Inelastic Final-State Interactions and Two-body Hadronic B decays into Single-Isospin channels

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Abstract

The role of inelastic final-state interactions in CP asymmetries and branching ratios is investigated in certain chosen single isospin two-body hadronic B decays. Treating final-state interactions through Pomeron and Regge exchanges, we demonstrate that inelastic final state interactions could lead to sizeable effects on the CP asymmetry.

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I. Introduction

It is well-known that CP asymmetries occur in two-body hadronic decays of B meson involving two distinct CKM angles and two differing strong phases[1]. The sources of the strong phases are several: perturbative penguin loops, final-state interaction (fsi) phases involving two different isospins, and inelastic final-state interactions involving a single isospin state.

In this paper we have studied the effect of interchannel mixing on B decays into two-body single-isospin channels. For reasons to follow, we have chosen to study the following decay modes: $B^{-} \rightarrow \eta_{c}\pi^{-}$, $B^{-} \rightarrow \eta_{c}K^{-}$ and $B^{-} \rightarrow \phi K^{-}$. The first of them, $B^{-} \rightarrow \eta_{c}\pi^{-}$, is a color-suppressed decay into a state with $I = 1$. At the tree-level, it proceeds through a CKM angle product $V_{cb}V_{cd}^\ast$. In absence of interchannel mixing, the CP asymmetry for this mode is known to vanish[2]. (Though ref.[2] does not include electromagnetic penguins, their inclusion does not alter this fact.) The color-favored decay channel $B^{-} \rightarrow D^{0}D^{-}$ with $I = 1$, also proceeds through a CKM product $V_{cb}V_{cd}^\ast$ at the tree-level, and has a nonvanishing CP asymmetry[2]. An inelastic coupling of $D^{0}D^{-}$ channel to $\eta_{c}\pi^{-}$ allows a two-step decay $B^{-} \rightarrow D^{0}D^{-} \rightarrow \eta_{c}\pi^{-}$ resulting in, as we show later, a significant CP asymmetry in $\eta_{c}\pi^{-}$ final state. The same can be argued for $B^{-} \rightarrow \eta_{c}K^{-}$, also a color-suppressed decay proceeding via $V_{cb}V_{cs}^\ast$ at the tree-level. It is also known to have a vanishing CP asymmetry[2] in absence of inelastic fsi. However, the interchannel mixing of $\eta_{c}K^{-}$ channel with $D^{0}D_{s}^{-}$, a color-favored channel, results in a nonvanishing CP asymmetry in $\eta_{c}K^{-}$ mode. Lastly, the mode $B^{-} \rightarrow \phi K^{-}$ involves $b \rightarrow s\bar{s}s$ decay and proceeds only through a penguin amplitude involving CKM angle products $V_{ub}V_{us}^\ast$ and $V_{cb}V_{cs}^\ast$. In absence of inelastic fsi, it is known to have a nonzero CP asymmetry. The strong phases here arise from the light quark
penguin loops. However, $\phi K^-$ channel can couple to $D^{*0}D_s^-$ and $D^0D_s^{*-}$ channels. The decays $B^- \to D^{*0}D_s^-$ and $B^- \to D^0D_s^{*-}$ being Cabibbo-favored, have branching ratios three orders of magnitude larger than that of the penguin process $B^- \to \phi K^-$. We have studied the effect of the inelastic coupling of the channels $B^- \to D^{*0}D_s^-$ and $D^0D_s^{*-}$ to $B^- \to \phi K^-$ channel on the branching ratio and CP asymmetry in the latter mode.

This paper is organized as follows: In section II, we describe the formalism and investigate the effect of inelastic fsi on the branching ratios and CP asymmetries in $B^- \to \eta_c\pi^-$, $B^- \to \eta_cK^-$ and $B^- \to \phi K^-$. The results are discussed in section III.

II. CP Asymmetry and Final State Interaction in $B^- \to \eta_c\pi^-$, $B^- \to \eta_cK^-$ and $B^- \to \phi K^-$

A. Definitions and Formalism

The effective Hamiltonian for $b \to s$ transition (for $b \to d$, replace $s$ by $d$) is given by [3, 4, 5]

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q = u, c} \left\{ V_{qb}V_{qs}^* \left[ C_1O_1^q + C_2O_2^q + \sum_{i=3}^{10} C_iO_i \right] \right\}. \quad (1)$$

The operators in Eq.(1) are the following;

$$O_1^q = (\bar{s}q)_{V-A}(\bar{q}b)_{V-A}, \quad O_2^q = (\bar{s}_\alpha q_\beta)_{V-A}(\bar{q}_\beta b_\alpha)_{V-A};$$

$$O_3 = (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}'q)_{V-A}, \quad O_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A},$$

$$O_5 = (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}'q)_{V+A}, \quad O_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A}; \quad (2)$$

$$O_7 = \frac{3}{2}(\bar{s}b)_{V-A} \sum_{q'} (e_{q'}q'_\alpha)_{V+A}, \quad O_8 = \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} (e_{q'}q'_\beta q'_\alpha)_{V+A},$$

$$O_9 = \frac{3}{2}(\bar{s}b)_{V-A} \sum_{q'} (e_{q'}q'_\alpha)_{V-A}, \quad O_{10} = \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} (e_{q'}q'_\beta q'_\alpha)_{V-A}.$$
$O_1$ and $O_2$ are the Tree Operators, $O_3, \ldots, O_6$ are generated by QCD Penguins and $O_7, \ldots, O_{10}$ are generated by Electroweak Penguins. Here $V \pm A$ represent $\gamma \mu (1 \pm \gamma_5)$, $\alpha$ and $\beta$ are color indices. $\sum_{q'}$ is a sum over the active flavors u,d,s and c quarks.

