Shape optimization of a flexible beam with a local shape feature based on ANCF

Gang HE*, Kang GAO**, Jun JIANG**, Ruifeng LIU** and Qian LI**

*College of Mechanical & Electrical Engineering, Hohai University
200 Jinling, Changzhou, 213022, China
E-mail: hegang@hhu.edu.cn

**College of Mechanical & Electrical Engineering, Hohai University
200 Jinling, Changzhou, 213022, China

Received: 14 September 2018; Revised: 10 June 2019; Accepted: 23 July 2019

Abstract
This paper proposes a shape optimization method of local shape features on a flexible beam modeled with the absolute nodal coordinate formulation (ANCF). A fully parameterized ANCF beam element is used to model the global geometry for flexible body dynamics analysis. The local shape feature of the ANCF element surfaces can be described by a nonuniform rational B-spline (NURBS) without the need for mesh refinement by changing the distribution of the integration points. A mapping between the ANCF element local coordinates and the NURBS localized geometric parameters is used for the consistent implementation of the overlapping methods. The selected shape parameters of the localized surface geometry are taken as the design variables for beam shape optimization, and the weighted summation of the compliance of the beam is taken as the objective to evaluate the influence of the dynamic response. Different weighting schemes are used to compare the effects of weighting. An example of an ANCF beam with a rib is given to validate our method. A two-stage shape optimization method based on SQP (Sequence quadratic program) is run to reduce the compliance of the beam. Because few design variables are used to represent the local and global shapes of the beam geometry, our shape optimization method based on the localized surface geometry has a significantly reduced computational burden and improved optimization efficiency.

Keywords: Shape optimization, Local feature, Absolute nodal coordinate formulation(ANCF), Flexible beam, Two-stage optimization, Equivalent static load

1. Introduction
Structural optimization has been widely used in many fields, including vehicle engineering, aerospace engineering, microelectromechanical systems (MEMSs) and robotics (Ambrosio and Neto, 2007; Goncalves and Ambrosio, 2003; Kang et al., 2005; Pi et al., 2012); these works are significant for realizing lightweight structures and improving their working performance, but most studies focus on linear static structural optimization. Recently, many studies have focused on the structural optimization of the flexible components of multibody dynamic systems, which is challenging in terms of computation capacity and optimization efficiency due to the simultaneous nonlinear and dynamic responses.

To deal with dynamic responses, the equivalent static load method is usually used in the structural optimization of flexible components in a multibody dynamic system to transform a dynamic structural optimization problem into a static one (Hong et al., 2010; Lee and Park, 2015). Lee (Lee and Park, 2015) presented a nonlinear dynamic topology optimization method by introducing transformed variables to update topology results in nonlinear dynamic analysis based on the density method. Hong (Hong et al., 2010) proposed an iteration method to link the results from multibody dynamic analysis and structural analysis software and optimized large-scale flexible multibody dynamic systems, within reasonable computation time and achieving acceptable results, using the equivalent static load method.

However, it is difficult to describe accurately the dynamic behavior of flexible bodies in traditional dynamic systems...
because many types of nonlinear problems exist, such as large deformation, large motion and nonlinear material. Absolute nodal coordinate formulation (ANCF), proposed by Shabana (Shabana and Dufva, 2005), is an approach of the nonincremental finite element method, and has the advantages of a constant mass matrix and zero Coriolis and centrifugal forces (Lan and Shabana, 2009; Shabana, 2015Shabana and Dufva, 2005). ANCF has obvious advantages, especially in problems addressing large motion and large deformation (Shabana, 2012), and can describe dynamic behaviors more accurately than other methods. Many researchers have used ANCF in the structural optimization of flexible components (Held and Seifried, 2013; Pi et al., 2012; Sun et al., 2016; Sun et al., 2017; Vohar et al., 2008). Pi (Pi et al., 2012) studied the shape optimization problem of an ANCF beam and proposed a deduced method of sensitivity to improve the computational efficiency of the direct differentiation method. Sun and Tian (Sun et al., 2016; Sun et al., 2017) modeled flexible multibody systems with ANCF and used the equivalent static load method and semi-implicit level set method together to optimize the topology of flexible components. Sun (Sun et al., 2015) and Li (Li et al., 2016) investigated the related problem of planar slider-crank mechanism with flexible component clearance joints based on ANCF and proposed a model reduction technique.

