Thermal stability analysis of eccentrically stiffened Sigmoid-FGM plate with metal – ceramic – metal layers based on FSDT

Pham Hong Cong and Nguyen Dinh Duc

Cogent Engineering (2016), 3: 1182098
Pham Hong Cong1,2 and Nguyen Dinh Duc2*

Abstract: This paper researches the thermal stability of eccentrically stiffened plates made of functionally graded materials (FGM) with metal – ceramic – metal layers subjected to thermal load. The equilibrium and compatibility equations for the plates are derived by using the first-order shear deformation theory of plates, taking into account both the geometrical nonlinearity in the von Karman sense and initial geometrical imperfections with Pasternak type elastic foundations. By applying Galerkin method and using stress function, effects of material and geometrical properties, elastic foundations, temperature-dependent material properties, and stiffeners on the thermal stability of the eccentrically stiffened S-FGM plates in thermal environment are analyzed and discussed.

Subjects: Materials Science; Mechanical Engineering; Structural Mechanical Engineering

Keywords: nonlinear stability; eccentrically stiffened thick S-FGM plates; first-order shear deformation; temperature-dependent materials properties

ABOUT THE AUTHOR
In Vietnam, Nguyen Dinh Duc is one of the well-known scientists in mechanical science. He is a full professor of Vietnam National University, Hanoi. Professor Nguyen Dinh Duc is the Head of Advanced Materials and Structures Laboratory of University of Engineering and Technology - Vietnam National University, Hanoi. He had graduated from Hanoi State University since 1984 and completed his Ph.D degree in 1991 at Moscow State University, Russia. Since 1997, he had become Doctor of Science (Dr. Habilitation) promoted by Russian Academy of Sciences. Webpage: http://uet.vn.edu.vn/~ducnd/, http://irgamme.uet.vn.edu.vn/gs-tskh-nguyen-dinh-duc/

PUBLIC INTEREST STATEMENT
In recent years, there has been significant interest in the development of functionally graded materials (FGMs) for engineering applications. FGM materials have been used in aerospace, nuclear, and microelectronics engineering applications, where the materials are required to work in extreme temperature environments. It is also important for these materials to maintain their structural integrity, with minimum failures due to material mismatch. The focus of this manuscript is on a theoretical analysis on the thermal stability of eccentrically stiffened plates made of FGMs with metal – ceramic – metal layers (S-FGM) subjected to thermal load. Both the FGM plate and the outside stiffeners are deformed under temperature and having temperature-dependent properties. The influences of the material and geometrical properties, elastic foundations, temperature-dependent material properties, and outside reinforced stiffeners on the thermal stability of the eccentrically stiffened S-FGM plates in thermal environment are analyzed and discussed. The outcomes from this work are important to composite engineers and designers.
1. Introduction
Since its first introduction in 1984 by a group of material scientists in Japan (Koizumi, 1997), functionally graded materials (FGMs) have attracted considerable attention in many engineering applications such as extremely high-temperature-resistant materials. To date, there have been a number of studies on the stability of eccentrically stiffened FGM plates. However, these studies have only been concentrated on thin structures using classical plate theory. Not much consideration has been given to eccentrically stiffened thick Sigmoid-FGM (S-FGM) plates with shear deformation behaviors, especially when material properties depend on temperature. An overview on studies that apply shear deformation theory to FGM plates is provided in the following part.

Wu (2004) has studied the thermal buckling and post-buckling behavior of simply supported FGM rectangular plates based on the first-order shear deformation plate theory. Ma and Wang (2004) studied the thermoelectric buckling behavior of functionally graded circular/annular plates based on first-order shear deformation plate theory. Duc and Tung studied the mechanical and thermal post-buckling of shear deformable FGM plates with temperature-dependent properties (Duc & Tung, 2010). Shen (1997) studied the thermal post-buckling analysis of imperfect laminated plates using a higher order shear deformation theory. Duc and Tung (2011) studied mechanical and thermal post-buckling of higher order shear deformable functionally graded plates on elastic foundations. Seren Akavci et al. used the first-order shear deformation theory for symmetrically laminated composite plates on elastic foundation (Seren Akavci, Yerli, & Dogan, 2007). Ghiasian et al. studied the thermal buckling of shear deformable temperature-dependent circular/annular FGM plates (Ghiasian, Kiani, Sadighi, & Eslami, 2014). Duc and Cong (2015) studied the nonlinear vibration of thick FGM plates on elastic foundation subjected to thermal and mechanical loads using the first-order shear deformation plate theory. In these studies (Seren Akavci et al., 2007; Duc & Cong, 2015; Duc & Tung, 2010, 2011; Ghiasian et al., 2014; Ma & Wang, 2004; Shen, 1997), the authors used shear deformation theory to study the nonlinear static stability of unstiffened FGM thick plates. Dung and Nga (2013) studied the nonlinear buckling and post-buckling of eccentrically stiffened functionally graded cylindrical shells surrounded by an elastic medium based on the first-order shear deformation theory but without temperature. Shen (2007) studied the thermal post-buckling behavior of shear deformable FGM plates with temperature-dependent properties. In Bich, Nam, and Phuong (2011), studied the nonlinear post-buckling of eccentrically stiffened functionally graded plates and shallow shells based on classical shell theory. The nonlinear stability of eccentrically stiffened functionally graded plates and shallow shells based on classical shell theory has been further studied by Dung and Thiem (2012). In (Duc, 2014; Duc & Cong, 2014), Duc and Cong studied the nonlinear post-buckling of imperfect eccentrically stiffened thin FGM plates with temperature-dependent material properties under temperature while resting on elastic foundations using a simple power-law distribution (P-FGM) and the classical plate theory. Swaminathan, Naveenkumar, Zenkour, and Carrera (2015) studied the stress, vibration, and buckling analyses of FGM plates–A state-of-the-art review. Thai and Kim (2015) studied a review of theories for the modeling and analysis of functionally graded plates and shells. Reddy and Chin (1998) studied the thermo-mechanical analysis of functionally graded cylinders and plates.

