EFFECT OF LOCAL SHEAR ON DRIFT FLUCTUATION DRIVEN TRANSPORT IN TOKAMAKS

J. KESNER (Massachusetts Institute of Technology, Cambridge, Massachusetts, United States of America)

ABSTRACT. A theory of electron thermal diffusivity has been proposed which is based on electron drift wave turbulence. When account is taken of the effect of local shear changes produced in high poloidal beta equilibria, this theory is found to predict the observed improvement of tokamak confinement with plasma current. Confinement is good at low poloidal beta, degrades as poloidal beta increases and then enters an improved transport region at high poloidal beta.

1. INTRODUCTION

Theoretical understanding of tokamak thermal transport is not well established and the fusion community has depended on empirical formulas. One empirical form proposed by Goldston for strongly heated tokamaks [1] is

\[ \tau_E = 0.037 \frac{I_p R^{1.75} \kappa^{1/2}}{a^{0.37} P_{aux}^{1/2}} \]  

where \( I_p \) is the plasma current in MA, \( \kappa \) is the ellipticity, \( R \) and \( a \) are the major and minor radii in m and \( P_{aux} \) is the auxiliary power in MW. It is evident from Eq. (1) that tokamak confinement improves with current and degrades with auxiliary power — a troublesome result. Only first-principle theory can give insight into the origin and implications of this result.

The electron thermal diffusivity resulting from both electromagnetic and electrostatic drift wave turbulence in a collisionless plasma has been evaluated by Horton et al. [2]. They assume the presence of a fixed amplitude spectrum of electron temperature gradient driven drift waves, the so-called \( r_e \) modes, and calculate the resulting particle transport. For the short wavelength electrostatic and electromagnetic waves, they choose a wave spectrum with \( \rho_e < 2\pi/k < c/\omega_{pe} \) (\( \omega_{pe} \) is the electron plasma frequency and \( \rho_e \) is the electron gyroradius). Stochasticity results from the resonance between the trapped electron circulation frequency, \( \Omega_e \), and the trapped electron bounce motion (for electromagnetic waves) or the wave difference frequency, \( \Delta \omega \) [3]. The circulation frequency is the \( E \times B \) drift frequency with which a trapped electron circulates around the wave potential island structures (in r-\( \theta \) space). The circulation frequency is only meaningful for trapped electrons. (The untrapped electrons will jump in the poloidal plane at an angle determined by the rotational transform with each transit around the torus and therefore cannot be trapped in the wave potential structure.) Horton et al. [2] found an electron thermal diffusivity of the form

\[ \chi_e = \epsilon^{1/2} \left( D_{em} \frac{c^2 \omega_{he}}{\omega_{pe}^2} + D_{es} \frac{\rho_i c T_e}{r_n eB} \right) \]  

where the \( D_{em} \) term comes from the short wavelength \( r_e \) modes and the \( D_{es} \) term comes from longer wavelength drift waves; \( \epsilon \) is the inverse aspect ratio, \( \epsilon = a/R \), \( \omega_{he} = \sqrt{e q_{he}}/R \) is the electron bounce frequency, \( q_{he} \) is the thermal speed, \( q \) is the safety factor, \( \rho_i \) is the ion gyroradius at the electron temperature, \( T_e \) is the electron temperature and \( r_n \) is the density gradient scale length. \( D_{em} \) and \( D_{es} \) are adjustable constants of order unity for
The local shear is reversed within the cross-hatched region. Thereafter, higher poloidal beta leads to more strongly reversed shear. The detailed behaviour of local shear with poloidal beta depends on the plasma current density profile as well as on plasma shaping, but reduction and reversal of local shear with increasing poloidal beta is always observed in calculations with reasonable (monotonically decreasing) current density profiles. For $\epsilon \beta_p \approx 1$, this effect is responsible for the 'second stability' phenomenon, which has been predicted to restabilize MHD ballooning modes [4]. We show here that it is through the $\epsilon \beta_p$ driven equilibrium changes of the shear appearing in the drift wave transport theory that an $\epsilon \beta_p$ dependence, and therefore a current dependence, enters $\chi_e$ and $\tau_{te}$.

