Quantum computation with three-electron double quantum dots at an optimal operation point

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The author analyzes quantum computation with the hybrid qubit (HQ) that is encoded using the three-electron configuration of a double quantum dot. All gate operations are controlled with electric signals, while the qubit remains at an optimal operation point that is insensitive to noise. An effective single-qubit description is derived, and two-qubit interactions are suggested using Coulomb and exchange interactions. Universal quantum control is described and numerically simulated using realistic parameters for HQs in Si and GaAs. High-fidelity quantum computing at the threshold of quantum error correction is possible if the Coulomb interactions between the HQs stay weak.

I. INTRODUCTION

Spin qubit quantum computers are promising platforms to achieve quantum computation.\textsuperscript{[4]} The electron spin of a gate-defined quantum dot (QD) naturally defines a two-level quantum system that encodes one bit of quantum information.\textsuperscript{[5]} Also a duo\textsuperscript{[6]} a trio\textsuperscript{[7]} and a quartet\textsuperscript{[8]} of such singly occupied QDs have been proposed as realizations of quantum bits. The so-called hybrid qubit (HQ) was introduced because the HQ is either a spin or a charge qubit depending on its operation principle. A spin qubit is well protected from charge noise, but typical spin qubits cannot be operated all-electrically. A charge qubit can be manipulated with electric fields, but it is therefore also susceptible to charge noise.

The HQ can be operated exclusively with electric signals similar to the triple QD (TQD) qubit that is coded using three singly occupied QDs.\textsuperscript{[9]} Because subnanosecond controls of the electrostatic potentials of QDs can be realized,\textsuperscript{[10,11]} electric manipulations of QD qubits are always more favorable than magnetic manipulations. Even though single spins can also be controlled with magnetic field pulses,\textsuperscript{[12,13]} it is difficult to selectively control a single spin with magnetic fields when this spin is in the vicinity of other spins.\textsuperscript{[14]} Electric fields can also modify single spins indirectly (e.g. via spin-orbit interactions or magnetic field gradients), but high-fidelity gates still remain challenging.\textsuperscript{[15,17]}

Most conveniently, the HQ is manipulated by the transfer between the $(n_{\text{QD}_1}, n_{\text{QD}_2}) = (1, 2)$ and (2,1) configurations,\textsuperscript{[18,21]} similar to a charge qubit.\textsuperscript{[22,23]} The transfer is tunable. To employ these gates experimentally, one has to build setups that have weak Coulomb interactions between the HQs, but it is much larger for the other one.

This paper studies DQDs with asymmetric addition energies in the layout of Fig. 1. QD\textsubscript{2} always has a small singlet-triplet energy difference, but the singlet-triplet energy difference difference is large for QD\textsubscript{1}. It will be shown that the HQ has a small energy difference in the (1, 2) configuration. When approaching the (2, 1) configuration, the qubit states pass through two avoided level crossings. It has been described theoretically\textsuperscript{[9]} and shown experimentally\textsuperscript{[19,20]} that these two anticrossings are sufficient for the single-qubit control of the HQ with the appropriate tuning protocols through the anticrossings.

It is also possible to operate the HQ in the vicinity of an anticrossing, where all the qubit operations are realized with microwave pulses of small amplitudes.\textsuperscript{[21]} An anticrossing is a sweet spot in the energy diagram where low-frequency noise in the control parameter does not dephase the qubit. Such sweet-spot operations have improved the coherence times for superconducting qubits.\textsuperscript{[25,27]} Note that microwave gates have also been realized for other spin qubit encodings, as for DQDs\textsuperscript{[25]} and TQDs.\textsuperscript{[28,29]}

For HQs, the entangling operations remain more challenging than the single-qubit gates. Ref. \textsuperscript{[9]} proposed that strong electrostatic couplings can be used to entangle two HQs. Also the rapid transfer of electrons between the HQs has been studied.\textsuperscript{[31]} In this paper, I show that the HQ can always be operated at its sweet spot while weak couplings between HQs enable two-qubit gates. I quantify the inter-qubit couplings with Coulomb and exchange interactions, and show that both interactions enable two-qubit gates.

The main findings of this paper are explicit manipulation protocols for HQs of high fidelities, while all the HQs are operated at their sweet spots. I simulate the gate operations with typical qubit parameters and show that these gates tolerate the dominant noise sources of HQs. At the sweet spot, the HQ keeps enough character of a spin qubit to be protected from charge noise, but it is already sufficiently close to a charge qubit to be rapidly tunable. To employ these gates experimentally, one has to build setups that have weak Coulomb interactions be-
The organization of the paper is as follows. Sec. II introduces the description of the HQ, and it specifies the magnitudes of the relevant parameters. The single-qubit control, the readout, and the initialization are also described. Sec. III derives the effective Hamiltonians for the couplings between HQs that are generated from Coulomb and exchange interactions. Sec. IV specifies two approaches to realize universal control of the HQ. Not only entangling operations are discussed, but it is also shown that single-qubit states are possible. Sec. V discusses the influence of electric and magnetic noise for HQs that are operated at their sweet spots. Sec. VI summarizes the findings.

II. SINGLE-QUBIT DESCRIPTION

The HQ is coded using a DQD that is occupied with three electrons, as shown in Fig. 1. The qubit manipulations require a much larger singlet-triplet energy splitting of a doubly occupied QD for QD1 compared to QD2. I consider only the subspace encoding in the $S = 1/2$, $s_z = 1/2$ spin configuration. Even though a global magnetic field is not required to realize single-qubit gates, it will be necessary to apply magnetic fields to initialize a HQ to $S = 1/2$, $s_z = 1/2$ QD1 can be used for the initialization of a singlet because a two-electron configuration quickly equilibrates to a singlet under thermal relaxation. In the presence of an external magnetic field, also the spin-up configuration at QD2 can be initialized because it has lower energy than the spin-down configuration.