Since general coordinates are used in ANCF, the total freedom of ANCF elements is much greater than that of traditional elements (Shabana, 2012), possibly resulting in considerable computational costs during dynamic analysis. When using a conventional method to model flexible bodies with localized surface geometry, such as holes, grooves and ribs, the computational costs may increase sharply with the number of elements. Therefore, improving the analysis efficiency of flexible bodies with such features remains a challenge in ANCF (Cho et al., 2005; Yu et al., 2016). Localized surface geometry exists in many structures, and ignoring such details leads to incorrect results in many dynamic analyses. Therefore, it is important to consider the effect of localized surface geometry in the implementation of finite element analysis. Recently, a new method proposed by He and Shabana (He et al., 2017) successfully described localized surface geometry by using NURBS; this method can accurately capture the surface geometry without significantly increasing the computational cost caused by mesh refinement.

This paper attempts to use the localized surface geometry integration method for the shape optimization design of flexible beams with ribs. The remainder of the paper is organized as follows. Section 2 briefly introduces the concepts and methodologies used in the definition of fully parameterized ANCF elements. Section 3 describes the presented method of defining localized surface geometry with ANCF elements and the corresponding subdomain integration method. In Section 4, the definition of the objective function and the shape optimization method of flexible bodies are addressed, considering the dynamic response. A numerical example of flexible bodies with local shape features is given to validate the proposed optimization method in Section 5. Finally, conclusions drawn from the investigation and the perspective for future research are outlined in Section 6.

2. Fully parameterized ANCF beam element

To describe a flexible body with detailed shape features, a fully parameterized ANCF beam element is used to model the global geometry of the flexible body. This section will briefly introduce a fully parameterized ANCF beam element and the related dynamic equation. A fully parameterized ANCF beam element is shown in Fig. 1. The global position vector \( r \) of the arbitrary point in the ANCF beam element can be described as (Shabana, 2012)

\[
r(x, t) = \mathbf{S}(x)e(t)
\]
where $\mathbf{x} = [x_1, x_2, x_3]$ is the set of local coordinates defined in the element local coordinate system, $\mathbf{e} = [\mathbf{e}^1 \mathbf{e}^2]^T$ is a vector of element nodal coordinates that consists of a global position vector and a gradient vector, where $\mathbf{e}^i = [(\partial e^i / \partial x_1)^T (\partial e^i / \partial x_2)^T (\partial e^i / \partial x_3)^T]$, $i = 1, 2$, $\mathbf{S} = [s_1 \mathbf{I} s_2 \mathbf{I} \cdots s_8 \mathbf{I}]$ is the shape function matrix, and $s_i$, $i = 1, 2, \ldots, 8$ are defined as follows:

\[
\begin{align*}
    s_1 &= 1 - 3\xi^2 + 2\eta \xi, \\
    s_2 &= 1 - \xi^2 + \eta \xi, \\
    s_3 &= 1 - \eta \xi, \\
    s_4 &= \xi, \\
    s_5 &= 3\xi^2 - 2\eta \xi, \\
    s_6 &= \xi^2 - 2\eta \xi, \\
    s_7 &= -\eta \xi, \\
    s_8 &= \eta \xi.
\end{align*}
\]

where $\xi = x_1 / l$, $\eta = x_2 / l$, $\zeta = x_3 / l$, and $l$ is the length of the beam element. The dynamic equations of the ANCF element can be expressed as (Shabana, 2012)

\[
\mathbf{M}\mathbf{e} + \mathbf{Q}_k = \mathbf{Q}_e \tag{3}
\]

where $\mathbf{M}$ is the mass matrix, $\mathbf{Q}_k$ is the vector of elastic forces, and $\mathbf{Q}_e$ is the vector of generalized external forces. External forces $\mathbf{Q}_e$ can be obtained from the virtual work by $\partial W_e = \mathbf{Q}_e \delta \mathbf{e}$, and the vector of elastic forces can be obtained from the strain energy $U$ by $\mathbf{Q}_k = (\partial U / \partial \mathbf{e})^T$. According to the continuum mechanics approach, the virtual work of the elastic forces in fully parameterized beam elements can be written as $\partial W_k = -\int_0^l \sigma_{p2} : \delta \mathbf{e} dV$. Here, $\sigma_{p2}$ is the second Piola–Kirchhoff stress tensor, and $\mathbf{e}$ is the Green–Lagrange strain tensor defined as $\mathbf{e} = (\mathbf{J}^T \mathbf{J} - \mathbf{I}) / 2$, where $\mathbf{J}$ is the matrix of position vector gradients.

