Full-Form Model-Free Adaptive Control for a Family of Multivariable System

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Abstract—This correspondence proposes a kind of model-free adaptive control (MFAC) on the basis of full-form equivalent-dynamic-linearization model (EDLM) for the multivariable nonlinear system. Compared with the current MFAC, i) this control law does not have denominator, which is stemmed from the norm of the inverse matrix and it inevitably misses the coupling relationships among the inputs and outputs (I/O) of systems. ii) the current restrictive assumption of a diagonally dominant matrix is reduced to extend its application. iii) the MFAC based on full-form EDLM is more general than the current MFAC based on partial-form and compact-form EDLM. At last, the convergence of tracking error and the BIBO stability of controlled system have been proved, which is one of the open questions in MFAC.

Index Terms—adaptive control, equivalent-dynamic-linearization, multivariable nonlinear systems.

I. INTRODUCTION

The developments in MFAC for single-input single-output (SISO) nonlinear systems have been witnessed recently. Hou has presented the MFAC controller on the basis of three kind of discrete-time EDLM: full-form EDLM, partial-form EDLM and compact-form EDLM [1]-[8]. The merits of MFAC are that the controller design only relies on the I/O data of systems, the model reduction, modeling error and unmodeled dynamics are avoided [9]. Since pseudo-gradient (PG) vector is the only requirement for the controller design and its components are the coefficients of the EDLM. The time-varying PG is only determined by the input and output data of the system [1]-[10]. In real applications, MFAC has performed in several fields, such as: pneumatic artificial muscle [11], autonomous parking systems [12], power system or microgrids system [13]-[15] and motor control system [16]. Since the discrete simple structure of the MFAC makes it easier be applied by computers.

For multi-input multi-output (MIMO) systems, owing to the couplings among the input and output, most of conclusions of MFAC in SISO systems can’t be extended to more complicated MIMO systems directly. Therefore, fewer results about MFAC for multivariable systems are available in the literature [17]. Furthermore, the stability analysis and the proof of convergence of the tracking error are still the open question for MFAC based on full-form EDLM in the MIMO system [1][9], which is relatively difficult to analyze the stability than the current MFAC based on partial-form EDLM and compact-form EDLM in MIMO system. In addition, the current MFAC methods in MIMO system change the inverse matrix into the reciprocal of matrix norm to help the stability analysis, however, this will lead to the loss of inputs and outputs coupling information. Meanwhile, current MFAC are based on a special kind of diagonally dominant matrix assumption for the pseudo Jacobian matrix (PJM), this note shows that the previous assumption turns out to be slightly more restrictive and can be extended wider.

Some major contributions of this correspondence lie as follows. 1) The full-form equivalent dynamical linearization model in MIMO systems is firstly proved, to the author’s best knowledge. 2) We reduce the assumption about the PJM, and the stability of the FFDL-MFAC in MIMO systems is still guaranteed. 3) Taking the norm of the inverse matrix part of the current MFAC may lose the coupling relationships of inputs and outputs, so the inverse calculations in the controller and estimation algorithm are still kept.

The remainder of this note is listed as following. Section II introduces the Full Form EDLM for a family of MIMO discrete-time nonlinear system. Section III presents the MFAC design and analyzes its stability. Section IV presents an example to illustrate the established results and Section V concludes the note. Finally, Appendix A gives the proof of the full-form EDLM in MIMO systems and Appendix B provides the system stability analysis in detail.

II. EDLM FOR A FAMILY OF MULTIVARIABLE NONLINEAR SYSTEMS

This section presents a kind of full-form EDLM for a family of multivariable system, which is basic tool for MFAC design and analysis in next section. The MIMO nonlinear system is shown as:

\[ y(i+1) = C(y(i), \ldots, y(i-n_y), u(i), \ldots, u(i-n_u)) \] (1)

where \( C(\cdots) = [C_1(\cdots), \ldots, C_m(\cdots)]^T \in \mathbb{R}^m \) is supposed to be the vector-valued function; \( u(i) \) and \( y(i) \) are the input vector and output vector of the system at the time of \( i \), respectively; \( n_y, n_u \in \mathbb{R} \) are the corresponding unknown orders.

