A Constrained Coupled Matrix-Tensor Factorization for Learning Time-evolving and Emerging Topics

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Abstract—Topic discovery has witnessed a significant growth as a field of data mining at large. In particular, time-evolving topic discovery, where the evolution of a topic is taken into account has been instrumental in understanding the historical context of an emerging topic in a dynamic corpus. Traditionally, time-evolving topic discovery has focused on this notion of time. However, especially in settings where content is contributed by a community or a crowd, an orthogonal notion of time is the one that pertains to the level of expertise of the content creator: the more experienced the creator, the more advanced the topic.

In this paper, we propose a novel time-evolving topic discovery method which, in addition to the extracted topics, is able to identify the evolution of that topic over time, as well as the level of difficulty of that topic, as it is inferred by the level of expertise of its main contributors. Our method is based on a novel formulation of Constrained Coupled Matrix-Tensor Factorization, which adopts constraints well-motivated for, and, as we demonstrate, are essential for high-quality topic discovery.

We qualitatively evaluate our approach using real data from the Physics and also Programming Stack Exchange forum, and we were able to identify topics of varying levels of difficulty which can be linked to external events, such as the announcement of gravitational waves by the LIGO lab in Physics forum. We provide a quantitative evaluation of our method by conducting a user study where experts were asked to judge the coherence and quality of the extracted topics. Finally, our proposed method has implications for automatic curriculum design using the extracted topics, where the notion of the level of difficulty is necessary for the proper modeling of prerequisites and advanced concepts.

Index Terms—Topic Discovery, Time-evolving, Tensors, Constrained Matrix-Tensor Factorization, Constrained Factorization

I. INTRODUCTION

Traditionally, topic modeling and discovery methods have focused on extracting high quality, interpretable topics that aim to succinctly represent the inherent latent structure within a corpus. Indicatively, there have been different schools of thought on topic extraction, ranging from factorization-based methods [14, 43] to probabilistic graphical models [12, 38].

Recently, there has been significant interest in studying the evolution of topics over time, and this has found particular applications in [26] and [19]. In general, taking time into account offers the advantage of putting an extracted topic into historical context and can enable the analysis to link the topic to external events that may be related to it.

To the best of our knowledge, the state-of-the-art in time-evolving topic extraction has focused on a notion of “time” that pertains to the particular moment that a topic emerged and how it evolved throughout its history within a corpus. However, when we are dealing with topic extraction from community and crowd based platforms, such as Stack Exchange, an additional notion of “time” arises. This notion of time is related to the evolution of the user who contributes the content: a relatively new user is more likely to contribute “entry-level” content, whereas an experienced user who has already contributed a significant amount of posts, is more likely to create more advanced content. Consider the two following questions posted in Stack Exchange Physics forum.

1. Why does centripetal acceleration have a magnitude? Assuming that the magnitude of velocity is constant. Why does centripetal acceleration have a magnitude? Since acceleration is the rate of change for velocity and its magnitude remains the same shouldn’m we express centripetal acceleration by the angle it changed in the vertical or horizontal over a period of time instead?

2. Will the Sun’s fast (but subluminal) removal cause gravitational waves? We cannot just remove the sun as it violates energy conservation. We can however let the Sun accelerate fast out of the solar system. Assuming this (unreasonable) scenario, will this fast disappearance of Sun cause any gravitational wave signature? Basically would an experiment such as LIGO be able to measure a gravitational signature of the Sun’s removal.

The first post is written by a new user who never answered to any others question in the Stack Exchange forum. The question is about “magnitude of velocity” which is not considered an advanced topic in physics. On the other hand the second post is written by an advanced user who has answer about 70 questions in the forum. The topic of the question is gravitational waves, an advanced topic in physics. Therefore, information about the “experience” of the user who contributed a piece of content in our corpus (measured by the number of post they have already contributed) provides useful information about the level of the content.

Previous work on topic detection has overlooked this notion of time, which relates to user maturity and experience, and which, as we showcase in this paper, can provide valuable insights on how advanced a particular topic is. In addition to being able to tease out latent concepts of varying levels, these insights are also useful in bootstrapping automated curriculum
design approaches such as [6] which require a set of concepts to be taught in a curriculum, as well as prerequisite relations for those concepts, which can be given via the user maturity dimension in our topic discovery.

In this paper we introduce an time-evolving topic discovery method, based on Constrained Coupled Matrix-Tensor Factorization, which effectively models time and user maturity/experience towards extracting interpretable topics, their temporal evolution, as well as their level of difficulty. Figure 1 shows a representative such topic detected by our algorithm. The topic corresponds to “Gravitational Waves Detection by Ligo Lab”; it is clearly an advanced Physics topic, and our method correctly infers its level of difficulty.

