Three Family $SU(5)$ GUT and Inverted Neutrino Mass Hierarchy

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Abstract

Supersymmetric $SU(5)$ GUT augmented with anomaly free $U(1)_F$ flavor symmetry is presented. Very economical field content and $U(1)_F$ charge assignment are obtained by specific construction. In particular, three families of $10+\bar{5}$ chiral matter, along the $SU(5)$ singlet states (some of which serve as right handed neutrinos) are obtained. Appealing texture zero Yukawa matrices provide natural understanding of hierarchies between charged fermion masses and mixings. The model predicts inverted hierarchical neutrino mass scenario with interesting implications.

1 Introduction

Although being very successful, the Standard Model is unable to resolve some puzzles. Among them is a problem of fermion flavor. The origin of hierarchies between charged fermion masses and CKM mixing angles is unexplained. Is there any underlying theory which might generate these hierarchies in a natural way? Moreover, in order to explain the neutrino data [1–3] some extension of the Standard Model, generating neutrino masses and mixings, is necessary. The number of fermion generations is a mystery. Do we have only three chiral families of quarks and leptons? Is any selection rule dictating the number of fermion generations?

Motivated by these questions, in this paper we address these issues within the framework of supersymmetric (SUSY) $SU(5)$ Grand Unified Theory (GUT). Latter’s motivation is to have unified description of electro-weak and strong interactions [4], while SUSY provides natural understanding of gauge hierarchy problem as well as the successful gauge coupling unification [5]. For understanding hierarchies between fermion masses and mixings, we apply Abelian flavor symmetry [6] $U(1)_F$ which by requirement is non-anomalous. The $U(1)_F$, in combination with SUSY $SU(5)$ GUT, due to anomaly constraint allows only three chiral families of matter $(10 + \bar{5})$-plets and few $SU(5)$
singlet states. We use some of these singlet states as right handed neutrinos, in order to build realist neutrino sector. Anomaly constraints fix $U(1)_F$ charge assignment in such a way that texture zero quark and lepton Yukawa matrices are generated. Together with natural understanding of hierarchies, model predicts inverted neutrino mass hierarchical scenario blending well with recent Daya Bay observation [2].

2 Three Family SUSY $SU(5) \times U(1)_F$

Consider SUSY $SU(5)$ GUT augmented with anomaly free $U(1)_F$ flavor symmetry. Setup with anomaly free $U(1)_F$, will allow to gauge $U(1)_F$ and remain within conventional 4-dimensional field theoretical framework (without need of discussing $U(1)$'s of a stringy origin [7]). In a recent work [8] the way of finding anomaly free $U(1)_F$ flavor symmetry within SUSY $SU(5)$ GUT was suggested. The finding was realized by embedding of $SU(5) \times U(1)_F$ in a single non-Abelian group with anomaly free field content. In this way, the $U(1)_F$ charge assignment can be fixed. Amongst several assignments, found in [8], there is one which also dictates the number of generations to be three. This, As will be shown below, leads to very economical and attractive scenario for fermion masses and mixings. Before showing this, we briefly discuss the way of finding of such $U(1)_F$.

The states non-trivial under $SU(5)$ group, we introduce, will be just those of minimal SUSY $SU(5)$. These are scalar superfields $\Sigma(24), H(5), \bar{H}(5)$ and three families of matter $(10 + \bar{5})$ supermultiplets. We assume that $\Sigma$ is not charged under $U(1)_F$ and thus does not contribute to the anomalies. Therefore, upon finding anomaly free $U(1)_F$ charge assignment we will deal with three 10-plets, one 5-plet (which is $H$) and four 5-plets (=three matter 5-plets plus $\bar{H}$). As already mentioned, we search $U(1)_F$ charge assignment by embedding of $SU(5) \times U(1)_F$ in non-Abelian group with anomaly free matter [8]. Let us consider $SU(7)$ group with chiral supermultiplets $35 + 2 \times 7$. This simple set is anomaly free [10]. Here 35 is three index antisymmetric representation and 7 is an anti-fundamental of $SU(7)$. Decomposition of these states via the chain $SU(7) \rightarrow SU(6) \times U(1)_7 \rightarrow SU(5) \times U(1)_6 \times U(1)_7$ looks

$$35 = 20_3 + 15_{-4} = (10_{-3} + \overline{10}_3)_3 + (10_2 + 5_{-4})_{-4},$$

$$7 = 6_{-1} + 1_6 = (5_{-1} + 1_5)_{-1} + (1_0)_6,$$

where subscripts inside and outside of parenthesis indicate $U(1)_6$ and $U(1)_7$ charges respectively. Note that $U(1)_6$ and $U(1)_7$ are coming from $SU(6)$ and $SU(7)$ respectively, with corresponding generators $Y_{U(1)_6} = \frac{1}{\sqrt{60}} \text{Diag}(1,1,1,1,1,-5)$ and $Y_{U(1)_7} = \frac{1}{\sqrt{84}} \text{Diag}(1,1,1,1,1,-6)$. The normalization factors $\frac{1}{\sqrt{60}}$ and $\frac{1}{\sqrt{84}}$ are omitted in Eq. (1). Now, in (1), without change of charge assignments, we replace the pair of $(\overline{10} + 5)$-plets by the pair $(10 + 5)$. With this replacement all anomalies (at the level of $SU(5)$ and $U(1)$'s) will remain intact, i.e. will still vanish. Thus, we will have the following anomaly free content

$$(10_{-3} + 10_3)_3 + (10_2 + \overline{5}_{-4})_{-4} + 2 \times [(5_{-1} + 1_5)_{-1} + (1_0)_6],$$

which involves three families (!) of matter $(10 + \overline{5})$ supermultiplets plus four $SU(5)$ singlets. Some of these singlets will be applied as right handed neutrinos (RHN). Worth noting that, in difference

\footnote{The way of this finding differs from those used earlier [9].}
from $SO(10)$ GUT, the $SU(5)$ does not involve (require) the RHN states. In the $SU(5)$ scenario, we have just built up, the RHNs are required for anomaly cancellation.

