Meson masses within the model of induced nonlocal quark currents.
Ja.V. Burdanov\textsuperscript{1}, G.V. Efimov\textsuperscript{2}, S.N. Nedelko\textsuperscript{2} and S.A. Solunin\textsuperscript{1}
\textsuperscript{1}Ivanovo State University, Department of Theoretical Physics, 153377 Ivanovo, Russia
\textsuperscript{2}Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia

Abstract

The model of induced quark currents formulated in our recent paper (Phys. Rev. D51, 174) is developed. The model being a kind of nonlocal extension of the bosonization procedure is based on the hypothesis that the QCD vacuum is realized by the (anti-)self-dual homogeneous gluon field. This vacuum field provides the analytical quark confinement. It is shown that a particular form of nonlocality of the quark and gluon propagators determined by the vacuum field, an interaction of quark spin with the vacuum gluon field and a localization of meson field at the center of masses of two quarks can explain the distinctive features of meson spectrum: Regge trajectories of radial and orbital excitations, mass splitting between pseudoscalar and vector mesons, the asymptotic mass formulas in the heavy quark limit: $M_{Q\bar{Q}} \to 2m_Q$ for quarkonia and $M_{Qq} \to m_Q$ for heavy-light mesons. With a minimal set of parameters (quark masses, vacuum field strength and the quark-gluon coupling constant) the model describes to within ten percent inaccuracy the masses and weak decay constants of mesons from all qualitatively different regions of the spectrum. 

PACS number(s): 12.39.-x, 11.10.Lm, 12.38.Aw, 14.40.Gx

1 Introduction

Achievements of the Nambu-Jona-Lasinio (NJL) model in description of meson masses, decay constants and so on are well-known \cite{1, 2, 3}. This success can be explained by the bosonization procedure which makes possible to extract collective modes and dynamical breaking of SU\textsubscript{L}(3)×SU\textsubscript{R}(3) and U\textsubscript{A}(1) symmetries. At the same time, an incorporation of quark confinement into consideration, description of heavy quarkonia and heavy-light mesons, radial and angular excitations of mesons, as well as different form-factors require an essential modification of the NJL model (e.g., see \cite{4}). Another general disadvantage is the nonrenormalizability of the local four-fermion interaction.

In recent paper \cite{5} we have suggested a model that in some sense can be considered as an extension of the standard NJL model. There are two crucial modifications. First, our model is based on the hypothesis that the QCD vacuum to be realized by the (anti-)self-dual homogeneous background gluon field. Second, the effective quark-quark coupling is described by the nonlocal four-quark interaction induced by the one-gluon exchange in presence of the (anti-)self-dual homogeneous gluon vacuum field. This vacuum field
ensures the analytical quark confinement and breaks the chiral symmetry. The model of induced quark currents gives a basis for investigating of all the above-mentioned problems from a general point of view. The main features of the model are as follows [5].

– There is the quark confinement. The quark propagator being an entire analytical function in the complex momentum plane [6] has the standard local ultraviolet behavior in the Euclidean region, and is modified essentially in the physical, i.e., Minkowski region. In contrast to purely chromomagnetic and chromoelectric configurations, the (anti-)self-dual homogeneous field is a stable configuration [7].

– The one-gluon exchange is decomposed into an infinite sum of current-current interaction terms, in which the quark currents are nonlocal, colorless and carry a complete set of quantum numbers including the orbital and radial ones. This effective quark-quark interaction generates a superrenormalizable perturbation expansion.

– The bosonization of the nonlocal four-quark interaction leads to ultraviolet finite effective meson theory. Mesons are treated as extended nonlocal objects.

– The model contains the minimal number of parameters: the quark masses, quark-gluon coupling constant and the tension of the background gluon field.

It was shown also, that the spectrum of the radial and orbital excitations is equidistant for sufficiently large angular momentum \( \ell \) or radial quantum number \( n \). In the heavy quark limit the mass of quarkonium tends to be equal to sum of the masses of constituent quarks.

In paper [3] the main attention was paid to mathematical details of obtaining nonlocal quark currents induced by the one-gluon exchange in presence of the vacuum field and to formulation of the bosonization procedure based on these nonlocal currents. A motivation and connection of the model with QCD has been discussed as far as possible. The present paper is concentrated on further development of the model and on application to systematic calculations of the weak decay constants and masses of mesons from different regions of the spectrum: the light mesons and their excited states, heavy quarkonia and heavy-light mesons.

With the minimal set of parameters (quark masses, vacuum field strength and one quark-gluon coupling constant) the model describes all qualitatively different regions of meson spectrum to within ten percent inaccuracy. The reasons driving this successful description can be easily recovered.

An interaction of quark spin with the vacuum gluon field, contained in the quark propagator, breaks the chiral symmetry and gives rise to the splitting between masses of the pseudoscalar and vector mesons with identical quark structure \((\rho − \pi, K − K^*)\). This spin-field interaction drives also the weak decay of pion and kaon.

Furthermore, the vacuum field produces three rigid asymptotic regimes for the spectrum of collective modes. The spectra of radial and orbital excitations of light mesons are equidistant for \( \ell \gg 1 \) or \( n \gg 1 \), i.e., they have Regge character. This is due to the specific form of nonlocality of the quark and gluon propagators determined by confining properties of the vacuum gluon field. After all, one concludes that the confinement is responsible for
Regge trajectories. Localization of meson field at the center of masses of a quark system
provides other two asymptotic regimes. In the limit of infinitely heavy quark, a mass
of quarkonium tends to be equal to a sum of the masses of constituent quarks, while a
mass of heavy-light meson approaches the mass of a heavy quark:

\[ M_{Q\bar{Q}} \to 2m_Q - \Delta_{Q\bar{Q}}, \]
\[ M_{Q\bar{q}} \to m_Q + \Delta_{Q\bar{q}}. \]

The next-to-leading terms \( \Delta_{Q\bar{Q}} \) and \( \Delta_{Q\bar{q}} \) do not depend on the heavy
quark mass. The same reasons provide the correct asymptotic behavior of the weak decay
constant for the heavy-light pseudoscalar mesons: \( f_P \sim 1/\sqrt{m_Q} \).

One can conclude that the (anti-)self-dual homogeneous background gluon field deter-
mines rather definitely a behavior of the masses and weak decay constants in all different
regions of the meson spectrum, and this behavior is quantitatively consistent with exper-
imental data.

Technically, these results are based on a decomposition of the bilocal colorless quark
currents into a series of nonlocal currents with complete set of quantum numbers: spin,
isospin, radial and orbital numbers. Tensor structure of these nonlocal currents is rep-
resented by the irreducible tensors of the four-dimensional Euclidean rotational group,
while their radial part is determined by a specific form of gluon propagator in the exter-
nal vacuum field. This decomposition provides a new point of view on the renormalization
problem: the Feynman diagrams appearing in each order of perturbation theory are ul-
traviolet finite due to the nonlocality of the meson-quark interaction.

The paper is organized as follows. In Sect. II we review the main points of the model
with some modifications, that relate to choosing the point of localization of meson field and
description of super-fine structure of the spectrum. These modifications reflect possibili-
ties missed in [5]. In Sect. III we consider the masses of light mesons and their excitations,
heavy quarkonia and heavy-light mesons, as well as the weak decay constants. The basic
approximations of the model and problems for further investigations are discussed in the
last section.

2 The model of induced nonlocal quark currents

2.1 Basic assumptions, approximations and notation

The representation of the Euclidean generating functional for QCD, in which the gluon and
ghost fields are integrated out, serves as a starting point for many models of hadronization.
Dyakonov and Petrov obtained this representation for the case of nontrivial vacuum gluon
field [8] (see also [1])

\[ Z = \int d\sigma_{vac} \int DqD\bar{q} \exp \left\{ \int d^4x \sum_{f} \bar{q}_f(x)(i\gamma_\mu \hat{\nabla}_\mu - m_f)q_f(x) + \sum_{n=2}^\infty L_n \right\}, \]

where \( N_F \) is the number of flavors corresponding to the SU\((N_F)\) flavor group and

\[ L_n = \frac{g^n}{n!} \int d^4y_1 \cdots d^4y_n j^{a_1}_{\mu_1}(y_1) \cdots j^{a_n}_{\mu_n}(y_n)G^{a_1 \cdots a_n}_{\mu_1 \cdots \mu_n}(y_1, \ldots, y_n | B), \]
\[ j^a_\mu(y) = \sum_f \bar{q}_f(x) \gamma_\mu t^a q_f(x), \]
\[ \hat{\nabla}_\mu = \partial_\mu - it^a B^a_\mu. \]

The function \( G^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n} \) is the exact (up to the quark loops) \( n \)-point gluon Green function in the external field \( B^a_\mu \). We will investigate the mesonic (\( q\bar{q} \))-collective modes and consider Eq. (1) with the quark-quark interaction truncated up to the term \( L_2 \)

\[ Z = \int d\sigma_{\text{vac}} \int D\bar{q} Dq \exp \left\{ \int d^4x \sum_f \bar{q}_f(x) (i\gamma_\mu \hat{\nabla}_\mu - m_f) q_f(x) \right. \]
\[ + \frac{g^2}{2} \int d^4x d^4y \ j^a_\mu(x) G^{ab}_{\mu\nu}(x,y | B) j^b_\nu(y) \right\}. \tag{2} \]

Representations (1) and (2) imply, that there exists some vacuum (classical) gluon field \( B^a_\mu(x) \), which minimizes the effective action (or effective potential) of the Euclidean QCD. In the general case, the vacuum field depends on a set of parameters \( \{\sigma_{\text{vac}}\} \), and the measure \( d\sigma_{\text{vac}} \) averages all physical amplitudes over a subset of \( \{\sigma_{\text{vac}}\} \), in respect to which the vacuum state is degenerate (for more details see [5] and references therein).

The quark-gluon interaction both in Eqs. (1) and (2) are local, and a decomposition over degrees of \( g^2 \) generates a renormalizable perturbation theory. It means, that an appropriate regularization is implied in Eqs. (1) and (2). This point has to be stressed here, since our final technical aim is a transformation of the interaction term in Eq. (2), which generates completely new superrenormalizable perturbation expansion of the functional integral (4).

