Path Tracking Controller Design for Surface Vessel Based on Sliding Mode Control Method with Switching Law

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ABSTRACT

In this paper, the path tracking controller for a surface vessel based on the sliding mode control (SMC) with the switching law is proposed. In order to have no restriction on movement and improved tracking performance, the proposed control system is developed as follows: First, the kinematic and dynamic models in Cartesian coordinates are considered to solve the singularity problem at the origin. Second, the new multiple sliding surfaces are designed with the SMC and approach angle concept to solve the under-actuated property. Third, the switching control system is designed to improve tracking performance. To prove the stability of the proposed switching system under the arbitrary switching, the Lyapunov stability analysis method with the common Lyapunov function is used. Finally, the computer simulations are performed to demonstrate the performance, effectiveness and stability of the proposed tracking controller of a surface vessel.

Key Words : Surface Vessel, Tracking Control, Sliding Mode Control, Switching Control System, Common Lyapunov Function

1. Introduction

There have been many researches and developments for tracking controller of a surface vessel. It is because that a surface vessel is a useful and suitable device for environmental surveying, seaside monitoring, military landings, and so on [1-5]. Despite its numerous useful applications, there are several challenging problems in the designing of tracking controller; a highly non-linear system models, stabilization, and under-actuated property, that is a system has smaller number of actuators than...
degrees of freedom. To solve the problems, many researches have been proposed as follows.

F. A. Papoulias employed the linearized dynamics to design a tracking controller for a surface vessel [1-2], but the proposed controller has the stability problems. To solve this stability problem, J. M. Godhavn proposed the controller without the linearization of dynamics [6], but it has the errors in tracking. To make up for this fault, the improved tracking controllers were proposed in some researches [3-5], but those also have stability problem in case of a straight trajectory. And to simplify the design process, some researchers used the state transformation method [7] which removes the difficulties caused by the off-diagonal terms of the system matrix. The dynamic surface control (DSC) technique [8] was also used to solve the explosion of complexity problem caused by the repeated differentiation of virtual controllers in the back stepping design procedure. In addition to the above mentioned methods, lots of methods have been proposed for the tracking controller of a surface vessel [9-18], and H. Ashrafiuon have focused on the Sliding Mode Control (SMC) for designing the tracking controller of surface vessels [19].

In this paper, I propose a method for designing the path tracking controller of a surface vessel based on the SMC with the switching law. In order to remove a movement restriction and improve a tracking performance, the proposed controller is developed as follows: First, the models of AUV, which has similar kinematic and dynamic behavior with an under-actuated surface vessel on a horizontal plane [21], in the Cartesian coordinates [20] are considered to remove the singularity problem at the origin. And the water flow disturbance is considered to design the practical tracking controller. Next, to solve the under-actuated property of a surface vessel, the new multiple sliding surfaces are designed with approach angle concept. And to improve the tracking performance, the switching system for tracking control is designed with switching algorithm and law [21-25]. For the stability analysis of the proposed switching control system under arbitrary switching, the Lyapunov stability analysis theory with the common Lyapunov function [26-30] is applied.

Finally, the numerical simulations are carried out to demonstrate the performance and effectiveness of the proposed control system.

2. System Modeling

In this chapter, for the kinematic and dynamic models of an under-actuated surface vessel, the models of AUV in the Cartesian coordinates is considered. It is because that AUV presents similar behavior to an under-actuated surface vessel and the AUV model in the polar coordinates has the singularity problem at the origin.

2.1 Kinematic and Dynamic Models

In this paper, the movement of a surface vessel on the horizontal plan is described in the earth fixed frame and the body fixed frame as shown in Fig. 1. To depict the posture of a surface vessel, it is defined that the position \( \mathbf{x}_c \) as the middle point, the heading angle \( \theta_c \) as the angle between the X-axis and the heading direction, the current posture as \( \mathbf{q}_c = [x_c, y_c, \theta_c]^T \) and the reference postures as \( \mathbf{q}_r = [x_r, y_r, \theta_r]^T \), respectively. Additionally, the surge velocity, the sway velocity, and the yaw velocity of a surface vessel are defined as \( u \), \( v \), and \( r \), respectively. With the defined terms, the kinematic model of a surface vessel is described as follows:

![Earth-Fixed and body-fixed frames of system](image)
Then the effect of water flow disturbance is considered with the following assumption.

