Precision determination of pion-nucleon coupling constants using effective field theory

P. Reinert¹, H. Krebs¹ & E. Epelbaum¹∗

¹Ruhr University Bochum, Faculty of Physics and Astronomy, Institute for Theoretical Physics II, D-44870 Bochum, Germany.

The pion-nucleon (πN) coupling constants determine the strength of the long-range nuclear forces and play a fundamental part in our understanding of nuclear physics. While the charged- and neutral-pion couplings to protons and neutrons are expected to be very similar, owing to the approximate isospin symmetry of the strong interaction, the different masses of the up- and down-quarks and electromagnetic effects may result in their slightly different values. Despite previous attempts to extract the πN coupling constants from pion-nucleon scattering data¹², experimental data on pionic atoms³⁴ as well as proton-antiproton⁵ and nucleon-nucleon⁶⁻¹¹ scattering data, our knowledge of their values is still deficient, and their first-principles determination from lattice quantum-chromodynamics (QCD) and quantum-electrodynamics (QED) calculations has so far been impractical¹². Here we use chiral effective field theory¹³⁻¹⁴ (EFT) to describe the low-energy interactions of pions, protons, neutrons and photons and to determine the πN coupling constants from a combined Bayesian analysis of neutron-proton and proton-proton scattering data at the per-cent level with fully controlled uncertainties. Our results are consistent with no significant charge dependence of the coupling constants. These findings increase our understanding of low-energy nuclear physics and mark an important step towards developing a precision theory of nuclear forces and structure.

The interaction of the nucleon with the isovector weak current $A_\mu^i$ is described in terms of the axial and induced pseudoscalar form factors $G_A(q)$ and $G_P(q)$. In the limit of exact isospin symmetry, the matrix element of $A_\mu^i(x = 0)$ between nucleon states can be parametrized via

$$
\langle N(p')|A_\mu^i(0)|N(p)\rangle = \bar{u}(p') \left[ \gamma^\mu G_A(p' - p) + \frac{(p' - p)^\mu}{2m_N} G_P(p' - p) \right] \gamma_5 \tau_i u(p), \tag{1}
$$

with $u(p)$ and $\bar{u}(p')$ the corresponding Dirac spinors, $\tau_i$ the isospin Pauli matrices and $m_N$ the nucleon mass. The form factors $G_A(q)$ and $G_P(q)$ carry important information about the internal structure of the nucleon. For example, the axial charge of the nucleon $g_A \equiv G_A(0) = 1.2724(23)$¹⁵ controls the decay rate of a neutron to a proton. Recently, this quantity was calculated from first principles at a per-cent level using lattice QCD¹⁶. While $G_A(q)$ is a smooth
function near $q^2 = 0$, the induced pseudoscalar form factor possesses a pion-pole contribution, $G_P(q) = 4m_N g_{\pi NN} F_\pi / (M_\pi^2 - q^2) + $ non-pole terms, whose residue is determined by the (pseudoscalar) $\pi N$ coupling constant $g_{\pi NN}$. The pion decay constant $F_\pi = (92.1 \pm 1.2)$ MeV determines the rate of weak decays $\pi^\pm \rightarrow \mu^\pm \nu_\mu$. The strong-interaction constant $g_{\pi NN}$ is connected to $g_A$ and $F_\pi$ entering weak processes via the celebrated Goldberger-Treiman relation $F_\pi g_{\pi NN} = g_A m_N (1 + \Delta_{GT})$, where the small Goldberger-Treiman discrepancy $\Delta_{GT}$ is driven by the non-vanishing masses of the up- and down-quarks. Away from the isospin limit and in the presence of QED, one has to distinguish between protons (p) and neutrons (n) and between the charged and neutral pions by introducing three coupling constants $g_{\pi^0 pp}$, $g_{\pi^0 nn}$ and $g_{\pi^\pm pn}$ or, equivalently, the corresponding pseudovector couplings $f_p \equiv f_{\pi^0 pp} = M_\pi^\pm g_{\pi^0 pp} / (2\sqrt{4\pi m_p})$, $f_n \equiv f_{\pi^0 nn} = M_\pi^\pm g_{\pi^0 nn} / (2\sqrt{4\pi m_n})$ and $f_c \equiv f_{\pi^\pm pn} = M_\pi^\pm g_{\pi^\pm pn} / (\sqrt{4\pi (m_p + m_n)})$. These constants determine the strength of the long- and intermediate-range nuclear forces originating from exchange of virtual pions. Their precise knowledge with controlled uncertainties is, therefore, of utmost importance for a quantitative understanding of nuclear physics.

In this Letter we use chiral EFT, an effective field theory of QCD, to describe the low-energy interactions between two nucleons, and employ the resulting NN potential to extract the $\pi N$ coupling constants from a combined Bayesian analysis of np and pp scattering data below pion-production threshold. Chiral EFT utilizes an expansion in powers of momenta and pion masses to describe interactions between pions and nucleons in a systematically improvable way. The corresponding effective Lagrangian contains all possible terms compatible with the symmetries of QCD. The non-perturbative dynamics of QCD is encoded in the so-called low-energy constants (LECs), which control the strength of the interactions in the effective Lagrangian and can be determined from experiments or lattice QCD calculations. Chiral EFT has also been extended to include virtual photons. Fig. 1 shows examples of contributions to the NN force in chiral EFT. The most important terms at leading order (LO) include one-pion exchange (Fig. 1b) and contact interactions (Fig. 1e). Two-pion exchange (Fig. 1d) and one-photon exchange (Fig. 1a) start to contribute at next-to-leading order (NLO), while pion-photon exchange (Fig. 1c) appears first at fourth order (N^3LO).

In recent years, the chiral expansion of the NN force has been pushed to fifth order (N^4LO). All relevant isospin-invariant $\pi N$ LECs have been reliably determined from a dispersion theory analysis of $\pi N$ scattering in ref.\cite{21}. Therefore, the long-range part of the NN interaction is parameter-free. To avoid distortions of the long-range forces due to a finite cutoff $\Lambda$, we introduced in ref.\cite{20} an improved local regulator which respects the analytic structure of the interaction. The LECs accompanying short-range operators (Fig. 1e) were determined in ref.\cite{20} from a fit to the
Figure 1: **Diagrammatic illustration of the NN interaction in chiral EFT.** Photons, pions and nucleons are shown by wavy, dashed and solid lines, respectively. Diagrams a, b, c, d and e are representative examples of the one-photon exchange, one-pion exchange (OPE), electromagnetic corrections to the OPE, two-pion exchange (TPE) and NN short-range contributions, respectively. The range of interactions decreases from the left to the right.

