Tomography of the Quark Gluon Plasma by Heavy Quarks

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Abstract. Using the recently published model [1, 2] for the collisional energy loss of heavy quarks in a Quark Gluon Plasma (QGP), based on perturbative QCD (pQCD), we study the centrality dependence of $R_{AA}$ and $R_{AA}(p_{T\text{min}})$, measured by the Phenix collaboration, and compare our model with other approaches based on pQCD and on Anti de Sitter/Conformal Field Theory (AdS/CFT).

1. Introduction

The analysis of the spectra of light hadrons, observed in ultrarelativistic collisions of Au nuclei at a center of mass energy of $\sqrt{s} = 200$ GeV, has revealed that in these collisions a new kind of strongly interacting matter is produced. One of the evidences is the observation that the spatial deformation of the overlap zone of projectile and target, quantified by the eccentricity $\epsilon$, is converted into an asymmetry in momentum space in azimuthal direction, called elliptic flow $v_2 = <\cos 2(\phi - \phi_{\text{reaction}})>$ [3]. The experimental $v_2$ is quantitatively described by ideal hydrodynamics. This means that the viscosity coefficient is small [5, 6]. Even if in the meantime a detailed analysis of the impact parameter dependence of the elliptic flow of different particles has revealed that the situation is a bit more complicated [4] a remarkable and unexpected degree of local thermalization is obtained in this new kind of matter, the plasma of quarks and gluons (QGP).

This small viscosity has the unwanted consequence that a local equilibrium among the constituents of the QGP, light quarks and gluons, is maintained until the phase transition. Hence those hadrons which contain only light quarks carry only information on plasma properties close to the phase transition. Therefore, most of the observed particles are not very useful to obtain the desired information on the creation and time evolution of the QGP and one has to concentrate on those few probes which do not come to an equilibrium with the expanding QGP. These probes include photons, jets and heavy mesons. The latter are an especially useful probe because a) due to the large mass of the heavy quarks the kinematic properties of heavy mesons are close to that of heavy quarks before hadronization, b) the initial momentum distribution of heavy quarks can be inferred from pp collisions and is therefore known. Consequently, comparing the $p_T$...
spectra of heavy mesons, obtained in heavy ion reactions, with that of pp collisions one has direct access to the momentum change which the heavy quarks suffer while traversing the plasma because the cross section for collisions of the heavy meson after hadronization is presumably small. For this purpose one defines $R_{AA} = d\sigma_{AA}/dp_T^2/(\langle N_c \rangle d\sigma_{pp}/dp_T^2)$, where $\langle N_c \rangle$ is the average number of initial binary collisions. If heavy quarks do not suffer from an energy loss while traversing the plasma $R_{AA}$ should be $\approx 1$ but this is not really true because the transverse momentum of the partons which create the heavy quark pair has been modified by the medium. This will be discussed below.

Because the mean free path of heavy quarks is shortest at the beginning of the expansion, the deviation of $R_{AA}$ from one encodes dominantly the interaction of the heavy quarks with the QGP at the beginning of the expansion. Initially the heavy quarks are isotropically distributed in azimuthal direction. They can get elliptic flow only by interactions with the light quarks and gluons. Because it takes time until the eccentricity is converted into elliptic flow the $v_2$ of the heavy mesons is sensitive to the interaction of the heavy quarks with the plasma at the end of the expansion of the plasma.

2. The Model

Recently we have advanced a model [1] which studies the creation of heavy quarks in a QGP, their interaction with the expanding plasma (described by ideal hydrodynamics) and how this interaction modifies the observed spectra of heavy mesons (or more precisely that of single non photonic electrons, the decay products of heavy mesons). The elementary interaction between the heavy quarks and the partons of the plasma, light quarks, $q$, and gluons, $g$, is described by pQCD where the density, the temperature and the average velocity of the partons is given by the hydrodynamical expansion. The time evolution of the distribution of the heavy quarks can either be calculated by a Boltzmann equation or by a Fokker-Planck equation. The results presented here are based on the solution of the Boltzmann equation and we use the Fokker Planck approach only to calculate drag and diffusion coefficients which can be compared with other approaches. The details of the model can be found in ref. [1, 2]. As compared to former approaches our approach differs in two respects:

- We employ a running nonperturbative coupling constant whose value remains finite at $t \rightarrow 0$ [8].
- We use an infrared regulator in the t-channel which is determined by hard thermal loop calculations, as done by Braaten and Thoma [9] in the case of QED. The details of how to extend their approach to QCD is found in the appendix of ref. [1].