By Abelian symmetries $U(1)_6$ and $U(1)_7$, with charges given in Eq. (2), we can build superposition $\tilde{a}Q_{U(1)_6} + \tilde{b}Q_{U(1)_7}$. This superposition is automatically anomaly free for arbitrary $\tilde{a}$ and $\tilde{b}$, because the orthogonal generators $Y_{U(1)_6}$ and $Y_{U(1)_7}$ originate from single $SU(7)$. Thus, using (2) we can write the anomaly free set

$$10_{−3â+3b} + 10_{3â+3b} + 10_{2â−4b} + 5_{−4â−4b} + 2 \times (5_{−a−b} + 1_{5â−b} + 1′_{6b}) ,$$

where subscripts denote charges. As it turns out, for building realistic phenomenology it is useful to add to this superposition another anomaly free $U(1)$, which can be found by similar procedure. For instance, consider $27$-plet of $E_6$ group with a chain $E_6 \rightarrow SO(10) \times U(1)_{E_6} \rightarrow SU(5) \times U(1)_{E_6}$ of decomposition [11]:

$$27 = 16_1 + 10_{−2} + 1′_4 = (10 + 5 + 1)_1 + (5 + 5′)_{−2} + 1′_4 .$$

Here subscripts denote $U(1)_{E_6}$ charges. In this content, we can replace $5_{−2}$ with $5_{−2}$ and at the same time add $10_p + 10_{−p}$. Moreover, we can add two $SU(5)$ singlets with $U(1)_{E_6}$ charges $k$ and $−k$ respectively. With this replacement and additions, the anomalies $SU(5)^3$, $SU(5)^2 \cdot U(1)_{E_6}$, etc, will be unchanged. This content allows to build superposition of three Abelian groups $U(1)_6$, $U(1)_7$ and $U(1)_{E_6}$: $\tilde{Q}_{sup} = \tilde{a}Q_{U(1)_6} + \tilde{b}Q_{U(1)_7} + \tilde{c}Q_{U(1)_{E_6}}$. Thus, the field content and $\tilde{Q}_{sup}$ charge assignment will look:

$$10_{−3â+3b+pâ} + 10_{3â+3b−pâ} + 10_{2â−4b+â} + 5_{−4â−4b−2â} + 5_{−a−b+â} + 5_{−a−b−2â}$$

$$\quad + 1_{5â−b+â} + 1_{5â−b−2â} + 1′_{6b+kâ} + 1′_{6b−kâ} , \quad \text{with} \quad 30 \tilde{a}(3 + 2p) = \tilde{c}(2k^2 + 10p^2 − 27) .$$

Relations between $\tilde{a}, \tilde{c}, k$ and $p$ (imposed for $\tilde{c} \neq 0$) given in Eq. (5) insures that all anomalies vanish. Clearly, with rational selection of $\tilde{a}, k$ and $p$ the value of $\tilde{c}$ also will be rational. The set given in Eq. (5) is one simple selection among several options and opens up many possibilities for model building with realistic phenomenology. We will identify these charges with the charges of $U(1)_F$ flavor symmetry.

### 3 Model: Quark and Charged Lepton Yukawa Textures

To the field content of Eq. (5) we add the pair $5_q + 5_{−q}$. This is needed to have, besides the matter fields, the Higgs supermultiplets $H + \bar{H}$. Thus, in total we have three 10-plets, one 5-plet and four 5-plets plus $SU(5)$ singlets. The $U(1)_F$ charge assignment ($q, −q$ for $5_q, 5_{−q}$ and for remaining 5-plets given in Eq. (5)) is not unique. We can exchange 5-plet’s $U(1)_F$ charge with one of the 5-plets’ charge. With this, all anomalies will still vanish. In addition, out of the four 5-plets, any of them can be identified with the Higgs superfield $\bar{H}$. As it turns out, for the charges of the pair $(H, \bar{H})$, we will have 13 possible options for $(Q_H, Q_{\bar{H}})$:

$$(Q_H, Q_{\bar{H}})^{(0)} = \{(q, −q), (q, −4\tilde{a} − 4\tilde{b} − 2\tilde{c}), (q, −\tilde{a} − \tilde{b} + \tilde{c}), (q, −\tilde{a} − \tilde{b} − 2\tilde{c}),$$

$$(-4\tilde{a} − 4\tilde{b} − 2\tilde{c}), (-\tilde{a} + \tilde{b} + \tilde{c}), (-4\tilde{a} − 4\tilde{b} − 2\tilde{c}), (-\tilde{a} − \tilde{b} + \tilde{c}), (-\tilde{a} − \tilde{b} − 2\tilde{c}), (-\tilde{a} − \tilde{b} + \tilde{c}),$$

$$(-\tilde{a} − \tilde{b} + \tilde{c}), (-4\tilde{a} − 4\tilde{b} − 2\tilde{c}), (-\tilde{a} − \tilde{b} + \tilde{c}), (-\tilde{a} − \tilde{b} − 2\tilde{c}), (-\tilde{a} − \tilde{b} + \tilde{c}),$$

$$(-\tilde{a} − \tilde{b} − 2\tilde{c}), (-4\tilde{a} − 4\tilde{b} − 2\tilde{c}), (-\tilde{a} − \tilde{b} − 2\tilde{c}), (-\tilde{a} − \tilde{b} + \tilde{c}), (-\tilde{a} − \tilde{b} − 2\tilde{c}), \} ,$$

(6)
with $l = 1, \cdots, 13$. Note that we have left out possibilities obtained from those given in (6) by the substitution $q \to -q$. These 13 options open up various possibilities for the model building [12]. Below we present one of them, which we found to have nice and attractive properties with interesting implications for fermion masses and mixings.

