Two Kinds of the Coexistent States in One-Dimensional Quarter-Filled Systems under Magnetic Fields

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The coexistent state of the spin density wave (SDW) and the charge density wave (CDW) in the one-dimensional quarter-filled system and with the Coulomb interaction up to the next-nearest sites under magnetic fields is studied. It is found that, in the coexistent state of 2k_F-SDW and 2k_F-CDW, the charge order is suppressed and 2k_F-CDW changes to 2k_F-SDW having the different alignment of spin under high magnetic fields, whereas, in the coexistent state of 2k_F-SDW and 4k_F-CDW, 4k_F-CDW still remains and 2k_F-SDW is suppressed. The critical temperature of the charge order is higher than that of the spin order when the inter-site Coulomb interaction is strong. These will be observed in experiments such as the X-ray scattering measurement and NMR.

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1. INTRODUCTION

Organic conductors such as (TMTSF)$_2$X and (TMTTF)$_2$X (X=ClO$_4$, PF$_6$, AsF$_6$, ReO$_4$, Br, SCN, etc.) are the quasi-one dimensional quarter-filled systems and exhibit various kinds of ground states, for example, spin-Peierls, spin density wave (SDW) and superconductivity. In (TMTSF)$_2$PF$_6$, the incommensurate SDW occurs at $T = 12$ K, where the wave vector is determined by NMR experiments as (0.5, 0.24, 0.06). Pouget and Ravy observed the coexistence of 2k_F-SDW and 2k_F-charge density wave (CDW) by the X-ray scattering measurement, where $k_F = \pi/4a$ is Fermi wave number and $a$ is the lattice constant. The critical temperature ($T_{CDW}$) at which 2k_F-CDW is observed is the same temperature as $T_{SDW}$.

On the other hand, it is found that SDW in (TMTTF)$_2$X (X=Br and SCN) has the commensurate wave vector, (0.5, 0.25,0), as observed in H-NMR experiments. From the angle dependence of satellite peak positions in H-NMR, the alignment of the spin moment along the conductive axis (a-axis) is obtained to be ($\uparrow$, 0, $\downarrow$), where the arrow means the spin moment. In (TMTTF)$_2$Br, 4k_F-CDW accompanied by 2k_F-SDW is found in X-ray scattering measurements, where $T_{CDW} \sim 100$ K and $T_{SDW} \sim 13$ K.

In the one-dimensional quarter-filled systems, the CDW-SDW coexistent state is shown to be caused by the interplay between the on-site Coulomb interaction ($U$) and the inter-site Coulomb interaction ($V$). The inter-site Coulomb interaction plays important role for the charge order. Mila has estimated $U/t \sim 5$ and $V/t \sim 2$, where $t$ is a transfer integral. In the ground state becomes the coexistent state of 2k_F-SDW and 4k_F-CDW in the one-dimensional extended Hubbard model with $V$. It is found by Kobayashi et al. and Mazumdar et al. that the coexistent state of 2k_F-SDW and 2k_F-CDW is stabilized when the next nearest neighbor Coulomb interaction ($V_2$) and the dimerization of the energy band are considered. Thus, two kinds of the coexistent states (2k_F-SDW-2k_F-CDW and 2k_F-SDW-4k_F-CDW), which are observed by the X-ray scattering measurements in (TMTSF)$_2$PF$_6$ and (TMTTF)$_2$Br, can be explained by Seo and Fukuyama and Kobayashi et al. and Mazumdar et al., respectively.

Recently, we have studied the effects of the magnetic field ($H$) on the coexistent state of 2k_F-SDW and 4k_F-CDW for both cases of strong and weak coupling of the correlation between electrons, where the one-dimensional quarter-filled extended Hubbard model with $V$ is used. It is found that although the spin order is suppressed at high fields, the charge order still remains in the strongly coupling systems ($U/t \sim 5$). When the coupling is small ($U/t \sim 1.5$), both orderings of 2k_F-SDW and 4k_F-CDW disappear at the critical magnetic field, which is the same as the Pauli paramagnetic field in the spin-singlet superconductivity.

