Evaluating the effect of immeasurable parameters of exoplanets on their habitability using latitudinal energy balance model

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Among different models for determining the habitable zone around a star, Latitudinal Energy Balance Model (LEBM) is very beneficial due to its parametricity, which keeps a good balance between complexity and simulation time. This flexibility makes it a good tool to assess the impact of physical parameters on the temperature and the habitability of a planet. Among different physical parameters of a planet, some of them, up until now, cannot be determined by any method, like the planet’s spin obliquity, diurnal period, ocean-land ratio, and pressure level. In this work, we apply this model to study the effect of these immeasurable parameters on the habitability of three exoplanets located in inner, outer and middle of their HZ, each served as a representation for their realm in HZ. Among the examined parameters, the impact of pressure is more straightforward. It has a nearly direct relation with temperature and also with the habitability in the case of a cold planet. The effect of other parameters is discussed with details. To quantify the effect of all these immeasurable parameters, we utilize a statistical interface, which provides us with the conditional probability on habitability status of each planet.

Keywords: Energy Balance Model, Astrobiology, Planetary systems, Exoplanet atmosphere, Habitable zone.

I. INTRODUCTION

The first confirmation of an exoplanet orbiting a main-sequence star was made in 1995 when a giant planet was found in a four-day orbit around the nearby star 51 Pegasi, named 51 Pegasi b afterward (Mayor et al., 1995). By the discovery of 51 Pegasi b, the field of exoplanet detection flourished and resulted in launching many projects, especially transit based ones. By now, over 2000 exoplanets have been detected by Kepler telescope and it will be increasing by other projects like TESS (Fischer et al., 2015).

Due to the key role of liquid water in the biochemistry of life on earth, habitable zone (HZ) is defined as spatial extent around a main-sequence star in which the planet can maintain liquid water on its surface in an extended period of time (Kasting et al., 1993). In addition to the above definition, which is called spatial HZ, there is another way of assessing the concept by using a climate model on large numbers of planets with different planetary orbital elements called orbital HZ (Forget, 2013). Energy Balance Model (EBM) is one of the above-mentioned climate models. After (Budyko et al., 1969) and (Sellers et al., 1969) Latitudinal EBM (LEBM) has been proved to be useful in climate science. After (Williams et al., 1997), (Franck et al., 2000) and (Gaidos et al., 2004) this model has been utilized in terrestrial exoplanets. Moreover, in (Spiegel et al., 2004) and (Vladilo et al., 2013) it was used for an earth-like planet with different rotational speed and various pressures respectively.

Some of the physical parameters, which affect habitability can be achieved by observation, like semi major axis and eccentricity. However, many parameters are still immeasurable to us. Among them are atmosphere pressure, diurnal period, spin obliquity, and ocean coverage.

In orbital HZ we are free to manipulate any parameter, which makes it a suitable tool to investigate situations like having different ocean coverages or different spin obliquities. It may cause the question that how changing the immeasurable parameters can change the habitable fraction of a planet and whether it can change the status of a planet from habitable to an inhabitable one or vice versa.

Moreover, the earth itself is not completely habitable based on LEBM, which can specify the importance of the habitability fraction of planets instead of jumping into a Boolean conclusion classifying the planets into habitable or inhabitable worlds.

This paper is organized as follows: in the section II we will introduce LEBM, the section III focuses on computational details and the section IV discusses the application of the model on real cases.

II. LATITUDINAL ENERGY BALANCE MODEL

Energy balance model relates the amount of energy receives and radiates by a planet to the energy that is stored in it; depending on assumptions for the planet to serves as a point-like, consisting of strips on its surface in which physical quantities are averaged on, or tiling the surface with small squares and including the effect of height, we end up with 0, 1, 2 and 3 Dimensional Energy Balance Model. Implementing higher dimensions costs more computational power and time. In order to balance between detail and computation time, LEBM has been chosen. One-dimensional or latitudinal energy bal-
The atmospheric heat capacity becomes:

\[ C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} [D(1 - x^2) \frac{\partial T}{\partial x}] \]

where \( C \) is the effective heat capacity, \( T \) is temperature, \( D \) is diffusion coefficient, which determines the efficacy of zonal exchange heat, \( x \) is related to the latitude such that \( \lambda = \sin^{-1} x \), \( I \) is Outgoing Long wave Radiation (OLR), \( S \) is insolation, and \( A \) represents the albedo of the layer. The \((1 - x^2)\) emerges due to the application of diffusion equation in spherical geometry.

