Study on Negative Resistance Mechanism and Elimination Method of Network Simplification

Lin Zhu¹, Dong Fu¹, Qiliang Sheng¹, Bei Wang¹ and Xinge Hu¹

¹School of Electric Power South China University of Technology Guangzhou, China

E-mail: 1061243886@qq.com

Abstract: The key of the further development of power system dynamic equivalence study is that correct understanding of negative resistance mechanism in network simplification and how to get rid of negative resistance. This paper analyzes the network simplification of power system, and it thoroughly studies the mechanism of equivalent branches with negative resistance. This paper also leads the definition of the constant impedance load and studies the property and the size of constant impedance load to further explain the mechanism of negative resistance production. This paper proposes a network transformation method which is based on power flow calculation results. This method transforms series branches which include negative resistance into PI-type branches. Finally the validity of this method is verified in sample cases from the China Southern Power Grid and IEEE 39 bus system. This method also solves the problem that some simulation software can’t model the branches with negative resistance.

1. Introduction

The dynamic equivalent of large-scale power systems can effectively solve the problem that large-scale power system simulates slowly. And it also helps people focus on particular power system research. Coherency equivalent method [1-3] is a common method of the large-scale power system dynamic equivalent. When people use CSR method [4-6] for network simplification, it finds that the value of equivalent branch resistance is negative. Because of negative resistance, it makes that power loss of the line is negative. It goes against power loss of actual power system, and it will reduce the credibility of the load flow results. For electromagnetic transient simulation software, the resistance only can set a value greater than or equal to zero. And the simulation software doesn’t have negative resistance model. Because of those reasons, people can’t analyze and calculate power network which has negative resistances.

Negative resistance researches just as the level of using power electronic devices to make negative resistances [7-8], but it not deeply studies virtual negative resistance in the power simulation software. Literature [9] analyzes and calculates a variety of non-linear models in network by point linearization method. And it deeply discusses the problem that in some operation mode equivalent parameters of the grid appear negative resistance, which is based on flow calculation tracking. But it didn’t thoroughly discuss the constant impedance load of non-linear load, and it also not given consideration to the constant impedance load impacts on Gaussian elimination method. Furthermore, it also not presented an effective method to eliminate negative resistance that it didn’t radically resolve the problem of negative resistance.

This paper deduces and analyzes the mechanism of negative resistance which appears after power system network simplification, and it also demonstrates the inevitability of an equivalent branch which
will appear negative resistance. This paper also discusses that the property and the value of the constant impedance load impact on the negative resistance appearance which further illustrates the negative resistance mechanism. This paper also presents a practical network transformation method that it makes a network with negative resistances transform into a network without negative resistances. Finally, the reasonability of mechanism is verified in a simple case from the IEEE 39 bus system. And the effectiveness of the network transformation method is verified in a simple case from the China Southern Power Grid.

2. Mechanism of negative resistance production

At present, the scale of power grids become more and more large, and it leads to power system dynamic equivalence [10-11] researches become more and more thoroughly. But it also lacks the related research work about negative resistance in network simplification. This paper will analyze the mechanism of negative resistance by introduces network simplification method. We set that node a is elimination node. Nodes which link up with node a are setting to $b_1, b_2, \cdots b_r$. So elimination node a and nodes $b_1, b_2, \cdots b_r$ make up the node admittance matrix,

$$
Y = \begin{bmatrix}
y_{11} & y_{12} & \cdots & y_{1(r+1)} \\
y_{21} & y_{22} & \cdots & y_{2(r+1)} \\
\vdots & \vdots & \ddots & \vdots \\
y_{(r+1)1} & y_{(r+1)2} & \cdots & y_{(r+1)(r+1)}
\end{bmatrix}
$$

(1)

In equation (1), $y_{11}'$ is the self-admittance of elimination node a, $y_{22}, y_{33}, \cdots y_{(r+1)(r+1)}$ are self-admittance of reserved nodes, $y_{ij}$ is transadmittance of node i and node j.