We suppose that the multivariable system (1) is on the basis of the Assumptions 1 and 2:

Assumption 1: The partial derivatives of \( C(\cdots) \) for all its elements are continuous.
Assumption 2: \( C(\cdot) \) is the generalized Lipschitz function, then we have
\[
|y(i+1) - y(i)| \leq a \|L(i) - L(i_j)\| \tag{2}
\]
where \( L(i) = \begin{bmatrix} Y(i) \\ U_{in}(i) \end{bmatrix} \) is a matrix that contains control input vector \( U_{in}(i) = [u^T(i), \ldots, u^T(i-l_0+1)]^T \) and system output vector \( Y(i) = [y(i), \ldots, y(i-l_y+1)]^T \) with its time window \([i-l_y+1, i] \) and \([i-l_y+1, i] \), respectively. The positive integers \( l_y \) and \( l_u \) are named pseudo orders of the system with \( 1 \leq l_y < n_y \), \( 1 \leq l_u < n_u \). Please refer to [1], [17] for more details about Assumption 1 and Assumption 2.

Theorem 1: Given system (1) satisfying above two assumptions, if \( \Delta L(i) \neq 0 \), \( 1 \leq l_y < n_y \), \( 1 \leq l_u < n_u \), there must exist a matrix \( \Phi(i) \) named pseudo Jacobian matrix, such that (1) is expressed by the following full-form EDLM:
\[
\Delta y(i+1) = \Phi(i) \Delta L(i) \tag{3}
\]
with \( \|\Phi(i)\| \leq a \) for any \( i \), where \( \Phi(i) = [\Phi_1(i), \ldots, \Phi_{l_k + 1}(i), \ldots, \Phi_{l_i} + 1 (i), \ldots, \Phi_{l_y + 1}(i), \ldots, \Phi_{l_u + 1}(i)] \),
\[
\Phi_1(i) = \begin{bmatrix}
\Phi_{11}(i) & \Phi_{12}(i) & \cdots & \Phi_{1m}(i) \\
\Phi_{21}(i) & \Phi_{22}(i) & \cdots & \Phi_{2m}(i) \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_{m1}(i) & \Phi_{m2}(i) & \cdots & \Phi_{mm}(i)
\end{bmatrix} \in R^{m \times m}
\]
k = 1, \ldots, l_y + l_u.
\[
\Delta L(i) = \begin{bmatrix}
\Delta Y_{o}(i) \\
\Delta U_{o}(i)
\end{bmatrix} \tag{4}
\]
And we suppose that \( \Delta y(i) = 0 \) and \( \Delta u(i) = 0 \) for all \( i \leq 0 \).

Proof: Refer to Appendix A.

Assumption 3: \( \Phi_{o+1}(i) \) is a nonsingular matrix with the sign of all the elements unchanged, and each matrix \( \Phi_k(i) \) is bounded with following sense \( \|\Phi_k(i)\| \leq a \), \( k = 1, \ldots, m \).

Remark 1: For more details about FFDL modeling method, please refer to [1], [17].

III. MFAC DESIGN WITH ITS STABILITY ANALYSIS
This section gives the controller design and stability analysis for the proposed method with some necessary Theorems and Lemmas.

A. Model-Free Adaptive Controller Design
We rewrite (3) into (4).
\[
y(i+1) = y(i) + \Phi(i) \Delta L(i) \tag{4}
\]
The cost function is given as follow:
\[
J = \left[ y^d(i+1) - y(i+1) \right]^T \left[ y^d(i+1) - y(i+1) \right] + \lambda \Delta u^T(i) \Delta u(i) \tag{5}
\]
Where, \( \lambda \) is the weighting constant; \( y^d(i+1) = \left[ y^d(i+1), \ldots, y^d(i+N) \right]^T \) is the set point vector.

We substitute (4) into (5) and minimize the (5) to have:
\[
\Delta u(i) = \left[ \lambda I + \Phi_{o+1}(i) \Phi_{o+1}(i)^T \right]^{-1} \Phi_{o+1}(i) \left( y^d(i+1) - y(i) \right)
\]
\[
= - \sum_{k=1}^{l_o+1} \Phi_k(i) \Delta y(i-k+1) - \sum_{k=1}^{l_o+1} \Phi_k(i) \Delta u(i-k+1) \tag{6}
\]
We choose one index function for unknown PJM \( \Phi(i) \) estimation as:
\[
J(\Phi(i)) = \left[ \Delta y(i) - \Phi(i) \Delta L(i-1) \right]^T \left[ \Delta y(i) - \Phi(i) \Delta L(i-1) \right] + \mu \left[ \Phi(i) - \Phi(i-1) \right]^T \left[ \Phi(i) - \Phi(i-1) \right] \tag{7}
\]
Where, \( \Phi(i) \) is the online estimated result of \( \Phi(i) \).