Our contributions are summarized as follows:

- **Novel Problem Formulation:** We introduce a novel method based on coupled matrix-tensor analysis to discover evolutionary topics and their level of difficulty in online communities.

- **Constrained Coupled Matrix-Tensor Model:** We propose a novel flexible constrained coupled matrix-tensor factorization model which incorporates sparsity, non-negativity, and orthogonality constraints which are motivated by our topic discovery goal and, as we demonstrate in the experimental evaluation, are essential for the accurate discovery of topics. We derive an efficient Alternating Least Squares algorithm for our proposed factorization model, and in order to promote reproducibility and further research, we release our code at https://github.com/ConCMTF/ConCMTF.

- **Evaluation on Real Data:** We qualitatively evaluate our method in comparison to the baseline approaches on real and public data from the Physics and also Programming forum of Stack Exchange. In particular, we demonstrate the power of the proposed method in discovering easy-to-interpret time-evolving topics and their level of difficulty.

- **User Study:** For the quantitative analysis of our method, we conduct a user study among 10 domain experts in Physics who judged the quality, interpretability, and coherence of our results.

**Roadmap** The paper is structured as follows. We start with a discussion of related works in Section III. In Section III we provide certain our notations and background on tensor factorization. We formally define our framework in Section IV. Our algorithm for the problem is described in Section V. Section VI presents the results and empirical evaluations. Finally, we conclude with a summary and directions for future work in Section VIII. A version of this paper appears in the Proceedings of the 2018 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining [10].

## II. Related Work

### A. Tensor Decomposition

Our problem is clearly related to a large body of work both in the domain of tensor decomposition as well as topic extraction. We give an overview of the existing work in each domain below.

**Tensor Decomposition:** The tensor decomposition technique was first introduced by Hitchcock [20] in 1927. Many variations of the tensor decomposition problem have been studied; among them, there are two particular tensor decompositions that can be considered as higher-order extensions of the matrix singular value decomposition: (1) CANDECOMP/PARAFAC (or CP for short), and (2) the Tucker decomposition. For an overview of tensor decomposition techniques, see Kolda et al. [22], and Papalexakis et al. [33].

Another tensor decomposition technique is Coupled Tensor Factorization which was introduced by Smilde et al. [36] in the area of Chemometrics. Since then there has been significant development of such coupled models, either when matrices are coupled together [35] or when matrices and tensors are coupled [15], [29]. A notable example of using the coupled tensor factorization is a recent work by Papalexakis et al. [31] which seeks to identify coherent regions of the brain using a (noun, voxel, person) tensor and a (noun, feature) matrix.

**Topic Extraction:** The topic extraction problem has been rigorously studied in the past. Among the existing methods for solving this problem, is the family of Markov chain-based topic-extraction methods [11], [42], [45], [34], [8]. In [11] a Dynamic Topic Modeling tool is proposed which captures the topic evolution in a collection of documents that is organized sequentially into several discrete time periods, and then within each period an LDA model is trained on its documents. The Gaussian distributions are used to tie a collection of LDAs by chaining the Dirichlet prior and the natural parameters of each topic.

Mei et al. [25] suggested a mixture model to extract the subtopics in weblog collections, to identify and track topics in time-stamped text data. In a similar work by Morinaga et al. [27] finite mixture model is used to represent documents at each discrete time interval. In their model, a topic changes on certain documents if the topic mixtures drift significantly from the previous ones. Kandylas et al. [21] analyzed the evolution
of knowledge communities using a method called Streemer which focuses time-evolving clusters.

In a work by Aggarwal et al. [11] on topic modelling of data streams, a fixed number of $k$ clusters (topics) are maintained over time. If a new document arrives that is far from all existing clusters, it can become a new cluster. Liu et al. [24] take a similar approach except instead of using single words as document features, they use multiword phrases as topic signatures. A drawback of these methods is they consider a-priori fixed feature space per topic.

Most works on extracting time-evolving topics, use post-discretized or pre-discretized time analysis. Post-discretized methods fit a topic modeling to documents without considering time and then documents are sorted in time by slicing them into discrete subsets [17]. However, in pre-discretized methods [41], the documents are first sliced into discrete time slices, and then a topic model is fitted to each slice separately.

