Manipulability of the Kondo effect in a T-shaped triple-quantum-dot structure

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(Dated: November 11, 2019)

We theoretically investigate the Kondo effect of a T-shaped triple-quantum-dot structure, by means of the numerical renormalization group method. It is found that at the point of electron-hole symmetry, the system’s entropy has opportunities to exhibit three kinds of transition processes for different interdot couplings, with the decrease of temperature. This leads to the different pictures of the Kondo physics, including the three-stage Kondo effect. Next when the electron-hole symmetry is broken or the structural parameters are changed, the Kondo resonance can also be observed in the conductance spectrum. However, it shows alternative dependence on the relevant quantities, i.e., the Coulomb interaction and interdot couplings. All these phenomena exhibit the abundant and interesting Kondo physics in this system. We believe that this work can be helpful for further understanding the Kondo effect in the triple-quantum-dot structures.

PACS numbers: 74.81.Fa, 74.25.F-, 74.45.+c, 74.50.+r
Keywords: Kondo effect; Triple quantum dots; Coulomb interaction; Interdot coupling

I. INTRODUCTION

The Kondo effect, which first originates from the correlation between the impurity spin and conduction spin of the host metal, results in the apparent increase of the resistance at sufficiently low temperature [1-3]. The underlying reason for this effect is explained as the spin-flip scattering processes between the impurity spin and conduction electrons [4-6]. The successful fabrication of the quantum dot (QD) introduce new physics to the Kondo effect. When the Kondo QD is coupled to two leads, the conductance plateau appears in electron transport through the system, instead of resistance enhancement [7]. Compared with the conventional Kondo effect, multiple spin-flip processes take place, which give rise to the additional tunneling phenomenon. If the system is below the Kondo temperature, the resonant tunneling will be achieved in the case of the incident electron energy consistent with the Fermi energy of the whole system. Therefore, the conductance plateau arises in the transport spectrum with \( G = \frac{2e^2}{h} \). The Kondo effect in QD systems has already been one subject of extensive studies for more than two decades [8-17].

QD has its advantages that multiple “atoms” can couple to one “molecule” in different manners, and also, the metallic leads are allowed to connect with different QDs. This makes the Kondo effect more complex and exotic in QD-molecule systems [18-35]. During the past years, the Kondo effects in these systems have attracted extensive attentions. And lots of interesting phenomena have been observed. Take the double QD systems as an example, the Kondo effect can drive various results, including the transformation of the Kondo resonance [21-24], the SU(4) Fermi liquid behaviors [25-26], the Kondo-assisted interference effects [27-30], the interplay between the spin and orbital Kondo effects [31-34], the Kondo Effect influenced by the presence of ferromagnetic leads [26-35]. Moreover, when the QD number in one molecule increases to three, the Kondo effect exhibit some special characteristics, such as the triple-anisotropic charge Kondo effect [36], the ferromagnetic Kondo Effect [37], and the symmetry-related Kondo effects [38]. In view of the QD molecules, the T-shaped QDs are important geometries for the Kondo effect and its-related quantum transport behaviors. One typical phenomenon is the Fano-Kondo effect influenced by the presence of ferromagnetic leads [36-39]. On the other hand, they are able to exhibit the two-stage Kondo effect, when the antiferromagnetic coupling between the QDs’ spins competes with the Kondo effect [30-40]. The two-stage nature of screening has been revealed especially in the temperature dependence of the conductance. With the decrease of temperature, for \( T \approx T_K \), the conductance increases due to the occurrence of the first-stage Kondo effect, but when temperature further decreases to lower than the other characteristic temperature \( T^* \), the conductance drops to zero, and the second-stage screening takes place.