### A. Model Parameters

Including the pressure, we follow the definitions of (Vladilo et al., 2013) for heat capacity, diffusion coefficient, albedo and OLR, which is as follows:

1. **Heat Capacity**

The atmosphere thermal inertia depends on the fraction of the planets surface, which is covered with ocean, land, and ice. Considering different levels of pressure, heat capacity of the atmosphere becomes:

\[ C_{atm} = \frac{c_p}{c_{p,o}} \left( \frac{P}{P_o} \right) C_{atm,o} \]  

where \( c_p \) and \( P \) are specific heat capacity and pressure of the atmosphere. Index \( o \) in this relation corresponds to the values for the earth. Heat capacity of the atmosphere, \( C_{atm,o} \) is set to be \( 10.1 \times 10^6 \text{J m}^{-2} \text{K}^{-1} \), and heat capacities of land \( C_l \), ocean \( C_o \), and ice \( C_i \) are defined as follows:

\[ C_l = 10^6 + C_{atm,o} \]
\[ C_o = 210 \times 10^6 + C_{atm,o} \]
\[ C_i = \begin{cases} 1.0 \times 10^6 + C_{atm} & \text{if } T < 263 \\ 43 \times 10^6 + C_{atm} & \text{if } 263 < T < 273 \end{cases} \]

And total heat capacity is equal to

\[ C = f_lC_l + f_o(1 - f_l)C_o + f_iC_i \]  

By assigning \( f_o \) we have

\[ f_l = 1 - f_o \]

And the fraction of the surface covered by ice is determined by

\[ f_i(T) = \max \{0, 1 - e^{-\frac{T-273}{10K}}\} \]  

2. **Diffusion Coefficient**

The diffusion coefficient \( D \) is defined such that a planet at 1 A.U. around a star of 1M⊙, with a rotational period of 1 day, will reproduce the average temperature profile measured on Earth. For a planet with different rotational period, \( D \) is proportional to rotational velocity \( \omega_o^{-2} \) and for different pressures, \( D \) is linearly proportional to the pressure. The diffusion coefficient is thus:

\[ D = \left( \frac{P}{P_o} \right) \left( \frac{c_p}{c_{p,o}} \right) \left( \frac{m}{m_o} \right)^2 \left( \frac{\omega}{\omega_o} \right)^{-2} D_o \]

where \( \omega \) is the rotational velocity of the planet and \( m \) is the mean molecular weight. Index \( o \) corresponds to the values for the earth.

3. **Albedo**

Albedo is defined based on the fraction of land, ocean, ice on land, ice on ocean, and the fraction of clouds on each of these surfaces.

\[ A = f_o \left\{ (1 - f_l) \left[ a_o(1 - f_{cw}) + a_i f_{cw} \right] + f_l \left[ a_{io}(1 - f_{ci}) + a_c f_{ci} \right] \right\} + f_i \left\{ (1 - f_i) \left[ a_i(1 - f_{ci}) + a_c f_{ci} \right] + f_l \left[ a_{il}(1 + f_{ci}) + a_c f_{ci} \right] \right\} \]

with parameters, which are defined as:

\[ a_o = \frac{0.026}{(1.1)^{1.7} + 0.065} + 0.15(\mu - 0.1)(\mu - 0.5)(\mu - 1.0) \]
\[ a_c = \max\{a_{io}, [\alpha + \beta Z]\} \]
\[ \alpha = -0.07, \beta = 8 \times 10^{-3}(\gamma)^{-1} \]
\[ a_{il} = 0.85, a_{io} = 0.62, a_i = 0.2 \]
\[ f_{cw} = 0.67, f_{cl} = 0.50, f_{ci} = 0.50 \]

Implementing this albedo instead of the temperature dependent relation in (Spiegel et al., 2009) prevents the globally frozen world when the simulation starts at Northern winter solstice or when the distance is changed from 1AU to 1.025 AU. In each configuration the planet freezes for no reason (figure 3, 4).
4. Outgoing Long wave Radiation

The Outgoing Long wave Radiation (OLR) is:

\[ I = \frac{\sigma T^4}{1 + 0.75 \tau_{IR}[T,P]} \quad (10) \]

Where \( \sigma \) is Boltzmann constant and \( \tau_{IR} \) is optical depth of the atmosphere, which for atmospheric pressures is defined as:

\[ \tau_{IR}(T,P) = 0.79 \left( \frac{T}{273} \right)^3 \quad (11) \]

Changing the pressure manifests itself in the optical depth, which governs the greenhouse effect. When the pressure has a value not equal to the atmospheric pressure, deriving an analytical function is not possible. For these cases, we utilized the procedure described in (Vladilo et al., 2013). The final result is a chart specifying \( \tau \) according to the pressure and temperature.