Our work is different from previous topic evolution methods as the majority of previous attempts have considered and defined a time span for each topic (e.g. [25]). In these methods, the extracted topic is highly dependent on how the time-spans are defined. In our work, we do not need to define a time-span. Another shortcoming of the vast majority of existing works is that a-priori fixed feature space is considered for each topic, whereas in our model we define a topic as a collection of words that appear together. Another advantage of our work is beside finding time-evolving topics, our method is capable of detecting bursty or conventional topics, such as “Japan tsunami” or “democratic convention” since we consider the tensor and the model, where \( f \) minimizes the Frobenius norm of the difference between the tensor and the model, which is super-diagonal, i.e., it only has non-zero values in \((i, j, k)\) where now the factor vectors are combined using a core tensor \( G \). The \((i, j, k)\) entry \( G_{ijk} \) of the core tensor is indicating the interaction between the \((i, j, k)\) latent factors. We can write Tucker 3 as a soft clustering that detects groups of words that tend to appear together in certain time intervals and specific post numbers. Thus, in this case co-clustering is taking advantage of the ternary relationship between words, time stamps, and post numbers which makes it a good candidate for topic modelling applications. In a similar work, Agrawal et al. [7], [3], [4], [5] used PARAFAC as a co-clustering to model the comparison between the results of different search engines, based on emerging latent topics. For a set of queries, they create a (query, keyword, date, search engine) tensor and use the CP decomposition to create latent representations of search engines in the same space.

### III. Preliminary Definitions

In this section, we provide the notation we will use throughout the paper. In addition, we explain two popular tensor decompositions canonical or PARAFAC and Tucker 3.

**Notations:** Vectors are denoted by boldface lower case letters, e.g. \( a \). Matrices are denoted by boldface Capital letters, e.g. \( A \). Tensors are denoted by Calligraphic letters, e.g. \( A \).

| Symbol | Definition |
|--------|------------|
| \( a_i \) | An entry of a vector (same for matrix and tensor) |
| \( T \) | Matricization of \( T \) in the first mode |
| \( A \otimes B \) | Kronecker product of two matrices |
| \( \times_n \) | The \( n \) mode product |
| \( \circ \) | Outer product of two vectors |
| \( \|A\|_F \) | Frobenius norm of matrix \( A \) |
| \( A^+ \) | Moore-Penrose Pseudoinverse of \( A \) |

**CP/PARAFAC Tensor Decomposition:** Given tensor \( T \), we can analyze it into a sum of \( F \) rank-one factors using the CP/PARAFAC decomposition. Typically, in order to compute the CP/PARAFAC decomposition, we solve the following optimization problem:

\[
\min_{a, b, c} \| T - \sum_f a_f \circ b_f \circ c_f \|_F^2,
\]

which minimizes the Frobenius norm of the difference between the tensor and the model, where \( a_f, b_f, \) and \( c_f \) are latent vectors that correspond to words, time, and post numbers.

\[ T \approx \sum_f a_f \circ b_f \circ c_f \]

To decide about the right number of latent factors \( F \), we used AutoTen [30] which allows us to find more structured latent embeddings in the data. Each latent factor of the embeddings defines a pattern in the forum meaning a set of words (topic) used in a specific time in specific post numbers. If a set of words only appear in a specific short time period (with a bursty distribution), we consider it a topic triggered by an external event at a specific time, Figure 1. On the other hand, if a set of words appear all the time but only specific post numbers we consider it as a part of the evolutionary topics. The post number of these topics determines the difficulty of the topic. For example, if a topic appears in low post numbers, it corresponds to basic topics only discussed by newbies, Figure 3a. Whereas Figure 3f shows that the topics mainly appearing in high post numbers written by advanced users who have contributed a lot in the past. PARAFAC is unique under mild conditions. This is important because it allows us to uniquely unravel a large number of possibly overlapping co-clusters that are hidden in the data. As a result, having a tensor with (word, time, post number) modes, one can view PARAFAC as a soft clustering that detects groups of words that tend to appear together in certain time intervals and specific post numbers. Thus, in this case co-clustering is taking advantage of the ternary relationship between words, time stamps, and post numbers which makes it a good candidate for topic modelling applications. In a similar work, Agrawal et al. [7], [3], [4], [5] used PARAFAC as a co-clustering to model the comparison between the results of different search engines, based on emerging latent topics. For a set of queries, they create a (query, keyword, date, search engine) tensor and use the CP decomposition to create latent representations of search engines in the same space.

**Tucker 3 Tensor Decomposition:** In addition to CP/PARAFAC, the other most widely used tensor decomposition model is the Tucker model [40]. In the original paper, Tucker introduced three models; in this paper we are going to focus on the third one, also known as Tucker 3, which can be seen as a generalization of CP/PARAFAC. In Tucker 3 the tensor is decomposed into \( T \approx \sum_i \sum_j \sum_k G_{ijk} \otimes a_i \circ b_j \circ c_k \) where now the factor vectors are combined using a core tensor \( G \). The \((i, j, k)\) entry \( G_{ijk} \) of the core tensor is indicating the interaction between the \((i, j, k)\) latent factors. We can write CP/PARAFAC as a special Tucker 3 model where the core is super-diagonal, i.e., it only has non-zero values in \((i, i, i)\).

