STATUS OF THE STRONG COUPLING CONSTANT

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Abstract

The current status of measurements of the strong coupling constant from different reactions is reviewed. Including new results presented at the 1996 ICHEP conference, a global average $\alpha_s(M_Z) = 0.118 \pm 0.003$ is obtained.

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1 Introduction

Over the past years significant progress has been made in the determination of the strong coupling constant $\alpha_s$. Next-to-leading order (NLO) theoretical predictions are generally available. For some inclusive quantities also the next-to-next-to-leading (NNLO) orders have been calculated and estimates of the next higher terms exist. In addition, resummations of leading-log (LL) and next-to-leading-logarithmic (NLL) corrections to all orders have been performed in some cases. Power law corrections are controllable in the framework of the operator-product-expansion (OPE) or the resummation of renormalon chains. On the experimental side information exists from many different reactions over a large range of both space-like and time-like momentum transfers. Reactions include neutrino- and lepton-nucleon scattering, proton-(anti)proton collisions, $e^+e^-$-annihilation and decays of bound states of heavy quarks. Observables are total cross section measurements, sum rules, scaling violations, branching ratios, global event shape variables and production rates of hadron jets, i.e. bundles of particles close by in phase space originating from isolated hard partons.

1.1 Theoretical Predictions

The QCD prediction for a cross section $\sigma(Q)$ at an energy scale $Q$ can be expressed as sum of perturbative terms $\delta_{\text{pert}}$, which vary logarithmically with energy, and non-perturbative power law corrections $\delta_{\text{np}}$. The perturbative part is of the form

$$\delta_{\text{pert}} = \alpha_s(\mu) A + \alpha_s^2(\mu) \left( B + A \frac{\beta_0}{4\pi} \ln \frac{\mu^2}{Q^2} \right) + \ldots$$

Here $\mu$ is an arbitrary renormalization scale used in the calculation and $\beta_0$ the leading order coefficient of the QCD beta-function. For $\mu \approx Q$ the perturbative expansion is well behaved, allowing for a measurement of the strong coupling constant. For $|\ln(\mu^2/Q^2)| \gg 1$ the effective expansion parameter becomes $\alpha_s(\mu) \ln(\mu^2/Q^2)$, which leads to an unstable theoretical prediction. The full theory does not depend on $\mu$, but the truncated perturbative expansion does. Variation of $\mu$ thus allows to probe the sensitivity of the theory to uncalculated higher orders and thereby to assess theoretical uncertainties.

In the following the nominal scale $\mu$ of an $\alpha_s$-measurement is identified with the physical scale of the respective process. For data covering a range of scales the geometric mean of high and low end is taken, unless a central scale is specified explicitly by the authors. Combined results from different measurements are quoted at the geometric mean of the individual scales. The geometric mean for the scales is motivated by the fact, that (in leading order) $\alpha_s$ is inversely proportional to the logarithm of the scale.

The beta function describes the renormalization scale dependence of the strong coupling,

$$\mu \frac{d \alpha_s(\mu)}{d \mu} = -\frac{\beta_0}{2\pi} \alpha_s^2(\mu) - \frac{\beta_1}{4\pi^2} \alpha_s^3(\mu) + \mathcal{O}(\alpha_s^4)$$

with $\beta_0 = 11 - 2n_f/3$ and $\beta_1 = 51 - 19n_f/3$ for QCD based on an SU(3)-colour gauge symmetry. The quantity $n_f$ denotes the number of active quark flavours. Measurements of $\alpha_s$ obtained at different energy scales can be compared, either by evolving backwards to the point $\Lambda$ where $\alpha_s$ diverges, or by evolving to a common reference energy, which in recent years has become the $Z$ mass. In the evolution $\alpha_s$ is continuous at flavour thresholds, which, within the still comparatively large uncertainties of the quark masses, can be taken at the $\overline{\text{MS}}$-running mass of the respective quark flavour. As an example consider the evolution of an $\alpha_s$-measurement with
$n_f = 3$ from the scale $\mu = m_t$ to $\mu = M_Z$. It proceeds by first going back to the charm-quark mass $m_c = 1.3 \pm 0.2$ GeV using the evolution equation for three flavours, then going up to $m_b = 4.3 \pm 0.2$ GeV with $n_f = 4$ and finally evolving all the way up to $\mu = M_Z$ with $n_f = 5$ active flavours. Many slightly different schemes are in use to perform the evolution numerically. Despite being formally equivalent to NLO the resulting differences in the value for $\alpha_s(M_Z)$ can be as large as $\delta \alpha_s(M_Z) = 0.001$. To be consistent, for the numbers presented in this paper the treatment of flavour thresholds described above has been applied together with Runge-Kutta integration of the NNLO-beta function.

1.2 Combination of Individual Results

Many measurements of the strong coupling constant are dominated by theoretical uncertainties. In order to be quantitative, it will be assumed that they represent 68% confidence intervals. Their non-statistical nature suggests that they should be interpreted in the Bayesian sense. Note that doing this also for the experimental uncertainties would justify to combine experimental and theoretical errors in quadrature.

