Orientation Parameters of the Cepheid System in the Galaxy

V.V. Bobylev

Pulkovo Astronomical Observatory, St. Petersburg, Russia
Sobolev Astronomical Institute, St. Petersburg State University, Russia

Abstract—Based on the distribution of long-period Cepheids, we have redetermined the orientation parameters of their principal plane in the Galaxy. Based on 299 Cepheids with heliocentric distances \( r < 20 \text{ kpc} \) and pulsation periods \( P \geq 5 \text{ d} \), we have found the directions of the three principal axes of the position ellipsoid: \( L_1 = 281.0 \pm 0.1^\circ \), \( B_1 = -1.9 \pm 0.1^\circ \), \( L_2 = 11.0 \pm 0.7^\circ \), \( B_2 = 0.2 \pm 0.1^\circ \) and \( L_3 = 275.9 \pm 0.7^\circ \), \( B_3 = 88.1 \pm 0.1^\circ \). Thus, the line of nodes \( l_\Omega = L_3 + 90^\circ = 5.9^\circ \) is very close to the direction to the Galactic center; the Cepheid symmetry plane is inclined to the Galactic plane approximately by \( -2^\circ \) in the direction of the first axis \( (L_1) \). The direction of the line of nodes found from old Cepheids \( (P < 5 \text{ d}) \) differs significantly and is \( l_\Omega = 298^\circ \). The elevation of the Sun above the Galactic plane has been estimated from 365 closer stars \( (r < 4 \text{ kpc}) \) without any constraint on the pulsation period to be \( h_\odot = 23 \pm 5 \text{ pc} \).

INTRODUCTION

Cepheids play a very important role in studying the Galactic structure. Their number increases; the calibration of the period–luminosity relation needed to determine their distances is improved. This has become possible owing to the fact that the trigonometric parallaxes have been measured for several Cepheids (Fouquè et al. 2007). Using infrared photometry allowed the interstellar extinction to be taken into account much more accurately (Berdnikov et al. 2000).

The layer of neutral hydrogen in the Galaxy is known to be warped at large Galactocentric distances (Westerhout 1957; Barton et al. 1988). Hydrogen rises above the Galactic plane in the second quadrant and goes below it in the third and fourths quadrants. The results of a study of this structure based on currently available data on the HI and HII distributions are presented in Kalberla and Dedes (2008) and Cersosimo et al. (2009), respectively. This structure is revealed from the spatial distribution of stars and dust (Drimmel and Spergel 2001), from the distribution of pulsars (Yusifov 2004), from OB stars (Miyamoto and Zhu 1998), and from the 2MASS red giant clump (Momany et al. 2006). The Cepheid system exhibits a similar feature (Fernie 1968; Berdnikov 1987).

Several models were proposed to explain the nature of the Galactic warp: (1) the interaction between the disk and a nonspherical dark matter halo (Sparke and Casertano 1988); (2) the gravitational influence from the closest satellites of the Galaxy (Bailin 2003); (3) the interaction of the disk with the circumgalactic flow of high-velocity hydrogen clouds.
produced by mass transfer between the Galaxy and the Magellanic Clouds (Olano 2004); (4) the intergalactic flow (López-Corredoira et al. 2002); and (5) the interaction with the intergalactic magnetic field (Battaner et al. 1990).

The goal of this paper is to redetermine the orientation of the Cepheid system in our Galaxy. For this purpose, we apply a method that allows this problem to be solved in a general form. Studying the dependence of the derived parameters on the stellar age and distance is also of great importance.

DATA

Here, we use Cepheids of the Galaxy’s flat component classified as DCEP, DCEPS, CEP(B), CEP in the GCVS (Kazarovets et al. 2009) as well as CEPS used by other authors. To determine the distance based on from the period–luminosity relation, we used the calibration from Fouqué et al. (2007): \( \langle M_V \rangle = -1.275 - 2.678 \log P \), where the period \( P \) is in days. Given \( \langle M_V \rangle \), taking the period-averaged apparent magnitudes \( \langle V \rangle \) and extinction \( A_V = 3.23E(\langle B \rangle - \langle V \rangle) \) mainly from Acharova et al. (2012) and, for several stars, from Feast and Whitelock (1997), we determine the distance \( r \) from the relation

\[
r = 10^{-0.2(\langle M_V \rangle - \langle V \rangle - 5 + A_V)}.
\]

For a number of Cepheids (without extinction data), we used the distances from the catalog by Berdnikov et al. (2000), which were determined from infrared photometry.

