Dispersive Bounds on The Shape Of $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}_l$ Formfactors

Debrupa Chakraverty\textsuperscript{1}, Triptesh De\textsuperscript{2}, Binayak Dutta-Roy\textsuperscript{3}

and

K. S. Gupta\textsuperscript{4}

\textit{Saha Institute of Nuclear Physics,}
\textit{1/AF Bidhannagar, Calcutta - 700 064, India}

Abstract

We derive a theoretically allowed domain for the charge radius $\rho$ and curvature $c$ of the Isgur-Wise function describing the decay $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}_l$. Our method uses crossing symmetry, dispersion relations and analyticity in the context of the Heavy Quark Effective Theory but is independent of the specifics of any given model. The experimentally determined values of the $\Upsilon$ masses have been used as input information. The results are of interest for testing different models employed to calculate the heavy baryon formfactors which are used for the extraction of $|V_{cb}|$ from the experimental data.

\textsuperscript{1}Electronic address: rupa@tnp.saha.ernet.in
\textsuperscript{2}Electronic address: td@tnp.saha.ernet.in
\textsuperscript{3}Electronic address: bnyk@tnpdec.saha.ernet.in
\textsuperscript{4}Electronic address: gupta@tnp.saha.ernet.in
1. INTRODUCTION

During the last few years, considerable progress has been made in the development of methodologies for the extraction of $|V_{cb}|$ [1-6] from both exclusive and inclusive semileptonic B meson decays, where the leptonic current is clearly separated from the matrix elements of the hadronic current. The CKM matrix element $V_{cb}$ has important implications for the investigation of rare decays and CP violation. It is therefore imperative to have as many independent and accurate determinations of $|V_{cb}|$ as possible. There are however some theoretical limitations in the determination of $|V_{cb}|$ from inclusive semileptonic decays due to issues related to validity of quark-hadron duality near the kinematic endpoint region [7]. On the other hand, the exclusive decays must be described in terms of a number of undetermined nonperturbative formfactors that contain the physics of the hadronisation process.

From the theoretical point of view, in the limit where the heavy quark mass tends to infinity, the analysis of heavy baryonic semileptonic decays are comparable to that of semileptonic decays of heavy mesons. This is due to the fact that heavy quark symmetry in the above limit predicts a single universal formfactor for both these types of decays known as the Isgur-Wise (IW) function [8]. It is therefore interesting to estimate $V_{cb}$ from the decays of heavy baryons. However, experimental data on the decay of heavy baryons is still sparse compared to the corresponding meson decays. Semileptonic decays of the $\Lambda_b$ baryon have up to now only been observed at LEP [9, 10]. It is however expected that in the near future more data would be available from the LEP as well as from the the forthcoming B factory. Once these results are available, the theoretical value for the semileptonic decay width of $\Lambda_b$ can be used to gain an independent determination of $|V_{cb}|$.

In semileptonic decays of both heavy mesons and baryons, the heavy quark symmetry predicts the value of the corresponding IW functions at zero recoil [8, 11], though the shapes of the IW functions are left unspecified and require detailed knowledge of the
non-perturbative strong interaction physics. Unfortunately, the differential decay width \( \frac{d\Gamma}{dq^2} \) vanishes at zero recoil and hence the known normalization of this function at that point is not enough to extract \( V_{cb} \) from the data. To overcome this difficulty various models are used to determine the functional form of the IW function and \( V_{cb} \) is extracted therefrom. However, the observed variations of \( |V_{cb}| \) obtained from different models turn out to be larger than the statistical and systematic uncertainties in the experiments. From the experimentally measured lepton invariant mass spectrum one may determine \( V_{cb} \) in a model independent way by extrapolating to the kinematical endpoint of maximal momentum transfer to the leptons corresponding to the zero recoil point. However, this extrapolation requires a large amount of data, very close to \( q^2_{\text{max}} \), which is difficult to assess experimentally. Thus measurement of \( |V_{cb}| \) requires a parametrization of the IW function. It is therefore necessary to put constraints as far as possible in a model-independent manner on this function.

