Normal charge densities in quantum critical superfluids

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Superfluidity arises from the spontaneous breaking of a U(1) symmetry – the condensate transports mass/charge without friction.

The order parameter can be modeled by a complex scalar with Mexican hat potential, which acquires a vev.

The vev of the condensate is given by the modulus, the phase is a gapless mode (no energy cost, linear dispersion relation) – the Goldstone boson.
The long-wavelength, low-energy dynamics of superfluids are well-described by the Landau-Tisza hydrodynamic model.

Consistent coupling of the Goldstone mode (superfluid phase) to the conserved densities of the system.

\[ \partial_{\mu} T^{\mu\nu} = F^{\mu\nu} , \quad j_{\mu} = 0 , \quad \partial_{\mu} j^{\mu} = 0 , \quad u^{\mu} \partial_{\mu} \varphi = \mu \]

Modified constitutive relations and thermodynamics compared to ordinary hydrodynamics

\[ T^{\mu\nu} = (\epsilon_n + P) u^{\mu} u^{\nu} + P \eta^{\mu\nu} + \frac{\rho_s}{\mu} \partial_{\mu} \varphi \partial^{\nu} \varphi , \quad j^{\mu} = \rho_n u^{\mu} + \frac{\rho_s}{\mu} \partial_{\mu} \varphi . \]

\[ \epsilon_n + P = T_s + \rho_n \mu , \quad \rho = \rho_n + \rho_s , \]

\[ dP = s dT + \rho d\mu - \frac{\rho_s}{2\mu} d(\partial_{\nu} \varphi \partial^{\nu} \varphi + \mu^2) . \]
Retarded Green’s functions can be computed by linearizing around equilibrium [Kadanoff&Martin].

Eg for the electric conductivity

\[ \sigma(\omega) = \frac{i}{\omega} G^{R}_{jx,jx}(\omega, 0) = \frac{i}{\omega} \left[ \frac{\rho_n^2}{\mu \rho_n + sT} + \frac{\rho_s}{\mu} \right]. \]

The second contribution to the \( \omega = 0 \) pole is the macroscopic manifestation of superfluidity.

In a real superconductor, translations are broken, the first term gives rise to a Drude peak. Hence \( \rho_s \neq 0 \) implies a diverging conductivity and vanishing resistivity \( \rho_{dc} = 0 \).

\[ \sigma(\omega) = \frac{\rho_n^2}{(\mu \rho_n + sT)(\Gamma - i\omega)} + \frac{\rho_s}{\mu} \frac{i}{\omega}. \]
Hydrodynamics cannot be used to solve for the equation of state. To compute the temperature dependence of e.g. $\rho_s$, $\rho_n$, a microscopic theory or an EFT is needed.

In BCS superconductors, the normal density is exponentially suppressed as $T \to 0$.

In $^4$He, $\rho_n \sim T^4$.

In the relativistic superfluid EFT of [Son'02], $\rho_n \sim T^{d+1}$ [Delacrétaz, Hofman & Mathys'19].

These theoretical results agree with experiments, and suggest that the system must be entirely superfluid at $T = 0$. 
In 2016, Bozovic et al. published a study of the superfluid density in very overdoped LSCO films.
They reported two surprising features

- The superfluid density is anomalously low.
- It has a linear behaviour with temperature, while standard ‘dirty’ BCS theory predicts $T^2$. 
Then [Mahmood et al'18] measured the ac conductivity of these films and reported a very modest loss of spectral weight below $T_c$. They conclude that this implies that $\rho_n(T=0) \equiv \rho_n^{(0)} \neq 0$, once again at odds with BCS.
So must $\rho_n^{(0)} = 0$ always?

How does this fit with the experiments on overdoped cuprates?

Let’s use holographic methods.

We will construct phases which have $\rho_n^{(0)} \neq 0$. 
Gravity in Anti de Sitter is dual to certain strongly-coupled Quantum Field Theories in one spatial dimension less \[\text{[Maldacena'97]}\].

Intuitive explanation why the entropy of black holes is the area of the horizon \[\text{[Bekenstein,Hawking]}\], not its volume.