For the self-admittance $y_{11}'$ of elimination node a, it makes of self-admittance $y_{11}$ and equivalent admittance $y_{21}$ which is transforming by the constant impedance load of node a, we have,

$$
y_{11}' = G_{11} + jB_{11}' = y_{21} + y_{11}
$$

(2)

In equation (2), $G_{11}'$ is revised self-conductance of node a, $B_{11}'$ is revised self-susceptance of node a.

Assumed that $Y_{11} = y_{11}'$, $Y_{22} = \begin{bmatrix} y_{22} & \cdots & y_{2(r+1)} \\
\vdots & \ddots & \vdots \\
y_{(r+1)2} & \cdots & y_{(r+1)(r+1)} \end{bmatrix}$, $Y_{12} = [y_{12} \cdots y_{1(r+1)}]$, when we use the Gaussian method to eliminate node a, and it sets the new node admittance matrix is that,

$$
Y_{22} = \begin{bmatrix} y_{22}' & \cdots & y_{2(r+1)}' \\
\vdots & \ddots & \vdots \\
y_{r2} & \cdots & y_{r(r+1)}' \end{bmatrix}
$$

(3)

In equation (3), the $y_{ij}'$ is given by, $y_{ij}' = y_{ij} - \frac{y_{ij}y_{ij}}{y_{11}}$. $i,j=2,3,\cdots,r+1$.

For the mutual conductance of equivalent branch, it can be calculated as,

$$
\text{real}(y_{ij}') = \text{real} \left( y_{ij} \cdot \frac{y_{ij}y_{ij}}{y_{11}} \right) = G_{ij}' = G_{ij} - \frac{(G_{ij}G_{ii} - B_{ij}B_{ii})G_{11}}{G_{11}^2 + B_{11}^2}
$$

(4)

Among in equation (4), $G_{ij}, G_{ii}$, $G_{ij}$ are mutual conductance between node a and node i, node a and node j, node i and node j respectively. $B_{ij}, B_{ii}$ are mutual susceptance between node a and node i, node a and node j respectively. For an actual power system, the resistance of branch is much less than the reactance of branch, so in equation (4), we can have,

$$
\text{ABS}[G_{ij}B_{ii} + G_{ii}B_{ij}] < \text{ABS}[-B_{ij}B_{ii}]
$$

(5)

$G_{11}'$ and $B_{11}'$ which in equation (4) are all including self-admittance of elimination node and equivalent admittance that transformed by constant impedance load of elimination node. For the
self-admittance, actual branch reactance is greater than the actual branch resistance, so self-conductance is much less than self-susceptance. And the value of self-susceptance is negative, the value of self-conductance is positive. For the equivalent admittance, it depends on non-linear load which pours into node. For a real power system, non-linear load of nodes is corresponding to the requirement of receiving-end grid. The receiving-end grid active load demand often greater than reactive load demand, so equivalent conductance is greater than equivalent susceptance. If we add two admittances, we have,

1) When the active load of node is large enough, the equivalent conductance is the major part, so the revised self-conductance is greater than the revised self-susceptance, and then it can be calculated that,

\[ |ABS[G_{11}]| \gg |ABS[B_{11}]| \] (6)

By equation (5), we can have,

\[-B_{ij}B_{ii}G_{11} + \left(G_{ij}B_{ii} + G_{ii}B_{ij}\right)B_{11} \approx -B_{ij}B_{ii}G_{11} \leq 0 \] (7)

2) When the active load of node is small enough, the self-susceptance is the major part, so the revised self-susceptance is greater than the revised self-conductance, and absolute values of \( G_{11} \) and \( B_{11} \) are given by,

\[ |ABS[G_{11}]| \ll |ABS[B_{11}]| \] (8)

By equation (5), we can have,

\[-B_{ij}B_{ii}G_{11} + \left(G_{ij}B_{ii} + G_{ii}B_{ij}\right)B_{11} \approx \left(G_{ij}B_{ii} + G_{ii}B_{ij}\right)B_{11} \geq 0 \] (9)

As to 1), when \( |ABS[-B_{ij}B_{ii}G_{11}]| \) is large enough, the value of \( G_{ij}^{-} \) which calculates by equation (6) is greater than or equal to zero, so the resistance value of equivalent branch is negative. As to 2), the value which calculates by equation (9) is greater than or equal to zero, calculated value \( G_{ij}^{-} \) is less than or equal to zero, so the resistance value of equivalent branch is positive.

