Longitudinal/Goldstone Boson Equivalence and Phenomenology
of Probing the Electroweak Symmetry Breaking

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Abstract

We formulate the Equivalence between the longitudinal weak-boson and the
Goldstone boson as a criterion for sensitively probing the electroweak symmetry
breaking mechanism and develop a precise power counting rule for chiral La-
grangian formulated electroweak theories. With these we semi-quantitatively
analyze the sensitivities to various effective operators related to electroweak
symmetry breaking via weak-boson scatterings at the CERN Large Hadron
Collider (LHC).

Recent LEP/SLC experiments can test the electroweak (EW) theory to the accu-

racy of one-loop corrections, and support the spontaneously broken \( SU(2) \times U(1) \)
gauge theory as the correct theory of the EW interactions. However, light Higgs boson
has not been found, and the current experiments are insensitive to the spontaneous
symmetry breaking (SSB) sector of the theory, compatible with a wide range of the
Higgs boson mass \( 60\text{GeV} \leq m_H \leq 1\text{TeV} \). So the SSB mechanism in the EW theory is
still a mystery, and it is thus important to probe all possible SSB mechanisms: either
weakly or strongly interacting.

We know that only the longitudinal component \( V_L^a \) of the weak-boson \( V^a \) \((W^\pm, Z^0)\)
(arising from “eating” the would-be Goldstone boson (GB)) is sensitive to the SSB

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sector, while the transverse component $V_T^a$ is not. The physical $V_L^a$ scattering amplitude is quantitatively related to the corresponding GB amplitude by the electroweak Equivalence Theorem (ET)\textsuperscript{1-3} which comes from the following ET identity
\begin{equation}
T[V_L^{a_1}, \ldots, V_L^{a_n}; \Phi_\alpha] = C \cdot T[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + B,
\end{equation}
\begin{equation}
C \equiv \prod_{a_1}^{a_n} \frac{C_{mod}}{C_{mod}^{a_1}} \cdot \ldots \cdot \frac{C_{mod}^{a_n}}{C_{mod}^{a_n}}, \quad B \equiv \sum_{l=1}^{l=1} \left( \prod_{a_1}^{a_n} \frac{C_{mod}^{a_l+1}}{C_{mod}^{a_l}} \cdot \ldots \cdot \frac{C_{mod}^{a_n}}{C_{mod}^{a_l}} \cdot T[v^{a_1}, \ldots, v^{a_l}, -i\pi^{a_l+1}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + \text{permutations of } v's \text{ and } \pi's \right),
\end{equation}
where $\pi^a$'s are GB fields, and $\Phi_\alpha$ denotes other possible physical in/out states.

For strongly interacting SSB models, the $V_L^a$-amplitude on the L.H.S. of (1) is experimentally measurable, while the GB-amplitude on the R.H. S. of (1), though not directly measurable, carries the information about the SSB mechanism. Similar to $V_T^a$, the $B$-term in (1) is not sensitive to the SSB mechanism. If, under certain conditions, the $B$-term can be neglected, (1) reveals the equivalence between the $V_L^a$-amplitude and the GB-amplitude. In this case the $V_L^a$-scattering experiments can be used to sensitively and unambiguously probe the SSB mechanism. When $B$ is not negligible, measurements of the $V_L^a$, $V_T^a$ and $B$ amplitudes with higher precision will be required for probing the SSB mechanism, and those experiments at LHC will be harder.

The conditions for neglecting the $B$-term in (1), i.e. the condition for the validity of the ET, is actually subtle. We first note that the spin-0 GB's are invariant under the proper Lorentz transformations, while, on the contrary, $V_L$, $V_T$ and $B$ are Lorentz non-invariant. Therefore the ratio of the $B$-magnitude relative to the GB-amplitude in (1) is Lorentz frame dependent. So neglecting $B$ makes sense only if the Lorentz frame belongs to a group of frames within which Lorentz transformation does not significantly enhance $B$. We call such frames safe frames. The condition for a Lorentz frame to be safe is given in Ref.3, which is
\begin{equation}
E_j \sim k_j \gg M_W, \quad (j = 1, 2, \ldots, n),
\end{equation}
where $E_j$ is the energy of the $j$-th external $V_L^a$-line. For a given process, $E_j$ can be easily obtained from the kinematics. So this condition is a convenient criterion for judging whether the experimental center of mass frame is safe or not for a given process, i.e. it can discriminate processes which are not sensitive for probing the SSB mechanism. With this consideration, the ET can be precisely formulated as
\begin{equation}
T[V_L^{a_1}, \ldots, V_L^{a_n}; \Phi_\alpha] = C \cdot T[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + O(M_W/E_j-suppressed), \quad (3a)
\end{equation}
\textsuperscript{3}See the example given in Ref.3.

See the example given in Ref.3.
\[ E_j \sim k_j \gg M_W, \quad (j = 1, 2, \ldots, n) \]  

\[ B \ll C \cdot T[-i\pi^a_1, \ldots, -i\pi^a_n; \Phi_\alpha]. \]  

(3b) and (3c) are the conditions for neglecting the \( B \)-term in (1) (for the validity of the ET), or the conditions for sensitively probing the SSB mechanism via \( V_L^a \)-scattering experiments. Here we see the profound physical content of the ET, i.e. ET is not merely a tool for simplifying calculations.

