Axion detection in the milli-eV mass range

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**Abstract**

We propose an experimental scheme to search for galactic halo axions with mass $m_a \sim 10^{-3}$eV, which is above the range accessible with cavity techniques. The detector consists of a large number of parallel superconducting wires embedded in a material transparent to microwave radiation. The wires carry a current configuration which produces a static, inhomogeneous magnetic field $\vec{B}_0(\vec{x})$ within the detector volume. Axions which enter this volume may convert to photons. We discuss the feasibility of the detector and its sensitivity.
The axion has remained a prime candidate for dark matter [1]. Current constraints on the axion allow masses between $10^{-3}$eV and $10^{-7}$eV. If the galactic halo is made up exclusively of axions, their density in the solar neighborhood is approximately $0.5 \times 10^{-24}$gr/cm$^3$ and their velocity dispersion is approximately $10^{-3}c$. Galactic halo axions can be detected by stimulating their conversion to photons in an electromagnetic cavity permeated by a strong magnetic field [2]. Detectors of this type are being built with increasing sensitivity [3]. However, it appears at present that these cavity detectors cannot cover the entire mass window. In particular, their range is limited in the direction of large axion masses by the complexities involved in segmenting a given magnetic volume into many small cavities. The most complex system envisaged so far would reach $m_a \simeq 1.6 \times 10^{-5}$eV [3]. Much larger masses ($\sim 10^{-3}$eV) are difficult for the cavity detector to access given presently available technology. In this letter, elaborating on earlier ideas [4], we propose an alternative approach which is specifically intended to address the possibility of larger axion masses.

The coupling of the axion to two photons is [1] ($\hbar = c = 1$)

$$L_{a\gamma\gamma} = \frac{\alpha}{8\pi} \frac{a}{f_a} \left[ \frac{N_e}{N} - \left( \frac{5}{3} + \frac{m_d - m_u}{m_d + m_u} \right) \right] F_{\mu\nu} \tilde{F}^{\mu\nu} = \frac{\alpha}{4\pi} \frac{a}{f_a} g_\gamma F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (1)$$

where $\alpha$ is the fine structure constant, $a$ is the axion field, $f_a$ is the axion decay constant, $m_u$ and $m_d$ are the up and down quark current masses, and $N$ and $N_e$ are model-dependent coefficients. In grand unified axion models, one has $N_e/N = 8/3$, hence $g_\gamma = m_u/(m_u + m_d) \simeq 0.36$. The axion mass is given by

$$m_a = \frac{f_\pi m_\pi}{f_a} \sqrt{m_u m_d} \simeq 0.6 \text{eV} \left( \frac{10^7 \text{GeV}}{f_a} \right). \quad (2)$$

Thus Eq.(1) can be rewritten

$$L_{a\gamma\gamma} = -g_\gamma \frac{\alpha}{\pi} \frac{m_a}{0.6 \times 10^{16}(\text{eV})^2} a \vec{E} \cdot \vec{B}. \quad (3)$$
Because of the coupling of Eq. (1), axions will convert to photons (and vice versa) in an externally applied magnetic field. The cross-section for \( a \rightarrow \gamma \) conversion in a region of volume \( V \) permeated by a static magnetic field \( \vec{B}_0(\vec{x}) \) is

\[
\sigma = \frac{1}{16\pi^2\beta_a} \left( \frac{\alpha g_\gamma}{\pi f_a} \right)^2 \int d^3k_\gamma \delta(E_a - \omega) \left| \int_V d^3x \, e^{i(\vec{k}_\gamma - \vec{k}_a) \cdot \vec{x}} \delta(\vec{E}_a - \omega, \vec{k}_a) \times \vec{B}_0(\vec{x}) \right|^2,
\]

where \((E_a, \vec{k}_a) = E_a(1, \vec{\beta}_a)\) is the axion 4-momentum, and \((\omega, \vec{k}_\gamma) = \omega(1, \vec{n})\) is the photon 4-momentum. \( E_a = \omega \) because the magnetic field is static. The momentum transfer \( \vec{q} = \vec{k}_\gamma - \vec{k}_a \), which is necessary because the photon is massless while the axion is massive, is provided by the inhomogeneity of the magnetic field. Galactic halo axions are non-relativistic \( (k_a \sim 10^{-3} m_a) \). Hence, to obtain resonant conversion, the magnetic field should be made inhomogeneous on the length scale \( m_a^{-1} \).

