Control Complexity in Bucklin, Fallback, and Plurality Voting: An Experimental Approach*

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Walsh [Wal10, Wal09], Davies et al. [DKNW10, DKNW11], and Narodytska et al. [NWX11] studied various voting systems empirically and showed that they can often be manipulated effectively, despite their manipulation problems being NP-hard. Such an experimental approach is sorely missing for NP-hard control problems, where control refers to attempts to tamper with the outcome of elections by adding/deleting/partitioning either voters or candidates. We experimentally tackle NP-hard control problems for Bucklin and fallback voting. Among natural voting systems with efficient winner determination, fallback voting is currently known to display the broadest resistance to control in terms of NP-hardness, and Bucklin voting has been shown to behave almost as well in terms of control resistance [ER10, EPR11, EFPR11]. We also investigate control resistance experimentally for plurality voting, one of the first voting systems analyzed with respect to electoral control [BTT92, HHR07].

Our findings indicate that NP-hard control problems can often be solved effectively in practice. Moreover, our experiments allow a more fine-grained analysis and comparison—across various control scenarios, vote distribution models, and voting systems—than merely stating NP-hardness for all these control problems.

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Electoral control [BTT92, HHR07] refers to attempts to tamper with the outcome of elections by adding/deleting/partitioning either the voters or the candidates. To protect elections against such control attempts and other ways of manipulation (see, e.g., the surveys [FHH10, FP10]), much work has been done recently to show that the attacker’s task can be computationally hard: Certain voting systems are resistant to manipulation [FHH10, FP10, CSL07] or control [BTT92, HHR07, ER10, EPR11] in certain scenarios. However, most of this work is concerned with NP-hardness results, which is a worst-case measure of complexity and leaves open the possibility that many elections can still be manipulated or controlled in a reasonable amount of time.

To avoid this disadvantage, manipulation and control problems have also been tackled from different angles. For example, from a theoretical perspective, Zuckerman et al. [ZPR09] proposed approximation algorithms for NP-hard manipulation problems and Faliszewski et al. [FFHR11] showed that restricting to single-peaked electorates may strip manipulation and control problems off their NP-hardness shields. From an experimental perspective, in a series of papers Walsh et al. [Wal10, Wal09, DKNW10, DKNW11] (see also [CT07]) studied various voting systems empirically, such as single transferable vote (STV), veto, and Borda, and showed that they can often be manipulated effectively, even though their manipulation problems are NP-hard. Such an experimental approach is sorely missing for NP-hard control problems to date.

This paper is the first attempt to tackle NP-hard control problems via an experimental analysis. Among natural voting systems with efficient winner determination, the system currently known to display the broadest resistance (NP-hardness) to control is fallback voting, proposed by Brams and Sanver [BS09] via combining approval with Bucklin voting. Erdélyi et al. [ER10, EPR11] showed that fallback voting is resistant to 20 out of the 22 standard types of control and that Bucklin voting behaves almost as good. Shortly after these results with all proofs were made public in a technical report dated March 11, 2011, [EFPR11], Menton [Men12] reported analogous results for normalized range voting (the version of his technical report that establishes a matching number of resistances is dated April 25, 2011).

We empirically investigate six voter control scenarios for Bucklin and fallback voting and two for plurality voting. Furthermore we investigate twelve candidate control scenarios for all three voting systems[1] i.e., while Walsh et al. [Wal10, Wal09, DKNW10, DKNW11] focused on constructive manipulation problems only (where the aim is to make a candidate win), we study both constructive and destructive control problems (the latter aiming at preventing some candidate’s victory). When generating random elections in our experiments, we consider two probability distributions: the Impartial Culture model (where votes are distributed uniformly and are drawn independently) and the Two Mainstreams model, introduced here to model two mainstreams in society by adapting the Polya Eggenberger urn model [Ber85].

After introducing the investigated voting systems and types of electoral controls in Section 2, we present the experimental setting and implemented algorithms in Sections 3 and 4. Section 5 summarizes some of our findings and observations for particular control scenarios. We conclude by providing a brief discussion of the main findings of our experiments in Section 6 which allow a more fine-grained analysis and comparison—across various control scenarios, vote distribution models, and voting systems—than merely stating NP-hardness for all these problems. A comprehensive presentation of all results can be found in the appendix.

Note that there are overall eight voter control scenarios but we only analyze those where for the corresponding control problem no deterministic polynomial-time algorithm is known. Furthermore we do not analyze two types of control by adding candidates, namely the case where the number of candidates that can be added is not limited, so that we investigate 18 of the 22 known types of electoral control.

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2. Preliminaries

2.1. Elections and Voting Systems

An election is a pair \((C, V)\) consisting of a finite candidate set \(C = \{c_1, c_2, \ldots, c_n\}\) and a finite list of voters \(V = \{v_1, v_2, \ldots, v_n\}\) expressing their preferences over the candidates in \(C\). How the votes are represented depends on the voting system used. A voting system \(\mathcal{E}\) determines how the voters’ ballots are cast and who has won a given election \((C, V)\), where the set \(W \subseteq C\) of winners may be empty or have one or more elements. We call an election with votes cast according to a voting system \(\mathcal{E}\) an \(\mathcal{E}\) election. Here we focus on the systems Bucklin voting, fallback voting, and plurality voting.

Bucklin voting is a preference-based voting system named after James W. Bucklin [HH26]. “Preference-based” means that the voters’ ballots are (strict) linear orders over all candidates in \(C\). For example, if \(C = \{c_1, c_2, c_3\}\) and a vote \(v\) is given by \(c_2 c_1 c_3\), then this voter \(v\) strictly prefers \(c_2\) to \(c_1\) and \(c_1\) to \(c_3\).

Let \((C, V)\) be a given Bucklin election. The level \(i\) score of a candidate \(c \in C\) \(\text{score}_{(C, V)}^i(c)\), for short) is the number of voters in \(V\) ranking \(c\) among their top \(i\) positions. Letting the strict majority threshold of a list \(V\) of votes be \(\text{maj}(V) = \left\lceil |V|/2 \right\rceil + 1\), the Bucklin score of \(c \in C\) is defined to be the smallest \(i\) such that \(\text{score}_{(C, V)}^i(c) \geq \text{maj}(V)\). Every candidate with the smallest Bucklin score (say \(\ell\)) and the highest level \(\ell\) score is a level \(\ell\) Bucklin winner (BV winner, for short). Note that there always exists a Bucklin winner, level 1 Bucklin winners are always unique, but on levels \(\ell \geq 2\) there can be more than one BV winner.

Fallback voting is a hybrid voting system introduced by Brams and Sanver [BS09]. It combines Bucklin voting with approval voting [BF83]. In a fallback election, each voter determines those candidates he or she approves of and provides a linear order of the approved candidates. So for \(C = \{c_1, c_2, c_3\}\) a vote in a fallback election could be of the form \(c_3 c_1\), meaning that this voter approves of \(c_3\) and \(c_1\), and disapproves of \(c_2\).

Winners are determined as follows in fallback voting: Given a fallback election \((C, V)\), the notions of level \(i\) score of a candidate \(c \in C\) and level \(i\) fallback winner are defined analogously as in Bucklin voting. If there is a level \(\ell\) fallback winner with \(\ell \leq |C|\), then he or she is the fallback winner in \((C, V)\). Otherwise (i.e., if no fallback winner exists in \((C, V)\)), every candidate with a highest approval score (which is the number of voters approving of this candidate) is a fallback winner in \((C, V)\). The second case can occur in fallback elections, since the voters can prevent the candidates from gaining points, and so it is possible that no candidate reaches or exceeds the strict majority threshold on any level.

Note that Bucklin elections can be seen as fallback elections where each voter approves of all candidates. So Bucklin voting is a special case of fallback voting.

In plurality voting, the most preferred candidate in each vote gains one point, and the candidates with the most points are the plurality winners. Note that there always exists at least one plurality winner. This voting rule is preference-based as well, even though the ranking of the candidates after the top candidate is irrelevant.

2.2. Electoral Control and Control Complexity

Electoral control is a way to tamper with the outcome of an election by changing the structure of the election itself [BTT92, HHR07]. These structural changes include adding, deleting, and partitioning voters or candidates. In the model of electoral control these changes are exerted by an external actor, the “chair,” having full knowledge of the voters’ preferences. For a detailed discussion of why and where this assumption is appropriate when investigating control complexity, see [HHR07]. Bartholdi et al. [BTT92] introduced the notion of constructive control where the chair’s goal is to make a distinguished candidate end up winning alone the resulting election. The case where the chair’s control action aims at preventing a given candidate from being a unique winner is called destructive control and has been introduced by Hemaspaandra et al. [HHR07].

To study the complexity of control in different scenarios, a decision problem is defined for each type of electoral control.
destructive cases, we simply ask whether it is possible to introduce the tie-handling rules “Ties Promote” (TP) in which all subelection winner, yielding the destructive control problems (“Ties Eliminate” (TE) in which only a unique winner from either subelection can move on to the runoff, and “Ties Eliminate” (TE) in which only a unique winner from either subelection can move on to the runoff (if there is more than one winner, none of them moves on).

Depending on the tie-handling rule used, we obtain the problems $\delta$-CCPV-TP and $\delta$-CCPV-TE. For the destructive cases, we simply ask whether it is possible to prevent the distinguished candidate from being a unique winner, yielding the destructive control problems $\delta$-DCDV, $\delta$-DCAV$\delta$-DCPV-TP, and $\delta$-DCPV-TE. Each of the four problems just defined models “two-district gerrymandering.”

| $\delta$-CONSTRUCTIVE CONTROL BY DELETING VOTERS ($\delta$-CCDV) |
|---------------------------------------------------------------|
| **Given:** An $\delta$ election $(C,V)$, a distinguished candidate $c \in C$, and a positive integer $k \leq \|V\|$. |
| **Question:** Is there a subset $V' \subseteq V$ with $\|V'\| \leq k$ such that $c$ is the unique $\delta$ winner of election $(C,V - V')$? |

| $\delta$-CONSTRUCTIVE CONTROL BY ADDING VOTERS ($\delta$-CCAV) |
|---------------------------------------------------------------|
| **Given:** An $\delta$ election $(C,V \cup V')$, where $V \cap V' = \emptyset$ and $V$ is the list of registered voters and $V'$ is the list of unregistered voters, a distinguished candidate $c \in C$, and a positive integer $k \leq \|V'\|$. |
| **Question:** Is there a subset $V'' \subseteq V'$ with $\|V''\| \leq k$ such that $c$ is the unique $\delta$ winner of election $(C,V \cup V'')$? |

| $\delta$-CONSTRUCTIVE CONTROL BY PARTITION OF VOTERS ($\delta$-CCPV) |
|---------------------------------------------------------------|
| **Given:** An $\delta$ election $(C,V)$ and a distinguished candidate $c \in C$. |
| **Question:** Is there a partition $(V_1,V_2)$ of $V$ such that $c$ is the unique $\delta$ winner of election $(W_i \cup W_2,V)$, where $W_i, i \in \{1,2\}$, is the set of $\delta$ winners of subelection $(C,V_i)$ surviving the tie-handling rule? |

| $\delta$-CONSTRUCTIVE CONTROL BY DELETING CANDIDATES ($\delta$-CCDC) |
|---------------------------------------------------------------|
| **Given:** An $\delta$ election $(C,V)$ and a distinguished candidate $c \in C$. |
| **Question:** Does there exist a subset $C' \subseteq C$ such that $\|C'\| \leq k$ and $c$ is the unique $\delta$ winner of election $(C - C',V)$? |

| $\delta$-CONSTRUCTIVE CONTROL BY ADDING CANDIDATES ($\delta$-CCAC) |
|---------------------------------------------------------------|
| **Given:** An $\delta$ election $(C \cup D,V)$, $C \cap D = \emptyset$, a distinguished candidate $c \in C$, and a nonnegative integer $k$. (C is the set of originally qualified candidates and $D$ is the set of spoiler candidates that may be added.) |
| **Question:** Does there exist a subset $D' \subseteq D$ such that $\|D'\| \leq k$ and $c$ is the unique $\delta$ winner of election $(C \cup D',V)$? |

| $\delta$-CONSTRUCTIVE CONTROL BY PARTITION OF CANDIDATES ($\delta$-CCPC) |
|---------------------------------------------------------------|
| **Given:** An $\delta$ election $(C,V)$ and a distinguished candidate $c \in C$. |
| **Question:** Is it possible to partition $C$ into $C_1$ and $C_2$ such that $c$ is the unique $\delta$ winner of election $(W_i \cup C_2,V)$, where $W_i$ is the set of $\delta$ winners of subelection $(C_i,V)$? |

| $\delta$-CONSTRUCTIVE CONTROL BY RUNOFF-PARTITION OF CANDIDATES ($\delta$-CCroPC) |
|---------------------------------------------------------------|
| **Given:** An $\delta$ election $(C,V)$ and a distinguished candidate $c \in C$. |
| **Question:** Is it possible to partition $C$ into $C_1$ and $C_2$ such that $c$ is the unique $\delta$ winner of election $(W_i \cup W_2,V)$, where $W_i, i \in \{1,2\}$, is the set of $\delta$ winners of subelection $(C_i,V)$? |
Summing up, we now have defined twelve candidate control problems and thus a total of 20 control problems. Note that the classic standard control scenarios include a version of control by adding candidates where the number of candidates that may be added is not bound by a constant given in the instance. With that we have fourteen candidate control problems and 22 different types of control but we do not consider these two cases in our experimental analysis.

Let $\mathcal{C}$ be a type of electoral control. Using the notions defined by Bartholdi et al. \cite{BTT92} (see also \cite{HHR07}), we say a voting system $\mathcal{E}$ is immune to $\mathcal{C}$ if the chair never succeeds in exerting control of type $\mathcal{C}$. If $\mathcal{E}$ is not immune to $\mathcal{C}$, it is susceptible to $\mathcal{C}$. If $\mathcal{E}$ is susceptible to a control type $\mathcal{C}$, we say it is vulnerable to $\mathcal{C}$ if the corresponding decision problem is decidable in deterministic polynomial time, and we say it is resistant to $\mathcal{C}$ if the corresponding decision problem is NP-hard.

### 2.3. Control Complexity in Bucklin, Fallback, and Plurality Voting

Plurality voting is one of the first voting systems for which the complexity of constructive control \cite{BTT92} and destructive control \cite{HHR07} has been studied in the above scenarios. Control in fallback voting and Bucklin voting has been previously studied by Erdélyi et al. \cite{ER10, EPR11} with respect to classical complexity and also with respect to parameterized complexity \cite{EF10}. In terms of NP-hardness, among natural systems with polynomial-time winner determination fallback voting has the most resistances to control (namely, 20 out of the 22 standard control types) and Bucklin voting behaves similarly well—just one case is open (Bucklin-DCPV-TP). Table 1 gives an overview of known complexity results for electoral control in these three systems.

### 3. Experimental Setting

In this section we describe the experimental setting. As stated in Section 2.2, the instances of control by adding and deleting both candidates and voters contain a parameter $k$ bounding the number of candidates/voters that can be deleted or added. In our experiments, we confine ourselves to the case of $k = \lfloor n/3 \rfloor$, where $n$ is the number of voters. Since every yes-instance for a given $k$ is also a yes-instance for $k' \geq k$, the number of yes-instances found in our experiments are a lower bound for the number of yes-instances when more voters can be deleted or added.

We randomly generated elections $(C,V)$ with $m = \|C\|$ and $n = \|V\|$ for all combinations of $n,m \in \{4, 8, 16, 32, 64, 128\}$. Each combination of $n$ and $m$ is one data point for which we evaluated 500 of these
elections, trying to determine for each given election whether or not control is possible. How the elections have been generated and how the algorithms are designed will be described below.

Before we specify the different distribution models underlying our election generation, we explain how random votes can be cast in Bucklin and fallback voting and how many different votes exist in both voting systems.

Assuming that the generated election has \( m \) candidates, in Bucklin voting a random vote can be obtained by generating a random permutation over the \( m \) different candidates. Clearly, the overall number of different votes in Bucklin elections is \( m! \).

In fallback voting random votes can be generated as follows: In a first step, a preference (i.e., linear order) \( p \) over all \( m \) candidates is drawn randomly under a certain distribution (see below) from all \( m! \) possible preferences. In a second step, the number of approved candidates, say \( \ell \), is drawn from the possible numbers \( \{0, 1, 2, \ldots, m\} \). The preference \( p \) and \( \ell \) are drawn independently. Then, the generated vote consists of the first \( \ell \) candidates in \( p \). Generalizing this, we know that there can be \( \sum_{\ell=0}^{m} \binom{m}{\ell} \ell! \) different votes in fallback elections with \( m \) candidates. With this in mind, we now specify the two distribution models we will be working with.

In the Impartial Culture model (IC model) we assume uniformly distributed votes and draw each vote independently out of all possible preferences.

In the second model, which we call the Two Mainstreams model (TM model), we adapt the Polya Eggenberger urn model (PE model, see [Ber83]) that has been used by Walsh [Wal10] in the following way: We draw two votes out of an urn containing all possible, say \( t \), votes (with \( t = m! \) or \( t = \sum_{\ell=0}^{m} \binom{m}{\ell} \ell! \), depending on the voting system). Each of these votes can be interpreted as a representative of one "main stream" in society (e.g., liberal and conservative). Then each vote is put back into the urn with \( k \) additional votes of the same form. Out of this urn the votes for the election are drawn randomly with replacement. So we have that each voter’s preference is with probability \( 1/3 \) from the first mainstream, with probability \( 1/3 \) from the second mainstream, and with probability \( 1/3 \) it is a different preference. The main difference to the above-mentioned Polya Eggenberger urn model is that the voters do not influence each other. We do have correlated votes in the sense that with a certain probability voters vote like other voters but there are no direct dependencies between the voters, whereas in the PE model the preference of the first voter influences the preference of the second voter, who in turn influences the preference of the third voter, and so on. Of course, an investigation of control in elections generated in this model could be interesting as well but we postpone this to future work.

Note that for control by adding voters, a second list of votes has to be generated, namely the ballots of the unregistered voters that may be added. In our setting, the list of unregistered voters is of the same size as the list of registered voters and both lists are generated with the same underlying distribution model.

4. A High-Level Description of the Algorithms

All algorithms for the different types of control share the same essential method of testing various subsets, and they differ only in the type of preordering and internal testing.

Before actually searching for a successful sublist of voters or subset of candidates, the algorithms check conditions that would indicate that the given instance is a no-instance. If the tested conditions do not hold the candidates or voters are preordered to ensure that the algorithm tries promising subsets or sublists first. Depending on the control type at hand some of the following conditions are tested.

**Condition 1:** The distinguished candidate is positioned on the last place in every vote, or is positioned on the last place or is disapproved by every voter if \((C, V)\) is a fallback election.

**Condition 2:** For each \( k' \leq k \), determine the smallest \( i \) and \( j \) such that

\[
\text{score}_{(C,V)}(c') \geq \left\lceil \frac{|V|-k'}/2 \right\rceil + 1 + k' \quad \text{and} \quad \text{score}_{(C,V)}(c) \geq \left\lceil \frac{|V|-k'}/2 \right\rceil + 1
\]

hold for \( c' \in C - \{c\} \). It holds that \( i \leq j - 1 \) for all \( k' \leq k \).
Condition 3: For each $k' \leq k$ determine the smallest $i$ and $j$ such that

$$\text{score}_{i(C,V)}(c') \geq \left\lfloor \frac{(\|V\| + k')/2}{2} \right\rfloor + 1$$

and $\text{score}_{i(C,V)}(c) \geq \left\lfloor \frac{(\|V\| + k')/2}{2} \right\rfloor - k'$

hold for $c' \in C - \{c\}$. It holds that $i \leq j - 1$ for all $k' \leq k$.

Condition 4: In the given election, the winner has a strict majority on the first level.

Condition 1 is tested for every constructive control type except the partition of candidates cases (with and without runoff).

In the following sections we will describe the different algorithms for the different types of electoral controls and the implemented preorderings.

4.1. Algorithms for Voter Control

We begin with the algorithms for the voter control cases. For both constructive control by adding and deleting voters, Condition 1 is tested. Note that for the adding voter cases this condition has to hold for both voter lists, the registered voters and the unregistered voters.

For constructive control by deleting voters, Condition 2 is additionally tested. If Condition 2 holds, $c$ cannot be made a unique winner by deleting at most $k$ voters because even if all $k$ voters would harm the strongest rival $c'$ of $c$ the most and $c$ not at all, the rival would still reach a strict majority on a lower level than $c$.

For constructive control by adding voters, Condition 1 and 3 are tested. If Condition 3 holds for the given election and the given distinguished candidate $c$, then even if all added voters helped only $c$ on the lowest level, there would still be at least one other candidate reaching a strict majority on a level lower than $c$.

For the voter-partition cases, we have Condition 4 indicating that control is not possible for both the constructive and destructive case, namely that in the given election there is a unique winner on the first level. It is easy to see that for every possible partition $(V_1, V_2)$ of $V$ a level 1 winner is also a level 1 winner in at least one of the subelections. Since level 1 winners are always unique, independent of the tie-handling model, this candidate always participates in the runoff and will therefore always be the unique level 1 winner of the resulting two-stage election. So no distinguished candidate can ever be made the unique winner by partitioning the voters. So the algorithms for destructive and constructive control by partition of voters first check Condition 4 where the latter checks Condition 1 as well.

After having excluded these trivial cases, the different algorithms search for a successful sublist of $V$ after having ordered the voters.

We will describe this procedure for constructive control by deleting voters where the voters are ordered ascending for $c$. That is, after the preordering $v_1$ is a voter positioning $c$ worst and $v_n$ is a voter positioning $c$ best among all voters. In fallback voting, the “worst position” for a candidate is to be not approved at all. The algorithm now starts with deleting those votes $c$ benefits least of. It follows the procedure of a depth-first search on a tree of height $k$ that is structured as shown in Figure[1].

In each node, it is tested whether deleting the voters on the path is a successful control action. So on the path $s \rightarrow 1 \rightarrow 2 \rightarrow 3$ the algorithm tests the sublists $(v_1)$, $(v_1, v_2)$, $(v_1, v_2, v_3)$ and then tracks back testing the sublists $(v_1, v_2, v_4)$, $(v_1, v_2, v_5)$, $(v_1, v_3)$, $(v_1, v_3, v_4)$, and so on. The branches on the left side are visited first and due to the preordering of the votes, these are the votes $c$ benefits least of.

For the adding-voters case, the voters in the list of unregistered voters are ordered in a descending order for the distinguished candidate and the algorithm proceeds similar to the algorithm for the deleting-voters case, trying to find a successful sublist for the control. With this preordering, the algorithm first tests those voters the distinguished candidate can benefit most from when these are added to the voter list.

For the partition cases the algorithm considers every possible sublist of the voter list up to size $k = \lfloor n/2 \rfloor$ as $V_1$, sets $V_2 = V - V_1$, and tests whether this is a successful control action or not. For the constructive cases the voters are preordered descendingly with respect to the distinguished candidate whereas for the destructive control cases no preordering is implemented.
4.2. Algorithms for Candidate Control

The algorithms for the candidate control scenarios test Condition 1 in the constructive cases except where the candidates are partitioned. For the destructive cases, on the other hand, Condition 4 is always tested. Note that for the adding candidates cases both conditions must hold in the election over both the registered and the unregistered candidates.

After testing for trivial instances the algorithms make use of the same approach of testing systematically preordered candidate subsets to find a successful control action. Here, the candidates are also ordered with respect to the distinguished candidate, where a descending order means that the first candidate has the most voters positioning him or her before the distinguished candidate and the last candidate has the fewest voters doing that. An ascending order is defined analogously. Again, in the adding candidates case, the votes over all candidates (including the unregistered) are considered.

The descending ordering is used for finding control actions for the constructive case of deleting candidates and the destructive cases of adding and partitioning candidates with and without runoff. In the constructive case of the deleting candidates scenario we want to make the distinguished candidate the winner, so the algorithm tries to delete those candidates whose deletion moves the distinguished candidate forward in as many votes as possible. On the contrary the algorithm for destructive control by adding candidates tries to prevent the distinguished candidate from winning, so candidates are added that move the distinguished candidate back in as many votes as possible. The algorithm for both destructive cases of partition of candidates assigns the distinguished candidate to the subset $C_1$ and tries to prevent him or her from moving to the final election. So the other participants in $C_1$ are chosen from the remaining candidates after having ordered them descending with respect to the distinguished candidate.

With analog arguments it is obvious that an ascending order of the candidates with respect to the distinguished candidate is used for the constructive case of deleting candidates, the destructive case of adding candidates, and the constructive case of partition of candidates. Note that the algorithm for constructive control by partition of candidates positions the distinguished candidate in $C_2$ and assigns those other candidates to $C_2$ that are positioned behind the distinguished candidate in as many votes as possible since those are direct rivals for the distinguished candidate in the final election.

Obviously, in the worst-case, the algorithms check all possible subsets of size at most $k$, so they have a worst-case running time of $\sum_{i=1}^{k} \binom{n}{i}$. To handle the worst-case scenarios, a time limit of ten minutes has been

\footnote{Recall that the candidates in $C_2$ participate directly in the final round whereas those candidates in $C_1$ have to compete against each other in a pre-round election.}
implemented such that the algorithms stop when exceeding this limit, indicating by the output that the search process is aborted unsuccessfully. Setting the time limit higher can only increase the number of yes-instances, so again, the results obtained in our experiments give a lower bound for the number of yes-instances in the generated elections. In our experiments we implemented the same timeout value for all investigated types of control. As our results in Section 5 will show, the different control types react differently to this constant timeout threshold, so tuning of the timeout-parameter would be an interesting issue for further experiments. Also, varying the timeout value with respect to the election size at hand might be an interesting approach.

The algorithms and data-generation programs are implemented in Octave 3.2 and the experiments were run on a 2.67 GHz Core-I5 750 with 8GB RAM.

5. Summary of Experimental Results

Tables 2 and 3 summarize our experimental results on control in Bucklin, fallback, and plurality voting. We investigated the three voting systems only for those control types they are not known to be vulnerable to, which is indicated by an R- or an S-entry in Table 1. That is, destructive control by adding and by deleting voters (DCAV and DCDV) are omitted in Tables 2 and 3. Also, since our algorithms use the parameter $k$ bounding the number of candidates to be added, constructive and destructive control by adding an unlimited number of candidates (CCAUC and DCAUC) are not considered either. For each combination of any of the remaining 18 control types, any of the two voting systems Bucklin and fallback voting, and any of the two distribution models (IC and TM), we tested a total of 18,000 = 36 · 500 elections, varying over the 36 data points with different values for $m$ and $n$, as explained above. This gives a total of 1,296,000 = 18 · 4 · 18,000 generated and tested elections. For plurality voting we investigated fourteen types of electoral control leading to 504,000 = 14 · 2 · 18,000 generated and tested elections.

Tables 2 and 3 give an overview of the percentage of timeouts for each such combination of control type/voting system/distribution model, and also the minimal and maximal percentage of yes-instances observed. We do not discuss the results for all these cases in detail here. Rather, we will focus on adding/deleting candidates/voters and partition of voters to very briefly discuss some observations from our experiments, to exemplify some of the numbers in Tables 2 and 3. For those cases that we discuss in detail, we provide plots giving the percentage of yes-instances, timeouts, and average computational costs for all different election sizes that were tested. Note that a comprehensive presentation of all results containing the above information for all cases can be found in the appendix.

Constructive Control by Deleting and by Adding Voters:

We here briefly discuss some results for control by deleting voters only, since those for control by adding voters are very similar, in both Bucklin and fallback voting.

In the IC model, increasing the number of candidates decreases the number of yes-instances in the generated Bucklin elections as can be seen in Figure 2 showing the results for control by deleting voters for Bucklin voting in the IC model. On the other hand, the number of yes-instances increases with the number of voters growing. In the Two Mainstreams model, the same correlations can be observed but here, again, the total number and percentage of yes-instances is smaller than in the IC model.

Fallback voting behaves very similarly, so for both distributions and both voting systems increasing the number of candidates makes successful actions of control by deleting voters less likely.

In both voting systems and in both distribution models, timeouts occur whenever the number of voters exceeds 32. If the number of candidates is 128, we have timeouts already with 16 voters. This can also be seen in the development of the computational costs shown in Figure 2 after the peak for $n = 16$. For bigger electorates the average computational costs drop since the number of timeouts increases with the number of no-instances diminishing.
### Table 2: Overview of experimental results on control in Bucklin and fallback voting. Key: The “min” and “max” columns give the minimal and maximal percentage of yes-instances observed in all tested instances for the given control type, including those elections where timeouts occurred; “to” gives the percentage of timeouts that occurred for the total of 18,000 elections tested in this control case.

| Control Type | IC | TM | IC | TM | IC | TM | IC | TM |
|--------------|----|----|----|----|----|----|----|----|
| CCAC         | 1  | 0  | 11 | 7  | 51 | 50 | 0  | 0  |
| DCAC         | 53 | 39 | 92 | 71 | 11 | 14 | 71 | 42 |
| CCDC         | 13 | 15 | 33 | 36 | 37 | 37 | 13 | 17 |
| DCDC         | 8  | 12 | 78 | 63 | 15 | 22 | 48 | 25 |
| CCPC-TE      | 0  | 0  | 19 | 18 | 62 | 64 | 1  | 0  |
| DCPC-TE      | 8  | 16 | 88 | 65 | 18 | 29 | 49 | 29 |
| CCPC-TP      | 1  | 0  | 17 | 17 | 62 | 64 | 1  | 0  |
| DCPC-TP      | 8  | 16 | 87 | 61 | 18 | 29 | 49 | 29 |
| CCRPC-TE     | 1  | 1  | 18 | 14 | 62 | 63 | 1  | 0  |
| DCRPC-TE     | 8  | 16 | 86 | 68 | 20 | 29 | 46 | 29 |
| CCRPC-TP     | 1  | 0  | 19 | 14 | 62 | 63 | 1  | 1  |
| DCRPC-TP     | 8  | 16 | 85 | 68 | 21 | 29 | 45 | 27 |
| CCAV         | 4  | 1  | 99 | 41 | 13 | 13 | 2  | 1  |
| CDDV         | 2  | 1  | 97 | 39 | 16 | 12 | 2  | 1  |
| CCPC-TE      | 2  | 0  | 97 | 34 | 9  | 45 | 2  | 0  |
| DCPV-TE      | 50 | 34 | 100| 88 | 4  | 16 | 64 | 40 |
| CCRPC-TP     | 1  | 1  | 53 | 20 | 40 | 50 | 1  | 0  |
| DCPV-TP      | 37 | 27 | 100| 87 | 6  | 17 | 60 | 39 |

### Table 3: Overview of experimental results on control in plurality voting. Key: The “min” and “max” columns give the minimal and maximal percentage of yes-instances observed in all tested instances for the given control type, including those elections where timeouts occurred; “to” gives the percentage of timeouts that occurred for the total of 18,000 elections tested in this control case.

| Control Type | IC | TM | IC | TM | IC | TM | IC | TM |
|--------------|----|----|----|----|----|----|----|----|
| CCAC         | 0  | 0  | 20 | 3  | 50 | 34 | 0  | 0  |
| DCAC         | 70 | 47 | 99 | 60 | 7  | 25 | 71 | 42 |
| CCDC         | 5  | 22 | 66 | 40 | 37 | 35 | 13 | 17 |
| DCDC         | 8  | 12 | 78 | 63 | 15 | 22 | 48 | 25 |
| CCPC-TE      | 0  | 0  | 60 | 21 | 58 | 65 | 1  | 0  |
| DCPC-TE      | 1  | 2  | 100| 59 | 22 | 41 | 64 | 40 |
| CCPC-TP      | 0  | 0  | 64 | 24 | 58 | 65 | 1  | 0  |
| DCPC-TP      | 1  | 3  | 100| 59 | 22 | 44 | 60 | 39 |
| CCRPC-TE     | 0  | 0  | 65 | 19 | 50 | 63 | 0  | 0  |
| DCRPC-TE     | 25 | 14 | 100| 61 | 12 | 37 | 60 | 39 |
| CCRPC-TP     | 0  | 0  | 65 | 21 | 50 | 61 | 0  | 0  |
| DCRPC-TP     | 23 | 14 | 100| 61 | 13 | 35 | 60 | 39 |

Table 2: Overview of experimental results on control in Bucklin and fallback voting. Key: The “min” and “max” columns give the minimal and maximal percentage of yes-instances observed in all tested instances for the given control type, including those elections where timeouts occurred; “to” gives the percentage of timeouts that occurred for the total of 18,000 elections tested in this control case.

Table 3: Overview of experimental results on control in plurality voting. Key: The “min” and “max” columns give the minimal and maximal percentage of yes-instances observed in all tested instances for the given control type, including those elections where timeouts occurred; “to” gives the percentage of timeouts that occurred for the total of 18,000 elections tested in this control case.
Control By Partition of Voters:

As mentioned in Section 2, control by partition of voters comes in four problem variants, where each case must be investigated separately.

For constructive control by partition of voters in model TP we made the following observations: Similarly to control by deleting or by adding voters, the number of controllable elections increases with the number of voters increasing. This was observed for all three voting systems investigated. We have seen that in at most 13% of the tested plurality elections in the TM model a successful control action can be found. Note that no timeouts occur for up to 32 candidates, so more than 87% of the elections tested are demonstrably not controllable in these cases. For both distribution models, plurality elections produce fewer timeouts than the corresponding fallback or Bucklin elections. This suggests that the control problem for plurality voting is easier to solve on average than for fallback or Bucklin voting. Using the tie-handling model TE instead of TP, in both Bucklin and fallback voting an increase of yes-instances in the constructive cases is evident. By contrast, in the destructive counterparts no significant difference can be observed with respect to the tie-handling rule used.

The most striking results are those obtained for the destructive cases. Here we have that, for all three voting
systems (and both tie-handling models for fallback and Bucklin voting) in the TM model, the average number of controllable elections is very high; and in the IC model, control is almost always possible, see Figure 3.

In light of the fact that for these cases the resistance proofs of Erdélyi et al. [ER10, EPR11] for fallback and Bucklin voting tend to be the most involved ones (yielding the most complex instances for showing NP-hardness), these results might be surprising at first glance. However, one explanation for the observed results can be found in exactly this fact: The elections constructed in these reductions have a very complex structure which seems to be unlikely to occur in randomly generated elections (at least in elections generated under the distribution models discussed in this paper). Another explanation is that the problems used to reduce from in [ER10, EPR11] tend to be easy to solve for small input sizes, but due to the complexity of the reduction, the resulting elections have many voters/candidates compared to the elections generated for the conducted experiments.

In the destructive cases, the number of timeouts is for all three voting systems the lowest among all control types investigated. In Bucklin elections with uniformly distributed votes and for destructive control by partition of voters in model TP, for only 3.32% of the elections no decision can be made within the time limit. As can be seen in the table, timeouts begin to occur for those elections where the number of voters exceeds 16. But, again, we have to emphasize that these values are very low compared to other types of con-

| m \ n | 4 | 8 | 16 | 32 | 64 | 128 |
|-------|---|---|----|----|----|-----|
| 4     | 0 | 0 | 0  | 18 | 40 | 61  |
| 8     | 0 | 0 | 0  | 3  | 15 | 29  |
| 16    | 0 | 0 | 0  | 1  | 5  | 15  |
| 32    | 0 | 0 | 0  | 0  | 3  | 9   |
| 64    | 0 | 0 | 0  | 0  | 1  | 5   |
| 128   | 0 | 0 | 0  | 0  | 0  | 1   |

Figure 3: Fallback voting in the IC model for DCPV-TP.
trol. This explains the plateaus all graphs show. On the one hand, increasing the number of voters increases the number of yes-instances. But on the other hand, for more than 16 voters timeouts begin to diminish the fraction of observed yes-instances. Also, the average running time of the algorithm for those instances where the time limit is not exceeded is rather low, compared to other types of control, see Figure 3c. The highest computational costs occur for those election sizes where the most no-instances were observed. As expected, in the corresponding constructive cases the number of timeouts is significantly higher and so are the average computational costs.

Control By Adding Candidates:

(a) Percentage of yes-instances.

(b) Percentage of timeouts.

(c) Average time the algorithm needs to give a definite output, instances where timeouts occur are excluded.

Figure 4: Fallback voting in the TM model for CCAC.

So far, the results for constructive control by adding candidates show the highest number of timeouts. For those election sizes where no timeouts occur (i.e., where the determination of yes- or no-instances is successful), we have that not many elections can be controlled successfully in either of the two voting systems. In Figure 4, we see the results for fallback voting in the TM model, exemplifying in Figure 4c the low number of yes-instances for this type of control. For example, in the “max” column in Table 2, the highest percentage of controllable fallback elections is 11% in the IC model and only 7% in the TM model. Plurality voting
shows similar results as fallback voting with at most 20% yes-instances in the IC model and less than 4% in the TM model.

Figure 4b gives the detailed occurrence of timeouts for the different election sizes and Figure 4c shows the average time needed to determine whether a given fallback election generated under the TM model can be controlled by adding candidates or not. Remember that in the latter figure the average values do not consider those elections where the algorithm exceeded the time limit of 600 seconds. Together with the timeout table we can see in Figure 4a that in elections with up to 16 candidates the number of non-controllable elections is very high and increases with the number of candidates increasing. When more than 16 candidates participate in an election the number of no-instances diminishes as drastically as the timeout rate grows. Looking at the computational costs in Figure 4c we can see this in the peaks for $m = 16$. Since by design our algorithm needs generally more time to determine that an instance is a no-instance than finding a yes-instance, the high number of no-instances for 16 candidates inflates the average computing time. Knowing that the average computing time for finding yes-instances is not particularly high for this type of control (see Figure 12 in the appendix) we might conjecture that for the bigger election sizes the instances where no distinction could be made by our algorithm are no-instances rather than yes-instances. This indicates that this control type presumably has the lowest overall number of yes-instances. Thus, even for small election sizes, this type of control seems to be hard to exert successfully.

Turning now to the destructive variant of control by adding candidates, for Bucklin elections generated with the IC model, 71% is a lower bound for the number of controllable elections. For up to 16 candidates, a successful control action can be found in nearly all elections. The results for the TM model reconfirm the observation made before, namely that the tendencies in both models are similar, but with at most 77% and at least 42% of controllable elections the overall numbers are again lower than in the IC model. The latter results hold for fallback elections generated in the TM model as well, whereas in the IC model at least 53% and no more than 92% of the fallback elections are controllable. In the tested plurality elections generated with the IC model, similarly to Bucklin voting, more than 70% and nearly up to 100% are controllable. In the TM model, roughly between 50% and 60% of yes-instances are found for those election sizes where no timeouts occur, so between 40% and 50% of these plurality elections are not controllable. In this control scenario, for about 46% of the elections no definite output is given in the constructive case, whereas in only about 8% of the elections timeouts occur in the destructive case.

6. Discussion and Conclusions

We have empirically studied the complexity of NP-hard control problems for plurality, fallback, and Bucklin voting in the most important of the common control scenarios. This is the first such study for control problems in voting and complements the corresponding results [Wal10, Wal09, DKNW10, DKNW11, NWX11, CT07] for manipulating elections. In general, our findings indicate that control can often be exerted effectively in practice, despite the NP-hardness of the corresponding problems. Our experiments also allow a more fine-grained analysis and comparison—across various control scenarios, vote distribution models, and voting systems—than merely stating NP-hardness for all these problems. Our experimental results allow a more fine-grained analysis and comparison—across various control scenarios, vote distribution models, and voting systems—than merely stating NP-hardness for all these problems. Tables 2 and 3 give an overview of our experimental results. A detailed analysis and discussion follows here, and a comprehensive presentation of all experimental data and results can be found in the appendix.

IC versus TM:

Comparing the results for the different distribution models, we see that in every voting system for all control types studied (except fallback voting in constructive control by deleting candidates) the overall number of yes-instances is higher in the IC than in the TM model. This may result from the fact that in elections with uniformly distributed votes, all candidates are likely to be approximately equally preferred by the voters. So both constructive and destructive control actions are easier to find. This also explains the observation that the IC model mostly produces fewer timeouts.
Constructive versus Destructive Control:

For all investigated types of control where both constructive and destructive control was investigated, we found that the destructive control types are experimentally much easier than their constructive counterparts, culminating in almost 100% of controllable elections for certain control types in the IC model. These findings confirm—and strengthen—the theoretical insight of Hemaspaandra et al. [HHR07] that the destructive control problems disjunctively truth-table-reduce to their constructive counterparts and thus are never harder to solve, up to a polynomial factor (see also the corresponding observation of Conitzer et al. [CSL07] regarding manipulation): In fact, destructive control tends to be even much easier than constructive control.

Comparison Across Voting Systems:

For constructive control, we have seen that fallback and Bucklin voting show similar tendencies and numbers of yes-instances, especially regarding voter control. We also observed that their constructive voter control problems are in general harder to solve than those for plurality voting. In all three voting systems, constructive control by adding candidates seems to be one of the hardest control problems showing the smallest numbers of yes-instances in the TM model. Only for the constructive partition-of-candidates cases higher numbers of timeouts were observed.

Adding Candidates versus Deleting Candidates:

Comparing control by adding candidates to control by deleting candidates in the constructive case we observed that the number of yes-instances for control by deleting candidates is significantly higher. These findings are perhaps not overly surprising, since in the voting systems considered here adding candidates to an election can only worsen the position of the designated candidate in the votes. That is, constructive control can be exerted successfully only if by adding candidates rivals of the designated candidate lose enough points so as to get defeated by him or her. This, in turn, can happen only if the designated candidate was already a highly preferred candidate in the original election.

Voter Control versus Candidate Control:

For fallback and Bucklin voting, we can also compare constructive candidate and voter control directly. In both voting systems and both distribution models, the number of yes-instances for constructive control by adding voters is around four times higher than the number of yes-instances in the corresponding candidate control type, which confirms the argument above, saying that adding candidates cannot push the designated candidate directly. Constructive control by deleting voters can be successfully exerted more frequently when votes are less correlated, whereas the proportion of successful control actions for deleting candidates is about the same for both considered distribution models. The observed differences between these types of voter and candidate control may result from the fact that adding or deleting candidates only shifts the position of the designated candidate, which may not influence the outcome of the election as directly as increasing or decreasing the candidates’ scores by adding or deleting voters does. This explains why voter control can be tackled more easily than candidate control by greedy approaches such as ours.

Concluding Remarks:

Just as Walsh [Wal09, Wal10] observes for manipulation in the veto rule and in STV, for all types of control investigated in our experiments, the curves do not show the typical phase transition known for “really hard” computational problems such as the satisfiability problem (see [GW95, CKT91] for a detailed discussion of this issue).

These observations raise the question of how other distribution models influence the outcome of such experiments. Furthermore, the algorithms implemented could be improved in terms of considering a higher
number of elections per data point, increasing the election sizes, or allowing a higher number of voters or candidates to be deleted or added in the corresponding control scenarios. Besides this, other voting systems can be analyzed since only their winner determination has to be implemented in addition to a few minor adjustments such as trivial-case checks for the investigated control scenarios tailored to the voting system at hand.

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A. Fallback Voting

A.1. Constructive Control by Adding Candidates

Figure 5: Results for fallback voting in the IC model for constructive control by adding candidates. Number of candidates is fixed.

|      | $m = 4$               | $m = 8$               | $m = 16$               | $m = 32$               | $m = 64$               | $m = 128$               |
|------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
|      | $n = 4$               | $4$                   | $8$                   | $16$                  | $32$                  | $64$                  | $128$                  |
|      | $n = 8$               | $13$                  | $15$                  | $27$                  | $26$                  | $22$                  | $20$                   |
|      | $n = 16$              | $16$                  | $31$                  | $29$                  | $35$                  | $33$                  | $34$                   |
|      | $n = 32$              | $487$                 | $485$                 | $473$                 | $474$                 | $478$                 | $480$                  |
|      | $n = 64$              | $484$                 | $469$                 | $471$                 | $465$                 | $467$                 | $466$                  |
|      | $n = 128$             | $0$                   | $0$                   | $0$                   | $0$                   | $0$                   | $0$                    |
|      | $m = 16$              | $4$                   | $8$                   | $16$                  | $32$                  | $64$                  | $128$                  |
|      | $m = 32$              | $16$                  | $27$                  | $51$                  | $52$                  | $40$                  | $40$                   |
|      | $m = 64$              | $484$                 | $473$                 | $449$                 | $448$                 | $460$                 | $0$                    |
|      | $m = 128$             | $0$                   | $0$                   | $0$                   | $0$                   | $0$                   | $460$                  |
|      | $n = 64$              | $453$                 | $477$                 | $485$                 | $487$                 | $483$                 | $483$                  |
|      | $n = 128$             | $466$                 | $495$                 | $495$                 | $494$                 | $497$                 | $496$                  |
|      | $m = 128$             | $472$                 | $497$                 | $497$                 | $499$                 | $497$                 | $499$                  |
|      | $m = 128$             | $21$                  |                      |                      |                      |                      |                        |
Figure 6: Results for fallback voting in the IC model for constructive control by adding candidates. Number of voters is fixed.

| m   | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| # cp | 13 | 16 | 16 | 11 | 6  | 2   |
| # ci  | 487 | 484 | 484 | 36 | 28 | 26  |
| # to  | 0  | 0  | 453 | 466 | 472 |
| n = 16 | 15 | 31 | 27 | 20 | 5  | 1   |
| # cp  | 27 | 29 | 51 | 15 | 5  | 3   |
| # ci  | 473 | 471 | 449 | 0  | 0  | 0   |
| # to  | 0  | 0  | 485 | 495 | 497 |
| n = 32 | 485 | 469 | 473 | 3  | 0  | 3   |
| # cp  | 22 | 33 | 40 | 17 | 3  | 3   |
| # ci  | 478 | 467 | 460 | 0  | 0  | 0   |
| # to  | 0  | 0  | 483 | 497 | 497 |
| n = 64 | 480 | 466 | 0  | 0  | 0   |
| # cp  | 20 | 34 | 40 | 17 | 4  | 1   |
| # ci  | 478 | 467 | 460 | 0  | 0  | 0   |
| # to  | 0  | 0  | 483 | 497 | 497 |
| n = 128 | 480 | 466 | 0  | 0  | 0   |

| m   | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| # cp | 13 | 16 | 16 | 11 | 6  | 2   |
| # ci  | 487 | 484 | 484 | 36 | 28 | 26  |
| # to  | 0  | 0  | 453 | 466 | 472 |
| n = 16 | 15 | 31 | 27 | 20 | 5  | 1   |
| # cp  | 27 | 29 | 51 | 15 | 5  | 3   |
| # ci  | 473 | 471 | 449 | 0  | 0  | 0   |
| # to  | 0  | 0  | 485 | 495 | 497 |
| n = 32 | 485 | 469 | 473 | 3  | 0  | 3   |
| # cp  | 22 | 33 | 40 | 17 | 3  | 3   |
| # ci  | 478 | 467 | 460 | 0  | 0  | 0   |
| # to  | 0  | 0  | 483 | 497 | 497 |
| n = 64 | 480 | 466 | 0  | 0  | 0   |
| # cp  | 20 | 34 | 40 | 17 | 4  | 1   |
| # ci  | 478 | 467 | 460 | 0  | 0  | 0   |
| # to  | 0  | 0  | 483 | 497 | 497 |
| n = 128 | 480 | 466 | 0  | 0  | 0   |
Figure 7: Results for fallback voting in the TM model for constructive control by adding candidates. Number of candidates is fixed.

|       | $m=4$ |       | $m=8$ |
|-------|-------|-------|-------|
| $n$   | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 14 12 17 21 31 25 | 9 19 13 25 27 23 |
| # ci  | 486 488 483 479 469 475 | 491 481 487 475 473 477 |
| # to  | 0 0 0 0 0 0 | 0 0 0 0 0 0 |

|       | $m=16$ |       | $m=32$ |
|-------|-------|-------|-------|
| $n$   | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 11 17 21 9 13 17 | 5 9 5 6 7 8 |
| # ci  | 489 483 479 491 487 0 | 82 26 6 0 0 0 |
| # to  | 0 0 0 0 0 0 | 413 465 489 494 493 492 |

|       | $m=64$ |       | $m=128$ |
|-------|-------|-------|--------|
| $n$   | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 1 5 1 4 0 1 | 1 0 2 0 0 0 |
| # ci  | 92 33 5 0 0 0 | 95 36 10 0 0 0 |
| # to  | 407 462 494 496 500 499 | 404 464 488 500 500 500 |
Figure 8: Results for fallback voting in the TM model for constructive control by adding candidates. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |   |
|---|---------|---|---------|---|
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 14 | 9 | 11 | 5 | 1 | 1 | 12 | 19 | 17 | 9 | 5 | 0 |
| # ci | 486 | 491 | 489 | 82 | 92 | 95 | 488 | 481 | 483 | 26 | 33 | 36 |
| # to | 0 | 0 | 0 | 413 | 407 | 404 | 0 | 0 | 0 | 465 | 462 | 464 |

|   | $n = 16$ |   | $n = 32$ |   |
|---|---------|---|---------|---|
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 17 | 13 | 21 | 5 | 1 | 2 | 21 | 25 | 9 | 6 | 4 | 0 |
| # ci | 483 | 487 | 479 | 6 | 5 | 10 | 479 | 475 | 491 | 0 | 0 | 0 |
| # to | 0 | 0 | 0 | 489 | 494 | 488 | 0 | 0 | 0 | 494 | 496 | 500 |

|   | $n = 64$ |   | $n = 128$ |   |
|---|---------|---|---------|---|
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 31 | 27 | 13 | 7 | 0 | 0 | 25 | 23 | 17 | 8 | 1 | 0 |
| # ci | 469 | 473 | 487 | 0 | 0 | 0 | 475 | 477 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 0 | 493 | 500 | 500 | 0 | 0 | 483 | 492 | 499 | 500 |
A.1.1. Computation Costs

Figure 9: Average time the algorithm needs to find a successful control action for constructive control by adding candidates in fallback elections in the IC model. The maximum is 194.7 seconds.