In this paper, we examine how the coexistent state of 2k_F-SDW and 2k_F-CDW and the one of 2k_F-SDW and 4k_F-CDW are affected by magnetic fields and temperatures. We use the one-dimensional extended Hubbard model with $V$ and $V_2$, where the dimerization is neglected for simplicity, because the coexistent states are stabilized without the dimerization. We use parameters, $U/t = 5.0$, which is indicated in (TMTTF)$_2$X ((TMTSF)$_2$X). In most cases we set $V/U \leq 1$ and $V_2 < V$. We study the case when the anisotropy of the spin is strong, because it is found that the easy axis of the spin in the quasi-one-dimensional organic conductors is b-axis. In order to clarify the ground state under fields at finite temperatures, we show the $V$-$H$, $V_2$-$H$, $V$-$T$ and $V_2$-$T$ phase diagrams, which enables to discuss the effect of pressure and the ordering temperatures of the SDW and CDW. It is expected that the Hubbard model with large $U/t \sim 5$ is qualitatively understood by the Ising model with the perpendicular field, as we will show below.

The $H$-dependence of the antiferromagnetic state in
one-dimensional Ising model has been studied in the mean field approximation, where the component of the spin along the c-axis is considered. When the magnetic field is applied perpendicular to the spin moment, \[ S_z(H,j)/S_z(0,j) = \sqrt{1 - (H/H^0)^2}, \] (1)

where \( S_z(H,j) \) is the amplitude of the spin moment at the site at \( H = 0 \) and \( H^0 \) is the critical field at which the ordering of the antiferromagnetic state disappears (\( H^0 \propto J \), where \( J \) is the coupling constant). 

II. FORMULATION

We study the one-dimensional extended Hubbard model,

\[ \hat{\mathcal{H}} = \hat{\mathcal{K}} + \hat{\mathcal{U}} + \hat{\mathcal{V}} + \hat{\mathcal{V}}_2, \] (2)

\[ \hat{\mathcal{K}} = -t \sum_{i,x} \left( \sigma^x_{i,x} c^\dagger_{i,x+1} \sigma + h.c. \right) + \frac{\mu_B g}{2} \sum_{i,x} \left( \sigma^x_{i,x} (H)_{\sigma\sigma'} c_{i,x} \sigma', \right), \] (3)

\[ \hat{\mathcal{U}} = U \sum_i n_{i,\uparrow} n_{i,\downarrow}, \] (4)

\[ \hat{\mathcal{V}} = V \sum_{i,x} \left( \sigma^x_{i,x} n_{i+1,\sigma} \right), \] (5)

\[ \hat{\mathcal{V}}_2 = V_2 \sum_{i,x} \left( \sigma^x_{i,x} n_{i+2,\sigma} \right), \] (6)

where \( \mu_B = e\hbar/2m_c \) is the Bohr magneton and \( c^\dagger_{i,x} \) is the creation operator of \( \sigma \) spin electron at \( i \) site, \( n_{i,\sigma} \) is the number operator, \( g = 2, i = 1, \cdots, N_S, N_S \) is the number of the total sites and \( \sigma = \uparrow \) and \( \downarrow \). The notation in this paper follows Seo and Fukuyama. The magnetic field is applied to the x-axis and \( (H)_{\sigma\sigma'} = H(\hat{\sigma}_x)_{\sigma\sigma'} \), where \( H \) is the strength of the magnetic field and \( \hat{\sigma}_x \) is Pauli spin matrix. We consider the quarter-filled case.

