An independent limit on the axion mass from the variable white dwarf star R548

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Abstract. Pulsating white dwarfs with hydrogen-rich atmospheres, also known as DAV stars, can be used as astrophysical laboratories to constrain the properties of fundamental particles like axions. Comparing the measured cooling rates of these stars with the expected values from theoretical models allows us to search for sources of additional cooling due to the emission of weakly interacting particles. In this paper, we present an independent inference of the mass of the axion using the recent determination of the evolutionary cooling rate of R548, the DAV class prototype. We employ a state-of-the-art code which allows us to perform a detailed asteroseismological fit based on fully evolutionary sequences. Stellar cooling is the solely responsible of the rates of change of period with time (\dot{\Pi}) for the DAV class. Thus, the inclusion of axion emission in these sequences notably influences the evolutionary timescales, and also the expected pulsational properties of the DAV stars. This allows us to compare the theoretical \dot{\Pi} values to the corresponding empirical rate of change of period with time of R548 to discern the presence of axion cooling. We found that if the dominant period at 213.13 s in R548 is associated with a pulsation mode trapped in the hydrogen envelope, our models indicate the existence of additional cooling in this pulsating white dwarf, consistent with axions of mass $m_a \cos^2 \beta \sim 17.1$ meV at a $2\sigma$ confidence level. This determination is in agreement with the value inferred from another well-studied DAV, G117–B15A. We now have two independent and consistent estimates of the mass of the axion obtained from DAVs, although additional studies of other pulsating white dwarfs are needed to confirm this value of the axion mass.

Keywords: Stars: white dwarfs, stars: oscillations, stars: asteroseismology, stars: evolution, astroparticle physics, axions

1 Introduction

Axions are weakly interacting hypothetical particles. Their existence was proposed about 35 years ago to solve the so-called “strong CP problem” in quantum chromodynamics, that is the violation of charge-parity symmetry in strong interactions [1–3]. Comprehensive accounts of the properties of axions, and of the astrophysical and cosmological searches aimed at detecting these elusive particles, or at least constraining their properties, can be found in Refs. [4–6]. Besides its relevance for the standard model, axions are also natural candidates to be part of the non-baryonic dark matter of the Universe. Notwithstanding, their contribution to the non-baryonic dark matter content of the Universe depends on their mass, which determines the intensity of the coupling with matter. However, the theory that predicts the existence of these particles does not provide any indication of their mass. Thus, the mass of the axion has to be determined either directly using sophisticated experimental facilities, or indirectly employing the observed properties of astronomical objects [4, 7]. In particular, white dwarf stars — see Refs. [8–10] for recent and comprehensive reviews of their properties — are excellent candidates to test the existence of weakly interacting particles like axions [4, 11]. The reason for this is that their evolution is a simple and well studied gravothermal process of cooling that can be accurately measured using either their luminosity function, or by studying the secular variations of the periods of variable white dwarfs.

Of the two types of axion models proposed so far — the so-called KVSZ model [12, 13], and the DFSZ model [14, 15] — in this paper we will focus on the DFSZ axions, which couple
to electrons in addition to photons and hadrons. The coupling strength of DFSZ axions to electrons is defined through the dimensionless coupling constant $g_{ae}$, which is related to the mass of the axion, $m_a$, through the relation:

$$g_{ae} = 2.8 \times 10^{-14} \frac{m_a \cos^2 \beta}{1 \text{ meV}},$$

(1.1)

where $\cos^2 \beta$ is a free, model-dependent parameter that is usually set equal to unity. Thus, the value of $m_a$ determines how strongly DFSZ axions couple to electrons, and hence how large the axion emissivity is. At the typical temperatures and densities found in the cores of white dwarfs, the emission of DFSZ axions takes place from the deepest regions of these stars, mostly through bremsstrahlung emission [16], the production rate being [17, 18]:

$$\epsilon_a = 1.08 \times 10^{23} \frac{g_{ae}^2 Z^2}{4\pi} T_4^4 F(T, \rho) \ [\text{erg/g/s}].$$

(1.2)