Let us identify all ingredients of these general formulas for the particular case of an homogeneous (anti-)self-dual vacuum field

\[ \hat{B}_\mu(x) = \hat{n} B_\mu(x), \quad B_\mu(x) = B_{\mu\nu} x_\nu, \]
\[ \hat{n} = n^a t^a, \quad n^2 = n^a n_a = 1. \]

The constant tensor \( B_{\mu\nu} \) satisfies the conditions:

\[ B_{\mu\nu} = -B_{\nu\mu}, \quad B_{\mu\nu} B_{\rho\sigma} = -B^2 \delta_{\mu\nu}, \]
\[ \hat{B}_{\mu\nu} = \frac{1}{2} \hat{\varepsilon}_{\mu\alpha\beta} B_{\alpha\beta} = \pm B_{\mu\nu}, \]

where \( B \) is a gauge invariant tension of the vacuum field. Since the chromomagnetic \( H \) and chromoelectric \( E \) fields relate to each other like \( H = \pm E \), two spherical angles \( (\varphi, \theta) \) define a direction of the field in the Euclidean space. In the diagonal representation of \( \hat{n} = n^a t^a \), an additional angle \( \xi \) is needed to fix a direction of the field in the color space

\[ \hat{n} = t^3 \cos \xi + t^8 \sin \xi, \quad 0 \leq \xi < 2\pi. \]
The one-loop calculations and some nonperturbative estimations of the effective potential for the homogeneous gluon field argue (do not prove) that the potential could have a minimum at nonzero value of the field tension \( B \neq 0 \) (e.g., see \([4, 7, 10]\)). We will assume, that the field under consideration realizes a nonperturbative QCD vacuum, and study the manifestations of this field in the spectrum of collective modes. Since the effective potential is invariant under the Euclidean rotations, parity and gauge transformations, this vacuum should be degenerate with respect to the directions of the field in the color and Euclidean space and should be the same for anti-self- and self-dual configurations. According to this argumentation, the field \( B^a_\mu \) in (2) corresponds to the tension \( B \) minimizing the effective potential, and the measure \( d\sigma_{vac} \) has the form

\[
\int d\sigma_{vac} = \frac{1}{(4\pi)^2} \int_0^\pi d\theta \sin \theta \int_0^{2\pi} d\varphi \int_0^{2\pi} d\xi \sum_{\pm},
\]

where the sign \( \sum_{\pm} \) denotes averaging over the self- and anti-self-dual configurations. To simplify calculations and to clarify the technical side of bosonization procedure in presence of the background field, we will omit the integral over \( \xi \) in (3) and fix a particular vector \( n^a = \delta^a_8 \). In the fundamental (matrix \( t^8 \)) and adjoint (matrix \( C^8 \)) representations of \( SU_c(3) \) one gets

\[
\hat{n} = t^8 = \text{diag}\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right), \quad \hat{B}_{\mu\nu}\hat{B}_{\rho\sigma} = -(t^8)^2 B^2 \delta_{\mu\nu},
\]

\[
\hat{n} = C^8 = \frac{\sqrt{3}}{2} K, \quad \hat{B}_{\mu\nu}\hat{B}_{\rho\sigma} = -\frac{3}{4} K^2 B^2 \delta_{\mu\nu},
\]

\[
K_{54} = -K_{45} = K_{76} = -K_{67} = i, \quad K^2 = \text{diag}(0, 0, 0, 1, 1, 1, 1, 0).
\]

The rest of elements of the matrix \( K \) are equal to zero. It is convenient to define the mass scale \( \Lambda^2 = \sqrt{3}B \):

\[
\hat{B}_{\mu\nu}\hat{B}_{\rho\sigma} = -\frac{1}{4} K^2 \Lambda^4 \delta_{\mu\nu},
\]

\[
\hat{B}_{\mu\nu}\hat{B}_{\rho\sigma} = -v^2 \Lambda^4 \delta_{\mu\nu}, \quad v = \text{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right).
\]

We would like to stress, that the averaging over directions of the background field in the color space should be incorporated into the formalism, and its role should be analyzed. But first of all, we would like to go as far as possible with the formalism, that is simpler as possible, and test, how the technique proposed in \([3]\) works in the meson phenomenology.

### 2.2 Quark and gluon propagators

The quark propagator \( S_f(x, y \mid B) \) in Eq. (2) satisfies the equation:

\[
(i\gamma_\mu \hat{\nabla}_\mu - m_f) S_f(x, y \mid B) = -\delta(x - y),
\]
and can be written in the form
\[ S_f(x, y \mid B) = e^{\frac{i}{2}(x \cdot B y)} H_f(x - y \mid B) e^{\frac{i}{2}(x \cdot B y)}, \]
\[ H_f(z) = \frac{i \tilde{\nabla}_\mu (z) \gamma_\mu + m_f}{\nabla^2 (z) + m_f^2 + (\sigma B)} \delta (z), \tag{4} \]
\[ \tilde{H}_f(p \mid B) = \frac{1}{2v \Lambda} \int_0^1 dt e^{-\frac{p^2}{2v^2 \Lambda^2} t} \left( \frac{1 - t}{1 + t} \right)^\alpha f \left[ \alpha f + \frac{1}{\Lambda} p_\mu \gamma_\mu + it \frac{1}{\Lambda} (\gamma f p) \right] \cdot \left[ P_\pm + P_\mp \frac{1 + t^2}{1 - t^2} - i \frac{1}{2} (\gamma f \gamma) \frac{t}{1 - t^2} \right], \tag{5} \]
where
\[ P_\pm = \frac{1}{2} (1 \pm \gamma_5), \quad \alpha f = \frac{m_f}{\Lambda}, \quad (xBy) = x_\mu B_{\mu \nu} y_\nu, \]
\[ (pf\gamma) = p_\mu f_{\mu \nu} \gamma_\nu, \quad f_{\mu \nu} = \frac{i^8}{v \Lambda^2} B_{\mu \nu}, \quad f_{\mu \nu} f_{\rho \sigma} = -\delta_{\mu \sigma}. \]

The function $\tilde{H}_f$ is the Fourier transformed $H_f$. The upper (lower) sign in the matrix $P_\pm$ corresponds to the self-dual (anti-self-dual) field.

The term $(\sigma B) = \sigma_{\alpha \beta} \tilde{B}_{\alpha \beta}$ in Eq. (4) (the second line in Eq. (4)) describes an interaction of a quark spin with the background field. One can see, that this spin-field interaction leads to the singularity $1/m_f$ for $m_f \to 0$, which is a manifestation of the zero mode (the lowest Landau level) of the massless Dirac equation in the external (anti-)self-dual homogeneous field. The mathematical point is that the spectrum of the operator $\gamma_\mu \partial_\mu$ is continuous, whereas the spectrum of the operator $\gamma_\mu \tilde{\nabla}_\mu (x)$ is discrete and the lowest eigen number is equal to zero. Simple calculations give for $m_f \to 0$
\[ \tilde{H}_f(p \mid B) = 2 e^{-\frac{p^2}{2v^2 \Lambda^2}} \left[ \frac{1}{m_f} + \frac{1}{m_f^2} (p_\mu \gamma_\mu + i(\gamma f p)) \right] \cdot \left[ P_\pm + \frac{i}{4} (\gamma f \gamma) \right] + O(1), \tag{6} \]
and
\[ \lim_{\varepsilon \to 0} \lim_{m_f \to 0} m_f \langle \tilde{q}_f(x) q_f(x + \varepsilon) \rangle_B = - \lim_{\varepsilon \to 0} \lim_{m_f \to 0} m_f \text{Tr} H_f(\varepsilon \mid B) \]
\[ = - \int \frac{d^4 p}{(2\pi)^4} \lim_{m_f \to 0} m_f \text{Tr} \tilde{H}_f(p \mid B) = - \frac{4}{3\pi^2 \Lambda^4}. \tag{7} \]
Due to the spin-field interaction the quark condensate is nonzero in the limit of vanishing quark mass. This indicates that the chiral symmetry is broken by the vacuum field in the limit $m_f \to 0$ (see also [3]). It will be clear below that just the spin-field interaction gives rise to the splitting between the masses of the pseudoscalar and vector mesons and provides a smallness of pion mass.
In terms of the variable $\zeta = p \gamma_{\mu}$, the propagator $\bar{H}_f(\zeta | B)$ is an entire analytical function in the complex $\zeta$-plane. There are no poles corresponding to the free quarks, which is treated as the confinement of quarks. The following asymptotic behavior takes place:

$$\bar{H}_f(\zeta | B) \rightarrow \begin{cases} \frac{m_f + \zeta}{\zeta^2 - p^2} & \text{if } \zeta \to \pm \infty \ (p^2 \to \infty) \\ O(\exp(\frac{\zeta^2}{2v^2})) = O(\exp(\frac{p^2}{2v^2})) & \text{if } \zeta \to \pm i \infty \ (p^2 \to -\infty). \end{cases}$$  \hspace{1cm} (8)

Equations (8) shows the standard local behavior of the fermion propagator in the Euclidean region ($p^2 \to \infty$), while in the physical region ($p^2 \to -\infty$) we see the exponential increase typical for nonlocal theories (for more details about the general theory of nonlocal interactions of quantized fields see [9, 11]). Below, the absence of the poles and the exponential increase will be referred as the confinement properties of a propagator.

Function $D_{ab \mu \nu}(x, y | B)$ in representation (2) is the exact gluon propagator for the pure gluodynamics in presence of the vacuum field $B^a_{\mu}$. This function is unknown, and some approximation has to be introduced. For instance, the local NJL model corresponds to the choice $D_{ab \mu \nu} = \delta_{ab} \delta_{\mu \nu} \delta(x - y)$. We go beyond this approximation and replace the function $D_{\mu \nu}^{ab}(x, y | B)$ by the confined part

$$D_{\mu \nu}(x, y | B) = \delta_{\mu \nu} K^2 e^{\frac{i}{2} (x \cdot \bar{B} y)} D(x - y | \Lambda^2) e^{\frac{i}{2} (x \cdot \bar{B} y)},$$  \hspace{1cm} (9)

of the gluon propagator

$$G^{ab}_{\mu \nu}(x, y | B) = D^{ab}_{\mu \nu}(x, y | B) + R^{ab}_{\mu \nu}(x, y | B),$$

which is a solution of the equation (for details see [5])

$$(\bar{\nabla}^2 \delta_{\mu \nu} + 4i \bar{B}_{\mu \nu}) G_{\nu \rho}(x, y | B) = -\delta_{\mu \rho} \delta(x - y).$$

The Fourier transform of the function $D(z | \Lambda^2)$ is an entire analytical function in the momentum space. It has the local behavior in the Euclidean region, but increases exponentially in the physical region. This function describes a propagation of the confined modes of the gluon field. Other terms $R^{ab}_{\mu \nu}$, that contain a contribution of the zero modes and an anti-symmetric part, will be omitted.

Thus, the central point of our extension of the NJL-model consists in taking into account the confining influence of the background field both on the quark and gluon propagators.

