A New Physically Motivated Clutter Model With Applications to Nondestructive Ultrasonic Testing

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Abstract—In this article, we construct a new physically motivated model for the grain scattering noise (clutter). We assume the clutter is the output of a random linear time-invariant (LTI) filter, the impulse response of which is considered to be a realization of a Gaussian wide sense stationary (WSS) random process, when the transmitted ultrasonic pulse is at the input. In an ultrasonic non-destructive testing (NDT) session, the clutter noise, which is signal-dependent and caused by the microstructure of the material, presents a substantial challenge in identifying defects in the material under testing. The model is used to aid in the detection of a defect in the material. The model incorporates, explicitly, many important physical characteristics of the generated clutter, such as the average grain size, the random shape, and orientation of the grains, and emphasizes the single scatterer assumption (Rayleigh region). The statistical properties of the model are formulated and derived. A comparison with the usual matched filter to indicate the model advantage is performed. An application to real ultrasonic data is conducted with excellent results. Furthermore, we explored the effect of the choice of the model parameters, and the model shows robustness toward parameter misspecification. We then tested the performance under a deviation from the single scatterer assumption, for a more complex target, using simulated noise and obtained promising results.

Index Terms—Clutter noise model, clutter suppression, flaw detection.

I. INTRODUCTION

The problem of detecting a target in signal-dependent noise has been researched extensively throughout the years. Signal-dependent noise is referred to as clutter in radar, backscattering noise in ultrasonic non-destructive testing (NDT) of materials, and reverberation in sonar. However, the term clutter has been used interchangeably with backscattering noise, and we will follow this convention here. In an ultrasonic NDT session, a piezoelectric transducer fires a short-duration pulse of ultrasound in a narrow beam into the material and any echoes coming back are received with the same transducer [1]. The goal of examining the collected data can be categorized into two subcategories, one is detection (detecting if a defect exists in the scanned material), and the other is classification (classify the type of defect, if it exists). A non-destructive test is the one in which there is no impairment of the properties and performance in the future use of the object under examination [2].

In this work, we construct a physically motivated clutter model as the output of a random linear time-invariant (LTI) filter, the impulse response of which can be assumed to be a realization of a Gaussian wide sense stationary (WSS) random process. The model is used to aid in the detection phase of examining the data. Modeling the clutter as the output of a random LTI filter, whose impulse response is assumed to be a WSS random process has been used before in radar problems [3]–[5]. The term physically motivated indicates that we took into account some of the important physical characteristics of the grains in the material that generated the clutter, such as the average grain size and the random orientation and shape of the grains. In addition, we emphasize the single scatterer assumption (Rayleigh region), in contrast to multi-scattering, which results when the wavelength of the test signal becomes very close to the size of the general diameter of the grains [6]. Also, we will assume that the material under test is homogeneous (uniform grain size throughout the materials) and isotropic (the material has identical property values in all directions). Excellent results from applying the model on real ultrasonic data were obtained, and the model exhibits robustness toward parameter misspecification.

The clutter noise in NDT is caused by the microstructure of the material under test. When an ultrasonic wave travels through a coarse-grained material, the traveled pulse hits the grain boundaries, which will cause some of its energy to propagate back to the transducer. Single grain size in such a material is usually very small, and the reflected amount of energy is minimal from each grain. However, a large number of grains in a scanned volume of the material, taking into account the randomness of their size, orientation, and shape, will result in an in-phase addition and a more considerable amount of energy to be reflected back to the transducer. Even though the flaw size is usually much larger than an individual
grain, in many cases, these reflected energies mask the echoed energy from a flaw located in the same path of the scanned volume. In general, in coarse-grained materials, acoustic wave propagation is a function of the microstructure, frequency, and wave mode [7]. An elaborate discussion for the ultrasonic NDT, acoustic wave propagation in materials, and clutter noise characteristics can be found in [1], [2], [8], [9], and the references within.

Many algorithms have been proposed to tackle the above problem in either the frequency domain, time domain, or both. Some of the proposed methods are simple filtering, Wiener filtering, split spectrum, wavelet filtering, spectrum equalization, empirical mode decompositions, and even the use of the neural network. Many of these methods are considered nonparametric such as split spectrum [10], [11] and some are considered parametric such as [12] and [13]. In the parametric approach, a model that depends on some parameters is chosen, and these parameters are estimated from the collected data (our approach will be of this type). On the other hand, the nonparametric approach does not make such an assumption and uses the collected data directly. Proposed models for the clutter noise in materials range in simplicity and complexity, depending on their applications. Many models have been provided for simulation purposes with no statistical framework [10], [14]–[16]. Some other models include a statistical framework such as [17], where the model is assumed to be delayed, distorted, and scaled versions of the transmitted pulse. However, the problem was tackled in the frequency domain and without any reference to the detection problem. A similar model to the previous one can be found in [12], where the grain noise is modeled as a Gauss–Markov random field and uses the likelihood ratio test as a detection tool and has shown good results in detecting flaws in stainless steel material based on ultrasound images. In [18], Sanie et al. focused on the amplitude variation statistics and not on the detection problem. Many models assume that the delays of the reflected signals are uniformly distributed over the scanned volume [10], [12], [17], [18], to account for the grain’s random orientation, shape, and size. However, such an assumption raises many improbable situations that are not physically possible in the material, such as having three or more scatterer centers less than an average grain size apart when the assumption to ignore the multi-scatterer is assumed. In contrast, in our model, we tried to emphasize the single scatterer assumption by considering a single scatterer center per average grain size, and later we explored the multi-scatterer assumption for a more complicated target.

In this study, we first state the detection problem and provide the model assumptions in Section II, followed by a performance comparison with the usual matched filter (MF) and the detector implementation in Section III. Next, in Section IV, we apply the model on real ultrasonic data and discuss the outcomes. In Section V, we explore the model parameter sensitivity and in Section VI we discuss the multi-scatterer centers for a more complicated target. Finally, conclusions are given in Section VII.