The interaction terms, \( \hat{\mathcal{U}}, \hat{\mathcal{V}} \) and \( \hat{\mathcal{V}}_2 \) are treated in the mean field approximation as

\[ \hat{\mathcal{U}} = \sum_{k_x} \sum_Q \left( \rho_{\uparrow\uparrow}(Q,T) C^\dagger(k_x,\downarrow) C(k_x - Q, \downarrow) \right) \]

\[ + \rho_{\uparrow\downarrow}^*(Q,T) C^\dagger(k_x - Q, \uparrow) C(k_x, \uparrow) \}

\[ \hat{\mathcal{V}} = \left( \frac{V}{U} \right) \sum_{k_x,\sigma,\sigma'} \sum_Q \left( e^{-iQa} \rho_{\sigma\sigma}(Q,T) C^\dagger(k_x,\sigma') C(k_x - Q, \sigma) \right) \]

\[ + \rho_{\sigma\sigma'}^*(Q,T) C^\dagger(k_x,\sigma) C(k_x - Q, \sigma) \}

\[ \hat{\mathcal{V}}_2 = \left( \frac{V_2}{U} \right) \sum_{k_x,\sigma,\sigma'} \sum_Q \left( e^{-2iQa} \rho_{\sigma\sigma}(Q,T) C^\dagger(k_x,\sigma') C(k_x - Q, \sigma) \right) \]

\[ + \rho_{\sigma\sigma'}^*(Q,T) C^\dagger(k_x,\sigma) C(k_x - Q, \sigma) \}

where \( I = U/N_S \). The self-consistent equation for the order parameter \( \rho_{\sigma\sigma}(Q,T) \) at finite temperature, \( T \), is given by

\[ \rho_{\sigma\sigma}(Q,T) = I \sum_{k_x} < C^\dagger(k_x,\sigma) C(k_x - Q, \sigma) > . \] (10)

In eqs. (7)-(10), we take the order parameters, \( \rho_{\sigma\sigma}(Q,T) \), only between electrons with same spins. We do not consider the case of the mean field, \( \rho_{\sigma\sigma}(Q,T) = I \sum_{k_x} < C^\dagger(k_x,\sigma) C(k_x - Q, \sigma) > \) with \( \sigma \neq \sigma' \). We neglect the effect of the spin tilting to the \( x \) - \( y \) plane by setting \( \rho_{\sigma\sigma}(Q,T) = 0 \) in this mean field approximation. This simplification corresponds to the assumption that the rotational symmetry in the spin-space is broken and that the \( z \)-axis is the easy axis although the spin in this Hubbard model is isotropic. The magnetic field is applied perpendicular to the easy axis when \( H \neq 0 \).

In the Fulde-Ferrell-Larkin-Ovchinnikov state in the superconductivity, the total moment of the Cooper pair is changed by the magnetic field. Similar situation may occur in SDW and CDW, i.e., the wave vector \( Q \) may be changed by the magnetic field. However, such state will be stabilized only in the low temperature and strong magnetic field region and will not change the essential feature studied in this paper. Therefore, we take the possible wave vectors of the order parameters as \( Q = Q_0, 2Q_0, 3Q_0 \) and \( 4Q_0 \) (equivalent to 0), where \( Q_0 = 2k_F \).

We evaluate the Helmholtz free energy by using the eigenvalue \( \epsilon_j \) and the unitary matrix \( U_{k\sigma,j} \) obtained by diagonalizing the mean field Hamiltonian \( \hat{\mathcal{K}} + \hat{\mathcal{U}}^M + \hat{\mathcal{V}}^M + \hat{\mathcal{V}}^2_2 \), where index \( j \) includes the degree of the spin freedom. We determine the chemical potential \( \mu(\rho_{\sigma\sigma}(Q,T)) \) from \( N = 1/4 \), where

\[ N = \sum_{k_x,\sigma} < C^\dagger(k_x,\sigma) C(k_x, \sigma) > \]

\[ = \frac{1}{2N_S} \sum_{j=1}^{2N_S} \left[ \exp \left( \frac{\epsilon_j - \mu}{T} \right) + 1 \right]^{-1}. \] (11)

The self-consistency condition (eq. (10)) is given by

\[ \rho_{\sigma\sigma}(Q,T) = I \sum_{k_x} \sum_j U^*_{(k_x,\sigma),j} U_{(k_x+Q,\sigma),j} f(\epsilon_j), \] (12)

where \( f(\epsilon_j) \) is the Fermi function.