From the above review, to the best of our knowledge, it has been shown that there are no publications on the nonlinear stability of a thick S-FGM plate (with metal – ceramic – metal layers) reinforced by stiffeners in a thermal environment using the first-order shear deformation plate theory. This paper will focus on studying the buckling and post-buckling of an eccentrically stiffened functionally graded thick plate on elastic foundations under thermal loads with both S-FGM plates and stiffeners having temperature-dependent properties and thermal deformations. The paper also analyzes and discusses the effects of material and geometrical properties, temperature, elastic foundations, and eccentric stiffeners on the buckling and post-buckling loading capacity of the functionally graded plate in thermal environments.

2. Functionally graded plates on elastic foundations
Consider a eccentrically stiffened thick S-FGM plate (metal – ceramic – metal) of length a, width b, and thickness h resting on an elastic foundation. A coordinate system (x, y, z) is established, in which
Figure 1. Geometry and coordinate system of an eccentrically stiffened S-FGM plate on an elastic foundation.

By applying a Sigmoid power-law distribution, Young's modulus and thermal expansion coefficient can be expressed in the form (Duc & Tung, 2010):

$$
\begin{align*}
E(z) &= E_m + E_{cm}\left\{ (2z+h)/h\right\}^N, \quad -h/2 \leq z \leq 0 \\
&= E_m + E_{cm}\left\{ (-2z+h)/h\right\}^N, \quad 0 \leq z \leq h/2
\end{align*}
$$

(1)

where $N$ is volume-fraction index, the subscripts $m$ and $c$ refer to the metal and ceramic constituents, respectively, Poisson's ratio ($v$) is assumed to be a constant and $E_{cm} = E_c - E_m$, $a_{cm} = a_c - a_m$.

A material property ($P_r$), such as the elastic modulus ($E$), and the thermal expansion coefficient ($\alpha$) can be expressed as a nonlinear function of temperature (Touloukian, 1967), as:

$$
P_r = P_0\left( P_{-1} T^{-1} + P_1 T^1 + P_2 T^2 + P_3 T^3 \right)
$$

(2)

where $T = T_0 + \Delta T(z)$ and $T_0 = 300$ K (room temperature); $P_{-1}$, $P_1$, $P_2$, $P_3$ are coefficients characterizing the constituent materials; and $\Delta T$ is the temperature rise from stress-free initial state. In short, T-D (temperature-dependent) will be used for the cases in which the material properties depend on temperature. Otherwise, T-ID will be used for the temperature-independent cases. The material properties for the latter scenario have been determined by Equation (2) at room temperature, i.e. $T_0 = 300$ K.

3. Theoretical formulation

The present study uses first-order shear deformation plate theory to establish the governing equations and determine the buckling loads and post-buckling paths of the eccentrically stiffened S-FGM plates.

The strains across the plate thickness at a distance $z$ from the middle surface are given by:

$$
\begin{align*}
\varepsilon_x &= \varepsilon_x^0 + z \chi_x, \quad \varepsilon_y = \varepsilon_y^0 + z \chi_y, \quad \gamma_{xy} = \gamma_{xy}^0 + z \chi_{xy}, \quad \gamma_{xz} = \gamma_{xz}^0 + \phi_x, \quad \gamma_{yz} = \gamma_{yz}^0 + \phi_y,
\end{align*}
$$

(3)

where

$$
\begin{align*}
\varepsilon_x^0 &= u_x + (w_x)^2/2, \quad \varepsilon_y^0 = v_y + (w_y)^2/2, \quad \gamma_{xy}^0 = u_y + v_x + w_x w_y, \\
\chi_x &= \phi_{xz}, \quad \chi_y = \phi_{yz}, \quad \chi_{xy} = \phi_{xy} + \phi_{yx}
\end{align*}
$$

(4)

where $\varepsilon_x^0$, $\varepsilon_y^0$ are normal strains, $\gamma_{xy}^0$ is shear strain on the mid-plane of the plate, $u$, $v$ are the displacement components along the $x$, $y$ directions; and $\phi_x$, $\phi_y$, are the rotations in the $(x, z)$ and $(y, z)$ planes, respectively.
Hooke’s law for an S-FGM plate under thermal conditions is defined as:

\[
\sigma_x = E \left( \varepsilon_x + \nu \varepsilon_y - (1 + \nu)\alpha \Delta T \right) / \left( 1 - \nu^2 \right), \quad \sigma_y = E \left( \varepsilon_y + \nu \varepsilon_x - (1 + \nu)\alpha \Delta T \right) / \left( 1 - \nu^2 \right) \tag{5}
\]

\[
\sigma_{xy} = G \gamma_{xy} / (2(1 + \nu)), \quad \sigma_{zx} = E \gamma_{zx} / (2(1 + \nu)), \quad \sigma_{yz} = E \gamma_{yz} / (2(1 + \nu))
\]

For stiffeners in thermal environments with temperature-dependent properties, its form proposed adapted from Duc and Cong (2014), is as follows:

\[
\sigma_x^* = E_0 \varepsilon_x - E_0 \alpha_0 \Delta T / (1 - 2\nu), \quad \sigma_y^* = E_0 \varepsilon_y - E_0 \alpha_0 \Delta T / (1 - 2\nu).
\tag{6}
\]

where \( E, \nu, \alpha \) are Young’s modulus, Poisson’s ratio, and thermal expansion coefficient of stiffeners, respectively.