5. Incoming Solar Radiation

Assuming a simple main sequence scaling for the luminosity, the total averaged diurnal insolation becomes:

\[ S = \frac{q_o H}{\pi} \bar{\mu} = \frac{q_o}{\pi} (H \sin \lambda \sin \delta + \cos \lambda \cos \delta \sin H) \quad (12) \]

Which we have the bolometric flux received at a distance of 1 A.U. as:

\[ q_o = 1.36 \times 10^6 \left( \frac{M_*}{M_\odot} \right)^4 \text{ ersg}^{-1} \text{ cm}^{-2} \quad (13) \]

And the radian half-day length as:

\[ \cos H = -\tan \lambda \tan \delta \quad (14) \]

\( \delta \) is the solar declination, which is calculated as follows:

\[ \sin \delta = -\sin \delta_o \cos(\phi_p - \phi_{peri} - \phi_a) \quad (15) \]

Which \( \delta_o \) is the obliquity, \( \phi_p \) is the current orbital longitude of the planet, \( \phi_{peri} \) is the longitude of periastron, and \( \phi_a \) is the longitude of winter solstice relative to the longitude of periastron.

B. Habitability Function, Habitability Fraction

The habitability function of each latitude is defined as:

\[ h[\lambda,t] = \begin{cases} 1 & \text{if } 273 \leq T[\lambda,t] \leq T_{\text{boil}}(p_o) \\ 0 & \text{otherwise} \end{cases} \quad (16) \]

and the fraction of the potentially habitable surface at time \( t \) is:

\[ H_{\text{area}}[\lambda] = \frac{\int_{-\pi/2}^{\pi/2} h[\lambda,t] \cos \lambda d\lambda}{2} \quad (17) \]

III. SIMULATION PROCEDURE AND VALIDATION

Our goal is to study the impact of different immeasurable parameters of an exoplanet on its temperature and habitability. These parameters include pressure, the fraction of ocean and land, obliquity and diurnal period. Usually, these parameters are set to be equal to the earth parameters. However, it is just one case among numerous cases, which have different values. We want to study as many as possible cases to see how the planet habitability behaves with a change in those parameters. In order to solve equation [4], we used staggered grid in which variables are calculated at the center of the grid cells and their derivatives at the cells borders. The spatial resolution is 1.25° (145 grid points from north to south pole) and temporal resolution is adapted stable time step condition for diffusion equation:

\[ \delta t < \frac{(\Delta x)^2 C}{2D(1 - x^2)} \quad (18) \]

And the boundary condition is \( \frac{dT}{d\lambda} = 0 \) at poles. Since the model is zonally averaged, it only can be utilized in situations that diurnal period is small compared to orbital period (Forgan., 2013). To define habitability status we use the equation (17), which calculates the habitable fraction of the surface. When a steady state is reached, we calculate mean \( \bar{H} \) and the standard deviation \( \sigma_H \) of the habitability fraction. Finally, the habitability status of the planet is determined according to the Table [11] in which we classify the status of the planet as habitable, hot, snowball, or a transient one (Forgan., 2013).

At first, we should assess the validity of the model by applying it to an earth-sun like system, comparing its result with real earth data. In figure [14] the green line shows the average taken from the NCEP/NCAR global temperature data and the red one is our model simulation for an earth-like planet orbiting a sun-like star. It is in good match except in the South Pole (Kistler et al. 1999; Kalnay et al. 1996).
FIG. 1: (a) Comparing real mean data of the temperature in different latitudes of the earth with the simulation data of a planet in a sun-earth like system. (b) Temperature profile of simulated data for different latitudes for one hundred year simulation.

