Evolution of the 7/2 fractional quantum Hall state in two-subband systems

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We report the evolution of the fractional quantum Hall state (FQHS) at even-denominator filling factor \( \nu = 7/2 \) in wide GaAs quantum wells in which electrons occupy two electric subbands. The data reveal subtle and distinct evolutions as a function of density, magnetic field tilt-angle, or symmetry of the charge distribution. When the charge distribution is strongly asymmetric, there is a remarkable persistence of a resistance minimum near \( \nu = 7/2 \) when two Landau levels belonging to the two subbands cross at the Fermi energy. The field position of this minimum tracks the 5/2 filling of the symmetric subband, suggesting a pinning of the crossing levels and a developing 5/2 FQHS in the symmetric subband even when the antisymmetric level is partially filled.

The fractional quantum Hall states (FQHSs) at the even-denominator Landau level (LL) filling factors \([1]\) have recently come into the limelight thanks to the theoretical prediction that these states might be non-Abelian \([2]\) and be useful for topological quantum computing \([3]\). This expectation has spawned a flurry of investigations, both experimental \([4\,10]\) and theoretical \([11\,13]\), into the origin and stability of the even-denominator states. Much of the attention has been focused on the \( \nu = 5/2 \) FQHS which is observed in very low disorder two-dimensional electron systems (2DESs) when the Fermi energy (\( E_F \)) lies in the spin-up, excited-state (\( N = 1 \)) LL of the ground-state (symmetric, S) electric subband of the 2DES, namely in the \( S1 \uparrow \) level. Here we examine the stability of the FQHS at \( \nu = 7/2 \), another even-denominator FQHS, typically observed when \( E_F \) is in the \( S1 \downarrow \) level (Fig. 1(a)). The \( \nu = 7/2 \) FQHS, being related to the 5/2 state through particle-hole symmetry, is also theoretically expected to be non-Abelian. Our study, motivated by theoretical proposals that the even-denominator FQHSs might be favored in 2DESs with “thick” wavefunctions \([11\,13]\), is focused on electrons confined to wide GaAs quantum wells (QWs).

In a realistic, experimentally achievable wide QW, however, the electrons at \( \nu = 7/2 \) can occupy the second (antisymmetric, A) electric subband when the subband energy spacing (\( \Delta \)) is comparable to the cyclotron energy \( \hbar \omega_c \) (Figs. 1(b-d)). Here we experimentally probe the stability of the \( \nu = 7/2 \) FQHS in wide QW samples with tunable density in the vicinity of the crossings (at \( E_F \)) between the S1 and the A0 LLs.

Our samples were grown by molecular beam epitaxy, and each consist of a wide GaAs QW bounded on each side by undoped \( Al_{0.2}Ga_{0.76}As \) spacer layers and Si \( \delta \)-doped layers. We report here data, taken at \( T \approx 30 \text{ mK} \), for three samples with QW widths of \( W = 37, 42, \) and \( 55 \text{ nm} \). The QW width and electron density \( (n) \) of each sample were designed so that its \( \Delta \) is close to \( \hbar \omega_c \) at the magnetic field position of \( \nu = 7/2 \). This enables us to make the S1 and the A0 LLs cross at \( E_F \) by tuning \( n \) or the charge distribution asymmetry, which we achieve by applying back- and front-gate biases \([7\,14\,16]\). For each \( n \), we measure the occupied subband electron densities \( n_S \) and \( n_A \) from the Fourier transforms of the low-field (\( B \leq 0.5 \text{ T} \)) Shubnikov-de Haas oscillations \([14\,15]\), and determine \( \Delta = (\pi \hbar^2/m^*)\left(n_S - n_A\right) \), where \( m^* = 0.067m_e \) is the GaAs electron effective mass. At a fixed total density, \( \Delta \) is smallest when the charge distribution is “balanced” (symmetric) and it increases as the QW is imbalanced. Our measured \( \Delta \) agree well with the results of calculations that solve the Poisson and Schroedinger equations to obtain the potential energy and the charge distribution self-consistently (see, e.g., Figs. 1(a,d)).

