$B \to D^{(*)}\tau\nu_\tau$ in the 2HDM with an anomalous $\tau$ coupling

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Abstract

The puzzle of $R(D^{(*)})$ associated with $B \to D^{(*)}\tau\nu$ decay is addressed in the two-Higgs-doublet model. An anomalous coupling of $\tau$ to the charged Higgs is introduced to fit the data from BaBar, Belle, and LHCb. It is shown that all four types of the model yield similar values for the minimum $\chi^2$. We also show that the newly normalized $R(D^{(*)})$ with the branching ratio of $B \to \tau\nu$ decay exhibits a much smaller minimum $\chi^2$. 

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I. INTRODUCTION

One of the most interesting puzzles in flavor physics in recent years has been the excess of the semitauninc $B$ decays, $B \to D^{(*)}\tau\nu$. The excess is well expressed in terms of the ratio

$$R(D^{(*)}) \equiv \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\ell\nu)},$$

where $\mathcal{B}$ is the branching ratio. The standard model (SM) prediction is $R(D)_{SM} = 0.300 \pm 0.008$, $R(D^*)_{SM} = 0.252 \pm 0.003$.

The BaBar Collaboration has reported that the measured $R(D)$ exceeds the SM prediction by $2.0\sigma$, while $R(D^*)$ exceeds the SM prediction by $2.7\sigma$, and the combined significance of the disagreement is $3.4\sigma$. BaBar analyzed the possible effect of a charged Higgs boson in the Type-II two-Higgs-doublet model (2HDM), and excluded the model at the 99.8% confidence level.

The Belle measurements of $R(D^{(*)})$ are slightly smaller than those of BaBar, but still larger than the SM expectations. Interestingly, Belle’s results are compatible with the Type-II 2HDM in the $\tan \beta/m_{H^\pm}$ region around $0.45\text{e}^2/\text{GeV}$ (where $\beta$ is the ratio of the two vacuum expectation values of the 2HDM) and zero, and recent measurements of $R(D^*)$ are consistent with the SM predictions. On the other hand, LHCb reported that $R(D^*)$ is larger than the SM predictions by $2.1\sigma$.

In this paper we try to fit the global data on $R(D^{(*)})$ with the 2HDM of all types. The 2HDM is a natural extension of the SM Higgs sector, so it has been tested to fit the $R(D^{(*)})$ puzzle. Out of all the types of 2HDM, the Type-II model is the most promising because the new physics (NP) effects are involved with the coupling of $\tan^2 \beta$ while for other types the couplings are 1 or $1/\tan^2 \beta$. As mentioned before, there is tension between BaBar and Belle regarding the compatibility of the Type-II 2HDM to the data. In this analysis we introduce an anomalous $\tau$ coupling to the charged Higgs. Since the NP effects are enhanced by new couplings and suppressed by the charged Higgs mass, the new couplings should be large enough to allow a heavy charged Higgs to fit the data. We also investigate possible roles of leptonic decay $B \to \tau\nu$ to solve the $R(D^{(*)})$ puzzle. It was suggested that the normalized $R(D^{(*)})$ with $\mathcal{B}(B \to \tau\nu)$, $R_\tau(D^{(*)})$ are consistent with the SM.


| Type   | $\xi^u_A$ | $\xi^d_A$ | $\xi^\ell_A$ |
|--------|-----------|-----------|--------------|
| Type-I | $\cot \beta - \cot \beta - \cot \beta$ |             |              |
| Type-II| $\cot \beta$  $\tan \beta$   $\tan \beta$ |             |              |
| Type-X | $\cot \beta - \cot \beta$  $\tan \beta$ |             |              |
| Type-Y | $\cot \beta$  $\tan \beta - \cot \beta$ |             |              |

TABLE I. $\xi$s for each type of 2HDM.

We implement the global $\chi^2$ fitting to $R(D^{(*)})$ as well as $R_\tau(D^{(*)})$ with the anomalous $\tau$ coupling.