II. Problem Statement and Model Assumptions

The model for the received signal shown in Fig. 1 is as follows:

\[
\mathcal{H}_0 : x[n] = w[n] + c[n] \\
\mathcal{H}_1 : x[n] = A_0 s[n-n_0] + c[n] + w[n].
\]

To formulate the problem in a statistical framework, we assume that the received signal is \( x[n] = c[n] + w[n] \) under hypothesis \( \mathcal{H}_0 \), where \( c[n] \) denotes the clutter noise, and \( w[n] \) denotes the ambient white noise. Under the hypothesis \( \mathcal{H}_1 \), we get \( x[n] = A_0 s[n-n_0] + c[n] + w[n] \), where the defect impulse response \( g[n] \) is modeled as a point target (only a shift and scaling of the transmitted signal \( s[n] \)). The amplitude \( A_0 \) and delay \( n_0 \) represent the scale factor and delay that correspond to the target geometry and location, respectively. The transmitted ultrasonic pulse \( s[n] \) is specified by the type of transducer excitation that is used for testing. The optimal signal design problem is beyond the scope of this article and will be addressed in a future work. We then modeled the clutter \( c[n] \) as the output of a random LTI filter with the impulse response \( h[n] \), where the input is the transmitted ultrasonic pulse \( s[n] \).

As shown in Fig. 1 there are two types of noise that will affect our received signal. First, the ambient white noise \( w[n] \) that is introduced in the instrumentation (circuitry, A/D conversion, and temperature variations) and other artifacts [7], and is assumed to be an independent identically distributed (IID) white Gaussian noise process with a probability density function (PDF) \( w \sim \mathcal{N}(0, \sigma_w^2 I) \), where the boldface \( w \) indicates the vector of ambient white noise samples, and the symbol \( \sim \) means “distributed as”. The ambient white noise can be reduced by simple averaging of multiple A-scans for the same scanned volume. Second, the clutter noise \( c[n] \) is caused by the material microstructure and is assumed to be a colored Gaussian noise process with a PDF \( c \sim \mathcal{N}(0, C_c) \), where \( C_c \) is the Toeplitz autocorrelation matrix of the clutter. The clutter noise \( c[n] \) and the impulse response \( h[n] \) will be explored in more detail in Section II-A. We note that the use of the discrete time representation indicates that the real continuous signals have been sampled with a sampling rate \( f_s = 1/T_s \) Hz. In addition, we described all signals as a function of time,
A. Random LTI Filter $h[n]$ and Clutter Noise $c[n]$

In this section, we show how to construct a new physically motivated model for the grain scattering as a Gaussian WSS random process, and determine its mean, autocorrelation sequence (ACS), and its power spectral density (PSD). The process $h[n]$ can be thought of as the product of two independent processes $a[n]$, and $u[n-m_0]$ (normalized) that is $h[n] = a[n](u[n-m_0]-\mu_a)$ (it can be shown that the product of two independent WSS processes is also WSS [19, p. 591]).

A portion of a sample realization of $a[n]$, $u[n]$, $u[n-m_0]$, and $h[n]$ is shown in Fig. 2. The amplitude variations of the reflected energy from the grains depend on many factors, such as grain size, orientation, shape, distance from the transducer, the transmitted pulse characteristics, and acoustic impedance. Such dependency makes these variations very random and is best described as a random process. The process $a[n]$ is assumed to be an IID Gaussian random process with a PDF $a \sim \mathcal{N}(0, \sigma_a^2)$. This is a practical assumption also used in [15] and [16]. The process $u[n]$ is an impulse train process with impulses that are randomly distributed as described in the following paragraph. Without loss of generality, let $I_i = [n : iM \leq n \leq iM + (M - 1)]$ (see Fig. 2). Then, the following hold.

1) In each interval $I_i$, a position is chosen at random for the spike. That is to say $\zeta_i \in [0, M - 1]$, where $\zeta_i$ is a discrete random variable taking on integer values with $\zeta_i \sim \mathcal{U}(0, M - 1)$, and $\mathcal{U}$ denotes the discrete uniform distribution.

2) All spike locations $\zeta_i$’s are IID. This forms the process $u[n]$.

3) Then, the process $u[n]$ is shifted to the right by $m_0$ samples, where $m_0 \sim \mathcal{U}[0, M - 1]$.

4) The random shift $m_0$ is independent of all $\zeta_i$’s, which leads to our new process $v[n] = u[n-m_0]$.

Since, in practice, the transducer width is much larger than a single grain size, we expect to see several overlapped spikes that come from all directions and violate the single spike per average grain size assumption. However, such an assumption is an important simplification that led to a deterministic robust model that performed well on real ultrasonic data as seen in Section IV. Moreover, the simplification of assuming one spike per average grain size can be thought of as if the axial distance between the transducer and the scanned volume is divided into small equally spaced sections (that is the average grain size) and we only allow one reflection per average grain size and the reflection represents the net results to all the reflections that arrive at that particular distance/time. As discussed before, many models assume that the scatterer delays are uniformly distributed over the total scanned volume, to account for the random orientation, shape, and size of the grains [12], [17], [18]. However, simulating uniform distributed delays over the total scanned volume will result in many improbable and physically impossible situations, such as having three or more scattering centers less than an average grain size apart. On the other hand, our model incorporates the single scatterer restriction per average grain size and utilizes the average grain size explicitly (see Assumption 1). It also accounts for the random shape and orientation of the grains (see Assumptions 1 and 3). Assumption 3 is a necessary step to transfer the process $h[n]$ from being a non-stationary to a WSS process [20, p.135]. Referring to Assumptions 1–3, $M$ is a parameter that represents the number of samples and is proportional to the average grain size of the material under testing, as well as the sampling rate. Assume the distance between the surface of the transducer and a certain grain is $L$ meter. In an NDT testing session, the sound will travel from the surface of the transducer to the first edge (boundary) of that grain and bounce back, and the total roundtrip distance traveled will be $2L$ meter. Next, the distance traveled from the surface of the transducer to the second edge is $2(L + D)$ meter, where $D$ is the average grain size in meter, and therefore the difference in the distance that these two reflections will have in a typical A-scan is $2D$ meter. As an example, if we have a material with an average grain size of $D = 100 \mu$m, a sampling rate of $F_s = 100$ MHz, and the speed of sound in the material is $c = 6000$ m/s, then using (2) we get $M = 4$, where $\lceil \cdot \rceil$ indicates the round up to the nearest integer and results from

