Unconditionally Secure Quantum Bit Commitment is Simply Possible

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Abstract

Mayers, Lo and Chau proved unconditionally secure quantum bit commitment is impossible. It is shown that their proof is valid only for a particular model of quantum bit commitment encoding, in general it does not hold good. A different model of unconditionally secure quantum bit commitment - both entanglement and disentanglement-based - is presented. Even cheating can be legally proved with some legal evidences. Unconditionally secure quantum bit commitment can be established on the top of unconditionally secure quantum coin tossing, which is also claimed to be two-way impossible.
The task of quantum key distribution (QKD) is to provide identical sequence of random bits for two distant parties — sender and receiver. Its security against eavesdropping is guaranteed by quantum mechanics. But the question is: Will they always get identical sequence? From conventional quantum key distribution (QKD) protocols [1,2], they cannot have identical sequence if one of them becomes dishonest. It seems to be a non-issue. If they want to communicate secretly there is no reason of being dishonest. One may even argue that secure communication between mistrusted parties is itself meaningless and therefore, honesty is the best policy in secure communication. But, in conventional QKD protocols, dishonesty is allowed by the protocol itself. This is a new thing.

To elucidate the issue, let us recall the BB-84 QKD protocol [1]. Like all other conventional QKD protocols it also works on two-step process. In the first step, sender transmits a sequence of 0°, 90°, 45° and 135° polarized single photons. The 0° and 45° single photons represent bit 0 and 90° and 135° single photons represent bit 1. Receiver could recover the bit values if sender gives the required information (basis of measurements) regarding the bit values. In the second step, sender reveals the required information to receiver. The problem is, sender can flip the bit value by changing the required information although he committed the bit value in the first step. This is cheating.

This particular type of cheating can be described as 180° shift from commitment. This shift may be accepted if receiver does not get ultimately cheated. Bennett and Brassard were aware about the problem and they observed that their BB-84 protocol is totally insecure against cheating if sender uses suitable entangled states instead of the said BB-84 states. To overcome this difficulty, the idea of bit commitment surfaced in the early 90’s. It was anticipated that if quantum bit commitment (QBC) is established on...
the top of QKD scheme then cheating could be detected. As if, in cryptographic communication quantum mechanics could resist a committed partner to be an imposter. If secure QBC protocol is found, it was thought that it could be the basis of other important cryptographic schemes such as secure quantum coin tossing, secure quantum oblivious transfer, secure two-party quantum computation. So, the security issue of quantum bit commitment has immense importance.

In 1995, on the basis of conventional model quantum cryptography, a QBC protocol [3], known as BCJL scheme, was proposed and claimed to be provably secure against all types of cheating. Mayers [4] followed by Lo and Chau proved [5] it incorrect. But message of their work is that there cannot have any secure bit commitment protocol, although they worked on a particular model of quantum bit commitment encoding. Recently Kent [6] has invaded this belief. He showed that secure classical bit commitment protocol exists. As the security of his protocol is based on special theory of relativity, it is still widely believed that their proof is valid for all unknown quantum bit commitment protocols [7], which will not use relativity to ensure security against cheating. If it be so, in cryptography relativity wins over quantum mechanics. We shall see, that the belief - quantum cryptography is too weak to realize bit commitment encoding- is misplaced.

We shall first discuss why their proof cannot be considered as a generalized result. Recall the reasoning of complete cheating. Complete cheating is possible when two density matrices associated with bit 0 and 1 are same i.e \( \rho_0 = \rho_1 \). Because of this equivalence of two density matrices, using entanglement, sender, after transmitting the state \( |0\rangle \), corresponding to bit 0, can alone apply unitary transformation \( U \) to convert \( |0\rangle \) to \( |1\rangle \), corresponding to bit 1 and vice versa, keeping the receiver in dark about this transformation.
But it does not necessarily mean whenever $\rho_0 = \rho_1$ cheating will be possible and successful.

Consider the following simple quantum coding technique [8-9].