Assumption 1: The water flow disturbance is described two-dimensionally in the earth-fixed frame with a velocity $V_d$ and a direction $\theta_d$ as follows:

$$ D_x = V_d \cos \theta_d, \quad D_y = V_d \sin \theta_d. \quad (2) $$

where $D_x$ is the water flow velocity in the X-axis direction, and $D_y$ is the water flow velocity in the Y-axis direction. Since the water flow velocity and direction are not constant, there exist the differences between the disturbance of current and reference position. To apply these disturbance difference to the system model, the following additional terms are defined as shown in Fig. 2;

$$ \begin{cases} u_c & \text{surge velocity} \\ v_c & \text{sway velocity} \\ r_c & \text{yaw velocity} \end{cases} \quad (3) $$

And the dynamics for a surface vessel is expressed as following differential equations [20] with assumption 2.

Assumption 2: The center of buoyancy coincides with the center of mass, and the mass is distributed homogeneously. The hydrodynamic drag terms, which have higher order than two, heave, pitch, and roll motions can be neglected.

Based on the posture error assignment in Fig. 2, the posture error $[x_c \, y_c \, \theta_c]^T$ of a surface vessel is derived as follows:

$$ \begin{bmatrix} x_c \\ y_c \\ \theta_c \end{bmatrix} = \begin{bmatrix} \cos \theta_c & -\sin \theta_c & 0 \\ \sin \theta_c & \cos \theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_c \\ v_c \\ r_c \end{bmatrix} + \begin{bmatrix} V_{dc} \cos \theta_{dc} \\ V_{dc} \sin \theta_{dc} \end{bmatrix}. \quad (4) $$

By differentiating and transforming (4) with the water disturbance equation and defined terms, the equation for the surge velocity $u$, the sway velocity $v$, and the yaw velocity $r$ of a surface vessel is then derived as follows:

$$ \begin{bmatrix} u_c \\ v_c \\ r_c \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ \theta_c \end{bmatrix} + \begin{bmatrix} x_{cr} \\ y_{cr} \\ \theta_{cr} \end{bmatrix} + \begin{bmatrix} \cos (\theta_c - \theta) - \sin (\theta_c - \theta) \cos (\theta_c - \theta) \sin (\theta_c - \theta) \\ \sin (\theta_c - \theta) \cos (\theta_c - \theta) \sin (\theta_c - \theta) \cos (\theta_c - \theta) \cos (\theta_c - \theta) \sin (\theta_c - \theta) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ r_r \end{bmatrix}. \quad (5) $$

Fig. 2. Coordinate assignments for system modeling

And the dynamics for a surface vessel is expressed as following differential equations [20] with assumption 2.
The linear and quadratic drag coefficients are $N_{\text{d}1}$ and $N_{\text{d}2}$, respectively.

In the dynamics of a surface vessel (6), the term $F_u$ denotes the control force along the surge $u$ motion of the body-fixed frame, and $F_v$ denotes the control moment for producing the angular motion around the $z$-axis of the body-fixed frame. In a surface vessel, there exists no side thruster to control the sway motion $v$, thus the sway motion is uncontrollable and a surface vessel is an under-actuated dynamic system.

3. Design of Tracking Controller with Switching

In this chapter, the tracking controller for a surface vessel is designed under the consideration of the under-actuated property and stability problems. For the under-actuated property of a surface vessel, the new multiple sliding surfaces are designed with an approach angle concept. Then, based on the proposed reaching law and sliding surfaces, the controllers are designed. Next, the switching control system is developed with the designed controllers and switching law to improve the tracking performance. Finally, the stability of the proposed control system is proved by using the Lyapunov stability theory with the common Lyapunov function.

3.1 Advanced Reaching Law

Before designing the appropriate sliding surfaces, the reaching law is designed to make the states converge to the proposed sliding surfaces with the condition as follows:

$$S \cdot \dot{S} < 0, \text{ for any } t$$  \hspace{1cm} (7)

Here, to satisfy the above condition, the reaching law is typically chosen as follows:

$$\dot{S} = -K \cdot \text{sgn}(S), \text{sgn}(S) = \begin{cases} 1 & \text{where } S > 0 \\ 0 & \text{where } S = 0 \\ -1 & \text{where } S < 0 \end{cases}$$  \hspace{1cm} (8)

where $K$ is the positive constant. From the above reaching law, the reaching time $t_r$, the necessary time for each state to converge to the sliding surface, is derived as follows:

$$t_r = \frac{|S(0)|}{K}.$$  \hspace{1cm} (9)

