Interplay between single particle coherence and kinetic energy driven superconductivity in doped cuprates

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Within the kinetic energy driven superconducting mechanism, the interplay between the single particle coherence and superconducting instability in doped cuprates is studied. The superconducting transition temperature increases with increasing doping in the underdoped regime, and reaches a maximum in the optimal doping, then decreases in both underdoped and overdoped regimes. By virtue of systematic studies using NMR and muon spin rotation techniques, particularly inelastic neutron scattering, it has been well established that AFSRC coexists with the SC-state in the whole SC regime, which implies a directive cooperative relation between AFSRC and superconductivity.

In the conventional metals, superconductivity results when electrons pair up into Cooper pairs, which is mediated by the interaction of electrons with phonons. It has been realized that this reduction in electron potential energy actually corresponds to a decrease in the intrinsic kinetic energy, thus providing a clear link between the pairing mechanism and phonons. As a natural consequence of this phonon-mediated pairing, the Cooper pairs in the conventional superconductors have an isotropic s-wave symmetry. In doped cuprates, the charge carriers form the Cooper pairs when they become superconductors as in the conventional superconductors. Although the possible doping dependent pairing symmetry has been suggested, the Cooper pairs in the optimally doped cuprate superconductors have a dominated d-wave symmetry, which is an indication of the unconventional SC mechanism. An alternative idea is that superconductivity in doped cuprates arises directly from the repulsive interactions between charge carriers. In particular, it has been suggested based on the non-Fermi liquid normal-state that the form of the Cooper pairs is determined by the need to reduce the frustrated kinetic energy. The normal-state exhibits a number of anomalous properties which is due to the charge-spin separation (CSS), while the SC-state is characterized by the charge-spin recombination.

Recently, the angle resolved photoemission spectroscopy (ARPES) measurements have shown that the SC transition temperature in doped cuprate is dependent of both gap parameter and weight of the coherent excitations in the spectral function. This strongly suggests that the single particle coherence plays an important role in superconductivity. Within the t-J model, one of us has discussed the kinetic energy driven SC mechanism in doped cuprates based on the CSS fermion-spin theory, where the dressed holons interact occurring directly through the kinetic energy by exchanging dressed spinon excitations, leading to a net attractive force between dressed holons, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground-state. The SC transition temperature is identical to the dressed holon pair transition temperature, and is proportional to the hole doping concentration in the underdoped regime. However, the single particle coherence in the system is not considered, which leads to an obvious weakness that the SC transition temperature is too high, and not suppressed in the overdoped regime. In this paper, we cure this weakness by considering the charge carrier single particle coherence, then we calculate explicitly the dynamical spin structure factor (DSSF) of cuprate superconductors in terms of the collective mode in the dressed holon particle-particle channel, and reproduce all main features found in inelastic neutron scattering experiments in the SC-state.

We start from the t-J model on a square lattice, $H = -t \sum_{i\sigma} C_{i\sigma}^\dagger C_{i+\hat{x}\sigma} + \mu \sum_{i\sigma} C_{i\sigma}^\dagger C_{i\sigma} + J \sum_{i\hat{\eta}} S_i \cdot S_{i+\hat{\eta}}$, with $\hat{\eta} = \pm \hat{x}, \pm \hat{y}$, $C_{i\sigma}^\dagger (C_{i\sigma})$ is the electron creation (annihilation) operator, $S_i = C_{i\sigma}^\dagger \hat{\sigma} C_{i\sigma}/2$ is spin operator with $\hat{\sigma} = (\sigma_+ \sigma_\downarrow)$ as Pauli matrices, and $\mu$ is the chemical potential. The t-J model is subject to an important on-site local constraint to avoid the double occupancy, i.e., $\sum_{\sigma} C_{i\sigma}^\dagger C_{i\sigma} \leq 1$. This local constraint can be treated properly in analytical calcula-
tions within the CSS fermion-spin theory, where the constrained electron operators are decoupled as, $C_t = h^\dagger_i S^+_i - C_i = h^\dagger_i S^+_i$, with the spinful fermion operator $h_{i\sigma} = e^{-i\Phi_{i\sigma}} h_i$ describes the charge degree of freedom together with the phase part of the spin degree of freedom (dressed holon), while the spin operator $S_i$ describes the amplitude part of the spin degree of freedom (dressed spinon), then the electron local constraint for the single occupancy, $\sum_{\sigma} C_i^{\dagger} C_i = S^+_i h_i h^\dagger_i S^-_i + S^-_i h_i h^\dagger_i S^+_i = h^\dagger_i (S^+_i S^-_i + S^-_i S^+_i) = 1 - h^\dagger_i h_i \leq 1$, is satisfied in analytical calculations, and the double spinful fermion occupancy, $h^\dagger_{i\sigma} h^\dagger_{i\sigma} = e^{i\Phi_{i\sigma}} h^\dagger_i h_i e^{i\Phi_{i\sigma}} = 0$, $h_{i\sigma} h_{i\sigma} = e^{-i\Phi_{i\sigma}} h_i h_i e^{-i\Phi_{i\sigma}} = 0$, are ruled out automatically. It has been shown that these dressed holon and spinon are gauge invariant, and in this sense, they are real. At the half-filling, the t-J model is reduced to an AF Heisenberg model, where there is no charge degree of freedom, and the real spinon excitation is described by the spin operator $S_i$. Although in common sense $h_{i\sigma}$ is not a real spinful fermion, it behaves like a spinful fermion. In this CSS fermion-spin representation, the low-energy behavior of the t-J model can be expressed as,