With the goals of our study in mind, we concluded that it was better not to use several stars lying higher than 2 kpc above the Galactic plane and those located deep in the Galaxy’s inner region. Thus, we used two limitations,

\[
|Z| < 2 \text{ kpc}, \quad X < 6 \text{ kpc},
\]

satisfied by 465 Cepheids. Their distributions in projections onto the Galactic XY, XZ, and YZ planes are shown in Figs. 1–3.

Several calibrations proposed to estimate the Cepheid ages are known. Here, we use the calibration by Efremov (2003),

\[
\log t = 8.50 - 0.65 \log P,
\]

derived from Cepheids belonging to open clusters of the Large Magellanic Cloud.

THE METHOD

We apply the well-known method of determining the symmetry plane of a stellar system with respect to the principal (in our case, Galactic) coordinate system. The basics of this approach were described by Polak (1935), and the technique for estimating the errors in the angles can be found in Parenago (1951) and Pavlovskaya (1971).

In the rectangular coordinate system centered on the Sun, the \( x \) axis is directed toward the Galactic center, the \( y \) axis is in the direction of Galactic rotation \( (l = 90^\circ, b = 0^\circ) \),
and the $z$ axis is directed toward the North Galactic Pole. Then,

$$
\begin{align*}
x &= r \cos l \cos b, \\
y &= r \sin l \cos b, \\
z &= r \sin b.
\end{align*}
$$

Let $m, n,$ and $k$ be the direction cosines of the pole of the sought-for great circle from the $x, y,$ and $z$ axes. The sought-for symmetry plane of the stellar system is then determined as the plane for which the sum of the squares of the heights, $h = mx + ny + kz,$ is at a minimum:

$$\sum h^2 = \text{min}.$$  

The sum of the squares

$$h^2 = x^2m^2 + y^2n^2 + z^2k^2 + 2yznk + 2xzk + 2xym$$

can be designated as $2P = \sum h^2.$ As a result, the problem is reduced to searching for the minimum of the function $P$:

$$2P = am^2 + bn^2 + ck^2 + 2fnk + 2ekm + 2dmn,$$

where the second-order moments of the coordinates $a = [xx], b = [yy], c = [zz], f = [yz], e = [xz], d = [xy], \text{written via the Gauss brackets, are the components of a symmetric tensor:}$

$$\begin{pmatrix}
a & d & e \\
d & b & f \\
e & f & c
\end{pmatrix},$$

whose eigenvalues $\lambda_{1,2,3}$ are found from the solution of the secular equation

$$\begin{vmatrix}
a - \lambda & d & e \\
d & b - \lambda & f \\
e & f & c - \lambda
\end{vmatrix} = 0,$$

and the directions of the principal axes $L_{1,2,3}$ and $B_{1,2,3}$ are found from the relations

$$\tan L_{1,2,3} = \frac{ef - (c - \lambda)d}{(b - \lambda)(c - \lambda) - f^2},$$

$$\tan B_{1,2,3} = \frac{(b - \lambda)e - df}{f^2 - (b - \lambda)(c - \lambda) \cos L_{1,2,3}}.$$  

The errors in $L_{1,2,3}$ and $B_{1,2,3}$ are estimated according to the following scheme:

$$
\begin{align*}
\varepsilon(L_2) &= \varepsilon(L_3) = \frac{\varepsilon(xy)}{a - b}, \\
\varepsilon(B_2) &= \varepsilon(\varphi) = \frac{\varepsilon(xz)}{a - c}, \\
\varepsilon(B_3) &= \varepsilon(\psi) = \frac{\varepsilon(yz)}{b - c}, \\
\varepsilon^2(L_1) &= \frac{\varphi^2 \varepsilon^2(\psi) + \psi^2 \varepsilon^2(\varphi)}{(\varphi^2 + \psi^2)^2}, \\
\varepsilon^2(B_1) &= \frac{\sin^2 L_1 \varepsilon^2(\psi) + \cos^2 L_1 \varepsilon^2(L_1)}{(\sin^2 L_1 + \psi^2)^2},
\end{align*}
$$

3
Figure 1: Distribution of Cepheids in projection onto the Galactic XY plane. The Sun is at the intersection of the dotted lines; the dashed line indicates the circle of radius $R_0 = 8$ kpc around the Galactic center. The filled circles are long-period Cepheids ($P \geq 5^d$); the small gray circles are short-period Cepheids ($P < 5^d$); the circles mark the three Cepheids with large $Z$ discussed in the text.

where

\[
\varphi = \cot B_1 \cos L_1, \quad \psi = \cot B_1 \sin L_1.
\]