The HQ is described in the basis of the lowest states with $S = 1/2$, $s_z = 1/2$. One considers $|x\rangle = |S\uparrow\rangle$ in the (2, 1) configuration, and the states $|1\rangle \propto \sqrt{2}(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle) - |S\uparrow\uparrow\rangle$ and $|0\rangle = |\uparrow\uparrow\rangle$ in the (1, 2) configuration. The singlet $|S\rangle \propto |\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle$ and the triplets $|T_+\rangle = |\uparrow\uparrow\rangle$, $|T_0\rangle \propto |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, and $|T_-\rangle = |\downarrow\downarrow\rangle$ describe the two-electron configurations at a QD. The three-electron states with $S = 1/2$, $s_z = 1/2$ are defined using the standard spin addition rules. The Hamiltonian for the HQ in the vicinity of $\epsilon_s$ has the form:

$$H_{\{|x\rangle, |1\rangle, |0\rangle\}} = \begin{pmatrix} \epsilon & -\Delta_1 & \Delta_0 \\ -\Delta_1 & \delta & 0 \\ \Delta_0 & 0 & 0 \end{pmatrix}. \quad (1)$$

The Coulomb repulsion is high for a doubly occupied QD in the triplet configuration, which raises the energy of $|1\rangle$ compared to $|0\rangle$ by $\delta$. $|x\rangle$ and $|1\rangle$ ($|x\rangle$ and $|0\rangle$) are coupled via the real parameter $\Delta_1 > 0$ ($\Delta_0 > 0$), which is called the tunnel coupling in the following. $\epsilon$ is the detuning parameter between the (2, 1) and (1, 2) configurations. The electron configuration in (2, 1) $|\{1\}\rangle$ has lower energy for $\epsilon < 0$ ($\epsilon > 0$), and (2, 1) and (1, 2) have equal energies at $\epsilon = 0$.

Eq. (1) describes a three level system with two anticrossings. Fig. 2 shows the energy levels for typical qubit parameters in the transition region between (2, 1) and (1, 2). The two states with the lowest energies encode the HQ. For $\epsilon \ll 0$, only the ground state is in (2, 1), while the first excited state is in (1, 2). This configuration is ideal for the readout of the HQ because a nearby charge detector can distinguish the two states of the HQ, similar to the readout of a singlet-triplet qubit. For $\epsilon \gg 0$, the HQ is in (1, 2) and the energy difference between the qubit states is unchanged when $\epsilon$ is slightly

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Figure 1. Setup of two HQs. Each HQ is coded using the three electron configuration of a DQD. The superscripts L and R label the positions of the HQs; the subscripts 1 and 2 label the QDs.

Figure 2. Energy diagram of the relevant states to describe the HQ in the $S = 1/2$, $s_z = 1/2$ spin configuration of a three-electron DQD, according to Eq. (1), for $\Delta_1/h = 14.5$ GHz, $2\Delta_0/h = 5.2$ GHz, and $\delta/h = 12.1$ GHz. $\epsilon$ models the energy detuning between the (2, 1) and (1, 2) charge configurations. The HQ is coded using the two states with the lowest energies. The HQ is in (1, 2) for $\epsilon \to \infty$ with the excited state $|\{1\}\rangle \propto \sqrt{2}(|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle)$ and the ground state $|\{0\}\rangle = |\uparrow\uparrow\rangle$. Only the ground state $|\{x\}\rangle = |S\uparrow\rangle$ is in (2, 1) for $\epsilon \to -\infty$, while the excited state remains in (1, 2). There are two states couplings $|\{1\}\rangle \leftrightarrow |\{x\}\rangle$ and $|\{0\}\rangle \leftrightarrow |\{x\}\rangle$ at $\epsilon \simeq 0$. The gray line shows the $S = 3/2$, $s_z = 1/2$ state (called $|\{y\}\rangle$) in (1, 2) that is uncoupled to (2, 1). All gate operations for the HQ are realized at the sweet spot $\epsilon_s$, where the qubit is insensitive to small variations in $\epsilon$. The dashed lines approximate the qubit levels in the vicinity of $\epsilon_s$ [see Eq. (3)]. The parameters $\Delta_1$ and $\Delta_0$ are defined in the text.
modified. This regime is highly protected from charge noise \[^{[9][10]}\]. Note that there is additionally one \(S = 3/2\), \(s_z = 1/2\) state in the \((1, 2)\) configuration (called \(|y\rangle\)) that is uncoupled to the \((2, 1)\) configuration.

I consider a HQ with similar magnitudes of the singlet-triplet energy splitting at QD\(_2\) \(\Delta\) and the tunnel couplings \(\Delta_1\) and \(\Delta_0\). \(\Delta_1 \geq \Delta_0\). The singlet-triplet energy splitting at QD\(_1\) is much larger than \(\Delta\) such that doubly occupied triplets at QD\(_1\) are neglected. Note that such HQs have been realized using Si DQDs \[^{[12][21]}\] where the singlet-triplet energy difference depends on the orbital and the valley energy splittings. The orbital energy splitting is determined by the confining potential of a QD, while the valley energy splitting is determined by the potential landscape at the atomic scale. A recent experiment showed that the valley splitting of a Si QD can be controlled electrically. \[^{[11]}\] Note that HQs can also be built using GaAs QDs, but these setups necessarily require asymmetric DQD potentials to operate them with the equivalent tunneling pulses. In this case, the smaller QD has a large singlet-triplet splitting; the larger or elliptic QD has a small singlet-triplet splitting. \[^{[12][13]}\] The parameters of previous experiments with Si HQs are used in the following: \(2\Delta_1/\hbar = 14.5\) GHz, \(2\Delta_0/\hbar = 5.2\) GHz, and \(\delta/\hbar = 12.1\) GHz (cf. Refs. \[^{[19][20][21]}\]).