$$H = -t \sum_{\langle ij \rangle} (h^\dagger_i S^+_i h^\dagger_{i+\eta} S^-_{i+\eta} + h_i S^-_i h^\dagger_{i+\eta} S^+_{i+\eta}) - \mu \sum_{\langle ij \rangle} h_{i\sigma} h_{i\sigma} + J_{\text{eff}} \sum_{\langle ij \rangle} S_{i\uparrow} \cdot S_{j\uparrow},$$

(1)

with $J_{\text{eff}} = (1 - x)^2 J$, and $x = \langle h^\dagger_{i\sigma} h_{i\sigma} \rangle = \langle h^\dagger_i h_i \rangle$ is the hole doping concentration. The order parameter for the electron Cooper pair in the CSS fermion-spin approach can be expressed as, $\Delta = \langle C^\dagger_{i\uparrow} C^\dagger_{i\downarrow} - C^\dagger_{i\downarrow} C^\dagger_{i\uparrow} \rangle = \langle h^\dagger_i h^\dagger_{i+\eta} S^+_i S^-_{i+\eta} - h_i h^\dagger_{i+\eta} S^+_i S^-_{i+\eta} \rangle$. In the doped regime without AFLRO, the dressed spinons form the disordered spin liquid state, where the dressed spinon correlation function $\langle S^+_i S^-_j \rangle = \langle S^-_i S^+_j \rangle$, then the order parameter for the electron Cooper pair can be written as $\Delta = -\langle S^+_i S^-_j \rangle \Delta_h$, with the dressed holon pairing order parameter $\Delta_h = \langle h^\dagger_i h^\dagger_{i+\eta} - h_i h^\dagger_{i+\eta} \rangle$. This shows that the SC order parameter is closely related to the dressed holon pairing amplitude, and is proportional to the number of doped holes, and not to the number of electrons. However, in the extreme low doped regime with AFLRO, where the dressed spinon correlation function $\langle S^+_i S^-_j \rangle \neq \langle S^-_i S^+_j \rangle$, then the conduct is disrupted by AFLRO. Therefore in this paper, we only focus on the case without AFLRO.

As shown in Ref.[14], the dressed holon-spinon coupling occurring in the kinetic energy term of the t-J model is quite strong. This interaction can induce the dressed holon pairing state by exchanging dressed spinon excitations in the higher power of the hole doping concentration $x$. In this case, the SC mechanism can be discussed in terms of Eliashberg’s strong coupling theory, and the self-consistent equations that satisfied by the full dressed holon diagonal and off-diagonal Green’s functions are obtained as

$$g(k) = g^{(0)}(k) + g^{(0)}(k)[\Sigma^{(h)}_1(k)g(k) - \Sigma^{(h)}_2(-k)\Sigma^{(h)}_3(k)],$$

$$\Sigma^{(h)}_1(k) = g^{(0)}(-k)[\Sigma^{(h)}_1(-k)\Sigma^{(h)}_3(-k) + \Sigma^{(h)}_2(-k)g(k)],$$

(2a)

respectively, where the four-vector notation $k = (k, i\omega_n)$, the dressed holon mean-field (MF) diagonal Green’s function $g^{(0)}(k) = i\omega_n - \xi_k$, the MF dressed holon excitation spectrum $\xi_k = Z\chi_k - \mu$, with $\gamma_k = (1/Z) \sum_\eta e^{i\eta k} Z$ is the number of the nearest neighbor sites, the dressed spinon correlation function $\chi = \langle S^+_i S^-_{i+\eta} \rangle$, and the dressed holon self-energies,

$$\Sigma^{(h)}_1(k) = (Zt)^2 \frac{1}{N^2} \sum_{\langle p,p' \rangle} \gamma^2_{p,p'+k} \frac{1}{\tilde{\beta}} \sum_{ip_m} g(p + k) \times \frac{1}{\tilde{\beta}} \sum_{ip_m} D^{(0)}(p') D^{(0)}(p' + p),$$

(3a)

$$\Sigma^{(h)}_2(k) = (Zt)^2 \frac{1}{N^2} \sum_{\langle p,p' \rangle} \gamma^2_{p,p'+k} \frac{1}{\tilde{\beta}} \sum_{ip_m} \Xi(-p - k) \times \frac{1}{\tilde{\beta}} \sum_{ip_m} D^{(0)}(p') D^{(0)}(p' + p),$$