$$M = \left\lceil \frac{2D}{cT_s} \right\rceil. \quad (2)$$

By making $M = 4$ for the previous example, it allows only one scatterer center per average grain size, which emphasizes the single scatterer assumption (Rayleigh region). The random shift $\zeta_i$ accounts for the different orientation and shape of the grains; therefore, the scatterer center will vary from grain to grain.

As for the clutter $c[n]$, the input to the random filter $h[n]$ is assumed to be the transmitted pulse $s[n]$, then
Since the convolution is a commutative operation, we can write \( c[n] = s[n] * h[n] \), where \( * \) denotes the convolution operator. Since the convolution is a commutative operation, we can write \( c[n] = h[n] * s[n] \), where the filter input is now \( h[n] \) and the filter impulse response is \( s[n] \). Since we assumed \( h[n] \) is a WSS Gaussian random process, let us denote its distribution by \( \mathcal{N}(0, \Sigma_h) \), where \( \Sigma_h \) is the Toeplitz autocorrelation matrix. We know that the distribution of \( c[n] \) will also be a WSS Gaussian random process with ACS equal to

\[
 r_c[k] = s[-k] * s[k] * r_h[k]
\]  

and PSD equal to

\[
P_c(f) = |S(f)|^2 P_h(f)
\]  

where \(-1/2 \leq f \leq 1/2\), \( r_h[k] \), and \( P_h(f) \) are the ACS and PSD for the process \( h[n] \), respectively, [20, p.602]. A full derivation of the mean, ACS, and PSD of \( h[n] \) is given in the Appendix. Letting \( \alpha = (1/M^2(M - 1)) \sigma_h^2 \), then the mean, ACS, and PSD of \( h[n] \) is given by \( \mu_h = 0 \), \( r_h[k] = a \delta[k] \), \( P_h(f) = a \), and for \( c[n] \) using (3) and (4), we get

\[
r_c[k] = a (s[-k] * s[k])
\]  

and a PSD of

\[
P_c(f) = a|S(f)|^2, \quad \frac{1}{2} \leq f \leq \frac{1}{2}.
\]  

Thus, we can denote the distribution of \( c \) as \( \mathcal{N}(0, \Sigma_c) \), where the elements of \( \Sigma_c \) can be calculated from \( [\Sigma_c]_{ij} = a[s[k] * s[-k] * \delta[k]]_{k=-j} \). As shown in (5), the clutter matrix \( \Sigma_c \) depends only on two parameters \( \sigma_h^2 \) and \( M \) (for a given transmitted ultrasonic signal \( s[n] \)). As indicated, the process \( h[n] \) has a flat spectrum; so, the output PSD \( P_c(f) \) of the random LTI system is just a scaled version of the energy spectral density (ESD) of the transmitted pulse \( s[n] \) [shown in (6)]. Moreover, the Gaussian assumption for the clutter \( c[n] \) is a reasonable one when the material is homogeneous (which we assumed) and the number of grains is large enough for the central limit theory to hold [21].

The non-stationary behavior of the clutter arises from the distance-dependent attenuation, frequency-dependent attenuation, absorption, and diffraction [17]. Even though the non-stationary mean and autocorrelation functions completely characterize the process, the analytical derivation of these statistics is demanding [11], and will not be considered here, though an attempt has been made in [22]. To compromise, we simplified the problem to be a WSS process by not taking the distance-dependent and frequency-dependent attenuation into account. However, it was determined that, for a short-time data window, any non-stationarities due to material attenuation are small and can be ignored [13]. Similar to speech processing, a window of 10–20 ms of the data can be assumed stationary even though the process is heavily non-stationary [23]. This is not the case for the non-stationarity caused by the transducer focusing effect [13]. Hence, we focused our simulation on the far-field area, which starts at approximately \( d^2/4 \lambda \) meters, where \( d \) is the transducer diameter in meters, and \( \lambda \) is the wavelength of the transmitted pulse in meters [24]. An adaptive implementation is possible using our model by segmenting the data into smaller time windows and then applying the model to each segment. What makes employing our model different from other similar adaptive implementations [10], [17] is that even by segmenting the data, only one parameter needs to be estimated for each segment, that is, \( \sigma_h^2 \), in comparison to others, where a full covariance matrix (row) needs to be estimated for each segment. Here, the tradeoff is between complexity and non-stationarity, on the one hand, and simplicity and WSS on the other hand.

