Modelling and analysis of three-layer floor slabs made of composite materials

K Z Khayrnasov
Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia
Email: kamilh@mail.ru

Abstract. A three-layer composite floor slab is considered, consisting of two external bearing layers made of a multilayer composite material, and a filler layer between the bearing layers made of polystyrene. Relations are given to determine the characteristics of multilayer composite materials with different locations of the base layers of the material. The technique of the optimal arrangement of the basis of the composite material in layers is given. Methodology for calculating three-layer structures consisting of external bearing layers and a filler layer between them. The analysis of the results and prospects of using three-layer composite floor slabs in construction.

1. Introduction
Floor slabs are used everywhere in housing construction and their geometric parameters have remained virtually unchanged for decades, despite the fact that programs have appeared that allow a detailed calculation of such products, taking into account geometric and physical nonlinearities, to take into account the structure of the material [1, 2]. Designs made of a material having several layers with different physical and mechanical characteristics have recently become very widespread. This is due to the advantages that are achieved when designing a material with specified parameters. This primarily relates to composite materials made of durable threads based on glass, boro, carbon, and other fibers bonded together. At the same time, the calculation of such structures is a rather difficult task, given that such a heterogeneity of the material requires an adequate mathematical apparatus that actually describes the behavior of the material and, ultimately, the structure under the action of loads. When describing the material, mainly idealized models of its behavior are used, or the characteristics of the material are averaged, and the calculation for the obtained orthotropic material is performed. Recent theoretical and experimental studies have made it possible to obtain a fairly accurate theory of calculating structures from composite material that actually describes its behavior [3, 4].

The combination of composite material and traditionally used in the manufacture of floor slabs, for example, of reinforced concrete, gives a significant advantage in strength, dynamic characteristics, and crack resistance compared to reinforced concrete slabs. Currently, there are a significant number of publications devoted to the experimental and theoretical study of reinforced concrete structures reinforced with composite materials [5-7].

However, the theoretical studies presented do not take into account the compatibility of deformations of the composite material and the reinforced concrete base, the physical and geometric nonlinearity of the materials used, the structure of concrete that accepts compressive loads is
significant (15 times) greater than tensile. In addition, the comparison of a three-layer composite and reinforced concrete floor slab was not investigated.

2. Methodology
There are two approaches to modeling a multilayer composite material consisting of layers with different orientations of the base layers. The first is when the reduced characteristics for the material are determined and the calculation is further carried out as for an orthotropic material with the calculated reduced characteristics of the multilayer composite material and the second when the number of equations depends on the number of layers of the composite material and the calculation is based on the characteristics of each layer [8-11]. In this study, the first method is used: the method of reduced characteristics. For the three-layer composite plate, considered in this paper, consisting of the outer bearing layers of the multilayer composite material and the filler layer, between the bearing layers consisting of polystyrene, which mainly perceive shear stresses [12-15]. Offsets in the outer support layers are to be determined. The displacements in the filler are determined by dotted laws from the displacements of the bearing layers. The displacements in the middle plane of the intermediate layer of the three-layer structure are defined as the arithmetic mean of the displacements of the bearing layers. The angle of inclination of the line of displacement along the thickness of the filler is the difference between the movements of the bearing layers divided by the thickness of the filler.

3. Main part
The connection matrix of stresses and strains for an orthotropic material, the orthotropic axes of which coincide with the axes of coordinates, in the plane-stressed state have the form

\[ \sigma = (E) \varepsilon, \tag{1} \]

where

\[
(E) = \begin{pmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{21} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{pmatrix}, \quad \varepsilon^T = (\varepsilon_s, \varepsilon_\theta, \varepsilon_{s\theta}), \quad \sigma^T = (\sigma_s, \sigma_\theta, \sigma_{s\theta}),
\]

\[
Q_{11} = E_s/(1 - v_{s\theta}v_{s\theta}), \quad Q_{12} = v_{s\theta}E_s/(1 - v_{s\theta}v_{s\theta}),
\]

\[
Q_{21} = v_{s\theta}E_s/(1 - v_{s\theta}v_{s\theta}), \quad Q_{22} = E_\theta/(1 - v_{s\theta}v_{s\theta}), \quad Q_{66} = G_{66}.
\]

