Particle physics with a laser-driven positronium atom

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A detailed quantum-electrodynamical calculation of muon pair creation in laser-driven electron-positron collisions is presented. The colliding particles stem from a positronium atom exposed to a superintense laser wave of linear polarization, which allows for high luminosity. The threshold laser intensity of this high-energy reaction amounts to a few $10^{22}$ W/cm$^2$ in the near-infrared frequency range. The muons produced form an ultrarelativistic, strongly collimated beam, which is explicable in terms of a classical simple-man’s model. Our results indicate that the process can be observed at high positronium densities with the help of present-day laser technology.

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Laser fields can pave new ways to high-energy physics [1–4]. By laser-matter interactions in the MeV regime, fundamental nuclear physics processes were realized in experiment [5] and new laser-induced nuclear phenomena have been predicted theoretically [6]. While inside the most powerful laser fields presently available [1], electrons temporarily acquire ponderomotive energies in the GeV range. Relativistic rectification techniques of the laser field based on laser-plasma interaction enable real particle acceleration and extraction of monoenergetic electron (ion) beams up to GeV (MeV) energies [7]. However, the electron’s temporary energy gain in the laser beam can directly be exploited without any rectification technique when particle collisions followed by particle reactions happen inside the laser beam [3, 8].

A further advantage of laser fields can be utilized when the interaction with single atoms rather than a plasma is considered. Then, apart from the high energies achievable, lasers can be used to generate well-controlled coherent collisions of the atomic constituent particles at microscopic impact parameters, which can lead to enormous luminosities [8, 9]. An atomic species of particular interest in this context is positronium (Ps), the bound state of an electron and a positron, since it exhibits unique dynamic properties in a strong laser field [10]: After instantaneous ionization the leptons oscillate in opposite directions along the laser electric field, which leads to periodic $e^+e^-$ collisions since both particles experience an identical ponderomotive drift motion due to their equal charge-to-mass ratios. Note that in ordinary atoms, the magnetically induced drift in the laser propagation direction suppresses recollisions at high intensities [2].

In the following we consider laser-induced muon pair creation from electron-positron annihilation (see Fig. 1). In contrast to the usual setup of colliding particle beams, however, we assume that an initially quiescent Ps atom is exposed to a superintense laser wave of linear polarization and the muons are produced in the laser-driven $e^+e^-$ collisions described above. The corresponding reaction rate is evaluated within the framework of laser-dressed quantum electrodynamics by employing relativistic Volkov states [11]. We show that the minimal laser intensity to ignite the process is determined by the relation $eA \geq M c^2$, with electron charge $-e$, muon rest energy $Me^2 \approx 106$ MeV, and amplitude of the laser’s vector potential $A$, as the collision energy comes from the transversal motion. For a laser wavelength $\lambda = 1$ μm, the threshold intensity amounts to $5.5 \times 10^{22}$ W/cm$^2$ which is almost reached by present most powerful laser systems [12]. Such a superintense laser wave supplies the electron-positron system with an energy of $2Mc^2$ in their center-of-mass (c.m.) frame, which coincides with the energetic threshold for muon production in the field-free case. Note that the required intensity thus lies several orders of magnitude below the critical value of $2.3 \times 10^{29}$ W/cm$^2$ where $e^+e^-$ pair creation from vacuum and other vacuum nonlinearities in laser fields are expected to appear [1, 2].

The rates for the laser-induced decay $Ps \rightarrow \mu^+\mu^-$ are shown to render experimental observation feasible at high Ps density ($\sim 10^{18}$ cm$^{-3}$ [13]) and laser repetition rate ($\sim$ Hz [4]).

Lepton-lepton interactions in a laser field have been studied in detail by the examples of laser-assisted $e^-e^-$ (Møller) and $e^+e^-$ (Bhabha) scattering [14]. In these cases the laser wave served as a background field which modified the field-free properties of the scattering reaction. Contrary to that, in the present scenario the laser wave is playing a vital role in initiating a process that would not take place in the absence of the laser field.