The above content has analyzed the mechanism of negative resistance. The main reason of negative resistance production is that non-linear load, which pours into elimination node. When the active part of non-linear load is too large, it makes equivalent conductance greater than equivalent susceptance. And the original susceptance is greater than conductance relationship turns into conductance is greater than susceptance, which is leading to calculate value of resistance is negative.

3. Influencing factors of negative resistance
Non-linear load usually uses ZIP model which is composed of constant impedance load, constant current load and constant power load. The previous section analyzes the generation of negative resistance which is caused by equivalent impedance, so this section will thoroughly discusses the constant impedance load.

As to the constant impedance load, it can be divided into active load and reactive load. The reactive part of constant impedance load also can be divided into capacitive load and induction load. When reactive power is inductive load, the equivalent susceptance which calculates by equation (3) is negative. And it will lead to absolute value of \( B_{11} \) is increasing, which will lead to decrease the absolute value that calculates by equation (7). Thereby, it will reduce chances of the negative resistance. When reactive power is capacitive load, the equivalent susceptance which calculates by equation (3) is positive. And it will lead to absolute value of \( B_{11} \) is decreasing, which will lead to increase the absolute value that calculates by equation (7). Thereby, it will increase chances of negative resistance.

We can set that \( f_{za}, f_{ia}, f_{pa} \) are active load coefficient of constant impedance, constant current, constant power respectively. And assumed that \( g_{za}, g_{ia}, g_{pa} \) are reactive load coefficient of constant impedance, constant current, constant power respectively. It also has \( f_{za} + f_{ia} + f_{pa} = 1, g_{za} + \)
\( g_{pa} + g_{la} = 1 \). Therefore, when each elimination node has constant total non-linear power, we can have,

- **Reactive part of constant impedance load is capacitive.** When active coefficient \( f_{za} \) of constant impedance load is certain, we increase the value of reactive coefficient \( g_{za} \), that is increasing capacitive reactive power, so absolute value of \( B_{11} \) is increasing that leads to produce negative resistance easily, otherwise it can avoid negative resistance effectively. When reactive coefficient \( g_{za} \) of constant impedance load is certain, we can increase the value of active coefficient \( f_{za} \), that is increasing active power, so absolute value of \( G_{11} \) is increasing and it leads to absolute value which calculates by equation (7) is increasing, which will increases the probability of negative resistance, otherwise it can avoid negative resistance effectively.

- **Reactive part of constant impedance load is inductive.** When active coefficient \( f_{za} \) of constant impedance load is certain, we decrease the value of reactive coefficient \( g_{za} \), that is decreasing inductive reactive power, so absolute value of \( B_{11} \) is decreasing that leads to produce negative resistance easily, otherwise it can avoid negative resistance effectively. When reactive coefficient \( g_{za} \) of constant impedance load is certain, we increase the value of active coefficient \( f_{za} \), that is increasing active power, so absolute value of \( G_{11} \) is increasing and absolute value which calculates by equation (7) is increasing, which will increases the probability of negative resistance, otherwise it can avoid negative resistance effectively.

The above content analyzed the relationship between the characteristics of constant impedance reactive load, the value of load coefficient and the value of branch resistance. When we increase the value of active part of constant impedance load, the value of capacitive reactive part of constant impedance load or decrease the value of inductive reactive part of constant impedance load, these measures will increase the probability of the generation of negative resistance.

