Quality Enhancement of a Complex Holographic Display Using a Single Spatial Light Modulator and a Circular Grating

Le Thanh Bang¹, Yan-Ling Piao¹, Jong-Jae Kim², and Nam Kim¹*

¹School of Electrical & Computer Engineering, Chungbuk National University, 1 Chungdae-ro, Seowon-gu, Cheongju 28644, South Korea
²Technical Research Institute, Korea Minting & Security Printing Corporation, 80-67 Gwahak-ro, Yuseong-gu, Daejeon 34132, South Korea

(Received October 15, 2015 : revised December 2, 2015 : accepted December 8, 2015)

This paper proposes an optical system for complex holographic display that enhances the quality of the reconstructed three-dimensional image. This work focuses on a new design for an optical system and the evaluation of the complex holographic display, using a single spatial light modulator (SLM) and a circular grating. The optical system is based on a $4f$ system in which the imaginary and real information of the hologram is displayed on concentric rectangular areas of the SLM and circular grating. Thus, this method overcomes the lack of accuracy in the pixel positions between two window holograms in previous studies, and achieves a higher intensity of the real object points of the reconstructed hologram than the original phase-reconstructed hologram. The proposed method provides approximately 30% less NMRS (Normal Root Mean Square) error, compared to previous systems, which is verified by both simulation and optical experiment.

Keywords: Holographic recording, Holography, Holographic interferometry, Displays

OCIS codes: (090.0090) Holography; (090.1995) Digital holography; (090.2880) Holographic interferometry; (120.2040) Displays

I. INTRODUCTION

A digital holographic display is a type of display technology that provides full depth information for real objects. The optical field of an object can be recorded and stored in a digital device such as a CCD camera or optical sensor, with the capability to transfer and display it at any time and place. In a digital hologram, reconstruction devices can only be used for a phase hologram or an amplitude hologram, but not both at the same time. Previous reports have proposed several methods that use a spatial light modulator (SLM) to reconstruct a complex hologram. Such complex reconstructed holography with an SLM may use either of two methods: two SLMs [1-3], or a single SLM [4-6]. The method with two SLMs uses a coupled phase and amplitude for each SLM set up on both sides of a beam splitter, for combination into a complex hologram. This solution provides a full-display complex hologram with high spatial resolution, but coupling the two displays with pixel accuracy is very difficult. Using the second method, with a single SLM, some researchers attempted to overcome the pixel-matching problem by using a phase-modulation hologram [4] or an amplitude-modulation hologram [5], or both imaginary and real parts of a complex hologram with two small windows and a $4f$ system that integrated the phase or amplitude sinusoidal grating to create a complex hologram where $4f$ is a common optical configuration in holographic recording [6]. However, this method only partially avoids the pixel-matching problem and did not yield an entirely complex hologram, with only phase or amplitude information.

In this paper we show how to overcome the constraints of complex holography by using a $4f$ system with an input plane with concentric rectangular data for imaginary and real information, and a circular grating. By mathematically calculating the difference between the two sizes of the concentric rectangular areas in the SLM, we achieve a complex hologram, instead of just adjusting the accuracy of the pixel positions on the SLM, as in previous studies. In our system the optical setup

*Corresponding author: namkim@chungbuk.ac.kr
Color versions of one or more of the figures in this paper are available online.
is simple, and the system provides a higher intensity for each reconstructed object point than do previous systems [2-13]. Section II describes the proposed method with size variation of concentric rectangular areas for better reconstruction quality. Our results are described in section III by simulation of and experiment with the newly designed optical system.

II. PROPOSED METHOD

The proposed method is based on a 4-f system using a circular grating to implement the display system of a complex hologram. The principle of this method is illustrated in Fig. 1. On the input plane depicted in Fig. 1(a), the SLM is divided into two concentric rectangles such that the outer rectangle contains the real part of the complex hologram, and the inner rectangle contains the imaginary part. The reflected light of the imaginary part from the SLM passes through a device to shift its phase by π/2, before it goes into the 4-f system (device $D_1$ in Fig. 1(e)). In the 4-f system in Fig. 1(e), a circular grating and a pinhole are used to filter the high frequencies.

The diffracted light after the circular grating is shown in Fig. 1(b). Prior to formulating the mathematical proof of our proposal, we calculated the Fourier transform of a circular cosine function.