(3b)

where $p = (p, ip_m)$, $p' = (p', ip'_m)$, the MF dressed spinon Green’s function $g^{(0)}(k) = (i(p_m)^2 - \omega_0^2) / B_p$, with $B_p = \lambda(2\chi_p i(\gamma_p - 1) - \gamma(\gamma_p - \epsilon))$, $\lambda = 2ZJ_{\text{eff}}$, $\epsilon = 2t(\gamma_{p_m})$, and the MF dressed spinon excitation spectrum $\omega_0^2 = A_1 \frac{\gamma^2_{p,p'}}{A_2} + A_2 \gamma_{p,p'} + A_3$, with $A_1 = \alpha\chi^2(\epsilon_{\chi}^2 + \chi/2)$, $A_2 = -\epsilon^2[\alpha(\chi_0^2 + \chi^2/2) + \alpha C_0 (1 - \alpha)/(4\chi_0^2) - \alpha\chi^2/(2\chi_0^2)] + \alpha C_0 (1 - \alpha)/(2\chi_0^2) - \alpha\chi^2/(2\chi_0^2) / 2\chi_0^2$, $A_3 = \alpha^2\gamma_{p,p'}^2(1 - \alpha)/(2\chi_0^2) - \alpha\chi^2/(2\chi_0^2) / 2\chi_0^2$, and the dressed holon particle-hole parameter $\phi = \langle h^\dagger_{i\sigma} h_{i+\eta\sigma} \rangle$, the dressed spinon correlation functions $\chi = \langle S^+_i S^-_{i+\eta} \rangle$, $C = (1/Z^2) \sum_{\eta,\eta'} \langle S^+_i S^-_{i+\eta} S^+_j S^-_{j+\eta'} \rangle$, $C_z = (1/Z^2) \sum_{\eta,\eta'} \langle S^+_i S^-_{i+\eta} S^+_{j+\eta} S^-_{j+\eta'} \rangle$. In order to satisfy the sum rule of the dressed spinon correlation function $\langle S^+_i S^-_j \rangle = 1/2$ in the case without AFLRO, the important decoupling parameter $\alpha$ has been introduced in the MF calculation, which can be regarded as the vertex correction. In the above calculations of the self-energies, the dressed spinon part has been limited to the MF level, i.e., the full dressed spinon Green’s function $D(p)$ in Eq. (3) has been replaced by the MF dressed spinon Green’s function, since the normal-state charge transport obtained at this level can well describe the experimental data.

Since the pairing force and dressed holon gap function have been incorporated into the self-energy function $\Sigma^{(h)}_2(k)$, then it is called as the effective dressed holon gap function. On the other hand, the self-energy function $\Sigma^{(h)}_1(k)$ renormalizes the MF dressed holon spectrum, and therefore it describes the dressed holon single particle coherence. In other words, $\Sigma^{(h)}_1(k)$ describes the dressed holon quantum fluctuation, and $\Sigma^{(h)}_2(k)$ describes the dressed holon pairing instability. Moreover, $\Sigma^{(h)}_2(k)$ is an
even function of $i\omega_n$, while $\Sigma^{(h)}(k)$ is not. In this case, it is convenient to break $\Sigma^{(1)}_1(k)$ up into its symmetric and antisymmetric parts as, $\Sigma^{(1)}_1(k) = \Sigma^{(s)}_1(k) + i\omega_n\Sigma^{(a)}_1(k)$, where $\Sigma^{(s)}_1(k)$ and $\Sigma^{(a)}_1(k)$ are both even functions of $i\omega_n$. Now we define the dressed holon renormalization coefficient (charge carrier weight of the coherent excitations in the spectral function) $Z_F(k) = 1 - \Sigma^{(h)}_1(k)$. As in the conventional superconductor\textsuperscript{17}, the retarded function $\text{Re}\Sigma^{(h)}_1(k)$ may be a constant, independent of $(k, \omega)$. It just renormalizes the chemical potential, and therefore can be neglected. Furthermore, we only study the static limit of the effective dressed holon gap function and dressed holon renormalization coefficient, i.e., $\Sigma^{(1)}_2(k) = \Delta_h(k)$, and $Z_F(k) = 1 - \Sigma^{(h)}_1(k)$. In this case, the dressed holon diagonal and off-diagonal Green’s functions in Eq. (2) can be rewritten explicitly as,

$$g(k) = \frac{1}{2Z_F(k)} \left( 1 + \frac{\xi_k}{E_k} \right) \frac{1}{i\omega_n - E_k} + \frac{1}{2Z_F(k)} \left( 1 - \frac{\xi_k}{E_k} \right) \frac{1}{i\omega_n + E_k},$$

$$\Sigma^{(1)}_1(k) = -\frac{1}{Z_F(k)} \frac{\Delta_{hZ}(k)}{2E_k} \left( \frac{i\omega_n - E_k}{i\omega_n + E_k} \right),$$