The complicated dynamics of strongly-coupled quantum matter can be described non-perturbatively by solving Einstein’s equations in Anti de Sitter.
A superfluid can be realized in the boundary by spontaneously breaking a U(1) symmetry. This was originally done [Gubser’08, Hartnoll, Herzog & Horowitz’08] by coupling a charged, complex scalar to gravity

\[ S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{1}{4} F^2 - |D\eta|^2 - V(|\eta|) \right] . \]

At low temperatures, \( \eta \) condenses close to the horizon, leading to a spacetime with a lump of charged scalar field sitting outside the horizon.
The original solutions constructed by [Hartnoll, Herzog & Horowitz’08] were shown to obey the Landau-Tisza model of superfluid hydrodynamics [Sonner & Withers’10].
By considering a quartic potential, [Gubser & Nellore’09, Horowitz & Roberts’09] showed that two types of IR geometries were allowed:

\[ ds^2_{IR} = -\frac{L_t^2}{r^{2z}} dt^2 + \frac{L_{IR}^2 dr^2 + L_x^2 d\vec{x}^2}{r^2} \]

Whether the AdS$_4$ or Lifshitz groundstate is selected depends on whether the gauge field is irrelevant at $T = 0$ close to the horizon or not, see Davison’s talk on Friday.
In the solutions just described, all the boundary charge at $T = 0$ is sourced by the condensate: $\rho_n^{(0)} = 0$.

This is a consequence of the vanishing horizon electric flux at $T = 0$. 
To go beyond this, consider a more general action [Adams, Crampton, Sonner & Withers’12]

\[ S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{Z(\phi)}{4} F^2 - |D\eta|^2 - \frac{1}{2} (\partial\phi)^2 - V(\phi, |\eta|) \right]. \]

We also want to consider more general groundstates

\[ ds^2_{\text{IR}} = r^{\frac{2}{d-2\theta}} \left[ -\frac{L_t^2}{r^{2z}} dt^2 + \frac{L_R^2 dr^2 + L_x^2 d\vec{x}^2}{r^2} \right] \]

They violate hyperscaling [Charmousis, Goutéraux et al’10, Goutéraux & Kiritsis’11, Huijse, Sachdev & Swingle’11]

\[ s \sim T^\frac{d-\theta}{z} \]
This holographic setup realizes the following scenario
\[ S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{Z(\phi)}{4} F^2 - |D\eta|^2 - \frac{1}{2} (\partial\phi)^2 - V(\phi, |\eta|) \right]. \]

- For small \( g \), the phase has \textbf{vanishing} horizon flux at \( T = 0 \).

\[ ds_{IR}^2 = r^2 d\theta \left[ -L_t^2 dt^2 + L_{IR}^2 dr^2 + L_x^2 d\vec{x}^2 \right] \]

\( z=1, \Theta \neq 0 \)
\[ S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{Z(\phi)}{4} F^2 - |D\eta|^2 - \frac{1}{2} (\partial\phi)^2 - V(\phi, |\eta|) \right]. \]

- For large \( g \), the phase has **non-vanishing** horizon flux at \( T = 0 \).

\[ ds^2_{IR} = r^{\frac{2}{d}} \theta \left[ - \frac{L_t^2}{r^{2z}} dt^2 + \frac{L_{IR}^2 dr^2 + L_x^2 d\vec{x}^2}{r^2} \right] \]
\[ S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{Z(\phi)}{4} F^2 - |D\eta|^2 - \frac{1}{2} (\partial\phi)^2 - V(\phi, |\eta|) \right]. \]

- For even larger $g$, the condensate disappears and the phase has **non-vanishing** horizon flux at $T = 0$. 

\[
s_{dR}^2 = r^2 d\theta \left[ -\frac{L_t^2}{r^{2z}} dt^2 + \frac{L_{IR}^2 dr^2 + L_x^2 d\vec{x}^2}{r^2} \right].
\]
Using the holographic dictionary, we compute $\rho_{n,s}$ numerically

Zero horizon flux: $\rho_n \simeq \frac{1 - c_{IR}^2}{c_{IR}^2} \frac{sT}{\mu} + ... \quad c_{IR} \equiv \frac{L_t}{L_x}$

Nonzero horizon flux: $\rho_n \simeq \frac{\rho_n^{(0)}}{\mu^2} + \# T^{1-\frac{\theta}{z}} + ...$

The temperature dependence is controlled by the scaling properties of the underlying normal groundstate.
Now, break translations weakly à la \cite{Andrade2013}:

\[
\sigma(\omega) = \frac{\rho_n^2}{(\mu \rho_n + sT)(\Gamma - i\omega)} + \frac{\rho_s i}{\mu \omega}.
\]

In the region with \(\rho_n^{(0)} \neq 0\), results qualitatively very similar to \cite{Bozovic2016, Mahmood2018}.

