Partial feedback linearization for a path control in a gantry crane used in lifting machinery in civil engineering

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Abstract. This paper presents an alternate form for the dynamic modelling of a mechanical system that simulates in real life a gantry crane type, using Euler Lagrange classical mechanics, which allows find the equations of motion that our model describes. In order to verify the theoretical results obtained, a comparison was made between solutions obtained by simulation in SimMechanics-Matlab and Euler Lagrange equations system, has been solved through Matlab libraries for solving equations systems of the type and order obtained. Based on the partial feedback linearization, an improved nonlinear controller is analyzed and designed for the movement of an overhead crane or industrial shed. Three control inputs composed of bridge movement forces, carriage displacement, and pendulum or load angle are used to handle the state variables consisting of bridge movement, carriage movement, load displacement, and angle of turn of the same. The objective is to bring the mass of the pendulum from one point to another with a specified distance without the oscillation from it, so that, the answer was overdamped. The asymptotic stability of Lyapunov is also studied and the control scheme is of position of the load, constituted by the non-linear feedback of the actuated and non-actuated states. To verify the quality of the control process, verify numerical simulations. The proposed controller asymptotically stabilizes all states in the system.

1. Introduction
A special type of crane that lifts the load by mounting on one or two beams, Gantry cranes are used in civil engineering machinery, can be placed indoors or outdoors, used for activities in steel yards, precast segment yards, outdoor construction and storage sites, docks, power plants, ports, railway stations and other places for cargo handling and installation operations, also in shipyards or in places such as steelworks, where height clearance may be a problem. Its metal frame is like a gantry frame, with two feet under the main beam, and it can walk directly on the ground track. Both ends of the main beam can have a stabilizing cantilever beam. The gantry crane has the characteristics of high site utilization rate, wide operating range, wide adaptability and great versatility and is used in the yard for loading and unloading of bulk material. The oscillatory behavior of cranes originates from the physical structure that all cranes use vertical suspension cables to support the payload. Such a structure thereby creates the possibility of pendulum-like payload oscillation. According to the primary dynamic properties of cranes, the location of the suspension cable connection point can be described by different coordinate systems. Gantry cranes with track located

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on the floor can be used where the overhead crane or bridge are not practical, they are more cost-effective compared to the overhead crane system.

Gantry cranes can be used when the existing factory structure cannot bear the loads of the overhead crane wheels; o loads should be handled outside the crane span and when relocation of the crane is required after completing work at a given site. This gantry crane works by rails traveling on the ground, the gantry crane lifting and trolley moving mechanisms are basically the same as for the bridge crane. Due to their long travel, gantry cranes are generally used respectively to prevent the crane from biasing and increasing strength, and even accidents. Due to the requirements of high positioning precision, small turning angle, short transport time and high safety, motion and stabilization control for a gantry crane system becomes an interesting topic in the field of control technology development. The main problems for the control design are determined by the non-linear dynamic behavior of the system, the time variation of the system, as well as the coupling of the dynamics between the different directions of movement of the crane.

To optimize the crane performance in terms of minimization of the working operation time, the accurate characterization of the dynamic response of containers during the lifting/lowering maneuvering phase is an essential requirement. Moreover, harbors are often located in windy areas, characterized by the occurrence of gust fronts that can affect the cranes working efficiency and, sometimes, compromise the safety of workers.

Numerical simulations, aimed to study the effects of non linearities arising from mechanical friction and air resistance, were performed in [1] where the results were also experimentally validated. In [2]-[3], modeling of a variety of pendular structures (such as gantry cranes) was proposed and suitable control strategies were thoroughly examined by highlighting the computational implications of the models on the control implementations.

The dynamics of the car-pendulum type gantry crane is more complex than the single pendulum-type overhead crane, so the system analysis and control algorithm design also is more difficult. In this paper, the gantry crane that exhibits car-pendulum dynamics is investigated by using the Euler-Lagrange equations [4]-[5].