Assume that the circular cosine grating function in Fig. 1(b) is

$$g(x, y) = \begin{cases} 
1 + \frac{m}{2} \cos \left(2\pi d\sqrt{x^2 + y^2}\right) & \text{for amplitude} \\
\exp \left(\frac{i}{2}\pi d\sqrt{x^2 + y^2}\right) & \text{for phase}
\end{cases}$$

or $g(r) = \cos(2\pi dr)$ in the general case, where $d$ is the radial spatial frequency of the grating. From previous research [7, 9, and 10], we have a Fourier transform as follows:

$$\frac{1}{r^\mu} J_\mu(2\pi f r) \leftrightarrow \pi^{\mu-1} f^{-\mu} \Gamma(\mu) \left(f^2 - q^2\right)^{-\mu-1} \text{rect} \left(\frac{q}{2f}\right)$$

where $r = \sqrt{x^2 + y^2}$ ≥ 0 and $q = \sqrt{u^2 + v^2}$. From Spiegel et al. [8], we have

$$\cos(r) = \frac{\pi r}{2} J_1(r)$$

Thus the Fourier transform of the circular cosine function is

$$\cos(2\pi dr) \leftrightarrow -\frac{1}{2\pi} \frac{d}{\left(d^2 - q^2\right)^{\frac{1}{2}}} \text{rect} \left(\frac{q}{2d}\right)$$

The principle of the proposed method is described in Fig. 1(e): a 4-f optical system with an amplitude SLM, lenses $L_1$ and $L_2$, the device $D_1$, and a circular grating.

On the SLM, the input hologram is divided into two parts, as shown in Fig. 1(a), with the outside area displaying the pixel values $H_i$ and the inside area displaying pixel values $H_r$. The pixel values on the SLM are described by these functions:

$$H_{r, \text{small}} = H_r \times \text{rect} \left(\frac{x}{a}\right) \times \text{rect} \left(\frac{y}{b}\right)$$

$$H_{i, \text{small}} = H_i \times \left(1 - \text{rect} \left(\frac{x}{a}\right) \times \text{rect} \left(\frac{y}{b}\right)\right)$$

with $a$ and $b$ being the height and width of $H_r$.

The reflected light from the $H_i$ components of the SLM...
From Eq. (4), we have the function of light after lens $L_1$ as:

$$H_i = H_{\text{small}} + H_{\text{r, small}}e^{i\pi/2}$$  \hspace{1cm} (7)

The function of the light after lens $L_1$ is

$$u_{\text{plane1}}(x_1, y_1) = \frac{1}{i\lambda f_1} F(H_i)$$

$$= ab \times \sin c(a f_x) \times \sin c(b f_y) 
\times \left[ i\delta(t) \frac{1}{i\lambda f_1} H_{\text{r, Fourier}}(f_x, f_y) + \delta(t) \frac{1}{i\lambda f_1} H_{\text{r, Fourier}}(f_x, f_y) \right]$$ \hspace{1cm} (8)

From this, the light after the circular grating has the following function:

$$\text{Grating Function} = u_{\text{plane1}}(x_1, y_1) \times \cos(2\pi f_x r)$$ \hspace{1cm} (9)

where $\sqrt{x_1^2 + y_1^2} \geq 0$.

The function for the light after lens $L_2$ is a Fourier transform of Eq. (9), so we have

$$u_{\text{plane2}}(x_2, y_2) = \frac{1}{2T} \text{rect} \left( \frac{x_2}{a} \right) \text{rect} \left( \frac{y_2}{b} \right) \left[ iH_i \left( \frac{x_2}{T} \right) \right]$$

$$+ \frac{f_2}{f_1} \times \left[ -f_2 \times \frac{1}{2\pi} \times \text{rect} \left( \frac{q}{2f_2} \right) \right]$$ \hspace{1cm} (10)

From Eq. (4), we have the function of light after lens $L_2$ as:

$$u_{\text{plane2}}(x_2, y_2) = \frac{1}{2T} \text{rect} \left( \frac{x_2}{a} \right) \text{rect} \left( \frac{y_2}{b} \right) \left[ iH_i \left( \frac{x_2}{T} \right) \right]$$

$$+ H \left( \frac{x_2}{T} \right) \left[ -f_2 \times \frac{1}{2\pi} \times \text{rect} \left( \frac{q}{2f_2} \right) \right]$$ \hspace{1cm} (11)

where $T = \frac{f_2}{f_1}$ and $q = \sqrt{x_2^2 + y_2^2}$.