Bit 0 $\rightarrow \{\psi, \phi, \psi, \phi, \psi, \phi, \psi, \phi, \psi, \phi, \ldots \ldots \ldots \ldots \}$$
Bit 1 $\rightarrow \{\phi, \phi, \psi, \psi, \phi, \psi, \phi, \psi, \phi, \psi, \ldots \ldots \ldots \ldots \}$$

These are two reasonably large sequences of two nonorthogonal quantum states $\psi$ and $\phi$ (they are strictly not orthogonal because it will be classical encoding with quantum states). Suppose these are two sequences of $0^\circ$ and $45^\circ$ (1:1) polarized single photons. So $\rho_0 = \rho_1$. Information regarding the above two sequences is shared between sender and receiver. Here cheating is not possible as receiver can alone recover the bit value from the information they initially shared. The simple method of recovery of bit values can be like this: Bob uses analyzer at $0^\circ$ and $45^\circ$ orientations for his measurements and wants to recover the bit values from exactly one half of the transmitted sequences without missing to detect a single state. In the first 50% events Bob measures according to the first (given above) sequence and in the last 50% events he measures according to the second sequence properly using his analyzers. Therefore, always he could statistically and deterministically recover the exact half of any of the above two sequences. If exact first half of the first sequence is recovered then bit is deterministically 0. Similarly if exact last half of the second sequence is recovered then the bit is deterministically 1. But this is a cheating-free single step QKD protocol not the two-step quantum bit commitment protocol. Similarly our single step entanglement-based QKD protocol [9] can be considered as a cheating-free protocol. It perhaps implies that cheating was possible in conventional QBC as because they did not share information of the two density matrices not because of the equivalence of density matrices though their encoding does not allow to do so.
So sharing of information can be a precondition to have a secure QBC. But this precondition is not enough to realize two-step bit commitment encoding. Next we shall see, single step cheating-free QKD protocol can be simply modified into two-step protocol to realize unconditionally secure quantum bit commitment. First we shall present a two-step entanglement-based QBC protocol.

Suppose Alice has n pairs of EPR particles. Taking one particle of each pair, she arranges them in a particular fashion and taking the partner particles she arranges them in another way with the help of quantum memory. Suppose the two arrangements are:

\[ S_0 = \{ A, B, C, D, E, F, G, H, \ldots \}, \]
\[ s_0 = \{ b, f, g, a, e, h, d, c, \ldots \}. \]

Here capital and its small letter stand for an entangled pair. That is, particle "A" and "a" form an EPR pair and particle "B" and "b" form another EPR pair and so on. These two arrangements represent bit 0. To represent bit 1, similarly she can arrange them in another two different ways:

\[ S_1 = \{ M, N, O, P, Q, R, S, T, \ldots \}, \]
\[ s_1 = \{ s, o, n, p, t, q, m, r, \ldots \}. \]

To avoid confusion we have used two sets of capital and small letters to denote entangled pairs. The entangled state can be represented as,

\[ |\psi\rangle_{i,j} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_i |\downarrow\rangle_j - |\downarrow\rangle_i |\uparrow\rangle_j) \]

where i and j denote the position of the EPR particles in \( S_{0/1} \) and \( s_{0/1} \) respectively. The above information about the two arrangements is secretly shared between them.

Bit commitment encoding can be executed in two-step process.
In the first step, Alice commits bit 0 by sending $S_0$ and in the second step, she reveals the required information just by sending $s_0$. Similarly, she can commit bit 1 by sending $S_1$ in the first step and reveals its value by sending $s_1$ in the second step. Instead of directly sending the 2nd sequence, the results of measurements on 2nd sequence in a pre-committed basis can also be revealed. From the first incoming sequence $S_0$ or $S_1$, Bob cannot recover the bit values. But he can alone recover Alice’s committed bit when he will get the partner sequence $s_0$ or $s_1$. He can measure the spin in a fixed direction. Measurements on the two sequences of EPR particles will produce correlated data. Bob’s task is to recover the bit values from these data and initially shared data. If dishonest Alice sends $s_1$ after $S_0$ or $s_0$ after $S_1$, then Bob could not identify any of the bit values because EPR correlation will be lost in the case of cheating. Thus cheating will be exposed.

