Area law corrections from state counting and supergravity

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Abstract. Modifications of the Bekenstein-Hawking area law for black holes are crucial in order to find agreement between the microscopic entropy based on state counting and the macroscopic entropy based on an effective field theory computation. We discuss this and related issues for the case of four-dimensional $N = 2$ supersymmetric black holes. We also briefly comment on the state counting for $N = 4$ and $N = 8$ black holes.

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1. Introduction

A microscopic derivation of the entropy of certain extremal black holes has recently become available in the context of string theory [1]-[6]. For four-dimensional extremal black holes in the limit of large electric/magnetic charges $Q$, the microscopic entropy is generically of the form
\[ S_{\text{micro}} \sim \sqrt{\frac{Q}{4}}. \]  
(1)

This result agrees with the one obtained from macroscopic calculations based on the corresponding effective field theories. Here one first constructs the associated black hole solution and then one computes the macroscopic entropy according to the Bekenstein-Hawking area law [7, 8].

In the context of string theory and M-theory, the microscopic entropy (1) is calculated by counting excitations of D-branes and M-branes. In the context of type-IIA compactifications on Calabi-Yau threefolds $CY_3$ extremal black holes are microscopically represented by wrapping a D4-brane on a smooth holomorphic four-cycle $P$ of the Calabi-Yau threefold and by considering its bound state with $|q_0|$ D0-branes. In M-theory compactifications on $CY_3 \times S^1$, on the other hand, they are represented by five-branes wrapped on $P \times S^1$, with $|q_0|$ quanta of lightlike momentum along the circle $S^1$. The massless excitations of the five-brane wrapped on $P$ are described by a $(0, 4)$ two-dimensional conformal theory [5], and the degeneracy of states of this conformal field theory yields the microscopic entropy according to Cardy’s formula,
\[ d(|q_0|, c_L) \approx \exp(S_{\text{micro}}) \approx \exp \left( 2\pi \sqrt{\frac{1}{6} |q_0| c_L} \right). \]  
(2)

Here $|q_0|$ is taken to be large and $c_L$ denotes the central charge for the left-moving sector. Evaluation of (2) for the case of a five-brane wrapped around $P \times S^1$ yields [3],
\[ S_{\text{micro}} = 2\pi \sqrt{\frac{1}{6} |q_0| \left( C_{ABC} p^A p^B p^C + c_{2A} p^A \right)}, \]  
(3)
where $C_{ABC}$ and $c_{2A}$ denote the triple intersection numbers and the second Chern class numbers of the Calabi-Yau threefold, respectively. The charges $p^A$ denote the expansion coefficients of the four-cycle $P$ in a homology basis $\Sigma_A$ of four-cycles, $P = p^A \Sigma_A$. In obtaining (3) the topological data of the four-cycle $P$ have been expressed in terms of topological data of the Calabi-Yau threefold.

The Calabi-Yau compactification reduces the number of supersymmetries to eight, so that the black hole solution has $N = 2$ supersymmetry at spatial infinity. One may also consider compactifications on $K3 \times T^2$ or on $T^6$ with the five-brane wrapping a four-cycle. In that case one has $N = 4$ or $N = 8$ supersymmetry at spatial infinity while the massless excitations are still described by a $(0, 4)$ two-dimensional conformal field theory. Hence one obtains entropy formulae similar to (3). We will return to them later.

Inspection of (3) shows that, for large charges $p^A$, there are subleading corrections (proportional to $c_{2A}$) to the microscopic entropy. It was argued in [3, 8] that these deviations in the entropy formula should, at the macroscopic level, arise from terms...
in the effective action proportional to the square of the Weyl tensor, with coefficients linearly related to the second Chern class of the Calabi-Yau threefold.