Figure 1 shows top and side views of the detector we propose. It consists of an array of parallel superconducting wires embedded in a microwave-transparent dielectric. The dielectric medium keeps the wires in place. The dimensions of the detector are \((L_x, L_y, L_z)\). \( \hat{y} \) is the common direction of the wires. The intersections of the wires with the \((x, z)\) plane form an array with unit cell size \( d < \sim m_a^{-1} \). We denote the location of a wire with the integers \((n_z, n_x)\), where

\[
\begin{align*}
n_z & \in (-N_z/2, N_z/2), & N_z d &= L_z; \\
n_x & \in (-N_x/2, N_x/2), & N_x d &= L_x.
\end{align*}
\]

Let the wires carry the following current configuration:

\[
I(n_z, n_x) = I(n_z) = I_0 \sin(n_z d q).
\]

Resonant conversion occurs when \( q \sim m_a^{-1} \). For an infinite distribution of current, the magnetic field generated is \( \vec{B}(z) = -\hat{x} B_0 \cos(qz) \), where \( B_0 = I_0/(qd^2) \). We expect the magnetic field to be dominated by this term, but since our detector has finite volume,
the field may be modified considerably by finite size effects. We first investigate these finite size effects.

The magnetic field at point \((z, x)\) inside the detector is dominated by its \(\hat{x}\) component

\[
B_x(z, x) = \frac{I_0}{2\pi} \sum_{n_z=-N_z/2}^{N_z/2} \sin(qdn_z) \sum_{n_x=-N_x/2}^{N_x/2} \frac{z-n_zd}{(z-n_zd)^2 + (x-n_xd)^2}.
\]  

Replacing sums with integrals, we find

\[
B_x(z, x) \simeq -B_0 \left[ f(x) \cos(qz) - \frac{1}{2} g(x, z) \cos \left( \frac{qL_z}{2} \right) + \mathcal{O} \left( \frac{1}{qL} \right) \right],
\]

for \(|x| \leq L_x/2\) and \(|z| \leq L_z/2\), where

\[
B_0 \equiv \frac{I_0}{qd^2},
\]

\[
f(x) \equiv 1 - e^{-qL_x/2} \cosh(qx),
\]

\[
g(x, z) \equiv \frac{1}{\pi} \left[ \arctan \left( \frac{L_x/2 - x}{L_z/2 - z} \right) + \arctan \left( \frac{L_x/2 - x}{L_z/2 + z} \right) + \arctan \left( \frac{L_x/2 + x}{L_z/2 - z} \right) + \arctan \left( \frac{L_x/2 + x}{L_z/2 + z} \right) \right].
\]

Note that \(f(x) = 1\) everywhere inside the detector volume, except within a distance \(\Delta x \sim q^{-1} \sim m_a^{-1}\) from the surface. Eq.(7) shows that the most important finite size effects occur when \(\cos(qL_z/2) \neq 0\). Figures 2 and 3 show the \(z\) and \(x\) dependences of \(B_x\) when \(|\cos(qL_z/2)| = 1\), the least favorable case. Both the exact (Eq.(6)) and the approximate (Eq.(7)) expressions for \(B_x(z, x)\) are plotted. There is excellent agreement between the two. Of course, the exact curve displays the kinks in \(B_x\) which result from the discreteness of the current distribution, whereas the other curve is smooth.

Henceforth, we will make the simplifying assumption that \(B_x\) depends only on \(z\). For \(m_a L_x, m_a L_y \gg 1\), Eq.(4) becomes

\[
\sigma = \sigma_0 \sum_{n_z=\pm 1} \left| \frac{1}{L_z} \int_{-L_z/2}^{L_z/2} dz \frac{B_x(z)}{B_0} e^{iE_a(n_z-\beta_0)z} \right|^2,
\]

Note that summing over \(n_x\) and \(n_y\) in Eq.(4) is unnecessary.