2013 Granada database of mutually compatible np and pp data. Furthermore, we have introduced a N\(^4\)LO\(^+\) NN potential, where the leading F-wave short-range interactions, formally appearing at sixth order, were taken into account in order to achieve a statistically satisfactory description of certain very precisely measured pp data, see also ref. \(^{19}\). This allowed us to achieve a description of NN data on par with or even better than that based on most precise phenomenological potentials, but with a much smaller number of adjustable parameters. However, the treatment of isospin-breaking (IB) effects in ref. \(^{20}\) was incomplete and limited to the one of the Nijmegen\(^{23}\) and Granada\(^{22}\) partial wave analyses (PWA).

In this Letter, we include the charge-independence-breaking (CIB) and charge-symmetry-breaking (CSB) IB NN interactions up through N\(^4\)LO. In particular, we employ the most general form of the OPE potential including the leading electromagnetic corrections\(^{24}\) and take into account the leading and subleading IB two-pion-exchange contributions\(^{25–27}\). These long-range interactions are expressed in terms of known LECs, the \(\pi N\) coupling constants \(f^2_p, f^2_c\) and \(f^2_0 \equiv f_p f_n\) to be determined, the nucleon mass difference \(\delta m = m_n - m_p \simeq 1.29\) MeV and its QCD contribution \(\delta m_{QCD} = 2.05(30)\) MeV\(^{28}\), see ref. \(^{29}\) for an update and ref. \(^{30}\) for a recent *ab initio* calculation using lattice QCD and QED. We also include short-range IB interactions in the \(^1S_0\), \(^3P_0\), \(^3P_1\) and \(^3P_2\) partial waves. Details of the employed NN interaction are given in Methods.

We end up with 33 parameters that need to be determined from NN data, comprising of 3 \(\pi N\) LECs \(f^2 \equiv \{f^2_p, f^2_c, f^2_0\}\) and 25+5 LECs \(C_i\) from isospin-invariant + IB short-range interactions, collectively denoted as \(C \equiv \{C_i\}\). For normally distributed errors, the likelihood of data \(D\) given
\( f^2, C \) and \( \Lambda \) is given by
\[
p(D|f^2 \Lambda C) = \frac{1}{N} e^{-\frac{1}{2} \chi^2}, \tag{2}\]
where \( N \) is a normalization constant. The data \( D \) employed in our analysis include mutually compatible np and pp scattering data according to our own selection as detailed in Methods. The definition of the \( \chi^2 \)-measure is provided in Methods. Using Bayes’ theorem to relate the probability density function (PDF) \( p(f^2 \Lambda C | D) \) of the parameters given the data to \( p(D|f^2 \Lambda C) \), and integrating over the nuisance parameters \( C \) and \( \Lambda \), we obtain the PDF of \( f^2 \) given \( D \)
\[
p(f^2|D) = \int d\Lambda dC \frac{p(D|f^2 \Lambda C)p(f^2 \Lambda C)}{p(D)}. \tag{3}\]
For the case at hand, \( p(D) \) is a (normalization) constant. Furthermore, we use independent priors for \( f^2, C \) and \( \Lambda \) so that \( p(f^2 \Lambda C) = p(f^2) p(C) p(\Lambda) \), and employ a Gaussian prior for \( C \) and uniform priors for \( \Lambda \) and \( f^2 \) specified in Methods. To determine \( f^2 \) we need to find the maximum of \( p(f^2|D) \) in Eq. (3). However, for each set of \( f^2 \), this requires integrating over a 31-dimensional space spanned by \( \Lambda \) and \( C \), which is not feasible. Instead, we employ the Laplace approximation by fitting \( C \) to \( D \) for fixed values of \( f^2 \) and \( \Lambda \) and expressing the likelihood \( p(D|f^2 \Lambda C) \) as
\[
p(D|f^2 \Lambda C) \approx \frac{1}{N} e^{-\frac{1}{2} [\chi^2_{\text{min}} + \frac{1}{2} (C-C_{\text{min}})^T H (C-C_{\text{min}}) ]}. \tag{4}\]
Here, \( \chi^2_{\text{min}} \equiv \chi^2_{\text{min}}(f^2, \Lambda) \) at \( C_{\text{min}} \equiv C_{\text{min}}(f^2, \Lambda) \) and the Hessian \( H \equiv H(f^2, \Lambda) \) is given by
\[
H_{ij} = \left. \frac{\partial^2 \chi^2}{\partial C_i \partial C_j} \right|_{C=C_{\text{min}}}. \]
Performing an analytical integration over \( C \) then allows us to cast Eq. (3) into a numerically tractable form, see Methods for details. The remaining integration over \( \Lambda \in [400, 550] \text{ MeV} \) is performed numerically. We emphasize that reducing the amount of information in the employed priors for \( C, f^2 \) and \( \Lambda \) has a negligible effect on our results as explained in Methods.

To account for the uncertainty inherent in the choice of the energy range of our PWA, we performed separate analyses of NN data up to the laboratory energies of \( E_{\text{lab}}^{\text{max}} = 220, 240, 260, 280 \) and 300 MeV. Further, to address the systematic error stemming from the truncation of the EFT expansion for IB interactions, we considered two additional models of the NN interaction that include IB pion-photon- and two-pion-exchange contributions beyond \( N^4\text{LO} \). Our final PDF for the \( \pi N \) coupling constants are obtained by performing Bayesian averaging over five values for \( E_{\text{lab}}^{\text{max}} \) and three models for IB interactions as detailed in Methods. For all considered cases, the self-consistency of our results is verified by comparing the quantiles of the residuals with those of the assumed normal distribution, see Extended Data Fig. 2 for representative examples. Although no further assumptions regarding the shape of the distributions \( p(f^2|D) \) have been made, the calculated PDF \( p(f^2|D) \) are found to follow a multivariate Gaussian distribution to a very high
Figure 2: Marginal posteriors for the central model and the energy range of $E_{\text{lab}} = 0 - 280$ MeV. a, b and c show the probability distributions $p(f_i^2|D)$ in units of $10^2$. d, e and f show the joint distributions $p(f_i^2, f_j^2|D)$ in units of $10^5$. Blue solid lines and filled contours are based on the exact numerical evaluation and subsequent marginalization of Eq. (13) of Methods, while orange dashed lines/contours represent its approximation by a multivariate Gaussian distribution as described in the text.

accuracy. This is exemplified in Fig. 2 for the case of the central model and the energy range of $E_{\text{lab}} = 0 - 280$ MeV. The distributions $p(f_i^2|D)$ can, therefore, be accurately characterized by the central values and errors of the $f_i^2$’s listed in Extended Data Table 4, along with the corresponding correlation coefficients given in Extended Data Table 5, which greatly facilitates their averaging.