The associated four-dimensional effective field theory is based on $N = 2$ supergravity coupled to a number of vector multiplets whose gauge fields are associated with electric and magnetic charges, denoted by $q_I$ and $p^I$, respectively. The theory incorporates, in a systematic fashion, the phenomenon of electric/magnetic duality, according to which the electric and magnetic charges can be interchanged and/or rotated and it includes higher-derivative couplings with among them a certain class of terms quadratic in the Weyl tensor. The effective theory is thus complicated and depends on many fields. How can the macroscopic description of extremal $N = 2$ black holes then be so constrained and systematic as to precisely reproduce the results from the counting of microstates such as (3)? The crucial ingredient that is responsible for the remarkable restrictions on the entropy formulae obtained on the basis of these complicated effective field theories is the enhancement to full supersymmetry at the horizon. The black hole solutions that we consider are static, rotationally symmetric solitonic interpolations between two $N = 2$ supersymmetric groundstates: flat Minkowski spacetime at spatial infinity and Bertotti-Robinson spacetime at the horizon [9, 10]. The interpolating solution preserves $N = 1$ supersymmetry so that we are dealing with a BPS configuration and the black hole is extremal. The interpolating solution depends, generically, on the electric and magnetic charges as well as on the values of the moduli fields at spatial infinity. The supersymmetry enhancement at the horizon is responsible for the fixed-point behaviour of the moduli forcing them to take certain values depending on the electric/magnetic charges at the horizon [11, 12]. The precise relation can be deduced from electric-magnetic duality considerations [13, 10]. The near-horizon geometry is thus entirely determined in terms of the charges carried by the black hole, and so is the entropy.

2. Supersymmetric black hole solutions

The supergravity Lagrangians that give rise to these extremal black hole solutions are based on the coupling of $n$ vector multiplets to $N = 2$ supergravity. They contain various other couplings, such as those associated with hypermultiplets, which play only a limited role in the following and will be omitted. The construction of the coupling of vector multiplets to $N = 2$ supergravity utilizes the so-called superconformal multiplet calculus [14] which enables one to straightforwardly include the interactions proportional to the square of the Weyl tensor. Let us recall that the covariant fields of a vector multiplet, the field strength of the vector gauge field, a complex scalar, a doublet of gaugini and a triplet of auxiliary scalar fields, constitute a restricted chiral multiplet. The complex scalar fields are denoted by $X^I$. We consider only abelian vector multiplets, which we label by $I = 0, 1, \ldots, n$. The extra vector multiplet is required to provide the graviphoton field of supergravity. The supersymmetric (Wilsonian) action is encoded in a holomorphic function $F(X)$ of the scalars (or, in superspace, of the corresponding
chiral superfields). Under electric/magnetic duality transformations the function $F(X)$ changes, but the corresponding equations of motion and Bianchi identities remain the same.

In the superconformal framework there is another multiplet, the so-called Weyl multiplet, which comprises the gravitational degrees of freedom, namely the graviton, two gravitini as well as various other superconformal gauge fields and also some auxiliary fields. One of these auxiliary fields is an anti-selfdual Lorentz tensor field $T^{abij}$, where $i, j = 1, 2$ denote chiral $SU(2)$ indices, which occurs in the gravitino transformation law according to

$$
\delta \psi_\mu^i = 2 D_\mu e^i - \frac{1}{8} \epsilon_{i}^{ab} \gamma_\mu \epsilon_j T^{abij} + \cdots
$$

The covariant quantities of the Weyl multiplet also reside in a reduced chiral multiplet, denoted by $W^{abij}$, from which one constructs the unreduced chiral multiplet $W^2 = (W^{abij} \epsilon_{ij})^2$ [12]. The lowest component field of $W^2$ is equal to $\hat{A} = (T^{abij} \epsilon_{ij})^2$. Because $W^2$ is also a chiral multiplet, we can simply include interactions between the vector multiplets and the Weyl multiplet by extending the holomorphic function to a function that depends both on $X^I$ and $\hat{A}$. However, this function must be homogeneous of degree two and thus satisfies

$$
X^I F_I + 2 \hat{A} F_{\hat{A}} = 2 F,
$$

where $F_I = \partial F/\partial X^I$, $F_{\hat{A}} = \partial F/\partial \hat{A}$. The most prominent interaction term that is induced by the $\hat{A}$-dependence is quadratic in the Weyl tensor and proportional to the derivative $F_{\hat{A}}$. Note that there are no terms proportional to the derivative of the Riemann tensor.

Our first task is to find all $N = 2$ supersymmetric field configurations (in the full off-shell theory) that are consistent with the static, spherically symmetric, black hole geometry, which in isotropic coordinates $(t, r, \phi, \theta)$ is described by the line element

$$
\text{d}s^2 = -e^{2g(r)} \text{d}t^2 + e^{2f(r)}(\text{d}r^2 + r^2 \text{d}\Omega^2).
$$