Since axions can (mostly) freely escape from the interior of white dwarfs, their existence would accelerate the cooling, with more massive axions producing a larger increase in the rate of cooling. Fortunately, some white dwarfs pulsate, and their pulsational properties depend sensitively on the rate of cooling. Specifically, there are two competing internal evolutionary processes that govern the rate of change of the pulsation period with time ($\dot{\Pi}$) for a single mode in pulsating white dwarfs. Cooling of the star increases the periods as a result of the increasing radius of the degeneracy boundary, whereas residual gravitational contraction decreases the periods [19]. Contraction is still significant for the hotter classes of pulsating white dwarfs, such as the so-called DOVs, but it has been concluded [20] that for the class of $g$(gravity)-mode pulsating white dwarfs with hydrogen atmospheres — known as DAVs or ZZ Ceti stars — the evolutionary $\dot{\Pi}$ is dictated solely by the rate of cooling. Therefore, a comparison of the theoretical models of pulsating white dwarfs with the observations allows us to derive useful constraints on the value of the mass of the axion. Moreover, ZZ Ceti stars, with over 148 members known [21], provide us with a relatively large observational data base to perform such studies.

The star G117−B15A is the benchmark member of the ZZ Ceti class, its pulsation periods ($\Pi$) being 215.20 s, 270.46 s and 304.05 s [22]. The most recent determination of the rate of change of the period at 215 s of this star is $\dot{\Pi} = (4.19 \pm 0.73) \times 10^{-15}$ s/s [23]. The possibility of employing the measured period drift of the largest amplitude mode of G117−B15A to derive a limit on the mass of axions was brought up almost two decades ago [11]. The evolution of DA white dwarf models with and without axion emission was considered, and the theoretical values of $\dot{\Pi}$ for increasing masses of the axion were compared with the observed rate of change of period with time of G117−B15A. Employing a semi-analytical treatment, $m_a \cos^2 \beta = 8.7$ meV was obtained. Subsequently, a detailed asteroseismological model for G117−B15A revealed $m_a \cos^2 \beta \leq 4.4$ meV [24]. A few years later, an upper limit of 13.5 − 26.5 meV was derived for the axion mass using an improved asteroseismological model for G117−B15A, and a better treatment of the uncertainties involved [25]. Armed with the latest asteroseismological model [26] and the most recently measured value of $\dot{\Pi}$ [23], the problem of determining the mass of the axion using white dwarf asteroseismology was revisited [27]. The 215 s period in white dwarf models of the star G117−B15A was associated with a pulsation mode trapped in the hydrogen envelope. The models strongly indicated the existence of an additional mechanism of energy loss in this pulsating white dwarf, consistent with axions of mass $m_a \cos^2 \beta \sim 17.4$ meV.
The star R548 — that is, ZZ Ceti itself, with an effective temperature $T_{\text{eff}} = 11990 \pm 200$ K, and surface gravity $\log g = 7.97 \pm 0.05$ [28] — is another DAV star that has been studied for the last few decades. Both G117−B15A and R548 display similar pulsational properties, share periods near 213−215 s and 272−274 s, have similar effective temperatures and masses, and are expected to be similar in structure. Since the discovery of R548 [29], there have been multiple attempts to measure the drift rate of its pulsation period at $\sim 213.13$ s [30–32]. The rate of change of this period with time has recently been measured for the very first time [33], and forms our underlying motivation to determine another independent constraint on the axion mass by using the best asteroseismological model for R548 [26].

The paper is organized as follows. In Sect. 2 we give a brief account of the measurements of the rate of change of period with time for the 213.13 s period in R548, while in Sect. 3 we succinctly present the asteroseismological model of R548. In Sect. 4 we derive a new constraint on the axion mass. Finally, in Sect. 5 we summarize our findings and present our concluding remarks.

