Velocity of Light in Dark Matter with Charge

Ichiro Oda

Department of Physics, Faculty of Science, University of the Ryukyus,
Nishihara, Okinawa 903-0213, Japan.

Abstract

We propose an interesting mechanism to reconcile the recent experiments of the Michelson-Morley type and slowdown of the velocity of light in dark matter with a fractional electric charge when the index of refraction of dark matter depends on the frequency of a photon. After deriving the formula for the velocity of light in a medium with the index of refraction \( n(\omega) \) in a relativistic regime, it is shown that the local anisotropy of the light speed is proportional to the second order in \( n(\omega) - 1 \). This result implies that the experiments of the Michelson-Morley type do not give rise to a stringent constraint on the slowdown of the velocity of light in dark matter with electric charge.

\[1\] E-mail address: ioda@phys.u-ryukyu.ac.jp
1 Introduction

It is nowadays well known that a large portion, about 23 percentages, of the total energy density in the present universe is occupied by unknown material, what we call "dark matter". The particles, of which dark matter is made, are usually thought to be weakly interacting massive ones, in other words, WIMPs, which are absent in the Standard Model (SM). These hypotheoretical particles are supposed to have mass of order ranging from GeV to TeV, and cross section of their annihilation process is comparable to that of the weak interaction. Thus they do not completely annihilate in the course of evolution of the universe and their mass density in the present universe could be of the order of the critical density $\rho_c \approx 0.52 \times 10^{-5} \text{GeV/cm}^3$ (more precisely, $\rho_{DM} \approx 1.2 \times 10^{-6} \text{GeV/cm}^3$ averaged over large distance scales) [1].

The difficulty of terrestrial experiments for detecting dark matter by direct or indirect methods makes it impossible to clarify its physical properties and therefore answer various questions associated with dark matter such as the mechanism of the generation in the early universe. On the other hand, the density of dark matter in clusters of galaxies is determined by various methods of measurement of the gravitational potential. For instance, mass distribution in a cluster is obtained by the method of gravitational lensing. The result is that most of the mass is due to dark matter distributed smoothly over the cluster. Put differently, dark matter, like the usual matter, is more dense in galaxies and it appears as if luminous matter is embedded into the cloud of dark matter of larger size-galactic halo. It is easy to understand this concentration of mass of dark matter near clusters of galaxies from a physical viewpoint since the gravitational interaction attracts dark matter near the clusters compared to empty outer space.

Given our current ignorance of the dark sector in the universe, it seems prudent not to restrict our interests only in WIMPs, though they must be most plausible, and open our mind to many possibilities. Indeed, though the particles consisting of WIMPs carry no electric charge, the notion of charged massive particles (CHAMPs) has also appeared periodically [2]-[6]. Since CHAMPs, which normally carry a full unit of electric charge, receive stringent constraints from searches for heavy hydrogen to direct detection in the underground experiments, the possibility of CHAMPs carrying a fractional charge has been recently investigated [7, 8].

In this article, we shall assume that the earth itself is embedded into a medium of dark matter with a fractional electric charge. Then, it is quite natural to think that the existence of such a dark matter manifests itself the index of refraction $n(\omega)$ depending on the (angular) frequency of light since the dark matter can interact with the gauge field in a direct way [9]. Moreover, we can ask ourselves if the recent experiments of the Michelson-Morley type [11, 12] shed some light on the possibility of detecting the dark matter on the earth through

\[ \text{Precisely speaking, an electrically-neutral particle propagating through matter is also refracted if it couples to a charged particle even indirectly. For instance, a neutrino can couple to an electron via weak interaction and its refractive property has an important effect on neutrino propagation when the neutrino is massive [10].} \]
measuring the local anisotropy of velocity of light or not.

One of motivations behind the present study comes from a recent claim \cite{13} that in case of the OPERA superluminal neutrinos \cite{14}, such an assumption shows a clear tension and might be in conflict with the most recent experiments of the Michelson-Morley type about the local anisotropy of velocity of light \cite{11, 12}. In order to relax this tension, it is useful to recall that the index of refraction of a medium is in general dependent on the frequency of light. If we take account of this fact together with its property under the Lorentz transformation in a proper manner, a term depending on the frequency provides an additional contribution to the velocity of light and then the resulting constraint would become consistent with the experimental results of the Michelson-Morley type.

