Nonlinear optical response in gapped graphene

S A Jafari

Department of Physics, Sharif University of Technology, Tehran 11155-9161, Iran
and
School of Physics, Institute for Research in Fundamental Sciences, Tehran 19395-5531, Iran

E-mail: jafari@sharif.edu

Received 9 February 2012, in final form 21 March 2012
Published 18 April 2012
Online at stacks.iop.org/JPhysCM/24/205802

Abstract
We present a formulation for the nonlinear optical response in gapped graphene, where the
low-energy single-particle spectrum is modeled by massive Dirac theory. As a representative
example of the formulation presented here, we obtain a closed form formula for the third
harmonic generation in gapped graphene. It turns out that the covariant form of the low-energy
theory gives rise to peculiar logarithmic singularities in the nonlinear optical spectra. The
universal functional dependence of the response function on dimensionless quantities indicates
that the optical nonlinearity can be largely enhanced by tuning the gap to smaller values.

1. Introduction

Graphene is the first example of the realization of a truly two-dimensional (2D) crystal made of carbon atoms [1].
This intriguing material in addition to promise for novel applications, from a fundamental point of view, provides the
condensed matter community with a low-energy laboratory of Dirac electrons on the table top [2]. Since its discovery
there has been remarkable success in pushing the idea of employing electronic, mechanical and various other
properties of graphene in technological applications [3, 4]. The chiral nature of Dirac charge carriers in graphene
makes them unstoppable which is mathematically manifested as the absence of back-scattering. This means that with
pristine graphene which contains massless Dirac fermions, there can be no ‘off’ state in electronic applications.
Therefore, to enable the use of graphene in electronic devices, one needs to open up a gap in the single-particle
spectrum [5] usually by statically reducing the symmetry via extrinsic effects [6], or by coupling to another field
to generate dynamical masses [7]. Massive Dirac fermions possess a single-particle gap which causes the graphene to
behave like a truly 2D semiconductor in some respects. Nevertheless the nature of such ‘relativistic’ massive theory
is drastically different from the usual parabolic bands in a semiconductor.

Before the synthesis of graphene, the problem of two dimensions was usually approached from the third
dimension by, for example, a geometrical confinement in heterostructures [8], or by an appropriately chosen B-field
which effectively confines the dynamics of carriers into two spatial dimensions [9]. In this respect, truly 2D gapped
graphene provides a novel platform for nonlinear optical applications. The optical properties of 2D graphene have been
extensively investigated at linear level [10, 11]. However, at nonlinear level, there has been a limited number of studies;
a classical theory for the electromagnetic (EM) response of pristine graphene was developed by Mikhailov et al
[12] who attributed strong nonlinear EM response to the massless nature of Dirac electrons in graphene. Nonlinear
response of massless Dirac fermions was studied by Wright et al, who used direct expansion of the wavefunction
in terms of multiples of frequency of the applied electric field [13]. They found a high triple-frequency current response
in massless graphene. On the experimental side, Hendry and collaborators used four-wave mixing to study the nonlinear
optical response in graphene flakes, where they found a remarkable large third-order optical response [14]. Also
broadband optical nonlinearity was observed by Wang et al in graphene dispersions [15].

Therefore it is timely to consider the problem of third-order optical response in the 2D lattice of graphene by generalizing it to the massive case. The problem of third-order optical response for 1 + 1-dimensional massive Dirac fermions has been previously considered by Wu [16]. So the present work can also be considered as a natural generalization of Wu’s work to 2 + 1 dimensions. In one spatial dimension, or for quasi-one-dimensional systems such as organic materials or carbon nanotubes, closed form expressions for the optical nonlinear responses can be found [17, 18]. We employ a formulation we have developed earlier for investigation of the nonlinear optical properties in the matrix form [20]. We find that the covariant form of the dispersion relation for massive Dirac fermions in 2 + 1 also allows for a simple closed form expression for the third-order response in general. We develop the general theory of nonlinear optical response for 2 + 1 Dirac fermions with arbitrary gap parameter in terms of Feynman diagrams. Then, as a representative example, we evaluate the line-shape of the third harmonic generation (THG). We obtain diagrams. Then, as a representative example, we evaluate the

\[ J_i = \sum_p \phi_p^\dagger V_p \sigma_i V_p \psi_p, \quad i = x, y \]

where the transformed Pauli matrices are given by,

\[ V_p \sigma_x V_p = \frac{-m \cos \phi_p}{\varepsilon_p} \sigma_x + \sin \phi_p \sigma_y + \frac{p \cos \phi_p}{\varepsilon_p} \sigma_z \]
\[ V_p \sigma_y V_p = \frac{-m \sin \phi_p}{\varepsilon_p} \sigma_x - \cos \phi_p \sigma_y + \frac{p \sin \phi_p}{\varepsilon_p} \sigma_z. \]