According to (9), as $K$ values increases, the reaching time decreases. However, large $K$ value increases the chattering phenomenon and tracking errors, and those worse the tracking performance. In order to improve the tracking performance, this paper considered the following reaching law:

$$\dot{S} = -K_p R(S) \text{sgn}(S),$$  \hspace{1cm} (10)

where $K_p = [k_{1r} k_{2r}]^T$, $k_{1r}$ and $k_{2r}$ are the positive constant, $K_p$ is the strictly positive offset which is less than 1. $\alpha$, $\beta$ are $\gamma$ strictly positive integers. With the above conditions, $R(S)$ is always strictly positive. Therefore, $R(S)$ does not affect the stability of controller. In the reaching law (10), as $|S|$ increases, $R(S)$ converges to $|S|^\alpha/K_p$, and consequently $t_r$ converges to $K_p |S(0)|/|K_p| |S|^\alpha$, which is smaller than $|S(0)|/K$ with large $|S|$ value. It means that each state converge to the proposed sliding surface faster. On the other hand, as $|S|$ decreases, $R(S)$ converges to $|S|^\alpha$, and consequently the reaching law $\dot{S}$ gradually decreases, which decreases the chattering phenomenon and the tracking errors. Therefore, the reaching law (10) is suitable to improve tracking performance of controller.

3.2 Sliding Surface with Approach Angle

In order to make a surface vessel follow the trajectory, the controller needs to make the posture error $(x_c, y_c, \theta_c)$ converge to zero as time elapses. To do this, the approach angle concept, respect to the defined coordinate as shown in Fig. 2, is applied. From Fig. 2, it is verified that $y_c$ converges to zero as the heading
angle $\theta_c$ converges to an approach angle $\theta$. Since the heading angle is tangential to the reference trajectory, the heading angle error $\theta_e$ converges to zero as a surface vessel follows the reference trajectory exactly. Therefore, the posture error $(x_e, y_e, \theta_e)$ converges to zero as $x_e$ and $\theta - \theta_e$ converge to zero. Then, the sliding surface with the first order filters is defined as follows:

$$s = \begin{bmatrix} s_x \\ s_\theta \end{bmatrix} = \begin{bmatrix} x_e + k_1 x_e \\ \dot{\theta}_e + k_2 \dot{\theta}_a \end{bmatrix},$$  

(11)

$$\theta_a = \theta - \theta_e, \quad \theta = \tan^{-1}(\frac{y_e - y_c}{x_e - x_c}).$$

where $k_1, k_2 > 0$. Based on the first order filter property, as $s_x$ and $s_\theta$ converge to zero, $x_e$ and $\theta_a$ converge to zero. In addition, the sliding surfaces (11) have different properties according to the values of $k_1$ and $k_2$. With the large $k_1$ and $k_2$ values, each state converge to the sliding surface rapidly. And the small $k_1$ and $k_2$ values allows each state converge to the sliding surface with small chattering phenomenon. Based on this property, the multiple sliding surfaces $S_1, S_2, \ldots, S_n$ which have different values of $k_1$ and $k_2$ are designed as follows:

$$S_n = \begin{bmatrix} s_{x_n} \\ s_{\theta_n} \end{bmatrix} = \begin{bmatrix} x_e + k_{1n} x_e \\ \dot{\theta}_e + k_{2n} \dot{\theta}_a \end{bmatrix}. $$

(12)

where $k_{11} \neq k_{12} \neq \ldots \neq k_{1n} > 0$ and $k_{21} \neq k_{22} \neq \ldots \neq k_{2n} > 0$. Then, by using the multiple sliding surfaces which have the different properties, the advanced tracking controller can be designed.

3.3 Control Law Design for Surface Vessel

The sliding surfaces $S_1, S_2, \ldots, S_n$ have a similar form, only have different values of $k_1$ and $k_2$, therefore the each controller for the proposed multiple sliding surfaces can be designed in the same method. Then, to design the appropriate control law for each sliding surfaces, I substitute (1) and (5) into each differentiated sliding surface as follows:

$$\dot{S}_n = -K_c R(S_n) sgn(S_n) = \begin{bmatrix} x_e + k_{1n} \dot{x}_e \\ \dot{\theta}_e + k_{2n} \theta_a \end{bmatrix}$$

$$= \begin{bmatrix} x_e + k_{1n} y_r \\ \dot{\theta}_e + k_{2n} \theta_a \end{bmatrix} - \begin{bmatrix} k_{1n} u_m \\ k_{2n} \dot{r}_m \end{bmatrix}$$

$$+ \begin{bmatrix} k_{1n} (u \cos(\theta - \theta_e) - v \sin(\theta - \theta_e)) \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} k_{1n} \sin(\theta_e (V_d \cos(\theta_e - \theta_d) - V_d \cos(\theta_d)) \\ 0 \end{bmatrix} + \begin{bmatrix} k_{1n} \cos(\theta_e (V_d \sin(\theta_e - V_d \sin(\theta_e - \theta_d)) \\ 0 \end{bmatrix}. $$