3
where we have dropped terms of $O(\beta_a^2)$, and $\sigma_0$ is defined by
\[
\sigma_0 = \frac{1}{4\beta_a} \left( \frac{\alpha g_a}{\pi f_a} \right)^2 L_x L_y L_z^2 B_0^2.
\] (10)

For $B_x(z) = -B_0 \cos(qz)$,
\[
\sigma = \frac{\sigma_0}{L_z^2} \left\{ \left( \frac{\sin \left\{ \frac{L_z}{2} [E_a(1 + \beta_{az}) - q] \right\}}{E_a(1 + \beta_{az}) - q} \right)^2 + \left( \frac{\sin \left\{ \frac{L_z}{2} [E_a(1 - \beta_{az}) - q] \right\}}{E_a(1 - \beta_{az}) - q} \right)^2 \right\},
\] (11)

where only the resonance terms are kept. The purpose of the detector is to search for the axion signal resonance by tuning the wave number $q$ of the current configuration. Eq.(11) shows that the bandwidth of the detector is $\Delta k_d \simeq 2/L_z$. On the other hand, the width of the axion signal in the same variable is $\Delta k_a \simeq 2 \times 10^{-3} m_a$. Provided that the axion signal falls entirely within the bandwidth of the detector ($|q - m_a \pm 10^{-3} m_a| < 1/L_z$), the power into the detector from $a \rightarrow \gamma$ conversion on resonance ($q \simeq m_a$) is
\[
P = \sigma \beta_a \rho_a = \frac{1}{8} \left( \frac{\alpha g_a}{\pi f_a} \right)^2 V L_z B_0^2 \rho_a \]
\[= 2 \times 10^{-21} \text{Watt} \left( \frac{V L_z}{\text{meters}^4} \right) \left( \frac{B_0}{8 \text{Tesla}} \right)^2 \left( \frac{m_a}{10^{-3} \text{eV}} \right)^2 \]
\[\times \left( \frac{\rho_a}{0.5 \times 10^{-24} \text{gr/cm}^3} \right) \left( \frac{g_\gamma}{0.36} \right)^2,
\] (12)

where $V = L_x L_y L_z$. This power must be collected and brought to the front end of a microwave receiver. Let $\zeta$ be the efficiency with which this can be done and let $T$ be the total (physical plus electronic) noise temperature of the receiver. The signal to noise ratio over the frequency bandwidth $\Delta f_s = 10^{-6} m_a/2\pi$ of the axion signal is
\[
\frac{s}{n} = \frac{\zeta P}{T} \sqrt{\frac{t}{\Delta f_s}},
\] (13)

where $t$ is the measurement integration time. The search can be carried out over the whole frequency bandwidth of the detector ($\Delta f_d \simeq 2/(2\pi L_z)$) simultaneously. Thus the search rate for a given signal to noise ratio is
\[
\frac{\Delta m_a}{t} = 2\pi \frac{\Delta f_d}{t} = 2\pi \frac{\Delta f_s}{\Delta f_d} \frac{\Delta f_d}{\Delta f_s} = 2\pi \left( \frac{n}{s} \right)^2 \left( \frac{\zeta P}{T} \right)^2 \frac{2 Q_a}{m_a L_z}
\]
\[ Q_a = 10^6 \]

where \( Q_a \) is the “quality factor” of the axion signal, i.e., the ratio of energy to energy spread of galactic halo axions.

Assuming that \( T \simeq 10\text{K} \) is realistic, Eq.(14) tells us that a detector of linear dimensions of order a couple of meters is required to search at a reasonable rate near \( m_a = 10^{-3}\text{eV} \). If \( L_x = L_y = L_z = 2\text{m} \) and \( d = 0.2\text{mm} \), there are \( 10^8 \) wires arranged in \( 10^4 \) planes of \( 10^4 \) wires each. To extend the search to smaller axion masses keeping a reasonably large search rate, successively larger (\( L \sim m_a^{-3/7} \)) but less fine grained (\( d \sim m_a^{-1} \)) detectors are needed. For a search near \( m_a = 10^{-5}\text{eV} \), assuming again \( T \simeq 10\text{K} \), the detector needs to be of order 10m in linear dimension but requires only \( 10^5 \) wires in a \( d = 2\text{cm} \) matrix.

We now turn to some practical concerns. The current configuration \( I_y(z) \) must be chosen such that its wave number \( q \) can be easily tuned. Since \( I_y(z) \) is periodic, the number of different-value currents needed is much smaller than the total number of wires in \( \hat{z} \) direction. If we replace the harmonic current function \( I_y(n_z) \) with a function having the same repeat length along \( z \) but with integer ratios of the currents, the number of current sources can be minimized by employing current dividers. Figure 4 shows the magnetic field produced by the square wave, triangular wave, and sine wave current configurations as functions of \( z \), given the same maximum current strength. Our numerical computation for the conversion cross section of axions into photons as a function of \( m_a \) shows the expected resonance peak (\( m_a = q \)) prominent for all three current configurations.

