High-utility itemset mining for subadditive monotone utility functions

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Abstract. High-utility Itemset Mining (HUIM) finds itemsets from a transaction database with utility no less than a user-defined threshold where the utility of an itemset is defined as the sum of the utilities of its items. In this paper, we introduce the notion of generalized utility functions that need not be the sum of individual utilities. In particular, we study subadditive monotone (SM) utility functions and prove that it generalizes the HUIM problem mentioned above. Moving on to HUIM algorithms, the existing algorithms use upper-bounds like “Transaction Weighted Utility” and “Exact-Utility, Remaining Utility” for efficient search-space exploration. We derive analogous and tighter upper-bounds for SM utility functions and explain how existing HUIM algorithms of different classes can be adapted using our upper bound. We experimentally compared adaptations of some of the latest algorithms and point out some caveats that should be kept in mind while handling general utility functions.

Keywords: High-utility itemset mining, Subadditive monotone function, Graph-based utility function

1 Introduction

High-utility itemset mining (HUIM) finds itemsets from a transaction database with utility no less than a user-defined threshold [1]. Every item in a transaction is associated with a quantity and a global importance, say profit. The utility of an item is defined as the product of its quantity and profit. The utility of an itemset in a transaction is the sum of the utilities of individual items. HUIM can be used to identify sets of items that generated large profit over many transactions.

HUIM has also been applied to extract co-expression patterns from gene-expression data [2]. With the penetration of data mining techniques in hitherto untrodden areas, we envisage scenarios where “utility” of an itemset (in a transaction) is not a simple addition of the utilities of the individual items. We define this problem as “high generalized-utility itemset mining” (HGUIM) and investigate this problem for utility functions that are subadditive and monotone. Our results can be seen as a generalization of HUIM since summation (of utilities) is such a function. While monotonicity appears to be a natural requirement, subadditivity stipulates the utility of a combination of items cannot be more than
the combination of individual utilities. Subadditivity can be observed at several places in the retail domain, e.g., customers frequently purchase a set of products at less price compared to the sum of the individual prices. Retail stores often offer discounts \[3\] to customers on purchasing a hamper of products.

Generalization of the utility function can lead to interesting applications of itemset mining techniques firstly by allowing novel mapping of quantities to utility and furthermore, by allowing integration of additional information (say, on items) for computing utility. For an example of the latter, consider a social network where the interests of every user are known. Suppose one transaction is created for every distinct interest, e.g., comprising of members who like “chess”, and suppose the utility of a set of members is some clustering score (e.g., network modularity \[4\]) of that group. Then, HGUIM will reveal groups of users that have a high overall modularity score across all interests.

With general utility functions in mind, we explored how they can be incorporated within existing HUIM algorithms. We ask the question if, when and how, can an existing high-utility itemset mining algorithm be adapted for any arbitrary subadditive monotone function? Will the existing bounds used for search-space exploration still work? Our specific contributions are summarized below.

1. We introduce the problem of high-utility itemset mining for a general utility function. We show that current HUIM problem is a special case of HGUIM using a subadditive and monotone utility function.
2. We show how to design subadditive and monotone utility functions that also consider additional information about relationship between the items, e.g., in the form a relationship graph.
3. We investigate how existing tree-based, list-based and projection-based HUIM algorithms can be adapted for subadditive monotone utility functions by deriving new and tight upper bounds.
4. We conduct extensive experiments with adaptations of existing HUIM algorithms on real dense and sparse datasets. Our results demonstrate that computation of utility can be a big factor in the runtime performance.

2 Related work

Several algorithms have been proposed in the literature for HUIM. Broadly these algorithms vary in terms of the number of stages they run for (one phase and two phase), their database representation (tree, utility list and projected database) and other data structures and heuristics used to efficiently prune non-candidates.