We can verify the statistical validity of our results by studying the likelihood at the optimal values of the parameters. E.g., fitting the LECs $C$ for $\Lambda = 463.5$ MeV (see Extended Data Fig. 1) and the central values of the $f^2$’s from Eq. (5) in the range of $E_{\text{lab}} = 0 - 280$ MeV yields $\chi^2 = 4950.72$ for $N_{\text{dat}} = 4926$, leading to $\chi^2/N_{\text{dat}} = 1.005$. The quantity $\chi^2/(N_{\text{dat}} - N_{\text{par}}) - 1 = 0.012$, where $N_{\text{par}} = 34$, is comparable to half of the standard deviation (s.d.), $\sqrt{2/(N_{\text{dat}} - N_{\text{par}})} = 0.020$, expected for a perfect model.

We have also investigated the robustness of our results with respect to the variation of input parameters. Specifically, the uncertainty from higher-order $\pi N$ LECs entering the two-pion-
exchange potential is quantified by generating 50 sets of these LECs based on the central values and covariance matrix from ref. [21]. Further, the uncertainty of the QCD contribution $\delta m_{\text{QCD}}$, even taking its conservative estimate of $\pm 0.30 \text{ MeV}$[28], is found to induce errors in $f_i^2$ that are negligibly small compared to the ones given below. Our final result for the $\pi N$ coupling constants after the Bayesian averaging reads

$$f_p^2 = 0.0770(5)^a(0.8)^b,$$
$$f_0^2 = 0.0779(9)^a(1.3)^b,$$
$$f_c^2 = 0.0769(5)^a(0.9)^b,$$  \hspace{1cm} (5)

where the first error (a) is obtained from the marginal posteriors $p(f^2|D)$ and includes the statistical and systematic errors due to the truncation of the EFT expansion, the choice of the energy range and the associated data selection. The second error (b) reflects the uncertainty in the higher-order $\pi N$ LECs.

Our results for $f_i^2$ are compared in Fig. 3 with selected earlier determinations. Our value for $f_c^2$ is consistent with all quoted determinations from the $\pi N$ system (at the 1.3$\sigma$ level). The results for $f_0^2$ and $f_c^2$ agree within errors with the recent determination by the Granada group[11], while for $f_p^2$ we obtain a slightly larger value. However, contrary to the Granada group that found evidence that the coupling of neutral pions to neutrons is larger than to protons, $f_0^2 - f_p^2 = 0.0029(10)$[11], our result $f_0^2 - f_p^2 = 0.0010(10)^a(2)^b$ is consistent with no charge dependence. Compared to the Granada analysis, we also do not find a large anticorrelation between $f_0^2$ and $f_c^2$, see Extended Data Table 5.

In summary, our Bayesian PWA of np and pp scattering data in the framework of chiral EFT yields new reference values for the $\pi N$ coupling constants with controlled uncertainties, which provide a solid basis for precision nuclear physics and establish important benchmarks for future first principles calculations using lattice QCD and QED.

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Figure 3: **Values for the πN coupling constants.** The data points show selected determinations of the πN coupling constants $f_0^2$, $f_p^2$ and $f_c^2$. The results were obtained using fixed-$t$ dispersion relations of πN scattering\[1-2\] (filled dots), πN scattering lengths in combination with the GMO sum rule\[3,4\] (filled squares), proton-antiproton PWA\[5\] (open triangle) and NN PWA\[6-10\] including the 2017 Granada PWA from Navarro Pérez et al.\[11\] (open diamonds). When provided separately, the statistical and systematic uncertainties are added in quadrature. The vertical bands show our full uncertainty. Uncertainties are one s.d.
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Methods

**Chiral effective field theory for the NN potential.** The functional form of the NN interaction employed in our analysis corresponds to the N^4LO+ potential of ref. [20], supplemented with the CIB and CSB interactions up to N^4LO as described below.

- We employ the most general form of the static OPE potential given by

  \[ V_{\text{pp}}^{\text{1p}} = f_p^2 V(M_{\pi^0}), \quad V_{\text{nn}}^{\text{1p}} = f_n^2 V(M_{\pi^0}), \quad V_{\text{np}}^{\text{1p}} = -f_0^2 V(M_{\pi^0}) + (-1)^{I+1} 2 f_c^2 V(M_{\pi^0}), \]

  \[ \text{where } I = 0, 1 \text{ is the isospin quantum number of the np system and } V(M_i) \text{ is given by} \]

  \[ V(M_i) = -\frac{4\pi}{M_{\pi^\pm}^2} \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}, \]

  Here, \( \vec{\sigma} \) are Pauli spin matrices and \( \vec{q} = \vec{p}' - \vec{p} \) is the momentum transfer of the nucleons, with \( \vec{p}' \) and \( \vec{p} \) denoting the final and initial momenta.

- Electromagnetic corrections to the OPE start to contribute at N^3LO. The expressions for the corresponding \( \pi\gamma \)-exchange potential at this order have been derived in ref. [24] and depend only on the known quantities \( \alpha, F_\pi, g_A \) and the pion masses. We employ the convention to absorb the pion-pole contribution to the \( \pi\gamma \)-exchange potential into a definition of \( f_c^2 \).