The paper is organized as follows. Section II introduces the 2HDM with the anomalous $\tau$ coupling to describe $B \to D^{(*)}\tau\nu$ and $B \to \tau\nu$ transitions. In Sec. III $R(D^{(*)})$ and $R_\tau(D^{(*)})$ are expressed in the 2HDM with the new coupling. Our results and discussions are given in Sec. IV, and conclusions follow in Sec. V.

II. 2HDM WITH ANOMALOUS $\tau$ COUPLINGS

The Yukawa interaction in the 2HDM is given by \cite{15}

$$
\mathcal{L}_{\text{Yukawa}} = - \sum_{f=u,d,\ell} \frac{m_f}{v} \left( \xi^f_h \bar{f}f h + \xi^f_H \bar{f}f H - i\xi^f_A \bar{f}\gamma_5 f A \right)
$$

$$
\left[ \frac{\sqrt{2}V_{ud}}{v} \bar{u} \left( m_u \xi^u_A P_L + m_d \xi^d_A P_R \right) dH^+ + \frac{\sqrt{2}\xi^\ell_A m_\ell}{v} \bar{\nu}_L \ell_R H^+ + \text{h.c.} \right],
$$

where $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV, $v_{1,2}$ are the vacuum expectation values (VEVs) of the scalar fields $\Phi_{1,2}$ of the 2HDM with $\tan \beta \equiv v_2/v_1$, and the $\xi$s are the couplings defined in Table I. Here we introduce an anomalous factor $\eta$ to enhance $\xi^\tau_A$ \cite{13}. The motivation is that $\tau$ is screened from the second Higgs VEV $v_2$ and the neutral component of $\Phi_2$ by a factor of $\eta$. In this case the tau mass is $\sim Y_{\text{Yukawa}}v_2/\eta$, effectively enhancing the Yukawa coupling of $\tau$ to $H^\pm$, while that of $\tau$ to neutral Higgses remains unchanged. This kind of model can be easily constructed within extra dimensions. For example, as in Refs. \cite{16,17}, the overlappings between the wave functions of $\tau$ and the neutral component of $\Phi_2$ over the extra dimension would determine the strength of the $\tau$ coupling to the neutral part of $\Phi_2$. We could simply assume that the overlapping of $\tau$ and the neutral $\Phi_2$ is rather weak compared to other cases. The enhancement occurs for Type-I and Type-Y models because
in these models leptons couple only to $\Phi_2$. The same thing could happen for $\tau$ and the neutral component of $\Phi_1$ to screen $\tau$ from $v_1$, resulting in $\eta$ enhancement for $\tau$-$H^\pm$ couplings in Type-II and X models. In this work we assume that phenomenologically $\tau$ couplings to $H^\pm$ are enhanced by a factor of $\eta$ for all types of the model,

$$\xi_A^\tau \rightarrow \eta \xi_A^\tau . \quad (4)$$

Now the effective Lagrangian for the $b \rightarrow c\ell\nu$ transition is

$$L(b \rightarrow c\ell\nu) = L(b \rightarrow c\ell\nu)_{\text{SM}} + L(b \rightarrow c\ell\nu)_{2\text{HDM}}$$

$$= \frac{G_F V_{cb}}{\sqrt{2}} \left[ \bar{c}\gamma^\mu (1 - \gamma_5) b \, \bar{\ell}\gamma_\mu (1 - \gamma_5) \nu_\ell \right]$$

$$+ \frac{V_{cb}}{m_{H^\pm}^2} \left[ \bar{c} \left( g_s^c + g_p^c \gamma_5 \right) b \, \bar{\ell} \left( f_s^\ell - f_p^\ell \gamma_5 \right) \nu_\ell \right] ,$$

where

$$g_s^c = \frac{m_u \xi_A^u + m_b \xi_A^d}{\sqrt{2}v} , \quad g_p^c = \frac{-m_u \xi_A^u + m_b \xi_A^d}{\sqrt{2}v} , \quad (6)$$