In the present work, we would like to investigate the Kondo effect of a T-shaped triple-QD (TTQD) structure. According to the previous works, the TTQDs exhibit some important geometry-related transport properties, especially when the strong correlation effects exist [41]. We then present the pictures of the Kondo physics in such a system from multiple aspects, by means of the numerical renormalization group (NRG) method. It is found that in this system, the Kondo resonance is allowed to

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electron with energy $\varepsilon$ interacts. It reads one level exists in each QD with finite Coulomb interaction. The Hamiltonian for the TTQDs is

$$H = H_C + H_D + H_T.$$

The first term is the Hamiltonian of the metallic leads, and it is written as

$$H_C = \sum_{\alpha k \sigma} \varepsilon_{\alpha k} a_{\alpha k \sigma}^\dagger a_{\alpha k \sigma}. \quad (1)$$

$a_{\alpha k \sigma}^\dagger (a_{\alpha k \sigma})$ is the operator to create (annihilate) an electron with energy $\varepsilon_{\alpha k}$ for lead-$\alpha$ ($\alpha = L, R$), where $k$ is the momentum quantum number of the free conduction electron. Next, $H_{QD}$ models the Hamiltonian for the TTQDs. We suppose that only one level exists in each QD with finite Coulomb interaction. It reads

$$H_{QD} = \sum_{j \sigma} \varepsilon_j d_{j \sigma}^\dagger d_{j \sigma} + \sum_{\sigma, j=1}^{2} t_j d_{j+1 \sigma}^\dagger d_{j \sigma} + h.c. + \sum_j U_j n_{j} n_{j}. \quad (2)$$

$d_{j \sigma}^\dagger (d_{j \sigma})$ is the operator to create (annihilate) an electron with energy $\varepsilon_j$ and spin $\sigma$ in QD-$j$ ($j = 1, 2$). $t_j$ is the interdot coupling coefficient, and $U_j$ indicates the strength of intradot Coulomb repulsion in the corresponding QD. The last term of $H$ denotes the coupling between QD-1 and the leads. For our considered system, it is directly written as

$$H_T = \sum_{\alpha k \sigma} (V_{\alpha k} a_{\alpha k \sigma}^\dagger d_{1 \sigma} + h.c.). \quad (3)$$

$V_{\alpha k}$ describes the QD-lead coupling coefficient. In the wide-band limit with bandwidth being $W_D = 2D$, the density of states can be viewed as $\rho_0 = \frac{\pi}{W_D}$. Accordingly, the coupling strength between QD-1 and leads can be defined as $\Gamma_\alpha = \pi |V_{\alpha}|^2 \rho_0$. This work only focuses on the case of symmetric QD-lead coupling with $\Gamma_\alpha = \Gamma$.

This work intends to utilize Wilson’s NRG method to study the properties of the system in the Kondo regime. In the NRG iteration process, we take the logarithmic discretization parameter of the leads to be $\Lambda = 4$, and keep 2000 states with the lowest energy in each iteration diagonalization step. The lowest temperature in the iteration process is $T_{min} = 10^{-24} D$. In addition, we use the $Z$-average method to eliminate the parity error in the iteration process.

In order to calculate the quantum transport properties governed by the electron correlation effects, we follow the theory pioneered by Meir and Wingreen. And then, the linear conductance in this system can be expressed as

$$G = \mathcal{G}_0 \pi \Gamma \rho_d(\omega)_{\omega = \mu}, \quad (4)$$

with $\mathcal{G}_0 = \frac{2e^2}{h}$, $\rho_d = \frac{1}{\pi} \text{Im} \sum_G G_{dd,\sigma}$ is the local density of states in QD-1. $\mu$ is the system’s chemical potential.

Through the calculation with the NRG method, it is known that some fixed points of the group flow are related to different electronic states. The entropy $S$ of the QDs of the whole system is determined by the microscopic states in the form of $S \propto \ln W$ (The Boltzmann constant $k_B$ has been assumed to be 1). Following the diagonalization of NRG iteration, the system temperature decreases and the corresponding degree of freedom (microscopic states) decreases. Therefore, we can study the variation of the QD states by the change of the QD’s entropy. Compared with the tunneling junction without QDs, the relationship between the contribution of QDs to the system’s entropy and temperature can be expressed as

$$S_\alpha(T) = \frac{(E - F)}{T} - \frac{(E - F)_0}{T}. \quad (5)$$

Footmark 0 denotes the case without QDs, in which the energy of the system is also denoted as $E = \langle H \rangle = \text{Tr}(He^{-H/k_B T})$ with $F = -k_B T \ln \text{Tr}(e^{-H/k_B T})$ being the system’s free energy.
III. NUMERICAL RESULTS AND DISCUSSIONS

With the theory in the above section, we next perform investigations about the Kondo effects in our system, by calculating the linear conductance, entropy, and susceptibility. In the context, the half bandwidth $D$ is taken to be the energy unit, and the QD-lead coupling strength is $\Gamma = 0.04$. Such an assumption is reasonable according to the current experiments[10].