- At the order considered, the IB two-pion-exchange potential involves the following structures

  \[ V_{2\pi} = (V_C + V_S \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}) \tau_1^3 \tau_2^3 + (W_C + W_S \vec{\sigma}_1 \cdot \vec{\sigma}_2 + W_T \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}) (\tau_1^3 + \tau_2^3), \]

  with the first and second lines showing the CIB and CSB operators, respectively. The leading IB TPE contributions appear at fourth order (N^3LO). The corresponding unregularized expressions for \( V_{C(4)} \) and \( W_{C(4)} \) are given in Eqs. (3.40) and (3.47) of ref. [27] with \( \Lambda = \infty \), while \( V_{T(4)} = V_{S(4)} = 0 \). The expressions at fifth order (N^4LO) are given in Eqs. (3.49), (3.52) of ref. [27] for \( V_{T(5)} \) and in Eq. (2.11) of ref. [31] for \( W_{C(5)} \). For the remaining IB TPE contributions at this order, we employ the expressions

  \[ W_{T(5)}(q) = -\frac{1}{q^2} W_{S(5)}(q) = -\left( \frac{g_A^2}{16\pi^2 F_\pi^4} \delta m c_4 + \frac{4 f_c^3}{M_{\pi^\pm}^4} (f_p - f_n) \right) L(q), \]

  \[ V_{C(5)}(q) = -\frac{f_c (2 f_c - f_p - f_n)}{24\pi^2 F_\pi^4 M_{\pi^\pm}^4} \left( \frac{16\pi^2 F_\pi^2 f_c^2}{4M_{\pi^\pm}^2 + q^2} (128 M_{\pi^\pm}^4 + 112 M_{\pi^\pm}^2 q^2 + 23 q^4) \right) \]

  \[ -(8 M_{\pi^\pm}^2 + 5 q^2) M_{\pi^\pm}^2 \right) L(q), \]

  \[ (9) \]
where \( q \equiv |\vec{q}| \), \( M_\pi = 2/3 M_\pi^+ + 1/3 M_\pi^0 \) and the function \( L(q) \) is given by \( L(q) = s/q \ln((s + q)/(2M_\pi)) \) with \( s = \sqrt{q^2 + 4M_\pi^2} \). The above expressions generalize the ones of ref. \(^{27}\) by including all TPE terms linear in the differences between \( f_i \)'s with no assumptions about the dominance of CSB over CIB.

The employed IB TPE depends on the known LECs \( F_\pi, g_A, c_1, c_2, c_3, c_4 \), pion masses, the neutron-proton mass differences \( \delta m \) and \( \delta m_{\text{QCD}} \) and the IB \( \pi N \) coupling constants \( f_p, f_n \) and \( f_c \) to be determined.

- Up to the considered order, the CIB and CSB short-range interactions contributing to the np and pp systems involve two terms in the \( ^1S_0 \) channel and one term in each of the \( ^3P_0 \), \( ^3P_1 \) and \( ^3P_2 \) partial waves, see ref. \(^{27}\) for explicit expressions. The corresponding LECs have to be extracted from NN data.

Regularization of the IB potential is performed in the same way as in ref. \(^{20}\). In particular, contact interactions are multiplied with a nonlocal Gaussian cutoff, while regularization of the long-range contributions is carried out by utilizing the spectral representation. For the contributions to the \( \pi \gamma \)-exchange potential and \( V_C^{(4)} \) that feature a pole at the branch point, the spectral representation is employed for the corresponding indefinite integral.

Having defined the functional form of the employed NN potential, we are now in the position to specify the numerical values of various parameters. The long-range interactions stemming from the pion, pion-photon and two-pion exchanges are completely determined by the \( \pi N \) coupling constants \( f_p^2, f_0^2 \) and \( f_c^2 \) treated as free parameters, the cutoff value \( \Lambda \) to be marginalized over and the known masses and constants specified in Extended Data Table 1. To account for the Goldberger-Treiman discrepancy, we replace \( g_A \) in all expressions with a rescaled effective axial vector coupling \( g_A^{\text{eff}} \). For consistency reasons, we employ the values for \( F_\pi \) and \( g_A^{\text{eff}} \) from the determination of the \( \pi N \) LECs in ref. \(^{21}\). For the QCD contribution to the nucleon mass difference, we use the value \( \delta m_{\text{QCD}} = 2.05(30) \text{ MeV} \) which is compatible with both the lattice QCD and QED result of ref. \(^{28}\) obtained by using the experimental value of \( \delta m = \delta m_{\text{QCD}} \simeq 2.16 \text{ MeV} \), and with the recent determination by the same authors \( \delta m_{\text{QCD}} \simeq 1.87(16) \text{ MeV} \). The short-range part of the potential involves 25 isospin-invariant contact interactions, see Eqs. (A.1) and (17) of ref. \(^{20}\), and 5 IB contact terms as explained above. The strengths \( C_i(\Lambda) \) of the corresponding interactions are treated as free parameters.

The NN potential specified above defines our central model (Model 1) used in the determination of \( f_2 \). To estimate the systematic uncertainty from the truncation of the EFT expansion of
the IB NN potential, we also consider two additional interaction models that include selected IB contributions at sixth order (i.e. N^5LO):

Model 2: Same as Model 1 but including the subleading \( \pi\gamma \)-exchange potential driven by the numerically large isovector magnetic moment \( \kappa_v \approx 4.706 \) of the nucleon, which has been worked out in ref. 32.

Model 3: Same as Model 1 but including the IB TPE potential \( \propto (f_j - f_k)c_i \). These contributions have not been considered before. The unregularized expressions read:

\[
V^{(6)}_T(q) = -\frac{1}{q^2}V^{(6)}_S(q) = -\frac{f_c(2f_c - f_p - f_n)c_4}{4F^2\pi^2 M^2_{\pi^\pm}} (4M^2_{\pi} + q^2) A(q),
\]

\[
W^{(6)}_C(q) = -\frac{f_c(f_p - f_n)}{2F^2\pi^2 M^2_{\pi^\pm}} (2M^2_{\pi} + q^2) (4M^2_{\pi}c_1 - c_3(2M^2_{\pi} + q^2)) A(q),
\]

(10)

where the function \( A(q) \) is given by \( A(q) = 1/(2q) \arctan(q/(2M_{\pi})) \).

Calculation of the scattering amplitude in the presence of electromagnetic interactions. In addition to the finite-range NN force described above, it is necessary to take into account the long-range electromagnetic interactions when calculating the scattering amplitude. Throughout this work, we include the so-called improved Coulomb potential\(^{33}\), the magnetic moment interaction\(^{34}\) as well as the vacuum-polarization potential\(^{35}\) as described in detail in ref. 20. These effects have also been taken into account in the PWAs by the Nijmegen\(^{23}\) and Granada\(^{11,22}\) groups. The nuclear amplitude is calculated by solving the Lippmann-Schwinger equation in the partial wave basis including all channels up to \( j = 20 \).

