The Central Term in 3D Simple Superalgebra

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Abstract

A matter self-interacting model with $N = 1$-supersymmetry in 3D is discussed in connection with the appearance of a central charge in the algebra of the supersymmetry generators. The result is extended to include gauge fields with a Chern-Simons term. The main result is that, for a simple supersymmetry, only the matter sector contributes to the central charge in contrast to what occurs in the $N = 2$ case.

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Ordinary and supersymmetric Abelian gauge models in three-dimensional space-times have been fairly-well investigated in various contexts over the past years [1]. Besides their relevance in connection with the possibility of getting non-perturbative results more easily, the ultraviolet finiteness of Yang-Mills (and gravity) Chern-Simons models is a remarkable feature of field theories defined in $D = (1 + 2)$ [2]. Also, 3D gauge theories seem to be the right way to tackle exciting topics of Condensed Matter Physics such as High-$T_c$ Superconductivity and Fractional Quantum Hall Effect [3].

Our purpose in this paper is to assess an Abelian three-dimensional gauge model with $N = 1$ supersymmetry, from the point-of-view of the algebra of supersymmetry generators. We actually wish to present here a few remarks on the connection between topologically non-trivial solutions, the Chern-Simons term, and the presence of a central charge operator in the supersymmetry algebra [7].

The super-Poincaré algebra in $(1 + 2)$ dimensions is generated by a real two-component spinorial charge, $Q_a$, whose operatorial relations are listed below:

$$\{Q_a, Q_b\} = 2P_{ab} \quad \text{e} \quad [Q_a, P_{ab}] = 0, \quad (1)$$

where $P_{ab}$ is the translation generator. We shall represent vectors in a twofold way: for Lorentz indices we will use greek letters, and for bi-spinorial indices we will use latin letters, bearing in mind the mapping $V_{ab} = V_{\mu}(\gamma^\mu)_{ab}$ [4]. The super-Poincaré algebra (1) for an extended supersymmetry with $N$ flavours is generalized to:

$$\begin{align*}
\{Q^i_a, Q^j_b\} &= 2\delta^{ij}P_{ab} + A^{ij}\epsilon_{ab}, \\
[Q^i_a, P_{ab}] &= 0, \\
[Q^i_a, A^{jk}] &= 0, \quad (2)
\end{align*}$$

where $i, j, k = 1, ... , N, \epsilon_{ab}$ is the Levi-Civita tensor in spinor space and $A^{ij} = -A^{ji}$ is the central charge that transforms under the symmetry group which defines an automorphism of the algebra of extended supersymmetry.

To find out if a quantum field theory exhibits supersymmetry, one might study the set of Ward identities among the Green’s functions, and then establish whether or not they are respected. In our work, we adopt another procedure, namely, we explicitly compute the equal-time current algebra associated to supersymmetry; such a method is able to automatically signal the eventual presence of central charge operators as originated from topologically non-trivial field configurations [7].

Adopting the metric tensor as $\eta^{\mu\nu} = (+; -, -)$, we shall choose the following representation for the $\gamma$-matrices:

$$\begin{align*}
(\gamma^0)^a_b &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\
(\gamma^1)^a_b &= \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \\
(\gamma^2)^a_b &= \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \\
C_{ab} &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad (3)
\end{align*}$$

2
where we have the “metric” tensor \( C_{ab} = -C_{ba} = C^{cd} \) and \( C_{ab}C^{cd} = \delta^c_{[a} \delta^d_{b]} \). We list below a number of algebraic relations among the \( \gamma \)-matrices that have been useful in our component-field calculations:

