Does Weak CP Phase Originate from a Certain Geometry?

Yong Liu

Lab of Numerical Study for Heliospheric Physics (LHP)
Chinese Academy of Sciences
P. O. Box 8701, Beijing 100080, P.R.China
E-mail: yongliu@ns.lhp.ac.cn

Abstract

We further investigate the probability that, weak CP phase originates in a certain geometry. We find that our postulation on weak CP Phase gives strict constraints on angle $\gamma$ in unitarity triangle $\Delta B$ and the element $V_{td}$ in Cabibbo-Kobayashi-Maskawa (CKM) matrix. The predicted $|V_{td}|$ is about 0.0086 and $\gamma$ about $75.3^0$ with the very narrow window respectively. These two parameters can be used to test our postulation precisely in near future.

PACS number(s): 11.30.Er, 12.10.Ck, 13.25.+m
Does Weak CP Phase Originate from a Certain Geometry?

As is well known, CP violation is one of the most important problems in particle physics [1-6]. People have been concerning it for more than thirty years. However, we still know little about it. With the running of the $B-$factories, we expect to know more about it in near future.

According to the standard CKM mechanism, CP violation originates from a phase present in the three by three quark mixing matrix [7-8]. Last winter, we have found that, the weak CP phase and the other three mixing angles in CKM matrix satisfy a certain geometry relation [9-10], so we postulated that, weak CP phase is a geometry phase. Although it is a ad hoc postulation, all the conclusions extracted from it are consistent with the present experimental results.

Our postulation in the standard parametrization can be expressed as [10]

$$
\sin \delta_{13} = \frac{(1 + s_{12} + s_{23} + s_{13})\sqrt{1 - s_{12}^2 - s_{23}^2 - s_{13}^2 + 2s_{12}s_{23}s_{13}}}{(1 + s_{12})(1 + s_{23})(1 + s_{13})}
$$

where $s_{ij}$ and $\delta_{13}$ are the parameters in the standard parametrization [11-12]

$$
V_{KM} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}s_{13} \\
    s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & -s_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}s_{13}
\end{pmatrix}
$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ for the "generation" labels $i,j = 1, 2, 3$. Here, the real angles $\theta_{12}, \theta_{23}$ and $\theta_{13}$ can all be made to lie in the first quadrant. The phase $\delta_{13}$ lies in the range $0 < \delta_{13} < 2\pi$. In following, we will make the three angles $\theta_{ij}$ lie in the first quadrant.

The central purpose of this work is to further investigate our postulation carefully. Firstly, we investigate the whole CKM matrix, secondly, we investigate the three angles in unitarity triangle DB, both with our postulation being used. In fact, if we notice that, the moduli of the elements in CKM matrix are squared moduli invariants, and the three angles in unitarity triangle are composed of squared and quartic invariants [13], we can see that is meaningful to do so.

1. About the Moduli of the Elements in CKM Matrix

We investigate the whole CKM matrix firstly, we hope to extract some useful information from our postulation. The programme is

a. For each group of given $V_{ud}$, $V_{ub}$ and $V_{tb}$, solve $s_{12}$, $s_{23}$, $s_{13}$ from the following equation

$$
V_{ud} = c_{12}c_{13} \quad V_{ub} = s_{13} \quad V_{tb} = c_{23}c_{13}.
$$

b. Substituting Eq.(1) into CKM matrix Eq.(2). Then, solve the moduli of all the elements.

c. Let $V_{ud}$, $V_{ub}$ and $V_{tb}$ vary in certain ranges. Repeat the steps a and b.
When we let $V_{ud}$, $V_{ub}$ and $V_{tb}$ vary in the ranges [11]

$$0.9745 \leq V_{ud} \leq 0.9760 \quad 0.0018 \leq V_{ub} \leq 0.0045 \quad 0.9991 \leq V_{tb} \leq 0.9993$$ (4)

$$0.9745 \leq V_{ud} \leq 0.9760 \quad 0.00045 \leq V_{ub} \leq 0.00585 \quad 0.9991 \leq V_{tb} \leq 0.9993$$ (5)

