Using the minimum spanning tree to trace mass segregation

Richard J. Allison,1⋆ Simon P. Goodwin,1 Richard J. Parker,1 Simon F. Portegies Zwart,2,3 Richard de Grijs1,4 and M. B. N. Kouwenhoven1

1Department of Physics and Astronomy, University of Sheffield, Sheffield S3 7RH
2Section Computational Science, University of Amsterdam, Kruislaan 403, Amsterdam, The Netherlands
3Astronomical Institute 'Anton Pannekoek', University of Amsterdam, Kruislaan 403, Amsterdam, The Netherlands
4NAOC-CAS, Beijing 100012, China

Accepted 2009 January 14. Received 2009 January 12; in original form 2008 December 5

ABSTRACT
We present a new method to detect and quantify mass segregation in star clusters. It compares the minimum spanning tree (MST) of massive stars with that of random stars. If mass segregation is present, the MST length of the most massive stars will be shorter than that of random stars. This difference can be quantified (with an associated significance) to measure the degree of mass segregation. We test the method on simulated clusters in both 2D and 3D and show that the method works as expected.

We apply the method to the Orion Nebula Cluster (ONC) and show that the method is able to detect the mass segregation in the Trapezium with a ‘mass segregation ratio (MSR)’ \( \Lambda_{\text{MSR}} = 8.0 \pm 3.5 \) (where \( \Lambda_{\text{MSR}} = 1 \) is no mass segregation) down to 16 \( \text{M}_\odot \), and also that the ONC is mass segregated at a lower level (\( \sim 2.0 \pm 0.5 \)) down to 5 \( \text{M}_\odot \). Below 5 \( \text{M}_\odot \) we find no evidence for any further mass segregation in the ONC.

Key words: methods: data analysis.

1 INTRODUCTION
It is thought that many young star clusters show ‘mass segregation’ – where the massive stars are more centrally concentrated than lower mass stars (see Section 2 for a review of current methods for detecting mass segregation).

There are two possible origins of mass segregation in star clusters. It may be dynamical, in that mass segregation evolves within an initially non-mass segregated cluster due to dynamical effects (Chandrasekhar 1942; Spitzer 1969). Or it may be primordial, where mass segregation is an outcome of the star formation process (Murray & Lin 1996; Bonnell & Bate 2006). Information about the presence and degree of mass segregation thus provides strong constraints on star cluster formation and evolution. However, there are currently no good methods by which mass segregation can be determined and quantified absolutely. This seriously limits the interpretation of observations of mass segregation, and especially our ability to compare different clusters.

In this paper we present a new method, based on the minimum spanning tree (MST), which allows us to quantify the degree of mass segregation in a cluster, and examine how mass segregation changes with stellar mass. In Section 2 we review the limitations of current methods of determining mass segregation. In Sections 3 and 4 we introduce our new method and test it against a number of artificial clusters and the Orion Nebula Cluster (ONC). In Section 5 we discuss the method’s limitations and some ways in which it might be improved, and we summarize our results in Section 6.

2 MASS SEGREGATION

2.1 What is mass segregation?
In this paper we will take as a working definition of mass segregation that the most massive stars are not distributed in the same way as other stars. In particular, that they have a more concentrated distribution.

This is not the only possible definition of mass segregation. It would be possible to define mass segregation as a significant difference in the mass functions (MFs) at different positions in a cluster. Indeed, this is often the way mass segregation is currently defined (see Section 2.2). However, we feel such a definition is problematic as it raises the question of how many stars need to be included in order to properly sample the MF and therefore tell if two MFs are different. Furthermore, how much do the mass ranges sampled

⋆E-mail: r.allison@sheffield.ac.uk

1 We deliberately distinguish between the present-day MF and the initial MF.
need to overlap in order to differentiate MF slopes? Both of these questions are difficult to answer without assuming a MF a priori.

2.2 Current methods for detecting mass segregation

The current methods can be loosely separated into two groups, those that involve fitting a density profile and characteristic radii to various mass ranges; and those that trace the variation of the MF with radius.

