Charm quark-antiquark correlations in photon-proton scattering

M. Łuszczak and A. Szczurek

1 Institute of Nuclear Physics
PL-31-342 Cracow, Poland
2 University of Rzeszów
PL-35-959 Rzeszów, Poland

Abstract

Correlation of charm quark - charm antiquark in $\gamma p$ scattering are calculated in the $k_t$-factorization approach. We apply different unintegrated gluon distributions (uGDF) used in the literature. The results of our calculations are compared with very recent experimental results from the FOCUS collaboration. The CCFM uGDF developed recently by Kwieciński et al. gives a good description of the data. New observables are suggested for future studies. Predictions and perspectives for the HERA energies are presented.

In recent years a lot of activity was devoted to the description of the photon-proton total cross section (or $F_2$ structure function) in terms of the unintegrated gluon distribution functions (uGDF) (see e.g.\cite{1,2} and references therein). In some of the analyses also inclusive charm quark (or meson) were considered\cite{3}. Although the formalism of uGDF is well suited for studying more exclusive observables, only very few selected cases were considered in the literature. A special example is azimuthal jet-jet correlations in photon-proton scattering\cite{4,5,6}. In this case one samples in a nontrivial way simultaneously the $x$ and $k_t$ dependences of uGDF. The H1 collaboration at HERA has measured very recently such correlations\cite{7}. Very similar analysis was performed in the past also for open charm production at somewhat lower energies\cite{8}. Recently the FOCUS collaboration at Fermilab provided new precise data for charm-anticharm correlations\cite{9}. It is our aim here to analyze the charm-anticharm correlations in terms of uGDF. In the present paper we wish to compare results for different uGDF available in the literature. While the total cross section depends on small values of $x$, the high-$p_t$ jets and/or heavy quark production test gluon distributions at somewhat higher $x$. The high-$p_t$ cross section depends on higher values of $x$, and this is the range where the uGDF give a good description of the data. This is unexpected since the uGDF are not designed to give a good description of the high-$x$ jet data. This is why it is important to compare the predictions for different uGDF.
larger \( x \). Only some approaches from the literature are applicable in this region. In particular, we wish to test results based on CCFM unintegrated parton distributions developed recently by Kwieciński et al.\[^{[10, 11]}\].

The total cross section for quark-antiquark production in the reaction \( \gamma + p \to Q + \bar{Q} + X \) can be written as \[^{[6]}\]

\[
\sigma_{\gamma p \to Q\bar{Q}}(W) = \int d\phi \int dp_{1,t}^2 \int dp_{2,t}^2 \int dz \frac{f_g(x_g, \kappa^2)}{\kappa^4} \cdot \tilde{\sigma}(W, \vec{p}_{1,t}, \vec{p}_{2,t}, z) . \tag{1}
\]

In the formula above \( f_g(x, \kappa^2) \) is the unintegrated gluon distribution with the convention from Ref.\[^{[6]}\]. \(^1\) The gluon transverse momentum is related to the quark/antiquark transverse momenta \( \vec{p}_{1,t} \) and \( \vec{p}_{2,t} \) as:

\[
\kappa^2 = p_{1,t}^2 + p_{2,t}^2 + 2p_{1,t}p_{2,t}\cos\phi . \tag{2}
\]

In Eq.(1) we have introduced:

\[
\tilde{\sigma}(W, \vec{p}_{1,t}, \vec{p}_{2,t}, z) = \frac{\alpha_{em}}{2} e^2 \frac{\alpha_s(\mu^2)}{\kappa^2} \left\{ \frac{1}{\vec{p}_{1,t}^2 + m_Q^2} + \frac{1}{\vec{p}_{2,t}^2 + m_Q^2} \right\}^2 \cdot m_Q^2 \left( \frac{1}{\vec{p}_{1,t}^2 + m_Q^2} + \frac{1}{\vec{p}_{2,t}^2 + m_Q^2} + \right)^2 . \tag{3}
\]

The unintegrated gluon distribution \( f_g \) is evaluated at

\[
x_g = \frac{M_t^2}{W^2} , \tag{4}
\]

where

\[
M_t^2 = \frac{p_{1,t}^2 + m_Q^2}{z} + \frac{p_{2,t}^2 + m_Q^2}{1 - z} . \tag{5}
\]

It is obvious that at larger transverse momenta of quarks and/or heavy quark-antiquark production one samples larger values of \( x_g \) than in the case of the total photon-proton cross section.