In this paper we present an analysis of the model independent constraints on heavy baryonic IW function arising in the description of semileptonic \( \Lambda_b \to \Lambda_c l \bar{\nu}_l \) decay. This is accomplished by first obtaining constraints on the charge radius and the convexity of the IW function for the elastic \( \Lambda_b \) formfactors using the techniques of dispersion relations and analyticity. Next, heavy quark symmetry is used to relate this IW function to the formfactors appearing in the semileptonic \( \Lambda_b \to \Lambda_c l \bar{\nu}_l \) decay. The dispersion theoretic method of extracting information on amplitudes is quite old \cite{12, 13, 14} and has been applied to the study of semileptonic decays of light mesons in a more contemporary language \cite{15, 16}. Its application to heavy quark systems has received much attention in more recent years \cite{7, 17 - 24}. In Section 2 we give a brief account of the general formalism for the decay of heavy baryons. The method for the estimation of model independent bounds on the baryonic IW function is described in Section 3. The numerical results are presented in Section 4, and Section 5 concludes the paper with some discussions and
future outlook.

2. GENERAL FORMALISM OF HEAVY BARYON DECAY

In this section we give a brief account of the semileptonic decay of $\Lambda_b$ baryon using heavy quark symmetry [26].

The hadronic matrix element for the $\Lambda_b \to \Lambda_c l \bar{\nu}_l$ decay is parametrized by the following general decomposition consistent with Lorentz invariance.

$$
\langle \Lambda_c(p')| J_\mu | \Lambda_b(p) \rangle = \bar{u}_{\Lambda_c} \{ [f_1(q^2)\gamma_\mu + if_2(q^2)\sigma_{\mu\nu}q^\nu + f_3(q^2)q_\mu] - [g_1(q^2)\gamma_\mu + ig_2(q^2)\sigma_{\mu\nu}q^\nu + g_3(q^2)q_\mu] \gamma_5 \} u_{\Lambda_b},
$$

(2.1)

where $J_\mu = (V_\mu - A_\mu) = \bar{c}\gamma_\mu(1 - \gamma_5)b$ and $q = (p - p')$ is the momentum transferred to the leptons. Here $u_{\Lambda_b}$ and $u_{\Lambda_c}$ are the Dirac spinors of $\Lambda_b$ and $\Lambda_c$ respectively. The $f$'s and $g$'s are the formfactors for the independent Lorentz covariants corresponding to the hadronic matrix elements of the vector and axial vector currents respectively.

When both baryons are heavy, it is convenient to parametrize the matrix element in terms of the four velocities $v$ and $v'$ [26]:

$$
\langle \Lambda_c(v')| J_\mu | \Lambda_b(v) \rangle = \bar{u}_{\Lambda_c} \{ F_1(\omega)\gamma_\mu + F_2(\omega)v_\mu + F_3(\omega)v'_\mu - [G_1(\omega)\gamma_\mu + G_2(\omega)v_\mu + G_3(\omega)v'_\mu] \gamma_5 \} u_{\Lambda_b},
$$

(2.2)

where the formfactors [written with corresponding uppercase $F$'s and $G$'s] are functions of $\omega = v \cdot v'$, the only non-trivial Lorentz scalar formed from $v$ and $v'$.

In terms of these formfactors, the differential scalar decay width in the zero mass approximation for charged lepton is given by

$$
\frac{d\Gamma}{dq^2} = \frac{|V_{cb}|^2 G_F^2 k t^\frac{3}{2}}{96\pi^3 m_{\Lambda_b}^3} \{ t_- [2t|F_1|^2 + |H_V|^2] + t_+ [2t|G_1|^2 + |H_A|^2] \},
$$

(2.3)

where

$$
H_V(q^2) = (m_{\Lambda_b} + m_{\Lambda_c})F_1 + \frac{t_+}{2} (\frac{F_2}{m_{\Lambda_b}} + \frac{F_3}{m_{\Lambda_c}}),
$$

(2.4)
$$H_A(q^2) = (m_{\Lambda_b} - m_{\Lambda_c})G_1 - \frac{t_-}{2}(\frac{G_2}{m_{\Lambda_b}} + \frac{G_3}{m_{\Lambda_c}}),$$  

(2.5)

with $t = q^2$, $t_\pm = (m_{\Lambda_b} \pm m_{\Lambda_c})^2 - t$, and $k = \frac{1}{2}\sqrt{\frac{t_-}{t}}$.