None of the current methods to examine mass segregation gives a model-independent, quantitative measure of the amount of mass segregation in a cluster (and most are not quantitative at all). The current methods of determining the presence of mass segregation also rely on a determination of the centre of the cluster, which in itself may be very difficult to define.

Density profiles are usually fitted with a model profile (e.g. a King 1966 model) or as a cumulative distribution function for different mass ‘bins’. This allows the variation in distribution to be easily seen, and the ‘degree’ of mass segregation can be measured by the change of the slope of the distribution function. From these distributions characteristic radii for each mass bin (e.g. core radius) can be found. This method has been used in many studies (Hillenbrand 1997; Pinfield, Jameson & Hodgkin 1998; Raboud 1999; Adams et al. 2001; Littlefair et al. 2003; Sharma et al. 2008). However, the accuracy of this method has been brought into question by Gouliermis et al. (2004), who show that this method is highly dependent on the number of mass bins, the size of the bins and on the models used to fit the density profile. Converse & Stahler (2009) use a method which is similar to comparing cumulative distributions. They study the amount of mass inside a projected radius and obtain a quantifiable measure of the degree of mass segregation from the area between this curve and a curve for a cluster with no mass segregation. However, this relies on having a model for a non-mass segregated cluster. Gouliermis, de Grijs & Xin (2009) use the variation of the ‘Spitzer radius’ (the Spitzer radius is the rms distance of stars in a cluster around the centre of mass) with luminosity to determine the presence of mass segregation.

A variation of the MF with radius will show the presence of mass segregation as a shallowing of the slope of the MF with radius (de Grijs et al. 2002a,b,c; Gouliermis et al. 2004; Harayama, Eisenhauer & Martins 2008; Kumar, Sagar & Melnick 2008; Sabbi et al. 2008). MFs are calculated for various annuli, such that in a mass segregated cluster the radii closest to the centre of the cluster will contain the most massive stars, and so they will have shallower slopes than those at larger radii. This method also suffers from a strong dependence on the choice of annuli and mass bins for the MF (Ascenso, Alves & Lago 2008). Gouliermis et al. (2004) find that a unique set of annuli to use for comparison is not simple to choose because of the variation of the shape and slope of the MF as the choice of bin size changes for individual clusters. A particular problem is that low-mass stars are often difficult to detect in the central regions leading to relatively small regions of overlap in the MFs at different radii (which are the slopes to be compared). A further complication is introduced by the fact that the cluster centre must be determined; it is generally chosen as where the luminosity is highest – i.e. where the massive stars are.

## 3 MINIMUM SPANNING TREE METHOD

In this section we detail our new method for defining and quantifying mass segregation using a MST.

### 3.1 The MST

The MST of a sample of points is the path connecting all points in a sample with the shortest possible pathlength, which contains no closed loops (see e.g. Prim 1957). The length of the MST is unique, but the exact path might differ in some special circumstances. For example, an equilateral triangle of stars named 1, 2 and 3 has two possible MSTs – one connecting stars 1 & 2 & 3, and one connecting stars 1 & 2 & 1 & 3. However, the lengths of both MSTs are identical.

We have used the algorithm of Prim (1957) (see also Cartwright & Whitworth 2004) to construct our MSTs. We construct an ordered list of the separations between all possible pairs of stars. Stars are then connected together in ‘nodes’ starting with the shortest separations and proceeding through the list in order of increasing separation, forming new nodes as long as the formation of the node does not create a closed loop.

### 3.2 Quantifying mass segregation

Following our definition of mass segregation (see Section 2.1) we can detect and quantify mass segregation by comparing the typical MST of cluster stars with the MST of the most massive stars.

