \langle \delta a X \rangle \quad (40)$$

$$= \int [d\phi][d\phi^a][d\tilde{\phi}][d\varphi_+][d\pi_a][dB][d\lambda] \delta a X \cdot e^{\frac{i}{\hbar}W_{BLT}(\phi - \varphi_+, \phi^{*Aa}, \tilde{\phi})} \cdot \tilde{V} \left[ e^{\frac{i}{\hbar}S_{\delta}} \right] \cdot e^{-\frac{i}{\hbar}S_{\delta A_1} M A B \lambda_B}. \quad (41)$$
Let us take \( a = 1 \). Then
\[
\delta_1 X = \frac{\delta X}{\delta \phi_A} \cdot \pi_{A1} + \frac{\delta X}{\delta \phi^{*A1}} (-1)^{\epsilon_A} M^{BA} \left[ \lambda_B - \frac{1}{2} \left( B_B + \frac{\delta R_{B1}}{\delta \phi_C} R_{C2} \right) \right] \\
+ \frac{\delta X}{\delta \phi_A} \cdot \frac{1}{2} M^{BA} \left[ -2 \phi_B^2 + \pi_{B1} - R_{B1}(\phi - \varphi_+) \right].
\]

We reintroduced the primes for the \( \phi^{*A'} \) in order to distinguish the antifields before and after the redefinition. Now it is important to notice that
\[
\delta_A \varphi_{A+} = \pi_{Aa} - R_{Aa}(\phi - \varphi_+)
\]
\(-\delta_2 \delta_1 \varphi_{A+} = -\delta_2(\pi_{A1} - R_{A1}(\phi - \varphi_+)) = B_A + \frac{\delta R_{A1}(\phi - \varphi_+)}{\delta \phi_B} R_{B2}(\phi - \varphi_+).
\)

Using this, and noticing that the Ward identities themselves allow us to ‘integrate by parts’ the (anti-)BRST operator, we see that
\[
\left\langle \frac{\delta X}{\delta \phi^{*A1'}} \left( B_B + \frac{\delta R_{B1}}{\delta \phi_C} R_{C2} \right) \right\rangle = \left\langle \delta_1 \delta_2 \left[ \frac{\delta X}{\delta \phi^{*A1}} \right] \varphi_{B+} \right\rangle
\]
\[
\left\langle \frac{\delta X}{\delta \phi_A} (\pi_{B1} - R_{B1}) \right\rangle = (-1)^{\epsilon_B+1} \left\langle \delta_1 \left[ \frac{\delta X}{\delta \phi_A} \right] \varphi_{B+} \right\rangle.
\]

Hence, both terms disappear from the Ward identity as \( \varphi_+ \) is fixed to zero by a delta function integration. Denote the complete measure of the path integral by \( d\mu \), then we can write the remaining Ward identity as
\[
0 = \int d\mu \left[ \frac{\delta X}{\delta \phi_A} \pi_{A1} + \frac{\delta X}{\delta \phi^{*A1}} (-1)^{\epsilon_A} M^{BA} \lambda_B + \phi^{*A2'} (-1)^{\epsilon_X} \frac{\delta X}{\delta \phi_A} \right] \\
e^{-\frac{\hbar}{\lambda} W_{BLT} (\phi - \varphi_+, \phi^{*A}, \delta)} \hat{V} \left[ e^{\phi \hat{\pi}_S} \right] e^{-\frac{\hbar}{\lambda} \hat{\pi}_A M^{AB} \lambda_B}.
\]

In the first term, the \( \varphi_{A+} \) can trivially be integrated out. Then, considering the expressions for \( \hat{V} \) and \( \hat{S}_S \), we see that \( \pi_{A1} \) can be replaced by \(-\frac{\hbar}{\lambda} \frac{\delta}{\delta \phi^{*A}_1} \).

