An Over-Modulated Model Predictive Current Control for Permanent Magnet Synchronous Motors

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ABSTRACT To improve the voltage utilization rate of DC bus and reduce the voltage vector tracking error, a finite-control-set modulated model predictive control (FCS-M2PC) with over modulation capacity for permanent magnet synchronous motor (PMSM) incorporating penalty function is proposed in this paper. Firstly, three modulation regions are determined according to the combined action of dead-beat control and model predictive control (MPC). Secondly, three, two and one voltage vectors are applied to linear modulation region (LMR), over modulation region I (OMR-I) and over modulation region II (OMR-II), respectively, to synthesize the reference voltage vector. Thirdly, to obtain better steady-state performance in the whole modulation region, the optimal vector dwell-times can be obtained from the uniform penalty function defined by a set of mode parameters to minimize the error between the current references and the current predictions, rather than applying a single voltage vector in the conventional finite-control-set model predictive control (FCS-MPC). The effectiveness and superiority of the proposed method are validated experimentally.

INDEX TERMS Model predictive current control, over modulation, penalty function, dead-beat control, steady-state performance.

I. INTRODUCTION
In last decade, three-phase permanent magnet synchronous motors (PMSMs) have attracted a lot of attention and been widely adopted in the industrial field due to their interesting features, including high power density, high efficiency, and high reliability [1]-[3]. Some applications like electrical vehicle demand such a traction system that is characterized with wide speed range and high DC bus utilization, which increases the difficulty of controller design [4]-[5].

Recently, a plenty of model predictive control (MPC) methods have been extensively developed in conjunction with active front-end-rectifiers, matrix converters and motor drive systems due to its expected dynamic response, feasible implementation of multi-objective optimization and stronger performance in terms of nonlinearities control [6]-[9]. Among them, the finite-control-set model predictive control (FCS-MPC) method has been a promising one thanks to the rapid and robust development of microprocessor technology. The cost function in FCS-MPC method includes the dynamic model and control variable of the system, and the voltage vector minimizing it is selected as the optimal vector [10]-[12]. A well-known drawback of the traditional FCS-MPC is the undesired steady state performance as only one voltage vector is decided and applied during each sampling period.

Several solutions have been proposed to overcome this drawback [13]-[27]. The steady-state performance can be enhanced by two-vector-based methods [13]-[16]. The optimal vector determined by the cost function is applied, in conjunction with the zero vector [13]-[14] and with another active vector or the zero vector [15]-[16]. The dwell-times of two vectors are calculated according to the error between the reference vector and two active vectors. Comparatively, owing to the use of two active vectors, a better steady state performance can be obtained by using the methods in [15]-[16], especially in the high-speed region.

In order to further improve the steady state performance, several enhanced FCS-MPC methods incorporating modulators have been studied, termed as finite-control-set...
modulated model predictive control (FCS-M2PC) [17]-[21]. In the conventional FCS-M2PC method proposed in [21], two active voltages and the zero vector are applied to the inverter. The two active voltages are selected by the cost function and deemed as the optimal and suboptimal active voltages, respectively. The dwell-times of three vectors are calculated according to their corresponding cost function values, and their applied sequences rely on the location of the reference vector. Using these FCS-M2PC methods, the current ripple and harmonics can be significantly reduced, while some issues are still left open at present.

One issue is that the calculated dwell-times may be negative as highlighted in [22]. For example, [23] proposes an FCS-M2PC for a doubly fed induction generator. In this method, three vectors are selected and applied, and their dwell-times are acquired based on a sinusoidal law derived from the geometric relationship of the vectors, which is rather similar to the well-known SVPWM. The resulted dwell-times will be impractical if they are out of the interval [0, T]. As a consequence, a secondary regulation in terms of the impractical dwell-times is required, subsequently leading to a large tracking error. To overcome this drawback, an FCS-M2PC considering dwell-time regulation is proposed in [24]. In particular, a hybrid control set is established, which is composed of six active vectors and 6 × Nm virtual vectors. All the vectors in the hybrid control set are dwell-time weighted, on this basis, an optimization problem associated with the dwell-times is posed. After transforming the original problem, both the optimal direction and dwell-time of the output vector can be obtained simultaneously via finite enumeration and evaluation online. However, the total number of the enumerations and evaluations during the optimization process is equal to Nm (the number of the virtual vector groups) + 6. Thus, it is very demanding for the real-time implementation of this method.