\[
\begin{align*}
(\gamma^\mu)^{ab}(\gamma^\nu)_{ab} &= \eta^{\mu\nu}, \\
(\gamma^\mu)^{ab}(\gamma^\nu)_{bd} &= \eta^{\mu\nu}\delta^a_d - i \epsilon^{\mu\nu\rho}(\gamma^\rho)^a_d, \\
(\gamma^\mu)_a (\gamma^\nu)^b_c &= -\eta^{\mu\nu}, \\
(\gamma^\mu)_a (\gamma^\nu)^b_c (\gamma^\rho)^d_a &= -i \epsilon^{\mu\nu\rho}(\gamma^\rho)^a_d, \\
(\gamma^\mu)_a (\gamma^\nu)^b_c (\gamma^\rho)^d_a &= -i \epsilon^{\mu\nu\rho}, \\
(\gamma^\mu)^{ab}(\gamma^\nu)^b_c (\gamma^\rho)^d_a &= i \epsilon^{\mu\nu\rho}C_{da} + \eta^{\rho\nu}(\gamma^\mu)^{da} - \eta^{\rho\mu}(\gamma^\nu)^{da} - \eta^{\mu\nu}(\gamma^\rho)^{da}.
\end{align*}
\] (5)

This paper is outlined as follows: in Section 1, a self-interacting scalar model is presented and the su.sy. algebra is written down with the explicit form for the central charge operator; the introduction of the gauge sector is discussed in Section 2. Finally, in Section 3, one discusses the supersymmetric version of a Chern-Simons term, and the connection it bears with the central charge is investigated. Our General Conclusions follow in Section 4.

1 Self–Interacting Scalar Model and Vortex Configurations

The component-field expansion for a scalar superfield reads

\[
\Phi(x, \theta) = A(x) + \theta^a \psi_a(x) - \theta^2 F(x);
\] (6)

where \( \theta \) is a (real) Grassmann-valued Majorana spinor and \( A(x) \) is a physical scalar, \( \psi_a(x) \) is a physical fermion and \( F(x) \) is an auxiliary field. The supersymmetry covariant derivative is represented as

\[
D_a = \partial_a + i \theta^b \partial_{ab},
\]

\[
\{D_a, D_b\} = 2P_{ab}.
\] (7) (8)

The most general \( N=1 \)-supersymmetric action with renormalizable matter self-interactions is given by

\[
S_{\text{scalar}} = \int d^3x d^2\theta \left\{ -\frac{1}{2} (D_a \Phi)^2 + \frac{1}{2} m \Phi^2 + \frac{\lambda}{8} \Phi^4 \right\},
\] (9)

and the supersymmetry transformations on the components \( A, \psi_a \) and \( F \) read as below:

\[
\begin{align*}
\delta A &= -\epsilon^a \psi_a, \\
\delta \psi_a &= -\epsilon^b (C_{ab} F + i \partial_{ab} A), \\
\delta F &= -\epsilon^b i \partial^a_b \psi_a.
\end{align*}
\] (10)
The action for the physical fields,
\[
S_{\text{scalar}} = \int d^3x \left\{ \frac{i}{2} \left[ \frac{1}{2} (\partial_a A)(\partial^a A) + \bar{\psi}^a i \partial_a \psi^b \right] + m \psi^2 + \frac{3}{2} \lambda \psi^2 A^2 + \right.
\]
\[
- \frac{1}{2} m^2 A^2 - \frac{1}{2} m \lambda A^4 - \frac{1}{8} \lambda^2 A^6 \right\},
\]
(11)
can be shown to be invariant under the non-linear “on-shell” transformations,
\[
\delta A = -\epsilon^a \psi_a, \\
\delta \psi_a = -\epsilon^b \left[ C_{ab} \left( -\frac{1}{2} m A - \frac{1}{2} \lambda A^3 \right) + i \partial_a A \right].
\]
(12)