\[
\Delta_1 \text{ in Eq. (1) is caused by the coupling to an excited orbital, which is larger than the ground state orbital that determines } \Delta_0. \text{ Tunnel couplings are exponentially suppressed with the distance between localized orbitals. Therefore, } \Delta_1 \text{ is significantly larger than } \Delta_0, \text{ and it is meaningful to analyze Eq. (1) in the eigenbasis of the larger anticrossing. The states are rotated to } |\vec{x}\rangle = \cos(\theta_e/2) |x\rangle + \sin(\theta_e/2) |1\rangle \text{ and } |\vec{y}\rangle = -\sin(\theta_e/2) |x\rangle + \cos(\theta_e/2) |1\rangle, \text{ with } \theta_e = -2\Delta_1/U_e, \cos(\theta_e) = (\epsilon - \Delta)/U_e, \text{ and } U_e = \sqrt{(2\Delta_1)^2 + (\epsilon - \delta)^2}. \text{ This basis rotation modifies Eq. (1) to}
\]

\[
H_{\{\vec{x}, \vec{y}\}, \{0\}} = \begin{pmatrix}
\frac{\epsilon + \delta - U_e}{2} + \frac{\Delta_0}{2} \\
\frac{-\epsilon - \delta - U_e}{2} - \frac{\Delta_0}{2} \\
\frac{\epsilon - \delta - U_e}{2} - \frac{\Delta_0}{2} \\
\frac{\epsilon + \delta + U_e}{2} - \frac{\Delta_0}{2}
\end{pmatrix}
\]

The coupling of \(|\vec{x}\rangle\) to \(|\{\vec{y}\}, \{0\}\rangle\) is neglected in Eq. (2) to describe the low-energy subspace that defines the HQ. It will be shown that the latter subspace has an anticrossing at \(\epsilon_* = \Delta_1^2/\delta\), while \(U_e \gg \Delta_0\). The two-level system \(|\{\vec{y}\}, \{0\}\rangle\) is approximated by:

\[
H_{\{\vec{y}, \vec{y}\}, \{0\}} \approx \frac{\Delta_0}{2} \sigma_z + A(\epsilon) \sigma_x,
\]

with \(\Delta_0 = 2\Delta_0 \sin(\theta_e/2)\). \(A(\epsilon) = \frac{\delta^2}{2(\delta^2 + \Delta_0^2)} (\epsilon - \epsilon_*)\), and \(\theta_e = \theta_{x*}\). To derive Eq. (3), the dependency of \(\sin(\theta_e/2)\) on \(\epsilon\) is neglected because this term varies slowly in the vicinity of \(\epsilon_*\). Then the qubit’s energy splitting \(\sqrt{(\epsilon + \delta - U_e/2)^2 + [\Delta_0 \sin(\theta_e/2)]^2}\) within \(|\vec{y}\rangle, |0\rangle\) is minimized with respect to \(\epsilon\), giving the first term in Eq. (3), with \(\Delta_0 = 2\Delta_0 \sin(\theta_e/2)\) for \(\epsilon_* \approx \Delta_1^2/\delta\). The second term in Eq. (3) is obtained from \(\partial_\epsilon [\frac{\epsilon - \delta - U_e}{2} + \partial_\epsilon [\Delta_0 \sin(\theta_e/2)]^2]_{\epsilon = \epsilon_*}\), giving \(\frac{\delta^2}{2(\delta^2 + \Delta_0^2)}\).

The basis in Eq. (3) has been rotated to \(|\vec{T}\rangle = \frac{1}{\sqrt{2}} \left[-\sin\left(\frac{\theta_e}{2}\right) |x\rangle + \cos\left(\frac{\theta_e}{2}\right) |1\rangle + |0\rangle\right]\), \(|\vec{U}\rangle = \frac{1}{\sqrt{2}} \left[\sin\left(\frac{\theta_e}{2}\right) |x\rangle + \cos\left(\frac{\theta_e}{2}\right) |1\rangle - |0\rangle\right]\), with the definitions of the Pauli operators \(\sigma_x = |\vec{T}\rangle \langle \vec{T}| - |\vec{U}\rangle \langle \vec{U}|\) and \(\sigma_y = |\vec{T}\rangle \langle \vec{U}| + |\vec{U}\rangle \langle \vec{T}|\). Note that \(|\vec{T}\rangle\) and \(|\vec{U}\rangle\) have finite contributions in \((2, 1)\) and \((1, 2)\). \(\theta_e \approx 1.34\pi\) is the mixing angle, with \(\cos^2(\theta_e/2) \approx 0.26\) and \(\sin^2(\theta_e/2) \approx 0.74\). Fig. (2) proves that the effective two-level system in Eq. (3) describes the HQ close to \(\epsilon_*\). The leakage state \(|\vec{x}\rangle\) is raised by \(\Delta_1 = \Delta_1^2/\delta\). \(\Delta_0 = 2\Delta_0 \sin(\theta_e/2)\) is the energy difference between \(|\vec{T}\rangle\) and \(|\vec{U}\rangle\).

Eq. (3) permits the usual Rabi control of the qubit with microwave drives of the detuning parameter \(\epsilon\).

If \(\epsilon\) is driven with small amplitudes around \(\epsilon_*\), \(A(\epsilon) \approx A(\cos(2\pi\Omega t/\hbar + \phi))\), then all possible single-qubit operations can be realized for \(\Omega = \Delta_0\) when the phase \(\phi\) is varied. Eq. (3) gives in the rotating frame with \(\frac{\Delta_0}{2} \sigma_z\) the static Hamiltonian

\[
H_{\text{raa}} = A(\cos(\phi) \sigma_x + \sin(\phi) \sigma_y).
\]

Eq. (6) uses implicitly a rotating wave approximation, which is valid for \(\Delta_0 \gg A\).