From Equation (4), the geometrical compatibility equation can be written as:

\[
\varepsilon_{x,xy}^0 + \varepsilon_{y,xx}^0 - \gamma_{xy,yy}^0 = \left( w_{,xy}^0 \right)^2 - w_{,xx}^0 w_{,yy}^0.
\tag{7}
\]

In order to investigate the FGM plates with stiffeners in the thermal environment, the materials’ moduli with temperature-dependent properties have been taken into account. In addition, it can be assumed that all elastic moduli of FGM plates and stiffener are temperature-dependent and they are deformed in the presence of temperature. Hence, the geometric parameters, the plate’s shape and stiffeners are varied through the deforming process due to the temperature change. Assuming that the thermal stress of stiffeners is subtle which distributes uniformly through the whole plate structure, it can be ignored. Lekhnitsky smeared stiffeners technique can be adapted from Bich et al. (2011) for eccentrically stiffened FGM plate under temperatures as follows:

\[
N_x = (A_{11} + E_0 A_1^1 / d_1^1) \varepsilon_x^0 + A_{12} \varepsilon_y^0 + (B_{11} + C_1^1) \phi_{x,x} + B_{12} \phi_{x,y} + \Phi_1,
\]

\[
N_y = A_{12} \varepsilon_x^0 + (A_{22} + E_0 A_2^1 / d_2^1) \varepsilon_y^0 + B_{12} \phi_{x,y} + (B_{22} + C_2^1) \phi_{y,x} + \Phi_1,
\]

\[
N_{xy} = A_{66} \gamma^0_{xy} + B_{66} \left( \phi_{y,y} + \phi_{x,x} \right),
\]

\[
M_x = (B_{11} + C_1^1) \varepsilon_x^0 + B_{12} \varepsilon_y^0 + (D_{11} + E_0 d_1^1 / d_1^1) \phi_{x,x} + D_{12} \phi_{x,y} + \Phi_2,
\]

\[
M_y = B_{12} \varepsilon_x^0 + (B_{22} + C_2^1) \varepsilon_y^0 + D_{12} \phi_{x,y} + (D_{22} + E_0 d_2^1 / d_2^1) \phi_{y,y} + \Phi_2,
\]

\[
M_{xy} = B_{66} \varepsilon_{xy}^0 + D_{66} \left( \phi_{y,y} + \phi_{x,x} \right), Q_x = A_{44} \left( w_{,x} + \phi_x \right), Q_y = A_{55} \left( w_{,y} + \phi_y \right),
\]

where \( A_1, A_2 \) are cross-section areas of stiffeners; \( d_1, d_2 \) are spacing of the longitudinal and transversal stiffeners; \( I_1, I_2 \) are second moments of cross-section areas; \( z_1, z_2 \) are eccentricities of stiffeners with respect to the middle surface of plate; \( b_1, b_2 \) are width of longitudinal and transversal stiffeners; \( h_1, h_2 \) are thickness of longitudinal and transversal stiffeners; and specific expressions of coefficients \( A_0, B_0, D_0 \) are given in Appendix A and

\[
\Phi_1 = -\frac{1}{1 - \nu} \int_{-h/2}^{h/2} E(z) \alpha(z) \Delta T dz, \quad \Phi_2 = -\frac{1}{1 - \nu} \int_{-h/2}^{h/2} E(z) \alpha(z) \Delta T dz,
\tag{9}
\]

After the thermal deformation process, the geometric shapes of stiffeners can be determined as follows (Duc & Cong, 2014).
Substituting Equation (11) into Equation (8), yields:

\[
\begin{align*}
M_x &= B_{11}^* N_x + B_{21}^* N_y + D_{11}^* \phi_{xx} + D_{12}^* \phi_{xy} + C_{12} \Phi_1 + \Phi_2, \\
M_y &= B_{12}^* N_x + B_{22}^* N_y + D_{12}^* \phi_{xx} + D_{22}^* \phi_{yy} + C_{22} \Phi_1 + \Phi_2, \\
M_{xy} &= B_{66}^* N_{xy} + D_{66}^* (\phi_{xy} + \phi_{yx}).
\end{align*}
\]

(12)

where coefficients \(A_{ij}', B_{ij}', C_{ij}', \text{ and } D_{ij}'\) are given in Appendix A.

The nonlinear equilibrium equations of an eccentrically stiffened S-FGM plate on elastic foundations, based on the first-order shear deformation plate theory (Reddy, 2004), are:

\[
\begin{align*}
N_{xx} + N_{xy,y} &= 0, N_{xy,x} + N_{yy,y} = 0, \\
Q_{xx} + Q_{y,y} + N_x w_{xx} + 2N_y w_{xy} + N_y w_{yy} - k_1 w + k_2 (w_{xx} + w_{yy}) &= 0, \\
M_{xx} + M_{xy,y} - Q_x &= 0, M_{xy,x} + M_{yy,y} - Q_y = 0.
\end{align*}
\]

(13a)

(13b)

(13c)

Considering Equation (13a), a stress function \(f(x, y)\) may be defined as:

\[
N_x = f_{yy}, \quad N_y = f_{xx}, \quad N_{xy} = -f_{xy}.
\]

(14)

The three equations (13b) and (13c) become:

\[
\begin{align*}
M_{xx} + 2M_{xy,y} + M_{yy,y} + N_x w_{xx} + 2N_y w_{xy} + N_y w_{yy} - k_1 w + k_2 (w_{xx} + w_{yy}) &= 0, \\
M_{xx} + M_{xy,y} - Q_x &= 0, M_{xy,x} + M_{yy,y} - Q_y = 0.
\end{align*}
\]

(15a)

(15b)

