Forward and Inverse Predictive Model for the Trajectory Tracking Control of a Lower Limb Exoskeleton for Gait Rehabilitation: Simulation modelling analysis

M A Zakaria, A P P A Majeed, Z Taha, M M Alim, K Baarah
Innovative Manufacturing, Mechatronics and Sport (iMAMS) Laboratory, Faculty of Manufacturing Engineering, Universiti Malaysia Pahang
maizzat@ump.edu.my

Abstract. The movement of a lower limb exoskeleton requires a reasonably accurate control method to allow for an effective gait therapy session to transpire. Trajectory tracking is a non-trivial means of passive rehabilitation technique to correct the motion of the patients’ impaired limb. This paper proposes an inverse predictive model that is coupled together with the forward kinematics of the exoskeleton to estimate the behaviour of the system. A conventional PID control system is used to converge the required joint angles based on the desired input from the inverse predictive model. It was demonstrated through the present study, that the inverse predictive model is capable of meeting the trajectory demand with acceptable error tolerance. The findings further suggest the ability of the predictive model of the exoskeleton to predict a correct joint angle command to the system.

1. Introduction
The growing demand for health services such as rehabilitation tends to outstrip the supply in a limited economic setting. Notwithstanding, rehabilitation services are essential to address disabilities that are associated with gait disorder. Gait, is the ability to maintain the walking pattern of a person. Gait abnormalities are the inability of a person to walk that may originate from cerebellar disease, neuromuscular disease, cognitive impairment, stroke, brain or spinal injury [1], [2]. Studies have shown that through the continuous and repetitive motion, the patient's walking pattern may be restored [3]. The assistance from physiotherapists are required to facilitate this form of activity, however, it is worth to note that it is labour and cost intensive. The shortcomings of conventional rehabilitation therapy sessions have led towards the engagement of robotics to mitigate the aforesaid issues.

Various types of methods have been researched previously to address the rehabilitation issues by means of the control system development for rehabilitation device. For instance, a 3-dof device developed by Rahman et. al. uses the integration of Proportional Derivative (PD) control with the integration of neuro fuzzy system [4]. Sliding mode control and nonlinear torque control which is proven to reduce the chattering of the system was employed by the author in [5]. Furthermore, several other methods using AFC-PD were addressed in [6] which provide the adaptability to the patient’s movement based on the torque and current feedback to the system.
This paper presents the mathematical modelling of the lower limb exoskeleton with actuators. The movement of the lower limb exoskeleton requires a reasonably precise positional control to track the prescribed trajectory to rehabilitate the lower limb joints. Contrary to the previous research, the forward and inverse kinematics are developed to predict the correct positional tracking of the exoskeleton. The actuator dynamics is also modelled in order to control the movement of the lower limb exoskeleton. A classical proportional-integral-derivative (PID) control architecture is used as a controller for this robotic system. The advantage of the inverse model control is that the predictive motion that can be estimated by the device without any additional sensors besides the encoders attached to the joint angles. This method is easier to implement and require less burden to the processing unit of the system as the solution of the joint angle is already derived and only require one line of the joint angle command to the system.

2. Methodology

Figure 1 depicts the overall block diagram of the exoskeleton system. The actuator and forward kinematics model are assigned as the plant of the system, whilst the inverse kinematics prediction model provides the solution to the required Cartesian coordinate input to the exoskeleton. The controller block is used to provide the error correction to the evaluated system.

The Denavit-Hartenberg (D-H) algorithm is used to construct the exoskeleton axis and its relevant parameters. Although the D-H method provides a long and complex mathematical solution, nonetheless, it is a systematic method to develop sophisticated robotic modelling. It is worth mentioning that there are other methods that are readily available to develop such model, however, it is beyond the scope of this paper. The readers are encouraged to refer to [7] for further detailed treatment of the other techniques. The methods involved in the present study are as follows:

1. The lower limb exoskeleton is shown in Figure 2. From the figure, the D-H parameter may be assigned to the degree of freedom of interest.
2. Then, the D-H parameters are tabulated as in Table 1 to ascertain its corresponding values.
3. The resultant D-H parameter is then shown in Equation 3.