3. Integration of the local shape feature
3.1 Geometry definition of local shape feature

According to the requirements of the localized surface geometry, the geometry can be defined using many methods. The most commonly used methods include the use of an algebraic function and a NURBS curve or surface, which will be introduced in the following section.

One method of defining localized surface geometry is to use the function $z = f(x, y)$ with fully parameterized ANCF elements, allowing the thickness of the element to vary with the two local coordinates. The position vector of an arbitrary point in an element can be defined as $\mathbf{r}(x, y, z) = \mathbf{S}(x, y, f(x, y))\mathbf{e}(t)$. Hence, a certain localized surface geometry can be defined in the ANCF element without further mesh refinement.

Another method of defining localized surface geometry is to use the NURBS curve and surface. The basic principle of NURBS representation is using the knot vector, control points and basis function to describe curves or surfaces. Here, we take NURBS curves as an example. The knot vector $U = \{u_0, u_1, \ldots, u_{n+p+1}\}$ is a nondecreasing sequence of parameter values, where $p$ is the degree of the NURBS curve, $n$ is the number of control points, and $u_t$ is the $t$th knot value. The NURBS curve is the weighted and rational form of the B-spline and can be expressed as follows (Piegl and Tiller, 1997):

\[
\mathbf{c}(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u)w_i \mathbf{P}_i}{\sum_{i=0}^{n} N_{i,p}(u)w_i} \tag{4}
\]

where $w_i$ is the weight of the $i$th control point, $\mathbf{P}_i$ is the control point matrix of the NURBS curve, and $N_{i,p}(u)$ is the $i$th B-spline basis function of degree $p$ (Piegl and Tiller, 1997). NURBS curves and surfaces can accurately describe
free geometry, and they can be adjusted for various shapes by intuitively changing the positions of the control points. Because of these advantages, NURBS is used to define the localized surface geometry studied in this investigation.

3.2 Overlapping local shape feature

For dealing with a protrusion feature such as a rib on an ANCF element, the overlapping method is used in this investigation. The basic principle of the overlapping method is to add a height value to the basic element shape to describe the protrusion feature (He et al., 2017). Because ANCF geometry is expressed by ANCF shape functions and the element nodal coordinates, the modified position field with a protrusion feature can be defined as (He et al., 2017)

$$\mathbf{r}(x, y, z) = \begin{cases} \mathbf{\bar{r}}(x, y, z) & (x, y) \notin \Omega_l \\ \mathbf{\bar{r}}(x, y, z) + \mathbf{h}(x, y) \mathbf{n} & (x, y) \in \Omega_l \end{cases}$$  \tag{5}

where $\mathbf{r}$ denotes the ANCF element geometry with protrusion features; $\mathbf{\bar{r}}$ denotes the original ANCF element geometry without protrusion features; $\mathbf{n}$ is the normal of the basic ANCF mid-surface, calculated as $\mathbf{n} = (\mathbf{\bar{r}}_y \times \mathbf{\bar{r}}_z) / |\mathbf{\bar{r}}_x \times \mathbf{\bar{r}}_z|$; $\mathbf{h}(x, y)$ is the thickness of the localized geometry represented by a NURBS curve or surface; and $\Omega_l$ is the element domain on which a protrusion feature is added. Therefore, a protrusion feature on the original ANCF element can be defined by assigning a positive value to its height, as shown in Fig. 2.

![Fig. 2 Overlapping of localized surface geometry coordinates for ANCF](image)

For ANCF elements with localized surface geometry represented by NURBS, there exist two sets of coordinates. One set is the local coordinates of the element and can be expressed as $(x, y, z)$, and the other is the parameters of the NURBS surface and can be expressed as $(u, v)$, which are two independent parameters over two-dimensional parametric domain. A mapping is used to transform the ANCF local coordinates to the parameters of NURBS. When the localized surface geometry is defined on the top surface of the ANCF element, the geometry domain can be defined along the longitudinal and lateral directions of the element as $x \in [0, 1]$ and $y \in [y_a, y_b]$, respectively. However, the function defining the geometry may change with $y$ only (He et al., 2017).

3.3 Subdomain integration method

Numerical integration is used in the simulation of ANCF, and the distribution of the integration point is very important to the accuracy of the analysis results. To balance the computation cost and the accuracy of the geometry’s presentation, the subdomain integration method is used to realize the numerical integration of the localized geometry.