The system model is built and the system properties are analyzed. An adaptive path control algorithm is proposed for the car-pendulum type gantry crane and the simulation results demonstrate the system dynamics and the effectiveness of the proposed control algorithm. The path controller is the most common form of feedback and control is often combined with logic, sequential functions, selectors, and simple function blocks to build the complicated automation systems used for energy production, transportation, and manufacturing. Many sophisticated control strategies, such as model predictive control, are also organized hierarchically. Path control is used at the lowest level; the multivariable controller gives the setpoints to the controllers at the lower level.

During the present work several aspects of study were taken into account, starting in the section 2, by the dynamic modeling of the system through the Euler-Lagrange equations in a conservative way. Next, in section 3, shows the non conservative modelling and path control. In the section 4, we analice the nonlinear controller design, the partial feedback linearization, the stability of Lyapunov and the system was modeled taking into account the non-conservative forces and the trajectory was defined and compare these results are shown. Finally, the section 5, we present the conclusions of the work.

2. Mathematical modelling and Euler-Lagrange motion equations
The crane type modeling system started from a model in which intervenes a spring at the point where the force is applied for the movement of the carriage on the rail, as shown in Figures 1, 2 and 4, the real gantry crane as shown in the Figure 3.
Compared with Newtonian mechanics, Lagrangian mechanics has no new physics. In Newtonian mechanics, the equations of motion are given by Newton's laws. The second law, net force equals mass times acceleration, is applied to each particle. Instead of forces, Lagrangian mechanics uses the energies in the system. The central quantity of Lagrangian mechanics is the Lagrangian, a function that summarizes the dynamics of the entire system. Overall, the Lagrangian has units of energy, but no single expression for all physical systems. Any function that generates the correct equations of motion, in agreement with physical laws, can be taken as a Lagrangian. It is nevertheless possible to construct general expressions for large classes of applications. Lagrangian mechanics is important not just for its broad applications but also for its role in advancing deep understanding of physics. Lagrangian mechanics is widely used to solve mechanical problems in physics and engineering. The proportional-integral control (PID) was proposed by [6] and positive results were obtained. For the PID techniques, the authors used a PID controller for position control and its the base for this work.

For this system the Lagrangian model was developed, in which initially the kinetic and potential energies are calculated. For this model, was defined as M is the mass of the carriage (object on the rail), m1 the mass of the extreme of the pendulum, m2 the mass of the pendulum rod, \(\theta\) the angle between the vertical and the pendulum, \(l\) the length and, \(x\) the displacement of the carriage.

The Lagrangian of any system is defined,

\[ L = T - U \]  

where, \(T\) is the kinetic energy and \(U\) the potential, therefore, for this model it is,

\[ L = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m_2 \left( \frac{1}{4} l \dot{\theta}^2 + l \dot{\theta} \dot{x} \cos \theta + \dot{x}^2 \right) + \frac{1}{2} m_1 \left( \frac{1}{4} l \dot{\theta}^2 + 2 l \dot{\theta} \dot{x} \cos \theta + \dot{x}^2 \right) + \frac{1}{24} m_2 l^2 \dot{\theta}^2 + \frac{1}{2} m_2 \dot{\theta}^2 \left( \frac{\dot{x}}{l} \right)^2 + g l \cos \theta \left( m_1 + \frac{m_2}{2} \right) - \frac{1}{2} K x^2 \]  

The Euler-Lagrange equations are given by the following equation [5],

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0 \]  

where, \(q\) represents the degrees of freedom (DOF) generalized, in this case, there are two DOF, \(\theta\) and \(x\), therefore, from equation (3), the Euler-Lagrange equation, is obtained,

\[ \ddot{\theta} l^2 A + (\ddot{x} l \cos \theta + g l \sin \theta) B = 0 \]  

\[ \ddot{x} C - (\ddot{\theta} l \cos \theta - l \dot{\theta}^2 \sin \theta) B + K_x = 0 \]  

where,
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\[
A = \left( m_1 + \frac{m_2}{2} \right) + \frac{2m_1r^2}{5} \quad (6)
\]

\[
B = m_1 + \frac{m_2}{2} \quad (7)
\]

\[
C = m_1 + m_2 + M \quad (8)
\]