The method is based on finding the MST of the $N$ most massive stars and comparing this to the MST of sets of $N$ random stars in the cluster. If the length of the MST of the most massive stars is significantly shorter than the average length of the MSTs of the random stars, then the massive stars have a different, and more concentrated, distribution – hence the cluster is mass segregated. The ratio of the average random MST length to the massive star MST length gives a quantitative measure of the degree of mass segregation with an associated error (i.e. how likely it is that the massive star MST is the same as that of a random set of stars).

The algorithm proceeds in the following way.

1. Determine the length of the MST of the $N_{\text{MST}}$ most massive stars; $l_{\text{massive}}$. The MST of a subset of the $N_{\text{MST}}$ most massive (or what ever subset is of interest) stars is constructed. This MST has length $l_{\text{MST}}$.

2. Determine the average length of the MST of sets of $N_{\text{MST}}$ random stars; $\langle l_{\text{rand}} \rangle$. Sets of $N_{\text{MST}}$ random stars are constructed, and the average length, $\langle l_{\text{norm}} \rangle$, of their MSTs is determined. There is a dispersion associated with the average length of the random MSTs which is roughly Gaussian and so can be quantified by the standard deviation of the lengths $\langle l_{\text{norm}} \rangle \pm \sigma_{\text{norm}}$.

Numerical experiments have shown that 50 random sets are sufficient to obtain a good estimate of the errors. However, using hundreds of random sets tends to result in smoother trends. We suggest using 500–1000 random sets for low $N_{\text{MST}}$ and at least 50 for high $N_{\text{MST}}$.

Note that the error $\sigma_{\text{norm}}$ on $\langle l_{\text{norm}} \rangle$ is always large for small $N_{\text{MST}}$ due to stochastic effects when randomly choosing a small number of stars (see Section 4.2).

3. Determine with what statistical significance $l_{\text{massive}}$ differs from $\langle l_{\text{norm}} \rangle$. We define the ‘mass segregation ratio (MSR)’ $\Lambda_{\text{MSR}}$ as the ratio between the average random path length and that of the massive stars:

$$\Lambda_{\text{MSR}} = \frac{\langle l_{\text{norm}} \rangle}{l_{\text{massive}}} \pm \frac{\sigma_{\text{norm}}}{l_{\text{massive}}}$$

where an $\Lambda_{\text{MSR}}$ of $\sim 1$ shows that the massive stars are distributed in the same way as all other stars, a $\Lambda_{\text{MSR}}$ significantly $>1$ indicates mass segregation, and a $\Lambda_{\text{MSR}}$ significantly $<1$ indicates...
inverse-mass segregation (i.e. the massive stars are more widely spaced than other stars). The more (significantly) different the $\Lambda_{\text{MSR}}$ is from unity, the more extreme is the degree of (inverse-)mass segregation.

(4). Repeat the above steps for different values of $N_{\text{MST}}$ to determine at what masses the cluster is segregated, and to what degree at each mass. Clearly, the choice of $N_{\text{MST}}$ is arbitrary, so the $\Lambda_{\text{MSR}}$ needs to be determined for many values of $N_{\text{MST}}$ to gain information on how the degree of mass segregation changes with mass.

This method has three major advantages over current methods. First, it gives a quantitative measure of mass segregation with an associated significance, allowing different clusters to be directly compared. Secondly, it does not rely on defining a cluster centre or any special location in a cluster (allowing it to be applied to highly substructured regions as well). Thirdly, it is applicable not just to the most massive stars, but to any subset of stars (or brown dwarfs) that one might think has a different distribution to the `norm'.

It should be noted that whilst we are applying our method to simulated data and therefore know the mass of each star, an absolute determination of the mass is not required, just a measure of the relative masses (i.e. luminosity).

4 THE METHOD IN ACTION

4.1 A non-mass segregated cluster

We tested the method on a large sample of Plummer spheres (Plummer 1911) consisting of 1000 single stars with masses sampled from a Kroupa initial MF (Kroupa 2002). The masses are assigned randomly to stars so these clusters should not be mass segregated (except by chance).

