Quark masses using domain wall fermions

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Due to the attractive features that domain wall fermions possess with respect to chiral symmetry, we continue our investigation of the light quark masses with this discretization. Achieving reliable results, especially for \((m_u + m_d)/2\), requires strict control of systematic uncertainties. Our present results were obtained on a quenched \(\beta = 6.0\) lattice with spatial volume \(\approx (1.5 \text{ fm})^3\). Consequently we remark on effects of finite volume as well as finite extent in the fictitious fifth dimension. We compute the renormalization factors nonperturbatively and compare to the one–loop perturbative result.

1. INTRODUCTION

Last year an exploratory calculation of the strange quark mass was completed using the domain wall fermion discretization within the quenched approximation [1]. At the 15–20% level the results were encouraging: the pion mass squared extrapolated to zero at the input mass \(m \to 0\), simulations at three different lattice spacings gave similar values for \(m_{\pi}^{\text{MS}}(2 \text{ GeV})\) and the values were in agreement with other lattice results. This work reports on the progress of the RIKEN/BNL/CU (RBC) Collaboration toward a precise calculation of the strange quark mass.

2. SPECTRUM

Using domain wall quarks we have computed light hadron masses on a \(16^3 \times 32\) quenched lattice with the following parameters: \(\beta = 6.0\) (plaquette action), domain wall height \(M = 1.8\), seven values of valence quark masses (\(am\)‘s), and number of sites in the extra dimension \(N_s = 16\). Results from 85 configurations for the pseudoscalar meson, vector meson, and nucleon masses are shown in Figures 1 and 2 (see also Ref. [2]).

With the light masses and improved statistics of this study compared to Ref. [3], it has become clear that the linear extrapolation of \((aM_{\pi})^2\) to \(am = 0\) gives a positive intercept, 0.018(2) (statistical error only), for these simulation parameters (see Fig. 1). Several effects could be responsible for this feature. First, the finite (and relatively small) volume of our lattice could increase the mass of the pion at a given \(am\), and these effects would be especially visible at the lighter masses; as shown below the smallest \(am\), 0.01, is roughly 1/4 the strange quark mass. Second, there is an intrinsic breaking of chiral symmetry due to the finite \(N_s\): the right– and left–handed surfaces states have some overlap within the fifth dimension and would result in a residual quark mass \(m_{\text{res}}\). Both finite \(V\) and \(N_s\) effects should be mostly \(am\)–independent. On the other hand studies by Columbia [4] at \(\beta = 5.7\) suggest that in the large \(N_s\) limit one cannot account for the entire \(M_{\pi}^2\) intercept by increasing the spatial volume by a factor 2\(^3\). Quenching effects predicted by quenched chiral perturbation theory or due to artificial instantons could play a role. The issue of residual mass is addressed in Ref. [3].

We cannot presently distinguish \(am\)–dependent finite \(V\) and \(N_s\) effects from any nonleading or logarithmic behavior of the pseudoscalar meson mass. For this analysis we assume that the above effects are all \(am\)–independent, so then the chiral limit is at \(am = -0.0038\). We discuss systematic uncertainties regarding this assumption below.

The \(am\) corresponding to the light quark mass is determined by extrapolating \((M_{\pi}/M_{\rho})^2\) to its physical ratio. There \(M_N/M_{\rho} = 1.37(5)\) compared to the physical ratio 1.22. The lattice spacing is then set using the \(\rho\) mass, and the strange quark mass is set using either the physical \(K\) or \(\phi\) mass.
mass. The bare quark masses \((m_q = am + 0.0038)\) are thus (with statistical errors only):

| bare mass | \(1/a = 1.91(0.04)\) GeV | \(m_l = 0.00166(0.00005)\) MeV | \(m_s(K) = 0.042(0.003)\) | \(m_s(\phi) = 0.053(0.004)\) |
|---|---|---|---|---|

The large difference in \(m_s\) due to whether the \(K\) or \(\phi\) meson is used to set the scale is a common feature to quenched lattice simulations. This is succinctly expressed by the parameter \(J \equiv M_K \cdot (dM_V/dM_{PS})\) \[1\]. The value obtained using physical meson masses is \(0.48 \pm 0.02\) (the spread is due to the fact that \((M_\phi - M_K)/M_K \neq 1\)); while our result is \(J = 0.34(7)\), comparable to calculations with Wilson fermions. In light of this difference, we do not average \(m_s(K)\) and \(m_s(\phi)\) but quote results for each input separately. The calculation of the \(\phi\) mass in quenched QCD is subject to systematic error since disconnected graphs are being neglected, and so in the rest of this work we consider only \(m_s(K)\).

