CAN WE DETECT THE ANISOTROPIC SHAPES OF QUASAR H II REGIONS DURING REIONIZATION THROUGH THE SMALL-SCALE REDSHIFTED 21 cm POWER SPECTRUM?

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ABSTRACT

Light-travel time delays distort the apparent shapes of H II regions surrounding bright quasars during early stages of cosmic reionization. Individual H II regions may remain undetectable in forthcoming redshifted 21 cm experiments. However, the systematic deformation along the line of sight may be detectable statistically, either by stacking tomographic 21 cm images of quasars identified, for example, by the James Webb Space Telescope, or as small-scale anisotropy in the three-dimensional 21 cm power spectrum. Here we consider the detectability of this effect. The anisotropy is largest when H II regions are large and expand rapidly, and we find that if bright quasars contributed to the early stages of reionization, then they can produce significant anisotropy, on scales comparable to the typical sizes of H II regions of the bright quasars (<30 Mpc). The effect therefore cannot be ignored when analyzing future 21 cm power spectra on small scales. If 10% of the volume of the intergalactic medium at \( z \simeq 10 \) is ionized by quasars with typical ionizing luminosity of \( S \gtrsim 5 \times 10^{46} \text{ s}^{-1} \), the distortions cause an \( \geq 10 \) percent enhancement of the 21 cm power spectrum in the radial (redshift) direction, relative to the transverse directions. The level of this anisotropy exceeds that due to redshift-space distortion and has the opposite sign. We show that ongoing experiments such as Murchison Widefield Array (MWA, formerly known as the Mileura Widefield Array) should be able to detect this effect. A detection would reveal the presence of bright quasars and shed light on the ionizing yield and age of the ionizing sources and the distribution and small-scale clumping of neutral intergalactic gas in their vicinity.

Subject headings: cosmology: theory — early universe — galaxies: evolution — galaxies: formation — galaxies: high-redshift — quasars: general

1. INTRODUCTION

How and when the intergalactic medium (IGM) was reionized is one of the long outstanding questions in cosmology, likely holding many clues about the nature of the first generation of light sources and the end of the cosmological dark age (see, e.g., Barkana & Loeb 2001 for a review). Observational breakthroughs in recent years have revealed our first clues about this epoch. The Thomson scattering optical depth, \( \tau_e = 0.09 \pm 0.03 \) (Page et al. 2007; Spergel et al. 2007), inferred from the polarization anisotropies in the cosmic microwave background (CMB) by the Wilkinson Microwave Anisotropy Probe (WMAP), suggests that reionization began at redshift \( z \sim 10 \). The detections of the so-called Gunn-Peterson (GP) troughs in the spectra of high-redshift quasars discovered in the Sloan Digital Sky Survey (SDSS; White et al. 2003; Fan et al. 2006b) and more detailed analyses of these spectra (Mesinger & Haiman 2004, 2007) suggest that reionization is ending at \( z \sim 6 \).

Despite this progress, little is known about the nature of the first ionizing sources. The dearth of bright quasars at \( z \sim 6 \) and a tight limit on the contribution of lower luminosity quasars to the unresolved soft X-ray background (Dijkstra et al. 2004) require that the ionizing background at \( z \approx 6 \) be dominated by stars, rather than quasars. On the other hand, luminous quasars may still contribute up to \( \sim 10\% \) of the ionizing background at \( z \approx 6 \) (e.g., Sbrinovsky & Wyithe 2007), and the contribution of the ionizing radiation from accreting quasar black holes may have been more significant at higher \( z \), during the earliest stages of reionization (Madau et al. 2004; Ricotti & Ostriker 2004; Oh 2001; Venkatesan et al. 2001). Overall, the present data allow a wide range of possible reionization histories, driven by different sources and various physical feedback mechanisms (e.g., Haiman & Holder 2003).

One of the promising future probes that could constrain the epoch of reionization, especially the early stages, is based on the detection of the redshifted 21 cm line of neutral hydrogen (see a recent review by Furlanetto et al. 2006). Forthcoming 21 cm surveys are expected to deliver tomographic maps of the brightness temperature of the 21 cm line and detect fluctuations corresponding to the topology of large discrete H II regions. Numerous recent studies have addressed various aspects of the power spectrum of such brightness temperature fluctuations (see references in Furlanetto et al. 2006).

In this paper, we examine one particular effect, arising from the distortion in the apparent shapes of H II regions due to the light-travel time delay between a light source and the ionization front it drives into the IGM. Since the distances from the Earth to various points on the surface of an H II region differ, different patches of an expanding H II region will be observed at different stages of their evolution, corresponding to different light-travel times. As pointed out in Cen & Haiman (2000), the delay can be ignored along the line of sight to individual sources (for example, when computing Ly absorption spectrum of a point source; see also White et al. 2003). On the other hand, the delay modifies the apparent expansion of a resolved H II region in the transverse direction, as could be observed, for example, in 21 cm studies (Wyithe & Loeb 2004b). The full equal arrival time surface in three dimensions has been computed in simple models of individual spherical quasar H II regions by Yu (2005) and shown to be highly anisotropic for young and bright sources.

In principle, the distortion could be measured directly in tomographic images of individual H II regions (Wyithe et al. 2005), although the immediate next generation of 21 cm instruments (such as PAST [Primeval Structure Telescope] or MWA) are unlikely to be able to achieve the required signal-to-noise ratio (S/N).

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for such direct imaging. Since individual H\textsc{ii} regions are likely to be intrinsically highly anisotropic, such a measurement can only be performed statistically. In this paper, we examine and quantify the possibility of measuring the corresponding anisotropy in the three-dimensional power spectrum of the 21 cm brightness temperature. This method does not require that individual H\textsc{ii} regions be detectable at high S/N.

Barkana & Loeb (2006) recently studied the anisotropy in the three-dimensional 21 cm power spectrum that arises from finite light-travel time across larger, joint H\textsc{ii} regions that have merged around galaxies. The growth of such superbubbles is dictated by the collapse rate of cosmic structures, and it produces an anisotropy that will be significant on larger scales (comparable to the sizes of the H\textsc{ii} superbubbles). In comparison, the effect we describe here arises from the finite rate of growth of isolated H\textsc{ii} regions around individual quasars. Finite speed of light effects during the lifetime of quasars can produce an appreciable additional smaller scale anisotropy only in a likely narrow window of redshifts during reionization: on the one hand, bright quasars need to present, which occurs only at relatively late stages; but on the other hand, once the ionized bubbles around these bright quasars percolate significantly, the small-scale anisotropy will diminish. Conversely, a detection of any small-scale anisotropy would reveal the presence of bright quasars and shed light on the ionizing yield and age of the ionizing sources and the distribution and small-scale clumping of neutral intergalactic gas in their vicinity. In this paper, we quantify the conditions under which a detection of this additional anisotropy, due to the presence of quasars, could be feasible with forthcoming experiments, with specifications similar to those proposed for MWA.

The rest of this paper is organized as follows. In § 2, we discuss the basic formalism to compute the 21 cm brightness temperature fluctuations, including the anisotropy from the light-travel time delay. In § 3, we present the expected level of anisotropy in a few simple models and discuss how it is expected to vary with various parameters. In § 4, we discuss instrumental noise and the corresponding limits on the detectability of the anisotropy. In § 5, we discuss our results and the implications of this work. Finally, in § 6, we summarize our main conclusions. Throughout this paper, we adopt the background cosmological parameters $\Omega_m = 0.29$, $\Omega_L = 0.71$, $\Omega_b = 0.047$, and $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$, with a power spectrum normalization $\sigma_{8_s} = 0.8$ and slope $n_s = 1$, consistent with the values measured recently by the WMAP experiment (Spergel et al. 2007). Unless noted otherwise, all lengths below are quoted in comoving units.

2. Brightness Temperature Fluctuations

Forthcoming radio observations will measure the brightness temperature at the redshifted wavelength $(1 + z)$ 21 cm. The brightness temperature is expected to be below that of the CMB before the first luminous sources turn on, so that cosmic H\textsc{i} can only be seen in absorption. Once the first sources turn on and ionize and heat the IGM, the brightness temperature can rise above that of the CMB, so that the 21 cm signal is seen in emission (see Furlanetto et al. 2006 for a review and references). Here we are interested in the epoch of reionization, when the spin temperature $T_s \gg T_{\text{CMB}}$. In this limit, the brightness temperature $T$ of the redshifted H\textsc{i} line is independent of $T_s$ and is fully determined by the local density of neutral H\textsc{i} (see, e.g., Field 1959; Madau et al. 1997; Sethi 2005). In this section, we outline how we compute the anisotropic correlation function of the spatially varying brightness temperature.