Before calculation, we would like to take the case of electron-hole symmetry to define the characteristic temperatures in this TTQD structure as follows:

(1) The first characteristic temperature $T_U$: In the case of high temperature, the respective states appear with equal probability, i.e., the empty state $|0\rangle$, the singly-occupied states $|\uparrow\rangle$ and $|\downarrow\rangle$, as well as the doubly-occupied states $|\uparrow,\downarrow\rangle$. Thus, the QDs are located in the free-orbit regime (FOR). We know that $4^3$ states are allowed to appear with equal probability, and then the entropy approximates the initial value $6 \ln 2$. With the decrease of temperature, the system has an opportunity to enter the local-moment regime (LMR), where the zero and double occupations are both forbidden. Two singly-occupied states appear with equal probability, so the state-function entropy is reduced by half to be $3 \ln 2$. Between the temperatures with these two entropy values, the first characteristic temperature can be defined, i.e., $T_U = \frac{1}{3}(T_{S_d=6\ln 2} + T_{S_d=3\ln 2})$.

(2) The second characteristic temperature $T_K$: When temperature is further reduced, the electrons of the QDs are screened by the conduction electrons in the leads, and the entropy tends to be zero. We can then define the second characteristic transition temperature. This temperature is exactly the Kondo temperature. In our system, its definition depends on the interdot-coupling strength. At the limit of weak interdot coupling, the Kondo screening only covers the QD in the main channel, there will be $T_K^{(1)} = \frac{1}{3}(T_{S_d=3\ln 2} + T_{S_d=0})$. For the case of strong interdot coupling, the Kondo effect is allowed to take place when QD-3 is screened. In such a case, the triple QDs form an antiferromagnetic chain, and then the whole TTQDs can be regarded as a large QD, which is equivalent to the Kondo impurity with the effective spin $\tilde{S} = \frac{1}{2}$. Thus the entropy of the system is $S_d = \ln(2\tilde{S} + 1) = \ln 2$, and the Kondo temperature can be defined as $T_K^{(2)} = \frac{1}{3}(T_{S_d=6\ln 2} + T_{S_d=0})$.

In Fig.2, we plot the curves of the entropy and conductance of the TTQDs, at the position of electron-hole symmetry. The pictures of the Kondo effect can be clearly observed. Firstly, Fig.2(a)-(b) show the results of the entropy and conductance, with the increase of the uniform interdot coupling coefficients (i.e., $t_c$). We see that for a relatively small $t_c$, e.g., $t_c = 0.001$, the entropy curve experiences two plateau regions before it decays to zero, with the decrease of temperature. The two plateaus correspond to FOR and LMR regions respectively, with the corresponding entropy values being $6 \ln 2$ and $3 \ln 2$. With the further decrease of temperature,
the QDs’ local magnetic moment can be completely screened, and the entropy value is equal to zero. In such a case, the conductance magnitude is enhanced and it reaches the unit value when $T \rightarrow T_K^{(1)} \sim 10^{-5}$, as shown in Fig. 2(b). On the other hand, if $t_c = 0.3$, the value of the mediate entropy plateau changes to be $\ln 2$. And then the Kondo resonance occurs at the case of $T \rightarrow T_K^{(2)} \sim 10^{-6}$. These results exactly prove our description in the above paragraph.