## 2 R548: Measuring the evolutionary cooling rate $\dot{\Pi}$

R548 was the third DAV star to be discovered [29] after HL Tau76 [34] and G44−32 [35], but it was chosen to be the class prototype as the first DAV to be deciphered. This star exhibits five independent pulsation eigenmodes, two of which are triplets. Their periods and corresponding (amplitudes), measured using the highest signal-to-noise data set from 2007, are as follows: 186.86 s (0.5 mma), 212.77 s (4.3 mma), 212.95 s (1.2 mma), 213.13 s (6.6 mma), 274.25 s (4.1 mma), 274.52 s (1.2 mma), 274.78 s (3.1 mma), 318.08 s (0.8 mma), and 333.64 s (0.8 mma). The first attempt to measure the rate of change of period with time of R548 was based on high-speed photometry acquired from 1970 to 1978 [30]. However, only an upper limit to the drift rate of the 213.13 s period of $2 \times 10^{-13}$ s/s could be found. Nevertheless, at that time this measurement was a considerable breakthrough, as R548 was considered to have the most stable period ever measured in a variable star at visual wavelengths.

The quest to measure the rate of change of period with time for R548 continued for several decades. Analyzing observations that spanned 31 years (from 1970 to 2001) an upper limit constraining the evolutionary drift rate of the 213 s period was established in 2003 [32]. This upper limit turned out to be $\dot{\Pi} \leq (5.5 \pm 1.9) \times 10^{-15}$ s/s after including the proper motion correction. A few years later, a value of $(2.1 \pm 1.2) \times 10^{-15}$ s/s was established [36] based on 37 years of observations (from 1970 to 2007). With additional data, the values of the drift rate have fluctuated up and down but the uncertainty of the determination has always reduced monotonically with the square of the elongating timebase. Very recently, a reliable measurement of the rate of change of period with time for the 213.13 s period in R548 has been finally obtained, $\dot{\Pi} = (3.3 \pm 1.1) \times 10^{-15}$ s/s, using 41 years of time-series photometry from 1970 to 2011 [33]. During the last ten years, the $\dot{\Pi}$ values so obtained have been consistent with the claimed measurement of $3.3 \times 10^{-15}$ s/s within uncertainties. This improves our confidence in utilizing the measured $\dot{\Pi}$ to constrain the mass of the axion. Additionally, it is worth mentioning that in the most recent study of pulsational properties of R548 [33] a measurement of the cooling rate for any other period of the 213 s triplet, or for any other mode, has not been claimed. Although in this work the $\dot{\Pi}$ values for the 212.78 s period have been determined at each juncture, these values show fluctuations that are significantly larger than the 1$\sigma$ uncertainties. Consequently, in the absence of a reliable
Table 1. Characteristics of R548 derived from a spectroscopic analysis and results of the best asteroseismological model. The quoted uncertainties in the asteroseismological model are the internal errors of our period-fitting procedure.

| Quantity                     | Spectroscopy | Asteroseismology |
|------------------------------|--------------|------------------|
| $T_{\text{eff}}$ [K]         | 11 990 ± 200 | 11 627 ± 390     |
| $M_*/M_\odot$               | 0.590 ± 0.026| 0.609 ± 0.012    |
| log $g$                      | 7.97 ± 0.05  | 8.03 ± 0.05      |
| log($R_*/R_\odot$)          | —            | −1.904 ± 0.015   |
| log($L_*/L_\odot$)          | —            | −2.594 ± 0.025   |
| $M_{\text{He}}/M_*$         | —            | 2.45 × 10$^{-2}$ |
| $M_\text{H}/M_*$            | —            | (1.10 ± 0.38) × 10$^{-6}$ |
| $X_C, X_O$ (center)         | —            | 0.26$^{+0.22}_{-0.09}$, 0.72$^{+0.09}_{-0.22}$ |

determination of $\dot{\Pi}$ for the 212.78 s period, in this paper we will only consider the measured $\dot{\Pi}$ for the 213.13 s.

The $\dot{\Pi}$ value obtained for R548 is completely consistent within uncertainties with the corresponding drift rate of the 215 s period of G117–B15A $\dot{\Pi} = (4.19 ± 0.73) \times 10^{-15}$ s/s. It is also compatible with the expected cooling of a $\sim 0.6 M_\odot$ white dwarf harboring a carbon-oxygen core. Consequently, the 213.13 s period of R548 can be used to derive constraints on the mass of the axion — see Sect. 4.