This article is organized as follows: In the next section, we derive a formula of propagation of light in moving media based on special relativity. In Section 3, it is shown that using this formula the local anisotropy of the light speed is proportional to the second order in $n(\omega) - 1$. Section 4 is devoted to discussion.

2 Velocity of propagation of light in moving media

In this section, we review special relativity \cite{15}, in particular, on the basis of both the addition law of relativistic velocities and the invariance of phase of a plane wave, we shall derive the velocity formula for propagation of light in moving media.

It is known that the Lorentz transformation for inertial coordinates can be used to derive the addition law of relativistic velocities. To show this fact explicitly, let us consider a physical setup where we have a moving point \( P \) whose three-dimensional velocity vector \( \vec{u}' \) has spherical coordinates \((u', \theta', \phi')\) in the inertial frame \( K' \), and the frame \( K' \) is moving with velocity \( \vec{v} = c_l \beta \) in the direction of the \( x_1 \) axis with respect to the inertial reference frame \( K \). Here \( c_l \) is a universal limiting speed, in other words, the maximum speed of all physical entities \cite{16}-\cite{21}.

Then, one wishes to calculate the velocity \( \vec{u} \) of the point \( P \) as seen from the inertial frame \( K \). To do so, let us first note that the Lorentz transformation takes the form for the differential expressions

\[
\begin{align*}
    dx_0 &= \gamma(dx'_0 + \beta dx'_1), \\
    dx_1 &= \gamma(dx'_1 + \beta dx'_0), \\
    dx_2 &= dx'_2, \\
    dx_3 &= dx'_3,
\end{align*}
\]

(1)

where we have defined \( \gamma \equiv \frac{1}{\sqrt{1-(\frac{v}{c_l})^2}} \). Since the components of velocity are defined by \( u'_i = c_l \frac{dx'_i}{dx_0} \) and \( u_i = c_l \frac{dx_i}{dx_0} \), they transform as

\[
\begin{align*}
    u_1 &= \frac{u'_1 + v}{1 + \frac{vu'_1}{c_l^2}}, \\
    u_2 &= \frac{u'_2}{\gamma(1 + \frac{vu'_1}{c_l^2})}, \\
    u_3 &= \frac{u'_3}{\gamma(1 + \frac{vu'_1}{c_l^2})}.
\end{align*}
\]

(2)
For later convenience, let us rewrite this transformation to a more general form

\[ u_|| = \frac{u'_|| + v}{1 + \frac{v_1 c}{c^2}}, \quad \vec{u}_\perp = \frac{\vec{u}'_\perp}{\gamma(1 + \frac{v_1 c}{c^2})}. \]  

(3)

where \( u_|| \) and \( \vec{u}_\perp \) indicate the components of velocity parallel and perpendicular to \( \vec{v} \), respectively. Moreover, it turns out that the magnitude \( u \) of \( \vec{u} \) and its polar angles in the two inertial frames are related as follows:

\[
\begin{align*}
\phi &= \phi', \\
\tan \theta &= \frac{u' \sin \theta'}{\gamma(u' \cos \theta' + v)}, \\
u &= \frac{\sqrt{u'^2 + v^2 + 2u'v \cos \theta' - \left(\frac{w' v \sin \theta'}{c_1}\right)^2}}{1 + \frac{w' v}{c_1} \cos \theta'}. 
\end{align*}
\]  

(4)

The inverse transformation for \( \vec{u}' \) in terms of \( \vec{u} \) can be easily obtained by interchanging primed and unprimed quantities and simultaneously changing the sign of \( v \).

In what follows, for simplicity, let us focus on the case of the parallel velocities, \( \theta' = 0 \). In this case, the magnitude of \( u \) takes the form

\[ u = \frac{u' + v}{1 + \frac{v_1 c}{c^2}}. \]  

(5)

If one particularly sets \( u' = c_l \), this expression reads \( u = c_l \), which simply means the postulate of a universal limiting speed, which is an alternative fundamental principle for the principle of invariant speed of light in special relativity [16, 17, 18, 19].

