Inclusive Higgs Production at Large Transverse Momentum

HONG ZHANG, ERIC BRAATEN

Department of Physics
The Ohio State University, Columbus, OH 43210, USA

We present a factorization formula for the inclusive production of the Higgs boson at large transverse momentum $P_T$ that includes all terms with the leading power of $1/P_T^2$. The cross section is factorized into convolutions of parton distributions, infrared-safe hard-scattering cross sections for producing a parton, and fragmentation functions that give the distribution of the longitudinal momentum fraction of the Higgs relative to the fragmenting parton. The infrared-safe cross sections and the fragmentation functions are perturbatively calculable. The factorization formula enables the resummation of large logarithms of $P_T/M_H$ due to final-state radiation by integrating evolution equations for the fragmentation functions. By comparing the cross section for the process $qar{q} \rightarrow Ht\bar{t}$ from the leading-power factorization formula at leading order in the coupling constants with the complete leading-order cross section, we infer that the error in the factorization formula decreases to less than 5% for $P_T > 600$ GeV at a future 100 TeV collider.

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1 Introduction

It is widely believed that the Higgs boson may reveal clues of the physics beyond the Standard Model. Among the basic properties of the Higgs are its production rates in high energy collisions. In hadron collisions, the Higgs is produced primarily with transverse momentum $P_T$ smaller than its mass $M_H$. However its production rate at much larger $P_T$ is important, because it may be more sensitive to physics beyond the Standard Model, such as the decay of a much heavier particle into the Higgs.

The difficulty in calculating the Higgs $P_T$ distribution is a result of its multi-scale nature. There are at least four scales in the calculation, including the partonic center-of-momentum energy $\sqrt{s}$, $P_T$, $M_H$ and $M_i$, where $M_i$ is the mass of the particle from which the Higgs is emitted. In regions where some of these scales are well separated, effective field theory may be used to separate these scales and simplify the calculation. The most successful effective field theory in Higgs physics is the HEFT, in which virtual top-quark loops are replaced by local interactions of the Higgs with the vector bosons. It is reliable in the region where $M_t$ is much larger than other scales in the problem. HEFT is very successful in explaining the low $P_T$ distribution and the total cross section, which is dominated by the low $P_T$ region.

However, it has been shown that HEFT is not effective in the large $P_T$ region. Comparison with complete NLO calculations indicates that the difference between the exact and HEFT results exceeds 5% at $P_T = 150$ GeV and increases with $P_T$ [1]. The effects of the dimension-7 operators in HEFT on the Higgs $P_T$ distribution have recently been considered [2, 3]. The expansion in the higher-dimension operators of HEFT breaks down for $P_T$ above about 150 GeV.

In this work [4], we present the leading-power (LP) factorization formula for inclusive production of the Higgs at $P_T$ much larger than its mass $M_H$. By separating the kinematic scales from the mass scales, the calculation of the Higgs $P_T$ distribution at large $P_T$ is greatly simplified. The theoretical error can be decreased by summing logarithms of $P_T/M_H$ to all orders by integrating the evolution equations.

2 Leading-power factorization formula

The differential cross section for the inclusive production of a Higgs boson in the collision of hadrons $A$ and $B$ can be written in a factorized form [5]:

$$d\sigma_{AB \rightarrow H+X}(P_A, P_B, P) = \sum_{a,b} \int_0^1 dx_a f_{a/A}(x_a, \mu) \int_0^1 dx_b f_{b/B}(x_b, \mu) \times d\hat{\sigma}_{ab \rightarrow H+X}(p_a = x_a P_A, p_b = x_b P_B, P; \mu),$$