Note that the second equation can be obtained from the first one by simply replacing \( \varphi \rightarrow -\varphi - \pi/2 \), which is a consequence of the vector character of the three Pauli matrices under SO(3) rotations.

3. Multi-current correlations

As detailed in [20], we are interested in calculation of multi-current loops to which photon propagators are attached. As a demonstration of this method, in the following we first calculate the two-current response which is connected to optical conductivity. After reproducing well-known results, along the same lines we proceed to calculate four-current correlations within our matrix diagrammatic formulation which is suitable for situations such as massive Dirac fermions in gapped graphene.

In general the fully retarded expectation value of nested commutators of various operators, e.g. \( A, B, C, Q \) schematically depicted in figure 1, can be Lehman represented as [20, 21],

\[ 2\pi \delta(v + v_\sigma) \sum_{abc} \sum_P \frac{A_{0a}B_{ab}C_{bc}Q_{c0}}{(v_1 - E_{0a})(v_2 - E_{ab})(-v_\sigma + E_{bc})} \]

where the sum of frequencies is \( \nu_\sigma = v_1 + v_2 + v_3 \), and \( \sum_{\rho} \) permutes \((A, v_1), (B, v_2), (C, v_3), (Q, -v_\sigma)\) around the current loop, and \( B_{ab}, \) etc stand for the matrix element \((a|b|b)\). Note that in the above formula, the substitution \( v \rightarrow v_+ = v + i0 \) for all frequencies is understood.
Matsubara summation, the two-current correlation function

Note that the sum over permutations of two $z$ around the loop produces a $2!$ where $i\vec{z}$ matter operator that couples to some power of the gauge field $A$ around a photon leg, and only two frequencies in the incident light denoted by wavy lines (photons). The frequency $\nu_0$ is the sum of all incoming frequencies $\nu_1 + \nu_2 + \nu_3$.

3.1. Linear response: optical conductivity

Within this formulation, the two-current response is given by [20],

\[
\langle J_x J_x \rangle = \text{Tr} \frac{1}{\beta L^2} \sum_{\vec{p}} \sum_{\nu_0} \left( \frac{-m \cos \varphi_0}{\epsilon_0} \sigma_x + \sin \varphi_x \sigma_y \right) \sum_{\nu_1, \nu_2} \left( \frac{-m \cos \varphi_1}{\epsilon_1} \sigma_x + \sin \varphi_x \sigma_y \right) \sum_{\nu_3} \left( \frac{-m \cos \varphi_3}{\epsilon_3} \sigma_x + \sin \varphi_x \sigma_y \right) \left( \frac{z_0 + \epsilon_0 \sigma_z}{z_0 - \epsilon_0} \right) \left( \frac{z_1 + \epsilon_1 \sigma_z}{z_1 - \epsilon_1} \right) \left( \frac{z_3 + \epsilon_3 \sigma_z}{z_3 - \epsilon_3} \right) \quad (10)
\]

where $i\nu$ is the external (photon) frequency. The corresponding diagram is simpler than the one in figure 1, where only two photons are attached, and only two frequencies $z_0 = i\nu_0 + i\nu_3$, $z_1 = i\nu_1 + i\nu_2$ are present. In the above equation, quantities in brackets are inter-band terms of the current operator and contain all necessary information about the matrix elements of the current operator between the valence and conduction band. The matrix multiplications required in the above expression are easily performed to simplify the result as

\[
\langle J_x J_x \rangle = 2! \text{Tr} \frac{1}{\beta L^2} \sum_{\vec{p}} \sum_{\nu_0, \nu_1, \nu_2} \left( \frac{z_0 - \epsilon_0^2}{z_0 + \epsilon_0^2} \right) \left( \frac{z_1 - \epsilon_1^2}{z_1 + \epsilon_1^2} \right) \left( \frac{z_2 - \epsilon_2^2}{z_2 + \epsilon_2^2} \right) \left( \frac{z_3 - \epsilon_3^2}{z_3 + \epsilon_3^2} \right) \left( \frac{z_0 \epsilon_0 \sigma_z}{z_0 - \epsilon_0^2} \right) \left( \frac{z_1 \epsilon_1 \sigma_z}{z_1 - \epsilon_1^2} \right) \left( \frac{z_3 \epsilon_3 \sigma_z}{z_3 - \epsilon_3^2} \right) \quad (11)
\]