An off-shell analysis which includes general interactions with the Weyl multiplet [10] reveals that there exists only one class of fully supersymmetric solutions, namely the Bertotti-Robinson spacetime corresponding to $adS_2 \times S^2$. This geometry is thus relevant for the black hole near the horizon or at spatial infinity (where the anti-de Sitter radius tends to infinity so that one is dealing with flat Minkowski spacetime). It is important to stress here that we do not explicitly use the action or field equations (except for the abelian gauge fields which are induced by the presence of the electric/magnetic black hole charges), as everything is encoded in the function $F(X, \hat{A})$. The use of an off-shell formulation is essential in view of the fact that the action is extremely complicated and generates an infinite sequence of higher-derivative interactions upon integrating out the auxiliary fields. In this way one obtains, for instance, an infinite series of terms proportional to the square of the Weyl tensor times powers of the field strengths associated with the vector multiplets. In view of the maximal supersymmetry the corresponding field equations must be satisfied.

The analysis of [10] shows that the $X^I$ and $\hat{A}$ must be constant (that is, in a certain gauge; in principle only appropriate ratios are determined in view of the invariance under local dilatations of the superconformal formulation) and that $e^{2g(r)} = e^{-2f(r)} = \cdots$
\begin{align*}
e^{-K}|Z|^{-2}r^2, \text{ where} \\
Z &= e^{K/2}(p^I F_I(X, \hat{A}) - q_I X^I), \\
e^{-K} = i[X^I F_I(X, \hat{A}) - F_I(X, \hat{A})X^I]. \tag{5}
\end{align*}

This shows that we are dealing with a spacetime geometry that is of the Bertotti-Robinson type. Note the dependence on the black hole magnetic and electric charges \((p^I, q_I)\). The quantities \(Z\) and \(K\) are both generically non-vanishing and constant. It was also found that 
\[T^{01}_{\ ij} = -i T^{23}_{\ ij} = 2 \epsilon_{ij} e^{-K/2} \tilde{Z}^{-1},\]
while all other components of \(T^{ab}_{\ ij}\) vanish. Therefore we have \(\hat{A} = -64 e^{-K} \tilde{Z}^{-2}\).

Hence the fully supersymmetric field configurations are characterized in terms of the (constant) moduli \(X^I\) (or rather, their ratios) and the electric/magnetic charges. However, when the field configuration satisfies the field equations and the Bianchi identities, which we know must be the case, then it must also be consistent with electric/magnetic duality. These equivalence transformations take the form of symplectic \(\text{SP}(2n + 2; \mathbb{Z})\) transformations. Now we observe that both \((p^I, q_I)\) and \((X^I, F_I)\) transform as symplectic vectors under duality transformations \([16]\). Since they are the only such vectors left in these supersymmetric configurations, they must satisfy a proportionality relation, which in principle determines the \(X^I\) in terms of the charges \([13, 10]\). Therefore fully supersymmetric field configurations are completely parametrized in terms of the charges. Observe that this observation already indicates that, also in the presence of higher-derivative interactions, the moduli will exhibit fixed-point behaviour at the horizon. An explicit proof of this will be presented elsewhere \([17]\).

What remains is to calculate the entropy for particular black hole solutions which interpolate between the two different fully supersymmetric field configurations at spatial infinity and at the horizon. Since the behaviour at the horizon is completely determined in terms of the charges, the resulting entropy formula will only depend on these charges. However, if one computes the macroscopic entropy for a black hole of the type considered in \([3, 4]\) by using the area law of Bekenstein and Hawking, then one discovers \([18]\) that the resulting expression does not agree with the expression for the microscopic entropy \((3)\). Thus, in order to obtain agreement with the counting of microstates provided by string theory, one is forced to depart from the area law. For that reason we adopt Wald’s proposal for the entropy which ensures the validity of the first law of black hole mechanics for more generic field theories. This proposal is based on the existence of a Noether charge associated with an isometry evaluated at the corresponding Killing horizon \([19]\). When evaluating this current subject to the field equations, current conservation becomes trivial and the current can be written as the divergence of an antisymmetric tensor. This tensor, sometimes called the Noether potential, is a local function of the fields and of the (arbitrary) gauge transformation parameters. Its integral over the horizon yields the macroscopic entropy.
3. Entropy as a Noether charge