where $P$ is the momentum of the Higgs, $P_A$ and $P_B$ are the momenta of the colliding hadrons, and $p_a$ and $p_b$ are the momenta of the colliding partons. The sums in Eq. [1]
are over the types of QCD partons, which consist of the gluon and the quarks and antiquarks that are lighter than the top quark. The integrals in Eq. (1) are over the longitudinal momentum fractions of the colliding QCD partons. The separation of the hard momentum scales of order \( M_H \) and larger from the nonperturbative QCD momentum scale \( \Lambda_{QCD} \) involves the introduction of an intermediate but otherwise arbitrary factorization scale \( \mu \). The parton distribution functions \( f_{a/A} \) and \( f_{b/B} \) are nonperturbative, but their evolution with \( \mu \) is perturbative. Their evolution equations can be used to sum logarithms of \( M_H^2/\Lambda_{QCD}^2 \) to all orders in perturbation theory. The integration range of \( x_a \) is from 0 to 1, up to kinematic constraints from \( d\hat{\sigma}_{ab\to H+X} \).

If the transverse momentum \( P_T \) of the Higgs is much larger than its mass \( M_H \), it is reasonable to expand the hard-scattering differential cross section \( d\hat{\sigma} \) in Eq. (1) in powers of \( 1/P_T^2 \). The factorization formula for the leading power can be inferred from the perturbative QCD factorization formula for inclusive hadron production at large transverse momentum [3]. The leading-power (LP) factorization formula for the hard-scattering differential cross section for the inclusive production of a Higgs in the collision of QCD partons \( a \) and \( b \) is

\[
d\hat{\sigma}_{ab\to H+X}(p_a, p_b, P) = \sum_i \int_0^1 dz \ d\hat{\sigma}_{ab\to i+X}(p_a, p_b, p_i = \tilde{P}/z; \mu) \ D_{i\to H}(z; \mu),
\]

where \( p_i \) is the momentum of the fragmenting parton and \( \tilde{P} \) is a light-like 4-vector whose 3-vector component is \( P \). The relation \( p_i = \tilde{P}/z \) is valid only when the fragmenting parton mass \( M_i \) can be ignored. The effect of nonzero \( M_i \) can be systematically considered with a more complicated factorization prescription [4]. The errors in the factorization formula in Eq. (2) are of order \( M_H^2/P_T^2 \). The sum over partons \( i \) is over the types of elementary particles in the Standard Model. The most important fragmenting partons for Higgs production are the weak vector bosons \( W \) and \( Z \), the top quark \( t \), the top antiquark \( \bar{t} \), the gluon \( g \), and the Higgs itself. The integral in Eq. (2) is over the longitudinal momentum fraction \( z \) of the Higgs relative to the fragmenting parton. The integration range of \( z \) is from 0 to 1, up to kinematic constraints from \( d\hat{\sigma}_{ab\to i+X} \). The fragmentation functions \( D_{i\to H}(z; \mu) \) are distributions for \( z \) that depend on \( M_H \) and on the mass \( M_i \) of the fragmenting parton. The only dependence on \( P_T \) in Eq. (2) is in the cross sections \( d\hat{\sigma} \) for producing the parton \( i \). In these cross sections, the Higgs mass \( M_H \) is set to 0, while the mass \( M_i \) of the fragmenting parton may or may not be at its physical value. Note that the hard-scattering cross section \( d\sigma \) on the left side of Eq. (2) has mass singularities in the limits \( M_H \to 0 \) and \( M_i \to 0 \). Since the parton production cross sections \( d\hat{\sigma} \) have no mass singularities, we will refer to them as infrared-safe cross sections. The infrared-safe cross sections \( d\hat{\sigma} \) and the fragmentation functions can all be calculated perturbatively as expansions in powers of \( \alpha_s \) and the other coupling constants of the Standard Model. The separation of the hardest scale \( P_T \) from the softer scales \( M_H \) and \( M_i \) involves the introduction of another arbitrary factorization scale \( \mu \).
There are important differences between the factorization formula in Eq. (2) for inclusive Higgs production at large $P_T$ and the analogous QCD factorization formula for inclusive hadron production. One difference is that a Higgs can be produced directly in the hard scattering. Thus the Higgs is included in the sum over fragmenting partons in Eq. (2). In contrast, a hadron at large $P_T$ cannot be produced directly in the hard scattering. Another difference is that the fragmentation functions for Higgs production are completely perturbative. They can be calculated order-by-order in the Standard Model coupling constants. In contrast, the fragmentation functions for hadron production are nonperturbative, although their evolution with the fragmentation scale is perturbative.

