Estimation of aircraft magnetic field model coefficient under Geomagnetic Gradient Change

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Abstract. Traditional airplane magnetic field models simplify geomagnetic field values as constants, overlooking changes to the model coefficient brought by gradually variational magnetic field which reduces the model accuracy. This paper puts forward an aircraft magnetic field model under geomagnetic gradient change. The geomagnetic gradient change is adopted the truncated singular value decomposition (TSVD) to solve multicollinearity in the model solving. It is proposed to use the coefficient obtained from the truncated singular value as initial conditions and utilize recursive damped least square (RDLS) to conduct adaptive adjustment to model coefficients aiming at the coefficient variation of induction field and Eddy Current Field caused by the geomagnetic field. The simulation outcome indicates that the method boasts a high magnetic compensation effect.

1. Introduction
The magnetic field of the aircraft and the field are induced by its motion exert an impact on magnetic anomaly detection of the aircraft. T-L model, the classical model of airborne magnetic interference, was first introduced by Tolles, Lawson and others [1,2] in 1943. As there is a certain correlation among variables of the model, the model displays strong multicollinearity which makes it very difficult to accurately estimate the model parameters. Tolles, Lawson did not put forward effective solutions to model solving and real-time compensation of the aircraft magnetic field in their paper. In 1961, Leliak invented a time-dependent method of magnetic interference between parameter estimation and aircraft maneuver based on T-L equation to make magnetic compensation [3]. His design has become a classic in the history of magnetic compensation and has been still a standard method of magnetic compensation till now. However, a excellent model solving has not been put forward. In 1979, Bickel put forward a Small-Signal magnetic compensation algorithm [4] which embodies the idea of a principal component estimation to a certain extent. It converts a parameter estimation of multivariate linear regression model with multicollinearity into an estimation of least square with the optimal solution under variance and has basically solved multicollinearity in the magnetic interference model at the expense of accuracy loss. Although the conventional solutions to multicollinearity, such as ridge estimate, principal components estimation, genetic algorithm [5-8], can evaluate parameters of a linear model, in case of ill-conditioned model linear equations, the deviation is quite big. An aircraft magnetic field compensation method based on neural network is proposed in Paper [9]. The method has slow convergence speed and is sometimes unstable.

At present, although there are various forms of aircraft magnetic interference equations, they still
derive from the 16 Tolles-Lawson coefficient compensation models without changing physical meaning of the essence. The geomagnetic field is assumed to be a constant in models, and models are built for three kinds of magnetic fields (constant, induced and eddy current magnetic field) generated by aircraft maneuver. In the real aircraft flight, the geomagnetic field varies with time and location, which will reduce accuracy of the aircraft magnetic interference model and lead to compensation deviation during magnetic compensation. This paper transforms the T-L equation by integrating the geomagnetic gradient change with coefficients of the induction and eddy current magnetic field and establish airplane magnetic field model under gradually variational condition. Secondly, TSVD [10,11] is adopted to evaluate the T-L equation coefficients. In the end, considering the variation of the induced and eddy current magnetic field coefficients caused by the gradual change of the geomagnetic field, estimated values obtained from the TSVD is taken as the initial coefficient, and recursive damped least square (RDSL) [12,13] is used to conduct adaptive adjustment to coefficients of the induction and eddy current magnetic field.

2. Another section of my paper

Magnetic field detected by probe of a total field magnetometer [1,2]:

\[
 H_{Total} = H_{pd} + H_{id} + H_{eq} + He
\]  

Since in the direction cosine of the aircraft coordinate system, there is:

\[
 \cos^2 X + \cos^2 Y + \cos^2 Z = 1
\]  

The geomagnetic field is

\[
 He(t) = He(t) \begin{pmatrix}
 \cos X \\
 \cos Y \\
 \cos Z
\end{pmatrix}
\]  

The induction field in the T-L model is:

\[
 H_{id} = \begin{pmatrix}
 \cos X & \cos Y & \cos Z
\end{pmatrix}
\begin{pmatrix}
 a_{11}He(t) & a_{12}He(t) & a_{13}He(t) \\
 a_{21}He(t) & a_{22}He(t) & a_{23}He(t) \\
 a_{31}He(t) & a_{32}He(t) & a_{33}He(t)
\end{pmatrix}
\begin{pmatrix}
 \cos X \\
 \cos Y \\
 \cos Z
\end{pmatrix}
\]  