By the optimizing (7), the estimation algorithm for the PJM is shown as follows:
\[
\Phi(i) = \Phi(i-1) + \left[ \Delta y(i) - \Phi(i-1) \Delta L(i-1) \right] \tag{8}
\]
\[
\cdot \Delta L^T(i-1) \left[ \mu I + \Delta L(i-1)^T \Delta L(i-1) \right]^{-1}
\]
Where, \( \mu, \eta > 0 \) are the adjustable parameters of estimation algorithm. The proof of blindness of PJM is similar to that in [17].

Then we introduce adjustable step factors \( \rho_k < 1 \) \( (k = 1, 2, \ldots, l_y + l_u) \) to have more tuning parameters and to prove the system stability. Then the controller vector is given by
\[
\Delta u(i) = \zeta_{o+1}(i) \rho_{o+1}(y^d(i+1) - y(i)) - \sum_{k=1}^{l_o+1} \rho_k \hat{\Phi}_k(i) \Delta y(i-k+1) - \sum_{k=1}^{l_o+1} \rho_k \hat{\Phi}_k(i) \Delta u(i-k+1) \tag{9}
\]
Where, \( \zeta_{o+1}(i) = [\zeta_{o+1}(i) \zeta_{o+1}(i+1) \ldots + \lambda \Delta y^T(i) \Delta u(i) \lambda \Delta y^T(i) \Delta u(i)]^{-1} \zeta_{o+1}(i+1) \).

B. Stability Analysis
This section gives the proof of stability of MFAC with some Lemma and assumptions.

Lemma 1 ([2][18]): Given \( M \in R^{m \times n} \), it has an induced consistent matrix norm such that
\[
\|M\| \leq s(M) + \varepsilon
\]
for any given \( \varepsilon > 0 \). \( s(\cdot) \) represents the spectral radius of \( \cdot \).

Assumption 4: We quote Assumption 3 and 4 in [9] for saving room in this note.

Theorem 2: If the system (1) satisfies Assumption 1-4 and is controlled by control law (8)-(9) with the reference vector \( y^d(i) = \text{const} \), there exists a \( \lambda_{\text{max}} \), when \( \lambda > \lambda_{\text{max}} \), it ensures that:
\[
i_0 \lim_{i \to \infty} |y(i+1) - y^d(i)| = 0 ; \text{ii)} \ y(i) \ and \ u(i) \ are \ bounded.
\]

Proof: Inspired by [1], [9], [17], we give the proof of Theorem 2 in Appendix B.
IV. SIMULATIONS

Example 1: An example in [1] is selected to make comparisons between the proposed MFAC and the current MFAC, and the multivariable system cited from [1] is shown as (10).

\[
\begin{align*}
y_1(i+1) &= \frac{2.5y_1(i)y_1(i-1) + 0.09u_1(i)u_1(i-1)}{1 + y_1^2(i) + y_1^2(i-1)} \\
&+ 1.2u_1(i) + 1.6u_1(i-2) + 0.9u_1(i)u_1(i-1) \\
&+ 0.5u_1(i) + 0.7\sin(0.5(y_1(i) + y_1(i-1))) \\
&\quad - \cos(0.5(y_1(i) + y_1(i-1))) \\
y_2(i+1) &= \frac{5y_2(i)y_2(i-1)}{1 + y_2^2(i) + y_2^2(i-1) + y_1^2(i-2)} + u_2(i) \\
&\quad + 1.1u_2(i-1) + 1.4u_2(i) + 0.5u_1(i)
\end{align*}
\]  

(10)

The given system is characterized with variable structure, discontinuous and is supposed unknown for controller design. The reference is

\[
y^d_1(i+1) = 5\sin(\pi i / 50) + 2\cos(\pi i / 20)
\]
\[
y^d_2(i+1) = 2\sin(\pi i / 50) + 5\cos(\pi i / 20)
\]

All the parameters and initial setting for both methods are all cited from [1]. The initial values are \(y_1(1) = y_1(3) = y_2(1) = y_2(3) = 0\) , \(y_1(2) = y_2(2) = 1\) , \(u_1(1) = u_1(2) = u_1(3) = 1\) , \(u_2(1) = 0\). The controller parameters are \(L_y = 1\) , \(L_u = 3\) , \(\eta = \rho_1 = \rho_2 = \rho_3 = 0.5\) , \(\mu = 1\) , \(\lambda = 1\) , \(\Phi(1) = \Phi(2) = \begin{bmatrix} 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\). The outputs of system controlled by proposed and current MFAC are given in Fig. 1 and Fig. 2. The outputs of both controller are given in Fig. 3. In this example, we apply the same estimation algorithm in [1], so Fig. 4 only shows the time-varying parameters in \(\hat{\Phi}_{w,1}(i)\) of the proposed MFAC for saving room.

