Gravitational Lensing is a unique tool to constrain the mass distribution of collapsed structures, this is particularly true for galaxies, either on a case by case basis using multiple images of background sources (such as quasars), or statistically using the so called galaxy-galaxy lensing technique. First, I will present the lensing theory, and then discuss the various methods applied to current observations. Finally, I will review the bright future prospects of galaxy lensing that will benefit of the development of high resolution, large, wide and deep (lensing) surveys.

1 The Theory: – What Do We Expect –

Although, Astrophysics is generally a question of detecting photons and making sense of them using various physical theories, Gravitational Lensing (GL) is generally a question of understanding the light paths of these photons which then allow to probe the intervening mass distribution. Thus GL must be seen as a useful tool (in a similar way as stellar dynamics) to probe the mass distribution of galaxies - objects of interest in this Conference.

1.1 Useful Gravitational Lensing Equations

A lens operates a transformation from Source plane to Image plane that is merely a simple 2D-mapping that can be describe by 3 equations relating the Source and Image properties:

1. Position:

   \[ \vec{\theta}_S = \vec{\theta}_I - \vec{\nabla} \varphi(\vec{\theta}_I) \]  

   this equation give the position of the source \( \vec{\theta}_S \) for an image at position \( \vec{\theta}_I \); \( \varphi \) is the lensing potential that relates to the projected Newtonian potential by:

   \[ \varphi = \frac{2}{c^2} \frac{D_S}{D_L D_S} \phi^{2D} \]

   Furthermore, as the mapping is purely geometrical, the surface brightness of the objects are conserved through the equation: \( S(\vec{\theta}_I) = S(\vec{\theta}_S) \).

2. Shape:

   \[
   \frac{d\vec{\theta}_S}{d\vec{\theta}_I} = A^{-1} = \begin{pmatrix}
   1 - \partial_{xx} \varphi(\vec{\theta}_I) & -\partial_{xy} \varphi(\vec{\theta}_I) \\
   -\partial_{yx} \varphi(\vec{\theta}_I) & 1 - \partial_{yy} \varphi(\vec{\theta}_I)
   \end{pmatrix}
   \equiv \begin{pmatrix}
   1 - \kappa - \gamma_1 & -\gamma_2 \\
   -\gamma_2 & 1 - \kappa + \gamma_1
   \end{pmatrix}
   \]
This equation defines the inverse of the amplification matrix $A^{-1}$ at the image position $\vec{\theta}_I$. The image magnification $\mu$ is defined as: $\mu^{-1} = \det(A^{-1})$ and the reduced shear (distortion induced by the mass distribution) is $\tilde{g} = \frac{2}{\kappa}$ where $2\kappa = \Sigma/\Sigma_{\text{crit}}$, with $\Sigma_{\text{crit}} = \frac{c_2}{4 G D_{\text{OS}} D_{\text{LS}}}$ the critical density. We also define the absolute shear $\bar{\gamma} = \frac{D_{\text{OS}} D_{\text{LS}}}{D_{\text{LS}}} \gamma$ which is independent on the redshift of the lens and the source and only depends on the mass distribution.

3. Time:

$$t_a \propto \frac{1}{2}(\vec{\theta}_S - \vec{\theta}_I)^2 - \varphi(\vec{\theta}_I)$$

This equation defines the arrival time of an image at position $\vec{\theta}_I$. The difference between two images gives the time-delay. Originally it was foreseen to use the time-delay to constrain the Hubble constant $H_0$, but it does strongly depend on the exact mass profile and distribution. Therefore, assuming a reasonable value for $H_0$, the measure of the time-delay can put strong constraints on the mass distribution, in particular in constraining the total convergence of the lens.

We will concentrate here on the determination of the mass distribution, thus assuming that we have measured the source and lens redshifts as well as determined/fixed the cosmology.

1.2 Case of a Circular Mass Distribution

All 3 equations depend on the lensing potential $\varphi$ either by its gradient (position), its second derivatives (shape/amplification) or its value (time-delay). For a circular mass distribution, it is easy to show that $\partial_r \varphi = \frac{m(r)}{r}$, and that all lensing equations can be written as functions of $\frac{m(r)}{r} \propto \Sigma(< r)$. Note that constraints are absolute in the case of multiple images but are only relative in terms of shear (galaxy-galaxy lensing) as in this case $\frac{m(r)}{r}$ can be expressed as an integral for which the limits are not always well defined [this is also true in cluster lensing and this effect is related to the mass sheet degeneracy].