Note that the sum over permutations of two $J_x$ operators around the loop produces a $2!$ factor in the right-hand side [20], as the above formula is symmetric with respect to fermion propagation frequencies $z_0, z_1$. With standard Matsubara summation, the two-current correlation function simplifies to

\[
\int \frac{2d^2 \vec{p}}{(2\pi)^2} \left[ \frac{1}{i\nu - 2\epsilon_0} - \frac{1}{i\nu + 2\epsilon_0} \right] \times m^2 + p^2 + 2m^2 \sin^2 \varphi_0 \quad (12)
\]

where the temperature is assumed to be zero so that the Fermi function is 1 only for negative energies, and is zero otherwise.

After analytic continuation, $i\nu \rightarrow \nu + i\eta$, the spectral function (imaginary part) corresponding to the absorption—i.e. the first term in the bracket—can be easily obtained. To compare with the existing results in the literature, we use the conductivity formula $\sigma_{xx}(\nu) = e^2 (J_x J_x)/(i\nu)$ and restore the fundamental constants to obtain

\[
\text{Re}\sigma_{xx}(\nu) = \frac{e^2}{8\hbar} \left[ 1 + 4m^2 \nu^2 \right] = \frac{\sigma_0}{2} \left[ 1 + 4m^2 \nu^2 \right] \quad (13)
\]

which after noting the fact that our calculation has been done for a single valley, agrees with, for example, equation (10) of [22]. Note that here $\sigma_0 = e^2/(4\hbar)$ is the conductance of ideal graphene.

In a similar way, one can calculate correlation functions like $\langle J_x J_y \rangle$. In simplifying such elements one should note that odd powers of $\sin \varphi$ or $\cos \varphi$ average to zero upon $\varphi$ integration. Moreover, summation over permutations $\mathcal{P}$ picks up the most symmetric part. In the case of two-current correlation the $xy$ component will become zero, unless a magnetic field is applied [23].

3.2. Four-current correlations

Now that we checked our formulation with well-known results, let us calculate higher order responses. But before proceeding to the question of higher order responses, we note that in a general material the higher order correlation between the current operators, i.e. $\langle J_{\alpha \beta} J_{\gamma \delta} \rangle$, which essentially contains four velocity vertices along $a, b, c, d \in \{x, y\}$ directions of space, is not the only important term when one is interested in the third-order response. In general, terms of the type $J_{\alpha \beta} \tau_{bc} J_{\gamma \delta}$ etc may also appear where the stress tensor $\tau_{bc}$ too contributes to the third-order response [21]. Similar to current vertices given by

\[
\nu_{\alpha} = \frac{\partial \varphi_{\alpha}}{\partial \epsilon_{\alpha}} \quad (14)
\]

even-parity vertices of the form

\[
\frac{\partial \varphi_{\alpha}}{\partial \epsilon_{\alpha}} \frac{\partial \varphi_{\beta}}{\partial \epsilon_{\beta}} \quad (15)
\]

may also enter the theory of nonlinear optical response for materials with arbitrary hand structure defined by $\epsilon_{\alpha}$. In the case of ideal graphene ($m = 0$), it is interesting to note that, due to the linear form of energy as a function of momentum $\vec{p}$, only velocity vertices will appear in the theory as we calculated them in equation (6). All higher order derivatives related to the stress tensor etc will be identically zero for massless Dirac fermions. In the case of massive Dirac fermions where the asymptotic behavior of the energy dispersion for photon energy scales much higher than the gap parameter $m$ becomes linear, the terms arising from the stress tensor will have negligible contributions, for energy scales much beyond those corresponding to band edge excitations.