Equation (2) predicts an increase in thermal diffusivity with electron temperature and therefore with auxiliary heating at constant current. This degradation of confinement is accentuated by the reduction of local shear that accompanies the increase in poloidal beta. However, we see that at sufficiently high poloidal beta the local shear reverses and then increases (in the reverse direction), leading to a recovery of confinement.

Thus we expect two regions of good confinement—at low and at high poloidal beta. Ohmically heated tokamaks always operate in the low poloidal beta region; more power requires more current and thereby reduces the poloidal beta. We would therefore not expect a degradation of confinement with Ohmic power. On the other hand, experiments with heating at constant current may be expected to exhibit a sharp deterioration of confinement to accompany increases in poloidal beta when $\beta_p \approx 1$.

2. THEORETICAL MODEL OF $\tau_{te}$ AND ITS DEPENDENCE ON $s_t$

A confinement time can be obtained from Eq. (2) by taking $\tau_{te} = a^2/4\chi$:

$$\tau_{te} = \frac{66}{f_t(\epsilon \beta_p) D_{em}} \frac{n_e a R^2}{\sqrt{T_e (1/q + \Lambda \beta_e)}}$$  \hspace{1cm} (3)

with

$$\Lambda = \frac{1}{2} \frac{D_{es}}{D_{em}} \frac{1}{\epsilon^{3/2}} \sqrt{\frac{m_e}{m_i}}$$  \hspace{1cm} (3a)

The dependence on local shear, $f_t(s_t)$, is exhibited explicitly in Eq. (3) and $s_t = s_t(\epsilon \beta_p)$. The confinement thus derived ignores profile factors and is approximate.
The dependence of the diffusion coefficient on magnetic shear is obtained numerically in Ref. [2] for electromagnetic modes. We fit the result of Ref. [2] with an exponential form and replace global shear by local shear to obtain

\[ f_r = 0.65 \exp(-3|s_r|) + 0.05 \]  

(4)

To estimate the local shear dependence on poloidal beta, we have performed equilibrium calculations using a version of the PEST code [5] that has been modified [6] to produce current profiles having minimum edge current density for a fixed edge safety factor \( q_e \). In the first series of calculations we fix \( q_0 = 1, q_a = 4, A = 3 \), for both a circular plasma and a D-shaped plasma (elongation \( \kappa = 1.5 \), triangularity \( \delta = 0.5 \)). The ellipticity and triangularity of the plasma boundary are defined by assuming the following form for the plasma boundary:

\[ Z(\theta) = \kappa a \sin(\theta - \delta \sin \theta) \]  

(5a)

\[ R(\theta) = R_0 + \xi(r) + a \cos \theta \]  

(5b)

where \( \xi(r) \) is the shift of the centre of the elliptic flux surface, and \( \kappa \) parameterizes the ellipticity and \( \delta \) the triangularity.

Figure 2 shows the local shear at the outer torus versus poloidal beta at the flux surface that is halfway out in poloidal flux. A similar trends of reduced and reversed local shear is observed in both cases, although the shear decreases sooner for the D-shaped...
equilibrium. The observed high beta shear reversal was seen to occur similarly for all reasonable current profiles.

Figure 3 is a plot of $\tau_{\text{E}}$ versus $T_e$ at fixed current from Eq. (3) (solid line) for a D-shaped tokamak ($q(a) = 4$, $\bar{n}_e = 3 \times 10^{13} \text{ cm}^{-3}$, $B_i = 1 \text{ T}$, $\epsilon = 0.33$, $\kappa = 1.5$, $\delta = 0.5$). We choose $D_{\alpha} = 1.5$ and $D_{\text{em}} = 0.6$, since these values minimize the mean square divergence from experimental D-III data in the database [7] discussed below. As the electron temperature increases, the confinement time is observed to decrease until a poloidal beta value is reached at which the outer torus local shear disappears. Thereafter, a further temperature rise leads to an approximately constant $\tau_{\text{E}}$. 