We then integrate by parts over \( \phi^{*A1} \), which leads to
\[
\frac{\hbar}{\lambda} (-1)^{\epsilon_X (\epsilon_A+1)} \frac{\delta}{\delta \phi^{*A1}} \left[ \frac{\delta X}{\delta \phi_A} e^{\frac{\hbar}{\lambda} W_{BLT}} \right] \hat{V} \left[ e^{\phi \hat{\pi}_S} \right]
\]
\[
= \left[ -i \hbar \Delta_1 X + \frac{\delta X}{\delta \phi_A} \cdot \frac{\delta W_{BLT}}{\delta \phi^{*A1}} \right] e^{\frac{\hbar}{\lambda} W_{BLT}} \hat{V} \left[ e^{\phi \hat{\pi}_S} \right].
\]

under the path integral. Here, we generalised that other operator well-known from BV:
\[
\Delta_a X = (-1)^{\epsilon_A+1} \frac{\delta}{\delta \phi^{*A}} \frac{\delta}{\delta \phi_A} X.
\]
For the second term we can proceed analogously by replacing $M^{AB}e^{-\frac{i}{\hbar}\phi_A}M^{AB}e^{-\frac{i}{\hbar}\phi_B}$ by $\left(-\frac{i}{\hbar}\right)\delta^{AB}e^{-\frac{i}{\hbar}\phi_A+M^{AB}\phi_B}$. Integrating by parts over $\phi_A$, we see that the derivative can only act on $W_{BLT}(\phi - \phi_+, \phi^a, \tilde{\phi})$, and we get under the path integral

$$\frac{\delta X}{\delta \phi^A} \frac{\hbar}{i} \frac{\delta}{\delta \phi^A} e^{\hat{V}W_{BLT}(\phi - \phi_+, \phi^a, \tilde{\phi})} \hat{V} \left[ e^{\frac{i}{\hbar}\hat{S}_\delta} \right] \delta(\phi_+). \tag{48}$$

Remembering that $\frac{d}{dx} f(x - y) = -\frac{d}{dy} f(x - y)$, this leads finally to

$$\int d\mu e^{\frac{i}{\hbar}\hat{V}W_{BLT}} \hat{V} \left[ e^{\frac{i}{\hbar}\hat{S}_\delta} \right]. \tag{49}$$

The complete Ward identity hence becomes, dropping the primes,

$$0 = \left( (X, W_{BLT})_1 - i\hbar \Delta X + (-1)^{\epsilon_X} \phi^a \phi^A \frac{\delta X}{\delta \phi^A} \right) \int [d\phi][d\phi^a][d\tilde{\phi}] \left( (X, W_{BLT})_1 - i\hbar \Delta X + (-1)^{\epsilon_X} \phi^a \phi^A \frac{\delta X}{\delta \phi^A} \right) \hat{V} \left[ e^{\frac{i}{\hbar}\hat{S}_\delta} \right]. \tag{50}$$

An analogous property is of course obtained by going through the same steps for the Ward identities $\langle \delta_2 X \rangle = 0$.

### 5.2 Quantum Master Equation

As in the case of the BV formalism, the fact that this Ward identity is valid for all $X(\phi, \phi^a, \tilde{\phi})$ leads to an equation on $W_{BLT}$, the so-called quantum master equation. Starting from the most general Ward identity, the purpose is of course to remove all derivative operators acting on $X$ by partial integrations. Again, we denote by $d\mu$ the measure of the path integral. We thus start from

$$0 = \int d\mu \left[ \hat{\delta} X \hat{\delta} W_{BLT} \right] - \hat{\delta} X \hat{\delta} W_{BLT} - i\hbar(-1)^{\epsilon_{A+1}} \hat{\delta} X \hat{\delta} W_{BLT} + (-1)^{\epsilon_X} \hat{\delta} X \hat{\delta} W_{BLT} + \hat{\delta} X \hat{\delta} W_{BLT} + \hat{\delta} X \hat{\delta} W_{BLT} \hat{V} \left[ e^{\frac{i}{\hbar}\hat{S}_\delta} \right]. \tag{51}$$

Notice that we have reexpressed the operator $\hat{V}$ as $e^{\frac{i}{\hbar}\hat{S}_\delta}$.

By integrating by parts over $\phi_A$ in the first term, we get the following two terms:

$$\int d\mu i\hbar X \Delta_a e^{\frac{i}{\hbar}W_{BLT}} e^{\frac{i}{\hbar}(S_\delta + \hat{S}_\delta)}$$

$$+ \int d\mu i\hbar X (-1)^{\epsilon_{A+1}} \frac{\hat{\delta} e^{\frac{i}{\hbar}W_{BLT}}}{\hat{\delta} \phi_A} \hat{\delta} \left[ e^{\frac{i}{\hbar}(S_\delta + \hat{S}_\delta)} \right]. \tag{52}$$
The second and third contribution to the Ward identity (51) can be combined to give
\[
\int d\mu (-i\hbar)(-1)^{\epsilon_A+1} \frac{\delta}{\delta \phi_A} \left[ \frac{\delta X}{\delta \phi^{*Aa}} e^{\hat{\pi} W_{BLT}} \right] e^{\hat{\pi} (S_{\phi}+\tilde{S}_{\delta})}.
\]