Theoretical errors for different $\alpha_s$-measurements are correlated via the underlying theory. Such correlations could be quantified by the derivatives of the predictions with respect to the various sources of uncertainty, e.g. unknown higher order coefficients. Instead, in many cases only total errors are available, leading to a situation where measurements are known to be correlated with very little information about the actual size of those correlations. To deal with this situation, the following averaging procedure\cite{1} has been employed.

For a set of measurements $x_i \pm \sigma_i$ the average is calculated as $\bar{x} = \sum x_i w_i$, with weights inversely proportional to the squares of the errors $\sigma_i$. This is a robust estimate of a common mean, which is also optimal if the single measurements are uncorrelated. To determine the error of the average and its $\chi^2$ the full covariance matrix is needed. Using only the diagonal terms, both are underestimated in the presence of positive correlations, i.e. a $\chi^2$ smaller than its expectation value is indicative of such terms. In this case a realistic error for the average can be obtained even if the size of the correlations is not known a priori, by introducing off-diagonal terms $\rho \sigma_i \sigma_j$ in the covariance matrix and adjusting the effective correlation coefficient $\rho$ such that $\chi^2$ attains its expectation value. Averages with errors determined according to this method will be referred to as “conservative averages” in the following. Measurements with asymmetric errors are shifted to the center of the error band and the error symmetrized prior to the averaging procedure.

2 Measurements of $\alpha_s$

This section contains an overview over the most important types of $\alpha_s$-measurements, presenting results from a large variety of reactions and observables.

2.1 Inclusive Measurements

Inclusive measurements of $\alpha_s$ depend only on one energy scale characterizing the process and are best understood theoretically. The perturbative prediction is known to $\mathcal{O}(\alpha_s^3)$, non-perturbative effects can be treated in the framework of the Operator Product Expansion [3](OPE).
The Ratios $R_\gamma$ and $R_l$

The quantities $R_\gamma$ and $R_l$ are defined by the ratios of the hadronic to the leptonic branching fractions of a virtual photon or the Z-boson. In both cases the hadronic system is formed from the electroweak (EW) coupling of a vector boson (Photon or Z) to a primary quark-antiquark pair. The sensitivity to the strong coupling comes from gluon radiation off the primary quarks. This radiation opens up new final states for the hadronic system, which increase the hadronic width with respect to the purely electroweak expectation. The QCD correction is dominated by the perturbative terms. The leading non-perturbative effects scale with $O(1/Q^4)$ and thus are negligible for $R_\gamma$ and $R_l$. Mass effects, in particular from the large top-bottom mass splitting, have been calculated [4] to $O(\alpha_s^2)$.

The first analyses [5] of $R_\gamma$ were still based on an erroneous third order correction [6]. A recent reanalysis [7] using the correct third order coefficient [8] and data from centre-of-mass energies between 5 GeV and 65 GeV yields $\alpha_s(\mu) = 0.175 \pm 0.023$ for a central scale $\mu = 18$ GeV, which corresponds to $\alpha_s(M_Z) = 0.128 \pm 0.012$.

A simple parametrization [9] for $R_l$ is available to third order in $\alpha_s(M_Z)$. Using the combined result from all four LEP-experiments [10] $R_l = 20.778 \pm 0.029$, a top-quark mass $m_t = 180$ GeV/$c^2$ and a Higgs-mass $M_H = 300$ GeV/$c^2$ one finds $\alpha_s(M_Z) = 0.124 \pm 0.005$. The statistical error is $\delta\alpha_s(M_Z)_{stat} = 0.004$, twice as large as the QCD theoretical uncertainty.

In addition to $R_l$ also the hadronic peak cross section of the Z and its width are sensitive to the strong coupling. Combining this information with all available electroweak data from LEP, SLC, collider measurements and Deep Inelastic Scattering in a global standard model fit, allows the simultaneous determination of $\alpha_s(M_Z)$ and the Higgs mass. The result is shown in Fig. [1]. One finds [11] a Higgs mass $M_H = 149$ GeV/$c^2$ and $\alpha_s(M_Z) = 0.1202 \pm 0.0033$. The standard model constraint thus not only gives a very precise measurement of $\alpha_s(M_Z)$, but also one perfectly consistent with other precision measurements.

![Figure 1: Combined measurement of $\alpha_s$ and the Higgs mass $M_H$ from QCD radiative corrections to the standard model.](image-url)
Measurement of $\alpha_s$ from $R_\tau$

An $\alpha_s$-measurement can also be obtained from $R_\tau = B_{\text{had}}/B_e$, the ratio of the hadronic to the electronic branching ratio of the tau lepton. Here QCD radiative corrections affect the hadronic final state from a W-decay. In contrast to $\alpha_s$-determinations from $R_\gamma$ or $R_l$ the mass of the hadronic system is not fixed in tau decays. This makes the quantity $R_\tau$ double inclusive, i.e. integrated over all hadronic final states at a given mass and integrated over all masses between $M_\pi$ and $m_\tau$.