Substituting the expressions of \(M_x, M_y, M_{xy}\) in Equation (12), and \(Q_x, Q_y\) in Equation (9) into Equation (15), we obtain:

\[
\begin{align*}
B_{21}^* f_{xxx} + (B_{11}^* - 2B_{21}^* + B_{22}^*) f_{xyy} + B_{12}^* f_{yyy} + D_{11}^* \phi_{xxx} + (D_{12}^* + 2D_{66}^*) \phi_{xyy} \\
+ (2D_{66}^* + D_{21}^*) \phi_{xyy} + D_{22}^* \phi_{yy} + f_{yy}(w_{xx} + w_{xx}) - 2f_y (w_{xy} + w_{xy}) \\
+ f_{xx} (w_{yy} + w_{yy}) - k_1 w + k_2 (w_{xx} + w_{yy}) &= 0, \\
B_{21}^* f_{xxx} + (B_{11}^* - B_{66}^*) f_{xyy} + D_{11}^* \phi_{xxx} + (D_{12}^* + D_{66}^*) \phi_{xyy} + D_{66}^* \phi_{yy} - A_{44} w_x - A_{44} \phi_x &= 0, \\
B_{21}^* f_{yyy} + (B_{22}^* - B_{66}^*) f_{xyy} + D_{22}^* \phi_{yy} + (D_{66}^* + D_{22}^*) \phi_{xyy} + D_{66}^* \phi_{yy} - A_{55} w_y - A_{55} \phi_y &= 0,
\end{align*}
\]

(16a)

(16b)

(16c)
where \( w' (x, y) \) is a known function representing the initial small imperfections of the plate.

The system of Equations (16) include four unknown functions \( (w, \phi_x, \phi_y, \text{and } f) \); so it is necessary to find the fourth equation relating to these functions using the compatibility equation (Equation 7). For this purpose, substituting the expressions of \( \varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0 \) from Equation (11) into Equation (7), we get:

\[
A_{11}^* f_{xxx} + (A_{66}^* - 2A_{12}^*) f_{xxyy} + A_{22}^* f_{yyyy} - B_{21}^* \phi_{xxx} - (B_{11}^* - B_{66}^*) \phi_{xxyy} - (B_{12}^* - B_{66}^*) \phi_{yyxx} - B_{12}^* \phi_{yxyy} - \left( \frac{w}{y} \right)^2 - 2w_x w_{xy} + w_{xx} w_{yy} + w_{xy} w_{yy} + w_{xx} w_{xy} = 0. \tag{17}
\]

Equations (16) and (17) are nonlinear equations in terms of the four dependent unknown functions \( (w, \phi_x, \phi_y, \text{and } f) \) used to investigate the buckling and post-buckling of an eccentrically stiffened functionally graded plate on elastic foundations subjected to compression, thermal and combined loads. The boundary condition (BC), will be considered:

The edges are simply supported and immovable (IM). The associated BCs are

\[
w = u = \phi_y = M_x = 0, \quad N_x = N_{xo} \text{ at } x = 0 \text{ and } x = a \]
\[
w = v = \phi_x = M_y = 0, \quad N_y = N_{yo} \text{ at } y = 0 \text{ and } y = b \tag{18}
\]

here, \( N_{xo}, N_{yo} \) are the pre-buckling force resultants in directions \( x \) and \( y \), respectively.

To solve Equations (16) and (17) for unknowns \( w, \phi_x, \phi_y, \text{and } f \), and with consideration of the BC (18), the following approximate solutions (Dung & Nga, 2013) are assumed:

\[
w = W \sin \alpha x \sin \beta y, \tag{19}
\]
\[
\phi_x = \lambda_1 \cos \alpha x \sin \beta y + \lambda_2 \sin 2\alpha x, \quad \phi_y = \lambda_3 \sin \alpha x \cos \beta y + \lambda_4 \sin 2\beta y, \tag{19}
\]
\[
f = F_1 \cos 2\alpha x + F_2 \cos 2\beta y + F_3 \sin \alpha x \sin \beta y + N_{xo} y^2 / 2 + N_{yo} x^2 / 2,
\]

where \( \alpha = m \pi / a, \beta = n \pi / b, m, n = 1, 2, \ldots \) are the number of half waves in the \( x, y \) directions, respectively; and \( W \) is the amplitude of deflection. Also, \( \lambda_i \) \( (i = 1−4) \) and \( F_i \) \( (i = 1−3) \) are coefficients to be determined.

Considering the BC (18), the imperfections of the plate are assumed as:

\[
w' = \mu h \sin \alpha x \sin \beta y, \tag{20}
\]

where the coefficient \( \mu \), varying between 0 and 1, represents the size of the imperfections.

After substituting Equations (19) and (20) into Equations (16b), (16c), and (17), the coefficients \( \lambda_i \) \( (i = 1−4) \) and \( F_i \) \( (i = 1−3) \) are found as:

\[
F_1 = f_1 W(W + 2\mu h), \quad F_2 = f_2 W(W + 2\mu h), \quad F_3 = f_3 W, \tag{21}
\]
\[
\lambda_1 = L_1 W, \quad \lambda_2 = L_2 W(W + 2\mu h), \quad \lambda_3 = L_3 W, \quad \lambda_4 = L_4 W(W + 2\mu h), \quad \lambda_i \quad (i = 1−4), \quad L_j \quad (j = 1−4, 5−7)
\]

and specific expressions of coefficients \( f_i \) \( (i = 1−3) \) and \( L_j \) \( (j = 1−4) \) are given in Appendix A.

Introduction of Equations (19) and (20) into Equation (16a), and applying the Galerkin method for the resulting equation yields:
where specific expressions of coefficients

Substitution of Equation (26) into Equation (25), and then the result into Equation (22), gives:

From Equations (4) and (11), one can obtain the following expressions in which Equation (14) and imperfection has been included:

Substitute Equations (19) and (20) into Equation (24), and then the result into Equation (23), gives:

From Equation (10), we have:

Substitution of Equation (26) into Equation (25), and then the result into Equation (22), gives:

where specific expressions of coefficients $e_i^p$ ($i = 1 - 4$) are given in Appendix A and $\overline{W} = W/h$. 
By setting $\mu = 0$, Equation (27) leads to an equation from which buckling temperature change of the eccentrically stiffened perfect FGM plates may be determined as $\Delta T_b = e_2^2$.

