Magnetic field effect on the laser-driven density of states for electrons in a cylindrical quantum wire: transition from one-dimensional to zero-dimensional behavior

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Abstract. The influence of a uniform magnetic field on the density of states (DoS) for carriers confined in a cylindrical semiconductor quantum wire irradiated by a monochromatic, linearly polarized, intense laser field is computed here non-perturbatively, following the Green’s function scheme introduced by some of the authors in a recent work (Lima et al 2009 Solid State Commun. 149 678). Besides the known changes in the DoS provoked by an intense terahertz laser field—namely, a significant reduction and the appearance of Franz–Keldysh-like oscillations—our model reveals that the inclusion of a longitudinal magnetic field induces additional blueshifts on the energy levels of the allowed states. Our results show that the increase of the blueshifts with the magnitude of the magnetic field depends only on the azimuthal quantum number $m$ ($m = 0, 1, 2, \ldots$), being more pronounced for states with higher values of $m$, which leads to some energy crossovers. For all states, we have obtained, even in the absence of a magnetic field, a localization effect that leads to a transition in the DoS from the usual profile of quasi-1D systems to a peaked profile typical of quasi-0D systems, as e.g. those found for electrons confined in a quantum dot.

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1. Introduction

In the past few years, the development of modern high-power, tunable laser sources—e.g. free-electron lasers (FELs)—has allowed the experimental study of the interaction of intense laser fields (ILFs) with charge carriers in semiconductors, mainly in the terahertz (THz) frequency range [1, 2]. In semiconductor nanostructures—quantum wells, wires, rings and dots—the motion of the charge carriers along at least one direction is blocked and limited to a few nanometers, which gives rise to the quantum confinement effect and the formation of a low-dimensional (low-D) system, which exhibits distinct optical and transport properties in comparison to bulk semiconductors [3, 4]. Interestingly, by illuminating low-D electronic systems with ILFs some new physical phenomena were theoretically anticipated and observed, such as THz resonant absorption [5], laser-modified magnetotransport [6], strong distortions in the optical absorption spectra [7–10] and remarkable changes in the density of states (DoS) in three-dimensional electron gases (3DEGs) [11], strictly 2DEGs [11], quasi-2DEGs [12] and quasi-1DEGs [13] via the dynamical Franz–Keldysh effect (DFKE) [14]. An important consequence of such changes in the DoS is the possibility of obtaining an optical control of the electronic density in the channel of modulation-doped semiconductor heterostructures [15], in which this quantity strongly influences both electron mobility and electron conductivity, thus determining its viability for device applications [3], including new optoelectronic devices capable of operating ultrafast modulation and switching optical signals [16–18].

In comparison to 2D electronic systems, the optical properties of 1D systems (e.g. electrons confined in semiconductor quantum wires (QWs) and quantum rings) irradiated by ILFs are less understood. Experimentally, this may be due, in part, to the difficulty of growing high-quality samples [19]. On the theoretical side, only recently some of the authors have developed a suitable non-perturbative approach for computing the effect of an intense THz laser field on the DoS for electrons confined in a cylindrical semiconductor QW [13]. This kind of approach is crucial for treating systems irradiated by ILFs since low-order perturbation theory estimates soon become invalid with the increase of laser field strength [1].

In addition, it is well known that the application of a magnetic field significantly alters the bound-state energy levels for electrons in atoms and molecules (Zeeman effect) [20]. For electrons in non-magnetic semiconductors, the application of a magnetic field introduces a

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This effect is similar to the usual Franz–Keldish effect, which describes the non-vanishing optical absorption below the band edge followed by characteristic oscillations above the band edge as a consequence of the application of a uniform electric field.
reduction in dimensionality due to the induced magnetic confinement. In fact, when a uniform magnetic field taken along, e.g., the z-axis direction and treated in the Landau gauge, as usual, is applied on a semiconductor, the charge carrier quantum states are affected by the appearance of two terms in the Hamiltonian operator: a parabolic magnetic potential, which confines the electronic wavefunction (along, say, the x-axis), and a first derivative term that couples the motions along directions perpendicular to the magnetic field (i.e. x and y), which is reminiscent of the Lorentz force [21]. As the vector potential associated with this field does not depend on y (in the Landau gauge), the original Schrödinger (SCD) equation can be separated into simpler differential equations. The equation for the motion along the x-axis is identical to that for a 1D harmonic oscillator and this causes the collapse of the DoS from a continuous, smooth function (horizontal steps for electrons in a quantum well) to a series of δ-functions called Landau levels. Of course, these levels are sharp only in an ideal system where electrons are never scattered by either other electrons or defects (impurities, phonons, etc). In a more realistic treatment, an electron typically survives for a finite time τ between successive scattering events, such that the Landau levels acquire a width \( \frac{\hbar}{\tau} \). For any assumed shape (e.g. Gaussian or Lorentzian), one does not expect significant changes in the DoS unless the separation of successive levels—namely, \( h \omega_c \), where \( \omega_c = eB/m^* \) is the cyclotron frequency, \( m^* \) being the carrier effective mass—exceeds their width, i.e. \( \omega_c \tau > 1 \), which means that an electron must survive for at least a complete orbit in the magnetic field before the DoS splits up. This separates semiclassical effects (seen at low magnetic fields) from quantum-mechanical effects (which arise at fields high enough to make the Landau levels well resolved). This more realistic picture is suitable for explaining more complex phenomena involving low-temperature experiments at high magnetic fields (i.e. those for which \( k_B T \ll \hbar \omega_c \)), such as the Shubnikov–de Haas and quantum Hall effects⁴. In other words, in the absence of other external fields, the effect of a uniform magnetic field on an electronic system is usually studied (within a quantum mechanical model) by charging the magnetic field contribution to the energy on the formation of discrete Landau levels, which changes the electron DoS in a peculiar manner (see e.g. [21], pp 223–6).