The low energy scale requires a good understanding of the non-perturbative contributions, which in the framework of the OPE are proportional to vacuum expectation values (condensates) of the QCD fields. These condensates can be determined from independent phenomenological analyses [12], or together with $\alpha_s$ from the higher moments of the mass spectrum of hadronic $\tau$-decays. [13] It turns out that the non-perturbative corrections to $R_\tau$ are surprisingly small, $\delta_{\text{np}} = -1.5 \pm 0.4\%$, and [14] that perturbative QCD is applicable down to mass scales below 1 GeV. As a consequence an $\alpha_s$ measurement from $R_\tau$ is potentially very accurate.

Assuming the validity of the completeness relation for the tau branching ratios into hadrons, electrons and muons, $B_{\text{had}}+B_e+B_\mu = 1$, and lepton universality, the ratio $R_\tau$ can be expressed as function of $B_e$ alone. Note that the larger mass of the muon leads to $B_\mu/B_e = 0.9726$. The branching ratio $B_e$ can be determined by direct measurements or, again assuming lepton universality, by comparing mass and lifetime of the tau lepton and the muon. From recent measurements [15] one obtains $R_\tau = 3.642 \pm 0.010$. A value for $\alpha_s(m_\tau)$ with a rather conservative error based on this number is quoted [14] as $\alpha_s(m_\tau) = 0.33 \pm 0.03$. This error is almost entirely due to uncertainties in the perturbative prediction $\delta_{\text{pert}}$, taken to be half the difference between Le Diberder-Pich resummation [16] and resummation of renormalon chains. [17] Other studies [18] suggest that it could be significantly smaller.

A combined analysis of $R_\tau$ and the leading moments of the hadronic mass spectrum has been performed by the ALEPH [19] and CLEO [20] collaborations. A three-sigma discrepancy between the two was due to the old 1994 PDG-value [21] for $B_e$ used in the CLEO analysis. The moments extracted from the hadronic mass spectrum are in good agreement. With an updated leptonic branching ratio [22] the CLEO result $\alpha_s(m_\tau) = 0.339 \pm 0.024$ becomes compatible with the ALEPH number $\alpha_s(m_\tau) = 0.353 \pm 0.022$. The conservative average of both $\alpha_s$-measurements is $\alpha_s(m_\tau) = 0.347 \pm 0.022 \pm 0.03$, where the second error [14] has been added to account for the fact that both analyses are based on Le Diberder-Pich resummation. Evolving up to the Z-mass and symmetrizing the larger one of the slightly asymmetric errors, one finally obtains $\alpha_s(Z) = 0.122 \pm 0.005$.

$\alpha_s$ from the Gross-Llewellyn-Smith Sum Rule

Determinations of the strong coupling constant based on sum rules are fully inclusive measurements at a very low $Q^2$-scale. In the Quark-Parton-Model (QPM) the Gross-Llewellyn-Smith sum rule (GLSR) counts the number of valence quarks in the nucleon. The perturbative QCD-correction [23] is known to $\mathcal{O}(\alpha_s^3)$. A measurement of the strong coupling constant taking into account also non-perturbative (“higher twist”) terms [24] has been performed by the NuTeV-Collaboration [25] at a scale $\mu = 1.73$ GeV, based on the old CCFR data. [26] The result $\alpha_s(\mu) = 0.260^{+0.041}_{-0.046}$ corresponds to $\alpha_s(Z) = 0.110^{+0.006}_{-0.009}$. As the input data have been re-calibrated recently, this number can be expected to be updated, too.
The Bjorken Sum Rule

Also for the Bjorken sum rule (BjSR) the perturbative QCD-correction to the QPM \(\mathcal{O}(\alpha_s^3)\) together with an estimate of the size of the non-perturbative effects. The BjSR is defined for charged-current neutrino-nucleon scattering or polarized lepton nucleon scattering. Measurements of the strong coupling constant so far only exist based on the spin-dependent structure functions of the nucleon. A first analysis \([29]\) had comparable experimental and theoretical errors, a recent update \([30]\) finds \(\alpha_s(\mu) = 0.320^{+0.031}_{-0.053}\) (exp) ± 0.016 (theo) for a scale \(\mu = 1.73\) GeV. Evolved to the Z-mass one obtains \(\alpha_s(M_Z) = 0.118^{+0.005}_{-0.007}\) with an error dominated by the experimental uncertainties.

The small size \(\delta\alpha_s(\mu) = 0.016\) of the theoretical uncertainties results from the stability of the analysis with respect to various ways of estimating missing higher order terms by means of Padé approximants (PA). The PA \([N/M]\) of a function is the ratio of two polynomials of order \(N\) and \(M\), which to order \(N + M\) has the same Taylor-expansion as the original function. Padé approximants thus offer a systematic way to guess how a perturbative expansion resums, by rewriting a perturbative series as a ratio of two polynomials. Compared to the original expression, the PA introduces poles in the coupling plane in a similar fashion as expected from renormalons. This may explain why PAs seem to be able to approximately resum the perturbative series, a finding corroborated e.g. by the observation \([31]\) that measurements of the strong coupling from global event shape variables become much more consistent when the second order prediction is replaced by its \([1/1]\)-PA.