The performance indexes for both are given in Tab I.

Through Fig. 1, Fig. 2 and Tab I, we can research the conclusion that the proposed MFAC controlled system has less tracking error than the current MFAC controlled one. Since the current MFAC takes the norm of the inverse matrix part of the proposed MFAC, this may lose the part of coupling relationships among inputs and outputs. To this end, we keep the inverse calculations in proposed MFAC unchanged.

![Figure 1](image1.png)

**Figure 1 Tracking performance of y1**

![Figure 2](image2.png)

**Figure. 2 Tracking performance of y2**

![Figure 3](image3.png)

**Fig. 3 Control inputs**

![Figure 4](image4.png)

**Fig. 4 Estimated parameters of PJM \(\hat{\Phi}_{w,1}(i)\)**

**TABLE I Performance Indexes for both MFAC**

|     | Proposed MFAC | Current MFAC |
|-----|---------------|--------------|
| \(e_{ITA}\) | 66.3846       | 121.8080     |
| \(y_1\) | 215.0012      | 233.3453     |

V. CONCLUSION

A kind of MFAC on the basis of full-form EDLM for a family of multivariable nonlinear systems is presented. By contraction mapping technique, we prove the tracking error astringency and analyze the BIBO stability of the multivariable system. The simulation is performed to verify the effectiveness of method.