As the DoS marks the maximum number of carriers that can occupy states with energies between \( \varepsilon \) and \( \varepsilon + d\varepsilon \), governing both the optical and transport properties of low-D systems and establishing its dimensionality [22], we find it interesting to investigate here in this work, within a non-perturbative scheme based on the Green’s function approach for non-resonant, ILFs [13], the effect of a longitudinal, uniform magnetic field on the laser-driven DoS for electrons in a semiconductor QW irradiated by a THz ILF.

2. Electron density of states in a semiconductor quantum wire

2.1. Electron density of states in the absence of external fields

In the absence of external electromagnetic (EM) fields, the discrete energy levels can be easily computed for quasi-1D electrons confined in a single semiconductor QW with a uniform 

⁴ This is done by relating the oscillations of some observable quantities (e.g. electrical resistivity), obtained by increasing the magnitude of the magnetic field, to the displacement of the Fermi level \( E_F \) over successive Landau levels and gaps. Most experiments are performed with a practically constant density of electrons, a case in which the number of occupied Landau levels decreases with the increase of \( B \). The DoS at \( \varepsilon = E_F \) is high whenever \( E_F \) lies within a Landau level, and a small change in \( E_F \) induces a large change in density (compressible electronic fluid), while the opposite behavior occurs when \( E_F \) lies within a gap, where the DoS is null (incompressible electronic fluid) [21].
cross-section. For simplicity, let us take into account a long cylindrical wire with radius \( a \) and length \( L (\gg a) \), surrounded by a thick, coaxial layer made up of another semiconductor that forms an impenetrable barrier.\(^5\) Within the effective-mass approximation, by choosing the \( z \)-axis as the wire axis itself, the time-independent SCD equation in *cylindrical coordinates* reads

\[
-h^2 \left( \frac{1}{m_z^{\star}} \nabla_z^2 + \frac{1}{m_r^{\star}} \frac{\partial^2}{\partial z^2} \right) \psi + V(r) \psi = \varepsilon \psi,
\]

(1)

where \( \nabla_z^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \), \( \psi = \psi(r, \theta, z) \) and \( m_z^{\star} (m_r^{\star}) \) is the effective mass for the motion perpendicular to (along) the \( z \)-axis direction. As we are considering that the electrons 'see' a confinement potential \( V(r) \) that is null for \( 0 \leq r < a \) and infinity for \( r > a \), the electron wavefunction must vanish at the heterointerface (and beyond it) and then the proper boundary condition is simply that \( \psi \) must be continuous (hence null) at \( r = a \). For wavefunctions in the form \( \psi(r, \theta, z) = \xi(r, \theta) \times \chi(z) \), the above partial differential equation (PDE) separates into

\[
-\frac{\hbar^2}{2m_z^{\star}} \nabla_z^2 \xi(r, \theta) + V(r) \xi(r, \theta) = E_{\perp} \xi(r, \theta)
\]

(2)

and

\[
-\frac{\hbar^2}{2m_r^{\star}} \frac{d^2}{dz^2} \chi(z) = E_{\parallel} \chi(z),
\]

(3)

where \( E_{\perp} (E_{\parallel}) \) is the kinetic energy related to the motion perpendicular to (along) the wire axis, with \( \varepsilon = E_{\perp} + E_{\parallel} \). The solution for the latter differential equation (i.e. that for the longitudinal motion) is simply \( \chi(z) = \exp(ik_z z) \), where \( k_z \equiv \sqrt{2m_z^{\star} E_{\parallel} / \hbar} \). As the potential does not depend on \( \theta \), the former differential equation can be further separated into radial and azimuthal parts. By assuming \( \xi(r, \theta) = R(r) \times W(\theta) \), one finds that \( d^2 W / d\theta^2 = -m_z^{\star} W \) for the azimuthal part, whose solution is \( W(\theta) = \exp(im\theta) \). The fact that this solution must be periodic, with a period \( 2\pi \), restricts \( m \) to integer values only. The resulting differential equation for the radial motion is

\[
\frac{d}{dr} \left( r \frac{dR}{dr} \right) + \frac{2m_z^{\star}}{\hbar^2} \left[ E_{\perp} - V(r) \right] r^2 R - m_z^{\star} R = 0.
\]

(4)

The boundary conditions for the requirement that \( R(r) \) must be finite at \( r = 0 \) and that \( R(r) \to 0 \) as \( r \to a^- \), being null for \( r \geq a \). This eigenvalue problem is analytically solvable in terms of Bessel functions, the solution being

\[
R(r) = \begin{cases} 
C J_m(k_{m,n} r), & \text{for } 0 \leq r \leq a, \\
0, & \text{for } r \geq a,
\end{cases}
\]

(5)

where \( J_m(x) \) is the (order \( m \)) Bessel function of the first kind. In this equation, \( k_{m,n} \equiv \sqrt{2m_z^{\star} E_{m,n}/\hbar} \), \( E_{m,n} \) being the bound-state energy eigenvalues, which form a discrete set. The boundary condition yields \( k_{m,n} = \xi_{m,n}/a \), where \( \xi_{m,n} \) is the \( n \)th zero of \( J_m(x) \). The normalization constant is \( C = [\sqrt{\pi L a} J_{m+1}(\xi_{m,n})]^{-1} \). With these zeros in hand, one finds the following energy eigenvalues:

\[
E_{m,n} = \frac{\hbar^2 k_{m,n}^2}{2m_z^{\star} a^2}
\]

(6)

\(^5\) This barrier arises as a consequence of the band offset between the inner and the outer semiconductor layers.