Next, let us choose \( u'_|| = \frac{\omega'}{n(\omega')} \), thereby giving rise to

\[ u_|| = \frac{\omega + v}{1 + \frac{v}{n(\omega') c_l}}. \]  

(6)

where \( \omega' \) is the (angular) frequency of light in the inertial frame \( K' \) and \( n(\omega') \) is the index of refraction of a medium. The important point is that the index of refraction of a medium in general depends on the magnitude of the frequency of the photon.

In order to determine \( n(\omega') \), we make use of the fact that the phase of a plane wave is an invariant quantity under the Lorentz transformation since the phase \( \phi \) can be identified with the mere counting of wave crests in a wave train, an operation that must be the same in every inertial frame

\[ \phi \equiv k_\mu x^\mu = \omega t - \vec{k} \cdot \vec{x} = \omega' t' - \vec{k}' \cdot \vec{x}'. \]  

(7)

With the frequencies \( \omega = c_l k_0 \) and \( \omega' = c_l k'_0 \), the Lorentz transformation for the wave-number vector \( k_\mu \) reads

\[
\begin{align*}
k'_0 &= \gamma(k_0 - \vec{\beta} \cdot \vec{k}), \\
k'_|| &= \gamma(k_|| - \beta k_0), \\
\vec{k}'_\perp &= \vec{k}_\perp.
\end{align*}
\]  

(8)
For light waves, $|\vec{k}| = k_0$ and $|\vec{k}'| = k'_0$, so using $\theta = 0$ coming from $\theta' = 0$ via Eq. (4) we have the relation

$$\omega' = \gamma (1 - \beta) \omega = \sqrt{\frac{1 - \beta}{1 + \beta}} \omega \approx (1 - \beta) \omega,$$

where the last relation holds when $\beta \equiv \frac{v}{c_l}$ is small. Thus, the index of refraction of a medium transforms to first order in $\beta$ as

$$n(\omega') = n(\omega) - \beta \omega \frac{dn}{d\omega}.$$  

Equation (10)

Plugging this expression into Eq. (6) (and taking $\pm v$ into consideration at the same time), it is easy to show that for medium flow at a speed $v$ parallel or antiparallel to the path of the light, the speed of the light, as observed in the laboratory ($K$-frame), is given to first order in $v$ by

$$u_{||}^{\pm} \approx \frac{c_l}{n(\omega)} \pm v \left[ 1 - \frac{1}{n(\omega)^2} + \frac{\omega}{n(\omega)^2} \frac{dn(\omega)}{d\omega} \right],$$

where superscripts $+$ and $-$ on $u_{||}$ denote the speed parallel and antiparallel to the light path, respectively. Formula (11) is the main result of this section and will be used in the next section.

### 3 The recent experiments of Michelson-Morley type

We wish to apply the formula (11) to the problem that the recent experiments of the Michelson-Morley type could or could not give us useful information on detection of dark matter with electric charge.

Before doing that, let us present some ideas behind our theory. Search for dark matter is underway, but no conclusive evidence has been obtained thus far. A direct way for the search is carried out in experiments trying to detect energy deposition in a detector caused by elastic scattering of dark matter off a nucleus. Besides the direct search, there are experiments to search for dark matter particles indirectly, which include the search for products of dark matter annihilation. One of the most promising indirect ways is to search for monoenergetic photons which are emitted in two-body annihilation processes $X + \bar{X} \to \gamma + \gamma, X + \bar{X} \to Z^0 + \gamma$ where $X, \bar{X}$ describe a dark matter particle and its antiparticle, respectively. By the crossing symmetry, these processes read $X + \gamma \to X + \gamma, X + \gamma \to X + Z^0$. If the photon energy is small compared to the weak energy scale (i.e., the energy of the photon beam is below the threshold for production of a single $Z^0$), only the former process is allowed to occur and it is nothing but the elastic Compton scattering.

At this stage, let us recall that there is a well-known connection between the index of refraction and the Compton scattering amplitude where the standard formula takes the form...
\[ n(\omega) = 1 + \frac{2\pi c^2 N F(\omega)}{\omega^2}, \]  
\[(12)\]

where \( N \) is the number of scattering centers per unit volume and \( F(\omega) \) is the forward Compton scattering amplitude for the photon scattering off the medium which is a function of the (angular) frequency \( \omega \) of a photon. Put differently, in this approach the dark matter with electric charge plays a role of a medium for the photons and provides a dispersive effect on light propagation.