### III. TWO-QUBIT INTERACTIONS

This section describes the interactions between two HQs. The superscripts L and R label the positions of the HQs. QD\(_2^L\) and QD\(_2^R\) are the neighboring QDs from HQ\(_L\) and HQ\(_R\) (cf. Fig. 1).

#### A. Capacitive Coupling

The charge configurations of two HQs couple capacitively via the Coulomb interaction. The dominant contribution is determined by the electron configurations at QD\(_2^L\) (n\(_{QD_2^L}\)) and QD\(_2^R\) (n\(_{QD_2^R}\)) according to

\[
C = \kappa n_{QD_2^L} n_{QD_2^R}.
\]

The magnitude of the coupling parameter \(\kappa\) depends on the layout of the experiment and the QD material. \(\kappa\) can be large; e.g., one can approximate its magnitude by the Coulomb interaction of two electric point charges that
are 250 nm apart, giving \( \kappa \approx \frac{e^2}{\epsilon_r \epsilon_0 |r|} \sim 500 \, \text{meV} \) for the dielectric constant \( \epsilon_r = 11.7 \) of silicon. This value agrees with the approximation of a few tenth of meV in Ref. [13].

The entangling operations that follow require much smaller \( \kappa \) than the naive estimate given above. The environment around the QDs and the metallic gates partly screen the capacitive couplings between the HQs, and thus they reduce \( \kappa \). Also the layout of the four QDs can be designed such that the Coulomb interactions between neighboring HQs are lowered [20]. Finally, bringing QD\(_L^1\) and QD\(_R^1\) further apart lowers \( \kappa \).

For two HQs operated at their sweet spots, \( \epsilon_L^0 \) and \( \epsilon_R^0 \), the projection of Eq. (7) to the subspace \( \{ | \uparrow \downarrow \rangle, | \uparrow \downarrow \rangle, | \downarrow \uparrow \rangle, | \downarrow \uparrow \rangle \} \) gives the two-qubit interaction

\[
\mathcal{C}_{2Q} = X_L \sigma^L_x \sigma^R_x, \tag{8}
\]

with \( X_L = -\frac{2}{5} \sin^2 \left( \theta^L/2 \right) \sin^2 \left( \theta^R/2 \right) \). Eq. (8) lowers the energies of the configurations \( | \uparrow \downarrow \rangle, | \uparrow \downarrow \rangle \) and \( | \downarrow \uparrow \rangle, | \downarrow \uparrow \rangle \) compared to \( | \uparrow \rangle \downarrow - | \downarrow \rangle \uparrow \) and \( | \downarrow \rangle \uparrow - | \uparrow \rangle \downarrow \). The first states have large weights in (2, 1, 2, 1) or (1, 2, 1, 2), while the latter states have large weights in (1, 2, 2, 1) or (2, 1, 1, 2).

Additionally to Eq. (8), there are also the single-qubit interactions

\[
\mathcal{C}_{1Q} = X_L \sigma^L_x \sigma^R_x + X_R \sigma^R_x, \tag{9}
\]

with \( X_L = -\frac{2}{5} \sin^2 \left( \theta^L/2 \right) \sin^2 \left( \theta^R/2 \right) \) and \( X_R = \frac{2}{5} \left[ 1 + \cos^2 \left( \theta^L/2 \right) \sin^2 \left( \theta^R/2 \right) \right] \).

**B. Exchange Coupling**

Encoded qubits in the multielectron regime have a large number of spin states that are not part of the computational subspace [9]. As a consequence, the time evolution out of the computational subspace, which is called leakage, must be considered [9]. Similar to TQDs, inter-qubit tunnel couplings between HQs can cause leakage because the subspaces of different local spin quantum numbers are coupled [9]. It will be shown that the system of two HQs at their sweet spots have large energy separations between leakage and computational states, which suppresses leakage events (cf. similar approaches for TQDs in Refs. [9, 51, 54]).

In the six-electron configuration, only states with total \( s_z = 1 \) are considered because the spin directions are conserved during tunneling events. Tab. I summarizes the spin states with the lowest energies. The computational subspace \( \{ | \uparrow \downarrow \rangle, | \uparrow \downarrow \rangle, | \downarrow \uparrow \rangle, | \downarrow \uparrow \rangle \} \) and the leakage states (called \( | \alpha \rangle, | \beta \rangle, | \gamma \rangle, | \delta \rangle \) and \( | \nu \rangle \) are considered. Only the states with \( S = 1/2 \) are able to couple the (2, 1) and (1, 2) configurations at QD\(_L^1\) and QD\(_R^1\) or at QD\(_L^0\) and QD\(_R^0\). These states have lower energies than the state with \( S = 3/2 \) of the same \( s_z \) (cf. Fig. 2: \( | \uparrow \rangle \) and \( | \downarrow \rangle \) have \( S = 1/2 \), \( | \gamma \rangle = S \) has \( S = 3/2 \)). As a consequence, all the computational states have lower energies than the leakage states (unless \( \delta^L, \delta^R \lesssim \frac{\Delta_0}{4} \)). Fig. 3 shows the energies of the single-qubit states and the leakage states for two identical HQs.

A tunnel coupling between QD\(_L^1\) and QD\(_R^1\) couples the (2, 1) and (1, 2) configurations at these HQs. Similar to the single-qubit interactions in Sec. I, phenomenologically introduce a state coupling between QD\(_L^0\) and QD\(_R^0\):

\[
\mathcal{H}' = - t_1 | \uparrow \rangle \langle \downarrow | \sqrt{\frac{3}{5}} \left( T_{+} \downarrow \right) - \sqrt{\frac{1}{5}} \left( T_{0} \uparrow \right) - t_0 | \uparrow \rangle \langle \downarrow | \langle S \uparrow | + \text{h.c.} \tag{10}
\]

Note that only the states with \( S = 1/2, s_z = 1/2 \) at QD\(_L^1\) and QD\(_R^1\) are coupled in Eq. (10). All triplets at QD\(_R^0\) are neglected because these states have much higher energies.

