LOCAL AND GLOBAL PROPERTIES OF THE WORLD

Key Words: Noncommutative geometry, Quantum gravity, Ultimate physical theory.

Abstract. The essence of the method of physics is inseparably connected with the problem of interplay between local and global properties of the universe. In the present paper we discuss this interplay as it is present in three major departments of contemporary physics: general relativity, quantum mechanics and some attempts at quantizing gravity (especially geometrodynamics and its recent successors in the form of various pregeometry conceptions). It turns out that all big interpretative issues involved in this problem point towards the necessity of changing from the standard space-time geometry to some radically new, most probably non-local, generalization. We argue that the recent noncommutative geometry offers attractive possibilities, and give us a
conceptual insight into its algebraic foundations. Noncommutative spaces are, in general, non-local, and their applications to physics, known at present, seem very promising. One would expect that beneath the Planck threshold there reigns a “noncommutative pregeometry”, and only when crossing this threshold the usual space-time geometry emerges.

1 Introduction

Editors of a book devoted to the Mach Principle write in the Introduction:

It is often not sufficiently appreciated how kind nature has been in supplying us with ‘subsystems’ of the universe which possess characteristic properties (literally in the sense ‘proper to the system’) that can be described and measured almost without recourse to the rest of the universe. (Barbour and Pfister 1995)

Physics started its triumphant progress when people like Galileo and Newton succeeded in isolating free fall of a stone from the network of interactions shaping the structure of the world. On the other hand, the question imposes itself: Is the “whole of the universe” a sum of its parts (or aspects) or perhaps “something more”, something that cannot be reconstructed by investigating only “local details”? It seems that the essence of the method of physics is inseparably connected with the problem of interplay between local and global aspects of the world’s structure. The aim of the present paper is to discuss this interplay as it reveals itself in three major departments of contemporary physics: general relativity, quantum mechanics and some attempts at quantizing gravity.

The very notion of local property is strictly connected with the concept of point and its neighbourhood. No wonder, therefore, that our analyses will focus on space-time structures. General relativity is par excellence a theory of space-time. Although its field equations, being differential equations, are defined locally, their local character is of a very peculiar nature: since Einstein’s equations themselves determine the structure of space-time on which they act, they are intimately connected with the topological structure of the
underlying manifold which in turn should be taken into account when solving
the boundary condition problem. All these questions have clearly a global
significance. We deal with them in Section 2.

Non-relativistic character of quantum mechanics manifests itself (among
others) in a strong asymmetry of space and time (in quantum mechanics there
is a position operator but there is no time operator). Some attempts to cure
this situation are physically interesting but, at least for the time being, math-
ematically non-satisfactory. The picture becomes even more complicated if
one takes into consideration the fact that two quantum systems (particles)
somehow know of each other, independently of the distance separating them,
as long as they were “correlated” in the past. This “non-separability” effect
strongly suggests that at the more fundamental level the ordinary space-time
geometry breaks down and some new aspects of the local-global interaction
should be expected. This set of problems is discussed in Section 3.

One of the most ambitious attempts to reconstruct the space-time geo-
metry (or its substitute) at the fundamental level was known under the name of
gemetrodynamics (Wheeler 1968, DeWitt 1967). The idea consisted in com-
bining together the spatio-temporal description of general relativity with the
probabilistic formalism of quantum mechanics. The set of all 3-geometries
.called superspace) forms the arena of this fluctuating geometry (or a quan-
tum foam), and the probability for a given 3-geometry to be the actual
state of the universe should be computed from the so-called Wheeler-DeWitt
equation. When this idea had met serious difficulties Wheeler (1980) pro-
posed a new program to recover the macroscopic space-time from what he
called pregeometry, a stuff of physics at the fundamental level. Various enti-
ties (shapeless collection of points, calculus of propositions, elementary acts
of measurements) were proposed as candidates for pregeometric elements.
These rather vague ideas gave the beginning to a number of mathemati-
cally sophisticated models. The situation in this field is critically reviewed
in Section 4.

The above signalled attempts at penetrating the fundamental level of
physics suggest that at this level the concepts of points and time instants
loose their usual meaning and should be replaced by some other mathem-
atical structure. One such mathematical structure has recently received the
growing interest, namely the so-called noncommutative geometry. It is a
vast generalization of the standard differential geometry allowing one to in-
vestigate spaces which so far were regarded as strongly pathological (e. g.

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non-Hausdorff spaces). This is possible owing to the astonishing parallelism between geometric and algebraic methods (discovered already by Descartes). It turns out that this parallelism can be extended to noncommutative algebras. In spite of the fact that noncommutative spaces are, in principle, purely global constructs, in which the concepts of points and their neighbourhoods lose their usual meaning, the authentic dynamics can be done on them (in terms of derivations of certain algebras as counterparts of the usual notion of vector fields). Although applications of noncommutative geometry to physics are still at their preliminary stage, the obtained results are very promising, and one branch of noncommutative geometry, the theory of quantum groups, is now in the focus of interest of many theorists. There are reasons to believe that at the fundamental level it is a non-local physics (based on noncommutative geometry) that governs the universe, and only above the Planck threshold the ordinary (commutative) space-time geometry emerges. In Section 5 we analyse the possibility of doing geometry without local concepts. Applications of such a geometry to fundamental physics are also briefly reviewed.

In Section 6 we comment on a philosophical significance of the above analyses.

2 Local and Global Aspects of the World in General Relativity

2.1 Mach’s Principle and General Relativity

As it is well known, Einstein, in his way towards the theory of general relativity, was greatly influenced by the set of ideas he read out of Mach’s writings, and which were called by him Mach’s Principle. Roughly speaking, Mach’s Principle asserts that physical properties, such as motion, inertia, centrifugal forces, must be fully determined by the global structure of the universe (distribution of masses in space). The following passages from Mach’s Science of Mechanics are often quoted as expressing this doctrine:

The universe is not twice given, with an earth at rest and an earth in motion; but only once, with relative motions, alone determinable (...) The principles of mechanics can, presumably,
by so conceived, that even for relative rotations centrifugal forces arise.

Newton’s experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative motion with respect to the mass of the earth and other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick (…)

When, accordingly, we say that a body preserves unchanged its direction and velocity in space, our assertion is nothing more or less than an abbreviated reference to the entire universe (…) (Mach 1960)

There are heated discussions, lasting to the present day (see Barbour and Pfister 1995), as to whether, or to what extent, Mach’s Principle has been incorporated onto general relativity. Since the outcome of these discussions depends on what does one precisely mean under the name of Mach’s Principle (there are many its formulations and some of them are rather fuzzy), we shall not try to multiply the possible answers. Instead, we shall adopt another strategy. There is no doubt that general relativity exhibits a subtle interplay of local and global properties of the universe, and that this interplay is encoded in the mathematical structure of this physical theory. Our goal will be to analyse the mathematical structure of general relativity in order to disentangle from it information about the interaction of local and global properties of the world.

2.2 The Structure of Field Equations

Field equations of general relativity are the result of the encounter of two powerful Einstein’s ideas. The first idea was nicely encapsulated by Hermann Weyl in his known saying: “space tells matter how to move, and matter tells space how to curve”. This, if suitably understood, is obviously a postulate concerning the interplay (a kind of feed-back) of the global properties of the world (structure of space or space-time, large scale distribution of matter) and its local properties (local curvature, description of motion with respect
to a local reference frame). The second idea is that of geometrization of gravity, i.e. of a “mapping of all the properties of the gravitational force and its influence upon physical processes onto the properties of a Riemann space” (Stephani 1982, p. 82). Of course, both ideas are not quite distinct: matter can tell space how to curve only if some physical processes have been “mapped” into the geometry of space.

Einstein’s field equations are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 3\kappa T_{\mu\nu}$$

where the left hand side is purely geometric (we keep the cosmological term $\Lambda g_{\mu\nu}$ for generality reasons) and the energy-momentum tensor $T_{\mu\nu}$ on the right hand side of these equations describes all forms of energy which can produce a gravitational field. The above field equations constitute a non-linear system of ten partial differential equations for determining ten components $g_{\mu\nu}$ of the metric tensor which are interpreted as gravitational potentials.

Einstein’s equations have a few properties which are important from our point of view.

First of all, even if we correctly choose the initial conditions these equations have no unique solution, since it is always possible to perform arbitrary coordinate transformations which do not influence physical meaning of a solution (technically, the ten above equations are not independent since the so-called (contracted) Bianchi identities must be satisfied).

Moreover, the field equations are not defined on an a priori given metric space. More precisely, the vanishing of divergence of the left hand side of Einstein’s equations enforces the vanishing of divergence of the energy momentum tensor $T_{\mu\nu}$ (this fact is physically interpreted as the local conservation law). But in order to compute the divergence of $T_{\mu\nu}$ one must know the metric components $g_{\mu\nu}$. This is not a vicious circle as it could look at first sight, but a deep aspect of the interplay of local and global properties of the world as they are encoded in the structure of the field equations.

