Parameters sensitivity Analysis of the Duncan-Chang Model Based on Orthogonal Test

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Abstract. This paper aimed at the problems of huge computation burdens and poor accuracy because of too many parameters in the embankment dam’s inversion. Authors designed a mixed-level test for eight parameters of the Duncan-Chang Model: $K$, $n$, $R_f$, $K_b$, $m$, $c$, $\varphi_0$, and $\Delta \varphi$ through an orthogonal test method and numerical analysis with FEM. Using the different directional displacement of two nodes and principal stress of element as the test index and through variance analysis, we can conclude that $K$, $K_b$ and $\varphi_0$ are sensitive parameters which control the strain and stress of dam, and which should be accurately obtained by back-analysis.

1. Introduction
When calculating the deformation and stability of embankment dam, the accuracy of the calculation depends primarily on the dam's physics and accurate selection of mechanical parameters. In actual engineering, the selection of material parameters relies primarily on the field or laboratory tests. On one hand, the actual condition of the dam material is very complex and the parameters are affected by physical properties, load size, loading methods, and stress history of the material, and other factors; on the other hand, due to the limitations of the existing test methods and laboratory equipment, the computed material parameters of the dam are often inaccurate. As a result, the ability to reasonably and accurately calculate the parameters of the dam has become the key issues in engineering calculations and safety evaluations. The parameter-inversion method of the embankment dam came into being under such conditions. Parameter inversion utilizes the field observation information of dams (such as dam displacement and stress values) as the basis for real parameter inversion of the dam. It is a comprehensive application of modern theories of planning, numerical analysis method, and observation techniques. These provide an effective means of obtaining more accurate material parameters of dams [1-2].

The constitutive relation of embankment dam material is complex and more parameters need to be inverted. There are nine parameters ($K$, $n$, $R_f$, $K_b$, $m$, $c$, $\varphi_0$, $\Delta \varphi$, $K_{ur}$) in the most used Duncan-Chang E-B model. If all of these parameters are utilized as the future inversion parameters of calculation, it will often result in significant amount of computation, and the accuracy of the inversion results cannot be guaranteed most of the time. Instead, when a sensitivity analysis of the parameters of the dam material constitutive model is conducted, the parameters which have greater impact (that is, larger sensitivity) on dam displacement and stress will be available to proceed inversion analysis. For those parameters of small sensitivity, they can be determined through experimentation or experience.
because they have less impact on dam displacement. This can greatly reduce the workload and provide the convenience of obtaining both accurate and reasonable dam parameters.\textsuperscript{13-5} The Orthogonal Test Method is an analysis method which arranges scientific and reasonable analysis of multi-factor experimentation programs. It is also able to test the results. The method selects representative points from a comprehensive test and then conducts the experiment. Those representative points have even and neat features. The design of orthogonal test method is based on fractional factorials from the variance analysis model and it has a high level of efficiency when the level is comparatively less. It is often used to arrange tests comprehensively to immediately find the influential degrees of different parameters on the results of the tests.\textsuperscript{[6]} This article first utilizes orthogonal design to conduct the analysis of the parameters of Duncan-Chang model on the embankment dam and the influential degrees of dam displacement and stress analysis.

2. Brief Introduction to Orthogonal Test
Orthogonal Test design is a scientific multi-factorial optimization method, which utilizes a normalized orthogonal array to design protocol. This method avoids huge workload problems caused by the combination of various test conditions for different factors and even avoids the possibility that the experiments are not doable. Adopting this method needs few tests while reflecting the internal disciplines of the complete combinations of experimental conditions. The Orthogonal Test is a scientific and efficient optimization of multi-factor experimental design method.

Key concepts of the Orthogonal testing design include test indexes, factors and levels. Test indexes are the attribute values which are selected to test or measure the experiment results for the purpose of testing. This article uses the vertical and horizontal displacements of the two nodes on the section and the principal stress as the test indexes. Possible existence causes or elements for the test indexes are called factors and the factors analyzed in this article are the eight parameters mentioned above. The different states of the factors in the test are called levels. Each factor has two or three levels in this article, see Table 1.

Sensitivity analysis is finding out the impact degree to the test indexes by the changes of factors. There are normally two ways to analyze the test results and they are range analysis and variance analysis. The range analysis is simple and only requires a minor calculation of the test results to identify the impact degree to test indexes by factors. But, the range analysis does not distinguish between the data fluctuations caused by the test condition in the process, nor provides criteria used to judge the role of factors of significance. The variance analysis makes up for the disadvantage of the range analysis and is an effective method to analyze the result of the Orthogonal.