In the subdomain integration method, the whole domain of an element is divided into $n_{d}$ nonoverlapping subdomains. There are $n_{\eta}$ integration points distributed in each subdomain, and the function $\phi(x_i, y_i, z_i)$ can be numerically integrated as

$$\Gamma_i = \sum_{j=1}^{n_{d}} \sum_{\eta=1}^{n_{\eta}} \phi(x_{ij}, y_{ij}, f(x_{ij}, y_{ij})) \Delta\eta_{ij}$$  \tag{6}
where $\overline{w}_{ji}$ is the weight of the integration point $\overline{q}_{ji}$. The calculation steps for the integration points, including the mapping process between ANCF and NURBS, are described in Fig. 3, where $\mathbf{B}=[0\ 1\ 0]^T$ and $\delta$ is a specified tolerance. The overlapping method is used to describe the global shape of the element’s surface geometry. Figure 4 shows the discretization of an element in the $y$ direction using the subdomain method, and the distribution of integration points is clearly shown. The domain is divided into five subdomains, where the middle three subdomains are used to define the local feature, and the corresponding integration points can closely approximate the localized surface geometry. Each subdomain has 3 integration points in both the $x$ and $y$ directions. Since the local geometry is described more accurately by locally placing the integration point, more accurate results of the dynamic analysis will be obtained through numerical integration without considerable increasing the computational cost (He et al., 2017).

4. Shape optimization of the localized surface geometry

To account for the presence of a localized surface geometry, the mesh should be refined to the scale of the local feature. Thus, the number of elements will sharply increase, and the computation and storage load will increase rapidly, especially for structural optimization. Based on the conventional shape optimization method and the proposed integration method for localized surface geometry, this investigation realizes the shape optimization of localized surface geometry and improves the efficiency of the optimization by decreasing the number of elements and design variables.

4.1 Design variables

The shape of a flexible beam with a rib is shown in Fig. 5; the geometry of the rib is described by NURBS curves, as shown in Fig. 5(a) and 5(b), in which the solid red dots are the control points of the NURBS curves. The number and location of control points can be selected and distributed according to the description requirement of the geometric features, and under the given accuracy, it is better to use fewer control points. The design domain is the shape of the rib on a flexible beam; therefore, choosing design variables is very important to the quality and efficiency of optimizing the shape of the rib.

Because the control points used to define the NURBS curves have a significant influence on the shape of the NURBS curve, their coordinates will be chosen as appropriate design variables. To reduce the computation capacity of the optimization, it is better to use as few design variables as possible. The local shape feature in the vertical cross section is
described by one NURBS curve defined by 7 control points. In the horizontal cross section, the localized geometry is described by two NURBS curves, and each NURBS curve is defined by 4 control points. The vertical direction has more influence on the rib details, and the distribution of the control points in the vertical cross section is assumed to be symmetrical; thus, the $z$ coordinates of the control points $\lambda_3$, $\lambda_4$ and $\lambda_5$ shown in Fig. 5(c) are selected as design variables, and the $y$ coordinates are assumed to be fixed. In Fig. 5(b), two parameters, $\lambda_1$ and $\lambda_2$, which describe the lateral width of the rib at the two ends, are selected as design variables to define the width of the rib.

![Model of a flexible beam with a rib](image1)

(a) Model of a flexible beam with a rib

![Horizontal section](image2)

(b) Horizontal section

![Vertical section](image3)

(c) Vertical section

Fig. 5 Descriptions of the rib shape and design variables of a flexible beam

### 4.2 Objective function

The compliance of the flexible body is taken as an objective function in this investigation. Minimizing the compliance is equivalent to maximizing the stiffness. The compliance is defined as (Sun et al., 2017)

$$f(\mathbf{e}, \lambda) = \mathbf{e}(\mathbf{e}, \lambda)^T \mathbf{e}(\mathbf{e}, \lambda)$$

(7)

It is difficult to evaluate compliance over the entire time domain when dealing with the dynamic response during optimization. The summation of the compliance of each time step during one dynamic analysis is taken as the objective function (Sun et al., 2017). However, this function cannot indicate changes in compliance during dynamic analysis. The compliance of a flexible beam changes with time, and several peaks may appear during the simulation. These peaks influence the optimization process and may cause numerical instability, which is a problem (Hong et al., 2010; Sun et al., 2017). To emphasize the influence of peaks in the compliance curve, an objective function for optimization of the nonlinear dynamic response is proposed (Lee and Park, 2015), as defined by the weighted summation of the compliances:

$$f(\mathbf{e}, \lambda) = \sum_{i=1}^{n} w_i (\mathbf{e}(\mathbf{e}, \lambda)^T \mathbf{e}(\mathbf{e}, \lambda))$$

(8)

where $n$ is the number of time steps and $w_i$ denotes the weighting for the compliance of the flexible body at the $i$th time step. The weight can be used to emphasize the influence of compliance at a certain time step. For example, if one of the peaks of the compliance occurs at the $i$th time step, and the value of peak needs to be limited, i.e., the weight assigned to $w_i$ should be increased. The optimization process can be more flexible and reliable than other processes because the influence of compliance at every time step on this objective function is considered by assigning different weights to different time steps.