The above property is strictly connected with the non-linearity of Einstein’s equations. Owing to it the combined gravitational effect of two bodies is not equal to the sum of the effects of each of these two bodies separately: the interaction of these two bodies with each other and with the generated gravitational field gives an essential contribution to the final effect. To see what does happen, one often uses the method of successive approximations:
one assumes that the space-time geometry is determined by a “part” of the energy-momentum tensor. The “rest” of it is called the test body; it is affected by the gravitational filed, but it does not contribute to it. However, one should remember that this is only an approximation. In fact, the universe as modelled by Einstein’s equations is a non-linear holistic system. This is clearly seen in the problem of defining gravitational energy in the framework of general relativity. It is typically a non-local entity: “gravitational potential energy contributes (negatively) non-locally to the total energy, and gravitational waves can carry (positive) non-local energy away from a system” (Hawking and Penrose 1996, p. 72).

In the next Subsection we shall discuss some local and non-local properties as they appear in general relativity.

2.3 Local and Global Problems in General Relativity

Another crucial property of the Einstein field equations is that they are hyperbolic partial differential equations (see, for instance, Choquet-Bruhat 1968). This property is closely related to the fact that the metric $g$, which is to be determined by Einstein’s equations, is a Lorentz metric (with the signature $-\,+,\,+,\,+$) rather than the more usual Riemann metric (with the signature $+\,,\,+,\,+,\,+$). Owing to this property space-time of general relativity (i.e. space-time the metric tensor of which satisfies Einstein’s field equations) is locally the Minkowski (or pseudoriemannian) space-time rather than the more standard Euclidean one. This is a strong constraint on the local structure of space-time coming from the very nature of (pseudo)Riemannian space: the tangent space at every point of the (pseudo)Riemannian space must be flat (pseudo)Euclidean independently of the global topological or metric structure of a given space.

This simple geometric property has important consequences for the physical interpretation. It is a geometric counterpart of Einstein’s Equivalence Principle: the fact that space-time is locally always flat (up to any desired precision) means that locally the gravitational field can always be transformed away, and consequently that the special theory of relativity is locally always valid.

However, the interaction of any locality with the global structure of space-time is not trivial. Only in the case, when the curvature of space-time vanishes, localities simply “add together” to form the Minkowski space-time (but
even in this case one can change the global topology by gluing together or cutting off certain parts of space-time).

In the next simple cases of space-times with constant curvature, or with space-sections of constant curvature, interesting phenomena can arise, such as the existence of closed timelike curves (in space-times with constant curvature) or light cones starting to reconverge (in space-times with space-sections of constant curvature). The study of the global (or large scale) structure of space-time has led to the formulation of many problems, for instance:

The problem of the chronological and causal structures of space-time. Two events \( p \) and \( q \) in space-time are said to be *chronologically* or *causally* related if they can be joined by an oriented (piece-wise) smooth timelike or non-spacelike curve from \( p \) to \( q \), respectively. Roughly speaking, a net of all such curves joining all possible events in space-time forms *chronological* and *causal structures* of space-time. The study of these two structures, interesting in itself, is an efficient tool in investigating topological and other global properties of space-times (Carter 1971; Hawking and Ellis 1973, chapter 6; Beem and Ehrlich 1981, chapter 2; Joshi 1993, chapter 4). It is interesting to notice that the causal structure of space-time is closely connected with the existence of a global time in the universe (i.e. time which would measure the entire history of the universe). As it is well known, there exist space-times which cannot be covered by a single coordinate system (e.g. space-times with the topology of sphere), and consequently no global time can be defined in such space-times. The necessary and sufficient condition for the existence of a global time is the causal stability of a given space-time. Space-time is said to be *causally stable* if a small perturbation of its Lorentz metric does not produce in it the appearance of the closed timelike curves (see Hawking and Ellis 1973, pp. 198-201).

The problem of Cauchy horizons and Cauchy developments. Owing to the hyperbolic character of Einstein’s equations the Cauchy data given at the initial hypersurface in space-time, in general, do not propagate throughout the entire space-time, but the region of their influence (the so-called *Cauchy development*) is limited by *Cauchy horizons*. Their existence is clearly connected with the possibility (or impossibility) to determine the solution to Einstein’s equations from the Cauchy data (the initial value problem) and with the deterministic properties of a given space-time (see, Hawking and Ellis 1973, chapter 7; Fischer and Marsden 1979).

The cosmological horizon problem. The existence of null-cones in
the tangent spaces at each event in space-time, interpreted as the existence of the limiting velocity of the propagation of physical signals, implies that various observers can influence (can observe), or be influenced by, in principle, limited subsets of events in space-time. Boundaries of these subsets are called (cosmological) horizons (one distinguishes particle-horizons, and past and future event horizons, see Rindler 1977, Tipler et al. 1980). The existence of horizons imposes severe constraints on the observational testing of cosmological models and creates the consistency problems for the standard cosmology (see, for instance, Kolb and Turner 1990, pp. 261-269; Roos 1994; Partridge 1995).

The singularity problem, perhaps the most difficult and most fundamental problem of all other problems. Roughly speaking, singularities are boundaries of space-time at which the manifold structure of space-time breaks down. The Big Bang singularity in the Friedman-Lemaître world models and the central singularity in the Schwarzschild solution are the most notable examples of singularities. More technically, singularities are defined in terms of incomplete non-spacelike (causal) geodesics: a space-time is singular if there is in it at least one non-spacelike incomplete geodesics. By using this definition (or rather a criterion of the existence of singularities), Penrose, Hawking and others have proven several theorems about the existence of singularities in a broad class of space-times satisfying rather tolerant conditions (see, Hawking and Ellis 1973, Beem and Ehrlich 1981, Tipler et al. 1980, Clarke 1993, Earman 1955).

Let us take a closer look at this problem since in it many aspects converge of the local and global structures of space-time. The root of the difficulty is connected with the fact that the Lorentz metric carried by space-time is not a metric in the topological sense. One can define a topology in terms of the chronological structure of space-time, the so-called Alexandrov topology, but without additional stronger assumptions it does not coincide with the manifold topology (in order to change it into the manifold topology the so-called strong causality condition must be assumed which asserts that no neighbourhood of any of points of a given space-time is intersected by a non-spacelike curve more than once, see Hawking and Ellis 1973, pp. 192-198, Lerner 1972). The natural idea would be to define a singular boundary of space-time as its Cauchy boundary (defined, as usual, in terms of Cauchy sequences), but this cannot be done because space-time does not carry the uniform structure which is necessary for doing so (see Gruszczak and Heller 1993). It was an
ingenious idea of Schmidt (1971) to define the Cauchy boundary of the total space of the frame fibre bundle over space-time (which carries the suitable uniform structure), and by “projecting it down” to space-time to construct the singular boundary of it, the so-called \textit{b-boundary} of space-time. This construction was regarded as an elegant and physically adequate definition of singularities. It came as a surprise when Bosshard (1976) and Johnson (1977) demonstrated that in the closed Friedman world model the initial and final singularities form the single point of the b-boundary, and that in the closed Friedman and Schwarzschild solutions their b-boundaries are not Hausdorff separated from the rest. Later on, strong indications were provided (Geroch et al. 1982) that this situation is fairly typical for a wider class of singular boundary constructions. These difficulties have led to “a tension between the noun and adjective” understanding of singularities. “The former attempts to conceive of singularities as entities that can be localized while the latter eschews localization and is content to speak of singular spacetimes when these spacetimes exhibit large-scale or global features” (Earman, p. 28).

The story has its continuation, some aspects of which will be touched upon in the next Subsection, but even now it is clear that in the situation when the standard structure of space-time breaks down it is the interaction between “local” and “global” that is severely perturbed (the beginning and the end of the world become the single point of the b-boundary, space-time loses its usual Hausdorff separability properties). Is this a pure pathology or perhaps an indication of some deeper regularities?

\section*{2.4 Global Formulations of General Relativity}

A strategy to solve at least some problems connected with the interaction between geometric structure of space-time and the large structure distribution of matter could consist in entirely eliminating the concept of space-time from the foundations of general relativity and deriving it on later stages of its construction. Such a possibility was suggested by Geroch (1972). Usually, the smooth manifold structure on a (non-empty) set \( M \) is defined in terms of a smooth atlas on \( M \). However, it is well known that it can equivalently be defined in terms of the algebra \( C^\infty(M) \) of smooth functions on \( M \). Moreover, the algebra \( C^\infty(M) \) can be regarded as a primary structure and the manifold \( M \) as a derived structure, namely as the set of characters of the
algebra $C^\infty(M)$. When this strategy is applied to general relativity, the smooth functions, elements of the algebra $C^\infty(M)$, can be identified with scalar fields. Moreover, as shown by Geroch (1072), the Einstein field equations can be written as functional equations in terms of $C^\infty(M)$. In this way, we have a global formulation of general relativity in which scalar fields are a primary concept and space-time a derived one.

Since, however, Geroch's formulation of general relativity is equivalent to its standard formulation we obtain nothing really new, except for the fact that Geroch's formulation is open for further generalizations.