Objective function. To determine the values of \( f^2_p, f^2_0 \) and \( f^2_c \) from np and pp scattering data, we employ the same augmented \( \chi^2 \) measure as in ref. 20. Specifically, for the \( j^{th} \) experiment consisting of \( n_j \) measurements \{\( O_{j,i}^{\text{exp}} \)\}_{i=1,...,n_j} of some observable and their corresponding statistical errors \{\( \delta O_{j,i}^{\text{exp}} \)\}_{i=1,...,n_j}, we define its \( \chi^2 \) measure as

\[
\chi^2_j = \sum_{i=1}^{n_j} \left( \frac{O_{j,i}^{\text{exp}} - Z_j O_{j,i}^{\text{theo}}}{\delta O_{j,i}} \right)^2 + \left( \frac{Z_j - 1}{\delta_{\text{norm},j}} \right)^2.
\]

(11)

Here, \{\( O_{j,i}^{\text{theo}} \)\}_{i=1,...,n_j} are the theoretical values of the observable calculated from the NN potential and \( \delta_{\text{norm},j} \) is the (systematic) normalization error of the dataset \( j \). If \( \delta_{\text{norm},j} \neq 0 \), the optimal norm is estimated by minimizing \( \chi^2_j \) with respect to the factor \( Z_j \), see ref. 20 for further information. Our total \( \chi^2 \) is the sum of the contributions of np and pp scattering data in a given energy range and
two additional data points for the deuteron binding energy $B_d$ and the np coherent scattering length $b_{np}$:

$$
\chi^2 = \sum_j \chi^2_j + \left( \frac{B_d - B_d^{\text{exp}}}{\delta B_d} \right)^2 + \left( \frac{b_{np} - b_{np}^{\text{exp}}}{\delta b_{np}} \right)^2,
$$

(12)

with $B_d^{\text{exp}} = 2.224575(9)$ MeV and $b_{np}^{\text{exp}} = -3.7405(9)$ fm. Following ref. [20], we use a slightly relaxed error for $B_d$, $\delta B_d = 5 \times 10^{-5}$ MeV, and employ the experimental uncertainty of $b_{np}$ for $\delta b_{np}$.

**Data selection.** Following the same criteria as refs. [21,22,38], we only consider NN data published in peer-reviewed journals and do not take into account pp total cross sections or quasi-elastic scattering data, i.e. data that have been extracted from three-body experiments. It is well known that not all available NN scattering data are mutually compatible within statistical errors. To determine mutually compatible np and pp data, we follow the standard iterative procedure to reject $3\sigma$-inconsistent data [11,22,38]. Specifically, we start from a fit to all considered NN data which yields $\chi^2$/datum $> 1$. If the residuals of the objective function are properly normal-distributed, then $\chi^2_j$ of Eq. (11) should follow a $\chi^2$-distribution with $n_j$ degrees-of-freedom ($n_j - 1$ for floated datasets, see ref. [20]). If for some dataset $\chi^2_j$ is either too high or too low, i.e. if the probability of obtaining that value is $\leq 0.27\%$, the dataset is rejected. The potential is refitted to the accepted data and the compatibility of all data is tested again, resulting in a new set of accepted datasets. This process is then repeated until the data selection has reached stability.

For the selection process, simultaneous fits of all contact LECs and the $\pi N$ constants $f_p^2$, $f_0^2$ and $f_c^2$ were performed using the central model (Model 1). We used a fixed cutoff value of $\Lambda = 450$ MeV which was shown in ref. [20] to yield the best description of NN data among the considered cutoffs and whose choice is verified a posteriori by being close to the optimal value $\Lambda = 463.5$ MeV of this analysis. For each value of $E_{\text{lab}}^{\text{max}}$ employed in this work, we have performed a separate data selection, and we found consistent results for $E_{\text{lab}}^{\text{max}} = 260 - 300$ MeV. Our data selection is largely in agreement with the Granada 2013 database from ref. [23] and, for the sake of brevity, we restrict ourselves to listing differences between our data selection at $E_{\text{lab}}^{\text{max}} = 300$ MeV and the Granada 2013 database in Extended Data Tables 2 and 3. As mentioned above, both tables also give our database for $E_{\text{lab}}^{\text{max}} = 260$ and 280 MeV if restricted to the corresponding energy ranges. For pp data with $E_{\text{lab}}^{\text{max}} \leq 240$ MeV, the 3 differential cross sections of PA(58) get rejected, while the 15 differential cross sections of PA(58) and CO(67) rejected at higher energies are compatible with the rest of the data based on the $3\sigma$ criterion. The CO(67) dataset, in particular, consists of measurements of remarkable precision that are sensitive to F-waves, see ref. [20] for details. While it is properly described by our potential within estimated truncation errors for all
If $E_{\text{lab}}^{\text{max}}$, its $\chi^2$ value is slightly too high to be statistically accepted for $E_{\text{lab}}^{\text{max}} \geq 260$ MeV. For np data with $E_{\text{lab}}^{\text{max}} \leq 240$ MeV, KA(63) gets rejected, and the dataset of ref. [20] is accepted for $E_{\text{lab}}^{\text{max}} = 240$ MeV only.

In addition, we have corrected minor misprints in ref. [22] for the data of refs. [20][21] and removed erroneously assigned normalization errors $\delta_{\text{norm,j}}$ to the datasets of refs. [22][44] which only contain a single data point.

**Calculation of the marginalized PDF $p(f^2|D)$**. In our Bayesian analysis, we employ an uninformative uniform prior for the cutoff $\Lambda$, namely $p(\Lambda) = (\Lambda_{\text{max}} - \Lambda_{\text{min}})^{-1}$ for $\Lambda \in [\Lambda_{\text{min}}, \Lambda_{\text{max}}]$, while 0 elsewhere. Similarly, we use a uniform prior for the $\pi N$ coupling constants $f_i^2$, which is non-zero in a sufficiently large region $B = \{ f^2 | f_{i,\text{min}}^2 \leq f_i^2 \leq f_{i,\text{max}}^2 \}$ around their previously obtained values. Finally, we employ a Gaussian prior for the LECs $C_i$, $p(C) = (\sqrt{2\pi} C)^{-n} \exp(-C^2/(2\tilde{C}^2))$, with $n = 30$ being the dimension of $\tilde{C}$, to encode the naturalness assumptions for their values. Here, the parameter $\tilde{C}$ to be specified below is dimension-less, and the (dimensionful) $C_i$’s in the spectroscopic notation are expressed in their natural units of $4\pi/(F^2 H^{2k})$ with $2k$ being the power of momenta and $\Lambda_b = 650$ MeV, see ref. [20].