$$f_s^\ell = f_p^\ell = -\frac{m_\ell \xi_A^\ell}{\sqrt{2}v} . \quad (7)$$

For $B \rightarrow \tau\nu$ decay,

$$L(B \rightarrow \tau\nu) = L(B \rightarrow \tau\nu)_{\text{SM}} + L(B \rightarrow \tau\nu)_{2\text{HDM}}$$

$$= \frac{G_F V_{ub}}{\sqrt{2}} \left[ \bar{u}\gamma^\mu (1 - \gamma_5) b \, \bar{\tau}\gamma_\mu (1 - \gamma_5) \nu_\tau \right]$$

$$+ \frac{V_{ub}}{m_{H^\pm}^2} \left[ \bar{u} \left( g_s^u + g_p^u \gamma_5 \right) b \, \bar{\tau} \left( f_s^\tau - f_p^\tau \gamma_5 \right) \nu_\tau \right] ,$$

where

$$g_s^u = \frac{m_u \xi_A^u + m_b \xi_A^d}{\sqrt{2}v} , \quad g_p^u = \frac{-m_u \xi_A^u + m_b \xi_A^d}{\sqrt{2}v} , \quad (9)$$

$$f_s^\tau = f_p^\tau = -\frac{m_\tau \xi_A^\tau}{\sqrt{2}v} . \quad (10)$$

Note that $\xi_A^\tau$ contains the enhancement factor $\eta$, $\xi_A^\tau = \eta \xi_A^{\ell=e,\mu}$.

### III. $B \rightarrow D^{(\ast)}\tau\nu$ AND $B \rightarrow \tau\nu$ DECAYS

The decay rates of $B \rightarrow D^{(\ast)}\ell\nu$ in the 2HDM can be expressed as

$$\Gamma^{D^{(\ast)}} = \Gamma_{\text{SM}}^{D^{(\ast)}} + \Gamma_{\text{mix}}^{D^{(\ast)}} + \Gamma_{H^\pm}^{D^{(\ast)}} . \quad (11)$$
The differential decay rates for $B \to D \ell \nu$ are given by

$$d\Gamma_{\text{SM}}^D = \frac{G_F^2 |V_{cb}|^2}{96\pi^3 m_B^2} \left\{ 4 m_B^2 P_D^2 \left(1 + \frac{m_B^2}{2s}\right) |F_1|^2 \right. + m_B^4 \left(1 - \frac{m_D^2}{m_B^2}\right)^2 \frac{3 m_D^4}{2s} |F_0|^2 \left\} \left(1 - \frac{m_\ell^2}{s}\right)^2 P_D ,$$

$$d\Gamma_{\text{mix}}^D = \frac{G_F}{\sqrt{2} m_{H^\pm}^2} \frac{g_s^c |V_{cb}|^2}{32\pi^3} \left( f_\ell^c + f_p^\ell \right) m_\ell \left(1 - \frac{m_B^2}{m_D^2}\right) \frac{m_B^2 - m_D^2}{m_B - m_c} |F_0|^2 \left(1 - \frac{m_\ell^2}{s}\right)^2 P_D ,$$

$$d\Gamma_{H^\pm}^D = \frac{(g_c^e)^2 |V_{cb}|^2}{64\pi^3 m_{H^\pm}^4 m_B^2} \left[ (f_\ell^c)^2 + (f_p^\ell)^2 \right] |F_0|^2 s \left(1 - \frac{m_\ell^2}{s}\right)^2 P_D ,$$

where $s = (p_B - p_D)^2$ is the momentum-transfer squared, and

$$P_D \equiv \frac{\sqrt{s^2 + m_B^4 + m_D^4 - 2(sm_B^2 + sm_D^2 + m_B^2 m_D^2)}}{2m_B} ,$$

is the momentum of $D$ in the $B$ rest frame. The form factors $F_0$ and $F_1$ are given by

