New and Simplified Manual Controls for Projection and Slice Tours, With Application to Exploring Classification Boundaries in High Dimensions

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ABSTRACT
This article describes new user controls for examining high-dimensional data using low-dimensional linear projections and slices. A user can interactively change the contribution of a given variable to a low-dimensional projection, which is useful for exploring the sensitivity of structure to particular variables. The user can also interactively shift the center of a slice, for example, to explore how structure changes in local subspaces. The Mathematica package as well as example notebooks are provided, which contain functions enabling the user to experiment with these new manual controls, with one specifically for exploring regions and boundaries produced by classification models. The advantage of Mathematica is its linear algebra capabilities and interactive cursor location controls. Some limited implementation has also been made available in the R package tourr. Supplementary materials for this article are available online.

1. Introduction

From a statistical perspective 3D is a rare data dimension, so unlike in most 3D rotation computer graphics applications, methods which work for arbitrary dimensions have more general utility. A good approach is to show projections from an arbitrary dimensional space to create dynamic data visualizations called tours. Tours involve views of high-dimensional data with low-dimensional projections. In his original paper on the grand tour, Asimov (1985) provided several algorithms for tour paths that could theoretically show the viewer the data from all sides. (Various videos of tours, e.g., Buja and Tukey (1987), Cook et al. (1993), on the ASA Statistical Graphics Section (2022), illustrate this concept.) Prior to Asimov’s work, there were numerous preparatory developments including Fisherkeller, Friedman, and Tukey’s (1974) PRIM-9. PRIM-9 had user-controlled rotations on coordinate axes, allowing one to manually tour through low-dimensional projections. (A video (Fisherkeller, Friedman, and Tukey 1973) illustrating the capabilities is available through the video library of ASA Statistical Graphics Section (2022).) Steering through all possible projections is impossible, unlike Asimov’s tours which allow one to quickly see many, many different projections. After Asimov, there have been many tour developments, which are summarized in Lee et al. (2022).

One such direction of work develops the ideas from PRIM-9, to provide manual control of a tour. Cook and Buja (1997) describe controls for 1D (or 2D) projections, respectively in a 2D (or 3D) manipulation space, allowing the user to select any variable axis, and rotate it into, out of, or around the projection through horizontal, vertical, oblique, radial or angular changes in value. Spyrison and Cook (2020) refined this algorithm and implemented it to generate radial tour animation sequences.

Manual controls are especially useful for assessing the sensitivity of structure to particular elements of the projection. There are many places where it is useful. In exploratory data analysis, where one sees clusters in a projection, one may ask whether some variables can be removed from the projection without affecting the clustering. For interpreting models, one can reduce or increase a variable’s contribution, to examine the variable importance. Having the user interact with a projection is extremely valuable for understanding high-dimensional data. However, these algorithms have two problems: (a) the preprocessing of creating a manipulation space overly complicates the algorithm, and (b) extending to higher dimensional control is difficult.

Another potentially useful manual control is to allow the user to choose the position of the center of a slice. The slice tour was introduced in L aa, Cook, and Valencia (2020). It operates by converting the projection plane into a slice, by removing or de-emphasizing points that are further than a fixed orthogonal distance from the plane. The projection plane is usually thought of as passing through the center of the data. Manual control would allow the user to change the position of the center point, by shifting it along a coordinate axis, while keeping the orientation of the projection plane fixed. The purpose would be to explore how or if the shape of the data, in the space orthogonal to the projection, changes as one gets away from the center. It would
also allow the user to interactively decide on the thickness of the slice.

This article explains the new manual controls for projection and slice tours. The next section describes the new algorithm for manual control, for both projections and slices. The use of these methods is illustrated to compare and contrast boundaries constructed by different classifiers. The software section describes a Mathematica (Wolfram Research, Inc. 2022) package that is used for the application and describes the interactive environment that would be desirable within R (R Core Team 2018) as new technology becomes available. The article is accompanied by an Appendix, supplementary materials with more details and adjustments to the manual controls, and three Mathematica notebooks that can be used to reproduce the applications.

2. How to Construct a Manual Tour

A manual tour allows the user to alter the coefficients of one (or more) variables, of \( p \), contributing to a \( d \)-dimensional projection. The initial ingredients are an orthonormal basis \( (A_{p \times d}) \) defining the projection of the data, and a variable id \( (m \in \{1, \ldots, p\}) \) specifying which coefficient will be changed: the \( m \)th row of \( A_{p \times d} \) which we denote by \( V_m \). A method to update the values of the component of the controlled variable \( V_m \) is then needed.