(13)

Then, by substituting (11) and (13) into (7), the control input $U_{an}(u_{an}, r_{an})$ for sliding surface $S_n$ is obtained as follows:

$$U_{an} = \begin{bmatrix} u_m \\ r_m \end{bmatrix} = \begin{bmatrix} 1/k_{1n} \\ 1/k_{2n} \end{bmatrix} K_c R(S_n) sgn(S_n)$$

$$+ \begin{bmatrix} x_e/k_{1n} + (2 V_d + 2 \dot{V}_{dr} + |u_r| + |v_r|) sgn(s_{x_n}) \end{bmatrix}$$

$$\dot{\theta}_e/k_{2n} + \dot{\theta}.$$

(14)

In a surface vessel, only $F_u$ and $F_r$ are the controllable inputs, thus the control input $U_{an}(F_u, F_r)$ for sliding surface $S_n$ is derived from (6) and (14) as follows:

$$U_{an} = \begin{bmatrix} F_u \\ F_r \end{bmatrix}$$

$$= \begin{bmatrix} m_1 \dot{u}_m - m_2 v_r \dot{r} + X_u \dot{u}_m + X_{u|u_m} u_m \\ m_3 \dot{v}_m - (m_{11} - m_{22}) u_m \dot{v}_m + N_r \dot{v}_m + N_{r|v} v_r \dot{r} \end{bmatrix}.$$  

(15)

where $u_m$ and $r_m$ are the control input values of $U_{an}$. Here each controllers have the different control input $U_{an}$ based on the sliding surface $S_n$. To operate those multiple controllers as the one unit controller, the switching algorithm for the tracking control of a surface vessel is proposed as shown in Fig. 3.
Based on the current posture \( q_c \), the reference posture \( q_r \), and the sliding surface \( S \), each controllers generates the appropriate control inputs which have different control properties. To use those multiple controllers efficiently, the proposed switching system estimates the posture errors with each control input as follows:

\[
E_{U_n} = (k_1 (x_r - x_{cm})^2 + k_2 (y_r - y_{cm})^2 + k_3 (\theta_r - \theta_{cm})^2)^{0.5},
\]

where \( x_{cm}, y_{cm}, \theta_{cm} \) are the estimated current posture with the control input \( U_{sn} \), and \( k_1, k_2, k_3 \) are the positive constant to weight importances for each error. With the defined switching law and control inputs, the control input \( U_s \) is designed as follows:

\[
U_s = U_{s1} \cdot s_{w1} + U_{s2} \cdot s_{w2} + \cdots + U_{sn} \cdot s_{wn},
\]

where \( s_{wi} = 1 \) when \( E_{U_{si}} \) is smallest, else \( s_{wi} = 0 \) for \( i \in 1, 2, \ldots, n \).

3.4 Stability Analysis with Common Lyapunov Function

In the switching system, the stability analysis under the arbitrary switching is one of the most difficult problems because even when all the controllers of switching system are exponentially stable, it is still possible that the overall system can be unstable for a certain switching signals. Thus, it is necessary to show the stability of switching system under the arbitrary switching, and to do so, the common Lyapunov function (CLF) method is used in this paper. It is because if there exists CLF for all its controllers and systems, then the stability of switching system is guaranteed under arbitrary switching.

**Theorem:** The control input (17) stabilizes the overall tracking control system under Assumption 1 and 2. Thus, the posture errors converge to zero.