Tables 1 and 2 show the fractions of random clusters that are found at $1\sigma$ significance to be either inverse-mass segregated (Inverse-MS: $\Lambda_{\text{MSR}} < 1$), mass segregated (MS: $\Lambda_{\text{MSR}} > 1$) or that showed no mass segregation (No MS: $\Lambda_{\text{MSR}} \sim 1$) for various numbers of members in the MST ($N_{\text{MST}}$). Table 1 uses the 3D positions of the stars, Table 2 is for a 2D projection as would be observed. In both the 2D and 3D clusters we find that false positives and negatives (i.e. $\Lambda_{\text{MSR}} > 1\sigma$ from unity) occur with equal frequency around 1/6th of the time – exactly what would be expected if the error is Gaussian.

Table 1. The percentage of Plummer spheres which show evidence of inverse-mass segregation (Inverse-MS), mass segregation (MS) and no mass segregation (No MS) for various numbers of members in the MST, $N_{\text{MST}}$, in 3D.

| $N_{\text{MST}}$ | 5 | 10 | 100 | 200 |
|-----------------|---|----|-----|-----|
| Inverse-MS      | 14.1 per cent | 15.5 | 17.1 | 15.2 |
| MS              | 12.6 per cent | 14.1 | 17.0 | 17.2 |
| No MS           | 73.3 per cent | 70.4 | 65.9 | 67.6 |

Table 2. As Table 1, but for clusters projected into 2D as would be observed.

| $N_{\text{MST}}$ | 5 | 10 | 100 | 200 |
|-----------------|---|----|-----|-----|
| Inverse-MS      | 13.1 per cent | 14.9 | 17.2 | 17.1 |
| MS              | 10.4 per cent | 12.2 | 17.0 | 19.1 |
| No MS           | 76.5 per cent | 72.9 | 65.8 | 63.8 |

4.2 Mass segregated clusters

We then test the method with mass segregated clusters. These are created by first producing Plummer spheres of 1000 stars in the same way as before. The positions of the most massive $x$ per cent of the stars are then swapped with randomly chosen stars positioned inside the $x$ per cent number radius, so only the most massive stars are inside this inner radius. The routine creates clusters in which the most massive stars are centrally concentrated, and so are mass segregated. This is not necessarily a realistic configuration as Nature might produce, but it is one in which we can easily control the degree of mass segregation.

Each cluster contains 1000 stars. Therefore, a 5 per cent MS places the 50 most massive stars within the inner 5 per cent number radius. For 10 per cent MS, it is the 100 most massive stars, and so on. We vary the radii in which the most massive stars are placed from the 5 per cent number radius to the 80 per cent number radius. Note that at large values the cluster is more like one in which the low-mass stars have been concentrated at the outskirts rather than one in which the high-mass stars are centrally concentrated.

Tables 3 and 4 show the percentage of clusters which the method finds to have mass segregation at $\sim 1\sigma$ significance for different numbers of stars in the MST ($N_{\text{MST}}$). In brackets after the percentage is the typical significance of the mass segregation. As before, Table 3 uses the 3D positions of the stars, while in Table 4 a 2D projection is used.

It is clear from Tables 3 and 4 that the ability of the method to detect mass segregation depends on a combination of the level of mass segregation and the number of stars in the MST. In the first column, when $N_{\text{MST}} = 5$ the method is able to detect mass segregation at roughly $2\sigma$ significance when the per cent MS is low. As it only uses the five most massive stars to look for mass segregation, it is not good at finding mass segregation when the per cent MS involves hundreds of stars.

In contrast, when $N_{\text{MST}} = 500$ (i.e. half of the stars in the cluster), the method is poor at spotting when the mass segregation only involves a few stars. However, it becomes extremely good when finding mass segregation involving many hundreds of stars.