Heavy Flavour Thresholds

Another potentially very precise \(\alpha_s\)-measurement is derived from the threshold behaviour of the \(b\bar{b}\)-production cross section in \(e^+e^-\)-annihilation. QCD sum rules for this process are dominated by the non-relativistic threshold behaviour and allow to extract simultaneously \(\alpha_s\) and the \(b\)-quark mass. A determination of the strong coupling \([32]\) at the BLM-optimized \([33]\) renormalization scale \(\mu = 3\) GeV yields \(\alpha_s(M_Z) = 0.110 \pm 0.001\). Naively one would set the renormalization scale to the \(b\)-mass, which corresponds to a shift \([34]\) \(\delta\alpha_s(M_Z) = +0.008\). Taking this as the theoretical error of the measurement one has \(\alpha_s(M_Z) = 0.110 \pm 0.008\) or \(\alpha_s(\mu) = 0.217^{+0.036}_{-0.030}\) for \(\mu = 3\) GeV, respectively.

2.2 Bound States of Heavy Quarks

The strong coupling constant has been determined from ratios of decay rates, which are described by perturbative QCD, and level splittings between different radial excitations of the bound system, which probe the QCD-potential between quark and antiquark.

Decays of Heavy Quarkonia

A precise \(\alpha_s\)-measurement is based on the ratio of the hadronic to the leptonic width of a heavy quark-antiquark pair. At Born-level this ratio of annihilation rate into three gluons over the rate into a lepton pair is proportional to \(\mathcal{O}(\alpha_s^3/\alpha^2_{em})\). Taking into account relativistic corrections proportional to the average \(<v^2/c^2>\) of the quarks, the theoretical prediction can be written in the form

\[
\frac{\Gamma(q\bar{q}\rightarrow\text{hadrons})}{\Gamma(q\bar{q}\rightarrow e^+e^-)} = R_{\text{pert}} \left( 1 + D \left\langle \frac{v^2}{c^2} \right\rangle \right).
\]

The perturbative prediction \(R_{\text{pert}}\) is known to NLO, \(D\) is a free parameter for the size of the non-perturbative relativistic corrections. Assuming \(D\) to be a universal constant, it has been
extracted together with $\alpha_s$ in a combined analysis of $\Upsilon$ and $J/\Psi$ decays. Theoretical uncertainties in $R_{pert}$ were studied by introducing ad hoc NNLO terms, by varying the renormalization scale and by Padé-like rewriting terms of the type $(1+B\alpha_s)/(1-B\alpha_s)$. The result at a scale $\mu = 10$ GeV is $\alpha_s(\mu) = 0.167^{+0.015}_{-0.011}$, dominated by the theoretical uncertainties.

The CLEO collaboration [36] has extracted a measurement of the strong coupling from the ratio $\Gamma(\Upsilon \rightarrow gg\gamma)/\Gamma(\Upsilon \rightarrow ggg)$, which to leading order is proportional to $\alpha_{em}/\alpha_s$. The result at the scale of the Upsilon mass $\mu = 9.7$ GeV is $\alpha_s(\mu) = 0.163 \pm 0.009(\text{exp}) \pm 0.010(\text{theo})$. The experimental error is dominated by systematic uncertainties in the photon background of purely hadronic Upsilon decays and roughly equal to the renormalization scale error. The conservative average of both measurements taken at the scale $\mu = 9.7$ is $\alpha_s(\mu) = 0.166 \pm 0.013$, which evolves to $\alpha_s(M_Z) = 0.112 \pm 0.006$.

Lattice Calculations

Very precise determinations of $\alpha_s$ were performed in the analysis of level splittings between the $S$- and the $P$-states of the $\Upsilon$-system by means of lattice calculations. The current level of precision is a result of reduced lattice spacing errors, a better understanding of the conversion from the lattice to the $\overline{\text{MS}}$ coupling measured elsewhere and the introduction of fermion loops on the lattice. Calculations exist with $n_f = 0$ and $n_f = 2$ dynamic fermions, which give only marginally different results and thus allow a safe extrapolation to the physical case of $n_f = 3$ light flavours. The most recent results are $\alpha_s(M_Z) = 0.118 \pm 0.003$ (NRQCD) and $\alpha_s(M_Z) = 0.116 \pm 0.003$ (FNAL). The conservative average is $\alpha_s(M_Z) = 0.117 \pm 0.003$, which at the typical scale of the calculations $\mu = 8.2$ GeV corresponds to $\alpha_s(\mu) = 0.184 \pm 0.008$.

