Orientation of cosmic web filaments with respect to the underlying velocity field

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ABSTRACT
The large-scale structure of the Universe is characterised by a web-like structure made of voids, sheets, filaments, and knots. The structure of this so-called cosmic web is dictated by the local velocity shear tensor. In particular, the local direction of a filament should be strongly aligned with $\hat{e}_3$, the eigenvector associated with the smallest eigenvalue of the tensor. That conjecture is tested here on the basis of a cosmological simulation. The cosmic web delineated by the halo distribution is probed by a marked point process with interactions (the Bisous model), detecting filaments directly from the halo distribution (P-web). The detected P-web filaments are found to be strongly aligned with the local $\hat{e}_3$; the alignment is within $30^\circ$ for $\sim 80\%$ of the elements. This indicates that large-scale filaments defined purely from the distribution of haloes carry more than just morphological information, although the Bisous model does not make any prior assumption on the underlying shear tensor. The P-web filaments are also compared to the structure revealed from the velocity shear tensor itself (V-web). In the densest regions, the P- and V-web filaments overlap well (90\%), whereas in lower density regions, the P-web filaments preferentially mark sheets in the V-web.

Key words: methods: statistical – methods: N-body simulations – large-scale structure of Universe.

1 INTRODUCTION
The large-scale matter distribution in the Universe represents a complex network of structure elements such as voids, filaments, sheets and knots, forming the so-called cosmic web (Jöeveer, Einasto & Tago 1978; Bond, Kofman & Pogosyan 1996). The cosmic web has attracted the attention of both observers and theoreticians and numerous studies have attempted to provide a quantitative description of the cosmic web, trying to translate the visual impression into rigorous mathematical algorithms. Motivation for constructing such methods ranges from the search for mathematical measures of the large scale structure that can be used as a discriminator between alternative cosmological models to the desire for a framework within which the environmental dependence of structure formation can be studied.

Currently, the studies concentrating on the large-scale environmental effects usually make implications simply from the density field (e.g., Blanton et al. 2005; Tempel et al. 2011; Lietzen et al. 2012), while various indications argue for a more intricate connection (Lee & Lee 2008). For example, it is known that the spin of dark matter (DM) haloes is correlated with the underlying web elements (e.g., Codis et al. 2012; Libeskind et al. 2012, 2013a) and there is observational evidence for the alignment of the rotation axes of galaxies along galaxy filaments (e.g., Lee & Erdogdu 2007; Tempel et al. 2013a). A more thorough insight into these relations requires definition algorithms for the large-scale structure.

Broadly speaking, web classifying algorithms follow one of two main streams. One is a classification based on the point process manifested by the distribution of galaxies or clusters of galaxies, treated as point objects. The other is based on the dynamics of the underlying density field and the velocity field it induces. This was pioneered by Hahn et al. (2007) who used the tidal tensor of the underlying mass distribution to classify the web (see also Forero-Romero et al. 2009). Hoffman et al. (2012) and Libeskind et al. (2012) classified the cosmic web by studying the velocity shear tensor of the underlying mass distribution, by looking at the number of the eigenvalues of the shear tensor above a threshold. This so-called V-web algorithm has been shown to improve the dynamical resolution with respect to the web based on the tidal tensor, enabling the classification of structures on the scale of few tens of kiloparsecs.
The V-web has also been used by Libeskind et al. (2013b) to quantify vorticity (the anti-symmetric part of the velocity deformation tensor) and its relationship to halo spins. A number of papers have dealt with the myriad other ways (principally geometric, rather than dynamical) to characterise the web (e.g. see Novikov et al. 2006; Aragón-Calvo et al. 2007; Sousbie 2011; Shandarin et al. 2012; Cautun et al. 2013).

The Bisous model (i.e. the marked point process with interactions, Stoica et al. 2005) provides a powerful tool for the construction of a network of filaments from the distribution of galaxies. The algorithm operates directly on the point-like distribution of galaxies, or their DM halo counterparts, without any explicit reference to the underlying dynamics in general, and the velocity shear tensor in particular. (From here on it is dubbed as the P-web). The V-web’s working hypothesis is that the shear tensor is the main driver that shapes the cosmic web. In particular, the model firmly predicts that the direction of filaments should be strongly aligned with the direction of $\hat{e}_3$, the eigenvector corresponding to the smallest eigenvalue of the shear tensor. This is a local relation between the two directions, namely it should be obeyed at any position on the filaments.

The aim of our paper is to test the filament-$\hat{e}_3$ hypothesis by studying a high-resolution DM-only ΛCDM simulation. The simulation will be probed by a halo finder, and the detected haloes will in turn be probed by a P-web finder. The velocity shear tensor is constructed on a cartesian grid by Clouds-in-Cells interpolation scheme; the tensor is diagonalised on each grid cell. The alignment of the filaments and the local direction of $\hat{e}_3$ are then compared.