### 2.3 Color singlet bilocal quark currents

Substituting gluon propagator (10) to the interaction term in representation (2), using the Fierz transformation of the color, flavor and Dirac matrices, and keeping only the scalar
$J^a_S$, pseudoscalar $J^a_P$, vector $J^a_V$ and axial-vector $J^a_A$ colorless currents, we arrive at the expression [5]

\begin{equation}
L_2 = \frac{g^2}{2} \sum_{bJ} C_{ij} \int \int d^4x d^4y J^{bJ}(x,y) D(x-y | \Lambda^2), J^{bJ}(y,x),
\end{equation}

\begin{equation}
J^{bJ}(x,y) = \bar{q}_f(x) M^b_{ff} \Gamma^J e^{i(x\hat{B}y)} q'_{f'}(y),
\end{equation}

\begin{align*}
\Gamma^S = 1, & \quad \Gamma^P = i\gamma_5, & \quad \Gamma^V = \gamma_{\mu}, & \quad \Gamma^A = \gamma_5\gamma_{\mu}, \\
C_S = C_P = \frac{1}{9}, & \quad C_V = C_A = \frac{1}{18}.
\end{align*}

Here $M^b_{ff'}$ are the flavor mixing matrices ($b = 0, \ldots, N_F^2 - 1$) corresponding to the SU($N_F$) flavor group. In the case of SU(2) and SU(3) they are given by the matrices $\tau^b$ and $\lambda^b$ respectively.

Due to the phase factor $\exp\{i(x\hat{B}y)\}$, bilocal quark currents ([14]) are the scalars under the local gauge transformations

\begin{align*}
q(x) & \rightarrow e^{-i\hat{\omega}(x)}q(x), \quad \bar{q}(x) \rightarrow \bar{q}(x)e^{i\hat{\omega}(x)}, \\
\hat{B}_\mu & \rightarrow e^{-i\hat{\omega}(x)}\hat{B}_\mu e^{i\hat{\omega}(x)} + \frac{i}{g}e^{-i\hat{\omega}(x)}\partial_\mu e^{i\hat{\omega}(x)}.
\end{align*}

Let us transform integration variables $x$ and $y$ in Eq. ([14]) to the coordinate system corresponding to the center of masses of quarks $q_f(x)$ and $q_{f'}(y)$

\begin{equation}
x \rightarrow x + \xi_f y, \quad y \rightarrow x - \xi_{f'} y, \quad \xi_f = \frac{m_f}{m_f + m_{f'}}, \quad \xi_{f'} = \frac{m_{f'}}{m_f + m_{f'}}.
\end{equation}

Corresponding transformation of the quark currents looks as

\begin{align*}
J^{bJ}(x,y) & \rightarrow \bar{q}_f(x) M^b_{ff} \Gamma^J e^{i(x\hat{B}y)} q'_{f'}(y) \\
\bar{q}_f(x + \xi_f y) M^b_{ff} \Gamma^J e^{i(x\hat{B}y)} q'_{f'}(x - \xi_{f'} y) = & \quad \bar{q}_f(x) M^b_{ff} \Gamma^J e^{i\hat{\omega}\hat{\nabla}_{f'}(x)} q'_{f'}(x) \overset{\text{def}}{=} J^{bJ}(x,y),
\end{align*}

where $\hat{\nabla}_{f'}$ is a linear combination of the left and right covariant derivatives

\begin{equation}
\hat{\nabla}_{f'}(x) = \xi_f (\hat{\partial} + i\hat{B}(x)) - \xi_{f'} (\hat{\partial} - i\hat{B}(x)).
\end{equation}

These covariant derivatives indicate, that the currents ([14]) are nonlocal and colorless. Interaction term ([15]) takes the form

\begin{equation}
L_2 = \frac{g^2}{8\pi^2} \sum_{bJ} C_{ij} \int \int d^4x d^4y \frac{1}{y^2} \exp \left\{ - \frac{\Lambda^2 y^2}{4} \right\} J^{bJ}(x,y) J^{bJ}(y,x),
\end{equation}

where we have made use of the representation ([3]). The currents are defined by Eq. ([14]). Transformation ([13]) turns out to be crucial for simultaneous description of the light-light, heavy-light and heavy-heavy mesons.
2.4 Decomposition of bilocal currents

An idea of our next step consists in a decomposition of the bilocal currents (14) over some complete set of orthonormalized polynomials in such a way, that the relative coordinate of two quarks \( y \) in Eq. (15) would be integrated out. One can see, that a particular form of this set is determined by the form of the gluon propagator (\( \exp\{-\Lambda^2 y^2/4\} \) in Eq. (15)). The propagator plays the role of a weight function in the orthogonality condition. The physical meaning of the decomposition consists in classifying a relativ motion of two quarks in the bilocal currents over a set of radial \( n \) and angular \( \ell \) quantum numbers. In other words, according to general principles of quantum mechanics the bilocal currents have to be represented as a set of quark currents with definite radial \( n \) and angular \( \ell \) quantum numbers. Thus, we are looking for a decomposition of the form

\[
J_{bJ}(x, y) = \sum_{n\ell} (y^2)^{\ell/2} f_{n\ell}^{bJ}(y) J_{bJn}(x),
\]

(16)

\[
f_{n\ell}^{bJ}(y) = L_{n\ell}(y^2) T_{\mu_1...\mu_\ell}(n_\ell),
\]

\( n_\ell = y/\sqrt{y^2} \).

The angular part of \( f_{n\ell}^{bJ} \) is given by the irreducible tensors of the four-dimensional rotational group \( T_{\mu_1...\mu_\ell}(n_\ell) \), which are orthogonal

\[
\int_{\Omega} \frac{d\omega}{2\pi^2} T_{\mu_1...\mu_\ell}(n_\ell) T_{\nu_1...\nu_\ell}(n_\ell) = \frac{1}{2^\ell(\ell+1)} \delta_{\ell_1\ell_2} \delta_{\mu_1\nu_1}...\delta_{\mu_\ell\nu_\ell},
\]

(17)

and satisfy the conditions:

\[
T_{\mu_1...\mu_\ell}(n_\ell) = T_{\mu_1...\nu_\ell}(n_\ell), \quad T_{\mu_1...\mu_\ell}(n_\ell) = 0,
\]

\[
T_{\mu_1...\mu_\ell}(n_\ell) T_{\mu_1...\mu_\ell}(n_{\ell'}) = \frac{1}{2^\ell} C_{\ell}^{(1)}(n_\ell n_{\ell'}).
\]

(18)

The measure \( d\omega \) in Eq. (17) relates to integration over the angles of unit vector \( n_\ell \), and \( C_{\ell}^{(1)} \) in Eq. (18) are the Gegenbauer’s (ultraspherical) polynomials. The polynomials \( L_{n\ell}(u) \) obey the condition:

\[
\int_0^\infty d\rho_\ell(u) L_{n\ell}(u) L_{n'\ell}(u) = \delta_{nn'},
\]

The weight function \( \rho_\ell(u) \) arising from the exponential term in (13) looks like

\[
\rho_\ell(u) = u^\ell e^{-u},
\]

hence \( L_{n\ell}(u) \) are the generalized Laguerre’s polynomials.
The details of calculation of the currents \( J_{bJℓn}(x) \) in Eq. (16) can be found in paper \([5]\). As a result, the interaction term \( L_2 \) takes the form

\[
L_2 = \frac{1}{2} \sum_{bJℓn} \left( \frac{G_{Jℓn}}{\Lambda} \right)^2 \int d^4x \left[ \mathcal{J}^{bJℓn}(x) \right]^2,
\]

\[
G_{Jℓn}^2 = C_J g^2 \frac{(ℓ + 1)}{2^n n!(ℓ + n)!},
\]

\[
\mathcal{J}^{bJℓn}_{µ_1...µ_ℓ}(x) = \bar{q}(x)V^{bJℓn}_{µ_1...µ_ℓ}(x)q(x),
\]

\[
V^{bJℓn}_{µ_1...µ_ℓ}(x) = V^{bJℓn}_{µ_1...µ_ℓ} \left( \frac{\hat{\nabla}(x)}{\Lambda} \right)
\]

\[
= M^\ell \Gamma^J \left\{ F_n(4s) \right\} \left( \frac{\hat{\nabla}^2(x)}{\Lambda^2} \right) T_\mu(\ell) \left( \frac{1}{i} \hat{\nabla}(x) \right),
\]

\[
F_n(4s) = s^n \int_0^1 dt t^{s+n} e^{st}.
\]

The doubled brackets in Eq. (21) mean that the covariant derivatives commute inside these brackets. Form-factors \( F_n(4s) \) are entire analytical functions in the complex \( s \)-plane, which is a manifestation of the gluon confinement.

The classification of the currents will be complete if we will decompose \( J^{aJℓn}_{α,µ_1...µ_ℓ} \) with \( J = V, A \) and \( ℓ > 0 \) into a sum of orthogonal currents \( J^{bJℓn}_{α,µ_1...µ_ℓ} \) with the different total angular momentum \( j = ℓ - 1, ℓ, ℓ + 1 \). Index \( α \) relates to the matrices \( \Gamma^V_α = γ_α \) and \( \Gamma^A_α = γ_5 γ_α \) in Eq. (21). This decomposition can be arranged by the following division

\[
\mathcal{J}^{bJℓn}_{α,µ_1...µ_ℓ} = \sum_{j=ℓ-1}^{ℓ+1} J^{bJℓn}_{α,µ_1...µ_ℓ, j},
\]

where

\[
J^{bJℓn}_{α,µ_1...µ_ℓ, j} = \begin{dcases}
\frac{1}{(ℓ+1)!} \mathcal{P}_{α,µ_1...µ_ℓ} \left[ \delta_{αµ_1} \mathcal{J}^{bJℓn}_{µ_1...µ_ℓ} \right], & j = ℓ - 1, \\
\frac{1}{ℓ+1} \sum_{i=1}^{ℓ} \left[ J^{bJℓn}_{α,µ_1...µ_1+1...µ_ℓ} - J^{bJℓn}_{µ_1,α...µ_1+1...µ_1+1...µ_ℓ} \right], & j = ℓ, \\
\frac{1}{ℓ+1} \mathcal{P}_{α,µ_1...µ_ℓ} \left[ J^{bJℓn}_{α,µ_1...µ_1+1...µ_ℓ} - ℓ+1 \delta_{αµ_1} J^{bJℓn}_{µ_1,µ_2...µ_ℓ} \right], & j = ℓ + 1.
\end{dcases}
\]

Symbol \( \mathcal{P}_{α,µ_1...µ_ℓ} \) in (24) denotes a cyclic permutation of the indices \( (αµ_1...µ_ℓ) \). Let \( s_J \) be defined as

\[
s_P = s_S = 0, \quad s_V = s_A = 1,
\]

then, using the orthogonality of the currents with different \( j \), we can rewrite interaction term \( L_2 \) as

\[
L_2 = \sum_{aJℓn} \sum_{j=[ℓ-s_J]}^{ℓ+s_J} \frac{1}{2Λ^2} G_{Jℓn}^2 \int d^4x \left[ J^{aJℓn}_j(x) \right]^2,
\]
where we have introduced the notation

\[ I_0^{bj0n} = J_0^{bj0n} \quad \text{for} \quad J = V, A, \quad \ell = 0, \]

\[ I_{\mu_1...\mu_\ell}^{bj\ell n} = J_{\mu_1...\mu_\ell}^{bj\ell n} \quad \text{for} \quad J = S, P, \quad \ell \geq 0. \]

This form is equivalent to Eq. (10), but now the interaction between quarks is expressed in terms of the nonlocal quark currents, that are elementary currents of the system in the sense of the classification over quantum numbers.