![Figure 1 Overall System Block Diagram](image-url)
Figure 2 Lower Limb Exoskeleton Coordinate Assignment

Table 1 D-H Exoskeleton Robotic Leg Parameters

| Link, i | Joint angle (degree), $\theta_i$ | Link offset (meter), $d_i$ | Link length (meter), $a_i$ | Twist angle (degree), $\alpha_i$ |
|---------|---------------------------------|---------------------------|--------------------------|---------------------------------|
| 1       | $\theta_1$                      | 0                         | 1.5                      | 0                               |
| 2       | $\theta_2$                      | 0                         | 0.5                      | 0                               |
| 3       | $\theta_3$                      | 0.2                       | 0                        | 0                               |
| 4       | $\theta_4$                      | 0.2                       | 0                        | 0                               |

2.1 Transformation Matrix

Once the D-H table has been tabulated; the robot transformation matrix can be obtained by substituting the D-H parameter into the Equation 3.

$$A_i^{-1} = \begin{bmatrix} c\theta_i & -ca_i s\theta_i & sa_i s\theta_i & a_i c\theta_i \\ s\theta_i & ca_i c\theta_i & -sa_i c\theta_i & a_i s\theta_i \\ 0 & sa_i & ca_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (1)

$$T_4^0 = A_1^0 A_2^1 A_3^2 A_4^3$$  \hspace{1cm} (2)

The total transformation matrix is as follows:

$$T_4^0 = \begin{bmatrix} C_{1,2,A} & -S_{1,2,A} & 0 & L_2 C_{1,2} + L_1 C_1 \\ S_{1,2,A} & c\alpha_i c\theta_i & 0 & L_2 S_{1,2} + L_1 S_1 \\ 0 & 0 & 1 & d_3 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (3)

Where $C_{i,j,k} = \cos(\theta_i + \theta_j + \theta_k)$ and $S_{i,j,k} = \sin(\theta_i + \theta_j + \theta_k)$.

2.2 Inverse Kinematics

The inverse must be obtained to derive the relationship between the required position and joint angles relationship. In order to derive this; the forward kinematic equation can be compared with the necessary inputs (position vectors and orientation vectors). The inverse kinematics is essential for the control system development. It allows the user to determine the required joint angles command in
order to reach specific position or configuration. The inverse kinematics utilises the ankle required position Px, Py, Pz as inputs to determine the joint angles, \( \theta_i \).

To calculate the desired angle of the exoskeleton, the general orientation, and position of the represented end effector can be defined as below after derivation using trigonometric derivations and rearrange the transformation matrix as in Equation 4. The further derivation details can be referred to [8]. If the dynamics of the exoskeleton is required, the readers are encouraged to refer to [6].

\[
A_3^{-1} A_2^{-1} A_1^{-1} T_4^{-1} = A_4^3
\] (4)

\[
\theta_1 = \tan^{-2}(-L_2 S_2 P_x + P_y (L_1 + L_2 C_2), P_x (L_1 + L_2 C_2) + P_y L_2 S_2)
\] (5)

\[
\theta_2 = \tan^{-2}(n_y, n_x) - \theta_1 - \theta_4
\] (6)

\[
\theta_4 = \tan^{-2}(-n_x S_{1,2} + n_y C_{1,2} n_x C_{1,2}, n_x C_{1,2} + n_y S_{1,2})
\] (7)

2.3 Actuator dynamics
The joints of the exoskeleton are controlled by actuators. Hence, the modelling of the actuators is non-trivial to understand the interaction issues between the control input and the resultant joint angles, namely the transient response of the actuators. More often than not, the torque, current and speed of the driving motor during transient time is studied to observe the performance of the system. The modelling of the actuators can be performed by applying Kirchoff’s Voltage Law to the motor model which yields the following equations:

\[
\frac{di}{dt} = \frac{V}{L} - \frac{R}{L} i - \frac{K}{L} \omega
\] (8)

\[
\frac{d\omega}{dt} = \frac{K}{J} i - \frac{b}{J} \omega
\] (9)

Where \( I, V, L, R, K, J, w \) are the motor current, voltage, inductance, internal resistance, motor constant, load inertia, and motor speed.

PID Control System
In this study, a conventional PID control system is used to move the actuator to the desired angles. The PID control law may be defined as:

\[
u(t) = K_p e + K_i \int e \, dt + K_d \frac{de}{dt}
\] (10)

Where \( K_p, K_i, K_d \) are the PID control gain, \( e \) is the PID error and \( u(t) \) is the control input.