with $\xi_k = \xi_k/Z_F(k)$, $\Delta_{hZ}(k) = \Delta_h(k)/Z_F(k)$, and the dressed holon quasiparticle spectrum $E_k = \sqrt{\xi_k^2 + |\Delta_{hZ}(k)|^2}$. Although $Z_F(k)$ is still a function of $k$, the wave vector dependence is unimportant, since everything happens at the electron Fermi surface. Therefore we need to estimate the special wave vector $k_0$ that guarantees $Z_F = Z_F(k_0)$ near the electron Fermi surface. In the present CSS fermion-spin framework\textsuperscript{15}, the electron diagonal Green’s function $G(i-j,t-t') = \langle \langle \xi(\mathbf{r}_i,t); \xi(\mathbf{r}_j,t') \rangle \rangle$ is a convoluted of the dressed spinon Green’s function $D(p)$ and dressed holon diagonal Green’s function $g(k)$, which reflects the charge-spin recombination\textsuperscript{12}, and can be calculated as\textsuperscript{18},

$$G(k) = \frac{1}{N} \sum_p \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{d\omega''}{2\pi} A_s(p,\omega') A_h(p - k, \omega'') \times n_F(\omega'') + n_B(\omega') \times \frac{1}{i\omega_n + \omega'' - \omega'},$$

where the dressed spinon spectral function $A_s(k,\omega) = -2\text{Im}D(k,\omega)$, the dressed holon spectral function $A_h(k,\omega) = -2\text{Im}g(k,\omega)$, and $n_F(\omega)$ and $n_B(\omega)$ are the boson and fermion distribution functions, respectively. This electron diagonal Green’s function has been used to extract the electron momentum distribution (then the electron Fermi surface) as\textsuperscript{18},

$$n_k = \frac{1}{2} - \frac{1}{N} \sum_p n_s(p) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A_h(p - k, \omega) n_F(\omega),$$

with $n_s(p) = \int_{-\infty}^{\infty} d\omega A_s(p,\omega)n_s(\omega)/2\pi$ is the dressed spinon momentum distribution. In the present case, this electron momentum distribution can be evaluated in terms of the MF dressed spinon Green’s function and dressed holon diagonal Green’s function (4a) as,

$$n_k = \frac{1}{2} - \frac{1}{N} \sum_p n_s^{(0)}(p) \frac{1}{2Z_F(p - k)} \times \left( 1 - \frac{\xi_p - k}{E_{p-k}} \tanh \left( \frac{1}{2} \beta E_{p-k} \right) \right),$$

with $n_s^{(0)}(p) = B_p \coth(\beta E_p/2)/(2\omega_p)$. Since the dressed spinons center around $[\pm\pi, \pm\pi]$ in the Brillouin zone in the MF level\textsuperscript{18}, then the electron momentum distribution (7) can be approximately reduced as $n_k \approx 1/2 - \rho_s^{(0)}[1 - \xi_{k\mathbf{A}} - k\tanh(\beta E_{k\mathbf{A}} - k/2)/E_{k\mathbf{A}} - k]/(2Z_F)$, with $k_\mathbf{A} = [\pi, \pi]$, and $\rho_s^{(0)} = (1/N) \sum_{p=\pm\pi} n_s^{(0)}(p)$. Therefore the Fermi wave vector from this electron momentum distribution is estimated\textsuperscript{18} as $k_F \approx [(1 - x)\pi/2, (1 - x)\pi/2]$, which evolves with doping. In this case, the wave vector $k_0$ is obtained as $k_0 = k_\mathbf{A} - k_F$, then we only need to calculate $Z_F = Z_F(k_0)$ as mentioned above. Since the charge-spin recombination from the convolution of the dressed spinon Green’s function and dressed holon diagonal Green’s function leads to form the electron Fermi surface\textsuperscript{12}, then the dressed holon single particle coherence $Z_F$ appearing in the electron momentum distribution also reflects the electron single particle coherence.

ARPES measurements\textsuperscript{19} have shown that in the real space the gap function and pairing force have a range of one lattice spacing, this indicates that the effective dressed holon gap function can be expressed as $\Delta_{hZ}(k) = \Delta_{hZ}^{(a)}(k).$ On the other hand, some experiments seem consistent with an s-wave pairing\textsuperscript{20}, while other measurements gave the evidence in favor of the d-wave pairing\textsuperscript{21,9}, therefore in the following discussions, we consider the cases of $\Delta_{hZ}^{(a)} = \Delta_{hZ}^{(d)}$, with $\gamma_k = \gamma_k^{(s)} = \gamma_k = (\cos k_x + \cos k_y)/2$, for the s-wave pairing, and $\Delta_{hZ}^{(a)} = \Delta_{hZ}^{(d)}$, $\gamma_k^{(a)} = \gamma_k^{(d)} = (\cos k_x - \cos k_y)/2$, for the d-wave pairing, respectively. In this case, the dressed holon effective gap parameter and renormalization coefficient in Eq. (3) satisfy the following equations\textsuperscript{14},

