A predictive scheme for neutrino masses

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Abstract

The solar and atmospheric data and possibly large value for the effective neutrino mass in neutrinoless double beta decay experiment together indicate that all the three neutrinos are nearly degenerate. A verifiable texture for the neutrino mass matrix is proposed to accommodate these results. This texture allows almost degenerate neutrino masses two of which are exactly degenerate at tree level. The standard model radiative corrections lift this degeneracy and account for the solar deficit. The solar scale is correlated with the effective neutrino mass $m_{ee}$ probed in neutrinoless double beta decay experiments. The model can accommodate a large value ($\sim \mathcal{O}(eV)$) for $m_{ee}$. Six observables corresponding to three neutrino masses and three mixing angles are determined in terms of only three unknown parameters within the proposed texture.

Introduction: Measurement of neutrinoless double beta decay is of considerable theoretical importance on two counts. Positive result would provide an unambiguous evidence for the non-conservation of lepton number. It would also give direct information on neutrino masses rather than on neutrino (mass)$^2$ differences which are probed in neutrino oscillation experiments.

Neutrinoless double beta decay experiments measure absolute value of an effective mass $m_{ee}$ for the electron neutrino defined as:

$$m_{ee} \equiv \sum u_{ei}^2 m_{\nu_i}.$$  

(1)

$U$ denotes here the neutrino mixing matrix and $m_{\nu_i}(i = 1, 2, 3)$ are neutrino mass values which can take either sign. The $m_{ee}$ is given by the 11 element of neutrino mass matrix in the basis with diagonal charged lepton masses.

The experimental bound on $m_{ee}$ is given by [1]

$$|m_{ee}| \leq 0.38 \text{ eV} \quad \text{at 95\% CL},$$

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where \( h \sim 0.6 - 2.8 \) denotes the uncertainty in nuclear matrix element [4]. Recent analysis [3] claims a positive evidence

\[
|m_{ee}| \approx 0.05 - 0.86 \text{eV} \quad \text{at 95\% CL (2)}
\]

taking account of uncertainty in the relevant nuclear matrix element.

The value of \( m_{ee} \) in this range if established in future [2] can have very important implications for particle physics and cosmology [4]. The implications of neutrinoless double beta decay measurements on neutrino mass hierarchies have been worked out in detail in a number of papers [2,3-5]. If there are only three light neutrinos with hierarchical masses then the result on the atmospheric neutrino deficit \((m_{\nu_3} \leq 0.07 \text{eV})\) and the negative results from CHOOZ \([7]\) \(|U_{e3}| \leq 0.12\) together imply \(m_{ee} \leq 10^{-3} \text{eV}\), which is substantially lower than the value quoted in eq.(2). The inverted mass hierarchy \(m_{\nu_1} \approx m_{\nu_2} \gg m_{\nu_3}\) would be allowed for \(m_{ee}\) less than or close to the atmospheric mass scale. In contrast, one would need almost degenerate masses [8] for all the three neutrinos if \(m_{ee}\) is substantially higher than the atmospheric scale.

The combined inference from the solar and atmospheric deficit and a possible large \(m_{ee}\) is a neutrino spectrum in which the common mass of any two neutrinos is much larger than their \((\text{mass})^2\) difference. One can distinguish two theoretical schemes which allow this [3]. Either two neutrinos have the same CP in which case they form two almost degenerate Majorana neutrinos or they have opposite CP and together form a pseudo-Dirac state.

A pseudo-Dirac neutrino corresponds to either of the following structures in case of two generations, \(\nu_e\) and \(\nu_x\) \((x = \mu\) or \(\tau\)):

\[
\begin{pmatrix}
\delta & m \\
m & \delta'
\end{pmatrix},
\]

or

\[
\begin{pmatrix}
a & b \\
b & -a
\end{pmatrix},
\]