2.3 Scaling Violations

Scaling violations are observed in Deep Inelastic Scattering (DIS) processes in the structure functions of the nucleon, and in $e^+e^-$-annihilation into hadrons in the fragmentation functions of the primary partons, i.e. in reactions with space- and time-like momentum transfers. In both cases perturbative QCD predicts a softening with increasing $Q^2$, $d\ln F/d\ln Q^2 \propto \alpha_s(Q)$ as described by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations. The softening of structure functions comes about because higher momentum transfers resolve more partons from vacuum fluctuations in the nucleon. For fragmentation functions the growing phase space permits additional gluon radiation and thus enhanced particle multiplicities in jets. The theory is known to next-to-leading order. In a determination of $\alpha_s$ perturbative and non-perturbative effects can be disentangled through their energy dependence. Here DIS processes are favoured because non-perturbative effects decrease rapidly with $1/Q^2$. In $e^+e^-$-annihilation they decrease only proportional to $1/Q^2$, which renders measurements of $\alpha_s$ from fragmentation functions less precise than the ones from DIS, despite the fact that they are performed at larger momentum transfers.

Scaling Violations in Structure Functions

Measurements of the strong coupling constant were performed in DIS with neutrino beams and charged leptons on targets of heavy and light nuclei. The most recent results for $\Lambda_{\overline{\text{MS}}}^{(4)}$ from various experiments are collected in Tab. 1 and displayed in Fig. 2. All numbers are in good agreement, although there is a slight trend towards larger values for more recent measurements. The two most precise results are from a combined analysis of SLAC-BCDMS measurements, $\Lambda_{\overline{\text{MS}}}^{(4)} = 263 \pm 42$ MeV, and a recent re-analysis of CCFR data.
Table 1: Measurements of the QCD scale from DIS in chronological order. The errors are the purely experimental uncertainties. The average is the weighted average from the lower part of the table.

which gives $\Lambda_{\overline{MS}}^{(4)} = 346 \pm 58$ MeV. For the latter the logarithmic derivatives of the structure functions together with the NLO QCD-fit are shown in Fig. 3. The value for $\Lambda_{\overline{MS}}^{(4)}$ moved up by 136 MeV compared to a previous publication [26] based on the same data, mainly as a consequence of an improved energy calibration for the detector. The weighted average based on the measurements published after 1990 yields $\Lambda_{\overline{MS}}^{(4)} = 287 \pm 31$ MeV. With a common theoretical uncertainty [42] of $\delta_\alpha_s(M_Z) = 0.004$ this corresponds to $\alpha_s(M_Z) = 0.116 \pm 0.005$, or equivalently $\alpha_s(\mu) = 0.200 \pm 0.016$ for an average scale $\mu = 5.4$ GeV. The total error from this NLO analysis is still dominated by theoretical uncertainties. First results from NNLO-analyses [45] indicate that the value of $\alpha_s$ is quite stable when going to higher orders, i.e. the theoretical error can be expected to decrease.

![Figure 2: Graphical representation of the results given in table 1. The vertical line marks the average $\Lambda_{\overline{MS}}^{(4)}$, the shaded area its uncertainty.](image-url)
Figure 3: Logarithmic derivatives of the structure functions $F_2$ and $xF_3$ as measured by the CCFR collaboration. The solid line is the result of a NLO QCD-fit.

Scaling Violations in Fragmentation Functions

The full NLO theoretical framework has also been used in two determinations of the strong coupling constant from scaling violations in fragmentation functions.\cite{16, 17} Both are based on inclusive and $uds$, $c$, $b$ and gluon-jet enriched data samples from hadronic $Z$ decays, combined with measurements from lower centre-of-mass energies down to $\sqrt{s} = 22$ GeV or $\sqrt{s} = 14$ GeV, respectively. The strong coupling constant was determined together with parametrizations for the fragmentation functions of the different parton types and a power-law correction describing non-perturbative effects. The latter was found to be small for the energy range under consideration. The results of these model independent analyses are\cite{16} $\alpha_s(M_Z) = 0.126 \pm 0.009$ and\cite{17} $\alpha_s(M_Z) = 0.121 \pm 0.012$, where the error is dominated by the QCD scale uncertainty. The larger error from the second analysis reflects a larger range for scale variations. The conservative average of both results is $\alpha_s(M_Z) = 0.124 \pm 0.010$, which for a central scale $\mu = 36$ GeV corresponds to $\alpha_s(\mu) = 0.146 \pm 0.014$.

Assuming that non-perturbative effects and differences in the fragmentation between quarks of different flavours and gluons are described correctly by Monte-Carlo models, one can also exploit the LUND matrix element model as the theoretical basis for a measurement of $\alpha_s$ from scaling violations. A result\cite{18} obtained by the DELPHI collaboration is consistent with the other determinations. Finally, it is worth noting that scale breaking effects in fragmentation functions can also be studied in DIS, using the momentum distribution of particles in the current jet.\cite{19} DIS thus offers the possibility to measure $\alpha_s$ from scale breaking effects both in structure functions and in fragmentation functions.