### 4.3 Constraint condition

The connection between the ANCF beam and the rigid body can be realized by applying position constraints in the multibody system (MBS). As Fig. 6 shows, the end node $\mathbf{r}^{ik}$ of the ANCF beam is constrained by point $\mathbf{q}^m$ on a rigid body with a spherical joint. The spherical joint constrains the relative displacement between node $\mathbf{r}^{ik}$ and point $\mathbf{q}^m$ of the rigid body, and the ANCF beam can rotate relative to the spherical joint. The constraint condition can be expressed by the following equation:
\[
C_q(r^{jk}, q^b) = [r^{jk}_x - r^{m}_x, r^{jk}_y - r^{m}_y, r^{jk}_z - r^{m}_z]^T = 0
\]  

(10)

![Fig. 6 Constraint condition of a spherical joint](image)

4.4 Shape optimization formulation

The shape optimization formulation of the flexible ANCF beam with localized surface geometry can be expressed as

\[
\text{find } \begin{bmatrix} \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n \end{bmatrix} \text{ design variables } \lambda \\
\text{to minimize } f(e, \lambda) \\
\text{subject to } \begin{align*}
M(e, \lambda)\ddot{e} + Q_q(e, \lambda) &= Q_e(e, \lambda) \\
C_q &= 0 \\
G_i(e, \lambda) &= 0 \\
G_j(e, \lambda) &\leq 0 \\
\lambda_i^l &\leq \lambda_i & \leq \lambda_i^u 
\end{align*}
\]

(11)

where \( \lambda = [\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n] \) is the design variable vector, \( f(e, \lambda) \) is the objective function, \( C_q \) is the constraint equation in the dynamic analysis, \( G(e, \lambda) = [g_1(e, \lambda), g_2(e, \lambda), \ldots, g_m(e, \lambda)] \) is the functional vector, \( G_i(e, \lambda) \) is the equality constraint, and \( G_j(e, \lambda) \) is the inequality constraint. Furthermore, \( \lambda_i^l \) and \( \lambda_i^u \) are the lower and upper bounds of the \( i \)-th design variable \( \lambda_i \), respectively.

Since the localized surface geometry is implemented at the integration level, it is difficult to deduce an explicit gradient formula, which limits the application of a conventional gradient-based optimization method. Of all the optimization methods proposed so far, sequential quadratic programming (SQP) is one of the most powerful algorithms for nonlinear optimization (Jin and Wang, 2010; Mo et al., 2006; Zhu et al., 2003), and it can transform the initial optimization problem into a series of quadratic optimization problems, possibly improving the optimization efficiency. The SQP method is used in this investigation, and the original shape optimization formulation can be transformed to solve the following quadratic optimization problem (Mo et al., 2006):

\[
\begin{align*}
\text{Find } \lambda^{(k)} \\
\text{to minimize } & \quad \nabla f(e, \lambda^{(k)})^T d + \frac{1}{2} d^T H^{(k)} d \\
\text{subject to } & \quad M(e, \lambda^{(k)})\ddot{e} + Q_q(e, \lambda^{(k)}) = Q_e(e, \lambda^{(k)}) \\
& \quad C_q = 0 \\
& \quad \nabla g_i(e, \lambda^{(k)})^T d + g_i(e, \lambda^{(k)}) = 0 \\
& \quad \nabla g_j(e, \lambda^{(k)})^T d + g_j(e, \lambda^{(k)}) \leq 0 \\
& \quad \lambda_i^{l(k)} \leq \lambda_i^{(k)} \leq \lambda_i^{u(k)}
\end{align*}
\]

(12)
where $V$ is the gradient and $H^{(k)}$ is a symmetric positive definite matrix. The iterative scheme of optimization is defined as

$$\lambda^{(k+1)} = \lambda^{(k)} + \alpha d^{(k)}$$

(13)

where $d$ is the search direction of the design variables and $\alpha$ is the step length obtained by the line search method based on a certain merit function (Jin and Wang, 2010; Mo et al., 2006). After assigning the initial values of $\lambda^{(0)}$ and $H^{(0)}$, the SQP method can be performed iteratively, and the convergent solution can be obtained. The detailed procedure of shape optimization is shown in Fig. 7, where $k$ is the iteration number and $\delta$ is the convergence tolerance.