1.3 Galaxy Mass Distributions - parametric vs. non-parametric

Parametric approaches have been favored from the beginning of lensing, as it provides simple formulae and gives analytical expressions of most or all the necessary lensing quantities. First, circular models were used like the point mass model; the singular isothermal sphere (SIS); the isothermal sphere with a core, with a truncation; the NFW model (Navarro et al 1997); and recently more general cuspy models have been proposed (Holley-Bockelmann et
The diversity of circular models has increased to allow more freedom in the radial profile of the mass distribution, in particular following the recent developments of numerical simulations of Dark Matter halos. By changing the slope of the mass profile we can for example fix the image position but then allow a wider range of acceptable time-delay or flux-ratio between multiple images as these quantities will effectively depend on the radial profile.

Of course, circularity is not likely to be the property of galaxy mass distribution, hence the need of elliptical mass distribution. However only the simplest mass profiles have an analytical expression for the elliptical mass distribution. For the more complex ones (such as the NFW model or the cuspy models), either pseudo-elliptical models (Golse & Kneib 2001) or numerically integrated expressions have been proposed (Munoz et al 2001).

The interest of parametric expressions is their easy use, and their predictive power. Furthermore, they are usually physically motivated and thus generally dynamically stable.

The non-parametric methods have been developed in the strong lensing regime to allow more freedom in the expression of the mass distribution. In general, the mass distribution is represented as a pixelated array in the mass plane or alternatively in the potential plane; the former being generally chosen as it allows to use linear expressions (Saha & Williams 1997). Non-parametric 1D and 2D methods have been developed for cluster weak lensing, however only the 1D mass reconstruction from the weak shear profile is of interest for galaxy lensing. Although interesting, these approaches are poorly predictive and generally proposed dynamically non-stable solutions.

Both methods should be explored as their results can be complementary, however in the future we ought to develop multi-scale/multi-component modeling which will take the best of the two current competing methods.

2 Observations: Where Do We Fight

Better are the observations in terms of position, shape, time-delay, better will be the constraints on the galaxy mass distribution that can be derived. However, strong and weak lensing constraints usually do not overlap, in the sense that strong lensing focus on the 1 to 10 kpc region, and the weak lensing on the 10 to 100 kpc region.

2.1 Strong Lensing: Constraints and External Shear

Because the number densities of background galaxies/quasars is small compared to the physical size of a lensing galaxies, we currently know only a small number of strong lensing systems.
For a multiple quasar system, if $N$ is the number of multiply images observed, we thus have (as a maximum): $2(N-1)$ constraints in position, $N-1$ constraints in amplification, $N-1$ constraints in time-delay. For a double system, it means a maximum of 4 constraints, but 12 for a quadruple system. Of course, external constraints are usually used, like the observed lensing galaxy center, its ellipticity and position angle (that is generally a maximum of 4). This is to compare to the description of the mass distribution of a galaxy which is represented at least by 6 or 7 parameters: $x, y, \varepsilon, \theta, \sigma_0, r_c, r_{cut}, \alpha ...$

One can see that for double quasar the number of constraints is of the order of the number of free parameters, and that generally quadruple system are over-constrained (assuming a one clump mass model).

Of course larger number of images can arise like the 6 image B1359+154 (Rusin et al 2001) or the 10 image radio-lens B1933+503 (Cohn et al 2000), but generally they also need more complex mass distribution to be properly understood. An interesting avenue, is to detect the host galaxy of multiple quasars, indeed, the larger the number of structures identified in the host galaxy the larger the number of constraints on the mass distribution. If the host is sufficiently extended, then it will form an Einstein ring as it has been recently observed and discussed by Kochanek et al (2001). However, lensing constraints are local and thus will only shade light on the mass distribution at the location of the images. Thus ideally, we would like to observe multiple images at different radius from the galaxy center, in order to probe accurately the mass profile.

The current situation has enable to show that rarely a perfect fit is obtained with only one mass clump centered on the main lensing galaxy (e.g. Keeton, Kochanek, Seljak 1996). The simplest way to improve the fit is to introduce what is called external shear: a mathematical tweak of 2 parameters (the intensity $\gamma_E$ and its orientation $\theta_E$). The only, but important drawback on the use of the external shear is that it is not physically motivated has it has no mass. Understanding the origin of the external shear is currently an important GL question.

There is four possible origins to the external shear: 1) the main galaxy itself (by allowing the radial mass profile to change for an elliptical mass distribution) [as e.g. in HE2149-27, Burud et al 2001]; 2) nearby galaxies [as in most systems?]; 3) nearby group of galaxies [as e.g. in PG1115, in HST14176]; 4) nearby cluster of galaxies [as e.g. in RXJ0911, Kneib et al 2001].

The external shear contribution increases with the size of the mass perturbing the system, and decreases with its distance to the lensing system. Clearly, nearby galaxies are the most likely origin, and clusters the less likely. However because most of strong lensing galaxies are ellipticals the presence of
a nearby group or cluster is not surprising, as ellipticals are usually found in dense environments.