In the infinite quark mass limit, the spin-flavour symmetry for the $b$ and $c$ quark relates all the formfactors to a single universal function $\xi(\omega)$, which is the IW function.

$$F_1(\omega) = G_1(\omega) = \xi(\omega),$$

and

$$F_2(\omega) = F_3(\omega) = G_2(\omega) = G_3(\omega) = 0.$$  

The IW function so defined is independent of heavy quark masses and normalized at zero recoil ($v = v'$) i.e. $\xi(1) = 1$.

The behaviour of the IW function close to zero recoil is of particular interest and it is customary to parametrize this behaviour through a Taylor’s series expansion about the zero recoil point namely,

$$\xi(\omega) = 1 - \rho^2(\omega - 1) + c(\omega - 1)^2 + O[(\omega - 1)^3],$$  

(2.6)

where $\rho$ and $c$ are referred to as the charge radius and the convexity parameter respectively.

The kinematic region, accessible to the semileptonic decay lies in such a small domain ($w = 1$ to 1.43) that a precise knowledge of the charge radius could basically determine the IW function in the physical region and the convexity parameter gives the corrections to the simple linear dependence. In the following section we derive bounds on $\rho$ and $c$ and thereby provide model independent constraints on the IW function.

### 3. DERIVATION OF THE BOUNDS

We consider the elastic formfactors of the $\Lambda_b$ baryon through the matrix elements of the flavour-conserving current $V_\mu = \bar{b}\gamma_\mu b$

$$\langle \Lambda_b(p')|V_\mu|\Lambda_b(p) \rangle = \bar{u}_{\Lambda_b}\{F_1^E(\omega)\gamma_\mu + F_2^E(\omega)(v + v')_\mu\}u_{\Lambda_b}.$$  

(3.1)
In the heavy quark limit, the b-number conserving elastic formfactors are related to the same IW function $\xi(\omega)$, which enters in the expression for the differential decay rate $\Lambda_b \to \Lambda_c l\bar{\nu}$. The short distance and the finite mass corrections are in this case much smaller than that for the formfactors involving flavour changing currents, corresponding to the decay $\Lambda_b \to \Lambda_c l\bar{\nu}$.

The method is based on the properties of the two-point function $\Pi(q^2)$ defined as

$$\Pi^{\mu\nu}(q^2) = i \int d^4x e^{iqx} \langle 0|T(V^\mu(x)V^\nu(0)|0 \rangle$$

$$= (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi(q^2), \quad (3.2)$$

with $V^\mu = b\gamma^\mu b$. The conservation of the current $V^\mu$ leads to the transverse nature of the two-point function. In QCD, the asymptotic nature of the two-point function $\Pi(q^2)$ is such that it satisfies a once subtracted dispersion relation

$$\chi(Q^2) = \frac{\partial \Pi(q^2)}{\partial q^2}|_{q^2=-Q^2} = \frac{1}{\pi} \int_0^\infty \frac{\text{Im}\Pi(t)}{(t + Q^2)^2} dt. \quad (3.3)$$

The absorptive part $\text{Im}\Pi(t)$ is obtained from the unitarity relation

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\text{Im}\Pi(q^2) = \frac{1}{2} \sum_{\Gamma} d\mu_{\Gamma}(2\pi)^4 \delta^{(4)}(q - p_{\Gamma})\langle 0|\langle V^\mu(0)|\Gamma\rangle\langle \Gamma|V^\nu(0)|0 \rangle, \quad (3.4)$$

where the summation is extended over all possible intermediate hadronic states with the quantum numbers of the $V^\mu$ current and is weighted by the corresponding phase space factor $d\mu_{\Gamma}$. A judicious choice of $\mu$ and $\nu$ ($\mu = \nu$) makes this a sum of positive definite terms and thus one can obtain a strict inequality by retaining only the term with intermediate $\Gamma = \Lambda_b\bar{\Lambda}_b$ state. From crossing symmetry, the $\Lambda_b\bar{\Lambda}_b \to$ vacuum matrix element is described by the same set of elastic formfactors, but relevant to the pair-production region ($4m^2_{\Lambda_b} \leq q^2 \leq \infty$) instead of the elastic region ($q^2 \leq 0$). This gives rise to an integral inequality

$$\chi(Q^2) \geq \frac{1}{24\pi^2} \int_{4m^2_{\Lambda_b}}^{\infty} \frac{dt}{(t + Q^2)^2} \sqrt{1 - \frac{4m^2_{\Lambda_b}}{t}[2t|F^E_1|^2 + |H^E|^2]}, \quad (3.5)$$

