On the effective operators for Dark Matter annihilations

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Abstract

We consider effective operators describing Dark Matter (DM) interactions with Standard Model fermions. In the non-relativistic limit of the DM field, the operators can be organized according to their mass dimension and their velocity behaviour, i.e. whether they describe s- or p-wave annihilations. The analysis is carried out for self-conjugate DM (real scalar or Majorana fermion). In this case, the helicity suppression at work in the annihilation into fermions is lifted by electroweak bremsstrahlung. We construct and study all dimension-8 operators encoding such an effect. These results are of interest in indirect DM searches.
1 Introduction

Significant experimental activity is currently devoted to the search for Dark Matter (DM) by looking at the excesses in cosmic ray production through DM annihilations (or decays) in the galactic halo. Detailed predictions for the fluxes largely depend on the particle physics model for the DM, e.g. the mass, the annihilation channels etc. It is therefore desirable to describe DM annihilations and their products within a general and model-independent framework and Effective Field Theory (EFT) provides such a tool.

Of course, the EFT is only applicable whenever there is a separation of scales between the process to describe (the annihilation of non-relativistic DM at a scale $\sim M_{\mathrm{DM}}$) and the underlying microscopic physics of the interactions (at a scale $\Lambda$). This may not always be the case, as, for example, in the case of supersymmetry with a compressed spectrum.

If we want to describe the annihilation of two non-relativistic DM particles, whose relative velocity is $v \sim 10^{-3}$ (in units of $c$) in our Galaxy today, it is convenient to expand the cross section in powers of $v$

$$v\sigma = a + bv^2 + \mathcal{O}(v^4),$$

(1.1)

where the first term corresponds to annihilation in the state of orbital angular momentum $L = 0$ ($s$-wave) while the second term describes $L = 1$ ($p$-wave). For the annihilation DM $\, \rightarrow \, f \bar{f}$ of a self-conjugate DM particle (real scalar or Majorana fermion) into SM fermions of mass $m_f$, helicity arguments lead to $a \propto (m_f/M_{\mathrm{DM}})^2$, and hence a very suppressed $s$-wave term for light final state fermions (e.g. leptons), while the $p$-wave is suppressed by $v^2$.

It is clear that the correct operator expansion must be done in terms of two parameters: mass dimension of the operator and relative velocity. The effective lagrangian would be generically given by an infinite series of non-renormalizable operators

$$\mathcal{L}_{\text{eff}} = \sum_{d>4} \frac{1}{\Lambda^{d-4}} \left( c^{(d,s)} \mathcal{O}^{(d,s)} + c^{(d,p)} \mathcal{O}^{(d,p)} \right),$$

(1.2)

where $\mathcal{O}^{(d,s\,\text{or}\,p)}$ indicates that the operator of dimension $d$ describes $s$-wave or $p$-wave annihilations, and we neglect annihilations in waves higher than $p$. There is no obvious ordering of the importance of the operators. An important role in this respect is played by ElectroWeak (EW) bremsstrahlung, which has received significant attention recently [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] (for earlier studies on the impact of gauge boson radiation on DM annihilations or cosmic ray physics, see [12, 13, 14]). Taking into account processes with the inclusion of EW radiation eludes the helicity suppression and opens up an $s$-wave contribution to the cross section [2, 3, 6, 7, 10, 12].

In this paper we will classify the operators (up to dimension 8) according to the $v$-behaviour of the amplitude connecting two self-conjugate particles – real scalars or Majorana fermions – in the initial state with the final state of two massless fermions and possibly a gauge boson.

The effects of lifting the helicity suppression by means of EW radiation has so far been studied within the context of explicit models [2, 3, 6, 7, 10]. The results of the present paper provide a model-independent approach to this problem and can be used to place robust constraints on the new physics responsible for the DM sector. Although several analyses in the literature place phenomenological constraints on the coefficients of the dimension-six operators (see e.g. Refs. [15, 16, 17, 18]), the important role of higher-dimensional operators is typically underestimated.
The remainder of the paper is organized as follows. In Section 2, we explain our methodology and construct the effective operators contributing to \( s \)-wave DM annihilations. We compute the differential and total annihilation cross sections for each operator in Section 3, and compare them to the contribution from the typical lowest-dimensional operator. We conclude in Section 4, mentioning phenomenological applications and possible analyses that could be carried out using these results. Finally, we collect some useful relations and identities in the Appendices, for the convenience of the reader.