\(^6\) The first few zeros of \( J_m(x) \) are tabulated in textbooks. A practical alternative to get these values with high numerical precision is to use mathematical software (e.g. Maple or Mathematica), as done in this work.
These energies mark the bottom of the 1D subbands, corresponding to the singularities of the DoS for the quasi-1D electronic system, which can be calculated from its general formulation
\[ \rho(\varepsilon) \equiv g_s \sum \delta (\varepsilon - \varepsilon_{m,n}(k_z)), \]
where \( g_s = 2 \) is the factor for spin degeneracy and \( \varepsilon_{m,n}(k_z) = E_{m,n} + \hbar^2 k_z^2 / (2 m^*_n) \) is the total electron energy. Since \( k_z \) is a quasi-continuum wavenumber, the summation on \( k_z \) can be converted into a ‘volume’ integral in \( k \)-space, yielding
\[
\rho_{\text{free}}(\varepsilon) = \frac{D_0}{L} \sum_{m,n} \Theta(\varepsilon - E_{m,n}) \sqrt{\varepsilon - E_{m,n}},
\]
where \( D_0 = g_s \sqrt{2 m^*_n} / (2 \pi \hbar) \) and \( \Theta(x) \) is the Heaviside unit-step function.

2.2. Laser-driven density of states for electrons under a magnetic field

In the simultaneous presence of intense EM radiation linearly polarized along the wire axis direction, denoted here as \( \mathbf{F} = F(r, t) \hat{z} \), and a uniform magnetic field \( \mathbf{B} = B \hat{z} \), the quantum states assumed by the quasi-1D carriers confined in a cylindrical QW can be studied within a non-relativistic model based on the following semiclassical, time-dependent Schrödinger–Pauli equation (a formulation of the SCD equation for spin-\( \frac{1}{2} \) particles that takes into account the interaction of the particle’s spin with an external magnetic field) [13, 23]:
\[
\left[ \frac{\mathbf{p}^2 + e \mathbf{A}}{2 m^*} + \frac{1}{2} g^* \mu_B B + V(r) \right] \Psi(r, t) = i\hbar \frac{\partial \Psi(r, t)}{\partial t},
\]
where \( \mathbf{p} = -i \hbar \mathbf{\nabla} \) is the electron momentum and \( \mathbf{A} = \mathbf{A}(r, t) \) is the vector potential associated with the external EM fields. The second term in the Hamiltonian operator is the Stern–Gerlach term, in which \( \mu_B = e \hbar / (2 m_0) \) is the Bohr magneton, \( m_0 \) being the free-electron rest mass, and \( g^* \) is the ‘Landau factor’. This term produces a splitting, known as the gyromagnetic spin splitting, between the spin-up (− sign) and spin-down (+ sign) electrons. For free electrons in a magnetic field one has \( g^* = 2 \), but in zincblende semiconductors spin–orbit coupling reduces the spin splitting, leading to a smaller effective value of \( g^* \) [29]. For conduction band electrons in bulk GaAs at low temperatures, one has \( g^* \approx -0.44 \) [30]. As pointed out by Harrison [23], even in a relatively high magnetic field of 10 T the difference in energy due to this term (i.e. \( g^* \mu_B B \)) is \( \approx 1 \) meV or less, which on the scale of typical nanostructures is relatively small and can be neglected in favor of a simpler PDE, namely
\[
\left[ \frac{\mathbf{p}_\perp^2 + e \mathbf{A}_\perp}{2 m^*_\perp} + \frac{(p_z + e A_z)^2}{2 m^*_z} + V(r) \right] \Psi(r, t) = i\hbar \frac{\partial \Psi(r, t)}{\partial t},
\]
where \( \mathbf{p}_\perp = -i \hbar \mathbf{\nabla}_\perp \) and \( \mathbf{A}_\perp \) account for the momentum and vector potential components perpendicular to the wire axis, respectively, whereas \( p_z = \hbar k_z \) and \( A_z \) are the momentum and vector potential components along the \( z \)-axis direction, respectively. In cylindrical coordinates, the transversal component of the gradient is \( \mathbf{\nabla}_\perp = \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} \). Since the quasi-1D electrons are simultaneously subjected to the fields \( \mathbf{F} \) and \( \mathbf{B} \), both polarized along the wire axis, it is suitable to try a separation of variables in the transversal and longitudinal coordinates. For this, let us adopt the symmetric gauge for the vector potential, in which \( \mathbf{A}_\perp = \mathbf{A}_\perp(r, \theta) = \frac{1}{2} r B \hat{\theta} \), which clearly accomplishes the Coulomb gauge (i.e. \( \mathbf{\nabla}_\perp \cdot \mathbf{A}_\perp = 0 \)). For the longitudinal component of the vector potential, in general \( A_z = A_z(x, y, t) \) since it should correspond only to ILF radiation polarized along the wire axis and propagating in a direction perpendicular to this axis.
Let us assume, for simplicity, that this field has a wavelength long enough for it to be described by the *dipole approximation* in the physically important region of space, such that \( A_z \approx A_z(t) \) is a good approximation. By writing \( \Psi(r, \theta, z, t) \) as the product \( \xi(r, \theta) \times \chi(z, t) \) and putting it back in (9), we find

\[
\frac{1}{\xi} \left\{ \frac{\mathbf{p}_r + e A_z(r, \theta)}{2 m_\perp^*} \right\}^2 + V(r) \xi = \frac{1}{\chi} \left\{ i \hbar \frac{\partial \chi}{\partial t} - \frac{[p_z + e A_z(t)]^2}{2 m_\parallel^*} \chi \right\}.
\]

This equality cannot be true except if both members are equal to a constant, namely \( E_{m,n} \). This allows us to separate the SCD equation (9) into two new PDEs (coupled by the constant \( E_{m,n} \)), namely

\[
\left\{ \frac{[\mathbf{p}_r + e A_z(r, \theta)]^2}{2 m_\perp^*} + V(r) \right\} \xi = E_{m,n} \xi,
\]

where the energy eigenvalues \( E_{m,n} \) are computed for a certain magnitude of the applied magnetic field, being unaffected by the laser field, and

\[
\left\{ \frac{[p_z + e A_z(t)]^2}{2 m_\parallel^*} + E_{m,n} \right\} \chi = i \hbar \frac{\partial \chi}{\partial t},
\]

where \( \chi(z, t) \) depends on the magnetic field only parametrically, via the energy \( E_{m,n} \).