**$U(1)_F$ symmetry breaking**

In order to break $U(1)_F$ gauge symmetry, we introduce $SU(5)$ singlet pair of flavon superfields $X + \bar{X}$ with $U(1)_F$ charges

$$Q(X) = -1, \quad Q(\bar{X}) = 1.$$  \hfill (7)

Without loss of generality, we have normalized flavons’ charges modulo to one. The scalar components of $X$ and $\bar{X}$ acquire VEVs

$$\langle |X| \rangle = \epsilon, \quad \langle |\bar{X}| \rangle = \bar{\epsilon},$$  \hfill (8)

where $M_{Pl} \approx 2.4 \cdot 10^{18}$ GeV is reduced Planck scale, which will be treated as natural cut off for all higher dimensional non-renormalizable operators. In our approach, top quark (and possibly bottom quark and tau lepton, in case of large $\tan \beta$) will get mass at renormalizable level. Yukawa couplings of light families emerge after $U(1)_F$ flavor symmetry breaking. Thus, the hierarchies between Yukawa couplings and CKM mixing angles will be expressed by powers of small parameters $\epsilon, \bar{\epsilon} \ll 1$.

**Yukawa textures**

In the charge assignment we need to fix the values of $\tilde{a}, \tilde{b}$ and $\tilde{c}$. If their ratios remain arbitrary there will be more than one extra $U(1)$ symmetry, and that we have to avoid. Together with fixing $\tilde{a}, \tilde{b}, \tilde{c}$, the values of $p, q$ and $k$ (in (5) and (6)) should be selected in such a way as to have phenomenologically viable quark and lepton Yukawa textures. It turns out, that one selection leading to attractive Yukawa sector, is the following:

$$\{\tilde{a}, \tilde{b}, \tilde{c}\} = \left\{-\frac{1}{2}, \frac{1}{6}, \frac{5}{3}\right\}, \quad p = q = k = 0.$$  \hfill (9)

In this case, in Eq. (6) we pick up $l = 3$, which fixes charges of $H, \bar{H}$ as $(Q_H, Q_{\bar{H}})^{l=3} = (0, 2)$. The charges of matter $(10 + \bar{5})$-plets (and also $SU(5)$ singlets) are also fixed and we will make the following identification: $Q_{10_i} = \{2, -1, 0\}, \quad Q_{\bar{5}_i} = \{0, -3, -2\}$, where $i = 1, 2, 3$ labels the flavor. The model’s field content and corresponding $U(1)_F$ charges are given in Table 1. With this assignment, $10 \cdot 10H$ and $10 \cdot \bar{5}\bar{H}$-type Yukawa couplings are given by

$$10_1 \left(\begin{array}{ccc} \epsilon^4 & \epsilon & \epsilon^2 \\ \epsilon & \epsilon^2 & \bar{\epsilon} \\ \epsilon^2 & \bar{\epsilon} & 1 \end{array}\right) H, \quad 10_1 \left(\begin{array}{ccc} \epsilon^4 & \epsilon & \epsilon^2 \\ \epsilon & \epsilon^2 & \bar{\epsilon} \\ \epsilon^2 & \bar{\epsilon} & 1 \end{array}\right) \bar{H},$$  \hfill (10)

\[\text{Table 1: Field Content and Corresponding}\] $U(1)_F$ $\text{Charges}$

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\[\text{Details of symmetry breaking, with general values of } \langle X \rangle \text{ and } \langle \bar{X} \rangle, \text{ is given in Ref. [8].}\]
where in front of each entry, dimensionless couplings ($\sim 1/5 - 5$) are assumed. As we will see shortly, good fit is achieved for $\bar{\epsilon} \sim 1/10$, $\epsilon \sim (0.05 - 0.2)\bar{\epsilon}^2$. On the other hand, with these values, the matrix elements $(1, 1), (1, 3), (3, 1)$ are so suppressed, that they are irrelevant and we can set them equal to zero. Thus, for all practical purposes, we can investigate the Yukawa matrices:

$$Y_{U, D, E} \propto \left( \begin{array}{ccc} 0 & \epsilon & 0 \\ \epsilon & \bar{\epsilon}^2 & \bar{\epsilon} \\ 0 & \bar{\epsilon} & 1 \end{array} \right),$$

with zero textures.

### Quark masses and mixings

Using the basis $q^T Y_U u^c h_u$ and $q^T Y_D d^c h_d$, without loss of generality we can parameterize up and down Yukawa matrices at GUT scale to have forms:

$$Y_U \simeq \left( \begin{array}{ccc} 0 & c\bar{\epsilon}^2 & 0 \\ c\bar{\epsilon}^2 & a_u\bar{\epsilon}^2 e^{i\xi_u} & \bar{\epsilon} \\ 0 & \bar{\epsilon} & 1 \end{array} \right) \lambda^0_t,$$

$$Y_D \simeq \left( \begin{array}{ccc} e^{i\varphi'} & 0 & 0 \\ 0 & e^{i\varphi} & 0 \\ 0 & 0 & 1 \end{array} \right) \left( \begin{array}{ccc} 0 & \lambda\bar{\epsilon}^2 & 0 \\ \lambda\bar{\epsilon}^2 & a_d\bar{\epsilon}^2 e^{i\xi_d} & b\bar{\epsilon} \\ 0 & b\bar{\epsilon} & 1 \end{array} \right) \lambda^0_b.$$

We have made field phase redefinitions in such a way that, in this basis, CKM matrix remains unity and in $Y_U$ only one phase $\xi_u$ appears. The phases $\varphi$ and $\varphi'$ will not contribute to the quark masses, but will be important for the CKM matrix elements. From Eqs. (12) and (13), in a fairly good approximation we obtain the following relations valid at GUT scale:

$$\frac{\lambda_c}{\lambda_t} \simeq \bar{\epsilon}^2 |a_u\bar{\epsilon}^2 e^{i\xi_u} - 1|, \quad c\lambda \simeq \sqrt{\frac{\lambda_u}{\lambda_c}} |a_u\bar{\epsilon}^2 e^{i\xi_u} - 1|,$$

$$\frac{\lambda_d}{\lambda_b} \simeq \bar{\epsilon}^2 \frac{|a_d\bar{\epsilon}^2 e^{i\xi_d} - bb'|}{1 + (b\bar{\epsilon})^2}, \quad \lambda\bar{\epsilon}^2 \sqrt{k} \simeq \sqrt{\frac{\lambda_d\lambda_s}{\lambda_b^2}} (1 + (b\bar{\epsilon})^2)^{3/4}.$$