2.1. Basic Formalism for Temperature Fluctuations

Observations will measure the brightness temperature $T$ at a given frequency $\nu$ and direction on the sky $\hat{n}$, but in a narrow frequency band, the signal is dominated by the emitting gas at the corresponding redshift $z$, or radial distance $r$ (with the relation between $r$ and observed frequency $\nu$ obtained from the appropriate cosmological model). Given the telescope beam $B(\hat{n}, \hat{n}'$, which for radio interferometers will correspond to the synthesized beam, we obtain

$$T(\hat{n}, r) = \int d\Omega' \int dr' T_0(r') W_r(r') \psi(\hat{n}', r') B(\hat{n}, \hat{n}')$$

where $T_0(r)$ is the mean brightness temperature at the redshift $z = z(r)$.

$$T_0 = \frac{3}{32\pi} \frac{h\lambda^2 A_{21} n_b(z)}{k H(z)} \approx 40 \left[ \frac{0.127}{\Omega_m h^2} \left( \frac{0.73}{h} \right)^2 \left( \frac{1 + z}{11} \right) \right]^{1/2} \text{mK},$$

with $\psi = x_H(r)[1 + \delta(r)]$, where $\delta$ and $x_H$ denote the spatially fluctuating overdensity and neutral hydrogen (H\textsc{i}) fraction $\{n_{H_1}/(n_{H_1} + n_{H_2})\}$, respectively. Finally, $W_r(r')$ is a window function, peaking at $r$, representing the bandwidth of the observations.

Assuming for simplicity that $T_0(r)$ does not evolve significantly within the bandwidth, it can be taken out of the integral. In this case,

$$T(\hat{n}, r) = T_0(r) \int d\Omega' \int dr' W_r(r') \psi(\hat{n}', r') B(\hat{n}, \hat{n}')$$

where $r$ corresponds (for example) to the center of the observed frequency band. The two-point correlation of the temperature fluctuation can then be written as

$$\langle \Delta T(\hat{n}_1, r_1) \Delta T(\hat{n}_2, r_2) \rangle = T_0(r_1) T_0(r_2)$$

$$\times \int dr \int dr' \int d\Omega \int d\Omega' \left\{ W_r(r) W_{r'}(r') B(\hat{n}, \hat{n}_1) B(\hat{n}', \hat{n}_2) \right\}$$

$$\times \left[ \langle \psi(\hat{n}_1, r) \psi(\hat{n}_2, r') \rangle - \langle \psi \rangle^2 \right]$$

Note that statistical homogeneity of the signal is assumed but not statistical isotropy; i.e., the correlation depends only on $r_1 - r_2$, and the angle brackets denote averages over (say) $r_1$. The expression above also assumes that the neutral fraction does not evolve significantly over the length scales of interest, so $\langle \psi \rangle \equiv \langle \psi \rangle_{r_1} \approx \langle \psi \rangle_{r_2}$. It is customary to assume a relatively broad bandwidth of $\lesssim 0.2 - 0.5$ MHz while computing the theoretical signal (e.g., Zaldarriaga et al. 2004). This choice is motivated by the fact that it roughly matches the length scales ($\lesssim 10$ Mpc) corresponding to the synthesized beam (4$'$) of the upcoming radio interferometers, such as MWA. However, the frequency resolution of future experiments is much narrower, $\lesssim 1$ kHz (by design, this is necessary to remove radio frequency interference [RFI]). This corresponds to scales much smaller than other length scales in the problem. Mellema et al. (2006) have shown that the power spectrum on the scales we will be interested in (i.e., on the characteristic size, $\lesssim 10$ Mpc, of the H\textsc{ii} regions) is insensitive to the frequency
resolution once it is well below the above value. Furthermore, for a statistical signal, increasing the bandwidth around a given frequency does not give any advantage in the S/N for the detection (see discussion in § 5 below). As shown in § 3, the scales of interest to us are generally greater than 20 Mpc and the effect of a finite beamwidth of 4' is unlikely to affect the signal significantly at those scales. Therefore, for simplicity, and in accordance with our objectives in this paper, we assume $W_y(r') = \delta_y(r' - r)$ and $B(\hat{n}, \hat{n}') = \delta_y(\hat{n} - \hat{n}')$. We do note, however, that the random placement of the resolution pixels relative to the boundaries of the quasar bubbles will cause a shot noise in estimates of the anisotropy that could become significant, and will certainly necessitate averaging over many bubbles to detect angular structure. Quantifying this shot noise term is beyond the scope of the present paper (averaging over a large number of bubbles will be required, in any case, for other reasons; see the discussion in § 3 below).

Under this approximation, the expression for the two-point correlation function is simplified to

$$C(r_{12}, \theta) \equiv \langle \Delta T(\hat{n}_1, r_1) \Delta T(\hat{n}_2, r_2) \rangle = T_0(r_1)T_0(r_2) \left[ \langle \psi(\hat{n}_1, r_1)\psi(\hat{n}_2, r_2) \rangle - \langle \psi \rangle^2 \right]. \quad (6)$$

Here $\theta$ is the angle between the line of sight $\hat{n}$ and the vector $r_1 - r_2$ separating the two points, and angle brackets denote averages over (say) $r_1$ (as noted above, the correlation function could be written equivalently as a function of the angle $\theta$ and the frequency difference $\nu_2 - \nu_1$). We caution the reader that, as defined above, the angle $\theta$ is different from the angle subtended on the sky. The latter is typically defined in studies involving angular correlation functions, but we find the angle relative to the line of sight, which directly follows the shapes of the quasar-driven bubbles, more convenient for our purposes.\(^3\)

The correlation function of $\psi$ can be expanded as

$$\langle \psi(r_1)\psi(r_2) \rangle = \left( \{x_H(r_1)[1 + \delta(r_1)]\} \{x_H(r_2)[1 + \delta(r_2)]\} \right)$$

$$= \{x_H(r_1)x_H(r_2)\} + 2\{x_H(r_1)\delta(r_1)x_H(r_2)\}$$

$$+ \{x_H(r_1)\delta(r_1)x_H(r_2)\}.$$ \quad (7)

The three- and four-point functions in the expression above are, in general, nonzero and cannot be easily computed as $x_H$ is not a Gaussian random variable (for details see, e.g., Furlanetto et al. 2004, hereafter FZH04).

In what follows, we drop the terms that contain cross-correlations between the density and the neutral fraction (we discuss the neglect of these terms in detail below). In this case, we obtain:

$$\langle \psi \psi \rangle - \langle \psi \rangle^2 = \xi_{\psi\psi}(\theta) = \xi_{\psi\psi}(\theta) + \xi_{\psi\psi} - \xi_{\psi\psi}^2.$$ \quad (8)

Here $\xi_{\psi\psi} = \langle \delta(r_1)\delta(r_2) \rangle$ is the two-point correlation function of the total density contrast (which we compute using the fitting formula for the linear spectrum in Eisenstein & Hu 1999), and $\xi_{\psi\psi} = \langle x_H(r_1)x_H(r_2) \rangle$, to be discussed in detail below, is the correlation function owing to the inhomogeneities of the neutral fraction.\(^4\) To make further progress, we need to make some assumptions about the spatial variation of the neutral fraction.

\(^3\) We note that finite instrumental resolution may, in practice, cause a degradation of the precision to which the anisotropy of the correlation function can be extracted from the data, on scales near the resolution, since the brightness temperature fluctuations will effectively be smeared with a filter that is defined in the observed $(r, \theta)$-coordinates, and not in the $(r', \theta')$-coordinates we use to specify the anisotropy.

\(^4\) Another commonly used definition of the correlation function is $\xi_{\psi\psi} = \langle x_H(r_1)x_H(r_2) \rangle - \langle x_H \rangle^2$. We caution the reader that the definition we adopt here is different.

Before we proceed with computing the anisotropy in the above power spectrum due to the nonspherical shapes of the individual H II regions, we note that a different source of anisotropy will already be produced by redshift-space distortions due to peculiar velocities (e.g., Bharadwaj & Ali 2004; Barkana & Loeb 2005). This effect is analogous to the redshift-space distortion of the matter power spectrum (Kaiser 1987) and is described in the linear regime by

$$\xi_{\psi\psi}(r_{12}, \theta, z) \approx \xi_{\psi\psi}(r_{12}, 0, z) \left( 1 + \frac{2}{3} + \frac{1}{3} \beta^2 \right)$$

$$+ \xi_{\psi\psi}(r_{12}, 2, z) \left( \frac{4}{3} \beta + \frac{4}{7} \beta^2 \right) P_2(\theta). \quad (9)$$

Here $\beta \approx \Omega_m^{0.6}/b$, and we use $b = 1$ throughout; $P_2(\theta)$ is the Legendre function with $l = 2$, and

$$\xi_{\psi\psi}(r, 0, z) = \frac{D_+(z)}{2\pi^2} \int dk k^2 P(k) j_0(kr), \quad (10)$$

$$\xi_{\psi\psi}(r, 2, z) = \frac{D_+(z)}{2\pi^2} \int dk k^2 P(k) j_2(kr). \quad (11)$$

Here $D_+(z)$ is the growing mode of density perturbations and $P(k)$ is the matter power spectrum. We have further neglected the fourth (and higher) moments of the correlation function, which make a negligible contribution (see, e.g., Hamilton 1998). The anisotropy we consider below, intrinsic to the correlation function of the neutral fraction, will be in addition to this redshift-space distortion anisotropy.