$$F_0 = \frac{\sqrt{m_B m_D}}{m_B + m_D} (w + 1) S_1 ,$$

$$F_1 = \frac{\sqrt{m_B m_D (m_B + m_D)}}{2m_B P_D} \sqrt{w^2 - 1} V_1 ,$$

where

$$V_1(w) = V(1) \left[ 1 - 8 \rho_D^2 z(w) + (51 \rho_D^2 - 10) z(w)^2 - (252 \rho_D^2 - 84) z(w)^3 \right] ,$$

$$S_1(w) = V_1(w) \left\{ 1 + \Delta \left[ -0.019 + 0.041 (w - 1) - 0.015 (w - 1)^2 \right] \right\} ,$$

with

$$w = \frac{m_B^2 + m_D^2 - s}{2m_B m_D} , \quad z(w) = \frac{\sqrt{w + 1} - \sqrt{2}}{\sqrt{w + 1} + \sqrt{2}} ,$$

$$\rho_D^2 = 1.186 \pm 0.055 , \quad \Delta = 1 \pm 1 .$$

For $B \to D^* \ell \nu$ decay,

$$d\Gamma_{\text{SM}}^{D^*} = \frac{G_F^2 |V_{cb}|^2}{96\pi^3 m_B^2} \left[ (|H_+|^2 + |H_-|^2 + |H_0|^2) \left( 1 + \frac{m_\ell^2}{2s} \right) + \frac{3 m_\ell^2}{2s} |H_s|^2 \right]$$

$$\times s \left(1 - \frac{m_\ell^2}{s}\right)^2 P_{D^*} ,$$

$$d\Gamma_{\text{mix}}^{D^*} = \frac{G_F m_\ell g_c^e |V_{cb}|^2}{\sqrt{2} 8\pi^3 m_{H^\pm}^4 m_b + m_c} \frac{f_\ell^c + f_p^\ell}{A_0^\ell} \frac{A_0^2}{A_0} \left(1 - \frac{m_\ell^2}{s}\right)^2 P_{D^*}^3 ,$$

$$d\Gamma_{H^\pm}^{D^*} = \frac{(g_c^e)^2 |V_{cb}|^2 (f_\ell^c)^2 + (f_p^\ell)^2}{16\pi^3 m_{H^\pm}^4 (m_b + m_c)^2} A_0^2 s \left(1 - \frac{m_\ell^2}{s}\right)^2 P_{D^*}^3 ,$$

5
where \( P_{D^*} = P_B(m_D \rightarrow m_{D^*}) \). The form factors are given by

\[
H_{\pm}(s) = (m_B + m_{D^*})A_1(s) \mp \frac{2m_B}{m_B + m_{D^*}} P_{D^*} V(s) ,
\]

\[
H_0(s) = \frac{-1}{2m_{D^*} \sqrt{s}} \left[ \frac{4m_B^2 P_{D^*}^2}{m_B + m_{D^*}} A_2(s) - (m_B^2 - m_{D^*}^2 - s)(m_B + m_{D^*}) A_1(s) \right] ,
\]

\[
H_s(s) = \frac{2m_B P_{D^*}}{\sqrt{s}} A_0(s) ,
\]

where

\[
A_1(w^*) = \frac{w^* + 1}{2} r_{D^*} h_{A_1}(w^*) ,
\]

\[
A_0(w^*) = \frac{R_0(w^*)}{r_{D^*}} h_{A_1}(w^*) ,
\]

\[
A_2(w^*) = \frac{R_2(w^*)}{r_{D^*}} h_{A_1}(w^*) ,
\]

\[
V(w^*) = \frac{R_1(w^*)}{r_{D^*}} h_{A_1}(w^*) ,
\]

with

\[
w^* = \frac{m_B^2 + m_{D^*}^2 - s}{2m_B m_{D^*}} , \quad r_{D^*} = \frac{2 \sqrt{m_B m_{D^*}}}{m_B + m_{D^*}} ,
\]

and

\[
h_{A_1}(w^*) = h_{A_1}(1) \left[ 1 - 8 \rho_{D^*}^2 z(w^*) + (53 \rho_{D^*}^2 - 15) z(w^*)^2 - (231 \rho_{D^*}^2 - 91) z(w^*)^3 \right] ,
\]

\[
R_0(w^*) = R_0(1) - 0.11(w^* - 1) + 0.01(w^* - 1)^2 ,
\]

\[
R_1(w^*) = R_1(1) - 0.12(w^* - 1) + 0.05(w^* - 1)^2 ,
\]

\[
R_2(w^*) = R_2(1) + 0.11(w^* - 1) - 0.01(w^* - 1)^2 .
\]