2.2.1. The transversal problem. As in the dipole approximation equation (11) is not affected by the laser field, we can solve it as if the electrons were under the action of the QW confinement potential \( V(r) \) and the magnetic field \( \mathbf{B} = B \hat{z} \) only. Again, since the potential \( V(r) \) is null for \( 0 \leq r < a \) and infinite for \( r > a \), the electron wavefunction must vanish at \( r \gg a \) and then the boundary condition is that \( \xi \) must be continuous (hence null) at \( r = a \). As \( V(r) = 0 \) inside the wire, we can expand the square in the kinetic term of the Hamiltonian in (11), which yields, by noting that \( \nabla_\perp \cdot (A_\perp \xi) = A_\perp \cdot \nabla_\perp \xi + \xi (\nabla_\perp \cdot A_\perp) = A_\perp \cdot \nabla_\perp \xi \) because \( \nabla_\perp \cdot A_\perp = 0 \) in the Coulomb gauge,

\[
\frac{1}{2 m_\perp^*} (-\hbar^2 \nabla_\perp^2 + e^2 A_\perp^2 + 2 e A_\perp \cdot \mathbf{p}_r) \xi = E_{m,n} \xi.
\]

In the symmetric gauge, \( A_\perp = \frac{1}{2} r B \hat{\theta} \), as pointed out above, and our PDE in cylindrical coordinates becomes, after some algebra,

\[
-\frac{\hbar^2}{2 m_\perp^*} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \xi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \xi}{\partial \theta^2} \right] - \frac{1}{2} i \hbar \omega_c \frac{\partial \xi}{\partial \theta} + \frac{1}{8} m_\parallel^* \omega_c^2 r^2 \xi = E_{m,n} \xi,
\]

where \( \omega_c = eB/m_\perp^* \). Now, by attempting another separation of variables, we write \( \xi(r, \theta) \) as the product \( R(r) \times W(\theta) \), which yields

\[
r^2 R'' + r \left( \frac{R'}{R} + \frac{2 m_\perp^*}{\hbar^2} \right) \left( \frac{1}{2} i \hbar \omega_c \frac{W'}{W} r^2 - \frac{1}{8} m_\parallel^* \omega_c^2 r^4 + E_{m,n} r^2 \right) = -W''.
\]

The azimuthal symmetry of our physical system implies that the probability of finding the electron at a given point—i.e. the square of the absolute value of the electron wavefunction \( |\Psi(r, \theta, z, t)|^2 \)—is independent of \( \theta \), so \( W(\theta) \) has the form \( \exp(\imath c \theta) \) (apart from a multiplicative factor).
constant). Therefore, the ratio \( W' / W = i c \) does not depend on \( \theta \) and then the left-hand side of (15) depends only on \( r \), whereas the right-hand side depends only on \( \theta \). This is possible only if both sides are equal to another constant. To find this constant, it is enough to note that the angular equation \( - W'' / W = \nu^2 \) easily solves \( W(\theta) = e^{i\nu \theta} \) for some real value of \( \nu \), as expected from the above symmetry argument. Indeed, as this angular solution is periodic (with period \( 2\pi \)) then \( \nu \) must be an integer, which is reinforced here by writing \( \nu = m \). Therefore, \( W(\theta) = e^{im \theta} \) and then the radial equation becomes

\[
R'' + \frac{R'}{r} + \frac{2m^2}{\hbar^2} \left( E_{m,n} - \frac{1}{2} m \hbar \omega_c - \frac{\hbar^2 m^2}{2m^2} - \frac{1}{8} m^2 \omega_c r^2 \right) R = 0. \tag{16}
\]

To solve this differential equation analytically, it is useful to make the change of variable \( x = \alpha r^2 \), with \( \alpha = eB / (2\hbar) = 1/(2 \ell_B^2) \), \( \ell_B \) being the magnetic length. With this change, \( R(r) \) becomes a function \( f(x) \), the derivatives being related by \( dR/dr = 2\alpha r df/dx \). This change reduces (16) to

\[
x^2 f'' + f' + \left( \frac{E_{m,n}}{\hbar \omega_c} - \frac{m^2}{2} - \frac{m^2}{4x} \frac{x}{4} \right) f(x) = 0. \tag{17}
\]

By taking into account the asymptotic properties of this differential equation, we find that, for large values of \( x \), the solution behaves as \( e^{-x/2} \), which suggests that we should try to reduce (17) to some known differential equation by writing the desired solution \( f(x) \) as the product \( e^{-\frac{x}{2}} x^{\frac{|m|}{2}} X(x) \). By substituting this product in (17), one finds

\[
x X'' + (|m| + 1 - x) X' + \left( \frac{E_{m,n}}{\hbar \omega_c} - \frac{m + |m| + 1}{2} \right) X(x) = 0. \tag{18}
\]