A side project pursued here is to compare the filaments constructed by the P- and the V-web. However, a comment of caution is due here. The cosmic web is an ill-defined structure. A close visual inspection reveals a smooth transition from knots to filaments, from filaments to sheets, and eventually from sheets to voids. There is no clear-cut principle that separates these classes and virtually all web finders have one or more free parameters that dictate the transition from one class to the other. Given the very different nature of the P- and the V-web algorithms one should not expect a high level of overlap of the two kinds of filament, even if they share a common alignment.

## 2 DATA AND METHODS

### 2.1 N-body simulation

A DM-only N-body cosmological simulation is run assuming the standard ΛCDM concordance cosmology (e.g. WMAP5, 2006). The simulation is probed by a halo finder, and the detected haloes will in turn be probed by a P-web finder.
in particular a flat universe with cosmological constant density parameter \( \Omega_\Lambda = 0.72 \), matter density parameter \( \Omega_m = 0.28 \), a Hubble constant parameterised by \( H_0 = 100 \, h \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) (with \( h = 0.7 \)), a spectral index of primordial density fluctuations given by \( n_s = 0.96 \), and mass fluctuations given by \( \sigma_8 = 0.817 \).

The simulations span a box of side length 64 \( h^{-1}\text{Mpc} \) with 1024\(^3\) particles, achieving a mass resolution of \( \sim 1.89 \times 10^7 \, h^{-1} \, M_\odot \) and a spatial resolution of 1 \( h^{-1} \, \text{kpc} \). The publicly available halo finder AHF \citep{Gill2004, Knollmann2009} is run on the particle distribution to obtain a halo catalogue. AHF identifies haloes and subhaloes in the simulation by searching the particle distribution for local density maxima and checking that particles within the virial radius are gravitationally bound to the host structure. Substructures are identified as haloes whose centres are located within the virial radius of a more massive parent halo. Only haloes more massive than \( 10^8 \, h^{-1} \, M_\odot \) are considered in this work.

The initial conditions of the simulation were constrained using the Hoffman-Ribak algorithm \citep{Hoffman1992} to reproduce the correct environment \citep[on scales of \( \sim 10 \, \text{Mpc} \)]{Libeskind2010} of the local group \citep[see][]{Libeskind2010}. The salient aspects are highlighted here, in brief. The cosmic velocity field is calculated using a “Clouds-in-Cell” (CIC) algorithm on a 256\(^3\) grid. The cell size is thus \( 250 \, h^{-1} \, \text{kpc} \) and the number of grid cells is chosen to be the finest mesh which ensures each cell contains at least one particle. The velocity (and density) fields are then smoothed with a gaussian kernel equal to at least one cell \citep[i.e. \( \text{r}_{\text{smooth}} = 250 \, h^{-1} \, \text{kpc} \)]{Tempel2013b} in order to get rid of the spurious artificial cartesian grid introduced by the CIC. In practice the smoothing sets the scale of the calculation and we use two smoothings throughout this paper: 500 and 1000 \( h^{-1} \, \text{kpc} \). The velocity shear tensor is defined as \( \Sigma_{\alpha\beta} = \frac{1}{2} \left( \frac{\partial v_\beta}{\partial x_\alpha} + \frac{\partial v_\alpha}{\partial x_\beta} \right) \) and is calculated by means of FFT in \( k\)-space. The velocity shear tensor is then diagonalised and its eigenvectors and eigenvalues are identified. Note that the velocity shear field is identical to the tidal field, defined as the Hessian of the potential, namely \( \nabla^2 \phi = \frac{\partial^2 \phi}{\partial r_\alpha \partial r_\beta} \), when smoothed on large enough \( (i.e. > 1 \, \text{Mpc}) \) scales.

In addition to being a universal characteriser of the velocity field, the velocity shear tensor can be used also for classifying the cosmic web. The eigenvector corresponding to the greatest eigenvalue of the shear tensor denotes the direction along which material is collapsing fastest (or expanding slowest), similarly for the intermediate and minor eigenvalues. In this way a web classification can be carried out by simply counting the number of axes that are collapsing: 0, 1, 2 or 3 for voids, sheets, filaments or knots, respectively. An axis is said to be “collapsing” if its eigenvalue is greater than some threshold (chosen to be 0.5 so as to accurately reproduce the visual impression of the cosmic web). Note that filaments are defined by two collapsing axes and an expanding one. The expanding axis thus has the lowest eigenvalue and corresponds to the orientation of the filament, being identical to the \( e_3 \) vector of the V-field.