5. Numerical Example

In this section, an example of a flexible beam with a rib is given to validate the shape optimization of the localized surface geometry. The example of a pendulum with a rib is analyzed, and the first node of the beam is constrained with a spherical joint. The total analysis time is 1 s, with 6000 integration steps. The length and width dimensions of the flexible beam are $1 \times 1$ m$^2$, and the beam thickness without considering the rib is 0.1 m. The Young's modulus of the flexible beam is 2 MPa, and its shear modulus is 1 MPa. The density of the flexible beam is 2000 kg/m$^3$.

There is only one integration domain in the longitudinal direction of the elements, and there are 4 integration points in the domain. According to the shape of the rib details, in the lateral direction, the ANCF element is divided into several subdomains with 2 integration points in each subdomain, and 3 integration points are used in the vertical direction of the elements in the following simulation.

Figure 8 shows the displacement curves of the end node of the flexible beam with a rib using both ANCF and ABAQUS, illustrated as lines with solid circle markers and triangular markers, respectively. The two sets of results are very similar, which demonstrate the effectiveness of ANCF in performing a dynamic analysis of flexible beams.

5.1 Shape optimization of the transverse section

To improve the efficiency of the optimization, the optimization process is divided into two stages: rib cross section...
shape optimization and rib longitudinal section shape optimization. In the first stage, the initial values of the design variables are set as $\lambda_1=0.12$, $\lambda_2=0.12$, $\lambda_3=0.02$, $\lambda_4=0.04$, and $\lambda_5=0.05$. $\lambda_1$ and $\lambda_2$ are set to vary in $[0.06,0.16]$, while $\lambda_3$, $\lambda_4$ and $\lambda_5$ are set to vary in $[0,0.1]$. The constraints of the design variables are set as $\lambda_3 \leq \lambda_4 \leq \lambda_5$.

Because the weighted summation is used to define the objective function, the selection of different weights for the optimization is important to the result. Table 1 and Table 2 give two different sets of weights. The comparison of the optimization results from the two weighted summations is shown in Fig. 9, which shows that the two weighted summations have approximately the same rate of convergence. Therefore, the weights of the objective function have little influence on convergence rate and result of the optimization.

![Fig. 8 Comparison of end node displacement using ANCF and ABAQUS](image)

![Fig. 9 Optimization convergence comparison between different weighted summations](image)

| Integration steps | (0,1000) | (1000,2000) | (2000,3000) | (3000,4000) | (4000,5000) | (5000,6000) |
|-------------------|----------|-------------|-------------|-------------|-------------|-------------|
| Weight            | 0.5      | 0.4         | 0.8         | 0.6         | 0.4         | 0.3         |

| Integration interval | (0, 500) | (500, 1000) | (1000, 1500) | (1500, 2000) | (2000, 2500) | (2500, 3000) | (3000, 3500) | (3500, 4000) | (4000, 4500) | (4500, 5000) | (5000, 5500) | (5500, 6000) |
|----------------------|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Weight               | 0.2      | 0.6         | 0.3         | 0.3         | 0.9         | 0.9         | 0.4         | 0.7         | 0.3         | 0.2         | 0.1         | 0.3         |

| Number of iterations | 168      |
|----------------------|----------|
| Objective function   | $3.402 \times 10^8$ m/N |
| Design variables     | $\lambda_1=0.06$, $\lambda_2=0.06$, $\lambda_3=0$, $\lambda_4=0.1$, $\lambda_5=0.1$ |
Table 3 gives the optimization results of the flexible beam with a rib after 168 iterations under the object function of weight array 1. Figure 10 shows the changes in the shape of the flexible beam with a rib during the iteration process, and the lateral and longitudinal sections clearly change during optimization. The shape of the rib details shows that the lateral size of the rib decreases while the vertical size increases after performing the optimization of this stage.