Figure 10: Average time the algorithm needs to determine no-instance of constructive control by adding candidates in fallback elections in the IC model. The maximum is 459.54 seconds.
Figure 11: Average time the algorithm needs to give a definite output for constructive control by adding candidates in fallback elections in the IC model. The maximum is 425.43 seconds.

Figure 12: Average time the algorithm needs to find a successful control action for constructive control by adding candidates in fallback elections in the TM model. The maximum is 159.22 seconds.
Figure 13: Average time the algorithm needs to determine no-instance of constructive control by adding candidates in fallback elections in the TM model. The maximum is 451.05 seconds.

Figure 14: Average time the algorithm needs to give a definite output for constructive control by adding candidates in fallback elections in the TM model. The maximum is 439.68 seconds.
A.2. Destructive Control by Adding Candidates

Figure 15: Results for fallback voting in the IC model for destructive control by adding candidates. Number of candidates is fixed.

|   | \( m = 4 \) |   | \( m = 8 \) |
|---|---|---|---|
| \( n \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 266 | 302 | 309 | 301 | 304 | 299 | 335 | 363 | 375 | 389 | 404 | 368 |
| \# ci | 234 | 198 | 191 | 199 | 196 | 201 | 165 | 137 | 125 | 111 | 96 | 132 |
| \# to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( m = 16 \) |   |   |   |   |   |   |   |   |   |   |   |   |
| \( n \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 356 | 418 | 433 | 456 | 452 | 459 | 372 | 414 | 418 | 434 | 442 | 423 |
| \# ci | 144 | 82 | 67 | 44 | 48 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# to | 0 | 0 | 0 | 0 | 0 | 41 | 128 | 86 | 82 | 66 | 58 | 77 |
| \( m = 32 \) |   |   |   |   |   |   |   |   |   |   |   |   |
| \( n \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 376 | 373 | 394 | 383 | 403 | 388 | 362 | 373 | 361 | 367 | 375 | 376 |
| \# ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# to | 124 | 127 | 106 | 117 | 97 | 112 | 138 | 127 | 139 | 133 | 125 | 124 |
Figure 16: Results for fallback voting in the IC model for destructive control by adding candidates. Number of voters is fixed.

| $m$ | $n = 4$ | $n = 8$ |
|-----|---------|---------|
| $n = 16$ | $n = 32$ | $n = 64$ | $n = 128$ |
| $n = 32$ | $n = 64$ | $n = 128$ |
| $n = 64$ | $n = 128$ |

| $m$ | $n = 4$ | $n = 8$ |
|-----|---------|---------|
| $n = 16$ | $n = 32$ | $n = 64$ | $n = 128$ |
| $n = 32$ | $n = 64$ | $n = 128$ |
| $n = 64$ | $n = 128$ |

| $m$ | $n = 4$ | $n = 8$ |
|-----|---------|---------|
| $n = 16$ | $n = 32$ | $n = 64$ | $n = 128$ |
| $n = 32$ | $n = 64$ | $n = 128$ |
| $n = 64$ | $n = 128$ |
Figure 17: Results for fallback voting in the TM model for destructive control by adding candidates. Number of candidates is fixed.
Figure 18: Results for fallback voting in the TM model for destructive control by adding candidates. Number of voters is fixed.

| m   | 4   | 8   | 16  | 32  | 64  | 128 | m   | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp| 194 | 212 | 266 | 255 | 272 | 267 | 211 | 257 | 290 | 296 | 280 | 284 |     |
| # ci| 306 | 288 | 234 | 123 | 118 | 135 | 289 | 243 | 210 | 99  | 108 | 105 |     |
| # to| 0   | 0   | 0   | 122 | 110 | 98  | 0   | 0   | 0   | 105 | 112 | 111 |     |

| m   | 4   | 8   | 16  | 32  | 64  | 128 | m   | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp| 261 | 296 | 323 | 304 | 322 | 338 | 254 | 315 | 352 | 329 | 334 | 353 |     |
| # ci| 239 | 204 | 177 | 62  | 58  | 58  | 246 | 185 | 148 | 27  | 19  | 20  |     |
| # to| 0   | 0   | 0   | 134 | 120 | 104 | 0   | 0   | 0   | 144 | 147 | 127 |     |

| m   | 4   | 8   | 16  | 32  | 64  | 128 | m   | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp| 276 | 305 | 352 | 348 | 342 | 337 | 266 | 319 | 355 | 350 | 335 | 336 |     |
| # ci| 224 | 195 | 148 | 5   | 4   | 6   | 234 | 181 | 13  | 7   | 6   | 0   |     |
| # to| 0   | 0   | 0   | 147 | 154 | 157 | 0   | 0   | 132 | 143 | 159 | 164 |     |
A.2.1. Computational Costs

Figure 19: Average time the algorithm needs to find a successful control action for destructive control by adding candidates in fallback elections in the IC model. The maximum is 18,53 seconds.

Figure 20: Average time the algorithm needs to determine no-instance of destructive control by adding candidates in fallback elections in the IC model. The maximum is 453, 8 seconds.
Figure 21: Average time the algorithm needs to give a definite output for destructive control by adding candidates in fallback elections in the IC model. The maximum is 46.48 seconds.

Figure 22: Average time the algorithm needs to find a successful control action for destructive control by adding candidates in fallback elections in the TM model. The maximum is 11.33 seconds.
Figure 23: Average time the algorithm needs to determine no-instance of destructive control by adding candidates in fallback elections in the TM model. The maximum is 366.67 seconds.

Figure 24: Average time the algorithm needs to give a definite output for destructive control by adding candidates in fallback elections in the TM model. The maximum is 113.63 seconds.
A.3. Constructive Control by Deleting Candidates

Figure 25: Results for fallback voting in the IC model for constructive control by deleting candidates. Number of candidates is fixed.

|     | $m = 4$ |     | $m = 8$ |     | $m = 16$ |     | $m = 32$ |     | $m = 64$ |     | $m = 128$ |
|-----|---------|-----|---------|-----|---------|-----|---------|-----|---------|-----|---------|
| $n$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 115 129 131 140 153 159 | 98 103 154 114 121 146 | 98 103 154 114 121 146 | 98 103 154 114 121 146 | 98 103 154 114 121 146 | 98 103 154 114 121 146 |
| # ci | 385 371 369 360 347 341 | 402 397 346 386 379 354 | 402 397 346 386 379 354 | 402 397 346 386 379 354 | 402 397 346 386 379 354 | 402 397 346 386 379 354 |
| # to | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |

|     | $m = 16$ |     | $m = 32$ |     | $m = 64$ |     | $m = 128$ |
|-----|---------|-----|---------|-----|---------|-----|---------|
| $n$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 106 133 158 167 166 152 | 109 129 144 123 139 148 | 109 129 144 123 139 148 | 109 129 144 123 139 148 | 109 129 144 123 139 148 | 109 129 144 123 139 148 |
| # ci | 394 367 342 333 334 348 | 33 2 0 0 0 0 | 33 2 0 0 0 0 | 33 2 0 0 0 0 | 33 2 0 0 0 0 | 33 2 0 0 0 0 |
| # to | 0 0 0 0 0 0 | 358 369 356 377 361 352 | 358 369 356 377 361 352 | 358 369 356 377 361 352 | 358 369 356 377 361 352 | 358 369 356 377 361 352 |

|     | $m = 64$ |     | $m = 128$ |
|-----|---------|-----|---------|
| $n$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 86 85 99 103 110 90 | 71 76 78 63 74 63 | 71 76 78 63 74 63 |
| # ci | 29 1 0 0 0 0 | 0 28 30 23 0 0 | 0 28 30 23 0 0 |
| # to | 385 414 401 397 390 410 | 429 396 392 414 426 437 | 429 396 392 414 426 437 |
Figure 26: Results for fallback voting in the IC model for constructive control by deleting candidates. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |   | $n = 16$ |   | $n = 32$ |   | $n = 64$ |   | $n = 128$ |
|---|---------|---|---------|---|----------|---|----------|---|----------|---|----------|
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 115 | 98 | 106 | 109 | 86 | 71 | 129 | 103 | 133 | 129 | 85 | 76 |
| # ci | 385 | 402 | 394 | 33 | 29 | 0 | 371 | 397 | 367 | 2 | 1 | 28 |
| # to | 0 | 0 | 0 | 358 | 385 | 429 | 0 | 0 | 0 | 369 | 414 | 396 |
|   | $n = 4$ |   | $n = 8$ |   | $n = 16$ |   | $n = 32$ |   | $n = 64$ |   | $n = 128$ |
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 131 | 154 | 158 | 144 | 99 | 78 | 140 | 114 | 167 | 123 | 103 | 63 |
| # ci | 369 | 346 | 342 | 0 | 0 | 30 | 360 | 386 | 333 | 0 | 0 | 23 |
| # to | 0 | 0 | 0 | 356 | 401 | 392 | 0 | 0 | 0 | 377 | 397 | 414 |
|   | $n = 4$ |   | $n = 8$ |   | $n = 16$ |   | $n = 32$ |   | $n = 64$ |   | $n = 128$ |
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 153 | 121 | 166 | 139 | 110 | 74 | 159 | 146 | 152 | 148 | 90 | 63 |
| # ci | 347 | 379 | 334 | 0 | 0 | 0 | 341 | 354 | 348 | 0 | 0 | 0 |
| # to | 0 | 0 | 0 | 361 | 390 | 426 | 0 | 0 | 0 | 352 | 410 | 437 |
Figure 27: Results for fallback voting in the TM model for constructive control by deleting candidates. Number of candidates is fixed.

|     | \(m = 4\)                  | \(m = 8\)                  |
|-----|-----------------------------|-----------------------------|
| \(n\) | 4 8 16 32 64 128           | 4 8 16 32 64 128           |
| \# cp | 116 138 162 145 161 176    | 104 132 134 134 153 140    |
| \# ci | 384 362 338 355 339 324    | 396 368 366 347 347 360    |
| \# to | 0 0 0 0 0 0                | 0 0 0 0 0 0                |
| \(m = 16\) |                       |                            |
| \# cp | 137 146 166 180 169 174    | 119 137 153 137 130 140    |
| \# ci | 363 354 334 320 331 326    | 66 29 0 0 0 0              |
| \# to | 0 0 0 0 0 0                | 315 334 347 363 370 360    |
| \(m = 32\) |                       |                            |
| \# cp | 106 96 124 102 105 109     | 82 106 86 77 91 89         |
| \# ci | 76 33 10 0 0 0             | 77 0 0 0 0 0               |
| \# to | 318 371 366 398 395 391    | 341 394 414 423 409 411    |
| \(m = 64\) |                       |                            |
| \# cp |                           |                            |
| \# ci |                           |                            |
| \# to |                           |                            |
| \(m = 128\) |                      |                            |
| \# cp |                           |                            |
| \# ci |                           |                            |
| \# to |                           |                            |

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Figure 28: Results for fallback voting in the TM model for constructive control by deleting candidates. Number of voters is fixed.

| $n$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 116 | 104 | 137 | 119 | 106 | 82  |
| # ci | 384 | 396 | 363 | 66  | 76  | 77  |
| # to | 0   | 0   | 0   | 315 | 318 | 341 |

| $n$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 162 | 132 | 166 | 153 | 124 | 86  |
| # ci | 338 | 368 | 334 | 0   | 0   | 0   |
| # to | 0   | 0   | 0   | 347 | 366 | 414 |

| $n$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 161 | 153 | 169 | 130 | 105 | 91  |
| # ci | 339 | 347 | 331 | 0   | 0   | 0   |
| # to | 0   | 0   | 0   | 370 | 395 | 409 |

| $n$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 161 | 153 | 169 | 130 | 105 | 91  |
| # ci | 339 | 347 | 331 | 0   | 0   | 0   |
| # to | 0   | 0   | 0   | 370 | 395 | 409 |
A.3.1. Computation Costs

Figure 29: Average time the algorithm needs to find a successful control action for constructive control by deleting candidates in fallback elections in the IC model. The maximum is 64.05 seconds.

Figure 30: Average time the algorithm needs to determine no-instance of constructive control by deleting candidates in fallback elections in the IC model. The maximum is 356.18 seconds.
Figure 31: Average time the algorithm needs to give a definite output for constructive control by deleting candidates in fallback elections in the IC model. The maximum is 250.39 seconds.

Figure 32: Average time the algorithm needs to find a successful control action for constructive control by deleting candidates in fallback elections in the TM model. The maximum is 49.05 seconds.
Figure 33: Average time the algorithm needs to determine no-instance of constructive control by deleting candidates in fallback elections in the TM model. The maximum is 347.06 seconds.

Figure 34: Average time the algorithm needs to give a definite output for constructive control by deleting candidates in fallback elections in the TM model. The maximum is 227.76 seconds.
A.4. Destructive Control by Deleting Candidates

Figure 35: Results for fallback voting in the IC model for destructive control by deleting candidates. Number of candidates is fixed.

|   | m = 4 |   | m = 8 |
|---|-------|---|-------|
| n | 4     | 8 | 16    | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 84 | 97 | 107 | 75 | 68 | 39 | 181 | 228 | 257 | 248 | 250 | 247 |
| # ci | 416 | 403 | 393 | 425 | 432 | 461 | 319 | 272 | 243 | 252 | 250 | 253 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| m = 16 | | | | | | | | | | | | |
| n | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 303 | 387 | 389 | 99 | 411 | 401 | 336 | 364 | 415 | 406 | 433 | 406 |
| # ci | 197 | 113 | 111 | 99 | 89 | 99 | 1 | 0 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| m = 32 | | | | | | | | | | | | |
| n | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 271 | 335 | 361 | 371 | 377 | 371 | 243 | 274 | 306 | 319 | 315 | 344 |
| # ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 229 | 165 | 139 | 129 | 123 | 129 | 257 | 226 | 194 | 181 | 185 | 156 |

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Figure 36: Results for fallback voting in the IC model for destructive control by deleting candidates. Number of voters is fixed.

| m  | n = 4 | | n = 8 | | n = 16 | | n = 32 | | n = 64 | | n = 128 |
|-----|-------| |-------| |-------| |-------| |-------| |-------|
|     | 4     | 8    | 16   | 32    | 64    | 128   | 4     | 8    | 16   | 32    | 64    | 128   |
| # cp | 84    | 181  | 303  | 336   | 271   | 243   | 97    | 228  | 387  | 364   | 335   | 274   |
| # ci | 416   | 319  | 197  | 1     | 607   | 0     | 403   | 272  | 113  | 0     | 0     | 0     |
| # to | 0     | 0    | 0    | 0     | 0     | 0     | 0     | 0    | 0    | 0     | 0     | 0     |
| n = 16 | | | | | | | | | | | | |
| m  | 4     | 8    | 16   | 32    | 64    | 128   | 4     | 8    | 16   | 32    | 64    | 128   |
| # cp | 107   | 257  | 389  | 415   | 361   | 306   | 75    | 248  | 401  | 405   | 371   | 319   |
| # ci | 393   | 243  | 111  | 0     | 0     | 0     | 425   | 252  | 99   | 0     | 0     | 0     |
| # to | 0     | 0    | 0    | 0     | 0     | 0     | 0     | 0    | 0    | 0     | 0     | 0     |
| n = 32 | | | | | | | | | | | | |
| m  | 4     | 8    | 16   | 32    | 64    | 128   | 4     | 8    | 16   | 32    | 64    | 128   |
| # cp | 68    | 250  | 411  | 433   | 377   | 315   | 39    | 247  | 401  | 406   | 371   | 344   |
| # ci | 432   | 250  | 89   | 0     | 0     | 0     | 461   | 253  | 99   | 0     | 0     | 0     |
| # to | 0     | 0    | 0    | 0     | 0     | 0     | 0     | 0    | 0    | 0     | 0     | 0     |
| n = 64 | | | | | | | | | | | | |
| m  | 4     | 8    | 16   | 32    | 64    | 128   | 4     | 8    | 16   | 32    | 64    | 128   |
| # cp | 84    | 181  | 303  | 336   | 271   | 243   | 97    | 228  | 387  | 364   | 335   | 274   |
| # ci | 416   | 319  | 197  | 1     | 607   | 0     | 403   | 272  | 113  | 0     | 0     | 0     |
| # to | 0     | 0    | 0    | 0     | 0     | 0     | 0     | 0    | 0    | 0     | 0     | 0     |
| n = 128 | | | | | | | | | | | | |

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Figure 37: Results for fallback voting in the TM model for destructive control by deleting candidates. Number of candidates is fixed.

|   | \(m = 4\) |                   | \(m = 8\) |                   |
|---|-----------|------------------|-----------|------------------|
|   | \(n\)    | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) |
|   | \# cp     | 60 69 67 85 120 116 | 127 130 163 180 209 193 | 60 69 67 85 120 116 | 127 130 163 180 209 193 |
|   | \# ci     | 440 431 433 415 380 384 | 373 370 337 320 291 307 | 440 431 433 415 380 384 | 373 370 337 320 291 307 |
|   | \# to     | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
|   | \(m = 16\) |         |                   | \(m = 32\) |                   |
|   | \# cp     | 189 252 276 286 305 300 | 238 302 293 305 316 254 | 189 252 276 286 305 300 | 238 302 293 305 316 254 |
|   | \# ci     | 311 248 224 214 195 200 | 153 104 81 34 21 15 | 311 248 224 214 195 200 | 153 104 81 34 21 15 |
|   | \# to     | 0 0 0 0 0 0 | 109 94 126 161 163 231 | 0 0 0 0 0 0 | 109 94 126 161 163 231 |
|   | \(m = 64\) |         |                   | \(m = 128\) |                   |
|   | \# cp     | 221 221 222 26 207 191 | 147 183 193 205 174 142 | 221 221 222 26 207 191 | 147 183 193 205 174 142 |
|   | \# ci     | 110 96 69 23 8 6 | 123 101 66 18 7 7 | 110 96 69 23 8 6 | 123 101 66 18 7 7 |
|   | \# to     | 169 183 209 231 285 303 | 230 216 241 277 319 351 | 169 183 209 231 285 303 | 230 216 241 277 319 351 |
Figure 38: Results for fallback voting in the TM model for destructive control by deleting candidates. Number of voters is fixed.

| m   | n = 4 | n = 8 |
|-----|-------|-------|
|     | cp    | ci    | to    | cp    | ci    | to    |
| 4   | 60    | 440   | 0     | 69    | 431   | 0     |
| 8   | 127   | 373   | 0     | 130   | 370   | 0     |
| 16  | 189   | 311   | 0     | 221   | 110   | 0     |
| 32  | 238   | 153   | 0     | 147   | 123   | 0     |
| 64  | 221   | 110   | 0     | 221   | 96    | 0     |
| 128 | 183   | 96    | 0     | 183   | 101   | 0     |

| m   | n = 16 | n = 32 |
|-----|--------|--------|
|     | cp    | ci    | to    | cp    | ci    | to    |
| 4   | 67    | 433   | 0     | 85    | 415   | 0     |
| 8   | 163   | 337   | 0     | 180   | 320   | 0     |
| 16  | 276   | 224   | 0     | 286   | 214   | 0     |
| 32  | 293   | 81    | 0     | 305   | 34    | 0     |
| 64  | 222   | 69    | 0     | 246   | 23    | 0     |
| 128 | 193   | 66    | 0     | 205   | 18    | 0     |

| m   | n = 64 | n = 128 |
|-----|--------|---------|
|     | cp    | ci    | to    | cp    | ci    | to    |
| 4   | 120   | 380   | 0     | 116   | 384   | 0     |
| 8   | 209   | 291   | 0     | 193   | 307   | 0     |
| 16  | 305   | 195   | 0     | 300   | 200   | 0     |
| 32  | 316   | 21    | 0     | 254   | 15    | 0     |
| 64  | 207   | 8     | 0     | 191   | 6     | 0     |
| 128 | 174   | 7     | 0     | 142   | 0     | 0     |

| m   | n = 4 | n = 8 |
|-----|-------|-------|
|     | cp    | ci    | to    | cp    | ci    | to    |
| 4   | 60    | 440   | 0     | 69    | 431   | 0     |
| 8   | 127   | 373   | 0     | 130   | 370   | 0     |
| 16  | 189   | 311   | 0     | 221   | 110   | 0     |
| 32  | 238   | 153   | 0     | 147   | 123   | 0     |
| 64  | 221   | 110   | 0     | 221   | 96    | 0     |
| 128 | 183   | 96    | 0     | 183   | 101   | 0     |

| m   | n = 16 | n = 32 |
|-----|--------|--------|
|     | cp    | ci    | to    | cp    | ci    | to    |
| 4   | 67    | 433   | 0     | 85    | 415   | 0     |
| 8   | 163   | 337   | 0     | 180   | 320   | 0     |
| 16  | 276   | 224   | 0     | 286   | 214   | 0     |
| 32  | 293   | 81    | 0     | 305   | 34    | 0     |
| 64  | 222   | 69    | 0     | 246   | 23    | 0     |
| 128 | 193   | 66    | 0     | 205   | 18    | 0     |

| m   | n = 64 | n = 128 |
|-----|--------|---------|
|     | cp    | ci    | to    | cp    | ci    | to    |
| 4   | 120   | 380   | 0     | 116   | 384   | 0     |
| 8   | 209   | 291   | 0     | 193   | 307   | 0     |
| 16  | 305   | 195   | 0     | 300   | 200   | 0     |
| 32  | 316   | 21    | 0     | 254   | 15    | 0     |
| 64  | 207   | 8     | 0     | 191   | 6     | 0     |
| 128 | 174   | 7     | 0     | 142   | 0     | 0     |
A.4.1. Computational Costs

Figure 39: Average time the algorithm needs to find a successful control action for destructive control by deleting candidates in fallback elections in the IC model. The maximum is 29.04 seconds.

Figure 40: Average time the algorithm needs to determine no-instance of destructive control by deleting candidates in fallback elections in the IC model. The maximum is 345.94 seconds.
Figure 41: Average time the algorithm needs to give a definite output for destructive control by deleting candidates in fallback elections in the IC model. The maximum is 75.5 seconds.

Figure 42: Average time the algorithm needs to find a successful control action for destructive control by deleting candidates in fallback elections in the TM model. The maximum is 63.71 seconds.
Figure 43: Average time the algorithm needs to determine no-instance of destructive control by deleting candidates in fallback elections in the TM model. The maximum is 290.26 seconds.

Figure 44: Average time the algorithm needs to give a definite output for destructive control by deleting candidates in fallback elections in the TM model. The maximum is 122.72 seconds.
### A.5. Constructive Control by Partition of Candidates in Model TE

Figure 45: Results for fallback voting in the IC model for constructive control by partition of candidates in model TE. Number of candidates is fixed.

|   | \(m=4\) | \(m=8\) | \(m=16\) | \(m=32\) | \(m=64\) | \(m=128\) |
|---|---|---|---|---|---|---|
| \(n\) | 10 | 26 | 27 | 16 | 11 | 39 | 68 | 87 | 77 | 72 | 71 |
| \# cp | 490 | 474 | 476 | 473 | 484 | 489 | 461 | 432 | 413 | 423 | 428 | 429 |
| \# ci | 52 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# to | 429 | 404 | 405 | 422 | 424 | 448 | 483 | 459 | 460 | 468 | 461 | 473 |

|   | \(m=16\) | \(m=32\) | \(m=64\) | \(m=128\) |
|---|---|---|---|---|
| \(n\) | 11 | 13 | 21 | 11 | 11 | 7 | 1 | 6 | 5 | 7 | 8 |
| \# cp | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# to | 489 | 487 | 479 | 489 | 489 | 489 | 493 | 499 | 494 | 495 | 493 | 492 |
Figure 46: Results for fallback voting in the IC model for constructive control by partition of candidates in model TE. Number of voters is fixed.

|   | $n = 4$                      | $n = 8$                      |
|---|------------------------------|------------------------------|
| $m$ | 4   8  16  32  64  128 | 4   8  16  32  64  128 |
| # cp | 10  39  71  11  17  11  7 | 26  68  96   41  13  1  |
| # ci | 490  461  0  0  0  0  0 | 474  432  0  0  0  0  0 |
| # to | 0  0  429  483  489  493 | 0  0  404  459  487  499 |

| $n = 16$                      | $n = 32$                      |
|---|------------------------------|------------------------------|
| $m$ | 4   8  16  32  64  128 | 4   8  16  32  64  128 |
| # cp | 24  87  95  40  21  6  6 | 27  77  78  32  11  5  |
| # ci | 476  413  0  0  0  0  0 | 473  423  0  0  0  0  0 |
| # to | 0  0  405  460  479  494 | 0  0  422  468  489  495 |

| $n = 64$                      | $n = 128$                      |
|---|------------------------------|------------------------------|
| $m$ | 4   8  16  32  64  128 | 4   8  16  32  64  128 |
| # cp | 16  72  76  39  11  7 | 11  71  52  27  11  8 |
| # ci | 484  428  0  0  0  0 | 489  429  0  0  0  0 |
| # to | 0  0  424  461  489  493 | 0  0  448  473  489  492 |
Figure 47: Results for fallback voting in the TM model for constructive control by partition of candidates in model TE. Number of candidates is fixed.

|       | m = 4          | m = 8          |
|-------|----------------|----------------|
| n     | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp  | 7  24 20 61 50 0    | 25 42 55 81 91 70 |
| # ci  | 493 476 471 439 450 | 475 458 445 419 409 430 |
| # to  | 0  0  0  0  0  0    | 0  0  0  0  0  0    |
|       | m = 16          | m = 32          |
| n     | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp  | 51 73 70 66 48 0    | 11 21 16 23 14 20 |
| # ci  | 0  0  0  0  0  0    | 0  0  0  0  0  0    |
| # to  | 449 427 430 440 452 | 489 479 484 477 486 480 |
|       | m = 64          | m = 128         |
| n     | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp  | 5  9 11 9 3 6      | 1  4  6  2  5  0  |
| # ci  | 0  0  0  0  0  0    | 0  0  0  0  0  0    |
| # to  | 495 491 489 491 497 494 | 499 496 494 498 495 500 |
Figure 48: Results for fallback voting in the TM model for constructive control by partition of candidates in model TE. Number of voters is fixed.

| n = 4 | n = 8 |
|-------|-------|
| m     |       |       |
| 4     | 8     | 16    | 32   | 64   | 128  | 4     | 8     | 16    | 32   | 64   | 128  |
| # cp  |       |       |       |       |      |       |       |       |       |      |      |      |
| 7     | 25    | 51    | 11    | 5     | 1    | 24    | 42    | 73    | 21   | 9    | 4    |
| # ci  |       |       |       |       |      |       |       |       |       |      |      |      |
| 493   | 475   | 0     | 0     | 0     | 0    | 476   | 458   | 0     | 0    | 0    | 0    |
| # to  |       |       |       |       |      |       |       |       |       |      |      |      |
| 0     | 0     | 449   | 489   | 495   | 499  | 0     | 0     | 427   | 479  | 491  | 496  |
| n = 16|       |       |       |       |      |       |       |       |       |      |      |      |
| m     |       |       |       |       |      |       |       |       |       |      |      |      |
| 4     | 8     | 16    | 32    | 64    | 128  | 4     | 8     | 16    | 32   | 64   | 128  |
| # cp  |       |       |       |       |      |       |       |       |       |      |      |      |
| 20    | 55    | 70    | 16    | 11    | 6    | 29    | 81    | 68    | 23   | 9    | 2    |
| # ci  |       |       |       |       |      |       |       |       |       |      |      |      |
| 480   | 455   | 0     | 0     | 0     | 0    | 471   | 419   | 0     | 0    | 0    | 0    |
| # to  |       |       |       |       |      |       |       |       |       |      |      |      |
| 0     | 0     | 430   | 484   | 489   | 494  | 0     | 0     | 432   | 477  | 491  | 498  |
| n = 32|       |       |       |       |      |       |       |       |       |      |      |      |
| m     |       |       |       |       |      |       |       |       |       |      |      |      |
| 4     | 8     | 16    | 32    | 64    | 128  | 4     | 8     | 16    | 32   | 64   | 128  |
| # cp  |       |       |       |       |      |       |       |       |       |      |      |      |
| 61    | 91    | 60    | 14    | 3     | 5    | 5     | 70    | 48    | 20   | 6    | 0    |
| # ci  |       |       |       |       |      |       |       |       |       |      |      |      |
| 439   | 409   | 0     | 0     | 0     | 0    | 495   | 430   | 0     | 0    | 0    | 0    |
| # to  |       |       |       |       |      |       |       |       |       |      |      |      |
| 0     | 0     | 440   | 486   | 497   | 495  | 0     | 0     | 452   | 480  | 494  | 500  |
A.5.1. Computational Costs

Figure 49: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TE in fallback elections in the IC model. The maximum is 160, 89 seconds.

Figure 50: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TE in fallback elections in the IC model. The maximum is 24, 34 seconds.
Figure 51: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TE in fallback elections in the IC model. The maximum is 425.43 seconds.

Figure 52: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TE in fallback elections in the TM model. The maximum is 211.22 seconds.
Figure 53: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TE in fallback elections in the TM model. The maximum is 23.89 seconds.

Figure 54: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TE in fallback elections in the TM model. The maximum is 211.22 seconds.
A.6. Destructive Control by Partition of Candidates in Model TE

Figure 55: Results for fallback voting in the IC model for destructive control by partition of candidates in model TE. Number of candidates is fixed.

| n  | 4 | 8 | 16 | 32 | 64 | 128 | m = 4 | 4 | 8 | 16 | 32 | 64 | 128 | m = 8 |
|----|---|---|----|----|----|-----|-------|---|---|----|----|----|-----|-------|
| # cp | 107 | 122 | 131 | 95 | 76 | 40 | 248 | 301 | 324 | 314 | 303 | 292 | 130 |
| # ci | 393 | 378 | 369 | 405 | 424 | 460 | 252 | 199 | 176 | 186 | 197 | 208 | 130 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| n  | 4 | 8 | 16 | 32 | 64 | 128 | m = 16 | 4 | 8 | 16 | 32 | 64 | 128 | m = 32 |
|----|---|---|----|----|----|-----|-------|---|---|----|----|----|-----|-------|
| # cp | 370 | 429 | 436 | 439 | 438 | 419 | 253 | 363 | 424 | 416 | 435 | 419 | 130 |
| # ci | 9 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 121 | 71 | 64 | 61 | 62 | 81 | 246 | 137 | 76 | 84 | 65 | 81 | 130 |

| n  | 4 | 8 | 16 | 32 | 64 | 128 | m = 64 | 4 | 8 | 16 | 32 | 64 | 128 | m = 128 |
|----|---|---|----|----|----|-----|-------|---|---|----|----|----|-----|-------|
| # cp | 224 | 279 | 346 | 393 | 402 | 390 | 205 | 273 | 314 | 341 | 357 | 361 | 130 |
| # ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 276 | 221 | 154 | 107 | 98 | 110 | 295 | 227 | 186 | 159 | 143 | 139 | 130 |
Figure 56: Results for fallback voting in the IC model for destructive control by partition of candidates in model TE. Number of voters is fixed.

| m  | 4  | 8   | 16  | 32  | 64  | 128 | n = 4       | 4  | 8   | 16  | 32  | 64  | 128 |
|----|----|-----|-----|-----|-----|-----|-------------|----|----|-----|-----|-----|-----|
| # cp | 107 | 248 | 370 | 253 | 224 | 205 | 122        | 301| 429 | 363 | 279| 273|     |
| # ci | 393 | 252 | 9   | 1   | 0   | 0   | 378        | 199| 0   | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 121 | 246 | 276 | 295 | 0          | 0  | 71  | 137 | 221| 227|     |

| m  | 4  | 8   | 16  | 32  | 64  | 128 | n = 16     | 4  | 8   | 16  | 32  | 64  | 128 |
|----|----|-----|-----|-----|-----|-----|------------|----|----|-----|-----|-----|-----|
| # cp | 131 | 324 | 436 | 424 | 346 | 314 | 95         | 314| 439 | 416 | 393| 341|     |
| # ci | 369 | 176 | 0   | 0   | 0   | 0   | 405        | 186| 0   | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 64  | 76  | 154 | 186 | 0          | 0  | 61  | 84  | 107| 159|     |

| m  | 4  | 8   | 16  | 32  | 64  | 128 | n = 64     | 4  | 8   | 16  | 32  | 64  | 128 |
|----|----|-----|-----|-----|-----|-----|------------|----|----|-----|-----|-----|-----|
| # cp | 76  | 303 | 438 | 435 | 402 | 357 | 40         | 292| 419 | 419 | 390| 361|     |
| # ci | 424 | 197 | 0   | 0   | 0   | 0   | 460        | 208| 0   | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 62  | 65  | 98  | 143 | 0          | 0  | 81  | 81  | 110| 139|     |
Figure 57: Results for fallback voting in the TM model for destructive control by partition of candidates in model TE. Number of candidates is fixed.

|       | m = 4                     | m = 8                     |
|-------|---------------------------|---------------------------|
| n     | m = 4                     | m = 8                     |
|       | 4     8     16   32   64   128 | 4     8     16   32   64   128 |
| # cp  | 82    88    88    105   163   138 | 178   187   215   258   286   277 |
| # ci  | 418   412   412   395   337   362 | 322   313   285   242   214   223 |
| # to  | 0    0    0    0    0   0   0   0 | 0    0    0    0    0   0   0   0 |

|       | m = 16                     | m = 32                     |
|       | 4     8     16   32   64   128 | 4     8     16   32   64   128 |
| # cp  | 229   279   318   307   326   304 | 151   222   228   228   264   226 |
| # ci  | 159   138   83    56    30    29 | 153   104   81    34    21    15 |
| # to  | 112   83    99    137   144   167 | 196   174   191   238   215   259 |

|       | m = 64                     | m = 128                    |
|       | 4     8     16   32   64   128 | 4     8     16   32   64   128 |
| # cp  | 144   159   169   208   212   198 | 123   177   154   195   203   173 |
| # ci  | 110   96    69    23    8    6  | 123   101   60    18    7    7  |
| # to  | 246   245   262   269   280   296 | 254   222   242   287   290   320 |

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Figure 58: Results for fallback voting in the TM model for destructive control by partition of candidates in model TE. Number of voters is fixed.

| m   | n = 4 |        | n = 8 |
|-----|-------|--------|-------|
|     | 4     | 8      | 16    | 32    | 64    | 128 | 4     | 8      | 16    | 32    | 64    | 128 |
| # cp| 82    | 178    | 229   | 151   | 144   | 123 | 88    | 187    | 279   | 222   | 159   | 177 |
| # ci| 418   | 322    | 159   | 153   | 110   | 123 | 412   | 313    | 138   | 104   | 96    | 101 |
| # to| 0     | 0      | 112   | 196   | 246   | 254 | 0     | 0      | 83    | 174   | 245   | 222 |
|     |       |        |       |       |       |     |       |        |       |       |       |     |
| m   | n = 16|       | n = 32|       |
|     | 4     | 8      | 16    | 32    | 64    | 128 | 4     | 8      | 16    | 32    | 64    | 128 |
| # cp| 88    | 215    | 318   | 228   | 169   | 154 | 105   | 258    | 307   | 228   | 208   | 195 |
| # ci| 412   | 285    | 83    | 69    | 60    | 395 | 242   | 56     | 34    | 23    | 18    | 18  |
| # to| 0     | 0      | 99    | 191   | 262   | 242 | 0     | 0      | 137   | 238   | 269   | 287 |
|     |       |        |       |       |       |     |       |        |       |       |       |     |
| m   | n = 64|       | n = 128|      |
|     | 4     | 8      | 16    | 32    | 64    | 128 | 4     | 8      | 16    | 32    | 64    | 128 |
| # cp| 163   | 286    | 326   | 264   | 212   | 203 | 138   | 277    | 304   | 226   | 198   | 173 |
| # ci| 337   | 214    | 30    | 21    | 8     | 7   | 362   | 223    | 29    | 15    | 6     | 7   |
| # to| 0     | 0      | 144   | 215   | 280   | 290 | 0     | 0      | 167   | 259   | 296   | 320 |
A.6.1. Computational Costs

Figure 59: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TE in fallback elections in the IC model. The maximum is 93.6 seconds.

Figure 60: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TE in fallback elections in the IC model. The maximum is 23.03 seconds.
Figure 61: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TE in fallback elections in the IC model. The maximum is 93.6 seconds.

Figure 62: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TE in fallback elections in the TM model. The maximum is 108.15 seconds.
Figure 63: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TE in fallback elections in the TM model. The maximum is 18.22 seconds.

Figure 64: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TE in fallback elections in the TM model. The maximum is 103.95 seconds.
A.7. Constructive Control by Partition of Candidates in Model TP

Figure 65: Results for fallback voting in the IC model for constructive control by partition of candidates in model TP. Number of candidates is fixed.

| m = 4 | m = 8 |
|-------|-------|
| n     |       |       |       |       |       |       |       |       |       |       |
|       | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 9     | 25    | 25    | 31    | 25    | 7     | 45    | 69    | 68    | 69    | 74    | 64    |
| # ci  | 491   | 475   | 475   | 469   | 475   | 493   | 455   | 431   | 432   | 431   | 426   | 436   |
| # to  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| m = 16|       |       |       |       |       |       |       |       |       |       |       |       |
| n     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 63    | 86    | 87    | 78    | 82    | 70    | 24    | 32    | 44    | 32    | 30    | 26    |
| # ci  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| # to  | 437   | 414   | 413   | 422   | 418   | 430   | 476   | 468   | 486   | 468   | 470   | 474   |
| m = 32|       |       |       |       |       |       |       |       |       |       |       |       |
| n     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 10    | 15    | 14    | 10    | 14    | 17    | 3     | 8     | 10    | 7     | 5     | 3     |
| # ci  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| # to  | 490   | 485   | 486   | 490   | 486   | 483   | 497   | 492   | 490   | 493   | 495   | 497   |
Figure 66: Results for fallback voting in the IC model for constructive control by partition of candidates in model TP. Number of voters is fixed.

|   | n = 4  |           |           |           |           | n = 8  |           |           |           |           |
|---|--------|-----------|-----------|-----------|-----------|--------|-----------|-----------|-----------|-----------|
| m | 4      | 8         | 16        | 32        | 64        | 128    | 4         | 8         | 16        | 32        |
|   | # cp   | 9         | 45        | 63        | 24        | 10     | 3         | 25        | 69        | 86        |
|   | # ci   | 491       | 455       | 0         | 0         | 0      | 0         | 475       | 431       | 0         |
|   | # to   | 0         | 0         | 437       | 476       | 490    | 497       | 0         | 0         | 414       |
| n = 16 | 128    |           |           |           |           |        |           |           |           |           |
| m | 4      | 8         | 16        | 32        | 64        | 128    | 4         | 8         | 16        | 32        |
|   | # cp   | 25        | 68        | 87        | 44        | 14     | 10        | 31        | 69        | 78        |
|   | # ci   | 475       | 432       | 0         | 0         | 0      | 0         | 469       | 431       | 0         |
|   | # to   | 0         | 0         | 413       | 456       | 486    | 490       | 0         | 0         | 422       |
| n = 32 | 128    |           |           |           |           |        |           |           |           |           |
| m | 4      | 8         | 16        | 32        | 64        | 128    | 4         | 8         | 16        | 32        |
|   | # cp   | 25        | 74        | 82        | 30        | 14     | 5         | 7         | 64        | 7         |
|   | # ci   | 475       | 426       | 0         | 0         | 0      | 0         | 493       | 436       | 0         |
|   | # to   | 0         | 0         | 418       | 470       | 486    | 495       | 0         | 0         | 493       |
Figure 67: Results for fallback voting in the TM model for constructive control by partition of candidates in model TP. Number of candidates is fixed.

|     | $m = 4$ |     | $m = 8$ |
|-----|---------|-----|---------|
| $n$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 9 24 24 44 47 50 | 35 64 62 59 83 86 |
| # ci | 491 476 476 456 453 450 | 465 436 438 441 417 414 |
| # to | 0 0 0 0 0 0 | 0 0 0 0 0 0 |

$m = 16$

|     | $m = 16$ |     | $m = 32$ |
|-----|---------|-----|---------|
| $n$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 53 74 54 66 53 40 | 20 14 20 23 22 21 |
| # ci | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to | 447 426 446 434 447 460 | 480 486 480 477 478 479 |

$m = 64$

|     | $m = 64$ |     | $m = 128$ |
|-----|---------|-----|---------|
| $n$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 6 6 8 14 5 4 | 7 6 4 2 2 2 |
| # ci | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to | 494 494 492 486 495 496 | 493 494 496 498 498 498 |
Figure 68: Results for fallback voting in the TM model for constructive control by partition of candidates in model TP. Number of voters is fixed.

|   | \( n = 4 \) | \( n = 8 \) |
|---|-------------|-------------|
|   | \( m = 4 \) | \( n = 4 \) | \( m = 8 \) | \( n = 8 \) | \( m = 16 \) | \( n = 16 \) | \( m = 32 \) | \( n = 32 \) | \( m = 64 \) | \( n = 64 \) | \( m = 128 \) | \( n = 128 \) |
| \( n = 4 \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 9 | 35 | 53 | 20 | 6 | 7 | 24 | 64 | 74 | 14 | 6 | 6 |
| # ci | 491 | 465 | 0 | 0 | 0 | 0 | 476 | 436 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 447 | 480 | 494 | 493 | 0 | 0 | 426 | 486 | 494 | 494 |
| \( n = 8 \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 24 | 62 | 54 | 20 | 8 | 4 | 44 | 59 | 66 | 23 | 14 | 2 |
| # ci | 476 | 438 | 0 | 0 | 0 | 0 | 456 | 441 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 446 | 480 | 492 | 496 | 0 | 0 | 434 | 477 | 486 | 498 |
| \( n = 64 \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 47 | 83 | 53 | 22 | 5 | 2 | 50 | 86 | 40 | 21 | 4 | 2 |
| # ci | 453 | 417 | 0 | 0 | 0 | 0 | 450 | 414 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 447 | 478 | 495 | 498 | 0 | 0 | 460 | 479 | 496 | 498 |

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Figure 69: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TP in fallback elections in the IC model. The maximum is 136.83 seconds.

Figure 70: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TP in fallback elections in the IC model. The maximum is 33.13 seconds.
Figure 71: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TP in fallback elections in the IC model. The maximum is 136.83 seconds.

Figure 72: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TP in fallback elections in the TM model. The maximum is 179.79 seconds.
Figure 73: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TP in fallback elections in the TM model. The maximum is 32.92 seconds.

Figure 74: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TP in fallback elections in the TM model. The maximum is 179.19 seconds.
A.8. Destructive Control by Partition of Candidates in Model TP

Figure 75: Results for fallback voting in the IC model for destructive control by partition of candidates in model TP. Number of candidates is fixed.

|     | m = 4   |     | m = 8   |     | m = 16  |     | m = 32  |     | m = 64  |     | m = 128 |
|-----|---------|-----|---------|-----|----------|-----|----------|-----|----------|-----|---------|
| n   | 4       | 8   | 16      | 32  | 64       | 128 | 4        | 8   | 16       | 32  | 64      | 128 |
| # cp| 107     | 122 | 131     | 95  | 76       | 40  | 248      | 301 | 324      | 314 | 303     | 292 |
| # ci| 393     | 378 | 369     | 405 | 424      | 460 | 252      | 199 | 176      | 186 | 197     | 208 |
| # to| 0       | 0   | 0       | 0   | 0        | 0   | 0        | 0   | 0        | 0   | 0       | 0   |

|     | m = 16  |     | m = 32  |     | m = 64  |     | m = 128 |
|-----|---------|-----|----------|-----|----------|-----|---------|
| n   | 4       | 8   | 16      | 32  | 64       | 128 | 4        | 8   | 16       | 32  | 64      | 128 |
| # cp| 367     | 426 | 432     | 433 | 434      | 416 | 250      | 356 | 415      | 412 | 434     | 416 |
| # ci| 9       | 0   | 0       | 0   | 0        | 0   | 1        | 0   | 0        | 0   | 0       | 0   |
| # to| 124     | 74  | 68      | 67  | 66       | 84  | 249      | 144 | 85       | 88  | 66      | 84  |

|     | m = 64  |     | m = 128 |
|-----|---------|-----|---------|
| n   | 4       | 8   | 16      | 32  | 64       | 128 | 4        | 8   | 16       | 32  | 64      | 128 |
| # cp| 224     | 279 | 346     | 393 | 402      | 390 | 205      | 273 | 314      | 341 | 357     | 361 |
| # ci| 0       | 0   | 0       | 0   | 0        | 0   | 0        | 0   | 0        | 0   | 0       | 0   |
| # to| 276     | 221 | 154     | 107 | 98       | 110 | 295      | 227 | 186      | 159 | 143     | 139 |
Figure 76: Results for fallback voting in the IC model for destructive control by partition of candidates in model TP. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |
|---|---------|---|---------|
| $m$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 107 248 367 250 224 205 | 122 301 426 356 279 273 |
| # ci | 393 252 9 1 0 0 | 378 199 0 0 0 0 |
| # to | 0 0 124 249 276 295 | 0 0 74 144 221 227 |
| $n = 16$ |   |   |
| $n = 32$ |   |   |
| $n = 64$ |   |   |
| $n = 128$ |   |   |
| $m$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 131 324 432 415 346 314 | 95 314 433 412 393 341 |
| # ci | 369 176 0 0 0 0 | 405 186 0 0 0 0 |
| # to | 0 0 68 85 154 186 | 0 0 67 88 107 159 |
| $n = 64$ |   |   |
| $n = 128$ |   |   |
| $m$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 76 303 434 434 402 357 | 40 292 416 416 390 361 |
| # ci | 424 197 0 0 0 0 | 460 208 0 0 0 0 |
| # to | 0 0 66 66 98 143 | 0 0 84 84 110 139 |
Figure 77: Results for fallback voting in the TM model for destructive control by partition of candidates in model TP. Number of candidates is fixed.

| $m$ | $n$ | 4 | 8 | 16 | 32 | 64 | 128 |
|-----|-----|---|---|----|----|----|-----|
| $m=4$ | # cp | 82 | 88 | 88 | 105 | 163 | 138 |
|      | # ci | 418 | 412 | 412 | 395 | 337 | 362 |
|      | # to | 0 | 0 | 0 | 0 | 0 | 0 |
| $m=8$ | # cp | 178 | 187 | 215 | 258 | 286 | 277 |
|      | # ci | 322 | 313 | 285 | 242 | 214 | 223 |
|      | # to | 0 | 0 | 0 | 0 | 0 | 0 |
| $m=16$ | # cp | 221 | 276 | 301 | 321 | 304 | 222 |
|      | # ci | 159 | 137 | 83 | 56 | 30 | 29 |
|      | # to | 120 | 87 | 111 | 143 | 149 | 167 |
| $m=32$ | # cp | 151 | 222 | 228 | 228 | 264 | 226 |
|      | # ci | 153 | 104 | 81 | 34 | 21 | 15 |
|      | # to | 196 | 174 | 191 | 238 | 215 | 259 |
| $m=64$ | # cp | 123 | 177 | 168 | 194 | 203 | 173 |
|      | # ci | 123 | 101 | 66 | 18 | 7 | 7 |
|      | # to | 254 | 222 | 266 | 288 | 290 | 320 |

$|m|_{n}$
Figure 78: Results for fallback voting in the TM model for destructive control by partition of candidates in model TP. Number of voters is fixed.

| m   | n = 4 |   | n = 8 |   | n = 16 |   | n = 32 |   | n = 64 |   | n = 128 |   |
|-----|-------|---|-------|---|--------|---|--------|---|--------|---|--------|---|
|     | 4     | 8 | 16    | 32| 64     | 128| 4      | 8 | 16     | 32| 64     | 128|
|     | 4     | 8 | 16    | 32| 64     | 128| 4      | 8 | 16     | 32| 64     | 128|
| # cp| 82    | 178| 221   | 151| 144    | 123| 88     | 187| 276    | 222| 159    | 177|
| # ci| 418   | 322| 159   | 153| 110    | 123| 412    | 313| 137    | 104| 96     | 101|
| # to| 0     | 0 | 120   | 196| 246    | 254| 0      | 0 | 87     | 174| 245    | 222|
| m   | 4     | 8 | 16    | 32| 64     | 128| 4      | 8 | 16     | 32| 64     | 128|
| # cp| 88    | 215| 306   | 228| 169    | 168| 105    | 258| 301    | 228| 208    | 194|
| # ci| 412   | 285| 83    | 81 | 69     | 66 | 395    | 242| 56     | 34 | 23     | 18 |
| # to| 0     | 0 | 111   | 191| 262    | 266| 0      | 0 | 143    | 238| 269    | 288|
| m   | 4     | 8 | 16    | 32| 64     | 128| 4      | 8 | 16     | 32| 64     | 128|
| # cp| 163   | 286| 321   | 264| 212    | 203| 138    | 277| 304    | 226| 198    | 173|
| # ci| 337   | 214| 30    | 21 | 8      | 7  | 362    | 223| 29     | 15 | 6      | 7  |
| # to| 0     | 0 | 149   | 215| 280    | 290| 0      | 0 | 167    | 259| 296    | 320|
A.8.1. Computational Costs

Figure 79: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TP in fallback elections in the IC model. The maximum is 93.92 seconds.