2.2. Correlation Function of the Neutral Fraction

While the fluctuations in the overdensity $\delta(r)$ are specified by the cosmological initial conditions, in order to describe the spatial variations in the neutral fraction $x_H(r)$, we need to make assumptions about the process of reionization. It is generally believed that this transition occurred by the percolation of ionized bubbles around individual sources, or clusters of sources, that formed in nonlinear halos. The nature of the sources of ionization is not clear; they could be either star-forming galaxies or QSOs, or both. Several works have proposed simple models for the correlation function $\xi_{\psi\psi}$ of the neutral fraction in this picture (e.g., Knox et al. 1998; Gruzinov & Hu 1998; Santos et al. 2003; FZH04; Zhang et al. 2007).

We follow the simplest model described in FZH04 (adopted from Knox et al. 1998), which assumes that the ionizing sources are randomly distributed in space. In reality, the sources are likely to be located at the peaks of the density field and therefore clustered. In general, this will increase $\xi_{\psi\psi}$ (by about an order of magnitude, for ionizing sources in dark matter halos that correspond to $2 - 3 \sigma$ peaks; e.g., Santos et al. 2003) and can also modify its anisotropy (see discussion in § 5 below). In the case of randomly distributed identical sources with space density $n$, each surrounded by an ionized bubble with volume $V$, we have the joint probability for the ionized fraction $x_i = 1 - x_H$,

$$\langle x_i(r_1)x_i(r_2) \rangle = \left( 1 - e^{-n V_o} \right) + e^{-n V_o} \left[ 1 - e^{-n(V - V_o)} \right]^2, \quad (12)$$

where $V_o = V_o(r_1 - r_2)$ is the volume of the overlap between two ionized bubbles located at $r_1$ and $r_2$. The first term in equation (12) represents the probability that both points are ionized.
by the same source; the second term describes the case when they are ionized by two different sources. Defining \( p_{\text{same}} \equiv 1 - \exp(-nV_o) \) as the probability that the two points belong to the same ionized bubble (i.e., when the overlap region contains at least one source), the correlation function for the neutral fraction, obtained from the above equation, can be conveniently rewritten (Barkana & Loeb 2006) as

\[
\xi_{xx} = \frac{\tilde{x}_H(r_1)\tilde{x}_H(r_2)}{(1 - p_{\text{same}})}, \quad (13)
\]

We note that equation (12) assumes that the bubble boundaries are static and not affected by overlap. A more realistic assumption would be that the total ionized volume is conserved, which would require expanding the joint boundaries of two bubbles that have merged. This would guarantee \( 1 - \tilde{x}_{II} = nV' \), but modifying equation (12) to take the bubble mergers into account would require further assumptions and would be overly complicated. Alternatively, one could start with the postulate that bubbles do not overlap (i.e., counting a merged bubble as a single object); however, this condition would not allow the bubbles to be randomly distributed on small scales.

The above ambiguity affects the correlation function on scales comparable to the bubble size, but we expect it would not significantly modify its predicted anisotropy. In any case, mergers between bubbles will likely dilute the anisotropy signal we discuss below, and therefore we focus on the earliest epochs of reionization, when the filling factor of the ionized bubbles is small. In this limit, \( nV' \ll 1 \), overlapping bubbles are rare, and the ambiguity is avoided: we can approximate \( p_{\text{same}} \approx nV_o \equiv nfV \approx (1 - \tilde{x}_{II})f \), where \( f \equiv V_o/V' \).

If the ionized bubbles were spherically symmetric with \( V = (4\pi/3)R^3 \), then we would have the simple expression

\[
f(r, R) = 1 - \frac{3r}{4R^3} \left( R^2 - \frac{1}{12}r^2 \right), \quad (14)
\]

for \( r \leq 2R \) and zero otherwise, and where \( r = |r_1 - r_2| \) (cf. eq. [18] in FZH04). Allowing the characteristic size \( R = R(z) \) and the mean neutral fraction \( \tilde{x}_H = \tilde{x}_{II}(z) \) to evolve with redshift introduces a new ambiguity in the redshift at which \( \tilde{x}_H \) and \( f \) are to be evaluated in \( p_{\text{same}} \equiv (1 - \tilde{x}_{II})f \). In practice, however, \( \tilde{x}_H \) and \( R \) should evolve only on long timescales, comparable to the Hubble time. Indeed, Barkana & Loeb (2006) discuss an anisotropy in the correlation function caused by this evolution and the finite speed of light. As discussed in §1, our focus here is on the anisotropies on smaller scales, comparable to the size of individual quasar bubbles. The anisotropy from the cosmic evolution is negligibly small on these scales (see Fig. 3 in Barkana & Loeb 2006), and we therefore ignore it here and use the simple expressions,

\[
\xi_{xx} = \frac{\tilde{x}_H^2}{1 - p_{\text{same}}}, \quad (15)
\]

\[
p_{\text{same}} = (1 - \tilde{x}_H)f(r, R). \quad (16)
\]

Note that the anisotropy we discuss below will arise entirely from the dependence of \( p_{\text{same}}(r_1, r_2) \) on orientation (i.e., the angle between \( r_1 - r_2 \) and the line of sight).

Let us consider various limits of the expressions in equations (15) and (16): (1) for \( r \) tending to zero \( f \) tends to unity; (2) as \( r \) tends to infinity \( p_{\text{same}} \) tends to zero and the correlation function approaches \( \tilde{x}_H^2 \), as they should; (3) as the average value of the neutral fraction tends to zero, the correlation function vanishes; and (4) when the average value of the neutral fraction tends to unity, \( \xi_{xx} \) should approach \( \tilde{x}_H^2 \), as indeed it does.

2.3. Light-Travel Delay Anisotropy of Individual H II Regions

Assuming that all ionizing sources carve out spherically symmetric Strömgren spheres in their rest frames (at least statistically; we discuss this assumption in detail below), it can be shown that as light from different points on the Strömgren sphere is observed simultaneously by an observer on Earth, the sphere will appear distorted. The effect is, roughly, an elongation along the line of sight; it is caused by the fact that photons observed at increasingly large impact parameters away from the line of sight to the source have traveled a longer path (Cen & Haiman 2000; White et al. 2003; Wyithe & Loeb 2004b; Yu 2005). As a result, at large impact parameters, we effectively observe the source at a younger age, when it had a smaller Strömgren radius.

Estimating the apparent distortion entails computing the locus of all points lying on the Strömgren sphere from which the light is received at the same time. Expressions for this equal arrival time surface, as a function of the angle relative to the line of sight, have been obtained in the case of a steady source (see Yu 2005 and references therein). We here adopt the results of Yu (2005) and include a somewhat different, brief derivation for completeness.

Let us follow the notation of Yu (2005) and refer to the location of the quasar (or any generic ionizing source) as “O” and to the point on the Strömgren sphere that intersects the line of sight to the quasar as “C.” Let \( \theta \) refer to the angle between OC and OB, where point B is some other location on the Strömgren sphere’s surface (see Fig. 1 for an illustration). The proper (not comoving) length of OC is given by

\[
R_{ \max } = \int_{t_i}^{t_i + t_{\max}} v(t') \, dt', \quad (17)
\]

where \( v \) is the proper propagation velocity of the ionization front in the rest frame of the quasar, \( t_i \) is the cosmic time at which the quasar switched on, and \( t_i + t_{\max} \) is the cosmic time at which the quasar’s photons reach point C. Here \( t_{\max} \) stands for the age of the H II region at the time it is observed. We note that since photons take a finite time to travel between the source and the edge of the H II region, this age is larger than the age \( t_q \) of the quasar, defined at the time it produced the photons that resulted in the H II region: \( t_{\max} = t_q + R_{ \max }/c \). Our aim is to find \( R(\theta) \), from the condition that photons traveling from point B reach Earth simultaneously with photons from point C. This requires that light leaves point B earlier than from point C by the difference in light-travel times,

\[
\Delta t = \frac{1}{c} [R_{ \max } - R(\theta) \cos(\theta)]. \quad (18)
\]

(2.3.1) Technically, these are not Strömgren spheres, since they likely do not reach equilibrium within the source lifetime, owing to the long recombination time in the IGM (e.g., Shapiro & Giroux 1987); however, we follow here the widespread use of this misnomer.
For reference, the dashed curve shows a circle with radius $R_{\text{max}}$. The notation follows Yu (2005).

