Statistical Entropy of an Extremal Black Hole with 0- and 6-Brane Charge

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Abstract

A black hole solution to low energy type IIA string theory which is extremal, non-supersymmetric, and carries 0- and 6-brane charge is presented. For large values of the charges it is metastable and a corresponding D-brane picture can be found. The mass and statistical entropy of the two descriptions agree at a correspondence point up to factors of order one, providing more evidence that the correspondence principle for black holes and strings of Horowitz and Polchinski may be extended to include black holes with more than one Ramond-Ramond charge.
1. Introduction

Over the past year, the entropies of several varieties of black holes in string theory have been explained on a microscopic level in terms of configurations of D-branes and strings. When the string coupling, $g$, is taken to zero, a black hole solution to a low-energy effective string theory becomes a weakly coupled system described by D-branes and strings in flat space. For BPS black holes, a non-renormalization theorem fixes the degrees of freedom as the string coupling is varied, so that the statistical entropy computed using the small $g$ picture exactly matches the Bekenstein-Hawking entropy of the black hole. Several of these black holes have been studied, and the small string coupling limit of all of them correctly describes the microscopic origin of the entropy [1,2,3].

Even more surprising is that this microscopic description of black holes has been shown to correctly describe the entropy of certain non-BPS black holes. In particular, some near-extremal black holes [4,6] have been analyzed, as well as some black holes which are far from BPS-states [7,8]. It is unclear in these cases precisely why this procedure is successful.

Recently a less exact but more widely applicable correspondence principle between the black hole and string pictures has been formulated [9]. The correspondence principle states that the mass and entropy of the two pictures should match only at a particular value of the string coupling: its value when, for fixed charges, the curvature at the black hole horizon is of order one in string units. This correspondence point is where the low energy effective theory breaks down; the string size is of the order of the Schwarzschild radius. Since the precise value of $g$ at which the transition to strong coupling occurs is unknown up to factors of order one, the mass and entropy of the two pictures must match only up to factors of order one. This was shown to apply to all black holes with less than two Ramond-Ramond (RR) charges, and an example with two RR charges was shown to work, suggesting that the correspondence principle may hold for all black holes.

In this paper a black hole solution to the low energy effective theory of type IIA string theory is presented. It is extremal, non-supersymmetric, and carries 0- and 6-brane charge. Despite the fact that 0- and 6-branes have been shown to repell each other, they do form such bound states. Furthermore, the correspondence principle will be shown to hold in this case, providing additional evidence that it is valid for black holes with two RR charges.

The paper is organized as follows. In Section 2 units and conventions are explained, and the black hole solution is presented in Section 3. In Section 4 interactions between 0- and 6-branes are explored. A definition of entropy is provided in the next section, followed
by a D-brane description of the black hole in Section 6. Some final remarks are then discussed.

2. Units, Conventions, and Quantization

The bosonic part of type IIA string theory has the low energy effective action

\[
S = \frac{1}{(2\pi)^7 g^2} \int d^{10}x \sqrt{-G} \left[ e^{-2\phi} \left( R + 4 (\nabla \phi)^2 \right) - g^2 F^2 \right]
\]

(2.1)

where all matter terms but the Ramond-Ramond (RR) 2-form field strength, \(F\), have been set to zero. The remaining terms are the string metric, \(G\), and the dilaton, \(\phi\), where \(\phi \to 0\) at spatial infinity. These quantities are expressed in string units: \(\sqrt{\alpha'} = 1\). \(g\) is the string coupling, and the ten-dimensional Newton’s constant, \(G_{10}^N\), is set to \(8\pi^6 g^2\) in accordance with S-duality considerations [10].

A solution to the equations of motion for this action can be obtained from a solution of the equations of motion of the 11-D supergravity action

\[
^{11}S = \frac{1}{(2\pi)^8 g^3} \int d^{11}x \sqrt{-^{11}g} \, ^{11}R
\]

(2.2)

of the form

\[
^{11}ds^2 = e^{4\phi/3} \left( dx^{11} + 2gA_\mu dx^\mu \right)^2 + e^{-2\phi/3} \, ^{10}ds^2
\]

(2.3)

by compactification in the \(x^{11}\) direction provided \(\frac{\partial}{\partial x^{11}}\) is a Killing vector of the solution [11]. The dilaton, \(\phi\), gauge potential, \(A_\mu\), and the 10-dimensional string metric, \(^{10}ds^2\), can be read off from the form of (2.3). The radius of the compactified \(x^{11}\) direction is required to be \(g\), the string coupling, so that \(\phi \to 0\) at spatial infinity. The fields in the 11-dimensional solution are then in 10-dimensional string units.