Since we assume in this article that dark matter carries a fractional electric charge associated with \( U(1)_{EM} \) symmetry, it must be a stable Dirac particle with spin \( \frac{1}{2} \) like protons. Then, the amplitude for the forward scattering of a photon from a polarization state \( \vec{e}_1 \) to the one \( \vec{e}_2 \) by the dark matter must be of the general form \[ F(\omega) = \psi_f^* \left[ F_1(\omega) \vec{e}_2^* \cdot \vec{e}_1 + i F_2(\omega) \vec{\sigma} \cdot (\vec{e}_2^* \times \vec{e}_1) \right] \psi_i, \]  
\[(13)\]

where \( \psi_i(\psi_f) \) is the wave function of the initial (final) dark matter and \( \vec{\sigma} \) is the spin matrix of the dark matter. In general, both \( F_1(\omega) \) and \( F_2(\omega) \) are complex and have dispersive and absorptive parts. If one averages over dark matter spins in the amplitude one is left only with \( F_1(\omega) \). The amplitudes \( F_1(\omega) \) and \( F_2(\omega) \) are separable if one can do experiments with polarized dark matters: \( F_1(\omega) \) corresponds to parallel and \( F_2(\omega) \) to perpendicular linear polarization vectors of the initial and final photons, respectively. Since we are interested in the spin-averaged forward amplitude, we will focus on only \( F_1(\omega) \) henceforth. (We therefore define \( F = F_1 \).)

With the assumption of causality and analyticity for the forward scattering amplitude \( F(\omega) \), the once-subtracted dispersion relation is given by

\[ \text{Re} F(\omega) = \text{Re} F(0) + \frac{2\omega^2}{\pi} P \int_{\omega_0}^{\infty} \frac{d\omega'}{\omega'} \frac{\text{Im} F(\omega')}{\omega'^2 - \omega^2}, \]  
\[(14)\]

where \( P \) denotes the principal value and \( \omega_0 \) is the threshold for producing a single dark matter which is approximately taken to be \( \omega_0 \approx M_X c_t^2 \approx 100 GeV \). Using the optical theorem, the imaginary part of the forward elastic scattering amplitude is related to the total cross section \( \sigma(\omega) \) by

\[ \sigma(\omega) = \frac{4\pi c_t}{\omega} \text{Im} F(\omega). \]  
\[(15)\]

Furthermore, it is known that the forward elastic scattering amplitude at \( \omega = 0 \) is real and given by the Thomson formula

\[ \text{Re} F(0) = -\frac{(ee)^2}{M_X c_t^2}, \]  
\[(16)\]
where $\varepsilon e$ and $M_X$ are respectively the charge and mass of the dark matter particle. Thus, one can rewrite $ReF(\omega)$ in Eq. (14) to the form

$$ReF(\omega) = -\frac{(\varepsilon e)^2}{M_X c_t^2} + \frac{\omega^2}{2\pi^2 c_t} P \int_{\omega_0}^{\infty} d\omega' \frac{\sigma(\omega')}{\omega'^2 - \omega^2}.$$  

(17)

Substituting this expression into the standard formula (12), we obtain

$$Ren(\omega) - 1 = -\frac{2\pi N(\varepsilon e)^2}{M_X} \frac{1}{\omega^2} + \frac{c_t N}{\pi} P \int_{\omega_0}^{\infty} d\omega' \frac{\sigma(\omega')}{\omega'^2 - \omega^2}.$$  

(18)

Since $Imn(\omega)$ does not appear any more and $n(\omega)$ in the previous section is in fact equivalent to $Ren(\omega)$ in this section, for simplicity we rewrite $Ren(\omega)$ as $n(\omega)$ from now on.

Now let us consider the case of the low photon energy of $\omega \ll \omega_0$. It is then sufficient to limit ourselves to an expansion up to the constant order in $\omega^2$

$$\delta(\omega) \equiv n(\omega) - 1 \approx \frac{\delta_0}{\omega^2} + \delta_2,$$  

(19)

where $\delta_0, \delta_2$ are defined as

$$\delta_0 = -\frac{2\pi N(\varepsilon e)^2}{M_X}, \quad \delta_2 = \frac{c_t N}{\pi} P \int_{\omega_0}^{\infty} d\omega' \frac{\sigma(\omega')}{\omega'^2}.$$  

(20)

Note that observations of $\gamma$-ray burst (GRB) emission from radio energies to tens of GeV [24] require us that the energy dependence of the speed of the photon is so small that $\delta_0$ must be very tiny. Moreover, the recent ICARUS result [25] implies that $\delta_2$ are very small as well.