In the next-to-leading-log calculation one works with effective Wilson coefficients $C_i^{eff}$, rather than the coefficients that appear in (1). The derivation of these effective coefficients is well known [3, 4, 5]. We simply quote their values

$$
C_1^{eff} = \bar{C}_1, \quad C_2^{eff} = \bar{C}_2, \quad C_3^{eff} = \bar{C}_3 - P_s/N_c, \quad C_4^{eff} = \bar{C}_4 + P_s,
$$
$$
C_5^{eff} = \bar{C}_5 - P_s/N_c, \quad C_6^{eff} = \bar{C}_6 + P_s, \quad C_7^{eff} = \bar{C}_7 + P_e,
$$
$$
C_8^{eff} = \bar{C}_8, \quad C_9^{eff} = \bar{C}_9 + P_e, \quad C_{10}^{eff} = \bar{C}_{10},
$$

with [6]

$$
\bar{C}_1 = 1.1502, \quad \bar{C}_2 = -0.3125, \quad \bar{C}_3 = 0.0174, \quad \bar{C}_4 = -0.0373, \quad \bar{C}_5 = 0.0104, \quad \bar{C}_6 = -0.0459,
$$
$$
\bar{C}_7 = -1.050 \times 10^{-5}, \quad \bar{C}_8 = 3.839 \times 10^{-4}, \quad \bar{C}_9 = -0.0101, \quad \bar{C}_{10} = 1.959 \times 10^{-3},
$$

and

$$
P_s = \frac{\alpha_s(\mu)}{8\pi} C_1(\mu) \left[ \frac{10}{9} + \frac{2}{3} \ln \frac{m_q^2}{\mu^2} - G(m_q, \mu, q^2) \right],
$$
$$
P_e = \frac{\alpha_{em}(\mu)}{3\pi} \left[ C_2(\mu) + \frac{C_1(\mu)}{N_c} \right] \left[ \frac{10}{9} + \frac{2}{3} \log \frac{m_q^2}{\mu^2} - G(m_q, \mu, q^2) \right],
$$

where

$$
G(m_q, \mu, q^2) = -4 \int_0^1 dx x(1-x) \ln \left[ 1 - x(1-x) \frac{q^2}{m_q^2} \right],
$$

$q^2$ is the momentum carried by the gluon or the photon in the penguin diagram and $m_q$ the mass of the quark q in the penguin loop. For $q^2 > 4m_q^2$, $G(m_q, \mu, q^2)$ becomes complex giving rise to strong perturbative phases through $P_s$ and $P_e$. The parameters
we employ are:

\[ m_u = 5\, \text{MeV}, m_s = 175\, \text{MeV}, m_c = 1.35\, \text{GeV}, m_b = 5.0\, \text{GeV}, \]

\[ CKM\, \text{angles}: A = 0.81, \lambda = 0.22, (\rho, \eta) = (-0.20, 0.45) \text{ and } (0.30, 0.42), \]

\[ f_D = 200\, \text{MeV}, f_{D_s} = f_{D_s^*} = f_\eta = 300\, \text{MeV}, f_\phi = 233\, \text{MeV}. \] (8)

Consider now each one of the decays \( B^- \to \eta_c \pi^- \), \( \eta_c K^- \) and \( \phi K^- \) in absence of inelastic FSI. In the factorization approximation, which we adopt, the decay amplitudes are:

\[ A(B^- \to \eta_c \pi^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cs}^* a_2 - V_{tb} V_{ts}^* (a_3 - a_5 - a_7 + a_9) \right\} \eta_c |(\bar{c}c)_{V-A}| 0 < \pi^- |(\bar{d}b)_{V-A}| B^- > \]

(10)

\[ A(B^- \to \eta_c K^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cs}^* a_2 - V_{tb} V_{ts}^* (a_3 - a_5 - a_7 + a_9) \right\} \eta_c |(\bar{c}c)_{V-A}| 0 < K^- |(\bar{s}b)_{V-A}| B^- > \]

(11)

\[ A(B^- \to \phi K^-) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ (1 + \frac{1}{N_c}) C_{2i}^{eff} + (1 + \frac{1}{N_c}) C_{4i}^{eff} + a_5 - a_7/2 - \frac{1}{2} (1 + \frac{1}{N_c}) C_{9i}^{eff} \right\} < \phi |(\bar{s}s)_{V-A}| 0 < K^- |(\bar{s}b)_{V-A}| B^- > \]

(12)

where we have used the unitarity relation \( \sum_{q=u,c} V_{qb} V_{qs}^{*} = -V_{tb} V_{ts}^{*} \) and defined

\[ a_{2i} = C_{2i}^{eff} + \frac{1}{N_c} C_{2i-1}^{eff}, \]

\[ a_{2i-1} = C_{2i-1}^{eff} + \frac{1}{N_c} C_{2i}^{eff}, \]

(13)

with \( i \) an integer. The strong phases appear through \( P_s \) and \( P_e \) defined in (5) and (6). However, in (10 ) and (11 ), odd coefficients \( a_3, a_5, a_7 \) and \( a_9 \) involve such combinations of \( C_{i}^{eff} \) as to cancel the effect of \( P_s \) and \( P_e \). Thus, strong phases do not appear in (10 ) and (11 ) but they do in (12 ). Hence, CP asymmetry vanishes for \( B^- \to \eta_c \pi^- \) and
\( \eta_c K^- \) but is nonzero for \( B^- \to \phi K^- \). This is true even if the electromagnetic penguins were ignored as in ref\[2\].