Let us focus on the process of $t_c$’s increment, where the plateau with value $\ln 2$ appears and the $3\ln 2$-valued plateau disappears gradually. Take the case of $t_c = 0.01$ as an example, one new phase transition takes place prior to the occurrence of the Kondo effect. Its corresponding characteristic temperature can be defined, i.e., $T_M = \frac{1}{2}(T_{S_d=3\ln 2} + T_{S_d=\ln 2})$. Such a temperature is labelled as the slow increase of the conductance magnitude, just as shown in Fig. 2(a)-(b). We attribute this phenomenon to the occurrence of the second-stage Kondo effect. In order to understand it, we present the result of T-shaped double QDs for comparison, corresponding to the insets of Fig. 2(a) and Fig. 2(b), respectively. It can be seen that T-shaped double-QD system exhibits the clear two-stage Kondo effect with the increase of interdot coupling. For the small interdot coupling, the entropy of the system decreases from $2\ln 2$ to $\ln 2$, and the first-stage Kondo effect occurs which causes the corresponding conductance to reach the unit value. But if the system’s temperature is lower than $T^* \sim T_K \exp(-T_K/J)$, the entropy of the system will decay from $\ln 2$ to 0. In such a case, the system screens the spin magnetic moment of the side-
coupled QD as well, and the conductance becomes close to zero due to the second-stage Kondo effect. Via comparison, we consider that for TTQD case, the Kondo effect is allowed to occur stage by stage, and the second-stage Kondo effect is relatively weak due to the existence of the QD-3, which is only manifested as the slow increase of the conductance with the decrease of temperature [see Fig.2(b)]. Until temperature reaches $T_K^{(2)}$, the third-stage Kondo effect occurs, proved by the appearance of the conductance plateau.

In order to strengthen the description above, we analyze the influence of the coupling between QD-2 and QD-3 (i.e., $t_2$) on the Kondo effect of our system by taking $t_1 = 0.02$. The results are shown in Fig.2(c)-(d). We find that in the case of $t_2 = 0.006$, the effect of QD-3 is very small, but it induces the appearance of the residual entropy, i.e., $S_2 = \ln 2$, because of the isolate spin in QD-3. In such a case, the conductance is suppressed completely by the second-stage Kondo effect of the T-shaped double QDs. Next, following the increase of $t_2$, the entropy decreases from the platform of $\ln 2$ to 0 at the low-temperature limit. The reason lies in that in this situation, the whole system enables to screen the spin magnetic moment of QD-3. Resultantly, the third-stage Kondo effect takes its effect and the conductance magnitude rises gradually, until the temperature reaches $T_K^{(2)}$, the third-stage Kondo effect is allowed to occur stage by stage, with the higher Kondo temperature. Up to now, the leading physics of the Kondo effect in the TTQDs has been clarified.

The following, we performed detailed analysis about the Kondo effect by considering different cases. Fig.3 shows the results of weak interdot coupling where $t_j = 0.001$. As for the QD levels and Coulomb interaction, we take $\varepsilon_j = \varepsilon$ and $U_j = U = 0.8$. For benefiting our description, an additional quantity $\delta$ is introduced, defined as $\delta = \varepsilon + U$. In Fig.3(a), we plot the conductance spectra of our system at different temperatures. Note that in the case of $\delta = 0$, the Kondo temperature is $T_K^{(1)}$. One can find in this figure that the conductance property is very similar to the experimental result in the single-QD case [17]. When temperature is relatively high, e.g., $T \sim 10^{-4}$, the system does not enter the Kondo regime. The conductance spectrum exhibits the Coulomb-blockade result between the two conductance peaks, where the electron transport is suppressed. As temperature gradually decreases, it has an opportunity to be below the Kondo temperature ($T_K^{(1)} \sim 10^{-5}$), and then the conductance magnitude rises gradually, until the conductance plateau shows up as the unit value. It can be seen that the positions corresponding to the half-peak width of the conductance plateau is where $\delta = \pm 0.4$, and it is exactly the position of $\pm \frac{U}{T}$.