**Proof:** In order to show the existence of CLF for the proposed switching system, the Lyapunov function selected as follows:

\[
V = \frac{1}{2} S_i^T S_i > 0 \quad (i \in 1, 2, \ldots, n)
\]

where \( V \) is positive definite. To prove the stability of the controllers with the Lyapunov function \( V \), (13) and (15) are substituted as follows:

\[
\dot{V} = S_i^T \dot{S}_i = s_x \dot{s}_x + s_{\theta} \dot{s}_{\theta},
\]

\[
= k_1 (u_x \cos (\theta_r - \theta_r) - v \sin (\theta_r - \theta_r)) + k_1 \sin \theta_r (V_{dx} \cos \theta_d - V_{dc})
\]

\[
+ k_1 \cos \theta_r (V_{dx} \sin \theta_d - V_{dc} \cos \theta_d) + x_r + k_1 (y_r - u_{ci}) + s_{\theta_i} (\dot{\theta}_r + k_2 \dot{\theta} - k_2 r_{ci})
\]

\[
= k_1 (s_{x} u_r \cos (\theta_r - \theta_r) - s_{x} | u_r | - s_{x} v \sin (\theta_r - \theta_r) - | v_r |)
\]

\[
+ s_{x} V_{dx} \sin \theta_d \cos \theta_r + \cos \theta_r \sin \theta_d) - | s_{x} | 2 V_{dx}
\]

\[
- s_{x} V_{dc} \sin \theta_d \cos \theta_r + \cos \theta_r \sin \theta_d) - | s_{x} | 2 V_{dc}
\]

\[
- k_1 R(s_{x}) | s_{x} | - k_2 R(s_{\theta}) | s_{\theta} |
\]

\[
< k_1 (s_{x} u_r - | s_{x} | u_r + s_{x} v_r - | v_r |) + s_{x} 2 V_{dx} - | s_{x} | 2 V_{dx} - | s_{x} | 2 V_{dc}
\]

\[
- k_1 R(s_{x}) | s_{x} | - k_2 R(s_{\theta}) | s_{\theta} | < 0.
\]

where \( i \in 1, 2, \ldots, n \). Since the Lyapunov function \( V \) is positive definite and \( \dot{V} \) is negative definite regardless, it is proven that all controllers are stable by the Lyapunov function stability theory with CLF, and the proposed switching system is stable under arbitrary switching. Based on the above results, the proposed switching system for a tracking control of a surface vessel makes the posture error \( (x_r, y_r, \theta_r) \) converge to zero stably with the control input.
4. Simulation Results

In this chapter, the simulation results for the given reference trajectories are presented to demonstrate the performance of the proposed controller for a surface vessel. For the simulations, the rigid body and hydrodynamic parameters [20] are selected as Table 1.

Table 1. Hydrodynamic parameters

| Symbol | Parameters          | Value | Unit |
|--------|---------------------|-------|------|
| \(m\)  | Mass                | 185   | kg   |
| \(I_z\)| Rotation inertia    | 50    | kgm² |
| \(X_u\) | Added mass          | -30   | kg   |
| \(Y_u\) | Added mass          | -80   | kg   |
| \(Y_r\) | Added mass          | -1    | kg   |
| \(N_r\) | Added mass          | -30   | kgm² |
| \(X_u\) | Surge linear drag   | 70    | kg/s |
| \(X_u|u|\) | Surge quadratic drag| 100  | kg/m |
| \(Y_u\) | Sway linear drag    | 100   | kg/s |
| \(Y_u|v|\) | Sway quadratic drag| 200   | kg/m |
| \(N_r\) | Yaw linear drag     | 50    | kgm²/s |
| \(N_r|r|\) | Quadratic yaw drag  | 100   | kgm² |
| \(x_0\) | Position of COG     | 0     | m    |

The initial conditions are selected as \([x_e, y_e, \theta_e] = [0, 0, 0]\) and \([x_r, y_r, \theta_r] = [5.5, \pi/4]\). Then, the control parameter values are chosen as follows:

\[
k_1 = 10, \quad k_{12} = 10, \quad k_{13} = 1, \quad k_{r1} = 0.4, \quad k_{r2} = 0.9, \quad K_p = 0.8, \quad \alpha = 0.2, \quad \beta = 0.3, \quad \gamma = 0.2
\]

For the water flow disturbance of a surface vessel, the current direction and velocity is designed as follows:

\[
V_{dc} = V_c \left| \sin \frac{x_e}{10} + \sin \frac{y_e}{10} \right| / 2,
\]

\[
V_{dr} = V_r \left| \sin \frac{x_r}{10} + \sin \frac{y_r}{10} \right| / 2,
\]

\[
\theta_{de} = \frac{\pi}{3} + \frac{\pi}{36} \left( \sin \frac{x_e}{10} + \cos \frac{y_e}{10} \right),
\]

\[
\theta_{dr} = \frac{\pi}{3} + \frac{\pi}{36} \left( \sin \frac{x_r}{10} + \cos \frac{y_r}{10} \right).
\]

where \(V_c = V_r = 1 \text{ m/s}\), \(\theta_{de}\), \(\theta_{dr}\) have minimum 50° and maximum 70° values. With the water flow equation and parameters, the disturbances are generated as shown in Figs. 4 and 5.