2.4 Results from Hadron Colliders

Prompt photon production in hard parton-parton scattering is a Compton-like process of $O(\alpha_s\alpha_{em})$. In the difference $\sigma(p\overline{p}\rightarrow\gamma X) - \sigma(pp\rightarrow\gamma X)$ the sea quark and gluon structure functions of the nucleon cancel, i.e. only the well known valence quark distribution is needed as external input for an $\alpha_s$-determination. The UA6 collaboration\cite{20} finds $\Lambda_{\overline{MS}}(4) = 235 \pm 106^{+146}_{-90}(\text{theo})$ MeV. Translated into $\alpha_s$ at the typical scale $\mu = 4$ GeV of the measurement one finds $\alpha_s(\mu) = 0.206^{+0.042}_{-0.033}$, which corresponds to $\alpha_s(M_Z) = 0.112^{+0.012}_{-0.010}$.

Heavy quarks not present in the initial state can be produced by quark-antiquark
annihilation or gluon–gluon fusion processes, which to leading order are of $\mathcal{O}(\alpha_s^2)$. The theory is developed to NLO. Experimentally those reactions can be tagged by the decay characteristics of the heavy hadrons. The most precise measurement from an analysis of $b\bar{b}$+jets production in $p\bar{p}$ collisions is $\alpha_s(\mu) = 0.138^{+0.028}_{-0.019}$, determined at a scale $\mu = 20$ GeV. Evolved to the $Z$ mass one finds $\alpha_s(M_Z) = 0.1097^{+0.016}_{-0.012}$.

The strong coupling constant has also been determined from QCD-radiative corrections to $W$ production at hadron colliders, based on the cross section ratio $R_W = \sigma(W+1\text{jet})/\sigma(W+0\text{jets})$. This ratio is sensitive to the product $\alpha_s F(x)$ of strong coupling constant and parton densities in the nucleon. The perturbative QCD-correction is known to NLO. Taking the nucleon structure functions from low energy data, $\alpha_s$ enters both in the matrix element for jet-production in $W$ decays and in the evolution of the structure functions to collider energies. A measurement $\alpha_s(M_W) = 0.123 \pm 0.025$, or equivalently $\alpha_s(M_Z) = 0.121 \pm 0.024$, has been obtained by the UA2-collaboration at a centre-of-mass energy $\sqrt{s} = 630$ GeV, with an error dominated by experimental uncertainties. The ratio $R_W$ has also been determined at the Tevatron, but it turns out to be rather insensitive to $\alpha_s$ when parton densities from low energy measurements are employed in the analysis. The reason is that for larger values of $\alpha_s$ the enhanced softening of the structure functions in the evolution almost compensates the increased jet production rates from the matrix element. In other words, the determination of the strong coupling from $W$-decays at the Tevatron requires parton densities measured at the same energy.

Another interesting prospect is the determination of the strong coupling constant from the inclusive jet cross section at hadron colliders. Here an extraction of $\alpha_s(E_T)$ with high precision seems feasible.

### 2.5 Global Event Shape Variables

To leading order the fraction of three-jet events in $e^+e^-$-annihilation is proportional to $\alpha_s$. Global event shape variables which are sensitive to the topology of multijet events exploit this fact for a measurement of the strong coupling constant. In order to be defined in perturbative QCD they must be constructed such, that they are insensitive to soft gluon radiation and that they remain unchanged if any of the final state particles splits into two collinear ones. An example is the variable thrust $T = \max_i(|\sum_p \vec{p} \cdot \vec{n}_i|)/(|\sum_p \vec{p}|)$, which measures the collimation of the momentum flow in an event. An ideal two-jet event without final state gluon radiation has $T = 1$. Gluon radiation decreases the value of $T$ until one finds $T = 1/2$ for a perfectly isotropic event.

Another example is $y_3$, defined by means of a jet clustering algorithm where initially each particle is considered its own jet. Then those two jets which are closest in phase space are combined by adding their 4-momenta. Iterating the procedure, $y_3$ is defined as that distance where the event makes the transition from three to two jets. Common measures for the distance between two jets $i$ and $j$ are the Durham-metric $D_{ij} = 2\min(E_i^2, E_j^2)(1 - \cos \theta_{ij})/s$ or the Jade-metric $y_{ij} = 2(E_i E_j)(1 - \cos \theta_{ij})/s$. In both cases $\theta_{ij}$ is the opening angle between the jets and $s$ denotes the total invariant mass of the hadronic system.

The theoretical prediction for all global event shape variables is known to NLO, based on a numerical integration of the ERT-matrix elements. For some variables also leading-logarithmic and next-to-leading logarithmic corrections, which dominate the cross section at high thrust, have been resummed to all orders. In the latter case an improved theoretical prediction is obtained by combining the resummed prediction with the second order matrix elements, which is exact to $\mathcal{O}(\alpha_s^2)$ over the whole phase space and contains the dominant terms for two-jet like configurations to all orders. There is a certain freedom in performing the matching, which gives rise to differences at $\mathcal{O}(\alpha_s^2)$. It thus can be employed as an alternative
to the variation of the renormalization scale to probe the sensitivity of an $\alpha_s$ measurement to unknown higher order perturbative corrections.