Figure 80: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TP in fallback elections in the IC model. The maximum is 31.75 seconds.
Figure 81: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TP in fallback elections in the IC model. The maximum is 93.92 seconds.

Figure 82: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TP in fallback elections in the TM model. The maximum is 108.35 seconds.
Figure 83: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TP in fallback elections in the TM model. The maximum is 24.8 seconds.

Figure 84: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TP in fallback elections in the TM model. The maximum is 104.12 seconds.
Figure 85: Results for fallback voting in the IC model for constructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

|      | m = 4        | m = 8        | m = 16 | m = 32 | m = 64 | m = 128 |
|------|--------------|--------------|--------|--------|--------|---------|
| n    | 4  8  16  32 | 4  8  16  32 | 4  8  16 32 | 4  8  16 32 | 4  8  16 32 | 4  8  16 32 |
| # cp | 8  27  23  29 | 23  47  60 65 | 60  64 128 | 60  64 128 | 60  64 128 | 60  64 128 |
| # ci | 492  473 471 | 477  487 487 | 477  453 440 | 435  440 460 | 435  440 460 | 435  440 460 |
| # to | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   |
|      | m = 16       | m = 32       | m = 64 | m = 128 | m = 64 | m = 128 |
| n    | 4  8  16  32 | 4  8  16  32 | 4  8  16 32 | 4  8  16 32 | 4  8  16 32 | 4  8  16 32 |
| # cp | 45  81  92  80 | 87  57  36  59 | 47  50  41  43 | 47  50  41  43 | 47  50  41  43 | 47  50  41  43 |
| # ci | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   |
| # to | 455  419 408 | 420  413 443 | 464  441 453 | 450  459 457 | 450  459 457 | 450  459 457 |
|      | m = 64       | m = 128      | m = 64 | m = 128 | m = 64 | m = 128 |
| n    | 4  8  16  32 | 4  8  16  32 | 4  8  16 32 | 4  8  16 32 | 4  8  16 32 | 4  8  16 32 |
| # cp | 11  21  25  33 | 22  19  7  8 | 10  15  9 10 | 10  15  9 10 | 10  15  9 10 | 10  15  9 10 |
| # ci | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   | 0  0  0  0   |
| # to | 489  479 475 | 467  478 481 | 493  492 490 | 485  491 490 | 485  491 490 | 485  491 490 |
Figure 86: Results for fallback voting in the IC model for constructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |   | $n = 16$ |   | $n = 32$ |   | $n = 64$ |   | $n = 128$ |
|---|---------|---|---------|---|---------|---|---------|---|---------|---|---------|
| $m$ | 4 8 16 32 64 128 | $m$ | 4 8 16 32 64 128 | $m$ | 4 8 16 32 64 128 | $m$ | 4 8 16 32 64 128 | $m$ | 4 8 16 32 64 128 | $m$ | 4 8 16 32 64 128 |
| # cp | 8 23 45 36 11 7 | # cp | 27 47 81 59 21 8 | # cp | 29 65 80 50 33 15 | # cp | 29 65 80 50 33 15 | # cp | 29 65 80 50 33 15 | # cp | 29 65 80 50 33 15 |
| # ci | 492 477 0 0 0 0 | # ci | 473 453 0 0 0 0 | # ci | 473 453 0 0 0 0 | # ci | 473 453 0 0 0 0 | # ci | 473 453 0 0 0 0 | # ci | 473 453 0 0 0 0 |
| # to | 0 0 455 464 489 493 | # to | 0 0 419 441 479 492 | # to | 0 0 419 441 479 492 | # to | 0 0 419 441 479 492 | # to | 0 0 419 441 479 492 | # to | 0 0 419 441 479 492 | # to | 0 0 419 441 479 492 |

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Figure 87: Results for fallback voting in the TM model for constructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

| $m$ | $n$ | 4 | 8 | 16 | 32 | 64 | 128 |
|-----|-----|---|---|----|----|----|-----|
|     | # cp | 5 | 15 | 21 | 34 | 56 | 47  |
|     | # ci  | 495| 485| 479| 466| 444| 453 |
|     | # to  | 0 | 0  | 0  | 0  | 0  | 0   |
| $m=4$ | $m=8$ | $m=16$ | $m=32$ | $m=64$ | $m=128$ |
|     | # cp | 26 | 37 | 59 | 68 | 63 | 52  |
|     | # ci  | 0  | 0  | 0  | 0  | 0  | 0   |
|     | # to  | 461| 463| 441| 432| 437| 448 |
| $m=16$ | $m=32$ | $m=64$ | $m=128$ |
|     | # cp | 8  | 15 | 10 | 18 | 11 | 11  |
|     | # ci  | 0  | 0  | 0  | 0  | 0  | 0   |
|     | # to  | 492| 485| 490| 482| 489| 489 |
| $m=64$ | $m=128$ | $m=128$ |
Figure 88: Results for fallback voting in the TM model for constructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

|      |  n = 4          |  n = 8          |
|------|-----------------|-----------------|
|      | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|      | 5  | 18  | 26  | 9   | 8   | 4   | 15  | 42  | 37  | 28  | 15  | 5   |
| # cp | 495 | 482 | 13  | 0   | 0   | 0   | 485 | 458 | 0   | 0   | 0   | 0   |
| # to | 0  | 0   | 461 | 491 | 492 | 496 | 0   | 0   | 463 | 472 | 485 | 495 |
|      | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|      | 21 | 43  | 59  | 34  | 10  | 5   | 34  | 54  | 68  | 33  | 18  | 4   |
| # cp | 479 | 457 | 0   | 0   | 0   | 0   | 466 | 446 | 0   | 0   | 0   | 0   |
| # to | 0  | 0   | 441 | 466 | 490 | 495 | 0   | 0   | 432 | 467 | 482 | 496 |
|      | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|      | 56 | 62  | 63  | 33  | 11  | 7   | 47  | 55  | 52  | 32  | 11  | 5   |
| # cp | 444 | 438 | 0   | 0   | 0   | 0   | 453 | 445 | 0   | 0   | 0   | 0   |
| # to | 0  | 0   | 437 | 467 | 489 | 493 | 0   | 0   | 448 | 468 | 489 | 495 |
A.9.1. Computational Costs

Figure 89: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TE in fallback elections in the IC model. The maximum is 214.19 seconds.

Figure 90: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TE in fallback elections in the IC model. The maximum is 20.2 seconds.
Figure 91: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TE in fallback elections in the IC model. The maximum is 214.19 seconds.

Figure 92: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TE in fallback elections in the TM model. The maximum is 229.26 seconds.
Figure 93: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TE in fallback elections in the TM model. The maximum is 592.3 seconds.

Figure 94: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TE in fallback elections in the TM model. The maximum is 229.26 seconds.
A.10. Destructive Control by Runoff Partition of Candidates in Model TE

Figure 95: Results for fallback voting in the IC model for destructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

|       | m = 4       |       | m = 8       |       |
|-------|-------------|-------|-------------|-------|
|       | n    | 4 | 8 | 16 | 32 | 64 | 128 | n    | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp  |      |   |   |    |    |    |     | # cp  |    |    |    |    |    |     |
|       | 102  | 105| 109| 90 | 73 | 40 |     | 236  | 285| 310| 292| 288| 270|     |
| # ci  | 398  | 395| 391| 410| 427| 460|     | 264  | 215| 190| 208| 212| 230|     |
| # to  | 0    | 0 | 0 | 0 | 0 | 0 |     | 0    | 0 | 0 | 0 | 0 | 0 |     |
|       | m = 16 |   |   |    |    |    |     |       |    |    |    |    |    |     |
| # cp  | 361  | 428| 432| 429| 416| 397|     | 292  | 371| 409| 407| 414| 381|     |
| # ci  | 9    | 0 | 0 | 0 | 0 | 0 |     | 1    | 0 | 0 | 0 | 0 | 0 |     |
| # to  | 130  | 72 | 68 | 71 | 84 | 103|     | 207  | 129| 91 | 93 | 86 | 119|     |
|       | m = 64 |   |   |    |    |    |     |       |    |    |    |    |    |     |
| # cp  | 241  | 284| 336| 370| 386| 335|     | 233  | 272| 287| 302| 319| 313|     |
| # ci  | 0    | 0 | 0 | 0 | 0 | 0 |     | 0    | 0 | 0 | 0 | 0 | 0 |     |
| # to  | 259  | 216| 164| 130| 114| 165|     | 267  | 228| 213| 198| 181| 187|     |
Figure 96: Results for fallback voting in the IC model for destructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

| m   | n = 4 |     |     |     |     |     | n = 8 |     |     |     |     |     |
|-----|-------|-----|-----|-----|-----|-----|-------|-----|-----|-----|-----|-----|
|     | 4     | 8   | 16  | 32  | 64  | 128 | 4     | 8   | 16  | 32  | 64  | 128 |
| # cp| 102   | 236 | 361 | 292 | 241 | 233 | 105   | 285 | 428 | 371 | 284 | 272 |
| # ci| 398   | 264 | 9   | 1   | 0   | 0   | 395   | 215 | 0   | 0   | 0   | 0   |
| # to| 0     | 0   | 130 | 207 | 259 | 267 | 0     | 0   | 72  | 129 | 216 | 228 |

| m   | n = 16 |     |     |     |     |     | n = 32 |     |     |     |     |     |
|-----|--------|-----|-----|-----|-----|-----|---------|-----|-----|-----|-----|-----|
|     | 4      | 8   | 16  | 32  | 64  | 128 | 4       | 8   | 16  | 32  | 64  | 128 |
| # cp| 109    | 310 | 432 | 409 | 336 | 287 | 90     | 292 | 429 | 407 | 370 | 302 |
| # ci| 391    | 190 | 0   | 0   | 0   | 0   | 410    | 208 | 0   | 0   | 0   | 0   |
| # to| 0      | 0   | 68  | 91  | 164 | 213 | 0      | 0   | 71  | 93  | 130 | 198 |

| m   | n = 64 |     |     |     |     |     | n = 128 |     |     |     |     |     |
|-----|--------|-----|-----|-----|-----|-----|---------|-----|-----|-----|-----|-----|
|     | 4      | 8   | 16  | 32  | 64  | 128 | 4       | 8   | 16  | 32  | 64  | 128 |
| # cp| 73     | 288 | 416 | 414 | 386 | 319 | 40      | 270 | 397 | 381 | 335 | 313 |
| # ci| 427    | 212 | 0   | 0   | 0   | 0   | 460    | 230 | 0   | 0   | 0   | 0   |
| # to| 0      | 0   | 84  | 86  | 114 | 181 | 0      | 0   | 103 | 119 | 165 | 187 |
Figure 97: Results for fallback voting in the TM model for destructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

|        | m = 4              | m = 8              |
|--------|--------------------|--------------------|
| n      | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp   | 79  84  80  148 126 | 168 180 204 247 280 271 |
| # ci   | 421 416 420 400 352 374 | 332 320 296 253 220 229 |
| # to   | 0  0  0  0  0  0    | 0  0  0  0  0  0    |

|        | m = 16             | m = 32             |
|--------|--------------------|--------------------|
| n      | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp   | 227 276 324 319 340 319 | 169 233 230 243 276 230 |
| # ci   | 159 137 83 30 29      | 153 104 81 34 21 15   |
| # to   | 114 87 93 125 130 152  | 178 163 189 223 203 255 |

|        | m = 64             | m = 128            |
|--------|--------------------|--------------------|
| n      | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp   | 149 167 171 205 217 183 | 126 163 162 179 172 153 |
| # ci   | 110 96 69 23 8 6      | 123 101 66 18 7 7    |
| # to   | 241 237 260 272 275 311 | 251 236 272 303 321 340 |
Figure 98: Results for fallback voting in the TM model for destructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

| m   | 4   | 8   | 16  | 32  | 64  | 128 | n = 4 | n = 8 |
|-----|-----|-----|-----|-----|-----|-----|-------|-------|
| # cp| 79  | 168 | 227 | 169 | 149 | 126 | 84    | 180   | 276   | 233   | 167   | 163   |
| # ci| 421 | 332 | 159 | 153 | 110 | 123 | 416   | 320   | 137   | 104   | 96    | 101   |
| # to| 0   | 0   | 114 | 178 | 241 | 251 | 0     | 0     | 87    | 163   | 237   | 236   |
| n = 16 | | | | | | | | | | | | |
| m   | 4   | 8   | 16  | 32  | 64  | 128 | n = 8 | n = 32 | n = 64 | n = 128 |
| # cp| 80  | 204 | 324 | 230 | 171 | 162 | 100   | 247   | 319   | 243   | 205   | 179   |
| # ci| 420 | 296 | 83  | 81  | 69  | 66  | 400   | 253   | 56    | 34    | 23    | 18    |
| # to| 0   | 0   | 93  | 189 | 260 | 272 | 0     | 125   | 223   | 272   | 303   |       |
| n = 64 | | | | | | | | | | | | |
| m   | 4   | 8   | 16  | 32  | 64  | 128 | n = 128 | n = 128 | n = 128 | n = 128 |
| # cp| 148 | 280 | 340 | 276 | 217 | 172 | 126   | 271   | 319   | 230   | 183   | 153   |
| # ci| 352 | 220 | 30  | 21  | 8   | 7   | 374   | 374   | 6     | 15    | 6     | 7     |
| # to| 0   | 0   | 130 | 203 | 275 | 321 | 0     | 0     | 311   | 255   | 311   | 340   |
A.10.1. Computational Costs

Figure 99: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TE in fallback elections in the IC model. The maximum is 59.44 seconds.

Figure 100: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TE in fallback elections in the IC model. The maximum is 20.16 seconds.
Figure 101: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TE in fallback elections in the IC model. The maximum is 59.44 seconds.

Figure 102: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TE in fallback elections in the TM model. The maximum is 58.6 seconds.
Figure 103: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TE in fallback elections in the TM model. The maximum is 15.84 seconds.

Figure 104: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TE in fallback elections in the TM model. The maximum is 56.03 seconds.
A.11. Constructive Control by Runoff Partition of Candidates in Model TP

Figure 105: Results for fallback voting in the IC model for constructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

| m = 4          | m = 8          |
|----------------|----------------|
| n              | # cp           | # cp           |
| 4              | 14 20 20 30 22 | 31 45 65 71 55 |
| 8              | 486 480 470 478| 469 455 435 429|
| 16             | 0 0 0 0 0 0    | 0 0 0 0 0 0    |
| 32             | 42 85 73 89 95 | 29 58 50 55 48 |
| 64             | 0 0 0 0 0 0    | 0 0 0 0 0 0    |
| 128            | 458 415 427 411| 471 442 450 445|
| m = 16         | # ci           | # ci           |
| 4              | 0 0 0 0 0 0    | 0 0 0 0 0 0    |
| 8              | 458 415 427 411| 471 442 450 445|
| 16             | 0 0 0 0 0 0    | 0 0 0 0 0 0    |
| 32             | 42 85 73 89 95 | 29 58 50 55 48 |
| 64             | 0 0 0 0 0 0    | 0 0 0 0 0 0    |
| 128            | 458 415 427 411| 471 442 450 445|
| m = 32         | # to           | # to           |
| 4              | 458 415 427 411| 471 442 450 445|
| 8              | 0 0 0 0 0 0    | 0 0 0 0 0 0    |
| 16             | 42 85 73 89 95 | 29 58 50 55 48 |
| 32             | 0 0 0 0 0 0    | 0 0 0 0 0 0    |
| 64             | 458 415 427 411| 471 442 450 445|
| 128            | 0 0 0 0 0 0    | 0 0 0 0 0 0    |
Figure 106: Results for fallback voting in the IC model for constructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

| m    | n = 4 |  |  |  |  |  |  | n = 8 |  |  |  |  |  |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| # cp | 140  | 31   | 42   | 29   | 12   | 5    | 20   | 45   | 85   | 58   | 20   | 9    | 20   | 45   | 85   |
| # ci | 486  | 469  | 0    | 0    | 0    | 0    | 480  | 455  | 0    | 0    | 0    | 0    | 480  | 455  | 0    |
| # to | 0    | 0    | 458  | 471  | 488  | 495  | 0    | 0    | 415  | 442  | 480  | 491  |

| m    | n = 16 |  |  |  |  |  |  | n = 32 |  |  |  |  |  |
|------|--------|------|------|------|------|------|------|--------|------|------|------|------|------|------|
| # cp | 20    | 65   | 73   | 50   | 21   | 11   | 30   | 71   | 89   | 55   | 19   | 10   | 20   | 65   | 73   |
| # ci | 480   | 435  | 0    | 0    | 0    | 0    | 470  | 429  | 0    | 0    | 0    | 0    | 480  | 435  | 0    |
| # to | 0    | 0    | 427  | 450  | 479  | 489  | 0    | 0    | 411  | 445  | 481  | 490  |

| m    | n = 64 |  |  |  |  |  |  | n = 128 |  |  |  |  |  |
|------|--------|------|------|------|------|------|------|--------|------|------|------|------|------|------|
| # cp | 22    | 55   | 95   | 48   | 23   | 4    | 11   | 43   | 59   | 46   | 19   | 9    | 22   | 55   | 95   |
| # ci | 478   | 445  | 0    | 0    | 0    | 0    | 489  | 457  | 0    | 0    | 0    | 0    | 478  | 445  | 0    |
| # to | 0    | 0    | 405  | 452  | 477  | 496  | 0    | 0    | 441  | 454  | 481  | 491  |
Figure 107: Results for fallback voting in the TM model for constructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

| $m$ | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|----|----|----|----|----|-----|
| # cp | 9  | 19 | 23 | 35 | 54 | 43  | 21 | 36 | 49 | 59 | 68 |     |
| # ci | 491| 481| 477| 465| 446| 457  | 479| 464| 457| 451| 441| 432 |
| # to | 0  | 0  | 0  | 0  | 0  | 0    | 0  | 0  | 0  | 0  | 0  | 0   |

$m = 16$

| $m$ | 32 | 53 | 52 | 69 | 69 | 61  | 14 | 29 | 27 | 44 | 32 | 31  |
| # cp | 23 | 0  | 0  | 0  | 0  | 0    | 0  | 0  | 0  | 0  | 0  | 0   |
| # ci | 445| 447| 448| 431| 431| 439  | 486| 471| 473| 456| 468| 469 |
| # to | 488| 482| 488| 485| 484| 487  | 500| 495| 493| 495| 494| 491 |

$m = 32$

| $m$ | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|----|----|----|----|----|-----|
| # cp | 12 | 18 | 15 | 16 | 13 | 13  | 0  | 5  | 7  | 5  | 6  | 9   |
| # ci | 0  | 0  | 0  | 0  | 0  | 0    | 0  | 0  | 0  | 0  | 0  | 0   |
| # to | 488| 482| 488| 485| 484| 487  | 500| 495| 493| 495| 494| 491 |
Figure 108: Results for fallback voting in the TM model for constructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

|   | n = 4 | n = 8 |
|---|-------|-------|
| m | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 9 21 32 14 12 0 | 19 36 53 29 18 5 |
| # ci | 491 479 23 0 0 0 | 481 464 0 0 0 0 |
| # to | 0 0 445 486 488 500 | 0 0 447 471 482 495 |

|   | n = 16 | n = 32 |
|---|-------|-------|
| m | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 23 43 52 27 12 7 | 35 49 69 44 15 5 |
| # ci | 477 457 0 0 0 0 | 465 451 0 0 0 0 |
| # to | 0 0 448 473 488 493 | 0 0 431 456 485 495 |

|   | n = 64 | n = 128 |
|---|-------|-------|
| m | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 54 59 69 32 16 6 | 43 68 61 31 13 9 |
| # ci | 446 441 0 0 0 0 | 457 432 0 0 0 0 |
| # to | 0 0 431 468 484 494 | 0 0 439 469 487 491 |
A.11.1. Computational Costs

Figure 109: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TP in fallback elections in the IC model. The maximum is 168.02 seconds.

Figure 110: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TP in fallback elections in the IC model. The maximum is 20.47 seconds.
Figure 111: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TP in fallback elections in the IC model. The maximum is 168.02 seconds.

Figure 112: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TP in fallback elections in the TM model. The maximum is 123.59 seconds.
Figure 113: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TP in fallback elections in the TM model. The maximum is 587.95 seconds.

Figure 114: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TP in fallback elections in the TM model. The maximum is 263.04 seconds.
Figure 115: Results for fallback voting in the IC model for destructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.
Figure 116: Results for fallback voting in the IC model for destructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

| $n = 4$ | $n = 8$ |
|--------|--------|
| $m$   | 4     | 8     | 16 | 32 | 64 | 128 | 4     | 8     | 16 | 32 | 64 | 128 |
| # cp  | 102   | 222   | 347 | 277 | 225 | 223 | 105   | 268   | 421 | 357 | 272 | 262 |
| # ci  | 398   | 278   | 9   | 1   | 0   | 0   | 395   | 232   | 0   | 0   | 0   | 0   |
| # to  | 0     | 0     | 144 | 222 | 275 | 277 | 0     | 0     | 79  | 143 | 228 | 238 |
| $n = 16$ | 4     | 8     | 16 | 32 | 64 | 128 | 4     | 8     | 16 | 32 | 64 | 128 |
| # cp  | 109   | 300   | 423 | 399 | 325 | 284 | 89    | 281   | 422 | 399 | 363 | 294 |
| # ci  | 391   | 200   | 0   | 0   | 0   | 0   | 411   | 219   | 0   | 0   | 0   | 0   |
| # to  | 0     | 0     | 77  | 101 | 175 | 216 | 0     | 0     | 78  | 101 | 137 | 206 |
| $n = 32$ | 4     | 8     | 16 | 32 | 64 | 128 | 4     | 8     | 16 | 32 | 64 | 128 |
| # cp  | 73    | 275   | 414 | 407 | 382 | 314 | 40    | 266   | 389 | 374 | 327 | 312 |
| # ci  | 427   | 225   | 0   | 0   | 0   | 0   | 460   | 234   | 0   | 0   | 0   | 0   |
| # to  | 0     | 0     | 86  | 93  | 118 | 186 | 0     | 0     | 111 | 126 | 173 | 188 |
Figure 117: Results for fallback voting in the TM model for destructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

| m = 4 | m = 8 |
|-------|-------|
| n     |       |       |
|       | 4     | 8     | 16    | 32    | 64    | 128   |
|       | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 79    | 84    | 80    | 100   | 148   | 126   |
| # ci  | 421   | 416   | 420   | 400   | 352   | 374   |
| # to  | 0     | 0     | 0     | 0     | 0     | 0     |

| m = 16 |
|--------|
| n      | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 212   | 268   | 320   | 317   | 339   | 319   |
| # ci  | 159   | 137   | 83    | 56    | 30    | 29    |
| # to  | 129   | 95    | 97    | 127   | 131   | 152   |

| m = 32 |
|--------|
| n      | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 141   | 163   | 169   | 202   | 216   | 182   |
| # ci  | 110   | 96    | 69    | 23    | 8     | 6     |
| # to  | 249   | 241   | 262   | 275   | 276   | 312   |

| m = 64 |
|--------|
| n      | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 122   | 159   | 159   | 178   | 171   | 152   |
| # ci  | 123   | 101   | 66    | 18    | 7     | 7     |
| # to  | 255   | 240   | 275   | 304   | 322   | 341   |

| m = 128 |
|---------|
| n       | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp   | 141   | 163   | 169   | 202   | 216   | 182   |
| # ci   | 110   | 96    | 69    | 23    | 8     | 6     |
| # to   | 249   | 241   | 262   | 275   | 276   | 312   |
Figure 118: Results for fallback voting in the TM model for destructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

| m   | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|-----|---|---|----|----|----|-----|---|---|----|----|----|-----|
| # cp | 79 | 159 | 212 | 156 | 141 | 122 | 84 | 172 | 268 | 228 | 163 | 159 |
| # ci | 421 | 341 | 159 | 153 | 110 | 123 | 416 | 328 | 137 | 104 | 96 | 101 |
| # to | 0 | 0 | 129 | 191 | 249 | 255 | 0 | 0 | 95 | 168 | 241 | 240 |

| m   | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|-----|---|---|----|----|----|-----|---|---|----|----|----|-----|
| # cp | 80 | 197 | 320 | 220 | 169 | 159 | 100 | 242 | 317 | 240 | 202 | 178 |
| # ci | 420 | 303 | 83 | 81 | 69 | 66 | 400 | 258 | 56 | 34 | 23 | 18 |
| # to | 0 | 0 | 97 | 199 | 262 | 275 | 0 | 0 | 127 | 226 | 275 | 304 |

| m   | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|-----|---|---|----|----|----|-----|---|---|----|----|----|-----|
| # cp | 148 | 278 | 339 | 276 | 216 | 171 | 126 | 271 | 319 | 230 | 182 | 152 |
| # ci | 352 | 222 | 30 | 21 | 8 | 7 | 374 | 229 | 29 | 15 | 6 | 7 |
| # to | 0 | 0 | 131 | 203 | 276 | 322 | 0 | 0 | 152 | 255 | 312 | 341 |
A.12.1. Computational Costs

Figure 119: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TP in fallback elections in the IC model. The maximum is 60.76 seconds.

Figure 120: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TP in fallback elections in the IC model. The maximum is 19.94 seconds.
Figure 121: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TP in fallback elections in the IC model. The maximum is 60.76 seconds.

Figure 122: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TP in fallback elections in the TM model. The maximum is 58.04 seconds.
Figure 123: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TP in fallback elections in the TM model. The maximum is 15.75 seconds.

Figure 124: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TP in fallback elections in the TM model. The maximum is 55.48 seconds.
A.13. Constructive Control by Adding Voters

Figure 125: Results for fallback voting in the IC model for constructive control by adding voters. Number of candidates is fixed.

|    | $m = 4$                                                                 | $m = 8$                                                                 | $m = 16$                                                                 | $m = 32$                                                                 | $m = 64$                                                                 | $m = 128$                                                                 |
|----|------------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|
| $n$ | $4$                       | $8$                       | $16$                      | $32$                      | $64$                      | $128$                      | $4$                       | $8$                       | $16$                      | $32$                      | $64$                      | $128$                      |
| # cp | 100                      | 154                      | 277                      | 395                      | 474                      | 492                      | 55                      | 119                      | 239                      | 365                      | 447                      | 494                      |
| # ci  | 400                      | 346                      | 223                      | 1                        | 0                        | 0                        | 445                      | 381                      | 261                      | 1                        | 0                        | 0                        |
| # to  | 0                        | 0                        | 104                      | 26                      | 8                        | 0                        | 0                        | 134                      | 53                      | 6                        | 0                        | 0                        |
Figure 126: Results for fallback voting in the IC model for constructive control by adding voters. Number of
voters is fixed.

| n = 4 | n = 8 |
|---|---|
| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 100 | 55 | 42 | 21 | 11 | 4 | 154 | 119 | 98 | 60 | 40 | 19 |
| # ci | 400 | 445 | 458 | 479 | 479 | 496 | 346 | 381 | 402 | 440 | 460 | 481 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| n = 16 | n = 32 |
| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 277 | 239 | 186 | 163 | 98 | 68 | 395 | 365 | 299 | 273 | 199 | 152 |
| # ci | 223 | 261 | 314 | 337 | 376 | 225 | 1 | 1 | 2 | 12 | 23 | 28 |
| # to | 0 | 0 | 0 | 0 | 26 | 207 | 104 | 134 | 199 | 215 | 278 | 320 |
| n = 64 | n = 128 |
| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 474 | 447 | 417 | 393 | 328 | 292 | 492 | 494 | 482 | 491 | 464 | 446 |
| # ci | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 26 | 53 | 83 | 107 | 172 | 207 | 8 | 6 | 18 | 9 | 36 | 54 |

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Figure 127: Results for fallback voting in the TM model for constructive control by adding voters. Number of candidates is fixed.

|       | $m = 4$ |       | $m = 8$ |
|-------|---------|-------|---------|
| $n$   | $4$     | $8$   | $16$    | $32$    | $64$    | $128$  |
| $m$   | $4$     | $8$   | $16$    | $32$    | $64$    | $128$  |
| # cp  | $75$    | $98$  | $142$   | $147$   | $187$   | $206$  |
| # ci  | $425$   | $402$ | $358$   | $157$   | $130$   | $151$  |
| # to  | $0$     | $0$   | $0$     | $196$   | $183$   | $143$  |
|       | $m = 16$|       | $m = 32$|
| $n$   | $4$     | $8$   | $16$    | $32$    | $64$    | $128$  |
| $m$   | $16$    | $32$  | $64$    | $128$   |
| # cp  | $27$    | $45$  | $69$    | $86$    | $114$   | $134$  |
| # ci  | $473$   | $455$ | $432$   | $291$   | $282$   | $266$  |
| # to  | $0$     | $0$   | $0$     | $123$   | $104$   | $100$  |
|       | $m = 64$|       | $m = 128$|
| $n$   | $4$     | $8$   | $16$    | $32$    | $64$    | $128$  |
| $m$   | $64$    | $128$ |
| # cp  | $9$     | $17$  | $19$    | $23$    | $31$    | $56$   |
| # ci  | $491$   | $483$ | $411$   | $390$   | $378$   | $377$  |
| # to  | $0$     | $0$   | $70$    | $87$    | $91$    | $67$   |
Figure 128: Results for fallback voting in the TM model for constructive control by adding voters. Number of voters is fixed.

|    | \(n = 4\) | \(n = 8\) | \(n = 16\) | \(n = 32\) | \(n = 64\) | \(n = 128\) |
|----|-----------|-----------|------------|------------|------------|------------|
| \(m\) | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| \# cp | 75 48 27 15 9 7 | 98 63 45 34 17 7 | 157 226 291 350 390 423 | 157 226 291 350 390 423 | 157 226 291 350 390 423 | 157 226 291 350 390 423 |
| \# ci | 425 452 473 485 491 493 | 402 437 455 466 483 493 | 402 437 455 466 483 493 | 402 437 455 466 483 493 | 402 437 455 466 483 493 | 402 437 455 466 483 493 |
| \# to | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \(m\) | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| \# cp | 142 116 69 44 19 10 | 147 119 86 41 23 8 | 147 119 86 41 23 8 | 147 119 86 41 23 8 | 147 119 86 41 23 8 | 147 119 86 41 23 8 |
| \# ci | 358 384 431 347 411 423 | 157 226 291 350 390 423 | 157 226 291 350 390 423 | 157 226 291 350 390 423 | 157 226 291 350 390 423 | 157 226 291 350 390 423 |
| \# to | 0 0 0 109 70 67 | 196 155 123 109 87 69 | 196 155 123 109 87 69 | 196 155 123 109 87 69 | 196 155 123 109 87 69 | 196 155 123 109 87 69 |
| \(m\) | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| \# cp | 187 161 114 76 31 18 | 206 169 134 95 56 28 | 206 169 134 95 56 28 | 206 169 134 95 56 28 | 206 169 134 95 56 28 | 206 169 134 95 56 28 |
| \# ci | 130 188 282 327 378 421 | 151 183 266 319 377 425 | 151 183 266 319 377 425 | 151 183 266 319 377 425 | 151 183 266 319 377 425 | 151 183 266 319 377 425 |
| \# to | 183 151 104 97 91 61 | 143 148 100 86 67 47 | 143 148 100 86 67 47 | 143 148 100 86 67 47 | 143 148 100 86 67 47 | 143 148 100 86 67 47 |
A.13.1. Computational Costs

Figure 129: Average time the algorithm needs to find a successful control action for constructive control by adding voters in fallback elections in the IC model. The maximum is 71,62 seconds.

Figure 130: Average time the algorithm needs to determine no-instance of constructive control by adding voters in fallback elections in the IC model. The maximum is 210,51 seconds.
Figure 131: Average time the algorithm needs to give a definite output for constructive control by adding voters in fallback elections in the IC model. The maximum is 110.45 seconds.

Figure 132: Average time the algorithm needs to find a successful control action for constructive control by adding voters in fallback elections in the TM model. The maximum is 74.8 seconds.
Figure 133: Average time the algorithm needs to determine no-instance of constructive control by adding voters in fallback elections in the TM model. The maximum is 233.48 seconds.

Figure 134: Average time the algorithm needs to give a definite output for constructive control by adding voters in fallback elections in the TM model. The maximum is 29.26 seconds.
A.14. Constructive Control by Deleting Voters

Figure 135: Results for fallback voting in the IC model for constructive control by deleting voters. Number of candidates is fixed.

|   | m = 4 | m = 8 |
|---|-------|-------|
| n | 4     | 8     |
|   | 16    | 32    |
|   | 64    | 128   |
| # cp | 100 | 203 |
|      | 418  | 442  |
|      | 463  | 487  |
| # ci | 400  | 297  |
|      | 82   | 0    |
|      | 0    | 0    |
| # to | 0    | 0    |
|      | 58   | 37   |
|      | 13   |      |

|   | m = 16 | m = 32 |
|---|-------|-------|
| n | 4     | 8     |
|   | 16    | 32    |
|   | 64    | 128   |
| # cp | 44  | 92   |
|      | 277  | 348  |
|      | 372  | 431  |
| # ci | 456  | 408  |
|      | 223  | 7    |
|      | 0    | 0    |
| # to | 0    | 0    |
|      | 145  | 128  |
|      | 69   |      |

|   | m = 64 | m = 128 |
|---|-------|---------|
| n | 4     | 8       |
|   | 16    | 32      |
|   | 64    | 128     |
| # cp | 17  | 43    |
|      | 151  | 198    |
|      | 258  | 348    |
| # ci | 483  | 457    |
|      | 349  | 29     |
|      | 0    | 0      |
| # to | 0    | 0      |
|      | 237  | 242    |
|      | 152  |        |

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Figure 136: Results for fallback voting in the IC model for constructive control by deleting voters. Number of voters is fixed.

|     | $n=4$ |     | $n=8$ |     |
|-----|------|-----|------|-----|
| $m$ | 4    | 8   | 16   | 32  | 64  | 128 |
| # cp| 100  | 57  | 44   | 25  | 17  | 8   |
| # ci| 400  | 443 | 456  | 475 | 483 | 492 |
| # to| 0    | 0   | 0    | 0   | 0   | 0   |

|     | $n=16$ |     | $n=32$ |     |
|-----|--------|-----|--------|-----|
| $m$ | 4      | 8   | 16     | 32  |
| # cp| 418    | 350 | 277    | 212 |
| # ci| 82     | 150 | 223    | 288 |
| # to| 0      | 0   | 2      | 7   |

|     | $n=64$ |     | $n=128$ |     |
|-----|--------|-----|--------|-----|
| $m$ | 4      | 8   | 16     | 32  |
| # cp| 463    | 422 | 372    | 307 |
| # ci| 0      | 0   | 0      | 0   |
| # to| 37     | 78  | 128    | 193 |
Figure 137: Results for fallback voting in the TM model for constructive control by deleting voters. Number of candidates is fixed.

|     | \( m = 4 \) |     | \( m = 8 \) |
|-----|-------------|-----|-------------|
| \( n \) | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 66 91 191 196 174 159 | 43 68 124 118 131 145 |
| # ci | 434 409 309 135 132 133 | 457 432 376 241 219 210 |
| # to | 0 0 0 169 194 208 | 0 0 0 141 150 145 |

|     | \( m = 16 \) |     | \( m = 32 \) |
|-----|-------------|-----|-------------|
| \( n \) | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 33 32 83 86 82 105 | 18 27 64 52 63 62 |
| # ci | 467 468 417 284 285 264 | 482 473 436 359 328 332 |
| # to | 0 0 0 130 133 131 | 0 0 0 89 109 106 |

|     | \( m = 64 \) |     | \( m = 128 \) |
|-----|-------------|-----|-------------|
| \( n \) | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 4 17 27 36 48 44 | 6 8 16 23 27 26 |
| # ci | 496 483 473 384 383 388 | 494 492 473 415 425 415 |
| # to | 0 0 0 80 69 68 | 0 0 11 62 48 59 |
Figure 138: Results for fallback voting in the TM model for constructive control by deleting voters. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |   | $n = 16$ |   | $n = 32$ |   | $n = 64$ |   | $n = 128$ |
|---|---------|---|---------|---|---------|---|---------|---|---------|---|---------|
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 66 | 43 | 33 | 18 | 4 | 6 | 91 | 68 | 32 | 27 | 17 | 8 |
| # ci | 434 | 457 | 467 | 482 | 496 | 494 | 409 | 432 | 468 | 473 | 483 | 492 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 191 | 124 | 83 | 64 | 27 | 16 | 196 | 118 | 86 | 52 | 36 | 23 |
| # ci | 309 | 376 | 417 | 436 | 473 | 473 | 135 | 241 | 284 | 359 | 384 | 415 |
| # to | 0 | 0 | 0 | 0 | 0 | 11 | 169 | 141 | 130 | 89 | 80 | 62 |
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 174 | 131 | 82 | 63 | 48 | 27 | 159 | 145 | 105 | 62 | 44 | 26 |
| # ci | 132 | 219 | 285 | 328 | 383 | 425 | 133 | 210 | 264 | 332 | 388 | 415 |
| # to | 194 | 150 | 133 | 109 | 69 | 48 | 208 | 145 | 131 | 106 | 68 | 59 |
A.14.1. Computational Costs

Figure 139: Average time the algorithm needs to find a successful control action for constructive control by deleting voters in fallback elections in the IC model. The maximum is 79.32 seconds.

Figure 140: Average time the algorithm needs to determine no-instance of constructive control by deleting voters in fallback elections in the IC model. The maximum is 114.99 seconds.
Figure 141: Average time the algorithm needs to give a definite output for constructive control by deleting voters in fallback elections in the IC model. The maximum is 88.57 seconds.

Figure 142: Average time the algorithm needs to find a successful control action for constructive control by deleting voters in fallback elections in the TM model. The maximum is 65.48 seconds.
Figure 143: Average time the algorithm needs to determine no-instance of constructive control by deleting voters in fallback elections in the TM model. The maximum is 42.8 seconds.

Figure 144: Average time the algorithm needs to give a definite output for constructive control by deleting voters in fallback elections in the TM model. The maximum is 42.28 seconds.
A.15. Constructive Control by Partition of Voters in Model TE

![Graph showing percentage of elections where control is possible vs. number of voters for different values of m and n.](image)

Figure 145: Results for fallback voting in the IC model for constructive control by partition of voters in model TE. Number of candidates is fixed.

|       | \( m = 4 \) |       | \( m = 8 \) |       | \( m = 16 \) |       | \( m = 32 \) |       | \( m = 64 \) |       | \( m = 128 \) |
|-------|--------------|-------|-------------|-------|--------------|-------|-------------|-------|--------------|-------|-------------|
| \( n \) | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
| \# cp | 125 | 311 | 451 | 464 | 434 | 353 | 97 | 276 | 457 | 485 | 470 | 448 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| \# ci | 375 | 189 | 49  | 0   | 0   | 0   | 403| 224 | 43  | 0   | 0   | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| \# to | 0   | 0   | 36  | 66  | 147 | 0   | 0  | 0   | 15  | 30  | 52  | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|       | \( m = 16 \) |       | \( m = 32 \) |       | \( m = 64 \) |       | \( m = 128 \) |
| \( n \) | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
| \# cp | 61  | 192 | 441 | 461 | 417 | 469 | 44 | 152 | 388 | 449 | 467 | 480 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| \# ci | 439 | 308 | 1   | 0   | 0   | 0   | 456| 348 | 0   | 0   | 0   | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| \# to | 0   | 0   | 58  | 39  | 29  | 31  | 0  | 0   | 112 | 51  | 33  | 20  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|       | \( m = 64 \) |       | \( m = 128 \) |
| \( n \) | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
| \# cp | 21  | 94  | 323 | 369 | 446 | 481 | 11 | 47  | 239 | 314 | 403 | 461 | 489| 453 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| \# ci | 479 | 406 | 0   | 0   | 0   | 0   | 0  | 0   | 261 | 186 | 97  | 39  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |

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Figure 146: Results for fallback voting in the IC model for constructive control by partition of voters in model TE. Number of voters is fixed.
Figure 147: Results for fallback voting in the TM model for constructive control by partition of voters in model TE. Number of candidates is fixed.

|        | \(m = 4\) | \(m = 8\) | \(m = 16\) | \(m = 32\) | \(m = 64\) | \(m = 128\) |
|--------|------------|------------|-------------|-------------|-------------|-------------|
| \(n\)  | \(4\)     | \(8\)     | \(16\)     | \(32\)     | \(64\)     | \(128\)     |
| # cp   | 70        | 126       | 172        | 164        | 143        | 130        |
| # ci   | 430       | 374       | 328        | 111        | 97         | 82         |
| # to   | 0         | 0         | 0          | 225        | 260        | 288        |

|        | \(m = 16\) | \(m = 32\) | \(m = 64\) | \(m = 128\) |
|--------|------------|------------|-------------|-------------|
| \(n\)  | \(4\)     | \(8\)     | \(16\)     | \(32\)     | \(64\)     | \(128\)     |
| # cp   | 34        | 68        | 104        | 95         | 100        | 124        |
| # ci   | 466       | 432       | 136        | 51         | 40         | 26         |
| # to   | 0         | 0         | 260        | 354        | 360        | 350        |

|        | \(m = 64\) | \(m = 128\) |
|--------|------------|-------------|
| \(n\)  | \(4\)     | \(8\)     | \(16\)     | \(32\)     | \(64\)     | \(128\)     |
| # cp   | 14        | 27        | 58         | 31         | 41         | 45         |
| # ci   | 486       | 473       | 63         | 29         | 8          | 7          |
| # to   | 0         | 0         | 379        | 440        | 451        | 448        |
Figure 148: Results for fallback voting in the TM model for constructive control by partition of voters in model TE. Number of voters is fixed.

| m  | n = 4 | n = 8 | n = 16 | n = 32 | n = 64 | n = 128 |
|----|-------|-------|--------|--------|--------|--------|
|    | # cp  | # ci  | # to   | # cp   | # ci   | # to   |
| 4  | 70    | 430   | 0      | 126    | 374    | 0      |
| 8  | 60    | 440   | 0      | 94     | 406    | 0      |
| 16 | 34    | 466   | 0      | 14     | 432    | 0      |
| 32 | 18    | 482   | 0      | 1        | 453    | 0      |
| 64 | 14    | 499   | 0      | 1       | 472    | 0      |
| 128| 1     | 374    | 0      | 1       | 484    | 0      |

| m  | n = 4 | n = 8 | n = 16 | n = 32 | n = 64 | n = 128 |
|----|-------|-------|--------|--------|--------|--------|
|    | # cp  | # ci  | # to   | # cp   | # ci   | # to   |
| 4  | 172   | 328   | 0      | 164    | 225    | 0      |
| 8  | 149   | 351   | 0      | 127    | 278    | 0      |
| 16 | 104   | 136   | 0      | 72     | 95     | 0      |
| 32 | 74    | 63    | 0      | 63     | 51     | 0      |
| 64 | 37    | 56    | 0      | 37     | 32     | 0      |
| 128| 37    | 111   | 0      | 111    | 29     | 0      |
|     |       | 95    | 0      | 95     | 9      | 0      |

| m  | n = 4 | n = 8 | n = 16 | n = 32 | n = 64 | n = 128 |
|----|-------|-------|--------|--------|--------|--------|
|    | # cp  | # ci  | # to   | # cp   | # ci   | # to   |
| 4  | 143   | 97    | 0      | 130    | 82     | 0      |
| 8  | 130   | 55    | 0      | 127    | 49     | 0      |
| 16 | 100   | 40    | 0      | 124    | 26     | 0      |
| 32 | 80    | 22    | 0      | 80     | 22     | 0      |
| 64 | 41    | 8     | 0      | 41     | 10     | 0      |
| 128| 28    | 2     | 0      | 28     | 2      | 0      |

| m  | n = 4 | n = 8 | n = 16 | n = 32 | n = 64 | n = 128 |
|----|-------|-------|--------|--------|--------|--------|
|    | # cp  | # ci  | # to   | # cp   | # ci   | # to   |
| 4  | 260   | 288   | 0      | 324    | 388    | 0      |
| 8  | 315   | 336   | 0      | 350    | 398    | 0      |
| 16 | 360   | 451   | 0      | 350    | 462    | 0      |
| 32 | 398   | 462   | 0      | 398    | 462    | 0      |
| 64 | 451   | 462   | 0      | 462    | 467    | 0      |
| 128| 462   | 467   | 0      | 462    | 467    | 0      |
A.15.1. Computational Costs

Figure 149: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TE in fallback elections in the IC model. The maximum is 53,51 seconds.

Figure 150: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TE in fallback elections in the IC model. The maximum is 586,56 seconds.
Figure 151: Average time the algorithm needs to give a definite output for constructive control by adding voters in fallback elections in the IC model. The maximum is 53.51 seconds.

Figure 152: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TE in fallback elections in the TM model. The maximum is 61.41 seconds.
Figure 153: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TE in fallback elections in the TM model. The maximum is 289.18 seconds.