5
where
\[
H_V^E(q^2) = 2m_{\Lambda_b}F_1^E + (4m_{\Lambda_b}^2 - t)\frac{F_2^E}{m_{\Lambda_b}}.
\] (3.6)

In the case of mesons, inclusion of other two-particle intermediate states related by spin-flavour symmetry were made to saturate the dispersion relation in order to find the relevant bound. For \(\Lambda_b\) baryon decay, there are no spin symmetric partners that contribute to the dispersion relation bound. However, the presence of more than one helicity amplitude fulfills in effect the same role.

For \(Q^2\) far from the resonance region \((2m_b\Lambda_{QCD} \ll 4m_b^2 + Q^2)\), the two-point function can be computed reliably from perturbative QCD. For large \(b\) quark mass, it is sufficient to take \(Q^2 = 0\). The one loop expression for \(\chi(0)\) is
\[
\chi(0) = \frac{3}{2\pi^2} \int_0^1 \frac{x^2(1-x^2)}{m_b^2} dx.
\] (3.7)
Perturbative \(\alpha_s\) corrections to this result are negligibly small at the physical \(m_{\Lambda_b}^2\) scale. The non-perturbative corrections \[27\] are included by expressing the two-point function as an operator product expansion and incorporating the leading nonperturbative gluonic and quark-antiquark vacuum condensate terms \(\langle G^2 \rangle\) and \(\langle q\bar{q} \rangle\). But these terms are suppressed by the fourth powers of a large mass scale for dimensional reasons.

The analysis simplifies by using a conformal transformation to map the full complex \(t\) plane onto the unit disc in the complex \(z\) plane,
\[
\frac{1 + z}{1 - z} = \sqrt{\frac{1 - \frac{t}{4m_{\Lambda_b}^2}}{}}.
\] (3.8)
The transformation maps the cut \([4m_{\Lambda_b}^2, \infty]\) on the unit \(z\) circle and the rest of the \(t\) plane into the open unit \(z\) disc. In terms of the new variables \(z\) the inequality eq. (3.5) gets translated to
\[
 \frac{1}{2\pi i} \oint \frac{dz}{z} \left[ |\phi_{F_1^E}(z)F_1^E(z)|^2 + |\phi_{H_1^E}(z)H_1^E(z)|^2 \right] \leq 1,
\] (3.9)
where the functions $\phi_{FE}$ and $\phi_{HV}$ are defined as

$$\phi_{FE} = \sqrt{\frac{5\pi}{96}} (1 + z) (1 - z)^{1/2}, \quad (3.10)$$

and

$$\phi_{HV} = \frac{1}{32} \sqrt{\frac{5\pi}{3}} (1 + z) (1 - z)^{3/2} m_{\Lambda_b}^{-1}. \quad (3.11)$$

These functions are analytic and nonzero inside the unit disc $|z| < 1$. Their moduli squared on the boundary are equal to the positive weights appearing in the integrals, multiplied by the Jacobian $|\frac{d\ell}{dz}|$ of the conformal transformation. To translate the bound into one that is valid in the semileptonic region, we need a function which is analytic inside the unit disc. The functions $\phi_i$ have no poles, branch cuts or zeros in the interior of the unit circle $|z| < 1$, but the formfactors $F_{FE}(q^2)$ and $H_{HV}(q^2)$ have poles due to the existence of spin-one $b\bar{b}$ states ($J^P = 1^-$) below the $\Lambda_b\bar{\Lambda}_b$ threshold. They also have branch cuts originating from non-resonant continuum contributions with the invariant masses below $\Lambda_b\bar{\Lambda}_b$ threshold, though their effects may be neglected [21, 23]. The relevant states giving rise to poles in this context are the $^3S_1$ and $^3D_1$ bottomonium states, i.e. $\Upsilon(nS)$ and $\Upsilon(nD)$, respectively. From the experimental values of the masses of the $\Upsilon$ resonances, the position of the poles are obtained at the points:

$$z_1 = -0.29, \ z_2 = -0.37, \ z_3 = -0.43, \ z_4 = -0.48, \ z_5 = -0.58, \ z_6 = -0.65. \quad (3.12)$$