Given the velocity $v(t)$ of the ionization front, $R(\theta)$ can be determined numerically from this relation (similar to the implicit relation in eq. [3] of Yu 2005). A solution exists as long as
\[ v = c \]
For $v = c$, a solution exists for $\theta \leq \pi/2$ but not for larger values of $\theta$, which just means we can only see the front part of the Strömgren sphere. The solution $R(\theta)$ tends to a constant at late times, and $R(\theta)$ becomes independent of $\theta$ as $v/c$ tends to zero (as should generally be the case at late times).

The physical velocity of the growth of the Strömgren sphere $v = dr/dt$ can be computed by (see, e.g., White et al. 2003; Wyithe & Loeb 2004a)
\[
(S + 4\pi r^2 n_{\text{HI}}) \left( \frac{dr}{dt} - H r \right) = c \left( S - \frac{4\pi}{3} r^3 n_{\text{HI}} C \alpha_B \right).
\]
(20)

Here $r$ is the physical (not comoving) radius of the Strömgren sphere, $S$ is the ionizing photon luminosity of the central source, $H = H(z)$ is the Hubble parameter at redshift $z$, $c$ is the speed of light, $n_{\text{HI}}$ is the neutral hydrogen number density, $\alpha_B$ is the case B recombination coefficient at the temperature $10^4$ K, and $C \equiv (n_{\text{HI}})/\langle n_{\text{HI}} \rangle$ is the small-scale clumping factor of ionized gas inside the Strömgren sphere. This formula gives the correct limit of $dr/dt \to c$ as $r \to 0$; i.e., the Strömgren sphere expands at the speed of light in the beginning.

In Figure 2, we show the comoving radius $(1 + z)R(\theta)$ for a set of fiducial parameters that could represent a bright quasar at $z = 10$, for four different ages of the H II region. As the figure shows, the anisotropy can be large (of order unity) if the source is young ($t_{\text{H II}} = \text{a few} \times 10^7$ yr).

As explained above, the effect of the $\theta$-dependence of $R$ on the correlation function is that $f(r, R)$ becomes a function of $\theta$. Geometrically, equation (14) represents the volume of overlap between two spheres of radius $R$ that are a distance $r$ apart, divided by the volume of a single sphere, $4\pi R^3$. This has to be replaced by the new function $f(r, \theta)$, which represents the normalized overlap volume between two objects whose shapes are described by equation (19). Note that the shape has azimuthal symmetry, but in general, the overlap volume does not. We numerically compute this overlap volume, with the line connecting the centers of the two objects (i.e., the positions $O_1$ and $O_2$ of the sources) of length $r$ and oriented at angle $\theta$ to the line of sight (divided, again, by the volume of a single object). The correlation function (eq. [6]) can now be computed using equations (8)–(16), but replacing $f(r, R)$ in equation (14) with the new $f(r, \theta)$.

3. RESULTS

The correlation function at a given redshift (eq. [6]), including its anisotropy caused by the light-travel delay, depends on such parameters as the properties of the sources (such as their typical age, luminosity, and number density) and of the IGM (such as its density, mean neutral fraction, and clumping factor). For a given reionization history, the correlation function will be determined by averaging over the distribution of the ages and luminosities of the sources that coexist in the IGM at any given redshift.

We first consider, in § 3.1, simple toy models, in which all ionizing sources (at a given redshift) are identical and propagate into a fully neutral IGM. These results are intended to illustrate the level of anisotropy expected from particular sources and the dependence of the anisotropy on various parameters.

In § 3.2, we present more realistic estimates of the possible level of anisotropy. These results differ from those in the toy-model case, in that (1) we consider coexisting sources with a range of luminosities, (2) we include the dilution of the anisotropy due to the presence of “fossil” H II regions, and (3) we consider the scenario in which the H II regions produced by bright quasars...
expand into a medium that is already partially ionized by preexisting galaxies (and/or lower luminosity, nonrelativistic quasars). Further complications will be discussed in § 5 below.

3.1. Illustrative Toy Models

In this section, we consider a population of identical sources, in order to isolate their contribution to the anisotropy in the more realistic models described in the next subsection. The most important source parameters are the luminosity and lifetime; large anisotropies are expected only for bright and short-lived sources, such as luminous quasars. As we show below, the anisotropy from ionized bubbles around galaxies or fainter quasars is expected to be negligible.

3.1.1. Quasar-like Sources

Quasars can have high photon luminosities [$\approx (0.5-5) \times 10^{57} \text{s}^{-1}$] and short ages (of a few $10^7$ yr; see, e.g., Haiman & Cen 2002; Martini 2004 for a review). In Figure 3 we plot the correlation function for different choices of these parameters. The results from Figure 3 can be briefly summarized as follows.

1. Level of anisotropy.—The distortion from light-travel delay can lead to substantial anisotropy in the correlation function. The level of anisotropy is $\sim 10^4$ or higher, if the source lifetimes are $\leq 2 \times 10^7$ yr, on scales $5 \text{Mpc} \leq r_{12} \leq 25$ Mpc. The particular distortions in the shapes of the Strömgren spheres translate to a characteristic $\theta$-dependence of the anisotropy.

2. Dependence on source lifetime.—For fixed source luminosity and fixed separation $r_{12}$, the anisotropy generally diminishes as the source lifetime gets longer. The sensitivity is very steep: for example, for $r_{12} \approx 35$ Mpc, the anisotropy for $t_{H_\downarrow} = 3.75 \times 10^7$ yr is about an order of magnitude smaller than for $t_{H_\downarrow} = 1.5 \times 10^7$ yr. The steep dependence of the anisotropy on the source lifetime is explained by the fact that the H II regions initially expand rapidly at relativistic speeds, but then become increasingly nonrelativistic as the source age increases. From equation (20), we can infer that the transition occurs sooner for less luminous sources: the critical radius is $r_{H_\downarrow} \approx (S/(4\pi n_{H_\downarrow}c))^1/2$, and as $r \approx ct$ during the relativistic phase, the corresponding critical age of the source at the transition scales as $t_{H_\downarrow} \propto S^{1/2}$. If sources have typical lifetimes that are independent of $S$ (or at least scale less steeply than $S^{1/2}$), then we can expect that bright sources with $t \gtrsim t_{H_\downarrow}$ will be in their relativistic phase, whereas fainter sources with $t \leq t_{H_\downarrow}$ will have nonrelativistic and isotropic bubbles, with $t_{H_\downarrow}$ given by

$$t_{H_\downarrow} = (4 \times 10^7 \text{yr}) \left(\frac{S}{2 \times 10^{56} \text{s}^{-1}}\right)^{1/2} \left(\frac{1 + z}{11}\right)^{-3/2} \left(\frac{n_{H_\downarrow}}{0.5}\right)^{-1/2}.$$

(21)

3. Dependence on scale.—The anisotropy shows a steady increase with increasing $r_{12}$ and then peaks at $r_{12} \approx 2R_{c}$, where $2R_{c} = R(0) + R(c)$ represents the diameter of a single H II bubble. For larger separations, the anisotropy decreases with increasing $r_{12}$.

4. Dependence on source luminosity.—Figures 3 and 4 show the change in anisotropy when the luminosity of the quasars is changed by a factor of 10. A comparison of these figures shows that a decrease in luminosity is generally associated with a decrease in the anisotropy at a given length scale, for a given age of the QSO. This is to be expected as the H II region of the QSO remains relativistic for a longer period if the luminosity is increased (eq. [21]).

5. Dependence on IGM clumping factor.—We have repeated the calculations in Figures 3 and 4 with the clumping factor $C$ varied in the expected range of $1 \leq C \leq 20$. The effect of an increased clumping factor is to slow down the expansion of the ionization fronts (see eq. [20]) and therefore to reduce the anisotropy. However, this effect is small unless the ionization front speed is limited by recombinations, which happens only at late times and only if the clumping is near the high end of the expected range ($C \sim 20$). Even in this case, we found that the reduction in the level of anisotropy is at most $\sim 50$%; the dependence on $C$ is negligibly small for $C \lesssim 10$ (however, the clumping of the IGM has the additional and more significant effect of determining the ratio of fossil to active H II regions; see discussion of this in § 3.2 below).

6. Comparison to redshift-space distortions.—It is useful to compare the anisotropy caused by quasars to the redshift-space distortion induced anisotropy. The two sources of anisotropies are, in fact, coupled through the first term of equation (8). To disentangle these two effects, we compute the correlation function in the density perturbations $\xi_{m}$ alone. The result is shown by the dashed curves in the top left panel of Figure 3. This panel shows that the redshift-space distortion induced anisotropy in $\xi_{m}$ is significant and at a comparable or larger magnitude than the anisotropy in $\xi_{c}$ caused by the quasars. However, the anisotropy in
In general, such sources are expected to be much less luminous than bright quasars, and therefore equation (21) suggests that they cannot cause any significant anisotropy. We can estimate the maximum photon luminosity of early galaxies by noting that $2 \times 10^{53}$–$10^{54}$ s$^{-1}$ and ages of $5 \times 10^6$–$5 \times 10^7$ yr that could be expected for the brightest star-forming galaxies and a wide range of parameters of the IGM, and indeed, we found that the anisotropy of the correlation function is dominated by the redshift-space distortion. Therefore, if the star-forming galaxies are the exclusive agents of the reionization process, the light-travel time delay anisotropy is unlikely to be detected. Conversely, this implies that any secure detection of the anisotropy would immediately reveal the presence of bright, quasar-blown bubbles.