In the 10-dimensional solution, electric charge under the field strength \(F\) is carried by 0-dimensional objects which have RR charge: 0-branes. Likewise, objects with magnetic charge under \(F\) are 6-branes [12]. Thus these charges should be quantized. One way to determine this quantization is to note how these charges arise from the 11-dimensional solution. Electric charge comes from momentum in the \(x^{11}\) direction of \(^{11}ds^2\) which, for compact \(x^{11}\) must be quantized in units of inverse radius, \(1/g\). Magnetic charge comes from a monopole or topological charge; discrete values are required to remove possible conical singularities in the 11-dimensional solution.
The 11-dimensional solution representing a 0-brane upon reduction to 10-dimensions can be written in the form of (2.3) with

\[ 10 \, ds^2 = - \left( 1 + \frac{c}{r^7} \right)^{-1/2} dt^2 + \left( 1 + \frac{c}{r^7} \right)^{1/2} [dr^2 + r^2 d\Omega^8] \]
\[ A_\mu dx^\mu = \frac{1}{2} \frac{c}{g} \frac{1}{r^7} + c dt \]
\[ e^{4\phi/3} = 1 + \frac{c}{r^7}. \]  

Setting the momentum in the \( x^{11} \) direction to \( 1/g \) requires the constant \( c \) to be \( c = 15 (2\pi)^2 g \). This implies that in 10-dimensions the integer normalized 0-brane charge is

\[ Q_0 = \frac{1}{2^6 \pi^7} \int_{S^8, r \to \infty} * F \]  

where \( * \) is the 10-D Hodge dual and the mass of one 0-brane is

\[ M_0 = \frac{1}{g}. \]  

To find the integer normalized magnetic charge, consider the 11-dimensional supergravity solution of the form (2.3) with

\[ 10 \, ds^2 = \left( 1 + \frac{m}{r} \right)^{-1/2} \left[ -dt^2 + dy_i dy^i \right] + \left( 1 + \frac{m}{r} \right)^{1/2} [dr^2 + r^2 d\Omega^2] \]
\[ A_\mu dx^\mu = \frac{m}{2g} (1 - \cos \theta) d\hat{\phi} \]
\[ e^{4\phi/3} = 1 + \frac{m}{r}, \]  

which, after reduction 10-dimensions, becomes the 6-brane solution. Here \( \hat{\phi} \) is the azimuthal angle of the two-spheres and the \( y_i, i = 1...6 \) are coordinates along the 6-brane. Consider the \( y_i \) directions compactified upon a 6-torus with volume \( V^6 \) at \( r \to \infty \) for convenience. Making this solution free of singularities requires a period for \( x^{11} \) of \( \Delta x^{11} = 4\pi m \) which means \( m = \frac{1}{2} g \). Thus the integer normalized 6-brane charge is found to be

\[ Q_6 = \frac{1}{\pi} \int_{T^6 \times S^2} F \]  

and the mass of a single 6-brane is

\[ M_6 = \frac{1}{g} \frac{V^6}{(2\pi)^6}. \]
3. Extremal Black Hole Solution

A solution to (2.2) in the form of (2.3) which after reduction to 10-D is an extremal black hole with 0- and 6-brane charge is

\[ 10ds^2 = -\left(1 - \frac{rq}{r}\right)^2 dt^2 + \left(1 - \frac{rq}{r}\right)^{-2} dr^2 + r^2 d\Omega^2 + dy_i dy^i \]

\[ A_\mu dx^\mu = \frac{1}{\sqrt{2}} q \left[ \frac{1}{r} dt + (1 - \cos \theta) d\phi \right] \]

\[ e^{A\phi/3} = 1. \]  

(3.1)

\( \hat{\phi} \) is the azimuthal angle of the two-sphere and \( y_i, i = 1...6 \) are flat directions which will be considered compactified on a six-torus of volume \( V^6 \). Note that the dilaton, \( \phi \), is constant and there is only one parameter, \( q \). This 11-D solution was previously found in [13] where a different compactification to 10-D was considered.