3 Asteroseismological model for R548

We have recently performed a detailed asteroseismological analysis of R548 [26] — among 43 other bright DAVs including G117–B15A — using a very large grid of DA white dwarf evolutionary models characterized by consistent chemical profiles for both the core and the envelope, and covering a wide range of stellar masses, thicknesses of the hydrogen envelope and effective temperatures. What is more important, these models were self-consistently generated with the LPCODE evolutionary code [37]. In particular, the evolutionary calculations were carried out from the ZAMS, through the thermally-pulsing and mass-loss phases on the AGB, and finally to the domain of planetary nebulae and white dwarfs. The effective temperature, the stellar mass and the mass of the H envelope of our DA white dwarf models vary within the ranges $14 000 \gtrsim T_{\text{eff}} \gtrsim 9 000$ K, $0.525 \lesssim M_* \lesssim 0.877 M_\odot$, $−9.4 \lesssim \log(M_\text{H}/M_*) \lesssim −3.6$, where the value of the upper limit of $M_\text{H}$ depends on $M_*$ and is fixed by the prior evolution. For the sake of simplicity, and also for coherence, the mass of the helium layer was kept fixed at the value predicted by the evolutionary calculations for each sequence [26].

The theoretical periods were assessed by means of a state-of-the-art pulsation code [38]. To find an asteroseismological model for R548, we searched for the model that minimizes a quality function that measures the distance between the theoretical ($\Pi^*$) and the observed ($\Pi^o$) periods [26]. A single best-fit model with the characteristics shown in Table 1 was found. The second column of Table 1 contains the spectroscopically determined values of $T_{\text{eff}}$ and log $g$ of R548 [28], and the stellar mass [26]. The parameters characterizing the asteroseismological model are shown in column 3. In Table 2 we compare the observed and the theoretical periods — along with the corresponding mode identification — and the expected rates of change of the periods with time. It is worth mentioning that the model
Table 2. The observed and theoretical periods of R548, along with the corresponding mode identification, and the subsequent rates of change of period with time (computed without including axion cooling). For the 213 s and 274 s triplets, we only list the periods corresponding to the central component.

| Π^o (s) | Π^t (s) | ℓ | k | Π^o (10^{-15}s/s) | Π^t (10^{-15}s/s) |
|---------|---------|---|---|----------------|----------------|
| 186.86  | 187.59  | 1 | 1 | —               | 2.51           |
| 212.95  | 213.40  | 1 | 2 | 3.3 ± 1.1       | 1.08           |
| 274.52  | 272.26  | 1 | 3 | —               | 3.76           |
| 318.08  | 311.36  | 2 | 8 | —               | 6.32           |
| 333.64  | 336.50  | 2 | 9 | —               | 8.80           |

nearly reproduces the observed periods, although the modes with periods at 318.08 s and 333.64 s remain relatively poorly matched. The most relevant result of Table 2 is that the observed rate of change of the 213 s period ($3.3 \times 10^{-15}$ s/s) is $\sim 3$ times larger than the theoretically expected value ($1.08 \times 10^{-15}$ s/s). Since the rate of change of period with time of this mode reflects the evolutionary timescale of the star, then the disagreement between the observed and theoretical values of $\dot{\Pi}$ is suggestive that R548 could be cooling faster than the cooling rate predicted by the standard theory of white dwarf evolution.

We note that in a previous work [39] it was found an average rate of change of the period with time of $(2.91 \pm 0.29) \times 10^{-15}$ s/s for the 213 s period of R548. However, this determination was based on identifying the 213 s period with a radial order $k = 1$ mode, instead of $k = 2$, which is our best-fit solution. Similar to our findings for the case of G117−B15A, the rate of change of period with time for the $\ell = 1$, $k = 2$ mode in our asteroseismological model is substantially smaller than for the dipole modes with $k = 1$ and 3, and much smaller than for the quadrupole ($\ell = 2$) modes with $k = 8$ and 9. We expect that the $k = 2$ mode has the smallest $\dot{\Pi}$ because it is a mode trapped in the outer hydrogen envelope in our model white dwarf. This also applies to the 215 s mode of G117−B15A [27]. Mode trapping reduces the rate of change of period with time by up to a factor of $\sim 3$ if the mode is trapped in the outer hydrogen envelope, because gravitational contraction — that is still appreciable in these regions — reduces the net increase in period from cooling [40]. Since the $k = 2$ mode is somewhat affected by gravitational contraction, it is less sensitive to the evolutionary cooling. However, the change of the period due to the increasing degeneracy resulting from cooling is still larger than the change due to residual contraction, and so, $\dot{\Pi} > 0$. As a result, the period of the $k = 2$ mode is still sensitive to cooling and will allow us to constrain the mass of the axion.