3. Sensitivity Experiment Design
This article adopts a homogeneous embankment dam for calculation. It is 100m in height, 8m in width at dam crest, 1:1.5 slope ratio in both upstream and downstream. Finite element division are shown in Figure 1.

![Finite element division of a cross](image)

In Figure 1, the values of nodes 95 and 71 were selected because they have larger displacement values (vertical displacement values of 95 and horizontal displacement values of 71) and the principal stress values of unit 8 for it has larger principal stress values (including maximum and minimum principal stresses) as the test indexes of the Orthogonal Test. There are nine parameters in total in Duncan-Chang Model: $K$, $n$, $R_f$, $K_h$, $m$, $c$, $\varphi_0$, $\Delta \varphi$, $K_{ur}$. According to the basic theory
of geotechnical principles and practical experience, $K_w = (1.2 \sim 3.0) K$, therefore, this article selects eight parameters in total as test factors. They are $K$, $n$, $R_f$, $K_b$, $m$, $c$, $\varphi_0$, and $\Delta \varphi$. Then select two levels of factor $\Delta \varphi$ and three levels of the other factors for analysis. Therefore, the test adopted in this article is an Orthogonal Test with mixed levels. Levels of test factors are shown in Table 1.

| Levels | $K$ | $n$ | $R_f$ | $K_b$ | $m$ | $c$ | $\varphi_0$ | $\Delta \varphi$ |
|--------|-----|-----|------|------|-----|-----|--------|-------------|
| 1      | 800 | 0.15| 0.6  | 500  | 0.15| 15  | 45     | 8           |
| 2      | 1000| 0.20| 0.7  | 600  | 0.20| 20  | 50     | 12          |
| 3      | 1200| 0.25| 0.8  | 700  | 0.25| 25  | 55     | 12          |

4. Result Analysis of Sensitivity Experiment

Orthogonal Test design is commonly expressed as $L_K (PJ)$. $L$ stands for the Orthogonal Table in the test protocol, $K$ means the number of test protocol or test conditions reflecting the number of orthogonal arrays, $P$ is the number of levels of the participated test factors in the trial, $J$ is the number of columns of the orthogonal table and the maximum number of test factors cannot exceed $J$. Based off the number of factors involved in the orthogonal test and their individual level, this article selects the orthogonal test protocol of mixed-levels as $L_{18} (2^1 \times 3^7)$ . Non-linear finite element calculations are conducted separately for 18 groups of tests and then the displacement values of nodes 95 and 71 and the principal stress values of unit eight are obtained. Sensitivity test design and test results are shown in Table 2.