For large Euclidean momentum the vertices \( \tilde{V}_a^{Jln} \) behave as \( 1/(p^2)^{1+\ell/2} \). Therefore, only the ”bubble” diagrams, shown in Fig. 1, are divergent. These divergencies can be removed by the counter-terms of the form \(-2\tilde{I}(x)\text{TrVS}\).

To avoid an unnecessary complication of notation, it is convenient to introduce condensed index \( \mathcal{N} \) enumerating the currents with all different combinations of the quantum numbers \( a, J, \ell, n \) and \( j \). The renormalized vacuum amplitude \( Z \) takes the form

\[
Z = \int d\sigma_{\text{vac}} \int Dq \bar{D}q \exp \left\{ \int \int d^4x d^4y \bar{q}(x) S^{-1}(x, y|B) q(y) \right. \\
+ \sum_{\mathcal{N}} \frac{1}{2\Lambda^2} \tilde{G}_{\mathcal{N}} \int d^4x \left[ I_{\mathcal{N}}(x) - \text{TrV}_{\mathcal{N}} S \right]^2 \right\}. \tag{25}
\]

2.5 Bosonization

By means of the standard bosonization procedure \([1, 2]\) applied to Eq. (25) the amplitude \( Z \) can be represented in terms of the composite meson fields \( \Phi_{\mathcal{N}} \): \(2\)

\[
Z = N \int \prod_{\mathcal{N}} D\Phi_{\mathcal{N}} \exp \left\{ \frac{1}{2} \int \int d^4x d^4y \Phi_{\mathcal{N}}(x) \left[ \left( \Box - M_{\mathcal{N}}^2 \right) \delta(x - y) \right. \\
- h_{\mathcal{N}}^2 \Pi_{\mathcal{N}}^R(x - y) \right] \Phi_{\mathcal{N}}(y) + I_{\text{int}}[\Phi] \right\}. \tag{26}
\]

\[
I_{\text{int}} = -\frac{1}{2} \int d^4x_1 \int d^4x_2 h_{\mathcal{N}} \Phi_{\mathcal{N}}(x_1) \left[ \Gamma_{\mathcal{N}\mathcal{N}'}(x_1, x_2) - \delta_{\mathcal{N}\mathcal{N}'} \Pi_{\mathcal{N}}(x_1 - x_2) \right] \Phi_{\mathcal{N}'}(x_2) \\
- \sum_{m=3}^{\infty} \frac{1}{m} \int d^4x_1... d^4x_m \prod_{k=1}^m h_{\mathcal{N}_k} \Phi_{\mathcal{N}_k}(x_k) \Gamma_{\mathcal{N}_1...\mathcal{N}_m}(x_1,...,x_m), \\
\Gamma_{\mathcal{N}_1...\mathcal{N}_m} = \int d\sigma_{\text{vac}} \text{Tr} \left\{ V_{\mathcal{N}_1}(x_1) S(x_1, x_2 | B)...V_{\mathcal{N}_m}(x_m) S(x_m, x_1 | B) \right\}. 
\]

Meson masses \( M_{\mathcal{N}} \) are defined by the equations

\[
1 + \left( \frac{G_{\mathcal{N}}}{\Lambda} \right)^2 \tilde{\Pi}_{\mathcal{N}}(\mathcal{N}) = 0, \tag{27}
\]

11
where $\tilde{\Pi}(N^2)$ is the diagonal part of the two-point function $\tilde{\Gamma}_{NN'}$, which corresponds to the diagram shown in Fig. 2.a. The fields $\Phi_N$ ($N = \{a, J, \ell, n, j\}$) with $j > 0$ satisfy the on-shell condition
\[ p_{\mu} \Phi_{N}^{\mu} (p) = 0, \text{ if } p^2 = -M^2_N, \]
which excludes all extra degrees of freedom of the field, so that the numbers $\ell$ and $j$ can be treated as the O(3) orbital momentum and total momentum, respectively.

The total momentum $j$ plays the role of an observable spin of the state with a given $N = \{b, J, \ell, n, j\}$.

The constants
\[ h_N = 1/\sqrt{\tilde{\Pi}(N^2)} \] 
play the role of the effective coupling constants of the meson-quark interaction.

The quark masses $m_f$, the scale $\Lambda$ (strength of the background field) and the quark-gluon coupling constant $g$ are the free parameters of the effective meson theory (26)-(28).

In the one-loop approximation, the interactions between mesons with given quantum numbers $N = \{b, J, \ell, n, j\}$ are described by the quark loops like the diagram in Fig. 2.b. The structure of diagrams in Fig. 2 is the same as in the standard NJL-model, but in our case the quarks propagate in the vacuum gluon field, and the meson-quark vertices are nonlocal. Due to this nonlocality the quark loops are ultraviolet finite. The whole diagram is averaged by integration over the measure $d\sigma_{\text{vac}}$.

Figure 3 illustrates the central idea of the method of induced nonlocal currents, that has been realized in this section. An effective four-quark interaction is represented as an infinite series of interactions between the nonlocal quark currents characterized by the complete set of quantum numbers $\{b, J, \ell, n, j\}$. The form of the currents is induced by a particular form of gluon propagator. This new representation of the four-quark interaction generates an expansion of any amplitude into the series of the partial amplitudes with a particular value of the quantum numbers. Each partial amplitude is ultraviolet finite at any order of expansion over degrees of the coupling constant $g$. The composite meson fields in Eq. (29) are nothing else but the "elementary" collective excitations, that are classified according to the complete set of quantum numbers of the relativistic two-quark system.

It should be stressed, that the model (29) satisfies all demands of the general theory of nonlocal interactions of quantum fields [14], which means that Eq. (29) defines a nonlocal, relativistic, unitary and ultraviolet finite quark model of meson-meson interactions.

Now we would like to test, how this formalism works in the meson phenomenology.

### 3 Meson spectrum and weak decay constants

Let us rewrite Eq. (27) in more detailed form
\[ \Lambda^2 + G^2_{Jen} \tilde{\Pi}(bJt;n; m_f, m_{f'}; \Lambda) = 0. \] (29)
The function $\tilde{\Pi}_{b\ell n_1 j}$ in Eq. (29) is given by the diagonal part in the momentum representation of the tensor

$$\Pi_{b\ell n_1 j}(x - y; m_f, m_{f'}; \Lambda) =$$

$$\int d\sigma_{\text{vac}} \text{Tr} \left[ V_{b\ell n_1 j}(x) S(x, y \mid B) V_{b\ell n_1 j}(y) S(y, x \mid B) \right].$$

(30)

Relation (28) is the master equation for meson masses. The function $\tilde{\Pi}$ can be calculated using the representations (4) and (5) for the quark propagator and (21) for the vertices. The only point, that requires a comment, is an averaging over the space directions of the vacuum field. Actually we have to average the tensors $f_{\mu\nu}$, $f_{\mu\nu}f_{\alpha\beta}$ and so on. The generating formula looks as

$$\langle \exp (i f_{\mu\nu} J_{\mu\nu}) \rangle = \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin \theta \exp (i f_{\mu\nu} J_{\mu\nu}) = \frac{\sin \sqrt{2} (J_{\mu\nu} J_{\mu\nu} \pm \tilde{J}_{\mu\nu} J_{\mu\nu})}{\sqrt{2} (J_{\mu\nu} J_{\mu\nu} \pm \tilde{J}_{\mu\nu} J_{\mu\nu})},$$

where $\varphi$, $\theta$ are the spherical angles, $J_{\mu\nu}$ is an anti-symmetric tensor, $\tilde{J}_{\mu\nu}$ is a dual tensor and $\pm$ corresponds to the self- and anti-self-dual vacuum field. In particular this general representation gives:

$$\langle f_{\mu\nu} \rangle = 0,$$

$$\langle f_{\mu\nu} f_{\alpha\beta} \rangle = \frac{1}{3} (\delta_{\alpha\mu} \delta_{\beta\nu} - \delta_{\alpha\nu} \delta_{\beta\mu} \pm \epsilon_{\alpha\beta\mu\nu}).$$

(31)

3.1 Light pseudoscalar and vector mesons

First of all, let us fit the free parameters of the model, taking the masses of $\pi$, $\rho$, $K$ and $K^*$ mesons as the basic quantities. Below, we will sometimes use a symbol of a given meson instead of the corresponding set of quantum numbers (for example, $\pi$ instead of $(3, P, 0, 0, 0)$).

Four equations for the masses of the basic mesons can be written in the form

$$2\tilde{\Pi}_\pi(-M_\pi^2; m_u, m_u; \Lambda) = \tilde{\Pi}_\rho(-M_\rho^2; m_u, m_u; \Lambda),$$

(32)

$$2\tilde{\Pi}_K(-M_K^2; m_s, m_u; \Lambda) = \tilde{\Pi}_{K^*}(-M_{K^*}^2; m_s, m_u; \Lambda),$$

(33)

$$2\tilde{\Pi}_\pi(-M_\pi^2; m_u, m_u; \Lambda) = \tilde{\Pi}_K(-M_K^2; m_s, m_u; \Lambda),$$

(34)

$$g^2 = -9\Lambda^2/\tilde{\Pi}_\pi(-M_\pi^2; m_u, m_u; \Lambda).$$

(35)

If $M_\pi$, $M_\rho$, $M_K$ and $M_{K^*}$ are taken to be equal to the experimental values, then the masses $m_u$ and $m_s$ of the $u$ and $s$ quarks as the functions of $\Lambda$ are defined by Eqs. (32), (33). Using $m_u(\Lambda)$ and $m_s(\Lambda)$ in (34), we find the value of $\Lambda$, that provides a simultaneous description
of the strange and nonstrange mesons. An optimal value of the coupling constant \( g \) is calculated by means of Eq. (35). By this way we arrive at the values:

\[
m_u = 198.28 \text{ MeV}, \quad m_s = 412.96 \text{ MeV}, \quad \Lambda = 319.46 \text{ MeV}, \quad g = 9.96.
\]  

Solution (36) is unique.

It is well-known, that there should be a special reason, which provides a small pion mass and splits the masses of pseudoscalar and vector mesons. Breaking of chiral symmetry due to the four-quark interaction and two independent coupling constants for pseudoscalar and vector mesons \( (g_p \neq g_v \text{ instead of our parameter } g) \) play the role of such reason in the local NJL-model. As has already been pointed out, an interaction of quark spin with the vacuum field leads to the singular behavior of the quark propagator in the massless limit and generates a non-zero quark condensate, which indicates breaking of the chiral symmetry by the vacuum gluon field. Now let us illustrate, that in our case the same spin-field interaction is responsible for small pion mass and for the mass-splitting between P- and V-mesons.