The residues of the formfactors at these poles are however unknown. To make up for this lack of information, one introduces a product of the functions of the form $(z - z_i)/(1 - \bar{z}_i z)$, known as Blaschke factors [28]

$$P(z) = \prod_{j=1}^{6} \frac{(z - z_j)}{(1 - \bar{z}_j z)}. \quad (3.13)$$

This function $P(z)$ has the virtue that it is analytic on the unit disc $|z_i| < 1$, and eliminates, through multiplication, the poles of $F_{iE}$ at each $z = z_j$. Since each term in $P(z)$
is contrived to be unimodular on the boundary, the inequality remains unchanged. Then the inequality (3.9) becomes

$$\frac{1}{2\pi i} \oint_{C} \frac{dz}{z} \left| \phi \left( \frac{z}{F_1(z)} P(z) F_1^E(z) \right) \right|^2 + \left| \phi \left( \frac{z}{H^E(z)} P(z) H^E(z) \right) \right|^2 \leq 1. \quad (3.14)$$

It is important to note that up to now the formfactors are treated as completely unknown functions, because heavy quark symmetry is quite inapplicable in the timelike domain near the pair-production threshold \[29\], though is only valid in the neighbourhood of the zero recoil point.

Both the functions $\phi_i(z)$ and $P(z) F_1^E(z)$ are now analytic in the unit disc. We can hence apply the well known results of interpolation theory \[28\] for vector-valued analytic functions to obtain the bounds on the formfactors at points inside the unit circle. Accordingly, we apply the inequality of the Schur-Caratheodory type \[28\] at the origin retaining terms up to the first derivative with respect to $z$

$$\phi_{F_1^E}^2(0) P^2(0) F_1^{2E}(0) + \phi_{H^E}^2(0) P^2(0) H^E_1(0) + (\phi_{F_1^E} P F_1^E)^2(0) + (\phi_{H^E} P H^E_1)^2(0) \leq 1, \quad (3.15)$$

where the prime denotes differentiation with respect to $z$. Through the conformal mapping, the point $\omega = 1$ is mapped to $z = 0$. We can hence use the prediction of heavy quark symmetry for the formfactors at the point of zero recoil, which leads to

$$F_1^E(z = 0) = \xi(z = 0) = 1, \quad H^E_1(z = 0) = 2m_{\Lambda_b} \xi(z = 0) = 2m_{\Lambda_b}. \quad (3.16)$$

The charge radius and convexity of the IW function are defined by

$$\rho^2 = -\left[ \frac{d\xi}{d\omega} \right]_{\omega=1} \quad c = -\left[ \frac{d^2\xi}{d\omega^2} \right]_{\omega=1}, \quad (3.17)$$

the derivatives here being with respect to $\omega$. They are related to derivatives with respect to $z$ through

$$\left[ \frac{d\xi}{dz} \right]_{z=0} = -8\rho^2, \quad \left[ \frac{d^2\xi}{dz^2} \right]_{z=0} = 128c - 32\rho^2. \quad (3.18)$$
From Eqs. (3.15 - 3.18), the upper and lower bounds on the charge radius are readily obtained to read

\[
\frac{b - \sqrt{b^2 - 4ad}}{a} \leq \rho^2 \leq \frac{b + \sqrt{b^2 - 4ad}}{a}
\]  (3.19)

with

\[
a = 64P^2(0)[\phi_{F1}^2 + \phi_{HV}^2],
\]  (3.20)

\[
b = 8P(0)[\phi_{F1}(0)(\phi_{F1} P)'_z(0) + \phi_{HV}(0)(\phi_{HV} P)'_z(0)],
\]  (3.21)

and

\[
d = P^2(0)[\phi_{F1}^2 + \phi_{HV}^2] + (\phi_{F1} P)'_z^2(0) + (\phi_{HV} P)'_z^2(0) - 1.
\]  (3.22)