The three quantities $\overline{x^2y^2}$, $\overline{x^2z^2}$ and $\overline{y^2z^2}$, should be calculated in advance. Then,

\[
\begin{align*}
\vartheta^2(xy) &= (x^2y^2 - d^2)/n, \\
\vartheta^2(xz) &= (x^2z^2 - e^2)/n, \\
\vartheta^2(yz) &= (y^2z^2 - f^2)/n,
\end{align*}
\]

where $n$ is the number of stars. Thus, the algorithm for solving the problem consists in setting up the function $2P (7)$, seeking for the roots of the secular equation (9), whose specific values are of no interest to us, and estimating the directions of the principal axes of the position ellipsoid from Eqs. (10)–(12). In the classical case, the problem was solved for a unit sphere ($r = 1$), but here we propose to use the distances (which act as the weights).

**RESULTS**

Based on the entire sample of Cepheids (465 stars), whose distribution in the Galaxy is shown in Figs. 1–3, we found the following directions of the principal axes of the position
Figure 2: Distribution of Cepheids in projection onto the Galactic XZ plane. The Galactic center is on the left (at X = 8 kpc). The notation is the same as that in Fig. 1.

ellipsoid:

\[
\begin{align*}
L_1 &= 278.96 \pm 0.05^\circ, & B_1 &= -1.33 \pm 0.00^\circ, \\
L_2 &= 8.93 \pm 0.43^\circ, & B_2 &= 1.41 \pm 0.04^\circ, \\
L_3 &= 232.37 \pm 0.43^\circ, & B_3 &= 88.06 \pm 0.07^\circ.
\end{align*}
\]

The mean age for this sample of Cepheids is \( t = 98 \) Myr. According to solution (13), the slope of the solid line in Fig. 3 corresponds to \((90^\circ - B_3)\). For comparison, we present the results of our calculations for the distribution on a unit sphere \((r = 1)\) obtained using the same stars:

\[
\begin{align*}
L_1 &= 283.32 \pm 0.02^\circ, & B_1 &= -0.66 \pm 0.00^\circ, \\
L_2 &= 13.32 \pm 0.67^\circ, & B_2 &= -0.37 \pm 0.03^\circ, \\
L_3 &= 312.95 \pm 0.67^\circ, & B_3 &= 89.24 \pm 0.03^\circ,
\end{align*}
\]

which differ significantly from the parameters of solution (13). Although the errors in the unknowns \( B_{1,2,3} \) in solution (14) are smaller, the angles \( B_{1,2,3} \) are also considerably smaller than those in solution (13). However, it can be clearly seen from Fig. 3 that the slope of \(2^\circ\) is better applicable to the data than \( \approx 0.5^\circ \) (as follows from solution (14)). This discrepancy decreases if the stars are considered in narrow distance ranges. Below, we consider only the results of the solutions with distances.

Distant stars make a major contribution to solution (13) (they have the largest weights). The solutions obtained from distant stars \((3 < r < 20 \text{ kpc})\) with different
Figure 3: Distribution of Cepheids in projection onto the Galactic $YZ$ plane. The slope of the solid line is $2^\circ$. The notation is the same as that in Fig. 1.

Pulsation periods (ages) are of interest. For the youngest stars,
\begin{align*}
L_1 &= 279.6 \pm 0.3^\circ, \quad B_1 = -2.1 \pm 0.1^\circ, \\
L_2 &= 9.9 \pm 1.3^\circ, \quad B_2 = 0.6 \pm 0.1^\circ, \\
L_3 &= 262.8 \pm 1.3^\circ, \quad B_3 = 87.8 \pm 0.3^\circ, \\
&\quad P \geq 9^d, \\
&\quad n_\star = 63, \\
&\quad \bar{t} = 54 \text{ Myr};
\end{align*}
(15)

for middle-age stars,
\begin{align*}
L_1 &= 284.5 \pm 0.2^\circ, \quad B_1 = -1.4 \pm 0.1^\circ, \\
L_2 &= 14.5 \pm 2.2^\circ, \quad B_2 = 0.3 \pm 0.1^\circ, \\
L_3 &= 272.3 \pm 2.2^\circ, \quad B_3 = 88.5 \pm 0.2^\circ, \\
&\quad 5^d \leq P < 9^d, \\
&\quad n_\star = 51, \\
&\quad \bar{t} = 96 \text{ Myr};
\end{align*}
(16)

for the oldest stars:
\begin{align*}
L_1 &= 261.6 \pm 0.4^\circ, \quad B_1 = -0.9 \pm 0.0^\circ, \\
L_2 &= 351.6 \pm 1.0^\circ, \quad B_2 = 3.2 \pm 0.4^\circ, \\
L_3 &= 188.0 \pm 1.0^\circ, \quad B_3 = 86.7 \pm 0.1^\circ, \\
&\quad P < 5^d, \\
&\quad n_\star = 63, \\
&\quad \bar{t} = 133 \text{ Myr}.
\end{align*}
(17)