Figure 154: Average time the algorithm needs to give a definite output for constructive control by partition of voters in model TE in fallback elections in the TM model. The maximum is 208.95 seconds.
A.16. Destructive Control by Partition of Voters in Model TE

Figure 155: Results for fallback voting in the IC model for destructive control by partition of voters in model TE. Number of candidates is fixed.

| n | m = 4 | m = 8 | m = 16 | m = 32 | m = 64 | m = 128 |
|---|-------|-------|--------|--------|--------|---------|
|   | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 251 391 478 474 405 287 | 369 477 500 493 454 406 | 432 491 499 479 458 | 465 497 500 499 482 466 | 480 498 500 494 474 | 489 500 500 499 497 493 |
| # ci | 249 109 22 0 0 0 | 131 23 0 0 0 0 | 68 9 1 0 0 0 | 35 3 0 0 0 0 | 20 2 0 0 0 0 | 11 0 0 0 0 0 |
| # to | 0 0 0 26 95 213 | 0 0 0 7 46 94 | 0 0 0 2 21 42 | 0 0 0 1 18 34 | 0 0 0 6 26 6 | 0 0 0 1 3 7 |
Figure 156: Results for fallback voting in the IC model for destructive control by partition of voters in model TE. Number of voters is fixed.

| n  | m  | cp  | ci  | to  |
|----|----|-----|-----|-----|
| 4  | 4  | 251 | 249 | 0   |
| 8  | 8  | 369 | 131 | 0   |
| 16 | 16 | 432 | 68  | 0   |
| 32 | 32 | 465 | 35  | 0   |
| 64 | 64 | 480 | 20  | 0   |
| 128| 128| 489 | 11  | 0   |

| n  | m  | cp  | ci  | to  |
|----|----|-----|-----|-----|
| 4  | 4  | 391 | 109 | 0   |
| 8  | 8  | 477 | 23  | 0   |
| 16 | 16 | 491 | 9   | 0   |
| 32 | 32 | 497 | 3   | 0   |
| 64 | 64 | 498 | 2   | 0   |
| 128| 128| 500 | 0   | 0   |

| n  | m  | cp  | ci  | to  |
|----|----|-----|-----|-----|
| 16 | 4  | 478 | 22  | 0   |
| 8  | 8  | 500 | 0   | 0   |
| 16 | 16 | 500 | 0   | 0   |
| 32 | 32 | 500 | 0   | 0   |
| 64 | 64 | 500 | 0   | 0   |
| 128| 128| 500 | 0   | 0   |

| n  | m  | cp  | ci  | to  |
|----|----|-----|-----|-----|
| 16 | 4  | 474 | 0   | 0   |
| 8  | 8  | 493 | 0   | 0   |
| 16 | 16 | 498 | 0   | 0   |
| 32 | 32 | 499 | 0   | 0   |
| 64 | 64 | 499 | 0   | 0   |
| 128| 128| 500 | 0   | 0   |

| n  | m  | cp  | ci  | to  |
|----|----|-----|-----|-----|
| 64 | 4  | 405 | 0   | 0   |
| 8  | 8  | 454 | 0   | 0   |
| 16 | 16 | 479 | 0   | 0   |
| 32 | 32 | 482 | 0   | 0   |
| 64 | 64 | 494 | 0   | 0   |
| 128| 128| 497 | 0   | 0   |

| n  | m  | cp  | ci  | to  |
|----|----|-----|-----|-----|
| 64 | 4  | 287 | 0   | 0   |
| 8  | 8  | 406 | 0   | 0   |
| 16 | 16 | 458 | 0   | 0   |
| 32 | 32 | 466 | 0   | 0   |
| 64 | 64 | 474 | 0   | 0   |
| 128| 128| 493 | 0   | 0   |

| n  | m  | cp  | ci  | to  |
|----|----|-----|-----|-----|
| 128| 4  | 213 | 95  | 0   |
| 128| 8  | 94  | 46  | 0   |
| 128| 16 | 42  | 21  | 0   |
| 128| 32 | 34  | 18  | 0   |
| 128| 64 | 26  | 14  | 0   |
| 128| 128| 7   | 6   | 0   |
Figure 157: Results for fallback voting in the TM model for destructive control by partition of voters in model TE. Number of candidates is fixed.

|  | \(m = 4\) |  | \(m = 8\) |  | \(m = 16\) |  | \(m = 32\) |  | \(m = 64\) |  | \(m = 128\) |
|---|---|---|---|---|---|---|---|---|---|---|
| \(n\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 178 | 237 | 247 | 218 | 169 | 153 | 217 | 294 | 325 | 321 | 231 | 229 |
| \# ci | 322 | 263 | 253 | 132 | 95 | 79 | 283 | 206 | 175 | 86 | 64 | 49 |
| \# to | 0 | 0 | 0 | 150 | 236 | 268 | 0 | 0 | 0 | 93 | 205 | 222 |
| \(m = 16\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 272 | 320 | 373 | 383 | 300 | 244 | 325 | 369 | 410 | 399 | 334 | 266 |
| \# ci | 228 | 180 | 110 | 48 | 41 | 35 | 175 | 131 | 63 | 35 | 16 | 17 |
| \# to | 0 | 0 | 17 | 69 | 159 | 221 | 0 | 0 | 27 | 66 | 150 | 217 |
| \(m = 64\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 352 | 375 | 414 | 425 | 333 | 258 | 343 | 384 | 437 | 440 | 354 | 283 |
| \# ci | 148 | 125 | 64 | 23 | 10 | 11 | 157 | 116 | 46 | 31 | 3 | 5 |
| \# to | 0 | 0 | 0 | 52 | 157 | 231 | 0 | 0 | 17 | 29 | 143 | 212 |
Figure 158: Results for fallback voting in the TM model for destructive control by partition of voters in model TE. Number of voters is fixed.

|       | $n = 4$ |       | $n = 8$ |
|-------|---------|-------|---------|
| $m$   | 4       | 8     | 16      | 32      | 64     | 128    | 4       | 8       | 16      | 32      | 64     | 128    |
| # cp  | 178     | 217   | 272     | 325     | 352    | 343    | 237     | 294     | 320     | 369     | 375    | 384    |
| # ci  | 322     | 283   | 228     | 175     | 148    | 157    | 263     | 206     | 180     | 131     | 125    | 116    |
| # to  | 0       | 0     | 0       | 0       | 0      | 0      | 0       | 0       | 0       | 0       | 0      | 0      |

|       | $n = 16$ |       | $n = 32$ |
|-------|---------|-------|---------|
| $m$   | 4       | 8     | 16      | 32      | 64     | 128    | 4       | 8       | 16      | 32      | 64     | 128    |
| # cp  | 247     | 325   | 373     | 410     | 414    | 437    | 218     | 321     | 383     | 399     | 425    | 440    |
| # ci  | 253     | 175   | 110     | 63      | 64     | 46     | 132     | 86      | 48      | 35      | 23     | 31     |
| # to  | 0       | 0     | 17      | 27      | 22     | 17     | 150     | 93      | 69      | 66      | 52     | 29     |

|       | $n = 64$ |       | $n = 128$ |
|-------|---------|-------|-----------|
| $m$   | 4       | 8     | 16       | 32       | 64     | 128    | 4       | 8       | 16      | 32      | 64     | 128    |
| # cp  | 169     | 231   | 300      | 334      | 333    | 354    | 153     | 229     | 244     | 266     | 258    | 283    |
| # ci  | 95      | 64    | 41       | 16       | 10     | 3      | 79      | 49      | 35      | 17      | 11     | 5      |
| # to  | 236     | 205   | 159      | 150      | 157    | 143    | 268     | 222     | 221     | 217     | 231    | 212    |
A.16.1. Computational Costs

Figure 159: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TE in fallback elections in the IC model. The maximum is 35.22 seconds.

Figure 160: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TE in fallback elections in the IC model. The maximum is 596.35 seconds.
Figure 161: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TE in fallback elections in the IC model. The maximum is 35.22 seconds.

Figure 162: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TE in fallback elections in the TM model. The maximum is 29.74 seconds.
Figure 163: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TE in fallback elections in the TM model. The maximum is 121, 13 seconds.

Figure 164: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TE in fallback elections in the TM model. The maximum is 48, 7 seconds.
A.17. Constructive Control by Partition of Voters in Model TP

Figure 165: Results for fallback voting in the IC model for constructive control by partition of voters in model TP. Number of candidates is fixed.

|        | $m = 4$ |        | $m = 8$ |        | $m = 16$ |        | $m = 32$ |        | $m = 64$ |        | $m = 128$ |
|--------|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|----------|
|        | n=4     | n=8    | n=16    | n=32   | n=64    | n=128  | n=4     | n=8    | n=16    | n=32   | n=64    |
| # cp   | 62      | 108    | 207     | 206    | 162     | 112    | 46      | 108    | 236     | 242    | 195     |
| # ci   | 438     | 392    | 293     | 0      | 0       | 0      | 454     | 392    | 232     | 0      | 0       |
| # to   | 0       | 0      | 294     | 338    | 388     | 0      | 0       | 32     | 258     | 305    | 371     |

|        | $m = 16$ |        | $m = 32$ |        | $m = 64$ |        | $m = 128$ |
|--------|---------|--------|---------|--------|---------|--------|----------|
|        | n=4     | n=8    | n=16    | n=32   | n=64    | n=128  | n=4     | n=8    | n=16    | n=32   | n=64    |
| # cp   | 34      | 117    | 263     | 252    | 166     | 122    | 30      | 91     | 201     | 231    | 166     |
| # ci   | 466     | 383    | 0       | 0      | 0       | 0      | 470     | 409    | 0       | 0      | 0       |
| # to   | 0       | 0      | 237     | 248    | 334     | 378    | 0       | 0      | 299     | 269    | 334     |

|        | $m = 64$ |        | $m = 128$ |
|--------|---------|--------|----------|
|        | n=4     | n=8    | n=16    | n=32   | n=64    | n=128  | n=4     | n=8    | n=16    | n=32   | n=64    |
| # cp   | 5       | 69     | 182     | 188    | 160     | 102    | 10      | 46     | 139     | 194    | 122     |
| # ci   | 495     | 431    | 0       | 0      | 0       | 0      | 490     | 454    | 0       | 0      | 0       |
| # to   | 0       | 0      | 318     | 312    | 340     | 398    | 0       | 0      | 361     | 306    | 378     |

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Figure 166: Results for fallback voting in the IC model for constructive control by partition of voters in model TP. Number of voters is fixed.

|      | \(n=4\) | \(n=8\) | \(n=16\) | \(n=32\) | \(n=64\) | \(n=128\) |
|------|----------|----------|-----------|-----------|-----------|-----------|
|      |          |          | \(m\) \(=4\) | \(m\) \(=8\) | \(m\) \(=16\) | \(m\) \(=32\) | \(m\) \(=64\) | \(m\) \(=128\) |
| \(#\ cp\) |          |          | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \(#\ ci\) |          |          | 438 | 454 | 466 | 470 | 495 | 490 | 392 | 392 | 383 | 409 | 431 | 454 |
| \(#\ to\) |          |          | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

|      | \(n=64\) | \(n=128\) |
|------|----------|-----------|
| \(m\) |          | \(m\) \(=4\) | \(m\) \(=8\) | \(m\) \(=16\) | \(m\) \(=32\) | \(m\) \(=64\) | \(m\) \(=128\) |
| \(#\ cp\) |          |          | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \(#\ ci\) |          |          | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \(#\ to\) |          |          | 338 | 305 | 334 | 334 | 340 | 378 | 388 | 371 | 378 | 383 | 398 | 369 |
Figure 167: Results for fallback voting in the TM model for constructive control by partition of voters in model TP. Number of candidates is fixed.

| n   | m = 4 | m = 8 |
|-----|-------|-------|
|     | 4     | 8     | 16   | 32   | 64   | 128  |
|     | 4     | 8     | 16   | 32   | 64   | 128  |
| # cp| 61    | 76    | 99   | 85   | 67   | 66   |
| # cp| 61    | 76    | 99   | 85   | 67   | 66   |
| # ci| 439   | 424   | 401  | 111  | 97   | 82   |
| # ci| 471   | 436   | 390  | 95   | 55   | 49   |
| # to| 0     | 0     | 304  | 336  | 352  | 0    |
| # to| 0     | 0     | 304  | 336  | 352  | 0    |

| n   | m = 16 | m = 32 |
|-----|--------|-------|
|     | 4      | 8     | 16   | 32   | 64   | 128  |
|     | 4      | 8     | 16   | 32   | 64   | 128  |
| # cp| 17     | 52    | 72   | 57   | 42   | 48   |
| # cp| 17     | 32    | 60   | 34   | 41   | 40   |
| # ci| 483    | 448   | 89   | 50   | 40   | 26   |
| # ci| 483    | 468   | 70   | 33   | 22   | 14   |
| # to| 0      | 0     | 304  | 336  | 352  | 0    |
| # to| 0      | 0     | 304  | 336  | 352  | 0    |

| n   | m = 64 | m = 128 |
|-----|--------|---------|
|     | 4      | 8       | 16     | 32     | 64     | 128    |
|     | 4      | 8       | 16     | 32     | 64     | 128    |
| # cp| 12     | 25      | 49     | 20     | 27     | 25     |
| # cp| 12     | 25      | 49     | 20     | 27     | 25     |
| # ci| 488    | 475     | 62     | 27     | 8      | 7      |
| # ci| 488    | 475     | 62     | 27     | 8      | 7      |
| # to| 0      | 0       | 389    | 453    | 465    | 468    |
| # to| 0      | 0       | 389    | 453    | 465    | 468    |
Figure 168: Results for fallback voting in the TM model for constructive control by partition of voters in model TP. Number of voters is fixed.

|      | $n=4$          |        | $n=8$          |        | $n=16$         |        | $n=32$         |        | $n=64$         |        | $n=128$        |        |
|------|----------------|--------|----------------|--------|----------------|--------|----------------|--------|----------------|--------|----------------|--------|
|      | $m$ 4 8 16 32 64 128 |        | $m$ 4 8 16 32 64 128 |        |
| # cp | 61 29 17 12 5 64 52 25 10 |        | 76 64 52 32 25 10 |        |
| # ci | 439 471 483 488 495 |        | 424 436 448 468 475 490 |        |
| # to | 0 0 0 0 0 0 0 0 0 |        | 0 0 0 0 0 0 0 0 0 |        |
|      | $n=16$          |        | $n=32$          |        |
| # cp | 99 96 72 60 49 31 85 76 57 34 20 6 |        | 85 76 57 34 20 6 |        |
| # ci | 401 390 89 70 62 57 111 95 50 33 27 23 |        | 111 95 50 33 27 23 |        |
| # to | 0 14 339 370 389 412 |        | 304 329 393 433 453 471 |        |
|      | $n=64$          |        | $n=128$         |        |
| # cp | 67 71 42 41 27 14 66 51 48 40 25 13 |        | 82 49 26 14 7 2 |        |
| # ci | 97 55 40 22 8 1 82 49 26 14 7 2 |        | 82 49 26 14 7 2 |        |
| # to | 336 374 418 437 465 476 |        | 352 400 446 446 468 485 |        |
A.17.1. Computational Costs

Figure 169: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TP in fallback elections in the IC model. The maximum is 89.37 seconds.

Figure 170: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TP in fallback elections in the IC model. The maximum is 564.65 seconds.
Figure 171: Average time the algorithm needs to give a definite output for constructive control by partition of voters in model TP in fallback elections in the IC model. The maximum is 301,87 seconds.

Figure 172: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TP in fallback elections in the TM model. The maximum is 106,58 seconds.
Figure 173: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TP in fallback elections in the TM model. The maximum is 393,18 seconds.

Figure 174: Average time the algorithm needs to give a definite output for constructive control by partition of voters in model TP in fallback elections in the TM model. The maximum is 336,56 seconds.
A.18. Destructive Control by Partition of Voters in Model TP

Figure 175: Results for fallback voting in the IC model for destructive control by partition of voters in model TP. Number of candidates is fixed.

\[
\begin{array}{c|cccccc}
\text{n} & 4 & 8 & 16 & 32 & 64 & 128 \\
\text{# cp} & 185 & 267 & 411 & 302 & 196 & \\
\text{# ci} & 315 & 233 & 79 & 0 & 0 & 0 \\
\text{# to} & 0 & 0 & 0 & 89 & 198 & 304 \\
\end{array}
\]

\[
\begin{array}{c|cccccc}
\text{m=16} & 4 & 8 & 16 & 32 & 64 & 128 \\
\text{# cp} & 407 & 480 & 498 & 473 & 425 & \\
\text{# ci} & 93 & 20 & 0 & 0 & 0 & 0 \\
\text{# to} & 0 & 0 & 2 & 4 & 27 & 75 \\
\end{array}
\]

\[
\begin{array}{c|cccccc}
\text{m=32} & 4 & 8 & 16 & 32 & 64 & 128 \\
\text{# cp} & 480 & 494 & 500 & 499 & 495 & 474 \\
\text{# ci} & 20 & 6 & 0 & 0 & 0 & 0 \\
\text{# to} & 0 & 0 & 0 & 1 & 5 & 26 \\
\end{array}
\]

\[
\begin{array}{c|cccccc}
\text{m=64} & 4 & 8 & 16 & 32 & 64 & 128 \\
\text{# cp} & 489 & 500 & 499 & 495 & 474 & 489 \\
\text{# ci} & 20 & 6 & 0 & 0 & 0 & 0 \\
\text{# to} & 0 & 0 & 0 & 1 & 3 & 7 \\
\end{array}
\]
Figure 176: Results for fallback voting in the IC model for destructive control by partition of voters in model TP. Number of voters is fixed.

| n   | m   | # cp | # ci | # to | n   | m   | # cp | # ci | # to |
|-----|-----|------|------|------|-----|-----|------|------|------|
|     | 4   | 8    | 16   | 32   | 64  | 128 | 4    | 8    | 16   |
|     |     | 4    | 8    | 16   | 32   | 64  | 128  |      |      |
|     |     | 185  | 310  | 407  | 455  | 480 | 489  | 267  | 435  |
|     |     | 315  | 190  | 93   | 45   | 20  | 11   | 233  | 65   |
|     |     | 0    | 0    | 0    | 0    | 0   | 0    | 0    | 0    |
| n = 16 |     | 421  | 497  | 498  | 500  | 500 | 500  | 411  | 484  |
| n = 16 |     | 79   | 3    | 0    | 0    | 0   | 0    | 0    | 0    |
| n = 16 |     | 0    | 0    | 2    | 0    | 0   | 0    | 0    | 0    |
| n = 16 |     | 89   | 16   | 4    | 2    | 1   | 1    |      |      |
| n = 32 |     |      |      |      |      |      |      |      |      |
| n = 64 |     |      |      |      |      |      |      |      |      |
| n = 64 |     |      |      |      |      |      |      |      |      |
| n = 64 |     |      |      |      |      |      |      |      |      |
| n = 128|     |      |      |      |      |      |      |      |      |
| n = 128|     |      |      |      |      |      |      |      |      |
| n = 128|     |      |      |      |      |      |      |      |      |

| n   | m   | # cp | # ci | # to | n   | m   | # cp | # ci | # to |
|-----|-----|------|------|------|-----|-----|------|------|------|
| 4   | 8   | 16   | 32   | 64   | 128 | 4   | 8    | 16   | 32   |
|     |     | 4    | 8    | 16   | 32   | 64  | 128  |      |      |
|     |     | 198  | 73   | 27   | 15   | 5   | 3    | 304  | 143  |
|     |     | 0    | 0    | 0    | 0    | 0   | 0    | 0    | 0    |
|     |     | 0    | 0    | 0    | 0    | 0   | 0    | 0    | 0    |
|     |     | 198  | 73   | 27   | 15   | 5   | 3    | 304  | 143  |
|     |     | 0    | 0    | 0    | 0    | 0   | 0    | 0    | 0    |
|     |     | 0    | 0    | 0    | 0    | 0   | 0    | 0    | 0    |
|     |     | 0    | 0    | 0    | 0    | 0   | 0    | 0    | 0    |
Figure 177: Results for fallback voting in the TM model for destructive control by partition of voters in model TP. Number of candidates is fixed.

|     | $m = 4$ |     | $m = 8$ | $m = 16$ |     | $m = 32$ | $m = 64$ |     | $m = 128$ |
|-----|---------|-----|---------|----------|-----|----------|----------|-----|----------|
| $n$ | 4  8   | 16 | 32     | 64  128  | 4  8 | 16  32   | 64  128  | 4  8 | 16  32   |
| # cp | 148  192  218  173  145  133 | 194  265  317  301  228  234 | 260  315  364  374  304  258 | 319  365  407  391  340  283 | 351  375  413  420  340  281 | 341  385  436  437  361  297 | 149  125  64  23  10  11 | 159  115  46  31  3  5 |
| # ci | 352  308  282  132  95  79  | 306  235  182  86  64  49  | 0  0  1  113  208  217  | 234  181  63  35  16  17  | 0  0  40  78  155  207  | 0  0  30  74  144  200  | 149  125  64  23  10  11 | 159  115  46  31  3  5 |
| # to | 0  0  195  260  288  | 0  0  1  113  208  217  | 260  315  364  374  304  258 | 319  365  407  391  340  283 | 351  375  413  420  340  281 | 341  385  436  437  361  297 | 149  125  64  23  10  11 | 159  115  46  31  3  5 |

|     | $m = 4$ |     | $m = 8$ | $m = 16$ |     | $m = 32$ | $m = 64$ |     | $m = 128$ |
|-----|---------|-----|---------|----------|-----|----------|----------|-----|----------|
| $n$ | 4  8   | 16 | 32     | 64  128  | 4  8 | 16  32   | 64  128  | 4  8 | 16  32   |
| # cp | 148  192  218  173  145  133 | 194  265  317  301  228  234 | 260  315  364  374  304  258 | 319  365  407  391  340  283 | 351  375  413  420  340  281 | 341  385  436  437  361  297 | 149  125  64  23  10  11 | 159  115  46  31  3  5 |
| # ci | 352  308  282  132  95  79  | 306  235  182  86  64  49  | 0  0  1  113  208  217  | 234  181  63  35  16  17  | 0  0  40  78  155  207  | 0  0  30  74  144  200  | 149  125  64  23  10  11 | 159  115  46  31  3  5 |
| # to | 0  0  195  260  288  | 0  0  1  113  208  217  | 260  315  364  374  304  258 | 319  365  407  391  340  283 | 351  375  413  420  340  281 | 341  385  436  437  361  297 | 149  125  64  23  10  11 | 159  115  46  31  3  5 |
Figure 178: Results for fallback voting in the TM model for destructive control by partition of voters in model TP. Number of voters is fixed.

|   | n = 4 |   | n = 8 |
|---|---|---|---|
|   | m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 148 | 194 | 260 | 319 | 351 | 341 | 192 | 265 | 315 | 365 | 375 | 385 |
| # ci | 352 | 306 | 240 | 181 | 149 | 159 | 308 | 235 | 185 | 135 | 125 | 115 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|   | n = 16 |   | n = 32 |
|   | m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 218 | 317 | 364 | 407 | 413 | 436 | 173 | 301 | 374 | 391 | 420 | 437 |
| # ci | 282 | 182 | 96 | 63 | 64 | 46 | 132 | 86 | 48 | 35 | 23 | 31 |
| # to | 0 | 1 | 40 | 30 | 23 | 18 | 195 | 113 | 78 | 74 | 57 | 32 |
|   | n = 64 |   | n = 128 |
|   | m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 145 | 228 | 304 | 340 | 340 | 361 | 133 | 234 | 258 | 283 | 281 | 297 |
| # ci | 95 | 64 | 41 | 16 | 10 | 3 | 79 | 49 | 35 | 17 | 11 | 5 |
| # to | 260 | 208 | 155 | 144 | 150 | 136 | 288 | 217 | 207 | 200 | 208 | 198 |
A.18.1. Computational Costs

Figure 179: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TP in fallback elections in the IC model. The maximum is 60.49 seconds.

Figure 180: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TP in fallback elections in the IC model. The maximum is 527.4 seconds.
Figure 181: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TP in fallback elections in the IC model. The maximum is 82.22 seconds.

Figure 182: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TP in fallback elections in the TM model. The maximum is 42 seconds.
Figure 183: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TP in fallback elections in the TM model. The maximum is 173.62 seconds.

Figure 184: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TP in fallback elections in the TM model. The maximum is 93.34 seconds.
B. Bucklin Voting

B.1. Constructive Control by Adding Candidates

Figure 185: Results for Bucklin voting in the IC model for constructive control by adding candidates. Number of candidates is fixed.

|     | \(m = 4\)       | \(m = 8\)       | \(m = 16\)      | \(m = 32\)      |
|-----|-----------------|-----------------|-----------------|-----------------|
| \(n\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) |
| \# cp | 40 | 62 | 73 | 77 | 78 | 107 | 42 | 48 | 77 | 78 | 85 | 95 | 42 | 48 | 77 | 78 | 85 | 95 |
| \# ci | 460 | 438 | 427 | 423 | 422 | 393 | 458 | 452 | 423 | 422 | 415 | 405 | 458 | 452 | 423 | 422 | 415 | 405 |
| \# to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

|     | \(m = 16\)      | \(m = 32\)      |
|-----|-----------------|-----------------|
| \(n\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) |
| \# cp | 29 | 61 | 70 | 108 | 115 | 117 | 17 | 25 | 30 | 38 | 44 | 64 | 17 | 25 | 30 | 38 | 44 | 64 |
| \# ci | 471 | 439 | 430 | 392 | 385 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# to | 0 | 0 | 0 | 0 | 0 | 0 | 483 | 475 | 470 | 462 | 456 | 436 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

|     | \(m = 64\)      | \(m = 128\)      |
|-----|-----------------|-----------------|
| \(n\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) | \(4\) | \(8\) | \(16\) | \(32\) | \(64\) | \(128\) |
| \# cp | 5 | 12 | 10 | 15 | 16 | 15 | 0 | 4 | 2 | 7 | 5 | 8 | 0 | 4 | 2 | 7 | 5 | 8 |
| \# ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# to | 495 | 488 | 490 | 485 | 484 | 485 | 500 | 496 | 498 | 493 | 495 | 492 | 500 | 496 | 498 | 493 | 495 | 492 |
Figure 186: Results for Bucklin voting in the IC model for constructive control by adding candidates. Number of voters is fixed.

| $n = 4$ | $n = 8$ |
|---------|---------|
| $m$     | # cp    | # ci    | # to    | # cp    | # ci    | # to    |
| 4       | 40      | 460     | 0       | 4       | 40      | 460     |
| 8       | 42      | 458     | 0       | 8       | 42      | 458     |
| 16      | 42      | 471     | 0       | 16      | 42      | 471     |
| 32      | 42      | 471     | 0       | 32      | 42      | 471     |
| 64      | 42      | 471     | 0       | 64      | 42      | 471     |
| 128     | 10      | 471     | 0       | 128     | 10      | 471     |

| $n = 16$ | $n = 32$ |
|---------|---------|
| $m$     | # cp    | # ci    | # to    | # cp    | # ci    | # to    |
| 4       | 40      | 460     | 0       | 4       | 40      | 460     |
| 8       | 42      | 458     | 0       | 8       | 42      | 458     |
| 16      | 42      | 471     | 0       | 16      | 42      | 471     |
| 32      | 42      | 471     | 0       | 32      | 42      | 471     |
| 64      | 42      | 471     | 0       | 64      | 42      | 471     |
| 128     | 10      | 471     | 0       | 128     | 10      | 471     |

| $n = 64$ | $n = 128$ |
|---------|---------|
| $m$     | # cp    | # ci    | # to    | # cp    | # ci    | # to    |
| 4       | 40      | 460     | 0       | 4       | 40      | 460     |
| 8       | 42      | 458     | 0       | 8       | 42      | 458     |
| 16      | 42      | 471     | 0       | 16      | 42      | 471     |
| 32      | 42      | 471     | 0       | 32      | 42      | 471     |
| 64      | 42      | 471     | 0       | 64      | 42      | 471     |
| 128     | 10      | 471     | 0       | 128     | 10      | 471     |

| $n = 4$ | $n = 8$ |
|---------|---------|
| $m$     | # cp    | # ci    | # to    | # cp    | # ci    | # to    |
| 4       | 40      | 460     | 0       | 4       | 40      | 460     |
| 8       | 42      | 458     | 0       | 8       | 42      | 458     |
| 16      | 42      | 471     | 0       | 16      | 42      | 471     |
| 32      | 42      | 471     | 0       | 32      | 42      | 471     |
| 64      | 42      | 471     | 0       | 64      | 42      | 471     |
| 128     | 10      | 471     | 0       | 128     | 10      | 471     |
Figure 187: Results for Bucklin voting in the TM model for constructive control by adding candidates. Number of candidates is fixed.

|     | \(m = 4\) |     | \(m = 8\) |     | \(m = 16\) |     | \(m = 32\) |     | \(m = 64\) |     | \(m = 128\) |
|-----|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|----------|
| \(n\) | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| \# cp | 33 33 39 43 53 | 19 28 31 38 40 50 | 0 0 0 0 0 | 0 0 0 0 0 | 0 0 0 0 0 | 0 0 0 0 0 |
| \# ci | 467 467 461 457 465 447 | 481 472 469 462 460 450 | 487 485 476 463 468 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \# to | 0 0 0 0 0 | 0 0 0 0 0 | 0 0 0 0 0 | 0 0 0 0 0 | 0 0 0 0 0 | 0 0 0 0 0 |
| \(m = 16\) | \(m = 32\) | \(m = 64\) | \(m = 128\) | \(m = 64\) | \(m = 128\) |
| \# cp | 13 15 24 37 32 32 | 10 20 12 8 8 4 | 5 4 1 3 3 1 | 0 1 2 2 0 1 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \# ci | 487 485 476 463 468 0 | 0 0 0 0 0 0 | 490 480 488 492 492 492 | 495 496 499 497 497 499 | 500 499 498 498 500 499 | 500 499 498 498 500 499 |
Figure 188: Results for Bucklin voting in the TM model for constructive control by adding candidates. Number of voters is fixed.

|       | $n=4$               |       | $n=8$               |       |
|-------|---------------------|-------|---------------------|-------|
| $m$   | 4  8  16  32  64  128 | $n=4$ | 4  8  16  32  64  128 | $n=8$ |
| # cp  | 33  19  13  10  5  0    | # cp  | 33  28  15  20  4  1    |       |
| # ci  | 467 481 487 0 0 0     | # ci  | 467 472 485 0 0 0     |       |
| # to  | 0 0 0 490 495 500   | # to  | 0 0 0 480 496 499   |       |
|       | $n=16$               |       | $n=32$               |       |
| $m$   | 4  8  16  32  64  128 | $n=16$ | 4  8  16  32  64  128 | $n=32$ |
| # cp  | 39  31  24  12  1  2   | # cp  | 43  38  37  8  3  2    |       |
| # ci  | 461 469 476 0 0 0     | # ci  | 457 462 463 0 0 0     |       |
| # to  | 0 0 0 488 499 498   | # to  | 0 0 0 492 497 498   |       |
|       | $n=64$               |       | $n=128$              |       |
| $m$   | 4  8  16  32  64  128 | $n=64$ | 4  8  16  32  64  128 | $n=128$ |
| # cp  | 35  40  32  8  3  0    | # cp  | 53  50  32  4  1  1    |       |
| # ci  | 465 460 468 0 0 0     | # ci  | 447 450 0 0 0 0     |       |
| # to  | 0 0 0 492 497 500   | # to  | 0 0 468 496 499 499   |       |
B.1.1. Computational Costs

Figure 189: Average time the algorithm needs to find a successful control action for constructive control by adding candidates in Bucklin elections in the IC model. The maximum is 148 seconds.

Figure 190: Average time the algorithm needs to determine no-instance of constructive control by adding candidates in Bucklin elections in the IC model. The maximum is 537.35 seconds.
Figure 191: Average time the algorithm needs to give a definite output for constructive control by adding candidates in Bucklin elections in the IC model. The maximum is 427.51 seconds.

Figure 192: Average time the algorithm needs to find a successful control action for constructive control by adding candidates in Bucklin elections in the TM model. The maximum is 85.81 seconds.
Figure 193: Average time the algorithm needs to determine no-instance of constructive control by adding candidates in Bucklin elections in the TM model. The maximum is 523.05 seconds.

Figure 194: Average time the algorithm needs to give a definite output for constructive control by adding candidates in Bucklin elections in the TM model. The maximum is 491.13 seconds.
B.2. Destructive Control by Adding Candidates

Figure 195: Results for Bucklin voting in the IC model for destructive control by adding candidates. Number of candidates is fixed.

| m = 4 | m = 8 |
|-------|-------|
| n     | m     | n     | m     |
| 4     | 357   | 4     | 405   |
| 8     | 374   | 8     | 426   |
| 16    | 360   | 16    | 444   |
| 32    | 377   | 32    | 456   |
| 64    | 371   | 64    | 468   |
| 128   | 389   | 128   | 447   |

| m = 16 | m = 32 |
|--------|--------|
| n     | m     | n     | m     |
| 4     | 430   | 4     | 597   |
| 8     | 463   | 8     | 453   |
| 16    | 483   | 16    | 468   |
| 32    | 492   | 32    | 484   |
| 64    | 493   | 64    | 486   |
| 128   | 497   | 128   | 485   |

| m = 64 | m = 128 |
|--------|---------|
| n     | m     | n     | m     |
| 4     | 399   | 4     | 487   |
| 8     | 420   | 8     | 449   |
| 16    | 427   | 16    | 409   |
| 32    | 456   | 32    | 409   |
| 64    | 456   | 64    | 435   |
| 128   | 459   | 128   | 438   |

| m = 16 | m = 128 |
|--------|---------|
| n     | m     | n     | m     |
| 4     | 101   | 4     | 113   |
| 8     | 80    | 8     | 89    |
| 16    | 73    | 16    | 91    |
| 32    | 44    | 32    | 91    |
| 64    | 44    | 64    | 65    |
| 128   | 41    | 128   | 62    |
Figure 196: Results for Bucklin voting in the IC model for destructive control by adding candidates. Number of voters is fixed.

| m   | n = 4 | n = 8 |
|-----|-------|-------|
|     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp | 357   | 405   | 430   | 397   | 399   | 387   | 374   | 426   | 463   | 453   | 420   | 411   |
| # ci | 143   | 95    | 70    | 1     | 0     | 0     | 126   | 74    | 37    | 0     | 0     | 0     |
| # to | 0     | 0     | 0     | 102   | 101   | 113   | 0     | 0     | 0     | 47    | 80    | 89    |

| m   | n = 16 | n = 32 |
|-----|--------|--------|
|     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp | 360   | 444   | 483   | 468   | 427   | 409   | 377   | 456   | 492   | 484   | 456   | 409   |
| # ci | 140   | 56    | 17    | 0     | 0     | 0     | 123   | 44    | 8     | 0     | 0     | 0     |
| # to | 0     | 0     | 0     | 32    | 73    | 91    | 0     | 0     | 0     | 16    | 44    | 91    |

| m   | n = 64 | n = 128 |
|-----|--------|---------|
|     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp | 371   | 468   | 493   | 486   | 456   | 435   | 389   | 447   | 497   | 485   | 459   | 438   |
| # ci | 129   | 32    | 7     | 0     | 0     | 0     | 111   | 53    | 0     | 0     | 0     | 0     |
| # to | 0     | 0     | 0     | 14    | 44    | 65    | 0     | 0     | 3     | 15    | 41    | 62    |
Figure 197: Results for Bucklin voting in the TM model for destructive control by adding candidates. Number of candidates is fixed.

| m  | n = 4 | m = 8 |
|----|-------|-------|
| 4  | 8     | 16    | 32   | 64   | 128  | 4    | 8    | 16   | 32   | 64   | 128  |
| # cp | 211  | 218  | 240  | 263  | 284  | 305  | 237  | 283  | 325  | 350  | 343  | 349  |
| # ci | 289  | 282  | 260  | 237  | 216  | 195  | 263  | 217  | 175  | 150  | 157  | 151  |
| # to | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |

| m  | n = 16 | m = 32 |
|----|--------|--------|
| 4  | 8      | 16     | 32    | 64    | 128   | 4    | 8    | 16   | 32   | 64   | 128  |
| # cp | 254  | 306   | 315   | 370   | 385   | 362  | 274  | 305  | 339  | 355  | 356  | 361  |
| # ci | 246  | 194   | 185   | 130   | 115   | 19   | 135  | 104  | 62   | 27   | 9    | 6    |
| # to | 0    | 0     | 0     | 0     | 0     | 119  | 91   | 91   | 99   | 118  | 135  | 133  |

| m  | n = 64 | m = 128 |
|----|--------|---------|
| 4  | 8      | 16     | 32    | 64    | 128   | 4    | 8    | 16   | 32   | 64   | 128  |
| # cp | 275  | 306   | 321   | 368   | 356   | 369  | 294  | 299  | 326  | 344  | 340  | 365  |
| # ci | 124  | 105   | 61    | 24    | 6     | 8    | 109  | 94   | 64   | 22   | 7    | 1    |
| # to | 101  | 89    | 118   | 108   | 138   | 123  | 97   | 107  | 110  | 134  | 153  | 134  |
Figure 198: Results for Bucklin voting in the TM model for destructive control by adding candidates. Number of voters is fixed.

|     | m   | n   | # cp  | # ci  | # to  |
|-----|-----|-----|-------|-------|-------|
| 4   | 8   | 16  | 32    | 64    | 128   |
| # cp| 211 | 237 | 254   | 274   | 275   |
| # ci| 289 | 263 | 246   | 135   | 124   |
| # to| 0   | 0   | 91    | 101   | 97    |
| 8   | 16  | 32  | 64    | 128   |       |
| # cp| 218 | 283 | 306   | 305   | 306   |
| # ci| 282 | 217 | 194   | 104   | 105   |
| # to| 0   | 0   | 91    | 89    |       |

|     | m   | n   | # cp  | # ci  | # to  |
|-----|-----|-----|-------|-------|-------|
| 4   | 8   | 16  | 32    | 64    | 128   |
| # cp| 240 | 325 | 315   | 339   | 321   |
| # ci| 26  | 175 | 185   | 62    | 61    |
| # to| 0   | 0   | 99    | 118   | 110   |
| 8   | 16  | 32  | 64    | 128   |       |
| # cp| 263 | 350 | 370   | 355   | 368   |
| # ci| 237 | 150 | 130   | 27    | 24    |
| # to| 0   | 0   | 99    | 118   | 108   |

|     | m   | n   | # cp  | # ci  | # to  |
|-----|-----|-----|-------|-------|-------|
| 4   | 8   | 16  | 32    | 64    | 128   |
| # cp| 284 | 343 | 385   | 356   | 356   |
| # ci| 216 | 157 | 115   | 9     | 6     |
| # to| 0   | 0   | 135   | 138   | 153   |
| 8   | 16  | 32  | 64    | 128   |       |
| # cp| 305 | 349 | 362   | 361   | 369   |
| # ci| 195 | 151 | 19    | 6     | 8     |
| # to| 0   | 0   | 119   | 133   | 123   |
B.2.1. Computational Costs

Figure 199: Average time the algorithm needs to find a successful control action for destructive control by adding candidates in Bucklin elections in the IC model. The maximum is 13,77 seconds.

Figure 200: Average time the algorithm needs to determine no-instance of destructive control by adding candidates in Bucklin elections in the IC model. The maximum is 532,06 seconds.
Figure 201: Average time the algorithm needs to give a definite output for destructive control by adding candidates in Bucklin elections in the IC model. The maximum is 13.77 seconds.

Figure 202: Average time the algorithm needs to find a successful control action for destructive control by adding candidates in Bucklin elections in the TM model. The maximum is 9.61 seconds.
Figure 203: Average time the algorithm needs to determine no-instance of destructive control by adding candidates in Bucklin elections in the TM model. The maximum is 444.22 seconds.

Figure 204: Average time the algorithm needs to give a definite output for destructive control by adding candidates in Bucklin elections in the TM model. The maximum is 107.28 seconds.
B.3. Constructive Control by Deleting Candidates

Figure 205: Results for Bucklin voting in the IC model for constructive control by deleting candidates. Number of candidates is fixed.

| m = 4 | m = 8 |
|-------|-------|
| n     | 4     | 8     | 16    | 32    | 64    | 128   |
| cp    | 124   | 187   | 154   | 205   | 186   | 198   |
| ci    | 376   | 313   | 346   | 295   | 314   | 302   |
| to    | 0     | 0     | 0     | 0     | 0     | 0     |
| m = 16|       |       |       |       |       |       |
| n     | 4     | 8     | 16    | 32    | 64    | 128   |
| cp    | 181   | 218   | 251   | 265   | 288   | 302   |
| ci    | 319   | 282   | 249   | 235   | 212   | 198   |
| to    | 0     | 0     | 0     | 0     | 0     | 0     |
| m = 32|       |       |       |       |       |       |
| n     | 4     | 8     | 16    | 32    | 64    | 128   |
| cp    | 132   | 149   | 136   | 122   | 151   | 137   |
| ci    | 0     | 0     | 0     | 0     | 0     | 0     |
| to    | 368   | 351   | 364   | 378   | 349   | 363   |
Figure 206: Results for Bucklin voting in the IC model for constructive control by deleting candidates. Number of voters is fixed.

|   | n = 4                  | n = 8                  |
|---|------------------------|------------------------|
| m | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp | 124 167 181 163 132 89 | 187 154 218 194 149 78 |
| # ci | 376 333 319 0 0 0     | 313 346 282 0 0 0     |
| # to | 0  0  0  337 368 411  | 0  0  0  306 351 422  |

|   | n = 16                 | n = 32                 |
|---|------------------------|------------------------|
| m | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp | 154 175 251 203 136 98 | 205 212 265 235 122 70 |
| # ci | 346 325 249 0 0 0     | 295 288 235 0 0 0     |
| # to | 0  0  0  297 364 402  | 0  0  0  265 378 430  |

|   | n = 64                 | n = 128                |
|---|------------------------|------------------------|
| m | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp | 186 185 288 254 151 67 | 198 221 302 245 137 81 |
| # ci | 314 315 212 0 0 0     | 302 279 198 0 0 0     |
| # to | 0  0  0  246 349 433  | 0  0  0  255 363 419  |
Figure 207: Results for Bucklin voting in the TM model for constructive control by deleting candidates. Number of candidates is fixed.

| n  | 4  | 8  | 16 | 32 | 64 | 128 | m = 4 | 4  | 8  | 16 | 32 | 64 | 128 | m = 8 |
|----|----|----|----|----|----|-----|-------|----|----|----|----|----|-----|-------|
| # cp | 124 | 172 | 191 | 197 | 206 | 185 | 151 | 138 | 146 | 166 | 189 | 182 | 151 | 138 | 146 | 166 | 189 | 182 |
| # ci | 376 | 328 | 309 | 303 | 294 | 315 | 349 | 362 | 354 | 334 | 311 | 318 | 349 | 362 | 354 | 334 | 311 | 318 |
| # to | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

| n  | 4  | 8  | 16 | 32 | 64 | 128 | m = 16 | 4  | 8  | 16 | 32 | 64 | 128 | m = 32 |
|----|----|----|----|----|----|-----|-------|----|----|----|----|----|-----|-------|
| # cp | 176 | 182 | 203 | 223 | 202 | 207 | 153 | 152 | 176 | 162 | 154 | 165 | 153 | 152 | 176 | 162 | 154 | 165 |
| # ci | 324 | 318 | 297 | 277 | 298 | 293 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| # to | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

| n  | 4  | 8  | 16 | 32 | 64 | 128 | m = 64 | 4  | 8  | 16 | 32 | 64 | 128 | m = 128 |
|----|----|----|----|----|----|-----|-------|----|----|----|----|----|-----|-------|
| # cp | 133 | 127 | 131 | 136 | 111 | 122 | 128 | 107 | 91 | 87 | 97 | 93 | 128 | 107 | 91 | 87 | 97 | 93 |
| # ci | 2   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| # to | 365 | 373 | 369 | 364 | 389 | 378 | 372 | 393 | 409 | 413 | 403 | 407 | 372 | 393 | 409 | 413 | 403 | 407 |
Figure 208: Results for Bucklin voting in the TM model for constructive control by deleting candidates. Number of voters is fixed.

|      | \( n = 4 \) | \( n = 8 \) |
|------|-------------|-------------|
| \( m \) | \( n = 16 \) | \( n = 32 \) | \( n = 64 \) | \( n = 128 \) | \( n = 64 \) | \( n = 128 \) | \( n = 64 \) | \( n = 128 \) |
| # cp | 124 151 176 153 133 128 | 172 138 182 152 127 107 | 191 146 203 176 131 91 | 197 166 223 162 136 87 |
| # ci | 376 349 324 0 2 0 | 328 362 318 0 0 0 | 309 354 297 0 0 0 | 303 334 277 0 0 0 |
| # to | 0 0 0 347 365 372 | 0 0 0 348 373 393 | 0 0 0 324 369 409 | 0 0 0 338 364 413 |
| # cp | 206 189 202 154 111 97 | 185 182 207 165 122 93 | 294 311 298 0 0 0 | 315 318 293 0 0 0 |
| # ci | 0 0 0 346 389 403 | 0 0 0 335 378 407 | 0 0 0 346 389 403 | 0 0 0 335 378 407 |
B.3.1. Computational Costs

Figure 209: Average time the algorithm needs to find a successful control action for constructive control by deleting candidates in Bucklin elections in the IC model. The maximum is 61.64 seconds.

Figure 210: Average time the algorithm needs to determine no-instance of constructive control by deleting candidates in Bucklin elections in the IC model. The maximum is 428.06 seconds.
Figure 211: Average time the algorithm needs to give a definite output for constructive control by deleting candidates in Bucklin elections in the IC model. The maximum is 178.54 seconds.

Figure 212: Average time the algorithm needs to find a successful control action for constructive control by deleting candidates in Bucklin elections in the TM model. The maximum is 42.25 seconds.
Figure 213: Average time the algorithm needs to determine no-instance of constructive control by deleting candidates in Bucklin elections in the TM model. The maximum is 415.02 seconds.

Figure 214: Average time the algorithm needs to give a definite output for constructive control by deleting candidates in Bucklin elections in the TM model. The maximum is 247.38 seconds.
B.4. Destructive Control by Deleting Candidates

Figure 215: Results for Bucklin voting in the IC model for destructive control by deleting candidates. Number of candidates is fixed.

\[
\begin{array}{cccccccc}
\text{\# cp} & 241 & 251 & 266 & 281 & 286 & 276 & 349 & 385 & 422 & 426 & 430 & 441 \\
\text{\# ci} & 259 & 249 & 234 & 219 & 214 & 224 & 151 & 115 & 78 & 74 & 70 & 59 \\
\text{\# to} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\text{\# cp} & 453 & 481 & 490 & 495 & 497 & 496 & 415 & 465 & 480 & 489 & 495 & 493 \\
\text{\# ci} & 47 & 19 & 10 & 5 & 3 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\
\text{\# to} & 0 & 0 & 0 & 0 & 0 & 0 & 84 & 35 & 20 & 11 & 5 & 7 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\text{\# cp} & 349 & 394 & 436 & 470 & 478 & 475 & 247 & 340 & 381 & 392 & 434 & 458 \\
\text{\# ci} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{\# to} & 151 & 106 & 64 & 30 & 22 & 25 & 253 & 160 & 119 & 108 & 66 & 42 \\
\end{array}
\]
Figure 216: Results for Bucklin voting in the IC model for destructive control by deleting candidates. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |
|---|---------|---|---------|
| $m$ | 4 8 16 32 64 128 | 4 8 16 32 64 | 128 |
| # cp | 241 349 453 415 349 247 | 251 385 481 465 394 340 |
| # ci | 259 151 47 1 0 0 | 249 115 19 0 0 0 |
| # to | 0 0 0 84 151 253 | 0 0 0 35 106 160 |
| $n = 16$ |   |   |
| $m$ | 4 8 16 32 64 128 | 4 8 16 32 64 | 128 |
| # cp | 266 422 490 480 436 381 | 281 426 495 489 470 392 |
| # ci | 234 78 10 0 0 0 | 219 74 5 0 0 0 |
| # to | 0 0 0 20 64 119 | 0 0 0 11 30 108 |
| $n = 32$ |   |   |
| $m$ | 4 8 16 32 64 128 | 4 8 16 32 64 | 128 |
| # cp | 286 430 497 495 478 434 | 276 441 496 493 475 458 |
| # ci | 214 70 3 0 0 0 | 224 59 4 0 0 0 |
| # to | 0 0 0 5 22 66 | 0 0 0 7 25 42 |
Figure 217: Results for Bucklin voting in the TM model for destructive control by deleting candidates. Number of candidates is fixed.

|       | \(m = 4\)                  | \(m = 8\)                  |
|-------|-----------------------------|-----------------------------|
| \(n\) | 4  8 16 32 64 128           | 4  8 16 32 64 128           |
| # cp  | 143 123 143 137 182 193     | 206 232 261 282 319 276     |
| # ci  | 357 377 357 363 318 307     | 294 268 239 218 181 224     |
| # to  | 0  0 0 0 0 0               | 0  0 0 0 0 0               |

|       | \(m = 16\)                  | \(m = 32\)                  |
|-------|-----------------------------|-----------------------------|
| \(n\) | 4  8 16 32 64 128           | 4  8 16 32 64 128           |
| # cp  | 296 304 354 379 386 354     | 297 314 344 352 316 322     |
| # ci  | 204 196 146 121 114 146     | 146 129 78 46 18 14         |
| # to  | 0  0 0 0 0 0               | 0  0 0 0 0 0               |

|       | \(m = 64\)                  | \(m = 128\)                 |
|-------|-----------------------------|-----------------------------|
| \(n\) | 4  8 16 32 64 128           | 4  8 16 32 64 128           |
| # cp  | 239 274 286 280 270 219     | 172 198 208 202 192 170     |
| # ci  | 137 91 68 29 11 10          | 114 89 75 23 9 4            |
| # to  | 124 135 146 191 219 271     | 214 213 217 275 299 326     |
Figure 218: Results for Bucklin voting in the TM model for destructive control by deleting candidates. Number of voters is fixed.

|   | $n = 4$       | $n = 8$       |
|---|--------------|--------------|
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 143 | 206 | 296 | 297 | 239 | 172 | 123 | 232 | 304 | 314 | 274 | 198 |
| # ci | 357 | 294 | 204 | 146 | 137 | 114 | 377 | 268 | 196 | 129 | 91 | 89 |
| # to | 0 | 0 | 0 | 57 | 124 | 214 | 0 | 0 | 0 | 57 | 135 | 213 |
| $n = 16$ | | | | | | | | | | | | |
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 143 | 261 | 354 | 344 | 286 | 208 | 137 | 282 | 379 | 352 | 280 | 202 |
| # ci | 357 | 239 | 146 | 78 | 68 | 75 | 363 | 218 | 121 | 46 | 29 | 23 |
| # to | 0 | 0 | 0 | 78 | 146 | 217 | 0 | 0 | 0 | 102 | 191 | 275 |
| $n = 32$ | | | | | | | | | | | | |
| $m$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 182 | 319 | 386 | 316 | 270 | 192 | 193 | 276 | 354 | 322 | 219 | 170 |
| # ci | 318 | 181 | 114 | 18 | 11 | 9 | 307 | 224 | 146 | 14 | 10 | 4 |
| # to | 0 | 0 | 0 | 166 | 219 | 299 | 0 | 0 | 0 | 164 | 271 | 326 |
B.4.1. Computational Costs

Figure 219: Average time the algorithm needs to find a successful control action for destructive control by deleting candidates in Bucklin elections in the IC model. The maximum is 25.59 seconds.