These relations help to find a good fit. With proper selection of input parameters $\bar{\epsilon}, \lambda, a_u, b, b', c, c', k, \xi_{u,d}, \varphi, \varphi'$ we can get desirable values for fermion mass hierarchies and CKM mixing angles at GUT scale. Then, using RG we can calculate these ratios at low scales:

$$\frac{\lambda_{u,c}}{\lambda_t} \bigg|_{M_t} = \eta^3_t \eta_b \frac{\lambda_{u,c}}{\lambda_t} \bigg|_{M_G}, \quad \frac{\lambda_{d,s}}{\lambda_b} \bigg|_{M_t} = \eta^3_t \eta_b \frac{\lambda_{d,s}}{\lambda_b} \bigg|_{M_G}, \quad \frac{\lambda_{e,\mu}}{\lambda_e} \bigg|_{M_t} = \eta^3_t \frac{\lambda_{e,\mu}}{\lambda_e} \bigg|_{M_G},$$

### Table 1: $U(1)_F$ charge assignment for the model’s states.

| $Q_{U(1)_F}$ | 10 | 10 | 10 | 5 | 5 | 2 | 5 | H(5) | H(5) | Σ(24) | X | X | 1 | 1 | 1 | 1 | 1 | 4 |
|---------------|----|----|----|---|---|---|---|------|------|-------|---|---|---|---|---|---|---|---|
|               | 2  | −1 | 0  | 0 | −3 | −2 | 0 | 2    | 0    | −1   | 1 | 1 | 1 | −1 | 4 |    |    |  |
where RG factors
\[
\eta_t = \exp\left(\frac{1}{16\pi^2} \int_{m_t}^{M_G} \lambda_t^2 d\ln\mu\right), \quad \eta_b = \exp\left(\frac{1}{16\pi^2} \int_{m_b}^{M_G} \lambda_b^2 d\ln\mu\right), \quad \eta_r = \exp\left(\frac{1}{16\pi^2} \int_{m_r}^{M_G} \lambda_r^2 d\ln\mu\right)
\]
are given in 1-loop approximation.

We will consider two cases with low/moderate and large values of the MSSM parameter \(\tan\beta\).

**Fit for \(\tan\beta = 5 - 15\)**

We take experimental value \(m_t(m_t) = 163.68\) GeV, determining top Yukawa coupling at weak scale, and with \(\tan\beta = 5 - 15\) we find \(\eta_t = 1.097, \eta_b \simeq \eta_r \simeq 1\). For this case, good fit is obtained for the following values of input parameters:

\[
\begin{align*}
\bar{c} &= 0.0847, \quad \lambda_t = 0.476, \quad a_u = 0.6, \quad a_d = 3.7, \\
b &= -0.798, \quad b' = -7.14, \quad c = 0.037, \quad k = 0.864, \\
\xi_u &= 0, \quad \xi_d = -0.065, \quad \varphi = -2.696, \quad \varphi' = -0.97.
\end{align*}
\]

These at GUT scale give
\[
\begin{align*}
|V_{us}| &= 0.2243, \quad |V_{cb}| = 0.0383, \quad |V_{ub}| = 0.00318, \quad \overline{\rho} = 0.118, \quad \overline{\eta} = 0.34,
\end{align*}
\]

where \(\overline{\rho} + \overline{\eta} = \frac{V_{ud}V_{ub}^*}{V_{cb}V_{tb}^*}\).

Performing renormalization (using (16) and [13]), at low scales we get (with input \(m_t(m_t) = 163.68\) GeV, \(m_b(m_b) = 4.24\) GeV):

\[
(m_u, m_d, m_s, m_c) (2\text{ GeV}) = (2.1, 4.64, 91.69, 1082)\text{ MeV}
\]

at \(\mu = M_Z\) : \(|V_{us}| = 0.2243, \quad |V_{cb}| = 0.042, \quad |V_{ub}| = 0.00349, \quad \overline{\rho} = 0.118, \quad \overline{\eta} = 0.34\). (19)

These values of masses and CKM matrix elements are in good agreement with experiments [14], [15].

**Fit for \(\tan\beta = 55\)**

In this case we have \(\eta_t = 1.114, \eta_b = 1.158, \eta_r = 1.105\). Input parameters are selected as:

\[
\begin{align*}
\bar{c} &= 0.0723, \quad \lambda_t = 0.53, \quad a_u = 0.545, \quad a_d = 6.81, \\
b &= -0.777, \quad b' = -11.58, \quad c = 0.0375, \quad k = 0.783, \\
\xi_u &= -0.055, \quad \xi_d = -0.0593, \quad \varphi = -2.73, \quad \varphi' = -0.98.
\end{align*}
\]

(20)
giving at GUT scale
\[
\mu = M_G : \frac{\lambda_u}{\lambda_t} = 4.5 \cdot 10^{-6}, \quad \frac{\lambda_c}{\lambda_t} = 0.002367, \quad \frac{\lambda_d}{\lambda_t} = 3.73 \cdot 10^{-4}, \quad \frac{\lambda_s}{\lambda_b} = 0.00723, \quad |V_{us}| = 0.2259, \quad |V_{cb}| = 0.0317, \quad |V_{ub}| = 0.00272, \quad \overline{\beta} = 0.135, \quad \overline{\gamma} = 0.345.
\]
The renormalization procedure (using (16) and [13]) gives at low scales (with input \(m_e(m_t) = 163.68\) GeV, \(m_b(m_b) = 4.24\) GeV):
\[
(m_u, m_d, m_s, m_e) (2\) GeV) = (2.08, 4.85, 93.86, 1096) MeV
\]
at \(\mu = M_Z : |V_{us}| = 0.2259, \quad |V_{cb}| = 0.0409, \quad |V_{ub}| = 0.00351, \quad \overline{\beta} = 0.135, \quad \overline{\gamma} = 0.345.
(21)
These agree well with experiments.