The setup can be tuned towards the \( (1, 1, 2, 2) \) configuration. Only states with a singlet at QD\(_R^0\) are considered, giving the four states \( \{ | \uparrow \uparrow S \rangle, | \uparrow \uparrow S \rangle, \langle T_{0} \downarrow \downarrow \rangle, \langle T_{0} \downarrow \downarrow \rangle \} \) with \( s_z = 1 \). If the \( (1, 1, 2, 2) \) spin configuration is only virtually excited, then \( \mathcal{H}' \) can be eliminated using Schrieffer-Wolff perturbation theory [52, 53]. Only the effective interactions on the qubit subspace are summarized in the following; Appx. III gives a detailed summary of their derivations.

There is an effective two-qubit interaction

\[
\mathcal{E}_{2Q} = \mathcal{X}_C \sigma^L_x \sigma^R_x + \mathcal{Z}_C \sigma^L_x \sigma^R_x, \tag{11}
\]
Table I. Relevant two-qubit states with total $s_z = 1$ for two HQs. All the states of a three-electron DQD with $S = 1/2$, $s_z = \pm 1/2$ have contributions in the $(2,1)$ and $(1,2)$ configurations. The computational subspace $\{ |T^L \rangle, |T^R \rangle, |\overline{T}^L \rangle, |\overline{T}^R \rangle \}$ is energetically separated from all the leakage states. I use the abbreviations $|v_1 \rangle$ and $|v_0 \rangle$ for the basis states in $S = 1/2$, $s_z = -1/2$ that are obtained from $|T \rangle$ and $|\overline{T} \rangle$ by flipping all the spins. The states $|\alpha \rangle$, $|\beta \rangle$, $|\gamma \rangle$, $|\delta \rangle$, and $|\nu \rangle$ label the leakage states with the energies $\frac{\Delta_0^L}{2} + \delta^L$, $-\frac{\Delta_0^L}{2} + \delta^R$, $\delta^L + \frac{\Delta_0^R}{2}$, $\delta^L - \frac{\Delta_0^R}{2}$, and $\delta^L + \delta^R$.

| state | $(s^L, s_z^L)$ | energy |
|-------|----------------|--------|
| $|T^L \rangle$ | $\left(\frac{1}{2}, \frac{1}{2}\right)$ | $\frac{\Delta_0^L + \Delta_0^R}{2}$ |
| $|T^R \rangle$ | $\left(\frac{1}{2}, \frac{1}{2}\right)$ | $-\frac{\Delta_0^L + \Delta_0^R}{2}$ |
| $|\overline{T}^L \rangle$ | $\left(\frac{1}{2}, \frac{1}{2}\right)$ | $\frac{\Delta_0^L}{2} + \delta^R$ |
| $|\overline{T}^R \rangle$ | $\left(\frac{1}{2}, \frac{1}{2}\right)$ | $-\frac{\Delta_0^L}{2} + \delta^R$ |

with $X_E = \left[\frac{1 - \epsilon_0^2}{4}\right] J_0 + \left[\frac{\epsilon_0^2 (3 - 11 \epsilon_0^2)}{108}\right] J_1$ and $Z_E = \frac{c_L (1 - c_L)}{6} \sqrt{J_0 J_1}$.

with $Z_E^L = \left[\frac{c_L (1 + c_L)}{6}\right] \sqrt{J_0 J_1}$, $X_E^R = \left[\frac{1 + \epsilon_0^2}{4}\right] J_0 - \left[\frac{\epsilon_0^2 (3 - 11 \epsilon_0^2)}{108}\right] J_1$, and $X_E^R = \left[\frac{1 - \epsilon_0^2}{4}\right] J_0 + \left[\frac{c_L (1 - c_L)}{6}\right] J_1$.

Besides Eq. (11) and Eq. (12), the effective Hamiltonian also contains contributions for the leakage states and between leakage and computational states. All these contributions will be included in the numerical calculations that contain inter-qubit exchange interactions. Assuming large energy separations between the computational and leakage states, these interactions introduce minor effects for the qubit’s time evolution, and they can be neglected.

### IV. UNIVERSAL QUBIT CONTROL

I discuss gate operations for the parameters (A) $\Delta_0^L \approx \Delta_0^R$ and (B) $\Delta_0^L \gg \Delta_0^R$. Note that the HQs are always operated at their sweet spots $\epsilon_0^L$ and $\epsilon_0^R$.

#### A. Nearly identical qubits $\Delta_0^L \approx \Delta_0^R$

I consider the Hamiltonian:

$$\mathcal{H}_A = \frac{\Delta_0^L}{2} + Z^L \sigma_z + \frac{\Delta_0^R}{2} \sigma_z + \frac{\Delta_0^L}{2} \sigma_z + \frac{\Delta_0^R}{2} \sigma_z.$$

If HQ$^L$ and HQ$^R$ are in resonance, $Z^L = \Delta_0^R - \Delta_0^L$, then HQ$^L$ and HQ$^R$ entangle under static time evolutions. In
After the time $t = h/(4\Delta X)$, adding a small tunnel coupling between QD$^L$ and QD$^R$ and tuning the spin configuration towards $(1,1,2,2)$ brings HQ$^L$ into resonance with HQ$^R$ (cf. Eq. (12), $Z_L^R = \Delta_R^L - \Delta_R^0$). The HQs entangle after $t = h/(4\Delta X)$ for $\chi = \chi_C$, $\chi_L$, and $\chi_R$. iSWAP shows the infidelity, according to Eq. (A1), of the entangling operation with the given parameters, which is compared to an ideal entangling operation according to Eq. (13). The qubits can be operated independently for $|\Delta_R^L - \Delta_R^0|/h > 20$ MHz. When the inter-qubit exchange interaction increases, HQ$^L$ and HQ$^R$ are brought into resonance to construct high-fidelity entangling operations. Note that also leakage errors increase with the inter-qubit exchange interactions, as discussed in the text.