Another issue of the existing FCS-M2PC methods is the lack of overmodulation capability [25]. When the reference vector lies outside the inscribed circle of the hexagon shaped by the endpoints of six active vectors, an overmodulation control is required. Several works concerning the overmodulation control have been carried out to enhance DC bus voltage utilization and hence the extend the speed range of the PMSM drive systems. Representatively, an overmodulated FCS-M2PC method is proposed in [26]. When overmodulation operation is required, the core idea of this method is to limit the reference vector into the linear modulation region (LMR) by adjusting the amplitude of reference vector. As such, the adjusted reference vector can also be synthesized using two active voltage vectors and the zero vector, as the same for the case in LMR. Differently, an FCS-M2PC with optimized overmodulation strategy is proposed in [27]. The dwell-times of three vectors are calculated according to the error between the measured and the desired currents. The operating region is determined on the basis of the number of the negative dwell-times, which results in extra calculations and a delay in control. In addition, [27] verifies the method using the output dwell-times results merely.

In this paper, an FCS-M2PC incorporating a uniform penalty function with the overmodulation capacity is proposed to accomplish the optimized response in both LMR and overmodulation regions. A similar cost function in the form of a quadratic sum of squares with regard to the identification of the reference voltage vector in [27] is used in the proposed method. Differently, the cost function is not only used to identify the reference voltage vector but also used to construct a standard convex optimization form to find the optimal solution. In the LMR, two active vectors are selected by minimizing the cost function, and employed along with the zero vector to synthesize the reference vector. In this way, the steady-state performance in the LMR can be extensively improved compared to that in conventional FCS-MPC [10]-[12] and two-vector-based FCS-MPC [13]-[16]. Then, in order to enhance the overmodulation capacity, the overmodulation region is further divided into the overmodulation region I (OMR-I) and the overmodulation region II (OMR-II). To this end, the reference vector is acquired in accordance with the dead-beat control concept, and then its amplitude is compared with the maximum output voltage amplitude of inverter in different modulation regions. Afterwards, a uniform penalty function is presented for all three modulation regions. Both the current tracking errors and the constraints of dwell-times are considered in the uniform penalty function by defining a set of mode parameters. The optimization problem associated with the penalty function is solved to calculate the dwell-times of three, two and one voltage vectors in the LMR, OMR-I and OMR-II, respectively, without requiring a second modification for duty times, as a consequence extra computation burden can be greatly alleviated.

The rest of this paper is structured as follows. The mathematical models of three-phase drive system incorporating two-level voltage source inverter (2L-VSI) are presented in Section II. In Section III, the proposed FCS-M2PC method is described in detail. The experiment results are given in Section IV. Finally, conclusions are made in Section V.

II. Mathematical Model of Three-Phase Drive and Conventional FCS-MPC

In this section, the mathematical model of PMSM driven by 2L-VSI as illustrated in Fig. 1 is presented. Meanwhile, the core principle of conventional FCS-MPC is introduced, which is a foundation of the proposed FCS-M2PC method.

A. Two-Level Voltage Source Inverter

The 2L-VSI provides eight switching states and each pattern corresponds to a voltage vector expressed as v_i, where i relies on the switching states and i ∈ {0, 1, 2, 3, 4, 5, 6, 7}. v_i is
defined by the binary number \([S_a, S_b, S_c]\), and \(S_i\) \((i = a, b, \text{or} c) \in \{0, 1\}\). The output voltage of inverter can be calculated by

\[
v_{\text{out}} = \frac{2v_{dc}}{3} \left( S_a + S_b e^{\frac{j\pi}{3}} + S_c e^{-\frac{j\pi}{3}} \right)
\]  

(1)

**FIGURE 1.** Schematic of two-level voltage source inverter and PMSM.

Graphically, six active vectors \(v_1(100), v_2(110), v_3(010), v_4(011), v_5(001), \) and \(v_6(101)\) are 60° apart and describe the vertices of a hexagon with a magnitude of \(2v_{dc}/3\), while two zero vectors, \(v_0(000)\) and \(v_7(111)\), are at the origin, as shown in Fig. 2. In what follows, the hexagon is called voltage hexagon (VH), and its inscribed circle with the radius of \(v_{dc}/\sqrt{3}\) and its circumscribed circle with the radius of \(2v_{dc}/3\) are tagged with IC and CC, respectively.