**Charged lepton sector**

Now let us discuss the charged lepton sector. Relevant Yukawa couplings originate from 10 \(\cdot\) 5 \(\cdot\) \(\tilde{H}\)-type interactions of Eq. (10) (while in practice \(Y_E\) has the structure of Eq. (11)). Without breaking the \(SU(5)\) symmetry in these interactions, one would get the asymptotic relation \(M_D = M_E^T\), which is unacceptable and is a well known problem for minimal SUSY \(SU(5)\) GUT. However, by some specific extension, care can be exercised to solve this problem [16]. Without specifying origin of \(SU(5)\) breaking in this sector, we assume that it happens (i.e. \(SU(5)\) symmetry breaking) in the sector of light families. Thus, in analogy of \(Y_D\) (see Eq. (13)), in a basis \(t^T Y_E c^T h_d\), we parameterize \(Y_E\) to have the following form
\[
Y_E \simeq \begin{pmatrix} 0 & k_e \lambda_e \bar{e}^2 & 0 \\ \lambda_e \bar{e}^2 & k_{22} a_d \bar{e}^2 e^{i \xi_e} & b \bar{e} \\ 0 & b \bar{e} & 1 \end{pmatrix} \lambda_r^0, \quad \text{with } \lambda_r^0 = \lambda_r^0.
(22)
\]
In \(Y_E\) only one complex phase \(\xi_e\) appears. Remaining phases are rotated away by proper phase redefinitions of the \(l\) and \(e^c\) states. With \(\{k_{22}, k_e, \lambda_e, \xi_e\} \neq \{1, k, \lambda, \xi_d\}\) we can avoid the relation \(m_u = m_d\), while keeping \(m_r^0 = m_r^0\) (at the GUT scale). Good fit can be obtained with
\[
\begin{align*}
&\text{for } \tan \beta = 5 - 15, \quad \lambda_e = 2.51, \quad k_e = 0.082, \quad k_{22} = 4.517, \quad \xi_e = -0.065, \\
&\text{for } \tan \beta = 55, \quad \lambda_e = 3.11, \quad k_e = 0.07656, \quad k_{22} = 3.3677, \quad \xi_e = -0.06.
\end{align*}
(23)
\]
and remaining parameters given in Eqs. (18) and (20) respectively. With these we obtain
\[
\begin{align*}
&\text{at } \mu = M_G, \quad \text{for } \tan \beta = 5 - 15, \quad \frac{\lambda_e}{\lambda_r} = 2.787 \cdot 10^{-4}, \quad \frac{\lambda_\mu}{\lambda_r} = 0.05883, \\
&\text{at } \mu = M_G, \quad \text{for } \tan \beta = 55, \quad \frac{\lambda_e}{\lambda_r} = 2.065 \cdot 10^{-4}, \quad \frac{\lambda_\mu}{\lambda_r} = 0.0436.
\end{align*}
(24)
\]
These lead to
\[
\begin{align*}
m_e(m_e) = 0.511 \text{ MeV}, \quad m_\mu(m_\mu) = 105.66 \text{ MeV}, \quad m_\tau(m_\tau) = 1.777 \text{ GeV},
\end{align*}
(25)
in agreement with experiments. The mixing angles originating from the charged lepton sector, for \(\tan \beta = 5 - 15\), are \(\{\theta_{23}, \theta_{12}, \theta_{13}\} \simeq \{31.5^o, 1.02^o, 0.61^o\}\). While for \(\tan \beta = 55\) we got \(\{\theta_{23}, \theta_{12}, \theta_{13}\} \simeq \{40.2^o, 0.93^o, 0.78^o\}\). Note that while \(\theta_{23}\) is large (but not sufficiently), the \(\theta_{12}\) and \(\theta_{13}\) are too small. This means that neutrino sector should be responsible for generating proper values of the lepton mixing angles.
4 Neutrino Sector

To build the realistic neutrino sector, we apply the singlet states $\mathbf{1}_{1,2,3}$ (with $U(1)_F$ charges given in Table 1) as right handed neutrinos. Their Dirac type couplings (to $\bar{5}_i$ states) and the mass matrix respectively are given by:

$$m_D \propto \begin{pmatrix} \frac{\bar{5}_1}{\epsilon} & \frac{\bar{5}_2}{\epsilon^2} & \frac{\bar{5}_3}{\bar{\epsilon}} \end{pmatrix} H , \quad M_R \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & \epsilon^2 \\ 1 & 3 & 0 \end{pmatrix} M_s ,$$

where $M_s$ is some mass scale and in the entries of these matrices the dimensionless couplings are omitted. Integration of heavy $\mathbf{1}_i$ states leads to $3 \times 3$ mass matrix for the light neutrinos:

$$M_\nu = m_DM^{-1}_Rm^T_D \propto \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \bar{\epsilon} \\ \epsilon^2 & \alpha^2 \epsilon^4 & \alpha \beta \epsilon^3 \\ \bar{\epsilon} \epsilon & \alpha \beta \epsilon^3 & \beta^2 \epsilon^2 \end{pmatrix} \bar{m} ,$$

with $\bar{m} \sim \frac{\langle h^{(0)} \rangle^2}{M_\nu}$ and $\alpha, \beta$ are some dimensionless couplings. Note that, $M_\nu$’s $2 - 3$ block’s determinant is zero. It is convenient to work in a basis where charged lepton mass matrix is diagonal, i.e. rotate whole lepton doublets by unitary matrix which diagonalizes the matrix $Y_E Y_E^\dagger$. In this basis, the weak leptonic current is diagonal and the neutrino mass matrix can be denoted by $\bar{M}_\nu$. The convenience of this basis is that the diagonalizing matrix $U$:

$$U^T \bar{M}_\nu U = M_\nu^{Diag}$$

will coincide with the lepton mixing matrix. The latter, in a standard parametrization, has the form:

$$U = P_1 \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\rho} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P_2$$

with $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. The phase matrices $P_{1,2}$ are given by:

$$P_1 = \text{Diag} \left( e^{-i\omega_1} , e^{-i\omega_2} , e^{-i\omega_3} \right) , \quad P_2 = \text{Diag} \left( 1 , e^{-i\rho/2} , e^{-i\rho/2} \right) ,$$

where $\omega_{1,2,3}, \rho_{1,2}$ are some phases.