Let us now consider the decay channel \( B^- \to D^0 D^- \). The decay amplitude is given by,

\[
A(B^- \to D^0 D^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cd}^{\ast} a_1 - V_{tb} V_{td}^{\ast} (a_4 + (a_6 + a_8) R_1 + a_{10}) \right\} < D^- | (\bar{d}c)_{V-A} | 0 > < D^0 | (\bar{c}b)_{V-A} | B^- >
\]

where

\[
R_1 = \frac{2m_D^2}{(m_b - m_c)(m_c + m_d)}.
\]

Note that the above decay is color-favored (tree diagram being proportional to \( a_1 \)) and that strong phases do not cancel in the even coefficients \( a_4, a_6, a_8 \) and \( a_{10} \). Hence CP asymmetry in \( B^- \to D^0 D^- \) is nonvanishing\[2\]. Throughout our calculations, we have used the formfactors from Bauer, Stech and Wirbel\[6\].

In the following section we discuss in detail the mixing of \( D^0 D^- \) and \( \eta_c \pi^- \) channels through inelastic fsi.

**B. Inelastic mixing of \( D^0 D^- \) and \( \eta_c \pi^- \) channels.**

Inelastic fsi have been discussed in the past\[7, 8, 9, 10, 11\] in the context of the K-matrix formalism. The desirable feature of this method is that unitarity of the S-matrix is ensured. The difficulty lies in the proliferation of K-matrix parameters, mostly unknown, with the number of channels. Moreover, in two-channel problems, the second channel (the inelastic channel) is assumed to reflect (through unitarity) all the inelastic channels. This is an oversimplification of reality. In ref.\[8\], the coupling of \( \eta_c \pi^- \) channel to \( \eta \pi^- \) and \( \eta' \pi^- \) is discussed in K-matrix formalism. We comment on this work in Section III.
In the calculations we present below, we make no effort to enforce two-channel unitarity. Rather, we couple the decay channel $\eta_c\pi^-$ to $D^0D^-$ using a Regge-exchange model. The model coupling constants are related to known coupling constants by approximations we explain in the text. The advantage of this procedure, in contrast to the K-matrix approach[7, 8, 9, 10, 11], is that the scattering parameters are determined more realistically. The shortcoming is that the relevant elements of the S-matrix being completely determined, the S-matrix itself does not satisfy two-channel unitarity. Yet, we think it is more realistic to treat the effect of inelastic channels one channel at a time rather than enforce two-channel unitarity on what is in fact a multi-channel problem.

We begin by establishing certain key equations for an arbitrary number of two-body channels. An $n \times n$ S-matrix for s-wave scattering satisfying unitarity can be written as [12]

$$S = (1 + ik^{\frac{1}{2}}Kk^{\frac{1}{2}})(1 - ik^{\frac{1}{2}}Kk^{\frac{1}{2}})^{-1},$$

where $k$ is a digonal momentum matrix and $K$ a real-symmetric matrix with $n(n+1)/2$ real parameters. The decay amplitudes for the B meson into $n$ two-body channels are inelastically coupled through

$$A = (1 - ik^{\frac{1}{2}}Kk^{\frac{1}{2}})^{-1}A^{(0)},$$

where $A^{(0)}$ is a column of uncoupled amplitudes. The coupled (unitarized ) amplitudes are assembled in the column $A$. From (16) and (17) it is easily shown that [11]

$$A = \frac{1 + S}{2}A^{(0)}.$$  

Let us label channels $\eta_c\pi^-$ and $D^0D^-$ as channels 1 and 2 respectively. $A_1^{(0)}$ and $A_2^{(0)}$ are given by (10) and (14) respectively. In order to calculate the effect of channel 2 on channel 1 and vice-versa, we need to calculate the elements $S_{11}$, $S_{12}$ and $S_{22}$ of the S-matrix. We describe their evaluation in the following.
We assume that Pomeron exchange dominates elastic scattering. The scatterings
\( \eta c \pi^- \to \eta c \pi^- \) and \( D^0 D^- \to D^0 D^- \) are then represented by amplitudes of the form\[13\]

\[
P(s,t) = \beta(t) \left( \frac{s}{s_0} \right)^{\alpha_P(t)} e^{i\alpha_P(t)/2}
\]  

(19)

where \( \sqrt{s_0} \) is an energy scale and the Pomeron trajectory is parameterized by

\[
\alpha_P(t) = 1.08 + 0.25t.
\]

(20)

The momentum transfer \( t \) is expressed in \( GeV^2 \) in the above. The Pomeron coupling strength, \( \beta(t) \), is assumed\[14, 15\] to have a \( t \)-dependence of the form \( \beta(t) = \beta(0)e^{2.8t} \).

In the additive quark model, \( \beta(0) = 4\beta(cu) \) for \( \eta c \pi^- \to \eta c \pi^- \), and \( \beta(0) = 2\beta(cu) + \beta(uu) + \beta(cc) \) for \( D^0 D^- \to D^0 D^- \) scattering. The residue \( \beta(uu) \) can be extracted from high energy \( pp \) and \( \pi p \) scattering data yielding \[14, 13\] \( \beta(uu) \approx 6.5 \). No experimental information exists for the determination of \( \beta(cu) \). We make the theoretical ansatz\[13\]:

\[
\beta(cu) \approx \frac{1}{10}\beta(uu),
\]

and assume \( \beta(cc) \) to be negligibly small\[13\].