Then:

\[
 H_{id} = \begin{pmatrix}
 \cos X & \cos Y & \cos Z
\end{pmatrix}
\begin{pmatrix}
 (a_{11} + 1)He(t) & a_{12}He(t) & a_{13}He(t) \\
 a_{21}He(t) & (a_{22} + 1)He(t) & a_{23}He(t) \\
 a_{31}He(t) & a_{32}He(t) & (a_{33} + 1)He(t)
\end{pmatrix}
\begin{pmatrix}
 \cos X \\
 \cos Y \\
 \cos Z
\end{pmatrix}
\]  

\[
 H_{Total} = H_{pd} + H_{id} + H_{eq} + He(t)
\]  

\[
 = (T L V)
\begin{pmatrix}
 \cos X \\
 \cos Y \\
 \cos Z
\end{pmatrix}
\]  

\[
 + (\cos X \cos Y \cos Z)
\begin{pmatrix}
 b_1(t) & b_2(t) & b_3(t) \\
 b_4(t) & b_5(t) & b_6(t) \\
 b_7(t) & b_8(t) & b_9(t)
\end{pmatrix}
\begin{pmatrix}
 \cos X \\
 \cos Y \\
 \cos Z
\end{pmatrix}
\]  

\[
 + (\cos X \cos Y \cos Z)
\begin{pmatrix}
 a_1(t) & a_2(t) & a_3(t) \\
 a_4(t) & a_5(t) & a_6(t) \\
 a_7(t) & a_8(t) & a_9(t)
\end{pmatrix}
\begin{pmatrix}
 \cos X \\
 \cos Y \\
 \cos Z
\end{pmatrix}
\]
Among which,
\[ b_j(t)' = b_j He(t), i=1,2,3, j=1,2,3 \]
\[ a_j(t)' = (a_j + 1)He(t), i=1,2,3 \]
\[ a_j(t)' = a_j He(t), i=1,2,3, j=1,2,3, i \neq j \]

It can be seen that in equation (6), the gradually variational value of the geomagnetic field is integrated with the T-L model, and there is no need to discuss its size and change, and because it can be directly solved in equations. Therefore, there is no need to separate the magnetic interference generated by the air platform from the total field intensity signal. However, as \( a_j(t)' \) and \( b_j(t)' \) vary with the geomagnetic field, an algorithm should be sought out to carry out adaptive adjustment to it.

It is noted that \( H_{ed} \) is assumed to be constant in the derivation of T-L model eddy current field \( He(t) \) [1,2]. There is
\[ H_{ed} = \frac{dHe}{dt}N \]  
(7)

among which \( N \) is the coefficient matrix that generates equivalent eddy current, and it is only essentially relevant to the nature of aircraft structure and material.

\[
\frac{dHe}{dt} = (u_i \frac{dHe}{dt} + He \dot{u}_i \dot{u}_i \frac{dHe}{dt} + He \ddot{u}_i + u_i \frac{dHe}{dt} + He \dot{u}_i) \\
= (\frac{dHe}{dt} \dot{u}_i \frac{dHe}{dt} \ddot{u}_i) \\
(8)
\]

In which \([u_1 \ u_2 \ u_3]^T \) is direction vector of the geomagnetic field.

The model deviation resulting from geomagnetic field change \( \Delta \) is:
\[ \Delta = \Delta H_{ed} \frac{He}{He} = (u_i \frac{dHe}{dt} + u_i \frac{dHe}{dt} + u_i \frac{dHe}{dt} + He \dot{u}_i)N \frac{He}{He} \]  
(9)

Since this paper assumes the geomagnetic field is varying gradually, \( \Delta \) can be supposed 0, and at this moment the magnetic field model of the airplane is still the T-L equation. This assumption will definitely lead to a certain coefficient estimation deviation, but the simulation outcome at the end of this paper implies the effect of the magnetic compensation is still perfect.

3. Coefficient estimation of the field model

3.1. TSVD parameter estimation algorithm

Suppose [14]:
\[ Kx = y + \varepsilon \]  
(10)

among which \( K \in \mathbb{R}^{m \times n} \), and \( \varepsilon \) is a random error. The matrix \( K \) can be decomposed as:
\[ K = U \Sigma V^T = \sum_{i=0}^{\delta} u_i \sigma_i v_i^T \]  
(11)

among which \( u_i, v_i \) are column vectors of the matrix, and \( \sigma_i \) is the singular value of the matrix \( K \). Then the least square solution of the equation can be written as:
In order to obtain a time solution, too small singular values should be truncated to create a stable solution space, and meanwhile make the space approximate to the original solution space as far as possible. The formula to evaluate $x$ via TSVD is as follows:

$$x_{\text{red}} = \sum_{i=1}^{k} \frac{u_i^T y}{\sigma_i} v_i (1 \leq k < n)$$

among which, $k$ is a truncated regularization parameter. In order to get a better estimation result, an appropriate parameter $k$ is in need. If $k$ is too big, a stable solution cannot be obtained whilst if too small, the approximate solution space will lose distortion. Here the generalized cross validation rule is applied [15,16]:

$$G(k) = \left(\frac{n}{n-k}\right) \frac{1}{k} \sum_{i=k+1}^{n} \left(\frac{1}{n-k} \sum_{i=k+1}^{n} (u_i^T y)^2 \right), k = 1, 2, ...$$

The $k$ makes $G(k)$ minimum is optimal and $x_{\text{red}}$ can be got by introducing it to equation (13).