The next logical step would be to take any functional algebra $C$, and to treat functions belonging to $C$ as smooth functions on the space of characters of $C$. Such a space (satisfying two additional conditions, namely the closeness with respect to localization and closeness with respect to composition with the Euclidean functions) is called a differential space (for details see Gruszczak et al. 1988). Owing to the two above mentioned conditions differential geometry can be done on differential spaces. In particular, all quantities required to define Einstein's equations (curvature, Ricci tensor...) can be defined in terms of $C$. Differential spaces satisfying Einstein's equations are called Einstein algebras (see Heller 1992). Since the concept of smoothness is here generalized as compared with the standard differential geometry, Einstein algebras are authentically more general than the usual theory of general relativity. In contrast with general relativity some weaker types of singularities can be fully described in terms of Einstein algebras.

The further generalization consists in replacing the functional algebra $C$ by a sheaf $\mathcal{C}$ of functional algebras; then differential space is replaced by what is called structured space (see Heller and Sasin 1995a); the corresponding sheaf $\mathcal{C}$ of functional algebras is called a differential structure on a given structured space. By defining Einstein algebras in terms of this differential structure one obtains the sheaf of Einstein algebras. It turns out that all sorts of singularities can be described in terms of structured spaces. Even if in some stronger types of singularities the structured space structure behaves badly, by using this approach one can fully analyse the situation.

As an example, let us consider the initial and final singularities (understood as the b-boundary points) in the closed Friedman world model.

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1Elements of the (algebraic) dual space $C^\infty(M)^*$ with respect to $C^\infty(M)$ are called characters of the algebra $C^\infty(M)$. 
Let $\bar{M} = 3D M \cup \partial_b M$ be the b-completed space-time of the closed Friedman world model, where $M$ is the space-time of this model, and $\partial_b M$ its b-boundary. $M$ is open and dense in $\bar{M}$. Since $M$ is a smooth manifold, one can easily describe it as a structured space with the corresponding differential structure $\mathcal{C}$. It turns out that the differential structure $\mathcal{C}$ can be extended to a differential structure $\bar{\mathcal{C}}$ on the b-completed space-time $\bar{M}$, but only in a trivial way, i. e. only constant functions smoothly (in the generalized sense) extend to $\bar{M}$. In the theory of structured spaces derivations of $\mathcal{C}$ play the role of vector fields. Of course, a derivation of a constant function vanishes. This means that only zero vector fields extend to $\bar{M}$, and consequently that the “bundle length” of all curves joining the initial and final singularities is equal to zero. The b-completed space-time $\bar{M}$ of the Friedman closed model shrinks to the single point (the Hausdorff separability breaks down). The global structure of the Friedman universe behaves in a strongly pathological way. However, locally (i. e. if one restricts oneself to $M$ or to its open subsets; we remember that $M$ is open in $\bar{M}$) everything is all right. (For the detailed analysis of this situation see Heller and Sasin 1955a, b; in these works such situations are called malicious singularities.)

The above analysis clearly shows that it is an interaction between local and global properties of space-time that is the main factor of the singular behaviour notoriously met in general relativity.

# 3 Quantum Mechanics: Towards New Conceptions of Time and Space?

## 3.1 Introductory Remarks

In this Section we analyse fundamental concepts of quantum mechanics. We show that they lead to some problems which force us to modify the usual notion of space-time. The first problem comes from the status of time in classical quantum mechanics. There exists in fact a deep conceptual asymmetry between space and time in quantum mechanics: space is quantized whereas time is not. Thus, time is “infinitely divisible”. This leads to “strange” consequences, for example to the so-called “Zeno’s paradox” (see below, Section 3.1). Its interpretation is difficult because it is deeply related to the process of measurement which is not completely understood in the framework
of the standard interpretation of Bohr’s school. This is partly due to the irreversibility of the measurement process which contradicts the reversibility of the evolution equation (Schrödinger’s equation, for example). To understand the irreversibility of time we should introduce, as in Prigogine’s work, a time operator. However, it seems that, although this work is intuitively very interesting, it is not entirely satisfactory from the mathematical point of view. Nevertheless, as we shall see, problems induced by the status of time in quantum mechanics suggest a modification of its mathematical nature. The second problem is related to the famous E.P.R. paradox which introduces the idea of non-locality or more precisely of non-separability with respect to space. In fact, in quantum mechanics space cannot be viewed as a set of isolated points. These problems lead to a deep modification of our representation of “quantum” space-time. It is interesting to notice that the above mentioned problems concerning the nature of time and the problem of the pointlike structure of space-time were in fact present in the debate between Einstein and Cartan.

### 3.2 Time and Quantum Mechanics

In quantum mechanics, every system is described by a wave function \( \Psi \) which obeys Schrödinger’s evolution equation

\[
H\Psi = \frac{i\hbar}{2\pi} \frac{\partial}{\partial t} \Psi
\]

where \( H \) is the Hamiltonian and \( \hbar \) the Planck constant. The Hamiltonian involves two terms: one is related to the kinetic energy and the second describes the potentials associated with the interactions. Thus, the Hamiltonian contains all dynamical information concerning the evolution of the system.

The Schrödinger equation allows us to connect the initial value of the wave function \( \Psi(0) \) with its value at an arbitrary time instant \( t \). This can be expressed by using the evolution operator \( U \)

\[
\Psi(t) = 3D \quad U(t)\Psi(0)
\]

\[
U(t) = 3D \exp\left(-\frac{2\pi i}{\hbar}Ht\right)
\]
The evolution of the wave function is thus completely deterministic. Furthermore, to each evolution described by the Schrödinger equation, we can associate a reversed evolution changing the sign of time but without changing the potential of the external forces (see Fer 1977). It is then possible to show that the probability density $\Psi^*\Psi$ goes back in time and that the mean value of the momentum has the opposite sign. This is what is called the “micoreversibility” of quantum mechanics. It is the consequence of the mathematical structure of the Schrödinger equation. If we choose a suitable evolution equation (for instance a nonlinear dynamical equation), the micoreversibility could immediately disappear. Louis de Broglie (1956, pp. 144-164) tried to introduce such nonlinear equations to avoid conceptual difficulties of the usual quantum theory but without real success.

The time variable appearing in quantum mechanics is thus completely reversible as it is in classical and in relativity theories. Furthermore, in quantum mechanics, time is not described by a Hermitian operator as it is the case as far as usual observables (position, momentum,...) are concerned; in other words, time is not quantized (time variable commutes with all observables; there is no operator canonically conjugated to time). Even in the relativistic quantum field theory, time remains reversible. These observations lead to a great difficulty related to the so-called “measurement problem”.

When a quantum system is not observed, it is described by the evolution operator $U(t)$ which is the unitary operator. But, when it is subject to a measure operation, the state of the system is obtained by the use of a “projector” (which is not a unitary operator) describing the “collapse” of the wave function (which is in general a linear superposition of states) to a particular state. The collapse is not a phenomenon which could be explained in the framework of the usual quantum mechanics. It should be regarded as a “trick” allowing one to obtain the state which results from the measurement process. But now comes the problem. We have seen that Schrödinger’s equation is time reversible but the collapse of the wave function is, from its very nature, irreversible. There is something strange in quantum mechanics! The impossibility of this physical theory to give a satisfactory interpretation of the measurement irreversibility problem=20 is at the root of some conceptual problems. We shall describe one of them called “Zeno’s paradox” (for a very interesting discussion of this paradox, see Omnès 1994; Zeno’s paradox was introduced for the first time by Misra and Sudarshan (1977)).

=20 The quantum version of Zeno’s paradox is related to a strange prop-
erty of the (quasi-)continuously observed systems. Let us explain the core of the argument proposed by Misra and Sudarshan.

We start with an unstable particle and we assume that it can be continuously observed. One could immediately object that this kind of observation is impossible both theoretically and practically. In fact, this objection is easily ruled out because on the one hand, as we have seen above, in the usual quantum mechanics time is not quantized. This means that we can consider time as a continuous variable. On the other hand, a (quasi-)continuous observation of an unstable particle can be actually performed — although only in an approximative sense — by using detection techniques as tracks in bubble chambers. This is in fact a weak argument, but it does not matter since after all we are considering the quantum Zeno paradox as a kind of Gedankenexperiment.