Now, taking into account that supersymmetry is a symmetry of the action (the Lagrangian density transforms as a total derivative), it can be shown that the Noether current associated to \(N=1\)-supersymmetry turns out to be:
\[
J^\mu_c = -i \psi^a (\gamma^\mu)_{ac} \left( m A + \frac{\lambda}{2} A^3 \right) - \frac{i}{2} \varepsilon^{\mu \nu \rho} \psi^b (\gamma_\rho)_{bc} \partial_\nu A + \\
+ \frac{1}{2} \psi_c \partial^\mu A + \frac{1}{2} A \partial^\mu \psi_c - \frac{i}{2} \varepsilon^{\mu \nu \rho} A \partial_\nu \psi^a (\gamma_\rho)_{ac}.
\]
(13)
The supercharge is defined as
\[
Q_c = \int d^2\vec{x} J^0_c
\]
\[
= \int d^2\vec{x} \left\{ -i \psi^a (\gamma^0)_{ac} \left( m A + \frac{\lambda}{2} A^3 \right) + \frac{1}{2} \psi_c \partial^0 A + \frac{1}{2} A \partial^0 \psi_c + \\
- \frac{i}{2} \varepsilon^{0 \nu \rho} \psi^a (\gamma_\rho)_{ac} \partial_\nu A + \frac{i}{2} \varepsilon^{0 \nu \rho} A \partial_\nu \psi^a (\gamma_\rho)_{ac} \right\}.
\]
(14)

With the help of the canonical commutation (and anticommutation) relations for the physical fields, a tedious calculation yields the following expression for the algebra of supersymmetry charges:
\[
\{Q_a, Q_b\} = \int d^2\vec{x} \times \\
-2i \left\{ \frac{1}{4} \left[ 2 i \psi^a (\gamma^0)_a^b \partial^0 \psi_b + (\partial^0 A)(\partial^0 A) + 2 i \psi^a (\gamma^i)_a^b \partial^i \psi_b + (\partial^i A)(\partial^i A) + \\
\psi^2 \left( m + \frac{3}{2} \lambda A^2 \right) + \left( \frac{1}{2} m^2 A^2 + \frac{1}{2} m \lambda A^4 + \frac{1}{4} \lambda^2 A^6 \right) \right] \right\} (\gamma^0)_{ab} + \\
-2i \left\{ \frac{1}{4} \left[ 2 i \psi^a (\gamma^0)_a^b \partial^0 \psi_b + (\partial^0 A)(\partial^0 A) \right] \right\} (\gamma_i)_{ab}.
\]
(15)

If we compare this expression with the 0\(\mu\) component of the “improved” energy-momentum tensor
\[
T_{\mu \nu} = \frac{1}{e} \frac{\delta S}{\delta e^\mu_{a'}} e^{(a')}_{a} = - \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu \nu}} \bigg|_{g^{\mu \nu} = \eta^{\mu \nu}},
\]
(16)
where $S$ in this expression indicates the action (11), we may rewrite (15) as
\[ \{Q_a, Q_b\} = 2i P^\mu (\gamma_\mu)_{ab} + 2i \epsilon^{ij} \int d^2 \vec{x} (\partial_i A) (\gamma_j)_{ab} \left( mA + \frac{\lambda}{2} A^3 \right). \] (17)

In terms of the chiral components of the supersymmetry charge, the algebra takes over the form:
\[ \{Q^+, Q^+\} = 2i (P^0 + P^1) - 2 \int d^2 \vec{x} \left( mA + \frac{\lambda}{2} A^3 \right) \partial_2 A, \] (18)
\[ \{Q^-, Q^-\} = 2i (P^0 - P^1) - 2 \int d^2 \vec{x} \left( mA + \frac{\lambda}{2} A^3 \right) \partial_2 A, \]
\[ \{Q^+, Q^-\} = -2i P^2 - 2 \int d^2 x \left( mA + \frac{\lambda}{2} A^3 \right) \partial_1 A. \] (19)

Expressions (17) and (19) sign the presence of a central charge that is sensitive to a topologically non-trivial behavior of the scalar sector at infinity:
\[ T_2 = \int dx_1 \int dx_2 \frac{\partial}{\partial x_2} \left( mA^2 + \frac{\lambda}{4} A^4 \right), \]
\[ T_1 = \int dx_2 \int dx_1 \frac{\partial}{\partial x_1} \left( mA^2 + \frac{\lambda}{4} A^4 \right), \] (20)

where we observe the topological character of the central charge terms, which has its origin in the mass and self-interacting terms of the scalar field in Lagrangian. Bearing in mind this result, we shall now consider the introduction of an Abelian gauge field with $N=1$–supersymmetry. We also know that such a coupling is fundamental to stabilize the soliton-like solutions in the form of magnetic vortices with finite energy.