To define the numerical model, the constant values are replaced [6],

| Constant | Value           |
|----------|-----------------|
| \( l \)  | 0.2 [m]         |
| \( g \)  | 9.81 [m/s²]     |
| \( K \)  | 100 [N/m]       |
| \( m_1 \) | 0.088338025 [Kg]|
| \( m_2 \) | 0.022245336 [Kg]|
| \( M \)  | 0.548069759 [Kg]|
| \( r \)  | 0.02 [m]        |

Table 1. Constant values in the model.

Substituting these values in equations (4) and (5),

\[
0.003844 \ddot{\theta} + 0.19514 \sin \theta + 0.01989 \ddot{x} \cos \theta = 0 \quad (9)
\]

\[
0.6586 \dddot{x} - 0.01989 \dddot{\theta} \sin \theta + 0.01989 \dddot{\theta} \cos \theta + 100x = 0 \quad (10)
\]

As can be seen, the equations (9) and (10) are coupled, that is to say, both depend on two variables to their second derivative, \( \dddot{\theta} \), \( \dddot{x} \), therefore, the initial conditions depend on the initial energy that shows this system, then, to find the initial conditions that can take, part of the potential energy is taken, which it is the energy that accumulates the spring and the pendulum when \( \theta \) is different from zero.

There are infinite energy values that could take the system, however, only interested in those who do not turn the model into a chaotic system, since it is assuming (until this moment) from conservative way, i.e., that no gain or loss of energy and therefore the energy level that present, is due to the initial position of the pendulum and spring’s compression. For spring it can be assumed values between \(-0.1\) and \(0.1\), and for the angular position of the pendulum \(\pi\) and \(-\pi\).

3. Non conservative modelling and path control
Concerning type overhead cranes, described a control structure by using the trolley motion and an additional translational actuator at the load suspension unit as system inputs. According to such a control structure, they established a decoupling between position and tilt angle of the load. State-space- based methods were used to design the decoupling control [8]. The procedure takes into account the unique properties of gantry cranes, such as, the single-mode dynamics, the known frequency range, and the standard deceleration period. To control the system described above, the spring is deleted, which exerts a force on the mass placed on the rail. The force is determined, but not as exerted by the spring, since this will be the variable being monitored.
The objective is to bring the mass of the pendulum from one point to another with a specified distance it to oscillate, so the answer is overdamped. In [13], this article then extended to incorporate a non collocated PID controller for control of sway angle of the pendulum. Implementation results of the response of the rotary crane system with the controllers are presented in time and frequency domains. The performances of the control schemes are assessed in terms of level of sway reduction, rotational angle tracking capability and time response specifications [6].

It is worth mentioning the important role of a class of continuously differentiable, monotonously increasing, and bounded functions. In order to preserve simplicity in the exposition and despite removing generality, treatment will be considered using one element of this class of functions: the hyperbolic tangent function. To control the system described above, the spring is deleted, which exerts a force on the mass placed on the rail. The force is determined, but not as exerted by the spring, since this will be the variable being monitored. The objective is to bring the mass of the pendulum from one point to another with a specified distance it to oscillate, so the answer is overdamped [11] - [12].

From the system described in Figure 4, the dynamic model is determined from the Euler-Lagrange equations for generalized force, this is:

$$q_1 = \theta \frac{d}{dt} \left( \frac{\partial C}{\partial \dot{q}_1} \right) - \frac{\partial L}{\partial q_1} = Q_i \quad i=1,2$$

(11)

where $q_1 = \theta$, $q_2 = x$, $Q_1 = Q_\theta$ and $Q_2 = Q_x$; these last two represent the generalized forces over the system, which will no longer be conservative, due to the existence of an external force $F$ and friction is assumed in the pendulum axis $(b_1 \dot{\theta})$ and carriage contact with the rail $(b_2 \dot{x})$, according to Figure 8. With this variation, the new Euler-Lagrange equations, are:

$$\ddot{\theta} D + [\ddot{x} l \cos \theta + g l \sin \theta] B = -b_1 \dot{\theta}$$

