Influence of «defocusing» on the correlation function in the problem of particles suspension accounting

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Abstract. This paper describes influence of such parameter as “defocusing” on the ratio of the peak of the correlation function to its pedestal, calculated for adjacent planes of the volume with suspended particles. This ratio is used in the method of statistical accounting of the particles in the volume. Numerical modelling and experimental validation are performed.

1. Introduction
Nowadays, digital image processing techniques find application in many fields of science and technology. They help to implement numerically different measurement methods of various characteristics of particle flows or distributions of suspended particles. Application of digital image processing of particles includes basic research: the study of the dynamics of particle fluxes (two-phase flow of gas, supersonic flow, etc.) [1, 2]; the study of transparent medium (to prevent the defects of glass, etc.) [3]. There are lots of methods of coherent processing of particle images, among which are laser anemometry of particle images (PIV, or particle image velocimetry) [4] and digital holography methods (DH) [5, 6]. The correlation technique has been used for the displacement [8] and deformation [9] measurement, as well as for roughness measurements [10]. Width [10] and position [8, 9] of correlation peak are analyzed usually to extract useful data, namely, information about roughness of the illuminated surface, simulated displacements on image and deformation field. This study, which is a continuation of [7], proposes calculation of the cross-correlation of images of two adjacent slices of a volume to estimate the characteristics of the distribution of the particles. We study particles in a volume of optically transparent medium. We examine the changing of peak to pedestal ratio when the concentration, size and other statistical parameters of the particle distribution are varied. It is considered to be more reliable, allowing to achieve higher speeds of calculations as compared to other methods.

2. Statement of the problem
We consider the case of scattering particles distributed in the volume of the optical medium. The purpose of this study is to estimate the efficiency and determinate the applicability of the particle image statistical description based on analysis of cross-correlation function parameters. We consider a simple case of the distribution of particles in a volume to identify and verify the basic properties and behavior of the correlation function. Such a property is correlation function peak separation in event of «defocusing» of the particles, peak broadening with an increasing size of repetitive elements, etc.

3. Algorithm based on cross-correlation calculation
We have developed a simulation model for the validation of the method of statistical accounting of the particles in the volume and evaluation of its effectiveness as a tool to determine the nature of the distribution of particles suspended in a volume. This simulation has been implemented using National
Instruments LabVIEW development environment. The modelling program consists of three consecutive blocks, which are examined and explained below.

3.1. Simulation of the distribution of particles suspended in the volume of optical medium
On the first stage, program simulates a suspension of particles in a volume with specified distribution, size and number of particles. The volume of the optical medium is defined as a set of segments with a specific distribution of scattering particles in each of them. We consider random distribution (equally probable in each segment) in order for the developed model of the volume of optical medium to be in accordance with the conditions that most frequently take place in the research problems of particles. The mathematical model of particle distribution in a volume is described in [11].

3.2. Modelling of the formation of particles volume images
Second stage is the image processing: program calculates two adjacent layers of the volume with the mathematical apparatus of the scalar diffraction theory. To calculate the electromagnetic field propagation from coherent light source through the volume of the optical medium, the method of propagation of the angular spectrum of plane waves implemented through two fast Fourier transform can be used [12]. We use an algorithm based on the edge-point linking and thresholding technique that was described in [13] to detect layers of volume with particles in it.

3.3. Cross-correlation analysis
Cross-correlation function is a method for assessing the degree of correlation between two sequences (or arrays in two-dimensional case). Consider two arrays \( f \) and \( g \). Cross-correlation is defined as:

\[
(f \ast g)_{i,j} = \sum_k f^*_k \cdot g_{i,j+k}
\]

Here \( i \) and \( j \) are indices that indicate coordinates of a point where correlation calculation is performed, \( k \) is a shift-indice and \( f^* \) indicates a complex conjugation on \( f \). In case of more efficient and faster computation using fast Fourier transform algorithm [14] cross-correlation function can be calculated as:

\[
f \ast g = \hat{F}^{-1}[(\hat{F}[f])^\dagger \cdot (\hat{F}[g])]
\]

Here \( f \) and \( g \) are two-dimensional arrays, \( \hat{F}[g] \) is the Fourier transform on \( g \) and \( \hat{F}^{-1}[g] \) is inverse Fourier transform. Cross-correlation function of two focused layers of particle distribution depends on many parameters. Among them are the distance between the layers \( \Delta z \), particles’ diameter \( D \), concentration \( C \) (depends on number of particles in each segment \( N_p \) and number of segments \( N \)), transparency of particles \( T \), wavelength of the laser source \( \lambda \), etc.

It is more convenient and faster to analyze two-dimensional case of correlation function. We can move from a three-dimensional correlation function to a two-dimensional one avoiding loss of information. Due to the symmetry of the particles themselves and of the distributions we study, cross-correlation turns to be symmetrical as well. The transition from 3D to 2D is illustrated in Fig. 1.
4. Study of the dependence of the correlation function form on parameters of the particles

4.1. Dependence of correlation on space between slices of volume
While \( \Delta z \) is not large enough to change the pattern of two images of different slices of volume, correlation peak is wide and has no inflections or borders. The difference between focused slices increases with a growth of \( \Delta z \) and the form of cross-correlation function changes: border between the peak and the pedestal of the cross-correlation function becomes distinguishable, videlicet, peak becomes narrower but pedestal remains the same. This effect of peak indication is caused by the nature of distinction between images, namely, the diffraction of scattered light. The larger is the distance between layers of particles \( \Delta z \), the greater is the effect of diffraction, and defocused particles are surrounded by diffraction rings. We performed the experimental validation and computational simulation of the mutual change between \( \Delta z \) and correlation function of two adjacent layers. Dependence of cross-correlation on thickness between slices of volume for each case is shown in Fig. 2, 3.