Using the approximation for the likelihood $p(D|f^2C\Lambda)$ in Eq. (4) and the explicit form of the priors $p(\Lambda)$, $p(f^2)$ and $p(C)$, the expression for the marginalized posterior in Eq. (3) takes the form

$$p(f^2|D) = \frac{1}{N} \int_{\Lambda_{\text{min}}}^{\Lambda_{\text{max}}} d\Lambda \frac{1}{\sqrt{\det A}} e^{-\frac{1}{2}(\chi^2_{\text{min}} + \frac{1}{\tilde{C}^2} C_{\text{min}}^T A^{-1} C_{\text{min}} - \frac{1}{\tilde{C}^2} C_{\text{min}}^T A^{-1} C_{\text{min}} \chi_B(f^2))},$$

(13)

where $A = \frac{1}{2} H + \frac{1}{\tilde{C}^2} \mathbb{1}$, the normalization constant $\tilde{N}$ is given by $\tilde{N} = NC^n p(D) (\Lambda_{\text{max}} - \Lambda_{\text{min}}) \prod_i (f_{i,\text{max}}^2 - f_{i,\text{min}}^2)$ and $\chi_B$ is the indicator function of the cuboid $B$.

It remains to specify the parameters entering the priors $p(\Lambda)$, $p(f^2)$ and $p(C)$. For the naturalness prior of the short-range LECs $C_i$ we choose the value of $\tilde{C} = 5$. Given the abundance of NN scattering data, the $C_i$’s are well-constrained by the likelihood. In our previous study[20], the $C_i$’s were found to be of natural size in the considered cutoff range of $\Lambda$, i.e. $|C_i| \sim 1$, even without imposing any naturalness constraints. In particular, none of the values of $|C_i|$ from ref. [20] exceed $\tilde{C} = 5$, see Fig. 7 of ref. [45]. We, therefore, expect the naturalness constraint $\tilde{C} = 5$ to have a negligible effect on the obtained results. The same Gaussian prior with $\tilde{C} = 5$ was also employed in ref. [25] in a fit to scattering phase shifts and was reported to have only minor impact. Next, for the $p(\Lambda)$-prior, we employ the values of $\Lambda_{\text{min}} = 400$ MeV and $\Lambda_{\text{max}} = 550$ MeV. Extended Data Figure [1] shows, as a representative example, the integrand of Eq. (13) at the central values for the $f_i^2$’s for the case of Model 1 and $E_{\text{lab}} = 0 - 280$ MeV in the employed cutoff range. The
maximum of the integrand is at $\Lambda \simeq 463.5$ MeV, and its contribution beyond $\Lambda = 430 - 510$ MeV has already dropped by more than five orders of magnitude compared to its maximum and is negligible. Thus, the cutoff is constrained by the likelihood and not by the prior. We emphasize that the preference of the $\Lambda = 430 - 510$ MeV region does not contradict the findings of ref. $^{20}$, where a weak $\Lambda$-dependence of the $\chi^2$/datum values was shown. Indeed, $\chi^2$ values have a logarithmic scale compared to the probability density and thus vary much less. Finally, for the uniform $p(f^2)$-prior, we choose the parameters to be $f^2_{p,\min} = f^2_{0,\min} = f^2_{c,\min} = 0.0729$, $f^2_{p,\max} = f^2_{c,\max} = 0.0812$ and $f^2_{0,\max} = 0.0827$. The chosen limits of $f^2$ only enter the marginal PDF, and the adequacy of their choice becomes obvious from Fig. 2. To summarize, we expect that making the priors $p(\Lambda)$, $p(f^2)$ and $p(C)$ less informative should have a negligible effect on our results.

The remaining integration over $\Lambda$ in Eq. (13) is performed numerically via Gaussian quadrature. Scaling up the number of integration points for the cutoff naively is computationally costly as each evaluation of the integrand in Eq. (13) requires a fit of 30 contact LECs. However, the logarithm of the integrand is a smooth function so that we can safely evaluate it at just 8 points in the range of $\Lambda = 400 - 550$ MeV and interpolate it by a polynomial.

**Systematic uncertainty due to the choice of the energy range in the fits and neglect of IB interactions beyond N$^3$LO.** To extract the constants $f^2$ from NN data, one needs to specify the maximum energy of data considered in the PWA $E_{\text{lab}}^{\text{max}}$. To stay unbiased, we perform determinations of $f^2$‘s at different values of $E_{\text{lab}}^{\text{max}}$ varied in steps of 20 MeV. Since our theoretical framework and the employed parametrization of the scattering amplitude are valid below the pion production threshold, the largest value of $E_{\text{lab}}^{\text{max}}$ we consider is $E_{\text{lab}}^{\text{max}} = 300$ MeV. On the other hand, when lowering $E_{\text{lab}}^{\text{max}}$, the number of experimental data gets reduced leading to larger statistical errors, and one runs the danger of overfitting. To verify consistency of our fit results at individual energies, we compare the resulting quantiles of the empirical distribution of residuals against the ones of the assumed normal distribution $N(0,1)$. In order to statistically quantify deviations from the Gaussian distribution of residuals, various confidence bands have been derived in the literature. In this work we employ the ones by Aldor-Noiman et al. $^{47}$ which are one of the most recent and most sensitive, especially towards the tails of the quantile-quantile plot. In Extended Data Fig. 2b, one observes that the corresponding graphical normality test is fulfilled at the $1\sigma$-level for the fit up to $E_{\text{lab}}^{\text{max}} = 300$ MeV. When the energy range of the fit is reduced to $\sim 200$ MeV, we start observing systematic distortions from the normal distribution of residuals pointing towards a possible overfitting issue. We, therefore, do not take into account fits with $E_{\text{lab}}^{\text{max}}$ below $E_{\text{lab}}^{\text{max}} = 220$ MeV. For this energy range, the distribution of empirical residuals is still statistically consistent with the normal one at the $2\sigma$-level, see Extended Data Fig. 2a. This leaves us with five energy ranges.
corresponding to \( E_{\text{lab}}^{\text{max}} = 220, 240, 260, 280 \) and 300 MeV.