To gain more information on the line of sight mass contribution, deep, wide multi-color images, followed by a redshift survey of the nearby structure, or a deep X-ray observation, are needed to quantify precisely the different mass components and to explain the origin of external shear.

2.2 Galaxy-Galaxy Lensing: Scaling Laws and Recovery Methods

As massive clusters in their outskirts, foreground galaxies distort background galaxies following the weak lensing equation that reads (in the weak regime approximation): 

\[ \vec{\epsilon}_I = \vec{\epsilon}_S + \vec{g} \]

where \( g = \frac{\mathbf{\gamma}}{1 - \kappa} \sim \gamma \) is the reduced shear. By averaging over the (a priori) random orientation of the sources the mean ellipticity of the images equals the reduced shear: 

\[ < \vec{\epsilon}_I > = \vec{g} \]

and the dispersion of the measurements is \( \sigma_g = \frac{\sigma_{\epsilon_S}}{\sqrt{N}} \). As \( \sigma_{\epsilon_S} \sim 0.25 \), ideally, for a 4-sigma measure the number of galaxies needed scales as \( N \sim g^{-2} \). However due to the measurement errors (circularisation and anisotropies of the PSF — see a number of contributions in this conference) the number of galaxies needed scales probably more as \( N \sim 2 - 3 \times g^{-2} \). As we are probing regions with \( g \sim 0.01 \) to \( 0.0001 \) a very large amount of quality data is required.

However this simple calculations over simplify the problem, indeed galaxies have different sizes, luminosities, masses and are not at the same redshift. Using a simple approach, one can only constrain the mass of the average galaxy which is not exactly what we aim to learn. Therefore, it is important to use scaling laws and tune them try to understand better the mass distribution of galaxies in their diversity.

The first critical scaling is the distance. Ideally, one wants to know the redshifts of the lensing galaxies as they will allow to define their angular diameter distances, and estimate their luminosities from their broad band magnitudes. It is thus clear that any weak lensing survey with multicolor and spectroscopic informations (for the brighter galaxies) is of critical importance when we want to relate the galaxy-galaxy lensing signal to galaxy mass distribution (e.g. the LCRS galaxy-galaxy lensing analysis: Smith et al 2001, and the recent SDSS analysis: McKay et al 2001). Not having a spectroscopic redshift, one can alternatively used photometry redshifts or any redshift informations that can be derived from the broad-band photometry (e.g. the early work of Brainerd et al 1996).

The second scaling is to assume that the mass distribution can be represented by a universal mass profile which depends only on a very small number of parameters. The general approach first proposed by Brainerd et al.
(1996) is to scale the velocity dispersion $\sigma$ and truncature radius $r_{\text{cut}}$ of an isothermal profile with the galaxy luminosity following some general prescription like the Faber-Jackson law: $\sigma = \sigma_\ast (L/L_\ast)^{1/4}$, the Kormendy relation: $r_{\text{cut}} = r_{\text{cut}}\ast (L/L_\ast)^{0.8}$ [implying $(M/L) \propto L^{0.3}$] or assuming $(M/L) = \text{cste}$ whatever the luminosity [implying $r_{\text{cut}} = r_{\text{cut}}\ast (L/L_\ast)^{0.5}$]. Of course, this is really the simplest one can assume, the mass distribution may depend on the effective radius of the galaxy and/or its morphological type, the exponent in the scaling relations may be different that the standard ones. Furthermore, the truncated isothermal sphere may not represent the correct universal mass profile, hence other mass profile such as cuspy models should be investigated.

Finally, galaxy mass distribution is likely not circular. Thus, one want to relate the ellipticity of the mass to the ellipticity of the light (by reason of symmetry they should have the same orientation). Either one can assume that mass and light have the same ellipticity or one can try to understand what is the scaling law relating the 2 ellipticities.

The simplest recovering technique is what we can call the direct averaging where we try to estimate the mass distribution directly from the PSF corrected measured ellipticities (e.g. Bridle et al in this conference proceeding). Basically this means we estimate the absolute shear for a galaxy pair (the foreground and background galaxies separated by a distance $r$) at a scaled distance $r/r_s$ ($r_s$ is estimated from the foreground galaxy properties, such as the half-light radius, or a luminosity scaled radius) by averaging background galaxy ellipticities:

$$\bar{\gamma}(r/r_s) = \frac{D_{OS}}{D_{LS}} \langle \epsilon_i(r) \rangle$$

where the ratio of the angular distances corrects from the redshift difference from one galaxy pair to another.