III. DETECTOR IMPLEMENTATION AND COMPARISON

A. MF Versus Generalized MF (GMF)

To show the advantage of using our model, it is important to compare it with other commonly used methods. However, the lack of a unified database that can be used to compare performances and the use of different metrics to evaluate these performances made the comparison a difficult one. In addition, the lack of appropriate test specimens has made the development and evaluation of detection techniques even more challenging [13]. For the above reasons, it was difficult to compare the performance of our model with other methods. Nevertheless, in [13], it was shown that the use of the colored Gaussian model for the clutter was more consistent than the split spectrum method [10], [11]. Interestingly enough, our model implementation leads to an easily implemented optimal detector, under the assumption of a known defect signal \( A_0 s[n + n_0] \) (\( A_0 \) and \( n_0 \) are known). A comparison of our model with the conventional MF, which is used heavily in similar applications, also in radar and sonar, was completed to show its advantage. Matched filtering uses only the diagonal part of the Toeplitz autocorrelation matrix (assuming the process is normalized, i.e., has zero mean), and thus the noise is considered white. To start, we assume that the signal model is known, the observation window is aligned with the actual defect location (\( n_0 \) is known), and the amplitude is known and positive (\( A_0 > 0 \) and known). In this manner, the problem becomes a classical known deterministic signal in colored Gaussian noise. For this problem, the optimal detector that maximizes the probability of detection for a fixed probability of false alarm can be obtained by using the Neyman–Pearson criterion [25]. Referring to Fig. 1 and (1), the optimal detector decides whether a defect is present (\( H_1 \)) if \( L_G(x) = (p(x; H_1))/(p(x; H_0)) > \gamma \), where \( L_G(x) \) is the likelihood ratio function, \( p(x; H_1) \) and \( p(x; H_0) \) are the PDFs under \( H_1 \) and \( H_0 \), respectively, and \( \gamma \) is a threshold constant. We have

\[
L_G(x) = \frac{1}{(2\pi)^{N/2} \Sigma_{c+s_0^2}^{-1/2}} \exp \left[ -\frac{1}{2} (x - s)^T (C_c + \sigma_0^2 I)^{-1} (x - s) \right]
\]

where \( s = [A_0 s[n_0] A_0 s[n_0 + 1] \cdots A_0 s[n_0 + L - 1]]^T \), and \( L \) is the number of samples that corresponds to the transmitted pulse length. Let \( R = C_c + \sigma_0^2 I \). After simplifying the above expression, we get

\[
T(x) = x^T R^{-1} s_0
\]
where \( T(x) \) is called the test statistic, and \( x_0 = [s[n_0], s[n_0 + 1], \ldots, s[n_0 + L - 1]]^T \). This form of the test statistic is called the GMF [we will call it \( T_{GMF}(x) \)] [25], since we are correlating the received signal \( x \) with a whitening version \( R^{-1}s \) of the transmitted pulse. As discussed before, the difference between the MF and the GMF is the use of the off diagonal terms in \( R \). For the MF, the test statistic becomes the following:

\[
T_{MF}(x) = \frac{x^T s_0}{\sigma_{R}^2}
\]

where \( \sigma_{R}^2 \) is the diagonal element of the matrix \( R \). Since we assumed all parameters are known, we can move \( \sigma_{R} \) to the threshold part of the test and the new test statistic becomes \( T_{MF}(x) = x^T s_0 \). The distributions of the test statistics for both the GMF and MF are the following [25]:

\[
T_{GMF}(x) \sim N(0, s_0^T R^{-1} s_0), \quad \text{under } H_0
\]

and

\[
T_{MF}(x) \sim N(0, s_0^T R s_0), \quad \text{under } H_0
\]

For these test statistics, the performance for the GMF is

\[
P_{FA} = Q \left( \frac{\gamma'}{s_0^T R^{-1} s_0} \right)
\]

\[
P_D = Q \left( Q^{-1}(P_{FA}) - A_0 \right) \frac{s_0^T R^{-1} s_0}{\sqrt{s_0^T R s_0}} \tag{7}
\]

where the threshold can be found as \( \gamma' = Q^{-1}(P_{FA}) (s_0^T R^{-1} s_0)^{1/2} \). \( P_{FA} \) is the probability of the false alarm, \( P_D \) is the probability of detection, and \( Q() \) is the right-tail probability of the standard normal Gaussian. For the MF, it is

\[
P_{FA} = Q \left( \frac{\gamma''}{s_0^T R s_0} \right)
\]

\[
P_D = Q \left( Q^{-1}(P_{FA}) - A_0 \right) \frac{\left(s_0^T s_0 \right)^2}{\sqrt{s_0^T R s_0}} \tag{8}
\]

where the threshold can be found as \( \gamma'' = Q^{-1}(P_{FA}) (s_0^T R s_0)^{1/2} \) for the MF. From (7) and (8), the probability of detection, for a given probability of false alarm depends on \( d_{GMF}^2 = A_0 (s_0^T R^{-1} s_0)^{1/2} \) for the GMF, and \( d_{MF}^2 = A_0 \left(s_0^T s_0 \right)^2 / s_0^T R s_0 \) for the MF, where \( d_{GMF}^2 \) and \( d_{MF}^2 \) are called the deflection coefficients [25]. To quantify the advantage of using our model over the usual MF, we can study the following ratio (which can be interpreted as the signal to noise ratio (SNR) gain of the GMF over MF):

\[
\text{SNR Gain} = \frac{d_{GMF}^2}{d_{MF}^2} = \frac{s_0^T R^{-1} s_0 s_0^T R s_0}{(s_0^T s_0)^2} \tag{9}
\]

where we will use 10LOG10(SNR Gain) dB in accordance with convention. The above ratio is always greater than 1.