$$1 = (Zt)^2 \frac{1}{N^3} \sum_{k,q,p} \gamma_{k+q}^2 \gamma_{k+q-k}^2 \left( \frac{B_q B_p}{Z_F^2 E_k \omega_p \omega_q} \times \left( \frac{F_1^{(1)}(k, q, p)}{(\omega_p - \omega_q)^2 - E_k^2} + \frac{F_1^{(2)}(k, q, p)}{(\omega_p + \omega_q)^2 - E_k^2} \right) \right) \times \frac{F_1^{(2)}(q, p)}{(\omega_p - \omega_q - E_{p+q-k})^2 + \frac{F_1^{(2)}(q, p)}{(\omega_p - \omega_q - E_{p+q-k})^2}};$$

$$Z_F = 1 + (Zt)^2 \frac{1}{N^2} \sum_{q,p} \gamma_{p+q}^2 \frac{1}{Z_F} \frac{B_q B_p}{4\omega_p \omega_q} \times \left( \frac{F_1^{(1)}(q, p)}{(\omega_p - \omega_q - E_{p+q-k})^2 + \frac{F_1^{(2)}(q, p)}{(\omega_p - \omega_q - E_{p+q-k})^2}} \right) + \frac{F_1^{(2)}(q, p)}{(\omega_p - \omega_q - E_{p+q-k})^2 \right).$$
simultaneously with other self-consistent equations

\[ \Delta^{(4)}(q, p) = \frac{F^{(4)}(q, p)}{(\omega_{q} + \omega_{p} + F_{p-q+k_{0}})^{2}} \]

\[ \frac{F^{(3)}(q, p)}{(\omega_{q} + \omega_{p} + F_{p-q+k_{0}})^{2}} \], \quad \text{(8b)}

respectively,

where

\[ F^{(1)}_{1}(k, q, p) = \langle \omega_{p} - \omega_{q} \rangle n_{B}(\omega_{q}) n_{B}(\omega_{p})[1 - 2n_{F}(E_{k})] + E_{k}[n_{B}(\omega_{q}) n_{B}(\omega_{p}) + n_{B}(\omega_{q}) n_{B}(\omega_{p})] \]

\[ F^{(2)}_{1}(k, q, p) = \langle \omega_{p} - \omega_{q} \rangle n_{B}(\omega_{q}) n_{B}(\omega_{p})[1 - 2n_{F}(E_{k})] + E_{k}[n_{B}(\omega_{q}) n_{B}(\omega_{p}) + n_{B}(\omega_{q}) n_{B}(\omega_{p})] \]

\[ F^{(2)}_{2}(q, p) = n_{F}(E_{p-q+k_{0}})[n_{B}(\omega_{q}) - n_{B}(\omega_{p})] - n_{B}(\omega_{q}) n_{B}(\omega_{p}) - n_{B}(\omega_{q}) n_{B}(\omega_{p}) \]

\[ F^{(2)}_{2}(q, p) = n_{F}(E_{p-q+k_{0}})[n_{B}(\omega_{q}) - n_{B}(\omega_{p})] + n_{B}(\omega_{p}) n_{B}(\omega_{q}) \]

\[ F^{(4)}(q, p) = n_{F}(E_{p-q+k_{0}})[n_{B}(\omega_{q}) - n_{B}(\omega_{p})] + n_{B}(\omega_{p}) n_{B}(\omega_{q}) \]

These two equations must be solved simultaneously with other self-consistent equations\textsuperscript{14}, then all order parameters, decoupling parameter \( \alpha \), and chemical potential \( \mu \) are determined by the self-consistent calculation\textsuperscript{18}.

In this case, the dressed holon pair order parameter is obtained in terms of the off-diagonal Green’s function (4b) as,

\[ \Delta^{(a)}(k) = (2/N) \sum_{p} \frac{\Delta^{(a)}[p - k]}{2F_{p-k}^{2}} \tanh[\beta F_{p-k}] \]

\[ \times \frac{B_{p}}{2\omega_{p}} \coth[\beta \omega_{p}], \quad \text{(9)} \]