If the usual quantum theory is complete, it must give the probability of decay of the unstable particle considered above when it is continuously observed. Quantum mechanics tells us that the probability of observing the decay of such an unstable particle during the time interval $t$ is proportional to $t^2$. Let us denote by $p(t)$ the probability that we do not observe any decay. Then $p(t)$ can be written as

$$p(t) = 3D1 - q(t) = 3D1 - at^2,$$

where $a$ is a real constant. If we make $n$ identical observations during the time interval $t$, we can express the probability of finding no decay after time $t$ in the following way

$$p(t, n) = 3D \left(1 - a\left(\frac{t}{n}\right)^2\right)^n.$$

Here we have assumed that each observation during the time interval $t/n$ is independent of all other such observations. Now, let us consider a continuous observation. We have to take the limit of $p(t, n)$ when $n$ tends to infinity. But here the paradox appears, because:

$$\lim_{n \to \infty} p(n, t) = 3D \lim_{n \to \infty} \exp\left(-\frac{t^2}{n}\right) = 3D1$$

This is surprising because it means that an unstable particle which is continuously observed will never decay. It is frozen in its initial state by the
fact of observation. The decay is proved to be impossible in the same way as the motion of the arrow was shown by Zeno to be impossible. Misra and Sudarshan have demonstrated that this paradox has a nice consequence for the so-called “Schrödinger’s cat experiment”. A cat is placed in a sealed box. In this box there is a system containing a lethal gas which can be diffused if an unstable atom decays. Usual quantum mechanics says that if we do not observe the cat, its wave function is a superposition of two states: “the cat alive” and “the cat dead”. When we open the box, the collapse of the wave function occurs and this superposition disappears: we have only one of the two states quoted above. Now, it turns out that this explanation is too simple because due to Zeno’s paradox, if the cat observes continuously the system, he can stay alive!

We could suspect that the difficulty of understanding Zeno’s paradox comes from the fact that we have no completely coherent explanation of what measurement is in quantum mechanics. In particular, we do not adequately understand the irreversibility implied by the measurement. The paradox shows that an irreversible process, such as the decay of a particle, is not possible in the framework of the continuous observation. But why is this so? There are in fact at least two possibilities: (1) The prediction coming from Zeno’s paradox is true, i.e. Zeno’s paradox is not really a paradox but a relevant theoretical result. But then we have to explain why observation forbids the decay. And this is really difficult since the act of measurement is not effectively described in the standard quantum mechanics. (2) There could be that (quasi-)continuously observed unstable systems effectively decay. In this case, the paradox would show that quantum mechanics is not complete because it cannot allow us to compute the decay probability.

The experimental test performed by Cook (1988) has shown that Zeno’s paradox is a true physical effect. Therefore, the second possibility has been ruled out. But in order to understand this effect, we have to consider not only an isolated system (the unstable particle) but also a system strongly coupled to its environment (see Joos 1996). This kind of approach is treated in the framework of the so-called “decoherence theories”. These theories try to explain the emergence of the classical world from the quantum one by a process which destroys the quantum coherence through a strong coupling of a system to its environment. Unfortunately, however, we are not sure that these theories offer any explanation of the reduction of the wave-packet, i.e. of the “measurement problem”.

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It is therefore reasonable to believe that Zeno’s paradox, and maybe other classical paradoxes ("Schrödinger’s cat", "Wigner’s friend",...) as well, give us a warning that one has to modify the mathematical nature of time in order to get the satisfactory interpretation of irreversible processes in the quantum context. These processes are to be considered at two different levels: physical irreversible processes (e.g. processes of decay), and measurement processes (collapse of the wave function). These two levels are conceptually distinct in the standard interpretation of quantum mechanics. We should notice that the understanding of what we call irreversible phenomena in classical mechanics does not directly imply the modification of the mathematical nature of time (this can be easily seen by inspecting simple models such as the “Kac ring model”, see Kac 1959, p. 99). But nevertheless, for the understanding of measurement process in quantum mechanics a modification of the mathematical nature of time (and perhaps also of its philosophical nature) would be required.

This modification could be implemented by introducing a time operator belonging to a noncommutative algebra. Even in the context of classical dynamical systems, Prigogine and its school have introduced such an operator (see Prigogine 1980; Prigogine and Stengers 1984; Prigogine 1995a); it does not commute with the evolution operator. Prigogine’s intuition is that the modification of the mathematical nature of time could explain the fundamental nature of the irreversible phenomena. The main idea of his theory is that we have to eliminate the concept of individual trajectory by using probabilistic distributions. This procedure has a nice consequence: it introduces a kind of non-locality in space-time. Prigogine and Elskens (1985) write

\[ \text{Irreversibility leads to a well-defined form of non-locality in which a point is replaced by an ensemble of points according to a new space-time geometry determined by the inclusion of the privileged arrow of time.} \]

Arguments used by Prigogine and his school are not yet completely satisfactory at the mathematical level (for a thorough discussion of Prigogine’s arguments see Bricmont 1995, pp. 159-208; Prigogine’s (1995b) answer to this paper can be found in the same issue of Physicalia Magazine, pp. 213-218), but his intuition is interesting: a modification of the mathematical nature of time, introduced in order to understand the irreversibility of some processes, could lead to a non-local character of space-time. We could think
that at the quantum level the introduction of a time operator would force us to consider space-time as a non-local entity, i.e. a geometrical object which is not definable starting from the concept of “point”.

### 3.3 Non-Separability of the Quantum World

First of all, we must distinguish “non-locality” and “non-separability”. Following Omnès (1994, p. 399), non-locality characterizes a connection between two physical systems which arises instantaneously irrespective of any distance. More generally, we could admit that this connection is realized via a space-like vector (but not necessarily instantaneously). The non-separability says nearly the same thing but here “one insists upon the impossibility of considering a particle independently of the other one, as long as they are strongly correlated in view of a common event in the past” (Omnès, 1994, p. 399).

From the well-known discussions around the E.P.R. paradox and the Bell inequalities (see Jammer 1974, pp. 302-312), we have learned that quantum mechanics involves a kind of non-separability. Two systems which have interacted in the past are correlated in the following sense. If we perform a measurement on one of these systems it immediately affects the second system independently of the distance between them. In other words, the collapse of the wave function is really instantaneous. Of course, this is due to the fact that the two systems are described by a same wave function which spreads over the whole space. The connection between two correlated systems is not the usual one, namely it is not a new kind of physical interaction which would transmit some information or energy. This non-separability occurs in all versions of quantum theory. For example, if we consider Bohm’s theory (see Jammer, pp. 278-296) or Nelson’s (1985) stochastic quantum theory, we are lead to non-local potential and non-local effects as well.

How is it possible to conceive such a non-separability between two systems having interacted in the past? If we want to save the relativity principle, i.e. the Lorentz covariance, and the usual causality, it is not possible to describe the E.P.R. correlations between the systems using the properties of (Minkowskian or even Riemannian) space-time. Therefore, we have two types of interactions between physical systems. One is described by the propagation of a signal on space-time according to the laws of relativity, and the second is an instantaneous correlation, whatever the distance separating
the systems, which affects only the systems having interacted in the past (following terminology proposed by Omnès (1994), we say that this type of correlation is selective). We would certainly feel more comfortably if all interactions between the systems were described by the same unified concept in the geometrical context of space-time.

To describe the instantaneous collapse of the wave function without introducing non-local influences (non-local potential as in Bohm’s theory), which would destroy Lorentz covariance, we could think about a deep modification of the geometrical structure of space. Let us suppose that space is no longer based on point-like entities. Then one could consistently imagine some type of non-separability which would be perfectly well described in geometrical terms. Of course, such a new theory should give the standard theory of general relativity as some sort of approximation.

As we have seen, quantum mechanics persuasively suggests the necessity of modifying the nature of time, and quantum correlations even more strongly compel us to look for a drastically new concept of space which would be able to render understandable instantaneous effects of the irreversible collapse of the wave function. Since, however, quantum correlations are not present between all physical systems but only between those systems which have interacted in the past, the new geometrical structure of space-time should unify local and non-local properties.

3.4 Back to the Past

It seems that the new geometrical framework, suggested by the problems arising in the standard formulation of quantum mechanics, should not be founded on the concept of point as its basic ingredient. Moreover, within the new framework one must be able to consistently describe the time irreversibility of the wave function collapse. Both these requirements remind us proposals which were put forward by Cartan in his work about manifolds endowed with absolute parallelism. It is known that Cartan and Einstein discussed some extensions of general relativity based on manifolds without curvature but with a non-vanishing torsion. It should be remembered that such manifolds admit different kinds of global parallelizations (a detailed discussion of the theories based on the absolute parallelism can be found in the book by Tonnellat 1965, pp. 274-288). In a famous note, Elie Cartan asks how is it possible to restrict the class of such manifolds in order to describe real
physical phenomena. He considers the case in which the fundamental equations of a given theory remain invariant only with respect to right-handed rectangular coordinates systems but not with respect to the left-handed ones. This implies a kind of fundamental irreversibility of the physical laws.

"On peut alors imaginer un système d'équations $E$ qui garderaient leur forme pour tous les systèmes de référence rectangulaires directs, mais qui changeraient de forme pour les repères inverses. Un tel système correspondrait à un Univers dans lequel l'ensemble des lois du champ gravitationnel-électromagnétique jouirait d'une espèce de polarisation: si on considère, par exemple, un système de charges électriques et leur évolution dans un certain intervalle de temps, cette évolution serait impossible si on renversait le sens de la durée: la physique serait irreversible. La théorie classique ne présente rien de pareil; mais il n'est pas interdit de penser que l'irréversibilité de la Physique échappe à notre expérience, à cause de la faiblesse des champs qui entrent dans notre domaine immédiat de connaissance (Cartan 1974, p. 127)."