The inelastic scattering \( \eta c \pi^- \to D^0 D^- \) is mediated by \( D^* \) Regge-exchange in the \( t \) channel (defined by \( t = (P_{D^0} - P_{\eta c})^2 \)). The amplitude is of the form \[16\]

\[
R(s,t) = \beta_R(t) \frac{1 - e^{i\pi\alpha(t)}}{\sin[\pi\alpha(t)]} \left( \frac{s}{s_0} \right)^{\alpha(t)}.
\]

(21)

For \( \beta_R(t) \) we adopt a \( t \) dependence\[13, 16, 17\]

\[
\beta_R(t) = \frac{\beta_R(0)}{\Gamma[\alpha(t)]}.
\]

(22)

The fact that \( \Gamma(z) \) has simple poles at \( z = 0, -1, -2, \ldots \), ensures that the Regge amplitude (21) does not develop nonsense poles at \( \alpha(t) = 0, -1, -2, \ldots \). We also note that in addition to \( R(s,t) \) of (21), there is a \( u \)-channel exchange amplitude \( R(s,u) \) generated by a charged \( D^* \) exchange.
Generally, $s_0$ is expected to be process-dependent. For light mesons and baryons it has been taken\cite{13} as $s_0 = \frac{1}{\alpha'} \approx 1 GeV^2$. However, for heavy mesons and baryons, the scale $s_0$ must reflect somehow a higher threshold for the scattering processes. Based on the work of \cite{17}, it is argued in \cite{16} that for $\pi D$ scattering mediated by $\rho$-trajectory, $s_{\pi D}^0 \approx \frac{2}{\alpha_R}$. We assume this value for $\eta_c \pi^- \to D^0 D^-$ scattering amplitude.

We determine $\beta_R(0)$ by taking the limit $\alpha(t) \to 1$ ($D^*$ pole) in (21) and comparing it with the perturbative t-channel $D^*$-pole diagram. For the latter we assume a $VPP$ vertex of form:

$$I_{\text{int}}^{VPP} = f_{ijk} g_{VPP} V_i^J P^j \partial^\mu P^k,$$

where $i$, $j$ and $k$ are SU(4) labels and $f_{ijk}$ the antisymmetric symbol.

The perturbative t-channel $D^*$-exchange graph yields,

$$R(s \to \infty, t \sim m_{D^*}^2) = g_{VPP}^2 \frac{s}{t - m_{D^*}^2}.$$  

Comparing the limiting case ($t \to m_{D^*}^2, \alpha(m_{D^*}^2) = 1$) of (21) with (24) results in

$$\beta_R(0) = \pi g_{VPP}^2.$$  

SU(4) symmetry allows us to determine $g_{VPP}$ from $\rho \to \pi \pi$ and $K^* \to K \pi$ decays\cite{18},

$$\frac{g_{VPP}^2}{4\pi} \approx 3.0.$$  

Heavy Quark effective Theory(HQET)\cite{19,20,21,22} could also have been used to determine $g_{D^*-D^0\pi}$ if the rate $\Gamma(D^* \to D\pi)$ were known. In absence of this information, authors of ref.\cite{19} fix this coupling by constraining it to yield the axial coupling of the nucleon, $g_A \approx 1.25$. This results in

$$g_{D^*-D^0\pi} = g_{D^*D^0\pi} = g_{D^0D^+K^-} \approx \frac{3\sqrt{m_{D^*} m_{D^-}}}{4f_\pi} \to \frac{\sqrt{m_{D^*} m_{D^-}}}{f_\pi},$$
where \( f_\pi = 131\, \text{MeV} \). This is a much larger coupling constant than that implied in \cite{18}, resulting in \( \Gamma(D^{*-} \to D^0\pi^-) = (100 - 180)\, \text{KeV} \). In contrast, the SU(4) symmetry scheme of ref\cite{18} obtains \( \Gamma(D^{*-} \to D^0\pi^-) = 16\, \text{KeV} \) using \( g_{VP} \) given in (26). In our calculations we use the SU(4) symmetry coupling given by (23) and (26) only.

From the scattering amplitudes we project out the elements of the S-wave scattering matrix

\[
S_{ij} = \delta_{ij} + \frac{i}{8\pi\sqrt{\lambda_i\lambda_j}} \int_{t_{min}}^{t_{max}} dt T(s,t),
\]

where \( T(s,t) \) is the total amplitude, \( \lambda_i \) and \( \lambda_j \) are the usual triangle functions \( \lambda(x,y,z) = (x^2 + y^2 + z^2 - 2xy - 2xz - 2yz)^{1/2} \) for channels \( i \) and \( j \) respectively and \( t_{max}, t_{min} \) are the limits of the momentum transfer. We also took into account the u-channel charged \( D^* \)-exchange in calculating the S-matrix elements. The resulting S-matrix elements (channel 1 = \( \eta_c\pi^- \), channel 2 = \( D^0D^- \)) are,

\[
S = \begin{pmatrix}
0.946 - 0.93 \times 10^{-3}i & 0.19 \times 10^{-2} + 0.068i & \ldots \\
0.19 \times 10^{-2} + 0.068i & 0.843 - 0.27 \times 10^{-2}i & \ldots \\
& & \ldots \\
& & & \ldots
\end{pmatrix}
\]

Clearly, two-channel unitarity is not satisfied but the S-matrix elements \( S_{ij}(i,j = 1,2) \) are completely determined. Calculation of the unitarized decay amplitudes proceeds by using \( A_0 \) from (10) and (14), and \( S \) from (28) in (18). The calculation of the branching ratios and CP asymmetry is then straightforward. We have chosen to perform the calculation for \( N_c = 3 \) and \( N_c = 2.4 \). The latter choice, suggested in \cite{23}, could be interpreted to reflect nonfactorization effects. The results are shown in Tables 1 and 2.