**FIGURE 2.** The modulation region.

**B. Mathematical Model of PMSM**

The mathematical model of three-phase PMSM is normally based on \(d-q\) frame, as follows

\[
\begin{align*}
\frac{d}{dt}i_d &= \frac{1}{L_d} \left( v_d - R_i i_d + \omega_e L_i i_q \right) \\
\frac{d}{dt}i_q &= \frac{1}{L_q} \left( v_q - R_i i_q - \omega_e \left( L_i i_d + \psi_f \right) \right)
\end{align*}
\]

(2)

\[
T_e = \frac{3}{2} p_s \mu \left[ i_d (L_d - L_q) + \psi_f \right]
\]

(3)

where \(v_d\) and \(v_q\) are the \(d\)- and \(q\)-axes components of stator voltage, respectively; \(i_d\) and \(i_q\) are the \(d\)- and \(q\)-axes components of stator currents, respectively; \(R_i\) is the armature resistance; \(L_d\) and \(L_q\) are defined as \(d\)- and \(q\)-axes inductances, respectively; \(\psi_f\) is the permanent magnet flux; \(T_e\) is the developed electromagnetic torque; \(p_s\) is the number of pole pairs.

Regarding the interior PMSM (IPMSM) that is concerned in this paper, it has \(L_d < L_q\), so \(i_d=0\) control scheme is adopted in order to acquire a maximum electromagnetic torque shown in (3).

**C. Conventional FCS-MPC**

In principle, the conventional FCS-MPC for PMSM drive system predicts the system behavior for each voltage vector and then selects an optimum voltage vector as the output. This prediction is based on the discretized model of the system, which is generally deduced by the following forward Euler equation, as

\[
\frac{di}{dt} \approx \frac{i(k+1) - i(k)}{T_s}
\]

(4)

where \(T_s\) is the sampling period. On this basis, (2) can be discretized to obtain the predicted current value at \((k+1)\)th as

\[
\begin{align*}
i_d(k+1) &= i_d(k) + \frac{T_s}{L_d} (v_d(k) - R_i i_d(k) + \omega_e L_i i_q(k)) \\
i_q(k+1) &= i_q(k) + \frac{T_s}{L_q} (v_q(k) - R_i i_q(k) - \omega_e (L_i i_d(k) + \psi_f))
\end{align*}
\]

(5)

The \(d\)- and \(q\)-axes \((k)\)th voltage \(v_d(k)\) and \(v_q(k)\) can be deduced from (5) as

\[
\begin{align*}
v_d(k) &= \frac{L_d}{T_s} (i_d(k+1) - i_d(k)) + R_i i_d(k) - L_q \omega_e i_q(k) \\
v_q(k) &= \frac{L_q}{T_s} (i_q(k+1) - i_q(k)) + R_i i_q(k) + L_d \omega_e i_d(k) + \psi_f \omega_e (k)
\end{align*}
\]

(6)

Afterwards, the stator current is predicted for each vector provided by the inverter, and the dwell-time of vector is assumed as the sampling period \(T_s\). The system’s future behavior is evaluated by a cost function given by

\[
g(v_i) = E_d^2(v_i) + E_q^2(v_i)
\]

(7)

with

\[
E_d(v_i) = i_d^2(k+1) - i_d^2(k) \\
E_q(v_i) = i_q^2(k+1) - i_q^2(k)
\]

(8)

(9)

where \(i_d^2\) and \(i_q^2\) are the \(d\)- and \(q\)-axes current references, respectively; \(i_d(k+1)\) and \(i_q(k+1)\) are the predicted \(d\)- and \(q\)-axes currents, respectively. Following this, by comparing the cost function values associated with all possible vectors, the vector minimizing the cost function is selected as the optimum vector and applied to the inverter.

**III. Proposed Over-modulated Method**

**A. Determination of Operating Modulation Region**
In order to optimize the system’s performance, different modulation strategies should be adopted for different modulation regions. Thus, the first objective is to determine the operating modulation region.