The next thing is to realize the quantitative meaning of the condition (3c). To a given order \( N \) in a perturbative expansion, the amplitude \( T \) can be written as \( T = \sum_{l=0}^{N} T_l \) with \( T_0 > T_1, \ldots, T_N \). Let \( T_{\text{min}} = \{T_0, \ldots, T_N\}_{\text{min}} \). Then, to the precision of \( T_{\text{min}} \), condition (3c) precisely implies\(^3\)

\[ B \approx O\left(\frac{M^2_W}{E^2_\pi}\right) T_0[-i\pi^a_1, \ldots, -i\pi^a_n; \Phi_\alpha] + O\left(\frac{M_W}{E_\pi}\right) T_0[V^a_1, -i\pi^a_2, \ldots, -i\pi^a_n; \Phi_\alpha] \]

\[ \ll T_{\text{min}}[-i\pi^a_1, \ldots, -i\pi^a_n; \Phi_\alpha]. \]  

(4)

In the chiral Lagrangian formulated EW theory (CLEWT), the \( O(E^2) \) leading amplitude \( T_0 \) is model-independent. Thus, for probing the SSB mechanism, we should take into account the next-to-leading \( O(E^4) \) amplitude \( T_1 \), i.e. \( T_{\text{min}} = T_1 \). By means of our power counting rule (6), we can estimate that for leading contributions, \( T_1 = O\left(\frac{E^4_\pi}{f^4_\pi} f^{4-n}_\pi\right) \) and \( B = O\left(g^2 f^{4-n}_\pi\right) \) \( ^b \). Thus condition (4) requires \( \frac{M^2_W}{E^2_\pi} \ll \frac{1}{4} \xi^2 \), or \( (0.7 \text{ TeV}/E)^4 \ll 1 \). So the probe is generally sensitive when \( E \geq 1 \) TeV which is possible at the LHC.

In the CLEWT, the Lagrangian can be written in the following form\(^6\)

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)} + \sum_{n=1}^{14} \mathcal{L}_n = \sum_n \ell_n \frac{f_\pi r_n}{\Lambda a_n} \mathcal{O}_n(W_{\mu\nu}, B_{\mu\nu}, D U, f, \bar{f}), \]

(5)

where \( \mathcal{L}_G, \mathcal{L}_F \) are the kinetic terms of the gauge fields and fermions. The explicit formula for \( \mathcal{L}_{\text{eff}} \) is given in Ref.5~6, in which \( \mathcal{L}^{(2)}, \mathcal{L}^{(2)\prime}, \mathcal{L}_{1-11} \) are \( CP \) conserving, and \( \mathcal{L}_{12-14} \) are \( CP \) violating. Here, the dimensionless coefficients \( \ell_n \)'s can be naturally regarded as of \( O(1) \). In Ref.5, we developed the following power counting rule in the CLEWT for the S-matrix element \( T \)

\[ T = c_T f_\pi D_T \left(\frac{f_\pi}{\Lambda}\right)^{N_\mathcal{O}} \left(\frac{E}{f_\pi}\right)^{D_{E0}} \left(\frac{E}{\Lambda}\right)^{D_{EL}} \left(\frac{M_W}{E}\right)^{E_\pi} H(\ln E/\mu), \]

\[ N_\mathcal{O} = \sum_n a_n, \quad D_{E0} = 2 + \sum_n V_n(d_n + \frac{1}{2} f_n - 2), \quad D_{EL} = 2L, \]

where the dimensionless coefficient \( c_T \) contains possible powers of gauge couplings \( (g, g') \) and Yukawa couplings \( (y_f) \) from the vertices in \( T \), which can be easily

\(^b\)In the CLEWT, \( f_\pi = 246 \text{ GeV} \) and the effective cut-off \( \Lambda \simeq 4 \pi f_\pi \simeq 3.1 \text{ TeV} \).
Table 1. Contributions of the model-dependent operators to the $W^\pm W^\pm \to W^\pm W^\pm$ amplitudes