We note from these Tables that the induced CP asymmetry in \( \eta_c\pi \) channel is large; in fact, as large as in channel \( D^0D^- \) to which it is coupled. The CP asymmetry in \( \eta_c\pi^- \) channel, however, depends almost linearly on \( g_{VP}^2 \). Thus, increasing (decreasing) \( g_{VP} \)
by a factor of 2 results in an increase (decrease) of CP asymmetry by approximately a factor of four. We defer the discussion of the results to Section III.

C. Inelastic Mixing of $D^0 D_{s}^−$ and $\eta_c K^−$ channels

In absence of interchannel coupling, the decay amplitude for $B^− \to D^0 D_{s}^−$ is given by

$$A(B^− \to D^0 D_{s}^−) = \frac{G_F}{\sqrt{2}} \{ V_{cb} V_{cs}^∗ (a_1 - (a_6 + a_8) R_2 + a_{10}) \} < D_{s}^− | (\bar{s}c)_{V−A} | 0 > < D^0 | (\bar{c}b)_{V−A} | B^− >,$$

(30)

where

$$R_2 = \frac{2m_{D_{s}^−}}{(m_b - m_c)(m_c + m_s)}.$$  

(31)

Let us label $\eta_c K^−$ and $D^0 D_{s}^−$ as channels 1 and 2 respectively. The pomeron-mediated elastic scattering now involves coupling constants $2\beta (cu) + 2\beta (cs)$ for $\eta_c K^−$ channel and $\beta (cc) + \beta (cu) + \beta (cs) + \beta (us)$ for $D^0 D_{s}^−$ channel. For $\beta (cs)$ we use the ansatz: $\beta (cs) \approx \frac{1}{10} \beta (us) \approx \frac{1}{15} \beta (uu)$. The Pomeron amplitude is then given as in (19).

The inelastic scattering $\eta_c K^− \to D^0 D_{s}^−$ is mediated by $D^{*0}$-exchange in the $t$ channel ($t = (P_{D^0} - P_{\eta_c})^2$), and by $D_{s}^{*}$-exchange in the $u$ channel. The calculation of the effect of inelastic coupling of $\eta_c K^−$ and $D^0 D_{s}^−$ channels parallels that of $\eta_c \pi^−$ and $D^0 D^−$ channels described in the previous section. The resulting $S$-matrix ($\eta_c K^− = \text{channel 1, } D^0 D_{s}^− = \text{channel 2}$) is

$$S = \begin{pmatrix} 0.954 - 0.8 \times 10^{-3} i & 0.85 \times 10^{-3} + 0.068i & \ldots \ldots \\ 0.85 \times 10^{-3} + 0.068i & 0.888 - 0.19 \times 10^{-2} i & \ldots \ldots \end{pmatrix}.$$  

(32)

The resulting branching ratios and CP asymmetries are shown in Tables 1 and 2. Again, we notice that CP asymmetry induced in channel $\eta_c K^−$ is comparable to that in channel $D^0 D_{s}^−$. Further discussion of the results is deferred to section III.
D. Inelastic coupling of $\phi K^-$ to $D^{*0}D_s^-$ and $D_s^{*-}D^0$ channels.

In absence of inelastic fsi, the decay amplitudes for $B^- \to D^{*0}D_s^-$ and $D_s^{*-}D^0$ are:

$$A(B^- \to D^{*0}D_s^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{cb}V_{cs}^* a_1 - V_{tb}V_{ts}^* (a_4 - (a_6 + a_8)R_3 + a_{10}) \right\} < D_s^- |(\bar{s}c)_V A| 0 > < D^{*0}_s |(\bar{c}b)_V A| B^- >,$$

$$A(B^- \to D^0D_s^{*-}) = \frac{G_F}{\sqrt{2}} \{ V_{cb}V_{cs}^* a_1 - V_{tb}V_{ts}^* (a_4 + a_{10}) \} < D^*_s^- |(\bar{s}c)_V A| 0 > < D^0 |(\bar{c}b)_V A| B^- >,$$

where

$$R_3 = \frac{2m^2_{D_s}}{(m_s + m_c)(m_c + m_b)}.$$

(35)

Inelastic fsi couple the amplitude for $B^- \to \phi K^-$, eq.(12), to the amplitudes in (33) and (34). The calculation of the S-matrix elements ($\phi K^- =$ channel 1, $D_s^-D^{*0} =$ channel 2, $D_s^{*-}D^0 =$ channel 3) is considerably simplified in the B rest-frame. This is because in this frame the vector meson can only have longitudinal helicity in a $B \to VP$ decay. Because of this fact, the Pomeron and Regge amplitudes involving only helicity 0$\to$helicity 0 transition are the same as in spin-less scattering. The Pomeron amplitude for elastic scattering is given by (19) with $\beta(0) = 2\beta(su) + 2\beta(ss)$ for $\phi K^-$ elastic scattering, and $\beta(0) = \beta(us) + \beta(cu) + \beta(cs) + \beta(cc)$ for $D_s^-D^{*0}$ and $D_s^{*-}D^0$ elastic scatterings. We assume $\beta(ss) \approx \frac{2}{3}\beta(su)$.