### 2.2 Velocity shear tensor – the “V-web”

As mentioned in the introduction, the cosmic web can be quantified by means of the velocity shear tensor. This method is described in detail \citep{Hoffman2012, Libeskind2012, Libeskind2013b}. The salient aspects are highlighted here, in brief. The cosmic velocity field is calculated using a “Clouds-in-Cell” (CIC) algorithm on a 256\(^3\) grid. The cell size is thus 250 \( h^{-1} \, \text{kpc} \) and the number of grid cells is chosen to be the finest mesh which ensures each cell contains at least one particle. The velocity (and density) fields are then smoothed with a gaussian kernel equal to at least one cell \citep[i.e. \( \text{r}_{\text{smooth}} = 250 \, h^{-1} \, \text{kpc} \)]{Tempel2013b} in order to get rid of the spurious artificial cartesian grid introduced by the CIC. In practice the smoothing sets the scale of the calculation and we use two smoothings throughout this paper: 500 and 1000 \( h^{-1} \, \text{kpc} \). The velocity shear tensor is defined as \( \Sigma_{\alpha\beta} = \frac{1}{2} \left( \frac{\partial v_\beta}{\partial x_\alpha} + \frac{\partial v_\alpha}{\partial x_\beta} \right) \) and is calculated by means of FFT in \( k\)-space. The velocity shear tensor is then diagonalised and its eigenvectors and eigenvalues are identified. Note that the velocity shear field is identical to the tidal field, defined as the Hessian of the potential, namely \( \nabla^2 \phi = \frac{\partial^2 \phi}{\partial r_\alpha \partial r_\beta} \), when smoothed on large enough \( (i.e. > \text{few Mpc}) \) scales.

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### 2.3 Point Processes – the “P-web”

We apply an object point process with interactions \citep[the Bisous process]{} to trace the filamentary network in the distribution of haloes. The method is applicable to an observed galaxy distribution as well as dark haloes in simulations – it just requires the spatial coordinates of the objects. The morphological and quantitative characteristics of the initially complex geometry of the filamentary network is obtained by sampling the probability density of the detected structures and by applying the methods of statistical inference. We use the Metropolis-Hastings (MH) algorithm to sample the model probabilities. A thorough explanation of the method can be found in \citep{Stoica2007, Stoica2010}. A realisation of the method used in this work is described in full detail \citep{Tempel2013b}.

In brief, the number density distribution of objects is probed by randomly placed segments \citep[in the present case by short thin cylinders]. The likelihood of a cylinder is considered higher, if being linked with another segment, thus forming an element of a filament.

After a large number of repetitions of the MH algorithm, a network of filaments emerges, where each filament being labelled with coordinates, direction, and detection probability. Altogether, we run 50 simulations and construct a filament detection probability field and the orientation field. Based on these fields, we extract the filament axes as described in \citep{Tempel2013b}. A movie, illustrating the Bisous process, the Metropolis-Hastings sampling, and the detection of filament axes is available at http://www.aai.ee/~elmo/PV-web/.
The advantages of this method are manifold. It is insensitive to observational systematics introduced by survey geometry and selection effects and is capable of recovering poorly sampled structures. Besides, since the Bisous model assigns a probability to each detected structure, we are directly supplied with the reliability of the proposed filaments.

The method requires a fixed scale for the filament elements. In the present study, we seek filaments at two scales: with radii in order of \( r = 0.5 \) and \( r = 1.0 \, h^{-1}\text{Mpc} \) – the scales roughly corresponding to galaxy groups. The detected filaments can be seen as bridges between galaxy clusters.

3 RESULTS

3.1 Filaments based on point process and velocity field

In order to illustrate and quantify the P- and V-web filaments, the density field of a thin (2 and 4 \( h^{-1}\text{Mpc} \) for upper and lower panels, respectively) slice of the whole computational box is shown in Fig. 1 (left panels), with a Gaussian smoothing with a 0.5 and 1.0 \( h^{-1}\text{Mpc} \) kernel for the upper and the lower panel, respectively. The detected P-web filaments are shown at two scales: \( r = 0.5 \) and \( r = 1.0 \, h^{-1}\text{Mpc} \) filaments in the upper and the lower panels, respectively. Visually, the filaments appear to delineate the underlying density field well. Some of the filaments are detected at both scales, others not. In places where the density field exhibits broader structures, only filaments of the scale \( r = 1.0 \, h^{-1}\text{Mpc} \) are detected and vice versa.

The middle panels in Fig. 1 show the web elements as found by the V-web and the axes of the filaments detected in the P-web. The size of the smoothing kernel of the velocity field corresponds to the scale of the P-web filaments and the P-web. The size of the smoothing kernel of the velocity field can be seen as bridges between galaxy clusters.