Figure 11 compares the displacement curves for the end node between the original model and the optimized model during the dynamic simulation time. The displacement of the optimized model at the last time step is slightly smaller than that of the original model. The changes in end node displacement due to the use of the optimization model may be caused by the decrease in the compliance, which causes the increase in the rigidity of the flexible beam with a rib. Figure 12 compares the compliance curves before and after the optimization of the transverse section. It can be found that the compliance has been reduced obviously after optimization, especially at the peaks of the two compliance curves. Figure 13 illustrates the comparison of the deformation results of the flexible beam at 4 steps from the dynamic analysis of the original model and optimized model. It is obvious that the resultant deformation of the optimized model is smaller than that of the original model, meaning that the optimized model has better rigidity, and the decrease in the compliance of the flexible beam with a rib is verified in detail. Hence, this example has demonstrated the effectiveness of our optimization method.

5.2 Shape optimization of the longitudinal cross section

In the second stage of the optimization process, the longitudinal section of the rib is optimized. Figure 14 shows the NURBS curve defining the longitudinal shape of the rib, and the related design variables are \([\lambda_6, \lambda_7, \lambda_8, \lambda_9]\). In this stage of optimization, all design variables are set to vary in \([0.0, 0.1]\). The purpose of this stage of optimization is to minimize the volume of the beam without reducing its rigidity.

Figure 15 presents a comparison of the objective function with different weights, which are given in Table 1 and Table 2. Notably, the iterations of the optimizations with the two objective functions have similar convergence properties. Table 4 gives the values of the design variables and object function after 50 iterations. Figures 16(a) and 16(b) illustrate a comparison of the shape of the flexible beam with a rib from the original model with that from the optimized model. Figure 16(c) shows the front view of the rib on the optimized model, clearly showing the changes in the longitudinal shape of the rib. Additionally, the height of the rib decreases from one end to the other end, changing the volume of the
beam from 1.202 m$^3$ to 1.092 m$^3$ after optimization.

Figure 17 shows the end node displacement from the dynamic analysis results before and after optimization; there is a slight difference between the two displacement curves. Figure 18 compares the compliance curves of the ANCF beam with rib before and after optimization of the longitudinal section. Most sections of the two compliance curves are similar, but it can be found that most peaks of the compliance curves after optimization are slightly higher than the compliance curves before optimization of the longitudinal section, which demonstrates that the rigidity of the flexible beam with a rib increases after optimization and that its volume is simultaneously reduced. Thus, the effectiveness of our optimization method can be validated by this example.

| Table 4 Optimization results of the flexible beam |
|-----------------------------------------------|
| **Objective function** | $2.793 \times 10^8$ m/N |
| **Design variables** | $\lambda_6=0.1$, $\lambda_7=0.03$, $\lambda_8=0.05$, $\lambda_9=0.011$ |

![Graphs showing deformation and compliance curves](image)

**6. Conclusion**

The structural optimization of a flexible body with local shape features in a multibody dynamic system is challenging because of the very high computational load and memory storage needed. Based on the integration method of localized surface geometry with fully parameterized ANCF elements, this paper proposes a shape optimization design method for flexible beams with local shape features. The proposed method can successfully describe complex shape features with fewer elements and design variables. ANCF finite elements are used to create the global geometry, the local shape features can be described by ANCF element surfaces without the need for mesh refinement during numerical integration, and a mapping between the ANCF local coordinates and NURBS localized geometric parameters is used for the consistent implementation of the overlapping methods. The geometric parameters of the rib in the ANCF beam are taken as design variables, and the method of weighted compliant summation of the beam is defined as the objective function to reflect
the influence of the dynamic response over the entire time domain. Two different weight arrays are used to compare the
effect of weighting. An example of an ANCF beam with a rib is given to demonstrate the validation of our method. The
two stages of the shape optimization methods, namely, optimization of the transverse and longitudinal cross sections, are
run successively. Approximately convergent results are obtained with SQP method. Because fewer design variables are
used, our method greatly reduces the computational burden and improves the efficiency of the optimization, and it can
also be extended to other ANCF elements. The next steps of our research include topology optimization of flexible bodies
with more complicated geometries and finding a faster and more stable optimization algorithm.

Fig. 16 The shape of the flexible beam with a rib from the original model and the optimized model

Fig. 17 Comparison of end node displacement before and after optimization of the longitudinal section

Fig. 18 Comparison of compliance before and after optimization of the longitudinal section

Acknowledgments

This work was supported by the National Natural Science Foundation of China [No. 51375141]. The authors thank
the reviewers for their valuable comments.