Liu et al. \[1\] developed a two-phase algorithm for mining high-utility itemsets from a transaction database. This algorithm generates a candidate set of high-utility itemsets in the first phase using transaction-weighted utilities \[5\]. A database scan is performed in the next phase to filter out the high-utility itemsets from the candidates however a lot of database scans are necessary during the candidate generation phase. Ahmed et al. \[6\] proposed the first recursive tree-based algorithm for mining high-utility itemsets for incremental and interactive pattern mining using only two database scans. Tseng et al. \[7\] proposed another
tree data structured called UP-Tree and several strategies for estimating tight utility estimates. Dawar et al. [8] proposed UP-Hist data structure to further reduce the number of candidates generated.

Liu et al. [9] proposed the first algorithm for mining high-utility itemsets in a single phase. The authors proposed an inverted-list data structure that stores the transaction identifier (TID), exact utility (EU), and remaining utility (RU) with each itemset. Fournier-Viger et al. [10] proposed an “Efficient Utility Co-occurrence Structure” (EUCS) strategy to reduce the number of join operations by keeping the TWU information for every pair of items and an algorithm called FHM to make use of EUCS. Zida et al. [11] proposed a fast and memory-efficient algorithm called EFIM that stores the transactions as a projected database recursively during the mining process. EFIM was about two to three orders of magnitude faster than UP-Growth+, FHM, and several other algorithms on dense datasets. Liu et al. [12] proposed a data structure called Chain of Accurate Utility Lists (CAUL) and an algorithm called D2HUP for efficiently mining high-utility itemsets from sparse datasets. Dawar et al. [13] proposed a hybrid algorithm that combined the benefits of tree-based and list-based algorithms by mining high-utility itemsets without candidate generation.

However, all of the above techniques work for a specific utility function and do not generalize to subadditive monotone utility functions.

3 HGUI Mining Problem

Consider the usual setting of HUIM on a transaction database \( D \) with \( n \) transactions \( \{T_1, T_2, \ldots, T_n\} \) over items \( I = \{i_1, i_2, \ldots, i_m\} \). Each transaction can be thought of as a subset of \( I \) along with a quantity (or weight) associated with every item \( i \in T \); we will use \( q(i, T) \) to denote this quantity and we will express \( T \) as \( \{(i_{j_1} : n_{j_1}), (i_{j_2} : n_{j_2}), \ldots, (i_{j_k} : n_{j_k})\} \) where \( i_j \) denotes an item in \( T \) and \( n_j = q(i_j, T) \) is its quantity in \( T \). By an “itemset \( X \) in \( T \)” we will mean a set of some items appearing in \( T \) and associated with the same quantity as that in \( T \). We will use “weighted itemset” to refer to such sets with quantities.

For this work we assume that there is some “generalized utility” function \( f(\cdot) \) that returns a numerical value for any weighted itemset. For example, \( f(\{(x_1 : n_1), (x_2 : n_2), \ldots, (x_k : n_k)\}) \) could be its cardinality \( k \), the addition function over the quantities \( \sum_j n_j \) or some complex function depending upon the items and their quantities.

Now, let \( T \) be some transaction; for any subset \( X = \{x_1, \ldots\} \) of items from \( T \), we can define the utility of \( X \) in \( T \), denoted by \( u(X, T) \), as the value of \( f(\cdot) \) on the weighted itemset \( \{(x_1, q(x_1, T)), \ldots\} \). The utility of \( X \) in the database is defined as in HUIM: \( u(X) = \sum_{X \subseteq T, T \in D} u(X, T) \).

**HGUIM problem:** An itemset \( X \) is called a high general-utility itemset if \( u(X) \) is no less than a given minimum user-defined threshold denoted by \( \theta \). Given a transaction database \( D \), a generalized-utility function \( f(\cdot) \) and a minimum user-defined threshold \( \theta \), the aim is to find all high-utility itemsets.

Current HUIM algorithms work by recursively growing an itemset (called as the prefix) by appending an item from the set of items to the prefix and then
determining whether the newly formed prefix (called β) is high-utility or not. For this, a projected database for β is constructed that contains all transactions with itemset β, and the algorithm recursively extends β. Since the search space of prefixes is exponential in the number of distinct items present in the database, the computational challenge is to efficiently determine whether there could be any high-utility itemset containing β. For this purpose, current HUIM algorithms use upper bound functions of \( u(\beta) \) to decide for further exploration.