We obtain the free energy per site

\[ F(\rho_{\sigma\sigma}(Q,T)) = 2\mu N - \frac{T}{N_S} \sum_{j=1}^{2N_S} \log \left\{ \exp \left( \frac{\mu - \epsilon_j}{T} \right) + 1 \right\} \]

\[ - \frac{1}{U} \sum_Q \rho_{\uparrow\uparrow}(Q,T) \rho_{\uparrow\uparrow}^*(Q,T), \]

\[ - \frac{V}{U^2} \sum_{Q,\sigma,\sigma'} \sum_j e^{-iQa} \rho_{\sigma\sigma}(Q,T) \rho_{\sigma\sigma'}^*(Q,T) \]

\[ - \frac{V_2}{U^2} \sum_{Q,\sigma,\sigma'} \sum_j e^{-2iQa} \rho_{\sigma\sigma}(Q,T) \rho_{\sigma\sigma'}^*(Q,T), \] (13)
At $T = 0$ it reduces to the ground state energy per site

$$E(H, \rho_{\sigma\sigma}(Q, 0)) = \frac{1}{N_s} \sum_{j=1}^{N_s} \varepsilon_j - \frac{1}{U} \sum_Q \rho_{\uparrow\uparrow}(Q, 0) \rho_{\downarrow\downarrow}(Q, 0),$$

$$-\frac{V}{U^2} \sum_{Q,\sigma,\sigma'} e^{-iQa} \rho_{\sigma\sigma}(Q, 0) \rho_{\sigma'\sigma'}(Q, 0)$$

$$-\frac{V_2}{U^2} \sum_{Q,\sigma,\sigma'} e^{-2iQa} \rho_{\sigma\sigma}(Q, 0) \rho_{\sigma'\sigma'}(Q, 0).$$

The electron density at the $j$ site, $n(j, T)$, its deviation from the mean value $(1/2)$, $\delta(j, T)$, and the spin moment at $j$ site, $S_z(j, T)$, are given by

$$n(j, T) = \frac{1}{U} \sum_Q \rho_{\sigma\sigma}(Q, T)e^{iQja} = \frac{1}{2} + \delta(j, T),$$

and

$$S_z(j, T) = \frac{1}{2U} \sum_Q (\rho_{\uparrow\uparrow}(Q, T) - \rho_{\downarrow\downarrow}(Q, T))e^{iQja}.$$

When $T = 0$, $n(j) \equiv n(j, 0)$, $S_z(j) \equiv S_z(j, 0)$ and $\delta(j) \equiv \delta(j, 0)$.

III. RESULTS AND DISCUSSIONS

A. $2k_F$-SDW and $4k_F$-CDW

In this subsection, we take $V_2 = 0$ and $T = 0$. It is known that the coexistent state of $2k_F$-SDW and $4k_F$-CDW are induced by $U$ and $V$. Figs. 1 and 2 are $S_z$ and $\delta$ as a function of $V/t$ at $U/t = 5.0$ and $H = 0$. For $0 \leq V \leq 0.39$, the antiferromagnetic order ($(\uparrow, \downarrow, \downarrow)$, i.e., $S_z(1) = S_z(2) = -S_z(3) = -S_z(4)$) is stabilized and there is no charge order $(\delta(1) = \delta(2) = \delta(3) = \delta(4) = 0)$. The spin order of $(\uparrow, \downarrow, \downarrow, \downarrow)$ has the wave vector of $2k_F$. For $V/t > 0.39$, the spin order becomes $(\uparrow, 0, \downarrow, 0)$ ($S_z(1) = -S_z(3)$, $S_z(2) = S_z(4) = 0$) and the charge order $(\delta, -\delta, -\delta, -\delta)$ coexists, where $\delta(1) = \delta(3) = \delta$ and $\delta(2) = -\delta$. These orders, $(\uparrow, 0, \downarrow, 0)$ and $(\delta, -\delta, -\delta, -\delta)$, mean $2k_F$-SDW and $4k_F$-CDW, respectively. These results were obtained by Seo and Fukuyama.