2 Effective operators

We will carry out our analysis under the following set of assumptions on the DM sector:

1. the DM is either a Majorana fermion or a real scalar field;
2. the DM is neutral under the SM gauge group;
3. there exists a \( Z_2 \) symmetry under which the DM is odd and the SM is even;
4. the DM couples only to the fermions in the SM spectrum, which are assumed to be massless.

Of course, any of these assumptions may not hold in reality and if this is the case our analysis requires modifications. Assumption 1 specifies the conditions under which the helicity suppression is effective, while assumption 2 is there to simplify the discussion, even though it is not strictly necessary and relaxing this assumption can also lead to interesting effects (see Refs. [7, 19]). Assumption 3 is commonly used in DM phenomenology to ensure the stability of the DM particle.

More clarifications about assumption 4 are in order. Considering \( m_f \neq 0 \) would introduce another mass scale into the problem and would render the operator classification much less transparent. Our analysis is still valid in the regime \( m_f/\Lambda \ll v \), which may not hold for the third-generation quarks. The other piece of information in assumption 4 is that the DM particle only couples to the fermion sector of the SM. This needs not to be true, of course. Allowing for DM interactions with the other SM particles, namely gauge bosons or the Higgs, the possibility of having additional operators contributing to DM annihilations in \( s \)-wave opens up. For instance for Majorana DM, \( \chi \), coupling to the Higgs doublet \( H \) or a generic field-strength \( F_{\mu\nu} \) one can have \((\bar{\chi}\gamma^5\chi)(H^\dagger H)\) and \((\bar{\chi}\gamma^5\chi)F_{\mu\nu}F^{\mu\nu}\), which are CP odd, and \((\bar{\chi}\gamma^5\chi)F_{\mu\nu}\tilde{F}^{\mu\nu}\), which is CP even. For a list of effective operators connecting DM to vector bosons and/or Higgs bosons see e.g. Refs.[20, 21]. We will not consider these possibilities here.

We want to classify the operators of dimension \( d = 6, 7, 8 \) according to the \( v \)-behaviour of the amplitude connecting two DM particles in the initial state with two massless SM fermions \( f \) and a gauge boson in the final state. We look for operators which are hermitian, gauge invariant (under \( SU(2)_L \otimes U(1)_Y \)) and giving a non-zero contribution to the amplitude for the annihilation of DM into two fermions and one gauge boson. We will further classify the operators according to their \( CP \) transformation properties (see Appendix A for the relevant transformation properties). The operators containing \( \partial f \) give no contribution to the process under consideration, due to the Equation Of Motion (EOM). In order to ensure manifest gauge invariance, we first introduce the
following notation for the covariant derivative
\[ \overrightarrow{D}_\mu f = \left( \overrightarrow{\partial}_\mu + igT^a W^a_\mu + ig Y f B_\mu \right) f, \] (2.1)
\[ \overleftarrow{D}_\mu = \overleftarrow{f} \left( \overrightarrow{\partial}_\mu - igT^a W^a_\mu - ig Y f B_\mu \right), \] (2.2)

where \( T^a = \sigma^a/2 \), with \( \sigma^a \) being the usual Pauli matrices, and the charge \( Q_f \) of the fermion \( f \) is related to its hypercharge \( Y_f \) by \( Q_f = T^3 + Y_f \). Furthermore, the field-strength \( W_{\mu\nu} \) of the SM gauge fields is related to the covariant derivatives as usual
\[ \overrightarrow{D}_\mu, \overrightarrow{D}_\nu f = (igT^a) W^a_{\mu\nu} f, \] (2.3)
\[ \overleftarrow{f} \overrightarrow{D}_\mu, \overrightarrow{D}_\nu f = \overleftarrow{f} (igT^a) W^a_{\mu\nu}. \] (2.4)

In the following, we will separately deal with the cases where the DM is a Majorana fermion or a real scalar. For simplicity, we will consider only left-handed SM fermions \( f_L \), but the analysis can be applied to right-handed fermions straightforwardly.