However, this smallness is not a real problem for the present purpose. The key point is that if we regard dark matter as a kind of usual medium for light, the frequency dependence must be always given by (19) at least for the low energy photons. Then, an important relation, which will be utilized shortly, can be obtained

$$\omega \frac{dn(\omega)}{d\omega} = -2(n - 1 - \delta_2).$$  

(21)

Next, following the procedure in Ref. [13], let us calculate the local anisotropy of the speed of light whose experimental upper bound has been already obtained in the experiments of the Michelson-Morley type [11, 12]

$$\left(\frac{\Delta c}{c}\right)_{Exp} \approx 1 \times 10^{-17}.$$  

(22)

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3In the most recent Michelson-Morley experiments [11, 12], photons with the order $100THz$ are utilized, which corresponds to the order $1eV$.

4Expanding in powers of $\omega$ is known to be a useful tool for obtaining various interesting low-energy cross sections in the analysis of the Compton scattering [23].

5Although $E = h\omega$ corresponds to the energy of a photon, we will regard it as a typical energy scale controlling the system under consideration, in other words, the energy of the ICARUS neutrinos.
In these experiments, the parallel average velocity is measured

\[ c_{||} = \frac{2L}{\frac{u_{||}}{u_{||}} + \frac{u_{||}}{u_{||}}}. \]  

(23)

Using Eqs. (11) and (21), this quantity can be evaluated to be

\[ c_{||} = \frac{c_l}{n} - \frac{v^2}{c_l} \frac{[(n - 1)^2 + 2\delta_2]^2}{n^4}. \]  

(24)

Then, the theoretical local anisotropy of the speed of light is calculated to be

\[ \frac{\Delta c}{c} = \frac{\frac{c_l}{n(\omega)} - c_{||}}{\frac{c_l}{n(\omega)}} \approx 4 \left( \frac{v}{c_l} \right)^2 \delta_2^2 \approx 4 \left( \frac{v}{c_l} \right)^2 \delta^2, \]  

(25)

where we have used \( \delta \approx 0, \beta \ll 1 \) and \( \delta \approx \delta_2 \). It is worthwhile to notice that compared to the result in Ref. [13] which is proportional to the linear order in \( \delta \), our result (25) becomes proportional to the second order in \( \delta \) since we have taken account of the frequency dependence and the Lorentz transformation of the index of refraction \( n(\omega) \).

In order to show concretely that Eq. (25) yields a very weak constraint on the local anisotropy of the speed of light, let us apply it to the OPERA result [14] together with the SN1987A result [26]. Here note that the OPERA result has been recently refuted by the ICARUS group [25]. The reason why we use the OPERA result is two-fold. First, Ref. [13] has discussed the OPERA result and shown that there is some tension between the recent Michelson-Morley experiments and the models based on CHAMPs owing to a large value of \( \delta \) in the OPERA result. But we wish to show that even in this large value of \( \delta \) there appears no tension between them since (25) becomes proportional to the second order in \( \delta \) in our calculation. And the second reason is that even if the OPERA main result, in particular the size of \( \delta \) might be wrong, the other result, i.e., \( \delta \) is independent of the neutrino energy, is consistent with that of GRB emission [24] and seems to be true.

The result of SN1987A implies that the velocity of a neutrino is almost the same as a universal limiting speed

\[ v_\nu \approx c_l. \]  

(26)

Then, we can obtain the following relation

\[ \frac{v_\nu - \frac{c_l}{n}}{\frac{c_l}{n}} \approx \frac{c_l - \frac{c_l}{n}}{\frac{c_l}{n}} = n - 1 = \delta \approx 2.37 \times 10^{-5}, \]  

(27)

where we have used the OPERA results in the last step. The OPERA result suggests that \( \delta \) is independent of the neutrino energy, so this equation implies

\[ \delta_2 \approx \delta \approx 2.37 \times 10^{-5}. \]  

