$H \to \ell\ell'$ in the Simplest Little Higgs Model

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Abstract

Little Higgs Models are promising constructs to solve the hierarchy problem affecting the Higgs boson mass for generic new physics. However, their preservation of lepton universality forbids them to account for the $H \to \tau\mu$ CMS hint and at the same time respect (as they do) the severe limits on $H \to \mu e$ inherited from the non-observation of $\mu \to e\gamma$. We compute the predictions of the Simplest Little Higgs Model for the $H \to \ell\ell'$ decays and conclude that the measurement of any of these decays at LHC (even with a much smaller rate than currently hinted) would, under reasonable assumptions, disfavor this model. This result is consistent with our earlier observation of very suppressed lepton flavor violating semileptonic tau decays within this model.

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1 Introduction

The discovery of a scalar boson at the LHC experiments ATLAS [1] and CMS [2] with properties remarkably close to the one proposed as agent of electroweak symmetry breaking in 1964 [3] has explained the origin of elementary particle masses (with maybe neutrinos aside) within the Standard Model (SM). Despite the
One way to do this is to take advantage of observables where both sharp predictions and accurate measurements are possible, in such a way that tiny deviations between them can become statistically significant and hint at the new dynamics. An alternative is to search for forbidden processes in the SM, where an observation must be due to new phenomena. Lepton flavor violating (LFV) processes are practically forbidden in the SM, but the severe limits on Higgs decays at Belle-II [18] would (using reasonable approximations) rule out this model, as it predicts decay rates four to five orders of magnitude smaller than current bounds. According to LHC results, H → ℓℓ′ decays are allowed at a rate of about 1%. However, the severe limits on μ → eγ constrain \( BR(H → μe) \lesssim 10^{-8} \) [11] and a strong suppression on \( BR(H → τμ) \) is also expected from the \( τ → μγ \) bounds [14]. Since the SLH model does not violate lepton universality (LU), confirmation of the CMS hint in \( H → τμ \) decays would therefore falsify the (S)LH Models. On the contrary, if no LFV Higgs decays are confirmed at the LHC and recent LHC breaking hints are refuted, these models with collective electroweak symmetry breaking will remain to be a promising alternative for the ultraviolet (UV) completion of the SM. Our aim in this paper is to make the extraordiary success of this theory, there remain some unanswered questions within the SM, such as the origin of dark matter or the explanation of the baryon asymmetry of the universe. These and other puzzles motivate the scrutiny of the SM predictions against measurements in the search for new physics.

In the next section we outline the main features of the SLH model, referring to our earlier work [17] for an extended discussion. In section 3 we compute the leading contributions to the H → ℓℓ′ decays within this model explaining our expansion in powers of the electroweak symmetry breaking scale (v) over that of the new particles of the extended model (f). We also confront the predictions of the SLH model for the considered decays to the current bounds, employing a parameter space of the SLH model which is consistent with current data and in section 4 we state our conclusions.

1Here, and throughout the letter, the notation \( BR(H → τμ) \) implies \( BR(H → τ⁻μ⁺) + BR(H → τ⁺μ⁻) \).

2CMS has just presented their update including 13 TeV data [8], \( BR(H → τμ) = 0.84_{-0.37}^{+0.39} \% \) (\( < 1.51\% \) at 95% CL) caused a big excitement in the community, although ATLAS [7] did not confirm it a few months later, setting the bound \( BR(H → τμ) < 1.85\% \) at the same confidence level [9].
2 The Simplest Little Higgs Model

We reviewed in some detail the SLH model in our previous work [17]. Here we will only recall its main features for the reader's convenience, but refer to [17] and references therein for an expanded account.

The SLH model extends the SM electroweak gauge group \( SU(2)_L \otimes U(1)_Y \) to \( SU(3)_L \otimes U(1)_X \) in a minimal way. Correspondingly, the \( SU(2)_L \) SM fermion doublets are enlarged to \( SU(3)_L \) triplets and five extra weak gauge bosons are added (three neutral and two of opposite unit charges which, after diagonalization, give rise to heavy copies of the W bosons that we call \( W^{\prime \pm} \).) \( SU(3) \) invariant interactions are written so as to reproduce all the SM dynamics when restricted to the SM fields. Among the new particles, only the heavy quasi-Dirac neutrinos \( N_k \) (\( k = 1, 2, 3 \) is the family index), which allow the LFV transitions, and the \( W^{\prime \pm} \) bosons play a role in our study. Within the SLH model, LFV is achieved because of the misalignment between the SM down-type lepton and the new heavy neutrino mass matrices.

