Chaos Analysis of Fractional-order System with Delay Based on Adomian Decomposition

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Abstract. In this paper, fractional-order Lü chaotic system with nonlinear delay term is proposed, and the fractional-order Lü chaotic system with time-delay is numerically decomposed by Adomian decomposition. At the same time, the influence of fractional-order number on the system is verified. Then the bifurcation diagram, complexity and attractor phase diagram of the system are analyzed by Matlab software. The numerical simulation results show that the 0.9-order system has rich dynamic characteristics, which provides a theoretical basis for parameter selection when fractional order delay system is applied to multimedia encryption.

1. Introduction
In 1695, German mathematician Leibniz first proposed the related concept of fractional-order to French mathematician L'Hôpital in his letter [1]. In 1977, the French-American mathematician Mandelbrot discovered that fractal characteristics are common in nature when studying the complex boundaries of coastlines [2], the fractal theory provides theoretical support for fractional order. In 1963, Lorenz, an American meteorologist, first proposed the chaos model. Since then, chaos theory has developed rapidly from dynamics and time series. In recent years, many scholars have proposed many fractional-order chaotic systems from integer-order chaotic systems, such as fractional-order Chen system [3-4], fractional-order Lü system [5], fractional-order Lorenz system [6], etc [7-11].

The research direction of fractional-order chaotic dynamic system is mainly focused on the chaotic dynamic characteristics of the system and synchronization control research. For example, in [10], the complex frequency domain method (FDM) is adopted, i.e. the basic principle of Laplace transform is adopted to convert the fractional-order chaotic system into a high-dimensional (high-order) integer-order chaotic system approximately. Such methods will take into account the error and the frequency range when converting the pull-type change, thus causing large errors. In [10], the nonlinear dynamics of fractional Lorenz system is analyzed. It is found that the bifurcation diagram and Lyapunov exponent spectrum of the system are different under different fractional orders. The fractional order system is realized by chain fractional order module circuit. The circuit realization result is basically consistent with the numerical simulation result. When Adomian decomposition method (ADM) is applied to fractional order system, the nonlinear term of the system needs to be decomposed, and the decomposed term is numerically solved by Matlab simulation. In [11-12], the dynamics of fractional order Lü and simplified fractional order Lorenz chaotic system are analyzed by ADM, and ADM algorithm is implemented by digital processor (DSP), which provides a new research idea for fractional order chaotic system to be applied to image, voice and signal encryption. In [4], the classical predictor-corrector method is used to solve and synchronize the fractional order Chen chaotic system,
and the synchronization control circuit of the fractional order chaotic system is designed. The circuit simulation results verify the feasibility of the synchronization control algorithm.

In this paper, fractional-order delay Lü system is taken as the research object, and the improved Adomian decomposition method is adopted to carry out numerical calculation and simulation on the system, and the phase diagram of the system is obtained. In order to study the characteristics of the system under the change of parameters, the bifurcation diagram and complexity diagram of the 0.9 order delay Lü system are drawn by Matlab software. The numerical simulation results lay a foundation for the application of the system to parameter adjustment in multimedia encryption.

2. Fractional Lü chaotic System with Nonlinear Delay Term

Lü Jinhu et al. proposed Lü system. Based on this, the dynamic equation of fractional delay Lü system is proposed as follows:

\[
\begin{align*}
\frac{d^{q_1}x}{dt^{q_1}} &= a(y - x) \\
\frac{d^{q_2}y}{dt^{q_2}} &= cy - xz \\
\frac{d^{q_3}z}{dt^{q_3}} &= x(t - \tau)y - bz
\end{align*}
\]  

(1)

Where \( x, y, z \) is variable and \( a, b, c \) is parameter of system.

The form of the solution of the equation is similar to that of the equation in reference [11], and is not given in this paper.