### 3.2. RDLS algorithm

Equation (6) can compensate for the magnetic interference caused by the geomagnetic field and the aircraft platform. Since the longer the duration $\Delta t$ is from calibration phase to compensation phase, the larger the gap between the estimated magnetic interference coefficient in the calibration phase and the actual magnetic interference coefficient in the compensation phase, finally the compensation effects deteriorates. Therefore, $\Delta t$ is needed to be reduced as much as possible. Here, suppose $\Delta t = 1$, namely, making use of the coefficient estimated from measured data from 0 to $t-1$ moment to compensate for the magnetic interference generated by the geomagnetic field and the aircraft platform during the $t$ moment. The RDSL method is applied to estimate the magnetic interference coefficient in real time. The optimized criterion function for the algorithm is [17]:

$$\min J = \sum_{i=1}^{N} [y(t) - \phi(t)\hat{\theta}(t)]^2 + \mu \| \theta(t) - \theta(t-1) \|^2$$

$y(t)$ is the output data for the magnetometer, $\theta(t)$ is a column vector of a magnetic interference coefficient to be estimated, $\phi(t)$ is a row vector constituted by direction cosine and its derivatives and $t = 1, 2, \cdots N$. The forgetting factor $\beta$ is to emphasize the significance of "up-to-date" data in order to solve coefficient time-varying. The damping factor $\mu$ is to reduce adverse effect of noise on a real-time solution coefficient in the “up-to-date” data. The recursion formula for model coefficients is:

$$\begin{align*}
\theta(t) &= \theta(t-1) + \beta \mu P(t)[\theta(t-1) - \theta(t-2)] + P(t)\phi(t)[y(t) - \phi^T(t)\theta(t-1)] \\
P(t)^{-1} &= \mu(1-\beta)I + \beta P(t-1)^{-1} + \phi(t)\phi(t)^T
\end{align*}$$

The recursion formula for $P(t)$ is:
among which $r_i$ is the successor vector of $r_i$, $r_i = [1, 0, ..., 0]$, $\mu = (1 - \beta)\mu / \beta$.

Since the constant field coefficient does not differ with variation of the geomagnetic field, only the induction field and eddy field coefficient can be adjusted adaptively, and the initial coefficient can be obtained through TSVD algorithm based on generalized crossing validation.

4. Simulation analysis

The Algorithm simulation is semi-physical. Nonmagnetic revolving table is used to simulate aircraft rocking, pitching and yawing, and it gathers maneuver data and installs ferromagnetic objects to simulate the interference field on the platform. The geomagnetic gradient change in the experiment is shown in figure 1. Figure 2 shows the total field value in the maneuver process and magnetic compensation outcome of T-L equation based on TSVD parameter estimation. The TSVD compensation results based on multi-group experimental data are shown in figure 3, and the figure of merit [3,4] (FOM) of several groups are still relatively big, which implies that TSVD algorithm based on generalized cross validation has overcome ill-condition, however, the compensation effect is still not ideal. It mainly caused by the T-L model time variation.

![Picture](image_url)

**Figure 1.** Gradually variational data of geomagnetic.
**Figure 2.** Magnetic compensation outcome of T-L equation based on TSVD parameter estimation.

**Figure 3.** Multi-group compensation results of gradually variational geomagnetic field model based on RDLS and TSVD.

**Figure 4.** Compensation results of gradually variational geomagnetic field model based on RDLS and TSVD.
The solutions obtained from TSVD is taken as initial conditions and introduced to the RDLS algorithm in order to estimate model parameters of airplane magnetic field under the gradually variational magnetic field and adaptively update induction field and eddy field parameters. Figures 3 and 4 are the RDLS compensation results of the experimental data for this time and multi-groups comparing with TSVD. The method is more stable in compensation outcome with the peak to peak value lower than 0.1nT and performance better than TSVD algorithm. According to the data, the proposed adaptive compensation algorithm of the airplane magnetic field model under the geomagnetic gradient change can better track the variations of the geomagnetic field and adaptively adjust coefficients, and achieve a better compensation effect.

5. Conclusions
During the process of aircraft flight, the geomagnetic field varies with the change of time and location, which will generate deviation when being in use of the T-L model to carry out magnetic compensation. In this paper the T-L model is transformed by integrating the geomagnetic field with the induction and eddy current field coefficient, based on which adaptive magnetic interference compensation method of the airplane magnetic field model parameter estimation is designed under gradually variational geomagnetic field and. Simulation results indicate that the method boasts the better parameter estimation accuracy and a better magnetic compensation effect. However, in deriving the T-L equation, this method does not improve the assumption that the geomagnetic field is constant, which will inevitably affect the magnetic compensation accuracy. This is the direction of further research.

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