On the Probability of the Development of the Diagnostic Feature of Decline the Bearing Capacity of Structures under Random Dynamic Loads

Sh Sh Iskhakov¹, F E Kovalev¹, S V Zarín¹
¹Department of Special facilities of rocket and space complexes, Military-Space academy in the name of A.F. Mozhayskiy, Zhdanovskaya st., Saint-Petersburg and 197198, Russia
E-mail: sergeyyarin27091989@gmail.com

Abstract. The article is devoted to the problem of identifying the diagnostic feature of the structure bearing capacity decrease under random dynamic loads. The diagnostic features that characterize the decrease of the bearing capacity of structures and foundation are identified. The model of the dynamic system that describes the process of the diagnosed building structure functioning is presented. The article shows the difficulties encountered in the process of vibration diagnostics of reducing the bearing capacity of structures and foundations. It is pointed out that the output vibration signals can be recorded in the mode of displacements, velocities and accelerations. But at the same time, the probability spread of detection of diagnostic feature of the bearing capacity decrease of building elements in the energy spectra of the output vibration signals can be significant. To develop a method for determining this probability, the authors propose to use a regression analysis with the solution of the problem at three levels of "regression". In the article the factors influencing the probability of detection the vibration diagnostic feature in bearing capacity decrease of building elements are analyzed. This probability characterizes the effectiveness of the vibration diagnostic method in monitoring systems of unique objects, including launch facilities. The authors have shown that the maximum efficiency is achieved by using systems for monitoring energy spectra of vibration accelerations at the output. At the same time, the authors conclude that the probability of effective functioning of vibration diagnostic systems is mainly influenced by such factors as the sensitivity of the vibration diagnostic method to the rigidity decrease of structures and bases. The equipment error of registration of the energy spectra of vibration accelerations is also to be taken into consideration.

1. Introduction
In the functional vibration diagnostics of technical condition of long-span structures, the reduction of their bearing capacity is connected with their flexural rigidity $C$. Diagnostic feature of decrease in their bearing capacity is known to be the changes in dynamic parameters of structures [1-5]. These dynamic parameters are contained in their transfer functions $\eta \omega$ [6] where $\omega$ is the circular frequency of the dynamic load impact $P t$ with the energy spectrum (spectral density function) $G_p \omega$ [7, 8].
In practice loads $P t$ are often random functions of space and time $t$ [6-8]. Examples of such loads are: wind loads [9, 10] for high-rise buildings and constructions; field of pressure pulsations of a gas...
jet of carrier rockets [3, 11, 12] for launching facilities. In these cases informative diagnostic feature of reduction of the bearing capacity structures is a reduction of its frequency (for the first fundamental form) of the vibrations \( \lambda_k \) by the same amount \( \Delta \lambda \) and the increase of the resonance extrema of the transfer function \( \eta(\omega) \) in some \( \nu \) times [1-4, 11-15] (Figure 1).

\[
D_r = \begin{cases} 
\Delta \lambda > 0; \\
\nu > 1,
\end{cases} \Rightarrow \text{decrease BC.} \tag{1}
\]

![Figure 1. The formation of the diagnostic features of the stiffness reduce C of span structures.](image1)

The ability to identify these \( D_r \) is due to the functioning of the dynamic system (diagnosed building structure) by the algorithm (Figure 2) [4, 6, 9]:

\[
G_p(\omega) \times \eta(\omega) = G_\xi(\omega), \tag{2}
\]

where the energy spectrum of the output vibrating signal \( G_\xi(\omega) \) in the general case may be determined on the implementations of the vibration displacements \( V_t \), velocities \( \dot{V} \) and acceleration \( \ddot{V} \) [16].

Diagnostic features (1) (Figure 1) can manifest itself in the spectrum of the output vibrating signal \( G_\xi(\omega) \) in the form of the extremum on the natural (resonant) frequency of vibrations of the structure \( \lambda_k \) in cases of sufficient energy input \( P_t \) with the spectrum \( G_p(\omega) \) for the functioning of the algorithm (2) with the graphical interpretation in Figure 3.