Figure 220: Average time the algorithm needs to determine no-instance of destructive control by deleting candidates in Bucklin elections in the IC model. The maximum is 420.05 seconds.
Figure 221: Average time the algorithm needs to give a definite output for destructive control by deleting candidates in Bucklin elections in the IC model. The maximum is 25.59 seconds.

Figure 222: Average time the algorithm needs to find a successful control action for destructive control by deleting candidates in Bucklin elections in the TM model. The maximum is 63.26 seconds.
Figure 223: Average time the algorithm needs to determine no-instance of destructive control by deleting candidates in Bucklin elections in the TM model. The maximum is 309.1 seconds.

Figure 224: Average time the algorithm needs to give a definite output for destructive control by deleting candidates in Bucklin elections in the TM model. The maximum is 106.02 seconds.
### B.5. Constructive Control by Partition of Candidates in Model TE

![Diagram](image)

Figure 225: Results for Bucklin voting in the IC model for constructive control by partition of candidates in model TE. Number of voters is fixed.

|     | m = 4         | m = 8         | m = 16         | m = 32         | m = 64         | m = 128        |
|-----|---------------|---------------|---------------|---------------|---------------|---------------|
| n   | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp| 42  93  114  127  150  172 | 147  205  231  284  284  300 | 353  295  269  216  216  200 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 |
| # ci| 458  407  386  373  350  328 | 353  295  269  216  216  200 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 |
| # to| 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 |

|     | m = 16        | m = 32        | m = 64        | m = 128       |
|-----|---------------|---------------|---------------|---------------|
| n   | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp| 196  256  252  223  200  191 | 41  39  62  63  66  55 | 0  0  0  0  0  0 | 0  0  0  0  0  0 |
| # ci| 304  244  248  277  300  309 | 459  461  438  437  434  445 | 0  0  0  0  0  0 | 0  0  0  0  0  0 |
| # to| 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 |

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Figure 226: Results for Bucklin voting in the IC model for constructive control by partition of candidates in model TE. Number of voters is fixed.

| n = 4  | n = 8  |
|--------|--------|
| m      |        |        |
| 4      | 8      | 16     | 32     | 64     | 128    |
| # cp   | 42     | 147    | 196    | 41     | 15     | 6      |
| # ci   | 458    | 353    | 0      | 0      | 0      | 0      |
| # to   | 0      | 0      | 304    | 459    | 485    | 494    |
| m      |        |        |        |        |        |        |
| 4      | 8      | 16     | 32     | 64     | 128    |
| # cp   | 114    | 231    | 252    | 62     | 11     | 3      |
| # ci   | 386    | 269    | 0      | 0      | 0      | 0      |
| # to   | 0      | 0      | 248    | 438    | 489    | 497    |
| m      |        |        |        |        |        |        |
| 4      | 8      | 16     | 32     | 64     | 128    |
| # cp   | 150    | 284    | 200    | 66     | 23     | 8      |
| # ci   | 350    | 216    | 0      | 0      | 0      | 0      |
| # to   | 0      | 0      | 300    | 434    | 477    | 492    |

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Figure 227: Results for Bucklin voting in the TM model for constructive control by partition of candidates in model TE. Number of candidates is fixed.

|       | $m = 4$ |       | $m = 8$ |
|-------|---------|-------|---------|
| $n$   | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 27 50 87 87 120 126 | 94 125 150 168 187 223 |
| # ci  | 473 450 413 413 380 374 | 406 375 350 332 313 277 |
| # to  | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| $m = 16$ |       | $m = 32$ |
| $n$   | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 105 126 142 99 114 80 | 27 32 40 36 29 120 |
| # ci  | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to  | 395 374 358 401 386 420 | 473 468 460 464 471 488 |
| $m = 64$ |       | $m = 128$ |
| $n$   | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 8 13 7 10 10 11 | 9 4 1 3 2 1 |
| # ci  | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to  | 492 487 493 490 490 489 | 491 496 499 497 498 499 |
Figure 228: Results for Bucklin voting in the TM model for constructive control by partition of candidates in model TE. Number of voters is fixed.

| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|---|---|---|----|----|----|-----|---|---|----|----|----|-----|
| \# cp | 27 | 94 | 105 | 27 | 8 | 9 | 50 | 125 | 126 | 32 | 13 | 4 |
| \# ci | 473 | 406 | 0 | 0 | 0 | 0 | 450 | 375 | 0 | 0 | 0 | 0 |
| \# to | 0 | 0 | 305 | 473 | 492 | 491 | 0 | 0 | 374 | 468 | 487 | 496 |
| n = 16 | n = 32 |
| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 87 | 150 | 142 | 40 | 7 | 1 | 87 | 168 | 99 | 36 | 10 | 30 |
| \# ci | 413 | 350 | 0 | 0 | 0 | 0 | 413 | 332 | 0 | 0 | 0 | 0 |
| \# to | 0 | 0 | 358 | 460 | 493 | 499 | 0 | 0 | 401 | 464 | 490 | 497 |
| n = 64 | n = 128 |
| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 120 | 187 | 114 | 29 | 10 | 2 | 126 | 223 | 80 | 12 | 11 | 1 |
| \# ci | 380 | 313 | 0 | 0 | 0 | 0 | 374 | 277 | 0 | 0 | 0 | 0 |
| \# to | 0 | 0 | 386 | 471 | 490 | 498 | 0 | 0 | 420 | 488 | 489 | 499 |
B.5.1. Computational Costs

Figure 229: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 286.05 seconds.

Figure 230: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 26.26 seconds.
Figure 231: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 286.05 seconds.

Figure 232: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 142.58 seconds.
Figure 233: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 26.14 seconds.

Figure 234: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 142.58 seconds.
B.6. Destructive Control by Partition of Candidates in Model TE

Figure 235: Results for Bucklin voting in the IC model for destructive control by partition of candidates in model TE. Number of candidates is fixed.

|        | m = 4 |        | m = 8 |
|--------|-------|--------|-------|
| n      | 4     | 8      | 16    | 32    | 64    | 128   | 4    | 8    | 16    | 32    | 64    | 128   |
| # cp   | 288   | 316    | 312   | 320   | 340   | 312   | 429  | 465  | 480   | 486   | 487   | 484   |
| # ci   | 212   | 184    | 188   | 180   | 160   | 188   | 71   | 35   | 20    | 14    | 13    | 16    |
| # to   | 0     | 0      | 0     | 0     | 0     | 0     | 0    | 0    | 0     | 0     | 0     | 0     |
| m = 16 |       |        |       |       |       |       |      |      |       |       |       |       |
| n      | 4     | 8      | 16    | 32    | 64    | 128   | 4    | 8    | 16    | 32    | 64    | 128   |
| # cp   | 488   | 496    | 500   | 499   | 498   | 499   | 353  | 438  | 478   | 494   | 494   | 495   |
| # ci   | 4     | 0      | 0     | 0     | 0     | 0     | 1    | 0    | 0     | 0     | 0     | 0     |
| # to   | 1     |        | 1     | 2     | 1     |       | 146  | 62   | 22    | 6     | 6     | 5     |
| m = 32 |       |        |       |       |       |       |      |      |       |       |       |       |
| n      | 4     | 8      | 16    | 32    | 64    | 128   | 4    | 8    | 16    | 32    | 64    | 128   |
| # cp   | 244   | 349    | 417   | 462   | 465   | 472   | 244  | 325  | 379   | 406   | 426   | 446   |
| # ci   | 0     | 0      | 0     | 0     | 0     | 0     | 0    | 0    | 0     | 0     | 0     | 0     |
| # to   | 356   | 151    | 83    | 38    | 35    | 28    | 256  | 175  | 121   | 94    | 74    | 54    |
| m = 64 |       |        |       |       |       |       |      |      |       |       |       |       |
| n      | 4     | 8      | 16    | 32    | 64    | 128   | 4    | 8    | 16    | 32    | 64    | 128   |
| # cp   |       |        |       |       |       |       |      |      |       |       |       |       |
| # ci   |       |        |       |       |       |       |      |      |       |       |       |       |
| # to   |       |        |       |       |       |       |      |      |       |       |       |       |
| m = 128|       |        |       |       |       |       |      |      |       |       |       |       |

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Figure 236: Results for Bucklin voting in the IC model for destructive control by partition of candidates in model TE. Number of voters is fixed.

|   | $n = 4$ |          | $n = 8$ |          |
|---|---------|----------|---------|----------|
| $m$ | 4  8  16  32  64  128 |         | 4  8  16  32  64  128 |         |
| # cp | 288  429  488  353  244  244 |         | 316  465  496  438  349  325 |         |
| # ci | 212  71  4  1  0  0 |         | 184  35  0  0  0  0 |         |
| # to | 0  0  8  146  256  256 |         | 0  0  4  62  151  175 |         |
| $n = 16$ |         |         |         |         |
| $m$ | 4  8  16  32  64  128 |         | 4  8  16  32  64  128 |         |
| # cp | 312  480  500  478  417  379 |         | 320  486  499  494  462  406 |         |
| # ci | 188  20  0  0  0  0 |         | 180  14  0  0  0  0 |         |
| # to | 0  0  0  22  83  121 |         | 0  0  1  6  38  94 |         |
| $n = 32$ |         |         |         |         |
| $m$ | 4  8  16  32  64  128 |         | 4  8  16  32  64  128 |         |
| # cp | 340  487  498  494  465  426 |         | 312  484  499  495  472  446 |         |
| # ci | 160  13  0  0  0  0 |         | 188  16  0  0  0  0 |         |
| # to | 0  0  2  6  35  74 |         | 0  0  1  5  28  54 |         |
Figure 237: Results for Bucklin voting in the TM model for destructive control by partition of candidates in model TE. Number of candidates is fixed.

| m | n  | 4 | 8 | 16 | 32 | 64 | 128 |
|---|----|---|---|----|----|----|-----|
| m = 4 | # cp | 213 | 174 | 199 | 197 | 250 | 280 |
|   | # ci | 287 | 326 | 301 | 303 | 250 | 220 |
|   | # to | 0   | 0   | 0   | 0   | 0   | 0   |
| m = 16 | # cp | 320 | 325 | 365 | 389 | 387 | 343 |
|   | # ci | 165 | 152 | 105 | 61  | 43  | 37  |
|   | # to | 15  | 23  | 3   | 50  | 70  | 120 |
| m = 64 | # cp | 160 | 193 | 237 | 240 | 246 | 213 |
|   | # ci | 137 | 91  | 68  | 29  | 11  | 10  |
|   | # to | 203 | 216 | 195 | 231 | 243 | 277 |
| m = 8 | # cp | 270 | 302 | 322 | 381 | 408 | 394 |
|   | # ci | 230 | 198 | 178 | 119 | 92  | 106 |
|   | # to | 0   | 0   | 0   | 0   | 0   | 0   |
| m = 16 | # cp | 216 | 247 | 273 | 291 | 282 | 285 |
|   | # ci | 146 | 129 | 78  | 46  | 18  | 14  |
|   | # to | 138 | 124 | 149 | 163 | 200 | 201 |
| m = 32 | # cp | 147 | 191 | 198 | 223 | 183 | 185 |
|   | # ci | 114 | 89  | 75  | 23  | 9   | 4   |
|   | # to | 239 | 220 | 227 | 254 | 308 | 311 |
Figure 238: Results for Bucklin voting in the TM model for destructive control by partition of candidates in model TE. Number of voters is fixed.

| n = 4 | n = 8 |
|-------|-------|
| m     | n = 16 | n = 32 | n = 64 | n = 128 |
| m     | n = 16  | n = 32  | n = 64  | n = 128  |
| # cp  | 213  270 320 216 160 147 | 174  302 325 247 193 191 |
| # ci  | 287  230 165 146 114 114 | 326  198 152 129 91  89  |
| # to  | 0  0  15  138 203 239 | 0  0  23  124 216 220  |
| m     | n = 64 | n = 128 |
| # cp  | 199  322 365 273 237 198 | 197  381 389 291 240 223  |
| # ci  | 301  178 105  78  68  75 | 303  119  61  46  29  23  |
| # to  | 0  0  30  149 195 227 | 0  0  50  163 231 254  |
| m     | n = 64  | n = 128  |
| # cp  | 250  408 387 282 246 183 | 280  394 343 285 213 185  |
| # ci  | 250  92  43  18  11  9 | 220  106  37  14  10  4  |
| # to  | 0  0  70  200 243 308 | 0  0  120 201 277 311 |
B.6.1. Computational Costs

Figure 239: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 36.79 seconds.

Figure 240: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 77.9 seconds.
Figure 241: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 77.9 seconds.

Figure 242: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 48.67 seconds.
Figure 243: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 36.79 seconds.

Figure 244: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 43.14 seconds.
B.7. Constructive Control by Partition of Candidates in Model TP

Figure 245: Results for Bucklin voting in the IC model for constructive control by partition of candidates in model TP. Number of candidates is fixed.

| $m=4$ | $m=8$ |
|-------|-------|
| $n$   | 4     | 8     | 16   | 32   | 64   | 128  | 4     | 8     | 16   | 32   | 64   | 128  |
| # cp  | 43    | 77    | 101  | 128  | 151  | 151  | 136   | 206   | 240  | 280  | 270  | 301  |
| # ci  | 457   | 423   | 399  | 372  | 349  | 349  | 364   | 294   | 260  | 220  | 230  | 199  |
| # to  | 0     | 0     | 0    | 0    | 0    | 0    | 0     | 0     | 0    | 0    | 0    | 0    |
| $m=16$ |       |       |      |      |      |      |       |       |      |      |      |
| $n$   | 4     | 8     | 16   | 32   | 64   | 128  | 4     | 8     | 16   | 32   | 64   | 128  |
| # cp  | 227   | 239   | 222  | 221  | 191  | 189  | 47    | 57    | 66   | 73   | 66   | 61   |
| # ci  | 0     | 0     | 0    | 0    | 0    | 0    | 0     | 0     | 0    | 0    | 0    | 0    |
| # to  | 273   | 261   | 278  | 279  | 309  | 311  | 453   | 443   | 434  | 427  | 434  | 439  |
| $m=32$ |       |       |      |      |      |      |       |       |      |      |      |
| $n$   | 4     | 8     | 16   | 32   | 64   | 128  | 4     | 8     | 16   | 32   | 64   | 128  |
| # cp  | 11    | 16    | 16   | 16   | 17   | 24   | 5     | 4     | 9    | 11   | 11   | 8    |
| # ci  | 0     | 0     | 0    | 0    | 0    | 0    | 0     | 0     | 0    | 0    | 0    | 0    |
| # to  | 489   | 484   | 484  | 484  | 483  | 476  | 495   | 496   | 491  | 489  | 489  | 492  |
Figure 246: Results for Bucklin voting in the IC model for constructive control by partition of candidates in model TP. Number of voters is fixed.

|     | \( n = 4 \) |     | \( n = 8 \) |
|-----|-------------|-----|-------------|
| \( m \) | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp | 43 136 227 47 11 5 | 77 206 239 57 16 4 |
| # ci | 457 364 0 0 0 0 | 423 294 0 0 0 0 |
| # to | 0 0 273 453 489 495 | 0 0 261 443 484 496 |
| \( n = 16 \) |     | \( n = 32 \) |
| # cp | 101 240 222 66 16 9 | 128 280 221 73 16 11 |
| # ci | 399 260 0 0 0 0 | 372 220 0 0 0 0 |
| # to | 0 0 278 434 484 491 | 0 0 279 427 484 489 |
| \( n = 64 \) |     | \( n = 128 \) |
| # cp | 151 270 191 66 17 11 | 151 301 189 61 24 8 |
| # ci | 349 230 0 0 0 0 | 349 199 0 0 0 0 |
| # to | 0 0 309 434 483 489 | 0 0 311 439 476 492 |
Figure 247: Results for Bucklin voting in the TM model for constructive control by partition of candidates in model TP. Number of candidates is fixed.

|     | $m = 4$ |     | $m = 8$ |
|-----|---------|-----|---------|
| $n$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 22 53 76 100 120 114 | 93 114 148 155 187 192 |
| # ci | 478 447 424 400 380 368 | 407 386 352 345 313 308 |
| # to | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| $m = 16$ |     | $m = 32$ |
| $n$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 113 136 132 108 109 86 | 35 26 30 25 25 32 |
| # ci | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to | 387 364 368 392 391 414 | 465 474 470 475 475 468 |
| $m = 64$ |     | $m = 128$ |
| $n$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 20 11 15 13 6 6 | 5 5 3 1 3 2 |
| # ci | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to | 480 489 485 487 494 494 | 495 495 497 499 497 498 |
Figure 248: Results for Bucklin voting in the TM model for constructive control by partition of candidates in model TP. Number of voters is fixed.

| $m$  | 4   | 8   | 16  | 32  | 64  | 128 | 4    | 8    | 16  | 32  | 64  | 128  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp | 22  | 93  | 113 | 35  | 20  | 5   | 53  | 114 | 136 | 26  | 11  | 5   |
| # ci | 478 | 407 | 0   | 0   | 0   | 0   | 447 | 386 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 387 | 465 | 480 | 495 | 0   | 0   | 364 | 474 | 489 | 495 |

| $m$  | 4   | 8   | 16  | 32  | 64  | 128 | 4    | 8    | 16  | 32  | 64  | 128  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp | 76  | 148 | 132 | 30  | 15  | 3   | 100 | 155 | 108 | 25  | 13  | 1   |
| # ci | 424 | 352 | 0   | 0   | 0   | 0   | 400 | 345 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 368 | 470 | 485 | 497 | 0   | 0   | 392 | 475 | 487 | 499 |

| $m$  | 4   | 8   | 16  | 32  | 64  | 128 | 4    | 8    | 16  | 32  | 64  | 128  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp | 120 | 187 | 109 | 25  | 6   | 3   | 114 | 192 | 86  | 32  | 6   | 2   |
| # ci | 38  | 313 | 0   | 0   | 0   | 0   | 386 | 308 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 391 | 475 | 494 | 497 | 0   | 0   | 414 | 468 | 494 | 498 |
B.7.1. Computational Costs

Figure 249: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 164.65 seconds.

Figure 250: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 36.09 seconds.
Figure 251: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 164.65 seconds.

Figure 252: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 155.73 seconds.
Figure 253: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 36.83 seconds.

Figure 254: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 155.73 seconds.
Figure 255: Results for Bucklin voting in the IC model for destructive control by partition of candidates in model TP. Number of candidates is fixed.

| m = 4 | m = 8 |
|-------|-------|
| n     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 288   | 316   | 312   | 320   | 340   | 312   | 429   | 465   | 480   | 486   | 487   | 484   |
| # ci  | 212   | 184   | 188   | 180   | 160   | 188   | 71    | 35    | 20    | 14    | 13    | 16    |
| # to  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| m = 16 | m = 32 |
| n     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 486   | 496   | 500   | 499   | 498   | 499   | 353   | 438   | 478   | 494   | 494   | 495   |
| # ci  | 4     | 0     | 0     | 0     | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 0     |
| # to  | 10    | 4     | 0     | 1     | 2     | 1     | 146   | 62    | 22    | 6     | 6     | 5     |
| m = 64 | m = 128 |
| n     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 244   | 349   | 417   | 462   | 465   | 472   | 244   | 325   | 379   | 406   | 426   | 446   |
| # ci  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| # to  | 256   | 151   | 83    | 38    | 35    | 28    | 256   | 175   | 121   | 94    | 74    | 54    |
Figure 256: Results for Bucklin voting in the IC model for destructive control by partition of candidates in model TP. Number of voters is fixed.

| $m$ | $n=4$ | $n=8$ | $n=16$ | $n=32$ | $n=64$ | $n=128$ |
|-----|-------|-------|--------|--------|--------|---------|
|     | # cp  | # ci  | # to   | # cp   | # ci   | # to    |
| 4   | 288   | 212   | 0      | 312    | 188    | 0       |
| 8   | 429   | 71    | 0      | 480    | 20     | 0       |
| 16  | 486   | 4     | 0      | 500    | 0      | 0       |
| 32  | 353   | 1     | 0      | 478    | 0      | 0       |
| 64  | 244   | 0     | 0      | 417    | 0      | 0       |
| 128 | 244   | 0     | 0      | 379    | 0      | 0       |
|     | 316   | 184   | 0      | 478    | 180    | 0       |
|     | 465   | 35    | 0      | 417    | 14     | 0       |
|     | 496   | 0     | 0      | 379    | 0      | 0       |
|     | 438   | 0     | 0      | 417    | 0      | 0       |
|     | 349   | 0     | 0      | 379    | 0      | 0       |
|     | 325   | 0     | 0      | 379    | 0      | 0       |

| $m$ | $n=16$ | $n=32$ | $n=64$ | $n=128$ |
|-----|--------|--------|--------|---------|
|     | # cp   | # ci   | # to   | # cp   | # ci   | # to    |
| 4   | 312    | 188    | 0      | 340    | 160    | 0       |
| 8   | 480    | 20     | 0      | 487    | 13     | 0       |
| 16  | 500    | 0      | 0      | 498    | 0      | 0       |
| 32  | 478    | 0      | 0      | 494    | 0      | 0       |
| 64  | 417    | 0      | 0      | 465    | 0      | 0       |
| 128 | 379    | 0      | 0      | 426    | 0      | 0       |
|     | 478    | 180    | 0      | 494    | 188    | 0       |
|     | 496    | 14     | 0      | 494    | 16     | 0       |
|     | 438    | 0      | 0      | 494    | 0      | 0       |
|     | 349    | 0      | 0      | 494    | 0      | 0       |
|     | 325    | 0      | 0      | 494    | 0      | 0       |
|     | 325    | 0      | 0      | 494    | 0      | 0       |
Figure 257: Results for Bucklin voting in the TM model for destructive control by partition of candidates in model TP. Number of candidates is fixed.

| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|---|---|---|----|----|----|-----|---|---|----|----|----|-----|
| n | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 213 | 174 | 199 | 197 | 250 | 280 | 270 | 302 | 322 | 381 | 408 | 394 |
| # ci | 287 | 326 | 301 | 303 | 250 | 220 | 230 | 198 | 178 | 119 | 92 | 106 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

m = 16

| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|---|---|---|----|----|----|-----|---|---|----|----|----|-----|
| n | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 319 | 324 | 365 | 389 | 387 | 343 | 216 | 247 | 273 | 291 | 282 | 284 |
| # ci | 165 | 152 | 105 | 61 | 43 | 37 | 146 | 129 | 78 | 46 | 18 | 14 |
| # to | 16 | 24 | 30 | 50 | 70 | 120 | 138 | 124 | 149 | 163 | 200 | 202 |

m = 32

| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|---|---|---|----|----|----|-----|---|---|----|----|----|-----|
| n | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 160 | 193 | 237 | 240 | 245 | 213 | 147 | 191 | 198 | 223 | 183 | 185 |
| # ci | 137 | 91 | 68 | 29 | 11 | 10 | 114 | 89 | 75 | 23 | 9 | 4 |
| # to | 203 | 216 | 195 | 231 | 244 | 277 | 239 | 220 | 227 | 254 | 308 | 311 |

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Figure 258: Results for Bucklin voting in the TM model for destructive control by partition of candidates in model TP. Number of voters is fixed.

| m | n = 4 | n = 8 |
|---|---|---|
| n | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 213 | 270 | 319 | 216 | 160 | 147 | 174 | 302 | 324 | 247 | 193 | 191 |
| # ci | 287 | 230 | 165 | 146 | 137 | 114 | 326 | 198 | 152 | 129 | 91 | 89 |
| # to | 0  | 0  | 16 | 138 | 203 | 239 | 0  | 0  | 24 | 124 | 216 | 220 |

| m | n = 16 | n = 32 |
|---|---|---|
| n | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 199 | 322 | 365 | 273 | 237 | 198 | 197 | 381 | 389 | 291 | 240 | 223 |
| # ci | 301 | 178 | 105 | 78  | 68  | 75  | 303 | 119 | 61 | 46 | 29 | 23 |
| # to | 0  | 0  | 30 | 149 | 195 | 227 | 0  | 0  | 50 | 163 | 231 | 254 |

| m | n = 64 | n = 128 |
|---|---|---|
| n | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 250 | 408 | 387 | 282 | 245 | 183 | 280 | 394 | 343 | 284 | 213 | 185 |
| # ci | 250 | 92  | 43  | 18  | 11  | 9   | 220 | 106 | 37 | 14 | 10 | 4 |
| # to | 0  | 0  | 70 | 200 | 244 | 308 | 0  | 0  | 120 | 202 | 277 | 311 |

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B.8.1. Computational Costs

Figure 259: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 75,81 seconds.

Figure 260: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 36,71 seconds.
Figure 261: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 75,81 seconds.

Figure 262: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 122,89 seconds.
Figure 263: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 10.36 seconds.

Figure 264: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 120.28 seconds.
B.9. Constructive Control by Runoff Partition of Candidates in Model TE

Figure 265: Results for Bucklin voting in the IC model for constructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

|    | m = 4 |    | m = 8 |
|----|-------|----|-------|
| n  |       | n  |       |
|    | 4     | 8  | 16    | 32   | 64   | 128  | 4     | 8     | 16    | 32   | 64   | 128  |
| # cp| 34    | 77  | 87   | 118  | 122  | 131  | 64    | 148   | 177   | 218  | 225  | 237  |
| # ci| 466   | 423 | 413  | 382  | 378  | 369  | 436   | 352   | 323   | 282  | 275  | 263  |
| # to| 0     | 0   | 0    | 0    | 0    | 0    | 0     | 0     | 0     | 0    | 0    | 0    |

|    | m = 16 |    | m = 32 |
|----|-------|----|-------|
| n  | 4     | 8  | 16    | 32   | 64   | 128  | 4     | 8     | 16    | 32   | 64   | 128  |
| # cp| 139   | 250 | 268  | 283  | 266  | 254  | 61    | 104   | 121   | 132  | 122  | 125  |
| # ci| 0     | 0   | 0    | 0    | 0    | 0    | 0     | 0     | 0     | 0    | 0    | 0    |
| # to| 361   | 250 | 232  | 217  | 234  | 246  | 439   | 396   | 379   | 368  | 378  | 375  |

|    | m = 64 |    | m = 128 |
|----|-------|----|--------|
| n  | 4     | 8  | 16    | 32   | 64   | 128  | 4     | 8     | 16    | 32   | 64   | 128  |
| # cp| 20    | 32  | 50   | 49   | 51   | 63   | 3     | 11    | 16    | 29   | 33   | 24   |
| # ci| 0     | 0   | 0    | 0    | 0    | 0    | 0     | 0     | 0     | 0    | 0    | 0    |
| # to| 480   | 468 | 450  | 451  | 449  | 437  | 497   | 489   | 484   | 471  | 467  | 476  |
Figure 266: Results for Bucklin voting in the IC model for constructive control by runoff—partition of candidates in model TE. Number of voters is fixed.

| m   | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|----|----|----|----|----|-----|
| # cp| 34 | 64 | 139| 61 | 20 | 3   | 77 | 148| 250| 104| 32 | 11  |
| # ci| 466| 436| 0  | 0  | 0  | 0   | 423| 352| 0  | 0  | 0  | 0   |
| # to| 0  | 0  | 361| 439| 480| 497 | 0  | 0  | 250| 396| 468| 489 |

| m   | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|----|----|----|----|----|-----|
| # cp| 87 | 177| 268| 121| 50 | 16  | 118| 218| 283| 132| 49 | 29  |
| # ci| 413| 323| 0  | 0  | 0  | 0   | 382| 282| 0  | 0  | 0  | 0   |
| # to| 0  | 0  | 232| 379| 450| 484 | 0  | 0  | 217| 368| 451| 471 |

| m   | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|----|----|----|----|----|-----|
| # cp| 122| 225| 266| 122| 51 | 33  | 131| 237| 254| 125| 63 | 24  |
| # ci| 378| 275| 0  | 0  | 0  | 0   | 369| 263| 0  | 0  | 0  | 0   |
| # to| 0  | 0  | 234| 378| 449| 467 | 0  | 0  | 246| 375| 437| 476 |
Figure 267: Results for Bucklin voting in the TM model for constructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

| $n$   | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-------|----|----|----|----|----|-----|----|----|----|----|----|-----|
| $m = 4$ |    |    |    |    |    |     |    |    |    |    |    |     |
| # cp  | 25 | 36 | 55 | 68 | 87 | 88  | 52 | 74 | 84 | 106| 111| 122 |
| # ci  | 475| 464| 445| 432| 413| 412 | 448| 426| 416| 394| 389| 378 |
| # to  | 0  | 0  | 0  | 0  | 0  | 0   | 0  | 0  | 0  | 0  | 0  | 0   |
| $m = 16$ |    |    |    |    |    |     |    |    |    |    |    |     |
| # cp  | 60 | 103| 105| 109| 105| 104 | 21 | 45 | 53 | 45 | 29 | 28  |
| # ci  | 0  | 0  | 0  | 0  | 0  | 0   | 0  | 0  | 0  | 0  | 0  | 0   |
| # to  | 440| 397| 395| 391| 395| 396 | 479| 455| 447| 455| 471| 472 |
| $m = 32$ |    |    |    |    |    |     |    |    |    |    |    |     |
| # cp  | 18 | 14 | 15 | 19 | 22 | 21  | 9  | 7  | 7  | 7  | 9  | 9   |
| # ci  | 0  | 0  | 0  | 0  | 0  | 0   | 0  | 0  | 0  | 0  | 0  | 0   |
| # to  | 482| 486| 485| 481| 478| 479 | 491| 493| 493| 493| 491| 491 |
| $m = 64$ |    |    |    |    |    |     |    |    |    |    |    |     |
| # cp  |    |    |    |    |    |     |    |    |    |    |    |     |
| # ci  |    |    |    |    |    |     |    |    |    |    |    |     |
| # to  |    |    |    |    |    |     |    |    |    |    |    |     |
| $m = 128$ |    |    |    |    |    |     |    |    |    |    |    |     |
| # cp  |    |    |    |    |    |     |    |    |    |    |    |     |
| # ci  |    |    |    |    |    |     |    |    |    |    |    |     |
| # to  |    |    |    |    |    |     |    |    |    |    |    |     |
Figure 268: Results for Bucklin voting in the TM model for constructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |   | $n = 16$ |   | $n = 32$ |   | $n = 64$ |   | $n = 128$ |
|---|---------|---|---------|---|----------|---|----------|---|----------|---|----------|
| $m$ | 4 8 16 32 64 128 |   | 4 8 16 32 64 128 |   | 4 8 16 32 64 128 |   | 4 8 16 32 64 128 |   | 4 8 16 32 64 128 |   | 4 8 16 32 64 128 |
| # cp | 25 52 60 21 18 9 |   | 36 74 103 45 14 7 |   | 464 426 0 0 0 0 |   | 397 455 486 493 |   | 453 426 0 0 0 0 |   | 432 394 0 0 0 0 |
| # ci | 475 448 0 0 0 0 |   | 464 426 0 0 0 0 |   | 397 455 486 493 |   | 453 426 0 0 0 0 |   | 432 394 0 0 0 0 |   | 432 394 0 0 0 0 |
| # to | 0 0 440 479 482 491 |   | 0 0 397 455 486 493 |   | 0 0 397 455 486 493 |   | 0 0 397 455 486 493 |   | 0 0 397 455 486 493 |   | 0 0 397 455 486 493 |
### B.9.1. Computational Costs

Figure 269: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 154.23 seconds.

Figure 270: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 22.8 seconds.
Figure 271: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 154.23 seconds.

Figure 272: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 128.41 seconds.
Figure 273: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 22.56 seconds.

Figure 274: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 128.41 seconds.
**B.10. Destructive Control by Runoff Partition of Candidates in Model TE**

![Figure 275](image)

Figure 275: Results for Bucklin voting in the IC model for destructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

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|       | $m = 4$ |       | $m = 8$ |
|-------|---------|-------|---------|
| $n$   | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 288 299 286 290 312 288 | 418 446 467 478 486 478 |
| # ci  | 212 201 214 210 188 212 | 82 54 33 22 14 22 |
| # to  | 0 0 0 0 0 0 | 0 0 0 0 0 0 |

---

|       | $m = 16$ |       | $m = 32$ |
|-------|---------|-------|---------|
| $n$   | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 490 498 500 499 500 499 | 378 451 480 492 494 488 |
| # ci  | 4 0 0 0 0 0 | 1 0 0 0 0 0 |
| # to  | 6 2 0 1 0 1 | 121 49 20 8 6 12 |

---

|       | $m = 64$ |       | $m = 128$ |
|-------|---------|-------|---------|
| $n$   | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 256 356 406 451 461 464 | 232 317 369 386 407 419 |
| # ci  | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to  | 244 144 94 49 39 36 | 268 183 131 114 93 81 |
Figure 276: Results for Bucklin voting in the IC model for destructive control by runoff—partition of candidates in model TE. Number of voters is fixed.

| m  | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|----|----|----|----|----|----|-----|----|----|----|----|----|-----|
| # cp | 288 | 418 | 490 | 378 | 256 | 232 | 299 | 446 | 498 | 451 | 356 | 317 |
| # ci | 212 | 82 | 4 | 1 | 0 | 0 | 201 | 54 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 6 | 121 | 244 | 268 | 0 | 0 | 2 | 49 | 144 | 183 |

| m  | 4  | 8  | 16 | 32 | 64 | 128 |
|----|----|----|----|----|----|-----|
| # cp | 286 | 467 | 500 | 480 | 406 | 369 |
| # ci | 214 | 33 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 20 | 94 | 131 | 0 | 0 | 1 | 8 | 49 | 114 |

| m  | 4  | 8  | 16 | 32 | 64 | 128 |
|----|----|----|----|----|----|-----|
| # cp | 312 | 486 | 500 | 494 | 461 | 407 |
| # ci | 188 | 14 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 6 | 39 | 93 | 0 | 0 | 1 | 12 | 36 | 81 |
Figure 277: Results for Bucklin voting in the TM model for destructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 4   | 4   | 213  | 287  | 0    |
| 4   | 8   | 169  | 307  | 0    |
| 4   | 16  | 193  | 307  | 0    |
| 4   | 32  | 193  | 254  | 0    |
| 4   | 64  | 246  | 229  | 0    |
| 4   | 128 | 271  | 236  | 0    |
| 8   | 4   | 169  | 287  | 0    |
| 8   | 8   | 193  | 307  | 0    |
| 8   | 16  | 193  | 307  | 0    |
| 8   | 32  | 246  | 229  | 0    |
| 8   | 64  | 271  | 236  | 0    |
| 8   | 128 | 293  | 264  | 0    |

$m = 16$

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 4   | 4   | 330  | 287  | 0    |
| 4   | 8   | 169  | 307  | 0    |
| 4   | 16  | 193  | 307  | 0    |
| 4   | 32  | 246  | 229  | 0    |
| 4   | 64  | 271  | 236  | 0    |
| 4   | 128 | 293  | 264  | 0    |
| 8   | 4   | 330  | 287  | 0    |
| 8   | 8   | 169  | 307  | 0    |
| 8   | 16  | 193  | 307  | 0    |
| 8   | 32  | 246  | 229  | 0    |
| 8   | 64  | 271  | 236  | 0    |
| 8   | 128 | 293  | 264  | 0    |

$m = 32$

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 4   | 4   | 330  | 287  | 0    |
| 4   | 8   | 339  | 307  | 0    |
| 4   | 16  | 382  | 307  | 0    |
| 4   | 32  | 418  | 254  | 0    |
| 4   | 64  | 411  | 229  | 0    |
| 4   | 128 | 379  | 236  | 0    |
| 8   | 4   | 330  | 287  | 0    |
| 8   | 8   | 339  | 307  | 0    |
| 8   | 16  | 382  | 307  | 0    |
| 8   | 32  | 418  | 254  | 0    |
| 8   | 64  | 411  | 229  | 0    |
| 8   | 128 | 379  | 236  | 0    |

$m = 64$

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 4   | 4   | 330  | 287  | 0    |
| 4   | 8   | 339  | 307  | 0    |
| 4   | 16  | 382  | 307  | 0    |
| 4   | 32  | 418  | 254  | 0    |
| 4   | 64  | 411  | 229  | 0    |
| 4   | 128 | 379  | 236  | 0    |
| 8   | 4   | 330  | 287  | 0    |
| 8   | 8   | 339  | 307  | 0    |
| 8   | 16  | 382  | 307  | 0    |
| 8   | 32  | 418  | 254  | 0    |
| 8   | 64  | 411  | 229  | 0    |
| 8   | 128 | 379  | 236  | 0    |

$m = 128$

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 4   | 4   | 169  | 91   | 29   |
| 4   | 8   | 193  | 91   | 29   |
| 4   | 16  | 240  | 236  | 29   |
| 4   | 32  | 236  | 239  | 11   |
| 4   | 64  | 239  | 208  | 10   |
| 4   | 128 | 208  | 114  | 4    |
| 8   | 4   | 169  | 91   | 29   |
| 8   | 8   | 193  | 91   | 29   |
| 8   | 16  | 240  | 236  | 29   |
| 8   | 32  | 236  | 239  | 11   |
| 8   | 64  | 239  | 208  | 10   |
| 8   | 128 | 208  | 114  | 4    |

$m = 256$

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 4   | 4   | 169  | 91   | 29   |
| 4   | 8   | 193  | 91   | 29   |
| 4   | 16  | 240  | 236  | 29   |
| 4   | 32  | 236  | 239  | 11   |
| 4   | 64  | 239  | 208  | 10   |
| 4   | 128 | 208  | 114  | 4    |
| 8   | 4   | 169  | 91   | 29   |
| 8   | 8   | 193  | 91   | 29   |
| 8   | 16  | 240  | 236  | 29   |
| 8   | 32  | 236  | 239  | 11   |
| 8   | 64  | 239  | 208  | 10   |
| 8   | 128 | 208  | 114  | 4    |

$m = 512$

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 4   | 4   | 169  | 91   | 29   |
| 4   | 8   | 193  | 91   | 29   |
| 4   | 16  | 240  | 236  | 29   |
| 4   | 32  | 236  | 239  | 11   |
| 4   | 64  | 239  | 208  | 10   |
| 4   | 128 | 208  | 114  | 4    |
| 8   | 4   | 169  | 91   | 29   |
| 8   | 8   | 193  | 91   | 29   |
| 8   | 16  | 240  | 236  | 29   |
| 8   | 32  | 236  | 239  | 11   |
| 8   | 64  | 239  | 208  | 10   |
| 8   | 128 | 208  | 114  | 4    |
Figure 278: Results for Bucklin voting in the TM model for destructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

|   | \(n = 4\) |   | \(n = 8\) |
|---|---|---|---|
| \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 213 | 264 | 330 | 231 | 169 | 146 | 169 | 293 | 339 | 258 | 193 | 179 |
| # ci | 287 | 236 | 165 | 146 | 137 | 114 | 331 | 207 | 152 | 129 | 91 | 89 |
| # to | 0 | 0 | 5 | 123 | 194 | 240 | 0 | 0 | 9 | 113 | 216 | 232 |
|   | \(n = 16\) |   | \(n = 32\) |
| \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 193 | 317 | 382 | 290 | 240 | 182 | 193 | 373 | 418 | 306 | 236 | 208 |
| # ci | 307 | 183 | 105 | 78 | 68 | 75 | 307 | 127 | 61 | 46 | 29 | 23 |
| # to | 0 | 0 | 13 | 132 | 192 | 243 | 0 | 0 | 21 | 148 | 235 | 269 |
|   | \(n = 64\) |   | \(n = 128\) |
| \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 246 | 404 | 411 | 295 | 239 | 168 | 271 | 389 | 379 | 290 | 208 | 158 |
| # ci | 254 | 96 | 43 | 18 | 11 | 9 | 229 | 111 | 37 | 14 | 10 | 4 |
| # to | 0 | 0 | 46 | 187 | 250 | 323 | 0 | 0 | 84 | 196 | 282 | 338 |
B.10.1. Computational Costs

Figure 279: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 43.52 seconds.

Figure 280: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 22.78 seconds.
Figure 281: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TE in Bucklin elections in the IC model. The maximum is 43.52 seconds.

Figure 282: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 80.81 seconds.
Figure 283: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 7.23 seconds.

Figure 284: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TE in Bucklin elections in the TM model. The maximum is 78.82 seconds.
B.11. Constructive Control by Runoff Partition of Candidates in Model TP

Figure 285: Results for Bucklin voting in the IC model for constructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

|       | m = 4     | m = 8     | m = 16    | m = 32    | m = 64    | m = 128   |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|
| n     | 4 8 16   | 4 8 16   | 4 8 16    | 4 8 16    | 4 8 16    | 4 8 16    |
| # cp  | 29 78 95 | 69 174 211| 431 326 289| 329 288 274|          |           |
| # ci  | 471 422 405 | 374 373 | 374 373 | 374 373 |           |           |
| # to  | 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 |           |           |
|       | m = 16    | m = 32    |           |           |           |           |
| n     | 4 8 16   | 4 8 16   | 4 8 16    | 4 8 16    | 4 8 16    | 4 8 16    |
| # cp  | 138 247 267 | 251 237 | 44 111 120 | 115 134 122 |          |           |
| # ci  | 0 0 0 0 0 0 0 0 0 0 0 0 0 0|           |           |           |           |           |
| # to  | 362 253 233 | 219 249 | 263 456 389 | 380 385 366 | 378 382 398 |           |
|       | m = 64    | m = 128   |           |           |           |           |
| n     | 4 8 16   | 4 8 16   | 4 8 16    | 4 8 16    | 4 8 16    | 4 8 16    |
| # cp  | 13 45 65 | 66 46   | 6 18 16 | 29 33 24 |           |           |
| # ci  | 0 0 0 0 0 0 0 0 0 0 0 0 0 0|           |           |           |           |           |
| # to  | 487 455 435 | 440 454 | 494 482 484 | 471 467 476 |           |           |

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Figure 286: Results for Bucklin voting in the IC model for constructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

| m   | n=4 |       | n=8 |       | n=16 | n=32 | n=64 | n=128 |
|-----|-----|-------|-----|-------|------|------|------|-------|
|     | 4   | 8     | 16  | 32    | 64   | 128  | 4    | 8     | 16    | 32    | 64    | 128   |
| # cp|     |       |     |       | 78   | 174  | 247  | 111  | 45   | 18   |
| # ci| 471 | 431   | 0   | 0     | 422  | 326  | 0    | 0    | 0    | 0    |
| # to| 0   | 0     | 362 | 456   | 487  | 494  | 0    | 0    | 253  | 389  | 455  | 482   |

| m   | n=16 |       | n=32 |       | n=64 | n=128 |
|-----|------|-------|------|-------|------|-------|
|     | 4    | 8     | 16   | 32    | 64   | 128   |
| # cp| 95   | 171   | 267  | 120   | 65   | 16    |
| # ci| 405  | 329   | 0    | 0     | 389  | 289   | 0    | 0    | 0    | 0    |
| # to| 0    | 0     | 233  | 380   | 435  | 484   | 0    | 0    | 219  | 385  | 448  | 471   |

| m   | n=64 |       | n=128 |
|-----|------|-------|-------|
|     | 4    | 8     | 16    | 32    | 64    | 128   |
| # cp| 126  | 212   | 251   | 134   | 60    | 33    |
| # ci| 374  | 288   | 0     | 0     | 0     | 373   | 274  | 0    | 0    | 0    |
| # to| 0    | 0     | 249   | 366   | 440   | 467   | 0    | 0    | 263  | 378  | 454  | 476   |
Figure 287: Results for Bucklin voting in the TM model for constructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

|   | $m = 4$ |   | $m = 8$ |   | $m = 16$ |   | $m = 32$ |   | $m = 64$ |   | $m = 128$ |
|---|---|---|---|---|---|---|---|---|---|---|
| n  | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|   |   |   |   |   |   |   |   |   |   |   |   |   |
| # cp | 19 | 43 | 58 | 79 | 87 | 82 | 51 | 73 | 84 | 98 | 120 | 125 |
| # ci | 481 | 457 | 442 | 421 | 413 | 418 | 449 | 427 | 416 | 402 | 380 | 375 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|   |   |   |   |   |   |   |   |   |   |   |   |   |
| n  | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|   |   |   |   |   |   |   |   |   |   |   |   |   |
| # cp | 71 | 104 | 101 | 118 | 104 | 103 | 25 | 46 | 51 | 61 | 43 | 23 |
| # ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 429 | 396 | 399 | 382 | 396 | 397 | 475 | 454 | 449 | 439 | 457 | 477 |
|   |   |   |   |   |   |   |   |   |   |   |   |   |
| n  | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|   |   |   |   |   |   |   |   |   |   |   |   |   |
| # cp | 13 | 11 | 14 | 27 | 20 | 15 | 7 | 10 | 9 | 7 | 10 | 5 |
| # ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 487 | 489 | 486 | 473 | 480 | 485 | 493 | 490 | 491 | 493 | 490 | 495 |
Figure 288: Results for Bucklin voting in the TM model for constructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

|   | \(n=4\) | \(n=8\) | \(n=16\) | \(n=32\) | \(n=64\) | \(n=128\) |
|---|---------|---------|---------|---------|---------|---------|
| \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 19 | 51 | 71 | 25 | 13 | 7 | 43 | 73 | 104 | 46 | 11 | 10 |
| \# ci | 481 | 449 | 0 | 0 | 0 | 0 | 457 | 427 | 0 | 0 | 0 | 0 |
| \# to | 0 | 0 | 429 | 475 | 487 | 493 | 0 | 0 | 396 | 454 | 489 | 490 |
| \(n=4\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 58 | 84 | 101 | 51 | 14 | 9 | 79 | 98 | 118 | 61 | 27 | 7 |
| \# ci | 442 | 416 | 0 | 0 | 0 | 0 | 421 | 402 | 0 | 0 | 0 | 0 |
| \# to | 0 | 0 | 399 | 449 | 486 | 491 | 0 | 0 | 382 | 439 | 473 | 493 |
| \(n=8\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 87 | 120 | 104 | 43 | 20 | 10 | 82 | 125 | 103 | 23 | 15 | 5 |
| \# ci | 413 | 380 | 0 | 0 | 0 | 0 | 418 | 375 | 0 | 0 | 0 | 0 |
| \# to | 0 | 0 | 396 | 457 | 480 | 490 | 0 | 0 | 397 | 477 | 485 | 495 |

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B.11.1. Computational Costs

Figure 289: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 152.45 seconds.

Figure 290: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 23.19 seconds.
Figure 291: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 152.45 seconds.

Figure 292: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 140.93 seconds.
Figure 293: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 22.53 seconds.