with \(\delta, \delta' \ll m\) and \(a \sim b\). These two textures differ from each other both conceptually and phenomenologically. For \(\delta, \delta' = 0\), both of them lead to neutrinos with equal and opposite masses. The former is invariant under a global \(L_e - L_x\) symmetry which needs to be broken by non-zero \(\delta, \delta'\) in order to generate splitting . In contrast, the texture (4) is not invariant under any combination of \(L_e\) and \(L_x\). As a result, there does not exist any symmetry to
To protect degeneracy and the standard charged current interactions automatically introduce a radiative splitting among neutrinos. At the phenomenological level, the texture in (3) implies almost maximal mixing while the mixing implied by (4) is arbitrary (\(\tan 2\theta = \frac{b}{a}\)). Finally, the neutrinoless double beta decay amplitude implied by eq.(3) is much smaller than the common mass (for \(\delta \leq m\)) while it is comparable to the neutrino mass (\(\sqrt{a^2 + b^2}\)) in case of eq.(4).

It follows from above that the texture in eq.(4) leads to almost degenerate pair of neutrinos with large neutrinoless double beta decay amplitude and very small splitting. It seems ideal for the description of neutrino masses if the latest result [3] are correct. The splitting introduced by the standard model radiative correction is of \(O\left(\frac{m_{ee} m_e^2}{16\pi^2 M_Z^2}\right)\). This would be in the correct range for the description of the solar neutrino scale. One needs to generalize the basic texture in eq.(4) to three generations in order to incorporate a solution to the atmospheric neutrino problem as well. In the following, we suggest an economical and very predictive scheme which provides a natural and coherent understanding of neutrino properties required on phenomenological ground.

**Proposed Texture:** Let us consider a CP conserving theory specified by a general 3 \times 3 real symmetric neutrino mass matrix \(M_\nu\). We require that above \(M_\nu\) leads to a pseudo Dirac neutrino. General conditions on \(M_\nu\) under which this happens were discussed in [11]. In particular, the \(M_\nu\) should satisfy

\[
tr(M_\nu) \sum_i \Delta_i = \det M_\nu ,
\]

where \(\Delta_i\) represents the determinant of the 2 \times 2 block of \(M_\nu\) obtained by blocking the \(i^{th}\) row and column. While many solutions to this constraint are possible [11], we study here a specific solution which meets the requirement demanded by the observed features of neutrino masses and mixing. The proposed solution to the above constraint corresponds to the following neutrino mass matrix in the basis with diagonal charged leptons:

\[
M_{0\nu} = \begin{pmatrix}
s & t & u \\
t & -s & 0 \\
u & 0 & -s
\end{pmatrix} .
\]

This ansatz is a direct generalization of eq.(4) and contains all the features mentioned in the 2 \times 2 case. It is given in terms of only three parameters \(s, t, u\) which after the known radiative corrections lead to three neutrino masses and three mixing angles making the scheme very predictive.
The $M_{0\nu}$ in eq.(3) is diagonalized to obtain

$$U_0^T M_{0\nu} U_0 = \text{Diag.}(m, -m, -s)$$

with

$$m = \sqrt{s^2 + t^2 + u^2} .$$

The tree level mixing matrix $U_0$ is given by

$$U_0 = \begin{pmatrix}
c_1 & -s_1 & 0 \\
s_1c_2 & c_2c_1 - s_2 \\
s_1s_2 & s_2c_1 & c_2
\end{pmatrix},$$

(6)

$s_i = \sin \theta_i$ and $c_i = \cos \theta_i$. The mixing angles are given by

$$\tan \theta_1 = \sqrt{\frac{m - s}{m + s}},$$

$$\tan \theta_2 = \frac{u}{t} .$$

(7)

The above mass matrix contains many desirable features at the tree level itself.