(12)

$$\ddot{x} C + [\ddot{x} l \cos \theta + g l \sin \theta] B = F - b_2 \dot{x}$$

(13)

$Ve$ of $Z$ is:

$$D = \left( \frac{2}{5} m_1 r^2 \right)$$

(14)

and $B, C$ are the same terms in the equations (7) and (8), respectively. To control the system, you must first find the state space,

$$Z = \begin{pmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{pmatrix}$$

(15)

Such that the derivative of $Z$ is:

$$Z = \begin{pmatrix} Z_2 \\ \frac{(l B \cos(Z_2)) \epsilon_1 + C \epsilon_2}{\epsilon_3} \\ Z_4 \\ \frac{(l^2 B + D) \epsilon_1 + (l B \cos(Z_1)) \epsilon_2}{-\epsilon_3} \end{pmatrix}$$

(16)

where:

$$\epsilon_1 = F - b_2 \dot{x} + \dot{\theta}^2 l B \sin(Z_1)$$,

$$\epsilon_2 = b_1 \dot{\theta} + g l B \sin(Z_1)$$,

$$\epsilon_3 = l^2 B^2 \cos^2(Z_1) - l^2 B C - DC$$
The equations in space of states (16) were implemented in Simulink to obtain the natural response of the system to a step input in $F$ and subsequently draw the desired trajectory. In the scheme of Simulink, the value for $b_1 = 0.1$, and $b_2 = 0.5$, was assumed; because the system is dimensionally small, the input for the force $F$ was 0.1, for a time of 2 seconds. The natural response to these values are shown in Figure 9 and Figure 10 for $\theta$, $x$ respectively.

![Figure 7. Output to an input $F = 0.1$, for 2 seconds.](image)

![Figure 8. Output to an input $F = 0.1$, for 2 seconds.](image)

A suitable trajectory for the variable $x$ is that in which the speed starts and ends at zero, the trigonometric function $\tanh$, so that the function of the desired trajectory is [6],

$$x^* = \frac{d}{2} \tanh \left[ a \left( t - \frac{T}{2} \right) + \frac{d}{2} \right]$$  \hspace{1cm} (17)

where $d$ is the distance to travel, $t$ represents time, $T$ the time and $\alpha$ a factor to vary the slope of the curve. The function type proposed is more than twice differentiable, this ensures a continuous acceleration along the way. Having defined the trajectory to $x$, these values are entered into equation (12), and trajectories for $\theta$ y $\dot{\theta}$ are found.

The output for $\theta$, $\dot{\theta}$ and $x$, $\dot{x}$ is registered in Figure 9 and Figure 10, respectively.

![Figure 9. Output for $\theta$ and $\dot{\theta}$.](image)

![Figure 10. Output for $x$ and $x'$.](image)

As seen in the previous two graphs, variables were controlled according to the desired trajectory. As for the position and velocity of the pendulum shown in Figure 9, it can be seen that the controller tries to keep the pendulum at its equilibrium point. It can be seen in the red line that the position has few oscillations, so that remains in quasi-equilibrium. The Figure 10 shows the position and speed of the cart under the action of the controller, it is observed that the cart arrives at the desired position by the trajectory defined in a non-abrupt way and without going back. As for the speed, we see that this increases when the cart is starting and decreases when it is reaching its stationary zone, so that the car remains with a null position error [6].