The existence of the core tensor \( G \) in Tucker 3 is key to our application. Due to this core, Tucker 3 is able to capture interactions between latent components which, as we will see in the rest of the paper, are important for topic discovery. Sparsity on core: Imposing sparsity constraint on core tensor, as in e.g., [16], improves the interpretability of the components since fewer interactions are included.

In tensor/matrix form, the Tucker 3 model can be written as \( T \approx G_{x_1 \times x_2 \times x_3} C_{x_1 \times x_3} B_{x_2 \times x_3} A_{x_1} \) where \( \times_N \) is the \( N \)-mode tensor-matrix product.
IV. PROPOSED CONSTRAINED COUPLED MATRIX-TENSOR FACTORIZATION

In a wide variety of applications, we have data that form a tensor and have side information or metadata that may form matrices or other tensors. For instance, suppose we have a (word, time, post number) tensor that indicates how many times a word was used in a specific time and specific post numbers. Usually, question answering platforms also have some metadata on the questions/answers, for instance tags of the questions, that can form a (words, tags) matrix. Thus we have a third-order tensor, \( T \in \mathbb{R}^{I \times J \times K} \) and a matrix \( Y \in \mathbb{R}^{1 \times F} \), coupled in the first mode of each and there is a one-to-one correspondence of elements in the first mode of the tensor and the matrix ("word" mode in our case). The coupled-matrix and tensor factorization (CMTF) algorithms jointly factorizes multiple data sets in the form of higher-order tensors and matrices by extracting a common latent structure from the shared mode. Imposing a Tucker model yields:

\[
\min \| T - G_{x_3} C_{x_2} B_{x_2} A_{x_1} \|_F^2 + \| Y - AD^T \|_F^2 (1)
\]

On the other hand if we impose a PARAFAC decomposition, we have

\[
\min_{a_{r}, b_{r}, c_{r}, d_{r}} \| T - \sum_r a_r \circ b_r \circ c_r \|_F^2 + \| Y - \sum_r a_r d_r^T \|_F^2 (2)
\]

The existing work on coupled-matrix tensor factorization only considers non-negativity constraints, e.g. \( A \geq 0 \). Non-negativity is an important feature of latent factors since many real world tensors have non-negative values and hidden components have a physical meaning only when non-negative. Although non-negativity improves interpretability, in many applications it is not enough to make sense of the data. When the goal of factorization is to find the latent topics within the tensor and the matrix, we would like to find as many non-overlapping structures as possible. Non-overlapping latent components directly imply that the latent topics are concise and hence interpretable. We can control the amount of overlap in latent components by imposing orthogonality constraint on each latent component. This means for the first mode, we would like the columns of the latent component \( A \) to be orthogonal, \( \forall i,j \quad A_i^T A_j \leq \epsilon_A \quad i \neq j \). If \( \epsilon_A \) is set to 0, this implies latent components should be completely orthogonal, while values greater than 0 means some overlap is allowed.

Furthermore, in practice we desire the factors to be sparse as well. Sparsity constraints improve parsimony and offer a simpler and hence more interpretable model. We can impose sparsity constraint on all factors and on the core tensor as well. We enforce the sparsity constraint by imposing constraint on \( \ell_1 \) norm of each column in factor matrices and on the core tensor. Enforcing sparsity on each column of the factor matrices means sparsity is imposed uniformly on each latent component for each mode. Sparsity becomes specially favorable when it is imposed on the core tensor; meaning only a few latent components interact with each other. This removes redundancy and achieve compact sparse representations of the core and hence the core tensor will be easily interpretable.

To the best of our knowledge we are the first to introduce the constraint coupled-matrix tensor factorization problem with non-negativity, sparsity and orthogonality constraints. Our intuition and constraints are captured in a formal definition as follows.

**Problem 1 ():** Given a tensor \( T \in \mathbb{R}^{I \times J \times K} \), auxiliary matrix \( Y \in \mathbb{R}^{1 \times F} \), and number of factors for each component \( R_1, R_2, R_3 \), find the components \( A, B, C, D \), and tensor \( G \) such that

\[
\min \| T - G_{x_3} C_{x_2} B_{x_2} A_{x_1} \|_F^2 + \| Y - AD^T \|_F^2,
\]

Subject to: For each factor \( F \in \{A, B, C, D\} \)

\[
F \geq 0 \\text{ and } \forall i \| F_i \|_1 \leq \epsilon_{F_1}, \text{ and } \forall i,j \quad F_i^T F_j \leq \epsilon_{F_2} \quad i \neq j
\]

For core tensor \( G \), \( G \geq 0 \) \( \| G \|_1 \leq \epsilon_{G} \).

For the sake of interpretation, it is enough for core to be sparse, having a few non-zero elements. Lifting orthogonality constraint from core means we allow interaction between same factors, but only a few factors to interact with each other.