In the case of T-D, both sides of Equation (27) are temperature-dependent, which makes it very difficult to solve. Fortunately, a numerical technique using an iterative algorithm has been applied to determine the buckling loads, as well as to determine the deflection-load relationship in the post-buckling period of the eccentrically stiffened FGM plate. Given more details, including the material parameter $N$, the geometrical parameter $(b/a, b/h)$, and the value of $W/h$, we can determine $\Delta T$ in Equation (27), $\Delta T_1$. This iterative procedure will stop at the $k$th step if $\Delta T_k$ satisfies the condition $|\Delta T - \Delta T_k| \leq \xi$. Here, $\Delta T$ is a desired solution for the temperature and $\xi$ is a tolerance used in the iterative steps. This is also an interesting point to be solved in this article.

4. Numerical results and discussion

In this section, the components of the material are silicon nitride $Si_3N_4$ (ceramic) and SUS304 stainless steel (metal). The material properties $(Pr)$ in Equation (2) are shown in Table 1 and Poisson’s ratio is chosen to be $v = 0.3$.

In particular, for the case of an S-FGM plate without stiffeners with the conditions: $A_1^T = A_2^T = 0$, $I_1^T = I_2^T = 0$ and ceramic – metal – ceramic layers which are compared the numerical results of unstiffened thick S-FGM plates with Duc and Tung (2010). As can be seen, a good agreement is obtained in this comparison (Figure 2).

Figure 3 illustrates the effect of eccentric stiffeners on the nonlinear response of S-FGM plates under thermal loads. It is clear that the stiffeners can enhance the thermal loading capacity for the imperfect and perfect S-FGM plates.

Figure 4 presents the effect of volume fraction index($N$) on the post-buckling of eccentrically stiffened S-FGM plates under thermal load. These post-buckling curves show that the loading ability of S-FGM plates become worse with the increase of $N$. It leads the plate’s stiffness to be decreased which results in the decrease of load-carrying of eccentrically stiffener S-FGM plate (the module elastic $E$ of metal is lower than that of ceramic, $E_c > E_m$).

Figure 5 shows the effects of the elastic foundations on the nonlinear response of eccentrically stiffened S-FGM plates with temperature-dependent material properties. Elastic foundations are recognized to have strong impact, as demonstrated by curves (1) and (2), which show that the ability of sustaining compression and thermal load will increase if the effects of elastic foundations enhance from $(K_1 = 0, K_2 = 0)$ to $(K_1 = 100, K_2 = 0)$. Furthermore, Pasternak’s elastic foundation $(K_1)$ is more powerful than Winkler’s foundation $(K_2)$, which is proven by curve (3) with $K_1 = 100, K_2 = 10$, and curve (4) with $K_1 = 50, K_2 = 20$.

Figure 6 shows the effects of imperfections on post-buckling response of the eccentrically stiffened S-FGM plates under thermal load. In the post-buckling period, the imperfections have actively

| Property | Material | $P_{1}$ | $P_{9}$ | $P_{1}$ | $P_{2}$ | $P_{3}$ |
|----------|----------|---------|---------|---------|---------|---------|
| $E (P_{1})$ | Silicon nitride | 0 | 348.43 $\times$ 10$^9$ | $-3.070 \times 10^{-4}$ | $2.160 \times 10^{-7}$ | $-8.946 \times 10^{-11}$ |
|         | Stainless steel | 0 | 201.04 $\times$ 10$^9$ | $3.079 \times 10^{-4}$ | $-6.534 \times 10^{-7}$ | 0 |
| $\alpha (1/K)$ | Silicon nitride | 0 | 5.8723 $\times$ 10$^{-6}$ | $9.095 \times 10^{-4}$ | 0 | 0 |
|         | Stainless steel | 0 | 12.330 $\times$ 10$^{-6}$ | $8.086 \times 10^{-4}$ | 0 | 0 |

Notes: $h_1 = 0.08 \text{ m}$, $h_2 = 0.08 \text{ m}$, $b_1 = 0.008 \text{ m}$, $b_2 = 0.008 \text{ m}$, $d_1 = 0.15 \text{ m}$, $d_2 = 0.15 \text{ m}$, $E_c = E_m$. 

|
affected the load bearing ability of the plate. In other words, the loading ability increases together with $\mu$.

Figure 7 shows the effects of temperature-dependent material properties on the nonlinear stability of eccentrically stiffened thick S-FGM plates under thermal load. There is a comparison between...
the thermal post-buckling curves of both perfect and imperfect FGM plates with T-D and T-ID material properties. It is apparent that T-D material properties make the S-FGM plate considerably weaker under thermal load.
5. Conclusions

This paper presents an analysis approach, in conjunction with an iterative procedure, to investigating the buckling and post-buckling behavior of the eccentrically stiffened S-FGM plates under thermal load. The formulation is based on the first-order shear deformation theory, accounting for both the von Karman nonlinearity and initial imperfections. The paper also analyzes and discusses the effects of material and geometrical properties, temperature, elastic foundations, and eccentric stiffeners on the buckling and post-buckling loading capacity of the eccentrically stiffened S-FGM plate in thermal environments.

Funding
This work was supported by the Grant of Newton Fund (UK) [grant number NRCP1516/1/68].

Author details
Pham Hong Cong1,2
E-mail: congph54@gmail.com
Nguyen Dinh Duc1
E-mail: ducnd@vnu.edu.vn
1 Centre for Informatics and Computing, Vietnam Academy of Science and Technology, 18 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam.
2 University of Engineering and Technology - Vietnam National University, 144 Xuan Thuy, Cau Giay, Hanoi, Vietnam.

Citation information
Cite this article as: Thermal stability analysis of eccentrically stiffened Sigmoid-FGM plate with metal – ceramic – metal layers based on FSDT, Pham Hong Cong & Nguyen Dinh Duc, Cogent Engineering (2016), 3: 1182098.

Cover image
Source: Author.