![Figure 2. Dynamical system.](image2)

![Figure 3. A graphical model of the formation of the output vibrating signal G_ξ(ω) by algorithm (2).](image3)

However, the specified extremum at frequency \( \lambda_k \) (Figure 3) in the spectrum of the output signal \( G_\xi(\omega) \) can not be manifested in certain parameters of the functions \( G_p(\omega) \), \( \eta(\omega) \) and \( G_\xi(\omega) \).
included in the algorithm (2). This circumstance is an obstacle to the detection of diagnostic feature (1) of reducing the bearing capacity of structures in the process of their long-term exploitation. This is especially true for buildings and constructions. When monitoring the technical condition by vibration method there is no possibility to register random input actions $P_t$ and their energy spectra $G_p(\omega)$, for example, it is typical for high-rise buildings and constructions [9] and for launching facilities when launching the space-purpose missiles [12, 13]. In this regard, when substantiating the expediency of using stationary monitoring systems [17-21], the probability estimation $\Pi$ (6) of the fact that the informative extremum with the actual ordinate $h_\lambda_k > h_\lambda_k$ exceeding the minimum (permissible) value $h_\lambda_k$ practically may have significant parameters for the effective application of monitoring systems on specific buildings and constructions is relevant (Figure 2).

$$\Pi[h_\lambda_k > h_\lambda_k] = f \land \varphi_i .$$  \hspace{1cm} (3)

2. Methods

To solve this problem it is advisable to use regressive proof (analysis), in which [22] "the course of reasoning goes from consequences to grounds".

The event interesting for us:

$$h_\lambda_k > h_\lambda_k,$$  \hspace{1cm} (4)

is the result of functioning of dynamic system (Figure 2) according to the algorithm (2). Thus, the probability $\Pi$ (3) of the event (4), according to the regression analysis [22], should be considered as a certain function, the basis of which is the product of functions $\varphi$. The functions $\varphi_i$ depend on the initial functions that are included in the algorithm (2):

$$f \land \varphi_i = \varphi_1[G_p, \omega] \cdot \varphi_2[\eta, \omega] \cdot \varphi_3[G_\varepsilon, \omega],$$  \hspace{1cm} (5)

where sign $\land$ is a logical symbol of "product" [22], and $\varphi_i$ are symbols of three functions, the product of which is written in the right part of equality (5).

3. Results

Recording (5) is the first level of regression analysis of the factors determining the probability $\Pi$ (3) of the event (4), i.e. the development of the resonance extremum in the spectrum of the output vibration signal $G_\varepsilon(\omega)$ (Figure 3).

The second level of "regression" [22] [that is, lowering the level of analysis to the basis of the function $f \land \varphi_i$ (5)] is the consideration for each function $\varphi_i$ of products of their constituent functions $\varphi_{i,j}$. So, for the function $\varphi_i[G_p, \omega]$ in (5) we can write the equality:

$$\varphi_i[G_p, \omega] = \varphi_{1,1}[\Delta\omega] \cdot \varphi_{1,2}[E_{\lambda_k}],$$  \hspace{1cm} (6)

where the function $\varphi_{1,1}[\Delta\omega]$ depends on the frequency band width $\Delta\omega$ (Figure 3), in which the energy spectrum of the input action $G_p(\omega)$ is formed. The function $\varphi_{1,2}[E_{\lambda_k}]$ depends on the energy $E_{\lambda_k}$ of the spectrum of the input load at the resonant frequency $\lambda_k$. Without this energy the occurrence of an event (4) in the form of the extremum in the spectrum of the output vibration signal $G_\varepsilon(\omega)$ [3, 13] is impossible (Figure 3).