Fig. 4. Water flow velocity in X–axis direction

Fig. 5. Water flow velocity in Y–axis direction

Then, to investigate the stability and performance of the proposed controller for a surface vessel, the values for the multiple sliding surfaces and reference condition are selected as follows:

\[
S_1 = \left[ \begin{array}{c} x_e + x_e \\ \dot{\theta}_a + \theta_a \end{array} \right], \quad S_2 = \left[ \begin{array}{c} x_e + 0.1x_e \\ \dot{\theta}_a + 0.1\theta_a \end{array} \right],
\]

\[
S_3 = \left[ \begin{array}{c} x_e + 0.1x_e \\ \dot{\theta}_a + 0.1\theta_a \end{array} \right], \quad S_4 = \left[ \begin{array}{c} x_e + 100x_e \\ \dot{\theta}_a + 100\theta_a \end{array} \right].
\]
Condition: For a complicated line, the linear, angular, surge, and yaw velocities of reference trajectory are selected as follows:

\[
\begin{align*}
0s \leq t \leq 3s : v_r = 4\text{m/s}, \quad w_r = 0\text{rad/s}, \\
3s \leq t \leq 11s : v_r = 4\text{m/s}, \quad w_r = -0.4\text{rad/s}, \\
11s \leq t \leq 14s : v_r = 4\text{m/s}, \quad w_r = 0\text{rad/s}, \\
14s \leq t \leq 22s : v_r = 4\text{m/s}, \quad w_r = 0.4\text{rad/s}, \\
22s \leq t \leq 30s : v_r = 4\text{m/s}, \quad w_r = 0\text{rad/s}.
\end{align*}
\]

Then, to confirm the performance and stability of the proposed switching control system, the simulations are performed with different number of sliding surfaces as follows.

Fig. 6 depicts the tracking control result of proposed switching control system with sliding surfaces \(S_1\sim S_4\). As shown in Fig. 6, a surface vessel stably and exactly follows the given reference trajectory from the origin.

Fig. 8. Tracking errors with sliding surfaces \(S_1\sim S_4\)

Fig. 9. Tracking errors with sliding surface \(S_1\)

Fig. 10. Tracking errors with sliding surfaces \(S_1\sim S_2\)
point, without a singularity problem. Figs. 7 and 8 show switching law result and the tracking errors while a surface vessel follows the given reference trajectory. Fig. 7 shows that the proposed system selects an appropriate controller from the multiple controllers based on the switching algorithm and law. And Fig. 8 shows that the posture errors converge to zero in short time with very small tracking errors and chattering phenomenon. Based on the Figs. 6-8, it is verified that the proposed control system is stable under arbitrary switching with remarkable tracking performance. And Fig. 9 depicts the tracking control result with only one controller (based on $S_1$), and it shows a large tracking errors. Fig. 10 is with two controllers (based on $S_1$ and $S_2$) and shows that the additional controller decreases the tracking errors. And by comparing Figs. 8-10, it is proved that the switching system with multiple controllers improves the tracking performance of proposed control system.

From the simulation results, I verify that the proposed switching control system for surface vessel tracking is stable under arbitrary switching, and make a surface vessel exactly follow the given reference trajectory with the decreased reaching time, tracking errors, and chattering phenomenon.

5. Conclusion

In this paper, a method for designing the path tracking controller for a surface vessel based on the sliding mode control with the switching law is proposed. In order to have no restriction on movement and improved tracking performance, the proposed controller is developed as follows:

First, the kinematic and dynamic model equations in the Cartesian coordinates are considered because the kinematic model in the polar coordinates has the singularity problem at the origin. In addition, the water disturbance is considered to design the practical tracking controller of a surface vessel. Second, in order to solve the under-actuated property, the new multiple sliding surfaces are design by using the SMC with an approach angle. Third, the switching system for tracking control is designed by connecting the multiple controllers with the switching algorithm and law to improve the tracking performance. To prove the stability of the proposed switching control system under the arbitrary switching, the Lyapunov stability analysis method with the common Lyapunov function is used. Finally, the numerical computer simulations are performed to demonstrate the performance and effectiveness of the proposed tracking control system of a surface vessel. From the simulation results, it is verified that the proposed tracking control system is stable under the arbitrary switching, and has the improved tracking performance.

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