Escape dynamics based on bounded rationality

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In an extreme case, such as escape panic, bounded rationality can have a direct impact on human behavior. A cellular automaton model is constructed for the escape dynamics in closed boundary, and the influence of bounded rational behavior on collective effects is investigated with mean field approximation. Analyzing the escape efficiency shows that under the premise of rationality, the bounded rational strategy can get higher performance. In the escape process, a possible metastable state appears, and the escape time is power-law dependent on system size.

I. INTRODUCTION

Public security is the cornerstone of national and social stability. In addition to the direct loss of life and property caused by natural disasters, crowd congestion in emergency situations often leads to disaster (e.g. clogging and stampede\cite{1,2,3}). It naturally becomes important to understand the collective behavior patterns in case of emergency. Recent models and experiments show that the characteristics of group movement emerge as short intermittent bursts\cite{4,5,6}. When the desire of escaping danger exceeds the idea of avoiding collision, people’s behavior pattern changes from order to disorder. However, our understanding of the transition is still limited. Even the research on escape dynamics in emergency situations has a long history\cite{7,8,9}, it was not until the 1990s that the dynamics on collective behavior attracts people’s attention\cite{3,9}. Subsequently, game theory, decision theory, communication model and queuing model had also been comprehensively applied\cite{10}. However, due to the lack of individual self-organization, the prediction of many results deviates from the actual situation. Some works start from the hydrodynamics model to study the collective behavior of the population\cite{2,11}. It successfully explained the group behavior during the pilgrimage to mecca. Also we can get some non-trival macroscopic patterns in these hydrodynamics models\cite{6,12}. However, the macro-model sometimes is coarse and misses indi-
individual information, while the micro-model represented by Social force model, Cellular automata model and Magnetic field force model\(^1,9,13-15\) can give a more detailed description of the escape dynamics. Simulating the collective behavior in emergency situations becomes more convenient with the increasing of computer capabilities. The simulating results, such as the "faster-is slower" phenomenon also can be verified by experiments on different group (e.g people, vehicles, ants, sheep, microbial populations, etc.)\(^4,16-19\).

With the development of sensor technology and the improvement of microchip computing power, collecting more redundant data and using more realistic methods to simulate the escape process becomes possible\(^20-24\). It is of practical significance that the disaster may happen in different complex environments\(^25\). In addition, some researches focus on behavior itself, for escape dynamics provides an extreme case to investigate collective behavior\(^15,21,26,27\). The diverse and fascinating collective behaviors occur in both virtual and real space\(^28,29\): social network, financial network and social norms, these virtual social connections naturally incubate the collective behavior; as for the real space, collective modes are common in urban dynamics, traffic flow and pedestrian dynamics. Therefore, the escape dynamics provides us with an extreme environment, in which human instinct dominates\(^23,30\). It makes us have chances to effectively study human behavior itself without complex social relations.

Based on the point, this work introduces bounded rationality\(^31,32\) from behavioral economics in the escape dynamics through replicator dynamics method\(^33,34\). A cellular automaton model is used to model the escape dynamics in closed boundary. And the influence of bounded rational behavior strategy on collective behavior is investigated by using mean field approximation\(^35\). The escape efficiency is affected by the environment and heterogeneous information processing. We also analyze the group and individual escape time, giving a possible picture to understand the connection between collective behavior and individual action.
II. ESCAPE DYNAMICS

A. Cellular Automaton Model

We construct a cellular automation model for simulating the pedestrian flow in a two-dimensional system. The underlying structure is a \( L \times L \) cell grid, where \( L \) is the system size. The state of cell can be empty, or occupied by one pedestrian exactly or wall. It’s a instructive sample, once we set the size of cell as \( 0.5m \times 0.5m \), which can simulate the escaping when some disaster happens. Model adopts the Moore neighbor, and pedestrians update their positions by transition matrices \( T(i, t) \),

\[
T(i, t) = \begin{pmatrix}
  P_{1,1}(i, t) & P_{1,2}(i, t) & P_{1,3}(i, t) \\
  P_{2,1}(i, t) & P_{2,2}(i, t) & P_{2,3}(i, t) \\
  P_{3,1}(i, t) & P_{3,2}(i, t) & P_{3,3}(i, t)
\end{pmatrix}
\]