Figure 294: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 140.93 seconds.
B.12. Destructive Control by Runoff Partition of Candidates in Model TP

Figure 295: Results for Bucklin voting in the IC model for destructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

| n  | 4  | 8  | 16 | 32 | 64 | 128 |
|----|----|----|----|----|----|-----|
| # cp | 288 | 299 | 286 | 290 | 312 | 288 |
| # ci | 212 | 201 | 214 | 210 | 188 | 212 |
| # to | 0   | 0   | 0   | 0   | 0   | 0   |

| m = 16 |
|--------|
| n  | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 479 | 494 | 499 | 499 | 500 | 499 |
| # ci | 4   | 0   | 0   | 0   | 0   | 0   |
| # to | 17  | 6   | 1   | 1   | 0   | 1   |

| m = 32 |
|--------|
| n  | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 235 | 340 | 397 | 445 | 456 | 459 |
| # ci | 0   | 0   | 0   | 0   | 0   | 0   |
| # to | 265 | 160 | 103 | 55  | 44  | 41  |

| m = 64 |
|--------|
| n  | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 223 | 304 | 359 | 379 | 401 | 413 |
| # ci | 0   | 0   | 0   | 0   | 0   | 0   |
| # to | 277 | 196 | 141 | 121 | 99  | 87  |

| m = 128 |
|---------|
| n  | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 400 | 427 | 467 | 474 | 485 | 478 |
| # ci | 100 | 73  | 33  | 26  | 15  | 22  |
| # to | 0   | 0   | 0   | 0   | 0   | 0   |
Figure 296: Results for Bucklin voting in the IC model for destructive control by runoff—partition of candidates in model TP. Number of voters is fixed.

|     | n = 4 |     | n = 8 |     | n = 16 |     | n = 32 |     | n = 64 |     | n = 128 |
|-----|-------|-----|-------|-----|--------|-----|--------|-----|--------|-----|--------|
| m   | 4     | 8   | 16    | 32  | 64     | 128 | 4      | 8   | 16     | 32  | 64     | 128 |
| # cp| 288   | 400 | 355   | 235 | 223    |     | 299    | 427 | 494    | 434 | 340    | 304 |
| # ci| 212   | 10  | 4     | 1   | 0      | 0   | 201    | 73  | 0      | 0   | 0      | 0   |
| # to| 0     | 0   | 17    | 144 | 265    | 277 | 0      | 0   | 6      | 66  | 160    | 196 |
| n = 16 |       |     |       |     |        |     |        |     |        |     |        |
| m   | 4     | 8   | 16    | 32  | 64     | 128 | 4      | 8   | 16     | 32  | 64     | 128 |
| # cp| 286   | 467 | 499   | 472 | 397    | 359 | 290    | 474 | 499    | 487 | 445    | 379 |
| # ci| 214   | 33  | 0     | 0   | 0      | 0   | 210    | 26  | 0      | 0   | 0      | 0   |
| # to| 0     | 0   | 1     | 28  | 103    | 141 | 0      | 0   | 1      | 13  | 55     | 121 |
| n = 32 |       |     |       |     |        |     |        |     |        |     |        |
| m   | 4     | 8   | 16    | 32  | 64     | 128 | 4      | 8   | 16     | 32  | 64     | 128 |
| # cp| 312   | 485 | 500   | 493 | 456    | 401 | 288    | 478 | 499    | 486 | 459    | 413 |
| # ci| 188   | 15  | 0     | 0   | 0      | 0   | 212    | 22  | 0      | 0   | 0      | 0   |
| # to| 0     | 0   | 0     | 7   | 44     | 99  | 0      | 0   | 1      | 14  | 41     | 87  |
Figure 297: Results for Bucklin voting in the TM model for destructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

| $n$  | 4 | 8 | 16 | 32 | 64 | 128 | $m = 4$ | 4 | 8 | 16 | 32 | 64 | 128 |
|------|---|---|----|----|----|-----|-------|---|---|----|----|----|-----|
| # cp | 213 | 169 | 193 | 193 | 246 | 271 | 256 | 286 | 316 | 373 | 404 | 389 |
| # ci | 287 | 331 | 307 | 307 | 254 | 229 | 244 | 214 | 184 | 127 | 96  | 111 |
| # to | 0  | 0  | 0  | 0  | 0  | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

| $n$  | 4 | 8 | 16 | 32 | 64 | 128 | $m = 16$ | 4 | 8 | 16 | 32 | 64 | 128 |
|------|---|---|----|----|----|-----|-------|---|---|----|----|----|-----|
| # cp | 316 | 334 | 380 | 417 | 411 | 379 | 216 | 248 | 276 | 302 | 293 | 289 |
| # ci | 165 | 152 | 105 | 61  | 43  | 37  | 146 | 129 | 78  | 46  | 18  | 14  |
| # to | 19 | 14 | 15 | 22  | 46  | 84  | 138 | 123 | 146 | 152 | 189 | 197 |

| $n$  | 4 | 8 | 16 | 32 | 64 | 128 | $m = 64$ | 4 | 8 | 16 | 32 | 64 | 128 |
|------|---|---|----|----|----|-----|-------|---|---|----|----|----|-----|
| # cp | 160 | 187 | 234 | 235 | 238 | 208 | 136 | 174 | 180 | 208 | 167 | 156 |
| # ci | 137 | 91  | 68  | 29  | 11  | 10  | 114 | 89  | 75  | 23  | 9   | 4   |
| # to | 203 | 222 | 198 | 236 | 251 | 282 | 250 | 237 | 245 | 269 | 324 | 340 |

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Figure 298: Results for Bucklin voting in the TM model for destructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

| $m$ | $n = 4$ | $n = 8$ | $n = 16$ | $n = 32$ | $n = 64$ | $n = 128$ |
|-----|---------|---------|---------|---------|---------|---------|
|     | 4       | 8       | 16      | 32      | 64      | 128     |
| # cp | 213     | 256     | 316     | 216     | 160     | 136     |
| # ci | 287     | 244     | 165     | 146     | 137     | 114     |
| # to | 0       | 0       | 19      | 138     | 203     | 250     |

| $m$ | $n = 16$ | $n = 32$ | $n = 64$ | $n = 128$ |
|-----|---------|---------|---------|---------|
|     | 4       | 8       | 16      | 32      |
| # cp | 193     | 316     | 380     | 276     |
| # ci | 307     | 184     | 105     | 78      |
| # to | 0       | 0       | 15      | 146     |

| $m$ | $n = 64$ | $n = 128$ |
|-----|---------|---------|
|     | 4       | 8       |
| # cp | 246     | 404     |
| # ci | 254     | 96      |
| # to | 0       | 0       |
B.12.1. Computational Costs

Figure 299: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 44.23 seconds.

Figure 300: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 22.47 seconds.
Figure 301: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TP in Bucklin elections in the IC model. The maximum is 44.23 seconds.

Figure 302: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 80.1 seconds.
Figure 303: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 7.15 seconds.

Figure 304: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TP in Bucklin elections in the TM model. The maximum is 78.09 seconds.
B.13. Constructive Control by Adding Voters

![Graph showing the percentage of elections where control is possible against the number of voters, n.](image)

Figure 305: Results for Bucklin voting in the IC model for constructive control by adding voters. Number of candidates is fixed.

|         | m = 4          | m = 8          |
|---------|----------------|----------------|
| n       | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp    | 87 159 280 409 481 497 | 71 124 280 357 461 485 |
| # ci    | 413 341 220 7 0 0 | 429 376 220 6 0 0 |
| # to    | 0 0 84 19 3 | 0 0 0 137 39 15 |

|         | m = 16        | m = 32        |
|---------|---------------|---------------|
| n       | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp    | 44 80 208 308 429 490 | 15 59 163 261 410 479 |
| # ci    | 456 420 292 21 0 0 | 485 441 337 35 0 0 |
| # to    | 0 0 0 171 71 10 | 0 0 0 204 90 21 |

|         | m = 64        | m = 128        |
|---------|---------------|---------------|
| n       | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp    | 12 42 121 197 359 479 | 9 20 103 146 324 457 |
| # ci    | 488 458 377 72 1 0 | 491 480 232 94 2 0 |
| # to    | 0 0 2 231 140 21 | 0 0 165 260 174 43 |
Figure 306: Results for Bucklin voting in the IC model for constructive control by adding voters. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |   | $n = 16$ |   | $n = 32$ |   | $n = 64$ |   | $n = 128$ |
|---|---------|---|---------|---|---------|---|---------|---|---------|---|---------|
| $m$ | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp | 87  71  44  15  12  9 | 159  124  80  59  42  20 | 341  376  420  441  458  480 | 280  280  208  163  121  103 | 409  357  308  261  197  146 | 481  461  429  410  359  324 | 497  489  490  479  479  457 | 280  280  208  163  121  103 |
| # ci | 413  429  456  485  488  491 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 | 0  0  0  0  0  0 |
| # to | 71  44  15  12  9  | 159  124  80  59  42  20 | 341  376  420  441  458  480 | 280  280  208  163  121  103 | 409  357  308  261  197  146 | 481  461  429  410  359  324 | 497  489  490  479  479  457 | 280  280  208  163  121  103 |
Figure 307: Results for Bucklin voting in the TM model for constructive control by adding voters. Number of candidates is fixed.

| $m$ | 4     | 8     | 16    | 32    | 64    | 128   |
|-----|-------|-------|-------|-------|-------|-------|
| $n$ |       |       |       |       |       |       |
|     | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp| 64    | 84    | 159   | 175   | 190   | 203   |
| # ci| 436   | 416   | 341   | 203   | 231   | 229   |
| # to| 0     | 0     | 0     | 122   | 79    | 68    |

| $m$ | 16    | 32    | 64    | 128   |
|-----|-------|-------|-------|-------|
| $n$ |       |       |       |       |
|     | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp| 23    | 45    | 69    | 79    | 85    | 139   |
| # ci| 477   | 455   | 431   | 357   | 352   | 314   |
| # to| 0     | 0     | 0     | 64    | 63    | 47    |

| $m$ | 64    | 128   |
|-----|-------|-------|
| $n$ |       |       |
|     | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp| 10    | 11    | 28    | 26    | 40    | 53    |
| # ci| 490   | 489   | 472   | 422   | 403   | 406   |
| # to| 0     | 0     | 0     | 52    | 57    | 41    |
Figure 308: Results for Bucklin voting in the TM model for constructive control by adding voters. Number of voters is fixed.

| n     | m  | # cp | # ci | # to |
|-------|----|------|------|------|
| 4     | 4  | 64   | 436  | 0    |
|       | 8  | 57   | 443  | 0    |
|       | 16 | 23   | 477  | 0    |
|       | 32 | 18   | 485  | 0    |
|       | 64 | 30   | 490  | 0    |
|       | 128| 10   | 497  | 0    |
|       | 4  | 84   | 416  | 0    |
|       | 8  | 66   | 434  | 0    |
|       | 16 | 45   | 455  | 0    |
|       | 32 | 30   | 470  | 0    |
|       | 64 | 11   | 489  | 0    |
|       | 128| 5    | 495  | 0    |
| 8     | 4  | 84   | 416  | 0    |
|       | 8  | 66   | 434  | 0    |
|       | 16 | 45   | 455  | 0    |
|       | 32 | 30   | 470  | 0    |
|       | 64 | 11   | 489  | 0    |
|       | 128| 5    | 495  | 0    |

| n     | m  | # cp | # ci | # to |
|-------|----|------|------|------|
| 16    | 4  | 159  | 341  | 0    |
|       | 8  | 92   | 408  | 0    |
|       | 16 | 69   | 431  | 0    |
|       | 32 | 30   | 470  | 0    |
|       | 64 | 28   | 472  | 0    |
|       | 128| 12   | 442  | 0    |
|       | 4  | 175  | 203  | 0    |
|       | 8  | 113  | 302  | 0    |
|       | 16 | 79   | 357  | 0    |
|       | 32 | 58   | 390  | 0    |
|       | 64 | 30   | 422  | 0    |
|       | 128| 26   | 438  | 0    |
| 32    | 4  | 175  | 203  | 0    |
|       | 8  | 113  | 302  | 0    |
|       | 16 | 79   | 357  | 0    |
|       | 32 | 58   | 390  | 0    |
|       | 64 | 30   | 422  | 0    |
|       | 128| 26   | 438  | 0    |
| 64    | 4  | 190  | 231  | 79   |
|       | 8  | 133  | 296  | 71   |
|       | 16 | 85   | 352  | 63   |
|       | 32 | 51   | 391  | 58   |
|       | 64 | 40   | 403  | 57   |
|       | 128| 22   | 442  | 36   |
|       | 4  | 203  | 229  | 68   |
|       | 8  | 142  | 298  | 60   |
|       | 16 | 139  | 314  | 47   |
|       | 32 | 80   | 376  | 44   |
|       | 64 | 53   | 406  | 41   |
|       | 128| 21   | 442  | 37   |
| 128   | 4  | 190  | 231  | 79   |
|       | 8  | 133  | 296  | 71   |
|       | 16 | 85   | 352  | 63   |
|       | 32 | 51   | 391  | 58   |
|       | 64 | 40   | 403  | 57   |
|       | 128| 22   | 442  | 36   |
|       | 4  | 203  | 229  | 68   |
|       | 8  | 142  | 298  | 60   |
|       | 16 | 139  | 314  | 47   |
|       | 32 | 80   | 376  | 44   |
|       | 64 | 53   | 406  | 41   |
|       | 128| 21   | 442  | 37   |
B.13.1. Computational Costs

Figure 309: Average time the algorithm needs to find a successful control action for constructive control by adding voters in Bucklin elections in the IC model. The maximum is 106.7 seconds.

Figure 310: Average time the algorithm needs to determine no-instance of constructive control by adding voters in Bucklin elections in the IC model. The maximum is 242.32 seconds.
Figure 311: Average time the algorithm needs to give a definite output for constructive control by adding voters in Bucklin elections in the IC model. The maximum is 184.92 seconds.

Figure 312: Average time the algorithm needs to find a successful control action for constructive control by adding voters in Bucklin elections in the TM model. The maximum is 29.3 seconds.
Figure 313: Average time the algorithm needs to determine no-instance of constructive control by adding voters in Bucklin elections in the TM model. The maximum is 66.06 seconds.

Figure 314: Average time the algorithm needs to give a definite output for constructive control by adding voters in Bucklin elections in the TM model. The maximum is 62.9 seconds.
B.14. Constructive Control by Deleting Voters

Figure 315: Results for Bucklin voting in the IC model for constructive control by deleting voters. Number of candidates is fixed.

| $n$ | $m=4$ | $m=8$ | $m=16$ | $m=32$ | $m=64$ | $m=128$ | $m=128$ |
|-----|-------|-------|--------|--------|--------|--------|--------|
|     |       |       |        |        |        |        |        |
| # cp | 88    | 214   | 346    | 441    | 484    | 497    | 514    |
| # ci | 412   | 286   | 154    | 10     | 0      | 427    | 360    |
| # to | 0     | 0     | 49     | 16     | 3      | 0      | 0      |

| $n$ | $m=64$ | $m=128$ |
|-----|--------|---------|
|     |        |         |
| # cp | 15     | 34      |
| # ci | 485    | 466     |
| # to | 0      | 0       |

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Figure 316: Results for Bucklin voting in the IC model for constructive control by deleting voters. Number of voters is fixed.

|       | n = 4    |       | n = 8    |
|-------|----------|-------|----------|
| m     | 4  8 16 32 64 128 | 4  8 16 32 64 128 |
| # cp  | 88 73 47 32 15 10  | 214 140 100 70 34 25 |
| # ci  | 412 427 453 468 485 490 | 286 360 400 430 466 475 |
| # to  | 0 0 0 0 0 0  | 0 0 0 0 0 0  |

|       | n = 16  |       | n = 32  |
|-------|----------|-------|----------|
| m     | 4  8 16 32 64 128 | 4  8 16 32 64 128 |
| # cp  | 346 323 244 178 139 90  | 441 394 369 266 197 144 |
| # ci  | 154 177 256 322 357 253 | 10 7 14 40 58 93 |
| # to  | 0 0 0 0 0 4 157 | 49 99 117 194 245 263 |

|       | n = 64  |       | n = 128 |
|-------|----------|-------|----------|
| m     | 4  8 16 32 64 128 | 4  8 16 32 64 128 |
| # cp  | 484 467 404 366 309 262 | 497 499 484 478 414 417 |
| # ci  | 0 0 0 0 2 2  | 0 0 0 0 0 0  |
| # to  | 16 33 96 134 189 236 | 3 1 16 22 86 83 |
Figure 317: Results for Bucklin voting in the TM model for constructive control by deleting voters. Number of candidates is fixed.

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 4   | 4   | 75   | 425  | 0    |
| 8   | 8   | 131  | 369  | 0    |
| 16  | 16  | 190  | 310  | 0    |
| 32  | 32  | 209  | 216  | 0    |
| 64  | 64  | 176  | 222  | 0    |
| 128 | 128 | 192  | 227  | 0    |
| 4   | 4   | 45   | 455  | 0    |
| 8   | 8   | 76   | 424  | 0    |
| 16  | 16  | 126  | 374  | 0    |
| 32  | 32  | 129  | 298  | 0    |
| 64  | 64  | 140  | 283  | 0    |
| 128 | 128 | 162  | 275  | 0    |

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 16  | 4   | 27   | 473  | 0    |
| 8   | 8   | 42   | 458  | 0    |
| 16  | 16  | 77   | 423  | 0    |
| 32  | 32  | 84   | 351  | 0    |
| 64  | 64  | 85   | 342  | 0    |
| 128 | 128 | 101  | 330  | 0    |
| 4   | 4   | 21   | 479  | 0    |
| 8   | 8   | 26   | 474  | 0    |
| 16  | 16  | 45   | 394  | 0    |
| 32  | 32  | 49   | 403  | 0    |
| 64  | 64  | 49   | 386  | 0    |
| 128 | 128 | 101  | 368  | 0    |

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 32  | 4   | 47   | 490  | 0    |
| 16  | 8   | 28   | 488  | 0    |
| 32  | 16  | 25   | 427  | 0    |
| 64  | 32  | 33   | 430  | 0    |
| 128 | 64  | 46   | 410  | 0    |
| 4   | 4   | 5    | 495  | 0    |
| 8   | 8   | 7    | 493  | 0    |
| 16  | 16  | 10   | 443  | 0    |
| 32  | 32  | 12   | 445  | 0    |
| 64  | 64  | 20   | 439  | 0    |
| 128 | 128 | 23   | 443  | 0    |

| $m$ | $n$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| 64  | 4   | 10   | 490  | 0    |
| 32  | 8   | 12   | 488  | 0    |
| 64  | 16  | 25   | 427  | 0    |
| 128 | 32  | 33   | 430  | 0    |
| 4   | 4   | 5    | 495  | 0    |
| 8   | 8   | 7    | 493  | 0    |
| 16  | 16  | 10   | 443  | 0    |
| 32  | 32  | 12   | 445  | 0    |
| 64  | 64  | 20   | 439  | 0    |
| 128 | 128 | 23   | 443  | 0    |
Figure 318: Results for Bucklin voting in the TM model for constructive control by deleting voters. Number of voters is fixed.

|   | n = 4 |   | n = 8 |
|---|-------|---|-------|
| m | 4     | 8 | 16    | 32  | 64  | 128 | 4  | 8  | 16  | 32  | 64  | 128 |
| # cp | 75 | 45 | 27 | 21 | 10 | 5 | 131 | 76 | 42 | 26 | 12 | 7 |
| # ci | 425 | 455 | 473 | 479 | 490 | 495 | 369 | 424 | 458 | 474 | 484 | 493 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| n = 16 |   |   |   |   |   |   |   |   |   |   |   |   |
| m | 4  | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 190 | 126 | 77 | 45 | 28 | 10 | 209 | 129 | 84 | 49 | 25 | 12 |
| # ci | 310 | 374 | 423 | 394 | 427 | 443 | 216 | 298 | 351 | 403 | 430 | 445 |
| # to | 0 | 0 | 0 | 61 | 45 | 47 | 75 | 73 | 65 | 48 | 45 | 43 |
| n = 32 |   |   |   |   |   |   |   |   |   |   |   |   |
| m | 4  | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 176 | 140 | 85 | 64 | 33 | 20 | 192 | 162 | 101 | 68 | 46 | 23 |
| # ci | 222 | 283 | 342 | 386 | 410 | 439 | 227 | 275 | 333 | 368 | 408 | 443 |
| # to | 102 | 77 | 73 | 50 | 57 | 41 | 81 | 63 | 66 | 64 | 46 | 34 |
| n = 64 |   |   |   |   |   |   |   |   |   |   |   |   |
| m | 4  | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 176 | 140 | 85 | 64 | 33 | 20 | 192 | 162 | 101 | 68 | 46 | 23 |
| # ci | 222 | 283 | 342 | 386 | 410 | 439 | 227 | 275 | 333 | 368 | 408 | 443 |
| # to | 102 | 77 | 73 | 50 | 57 | 41 | 81 | 63 | 66 | 64 | 46 | 34 |
B.14.1. Computational Costs

Figure 319: Average time the algorithm needs to find a successful control action for constructive control by deleting voters in Bucklin elections in the IC model. The maximum is 46.5 seconds.

Figure 320: Average time the algorithm needs to determine no-instance of constructive control by deleting voters in Bucklin elections in the IC model. The maximum is 247.02 seconds.
Figure 321: Average time the algorithm needs to give a definite output for constructive control by deleting voters in Bucklin elections in the IC model. The maximum is 188.16 seconds.

Figure 322: Average time the algorithm needs to find a successful control action for constructive control by deleting voters in Bucklin elections in the TM model. The maximum is 158.2 seconds.
Figure 323: Average time the algorithm needs to determine no-instance of constructive control by deleting voters in Bucklin elections in the TM model. The maximum is 37.76 seconds.

Figure 324: Average time the algorithm needs to give a definite output for constructive control by deleting voters in Bucklin elections in the TM model. The maximum is 35.6 seconds.
B.15. Constructive Control by Partition of Voters in Model TE

Figure 325: Results for Bucklin voting in the IC model for constructive control by partition of voters in model TE. Number of candidates is fixed.

|       | \( m = 4 \)          | \( m = 8 \)          | \( m = 16 \)         | \( m = 32 \)         | \( m = 64 \)         | \( m = 128 \)        |
|-------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| \( n \) | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| \# cp | 97 282 431 471 398 355 | 84 284 468 489 458 437 | 0 0 29 102 145 0 0 11 42 63 | 0 0 29 102 145 0 0 11 42 63 | 0 0 29 102 145 0 0 11 42 63 | 0 0 29 102 145 0 0 11 42 63 |
| \# ci | 403 218 69 0 0 0 416 216 32 0 0 0 416 216 32 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 |
| \# to | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 |
Figure 326: Results for Bucklin voting in the IC model for constructive control by partition of voters in model TE. Number of voters is fixed.

|       | n = 4 |       | n = 8 |
|-------|-------|-------|-------|
| m     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 97    | 84    | 32    | 23    | 12    | 9     | 282   | 284   | 219   | 162   | 87    | 41    |
| # ci  | 403   | 416   | 468   | 477   | 488   | 491   | 218   | 216   | 281   | 338   | 413   | 459   |
| # to  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
|       | n = 16|       | n = 32|
| m     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 431   | 468   | 444   | 408   | 322   | 239   | 471   | 489   | 491   | 471   | 471   | 429   | 380   |
| # ci  | 69    | 32    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| # to  | 0     | 0     | 56    | 92    | 178   | 261   | 29    | 11    | 29    | 71    | 71    | 120   |
|       | n = 64|       | n = 128|
| m     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 398   | 458   | 480   | 475   | 471   | 465   | 355   | 437   | 456   | 469   | 471   | 476   |
| # ci  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| # to  | 102   | 42    | 20    | 25    | 29    | 35    | 145   | 63    | 44    | 31    | 29    | 24    |

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Figure 327: Results for Bucklin voting in the TM model for constructive control by partition of voters in model TE. Number of candidates is fixed.

|   | \(m = 4\) |   | \(m = 8\) |
|---|-----------|---|-----------|
| \(n\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 54 | 130 | 142 | 149 | 128 | 158 | 45 | 86 | 145 | 120 | 120 | 125 |
| \# ci | 446 | 370 | 358 | 189 | 159 | 115 | 455 | 414 | 355 | 106 | 84 | 67 |
| \# to | 0 | 0 | 162 | 213 | 227 | 0 | 0 | 0 | 274 | 296 | 308 |

|   | \(m = 16\) |   | \(m = 32\) |
|---|-----------|---|-----------|
| \(n\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 17 | 51 | 117 | 81 | 99 | 115 | 15 | 36 | 66 | 58 | 75 | 79 |
| \# ci | 483 | 449 | 97 | 54 | 26 | 34 | 485 | 464 | 63 | 35 | 14 | 18 |
| \# to | 0 | 0 | 286 | 365 | 375 | 351 | 0 | 0 | 371 | 407 | 411 | 403 |

|   | \(m = 64\) |   | \(m = 128\) |
|---|-----------|---|-----------|
| \(n\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 0 | 21 | 49 | 28 | 46 | 57 | 6 | 5 | 28 | 17 | 25 | 45 |
| \# ci | 500 | 479 | 60 | 28 | 5 | 8 | 494 | 495 | 40 | 20 | 6 | 0 |
| \# to | 0 | 0 | 392 | 444 | 449 | 435 | 0 | 0 | 432 | 463 | 469 | 455 |
Figure 328: Results for Bucklin voting in the TM model for constructive control by partition of voters in model TE. Number of voters is fixed.

| $n$ | $m$ | $4$ | $8$ | $16$ | $32$ | $64$ | $128$ | $4$ | $8$ | $16$ | $32$ | $64$ | $128$ |
|-----|-----|-----|-----|-----|-----|-----|------|-----|-----|-----|-----|-----|------|
|     | # cp | 54  | 45  | 17  | 15  | 0    | 6    | 130 | 86  | 51  | 36  | 21  | 5    |
|     | # ci  | 446 | 455 | 483 | 485 | 500  | 494  | 370 | 414 | 449 | 464 | 479 | 495  |
|     | # to  | 0   | 0   | 0   | 0   | 0    | 0    | 0   | 0   | 0   | 0   | 0   | 0    |
| $n=16$ | $n=32$ | $n=64$ | $n=128$ |
|     | # cp | 142 | 145 | 117 | 66  | 49   | 28   | 149 | 120 | 81  | 58  | 28  | 17   |
|     | # ci | 358 | 355 | 97  | 63  | 60   | 40   | 189 | 106 | 54  | 35  | 28  | 20   |
|     | # to | 0   | 0   | 286 | 371 | 392  | 432  | 162 | 274 | 365 | 407 | 444 | 463  |
| $n=64$ | $n=128$ | $n=256$ | $n=512$ |
|     | # cp | 128 | 120 | 99  | 75  | 46   | 25   | 158 | 125 | 115 | 79  | 57  | 45   |
|     | # ci | 159 | 84  | 26  | 14  | 5    | 6    | 115 | 67  | 34  | 18  | 8   | 0    |
|     | # to | 213 | 296 | 375 | 411 | 449  | 469  | 227 | 308 | 351 | 403 | 435 | 455  |
B.15.1. Computational Costs

Figure 329: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TE in Bucklin elections in the IC model. The maximum is 47 seconds.

Figure 330: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TE in Bucklin elections in the IC model. The maximum is 475.79 seconds.
Figure 331: Average time the algorithm needs to give a definite output for constructive control by partition of voters in model TE in Bucklin elections in the IC model. The maximum is 47 seconds.

Figure 332: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TE in Bucklin elections in the TM model. The maximum is 57.39 seconds.
Figure 333: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TE in Bucklin elections in the TM model. The maximum is 274.43 seconds.

Figure 334: Average time the algorithm needs to give a definite output for constructive control by partition of voters in model TE in Bucklin elections in the TM model. The maximum is 203.24 seconds.
B.16. Destructive Control by Partition of Voters in Model TE

Figure 335: Results for Bucklin voting in the IC model for destructive control by partition of voters in model TE. Number of candidates is fixed.

| n  | m = 4 | m = 8 | m = 16 | m = 32 | m = 64 | m = 128 |
|----|-------|-------|--------|--------|--------|---------|
|    | cp    |        |        |        |        |         |
| 4  | 339   | 403    | 480    | 464    | 351    | 319     |
| 8  | 427   | 479    | 496    | 494    | 444    | 424     |
| 16 | 73    | 21     | 4      | 0      | 0      | 0       |
| 32 | 0     | 0      | 30     | 149    | 181    |         |
| 64 | 0     | 0      | 0      | 6      | 56     | 76      |
| 128|       |        |        |        |        |         |
|    | ci    |        |        |        |        |         |
| 4  | 161   | 97     | 20     | 6      | 0      | 0       |
| 8  | 73    | 21     | 4      | 0      | 0      | 0       |
| 16 | 0     | 0      | 30     | 149    | 181    |         |
| 32 | 0     | 0      | 6      | 56     | 76     |         |
| 64 | 0     | 0      | 0      | 6      | 56     | 76      |
| 128|       |        |        |        |        |         |
|    | to    |        |        |        |        |         |
| 4  | 0     | 0      | 0      | 30     | 149    | 181     |
| 8  | 0     | 0      | 0      | 6      | 56     | 76      |
| 16 | 0     | 0      | 0      | 6      | 56     | 76      |
| 32 | 0     | 0      | 0      | 6      | 56     | 76      |
| 64 | 0     | 0      | 0      | 6      | 56     | 76      |
| 128|       |        |        |        |        |         |

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Figure 336: Results for Bucklin voting in the IC model for destructive control by partition of voters in model TE. Number of voters is fixed.

|      | \( n = 4 \) | \( n = 8 \) | \( n = 16 \) | \( n = 32 \) | \( n = 64 \) | \( n = 128 \) |
|------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( m \) | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| \# cp | 339 427 476 478 491 498 | 403 479 488 496 500 500 | 480 496 499 500 500 500 | 464 494 497 499 500 500 | 351 444 479 488 492 492 | 319 424 465 473 485 485 |
| \# ci | 161 73 24 22 9 2 | 97 21 12 4 0 0 | 20 4 0 0 0 0 | 6 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \# to | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
|      | \( n = 16 \) | \( n = 32 \) | \( n = 64 \) | \( n = 128 \) |
| \( m \) | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| \# cp | 480 496 499 500 500 500 | 464 494 497 499 500 500 | 351 444 479 488 492 492 | 319 424 465 473 485 485 |
| \# ci | 20 4 0 0 0 0 | 6 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \# to | 0 0 1 0 0 0 | 30 6 3 1 1 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
|      | \( n = 64 \) | \( n = 128 \) |
| \( m \) | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| \# cp | 351 444 479 488 492 492 | 319 424 465 473 485 485 |
| \# ci | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \# to | 149 56 21 12 8 8 | 181 76 35 27 15 15 |
Figure 337: Results for Bucklin voting in the TM model for destructive control by partition of voters in model TE. Number of candidates is fixed.

|   | \( m = 4 \) | \( m = 8 \) | \( m = 16 \) | \( m = 32 \) | \( m = 64 \) | \( m = 128 \) |
|---|-------------|-------------|-------------|-------------|-------------|-------------|
| \( n \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 203 | 208 | 230 | 216 | 199 | 205 | 261 | 280 | 321 | 352 | 303 | 288 | 203 | 208 | 230 | 216 | 199 | 205 | 261 | 280 | 321 | 352 | 303 | 288 |
| \# ci | 297 | 292 | 270 | 205 | 170 | 120 | 239 | 220 | 179 | 92 | 83 | 69 | 297 | 292 | 270 | 205 | 170 | 120 | 239 | 220 | 179 | 92 | 83 | 69 |
| \# to | 0 | 0 | 0 | 79 | 131 | 175 | 0 | 0 | 0 | 56 | 114 | 143 | 0 | 0 | 0 | 79 | 131 | 175 | 0 | 0 | 0 | 56 | 114 | 143 |
| \( m = 16 \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 | 350 | 367 | 402 | 432 | 409 | 365 | 350 | 367 | 402 | 432 | 409 | 365 |
| \# cp | 326 | 337 | 374 | 390 | 352 | 328 | 350 | 367 | 402 | 432 | 409 | 365 | 326 | 337 | 374 | 390 | 352 | 328 | 350 | 367 | 402 | 432 | 409 | 365 |
| \# ci | 174 | 163 | 94 | 66 | 35 | 30 | 150 | 133 | 82 | 25 | 13 | 19 | 174 | 163 | 94 | 66 | 35 | 30 | 150 | 133 | 82 | 25 | 13 | 19 |
| \# to | 0 | 0 | 32 | 44 | 113 | 142 | 0 | 0 | 16 | 43 | 78 | 116 | 0 | 0 | 32 | 44 | 113 | 142 | 0 | 0 | 16 | 43 | 78 | 116 |
| \( m = 64 \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 | 363 | 381 | 443 | 438 | 408 | 365 | 363 | 381 | 443 | 438 | 408 | 365 |
| \# cp | 361 | 371 | 415 | 443 | 416 | 379 | 363 | 381 | 443 | 438 | 408 | 365 | 361 | 371 | 415 | 443 | 416 | 379 | 363 | 381 | 443 | 438 | 408 | 365 |
| \# ci | 139 | 129 | 71 | 27 | 8 | 6 | 137 | 119 | 45 | 24 | 4 | 5 | 139 | 129 | 71 | 27 | 8 | 6 | 137 | 119 | 45 | 24 | 4 | 5 |
| \# to | 0 | 0 | 14 | 30 | 76 | 115 | 0 | 0 | 12 | 38 | 88 | 130 | 0 | 0 | 14 | 30 | 76 | 115 | 0 | 0 | 12 | 38 | 88 | 130 |
Figure 338: Results for Bucklin voting in the TM model for destructive control by partition of voters in model TE. Number of voters is fixed.

|       | \( n = 4 \) |       | \( n = 8 \) |       | \( n = 16 \) |       | \( n = 32 \) |       | \( n = 64 \) |       | \( n = 128 \) |
|-------|------------|-------|------------|-------|------------|-------|------------|-------|------------|-------|------------|
|       | 4          | 8     | 16         | 32    | 64         | 128   | 4          | 8     | 16         | 32    | 64         | 128   |
| \# cp | 203        | 261   | 326        | 350   | 361        | 363   | 208        | 280   | 337        | 367   | 371        | 381   |
| \# ci | 297        | 239   | 174        | 150   | 139        | 137   | 292        | 220   | 163        | 133   | 129        | 119   |
| \# to | 0          | 0     | 0          | 0     | 0          | 0     | 0          | 0     | 0          | 0     | 0          | 0     |
|       | 4          | 8     | 16         | 32    | 64         | 128   | 4          | 8     | 16         | 32    | 64         | 128   |
| \# cp | 230        | 321   | 374        | 402   | 415        | 443   | 216        | 352   | 390        | 432   | 443        | 438   |
| \# ci | 270        | 179   | 94         | 82    | 71         | 45    | 205        | 92    | 66         | 25    | 27         | 24    |
| \# to | 0          | 0     | 32         | 16    | 14         | 12    | 79         | 56    | 44         | 43    | 30         | 38    |
|       | 4          | 8     | 16         | 32    | 64         | 128   | 4          | 8     | 16         | 32    | 64         | 128   |
| \# cp | 199        | 303   | 352        | 409   | 416        | 408   | 205        | 288   | 328        | 365   | 379        | 365   |
| \# ci | 170        | 83    | 35         | 13    | 8          | 4     | 120        | 69    | 3          | 19    | 6          | 5     |
| \# to | 131        | 114   | 113        | 78    | 76         | 88    | 175        | 143   | 142        | 116   | 115        | 130   |
B.16.1. Computational Costs

Figure 339: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TE in Bucklin elections in the IC model. The maximum is 22,15 seconds.

Figure 340: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TE in Bucklin elections in the IC model. The maximum is 325,26 seconds.
Figure 341: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TE in Bucklin elections in the IC model. The maximum is 21.87 seconds.

Figure 342: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TE in Bucklin elections in the TM model. The maximum is 17.19 seconds.
Figure 343: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TE in Bucklin elections in the TM model. The maximum is 57.94 seconds.

Figure 344: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TE in Bucklin elections in the TM model. The maximum is 24.63 seconds.
B.17. Constructive Control by Partition of Voters in Model TP

Figure 345: Results for Bucklin voting in the IC model for constructive control by partition of voters in model TP. Number of candidates is fixed.

| m = 4 | m = 8 | m = 16 | m = 32 | m = 64 | m = 128 |
|-------|-------|--------|--------|--------|---------|
| n     | 4     | 8      | 16     | 32     | 64      | 128     |
| # cp  | 49    | 151    | 231    | 239    | 199     | 139     |
| # ci  | 451   | 349    | 269    | 0      | 0       | 0       |
| # to  | 0     | 0      | 261    | 301    | 361     | 0       |

| m = 4 | m = 8 | m = 16 | m = 32 | m = 64 | m = 128 |
|-------|-------|--------|--------|--------|---------|
| n     | 4     | 8      | 16     | 32     | 64      | 128     |
| # cp  | 19    | 154    | 307    | 362    | 283     | 196     |
| # ci  | 481   | 346    | 0      | 0      | 0       | 0       |
| # to  | 0     | 0      | 193    | 138    | 217     | 304     |

| m = 4 | m = 8 | m = 16 | m = 32 | m = 64 | m = 128 |
|-------|-------|--------|--------|--------|---------|
| n     | 4     | 8      | 16     | 32     | 64      | 128     |
| # cp  | 11    | 80     | 245    | 319    | 281     | 199     |
| # ci  | 489   | 420    | 0      | 0      | 0       | 0       |
| # to  | 0     | 0      | 255    | 181    | 219     | 301     |
Figure 346: Results for Bucklin voting in the IC model for constructive control by partition of voters in model TP. Number of voters is fixed.

|       | n = 4   | n = 8   | n = 16  | n = 32  | n = 64  | n = 128 |
|-------|---------|---------|---------|---------|---------|---------|
| m     | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp  | 49 34 19 20 11 5   | 151 158 154 108 80 38 | 349 342 346 392 420 462 | 231 331 307 287 245 183 | 239 331 362 357 319 279 | 199 246 283 263 281 269 |
| # ci  | 451 466 481 480 489 495 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to  | 0 0 0 0 0 0   | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
|       | n = 64  | n = 128 | n = 64  | n = 128 | n = 64  | n = 128 |
| m     | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp  | 199 246 283 263 281 269 | 139 204 196 208 199 222 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # ci  | 0 0 0 0 0 0   | 0 0 0 0 0 0 | 361 296 304 292 301 278 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to  | 301 254 217 237 219 231 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
Figure 347: Results for Bucklin voting in the TM model for constructive control by partition of voters in model TP. Number of candidates is fixed.

| $n$ | $m = 4$ | $m = 8$ | $m = 16$ | $m = 32$ | $m = 64$ | $m = 128$ |
|-----|---------|---------|----------|----------|----------|----------|
|     |         |         |          |          |          |          |
|     | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp | 28 91 93 109 100 104 | 25 67 113 84 80 88 | 25 67 113 84 80 88 | 25 67 113 84 80 88 | 25 67 113 84 80 88 | 25 67 113 84 80 88 |
| # ci | 472 409 407 189 159 151 | 475 433 319 106 84 67 | 475 433 319 106 84 67 | 475 433 319 106 84 67 | 475 433 319 106 84 67 | 475 433 319 106 84 67 |
| # to | 0 0 0 202 241 281 | 0 0 0 68 310 336 | 0 0 0 68 310 336 | 0 0 0 68 310 336 | 0 0 0 68 310 336 | 0 0 0 68 310 336 |

| $n$ | $m = 4$ | $m = 8$ | $m = 16$ | $m = 32$ | $m = 64$ | $m = 128$ |
|-----|---------|---------|----------|----------|----------|----------|
|     |         |         |          |          |          |          |
|     | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp | 18 45 100 45 53 66 | 12 30 67 40 42 63 | 12 30 67 40 42 63 | 12 30 67 40 42 63 | 12 30 67 40 42 63 | 12 30 67 40 42 63 |
| # ci | 482 455 97 54 26 34 | 488 470 63 35 14 18 | 488 470 63 35 14 18 | 488 470 63 35 14 18 | 488 470 63 35 14 18 | 488 470 63 35 14 18 |
| # to | 0 0 303 401 421 400 | 0 0 370 425 444 419 | 0 0 370 425 444 419 | 0 0 370 425 444 419 | 0 0 370 425 444 419 | 0 0 370 425 444 419 |

| $n$ | $m = 4$ | $m = 8$ | $m = 16$ | $m = 32$ | $m = 64$ | $m = 128$ |
|-----|---------|---------|----------|----------|----------|----------|
|     |         |         |          |          |          |          |
|     | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp | 2 19 37 24 24 28 | 1 13 22 7 12 20 | 1 13 22 7 12 20 | 1 13 22 7 12 20 | 1 13 22 7 12 20 | 1 13 22 7 12 20 |
| # ci | 498 481 60 28 5 8 | 499 487 40 20 6 0 | 499 487 40 20 6 0 | 499 487 40 20 6 0 | 499 487 40 20 6 0 | 499 487 40 20 6 0 |
| # to | 0 0 403 448 471 464 | 0 0 438 473 482 480 | 0 0 438 473 482 480 | 0 0 438 473 482 480 | 0 0 438 473 482 480 | 0 0 438 473 482 480 |
Figure 348: Results for Bucklin voting in the TM model for constructive control by partition of voters in model TP. Number of voters is fixed.

| m   | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp| 28  | 25  | 18  | 12  | 2   | 1   | 91  | 67  | 45  | 30  | 19  | 13  |
| # ci| 472 | 475 | 482 | 488 | 498 | 499 | 409 | 433 | 455 | 470 | 481 | 487 |
| # to| 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

| m   | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp| 93  | 113 | 100 | 67  | 37  | 22  | 109 | 84  | 45  | 40  | 24  | 7   |
| # ci| 407 | 319 | 97  | 63  | 40  | 4   | 189 | 106 | 54  | 35  | 28  | 20  |
| # to| 0   | 68  | 303 | 370 | 438 | 438 | 202 | 310 | 401 | 425 | 448 | 473 |

| m   | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp| 100 | 80  | 53  | 42  | 24  | 12  | 104 | 88  | 66  | 63  | 28  | 20  |
| # ci| 159 | 84  | 26  | 14  | 5   | 6   | 115 | 67  | 34  | 18  | 8   | 0   |
| # to| 241 | 336 | 421 | 444 | 471 | 482 | 281 | 345 | 400 | 419 | 464 | 480 |

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Figure 349: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TP in Bucklin elections in the IC model. The maximum is 71,61 seconds.

Figure 350: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TP in Bucklin elections in the IC model. The maximum is 588.49 seconds.
Figure 351: Average time the algorithm needs to give a definite output for constructive control by partition of voters in model TP in Bucklin elections in the IC model. The maximum is 259.19 seconds.

Figure 352: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TP in Bucklin elections in the TM model. The maximum is 76.56 seconds.
Figure 353: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TP in Bucklin elections in the TM model. The maximum is 308,87 seconds.

Figure 354: Average time the algorithm needs to give a definite output for constructive control by partition of voters in model TP in Bucklin elections in the TM model. The maximum is 243,35 seconds.
B.18. Destructive Control by Partition of Voters in Model TP

Figure 355: Results for Bucklin voting in the IC model for destructive control by partition of voters in model TP. Number of candidates is fixed.

| m = 4          | m = 8          | m = 16         | m = 32         | m = 64         | m = 128        |
|----------------|----------------|----------------|----------------|----------------|----------------|
| n              | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
| # cp           | 299 | 328 | 460 | 429 | 339 | 326 | 402 | 448 | 495 | 486 | 462 | 442 |
| # ci           | 201 | 172 | 40  | 6   | 0   | 0   | 98  | 52  | 5   | 0   | 0   | 0   |
| # to           | 0   | 0   | 65  | 161 | 174 |     | 0   | 0   | 14  | 38  | 58  |     |
| m = 16         | m = 32         | m = 64         | m = 128        |
| n              | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
| # cp           | 469 | 481 | 499 | 499 | 489 | 474 | 473 | 496 | 500 | 500 | 493 | 490 |
| # ci           | 31  | 19  | 0   | 0   | 0   | 0   | 27  | 4   | 0   | 0   | 0   | 0   |
| # to           | 0   | 0   | 1   | 11  | 26  |     | 0   | 0   | 7   | 10  |     |     |
| m = 64         | m = 128        |
| n              | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
| # cp           | 491 | 500 | 500 | 500 | 497 | 487 | 496 | 500 | 500 | 500 | 494 | 490 |
| # ci           | 1   | 0   | 0   | 0   | 0   | 0   | 4   | 0   | 0   | 0   | 0   | 0   |
| # to           | 0   | 0   | 0   | 3   | 13  |     | 0   | 0   | 0   | 6   | 10  |     |
Figure 356: Results for Bucklin voting in the IC model for destructive control by partition of voters in model TP. Number of voters is fixed.

| m  | n = 4 | n = 8 |
|----|-------|-------|
| # cp | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
| # ci | 299 | 402 | 469 | 473 | 491 | 496 | 328 | 448 | 481 | 496 | 500 | 500 |
| # to | 201 | 98  | 31  | 27  | 9   | 4   | 172 | 52  | 19  | 4   | 0   | 0   |

| m  | n = 16 | n = 32 |
|----|--------|--------|
| # cp | 460   | 495   | 499  | 500  | 500  | 500  | 429 | 486 | 499 | 500 | 500 | 500 |
| # ci | 40    | 5     | 0    | 0    | 0    | 0    | 6   | 0   | 0   | 0   | 0   | 0   |
| # to | 0     | 0     | 1    | 0    | 0    | 0    | 65  | 14  | 1   | 0   | 0   | 0   |

| m  | n = 64 | n = 128 |
|----|--------|--------|
| # cp | 339   | 462   | 489  | 493  | 497  | 494  | 326 | 442 | 474 | 490 | 487 | 490 |
| # ci | 0     | 0     | 0    | 0    | 0    | 0    | 0   | 0   | 0   | 0   | 0   | 0   |
| # to | 161   | 38    | 11   | 7    | 3    | 6    | 174 | 58  | 26  | 10  | 13  | 10  |
Figure 357: Results for Bucklin voting in the TM model for destructive control by partition of voters in model TP. Number of candidates is fixed.

| m = 4 | m = 8 |
|-------|-------|
| n     | 4     8     16   32   64   128 | 4     8     16   32   64   128 |
| # cp  | 195 203 228 212 202 212 | 254 278 315 345 307 300 |
| # ci  | 305 297 272 205 170 120 | 246 222 185 92  83   69 |
| # to  | 0    0    0    83  128  168 | 0    0    0    63  110  131 |

| m = 16 | m = 32 |
|-------|-------|
| n     | 4     8     16   32   64   128 | 4     8     16   32   64   128 |
| # cp  | 321 336 379 386 348 337 | 349 364 401 424 407 377 |
| # ci  | 179 164 94   66  35   30 | 151 136 82   25  13   19 |
| # to  | 0    0    0    27  48   117 | 0    0    17  51   80  104 |

| m = 64 | m = 128 |
|-------|-------|
| n     | 4     8     16   32   64   128 | 4     8     16   32   64   128 |
| # cp  | 361 372 414 443 418 385 | 363 381 442 438 412 373 |
| # ci  | 139 128 71   27  8    6  | 137 119 45   24  4    5  |
| # to  | 0    0    15   30  74   109 | 0    0    13  38   84  122 |
Figure 358: Results for Bucklin voting in the TM model for destructive control by partition of voters in model TP. Number of voters is fixed.

|   | \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # cp | 195 | 254 | 321 | 349 | 361 | 363 | 203 | 278 | 336 | 364 | 372 | 381 |
| # ci | 305 | 246 | 179 | 151 | 139 | 137 | 297 | 222 | 164 | 136 | 128 | 119 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

|   | \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # cp | 228 | 315 | 379 | 401 | 414 | 442 | 212 | 345 | 386 | 424 | 443 | 438 |
| # ci | 272 | 185 | 94 | 82 | 71 | 45 | 205 | 92 | 66 | 25 | 27 | 24 |
| # to | 0 | 0 | 27 | 17 | 15 | 13 | 83 | 63 | 48 | 30 | 30 | 38 |

|   | \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # cp | 202 | 307 | 348 | 407 | 418 | 412 | 212 | 212 | 300 | 377 | 385 | 373 |
| # ci | 170 | 83 | 35 | 13 | 8 | 4 | 120 | 120 | 69 | 19 | 6 | 5 |
| # to | 128 | 110 | 117 | 80 | 74 | 84 | 168 | 168 | 131 | 104 | 109 | 122 |
B.18.1. Computational Costs

Figure 359: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TP in Bucklin elections in the IC model. The maximum is 26,14 seconds.

Figure 360: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TP in Bucklin elections in the IC model. The maximum is 452,32 seconds.
Figure 361: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TP in Bucklin elections in the IC model. The maximum is 30.3 seconds.

Figure 362: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TP in Bucklin elections in the TM model. The maximum is 13.68 seconds.
Figure 363: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TP in Bucklin elections in the TM model. The maximum is 89.22 seconds.