If the exchange interactions between HQ$^L$ and HQ$^R$ $|Z_L^R|$ in Eq. (12) and $Z_L^L$ in Eq. (13) cancel the detuning between the HQs, then the entangling gate is realized. Simultaneously with the single-qubit energy shift, also $X_L^R$, $X_L^R$, and $X_L^R$ from Eq. (12) increase. These interactions cause systematic gate errors. Fig. 4 shows that the gate errors of the iSWAP below 1% can be realized for $|\Delta_R^L - \Delta_R^0|/h < 20$ MHz. Leakage errors are also caused by the exchange couplings between HQ$^L$ and HQ$^R$, and these leakage events significantly contribute to the gate infidelities. Fig. 5 extracts the leakage errors during an entangling gate with finite tunnel couplings between HQ$^L$ and HQ$^R$. The leakage probability is extracted from the time evolution $\mathcal{U}$ by taking the norm of the matrix $|\mathcal{U}_{P_C,P_L}|^2$ between the states from the computational subspace $P_C$ and the leakage subspace $P_L$.

If $Z_L^L$ is reduced, then HQ$^L$ and HQ$^R$ decouple. Fig. 4 compares the time evolution of Eq. (13) with the time evolution of

$$\mathcal{H}_{\text{ideal}}^{\text{L}} = \frac{X_L^L}{2} \sigma_x^L + \frac{X_L^R}{2} \sigma_y^R. \quad (15)$$

The differences rise significantly if $|\Delta_R^L - \Delta_R^0|$ decreases; but the infidelities stay below 1% for $|\Delta_R^L - \Delta_R^0|/h > 20$ MHz.

B. Distinct qubits $\Delta_R^L \gg \Delta_R^R$

Highly detuned HQs (e.g. $\Delta_R^L \gg \Delta_R^R$) can be operated exclusively with microwave signals. The $\sigma_z^L \sigma_z^R$ interaction between HQ$^L$ and HQ$^R$ can be neglected without drivings, and if both qubits are driven with their own

---

**Figure 4.** Gate operations for two HQs with similar eigenfrequencies $\Delta_R^L \approx \Delta_R^0$ that use the static Coulomb interaction according to Eq. (3), with $\chi_C/h \approx 1.3$ MHz, and the tunable exchange interactions from Eq. (11)- (12). A small detuning $\Delta_R^L - \Delta_R^0 \gg \chi_C$ is sufficient to neglect the interactions between HQ$^L$ and HQ$^R$. In the configurations of Fig. 1, $\Delta_R^L \gg X_R$. For $\Delta_R^L = 0$, all the single-qubit gates can be realized with resonant drivings of the qubits at their eigenfrequencies. For $\Delta_R^L = \Delta_R^0 - \Delta_R^L$, the HQs entangle under static time evolutions.

In the configurations of Fig. 1, $\chi$ in Eq. (13) has contributions from the capacitive couplings between the charge...
of HQ according to Eq. (A1), is lower than 1%. A similar drive ideal NOT one qubit is driven at the eigenfrequency of the other high-fidelity CPHASE gates.

The resulting gate operation is equivalent to a CPHASE gate with the frequency detunings between their eigenfrequencies. In the simulations, the gate times needed, which would require QDs in close vicinity, one can reduce κ in Eq. (7) by sufficiently separating the DQDs. In this case, there are no leakage errors (see Sec. IIIA) because the electron transfer between the QDs is forbidden. I simulate the time evolutions according to Eq. (16) without any rotating wave approximations. Besides the dominant time evolution from Eq. (18), there are rapidly oscillating terms that were neglected in the rotating wave approximations of Eq. (16) and Eq. (18).

Fig. 5 shows the errors of entangling operations for X = 0.55 μeV, driving amplitudes A/h = 0.1 GHz, and the HQ parameters defined earlier. To realize high-fidelity CPHASE operations, the deviations of the Makhlin invariants from their ideal values (|G1| + |G2| = 0) [60] permits high-fidelity CPHASE gates.

resonance frequencies. In the cross-resonance protocol, one qubit is driven at the eigenfrequency of the other qubit e.g.,

\[ H_B = \frac{\Delta_0^L}{2} \sigma_z^L + A \cos \left( \frac{\Delta_0^R}{h} \right) \frac{\Delta_0^R}{2} \sigma_z^R + X \sigma_x^L \sigma_x^R. \]

(16)

Transforming Eq. (16) to the rotating frame with \( H_{B}^{\text{rot}} = \frac{\Delta_0^L}{2} \sigma_z^L + A \frac{\Delta_0^R}{2} \sigma_z^R + X \sigma_x^L \sigma_x^R \) gives after a rotating wave approximation

\[ H_{B}^{\text{rot}} = \frac{\Delta_0^L - \Delta_0^R}{2} \sigma_z^L + A \frac{\Delta_0^R}{2} \sigma_z^R + X \frac{1}{2} \sigma_x^L \sigma_x^R + \sigma_y^L \sigma_y^R. \]

(17)

Another rotating wave approximation gives the effective interaction

\[ H_{B}^{\text{rot2}} = \frac{X}{2} \frac{A}{\sqrt{\left( \frac{\Delta_0^L - \Delta_0^R}{h} \right)^2 + A^2}} \times \left[ \cos (\vartheta) \sigma_z^L + \sin (\vartheta) \sigma_x^L \right] \sigma_x^R \]

(18)

in the rotating frame with \( \Delta_0^L - \Delta_0^R \) and \( \frac{X}{2} \sigma_x^L \). I use the abbreviations \( \cos (\vartheta) = \frac{\Delta_0^L}{\sqrt{\left( \frac{\Delta_0^L - \Delta_0^R}{h} \right)^2 + A^2}} \) and \( \sin (\vartheta) = \frac{X}{\sqrt{\left( \frac{\Delta_0^L - \Delta_0^R}{h} \right)^2 + A^2}} \).