Rotation of the axes of coordinates at an angle \( \theta \) converts the matrix of elastic coefficients to the form

\[
(\bar{E}) = \begin{pmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66}
\end{pmatrix},
\]

where

\[
\bar{Q}_{11} = c^4Q_{11} - s^4Q_{22} + 2(Q_{12} + 2Q_{66})s^2c^2,
\]

\[
\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + (s^2 + c^2)Q_{22}, \quad \bar{Q}_{16} = (c^2Q_{11} - s^2Q_{12} + (Q_{12} + 2Q_{66})(s^2 - c^2))sc,
\]

\[
\bar{Q}_{22} = s^4Q_{11} - c^4Q_{22} + 2(Q_{12} + 2Q_{66})s^2c^2, \quad \bar{Q}_{26} = (s^2Q_{11} - c^2Q_{12} - (Q_{12} + 2Q_{66})(s^2 - c^2))sc,
\]

\[
\bar{Q}_{66} = (Q_{11} - 2Q_{12} + Q_{22})s^2c^2 + (s^2 - c^2)Q_{66}, \quad s = \sin \theta, \quad c = \cos \theta.
\]

Layer deformations at a distance \( z \) from the middle surface take the form

\[ \varepsilon = \varepsilon^0 + z \chi^0, \]

where \( \varepsilon^0 \) are the deformations of the middle surface, \( \chi^0 \) are the curvature changes. Substituting these relations into equation (1) we get

\[ \sigma = (\bar{Q}) \varepsilon^0 + z(\bar{Q}) \chi^0. \]
If the stresses are expressed through normal efforts and moments
\[ \mathbf{N} = \int_{-h/2}^{h/2} \sigma dz, \quad \mathbf{N}' = (N_s, N_\theta, N_{s\theta}), \quad \mathbf{M} = \int_{-h/2}^{h/2} \sigma z dz, \quad \mathbf{M}' = (M_s, M_\theta, M_{s\theta}). \]  
(2)

Here is designated \( \mathbf{N} \) – are the membrane forces, \( \mathbf{M} \) – are the bending moments.

Conducting the integration of expressions (2) we have

The generalized characteristics of a multilayer composite material with layers of different angular locations of the base layers are obtained from the following relations.

The relationship between the forces and strain can be represented as
\[ (\mathbf{N}, \mathbf{M}) = (A \mathbf{B}) (\mathbf{\varepsilon}^a). \]

Here is designated \( \mathbf{N} \) – are the membrane forces, \( \mathbf{M} \) – are the bending moments.

Denoted here
\[ A_{ij} = \sum_{k=1}^{n} \bar{Q}_{ij}(h_k - h_{k-1}), \quad i, j = 1, 2, 3, \]
\[ B_{ij} = \sum_{k=1}^{n} \bar{Q}_{ij}(h_k^2 - h_{k-1}^2), \quad i, j = 1, 2, 3, \]
\[ D_{ij} = \sum_{k=1}^{n} \bar{Q}_{ij}(h_k^3 - h_{k-1}^3), \quad i, j = 1, 2, 3. \]

(3)

The designations in the formula (3) are shown in figure 1.

\[ \text{Figure 1. Multilayer composite structure} \]

A linear change in the transverse shear deformations in thickness complements, in the expression of the Hooke law, matrices of elastic coefficients by matrices stiffness of the shear
\[ \begin{pmatrix} \sigma_4^{(k)} \\ \sigma_5^{(k)} \end{pmatrix} = \begin{pmatrix} \bar{Q}_{44} & 0 \\ 0 & \bar{Q}_{55} \end{pmatrix} \begin{pmatrix} \bar{\varepsilon}_4^{(k)} \\ \bar{\varepsilon}_5^{(k)} \end{pmatrix}, \]

где \( \bar{Q}_{44} = G_{13} \), \( \bar{Q}_{55} = G_{23} \), \( G_{13}, G_{23} \) - модули сдвига.

Consider the kinematic model of a multilayer shell, when the number of resolving equations depends on the number of layers of material. In this case, it is assumed that the displacements of the carrier layers are to be determined and are independent parameters, while the displacements of the layers connecting the carrier layers are determined based on the displacements of the carrier layers, according to one or another law and are dependent parameters.

Thus, the system of equations with \( n \) bearing layers has an order of magnitude \( n \) times greater than the system of resolving equations in the reduced stiffness method. For simplicity,
we consider a three-layer packet of thickness $t$ with two bearing layers of thickness $t_1, t_3$ and a filler between the bearing layers of thickness $t_2$. In this work, displacements along the thickness of the packet are distributed according to the law of a broken line.

Consider the three-layer structure of a composite material, which consists of two external carrier layers and a layer of aggregate between them. For carrier layers apply the equations discussed above. The displacements of the filler are defined as the arithmetic average of the displacements of the carrier layers. Angle of rotation as the difference in the displacements of the bearing layers divided by the thickness of the composite material.