Here

\[
\rho_{D^*}^2 = 1.207 \pm 0.028 , \quad R_0(1) = 1.14 \pm 0.07 ,
\]

\[
R_1(1) = 1.401 \pm 0.033 , \quad R_2(1) = 0.854 \pm 0.020 .
\]

For the leptonic two-body decay \( B \rightarrow \tau \nu \), the branching ratio is

\[
\mathcal{B}(B \rightarrow \tau \nu) = \mathcal{B}(B \rightarrow \tau \nu)_{\text{SM}} (1 + r_{H^\pm})^2 ,
\]

where

\[
\mathcal{B}(B \rightarrow \tau \nu)_{\text{SM}} = \frac{G_F^2 |V_{ub}|^2 m_B^2 m_B}{8 \pi} f_B^2 \left( 1 - \frac{m_{\tau}^2}{m_B^2} \right)^2 \tau_B ,
\]

\[
r_{H^\pm} = \frac{(m_u/m_b) \xi_A^u - \xi_A^d}{1 + m_u/m_b} \xi_A^d \left( \frac{m_B}{m_{H^\pm}} \right)^2 .
\]
Here $f_B$ and $\tau_B$ are the decay constant and lifetime of $B$, respectively.

The experimental data are summarized in Table II [14]. At first we try to fit the data of Table II by minimizing $\chi^2$. BABAR results [3] already ruled out the Type-II 2HDM. We introduce an anomalous $\tau$ coupling for all types of 2HDM, which will be shown to significantly reduce the $\chi^2$ minimum.

In addition, it was suggested that the ratio

$$R_\tau(D^{(*)}) \equiv \frac{R(D^{(*)})}{B(B \to \tau\nu)} \, , \quad (43)$$

has some advantages in this analysis [14]. First of all the $\tau$ detection systematics is canceled in the ratio. But it should be noted that the ratio of Eq. (43) introduces the theoretical error on $V_{ub}$. We use the values of $R_\tau(D^{(*)})$ in Table III for the fit.
FIG. 1. $\chi^2$ vs $R(D)$ for (a) free anomalous couplings and (b) $\eta = \tan^2 \beta$ at the 1$\sigma$ level. In panel (b) Type-I with $\eta = -\tan^4 \beta$ and Type-II with $\eta = -\tan^2 \beta$ are also shown. In panel (a), Type-I and Type-X are overlapped; in panel (b), Type-I($\eta = -\tan^4 \beta$) and Type-X($\eta = \tan^2 \beta$) are overlapped.

IV. RESULTS AND DISCUSSIONS

In our analysis $\tan \beta$ and $M_{H^\pm}$ are by default the fitting parameters to minimize $\chi^2$, defined by

$$\chi^2 = \sum_i \frac{(x_i - \mu_i)^2}{(\delta \mu_i)^2}, \quad (44)$$

where the $x_i$s are model predictions and the $(\mu_i \pm \delta \mu_i)$s are experimental data. Figure 1 shows the $R(D)$ values vs $\chi^2$ with the anomalous $\tau$ coupling $\eta$. In Fig. 1(a), $\eta$ is set to be an additional fitting parameter, $-1000 \leq \eta \leq 1000$. Plots for the Type-I and Type-X models are overlapped. As can be seen from Eqs. (13) and (14), the 2HDM contributes to $R(D)$ as

$$R(D^{(*)})_{H^\pm} \sim \frac{g_{s,p} f^F_s}{m^2_{H^\pm}} + \left(\frac{g_{s,p} f^F_s}{m^2_{H^\pm}}\right)^2, \quad (45)$$

where the coefficients are omitted for simplicity. For free $\eta$, Types I and X behave similarly because $\xi^u_A$ and $\xi^d_A$ are the same (see Table I). This is also true for Types II and Y. We also consider the case of fixed $\eta \equiv \tan^2 \beta$ as in Ref. [13] in Fig. 1(b). The dominant contribution
TABLE IV. The best-fit $R(D^{(*)})$ values with free $\eta$ for different Types of the model.