Inelastic scatterings, $\phi K^- \to D_s^*-D^{*0}$ and $D_s^{*-}D^0$, are mediated by $D_s^*$ exchange in the t channel ($t = (P_{D_s} - P_\phi)^2$ and $(P_{D_s^*} - P_\phi)^2$ respectively). There are no u channel exchanges. $D_s$-trajectory, being a lower-lying trajectory, makes a smaller contribution and we neglect it. The Regge-amplitude is assumed to be of the form given in (21). In order to determine the coupling $\beta(0)$, we equate the limiting form of (21) for $t \to m^2_{D_s^*}$ with the perturbative expressions for $\phi K^- \to D_s^-D^{*0}$ and $D_s^{*-}D^0$ with $D_s^*$ exchange. We adopt the following definitions,
\( D_s^* - \text{trajectory} : \quad \alpha_{D_s^*} \approx -1.23 + 0.5t, \) \quad (36)

\( VVP - \text{vertex} : \quad L_{\text{int}}(VVP) = g_{VVP}d_{ijk} \epsilon_{\mu \nu \rho \sigma} \partial_\mu V^i_\nu \partial_\rho V^j_\sigma P^k, \) \quad (37)

\( VVV - \text{vertex} : \quad L_{\text{int}}(VVV) = g_{VVV}f_{ijk}(\partial_\mu V^i_\nu - \partial_\nu V^i_\mu)V^j_\mu V^k_\nu, \) \quad (38)

where \( d_{ijk} \) and \( f_{ijk} \) are SU(4) indices. The coupling \( g_{VVP} \) has dimension \((\text{mass})^{-1}\) while \( g_{VVV} \) is dimensionless. \( D_s^* \) trajectory is assumed to be parallel to \( D^* \) trajectory with a slope as in [14].

The evaluation of perturbative \( D_s^* \)-exchange diagram was done numerically. The calculation was made simpler by the fact that the vector particles could only be longitudinally polarized in B rest-frame. For large \( s \) we obtain for the diagrams shown in Figs.1 and 2,

\[
T^{\text{Fig.1}}(s \to \infty, t \sim m_{D_s^*}^2) = 0.7 GeV^2 g_{D^*D_s^*K} g_{D_s^*D_s^*f} \frac{s}{t - m_{D_s^*}^2},
\]

\[
T^{\text{Fig.2}}(s \to \infty, t \sim m_{D_s^*}^2) = 0.5 g_{DD_s^*K} g_{D_s^*D_s^*f} \frac{s}{t - m_{D_s^*}^2}. \quad (39)
\]

The corresponding Regge-exchange amplitude yields

\[
T(s \to \infty, t \sim m_{D_s^*}^2) = \frac{\beta(0)}{\pi} \frac{s}{t - m_{D_s^*}^2}, \quad (40)
\]

where we have used \( s_0 = 2/\alpha' \).

A comparison of (40) with (39) yields \( \beta(0) \). The coupling constants in (39) are determined as follows: In HQET [19], where light pseudoscalar mesons are introduced as nonlinear realization of \( SU(3)_L \times SU(3)_R \), one obtains \( (f_\pi = 131\,\text{MeV}) \), and we are using the parameter [19] \( f = -1.5 \),

\[
g_{D^*oD_s^*-K^-} \approx \frac{3}{4f_\pi}. \quad (41)
\]
Light vector and axial-vector mesons can also be introduced in HQET \([20, 21, 22]\) allowing a \(D_s^*D_s\phi\) coupling. However, HQET does not by itself permit an evaluation of this coupling constant. Use of vector dominance in the radiative decays of light mesons determines \([24]\) \(g_{VV\pi} \approx 6\text{GeV}^{-1}\). Using SU(4) symmetry, one obtains \(g_{D_s^*-D_s\phi} \approx 6/\sqrt{2}\text{GeV}^{-1}\) which is a little lower than the value given in (41). For want of a better choice we assume \(g_{D_s^*-D_s\phi} = g_{D_s^*0D_s^*-K^-}\).

HQET \([20, 21, 22]\) allows us to calculate the VVV coupling \(g_{D_s^*D_s^*\phi}\) provided an assumption is made as to how the flavor singlet and the flavor octet of the light vector meson couple to \(D_s^*D_s^*\). In nonet symmetry, we obtain

\[
\begin{align*}
  g_{D_s^*D_s^*\omega} &= 0 \\
  g_{D_s^*D_s^*\phi} &= g_{VV\pi} = g \text{ (of [22])} \approx 4.3
\end{align*}
\]  

As for the VPP coupling, we adopt the value in (26).

To calculate the effect of channels \(D_s^*D_s^0\) (channel 2) and \(D_s^-D_s^0\) (channel 3) on \(\phi K^-\) (channel 1), we need the elements \(S_{11}, S_{12}, S_{13}\) of the S-matrix. The decay amplitude for \(B^- \to \phi K^-\) is then given by,

\[
A(B^- \to \phi K^-) = \frac{1+S_{11}}{2}A^{(0)}(B^- \to \phi K^-)+\frac{S_{12}}{2}A^{(0)}(B^- \to D_s^0D_s^-)+\frac{S_{13}}{2}A^{(0)}(B^- \to D_s^0D_s^-),
\]

(44)

The relevant elements of the S-matrix are calculated to be,

\[
\begin{align*}
  S_{11} &= 0.784 - 0.37 \times 10^{-2}i \\
  S_{12} &= -0.53 \times 10^{-3} + 0.12 \times 10^{-2}i \\
  S_{13} &= -0.33 \times 10^{-3} + 0.70 \times 10^{-3}i
\end{align*}
\]  

(45)

Since \(B^- \to \phi K^-\) is a penguin mediated process, its branching ratio in absence of inelastic coupling is small \((\sim 10^{-6} \text{ to } 10^{-5})\)\([2]\). In contrast, the inelastic channels
$D^{*0}D_s^−$ and $D^{0}D_s^{*-}$ to which it couples are Cabibbo-favored and have branching ratios of the order of $10^{-3}$ to $10^{-2}$[2]. Thus, whereas the two channels $D^{*0}D_s^-$ and $D^{0}D_s^{*-}$ can influence the branching ratios and CP asymmetries in $\phi K^−$, we do not expect the $\phi K^−$ channel to significantly effect the channels $D^{*0}D_s^−$ and $D^{0}D_s^{*-}$. Further, the inelastic coupling of $D^{*0}D_s^−$ channel to $D^{0}D_s^{*-}$ occurs via the exchange of $(c\bar{c})$ mesonic trajectories, $\eta_c$ and $\psi$. As both of these trajectories are low-lying with large and negative intercepts $\alpha(0)$, their contribution to the inelastic scattering $D^{*0}D_s^− \rightarrow D^{0}D_s^{*-}$ is expected to be highly suppressed. We, therefore, do not expect one of these channels to effect the other significantly either. For these reasons we have displayed in Tables 1 and 2 only the effect on the channel $\phi K^-$.