The non-perturbative transition from partons to hadrons gives rise to power-law corrections, which in contrast to the case of inclusive variables cannot be handled in the framework of the OPE and thus usually are estimated by means of Monte Carlo models. This dependence on phenomenological models introduces an additional “hadronization uncertainty” into an $\alpha_s$-measurement. Only recently analytic calculations became available, based on the study of the long-distance behaviour of leading order matrix elements, which in some respect can be understood as generalizations of the OPE. Identifying different classes of power-law corrections, they introduce a small number of universal parameters, e.g. effective values for integrals over powers of the strong coupling at low energies, which relate non-perturbative corrections for different event shape variables.

The formalism has been worked out for mean values of some global event shape variables. From a measurement of the energy dependence of those mean values, a model independent determination of the strong coupling is obtained by fitting $\alpha_s$ together with the non-perturbative power law corrections. The result of a first analysis of this kind is shown in Fig. 4.

![Figure 4: Measurement of $\alpha_s$ based on mean values of global event shape variables measured between $\sqrt{s} = 14$ GeV and $\sqrt{s} = 133$ GeV.](image)

Measurements of $\alpha_s$ based on global event shape variables from centre-of-mass energies between $\sqrt{s} = 10.53$ GeV and $\sqrt{s} = 133$ GeV are listed in Tab. 4. Wherever available, the numbers are single experiment averages over several variables. Combining the
partial averages from energies in the range \(10.53 \text{ GeV} \leq \sqrt{s} \leq 133 \text{ GeV}\) into one global mean value finally yields \(\alpha_s(M_Z) = 0.121 \pm 0.005\).

### Table 2: Measurements of the strong coupling constant from global event shape variables.

The errors are the total errors of the individual measurements, which are dominated by the theoretical uncertainties. The averages were formed as described in the introduction. The results from AMY and VENUS are based on theoretical predictions calculated by the NLLJET Monte Carlo. The DELPHI-analysis at \(\sqrt{s} = 133 \text{ GeV}\) uses the second order prediction from perturbative QCD together with an analytical ansatz for non-perturbative effects.

| Theory     | \(\mu/\text{GeV}\) | \(\alpha_s(\mu)\) |
|------------|---------------------|--------------------|
| CLEO       | NLO                 | 10.53              | 0.164 \pm 0.015 |
| TPC/2\(\gamma\) | NLO+NLL            | 29.0               | 0.160 \pm 0.012 |
| MAC        | NLO                 | 34.0               | 0.130 \pm 0.032 |
| MARKII     | NLO                 | 34.0               | 0.153 \pm 0.032 |
| PLUTO      | NLO                 | 34.0               | 0.108 \pm 0.038 |
| TASSO      | NLO                 | 34.0               | 0.149 \pm 0.026 |
| MARK-J     | NLO                 | 34.0               | 0.126 \pm 0.013 |
| CELLO      | NLO                 | 34.0               | 0.144 \pm 0.026 |
| JADE       | NLO                 | 34.0               | 0.162 \pm 0.043 |
| Average    |                     | 34.0               | 0.134 \pm 0.019 |
| AMY        | NLLJET              | 58.0               | 0.130 \pm 0.006 |
| TOPAZ      | NLO+NLL             | 58.0               | 0.132 \pm 0.008 |
| TOPAZ      | NLO+NLL             | 58.0               | 0.139 \pm 0.008 |
| VENUS      | NLLJET              | 58.0               | 0.129 \pm 0.006 |
| Average    |                     | 58.0               | 0.132 \pm 0.006 |
| ALEPH      | NLO+NLL             | 91.2               | 0.125 \pm 0.005 |
| DELPHI     | NLO+NLL             | 91.2               | 0.123 \pm 0.006 |
| L3         | NLO+NLL             | 91.2               | 0.124 \pm 0.007 |
| OPAL       | NLO+NLL             | 91.2               | 0.120 \pm 0.006 |
| SLD        | NLO+NLL             | 91.2               | 0.120 \pm 0.008 |
| Average    |                     | 91.2               | 0.123 \pm 0.006 |
| ALEPH      | NLO+NLL             | 133.               | 0.119 \pm 0.008 |
| DELPHI     | NLO+NP              | 133.               | 0.119 \pm 0.009 |
| L3         | NLO+NLL             | 133.               | 0.107 \pm 0.010 |
| OPAL       | NLO+NLL             | 133.               | 0.110 \pm 0.008 |
| Average    |                     | 133.               | 0.114 \pm 0.007 |