## 2 On the $N=1$ Super–QED$_3$

The $N=1$–supersymmetric version of QED$_3$ is achieved upon the complexification of the scalar superfield in eq. (9) and the gauge–covariantization of the spinor derivative:
\[ \nabla_a \equiv D_a \mp i \Gamma_a, \] (21)
where $\Gamma_a$ is a gauge superconnection with super–helicity $h = \frac{1}{2}$, and the signs $–$ and $+$ indicate that the derivative is acting in the superfields $\Phi$ and $\bar{\Phi}$ respectively ($\bar{\Phi} \equiv \Phi^*$). $\Gamma_a$ admits the following $\theta$–expansion:
\[ \Gamma_a = \chi_a + \theta^b (C_{ab} B + i V_{ab}) + \theta^2 (2\lambda_a - i \partial_a \chi_b), \] (22)
where $\lambda_a$ is the gaugino field, $V_{ab}$ is the usual gauge field; $B$ and $\chi_a$ are compensating component–fields. In the so–called Wess–Zumino gauge [3], $\Gamma_a$ is reduced to:
\[ \Gamma_a = i \theta^b V_{ab} - 2 \theta^2 \lambda_a, \] (23)
where the supersymmetry transformations read:

$$\delta V_{ab} = i \epsilon_{(b} \lambda_{b)} ,$$
$$\delta \lambda_a = \frac{i}{2} \epsilon^c \partial_{c(a} V_{c)b} .$$  \hfill (24)

The covariantized vectorial derivative is written as

$$\nabla_{ab} = D_{ab} \mp i \Gamma_{ab} ,$$  \hfill (25)

where $\Gamma_{ab}$ is the vector superconnection. As we know, in order to have irreducible representations of symmetry, we need constraints in the model. In the supersymmetric case, we have the so-called conventional constraint, that acts in such a way that the supersymmetric algebra of the spinor derivatives, \(\{\nabla_a, \nabla_b\} = 2i \nabla_{ab} + F_{ab}\), will have $F_{ab} = 0$. Then, we easily compute that

$$\Gamma_{ab} = - \frac{i}{2} D_{(a} \Gamma_{b)} ,$$  \hfill (26)

implying that in the Wess–Zumino gauge we have

$$\Gamma_{ab} = V_{ab} + i \theta_{(a} \lambda_{b)} - \frac{i}{2} \theta^2 \partial_{c(a} V_{b)c} .$$  \hfill (27)

By the graded Bianchi identity, we redefine the gauge field as

$$W_a = \frac{1}{2} D^b D_a \Gamma_b ,$$  \hfill (28)

with the constraint $D^a W_a = 0$ ($D^a D_b D_a = 0$), implying as in the usual Lorentz gauge that exists only one independent component of the field $W_a$. Using the projector method, we write

$$W_a| = \lambda_a , \quad D_a W_b| = \frac{1}{2} (\partial_{ca} V_{b}^c + \partial_{cb} V_{a}^c) \equiv f_{ab} ,$$  \hfill (29)

with $f_{ab}$ the usual gauge field strength. Another relations that will be very important and that may straightforwardly be obtained are (conf. ([4])):

$$\nabla_a \nabla^2 = i \nabla^b \nabla_b \pm i W_a\ e^c (\nabla^2)^c = \Box \mp i W^a \nabla_a ,$$  \hfill (30)

where $\Box$ means the gauge–covariant d’Alembertian. Now, we are ready to discuss the supersymmetry algebra in the framework of $N=1$ Super-QED$_3$.