So far we have not completed an exhaustive study of the systematic errors at this lattice spacing. Therefore, for the present time we give rough estimates. Since \(M_N/M_\rho\) is 10% higher than the physical value, our determination of the lattice spacing has an inherent uncertainty of at least 10%. In this work we have assumed that finite \(V\) and \(N_s\) effects gave a positive \(m\)-independent contribution to the pion mass. If we instead assume such effects account only for half of the nonzero intercept \[3\] then the quark masses above would all decrease by 4 MeV. This is only a 5% change to the strange quark mass, but clearly the extrapolation to the average up and down quark mass is untrustworthy. A better understanding of the systematic uncertainties is necessary before a reliable calculation of \(m_t\) can be completed. The bare strange mass from this work is \(m_s(\beta = 6.0) = 80 \pm 6\) (stat.) \(\pm 10\) (sys.) MeV.

3. NONPERTURBATIVE RENORMALIZATION

The mass renormalization constant has been computed perturbatively to one–loop for domain wall fermions \[13\]. In this work we compute the renormalization constant nonperturbatively using the regularization independent momentum subtraction (RI/MOM) scheme as advocated by the Rome–Southampton group \[6\].

To determine the renormalization constant for an operator \(O\), we impose the following renormalization condition:

\[
\frac{Z_O(\mu a)}{Z_q(\mu a)} \mathrm{Tr} \left[ P_O \Lambda_O(p a) \right]_{p^2 = \mu^2} = 1, \quad (1)
\]

where \(P_O\) projects out the tree–level spin structure of \(O\), \(\Lambda_O\) is the amputated Greens function of \(O\) in momentum space, and \(Z_q\) is the quark wavefunction renormalization. For fermion bilinear operators \(O(x) = \bar{q}(x)\Gamma q(x)\) only a single momentum space propagator is needed to compute the amputated Greens function

\[
\Lambda_T(p) = S^{-1}(p) \left\langle S(p) \Gamma(p) S^{-1}(p) S(p) \right\rangle S^{-1}(p) \quad (2)
\]
where \( S(p) = \sum_y \exp(-ipt)q(y)\bar{q}(0) \). In this work we compute momentum space propagators on 52 configurations \([1]\). Eqs. (1) and (2) together give \( Z_t/Z_q \) at any momentum \( p \). In order for the RI/MOM method to work reliably the momentum should satisfy \( \Lambda_{QCD} \ll p \ll 1/a \) so that nonperturbative and discretization effects, respectively, are small; however with presently accessible lattices this range is narrow. The approach which we follow is to fit to the leading \( (ap)^2 \) errors \([8]\). First, we compute \( Z_q \) through

\[
Z_q' = -i \sum_\mu \gamma_\mu p_\mu S^{-1} \frac{1}{12} \sum_\mu p_\mu^2
\]

which differs from \( Z_q \) at \( O(g^4) \) in Landau gauge \([9]\), then multiply \( Z_t/Z_q \) by \( Z_q' \) to give \( Z_t' \) and divide by the two–loop running \( c_\Gamma \) resulting in

\[
Z_{t,\text{RGI}}(\mu) = \frac{Z_t'(\mu a)}{c_\Gamma(\mu a)} = \frac{Z_{t,\text{MS}}'(\mu a)}{c_{\text{MS}}(\mu a)}
\]

which is scale independent through two–loop order. The result for the scalar density is shown in Fig. 3. The solid line is a linear fit to \( a^2p^2 \) discretization effects.

The result for \( Z_{t,\text{MS}}'(2 \text{ GeV}) = 1.32 \) for these simulation parameters.

Taking the lattice \( m_s(K) \) from the previous section we find at \( \beta = 6.0 \)

\[
m_{\text{MS}}^s(2 \text{ GeV}) = 130 \pm 11 \pm 18 \text{ MeV}
\]

where the first error is the statistical uncertainty and the second is the systematic uncertainty.

4. CONCLUSIONS

We have completed a calculation of the light and strange quark masses at one lattice spacing and volume within the quenched lattice spacing. The increased precision has exposed finite \( N_s \) and \( V \) effects and these lead to errors of roughly 5% in the strange quark mass. We will soon have a calculation of \( m_s \) on a coarser \( (\beta = 5.85) \) lattice, and of course further work at larger volumes, larger \( N_s \), and smaller lattice spacing will reduce systematic uncertainties and yield a precise determination of the strange quark mass.

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