The choice of \( \mu_r^2 \) is not essential for the discussion in the present paper and will be discussed elsewhere. In the present calculation the scale of running coupling constant in (3) is taken to be \( \kappa^2 \) and the freezing prescription from \[^{[12]}\] is used. The latter prescriptions are not very important when studying correlations. They may be important, however, for the integrated cross sections, to be studied elsewhere \[^{[13]}\].

Thus the basic ingredient of our approach are unintegrated gluon distributions. Different models of uGDF have been proposed in the literature \[^{[1]}\].\[^{[2]}\]
(see for instance [1] [2] and references therein). The main effort has been concentrated on the small-x region. While the total cross section is the genuine small-x phenomenon \(x < 10^{-3}\), the production of charm and bottom quarks samples rather the intermediate-x region \(x \sim 10^{-2} - 10^{-1}\) even at the largest available energies at HERA. It is not obvious a priori if the methods used are appropriate for the intermediate values of x. In the present approach we shall present results for a few selected gluon distributions from the literature. For illustration we shall consider the simple BFKL [13], the saturation model used to study HERA photon-proton total cross sections [16] (GBW), and the saturation model being often used recently to calculate particle production in hadron-hadron collisions [17] (KL). These three model approaches are expected to be valid for small, not very well specified, values of x. At somewhat larger values of x all these models are expected to break. This may happen already at the FOCUS energy \(W = 18.4\) GeV. Clearly an extrapolation may be needed. As in Ref.[14] one can try to extend the applicability of these small-x models by multiplying the model distributions by a phenomenological factor \((1 - x)^n\). In principle, the value of \(n\) could be adjusted to inclusive spectra at lower energies. The choice of \(n\) is, however, marginal for the correlations studied in the present paper. We shall not discuss here the details of the different approaches. A more detailed discussion can be found in Ref.[14].

In addition, we shall consider two other approaches adequate for intermediate-x region. The CCFM approach seems to be the best tailored for this purpose. It was shown in Refs.[10, 11] how to solve the one-loop CCFM equation in the impact parameter representation.

The unintegrated parton distributions used in the present paper were obtained by solving the Kwieciński CCFM equations [10, 11] using LO GRV98 collinear distributions [22] as the input for the evolution. By construction this procedure assures that our uPDF provide a good description of the \(F_2\) structure function data.

The solution of the CCFM equation depends on three variables \(\tilde{f}_q = \tilde{f}_q(x, b, \mu^2)\). \(^2\) The familiar momentum representation unintegrated gluon distribution can be obtained via Fourier-Bessel transform

\[
f_q(x, \kappa^2, \mu^2) = \frac{1}{2\pi} \int \exp\left(i\kappa \vec{b}\right) \tilde{f}_q(x, b, \mu^2) \, d^2b.
\]  

(6)

As already mentioned in the introduction, it is our intention here to use uGDFs \(\tilde{f}_g^{CCFM}(x, b, \mu^2)\) which fulfill the b-space (one-loop) CCFM equations [10] [11]. However, the perturbative solution \(\tilde{f}_g^{CCFM}(x, b, \mu^2)\) does not include

\(^2\)In the present paper we shall use the notation \(\tilde{f}\) instead of \(\bar{f}\) as in Refs.[18] [19].
nonperturbative effects such as, for instance, intrinsic momentum distribution of partons in colliding hadrons. In order to include such effects we propose to modify the perturbative solution $\tilde{f}_g^{CCFM}(x, b, \mu^2)$ and write the modified gluon distribution $\tilde{f}_g(x, b, \mu^2)$ in the simple factorized form

$$\tilde{f}_g(x, b, \mu^2) = \tilde{f}_g^{CCFM}(x, b, \mu^2) \cdot F_g^{NP}(b).$$  \hspace{1cm} (7)