Polarization function \( \Pi_J (\ell = 0, n = 0, J = P, V) \) can be represented in the form

\[
\Pi_J(-M^2; m_f, m_{f'}; \Lambda) = \frac{\Lambda^2}{4\pi^2} \text{Tr}_v \int_0^1 dt_1 \int_0^1 dt_2 \int_0^1 ds_1 \int_0^1 ds_2 \left\{ \frac{m_f^2}{4\Lambda^2} \left( \frac{1 - s_1}{1 + s_1} \right) \frac{m_{f'}^2}{4\Lambda^2} \left( \frac{1 - s_2}{1 + s_2} \right) \right\} + \frac{M^2 F_1^{(j)}(t_1, t_2, s_1, s_2)}{\Lambda^2 \Phi_1(t_1, t_2, s_1, s_2)} + \frac{m_f m_{f'} F_2^{(j)}(s_1, s_2)}{\Lambda^2 (1 - s_1^2)(1 - s_2^2)\Phi_2(t_1, t_2, s_1, s_2)} + \frac{2v(1 - 4v^2 t_1 t_2) F_3^{(j)}(s_1, s_2)}{\Phi_3(t_1, t_2, s_1, s_2)} \exp \left\{ \frac{M^2}{2v\Lambda^2} \Phi(t_1, t_2, s_1, s_2) \right\},
\]

where

\[
\Phi = \frac{\Phi_1(t_1, t_2, s_1, s_2)}{\Phi_2(t_1, t_2, s_1, s_2)}, \quad \Phi_1 = 2v(t_1 + t_2)[s_1 \xi_1^2 + s_2 \xi_2^2] + s_1 s_2[1 + 4v^2 t_1 t_2(\xi_1 - \xi_2)^2], \quad \Phi_2 = 2v(t_1 + t_2)(1 + s_1 s_2) + (1 + 4v^2 t_1 t_2)(s_1 + s_2), \\
F_1^{(P)} = (1 + s_1 s_2)[A_1 A_2 + 4v^2(t_1 - t_2)^2 \xi_1 \xi_2 s_1 s_2], \quad F_1^{(V)} = \frac{1}{3}[(3 - s_1 s_2)A_1 A_2 + 4v^2(t_1 - t_2)^2 \xi_1 \xi_2 s_1 s_2(1 - 3s_1 s_2)], \\
A_1 = [1 - 4v^2 t_1 t_2(\xi_1 - \xi_2)]s_1 + 2v(t_1 + t_2)\xi_2, \quad A_2 = [1 + 4v^2 t_1 t_2(\xi_1 - \xi_2)]s_2 + 2v(t_1 + t_2)\xi_1, \\
F_2^{(P)} = (1 + s_1 s_2)^2, \quad F_2^{(V)} = 1 - s_1^2 s_2^2, \quad F_3^{(P)} = 2(1 + s_1 s_2), \quad F_3^{(V)} = 1 - s_1 s_2.
\]
Equations (37)-(39) shows, that singularity \((1 - s_1)^{-1}(1 - s_2)^{-1}\), arising from the spin-field interaction (see the second line of Eq. (38)), is accumulated in the term of Eq. (37) proportional to the quark masses. Other terms are free from this singularity, although the spin-field interaction contribute to them either. This is due to the structure of the trace of the Dirac matrices.

Let us compare a behavior of pion and \(\rho\)-meson polarization functions in the limit

\[
m_f = m_{f'} = m_u \ll \Lambda.
\]

Using the singularity of the integrand in Eq. (37) at \(s_1 \to 1\) and \(s_2 \to 1\), one can check, that the pion polarization function is singular in this limit and behaves as \(1/m_u^2\):

\[
\tilde{\Pi}_\pi(-M^2; m_u, m_u; \Lambda) = - \frac{4v^2\Lambda^4}{\pi^2 m_u^2} \int_0^1 \int_0^1 \frac{dt_1 dt_2}{\Phi_2^2(t_1, t_2, 1, 1)} \exp \left\{ \frac{M^2}{2v\Lambda^2} \Phi(t_1, t_2, 1, 1) \right\} + O(1). \tag{40}
\]

On the contrary, the \(\rho\)-meson polarization is regular at \(m_u = 0\) and looks like

\[
\tilde{\Pi}_\rho(-M^2; m_u, m_u; \Lambda) = - \frac{\Lambda^2}{4\pi^2} \text{Tr} V \int_0^1 dt_1 \int_0^1 dt_2 \int_0^1 ds_1 \int_0^1 ds_2 \left\{ \exp \left\{ \frac{M^2}{2v\Lambda^2} \Phi(t_1, t_2, s_1, s_2) \right\} \right\} \cdot \left[ \frac{M^2 F_2^{(V)}(t_1, t_2, s_1, s_2)}{\Lambda^2 \Phi_2^2(t_1, t_2, s_1, s_2)} + \frac{2v(1 - 4v^2 t_1 t_2) F_3^{(V)}(s_1, s_2)}{\Phi_2^2(t_1, t_2, s_1, s_2)} \right] + \exp \left\{ \frac{M^2}{2v\Lambda^2} \Phi(t_1, t_2, s_1, 1) \right\} \frac{2v}{\Phi_2^2(t_1, t_2, s_1, 1)} + O(m_u). \tag{41}
\]

This difference appears owing to the form of factors \(F_2^{(P)}\) and \(F_2^{(V)}\) in Eq. (39) and leads to the inequality

\[
|\tilde{\Pi}_\pi(-M^2; m_u, m_u; \Lambda)| \gg |\tilde{\Pi}_\rho(-M^2; m_u, m_u; \Lambda)|.
\]

This relation shows that the masses of pion and \(\rho\)-meson, satisfying Eq. (29), are strongly splitted and \(M_\rho^2 \gg M_\pi^2\) when the quark mass goes to zero. A similar picture takes place for \(K\) and \(K^*\) mesons, but since the strange quark mass is not so small, the effect is more smooth.

Above consideration is illustrated in Table 3. The pion mass is much larger, and difference in the masses of pseudoscalar and vector mesons is smaller, if the spin-field interaction in the quark propagator is eliminated (compare the first and the last lines in the table).

Thus, we can conclude, that the large splitting between the masses of P- and V-mesons is explained in our case by the spin-field interaction. This splitting is the reason, why Eqs. (34) have appropriate solution (36).
It should be noted, that the scalar polarization function $\tilde{\Pi}_S$ differs from the pseudoscalar $\tilde{\Pi}_P$ only by the sign before $m_f m_{f'}$ in Eq. (39). Due to the above-mentioned singularity, the term proportional to $m_f m_{f'}$ is leading, and $\tilde{\Pi}_S$ is positive for a wide range of parameter values. As a result, Eq. (29) has no real solutions for the case of scalar mesons. It looks interesting, that the scalar $(q\bar{q})$ bound states do not appear due to the same spin-field interaction, that diminishes the pion mass and provides nonzero quark condensate.

Consideration of the $SU_F(3)$ singlet and the eighth octet states shows an ideal mixing both for vector and pseudoscalar mesons. The masses of $\omega$ and $\phi$, calculated with the parameter values (36), are

$M_\omega = M_\rho = 770$ MeV, \hspace{1cm} $M_\phi = 1034$ MeV,

which is in a good agreement with the experimental values. An ideal mixing of the pseudoscalar states is not the case, that can provide an appropriate description of $\eta$ and $\eta'$ mesons. It is well known, that the problem of $\eta$ and $\eta'$ masses can be solved by taking into account another Euclidean gluon configuration - the instanton vacuum field [12]. The instantons can be incorporated into our formalism without any principal problems, and we hope to realize this idea in forthcoming publications.

Now let us consider the weak decays of $\pi$ and $K$ mesons. In the lowest approximation, an amplitude of the decay $P \rightarrow l\bar{\nu}$ is given by the diagram in Fig 2.c. The weak decay constant $f_P$ is defined by the standard formulas

$$f_P = h_P F(-M^2),$$

where the meson-quark coupling constant $h_P$ is calculated via Eq. (28), and $K$ is the KKM matrix element corresponding to a given meson. For arbitrary pseudoscalar meson the diagram in Fig. 2.c gives the following expression for $f_P$

$$f_P = h_P \frac{1}{4\pi^2} \text{Tr}_v \int_0^1 \int_0^1 \frac{dtds_1ds_2(1 + s_1 s_2)}{[2vt(1 + s_1 s_2) + s_1 + s_2]^3 \left(1 - s_1 \right) \left(1 + s_1 \right) \frac{m_f^2}{4v \Lambda^2} \left(1 - s_2 \right) \frac{m_{f'}^2}{4v \Lambda^2}} \left[ m_f \frac{s_1 + 2vt[1 - \xi_1(1 + s_2^2)]}{1 - s_1^2} + m_{f'} \frac{s_2 + 2vt[1 - \xi_2(1 + s_1^2)]}{1 - s_2^2} \right] \exp \left\{ \frac{M^2}{2v \Lambda^2} \Psi(t, s_1, s_2) \right\},$$

$$\Psi = \frac{s_1 s_2 + 2vt(s_1 \xi_1^2 + s_2 \xi_2^2)}{2vt(1 + s_1 s_2) + s_1 + s_2}.$$

The singularity of the integrand of Eq. (43) at $s_1 \rightarrow 1$ and $s_2 \rightarrow 1$ appears from above-mentioned spin-field interaction in the quark propagator and plays the main role in regulating the value of $f_P$ for the light mesons. Calculation of pion and kaon decay constants by means of Eq. (43) with the values of parameters (36) gives

$$f_\pi = 126 \text{ MeV}, \hspace{1cm} f_K = 145 \text{ MeV}.$$
Note, that the coupling constants $h_\pi$ and $h_K$ depend on the meson mass, quark masses and parameter $\Lambda$ (see Eq. (28) and Table 3).

One could get a definite impression, that simultaneous description of the masses of $\pi$, $K$, $\rho$, $K^*$, $\omega$ and $\phi$ mesons, and quite accurate values of $f_\pi$ and $f_K$ are obtained mostly due to the breakdown of chiral symmetry by the spin-field interaction (see also Eq. (7)).

In order to clarify a status of this impression, one needs to investigate the chiral limit of the model. Whether or not, and in what form the Goldberger-Treiman and Oakes-Reiner-Treiman relations are fulfilled in this limit? Naively, it seems that the chiral limit corresponds to the case $m_u \ll \Lambda$. However, the situation is much more complicated.