The mass, integer normalized 0- and 6-brane charges, and the Bekenstein-Hawking entropy can be determined in terms of the parameter \( q \) to be

\[ M = \frac{1}{g} \frac{V^6}{(2\pi)^6} 8q \]

\[ Q_0 = \frac{q}{\sqrt{2}} \frac{V^6}{(2\pi)^6} 4 \]

\[ Q_6 = \frac{q}{\sqrt{2}} 4 \]

\[ S_{BH} = 8\pi q^2 \frac{V^6}{(2\pi)^6}. \]

(3.2)

The mass of the black hole can be expressed in terms of the integer normalized charges, the mass of a single 0-brane (2.6), and the mass of a single 6-brane (2.9):

\[ M = \sqrt{2} (Q_o M_o + Q_6 M_6). \]

(3.3)

In terms of the integer normalized charges, the Bekenstein-Hawking entropy is

\[ S_{BH} = \pi Q_0 Q_6. \]

(3.4)

\( Q_0 \) and \( Q_6 \) are related by \( Q_0/Q_6 = V^6/(2\pi)^6 \). One can consider arriving at this solution by directly solving the 10-D field equations. Then, requiring that the dilaton be constant restricts the charges to be related in this way, and choosing an extremal solution fixes the
mass in terms of the charges. For $gQ_0 >> 1$ the solution has small curvature everywhere outside the horizon, so the low energy effective description is valid in the region of interest.

Several more features of this solution are worthy of note. First, the mass is greater than that of the constituent D-branes by a factor of $\sqrt{2}$. That the mass of a bound state of 0- and 6-branes is greater than the mass of separate 0- and 6-branes is not unusual given that 0- and 6-branes repel at large distances [14]: energy must be added to force the D-branes together. However, because the “ground state” of infinitely separated D-branes exists, it is expected that this black hole should be unstable to decay of 0- and 6-branes.

Even extreme black holes, which have zero Hawking temperature, can decay semiclassically if there exist particles in the theory with greater charge than mass [15]. The added gauge interaction allows negative energy orbits outside the event horizon producing an effective ergosphere. Because the black hole considered here has no dilaton charge, this is the only requirement for there to be radiation. In 10-D planck units, both the 0-brane and 6-brane have charge equal to twice their mass, so the black hole can radiate both kinds of D-branes. For large black holes, where the black hole charges are effectively unchanged by the radiation, a WKB approximation [15] estimates the decay rate into either type of D-brane to be $e^{-kQ}$ where $Q$ is $Q_0$ ($Q_6$) for 0-brane (6-brane) radiation and $k$ is a constant of order one. Thus, large black holes of this type are metastable.

4. Effective Description of 0-6 String

In order to construct a D-brane description of this black hole one needs to understand the simpler system of one 0-brane and one 6-brane with small separation. It has been argued that the short distance physics of such D-brane configurations is best described by massless open string modes connecting the branes [16]. Thus, understanding this simpler system reduces to understanding the 0-6 open string stretched between the branes.

The effective potential between two D-branes can be found from the open string loop amplitude in the D-brane background which has been calculated for stationary D-branes in [14]. The short distance limit of this amplitude is the infinite world sheet time limit of the integral. For the 0-6 brane system this yields an effective potential of

$$V(R) \propto K - R$$

where $R$ is the transverse separation of the branes and $K$ is a constant of order one in string units. The constant contribution to the potential can be thought of as due to massive open
string modes between the branes. This shows that there is a repulsive force between 0- and 6-branes at short distance, which should match smoothly to the repulsive $1/R^2$ force found at large distances in [14], reflecting the fact that this system has no supersymmetry.

It will be important to know how many and what kinds of degenerate modes of the string contribute to this effective result. To determine this, we first consider the branes coincident with no velocity and analyze the massless 0-6 string modes. The Neveu-Schwarz sector has a positive zero point energy [17] and so contributes no massless modes. The Ramond sector bosons are massive, so the massless modes to consider are R fermions. These arise from fields with same-type boundary conditions, either Neumann on both sides or Dirichlet on both sides. There are four such directions: the time direction and the three spatial directions transverse to the branes. Label these directions by the index $\mu = 0, 1, 2, 3$. The rotational symmetry in these directions is preserved, and so the zero modes of these R sector fermions are the same as for the open superstring in four dimensions; there are four states which can be represented as a four-component Dirac spinor. The GSO projection removes half the states, leaving one two-component Weyl spinor.