4. Partial feedback linearization

A useful technique for crane control is called partial feedback linearization, which is a method providing a natural global change of coordinates that transforms the system into a strict feedback form, and the control method can be easily applied to the new form of the system.
There are two partial feedback linearization techniques presented as collocated and non-collocated partial feedback linearization methods, respectively. Take the non conservative motion equations (12) and (13), we have:

\[
(B l \cos \theta + C) \ddot{x} + B g l \sin \theta + b_2 \dot{x} = F
\]  
(18)

and,

\[
B l \cos \theta \ddot{x} + D \ddot{\theta} + B g l \sin \theta + b_1 \dot{\theta} = 0
\]  
(19)

The action variable is defined as [5]:

\[
S = \int L(q, \dot{q}, t) \, dt
\]

take the motion equation (19), into the action, we have:

\[
S = \int L(q, \dot{q}, t) \, dt = B l \cos \theta \ddot{x} + D \ddot{\theta} + b_1 \dot{\theta}
\]

the action takes the form,

\[
S = k_1 \theta + k_2 x + k_3 (B l \cos \theta \ddot{x} + D \ddot{\theta} + b_1 \dot{\theta})
\]  
(20)

where, \(k_1, k_2\) and \(k_3\) are positive constants. Take the derivate of (20) respect to the time,

\[
\dot{S} = k_1 \dot{\theta} + k_2 \dot{x} + k_3 (B l \ddot{x} \cos \theta + D \ddot{\theta} + b_1 \dot{\theta} - B l \dot{x} \dot{\theta} \sin \theta)
\]

But, the (19):

\[
B l \ddot{x} \cos \theta + D \ddot{\theta} + b_1 \dot{\theta} - B g l \sin \theta
\]

now, the derivate of action, take the form:

\[
\dot{S} = k_1 \dot{\theta} + k_2 \dot{x} - k_3 (B g l \sin \theta + B l \dot{x} \dot{\theta} \sin \theta)
\]  
(21)

Reemplazing the equation (19) in (18), we have the expression:

\[
\ddot{x} = \frac{-D}{B l} \dot{\theta} \sec \theta - g \tan \theta - \frac{b_1}{B l} \dot{\theta} \sec \theta;
\]

reemplazing the after expression in (18), the force \(F\), take the form:

\[
F = b_2 \dot{x} - b_1 \dot{\theta} - D \ddot{\theta} - C_\theta \tan \theta - \frac{CD}{B l} \ddot{\theta} \sec \theta - \frac{C b_1}{B l} \dot{\theta} \sec \theta
\]

In the space state, we defined \(u = \dot{\theta}\), and

\[
\ddot{x} = \frac{-D}{B l} u \sec \theta - g \tan \theta - \frac{b_1}{B l} \dot{\theta} \sec \theta
\]

And the force:

\[
F = \left(-\frac{D}{B l} \sec \theta\right) u - \left(b_1 + \frac{C b_1}{B l} \sec \theta\right) \dot{\theta} - C_\theta \tan \theta + b_2 \dot{x}
\]  
(22)
The Lyapunov function candidate is a quadratic form:

$$V = \frac{1}{2}S^2$$

and the derivative with respect to time is:

$$\dot{V} = S\dot{S}$$

where, $\dot{S} = \epsilon - b_3S$, to guarantee the derivative of Lyapunov function is negative or $\dot{V} < 0$.

Now:

$$\dot{V} = \dot{S} = \epsilon - b_3S$$

or,

but, $\dot{S}$ is the equation (21):

$$\epsilon = k_1\dot{\theta} + k_2\dot{x} - k_3(Bgl\sin\theta + Bl\dot{x}\dot{\theta}\sin\theta) + b_3S$$

and the derivative with respect to the time is:

$$\dot{\epsilon} = k_1\ddot{\theta} + k_2\dot{x} - k_3(Bgl\dot{\theta}\cos\theta + Bl\dot{x}\dot{\theta}\sin\theta + Bl\dot{\theta}\dot{x}\sin\theta + Bl\dot{\theta}\dot{x}\dot{\theta}\cos\theta) + b_3S \tag{23}$$