\[ \text{FIG. 4. } \tau_{\text{E}} \text{ versus plasma current for fixed plasma pressure, } \beta = 3\%, \bar{n}_e = 3 \times 10^{13} \text{ cm}^{-3}, B_i = 1 \text{ T}, \epsilon = 0.33, \kappa = 1.5, \delta = 0.5. \]

\[ \text{FIG. 5. Experimentally measured } \tau_{\text{E}} \text{ versus theoretical value for circular boundary tokamaks in the database of Ref. [7].} \]
as the increasing reversed shear offsets the $T_e$ driven degradation. If we had assumed a larger $q_a$ value (lower plasma current), the confinement would have been reduced in the low temperature ($T_e \leq 1.4$ keV) region but it would have been improved at higher $T_e$.

Figure 4 is a plot of $\tau_{Fe}$ versus plasma current for fixed plasma pressure ($\beta = 3\%$). Above 300 kA, the confinement increases approximately linearly with current because of the increasing local shear that accompanies decreasing $\beta_p$. Below 300 kA, however, we are in the reversed shear region and the confinement increases with decreasing current. In fact, the confinement time is equal at 200 kA and at 600 kA. The region of minimum confinement corresponds to where the local shear goes through zero, and this generally corresponds to a region of MHD (ballooning) instability. Thus, both a stability barrier and a transport barrier separate the low beta region, where confinement scales approximately linearly with current from the high beta region where confinement is expected to scale inversely with current.

In Fig. 5, the scaling law obtained from Eq. (2) is compared with the Kaye--Goldston database of auxiliary heated tokamaks [7]. Because of the complicated shear dependence on plasma shaping, we consider only data from the the circular cross-section machines, ASDEX, DITE, PDX, TFR and TFTR. The database includes only data from gettered L-mode experiments with $P_{inj} \approx 2P_{OH}$. We assume that the electron loss channel is dominant (and therefore $\tau_{Fe} \approx 2\tau_{Fe}$). The database contains the measured $\tau_{Fe}$ values, the plasma current, the total input power and geometric parameters.

To quantify the comparison, we define $\nu^2 = (\tau_{Fe} - \tau_{exp})^2/\tau_{Fe}^2$. We find that $\nu$ attains a minimum value ($\nu = 0.23$) by setting $D_{es} = 8.5$ and $D_{es}/D_{em} = 2.7$ in Eq. (3). If we would leave out the local shear dependence (i.e. set $f_p$ = constant) we would obtain $\nu = 0.27$. For comparison, the Goldston L-mode fit gives $\nu = 0.24$. Thus we see that in this database the local shear leads to an observable improvement of the fit to the measured values.

The agreement between theory and experiment is good, indicating that electron drift wave transport might indeed be the dominant loss channel. Including local shear yields only a modest improvement since the database is confined to relatively low poloidal beta shots. Other physical processes, including profile effects and losses in the ion channel, undoubtedly account for some of the scatter.

We have compared Eq. (3) with a D-III database [7]. We did not carry out the appropriate equilibrium calculations for DIII-D, but instead used the local shear from the $\kappa = 1.5$ series of calculations discussed above; $\nu^2$ was again minimized for $D_{es}/D_{em} = 2.7$, with $D_{es} = 1.5$.

3. DISCUSSION AND CONCLUSIONS

We have shown that a self-consistent treatment of the global equilibrium by including local transport may be a key to understanding tokamak transport. Taking $\chi_e$ [2] for electron drift waves and postulating that the local shear factor modelled by Eq. (4) should be in front of $\chi_e$, we come to the conclusion that confinement can be improved or degraded via the $e\beta_p$ dependent local shear. We find that two regions of good confinement exist — at low and at high poloidal beta. Ohmically heated tokamaks always operate in the low poloidal beta region; more power requires more current and thereby reduces the poloidal beta. Thus an increase of $T_e$ is offset by a decrease in poloidal beta.

Since we have only discussed the electron channel, the improvement or deterioration of this channel is only meaningful if the ion channel does not dominate transport. Furthermore, our conclusions are only valid if the plasma is collisionless.