The other two channels, $D^{*0}D_s^-$ and $D^{0}D_s^{*-}$, are left largely unaffected by fsi.

Tables 1 and 2 show that though the effect of inelastic fsi on the branching ratio for $\phi K^-$ is small (due to the small size of $S_{12}$ and $S_{13}$ in (45)), the effect on the CP asymmetry is significant. The results are discussed in the following section.

### III. Results and Discussion

In absence of inelastic fsi, CP asymmetries in $B^- \rightarrow \eta_c\pi^-$ and $\eta_c K^-$ channels vanish[2]. We have shown that an inelastic coupling of $\eta_c\pi^-$ channel to $D^{0}D^-\eta_c$ and that of $\eta_cK^-$ to $D^{0}D_s^-$, leads to substantial CP asymmetries in $B^- \rightarrow \eta_c\pi^-$ and $\eta_c K^-$ decays. We have used $N_c = 3$ and 2.4 and two sets of values for $(\rho, \eta)$. The CP asymmetries depend sensitively on the coupling constant $g_{\nu PP}$ and the energy-scale parameter $s_0$. An increase (decrease) of $g_{\nu PP}$ by a factor of two increases (decreases) the CP asymmetry by roughly a factor of 4. Similarly, increasing $s_0$ from $1GeV^2$ to $2/\alpha'_R$ enhances the CP asymmetry by raising the values of the off-diagonal elements of the S-matrix. In addition, the calculated CP asymmetry will also depend on the
effective $q^2$ employed. We have used $q^2 = m_B^2/2$.

Ref. 8 discusses a three-channel problem, $B^- \to \eta \pi^-, \eta' \pi^-$ and $\eta_c \pi^-$, in the K-matrix formalism and demonstrates that a CP asymmetry of the order of $\sim 1\%$ can be generated in $\eta_c \pi^-$ channel through inelastic fsi. Our work differs from 8 in several respects. Most importantly, we couple $\eta_c \pi^-$, a color-suppressed channel, to $D^0 D^-$, a color-favored channel. The channels, $\eta \pi^-$ and $\eta' \pi^-$, invoked in 8 are both color-suppressed, and thus, not expected to be as important as $D^0 D^-$. In addition, we do not require the S-matrix to be unitary at two or three channel level. Another important difference between our work and that of 8 lies in how the strong interaction fsi parameters are determined. We use Pomeron and Regge phenomenology, presumably applicable at $\sqrt{s} \sim m_B$, while 8 determines the K-matrix elements through low-energy phenomenology. For example, the diagonal and off-diagonal elements of the K-matrix are evaluated using a contact $\phi^4$-interaction which allows for only S-wave scattering. This is expected to be a reasonable approximation at threshold but hardly likely to hold at $\sqrt{s} \sim m_B$. Despite these differences, we emphasize that the important conclusion of 8 was that a significant CP asymmetry in $\eta_c \pi^-$ could be generated by coupling it to $\eta \pi^-$ and $\eta' \pi^-$ channels. However, the fact that they also found asymmetries of the order of 10% and 20% in $\eta \pi^-$ and $\eta' \pi^-$ channels has little to do with inelastic fsi; asymmetries of this magnitude are generated in these channels in absence of inelastic fsi 2.

We have found that the CP asymmetry in $B^- \to \phi K^-$(a penguin driven process) is significantly effected by a coupling to Cabibbo-favored channels $D^{*0} D_s^-$ and $D^0 D_s^-$. Due to the smallness of the off-diagonal elements of the S-matrix, the effect on the branching ratio is not as large as on CP asymmetry. Again, the CP asymmetry depends sensitively on the coupling constants and the value of $s_0$.  

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Table 1: Branching ratios and CP asymmetries with $q^2 = m_b^2/2$ and $N_c = 3$ (entries in bracket correspond to the uncoupled case)

| CKM Matrix               | $\rho = 0.30, \eta = 0.42$ | $\rho = -0.20, \eta = 0.45$ |
|--------------------------|---------------------------|-----------------------------|
| Decay channel            | BR  | $a_{CP}$(%) | BR  | $a_{CP}$(%) |
| $B^- \rightarrow \eta_c\pi^-$ | $2.85 \times 10^{-6}$ | 3.97 | $2.87 \times 10^{-6}$ | 4.24 |
|                          | $(2.32 \times 10^{-6})$ | (0.00 ) | $(2.33 \times 10^{-6})$ | (0.00 ) |
| $B^- \rightarrow D^0D^-$  | $3.55 \times 10^{-4}$ | 3.94 | $3.22 \times 10^{-4}$ | 4.69 |
|                          | $(4.18 \times 10^{-4})$ | (3.97) | $(3.80 \times 10^{-4})$ | (4.72) |
| $B^- \rightarrow \eta_cK^-$ | $7.42 \times 10^{-5}$ | -0.25 | $7.42 \times 10^{-5}$ | -0.27 |
|                          | $(5.54 \times 10^{-5})$ | (0.00 ) | $(5.54 \times 10^{-5})$ | (0.00 ) |
| $B^- \rightarrow D^0D_s^-$ | $1.47 \times 10^{-2}$ | -0.21 | $1.48 \times 10^{-2}$ | -0.23 |
|                          | $(1.65 \times 10^{-2})$ | (-0.21 ) | $(1.66 \times 10^{-2})$ | (-0.23 ) |
| $B^- \rightarrow \phi K^-$ | $6.21 \times 10^{-6}$ | 0.74 | $5.97 \times 10^{-6}$ | 0.83 |
|                          | $(7.75 \times 10^{-6})$ | (0.58) | $(7.44 \times 10^{-6})$ | (0.65 ) |
Table 2: Branching ratios and CP asymmetries with $q^2 = m_b^2/2$ and $N_c = 2.4$ (entries in bracket correspond to the uncoupled case)