Now, in the space of state representation:

$$\dot{\epsilon} = \left(k_1 - \frac{Bk_2}{Bl}\sec\theta - k_3Bl\dot{x}\sin\theta + k_3D\dot{\theta}\tan\theta\right)u - k_2g\tan\theta - \frac{k_2b_1}{Bl}\sec\theta - k_3Bgl\dot{\theta}\cos\theta + k_3Bgl\dot{\theta}\sin\theta\tan\theta + k_3b_1\tan\theta\dot{\theta}^2 - k_3Bl\cos\theta\dot{x}\dot{\theta}^2 + b_3S \tag{24}$$

Now, redefined the Lyapunov function as:

$$V = \frac{1}{2}S^2 + \frac{1}{2}\epsilon^2$$

and the derivative with respect to time is:

$$\dot{V} = S\dot{S} + \epsilon\dot{\epsilon}$$

where, remember $\dot{S} = \epsilon - b_3S$,

$$\dot{V} = S(\epsilon - b_3S) + \epsilon\dot{\epsilon} = -b_3S^2 + (S + \dot{\epsilon})\epsilon$$

here, we defined $S + \dot{\epsilon} = -b_4\epsilon^2$, to guarantee to derive of Lyapunov function is negative or $\dot{V} < 0$.

and the derivative of Lyapunov function take the form as:

$$\dot{V} = -b_3S^2 - b_4\epsilon^2 \tag{25}$$

and,

$$u = \frac{1}{\rho}\left(-b_4\epsilon - S - b_3\dot{S} + k_3Bl\cos\theta\dot{x}\dot{\theta}^2 - k_3b_1\tan\theta\dot{\theta}^2 - k_3Bgl\dot{\theta}\sin\theta\tan\theta + k_3Bgl\dot{\theta}\cos\theta + k_2g\tan\theta + \frac{k_2b_1}{Bl}\dot{\theta}\sec\theta\right) \tag{26}$$
where:

\[ \gamma = k_1 - \frac{Dk_2}{Bl} \sec \theta - k_3 Bl \dot{x} \sin \theta + k_3 D \dot{\theta} \tan \theta \]

The gantry is stable under (22) and (26) conditions, with \( k_1, k_2, k_3, b_3 \) and \( b_4 \) positive constants.

4.1. Theorem 1. Stability analysis of all sliding surfaces
Take the equation (25):

\[ \dot{V} = -b_3 S^2 - b_4 \epsilon^2 = \frac{d}{d\tau} \]

integrate;

\[ \int_0^t \dot{V} d\tau = \int_0^t (-b_3 S^2 - b_4 \epsilon^2) d\tau \]

evaluate;

\[ V(t) = \frac{1}{2} S^2 + \frac{1}{2} \epsilon^2 = V(0) + \int_0^t (-b_3 S^2 - b_4 \epsilon^2) d\tau \leq V(0) < \infty \]

so, \( S \in L_{\infty} \), and \( \epsilon \in L_{\infty} \), i.e.

\[ |S|_{t \geq 0} = \|S\|_{\infty} < \infty \quad (27) \]

\[ |\epsilon|_{t \geq 0} = \|\epsilon\|_{\infty} < \infty \quad (28) \]

at the same time

\[ \dot{\Psi} = SS + \epsilon \dot{\epsilon} = -b_3 S^2 - b_4 \epsilon^2 < \infty \]

With this equation we have obtained the control law to imply \( \dot{S} \in L_{\infty} \) and \( \dot{\epsilon} \in L_{\infty} \), i.e.

\[ |S|_{t \geq 0} = \|S\|_{\infty} < \infty \quad (29) \]

and;

\[ |\epsilon|_{t \geq 0} = \|\epsilon\|_{\infty} < \infty \quad (30) \]

Now, the hierarchical sliding mode controller design.

