An Evaluation for Phase and Gain Imbalance Compensation in Direct-Conversion Receivers

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Abstract. In this paper, we present a modified compensation method for phase and gain in direct-conversion software-defined radio (SDR) receivers. In direct-conversion receivers, degradation owing to the phase and gain imbalances caused by analog devices. To solve this problem, we present a modified blind compensation techniques based on digital signal processing using a feedback control loop with a practical computation process. The modified method can reduce the complexity when a hardware logic circuit is used, like an FPGA. The simulation results verify that the modified method achieves better BER performance.

1 Introduction

A recent approach to increase the number of highly flexible and adaptable systems is termed software-defined radio (SDR). One of the key functions of an SDR is its multiband and multimode operation. An ideal multiband and multimode receiver consists of an analog-to-digital converter (ADC) placed immediately after the antenna to directly digitize the radio frequency (RF) signal. However, this approach is not practical because it requires high-resolution and high-sampling ADCs with a high clock frequency and high performance digital signal processors. Therefore, more realistic SDR receiver architectures, such as the direct-conversion (zero-IF) [1] receivers, have been proposed.

In direct-conversion receivers, the bit error rate (BER) performance is limited by analog devices such as mixers and $\pi/2$ phase-shift circuits, which have their own phase and gain imbalances. To solve this problem, [2] proposes an improved method in which the test signal is required. Further, for the method without using the test signal, [3] utilizes feedforward processing to achieve blind detection and compensation. However, practical compensation processes were not discussed in these studies. Moreover, feedforward processing cannot treat a time-varying signal in [3].

To solve these problems, blind compensation methods for phase and gain imbalances using a feedback control loop have been discussed in [4]. However, these methods [2-4] have a large computational cost or they cannot be implemented for practical applications. In this paper, we present a modified blind compensation method by using a feedback control loop with small computational complexity and practical implementation. In these approaches [2-4], implementation complexities have to be reduced. Therefore, to overcome the abovementioned drawbacks, the modified method utilizes a feedback control loop based on digital signal processing for blind detection and compensation of both phase and gain imbalances with only the input signal. In Sections 2, we will theoretically present the
modified method. We show that this method simplifies digital signal processing and makes it easier to implement in the direct-conversion receiver. Furthermore, to verify the validity of the modified method, various simulation results are presented.

2. Modified Gain and Phase Imbalance Compensation Methods

Figure 1(a) shows the block diagram of a direct-conversion receiver. Figure 1(b) shows the block diagram of a gain and phase imbalance compensation method in the direct-conversion receiver. As can be seen, the phase and gain compensation processing units are serially cascaded. In the followings, for reducing computational complexity from a practical viewpoint, we present and analyze a modified method for phase and gain compensation in the direct-conversion receiver.

Here, we suppose that the RF input signals have the received angular frequency components, where \( \omega_2 = 2\pi f_L \) is the local oscillator (LO) angular frequency and the relation is \( \omega_D = \omega_2 \cdot A_1 \) and \( A_2 \) are the in-phase gain (shown as MIXi and LPFi in Figure 1) and the quadrature gain (shown as Mixq and LPFq in Figure 1), respectively. Moreover, \( g ( = A2/A1 ) \) and \( \Delta \phi \) are the gain imbalance ratio and phase difference, respectively. Hence, the output signal of MIXi (mixer for in-phase), with a received input, is
\[
\cos(\omega_D t + \theta_D) \cos(\omega_2 t) = \cos(-\theta_D) \]
and the output signal of MIXq (mixer for quadrature), with a received input, is
\[
\cos(\omega_D t + \theta_D) \left[ -g \sin(\omega_2 t + \Delta \phi) \right] = -g \sin(\Delta \phi - \theta_D).
\]
The sum of the frequency components is filtered by each low-pass filter (LPF). Consequently, the output signal of each LPF is converted into a complex signal as shown below:

\[
y(t) = \cos(\omega_D t + \theta) \left[ \cos(\omega_2 t) - j g \sin(\omega_2 t + \Delta \phi) \right] = (1/2) \left[ \cos(\theta_D) - j g \sin(\Delta \phi - \theta_D) \right].
\]