When terms up to the second derivatives are included the inequality becomes

\[
\phi_{F1}^2(0)P^2(0)F_{1E}^2(0) + \phi_{HVE}^2(0)P^2(0)H_{V}^2E(0) + (\phi_{F1} P F_{1}^E)'_z^2(0)
\]
\[
+ (\phi_{HV} P H_{V}^E)'_z^2(0) + (\phi_{F1} P F_{1}^E)'_z^2(0) + (\phi_{HV} P H_{V}^E)'_z^2(0) \leq 1.
\]  (3.23)

In this case the domain of the allowed values for the charge radius and the convexity is the interior of an ellipse in the \(\rho^2 - c\) plane of the form

\[
(\rho^2 - \bar{\rho}^2)^2 + S[(c - \bar{c}) - T(\rho^2 - \bar{\rho}^2)]^2 = K^2,
\]  (3.24)

where \(\bar{\rho}^2, \bar{c}, S, T\) and \(K\) are readily determined through a comparison with eq. (3.23).

### 4. RESULTS

In this section we discuss the numerical results of our analysis. The lower and upper bounds on the charge radius \(\rho^2\) is obtained from Eq.(3.19) with basically reliable inputs. Actually, most of the predicted values of the charge radius from various models are well below the upper bound \(\rho^2 \leq 4.5\). The lower bound \(\rho^2 \geq -1.9\) is not interesting, since from Bjorken’s sum rule [30], the positivity of the charge radius is expected. On the other hand,
Eq.(3.24) gives the correlation between the charge radius $\rho^2$ and the convexity parameter $c$. In Eq.(3.24), the numerical values of the parameters are:

$$\rho^2 = 1.31, \ S = 256, \ \bar{c} = 1.45, \ T = 1.56 \ \text{and} \ K = 3.22.$$  \hspace{1cm} (4.1)

With these parameters, the solid curve in Fig. 1 restricts the allowed range for the charge radius and the convexity parameter of the IW function to lie within the interior of an ellipse in the $\rho^2 - c$ plane.

This result can be used to test the compatibility of some phenomenological models for the IW function. Each model leads to definite values for the charge radius and the convexity. For illustration, we consider the following models.

1. The MIT Bag model \cite{31} provides the form of IW function as

$$\xi(\omega) = \left(\frac{2}{\omega + 1}\right)^{3.5 + \frac{42}{\omega}}.$$  \hspace{1cm} (4.2)

This model gives the values of charge radius $\rho^2$ and convexity parameter $c$ as 2.35 and 3.95 respectively.

2. In Simple Quark Model \cite{32}, the IW function is well approximated by the formula

$$\xi(\omega) = \left(\frac{2}{\omega + 1}\right)^{1.32 + \frac{0.79}{\omega}},$$  \hspace{1cm} (4.3)

which predicts the computed value of the charge radius and the convexity parameter to be 1.0 and 1.11 respectively.

3. QCD Sum Rule \cite{33} predicts

$$\xi(\omega) = \left(\frac{2}{\omega + 1}\right)^{\frac{1}{2}}\exp[-0.8\frac{\omega - 1}{\omega + 1}].$$  \hspace{1cm} (4.4)

The value of the charge radius $\rho^2 = 0.65$ predicted in this model, gives the corresponding value of the convexity to be 0.47.

4. The IW function in the Skyrme model \cite{34} has the form

$$\xi(\omega) = 0.99\exp[-1.3(\omega - 1)],$$  \hspace{1cm} (4.5)
with \( \rho^2 = 1.3 \) and \( c = 0.85 \).

5. The **Relativistic Three Quark Model** \([35]\) calculation gives the approximate form of IW function near the zero recoil point to be

\[
\xi(\omega) = \left( \frac{2}{\omega + 1} \right)^{1.7 + \frac{1}{2}},
\]

which indicates the numerical values \( \rho^2 = 1.35 \) and \( c = 1.75 \).