In Ref.\[20\], two different functional forms for the nonperturbative form factor

$$F_g^{NP}(b) = F_{NP}(b) = \exp \left( -\frac{b^2}{4b_0^2} \right) \text{ or } \exp \left( -\frac{b}{b_e} \right)$$  \hspace{1cm} (8)

identical for all species of partons were used. In Eq.(8) $b_0$ (or $b_e$) is the only free parameter. The parameters were roughly adjusted in \[20\] to describe production of W and Z bosons in nucleon-nucleon collisions. In the present note we shall show only results for the Gaussian form factor. The dependence on the choice of the form factor will be studied elsewhere.

The resummation formulae \[21\] and the unintegrated parton distribution formulae for Higgs \[19\] and gauge boson \[20\] have identical structure if the following formal assignment is made:

$$\tilde{f}_g^{SGR}(x, b, \mu^2) = \frac{1}{2} F_g^{NP}(\mu, b, x) \left[ xg(x_1, \mu(b)) + ... \right] \exp \left( \frac{1}{2} S_g(b, \mu) \right),$$

$$\tilde{f}_q^{SGR}(x, b, \mu^2) = \frac{1}{2} F_q^{NP}(\mu, b, x) \left[ xq(x_1, \mu(b)) + ... \right] \exp \left( \frac{1}{2} S_q(b, \mu) \right),$$

$$\tilde{f}_{\bar{q}^2}^{SGR}(x, b, \mu^2) = \frac{1}{2} F_{\bar{q}^2}^{NP}(\mu, b, x) \left[ x\bar{q}(x, \mu(b)) + ... \right] \exp \left( \frac{1}{2} S_{\bar{q}^2}(b, \mu) \right).$$  \hspace{1cm} (9)

The index $SGR$ above stands for “soft-gluon resummation”. The explicit expressions for $S_g$, $S_q$ and $S_{\bar{q}^2}$ can be found in Ref.\[19, 20\], where in addition similarities and differences between Kwieciński CCFM and soft gluon resummation are discussed.

The $k_t$-dependent unintegrated distributions of gluons corresponding to the b-space resummation can be then obtained through the Fourier-Bessel transform

$$f_g^{SGR}(x, \kappa^2, Q^2) = \int dbb J_0(kb) \tilde{f}_g^{SGR}(x, b, Q^2).$$  \hspace{1cm} (10)

With the simple Ansatz \[3\] for $F_g^{NP}$ the whole scale $\mu^2$ dependence resides exclusively in the Sudakov-like form factor. For brevity, we shall call the gluon distribution in Eq.(10) the “resummation gluon distribution”.

Before we go to charm quark - charm antiquark correlations we wish to show the results for inclusive spectra of $c$ or $\bar{c}$. In Fig.\[1\] we show the
distributions $\frac{d\sigma}{d\phi}$ for different uGDF for low ($W=18.4$ GeV, left panel) and high ($W=200$ GeV, right panel) energies. At the lower energy the GBW uGDF gives somewhat steeper $p_t^2$ distribution than the other uGDF. At the high energy the slope of the $p_t^2$ distributions decreases. Quite similar slopes are obtained for different uGDF. Therefore this observable is not the best one to test models of uGDF. We shall show below that more exclusive correlation observables are more sensitive tests of models/parametrizations of uGDF.

In the present analysis we do not put any restrictions on heavy quark or heavy antiquark transverse momenta $\vec{p}_{1,t}$ and $\vec{p}_{2,t}$. A detailed analysis of the effect of such cuts on the results will be presented elsewhere [13].