An important point in our analysis is to estimate the uncertainties affecting the value of $\dot{\Pi}$ for the $k = 2$ mode in our asteroseismological model, because they directly translate into uncertainties in the derived axion mass. Here, we will adopt the same approach we used previously for G117−B15A [27]. We estimate an uncertainty $\sim 0.03 \times 10^{-15}$ s/s for $\dot{\Pi}^t$ of the 213 s period due to the uncertainties in the $^{12}$C(α, γ)$^{16}$O reaction rate. Since the $k = 2$ mode is trapped in the hydrogen envelope of our model white dwarf, the precise abundances of carbon and oxygen in the core do not significantly affect the calculated period and the corresponding rate of change of period with time. We also estimate an uncertainty in the theoretical $\dot{\Pi}$ of $\sim 0.06 \times 10^{-15}$ s/s due to the internal errors in fitting the period. Fortunately,
these uncertainties are small and do not contribute significantly to the uncertainties in the derived axion mass. As shown below, the uncertainties in the inferred value of \( m_a \) are dominated by the uncertainties in the observed rate of change of period with time of the 213 s period.

4 Axion emission and inference of the axion mass

We have found that the theoretically expected rate of change of period with time for the \( k = 2 \) mode is distinctly smaller than the value measured for R548, suggesting the existence of some additional cooling mechanism in this star. Here, we assume that this additional cooling can be entirely attributed to the emission of axions. Similar to it was done in our previous papers [24, 27], we have computed a set of DA white dwarf cooling sequences incorporating the emission of axions. This has been done considering different axion masses.
and the same structural parameters \((M_*, M_\odot)\) as the asteroseismological model in Table 1. We have adopted a range of values for the mass of the axion \(0 \leq m_a \cos^2 \beta \leq 30\, \text{meV}\), and also employed the most up-to-date axion emission rates \([17, 18]\). The evolutionary calculations including the emission of axions were started at evolutionary stages long before the ZZ Ceti phase to ensure that the cumulative effect of axion emission has reached an equilibrium value before the models reach the effective temperature of R548.

We found that the pulsation periods for the modes with \(\ell = 1, k = 1, 2, 3, \) and \(\ell = 2, k = 8, 9\) of the asteroseismological model experience negligible variations with increasing values of the mass of the axion, \(m_a\). This is because the periods themselves are not appreciably affected by the small changes in the structure of the white dwarf produced by the emission of axions. In contrast, the rates of the change of period with time are strongly affected by axion emission, substantially increasing for increasing values of \(m_a\). In particular, for the mode of interest \((\ell = 1, k = 2)\) the rate of change of period increases by a factor of order of 10 when \(m_a\) goes from 0 to 30 meV. This convincingly shows that, even though this mode is less sensitive to the evolutionary cooling of the star compared to the modes with \(k = 1\) and \(k = 3\), it is still a useful tool to constrain the mass of the axion.

In Fig. 1 we display the theoretical value of \(\dot{\Pi}\) corresponding to the period \(\Pi = 213\, \text{s}\) for increasing values of the axion mass (red solid curve). The dashed curves embracing the solid curve represent the uncertainty in the theoretical value of \(\dot{\Pi}\), \(\epsilon_{\dot{\Pi}} = 0.09 \times 10^{-15}\, \text{s/s}\). This value has been obtained considering the uncertainty introduced by our lack of precise knowledge of the \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction rate and by the internal uncertainties in fitting the asteroseismological model. Additionally, we assume that both sources of uncertainties do not depend on the mass of the axion. The horizontal (blue) solid line indicates the observed value of \(3.3 \times 10^{-15}\, \text{s/s}\), whilst its corresponding 1\(\sigma\) and 2\(\sigma\) uncertainties \([33]\) are shown using dashed and dotted lines, respectively. If one standard deviation from the observational value is considered, we conclude that the mass of the axion is \(m_a \cos^2 \beta = (17.1^{+4.3}_{-5.8})\, \text{meV}\). This value is in complete agreement with the axion mass inferred using G117–B15A, \(m_a \cos^2 \beta = (17.4^{+2.4}_{-2.7})\, \text{meV}\) \([27]\). Note, however, that within 2\(\sigma\), our results are compatible with \(m_a = 0\).