### TABLE I: Habitability status

| Condition | State                        |
|-----------|------------------------------|
| $H > 0.1 \text{ and } \sigma_H < 0.1H$ | Habitable Planet             |
| $H < 0.1 \text{ and } T > 373K \text{ for all seasons}$ | Hot Planet                   |
| $H > 0.1 \text{ and completely frozen}$ | Snowball Planet               |
| $H > 0.1 \text{ and } \sigma_H > 0.1H$ | Transient Planet             |

Figure 1b shows temperature profile in different latitudes for a hundred year simulation run.

The decreasing temperature from the equator to the poles and the effect of season changing are visible in these figures.

## IV. EXOPLANETS

To show the effect of immeasurable parameters on the habitability and temperature, we choose three planets $\tau$ Ceti e, Kepler-22b, and Kepler-62f among the lots of discovered exoplanets. These three planets according to the (Kopparapu et al., (2013) and (2014)) are respectively on inner, middle, and outer edge of optimistic habitable zone. Therefore, each one serves as a representation of other exoplanets located in the same approximate location in their star HZ.

The planet $\tau$ Ceti e on inner edge of optimistic HZ orbits around a star with mass 0.783±0.012 M$_\odot$, temperature $T$=5.344 ± 50 K, with a semi major axis of 0.552 AU, eccentricity of 0.05, and orbital period of 162.87 days (Feng et al., 2017). The planet Kepler-22b (designated as KOI-087.01) orbits within the optimistic HZ around the sun-like star Kepler 22 with mass 0.970 M$_\odot$ and effective temperature $T_{\text{eff}} = 5581 \pm 44K$. The planet is located on an orbit with semi-major axis of 0.849 AU, zero eccentricity, and orbital period of 286.89 days (Borucki et al., 2012). The planet Kepler-62f (KOI-701) is located on the outer edge of optimistic HZ with semi major axis of 0.718 AU, zero eccentricity and orbital period of 267 days (Borucki et al., 2013).

In simulation, we varied the spin obliquity (O.b.) to $(0.0, 22.5, 45.0, 67.5, 90.0)$ degree, diurnal period (T) to $(0.5, 0.75, 1.0, 1.25, 1.5)$ day, ocean fraction (O.F.) to $(0.1, 0.325, 0.55, 0.775, 1.0)$, pressure (P) to $(0.1, 0.4, 0.7, 1.0, 1.3)$ atm for $\tau$ Ceti e and Kepler-22b, and $(0.5, 1.5, 2.5, 3.5, 4.5)$ atm for Kepler-62f. The reason for choosing different values of pressure for Kepler-62f was that in the range of other planets’ pressures, there was not a notable change in temperature nor the habitability of Kepler-62f. Then, raising the pressure to the mentioned values showed its effect on temperature and habitability.

To present the effect of changing the parameters on the temperature and habitability, we monitor the changes at the end of the determined interval, fixing each parameter to its minimum and maximum and let our parameter of interest to vary in all of its values.

The effect of each parameter of interest on minimum, mean, and maximum temperature and habitability is generally non-linear. Thus, the interpretation is not straightforward. However, an overall description, tolerating a few deviation, is possible, which we explain it in the next sections.

### A. Changing Pressure

Increasing the pressure results in increasing diffusion. Therefore, heat transfers between zonal strips more effectively. The result is that the ice creation would reduce and it would lower the albedo which in turn increases...
the temperature. This effect is in accordance with the plots of figure 5. For a planet like τ Ceti e, due to its location on inner edge of HZ, it is prone to be hot. Therefore, increasing the pressure could decrease the habitability. However, in other two cases (Kepler-22b, Kepler-62f) what is observed is increasing the habitability, especially in the case of Kepler-62f, which the planet is inhabitable at low pressures.

B. Changing Ocean Fraction

According to equation 4, as the water has higher amount of heat capacity in comparison to the land, increasing the ocean fraction will increase the total heat capacity. The main effect of heat capacity is resisting against changing the temperature. In fact, it decreases the speed of such changing. However, if we give enough time to the planet to evolve, this property has no contribution in the final temperature profile of the planet.

In low spin obliquities, increasing the ocean fraction generally results in slightly increasing the temperature due to the capability of maintaining more heat. This temperature increment makes planets more habitable in two ways. First, Habitability fraction increases for Kepler-22b, located in the middle region of HZ (See figure 6). Second, in addition to slightly habitability fraction increment for τ Ceti e, its habitability status changes from transient to habitable in some cases. Look at figure 9 for these special cases.