Even if the current HUIM algorithms are adapted for HGUIM, it is unclear if the current bounds will hold true for a different, general, utility function. Since a correct, and preferably tight, bound is crucial for efficient pruning of search space and also depends on the data-structures used, careful attention must be given to coming up with an appropriate bound function. We undertake this task for certain types of utility functions in the next few sections.

4 Subadditive and monotone utility functions

Designing a generic yet efficient HGUIM algorithm is tricky since it has to work for all, including badly designed, utility functions. We leave that question for future but in this work show how to design HGUIM algorithms for any subadditive and monotone utility function — such functions include the \( \text{Sum}() \) utility function (defined below) that is used to define HUIM. For the definitions below, \( I \) is the universe of items and \( T \) is some transaction, essentially a weighted itemset.

**Definition 1 (subadditive and monotone function).** Consider functions that map weighted itemsets over \( I \) to integers. Such a function \( f() \) is subadditive if \( \forall X, Y \subseteq T, f(X \cup Y) \leq f(X) + f(Y) \). \( f() \) is monotone if \( \forall X \subseteq Y \subseteq T, f(X) \leq f(Y) \). A utility function \( u(\cdot,T) \) is subadditive and monotone if satisfies the above for every transaction.

It should be noted that we require \( u(X,T) \) to be subadditive and monotone for any \( T \) but no such property may hold for \( u(X) \).

We now give a few examples of subadditive and monotone utility functions. First is \( \text{Sum}({x_1 : n_1}, \ldots, {x_k : n_k}) = \sum_j n_j \), essentially returning the total quantity of items in an itemset. One should note that this is the utility function used in HUIM. Two other examples are \( \log(\prod_j n_j) \) and \( \sqrt[\sum_j n_j]{} \). It is known that the last function is subadditive and monotone \(^{[14]}\) and these properties are easy to verify for the first two functions as well.

4.1 Graph-based utility function

The examples of utility functions on weighted itemsets stated earlier depend on the quantities of included items but do not depend upon a particular grouping of items. Now we describe another utility function that not only depends upon the items but also allows us to incorporate any additional information on the relationship between the items into the utility function. Referring to the applications discussed in Section II such knowledge may be helpful in mining retail logs, and finding active communities in a social network.
Fig. 1: (a) Transaction database for coverage utility (b) Graph over items.

Given an undirected unweighted graph $G$ with vertices $V$, graph coverage ($Co(U)$) of a subset of vertices $U$ is defined as the cardinality of the set containing $U$ and the immediate neighbors of the vertices in $U$. It is known that $Co()$ is a subadditive and monotone function [13]. For example, coverage of \{A, C\} in the graph shown in Figure 1(b) is $|\{A, B, C, D\}| = 4$.

Coming back to high-utility itemset mining, suppose in addition to a transaction database we are also given an additional graph $G$ over the items (objects) that capture their pairwise relationships (e.g., see Figure 1), say friendship. Assume that a transaction defines a set of objects along with their frequency of activities on a particular day. Now we define a utility function named $ucov()$ that combines the quantity information in the database and relationship among the objects in terms of the coverage information in $G$. The function captures the notion of minimal coverage/influence of a set of objects and mined patterns can be used as a target for marketing purposes.

**Definition 2 (coverage utility of an itemset).** Let $T$ be some transaction and let $X$ be a weighted itemset with $k$ items from that transaction: $X = \{ (x_1 : q_1), (x_2 : q_2), \ldots, (x_k : q_k) \}$. Suppose that the items are ordered such that $q_i \leq q_{i+1}$ for all $i$. We define the coverage utility of $X$ in $T$ in the following manner:

$$
ucov(X, T) = q_1 \times Co(\{x_1 \cdots x_k\}) + \sum_{j=2}^{k} (q_j - q_{j-1}) \times Co(\{x_j \cdots x_k\})
$$

For example, $ucov(ACD; T_1)$ in the database in Figure 1 can be computed as $2 \times Co(\{DAC\}) + 3 \times Co(\{AC\}) + 5 \times Co(\{C\}) = 2 \times 5 + 3 \times 4 + 5 \times 3 = 37$.