We show $S_z$ and $\delta$ for $U/t = 5.0$ and $V/t = 0 \sim 5.0$ as a function of perpendicular field ($h_x \equiv \mu_B H/t$) in Figs. 3 and 4. The antiferromagnetic state is gradually suppressed up to the critical field ($h_c^1 = 1.3, 1.3, 1.5, 1.8, 2.4$ for $V/t = 0, 1.0, 2.0, 2.5, 5.0$, respectively), as shown in Fig. 3, where $S_z(1) = -S_z(3)$ and $S_z(2) = S_z(4) = 0$. Upon increasing $h_x$ the charge order decreases and becomes zero when $V/t = 1.0$, and the charge order becomes constant for $h_x > h_c^1$ when $V/t > 1.5$, as shown in Fig. 4, where $\delta(1) = \delta(3) = \delta$ and $\delta(2) = -\delta$. The $h_x$-dependence of the amplitude of $S_z$ is almost the same as eq. (1) when we set $H^2_0$ as $H^2_0 = th_c^1/\mu_B$, which is shown by solid lines in Fig. 3. From Figs. 3 and 4, we make the $V$-$h_x$ phase diagram, as shown in Fig. 5, where $(0,0,0,0)$ means that the spin and charge orders do not exist. The dotted lines show the second order transition lines and the solid line show...
the first order transition lines. In the region of large $h_x$ and $V$, the state of $4k_F$-CDW, $(\delta, -\delta, -\delta, -\delta)$, is stabilized.

The $h_x$-dependence of the spin order can be understood by the mean field solutions of the Ising model mentioned in the introduction. [23,24]

![Figure 3](image3.png)

**FIG. 3.** $2S_z(1)$ as a function of $h_x$. The solid line is given by eq. (1).

![Figure 4](image4.png)

**FIG. 4.** $\delta$ as a function of $h_x$.

**B. $2k_F$-SDW and $2k_F$-CDW**

Here, $V_2$ is introduced, which favors the coexistent state of $2k_F$-SDW and $2k_F$-CDW. [11][12] We search the most stable solution at $U/t = 5.0$, $V/t = 2.0$, $T = 0$ and $H = 0$ by changing $V_2$. These $V_2$-dependences of the spin density and charge density are shown in Figs. 6 and 7. For $0 \leq V_2/t \leq 1.4$, the coexistent state of $2k_F$-SDW and $4k_F$-CDW is stable, while the state changes to the coexistent state of $2k_F$-SDW and $2k_F$-CDW ($(\uparrow, \downarrow, \downarrow)$), i.e., $S_z(1) = -S_z(4) < S_z(2) = -S_z(3)$ and $((-\delta, \delta, \delta, -\delta), \delta(1) = \delta(4) = -\delta$ and $\delta(2) = \delta(3) = \delta)$ for $V_2/t > 1.4$. This transition is a first order transition.

The coexistent state of $2k_F$-SDW and $2k_F$-CDW obtained by Kobayashi et al. [13] is $(\uparrow, \downarrow, \downarrow)$ and $(-\delta, \delta, -\delta, -\delta)$, which is different from the coexistent state of $2k_F$-SDW and $2k_F$-CDW ($(\uparrow, \downarrow, \downarrow)$ and $(-\delta, \delta, \delta, -\delta)$). Since they limited the freedom of the order parameters, they could not find this coexistent state of $2k_F$-SDW and $2k_F$-CDW ($(\uparrow, \downarrow, \downarrow)$ and $(-\delta, \delta, \delta, -\delta)$). They also indicated that the coexistent state of $2k_F$-SDW and $2k_F$-CDW ($(\uparrow, \downarrow, \downarrow)$ and $(-\delta, \delta, \delta, -\delta)$) is suppressed when the dimerization is not included. In fact, we could not find the coexistent state of $2k_F$-SDW and $2k_F$-CDW of $(\uparrow, \downarrow, \downarrow)$ and $(-\delta, \delta, \delta, -\delta)$ in the parameter region, $(U/t = 5.0, V/t = 2.0$ and $V_2/t < 2.0$). However, as we show here, the coexistent state of $2k_F$-SDW and $2k_F$-CDW ($(\uparrow, \downarrow, \downarrow)$ and $(-\delta, \delta, \delta, -\delta)$) is stabilized even in the absence of the dimerization.