Regardless of control methods, tracking current references are always the primary goal. By means of dead-beat control, the goal can be expressed as

\[
\begin{align*}
i_d(k+1) &= i_d^* \\
i_q(k+1) &= i_q^*
\end{align*}
\]

(10)

Substituting (10) into (6), the d- and q-axes (k)th voltage reference \(v_d^*\) and \(v_q^*\) can be deduced as

\[
\begin{align*}
v_d^* &= \frac{L_d}{T_s} \left( i_d^* - i_d(k) \right) + R_{d} i_d(k) - L_{q} \omega_{q}(k) i_q(k) \\
v_q^* &= \frac{L_q}{T_s} \left( i_q^* - i_q(k) \right) + R_{q} i_q(k) + L_{d} \omega_{q}(k) i_d(k) + \psi_{d} \omega_{q}(k)
\end{align*}
\]

(11)

Accordingly, the amplitude of the reference voltage can be expressed by

\[
\left| v_{ref} \right| = \sqrt{\left( v_d^* \right)^2 + \left( v_q^* \right)^2}
\]

(12)

where \(v_{ref}\) is the reference voltage.

As shown in Fig. 2, three modulation regions can be determined according to the obtained amplitude of \(v_{ref}\), namely LMR, OMR-I and OMR-II. To be precise, the system is operated in LMR when \(v_{ref}\) is smaller than or equal to \(v_{dc}/\sqrt{3}\). Otherwise, the system is in OMR, which can be further divided into OMR-I when \(v_{ref}\) is larger than \(v_{dc}/\sqrt{3}\) and smaller than or equal to \(2v_{dc}/3\) and OMR-II when \(v_{ref}\) is larger than \(2v_{dc}/3\) and smaller than or equal to \(2v_{dc}/\sqrt{3}\).

B. Calculation of vector dwell-times for whole modulation origin

MPC method considering LMR is studied, where two active voltage vectors along with a zero voltage vector are applied to inverter during each sampling period. Their dwell-times are calculated according to cost function values corresponding to the three vectors, namely

\[
\begin{align*}
g_0 &= E_d^2 \left( v_0 \right) + E_q^2 \left( v_0 \right) \\
g_1 &= E_d^2 \left( v_{opt} \right) + E_q^2 \left( v_{opt} \right) \\
g_2 &= E_d^2 \left( v_{sub} \right) + E_q^2 \left( v_{sub} \right)
\end{align*}
\]

(13)

where \(v_{opt}\) and \(v_{sub}\) are the optimum and suboptimum vectors, respectively, which produce the smallest and second-smallest cost function values, respectively.

In order to optimize the system performance in different modulation origins with a uniform penalty function, a set of mode parameters \((\lambda_0, \lambda_1, \lambda_2)\) is defined as

\[
\begin{align*}
(1,1,1), & \text{if } 0 < \left| v_{ref} \right| < \frac{v_{dc}}{\sqrt{3}} \text{, LMR} \\
(0,1,1), & \text{if } \frac{v_{dc}}{\sqrt{3}} \leq \left| v_{ref} \right| \leq \frac{2v_{dc}}{3} \text{, OMR-I} \\
(0,0,1), & \text{if } \frac{2v_{dc}}{3} < \left| v_{ref} \right| \leq \frac{2v_{dc}}{\sqrt{3}} \text{, OMR-II}
\end{align*}
\]

(14)

On this basis, the current error can be minimized during each sampling period by modulating the selected three, two or one voltage vectors. The problem is then translated into obtaining optimal dwell-times \(t_j\) with \(j \in \{0,1,2\}\) that minimize the following error function (15) under the constraint of \(\lambda_{d0} + \lambda_{d1} + \lambda_{d2} - T_s = 0\), as

\[
\begin{align*}
\text{minimize } &: f_{0}(t) = \lambda_{d0} g_{d0} + \lambda_{d1} g_{d1} + \lambda_{d2} g_{d2} \\
\text{subject to } &: h_{1}(t) = \lambda_{d0} t_0 + \lambda_{d1} t_1 + \lambda_{d2} t_2 - T_s = 0
\end{align*}
\]

(15)

which is of a standard convex optimization form, since \(f_{0}(t)\) is convex and \(h_{1}(t)\) is affine.

To solve the fore-mentioned optimization problem, a penalty function associated with (15) is defined as

\[
p(t, \mu) = f_{0}(t) + \mu \left( \lambda_{d0} t_0 + \lambda_{d1} t_1 + \lambda_{d2} t_2 - T_s \right)^2
\]

(16)

where \(\mu > 0\) is the penalty factor. The second term in \(p\) penalizes deviations of \(t\) from feasibility. If the penalty factor \(\mu\) is sufficiently large, solutions of (16) is also applicable to the original problem shown in (15).