**Lemma 1.** Assume two sets $S = \{(A_1, q_1), \ldots, (A_n, q_n)\}$ and $T = \{(B_1, r_1), \ldots, (B_m, r_m)\}$ such that $q_1 \leq \cdots \leq q_n \leq r_1 \leq \cdots \leq r_m$. Let $S \cup T = \{(A_1, q_1), \ldots, (A_n, q_n), (B_1, r_1), \ldots, (B_m, r_m)\}$. Then, $ucov(S) + ucov(T) \geq ucov(S \cup T)$.

**Proof.** Let us analyze the terms in $ucov(S)$, $ucov(T)$, and $ucov(S \cup T)$. The first term in $ucov(T)$ is $(r_1) \times Co(\{T\})$. The first term in $ucov(T)$ can be expanded such that $ucov(S) + ucov(T) \geq ucov(S \cup T)$ for the first $n+1$ terms in $S \cup T$ as shown in Table 2 as $(r_1) \times Co(\{T\}) = (q_1) \times Co(\{T\}) + (q_2 - q_1) \times Co(\{T\}) + \cdots + (r_1 - q_n) \times Co(\{T\})$. The remaining terms in $ucov(T)$ and $ucov(S')$ are equal. □

**Theorem 1.** The function $ucov(I, T)$ is subadditive.
The remaining terms are equal. Hence, $ucov(I,T)$ is a monotone function.

\[ \text{consider two sets } S = \{(A_1, q_1) \cdots (A_n, q_n)\} \text{ and } T = \{(B_1, r_1) \cdots (B_m, r_m)\} \text{ such that } q_1 \leq \cdots \leq q_n \text{ and } r_1 \leq \cdots \leq r_m. \text{ } ucov(S \cup T) \text{ will contain a mix of elements from } S \text{ and } T. \text{ The sequence of terms in } ucov(S \cup T) \text{ can be divided into blocks such that a sequence of terms from one set (S or T) are followed by a term from the other set (T or S). Using Lemma 1, the term at the end of the block can be expanded such that } ucov(S) + ucov(T) \geq ucov(S \cup T). \text{ Hence, } ucov(I,T) \text{ is a subadditive function.} \]

\[ \text{Table 1: Comparison of } ucov(S) + ucov(T) \text{ with } ucov(S') \text{ (Lemma 1)} \]

| Quantity | $ucov(S)$ | $ucov(T)$ | $ucov(S')$ |
|----------|-----------|-----------|-----------|
| $m_1 = q_1$ | $m_1 \times Co(A_1 \cdots A_i)$ | $m_1 \times Co(T)$ | $m_1 \times Co(T)$ |
| $m_2 = q_2 - q_1$ | $m_2 \times Co(A_2 \cdots A_i) + m_2 \times Co(T)$ | $m_2 \times Co(T)$ | $m_2 \times Co(T)$ |
| $m_n = q_n - q_{n-1}$ | $m_n \times Co(A_n)$ | $m_n \times Co(T)$ | $m_n \times Co(T)$ |
| $m_{n+1} = r_1 - q_n$ | $m_{n+1} \times Co(T)$ | $m_{n+1} \times Co(T)$ | $m_{n+1} \times Co(T)$ |
| $m_{n+2} = r_2 - r_1$ | $m_{n+2} \times Co(T)$ | $m_{n+2} \times Co(T)$ | $m_{n+2} \times Co(T)$ |
| $m_{n+m} = r_m - r_{m-1}$ | $m_n \times Co(T)$ | $m_n \times Co(T)$ | $m_n \times Co(T)$ |