Fig.3(b) shows the curves of the ground-state energies in the subspaces of the triple QDs without coupling to the leads, marked by the good quantum numbers $(Q, S_z)$. It is found that the positions of $\delta = \pm 0.4$ are the crossing points of the ground states $(0, \pm \frac{1}{2})$ and $(\pm 3, 0)$. It is clearly known that the Kondo physics and the corresponding transport behaviors in this case are the same as those in the single-QD result. However, the thermodynamic quantities, such as the entropy and susceptibility, are different, as shown in Fig.3(c)-(d). In Fig.3(c), we give the entropy curve of the single QD with the change of temperature (see the -V- line). It shows that although the entropies of the TTQDs and the single QD are close to zero in the end, their high-temperature plateaus exhibit different values. This is exactly caused by the difference of the total spin between the TTQDs and single QD. Similarly, the susceptibility curves of the two systems in Fig.3(d) tend to zero from different values when the whole system arrives at the Kondo temperature. We then understand the difference between the weak-coupled TTQDs and the single QD cases.

We next turn to the case of strong interdot coupling by taking $t_j = 0.3$, and present the numerical results in Fig.4. Fig.4(a)-(d) are the spectra of the conductance, ground-state energies in respective subspaces, particle number, and spin correlations. Firstly, from Fig.4(a) we see that when the system is above the Kondo temperature, the Coulomb-blockade phenomenon is apparent, and three isolated Coulomb-blockade regions exist in the conductance spectrum. With the decrease of temperature, their Kondo transitions are almost the same. In the case of $T \sim 10^{-8}$, the Coulomb blockade changes to be the Kondo resonance, with the appearance of three Kondo plateaus. In the addition to the Kondo resonance around the electron-hole-symmetry point, the other Kondo resonances can be understood with the help of the results in Fig.4(b)-(c). One sees that as the temperature decreases to the Kondo temperature, the additional conductance plateaus appear in the positions where the particle number $N$ is equal to 1 and 5, respectively. The occupation of odd electron number is certainly causes the redundant spin to induce the Kondo effect. In Fig.4(d), it shows that when the system enters the Kondo regime, the TQDs construct one antiferromagnetic chain in the region of $N = 3$. This verifies our description at the beginning of this section. Namely, in the strong-coupling case, the QD chain transforms into a large QD, and the whole system is equivalent to the Kondo model with its effective spin $\frac{S}{2}$.

After the discussions above, we would like to investigate the Kondo physics of our system in the case of $t_1 \neq t_2$. Our purpose is to further clarify the role of the interdot couplings. Firstly, we take $t_1 = 0.3$ and change the value of $t_2$ to observe the transport behaviors. The numerical results are shown in Fig.5. It is found that for a relatively small
we present the local density of states of QD-1 in the case of $t_2 = 0.3$ and $\delta = 0$. The inset of Fig.5(b) describes the variation of the local density of states of QD-1 when $t_2$ increases gradually. It shows that for a small $t_2$, no Kondo peak emerge in the spectrum of the local density of states in the case of $\delta = 0$. Under the strong-coupling condition, the two conductance peaks get close gradually, and then the local density of states exhibits the Kondo peak. The entropy and susceptibility curves in Fig.5(c)-(d) show that when $t_2$ is small enough, the Kondo temperature $T_{K}^{(2)} = \frac{1}{2}(T_{S_{z}=\ln 2} + T_{S_{z}=0})$ near the position of $\delta = 0$ is very low. If it is lower than the lowest temperature $T_{\text{min}}$ in the NRG iteration process, the local magnetic moment will not be screened and the Kondo effect does not occur, thus the conductance will be very small. At the case of $t_2 = 0.3$, the Kondo temperature approaches $10^{-6}$, and the Kondo effect takes place, so the conductance plateau appears with its magnitude manifested as the unit value.

In contrast with the case in Fig.5, we in Fig.6 take $t_2 = 0.3$ and change $t_1$ to present the conductance variation behaviors governed by the Kondo effect. It is found in Fig.6(a) that in this case, the conductance plateau is robust in the center of the conductance spectrum, and it is a bit narrowed with the increase of $t_1$. This can be attributed to the decrease of the Kondo temperature in this process, as discussed in Fig.2(b). However, for the cases of $\delta = \pm 0.5$ [where $N = 1(5)$], it shows that two conductance valleys are formed in the case of small $t_1$. Only when $t_1 = 0.1$, the conductance plateaus begin to come up. Besides, from the inset of Fig.6(a), we ascertain that as $t_1$ increases, the Coulomb-blockade phenomenon disappears gradually, in the low-energy and high-energy regions. As a result, three conduc-
tance plateaus exist in the conductance spectrum, and they are separate from each other by the conductance valley. Also, with the increase of $t_1$, the whole conductance spectrum is widened accordingly. In Fig.6(b), we present the conductance spectra in the case of $t_1 = 0.1$ and $t_2 = 0.3$. It clearly shows that in the case of small $t_1$, the Kondo resonance in the vicinity of $\delta = \pm 0.6$ corresponds to a smaller $T_K$. For instance, when the system’s temperature decreases to $10^{-24}$, the conductance plateau can also be observed due to the occurrence of the Kondo effect. So far, from Fig.5 and Fig.6, we have known the roles of $t_1$ and $t_2$ in adjusting the Kondo effect and its-related conductance properties.