APPENDIX A

Proof of Theorem 1:
Proof:
From (1), we have
\[
\Delta y(i+1) = C(y(i), y(i-n_y), u(i), \ldots, u(i-n_u)) \\
- C(y(i-l_y), y(i-l_y-n_y), u(i-l_y), \ldots, u(i-l_y-n_u)) \\
= C(y(i), y(i-l_y), y(i-l_y-n_y), \ldots, y(i-n_y), u(i), \ldots, u(i-n_u)) \\
- C(y(i-l_y), y(i-l_y-n_y), u(i-l_y), \ldots, u(i-n_u)) \\
- C(y(i-l_y), y(i-l_y-n_y), u(i-l_y), \ldots, u(i-n_u)) \\
\]
\[
+ C(y(i-l_y), y(i-l_y-n_y), u(i-l_y), \ldots, u(i-n_u)) \\
\]
\[\tag{11}\]
Let
\[
\Psi(i) = C(y(i-l_y), y(i-l_y-n_y), u(i-l_y), \ldots, u(i-n_u)), \\
\]
\[\tag{12}\]
Due to the Assumption 1, (11) is transformed as (13) by the mean value theorem,
\[
\Delta y(i+1) = \frac{\partial C}{\partial y(i-k)} \Delta y(i) + \cdots + \frac{\partial C}{\partial y(i-l_y)} \Delta y(i-l_y+1) \\
+ \frac{\partial C}{\partial u(i)} \Delta u(i) + \cdots + \frac{\partial C}{\partial u(i-l_y)} \Delta u(i-l_y) + \Psi(i) \\
\]
\[\tag{13}\]
Where,
\[
\frac{\partial C}{\partial y(i-k)} = \begin{bmatrix}
\partial C_1 \\
\partial y_1(i-k) \\
\vdots \\
\partial C_m \\
\partial y_m(i-k)
\end{bmatrix}, \\
\frac{\partial C}{\partial u(i-j)} = \begin{bmatrix}
\partial C_1 \\
\partial u_1(i-j) \\
\vdots \\
\partial C_m \\
\partial u_m(i-j)
\end{bmatrix}, \\
\]
\[\text{for } k=0, \ldots, l_y-1, \\
\text{and } j=1, \ldots, l_u-1.\]
\[
\frac{\partial C}{\partial y(i-k)} \text{ and } \frac{\partial C}{\partial u(i-j)} \text{ mean the partial derivative of } C_p \text{ at one point in range of}
\]
\[
[y_q(i-k), y_q(i-k-1)], \quad k=0, \ldots, l_y-1 \quad \text{and} \quad [u_e(i-j), u_e(i-j-1)], j=0, \ldots, l_u-1, \text{respectively.}
\]
Then the below equation with variables matrix \( \eta(i)_{\text{nom}(l_y+l_u)} \) at any time \( i \) is considered
\[
\Psi(i) = \eta(i) \Delta L(i) \tag{14}
\]
There must exist no less than one solution \( \eta^*(i) \) for equation (14), when \( \Delta L(i) \neq 0 \).
Let
\[
\Phi(i) = \begin{bmatrix}
\frac{\partial C}{\partial y(i)} \\
\frac{\partial C}{\partial y(i-l_y)} \\
\frac{\partial C}{\partial u(i)} \\
\end{bmatrix}, \\
\]
\[\tag{15}\]
We obtain
\[
\Delta y(i+1) = \Phi(i) \Delta L(i) \tag{16}
\]
\[\|\Phi(i)\| \leq a \text{ is a direct result in Assumption 2.}
\]
APPENDIX B
Proof of Theorem 2:
Proof: This section gives proofs of the stringing error and stability of the controlled system.
We define the vector \( \Delta R(i) \)
\[\text{matrices}
\]
\[
\Psi_1(i) = \begin{bmatrix}
\Phi_{ly+2}(i), \Phi_{ly+3}(i), \ldots, \Phi_{ly+u}(i), 0
\end{bmatrix}_{\text{m column}}, \\
\Psi_2(i) = \begin{bmatrix}
\Phi(i), \Phi_2(i), \ldots, \Phi_{ly}(i)
\end{bmatrix}_{\text{m column}}, \\
\]
Then we have
\[
\Delta y(i) = \left[\Delta u^T(i), \ldots, \Delta u^T(i-l_u+1), \Delta y^T(i), \ldots, \Delta y^T(i-l_y+1)\right]^T \\
\]
\[\hat{\Phi}_{ly+1}(i)[\rho_{ly+1}(y(i+1) - y(i)) - \sum_{k=1}^{ly+1} \rho_{ly} \Phi_{ly}(i) \Delta y(i-k+1)] \\
= \begin{bmatrix}
\Delta u^T(i-l_u+1) \\
\Delta y^T(i-l_y+1) \\
\end{bmatrix}\]
\[\gamma_1 = \left[\Delta u^T(i-1), \ldots, \Delta u^T(i-l_u+1), \Delta y^T(i), \ldots, \Delta y^T(i-l_y+1)\right]^T \\
= PD(i-1) \Delta R(i-1) \tag{17}
\]
\[ A(i)_{m(l+y+a)s(m(l+y+a))} = \begin{bmatrix} -p_{by+2} & \cdots & -p_{by+la} & -p_1 & \cdots & -p_y & 0 \\ I \\ 0 & \ddots & 0 \\ I \\ \vdots & \ddots & \vdots \\ I \\ 0 & 0 \end{bmatrix} \]

\[ p_k = \rho_k \hat{\xi}_{by+1}(i) \hat{\Phi}_k(i), \ k = 1, \ldots, l_y, l_y + 2, \ldots, l_y + l_u. \]

\[ F = \begin{bmatrix} I \\ \vdots \\ I \\ 0 & I \\ \vdots & \ddots & \vdots \\ I \\ 0 & 0 \end{bmatrix} \]

\[ E = \begin{bmatrix} I_{new} \\ \vdots \\ I_{new} \end{bmatrix} \]

\[ D(i-1) = \begin{bmatrix} I \\ \vdots \\ I \end{bmatrix} \]

\[ \Phi_{by+1} \cdots \Phi_{by+ln} \Phi_1 \cdots \Phi_{l_y-1} \Phi_{l_y} \]

\[ \sum_{k=1}^{l_y+ln} \| \hat{\Phi}_k(i) \|_\infty + \sum_{k=1}^{l_y} \| \rho_k \hat{\xi}_{by+1}(i) \|_\infty \leq (\max_{k=1, \ldots, l_y+ln, \ldots, l_y+ln} \rho_k) \| \phi_{by+1}^T(i) \hat{\Phi}_{by+1}(i) + \lambda I \|_2 \| \phi_{by+1}^T(i) \|_\infty \]

\[ \sum_{k=1}^{l_y+ln} \| \hat{\Phi}_k(i) \|_\infty \leq (\max_{k=1, \ldots, l_y+ln, \ldots, l_y+ln} \rho_k) \sqrt{m} \| \phi_{by+1}^T(i) \hat{\Phi}_{by+1}(i) + \lambda I \|_2 \| \phi_{by+1}^T(i) \|_\infty \]