The starting global symmetry of the model, \( [SU(3) \otimes U(1)]_1 \otimes [SU(3) \otimes U(1)]_2 \), is spontaneously broken down to \( [SU(2) \otimes U(1)]_1 \otimes [SU(2) \otimes U(1)]_2 \) by the non-vanishing aligned vacuum expectation values (vevs) acquired by the two scalar triplets of the model \( \mathbf{5} \) (the scalar sector is a non-linear sigma model). These transform as \( (3, 1) \) and \( (1, 3) \) under \( SU(3)_1 \otimes SU(3)_2 \), respectively, and include the SM Higgs doublets as well as new pseudo-Goldstone bosons (pGBs). Both vevs, \( f_1 \) and \( f_2 \), are of order TeV and determine the high-energy scale of the model, \( f \), through \( f_1^2 + f_2^2 = f^2 \). The gauged diagonal subgroup, \( SU(3)_L \otimes U(1)_X \), breaks down to the SM electroweak gauge group via these scalar vacuum condensates. The gauge interactions of the model are fixed by gauge invariance and given in terms of the SM couplings. The spontaneous symmetry breaking produces 5 pGBs for each scalar. Since one \( SU(3) \) is weakly gauged, 5 pGBs give -through the Higgs mechanism- masses of \( O(f) \) to the new gauge bosons, while the 5 orthogonal combinations are pGBs (including the Higgs boson and, particularly, alleviating the hierarchy problem on its mass, \( M_H \sim O(v) \), due to the structure of the SLH model). Since the mass of the new gauge bosons is proportional to the high scale, \( f \): \( M_W = \frac{g_f^2}{2} \left( 1 + O\left( \frac{v^2}{f^2} \right) \right) \), this suppresses their contribution at low energies. Light and the new heavy neutrinos do mix as regulated by the small parameter \( \delta_v = \frac{v_2}{\sqrt{2} \tan \beta f} \), where \( \tan \beta = f_1/f_2 \) is the ratio between both vevs, and heavy neutrino masses are also set by the large scale \( f \). In our computation we will exploit the fact that the ratio \( v/f \) is small (\( \lesssim 0.1 \)) to expand our amplitudes in it and keep only the leading contribution.

3 Results

LFV Higgs decays arise at one-loop level in the SLH model and are possible because the “little” heavy neutrinos \( N_\ell \) couple to either charged lepton, \( \ell, \) irrespective of its flavor. In the considered \( H \rightarrow \ell \ell' \) decays only the topologies sketched in Fig. 1 contribute at this order. Since the Higgs boson couples not only to a pair of \( W^{(\prime)} \) but also to \( WW' \), the first topology gives rise to four different diagrams\(^3\). In the second topology, the loop mediating the LFV transition may as well be placed in the \( \ell' \) leg, and in the last one the exchange \( N_\ell \leftrightarrow \nu \) yields an inequivalent contribution. In all, this makes 12 different diagrams at this order working in the unitary gauge, where only physical degrees of freedom appear and the number of diagrams is reduced. Although each diagram diverges stronger in this gauge than in that of ’t Hooft-Feynman, the individual divergences cancel, as they must, to ensure a finite result.

The contributions of self-energy type (second diagram) are proportional to \( m_\nu \) and will thus be neglected for \( \ell = e, \mu \). Along the computation we have neglected powers of the ratios of lepton masses over gauge boson \( (W, W') \) and heavy neutrino \( (N) \) masses. For definiteness we include our results for the \( H \rightarrow \tau \ell \) decay. There is an overall dependence on the heaviest final-state lepton mass, which shows that the decay rate \( H \rightarrow \ell \ell' \)

\(^3\)In simple group models (like SLH), where the SM gauge group emerges from the diagonal breaking of a larger simple group, at least two sigma-model multiplets are required [15].