The azimuthal correlation functions $w(\phi)$ defined as:

$$w(\phi; W) \equiv \frac{\frac{d\sigma}{d\phi}(W)}{\int \frac{d\sigma}{d\phi}(W) \, d\phi},$$  \hspace{1cm} (11)

where

$$\frac{d\sigma}{d\phi}(W) = \int dp_{1,t}^2 \int dp_{2,t}^2 \int dz \frac{f_g(x_g, \kappa^2)}{\kappa^4} \cdot \tilde{\sigma}(W, \vec{p}_{1,t}, \vec{p}_{2,t}, z)$$  \hspace{1cm} (12)

in order that

$$\int w(\phi; W) \, d\phi = 1$$  \hspace{1cm} (13)

for two energies of $W = 18.4$ GeV (FOCUS) and $W = 200$ GeV (HERA) are shown in Fig.2. The GBW-glue (thin dashed) gives too strong back-to-back correlations for the lower energy. Another saturation model (KL, [17]) provides more angular decorrelation, in better agreement with the experimental data. The BFKL-glue (dash-dotted) provides very good description of the data. The same is true for the CCFM-glue (thick solid) and resummation-glue (thin solid). The latter two models are more adequate for the lower energy. The renormalized azimuthal correlation function [13] for BFKL, GBW and KL models are almost independent of the power $n$ in extrapolating to larger values of $x_g$. In the present paper, in calculating the cross section with the Kwieciński CCFM uGDF for simplicity we have fixed the scale for $\mu^2 = 4 m_c^2$. Allowing for dependence of the scale $\mu^2$ on kinematical variables such as $x$ or $\kappa^2$ would make the calculation very time consuming. The sensitivity to the choice of the scale will be discussed in detail elsewhere. For comparison in panel (b) we present predictions for $W = 200$ GeV. Except of the GBW model, there is only a small increase of decorrelation when going from the lower fixed-target energy region to the higher collider-energy region.
In the present paper we completely ignore the resolved photon component \[23\]. The latter should be, however, negligible for the FOCUS fixed target experiment \[9\].

Not only azimuthal correlations are interesting. In general, the integrand of Eq. (1) depends on four independent kinematical variables \(\phi, p_{1,t}^2, p_{2,t}^2, t\) (other combinations of the kinematical variables are also possible). In particular, the formula (1) can be rewritten in the form

\[
\sigma^{e^+ p \rightarrow \bar{Q} \bar{Q}}(W) = \int dp_{1,t}^2 \int dp_{2,t}^2 w(p_{1,t}^2, p_{2,t}^2; W) \tag{14}
\]

where the two-dimensional correlation function

\[
w(p_{1,t}^2, p_{2,t}^2; W) = \int d\phi \, dz \frac{f_g(x_g, \kappa^2)}{\kappa^4} \cdot \bar{\sigma}(W, \vec{p}_{1,t}, \vec{p}_{2,t}, z). \tag{15}\]

In Fig.3 we present some examples of \(w(p_{1,t}^2, p_{2,t}^2)\) at \(W=18.4\) GeV for different uGDF. The maps for different uGDF differ in details. The distribution for the GBW gluon distribution is concentrated along the diagonal \(p_{1,t}^2 = p_{2,t}^2\) and in this respect resembles the familiar collinear leading-order result. The other three distributions have a sizeable strength at the phase-space borders for \(p_{1,t}^2 \approx 0\) or \(p_{2,t}^2 \approx 0\). The KL gluon distribution gives in addition some enhancement at \(p_{1,t}^2 \approx p_{2,t}^2\), especially at large transverse momenta. Experimental studies of such maps could open an interesting new possibility to test models of uGDF in a more detailed differential fashion. In principle, such studies will be possible with HERA II runs at DESY. The leading-order approach of Kwiecinski contains the higher-order corrections via evolution equations. However, in contrast to NLO collinear approach it can be applied even in the region \(p_{1,t} = p_{2,t}\).