Irreversibility is thus connected to the intrinsic structure of the space-time manifold. Now, Cartan has noticed that in Einstein's theory of absolute parallelism there exist situations in which it is impossible to give any meaning to the concept of isolated physical corpuscle. In these situations we are forced to abandon the individuality of physical points and we are lead to a form of non-separability. As Cartan says it very clearly

"...cette théorie sera obligée de nier l'individualité physique des différents points qui constituent le fluide électrique ou matériel supposé à l'état continu. Le point matériel était abstrac- tion mathématique dont nous avions pris l'habitude et à laquelle nous avions fini par attribuer une réalité physique. C'est encore une illusion que nous devons abandonner si la théorie unitaire du champ arrive à s'établir (p. 128)."

The theories with nonvanishing torsion developed by Einstein and Cartan do not seem today very satisfactory, but they show something which is intuitively very interesting: the irreversibility and the non-separability can
be obtained in a purely geometrical setting. It is thus not unreasonable to think that quantum correlations and all paradoxes related to the irreversible wave function collapse could not be understood without a deep change of geometrical ideas which lay at the basis of special and general relativity theories in their standard formulations. A non-local geometry would probably be needed to unify the ideas of quantum mechanics with the theory of gravitational field.

4 Pregeometry

4.1 Quantum Geometrodynamics and Superspace

In the framework of general relativity, space-time is treated as a continuum, i.e. as a four-dimensional pseudo-Riemannian manifold. The Hamiltonian formulation of Einstein’s geometrodynamics emphasizes the role of what Wheeler (1968) has called *superspace*, the arena wherein the curved space geometry unfolds. Superspace is an infinite-dimensional space each point of which constitutes a Riemannian 3-space representing the space geometry of a relativistic space-time that is to say a space-time which is a solution of Einstein’s classical field equations. The time evolution of such a space-time, e.g. a cosmological model, appears in superspace as a continuous curve (with the precise beginning and end if the model is closed). As strongly and repeatedly emphasized by Wheeler, “the dynamic object is not space-time: it is *space*” (Misner et al. 1973, p. 1181).

However, when dealing with distances of the order of the Planck length, \( L_{Pl} = \left( \frac{\hbar G}{c^3} \right)^{1/2} = 3D \times 10^{-33} \) cm, and less — and we know that this could happen at the very last stages of gravitational collapse or in the close neighborhood of the initial or final cosmological singularities — quantum fluctuations take place in the geometry of space and become predominant: accordingly, classical geometrodynamics is superseded by *quantum geometrodynamics*, initiated by Wheeler (1968) and DeWitt (1967). Due to Heisenberg’s uncertainty principle, in the same way as in the usual quantum mechanics it is impossible to know the position and the velocity of a particle at the same time, one cannot know, in the framework of quantum geometrodynamics, the precise 3-geometry and its rate of change at the same time instant. Space-time as a purely classical concept loses its meaning and
simply does not exist in the quantum gravity regime.

In cosmology, this implies some fuzziness in the geometry of the universe which can now be described as a *quantum foam* — a collection of quantum fluctuations (at the Planck scale), continuously created and annihilated. In such conditions, elementary particles of our familiar world (protons, neutrons, electrons...) should be regarded as gigantic excited states travelling through the quantum foam, a picture as if directly borrowed from Clifford’s (1879) anticipative *Space Theory of Matter*.

The deterministic classical history of space evolving in time is now deprived of any meaning; one has to use explicitly the language of probabilities and to speak of the probability that the universe has actually such or such 3-geometry. This probability (more precisely, the wave function of the universe) obeys the fundamental *Wheeler-DeWitt equation*, the gravitational counterpart of the famous Schrödinger’s equation of the usual quantum mechanics. The Wheeler-DeWitt equation is a functional equation notoriously difficult to solve: its exact solutions are known only for very symmetric cosmological models (such as spatially homogeneous and isotropic Friedmann-Lemaître models or anisotropic models of Bianchi type) for which this equation reduces to a partial differential equation. Even in these simplified situations, the answers to important questions, such as the existence of the cosmological singularity, remain ambiguous. This is due to considerable technical difficulties within the formalism itself (see Misner 1969; Gotay and Demaret 1983), to the ignorance of the correct boundary conditions which should be imposed on the wave function of the universe (see Hartle and Hawking 1983; Vilenkin 1988) and, not the least, complex — and still unsolved — interpretative problems of the quantum formalism when it is applied to the universe (see, for instance, DeWitt and Graham 1973).

Within the formalism of quantum geometrodynamics global time does not exist any more: notions like “before” and “after” loose any meaning, and the concepts of space-time and time appear only as valid in the classical approximation. Consequently, they are secondary ideas in the formulation of a fundamental physical theory.

Moreover, at a submicroscopic scale (more precisely below Planck’s scale), due to the inescapability of quantum fluctuations in the 3-geometry, this geometry itself is not deterministic any more; as expressed by Wheeler, “it ‘resonates’ between one configuration (3-geometry) and another and another” (Misner et al. 1973, p. 1193). Only when one performs observations at a
much larger scale do these quantum fluctuations fit into a single space-time manifold, ruled by Einstein’s classical field equations.

4.2 Towards Pregeometry

Wheeler’s dream of building all physics on a purely geometric basis, more precisely on quantum geometrodynamics, collapsed when it became clear to himself that there was no natural place inside this geometric formalism for spin 1/2 and in particular for neutrinos (see Wheeler 1962, 1968). Indeed, elementary processes, such as pair creation, require a change in the topology of 3-geometries (the topology of the initial 3-geometry should develop a new wormhole to accommodate the new spin structures associated with the created particles). However, such a change in the topology of 3-geometries is totally forbidden within the formalism based on classical differential geometry, whose axioms are incompatible with the required multiple-connectedness of space at Planck’s scale.

In this way, the idea that geometry should constitute “the magic building material of the universe” had to collapse on behalf of what Wheeler has called pregeometry (see Misner et al. 1973, pp. 1203-1212; Wheeler 1980), a somewhat indefinite term which expresses “a combination of hope and need, of philosophy and physics and mathematics and logic” (Misner et al. 1973, p. 1203).

This fundamental change of perspective about the role of geometry in the description of the physical universe is not without link with considerations put forward by Sakharov as early as in 1967. His point of view was that geometry should be to elementary particle physics what elasticity is to atomic physics. As elasticity cannot explain atoms, but, on the contrary, atoms explain elasticity, geometrodynamics is not able to explain particles: a particle built out of geometry would look as queer as an atom made of elasticity. At a deeper level, there should exist something — call it pregeometry — which should account for geometry and which should certainly be as removed from geometry as the quantum mechanics of atomic and molecular systems is from elasticity.

Being deprived of any reference to the fundamental geometric notions which constitute the heart of the theoretical description of everyday physical reality, i.e. to space and time, pregeometry — whatever its precise formulation could be — is essentially a non-local concept.
4.3 Many Faces of Pregeometry

There is up to now no definite theoretical formulation of the idea of pregeometry, but only a large variety of tentative models more or less mathematically sophisticated (for a bibliographical review of the fundamental properties of these pregeometry models, see Gibbs (1995)).

However, many of these models do not consider, contrary to Wheeler’s point of view, space-time as an approximation to a deeper and more fundamental substratum of a quite different nature: they keep the idea of a preexisting space-time but view it as a lattice, i.e. as a discrete structure with the minimum length of the order of Planck length. Philosophical and theoretical motivations for subscribing to such an idea of a discrete space-time are quite diverse.

It is surely the advent of quantum mechanics, and especially the discovery of the uncertainty principle, that led some physicists, as early as in the 1930’s, to speculate that space-time could be discrete at the fundamental level. Heisenberg (1930) himself had considered a lattice geometry to try to get rid of the self-energy difficulty which plagued at that time the electron theory, but he soon rejected it. Some years later, Einstein (1936) expressed the following opinion:

...perhaps the success of the Heisenberg method points to a purely algebraic method of description of nature, that is, to the elimination of continuous functions from physics. Then, however, we must give up, by principle, the space-time continuum...

Technical difficulties within the process of renormalization, developed to eliminate the ultraviolet divergences present in quantum field theory, have also reinforced the belief, held by many physicists, in a natural cutoff at a very small length scale.

A way of introducing the minimum length into physics has been proposed by Snyder (1947). The replacement of space and time coordinates by non-commutative operators leads to a quantization of space-time, in consequence of the discrete nature of the spectrum of these operators. Unfortunately, this model, although Lorentz invariant, breaks the translation invariance. Similar methods have been proposed later but without great success, because of the difficulty of discretizing the full Poincaré group.
Other attempts at building discrete space-time introduce non-pointlike particles (superstring theory which views particles as one-dimensional strings of Planck length might in this respect be considered as the most advanced realization of this program), or try to formulate the field theory on a discrete lattice (for references, see Gibbs (1995)).

Renewed interest in the possibility of quantizing space-time has arisen in the framework of recent developments in the theory and physical applications of quantum groups, algebraic structures which appear as deformations of the classical notion of group (Gibbs 1995, pp. 25-28).