2.1. Existing Methods

The methods for updating component values in Cook and Buja (1997) (and used in Spyrison and Cook 2020) are prescribed primarily for a 2D projection, to take advantage of (then) newly developed 3D trackball controls made available for computer gaming. The first step was to construct a 3D manipulation space from a 2D projection. In this space, the coefficient of the controlled variable ranges between \(-1\) and \(1\). Movements of a cursor are recorded and converted into changes in the values of \( V_m \) thus, changing the displayed 2D projection. The algorithm also provided constraints to horizontal, vertical, radial, or angular motions only. The construction of the manipulation space overly complicates the manual controls, especially when considering possible techniques that will apply to arbitrary \( d \).

2.2. A New Simpler and Broadly Applicable Approach

The new approach emerged from experiments on the tour using linear algebra capabilities, and a relatively new interactive graphics interface, available in Mathematica. The components corresponding to \( V_m \) are directly controlled by cursor movement, which updates row \( m \) of \( A \). The updated matrix is then orthonormalized.

2.2.1. Algorithm

1. Provide \( A \) and \( m \). (Note that \( m \) could also be automatically chosen as the component that is closest to the cursor position.)

2. Change values in row \( m \), for example, if \( d = 2 \)

\[
A^* = [a_1^* \ a_2^*] = \begin{bmatrix}
    a_{11} & a_{12} \\
    \vdots & \vdots \\
    a_{m1} & a_{m2} \\
    \vdots & \vdots \\
    a_{p1} & a_{p2}
\end{bmatrix}.
\]

A large change in these values would correspond to making a large jump from the current projection. Small changes would correspond to tracking a cursor, making small jumps from the current projection.

3. Orthonormalize \( A^* \), using Gram-Schmidt.

   i. Normalize \( a_1^* \).

   ii. \( a_2^* = a_2^* - (a_2^* \cdot a_1^*)a_1^* \).

   iii. Normalize \( a_2^* \).

This algorithm will produce the changes to a projection as illustrated in Figure 1. The controlled variable, \( V_m \), corresponds to the dark line, and sequential changes to row \( m \) of \( A \) can be seen to roughly follow a specified position (dot). Changes in the other components happen as a result of the orthonormalization but are uncontrolled.

The algorithm will also allow for more than one variable to be controlled. This is more difficult to implement interactively, but is easy with scripting, as is available in the R implementation.

2.3. Refinements to Enforce Exact Position

The problem with the new simple method is that it is not faithful to the precise values for \( V_m \) because the orthonormalization will change them. Even though these changes are for the most part imperceptible, one may wish to avoid them and there are numerous ways that this can be enforced, a few are detailed in the Appendix, supplementary materials. These primarily differ in how the remaining variables are adjusted during orthonormalization.

Figure 1. Sequence of projections where the contribution of one variable is controlled (dark). The dot indicates the chosen values for the controlled variable, \( V_m \). The change is tracking the chosen value after unconstrained orthonormalization. It can be seen that the actual axis does not precisely match the chosen position, but it is close.
2.4. Manual Control for Slices

To better explore the space we combine the manual controls for the projection with manual controls for slicing. A slice is a section of the data that is defined by a projection, a center point that is anchoring it in the high-dimensional space, and the slice thickness $h$ (Laa, Cook, and Valencia 2020). A data point is inside the slice if its orthogonal distance from the projection plane (passing through the center point) is below the thickness $h$. This orthogonal distance is computed in terms of the component that is normal on the projection plane. For $x_i$ a $p$ dimensional data point and $c$ the center point (in the same $p$ dimensional space) we compute the orthogonal distance as

$$v_i^2 = ||x_i^\prime - c^\prime||^2 = x_i^2 + c^2 - 2x_i^\prime \cdot c^\prime,$$  \hspace{1cm} (1)

with $c^\prime = c - (c \cdot a_1)a_1 - (c \cdot a_2)a_2$, $x_i^\prime = x_i - (x_i \cdot a_1)a_1 - (x_i \cdot a_2)a_2$ and $a_{k}, k = 1,2(=d)$ denoting the columns of the projection matrix, $A = (a_1, a_2)$. Although not required, manual control of slices works more understandably if the data is first standardized.

2.4.1. Shifting the Center

A natural starting point is to place $c$ in the center of the data distribution, but shifting it away from the mean can provide additional insights. In the case of a single orthogonal direction on the projection plane we can pick a sequence of center points $c$ in steps along that direction to move the slice and fully cover the data space. This no longer works in higher-dimensional spaces, and we can think of picking one direction and shifting the slice along the component orthogonal to the projection plane.