In Extended Data Table 4, we show the extracted values of the constants \( f^2 \) for three different interaction models and five energy ranges. In each case, we have determined the maximum of the marginal posteriors \( p(f^2|D) \) using the mutually consistent np and pp data from the independent data selection up to the given value of \( E_{\text{lab}}^{\text{max}} \) for Models 1, 2 and 3 by numerically evaluating the integral in Eq. (13). Around 50 evaluations of the marginal posterior turn out to be sufficient to accurately determine the maximum of \( p(f^2|D) \). For the employed grid of 8 \( \Lambda \)-values, we thus had to carry out \( \sim 400 \) determinations of the set of 30 LECs \( C \) for every of the 15 considered cases.

While the central values of the \( f^2 \) can be conveniently determined in this way, numerically obtaining the full marginal posteriors on a sufficiently dense grid is computationally costly, and it was only done for our central model with \( E_{\text{lab}}^{\text{max}} = 280 \) MeV as shown in Fig. 2. This required performing 8000 fits of the contact LECs \( C \). As discussed in the text, the marginal posterior can be very well approximated by a multivariate Gaussian distribution, whose central values are given by the maximum of \( p(f^2|D) \) and whose covariance matrix can be inexpensively calculated from the \( \sim 50 \) points evaluated during the determination of the maximum. The covariance matrices then yield the uncertainties quoted in Extended Data Table 4 and, for the sake of completeness, we also give the corresponding correlation coefficients in Extended Data Table 5. In order to arrive at our final recommended values for the \( \pi N \) couplings \( f^2_i \), we perform a Bayesian averaging of the 15 multivariate Gaussian distributions obtained above

\[
p_{\text{avg}}(f^2|D) = \frac{1}{15} \sum_{E_{\text{lab}}^{\text{max}}} \sum_{M} p(f^2|D E_{\text{lab}}^{\text{max}} M),
\]

where \( p(f^2|D E_{\text{lab}}^{\text{max}} M) \) is given by Eq. (13) with \( E_{\text{lab}}^{\text{max}} = 220, 240, 260, 280, 300 \) MeV and \( M \) is the set of the three potential models. Therefore, the averaging corresponds to both a marginalization over the discrete parameter \( E_{\text{lab}}^{\text{max}} \) (with uniform prior) and a model averaging over Models 1-3. The final results for the central values and errors (a) of the \( f^2_i \) given in Eq. 5 have been obtained by approximating \( p_{\text{avg}}(f^2|D) \) with a single multivariate Gaussian around its maximum.

**Sensitivity to the values of the higher-order \( \pi N \) LECs.** To propagate the statistical errors in the values of the higher-order \( \pi N \) LECs \( c_i, \tilde{d}_i \) and \( \tilde{e}_i \) determined from the Roy-Steiner equation analysis of ref. 21, see Extended Data Table 1, we have generated a sample of 50 sets of normally distributed values of these LECs. For each set, the determination of \( f^2_i \) is repeated for the central model (albeit with different values of the corresponding \( \pi N \) LECs each time) and \( E_{\text{lab}}^{\text{max}} = 280 \) MeV. The resulting uncertainties of the \( f^2_i \) is then obtained by taking the standard deviation of the these 50 values of the \( f^2_i \).
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Extended Data Figure 1: The integrand of Eq. (13) as a function of the cutoff $\Lambda$ for Model 1. $\bar{f}_i^2$ refer to the central values of the LECs $f_i^2$ given in Eq. (5). The data $D$ correspond to the mutually compatible NN scattering data according to the own selection in the range of $E_{\text{lab}} = 0 - 280$ MeV.
Extended Data Figure 2: Tail-sensitive rotated quantile-quantile plots for Model 1 and $\Lambda = 463.5$ MeV. The plots a and b show the results of the fits in the energy range of $E_{\text{lab}} = 0 - 220$ MeV and $E_{\text{lab}} = 0 - 300$ MeV, respectively. The distribution of residuals, shown by blue crosses, is consistent with the normal distribution $\mathcal{N}(0, 1)$ at a given confidence level if all points lie inside the corresponding confidence band. Dashed and solid lines mark the 68% and 95% confidence bands, respectively.
Extended Data Table 1: **Parameters for the long-range part of the NN potential**

| Quantity                                      | Employed value          |
|-----------------------------------------------|-------------------------|
| **Masses and mass differences:**              |                         |
| $M_{\pi^\pm}$                                | 139.57 MeV              |
| $M_{\pi^0}$                                  | 134.98 MeV              |
| $m_p$                                        | 938.272 MeV             |
| $m_n$                                        | 939.565 MeV             |
| $\delta m_{QCD}$, ref. [28]                  | 2.05(30) MeV            |
| **Fine-structure constant $\alpha$:**         | 1/137.036               |
| **Pion decay constant $F_{\pi}$:**           | 92.2 MeV                |
| **Effective axial-vector coupling of the nucleon $g_A^{\text{eff}}$:** | 1.289                   |
| **Pion-nucleon LECs from ref. [21]:**         |                         |
| $c_1$                                        | $-1.10(3)$ GeV$^{-1}$   |
| $c_2$                                        | 3.57(4) GeV$^{-1}$      |
| $c_3$                                        | $-5.54(6)$ GeV$^{-1}$   |
| $c_4$                                        | 4.17(4) GeV$^{-1}$      |
| $\bar{d}_1 + \bar{d}_2$                     | 6.18(8) GeV$^{-2}$      |
| $\bar{d}_3$                                  | $-8.91(9)$ GeV$^{-2}$   |
| $\bar{d}_5$                                  | 0.86(5) GeV$^{-2}$      |
| $\bar{d}_{14} - \bar{d}_{15}$                | $-12.18(12)$ GeV$^{-2}$ |
| $\bar{e}_{14}$                               | 1.18(4) GeV$^{-3}$      |
| $\bar{e}_{17}$                               | $-0.18(6)$ GeV$^{-3}$   |
Extended Data Table 2: Differences in the np data selection of this work with $E_{\text{max}} = 300$ MeV in comparison to the Granada 2013 database\textsuperscript{22}

| $E_{\text{lab}}$ [MeV] | Ref. Code | Obs.     | $n$ | comment | Ref. |
|-------------------------|-----------|----------|-----|---------|------|
|                         |           | $\sigma_{\text{tot}}$ |     |         |      |
| 0.509-2.003             | PO(82)    |           | 3   |         | 48   |
| 1.005-2.530             | FI(54)    |           | 2   |         | 49   |
| 2.535                   | DV(71)    | $\sigma_{\text{tot}}$ | 1   |         | 50   |
| 25.8                    | OC(91)    | $D_t$     | 1   | (a)     | 42   |
| 77.0                    | WH(60)    | $P$       | 8   | (a)     | 51   |
| 77.0                    | WH(60)    | $P$       | 9   | (a)     | 51   |
| 199.0                   | TH(68)    | $P$       | 8   |         | 52   |
| 200.0                   | KA(63)    | $d\sigma/d\Omega$ | 19  | (b)     | 53   |
| 300.0                   | DE(54)    | $d\sigma/d\Omega$ | 15  |         | 54   |