In all these tentative methods of developing pregeometry models, one has to accept in advance a preexisting form of space-time. This is not satisfactory from the point of view of Wheeler’s conception of pregeometry for whom the features of the conventional space-time, such as its continuity, dimensionality, and even causality and topology, should not be present from the beginning but should naturally emerge in the transition process from pregeometry to the usual space-time dynamics of our conventional physical theories. The choice of appropriate fundamental building blocks from which pregeometry is to be made remains unspecified; this explains the large variety of pregeometry models which have been proposed in the last decades. Below we shall briefly describe some of the most important and original of these models.

**Wheeler’s “bucket of dust”**. In his first attempt to formulate the concept of pregeometry, Wheeler (1964) discussed the idea of “dimensionality without dimensionality”. More precisely, he asked whether geometry can be constructed out of more basic elements, i.e. out of a Borel set (a collection of points (“bucket of dust”) devoid of any specific dimensionality), when using the quantum principle. The hope would be to ascribe a probability amplitude to each possible configuration of points in the Borel set and, in this way, perhaps be able to explain why the dimensionality three would be distinguished rather than any other dimensionality. But the possibility of defining such a mathematical concept rests on some notion of distance between two points, i.e. on a metric, which is completely foreign to the idea of pregeometry. As noticed by Wheeler (1980, p. 3): “Here also too much geometric structure is presupposed to lead to a believable theory of geometric structure.”

**Pregeometry as the calculus of propositions**. Afterwards Wheeler (see Misner et al. 1973, pp. 1208-1212) explored the idea of using propositional logic (the logic of *and*, *or* and *not* statements) as the fundamental
building block of pregeometry, space and time — i.e. the continuum of everyday physics — hopefully emerging from the statistics of large numbers of complex logical propositions. Why logic? Because, as stated by Wheeler (Misner et al. 1973, p. 1212): “Logic is the only branch of mathematics that can ‘think about itself’.”

However, as shown somewhat later, this idea was not very fruitful: mathematical logic does not appear as the natural foundation for pregeometry: in order to give an account of space-time, it is difficult to imagine how one could do without any reference to the central principle of all physics, namely to the quantum principle (Patton and Wheeler 1975).

Wheeler’s self-reference cosmogony. Wheeler’s latest conception of pregeometry is deeply connected with the existence of observers. Since the advent of quantum mechanics the central role of the act of observation has been recognized: the only way to say that an object exists or that a process is taking place is to observe it (“No elementary phenomenon is a phenomenon until it is an observed (registered) phenomenon” (Wheeler 1979)). But the ultimate nature of any measurement is a yes/no question posed by an observer: in the case of the click of a counter, the information one deals with in one yes/no bit of information (bit or binary digit is the basic unit of information), while in other types of measurement large numbers of bits can be gathered (think, for instance, of the registration of an interference pattern on a screen). According to Wheeler, the universe is information theoretical in nature, i.e. defined via discrete bits of information. He expresses this idea in the following way: “… every physical quantity, every it, derives its ultimate significance from bits, a conclusion which we epitomize in the phrase It from Bit” (Wheeler 1990).

But, in every measurement process, the observer acts on the system he is studying and, in this way, he must play a role in its future evolution. The pure observer has been converted into a participator: “…in the elementary quantum phenomenon the observer-participator converts conceivability into actuality” (Wheeler 1980, p. 5). Accordingly, the universe is participatory in its nature and the human observer is endowed with an active and capital role of a participant in the genesis of the universe (“Is observership the ‘electricity’ that powers genesis?” (Wheeler 1977, p. 21)). Such a model is called by Wheeler self-excited and the corresponding cosmogony is known as the self-reference cosmogony: the universe gives birth to communicating participators and communicating participators give meaning to the universe through their
continuous exchange of information (Patton and Wheeler 1975, p. 565). This conception of the world is obviously very near to the one advocated by the French philosopher Maurice Blondel (1927): “La pensée créée n’existe pas sans la nature, et la nature elle-même se suspend à la pensée comme à sa raison d’être”. These ideas are also manifestly deeply related to the Strong Anthropic Principle (Barrow and Tipler 1985; Demaret and Lambert 1994) and to the line of thought of the philosophical idealistic school with its famous representatives: Parmenides of Elea, George Berkeley and the French philosopher Octave Hamelin who tried to prove that the internal laws of the human cerebral activity had necessarily to give birth to the ensemble of spatial, temporal, causal,... relations which constitute what we call the “external world” (see, for instance, Grégoire 1969, p. 54).

However, Wheeler’s most recent conception of pregeometry should not be too easily identified with the idealistic thought which assigns the proper existence only to the mind. Wheeler’s world probably possesses some consistency on its own right; only very few physicists would deny some reality to the world.

Up to now, nobody has succeeded in constructing a full realization of Wheeler’s proposal of pregeometry, i.e. an information theoretical world defined by the participatory observer. Indeed, such a task seems to be beyond our present possibilities.

Other proposals for pregeometry emphasize the relational nature of space-time. The basic assumption common to all of them is the hypothesis that there exist fundamental objects which can be of different types: n-units (Penrose), preparticles (Bunge and García-Maynez, García Sucre), quantum processes (von Weizsäcker, Finkelstein),... Space-time would then consist of the network of relations among these fundamental objects. We give below very brief comments on these relational theories of space-time (we refer to Lorente (1993, 1995) and Gibbs (1995) for more details).

The starting point of Penrose’s (1971) model is an ensemble of elementary objects called n-units, each characterized by the well-defined total angular momentum $n \times \hbar/2$. The interaction of all these objects between themselves gives rise to the spin network. There is no need for an underlying space-time to begin with; space-time comes out at the end. Penrose’s ideas have been later elaborated by Ponzano and Regge (1968) as well as by Hasslacher and Perry (1981) who have shown that the quantum theory of gravity in three dimensions can be described by means of the evaluation of spin net-
works by explicitly using diagrammatic methods. Recently, LaFave (1993) has proposed a way of extending Ponzano-Regge model to four dimensions by reinterpreting this theory in the light of Wheeler’s latest pregeometry philosophy.

Bunge and García-Maynez (1977) and García Sucre (1985) have chosen as primitive concepts “things” not located in space (which can be called preparticles) acting among themselves, the result of these interactions being identified with the temporal and spatial structure of the world.

Von Weizsäcker (1986) has considered the set of relations among binary alternatives, called urs (equivalent to yes/no experiments), at the basis of all quantum processes as well as of space-time. In the same line of thought, Finkelstein (1969-1974), in his series of papers about “space-time code”, has considered the world as a network of quantum processes, which he calls monads, giving rise — through their interactions — to space-time.

These relational theories of space-time are somewhat reminiscent of Leibniz’s conception of space and time as expressed in his Monadology. According to Leibniz, space is but a set of all “points” (monads) and of relations between them.

Some other pregeometric type of models are characterized by abstract algebraic elements, the classical features of the world emerging from this abstract system. In the model studied by Cahill and Klinger (1996), called Heraclitean Quantum System (Heraclitus of Ephesus argued that the world is in the state of flux and that the common sense is mistaken in regarding that the universe is made of stable things), the algebra is taken to be a Grassmann algebra (such an algebra is well-known from modelling the fermionic sector of the standard model of elementary particle physics).

Another purely algebraic attempt at modelling pregeometry which has received a great deal of attention in the last years is based on noncommutative geometry. The key idea of this model is that the topological structure of space-time can be understood in terms of essentially non-local mathematical concepts, i.e. in terms of a noncommutative algebra which would play analogous role to the algebra of smooth functions on the usual manifold. The next Section of this essay will be devoted to the overview of the foundations of this attractive new field of mathematics which seems to be very promising for the study of the quantum gravity regime of the universe below the Planck threshold.
5 Non-Local Geometry and Non-Local Physics

5.1 Introductory Remarks

As we have seen in the preceding sections, in contemporary physics many signals appear suggesting that on the fundamental level time and space in their usual form might not exist and, consequently, that the “beginning” of the universe might be aspatial and atemporal. However, the models known so far said very little how physics with no space and no time could be like. In particular, no mathematical structures were known which could adequately be able to model such situations in their full generality. All models used so far by physicists in this domain were either approximate or toy models, or were based on a non-fully understood mathematics (or both). The state of the art has significantly changed after Alain Connes (together with his co-workers) has elaborated a bundle of mathematical results known under the common name of noncommutative geometry. Although its applications to physics are still at their preliminary stage, at least we have a sound mathematical theory which is able to deal with entirely non-local situations. The aim of the present section is to give a conceptual insight into mathematical foundations of noncommutative geometry. This is important from the philosophical point of view since by penetrating into foundations of this geometry we could understand how physics is possible with no points in space and no instants in time. Actual physical models based on noncommutative geometry are for us here of secondary interest; they will be only briefly mentioned in subsection 5.4.