![Feynman graph for muon pair creation from electron-positron annihilation in a laser field. The arrows are labeled by the particle’s free momenta ($p_{\pm}$, $P_{\pm}$) outside and the effective momenta ($q_{\pm}$, $Q_{\pm}$) inside the laser field.](image-url)
Within a strong-field approximation, the amplitude for $e^+e^- \rightarrow \mu^+\mu^-$ from a laser-driven Ps atom can be written as a superposition integral [15]

$$S_{\text{Ps}} = \frac{1}{\sqrt{V}} \int \frac{d^3p}{(2\pi)^3} \tilde{\Phi}(p) S_{e^+e^-}$$  \hspace{1cm} (1)

with a normalization volume $V$ and the Fourier transform $\tilde{\Phi}(p)$ of the Ps ground-state wave-function which depends on the relative momentum $p$ of the $e^+e^-$ two-body system. $\Phi(p)$ represents the probability amplitude for finding an exciton of momentum $p_+ = p$ and, correspondingly, an electron of momentum $p_- = -p$ within the Ps ground-state wave-packet. Furthermore,

$$S_{e^+e^-} = -i\alpha \int d^4x d^4y \tilde{\Psi}_{p_+,s_+}(x)\gamma^\mu \Psi_{p_-,s_-}(x)$$

$$\times D_{\mu\nu}(x-y)\tilde{\phi}_{p_-,s_-}(y)\gamma^\nu \Psi_{p_+,s_+}(y)$$  \hspace{1cm} (2)

denotes the amplitude for the process $e^+e^- \rightarrow \mu^+\mu^-$ in a laser wave (cf. Fig. 1). Note that we use relativistic units with $\hbar = c = 1$. In Eq. (2), $\alpha$ is the fine structure constant, $\Psi_{p_+,s_+}$ ($\Psi_{p_-,s_-}$) are the Volkov states [11] of the $e^\pm$ ($\mu^\pm$) depending on the free four-momenta $p_\pm$ ($P_\pm$) and spin states $s_\pm$ ($S_\pm$) of the particles, and $D_{\mu\nu}$ is the coordinate-space representation of the free photon propagator. The integrations in Eq. (2) are performed in the usual way by Fourier series expansion where the Fourier coefficients are given by generalized Bessel functions [16]. At the electron-positron vertex they read $J_n(u,v)$, with the index $n$ corresponding to the number of laser photons absorbed or emitted, and the arguments given by

$$u = \frac{eA(p_-)}{(kp_-)} - \frac{eA(p_+)}{(kp_+)}; \quad v = -\frac{e^2A^2}{8} \left[ \frac{1}{(kp_-)} + \frac{1}{(kp_+)} \right].$$  \hspace{1cm} (3)

Here, $k^\mu = (\omega(1,0,0,1), A^\mu(x) = e^{\mu} A \cos(kx)$ with $\mu = (0,1,0,0)$, and $\xi = eA/(mv^2)$ are the laser’s wave four-vector, four-potential, and normalized vector potential, respectively. Note that $v \approx v_0 \equiv -m\xi^2/(2\omega)$ is practically constant for the relevant values of $|p| \sim m\alpha$. Similar generalized Bessel functions $J_N(U,V)$ appear at the muon-antimuon vertex, where $U, V$ are given by Eq. (3) with the corresponding momenta of muon and antimuon. Due to the properties of the Bessel functions, the typical photon number is $n \approx -m\xi^2/\omega$ so that the intermediate photon four-momentum satisfies

$$q^2 = (p_+ + q_+ - n\epsilon k)^2 \approx 4m^2 - 4\omega \epsilon n \approx 8m^2\xi^2,$$  \hspace{1cm} (4)

implying that the process proceeds nonresonantly (i.e., $q^2 \neq 0$) [14]. Here, $q_\pm$ denotes the effective $e^\pm$ momenta inside the laser field and $m_\epsilon = m(1 + \xi^2)^{1/2}$ is the laser-dressed mass [11]. By virtue of an addition theorem for Bessel functions [16], Eq. (2) adopts the structure

$$S_{e^+e^-} = -i\alpha \sum_{r \geq r_{\text{min}}} J_n(U-u,V-v) T_r$$

$$\times \delta(q_+ + q_- - Q_+ - Q_- + rk)$$  \hspace{1cm} (5)

where $r = N - n$ is the total number of absorbed laser photons, $r_{\text{min}} = (M^2 - m^2)/(\omega m)$, and $T_r$ are rather complicated functions of the particle momenta and laser parameters. The main $p$ dependence in Eq. (1) is contained in oscillatorily damped integrals of the form

$$I_r = \int \frac{d^3p}{(2\pi)^3} \tilde{\Phi}(p)J_r(U-u,V-v)$$

$$\approx \frac{J_r(U,\Delta V)}{\sqrt{(2\pi)^3|x_0^6/\alpha^2}} \left( \frac{\omega}{m\alpha^2\xi} \right)^{2/3} \frac{2^6(\xi\Delta V)^{1/3}}{\alpha^{2/3}|v_0|^2},$$  \hspace{1cm} (6)

with $\Delta V = V - v_0$ and the Ps Bohr radius $a_0$. The remaining $p$ dependence of $S_{e^+e^-}$, neglected in Eq. (6), is very weak because of the nonrelativistic momenta contained in the Ps ground state [15]. According to the factor $[\omega/(m\alpha^2\xi)]^{2/3} \sim [a_0/(\alpha\xi)]^{2/3} \ll 1$ in Eq. (6), the average over the Ps momentum distribution leads to destructive interference of the partial waves constituting the bound state, which can equivalently be described as wave packet spreading [cf. Eq. (12) below].