2.4.2. Changing the Thickness

In addition, it is also useful to interactively change the slice thickness $h$ (also called the slice radius), in particular, to find the preferred value for exploring the input data. For guidance the estimates of the number of points inside the slice as a function of the original sample size $N$ and the number of dimensions $p$ from Laa et al. (2022) can be used: in case of a uniform distribution inside a sphere of radius $R$ a slice with thickness $h$ will contain $N_p$ points, with

$$N_p(h, p, R, N) = \frac{N}{2} \left( \frac{h}{R} \right)^{p-2} \left( p - (p - 2) \left( \frac{h}{R} \right)^2 \right).$$  \hspace{1cm} (2)

3. Software

The implementation of the manual tour as suggested here requires the visualization of the current projection in terms of an axis display (see Figure 1 for an example). This display should be able to track the mouse position and adjust the projection based on the user’s cursor position. The user can select one axis (corresponding to one variable) by clicking on it, and then adjust the position of that axis by dragging it. In practice, the closest axis will be selected, and the dragging results in tracking the mouse position which will update the projection in small steps, such that no further interpolation is required.

The interactions in the axis display need to be mapped back onto the current projection matrix, which will then be orthonormalized and redisplayed. A second, linked display shows the projected data in sync with the updates from the axis display.

In addition, we also want to be able to look at slices of the data and select slicing parameters interactively. Here we have implemented a switch to change between projection and slice view, a slider that adjusts the slice thickness, $h$ (named height in the Mathematica interface), and a numeric input to specify the center point $c$ explicitly. The display will update based on these input values, in particular, the view will jump to a new slice when $c$ is changed.

Within R, the natural implementation for an interactive interface would use shiny (Chang et al. 2023) with plotly (Sievert 2020) drawing. However, the plotly package is designed to add to ggplot2 (Wickham 2016) graphics, not to be a programmable interactive graphics system. It lacks the capability to implement the primary tool needed for a manual tour, which is the tracking of the cursor position in small increments. In theory, the historical locator() function of base R should provide the necessary capability, but its behavior is not consistent between graphical devices. The cursor position control is possible for 3D plots with rgl (Murdoch and Adler 2022) and plotly but not for the arbitrary dimension needed for tours. The level of interactivity would have been possible with the cranvas system (Xie, Hofmann, and Cheng 2014) or rJava (Urbanek 2022) but neither has a graphics toolkit that currently operates across platforms. What we have chosen to do is to use the interactivity of Mathematica to experiment, and to implement the resulting ideas as an animation in the tourr (Wickham et al. 2011) package. There is hope that better interactivity will be available for R soon, perhaps in javascript (e.g., Hart and Wang 2022; Harrison 2022), which might allow for the experimentation to develop new techniques.

3.1. Mathematica Package

The Mathematica implementation relies on the inbuilt function LocatorPane. This function creates a region on the screen where the position of the cursor is captured and then converted to input that updates the graphics functions, enabling the manual navigation of the tour. We then use the graphics function ListPlot embedded in dynamic objects via DynamicModule to produce the display. User interface objects, such as sliders, locator panes, and input fields, are used to provide user control of slice thickness, center, scale, and point size.

The examples presented in the application section below (and attached in notebook format in the supplementary material) all use our primary new function, SliceDynamic. This function typically accepts grouped data in the form of a matrix where the second last column details the name of the group and the last column details the group index (which can be one if there is only one group). After specifying the initial slice thickness and the slice range, the user is presented with an interactive display in which the control objects appear on the left and the slice visualization appears on the right. The user can change the orientation of the slice via the locator pane, which changes the projection matrix; a slider controls the slice thickness; and there is an input field or an interactive star glyph, to change the slice center. The user can also change the appearance of the plot by zooming into the center or changing the point size with the
sliders provided. It is worth noting that this zoom works best with data that has been centered and scaled. The projected data can be displayed to contrast it with the slice by ticking a box. The explicit projection matrix can also be displayed via another checkbox. This is especially useful when we wish to import a projection that was identified as interesting in the manual exploration into a later study, as shown in the applications below. More details about the implementation and usage instructions are given in the Appendix, supplementary materials.

3.2. Extensions to the R Package tourr

The R package tourr provides numerous types of tours, displayed as animations.

The new function, radial_tour() has been constructed that will generate a sequence of projections that will decrease the coordinates for $V_m$ to 0 and back to the original values. It is applicable for any projection dimension, $d$. The script interface has one advantage over an interactive interface, in that two or more variables can be controlled simultaneously.

