Mapping corrosion of metallic slab by thermography

P. Bison¹, M. Ceseri², G. Inglese³

¹CNR - ITC, C.so Stati Uniti 4, 35127, Padova, Italy
²Dipartimento di Matematica, Università di Firenze, Firenze, Italy
³CNR - IAC, 50019 Sesto Fiorentino (FI), Italy

E-mail: paolo.bison@itc.cnr.it

Abstract. Thermography is used to detect corrosion on a aluminum specimen. Two identical aluminum plates are extracted from the same base material. One of them is machined on one side, in such a way to simulate a material loss. Both the sound and damaged plate are heated on the undamaged side by a sine modulated heating source. A thermographic camera records a sequence of images of the temperature surface of both the sound and damaged sample on the heated (undamaged) sides. Several sequences are recorded with different modulation periods. By a suitable data reduction procedure, the thermographic sequence is reduced to a couple of images representing amplitude and phase of the oscillating temperature field. A perturbative method is used to solve iteratively the direct problem in the corroded domain that is confronted with the experimental data until an optimum matching is reached.

1. Introduction

Corrosion effects are precluded from direct inspection in many practical situations as, for example, in material loss inside pipelines or in the heat exchanger of power plants, or in aircraft fuselage in correspondence of screws and bolts. Thermography is often used in corrosion detection together with three main heating methods based respectively on pulsed [1], modulated [2] and moving line configurations [3]. The heating is applied on the accessible surface, while the corrosion is developed on the other side of the material, inaccessible to the observer. Concerning the modeling of the heat diffusion, the 1D approach is widely used. In Ref. 1, the contrast generated by the material loss is modeled as the temperature difference of two infinitely extended slabs of different thicknesses. Top hat shape approximation may be applied when the depth of the corrosion defect is much smaller than its diameter. Such a problem was addressed in Ref. 4. Evidently, such an approach is not able to recover the complicated shape of a real corrosion where thickness is varying point by point. Recently, this more general problem has been addressed [5], and the same approach is used here: a) the direct problem is solved for the undamaged plate where the thermal parameters of the material are assumed to be homogeneous and isotropic; b) the corroded plate is considered and virtually stretched by a suitable change of spatial variables in the zone of damage, in such a way to make it equal to the undamaged one; c) the coordinate transformation is applied at the expense of the local thermal parameters variation; d) the thermal problem in the undamaged domain, but with variable and eventually anisotropic thermal parameters is solved by a perturbative approach; e) the inversion and the recovery of the shape of damage are obtained by minimizing the difference between the experimental data and the direct problem solution with suitable parameters.
2. The mathematical model

The problem is described in 2-D (see Fig. 1 for a scheme of the domain that is a rectangle of height \( h \) and length \( l \)), but it can trivially extended to the 3-D case. The heating function is assumed sinusoidal and the temperature oscillates at the frequency of the stimulus at the steady-periodic conditions. The stationary part of the temperature field is here neglected. It will be filtered out in the data reduction. The body under test is made of aluminum with high thermal conductivity and therefore the conduction is dominant over the exchange with the environment. The boundary conditions are considered therefore adiabatic. The thermal properties (thermal conductivity \( \lambda = 164 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1} \) and volumetric heat capacity \( C_0 = 2460921 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} \)) are homogeneous and isotropic.

Being

\[
T_0(x,y,t) = \Theta_0(x,y,\omega) \cdot \exp(i\omega t)
\]

the temperature of the specimen, it solves the Helmholtz equation:

\[
\Delta \Theta_0(x,y,\omega) = \sigma^2(\omega) \cdot \Theta_0(x,y,\omega)
\]

with

\[
\sigma(\omega) = \sqrt{\frac{i\omega}{\alpha}} = (1 + i)\sqrt{\frac{\omega}{2\alpha}}
\]

where \( \alpha = \lambda/C_0 \) is the thermal diffusivity and \( \omega = 2\pi f \) the angular frequency (\( f \) is the frequency).