The total rate for laser-driven Ps decay into muons is found from the square of the amplitude (1), averaged and summed over the particle spins and integrated over the outgoing muon momenta:

$$R_{\text{Ps}} \approx \frac{\alpha^2}{2^6 m^4\xi^4} \int \frac{d^3P_+}{(2\pi)^3} \int \frac{d^3P_-}{(2\pi)^3} \sum_{s_+,s_-} \sum_{r \geq r_{\text{min}}} |I_r|T_r|^2$$

$$\times \delta(q_+ + q_- - Q_+ - Q_- + rk).$$  \hspace{1cm} (7)

Via the energy-momentum conserving $\delta$-function in Eq. (7), the muon kinematics can be analyzed. One finds that the laser intensity parameter needs to satisfy

$$\xi \geq \xi_{\text{min}} = \frac{M}{m\sqrt{2}} \approx 150.$$  \hspace{1cm} (8)

It is remarkable that Eq. (8) agrees with the naive estimate of the threshold intensity given above, although the muons have to be created with their laser-dressed mass $M_\epsilon = M(1 + (m\xi/M)^2)^{1/2}$ which is significantly larger than their bare mass $M$. The energetic difference $\Delta M^2 \equiv 2(M_\epsilon^2 - M^2)$ is supplied by the laser photons absorbed at the scattering event. Close to threshold, the muons have typical momenta of

$$P_+ \approx M, \quad P_\mu \approx 0, \quad P_\epsilon \approx M/2.$$  \hspace{1cm} (9)

The total number of absorbed laser photons is $r \approx 2M^2/(\omega m)$, which is of order $10^{10}$ at $\omega = 1$ eV and in agreement with the energy conservation law: $r\omega \approx 2(P_0 - m)$. Note that, even at threshold, the muons are highly relativistic so that their life time in the lab frame is increased from $2.2 \mu s$ to $\sim ms$ due to time dilation. The typical muon momenta in Eq. (9) can also be deduced from a classical simple-man’s model. To this end, assume that at the reaction threshold ($\xi = \xi_{\text{min}}$), the muons are created with zero momentum in the (primed)
From a single laser-driven Ps atom the total rate of muon production can be estimated as

\[ R_{Ps} \approx \frac{27}{2} \frac{\alpha^2}{\pi^2 m^2 \xi^2 a_0} \left[ 1 - \frac{\epsilon^2}{\xi^2} \left( \frac{a_0}{\alpha \xi \lambda} \right)^3 \left( \frac{m}{\omega \xi^4} \right)^{1/3} \right]. \tag{10} \]

The analytical estimate (10) is confirmed by direct numerical evaluation of Eq. (7), as shown in Fig. 2. The high-order Bessel function in Eq. (6) was evaluated with numerical evaluation of Eq. (7), as shown in Fig. 2. The solid line shows the analytical estimate in Eq. (10); the black squares result from numerical calculations based on Eq. (7). The muon production rate from a single Ps atom attains a maximum value of order \(10^{-10}\) s\(^{-1}\) at a laser intensity of \(10^{23}\) W/cm\(^2\) (see Fig. 2). Since high laser intensities are obtained by short, tightly focussed pulses, the exponential observation of the predicted process requires dense Ps targets. The highest Ps density achieved so far is of order \(10^{15}\) cm\(^{-3}\) [13]; recent proposals aim at \(10^{18}\) cm\(^{-3}\) with regard to antihydrogen experiments [18] or generation of a Ps Bose-Einstein condensate [13]. At the latter density, a typical laser focal volume \(V_l \approx (10 \lambda)^3\) would contain about \(10^9\) Ps atoms, so that the total rate roughly amounts to \(1\) s\(^{-1}\) which still seems too small to be measured in view of the short duration of strong laser pulses (~ fs – ns). However, the wave packet spreading which has a detrimental impact on the reaction rate, can be controlled by applying a second counterpropagating laser.