3.1.2. Star-forming Galaxies

As stated above, a more realistic model would have to include averaging over a range of luminosities and ages at a given redshift. The simple models shown in Figures 3 and 4, however, already allow us to draw some basic conclusions. First, since the anisotropy decreases steeply with source lifetime, if all of the ionizing sources have ages far exceeding a few $10^7$ yr that could be expected for the brightest star-forming galaxies and a wide range of parameters of the IGM, and indeed, we found that the anisotropy of the correlation function is dominated by the redshift-space distortion. Therefore, if the star-forming galaxies are the exclusive agents of the reionization process, the light-travel time delay anisotropy is unlikely to be detected. Conversely, this implies that any secure detection of the anisotropy would immediately reveal the presence of bright, quasar-blown bubbles.

3.2. Possible Anisotropy in More Realistic Scenarios

As stated above, a more realistic model would have to include averaging over a range of luminosities and ages at a given redshift. The simple models shown in Figures 3 and 4, however, already allow us to draw some basic conclusions. First, since the anisotropy decreases steeply with source lifetime, if all of the ionizing sources have ages far exceeding a few $10^7$ yr and/or if they have ionizing luminosities significantly below a few $10^{56}$ s$^{-1}$, then the correlation function anisotropy will be negligible ($\lesssim 1\%$). This critical value for the lifetime coincides with expectations for bright quasars (see the review by Martini 2004), whereas the critical luminosity is about an order of magnitude below those of the already known SDSS quasars at $z > 6$.

Quasars of the required brightness almost certainly appeared only in the late stages of reionization. These relevant later parts of the reionization history likely involved a mixture of both types of sources discussed above. The process of reionization was likely initiated by low-luminosity sources, such as star-forming galaxies
(with masses \(\lesssim 10^8 M_\odot\)), or smaller black holes (with masses \(\lesssim 10^6 M_\odot\); Madau et al. 2004; Ricotti & Ostriker 2004), at high redshift (\(z \gtrsim 10\)). However, at somewhat lower redshifts, but well before reionization is completed, bright quasars could have formed in higher mass dark matter halos (\(\sim 10^{12} M_\odot\)) corresponding to the rarer high-\(\sigma\) peaks of the underlying density distribution. The unresolved soft X-ray background puts a tight limit on the contribution of quasars to reionization at \(z \sim 6\)–\(20\) but does not rule out the above scenario, in which quasars contribute a few ionizing photons per H atom at \(z \gtrsim 6\) (see Fig. 1 in Dijkstra et al. 2004).

The toy models above suggest that if bright quasars contribute a nonnegligible fraction of the ionizing photons produced at \(z \gtrsim 6\), then the anisotropy of their Strömgren spheres could be observable via the measurements of the 21 cm correlation function. In the rest of this section, we present somewhat more elaborate models, to quantify the anisotropy in a scenario in which the universe is partially ionized by faint sources (star-forming galaxies and/or low-luminosity quasars) and the H \(\alpha\) regions of bright, relativistic QSOs, with a range of luminosities, are expanding into this partially ionized medium.

More specifically, we make the following modifications to the toy models of § 3.1.

1. *Preionization by nonrelativistic sources.*—First, we assume that at the epoch of interest, nonrelativistic sources (galaxies or low-luminosity quasars) had already ionized 60% of the IGM. This is consistent with current constraints on the neutral fraction around \(z \sim 6\) (e.g., Fan et al. 2006a). The size of the H \(\alpha\) regions of these bright QSOs (and thus the H \(\alpha\) regions of other sources) is likely to be larger than any other scale in the problem (e.g., compared to the clustering scale of H \(\alpha\) regions of other sources), and therefore one can assume that at the scales comparable to the H \(\alpha\) region of the bright QSO, the only source of ionization inhomogeneity is owing to the presence of the bright QSOs. Alvarez & Abel (2007) and Lidz et al. (2007) present models which might bear out this picture (although, as these authors argue, fluctuations from galaxies can be important in other contexts involving smaller scales, such as interpreting Ly\(\alpha\) absorption spectra). For our purposes, the partial preionization can be regarded as uniform, and we therefore assume that its sole effect is to reduce the H \(\alpha\) density in the IGM (in eq. [20]).

2. *Additional ionization by bright quasars.*—The ionization fraction caused by bright QSOs could be a small fraction (say, 5%–10%) compared to the other sources (say, 60%, as adopted above). To be explicit, we set \(x_{\alpha} = 0.05\) as our fiducial choice for the quasar contribution, reduced by a factor of 10 from the toy models. To motivate this choice, we note that each quasar with the smallest luminosity of interest (\(\sim 5 \times 10^{56} \text{s}^{-1}\)) would contribute a newly ionized volume of \(\sim 10^4 \text{Mpc}^3\). With the steepest allowed log-logistic slope for the quasar luminosity function (LF) at \(z \sim 6\) (see discussion in § 5), we expect a space density of \(\sim 10^{-7} \text{Mpc}^{-3}\) for these quasars. The choice of 5%–10% for the fraction of the volume ionized by these relativistic sources is therefore about the maximum allowed by the upper limits on their space density. For reference, it is also useful to note that at \(z \sim 5–6\), the known population of bright and detectable quasars contribute only about 1% of the ionizing background (e.g., Madau et al. 1999; Fan et al. 2001). Sbrinovsky & Wyithe (2007) used the observed quasar LF to explicitly find an upper limit of 14.5% on the contribution of luminous (but below the detection threshold of SDSS) quasars at \(z \sim 6\) to the ionizing background. In practical terms, this means that \(p_{\text{same}} (x; 16)\) and the correlation function will now be defined with respect to the ionization fraction caused by the bright QSOs alone (e.g., 5%). The correlation function of the H \(\alpha\) fluctuations at the scale of the H \(\alpha\) regions (\(\gtrsim 10 \text{Mpc}\)) of bright QSOs will be dominated by the ionization inhomogeneities caused by these H \(\alpha\) regions (the other component will come from the density inhomogeneities which, as follows from Fig. 3, will contribute negligibly). This also means that the light-travel time anisotropy at these scales will be mostly owing to the presence of relativistic H \(\alpha\) regions of the bright QSOs, even though they contribute a small fraction to the ionization fraction.

3. *A finite spread in quasar luminosities.*—It is obviously unrealistic to assume that all quasars have the same luminosity. To relax this assumption, we extend our analysis to include H \(\alpha\) regions with a range of sizes. H \(\alpha\) regions with volume \(V\) then contribute an ionized fraction \(nV\), and one must compute the anisotropy by summing over the contribution from different H \(\alpha\) regions. As shown in FZH04, the existence of different bubble sizes can easily be incorporated by redefining the following quantities:

\[
x_i = \sum_R x_i(R),
\]

\[
p_{\text{same}}(r, \theta) = \sum_R p_{\text{same}}(r, R, \theta).
\]

Here \(R\) is the radius of the H \(\alpha\) region corresponding to a particular type of source, determined by the luminosity and lifetime of the objects (note that this expression is strictly valid in the limit we consider here, i.e., \(nV \ll 1\)). For simplicity, we consider here only a finite spread of luminosities, in the range \(2 \times 10^{56} \text{s}^{-1} \lesssim S \lesssim 2 \times 10^{57} \text{s}^{-1}\). Quasars brighter than this range are already known to be rare. There could, of course, be additional quasars below this luminosity range, but we assume their H \(\alpha\) regions would be nonrelativistic, and we implicitly lump any such quasars together with the galaxy preionization discussed above.

We keep the lifetime fixed at \(t_{fH\alpha} = 3 \times 10^7 \text{yr}\) (in principle, one should also allow for a spread in ages between 0 yr < \(t_{fH\alpha} < 3.75 \times 10^7 \text{yr}\), which would slightly increase the anisotropy). Within the \(2 \times 10^{56} \text{s}^{-1} \lesssim S \lesssim 2 \times 10^{57} \text{s}^{-1}\) range, we consider either a flat or a steep luminosity function, \(n(L) \propto L^{\alpha_0}\) or \(n(k) \propto (L^0) \propto L^{\alpha}\). The latter slope is motivated by current empirical limits on the slope of the quasar LF at \(z \sim 6\) (e.g., Fan et al. 2006a), while the former (flat) slope is motivated by the possibility that the high-z quasar LF turns over and flattens not too far below the SDSS detection threshold.