In the LMR, the minimal value of (16) is found by

\[
\begin{align*}
\frac{\partial p}{\partial t_0} &= 2 \left[ \lambda_{d0} g_{d0} + \mu \left( \lambda_{d0} t_0 + \lambda_{d1} t_1 + \lambda_{d2} t_2 - T_s \right) \right] = 0 \\
\frac{\partial p}{\partial t_1} &= 2 \left[ \lambda_{d1} g_{d1} + \mu \left( \lambda_{d0} t_0 + \lambda_{d1} t_1 + \lambda_{d2} t_2 - T_s \right) \right] = 0 \\
\frac{\partial p}{\partial t_2} &= 2 \left[ \lambda_{d2} g_{d2} + \mu \left( \lambda_{d0} t_0 + \lambda_{d1} t_1 + \lambda_{d2} t_2 - T_s \right) \right] = 0
\end{align*}
\]

(17)

Then, the solution of (17) can be calculated by Elimination method, as

\[
\begin{align*}
t_0 &= \frac{\mu g_{d0} g_{d2}}{\lambda_{d0} g_{d1} g_{d2} + \mu \lambda_{d1} \left( g_{d1} g_{d2} + g_{d0} g_{d2} + g_{d2}^2 \right)} T_s \\
t_1 &= \frac{\mu g_{d0} g_{d1}}{\lambda_{d0} g_{d1} g_{d2} + \mu \lambda_{d1} \left( g_{d1} g_{d2} + g_{d0} g_{d2} + g_{d2}^2 \right)} T_s \\
t_2 &= \frac{\mu g_{d0} g_{d1} g_{d2}}{\lambda_{d0} g_{d1} g_{d2} + \mu \lambda_{d1} \left( g_{d1} g_{d2} + g_{d0} g_{d2} + g_{d2}^2 \right)} T_s \\
t_0 &= T_s - t_1 - t_2
\end{align*}
\]

(18)

According to the definition of penalty function, \(\mu\) is a monotonically increasing sequence, which means \(\mu \rightarrow \infty\). Hence, (18) can be further simplified as
where \( t_0, t_1 \) and \( t_2 \) are the dwell-time of \( v_0(v_7) \), \( v_{opt} \) and \( v_{sub} \) respectively and all of them are non-negative according to the constraint and results.

In OMR-I, considering the minimum current error, two active vectors (the optimum and suboptimum vectors) are modulated to synthesize \( v_{opt} \). The minimal current error can be achieved through the collaborative action of \( v_{opt} \) and \( v_{sub} \) in a single sampling period. In this case, \( \lambda_0 = 0 \) and the optimal dwell-times allocation of the \( v_{opt} \) and \( v_{sub} \) can be deduced as

\[
\begin{align*}
  t_i &= T_i - t_0 - t_f,
  t_1 &= T_1 - T_2 = \frac{g_2}{g_1 + g_2} T_s, \\
  t_2 &= T_2 - T_1 = \frac{g_1}{g_1 + g_2} T_s.
\end{align*}
\]

(20)

Moreover, for a large load transient or a DC bus voltage fluctuation could incur a deeper overmodulation, in which manner, the system operates in OMR-II. In this case, \( v_{ref} \) is beyond the boundary of hexagon. In this region, in order to further improve voltage utilization rate, one active vector (the optimal vector) is applied with the entire sampling period to synthesize \( v_{ref} \), the optimal dwell-time allocation of the \( v_{opt} \) is defined as

\[
t_1 = T_i
\]

(21)

In this way, the operation smoothly transitions from PWM modulation at small current errors to two-vector overmodulation in OMR-I and then to FCS-MPC performance in OMR-II. This ensures the optimum performance in all modes of operation.

C. PWM generation for whole modulation origin

After obtaining the vectors and dwell-times, the exemplified switching sequence of each phase for applying the selected voltage vectors to the inverter are shown in Fig. 3. For exemplification, the zero vector is divided into \( v_0 \) and \( v_7 \) evenly according to the duty time and \( v_4 \) and \( v_6 \) are selected as \( v_{sub} \) and \( v_{opt} \) respectively in LMR. Similarly, \( v_4 \) and \( v_6 \) are selected as \( v_{sub} \) and \( v_{opt} \) respectively in OMR-I and \( v_6 \) is selected as \( v_{opt} \) in OMR-II. The switching sequence of each phase in LMR, OMR-I and OMR-II are shown in Fig. 3(a), Fig. 3(b) and Fig. 3(c), respectively. Particularly, to reduce the current ripple and current harmonics, the three vectors selected in LMR are applied to the inverter with a symmetrical structure with regard to the period. As shown in Fig. 3(a), \( t_0, t_1 \) and \( t_2 \) are configured to have a symmetrical structure with respect to the time axis. In order to reduce switching times, a symmetrical structure is also used in OMR-I. While in OMR-II, only one voltage vector is applied to inverter, so the switching sequence is given directly without special design.