### 2.1 Scalar Superaction with a Background Gauge Field

The U(1)–invariant superfield action without kinetic term for the gauge sector is given as below:

$$S_{scalar} = - \frac{1}{2} \int d^3x d^2\theta \left\{ (\nabla^a \Phi)(\nabla_a \Phi) \right\} ,$$  \hfill (31)
Redefining the component field, (we can always do this) by the projections
\[ \Phi = A, \quad \bar{\Phi} = \bar{A}, \]
\[ \nabla_a \Phi = \psi_a, \quad \nabla_a \bar{\Phi} = \bar{\psi}_a, \]
\[ \nabla^2 \Phi = F, \quad \nabla \bar{\Phi} = \bar{F}, \]
the gauge–field component action takes the form:
\[ S = \frac{1}{2} \int d^3 x \left\{ \bar{\psi}^a D_a^b \psi_b + \psi^a D_a^b \bar{\psi}_b + A \square \bar{A} + \bar{A} \square A \right\}, \tag{33} \]
with the “on shell” supersymmetric transformations:
\[ \delta \bar{\psi}^a = \epsilon_b i D^{ab} \bar{A}, \quad \delta \psi_b = -\epsilon^c i D_{bc} A, \]
\[ \delta A = -\epsilon^a \psi_a, \quad \delta \bar{A} = -\epsilon^a \bar{\psi}_a. \tag{34} \]

The Noether current associated to N=1–supersymmetry is now:
\[ J^\mu_c = i \epsilon^\mu_{\nu\rho} [\bar{\psi}^a (\gamma_\rho)_{ac} \partial_\nu A + \psi^a (\gamma_\rho)_{ac} \partial_\nu \bar{A}] - \frac{1}{2} (\psi_c \partial^\mu \bar{A} + \bar{\psi}_c \partial^\mu A) + \frac{1}{2} (\bar{A} \partial^\mu \psi_c + A \partial^\mu \bar{\psi}_c) + i \frac{1}{2} \epsilon^\mu_{\nu\rho} (\bar{A} \partial_\nu \psi^a + A \partial_\nu \bar{\psi}^a)(\gamma_\rho)_{ac}, \tag{35} \]
yielding the supercharge
\[ Q_c = \int d^2 \vec{x} J^0_c = \int d^2 \vec{x} \frac{1}{2} \left\{ - (\psi_c \partial^0 \bar{A} + \bar{\psi}_c \partial^0 A) + i \epsilon^{0ij} [\bar{\psi}^a (\gamma_j)_{ac} \partial_i A + \psi^a (\gamma_j)_{ac} \partial_i \bar{A}] + (\bar{A} \partial^0 \psi_c + A \partial^0 \bar{\psi}_c) + i \epsilon^{0ij} (\bar{A} \partial_i \psi^a + A \partial_i \bar{\psi}^a)(\gamma_j)_{ac} \right\} \tag{36} \]

The canonical conjugate momenta that will be necessary for the supercharge algebra are
\[ \Pi_{\psi_d} = -i \bar{\psi}^a (\gamma_0)^a_d, \quad \Pi_{\bar{\psi}_d} = -i \psi^a (\gamma_0)^a_d, \]
\[ \Pi_A = \langle D^0 \bar{A} \rangle, \quad \Pi_{\bar{A}} = \langle D^0 A \rangle, \tag{37} \]
giving the canonical commutation and anticommutation relations
\[ \{ \psi^d, \bar{\psi}^a \} = i (\gamma^0)^{ad} \delta^2(\vec{x} - \vec{y}) , \quad \{ \bar{\psi}^d, \psi^a \} = i (\gamma^0)^{ad} \delta^2(\vec{x} - \vec{y}), \]
\[ [A, D^0 \bar{A}] = \delta^2(\vec{x} - \vec{y}) , \quad [\bar{A}, D^0 A] = \delta^2(\vec{x} - \vec{y}). \tag{38} \]

After a lengthy computation, using the \( \gamma \)-matrices Clifford algebra, we reach the result
\[ \{ Q_a, Q_b \} = -2i P^\mu (\gamma_\mu)_{ab}, \tag{39} \]
where the momentum operator \( P^\mu \) appearing in the RHS includes now contributions from the gauge field minimally coupled to matter through (33). Nevertheless, no term in the form of a central charge arises from the action (33); this means that the central charge operator of eq. (17) is not modified by the introduction of the \( U(1) \) gauge superfield. The role of the latter is to stabilize the topological configurations associated to the action (33) in the form of vortex-like solitons, as already known from the works quoted in ref. [8].
3 The Supersymmetric Chern–Simons Term