In contrast to the results of solutions (15)–(16), which, on the whole, agree between themselves, the oldest Cepheids give a significantly different orientation of the line of
nodes, \( l_\Omega = L_3 + 90^\circ = 278^\circ \). This is no surprise, because old Cepheids have had time to recede from their birthplace, they made more than half of their revolution around the Galactic center, i.e., they were formed in a different part of the Galaxy (for example, with respect to the Magellanic Clouds). The surprising thing is that a slope of \( \approx 3^\circ \) is present in their distribution (the angles \( B_2 \) and \( B_3 \)). This finding may imply that the disk warp can be a long-lived structure, at least older than \( \approx 150 \text{ Myr} \).

The roots of the secular equation (9) describe the shape of the ellipsoid but provide no information about the coordinates of its center. The shift along the \( z \) coordinate, \( h_\odot = -\pi \), is most interesting.

Three stars with very large \( Z \) are marked in Figs. 1–3. These are two long-period Cepheids, DR Cep (\( P = 19.8^d, Z = 1.9 \text{ kpc} \)) and FQ Lac (\( P = 11.3^d, Z = -1.3 \text{ kpc} \)), and one short-period Cepheid, IT Lac (\( P = 2.6^d, Z = -1.0 \text{ kpc} \)). Unfortunately, as yet no information about their radial velocity measurements is available for them. Their proper motions from the UCAC4 catalog (Zacharias et al. 2013) are very unreliable. This is because at such large distances (\( r \approx 15 \text{ kpc} \)), a typical error \( e_\mu \approx 2 \text{ mas yr}^{-1} \) will contribute \( 4.74re_\mu \approx 140 \text{ km s}^{-1} \) to the space velocity, which is very much. Therefore, it is not yet possible to judge the character of their velocities. These stars most likely have peculiar velocities. For this reason, we decided not to use these stars to determine the orientation parameters.

Since the results of solutions (15)–(16) do not differ greatly, the interval of periods can be combined. Based on 299 stars (\( r < 20 \text{ kpc} \)) with pulsation periods \( P \geq 5^d \) (\( \bar{t} = 77 \text{ Myr} \)), we have now found

\[
\begin{align*}
L_1 &= 281.0 \pm 0.1^\circ, \quad B_1 = -1.9 \pm 0.1^\circ, \\
L_2 &= 11.0 \pm 0.7^\circ, \quad B_2 = 0.2 \pm 0.1^\circ, \\
L_3 &= 275.9 \pm 0.7^\circ, \quad B_3 = 88.1 \pm 0.2^\circ,
\end{align*}
\]

(18)

the line of nodes \( l_\Omega = L_3 + 90^\circ = 5.9^\circ \) is close to the direction to the Galactic center. The parameters (18) agree satisfactorily with the results of analyzing the layer of neutral hydrogen (Kalberla and Dedes 2008). It should be noted that the distances to hydrogen clouds are estimated from the radial velocities (kinematic distances) with a low accuracy; in our case, however, the accuracy is higher, because the distance error is, on average, 10–15%. Therefore, our results are of indubitable interest.

Based on 163 stars (\( r < 20 \text{ kpc} \)) with pulsation periods \( P < 5^d \) (\( \bar{t} = 138 \text{ Myr} \)), we found

\[
\begin{align*}
L_1 &= 249.5 \pm 0.4^\circ, \quad B_1 = -2.1 \pm 0.1^\circ, \\
L_2 &= 339.4 \pm 1.9^\circ, \quad B_2 = 1.9 \pm 0.2^\circ, \\
L_3 &= 208.1 \pm 1.9^\circ, \quad B_3 = 87.2 \pm 0.1^\circ,
\end{align*}
\]

(19)

the direction of the line of nodes is \( l_\Omega = 298^\circ \).