With this point in mind, in this paper we focus on four-current correlations of the form $\langle J_x J_y J_x J_y \rangle$. From the rightmost current operator, only the inter-band terms contribute which lead to an excitation into the conduction
band. Then the second operator from the right will have to create intra-band excitation. However, for the third and fourth operator, there are two different possibilities depicted in figure 2. In the first, the operator number 3 creates an intra-band excitation as in part (i) of figure 2, in which case the operator number 4 must create an inter-band excitation. Such a sequence of operators can be represented by $-++--$, where $-$ denotes inter-band vertex and $+$ stands for an intra-band vertex. With this notation, the second possibility denoted in part (ii) of figure 2 can be summarized as $+--+$, i.e. the third operator returns the system to the valence band, and operator number 4 has to create an intra-band excitation. Note that a sequence like $-+--$ is not a genuine four-operator correlation, but rather a product of two-operator correlations and can be extracted from linear responses as well. All other possibilities can be generated by the sum over the permutation $\sum p$. Therefore the above sequences are two general families of excitation patterns which by the cyclic property of the trace involved in the loop diagram of figure 1 include all possibilities. These are summarized in figure 2.

Now let us calculate the contribution of two sequences of excitations (i) and (ii). First note that the only terms surviving the trace operation are those proportional to the unit matrix. Adding the above terms gives

$$2\sum_p \frac{1}{\beta} \sum_{i\omega_n} \int \frac{d^2p}{(2\pi)^2} \left[ \frac{1}{\sum_{\alpha=0} \frac{\varepsilon_{\alpha}^2}{\varepsilon_p^2} - \varepsilon_p^2} \right] \times p^2 \cos^2 \psi_p \left[ \frac{m^2 \cos^2 \psi_p}{\varepsilon_p^2} + \sin^2 \psi_p \right] \times [\varepsilon_p^2(z_2 - z_3)(z_0 - z_1) - (\varepsilon_p^2 - z_2z_3)(\varepsilon_p^2 - z_0z_1)],$$

where frequencies $z_\alpha, \alpha = 0-3$ are defined in figure 1. Similarly for the class (ii) terms we obtain,

$$2\sum_p \frac{1}{\beta} \sum_{i\omega_n} \int \frac{d^2p}{(2\pi)^2} \left[ \frac{1}{\sum_{\alpha=0} \frac{\varepsilon_{\alpha}^2}{\varepsilon_p^2} - \varepsilon_p^2} \right] \times p^2 \cos^2 \psi_p \left[ \frac{m^2 \cos^2 \psi_p}{\varepsilon_p^2} + \sin^2 \psi_p \right] \times [\varepsilon_p^2(z_0z_1 + z_0z_2 - z_1z_2 - \varepsilon_p^2)].$$

Now we can evaluate the Matsubara sums, and at the end the result will be symmetrized with respect to exchange of the frequencies of the attached photon vertices. Using the angular averages

$$\langle \cos^4 \psi \rangle = \frac{5}{8}, \quad \langle \cos^2 \psi \sin^2 \psi \rangle = \frac{1}{8},$$

the angular integration can be simplified to,

$$\sum \sum_{i\omega_n} \int \frac{d\rho}{2\pi} \left[ \frac{1}{\sum_{\alpha=0} \frac{\varepsilon_{\alpha}^2}{\varepsilon_p^2} - \varepsilon_p^2} \right] \times p^2 \left[ \frac{m^2 \rho^2}{\varepsilon_p^2} + 1 \right] [z_0z_1 + z_0z_2 - z_1z_2 - \varepsilon_p^2].$$

Up to this point, the formulation is quite general. At this point, let us specialize to the case of THG, where $\nu_1 = \nu_2 = i\nu_3 = \nu_1 + i\nu_3 = 3i\nu$. In this case, the Feynman diagram shown in figure 1 must be labeled with $z_\alpha = i\omega_n + \alpha i\nu$ for $\alpha = 0-3$. Permuting the external vertices amounts to moving the outgoing $3i\nu$ photon frequency around the loop. Such permutation can be accounted for by a cyclic permutation of the set $\{z_0, z_1, z_2, z_3\}$. Performing the $\sum_p$ will only affect the last term in equation (20) as,

$$\sum_p [z_0z_1 + z_0z_2 - z_1z_2 - \varepsilon_p^2] = 2[z_0z_2 + z_1z_3 - 2\varepsilon_p^2],$$

where the THG identification, $z_\alpha = i\omega_n + \alpha i\nu$ with $\alpha = 0-3$, is understood. Let us emphasize that the above result for the sum over permutation holds only for THG. Therefore the THG susceptibility will become

$$\chi_{\text{THG}} = \frac{1}{\beta} \sum_{i\omega_n} \int \frac{d\rho}{2\pi} \left[ \frac{1}{\sum_{\alpha=0} \frac{\varepsilon_{\alpha}^2}{\varepsilon_p^2} - \varepsilon_p^2} \right] \times p^2 \left[ \frac{m^2 \rho^2}{\varepsilon_p^2} + 1 \right] [z_0z_2 + z_1z_3 - 2\varepsilon_p^2].$$