and

$$0.97375 \leq V_{ud} \leq 0.97675 \quad 0.00045 \leq V_{ub} \leq 0.00585 \quad 0.99895 \leq V_{tb} \leq 0.99955$$ (6)

we get the magnitudes of the elements of the complete matrix as

$$\begin{pmatrix}
0.9745 \sim 0.9760 & 0.2182 \sim 0.2244 & 0.0018 \sim 0.0045 \\
0.2180 \sim 0.2242 & 0.9736 \sim 0.9752 & 0.0371 \sim 0.0424 \\
0.0079 \sim 0.0093 & 0.0365 \sim 0.0415 & 0.9991 \sim 0.9993
\end{pmatrix}$$ (7)

$$\begin{pmatrix}
0.9745 \sim 0.9760 & 0.2182 \sim 0.2244 & 0.00045 \sim 0.00585 \\
0.2180 \sim 0.2242 & 0.9736 \sim 0.9757 & 0.0296 \sim 0.0424 \\
0.0079 \sim 0.0093 & 0.0364 \sim 0.0415 & 0.9991 \sim 0.9993
\end{pmatrix}$$ (8)

and

$$\begin{pmatrix}
0.97375 \sim 0.97675 & 0.2143 \sim 0.2276 & 0.00045 \sim 0.00585 \\
0.2142 \sim 0.2275 & 0.9727 \sim 0.9762 & 0.0296 \sim 0.0458 \\
0.0072 \sim 0.0102 & 0.0337 \sim 0.0448 & 0.99895 \sim 0.99955
\end{pmatrix}$$ (9)

respectively.

Compare with that given by [11]

$$\begin{pmatrix}
0.9745 \sim 0.9760 & 0.217 \sim 0.224 & 0.0018 \sim 0.0045 \\
0.217 \sim 0.224 & 0.9737 \sim 0.9753 & 0.036 \sim 0.042 \\
0.004 \sim 0.013 & 0.035 \sim 0.042 & 0.9991 \sim 0.9993
\end{pmatrix}$$ (10)

we find that, the predicted results are well in agreement with that given by data book. In the meantime, $|V_{td}|$ is not sensitive to the variations of the inputs. It lies in a very narrow window with the central value about ($\sim 0.0086$), even if we take a little more large error ranges for the inputs.

On the other hand, the relevant result extracted from the experiment on $B^0_d - \overline{B^0_d}$ mixing is [11]

$$|V_{tb}^* \cdot V_{td}| = 0.0084 \pm 0.0018.$$ (11)

we find that, the prediction about $|V_{td}|$ based on our postulation, not only coincide with the experimental result very well, but also gives a more strict constraint.

2. About the Three Angles of the Unitarity Triangle
The three angles $\alpha$, $\beta$ and $\gamma$ in the unitarity triangle $DB$ defined as [6]

$$
\alpha \equiv \arg\left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) \quad \beta \equiv \arg\left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right) \quad \gamma \equiv \arg\left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right)
$$

(12)

If a small change being made, it is easy to see that, the defined angles are composed of squared and quartic invariants. For example,

$$
\alpha \equiv \arg\left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) = \arg\left( -\frac{V_{td}V_{ub}^*V_{ub}V_{tb}^*}{V_{ud}V_{ad}V_{ab}V_{tb}^*} \right)
$$

the numerator in the definition is a quartic invariant and the denominator is a product of two squared invariants.

Now, we investigate the three angles. The programme is similar to that in section 1.

a. For each group of given $V_{ud}$, $V_{ub}$ and $V_{tb}$, solve $s_{12}$, $s_{23}$, $s_{13}$ from Eq.(3).

b. Substituting Eq.(1) into CKM matrix Eq.(2). Then, solve all of the elements with the results of $a$ being used.

c. Solve $\alpha$, $\beta$ and $\gamma$ according to the definition Eq.(12).

d. Let $V_{ud}$, $V_{ub}$ and $V_{tb}$ vary in certain ranges. Repeat the steps $a$, $b$ and $c$.