Figure 1 shows a series of longitudinal (\( R_{xx} \)) and Hall (\( R_{xy} \)) resistance traces in the range \( 3 < \nu < 4 \) for a 44 nm-wide QW sample, taken at different \( n \) from 2.13 to 2.96 \( \times 10^{11} \text{ cm}^{-2} \) while keeping the total charge distribution balanced. As \( n \) is increased in this range, \( \Delta \) decreases from 64 to 54 K while \( \hbar \omega_c \) at \( \nu = 7/2 \) increases from 50 K to 70 K, so we expect crossings between the S1 and A0 levels, as illustrated in Figs. 1(a-d). These crossings manifest themselves in a remarkable evolution of the FQHSs as seen in Fig. 1. At the lowest \( n \), which corresponds to the LL diagram shown in Fig. 1(a), \( R_{xx} \) shows a reasonably deep minimum at \( \nu = 7/2 \), accompanied by a clear inflection point in \( R_{xy} \) at \( 7/2(h/e^2) \), and a weak minimum near \( \nu = 10/3 \). These features are characteristic of the FQHSs observed in high-quality, standard (single-subband) GaAs 2DESs, when \( E_F \) lies in the \( S1 \downarrow \) LL \([1\,2]\). As \( n \) is raised, we observe an \( R_{xx} \) spike near \( \nu = 7/2 \), signaling a crossing of \( S1 \downarrow \) and \( A0 \uparrow \). At \( n = 2.51 \times 10^{11} \text{ cm}^{-2} \), these levels have crossed, and \( E_F \) is now in \( A0 \uparrow \) (Fig. 1(b)). There is no longer a minimum at \( \nu = 7/2 \), and instead, there are very strong minima at \( \nu = 10/3 \) and \( 11/3 \). Further increasing \( n \) causes a crossing of \( S1 \uparrow \) and \( A0 \uparrow \), and at \( n = 2.63 \times 10^{11} \text{ cm}^{-2} \), \( E_F \) at \( \nu = 7/2 \) lies in \( S1 \uparrow \) (Fig. 1(c)). Here the \( R_{xx} \) minimum and \( R_{xx} \) inflection point at \( \nu = 7/2 \) reappear, signaling the return of a FQHS. As we increase \( n \) even further, \( S1 \uparrow \) and \( A0 \downarrow \) cross, and, at \( n = 2.96 \times 10^{11} \text{ cm}^{-2} \), when \( E_F \) at \( \nu = 7/2 \) lies in \( A0 \downarrow \), there is again no \( \nu = 7/2 \) minimum but there are strong FQHSs at \( \nu = 10/3 \) and \( 11/3 \).

The above observations provide clear and direct evidence that the even-denominator \( \nu = 7/2 \) FQHS is stable.
FIG. 1. (color online) Left panel: Waterfall plot of $R_{xx}$ and $R_{xy}$ traces at different densities for a 42-nm-wide GaAs QW. (a-d) LL diagrams at $\nu = 7/2$ for different densities. Self-consistently calculated charge distributions are shown in the insets to (a) and (d) for $n = 2.13$ and $2.96 \times 10^{11}$ cm$^{-2}$. when $E_F$ is in an excited ($N = 1$) LL but not when $E_F$ lies in a ground-state ($N = 0$) LL [7]. Examining traces taken at numerous other $n$, not shown in Fig. 1 for lack of space, reveal that the appearances and disappearances of the $\nu = 7/2$ FQHS are sharp, similar to the behavior of the $5/2$ FQHS at a LL crossing [17]. It is noteworthy that when the two crossing levels have antiparallel spins, a “spike” in $R_{xx}$ at the crossing completely destroys the FQHS at $\nu = 7/2$ and nearby fillings. At the crossing of two levels with parallel spins, on the other hand, there is no $R_{xx}$ spike. These behaviors are reminiscent of easy-axis and easy-plane ferromagnetism for the antiparallel- and parallel-spin crossings, respectively [16, 18].