| CKM Matrix | ρ = 0.30, η = 0.42 | ρ = −0.20, η = 0.45 |
|------------|---------------------|---------------------|
| Decay channel | BR | $a_{CP}$ (%) | BR | $a_{CP}$ (%) |
| $B^- \to \eta_c \pi^-$ | $1.31 \times 10^{-5}$ | 1.80 | $1.32 \times 10^{-5}$ | 1.90 |
| | $(1.29 \times 10^{-5})$ | (0.00 ) | $(1.29 \times 10^{-5})$ | (0.00 ) |
| $B^- \to D^0 D^-$ | $3.38 \times 10^{-4}$ | 3.74 | $3.06 \times 10^{-4}$ | 4.45 |
| | $(3.98 \times 10^{-4})$ | (3.80) | $(3.61 \times 10^{-4})$ | (4.52) |
| $B^- \to \eta_c K^-$ | $3.12 \times 10^{-4}$ | -0.12 | $3.11 \times 10^{-4}$ | -0.13 |
| | $(3.08 \times 10^{-4})$ | (0.00 ) | $(3.08 \times 10^{-4})$ | (0.00 ) |
| $B^- \to D^0 D_s^-$ | $1.29 \times 10^{-2}$ | -0.19 | $1.30 \times 10^{-2}$ | -0.20 |
| | $(1.45 \times 10^{-2})$ | (-0.19 ) | $(1.46 \times 10^{-2})$ | (-0.21) |
| $B^- \to \phi K^-$ | $7.36 \times 10^{-6}$ | 0.58 | $7.06 \times 10^{-6}$ | 0.65 |
| | $(9.14 \times 10^{-6})$ | (0.43) | $(8.75 \times 10^{-6})$ | (0.48 ) |
References

[1] See, for example, G.Kramer, W.F.Palmer and H.Simma, Nucl.Phys.B428, 77(1994).

[2] G.Kramer, W.F.Palmer and H.Simma, Z.Phys.C66, 429(1995).

[3] N.G.Deshpande and X.G.He, Phys.Rev.Lett. 74, 26(1995); Phys.Lett.B336, 471(1994).

[4] R.Fleischer, Z.Phys.C62, 81(1994).

[5] A.J.Buras, M.Jamin, M.Lautenbacher and P.Weisz, Nucl.Phys.B400, 37 (1993); A.J.Buras, M.Jamin and M.Lautenbacher, ibid. 75 (1993); M.Ciuchini, E.Franco, G.Martinelli and L.Reina, Nucl.Phys.B415, 403(1994).

[6] M.Bauer, B.Stech and M.Wirbel, Z.Phys.C34, 103(1987).

[7] A.N.Kamal, N.Sinha and R.Sinha, Z.Phys.C41, 207(1988).

[8] S.Barshay, D.Rein and L.M.Sehgal, Phys.Lett.B259,475(1991).

[9] M.Wanninger and L.M.Sehgal, Z.Phys.C50, 47(1994).

[10] A.N.Kamal, Int.J.Mod.Phys.A7, 3515(1992).

[11] A.N.Kamal and C.W.Luo, Alberta-Thy-02-97, hep-ph/9702289 (unpublished).

[12] R.G.Newton, *Scattering Theory of Waves and Particles*, Mc-Graw-Hill, N.Y.(1996).

[13] P.D.B.Collins, *Introduction to Regge Theory and High Energy Physics* (Cambridge Univ.Press, 1977).
[14] H.Q.Zheng, Phys.Lett.B356,107(1995).

[15] G.Nardulli and T.N.Pham, Phys.Lett.B391, 165 (1997).

[16] B.Blok and I.Halperin, Phys.Lett.B385, 324 (1996).

[17] P.E.Volkovitsky and A.B.Kaidalov, Yad.Fiz.35,1231, 1556(1982).

[18] R.L.Thews and A.N.Kamal, Phys.Rev.D32, 810(1985).

[19] T.M.Yan, H.Y.Cheng, C.Y.Cheung, G.L.Lin, Y.C.Lin and H.L.Yu, Phys.Rev.D46, 1148(1992).

[20] J.Schechter and A.Subbaraman, Phys.Rev.D48, 332(1993).

[21] R.Casalbuoni, A.Deandrea, N.D.Bartolomeo, R.Gatto, F.Feruglio and G.Nardulli, Phys.Lett.B299, 139(1993).

[22] A.N.Kamal and Q.P.Xu, Phys.Rev.D49, 1526(1994).

[23] A.N.Kamal and T.N.Pham, Phys.Rev.D50, 395(1994).

[24] B.J.Edwards, *Radiative Decay of Mesons*, Ph.D thesis(1978), University of Alberta
Figure Captions

Fig. 1: Inelastic scattering $B^- \rightarrow D^{*0}D_s^- \rightarrow \phi K^-$ through $D_s^*$ exchange

Fig. 2: Inelastic scattering $B^- \rightarrow D^0D_s^- \rightarrow \phi K^-$ through $D_s^*$ exchange