We still let $V_{ud}$, $V_{ub}$ and $V_{tb}$ vary in the ranges given by Eq.(4), Eq.(5) and Eq.(6). The corresponding results are

$$
73.3^0 \leq \alpha \leq 94.4^0 \quad 10.6^0 \leq \beta \leq 31.3^0 \quad 74.9^0 \leq \gamma \leq 75.6^0
$$

(13)

$$
70.3^0 \leq \alpha \leq 98.9^0 \quad 6.1^0 \leq \beta \leq 34.2^0 \quad 74.8^0 \leq \gamma \leq 75.6^0
$$

(14)

and

$$
66.9^0 \leq \alpha \leq 99.8^0 \quad 5.6^0 \leq \beta \leq 34.2^0 \quad 74.5^0 \leq \gamma \leq 76.0^0
$$

(15)

respectively.

The recent analysis by Buras gives [14]

$$
35^0 \leq \alpha \leq 115^0 \quad 11^0 \leq \beta \leq 27^0 \quad 41^0 \leq \gamma \leq 134^0
$$

(16)

or more strictly

$$
70^0 \leq \alpha \leq 93^0 \quad 19^0 \leq \beta \leq 22^0 \quad 65^0 \leq \gamma \leq 90^0
$$

(17)

It is easy to find, similar to that on $V_{td}$ in above, starting from our postulation, we obtain a more strict constraint on $\gamma$. We predict a very narrow window for $\gamma$ with the central value about ($\sim 75.3^0$). Furthermore, all the predictions about $\alpha$, $\beta$ and $\gamma$ coincide with the relevant analysis [14].

3. Conclusions and Discussions

In conclusion, we have further investigated the postulation that, weak CP phase originates in a certain geometry. Based on this postulation, we obtain two strict constraints on the
magnitude of CKM matrix element $V_{td}$ and angle $\gamma$ respectively. These can be put to the more precisely test on our postulation in $B-$factory in near future.

The significance of Eq.(1) is evident. If it can be further verified in the future, we can at least remove a uncertainty coming from the weak interaction. Through the study on heavy flavors, with Eq.(1) being considered, we can then extract more informations on strong interaction. For instance, it becomes possible to extract the CP phase related to the part of strong interaction of final states. The further work along this direction is under way.

References

[1] J.H.Christenson, J.W.Cronin, V.L.Fitch and R.Turlay, Phys.Rev.Lett.13,138(1964).

[2] L.L.Chau, Phys.Rept.95,1(1983).

[3] E.A.Paschos and U.Turke, Phys.Rept.4,145(1989). E.A.Paschos, CP Violation: Present and Future, DO-TH 96/01, FERMILAB-Conf-96/045-T.

[4] CP Violation Ed. L.Wolfenstein, North-Holland, Elsevier Science Publishers B.V. 1989.

[5] CP Violation Ed. C.Jarlskog, World Scientific Publishing Co.Pte.Ltd 1989.

[6] V.Gibson, J.Phys.G 23, 605(1997).

[7] N.Cabibbo, Phys.Rev.Lett.10,531(1963).

[8] M.Kobayashi and T.Maskawa, Prog.Theor.Phys.42,652(1973).

[9] J.L.Chen, M.L.Ge, X.Q.Li and Y.Liu, New Viewpoint to the Source of Weak CP Phase, Preprint hep-ph/9711330. Jing-Ling Chen, Mo-Lin Ge, Xue-Qian Li and Yong Liu, A Possible Hidden Symmetry and Geometrical Source of The Phase in The CKM Matrix, Preprint hep-ph/9809527.

[10] Yong Liu, Cabibbo-Kobayashi-Maskawa Matrix, Unitarity Triangle and Geometry Origin of the Weak CP Phase, Preprint hep-ph/9811508.

[11] C.Caso et al, The Europ.Phys.J.C 3 1(1998).

[12] L-L.Chau and W-Y.Keung, Phys.Rev.Lett.,53,1802(1984).

[13] J.F.Donoghue, E.Golowich and B.R.Holstein, Dynamics of the Standard Model Cambridge University Press, 1992.

[14] A.J.Buras, CKM Matrix: Present and Future, hep-ph/9711217.