(1)

where \( P_{m,n}(i, t) \) means the possibility that the pedestrian \( i \) moves from \( t \) time at position \( (x(i, t), y(i, t)) \) to next time-step position. The neighbors’ directions are labeled by \( (m, n) \), where \( m, n = 1, 2, 3 \). Each cell can either be empty, or occupied by wall or exactly one pedestrian. Every time-step pedestrian can choose to move into a new position or stop. Once we have chosen the location of the exit, the synchronously updated cellular automaton can imitate the escape process\[13, 36\].

B. Heterogeneous Information and Bounded Rationality

Bounded rationality is formalized with such major variables as incomplete information, information processing and the non-traditional policy-maker target function\[32\]. Heterogeneous information could be the reason why people shows irrationality\[20, 30, 33, 37\]. The extreme situation of escaping from disasters constrains people’s behavior, in which only intuition or social habits remains, no long term trade-off. The replicator dynamics modeling\[33, 34\] can link the different behaviors, whether practical or spiritual, during the escaping process. The transition possibility \( P(i, t) \) derives from the follow definition,

\[
P_{m,n}(i, t) = \frac{B_{m,n}(i, t)R_{m,n}(i, t)}{\sum B(i, t)R(i, t)}
\]

(2)
where $R(i,t), B(i,t)$ means the weight from rational and bounded rational part respectively. The definition of the components in matrix $R_{m,n}(i,t) = O_{m,n}(i,t)E_{m,n}(i,t)$,

$$O_{m,n}(i,t) = \begin{cases} 1 & \text{empty} \\ \epsilon & \text{occupied} \end{cases}, E_{m,n}(i,t) = \begin{cases} \alpha & \text{exit} \\ \epsilon & \text{nothing} \end{cases}$$

(3)

which means if the position $(m,n)$ around the individual $i$ at $t$ time is empty, the $O_{m,n}(i,t) = 1$, whereas the value is $\epsilon$. And the $E_{m,n}(i,t) = \alpha$ only holds when the exit direction is indicated by $(m,n)$, if not take the value $\epsilon$. The value $\epsilon$ is a minimum value that the calculation accuracy can reach. The parameter $\alpha$ represents the attraction of the exit to persons want to escape, or the importance of the information of the exit position.

The definition of the bounded rational part $B_{m,n}$ relies on the heterogeneous information from the crowd. As the transport model of statistical physics inspired us, the escape dynamics needs more information that persons’ position and velocity distribution, the basic variables of the transport theory. Considering the full information cannot easily be achieved by individuals, the mean-field approximate can provide a global perception for the people on move, which shows as the follow,

$$B_{m,n}(i,t) = \begin{cases} 1 & \text{rational} \\ n_{m,n}(i,t) & \text{crowd} \\ v_{m,n}(i,t) & \text{follower} \end{cases}$$

(4)

The \textit{rational} indicates the transition possibility only decided by $R(i,t)$, the neighbor occupied state and the direction of exit, or the objective environment. The \textit{crowd} defines $n_{m,n}(i,t) = \sum_{m,n} N(i,t)/\sum_{All} N(i,t)$, where $N(i,t)$ is the population distribution at $t$ time. The definition shows the proportion of individuals in $(m,n)$ orientation as mean-field approximation, and people will be attracted to the direction with more density. We use it to mimic the “crowd” behavior for individuals, which also means people can potentially get more population density information. As for the \textit{follower}, $v_{m,n}(i,t) = \sum_{m,n} N(i,t)\|v(i,t)\|_{(m,n)}/\sum_{All} N(i,t)$, where $\|v(i,t)\|$ is the velocity distribution at $t$ time. The proportion of individuals move to $(m,n)$ orientation has been extracted, and people will follow others as the weight. It transfers more potential velocity information to people. The latter two strategies shows people can process the heterogeneous information, even it looks irrational intuitively.
C. Evolution Rules