Now, by having \( \tilde{a} \equiv -\frac{E_{m,n}}{\hbar \omega_c} + \frac{m + |m| + 1}{2} \) and \( b \equiv |m| + 1 \), this equation is reduced to the form of Kummer’s differential equation, whose analytical solution \( X(x) \) is simply a linear combination of Kummer’s functions \( M(\tilde{a}, b, x) \) and \( U(\tilde{a}, b, x) \) [28]. As the function \( U(\tilde{a}, b, x) \) diverges as \( x \to 0^+ \), this part of the solution must be ignored and then our solution for the radial equation (17) is simply \( f(x) = \tilde{C} e^{-\frac{x}{2}} x^{\frac{|m|}{2}} M(\tilde{a}, b, x) \), which corresponds to

\[
R(r) = \tilde{C} e^{-\frac{r^2}{2}} r^{|m|} M(\tilde{a}, b, \alpha r^2). \tag{19}
\]

The corresponding energy eigenvalues are easily obtained by searching for the roots of \( R(a) = 0 \) numerically. Note that the solution is not the same for \( +m \) and \( -m \), since the value of parameter \( \tilde{a} \) depends on the sign of \( m \).  \(^7\)

2.2.2. The longitudinal problem. The longitudinal motion is ruled by PDE (12). Hereafter, we shall restrict ourselves to a monochromatic radiation field for which \( A_z(t) = A_0 \sin(\omega t) \), where \( \omega \) is the angular frequency and \( A_0 = F_0 / \omega \), \( F_0 \) being the laser field strength. By noting that \( p_z = \hbar k_z \), equation (12) becomes

\[
\left\{ \frac{\hbar k_z + eA_z(t)}{2m^*} \right\}^2 + E_{m,n} \chi = i\hbar \frac{\partial \chi}{\partial t}, \tag{20}
\]

\(^7\) In fact, for a given value of the magnetic field strength, the dependence of \( \tilde{a} \) on the sign of \( m \) (not only on \( |m| \)) yields the simultaneous formation of blueshifted \( (m \geq 0) \) and redshifted \( (m < 0) \) states.
where \( \chi = \chi(z, t) \) and \( E_{m,n} \) is any eigenvalue whose numerical value is easily found by demanding that the solution in (19) be null at \( r = a \) (the boundary condition), for a given magnetic field. As the laser field has only a \( z \)-component, the SCF equation for the radial and azimuthal parts, equation (11), is not altered by the presence of the laser field; thus the eigenvalues \( E_{m,n} \) and the corresponding eigenfunctions \( \xi(r, \theta) = R(r) e^{i\alpha \theta} \), obtained in the previous subsection, remain the same. Therefore, by integrating (20) in \( t \), one finds

\[
\chi(z, t) = \chi(z, 0) e^{-\left(\frac{\gamma^2}{2m^*} + E_{m,n} \right) t + i\alpha y} e^{-i\alpha z [1 - \cos(\phi)]} e^{iy \sin(2\omega t)},
\]

where \( \gamma \equiv e^2 F_0^2 / (8 m^* \omega^3) \) (dimensionless) and \( \alpha_0 \equiv e F_0 / (m^* \omega^3) \) is the so-called laser-dressing parameter\(^8\). The term \( 2\gamma \omega = e^2 F_0^2 / (4 m^* \omega^2) \) is the energy blueshift induced by the laser field (i.e. the DFKE blueshift) \(^9\). The factor \( e^{-\left(\frac{\gamma^2}{2m^*} + E_{m,n} \right) t} \) is related to the probability amplitude of a process in which one adds an electron in a state \( |m', n', k_z'\rangle \) at time \( t' \) to the system and then it evolves to a state \( |m, n, k_z\rangle \) at time \( t (t > t') \). This probability is given by

\[
P_{|m', n', k_z'\rangle \rightarrow |m, n, k_z\rangle} = \int \Psi_{m', n', k_z'}^*(\mathbf{r}, t') \Psi_{m, n, k_z}(\mathbf{r}, t) \, d^3r = h_{m, n, k_z}(t', t) \delta_{m', m} \delta_{n', n} \delta_{k_z', k_z},
\]

where

\[
h_{m, n, k_z}(t', t) = e^{-i \left[ E_{m,n} (k_z) + 2\gamma \omega t \right]} e^{i \alpha z K \cos(\phi - \omega t)} e^{iy \sin(2\omega t)}.
\]

Let us now introduce the retarded propagator (or Green’s function) for non-interacting electrons \(^{24}\):

\[
G^+(m', n', k_z'; m, n, k_z; t > t') = \delta_{m', m} \delta_{n', n} \delta_{k_z', k_z} \times G^+_{m,n,k_z}(t > t'),
\]

where \( G^+_{m,n,k_z}(t > t') = -i / \hbar \Theta(t - t') h_{m,n,k_z}(t, t') \). This two-time Green’s function is the solution of

\[
\left[ i\hbar \frac{\partial}{\partial t} - \frac{(p + eA)^2}{2m^*} - E_{m,n} \right] G^+_{m,n,k_z}(t > t') = \delta(t - t')
\]

in \( (m, n, k_z; t) \) space and, in real space,

\[
\left[ i\hbar \frac{\partial}{\partial t} - \frac{(p + eA)^2}{2m^*} - V(r) \right] G^+_{m,n,k_z}(t > t') \Psi(r, 0) = \delta(t - t') \Psi(r, 0),
\]

where \( \Psi( r, 0) = \Psi_{m,n,k_z}( r, 0) \), the parameters \( m, n \) and \( k_z \) being quantum numbers—i.e. \( G^+_{m,n,k_z}(t > t') \) is the actual Green’s function in \( (m, n, k_z; t) \) space\(^9\). The Fourier transform of \( G^+_{m,n,k_z}(t > t') \), which is equivalent to the average over \( t - t' \), is

\[
\Phi^+_{m,n,k_z}(\tilde{r}, t') = \int_{-\infty}^{\infty} e^{i(e+\eta) \tilde{r}} \times G^+_{m,n,k_z}(\tau > 0) \, d\tau,
\]