Table 3. The frequency of a positive detection of mass segregation for various $N_{\text{MST}}$ and various degrees of mass segregation (per cent MS) in a 3D cluster. The numbers in parentheses denote the typical significance ($\sigma$) at which mass segregation is detected.

| Per cent MS | 5   | 10  | $N_{\text{MST}}$ |
|-------------|-----|-----|-----------------|
| 5           | 100 (2) | 100 (3) | 100 (3) | 90 (2) | 40 (1) |
| 10          | 100 (2) | 100 (3) | 100 (7) | 100 (4) | 70 (1) |
| 20          | 100 (2) | 100 (3) | 100 (7) | 100 (10) | 100 (3) |
| 50          | 80 (2)  | 100 (2) | 100 (5) | 100 (8) | 100 (15) |
| 80          | 30 (1)  | 80 (1)  | 100 (4)  | 100 (6) | 100 (10) |

Table 4. As Table 3, but for a 2D projection as would be observed.

| Per cent MS | 5   | 10  | $N_{\text{MST}}$ |
|-------------|-----|-----|-----------------|
| 5           | 100 (2) | 100 (2) | 100 (3) | 70 (2) | 30 (2) |
| 10          | 100 (2) | 100 (2) | 100 (6) | 100 (2) | 50 (2) |
| 20          | 100 (2) | 100 (2) | 100 (6) | 100 (9) | 80 (2) |
| 50          | 70 (1)  | 100 (2) | 100 (5) | 100 (7) | 100 (12) |
| 80          | 0 (–)   | 60 (1)  | 100 (3) | 100 (6) | 100 (10) |
Figure 1. The evolution of $\Lambda_{\text{MSR}}$ with $N_{\text{MST}}$ for a Plummer sphere with an initial 5 per cent MS. The dashed line indicates $\Lambda_{\text{MSR}} = 1$, i.e. No MS.

Looking across Tables 3 and 4, the highest significances for finding mass segregation (up to 10 or 15 $\sigma$ results) occur when $N_{\text{MST}}$ is equal to the number of stars that have been mass segregated. The best results are for large $N_{\text{MST}}$ as the variance in $\langle l_{\text{norm}} \rangle$ is smallest when $N_{\text{MST}}$ is large. Note that, as mentioned above, when $N_{\text{MST}}$ is small the variance in $\langle l_{\text{norm}} \rangle$ is large due to stochastic effects. Therefore, it is vital to vary $N_{\text{MST}}$ in order to examine mass segregation. However, this is an important diagnostic tool, as variations of $\Lambda_{\text{MSR}}$ and its significance with $N_{\text{MST}}$ contain information about how, and down to which mass segregation is found.

In Fig. 1 we show the variation of the $\Lambda_{\text{MSR}}$ with $N_{\text{MST}}$ for a 5 per cent level of mass segregation (i.e. the 50 most massive stars are segregated). The $\Lambda_{\text{MSR}}$ is around 9 for $N_{\text{MST}} < 50$, and then drops sharply toward unity when $N_{\text{MST}} > 50$. This shows two features of mass segregation that can be extracted using this method.

First, it shows that only the 50 most massive stars are mass segregated (as described above). However, it also shows that the 50 most massive stars are equally mass segregated. That is, the method places the 50 most massive stars at the centre of the cluster but does not order them in any specific way within the centre.

To illustrate this we show in Fig. 2 the variation of the $\Lambda_{\text{MSR}}$ with $N_{\text{MST}}$ for a cluster in which the 10 most massive stars are placed in the inner 1 per cent radius, and the 11th to 70th most massive stars in the 1–7 per cent radius. The method clearly detects that there are two levels of mass segregation – one involving the 10 most massive stars, and one which involves the 70 most massive stars.

4.3 A complex mass segregated cluster

We also apply the method to a more realistic and complex cluster. In Fig. 3 we show a complex 1000-body cluster.\(^2\) The triangles show the positions of the five most massive stars and the squares the positions of the sixth to 12th most massive stars. The cluster has collapsed into a configuration in which the five most massive stars form a dense Trapezium-like cluster to one side, and the sixth to 12th most massive stars have formed a separate ‘clump’ on the other side. For this reason the cluster has no well-defined centre which serves to illustrate the strength of the MST method in dealing with unusual and clumpy clusters.