In the following, a HQ is analyzed in the eigenbasis \( \{ |T\rangle, |B\rangle \} \) according to Eq. (3) because it is operated at its sweet spot εz.

V. NOISE DISCUSSION

This section shows that noise from fluctuating magnetic and electric fields only causes small errors for HQs. In the following, a HQ is analyzed in the eigenbasis \( \{ |T\rangle, |B\rangle \} \) according to Eq. (3) because it is operated at its sweet spot εz.

A. Hyperfine Interactions

Fluctuating local magnetic fields were identified as a natural problem for spin qubits. The nuclear magnetic fields of the host’s nuclei couples via the contact hyperfine interaction to the spin of an electron. For localized electrons, the contact hyperfine interaction can be described by slowly-fluctuating local magnetic fields at the QDs: \( \delta B^{\text{QD1}} \) and \( \delta B^{\text{QD2}} \).

For the HQ, a noise term \( \mathcal{H}^{\text{B}} = \frac{g_B}{2} \left( \delta B^{\text{QD1}} \cdot \sigma^{\text{QD1}} + \delta B^{\text{QD2}} \cdot \sigma^{\text{QD2}} \right) \) describes the fluctuating magnetic fields,
where \( \sigma_{QD_i} = (\sigma_x^{QD_i}, \sigma_y^{QD_i}, \sigma_z^{QD_i}) \) are the Pauli operators for the electrons at QD_i. For the HQ, this term gives the contribution
\[
\mathcal{H}_B^{\delta B} = \{ \hat{T}_i, |\varphi_i\rangle \} = \delta B \sigma_x,
\]
with \( \delta B = \frac{g_{\mu_B} \mu_B}{2} \left[ -\frac{3+\cos^2(\varphi)}{6} \delta B_{z_{QD_1}} + \frac{8 \cos^2(\varphi)}{6} \delta B_{z_{QD_2}} \right] \)
\( \sigma_x = \ket{\varphi} \bra{0} + \ket{0} \bra{\varphi} \).

The nuclear magnetic field can be treated as static during a single gate operation, but it fluctuates between successive measurements. Typical magnitudes of \( \delta B_{z_{QD_1}} \) are 5 mT for GaAs QDs and 100 \( \mu \)T for Si QDs. The associated frequencies are \( g_{\mu_B} \delta B_{z_{QD_1}} / h \approx 30 \text{ MHz} \) for GaAs QDs and \( g_{\mu_B} \delta B_{z_{QD_2}} / h \approx 3 \text{ MHz} \) for Si QDs, while \( \Delta_0 / h \) reaches several GHz. Therefore, one can treat Eq. (19) as a small perturbation to Eq. (3).

**VI. SUMMARY AND CONCLUSION**

This paper has derived an effective qubit description at the sweet spot. Two HQs can be coupled via Coulomb and exchange interactions. If both HQs are operated at their sweet spots, then the Coulomb interaction stays constant but the inter-qubit exchange interaction can be manipulated quickly. In a first approach, two qubits of similar eigenfrequencies are analyzed. The exchange interaction can bring the qubits in and out of resonance. Two qubits of identical eigenfrequencies entangle under static time evolutions, but two distinct qubits evolve independently. In a second approach, two highly distinct qubits are analyzed. The Coulomb interaction stays constant, and only resonant electric signals are needed to realize single-qubit and two-qubit gates.

This paper has simulated HQs in Si and GaAs with realistic parameters that are extracted from experiments. Most critically, the gate operations require Coulomb interactions between HQs that are small compared to the naive estimates for the setup. Especially, if inter-qubit exchange interactions are used for the entangling operations, then the inter-qubit Coulomb couplings should be small. This paper has discussed how weak inter-qubit Coulomb couplings can be realized. It should be possible to realize such weak inter-qubit couplings with a careful design of the QD layout.

Fault-tolerant quantum computation requires high-fidelity quantum gates with error probabilities below 1%. My simulations showed that quantum computation with this infidelities is possible if multi-qubit arrangements of HQ with the described parameters can be fabricated. Nuclear spin noise and charge noise are less critical. Additionally, these statistical errors can be reduced with refocusing protocols with similar approaches as for DQDs and TQDs. A reduction of the nuclear spin fluctuation can be realized by preparing the nuclear spin bath or with QD materials that contain nuclei of zero spin if the need arises.

A recent experiment also realized resonant single-qubit gates for the HQ. In this case, the HQ is operated deep in (1,2) which reduces the influence of charge noise. Resonant single-qubit gates are possible when the transition region to (2,1) is approached, while one still stays away from the anticrossings. Two-qubit gates with exchange interactions cannot be used to entangle HQs in Ref. in the same way as in the study of this paper because many leakage states are degenerate with the computational states. In Ref., the \( S = 3/2, s_z = 1/2 \) state is nearly degenerate with the qubit states (cf. Fig. 3). The Coulomb interaction can still be used to entangle HQs, similar to Sec. while the interaction Hamiltonian is \( \propto \sigma_z \sigma_z \). Universal qubit control likely requires DC control of the interaction Hamiltonian. Either two HQs are isolated from each other, or they are coupled via \( \sigma_z \sigma_z \) (in contrast to the approach in Sec. where single-qubit and two-qubit gates are possible without changing the operation points of the HQs).