The upper panel in Fig. 2 shows the volume/mass filling fraction for the V-web elements and the P-web filaments. Since the plot is shown as a function of density, the volume and mass filling fractions are one and the same. We see that from V-web, filaments are more likely detected in higher density regions, as illustrated also by Libeskind et al. (2012) and Fig. 1, whereas the chances of finding filaments with the P-web are much less dependent on the underlying density. Interestingly, the maximum volume filling fraction of both classification methods occurs at the same density level, namely 10\( \rho_{\text{mean}} \), although filaments exist at all density levels.

The lower panel of Fig. 2 shows the fractions of the P-web filaments detected as different kinds of V-web elements. We note that these fractions follow rather closely the volume/mass filling fraction of the V-web filaments. This is not surprising, since filaments in the halo distribution are also found in lower density environments, where the fraction of V-web filaments is very low. P-web filaments are detected also in low density environments because the classification/detection based on point process is built to be independent of the number density and the procedure seeks defined structures. In general, the volume/mass filling fraction of P-web filaments is less density dependent, indicating that filaments with some fixed scale exist everywhere.

The filament finder based on the point process also detects filaments that are located in V-web sheets: if a sheet has a filament-like overdensity, the P-web will detect it as a filament. On the other hand, since the transition from sheets to filaments is smooth, it is hard to make difference between filaments and sheets using the velocity field alone.

The differences described above make it difficult to compare these two web classification methods directly. Clearly, within density range of 4–30\( \rho_{\text{mean}} \), the filaments detected by the two methods are roughly the same, and the maximal overlap is \( \sim 90\% \). At lower and higher densities, the comparison is not possible, since the detected structures are of different nature. However, the P-web filaments can be directly compared to the underlying velocity field.

3.2 Correlation between filaments and velocity shear tensor

Both the P-web and the V-web return directions which are directly related to the large scale structure. In the P-web’s case this is the direction of each filament. In the V-web’s case this is the direction of the eigenvectors of the shear tensor.
In this section we compare the direction of filaments defined by the P-web with that of \( \hat{\mathbf{e}}_3 \), the axis of least collapse.

The right panels in Fig. 1 show the filament detection probability field that is based on point process. The filament detection field is overlapped with the projected \( \hat{\mathbf{e}}_3 \) vector of velocity shear tensor. From this Figure, we see that the detected filaments and the underlying velocity field are very strongly correlated. The correlation is visible from low-density environments to high-density environments.

To quantify the visible correlation between the P-web filaments and underlying velocity field, in Fig. 2 (upper row) we show the distribution of cosine of the angle between the P-web filament axis and the orientation of \( \hat{\mathbf{e}}_3 \) vector of velocity shear tensor. For that we use the CIC shells that are less than a filament radius away from the P-web filament axes. Distributions are shown for various web types defined by velocity shear tensor classification. The velocity shear tensor is very strongly aligned parallel to the filaments detected by P-web: the alignment is within \( 30^\circ \) for \( \sim 80\% \) of the detected elements (excluding knots). The correlation is roughly the same for voids, sheets, and filaments, being slightly larger for the latter. The correlation strength is independent of the defined filament scale. The same correlation shows that filaments from point process classified as voids, sheets or filaments by velocity field are dynamically the same structures: they just live in different density environments. The filaments defined by the P-web are therefore intimately related to the underlying velocity field as characterised by the shear.

The lower row in Fig. 2 shows the distribution between P-web filaments and the tidal field (the “linear” velocity shear tensor). The correlation for the tidal field is slightly weaker than for the V-web, showing that P-web filaments are dynamically stronger than linear perturbations predict.

4 SUMMARY AND CONCLUSIONS

A cosmological DM N-body simulation has been used to compare the filaments detected from the halo distribution to the velocity field. The so-called V-web algorithm, here used to probe the underlying velocity field, was initially developed for the purpose of cosmic web identification in numerical simulations, while the P-web generation algorithm, functioning as a marked point process, was developed for the identification of filaments in large sky surveys. Although employing completely different techniques, philosophies and motivations, these two methods show some remarkable similarities. Most importantly, the direction of filaments defined using point-processes applied to the halo distribution matches very well the eigenvector of the velocity shear tensor corresponding to slowest collapse. This is our main result.

On the other hand, the intimate correspondence between filaments found in the two methods (using the velocity shear tensor and the point process based on halo distribution) are somewhat different: filaments defined from the velocity shear tensor are mostly located in higher density environments, while filaments in point process are also found in lower density environments and are in general less density-dependent. In principle the V-web could be “tuned” to reduce this density dependency (by lowering the dynamical threshold above which axes are considered to be collaps-

An exact filament-by-filament comparison between the two classification methods requires a more in depth study, which should take into account the hierarchical nature of these structures. For example, the V-web simultaneously identifies all web types (knots, filaments, sheets, and voids) while the point process is designed only to identify filaments, thus a more sophisticated comparison is complex. That said, it should perhaps be considered as a success (and sanity check) that both of these methods, developed with completely different techniques, aims, and philosophies – find similar objects with similar orientations.

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