Adiabatic Quantum Transistors

Dave Bacon, Gregory Crosswhite
University of Washington

&

Steve Flammia
Perimeter Institute

Workshop on Quantum Algorithms, Computational Models and Foundations of Quantum Mechanics
UBC, Vancouver, 23 July 2010
A zoo of quantum computational models

- Topological
- Adiabatic
- Holonomic
- Measurement-based
- Circuit model
- Holonomic
A zoo of quantum computational models

Which one (if any!) will lead to an actual quantum computer?

This talk: try to combine aspects of all of these models to devise a new architecture for quantum computing
A review of the zoo

- Adiabatic evolution offers **robustness** to timing and control errors that exist in the circuit model.
- Errors are **suppressed** by the **spectral gap**.
- It is unknown if it is **fault tolerant** (without additional assumptions) and lack of **modularity** makes it difficult to analyze theoretically.
Holonomic QC is also robust to timing errors, and some (fewer) types of control errors. Can be made fault tolerant. Typically requires simultaneous control of multiple parameters to achieve non-trivial geometric phases.
A review of the zoo

- Topological quantum phases are insensitive to local perturbations
  Bravyi Hastings Michalakis 2010
- Naturally long-lived quantum memory
- Sensitive to finite temperature, and still requires active error correction. Also, initialization is difficult.
A review of the zoo

- Very minimal requirements: only local measurements, which every scheme uses anyway.
- Simple initial states (relatively speaking) can be used as the entangled resources.
- There is absolutely nothing disadvantageous about measurement-based QC.
A review of the zoo

- Circuit model provides the most natural language for programming quantum computers and designing quantum algorithms.

- Direct implementation involves pulsed gates and a huge amount of control... very challenging, to say the least.
Adiabatic teleportation

single qubit

|ψ⟩

Bell pair

|Φ⟩

This is a ground state of $H_i = -X_2X_3 - Z_2Z_3$

(could also use the exchange interaction)

Bacon STF, PRL 2009

Related: Oreshkov Brun Lidar, PRL 2009; Oreshkov, PRL 2009
Adiabatic teleportation

Bell pair

$|\Phi\rangle$

$|\psi\rangle$

This is a ground state of $H_f = -X_1 X_2 - Z_1 Z_2$
Adiabatic teleportation

\[ |\psi\rangle \]

\[ H(t) = (1 - t)H_i + tH_f \]
Adiabatic teleportation

\[ H(t) = (1 - t)H_i + tH_f \]
Adiabatic teleportation

\[ H(t) = (1 - t)H_i + tH_f \]

\[ \mathcal{T} \exp \left( -i \int_0^T d\tau H(\tau) \right) \]
Adiabatic teleportation

\[ H(t) = (1 - t)H_i + tH_f \]
Adiabatic teleportation

\[ H(t) = (1 - t)H_i + tH_f \]

Notice that the ground space is stabilized by XXX and ZZZ for all t.
Adiabatic teleportation

\[ H(t) = (1 - t)H_i + tH_f \]

Notice that the ground space is stabilized by XXX and ZZZ for all t.
The adiabatic evolution acts like a post-selected teleportation!

Notice that the ground space is stabilized by XXX and ZZZZ for all t.
Adiabatic gate teleportation

\[ |\psi\rangle \]

\[ U_3 H(t) U_3^\dagger = (1 - t) U_3 H_i U_3^\dagger + t H_f \]

Gottesman Chuang 1999
Adiabatic gate teleportation

\[ U_3 H(t) U_3^\dagger = (1 - t) U_3 H_i U_3^\dagger + t H_f \]

Gottesman Chuang 1999
Adiabatic gate teleportation

\[ U_3 H(t) U_3^\dagger = (1 - t) U_3 H_i U_3^\dagger + t H_f \]

Gottesman Chuang 1999
Adiabatic gate teleportation

Now the adiabatic evolution teleports the unitary onto the qubit.

\[
U_3 H(t) U_3^\dagger = (1 - t) U_3 H_i U_3^\dagger + t H_f
\]