### 4. Negative resistance elimination method research

In an actual power system, each node has constant injection power, which inevitably results in negative resistance. The negative resistant of a branch determines the active loss of the branch which is a part of the network load flow and influences the voltage drop between the nodes. If the negative resistance is ignored before the load flow re-calculation, it will lead to the redistribution and even worse, none convergence of the power system load flow. Therefore, the transformation of simplified network is the only way to eliminate the negative resistance of branches. This paper presents a practical network transform method which can eliminate the negative resistance without affecting the calculation result of power system load flow.

Figure 1 is an equivalent branch after network simplification, and \( V_1 \angle \theta_1, V_2 \angle \theta_2 \) denotes the voltage magnitude and phase angle of bus 1 and bus 2, respectively. \( R \) and \( X \) are the negative resistance and the reactance of the branch.

![Figure 1. Relevant parameters of new line](image)

Then the series branch is transformed into a parallel branch which is given that the current flow through the branch is constant, thus, we have,

\[
R_1 = \frac{X^2 + R^2}{R}, \quad X_1 = \frac{X^2 + R^2}{X} \quad (10)
\]

After the transformation, though, the value of \( R_1 \) still remains negative. According to the current through the circuit is constant, \( R_1 \) can be transformed into two grounded branches. After the transformation, \( Z_2, Z_3 \) can be obtained as,
\[ \frac{V_1 \angle \theta_1 - 0}{Z_2} = \frac{V_1 \angle \theta_1 - V_2 \angle \theta_2}{R_1}, \quad \frac{0 - V_2 \angle \theta_2}{Z_3} = \frac{V_1 \angle \theta_1 - V_2 \angle \theta_2}{R_1} \] (11)

The resistance values \( R_2 \) and \( R_3 \) in \( Z_2, Z_3 \) are given by,
\[ R_2 = \text{real} \left( \frac{V_1 \angle \theta_1 \ast R_1}{V_1 \angle \theta_1 - V_2 \angle \theta_2} \right), \quad R_3 = \text{real} \left( \frac{-V_2 \angle \theta_2 \ast R_1}{V_1 \angle \theta_1 - V_2 \angle \theta_2} \right) \] (12)

For a long-distance transmission line, we can approximately have \( \theta_1 \approx \theta_2 \). Therefore either one of the values in equation (12) must be negative. Assumed that the value of \( R_2 \) is negative, the active loss produced by \( R_2 \) is equal to that by \( R_1 \). When the branch \( Z_2 \) is transformed into a parallel branch, we can have,
\[ R_4 = \frac{X_2^2 + R_2^2}{R_2} \] (13)

If the terminal voltages of the grounded branch are given as \( V_1 \angle \theta_1 \) and zero respectively and the resistance of ground branch is \( R_4 \), the active loss of this branch can be calculated as,
\[ \Delta P = \frac{V_1^2}{R_4} \] (14)

Due to the constant active loss from branch \( R_4 \), it can be equivalent as a power source of which the output power is \( \Delta P \). Then if this power source is merged with the nonlinear loads of node 1, resistance \( R_4 \) can thus be eliminated. The similar transformation is conducted on branch \( Z_3 \) and finally, we have a PI-type branch which only contains the reactance.

5. Case analysis
This paper uses IEEE 39 bus system and China Southern Power Grid analysis examples. For the IEEE 39 bus system, the network simplification is eliminating node 28 and producing branch 26-29. Constant impedance active load coefficient, constant current active load coefficient, constant power active load coefficient of node 28 are \( f_{za} = 1 \), \( f_{za} = 0 \), \( f_{pa} = 0 \) and Constant impedance reactive load coefficient, constant current reactive load coefficient, constant power reactive load coefficient of node 28 are \( g_{za} = 0.5 \), \( g_{pa} = 0.5 \), \( g_{za} = 0 \). We use transformation method which is proposed in this paper. Simulation experiments are using BPA software.