To make a qualitative comparison, we computed the autocorrelation matrix \( R = C_c + \sigma_{w}^2 I \) for two different scenarios. First, where the clutter noise dominates the ambient white noise and second, where the ambient white noise dominates the clutter noise. Let clutter to ambient noise (CNR) (dB) = 10LOG10\( (r_c[0]/r_w[0]) \), where \( r_c[0] \) is the total average power of the clutter noise and \( r_w[0] \) is the total average power of the ambient white noise. We know that \( r_c[0] = a E_s \) and \( r_w[0] = \sigma_w^2 \), where \( E_s \) is the energy of the transmitted ultrasonic pulse and \( a = 1/(M^2(M - 1)) \sigma_A^2 \). The term CNR (dB) is what is commonly called clutter to ambient noise ratio. The quantity CNR (dB) indicates which noise is dominating the overall noise for a given transmitted pulse. Even though the matrix \( R \) also depends on the energy of the transmitted pulse \( s[n] \), as mentioned before, we will not pursue that in this work and, without loss of generality in computing (9), we can assume \( E_s = 1 \). It is clear from Fig. 3 that the performance of the GMF surpasses the MF as the ratio CNR increases, which translates into the first scenario where the clutter noise dominates the ambient white noise. As for the second scenario, where the ambient white noise dominates the clutter noise, it is shown in Fig. 3 that the improvement in performance is less, as expected. However, our detector performance always surpasses the MF. Note that in NDT application, the CNR usually falls in the region shown in Fig. 3 to the right, so, we expect an advantage of almost 10 dB for using the GMF over the MF. Next, the receiver operating characteristic (ROC) curve is used as another evaluation tool. We chose the following parameters \( \sigma_A^2 = 0.035 \) and \( \sigma_w^2 = 0.0001 \) (these values were chosen to match the real data). The values of the amplitude reflect cases from low SNR to high SNR. Evidently, our model shows a significant performance advantage in the low SNR region and it improves as the SNR increases as shown in Fig. 4.

B. Practical Detector Implementation-Generalized Likelihood Ratio Test (GLRT)

To apply the detector to the problem at hand, since we need to estimate \( -\infty < A_0 < \infty \) and \( n_0 \). In practice, we have two approaches, the GLRT approach and the Bayesian approach. The GLRT appears to be more widely used for its ease of implementation and less restrictive assumptions [25], and it will be the approach we follow here. The detector that works well in practice and usually maximizes the probability
of detection for a given probability of false alarm for such a problem can be achieved by implementing the GLRT, which determines $\mathcal{H}_1$ if $\ln(L_G(x)) = \ln(p(x, \hat{A}_0, \hat{n}_0, \mathcal{H}_1)/p(x; \mathcal{H}_0)) > \ln(\gamma)$, where $\ln$ indicates the natural logarithm (a monotonically increasing function), $A_0$ and $n_0$ is the maximum likelihood estimator for $A_0$ and $n_0$, respectively, [26]. For that, we get the following results:

$$
\ln L_G(x) = \ln \frac{p(x; \hat{A}_0, \hat{n}_0, \mathcal{H}_1)}{p(x; \mathcal{H}_0)}
\ln L_G(x) = \max_{n_0 \in [0:N-L]} \left\{ \frac{(x^T R^{-1} s_0)^2}{s_0^T R^{-1} s_0} \right\}.
$$

We decide $\mathcal{H}_1$ (a defect is present) if

$$
T(x) = \max_{n_0 \in [0:N-L]} \frac{(x^T R^{-1} s_0)^2}{s_0^T R^{-1} s_0} > \gamma'.
$$

Note to get (10), we used the MLE for the amplitude $\hat{A}_0 = (x^T R^{-1} s_0/s_0^T R^{-1} s_0)$. To get the theoretical detection performance of the above test statistic is a difficult task, since we need to determine the PDF of the maximum of $N - L + 1$ correlated Gaussian random variables and will not be pursued here. The interested reader can refer to [25, p. 260] and [27] for more insights on how to obtain such a PDF. The option to determine the performance by simulation is a practical one and will be followed here. We next summarize the steps to implement our detector.

1. **Estimate $\sigma_A^2$:** This can be done in the frequency domain, if a backwall signal is available or could be attained empirically.
2. **Estimate $\sigma_C^2$:** This can be done empirically by fitting the simulated noise to the measured noise [15], [16]. If $M$ is unknown, we can use the same empirical fitting. Our analysis showed that this is not a difficult task. In addition, our model exhibited robustness even if those parameters were misspecified.
3. **After estimating $\sigma_C^2$, $\sigma_A^2$, and $M$, we can generate our clutter matrix $C_\gamma$, then construct the matrix $R$ and take its inverse to get $R^{-1}$.**
4. **Next, we are ready to implement the GLRT. We estimate $n_0$ by using a grid search for $\hat{n}_0$ (using a sliding window).**
5. **Finally, we use (10) and compare the test statistic to a threshold, which is application-dependent.**

In estimating $n_0$, we can write the sliding window samples as follows

$$s_\xi = [s[\xi] s[\xi + 1] \cdots s[\xi + L - 1]]^T,$$

where $\xi \geq 0$ represents the amount of shift applied to the transmitted signal. It should be borne in mind that the increment $\xi$ can have depends on the shape of the transmitted signal, which we will cover in future work, and the sampling rate. However, the sampling rate in such applications is considered high enough and usually will not cause a problem. A poor choice of the increment of $\xi$ will lead to a degradation of the detector performance since we will miss the maximum correlation point between the deterministic known transmitted signal and the unknown defect reflection signal. In our simulation, an increment of one sample was adopted since the length of the data record was reasonable. If the data record is long, a larger increment with an overlapping window will be more computationally efficient.

In this work, we determined the threshold by simulation, since the real test statistic PDF under $\mathcal{H}_0$ could not be achieved analytically. In practice, samples of defect-free blocks can be first used to get an estimate for the PDF under $\mathcal{H}_0$.