In view of the above results, we next present the quantitative discussion about the Kondo temperature which is determined by the interdot couplings. To begin with, we employ the Schrieffer-Wolff transformation to solve the Kondo temperature, and the detailed process can be seen in the appendix. Through the complicated deductions, the Kondo temperature is estimated by the Haldane's expression\[48–50\], i.e.,

$$T_K = 0.182U\sqrt{J_{p0}}e^{-1/(J_{p0})}$$  \hspace{1cm} (6)$$

where

$$J_{p0} = \frac{2\Gamma}{\pi} \left( \frac{(|GS|d_k^{\uparrow}|\mu|)^2}{E_{GS} - E_{\mu}} + \frac{|\nu|d_k^{\downarrow}|GS|)^2}{E_{GS} - E_{\nu}} \right).$$

With the help of Eq.(6), we consider the case of $|\delta| \leq 0.3$ where $N = 3$ and present the curves of the Kondo temperatures corresponding to the central conductance plateaus in Fig.4 and Fig.6. The numerical results are shown in Fig.7(a)-(b), respectively. It can be seen from the figure that the Kondo temperature calculated by the theoretical formula (shown by dotted lines) is very close to the characteristic transition temperature (expressed by $-\Delta-$ lines and $-\circ-$ lines respectively) obtained from the measurement of entropy. The Kondo temperature is above $10^{-6}$, greater than the minimum temperature of the iteration diagonalization. So, the Kondo plateau comes into being. However, on both sides of $|\delta| > 0.2$, the curves deviate from each other gradually. This is because the total number of particles in the system deviates from $N = 3$ gradually, and the whole system can no longer be regarded as an effective Kondo model with effective spin $\hat{S} = \frac{1}{2}$. As a consequence, the errors between them begin to take effect gradually.

Finally, we try to consider the influence of different Coulomb interactions in the QDs on the Kondo effect, as shown in Fig.8. Fig.8(a) shows the conductance spectra of $t_2 = 0.3$ with different values of $t_1$. As for the Coulomb interactions in the QDs, we set $U_1 = U_3 = 0.8$ and $U_2 = 0$. We find that no matter how $t_1$ changes, the conductance spectrum of the system shows three Kondo peaks. Compared with Fig.6(a), the difference between these two cases

![FIG. 8: (a) Conductance influenced by $t_1$ in the case of $U_2 = 0$ and $t_2 = 0.3$. (b) Result of $U_3 = 0$ and $t_1 = 0.3$ for different $t_2$. (c) Local density of states of QD-1 when $U_3 = 0$, $t_1 = 0.3$, and $\delta = 0$.](image-url)
The ground state energies in the special subspaces, corresponding to the cases of (a)-(c).