| Operators | $T_1[4\pi]$ | $T_1[3\pi, W_T]$ | $T_1[2\pi, 2W_T]$ | $T_1[\pi, 3W_T]$ | $T_1[4W_T]$ |
|-----------|-------------|-----------------|-----------------|-----------------|-------------|
| $\mathcal{L}^{(2)\prime}$ | $\ell_0 \frac{E^2}{\Lambda}$ | $\ell_0 g \frac{L}{E} \frac{E}{\Lambda}$ | $\ell_0 g^2 \frac{L}{E} \frac{E}{\Lambda}$ | $\ell_0 g^3 \frac{L}{E} \frac{E}{\Lambda}$ | / |
| $\mathcal{L}_{1,13}$ | / | $\ell_{1,13} c^3 g \frac{L}{E} \frac{E}{\Lambda}$ | $\ell_{1,13} c^4 \frac{L}{E} \frac{E}{\Lambda}$ | $\ell_{1,13} c^5 g \frac{L}{E} \frac{E}{\Lambda}$ | $\ell_{1,13} c^6 g^2 \frac{L}{E} \frac{E}{\Lambda}$ |
| $\mathcal{L}_2$ | $\ell_2 c^3 \frac{E}{\Lambda}$ | $\ell_2 c^4 \frac{E}{\Lambda}$ | $\ell_2 c^5 \frac{E}{\Lambda}$ | $\ell_2 c^6 \frac{E}{\Lambda}$ | $\ell_2 c^7 \frac{E}{\Lambda}$ |
| $\mathcal{L}_3$ | $\ell_3 c^2 \frac{E}{\Lambda}$ | $\ell_3 c^3 \frac{E}{\Lambda}$ | $\ell_3 c^4 \frac{E}{\Lambda}$ | $\ell_3 c^5 \frac{E}{\Lambda}$ | $\ell_3 c^6 \frac{E}{\Lambda}$ |
| $\mathcal{L}_{4,5}$ | $\ell_{4,5} \frac{E}{\Lambda}$ | $\ell_{4,5} \frac{E}{\Lambda}$ | $\ell_{4,5} \frac{E}{\Lambda}$ | $\ell_{4,5} \frac{E}{\Lambda}$ | $\ell_{4,5} \frac{E}{\Lambda}$ |
| $\mathcal{L}_{6,7,10}$ | / | / | / | / | / |
| $\mathcal{L}_{8,14}$ | / | $\ell_{8,14} c^3 \frac{E}{\Lambda}$ | $\ell_{8,14} c^4 \frac{E}{\Lambda}$ | $\ell_{8,14} c^5 \frac{E}{\Lambda}$ | $\ell_{8,14} c^6 \frac{E}{\Lambda}$ |
| $\mathcal{L}_9$ | $\ell_9 c^2 \frac{E}{\Lambda}$ | $\ell_9 c^3 \frac{E}{\Lambda}$ | $\ell_9 c^4 \frac{E}{\Lambda}$ | $\ell_9 c^5 \frac{E}{\Lambda}$ | $\ell_9 c^6 \frac{E}{\Lambda}$ |
| $\mathcal{L}_{11,12}$ | / | $\ell_{11,12} c^3 \frac{E}{\Lambda}$ | $\ell_{11,12} c^4 \frac{E}{\Lambda}$ | $\ell_{11,12} c^5 \frac{E}{\Lambda}$ | $\ell_{11,12} c^6 \frac{E}{\Lambda}$ |

determined from the vertices. $H$ is a function of $\ln(E/\mu)$ insensitive to $E$ where $\mu$ denotes the relevant renormalization scale. $d_n$ is the number of derivatives in the type-$n$ vertex, $\nu_n$ is the number of type-$n$ vertices in $T$, $f_n = 2i_F + e_F$ is the number of fermion fields.

With this counting rule, we can estimate the sensitivities to probing specific operators in (5) via various $W-W$ scattering amplitudes. In Table-1, we list the results in the important $W^\pm W^\pm$ channel as a typical example. We first see that $\mathcal{L}_{6,7,10}$ do not contribute to this channel. Table-1 then shows that the $4W_L^\pm$ channel can probe $\mathcal{L}_{4,5}$ most sensitively, while the contributions of $\mathcal{L}^{(2)\prime}, \mathcal{L}_{2,3,9}$ to this channel lose $E$-power dependence by a factor-$2$. This channel cannot probe $\mathcal{L}_{1,8,11-14}, \mathcal{L}_{1,8,11-14}$ can only be probed via channels with $W_T^\pm$’s, among which $\mathcal{L}_{11,12}$ are most dominant though they are still suppressed by a factor $g f_\pi/E$ relative to the leading contributions to the $4W_L^\pm$ channel. $\mathcal{L}_{1,8,13,14}$ are generally suppressed by higher powers of the factor $g f_\pi/E$ and are thus less sensitive. For a more complete classification, see Table-3 in Ref.5.

We have further calculated the number of events per $[100\text{fb}^{-1} \cdot \text{GeV}]$ at the LHC from our counting rule (6) combined with the effective-$W$ approximation. We have compared them with the corresponding available explicit calculations in Ref.8 for a few typical examples. The comparison shows that the deviations are reasonably within a factor-2 which is of the same order as the uncertainty of the effective-$W$ approximation. Therefore our power counting rule does give correct semi-quantitative results and is thus very useful and convenient for making a systematical analysis for the sensitivities to probing the SSB mechanism at the LHC and future linear colliders. In the typical case with $\ell_n \sim O(1)$, the number of the LHC events for the $W^+W^+$ channel are shown in Fig.1. By comparing with the events
from $B$, we see that the probe of $L_{4,5}$ are most sensitive, that of $L_{3,9,11,12}$ are marginal, and that of $L^{(2)}, L_{1,2,8,13,14}$ are insensitive. More of the details are given in Ref.5.

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Fig. 1 Sensitivities of operators $\mathcal{L}^{(2)}/\mathcal{L}_{1,14}$ with $\ell_n \sim O(1)$, at the 14 TeV LHC.