After we finished this study, we know the very recent study at $T = 0$ [28] and $T \neq 0$ [29] in the absence of the magnetic field by Tomio and Suzumura. They calculate the mean field solution using the extended Hubbard model with $V$ and $V_2$. They show that the state of
2$k_F$-SDW and 2$k_F$-CDW (($\uparrow\uparrow\uparrow\downarrow\downarrow$) and ($-\delta, \delta, -\delta$)) coexists even if the dimerization is not included, too. 

We analyze the coexistent state of 2$k_F$-SDW and 2$k_F$-CDW under magnetic fields at $0 \leq V_2/t \leq 2.0$. In the case of the perpendicular field ($h_x$), the state of ($\uparrow\uparrow\uparrow\downarrow\downarrow$) and ($-\delta, \delta, -\delta$) becomes the one of ($\uparrow\uparrow\uparrow\downarrow\downarrow$) and (0,0,0,0) ($\delta(1) = \delta(2) = \delta(3) = \delta(4) = 0$) at $0.75 < h_x < 1.25$, which is shown in Figs. 8 and 9. We can see that the coexistent state of 2$k_F$-SDW and 2$k_F$-CDW changes to the state of 2$k_F$-SDW. This transitions is the second order phase transition, since $|S_z(1)|$ and $|S_z(3)|$ ($|S_z(2)|$ and $|S_z(4)|$) increase (decrease) gradually, and ($\uparrow\uparrow\uparrow\downarrow\downarrow$) changes to ($\uparrow\uparrow\uparrow\downarrow\downarrow$) smoothly. When $h_x > 1.25$, the 2$k_F$-SDW state disappears. For $0 \leq V_2/t \leq 1.4$, as $h_x$ increases, the spin density in the coexistent state of 2$k_F$-SDW and 4$k_F$-CDW decrease and becomes zero at $h_x = 1.3, 1.3, 1.3, 1.3, 1.3, 1.3$ and 1.5 for $V_2/t = 1.4, 1.0, 0.5$ and 0, as shown in Fig. 10, where $S_z(1) = -S_z(3)$ and $S_z(2) = S_z(4) = 0$. When we set $H_0^x$ as $H_0^x = t\hbar^2/\mu_B$, the $h_x$-dependence of $S_z$ is in agreement with eq. (1), which is shown by solid line in Fig. 10. The order of 4$k_F$-CDW becomes nonzero at the higher field for $V_2/t < 0.5$, as shown in Fig. 11. We show the $V_2-h_x$ phase in Fig. 12, where the solid line is for the first order transition and the dotted lines are for the second order transitions.
We show $S_z$ and $\delta$ as a function of $T$ at $H = 0$. At $U/t = 5.0$ and $V_z/t = 0$, $S_z$ and $\delta$ as a function of $T$ for various values of $V$ are shown in Figs 13 and 14, where $S_z(1, T) = S_z(2, T) = -S_z(3, T) = -S_z(4, T)$ for $V/t = 0$ and $S_z(1, T) = -S_z(3, T), S_z(2, T) = S_z(4, T) = 0$, $\delta(1, T) = \delta(3, T) = \delta(T)$ and $\delta(2, T) = \delta(4, T) = -\delta(T)$ for $V/t = 0.5 \sim 5.0$. As $V$ increases, the critical temperatures ($T_{SDW}$ and $T_{CDW}$) at which $S_z$ and $\delta$ become zero increase. The $T$-dependences of $n$ for $V/t = 3.0, 4.0$ and $5.0$ have a dip at $T_{SDW}$ for $V/t = 3.0, 4.0$ and $5.0$, which can be seen in Fig. 14. We show the $V$-$T$ phase diagram in Fig. 15, where the solid and dotted lines are for the first and the second order transitions, respectively. Note that $T_{CDW} > T_{SDW}$ for $V/t > 2.0$, that is, the charge order remains even if the spin order disappears under high temperatures.