Figure 2(a) shows the modified phase imbalance compensation method shown in the block diagram of the direct-conversion receiver. From Equation (1), the real part, \( y_r(t) \), and the imaginary part, \( y_i(t) \), of the input signal before compensation are given by
\[
\begin{align*}
y_{in}(t) = y_r(t) + j \cdot y_i(t) &= (1/2) \left[ \cos(\theta_D) \right] - j (1/2) \sin[\Delta \phi - \theta_D] \\
&= (1/2) \left[ \cos(-\theta_D) \right] - j (1/2) \left[ \sin(\Delta \phi) \cos(\theta_D) + \cos(\Delta \phi) \sin(-\theta_D) \right] \\
&= (1/2) \left[ \cos(-\theta_D) \right] - j (1/2) \left[ \sin(\Delta \phi) \cos(\theta_D) + \sin(-\theta_D) \right] \\
&= 1 - j \left[ \sin(\Delta \phi) \cos(\theta_D) + Q \right].
\end{align*}
\]
where \( \cos \Delta \phi = 1 \), \( I \) is the in-phase input signal without compensation, and \( Q \) is the quadrature input signal without compensation. The gain imbalance \( g \) in Equation (1) is set to 1. Further, we assume that the amplitude of each signal is normalized as shown in Equation (1). The in-phase signal, \( y_r(t) \), is multiplied by the quadrature signal, \( y_i(t) \), to calculate the following equation:

\[
y_{pd}(t) = (1/4) \cos(-\theta_D) \cdot \sin(\theta_D + \Delta \varphi) \\
= (1/8) \sin(\Delta \varphi).
\]  

We assume that the gain imbalance ratio \( g \) varies slowly. As can be seen in Figure 2(a), the multiplied signal is fed into the LPF. The LPF output signal is then fed into the integrator, which is a perfect integrator \((1/S)\), with the loop constant \((G_p)\) as a reciprocal of the time constant \((\tau_p = 1/G_p)\) in the loop. Thereafter, the output of the integrator is multiplied by the loop constant \((G_p)\).

Next, the output of the abovementioned multiplier is multiplied by the in-phase input signal. This multiplied signal is subtracted from the quadrature input signal. The output of the phase compensation section, \( y_{out}(t) \), produces the following equations:

\[
y_{out}(t) = I - j[Q' - I \sin(\Delta \varphi)] = I - j[I \sin(\Delta \varphi) - I \sin(\Delta \varphi) + Q]
= I - jQ,
\]  

where \( Q' = I \sin(\Delta \varphi) + Q \). Finally, phase imbalance compensation is performed from Equation (4).

Figure 2(b) shows the modified gain imbalance compensation method shown in the block diagram of the direct-conversion receiver. We assume that the phase difference due to imbalance \((\Delta \varphi)\) varies slowly. In Figure 2(b), the loop constant \((G_g)\) is the reciprocal of the time constant \((\tau_g = 1/G_g)\) in the loop, and \(1/S\) denotes a perfect integrator. From Equation (1), the real part of the input, \( y_r(t) \), and the imaginary part, \( y_i(t) \), of the input signal before compensation are given by

\[
y_r(t) = (1/2) \cos(\theta_D)
\]  
\[
y_i(t) = (1/2) [(g) \sin(\Delta \varphi - \theta_D)].
\]  

Following the squaring operation, we obtain the real part, \( z_r(t) \), and the imaginary part,
\( z_r(t) \), as follows:

\[
z_r(t) = \left(1/8 \right) \left[ 1 + \cos (2\theta_D) \right]
\]

\( z_i(t) = \left( g \right)^2 \left(1/8 \right) \left[ 1 - \cos \left( 2(\Delta \varphi - \theta_D) \right) \right]
\]

(7)

(8)

Let the gain imbalance ratio \((g)\) be replaced with \((1 + A)\). Here, 1 implies that the amplitudes of both the signals are normalized. Further, we assume that \(1 > \Delta A\), and \((\Delta A)^2\) is negligible. In Equations (7) and (8), the double frequency term becomes zero because of the LPF. By using Equations (7) and (8), the result of \( z_r(t) \) minus \( z_i(t) \) is \((1/8) \left[ 1 - (g)^2 \right]\) as the input of the LPF. The approximate part of the abovementioned result for \( y_{gd} \) is \((1/8) \left[ 2\Delta A + (\Delta A)^2 \right]\) by leaving out the term \((1/8)(\Delta A)^2\); therefore, the output of the LPF is given by the following equation:

\[
y_{gd} = \Delta A/4.
\]

(9)

The residual error results from the term \((1/8)(\Delta A)^2\). However, the error is decreasing in the control loop.