References
Bich, D. H., Nam, V. H., & Phuong, N. T. (2011). Nonlinear postbuckling of eccentrically stiffened functionally graded plates and shallow shells. Vietnam Journal of Mechanics, 33, 131–147.
Duc, N. D. (2014). Nonlinear static and dynamic stability of functionally graded plates and shells. Hanoi: Vietnam National University Press.
Duc, N. D., & Cong, P. H. (2014). Nonlinear post-buckling of an eccentrically stiffened thin FGM plate resting on elastic foundations in thermal environments. Thin-Walled Structures, 75, 103–112.
Duc, N. D., & Cong, P. H. (2015). Nonlinear vibration of thick FGM plates on elastic foundation subjected to thermal and mechanical loads using the first-order shear deformation plate theory. Cogent Engineering, 2, 104522.
Duc, N. D., & Tung, H. V. (2010). Mechanical and thermal postbuckling of shear-deformable FGM plates with temperature-dependent properties. Mechanics of Composite Materials, 46, 461–476.
Dung, D. V., & Ngo, N. T. (2013). Nonlinear buckling and post-buckling of eccentrically stiffened functionally graded cylindrical shells surrounded by and elastic medium based on the first order shear deformation theory. Vietnam Journal of Mechanics, 35, 285–298.
Dung, D. V., & Thiem, H. T. (2012). On the nonlinear stability of eccentrically stiffened functionally graded imperfect plates resting on elastic foundation. In Proceedings of the 2nd international conference on engineering mechanics and automation (ICEMA2) (pp. 216–225). Hanoi.
Ghasian, S. E., Kiani, Y., Sadighi, M., & Eslami, M. R. (2014). Thermal buckling of shear deformable temperature dependent circular/annular FGM plates. International Journal of Mechanical Sciences, 81, 137–148.
http://dx.doi.org/10.1016/j.ijmecsci.2014.02.007
Koizumi, M. (1997). FGM activities in Japan. Composites part B: Engineering, 28(1), 1–4.
http://dx.doi.org/10.1016/S1355-8368(96)00016-9
Mo, L. S., & Wang, T. J. (2004). Buckling of functionally graded circular/annular plates based on the first-order shear deformation plate theory. Key Engineering Materials, 261–263, 609–614.
http://dx.doi.org/10.4028/www.scientific.net/KEM.261-263
Reddy, J. N. (2004). Mechanics of laminated composite plates and shells: Theory and analysis. Boca Raton, FL: CRC Press.
Reddy, J. N., & Chin, C. D. (1998). Thermomechanical analysis of functionally graded cylinders and plates. Journal of Thermal Stresses, 21, 593–626.
http://dx.doi.org/10.1080/01495739791856165
Seren Akavci, S., Yerli, H. R., & Dogan, A. (2007). The first order shear deformation theory for symmetrically laminated composite plates on elastic foundation. The Arabian Journal for Science and Engineering, 32, 341–348.
Shen, H. S. (1997). Thermal post-buckling analysis of imperfect laminated plates using a higher-order shear deformation theory. International Journal of Non-Linear Mechanics, 32, 1035–1050.
http://dx.doi.org/10.1016/S0020-7462(96)00135-7
Shen, H. S. (2007). Thermal post-buckling behaviour of shear deformable FGM plates with temperature-dependent properties. International Journal of Mechanical Sciences, 49, 466–478.
Swaminathan, K., Naveenkumar, D. T., Zenkour, A. M., & Carrera, E. (2015). Stress, vibration and buckling analyses of FGM plates–A state of the art review. Composite Structures, 120, 10–31.
http://dx.doi.org/10.1016/j.compstruct.2014.09.070
Thai, H. T., & Kim, S. E. (2015). A review of theories for the modeling and analysis of functionally graded plates and shells. Composite Structures, 128, 70–84.
http://dx.doi.org/10.1016/j.compstruct.2015.03.010
Touloukian, Y. S. (1967). Thermophysical properties of high temperature solid materials. New York, NY: Macmillan.
Wu, L. (2004). Thermal buckling of a simply supported moderately thick rectangular FGM plate. Composite Structures, 64, 211–218.
http://dx.doi.org/10.1016/j.compstruct.2003.08.004
Appendix A

\[ A_{11} = A_{22} = E_1/(1 - v^2), \quad A_{12} = E_1 v/(1 - v^2), \quad A_{66} = E_1/(2(1 + v)), A_{44} = \chi_1 E_1 (2(1 + v)), \]
\[ A_{55} = \chi_1 E_1/(2(1 + v)), B_{11} = B_{22} = E_2/(1 - v^2), \quad B_{12} = E_2 v/(1 - v^2), \quad B_{66} = E_2/(2(1 + v)), \]
\[ D_{11} = D_{22} = E_3/(1 - v^2), \quad D_{12} = E_3 v/(1 - v^2), \quad D_{66} = E_3/(2(1 + v)), \]

where \( \chi_1, \chi_2 \) are the shear correction factor, \( \chi_1 = \chi_2 = 4/3 \).