Gottesman Chuang 1999
Universality

$U_\alpha | \psi \rangle$
Universality

$U_b U_a |\psi\rangle$
Universality

$U_b U_a \ket{\psi}$

Etc...

but what about two qubit gates?
Universality
Universality
Universality

Two-qubit gates introduce 3-body terms...
Universality

Two-qubit gates introduce 3-body terms...
Two-qubit gates introduce 3-body terms...

to get rid of them, use perturbation gadgets.
Universal, 2-body

Our perturbation gadgets: Bartlett & Rudolph 2006
Universal, 2-body

Our perturbation gadgets: Bartlett & Rudolph 2006
Universal, 2-body

Qubit encoded in subspace $|00\rangle, |11\rangle$

Our perturbation gadgets: Bartlett & Rudolph 2006
Universal, 2-body

Our perturbation gadgets: Bartlett & Rudolph 2006
Universal, 2-body

Now qubits are encoded locally

Our perturbation gadgets: Bartlett & Rudolph 2006
Universal, 2-body

Gate fidelity $= 1 - \Theta(\lambda^2)$

Gap $= \Theta(\lambda)$

Ratio of energy scales $= \lambda$

Our perturbation gadgets: Bartlett & Rudolph 2006
1-d architecture
Adiabatic Code Deformation

Energy

Quantum error-correcting codespace

Time

Quantum error-correcting codespace

Must be degenerate throughout the entire evolution; any splittings are errors that need to be coded for and corrected.

Bombin Delgado, J. Phys. A 2009

“Open-loop” holonomy, Kult Aberg Sjoqvist, PRA 2006
Adiabatic holonomic evolution offers robustness to timing and control errors that exist in the circuit model.

Excitations are suppressed by the constant gap.

“Ground state” errors can be corrected via coding.

It is modular, and hence as easy to program as the circuit model.

Uses only control between subsystems, not levels.

Gates are prepared offline, leading to fewer errors.

It leads to more results of interest to theorists...
One Way QC

Create Entangled State

Raussendorf Briegel, PRL 2001
One Way QC

Adaptively measure to enact circuit

Raussendorf Briegel, PRL 2001
One Way QC

Adaptively measure to enact circuit

Raussendorf Briegel, PRL 2001
One Way QC

Adaptively measure to enact circuit

Raussendorf Briegel, PRL 2001
One Way QC

Adaptively measure to enact circuit

Raussendorf Briegel, PRL 2001
Cluster State Hamiltonian

\[ S_v = X_v \prod_{w \text{ adjacent to } v} Z_w \]

Cluster state is ground state of \( H_C = -\Delta \sum_v S_v \)

Again, it’s possible to use gadgets to make only 2-qubit interactions

Bartlett Rudolph, PRA 2006
Adiabatic One-way QC

\[ H = -Z_{n-1}X_n - \sum_{j=2}^{n-1} S_j \]

\[ S_j = Z_{j-1}X_jZ_{j+1} \]

Suppose we prepare \(|+\rangle\) on the first physical qubit.

Turn on -X fields and turn off cluster state coupling.

Bacon Flammia, arXiv:0912.2098
Adiabatic One-way QC

\[ H = -Z_{n-1}X_n - \sum_{j=2}^{n-1} S_j \]

Suppose we prepare \(|+\rangle\) on the first physical qubit.

Turn on -X fields and turn off cluster state coupling.

Bacon Flammia, arXiv:0912.2098
Adiabatic One-way QC

\[ H = -Z_{n-1}X_n - \sum_{j=2}^{n-1} S_j \]

\[ S_j = Z_{j-1}X_j Z_{j+1} \]

Suppose we prepare \(|+\rangle\) on the first physical qubit.

Turn on -X fields and turn off cluster state coupling.