2.1. The corroded domain as a perturbation of the original one

The corroded domain is shown in Fig. 1, where the bottom continuous line is now transformed in the dashed one. We introduce now a new system of coordinates in order to transform the damaged region in the original one:

\[
\xi = \Phi_1(x,y) = x
\]

\[
\eta = \Phi_2(x,y) = y + \epsilon \theta(x) \psi(y)
\]

Here \( 0 < \epsilon \theta < h, \theta(x) \) represents the profile of the corroded specimen, while \( \psi \) is a working variable that modulates the stretching of the domain with \( \psi(0) = 0, \psi(h-\epsilon) = 1, \psi(h) = 1, \psi(y) \equiv 0 \) and \( \psi(0) = 0 \).

After the change of variable \( \Phi \), heat equation for \( \Theta(\xi, \eta, \omega) \equiv \Theta(x,y,\omega) \) becomes

\[
\nabla \cdot \lambda \left( J \Phi \right) (J \Phi)^T \nabla \Theta(\xi, \eta, \omega) = \frac{C_0}{|J \Phi|} i\omega \cdot \Theta(\xi, \eta, \omega)
\]

Here \( J \Phi \) is the Jacobian matrix of \( \Phi \) and \( |J \Phi| \) is the Jacobian determinant. \( \epsilon \) appears in the coordinates system transformation as the parameter of the perturbation. After the change of coordinates it affects both thermal parameters and temperature field. When \( \epsilon \) is zero we obtain the unperturbed solution. When \( \epsilon \) is greater than zero it describes the amount of the perturbation. It is possible to expand every quantity as a Taylor series around \( \epsilon = 0 \) as follows:

\[
\kappa = \lambda \cdot (I + \epsilon \cdot k_1 + \ldots)
\]

\[
\kappa = C_0 \cdot (1 - \epsilon \cdot \theta \psi' + \ldots)
\]

\[
\Theta = \tilde{\Theta}_0 + \epsilon \cdot \tilde{\Theta}_1 + \ldots
\]

\[
k_1 = \begin{pmatrix}
-\theta \psi' & \theta \psi \\
\theta \psi & \theta \psi
\end{pmatrix}
\]

and neglecting terms of the order of \( \epsilon^2 \) and greater, the contrast map (difference between the sound specimen and the corroded one) solves the following differential equation:

\[
\Delta \tilde{\Theta}_1 - \sigma^2 \tilde{\Theta}_1 = -\left( \theta \psi \tilde{\Theta}_0 + \nabla \cdot k_1 \nabla \tilde{\Theta}_0 \right)
\]

that is a Helmholtz equation for the approximation of the contrast function \( \tilde{\Theta}_1 \) plus a source term that depends on the unperturbed solution, the corrosion profile and the working variable \( \psi \). It is worth noting that the boundary condition for the contrast \( \tilde{\Theta}_1 \) on the heated side becomes simply adiabatic,
These two experimental quantities permit to build the following temperature complex map:

\[ \Theta_0(y, \omega) = \frac{Q}{\lambda \sigma} \frac{\cosh(y-h)}{\sinh(\sigma h)} \Theta_0(\eta, \omega) \]  

(8)

with \( Q \) the power of the heat source. Once the shape of the corrosion and the \( \psi \) are given the right hand of the equation (7) can be computed. Let us call \( f(x,y) \) the inhomogeneous term of eq. (7). Defining \( G(\xi, \eta, \omega) \) the Green function of the homogeneous part of equation (7), the solution of the inhomogeneous one is [6-8]:

\[ \tilde{\Theta}_1(\xi_0, \eta_0, \omega) = \int \limits_{\text{domain}} G(\xi_0 - \xi, \eta_0 - \eta, \omega) f(\xi, \eta, \omega) d\xi d\eta \]  

(9)