Next, we examine the evolution of the $\nu = 7/2$ FQHS in the presence of a parallel magnetic field component $B_{||}$, introduced by tilting the sample so that its normal makes an angle $\theta$ with the total field direction (Fig. 2(b)). Figure 2(a) captures this evolution for electrons confined to a symmetric, 37-nm-wide QW [19]. This QW is narrower so that, at $n = 2.34 \times 10^{11}$ cm$^{-2}$, its $\Delta$ ($= 82$ K) is well above $h\omega_c$ ($= 55$ K). The $\theta = 0$ trace then corresponds to $E_F$ lying in S1↓, as shown in Fig. 2(d). As $\theta$ is increased, we observe only a gradual change in the strength of the $\nu = 7/2$ FQHS, until it disappears at large $\theta > \sim 55^{\circ}$. This is not surprising since, in a two-subband system like ours, we expect a severe mixing of the LLs of the two subbands with increasing $\theta$ [20] rather than sharp LL crossings as manifested in Fig. 1 data.

We highlight three noteworthy features of Fig. 2 data. First, the $\nu = 7/2$ $R_{xx}$ minimum persists up to relatively large $\theta$ (up to $50^{\circ}$), and it even appears that the $R_{xy}$ plateau is better developed at finite $\theta$ (up to $\theta = 42^{\circ}$) compared to $\theta = 0$, suggesting a strengthening of the $7/2$ FQHS at intermediate angles. Second, deep $R_{xx}$ minima develop with increasing $\theta$ at $\nu = 10/3$ and $11/3$, suggesting the development of reasonably strong FQHSs at these fillings. This is consistent with the results of Xia et al. who report a similar strengthening of the $7/3$ and $8/3$ states - the equivalent FQHSs flanking the $\nu = 5/2$ state in the S1↑ level - when a wide QW sample is tilted in field [9]. Third, the large magnitude of $B_{||}$ at the highest angles appears to greatly suppress $\Delta$ [21], rendering the electron system essentially into a bilayer system. This is evidenced by the dramatic decrease in the strength of the $\nu = 3$ QHS and the disappearance of the $\nu = 11/3$ $R_{xx}$ minimum at $\theta = 79^{\circ}$; note that a FQHS should not exist at $\nu = 11/3$ in a bilayer system with two isolated 2DESs as it would correspond to $11/6$ filling in each layer.

We now focus on data taken on a 55-nm-wide QW where we keep the total $n$ fixed and change the charge distribution symmetry by applying back- and front-gate biases with opposite polarity. In Fig. 3(a) we show a set of $R_{xx}$ traces, each taken at a different amount of asym-
FQHS and instead we observe strong FQHSs at $E_R$ and we observe a very strong \( \nu \), the one shown in Fig. 1(d). Consistent with this LL diagram, \( \Delta \) is much smaller than \( \bar{\Delta} \). When the charge distribution is symmetric or nearly symmetric, the in-field charge distribution is given by \( \Delta(B) = \hbar \omega_c - E_2 \). This expression ensures that \( \Delta(B) \) is fixed at a given \( B \), consistent with the pinning of the S1$^\uparrow$ and A0$^\downarrow$ levels. We then perform in-field self-consistent calculations for a series different QW asymmetries. For each QW asymmetry, the in-field charge distribution is given by:

\[
\rho(B) = e (eB/h) [\nu_S \cdot |\psi_S(B)|^2 + \nu_A \cdot |\psi_A(B)|^2].
\] (1)