Figure 364: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TP in Bucklin elections in the TM model. The maximum is 36.57 seconds.
### C. Plurality Voting

#### C.1. Constructive Control by Adding Candidates

Figure 365: Results for plurality voting in the IC model for constructive control by adding candidates. Number of candidates is fixed.

|  | \( n \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \# cp | \( m = 4 \) | 22 | 27 | 34 | 20 | 24 | 15 | 3 | 32 | 37 | 47 | 39 | 52 |
| \# ci | 478 | 473 | 466 | 480 | 476 | 485 | 497 | 468 | 463 | 453 | 461 | 448 |
| \# to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# cp | \( m = 16 \) | 0 | 10 | 21 | 49 | 72 | 100 | 0 | 5 | 13 | 13 | 17 | 26 |
| \# ci | 500 | 490 | 479 | 451 | 428 | 310 | 0 | 0 | 0 | 0 | 0 | 7 |
| \# to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# cp | \( m = 32 \) | 0 | 0 | 2 | 6 | 8 | 5 | 0 | 0 | 1 | 0 | 1 | 1 |
| \# ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \# to | 500 | 500 | 498 | 496 | 492 | 495 | 500 | 500 | 499 | 500 | 499 | 499 |

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Figure 366: Results for plurality voting in the IC model for constructive control by adding candidates. Number of voters is fixed.

|   | $n=4$ | $n=8$ | $n=16$ | $n=32$ | $n=64$ | $n=128$ |
|---|---|---|---|---|---|---|
| $m$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 22 3 0 0 0 0 | 27 32 10 5 0 0 | 27 32 10 5 0 0 | 27 32 10 5 0 0 | 27 32 10 5 0 0 | 27 32 10 5 0 0 |
| # ci | 478 497 500 0 0 0 | 473 468 490 0 0 0 | 473 468 490 0 0 0 | 473 468 490 0 0 0 | 473 468 490 0 0 0 | 473 468 490 0 0 0 |
| # to | 0 0 0 500 500 500 | 0 0 0 495 500 500 | 0 0 0 495 500 500 | 0 0 0 495 500 500 | 0 0 0 495 500 500 | 0 0 0 495 500 500 |

|   | $n=16$ | $n=32$ | $n=64$ | $n=128$ |
|---|---|---|---|---|
| $m$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 34 37 21 13 2 1 | 20 47 49 13 6 0 | 20 47 49 13 6 0 | 20 47 49 13 6 0 |
| # ci | 466 463 479 0 0 0 | 480 453 451 0 0 0 | 480 453 451 0 0 0 | 480 453 451 0 0 0 |
| # to | 0 0 0 487 498 499 | 0 0 0 487 494 500 | 0 0 0 487 494 500 | 0 0 0 487 494 500 |

|   | $n=64$ | $n=128$ |
|---|---|---|
| $m$ | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 24 39 72 17 8 1 | 15 52 100 26 5 1 |
| # ci | 476 461 428 0 0 0 | 485 448 310 7 0 0 |
| # to | 0 0 0 483 492 499 | 0 0 90 467 495 499 |
Figure 367: Results for plurality voting in the TM model for constructive control by adding candidates. Number of candidates is fixed.

|   | $m = 4$ |   | $m = 8$ |   | $m = 16$ |   | $m = 32$ |   | $m = 64$ |   | $m = 128$ |   |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 13 | 17 | 12 | 9 | 10 | 3 | 1 | 6 | 5 | 5 | 3 | 1 |
| # ci | 487 | 483 | 488 | 491 | 490 | 497 | 499 | 494 | 494 | 495 | 497 | 499 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $m = 16$ |   |   |   |   |   |   |   |   |   |   |   |   |
| # cp | 0 | 3 | 1 | 0 | 3 | 6 | 0 | 0 | 1 | 0 | 0 | 0 |
| # ci | 500 | 497 | 499 | 500 | 497 | 487 | 1 | 0 | 28 | 452 | 482 | 485 |
| # to | 0 | 0 | 0 | 0 | 0 | 7 | 499 | 500 | 471 | 48 | 18 | 14 |
| $m = 32$ |   |   |   |   |   |   |   |   |   |   |   |   |
| # cp | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # ci | 0 | 0 | 0 | 6 | 447 | 491 | 0 | 0 | 0 | 0 | 0 | 456 |
| # to | 500 | 500 | 500 | 494 | 52 | 9 | 500 | 500 | 500 | 500 | 500 | 44 |
Figure 368: Results for plurality voting in the TM model for constructive control by adding candidates. Number of voters is fixed.

|     | \( n = 4 \) |     | \( n = 8 \) |
|-----|-------------|-----|-------------|
| \( m \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 13 | 1 | 0 | 0 | 0 | 0 | 17 | 6 | 3 | 0 | 0 | 0 |
| \# ci | 487 | 499 | 500 | 1 | 0 | 0 | 483 | 494 | 497 | 0 | 0 | 0 |
| \# to | 0 | 0 | 0 | 499 | 500 | 500 | 0 | 0 | 0 | 500 | 500 | 500 |

\( n = 16 \)

|     | \( n = 4 \) |     | \( n = 8 \) |
|-----|-------------|-----|-------------|
| \( m \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 12 | 6 | 1 | 1 | 0 | 0 | 9 | 5 | 0 | 0 | 0 | 0 |
| \# ci | 488 | 494 | 499 | 28 | 0 | 0 | 491 | 495 | 500 | 452 | 6 | 0 |
| \# to | 0 | 0 | 0 | 471 | 500 | 500 | 0 | 0 | 0 | 48 | 494 | 500 |

\( n = 32 \)

|     | \( n = 4 \) |     | \( n = 8 \) |
|-----|-------------|-----|-------------|
| \( m \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 10 | 3 | 3 | 0 | 1 | 0 | 3 | 10 | 6 | 1 | 0 | 0 |
| \# ci | 490 | 497 | 497 | 482 | 447 | 0 | 497 | 490 | 487 | 485 | 491 | 456 |
| \# to | 0 | 0 | 0 | 18 | 52 | 500 | 0 | 0 | 7 | 14 | 9 | 44 |

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Figure 369: Average time the algorithm needs to find a successful control action for constructive control by adding candidates in plurality elections in the IC model. The maximum is 562.76 seconds.

Figure 370: Average time the algorithm needs to determine no-instance of constructive control by adding candidates in plurality elections in the IC model. The maximum is 178.74 seconds.
Figure 371: Average time the algorithm needs to give a definite output for constructive control by adding candidates in plurality elections in the IC model. The maximum is 562.76 seconds.

Figure 372: Average time the algorithm needs to find a successful control action for constructive control by adding candidates in plurality elections in the TM model. The maximum is 71.66 seconds.
Figure 373: Average time the algorithm needs to determine no-instance of constructive control by adding candidates in plurality elections in the TM model. The maximum is 47,64 seconds.

Figure 374: Average time the algorithm needs to give a definite output for constructive control by adding candidates in plurality elections in the TM model. The maximum is 47,35 seconds.
C.2. Destructive Control by Adding Candidates

Figure 375: Results for plurality voting in the IC model for destructive control by adding candidates. Number of candidates is fixed.

|     | \(m = 4\) |     | \(m = 8\) | \(m = 16\) | \(m = 32\) | \(m = 64\) | \(m = 128\) |
|-----|-----------|-----|-----------|-----------|-----------|-----------|-----------|
| \(n\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 351 | 356 | 376 | 364 | 386 | 380 | 380 | 417 | 440 | 450 | 462 | 466 |
| # ci | 149 | 144 | 124 | 136 | 114 | 120 | 120 | 83 | 60 | 50 | 38 | 34 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

|     | \(m = 16\) |     | \(m = 32\) | \(m = 64\) | \(m = 128\) |
|-----|-----------|-----|-----------|-----------|-----------|
| \(n\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 394 | 426 | 445 | 466 | 493 | 497 | 379 | 404 | 435 | 464 | 470 | 489 |
| # ci | 106 | 74 | 55 | 34 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 121 | 96 | 65 | 36 | 30 | 11 |

|     | \(m = 64\) |     | \(m = 128\) |
|-----|-----------|-----|-----------|
| \(n\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 374 | 398 | 436 | 450 | 470 | 473 | 352 | 393 | 407 | 448 | 461 | 460 |
| # ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 126 | 102 | 64 | 50 | 30 | 27 | 148 | 107 | 93 | 52 | 39 | 40 |
Figure 376: Results for plurality voting in the IC model for destructive control by adding candidates. Number of voters is fixed.

| m      | n = 4 | n = 4 | n = 8 | n = 8 | n = 8 | n = 8 | n = 8 | n = 8 |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| # cp   |       |       |       |       |       |       |       |       |
| # ci   |       |       |       |       |       |       |       |       |
| # to   |       |       |       |       |       |       |       |       |

| m      | n = 16 | n = 32 | n = 64 | n = 128 | n = 128 | n = 128 | n = 128 | n = 128 |
|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| # cp   |       |       |       |         |         |         |         |         |
| # ci   |       |       |       |         |         |         |         |         |
| # to   |       |       |       |         |         |         |         |         |
Figure 377: Results for plurality voting in the TM model for destructive control by adding candidates. Number of candidates is fixed.

| $n$ | $m = 4$ | $m = 8$ | $m = 16$ | $m = 32$ | $m = 64$ | $m = 128$ |
|-----|---------|---------|-----------|-----------|-----------|-----------|
|     | $n$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 302 | 289 | 275 | 289 | 269 | 298 | 286 | 270 | 288 | 278 | 283 | 286 |
| # ci | 198 | 211 | 225 | 211 | 231 | 202 | 214 | 230 | 212 | 222 | 217 | 214 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $m = 16$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 276 | 273 | 281 | 274 | 302 | 287 | 260 | 271 | 262 | 255 | 279 |
| # ci | 224 | 227 | 219 | 219 | 226 | 0 | 6 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 0 | 0 | 0 | 198 | 207 | 240 | 229 | 238 | 245 | 221 |
| $m = 32$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 260 | 256 | 249 | 277 | 255 | 260 | 264 | 275 | 236 | 260 | 264 | 261 |
| # ci | 11 | 0 | 0 | 0 | 0 | 8 | 0 | 0 | 0 | 0 | 0 |
| # to | 229 | 244 | 251 | 223 | 245 | 240 | 228 | 225 | 264 | 240 | 236 | 239 |
Figure 378: Results for plurality voting in the TM model for destructive control by adding candidates. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |
|---|---------|---|---------|
|   | 4 8 16 32 64 128 |   | 4 8 16 32 64 128 |
| # cp | 302 286 276 287 260 264 | # cp | 289 270 273 260 256 275 |
| # ci | 198 214 224 6 11 8 | # ci | 211 230 227 0 0 0 |
| # to | 0 0 0 229 229 228 | # to | 0 0 0 240 244 225 |
|   | $n = 16$ |   | $n = 32$ |
| # cp | 275 288 281 271 249 236 | # cp | 289 278 281 262 277 260 |
| # ci | 225 212 219 0 0 0 | # ci | 211 222 219 0 0 0 |
| # to | 0 0 0 229 251 264 | # to | 0 0 0 238 223 240 |
|   | $n = 64$ |   | $n = 128$ |
| # cp | 269 283 274 255 255 264 | # cp | 298 286 302 279 260 261 |
| # ci | 231 217 226 0 0 0 | # ci | 202 214 0 0 0 0 |
| # to | 0 0 0 245 245 236 | # to | 0 0 198 221 240 239 |
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Figure 379: Average time the algorithm needs to find a successful control action for destructive control by adding candidates in plurality elections in the IC model. The maximum is 16.32 seconds.

Figure 380: Average time the algorithm needs to determine no-instance of destructive control by adding candidates in plurality elections in the IC model. The maximum is 356.7 seconds.
Figure 381: Average time the algorithm needs to give a definite output for destructive control by adding candidates in plurality elections in the IC model. The maximum is 16, 32 seconds.

Figure 382: Average time the algorithm needs to find a successful control action for destructive control by adding candidates in plurality elections in the TM model. The maximum is 4, 23 seconds.
Figure 383: Average time the algorithm needs to determine no-instance of destructive control by adding candidates in plurality elections in the TM model. The maximum is 356.7 seconds.

Figure 384: Average time the algorithm needs to give a definite output for destructive control by adding candidates in plurality elections in the TM model. The maximum is 287.86 seconds.
C.3. Constructive Control by Deleting Candidates

Figure 385: Results for plurality voting in the IC model for constructive control by deleting candidates. Number of candidates is fixed.

|   | m = 4 |   | m = 8 |
|---|---|---|---|
| n | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 165 | 139 | 182 | 193 | 199 | 199 | 146 | 151 | 174 | 194 | 207 | 209 |
| # ci | 335 | 361 | 361 | 307 | 301 | 301 | 354 | 349 | 326 | 306 | 293 | 291 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| m = 16 |   |   |   |   |   |   |   |   |   |   |   |   |
| n | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 176 | 251 | 303 | 298 | 317 | 328 | 124 | 218 | 223 | 252 | 220 | 215 |
| # ci | 324 | 249 | 197 | 202 | 183 | 172 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 376 | 282 | 277 | 248 | 280 | 285 |
| m = 32 |   |   |   |   |   |   |   |   |   |   |   |   |
| n | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 66 | 108 | 123 | 110 | 107 | 86 | 31 | 35 | 47 | 42 | 34 | 27 |
| # ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 434 | 392 | 377 | 390 | 393 | 414 | 469 | 465 | 453 | 458 | 466 | 473 |

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Figure 386: Results for plurality voting in the IC model for constructive control by deleting candidates. Number of voters is fixed.

|    | $n = 4$ |    | $n = 8$ |    | $n = 16$ |    | $n = 32$ |    | $n = 64$ |    | $n = 128$ |
|----|---------|----|---------|----|---------|----|---------|----|---------|----|---------|
| $m$ | 4       | 8  | 16      | 32 | 64      | 128| 4       | 8  | 16      | 32 | 64      |
| # cp | 165     | 146 | 176     | 124 | 66      | 31 | 139     | 151| 251     | 218| 108     | 35 |
| # ci | 335     | 354 | 324     | 0   | 0       | 0  | 361     | 349| 249     | 0  | 0       | 0  |
| # to | 0       | 0   | 0       | 376 | 434     | 469| 0       | 0  | 0       | 282| 392     | 465|
| $n = 16$ |    |    |         |    |         |    | $n = 32$ |    |         |    |         |    |
| $m$ | 4       | 8  | 16      | 32 | 64      | 128| 4       | 8  | 16      | 32 | 64      |
| # cp | 182     | 174 | 303     | 223 | 123     | 47 | 193     | 194| 398     | 252| 110     | 42 |
| # ci | 318     | 326 | 197     | 0   | 0       | 0  | 307     | 306| 202     | 0  | 0       | 0  |
| # to | 0       | 0   | 0       | 277 | 377     | 453| 0       | 0  | 0       | 248| 390     | 458|
| $n = 64$ |    |    |         |    |         |    | $n = 128$ |    |         |    |         |    |
| $m$ | 4       | 8  | 16      | 32 | 64      | 128| 4       | 8  | 16      | 32 | 64      |
| # cp | 199     | 207 | 317     | 220 | 107     | 34 | 199     | 209| 328     | 215| 86      | 27 |
| # ci | 301     | 293 | 183     | 0   | 0       | 0  | 301     | 291| 172     | 0  | 0       | 0  |
| # to | 0       | 0   | 0       | 280 | 393     | 466| 0       | 0  | 0       | 285| 414     | 473|
Figure 387: Results for plurality voting in the TM model for constructive control by deleting candidates. Number of candidates is fixed.

|   | $m=4$ |   | $m=8$ |   | $m=16$ |   | $m=32$ |   | $m=64$ |   | $m=128$ |   |
|---|-------|---|-------|---|--------|---|--------|---|--------|---|--------|---|
| $n$ | 4    | 8  | 16   | 32 | 64    | 128 | 4     | 8  | 16     | 32 | 64     | 128 |
| # cp | 149  | 163 | 172  | 188 | 173   | 183 | 124   | 159 | 161   | 163 | 162    | 173 |
| # ci | 351  | 337 | 328  | 312 | 327   | 317 | 376   | 341 | 339   | 337 | 338    | 327 |
| # to | 0    | 0   | 0    | 0   | 0     | 0   | 0     | 0   | 0     | 0   | 0      | 0   |
| $m=16$ |       |   |       |   |       |   |       |   |       |   |       |   |
| $n$ | 4    | 8  | 16   | 32 | 64    | 128 | 4     | 8  | 16     | 32 | 64     | 128 |
| # cp | 161  | 166 | 198  | 186 | 201   | 184 | 124   | 131 | 162   | 165 | 185    | 164 |
| # ci | 339  | 334 | 302  | 314 | 299   | 316 | 1     | 0   | 0     | 0   | 0      | 0   |
| # to | 0    | 0   | 0    | 0   | 0     | 0   | 375   | 369 | 338   | 335 | 315    | 336 |
| $m=32$ |       |   |       |   |       |   |       |   |       |   |       |   |
| $n$ | 4    | 8  | 16   | 32 | 64    | 128 | 4     | 8  | 16     | 32 | 64     | 128 |
| # cp | 117  | 149 | 164  | 171 | 140   | 157 | 110   | 168 | 151   | 143 | 137    | 156 |
| # ci | 1    | 0   | 0    | 0   | 0     | 0   | 0     | 0   | 0     | 0   | 0      | 0   |
| # to | 382  | 351 | 336  | 329 | 360   | 343 | 390   | 332 | 349   | 357 | 363    | 344 |
Figure 388: Results for plurality voting in the TM model for constructive control by deleting candidates. Number of voters is fixed.

|   | \(n = 4\) |   | \(n = 8\) |
|---|---|---|---|
| \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 149 | 124 | 161 | 124 | 117 | 110 | 163 | 159 | 166 | 131 | 149 | 168 |
| \# ci | 351 | 376 | 339 | 1 | 1 | 0 | 337 | 341 | 334 | 0 | 0 | 0 |
| \# to | 0 | 0 | 0 | 375 | 382 | 390 | 0 | 0 | 0 | 369 | 351 | 332 |

|   | \(n = 16\) |   | \(n = 32\) |
|---|---|---|---|
| \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 172 | 161 | 198 | 162 | 164 | 151 | 188 | 163 | 186 | 165 | 171 | 143 |
| \# ci | 328 | 339 | 302 | 0 | 0 | 0 | 312 | 337 | 314 | 0 | 0 | 0 |
| \# to | 0 | 0 | 0 | 338 | 336 | 349 | 0 | 0 | 0 | 335 | 329 | 357 |

|   | \(n = 64\) |   | \(n = 128\) |
|---|---|---|---|
| \(m\) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| \# cp | 173 | 162 | 201 | 185 | 140 | 137 | 183 | 173 | 184 | 164 | 157 | 156 |
| \# ci | 327 | 338 | 299 | 0 | 0 | 0 | 317 | 327 | 316 | 0 | 0 | 0 |
| \# to | 0 | 0 | 0 | 315 | 360 | 363 | 0 | 0 | 0 | 336 | 343 | 344 |
C.3.1. Computational Costs

Figure 389: Average time the algorithm needs to find a successful control action for constructive control by deleting candidates in plurality elections in the IC model. The maximum is 153.26 seconds.

Figure 390: Average time the algorithm needs to determine no-instance of constructive control by deleting candidates in plurality elections in the IC model. The maximum is 283.07 seconds.
Figure 391: Average time the algorithm needs to give a definite output for constructive control by deleting candidates in plurality elections in the IC model. The maximum is 153.26 seconds.

Figure 392: Average time the algorithm needs to find a successful control action for constructive control by deleting candidates in plurality elections in the TM model. The maximum is 24.32 seconds.
Figure 393: Average time the algorithm needs to determine no-instance of constructive control by deleting candidates in plurality elections in the TM model. The maximum is 279.1 seconds.

Figure 394: Average time the algorithm needs to give a definite output for constructive control by deleting candidates in plurality elections in the TM model. The maximum is 177.03 seconds.
C.4. Destructive Control by Deleting Candidates

Figure 395: Results for plurality voting in the IC model for destructive control by deleting candidates. Number of candidates is fixed.

|      | m = 4        | m = 8        | m = 16       | m = 32       | m = 64       | m = 128      |
|------|--------------|--------------|--------------|--------------|--------------|--------------|
| n    | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 220 218 208 217 243 224 | 239 334 374 404 415 420 | 261 166 126 96 85 80 | 245 471 469 485 485 487 | 244 29 31 15 15 13 | 34 284 454 457 474 444 |
| # ci | 280 282 292 283 257 276 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to | 0 0 0 0 0 0 | 280 282 292 283 257 276 | 261 166 126 96 85 80 | 245 471 469 485 485 487 | 244 29 31 15 15 13 | 34 284 454 457 474 444 |

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Figure 396: Results for plurality voting in the IC model for destructive control by deleting candidates. Number of voters is fixed.

| n = 4 | n = 8 |
|-------|-------|
| m     | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp  | 220 239 336 245 90 34 | 218 334 467 471 461 284 |
| # ci  | 280 261 164 11 5 3 | 282 166 33 0 0 0 |
| # to  | 0 0 0 244 405 463 | 0 0 0 29 39 216 |
| n = 16|       |               |
| m     | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp  | 208 374 483 469 465 454 | 217 404 488 485 478 457 |
| # ci  | 292 126 17 0 0 0 | 283 96 12 0 0 0 |
| # to  | 0 0 0 31 35 46 | 0 0 0 15 22 43 |
| n = 32|       |               |
| m     | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp  | 243 415 497 485 475 474 | 224 420 497 487 474 444 |
| # ci  | 257 85 3 0 0 0 | 276 80 3 0 0 0 |
| # to  | 0 0 0 15 25 26 | 0 0 0 13 26 56 |
| n = 64|       |               |
| m     | 4  8  16 32 64 128 | 4  8  16 32 64 128 |
| # cp  | 243 415 497 485 475 474 | 224 420 497 487 474 444 |
| # ci  | 257 85 3 0 0 0 | 276 80 3 0 0 0 |
| # to  | 0 0 0 15 25 26 | 0 0 0 13 26 56 |
| n = 128|      |               |
Figure 397: Results for plurality voting in the TM model for destructive control by deleting candidates. Number of candidates is fixed.

| m = 4       | m = 8       |
|-------------|-------------|
| n           | m           | n           | m           |
| 4           | 4           | 4           | 4           |
| 8           | 8           | 8           | 8           |
| 16          | 16          | 16          | 16          |
| 32          | 32          | 32          | 32          |
| 64          | 64          | 64          | 64          |
| 128         | 128         | 128         | 128         |
| # cp        | # cp        | # cp        | # cp        |
| 145         | 143         | 93          | 125         |
| 93          | 90          | 80          | 117         |
| 80          | 73          | 70          | 131         |
| 73          | 80          | 80          | 145         |
| 80          | 117         | 131         | 145         |
| 117         | 145         | 145         | 130         |
| # ci        | # ci        | # ci        | # ci        |
| 355         | 357         | 407         | 375         |
| 407         | 420         | 405         | 383         |
| 420         | 427         | 420         | 369         |
| 427         | 420         | 420         | 355         |
| 420         | 375         | 383         | 370         |
| 375         | 357         | 375         | 383         |
| # to        | # to        | # to        | # to        |
| 0           | 0           | 0           | 0           |
| 0           | 0           | 0           | 0           |
| 0           | 0           | 0           | 0           |
| 0           | 0           | 0           | 0           |
| m = 16      | m = 32      | m = 16      | m = 32      |
| n           | m           | n           | m           |
| 4           | 4           | 4           | 4           |
| 8           | 8           | 8           | 8           |
| 16          | 16          | 16          | 16          |
| 32          | 32          | 32          | 32          |
| 64          | 64          | 64          | 64          |
| 128         | 128         | 128         | 128         |
| # cp        | # cp        | # cp        | # cp        |
| 249         | 161         | 193         | 145         |
| 193         | 156         | 190         | 156         |
| 190         | 138         | 199         | 138         |
| 199         | 122         | 182         | 122         |
| 182         | 126         | 195         | 126         |
| # ci        | # ci        | # ci        | # ci        |
| 251         | 192         | 307         | 141         |
| 307         | 318         | 310         | 66          |
| 310         | 305         | 301         | 41          |
| 301         | 305         | 318         | 19          |
| 318         | 310         | 301         | 17          |
| # to        | # to        | # to        | # to        |
| 0           | 0           | 0           | 0           |
| 0           | 0           | 0           | 0           |
| 0           | 0           | 0           | 0           |
| 0           | 0           | 0           | 0           |
Figure 398: Results for plurality voting in the TM model for destructive control by deleting candidates. Number of voters is fixed.

| $n = 4$ | $n = 8$ |
|---------|---------|
| $m$     | $n = 16$ | $n = 32$ | $n = 64$ | $n = 128$ | $n = 4$ | $n = 8$ | $n = 32$ | $n = 64$ | $n = 128$ |
| $m$     | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp    | 145 143 249 161 44 20 | 93 125 193 145 70 43 | 407 375 307 141 122 106 | 0 0 0 214 308 351 |
| # ci    | 355 357 251 192 169 177 | 407 375 307 141 122 106 | 193 153 118 41 28 25 | 32 311 395 419 |
| # to    | 0 0 0 147 287 303 | 0 0 0 214 308 351 | 0 0 0 214 308 351 | 0 0 0 214 308 351 |

| $n = 64$ | $n = 128$ |
|---------|---------|
| $m$     | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp    | 80 117 190 156 79 33 | 95 131 199 138 77 56 |
| # ci    | 420 383 310 66 77 43 | 405 369 301 41 28 25 |
| # to    | 0 0 0 278 344 424 | 0 0 0 321 395 419 |

| $n = 64$ | $n = 128$ |
|---------|---------|
| $m$     | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp    | 73 145 182 122 87 30 | 80 130 195 126 95 25 |
| # ci    | 427 355 318 19 10 5 | 420 370 305 17 4 3 |
| # to    | 0 0 0 359 403 465 | 0 0 0 357 401 472 |
C.4.1. Computational Costs

Figure 399: Average time the algorithm needs to find a successful control action for destructive control by deleting candidates in plurality elections in the IC model. The maximum is 100.99 seconds.

Figure 400: Average time the algorithm needs to determine no-instance of destructive control by deleting candidates in plurality elections in the IC model. The maximum is 283.09 seconds.
Figure 401: Average time the algorithm needs to give a definite output for destructive control by deleting candidates in plurality elections in the IC model. The maximum is 96.65 seconds.

Figure 402: Average time the algorithm needs to find a successful control action for destructive control by deleting candidates in plurality elections in the TM model. The maximum is 137.95 seconds.
Figure 403: Average time the algorithm needs to determine no-instance of destructive control by deleting candidates in plurality elections in the TM model. The maximum is 247.8 seconds.

Figure 404: Average time the algorithm needs to give a definite output for destructive control by deleting candidates in plurality elections in the TM model. The maximum is 160.46 seconds.
C.5. Constructive Control by Partition of Candidates in Model TE

Figure 405: Results for plurality voting in the IC model for constructive control by partition of candidates in model TE. Number of voters is fixed.

|        | \(m = 4\)          | \(m = 8\)          | \(m = 16\)          | \(m = 32\)          | \(m = 64\)          | \(m = 128\)         |
|--------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| \(n\)  | \(4\)             | \(8\)             | \(16\)            | \(32\)            | \(64\)            | \(128\)           |
| \# cp  | 23 60 115 100 122 139 | 98 164 254 268 298 314 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \# ci  | 477 440 385 400 378 361 | 402 336 246 232 202 186 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \# to  | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \(m = 16\) | \(m = 32\)          | \(m = 64\)          | \(m = 128\)         |
| \(n\)  | \(4\)             | \(8\)             | \(16\)            | \(32\)            | \(64\)            | \(128\)           |
| \# cp  | 102 208 235 204 210 181 | 12 37 45 47 68 67 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \# ci  | 398 292 265 296 290 319 | 488 463 455 453 432 433 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| \# to  | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
Figure 406: Results for plurality voting in the IC model for constructive control by partition of candidates in model TE. Number of voters is fixed.

| m   | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp | 23  | 98  | 102 | 12  | 0   | 0   | 60  | 164 | 208 | 37  | 5   | 3   |
| # ci | 477 | 402 | 0   | 0   | 0   | 0   | 440 | 336 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 398 | 488 | 500 | 500 | 0   | 0   | 292 | 463 | 495 | 497 |

| m   | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|
| # cp | 115 | 254 | 235 | 45  | 5   | 3   |
| # ci | 385 | 246 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 265 | 455 | 495 | 497 |

| m   | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|
| # cp | 122 | 298 | 210 | 68  | 22  | 5   |
| # ci | 378 | 202 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 290 | 432 | 478 | 495 |

| m   | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|
| # cp | 139 | 314 | 181 | 67  | 23  | 8   |
| # ci | 361 | 186 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 319 | 433 | 477 | 492 |
Figure 407: Results for plurality voting in the TM model for constructive control by partition of candidates in model TE. Number of candidates is fixed.

| $m = 4$ | $m = 8$ |
|---------|---------|
| $n$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 6 | 21 | 35 | 43 | 54 | 57 | 15 | 47 | 70 | 69 | 101 | 93 |
| # ci | 494 | 479 | 465 | 457 | 446 | 443 | 485 | 453 | 430 | 431 | 399 | 407 |
| # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$m = 16$

| $n$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 29 | 62 | 53 | 43 | 46 | 34 | 3 | 12 | 8 | 15 | 17 | 8 |
| # ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 471 | 438 | 447 | 457 | 454 | 466 | 497 | 488 | 492 | 485 | 483 | 492 |

$m = 32$

| $n$ | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 2 | 1 | 4 | 2 | 4 | 2 | 0 | 0 | 3 | 2 | 2 | 1 |
| # ci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # to | 498 | 499 | 496 | 498 | 496 | 498 | 500 | 500 | 497 | 498 | 498 | 499 |
Figure 408: Results for plurality voting in the TM model for constructive control by partition of candidates in model TE. Number of voters is fixed.

| m    | n=4 | n=8 |
|------|-----|-----|
|      | 4   | 8   | 16  | 32  | 64  | 128 |
|      | 4   | 8   | 16  | 32  | 64  | 128 |
| # cp | 6   | 15  | 29  | 3   | 2   | 0   |
| # cp | 6   | 15  | 29  | 3   | 2   | 0   |
| # ci | 494 | 485 | 0   | 0   | 0   | 0   |
| # ci | 494 | 485 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 471 | 497 | 498 | 500 |
| # to | 0   | 0   | 471 | 497 | 498 | 500 |
|      | n=16|     |     |     |     |     |
|      |     |     |     |     |     |     |
|      |     |     |     |     |     |     |
|      |     |     |     |     |     |     |
|      |     |     |     |     |     |     |
| # cp | 35  | 70  | 53  | 8   | 4   | 3   |
| # cp | 35  | 70  | 53  | 8   | 4   | 3   |
| # ci | 465 | 430 | 0   | 0   | 0   | 0   |
| # ci | 465 | 430 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 447 | 492 | 496 | 497 |
| # to | 0   | 0   | 447 | 492 | 496 | 497 |
|      | n=64|     |     |     |     |     |
|      |     |     |     |     |     |     |
|      |     |     |     |     |     |     |
|      |     |     |     |     |     |     |
|      |     |     |     |     |     |     |
| # cp | 54  | 101 | 46  | 17  | 4   | 2   |
| # cp | 54  | 101 | 46  | 17  | 4   | 2   |
| # ci | 446 | 399 | 0   | 0   | 0   | 0   |
| # ci | 446 | 399 | 0   | 0   | 0   | 0   |
| # to | 0   | 0   | 454 | 483 | 496 | 498 |
| # to | 0   | 0   | 454 | 483 | 496 | 498 |
C.5.1. Computational Costs

Figure 409: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TE in plurality elections in the IC model. The maximum is 158.73 seconds.

Figure 410: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TE in plurality elections in the IC model. The maximum is 27.99 seconds.
Figure 411: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TE in plurality elections in the IC model. The maximum is 158, 11 seconds.

Figure 412: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TE in plurality elections in the TM model. The maximum is 188, 89 seconds.
Figure 413: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TE in plurality elections in the TM model. The maximum is 27.64 seconds.

Figure 414: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TE in plurality elections in the TM model. The maximum is 188.89 seconds.
C.6. Destructive Control by Partition of Candidates in Model TE

Figure 415: Results for plurality voting in the IC model for destructive control by partition of candidates in model TE. Number of candidates is fixed.

|   | m = 4 |   | m = 8 |
|---|-------|---|-------|
| n | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 288 282 290 293 308 294 | 379 445 459 477 476 485 |
| # ci | 212 218 210 207 192 206 | 121 55 41 23 24 15 |
| # to | 0 0 0 0 0 0 | 0 0 0 0 0 0 |

|   | m = 16 |   | m = 32 |
|---|-------|---|-------|
| n | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 394 485 494 498 499 | 19 332 375 444 473 485 |
| # ci | 0 0 0 0 0 0 | 11 0 0 0 0 0 |
| # to | 106 15 6 2 2 1 | 470 168 125 56 27 15 |

|   | m = 64 |   | m = 128 |
|---|-------|---|-------|
| n | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp | 2 133 343 367 411 432 | 2 43 266 293 352 395 |
| # ci | 5 0 0 0 0 0 | 3 0 0 0 0 0 |
| # to | 493 367 157 133 89 68 | 495 457 234 207 148 105 |
Figure 416: Results for plurality voting in the IC model for destructive control by partition of candidates in model TE. Number of voters is fixed.

|   | $n = 4$ |   | $n = 8$ |   | $n = 16$ |   | $n = 32$ |   | $n = 64$ |   | $n = 128$ |
|---|---------|---|---------|---|---------|---|---------|---|---------|---|---------|
| $m$ | 4   | 8   | 16  | 32  | 64  | 128 | 4    | 8    | 16   | 32  | 64   | 128 |
| # cp  | 288  | 379  | 394 | 19   | 2   | 2   | 282 | 445  | 485  | 332  | 133  | 43  |
| # ci  | 212  | 121  | 0   | 11   | 5   | 3   | 218 | 55   | 0    | 0    | 0    | 0   |
| # to  | 0    | 0    | 106 | 470  | 493 | 495 | 0   | 0    | 15   | 168  | 367  | 457 |
| $n = 16$ |   |   |   |   |   |   |   |   |   |   |   |
| $m$ | 4    | 8   | 16  | 32  | 64  | 128 | 4    | 8    | 16   | 32  | 64   | 128 |
| # cp  | 290  | 459  | 494 | 375  | 343 | 266 | 293 | 477  | 498  | 444  | 367  | 293 |
| # ci  | 210  | 41   | 0   | 0    | 0   | 0   | 207 | 23   | 0    | 0    | 0    | 0   |
| # to  | 0    | 0    | 6   | 125  | 157 | 234 | 0   | 0    | 2    | 56   | 133  | 207 |
| $n = 32$ |   |   |   |   |   |   |   |   |   |   |   |
| $m$ | 4    | 8   | 16  | 32  | 64  | 128 | 4    | 8    | 16   | 32  | 64   | 128 |
| # cp  | 308  | 476  | 498 | 473  | 411 | 352 | 294 | 485  | 499  | 485  | 432  | 395 |
| # ci  | 192  | 24   | 0   | 0    | 0   | 0   | 206 | 15   | 0    | 0    | 0    | 0   |
| # to  | 0    | 0    | 2   | 27   | 89  | 148 | 0   | 0    | 1    | 15   | 68   | 105 |
Figure 417: Results for plurality voting in the TM model for destructive control by partition of candidates in model TE. Number of candidates is fixed.

| m = 4 | m = 8 |
|-------|-------|
| n     | # cp  | # ci | # to | n     | # cp  | # ci | # to |
| 4     | 188   | 312  | 0    | 4     | 234   | 366  | 0    |
| 8     | 147   | 353  | 0    | 8     | 200   | 378  | 0    |
| 16    | 130   | 370  | 0    | 16    | 137   | 384  | 0    |
| 32    | 116   | 363  | 0    | 32    | 116   | 378  | 0    |
| 64    | 122   | 384  | 0    | 64    | 122   | 378  | 0    |
| 128   | 234   | 353  | 0    | 128   | 234   | 378  | 0    |
| m = 16| # cp  | # ci | # to | m = 32| # cp  | # ci | # to |
| n     | 295   | 161  | 44   | n     | 295   | 161  | 44   |
| 4     | 244   | 154  | 102  | 8     | 244   | 154  | 102  |
| 8     | 242   | 105  | 153  | 16    | 242   | 105  | 153  |
| 16    | 243   | 70   | 187  | 32    | 243   | 70   | 187  |
| 32    | 237   | 105  | 244  | 64    | 237   | 105  | 244  |
| 64    | 237   | 105  | 244  | 128   | 237   | 105  | 244  |
| 128   | 237   | 105  | 244  | 128   | 237   | 105  | 244  |

| m = 64| m = 128|
|-------|--------|
| n     | # cp  | # ci | # to | n     | # cp  | # ci | # to |
| 4     | 10    | 169  | 321  | 4     | 10    | 169  | 321  |
| 8     | 19    | 122  | 345  | 8     | 19    | 122  | 345  |
| 16    | 33    | 28   | 425  | 16    | 33    | 28   | 425  |
| 32    | 10    | 47   | 432  | 32    | 10    | 47   | 432  |
| 64    | 4     | 58   | 438  | 64    | 4     | 58   | 438  |
| 128   | 4     | 58   | 438  | 128   | 4     | 58   | 438  |
| m = 64| # cp  | # ci | # to | m = 128| # cp  | # ci | # to |
| n     | 10    | 169  | 321  | n     | 10    | 169  | 321  |
| 4     | 19    | 122  | 345  | 4     | 19    | 122  | 345  |
| 8     | 33    | 28   | 425  | 8     | 33    | 28   | 425  |
| 16    | 10    | 47   | 432  | 16    | 10    | 47   | 432  |
| 32    | 4     | 58   | 438  | 32    | 4     | 58   | 438  |
| 64    | 4     | 58   | 438  | 64    | 4     | 58   | 438  |
| 128   | 4     | 58   | 438  | 128   | 4     | 58   | 438  |
| m = 64| # cp  | # ci | # to | m = 128| # cp  | # ci | # to |
| n     | 10    | 169  | 321  | n     | 10    | 169  | 321  |
| 4     | 19    | 122  | 345  | 4     | 19    | 122  | 345  |
| 8     | 33    | 28   | 425  | 8     | 33    | 28   | 425  |
| 16    | 10    | 47   | 432  | 16    | 10    | 47   | 432  |
| 32    | 4     | 58   | 438  | 32    | 4     | 58   | 438  |
| 64    | 4     | 58   | 438  | 64    | 4     | 58   | 438  |
| 128   | 4     | 58   | 438  | 128   | 4     | 58   | 438  |
Figure 418: Results for plurality voting in the TM model for destructive control by partition of candidates in model TE. Number of voters is fixed.

| $n$ | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|----|----|----|----|----|-----|
| $m$ | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
| # cp | 188 | 234 | 295 | 29 | 10 | 12 | 147 | 200 | 244 | 57 | 33 | 29 |
| # ci | 312 | 266 | 161 | 192 | 169 | 177 | 353 | 300 | 154 | 141 | 122 | 106 |
| # to | 0 | 0 | 44 | 279 | 321 | 311 | 0 | 0 | 102 | 302 | 345 | 365 |
| $n = 16$ | | | | | | | | | | | | |
C.6.1. Computational Costs

Figure 419: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TE in plurality elections in the IC model. The maximum is 172.19 seconds.

Figure 420: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TE in plurality elections in the IC model. The maximum is 28.26 seconds.
Figure 421: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TE in plurality elections in the IC model. The maximum is 172,19 seconds.

Figure 422: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TE in plurality elections in the TM model. The maximum is 115,89 seconds.
Figure 423: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TE in plurality elections in the TM model. The maximum is 20.15 seconds.

Figure 424: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TE in plurality elections in the TM model. The maximum is 97.65 seconds.
C.7. Constructive Control by Partition of Candidates in Model TP

Figure 425: Results for plurality voting in the IC model for constructive control by partition of candidates in model TP. Number of candidates is fixed.

|     | $m = 4$ |     | $m = 8$ |     | $m = 16$ |     | $m = 32$ |     | $m = 64$ |     | $m = 128$ |
|-----|---------|-----|---------|-----|---------|-----|---------|-----|---------|-----|---------|
|     | 4    | 8    | 16   | 32   | 64   | 128 | 4    | 8    | 16   | 32   | 64   | 128 |
| # cp | 17   | 54   | 120  | 100  | 132  | 147 | 82   | 164  | 241  | 246  | 275  | 318 |
| # ci | 483  | 446  | 380  | 400  | 368  | 353 | 418  | 336  | 259  | 254  | 225  | 182 |
| # to | 0    | 0    | 0    | 0    | 0    | 0   | 0    | 0    | 0    | 0    | 0    | 0   |
|     | 16   | 64   | 128  | 4    | 8    | 16   | 32   | 64   | 128  | 4    | 8    | 16   |
| # cp | 90   | 219  | 254  | 231  | 212  | 202 | 18   | 42   | 45   | 56   | 57   | 57   |
| # ci | 0    | 0    | 0    | 0    | 0    | 0   | 0    | 0    | 0    | 0    | 0    | 0   |
| # to | 410  | 281  | 246  | 269  | 288  | 298 | 482  | 458  | 455  | 444  | 443  | 443  |
|     | 64   | 128  | 4    | 8    | 16   | 32   | 64   | 128  | 4    | 8    | 16   | 32   |
| # cp | 4    | 8    | 12   | 15   | 23   | 17  | 0    | 1    | 3    | 3    | 10   | 7   |
| # ci | 0    | 0    | 0    | 0    | 0    | 0   | 0    | 0    | 0    | 0    | 0    | 0   |
| # to | 496  | 492  | 488  | 485  | 477  | 483 | 500  | 499  | 497  | 497  | 490  | 493 |
Figure 426: Results for plurality voting in the IC model for constructive control by partition of candidates in model TP. Number of voters is fixed.
Figure 427: Results for plurality voting in the TM model for constructive control by partition of candidates in model TP. Number of candidates is fixed.

|     | $m=4$          | $m=8$          | $m=16$         | $m=32$         | $m=64$         | $m=128$        |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|
| $n$ | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp | 7  23  35  46  66 | 21  48  77  69  99  117 | 21  48  77  69  99  117 | 21  48  77  69  99  117 | 21  48  77  69  99  117 | 21  48  77  69  99  117 |
| # ci | 493  477  465  465  454  434 | 479  452  423  431  401  383 | 479  452  423  431  401  383 | 479  452  423  431  401  383 | 479  452  423  431  401  383 | 479  452  423  431  401  383 |
| # to | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 |
|     | $m=16$          | $m=32$          | $m=64$         | $m=128$        | $m=64$         | $m=128$        |
| $n$ | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp | 26  54  48  49  48  48 | 2  13  6  14  14  14 | 2  13  6  14  14  14 | 2  13  6  14  14  14 | 2  13  6  14  14  14 | 2  13  6  14  14  14 |
| # ci | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 |
| # to | 474  446  452  451  452  452 | 498  487  494  486  486  496 | 498  487  494  486  486  496 | 498  487  494  486  486  496 | 498  487  494  486  486  496 | 498  487  494  486  486  496 |
|     | $m=64$          | $m=128$         | $m=64$         | $m=128$        | $m=64$         | $m=128$        |
| $n$ | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 | 4  8  16  32  64  128 |
| # cp | 0  2  3  6  2  2 | 0  0  0  1  2  2 | 0  0  0  1  2  2 | 0  0  0  1  2  2 | 0  0  0  1  2  2 | 0  0  0  1  2  2 |
| # ci | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 | 0  0  0  0  0 |
| # to | 500  498  497  494  498  498 | 500  500  500  499  498  498 | 500  500  500  499  498  498 | 500  500  500  499  498  498 | 500  500  500  499  498  498 | 500  500  500  499  498  498 |
Figure 428: Results for plurality voting in the TM model for constructive control by partition of candidates in model TP. Number of voters is fixed.

| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $#\,cp$ | 7  | 21 | 26 | 2  | 0  | 0   |
| $#\,ci$ | 493 | 479 | 0  | 0  | 0  | 0   |
| $#\,to$ | 0  | 0  | 474 | 498 | 500 | 500 |

$n = 4$

| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $#\,cp$ | 35 | 77 | 48 | 6  | 3  | 0   |
| $#\,ci$ | 465 | 423 | 0  | 0  | 0  | 0   |
| $#\,to$ | 0  | 0  | 452 | 494 | 497 | 500 |

$n = 16$

| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $#\,cp$ | 46 | 99 | 48 | 14 | 2  | 2   |
| $#\,ci$ | 454 | 401 | 0  | 0  | 0  | 0   |
| $#\,to$ | 0  | 0  | 452 | 486 | 498 | 498 |

$n = 64$

| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $#\,cp$ | 23 | 48 | 54 | 13 | 2  | 0   |
| $#\,ci$ | 477 | 452 | 0  | 0  | 0  | 0   |
| $#\,to$ | 0  | 0  | 446 | 487 | 498 | 500 |

$n = 8$

| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $#\,cp$ | 40 | 69 | 49 | 14 | 6  | 1   |
| $#\,ci$ | 460 | 431 | 0  | 0  | 0  | 0   |
| $#\,to$ | 0  | 0  | 451 | 486 | 494 | 499 |

$n = 32$

| $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|----|----|----|----|----|-----|
| $#\,cp$ | 66 | 117 | 48 | 4  | 2  | 2   |
| $#\,ci$ | 434 | 383 | 0  | 0  | 0  | 0   |
| $#\,to$ | 0  | 0  | 452 | 496 | 498 | 498 |

$n = 128$
C.7.1. Computational Costs

Figure 429: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TP in plurality elections in the IC model. The maximum is 174.14 seconds.

Figure 430: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TP in plurality elections in the IC model. The maximum is 27.68 seconds.
Figure 431: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TP in plurality elections in the IC model. The maximum is 174, 14 seconds.

Figure 432: Average time the algorithm needs to find a successful control action for constructive control by partition of candidates in model TP in plurality elections in the TM model. The maximum is 294, 18 seconds.
Figure 433: Average time the algorithm needs to determine no-instance of constructive control by partition of candidates in model TP in plurality elections in the TM model. The maximum is 28.05 seconds.

Figure 434: Average time the algorithm needs to give a definite output for constructive control by partition of candidates in model TP in plurality elections in the TM model. The maximum is 294.18 seconds.
C.8. Destructive Control by Partition of Candidates in Model TP

Figure 435: Results for plurality voting in the IC model for destructive control by partition of candidates in model TP. Number of candidates is fixed.

|       | m = 4       | m = 8       |
|-------|-------------|-------------|
| **n**| 4 8 16 32 64 128  | 4 8 16 32 64 128  |
| **# cp** | 288 282 290 293 308 294 | 379 445 459 477 476 485 |
| **# ci** | 212 218 210 207 192 206 | 121 55 41 23 24 15 |
| **# to** | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| **m = 16** | m = 32     | m = 64      |
| **n** | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| **# cp** | 393 484 494 498 498 499 | 19 332 375 444 473 485 |
| **# ci** | 28 2 0 0 0 0 | 11 0 0 0 0 0 |
| **# to** | 79 14 6 2 2 1 | 470 168 125 56 27 15 |
| **m = 64** | m = 128     | m = 128     |
| **n** | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| **# cp** | 2 133 343 367 411 432 | 2 43 266 293 352 395 |
| **# ci** | 5 0 0 0 0 0 | 3 0 0 0 0 0 |
| **# to** | 493 367 157 133 89 68 | 495 457 234 207 148 105 |
Figure 436: Results for plurality voting in the IC model for destructive control by partition of candidates in model TP. Number of voters is fixed.

| m  | n = 4 | n = 8 |
|-----|-------|-------|
|     | 4     | 8     | 16 | 32 | 64 | 128 | 4     | 8     | 16 | 32 | 64 | 128 |
| # cp | 288   | 379   | 393 | 19 | 2   | 2   | 282   | 445   | 484 | 332 | 133 | 43   |
| # ci | 212   | 121   | 28  | 11 | 5   | 3   | 218   | 55    | 2   | 0   | 0   | 0    |
| # to | 0     | 0     | 79  | 470| 493 | 495 | 0     | 0     | 14  | 168 | 367 | 457  |

| m  | n = 16 | n = 32 |
|-----|--------|--------|
|     | 4     | 8     | 16 | 32 | 64 | 128 | 4     | 8     | 16 | 32 | 64 | 128 |
| # cp | 290   | 459   | 494 | 375 | 343 | 266 | 293   | 477   | 498 | 444 | 367 | 293  |
| # ci | 210   | 41    | 0   | 0   | 0   | 0   | 207   | 23    | 0   | 0   | 0   | 0    |
| # to | 0     | 0     | 6   | 125 | 157 | 234 | 0     | 0     | 2   | 56  | 133 | 207  |

| m  | n = 64 | n = 128 |
|-----|--------|---------|
|     | 4     | 8     | 16 | 32 | 64 | 128 | 4     | 8     | 16 | 32 | 64 | 128 |
| # cp | 308   | 476   | 498 | 473 | 411 | 352 | 294   | 485   | 499 | 485 | 432 | 395  |
| # ci | 192   | 24    | 0   | 0   | 0   | 0   | 206   | 15    | 0   | 0   | 0   | 0    |
| # to | 0     | 0     | 2   | 87  | 148 | 0   | 0     | 0     | 1   | 15  | 68  | 105  |
Figure 437: Results for plurality voting in the TM model for destructive control by partition of candidates in model TP. Number of candidates is fixed.

|        | $m = 4$ |        | $m = 8$ |
|--------|---------|--------|---------|
| $n$    | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp   | 188 147 130 137 116 122 | 234 200 230 232 260 230 |
| # ci   | 312 353 370 363 384 378 | 266 300 270 268 240 270 |
| # to   | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
|        | $m = 16$ |        | $m = 32$ |
| $n$    | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp   | 295 244 242 243 217 237 | 29 57 74 88 91 97 |
| # ci   | 161 154 105 70 39 36 | 192 141 66 41 19 17 |
| # to   | 44 102 153 187 244 227 | 279 302 360 371 390 386 |
|        | $m = 64$ |        | $m = 128$ |
| $n$    | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp   | 10 33 34 47 58 58 | 12 29 13 31 31 34 |
| # ci   | 169 122 77 28 10 4 | 177 106 43 25 5 3 |
| # to   | 321 345 389 425 432 438 | 311 365 444 464 464 463 |
Figure 438: Results for plurality voting in the TM model for destructive control by partition of candidates in model TP. Number of voters is fixed.

| m   | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp | 188 | 234 | 295 | 29  | 10  | 12  | 147 | 200 | 244 | 57  | 33  | 29  |
| # ci | 312 | 266 | 161 | 192 | 169 | 177 | 353 | 300 | 154 | 141 | 122 | 106 |
| # to | 0   | 44  | 279 | 321 | 311 | 62  | 0   | 102 | 302 | 345 | 365 | 128 |

| m   | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp | 130 | 230 | 242 | 74  | 34  | 13  | 137 | 232 | 243 | 88  | 47  | 31  |
| # ci | 370 | 270 | 105 | 66  | 77  | 43  | 363 | 268 | 70  | 41  | 28  | 25  |
| # to | 0   | 153 | 360 | 389 | 444 | 0   | 0   | 187 | 371 | 425 | 444 | 0   |

| m   | 4   | 8   | 16  | 32  | 64  | 128 | 4   | 8   | 16  | 32  | 64  | 128 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| # cp | 116 | 260 | 217 | 91  | 58  | 31  | 122 | 230 | 237 | 97  | 58  | 34  |
| # ci | 384 | 240 | 39  | 19  | 10  | 5   | 378 | 270 | 36  | 17  | 4   | 3   |
| # to | 0   | 244 | 390 | 432 | 464 | 0   | 0   | 227 | 386 | 438 | 463 | 0   |
C.8.1. Computational Costs

Figure 439: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TP in plurality elections in the IC model. The maximum is 174.36 seconds.