In order to discuss the central conductance peak in Fig.8(b), in Fig.9(a)-(c) we present the dependence of the Kondo temperature on $\delta$ in the region of the half width of the peak. It can be seen that the half-peak width increases with the increment of $t_2$, but the Kondo temperature $T_K$ near the peak of $\delta = 0$ decreases. This phenomenon can be understood by the results in Fig.9(d)-(f). When the number of particles is $N = 3$, the lowest energy ground state is located in the subspace $(0, \pm \frac{1}{2})$, and the Kondo temperature in Eq.(6) is only relevant to the imaginary excitation from states $(0, \pm \frac{1}{2}, 0)$ to $(\pm 1, 0, 0)$, i.e., $(\pm 1, 0, 0)$. Therefore in Fig.9(d)-(f), the ground-state energy curves of these special subspaces are shown in the vicinity of $\delta = 0$. It shows that the three curves in the figure can be enclosed in an isosceles triangle. As $t_2$ increases, the length of the bottom edge of the isosceles triangle increases gradually, and the vertexes correspond exactly to the positions shown by the two dotted arrows in Fig.9(a)-(c), which are exactly the positions of the half-peak width of the central conductance peak in Fig.8(b). After comparing the heights of the triangles formed by the ground state $(0, \pm \frac{1}{2}, 0)$ and the state $(\pm 1, 0, 0)$, one can ascertain the energy of the virtual excitation for the Kondo effect, i.e. $\Delta E$. It shows that when $t_2$ increases from 0.1 to 0.3, $\Delta E$ increases from $2.365 \times 10^{-3}$ to $8.126 \times 10^{-2}$ monotonously. This change means the reduced probability of virtual excitation related to the Kondo effect. Accordingly in Fig.9(a)-(c), the Kondo temperature $T_K$ in the vicinity of $\delta = 0$ decreases monotonously.

IV. SUMMARY

In summary, we have performed theoretical investigations about the Kondo effect of the TTQD structure, by means of the NRG method. As a result, it has been found that this system exhibits the intricate Kondo physics, because of the various structural parameters, e.g., the interdot coupling coefficients and intradot Coulomb interactions. One important result is that at the point of electron-hole symmetry, multiple-stage Kondo pictures have opportunities to come into being, which are related to the interdot coupling. Such a phenomenon is described by the different transition processes of the system’s entropy, following the decrease of temperature. In addition, when the system departs from the position of electron-hole symmetry, the Kondo resonance can also be observed, and it experiences alternative variation manners when the system’s parameters are taken in different ways. Therefore, in this TTQD structure, intricate Kondo physics can be realized. Based on these results, we think that this work can be helpful for further understanding the Kondo effect in the TTQD structures.
Acknowledgments

This work was financially supported by the Fundamental Research Funds for the Central Universities (Grants No. N180503020 and N182410008-1) and the National Natural Science Foundation of China (Grants No. 11604221 and 11905027). Our numerical results are obtained via “NRG Ljubljana”—open source numerical renormalization group code.

Appendix A: Appendix

For our system, it Hamiltonian can be rewritten as

$$H_{\text{eff}} = e^SHe^{-S} \approx (H_0 + H_T) + ([S, H_0] + [S, H_T]) + \frac{1}{2} [S, [S, H]] + \ldots \quad (A1)$$

To cut off the mixed terms, it is necessary to satisfy $H_T + [S, H_0] = 0$. With this condition, we obtain

$$H_{\text{eff}} = H_0 + \frac{1}{2} [S, H_T] + \frac{1}{2} [S, [S, H_T]] + \ldots$$

$$= H_0 + \frac{1}{2} [S, H_T] + \ldots \quad (A2)$$

For the part of $H_0$, there exists $H_0|m\rangle = E_m|m\rangle$, where $|m\rangle$ and $E_m$ are the eigenstate and eigenenergy, respectively. Thus,

$$\langle n|H_T|m\rangle = \langle n|SH_0|m\rangle - \langle n|HS_0|m\rangle$$

$$\rightarrow \langle n|H_T|m\rangle = E_m\langle n|S|m\rangle - E_n\langle n|S|m\rangle$$

$$\rightarrow \langle n|S|m\rangle = \frac{\langle n|H_T|m\rangle}{E_m - E_n}. \quad (A3)$$

In the case of $\frac{U}{\Gamma} \ll 1$ and $\Gamma \ll |\varepsilon_{\pm}| (\varepsilon_{\pm} = \varepsilon + \frac{U}{2} \pm \frac{U}{2})$,

$$\langle n|H_{\text{eff}}|m\rangle \approx E_m\delta_{mn} + \frac{1}{2} \langle n|S, H_T|m\rangle$$

$$= E_m\delta_{mn} + \frac{1}{2}\left(\langle n|SH_T|m\rangle - \langle n|H_T S|m\rangle\right)$$

$$= E_m\delta_{mn} + \frac{1}{2}\sum_{\phi} \langle n|S|\phi\rangle \langle \phi|H_T|m\rangle - \langle n|H_T|\phi\rangle \langle \phi|S|m\rangle).$$