### 2.6 Jets from processes with variable \(Q^2\)

One contribution to multijet production in \(ep\)-collisions is gluon radiation from a quark scattered off a virtual photon with large \(Q^2\). Quark and gluon emerge as two jets in addition to the jet from the proton remnant. The production rate \(R_{2+1}\) of those (2+1)-jet final states is known to NLO for jets defined by the JADE algorithm. Tagging the scattered electron allows to control the \(Q^2\) of the process and thus to establish the running of the strong coupling constant within a single experiment. First measurements have been published by the H1 and ZEUS collaborations.\(^7\) The conservative average of both results is \(\alpha_s(M_Z) = 0.119 \pm 0.013\), which corresponds to \(\alpha_s(\mu) = 0.156 \pm 0.022\) for a central scale \(\mu = 19.6 \text{ GeV}\) of the two measurements.
The error is dominated by the theoretical uncertainties. The published data still suffer from too low statistics for a convincing demonstration of the running of $\alpha_s$, but improved results can be expected within the near future.\[71\]

Using radiative $Z$-decays, $Z\rightarrow q\bar{q}\gamma$, the running of the strong coupling can also be seen in $e^+e^-$-annihilation by a single experiment. A first measurement with still rather low statistical precision has been presented by the L3-collaboration.\[72\] The strong coupling extracted at six different scales below the $Z$-mass was found to be consistent with the running as expected by QCD. The combined result is $\alpha_s(M_Z) = 0.119 \pm 0.007$, which corresponds to $\alpha_s(\mu) = 0.131 \pm 0.008$ for an average scale $\mu = 50.8$ GeV.

### 3 A Global Average for $\alpha_s$

A compilation of the results discussed in the preceding section is given in Tab. 3. The $\alpha_s$-values from the different types of measurements evolved to the scale of the $Z$-mass are displayed in Fig. 5. Over an energy range from $\mu = 1.73$ GeV to $\mu = 133$ GeV all results appear to be consistent with one common mean for $\alpha_s(M_Z)$. The weighted average, assuming uncorrelated errors, is $\alpha_s(M_Z) = 0.1177 \pm 0.0015$ with $\chi^2/\text{ndf} = 6.7/14$. The conservative average is $\alpha_s(M_Z) = 0.1177 \pm 0.0037$, where this error is biased towards large values through the inclusion of some less precise results. Restricting the average to measurements with error $\delta \alpha_s(M_Z) < 0.008$, one finally obtains $\alpha_s(M_Z) = 0.1183 \pm 0.0032$.

The value of the strong coupling constant at the physical scale of the respective measurement is shown in Fig. 6. The running in agreement with the QCD prediction is evident. Note that many different types of reactions with different numbers of active quark flavours are consistent, in agreement with the expectation that $\alpha_s$ is flavour independent.

| Measurement          | $\mu$/GeV | $\alpha_s(\mu)$ | $\alpha_s(M_Z)$ |
|----------------------|-----------|-----------------|-----------------|
| BjSR                 | 1.732     | 0.320 (35)      | 0.118 (66)      |
| GLSR                 | 1.732     | 0.260 (41)      | 0.110 (66)      |
| $R_{\tau}$           | 1.777     | 0.347 (37)      | 0.122 (05)      |
| $b\bar{b}$ threshold | 3.        | 0.217 (36)      | 0.110 (08)      |
| prompt $\gamma$      | 4.0       | 0.206 (42)      | 0.112 (10)      |
| DIS                   | 5.4       | 0.200 (16)      | 0.116 (05)      |
| LGT                   | 8.2       | 0.184 (08)      | 0.117 (03)      |
| $c\bar{c}, b\bar{b}$ decays | 9.7 | 0.166 (13)    | 0.112 (06)      |
| $R_{R}$               | 18.0      | 0.175 (23)      | 0.128 (13)      |
| $ep\rightarrow$Jets  | 19.6      | 0.156 (22)      | 0.119 (13)      |
| $p\bar{p}\rightarrow$ $b\bar{b}$+Jets | 20.0 | 0.138 (26)    | 0.109 (10)      |
| $e^+e^-$-fragm.      | 36.0      | 0.146 (14)      | 0.124 (10)      |
| $pp\rightarrow$W+Jets | 80.2     | 0.123 (25)      | 0.121 (24)      |
| SM constraints        | 91.2      | 0.120 (03)      | 0.120 (03)      |
| event shapes          | $\leq 133$ |                | 0.121 (05)      |

Table 3: Compilation of measurements of the strong coupling constant. The total errors in the last two digits as given in parenthesis in most cases are dominated by the theoretical uncertainties.
4 Summary

In the past year significant progress has been made in the determination of the strong coupling constant by an improved understanding of power law corrections and perturbative higher orders on the theoretical side, and by precise new measurements from DIS, lattice gauge theories and standard model fits. All results are perfectly compatible with a common mean. Based on data covering the energy range $1 \text{ GeV} < \mu < 133 \text{ GeV}$, an average value $\alpha_s(M_Z) = 0.118 \pm 0.003$ is obtained. The error takes into account the possibility of positive correlations between the measurements. From the RMS-scatter of the individual results an even smaller error would be derived.

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Figure 6: Running of the strong coupling constant established by various types of measurements at different scales, compared to the QCD prediction for $\alpha_s(M_Z) = 0.118 \pm 0.003$. The open dots are results based on global event shape variables.

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