3 Fragmentation functions

One of the most important partons that fragments into the Higgs is the Higgs boson itself. The LO fragmentation function for Higgs into Higgs is a delta function:

$$D_{H \to H}(z) = \delta(1-z) + \mathcal{O}(g_W^2, y_t^2).$$  \hspace{1cm} (3)$$

The leading corrections are of order $g_W^2$ from the coupling of the Higgs to weak vector bosons and of order $y_t^2$ from the Yukawa coupling of the Higgs to the top quark.

The leading-order contribution to the fragmentation function for top quark into Higgs comes from the tree-level process $t^* \to H + t$. The fragmentation function can be expressed as an integral over $Q^2$, the square of the invariant mass of the final state $H + t$, from $M^2_H/z + M^2_t/(1-z)$ to $\infty$. The integral over $Q^2$ is divergent. In the $\overline{\text{MS}}$ renormalization scheme, the LO fragmentation function for $t \to H$ is

$$D_{t \to H}(z, \mu) = \frac{y_t^2}{16\pi^2} z \left( \log \frac{\mu^2}{M^2_t} - \log [z^2 + 4 \zeta_t (1-z)] + 4 (1-\zeta_t) \frac{1-z}{z^2 + 4 \zeta_t (1-z)} \right),$$  \hspace{1cm} (4)$$

where $\zeta_t \equiv M^2_H/(4M^2_t) \approx 0.13$. The fragmentation function for $\bar{t} \to H$ is the same.

In Ref. [4], the top quark fragmentation function is also calculated in the invariant-mass-cutoff (IMC) scheme, in which the integral over $Q^2$ extends only up to $\mu^2$. The fragmentation functions for the $W$ and $Z$ bosons into Higgs are calculated at LO in both the $\overline{\text{MS}}$ and IMC renormalization schemes. The effect of evolution on all these fragmentation functions is also studied.

4 Comparison with a complete LO calculation

In this section, we apply the LP factorization formula to the process $q\bar{q} \to H t\bar{t}$ at LO, where $q$ is a massless quark. At LO, the LP factorization formula in Eq. (2) for
the process $q\bar{q} \rightarrow Ht\bar{t}$ has two terms:

$$
\frac{d^2\tilde{\sigma}_{q\bar{q}\rightarrow Ht\bar{t}}}{dP_T^2dy} = \frac{d^2\tilde{\sigma}_{q\bar{q}\rightarrow H+t\bar{t}}}{dP_T^2dy} + 2\int_0^1 \frac{dz}{z^2} \frac{d^2\tilde{\sigma}_{q\bar{q}\rightarrow t+t\bar{t}}}{dp_T^2dy}(p = \tilde{P}/z) D_{t\rightarrow H}(z).
$$

(5)

where $P$ is the momentum of the Higgs with mass $M_H$, $\tilde{P}$ is the corresponding momentum for a massless Higgs, and $p$ is the momentum of the fragmenting top quark. The subscripts $T$ represent the transverse components, and $\tilde{y}$ is the Higgs rapidity in the partonic center-of-momentum frame. In Eq. (5), we have explicitly used Eq. (3) and the fact that the fragmentation contributions from $t$ and $\bar{t}$ are the same. The relevant fragmentation function is given in Eq. (4).

The infrared-safe cross sections $d^2\tilde{\sigma}$ can be calculated using different prescriptions. In Ref. [4], we use both the zero-mass-top-quark (ZMTQ) prescription and the hybrid prescription. In the ZMTQ prescription, both the top quark mass and the Higgs mass are set to zero in the infrared-safe cross sections in Eq. (5). Consequently the error of the LP factorization formula is $O(M_t^2/P_T^2)$. In the hybrid prescription, the top quark mass is kept at its physical value in the infrared-safe cross section for $q\bar{q} \rightarrow H + t\bar{t}$. This decreases the error to $O(M_H^2/P_T^2)$.

