Correspondence

Letter to the editor of Heliyon re: new considerations on the validity of the Wiener-Granger causality test [Heliyon 6(10) (2020) e05208]

Sir/Madam,

Dr. Martina Chvosteková and her collaborator Dr. Anna Krakovská have criticized the conclusion of the recently published article (Grassmann, 2020), which states: "...not even the most basic requirement underlying any possible definition of causality is met by the Granger causality test... ". I am very glad about their contribution, since the topic is important and will profit from as much discussion as possible.

In the following, an answer to their criticism will be presented:

1. The presented serious claim that "...not even the most basic requirement underlying any possible definition of causality is met by the Granger causality test... " is in fact based on a questionable numerical testing of two filtered neural signals.

The conclusion presented in the paper is not based on a numerical testing, but on considerations of general validity: the chapters related to the Fourier transform and the Newton method, which constitute the core of our study, refer to any kind of time series. The two neural signals in question were chosen only as an example and mentioned since they triggered this work.

2. Instead of the F-test (or any statistical test), the causal link between time series is inferred after just comparing the standard deviation estimates of the predictive errors to each other.

Aim of the paper is not to discuss the best method to measure how significantly a time series approximates another one. Its main point is that the AR and ARMA algorithms, used by Granger to determine the causality between time series, are not able to give any evidence of such a causality.

In order to better explain this point, a study of the Granger causality without any reference to the RMS distribution has been included in the second part of this answer.

3. The Granger causality test involved only stochastic stationary processes which can be fitted by a linear autoregressive model.

As already said, any couple of signals could have been chosen for this study: the conclusions about the suitability of the ARMA method to assess causality do not depend on the stationarity of the time series.

4. It was found that the future of signal A (i.e., the time inverted signal A in meaning of the author) is helpful for the prediction of the past B.

The paper does not state that the future of signal A is "helpful" for the prediction of the past B, but rather that the future of signal A can be used as well as the past of signal A. The two statements are very different, because the latter demonstrates that the method of Granger - which is the only subject of the study - is not sensitive to the cause of a signal.

They criticize in particular the signals employed for the study and the use of RMS.

To be as clear as possible and show how these choices are of no relevance for the conclusion, the main points of (Grassmann, 2020) (which were not questioned by Dr. M. Chvosteková and Dr. A. Krakovská) will be presented in the following, without making use of the RMS of the neural signals at all:

Figure 1 shows the three toy functions that have been arbitrarily built and their AR(2) reconstruction, where the AR(2) model is given by:

\[ y'_k = a_1 y_{k-1} + a_2 y_{k-2} \]  

(1)

For all the three functions, the parameters resulting from the AR(2) model are \( a_1 = 2 \) and \( a_2 = -1 \). This can be explained with the differential calculus introduced by Newton:

\[ y_k = y_{k-1} + \frac{dy_{k-1}}{dx} \]

\[ y_k = y_{k-1} + (y_{k-1} - y_{k-2}) \]

\[ 2y_{k-1} - y_{k-2} \]

\[ a_1 y_{k-1} + a_2 y_{k-2} \]

(2)

where indeed \( a_1 = 2 \) and \( a_2 = -1 \).

This means that, provided the bin width is sufficiently small, the value of any function in a given bin can be reconstructed starting from the values in the two previous bins using the two coefficients 2 and -1, independently on the identity of the function. An absolute scale defining whether the bin width is sufficiently small is given by the Fourier mechanism. Every function can be expressed as a sum of sinusoidal functions with different frequencies. For each sinusoidal function, Eq. (3) is always valid.

\[ C \sin(x + \varphi) = A \sin(x) + B \sin(x + \theta) \]

(3)

where \( C = \sqrt{A^2 + B^2 + 2AB \cos(\theta)} \) and \( \varphi = \arctan2(B \sin(\theta), A + B \cos(\theta)) \). In other words, from any two points of that function, a third one...
can be reconstructed. This means that we can reconstruct the value of a function at a bin from the two preceding bins multiplied by the analytical coefficients of Eq. (3), and this reconstruction is analytically precise. These coefficients will deviate from the Newton's ones if $\Delta x$ is not small compared to the wavelength of any given Fourier component: in order to achieve the desired agreement between the Fourier and Newton coefficients $\Delta x$ must be small compared to the wavelength of that sinusoidal component. Consequently, the required bin-width depends on the Fourier spectrum.

For small bin width the AR(2) algorithm results in coefficients that, for decreasing bin width, tend to the Newton's ones. This is true for any type of function: for small bin widths (or lower frequencies) the AR(2) results in a good reconstruction with coefficients that tend to 2 and -1. The precision of the Newton methods can be improved by using the second, third and higher derivatives, including more bins. This means that the AR algorithm performs a simple mathematical reconstruction and is not related nor to causality neither to the information content of the function.

As a next step, the MA model has been studied. The toy functions mentioned above can be again considered: the MA(6) reconstruction of function 1 from function 2 and function 3 are shown in Figure 2A and B respectively.

![Figure 1](image1.png)

Figure 1. A) Toy function 1 (in blue) and the corresponding AR(2) (in red). B) Toy function 2 (in blue) and the corresponding AR(2) (in red). C) Toy function 3 (in blue) and the corresponding AR(2) (in red).

![Figure 2](image2.png)

Figure 2. A) Blue line: toy function 1. Red line: MA(6) to reconstruct function 1 from function 2. B) Blue line: toy function 1. Red line: MA(6) to reconstruct function 1 from function 3.

![Figure 3](image3.png)

Figure 3. A) Blue line: toy function 4. Red line: AR(2). B) Blue line: toy function 4. Red line: AR(6).
Figure 2 shows that the MA algorithm can reconstruct any function from any other function. Again, it can be concluded that the MA algorithm as well is not related to causality.

After having studied AR and MA, one needs to study their composition, the ARMA method. However, the AR algorithm by itself has already given such a perfect description of each of the toy functions that it cannot be improved in a meaningful way. The high quality of the AR prediction was due to the small bin-width that has been used and the low frequency Fourier components which have been used for the toy functions:

\[
\text{function } 1 = 0.1 \sin(t) + 0.2 \sin(2t) + 0.3 \sin(3t) + 0.4 \sin(4t)
\]

\[
\text{function } 2 = 0.4\cos(t) + 0.3\cos(2t) + 0.2\cos(3t) + 0.1\cos(4t)
\]

\[
\text{function } 3 = 0.4\sin(t) - 0.3\sin(2t) - 0.2\cos(3t) + 0.05\sin(4t)
\]

One can therefore increase the bin width and create a new function 4 (obtained by adding to function 1 some sinusoidal functions with higher frequencies):

\[
\text{function } 4 = \text{function } 1 + 0.2\sin(8t) + 0.2\sin(10t) + 0.2\sin(12t).
\]

Figure 3 shows how in this case an AR(6) model gives a better reconstruction compared to the AR(2) model, as expected.

Indeed for higher frequencies (or wider bin) the reconstruction can be improved by considering more past values of the function (as we did with the AR(6) model), or adding the past values of a second function (ARMA algorithm).

Figure 4A and B show the reconstruction of function 4 from function 2 and 3 respectively.

These Figures show that function 4 can be reconstructed by making use of function 2 as well as function 3, despite them being different ones, unrelated to function 4.

This observation can be done whether one makes use of RMS, RAMS or F-test. Therefore this improvement can not be interpreted as causality.

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Greta Grassmann: Wrote the paper.

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Reference

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