![Figure 1](image1.png)

**Figure 1.** Transition from normalized 3D cross-correlation function to its 2D cross-section. The cut place is indicated

![Figure 2](image2.png)

**Figure 2.** Normalized cross-correlation sections \{g, h, i\} between 2 slices \{a, c, e\} – first for observer, \{b, d, f\} – second for observer) of volume of real particles where \( D \approx 0.42 \) mm, \( N_p = 2 \) on the first slice, \( N_p = 1 \) on second slice and \( \Delta z \) varies. Sizes of detected slices are \( 1,062 \times 1,062 \) mm, \( \lambda = 532 \) nm. The distance from the object to the CCD equals to 6.7 mm. The difference in diffraction rings of defocused particles is shown in \{j, k, l\}
Figure 3. Normalized cross-correlation sections \{g, h, i\} between 2 slices \{a, c, e\} – first for observer, \{b, d, f\} – second for observer) of volume of spherical particles. The difference in diffraction rings of defocused particles is shown in \{j, k, l\}. All parameters are similar to Fig. 2.

The dashed circles are given to indicate focused particles, closer examination of the segments is marked with square frames. In the experimental and simulation cases we can see a separation of the peak of the correlation function from the pedestal due the diffraction of light.

4.2. Dependence of correlation on diameter of the particles

Correlation between two focused layers increases with increasing particles’ size. It is understandable from the definition of cross-correlation function as a sum of different shifts between arrays in each pair of coordinates as shown in equation (1). The larger is the size of similar objects on images (focused and defocused particles), the greater is the shift between them will be to contribute to the peak of correlation function. We performed a simulation of particles with the same coordinates in each layer and increasing diameters \(D\) to reach maximum accuracy of the estimation of this dependence. The result of it is shown in Fig. 3. Peak broadening due to the increased size of the particles is observed.

Figure 3. Random distribution particles \{a, b, c\} with \(\Delta z = 0.15\) mm, \(N = 3, N_p = 10, T = 0\) and their cross-correlations \{d, e, f\} calculated for 2nd and 3rd slices (from observer). Sizes of detected slices are 3.072 x 3.072 mm, \(\lambda = 632.8\) nm.
4.3. Dependence of correlation on concentration of the particles in a volume

Correlations between the pixels of adjacent segments of the volume are increasing with the increase of the concentration, or quantity of particles in each layer in volume. This effect can be explained by the increase of repetitive patterns with the increase of concentration in the case of equiprobable distribution of particles. This is evident in the reduction of the pedestal and the increase of the peak of the cross-correlation function (Fig. 4). Here a rapid decrease of the pedestal and it’s detachability from the increased peak can be seen. Border between peak (p) and pedestal (b) are indicated.

![Image of particles distribution](image)

**Figure 4.** Random distribution of particles {a, b, c} with \( \Delta z = 0.15 \) mm, \( D = 0.054 \) mm, \( N = 10 \), \( T = 0.01 \) and increasing concentration \( C \) and cross-correlation {d, e, f} calculated for 2\(^{nd}\) and 3\(^{rd}\) slices (from observer). Sizes of detected slices are 3.072 x 3.072 mm, \( \lambda = 632.8 \) nm

Using the last stage of algorithm, namely, the correlation analysis of these images and the peak to the pedestal ratio calculation we can describe the distribution and concentration of particles in the volume. The dependence of the peak to the pedestal ratio on number of particles in each slice turns to be linear for random distribution.

5. Conclusion

In this study, we demonstrated an approach to the analysis of opaque particles suspended in a volume of the optical medium, which uses the method of digital holography and calculation of cross-correlation function. Numerically propagating the field that has passed through the investigated volume and comparing adjacent layers of it with the instrument of correlation analysis, we did a basic research of correlation function in addition to the study of particles. For validation of this method we investigated the basic characteristics of the cross-correlation function in the case of random distribution of the particles in a volume. Thus, we suggested and confirmed with the help of simulation and experiment that with the increase of the distance between the layers of particles, peak of correlation separates from the pedestal. Moreover, we have confirmed a consequence of the definition of the correlation function, namely, that an increase of the diameter of the particles results in the correlation peak broadening. Finally, we found that the ratio of the peak of the correlation function to its pedestal is linearly dependent on the concentration of particles in the volume. To obtain the results listed above a numerical simulation model has been developed. Identified characteristics and patterns can later be used to develop more accurate and efficient methods of characterization of opaque particles.
6. Acknowledgement
T. A. V. and T. Yu. N. acknowledge financial support from Government of Russian Federation, Grant No. 074U01. N. V. P. acknowledges financial support from Russian Ministry of Education and Science project within the state mission for institutions of higher education, agreement №2014/190.

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