In Fig. 4 we show the change of the 2D $\Lambda_{\text{MSR}}$ with $N_{\text{MST}}$ for the cluster illustrated in Fig. 3. Fig. 4 shows that in this cluster there are three ‘levels’ of mass segregation. First, there is the presence of a Trapezium-like system formed from the five most massive stars.

\(^2\) The cluster has been evolved from a cold fractal distribution (see Goodwin & Whitworth 2004); we will discuss simulations like this in detail in a later paper, for now the cluster is merely used as an illustration of the method.
Using the MST to trace mass segregation

4.4 The Orion Nebula Cluster

Finally, we apply the method to real data, specifically the ONC data of Hillenbrand (1997). We use the 900 stars for which Hillenbrand (1997) provides masses. We note that this is not an ideal data set as many (presumably low-mass) stars lack masses. However, it serves to illustrate the method.

Hillenbrand & Hartmann (1998) show, using cumulative distribution function and mean stellar mass as a function of radius, that the ONC is mass segregated. They find evidence for mass segregation in the ONC for stars more massive than 5 $M_{\odot}$ within 1 pc, with less compelling evidence beyond this radius.

Fig. 5 shows the change of $\Lambda_{\text{MSR}}$ with $N_{\text{MST}}$ for the ONC. The MST method clearly picks out the mass segregation of the Trapezium system at $N_{\text{MST}} = 4$ with $\Lambda_{\text{MSR}} = 8.0 \pm 3.5$. However, the MST method also shows that there appears to be a secondary level of mass segregation involving the nine most massive stars ($>5 M_{\odot}$) in the ONC in agreement with Hillenbrand & Hartmann (1998). Whilst the significance of the $\Lambda_{\text{MSR}}$ is low (around 2.0 $\pm$ 0.5), there is a clear trend that each of the fifth to ninth stars show the same increased level of mass segregation. However, unlike Hillenbrand & Hartmann (1998) we find no evidence of mass segregation below 5 $M_{\odot}$.

5 FURTHER APPLICATIONS AND FUTURE WORK

In future papers we will apply our new method to more observational and theoretical data to look for mass segregation, and also to examine if low-mass stars have different distributions to brown dwarfs and/or high-mass stars.

Binaries. In our tests we have not included binary stars, and they will have an effect on the lengths of MSTs. If a binary is resolved, then it is very likely that the two components will be linked as a node (if this is not the case it would be unclear that the system really was a binary). Indeed, triples and quadruples will also be linked as a subset within the MST. This raises the possibility that MSTs could be very useful in locating binary and multiple systems by looking for short links within the MST. However, as noted by Cartwright & Whitworth (2004) care must be taken in dealing with binaries.

Incompleteness. In many clusters there is significant incompleteness, especially in regions around the most massive stars where low-mass stars cannot be observed (if they are present). This presents a significant problem as it is impossible to know if there are many low-mass stars in the ‘central’ regions. If there are, then an average MST length would be short. If not, it would be long (this is exactly the same problem faced by other methods). We will explore this problem in a future paper by creating and analysing synthetic observational data sets.

6 SUMMARY

We have outlined a new method of determining and quantifying mass segregation in a cluster by using a MST. The algorithm proceeds in the following way.
(1) Determine the length of the MST of the $N_{\text{MST}}$ most massive stars; $l_{\text{massive}}$

(2) Determine the average length of the MST of sets of $N_{\text{MST}}$ random stars; $\langle l_{\text{norm}} \rangle$

(3) Determine with what statistical significance $l_{\text{massive}}$ differs from $\langle l_{\text{norm}} \rangle$:

$$\Lambda_{\text{MSR}} = \frac{\langle l_{\text{norm}} \rangle}{l_{\text{massive}}} \pm \frac{\sigma_{\text{norm}}}{l_{\text{massive}}}$$

(4) Repeat the above steps for different values of $N_{\text{MST}}$ to determine at what masses the cluster is segregated, and to what degree at each mass.