In order to elucidate Wald’s Noether charge proposal let us first briefly consider a simple three-dimensional abelian gauge theory, with a gauge-invariant Lagrangian depending on the field strength $F_{\mu\nu}$, its derivatives $\partial_\rho F_{\mu\nu}$, as well as on matter fields $\psi$ and first derivatives thereof. Furthermore we add a Chern-Simons term (which acts as a topological mass term), so that the total Lagrangian takes the form

$$L_{\text{total}} = L_{\text{inv}}(F_{\mu\nu}, \partial_\rho F_{\mu\nu}, \psi, \nabla_\mu \psi) + c \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho,$$

where $\nabla_\mu \psi$ is the covariant derivative of $\psi$ and $c$ is some constant. This Lagrangian is not gauge invariant but changes into a total derivative,

$$\delta_\xi L_{\text{total}} = \partial_\mu N^\mu(\phi, \xi) = c \varepsilon^{\mu\nu\rho} \partial_\mu \xi \partial_\nu A_\rho,$$

where generically $\phi$ denotes all the fields and $\xi$ denotes the transformation parameter. For field configurations that satisfy the equations of motion, the corresponding Noether current can be written in terms of a so-called Noether potential $Q^{\mu\nu}$, which in the case at hand reads

$$Q^{\mu\nu}(\phi, \xi) = 2 L^{\mu\nu} \xi - 2 \partial_\rho L^{\rho, \mu\nu} \xi + L^{\rho, \mu\nu} \partial_\rho \xi + 2 c \varepsilon^{\mu\nu\rho} A_\rho \xi.$$  

Here $L^{\mu\nu}$ and $L^{\rho, \mu\nu}$ denote the derivatives of the action with respect to $F_{\mu\nu}$ and $\partial_\rho F_{\mu\nu}$, respectively. Observe that the Bianchi identity implies $L^{[\rho, \mu\nu]} = 0$. The Noether potential, whose definition is not unambiguous, is a local function of the fields and of the transformation parameter $\xi$. Observe that $Q_{\mu\nu}$ does not have to vanish for field configurations that are invariant (in the case at hand, this would imply $\partial_\mu \xi = \xi \psi = 0$). Modulo equations of motion the corresponding Noether current equals

$$J^\mu(\phi, \delta_\xi \phi) = \partial_\nu Q^{\mu\nu}(\phi, \xi).$$

Note that the current depends only on the gauge parameter through the gauge variations $\delta_\xi \phi$. This property is not automatic and in order to realize this we made a particular choice for the Noether potential, exploiting the ambiguity in its definition [20].

Integration of the Noether potential $Q^{\mu\nu}$ over the boundary of some (spacelike) hypersurface leads to a surface charge, which, when restricting the gauge transformation parameters to those that leave the background invariant, is equal to the Noether charge in the usual sense. In the case at hand this surface charge remains constant under variations that continuously connect solutions of the equations of motion. Here we consider a continuous variety of solutions of the field equations which are left invariant under a corresponding variety of residual gauge transformations. Hence, the parameters $\xi$ that characterize the residual symmetry may change continuously with the solution. Denoting the combined change of the solution $\phi$ and of the symmetry parameters $\xi$ by the variation $\delta$, one may thus write

$$\hat{\delta}(\delta_\xi \phi) = 0.$$  

In our example the Noether current can be written as a function of $\phi$ and $\delta_\xi \phi$, so that one knows that $\hat{\delta}J(\phi, \delta_\xi \phi)$ remains proportional to $\delta_\xi \phi$ and must therefore vanish for the
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symmetric configurations. Consequently $\delta Q^{\mu\nu}(\phi, \xi)$ must vanish up to a closed form, $\partial_{[\mu}^{[\nu} Q_{\rho]}^{\rho]}$, so that the surface charge obtained by integration over a Cauchy surface $C$ with volume element $d\Omega^\mu$,

$$\int_C d\Omega_\mu J^\mu(\phi, \delta \xi \phi) = \oint_{\partial C} d\Sigma_{\mu\nu} Q^{\mu\nu}(\phi, \xi),$$

is constant under the variations induced by $\delta$. Observe that the integrand on the right-hand side is in principle nonvanishing and nonconstant, so that the constancy of the total surface charge represents a nontrivial result.