V. PROPOSED ALGORITHM

Even though, tensor decomposition is an NP-hard problem, here we provide an Alternating Least Squares (ALS) algorithm which solves the Constrained Coupled-Matrix Tensor factorization problem and converges to a locally optimal solution (as a property of the family of ALS algorithms). Alternating Least Squares is probably the most widely used algorithm and dates back to the original papers by Carroll et.al [13] and Harshman [18]. Alternating Least Squares method has been shown to be very efficient and competitive for PARAFAC decomposition [39] and in practice works well [22]. This approach has the advantage that it can be applied to large scale problems. Using ALS method, we solve for each factor at a time while fixing all other factors. If we seek to estimate \( A \), it turns out that we need to concatenate the two pieces of the data \( T \) and \( Y \), whose rows refer to matrix \( A \), that is the matricized tensor \( T_A \) and matrix \( Y \), and we can then solve for \( A \) as

\[
A = \left[ T_A^T Y \right]^T \left( G_{A}(B \otimes C) + D \right)^{-1}^T
\]

Algorithm [1] shows our ALS algorithm to solve the constrained coupled-matrix tensor factorization, ConCMTF-ALS. These constraints include non-negativity, sparsity and orthogonality imposed by \( A \geq 0 \), \( \forall i \| A_i \|_1 \leq \epsilon_A \) and \( \forall i,j \quad A_i^T A_j \leq \epsilon_A \quad i \neq j \) respectively. Rather than alternating to solve each factor completely, we solve for each column of each factor independently. This is possible since the columns of each factor are independent and the constraints we consider can be specified to each column as well. A column in the factor of first mode, \( A \), indicates a group of words and a column in \( B \) indicates a specific weeks in the lifetime of the forum. It is important to note the
effect of specifying sparsity constraints on the columns rather than the whole matrix. This means sparsity will be spread uniformly across the whole matrix. It is worth mentioning that our algorithm can allow any convex constraints to be placed for each factor. Another advantage of our algorithm is that it can be easily used for PARAFAC decompositions instead of Tucker3 with minimal changes. To achieve this, instead of initializing core to random values in Line [1] we set the core tensor to a super diagonal tensor. In addition, there is no need to estimate core tensor in each iteration and hence Line [19] and [20] can be removed from the algorithm.

Algorithm 1 The Alternating Least Squares for Constrained Coupled Matrix-Tensor Factorization ConCMTF-ALS

Input: The tensor \( T \in \mathbb{R}^{I \times J \times K} \) and auxiliary matrix \( Y \in \mathbb{R}^{I \times F} \)

Output: Coupled Decompositions \( A \in \mathbb{R}^{I \times R_1}, B \in \mathbb{R}^{J \times R_2}, C \in \mathbb{R}^{K \times R_3}, D \in \mathbb{R}^{F \times R_1} \)

1: Initialize \( A, B, C, D, G \) to non-negative random values
2: while convergence criterion is not met do
3: \( A \leftarrow \arg\min \| T A Y - A [g_A(C \otimes B)^T D^T] \|_{F, \rho} \)
4: Subject to: \( A \geq 0 \text{ and } \forall i, A_{i,j} \leq \epsilon_A \)
5: \( \text{and } \forall i,j, A_{i,j} A_{i,j} \neq 0 \)
6: Normalize the columns of \( A \)
7: \( B \leftarrow \arg\min \| T B - B [g_B(C \otimes A)^T D] \|_{F, \rho} \)
8: Subject to: \( B \geq 0 \text{ and } \forall i, B_{i,j} \leq \epsilon_B \)
9: \( \text{and } \forall i,j, B_{i,j} B_{i,j} \neq 0 \)
10: Normalize the columns of \( B \)
11: \( C \leftarrow \arg\min \| T C - C [g_C(B \otimes A)^T D] \|_{F, \rho} \)
12: Subject to: \( C \geq 0 \text{ and } \forall i, C_{i,j} \leq \epsilon_C \)
13: \( \text{and } \forall i,j, C_{i,j} C_{i,j} \neq 0 \)
14: Normalize the columns of \( C \)
15: \( D \leftarrow \arg\min \| T D - D [g_D(Y - A D^2)^T] \|_{F, \rho} \)
16: Subject to: \( D \geq 0 \text{ and } \forall i, D_{i,j} \leq \epsilon_D \)
17: \( \text{and } \forall i,j, D_{i,j} D_{i,j} \neq 0 \)
18: Normalize the columns of \( D \)
19: \( G \leftarrow \arg\min \| \text{vec}(T) - (C \otimes B \otimes A) \text{vec}(G) \|_{F, \rho} \)
20: Subject to: \( G \geq 0 \text{ and } \forall i, G_{i,j} \leq \epsilon_G \)
21: return \( A, B, C, D, G \)