**Theorem 2.** $ucov(I,T)$ is a monotone function.

\[ \text{Proof. Let us consider a set } S = \{(A_1, q_1) \cdots (A_n, q_n)\} \text{ and another set } T = \{(B_1, r_1)\}. \text{ Let us consider a set } S' = S \cup T. \text{ Assume that set } S \text{ is ordered such that } q_i \leq q_j | i < j \text{ i.e } q_1 \text{ is the minimum and } q_n \text{ is the maximum weight in } S. \text{ Let the tuple } (B_1, r_1) \text{ be inserted at the } i^{th} \text{ position from the beginning in } S'. \text{ } S' \text{ is also ordered like } S. \text{ It can be quickly verified that the elements from the } 1^{st} \text{ till } i - 1^{th} \text{ position in } ucov(S) \text{ is less than or equal to } ucov(S'). \text{ Let us consider the } i^{th} \text{ position where element from } T \text{ gets inserted. It can be observed that the next term from } ucov(S) \text{ can be expanded i.e. } (q_i - q_{i-1}) \times Co(A_1 \cdots A_n) \times Co(T) = (q_i - q_{i-1}) \times Co(A_1 \cdots A_n) \times Co(T). \text{ The remaining terms are equal. Hence, } ucov(I,T) \text{ is a monotone function.} \]

\[ \text{Table 2: Comparison of } ucov(S) \text{ with } ucov(S') \text{ (Theorem 2)} \]

| Quantity | $ucov(S)$ | $ucov(T)$ | $ucov(S')$ |
|----------|-----------|-----------|-----------|
| $m_1 = q_1$ | $m_1 \times Co(A_1 \cdots A_i)$ | $m_1 \times Co(T)$ | $m_1 \times Co(T)$ |
| $m_2 = q_2 - q_1$ | $m_2 \times Co(A_2 \cdots A_i)$ | $m_2 \times Co(T)$ | $m_2 \times Co(T)$ |
| $m_{n-1} = q_{n-2} - q_{n-2}$ | $m_{n-1} \times Co(A_{n-1} \cdots A_i)$ | $m_{n-1} \times Co(T)$ | $m_{n-1} \times Co(T)$ |
| $m_{n} = r_1 - q_{n-1}$ | $m_{n} \times Co(A_1 \cdots A_i)$ | $m_{n} \times Co(T)$ | $m_{n} \times Co(T)$ |
| $m_{n+1} = q_{n+1} - r_1$ | $m_{n+1} \times Co(A_1 \cdots A_i)$ | $m_{n+1} \times Co(T)$ | $m_{n+1} \times Co(T)$ |
| $m_{n+2} = q_{n+2} - q_{n+1}$ | $m_{n+2} \times Co(A_{n+1} \cdots A_i)$ | $m_{n+2} \times Co(T)$ | $m_{n+2} \times Co(T)$ |
| $m_{n+m} = q_{n-m} - q_{n-m+1}$ | $m_{n+m} \times Co(A_n)$ | $m_{n+m} \times Co(T)$ | $m_{n+m} \times Co(T)$ |

We want to note that Theorems 1 and 2 hold true when we replace $Co()$ in Definition 2 by any subadditive and monotone function. So Definition 2 allows us to construct utility functions on weighted itemsets by using a subadditive and monotone function on itemsets (without quantities). For example, let $card()$ be the set cardinality function. It can be easily verified that $ucov()$ defined using $card()$ (in place of $Co()$) is identical to the $Sum()$ function defined earlier.
Algorithms for subadditive and monotone utility

The subset of a low-utility itemset can have high-utility as utility functions do not necessarily follow the downward-closure property \[16\]. Hence to avoid searching all itemsets, HUIM algorithms define bounds such as transaction utility(TU), transaction weighted utility(TWU) \[5\], exact utility and remaining utility \[9\]. We now discuss these bounds in the context of a general subadditive and monotone utility function \(u(X,T)\).

5.1 TU and TWU bounds

The TU of a transaction is the sum of the utilities of its items. TWU of an itemset \(X\) is the sum of TU for transactions that contains \(X\). The transaction-weighted downward closure property of TWU ensures that if TWU(\(X\)) is less than the threshold, \(X\) and its super-sets can not be high-utility itemsets \[5\]. We now define a related concept of transaction subadditive-monotone utility.