Both these new ingredients enhance the elastic cross section in the $qQ \rightarrow qQ$ as well as in the $gQ \rightarrow gQ$ channel. This can be seen in fig. [4] which shows the total elastic cross section of a c-quark with an energy of 10 GeV which traverses a $T = 400$ MeV plasma, left for the collisions with light quarks, right for the collisions with gluons for different assumptions on the coupling constant and the infrared regulator. We see that the lower
infrared regulator as well as the running coupling constant increase the cross section at low $t$ as compared to the one with the standard choices $\alpha(2\pi T)$ and $\mu = m_D$, where $m_D$ is the Debye mass. Other approaches use a temperature independent coupling constant and/or $\mu = km_D$ where $k$ varies between 0.3 and 1.

\[
\frac{\alpha(2\pi T), \mu^2 = m_D^2(T)}{\text{(standard)}} \quad \frac{\alpha(2\pi T), \mu^2 = 0.15m_D^2(T)}{\text{(model C)}} \quad \frac{\alpha(2\pi T), \mu^2 = 0.2m_D^2(T)}{\text{(model E)}}
\]

\[
\frac{\langle x^2(t) \rangle}{6t} = \frac{T}{\eta_D / s}
\]

The partons are part of a heat bath of a temperature of 400 MeV and the $c$-quark has an energy of 10 GeV.

Energy loss by radiation is not taken into account yet in this model. We introduce therefore a K-factor, i.e. a multiplication factor which is applied to the cross section. With a running coupling constant and an infrared regulator of $\mu = 0.2m_D$, dubbed "model E" in ref. [1] and shown as the thick (red) line in fig. 1, a K-factor of 1.8 describes the central as well as the minimum bias data for $R_{AA}$ and $v_2$ published by the STAR [10] and the Phenix [11] collaboration, see ref. [1].

In order to compare our model with other approaches we calculate the diffusion constant in space, $D_S = \frac{\langle x^2(t) \rangle}{6t}$, which is related to the drag coefficient $\eta_D$ by $D_S = T / (M_Q \eta_D)$ [14]. The drag coefficient can be connected to the ratio of viscosity and entropy density, $\eta/s$, one of the key quantities of the present discussion. The relation is, however, different in the different models and ranges from $\eta/s = D_S T / 6$ [14] to $\eta/s = D_S T / 2$ in the AdS/CFT approach. Fig. 2 displays this quantity for $b$ and $c$-quarks as a function of the plasma temperature. $D_S$ is very similar for $c$- and $b$-quarks in our approach and is for small temperatures close to the quantal limit.

\[\eta/s = \frac{D_S T}{6} \quad \text{model}\ a - \text{running}, \ K = 1.8\]

Figure 1. (Color online) Differential elastic cross section of $cq \rightarrow cq$ (left) and $cg \rightarrow cg$ (right) for different choices of the strong coupling constant and of the infrared regulator. The partons are part of a heat bath of a temperature of 400 MeV and the $c$-quark has an energy of 10 GeV.

Figure 2. (Color online) $D_S$, the spatial diffusion coefficient for $c$- and $b$-quarks, as a function of the temperature.
3. Centrality dependence

In ref. [1] we have compared our results with central and minimum bias data. As discussed in [2], the interaction between the heavy quarks and the plasma is a quite complicated process in which the spatial geometry plays an essential role and consequently the impact parameter (or centrality) dependence of the results is highly non-trivial. Therefore, it is useful to exploit the whole selection of centralities provided by the Phenix collaboration.

If hadrons have scattered before they create a heavy quark pair their transverse momentum distribution is modified. This so called Cronin effect yields a broadening of \( \Delta P^2_T = n_{\text{coll}}(\vec{r}_\perp) \sigma^2 \), where \( \sigma^2 \) is the broadening of the squared transverse momentum in a single NN collision and \( n_{\text{coll}} \) is the number of prior collisions. We parameterize this distribution by a Gaussian function with a variance of \( \Delta P^2_T \). The consequences for \( R_{dAu} \) for different values of \( \sigma^2 \) are shown in fig. 3. For later calculations we use \( \sigma^2 = 0.2 \text{ GeV}^2 \).

The result of our approach for the different centrality classes, as compared to the Phenix data, is shown in fig. 4. We see that for all centralities the general trend is well reproduced. For the most peripheral events the Phenix data show an decrease of \( R_{AA} \) which is not reproduced in our approach and known mechanisms do not account for this behavior.

Another way to present the data is the centrality dependence of the \( p_T^\text{min} \) integrated \( R_{AA} \) defined as

\[
R_{AuAu}(p_T^{\text{min}}) = \frac{\int_{p_T^{\text{min}}}^{\infty} dN_{AuAu}/dp_T}{\langle N_c \rangle \int_{p_T^{\text{min}}}^{\infty} dN_{pp}/dp_T}.
\] (1)

In fig. 5 we present \( R_{AuAu}(p_T^{\text{min}}) \) as a function of the participant number, \( N_{\text{part}} \), and for 2 different lower bounds of the integration, \( p_T^{\text{min}} \), in comparison with the results of the Phenix collaboration [11]. Because all heavy quarks are finally converted into heavy hadrons this presentation allows to study directly the average jet quenching as a function of the centrality of the reaction. Also these results are in good agreement with the data.
Figure 4. (Color online) $R_{AA}$ as a function of $p_T$ for different centrality bins in comparison with PHENIX data [11]. We display $R_{AA}$ for the model E [1] which uses a running coupling constant and an infrared regulator determined by the hard thermal loop approach.