Just for illustration of this statement, consider the pion mass in the limit $m_u \ll \Lambda$. Integrals in Eq. (40) can be evaluated, and we get the following asymptotic form of equation (29) for pion mass

$$1 - g^2 \frac{16 \Lambda^6}{9\pi^2 m_u^2 M^4_\pi} \text{Tr}_v v^2 \left[ \exp \left\{ \frac{M^2_\pi}{8v \Lambda^2} \right\} - \exp \left\{ \frac{M^2_\pi}{8v(1+2v)\Lambda^2} \right\} \right]^2 = 0. \quad (44)$$

For any values of $g$ there is a real positive or negative solution $M^2_\pi$ to Eq. (44). The pion mass is equal to zero, if the values of $m_u$, $g$ and $\Lambda$ satisfy the relation

$$\frac{m^2_u}{\Lambda^2} = \frac{g^2}{9\pi^2} \text{Tr}_v \frac{v^2}{(1+2v)^2} \approx \left( \frac{2g}{15\pi} \right)^2.$$ 

For a fixed value of $g$ and $m_u/\Lambda \to 0$, the solution $M_\pi$ to Eq. (44) is purely imaginary and behaves as

$$\frac{M_\pi}{\Lambda} = i \frac{4}{3} \sqrt{7 \ln \frac{\Lambda}{m_u}} + O \left[ \frac{\ln\ln(\Lambda/m_u)}{\sqrt{\ln(\Lambda/m_u)}} \right],$$

which has nothing physically reasonable interpretation, but just indicates, that the limit $g = \text{const}$, and $m_u/\Lambda \to 0$ is ill-defined.

From our point of view, a correct transition to the chiral limit has to be based on a simultaneous changing of $m_u$, $g$ and $\Lambda$ as functions of an actual physical parameter like the temperature or particle density. The quark masses, coupling constant and the vacuum field strength as the functions of this parameter have to be extracted from consideration of QCD dynamics at nonzero temperature and density. Unfortunately, this is a quite complicate problem, and we leave it for further investigations.

The main result of this subsection is very simple: we have demonstrated by the explicit model calculations, that the spin-field interaction contained in the quark propagator (in presence of the homogeneous (anti-)self-dual vacuum gluon field) can be responsible for observable masses of the light pseudoscalar and vector mesons (by exception of $\eta$, $\eta'$), and for the values of the weak decay constants of pion and kaon. All numerical results are given in Tables 1 and 2.
3.2 Regge trajectories

It has been shown in our previous paper \[5\], that the spectrum of radial and orbital excitations of the light mesons is asymptotically equidistant:

\[
M_{aJ\ell n}^2 = \frac{8}{3} \ln \left( \frac{5}{2} \right) \cdot \Lambda^2 \cdot n + O(\ln n), \text{ for } n \gg \ell, \tag{45}
\]

\[
M_{aJ\ell n}^2 = \frac{4}{3} \ln 5 \cdot \Lambda^2 \cdot \ell + O(\ln \ell), \text{ for } \ell \gg n. \tag{46}
\]

Technically this result is based on the exponential behavior of the quark propagator (8) and vertex function \( F_{n\ell} \) (22) in the Minkowski region \( (p^2/\Lambda^2 \to -\infty) \) like

\[
\tilde{H}_f(p \mid B) \to O\left(\exp\left\{ \frac{|p^2|}{2v\Lambda^2} \right\} \right), \quad F_{n\ell}(p^2) \to O\left(\exp\left\{ \frac{|p^2|}{4\Lambda^2} \right\} \right), \tag{47}
\]

and on the specific dependence of the coupling constant \( G_{J\ell n} \) (see Eq. (19)) on the orbital and radial quantum numbers \( \ell, n \), arising from the decomposition of the bilocal quark currents over the generalized Laguerre polynomials, which is determined in its order by the form of gluon propagator (9) (for details see sect. 2.4). In general words, the Regge character of the spectrum is determined in our model by the confining properties of the vacuum field.

Numerical calculation of masses of the first orbital excitations of \( \pi, K, \rho \) and \( K^* \) mesons by means of Eq. (29) with the parameters (36) gives the masses shown in Table 3. The super-fine structure of the excited states of \( \rho \) and \( K^* \) mesons coming from classification of currents over total momentum (Eq. (23)) is qualitatively correct. Super-fine splitting of the levels with \( \ell = 1 \) is not very large.

3.3 Heavy quarkonia

Exponential behavior of the quark propagator and vertices (47) is responsible for the following relation between the masses of heavy quarkonia \( M_{Q\bar{Q}} \) and heavy quark \( m_Q \) in the leading approximation \[5\]:

\[
M_{Q\bar{Q}} = 2m_Q, \quad \text{for } m_Q \gg \Lambda.
\]

Now let us calculate the next-to-leading term in the mass formula. In other words, we have to solve Eq. (29) with the polarization function \( \tilde{\Pi}_f \) defined by Eq. (37) with

\[
m_f = m_f' = m_Q \gg \Lambda, \quad M_{Q\bar{Q}} = 2m_Q - \Delta_{Q\bar{Q}} \tag{48}
\]

in the next-to-leading approximation over \( 1/m_Q \). Since the masses of quarks are equal to each other, we have (see Eq. (13))

\[
\xi_1 = \xi_2 = 1/2.
\]
which means that the composite quarkonium field $\Phi_{QQ}(x)$ is localized at the center of masses of two heavy quarks (in the Euclidean four-dimensional space). It is convenient to transform variables $s_1$ and $s_2$ in Eq. (37):

$$r_1 = (s_1 + s_2)/\sqrt{2}, \quad r_2 = (s_1 - s_2)/\sqrt{2}. \quad (49)$$

The term with $F_3^{(J)}$ in Eq. (37) does not contribute to the leading and next-to-leading behavior of the integral and can be omitted. After the transformation we arrive at the expression:

$$\tilde{\Pi}_f(-M^2; m_Q, m_Q; \Lambda) = -\frac{m_Q^2}{4\pi^2} \text{Tr}_v \int_0^1 dt_1 dt_2 \left( \int_0^{1/\sqrt{2}} dr_1 \int_{-r_1}^{r_1} dr_2 + \int_{1/\sqrt{2}}^{\sqrt{2}} dr_1 \int_{-\sqrt{2} - r_1}^{\sqrt{2} - r_1} dr_2 \right) \cdot \left[ \frac{R_1^{(J)}(t_1, t_2, r_1, r_2)}{\varphi_2(t_1, t_2, r_1, r_2)} + \frac{R_2^{(J)}(r_1, r_2)}{(2 - (r_1 - r_2)^2)(1 - (r_1 + r_2)^2)\varphi_2(t_1, t_2, r_1, r_2)} \right] \exp \left\{ \frac{M^2}{4v\Lambda^2} \varphi(t_1, t_2, r_1, r_2) \right\} + O \left( \frac{\Lambda}{m_Q} \right), \quad (50)$$

where

$$\varphi = \varphi_1(t_1, t_2, r_1, r_2) - \frac{m_Q^2}{M^2} \ln \frac{(\sqrt{2} + r_1)^2 - r_2^2}{(\sqrt{2} - r_1)^2 - r_2^2}, \quad (51)$$

$$\varphi_1 = \sqrt{2}v(t_1 + t_2)r_1 + r_1^2 - r_2^2$$

$$\varphi_2 = v(t_1 + t_2)(2 + r_1^2 - r_2^2) + \sqrt{2}(1 + 4v^2t_1t_2)r_1,$$

$$R_1^{(P)} = \frac{1}{4}(2 + r_1^2 - r_2^2)[A_1A_2 + v^2(t_1 - t_2)^2(r_1^2 - r_2^2)],$$

$$R_1^{(V)} = \frac{1}{12}[(6 - r_1^2 + r_2^2)A_1A_2 + v^2(t_1 - t_2)^2(r_1^2 - r_2^2)(1 - 3r_1^2 + 3r_2^2)],$$

$$A_1 = r_1 - r_2 + \sqrt{2}v(t_1 + t_2), \quad A_2 = r_1 + r_2 + \sqrt{2}v(t_1 + t_2),$$

$$R_2^{(P)} = (2 + r_1^2 - r_2^2)^2, \quad R_2^{(V)} = 4 - (r_1^2 + r_2^2)^2.$$

An asymptotic value of the integral over $r_2$ in the limit $M \gg \Lambda$ can be evaluated by the Laplace method. One can check, that the function $\varphi$ has a maximum at the point $r_2 = 0$ for any values of $r_1, t_1, t_2$:

$$\frac{\partial}{\partial r_2} \varphi(t_1, t_2, r_1, r_2)|_{r_2=0} = 0,$$

$$\frac{\partial^2}{\partial r_2^2} \varphi(t_1, t_2, r_1, r_2)|_{r_2=0} = -\frac{m_Q^2}{M^2} \frac{8\sqrt{2}r_1}{(2 - r_1^2)^2} - \frac{4v(t_1 + t_2) + \sqrt{2}r_1(1 - v^2(t_1 - t_2)^2)}{\varphi_2(t_1, t_2, r_1, 0)}, \quad (52)$$

$$\varphi(t_1, t_2, r_1, 0) = \frac{r_1[r_1 + 2\sqrt{2}v(t_1 + t_2)]}{\varphi_2(t_1, t_2, r_1, 0)} - \frac{2m_Q^2}{M^2} \ln \frac{\sqrt{2} + r_1}{\sqrt{2} - r_1}.$$
which means that the leading terms can be obtained by evaluating the Gaussian integral over \( r_2 \). Furthermore, one can see, that the largest value of the function \( \varphi(t_1, t_2, r_1, 0) \) in the interval \( r_1 \in [0, \sqrt{2}] \) corresponds to \( r_1 = 0 \) for any \( t_1, t_2 \), moreover:

\[
\frac{\partial}{\partial r_1} \varphi(t_1, t_2, r_1, 0) \bigg|_{r_1=0} = -\frac{1}{\sqrt{2}} \left( \frac{4m_Q^2}{M^2} - 1 \right) < 0,
\]  

(53)

and, in the leading approximation, the integrand is reduced to the exponential function in \( r_1 \). Using Eqs. (52) and (53), and taking into account Eq. (48), one can integrate over \( r_2 \) and \( r_1 \) with the result:

\[
\tilde{\Pi}_J(-M^2; m_Q, m_Q; \Lambda) = \frac{3\Lambda^3}{4\pi \sqrt{\pi} \Delta_{QQ}} \int_0^1 \frac{dt_1 dt_2}{\sqrt{t_1 + t_2}} + O \left( \frac{\Lambda}{m_Q} \right). 
\]  

(54)

Integrating over \( t_1 \) and \( t_2 \) in Eq. (54) and substituting the result to Eq. (29), one can find

\[
\Delta_{QQ}^{(J)} = \frac{2(\sqrt{2} - 1)}{\pi \sqrt{\pi}} C_J g^2 + O \left( \frac{\Lambda}{m_Q} \right),
\]  

(55)

where \( C_P = 1/9, C_V = 1/18 \) (see Eq. (13)). It should be stressed, that the difference in the constants

\[
\Delta_{QQ}^{(P)} = 2\Delta_{QQ}^{(V)}
\]  

(56)

originates from the Fierz transformation of the Dirac matrices in the interaction term \( L_2 \) in representation (2). Relation (56) means that the vector quarkonium state is always heavier than the pseudoscalar one.