### IV. Experimental Results With Real Data

The A-scans were obtained from 2-in diameter-type 303 austenitic stainless steel cylindrical rods. The rods were heat-treated for 1 h at temperatures of 1350 °C, 1370 °C, and 1387 °C to obtain various grain sizes. In each case, the heat-treated samples were put out rapidly in water. After that, the samples were prepared to simulate a suitable flat-bottom hole. The initial holes had to be made by means of an electro-discharge machine and later enlarged by drilling. The hole dimension was selected so that the reflected ultrasonic signal from both the grain and the flaw would be the same. In order to determine the average grain size, sections were cut along the diameter of the heat-treated samples and were examined myelographically in both the axial and radial directions. The grain size of the heat-treated samples was analyzed by the intercept method. The heat-treated samples resulted in grain sizes of 86, 106, and 160 μm. To increase the accuracy of the estimates, grain count was obtained for two perpendicular line segments for each surface. We focused our simulation only on the heat-treated samples with the grain size of 160 μm, since it is the most difficult case, and there were four samples. The flat-bottom hole with a diameter of 4.22 mm was simulated at a different location from the front face of the cylindrical blocks. The locations are as follows: 29, 36, 43, and 29 mm. The stainless steel samples were placed in a water bath and scanned with a KB-Aerotech Alpha transducer (0.5-in diameter unfocused), where the surface of the flat-bottom hole was perpendicular to the axial transducer field. The transducer has a center frequency of 5 MHz and a Gaussian-shaped spectrum with 2.5-MHz bandwidth. The received signals were then sampled at a sampling rate of 100 MHz. Each A-scan was repeated 200 times and then averaged to attenuate the ambient white noise. Also, the estimated speed of sound in the material was 5790 m/s [17]. Based on the grain size and the center frequency of the transducer, the scatterers fall into the two regions Rayleigh and stochastic, and hence, violate the single scatterer assumption. However, the model still
performed effectively even with the deviation from the single scatter assumption. The data are shown in Fig. 5. Following the steps in Section III-B. For steps 1 and 2, we obtained $\sigma^2 = 0.0001$ and $\sigma_A^2 = 0.035$. Using (2), we obtained $M = 6$. The transmitted pulse $s[n]$ and the spectrum of the transducer has been approximated by a Gaussian shape pulse with 5-MHz center frequency and 2.5-MHz bandwidth. For the duration of the transmitted pulse, we followed the same criterion as in [17]. Since the real data were the same, we obtained a duration that corresponds to $L = 65$ samples at a sampling rate of 100 MHz. The phase of the transmitted pulse was set to zero, since the pulse is assumed to be Gaussian. For more details refer to [17, Section V]. Once we had $M$, $\sigma^2$ and $\sigma_A^2$, we evaluated $C_t$, then $R$ and $R^{-1}$. Next, we estimated $n_0$ and then implemented the GLRT. Results are shown in Fig. 6, where $T_{n_0}(x) = T(x)|_{n=n_0} = (s^T R^{-1}s_0)^{\frac{1}{2}} / (s_0^T R^{-1} s_0)$. The last step is to decide the threshold $\gamma$, which depends on the optimality criterion of interest. We simulated 1000 defect-free realizations, using the estimated parameters. Next, we estimated the PDF under $H_0$ to obtain a threshold. We chose a threshold that gives us a $P_{FA} = 0.001$, which we found to be $\gamma = 64$ (indicated by a red dotted line in Fig. 6). Using this threshold, we decide $H_1$ if $T_{n_0}(x) > \gamma$ and referring back to Fig. 6, we detected all the four defects since $T_{n_0}(x)$ of the four signals was greater than $\gamma = 64$. As a comparison, we show the results of the MF implementation in Fig. 7, which indicates that the MF was not able to detect any of the simulated defects.

V. PARAMETER SENSITIVITY

An advantage of using the proposed model is that even if the value of the average grain size is unknown (which indicates that the number of samples $M$ is unknown), our model is fairly robust to parameter errors. Since $M$ and $\sigma_A^2$, in this case, are estimated empirically (assuming we have an estimate for the ambient white noise variance $\sigma_w^2$), we are expecting to see different combinations of $M$ and $\sigma_A^2$ that lead to an approximate result that match our real data. To that end, we assumed that the average size for the real data was unknown ($M$ is unknown) and we estimated $M$ and $\sigma_A^2$ empirically. The results are shown in Table I, where we followed the steps in Section III-B. For each combination of $M$ and $\sigma_A^2$, the estimate of $n_0$ where the maximum located for all four test signals were at the same location as in Fig. 6. The maximum test statistic values are shown on the right columns of Table I. If we assumed that the true PDF under $H_0$ is the one we estimated in Section IV, then, it is clear from Table I that we were able to detect all four defects when they are compared with the threshold $\gamma = 64$. An explanation for this is that the performance of our model only depends...
on $\alpha$, which depends on the ratio between $M$ and $\sigma^2_A$ rather than their individual values. However, to see the detector performance where $\alpha$ increases, which translate to cases when the single-scatterer assumption is no longer valid, we plotted the deflection coefficient (dB) versus the CNR (dB) as shown in Fig. 8. Note that for NDT applications, the CNR usually falls in the region between 25 and 35 dB.

VI. MULTISCATTERING FOR A MORE COMPLICATED TARGET

In many situations, a defect can have a very complicated geometrical shape, which causes the defect to have more than one scatterer surfaces in one resolution cell. A resolution cell refers to the volume of the scanned material, illuminated by the transducer, within which only one defect scatterer can be identified [17]. In our work, the resolution cell is assumed to be approximately equal to the transmitted ultrasonic pulse $s[n]$ duration, which corresponds to $L = 65$ samples. Since the speed of sound of the tested stainless steel material was 5790 m/s; then, the axial distance of one resolution cell is approximately 2 mm. If the defect axial distance is contained in one resolution cell, this will cause the defect to have two or more scattering surfaces within one resolution cell.