|                         |           | $\sigma_{\text{tot}}$, $\Delta\sigma_T$, $\Delta\sigma_L$ |     |         |      |
| 0                      | LO(74)    | $\sigma_{\text{tot}}$ | 1   | (c)     | 55   |
| 0.155-0.795            | DA(13)    | $\sigma_{\text{tot}}$ | 65  |         | 56   |
| 22.5                   | FL(62)    | $\sigma_{\text{tot}}$ | 1   | (d)     | 40   |
| 50.0                   | FI(80)    | $P$       | 4   |         | 57   |
| 135.0                  | LE(63)    | $A$       | 5   | (e)     | 59   |
| 137.0                  | LE(63)    | $R$       | 5   | (e)     | 58   |
| 197.0                  | SP(67)    | $D_t$     | 3   | (e)     | 59   |
| 295.0                  | GR(82)    | $\sigma_{\text{tot}}$, $\Delta\sigma_T$, $\Delta\sigma_L$ | 3   | (f)     | 60   |

(a) data not considered in Granada database selection process
(b) the outlier at 97.0° has been removed due to too high individual $\chi^2$
(c) LO(74) cites the data of ref. \textsuperscript{61}, which is already included in the database
(d) the original publication explicitly mentions that no total cross section has been measured
(e) data from quasi-elastic scattering
(f) dispersion relation prediction
Extended Data Table 3: **Same as Table 2 but for pp data**

| $E_{\text{lab}}$ [MeV] | Ref. Code | Obs. | $n$ | comment | Ref. |
|-------------------------|-----------|------|-----|---------|------|
| **additionally included data** |           |      |     |         |      |
| 137.0                   | PA(58)    | $d\sigma/d\Omega$ | 3   |         | 52   |
| 239.9                   | AL(04)    | $d\sigma/d\Omega$ | 17  | (a)     | 63   |
| 255.2                   | AL(04)    | $d\sigma/d\Omega$ | 18  | (a)     | 63   |
| 270.8                   | AL(04)    | $d\sigma/d\Omega$ | 19  | (a)     | 63   |
| 286.7                   | AL(04)    | $d\sigma/d\Omega$ | 20  | (a)     | 63   |
| **additionally rejected data** |           |      |     |         |      |
| 144.0                   | JA(71)    | $d\sigma/d\Omega$ | 27  |         | 64   |
| 144.1                   | CO(67)    | $d\sigma/d\Omega$ | 15  |         | 65   |
| 147.0                   | PA(58)    | $d\sigma/d\Omega$ | 15  |         | 52   |
| 217.0                   | TI(61)    | $P$  | 6   | (e)     | 65   |
Extended Data Table 4: Central values and statistical errors of the $\pi N$ coupling constants $f_i^2$ for all considered interaction models and $E_{\text{lab}}^{\text{max}}$ values

|                | 220 MeV | 240 MeV | 260 MeV | 280 MeV | 300 MeV |
|----------------|---------|---------|---------|---------|---------|
| **Model 1:**   |         |         |         |         |         |
| $f_p^2$        | 0.0772(5) | 0.0772(5) | 0.0769(4) | 0.0768(4) | 0.0769(4) |
| $f_0^2$        | 0.0782(10) | 0.0783(9) | 0.0781(8) | 0.0780(8) | 0.0782(8) |
| $f_c^2$        | 0.0770(5) | 0.0770(5) | 0.0769(4) | 0.0767(4) | 0.0766(4) |
| **Model 2:**   |         |         |         |         |         |
| $f_p^2$        | 0.0772(5) | 0.0772(5) | 0.0769(4) | 0.0768(4) | 0.0769(4) |
| $f_0^2$        | 0.0782(9) | 0.0782(9) | 0.0780(8) | 0.0779(8) | 0.0782(8) |
| $f_c^2$        | 0.0771(5) | 0.0771(5) | 0.0770(4) | 0.0767(4) | 0.0767(4) |
| **Model 3:**   |         |         |         |         |         |
| $f_p^2$        | 0.0772(5) | 0.0772(5) | 0.0770(4) | 0.0768(4) | 0.0770(4) |
| $f_0^2$        | 0.0778(9) | 0.0779(9) | 0.0776(8) | 0.0775(7) | 0.0777(7) |
| $f_c^2$        | 0.0772(5) | 0.0771(5) | 0.0771(4) | 0.0768(4) | 0.0768(4) |
Extended Data Table 5: Correlation coefficients between $f_i^2$’s for all considered interaction models and $E_{\text{lab}}^{\text{max}}$ values

|           | 220 MeV | 240 MeV | 260 MeV | 280 MeV | 300 MeV |
|-----------|---------|---------|---------|---------|---------|
| **Model 1:** |         |         |         |         |         |
| $\text{corr}(f_p^2,f_0^2)$ | -0.05   | -0.04   | -0.01   | -0.04   | -0.04   |
| $\text{corr}(f_0^2,f_c^2)$ | -0.13   | -0.08   | -0.12   | -0.14   | -0.15   |
| $\text{corr}(f_c^2,f_p^2)$ | 0.32    | 0.28    | 0.35    | 0.28    | 0.27    |
| **Model 2:** |         |         |         |         |         |
| $\text{corr}(f_p^2,f_0^2)$ | -0.04   | -0.04   | -0.01   | -0.04   | -0.04   |
| $\text{corr}(f_0^2,f_c^2)$ | -0.10   | -0.09   | -0.12   | -0.14   | -0.12   |
| $\text{corr}(f_c^2,f_p^2)$ | 0.29    | 0.29    | 0.35    | 0.28    | 0.27    |
| **Model 3:** |         |         |         |         |         |
| $\text{corr}(f_p^2,f_0^2)$ | 0.00    | 0.00    | 0.04    | 0.06    | 0.06    |
| $\text{corr}(f_0^2,f_c^2)$ | -0.03   | -0.01   | -0.05   | -0.07   | -0.04   |
| $\text{corr}(f_c^2,f_p^2)$ | 0.29    | 0.28    | 0.33    | 0.28    | 0.25    |