The results of numerical calculation of the masses of different heavy quarkonia states are summarized in Tables 4 and 5. The parameters \( \Lambda \) and \( g \) are equal to the values (34) fitting the light meson masses, and \( m_c = 1650 \) MeV, \( m_b = 4840 \) MeV. An agreement with the experimental values is rather satisfactory. The super-fine splitting (\( \chi_{c0}, \chi_{c1}, \chi_{c2} \) and so on) is very small, since it is regulated by the terms \( O(1/m_Q) \) in Eq. (29). Its description is qualitatively correct. The splitting is generated in our model by dividing the quark currents with \( \gamma_\mu \) and \( \ell > 0 \) into the antisymmetric, symmetric traceless and diagonal parts (see Eq. (23)), which extracts the states with different total angular momentum, mixed in the currents \( \bar{q} M \gamma_\alpha T^{(f)}_{\mu_1...\mu_f} F_{\alpha f} q \).

We conclude that the correct description of the heavy quarkonia in our model is provided by the specific form of nonlocality of the quark and gluon propagators induced by the vacuum field, localization of meson field at the center of masses of constituent quarks and by a separation of the nonlocal currents with different total momentum. In general, the spectrum is driven by the rigid asymptotic formulas (48) and (52).
3.4 Heavy-light mesons

Another interesting sector of meson spectrum is the heavy-light mesons, characterized by a rich physics [13, 14]. In this subsection we will consider the masses and weak decay constants of heavy-light mesons. First of all, let us get the asymptotic formulas in the limit of infinitely heavy quark. Namely, we have to investigate the behavior of the polarization function $\tilde{\Pi}_J(-M; m_Q, m_q; \Lambda)$ (Eq. (37)) and the weak decay constant $f_P$ (Eq. (43)) in the case

$$m_f = m_Q \gg \Lambda, \quad m_f' = m_q \sim O(\Lambda),$$

$$\xi_f = \frac{m_Q}{m_Q + m_q} = 1 + O(m_q/m_Q), \quad \xi_f' = \frac{m_q}{m_Q + m_q} = O(m_q/m_Q). \quad (57)$$

Equations (57) indicate, that in the heavy quark limit the composite meson field $\Phi_{Q\bar{q}}(x)$ is localized at the point, in which the heavy quark $Q$ is situated.

Let us show, that in the limit (57) the leading and next-to-leading terms of the solution to Eq. (29) read

$$M_{Q\bar{q}} = m_Q + \Delta^{(J)}_{Q\bar{q}} + O(1/m_Q), \quad (58)$$

where the next-to-leading term $\Delta^{(J)}_{Q\bar{q}}$ does not depend on the heavy quark mass $m_Q$. This term is a function of a light quark mass $m_q$ and coupling constant $G_{J00}$ (see Eq. (19)).

Omitting the term with $F_3^{(J)}$, which does not contribute to the leading and next-to-leading behavior of the integral and taking into account conditions (57), one can rewrite Eq. (37) in the form

$$\tilde{\Pi}_J(-M^2; m_Q, m_q; \Lambda) = -\frac{1}{4\pi^2} \text{Tr}_v \int_0^1 dt_1 dt_2 ds_1 ds_2 \left( \frac{1 - s_2}{1 + s_2} \right)^{m_Q^2/m_Q^2} \left\{ \left( 1 - 4v^2 t_1 t_2 \right) Y(t_1, t_2, s_2) T^{(J)}_1(s_1, s_2) s_1 M^2 \right\}$$

$$+ \left[ s_1 X(t_1, t_2, s_2) + Y(t_1, t_2, s_2) \right]^4$$

$$\frac{T^{(J)}_2(s_1, s_2) m_Q m_q}{(1 - s_1^2)(1 - s_2^2) [s_1 X(t_1, t_2, s_2) + Y(t_1, t_2, s_2)]^2} \exp \left\{ \frac{M^2}{2v \Lambda^2} \phi(t_1, t_2, s_1, s_2) \right\} + O\left( \frac{\Lambda}{m_Q} \right), \quad (59)$$

where

$$\phi = \frac{s_1 Y(t_1, t_2, s_2)}{s_1 X(t_1, t_2, s_2) + Y(t_1, t_2, s_2)} - \frac{m_Q^2}{2M^2} \ln \frac{1 + s_1}{1 - s_1},$$

$$X = 1 + 4v^2 t_1 t_2 + 2v(t_1 + t_2)s_2$$

$$Y = 2v(t_1 + t_2) + (1 + 4v^2 t_1 t_2)s_2$$

21
\[ T_1^{(P)} = 1 + s_1 s_2, \quad T_1^{(V)} = \frac{1}{3} (3 - s_1 s_2), \]
\[ T_2^{(P)} = (1 + s_1 s_2)^2, \quad T_2^{(V)} = 1 - s_1^2 s_2. \]

One can check, that for any \( t_1, t_2 \) and \( s_2 \) the function \( \phi(t_1, t_2, s_1, s_2) \) has a maximum at \( s_1 = s_1^{\text{max}} \):
\[
\begin{align*}
s_1^{\text{max}} &= \frac{Y(t_1, t_2, s_2)}{2X(t_1, t_2, s_2)} \left( 1 - \frac{m_Q^2}{M^2} \right) + O \left( \frac{\Lambda^2}{m_Q^2} \right), \\
\phi(t_1, t_2, s_1^{\text{max}}, s_2) &= \frac{Y(t_1, t_2, s_2)}{2X(t_1, t_2, s_2)} \left( 1 - \frac{m_Q^2}{M^2} \right)^2 + O \left( \frac{\Lambda^2}{m_Q^2} \right), \\
\frac{\partial^2}{\partial s_1^2} \phi(t_1, t_2, s_1, s_2) &= -\frac{2X(t_1, t_2, s_2)}{Y(t_1, t_2, s_2)} \left( 1 - \frac{m_Q^2}{M^2} \right)^2 + O \left( \frac{\Lambda^2}{m_Q^2} \right). \tag{60}
\end{align*}
\]

Therefore, we can write
\[
\Pi_J(-M^2; m_Q, m_q; \Lambda) = \quad -\frac{1}{4\pi^2} \operatorname{Tr}_v \iint_0^1 dt_1 dt_2 ds_2 \left( 1 - \frac{s_2}{1 + s_2} \right) \frac{m_q^2}{4v\Lambda^2} \exp \left\{ \frac{M^2}{4v\Lambda^2} \left( 1 - \frac{m_Q^2}{M^2} \right)^2 \frac{Y(t_1 t_2, s_2)}{X(t_1, t_2, s_2)} \right\} \frac{M^2}{2v^2 Y(t_1, t_2, s_2)} + O \left( \frac{\Lambda}{m_Q} \right). \tag{61}
\]

Integrating out the variable \( s_1 \) in Eq. (61), substituting \( M = m_Q + \Delta_Q^{(j)} \) to the resulting expression and using Eq. (62), we arrive at the equation
\[
1 = g^2 C_J \frac{1}{(2\pi)^3} \operatorname{Tr}_v \sqrt{0} \iint_0^1 dt_1 dt_2 ds_2 \left[ (1 - s_2)/(1 + s_2) \right]^{m_q^2/4v\Lambda^2} \frac{X(t_1, t_2, s_2) m_q}{1 - s_2^2} \frac{\Delta_Q^{(j)}}{\Lambda} \exp \left\{ \frac{\Delta_Q^{(j)}}{v\Lambda^2} \frac{Y(t_1 t_2, s_2)}{X(t_1, t_2, s_2)} \right\} + O \left( \frac{\Lambda}{m_Q} \right). \tag{62}
\]

Equation (62) describes dependence of \( \Delta_Q^{(j)} \) in mass formula (68) on the coupling constant \( g \), the light quark mass \( m_q \) and vacuum field strength \( B (\Lambda) \). There is a single real solution to Eq. (62) for any positive \( g \), \( m_q \) and \( \Lambda \). In particular, for the values (68) we get
\[
\begin{align*}
\Delta_Q^{(P)} &= 20 \text{ MeV}, \quad \Delta_Q^{(V)} = 155 \text{ MeV}, \\
\Delta_Q^{(P)} &= 63 \text{ MeV}, \quad \Delta_Q^{(V)} = 191 \text{ MeV}.
\end{align*}
\]
As is seen from Eq. (62), the difference between pseudoscalar $\Delta^{(P)}_{Qq}$ and vector $\Delta^{(V)}_{Qq}$ is due to the constant $C_J$, that appears from the Fierz transformation of the Dirac matrices. This is the same situation as in the case of heavy quarkonia (see Eq. (53)).

Table 3 demonstrates reasonably good agreement between the experimental data and the masses of the heavy-light mesons calculated by means of Eq. (29) with the parameters (36). The masses of heavy quarks are the same as in the description of the heavy quarkonia (see Table 1).

Now let us turn to the calculation of the weak decay constant for the pseudoscalar heavy-light mesons. Under conditions (57), the integral over $s_1$ in Eq. (43) can be evaluated by the Laplace method. The result is

$$f_P = h_P \frac{\Lambda^2}{m_Q} A_f \left( \frac{\Delta^{(P)}_{Qq}}{\Lambda}, \frac{m_q}{\Lambda} \right),$$  

(63)

where

$$A_f = \text{Tr}_v \sqrt{v} \frac{1}{(2\pi)^{3/2}} \int_0^1 dt d s_2 \left[ \frac{(1-s_2)/(1+s_2)}{[s_2 + 2vt]^{3/2}} \left[ 1 + 2vts_2 \right]^{3/2} \right] \times$$

$$\left[ \frac{\Delta^{(P)}_{Qq}}{\Lambda} + \frac{m_q}{\Lambda} \frac{1 + 2vts_2}{1 - s_2^2} \right] \exp \left\{ \frac{[\Delta^{(P)}_{Qq}]^2}{v^2} \frac{s_2 + 2vt}{1 + 2vts_2} \right\} + O \left( \frac{\Lambda}{m_Q} \right),$$  

(64)

and the difference $\Delta^{(P)}_{Qq}$ between the masses of heavy quark and heavy-light meson is given by Eq. (52). The procedure for obtaining Eqs. (63) and (64) is very similar to the calculations providing Eq. (52). To get the final formula for $f_P$, the asymptotic form of meson-quark coupling constant $h_P$ has to be defined in the case (57). Performing calculations analogous to that ones, which lead to Eq. (62), we arrive at

$$h_P = \sqrt{\frac{m_Q}{\Lambda}} A_h^{-1} \left( \frac{\Delta^{(P)}_{Qq}}{\Lambda}, \frac{m_q}{\Lambda} \right),$$

$$A_h^2 = \frac{\Delta^{(P)}_{Qq}}{2(2\pi)^{3/2}} \text{Tr}_v \frac{1}{\sqrt{v}} \int_0^1 dt \int d s_2 \left[ \frac{[(1-s_2)/(1+s_2)]}{[1 + 2vts_2]^{3/2}} \right] \times$$

$$\left[ \frac{\Delta^{(P)}_{Qq}}{\Lambda} + \frac{X(t_1,t_2,s_2)}{1 - s_2^2} \frac{m_q}{\Lambda} \right] \exp \left\{ \frac{[\Delta^{(P)}_{Qq}]^2}{v^2} \frac{Y(t_1,t_2,s_2)}{X(t_1,t_2,s_2)} \right\} + O \left( \frac{\Lambda}{m_Q} \right).$$  

(65)

One can see, that Eqs. (63)-(65) give the following asymptotic relation in the heavy quark limit (57)

$$f_P = \frac{\Lambda^{3/2}}{\sqrt{m_Q}} A_f,$$

(66)
where $A_f$ and $A_h$ do not depend on the heavy quark mass in the leading approximation over $\Lambda/m_Q$, as is indicated in (64),(65). Relation (66) agrees with the accepted notion about behavior of the weak decay constants of the heavy-light mesons [14]. Results of numerical calculation of the weak decay constants for different pseudoscalar mesons are given in Table 6.