![Image](image.png)

**FIGURE 3.** PWM signals for \( v_0, v_4, v_6 \) in different modulation origins. (a) LMR, (b) OMR-I, (c) OMR-II.

D. Summary of the Proposed Method

The proposed control method can be described by a control diagram as presented in Fig. 4. The steps are described as follows.

1. **Step 1.** Measure DC bus voltage, stator currents, machine speed and position at \( \theta \).

2. **Step 2.** Current prediction: predict the current at \((k+1)\) \(t\) using equation (2), (4) and (5).
3) Step 3. Cost function evaluation: evaluate the cost function (7) for all possible vectors of the power converter to find \( V_{op} \) and \( V_{opb} \).

4) Step 4. Voltage command calculation: calculate the amplitude of \( V_{opf} \).

5) Step 5. Partition modulation region: determine the operating modulation region of the system by comparing the amplitudes of \( V_{opf} \) and maximum output voltage of inverter in different modulation regions and determine the values of the set of mode parameters.

6) Step 6. Penalty function evaluation: Select different number of voltage vectors in accordance with the operating modulation region of the system, and consequently design the uniform penalty function as shown in (16), and calculate the dwell-times of each voltage vector by (19), (20) and (21).

7) Step 7. Vector dwell-times: apply the determined voltage vectors to the inverter with their optimal dwell-times.

### IV. Experimental Results

In order to further verify the effectiveness and superiority of the proposed method, an experimental setup is conducted as shown in Fig. 5. An IPMSM with the rated power of 2.3kW is chosen. The main parameters of the machine are presented in Table 1. The 2L-VSI consists of three FF300R12ME4 (Infineon) modules and is powered by an adjustable dc power supply, which has a maximum voltage of 300V, whereas the rated power is 3kW. Furthermore, the stator currents are sampled by three current sensors WHB25LSP3S1 and the DC bus voltage is sampled by a voltage sensor WHV05AS3S6. In addition, a digital signal processor (DSP) TMS320F28335 from TI company is employed to implement the real-time control code that is developed with C language in the Code Composer Studio 7.2. Besides, to record some waveforms like \( dq \)-axes currents, electromagnetic torque and speed, a D/A chip AD5344BRU is employed.

![Experimental setup](image)

**FIGURE 5.** Experimental setup.

| Parameters          | Units | Values |
|---------------------|-------|--------|
| Stator resistance   | \( \Omega \) | 0.8    |
| \( d \)-axis inductance | mH  | 3.465  |
| \( q \)-axis inductance | mH  | 3.93   |
| Stator flux         | Wb    | 0.272  |
| Rotary inertia      | kg\( \bullet \)m\(^2\) | 0.0028 |
| Number of pole pairs |       | 4      |

**TABLE 1.** Main parameters of the PMSM

At first, three experiments are carried out to attest the steady-state and dynamic performance in LMR of the proposed FCS-M2PC. Fig. 6 shows the steady-state responses of \( i_a \), speed \((N)\), \( i_d \) and \( i_q \) at 400 rpm with 5 N·m load for the proposed FCS-M2PC. As seen, the \( dq \)-axes currents fluctuation in proposed method are about 0.2A, 0.3A respectively. In the proposed method, the \( q \)-axis current can be validly regulated with neither an observable steady-state error nor a significant ripple. Obviously, the proposed method performs a good steady-state performance in term of the current ripple and harmonics.

![Steady-state responses at 400 rpm with 5 N·m load for proposed FCS-M2PC](image)

**FIGURE 6.** Steady-state responses at 400 rpm with 5 N·m load for proposed FCS-M2PC.

![Speed mutation responses from 600 rpm to 800 rpm with 5 N·m load for proposed FCS-M2PC](image)

**FIGURE 7.** Speed mutation responses from 600 rpm to 800 rpm with 5 N·m load for proposed FCS-M2PC.

![Load mutation responses from 0 to 5 N·m load at 800 rpm for proposed FCS-M2PC](image)

**FIGURE 8.** Load mutation responses from 0 to 5 N·m load at 800 rpm for proposed FCS-M2PC.

Fig. 7 shows the dynamic performance of the proposed method in a manner where the speed command is suddenly changed from 600 rpm to 800 rpm and the load keeps at 5 N·m. It can be observed that a smooth speed curve without
overshoot can be obtained, and the response process roughly spend 20 ms. Fig. 8 shows the dynamic responses under the load change condition, where the load changes from 0 to 5 N·m while the speed is at 800 rpm. As illustrated, $i_q$ can immediately track the load command and the speed shows an acceptable drop. The transient process is about 280 ms which is mainly caused by the output torque lag of the magnetic power brake.