\[ E_1 = E_m h + E_{cm} h/(N + 1), \quad E_2 = 0, \]
\[ E_3 = E_m h^3/12 + E_{cm} h^3 (1/(N + 3) - 1/(N + 2) + 1/(4N + 4)), \]
\[ \Phi_1 = -1/ \left( 1 - v \right) \int_{-h/2}^{h/2} E(z)\sigma(z)\Delta T dz, \quad \Phi_2 = -1/ \left( 1 - v \right) \int_{-h/2}^{h/2} E(z)\sigma(z)\Delta T dz, \]
\[ (11) \]
\[
\begin{align*}
\Delta &= (A_{11} + E_0 A_{11}'/d_1') (A_{22} + E_0 A_{22}'/d_2') - A_{12}, A_{11}' = (A_{11} + E_0 A_{11}'/d_1'), \\
A_{22}' &= (A_{22} + E_0 A_{22}'/d_2')/\Delta, \quad A_{12}' = A_{12}/\Delta, \quad A_{66}' = 1/A_{66}, \\
B_{11}' &= A_{11}'(B_{11} + C_1') - A_{12}' B_{12}, B_{22}' = A_{11}'(B_{22} + C_2') - A_{12}' B_{12}, \\
B_{12}' &= A_{12}' B_{12} - A_{12}'(B_{22} + C_2'), B_{21}' = A_{11}' B_{12} - A_{12}'(B_{11} + C_1'), \\
B_{66}' &= B_{66}/A_{66}, D_{11}' = D_{11} + E_0 A_{11}'/d_1' - B_{11}'(B_{11} + C_1') - B_{21}' B_{12}, \\
D_{22}' &= D_{22} + E_0 A_{22}'/d_2' - B_{22}'(B_{22} + C_2') - B_{12}' B_{12}, \\
D_{12}' &= D_{12} - B_{12}'(B_{11} + C_1') - B_{22}' B_{12}, D_{21}' = D_{21} - B_{21}'(B_{22} + C_2') - B_{11}' B_{12}, \\
D_{66}' &= D_{66} - B_{66} B_{66}, C_{11}' = (A_{12} - A_{22} - E_0 A_{12}'/d_1')/\Delta; \\
C_{22}' &= (A_{12} - A_{11} - E_0 A_{11}'/d_1')/\Delta, C_{12}' = C_{11}'(B_{11} + C_1') + C_{22}' B_{12}, C_{21}' = C_{22}'(B_{22} + C_2') + C_{11}' B_{12}.
\end{align*}
\]
\[
f_1 = \left( A_{44} B_2' + 4B_0 D_{11}' m^2 \pi^2 \right) h n/ \left\{ 32m^2 B_0' \left[ A_{11}'(A_{44} B_2' + 4m^2 \pi^2 B_0' D_{11}') + 4m^2 \pi^2 B_2' D_{21}' \right] \right\},
\]
\[
f_2 = \left( 4n^2 \pi^2 D_{22}' + A_{55} B_2^2 \right) m^2 B_0^2 / \left\{ 32n^2 \left[ A_{22}'(4n^2 \pi^2 D_{22}' + A_{55} B_2^2) + 4n^2 \pi^2 B_{12}' \right] \right\},
\]
\[
f_3 = h^2 (\bar{a}_{31}/\bar{a}_{33} L_1 + \bar{a}_{32}/\bar{a}_{33} L_3),
\]
\[
L_1 = - \left[ \left( \bar{a}_{12} \bar{a}_{35} + \bar{a}_{13} \bar{a}_{35} \right) L_3 + \bar{a}_{33} A_{44} m_\pi B_0 / \left( B_0 \left( \bar{a}_{11} \bar{a}_{33} + \bar{a}_{13} \bar{a}_{31} \right) \right) \right] / \left( \left( \bar{a}_{11} \bar{a}_{33} + \bar{a}_{13} \bar{a}_{31} \right) h \right),
\]
\[
L_2 = 8m^2 \pi^2 B_0' D_{21}' / \left( B_0 \left( A_{44} B_2' + 4m^2 \pi^2 B_0' D_{11}' \right) h^2 \right) L_1,
\]
\[
L_3 = \frac{\bar{a}_{33} (\bar{a}_{11} \bar{a}_{33} + \bar{a}_{13} \bar{a}_{31}) A_{55} m_\pi - \bar{a}_{33} (\bar{a}_{22} \bar{a}_{33} + \bar{a}_{23} \bar{a}_{32}) A_{44} m_\pi B_0}{B_0 \left( \bar{a}_{11} \bar{a}_{33} + \bar{a}_{13} \bar{a}_{31} \right) \left( \bar{a}_{22} \bar{a}_{33} + \bar{a}_{23} \bar{a}_{32} \right) - \left( \bar{a}_{22} \bar{a}_{33} + \bar{a}_{23} \bar{a}_{32} \right) \left( \bar{a}_{11} \bar{a}_{33} + \bar{a}_{13} \bar{a}_{31} \right) h},
\]
\[
L_4 = \frac{8n^4 \pi^2 B_0 B_0}{B_0 (4n^4 \pi^2 D_{12} + B_0^2 A_{11})} \overline{f}_1 \frac{1}{h^2} \overline{f}_1 = \frac{f_2}{h}, \overline{f}_2 = \frac{f_2}{h},
\]

\[
\overline{f}_3 = \frac{f_3}{h}, \overline{f}_4 = L_1 h, \overline{f}_5 = L_3 h, \overline{f}_6 = L_4 h^2,
\]

\[
\overline{a}_{11} = \frac{m^4 \pi^4 B_0^4}{B_0^4} \bar{D}_{11} + \frac{n^2 \pi^4}{B_0^4} \bar{D}_{66} + A_{44}, \overline{a}_{12} = \frac{B_0 mn \pi^4}{B_0^2} (\bar{D}_{11} + \bar{D}_{66}),
\]

\[
\overline{a}_{13} = B_0^3 \pi^3 \frac{\bar{B}_3}{B_0^2} (\bar{B}_{11} - \bar{B}_{66}),
\]

\[
\overline{a}_{21} = mn \pi^2 B_0 (\bar{D}_{11} + \bar{D}_{66}) / B_0^2, \overline{a}_{22} = n^2 \pi^2 / B_0^2 \bar{D}_{22} + m^2 \pi^2 B_0^2 / B_0^2 \bar{D}_{66} + A_{55},
\]

\[
\overline{a}_{23} = n^3 \pi^3 / B_0^2 \bar{B}_{11} + m^2 n \pi^3 B_0^2 / B_0^2 (\bar{B}_{11} - \bar{B}_{66}),
\]

\[
\overline{a}_{31} = \left[ m^2 B_0^2 \bar{D}_{11} + n^2 \bar{B}_{11} - \bar{B}_{66} \right], \overline{a}_{32} = [B_0^2 mn \pi^4 (\bar{B}_{22} - \bar{D}_{66}) + n^3 \pi^3 \bar{B}_{12}] / B_0^3,
\]