- The effective neutrino mass probed in the neutrinoless double beta decay is given by

$$m_{ee}^0 = s$$

- At the tree level, there is only one ($\text{mass})^2$ difference which provides the atmospheric scale

$$\Delta_{0A} = m^2 - s^2 .$$

(8)

The corresponding mixing angle ($\equiv \theta_{0A}$) coincides with $\theta_2$ and is large when $t \sim u$:

$$\sin^2 2\theta_{0A} = \sin^2 2\theta_2 .$$

(9)

- Eq.(3) already incorporates the constraint of CHOOZ [7] since it predicts $(U_0)_{e3} = 0$. This prediction would receive a calculable radiative correction making it possible to predict $U_{e3}$. 
There is no solar splitting at this stage but the would-be solar mixing angle is given by

$$\tan^2 \theta_{0S} = \tan^2 \theta_1 = \frac{\sqrt{\Delta_{0A} + (m_{ee}^0)^2} - m_{ee}^0}{\sqrt{\Delta_{0A} + (m_{ee}^0)^2} + m_{ee}^0},$$

(10)

It is seen that the solar mixing angle is determined in terms of the atmospheric scale and the effective neutrino mass $m_{ee}^0$ at tree level. We will see that the radiative corrections generate the solar splitting but do not change the above angle appreciably.

The presence of equal and opposite neutrino masses implies a Dirac neutrino and hence a $U(1)$ symmetry at tree level. This symmetry is however broken by the standard charged current interactions as can be seen from the general expressions given in [11]. Thus the degenerate pair would split due to radiative interactions. This splitting can be easily obtained [11] using the relevant renormalization group equations [12]. The consequences of these equations have been discussed in a number of papers [13].

The radiatively corrected neutrino mass matrix is given by

$$M_{\nu} = I_g I_t \left( \begin{array}{ccc} I_{\tilde{e}} & 0 & 0 \\ 0 & I_{\tilde{\mu}} & 0 \\ 0 & 0 & I_{\tilde{\tau}} \end{array} \right) M_{0\nu} \left( \begin{array}{ccc} I_{\tilde{e}} & 0 & 0 \\ 0 & I_{\tilde{\mu}} & 0 \\ 0 & 0 & I_{\tilde{\tau}} \end{array} \right),$$

(11)

where

$$I_{\tilde{\alpha}} \equiv 1 + \delta_{\alpha},$$

with

$$\delta_{\alpha} \approx c \left( \frac{m_{\alpha}}{4\pi v} \right)^2 \ln \frac{M_X}{M_Z}.$$  

(12)

$M_X$ here corresponds to a large scale, $c = \frac{1}{2}, -\frac{1}{\cos^2 \beta}$ in respective cases of the standard model and the minimal supersymmetric standard model [13] and $\alpha = e, \mu, \tau$. $I_{g,t}$ are calculable coefficients summarizing the effect of the gauge and the top quark corrections.

Apart from the overall factor $I_g I_t$, the radiative corrections are largely dominated by the $\tau$ Yukawa couplings and it is easy to determine neutrino mixing angle and masses keeping only $\delta_{\tau}$ corrections and working to the lowest order in $\delta_{\tau}$. We now have

$$U^T M_{\nu} U = \text{Diag.}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}),$$
with

\[ m_{\nu_1} \approx I_g I_t \left( m + \frac{\delta_r u^2}{m + s} \right), \]
\[ m_{\nu_2} \approx I_g I_t \left( -m - \frac{\delta_r u^2}{m - s} \right), \]
\[ m_{\nu_3} \approx I_g I_t \left( -s - \frac{2\delta_r s l^2}{m^2 - s^2} \right). \tag{13} \]

The tree level mixing matrix \( U_0 \) gets modified to a general \( U \):

\[
U = \begin{pmatrix}
    c_1 c_3 & -s_1 c_3 & s_3 \\
    c_1 s_2 s_3 + c_2 s_1 & c_2 c_1 - s_1 s_2 s_3 & -s_2 c_3 \\
    -c_1 c_2 s_3 + s_1 s_2 & s_1 s_3 c_2 + s_2 c_1 & c_2 c_3
\end{pmatrix}, \tag{14}
\]

As before the angles, \( \theta_1, \theta_2 \) respectively correspond to the solar and atmospheric mixing angles. These are now given by

\[
\tan \theta_A \approx \tan \theta_{0A} (1 - \delta_r), \\
\tan^2 \theta_S \approx \tan^2 \theta_{0S}, \tag{15}
\]

where \( \theta_{0A} \) (eq.(9) and \( \theta_{0S}(eq.(10)) \) denote the tree level solar mixing angle. The effective neutrino mass \( m_{ee}^0 \) is now corrected to

\[
m_{ee} = I_g I_t \ s. \tag{16}\]