Now, we shall add a supersymmetric Chern–Simons (CS) term to the action eq. (33) and we will verify how it modifies the supercharge algebra. For this purpose, we begin with the (gauge–invariant) CS term in superspace

\[ S_{CS} = \frac{M}{g^2} \int d^3x \, d^2\theta \, \Gamma^a W_a , \]  

(40)

where \( M \) is a mass parameter and \( g \) is the gauge coupling constant. In components, using the Wess–Zumino gauge, the action eq. (40) leads to the expression

\[ S_{CS} = \frac{M}{g^2} \int d^3x \left[ i V^{ab} (\partial_a V^c) + 4 \lambda^2 \right] , \]  

(41)

where the first term in the r.h.s. is the well–known CS term. Now, including the term (40) in the action (33), and then taking into account the equations of motion for the \( F \), \( \bar{F} \) (which are not affected by the CS term) and of the \( \lambda_a \)–field, the complete Lagrangian reads:

\[ L_{din} = \frac{i}{2} \bar{\psi}^a (\gamma^\mu)_a \partial_\mu \psi_b + \frac{i}{2} \bar{\psi}^a (\gamma^\mu)_a \partial_\mu \bar{\psi}_b - \frac{1}{2} (\partial^\mu \bar{A})(\partial_\mu A) + \frac{i}{2} (\partial^\mu \bar{A}) V^\mu A + \]  

\[ -\frac{i}{2} V^\mu \bar{A}(\partial_\mu A) + \frac{iM}{g^2} \epsilon^{\mu\nu\rho} V_\mu \partial_\nu V_\rho , \]  

(42)

with the “on shell” supersymmetric transformations:

\[ \delta \bar{\psi}^a = i \epsilon_b D^{ab} \bar{A} , \quad \delta \psi_b = -i \epsilon_c D_{bc} A , \]  

\[ \delta A = -\epsilon^a \psi_a , \quad \delta \bar{A} = -\epsilon^a \bar{\psi}_a , \]  

\[ \delta V_{ab} = i \epsilon_{(a} \lambda_{b)} . \]  

(43)

Following again the procedure to read off the Noether current associated to the transformations (43), it can be found out that the contribution of the CS term yields:

\[ (J_{SCS})^a_0 = \frac{i}{2} V^\mu \left( \bar{\psi}^a \bar{A} - \bar{\psi}_a A \right) , \]  

(44)

with the supercharge

\[ (Q_{SCS})^a_0 = \int d^2 \vec{x} (J_{SCS})^a_0 = \int d^2 \vec{x} \left\{ \frac{i}{2} V^\mu \left( \bar{\psi}^a \bar{A} - \bar{\psi}_a A \right) \right\} , \]  

(45)

whence

\[ \{ Q_a , Q_b \}_{SCS} = 2 i P^\mu [V^0] (\gamma^\mu)_{ab} , \]  

(46)

where \( P^\mu [V^0] \) means a functional that depends only on the time–component of the potential vector.
What we observe is that the $V^0$ potential field is completely eliminated from the algebra, implying that the “corrected” Chern-Simons $T^{0\mu}$ component of energy-momentum tensor, defined as “new” $P^\mu$ becomes independent on the potential gauge field. It is possible to say that the the conjugate momenta of the $A$ and $\bar{A}$ fields are in fact “corrected” by the CS term to become $\Pi_A \alpha \partial^0 \bar{A}$ and $\Pi_{\bar{A}} \alpha \partial^0 A$. This indicates that the CS term plays a role similar to a partial gauge fixing, eliminating one degree of freedom of the gauge field, referring to the algebra.