At present experimental luminosities (statistics) one may have a problem to explore the whole two-dimensional maps shown in Fig.3. In this case a more global variable could be useful. In order to quantify the spread over \(p_{1,t}^2 \times p_{2,t}^2\) plane and/or departure from the diagonal (LO collinear approach) we propose a new variable which can be interpreted as a measure of deviations from the equal-length momenta defined as:

\[
f(p_{\text{max}}^2 > k \, p_{\text{min}}^2; W) = \frac{\sigma(p_{\text{max}}^2 > k \, p_{\text{min}}^2; W)}{\sigma(W)}, \tag{16}\]

where \(p_{\text{max}}^2 = \max(p_{1,t}^2, p_{2,t}^2)\) and \(p_{\text{min}}^2 = \min(p_{1,t}^2, p_{2,t}^2)\). For example with \(k = 2\) one obtains respectively: GBW: 0.01, KL: 0.45, BFKL: 0.63, K(CCFM): \footnote{It is well known that the collinear NLO calculation is not reliable for \(p_{1,t} = p_{2,t}\).}
The quantity $f(p_{\text{max}}^2 > kp_{\text{min}}^2; W)$ is sensitive to both perturbative ($p_{1,t} \neq p_{2,t}$) and nonperturbative ($p_{1,t} \approx p_{2,t}$) processes and therefore reflects their interplay. We believe that the FOCUS collaboration could reprocess their present data in order to obtain analogous fractions for their $(p_{t,D}^2, p_{t,D}^2)$ distributions.

In the leading-order collinear approach (without parton showers included) the transverse momenta of two jets add up to zero. It is not the case for our leading-order $k_t$-factorization approach. In Fig.4 we present normalized to unity distribution in $p_+^2$, where $p_+ = \vec{p}_1 + \vec{p}_2$. Due to momentum conservation, in our approach the sum of transverse momenta is directly equal to transverse momentum of the gluon ($\vec{p}_+ = \vec{k}$). This means that the distribution in $p_+^2$ directly probes the transverse momentum distribution of gluons. The situation here is very similar to the situation for the azimuthal angle distribution. The GBW gluon distribution which describes very well the total $\gamma^* - p$ cross section gives very steep distribution in $p_+^2$ in comparison to other uGDF. We expect that the inclusion of QCD evolution effects, like in Ref. [24] for instance, should change this result and lead to somewhat broader distributions. The Kwieciński CCFM gluon distribution gives the best description of the FOCUS data [9]. We expect that this approach is suitable for $x > 0.01$. Although the other models give also reasonable description of the FOCUS $w(p_+^2)$ data, one should remember that their application for the low-energy data is somewhat unsure.

For completeness in Fig.5 we present normalized to unity distributions in the square of $\vec{p}_- = \vec{p}_1 - \vec{p}_2$. Here, however, the differences between different uGDF are smaller than in the previous two cases. Nevertheless the corresponding data would be a new possible observable to verify the unintegrated gluon distributions.

In summary, we have shown that the analysis of kinematical correlations of charm quarks and antiquarks opens new possibilities for verifying models of uGDF. The recently measured data of the FOCUS collaboration at Fermilab allows one to study the unintegrated gluon distribution in the intermediate-$x$ region. Many models of uGDF used in the literature are constructed rather for small values of $x$ and its application in the region of somewhat larger $x$ ($x > 0.05$) is questionable. The unintegrated gluon (parton) distribution which fulfill the CCFM equations, developed recently by Kwieciński et al., describe the data fairly well. It can be expected that the correlation data from the HERA II runs will give a new possibility to verify the different models of unintegrated gluon distributions in a more detailed way.
Acknowledgments We are indebted to Krzysztof Golec-Biernat and Hannes Jung for a discussion and Erik Gottschalk for providing us files with the FOCUS experimental data.

References

[1] Bo Anderson et al. (Small-x collaboration), Eur. Phys. J. C25 (2002) 77.