4.2. Theorem 2. Barbala’t theorem

\[ \lim_{t \to \infty} S_1 = 0 \]

and:

\[ \lim_{t \to \infty} S_2 = 0 \]
where, $S_1 = S_1(\theta)$ and $S_2 = S_2(x)$ to applied the equation (2):

$$S_1(\theta) = \theta + \frac{k_3 D}{k_1 + k_3 b_1} \dot{\theta}$$

and:

$$S_2(x) = x + \frac{k_3 Bl \cos \theta}{k_2} \dot{x};$$

applied to Barbala’ theorem, we have:

$$\lim_{t \to \infty} S_1(\theta) = \lim_{t \to \infty} \left( \theta + \frac{k_3 D}{k_1 + k_3 b_1} \dot{\theta} \right) = 0$$

to imply:

$$\theta + \frac{k_3 D}{k_1 + k_3 b_1} \dot{\theta} = 0$$  \hspace{1cm} (31)

and the similar form:

$$\lim_{t \to \infty} S_2(x) = \lim_{t \to \infty} \left( x + \frac{k_3 Bl \cos \theta}{k_2} \dot{x} \right) = 0$$

to imply:

$$\ddot{x} + \frac{k_3 Bl \cos \theta}{k_2} \dot{x} = 0$$  \hspace{1cm} (32)

Now, we have applicated the global stability of Lyapunov to the linearized equations. Take the (31), the quadratic function of Lyapunov take the form:

$$V_\theta = \frac{1}{2} \theta^2$$

And its derivate is:

$$\dot{V}_\theta = \theta \dot{\theta}$$

but:

$$\theta = -\frac{k_3 D}{k_1 + k_3 b_1} \dot{\theta};$$

so:

$$\dot{V}_\theta = \theta \dot{\theta} = -\frac{k_3 D}{k_1 + k_3 b_1} \dot{\theta}^2 < 0$$  \hspace{1cm} (33)

The similar form:

$$V_x = \frac{1}{2} x^2$$
And his derivate is:

$$\dot{V}_x = x \ddot{x}$$

but;

so;

$$x = - \frac{k_3 B \cos \theta}{k_2} \dot{x};$$

$$\dot{V}_x = x \ddot{x} = - \frac{k_3 B \cos \theta}{k_2} \dot{x}^2 < 0$$  (34)

The functions (33) and (34) are negative definite, therefore, the gantry crane system is asymptotically stable to the equilibrium point \(s, x^* , J, B, L = (0, 0, 0, 0)\). Summarily, the advantage of collocated and non-collocated partial feedback linearization methods is a conceptual and structural simplification of control problem, i.e., they are always utilized as an initial simplifying step for crane control problems.

5. Conclusions

Overhead crane systems are under actuated because they have a lower number of control inputs than the number of degrees of freedom to be controlled. Such an inherent property challenges their control design. A path mode control is recognized as one of the efficient tools to design controllers for complex nonlinear systems. Concerning the based design for overhead crane systems, it is one of the most active fields of research in control community. The physical characteristic is that an overhead crane is composed of several subsystems. The Euler-Lagrange formalism allows dynamic modeling of crane-type system in a simple way, thanks to the formalism is based on energy principles.

The feedback linearization method provides an effective design tool for the control of a class of under-actuated mechanical systems such as an overhead crane system. Based on this technique, a quality nonlinear controller was designed and the Lyapunov stability was studied. Both simulation and experiment results show that the proposed controller stabilizes the overhead crane system. All system responses asymptotically reach the desired values within a short period; the bridge and trolley were controlled to move them to the desired position, and the cargo was lifted up to the reference point precisely. Moreover, the cargo swings were kept small during the transfer process and then suppressed at the load destruction.

The path is a type of controller in state space that responds well to perturbations. In addition, this driver guarantees us a null position error.

The controller responds correctly, since it takes the carriage to the desired position in a good way following the path defined in (17). Any real overhead crane systems are subject to uncertainties. Uncertainties can be categorized by matched uncertainties and unmatched ones, this of invariance again matched uncertainties, uncertainties involved in crane dynamics cover both matched and unmatched uncertainties. In this representation, known as representation in the state space, it shows the variation of a vector \(Z\), known as state vector and contains the most important system information, as a function of the same \(Z\) vector. From this formulation, it was possible to synthesize laws nonlinear control.

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