Bacon Flammia, arXiv:0912.2098
Adiabatic One-way QC

\[ H = -Z_{n-1}X_n - \sum_{j=2}^{n-1} S_j \]

\[ S_j = Z_{j-1}X_j Z_{j+1} \]

Suppose we prepare \(|++\rangle\) on the first physical qubit

Turn on -X fields and turn off cluster state coupling

Turn on -X fields and turn off cluster state coupling

Bacon Flammia, arXiv:0912.2098
Adiabatic One-way QC

\[ H = -Z_{n-1}X_n - \sum_{j=2}^{n-1} S_j \]

\[ S_j = Z_{j-1}X_j Z_{j+1} \]

Suppose we prepare \(|+\rangle\) on the first physical qubit

Turn on -X fields and turn off cluster state coupling

Rotating the X fields in X-Y plane to make it universal

Bacon Flammia, arXiv:0912.2098
Adiabatic One-way QC

\[ H = -Z_{n-1}X_n - \sum_{j=2}^{n-1} S_j \]

\[ S_j = Z_{j-1}X_j Z_{j+1} \]

Suppose we prepare \(|+\rangle\) on the first physical qubit

Turn on -X fields and turn off cluster state coupling

Rotating the X fields in X-Y plane to make it universal

The gap is still constant

Bacon Flammia, arXiv:0912.2098
Classical Transistors

An “identity gate”

Problem: quantum information cannot be cloned
Quantum Transistors?

1. Many-body system in its ground state
Quantum Transistors?

1. Many-body system in its ground state
2. Qubits localized on one side of the device

|ψ⟩
Quantum Transistors?

1. Many-body system in its ground state
2. Qubits localized on one side of the device
3. Apply a strong 1-qubit external field to device

|ψ⟩
Quantum Transistors?

1. Many-body system in its ground state
2. Qubits localized on one side of the device
3. Apply a strong 1-qubit external field to device
4. Qubits now localized on other side of device with a quantum circuit applied to the qubits

Bacon Crosswhite Flammia, in preparation
Adiabatic Quantum Transistors

What if we turn on the fields all at once?
Adiabatic Quantum Transistors

This is the transverse-field Ising model (with funny BCs)

The gap is $\Theta(1/n)$

In analogy with transistors:
An applied field induces a quantum phase transition between an insulating and a “quantum logic” phase.

$$H(t) = (1 - t)H_C + tH_X$$
$R(\theta) = \exp(-i\theta Z/2)$
Example

How *slowly* must we turn on the external field in order for the device to successfully *quantum compute*?
Gap Scaling: 1D Numerics

Gap for 1D circuit with random angle rotations...
Gap Scaling: 2D Numerics

Gap for 2D circuit with random angle rotations
How slowly must we turn on the external field in order for the device to successfully quantum compute?

- In 1D with no twists, we have rigorously proven the gap is inverse polynomial in the circuit size.
- In 1D with twists, we have extremely strong evidence of polynomial scaling in the circuit size.
- For the 2D case with twists, we have some evidence of polynomial scaling in the size of circuit.
- We have not yet simulated the case with gadgets.
Fault-Tolerance

1D untwisted Hamiltonian decouples into two 1D Ising chains with a transverse field.

Two types of errors:
- Those that change energy
- Those within the degeneracy
- Includes system-bath couplings and Hamiltonian perturbations

- Assume the excitations obey detailed balance and are suppressed by a Boltzmann factor.
- Adiabatic evolution preserves eigenstates, so excitations can be (mathematically) dragged back to the beginning.
- Straightforward stabilizer arguments shows that these are correctible local and independent Pauli errors
1D untwisted Hamiltonian decouples into two 1D Ising chains with a transverse field.

Two types of errors:
- Those that change energy
- Those within the degeneracy
- Includes system-bath couplings and Hamiltonian perturbations

- Quantum info is susceptible to decoherence near the beginning and end, but in the middle, string-like stabilizer operators give us topological protection from local errors.
- We can reschedule the adiabatic evolution so that we only spend a constant amount of time in the bad regime, and these errors are local and independent there.
Conclusion

Adiabatic Gate Teleportation can be combined with cluster states to build robust adiabatic quantum logic elements.