| Types | I   | II  | X   | Y   |
|-------|-----|-----|-----|-----|
| $R(D)$ | 0.342 | 0.362 | 0.342 | 0.342 |
| $R(D^*)$ | 0.255 | 0.253 | 0.255 | 0.254 |
| $\chi^2_{\text{min}}/\text{d.o.f.}$ | 2.881 | 2.813 | 2.881 | 2.861 |

Since the first term is negative for Types I and II for $\eta \equiv \tan^2 \beta > 0$, the $\chi^2$ values are very poor compared to those for Types X and Y, as shown in Fig. 1(b). If we allow $\eta = -\tan^4 \beta$ for the Type-I model, the $\chi^2$ distribution over $R(D)$ overlaps with that for the Type-X model. Similar things happen for the Type-II model with $\eta = -\tan^2 \beta$ and the Type-Y model. In this case, Eq. (45) is not the same for Types II and Y; the sign of $\eta$ is more relevant to the $\chi^2$ distribution than the power of $\eta$. We can see that introducing the anomalous $\tau$ coupling improves the $\chi^2$ fitting, and any Type of 2HDM model is as good (or bad) as another. The best-fit values of $R(D^{(*)})$ and the corresponding minimum $\chi^2$ per degree of freedom (d.o.f.) are given in Table IV and the allowed region of $R(D)$ and $R(D^*)$ at the 1$\sigma$ level is given in Fig. 2.

Figure 3 shows the allowed region of $m_{H^\pm}$ vs $\tan \beta$. In Fig. 3 (a), $\eta$ is a free parameter within $-1000 \leq \eta \leq 1000$. In this case $m_{H^\pm}$ cannot be large enough because the $R(D)_{H^\pm}$ term of Eq. (46) gets smaller and cannot fit the data. One exception is the Type-II model. As shown in Eq. (46), there is a $\tan^2 \beta$ enhancement for $R(D)_{H^\pm}$, which allows $m_{H^\pm}$ to be large. If we require that the charged Higgs mass is $m_{H^\pm} \gtrsim 500$ GeV, only the Type-II model survives in Fig. 3(a). In Fig. 3 (b) we fix $\eta \equiv \pm \tan^\alpha \beta$ for some $\alpha$. For Types X and Y, the allowed stripe stretches to larger $m_{H^\pm}$ with smaller $\tan \beta$ as $\alpha$ goes from 2 to 3. This is
FIG. 2. Allowed region in the $R(D)-R(D^*)$ plane at the 1σ level with free $\eta$. Vertical and horizontal lines are the best-fit points.

FIG. 3. $R(D^{(*)})$-fitting results of $\tan\beta$ vs $m_{H^\pm}$ for (a) free anomalous couplings within $-1000 \leq \eta \leq 1000$ and (b) $\eta = (\pm)\tan^\alpha\beta$ for some $\alpha$ for different Types of the model, at the 1σ level. Regions for Type-I with $\eta = -\tan^4\beta$, Type-X with $\eta = \tan^2\beta$, and Type-Y with $\eta = \tan^2\beta$ are overlapped; regions for Type-X with $\eta = \tan^3\beta$ and Type-Y with $\eta = \tan^3\beta$ are also overlapped.
FIG. 4.  $\chi^2$ vs $R_\tau(D)$ for (a) free anomalous couplings and (b) $\eta = \tan^2 \beta$ at the 1$\sigma$ level. In panel (b) Type-I with $\eta = -\tan^4 \beta$ and Type-II with $\eta = -\tan^2 \beta$ are also shown. In panel (a), Type-I is overlapped with Type-X; in panel (b), grey-green, cyan-blue, and magenta-red curves are overlapped, respectively.