To get some feeling about the results, obtained from the neutrino mass matrix, let us first ignore $\theta_{12}^e$ and $\theta_{13}^e$ mixings. Since these angles are small, the picture qualitatively will remain unchanged. (Effects of these mixing angles are discussed in detail in an Appendix). With $\theta_{12}^e, \theta_{13}^e \ll 1$, in a good approximation $\bar{M}_\nu$ can be written as:

$$\bar{M}_\nu \simeq \begin{pmatrix} e & c & d \\ c & b^2 & ab \\ d & ab & a^2 \end{pmatrix} .$$
Figure 1: Region (i): Needed values of $K$, realizing normal hierarchical neutrino masses. Region (ii): Values of $K$ within considered scenario with normal ordering of neutrino masses of Eq. (56).

Note that $2 - 3$ block’s determinant of the matrix (31) is also zero: $\bar{M}_\nu^{(2,2)} \bar{M}_\nu^{(3,3)} - (\bar{M}_\nu^{(2,3)})^2 = 0$. This, using (29), leads to the following interesting relation$^4$

$$
\tan^2 \theta_{13} \simeq \frac{m_3}{m_2} \left| s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1} c_{12}^2 \right|,
$$

(32)

Using recent results from the neutrino experiments $[1-3]$, we can easily verify that the relation of Eq. (32) is incompatible with normal hierarchical neutrino masses. This conclusion remains robust taking into account the effects of $1 - 2$ and $1 - 3$ rotations coming from the charged lepton sector. With these, instead of Eq. (32) we have the exact expression

$$
\tan^2 \theta_{13} = \left| \frac{m_3}{m_2} s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1} c_{12}^2 \right| + \frac{K^2}{m_1 m_2} e^{i\kappa},
$$

(33)

where $K$ is real and $\kappa$ is some phase. Derivation of (33) and forms of $K, \kappa$ are given in Appendix (see Eqs. (53)-(55)). One can investigate for what values of $K$, desirable values of $\theta_{13}$ are obtained. In Fig. 1, region (i) corresponds to the values of $K$ as a function of $m_3$, which give $\theta_{13} \simeq 8.9^\circ$. On the other hand, region (ii) shows values of $K$ obtained within considered scenario for $\theta_{12}^e = 0.016$ and $\theta_{13}^e = 0.0136$). We see that points of region (ii) are well below from points of region (i). While Fig. 1 corresponds to the best fit values of the neutrino oscillation parameters [1], the conclusion is same by taking them within $8\sigma$ error bars. This demonstrates that within considered model, the normal hierarchical neutrino mass scenario can not be realized.

On the other hand, inverted hierarchy in neutrino masses is possible within considered $SU(5) \times U(1)_F$ model.$^5$ This is demonstrated in Fig. 2. Green dashed region includes points captured

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$^4$See Appendix for exact expression, detailed derivation and related discussion.

$^5$Worth pointing that it is usually hard to get inverted neutrino mass scenario within GUTs [17]. See however [18] (within $SO(10)$) and [19] (within $E_6$ GUT) with inverted hierarchical neutrino masses.
Figure 2: Inverted hierarchical neutrino mass scenario. Green dashed region shows allowed values of \((m_3, \theta_{13})\). Red bold curves (a) and (b) represent the dependance of \(\theta_{13}\) on \(m_3\) for phases \(\{\rho_1, \delta + \rho_2, \omega_3 - \omega_2, \kappa\} \simeq \{0, 0, 0, 0\}\) and \(\{\pi, \pi, 0, \pi\}\) respectively. Curve (a) gives largest possible values of \(\theta_{13}\), while curve (b) - lowest ones. Two horizontal lines are upper and low experimental bounds of \(\theta_{13}\) within the 1\(\sigma\). Dashed curves would had been obtained (instead of bold ones) with \(K \to 0\) in Eq. (33).

by two border bold curves (obtained via Eq. (33) within our model) and two horizontal lines (corresponding to the experimental values of \(\theta_{13}\) within 1\(\sigma\)). This figure corresponds to the best fit values of \(\theta_{12}, \theta_{23}, \Delta m^2_{\text{sol}}\) and \(\Delta m^2_{\text{atm}}\), while free phases (see Eqs. (54), (55)) are varied within full ranges. Dashed lines correspond to the case with \(K \to 0\). Thus, inclusion of the charged lepton sector somewhat extents the allowed region. All this demonstrates that inverted hierarchical scenario is easily realized. Fig. 2 shows that, the allowed region for \(m_3\) is fixed as:

\[
0.0008 \text{ eV} \lesssim m_3 \lesssim 0.0044 \text{ eV},
\]

and using (58) we get:

\[
m_1 \simeq 0.04852 \text{ eV} \times \left(1 + \left(\frac{m_3}{0.04852 \text{ eV}}\right)^2\right)^{1/2},
\]

\[
m_2 \simeq 0.0493 \text{ eV} \times \left(1 + \left(\frac{m_3}{0.0493 \text{ eV}}\right)^2\right)^{1/2}.
\]

These imply \(\sum m_i \approx 0.1 \text{ eV}\), satisfying the current bound [20] obtained from cosmology. Moreover, for neutrino less double \(\beta\)-decay parameter \(m_{\beta\beta} = |\sum U^2_{ei}m_i|\) we obtain:

\[
m_{\beta\beta} \simeq |s^2_{12}m_1 + s^2_{12}m_2e^{-im_1}|,
\]

leading to:

\[
0.011 \text{ eV} \lesssim m_{\beta\beta} \lesssim 0.05 \text{ eV}.
\]
Future experiments will be able to test viability of this scenario [21].

In summary, we have presented supersymmetric $SU(5)$ GUT supplemented with non-anomalous $U(1)_F$ flavor symmetry. Anomaly cancellation condition restricted the field content (dictated three families of $10 + \bar{5}$ matter), as well as $U(1)_F$ charge assignment. Texture zero Yukawa matrices gave natural understanding of hierarchies between charged fermion mass and mixings. Model automatically involves $SU(5)$ singlet states utilized as right handed neutrinos. Inverted hierarchical neutrino mass scenario is predicted within considered model. Other phenomenological issues, such as doublet-triplet splitting, proton decay etc., left beyond the scope of this paper, will be addressed elsewhere within more general class of models [12] supplemented by anomaly free $U(1)_F$ symmetry [8].