To simulate such a scenario, and since we assumed that the received signals are a distorted and delayed version of the transmitted signal, we added one additional reflected signal, which corresponds to the hypothetical second surface of the defect, to the original signals in Fig. 5. Also, white noise samples have been added and attenuation to the amplitude has been applied. The amplitude is chosen to be 50% of the estimated amplitude of the defect reflected signal that has been estimated for all the four signals in Section IV. The added signal is the approximated transmitted Gaussian shape signal and the shift corresponds to the mid-point where the peak is located (since the Gaussian pulse is symmetric). As for the location, a 1-mm distance translates to 34 samples (the speed of sound is 5790 m/s). However, the location was varied to create the difficult case where the combination of two scatterer surface reflected signals is of the destructive type and not a constructive type to cause the overall defect reflected signal to be attenuated. The results after adding the hypothetically simulated reflected signals from the second scatter surface are shown in Fig. 9. By following the steps in Section III-B and implementing the GLRT, we obtained the results shown in Fig. 10. From Fig. 10, it is clear that the detector still worked well. It was apparent that as we proceeded to simulate the same scenario but with higher amplitude levels that as long as the reflected signal from the first scattering surface dominates the second one, our model still worked well, even with the deviation from the single scattering assumption.
VII. Conclusion

A new physically motivated model for the grain scatter noise (clutter) has been proposed and constructed to aid in the detection of a defect in the material under testing. The proposed model incorporates multiple physical characteristics, such as the average grain size, random orientation, and shape of the grains, and emphasizes the single scatterer assumption (Rayleigh region). It has been shown that the proposed model exhibited a large advantage over the MF by almost 10 dB, especially for applications where the clutter noise dominates the ambient noise. A detector implementation on real ultrasonic data has been carried out and the results indicate the excellent performance of the proposed model. Moreover, the model demonstrates robustness toward the choice of the parameters. The simulation under the multiple scatterer assumption has been carried out and the model still led to good detectability. It is good to bear in mind that even though the model showed excellent results on real ultrasonic data, the underlying assumption focused on the single-scatterer assumption (Rayleigh). Also, the model did not consider the distance-dependent and frequency-dependent situations, which becomes more relevant as the defect size decreases and gets closer to the grain size. Future work will focus on applying the model on defects with different sizes, geometrical shapes, and materials. Also, the case of having two defects. Finally, the test on different materials that have different intrinsic properties such as inhomogeneity level, coarse level, and isotropic level need to be explored.

APPENDIX

First- and Second-Order Statistics and the PSD of the Physical Model

In Figs. 11 and 12, a spike process realization before and after the shift are shown. The intervals are defined as \( I_i = \{ n : iM \leq n \leq iM + (M - 1) \} \) for all \( i \). The model assumptions are stated in Section II-A. We next derive the first- and second-order PMFs.

First-Order PMF:

Consider an arbitrary sample point \( n \) and assume that it lies in \( I_i \) so that \( n = iM + u \), where \( u \) can take on integer values \( 0 \leq u \leq M - 1 \). Note that \( u \) refers to the position from the start of the \( i \)th interval. Then, there will be a spike at \( n \) if either \( iM + \tilde{\zeta}_i + m_0 = n \) (spike at \( n \) is due to the spike position being chosen as \( \tilde{\zeta}_i \) before the shift) or if \( (i - 1)M + \tilde{\zeta}_{i-1} + m_0 = n \) (spike from a previous interval has been shifted to the right into the interval under consideration). These two possibilities are mutually exclusive. Hence, we must have either \( \tilde{\zeta}_i + m_0 = n \) or \( \tilde{\zeta}_{i-1} + m_0 = n + M \). That these events are mutually exclusive follows from subtracting them to yield \( \tilde{\zeta}_i = \tilde{\zeta}_{i-1} - M \), which is not possible. Thus, we have that

\[
P[v[n] = 1] = \sum_{k=0}^{M-1} P[\tilde{\zeta}_{i-1} = u + M - k | m_0 = k] P[m_0 = k].
\]

But, \( \tilde{\zeta}_i \) and \( \tilde{\zeta}_{i-1} \) are both independent of \( m_0 \) so that we have

\[
P[v[n] = 1] = \sum_{k=0}^{M-1} P[\tilde{\zeta}_i = u - k | m_0 = k] P[m_0 = k]
+ \sum_{k=0}^{M-1} P[\tilde{\zeta}_{i-1} = u + M - k | m_0 = k] P[m_0 = k]
= \sum_{k=0}^{M-1} P[\tilde{\zeta}_i = u - k | m_0 = k]
+ \sum_{k=0}^{M-1} P[\tilde{\zeta}_{i-1} = u + M - k | m_0 = k]
= \sum_{k=0}^{M-1} P[\tilde{\zeta}_i = u - k] \frac{1}{M}
+ \sum_{k=0}^{M-1} P[\tilde{\zeta}_{i-1} = u + M - k] \frac{1}{M}
= \frac{1}{M} \left( \sum_{l=0}^{u} P[\tilde{\zeta}_i = l] + \sum_{l=u+1}^{M-1} P[\tilde{\zeta}_{i-1} = l] \right)
= \frac{1}{M} \left( \sum_{l=0}^{u} \frac{1}{M} + \sum_{l=u+1}^{M-1} \frac{1}{M} \right)
= \frac{1}{M}.
\]

Therefore, \( E[v[n]] = 1/M \).

Second-Order PMF:

Since \( v[n] = 1 \) or \( v[n] = 0 \), the second-order moment is \( E[v[n_1]v[n_2]] = P[v[n_1] = 1, v[n_2] = 1] \). Letting \( n_2 = n_1 + \Delta \) for \( \Delta \geq 0 \), we next determine \( P[v[n_1] = 1, v[n_2] = 1] = 1/M \).