because $R(D)_{H\pm} \sim \eta m_0 m_\tau / m_{H\pm}^2 + (\eta m_0 m_\tau / m_{H\pm}^2)^2$. Also shown in Fig. 3(b) are the Type-I model with $\eta = -\tan^4 \beta$ and the Type-II model with $\eta = -\tan^2 \beta$ for comparison. It would be expected from Eq. (46) that stripes for the Type-X and Y models are coincident up to $\sim \mathcal{O}(m_c/m_b)$. They also overlap with the stripe of the Type-I model with $\eta = -\tan^4 \beta$. The stripe for the Type-II model with $\eta = -\tan^2 \beta$ lies in the lowest region of $\tan \beta$ since there is already a $\tan^2 \beta$ term in $R(D)_{H\pm}$.

Now we turn to the $R_\tau(D^{(*)})$. Figure 4 shows $R_\tau(D)$ vs $\chi^2$. Note that the minimum $\chi^2$ reduces significantly compared to Fig. 1, $\chi^2_{\text{min}}$/d.o.f. = 0.623 (Type-I, X), 0.614 (Type-II) 0.615 (Type-Y) for free $\eta$ in Fig. 4(a). As discussed in Ref. [14], $R_\tau(D^{(*)})$ values from the BABAR and Belle results are consistent with each other and not so far from the SM predictions [14],

$$R_\tau(D)_{\text{SM}} = (3.136 \pm 0.628) \times 10^3, \quad (47)$$

$$R_\tau(D^{*})_{\text{SM}} = (2.661 \pm 0.512) \times 10^3. \quad (48)$$

In Fig. 4(b) we fix $\eta = \pm \tan^\alpha \beta$ for some $\alpha$. Any Type of the model is as good as another.
\[ R_\tau(D) \times 10^{-3} \]
| Types          | I   | II  | X   | Y   |
|----------------|-----|-----|-----|-----|
| \( R_\tau(D) \) | 2.828 | 2.885 | 2.828 | 2.915 |
| \( R_\tau(D^*) \) | 2.223 | 2.188 | 2.223 | 2.223 |
| \( \chi^2_{\text{min}}/\text{d.o.f.} \) | 0.623 | 0.614 | 0.623 | 0.615 |

TABLE V. The best-fit \( R_\tau(D^{(*)}) \) values with free \( \eta \) for different Types of the model.

\[
R_\tau(D^*) \text{ vs } \chi^2 \text{ shows similar behavior. The new contribution to } B(B \to \tau \nu) \text{ is}
\]
\[
r_{H^\pm} \sim -\xi^d A^\tau \left( \frac{m_B}{m_{H^\pm}} \right)^2 \begin{cases} 
(-\eta \cot^2 \beta) \left( \frac{m_B}{m_{H^\pm}} \right)^2 & \text{for Type-I} \\
(-\eta \tan^2 \beta) \left( \frac{m_B}{m_{H^\pm}} \right)^2 & \text{for Type-II} \\
\eta \left( \frac{m_B}{m_{H^\pm}} \right)^2 & \text{for Type-X} \\
\eta \left( \frac{m_B}{m_{H^\pm}} \right)^2 & \text{for Type-Y,} 
\end{cases} \tag{49}
\]

where terms of \( \mathcal{O}(m_u/m_b) \) are neglected. As in Eq. (46), only the combination of \( \xi_A^d A^\tau \) is relevant, and thus the Type-I model with \( \eta = -\tan^4 \beta \) looks much like the Type-X models with \( \eta = \tan^2 \beta \), and so on.

Table V shows the best-fit values of \( R_\tau(D^{(*)}) \) and \( \chi^2_{\text{min}}/\text{d.o.f.} \), and Fig. 5 shows the allowed region of \( R_\tau(D) \) and \( R_\tau(D^*) \) at the 1σ level.