The protocol, described above, is an entanglement-based QBC protocol. Using two sequences of deliberately prepared superposition states and following the same operational procedure disentanglement based QBC protocol can be given. Suppose the superposition states are:

\begin{align*}
|A\rangle_i &= 1/\sqrt{2}(|\leftrightarrow\rangle^r_0 + |\leftrightarrow\rangle^s_0), \\
|B\rangle_i &= 1/\sqrt{2}(|\leftrightarrow\rangle^r_0 + |\uparrow\rangle^s_0), \\
|C\rangle_i &= 1/\sqrt{2}(|\leftrightarrow\rangle^r_1 + |\uparrow\rangle^s_1), \\
|D\rangle_i &= 1/\sqrt{2}(|\leftrightarrow\rangle^r_1 + |\downarrow\rangle^s_1).
\end{align*}

The sequence $(Q_0)$ of the states $|A\rangle_i$ and $|B\rangle_i$ represent bit value 0 and the sequence $(Q_1)$ of states $|C\rangle_i$ and $|D\rangle_i$ represents bit value 1. The preparation procedure of these superposition states has been discussed in ref 8. To commit the bit value, say 0, Alice in the 1st step, splitting each state $|\rangle$ of the shared sequence $(Q_0)$ of states sends the sequence $(S_0)$ of the truncated state $|\rangle_r$ which does not
contain the bit value. The path \( s \) is the bit-carrying path. Alice keeps the remaining sequence \( (s_0) \) of states \( |s⟩ \) in quantum memory (using delay). In the second step, Alice transmits the sequence \( (s_0) \) of the states \( |s⟩ \) to Bob. Note that, positions of the complete state and truncated states in their respective sequences are same. In that sense \( Q_0 = S_0 = s_0 \) and \( Q_1 = S_1 = s_1 \), where \( S_1 \) and \( s_1 \) are wave function-split sequences representing the bit 1. Bob can alone recover the bit values from the second sequence because it carries the bit values. The simple method of recovery of bit value from the second sequence is discussed below.

Suppose in the sequence \( Q_0 \) the \( |A⟩ \)\( s \) are at odd positions and \( |B⟩ \)\( s \) are at even positions but \( |C⟩ \)\( s \) and \( |D⟩ \)\( s \) have no such regularity in \( Q_1 \). Now Bob uses 90° analyzer to measure on the second sequence of states \( |s⟩ \). He gets a sequence of "yes" and "no" results. If the results "yes" come only at even positions, then the bit is 0. If Alice transmits \( s_1 \) after transmitting \( S_0 \) or \( s_0 \) after \( S_1 \) Bob will certainly be aware of such improper execution of the protocol. Bob will have to go through the dual measurements on both the sequences (need not be at the same time), if he wants to know whether Alice is cheating or not. The probability of dual occurrence of result "yes" is given in table 1, considering Alice transmits \( s_0 \) after \( S_1 \) or \( s_1 \) after \( S_0 \) and Bob uses both analyzers at 0°. One cannot get double "yes" from a single particle. It implies that not only Bob but also any third party, who does not know anything about their shared information, could spot the cheating. It implies Bob could prove before the court that Alice tried to cheat him provided some legal evidences help him. This is also true for entanglement-based QBC, but Bob has to reveal their shared secret before the court. Of course before going to the court Bob has to be certain that there is no meaningful correlation in the data sets since Alice can transmit her two private sequences at random to defame Bob before the court by disapproving the Bob's revealed data as their
shared data. This is an interesting thing - we are tempted to say that honesty is the only policy in quantum communication.

The above protocol reveals another interesting thing: due to superposition principle it is possible to commit the bit value without sending the actual bit-carrying part of the wave function. The situation can be thought as a case of commitment prior to commitment. On the other hand, in our entanglement-based QBC protocol both first and second sequences are required to recover the bit value. So the significant difference between our entanglement-based and disentanglement-based QBC is that cheating is possible (although it will be unsuccessful) in disentanglement-based QBC but cheating is totally impossible in entanglement-based QBC.