Figure 2 shows a three-layer composite structure with two outer carrier layers and a layer of aggregate between them.

The displacements of the neutral line of the aggregate and the angle of rotation of the normal during deformation are determined from the relations:

$$u_0 = (\bar{u}_1 + \bar{u}_2)/2, \quad \varphi_0 = (\bar{u}_1 - \bar{u}_2)/h_3, \quad \bar{u}_1 = u_1 - h_1 e_{13}/2, \quad \bar{u}_2 = u_2 + h_2 e_{23}/2$$

where $u_0$ is the displacement and $\varphi_0$ is the rotation angle of the neutral filler line, $r$ are the shell radius, $e_{13}, e_{23}$ are the rotation angles around the coordinate lines, $\varphi_s, \varphi_\theta$ are the angles of inclination of the meridian to the axis of the shell and the angle in the circumferential direction

$$e_{13} = \left(\frac{\partial w}{\partial s} + u \frac{\partial \varphi}{\partial s}\right); \quad e_{23} = -\frac{1}{r} \left(\frac{\partial w}{\partial \theta} - v \sin \varphi\right).$$

Here denoted $u$ is the indicated by displacement in the meridional direction, $w$ is the displacement in the normal direction, $v$ is the displacement in the circumferential direction of the shell.

Displacements in the placeholder at a distance $z$ from the neutral axis can be written as

$$v_0(z) = v_0 + z \varphi_0 = 0.5(v_1 - h_1 e_{13}/2 + v_2 + h_2 e_{23}/2) + z(v_1 - h_1 e_{13}/2 - v_2 - h_2 e_{23}/2)/h_3,$n_0(z) = u_0 + z \varphi_0 = 0.5(u_1 - h_1 e_{13}/2 + u_2 + h_2 e_{23}/2) + z(u_1 - h_1 e_{13}/2 - u_2 - h_2 e_{23}/2)/h_3,$n_0(z) = w_0 + z(w_1 - w_2)/h_3.$n_0(z) = w_0 + z(w_1 - w_2)/h_3.$

Thus, one can consider a model of a shell structure consisting of several layers. The designations in the formula (4) are shown in figures 2.

![Figure 2. Three-layer composite structures](image)
When calculating and determining the stress-strain state of a three-layer floor slab, it is necessary to arrange the basis of the composite carrier layer along the lines of maximum stresses. The location of the basis of the composite material along the paths of maximum stresses allows you to get the most optimal in terms of stress-strain floor slab. To determine the paths of maximum stresses, it is necessary to calculate the stress-strain state of a three-layer floor slab of a homogeneous material under the action of operational loads. To determine the paths of maximum stresses, it is necessary to calculate the stress-strain state of a three-layer floor slab of a homogeneous material under the action of workloads. Having determined the paths of maximum stresses, we locate the basis of the bearing layers along the lines of maximum stresses and determine the generalized characteristics of the bearing layers of the multilayer composite material. Subsequently, after calculating the stress-strain state of the three-layer composite floor slab, adjust the location of the base of the bearing layers of the three-layer floor slab. We take into account that the composite material has low shear characteristics. After several such iterations, we obtain the optimal floor slab in terms of the stress-strain state [16, 17].

4. Results.
As a result of the study, a three-layer floor slab was calculated, consisting of external bearing layers made of multilayer fiberglass and a filler layer between the bearing layers of polystyrene (Figure 3). The above characteristics of the bearing layers of the composite material were calculated by the method described above. The aggregate layer perceived mainly shear deformations. A three-layer floor slab with bearing layers 20 mm thick was considered. The total thickness of the plate was 250mm. The dimensions of the floor slab in the plan were 6 meters and 1.5 meters. We considered a floor slab supported on a contour. The normal load was the allowable load on reinforced concrete floor slabs: 800 kg / m2. Calculation results: tensile strength was more than 10, deflection 6.45 mm.

Figure 3. Stress-strain state of a three-layer floor slab

5. Conclusion.
The analysis of the use of composite materials in construction for the repair and strengthening of building structures. A technique has been developed for determining the reduced characteristics of multilayer composite materials, which allows one to take into account multilayer composites with different positions of the composite materials in the layers. The technique of the optimal arrangement of the basis of composite materials along the lines of maximum stresses is considered. A method for calculating three-layer floor slabs consisting of external supporting multilayer composite layers and a filler layer between the bearing layers, which mainly accepts shear stresses and prevents the convergence of the bearing layers, is presented.
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