From Equation (1), the input of the compensation block in the loop, \( y_{in}(t) \), is given by the following equation:

\[
y_{in}(t) = \left(1/2 \right) \left[ \cos (-\theta_D) \right] - j \left(1/2 \right) (1 + \Delta A) \left[ \sin (-\theta_D) \right].
\]

(10)

Using Equation (10) and the imaginary part of Equation (11) multiplied by \((1-\Delta A)\), the output of the compensation block in the loop, \( y_{out}(t) \), is given as

\[
y_{out}(t) = \left(1/2 \right) \left[ \cos (-\theta_D) \right] - j \left(1/2 \right) (1 + \Delta A) \left[ 1 - (\Delta A) \left[ \sin (-\theta_D) \right] \right]
\]

\[
= \left(1/2 \right) \left[ \cos (-\theta_D) \right] - j \left(1/2 \right) \left[ \sin (-\theta_D) \right].
\]

(11)

In Equation (11), the gain compensation is carried out by multiplying \((1+\Delta A)\) by \((1-\Delta A)\). Instead of a division process, the multiplication process is used in the above equation. We assume that the abovementioned compensation methods can be applied to the receiver with an automatic gain control (AGC) operation.

## 4 Simulation Results and Evaluations

For the simulation of the phase and gain compensation, we used the modified algorithms presented in Figure 2. Figures 3(a) and 3(b) show the simulation block diagram for phase and gain compensation.

**Fig. 3** Simulation block diagram for (a) phase compensation and (b) gain compensation.

Figures 4(a) to 4(d) depict the simulation results of the phase and gain compensation responses, with detected phase and gain imbalances. The parameters and block diagrams used for the simulation are shown in Table 1 and Figures 3(a) and 3(b), respectively. In this
simulation, the basic characteristics of phase and gain compensation are evaluated from Equations (4) and (11). The 16-bit fixed-point process is employed for the compensation algorithm. When Equations (3) and (4) (for phase compensation) and Equations (9) and (11) (for gain compensation) are approximated, the simulation results show that the feedback loop locks and converges to the initial values of the imbalance. The computational complexity can be reduced by using the abovementioned methods (particularly for the division process in gain compensation).

The simulation results of the correction values for the approximate and the exact computations are shown in Figures 4(c) and 4(d), respectively. Other than the initial imbalances (10° and 6 dB), the simulation parameters are the same as those in Table 1.

| Table 1 Simulation parameters. |
|-------------------------------|
| Modulation of signal          | QPSK (Fig.4), 16/64QAM (Fig.5) |
| Filter of desired signal      | root-raised cosine (α=0.5)     |
| Fixed-point process bit       | 16 bit                         |
| Initial phase imbalance       | 1, 2, 4, 10 (Fig.4), 10 (Fig.5)° |
| Initial gain imbalance        | 0, 5, 1, 3, 6 (Fig.4), 1 (Fig.5) dB |
| Time constant in the loop (τp, τg) | 65536 (Fig.4), 16384 and 131072(Fig.5) samples |

Figure 5 shows the simulation results of the BER versus carrier to noise power ratio (C/N) performance of the modified phase and gain compensation blocks in Figures 2(a) and 2(b), with and without compensation. The block diagram and parameters used in the simulation are shown in Figure 6 and Table 1, respectively. In this simulation, 16 quadrature amplitude
modulation (QAM) /64QAM signals are randomly generated. Moreover, oversampling is
set to 16 times, and the receive filter is assumed to be a root-raised cosine (α = 0.5).
Additionally, 16-bit fixed-point processing is employed, and the initial phase and gain
imbalances are set to be 10° and 1 dB (assuming the imbalance is due to mixer, phase shift
circuit, amplifier, and LPF), respectively. The results show that the BER performance of the
modified compensation method is better than that of the method without compensation. In
this simulation, the time constant in the loop is set to be 16384 and 131072 samples. The
results show that the BER performance is directly related to the time constant. When the
time constant equals 131072 samples, the overall characteristics asymptotically approach
those of the case without imbalance.

5 Conclusions

In this paper, we presented an evaluation for an imbalance compensation method of the
direct-conversion receiver, using blind imbalance detection in the control loop. In the
control loop, our approximations reduce the complexity of digital signal processing.
The modified method that applies the approximations achieves better BER
performance. In the modified method, the phase and gain compensation processing blocks
are serially cascaded. The performance of this configuration is verified by simulation
results for 16/64 QAM modulated waves. Moreover, a sufficient improvement in the
convergence time in the loop can be achieved by selecting the time constant on the basis of
the amount of the detected error in the first-order loop. We can easily apply the modified
method using a digital signal processor such as a field programmable gate array (FPGA) for
direct-conversion receivers. For avoiding the division operation and reducing the number of
multiplications, the modified method can achieve reduction in complexity for practical
operations by using the FPGA. Furthermore, the modified method is expected to be
implemented in practical systems (modulation type, signal bandwidth, and sampling
frequency) to enhance their performance.

References

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