The atmospheric scale also receives radiative corrections and is now given by

\[
\Delta_A \equiv \frac{1}{2} (m_{\nu_1}^2 + m_{\nu_2}^2) - m_{\nu_3}^2 = I_g^2 I_t^2 \left( \Delta_{0A} + \delta_r (2 \sin^2 \theta_{0A} \Delta_{0A} - m_{ee}^0 \cos 2\theta_{0A}) \right). \tag{17}
\]

It is seen that all the tree level predictions receive small radiative corrections. Hence the basic ansatz is stable against radiative corrections unlike some of the ansatz considered in [13]. The non-trivial effect of the radiative corrections is generation of the solar splitting and a non-zero \( U_{e3} \):

\[
\Delta_S \equiv m_{\nu_2}^2 - m_{\nu_3}^2 \approx 2\delta_r m_{ee} \sqrt{\Delta_A + m_{ee}^2} \sin^2 2\theta_{0A}, \\
s_3 \equiv U_{e3} \approx \frac{\delta_r m_{ee}}{\sqrt{\Delta_A}} \sin 2\theta_{0A}. \tag{18}
\]

Both \( \Delta_S \) and \( U_{e3} \) are correlated with the effective neutrino mass \( m_{ee} \). This correlation is easy to understand. The neutrino mass matrix in eq.(5) coincides with \( L_e - L_\mu - L_\tau \).
symmetric structure proposed in many works \cite{13} when $s$ (and hence $m_{ee}$) is zero. Since this symmetry is also preserved by the diagonal charged lepton masses one cannot generate the solar splitting radiatively in this case. The presence of $s$ in the ansatz simultaneously leads to non-zero $m_{ee}$ and $\Delta_S$ resulting in their correlation.

**Phenomenology:** It is possible to subject our ansatz to stringent phenomenological tests since three basic parameters predict six observables namely, $m_{ee}$, $U_{e3}$, $(\Delta_A, \theta_A)$ and $(\Delta_S, \theta_S)$. The oscillation interpretation of the atmospheric neutrinos constrain $(\Delta_A, \theta_A)$ over a narrow range

$$\Delta_A \approx (1.5 - 5.0) \cdot 10^{-3} \text{eV}^2 \quad ; \quad \sin^2 2\theta_A \approx 0.8 - 1$$

The presently available results on solar neutrinos allow various possibilities \cite{14}. The most preferred solution based on the global analysis of data corresponds to large mixing angle solution (LMA) which involves relatively larger $\Delta_S$ and $\theta_S$. The next best one corresponds to LOW and quasi-vacuum oscillation (QVO) region. The small mixing solution is least preferred. The standard model restricted fit to all the solar data rules out this solution at 99.73% CL but this it is still allowed in a more general analysis including variations in the boron and/or hep neutrino fluxes. We summarize below inference based on the analysis of the observed rates in various experiments \cite{14}.

| No | Solution  | $\Delta_S$                      | $\tan^2 \theta_S$ |
|----|-----------|---------------------------------|--------------------|
| 1  | LMA       | $(1 - 50) \cdot 10^{-5} \text{eV}^2$ | 0.2 - 0.7          |
| 2  | SMA       | $(5 - 10) \cdot 10^{-6} \text{eV}^2$ | $(8 - 20) \cdot 10^{-4}$ |
| 3  | LOW-QVO   | $(3 \cdot 10^{-9} - 3 \cdot 10^{-7}) \text{eV}^2$ | 0.6-2.0            |
| 4  | VAC       | $(1 - 2) \cdot 10^{-10} \text{eV}^2$ | 0.2-0.6; 1.2-1.5   |

**Table:** The allowed ranges in parameters $\Delta_S$ and $\tan^2 \theta_S$ following from analysis of the solar rates \cite{14}. All the quoted ranges are at 95% CL.