A. Running Time:

Each step of our algorithm can be solved by any convex or least squares (LS) solvers; If we chose a LS solver or a non-negative least squares (NNLS) solver such as the one in the N-way Matlab Toolbox [2], we would subsequently need to transform the unconstrained NNLS solution into a constrained one by using projected gradient descent. However, For flexibility, we used CVX, a convex solver. In practice we observed that CVX is faster than using Least Squares solvers and projected gradient decent method combined. Each iteration of our algorithm has linear complexity with respect to the number of factors, and number of modes. In addition, our algorithm solves for each column of each matrix independently and hence makes our algorithm faster. More precisely, assuming a Parafac decomposition on a tensor of size \([n, n, n]\) with \(m\) factors, the running of to solve for each factor at each iteration will be \(O(m \times n^3)\) in which \(m\) is the number of factors and \(n\) is the number of variables in each column. In theory, the convex programming can be solved in the cubic number of variables \((O(n^3))\) [44] but in practice it runs faster. So the total running time of our algorithm is \(\text{iterations} \times O(m \times n^3)\). In addition our algorithm converges in less than a few tens of iterations. Thus, the whole running time is reasonable.

VI. RESULTS

We focus on question and answers related to the field of physics and python programming in Stack Exchange. We start by discussing our datasets in details, and then we present how we apply our tensor decomposition techniques on this datasets to find topics.

A. Data

Stack Exchange is a question answering website created in 2008. It features questions and answers on a wide range of topics from mathematics and programming to cooking and movies. Stack Exchange allows each question to be annotated with one or more terms (tags) indicating the subject matter of the question. We used the latest Physics and programming Data Dump in Stack Exchange. We only consider the questions which have at least one tag (almost 30 000 questions), and we only considered frequent words which appeared more than 100 times in the forum. We also stemmed all the words.

1) Physics Stack Exchange: From the physics forum data, we created a tensor (multi-way array) \( T \) with three modes (word, time, post number) of size \(1351 \times 304 \times 9\). When a user \(\mathbf{u}\) uses word \(w\) at week \(\mathbf{t}\) in his \(p^{th}\) post, we will increase \(T(w, \mathbf{t}, \log(p))\). Thus, the \((i, j, k)\) value of Tensor \( T \) indicates how many times word \(i\) was used at week \(j\) in \(\log(k^{th})\) posts of all users. Note that post number is relative to each users’ sign up date. Hence, if a user signs up and writes a question or answer, her post number would be 1.

An important aspect of considering time along with topics is the fact that the temporal information of the topics helps us understand the topic better. For example, the word “jobs” relates to employment, but after October 5, 2011, the word jobs may refer to "Steve Jobs". This is the key reason for why we use time as another dimension, we can get more insight about topics and distinguish evolutionary topics from event driven topics which follow a bursty distribution.

In our application, beside the words, post numbers and time stamps, we also have the tags associated to each question by the users. We can use question tags as a side information or metadata as a word-tag matrix. This matrix indicates how many times each word has been used for a specific tag. We denote this matrix by \(Y\). Our auxiliary matrix \(Y\) has two modes (words and tags) of size \(1351 \times 527\). \(Y_{i,j}\) indicates the number of times word \(i\) has been used in a post with tag \(j\).

2) Programming Stack Exchange: The programming Stack Exchange forum includes all the questions related to programming including Java, Python, C, R and other programming languages. We decided to only include the questions including "python" tag. From the programming questions having python tag, we created a tensor (multi-way array) \( T \) with
Fig. 2: An example of two components extracted by PARAFAC–NS algorithm on Physics dataset. The two components are similar in word, time and post number modes. The words gravity, time, light, speed, wave, particle, and energy are frequent in both components.

Fig. 3: An example of four components extracted by ConCMTF–ALS algorithm on physics dataset. All components have distinct set of words and distinct post numbers.

Fig. 4: An example of two components extracted by PARAFAC–NS algorithm on programming dataset. The two components are similar in word, time and post number modes.
three modes (word, time, post number) of size $432 \times 411 \times 50$. When a user $u$ uses word $w$ at week $t$ in his $p^{th}$ post, we will increase $T(w, t, p)$. Beside the words, post numbers and time stamps, we also have the tags (except python tag) associated to each question by the users. Similar to Physics data, we created an auxiliary word-tag matrix which indicates how many times each word has been used for a specific tag. The auxiliary matrix $Y$, has two modes (words and tags) of size $432 \times 30$ (we only kept top 30 tags).