As we have shown above, the coexistent state of 2$k_F$-SDW and 2$k_F$-CDW ($\uparrow \uparrow \downarrow \downarrow$) and ($\delta, -\delta, -\delta, -\delta$) are stabilized for $V_z/t > 1.4$ at $T/t = 0$ (see Figs. 6 and 7). At $T/t \neq 0$, for example, for $V_z/t = 1.6$ as $T$ increases, the magnitudes of $S_z$ and $n$ decreases and the second order transition to the state of 2$k_F$-SDW ($\uparrow \uparrow \downarrow \downarrow$) occurs, as shown in Figs. 16 and 17. Namely, $T_{CDW} \leq T_{SDW}$ in the coexistent state of 2$k_F$-SDW and 2$k_F$-CDW. For $V_z/t < 1.4$, $T_{CDW}$ in the coexistent state of 2$k_F$-SDW and 4$k_F$-CDW is the same as $T_{SDW}$, as shown in Figs. 18 and 19, where $S_z(1, T) = -S_z(3, T), S_z(2, T) = S_z(4, T) = 0$, $\delta(1, T) = \delta(3, T) = \delta(T)$ and $\delta(2, T) = \delta(4, T) = -\delta(T)$. We show the $V_z$-$T$ phase diagram in Fig. 20, where the solid and dotted lines are for the first and the second order transition lines, respectively.

Our phase diagrams ($V$-$T$ and $V_z$-$T$) are in good agreement with those obtained by Tomio and Suzumura. [29]
FIG. 13. $2S_z(T) = 0$ as a function of $T/t$.

FIG. 15. $V - T$ Phase diagram.

FIG. 14. $\delta(T)$ as a function of $T/t$.

FIG. 16. $2S_z(j, T)$ $(j = 1 \cdots 4)$ as a function of $T/t$. 
FIG. 17. $\delta(1,T)$ and $\delta(2,T)$ as a function of $T/t$.

FIG. 18. $2S_z(1,T)$ as a function of $T/t$.

FIG. 19. $\delta(T)$ as a function of $T/t$.

FIG. 20. $V_2 - T$ Phase diagram.

IV. CONCLUSIONS

We have shown the different dependences of the magnetic field and temperatures on the $2k_F$-SDW-$2k_F$-CDW coexistent state and $2k_F$-SDW-$4k_F$-CDW coexisting state.

When $V_2$ is absent or small, the coexistent state of $2k_F$-SDW and $4k_F$-CDW is stable. Then the charge order survives at high fields in the case of the perpendicular magnetic field to the easy axis when $V/t > 2.0$. This situation may be realized in $(TMTTF)_2X$, where $4k_F$-CDW is expected to be observed in X-ray scattering measurements under high fields.
On the other hand, if $V_2$ is large enough, the coexistent state of $2k_F$-SDW and $2k_F$-CDW is stabilized. When the magnetic field is applied perpendicular along the easy axis, the coexistent state of $2k_F$-SDW and $2k_F$-CDW changes to $2k_F$-SDW state. This transition is the second order phase transition, and $(\uparrow,\uparrow,\downarrow,\downarrow)$ becomes $(\uparrow,\uparrow,\downarrow,\downarrow)$. This transition may be observed in the angle dependence of satellite peak positions of NMR in (TMTSF)$_2$PF$_6$.

The critical temperature of the charge order ($T_{CDW}$) is higher than that of the spin order ($T_{SDW}$) in the coexistent phase of $2k_F$-SDW and $4k_F$-CDW when $V/t > 2.0$. This is in good agreement with $T_{SDW} \sim 13$ K and $T_{CDW} \sim 100$ K observed in X-ray scattering in (TMTTF)$_2$Br. Since $T_{CDW}$ is convergent to $T_{SDW}$ for $V/t < 2.0$, we expect that the critical temperatures of $2k_F$-SDW and $4k_F$-CDW will become the same when the pressure is applied to (TMTTF)$_2$Br.

In the coexistent phase of $2k_F$-SDW and $2k_F$-CDW, we find that $T_{CDW} \leq T_{SDW}$, which is consistent with the X-ray scattering measurement in (TMTSF)$_2$PF$_6$, where $T_{CDW} \approx T_{SDW}$. [3]

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