The radius \( b \) of the wires must be chosen neither too large nor too small. If \( b \) is too small, the magnetic field at the surface of the wires will exceed the critical field strength \( B_c \) for the breaking down of superconductivity even when the maximum average field \( B_0 \)
is still far below $B_c$. The contribution $B_b = I/(2\pi b)$ on the surface of a wire, which is due to the current $I$ in the wire itself, must be less than $B_c$ for all $I$ up to $I_0 = qd^2B_0$.

The resulting requirement on the wire radius is:

$$b > b_{\text{min}} = 9.5 \mu m \,(qd) \left( \frac{d}{0.2\text{mm}} \right) \left( \frac{B_0/B_c}{0.3} \right). \quad (15)$$

If the wire radius is sufficiently large compared with $b_{\text{min}}$, the magnetic field at the surface of a wire is dominated by the contribution from other wires (i.e., $B_0$), and $B_c$ constrains $B_0$ directly.

If $b$ is too large, the signal from $a \rightarrow \gamma$ conversion is lost to photon scattering by the wires. The photons emitted by axion decay all have momenta very nearly parallel to the $z$ axis and polarization perpendicular to the superconducting wires ($\vec{E} \perp \hat{y}$, see Eq.(3)).

The total scattering cross section per unit length of wire for such photons is

$$\sigma_T \simeq \frac{3\pi^2}{4} b(qb)^3.$$

where $q$ is the wavenumber of the photons. For $b = 10 \mu m$, $d = 0.2\text{mm}$, and $L = 2\text{m}$, the loss of the signal power at the very worst (when $qd = 1$) is less than 20%. The polarization with $\vec{E} \parallel \hat{y}$ is strongly reflected by the wires. If the typical deviation from parallelism in the orientation of individual wires in the matrix is $\Delta \theta$, the loss of signal power is on the order of $(\Delta \theta)^2$. This loss is less than that due to the scattering of photons by the wires if $\Delta \theta < 0.5 \times 10^{-2}$, which is easy to satisfy.

For $d = 0.2\text{mm}$ and $qd = 1$, $B_0 = 10$ Tesla implies $I_0 = 1591$ Ampere. For $L = 2\text{meters}$, the maximum force on a wire would be $\sim 3 \times 10^4\text{N}$; assuming $b = 10 \mu m$, this corresponds to a pressure of 0.25 GPa, safely below the strength limit of a number of casting resins. The requirements are similar for other values of $q$ and $d$.

Finally, plane-polarized, well-collimated photons will emerge from the ends of the detector in the $\pm \hat{z}$ direction. These can be focussed with a parabolic mirror or channeled
into a waveguide and brought to the input of a low-noise microwave receiver. The signal consists of noise (both emitted by the detector and generated by the receiver front end electronics) along with excess power resulting from axion conversion. As in the case of cavity detectors, the signal will be spectrum analyzed to search for the axion resonance as the detector’s operating frequency is swept. The electronics and signal analysis techniques would be similar to that of the cavity detectors.

In summary, we have described a detector for dark-matter axions that could be used to search the part of the allowed range of axion masses that is unreachable in the foreseeable future by cavity detectors.

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Figure Captions

Fig.1. Top and side views of the detector, showing the arrangement of wires.

Fig.2. $B_x(z, x)$ versus $z$ for $|x| = L_x/12$, $I = I_0 \sin(n_z dq)$. The spiked thin line is numerical and exact, while the smooth bold line is the analytical result of Eq.(7). $qd = \pi/20$, $L_x = L_z = 600d$.

Fig.3. $B_x(z, x)$ versus $x$ for $|z| = L_z/6$, $I = I_0 \sin(n_z dq)$. The spiked thin line is numerical and exact, while the smooth bold line is the analytical result of Eq.(7). $qd = \pi/20$, $L_x = L_z = 600d$.

Fig.4. $B_x(z)$ versus $z$ produced by the sine (middle bold line), square (outer thin line) and triangular (inner thin line) wave current configurations. $qd = \pi/20$, $L_x = L_z = 500d$. 