Changing the slice center manually can be accomplished with the new function, manual_slice(). This changes the value of the center point ($c$) of the slice, along a selected variable axis. It slices from the center of the data out to one side, back to the center, out to the opposite side, and then back to the center.

4. Application

To illustrate the usefulness of the manual controls we use the 4D penguins data (Horst, Hill, and Gorman 2020) and look at classification models following Wickham, Cook, and Hofmann (2015). We will show how classification boundaries can be explored and better understood on projections and slices through 4D space. Figure 2 shows a scatterplot matrix of this data. There are four variables ($bl =$ bill_length_mm, $bd =$ bill_depth_mm, $fl =$ flipper_length_mm, $bm =$ body_mass_g) measuring the size of the penguins from three species (Adelie, Chinstrap, and Gentoo). The scatterplot matrix shows that the three species appear to be likely separable and that at least the Gentoo can be distinguished from the other two species when $bd$ is paired with $fl$ or $bm$. The steps for exploring boundaries in this example are as follows:

1. Build your classification model.
2. Predict the class for a dense grid of values covering the data space.
3. Examine projections, using a manual tour so that the contribution of any variable is controlled.
4. Slice through the center, to explore where the boundaries will likely meet.
5. Move the slice by changing the center in the direction of a single variable to explore the extent of a boundary for a single group relative to a variable.

4.1. Constructing the 4D Prediction Regions

We use the classify package (Wickham 2022) to generate predictions across the 4D cube spanned by the data, with two classification models: Linear Discriminant Analysis (LDA) and Random Forest (RF). Both the data and the model points in the grid are centered and scaled (standard deviation $= 1$).

4.2. Exploring Projections Manually

We start by exploring the projections of the model prediction. Figure 3 summarizes the process. Rotating manually the view, we can visualize the location of predictions for each of the three species, and also get a sense of the difference between the two models. To illustrate this difference, we have manually rotated the projection for the RF model (left plot) to identify a view that shows the nonlinear but block-type structure that is typical of this type of model. This particular projection ($A_1$) is exported so that it can later be used to show the LDA model (middle plot) and the actual data (right plot). What can be seen is the linearity of the LDA model, where the boundaries are linear and oblique to the variable axes. And, interestingly, this particular projection of the original data shows very distinct clusters of the three species. That means the obscuring of the boundaries between groups for both of the models is driven by what is happening in the orthogonal space to the plane of the selected projection.

This example can be reproduced with the code in penguins_exploring_manually.nb and a run-through is shown in the video at https://vimeo.com/747585410.

4.3. Slicing through the Center

We now continue the investigation by slicing orthogonally to the projection $A_1$. For both models we look at a thin ($h =$
Figure 3. A projection identified using the manual tour because it shows differences in classification boundaries induced by different models. An RF model produces blocky, rectangular boundaries, while the LDA model produces oblique linear boundaries. For comparison, in this projection, the data shows the three groups are almost perfectly separable.

Figure 4. Comparing slices from two different projections $A_1$ (top row) and $A_2$ (bottom row), to compare and contrast classification boundaries induced by two models RF and LDA. A thin slice through the center of the data is shown. The plot of the data shows that the Gentoo group (light gray/green) is less separated in $A_1$, and the rotation to $A_2$ produces a bigger gap. The boundaries between groups are clear in $A_2$ also. The LDA boundary is more oblique between Gentoo and Chinstrap than the RF, which is too close to the Chinstrap group. On the other hand between Adelie and Gentoo, the RF is better situated about halfway between the two groups. Interestingly, the RF boundaries, which can be highly nonlinear, albeit blocky, are relatively linear for this problem.

0.5) slice through the center, $c^0 = (0, 0, 0, 0)$. At first, we explore how changing the projection away from $A_1$ can help with understanding the boundary better. For our example, notice that $A_1$ does not contain any contribution from the second variable ($bd$), so we will first rotate this variable into the view.

Figure 4 shows snapshots of the exploration. The top row is the initial projection ($A_1$ and $c^0$), and the bottom row is a later projection containing more of $bd$ (new projection $A_2$ while keeping the center fixed at $c^0$). The columns correspond to the two models, with RF on the left and LDA on the right.

Both models have some overlap at the center for the thin slice based on $A_1$ (despite the small value for $h$). The second slice (based on $A_2$) mostly resolves this overlap and reveals the primary differences between the models. The boundary between Adelie and Chinstrap is very similar for both models. The boundary between Chinstrap and Gentoo is where they differ.