4. *Fossil H \(\alpha\) regions.*—An important effect that will decrease the anisotropy we predict for short-lived quasars is the presence of fossil H \(\alpha\) regions around dead quasars. At redshift \(z = 10\), for our assumed \(\Omega_{\text{m}}h^2\), the recombination time in the IGM is \(\sim 10^4(C/5)\) yr. This is longer than the ages of the active H \(\alpha\) regions by a factor of \(5(t_{fH\alpha}/2 \times 10^7 \text{yr})^{-1} (C/5)^{-1}\). The implication is that for \(t_{fH\alpha} = 2 \times 10^7 \text{yr}\) and for \(C = 5\), at any given time, there would be approximately five fossil H \(\alpha\) regions, which have not yet fully recombined, for every actively ionized Strömgren sphere. Since these fossil H \(\alpha\) regions are not expanding, they will not produce an apparent anisotropy, and they will dilute the predicted level of anisotropy by a factor of \(\sim 5\). Several authors have attempted to compute the gas clumping factor \(C(z)\) from first principles, using numerical simulations or semi-analytic arguments. The overall clumping factor at high \(z\) is dominated by dense gas in collapsed halos (Haiman et al. 2001), and the resulting small-scale clumping is poorly understood. Iliev et al. (2005) find in a simulation that the contribution from the low-density IGM to the clumping at \(z = 10\) gives \(C = 8\); the dense gas inside halos can, however, increase this value by a factor of a few, so that the
clumping factor may be as high as $C = 20$ (Haiman et al. 2001; Iliev et al. 2005). On the other hand, the time taken by the fossil gas to recombine will actually vary with the density, implying that the dense parts of the fossil will recombine faster, while less dense parts will linger longer. Furthermore, the precise value of the relevant clumping factor will also be modified due to clustering of gas near quasars (one expects local overdensities, at least in the central regions of the fossil) and will also depend on the instrumental specifications. The active-to-fossil bubble abundance ratio will also be modified if the formation of quasars, within the last recombination time, is skewed to more recent epochs. In our fiducial model, we assume $C = 7$, which, for our choice of $t_{\text{HII}} = 3 \times 10^7$ yr$^{-1}$, implies that fossils outnumber active bubbles by a factor of $\sim 3$. We include the presence of these fossils in our calculations explicitly, treating them exactly as active H II regions, except that we assume that they are isotropic (i.e., setting $p_{\text{same}}$ to be isotropic). Note that the clumping factor, for the purpose of computing the speed of the ionization front (in eq. [20]) is different, and is still taken to be $C = 10$.

The results of the calculations that take into account these modifications are shown in Figure 5. The modifications 1–4 introduce several competing effects relative to our toy models. On the one hand, there is an increase in anisotropy owing to the more rapid expansion of the bubbles in an already pre-reionized medium. On the other hand, there is a decrease in the overall signal coming from these bubbles, as they contribute only a small fraction of the total ionized fraction, and this contribution is further diluted by the presence of fossil regions. We find that the effects of the finite spread of luminosities is less important, although the steep LF within the luminosity range considered produces a factor of $\sim 2$ lower anisotropy (because the fainter, less relativistic sources will have a higher contribution). Overall, we find that in the range of scales we considered, the level of the anisotropy can still reach above $\sim 10\%$. The detectability of this level of anisotropy will be discussed in the next section.

4. NOISE AND DETECTABILITY

The noise characteristics of interferometric experiments for the detection of surface brightness fluctuations have been discussed in detail in Fourier space in different contexts by various authors (e.g., Bowman et al. 2006; Zaldarriaga et al. 2004; Bharadwaj & Sethi 2001; White et al. 1999; McQuinn et al. 2006). The anisotropy computation lends itself more readily to interpretation in real space; we therefore estimate here the noise characteristics for ongoing and future interferometric experiments in real space. The surface brightness sensitivity $\Delta T_B$ for detecting an extended source (covering a solid angle equal to or larger than the synthesized beam) is

$$\Delta T_B = \Delta T_p \frac{\Omega_p}{\Omega_S}. \tag{24}$$

Here $\Omega_p$ and $\Omega_S$ are the solid angles of the primary and the synthesized beams, respectively; $\Delta T_p$, the antenna temperature sensitivity, is

$$\Delta T_p \simeq \frac{\sqrt{2} T_s}{\sqrt{\Delta v \Delta N^2}}. \tag{25}$$
Here $T_s$ is the system temperature, $\Delta \nu$ is the channel width for the measurement, $\Delta t$ is the total integration time, and $N$ is the total number of antennas. As the noise computation is valid for both filled antennas and dipole arrays, we refine our definitions relevant to ongoing and future experiments. The primary beam for a filled-aperture experiment, such as the Giant Metrewave Radio Telescope (GMRT), is the usual field of view. For an interferometer such as MWA, the primary beam is the beam of each "tile," which contains sixteen dipoles, and therefore $N$ is the total number of such tiles. Another assumption we make below is that the noise in each synthesized beam is uncorrelated. This requires uniform coverage in the visibility plane. Both GMRT and MWA contain long baselines that can give a small synthesized beam, but the noise in each pixel (synthesized beam) will then be correlated. Therefore, in both cases, the noise computation here is applicable only to antennas within baselines $\leq 1$ km.

The total number of independent (i.e., uncorrelated) pixels for a frequency channel is $N_m \approx \Omega_P/\Omega_S$. Assuming $N_m$ frequency channels, the total number of independent pixels in the data cube is $N_t \approx N_m N_p$. The quantity we wish to estimate is the two-point correlation function of the brightness temperature fluctuations,

$$\xi_{12} = \langle \Delta T_B(r_1) \Delta T_B(r_2) \rangle. \quad \text{(26)}$$

The observed temperature fluctuation $\Delta T_B(r_1)$ contains contributions from both signal and noise, but since noise is uncorrelated between two pixels, for noise, $\xi_{12} = 0$, and therefore the measured $\xi_{12}$ gives an unbiased estimator of the signal. We further assume that the measured $\Delta T_B$ is dominated by noise (or in other words, the signal cannot be directly imaged in the given integration time; see discussion in $\S$ 5), and in our computations below, we equate $\Delta T_B$ with the noise. The quantity of interest is the variance of the two-point function (eq. (26)). This variance can be computed by using the fact that each pair of measurements yields an uncorrelated random variable. The variance in the average of $n$ such pairs is the variance of each random variable, divided by the number of pairs (see, e.g., Papoulis 1984),

$$\delta^2 \xi_{12} = \frac{\sigma^2 \xi}{n(r_{12}, \theta)}.$$ \quad \text{(27)}$$

Here $\sigma^2 \xi = \langle \Delta T_B^2 \rangle$ is the variance of a single pair, and $n(r_{12}, \theta)$ is the number of pairs for a fixed $r_{12}$. Note that we explicitly write $n$ as a function of $r_{12}$ and the angle $\theta$, to take into account all possible correlations in the three-dimensional cube. The total number of pairs for all the pixels in the data cube, $N_r$, is $N_r, \text{pair} = N_r(N_r - 1)/2$, which allows us to write $n(r_{12}, \theta) = N_r, \text{pair} f(r_{12}, \theta)$, where $f(r_{12}, \theta)$ is the fraction of pairs within a given range of $r_{12} + \Delta r_{12}/2$ and $\theta + \Delta \theta/2$. This gives

$$\delta^2 \xi_{12} = \frac{2^{1/2} \Delta T_A^2}{N_r f(r_{12}, \theta)^{1/2}} \left( \frac{\Omega_P}{\Omega_S} \right). \quad \text{(28)}$$

The typical value of $f(r_{12}, \theta)$ can be roughly estimated by the following arguments. By definition, $f(r_{12}, \theta)$ is the fraction of pairs with a given $r_{12}$ and $\theta$. In practice, one is likely to use, at least in the initial stages of an experiment with small integration times, a small number of broad bins centered on a set of values of $r_{12}$ and $\theta$, to estimate the signal. For example, one may choose to divide the range of lengths available with a particular instrumental configuration into 10 bins with roughly equal width $\Delta r_{12}$, and likewise divide the $\pi/2$ angular range into 10 bins of equal $\Delta \theta$. Using 100 such bands, a typical value of $f(r_{12}, \theta)$ will then be $\approx 10^{-2}$. Of course, in reality, $f(r_{12}, \theta)$ will depend on $r_{12}$ and $\theta$, but this dependence can only be computed once the instrumental configurations and the choices of the bins are specified.