Where \( c_i^j = c_i^j(t - \tau) \), let \( m = \left[ \frac{t}{h} \right] \), when \( t_i \leq mh \),

\[
c_i^j = (1 - m + \frac{t}{h})c_i^j(t_{i-m}) + (m - \frac{t}{h})c_i^j(t_{i-m})
\]  

(2)

When \( t_i > mh \),

\[
c_i^j = (1 - m + \frac{t}{h})c_i^j(t_{i-m}) + (m - \frac{t}{h})c_i^j(t_{i-m-1})
\]  

(3)

Therefore, the analytical solution of the system is obtained as follows:

\[
x_j(n) = \sum_{i=0}^{6} c_i^j(t - t_0)^{i} i! q^i
\]  

(4)

When \( a = 30, b = 2.93, c = 22.2, q_1 = q_2 = q_3 = 0.95, \tau = 0.02 \), According to equation (4), the analytical solution of the system can be obtained, and numerical simulation is carried out by Matlab. It is found that chaotic attractors exist in the phase diagram of the system (1), as shown in Figure 1.

\[\begin{align*}
-30 &-20 \\
-10 &0 \\
10 &20 \\
30 &40
\end{align*}\]

(a) Phase diagram of \( x - y \)

\[\begin{align*}
-30 &-20 \\
-10 &0 \\
10 &20 \\
30 &40
\end{align*}\]

(b) Phase diagram of \( x - y \)

Figure 1. The chaotic attractor of system
3. Bifurcation Diagram and Complexity of Fractional Delay Lü System

In order to study the influence of system parameters on fractional delay Lü chaotic system, this paper uses bifurcation diagram and system complexity method to study the chaotic characteristics of the system.

3.1 Variation of parameter q

Fixed parameters $a = 30, b = 2.93, c = 22.2, \tau = 0.02, q \in [0.7, 1]$, the bifurcation diagram and complexity of the system are shown in Figure 2.

As can be seen from figure 2, when $q \in [0.7, 0.73]$ the fractional order delay Lü system is in a divergent state, and the bifurcation diagram cannot obtain the maximum value in this interval, so there is a blank and the corresponding complexity in this region cannot be analyzed. The systems in other regions are all in chaotic state, and the corresponding values of system complexity are relatively high. At the same time, it can be seen that the overall system complexity is in a process of decreasing as the fractional order of the system increases. In the application of image encryption or communication encryption, the system with high complexity has better encryption effect, so fractional order system is the best choice.

![Bifurcation diagram](image)

Figure 2. Bifurcation Diagram and Complexity of System (1) with q Variation

3.2 Variation of parameter a

Fixed parameters $b = 2.93, c = 22.2, q_1 = q_2 = q_3 = q = 0.95, \tau = 0.02, a \in [20, 50]$, the bifurcation diagram and complexity of the system are shown in Figure 3.

As can be seen from figure 3, fractional Lü system with time delay enters chaotic region in the form of standard period doubling bifurcation (PDB). When $a \in [20, 27.9]$, it belongs to a periodic state, as can be seen from the bifurcation diagram, the complexity of the system at this time is relatively small. When $a \in [27.9, 43]$, the system is chaotic, and the complexity of the system is about 0.65 at this time. When $a \in (43, 50]$, the system is in a periodic state, Figure 4(a) is period 1, figure 4(b) is period 2.
4. Conclusion

Based on the Adomian decomposition method, this paper studies the dynamics of fractional delay Lü chaotic systems. The Adomian decomposition method is used to analyze the numerical simulation of attractor, bifurcation diagram and complexity of the system. The mixed oscillation characteristics of 0.95 order delay Lü system are studied. At the same time, it is concluded that the complexity of fractional-order delay systems decreases with the increase of fractional-order in a certain range, thus further verifying the significance of studying fractional-order complexity. The probability of chaos in fractional-order systems is greater than that in integer order systems. This conclusion lays a foundation for the control of fractional-order delay Lü systems and their applications in chaotic cryptography, digital circuits and other fields.

Acknowledgments

This work is supported the Natural Science Foundation of Shandong Province (Grant No. ZR2017PA008), National natural science foundation (Grant No.61371163), Shandong Province Key Research and Development Plan (Grant No.2019GGX104092).

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