6. In **Infinite Momentum Frame Quark Model** \([36]\), the overlap integrals for the hadronic wave functions of parent and daughter baryon calculated in the infinite momentum frame leads to the following form of the IW function

\[
\xi(\omega) = \left( \frac{1}{\omega} \right)^{2m+1/2} \exp\left[ -\kappa^2 \frac{\omega - 1}{2\omega} \right] \frac{H_{2m}(\kappa \frac{\sqrt{\omega} + 1}{2\omega})}{H_{2m}(\kappa)},
\]

where

\[
H_l(x) = \int_{-\infty}^{\infty} dy (y + x)^l e^{-z^2},
\]

with \( m = 1 \) and \( \kappa = 1.5 \). This gives the corresponding values of \( \rho^2 \) and \( c \) to be 3.04 and 6.81 respectively.

In Fig. 1, the corresponding values of charge radius and convexity parameter in these models are plotted as points. In Table I, the predicted value of \( \rho^2 \) and the computed value of \( c \) parameter in these phenomenological models are shown. The fourth column of the table gives the allowed range of \( c \) for the predicted value of \( \rho^2 \) for each model to test the compatibility of the corresponding model with the dispersive bound.

The predicted values of the charge radius and convexity parameter in the Simple Quark Model and QCD sum rule lie well within the allowed domain, provided by dispersive approach, the others lie close to the periphery or quite outside this region.
5. CONCLUSION

In this paper we have related the two-point function in QCD corresponding to the $\Lambda_b \bar{\Lambda}_b$ pair production (evaluated at one loop) to the $\Lambda_b$ elastic formfactors via analyticity, crossing symmetry and dispersion relations. Some techniques of complex analysis, i.e. conformal mapping, introduction of Blaschke factor and the Schur-Caratheodory type inequality are employed to derive constraints on a simple parametrization of the elastic formfactors. In the heavy quark limit, these elastic formfactors and the formfactors for the semileptonic decay $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$ are connected to a single IW function through heavy quark symmetry. As a result, we have obtained model independent dispersive bounds on the charge radius and the curvature of the IW function in an expansion around the zero recoil point $\omega = 1$. These bounds turn out to have nontrivial implications on the compatibility of a number of models, which we have depicted in Table I. The constraints can be made more stringent with the inclusion of higher derivatives, which would give stronger correlation between the charge radius and the convexity parameter of the IW function by reducing the allowed domain in $\rho^2 - c$ plane.

In previous works [23, 24], this dispersive approach has been applied directly to the formfactors for the $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$ decay. But the power in that approach was somewhat limited due to the possible presence of $b\bar{c}$ bound states below the threshold for $\Lambda_b\bar{\Lambda}_c$ production, which have not been observed experimentally and thus require inputs from potential models. Our approach on the other hand involves only the elastic formfactors and the use of experimentally known $b\bar{b}$ bound states ($\Upsilon$s) enables us to impose more stringent bounds on the IW function without having to rely much on specific models. This approach can also be employed to study heavy baryon lifetimes and such works are in progress.

References

12
[1] M. Neubert, Phys. Lett. B 264, 455 (1991); Phys. Lett. B 338, 84 (1994).

[2] CLEO Collaboration, B. Barish et al., Phys. Rev. D 51, 1014 (1995).

[3] ARGUS Collaboration, H. Albrecht et al., Z. Phys. C 57, 533 (1993); Phys. Rep. 276, 223 (1996).

[4] ALEPH Collaboration, D. Buskulic et al., Phys. Lett. B 395, 373 (1997).

[5] DELPHI Collaboration, P. Abreu et al., Z. Phys. C 71, 539 (1996).

[6] OPAL Collaboration, K. Ackerstaff et al., Phys. Lett. B 395, 128 (1997).

[7] C. G. Boyd, B. Grinstein, and R. F. Lebed, Nuovo Cim. 109 A, 863 (1996).

[8] N. Isgur, and M. Wise, Phys. Lett. B232 113 (1989); Phys. Lett. B237 523 (1990).

[9] Particle Data Group, Phys. Rev. D54, 1 (1996).

[10] OPAL Collaboration, K. Ackerstaff et al., Z. Phys. C 74, 423 (1997).