Substituting Eq.(9) into the above formula, there will be

$$\langle n|H_{\text{eff}}|m\rangle$$

$$= E_m\delta_{mn} + \frac{1}{2} \sum_{\phi} \frac{\langle n|H_T|\phi\rangle \langle \phi|H_T|m\rangle}{E_m - E_\phi}$$

$$= E_m\delta_{mn} + \frac{1}{2} \sum_{\phi} \frac{\langle n|H_T|\phi\rangle \langle \phi|H_T|m\rangle}{E_m - E_\phi}$$

$$= \left(\frac{1}{E_m - E_\phi} + \frac{1}{E_n - E_\phi}\right). \quad (A4)$$

According to the existed research results, the charge fluctuation between QDs has no effect on the Kondo temperature. We can mainly consider the effective exchange interaction between QDs and the leads. For an isolated triple-QD system, there are 4^n eigenvectors, which can be divided into different subspaces according to the good quantum number ($Q, S_z$). Suppose we study the $S = \frac{1}{2}$ Kondo effect near the position of $\delta = 0$, which is half occupied with the ground-state spin being $\sigma = \frac{1}{2}$. The ground state can be expressed as $|0, \sigma, 0\rangle$ (i.e., the ground state of the subspace with $Q = 0$ and $\sigma = \frac{1}{2}$). We then concentrate on the virtual excitation from the global ground state to states $|\mp 1, 0, 0\rangle$ which are the ground states in the subspaces ($\mp 1, 0$). For the former case, the result in Eq.(10) is given as

$$\frac{1}{2}\langle n|H_T|\mu\rangle \langle \mu|H_T|m\rangle (\frac{1}{E_m - E_\mu} + \frac{1}{E_n - E_\mu}),$$

where $|\mu\rangle = |-1, 0, 0\rangle, |m\rangle$ and $|n\rangle$ correspond to the ground state ($GS$) = $|0, 1/2\rangle$, whose energy is $E_{GS}$. And then, Eq.(10) has its more compact form, i.e.,

$$\langle GS|H_T|\mu\rangle^2 \frac{1}{E_{GS} - E_\mu}.$$ 

By a same token, in the latter case where $|\nu\rangle = |1, 0, 0\rangle$, we have $\langle GS|H_T|\nu\rangle^2 \frac{1}{E_{GS} - E_\nu}$. Therefore,

$$\langle n|H_{\text{eff}}|m\rangle = E_m\delta_{mn} + \langle GS|H_T|\mu\rangle^2 \frac{1}{E_{GS} - E_\mu} + \langle GS|H_T|\nu\rangle^2 \frac{1}{E_{GS} - E_\nu}. \quad (A5)$$

Considering the expression of $H_T$, the Kondo expression of our system can be given as

$$H_k = \sum_{k\sigma} J_{k\sigma} \frac{\sigma\sigma'\mu}{2} c_{k\sigma'\mu} \cdot S,$$

where

$$J = |V|^2 \sum_{\sigma} \langle (GS|d_{k\sigma}^\dagger|\mu\rangle \frac{1}{E_{GS} - E_\mu} + \langle GS|d_{k\sigma}^\dagger|GS\rangle \frac{1}{E_{GS} - E_\nu}. \quad (A6)$$

After a simple deduction, one can find that $J_{P_D} = \sqrt{|V|^2 \sum_{\sigma} \langle (GS|d_{k\sigma}^\dagger|\mu\rangle \frac{1}{E_{GS} - E_\mu} + \langle GS|d_{k\sigma}^\dagger|GS\rangle \frac{1}{E_{GS} - E_\nu}}$, with $\Gamma = \pi |V|^2 \rho_0$. Accordingly, the Kondo temperature in our system is expressed as

$$T_K = 0.182U \sqrt{J_P} \exp\left(-\frac{1}{J_P}\right). \quad (A6)$$
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