Note that the angle $\tan^2 \theta_S$ is required to be less than one for most solutions. This is crucial in distinguishing two cases namely the standard model and the MSSM. These two cases give different signs for $\Delta_S$ due to difference in signs of the radiative corrections in these cases. $\Delta_S$ is positive (negative) in case of the SM (MSSM) when $m_{ee}$ is positive. Conventionally, the
\( \tan^2 \theta_S \) is allowed to be greater than one in the analysis of the solar data but \( \Delta_S \) is assumed positive. In the case of MSSM, positive values of \( \Delta_S \) correspond to \( m_{ee} < 0 \) and negative \( m_{ee} \) implies \( \tan^2 \theta_S > 1 \) from eq. (10). Because of this reason, the radiative corrections in MSSM are not suitable in describing the solar data and we will restrict ourselves to the case of the SM in the following.

We show in Fig. 1 the predicted values for \( \Delta_S, \tan^2 \theta_S \) and \( |U_{e3}| \) as a function of the effective mass \( m_{ee} \) for maximal atmospheric neutrino mixing and for two extreme values for the atmospheric mass scale. The numerical predictions include sub-dominant corrections from the electron and muon couplings also. We have varied \( m_{ee} \) from the maximum value of \( \mathcal{O}(\text{eV}) \) to the value \( \sim 10^{-3} \) which the future experiment [16] will probe. There are two regions corresponding to \( m_{ee} \approx 0.001 - 0.05 \text{eV} \) and \( m_{ee} \approx 0.05 - 1.0 \text{eV} \). The former region corresponds to \( \Delta_S \approx 10^{-9} - 10^{-7} \text{eV}^2 \) and \( \tan^2 \theta_S \approx 0.7 - 1.0 \). This region encompasses the LOW-QVO and vacuum oscillation solutions. The other region of \( m_{ee} \) corresponds to the range reported in [3]. In this region only the SMA solution can be realized. As follows from eqs. (10,18), \( \Delta_S (\tan^2 \theta_S) \) decreases (increases) with decreasing \( m_{ee} \). As a result, there does not exist a value for \( m_{ee} \) which can simultaneously reproduce the \( \Delta_S \) and \( \tan^2 \theta_S \) required to realize the most preferred LMA solution. The predicted values for \( U_{e3} \) is below the expected experimental sensitivity for the entire range in \( m_{ee} \).

**Summary:** Possibly large value for the effective majorana mass for the electron neutrino observed in the neutrinoless double beta decay experiment points to non-hierarchical neutrino masses which do not arise in many of the conventional schemes such as the seesaw model or supersymmetric model with \( R \) parity violation. We have proposed an economical ansatz for the neutrino mass matrix to describe almost degenerate neutrino masses [5]. Justification of this ansatz from simple symmetry may require an elaborate model. The proposed ansatz differs in spirit from many textures discussed recently [2,4]. Unlike in these works, we discuss a well-defined mechanism built into our ansatz and leading to generation of the solar scale and \( U_{e3} \) through the standard model radiative corrections. This makes the ansatz testable. The ansatz has stringent predictions for the solar neutrino solution which can be used to rule it out. These predictions correspond to SMA solution if \( m_{ee} \) is in the range reported in [3] or to vacuum or the LOW-QVO solution if \( m_{ee} \) is much smaller. Observation of sizable \( U_{e3} > 10^{-3} \) would also go against the ansatz.

\(^1 \)For positive \( m_{ee} \) one needs to reverse the role of \( \nu_1 \) and \( \nu_2 \). The relevant \( \tan^2 \theta_S \) is inverse of eq. (10) and is also greater than 1.
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FIG. 1. $10^6 \Delta S$ in (eV$^2$) (dotted line), $\tan^2 \theta_S$ (solid line) and $10^3|U_{e3}|$ (dashed line) shown as a function of $m_{ee}$ (in eV). The labels A and B correspond to $\sqrt{\Delta A} = 0.03$ eV and 0.07 eV respectively. The atmospheric mixing is assumed maximal.