Figure 440: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TP in plurality elections in the IC model. The maximum is 28.46 seconds.
Figure 441: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TP in plurality elections in the IC model. The maximum is 162.76 seconds.

Figure 442: Average time the algorithm needs to find a successful control action for destructive control by partition of candidates in model TP in plurality elections in the TM model. The maximum is 117.3 seconds.
Figure 443: Average time the algorithm needs to determine no-instance of destructive control by partition of candidates in model TP in plurality elections in the TM model. The maximum is 20.17 seconds.

Figure 444: Average time the algorithm needs to give a definite output for destructive control by partition of candidates in model TP in plurality elections in the TM model. The maximum is 96.35 seconds.
C.9. Constructive Control by Runoff Partition of Candidates in Model TE

Figure 445: Results for plurality voting in the IC model for constructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

|       | \( m = 4 \) |       | \( m = 8 \) |       | \( m = 16 \) |       | \( m = 32 \) |       | \( m = 64 \) |       |
|-------|-------------|-------|-------------|-------|-------------|-------|-------------|-------|-------------|-------|
| \( n \) | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 17 35 103 91 112 131 | 71 109 192 194 230 249 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # ci  | 483 465 397 409 388 369 | 429 391 308 306 270 251 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to  | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |

|       | \( m = 16 \) |       | \( m = 32 \) |       | \( m = 64 \) |       |
|-------|-------------|-------|-------------|-------|-------------|-------|
| \( n \) | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 74 223 310 310 321 301 | 11 79 124 160 167 152 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # ci  | 426 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to  | 0 277 190 190 179 199 | 489 421 376 340 333 348 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |

|       | \( m = 64 \) |       | \( m = 128 \) |
|-------|-------------|-------|
| \( n \) | 4 8 16 32 64 128 | 4 8 16 32 64 128 |
| # cp  | 3 14 47 50 55 51 | 0 1 16 18 19 11 |
| # ci  | 0 0 0 0 0 0 | 0 0 0 0 0 0 |
| # to  | 497 486 453 450 445 449 | 500 499 484 482 481 489 |
Figure 446: Results for plurality voting in the IC model for constructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

| m  | n = 4 | n = 8 | n = 16 | n = 32 | n = 64 | n = 128 |
|----|-------|-------|--------|--------|--------|--------|
|    | 4     | 8     | 16     | 32     | 64     | 128    |
|    |       |       |        |        |        |        |
| # cp | 17    | 71    | 74     | 11     | 3      | 0      |
| # ci | 483   | 429   | 426    | 0      | 0      | 0      |
| # to | 0     | 0     | 0      | 489    | 497    | 500    |
| n = 16 | 35   | 109   | 223    | 79     | 14     | 1      |
| n = 32 | 465   | 391   | 0      | 0      | 0      | 0      |
| n = 64 | 0     | 0     | 277    | 421    | 486    | 499    |
| n = 128 |       |       |        |        |        |        |
| m  | n = 4 | n = 8 | n = 16 | n = 32 | n = 64 | n = 128 |
|    | 4     | 8     | 16     | 32     | 64     | 128    |
|    |       |       |        |        |        |        |
| # cp | 103   | 192   | 310    | 124    | 47     | 16     |
| # ci | 397   | 308   | 0      | 0      | 0      | 0      |
| # to | 0     | 0     | 190    | 376    | 453    | 484    |
| n = 16 | 91    | 194   | 310    | 160    | 50     | 18     |
| n = 32 | 409   | 306   | 0      | 0      | 0      | 0      |
| n = 64 | 0     | 0     | 190    | 340    | 450    | 482    |
| n = 128 |       |       |        |        |        |        |
| m  | n = 4 | n = 8 | n = 16 | n = 32 | n = 64 | n = 128 |
|    | 4     | 8     | 16     | 32     | 64     | 128    |
|    |       |       |        |        |        |        |
| # cp | 112   | 230   | 321    | 167    | 55     | 19     |
| # ci | 388   | 270   | 0      | 0      | 0      | 0      |
| # to | 0     | 0     | 179    | 333    | 445    | 481    |
| n = 16 | 131   | 249   | 301    | 152    | 51     | 11     |
| n = 32 | 369   | 251   | 0      | 0      | 0      | 0      |
| n = 64 | 0     | 0     | 199    | 348    | 449    | 489    |
| n = 128 |       |       |        |        |        |        |
Figure 447: Results for plurality voting in the TM model for constructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

| $n$   | 4 | 8 | 16 | 32 | 64 | 128 | $n$   | 4 | 8 | 16 | 32 | 64 | 128 |
|-------|---|---|----|----|----|-----|-------|---|---|----|----|----|-----|
| $m=4$ |   |   |    |    |    |     | $m=8$ |   |   |    |    |    |     |
| # cp  | 7 | 21| 34 | 46 | 44 | 66  | 15    | 43| 68| 65 | 88 | 95 |     |
| # ci  | 493| 479| 466| 454| 456| 434 | 485   | 457| 432| 435| 412| 405|     |
| # to  | 0 | 0 | 0  | 0  | 0  | 0   | 0     | 0 | 0 | 0  | 0  | 0  | 0   |
| $m=16$|   |   |    |    |    |     | $m=32$|   |   |    |    |    |     |
| # cp  | 21| 62| 64 | 75 | 75 | 64  | 3     | 20| 26| 19 | 19 | 35 |     |
| # ci  | 88| 0 | 0  | 0  | 0  | 0   | 0     | 0 | 0 | 0  | 0  | 0  | 0   |
| # to  | 391| 438| 436| 425| 425| 436 | 497   | 480| 474| 481| 481| 465|     |
| $m=64$|   |   |    |    |    |     | $m=128$|   |   |    |    |    |     |
| # cp  | 0 | 6 | 17 | 3  | 6  | 8   | 0     | 1 | 5 | 2  | 6  | 2  |     |
| # ci  | 0 | 0 | 0  | 0  | 0  | 0   | 0     | 0 | 0 | 0  | 0  | 0  | 0   |
| # to  | 500| 494| 483| 497| 494| 492 | 500   | 499| 495| 498| 494| 498|     |
Figure 448: Results for plurality voting in the TM model for constructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

|   | \( n = 4 \) | \( n = 8 \) |
|---|---|---|
| \( m \) | \( n = 16 \) | \( n = 32 \) | \( n = 64 \) | \( n = 128 \) | \( n = 16 \) | \( n = 32 \) | \( n = 64 \) | \( n = 128 \) |
| \( \# \text{ cp} \) | 7 | 15 | 21 | 3 | 0 | 0 | 21 | 43 | 62 | 20 | 6 | 1 |
| \( \# \text{ ci} \) | 493 | 485 | 88 | 0 | 0 | 0 | 479 | 457 | 0 | 0 | 0 | 0 |
| \( \# \text{ to} \) | 0 | 0 | 391 | 497 | 500 | 500 | 0 | 0 | 438 | 480 | 494 | 499 |

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C.9.1. Computational Costs

Figure 449: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TE in plurality elections in the IC model. The maximum is 378.02 seconds.

Figure 450: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TE in plurality elections in the IC model. The maximum is 495.26 seconds.
Figure 451: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TE in plurality elections in the IC model. The maximum is 430.08 seconds.

Figure 452: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TE in plurality elections in the TM model. The maximum is 265.22 seconds.
Figure 453: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TE in plurality elections in the TM model. The maximum is 562.86 seconds.

Figure 454: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TE in plurality elections in the TM model. The maximum is 469.62 seconds.
C.10. Destructive Control by Runoff Partition of Candidates in Model TE

Figure 455: Results for plurality voting in the IC model for destructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

| m = 4 | m = 8 |
|-------|-------|
| n     | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp  | 291   | 276   | 269   | 275   | 284   | 274   | 379   | 442   | 441   | 467   | 466   | 474   |
| # ci  | 209   | 224   | 231   | 225   | 216   | 226   | 121   | 58    | 59    | 33    | 34    | 26    |
| # to  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |

| m = 16 | m = 32 |
|--------|--------|
| n      | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp   | 414   | 495   | 494   | 500   | 499   | 500   | 196   | 439   | 444   | 484   | 489   | 491   |
| # ci   | 86    | 2     | 0     | 0     | 0     | 0     | 11    | 0     | 0     | 0     | 0     | 0     |
| # to   | 0     | 3     | 6     | 0     | 1     | 0     | 293   | 61    | 56    | 16    | 11    | 9     |

| m = 64 | m = 128 |
|--------|---------|
| n      | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp   | 161   | 332   | 433   | 440   | 455   | 461   | 123   | 278   | 403   | 406   | 434   | 445   |
| # ci   | 5     | 0     | 0     | 0     | 0     | 0     | 3     | 0     | 0     | 0     | 0     | 0     |
| # to   | 334   | 168   | 67    | 60    | 45    | 39    | 374   | 222   | 97    | 94    | 66    | 55    |

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Figure 456: Results for plurality voting in the IC model for destructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

| $n$ | $m$ | # cp | # ci | # to |
|-----|-----|------|------|------|
| $n=4$ | $m=4$ | 4 | 291 | 209 | 0 |
|      |     | 8 | 379 | 121 | 0 |
|      |     | 16 | 414 | 86 | 0 |
|      |     | 32 | 196 | 11 | 0 |
|      |     | 64 | 161 | 5 | 0 |
|      |     | 128 | 123 | 0 | 0 |
| $n=8$ | $m=4$ | 4 | 276 | 224 | 0 |
|      |     | 8 | 442 | 58 | 0 |
|      |     | 16 | 495 | 2 | 0 |
|      |     | 32 | 439 | 0 | 0 |
|      |     | 64 | 332 | 0 | 0 |
|      |     | 128 | 278 | 0 | 0 |

| $n=4$ | $m=16$ | 4 | 196 | 224 | 0 |
|      |     | 8 | 442 | 58 | 0 |
|      |     | 16 | 495 | 2 | 0 |
|      |     | 32 | 439 | 0 | 0 |
|      |     | 64 | 332 | 0 | 0 |
|      |     | 128 | 278 | 0 | 0 |

| $n=8$ | $m=16$ | 4 | 276 | 224 | 0 |
|      |     | 8 | 442 | 58 | 0 |
|      |     | 16 | 495 | 2 | 0 |
|      |     | 32 | 439 | 0 | 0 |
|      |     | 64 | 332 | 0 | 0 |
|      |     | 128 | 278 | 0 | 0 |

| $n=4$ | $m=32$ | 4 | 196 | 224 | 0 |
|      |     | 8 | 442 | 58 | 0 |
|      |     | 16 | 495 | 2 | 0 |
|      |     | 32 | 439 | 0 | 0 |
|      |     | 64 | 332 | 0 | 0 |
|      |     | 128 | 278 | 0 | 0 |

| $n=8$ | $m=32$ | 4 | 276 | 224 | 0 |
|      |     | 8 | 442 | 58 | 0 |
|      |     | 16 | 495 | 2 | 0 |
|      |     | 32 | 439 | 0 | 0 |
|      |     | 64 | 332 | 0 | 0 |
|      |     | 128 | 278 | 0 | 0 |

| $n=4$ | $m=64$ | 4 | 196 | 224 | 0 |
|      |     | 8 | 442 | 58 | 0 |
|      |     | 16 | 495 | 2 | 0 |
|      |     | 32 | 439 | 0 | 0 |
|      |     | 64 | 332 | 0 | 0 |
|      |     | 128 | 278 | 0 | 0 |

| $n=8$ | $m=64$ | 4 | 276 | 224 | 0 |
|      |     | 8 | 442 | 58 | 0 |
|      |     | 16 | 495 | 2 | 0 |
|      |     | 32 | 439 | 0 | 0 |
|      |     | 64 | 332 | 0 | 0 |
|      |     | 128 | 278 | 0 | 0 |

| $n=4$ | $m=128$ | 4 | 196 | 224 | 0 |
|      |     | 8 | 442 | 58 | 0 |
|      |     | 16 | 495 | 2 | 0 |
|      |     | 32 | 439 | 0 | 0 |
|      |     | 64 | 332 | 0 | 0 |
|      |     | 128 | 278 | 0 | 0 |

| $n=8$ | $m=128$ | 4 | 276 | 224 | 0 |
|      |     | 8 | 442 | 58 | 0 |
|      |     | 16 | 495 | 2 | 0 |
|      |     | 32 | 439 | 0 | 0 |
|      |     | 64 | 332 | 0 | 0 |
|      |     | 128 | 278 | 0 | 0 |
Figure 457: Results for plurality voting in the TM model for destructive control by runoff-partition of candidates in model TE. Number of candidates is fixed.

| $m$ | $n$ | 4 | 8 | 16 | 32 | 64 | 128 | $m$ | $n$ | 4 | 8 | 16 | 32 | 64 | 128 |
|-----|-----|---|---|----|----|----|-----|-----|-----|---|---|----|----|----|-----|
|     | # cp | 188 | 144 | 124 | 129 | 106 | 115 | 234 | 197 | 214 | 218 | 239 | 219 |
|     | # ci | 312 | 356 | 376 | 371 | 394 | 385 | 266 | 303 | 286 | 282 | 261 | 281 |
|     | # to | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| m = 16 | # cp | 305 | 281 | 276 | 258 | 257 | 265 | 130 | 139 | 138 | 119 | 119 | 130 |
|     | # ci | 195 | 154 | 105 | 70 | 39 | 36 | 192 | 141 | 66 | 41 | 19 | 17 |
|     | # to | 0 | 65 | 119 | 172 | 204 | 199 | 178 | 220 | 296 | 340 | 362 | 353 |
| m = 32 | # cp | 93 | 95 | 82 | 87 | 96 | 86 | 88 | 106 | 71 | 86 | 75 | 68 |
|     | # ci | 169 | 122 | 77 | 28 | 10 | 4 | 177 | 106 | 43 | 25 | 5 | 3 |
|     | # to | 238 | 283 | 341 | 385 | 394 | 410 | 235 | 286 | 386 | 389 | 420 | 429 |

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Figure 458: Results for plurality voting in the TM model for destructive control by runoff-partition of candidates in model TE. Number of voters is fixed.

| m   | n = 4 | n = 8 |
|-----|-------|-------|
|     | 4     | 8     | 16   | 32   | 64   | 128  | 4     | 8     | 16   | 32   | 64   | 128  |
| # cp |       |       |      |      |      |      |       |       |      |      |      |      |
|      | 188   | 234   | 305  | 130  | 93   | 88   | 144   | 197   | 281  | 139  | 95   | 106  |
| # ci | 312   | 266   | 195  | 192  | 169  | 177  | 356   | 303   | 154  | 141  | 122  | 106  |
| # to | 0     | 0     | 178  | 238  | 235  |      | 0     | 65    | 220  | 283  | 288  |      |
| n = 16 |       |       |      |      |      |      |       |       |      |      |      |      |
| m   | n = 32 |       |      |      |      |      |       |       |      |      |      |      |
|     | 4     | 8     | 16   | 32   | 64   | 128  | 4     | 8     | 16   | 32   | 64   | 128  |
| # cp |       |       |      |      |      |      |       |       |      |      |      |      |
|      | 124   | 214   | 276  | 138  | 82   | 71   | 129   | 218   | 258  | 119  | 87   | 86   |
| # ci | 376   | 286   | 105  | 66   | 77   | 43   | 371   | 282   | 70   | 41   | 28   | 25   |
| # to | 0     | 0     | 178  | 238  | 235  |      | 0     | 65    | 220  | 283  | 288  |      |
| n = 64 |       |       |      |      |      |      |       |       |      |      |      |      |
| m   | n = 128 |       |      |      |      |      |       |       |      |      |      |      |
|     | 4     | 8     | 16   | 32   | 64   | 128  | 4     | 8     | 16   | 32   | 64   | 128  |
| # cp |       |       |      |      |      |      |       |       |      |      |      |      |
|      | 106   | 239   | 257  | 119  | 96   | 75   | 115   | 219   | 265  | 130  | 86   | 68   |
| # ci | 394   | 261   | 39   | 19   | 15   | 10   | 385   | 281   | 36   | 17   | 4    | 3    |
| # to | 0     | 0     | 204  | 362  | 394  | 420  | 0     | 199   | 353  | 410  | 429  |      |

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C.10.1. Computational Costs

Figure 459: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TE in plurality elections in the IC model. The maximum is 35.3 seconds.

Figure 460: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TE in plurality elections in the IC model. The maximum is 348.65 seconds.
Figure 461: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TE in plurality elections in the IC model. The maximum is 73.6 seconds.

Figure 462: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TE in plurality elections in the TM model. The maximum is 75.31 seconds.
Figure 463: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TE in plurality elections in the TM model. The maximum is 89.69 seconds.

Figure 464: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TE in plurality elections in the TM model. The maximum is 65.39 seconds.
C.11. Constructive Control by Runoff Partition of Candidates in Model TP

Figure 465: Results for plurality voting in the IC model for constructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

| m = 4 | m = 8 |
|-------|-------|
| n     |       |       |
|       | 4     | 8     | 16   | 32   | 64   | 128  |       |
| cp    | 15    | 46    | 100  | 97   | 118  | 124  | 69    | 138  | 201  | 188  | 228  | 243  |
| ci    | 485   | 454   | 400  | 403  | 382  | 376  | 431   | 362  | 299  | 312  | 272  | 257  |
| to    | 0     | 0     | 0    | 0    | 0    | 0    | 0     | 0    | 0    | 0    | 0    | 0    |
| m = 16 |       |       |
| n     |       |       |      |      |      |      |       |      |      |      |      |      |
| cp    | 67    | 222   | 317  | 322  | 316  | 295  | 12    | 63   | 147  | 172  | 161  | 140  |
| ci    | 433   | 0     | 0    | 0    | 0    | 0    | 0     | 0    | 0    | 0    | 0    | 0    |
| to    | 0     | 278   | 183  | 178  | 184  | 205  | 488   | 437  | 353  | 328  | 339  | 360  |
| m = 32 |       |       |
| n     |       |       |      |      |      |      |       |      |      |      |      |      |
| cp    | 4     | 16    | 41   | 55   | 48   | 52   | 0     | 0    | 13   | 13   | 13   | 24   |
| ci    | 0     | 0     | 0    | 0    | 0    | 0    | 0     | 0    | 0    | 0    | 0    | 0    |
| to    | 498   | 484   | 459  | 445  | 452  | 448  | 500   | 500  | 487  | 487  | 487  | 476  |
Figure 466: Results for plurality voting in the IC model for constructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

| $n$ | $m$ | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|-----|----|----|----|----|----|-----|----|----|----|----|----|----|
|     | $\# cp$ | 15 | 69 | 67 | 12 | 2  | 0   | 46 | 138 | 222 | 63 | 16 | 0   |
|     | $\# ci$ | 485| 431| 433| 0  | 0  | 0   | 454| 362 | 0   | 0  | 0  | 0   |
|     | $\# to$ | 0  | 0  | 0  | 488| 498| 500 | 0  | 0   | 278 | 437| 484| 500 |

| $n$ | $m$ | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|-----|----|----|----|----|----|-----|----|----|----|----|----|----|
|     | $\# cp$ | 100| 201| 317| 147| 41 | 13  | 97 | 188 | 322 | 172| 55 | 13  |
|     | $\# ci$ | 400| 299| 0  | 0  | 0  | 0   | 403| 312 | 0   | 0  | 0  | 0   |
|     | $\# to$ | 0  | 0  | 0  | 183| 353| 459 | 487| 0   | 178 | 328| 445| 487 |

| $n$ | $m$ | 4  | 8  | 16 | 32 | 64 | 128 | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|-----|----|----|----|----|----|-----|----|----|----|----|----|----|
|     | $\# cp$ | 118| 228| 316| 161| 48 | 13  | 124| 243 | 295 | 140| 52 | 24  |
|     | $\# ci$ | 382| 272| 0  | 0  | 0  | 0   | 376| 257 | 0   | 0  | 0  | 0   |
|     | $\# to$ | 0  | 0  | 0  | 184| 339| 452 | 487| 0   | 205 | 360| 448| 476 |

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Figure 467: Results for plurality voting in the TM model for constructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

|     | m = 4 |     | m = 8 |     | m = 16 |     | m = 32 |     | m = 64 |     | m = 128 |
|-----|-------|-----|-------|-----|--------|-----|--------|-----|--------|-----|---------|
| n   | 4     | 8   | 16    | 32  | 64     | 128 | 4      | 8   | 16     | 32  | 64      | 128 |
| # cp| 5     | 22  | 33    | 52  | 65     | 12  | 35     | 35  | 75     | 76  | 103     |     |
| # ci| 495   | 478 | 467   | 457 | 448    | 435 | 488    | 465 | 465    | 425 | 424     | 397 |
| # to| 0     | 0   | 0     | 0   | 0      | 0   | 0      | 0   | 0      | 0   | 0       |     |

|     | m = 16 |     | m = 32 |     | m = 64 |     | m = 128 |
|-----|--------|-----|--------|-----|--------|-----|---------|
| n   | 4      | 8   | 16     | 32  | 64     | 128 | 4      | 8   | 16     | 32  | 64      | 128 |
| # cp| 18     | 61  | 65     | 78  | 80     | 69  | 0      | 23  | 15     | 29  | 20      | 23  |
| # ci| 482    | 0   | 0      | 0   | 0      | 0   | 0      | 0   | 0      | 0   | 0       |     |
| # to| 0      | 439 | 435    | 422 | 420    | 431 | 500    | 477 | 485    | 471 | 480     | 477 |

|     | m = 64 |     | m = 128 |
|-----|--------|-----|---------|
| n   | 4      | 8   | 16     | 32  | 64     | 128 | 4      | 8   | 16     | 32  | 64      | 128 |
| # cp| 1      | 4   | 1      | 8   | 9      | 8   | 0      | 4   | 0      | 2   | 5       | 7   |
| # ci| 0      | 0   | 0      | 0   | 0      | 0   | 0      | 0   | 0      | 0   | 0       |     |
| # to| 499    | 496 | 499    | 492 | 491    | 492 | 500    | 496 | 500    | 498 | 495     | 493 |
Figure 468: Results for plurality voting in the TM model for constructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

| n | m | cp | ci | to | m | cp | ci | to |
|---|---|----|----|----|---|----|----|----|
| n=4 | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| 4 | 8 | 16 | 32 | 64 | 128 | 5 | 12 | 0 | 2 | 5 | 7 | 22 | 35 | 61 | 23 | 4 | 4 |
| n=8 | 0 | 0 | 0 | 0 | 0 | 0 | 478 | 465 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| n=16 | 0 | 496 | 500 | 498 | 495 | 493 | 0 | 0 | 439 | 477 | 496 | 496 | n=32 | n=64 | n=128 |
| m | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| 5 | 12 | 0 | 2 | 5 | 7 | 22 | 35 | 61 | 23 | 4 | 4 | 43 | 56 | 78 | 29 | 8 | 2 |
| n=32 | n=64 | n=128 |
| m | 0 | 0 | 435 | 485 | 499 | 500 | 0 | 0 | 422 | 471 | 492 | 498 | 0 | 0 | 431 | 477 | 492 | 493 |
C.11.1. Computational Costs

**Figure 469:** Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TP in plurality elections in the IC model. The maximum is 130,91 seconds.

**Figure 470:** Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TP in plurality elections in the IC model. The maximum is 518,67 seconds.
Figure 471: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TP in plurality elections in the IC model. The maximum is 130.91 seconds.

Figure 472: Average time the algorithm needs to find a successful control action for constructive control by runoff-partition of candidates in model TP in plurality elections in the TM model. The maximum is 114.84 seconds.
Figure 473: Average time the algorithm needs to determine no-instance of constructive control by runoff-partition of candidates in model TP in plurality elections in the TM model. The maximum is 514.44 seconds.

Figure 474: Average time the algorithm needs to give a definite output for constructive control by runoff-partition of candidates in model TP in plurality elections in the TM model. The maximum is 497.07 seconds.
C.12. Destructive Control by Runoff Partition of Candidates in Model TP

Figure 475: Results for plurality voting in the IC model for destructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

|      | m = 4 |              |      | m = 8 |              |
|------|-------|--------------|------|-------|--------------|
|      |       |              |      |       |              |
| n    | 4     | 8            | 16   | 32    | 64           | 128 |
|      |       |              |      |       |              |
| # cp | 288   | 276          | 269  | 275   | 284          | 274 |
|      |       |              |      |       |              |
| # ci | 212   | 224          | 231  | 225   | 216          | 226 |
|      |       |              |      |       |              |
| # to | 0     | 0            | 0    | 0     | 0            | 0   |
|      |       |              |      |       |              |
|      |       |              |      |       |              |
|      |       |              |      |       |              |
| n    | 4     | 8            | 16   | 32    | 64           | 128 |
|      |       |              |      |       |              |
| # cp | 414   | 491          | 490  | 499   | 499          | 500 |
|      |       |              |      |       |              |
| # ci | 86    | 2            | 0    | 0     | 0            | 0   |
|      |       |              |      |       |              |
| # to | 0     | 7            | 10   | 1     | 1            | 0   |
|      |       |              |      |       |              |
|      |       |              |      |       |              |
|      |       |              |      |       |              |
| n    | 4     | 8            | 16   | 32    | 64           | 128 |
|      |       |              |      |       |              |
| # cp | 154   | 316          | 409  | 431   | 445          | 450 |
|      |       |              |      |       |              |
| # ci | 5     | 0            | 0    | 0     | 0            | 0   |
|      |       |              |      |       |              |
| # to | 341   | 184          | 91   | 69    | 55           | 50  |
Figure 476: Results for plurality voting in the IC model for destructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

| m  | n = 4 | n = 8 | n = 16 | n = 32 | n = 64 | n = 128 |
|----|-------|-------|--------|--------|--------|---------|
| # cp | 288 | 379 | 414 | 177 | 154 | 116 |
| # ci | 212 | 121 | 86 | 11 | 5 | 3 |
| # to | 0 | 0 | 0 | 312 | 341 | 381 |
| m  | n = 16 | n = 32 | n = 64 | n = 128 |
| # cp | 269 | 424 | 490 | 441 | 409 | 389 |
| # ci | 231 | 76 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 0 | 35 | 14 | 28 |
| m  | n = 64 | n = 128 |
| # cp | 284 | 466 | 499 | 486 | 445 | 419 |
| # ci | 216 | 34 | 0 | 0 | 0 | 0 |
| # to | 0 | 0 | 0 | 14 | 55 | 81 |

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Figure 477: Results for plurality voting in the TM model for destructive control by runoff-partition of candidates in model TP. Number of candidates is fixed.

|       | $m=4$ |       | $m=8$ |       | $m=16$ |       | $m=32$ |       | $m=64$ |       | $m=128$ |
|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|
| $n$   | 4      | 8     | 16     | 32    | 64     | 128   | 4      | 8     | 16     | 32    | 64     | 128   |
| # cp  | 188    | 143   | 124    | 129   | 106    | 115   | 234    | 194   | 210    | 214   | 236    | 216   |
| # ci  | 312    | 357   | 376    | 371   | 394    | 385   | 266    | 306   | 290    | 286   | 264    | 284   |
| # to  | 0      | 0     | 0      | 0     | 0      | 0     | 0      | 0     | 0      | 0     | 0      | 0     |
|       | $m=16$ |       | $m=32$ |       | $m=64$ |       | $m=128$ |       |         |       |         |       |
| $n$   | 4      | 8     | 16     | 32    | 64     | 128   | 4      | 8     | 16     | 32    | 64     | 128   |
| # cp  | 305    | 277   | 271    | 256   | 257    | 265   | 130    | 133   | 136    | 115   | 117    | 126   |
| # ci  | 195    | 154   | 105    | 70    | 39     | 36    | 192    | 141   | 66     | 41    | 19     | 17    |
| # to  | 0      | 69    | 124    | 174   | 204    | 199   | 178    | 226   | 298    | 344   | 364    | 357   |
|       | $m=64$ |       | $m=128$ |       |         |       |         |       |         |       |         |       |
| $n$   | 4      | 8     | 16     | 32    | 64     | 128   | 4      | 8     | 16     | 32    | 64     | 128   |
| # cp  | 93     | 94    | 82     | 85    | 95     | 84    | 88     | 106   | 70     | 86    | 75     | 67    |
| # ci  | 169    | 122   | 77     | 28    | 10     | 4     | 177    | 106   | 43     | 25    | 5      | 3     |
| # to  | 238    | 284   | 341    | 387   | 395    | 412   | 235    | 288   | 387    | 389   | 420    | 430   |

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Figure 478: Results for plurality voting in the TM model for destructive control by runoff-partition of candidates in model TP. Number of voters is fixed.

|     | n = 4          |     | n = 8          |
|-----|----------------|-----|----------------|
| n   | 4 8 16 32 64 128 | n   | 4 8 16 32 64 128 |
| # cp| 188 234 305 130 93 88 | # cp| 143 194 277 133 94 106 |
| # ci| 312 266 195 192 169 177 | # ci| 357 306 154 141 122 106 |
| # to| 0 0 0 178 238 235 | # to| 0 0 69 226 284 288 |
| m   |              |     |              |
| m   |              |     |              |
| m   |              |     |              |
| m   |              |     |              |
| m   |              |     |              |

|     | n = 16        |     | n = 32        |
|-----|---------------|-----|---------------|
| m   | 4 8 16 32 64 128 | m   | 4 8 16 32 64 128 |
| # cp| 124 210 271 136 82 70 | # cp| 129 214 256 115 85 86 |
| # ci| 376 290 105 66 77 43 | # ci| 371 286 7 41 28 25 |
| # to| 0 0 124 298 341 387 | # to| 0 0 174 344 387 389 |
| n   |              |     |              |
| n   |              |     |              |
| n   |              |     |              |
| n   |              |     |              |
| n   |              |     |              |

|     | n = 64        |     | n = 128       |
|-----|---------------|-----|---------------|
| m   | 4 8 16 32 64 128 | m   | 4 8 16 32 64 128 |
| # cp| 106 236 257 117 95 75 | # cp| 115 216 265 126 84 67 |
| # ci| 394 264 39 19 10 5 | # ci| 385 284 36 17 4 3 |
| # to| 0 0 204 364 395 420 | # to| 0 0 199 357 412 430 |
C.12.1. Computational Costs

Figure 479: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TP in plurality elections in the IC model. The maximum is 42.85 seconds.

Figure 480: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TP in plurality elections in the IC model. The maximum is 348.43 seconds.
Figure 481: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TP in plurality elections in the IC model. The maximum is 78.7 seconds.

Figure 482: Average time the algorithm needs to find a successful control action for destructive control by runoff-partition of candidates in model TP in plurality elections in the TM model. The maximum is 78.26 seconds.
Figure 483: Average time the algorithm needs to determine no-instance of destructive control by runoff-partition of candidates in model TP in plurality elections in the TM model. The maximum is 89.53 seconds.

Figure 484: Average time the algorithm needs to give a definite output for destructive control by runoff-partition of candidates in model TP in plurality elections in the TM model. The maximum is 67.95 seconds.
C.13. Constructive Control by Partition of Voters in Model TP

Figure 485: Results for plurality voting in the IC model for constructive control by partition of voters in model TP. Number of candidates is fixed.

|        | $m=4$ |        | $m=8$ |        | $m=16$ |        | $m=32$ |        | $m=64$ |        | $m=128$ |
|--------|-------|--------|-------|--------|--------|--------|--------|--------|--------|--------|--------|
|        | $n=4$ | 8      | 16    | 32     | 64     | 128    | 4      | 8      | 16     | 32     | 64     | 128    |
| # cp   | 0     | 68     | 170   | 206    | 188    | 193    | 0      | 94     | 169    | 229    | 256    | 248    |
| # ci   | 500   | 432    | 330   | 0      | 0      | 0      | 500    | 406    | 331    | 5      | 0      | 0      |
| # to   | 0     | 0      | 294   | 312    | 307    | 0      | 0      | 0      | 266    | 244    | 252    |        |
|        | $n=16$ | 4      | 8     | 16     | 32     | 64     | 128    | 4      | 8      | 16     | 32     | 64     | 128    |
| # cp   | 0     | 63     | 173   | 150    | 237    | 271    | 0      | 29     | 126    | 106    | 156    | 225    |
| # ci   | 500   | 437    | 327   | 73     | 8      | 0      | 500    | 471    | 302    | 195    | 64     | 9      |
| # to   | 0     | 0      | 277   | 277    | 229    | 0      | 0      | 72     | 199    | 280    | 266    |        |
|        | $n=32$ | 4      | 8     | 16     | 32     | 64     | 128    | 4      | 8      | 16     | 32     | 64     | 128    |
| # cp   | 0     | 6     | 64    | 58     | 103    | 151    | 0      | 4      | 22     | 29     | 62     | 109    |
| # ci   | 500   | 494    | 401   | 323    | 179    | 77     | 500    | 496    | 429    | 387    | 306    | 188    |
| # to   | 0     | 0      | 35    | 119    | 218    | 272    | 0      | 0      | 44     | 84     | 133    | 204    |
Figure 486: Results for plurality voting in the IC model for constructive control by partition of voters in model TP. Number of voters is fixed.

| m    | 4   | 8   | 16  | 32  | 64  | 128 | n = 4 |  | m    | 4   | 8   | 16  | 32  | 64  | 128 | n = 8 |
|------|-----|-----|-----|-----|-----|-----|-------| |      |-----|-----|-----|-----|-----|-----|-------|
| # cp | 0   | 0   | 0   | 0   | 0   | 0   | 68    | | # cp | 0   | 0   | 0   | 0   | 0   | 0   | 6     |
| # ci | 500 | 500 | 500 | 500 | 500 | 500 | 432   | | # ci | 432 | 406 | 437 | 471 | 494 | 496 | 500   |
| # to | 0   | 0   | 0   | 0   | 0   | 0   | 0     | | # to | 0   | 0   | 0   | 0   | 0   | 0   | 0     |

| m    | 4   | 8   | 16  | 32  | 64  | 128 | n = 16 |  | m    | 4   | 8   | 16  | 32  | 64  | 128 | n = 32 |
|------|-----|-----|-----|-----|-----|-----|--------| |------|-----|-----|-----|-----|-----|-----|--------|
| # cp | 170 | 169 | 173 | 126 | 64  | 22  | 206    | | # cp | 150 | 106 | 58  | 29  | 0   | 0   | 72     |
| # ci | 330 | 331 | 327 | 302 | 401 | 429 | 302    | | # ci | 429 | 0   | 5   | 73  | 195 | 323 | 302   |
| # to | 0   | 0   | 72  | 35  | 44  | 294 | 0      | | # to | 294 | 266 | 277 | 199 | 119 | 84  | 0     |

| m    | 4   | 8   | 16  | 32  | 64  | 128 | n = 64 |  | m    | 4   | 8   | 16  | 32  | 64  | 128 | n = 128 |
|------|-----|-----|-----|-----|-----|-----|--------| |------|-----|-----|-----|-----|-----|-----|--------|
| # cp | 188 | 256 | 237 | 156 | 103 | 62  | 193    | | # cp | 271 | 225 | 151 | 109 | 0   | 0   | 0     |
| # ci | 0   | 0   | 8   | 64  | 179 | 306 | 0      | | # ci | 77  | 188 | 0   | 0   | 77  | 188 | 0     |
| # to | 312 | 244 | 255 | 280 | 218 | 133 | 307    | | # to | 252 | 229 | 266 | 272 | 204 | 0   | 0     |
Figure 487: Results for plurality voting in the TM model for constructive control by partition of voters in model TP. Number of candidates is fixed.

|   | \( m = 4 \) | \( m = 8 \) |
|---|---|---|
| \( n \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 0 | 29 | 49 | 41 | 52 | 65 | 0 | 20 | 32 | 38 | 50 | 51 |
| # ci | 500 | 471 | 451 | 198 | 158 | 131 | 500 | 480 | 468 | 191 | 89 | 68 |
| # to | 0 | 0 | 0 | 261 | 290 | 304 | 0 | 0 | 0 | 271 | 361 | 381 |
|   | \( m = 16 \) | \( m = 32 \) |
| \( n \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 0 | 18 | 32 | 26 | 34 | 48 | 0 | 9 | 23 | 18 | 28 | 33 |
| # ci | 500 | 482 | 468 | 266 | 144 | 59 | 500 | 491 | 416 | 335 | 254 | 140 |
| # to | 0 | 0 | 0 | 208 | 322 | 393 | 0 | 0 | 0 | 61 | 147 | 218 |
|   | \( m = 64 \) | \( m = 128 \) |
| \( n \) | 4 | 8 | 16 | 32 | 64 | 128 | 4 | 8 | 16 | 32 | 64 | 128 |
| # cp | 0 | 1 | 13 | 7 | 15 | 22 | 0 | 2 | 2 | 3 | 9 | 19 |
| # ci | 500 | 499 | 457 | 423 | 344 | 250 | 500 | 498 | 479 | 455 | 433 | 350 |
| # to | 0 | 0 | 3 | 7 | 141 | 280 | 0 | 0 | 19 | 42 | 58 | 131 |
Figure 488: Results for plurality voting in the TM model for constructive control by partition of voters in model TP. Number of voters is fixed.

| $n$ | $m$ | 4  | 8  | 16 | 32 | 64 | 128 |
|-----|-----|----|----|----|----|----|-----|
| $n=4$ | 4 | 0  | 0  | 0  | 0  | 0  | 0   |
|     | 8 | 29 | 20 | 18 | 9  | 1  | 2   |
|     | 16| 500| 500| 500| 500| 500| 500 |
|     | 32| 0  | 0  | 0  | 0  | 0  | 0   |
|     | 64| 0  | 0  | 0  | 0  | 0  | 0   |
|     | 128| 0 | 0  | 0  | 0  | 0  | 0   |
| $n=8$ | 4 | 49 | 32 | 32 | 23 | 13 | 2   |
|     | 8 | 41 | 38 | 26 | 18 | 7  | 3   |
|     | 16| 451| 468| 468| 416| 457| 479 |
|     | 32| 198| 191| 266| 335| 423| 455 |
|     | 64| 0  | 0  | 61 | 3  | 19 |     |
|     | 128| 261| 271| 208| 147| 7  | 42  |
| $n=16$ | 4 | 49 | 32 | 32 | 23 | 13 | 2   |
|     | 8 | 41 | 38 | 26 | 18 | 7  | 3   |
|     | 16| 451| 468| 468| 416| 457| 479 |
|     | 32| 198| 191| 266| 335| 423| 455 |
|     | 64| 0  | 0  | 61 | 3  | 19 |     |
|     | 128| 261| 271| 208| 147| 7  | 42  |
| $n=32$ | 4 | 52 | 50 | 34 | 28 | 15 | 9   |
|     | 8 | 65 | 51 | 48 | 33 | 22 | 19  |
|     | 16| 158| 144| 254| 344| 433| 131 |
|     | 32| 131| 68 | 59 | 140| 250| 350 |
|     | 64| 304| 381| 393| 327| 228| 131 |
| $n=64$ | 4 | 52 | 50 | 34 | 28 | 15 | 9   |
|     | 8 | 65 | 51 | 48 | 33 | 22 | 19  |
|     | 16| 158| 144| 254| 344| 433| 131 |
|     | 32| 131| 68 | 59 | 140| 250| 350 |
|     | 64| 304| 381| 393| 327| 228| 131 |
| $n=128$ | 4 | 52 | 50 | 34 | 28 | 15 | 9   |
|     | 8 | 65 | 51 | 48 | 33 | 22 | 19  |
|     | 16| 158| 144| 254| 344| 433| 131 |
|     | 32| 131| 68 | 59 | 140| 250| 350 |
|     | 64| 304| 381| 393| 327| 228| 131 |
C.13.1. Computational Costs

Figure 489: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TP in plurality elections in the IC model. The maximum is 126.26 seconds.

Figure 490: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TP in plurality elections in the IC model. The maximum is 329.07 seconds.
Figure 491: Average time the algorithm needs to give a definite output for constructive control by partition of voters in model TP in plurality elections in the IC model. The maximum is 226.68 seconds.

Figure 492: Average time the algorithm needs to find a successful control action for constructive control by partition of voters in model TP in plurality elections in the TM model. The maximum is 246.85 seconds.
Figure 493: Average time the algorithm needs to determine no-instance of constructive control by partition of voters in model TP in plurality elections in the TM model. The maximum is 156.07 seconds.

Figure 494: Average time the algorithm needs to give a definite output for constructive control by partition of voters in model TP in plurality elections in the TM model. The maximum is 149.9 seconds.
C.14. Destructive Control by Partition of Voters in Model TP

Figure 495: Results for plurality voting in the IC model for destructive control by partition of voters in model TP. Number of candidates is fixed.

| n   | m = 4 | m = 8 |
|-----|-------|-------|
| 4   | 276   | 352   |
| 8   | 273   | 410   |
| 16  | 334   | 455   |
| 32  | 315   | 464   |
| 64  | 308   | 451   |
| 128 | 352   | 442   |

| n   | m = 4 | m = 8 |
|-----|-------|-------|
| 4   | 224   | 148   |
| 8   | 227   | 90    |
| 16  | 147   | 45    |
| 32  | 0     | 0     |
| 64  | 0     | 0     |
| 128 | 166   | 36    |
| 256 | 185   | 49    |
| 512 | 192   | 58    |

| n   | m = 4 | m = 8 |
|-----|-------|-------|
| 4   | 136   | 131   |
| 8   | 0     | 8     |
| 16  | 0     | 0     |
| 32  | 12    | 16    |
| 64  | 16    | 27    |
| 128 | 0     | 0     |
| 256 | 0     | 7     |
| 512 | 0     | 6     |
| 1024| 0     | 10    |

| n   | m = 4 | m = 8 |
|-----|-------|-------|
| 4   | 365   | 372   |
| 8   | 500   | 499   |
| 16  | 500   | 498   |
| 32  | 497   | 497   |
| 64  | 496   | 497   |
| 128 | 497   | 498   |

| n   | m = 4 | m = 8 |
|-----|-------|-------|
| 4   | 135   | 128   |
| 8   | 0     | 1     |
| 16  | 0     | 0     |
| 32  | 0     | 0     |
| 64  | 0     | 0     |
| 128 | 3     | 2     |
| 256 | 4     | 3     |
| 512 | 3     | 2     |

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Figure 496: Results for plurality voting in the IC model for destructive control by partition of voters in model TP. Number of voters is fixed.

| m  | n = 4 | n = 8 |
|----|------|------|
|    | 4    | 8    | 16   | 32   | 64   | 128  | 4    | 8    | 16   | 32   | 64   | 128  |
| # cp | 276  | 352  | 364  | 365  | 372  | 273  | 410  | 482  | 492  | 500  | 499  |
| # ci | 224  | 148  | 136  | 131  | 135  | 127  | 90   | 18   | 8    | 0    | 1    |
| # to | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |

| m  | n = 16 | n = 32 |
|----|--------|--------|
|    | 4      | 8      | 16    | 32    | 64    | 128   | 4    | 8    | 16   | 32   | 64   | 128   |
| # cp | 353    | 455    | 498   | 500   | 500   | 334   | 464  | 488  | 493  | 497  | 498   |
| # ci | 147    | 45     | 2     | 0     | 0     | 0     | 0    | 0    | 0    | 0    | 0     |
| # to | 0      | 0      | 0     | 0     | 0     | 0     | 0    | 0    | 0    | 0    | 0     |

| m  | n = 64  | n = 128 |
|----|---------|---------|
|    | 4       | 8       | 16    | 32    | 64    | 128   | 4    | 8    | 16   | 32   | 64   | 128   |
| # cp | 315    | 451    | 484   | 494   | 496   | 497   | 308  | 442  | 473  | 490  | 497  | 498   |
| # ci | 0      | 0      | 0     | 0     | 0     | 0     | 0    | 0    | 0    | 0    | 0    | 0     |
| # to | 185    | 49     | 16    | 6     | 4     | 3     | 192  | 58   | 27   | 10   | 3    | 2     |
Figure 497: Results for plurality voting in the TM model for destructive control by partition of voters in model TP. Number of candidates is fixed.

| n  | m = 4 | m = 8 |
|----|-------|-------|
|    | 4     | 8     | 16    | 32    | 64    | 128   | 4     | 8     | 16    | 32    | 64    | 128   |
| # cp | 174   | 151   | 160   | 100   | 73    | 80    | 190   | 209   | 219   | 116   | 121   | 97    |
| # ci | 326   | 349   | 340   | 208   | 168   | 118   | 310   | 291   | 281   | 119   | 81    | 60    |
| # to | 0     | 0     | 0     | 192   | 259   | 302   | 0     | 0     | 0     | 265   | 298   | 343   |

| n  | m = 16 | m = 32 |
|----|--------|--------|
|    | 4      | 8      | 16     | 32    | 64    | 128   | 4     | 8     | 16     | 32     | 64     | 128    |
| # cp | 222   | 244   | 273   | 134   | 127   | 138   | 241   | 278   | 273   | 150   | 125   | 135   |
| # ci | 278   | 256   | 227   | 64    | 41    | 26    | 259   | 222   | 92    | 46    | 20    | 14    |
| # to | 0     | 0     | 0     | 302   | 332   | 336   | 0     | 0     | 135   | 304   | 355   | 351   |

| n  | m = 64 | m = 128 |
|----|--------|---------|
|    | 4      | 8       | 16     | 32     | 64     | 128   | 4     | 8     | 16     | 32     | 64     | 128    |
| # cp | 237   | 283   | 292   | 137    | 142    | 121   | 266   | 295   | 232    | 146    | 128    | 133    |
| # ci | 263   | 217   | 71    | 28     | 7      | 9     | 234   | 205   | 66     | 22     | 10     | 4      |
| # to | 0     | 0     | 137   | 335   | 351   | 370   | 0     | 0     | 202   | 332   | 362   | 363    |
Figure 498: Results for plurality voting in the TM model for destructive control by partition of voters in model TP. Number of voters is fixed.

| m  | 4  | 8  | 16 | 32 | 64 | 128 |
|----|----|----|----|----|----|-----|
| # cp | 174 | 190 | 222 | 241 | 237 | 266 |
| # ci | 326 | 310 | 278 | 259 | 263 | 234 |
| # to | 0   | 0   | 0   | 0   | 0   | 0   |

| m  | 4  | 8  | 16 | 32 | 64 | 128 |
|----|----|----|----|----|----|-----|
| # cp | 160 | 219 | 273 | 273 | 292 | 232 |
| # ci | 340 | 281 | 227 | 92  | 71  | 66  |
| # to | 0   | 0   | 0   | 135 | 137 | 202 |

| m  | 4  | 8  | 16 | 32 | 64 | 128 |
|----|----|----|----|----|----|-----|
| # cp | 73  | 121 | 127 | 125 | 142 | 128 |
| # ci | 168 | 81  | 41  | 20  | 7   | 0   |
| # to | 259 | 298 | 332 | 355 | 351 | 362 |

| m  | 4  | 8  | 16 | 32 | 64 | 128 |
|----|----|----|----|----|----|-----|
| # cp | 118 | 60  | 26  | 14  | 9   | 0   |
| # ci | 302 | 343 | 336 | 351 | 370 | 363 |

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Figure 499: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TP in plurality elections in the IC model. The maximum is 29,37 seconds.

Figure 500: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TP in plurality elections in the IC model. The maximum is 550,39 seconds.
Figure 501: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TP in plurality elections in the IC model. The maximum is 91.41 seconds.

Figure 502: Average time the algorithm needs to find a successful control action for destructive control by partition of voters in model TP in plurality elections in the TM model. The maximum is 114 seconds.
Figure 503: Average time the algorithm needs to determine no-instance of destructive control by partition of voters in model TP in plurality elections in the TM model. The maximum is 289.7 seconds.

Figure 504: Average time the algorithm needs to give a definite output for destructive control by partition of voters in model TP in plurality elections in the TM model. The maximum is 168.63 seconds.