The Green function for the homogeneous equation (7) is:

\[ G(\xi_0 - \xi, \eta_0 - \eta, \omega) = \sum_{m,n=\pm \infty} \frac{1}{2 \pi \lambda^2} \left[ K_0(\alpha R_{1,m,n}) + K_0(\alpha R_{2,m,n}) + K_0(\alpha R_{3,m,n}) + K_0(\alpha R_{4,m,n}) \right] \]

(10)

\[ R_{1,m,n} = \sqrt{\left(2ml + \xi_0 - \xi\right)^2 + \left(2nh + \eta_0 - \eta\right)^2} \]

\[ R_{2,m,n} = \sqrt{\left(2ml - \xi_0 - \xi\right)^2 + \left(2nh + \eta_0 - \eta\right)^2} \]

\[ R_{3,m,n} = \sqrt{\left(2ml + \xi_0 - \xi\right)^2 + \left(2nh - \eta_0 - \eta\right)^2} \]

\[ R_{4,m,n} = \sqrt{\left(2ml - \xi_0 - \xi\right)^2 + \left(2nh - \eta_0 - \eta\right)^2} \]

with \( K_0 \) the modified Bessel function of the second kind with zeroth order.

3. Experimental

A metallic plate is milled on one side in such a way to produce a localized loss of material with constant profile in every cross section. This permits us to consider the heat diffusion problem as two-dimensional. On the other side, two lamps make as uniform as possible the heat flux on the surface of the plate. Their intensity is modulated by a suitable power supply. A computer controls the frequency of the modulation. The infrared camera records temperature maps of the surface of the specimen at regular intervals. See Fig. 2 for a scheme of the experimental layout. Each pixel in the field of view is considered. Its temperature oscillates in time at the frequency of the heating source. Sequences of IR images at different frequencies have been produced.

4. Data reduction

From the temperature data of the IR sequence, the amplitude and phase of the oscillating part is recovered for each pixel in the field of view. It is supposed that the temperature of each pixel follows the equation:

\[ T(t) = A_0 + A_1 \cdot \cos(\omega t) + B_1 \cdot \sin(\omega t) \]  

(11)

Therefore we need to solve the linear system according to the mean square criterion:

\[ \begin{bmatrix} 1 & \cos(\omega t_0) & \sin(\omega t_0) \\ 1 & \cos(\omega t_1) & \sin(\omega t_1) \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} T_0 \\ T_1 \\ \vdots \end{bmatrix} \]  

(12)

\( A_0, A_1 \) and \( B_1 \) are the estimating parameters and \( T_0, T_1, \ldots \) the temperature data. The amplitude of the considered pixel is finally given by \( A(x) = (A_1^2 + B_1^2)^{1/2} \) while the phase is given by \( \phi(x) = \arctan(B_1/A_1) \). These two experimental quantities permit to build the following temperature complex map:
In Fig. 3 we show amplitude and phase of the experimental contrast obtained as the complex subtraction of data from corroded and sound specimens. Through a minimization procedure, the experimental map $\Theta_E$ is iteratively compared with the solution of eq. 7 that depends on $\theta(x)$. The iteration procedure varies the corrosion shape $\theta(x)$ until the difference between the solution of eq. 7 and the experimental temperature $\Theta_E$ is minimum (see Fig. 4):

$$\min_{\theta(x)} \| \Theta_E(x, \omega) - \tilde{\Theta}_i(x, \theta(x), \omega) \|^2$$

The blob-shaped image of corrosion (especially visible in the phase map of Fig. 3) shows that the 2D analysis is not totally appropriate in this case. Indeed, the high thermal conductivity of aluminum generates strong 3D heat fluxes in and around the zone of corrosion. The extension to the 3D case will be the argument of a future work.

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Figure 1. The domain is a rectangle with height $h$ and length $l$. $\theta(x)$ represents the shape of the material loss due to corrosion.
Figure 2. Layout of the experiment.

Figure 3. Amplitude and phase of the experimental contrast at 16 s period.

Figure 4. Reconstruction of the corrosion profile at 16s period, along the dashed line of Fig. 3