this shows that the symmetry of the electron Cooper pair is determined by the symmetry of the dressed holon pair, and therefore the SC gap function can be written as \( \Delta^{(a)}(k) = \Delta^{(a)} \gamma^{(a)}_{k} \), with the SC gap parameter is evaluated\textsuperscript{14} in terms of the dressed holon pair order parameter and Eq. (9) as \( \Delta^{(a)} = -\chi \Delta^{(b)} \). The present result in Eq. (9) also shows that the SC transition temperature \( T^{(a)} \) is identical to the dressed holon pair transition temperature occurring in the case of the effective dressed holon pairing gap parameter \( \tilde{\Delta}_{AZ} = 0 \). The SC transition temperature \( T^{(a)} \), as a function of the hole doping concentration \( x \) in the s-wave symmetry (solid line) and d-wave symmetry (dashed line) for \( t/J = 2.5 \) is plotted in Fig. 1 in comparison with the experimental result\textsuperscript{4} (inset). For the s-wave symmetry, the maximal SC transition temperature \( T^{(s)}_{c} \) is around a particular doping concentration \( x \approx 0.11 \), and then decreases for both lower doped and higher doped regimes. However, for the d-wave symmetry, the maximal SC transition temperature \( T^{(d)}_{c} \) is around the optimal doping concentration \( x \approx 0.18 \), and then decreases for both underdoped and overdoped regimes. Although the SC pairing symmetry is doping dependent, the SC state has the d-wave symmetry in a wide range of doping, in qualitative agreement with the experiments\textsuperscript{10,22}. Furthermore, \( T^{(d)}_{c} \) in the underdoped regime (\( T^{(s)}_{c} \) in the lower doped regime) is proportional to the hole doping concentration \( x \), and therefore \( T^{(d)}_{c} \) in the underdoped regime (\( T^{(s)}_{c} \) in the lower doped regime) is set by the hole doping concentration, which reflects that the dressed holon density directly determines the superfluid density in the underdoped regime for the d-wave case (the lower doped regime for the s-wave case). Using an reasonable estimation value of \( J \sim 800K \) to \( 1200K \) in doped cuprates, the SC transition temperature in the optimal doping is \( T^{(d)}_{c} \approx 0.2J \approx 160K \sim 240K \), also in qualitative agreement with the experimental data\textsuperscript{4,22}.

In the framework of the kinetic energy driven superconductivity\textsuperscript{14}, \( \Sigma^{(h)}_{10}(k) \) (then \( Z_{P} \)) describes the single particle coherence, which favors the single dressed holon motion in the background of the dressed spinon fluctuation, while \( \Sigma^{(h)}_{2}(k) \) describes the effective dressed holon pairing gap parameter, which measures the strength of the binding of dressed holons into dressed holon pairs and favors the dressed holon pair motion, therefore there is a competition between the single particle coherence and SC instability. In the underdoped and
optimally doped regimes, both superfluid density and $Z_F$ increase with increasing doping, this leads to that the SC transition temperature increases with increasing doping, and is proportional to the hole doping concentration\cite{14}. In the overdoped regime, although the superfluid density still increases with increasing doping\cite{14}, $Z_F$ is slows down with increasing doping\cite{13}, which leads to that the SC transition temperature decreases with increasing doping in the overdoped regime. However, as a result of the competition and self-consistent motion of the dressed holons, dressed holon pairs, and dressed spinons in the whole SC regime, the SC transition temperature is suppressed to the lower temperature due to the single particle coherence, this is why the SC transition temperature is so low in doped cuprates.

Now we turn to discuss the convergence of energy dependent incommensurate (IC) scattering to commensurate resonance, which is one of the most striking features of cuprate superconductors\cite{1,2,3}. Experimentally NMR and inelastic neutron scattering have provided rather detailed information on the spin fluctuation\cite{3,5,6}, where the distinct phenomena are the presence of the IC scattering peaks at low energies and commensurate resonance peak at relatively high energies, i.e., the IC scattering peaks are shifted from the AF wave vector $[\pi, \pi]$ to four points $[\pi(1 \pm \delta), \pi]$ and $[\pi, (1 \pm \delta)\pi]$ (in units of inverse lattice constant) at low energies with $\delta$ as the incommensurability parameter, which depends on both hole doping concentration and energy, then a sharp resonance peak at the commensurate AF wave vector $[\pi, \pi]$ is observed at relatively high energies. Although some of these magnetic properties have been observed in doped cuprates in the normal-state, these IC scattering and commensurate resonance are the main new feature that appears into the SC-state\cite{3,5}.

Within the CSS fermion-spin theory, the IC scattering and integrated spin response in the normal-state have been discussed\cite{15}, and the results of the doping dependence of the IC parameter $\delta$ and integrated dynamical spin susceptibility are consistent with experiments in the normal-state\cite{3,5,6}. Since the AF fluctuation is dominated by the scattering of the dressed spinons in the CSS fermion-spin theory\cite{15}, while in the present case in the SC state, this AF fluctuation has been incorporated into the electron off-diagonal Green’s function (hence the electron Cooper pair) in terms of the dressed spinon Green’s function, therefore there is a coexistence of the electron Cooper pair and AFSC, and then AFSC can persist into superconductivity\cite{14}. Following the previous discussions for the normal-state case\cite{15}, DSSF in the SC-state with the d-wave symmetry can be obtained as,

$$S(\mathbf{k}, \omega) = -2[1 + n_B(\omega)]\text{Im}D(\mathbf{k}, \omega) = 2[1 + n_B(\omega)]$$

$$\times \frac{B^2_k\text{Re}\Sigma^{(s)}(\mathbf{k}, \omega)}{[\omega^2 - \omega_{\mathbf{k}}^2 - B_k \text{Re}\Sigma^{(s)}(\mathbf{k}, \omega)]^2 + [B_k \text{Im}\Sigma^{(s)}(\mathbf{k}, \omega)]^2} \quad (10)$$