This mode should have an effective description in terms of gauge theories on the D-branes. It reduces to fermionic quantum mechanics on the 0-brane world line with the transverse rotational symmetry as an internal gauge symmetry. If the D-branes have a small separation, $Y^\mu$, then it is reasonable to assume that the lowest mass state of the string stretched between the branes is a minimally stretched string mode which approaches zero energy as the separation goes to zero. Thus, this mode can be described in terms of the effective gauge theory by adding to the effective action for the Weyl fermion a mass term proportional to $Y^\mu$; let

$$S_{eff} = \int dt \left( i \bar{\psi} \partial_t \psi - \bar{\psi} Y^\mu \sigma_\mu \psi \right)$$

(4.2)

where $\psi$ is the Weyl spinor representing the zero mode and the $\sigma^\mu$ are the Pauli matrices. This interaction term is the only linear mass term which can be constructed from $Y^\mu$.

The states of this effective theory can now be determined. For simplicity, choose coordinates so that the branes are separated in the $z$ direction. This system then contains two two-level systems composed of the two components of $\psi$ and their complex conjugates, so there are four states. However, because there is no field strength for the gauge potential, $Y_0$, the equation of motion for $Y_0$ just requires that all states be charge neutral. Note for the case of only one 0-, 6-brane pair this implies that the net 6-brane gauge charge is
also zero. This constraint eliminates two states, leaving two physical states. The ground state is tachyonic with mass \(-Y_z\), and describes the linear repulsive potential found in (4.1). However, the excited state has mass \(Y_z\) and so results in a linear attractive potential between the D-branes.

Consider now the thought experiment of slowly moving a 0-brane by a stationary 6-brane by the application of some external force. We have seen that the interaction is effectively described by a two level system with mass proportional to the brane separation. This experiment is then analogous to the 0-0 brane scattering experiment considered in [16]. If the 0-brane approaches the 6-brane from the +z direction with some small impact parameter, then after the branes cross, assuming that no other modes effect the interaction, the separation coordinate, \(Y^z\), changes sign. Thus the ground state mass is positive after the scattering event. This is interpreted as an excited state of “out” modes defined with the 0-brane infinitely separated in the \(-z\) direction. Therefore the two levels cross when the branes have zero separation.

Thus, in analogy to the 0-0 brane scattering case, it would seem that a fermionic string mode is usually created when 0- and 6-branes collide. This fermionic mode is long lived, assuming that the 0-brane has small enough velocity that the lifetime estimates made in [16] apply. The excited mode then causes the two D-branes to attract and recollide. The D-branes will recollide many times until the excited fermionic mode decays, allowing them to escape to infinite separation. This unstable state is the analog of the 0-0 brane metastable state of [16]. Of course, the 0- and 6-brane can not collide from infinite separation at small velocity initially because of the repulsive force between them. However, this unstable state can be constructed by imposing initial conditions in which the D-branes are closely separated with small relative velocity and the fermionic mode is in the excited state.

The interest in this configuration here is in bound states of 0- and 6-branes which could correspond to the black hole above. Since this black hole is extremal, one would like to find a metastable state where the D-branes have minimal velocities. What is the lower limit of the 0-brane velocity, \(v\), for this state to exist? It is consistent for the 0-brane to have zero velocity. In the two 0-brane case, because of supersymmetry, the leading order effective potential between the branes is proportional to \(v^4\). Thus some velocity was necessary in order to bind them together with open strings. Also, because there is enhanced symmetry when the two 0-branes coincide, additional massless modes exist for some range of close separations. In order to ignore the effects of these modes, a lower limit
on the velocity was required. For the 0-6 brane case, there is no enhanced symmetry of this type when the branes coincide, and so such a lower limit on the velocity cannot be made.

The upper limit on the velocity is $v^2 < R$, where $R << 1$ is the transverse separation in string units, so that velocity-dependent contributions to the effective potential can be ignored. Therefore a bound state might be formed by having 0- and 6-branes coincident with zero relative velocity but enough fermionic zero modes excited so that the D-branes experience an attractive potential. This is the sort of D-brane picture which will be presented in Sec. 6.