(53)

Integrating by parts twice, first over \(\phi_A\), then over \(\phi^{*Aa}\) gives us the terms:
\[
\int d\mu i\hbar (-1)^{\epsilon_A} X e^{\hat{\pi} W_{BLT}} \frac{\delta}{\delta \phi^{*Aa}} \left[ e^{\hat{\pi} (S_{\phi}+\tilde{S}_{\delta})} \right] \frac{\delta}{\delta \phi_A} \left[ e^{\hat{\pi} (S_{\phi}+\tilde{S}_{\delta})} \right].
\]

(54)

Notice that the second term of (52) cancels the second term of (54).

Also in the fourth term of (51), we have to integrate by parts, over \(\bar{\phi}_A\). This gives us again two terms:
\[
- \int d\mu X \epsilon_{ab} \frac{\delta}{\delta \bar{\phi}_A} \frac{\delta}{\delta \phi^{*Aa}} e^{\hat{\pi} W_{BLT}} \frac{\delta}{\delta \phi_A} \frac{\delta}{\delta \bar{\phi}^*_{Aa}} \left[ e^{\hat{\pi} (S_{\phi}+\tilde{S}_{\delta})} \right].
\]

(55)

It is possible to show that the first term in (54) and the second term in (55) cancel. Working out the two derivatives and using the explicit form of \(\tilde{S}_{\delta}\), we rewrite the first term of (54) as
\[
\int d\mu (i\hbar)(\frac{i}{\hbar})^2 X e^{\hat{\pi} W_{BLT}} \frac{\delta}{\delta \bar{\phi}_A} \frac{\delta}{\delta \phi^{*Aa}} \left[ e^{\hat{\pi} (S_{\phi}+\tilde{S}_{\delta})} \right] \frac{\delta}{\delta \phi_A} \frac{\delta}{\delta \bar{\phi}^*_{Aa}} \hat{V} e^{\hat{\pi} \hat{S}_{\delta}}.
\]

(56)

Now, we know that \(\delta_a S_{\phi} = 0\), which allows us to replace \(\frac{\delta}{\delta \phi_A} \pi_{Aa}\) by \(-\frac{\delta}{\delta \pi_{Ab}} \epsilon_{ab} B_A\).

Using the explicit form of \(\hat{S}_{\delta}\) again, this is
\[
- \int d\mu (i\hbar) X e^{\hat{\pi} W_{BLT}} \frac{\delta}{\delta \pi_{Ab}} \frac{\delta}{\delta \phi^{*Aa}} e^{\hat{\pi} \hat{S}_{\delta}} \frac{\delta}{\delta \bar{\phi}_A} \frac{\delta}{\delta \phi^*_{Aa}} \epsilon_{ab}.
\]

(57)

One more partial integration, over \(\pi_{Ab}\), is needed to see that the terms do cancel as mentioned above.

Summing all this up, we see that the Ward identities are equivalent to
\[
0 = \int d\mu X \left[ i\hbar \Delta_a - \epsilon_{ab} \phi^{*Aa} \frac{\delta}{\delta \phi^*_{Aa}} \frac{\delta}{\delta \phi_A} \left[ e^{\hat{\pi} W_{BLT}} \hat{V} e^{\hat{\pi} \hat{S}_{\delta}} \right] \right].
\]

(58)

As this is valid for all possible choices for \(X(\phi, \phi^{*a}, \bar{\phi})\), we see that \(W_{BLT}\) has to satisfy the quantum master equation
\[
\left[ i\hbar \Delta_a - \epsilon_{ab} \phi^{*Aa} \frac{\delta}{\delta \phi^*_{Aa}} \frac{\delta}{\delta \phi_A} \right] e^{\hat{\pi} W_{BLT}} = 0.
\]

(59)
This is equivalent to

$$\frac{1}{2}(W_{BLT}, W_{BLT})_a + \epsilon_{ab}\phi^*_{Ab} \frac{\delta W_{BLT}}{\delta \phi^A} = i\hbar \Delta_a W_{BLT}. \quad (60)$$

Remember that these are two equations, $a = 1, 2$. By doing the usual expansion $W_{BLT} = S_{BLT} + \hbar M_1 + \hbar^2 M_2 + \ldots$, we recover the classical master equation (37) for $S_{BLT}$.