[11] M. Neubert, Phys. Rep. 245, 259 (1994); Int. J. Mod. Phys. A 11, 4173 (1996).

[12] N. N. Meiman, Sov. Phys. JETP 17, 830 (1963).

[13] S. Okubo, and I. Fushih, Phys. Rev. D 4, 2020 (1971).

[14] V. Singh, and A. K. Raina, Fortschrritte der Physik 27, 561 (1979).

[15] C. Bourrely, Nucl. Phys. B 43, 434 (1972); Nucl. Phys. B 53, 289 (1973).

[16] C. Bourrely, B. Machet, and E. de Rafael, Nucl. Phys. B 189, 157 (1981).

[17] E. de Rafael, and J. Taron, Phys. Lett. B 282, 215 (1992); Phys. Rev. D 50, 373 (1994).
[18] I. Caprini, Z. Phys. C 61, 651 (1994); Phys. Lett. 339, 187 (1994); Phys. Rev. D 52, 6349 (1995); Phys. Rev. 53, 4082 (1996).

[19] I. Caprini, and M. Neubert, Phys. Lett. B 380, 376 (1996).

[20] I. Caprini, and C. Macesanu, Phys. Rev. D 54, 5686 (1996).

[21] C. G. Boyd, B. Grinstein, and R. F. Lebed, Nucl. Phys. B461, 493 (1996).

[22] C. G. Boyd, B. Grinstein, and R. F. Lebed, Phys. Lett. 353, 306 (1995).

[23] C. G. Boyd, and R. F. Lebed, Nucl. Phys. B485, 275 (1997).

[24] C. G. Boyd, B. Grinstein, and R. F. Lebed, Phys. Rev. D 56, 6895 (1997).

[25] I. Caprini, L. Lellouch and M. Neubert, hep-ph/9712417 (1997).

[26] H. Y. Cheng, and B. Tseng, Phys. Rev. D 53, 1457 (1996).

[27] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B 147, 385, 448, 519 (197985, 448, 519 (1979).

[28] P. Duren, Theory of $H^p$ Spaces, Academic Press, New York, 1970.

[29] P. Ball, H. G. Dosch, and M. A. Shifman, Phys. Rev. D 47, 4077 (1993).

[30] N. Isgur, M. B. Wise, and M. Youssefmir, Phys. Lett. B 254, 215 (1991).

[31] M. Sadzikowski, and K. Zalewski, Z. Phys. C 59, 667 (1993).

[32] B. Holdom, M. Sutherland, and J. Mureika, Phys. Rev. D 49, 2359 (1994).

[33] Y. B. Dai, C. S. Huang, M. Q. Huang, and C. Liu, Phys. Lett. B 387, 379 (1996).

[34] E. Jenkins, A. Manohar, and M. B. Wise, Nucl. Phys. B 396, 38 (1996).
[35] M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner, and P. Kroll, Phys. Rev. D 56, 348 (1997).

[36] B. König, J. G. Körner, M. Krämer, and P. Kroll, Phys. Rev. D 56, 4282 (1997).
Figure Captions

1. The charge radius $\rho^2$ and the convexity parameter $c$ are constrained to lie within the ellipse shown in figure. Values of $\rho^2 < 0$ are excluded by Bjorken’s Sum Rule. The points in the figure denote the numerical values of the charge radius and the convexity, predicted in different phenomenological models. The numbers appearing next to the points in the figure correspond to the models indicated by bold faced letters in Section 4.
| Model                                    | Value of $\rho^2$ | Value of $c$ | Allowed range of $c$ in $\rho^2 - c$ plane |
|------------------------------------------|-------------------|--------------|---------------------------------------------|
| MIT bag Model                            | 2.35              | 3.95         | 2.89 - 3.27                                 |
| Simple Quark Model                       | 1.01              | 1.11         | 0.79-1.19                                   |
| QCD Sum Rule                             | 0.65              | 0.47         | 0.23-0.63                                   |
| Skyrme Model                             | 1.30              | 0.85         | 1.25 - 1.64                                 |
| Relativistic Three-Quark Model           | 1.35              | 1.75         | 1.32-1.72                                   |
| Infinite Momentum Frame Quark Model      | 3.04              | 6.81         | 3.98-4.32                                   |