In the above two schemes, bit commitment encoding is two-step process. The QBC can be realized through multi-step procedure. Alice can commit through many steps and reveals the commitment after that (it can also be thought as a single-step commitment followed by multi-step disclosure). Yet the commitment is secure. The encoding is same except we need higher dimensional Hilbert space (for fixed n) to execute multi-step QBC. As for example, they can take GHZ state $|\psi_{GHZ}\rangle = 1/\sqrt{2}(|\uparrow_G \uparrow_H \uparrow_Z\rangle + |\downarrow_G \downarrow_H \downarrow_Z\rangle)$[11]. The n copies of three entangled particles (denoted by G, H, and Z) can be arranged in three different ways to represent bit 0. The arrangements are denoted by $G_0$, $H_0$ and $Z_0$. Similarly Alice can arrange them in another three different ways, denoted by $G_1$, $H_1$ and $Z_1$, to represent bit 1. Alice in the first step commits bit 0 by sending $G_0$ and reveals the commitment by sending $H_0$ and $Z_0$ in the next two steps. Similarly she can commit the bit 1. If they want to have a multi-step disentanglement-based QBC scheme they can use a linear chain of superposition state of our earlier protocol. For three-step disentanglement-based QBC, the superposition states are (see ref 8):
\[
|A\rangle_i = 1/\sqrt{3}(|\leftrightarrow\rangle_0^r + |\leftrightarrow\rangle_0^t + |\leftrightarrow\rangle_0^s)
\]
\[
|B\rangle_i = 1/\sqrt{3}(|\leftrightarrow\rangle_0^r + |\leftrightarrow\rangle_0^t + |\uparrow\rangle_0^s)
\]
\[
|C\rangle_i = 1/\sqrt{3}(|\leftrightarrow\rangle_1^r + |\leftrightarrow\rangle_1^t + |\uparrow\rangle_1^s)
\]
\[
|D\rangle_i = 1/\sqrt{3}(|\leftrightarrow\rangle_1^r + |\leftrightarrow\rangle_1^t + |\downarrow\rangle_1^s)
\]

Again sequence \((Q_0)\) of states \(|A\rangle_i\) and \(|B\rangle_i\) represent bit 0 and sequence \((Q_1)\) of states \(|C\rangle_i\) and \(|D\rangle_i\) represent bit 1. Alice commits the bit value by sending the sequence of states \(|\leftrightarrow\rangle^r\) and reveals the commitment by sending the sequence of states \(|\uparrow\rangle^t\) first and then by sending the actual bit-value-carrying sequence of states \(|\downarrow\rangle_s\).

To prove unconditional security the effect of noise is excluded. We shall consider it. Due to noise some of the Bob’s measured data will be corrupted. Manipulating noise (bringing noise level down) Alice can execute the protocol dishonestly up to the noise level. Nevertheless Bob can statistically faithfully recover the bit value in presence of noise. The main advantage of initial sharing of information of bit preparation is that we will not have to be worried about any unknown attacks. Note that, sharing means pre-commitment and this can give security against cheating even for unknown attacks. The BCJL scheme [3] failed because presently known attack was not clearly known to the authors.

In our alternative QBC protocols, the probability of the success in cheating is always zero. Security does not depend on time, space, technology, noise, and unknown attacks. Therefore, protocols can be safely claimed as absolutely secure protocols against cheating. It can be mentioned that security is not coming from quantum mechanics; it only allows us to perform quantum bit commitment. It is interesting to note, conventional QBC protocol totally fails because of entanglement. The same entanglement provides us secure QBC, although bit commitment is not the problem of alternative
QKD. Regarding bit commitment issue, entanglement is not our enemy rather our friend indeed.