\(^8\) For a given laser source, whose frequency is \( \nu \) (in THz) and output power is \( P \) (in kW cm\(^{-2}\)), the following practical formulae are useful: \( F_0 \) (in kV cm\(^{-1}\)) \approx 0.87 \sqrt{T} / \sqrt{T} \gamma \) and \( \alpha_0 \) (in effective units) \approx 7.31 \epsilon_{\nu}^{-5/4} \sqrt{T} / \nu^2 \). We adopted here the high-frequency value for the dielectric constant \( \epsilon_{\nu} \approx 10.9 \) (in GaAs).

\(^9\) The reader should note that \( d\Theta(t - t') / dt = \delta(t - t') \).

\(\textit{New Journal of Physics} 13 (2011) 073005 (http://www.njp.org/)\)
where \( \tau \equiv t - t' \). This averaging step is important as we are interested in the steady-state DoS. By converting \( e^{i x \cos y} \) and \( e^{i x \sin y} \) into Bessel functions, one finds

\[
\Theta^+_{m,n,k}(\epsilon, t') = \sum_{j=-\infty}^{+\infty} \mathfrak{g}_j(k_z, t') \frac{1}{\epsilon - \epsilon_{m,n}(k_z) - 2\gamma \hbar \omega - j\hbar \omega + i\eta},
\]

where the infinitesimal \( i\eta \) has been introduced to avoid divergences. The auxiliary function is

\[
\mathfrak{g}_j(k_z, t') \equiv (-1)^j f_j(k_z) \sum_{\ell=-\infty}^{+\infty} i^\ell J_{\ell+\ell}(\alpha_0 k_z) \times e^{i(\epsilon t' - \gamma \sin(2\omega t'))},
\]

where \( f_j(k_z) = \sum_{q=0}^{\infty} J_q(\gamma)/(1 + \delta_{q,0}) \times [J_{2q-j}(\alpha_0 k_z) + (-1)^q J_{2q+j}(\alpha_0 k_z)] \). Now, by averaging over the initial time \( t' \) (over the period \( 2\pi/\omega \) of the radiation field) [11], i.e. by computing \( \omega/(2\pi) \times \int_{-\pi/\omega}^{+\pi/\omega} \Theta^+_{m,n,k}(\epsilon, t') \, dt' \), we found

\[
\overline{\Theta}_{m,n,k}(\epsilon) = \sum_{j=-\infty}^{+\infty} \frac{f_j^2(k_z)}{\epsilon - \epsilon_{m,n}(k_z) - 2\gamma \hbar \omega - j\hbar \omega + i\eta}.
\]

Here, the physical meaning of \( j \) is that \(|j|\) photons are absorbed (emitted) if \( j \) is a strictly positive (negative) integer. When only non-resonant photons take part in the optical process, as we are interested in here, the electron–phonon interactions are elastic and no absorption/emission processes are allowed. This simplifies our calculations since only the value \( j = 0 \) has to be considered, reducing the above summation to

\[
\overline{\Theta}_{m,n,k}(\epsilon) = \frac{f_0^2(k_z)}{\epsilon - \epsilon_{m,n}(k_z) - 2\gamma \hbar \omega + i\eta},
\]

where \( f_0(k_z) = J_0(\gamma) J_0(\alpha_0 k_z) + 2 \sum_{q=1}^{\infty} J_{2q}(\gamma) J_{2q}(\alpha_0 k_z) \).

The laser-driven DoS in each 1D subband as a function of electron energy, i.e. \( D_{m,n}(\epsilon) \), can be determined from the imaginary part of the Fourier transform of this Green’s function through \(-g_s/\pi \times \sum_k \text{Im}[\overline{\Theta}_{m,n,k}(\epsilon)]\). As \( k_z \) is a quasi-continuum wavenumber, the usual conversion of the \( k \)-summation into a ‘volume’ integral in \( k \)-space applies, yielding

\[
\frac{D_{m,n}(\epsilon)}{L} = \frac{g_s}{2\pi} \int \delta \left( \epsilon - \frac{\hbar^2 k_z^2}{2m^*} - E_{m,n} - 2\gamma \hbar \omega \right) \times f_0^2(k_z) \, dk_z.
\]

By taking the mathematical properties of the Dirac-\( \delta \) function into account, we find the following expression for the overall DoS per unit length:

\[
\rho(\epsilon) = \frac{D_0}{L} \sum_{m,n} \Theta \left( \epsilon - E_{m,n} - 2\gamma \hbar \omega \right) f_0^2 \left( \frac{\sqrt{2m^*}}{\hbar} \left( \epsilon - E_{m,n} - 2\gamma \hbar \omega \right) \right).
\]

Note that this DoS depends on both laser field strength and frequency through the parameters \( \gamma \) and \( \alpha_0 \), the latter being present implicitly on the function \( f_0(x) \), as well as on the magnitude of the magnetic field, through the eigenvalues \( E_{m,n} \). Note also that, when we take the limit of low intensities or high frequencies for the laser field, \( \gamma \to 0 \) and then \( f_0(x) \to 1 \) since \( J_q(0) = \delta_{q,0} \), such that our laser-driven DoS recovers the inverse-square-root shape found for 1D systems in the absence of laser fields, as stated in equation (7). On the other hand, for a given (non-null) ILF, when one reduces the magnetic field towards zero the magnetic blueshift tends to zero and the DoS returns to its laser-driven profile given by equation (32) with the energy eigenvalues \( E_{m,n} \) in the absence of magnetic fields given in equation (6).
Figure 1. Dependence of the energy eigenvalues $E_{m,n}$ on the applied magnetic field $B$ for electrons in a cylindrical GaAs QW with radius $a = 100 \, \text{Å}$. The labels $(m, n)$ identify each 1D subband, $m$ being the azimuthal quantum number. The solid lines are for $m = 0$, the dashed lines are for $m = 1$, and the dash-dotted lines are for $m = 2$. Note that all curves with a given value of $m$ have practically the same slope. Note that greater values of $m$ can be associated with greater slopes. Therefore, with the increase of $B$, lower-lying states with $m > 0$ will cross higher-lying states with smaller values of $m$.