The $1/\beta\sum_{i\omega_n}$ can be performed with standard contour integration techniques producing Fermi functions $f(\pm \varepsilon_p)$.
corresponding to poles at conduction and valence bands, respectively. Assuming the temperature to be zero, and for an undoped massive Dirac spectrum, only contributions from poles at the valence band will be left for which the residues can be calculated to give,

\[ \chi_{THG}(v_+) = \int \frac{p \, dp}{2\pi} \left[ \frac{m^2}{\varepsilon_p^2} + 1 \right] \]

\[ \times \frac{3}{2} \frac{4\varepsilon_p^2 + v_+^2}{\varepsilon_p(v_+^2 - \varepsilon_p^2)(v_+^2 - 4\varepsilon_p^2)(v_+^2 - 9\varepsilon_p^2)}. \]  

\[ (22) \]

Note that the analytic continuation \( iv \rightarrow v_+ = v + i0 \) has been performed in the above equation [20].

4. Result and discussions

The radial momentum integration in equation (22) can be performed with the change of variable to \( \varepsilon = p^2 + m^2 \) which gives the following closed form result

\[ \chi_{THG}(v_+) = -\frac{m^3}{2\pi v_+^4} + \frac{1}{192\pi v_+^5} \]

\[ \times \left[ -24(6m^2 + v_+^2)(m^2 - v_+^2) \ln \frac{m + v_+}{m - v_+} \right. \]

\[ + 3(4m^2 - v_+^2)(24m^2 + v_+^2) \ln \frac{2m + v_+}{2m - v_+} \]

\[ + (4m^2 - 9v_+^2)(8m^2 + 3v_+^2) \ln \frac{2m + 3v_+}{2m - 3v_+} \].

\[ (23) \]

Let us emphasize that the above result is obtained for zero temperature and zero chemical potential. First of all, note that the above expression when the fundamental constants are restored will be proportional to \( e^4/\hbar^3 \). Therefore the above expression shows the results in the natural units. A remarkable feature of the above expression is that the quantity \( m\chi_{THG} \) is only a function of \( v/m \). This functional form is a universal characteristic of \( 2 + 1 \)-dimensional Dirac fermions. Hence in figure 3, we have plotted the real part and the magnitude of THG line-shape incorporating this observation. This peculiar scaling form indicates that in the limit where \( m \rightarrow 0 \), and the system approaches the gapless limit, since \( m\chi_{THG} \) in the natural units employed here will have to remain on the scale of unity, the \( \chi_{THG} \) itself will grow inversely proportional to the gap parameter \( m \). This sheds a new light onto why pristine (gapless) graphene is expected to display large optical nonlinearity [14].

Apart from the zero frequency divergence, which is a general feature of optical response functions, an interesting point in the above expression which can be noticed is that it is precisely the covariant (relativistic) form of the dispersion relation \( \varepsilon = p^2 + m^2 \), which after changing integration variables from momentum \( p \) to energy \( \varepsilon \) leads to logarithmic dependence at finite frequencies \( v_+/m = 2/\ell \), where \( \ell \) is an integer \( \ell = 1–3 \). This would not be the case for a typical parabolic dispersion relation in ordinary semiconductors. The main THG peak corresponding to \( \ell = 1 \) and the second one corresponding to \( \ell = 2 \) are clearly seen in the THG spectrum depicted in figure 3. The feature corresponding to \( \ell = 3 \) is the weakest feature. Note that as we calculated in equation (13), such logarithmic dependence does not show up in the linear optical response. Therefore it can be considered as valuable information which is contained only in higher order optical responses. Moreover, this logarithmic dependence is not restricted to the THG spectrum, but it also appears in all other higher order responses. To see this, we note in equation (20) that in the special case of THG, only the functional form of \( \sum_{\rho} \) as a function of loop frequencies \( z_\rho \) will be affected. But the dimensional form of the summand will generally retain a similar form.