### 2.1 Majorana fermion DM

Let us first suppose the DM particle is a Majorana fermion \( \chi \). It is possible to build several operators containing two DM fields, two SM fermion fields and zero or one gauge bosons. They can be built in full generality requiring gauge invariance and hermiticity, and further classified according to their \( CP \) properties. The Majorana-flip properties and the chiralities of the SM fermions make several structures identically zero. The only two Majorana fermion bilinears that are non-vanishing in the limit \( v \to 0 \) are \( \bar{\chi} \gamma^5 \chi \) and \( \bar{\chi} \gamma^\mu \gamma^5 \chi \) (see Appendix B). As we are interested in \( s \)-waves only, we will limit our analysis to these two bilinears for the Majorana fermions. For the pseudo-scalar bilinear \( \bar{\chi} \gamma^5 \chi \) no contractions with SM fermions can be built, as they would vanish either by chirality or by the EOM of fermions, so we are left with the axial-vector bilinear.

The lowest-dimensional terms that can be written out of two \( \chi \)'s and two \( f_L \)'s are of dimension 6. At this level, there is only one non-vanishing operator satisfying all criteria:
\[ \mathcal{O}_M = (\bar{\chi} \gamma^5 \gamma^\mu \chi) \left[ \bar{f}_L \gamma_\mu f_L \right]. \] (2.5)

The \( \mu = 0 \) component of this operator could \textit{a priori} give a \( v \)-independent contribution to the scattering amplitude, but it actually vanishes because of the identity \( u_f^\dagger (p) v_f (-p) = 0 \). Therefore this dimension-6 operator only contributes to the \( p \)-wave, as noted in Ref. [3], due to the helicity suppression, which cannot be removed by simply radiating a gauge boson from the external final state leg. In order to look for \( s \)-wave terms, we need to consider higher-dimensional operators with one EW gauge boson whose radiation in the annihilation process lifts the helicity suppression.

At dimension 7, there are no operators contributing to the \( s \)-wave cross section. In fact, all possible structures vanish either due to Majorana-flip properties or the chirality of the SM fermions, or because of the EOM of the \( f_L \)'s. A priori, the \( \mu = 0 \) component of the operator \((\bar{\chi} \gamma^5 \partial_\mu \chi) [\bar{f}_L \gamma^\mu f_L] \) would give a \( v \)-independent contribution to the scattering amplitude, but it vanishes again because of the identity \( u_f^\dagger (p) v_f (-p) = 0 \).

At the level of dimension 8, there are several structures that can be built requiring gauge invariance and hermiticity. They contain two \( \chi \)'s (in the bilinear \( \bar{\chi} \gamma^\mu \gamma^5 \chi \)), two \( f_L \)'s and two covariant
Table 1: List of dimension-8 operators contributing to the $s$-wave cross section for the annihilation of Majorana DM into two fermions and a gauge boson.

| Name | Operator | CP |
|------|----------|----|
| $O_{M1}$ | $(\overline{\chi}\gamma^5\gamma^\mu\chi) \left[ (\overline{f}_L \overrightarrow{D}_\rho) \gamma_\mu (\overrightarrow{D}_\rho f_L) \right]$ | + |
| $O_{M2}$ | $i\epsilon_{\mu\nu\rho\sigma} (\overline{\chi}\gamma^5\gamma^\mu\chi) \left[ (\overline{\gamma}_L \overrightarrow{D}^\nu) (\overrightarrow{D}_\sigma f_L) - (\overline{\gamma}_L \overrightarrow{D}^\sigma) (\overrightarrow{D}_\rho \gamma_\nu f_L) \right]$ | + |
| $O_{M3}$ | $i\epsilon_{\mu\nu\rho\sigma} (\overline{\chi}\gamma^5\gamma^\mu\chi) \left[ (\overline{\gamma}_L \overrightarrow{D}^\nu) \gamma_\rho (\overrightarrow{D}_\sigma f_L) - (\overline{\gamma}_L \overrightarrow{D}^\sigma) \gamma_\rho (\overrightarrow{D}_\nu f_L) \right]$ | + |
| $O_{M4}$ | $i (\overline{\chi}\gamma^5\gamma^\mu\chi) \left[ (\overline{f}_L \overrightarrow{D}_\rho) (\overrightarrow{D}_\rho f_L) - (\overline{f}_L \overrightarrow{D}_\mu) (\overrightarrow{D}_\rho f_L) \right]$ | - |
| $O_{M5}$ | $i (\overline{\chi}\gamma^5\gamma^\mu\chi) \left[ (\overline{f}_L \gamma_\mu \overrightarrow{D}_\rho) (\overrightarrow{D}_\rho f_L) - (\overrightarrow{f}_L \gamma_\mu \overrightarrow{D}_\rho) (\overrightarrow{D}_\rho f_L) \right]$ | - |