5. Definition of Entropy and Correspondence

Before one can consider finding a statistical description of the entropy of this black hole in terms of D-branes, one must define entropy for a non-equilibrium state. If a state is near an equilibrium state in some sense, one may define the entropy of the non-equilibrium state to be approximately that of the equilibrium state it approaches in some limit. Consider the black hole solution and its thermodynamic entropy. It is unstable, but for large charges its decay rate is exponentially suppressed. Thus, in the limit of infinite charges it is stable and one could compute the Bekenstein-Hawking entropy in the usual way: the horizon area over four. For charges large but not infinite, the black hole is approximately in an equilibrium state if one “turns off” Hawking radiation. This is usually accomplished by considering putting the black hole in a box with thermal gasses of 0- and 6- branes at an appropriate temperature to counteract the Hawking radiation, and considering the black hole as a sub-system. Operationally this is the same as ignoring the Hawking radiation and claiming that the entropy calculated in the usual way has meaning because the state is close to equilibrium. This is what I wish to do here because it will allow a reasonable definition of entropy for the D-brane configuration in terms of turning off a small quantum effect.

The correspondence principle, were it to apply in this case, would relate the masses and entropies of the black hole and D-brane picture up to factors of order one at the correspondence point. For fixed charges, this is the value of the string coupling at which the curvature at the black hole event horizon becomes one. In this case, for large charges $Q_0 \approx Q_6 = Q$ this value is $g = 1/Q$. At this point the black hole mass is $M \approx Q^2$ and the entropy is $S_{BH} \approx Q^2$ up to factors of order one.
6. D-Brane Picture: Microstate Counting

In this section I will argue for an effective D-brane picture of the black hole, identifying the low energy degrees of freedom which lead to a statistical description of the entropy and show that the mass and entropy calculated in this way indeed match the black hole results (3.3) and (3.4) at the correspondence point.

Consider a system of $Q_0$ 0-branes and $Q_6$ 6-branes close together with small relative velocities. This is an unstable configuration. Because the energy of the 0-6 fermionic string modes is much less than the energy in the D-branes, the number of low energy 0-6 fermionic string modes attached to a particular D-brane will fluctuate, and unless at least half are in their excited state, there is a net repulsive effective force on the D-brane. The D-brane will then either escape to infinite distance from the rest of the system if there are no excited open string modes attached to it, or be bound at a separation of order one if there are. This is because the long range effective potential between 0- and 6-branes falls off as $1/R$, a weaker repulsive force than the attractive force the string mode creates. If it becomes bound by long open string modes, then eventually these will decay, releasing the D-brane to infinite separation. Either way, the collection of D-branes will eventually disperse.

However, for large numbers of D-branes, each D-brane is connected by many different open strings to other D-branes. Thus the likelihood that the number of fermionic open string modes in the excited state attached to a particular D-brane will fluctuate to a low enough value for a long enough time that the D-brane can escape the short separation regime is very small. Therefore the state with many D-branes is metastable.

This metastable state is “close” to an equilibrium configuration in which the small quantum effect of open string mode decay is neglected. If such an equilibrium configuration can be consistently constructed, then the logarithm of its ground state degeneracy will closely approximate the statistical entropy of the D-brane picture corresponding to the black hole under investigation.

How can one consistently describe such an equilibrium state? Since open string mode decay is neglected, the D-branes do not need relative velocities to create more modes. It is sufficient to have all the D-branes coincident at zero velocity and choose the configuration of the fermionic open string modes. This is clearly a minimal energy configuration, since there is no D-brane kinetic energy and all the open string modes are degenerate and massless. To make this a stable configuration, at least half the modes on each D-brane
must be in an excited state; then if one considers small perturbations of the positions of the branes, the effective potential is always attractive. The remaining modes can be chosen arbitrarily.

The physical states of this system are those configurations of modes in the effective gauge theory which have zero net 0-brane gauge charge on each 0-brane and zero net 6-brane gauge charge on each 6-brane. The 6-brane gauge charge must be conserved because the 6-branes are wrapped around compact directions so no flux can escape to infinity. It was shown above that this constraint can be met for each individual 0-brane, 6-brane pair leaving two physical states per pair.