References

Ambrosio, J.A.C., Neto, M.A. and Leal, R.P., Optimization of a complex flexible multibody systems with composite
materials, Multibody System Dynamics, Vol.18, No.2 (2007), pp.117-144.
Cho, J.R., Kim, K.W., Jeon, D.H. and Yoo, W.S., Transient dynamic response analysis of 3-D patterned tire rolling over
Goncalves, J.P.C. and Ambrosio, J.A.C., Optimization of vehicle suspension systems for improved comfort of road vehicles using flexible multibody dynamics, Nonlinear Dynamics, Vol.34, No.1-2 (2003), pp.113-131.

He, G., Patel, M. and Shabana, A., Integration of localized surface geometry in fully parameterized ANCF finite elements, Computer Methods in Applied Mechanics and Engineering, Vol.313, (2017), pp.966-985.

Held, A. and Seifried, R., 2013. Gradient-based optimization of flexible multibody systems using the absolute nodal coordinate formulation, Proceedings of the ECCOMAS Thematic Conference on Multibody Dynamics (2013), University of Zagreb, Zagreb, Croatia, pp. 957-964.

Hong, E.P., You, B.J., Kim, C.H. and Park, G.J., Optimization of flexible components of multibody systems via equivalent static loads, Structural and Multidisciplinary Optimization, Vol.40, No.1-6 (2010), pp.549-562.

Jin, Z. and Wang, Y., A type of efficient feasible SQP algorithms for inequality constrained optimization, Applied Mathematics and Computation, Vol.215, No.10 (2010), pp.3589-3598.

Kang, B.S., Park, G.J. and Arora, J.S., Optimization of flexible multibody dynamic systems using the equivalent static load method, Aiaa Journal, Vol.43, No.4 (2005), pp.846-852.

Lee, H.-A. and Park, G.-J., Nonlinear dynamic response topology optimization using the equivalent static loads method, Computer Methods in Applied Mechanics and Engineering, Vol.283, (2015), pp.956-970.

Li, Y., Chen, G., Sun, D., Gao, Y. and Wang, K., Dynamic analysis and optimization design of a planar slider-crank mechanism with flexible components and two clearance joints, Mechanism and Machine Theory, Vol.99, (2016), pp.37-57.

Mo, J., Zhang, K. and Wei, Z., A variant of SQP method for inequality constrained optimization and its global convergence, Journal of Computational and Applied Mathematics, Vol.197, No.1 (2006), pp.270-281.

Pi, T., Zhang, Y.Q. and Chen, L.P., First order sensitivity analysis of flexible multibody systems using absolute nodal coordinate formulation, Multibody System Dynamics, Vol.27, No.2 (2012), pp.153-171.

Piegl L., Tiller, W., The NURBS Book, 2nd Edition(1997), Springer-Verlag.

Shabana, A.A., Computational Continuum Mechanics, second ed.(2012). Cambridge.

Shabana, A.A., Definition of ANCF Finite Elements, Journal of Computational and Nonlinear Dynamics, Vol.10, No.5 (2015), pp.054506.

Shabana, A.A. and Dufva, K., Analysis of Thin Plate Structures Using the Absolute Nodal Coordinate Formulation, Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, Vol.219, No.4 (2005), pp.345-355.

Sun, D., Chen, G., Shi, Y., Wang, T. and Sun, R., Model reduction of a flexible multibody system with clearance, Mechanism and Machine Theory, Vol.85, (2015), pp.106-115.

Sun, J.L., Tian, Q. and Hu, H.Y., Structural optimization of flexible components in a flexible multibody system modeled via ANCF, Mechanism and Machine Theory, Vol.104, (2016), pp.59-80.

Sun, J.L., Tian, Q. and Hu, H.Y., Topology optimization based on level set for a flexible multibody system modeled via ANCF, Structural and Multidisciplinary Optimization, Vol.55, No.4 (2017), pp.1159-1177.

Vohar, B., Kegl, M. and Ren, Z., Implementation of an ANCF beam finite element for dynamic response optimization of elastic manipulators, Engineering Optimization, Vol.40, No.12 (2008), pp.1137-1150.

Yu, Z.Q., Liu, Y.G., Tinsley, B. and Shabana, A.A., Integration of Geometry and Analysis for Vehicle System Applications: Continuum-Based Leaf Spring and Tire Assembly, Journal of Computational and Nonlinear Dynamics, Vol.11, No.3 (2016), pp.031011-1-11.

Zhu, Z., Zhang, K. and Jian, J., An improved SQP algorithm for inequality constrained optimization, Mathematical Methods of Operations Research, Vol.58, No.2 (2003), pp.271-282.