The elevation of the Sun above the Galactic plane \( h_\odot \) depends on the heliocentric distance, which is clearly seen from Figs. 2, 3. Our calculations show that a sample with a radius of 4–5 kpc is optimal (the error in \( h_\odot \) is smallest). For example, based on a sample of the closest (71 stars) Cepheids from the range \( r \leq 1 \text{ kpc} \) (with the rejection according to the 3\( \sigma \) criterion), we found

\[ h_\odot = 30 \pm 9 \text{ pc}, \]

(20)
but the influence of nonuniformities in the distribution of stars is great here. Based on 365 stars from the range \( r \leq 4 \) kpc, we found

\[
h_{\odot} = 23 \pm 5 \text{ pc},
\]

while based on the remaining 100 stars from the range \( 4 \text{ kpc} < r < 20 \text{ kpc} \), we found

\[
h_{\odot} = 45 \pm 39 \text{ pc}.
\]

**DISCUSSION**

Fernie (1968) estimated the direction of the line of nodes for the Cepheid system, \( 7^\circ \pm 4^\circ \), the inclination \(-0.8^\circ \pm 0.2^\circ \) in the direction \( l = 277^\circ \), and the elevation of the Sun above the Galactic plane, \( h_{\odot} = 45 \pm 15 \) pc from 328 stars. Since the sample of stars in Fernie (1968) probably contained quite a few old Cepheids, the inclination turned out to be small, and the determination of \( h_{\odot} \) was affected by distant Cepheids. Berdnikov (1987) found \( h_{\odot} = 26 \pm 6 \) pc from 363 stars, in good agreement with our result (21). Our value of \( h_{\odot} \) (21) is also in good agreement with \( h_{\odot} = 17 \pm 3 \) pc obtained by Joshi (2007) from young open star clusters and OB stars.

A discussion of the results of analyzing the warp of the hydrogen layer obtained from neutral, ionized, and molecular hydrogen can be found, for example, in Cersosimo et al. (2009). Hydrogen reaches its maximum elevations above the Galactic plane \( z = \) from +300 to +400 pc (at \( R \approx 12 \text{ kpc} \)) in the first and second quadrants and \( z = \) from −150 to −200 pc in the third and fourth quadrants (i.e., the warp is nonlinear). It can be seen from Fig. 3 that even after the elimination of the above three stars, the dispersion of the positions is larger at positive \( y \) (on the left in the Fig. 3); on average, the heights of the stars are larger than those of hydrogen. At positive \( y \), there are also Cepheids with negative \( z \). This can be related to their ages; having been formed \( \approx 50 \) Myr ago, they could be displaced below the plane in such a time. To confirm this assumption, it is necessary to analyze the space velocities of Cepheids, which we plan to do in another paper.

On the whole, we can conclude that the connection of Cepheids with the Galactic warp is beyond doubt.

**CONCLUSIONS**

Based on the distribution of Cepheids, we redetermined the orientation parameters of their principal plane in the Galaxy. Based on 299 stars at heliocentric distances \( r < 20 \text{ kpc} \) with pulsation periods \( P \geq 5^d \) we found the directions of the three principal axes of the position ellipsoid (solution (18)):

\[
L_1 = 281.0 \pm 0.1^\circ, B_1 = -1.9 \pm 0.1^\circ, L_2 = 11.0 \pm 0.7^\circ, B_2 = 0.2 \pm 0.1^\circ \text{ and } L_3 = 275.9 \pm 0.7^\circ, B_3 = 88.1 \pm 0.1^\circ. \]

The line of nodes \( l_\Omega = L_3 + 90^\circ = 5.9^\circ \) is very close to the direction to the Galactic center; the Cepheid symmetry plane is inclined to the Galactic plane approximately by \(-2^\circ \) in the direction of the first axis \( (L_1) \).

The oldest Cepheids (163 stars at \( r < 20 \text{ kpc} \) with pulsation periods \( P < 5^d \)) give a significantly different orientation of the line of nodes (solution (19)):

\[
L_1 = 249.5 \pm 0.4^\circ, B_1 = -2.1 \pm 0.1^\circ, L_2 = 339.4 \pm 1.9^\circ, B_2 = 1.9 \pm 0.2^\circ \text{ and } L_3 = 208.1 \pm 1.9^\circ, B_3 =
\]
$87.2 \pm 0.1^\circ$, The direction of the line of nodes $l_\Omega = 298^\circ$ differs approximately by $65^\circ$ from that obtained from a sample of younger Cepheids.

The Sun’s elevation above the Galactic plane was estimated from 365 stars at $r < 4 \text{ kpc}$ without any constraint on the pulsation period $P$ to be $h_\odot = 23 \pm 5 \text{ pc}$.

In future, the method considered here can be useful for analyzing large amounts of data, for example, those from the GAIA space experiment or masers with their trigonometric parallaxes measured by means of VLBI.

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