where the full dressed spinon Green’s function, $D^{-1}(\mathbf{k}, \omega) = D^{(0)-1}(\mathbf{k}, \omega) - \Sigma^{(s)}(\mathbf{k}, \omega)$, with $\text{Im}\Sigma^{(s)}(\mathbf{k}, \omega)$ and $\text{Re}\Sigma^{(s)}(\mathbf{k}, \omega)$ are the imaginary and real parts of the second order spinon self-energy, respectively, obtained from the dressed holon bubble in the dressed holon particle-particle channel as,

$$\Sigma^{(s)}(\mathbf{k}, \omega) = (Zt)^2 \frac{1}{N^2} \sum_{\mathbf{p}, \mathbf{q}} \left( \frac{\delta^2_{\mathbf{p}+\mathbf{q}+\mathbf{k}} + \delta^2_{\mathbf{p}-\mathbf{k}}}{\omega_{\mathbf{q}+\mathbf{k}} - \omega_{\mathbf{p}+\mathbf{q}} + 4Z_F^2 E_p E_{\mathbf{p}+\mathbf{q}}} \right) \times \left( \frac{F^{(1)}_s(\mathbf{k}, \mathbf{p}, \mathbf{q})}{\omega^2 - (E_p - E_{\mathbf{p}+\mathbf{q}} + \omega_{\mathbf{q}+\mathbf{k}})^2} \right)$$

$$+ \frac{F^{(2)}_s(\mathbf{k}, \mathbf{p}, \mathbf{q})}{\omega^2 - (E_p + E_{\mathbf{p}+\mathbf{q}} + \omega_{\mathbf{q}+\mathbf{k}})^2}$$

$$+ \frac{F^{(3)}_s(\mathbf{k}, \mathbf{p}, \mathbf{q})}{\omega^2 - (E_p + E_{\mathbf{p}+\mathbf{q}} - \omega_{\mathbf{q}+\mathbf{k}})^2}$$

$$+ \frac{F^{(4)}_s(\mathbf{k}, \mathbf{p}, \mathbf{q})}{\omega^2 - (E_p + E_{\mathbf{p}+\mathbf{q}} - \omega_{\mathbf{q}+\mathbf{k}})^2} \bigg), \quad (11)$$

with

$$F^{(1)}_s(\mathbf{k}, \mathbf{p}, \mathbf{q}) = (E_p - E_{\mathbf{p}+\mathbf{q}} + \omega_{\mathbf{q}+\mathbf{k}}) [n_B(\omega_{\mathbf{q}+\mathbf{k}})[n_F(E_p) - n_F(E_{\mathbf{p}+\mathbf{q}})] - n_F(E_{\mathbf{p}+\mathbf{q}}) n_F(-E_p)], \quad F^{(2)}_s(\mathbf{k}, \mathbf{p}, \mathbf{q}) = (E_p - E_{\mathbf{p}+\mathbf{q}} + \omega_{\mathbf{q}+\mathbf{k}}) [n_B(\omega_{\mathbf{q}+\mathbf{k}})[n_F(E_{\mathbf{p}+\mathbf{q}}) - n_F(E_p)] - n_F(E_{\mathbf{p}+\mathbf{q}}) n_F(-E_p) - n_F(E_{\mathbf{p}+\mathbf{q}}) n_F(-E_p) - n_F(E_p)]$$

$$F^{(3)}_s(\mathbf{k}, \mathbf{p}, \mathbf{q}) = (E_p + E_{\mathbf{p}+\mathbf{q}} + \omega_{\mathbf{q}+\mathbf{k}}) [n_B(\omega_{\mathbf{q}+\mathbf{k}})[n_F(-E_p) - n_F(E_{\mathbf{p}+\mathbf{q}}) + n_F(E_{\mathbf{p}+\mathbf{q}}) n_F(-E_p)]$$

$$F^{(4)}_s(\mathbf{k}, \mathbf{p}, \mathbf{q}) = (E_p + E_{\mathbf{p}+\mathbf{q}} - \omega_{\mathbf{q}+\mathbf{k}}) [n_F(\omega_{\mathbf{q}+\mathbf{k}})[n_F(E_{\mathbf{p}+\mathbf{q}}) - n_F(E_p)] - n_F(E_{\mathbf{p}+\mathbf{q}}) n_F(E_p) - n_F(E_{\mathbf{p}+\mathbf{q}}) n_F(E_p)]$$

In Fig. 2, we plot $S(\mathbf{k}, \omega)$ in the $(k_x, k_y)$ plane at doping $x = 0.15$ in temperature $T = 0.002J$ and energy $\omega = 0.11J$ for parameter $t/J = 2.5$, which shows that the IC spin fluctuation pattern occurs with doping, and the IC peaks are located at $(1/2, 1/2)$ and