5.2 The Concept of Point

It is sometimes said that space is collection of points. This saying is misleading since it suggests that the concept of point is not analysable, and this is not true. In the traditional geometry the concept of point can be introduced (at least) in four different ways. Although all these ways are equivalent, it is worthwhile to enumerate them all, since in noncommutative geometry they can lead to different generalizations.

A. Let $M$ be a smooth manifold. Usually $M$ is defined in terms of a smooth atlas on a certain set but, as it is well known, the entire smooth
manifold structure is encoded in the algebra $C^\infty(M)$ of smooth (real) functions on $M$, and the manifold can be equivalently defined in terms of this algebra. Let $x \in M$ and let $F_x$ be the set of all functions $f \in C^\infty(M)$ which vanish at $x$. The sets $F_x$, for every $x$, are maximal ideals in the algebra $C^\infty(M)$. It can be demonstrated that the existence of points in $M$ is equivalent to the existence of maximal ideals in $C^\infty(M)$.

B. An $\ast$-homomorphism (an involutive homomorphism) $\chi : A \to \mathbb{C}$ from an algebra $A$ into the field of complex numbers $\mathbb{C}$ is said to be a character on the algebra $A$ (here and in what follows we consider only involutive associative algebras with units). It can be shown that exists a one-to-one correspondence between characters on the algebra $C^\infty(M)$ and the maximal ideals of this algebra and, consequently, the points of $M$ are also determined by characters on $C^\infty(M)$.

C. A linear functional $f$ on a $\ast$-algebra (involutive algebra) $A$ is called positive if $f(aa^\ast) \geq 0$ for every $a \in A$. If, moreover, $f(1) = 3D1$, $f$ is called a state on the algebra $A$. States which cannot be presented as convex combinations of other states are said to be pure states on $A$. It turns out that also pure states on the algebra $C^\infty(M)$ uniquely determine the points of $M$.

D. To every state on the algebra $A$ there corresponds a probability measure. This probability measure for the algebra $C^\infty(M)$ is of the Dirac’s delta type. As it can be easily seen, it uniquely determines the points in $M$.

Clearly, the algebra $C^\infty(M)$ is commutative (since pointwise multiplication of functions belonging to $C^\infty(M)$ is a commutative operation), and it is precisely this property of $C^\infty(M)$ that is closely connected with the above methods of defining points in the manifold $M$. Moreover, if an abstract algebra $A$ has maximal ideals (or, equivalently, characters, pure states or Dirac’s probability measures), on the strength of the Gel’fand-Neimark-Segal (GNS) theorem it is isomorphic to the functional algebra on a space $M$, and the points of $M$ can be determined be either of methods (A) – (D).

Dealing with commutative algebra $C^\infty(M)$ rather than directly with the set $M$ opens the way for generalization. It is natural to ask whether a noncommutative algebra $A$ could also be interpreted as containing geometric information on a certain space.
5.3 Pointless Spaces

As shown by Connes and his co-workers, the answer to the last question of the preceding subsection is positive, although the mathematics which must be invested in order to decipher the geometric information contained in a noncommutative algebra is rather complex. In the present subsection we shall first see how the concept of point can disappear in noncommutative spaces, and then how differential geometry and physics can be done on such pointless spaces.

In the case of a commutative algebra \( A \), there exists the GNS isomorphism (for continuous functions) \( A \cong C^0(\text{Max } A) \), where Max \( A \) is the set of maximal ideals of the algebra \( A \), given by \( a \mapsto \hat{a} \), for every \( a \in A \), \( \hat{a} \) being the mapping which sends each \( a \in A \) to the mapping

\[
\hat{a} : \text{Max} A \to \mathbb{C}
\]

defined by

\[
\hat{a}(I) = 3Da + I \in A/I,
\]

where \( I \in \text{Max} M \). In the general case, when \( A \) is a noncommutative algebra, maximal ideals must be replaced by primitive ideals, i.e. by kernels of irreducible representations of \( A \) in a Hilbert space. A representation of an algebra \( A \) in a Hilbert space \( \mathcal{H} \) is a mapping \( \rho : A \to \text{End } \mathcal{H} \) of the algebra \( A \) into the set of linear transformations of the Hilbert space \( \mathcal{H} \) (such transformations are called endomorphisms of \( \mathcal{H} \) or operators acting on \( \mathcal{H} \)) preserving essential properties of the algebra (addition and multiplication of elements of \( A \), and their multiplication by scalars). A representation \( \rho \to \text{End } \mathcal{H} \) is said to be irreducible if only invariant subspaces of \( \mathcal{H} \) are \{0\} and \( \mathcal{H} \) itself, where by an invariant subspace of \( \mathcal{H} \) one understands a subspace \( \mathcal{H}_0 \subset \mathcal{H} \) such that, for any endomorphism \( \rho(a) \in \text{End } \mathcal{H} \), \( a \in A \), one has \( \rho(a)\mathcal{H}_0 \subset \mathcal{H}_0 \). And finally, the kernel of the representation \( \rho \to \text{End } \mathcal{H} \), \( \text{Ker}\rho \), is defined as \( \text{Ker}\rho := 3D\{a \in A : \rho(a) = 3D0\} \). Let us notice a certain similarity between the concept of maximal ideals and that of primitive ideals: in defining maximal ideals we require vanishing of functions at certain points of a set; in defining primitive ideals we require vanishing of representation mappings on certain elements of the algebra.

Let us denote the set of all primitive ideals of \( A \) by \( \text{Prim} A \). If \( A \) is commutative then \( \text{Prim} A = 3D\text{Max} A \), and we go back to the previous
construction. If \( A \) is noncommutative we also have a mapping

\[
\hat{a} : \text{Prim } A \rightarrow A/P,
\]

for \( P \in \text{Prim } A \), given by

\[
\hat{a}(P) = 3Da + P \in A/P,
\]

but the quotient algebra \( A/P \) can be very complicated, for instance the dimension of \( A/P \) can change as \( P \) changes. In the case when \( \text{Prim } A \) is a Hausdorff space, there is a counterpart of the GNS isomorphism (see Dupré 1978). To articulate it let us construct a disjoint union of the quotient algebras

\[
E := 3D\bigcup_P \{A/P : P \in \text{Prim } A\}
\]

with a suitable topology, and define the bundle \( \Omega := 3D\{E, \text{Prim } A, \pi\} \) where \( \pi : E \rightarrow \text{Prim } A \) is an obvious projection (it is, in fact, a Banach bundle). It can be shown that the set \( \Gamma^0(\Omega) \) of (bounded) continuous cross-sections of \( \Omega \) forms a \( C^* \)-algebra, and one obtains the isomorphism of \( A \) onto the \( C^* \)-algebra \( \Gamma^0_c(\Omega) \) of compactly supported cross-sections of \( \Omega \), \( A \cong \Gamma^0_c(\Omega) \).

For non-Hausdorff spaces more difficulties arise and the construction is not that obvious (see Dupré 1978).

As we can see, in the case of noncommutative spaces (even for Hausdorff topologies), the idea of a family of continuous functions vanishing at a given point (maximal ideal of the algebra \( A \)) is replaced by the kernel of an irreducible representation of the algebra \( A \) in a Hilbert space (primitive ideal). However, we must remember that in many applications the elements of the algebra \( A \) are cross-sections of a Banach bundle, and consequently they are global entities.

In the noncommutative case, we have the correspondence between representations of the algebra \( A \) in a Hilbert space \( \mathcal{H} \) and states on the algebra \( A \), and between irreducible representations of \( A \) in \( \mathcal{H} \) and pure states on \( A \), but of course the existence of pure states is no longer equivalent to the existence of points in the considered space (with a certain degree of tolerance, pure states could be regarded as generalizations of points).

However, it should be noticed that in some rather special cases, a noncommutative algebra \( A \) can admit maximal ideals. In such a case, if \( \mathcal{I} \) is a (two-sided) maximal ideal of \( A \), then the quotient \( A/\mathcal{I} \) is a simple algebra
(i.e. it has no non-trivial two-sided ideals). These maximal ideals can be regarded as “points” of the noncommutative space determined by the algebra $A$, but such “points” can have rich “internal structure”. One says that they “take their values in a simple algebra”. This remains in contrast with the commutative case where, for a maximal ideal $I$ of $A$, one has $A/I \cong C$, and one says that the points of the corresponding space “take their values in $C$”. The last property is the algebraic counterpart of the fact that in the commutative space points have no “internal structure”. (See Masson 1996, pp. 91-97.)

As an example of a space with “structured points” let us consider the algebra $A = 3DC^\infty(V) \otimes M(n, C)$ of smooth functions on a differentiable manifold $V$ with values in the matrices $M(n, C)$. All such functions vanishing at $p \in V$ form a maximal ideal $I$ of $A$ which can be identified with a point in a noncommutative space. Points of these space “take their values in the simple algebra $A/I$”.

The existence of spaces with “structured points” opens new possibilities as far as applications to physics are concerned (see, for instance, a noncommutative version of the Kaluza-Klein theory, Madore 1995, pp. 180-187).