This can be rewritten as follows:

$$u_{\text{plane2}}(x_2, y_2) = \frac{f_1}{4\pi} \times \frac{1}{(f_2^2 - x_2^2 - y_2^2)^2} \times \text{rect} \left( \frac{x_2}{a} \right) \times \text{rect} \left( \frac{y_2}{b} \right)$$

$$\times \left[ iH_i \left( \frac{x_2}{T} \right) + H \left( \frac{x_2}{T} \right) \right]$$ \hspace{1cm} (12)

Then

$$u_{\text{plane2}}(x_2, y_2) = G(x_2, y_2) \times \left[ iH_i \left( \frac{x_2}{T} \right) + H \left( \frac{x_2}{T} \right) \right]$$ \hspace{1cm} (13)

From this function, we see that $u_{\text{plane2}}(x_2, y_2)$ has two components: $G(x_2, y_2)$ and the complex hologram $\left[ iH_i \left( \frac{x_2}{T} \right) + H \left( \frac{x_2}{T} \right) \right]$.

Therefore, if the output of this system is a complex hologram, then the parameter $G(x_2, y_2)$ must be a nonzero constant. From Eq. (13), $G(x_2, y_2)$ has three components:

$$G_i(x_2, y_2) = \frac{1}{(f_2^2 - x_2^2 - y_2^2)^2}$$ \hspace{1cm} (14)

$$G_j(x_2, y_2) = \text{rect} \left( \frac{x_2}{a} \right) \text{rect} \left( \frac{y_2}{b} \right)$$ \hspace{1cm} (15)

$$G_k(x_2, y_2) = \text{rect} \left( \frac{\sqrt{x_2^2 + y_2^2}}{2f_2} \right)$$ \hspace{1cm} (16)

Among these three components, the function $G_i(x_2, y_2)$ must satisfy the condition $f_2^2 - q^2 \geq 0 \iff |q| \leq f_2$, which means that only the coordinates of a hologram in a circle with radius $f_2$ satisfy the condition. The function $G_l(x_2, y_2)$ is described in one and two dimensions in Fig. 2.

Because $G(x_2, y_2)$ is a nonzero constant, the parameters $a$ and $f_2$ must satisfy the following conditions:

- First case: $a \geq f_2$ encircles function $G_i(x_2, y_2)$ inside the rectangle of function $G_j(x_2, y_2)$. The function $u(x_2, y_2)$ becomes

$$u_{\text{plane2}}(x_2, y_2) = AX \left[ iH_i \left( \frac{x_2}{T} \right) + H \left( \frac{x_2}{T} \right) \right]$$ \hspace{1cm} (17)

where $A = G_i(x_2, y_2) \times G_j(x_2, y_2) = (x_2, y_2)$ in the circle with radius $f_2$. 

Quality Enhancement of a Complex Holographic Display Using a Single · · · - Le Thanh Bang et al.

3.1. Simulation

In this section we give simulation results from the Matlab program. The simulation parameters are described as follows: The object was created in Matlab as a spherical object or polyhedron, as shown in Fig. 4, with 104 object points, 5×104 object points, 105 object points, and 5×105 object points. The wavelength of the laser was 532 nm. The size of the hologram was 1200×1920 pixels. The focal length of lens $L_1$ was 10 cm. The size of the SLM screen was 16.39×10.56 mm. The spectral range of the circular grating was 300 nm to less than 16 µm.

III. RESULTS

3.1. Simulation

In this section we give simulation results from the Matlab program. The simulation parameters are described as follows: The object was created in Matlab as a spherical object or polyhedron, as shown in Fig. 4, with 104 object points, 5×104 object points, 105 object points, and 5×105 object points. The wavelength of the laser was 532 nm. The size of the hologram was 1200×1920 pixels. The focal length of lens $L_1$ was 10 cm. The size of the SLM screen was 16.39×10.56 mm. The spectral range of the circular grating was 300 nm to less than 16 µm.