There are actually more physical states than this because it is possible to satisfy the charge neutral constraint on a particular D-brane using modes from different D-branes. However, including these extra states does not change the leading order behavior of the number of degenerate configurations. Because the statistical entropy must match the thermodynamic entropy only up to factors of order one, the leading order behavior of the number of degenerate configurations is all that is required. Thus it suffices to restrict attention to the $2Q_0Q_6$ states found above.

Note that only modes from 0-6 strings have been considered; the open strings between like D-branes have been ignored. It seems likely that the presence of both 0- and 6-branes will break the enhanced symmetry usually found when many like D-branes are superimposed. In addition, the usual modes will be velocity suppressed in the physical non-equilibrium picture. In any case, these modes can not change the leading order behavior of the mass or entropy, so it is consistent to omit them from consideration.

Now the number of consistent degenerate configurations can be determined to leading order. The contribution to the number of configurations due to ways of choosing half of the two-level systems associated with each D-brane to be in the excited state is sub-leading. Therefore, the leading order behavior is determined by choosing the level of the remaining half of the two-level systems arbitrarily. For large charges, $Q_0 \approx Q_6 = Q$, the number of microstates to leading order at correspondence is

$$N \approx 2^{Q^2}$$

(6.1)

where the numerical factor in the exponent has been dropped since it will contribute a factor of order one to the entropy. This gives an approximate statistical entropy at correspondence of

$$S_{stat} \approx \log N \approx Q^2,$$

(6.2)
which matches the black hole entropy $S_{BH}$ up to factors of order one.

The total mass of this system is composed of three parts: the masses of the constituent D-branes, the 0-point energy due to massive 0-6 string modes, and the mass of the light open string fermionic modes. The following computations are valid at the correspondence point. The total mass of the constituent D-branes is $\approx Q/g \rightarrow Q^2$. The 0-point energy is order one in string units, and there are $\approx Q^2$ open strings, so the total 0-point energy contribution to the mass is $\approx Q^2$. As the string coupling is increased to the correspondence point, the D-branes “spread out” quantum-mechanically to about the string scale. Thus, even though the low energy fermionic open string modes are massless as $g \rightarrow 0$, they may contribute some mass at the correspondence point. The stretched open strings must remain shorter than order one, so their mass is at most of order one. Note that this fact prevents the open strings from being longer than the Schwarzschild radius at correspondence. There are $\approx Q^2$ of these strings, so their contribution to the total mass is again $\approx Q^2$. Thus the total mass of this configuration is $\approx Q^2$ which agrees with the black hole result (3.3) at correspondence.

7. Discussion

An extremal, non-BPS black hole solution to the low energy effective action of type IIA string theory with 0- and 6-brane charge has been presented. It is interesting that a non-supersymmetric bound state of 0- and 6-branes exists since the effective force between them is, at first glance, repulsive at all scales. The black hole is unstable to decay of 0- and 6-branes, but for large charges it is a long-lived state. The description of this state in terms of D-branes is that $Q_0$ slowly moving 0-branes and $Q_6$ slowly moving 6-branes are bound by excited but low energy fermionic modes of strings connecting different branes. The zero point energy of these connecting strings, due to massive modes, contributes enough mass to the configuration to account for the black hole's excess mass over the mass of the constituents at correspondence. The number of configurations of the approximately massless fermionic modes gives the microstates from which the statistical entropy of the system can be computed, and this entropy matches the thermodynamic entropy of the black hole at correspondence. This system is then more evidence that the correspondence principle of [9] is valid for black holes with two different RR charges.

There are two obvious generalizations of this solution. The first is just the non-extremal version with constant dilaton and the second is the class which carries dilaton
charge. For solutions far from extremality, the charges become unimportant and the analysis for Schwarzschild in [1] applies. For solutions near extremality, one problem is that non-extremal black hole solutions can radiate closed strings as well as D-branes and it is unclear how to generalize this D-brane picture to include such processes. The solutions which carry dilaton charge can be determined by modifying the 5-D Kaluza-Klein black hole solutions with non-constant dilaton found in [18].

Acknowledgements
I would like to thank G. Horowitz, J. Polchinski, M. Douglas, J. Pierre, M. Srednicki, and P. Pouliot for discussions. It was supported in part by NSF Grant PHY95-07065.
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