The model escape rules gives as follows,

**Step1. Initialization.** Set the position of exit \((x, y)\) and generate \(N(i, t)\) population distribution at the \(L \times L\) lattice. At the \(t = 0\) time, disaster turns out and individuals begin to move;

**Step2. Evolution.** At the \(t\) time step, the individual \(i\) move to the next position as transition matrices \(T(i, t)\) at \(t + 1\) time step. Update all individuals synchronously, and the conflict will be handled by compared the transition possibility;

**Step3. Escape.** For the individual whose destination is exit at the next time step, escape successfully, and remove it from the lattice and reduce population as \(N(t + 1) = N(t) - 1\). If \(N(t) = 0\) the escape stops.

**Step4. Update.** Update the transition matrices as above strategies, turn to **Step2.** The \(t\) time step escape finished.

III. COLLECTIVE BEHAVIOR MODES

A. Dynamics Simulation

![Evolution Sample](image1)

**FIG. 1:** Evolution Sample. Initial population ratio \(\rho_0 \approx 0.3\), lattice size \(L = 20\), the exit size is 2, and the parameter \(\alpha = 10\) with the rational strategy.

![Arch-like blocking](image2)

**FIG. 2:** Arch-like blocking. Initial population ratio \(\rho_0 \approx 0.3\), lattice size \(L = 50\), the exit size is 2, and the parameter \(\alpha = 10\), at time step \(t = 80\) with the rational strategy.

Firstly we build an escape dynamics simulation frame based on the escape rules, in which three
different strategies for processing heterogeneous information have been added. As an example, Fig. 1 depicts a typical escape process: at the beginning individuals distribute in the lattice $L \times L$ randomly with initial population ratio $\rho_0$; then they move into the exit direction as the parameter $\alpha$ which shows how important the exit information is for them; at the $t = 150$ time step, people escape from the disaster area. In the Fig. 2 we show the arch-like blocking as the other simulations and experiments found [4, 16, 18, 19], which indicates the escape dynamical model catches the key points for the flock clogging problem.

$$\rho_0 \approx 0.12, \alpha = 10$$

$$\rho_0 \approx 0.51, \alpha = 10$$

$$\rho_0 \approx 0.3, \alpha = 10$$

$$\rho_0 \approx 0.3, \alpha = 1$$

FIG. 3: Escape Efficiency

Fig. 3 shows the escape efficiency in the $t \in [0, 100]$ time steps at the parameter, lattice size $L = 30$ and the exit size is 2. Over time, all three behavior strategies are driving groups to flee disaster areas. Figs. 3a to 3c have different $\rho_0$ and the high initial population case can escape faster than the lower. It’s because the low initial population case has more empty space, which reduces the escape chances. As for the three behavior strategies, they have similar escape efficiency in the higher $\rho_0$ case, where the population decreasing is almost same in 100 time steps. The three
behavior strategies show relative large differences in low density situations. During the same time, the crowd made more people escape, the follower followed, and the relative worst is the rational strategy. The above results illustrate that the “Bounded Rationality” strategies can refine information from environment, which makes people work better in low density case. Figs. 3c and 3d have different $\alpha$ and the high $\alpha$ case can escape faster than the lower. It makes sense that $\alpha$ represents the importance of exit, and it could be the intensity of exit signs or how clear people knows the exit information. In the Fig. 3d, the exit information parameter $\alpha = 1$, it makes people random walks at some time and the crowd strategy shows more powerful capability of driving people to the exit.