Here we list again the assumptions we have made to derive the above expression. The main assumption is that the noise is uncorrelated between pixels, or more specifically, the entire primary beam is assumed to be filled with uncorrelated "pixels" of synthesized beam. This, as mentioned above, requires uniform sampling in the visibility plane with a small enough grid to sample the entire primary beam. Whether this can be achieved depends on the array configuration. As mentioned above, the current GMRT or the upcoming MWA might be able to achieve such uniform sampling for baselines $\leq 1$ km. (It should be noted that according to the present GMRT strategy, the baseline distribution is not expected to be uniform, as we implicitly assume for our estimates, but rather weighted toward smaller baselines [roughly a $1/r^2$ distribution with a core of 10 m; Bowman et al. 2006]. McQuinn et al. [2006] show that the noise level expected for the weighted distribution is lower than for the uniform distribution of baselines [Fig. 6 of McQuinn et al. 2006]. Therefore, our calculations here give a slight overestimate of the expected noise levels.) However, the statistical homogeneity and isotropy of the reionization signal will allow one to average over correlation function measurements from different primary beams (i.e., over different patches of the sky). As a result, even when pixels in a single primary beam contain correlations, it is possible to increase the number of uncorrelated correlation function measurements and therefore obtain the requisite number of uncorrelated pixels. Ongoing and future experiments, such as PAST (Pen et al. 2004) and LOFAR, use a large number of dipole antennas ($\approx 10^5$), and our noise estimate can be applied to these missions as well.

We now give numerical estimates of the noise, using equation (28), for GMRT and MWA; similar estimates can be made for missions such as PAST and LOFAR. We assume the following parameters common to GMRT and MWA: system temperature $T_s = 440$ K (this system temperature corresponds to $z \approx 8$ [Bowman et al. 2006]; we use this value throughout), total bandwidth $N_v \Delta \nu = 8$ MHz (note that the expected sensitivity depends only on the total bandwidth and not on the channel width), total observing time $\Delta t = 10^8$ s, and $f(r_{12}, \theta) = 10^{-2}$. For GMRT, we adopt a primary beam of $\approx 4^\circ$ and a synthesized beam of $\approx 0.45^\circ$ and assume 15 antennas, which gives $\Delta T_B \approx 0.35$ K and $\xi_{12} \approx 2 \times 10^{-5}$ K$^2$. For MWA, we assume a primary beam of $20^\circ$, a synthesized beam of $4^\circ$, and 500 antennas (or tiles), which gives $\Delta T_B \approx 0.25$ K and $\xi_{12} \approx 2.5 \times 10^{-7}$ K$^2$ (Bowman et al. 2006).

On comparing these expected noise levels with the signal strength shown in Figure 5, we conclude that while GMRT cannot detect the correlation function anisotropies, MWA has the capability to do so. For instance, the expected signal at $r_{12} \approx 35$ Mpc is larger than the expected noise levels of MWA by a factor of nearly 8. The detection of the anisotropy at the level of 2%–10% will require noise levels smaller by a further factor of nearly 10–50, which would require integration times between $0.2 \times 10^7$ and $2 \times 10^7$ s. These expectations can be considerably improved by using the expected noise levels of MWA$5000$ and SKA (Square Kilometre Array; Bowman et al. 2006; McQuinn et al. 2006).

One can compare the degree of difficulty involved in detecting the anisotropy associated with finite light speed effects with the anisotropy resulting from redshift-space distortion. As seen in Figure 3, even though the redshift-space anisotropy can be large, it is generally associated with the subdominant component (density fluctuations) of the overall signal. At scales $\approx 35$ Mpc, density fluctuations...
perturbation induced signal is more than an order of magnitude smaller than the signal in Figure 5. The signal owing to density perturbations increases at smaller scales. However, the forthcoming interferometers LOFAR and MWA are better suited for detecting the signal above scales $\lesssim 5$ Mpc (see, e.g., Fig. 6 in McQuinn et al. 2006); a future instrument such as SKA, which is likely to have much better sensitivity at smaller scales, is needed for detecting the redshift-space distortion (McQuinn et al. 2006). The anisotropy we discuss in this paper, however, is dominant at large scales and, in particular, MWA is ideally suited for detecting such a signal as it has much better sensitivity at large scales, owing to its large primary beam.

The S/N estimate above asks whether the instrument can measure the anisotropy on a particular scale $r_{12}$ and in a particular direction $\theta$. This estimate may be significantly more demanding than a mere detection, or a crude characterization, of the anisotropy. In reality, one can define much cruder measures of anisotropy to look for—say, dividing the angular range into a few quadrants and combining the independent power spectrum measurements within each quadrant and for several different $r_{12}$. The effective number of independent $(r, \theta)$ combinations that one can combine will depend on the actual observational strategy and on the band power used, and we leave more precise estimates to future work.

5. DISCUSSION

In deriving the anisotropy of the correlation function due to light-travel time delay, we assumed that the H II regions are spherically symmetric around a given source of UV photons. But there are many reasons why the H II regions will deviate from a spherical shape, including (1) density inhomogeneities around the source, (2) anisotropic emission from the source, and (3) mergers of H II regions. We briefly discuss each of these in detail below.

Density inhomogeneities.—From equation (20), it follows that during the initial ultrarelativistic phase of expansion, the velocity of the H II front is independent of density. As discussed above, the H II front makes the transition from the relativistic to the nonrelativistic phase at the radius $r \sim n_{HII}^{-1}(\chi)$. As a result, the H II region will expand more slowly into higher density regions and develop an intrinsic anisotropy. However, as the density field constitutes a homogeneous and isotropic random process, averaging over a large number of ionizing centers—as will be required to measure the correlation function—will tend to cancel this effect. Combinations and radiative transfer effects will cause further intrinsic anisotropies owing to density inhomogeneities (e.g., Bolton & Haehnelt 2007; Maselli et al. 2007), but these do not become important until much later, well into the nonrelativistic phase, when the light-travel time delay anisotropy is negligible.

Anisotropic emission.—Arguments similar to the previous case apply equally to beamed or otherwise anisotropic emission, as well: any intrinsic anisotropy in the source emission will be uncorrelated with its orientation relative to the line of sight. As a result, averaging over a volume containing a large number of ionizing centers will diminish this effect (producing no signal in the limiting case of a very large survey volume).

Anisotropy owing to bubble mergers.—Even if individual H II regions are spherical, their mergers will result in strong asymmetries, at least on scales comparable to the interdistance between the ionizing sources in a single bubble. The intrinsic shape after the mergers of H II regions, or the level of its anisotropy, is difficult to assess analytically and would require numerical simulations, which is beyond the scope of our work. However, any anisotropy caused by mergers also constitutes a homogeneous and isotropic random process, uncorrelated with the line of sight. This means that this anisotropy will be diluted as well, if measurements average over many clusters of ionizing centers.

Mergers, however, will have some additional implications. First, the mergers will increase the characteristic bubble size and therefore also the length scale where the correlations (and their anisotropy) peak (e.g., FZH04). Second, the light-travel time delay anisotropy will depend on the typical degree of synchronization between the sources in a single superbubble. Predicting this synchronization would require new assumptions about the source population and additional modeling, but the limiting cases are easily envisioned. If all sources in a bubble turned on simultaneously, this would be similar to a single but more luminous and anisotropic source, which will boost the predicted signal (additional travel time delays between the actual sources will again average out, if the sources are isotropically distributed within the superbubble). On the other hand, if the sources are turned on in a perfect sequence, one after the other, then the effect would be similar to increasing the typical source lifetime (by a factor that equals the average number of sources in a single bubble), without changing the luminosity. This will diminish the anisotropy signal, as discussed above. In particular, if the sources in a bubble maintain a luminosity similar to that of a single bright source for more than $\approx 5 \times 10^7$ yr, the signal will become undetectable. Whether this occurs would be interesting to work out in specific models for bubble growth. The situation should be possible to avoid when the filling factor of ionized bubbles is small.

In summary, the anisotropy of the correlation function does not depend crucially on the assumption of the sphericity of the H II region around a source. Detailed changes in the correlation function and its anisotropy, owing to merging of sources, are hard to assess but will generally lead to an increase in correlation length. The signal can also be significantly suppressed if more than a few bright quasar-like sources turn on, synchronized to within $\Delta t \approx t_q$ (the lifetime of a single source), and cluster together in a single bubble whose expansion is maintained for $\approx 5 \times 10^7$ yr. On the other hand, if the synchronization is either $\Delta t < t_q$ or $\gg t_q$, then we expect the anisotropy to increase or remain unaffected, respectively.

In our models, we also implicitly assumed that the edges of the H II regions are sharp. In reality, the edges can be blurred if quasar spectra are hard (Zaroubi & Silk 2005; although see R. Kramer et al. [in preparation] for a different conclusion) or if galaxies that are clustered around the quasar contribute significant ionizing flux spread spatially over an extended region (Wyithe & Loeb 2007). In practice, however, as long as the H II region boundaries are sharp enough to be well defined, such blurring should not significantly affect our conclusions. We also note that at least one of the $z > 6$ quasars appears to be surrounded by a sharp H II region, with a thickness $\lesssim 1$ (proper) Mpc for a radius of $\approx 5$ Mpc (Mesinger & Haiman 2004). On the other hand, the random placement of the instrumental resolution elements relative to the boundaries of the quasar bubbles will effectively further blur the edges of quasar bubbles and will also add shot noise to estimates of the anisotropy.