3.1.1. First Case : $a \leq f_2^2$

The width of the SLM was 16.39×0.56 mm. Figure 5(a) describes a normal root mean square (NRMS) error for reconstructing the object, depending on the parameters $a$ and $b$, fixing the value of $f_2$ at 5 mm. The experiment revealed that the NRMS error changed very little when $a$ and $b$ changed from 5 to 10 mm, or from 5 to 16 mm. Figure 5(b) plots the changes in the NRMS error when $a = 9$ mm and $b = 14$ mm as $f_2$ changed from 9 to 2 mm. At $f_2 = 5.4$ mm, the NRMS error was at its lowest value; at this position, the NRMS error was decreased by 30% compared to a reconstructed original phase hologram.

3.1.2. Second Case: $a^2 + b^2 \leq f_2^2$

If the ratio of the rectangle depends on the width and length of the SLM to satisfy this condition, the maximum values are $b = 8.8$ mm and $a = 5.4$ mm. Therefore, values of $a$ from 5.4 to 0.44 mm and $b$ from 8.8 to 3.8 mm are chosen, with steps of 200 µm. Figure 6(a) describes the NRMS error when $f_2 = 10$ mm, changing the size of the rectangle from 5.4×8.8 mm to 0.4×3.8 mm. When $a = 4.8$ mm and $b = 8.2$ mm the NRMS error reaches its lowest value, decreasing by about 40% compared to a reconstructed phase hologram. Figure 6(b) plots the NRMS error when $a \times b = 3.2\times6.6$ mm, changing $f_2$ from 8 to 10 mm. This figure shows that the NRMS error changes very little due to the rectangle inside the circle, as expressed by Eq. (18).

![Figure 2](image2.png)

FIG. 2. (a) Schematic plot of a cross section through the spectrum of the function $G_1(x_2, y_2)$. (b) Function $G_1(x_2, y_2)$ in two dimensions.

![Figure 3](image3.png)

FIG. 3. The position of function (a) $G_2(x_2, y_2)$ and (b) $G_3(x_2, y_2)$, with the size of the SLM.

![Figure 4](image4.png)

FIG. 4. Objects for the experiment: (a) spherical object, (b) polyhedral object with viewing angle 0°, and (c) polyhedral object with viewing angle 90°.

![Figure 5](image5.png)

FIG. 5(a) describes a normal root mean square (NRMS) error for reconstructing the object, depending on the parameters $a$ and $b$, fixing the value of $f_2$ at 5 mm. The experiment revealed that the NRMS error changed very little when $a$ and $b$ changed from 5 to 10 mm, or from 5 to 16 mm. Figure 5(b) plots the changes in the NRMS error when $a = 9$ mm and $b = 14$ mm as $f_2$ changed from 9 to 2 mm. At $f_2 = 5.4$ mm, the NRMS error was at its lowest value; at this position, the NRMS error was decreased by 30% compared to a reconstructed original phase hologram.

![Figure 6](image6.png)

FIG. 6(a) describes the NRMS error when $f_2 = 10$ mm, changing the size of the rectangle from 5.4×8.8 mm to 0.4×3.8 mm. When $a = 4.8$ mm and $b = 8.2$ mm the NRMS error reaches its lowest value, decreasing by about 40% compared to a reconstructed phase hologram. Figure 6(b) plots the NRMS error when $a \times b = 3.2\times6.6$ mm, changing $f_2$ from 8 to 10 mm. This figure shows that the NRMS error changes very little due to the rectangle inside the circle, as expressed by Eq. (18).

![Figure 7](image7.png)

The simulation of the reconstructed hologram is shown in Fig. 7, for a spherical object of 104 points.
3.2. Experiment

3.2.1. Using a Quarter-wave Plate for $D_1$

In the experiment, a laser of wavelength 532 nm was used; the focal lengths of lenses $L_1$ and $L_2$ were set to 10 cm and 10 mm respectively; and the size of the rectangle on the SLM was 4.8×8.2 mm$^2$. The SLM had the following characteristics: pixel size 8.1 μm, total size 1200×1920 pixels, and pinhole size 2 mm. The radius of the quarter-wave plate used before $L_1$ was 8.5 mm (the device $D_1$ in Fig. 1(e)). The reconstructed hologram is shown in Fig. 8.