6 Open Algebras

In section 2, we pointed out how combining the collective field approach and the recipe of [16], one is naturally lead to the construction of an extended action that contains terms of quadratic and higher order in the antifields. As we do not have a principle analogous to [16] for constructing a gauge fixed action that is invariant under extended BRST symmetry for the case of an open algebra, we will have to take the other point of view advocated there.

Although we may compare the collective field method to a method sometimes employed in French cuisine: a piece of pheasant meat is cooked between two slices of veal, which are then discarded [17], the collective fields again play an important part. Like in the case of ordinary BRST collective field quantisation, the introduction of the collective fields allow us to shift the problem of the off-shell non-nilpotency to the (anti-)BRST transformations of the collective fields. Indeed, $\delta_a \phi_A = \pi_{Aa}$, $\delta_a \pi_{Ab} = \epsilon_{ab} B_A$ and $\delta_a B_A = 0$ guarantee that $\delta^2 \phi_A = 0$ and that $(\delta_1 \delta_2 + \delta_2 \delta_1) \phi_A = 0$. Therefore, the originally present gauge symmetry can be fixed in an extended BRST invariant way like for closed algebras, i.e. by adding $S_\Psi = \frac{1}{2} \epsilon^{ab} \delta_a \delta_b \Psi$ to an extended BRST invariant action, $S_{inv}$. This way, Ward’s identities guarantee that whatever way we choose to construct $S_{inv}$, the partition function will be invariant of the gauge choice if $S_{inv}$ is extended BRST invariant.

Another requirement that has to be satisfied by a good quantisation procedure, is that when the gauge fixing is omitted, one gets the original, ill-defined partition function back. It is clear that by decomposing $S_{inv} = S_{BLT} + \tilde{S}_\delta$, with the familiar form for $\tilde{S}_\delta$ and with $S_{BLT} = S_0 + \ldots$ where the dots stand for terms of at least first order in the antifields $\phi^* A_a$ and $\tilde{\phi}^A$, does satisfy that requirement. Imposing that the SD equations are derivable as Ward identities again restricts us to this form. The naive point of view is then that before we add the gauge fixing $S_\Psi$, the antifields are fixed to zero by $\tilde{S}_\delta$, and we can hence add whatever terms proportional to them.

As far as the invariance of $S_{inv}$ under extended BRST transformations is concerned, we know that that is indeed satisfied if we take $S_{BLT}$ to be a solution of the classical master equation (37) and take

$$\delta_a \varphi_{Ab} = \delta_{ab} \left[ \pi_{Aa} - \epsilon_{ac} \phi^*_{Ac} - \frac{\delta S_{BLT}(\phi - \varphi_+)}{\delta \phi^*_{Aa}} \right] + (1 - \delta_{ab}) \epsilon_{ac} \phi^*_{Ac}$$
\[
\delta_a \phi^b_A = -\delta^b_a \left[ (-1)^a \lambda_A + \frac{1}{2} (B_A + \frac{\delta S_{BLT}(\phi - \varphi_+)}{\delta \phi^A}) \right].
\] (61)

Hence, we see that the question whether open algebras can be quantised in an extended BRST invariant way, reduces to the fact whether a solution to (37) can be found for open algebras with the boundary condition that \( S_{BLT} = S_0 + \phi^* A_a R_{Aa} + \ldots \) It has been proved that such solutions exist [11, 18, 19].

As far as the treatment of reducible gauge algebras is concerned, the collective field formalism does not define the ghost spectrum that has to be introduced for a correct quantisation. As was pointed out already in [6], once the configuration space is constructed correctly for a reducible gauge algebra, one is left with a nilpotent or on-shell nilpotent set of extended BRST transformation rules. Both cases are in fact the ones treated above.

7 Conclusion

In this paper, we modified the collective field approach to quantisation of gauge theories in order to derive an antifield formalism for extended BRST invariant quantisation. We introduced two collective fields for every field. This way, we have two ghost-antighost pairs associated with the two shift symmetries. The antighost field of the first pair acts as a source for the BRST transformations, the ghost field of the second as a source for the anti-BRST transformations. The remaining ghost and antighost naturally lead to a representation of the gauge fixing. The sum of the two collective fields can be integrated out trivially, while their difference is needed as a source term for the composition of a BRST and and anti-BRST transformation. We argue that this approach does allow for the extended BRST invariant treatment of open algebras, stressing the importance of the part played by the collective fields.

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