4 Discussion

For conclusion, we would like to point out several problems, that require more profound studying.

We have assumed from the very beginning, that the nonperturbative QCD vacuum is characterized by a nonzero background (anti-)self-dual homogeneous field. In other words, a minimum of the QCD effective potential (the free energy density) for this gluon configuration is assumed to be at nonzero field strength. Different estimations of the effective potential indicates, that this situation can be realized [10]. Although these estimations cannot be used as a basis for more or less rigorous proof, they underline the key role of the asymptotic freedom in forming the effective potential for an homogeneous gluon field. Just the asymptotic freedom is the distinctive feature of nonabelian gauge theories like QCD. Gluon self-interaction is manifested also in the nontrivial form of gluon propagator (9). Although the background field is quasi-abelian, the nonabelian nature of the gluon field plays the crucial role for above considered model.

In order to clarify the basic assumption of this paper, one needs to get a reliable nonperturbative estimation of the free energy density or effective potential of QCD for the background field under consideration. Lattice calculations seem to be the most promising approach to this problem.

In this paper, we have demonstrated how the singularity $1/m_f$ of the quark propagator affects the masses and weak decay constants of light mesons. However, more detailed consideration of the chiral symmetry breaking by the background field is needed. This can be achieved by investigating the Dirac equation in presence of the homogeneous (anti-)self-dual field. Another source of violation of the chiral symmetry is the effective four-quark interaction. The divergent diagram in Fig. 1 should play the key role in studying of this mechanism of the symmetry breaking. One can expect that additional breakdown of chiral symmetry by the four-quark interaction could diminish the quark masses. They come into our formalism as the current masses, but their fitted values (see Table 1) are close to the constituent quark masses rather than to the current ones.

One could see, that the coupling constant $g$ in Table 1 is rather large. A possible origin of this unpleasant feature could be covered in elimination of some terms of the gluon propagator (9) (for more details see [1]). In other words, some truncations in the gluon propagator was compensated by the rising of the coupling constant. This point also
has to be investigated carefully.

Acknowledgments

We would like to thank J. Hüfner, F. Lenz and F. Schöberl for interesting discussions. We are also grateful to our colleagues A. Dorokhov, A. Efremov, S. Gerasimov, M. Ivanov, N. Kochelev, V. Liubovitsky, S. Mikhailov and O. Teriaev for many stimulating questions they asked at the seminars in the Bogoliubov Laboratory of Theoretical Physics. One of the authors (S.N.N.) would like to thank H. Leutwyler for discussion and valuable comments.

This work was supported by the Russian Foundation for Basic Research under grant No. 94-02-03463-a and by the Russian University Foundation under grant No. 94-6.7-2042.

References

[1] S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).
[2] T. Hatsuda and T. Kunihiro, Phys. Rep. 247, 221 (1994).
[3] T. Eguchi, Phys. Rev. D14, 2755 (1976); T. Goldman and R. W. Haymaker, Phys. Rev. D24, 724 (1981).
[4] C. D. Roberts and R. T. Cahill, Aust. J. Phys. 40, 499 (1987); J. Praschifka, C. D. Roberts, and R. T. Cahill, Phys.Rev.D36, 209 (1987); 41 627 (1990); Ann. Phys. (N.Y.) 188, 20 (1987).
[5] G.V. Efimov and S.N. Nedelko , Phys. Rev. D51, 176 (1995).
[6] H. Leutwyler, Phys. Lett. 96B, 154 (1980).
[7] H. Leutwyler, Nucl. Phys. B179, 129 (1981).
[8] D. I. Dyakonov and V. Yu. Petrov, Nucl. Phys. B245, 259 (1984).
[9] G. V. Efimov and M. A. Ivanov, The Quark Confinement Model of Hadrons (IOP Publishing, Bristol and Philadelphia, 1993).
[10] S. G. Matinyan and G. K. Savvidy, Nucl. Phys. B134, 539 (1978); J. Finjord, Nucl. Phys. B194, 77 (1982); E. Elizalde, Nucl. Phys. B243, 398 (1984); E. Elizalde and J. Soto, ibid B260, 136 (1985).
[11] G. V. Efimov, Nonlocal interactions of quantized fields (Nauka, Moscow, 1977); G.V. Efimov and V.A. Alebastrov, Comm. Math. Phys. 31, 1 (1973); G.V. Efimov and O.A. Mogilevsky, Nucl.Phys. B44, 541 (1972).
[12] G. ’t Hooft, Phys. Rep. 142, 357 (1986).

[13] N. Isgur and M. Wise, Phys. Lett. B232, 113 (1989); ibid 237, 527 (1990).

[14] M. Nuebert, Phys. Rep. 245, 259 (1994).
Table 1: Parameters of the model.

| Parameter | Value  |
|-----------|--------|
| $m_u$ (MeV) | 198.3 |
| $m_d$ (MeV) | 198.3 |
| $m_s$ (MeV) | 413 |
| $m_c$ (MeV) | 1650 |
| $m_b$ (MeV) | 4840 |
| $\Lambda$ (MeV) | 319.5 |
| $g$ | 9.96 |

Table 2: The masses(MeV), weak decay constants (MeV) and meson-quark coupling constants $h$ of the light mesons. $M^*$ - calculation without taking into account the spin-field interaction.

| Meson | $\pi$ | $\rho$ | $K$ | $K^*$ | $\omega$ | $\phi$ |
|-------|-------|--------|-----|-------|---------|--------|
| $M$   | 140   | 770    | 496 | 890   | 770     | 1034   |
| $M^{exp}$ | 140  | 770    | 496 | 890   | 786     | 1020   |
| $f_P$  | -     | 145    | -   | -     | -       | -      |
| $f_P^{exp}$ | 132 | -      | 157 | -     | -       | -      |
| $h$    | 6.51  | 4.16   | 7.25| 4.48  | 4.16    | 4.94   |
| $M^*$  | 630   | 864    | 743 | 970   | 864     | 1087   |
Table 3: The masses (MeV) of orbital excitations of $\pi$, $K$, $\rho$ and $K^*$ mesons. Super-fine structure of the $\ell = 1$ excitation of $\rho$ and $K^*$ is shown ($\ell$ is the orbital momentum and $j$ is the total momentum (an observable spin) of a state).

| Meson | $\ell$ | $j$ | $M$  | $M^{\text{exp}}$ |
|-------|--------|-----|------|-----------------|
| $\pi$ | 0      | 0   | 140  | 140             |
| $b_1$ | 1      | 1   | 1252 | 1235            |
| $K$   | 0      | 0   | 496  | 496             |
| $K_1(1270)$ | 1 | 1  | 1263 | 1270         |
| $\rho$ | 0      | 1   | 770  | 770             |
|        | 1      | 0   | 1238 |                 |
| $a_1$ | 1      | 1   | 1311 | 1260            |
| $a_2$ | 1      | 2   | 1364 | 1320            |
| $K^*$ | 0      | 1   | 890  | 890             |
|        | 1      | 0   | 1274 |                 |
| $K_1(1400)$ | 1 | 1  | 1342 | 1400         |
| $K_2^*$ | 1     | 2   | 1388 | 1430            |

Table 4: The spectrum of charmonium.

| Meson | $\eta_c$ | $J/\psi$ | $\chi_{c0}$ | $\chi_{c1}$ | $\chi_{c2}$ | $\psi'$ | $\psi''$ |
|-------|-----------|----------|--------------|--------------|--------------|---------|---------|
| $n$   | 0         | 0        | 0            | 0            | 0            | 1       | 2       |
| $\ell$ | 0        | 0        | 1            | 1            | 1            | 0       | 0       |
| $j$   | 0         | 1        | 0            | 1            | 2            | 1       | 1       |
| $M$ (MeV) | 3000 3161 3452 3529 3531 3817 4120   |
| $M^{\text{exp}}$ (MeV) | 2980 3096 3415 3510 3556 3770 4040   |

Table 5: The spectrum of bottomonium.

| Meson | $\Upsilon$ | $\chi_{b0}$ | $\chi_{b1}$ | $\chi_{b2}$ | $\Upsilon'$ | $\chi'_{b0}$ | $\chi'_{b1}$ | $\chi'_{b2}$ | $\Upsilon''$ |
|-------|-------------|--------------|--------------|--------------|-------------|--------------|--------------|--------------|--------------|
| $n$   | 0           | 0            | 0            | 0            | 1           | 1            | 1            | 1            | 2            |
| $\ell$ | 0         | 1            | 1            | 1            | 0           | 1            | 1            | 1            | 0            |
| $j$   | 1           | 0            | 1            | 2            | 1           | 0            | 1            | 2            | 1            |
| $M$ (MeV) | 9490 9767 9780 9780 10052 10212 10215 10215 10292   |
| $M^{\text{exp}}$ (MeV) | 9460 9860 9892 9913 10230 10235 10255 10269 10355   |
Table 6: The masses and weak decay constants (MeV) of heavy-light mesons.

| Meson | $D$  | $D^*$ | $D_s$ | $D_s^*$ | $B$  | $B^*$ | $B_s$ | $B_s^*$ |
|-------|------|-------|-------|---------|------|-------|-------|---------|
| $M$   | 1766 | 1991  | 1910  | 2142    | 4965 | 5143  | 5092  | 5292    |
| $M_{\text{exp}}$ | 1869 | 2010  | 1969  | 2110    | 5278 | 5324  | 5375  | 5422    |
| $f_P$ | 149  | -     | 177   | -       | 123  | -     | 150   | -       |

FIG.1. Divergent bubble diagram.
FIG. 2. Diagrams describing different processes in effective nonlocal meson theory in the lowest (one-loop) order.
\[ \sum_{aJlnj} \]  

- - - - Gluon propagator in the vacuum field

\[ \sum_{aJlnj} \]

\[ \]  

Local quark current  

Nonlocal quark currents

FIG. 3. The decomposition of the four-quark interaction.