We have not attempted to match the radial dependence of $\chi_e$ with experimental data. To do this properly, the radial temperature and density dependences must be known as well as the radial dependence of the local shear. (The reduction of local shear will tend to make $\chi_e$ increase locally with radius.)

In strongly auxiliary heated discharges with $\varepsilon\beta_p \geq 1$, we predict that adding power will decrease $\tau_{Fe}$ and that adding current will normally increase $\tau_{Fe}$. The increase of $\tau_{Fe}$ with current results from a reduction of the poloidal field driven shear degradation at the outer torus. In the opposite limit of high poloidal beta ($\varepsilon\beta_p \lessgtr 1$), reducing plasma current would improve confinement. Additionally, operating in the low current regime is desirable because high $\beta_p$ may eliminate MHD ballooning modes (i.e. second stability) and low current will reduce volt-second requirements and/or current drive requirements.

Since a strong degradation of confinement accompanies auxiliary heating in high current discharges, a transport barrier must be overcome for entrance to the second transport regime. Recent experiments on the Versator II tokamak [8] have shown that high beta equilibrium ($\varepsilon\beta_p = 1$) can be achieved in low current discharges in which the thermal plasma pressure remains low and current drive is employed. Both the plasma current and the pressure are carried by energetic passing electrons.
which would not take part in the above mentioned transport. This leads to the possibility of accessing the high confinement region by starting with a high poloidal beta discharge and increasing the current (using RF current drive techniques) in proper proportion to the pressure during the heating phase so as to remain in the high confinement regime.

For our comparison of the scaling law (Eq. (2)) with the database for strongly auxiliary heated experiments (Fig. 5), it must be kept in mind that we calculate only one loss process, namely turbulent transport in the electron channel. The good agreement obtained implies that electron channel drift wave turbulence may indeed be the dominant loss process.

If we consider strongly auxiliary heated devices and therefore only keep the high beta term in the denominator of Eq. (3), we can apply the power balance (Eq. (6)) to show that $\tau_{\text{Ee}} \propto P_{\text{max}}^{0.6}$. Thus we obtain a power degradation similar to that observed by Kaye and Goldston [7]. The improvement of $\tau_{\text{E}}$ with current in these empirical fits can be compared with Fig. 4, where an approximately linear improvement of $\tau_{\text{Ee}}$ with current in the high current regime is evident.

ACKNOWLEDGEMENTS

The author would like to thank S. Kaye (PPPL) for supplying the database for the auxiliary heated experiments. Thanks are also due to P. Hakkarainen for permission to use his modified version of PEST, as well as to J. Freidberg, B. Lane, D. Sigmar (MIT) and L. Woods (Oxford Univ.) for helpful and stimulating discussions.

This work was supported by the United States Department of Energy, under Contract No. DE-AC02-78ET-51013.

REFERENCES

[1] GOLDSTON, R.J., Plasma Phys. Controll. Fusion 26 (1986) 87.
[2] HORTON, W., CHOI, D.-I., YUSHMANOV, P.N., PARAIL, V.V., Plasma Phys. Controll. Fusion 29 (1987) 901.
[3] HORTON, W., Plasma Phys. Controll. Fusion 27 (1985) 937.
[4] COPPI, B., CREW, G.B., RAMOS, J.J., Comments Plasma Phys. Controll. Fusion 6 (1981) 109.
[5] GRIMM, R.C., DEWAR, R.L., MANICKAM, J., J. Comput. Phys. 4 (1983) 94.
[6] HAKKARAINEN, S.P., FREIDBERG, J.P., Bull. Am. Phys. Soc. 32 (1987) 1772.
[7] KAYE, S.M., GOLDSTON, R.J., Nucl. Fusion 25 (1985) 65.
[8] LUCKHARDT, S.C., CHEN, K.I., CODA, S., et al., Phys. Rev. Lett. 62 (1989) 1508.

(Manuscript received 14 November 1988
Final manuscript received 17 April 1989)