**Definition 3 (TSMU and TSMWU).** The transaction subadditive monotone utility of a transaction \(T\) is defined as \(TSMU(T) = u(T,T)\). The transaction subadditive monotone weighted utility (TSMWU) of an itemset \(X\) is the sum of TSMU(\(T\)) for all the transactions containing \(X\).

Now we show that TSMWU satisfies the downward-closure property making it useful in mining algorithms and a tighter bound compared to TWU.

**Lemma 2.** If TSMWU(\(X\)) is less than threshold, \(X\) and its super-sets can not be high-utility itemsets.

*Proof.* For any transaction \(T\) and any \(X \subseteq X' \subseteq T\), \(u(X',T) \leq TSMU(T)\) as \(u(\cdot,T)\) is a monotone function. Therefore, \(u(X') \leq TSMWU(X)\) for all \(X \subseteq X'\). Hence, TSMWU(\(X\)) satisfies the downward closure property. \(\square\)

**Lemma 3.** TSMWU(\(X\)) is a tighter upper-bound compared to TWU(\(X\)).

*Proof.* This can be easily proved since \(TU(T) = \sum_{x \in T} u(x,T) \geq u(T,T) = TSMU(T)\) due to subadditivity. Therefore, TSMWU(\(X\)) \(\leq\) TWU(\(X\)). \(\square\)

5.2 Exact utility (EU) and remaining utility (RU) bounds

List-based \[9\,10\] and projection-based \[11\,12\] HUIM algorithms use the exact utility (EU) and remaining utility (RU) bounds that are tighter compared to the TU bound explained earlier. These algorithms process items according to some fixed order and items in each transaction are sorted accordingly; let \(X\) be some itemset that is being processed, \(T\) be some transaction containing \(X\) and \(T/X\) be the items appearing after \(X\) in \(T\). One should note that \(X \cup (T/X)\) is not \(T\), in particular, items appearing in \(T\) before \(X\) in the processing order are not in \(T/X\). The exact utility \(EU(X,T)\) is the sum of the utilities of the items in \(X\) in \(T\) and the remaining utility \(RU(X,T)\) is defined as the sum of the utilities of the items in \(T/X\). It can be shown that \(X\) and its extensions (according to the processing
order) cannot be high utility if \( EURU(X) = \sum_{T \supseteq X} EU(X, T) + RU(X, T) \) is less than the threshold \([9]\); this fact is used in HUIM algorithms to decide whether to examine extensions of the currently explored itemset \( X \). Now we define combined utility in an analogous manner for subadditive monotone utility functions and analyse its applicability for itemset mining.

**Definition 4 (Combined utility (CU)).** Given an itemset \( X \) and a trans-
action \( T \) with \( X \subseteq T \), the combined utility of \( X \) is defined as 
\[
CU(X,T) = \text{Sum}(X,T) + \text{Sum}(T/X,T)
\]
and
\[
CU(X) = \sum_{T \supseteq X} CU(X,T).
\]

**Lemma 4.** If \( CU(X) \) is less than a threshold, any extension \( X' \) of \( X \) in the pro-
cessing order of items is not a high-utility itemset.

**Proof.** Let \( X' \) be some extension of \( X \) in the processing order of items; since transactions are also (implicitly) stored in the same order, \( X' \subseteq X \cup T/X \). The proof of the lemma follows easily since monotonicity implies that 
\[
u(X',T) \leq \text{Sum}(X,T,X',T) = CU(X,T).
\]

**Lemma 5.** \( CU(X) \) is a tighter upper-bound compared to \( EURU(X) \).

**Proof.** First observe that subadditivity implies that 
\[
EU(X,T) = \sum_{x \in X} u(x,T) \geq u(X,T).
\]
Similarly it follows that \( RU(X,T) \geq u(T/X,T) \). Therefore, we immediate-
ly get 
\[
EU(X,T) + RU(X,T) \geq u(X,T) + u(T/X,T) \geq u(X \cup T/X,T) = CU(X,T)
\]
(second inequality is again due to monotonicity). Therefore it follows that 
\[
EURU(X) \geq CU(X).
\]

It should be noted that \( CU(X) = EURU(X) \) when \( \text{Sum}(\cdot) \) (defined in Sec-
tion \([4]\) is used as the utility function. The results of Subsections \([5.1]\) and \([5.2]\) show that we can use \( TSMWU \) in place of \( TWU \) and \( CU \) in place of \( EURU \) in HGUIM algorithms. We illustrate these bounds in our example transaction database presented in Figure \([4]\) for the \( \text{ucov}(\cdot) \) utility function. In the following subsections we show how to adapt existing HUIM algorithms to our general setting and highlight important changes that must be incorporated in them.