Figure 8 shows the reconstructed holograms of spherical objects with 104 object points (Fig. 8(a)), 5×104 object points (Fig. 8(b)), 105 object points (Fig. 8(c)), and 5×105 object points (Fig. 8(d)). Moreover, these reconstructed holograms shown in Figs. 8 (a), (b), (c), and (d) were obtained by applying respectively the techniques of the first case, second case, previous 4-$f$ method with a sinusoidal grating, and original phase hologram.

Figure 8(d) shows that the reconstructed original phase hologram has better overlap with the object points than does either the proposed method or the previous 4-$f$ method with a sinusoidal grating. On the other hand, in Fig. 8(a) with parameter $f_2 = 6$ mm for the first case, and in Fig. 8(b) with parameter $a \times b = 3.2 \times 6.6$ mm$^2$ for the second
quality enhancement of a complex holographic display using a single ... le thanh bang et al.

fig. 8. reconstructed spherical hologram: (a) proposed method in the first case with \( f_2 = 6 \) mm, (b) proposed method in the second case with \( a \times b = 3.2 \times 6.6 \) mm², (c) previous method using a sinusoidal grating, and (d) original phase hologram.

case, the quality and intensity of the object points of the reconstructed hologram are also better than both the previous method as shown in fig. 8(c), and the original phase hologram as shown in fig. 8(d).

3.2.2. Using Thick Glass for \( D_1 \)

A thick glass device changes the phase of light that passes through it. We used this property to calculate the phase change between light passing through thick glass and light not passing through it. We assume that the thick glass has a thickness \( d \) as in fig. 9.

In this figure we see that the light passing through the thick glass travels a longer distance than the light that does not pass through it. This distance is described as

\[
AB = \frac{n \times d}{\sqrt{n^2 + \sin^2 \theta}} \Leftrightarrow AB - d = \frac{n \times d}{\sqrt{n^2 + \sin^2 \theta}} - d
\]

(19)

For incident light passing through the thick glass and light not passing through to have a phase difference of \( \pi/2 \), the distances traveled must differ by an odd number of half-wavelengths:

\[
AB - d = \frac{n \times d}{\sqrt{n^2 + \sin^2 \theta}} - d = K \times \frac{\lambda}{2}
\]

(20)

\[\sin \theta = \frac{n}{K \times \lambda + \frac{2}{d}} \sqrt{\frac{K \times \lambda}{2} \left( \frac{K \times \lambda}{2} + 2 \times d \right)} \]

(21)

From eq. (21), when \( K \) is given, the angle of incident light
is calculated to yield a phase difference of $\pi/2$ between the light passing through the thick glass and the light that does not pass through it.

In Fig. 10 an experiment was conducted with the wavelength of the laser at 532 nm, where the refractive index of the glass was $n = 1.524$. Thick glass was placed before lens $L_1$ with a size of $4.8 \times 8.2$ mm$^2$, thickness of 2 mm, and angle of inclination 1.9256 degrees with respect to the reference beam. The size of the rectangle on the SLM was $4.8 \times 8.2$ mm$^2$. The relative measurement error due to angle adjustment was $5 \times 10^{-6}$.

Figure 10 shows the reconstructed hologram of a polyhedral object with two different viewing angles of 0° and 90°. The parameters in the first and second cases were similar to those for a spherical object. This shows that the quality of the reconstructed hologram was better than with either the previous method or the original phase hologram.

Figure 11 shows the average of the intensity of ten object-point layers from 121 to 132 of the reconstructed hologram from three methods: the reconstructed original phase hologram, a complex hologram using a sinusoidal grating method, and the proposed method. Figure 11(a) shows the average intensity of the object points from layers 121 to 132 of the reconstructed hologram for a spherical object using quarter wave plate. It can be seen that the average intensity of object points with the phase hologram was mainly in the range from 30 to 40, while with the sinusoidal grating method it was mainly in the range from 50 to 60, and from 80 to 100 with our proposed method in the second case.

Figure 11(b) shows the average intensity of object-point layers from 121 to 132 of the reconstructed hologram of a polyhedral object. In this figure the intensity of the object points in the original phase hologram was mainly in the range from 10 to 20, whereas with the sinusoidal grating method it was in the range from 20 to 40, and in our proposal method using thick glass with $K = 1$, the range was from 50 to 70.