5.1. IEEE 39 bus system
Figure 2 is IEEE 39 bus system. And in the figure, node 28 which is PQ bus connects with node 26 and 29. To verify that the constant impedance load of node 28 impacts on resistant of equivalent branch 26-29, we can change non-linear load of node 28 and observe resistance of equivalent branch 26-29 after network simplification. It will change active load of node 28 and keep up reactive load constant. The load value of node 28 and the admittance value of branch 26-29 show in table 1. Initial non-linear load of node 28 is \( 206+j27.6 \text{MVA} \). Branch resistance reduces gradually when added active load of node 28. Branch produced negative resistance when the active load of node 28 was 1000MW and the absolute value of resistance gradually increases when continued to increase the active load. It will change the reactive load of node 28 and keep up active load constant. The load value of node 28 and the admittance value of branch 26-29 show in table 1. When we increase reactive load of node 28 and reactive load is transformed from inductive load into capacity load, branch resistance gradually increases. But the extent of resistance decrease is small. It means that reactive load is not the main cause of negative resistance production.
6. Conclusion

This paper discusses the mechanism that the equivalent branches produce negative resistance in simplification network. This paper also deeply studies what the nature and size of constant impedance load impact on the negative resistance value. Finally, this paper proposes a practical and efficient transformation method. This method is transforming the branch with negative resistance into grounded branch with negative resistance. Finally the grounded branches equivalent as a power source and then this source merges with the nonlinear loads of nodes. The correction of the mechanism analysis and the effectiveness of the transformation method are verified in sample cases form the IEEE 39 bus system and the China Southern Power Grid respectively.

References

[1] HE Gui-xiong, CHAO Qin, and TIAN Yi-zhi. Research on the dynamic equivalent parameter aggregation of fixed-speed wind turbines in wind farm [J]. Renewable Energy Resources, 2009, 27(1): 14-18, 22.

[2] LIU Li-xia, LUO Min, LI Xiao-hui. Comparison and Improvement of Common Methods of Dynamic Equivalence in Power System [J]. Proceedings of the CSU-EPSA, 2011, 23(1): 149-154.

[3] Hu Jie, YU Yi-xin. A Practical Method of Parameter Aggregation for Power System Dynamic Equivalence [J]. Power System Technology, 2006, 30(24): 26-30.
[4] Tinney W.F, Powell W L. The REI Approach to Power Network Equivalents. Proc of PICA Conference. 1997.

[5] DyLiacco T.E. et al. An On-Line Topological Equivalent of a Power System. IEEE Trans on Power Apparatus and Systems. 1978. PAS-97 (5): 1550-1563.

[6] Dopazo J.F, Irisarri G, Sasson A M. Real-time External System Equivalent for On-Line Contingency Analysis, IEEE Trans on Power Apparatus and Systems, 1979, PAS-98 (6): 2153-2171.

[7] HE Hong-yu, ZHENG Xue-ren. A Fully Differential Operational Amplifier with Negative Resistance Gain Enhancement [J]. Microelectronics and Computer, 2011, 28(2): 178-180.

[8] ZOU Xue-cheng, LU Li, ZHANG Cheng-long. Constant gain amplifier using negative-resistance technology [J]. Huazhong Univ. of Sci. and Tech, 2008, 36(6): 54-56.

[9] LI Juan, JI Yan-chao, MA Zhi-teng. The Discussion About Negative Resistance of the Trancking Equivalent Parameters for Power System [J]. Journal of Northeast China Institute of Electric Power Engineering, 2003, 23(1): 39-42.

[10] CHEN Shu-yong, WANG Cong, SHEN Hong. Dynamic Equivalence for Wind Farms Based on Clustering Algorithm [J]. Proceedings of the CSEE, 2012, 32(4): 11-19.

[11] MI Zeng-qiang, SU Xun-wen, YANG Qi-xun. Multi-Machine Representation Method for Dynamic Equivalent Model of Wind Farms [J]. Transactions of China Electrotechnical Society, 2010, 25(5): 162-169.