### B. Experimental Evaluation

In this part, we evaluate our algorithms under CP/PARAFAC and Tucker3 decomposition models for CMTF. We implemented our algorithm in Matlab and used CVX package to solve each step of Algorithm [1]. All experiments were carried out on a machine with a 2.4 GHZ CPU, 16 GB RAM, running CentOS Linux 7. Our dataset and our code are immediately and freely available for download [2]. We compare our results to non-negative PARAFAC decomposition [2] and sparse non-negative Tucker3 [22]. We refer to them as PARAFAC–NS and TUCKER3–NS respectively. To decide the right number of latent factors ($F$) to be extracted in each algorithm, we used AutoTen [30] which allows us to find more structured latent embeddings in the data. For each component that is obtained from each decomposition, we do 2-means clustering on the vector associated with the word mode. Then, we take the cluster with the maximum mean and choose the cut-off value to be equal to the smallest value in that cluster, such that any value below that threshold is zeroed out. In this way, we avoid interpreting the noise words (i.e. those with very small values) as part of the topics.

**PARAFAC–NS vs. ConCMTF–ALS with PARAFAC:** Figure 2 shows two components selected from obtained components using PARAFAC–NS algorithm on Physics dataset. We observe that in these two decompositions, there are overlaps in the set of words found by PARAFAC–NS as well as overlap in time and post number modes. In fact, post numbers have identical trends and the words gravity, time, light, speed, wave, fraction, particle, and energy are among frequent words in both components. Moreover, the set of words in both components include a (relatively) large number of words and the word factors are very dense. If the goal of factorization is to find latent structure and patterns in the data, these two components are very similar and hence give us the same structure and little information.

We also used our algorithm, ConCMTF–ALS, assuming a CP/PARAFAC decomposition. For this decomposition, we only imposed non-negativity and orthogonality constraint on components $A$, $B$, $C$, and $D$ with $\epsilon_A = 0.05$, $\epsilon_B = 0.6$, $\epsilon_C = 0.2$, and $\epsilon_D = 0.2$. The intuition behind this is that we would like to find components which are distinct in their set of words and the level of maturity (post number values). However, we allow decompositions to have overlap in the time mode as we seek patterns in any period of forums lifetime.

Figure 3 illustrates the components produced by ConCMTF–ALS on Physics dataset. The set of words in each component are sparse and they do not share many words as it was in the case in PARAFAC–NS components. The post numbers of each component are non-overlapping as well. The first word component depicts the words “mass”, “wave”, “equation”, “velocity”, “particle”, “time”, “angular”, “slow”, and “oscillation” which were used in very low post number (i.e. by new users). These are in fact basic topics in physics. The second component covers topics related to harmonic motion and waves topics. Compared to the first component these words appear in larger post numbers, i.e. they are posted by more advanced users. The topic of the third component is mainly the first law of thermodynamics discussed mainly by advanced users with large post numbers. The last component included the words “inductor”, “flux”, “ring”, “diameter”, “transform”, “collide”, “magnetic”, “field”, and “circular”. These words are related to “Toroidal inductors and transformers” which only appeared in very large post number and by very advanced users.

Figure 4 is an example of a component which only appeared in a specific time period and moreover in specific post numbers (large post numbers). This pattern indicates words discussed in response to an external event. The set of words in this component consists of “gravity”, “Ligo”, “detection”, “laser”, “hole”, “theory”, “space”, “time”, “mass”, etc. The peak in time mode corresponds to Feb, 2016. This is the time that the detection of gravitational waves was announced by Ligo lab.

Figure 5 shows two components selected from obtained components using PARAFAC–NS algorithm on Programming dataset. As illustrated, these two decompositions, there

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1. https://github.com/ConCMTF/ConCMTF

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are overlaps in the set of words found by PARAFAC–NS as well as overlap in time and post number modes. On the other hand, figure 5f shows two components extracted by our algorithm. As illustrated in the figure, the set of words in each component are sparse and they do not share many words and each component shows semantically coherent topics. The first topic, figure 5a, includes words related to multiprocessing in python such as thread, multiprocessing subprocess, queue, virtualenv, asynchronous, and etc. Figure 5b shows multiprocess topics have a presence across various post numbers. This reveals that such a topic is of interest regardless the expertise of users. The second topic, figure 5c includes topics related to web crawling such as selenium, web-driver, Firefox, Flask, Twitter, url, css, and etc. The associated post number, related to web crawling such as selenium, web-driver, Firefox, Flask, Twitter, url, css, and etc. The associated post number, shows that such words are frequent across very spread post numbers. This topic clearly shows that such a topic is of interest regardless the expertise of users. If we look at the overlap of the words, figure 5d reveals that this topic is of interest regardless the expertise of users. The second topic, figure 5c, includes topics related to web crawling such as selenium, web-driver, Firefox, Flask, Twitter, url, css, and etc. The associated post number, shows that such words are frequent across various post numbers. This reveals that such a topic is of interest regardless the expertise of users.