By examining the difference between the MST of a subset of (massive) stars and an equal number of random stars it is possible to quantify the level of (inverse-)mass segregation in a cluster. Tests on artificial clusters show that the method behaves as expected and can identify mass segregation.

We apply the method to the ONC and find that the Trapezium is mass segregated with a $\Lambda_{\text{MSR}}$ of 8.0 $\pm$ 3.5. We also find that the fifth to 12th most massive stars down to around $5 M_\odot$ also show evidence of being mass segregated with an $\Lambda_{\text{MSR}}$ of 2.0 $\pm$ 0.5. Below $5 M_\odot$ we find no evidence for mass segregation.

ACKNOWLEDGMENTS

We thank Stuart Littlefair for useful discussions. RJA and RJP acknowledge financial support from STFC. MBNK was supported by PPARC/STFC under grant number PP/D002036/1. SFPZ is grateful for the support of the Netherlands Advanced School in Astrophysics (NOVA), the LKBF and the Netherlands Organization for Scientific Research (NWO). We acknowledge the support and hospitality of the International Space Science Institute in Bern, Switzerland, where part of this work was done as part of an International Team Programme.

REFERENCES

Adams J. D., Stauffer J. R., Monet D. G., Skrutskie M. F., Beacham C. A., 2001, AJ, 121, 2053
Ascenso J., Alves J., Lago M. T. V. T., 2009, A&A, 495, 147
Bonnell I. A., Bate M. R., 2006, MNRAS, 370, 488
Cartwright A., Whitworth A. P., 2004, MNRAS, 348, 589
Chandrasekhar S., 1942, Principles of Stellar Dynamics. Physical Sciences Data, Univ. Chicago Press, Chicago
Converse J. M., Stahler S. W., 2008, ApJ, 678, 431
de Grijs R., Johnson R. A., Gilmore G. F., Frayn C. M., 2002a, MNRAS, 331, 228
de Grijs R., Gilmore G. F., Johnson R. A., Mackey A. D., 2002b, MNRAS, 331, 245
de Grijs R., Gilmore G. F., Mackey A. D., Wilkinson M. I., Beaulieu S. F., Johnson R. A., Santiago B. X., 2002c, MNRAS, 337, 597
Goodwin S. P., Whitworth A. P., 2004, A&A, 413, 929
Gouliermis D., Keller S. C., Kontizas M., Kontizas E., Bellas-Velidis I., 2004, A&A, 416, 137
Gouliermis D. A., de Grijs R., Xin Y., 2009, ApJ, 692, 1678
Harayama Y., Eisenhauer F., Martins F., 2008, ApJ, 675, 1319
Hillenbrand L. A., 1997, AJ, 113, 1733
Hillenbrand L. A., Hartmann L. W., 1998, ApJ, 492, 540
King I. R., 1966, AJ, 71, 64
Kroupa P., 2002, Sci, 295, 82
Kumar B., Sagar R., 2008, MNRAS, 386, 1380
Littlefair S. P., Naylor T., Jeffries R. D., Devey C. R., Vine S., 2003, MNRAS, 345, 1205
Murray S. D., Lin D. N. C., 1996, ApJ, 467, 728
Pinfield D. J., Jameson R. F., Hodgkin S. T., 1998, MNRAS, 299, 955
Plummer H. C., 1911, MNRAS, 71, 460
Prim R. C., 1957, Bell Syst. Tech. J., 36, 1389
Raboud D., 1999, Rev. Mex. Astron. Astrofis. Ser. Conf., 8, 107
Sabb E. et al., 2008, AJ, 135, 173
Sharma S., Pandey A. K., Ogura K., Aoki T., Pandey K., Sandhu T. S., Sagar R., 2008, AJ, 135, 1934
Spitzer L. J., 1969, ApJ, 158, L139

This paper has been typeset from a \TeX\LaTeX file prepared by the author.