![Dynamic response diagram](image)

**FIGURE 9.** Steady-state responses in overmodulation regions. (a) steady-state responses in OMR-I, (b) steady-state responses in OMR-II, (c) THD of $i_a$ generated in OMR-I, (d) THD of $i_a$ generated in OMR-II.

![Dynamic response diagram](image)

**FIGURE 10.** Dynamic responses from OMR-I to OMR-II.

![Current trajectory diagram](image)

**FIGURE 11.** Current trajectory. (a) LMR, (b) OMR-I, (c) OMR-II.

Subsequently, the effectiveness of the proposed method in overmodulation regions are verified. Fig. 9 shows the steady-state performance of the proposed method in OMR-I and -II. The DC bus voltage is set as 100 V, and the load is constant at 5 N·m, while the speed commands for two OMRs are given as 700 rpm and 730 rpm, respectively. The two commands correspond to the OMR-I and OMR-II, as shown in Fig. 9 (a) and Fig. 9 (b), respectively. It is confirmed that the stator current is obviously distorted in both OMR-I and OMR-II, with the THD of 16.32% and 23.25% as presented in Fig. 9(c) and Fig. 9(d). Nevertheless, the running speed can reach the given one, manifesting the effective overmodulation operation of the proposed method.

Fig. 10 presents the dynamic responses of the proposed method from the operation in OMR-I into OMR-II, by altering the speed command from 700 rpm to 730 rpm. It is observed that the current curve changes smoothly, without an obvious distortion. Meanwhile, the speed can quickly track its command and the response process last for about 20 ms. This result indicates the good dynamic-state performance of the proposed method in OMRs. Then, the current trajectories obtained in LMR, OMR-I and OMR-II are drawn as shown in Fig. 11 (a), (b) and (c), respectively. It can be demonstrated that the current trajectory is a circular in LMR, and transforms to a regular polygon which consists of straight lines and arcs in OMR-I and then changes to a hexagon in OMR-II. This illustrates that the operating range of the voltage vector is extended to the VH by using the proposed method.

The voltage utilization rate of DC bus voltage, defined by the ratio of the fundamental wave amplitude of the inverter output line voltage with respect to the DC bus voltage, under different DC bus conditions is detected, as shown in Fig. 12. The DC bus voltage is adjusted from 50 V to 300 V with a step of 50 V. As seen, the obtained the voltage utilization rate in the overmodulation regions is higher than that in the LMR, which means that the voltage utilization rate is highly enhanced by the proposed overmodulation strategy.

![Voltage utilization rate diagram](image)

**FIGURE 12.** Voltage utilization rate in LMR and OMR.

Finally, in order to further prove the effectiveness and superiority of the proposed method, three comparative tests are conducted. Comparative test 1 with conventional FCS-MPC proposed in [10] and two-vector-based FCS-MPC proposed in [15] is conducted to attest the performance of proposed FCS-M2PC. Test 1 is implemented in such a way that the speed reversal is carried out at 400 rpm, as presented in Fig. 13. It can be found that both methods yield smooth speed curve. Also, the response process of proposed FCS-M2PC, that takes about 100ms, is nearly in consistent with that of conventional FCS-MPC and two-vector-based FCS-MPC. The THD of $i_a$ under three methods are shown in Fig.
14. The THDs of the conventional FCS-MPC, the two-vector-based MPC and the proposed method are 28.27%, 12.54% and 8.00%, respectively. Obviously, the proposed method performs a better steady-state performance in term of the current ripple and harmonics.

![Figure 13](image)

**FIGURE 13.** Test 1: Speed reversal operation from 400 to -400rpm (a) conventional FCS-MPC, (b) two-vector-based FCS-MPC, (c) proposed FCS-M2PC.