First Case: \( \Delta = 0 \). Here, \( n_1 = n_2 \) and therefore

\[
P[v[n_1] = 1, v[n_2] = 1] = P[v[n_1] = 1] = \frac{1}{M}.
\]

Second Case: \( \Delta \geq M \). In this case, \( n_1 \) and \( n_2 \) must be in different intervals as shown by the dashed boxes in Fig. 11. As a result, the spike events must be conditionally independent, that is, for a given \( m_0 \), the spikes within the dashed boxes are independent by Assumption 1. Therefore,

\[
P[v[n_1] = 1, v[n_2] = 1]
= \sum_{k=0}^{M-1} P[u[n_1 - k] = 1, u[n_2 - k] = 1 | m_0 = k] P[m_0 = k]
= \sum_{k=0}^{M-1} P[u[n_1 - k] = 1, u[n_2 - k] = 1] P[m_0 = k]
\]
Consideration for

Fig. 14. Case 3b. Therefore, as a result, we have, finally, that

and \( n_1 + \Delta - m_0 \in I_1 \) and therefore

\[
0 \leq n_1 - m_0 \leq M - 1
\]

\[
M \leq n_1 + \Delta - m_0 \leq 2M - 1
\]

or

\[
-(M - 1) \leq m_0 - n_1 \leq 0
\]

\[
-(2M - 1) \leq m_0 - (n_1 + \Delta) \leq -M
\]

or

\[
n_1 - (M - 1) \leq m_0 \leq n_1
\]

\[
n_1 - (M - 1) + (\Delta - M) \leq m_0 \leq n_1 + (\Delta - M).
\]

But, \((\Delta - M) < 0\) so that the larger of the two lower limits is \( n_1 - (M - 1) \) and the smaller of the two upper limits is \( n_1 + (\Delta - M) \). As a result, we have that \( n_1 - (M - 1) \leq m_0 \leq n_1 + (\Delta - M) \). Also, since it was assumed that \( n_1 \in I_0 \), \( n_1 - (M - 1) \leq 0 \), which becomes the lower limit, and hence

\[
0 \leq m_0 \leq n_1 + \Delta - M.
\]

The upper value is \( \geq 0 \) since \( n_1 + \Delta \geq M \) but also \( \leq M - 1 \) since \( n_1 + \Delta \leq 2M - 1 \). Hence, all possible values of \( m_0 \) determined by the inequality have a probability of \( 1/M \). There are a total of \( n_1 + \Delta - M + 1 \) terms. Next, consider case 3b, which proceeds in a similar manner. Then we must have \( n_1 - m_0 \in I_{-1} \) and \( n_1 + \Delta - m_0 \in I_0 \) and therefore

\[
-M \leq n_1 - m_0 \leq -1
\]

\[
0 \leq n_1 + \Delta - m_0 \leq M - 1
\]

or

\[
1 \leq m_0 - n_1 \leq M
\]

\[
-(M - 1) \leq m_0 - (n_1 + \Delta) \leq 0
\]

or

\[
n_1 + 1 \leq m_0 \leq n_1 + M
\]

\[
(n_1 + \Delta) - (M - 1) \leq m_0 \leq n_1 + \Delta.
\]

But, \( 1 \leq \Delta \leq M - 1 \) so that the upper limit must be \( n_1 + \Delta \). Also, \((n_1 + \Delta) - (M - 1) = n_1 + 1 + (\Delta - M) \leq n_1 + 1 \) so that the lower limit becomes \( n_1 + 1 \). We thus have

\[
n_1 + 1 \leq m_0 \leq n_1 + \Delta.
\]

But, it was assumed that \( n_1 + \Delta \geq M \) and therefore the upper limit must be \( M - 1 \) for a nonzero probability of \( m_0 \). Also, note that \( n_1 \neq 0 \) since \( \Delta \) would need to be \( M \) for \( n_1 + \Delta \in I_2 \).

Therefore, as a result, we have, finally, that \( n_1 + 1 \leq m_0 \leq M - 1 \), and the number of terms is \( M - 1 - (n_1 + 1) + 1 = M - 1 - n_1 \). Combining this with the number of terms for case 3a, which is \( n_1 + \Delta - M + 1 \), yields a total number of terms of \( \Delta \).

Now, we have

\[
P[v[n_1]] = 1, \quad v[n_2] = 1 = P[u[n_1 - m_0] = 1, u[n_2 - m_0] = 1]
\]

\[
= \sum_k P[u[n_1 - k] = 1, u[n_2 - k] = 1|m_0 = k]P[m_0 = k]
\]
\[ \sum_k P[u[n_1-k] = 1, u[n_2-k] = 1]P[m_0 = k] \]
\[ = \sum_k P[u[n_1-k] = 1]P[u[n_2-k] = 1]P[m_0 = k] \]
\[ = \sum_k \frac{1}{M^2} \frac{1}{M} = \frac{\Delta}{M^3}. \]

In summary, we have for the second-order PMF
\[ P[v[n_1] = 1, v[n_1 + \Delta] = 1] = \begin{cases} \frac{1}{M}, & \Delta = 0 \\ \frac{\Delta}{M^3}, & 1 \leq \Delta \leq M - 1 \\ \frac{1}{M^2}, & \Delta \geq M. \end{cases} \]

Note also that it is only dependent on the spacing between samples, and hence, this will produce a WSS random process for \( v[n] \).

**Moments of \( v[n] \):**

As already shown \( E[v[n]] = 1/M \) for all \( n \). To compute the ACS, we have \( r_v[k] = E[v[n]v[n+k]] = 1 \cdot P[v[n] = 1, v[n+k] = 1] \), and from the above result with \( k \) replacing with \( \Delta \), and since \( c_v[k] = r_v[k] - E^2[v[n]] \) then
\[ c_v[k] = \begin{cases} \frac{1}{M} \left( 1 - \frac{1}{M} \right), & k = 0 \\ -\frac{1}{M^2} \left( 1 - \frac{k}{M} \right), & 0 < k \leq M. \end{cases} \]

Hence, the ACS and PSD for the process \( h[n] = a[n]v'[n] \), where \( v'[n] = v[n] - \mu_v \), and \( \mu_v = 1/M \), are
\[ r_h[k] = \begin{cases} \frac{1}{M^2} (M-1) \sigma_A^2, & k = 0 \\ 0, & 1 \leq k \leq M - 1 \\ 0, & k \geq M \end{cases} \]
and
\[ P_h(f) = \frac{1}{M^2} (M-1) \sigma_A^2 \]
where \((-1/2) \leq f \leq (1/2)\).

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