The information given above on the muon kinematics and the typical photon numbers is also corroborated by numerical calculations. Figure 3 shows partial creation rates, i.e. the rates \(R_r\) for muon production by absorption of a certain number \(r\) of laser photons, their sum yielding the total reaction rate: \(R_{Ps} = \sum_r R_r\). The partial rates reflect the energy distribution of the created particles and agree with the analytical estimates. The muon production rate from a single Ps atom attains a maximum value of order \(10^{-10}\) s\(^{-1}\) at a laser intensity of \(10^{23}\) W/cm\(^2\) (see Fig. 2). Since high laser intensities are obtained by short, tightly focussed pulses, the exponential observation of the predicted process requires dense Ps targets. The highest Ps density achieved so far is of order \(10^{15}\) cm\(^{-3}\) [13]; recent proposals aim at \(10^{18}\) cm\(^{-3}\) with regard to antihydrogen experiments [18] or generation of a Ps Bose-Einstein condensate [13]. At the latter density, a typical laser focal volume \(V_l \approx (10 \lambda)^3\) would contain about \(10^9\) Ps atoms, so that the total rate roughly amounts to \(1\) s\(^{-1}\) which still seems too small to be measured in view of the short duration of strong laser pulses (~ fs – ns). However, the wave packet spreading which has a detrimental impact on the reaction rate, can be controlled by applying a second counterpropagating laser.
This kind of laser configuration is realized, e.g., by the Astra Gemini system at the Rutherford Appleton Laboratory, U.K. or the JETI "photon collider" at the University of Jena, Germany [19]. Then the spreading factor \( \langle a_0 / (\alpha \lambda) \rangle^4 \) in Eq. (10) is absent and the muon yield could be increased by \( \sim 10 \) orders of magnitude. Under these circumstances, with kJ laser systems of high repetition rates (\( \sim \text{Hz} \)) [1] the observation of laser-driven muon creation from Ps is expected to be feasible. The problem of wave packet spreading can be avoided by laser-driven collisions of free electrons and positrons, e.g., in an \( e^\pm \) plasma. Then integrals like the one in Eq. (6) are absent. Based on Eq. (2), the muon creation rate in this situation is approximately obtained as follows

\[
R_{e^+e^-} \approx \frac{1}{23\pi^2 m^2} \sqrt{1 - \frac{e^2_{\text{min}}}{\xi^2}} \frac{N_+N_-}{V_{\text{int}}}.
\]

Here, \( N_{\pm} \) is the number of \( e^\pm \) contained in the interaction volume \( V_{\text{int}} \). By employing realistic values of \( V_{\text{int}} = (10\lambda)^3 \) and \( N_{\pm} = 10^{17} \) (corresponding to a plasma density of \( 10^{16} \text{ cm}^{-3} \) [18]), we obtain \( R_{e^+e^-} \lesssim 10^{-2} \text{ s}^{-1} \), which is much smaller than \( R_{\text{ps}} \) in the crossed-beams setup. This corroborates the high luminosity achievable from Ps by laser-guided microscopic collisions.

Finally we note that the process \( \text{Ps} \rightarrow \mu^+\mu^- \) is heavily suppressed in a circularly polarized laser field [15]. This is expressed by a damping factor \( \langle a_0 / (\lambda \xi) \rangle^4 \sim 10^{-25} \) arising from the classical motion of the \( e^+ \) and \( e^- \) that co-rotate in the polarization plane at a macroscopic distance \( \sim \lambda \xi \). Contrary to that, in linearly polarized fields the classical \( e^\pm \) trajectories periodically meet and the rate damping results from quantum mechanical dispersion. Equation (12) thus describes muon creation in the collision of two extended but localized quantum wave packets. The reaction rate, being several orders of magnitude larger than for circular laser polarization, clearly indicates the distinct physical nature of the collision process. In conclusion, we have studied the high-energy reaction of muon pair creation by a nonrelativistic Ps atom subject to a superintense laser field of linear polarization. Due to the threshold intensity \( \approx 10^{23} \text{ W/cm}^2 \), the experimental investigation of this process which involves billions of photons would naturally be a goal for the next generation of high-power laser systems [20]. The measurement, though challenging, is facilitated by the ultrarelativistic kinematics of the created muons and requires high Ps densities which might be provided by other fields of physics. The results of the present study also hold for similar high-energy reactions like, e.g., pion or rho-meson production. Microscopic \( e^+e^- \) collisions arising from Ps in strong laser fields thus have the potential of paving an alternative route to laser particle physics.