Figure 8, 10, and 11 confirm that the quality of the reconstructed hologram for a spherical object using the proposed method is better than the quality of either the original phase hologram or the hologram obtained by the previous method using the 4-f system with a sinusoidal grating.

IV. CONCLUSION

In this study we proposed a method to display a full-display complex hologram with a single SLM and a circular grating, and demonstrated it experimentally. By calculating the difference between the size of the area of the changing phase of the hologram and the SLM, we were able to generate a complex hologram through a 4-f system with a circular grating. This method does not require an adjustment of the accuracy of pixel positions on the SLM, as in previous methods, and provides approximately 30% less NMRS...
error compared to previous systems. Also, the proposed method features ease of implementation and stability of the optical system. In this experiment, the reconstructed hologram with good quality has been achieved from the two optimized cases for focal length of the lens $L_2$, where 9 to 16 mm in the first case, and from 8 to 4 mm in the second case.

ACKNOWLEDGMENT

This research was supported by the MSIP (Ministry of Science, ICT and Future Planning), Korea, under the ITRC (Information Technology Research Center) support program (IITP-2015-R0992-15-1008) supervised by the IITP (Institute for Information & communications Technology Promotion) and 'The Cross-Ministry Giga KOREA Project' grant from the Ministry of Science, ICT and Future Planning, Korea.

REFERENCES

1. C. Stolz, L. Bigue, and P. Ambs, “Implementation of high-resolution diffractive optical elements on coupled phase and amplitude spatial light modulators,” Appl. Opt. 40, 6415-6424 (2001).
2. R. Tudela, E. Badosa, I. Labastida, and A. Carnicer, “Full complex Fresnel holograms displayed on liquid crystal devices,” J. Opt. A: Pure Appl. Opt. 5, S1-S6 (2003).
3. L. Neto, D. Roberge, and Y. Sheng, “Full-range, continuous, complex modulation by the use of two coupled-mode liquid-crystal televisions,” Appl. Opt. 35, 4567-4576 (1996).
4. N. Arellano, G. Zurita, C. Fabian, and J. Castillo, “Phase shifts in the Fourier spectra of phase gratings and phase grids: an application for one-shot phase-shifting interferometry,” Opt. Express 16, 19330-19341 (2008).
5. E. Ulusoy, L. Otural, and H. Ozaktas, “Full-complex amplitude modulation with binary spatial light modulators,” J. Opt. Soc. Am. A 28, 2310-2321 (2011).
6. J. Liu, W. Hsieh, T. Poon, and P. Tsang, “Complex Fresnel hologram display using a single SLM,” Appl. Opt. 50, H128-H135 (2011).
7. S. Reichelt, R. Hauser, G. Futterer, N. Leister, H. Kato, N. Usukura, and Y. Kanbayashi, “Full-range, complex spatial light modulator for real-time holography,” Opt. Lett. 37, 1955-1957 (2012).
8. M. Spiegel, S. Lipschutz, and J. Liu, Mathematical Handbook of Formulas and Tables (McGraw Hill, NY, USA, 2009), p. 155.
9. J. W. Goodman, Introduction to Fourier Optics (McGraw-Hill, Singapore, 1996), pp. 32-55.
10. M. Stein and G. Weiss, Introduction to Fourier Analysis on Euclidean Spaces (Princeton University Press, NJ, USA, 1971), pp. 133-172.
11. K. Lee, S. Jeung, and N. Kim, “Holographic demultiplexer with low polarization dependence loss using photopolymer diffraction gratings,” J. Opt. Soc. Korea 11, 51-54 (2007).
12. J. Park, M. Kim, B. Ganbat, and N. Kim, “Fresnel and Fourier hologram generation using orthographic projection images,” Opt. Express 17, 6320-6334 (2009).
13. G. Mills and I. Yamaguchi, “Effects of quantization in phase-shifting digital holography,” Appl. Opt. 44, 1216-1225 (2005).
14. B. Javidi and E. Tajahuerce, “Three-dimensional object recognition by use of digital holography,” Opt. Lett. 25, 610-612 (2000).
15. J. Li, Y. Li, Y. Wang, K. Li, R. Li, J. Li, and Y. Pan, “Two-step holographic imaging method based on single-pixel compressive imaging,” J. Opt. Soc. Korea 18, 146-150 (2014).