derivatives. It is possible to reduce the number of independent operators, contributing to the cross section for the process under consideration, by using EOM and the identities in Appendix C. In addition, some structures can be related to each other by terms (like $(\overline{\chi}\gamma^5\gamma^\mu\chi) \partial^2 [\overline{f}_L \gamma_\mu f_L]$), which do not contribute to the $s$-wave annihilation into two fermions and a gauge boson. Therefore they contribute in exactly the same way to the amplitude for the process we are interested in. There remain only five independent operators of dimension 8 contributing to the $s$-wave annihilation of DM into two SM fermions and a gauge boson, listed in Table 1 together with their $CP$ conjugation properties. We remain agnostic about the presence or absence of $CP$ violation in the Dark Matter sector, which can possibly induce $CP$ violation in the SM at loop level and therefore be further constrained. All other operators have either a larger dimensionality or produce more powers of $v^2$ in the annihilation cross section. Notice that we chose to keep a Lorentz-covariant formalism, despite looking at the non-relativistic limit. This implies that the same operator can lead to both $v$-independent and $v$-dependent terms in the amplitudes; for example, the operator $O_{M1}$ also gives a contribution to the $p$-wave cross section, but we will not consider it as it is very suppressed.

To summarize, the dimension-8 operator contributing to the $s$-wave annihilation cross section of Majorana DM into SM fermions is given by the sum of the operators in Table 1, $O_{M(8,s)}^\text{eff} = \sum_{i=1}^{5} c_i^{(8,s)} O_{Mi}$; the leading interactions are therefore described in terms of only a few operators

$$L_{\text{eff}} = \frac{1}{\Lambda^2} c^{(6,p)} O_M + \frac{1}{\Lambda^4} \sum_{i=1}^{5} c_i^{(8,s)} O_{Mi} + \text{higher-dim}. \quad (2.6)$$

In absence of $CP$-violation in the DM sector, only the first three operators in the sum need to be considered.

### 2.2 Real scalar DM

Next, let us consider the case where the DM particle is a real scalar $\phi$. By angular momentum conservation, two real scalars cannot annihilate into two massless fermions in the configuration with
by integration by parts, using the EOM of the DM particle or the identities (2.3)-(2.4). For instance, in absence of $CP$-violation in the DM sector, only the first four operators in the sum need to be considered.

Nevertheless, at the level of dimension 8, several gauge invariant hermitian operators can be built out of two $\phi$’s, two $f_L$’s and covariant derivatives. As discussed already for the Majorana case, it is possible to reduce the number of independent operators, contributing to the cross section for the process under consideration, by using the EOM and the identities in Appendix C. We are left with four $CP$-even operators and three $CP$-odd operators, listed in Table 2. A $v$-dependent annihilation $\phi\phi \rightarrow \bar{f}_Lf_L$ is mediated by the operator $O_{R4}$, while the $s$-wave annihilation of two $\phi$’s can proceed by switching on the emission of a gauge boson in the final state.