![Figure 1: Left panel: Differential cross section for inclusive Higgs production at central rapidity from the parton process $q\bar{q} \rightarrow t\bar{t}H$ at a 100 TeV $pp$ collider. Right panel: Fractional error in the differential cross section for inclusive Higgs production. The thick (thin) lines are the LP factorization results in the $\overline{\text{MS}}$ (IMC) renormalization schemes. See text for more details.](image)

On the left panel in Fig. [4] we show the differential cross section for inclusive Higgs production at central rapidity from the parton process $q\bar{q} \rightarrow t\bar{t}H$ at a 100 TeV $pp$ collider as a function of the Higgs transverse momentum $P_T$. The LP factorization results are calculated with top quark fragmentation functions in the $\overline{\text{MS}}$ renormalization scheme. For comparison, we also show another approximation obtained by setting the Higgs mass to zero in the complete LO result. On the right panel in Fig. [4] we show the fractional error, which is the difference between the approximate
result and the complete LO result divided by the complete LO result. The fractional error of the LP factorization formula with the top quark fragmentation function in the IMC renormalization scheme is also plotted.

Fig. 1 shows that the LP factorization formula with the ZMTQ and hybrid prescriptions both give increasingly good approximations to the complete LO result at large $P_T$, with the errors decreasing to below 5% for $P_T > 600$ GeV. In contrast, the fractional error for the complete LO result with $M_H = 0$ does not go to zero at large $P_T$. This result indicates that the LP factorization formula is a systematic method to separate the mass scales from $P_T$, with the error decreasing with $P_T$, while the complete LO with $M_H = 0$ is not a good approximation. More detailed analysis shows that setting $M_H = 0$ in the region where Higgs is produced collinearly to the top quark results in an error $O(M_H^2/M_t^2)$, which does not decrease with increasing $P_T$.

5 Summary and outlook

We have presented the leading-power (LP) factorization formula for Higgs production with transverse momentum $P_T$ much larger than the mass $M_H$ of the Higgs. We have calculated the LO fragmentation functions for Higgs, top quark, $W$ and $Z$ into Higgs. By comparing the LP factorization formula with the complete calculation at LO for the process $qar{q} \rightarrow Ht\bar{t}$, we find that the error in the LP factorization formula decreases to less than 5% for $P_T > 600$ GeV at a future 100 TeV collider.

The LP factorization formula can be applied to other Higgs production processes, as well as the production of other particles at $P_T$ much larger than their masses. The separation of scales in the LP factorization formula simplifies the higher order calculations. Moreover, the factorization formula can be extended to the next-to-leading-power (NLP) $[6, 7, 8]$, reducing the error to $O(M_H^4/P_T^4)$, which is about 6% at $P_T = 250$ GeV. Thus the NLP factorization formula could be useful even at the Large Hadron Collider.

References

[1] E. Bagnaschi, G. Degrassi, P. Slavich and A. Vicini, JHEP 1202, 088 (2012)
[2] R. V. Harlander and T. Neumann, Phys. Rev. D 88, 074015 (2013)
[3] S. Dawson, I. M. Lewis and M. Zeng, Phys. Rev. D 90, no. 9, 093007 (2014)
[4] E. Braaten and H. Zhang, arXiv:1510.01686 [hep-ph].
[5] J. C. Collins, D. E. Soper and G. F. Sterman, Adv. Ser. Direct. High Energy Phys. 5, 1 (1989)

[6] Z. B. Kang, J. W. Qiu and G. Sterman, Phys. Rev. Lett. 108, 102002 (2012)

[7] S. Fleming, A. K. Leibovich, T. Mehen and I. Z. Rothstein, Phys. Rev. D 86, 094012 (2012)

[8] Z. B. Kang, Y. Q. Ma, J. W. Qiu and G. Sterman, Phys. Rev. D 90, no. 3, 034006 (2014)