Coin tossing is another important cryptographic primitive: two distant mistrusted parties want to generate faithful random bits to authenticate the channel either by classical coin tossing or by quantum coin tossing (QCT). We can think of two types of coin tossing - ideal and nonideal. It should be mentioned that if one of them does not want to simulate the real coin tossing there is no physical law which can compel him/her to do so. The question is, how far the can the generated bits be considered secure against cheating? Very simple unconditionally secure classical coin tossing protocol exists [13] Lo and Chau claimed that [12] secure ideal QCT is impossible. Their proof is based on the assumptions: ii) shared entanglement cannot be proven genuine ii) entanglement is a necessary condition for secure ideal QCT. We have shown how to check [9] the authenticity of the shared entangled states. Therefore, simulation of ideal QCT is simply possible. It is well known that QCT can be based on QBC protocol. So, we are getting second QCT from our QBC. In addition to that, our QKD protocols are basically QCT protocols. Alternative QCT protocols can be ideal or non-ideal QCT protocols. That is, every bits are secure. These three types of QCT are unconditionally secure against cheating. Yet they cannot be used for authentication until they are proved absolutely secure in presence of noise.

The power of different cryptographic primitives is itself a subject of interest. Recently Kent has claimed that QCT cannot be built on the top of QBC and therefore it is weaker than QBC. We have already seen that our QKD can be thought as QCT on which we can implement our QBC. So Kent’s proof cannot encompass our model. We have seen that all QKD/QCT protocols are not QBC protocol. Is the reverse true? The reverse will not be true if there
is regularity in each of the two operating sequences. This type of QBC cannot be used as secure QCT for authentication. We conclude: 1. unconditionally secure QBC can be implement on the top of unconditionally secure QCT. 2. unconditionally secure QCT can be implement on the top of unconditionally secure QBC. 3. Every QBC is not QCT scheme. 4. Every QCT is not QBC scheme.

Yao has proved [14] that secure quantum oblivious is possible if secure quantum bit commitment is found. On the other hand Killian [15] has proved that secure oblivious scheme can be the basis of secure one-sided two-party computation. Applying classical reduction theory it has been argued that secure quantum computation scheme can be derived from the secure quantum bit commitment scheme. Now we have got secure quantum bit commitment scheme, can we hope for such secure quantum computation scheme? The problem is, Lo has already proved [16] that secure one-sided two-party quantum computation is impossible. We are in a fix. Either Lo’s proof is not a generalized result or the chain of logic is partly or totally incorrect. At least both cannot be right. This puzzle deserves further investigation.

There is another misleading analysis on conventional quantum cryptographic model. For a particular eavesdropping attack, it is stated that optimal information gain of the eavesdropper versus introduced error by him/her is bounded by the laws of quantum mechanics. This is true if there is only one eavesdropper. If we consider many eavesdroppers then optimal gain of information of any eavesdropper will depend on the co-operation of other eavesdroppers which quantum mechanics cannot dictate. Considering the many eavesdropping issue one can even lead to the conclusion: two eavesdroppers are more acceptable than one eavesdropper if one has to accept eavesdropping and can tolerate error. But this discussion is beyond the scope of this paper. One may wonder: why
so much shortcomings? Perhaps topics demands so.

In conclusion, as QBC issue tells us how to distribute cheating-free information at different time, it might have different applications in public and private life. And this is possible to implement within the present technology because, without storing the quantum states the results of measurements can be stored and revealed later instead of storing quantum states and sending them later to execute QBC.

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Table 1. Joint probabilities when $DA$ at $(0^\circ : 0^\circ)$

| committed state $\rightarrow$ revealed state | $P_{(\sqrt{r}:\sqrt{s})}$ | $P_{(\sqrt{r}:x_0)}$ | $P_{(x_0:\sqrt{s})}$ | $P_{(x_0:x_0)}$ |
|---------------------------------------------|---------------------------|-----------------------|-----------------------|------------------|
| $|\leftrightarrow\rangle_0 \rightarrow |\checkmark\rangle_{1}^{s}$ | 1/32                      | 1/32                  | 1/32                  | 1/32              |
| $|\leftrightarrow\rangle_0 \rightarrow |\wedge\rangle_{1}^{s}$ | 1/32                      | 1/32                  | 1/32                  | 1/32              |
| $|\leftrightarrow\rangle_{1}^{r} \rightarrow |\leftrightarrow\rangle_{0}^{s}$ | 1/16                      | 1/16                  | 1/16                  | 1/16              |
| $|\leftrightarrow\rangle_{1}^{r} \rightarrow |\uparrow\rangle_{0}^{s}$ | 0                         | 1/16                  | 0                     | 1/16              |