When the obliquity is very high, increasing the ocean fraction decreases the difference between maximum and minimum temperatures resulting in a more even temperature profile. In these cases, because of the special configuration of the planet in its orbit, the ice belts created on the equator are weaker than ice caps in the case of zero obliquity. The reason is that while the star height is always zero in zero obliquity case, it changes from zero to 90 degree with a 12 hours day cycle in the case of 90 degree obliquity. When the planet is semi-habitable, meaning having a habitability fraction between zero and one, this ocean fraction increment leads to an increase in the habitability of the planet.

C. Changing Spin Obliquity

When spin obliquity is zero, high latitudes receive negligible amount of solar energy. Therefore, ice poles are created and spread into other longitudes until the received insolation compensates the negative effect of albedo and OLR on the stored energy.

By increasing the spin obliquity, the poles receive more insolation, which in turn decreases the area of ice caps. In the median obliquities around 45 degree, as all latitudes receive moderate amount of insolation, the temperature profile experience less diversity.

When this increment reaches to more radical values like 90 degree, ice belt are created on the equator since they are the locations with the least insolation. Moreover, an increment in the maximum temperature is expected because of the same reason mentioned in the previous subsection; ice belt on the equator are weaker than ice caps on the poles, since they receive more insolation yearly in high obliquity planets in comparison to poles in low obliquity planets (See figure 7). Habitability fraction follows a more complicated process. When the planet has relatively moderate or high temperature, making the temperature profile to a more uniform one by increasing the obliquity, cause the habitability to increases; since there are more regions with a moderate temperature. Reaching more radical values of obliquity makes temperature profile again more diverse with a higher density of frozen and boiling regions. Therefore, habitability decreases.

When the planet has a relatively low temperature, a reverse process occurs. In low obliquities, parts of the planet, which receive more insolation are habitable. When spin obliquity increases, it again makes the temperature profile more uniform. This uniformity results in lowering temperature in regions with above zero temperature and raising temperature in frozen regions. However, as the insolation is not high enough, the overall effect is turning more regions to frozen than turning others to habitable. Therefore, the habitability decreases. In high obliquities, a similar state of low obliquity occurs; there are frozen parts with high insolation, which starts to melt. These areas are responsible for increasing the habitability.

There are some cases, which changing obliquity has no significant effect on habitability. These cases might be completely frozen in any spin degree or the dominant parameter is a parameter other than obliquity like pressure or diurnal period. Moreover, changing the status of some cases from transient to habitable or vice versa is considerable in figures 9 and 10. By increasing obliquity, planet τ Ceti e experiences transformation from habitable to transient in low pressures and vice versa in high pressures and the planet Kepler-22b experiences transformation from habitable to transient in moderate pressures.

D. Changing the Diurnal Period

According to the aforementioned debate, due to the limitation of the model, the diurnal period should be small enough with respect to the orbital period. According to equation 7, increasing the rotational velocity results in decreasing the diffusion coefficient, which means heat cannot transfer effectively between different latitudes. In fact, we expect that the main impact of the diurnal period should be changing the latitudinal temperature profile in which the difference between the maximum and minimum temperatures decreases. However, the effect of changing this parameter on mean tempera-
ture is intangible.

Since the main impact of changing this parameter is on the maximum and minimum temperature, what is expected is that if there is an effect on habitability, that effect would be increasing the habitability for making too hot regions colder and too cold regions warmer, which is in accordance with the figure 8.

![FIG. 2: Increasing the diurnal period results in a more homogeneous temperature distribution](image)

V. STATISTICAL INTERFACE TO THE IMMEASURABLE PARAMETERS

Up until now, we investigated the signature of the mentioned immeasurable parameters on the habitability of an exoplanet. As we see, they have a variety effects on the temperature and habitability and therefore should be taken into account carefully. The problem with these parameters is that we only know a possible range of them and not their exact values. Therefore, quantifying our lack of knowledge using a statistical approach is inevitable.

With the values for the parameters introduced in section IV, the simulation’s phase space contains 625 points. To have a better perception of how the temperature and habitability of a planet vary when a selected parameter or a combination of parameters changes, we encapsulated the result of all of the diagrams in one figure called “Map of parameters”. There are three of these figures (figure 9 to 11), each for one of the three planets. In these figures, by moving to the right, the pressure increases in each little square and the obliquity increases by jumping between the large squares. Similarly, by moving to the top, the diurnal period increases in each little square and the ocean fraction increases by jumping between the large squares. The habitability status and mean temperature of each case could be determined using the guidance next to each figure.