Then at the third level of regression analysis for each function $\varphi_i$ it is possible to consider specific possible values of $k_{i,j}$ of function $\varphi_{i,j}$ as probabilities in the range of values \{0,...,1\} of the event realization (4). So, if the natural oscillation frequency $\lambda_k$ enters the range $\Delta\omega$ of the spectrum of
external action $G_p \omega$ (Figure 3), then for the parameter of possible values of $\varphi_{1,1} \Delta \omega$ function in (6) [probability of the event realization (4)] the value equals one: $k_{1,1} = 1$. If $\lambda_k$ is not included in the frequency range $\Delta \omega$ of the input spectrum $G_p \omega$, then the implementation of the algorithm (2) and hence the event (4) is impossible, and therefore the value of the parameter $k_{1,1}$ should be taken as zero: $k_{1,1} = 0$. In this case the specified values of the parameter $k_{1,1}$ should follow from the real full-scale data for the diagnosed B&S.

For launching facilities it was established [12, 13] that the values of parameter $k_{1,1}$ of function $\varphi_{1,1} \Delta \omega$ pointed out should be consistent with the output vibrational signals $\xi t$ for which the energy spectrum $G_{\xi} \omega$ is used: for vibrational displacements $V t$, velocities $\dot{V}(t)$ or accelerations $\ddot{V}(t)$ (see Introduction). Full-scale data on the launching facilities show [12, 13] that the spectra of displacements $G_{\xi} \omega$ have a range $\Delta \omega$ to 20 Hz ($\Delta \omega \leq 20$ Hz). At the same time for the main bearing reinforced concrete structures (RCS) launching facilities their own time frequencies are in the frequency range $\lambda_k \geq 70$ Hz. In this case $k_{1,1} = 0$ is to be used.

In contrast, the spectra of vibration accelerations $G_{\ddot{V}} \omega$ on the launching facilities bearing structures have the frequency range $\Delta \omega = 120$ Hz [12, 13], which allows them to accept the parameter $k_{1,1}$ equal to one ($k_{1,1} = 1$). But in this case it is necessary to analyze the parameter $k_{1,2}$ of function $\varphi_{1,2} E_{\lambda_k}$ in (9) which can take the value $k_{1,2} = 1$, if the energy $E_{\lambda_k}$ in the spectrum of input action $G_p \omega$ is enough to generate an extremum at frequency $\lambda_k$ in the spectrum of vibrational accelerations $G_{\ddot{V}} \omega$ (Figure 3). Conversely, $k_{1,2} = 0$ if the energy $E_{\lambda_k}$ of the input action is not enough to develop the event (4) [2]. Thus, the $\varphi_1 G_p \omega$ (6) function can be either zero or one.

Similarly, at the second level of regression analysis in (5), the $\varphi_2 \eta \omega$ function can be considered for the transfer function $\eta \omega$ (Figure 1) of the dynamic system (Figure 2) which in general can depend on the product of three functions $\varphi_{2,1} C \cdot \varphi_{2,2} M$ and $\varphi_{2,3} K$, where $C$ and $M$, respectively, the rigidity and weight of the span reinforced concrete construction [17], and the parameter $K$ – a coefficient depending on the conditions of fixing the ends of the span reinforced concrete construction.

In the third and the last function $\varphi_3 \theta G_{\ddot{V}} \omega$ in (58) the symbol $\theta$ means the error [19] of recording the spectrum $G_{\ddot{V}} \omega$ of the output signals $\xi t = V t, \dot{V} t, \ddot{V} t$. It depends on the product of two functions:

$$\varphi_3 \theta_G = \phi_{3,1} \mu * \phi_{3,2} \theta_A,$$

where $\phi_{3,1} \mu$ – is a function depending on the sensitivity $\mu$ [23] of the method of functional vibration diagnostics [24];

$\phi_{3,2} \theta_A$ – function depending upon the registration equipment error $\theta_A$ of spectra of the output vibration signals $G_{\ddot{V}} \omega$ [23].