Overall, the HQ is an interesting candidate for further experimental and theoretical studies. It has many
characteristics of a charge qubit, especially with its fast operation times. On the other hand, it can be protected from charge noise similar to a spin qubit. The described sweet spot manipulations classify the HQ as a mixture of a charge qubit and a spin qubit that has advantages from both setups. This paper has shown that universal gate operations can be realized for the HQ at the threshold of quantum error correction, which should further motivate the search for optimal manipulation protocols of HQs.

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Appendix A: Gate Fidelities

The fidelity that is used in the paper should be defined in the following. To characterize gate fidelities, the time evolution \( U \) is compared to the ideal time evolution \( U_I \). The state space is doubled to two identical Hilbert spaces \( R \) and \( S \). The entanglement fidelity* is a measure for the gate performance. Eq. (A1) compares the time evolution \( [U_I^{-1}U]^S \) of \( S \) with a reference system \( R \) that is unchanged. \( \rho^{RS} = |\psi\rangle \langle \psi| \) is a maximally entangled state of the combined Hilbert space, e.g., \(|\psi\rangle = (|0000\rangle + |0110\rangle + |1001\rangle + |1111\rangle)/2 \).

\[
F = \text{tr} \left\{ \rho^{RS} U^{R \otimes [U_I^{-1}U]^S} \rho^{RS} U^{R \otimes [U_I^{-1}U]^S} \right\} \tag{A1}
\]

Appendix B: Effective Inter-Qubit Exchange Hamiltonian

This section describes the derivation of the effective interactions from Sec. 3 for weak tunnel couplings between HQ\(_L\) and HQ\(_R\). The HQs are operated at their sweet spots. The dominant Hamiltonian \( \mathcal{H} \) is determined by Eq. (1) with the approximations in Eq. (3). Eq. (10) defines the coupling between HQ\(_L\) and HQ\(_R\) (called \( \mathcal{H}' \)). The system is tuned towards \((1,1,2,2)\), while the computational states still remain in the ground states configurations. The low-energy subspace \( \mathcal{P} = \mathcal{P}_Q + \mathcal{P}_L \) contains the qubit states \( \mathcal{P}_Q = \{ |T^L+R⟩, |T^L−R⟩, |O^L+R⟩, |O^L−R⟩ \} \). The leakage subspace contains the states \( \mathcal{P}_L = \{ |α), [β), [γ), [δ), [ν) \} \) that are defined in Tab. II.

Only states that have a singlet at HQ\(_R\) are included in \((1,1,2,2)\): \( Q = \{ |↑↑ S⟩, |↑↓ S T_0⟩, |↑↑ S T_+⟩, |↓↓ S T_−⟩ \} \). A weak tuning towards \((1,1,2,2)\) only occupies \( Q \) virtually. Eq. (10) couples states in \((1,2,1,2)\) and \((1,1,2,2)\). Schrieffer-Wolff perturbation theory constructs an effective Hamiltonian on \( P \), assuming that the coupling between \( P \) and \( Q \) is weak. Additionally, \( Q \) has higher energy than \( P \).

The effective interaction on \( P \) is defined in second-order Schrieffer-Wolff perturbation theory by

\[
\mathcal{H}_{P}^{ij} = \mathcal{H}_{P}^{ij} + \frac{1}{2} \sum_{k \in Q} (\mathcal{H}^{ij}_{PQ})^{ik} \left( \frac{1}{E_i - E_k} + \frac{1}{E_j - E_k} \right) (\mathcal{H}^{PQ})^{ik} \tag{B1}
\]

Eq. (B1) uses the transition matrix element between the states \(|i⟩\) and \(|j⟩\) from \( P \): \( \mathcal{H}_{P}^{ij} = ⟨i | \mathcal{H} | j⟩ \). \( (\mathcal{H}^{ij}_{PQ})^{ik} \) and \( (\mathcal{H}^{PQ})^{ki} \) are the transition matrix elements between the states \(|i⟩\) from \( P \) and \(|k⟩\) from \( Q \). \( E_i \) is the energy of the state \(|i⟩\).

In the whole paper, it is assumed that the energy difference between \( P \) and \( Q \) is large, especially much larger than the energy differences between states in \( P \) or between states in \( Q \). One can therefore use for all \(|i⟩ \in P \) and \(|k⟩ \in Q \): \( E_i - E_k \approx E_{(1,2,1,2)} - E_{(1,1,2,2)} \).

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The CPHASE gate can be obtained using the sequence

\[
e^{-i\frac{\pi}{4} \sigma_z^L} e^{-i\frac{\pi}{4} \sigma_z^R} \text{iSWAP} e^{i\frac{\pi}{4} \sigma_z^L} \text{iSWAP} e^{-i\frac{\pi}{4} \sigma_z^R} e^{-i\frac{\pi}{4} \sigma_z^L}
\]

which is in a rotated basis compared to \( \sigma^L \sigma^R \) but it generates the equivalent entangling gate. It is well known that \( \sigma^L \sigma^R \) is maximally entangling with

\[
e^{-i\frac{\pi}{4} \sigma_z^L} \sigma^R e^{i\frac{\pi}{4} \sigma_z^L} = e^{i\frac{\pi}{4} \sigma_z^L} e^{-i\frac{\pi}{4} \sigma_z^L} e^{-i\frac{\pi}{4} \sigma_z^L} \text{CPHASE}
\]

such that two iSWAPs construct one CPHASE.

59) The CPHASE gate can be obtained using the sequence

\[
e^{-i\frac{\pi}{4} \sigma_z^L} e^{-i\frac{\pi}{4} \sigma_z^R} \text{iSWAP} e^{i\frac{\pi}{4} \sigma_z^L} \text{iSWAP} e^{-i\frac{\pi}{4} \sigma_z^R} e^{-i\frac{\pi}{4} \sigma_z^L}
\]

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