### 5.3 Adaptation of a tree-based algorithm

A tree-based algorithm like UP-Growth and UPGrowth+ \([7]\) creates a tree data
structure from transaction database which is then used to get potential high
utility candidates. In the tree construction process as well as mining of potential
candidates, a couple of heuristics are used to tighten the utility estimates and
hence reduce the number of potential candidates to be verified in the second
phase. The heuristics used during the potential candidates mining process are
called local-utility based heuristics whereas the heuristics used during global tree
construction are called global heuristics. We observe that the local heuristics
being dependent on estimates are not applicable for HGUIM scenarios.

Consider two sets of items \( S = \{(u_1,q_1)\} \) and \( T = \{(u_1,q_1),...(u_n,q_n)\} \). Let
us assume that the item \( u_1 \) is unpromising and its contribution in the utility
estimate for set \( T \) has to be removed. It can be easily verified that 
\[
f(S) + f(T) =
$f(S \cup T)$ when $U(\cdot)$ is the $\text{Sum}(\cdot)$ defined for HUIM. Therefore, $f(T) = f(S \cup T) - f(S)$ gives a correct bound for HUIM. However, local-heuristic strategies can result in incorrect upper bound estimate i.e. resulting in false negatives for those functions where $f(S) + f(T) > f(S \cup T)$.

Tree-based algorithms for HUIM can be adapted with minimal change for an arbitrary subadditive monotone function. There is no change in the tree data structure and only local-utility based heuristics must be disabled during the candidate generation phase. The tree construction and verification phase remain the same. This leads to direct use of utility values associated with the tree nodes in the mining process. However, absence of local heuristics for unpromising items results into generation of a large set of potential candidates. This algorithm is similar to IHUP-tree algorithm \cite{6} with global heuristics of UP-Growth+, and computes TSMWU bound instead of TWU.

### 5.4 Adaptation of a list-based algorithm

List-based algorithms \cite{9,10} for HUIM maintains a utility-list consisting of tuples (Transaction id, Exact utility, Remaining utility) with each itemset. We adapt the data structure to store tuples of the form (Transaction id, Exact utility, Combined utility). The exact utility and combined utility fields store the quantities of items instead of the sum value stored for HUIM. List-based algorithms construct the utility for a $k$-itemset by intersecting lists of two $k-1$ itemsets and the prefix itemset. We observe that there is no need to scan the prefix utility-list while constructing the utility-list for a $k$-itemset. Assume that we want to construct the utility-list for itemset $\{ABC\}$. The utility-list of $\{AB\}$ and $\{BC\}$ is combined. For the transactions common in $\{AB\}$ and $\{BC\}$ lists, quantity for $\{ABC\}$ can be combined easily by taking $\{AB\}$ from the first list and only $\{C\}$ from the second list. Our observation is true for any $k$-itemset utility-list construction. We call the algorithm FHM with the above-defined construction method for utility-lists as FHM* in rest of the paper.

### 5.5 Adaptation of a projection-based algorithm

Projection-based algorithms like EFIM \cite{11} merge two identical transactions to reduce the cost for database scan during the construction of projected database for an itemset. Consider two transactions $S = \langle (i_1, q_1), \ldots (i_n, q_n) \rangle$ and $T = \langle (i_1, r_1), \ldots (i_n, r_n) \rangle$. $S \cup T = \langle (i_1, q_1 + r_1), \ldots (i_n, q_n + r_n) \rangle$. Merging transactions does not change the utility of the itemset $\langle i_1, i_2, \ldots, i_n \rangle$ in the database for the $\text{Sum}(\cdot)$ defined by HUIM. However, utility of an itemset in the merged transaction can be less than the sum of its utility in individual transactions for an arbitrary subadditive function. We disable transaction merging when adapting a projection-based algorithms like EFIM for an arbitrary subadditive monotone function.