B. Collective Behavior and Meta-stable State

To investigate the factors affecting collective behavior more quantitatively, we define the escape ratio $p$, the corresponding group escape time $t_p$ and individual escape time $t_i$. The $t_p$ is the time taken to evacuate $p$ of the population and the $t_i = \frac{\sum_{n=1}^{N_0} t_n}{N_0}$ is the average time, where the $t_n$ is the time taken to evacuate $n$th person. Figs. 4 and 5 shows the escape time in crowd strategy at different parameter ($\alpha, p$), lattice size $L = 30$, initial population ratio $\rho_0 = 0.5$ and the exit size is 2. The parameter $\alpha$ has played a key role in the escape dynamics, as Fig. 4 shows, the escape time decays exponentially with this exit parameter. At small $\alpha$ region, the $\alpha$ increasing would push people escape more faster; at the large region, the effect of increasing is limited. The results reflect that there is a effective range for the relative importance of exit information in the dynamics processing.
The other statistical result gets in Fig. 5, where group escape time \( t_p \) increases with escape ratio near exponentially and the individual escape time \( t_i \) increases near linearly. It implies that the time taken to rescue more people rises sharply, which reveals the conflict of interest between individual and group. The above results are independent of initial conditions, for every case runs 20 times in different random initial population distribution.

\[
\begin{align*}
  t_p &= 0.6642 L^{1.794} \\
  t_i &= 0.2385 L^{1.834}
\end{align*}
\]

FIG. 6: The scale dependent of escape time

FIG. 7: The parabola fitting for the edge

The above nonlinear dependence inspired us to study the influence of system size on escape time. The results exhibit in Fig. 6, the other system parameters are: initial population ratio \( \rho_0 = 0.5 \), the parameter \( \alpha = 10 \), \( p = 0.9 \) and the exit size is same as before. Both the group and individual escape time have \( \ln(t_{pi})/\ln(L) \approx 1.8 \), and the exponential coefficient deviating from system area \( L^2 \) slightly. It can be treated as a signal of the critical self-organizing behavior for the escape dynamics [27, 28]. At the same time, we notice that the arch-like blocking is a corresponding phenomena during the critical process. It can be understood as a meta-stable state, for the empty is the stable final state. We extract the edge curve as parabola fitting \( y = a_2(t)x^2 + a_1(t) \) from the escape maps (e.g. Fig. 7 as a sample at \( L = 40, \alpha = 10, \rho_0 = 0.3 \)), and this meta-stable state will emerge in the mid-time term \( t \in [20, 120] \) for this sample). Where the pixel positions of corresponding evolution patterns represented by coordinate axes \( (x, y) \), and the position of exit is \((0,200)\). As Fig. 7 shows, the edge of population will push forward and become narrower. It can be describe by the two fitting parameters: \( a_1(t) = 0.1512t + 27.53, \quad a_2(t) = 3.986 \times 10^{-4}t + 0.04354 \) in this sample. As time goes, the opening of parabola becomes narrow until the meta-stable state disappears. Even parabola fitting is not enough accurate, it still reflects such a metastable propulsion process.
IV. CONCLUSIONS

In this work, we construct an escape dynamics frame with cellular automata method, and can generate the space-time escape map similar to the actual situation. We also use the Replicator Dynamics to combine the bounded rational behavior into the escape. And three different behavior strategies were compared: the rational, crowd and follower. The difference among these three strategies lies in the completeness of information. Our results show that under the premise of rationality, the bounded rational behavior of the crowd can get higher evacuation efficiency. Subsequently, the influence of escape ratio, exit parameter and system size on the collective behavior pattern was further studied by using parametric method in the crowd behavior strategy. We also introduced group escape time \( t_p \) and average escape time \( t_i \) to study the behavior patterns of people fleeing disaster areas, and found that changes in external environment and individual rationality will have non-trivial effects on it. The increase in the importance of exit information \( \alpha \) will help to improve the efficiency of escape, while conflicts of interests between individuals and group occur in the process of increasing the escape ratio \( p \). In addition, escape time is power-law dependent on system size. It can be treated as a signal of the critical self-organizing behavior for the escape dynamics. Finally we extract the edge curve as parabola fitting from the escape dynamics, which reflects such a metastable propulsion process.

In the extreme case, this work uses the replicator dynamics for reference, introduces bounded rationality, and adopts mean-field approximation to study the collective behavior. Furthermore, we can use more abundant tools to carry out more in-depth research on such problems: for example, using the state-of-the-art deep learning methods to recognize potential collective behavior and avoid trampling; make online games \([38, 39]\) based on the this model, measuring the parameter \( \alpha \) as a group rational scale; look for the possible order parameters in such a critical phenomena, etc.
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