Since \( \phi_{by+1}^T(i) \hat{\Phi}_{by+1}(i) \) is a symmetric semi-positive matrix, \( \phi_{by+1}^T(i) \hat{\Phi}_{by+1}(i) + \lambda I \) is also a symmetric positive matrix and we can obtain that

\[ \sum_{k=1}^{l_y+ln} \| \rho_k \hat{\xi}_{by+1}(i) \hat{\Phi}_k(i) \|_\infty + \sum_{k=1}^{l_y} \| \rho_k \hat{\xi}_{by+1}(i) \hat{\Phi}_k(i) \|_\infty \leq (\max_{k=1, \ldots, l_y+ln, \ldots, l_y+ln} \rho_k) M_1^{by+ln-1} < 1 \]

where, \( ||\cdot||_\infty \) represents the maximum row sum norm matrix, \( ||\cdot||_2 \) represents the spectral norm of matrix. Given the eigenvalues of \( \hat{\Phi}_{by+1}^T(i) \hat{\Phi}_{by+1}(i) \) are \( s_k \geq 0, \ k = 1, \ldots, N_y \), the corresponding eigenvalues of \( \hat{\Phi}_{by+1}^T(i) \hat{\Phi}_{by+1}(i) + \lambda I \) will be \( \lambda + s_k > 0 \), which implies the eigenvalues of \( \hat{\Phi}_{by+1}^T(i) \hat{\Phi}_{by+1}(i) + \lambda I \) will be \( \frac{1}{\lambda + s_k} > 0 \). Therefore, we obtain

\[ \| \hat{\Phi}_{by+1}(i) \hat{\Phi}_{by+1}(i) + \lambda I \|_2^2 \]

\[ = \sqrt{\| (\hat{\Phi}_{by+1}(i) \hat{\Phi}_{by+1}(i) + \lambda I)^{-1} \| \| (\hat{\Phi}_{by+1}(i) \hat{\Phi}_{by+1}(i) + \lambda I)^{-1} \| } \]

\[ = \sqrt{\| (\hat{\Phi}_{by+1}(i) \hat{\Phi}_{by+1}(i) + \lambda I)^{-1} \|^2 } = \frac{1}{\min \{ \lambda + s_k \}} \]

Combining (19) and (20), we have

\[ \sum_{k=1}^{l_y+ln} \| \hat{\Phi}_k(i) \|_\infty + \sum_{k=1}^{l_y} \| \rho_k \hat{\xi}_{by+1}(i) \|_\infty \leq 2(l_y + l_u - l_a) \]

According to Assumption 3, we have \( \| \hat{\Phi}_{by+1}(i) \|_\infty < a_1 \) and

\[ \sum_{k=1}^{l_y+ln} \| \hat{\Phi}_k(i) \|_\infty \leq 2(l_y + l_u - l_a) a_1. \]

Consequently, there is a positive\( \lambda_{man1} \), if \( \lambda > \lambda_{man1} \), it yields the following inequation:

\[ \sum_{k=1}^{l_y} \| \hat{\Phi}_k(i) \|_\infty + \sum_{k=1}^{l_y+ln} \| \hat{\Phi}_k(i) \|_\infty \leq \frac{1}{\min \{ \lambda + b_j \} \} \]

\[ \sum_{k=1}^{l_y} \| \hat{\Phi}_k(i) \|_\infty \leq \sqrt{m} \]

\[ \leq M_1 < 1 \]