| Test No. | $K$  | $n$  | $R_f$ | $K_b$ | $m$  | $c$  | $\varphi_0$ | $\Delta \varphi$ | Displacement of Node 95 | Displacement of Node 71 | Max Principal Stress of Unit 8 | Min Principal Stress of Unit 8 |
|----------|------|------|-------|-------|------|------|-------------|-----------------|--------------------------|--------------------------|-----------------------------|-----------------------------|
| 1        | 800  | 0.15 | 0.6   | 500   | 0.15 | 15   | 45         | -0.3659         | 0.1573                  | 1564.95                  | 641.70                     |
| 2        | 1000 | 0.15 | 0.7   | 600   | 0.20 | 20   | 50         | -0.2814         | 0.1109                  | 1572.15                  | 633.95                     |
| 3        | 1200 | 0.15 | 0.8   | 700   | 0.25 | 25   | 55         | -0.2240         | 0.0457                  | 1579.09                  | 627.14                     |
| 4        | 1200 | 0.20 | 0.6   | 500   | 0.20 | 20   | 55         | -0.2651         | 0.0457                  | 1596.89                  | 418.36                     |
| 5        | 800  | 0.20 | 0.7   | 600   | 0.25 | 25   | 45         | -0.3143         | 0.1744                  | 1555.26                  | 739.92                     |
| 6        | 1000 | 0.20 | 0.8   | 700   | 0.15 | 15   | 50         | -0.2655         | 0.1251                  | 1556.92                  | 652.98                     |
| 7        | 1200 | 0.25 | 0.6   | 600   | 0.15 | 25   | 50         | -0.2477         | 0.0668                  | 1580.94                  | 482.97                     |
| 8        | 800  | 0.25 | 0.7   | 700   | 0.20 | 15   | 55         | -0.2459         | 0.1298                  | 1551.98                  | 720.48                     |
| 9        | 1000 | 0.25 | 0.8   | 500   | 0.25 | 20   | 45         | -0.3306         | 0.1362                  | 1572.36                  | 611.85                     |
| 10       | 800  | 0.15 | 0.6   | 700   | 0.25 | 20   | 50         | -0.2708         | 0.1535                  | 1555.33                  | 786.63                     |
| 11       | 1000 | 0.15 | 0.7   | 500   | 0.15 | 25   | 55         | -0.3007         | 0.0771                  | 1584.97                  | 503.83                     |
| 12       | 1200 | 0.15 | 0.8   | 600   | 0.20 | 15   | 45         | -0.2983         | 0.1261                  | 1570.98                  | 634.88                     |
| 13       | 1000 | 0.20 | 0.6   | 600   | 0.25 | 15   | 55         | -0.2456         | 0.0822                  | 1578.30                  | 585.82                     |
| 14       | 1200 | 0.20 | 0.7   | 700   | 0.15 | 20   | 45         | -0.2584         | 0.1090                  | 1565.32                  | 615.15                     |
| 15       | 800  | 0.20 | 0.8   | 500   | 0.20 | 25   | 50         | -0.3439         | 0.1463                  | 1566.65                  | 637.64                     |
| 16       | 1000 | 0.25 | 0.6   | 700   | 0.25 | 20   | 45         | -0.2502         | 0.1167                  | 1558.96                  | 651.68                     |
| 17       | 1200 | 0.25 | 0.7   | 500   | 0.25 | 15   | 50         | -0.2807         | 0.0712                  | 1589.51                  | 490.47                     |
| 18       | 800  | 0.25 | 0.8   | 600   | 0.15 | 20   | 55         | -0.2869         | 0.1250                  | 1556.73                  | 619.91                     |

According to the variance analysis theory, sensitivity analysis of the test result is then conducted. The results are shown in Table 3 and Table 4.

Table 3 Sensitivity Analysis Results of Displacement
Factors & Vertical Displacement Values at Node 95 & Horizontal Displacement Values at Node 71

| Factors | Sum of Squares of Deviations | degree of freedom | Mean Square | Value F | Sum of Squares of Deviations | degree of freedom | Mean Square | Value F |
|---------|-----------------------------|------------------|-------------|---------|-----------------------------|------------------|-------------|---------|
| $K$     | 0.00544                     | 2                | 0.00272     | 1200.90 | 0.01268                     | 2                | 0.00634     | 213.60  |
| $n$     | 0.00081                     | 2                | 0.00041     | 183.71  | 0.00031                     | 2                | 0.00015     | 5.2135  |
| $R_f$   | 0.00093                     | 2                | 0.00046     | 208.07  | 0.00116                     | 2                | 0.00058     | 19.52   |
| $K_b$   | 0.01163                     | 2                | 0.00581     | 2609.70 | 0.00056                     | 2                | 0.00028     | 9.46    |
| $m$     | 0.00030                     | 2                | 0.00015     | 68.15   | 0.00012                     | 2                | 6.12e-5     | 2.06    |
| $c$     | 3.78e-5                     | 2                | 1.89e-5     | 8.49    | 7.38e-5                     | 2                | 3.69e-5     | 1.24    |
| $\varphi_0$ | 0.00518               | 2                | 0.00259     | 1163.50 | 0.00649                     | 2                | 0.00325     | 109.36  |
| $\Delta \varphi$ | 1.28e-6            | 1                | 1.28e-6     | 0.57    | 2.22e-5                     | 1                | 2.22e-5     | 0.75    |
| Error   | 4.45e-6                     | 2                | 2.29e-6     | 5.94e-5 | 2                            | 2.97e-5          |             |         |

Table 4 Sensitivity Analysis Results of Stress

| Factors | Max Principal Stress | Min Principal Stress |
|---------|----------------------|----------------------|
| $K$     | 1453.40              | 32323.25             | 94.93 |
| $n$     | 24.12                | 2775.40              | 8.15  |
| $R_f$   | 88.76                | 2010.44              | 5.90  |
| $K_b$   | 972.68               | 23470.31             | 68.93 |
| $m$     | 33.96                | 4426.58              | 13.00 |
| $c$     | 14.63                | 288.03               | 0.85  |
| $\varphi_0$ | 302.61             | 7337.69              | 21.55 |
| $\Delta \varphi$ | 0.80              | 0.62                 | 0.002 |
| Error   | 3.48                 | 340.50               |       |