Consequence of the quantum critical properties of the underlying normal groundstate, not of disorder.
The specific heat has also been measured for very overdoped LSCO

\[\text{[Wen et al, PRB’04, Wang et al, PRB’04]}\]

\[C = \gamma_0 T + \ldots\]

The rapid drop is a consequence of the decreasing horizon flux/normal charge carriers as ‘doping’ is decreased in the holographic model.
Zero horizon flux: $\rho_n \simeq \frac{1 - c_{IR}^2}{c_{IR}^2} \frac{sT}{\mu} + \ldots \sim T^{d+1}, \quad c_{IR} \equiv L_t/L_x$

- Recall that for vanishing horizon flux, $z = 1$: emergent Lorentz symmetry.

- Our result exactly matches the relativistic superfluid EFT computation [Delacrétaz, Hofman & Mathys’19], with the lightcone velocity given by $c_{IR}$.

$^4$He: $\rho_n \sim T^4$. Emergent relativistic symmetry with the lightcone velocity given by the phonon velocity.

- Ultimately, these results follow from the presence of a linearly dispersing mode at $T = 0$ (the Goldstone).
- How about Lifshitz phases? There we expect 
  \[ c_{IR} \equiv \frac{L_t}{L_x} r_h^{1-z} \sim T^{1-1/z} \] and \( \omega \sim k^z \) at \( T = 0 \).

- Let’s revisit the original holographic superconductors (no extra neutral scalar \( \phi \))

\[
S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{1}{4} F^2 - |D\eta|^2 - V(|\eta|) \right].
\]
We find that $\rho_n^{(0)} = 0$ only if

$$\lim_{T\to 0} \frac{sT}{c_I^2 c_R} = 0 \quad \Rightarrow \quad z < d + 2 - \theta$$

Setting $d = 2$ and $\theta = 0$, $\rho_n^{(0)} = 0$ only if $z < 4$. This explains some earlier results by [Herzog & Yarom’09]
What are the consequences on the hydrodynamic (superfluid sound) modes?

Superfluid second sound mode:

\[ c_2^2 = \left( \frac{s}{\rho} \right)^2 \frac{\rho_s}{(sT + \mu \rho_n)(\partial [s/\rho]/\partial T)_\mu}. \]

Zero horizon flux: \[ c_2^2 = \frac{z}{(d-\theta)} c_{IR}^2 \sim T^{2-\frac{2}{z}} \]

Nonzero horizon flux: \[ c_2^2 \sim sT \]

Superfluid second sound vanishes at \( T = 0! \)

Fourth sound (no normal velocity)

\[ c_4^2 = \frac{\rho_s}{\mu \left( \frac{\partial \rho}{\partial \mu} \right)_s} \approx \frac{\rho_s}{d\rho}. \]

Crisp diagnostic of \( \rho_n^{(0)} \neq 0 \). Non-vanishing at \( T = 0 \).
In summary

- $\rho_n^{(0)}$ need not vanish, depending on whether there is residual horizon electric flux at $T = 0$ or $z < d + 2 - \theta$.

- $\rho_n^{(0)} \neq 0$ leads to phenomenology qualitatively in agreement with experiments on overdoped cuprates.

- Disorder does not play a role, everything is controlled by the scaling properties of the underlying normal groundstate. Superconductivity is an irrelevant deformation, even though there is a finite condensate at $T = 0$.

- We found

  $$\rho_s \simeq \rho_s^{(0)} + \# T^{1-\frac{\theta}{2}} \ldots$$

  Set $\theta = 0$: linear in $T$ in behaviour, cf [Bozovic et al'16].

- Lifshitz superfluid effective field theory?