Given \( 0 < \rho_1 < 1, \ldots, 0 < \rho_l < 1, 0 < \rho_{by+1} < 1, \ldots, 0 < \rho_{by+ln} < 1 \), it has \( \max_{l=1, \ldots, l_y+ln} \rho_l < 1 \). The we obtain

\[ \sum_{k=1}^{l_y+ln} \| \rho_k \hat{\xi}_{by+1}(i) \hat{\Phi}_k(i) \|_\infty + \sum_{k=1}^{l_y} \| \rho_k \hat{\xi}_{by+1}(i) \hat{\Phi}_k(i) \|_\infty \leq (\max_{l=1, \ldots, l_y+ln} \rho_l) M_1^{by+ln-1} < 1 \]
According to [1] and [17], we will have the characteristic equation of $A(k)$:
\[
z^{m(i+y-k+1)} + z^{i+y-1} + \rho_0 + \rho_1 z^{i+y-1} + \cdots + \rho_2 z^{i+y-1} + \cdots = 0
\]  
(24)
Based on (24), we have (25).
\[
|z|^{i+y-1} \leq (\max_{i=1,2,\cdots,i+y-1} \rho_i) \left( \sum_{i=1}^{N} \| \hat{\Phi}_{i+1} \|_0 \right) + \sum_{i=1}^{N} \| \hat{\Phi}_i \|_0 < 1
\]
(25)
which means $|z| \leq (\max_{i=1,2,\cdots,i+y-1} \rho_i)^{i+y-1} M_1 < 1$. In light of Lemma 1 and (25), there will be an arbitrarily small positive $\varepsilon$ and a compatible norm $\| \cdot \|_0$ obtaining the following inequation.
\[
\| A(i) \|_0 \leq s(A(i)) + \varepsilon \leq (\max_{i=1,2,\cdots,i+y-1} \rho_i)^{i+y-1} M_1 + \varepsilon < 1
\]
(26)
where $\| \cdot \|_0$ represents the compatible norm of $\cdot$. Let
\[
d_1 = (\max_{i=1,2,\cdots,i+y-1} \rho_i)^{i+y-1} M_1.
\]
By the definition of spectral radius, [9] has achieved the following inequation
\[
\| A(i) \|_0 \leq s(A(i)) + \varepsilon \leq (\max_{i=1,2,\cdots,i+y-1} \rho_i)^{i+y-1} M_1 + \varepsilon < 1
\]
(27)
Similarly to (21), we have
\[
\| \hat{\Phi}_{i+1} \|_0 = \| \hat{\Phi}_{i+1} \|_0 \leq \| \Phi_{i+1} \|_0 \leq \| \Phi_{i+1} \|_0 < 1
\]
(28)
Therefore, it has positive $\lambda_{\min 2}$ and $M_2$, when $\lambda > \lambda_{\min 2}$, we obtain (29) and (30).
\[
0 < \| \hat{\Phi}_{i+1} \|_0 \leq \| \Phi_{i+1} \|_0 \leq M_2 < 1
\]
(29)
\[
d_3 = \rho_0 M_2 \| X \|_0 < 0.5
\]
(30)
where, $X = \begin{bmatrix} I_{b} \end{bmatrix}$, $b$ is a constant according to the equivalence theorem of matrix norm [18].

We combine (27), (29) and the norm of (17) together to obtain
\[
\| A(i) \|_0 + \| \hat{\Phi}_{i+1} \|_0 + \rho_0 M_2 \| e(i) \|_0
\]
\[
\| \hat{\Phi}_{i+1} \|_0 + \rho_0 M_2 \| e(i) \|_0
\]
\[
\vdots
\]
\[
\| \hat{\Phi}_{i+1} \|_0 + \rho_0 M_2 \sum_{k=1}^{i} d_3^{i-k} \| e(k) \|
\]
\[
< \rho_0 M_2 \sum_{k=1}^{i} d_3^{i-k} \| e(k) \|
\]
(31)
We combine (3) and (18) together to have
\[
e(i+1) = y(i+1) - (y(i) - \Phi(i) A(k)FD(i-1) A(i+1) \| y(i-1) + \rho_0 M_2 \| e(i) \|_0
\]
(32)
Since
\[
\| \hat{\Phi}_{i+1} \|_0 + \rho_0 M_2 \| e(i) \|_0
\]
(33)
then we have
\[
\| \hat{\Phi}_{i+1} \|_0 + \rho_0 M_2 \| e(i) \|_0
\]
(34)
Similarly, there exists the adjustable parameter $\rho_0$ and positive $\lambda_{\min 3}$, such that $\lambda > \lambda_{\min 3}$, then we have the following inequation.
\[
d_4 < 0.5 < d_3 = \| \hat{\Phi}_{i+1} \|_0 + \rho_0 M_2 \| e(i) \|_0
\]
(35)
We take the norm of (32) and use the $\| \Delta R(0) \|_0 = 0$ (Theorem 1) to get
\[
\| e(i+1) \|_0 + \| \hat{\Phi}_{i+1} \|_0 + \rho_0 M_2 \| e(i) \|_0
\]
(36)
Then (36) becomes
\[
\| e(i+1) \|_0 < d_3^{-1} \| e(2) \| + d_2 d_3^{-1} \| X \|_0 \| e(0) \|_0
\]
(37)
Let
\[
g(i+1) = d_3^{-1} \| e(2) \| + d_2 d_3^{-1} \| X \|_0 \| e(0) \|_0
\]
(38)
Then inequality (37) may be rewritten as below
\[
\| e(i+1) \|_0 < g(i+1) \| e(2) \| + d_3 d_2 d_3^{-1} \| X \|_0 \| e(0) \|_0
\]
(39)
where, $g(2) = d_3^{-1} \| e(2) \|$. According to (38) and (39), we have
g(i + 2) = d_2 \| e(2) \| + d_{d_2} \sum_{i=1}^{2} d_i^{2-i} \| e(j) \| + d_{d_3} g(i + 1) + d_{d_4} g(i)