In general relativity one follows the same approach as in the above example. The gauge transformations then take the form of diffeomorphisms and the residual gauge symmetries are associated with Killing vectors. The Lagrangian is not invariant but transforms as a density, which implies that $N^\mu(\phi, \xi) \propto \xi^\mu L$. Proceeding as in the example discussed above, the associated Noether current gives rise to a Noether potential. However, in this case there are a number of complications when considering variations of the surface charge. Another essential ingredient is that the boundary decomposes into two disconnected parts for black hole solutions, one associated with spatial infinity and one with the horizon. After identifying a surface charge that is constant under variations within a continuous variety of solutions of the equations of motion, the contributions coming from variations at spatial infinity must cancel against those coming from the horizon. It is this phenomenon that ensures the validity of the first law of black hole mechanics: the contributions originating from spatial infinity are related to variations of the black hole mass and angular momentum, while the contributions originating from the horizon are identified with the change of the black hole entropy (see [21, 22, 23] for a review). In this way one establishes a formula for the black hole entropy in terms of the surface charge of the Noether potential over the horizon. When the Lagrangian depends arbitrarily on the Riemann tensor $R_{\mu\nu\rho\sigma}$ (but not on its derivatives) and on matter fields and their first-order derivatives, one can show that the entropy of a static black hole is given by [24, 21, 22]

$$S_{\text{macro}} = \frac{1}{16} \int_{S^2} \epsilon_{ab} \epsilon_{cd} \frac{\partial (8\pi L)}{\partial R_{abcd}},$$

where the epsilon tensors act in the subspace orthogonal to the horizon associated with the time and the radial coordinates; the factor $8\pi$ is related to our normalization conventions for the Lagrangian.

With these results one can compute the macroscopic entropy of static, spherically symmetric, $N = 2$ supersymmetric black hole solutions in the presence of higher-derivative interactions. In view of the homogeneity of the function $F$, it is convenient to introduce rescaled variables $Y^I = e^{K/2} Z X^I$ and $\Upsilon = e^K Z^2 \hat{A}$. At the horizon we must have $\Upsilon = -64$. It then follows that the relation between the $Y^I$ and the electric/magnetic charges of the black hole is given by $Y^I - \bar{Y}^I = ip^I$ and $F_I(\Upsilon, \bar{\Upsilon}) = F_I(\bar{Y}, \Upsilon) = iq_I$. On the other hand, it follows from (3) that $|Z|^2 = p^IF_I(\Upsilon, \Upsilon) - q_I Y^I$, which determines the value of $|Z|$ in terms of $(p^I, q_I)$. Subsequently one establishes that
the expression for the entropy takes the remarkably concise form \[10\],

\[ S_{\text{macro}} = \pi \left| Z \right|^2 + 4 \text{Im} \left( \Upsilon F(Y, \Upsilon) \right), \]

where \( \Upsilon = -64 \).

In this formula the first term originates from the Bekenstein-Hawking entropy contribution associated with the area, whereas the second term is due to Wald’s modification induced by the presence of higher-derivative terms. Here we point out that this modification does not actually originate from the terms quadratic in the Weyl tensor, because the Weyl tensor vanishes at the horizon, but from a term in the Lagrangian proportional to the product of the Ricci tensor with the tensor field \( T^a_{bijkl} T_{cdkl} \). Note that when switching on higher-derivative interactions the value of \( |Z| \) changes and hence also the horizon area changes. There are thus two ways in which the presence of higher-derivative interactions modifies the black hole entropy, namely by a change of the near-horizon geometry and by an explicit deviation from the Bekenstein-Hawking area law. Also note that the entropy \( [13] \) is entirely determined in terms of the charges carried by the black hole, \( S = S(q, p) \). Because of the homogeneity property of the function \( F(Y, \Upsilon) \) one can show that the macroscopic entropy \( [13] \) must be an even function of the charges.

4. An \( N = 2 \) example

Let us then determine the macroscopic entropy of black hole solutions arising in type-IIA string theory compactified on a Calabi-Yau threefold, in the limit where the volume of the Calabi-Yau threefold is taken to be large, and let us compare it with the result for the microscopic entropy \( [3] \) obtained via state counting. The associated homogeneous function \( F(Y, \Upsilon) \) is given by (with \( I = 0, \ldots, n \) and \( A = 1, \ldots, n \))

\[ F(Y, \Upsilon) = -\frac{C_{ABC} Y^A Y^B Y^C}{6 Y^0} - \frac{1}{24 \times 64} c_{2A} \frac{Y^A}{Y^0} \Upsilon. \] \( [14] \)

The Lagrangian associated with this homogeneous function contains a term proportional to the square of the Weyl tensor with coefficient \( c_{2A} \text{Im} z^A \), where \( z^A = Y^A/Y^0 \).