Whether the various intrinsic anisotropies can be averaged out depends primarily on the number of bright sources within the survey volume. Roughly, one expects that in order to detect a $\Delta \xi/\xi = 10\%$ “systematic” anisotropy, in the presence of random intrinsic anisotropies of order unity, the survey has to contain at least $\approx (\Delta \xi/\xi)^{-2} = 100$ bubbles. The number of bubbles is highly uncertain, but we note that the lowest photon luminosity of interest ($\approx 2 \times 10^{56}$ s$^{-1}$) corresponds to a luminosity that is approximately a factor of 10–50 below that of the bright quasars detected in the SDSS (e.g., Wyithe & Loeb 2004a). The slope of
the quasar luminosity function at \( z > 6 \) is expected to be steep but poorly constrained observationally. Richards et al. (2006) placed an upper limit on the slope at \( 4 < z < 5.4 \) from the (lack of) gravitational lensing and found it to be flatter than \( d \log n / d \log L \approx -4 \). This limit, applied to \( z \approx 6 \), would allow the existence of up to \( \sim 100 \) \( \text{deg}^{-2} \) quasars that are sufficiently bright to cause detectable anisotropy, or up to \( 3 \times 10^4 \) sources in one of MWA’s 300 deg\(^2\) primary beam.

To provide another rough estimate of the possible number of sufficiently bright sources, in Figure 6 we show the number of dark matter halos per unit redshift and solid angle, at two different redshifts \( (z = 6 \) and \( 10) \), using the fitting formula from Jenkins et al. (2001). The abundance of the SDSS quasars implies that they have host halos masses of \( \sim 10^{12.8} M_\odot \) (e.g., Haiman & Loeb 2001). Figure 6 shows that the slope of the halo mass function at these high masses is \( d \log N / d \log M \sim -4 \), similar to (only slightly flatter than) the upper limit on the slope of the quasar LF. If we applied this scaling, we would expect \( \sim 10^{4.3} \) sources in the 300 deg\(^2\) area of one MWA primary beam. Because of the steepness of the halo mass function, however, this estimate is very sensitive to the \( M_{BH}\)-\( M_{halo} \) relation at high redshift. For example, if the quasar luminosity scales linearly with black hole mass \( L_q \propto M_{BH} \), but we had the steeper scaling \( M_{BH} \propto M_{halo}^{0.6} \) inferred for inactive galaxies at lower redshifts (e.g., Ferrarese 2002), then the expected number of sources would be reduced to \( \sim 10^{2.3} \). This would still allow the detection of about a few percent anisotropy, in the presence of order unity variations in the shapes of individual bubbles, but only marginally.

In our analysis above, we assumed, for simplicity, that the ionizing sources are randomly distributed in space. In reality, the sources are likely to be located at the peaks of the density field and therefore clustered. As mentioned above, this will increase the contribution of bright quasars to \( \xi_{xx} \). The clustering of the sources and the corresponding magnitude of the increase could be explicitly computed in a refined version of our model (see, e.g., eq. [19] and related discussion in FZH04). Here we simply note that the increase for rare sources should roughly trace their linear halo bias (Sheth & Tormen 1999), and we expect it to be about an order of magnitude for sources residing in dark matter halos that correspond to \( 2 \)–\( 3 \sigma \) peaks (for explicit calculations of the impact of source bias on the correlation function, see, e.g., Santos et al. 2003). More interesting in the present context is the fact that source clustering can also modify the apparent anisotropy. The three-dimensional spatial correlation function of the sources themselves will appear isotropic (neglecting peculiar velocities). However, the probability that a point is ionized is given by an integral of the space density of ionizing sources, around the given point, over a volume whose shape is anisotropic (given by eq. [19]). The probability distribution of sources within this volume will depend on both the distance and the angle with respect to the line of sight from another ionizing source. This effect may not be negligible, since the correlation length of the quasar distribution may approach the typical sizes of ionized bubbles (quasars at \( 3.5 < z < 5.4 \) already appear to have a correlation length as large as 25 Mpc; Shen et al. 2007) and will have to be included in future work and in analyzing actual 21 cm data.

As mentioned above, in our analysis we neglected terms containing the cross-correlation between density and the ionized fraction (eq. [6]). FZH04 argued that these terms are likely to be small compared to the other two terms that we retain, \( \xi_{xx} \xi_{\delta e} \) and \( \xi_{xx} - \xi_{\delta e}^2 \) (in eq. [8]). This is expected on the scale in question (\( \sim 20 \) Mpc) as the leading terms we retain are of order \( 0(x_{\delta e}) \) or \( 0(x_{\delta e}) \) depending on the value of \( f(r,R) \). The density-ionization fraction cross-correlation terms are of order \( 0(x_{\delta e}) \), which is expected to be smaller than the terms retained as \( \delta \ll 1 \) and \( \delta / x_{\delta e} \ll 1 \) at scales of interest. The exact ratio of the cross terms to the retained terms can nevertheless only be reliably calculated with numerical simulations. In practice, within our model, \( \xi_{xx} - \xi_{\delta e}^2 \) dominates over \( \xi_{xx} \xi_{\delta e} \). In their updated bubble-growth model, McQuinn et al. (2005) explicitly compared \( \xi_{xx} - \xi_{\delta e}^2 \) and \( \xi_{\delta e} \), over a range of redshifts and length scales. They found that the cross term is indeed subdominant, although only by a factor of \( \sim 2 \) at early stages of reionization and on small scales (see, e.g., their Fig. 2). We conclude that the cross terms will not affect our estimates by more than a factor of \( \sim 2 \), but they will have to be included in a more careful analysis of actual data.

In the context of the light-travel time delay anisotropy, retaining the density-ionized fraction cross-correlation can give rise to a new anisotropy, owing to correlation between the redshift-space distortion and the anisotropy due to light-travel time delay. In linear theory, the redshift-space distortion can readily be expanded into moments of a Legendre transformation, with only three of these moments nonvanishing (see, e.g., Hamilton 1998, eq. [9]). The light-travel time delay anisotropy can similarly be expanded into moments of Legendre transformation (this series will in general have a larger number of nonvanishing moments), and the correlation with the redshift-space distortion could be computed.

Finally, we note that the bright quasars producing the anisotropy in the power spectrum should be directly detectable with a sensitive future instrument, such as the James Webb Space Telescope (e.g., Haiman & Loeb 1998). Indeed, the 21 cm maps may help identify such quasars to begin with. With the help of such identification, an alternative method to search for the light-travel time delay anisotropy would be to stack the noisy 21 cm tomographic images centered on these quasars. This method will require additional modeling, in order to rescale the sizes of the \( H \) regions before they are stacked. In addition, if the emission of quasars is anisotropic, then this can introduce a selection effect: the optical selection will preferentially detect those QSOs that appear brighter along the line of sight toward us. This selection effect, if
unaccounted for, will mimic the effect of the anisotropy, since the transverse directions around the quasar may see systematically lower fluxes. We leave a more detailed discussion of this stacking approach to future work.

6. CONCLUSIONS

The time delay caused by finite light-travel time across cosmological H regions distorts their apparent shapes. This effect may be detectable in future redshifted 21 cm observations of bright ionizing sources during the various stages of reionization and yield constraints on the luminosity and ages of the sources and the neutral hydrogen density distribution in their surroundings (Wyithe & Loeb 2004b; Yu 2005). In principle, the distortion could be measured directly in tomographic images of individual H regions (Wyithe et al. 2005). Direct imaging of hundreds of sources at sufficient S/N is, however, unlikely to be achieved in forthcoming experiments such as LOFAR, PAST, or MWA, and may have to await the construction of SKA.

In this paper, we consider the detectability of this effect statistically, through measuring the anisotropy in the three-dimensional 21 cm power spectrum on a range of scales. We find that the anisotropy is largest when H regions expand at relativistic speeds. Our results indicate that if bright quasars contributed significantly (i.e., around 10% of the ionized fraction) to some stage of reionization, then the finite light speed effect could be observable in the anisotropy of the correlation function of the H ionization. For quasar luminosities \( \geq 5 \times 10^{56} \, \text{s}^{-1} \) and ages \( \leq 4 \times 10^{7} \, \text{yr} \), we expect an anisotropy of \( \geq 10\% \) in the correlation function, as shown in our results in Figures 3 and 4. We also compare this theoretical signal with the noise levels expected in ongoing and future radio interferometers that seek to detect this signal. We show that ongoing missions, such as MWA, might be able to detect this effect. A detection of this anisotropy would shed light on the ionizing yield and age of the ionizing sources and the distribution and small-scale clumping of neutral intergalactic gas in their vicinity. In particular, a secure detection of this anisotropy would immediately reveal the presence of a significant number of bright quasars. These sources should be directly detectable with a sensitive future instrument, such as the James Webb Space Telescope (see, e.g., Haiman & Loeb 1998), and indeed, the 21 cm maps may help identify such quasars to begin with. With the help of such identification, an alternative method to search for the light-travel time delay anisotropy would be to stack the noisy 21 cm tomographic images centered on these quasars.

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