= d_2 g(i + 1) + d_{d_2} \sum_{i=1}^{2} d_i^{2-i} \| e(j) \| + d_{d_3} g(i + 1) + d_{d_4} g(i)
(40)

\leq d_2 g(i + 1) + d_{d_2} \sum_{i=1}^{2} d_i^{2-i} \| e(j) \| + d_{d_3} [d_i^{2-i} e(2)]
+ \frac{d_{d_4}}{2} \sum_{i=1}^{2} d_i^{2-i} \| e(j) \| + d_{d_5} g(i)
(41)

Let

h(i) = d_2 d_3 \sum_{j=1}^{i-1} d_j^{i-j-1} \| e(j) \| + d_{d_4} g(i)

Then (41) combines with (35), we have

\begin{align*}
&h(i) < d_2 d_3 \sum_{j=1}^{i-1} d_j^{i-j-1} \| e(j) \| + d_{d_4} d_5 \| e(2) \|
+ \frac{d_{d_4}}{2} \sum_{j=1}^{i-1} d_j^{i-j-1} \| e(j) \|
+ d_{d_4} d_5 g(i)
\end{align*}
(42)

\begin{align*}
&= d_2 d_3 g(i + 1) + d_3 \left( \max_{i, j} d_{d_5} \right)^{i-j-1} M_1 < 1
\end{align*}
(43)

On the basis of 0 < \rho_k < 1, (k = 1, 2, ..., l_1, l_2, ..., l_7), we can get

\begin{align*}
0 < \left( \max_{i, j} \left( \frac{\rho_k}{1-l_1} \right)^{i-j-1} M_1 \right) < 1
\end{align*}
(44)

At last, we have the following inequality by substituting (44) into (43).

\begin{align*}
\lim_{i \to \infty} g(i + 2) < \lim_{i \to \infty} (d_4 + d_j) g(i + 1) < \ldots < \lim_{i \to \infty} d_4 \| g(2) \| = 0
\end{align*}
(45)

Theorem 2 is the direct conclusion of (45) and (39) under the condition that \lambda > \lambda_{\text{min}} = \max \{ \lambda_{\text{min}1}, \lambda_{\text{min}2}, h_{\text{min}} \}.

Since \gamma(k) is a vector which only contains the incremental inputs and incremental outputs of system. Therefore, the BIBO stability of the system can be achieved by proving \gamma(k) is bounded.

Combine (31), (39) and (44) together, then we will have

\begin{align*}
\| p(i) \| \leq \sum_{k=0}^{i} \| \Delta p(k) \| &\leq \sum_{j=1}^{i} \rho_{b_{j+1}} M_j \sum_{k=1}^{j} d_i^{j-k} \| e(k) \|
\leq \frac{\rho_{b_{i+1}} M_i}{1-d_2} \sum_{j=1}^{i} \| e(j) \| \leq \frac{\rho_{b_{i+1}} M_i}{1-d_2} \sum_{j=1}^{i} g(j)
\leq \frac{\rho_{b_{i+1}} M_i}{1-d_2} g(2) \leq \frac{\rho_{b_{i+1}} M_i}{1-d_2} (1-d_2-d_3)
\end{align*}
(46)

The boundedness of \| p(i) \| is the direct result of (46). It also shows that the system controlled by MFAC is guaranteed BIBO stability. Theorem 2 is proved.

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