The statistics of full-scale data on the launching facilities for the systems of Testing and long-term monitoring [13] allows to obtain the values of parameters $k_{i,j}$ in the above functions $\varphi_2 \eta \omega$ and $\varphi_3 \theta G_{\ddot{V}} \omega$ at the third level of regression analysis. Under different operating conditions of bearing reinforced concrete construction [25-27] and when using different spans, specific stiffness and
under different conditions of fixing the ends and in the case of errors in recording vibration parameters for the probability \( \Pi \) (3), (5) (Fig. 3) of identification of diagnostic feature of reducing the bearing capacity of reinforced concrete construction on launching facilities (1) (Figure 1) they give the following results [13]:

\[
\Pi \rightarrow 0 \text{ at } \xi = V t ; \\
\Pi = 0.32 \pm 0.93 \text{ at } \xi = \hat{V} t .
\]

### 4. Conclusions

The result (8) indicates the low efficiency of the spectra of vibration movements for the purpose of diagnostic feature identifying of the bearing capacity reduce of bearing reinforced concrete construction the launching facilities (1) (Figure 1). However, in situations where there is no possibility of recording the energy spectra of the input dynamic loads in the operated buildings and constructions, the energy spectra of vibration displacements \( G_r \omega \) allow, on the basis of the algorithm (2), to determine the energy spectra of the input dynamic effects \( G_p \omega \) with a priori known transfer functions \( \eta \omega \) of the bearing elements of buildings and constructions:

\[ G_p \omega = G_r \omega \eta \omega . \]

The result (9) indicates that the effectiveness of the method of functional vibration diagnostics [1-3, 13, 17-20] to identify diagnostic feature (1) when using the output system of vibration acceleration spectra (Figure 2) may change widely in practice. In this case, as shown above, under favorable conditions, the functions \( \varphi_1 \left[ G_r \omega \right] \) and \( \varphi_2 \left[ \eta \omega \right] \) in (5) can reach values in the probability of detecting diagnostic feature (1) close to one. In this case, the probability \( \Pi \) (3) of the event (4) (Figure 1), (Figure 3) depends mainly on the values of the functions included in (7), that is, on the sensitivity \( \mu \) of the method of functional vibration diagnostics and the equipment error \( \theta \) of the vibration measuring means [13, 23, 24].

The results of studies in the Military-Space academy in the name of A.F. Mozhayskiy [1-3, 11-13] show that the value of the function \( \varphi_{3.1} \mu \) in (7) is interconnected with the parameters of the function \( \varphi_2 \left[ \eta \omega \right] \) in (5) and that the sensitivity of the method \( \mu \) vibration diagnostics increases sharply with the increase in the overall stiffness of the reinforced concrete structures [24, 28] (see the function \( \varphi_{2.1} C \) as a function \( \varphi_2 \left[ \eta \omega \right] \). This is especially true for the span concrete structures fixed rigidly, when the value of the function \( \varphi_{2.3} K \) increases sharply in comparison with the hinged structures [28]. In this case, the error of the vibration diagnostics method itself is a fraction of a percent [13], and then the function values (7) are determined mainly only by the equipment errors of energy spectra registration of vibrational accelerations, which in modern technical means may not exceed 5%. In this case, the confidence probability of the method of functional vibration diagnostics [1] in comparison with the result (9) may increase to the desired values 0.95 in the construction practice for carriers of reinforced concrete construction [25].

Each figure should have a brief caption describing it and, if necessary, a key to interpret the various lines and symbols on the figure.

### 5. References

[1] Gusev N N and Iskhakov Sh Sh 2010 Method of Functional Vibration Diagnostics of Change of Bearing Capacity of Soil Foundation and Building Structures of Buildings and Constructions A. s. RU (21) 2008. 147445/28(13)A

[2] Iskhakov Sh Sh 2008 Collected Reports of the Jubilee Scientific Readings of White night part 2 (Saint-Petersburg: IAEIPS) pp 346-50

[3] Iskhakov Sh Sh 2011 Vibration Test-Functional Diagnostics of the State of Building Structures
in the Monitoring of Buildings and Structures to Prevent Emergencies (Saint-Petersburg: Mozhaiskiy MSA) p 163