It turns out that noncommutative spaces, as defined by general noncommutative algebras, are quite manageable provided we have at our disposal rather sophisticated mathematical tools. For instance, as has been demonstrated by Connes (1995), the measure theory on noncommutative spaces is replaced by the theory of von Neumann algebras, and many features usually dealt with by using topological methods are captured by the K-theory. There are also several ways of introducing differential calculus on noncommutative spaces (they are transparently discussed by Dubois-Violette (1995)). Unfortunately, we cannot enter here into these interesting topics. The problem which is now important for us is how physics can be done in terms of geometry in which there could be no points in space and no instants in time.

The essential thing for physics is dynamics, and any dynamics is thought to be a process evolving in time. The standard way to mathematically model dynamical processes is in terms of vector fields. Solutions of the corresponding system of differential equations (called dynamical system) give integral curves of these vector fields which in turn are interpreted as histories of the process. The value of the vector field at a given time instant of the history is a vector (tangent to this history) describing the “behaviour” of the system at the given time instant. It should be noticed that although the concept of
vector is a local concept, and as such it could have no counterpart in noncommutative geometry, the concept of vector field is a global concept and it survives the generalization to noncommutative geometry. It turns out to be enough to have a “generalized dynamics” in this new conceptual framework.

A counterpart of a vector field in noncommutative geometry is a derivation of the algebra $A$, i.e. a mapping $V : A \to A$ satisfying the Leibniz rule. In fact, one can do differential geometry in terms of such derivations. In particular, connection, curvature, Ricci tensor, and consequently Einstein (dynamical) equations can be defined (see, Sasin and Heller 1995). However, one must remember that all these concepts are non-local. For instance, curvature should not be imagined as a “curved space” but rather as a certain abstract operation on derivations of a given algebra. In the noncommutative framework, one can also do differential geometry in terms of (generalized) abstract differential forms rather than in terms of derivations (see Madore 1995). In the cases when both methods (in terms of derivations and in terms of forms) are applicable one must adapt one’s choice to the actual situation.

Non-commutative geometries become especially effective tool in dealing with various “pathological” or “singular” spaces (for instance, Penrose’s tilings, foliated spaces) if the algebra in question is a $C^*$-algebra. Connes (1995, chapter 2) has elaborated a method which allows one to convert a broad class of noncommutative algebras into $C^*$-algebras. The method consists in constructing a bundle the cross-sections of which form an algebra. The suitable completion of this algebra changes it into a $C^*$-algebra. Algebras of observables in the standard formulation of quantum mechanics are the prototype of $C^*$-algebras (in fact, every $C^*$-algebra can be represented as a subalgebra of the algebra of such observables, i.e. as a subalgebra of the algebra of bounded operators on a Hilbert space). Exactly, because of that the theory of $C^*$-algebras has been well developed and could be regarded as a link between traditional mathematics and its noncommutative generalizations.

### 5.4 Some Applications to Physics

As we have noticed at the beginning of this section, applications of noncommutative geometry to physics are at their preliminary stage of development, but even at this stage they are more than encouraging. One of the most important of these applications is the result obtained by Connes and Lott (1990) consisting in geometrizing the standard model of physical interactions. As
is well known, quantum electrodynamics with the Maxwell-Dirac lagrangian
gives a very elegant and very efficient description of electromagnetic interaction.
The standard model generalizes this description to other interactions
with the exception of gravity. This model works very well but is not elegant
from the mathematical point of view: Its lagrangian is a juxtaposition (a
sum) of five terms, each of them describing a contribution coming from a
different source. Connes and Lott have demonstrated that one can obtain an
elegant Maxwell-Dirac-like lagrangian for the standard model provided one
assumes that the underlying space-time, at the length scale of the order of
$10^{-16}$ cm, has the structure of a noncommutative space, namely the structure
of $M^4 \times F$ where $M^4$ is a 4-dimensional manifold and $F$ is a space consisting
of two points, $F = 3D\{a, b\}$ (in the framework of noncommutative geometry
this space can be given a metric structure).

Since the number of applications of noncommutative geometry to physics
rapidly increases, let us enumerate only some of them. For example,
methods of noncommutative geometry have been applied to gauge theories
(Dubois-Violette et al. 1990, Chamseddine et al. 1992), unification theories
(Chamseddine et al. 1993a, Chamseddine and Fröhlich 1994a), supersymmetry
theories (Chamseddine 1994), Chern-Simons theory (Chamseddine and
Fröhlich 1994b), and to the hamiltonian formalism (Kalau 1996, Hawkins
1996). One of the present authors together with his co-worker (Heller and
Sasin 1996a, b) has used noncommutative methods to study the problem of
classical singularities in general relativity.

The most obvious idea would be to speculate that geometry beneath the
Planck threshold (i.e. in the quantum gravity regime) is noncommutative
with no space and no time in the usual sense, and only by going to larger
scales one would obtain, via a kind of symmetry breaking, the standard commutative
gometry of space-time. Unfortunately, before implementing this
attractive idea into a working mathematical model some conceptual difficulties
must be overcome. For the time being some work has been done to generalize
general relativity to the noncommutative framework (Chamseddine
et al. 1993b), and to couple gravity to the standard model of fundamental
interactions (Chamseddine and Connes 1996a, b).
6 Concluding Remarks

As we have noticed in the Introduction, the very existence of physics is strictly connected with the possibility of isolating simple “local subsystems” from the net of entanglements constituting the structure of the universe. Enormous successes of the empirical method, based on this property, have somehow overshadowed the fact that the strategy of isolating “local subsystems” can be but an approximation to the more adequate approach in the study of the world. Although the “universe as a whole” always was a subject-matter of interest for many physicists and astronomers, it was commonly believed that its structure could be disclosed by investigating local physics as a “fair sample” of the rest. Even the beginnings of relativistic cosmology were strongly biased by this prejudice. There were modern mathematical methods, used in general relativity, quantum mechanics and quantum field theories, that gradually enforced the new perspective.

The typical feature of the 20th century mathematics is changing from local methods, characteristic of the older approach, to global methods of treating mathematical objects. Topology and functional analysis, which have become standard tools of doing mathematics, are global from the very beginning, and when employed in other branches of mathematical investigation they immediately produce problems with a pronounced global component. This is especially evident in the domain of differential geometry in which traditional “differential methods” give often misleading results unless certain global conditions are guaranteed. This effect is so strong that the usual stage for differentially geometric investigation is nowadays not a differential manifold itself but rather some “larger spaces” considered as global structures constructed over a given manifold, such as foliations, fibre bundles or even families of bundles (for instance, K-theory).

It seems that noncommutative geometry is a theory in which the above “globalization process” is at its apex. In a certain sense, localities have been engulfed by the global structure of noncommutative spaces, and they can only be recovered by restricting the corresponding noncommutative algebras to some of their subalgebras (for instance to their centers).

It goes without saying that such methods had sooner or later to find their place in theoretical physics. The theory of general relativity was perhaps the first physical theory upon which the global approach has been enforced, but soon other physical theories surrounded themselves to this new strategy. We
have observed the results of this process in the preceding sections.

As we have also seen, there are reasons to believe that at the fundamental level physics might be based on a noncommutative mathematics. Preliminary results in this direction are encouraging, and also “philosophy” of such an approach offers attractive interpretative possibilities. Let us only mention two widely discussed issues, which could find their unexpected clarification within the framework of physics based on noncommutative geometry, namely the Mach Principle problem (see above section 2) and the non-separability of events in quantum mechanics (section 3).

All stronger formulations of Mach’s Principle require that, roughly speaking, local physics should be entirely determined by the global structure of the universe, and general relativity (and other similar theories as well) stubbornly fail to satisfy this requirement. Noncommutative approach to physics at the fundamental level neatly clarifies the situation. Beneath the Planck threshold there would be no space-time in the usual sense but only a “non-commutative pregeometry” with no non-trivial “local neighbourhoods” at all (by “trivial local neighbourhoods” we mean those connected with eventual commutative subalgebras of the corresponding noncommutative algebra). In such circumstances, there would be only the fully Machian physics entirely determined by the global structure of the world. This “Machian property” should be regarded as incorporated into the primordial symmetry, and the present non-Machian physics as the result of the first symmetry breaking in the history of the universe, i.e. of the transition from noncommutative pregeometry to the usual commutative space-time geometry.

It is legitimate to assume that some “fragments” of the “old phase” would remain frozen into the present structure of the world. It seems reasonable to look for such vestiges of the primordial non-local symmetry in the domain of microphysics which is expected best to remember the broken primordial symmetries. No wonder that they would somehow be encoded into the structure of the phase space of quantum mechanics: all information about two particles which once interacted with each other is indeed contained in the same vector of the corresponding Hilbert space. And this is irrespectively of how great distance in space is separates them. After all, space distance is the later concept which was not present in the original symmetry.

At the end a word of warning seems indispensable. Many beautiful philosophies have collapsed because they were unable to find their support in a solid physical theory. Whether the looked for theory of ultimate physics
will really be based on noncommutative mathematics remains to be seen. The present preliminary results, although far from being conclusive, do not discourage such a belief.

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