It is understandable from these maps that a planet could have different states based on the value of its four immeasurable parameters. Therefore, setting these values to those of the earth do not give us an accurate understanding of the habitability of an exoplanet. For instance, figure 10 shows that 333 out of 625 cases (slightly more than 50 percent) have a habitability more than 0.4. However, it is still an inaccurate inference to say that the planet is likely to be habitable rather than inhabitable.

To access this issue more accurately, we need to meticulously determine our phase space. For the spin obliquity, all the possible values are covered in the simulation. Ocean fraction takes the values from zero to unit amount with the exception of 0.0 since the planet with no water is inhabitable in our assumption of habitability. Pressure 0.0 atm indicates no atmosphere on the planet and should be excluded from the possible range. The upper-limit of 1,100 atm for the pressure is a good choice based on the endurance of extremophiles in extreme environment (Stan-Lotter, 2007). Imposing a condition that desired outcome resembles to life on earth we dont consider as wide range as possible and set the upper-limit of pressure to 1.0 atm. Diurnal period depends on many parameters and has no apparent bounds. However, based on the escaping velocity, rotational speed of the planet imposes a lower limit of 0.1 hours to the diurnal period (Miguel & Brunini, 2010). The upper-limit could not be bounded. However, given enough time all planets end up to be tidally locked (Gladman, B., 1996). Using the probability distribution for primordial diurnal period introduced in (Miguel & Brunini, 2010) we can discuss the habitability probability with the help of conditional prob-
ability, given that the probability of diurnal period be in a specific range, which is the only way of reconciliation of probability scheme and LEBM inherent limitation on diurnal period.

Considering above discussion, we set the range (0.0 - 1.0) atm for pressure, (0.0 - 1.0) for ocean fraction, [0.0 - 90.0] degrees for spin obliquity, and (0.0042 - 1.5) days for diurnal period. Using (Miguel & Brunini, 2010) probability of diurnal period being the mentioned range can be calculated and is equal to 0.1226.

Since the parameters are continuous, to calculate the probabilities we need the area of the region, which gives us the desired outcome. Running the simulation for every and each point is not possible, thus, to overcome the situation we use Monte Carlo method. We generate many random coordinates in the domain of possible values. Then, we use a weighted average according to the simulated point to estimate the habitable fraction and habitable status.

To estimate those values in the 4D space of the parameters by using a weighted average, first we determine the nearest neighbor simulated coordinate and the 4D cube in which the random value is located. Then, we calculate the average according to this equation:

\[
\text{avg}(v) = \frac{\sum_{xyzw} v_{xyzw} e^{-\alpha d_{xyzw}}}{\sum_{xyzw} e^{-\alpha d_{xyzw}}}
\] (19)

where \(v_{xyzw}\) is the simulated value for each corner of the 4D cube and \(d_{xyzw}\) is the normalized-to-step-size euclidean distance from each nearest neighbors:

\[
d_{xyzw} = \left( \left( \frac{P - P_{\text{min}}}{P_{\text{max}} - P_{\text{min}}} \right)^2 + \left( \frac{O.F. - O.F_{\text{min}}}{O.F_{\text{max}} - O.F_{\text{min}}} \right)^2 \right) + \left( \frac{O.b. - O.b_{\text{min}}}{O.b_{\text{max}} - O.b_{\text{min}}} \right)^2 + \left( \frac{T - T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}} \right)^2 \right)^{\frac{1}{2}}
\] (20)

and coefficient \(\alpha\) is set to be the neperian logarithm of \(2.220446 \times 10^{-16}\) (the smallest positive number in floating-point representation with double precision).

With this value for distance equal to edge of cube, the exponential value becomes technically zero in calculation and so if the random value is located on the vertices’s, the average gives the simulated value.

Habitability fraction is a continuous variable, which we can estimate its value using the equation 19. Habitable status, on the other hand, is a categorical variable, which to average it, we assign +1, 0, and -1 respectively for habitable, transient and inhabitable. The averaged value will be rounded to the nearest category number.