The RF is almost straight in this view, as something we might expect from a tree model where splitting occurs on single variables only. However, it is straight in a combination of the second and third variables ($bd$ and $fl$). By examining the scatterplot matrix, we can understand how this boundary was built: on each of $bd$ and $fl$, it is possible to cut on a value where most of the Gentoo penguins are different from the other two species for any tree, and likely only one is needed. Thus, the forest construction is providing a sample of trees that use one variable or the other variable to split, producing the blocky boundary on the combination of the two.

In contrast, the LDA boundary has an oblique split between the two and roughly divides the space into three similarly sized areas. It is almost like a textbook illustration of how LDA works for two variables (2D) where by assuming the clusters have equal variance-covariance, it places a boundary halfway between the group means.
Finally, to determine which model describes the boundaries better, compare them with the $A_2$ projection of the data (bottom row of Figure 4). Gentoo is more separated from the other two in this projection, and one can imagine that the trees in the forest have greedily grasped any one of many places to make a split to separate the group. It might be argued though the that RF boundary is cut too close to the Chinstrap species, and might lead to some unnecessary misclassification with new data. The LDA boundary is better placed for all species.

This example can be reproduced with the code in penguins_slicing_through_center.nb and a run-through is shown in the first part of the video at https://vimeo.com/747590472.

4.4. Shifting the Slice Center

We have seen that starting from $A_1$ using the manual controls to change the contribution of the second variable we could find a clear separation boundary indicating the relation between this variable (bill depth) and the Gentoo penguin species. Instead of rotating to a different projection, we might also change the view by moving the slice along one axis in the 4D space. Here we will continue our exploration of the dependence on $bd$ and move the slice defined by $A_1$ to either large positive or negative values ($c_{bd} = \pm 1.5$ after centering and scaling). Thus, we define $c_{\pm} = (0, c_{bd}, 0, 0)$. Here we will also look at slices of the observed data points, using a thicker slice ($h = 1.5$) to capture enough points in a given view.

We start with a comparison of the two models and the data distribution shifting to $c_{\pm}$, thus, the slice is localized toward high values of bill depth in the top row of Figure 5. The star glyph at the right indicates the slice center value, which shows that it is higher on $bd$ and at the middle value for the other variables. This slice center guide is as proposed in Hurley, O'Connell, and Domijan (2022), and more details can be found in the Appendix, supplementary materials. We can see that all three slices (the two models and the data) contain almost no points from the third class (Gentoo) and that the decision boundary between the two models is very similar.

A more interesting comparison is found for $c_{-}$, thus, the slice localized toward low values of bill depth, shown in the bottom row of Figure 5. (The star glyph at right indicates the slice center value, which shows that it is lower on $bd$ and at the middle value for the other variables.) The RF model (left) predicts all three species within this slice, with an interesting boundary for the third class (Gentoo). On the other hand, the LDA model (middle) predominantly predicts the third class within the slice, this appears to be enforced through the linear structure of the model. Looking finally at the thick slice through the data we see that there are primarily observations from this class within the slice we can conclude that the two models have filled in the “empty” space (where we do not have any training observations) in very different ways and according to what we might expect given the model structure.

Finally, it is interesting to compare the slice views to the projection of the models seen in Figure 3 to better understand how the boundaries change along the $bd$ direction and where the differences in the projections come from.

This example can be reproduced with the code in penguins_shifting_slice_center.nb and a run-through is shown in the second part of the video at https://vimeo.com/747590472.

5. Discussion

This short note has described new technology for manually interacting with tours. The manual control of projection coefficients is most useful for assessing variable importance to the perceived structure but can be generally used for steering a
Supplementary Materials

The source material and animated gifs for this article are available at https://github.com/uschiLaa/mmtour. The supplementary materials include:

- The Mathematica source code defining the new functions in mmtour.wl.
- Two Mathematica notebooks with the code used in the application.
- Appendix with additional details about manual controls, and Mathematica functions.
- Data files for the examples.
- R code to reproduce the example data, Figures 1 and 2 in the article and Figures 1 and 2 in the appendix.
- Animations illustrating the manual tour and slicing are also available at https://vimeo.com/747585410 and https://vimeo.com/747590472.

Acknowledgments

The article was written in rmarkdown (Xie, Allaire, and Grolemund 2018) using knitr (Xie 2015). The scatterplot matrix of the penguins’ data was produced by the ggally (Emerson et al. 2013) package built on ggplot2 (Wickham 2011). We thank the Institute of Statistics, BOKU, for their hospitality while part of this work was conducted.

Disclosure Statement

The authors report there are no competing interests to declare.

Funding

The authors gratefully acknowledge the support of the Australian Research Council and the ResearchFirst undergraduate research program at Monash University.

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