Although this direct technique is very simple and robust, it allows only simple scaling for the mass, and the non-trivial contribution of galaxy clustering is directly included in the results, giving more an estimated of the galaxy-mass correlation function than the exact mass distribution of an average galaxy. Furthermore, 1) the mass derived from the shear $\gamma$ suffers from the so-called mass sheet degeneracy hence making difficult to derive any absolute mass estimate; 2) the direct average signal is washed out by any large scale mass distribution and thus should not be applied directly in galaxy cluster fields (Natarajan & Kneib 1997).

The alternative to the direct approach, are inverse methods, such as the maximum likelihood methods presented in Schneider & Rix (1996) and Natarajan & Kneib (1997). In these methods, we consider each (background) galaxy $i$ lensed by the nearby (foreground) galaxies. Assuming some scaling laws (see
above) one can predicts the expected induced distortion on each (background) galaxy and thus compute its intrinsic ellipticity $\vec{\epsilon}_{Si}$. Then by maximizing the likelihood $L = \Pi p_S(\vec{\epsilon}_{Si})$, where $p_S$ is the unlensed galaxy ellipticity distribution, one will be able to derive the best model that fits the observed data.

This strategy, although complex, allows to 1) probe various scaling laws, 2) test different form for the mass distribution profile, 3) use elliptical mass distribution, 4) model higher density environments like groups or clusters; thus this is the one to select specially with good quality data which is likely to be the case for current and future surveys.

2.3 Strong and Weak Lensing: Galaxy in Clusters and Mass Evolution

It has been realized (e.g. Kneib et al 1996) that it is compulsory to take into account the mass distribution of galaxies in clusters to accurately model the lensing distortion. In fact as shown by Natarajan & Kneib (1997) the presence of a large scale mass distribution boost the galaxy-galaxy signal making it easier to detect if one used an adequate method. This is opening prospects to try to understand how the mass distribution is (re)distributed from small scale to large scale as a function of time and local density. Such results will be of great interest and will be important to compare to numerical simulations. Such results are just coming along (Natarajan et al 2000) and are likely to be of great interest with the development of weak lensing surveys.

3 The Future: – How Will We Do –

To better understand the higher mass densities of galaxies, we will need to enlarge the number multiple image systems. This will come either by current facilities - for example searching for small separation multiple quasars in the new quasar surveys (SDSS, 2dF) or by future surveys (ACS/SNAP/NGST/radio). When we have increased the number of systems from the current $\sim 30$ to more than one thousand, we should be able to probe accurately the galaxy mass distribution vs. galaxy type, environment and redshift. We also need to better constrain the current multiple image systems, this is possible by probing more accurately the line-of-sight mass distribution (origin of the external shear, measure of the time-delay, accurate redshifts) and by applying strong+weak lensing techniques.

Galaxy-galaxy lensing is likely to become sort of an industry with the developments of high quality imaging and spectroscopic surveys and will allow to test the various scaling laws and possible universal mass distribution. When applied to cluster survey, galaxy-galaxy lensing will allow to test the stripping
efficiency on galaxy scale in higher densities environment. A possible interesting avenue, will also to conduct quasar-galaxy lensing survey to probe the weight of QSOs and their hosts and compare this results to normal galaxy mass distribution.

In short, there are good prospects to learn more on galaxy mass distribution (baryonic and dark matter) in the near future!

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References

1. Brainerd, T. G., Blandford, R. D., & Smail, I. 1996, ApJ, 466, 623
2. Bridle et al 2001, this conference
3. Burud et al 2001, A&A, in press
4. Cohn, J. D., Kochanek, C. S., McLeod, B. A., & Keeton, C. R. 2001, ApJ, 554, 1216
5. Holley-Bockelmann, K., Mihos, J. C., Sigurdsson, S., & Hernquist, L. 2001, ApJ, 549, 862
6. Golse & Kneib, 2002, A&A, submitted
7. Kneib, J.-P., Ellis, R. S., Smail, I., Couch, W. J., & Sharples, R. M. 1996, ApJ, 471, 643
8. Kneib, J., Cohen, J. G., & Hjorth, J. 2000, ApJL, 544, L35
9. Kochanek, C. S., Keeton, C. R., & McLeod, B. A. 2001, ApJ, 547, 50
10. McKay et al 2001, ApJ, submitted, astro-ph/0108013
11. Muñoz, J. A., Kochanek, C. S., & Keeton, C. R. 2001, ApJ, 558, 657
12. Natarajan, P. & Kneib, J. 1997, MNRAS, 287, 833
13. Natarajan et al 2000, in “Gravitational Lensing: Recent Progress and Future Goals”, Boston University, July 1999, ed. T.G. Brainerd and C.S. Kochanek, astro-ph/9909349
14. Rusin, D. et al. 2001, ApJ, 557, 594
15. Saha, P. & Williams, L. L. R. 1997, MNRAS, 292, 148
16. Smith, D. R., Bernstein, G. M., Fischer, P., & Jarvis, M. 2001, ApJ, 551, 643