### VI. CONCLUSION

We applied latitudinal energy balance model (LEBM) to study the effect of immeasurable parameters of an exoplanet, which cannot be obtained by observation. These parameters include surface pressure (P), ocean fraction (O.F.), spin obliquity (O.b.), and diurnal period (T). Starting with the diffusion equation, we tested its reliability by applying it to the earth data. Comparing the result of our simulation with the data of NCEP/NCAR global temperature data shows that the model, despite its simplicity, is precise enough to make it suitable for our survey. The main improvement of our simulation compared with (Foragn 2013 and Spiegel 2009) is defining it based on (Vladilo 2013). When the albedo is defined by relating it to only temperature, a global freezing problem, which is dependent on the starting point of the planet in its orbit occurs. For instance, by increasing the semi major axis of the earth to 1.025 AU, based on the defined starting point, the whole surface of the earth could turn into ice. This problem is solved when the equation 5 which uses the albedo of land, ocean, ice, and cloud portions on each of those surfaces is applied. To study the impact of each parameter on the temperature and habitability, three maps of parameters for each exoplanet consisting 625 different sets of parameters were formed. The effect of a parameter in particular is shown in four different sets of parameters extracted from the maps in which one parameter is free, changing from a minimum to a maximum value. These effects are shown in figures 5 to 8.

The pressure has a strong nearly linear impact on temperature. It is the dominant parameter in the examined range of our simulations. When it is increased, the temperature of the planet is also increased. Habitability, on the other hand, could undergo different scenarios. Depending on the mean temperature of the planet at the starting point, it could increase when more frozen regions start to melt or decrease when more regions go beyond

| Exoplanet | Habitable fraction | Transient fraction | Inhabitable fraction |
|-----------|-------------------|-------------------|---------------------|
| Kepler-22b | 0.836             | 0.408             | 0.0                 |
| Kepler-62f | 0.121             | 0.419             | 0.0                 |
| Ceti e    | 0.043             | 0.173             | 1.0                 |

There are 30000 generated random points, which is high enough such that repeating the Monte Carlo process changes the value of the outcomes less than 0.1%. The results for our three surveyed planets are summarized in table II. To calculate the probability of habitability we need just to multiply the probability of diurnal period being in the range of (0.0042 - 1.5) which equal to 0.1226.

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1 parentheses for not including and brackets for including the initial or final value of the intervals.
the boiling point of water.

Since the ocean has a much higher thermal inertia, the effect of increasing its ratio is to increase the total heat capacity of the surface. In a short period, it prevents abrupt changes in the temperature, however, in a long term this feature does not play a role as the planet reaches to its stable condition. The other impact of increasing the ocean fraction is reducing the difference between maximum and minimum temperatures. These effects make the planet more habitable by increasing the habitability fraction or changing the status of the planet from transient to habitable.

Spin obliquity is changed from 0 to 90 degrees. In low obliquities, ice caps are created and therefore the difference between maximum and minimum temperatures is high. As the obliquity increases, ice caps start to melt and temperatures approach each other. In very high obliquities, which equators receive the least insolation, ice belts are formed on the equator and thus the difference between maximum and minimum temperatures starts to increase.

The main impact of increasing diurnal period is a better heat transfer between latitudes which in turn decreases the difference between maximum and minimum temperature of the planet. Diurnal period does not have a noticeable effect on the mean temperature and so the habitability does not change under varying this parameter.

As we covered three exoplanets located in different locations of HZ, it is expected to obtain nearly same results for other exoplanets. However, repeating the procedure for each individual exoplanet would give us the exact result of the temperature and habitability.

A better understanding of the changing of immeasurable parameters simultaneously could be achieved by probing Map of Parameters for each planet in figures 9 and 11. To make and statistical inference from this maps we need to have the area of the phase space corresponds to our desired outcome, utilizing a Monte Carlo process, we created large number of points in the phase space and estimate the value of each point with a weighted averaging method, this approach is inevitable, since the random value is continuous and it not possible to run simulation for every and each point. As mentioned in the section VII there is no upper-bound for the diurnal period. Thus, we utilized conditional probability using (Miguel & Brunini, 2010). We set the probability of diurnal period to be in the range of (0.0042 - 1.5] days, and calculated the conditional probability of habitability of the planet, having the ranges of (0.0 - 1.0] atm for pressure, (0.0 - 1.0] for ocean fraction, [0.0 - 90.0] degrees for spin obliquity, given that the diurnal period is in the aforementioned range. To calculate the probability desired outcome we need just multiply the conditional probability to the probability of diurnal period being in that specific range.