3. Results and discussions

The mathematical dependence of the laser-driven DoS on energy established in equation (32) allows us to deduce some general features. Firstly, the coupling of the ILF to the quasi-1DEG results in a uniform blueshift in the DoS profile as a whole with respect to the laser-free DoS, due to the DFKE term $2\gamma \hbar \omega$ in both the denominator and the $\Theta$-function. As a consequence, the singularities in the laser-free DoS (located at the eigenvalues $E_{m,n}$) are also blueshifted by $2\gamma \hbar \omega$. Secondly, as we are multiplying each $D_{m,n}(\varepsilon)$ by $J_2^2(x)$, which is much smaller than 1 for large values of the argument $x$, then the laser-driven DoS is strongly reduced for energies much above each blueshifted singularity. Indeed, the oscillatory nature of the Bessel functions $J_2(x)$ leads to the emergence of oscillations in the DoS with the increase of laser intensity. These features are clear signatures of the DFKE effect for electrons, as already obtained in 2DEGs [12, 15, 25].

Our results for the electron DoS as a function of energy are for a single cylindrical GaAs QW with a radius of $10 \, \text{nm}$. We chose this material because of its great importance in modern device applications, and because its band parameters are well established in the literature [26, 27] and its conduction band (for electrons with small wavenumbers) is almost parabolic, which makes its modeling easy [26]. For GaAs, it is a good approximation to assume $m_{\perp}^{*} \approx m_{||}^{*} = 0.067 \, m_0$.10 The bound-state energy eigenvalues for this QW in the absence of

10 These effective masses are generally quite distinct (as e.g. in silicon-based heterostructures) [26].
Figure 2. DoS per unit of length (in units of $D_0 = g_s\sqrt{2m_\parallel^* / (2\pi\hbar)}$) as a function of energy for electrons in a GaAs cylindrical quantum wire (radius $a = 100$ Å) irradiated by a linearly polarized, THz laser field, in the absence of magnetic fields (i.e. $B = 0$). The dotted line is for $F_0 = 0$ (i.e. the laser-free DoS reference). The other lines are for $F_0 = 2$ (dashed), 5 (dash-dotted) and 10 kV cm$^{-1}$ (solid). Note the reduction of the DoS and the increase of the DFKE (uniform) blueshift with the increase of $F_0$. Note also the appearance of Franz–Keldysh oscillations for energies just above the discontinuities. The labels in the circles identify each 1D subband.

external fields are easily computed by taking into account the zeroes of the Bessel function (see footnote 6), as established in (6). The lowest few eigenvalues are $E_{0,1} = 33.13$ meV (ground state), $E_{1,1} = 84.12$, $E_{2,1} = 151.11$, $E_{0,2} = 174.58$, $E_{3,1} = 233.22$, $E_{1,2} = 281.99$, $E_{4,1} = 329.91$, $E_{2,2} = 405.92$ and $E_{0,3} = 429.05$ meV. These are the initial values of the curves obtained for $E_{m,n}$ as a function of $B$, as seen in figure 1. From equation (7), it is easy to see that these energy levels mark the positions of the divergences in the DoS in the absence of external fields, as shown by the dotted curve in figure 2.

In figure 1, the dependence of the energy eigenvalues $E_{m,n}$ on the magnitude of the applied magnetic field is depicted. For clarity, we omitted the curves for negative values of $m$. The solid lines are for $m = 0$, the dashed lines are for $m = 1$, and the dash-dotted lines are for $m = 2$. Clearly, the energy eigenvalues significantly increase (i.e. are blueshifted) with the increase of magnetic field (see footnote 7). Note that all curves for a given value of $m$ have practically the same slope dependence on $B$, the greater values of $m$ corresponding to greater slopes. Therefore, with the increase of $B$, lower-lying states with large $m$ values will cross higher-lying states with smaller values of $m$. So, the magnetic shift is not uniform, which makes the analysis of the DoS dependence on external fields somewhat more complex than the laser-driven DoS in the absence of magnetic fields, for which the DFKE blueshift is uniform, as shown in one of our recent papers [13].

Figure 2 shows the change of the electron DoS profile in the absence of magnetic fields (i.e. $B = 0$) provoked by linearly polarized, THz ILFs with increasing values of the field strength $F_0$. 

Figure 3. Changes of the laser-driven DoS per unit of length (in units of \( D_0 = g_s \sqrt{2m_v} / (2\pi\hbar) \)) as a function of energy, for increasing values of the applied magnetic field. The confined electrons are irradiated by a 1 THz ILF with a fixed strength of 5 kV cm\(^{-1}\). The dash-dotted curve is for \( B = 0 \) (a reference). The other curves are for \( B = 10 \) T (dashed) and 50 T (solid). For increasing values of \( B \), the energy eigenvalues \( E_{m,n} \) with higher \( m \) values increase more rapidly and then the magnetically induced blueshifts are not the same for distinct states. When we pass from \( B = 0 \) to \( B = 10 \) T, no energy crossover is observed. However, when \( B \) increases from 10 to 50 T the state (2, 1) crosses state (0, 2) and the state (3, 1) crosses state (1, 2), as indicated by the crossing (dashed) arrows. The solid arrows are just for identifying some states.