In test 2 and test 3, the conventional FCS-M2PC proposed in [21] and the overmodulated FCS-M2PC proposed in [26] are chosen for comparison. Test 2 is implemented in a manner where the speed reference is suddenly changed from 700 to 800 rpm with the same load of 5 N-m and the same DC bus voltage of 120 V. As shown in Fig. 15, it is clear that the conventional FCS-M2PC fails to accomplish the acceleration. The machine is out of control with extensively distorted stator current and highly fluctuating q-axis current. Meanwhile, the overmodulated FCS-M2PC also fails to accomplish the acceleration. Nevertheless, thanks to the overmodulation strategy, it can accelerate the machine to about 730 rpm, which is a higher speed than the result obtained by the conventional FCS-M2PC. Comparatively, in control of the proposed method, the machine can successfully accelerate to 800 rpm. The speed and voltage utilization are further improved in proposed method compared to conventional FCS-M2PC and overmodulated FCS-M2PC. Meanwhile, the q-axis current is stable although its fluctuation is increased, and the stator current is in a form of sine wave with some low-frequency harmonics.

![Figure 14](image)

**FIGURE 14.** THD of $i_q$ of at 400rpm with 5 N-m load. (a) conventional FCS-MPC, (b) two-vector-based FCS-MPC, (c) proposed FCS-M2PC.

Test 3 is conducted in such a way that the DC bus voltage suddenly drops from 150 V to 85 V. The load and speed reference are set as 5 N-m and 600 rpm, respectively. During the decrease of DC bus voltage, the PMSM operates from the linear region to the overmodulation region. As seen in Fig. 16(a), in conventional FCS-M2PC, the machine is obviously inoperable when the DC bus voltage is dropped, with highly distorted and noisy stator current and q-axis current observed. This so since means that 85 V voltage fails to operate the machine in LMR. As shown in Fig. 16(b), in overmodulated FCS-M2PC, although the speed of machine is dropped, it can still reach a steady 520 rpm without lag current tracking error by means of its overmodulation strategy. Comparatively, the proposed method shows an evident enhancement in terms of the voltage utilization rate, as shown in Fig. 16(c). The stator current maintains sinusoidal in shape and the q-axis current is still stable. It should be
mentioned that the dynamic process consists of two portions. One spends about 240 ms to discharge the dc-link capacitor within the DC power supply as shown in Fig. 16(c), which is inevitable and limited by the used DC power supply. The overshoot of DC bus voltage below to 85 V (65 V as marked in Fig. 16(c)) and the drop of speed are also occurred due to this discharging. Another part takes about 45 ms, which is responsible for shifting between LMR and OMRs of proposed method. It is foreseeable that the dynamic process will less than 45 ms if an ideal DC voltage drop can be achieved. All in all, the comparative test 2 and test 3 manifest that proposed method performs an outstanding overmodulation capacity in terms of both the speed region and the voltage utilization rate.

![Figure 15](image1)

FIGURE 15. Test 2: acceleration operation from 700 to 800rpm. (a) conventional FCS-M2PC, (b) overmodulated FCS-M2PC, (c) proposed FCS-M2PC

![Figure 16](image2)

FIGURE 16. Test 3: DC bus voltage decline. (a) conventional FCS-M2PC, (b) overmodulated FCS-M2PC, (c) proposed FCS-M2PC.

V. CONCLUSION

In this work, an FCS-M2PC with overmodulation capacity incorporating penalty function is proposed and successfully conducted in an IPMSM drive system. The contributions of this work are concluded as following.

1. Dead-beat control is employed to effectively calculate the amplitude of \( v_{ref} \) and then the modulation origins are divided into OMR-I and OMR-II according to amplitude comparison quickly.

2. Different number of voltage combinations are selected to obtain better steady-state performance not only in the LMR but also overmodulation regions. The overmodulation capacity of FCS-M2PC is enhanced significantly with this method.

3. A uniform penalty function considering both current tracking errors and the constraints of dwell-times is defined by a set of mode parameters to calculate the optimal dwell-
times for applied voltage vectors. The second modification is eliminated with this method.

All the merits of the proposed method are demonstrated by experiments. In particular, by means of the proposed method, the voltage utilization rate and the speed region are significantly improved for a fixed voltage. The proposed method may lead to greater noise in ultra-high speed motor applications as well as a smaller speed extension range. However, its use in medium and high speed motors is to be expected. Furthermore, in order to further expand the study of overmodulation, the reduction of harmonic content in the overmodulated region and other over-modulation control strategies that are not just based on model predictive control will be the focus of our future research.

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