3. Results
To verify the model, the exoskeleton is assumed to be running in all quadrant operations to ensure the correct values are obtained. All quadrant operations are illustrated into the desired input as shown in Table 2.

| Quadrant Working Operations |
|-----------------------------|

Table 2 Quadrant Working Operations
The trajectory tracking response is observed in terms of its error and desired angle calculated by the inverse kinematics. In the present study, the Symmetric Mean Absolute Percentage Error (SMAPE) is employed to compute the relative error of the response. The formula is defined in Equation 1.

\[
SMAPE(\%) = \frac{100}{n} \sum_{t=1}^{n} \frac{|F_t - A_t|}{|A_t| + |F_t|} 
\]

(15)

Where \(F_t, A_t, n\) are the desired value, actual value and sample count respectively. The tracking percentage error and performance is shown in Table 3, respectively. From the table, it can be seen that the robotic exoskeleton is able to reach the correct quadrant with less than 5% angular tracking error. The positional tracking error \((P_x, P_y, P_z)\) is acceptable in Quadrant 1, 2, and 3. In the quadrant 4 operation, the exoskeleton contain a larger relative error percentage (18.77%) in \(P_y\) position. However, the orientation and direction of the quadrant operation is still in correct position. Furthermore, the angular relative error for the 4th quadrant is still relatively small with no additional correction although some fine tuning may be required to improved the tracking error percentage. The mean response time of the exoskeleton to reach the steady state is 10.75 seconds which is acceptable.

**Figure 3** shows the trajectory tracking error of the exoskeleton control. The error is measured from the distance of the actual exoskeleton position compared with the desired position. From the figure, it can be seen that the error converged to smaller value over time. The largest positional tracking error can be seen in Quadrant 4 with a steady state error of 0.2 meters offset from the actual position.

| Quadrant | Desired \(P_x\) | Desired \(P_y\) | Desired \(P_z\) | \(\theta_1\) | \(\theta_2\) | \(\theta_3\) | \(\theta_4\) | Response time, t |
|----------|--------------|--------------|--------------|-----------|-----------|-----------|-----------|----------------|
| 1st      | 1.5          | 0.01         | 0.4          | 1.604     | -3.12     | -3.12     | 7 seconds  |
|          | Actual       | 1.499        | 0.0154       | 0.4       | 1.676     | -3.263    | -3.263    |
|          | % error      | 0.07         | 0.54         | 4.4       | 4.4       | 4.4       |
| 2nd      | -0.3         | 0.5          | 0.4          | -147.14   | -146.14   | -146.14   | 10 seconds |
|          | Actual       | -0.3869      | 0.471        | 0.4       | -152.51   | -151.51   | -151.51   |
|          | % error      | 25.3         | 5.97         | 3.69      | 3.69      | 3.69      |
| 3rd      | -0.5         | -0.4         | 0.4          | -151.64   | 147.5     | 147.5     | 14 seconds |
|          | Actual       | -0.4237      | -0.4242      | 0.4       | -157.34   | 152.9     | 152.9     |
|          | % error      | 16.20        | 5.87         | 3.59      | 3.59      | 3.59      |
| 4th      | 1.0          | -0.8         | 0.4          | -67.84    | -65.14    | -65.14    | 12 seconds |
|          | Actual       | 0.8369       | -0.9658      | 0.4       | -70.28    | 67.47     | 67.47     |
|          | % error      | 17.75        | 18.77        | 3.51      | 3.51      | 3.51      |

Table 3 Trajectory Tracking Performance
4. Conclusion and Future Works
In this paper, the lower limb exoskeleton system model is validated in all quadrants of operations. It has been demonstrated that the PID controller with inverse prediction model is able to track the required position and angular position of the system. However, it is worth to mention that some positioning error does exist, in which improvement in the overall closed loop system is required. This may include the inclusion of an external checking mechanism which periodically monitors whether the minimisation of the positional error has been achieved through an additional forward predictive kinematics in front of the required inputs.

Acknowledgments
The authors would like to thank Universiti Malaysia Pahang for the financial aid under research grant RDU160343 and RDU170731.

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