The dotted line is for \( F_0 = 0 \) (i.e. the laser-free DoS, which is a reference). The other lines are for \( F_0 = 2 \) (dashed), 5 (dash-dotted) and 10 kV cm\(^{-1}\) (solid). The corresponding DFKE blueshifts are 0.7, 4.2 and 16.7 meV, respectively. Note the reduction of the DoS and the increase of the DFKE blueshift with the increase of \( F_0 \). Note also the appearance of Franz–Keldysh oscillations for energies above the singularities. As seen in figure 2, these effects are more pronounced in the high laser intensity regime, in which the DoS changes from the inverse-square-root shape usually presented by quasi-1D electronic systems to a peaked distribution that resembles that exhibited by electrons in quasi-0D systems, such as e.g. semiconductor quantum dots [3]. Further, the THz laser field polarized along the wire axis direction couples strongly to the quasi-1D carriers, changing the \( k_z \) dependence of the wavefunction from the free-particle form (i.e. \( \exp(ik_zz) \)), found in the absence of external fields, to a complex oscillatory form, as described in equations (27)–(30). This drives the electrons out of certain kinetic energy regimes, resulting in a reduction of the DoS. This corresponds to a simple physical picture. The rapid oscillation of the external ac electric field (with which a linear potential \( eF(t)z \) is associated, at a given time \( t \)) pushes and pulls the electrons against the field direction and, when averaged in time, it yields a potential that restricts the (previously free) motion along the wire axis direction. This restriction causes an additional reduction in the dimensionality of the quasi-1D electronic system, which explains the quasi-0D-like aspect of the laser-driven DoS.
Figure 4. As in the previous figure, but for energies from 150 to 450 meV. When $B$ increases from 10 to 50 T the state $(2, 1)$ crosses state $(0, 2)$ and the state $(3, 1)$ crosses state $(1, 2)$, as indicated by the crossing (dashed) arrows. The solid arrows are just for identifying some states. For clarity, the curves for the state $(0, 3)$, which arises at 429.05 meV, were omitted.

The changes in the DoS profile for electrons under the combined action of a THz laser field with a fixed strength ($F_0 = 5 \text{kV cm}^{-1}$) and a uniform magnetic field $B$ are indicated in figure 3. The dash-dotted curve is for $B = 0$ (only a reference). The other curves are for $B = 10$ T (dashed) and 50 T (solid). As already pointed out, for increasing values of $B$, the energy eigenvalues $E_{m,n}$ with higher $m$ values increase more rapidly than those with smaller values of $m$; hence the magnetically induced blueshifts are not the same for distinct $m$-states. When we pass from $B = 0$ to $B = 10$ T, no energy crossover is observed. However, when $B$ increases from 10 T to 50 T some energy crossovers are observed, as indicated in the figure by the crossing (dashed) arrows. The state $(2, 1)$ crosses state $(0, 2)$ at $B = 13.2$ T and the state $(3, 1)$ crosses state $(1, 2)$ at $B = 26.5$ T. The energy crossovers are clearer in figure 4, where we focused our attention in the interval from 150 to 450 meV. Additional energy crossovers are seen in figure 1 for eigenstates with even greater energies.

4. Conclusions

In summary, in this work we developed a systematic investigation of the changes in the electron energy eigenvalues and the steady-state DoS for non-interacting electrons confined in a GaAs cylindrical QW with a nanometric radius and exposed to a non-resonant, linearly polarized, THz laser field and a uniform longitudinal magnetic field. We followed a Green’s function formalism in our calculations, so as to treat the electron–photon interaction exactly. Our results show that the effects of these external fields on the electron-DoS are (i) a non uniform blueshift induced solely by the applied magnetic field, which is greater for states with greater values of $m$ (the azimuthal quantum number) (see footnote 7); (ii) a uniform blueshift of $2\gamma \hbar \omega$ in the DoS profile as a whole (with respect to the laser-free DoS), induced by the laser field (DFKE blueshift);
(iii) a strong reduction for energies much greater than the blueshifted divergences (a consequence of the oscillatory nature of the Bessel functions); and (iv) the appearance of Franz–Keldysh-like oscillations. The latter effects were analyzed here in the light of the DFKE effect, as introduced by Jauho and Johnsen for 3DEGs and strictly 2DEGs (see [11]) and extended to quasi-2DEGs by Xu [12]. As seen in figure 2, these effects are more pronounced in the high laser intensity regime, in which the DoS changes from the usual inverse square-root profile exhibited by quasi-1D electrons to a peaked distribution that resembles that found for quasi-0D electrons in quantum dots [3].

The modifications of the laser-driven DoS studied here are strongly dependent on both the magnetic field and laser intensities, which suggests a simple mechanism for tuning the Fermi level and the density of carriers in each 1D subband, with obvious implications for all relevant physical properties [13]. For a QW with a fixed number of carriers, a detectable increase in the Fermi energy, with the occupancy of higher (previously empty) 1D subbands, is expected as a consequence of the DoS reduction, which certainly will affect both the optical and transport properties of nanostructures, so understanding the effects of non-resonant THz laser fields and magnetic fields on the DoS of quasi-1D electronic systems seems to offer a way of tuning such quantities. This kind of magneto-optical control may be of great interest for those working with electronic devices based on semiconductor nanostructures, a research field that has been stimulated by the evolution of modern crystal growth techniques, as well as the development of new sources of high magnetic fields (above 40 T, as pointed out in [31]) and high-quality, high-power laser radiation (mainly FELs).

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