Average displacement values of different factors under various levels are expressed in the factor-index diagram. The changes of node displacements and unit stresses then can be captured while factors move from different levels. These concisely reflect the influential degree of the parameters to displacement and stress. Shown as Fig. 2, 3, 4 and 5:

Fig 2 Change of vertical displacement of 95 point when each factor alters at different level

Fig 3 Change of horizontal displacement of 71 point when each factor alters at different level
As shown in Fig. 3, when parameter sensitivity is analysed with the vertical displacement values of node 95, statistic F will show that $K_b$, $K$, and $\varphi_0$ are most sensitive, $R_f$, $n$, and $m$ are less sensitive and $c$ and $\Delta \varphi$ are least sensitive. When horizontal displacement of node 71 is analysed, parameters $K$ and $\varphi_0$ are the most sensitive, $R_f$ and $K_b$ are less sensitive, $n$, $m$, $c$ and $\Delta \varphi$ are least sensitive. Figure 2 and Figure 3 show the changes of vertical and horizontal displacement of the dam when the parameter factors change between different levels. This verifies the results of the variance analysis in a more intuitive way. As a result, no matter if it is vertical or horizontal displacement, $K$, $K_b$ and $\varphi_0$ are the most sensitive and they determine the displacement of the dam. Meanwhile, $R_f$, $n$, $m$, $c$ and $\Delta \varphi$ are less sensitive and they have a minor effect on the displacement of the dam. In Fig. 4, the sensitivity of parameters in the model to the dam. The result: the parameters with highest sensitivity to the maximum principal stress are $K$, $K_b$, $\varphi_0$, $R_f$, $m$, and $n$ are less sensitive, $c$ and $\Delta \varphi$ are least sensitive. Also, the parameters with strong sensitivity to the minimum principal stress are $K$, $K_b$, $\varphi_0$, $m$, $n$, and $R_f$ are less sensitive, $c$ and $\Delta \varphi$ are least sensitive. Fig. 4 and Fig. 5 respectively reflect the changes of the maximum and minimum principal stress of the dam when factors of parameters change under different levels. In conclusion, $K$, $K_b$ and $\varphi_0$ also determine the principal stress of the dam (including the maximum and minimum).

Based on Table 3, Table 4, Fig. 2, Fig. 3, Fig. 4, Fig. 5 and the analysis above, when applying the Duncan-Chang Model to conduct stress and strain calculations on the embankment dam, $K$, $K_b$ and $\varphi_0$ are highly sensitive to both the displacement and stress of the dam and they are the determinant parameters. $R_f$, $n$, $m$, $c$ and $\Delta \varphi$ are not as sensitive to the displacement and stress and they don’t affect the calculation in a significant way. This conclusion is consistent to the analytical results from [3] and [18]. As a result, when parameter reversion analysis of Duncan-Chang Model is conducted, we only need the determinant parameters $K$, $K_b$ and $\varphi_0$ of the stress and strain calculation as the parameters for inversion. $R_f$, $n$, $m$, $c$, and $\Delta \varphi$ can be obtained through trials and experience. In this way, not only can the amount of computation be greatly reduced, but the accuracy of the reversion analysis results can also be guaranteed.
5. Conclusion
The article first utilized mixed levels of the orthogonal experiment method combined with the finite element numerical simulation to analyze the sensitivity of 8 parameters $K \cdot n \cdot R_f \cdot K_b \cdot m \cdot c \cdot \phi_0$ and $\Delta \phi$ in E-B model to displacement and stress of the dam, which led to the determinant parameters from the dam in stress strain computation. Through the analysis of the computational example, the following conclusion was reached:

(1) Parameters $K \cdot K_b$ and $\phi_0$ have stronger sensitivity to dam displacement (including vertical and horizontal displacement) and dam stress (including maximum and minimum). They are the determinant parameters in the numerical calculation in the embankment dams. Parameters $R_f \cdot n \cdot m \cdot c$ and $\Delta \phi$ have weaker sensitivity to dam displacement and stress.

(2) When utilizing the field observed information for parameter inversion, only $K \cdot K_b$ and $\phi_0$ are needed for back analysis computation. Parameters $R_f \cdot n \cdot m \cdot c$ and $\Delta \phi$ can be identified through experiments or experience.

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