Dynamics of anti-vibration mount with quasi-zero stiffness

V N Sorokin¹, B A Kalashnikov¹ and I Y Efimov²
¹Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia
²Federal state unitary enterprise "FNPC "Progress", Omsk, Russia

Abstract. The paper considers the dynamics of the anti-vibration mount of a new design with the quasi-zero stiffness effect, in which the mounting element is an elastic rubber-cord casing of type I-09, and a load adding element (corrector) is a toroid-shape rubber-cord shell. A mathematical model is developed and motion graphs of vibro-isolated object in the range of 1-10 Hz are built.

Keywords: air spring, rubber-cord shell, load-adding element, object vibration isolation, systems with quasi-zero stiffness.

1. Introduction

Vibration isolation of process equipment that uses vibration and shock processes, as well as moving the unbalanced elements is an actual problem.

For vibration isolation of vibro-active appliances and equipment both active and passive systems are used. For protection against harmful vibrations it is expedient to apply passive systems as more simple and cost-effective. One of the main characteristics of the vibration isolator is a frequency of its own oscillations. The lower this frequency, the wider the range of frequencies of disturbing forces, in which the vibration isolator is effective, is. To construct vibration isolation systems with a low natural frequency it is possible to use systems with quasi-zero stiffness [1, 2]. These systems are characterized by the fact that in the operating range they have a gently sloping area of the power characteristics and thus have a low rigidity, while maintaining a high load-bearing capacity in equilibrium, as well as the possibility to reconfigure when changing the mount static loads without shift of the working point from the middle of this area. This allows us to consider them as a means of vibration isolation of objects of large mass and low frequencies.

Practice has proved the efficiency of vibration protection systems with quasi-zero stiffness.

2. Statement of the problem

For vibration isolation of various vibro-active equipment, a mount with the effect of quasi-zero stiffness is offered (Figures 1 and 2). Vibro-active mass 1 is mounted on the bearing air spring 2, for example, on the basis of rubber-cord shell I-09, which bears the static load of the mount. In addition, the mount has a stiffness corrector, which includes, under pressure, toroidal rubber shell 3, which is based on four identical mount segments 4. Each segment represents a quarter of the annular pipe, cut in two parts by the vertical cylindrical plane. The outer parts of each segment are connected by means of hinges 5 to the legs 6 mounted on the mount base 7. The inner parts of each segment are also connected with mass 1 by means of hinges.
Under fluctuations of vibro-active mass 1, the deformation of the air springs 2 occurs and the stiffness mount corrector turns on. Joint hinge of segments 4 allows, under the oscillations of the mass, to deviate from horizontal the resultant of the elastic force generated by a toroid-shape shell 4. In this case, the vertical component of the force generated by a toroid-shape shell, is always directed opposite to the force generated by the carrier air spring 2, that is, compensates for it, creating the effect of quasi-zero stiffness within a certain range of the mass shift.

For creating a stiffness corrector, the application of a toroidal shell under pressure, in comparison with horizontally mounted and hinged air springs on the basis of the cylinder with the piston [2], allows to reduce the dimensions of mount and eliminate the sliding friction force of the piston in the cylinder, which impairs the damping properties of the mount.
The design of the mount allows changing it, within certain limits, when changing static load without a shift of the working point from the middle of the quasi-zero stiffness area. To do this, it is necessary to change the pressure in the carrier air spring and in a toroid-shape shell.

3. Theory

To assess the dynamics and efficiency of the proposed vibration isolation mount, let us firstly define the elastic force of the carrier air springs I-09 and a vertical component of the elastic force of the corrector with the assumption that the mount performs translational motion along the vertical axis. The elastic force of the carrier air springs is

\[ P_{\text{spr}}(x) = (p_{\text{overpr}} + p_A) \left( \frac{V_0}{V_0 - F_p(x) \cdot x} \right)^n \left( F_p(x) - mg - p_A F_p(x) \right), \]  

(1)

where \( m \) is the mass of one mount;
\( p_{\text{overpr}} \), \( p_A \) is the overpressure in the air spring and atmospheric pressure; 
\( V_0 \) is the volume of the air spring in the equilibrium position of the mount; 
\( n \) is the polytropic exponent (for low frequencies of 2 – 10 Hz \( n = 1.3 – 1.4 \) can be taken) [8, 9].

\( F_{sp}(x) \) is the effective area of the air springs, which generally is a function of the shift of its top head \( x \) with a stationary lower bottom (for rubber-cord shell of a diaphragm type it is constant).

For the considered air springs on the basis of rubber-cord shell I-09 on the according to experimental data it is possible to take \( V_0 = 0.00073 \ m^3 \),

\[ F_p(x) = F_{p0} \left( 1 + \frac{k_1}{F_{p0}} x + \frac{k_2}{F_{p0}} x^2 \right), \]  

(2)

where \( F_{p0} = 0.0064 \ m^2 \) is the effective area at the position of equilibrium;
\( k_1 = 0.035 \ m, \ k_2 = 0.75, \) are the ratios of the effective area change of the air spring travel.