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Appendix: Effects of $\theta^e_{ij}$ on the Neutrino Sector

In this appendix we work out the details of contributions from the charged lepton sector to the neutrino sector. In particular, as was pointed out in Sect. 4, we study impact of $\theta^e_{12}$ and $\theta^e_{13}$ mixing angles. With this study, we prove that within considered SUSY $SU(5) \times U(1)_F$ scenario, only inverted hierarchical neutrino mass scenario is realized.

Charged lepton mass terms $e^T M_E e$ get diagonalized by transformations $e = L_e e', e^c = R_e e^{c'}$, where $L_e, R_e$ are unitary matrices such that

$$L_e^\dagger M_E R_e = M_E^{\text{Diag}}. \quad (38)$$

Let us rotate the neutrino states $\nu$ by the same unitary transformation as $e$-states: $\nu = L_\nu \nu'$. With this, the weak current remains diagonal: $\bar{e}\gamma_\mu \nu = \bar{e}'\gamma_\mu \nu'$. On the other hand, the neutrino mass couplings $\frac{1}{2} \nu^T M_\nu \nu$ become $\frac{1}{2} \nu'^T \bar{M}_\nu \nu'$ with

$$M_\nu = L_e^\dagger M_\nu L_e^\ast. \quad (39)$$

Upon transformation $\nu' = U \nu''$, the neutrino couplings can be diagonalized, i.e.

$$U^\dagger \bar{M}_\nu U = M_\nu^{\text{Diag}}, \quad (40)$$

and finally the weak current will be $\bar{e}'\gamma_\mu U \nu''$. Thus, the matrix $U$ in (40) coincides with the lepton mixing matrix.

From Eqs. (39) and (40) we obtain

$$L_e^\dagger M_\nu L_e^\ast = U^\ast M_\nu^{\text{Diag}} U^\dagger. \quad (41)$$
Unitary matrix $L_\nu$ can be written as

$$L_\nu = P_1^d L_{23} L_{12} P_2^d,$$  \hfill (42)  

where $P_{1,2}$ are some diagonal phase matrices, $L_{12}$, $L_{23}$ and $L_{13}$ correspond to the rotation angles $\theta_{12}^e$, $\theta_{23}^e$ and $\theta_{13}^e$ respectively. Without loss of generality, the unitary matrices $L_{12}$ and $L_{23}$ can be taken to be real orthogonal matrices. Since within our scenario $\theta_{12}^e$ and $\theta_{13}^e$ are small, we can write

$$L_{13} L_{12} \approx P_1^d (1 + \Gamma) P_1^\nu,$$  \hfill (43)  

where $\Gamma$ is real:

$$\Gamma = \begin{pmatrix} 0 & s_{12}^e & s_{13}^e \\ -s_{12}^e & 0 & 0 \\ -s_{13}^e & 0 & 0 \end{pmatrix}, \quad \text{with } s_{12}^e \equiv \sin \theta_{12}^e, \quad s_{13}^e \equiv \sin \theta_{13}^e.  \hfill (44)$$

Without restricting any generality, we can take $P_2^d = P_1^{\nu*}$ and using (42), (43) in (41), we obtain

$$(1 + \Gamma^T) \hat{M}_\nu^* (1 + \Gamma) = U M_\nu^{\text{Diag}} U^T,$$  \hfill (45)  

with

$$\hat{M}_\nu^* = P_1^* L_{23} P_1^* M_\nu P_{23}^* P_L^*.  \hfill (46)$$

Because of smallness of $s_{12}^e$ and $s_{13}^e$, further we use approximation and keep first powers of these angles (and thus first powers of the matrix $\Gamma$). With this, from (45) we get

$$\hat{M}_\nu^* + \Gamma^T \hat{M}_\nu^* + \hat{M}_\nu^* \Gamma = T, \quad \text{with} \quad T \equiv U M_\nu^{\text{Diag}} U^T.  \hfill (47)$$

Note, that since 2–3 block’s determinant of $M_\nu$ is zero, similar applies to the 2–3 block of the matrix $\hat{M}_\nu$ (see Eq. (46)). Thus, $\hat{M}_\nu^*$ can be parameterized as

$$\hat{M}_\nu^* = \begin{pmatrix} \hat{\mathcal{e}} & \hat{\mathcal{c}} & \hat{\mathcal{d}} \\ \hat{\mathcal{c}} & \hat{\mathcal{b}}^2 & \hat{\mathcal{a}} \hat{\mathcal{b}} \\ \hat{\mathcal{d}} & \hat{\mathcal{a}} \hat{\mathcal{b}} & \hat{\mathcal{a}}^2 \end{pmatrix}.  \hfill (48)$$

With this, using matrix relation in Eq. (47), we derive

$$\hat{\mathcal{e}} - 2(\hat{\mathcal{c}} s_{12}^e + \hat{\mathcal{d}} s_{13}^e) = T_{11}, \quad \hat{\mathcal{c}} + \hat{\mathcal{e}} s_{12}^e - \hat{\mathcal{b}}(\hat{\mathcal{c}} s_{12}^e + \hat{\mathcal{d}} s_{13}^e) = T_{12} = T_{21},$$

$$\hat{\mathcal{b}}^2 + 2\hat{\mathcal{c}} s_{12}^e = T_{22}, \quad \hat{\mathcal{d}} + \hat{\mathcal{e}} s_{12}^e - \hat{\mathcal{a}}(\hat{\mathcal{b}} s_{12}^e + \hat{\mathcal{d}} s_{13}^e) = T_{13} = T_{31},$$

$$\hat{\mathcal{d}}^2 + 2\hat{\mathcal{d}} s_{13}^e = T_{33}, \quad \hat{\mathcal{a}} \hat{\mathcal{b}} + \hat{\mathcal{c}} s_{13}^e + \hat{\mathcal{d}} s_{12}^e = T_{23} = T_{32}.  \hfill (49)$$

By iteration (keeping $O(s_{12}^e)$ and $O(s_{13}^e)$) we obtain from (49):

$$\hat{\mathcal{d}}^2 = T_{33} - 2s_{13}^e T_{13}, \quad \hat{\mathcal{b}}^2 = T_{22} - 2s_{12}^e T_{12},$$

$$\hat{\mathcal{a}} \hat{\mathcal{b}} = T_{23} - s_{12}^e T_{13} - s_{13}^e T_{12}.  \hfill (50)$$