\(^4\)We point out that exchanging \( W \leftrightarrow W' \) in the diagrams built with the \( HWW' \) vertex yields two different results, as can easily be shown.
Figure 1: Feynman diagrams for $H \to \ell \ell'$ decays in the SLH model.

vanishes in the limit of massless decay products. Therefore, and in absence of a mechanism of LU violation in the SLH model, we will have \( BR(H \to \tau \mu) = BR(H \to \tau e) = \frac{m_\mu^2}{m_\tau^2} BR(H \to \mu e) \), which suppresses the latter decay rate by a factor $\sim 283$. Given this trivial proportionality, we will only be plotting $BR(H \to \tau \ell)$ in figs. 2-5.

Within this setting, there are two different mass scales in the problem: those of $O(v)$ ($M_W$ and $M_H$) and those of $O(f)$ ($M_N$ and $M_{W'}$). In Ref. [17] we used $\omega = \frac{m_W^2}{M_{W'}} \sim \frac{v^2}{f^2} << 1$ to characterize the ratio between two separated scales and $\chi_j = \frac{M_N^2}{M_{W'}} \sim O(1)$ for that of two high scales. Since there were only three mass scales in the study of semileptonic LFV tau decays within this model, all mass ratios could be expressed in terms of $\omega$ and $\chi_j$ (unless there is a very strong hierarchy between the different heavy neutrino flavors). In the present study, there is $M_H$, as well. This entails the appearance of four small ratios between a light and a heavy particle mass: $\frac{M_H^2}{M_N^2} \sim \frac{M_{W'}^2}{M_N^2} \sim \frac{M_{W'}^2}{M_{W'}} = \omega \sim \frac{v^2}{f^2} << 1$ (we recall that $\delta_\nu \sim \frac{v}{f}$ as well) and two involving particles with similar masses: $\frac{M_H^2}{M_N^2} \sim \frac{M_{W'}^2}{M_{W}} = \chi_j \sim O(1)$.

Our analytical expressions are simplified in the limit of only two heavy neutrinos that we follow [17]. We will however, consider also the case with three heavy neutrinos for completeness after eq. (6). Let us discuss our choices for defining the parameter space of the model before presenting our expressions. In the numerical analysis we have stick to the choices argued in our previous work [17], in such a way that a considerable portion of the points generated randomly in the ranges fixed a priori fulfils the constraints coming from $\mu \to e\gamma$, $\mu \to eee$ and $\mu - e$ conversion in nuclei [21] and also those on $\tau \to \mu\gamma$ [17].

We recall in the following the a priori range of variation that we are allowing for the independent model parameters in our parameter space scan:

- We have varied the scale of compositeness between 2 and 10 TeVs. Lower values are in tension with electroweak precision observables and larger figures enter the region where a UV completion of the SLH model (that would become strongly coupled) starts to be expected [20].
- The LFV processes are possible in the SLH model because of the presence of the heavy neutrinos. The dependence of the amplitude on their contribution is

$$\mathcal{T} = \sum_j V^{\mu*}_\ell V^{\tau}_\ell A(\chi_j) \, .$$  \hspace{1cm} (1)

Assuming two families and one mixing angle, this can be written

$$\mathcal{T} = \frac{\sin 2\theta}{2} [A(\chi_1) - A(\chi_2)] \, .$$  \hspace{1cm} (2)
This simplification will be used to write eq.(3) below. Particularly, since the terms with log(\omega) are independent on \chi_j, they do not contribute in this limit (see, however, eq. (5)). This, by the way, prevents the appearance of a moderately large log, log(\chi_j/\omega), in the two heavy neutrino scenario. In the numerics, we will use the limits 0 \leq \chi_1 \leq 0.25 and 1.1 \chi_1 \leq \chi_2 \leq 10\chi_1 and sin 2\theta \leq 0.25, consistent with current data \cite{21, 22}.

- Finally, the ratio of the two vevs in the model, tan \beta, is also a free parameter of the SLH model. We will take the range 1 < tan \beta < 10 for it, consistent with the known limits for the mixing between a 'little' and a light neutrino encoded in \delta_\nu, which should be \lesssim 0.05 \cite{21, 22}.