[4] Sokolov V, Musorina T, Starshinova E and Popovych I 2016 MATEC Web of Conferences (Electronic Materials vol 73) Article ID 04016

[5] Yun H and Reddy L 2011 Advances in Civil Engineering vol 2011 Article ID 275270

[6] Bolotin V V 1971 Application of Methods of Probability Theory and Reliability Theory in Calculations of Constructions (Moscow: Stroyizdat) p 255

[7] Pugachyov V S 1957 Theory of Random Functions and Their Application to Automatic Control Problems (Moscow: Gostehizdat) p 627

[8] Bendat J and Pirsel A 1974 Measurement and Analysis of Random Processes (Moscow: Mir) p 464

[9] Barshtein M F 1972 Guide to the Dynamics of Structures of Dynamic Calculation of High Buildings on the Wind ed B G Korneev and I M Rabinovich (Moscow: Stroyizdat) pp 286–321

[10] Iskhakov Sh Sh 2012 J. Science and security №3 p 12

[11] Kozin P A, Iskhakov Sh Sh, Kovalev F E and Vaskevich V M 2010 Collection of Scientific Works Accident of Prevention of Buildings and Structures (Moscow: WELD) pp 35–47

[12] Iskhakov Sh Sh, Kovalev F E and Vaskevich V M 2012 Security of Russia. Safety of the Building Complex ed N A Mahutov, O I Lobkov et al pp 711-9

[13] Iskhakov Sh Sh, Kovalev F E, Mohnatkin A P and Starchukov D S 2015 Formation and Development of Vibration Monitoring Systems of the Technical Condition of Bearing Elements of Launch Facilities (Saint-Petersburg: Mozhaiskiy MSA) p 110

[14] Lyapin A and Shatilov Y 2016 J. Procedia Engineering vol 150 pp 1867–71

[15] Zhou Z 2008 VDM Verlag pp 55-100

[16] Iorish Y I 1963 Vibrometry (Moscow: Mashgiz) p 743

[17] Telichenko V I, Khlystunov M S, Korol and Prokofiev V I 2009 Collection of Scientific Works of Accident Prevention of Buildings and Structures ed K I Eremin (Moscow: WELD) No8 pp 27–42

[18] Iskhakov Sh Sh, Kovalev F E and Vaskevich V M 2011 Seismological Studies in Arctic and Near-Arctic Regions ed F N Yudahin (Yekaterinburg: Ural branch of the RAS) pp 220–34

[19] Savin S N, Demishin S V and Sitnikov I V 2011 Mag. of Civil Engineering No7 (25) pp 33–9

[20] Savin S N 2012 Mag. of Civil Engineering №7 (33) pp 58–62

[21] Iskhakov Sh Sh, Kovalev F E and Vaskevich V M 2013 Design and application of monitoring systems for engineering (construction) structures of buildings and structures (Saint-Petersburg: Mozhaiskiy MSA) p 158

[22] Kondakov N I 1975 Logical Dictionary-Reference Book (Moscow: Nauka) p 717

[23] Zemlyanskiy A A 2001 Inspection and Testing of Buildings and Structures (Moscow: ASV) p 240

[24] Iskhakov Sh Sh, Kovalev F E and Mohnatkin A P 2013 Proc.of the Mozh. Mil. Space Acad. №641 pp 159-66

[25] SP 63.13330.201: Set of rules. Concrete and reinforced concrete structures. Fundamentals. The updated edition of SNiP 52-01-2003 (Moscow: Minregion RF) p 147

[26] International Organization for Standardization 2015 ISO 2394 General principles on reliability for structures (Geneva: International Organization for Standardization)

[27] European Committee for Standardization 2002 EN 1990:2001 Eurocode. Basis of structural design (Brussels: CEN) p 89

[28] Iskhakov Sh Sh, Kovalev F E and Mohnatkin A P 2013 Proc.of the Mozh. Mil. Space Acad. No639 pp 39-44 №639