These three expressions, by eliminating $\hat{\mathcal{a}}$ and $\hat{\mathcal{b}}$, give the following relation:

$$T_{23}^2 - T_{22} T_{33} = 2 s_{12}^e (T_{23} T_{13} - T_{12} T_{33}) + 2 s_{13}^e (T_{23} T_{12} - T_{22} T_{13}).  \hfill (51)$$
Substituting $T_{ij}$ elements (see Eq. (47)) in (51) we obtain

$$m_1m_2(U_{21}U_{32} - U_{22}U_{31})^2 + m_1m_3(U_{21}U_{33} - U_{23}U_{31})^2 + m_2m_3(U_{22}U_{33} - U_{23}U_{32})^2 = -2s_{12}^2K_1 - 2s_{13}^2K_2$$

(52)

with

$$K_1 = \frac{1}{2} \left( m_1m_2 \sin 2\theta_{13} s_{23} e^{i(\delta + \rho_2)} + (m_1 e^{i\rho_1} - m_2) m_3 \sin 2\theta_{12} c_{23} \right) e^{-i(\rho_1 + \rho_2 + \omega_1 + \omega_2 + 2\omega_3)},$$

$$K_2 = \frac{1}{2} \left( m_1m_2 \sin 2\theta_{13} c_{23} e^{i(\delta + \rho_2)} - (m_1 e^{i\rho_1} - m_2) m_3 \sin 2\theta_{12} s_{23} \right) e^{-i(\rho_1 + \rho_2 + \omega_1 + 2\omega_2 + \omega_3)}.$$  (53)

Using the form of $U$ of Eq. (29) in left hand side of (52), after some simplifications we obtain

$$-\tan^2 \theta_{13} e^{i(2\delta + \rho_2)} = \frac{m_3}{m_2} (s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1} c_{12}^2) + \frac{2}{m_1m_2c_{13}} (s_{12}^2 K_1 + s_{13}^2 K_2) e^{i(\rho_1 + \rho_2 + 2\omega_2 + 2\omega_3)}. \quad (54)$$

Introducing notations

$$K^2 = \frac{2}{c_{13}} |s_{12}^e K_1 + s_{13}^e K_2|,$$

$$\kappa = \rho_1 + \rho_2 + 2\omega_2 + 2\omega_2 + \text{Arg} (s_{12}^e K_1 + s_{13}^e K_2) - \text{Arg} (m_1 s_{12}^e e^{i\rho_1} + m_2 c_{12}^2),$$

from (54) we get Eq. (33) - the expression for $\tan^2 \theta_{13}$.

Having (54), we can now examine possibilities of realizing normal and inverted hierarchical neutrino mass scenarios within our model.

**Excluding normal hierarchical neutrino mass scenario**

Let us first see if normal hierarchical neutrino masses are possible. In this case, $m_3 > m_2 > m_1$ and observed mass square differences are $\Delta m_{\text{sol}}^2 = m_2^2 - m_1^2$ and $\Delta m_{\text{atm}}^2 = m_3^2 - m_2^2$. Thus, two masses, say $m_1$ and $m_2$, can be expressed as

$$m_1 = \sqrt{m_3^2 - \Delta m_{\text{sol}}^2 - \Delta m_{\text{atm}}^2}, \quad m_2 = \sqrt{m_3^2 - \Delta m_{\text{atm}}^2}.$$  (56)

While $\Delta m_{\text{sol}}^2 = m_2^2 - m_1^2$, $\Delta m_{\text{atm}}^2 = m_3^2 - m_2^2$ are both measured, the $m_3$ is unknown yet and we will treat it as a free parameter. Taking into account (52), we can easily verify that $m_1 = 0$ is excluded. Thus, $m_3 > \sqrt{\Delta m_{\text{sol}}^2 + \Delta m_{\text{atm}}^2} \simeq 0.05$ eV. On the other hand, we can also have an upper bound for $m_3$, set from the cosmological bound on a sum of three neutrino masses $\sum m_i < 1$ eV [20, 21]. This, taking into account (56), gives $m_3 < 0.34$ eV. Therefore, we will vary $m_3$ in a range

$$0.05 \text{ eV} < m_3 < 0.34 \text{ eV}.$$  (57)

With help of (33) and using the best fit values of quantities $\Delta m_{\text{sol}}^2$, $\Delta m_{\text{atm}}^2$ and neutrino mixing angles [1], we can see what values of $K$ are needed. In Fig. 1, dashed region (i) represents such values of $K$ (versus $m_3$). For fixed value of $m_3$, the multiple values of $K$ are obtained because of free phases appearing in (33). On the other hand, with (55) and (53) within our model with $s_{12}^e = 0.016$, $s_{13}^e = 0.0136$ we can calculate $K$ for different $m_3$ and remaining phases. Region (ii) of Fig. 1 corresponds to this. We see that regions (i) and (ii) do not overlap and therefore conclude that normal hierarchical neutrino mass scenario is not realized within considered model. This conclusion remains robust even varying the values of $\Delta m_{\text{sol}}^2$, $\Delta m_{\text{atm}}^2$, $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ within $8\sigma$ error bars.
Compatibility with inverted hierarchical neutrino masses

As turns out, the inverted hierarchical neutrino masses blend well with relation (33). In this case, \( \Delta m^2_{\text{sol}} = m_2^2 - m_1^2 \) and \( \Delta m^2_{\text{atm}} = m_2^2 - m_3^2 \) and thus:

\[
m_1 = \sqrt{m_3^2 + \Delta m^2_{\text{atm}} - \Delta m^2_{\text{sol}}}, \quad m_2 = \sqrt{m_3^2 + \Delta m^2_{\text{atm}}},
\]

(58)

If we set \( K \to 0 \) in (33), we can easily see that for certain values of \( m_3 \) and \( \rho_1 \) all observable can be obtained within experimentally preferred ranges. Inclusion of \( K \) do not change this positive result, but just offers slightly different choices of \( m_3 \) and various phases. For illustration see Fig. 2, with corresponding discussion starting in a paragraph right before Eq. (34).

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