Figure 6 shows the allowed region in the \( m_{H^\pm}-\tan \beta \) plane to fit the \( R_\tau(D^{(*)}) \). In Fig. 6(a), \( \eta \) is a free parameter. For Types X and Y, almost the entire region is allowed. The different behaviors of the Type I and II models are due to the factors of \( \eta/\tan^2 \beta \) (Type-I) and \( \eta \tan^2 \beta \) (Type-II) in Eq. (46).

In Fig. 6(b), \( \eta \equiv \pm \tan^\alpha \beta \) for some \( \alpha \). Compared to Fig. 3(b), each Type shows similar behavior, but with broader bands. The reason is that the \( R_\tau(D^{(*)}) \) values are more consistent with each other than \( R(D^{(*)}) \) ones, and thus more points in the \( m_{H^\pm}-\tan \beta \) plane are allowed around \( \chi^2_{\text{min}} \). And the bands for Types X and Y with \( \eta = \tan^2 \beta \) stretch to the region of \( m_{H^\pm} = 1000 \text{ GeV} \).

V. CONCLUSIONS

In this work we tried to solve the puzzle of \( R(D^{(*)}) \) in the 2HDM. We introduced \( \eta \) as an anomalous \( \tau \) coupling to \( H^+ \) to fit the data through minimizing \( \chi^2 \). To fit the excess of the
FIG. 5. Allowed region in the $R_\tau(D) - R_\tau(D^*)$ plane at the 1σ level with free $\eta$. Vertical and horizontal lines are the best-fit points.

FIG. 6. $R_\tau(D^{(*)})$-fitting results of $\tan \beta$ vs $m_{H^\pm}$ for (a) free anomalous couplings within $-1000 \leq \eta \leq 1000$ and (b) $\eta = (\pm) \tan^\alpha \beta$ for some powers of $\alpha$ for different Types of the model, at the 1σ level. Regions for Type-I with $\eta = -\tan^4 \beta$, Type-X with $\eta = \tan^2 \beta$, and Type-Y with $\eta = \tan^2 \beta$ are overlapped; regions for Type-X with $\eta = \tan^3 \beta$ and Type-Y with $\eta = \tan^3 \beta$ are also overlapped.
data over the SM predictions it needs to enhance the charged Higgs contributions, which come in the form of $R(D^{(*)})_{H^\pm} \sim \eta m_b m_\tau / m_{H^\pm}^2 + (\eta m_b m_\tau / m_{H^\pm}^2)^2$. Thus, for small values of $\eta$, $m_{H^\pm}$ cannot be large enough to avoid detection. For the Type-II the situation is alleviated because there is already a factor of \(\tan^2 \beta\) (but with opposite sign) in $R(D^{(*)})_{H^\pm}$. As shown in Fig. 3(b), a large $m_{H^\pm} \sim 1000$ GeV is allowed if $R(D^{(*)})_{H^\pm} \sim (m_b m_\tau / m_{H^\pm}^2) \tan^3 \beta + (m_b m_\tau / m_{H^\pm}^2)^2 \tan^6 \beta$ in any Type of 2HDM model.

The new ratios $R_\tau(D^{(*)})$ fit much better. Contributions of the form \(\sim (m_b m_\tau / m_{H^\pm}^2) \tan^2 \beta + (m_b m_\tau / m_{H^\pm}^2)^2 \tan^4 \beta\) allow a large $m_{H^\pm} \sim 1000$ GeV as in Fig. 6(b), which is not true for the $R(D^{(*)})$ fitting. In both cases of $R(D^{(*)})$ and $R_\tau(D^{(*)})$ fitting, any type of 2HDM is as good as another with an appropriate $\eta$. For a sufficiently large $m_{H^\pm} \gtrsim 1000$ GeV, new contributions of the form \(\sim (m_b m_\tau / m_{H^\pm}^2) \tan^k \beta\) with $k = 2$ fit the data well for $R_\tau(D^{(*)})$, while $k \geq 3$ for $R(D^{(*)})$. It should be noted that the errors in $R_\tau(D^{(*)})$ are still large.

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