Consider, in particular, black holes carrying charges \( q_0 \) and \( p^A \), only. Solving the associated stabilization equations for \( Y^I = Y^I(q, p) \) and substituting the result into \( [13] \) yields \( [10] \)

\[ S_{\text{macro}} = 2\pi \sqrt{\frac{1}{6} \left| q_0 \right| \left( C_{ABC} p^A p^B p^C + c_{2A} p^A \right)}, \]

in exact agreement with the microscopic entropy formula \( [3] \). Thus, we see that the entropy obtained via state counting \( [3] [4] \) is in accord with Wald’s proposal for the macroscopic entropy which deviates from the area law.

5. State counting for \( N = 4, 8 \) black holes

Let us now finally turn to black hole solutions occurring in type-IIA compactifications on \( K3 \times T^2 \) and on \( T^6 \) and let us discuss the associated microstate counting. In the M-theory picture we consider then a five-brane wrapped around a holomorphic four-cycle
$P$ in either one of these spaces. When proceeding with the counting of zero modes, as in the Calabi-Yau threefold case described in [5], the left- and right-moving bosonic and fermionic degrees of freedom are given in terms of the Hodge numbers of $P$ by:

\begin{align}
N_{\text{bosonic}}^{\text{left}} &= 2h_{2,0}(P) + h_{1,1}(P) + 2 - 2h_{1,0}(P), \\
N_{\text{fermionic}}^{\text{left}} &= 4h_{1,0}(P), \\
N_{\text{bosonic}}^{\text{right}} &= 4h_{2,0}(P) + 4 - 2h_{1,0}(P), \\
N_{\text{fermionic}}^{\text{right}} &= 4[h_{2,0}(P) + h_{0,0}(P)].
\end{align}

(16)

The effective two-dimensional theory describing the collective modes of a BPS black hole is a $(0,4)$ supersymmetric sigma-model. Therefore, the number of right-moving bosons and fermions has to match. Moreover the right-moving scalars are expected to parametrize a quaternionic manifold and therefore the number of right-moving real bosons should be a multiple of four. Inspection of (16) shows that in the case of a hole is a $(0,4)$ supersymmetric sigma-model. Therefore, the number of right-moving modes is consistent with $(0,4)$ supersymmetry, whereas this is not the case for $K3 \times T^2$ and for $T^6$, for which $h_{1,0}(P) = 1$ and $h_{1,0}(P) = 3$, respectively. This implies that the zero-mode counting for $K3 \times T^2$ and for $T^6$ has to deviate from the one described above.

Using (16) the central charges of the left- and right-moving sector are computed to be

\begin{align}
c_L &= N_{\text{bosonic}}^{\text{left}} + \frac{1}{2}N_{\text{fermionic}}^{\text{left}} = C_{ABC} p^A p^B p^C + c_{2A} p^A + 4h_{1,0}(P), \\
c_R &= N_{\text{bosonic}}^{\text{right}} + \frac{1}{2}N_{\text{fermionic}}^{\text{right}} = C_{ABC} p^A p^B p^C + \frac{1}{2}c_{2A} p^A + 4h_{1,0}(P),
\end{align}

(17)

which then via (2) leads to the following result for the microscopic entropy,

\[ S_{\text{micro}} = 2\pi \sqrt{\frac{1}{6} |q_0| (C_{ABC} p^A p^B p^C + c_{2A} p^A + 4h_{1,0}(P))}. \]

(18)

Now we note that the sub-subleading third term in this expression proportional to $h_{1,0}$ is not consistent with the macroscopic computation of the entropy based on $N = 2$ supergravity. As mentioned above, the entropy should be even in terms of the charges.

How is the zero-mode counting for $K3 \times T^2$ and for $T^6$ to be modified in order to remove the inconsistencies mentioned above? Let us recall that there are $b_1 = 2h_{1,0}(P)$ nondynamical gauge fields present. If we assume that the zero-modes are charged and couple to these gauge fields, then the following mechanism suggests itself. Due to gauge invariance, the number of left- and right-moving scalar fields is reduced by $b_1$, so that the number of right-moving scalar fields is indeed a multiple of four. Due to supersymmetry this must be accompanied by the removal of $2b_1$ right-moving fermionic real degrees of freedom. If, in addition, we assume that the removal of fermionic degrees of freedom is left-right symmetric, then the actual number of left-moving fermionic degrees of freedom is zero. The central charge in the left-moving sector is now computed to be $c_L = C_{ABC} p^A p^B p^C + c_{2A} p^A$, which is odd in the charges. The resulting microscopic entropy formula is then in full agreement with the macroscopic computation, and it is also consistent with anomaly inflow arguments [24].
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