Design scheme of the mount with a corrector is shown in Figure 3. Determine the expression for the vertical component of the elastic force generated by the four segments of the corrector and applied to vibro-isolation object.

\[ P_{\text{cor}}(x) = \left[ (p_0 + p_A) \left( \frac{V_{\text{cor}0}}{V_{\text{cor}0} + F \cdot \Delta l} \right)^n - p_A \right] F \cdot Sin \alpha = \]

\[ = \frac{F}{\sqrt{l^2 + x^2}} \left[ (p_0 + p_A) \left( \frac{V_{\text{cor}0}}{V_{\text{cor}0} + F \cdot \Delta l} \right)^n - p_A \right] x, \]  

(3)

where \( p_0 \) is the pressure in the toroidal shell of the corrector;
\( V_{\text{cor}0} \) is the static volume of the toroid-shape shell of the corrector;
\( l \) is the distance between hinges pins of the corrector segments in the equilibrium position;
\( \Delta l = \sqrt{l^2 + x^2} - l \) is the increase in this distance by moving the object from its equilibrium position vertically by \( x \);
\( \alpha \) is the angle of the line connecting hinges pins of the corrector segments with the horizontal in its current position;
\( F \) is the effective area of the corrector toroid-shape shell:
\[ F = \pi D_{ave} d_{inn}, \]

where \( D_{ave} \) is the average diameter of the toroidal shell of the corrector; \( d_{inn} \) is the inner diameter of the circular cross-section of the corrector toroid-shape shell;

\[ \text{Figure 3. The design of the mount with quasi-zero stiffness effect} \]

The equation of mass motion \( m \) in this case can be written as:

\[ m\ddot{x} + b\dot{x} + P_{spr}(x) - P_{cor}(x) = P_0 \sin \omega t, \]

where \( b \) is the damping coefficient in the mount; \( P_0, \omega \) is the amplitude and frequency of the vibration exciting force.

In order to obtain the load characteristics of the mount a zero stiffness in the equilibrium position it is necessary to equate the static stiffness of the carrier air spring \( I = 0.9 \) to the corrector static stiffness, which is obtained from the expression (3).

\[ C_{cor}^{\text{stat}} = \frac{P_0 F}{l}, \]

The expression for the static stiffness of the carrier air spring \( I = 0.9 \) can be obtained by differentiation of the expression (1) with respect to \( x \) regarding (2) and by substituting the resulting expression in \( x = 0 \). As a result, we obtain:

\[ C_{stat} = (p_{overpr} + p_A) \frac{nF^2 p_0}{V_0} + k_1 p_{overpr}, \]

where the overpressure in carrier air spring can be defined from the following expression:

\[ P_{overpr} = \frac{mg}{F_p0}. \]

Equating expressions (6) and (7) and taking into account expression (8), it is possible to determine the static overpresssure \( p_0 \), which is necessary to be provided in the toroidal shell of the corrector to create, on a load characteristic, a mount with the zero stiffness in the equilibrium position.
\[ p_0 = \left( p_{\text{overpr}} + p_A \right) \frac{nF_{p0}}{V_0} + k l p_{\text{overpr}} \frac{\lambda l}{F}, \]

where \( \lambda \) is the coefficient that takes into account the parameters of the corrective shell.

Figure 4 shows a graph of the pressure \( p_0 \) depending on the load with mass \( m \) at one mount, built for air springs I-09. For a toroid-shape shell of the corrector, following dimensions are assumed: \( D_{\text{aver}} = 0.147 \text{ m}, d_{\text{ext}} = 0.051 \text{ m}, \) the distance between the axes of the corrector segments hinges in equilibrium \( l = 0.081 \text{ m}. \) Graph in figure 4 shows that the pressure values in the shell of the corrector, depending on the mass of the object, are in the range from 0.04 to 0.126 MPa.

![Graph showing pressure vs mass](image)

**Figure 4.** Static pressurization curve in a corrector toroid-shape shell \( p_0 \) depending on the mass load \( m \) on one mount, built for air springs I-09

### 4. Experimental results

Solution of the equations is performed on a computer in Matlab with Simulink extension. According to the values obtained as a result of solving differential equation (5) motion graphs of vibro-isolated object in the range of 1-10 Hz are built.

Figures 5 and 6 show a graph of the carrier elastic force of air spring I-09, a graph of the vertical component of the elastic force produced by a corrector, and a graph of the resulting elastic force of the mount with an area of quasi-zero stiffness for the mass of 50 and 100 kg.
Figure 5. A graph of the force exerted by the mount for the mass of 50 kg, depending on its shift from the equilibrium position: for the carrier air springs I-09 ($P_{spr}$); for the corrector ($P_{cor}$); for the resultant force ($P_{res}$).

Figure 6. A curve of the force, acting from the mount to the mass (100 kg) depending on its shift from the equilibrium position for carrier air spring I-09 ($P_{spr}$); for a corrector ($P_{cor}$); for the resulting force ($P_{res}$).
Figure 7 shows a deformation curve of the toroid-shape shell $\Delta l$ depending on the mass shift from the equilibrium position $x$.

5. Discussion

From figure 4 it follows that with increasing mass of the vibro-insulated object, the pressure in the corrective should be also increased linearly. The graphs of the resultant force acting from the mount side on objects with a mass of 50 and 100 kg in the low frequency range oscillations (1 – 10 Hz) have substantially horizontal sections (figures 5 and 6), providing quasi-zero stiffness. Load shell deformation in the process of a mount work is small and balanced by its elastic properties (figure 7).

6. Conclusion

The use, together with carrier air spring on the basis of rubber-cord shell, the stiffness corrector designed on the basis of toroidal rubber shell under excessive pressure, allows to obtain a mount with the effect of quasi-zero stiffness, and thus to reduce the natural frequency of the suspension and to improve the vibration isolation of different vibro-active objects.

Application of the toroidal shell for designing a stiffness corrector, in comparison with horizontally mounted and hinged air springs on the basis of the pneumatic cylinder with a piston, allows to considerably reduce the dimensions of a mount and eliminate the sliding friction force of the piston in the cylinder, which reduces the damping characteristics of the mount.

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