In these LFV Higgs decays there is at least a lepton which can be considered massless and, thus, with fixed helicity. Then, the matrix element for the H → \tau\ell decays can be written as

\[ \mathcal{M}_H = -\frac{i e^2 \delta_\nu \sum_j V_{ij} V^{*}_{ij}}{s^2_\nu M_W^2} [O \log \chi_j - P(\chi_j)] \bar{u}(p') P_H u(q, m_\tau), \]  

where:

\[ O = \frac{\delta_\nu M_H^2 - 13 M_W^2}{16 M_W^2} \]

and

\[ P(\chi_j) = \cot (2\beta) \frac{M_H^2}{8 M_W^2 M_W \chi_j (\chi_j - 1)} \left[ \frac{3 \chi_j^3 - 9 \chi_j^2 + 8 \chi_j - 2}{8 M_W^2 M_W \chi_j (\chi_j - 1)} + \frac{\omega (2 \chi_j^3 + 74 \chi_j^2 + 35 \chi_j - 1)}{48 (\chi_j - 1)^2} \right] \]

In our computation we kept terms of subleading order (v^3/f^3) and check for accidental numerical enhancements of these before neglecting them. After checking its irrelevance, we omitted one such a term in O and another one in P(\chi_j).

The corresponding branching ratio is

\[ BR (H \rightarrow \tau\mu) = \frac{(M_H^2 - m_t^2)^2 \alpha^4 v^2 s^2_\nu M_W^4}{16 \pi M_H^2 \Gamma_H M_W^4 s^2_\nu} \left( \frac{\sin 2\theta}{2} \right)^2 \left[ O \log \frac{\chi_1}{\chi_2} + P(\chi_2) - P(\chi_1) \right]^2. \]  

We have performed a scan of the parameter space limited by the above a priori restrictions verifying that the constraints from low-energy processes are respected and plotted BR(H → \tau\ell) (%) as a function of one parameter in turn (f, M_{N_1}, tan \beta and sin 2\theta) in the left pannel of figs. 2-5. Roughly 23% of the 5 \cdot 10^4 randomly generated points satisfied the low-energy restrictions.

Since these branching fractions are very suppressed, we have considered next the case with three heavy neutrinos, N_k, expecting that the increase of degrees of freedom allows to satisfy the low-energy contraints on LFV processes and, at the same time, yield H → \ell\ell' with larger decay rates. In this general case we have considered the PDG \cite{23} parametrization for the mixing between charged leptons and heavy neutrinos assuming only a vanishing CP violating phase for simplicity. Therefore

\[ V_{\ell}^{cf} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \]  

The assumption of two heavy neutrinos has already been used.
where we have employed the usual notation \( c_{ij} \equiv \cos \theta_{ij} \), \( s_{ij} \equiv \sin \theta_{ij} \), \( i = 1, 2, 3 \) and \( f = e, \mu, \tau \). In this case the replacement needed in eq. (6) is
\[
\left( \frac{\sin 2\theta}{2} \right)^2 \left[ O \log \left( \frac{\chi_1}{\chi_2} \right) + P(\chi_2) - P(\chi_1) \right]^2 \rightarrow \sum_{i=1}^{3} V_{\ell f}^{i\mu} V_{\ell f}^{i\tau} \left[ O \log \left( \frac{\chi_1}{\chi_2} \right) + P(\chi_i) \right]^2 ,
\]
where the assumed CP conservation of \( V_{\ell f} \) was used.

We have verified that restricting the maximum values of \( |V_{\ell f}^{i\mu} V_{\ell f}^{i\tau}| \) to reasonable upper limits does not yield additional constraints than requiring the fulfilment of the experimental bounds on low-energy LFV processes. Consequently, we have scanned over \(-1 \leq s_{ij} \leq 1\) ensuring the low-energy restrictions. Constraints in the case with three heavy neutrinos are milder than in the previous case where two heavy neutral leptons were considered. We take advantage of that to slightly modify our choices of their parameters aiming to increase the predicted \( H \rightarrow \tau \ell \) branching ratios. For the masses of the three heavy neutrinos we have followed the educated guess \( 0.1 \leq \chi_1 \leq 0.25 \), \( 1.1 \chi_1 \leq \chi_2 \leq 100 \chi_1 \) and \( 1.1 \chi_2 \leq \chi_3 \leq 100 \chi_2 \) (we recall that \( \chi_i \) depends quadratically on the \( N_i \) mass), while \( f \) and \( \tan(\beta) \) have been varied in the same way as in the two heavy neutrino scenario. In this way we have obtained the plots depicted on the right pannels of figures 2-5. The comparison of the left and right plots of these figures shows that, indeed, allowing for an extra heavy neutrino gives us enough freedom to increase the predicted decay rates by some four orders of magnitude. However, these still remain well below the CMS hint on \( H \rightarrow \tau \mu \) and the detectability level at LHC.

