Effect of non zero cosmological constant on the motion of light ray

Sarani Chakraborty, A. K. Sen

Department Physics, Assam University, Silchar-788011, India

E-mail: sarani.chakraborty.phy@gmail.com, asokesen@yahoo.com

Abstract

Cosmological constant, the value of the energy density of the vacuum of space, was introduced by Einstein as an extension of general relativity. Recent studies show that cosmological constant has a noticeable effect on the shape of space-time i.e. it will influence the path of light ray. In this study the effect of non-zero cosmological constant on the path of light ray in three different types of space-time geometry namely Schwarzschild de Sitter, Kerr de Sitter and Kerr- Newman de Sitter has been discussed.

1. Introduction

Einstein explained gravitation as a demonstration of geometry of space-time and gave the final formulation in 1915 [1]. General relativity predicted that the universe must either expand or contract. Einstein was a believer of static universe and to make the universe static he introduced an anti-gravity force by adding the cosmological constant. Later when Hubble gave the proof of expanding universe, he dropped that idea and called it as the biggest mistake of his life. Up to 1990, scientists thought that the rate of expansion of the universe slows over time for gravitational pull of matter of the universe. But observation indicates that the universal expansion is speeding up. So there must be force acting against gravity and cosmological constant is an example of such type of force. Hence the idea of cosmological constant came back.

One of the most important predictions of general relativity is the deflection of light ray in presence of gravitational mass. The first order contribution of mass on the path of light ray was calculated by Einstein himself. After Einstein, higher order contribution of mass towards light deflection angle was calculated by Keeton and Petters [2]. Recently, Iyer and Petters [3] calculated it for strong field and found that under weak field approximation their expression matches with that of Keeton and Petters [2].

The light deflection angle for a rotating mass (Kerr mass) was calculated by Iyer and Hansen [4]. Bozza [5] obtained the lensing formula and calculated all other components related to lensing. Azzami et. al [6, 7] calculated the two individual components (parallel and perpendicular to equatorial plane) of light deflection angle in quasi-equatorial regime. Chakraborty and Sen [8] have recently obtained the light deflection angle for a charged, rotating body in the equatorial plane and showed how deflection angle changes with charge. The off equatorial light deflection for Kerr geometry was also studied by Chakraborty and Sen [9].

On the other hand, some authors have used the material medium approach, where the gravitational effect on the light ray was calculated by assuming some effective refractive index assigned to the medium through which light is propagating [8]. Sen [10] and Roy and Sen [11] used this method to calculate the gravitational deflection of light without any weak field approximation for Schwarzschild
and Kerr geometry. Nandi and Islam [12] used the optical-mechanical analogy of general relativity (based on Fermat’s principle) and obtained the refractive index.

All these above mentioned work were done without considering the effect of cosmological constant and as we already discussed, it is more realistic to study the trajectory of light ray by considering non-zero cosmological constant (de-Sitter universe).

Here we will review and discuss the effect of non-zero cosmological constant on the path of light ray in three different types of space-time geometry namely Schwarzschild de Sitter, Kerr de Sitter and Kerr Newman de Sitter. We will also make a comparison of effect of not zero cosmological constant among the three above mentioned line elements and draw a set of conclusions.

2. Schwarzschild de Sitter line element.

Schwarzschild de Sitter space-time represents the exterior of a static mass under de Sitter background. The metric for the Schwarzschild de Sitter space-time is given by (using units such that $G = c =1$) [13],

$$ds^2 = -f_r(r)dt^2 + \frac{dr^2}{f_r(r)} + r^2(d\vartheta^2 + \sin^2 \vartheta d\varphi^2)$$

where, $f_r = 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}$ and $\Lambda$ is the Cosmological constant with $m$ being the mass of the lens object.

3. Kerr de Sitter line element.

In general relativity the geometry of space time around rotating massive body can be described by Kerr metric. According to this metric, frame dragging, an unusual prediction of the general relativity is exhibited by such rotating bodies. The prediction of this effect is that all objects coming close to a rotating mass would be entrained to participate in its rotation, not because of any applied force or torque that can be felt, but because of the curvature of space time associated with the body [14]. At close enough distance all object even light must rotate with the body. According to General relativity, rotating bodies drag space time around them, a phenomenon known as frame dragging. The deflection produced in presence of a rotating black hole explicitly depends on the direction of light ray. Compared to the zero spin Schwarzschild case, the bending angle is greater for direct orbit and smaller for retrograde orbits [4].