4 Conclusions

The basic motivation of this paper was to analyze the 3-dimensional counterpart of a well-known result by Olive and Witten [7], namely, the appearance of a central charge in the algebra of simple supersymmetry as originated from non-trivial topological field configurations. To this aim we have analyzed the full supersymmetric model in 3D. We have computed the Noether supersymmetric charges, and the “improved” energy-momentum tensor using the gravitation approach. From the canonical commutations (and anti-commutations) relations of the superfields we obtained the supercharge algebra. Here, with and without gauge fields, we could conclude that vortex-like field configurations are responsible for a central charge in the supersymmetry algebra, even in the case of a $N = 1$-supersymmetry. It is worthwhile to mention the results obtained by Lee, Lee and Weinberg [9], where a central charge comes out in context of an $N = 2$ extended supersymmetric model. We would like to point out that the calculations of Section 3 recall that the Chern-Simons term for the gauge field does not give contribution to the central charge appearing in the algebra. In fact it could represent a sample of gauge fixing eliminating the time direction of the vector potential in the algebra. So the central charge of this model arises exclusively from the matter sector and its existence to the vortex configurations of the scalar fields. Clearly, the role of the gauge fields is to render finite the vortex energy [8]. The analysis of the BPS bounds and its consequences will appear in a forthcoming paper.

Next, it might be of relevance to analyze the presence of central charges in the models proposed by Dorey and Mavromatos [10] to study $P,T$ conserving superconducting gauge models whenever the latter are supersymmetrized. One could perhaps understand whether or not central charges may be related to some physical aspects of superconductivity.

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References
[1] A. Linde, *Rep. Progr. Phys.* **42** (1979) 389,  
S. Deser, R. Jackiw and S. Templeton, *Phys. Rev. Lett.* **48** (1982) 975,  
S. Deser, *Three Topics in Three Dimensions*, in the proceedings of the Trieste Conference on Supermembranes and Physics in 2+1 Dimensions, eds. M.J. Duff *et al.*, World Scientific (Singapore, July 1989);  
M.A. De Andrade, O.M. Del Cima, L.P. Colatto, *Phys. Lett.* **370B** (1996) 59,  
L.P. Colatto, M.A. De Andrade, O.M. Del Cima, D.H.T. Franco, J.A. Helayel-Neto, O. Piguet, *Jour. Phys.* **G24** (1998) 1301,  
[2] F. Delduc, C. Lucchesi, O. Piguet and S.P. Sorella, *Nucl. Phys.* **B346** (1990) 313,  
A. Blasi, O. Piguet and S.P. Sorella, *Nucl. Phys.* **B356** (1991) 154,  
C. Lucchesi and O. Piguet, *Nucl. Phys.* **B381** (1992) 281,  
O.M. Del Cima....  
[3] J. Schonfeld, *Nucl. Phys.* **B185** (1981) 157,  
[4] S. J. Gates, M. T. Grisaru, M. Roček and W. Siegel, *Superspace*,  
Benjamin/Cummings Publishing Company (1983),  
[5] S. Coleman and J. Mandula, *Phys. Rev.* **159** (1967) 1251,  
R. Haag, J. T. Lopuszanski and M. Sohnius, *Nuc. Phys.* **B88** (1975) 257;  
[6] R. Jackiw, *Field Theoretic Investigation in Current Algebra, Current Algebra and Anomalies*, S. B. Treiman, R. Jackiw, B Zumino and E. Witten (eds.), World Scientific Publishing Co. (1985),  
[7] E. Witten and D. Olive, *Phys. Lett.* **B78** (1978) 97,  
P. Di Vecchia and S. Ferrara, *Nuc. Phys.* **B130** (1977) 93,  
A. D’Adda and P. Di Vecchia, *Phys. Lett.* **B73** (1978) 162,  
[8] Lewis H. Ryder, *Quantum Field Theory* - Chap. 9, Cambridge University Press, 1985,  
[9] C. Lee, K. Lee and E. Weinberg, *Phys. Lett.* **B243** (1990) 105,  
[10] N. Dorey and N. E. Mavromatos, *Nuc. Phys.* **B386** (1992) 614;  
N. E. Mavromatos, *Superconducting Gauge Theories in (2+1)-Dimensions*, CERN preprint, CERN-TH.6331/91 (1991).