Figure 5. (Color online) Integral $R_{AA}$ (see text) as a function of the centrality.
4. Comparison with other models

The theoretical values of the two experimentally measured quantities, $R_{AA}$ and $v_2$, depend in a sensitive way on the two conceptually different ingredients of the theory, the interaction of the heavy quarks with the plasma and the expansion of the plasma itself. If one wants to compare different theories it is very helpful to separate both ingredients. This is possible by the definition of transport coefficients which can be calculated in every approach to the heavy quark-plasma interaction. These transport coefficients depend on the interaction of the heavy quark with the plasma but are independent of the expansion of the plasma. The most interesting of these coefficients is the drag coefficient, $\eta_D$. It describes the time evolution of the mean momentum $\frac{dp}{dt} = \eta_D p$ of the heavy quark $^{[12, 13]}$. It can be calculated from the microscopic interaction with help of eq. 2 of ref. $^{[1]}$ or from classical Langevin type approaches. In some of the models it is given as an input variable. Fig. 6 displays $\eta_D$ for different theoretical approaches, on the left hand side for c-quarks and on the right hand side for b-quarks. There we have assumed that the heavy quarks interact with a plasma of a temperature of 300 MeV. For all calculations we use the default values of the coupling constant. M&T refers to Moore and Teaney (eq. B31 with $\alpha_s = 0.3$) of $^{[14]}$, VH&R to van Hees and Rapp $^{[15, 16, 17]}$ (with a resonance width of $\Gamma = 400$ MeV), P&P to Peshier and Peigne $^{[18]}$, and AdS/CFT to the drag coefficient calculated in the framework of the anti de Sitter/ Conformal field theory by Gubser $^{[19, 20]}$. C (with $\alpha_s = 0.3$) and E refer to two parameter sets of our model, defined in $^{[1]}$.

![Figure 6](image-url)

**Figure 6.** (Color online) The drag coefficient $\eta_D$ for c-quarks (left) and b-quarks (right) as a function of the quark momentum. The temperature of the scattering partners is 300 MeV.

The largest drag we observe for the AdS/CFT approach. In this theory the drag coefficient is momentum independent. All (p)QCD based drag coefficients decrease with increasing $p_T$ and hence with increasing momentum the plasma becomes more transparent. Nevertheless, the pQCD based drag coefficient vary quite substantially due to different assumptions on the cut-offs and due to different ingredients, especially the presence of qQ resonances in the plasma. One may ask the question whether such difference does not have a consequences on the predictions of experimentally accessible quantities. In order to study this question we compare the results of two calculations: those of our model E with those in which the drag coefficient of model E is replaced by that of van Hees and Rapp. The expansion of the plasma, described by the hydrodynamical approach of Kolb and Heinz $^{[5]}$, is identical in both calculations. Fig.
shows the results. The top panels display $R_{AA}$ as a function of $p_T$ for c- and b-quarks, left (right) without (with) $p_T$ broadening due to the Cronin effect. In our model, which agrees with the data, the deviation of $R_{AA}$ from one is twice as large for large $p_T$ as if we use the drag coefficient of van Hees and Rapp in an otherwise unchanged model. A similar observation can be made for $v_2$, see bottom panel. For the impact parameter which has been used to simulate minimum bias events, the elliptic flow $v_2$ is reduced by a factor of two if we replace in our model the drag coefficient by that of van Hees and Rapp without changing the model for the plasma expansion. In their original publication van Hees and Rapp have described the data quite well. It is therefore interesting to explore whether the different models for the expansion of the plasma are at the origin of the difference. If this were the case it would stress another time the fact that the description of the experimental $R_{AA}$ and $v_2$ spectra is a double challenge: that to describe the Qq and Qg interactions and that to describe the expansion of the plasma. If this were not the case, the heavy quarks would not tell us something about the plasma properties during the expansion.

**Figure 7.** (Color online) $R_{AA}$ without (top,left) and with (top,right) Cronin effect and $v_2$ (bottom) for Au+Au collisions at $\sqrt{s}=200$ AGeV, $b=7$ fm as a function of $p_T$ for c- and b- quarks and for two different approaches: our model E and a calculation in which our drag coefficient has been replaced by that of van Hees and Rapp.

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