Here we will discuss the Kerr space time under de-Sitter background i.e. Kerr de-Sitter space time. The Kerr de-Sitter line element is given by [15],

$$ds^2 = \rho^2\left(\frac{dr^2}{\Delta} + \frac{d\vartheta^2}{\Delta}\right) + \sin^2 \vartheta \Delta_a \left(\frac{a dt - (r^2 + a^2) d\varphi}{\rho^2} \right)^2 - \frac{\Delta_a}{\rho^2} \left(\frac{dt - a \sin^2 \vartheta d\varphi}{1 + \frac{\Lambda}{3} a^2}\right)^2$$

$$\rho^2 = r^2 + a^2 \cos^2 \vartheta$$

Where,

$$\Delta = r^2 - 2mr + a^2 - \frac{\Lambda r^2}{3} (r^2 + a^2)$$

$$\Delta_a = 1 + \frac{\Lambda a^2 \cos^2 \vartheta}{3}$$

$a = J/m, J$ is the angular momentum of the gravitating body.

4. Kerr-Newman de Sitter line element

In the general theory of relativity, the Kerr–Newman line element represents the most generalized
form of the space time curvature, where all three factors (mass distribution, rotation, charge) have their contribution. Here we will discuss the Kerr–Newman line element under de-Sitter back ground i.e. Kerr-Newman de-Sitter line element [16],

$$ds^2 = \frac{\Delta_{KN}^2}{\Sigma^2} (dr^2 - a^2 \sin^2 \phi d\theta^2) - \frac{\rho^2}{\Delta_{KN}} dr^2 - \frac{\rho^2}{\Delta_\phi} d\phi^2$$

$$- \frac{\Delta_\phi \sin^2 \phi}{\Sigma^2} (adt - (r^2 + a^2) d\phi)^2$$

$$\Delta_\phi = 1 + \frac{a^2 \Lambda}{3} \cos^2 \phi$$

Where,

$$\Delta_{KN}^2 = (1 - \frac{\Lambda}{3} r^2)(r^2 + a^2) - 2mr + e^2$$

And e is the static charge.

5. Bending angle due to Schwarzschild de Sitter line element.

The concept of measurement of the effect of the cosmological constant on the light deflection angle was first introduced by Rindler and Ishak [17]. They derived the formula for light deflection angle under de Sitter back ground. According to their work a positive cosmological constant diminishes the bending angle.

Bhadra et al [13] obtained the light deflection angle in the Schwarzschild de Sitter (SDS) geometry. In their paper they computed the bending angle in the SDS space-time, taking the proper cosmological constant involved in the solution of the light trajectory. They showed that when the light path from the reference source is taken into consideration, the resultant bending in the SDS geometry will appear to increase rather than decrease due to the cosmological constant effect. They also discussed the possibility of the detection of the effect from the bending angle measurement. Their expression for bending angle is given by,

$$\alpha = \frac{4m}{r_0} - 2mr_0 \left( \frac{1}{d_{LS}} + \frac{1}{d_{OL}} \right) - \frac{\Lambda r_0}{6} \left( d_{OL} + d_{LS} \right) + \frac{\Lambda r_0^3}{6} \left( \frac{1}{d_{LS}} + \frac{1}{d_{OL}} \right)$$

Here $r_0$ is the closest approach of the light ray towards the gravitating body. $d_{LS}$ and $d_{OL}$ are the distance between lensing object and source and lensing object and observer. Their work shows that cosmological constant does effect the gravitational deflection of light.

6. Bending angle due to Kerr de Sitter line element.

Kraniotis [18] derived the frame dragging and deflection angle in terms of the Appell and Lauricella hypergeometric functions and the Weierstrass modular form for a Kerr and Kerr–(anti) de Sitter space times.

Sultana [15] in his paper used the method of Rindler and Ishak to obtain a weak field approximation for the bending angle in the Kerr–de Sitter space time. He derived the null geodesic equations for the Kerr–de Sitter metric and obtained the bending angle which includes contributions from the cosmological constant in the equatorial plane.
Here $b$ is the impact parameter.

7. **Bending angle due to Kerr-Newman de Sitter line element.**

Kraniotis [16] obtained the solution of null geodesics that describe photon orbits in the space time of a rotating electrically charged black hole (Kerr-Newman) including the contribution from the cosmological constant.

He derived the analytic solutions of the lens equations in terms of Appell and Lauricella hypergeometric functions and the Weierstrass modular form. He used the analytical solution to obtain the exact expression of the deflection angle of an equatorial light ray in the gravitational field of a Kerr-Newman de Sitter black hole.

8. **Conclusion.**

From the above discussion we can conclude the following points,

1. Cosmological constant has a noticeable effect on the path of the light ray. When compared with Schwarzschild expression for bending, we find that there are some extra terms in the expression for deflection which occur due to the presence of cosmological constant. If we set the cosmological constant to zero, deflection angle will reduce to that of Schwarzschild, Kerr or Kerr-Newman deflection angle.

2. The positive value of cosmological constant reduces the amount of deflection [17]. However when the light path from the reference source is taken into consideration, the resultant bending angle in the SDS geometry will appear to increase rather than decrease due to the effect of cosmological constant [13].

3. One should do the correction for cosmological constant to the expression of light deflection angle to learn the exact geometry of space time curvature.

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