5 Discussion and conclusions

In this paper we have derived an improved value of the mass of axions, assuming that the enhanced rate of cooling of the pulsating white dwarf R548 is entirely due to the emission of axions. In doing so we have employed a detailed asteroseismological model for R548 — the prototype of the DAV stars — obtained using full evolutionary calculations of DA white dwarf models \([26]\). Our calculations used the recent determination of the rate of change of period with time for the largest amplitude mode of this star \([33]\). We found that if the 213 s period in R548 is associated with a mode trapped in the H envelope, our theoretical models indicate the existence of an additional cooling mechanism in this pulsating white dwarf, consistent with axions of mass \(m_a \cos^2 \beta = (17.1^{+4.3}_{-5.8})\, \text{meV}\) at the 1\(\sigma\) level, or \(m_a \cos^2 \beta = (17.1^{+7.2}_{-17.1})\, \text{meV}\) at the 2\(\sigma\) level. Equivalently, in terms of the constant coupling, this mass of the axion can be expressed as \(g_{ae} = 4.8 \times 10^{-13}\). Our value for the axion mass is in excellent agreement with that inferred from the pulsating white dwarf G117–B15A, but considerably larger than that obtained from the hot branch of the white dwarf luminosity function, \(m_a \sim 5\, \text{meV}\) \([41, 42]\). Indeed, a detailed analysis of the hot branch of the white dwarf luminosity function suggests that values larger than 10 meV can be safely excluded. However, at the 2\(\sigma\) level our determination is consistent with that obtained using the white dwarf luminosity function.
Moreover, both techniques agree in finding that an anomalous rate of cooling of white dwarfs in this luminosity range exists. If this anomalous rate of cooling can be entirely attributed to the emission of axions deserves further scrutiny. It is, nevertheless, worth stressing that both methods are complementary and equally sensitive to the emission of axions in white dwarfs, and that both suggest that axions do exist, with a mass on the order of a few meV.

Another bound on the axion-electron coupling comes from helium ignition in low-mass stars on the red giant branch. Using this method an upper limit $m_a \cos^2 \beta \lesssim 9$ meV was obtained \cite{43}. The axion mass derived from pulsating white dwarfs is substantially larger than this upper limit, and axions with the mass derived in this paper would modify the tip of the red giant branch, although the uncertainties involved in the method of \cite{43} are comparable to those affecting our procedure. Finally, it is worth mentioning that axions with masses in the range of values derived here would provide a strong energy loss channel for core collapse supernovae and neutron stars \cite{44, 45}. However, we emphasize that again our results are compatible with these upper bounds at the $2\sigma$ level.

With the inference of the axion mass from R548 reported in this paper, we now have two independent determinations of the mass of the axion from two different pulsating white dwarfs, which agree with each other. These determinations imply that axions couple with electrons, and therefore must be of the DFSZ type. Although these results are encouraging, it is worth considering that both stars have comparable observational properties — with similar effective temperatures and gravities — and also share very similar pulsational characteristics, i.e., the periods of their dominant modes. In addition, the rates of change of period with time used to derive the mass of the axion are associated with the same $\ell = 1, k = 2$ pulsation mode in both stars. Therefore, we can expect to yield the same value of the mass of the axion based on the $\Pi$ of the same eigenmode in two similar stars. Hence, it would be desirable to have a measurement of the rate of change of period with time for another DAV star with pulsational and spectroscopic characteristics very different from those of G117–B15A and R548. Examples of such white dwarfs are L19-2 \cite{46} and G226–29. Also, a measurement of the rate of change of period with time for white dwarfs with hydrogen deficient envelopes — like EC20058–5234 \cite{47} and KIC 8626021 \cite{48} — will allow us to obtain another independent bound on the axion mass.

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