(28)
Substituting this value and the equatorial rotation speed of the earth at the observed place, \( \beta \equiv \frac{\nu}{c_l} = 1.55 \times 10^{-6} \) [13] into Eq. (25), one arrives at the result

\[
\left( \frac{\Delta c}{c} \right)_{\text{OPERA}} \approx 4.6 \times 10^{-21}.
\] (29)

Note that this value is much smaller than the experimental upper bound (22), so it is not in conflict with the recent experimental results of the Michelson-Morley type. Of course, our result does not refute the result obtained in Ref. [13] directly since in our approach at hand the index of refraction depends on the frequency of a photon whereas in the approach considered in Ref. [13] the index of refraction is assumed to be independent of the photon frequency. Actually, when dark matter does not carry an electric charge like a neutrino, Eq. (20) means \( \delta_0 = 0 \), thereby making it impossible to get Eq. (21) owing to \( \omega \frac{dn(\omega)}{d\omega} = 0 \).

For completeness, let us move on to a general case of the arbitrary photon energy. In this case, we can proceed the argument in a perfectly similar way to the case of the low energy photons. The relation (21) is now replaced with

\[
\omega \frac{dn(\omega)}{d\omega} = -2(n - 1 - \tilde{\delta}),
\] (30)

where \( \tilde{\delta} \) is defined as

\[
\tilde{\delta} = \frac{c_l N}{\pi} P \int_{\omega_0}^\infty d\omega' \frac{\omega'^2 \sigma(\omega')}{(\omega'^2 - \omega^2)^2}.
\] (31)

Then, it turns out that local anisotropy of the speed of light is exactly evaluated to be

\[
\frac{\Delta c}{c} = \left( \frac{\nu}{c_l} \right)^2 \left[ \frac{2\tilde{\delta}}{n} + \frac{n^2}{n^2} \right]^2.
\] (32)

With the reasonable assumptions as before, \( \delta \equiv n - 1 \approx 0, \beta \ll 1 \) and \( \delta \approx \tilde{\delta} \), we can obtain a similar result to Eq. (25)

\[
\frac{\Delta c}{c} \approx 4 \left( \frac{\nu}{c_l} \right)^2 \tilde{\delta} \approx 4 \left( \frac{\nu}{c_l} \right)^2 \delta^2.
\] (33)

Compared to the low energy case, a slight modification here appears in the \( \tilde{\delta} \) which is dependent on the photon frequency as can be seen in (31). However, except this fact, the essential feature remains unchanged so the conclusion obtained in the case of the low energy photons still holds even in this general case.

### 4 Discussion

Since the introduction of dispersion relations into elementary particle physics, originally within the context of quantum field theory [22], a large number of literatures have grown up on their
theoretical basis, on extensions and applications to new processes, and on their comparison with experiment with great success. The most advantageous point is that dispersion relations are very universal in the sense that they are formulated only on the basis of fundamental principles of quantum field theory such as causality, the Lorentz invariance and analyticity.

As a simple application of the dispersion relations, in this article, we have studied the velocity of light in dark matter with a fractional electric charge and found that the recent experiments of the Michelson-Morley type [11, 12] do not provide a stringent condition on the slowdown of the velocity of light if the index of refraction of dark matter depends on the frequency of a photon. The experiments of the Michelson-Morley type have played a critical role in proving the non-existence of the aether, i.e., a medium that was once supposed to fill all space and to support the propagation of electromagnetic waves, and might reveal a breakdown of the Lorentz invariance at the Planck scale in future. But they do not seem to shed some light on the change of the velocity of light in dark matter with charge. Perhaps, if the ICARUS result is correct, this situation will not change even in the future experiments of the Michelson-Morley type.\(^6\)

Nevertheless, the experiments using the photons will provide important insight into the properties of CHAMPs in future. For instance, polarized Compton scattering will yield information on spin-structure of the dark matter. Moreover, if one increases the energy of the incoming photon beam, one can probe the internal structure of the dark matter [23].

Finally, we wish to close this article by mentioning two future problems to be solved. An interesting application of the refractive effect in our model is calculation of the force acting on a macroscopic body when dark matter flux passes through. Simple calculation turns out to give a force proportional to \(|n(\omega) - 1|\) which could be measured in the Eotvos-Dicke experiment. Furthermore, the refractive effect in a high-temperature background such as in the early universe deserves study in future. It is of interest to notice that the presence of the electric charge might ensure the stability of the dark matter from another angle.

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