VII. REFERENCES

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FIG. 3: Changing the distance of the earth to sun from 1 A.U. to 1.025 A.U. in an sun-earth like system using LEBM. 
Top row: Global freezing using albedo in (Spiegel et al., 2009). 
Bottom row: No global freezing happened using albedo in equation 8
FIG. 4: Dependence to initial orbital position
Top row: Global freezing occurred when the simulation started at northern solstice using albedo in (Spiegel et al., 2009)
Bottom row: No global freezing happened using albedo in equation 8
(a) O.F. = 0.1, T = 0.5, O.b. = 90.0, Pressure = varies

(b) O.F. = 0.1, T = 0.5, O.b. = 0.0, Pressure = varies

(c) O.F. = 0.1, T = 0.5, O.b. = 0.0, Pressure = varies

(d) O.F. = 0.9, T = 0.5, O.b. = 0.0, Pressure = varies

FIG. 5: Pressure varies; for planet τ Ceti e (tCe), Kepler-22b (k22b), p = (0.1, 0.4, 0.7, 1.0, 1.3) atm; and for Kepler-62f (k62f), p = (0.5, 1.5, 2.5, 3.5, 4.5) atm.
FIG. 6: Ocean fraction varies; for planet τ Ceti e (tCe), Kepler-22b (k22b), $p_1 = 0.1$, $p_5 = 1.3$; and for Kepler-62f (k62f) $p_1 = 0.5$, $p_5 = 4.5$
FIG. 7: Spin obliquity varies; for planet τ Ceti e (tCe), Kepler-22b (k22b) $p_1 = 0.1$, $p_5 = 1.3$; and for Kepler-62f (k62f) $p_1 = 0.5$, $p_5 = 4.5$
FIG. 8: Diurnal period varies; for planet τ Ceti e (tCe), Kepler-22b (k22b) $p_1 = 0.1$, $p_5 = 1.3$; and for Kepler-62f (k62f) $p_1 = 0.5$, $p_5 = 4.5$
FIG. 9: τ Ceti e parameters map - All parameters are in an increasing order. Pressure values are (0.1, 0.4, 0.7, 1, 1.3) atm, diurnal periods are (0.5, 0.75, 1.0, 1.25, 1.5) days, obliquity values are (0.0, 22.5, 45.0, 67.5, 90.0) degree, and ocean fractions are (0.1, 0.325, 0.55, 0.775, 1.0). The right column shows the color chart of temperature.
FIG. 10: Kepler-22b parameters map - All parameters are in an increasing order. Pressure values are (0.1, 0.4, 0.7, 1, 1.3), diurnal periods are (0.5, 0.75, 1.0, 1.25, 1.5), obliquity values are (0.0, 22.5, 45.0, 67.5, 90.0) degree, and ocean fractions are (0.1, 0.325, 0.55, 0.775, 1.0). The right column shows the color chart of temperature.

| $Ob_1$ | $Ob_2$ | $Ob_3$ | $Ob_4$ | $Ob_5$ |
|--------|--------|--------|--------|--------|
| $P_1$  | $P_2$  | $P_3$  | $P_4$  | $P_5$  |
| $P_1$  | $P_2$  | $P_3$  | $P_4$  | $P_5$  |
| $P_1$  | $P_2$  | $P_3$  | $P_4$  | $P_5$  |
| $P_1$  | $P_2$  | $P_3$  | $P_4$  | $P_5$  |
| $P_1$  | $P_2$  | $P_3$  | $P_4$  | $P_5$  |

Habitability fraction

| N | H | T |
|---|---|---|
| N | Non-habitable |
| H | Habitable |
| T | Transient |

The table and diagram illustrate various parameters and their ranges for Kepler-22b, with the color chart indicating temperature variations.
FIG. 11: Kepler-62f parameters map - All parameters are in an increasing order. Pressure values are (0.5, 1.5, 2.5, 3.5, 4.5) atm, diurnal periods are (0.5, 0.75, 1.0, 1.25, 1.5) days, obliquity values are (0.0, 22.5, 45.0, 67.5, 90.0) degree, and ocean fractions are (0.1, 0.325, 0.55, 0.775, 1.0). The right column shows the color chart of temperature.