Figure 2: Dependence of the scale of compositeness, \( f \), of the branching ratio (%) of the \( H \rightarrow \tau \ell \) decays in the SLH model with two (left) and three (right) heavy neutrinos. The red line shows the 95% CL upper bound by CMS.

The general trend is that the SLH model produces \( H \rightarrow \tau \ell \) decay widths which are at least six orders of magnitude smaller than the \( BR \sim \mathcal{O}(\%) \) hinted by CMS (see also figure 6). A similarly strong suppression of \( BR(H \rightarrow \tau \ell) \) is also found in a recent analysis within the Little Higgs Model with T-parity including constraints from other charged lepton flavor violating processes [14]. Finally, in figure 8 we show that there is basically no correlation between \( BR(H \rightarrow \tau \ell) \) and \( BR(\mu \rightarrow e\gamma) \) (the most restrictive low-energy search) in the case with three heavy neutrinos. There are not sizable correlations between the pairs \( BR(H \rightarrow \tau \ell) \), \( BR(\tau \rightarrow \ell \gamma) \) and \( BR(H \rightarrow \mu e) \), \( BR(\mu \rightarrow e\gamma) \) either.
Figure 3: Dependence on the lightest mass of the heavy neutrinos, $M_{N_1}$, of the branching ratio (%) of the $H \to \tau \ell$ decays in the SLH model with two (left) and three (right) heavy neutrinos. The red line shows the 95% CL upper bound by CMS.

Figure 4: Dependence on the ratio of the two vevs, $\tan \beta$, of the branching ratio (%) of the $H \to \tau \ell$ decays in the SLH model with two (left) and three (right) heavy neutrinos. The red line shows the 95% CL upper bound by CMS.

As expected, if the SLH model is to satisfy the bounds on $H \to \mu e$ set by $\mu \to e\gamma$ ($BR(H \to \mu e) \lesssim 10^{-8}$ [11]) -as it does-, it must fall way too short to explain the CMS hint, as a consequence of its LU. It must
be pointed out, however, that we restricted the Yukawa interactions in the lepton sector up to operators of dimension 5. Given our ignorance of the flavor structure of the theory at its cut-off, it could be possible that the contribution of higher-dimensional operators (see, e.g. Harnik et. al. in [11]) could change the results we presented. Finally, we point out that very mild variations are appreciated in figs. 2-5 with respect to the independent variables. Decay probabilities are slightly larger for smaller $f$ and $M_{N_1}$ and for larger $\tan \beta$ and $\sin 2\theta$ (max$|V_{ei}^\mu V_{ei}^\tau|$).

4 Conclusions

Flavor violation has shown up in the quark and neutrino sectors. Although extremely suppressed in the SM extended with right-handed neutrinos, it may appear at measurable rates in several well-motivated new physics models. On the other hand, the discovery of the Higgs boson at the LHC has brought a new scenario to search for LFV in the decays of this scalar. An elegant solution to the hierarchy problem on the Higgs mass is provided by the LH models. Here, we have considered the SLH model (one of the simplest realizations of these ideas) against the ATLAS and CMS limits on $BR(H \rightarrow \tau \mu)$, of order percent. Given the LU of the SLH model, it is not surprising that the model cannot simultaneously account for the tiny rate at which the $H \rightarrow \mu \nu$ decays must proceed ($\lesssim 10^{-8}$) and also for a measurable signal at LHC. We have found that a $BR(H \rightarrow \mu e)$ as low as $10^{-10}$ is obtained naturally (even allowing for three heavy neutrinos, which increases the predicted decay rates with respect to the scenario with two heavy neutral leptons), with $BR(H \rightarrow \tau \ell)$ only enhanced by a factor of order 300. Thus, the confirmation of the CMS hint would disfavor the SLH model, as it will do a measurement at Belle-II of semileptonic LFV tau decays [17].
Figure 6: The correlation between the $H \rightarrow \tau \ell$ and $\mu \rightarrow e\gamma$ decays is illustrated within the SLH model in the case with three heavy neutrinos, $N_k$. The x-axis is cut at the current upper limit (UL) at 90% C.L. of $BR(\mu \rightarrow e\gamma)$, $5.7 \times 10^{-13}$ [23].

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