[2] J. Andersen et al. (Small-x collaboration), [hep-ph/0312333]

[3] V.A. Saleev and N.P. Zotov, Mod. Phys. Lett. A11 (1996) 25; S.P. Baranov and N.P. Zotov, Phys. Lett. B458 (1999) 389; S.P. Baranov and N.P. Zotov, Phys. Lett. B491 (2000) 111; A.V. Lipatov, V.A. Saleev and N.P. Zotov, Mod. Phys. Lett. A15 (2000) 1727; S.P. Baranov, H. Jung, L. Jönson, S. Padhi and N.P. Zotov, Eur. Phys. J. C24 (2002) 425; C. B. Mariotto, M.B.G. Ducatti and M.V.T. Machado, Phys. Rev. D66 (2002) 114013; L. Motyka and N. Timneanu, Eur. Phys. J. C27 (2003) 73; H. Jung, Phys. Rev. D65 (2002) 034015.

[4] J.R. Forshaw and R.G. Roberts, Phys. Lett. B335 (1994) 494.

[5] A.J. Askew, D. Graudenz, J. Kwieciński and A.D. Martin, Phys. Lett. B338 (1994) 92.

[6] A. Szczurek, N. Nikolaev, W. Schäfer and J. Speth, Phys. Lett. 500 (2001) 254.

[7] A. Aktas et al., [hep-ex/0310019]

[8] M.P. Alvarez et al. (NA14/2 Collaboration), Phys. Lett. 278 385; P.L. Frabetti et al., Phys. Lett. 308 (1993) 193.

[9] J.M. Link et al. (FOCUS collaboration), Phys. Lett. 566 (2003) 51.

[10] J. Kwieciński, Acta Phys. Polon. B33 (2002) 1809.

[11] A. Gawron and J. Kwieciński, Acta Phys. Polon. B34 (2003) 133.

[12] D.V. Shirkov and I.L. Solovtsov, Phys. Rev. Lett. 79 (1997) 1209.
[13] M. Luszczak and A. Szczurek, in preparation.
[14] A. Szczurek, Acta Phys. Polon. B34 (2003) 3191.
[15] A. J. Askew, J. Kwieciński, A.D. Martin and P.J. Sutton, Phys. Rev. D49 (1994) 4402.
[16] K. Golec-Biernat and M. Wüsthoff, Phys. Rev. D60 (1999) 114023-1.
[17] D. Kharzeev and E. Levin, Phys. Lett. B523 (2001) 79.
[18] A. Gawron, J. Kwieciński and W. Broniowski, Phys. Rev. D68 (2003) 054001.
[19] A. Gawron and J. Kwieciński, hep-ph/0309303
[20] J. Kwieciński and A. Szczurek, hep-ph/0311290, Nucl. Phys. B680 (2004) 164.
[21] J.C. Collins, D. Soper and G. Sterman, Nucl. Phys. B250 (1985) 199.
[22] M. Glück, E. Reya and A. Vogt, Eur. Phys.J. C5 (1998) 461.
[23] A. Szczurek, Eur. Phys. J. C26 (2002) 183.
[24] J. Bartels, K. Golec-Biernat and H. Kowalski, Phys. Rev. D66 (2002) 014001.
Figure 1: Inclusive distribution of charm quarks/antiquarks as a function of the corresponding transverse momentum. We present results for GBW (thin dashed), KL (thick dashed), BFKL (dash-dotted) and Kwieciński CCFM (solid).

Figure 2: Azimuthal correlations between $c$ and $\bar{c}$. The theoretical results are compared to the recent results from [9] (fully reconstructed pairs). The notation is the same as in Fig.1.
Figure 3: Some examples of the two-dimensional maps $w(p_{1,t}^2, p_{2,t}^2)$ for $W = 18.4$ GeV: a) GBW, b) KL, c) BFKL, d) CCFM, e) resummation.
Figure 4: $p_\perp^2$ distribution of $c - \bar{c}$. The theoretical results are compared to the recent results from [9] (fully reconstructed pairs). The notation is the same as in Fig. 7.

Figure 5: $p_\perp^2$ distribution of $c - \bar{c}$. The notation is the same as in Fig. 7.