\[
\overline{a}_{33} = \left[ B_0^2 m^4 \pi^4 \bar{A}_{11} + B_0^2 m^2 \pi^4 \bar{A}_{12} - 2 \bar{A}_{12} \right] / B_0^3,
\]

\[
\overline{A}_{11} = A_{11} / h, \overline{A}_{12} = A_{12} / h, \overline{A}_{66} = A_{66} / h,
\]

\[
\overline{A}_{44} = A_{44} / h, \overline{A}_{55} = A_{55} / h, \overline{A}_{11} = B_{11} / h, \overline{A}_{22} = B_{22} / h, \overline{A}_{12} = B_{12} / h,
\]

\[
\overline{B}_{11} = B_{11} / h, \overline{B}_{66} = B_{66} / h,
\]

\[
\overline{C}_{11} = C_{11}, \overline{C}_{22} = C_{22}, \overline{D}_{11} = D_{11}, \overline{D}_{22} = D_{22}, \overline{D}_{12} = D_{12}, \overline{D}_{21} = D_{21}, \overline{D}_{66} = D_{66}.
\]

\[
e_2^2 = \frac{8 mn \pi^2 B_0 B_0}{3 B_0^2 P_1 H} - B_0^2 \left\{ 2 m^2 n^2 \pi^2 \bar{B}_0 B_0 / B_0^2 \left( \bar{A}_{11}^2 - \bar{A}_{12} \bar{A}_{22} \right) \right\} \overline{f}_1 / B_0^2
\]

\[
+ \frac{m \pi B_0 / B_0^2}{B_0^2} \left[ m^2 \pi^2 \bar{B}_0 (\bar{B}_{11} A_{11}^2 + \bar{B}_{21} A_{12}^2 + n^2 \pi^2 \bar{B}_{11} A_{11}^2 + \bar{B}_{21} A_{12}^2) \right] \overline{L}_1
\]

\[
+ \frac{n \pi / B_0^2}{B_0^2} \left[ m^2 \pi^2 \bar{B}_0 (\bar{B}_{12} A_{11}^2 + \bar{B}_{22} A_{12}^2) \right] \overline{L}_1
\]

\[
+ \frac{m \pi / B_0^2}{B_0^2} \left[ m^2 \pi^2 \bar{B}_0 (\bar{B}_{11} A_{11}^2 + \bar{B}_{21} A_{12}^2) \right] \overline{L}_1
\]

\[
+ \frac{n \pi / B_0^2}{B_0^2} \left[ m^2 \pi^2 \bar{B}_0 (\bar{B}_{12} A_{11}^2 + \bar{B}_{22} A_{12}^2) \right] \overline{L}_1
\]

\[
+ \frac{m \pi / B_0^2}{B_0^2} \left[ m^2 \pi^2 \bar{B}_0 (\bar{B}_{11} A_{11}^2 + \bar{B}_{21} A_{12}^2) \right] \overline{L}_1
\]

\[
+ \frac{n \pi / B_0^2}{B_0^2} \left[ m^2 \pi^2 \bar{B}_0 (\bar{B}_{12} A_{11}^2 + \bar{B}_{22} A_{12}^2) \right] \overline{L}_1
\]

\[
+ \frac{mn \pi^2}{B_0^2} \left( \bar{A}_{12}^2 - \bar{A}_{11} A_{12} \right) P_1 H B_0^2,
\]

\[
e_2^2 = B_0^2 \left\{ n^4 \pi^4 B_0^2 \bar{B}_{11} - m^2 \pi^4 B_0^2 (\bar{B}_{11} - 2 \bar{B}_{66} + \bar{B}_{22}) + n^4 \pi^4 B_0^2 \right\} \overline{f}_1 / B_0^2
\]

\[
+ \left[ m^2 \pi^2 \bar{B}_0 (\bar{B}_{11} A_{11}^2 + \bar{B}_{21} A_{12}^2) \right] \overline{L}_1 / B_0^3
\]

\[
+ \left[ m^2 \pi^2 \bar{B}_0 (\bar{B}_{12} A_{11}^2 + \bar{B}_{22} A_{12}^2) \right] \overline{L}_1 / B_0^3
\]

\[
- K_1 \overline{D}_{12} B_0^2 / B_0^2 - (B_0^2 m^2 \pi^2 + n^2 \pi^2) \overline{D}_{12} B_0^2 / B_0^2, \right\} P_1 B_0^2 / (4 B_0^2 P_1 H),
\]

\[
e_2^2 = 32 \left( m^2 \pi^2 B_0^2 \bar{D}_{11} L_2 + n^2 \pi^2 B_0^2 \bar{D}_{22} L_2 - 2 m^2 \pi^2 B_0^2 \bar{D}_{21} L_2 - 2 n^2 \pi^2 B_0^2 \bar{D}_{12} L_2 \right) / (3 m n \pi^2 P_1 H B_0^2),
\]

\[
e_2^2 = 32 \left( m^2 \pi^2 B_0^2 \bar{D}_{11} L_2 + n^2 \pi^2 B_0^2 \bar{D}_{22} L_2 - 2 m^2 \pi^2 B_0^2 \bar{D}_{21} L_2 - 2 n^2 \pi^2 B_0^2 \bar{D}_{12} L_2 \right) / (3 m n \pi^2 P_1 H B_0^2),
\]

\[
- m^2 n^2 \pi^2 B_0 \left( \overline{f}_1 + \overline{f}_2 \right) / (2 B_0^2 P_1 H),
\]

\[
P_1 = \pi^2 \left\{ - \left( C_{11} A_{11}^2 + C_{22} A_{22}^2 \right) n^2 - \left( C_{22} A_{11}^2 + C_{11} A_{11} \right) B_0^2 m^2 \right\} \left( 4 B_0 \left( A_{12}^2 - A_{11} A_{22} \right) \right) .
\]