### 6 Experiments and results

In this section, we compare the performance of state-of-the-art algorithms from the category of tree-based, list-based, and projection-based algorithms. We specif-
Fig. 2: Performance evaluation on real datasets.

We choose UP-Growth+ [7] from tree-based, FHM [10] from list-based, EFIM [11] and D2HUP [12] from projection-based algorithms. We obtained the Java source code of UP-Growth+, FHM, EFIM and D2HUP algorithm from the SPMF library [17]. We conduct the experiments by adapting different algorithms for the Coverage utility function (Definition 2). The experiments were performed on an Intel Xeon(R) CPU=26500@2.00 GHz with 64 GB RAM and Windows Server 2012 operating system. We conduct experiments on the ChainStore, Kosarak, Mushroom, and Accidents datasets obtained from the SPMF library [17]. The datasets vary in the number of transactions, the number of items, and the average transaction length. The coverage graph was constructed from the transaction database by taking the set of distinct items present in the database as vertices and linking them by edges. For each vertex
v, an edge is formed between vertex v and every other vertex with probability 0.1. This process is terminated as soon as the degree of vertex v becomes four. The metrics for performance measure are total execution time, the number of explored candidates and the number of utility function calls (UFC). Every algorithm is executed five times, and an average is taken for the total execution time. In our experiments, the utility values are expressed in terms of percentage of the utility threshold at which at least one candidate itemset is reported.

**Result-1: Which category algorithm performs better on Sparse datasets?** We observe that tree-based algorithm UP-Growth+ and list-based algorithm FHM* perform better than projection based algorithms EFIM and D2HUP on Sparse datasets. The reason for this behavior is less number of utility function calls by UP-Growth+. The UP-Growth+ algorithm does not call the utility function during the candidate generation phase. The function calls are made only during the tree construction and verification phase. However, EFIM and D2HUP identify promising items during every recursive call, unlike FHM* and UP-Growth+. Hence, the number of function calls by EFIM, D2HUP is more compared to FHM* and UP-Growth+. There is an outlier observation at 14% threshold on Kosarak dataset which is due a large number of candidates being generated by UP-Growth+ and the overhead for candidate verification dominates the execution time.

**Result-2: Which category algorithm performs better on Dense datasets?** FHM* performs the best on the Mushroom dataset and EFIM performs the best on Accidents dataset. UP-Growth+ performs the worst for Mushroom and Accidents dense datasets as it generates lots of candidates and the verification phase takes a lot of time. The execution for UP-Growth+ didn’t complete on Accidents dataset even after 24 hours and was terminated. EFIM performs the best for Accidents dataset as it generates the least number of candidates as well as the number of calls to the utility computation function. FHM* performs the best for Mushroom dataset followed by EFIM.

**Result-3: Are the performance trend of algorithms same as HUIM scenario?** The performance trend of algorithms is different from HUIM scenario. Projection-based algorithms are known to perform an order of magnitude better than other algorithms for dense and sparse datasets. It is expected that D2HUP performs the best on Sparse and EFIM performs the best on dense datasets. However, we observe that tree-based and list-based algorithms perform better compared to projection-based algorithms on sparse datasets. FHM* competes with EFIM for the best performance on dense datasets. However, we observe that the total execution time of the algorithms is more correlated with the number of utility function calls compared to the number of candidates generated during the mining process.

7 Conclusion

In this paper, we formalized the problem of high-utility itemset mining for a general utility function. We adapted the existing high-utility itemset mining algorithms for arbitrary subadditive monotone functions. We defined a utility
function which captures the domain knowledge in the form a relationship graph between items. We observe that the performance of algorithms is mainly dependent on two features namely, number of utility function calls and the number of generated candidates. In future, we will explore some novel applications for the coverage utility function defined in this paper.

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