Now, for the different points on the boundary, \( \nu_S \) and \( \nu_A \) have specific and well-defined values. For example, at \( \nu = 4 \) (\( B = 3.75 \) T), for which we show the results of our self-consistent calculations in Figs. 3(c) and (e), we have \( \nu_S = 3 \) and \( \nu_A = 1 \) for the upper boundary and \( \nu_S = \nu_A = 2 \) for the lower one. Focusing on the upper boundary, i.e., using \( \nu_S = 3 \) and \( \nu_A = 1 \) in Eq. (1),
among all the QW asymmetries for which we perform the self-consistent calculations, one in particular has a subband separation which is equal to $\Delta(B) = 56$ K for $B = 3.75$ T. This particular QW asymmetry gives the upper boundary at $B = 3.75$ T. Then we calculate the zero-field subband separation for this asymmetry, which turns out to be $\Delta = 68$ K, and mark it in Fig. 3(b) as the upper boundary for the pinning at $B = 3.75$ T. For the lower boundary at $B = 3.75$ T, we repeat the above calculations using $\nu_S = \nu_A = 2$. The QW asymmetry that gives $\Delta(B) = 56$ K yields a zero-field $\Delta$ of 23 K which we mark in Fig. 3(b) as the lower boundary at 3.75 T. The rest of the boundary in Fig. 3(b) is determined in a similar fashion. For example, the upper boundary at $\nu = 7/2$ corresponds to ($\nu_S = 2.5, \nu_A = 1$) and the lower boundary to ($\nu_S = 2, \nu_A = 1.5$).

It is clear that the calculated boundary marked by the solid lines in Fig. 3(b) matches well the region (in $\Delta$ vs. $B$ plane) in which we experimentally observe a disappearance of the $\nu = 4$ $R_{xx}$ minimum and the appearance of $R_{xx}$ minima at anomalous fillings. This matching is particularly remarkable, considering that there are no adjustable parameters in our simulations, except for using a single value (7.3) for the enhanced $g$-factor $g^*$. In Fig. 3(b) we also include a dashed line representing the values of $B$ at which, according to our calculations, the $S_1^+$ level is exactly half-filled, i.e., $\nu_S = 5/2$ and $\nu_A = (\nu - 5/2)$. This dashed line tracks the positions of the observed $R_{xx}$ minima marked by vertical arrows in (a) very well, suggesting that these minima indeed correspond to $\nu_S = 5/2$. This is an astonishing observation, as it implies that there is a developing FQHS at 5/2 filling of the symmetric subband even when a partially filled A0↓ level is pinned to the half-filled S1↑ level at $E_F$!

In Fig. 3(b) we also show a boundary, marked by dotted lines, which is based on a simple, analytic model. Note that the simulations shown in Figs. 3(c-e) indicate that the in-field charge distributions, calculated self-consistently at $\nu = 4$, are nearly the same as the $B = 0$ distributions. In our simple model, we assume that the in-field wavefunctions $\psi_S(B)$ and $\psi_A(B)$ are just linear combinations of the $B = 0$ wavefunctions $\psi_S(0)$ and $\psi_A(0)$. We then set the total in-field charge distribution, given by Eq. (1), equal to its $B = 0$ value, $\rho(0) = e \nu_S |\psi_S(0)|^2 + e \nu_A |\psi_A(0)|^2$, and find:

$$\Delta_0^2 = (\nu_S - \nu_A)(eB/h)(\pi \hbar^2/m^*)\Delta(B).$$  \hspace{1cm} (2)

For a given value of $B$ and therefore $\Delta(B) = \hbar \omega_c - E_Z$, Eq. (2) gives the $B = 0$ subband separation $\Delta_0$ which corresponds to the onset of pinning/depinning of the relevant LLs. For example, to find the upper boundary at $B = 4$ T ($\nu = 3.75$), we use $\Delta(B) = 60$ K, $\nu_A = 1$ and $\nu_S = 2.75$, and solve Eq. (2) to find $\Delta_0 = 65$ K. To find the lower boundary at $B = 4$ T, we use $\nu_S = 2$ and $\nu_A = 1.75$ and find $\Delta_0 = 25$ K. As seen in Fig. 3(b), the dotted line given by the simple, analytic expression (2) matches the boundary determined from in-field self-consistent calculations reasonably well except for the lower points where $\nu S = \nu A \approx 2$ leads to $\Delta_0 \approx 0$.

In summary our results reveal distinct metamorphoses of the ground-state of two-subband 2DESs at and near $\nu = 7/2$ as either the field is tilted, or the density or the charge distribution symmetry are varied. Most remarkably, we observe a developing FQHS when a half-filled S1↑ level is pinned to a partially-filled A0↓ level [23].

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