![FIG. 2. The dynamical spin structure factor $S(k, \omega)$ in the $(k_x, k_y)$ plane in the superconducting-state at $x = 0.15$ in $T = 0.002J$ and $\omega = 0.11J$ for $t/J = 2.5$.](image-url)
For considering the resonance at relatively high energies we have made a series of scans for \( S(\mathbf{k}, \omega) \) at different energies, and the result at \( x = 0.15 \) for \( t/J = 2.5 \) in \( T = 0.002J \) and \( \omega = 0.33J \) is shown in Fig. 3. Comparing it with Fig. 2 for the same set of parameters except for \( \omega = 0.33J \), we find that IC peaks are energy dependent, i.e., although these scattering peaks retain the IC pattern at \([1/(1 + \delta)/2, 1/2] \) and \([1/2, (1 + \delta)/2] \) in low energies, the positions of IC peaks move towards \([1/2, 1/2] \) with increasing energy, and then the commensurate \([1/2, 1/2] \) resonance peak appears at relatively high energies \( \omega_{r} = 0.33J \). Moreover, the resonance energy is doping dependent, and is proportional to \( x \) in the underdoped regime\(^{23} \). Our these results are in qualitative agreement with experiments of doped cuprates in the SC-state\(^5 \).

As in the normal-state case\(^{15} \), the physics of the convergence of the IC magnetic scattering peaks at lower energies to commensurate resonance at higher energies in the SC-state also can be understood from the properties of the renormalized dressed spinon excitation spectrum \( \Omega_{\mathbf{k}}^{c} = \omega_{\mathbf{k}}^{2} + \text{Re} \Sigma^{(s)}(\mathbf{k}, \Omega_{\mathbf{k}}) \), which is doping and energy dependent. DSSF in Eq. (10) has a well-defined resonance character, where \( S(\mathbf{k}, \omega) \) exhibits peaks when the incoming neutron energy \( \omega \) is equal to the renormalized spin excitation (the collective mode in the dressed holon particle-particle channel), i.e., \( W(\mathbf{k}, \omega) \equiv \left[ \omega^{2} - \omega_{\mathbf{k}}^{2} - \delta_{\mathbf{k}} \text{Re} \Sigma^{(s)}(\mathbf{k}, \Omega_{\mathbf{k}}) \right]^{2} \approx 0 \) for certain critical wave vectors \( \mathbf{k}_{c} \), then the weight of these peaks is dominated by \( 1/\text{Im} \Sigma^{(s)}(\mathbf{k}_{c}, \omega) \). In this case, the positions of the magnetic scattering peaks are determined by both functions \( W(\mathbf{k}, \omega) \) and \( \text{Im} \Sigma^{(s)}(\mathbf{k}, \omega) \). Within the kinetic energy driven superconductivity, as a result of self-consistent motion of the dressed holon pairs and spinons, the IC scattering is developed beyond certain critical doping at low energies, this reflects that the low energy spin excitations drift away from the AF wave vector, or the zero point of \( W(\mathbf{k}, \omega) \) is shifted from \([1/2, 1/2] \) to \( \mathbf{k}_{c} \). With increasing energy, the spin excitations move towards to \([1/2, 1/2] \), i.e., the zero point of \( W(\mathbf{k}, \omega) \) in \( \mathbf{k}_{c} \) turns back to \([1/2, 1/2] \), then the commensurate \([1/2, 1/2] \) resonance appears at relatively high resonance energy \( \omega_{r} \). Since the essential physics is dominated by the dressed spinon self-energy renormalization due to the dressed holon bubble in the dressed holon particle-particle channel, then in this sense the mobile dressed holon pairs are the key factor leading to the convergence of the IC scattering peaks at lower energies to commensurate resonance at higher energies, i.e., the mechanism of the IC scattering peaks and commensurate resonance in the SC state is most likely related to the motion of the dressed holon pairs. This is why the position of the IC magnetic scattering peaks and commensurate resonance in the SC-state can be determined in the present study within the \( t-J \) model, while the dressed spinon energy dependence is ascribed purely to the self-energy effects which arise from the the dressed holon bubble in the dressed holon particle-particle channel.

In summary, within the CSS fermion-spin theory, we have discussed the interplay between the single particle coherence and kinetic energy driven SC instability in doped cuprates. The dressed holon pair instability is caused directly through the kinetic energy by exchanging dressed spinon excitations, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground-state\(^{14} \). The SC transition temperature \( T_{c} \) is determined by the dressed holon pair transition temperature, and is suppressed to low temperature due to the single particle coherence. Although the symmetry of the SC-state is doping dependent, the SC-state has the d-wave symmetry in a wide range of doping. Moreover, the maximal SC transition temperature \( T_{c}^{(d)} \) occurs around the optimal doping concentration \( x \approx 0.18 \), and then decreases in both underdoped and overdoped regimes, in agreement with the experiments\(^4 \). Within this SC mechanism, we have calculated DSSF of cuprate superconductors in terms of the collective mode in the dressed holon particle-particle channel, and reproduce all main features of inelastic neutron scattering experiments in the SC-state\(^5 \), including the energy dependence of the IC scattering peaks at low energies and commensurate resonance peak at relatively high energies.

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