The calculation of THG for massive Dirac fermions in \( 1 + 1 \) dimension by Wu [16] indicates a characteristic inverse square root divergence at the main THG peak \( v_{\ell=1} \) and its harmonics. In the case of small clusters (of correlated 2D nature), a numerical investigation of the THG spectrum suggests a line-shape composed of a superposition of resonances corresponding to simple poles [24]. For an extended 2D system with broken symmetry, it was shown that the nesting underlying spin density wave instability can produce a peculiar \( z^{-1/2} \ln(z) \) form of singularity in the nonlinear optical spectra [20]. With the above examples in mind, the gapped graphene as a first example of a truly 2D system provides us with a unique example of a 2D system where optical nonlinearity manifests in a universal logarithmic function of the dimensionless quantity \( v/m \).

It is interesting to check the limit of gapless graphene by taking the limit \( m \rightarrow 0 \) in equation (23), which gives

\[ m\chi_{THG}(v) = \frac{54i}{192} \frac{m}{v_+^4}. \]

The coefficient of the above expression is universal and can be regarded as a hallmark of the ideal graphene in the THG spectroscopy.

Acknowledgment

This work was supported by the National Elite Foundation (NEF) of Iran.
References

[1] Novoselov K S, Geim A K, Morozov S V, Jiang D, Zhang Y, Dubonos S V, Grigorieva I V and Firsov A A 2004 Science 306 666

Novoselov K S, Geim A K, Morozov S V, Jiang D, Katsnelson M I, Grigorieva I V, Dubonos S V and Firsov A A 2005 Nature 438 197

[2] For a review see: Castro Neto A H, Guinea F, Peres N M R, Novoselov K S and Geim A K 2009 Rev. Mod. Phys. 81 109

[3] Lin Y-M, Jenkins K A, Valdes-Garcia A, Small J P, Farmer D B and Avouris P 2009 Nano Lett. 9 422

[4] Geim A K 2009 Science 324 1530

[5] Han M Y, Ozyilmaz B, Zhang Y and Kim P 2007 Phys. Rev. Lett. 98 206805

[6] Zhou S Y, Gweon G-H, Fedorov A V, First P N, de Heer W A, Lee D-H, Guinea F, Castro Neto A H and Lanzara A 2007 Nature Mater. 6 770

[7] Kibis O V 2010 Phys. Rev. B 81 165433

[8] Bastard G 1988 Wave Mechanics Applied to Semiconductor Heterostructures (Paris: Les Editions de Physique)

[9] Meier T, Rossi F, Thomas P and Koch S W 1995 Phys. Rev. Lett. 75 2558

[10] Mak K F, Sfeir M Y, Wu Y, Lui C H, Misewich J A and Heinz T F 2008 Phys. Rev. Lett. 101 196405

[11] Pedersen T G, Jauho A-P and Pedersen K 2009 Phys. Rev. B 79 113406

[12] Mikhailov S A 2007 Eur. Phys. Lett. 79 27002

Mikhailov S A and Ziegler K 2008 J. Phys.: Condens. Matter 20 384204

[13] Wright A R, Xu X G, Cao J C and Zhang C 2009 Appl. Phys. Lett. 95 072101

[14] Hendry E, Hale P J, Moger J, Savchenko A K and Mikhailov S A 2010 Phys. Rev. Lett. 105 097401

[15] Wang J, Hernandez Y, Lotya M, Coleman J N and Blau W J 2009 Adv. Mater. 21 2430

[16] Wu W 1988 Phys. Rev. Lett. 61 1119

[17] Margulis V A and Sizikova T A 1998 Physica B 245 173

[18] Zarifi A, Fisker C and Pedersen T G 2007 Phys. Rev. B 76 045403

[19] Desloge E A 1994 Am. J. Phys. 62 216

[20] Jafari S A, Tohyama T and Maekawa S 2006 J. Phys. Soc. Japan 75 054703

[21] Jafari S A 2009 Opt. Commun. 282 317

[22] Kotov V N, Pereira V M and Uchoa B 2008 Phys. Rev. B 78 075433

[23] Gusynin V P and Sharapov S G 2006 Phys. Rev. B 73 245411

[24] Takahashi M, Tohyama T and Maekawa S 2002 Phys. Rev. B 66 125102