TUCKER3–NS vs ConCMTF–ALS with Tucker3: Figure 6 illustrates the decompositions obtained by TUCKER3–NS and ConCMTF–ALS when assuming a Tucker decomposition with 12, 4, and 4 latent factors for each mode. We can observe that the words produced in TUCKER3–NS algorithm are not distinct and have a big overlap (Fig. 6a and 6b). On the other hand, our algorithm ConCMTF–ALS, is able to find more distinct and coherent set of words (Fig. 6c and 6d). Moreover, using TUCKER3–NS, the core tensor has values in almost 55% of the core entries, making the interpretation difficult. On the contrary, only 20% of the core entries have values using ConCMTF–ALS.

VII. USER STUDY

To evaluate the quality of the topics found by our algorithm, we conducted a user study with two goals: 1) evaluate the cohesion of each learning unit, and 2) evaluate the ordering of the units. In the following sub-sections we present the details of our conducted user-study and the results of our study.

Survey Participants: We recruited 10 volunteers to participate in our survey. Six had a PhD in computer science. Two with a PhD in Physics and Space Physics respectively, and finally two participants had mechanical engineering background. None of the authors provided any judgements.

Survey Design: We selected 5 components from the set of all components resulted from ConCMTF–ALS assuming a CP/PARAFAC decomposition. The reason we decided to include a limited number of components in the study is that we noticed some set of the words associated with some components are quite advanced and the participants are most likely not familiar with them. For example, the following component was excluded from the survey: “bra, preserve, ket, property, configure, rigor, Hilbert, hermitian, electrodynamic”. This set of words refer to quantum mechanics and Bra-ket notation. To determine which topics are advanced and not familiar to our participants, we asked a scientist with a PhD in physics (who did not participate in our survey) to identify the less known topics. We showed the remaining set of words to our participants and asked them to count number of odd words in each set of words produced by ConCMTF–ALS assuming PARAFAC decomposition.

We asked the following question to our volunteers: Q1: Count the number of odd words in each topic.

Survey Results: Table I summarizes the results of our survey including the min, max, mean, and the median of values that our participants reported as the number of odd words in each topic (unit). In general, we observe that for the all of our units, the number of odd words is very low, demonstrating good cohesion in each set of words. Unit 3 has the most number of odd words on average. This unit consists of the words such as “inductor, flux, ring, difficulty, collide, avoid, wirewind, field, magnetic, diameter, illustrate, circular”. Most of our participants indicated “difficulty, avoid, illustrate, and collide” as odd words. These are the words that do not correspond to physics concepts and were not excluded from the data. The third unit has the least number of odd words. This unit consists of words such as mass, wave, equation, velocity, particle, time, psi, angular, slow, length, and transmit.

It is also important to evaluate the inter-judge agreement in a survey like ours. Due to the nature of the ratings, an appropriate way of analyzing the agreement is by using Krippendorff’s α statistical measure [23], which is applicable to the current scenario of judges assigning a value to a specific variable. The overall agreement measured by Krippendorff’s α for our ten judges turns out to be 0.32. This indicates that there is fair but imperfect agreement. When we look at the agreement of judges within the same background group, we observe the agreement between the second group with Physics and space physics background is 0.56 which shows our participants with physics background have a high level agreement on the number of odd topics.

A. Applicability to Curriculum Design

As we mention in the Introduction, our proposed topic discovery has implications to curriculum design, since it is
able to identify topics along with their level of difficulty; those levels of difficulty are key in determining prerequisite and co-requisite (i.e., concepts that must be taught at the same time) relations between concepts in the syllabus. Here, we demonstrate this applicability of our topic discovery to automated curriculum design, along the lines of recently proposed work of [6], [2]. In order to achieve this, first, we identify events triggered by external events which are not part of topics evolution and exclude them from the curriculum. We then order the topics based on their relevant difficulty, as inferred by our topic discovery, and as we mentioned previously, this ordering determines the arrangement of the topics in the curriculum. What follows is a curriculum we obtained from online discussion after removing all non-physics terms.

This curriculum is consistent with majority of curricula taught in basic physics courses in online/traditional classrooms. See [38] for more details.

VIII. Conclusion

In this paper, we proposed a novel time-evolving topic discovery method which is able to identify the level of difficulty of the extracted topics. Our approach is powered by a novel Constrained Coupled Matrix-Tensor Factorization for which we provide our code at [4]. We evaluate our resulting topics both qualitatively and quantitatively via a user study of expert judges, and we demonstrate the effectiveness of the proposed method in discovering high-quality, interpretable topics, their temporal evolution, and their level of difficulty. Finally, we highlight the implications of our approach in education-related applications.

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