Other operators involving $\phi \partial_\mu \phi$ or a gauge field-strength $F^{\mu \nu}$ can be obtained from the listed ones by integration by parts, using the EOM of the DM particle or the identities (2.3)-(2.4). For instance, the operator $\phi^2 \partial_\mu [\bar{f}_L \gamma_\mu f_L] F^{\mu \nu}$, considered in Ref. [22], is expressed in this basis as $1/g(O_{R1} - O_{R2})$.

To summarize, the dimension-8 operator contributing to the $s$-wave annihilation cross section of real scalar DM into SM fermions is given by the sum of the operators in Table 2: $O_{R}^{(8,s)} = \sum_{i=1}^{7} c_i^{(8,s)} O_{Ri}$; the leading interactions are therefore described in terms of only a few operators

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \sum_{i=1}^{7} c_i^{(8,s)} O_{Ri} + \text{higher-dim}. \quad (2.7)$$

In absence of $CP$-violation in the DM sector, only the first four operators in the sum need to be considered.
3 Annihilation cross sections

In this section we show analytical results for the annihilation cross sections due to the operators found above. For simplicity, we restrict ourselves to considering left-handed SM fermions only, but the results can be easily adapted to account for annihilations into right-handed fermions as well. We consider the process

\[ \text{DM}(k_1) \text{DM}(k_2) \to f_{i,L}(p_1) \bar{f}_{j,L}(p_2) V(k), \quad (3.1) \]

where light fermions in the final state - described here by the generic \( SU(2)_L \) doublet \( F = (f_1, f_2)^T \) - can be both leptons and quarks, and where \( V = W^\pm, Z, \gamma \). Note that diagrams with gauge boson emission from the final state legs have to be included in order to compute a gauge invariant amplitude (see Fig. 1).

It is convenient to introduce the kinematical variables \( y, z \) defined by

\[
\begin{align*}
p_0^1 &= (1 - y)\sqrt{s}/2, \\
p_0^2 &= (1 - z)\sqrt{s}/2, \\
k^0 &= (y + z)\sqrt{s}/2,
\end{align*}
\]

which are subject to the following phase space constraints

\[
\frac{m_V^2}{s} \leq y \leq 1, \quad \frac{m_V^2}{s} \leq z \leq 1 - y + \frac{m_V^2}{s},
\]

where \( s = (k_1 + k_2)^2 = 4M_{DM}^2/(1 - v^2/4) \). The scattering amplitudes will be proportional to a coefficient \( A \) containing the correct gauge couplings according to the different possible final states and their \( SU(2)_L \otimes U(1)_Y \) quantum numbers; more explicitly,

\[
\begin{align*}
A(f_1 f_1 \gamma) &= -\frac{g s_W}{2} (1 + y_f), & A(f_2 f_2 \gamma) &= +\frac{g s_W}{2} (1 - y_f), \\
A(f_1 f_1 Z) &= -\frac{g}{2 c_W} [1 - (1 + y_f) s_W^2], & A(f_2 f_2 Z) &= +\frac{g}{2 c_W} [1 - (1 - y_f) s_W^2], \\
A(f_1 f_2 W^+) &= -\frac{g}{\sqrt{2}}, & A(f_2 f_1 W^-) &= -\frac{g}{\sqrt{2}},
\end{align*}
\]

where \( Y_f = y_f/2 \) (e.g. \( y_L = -1, y_Q = 1/3 \)), \( g \) is the gauge coupling and \( s_W, c_W \) are the sin and cos of the weak angle, respectively.

The double-differential annihilation cross section is given in terms of the squared amplitude \( |\mathcal{M}|^2 \) (averaged over the initial spins and summer over the final ones) as

\[
v \frac{d^2 \sigma}{dydz} = \frac{|\mathcal{M}|^2}{128 \pi^3},
\]

6
while the total cross section is obtained by integrating over the kinematical variables on the phase space domain defined by Eq. (3.5). Let us now show the results of the computation of these cross sections for each of the operators found in the previous section.

3.1 Majorana fermion DM

In the zero-velocity approximation, \( s = 4M_{\text{DM}}^2 \) and the double-differential cross sections for the annihilation process (3.1) mediated by the dimension-8 operators \( O_{M1}, \ldots, O_{M5} \) in Table 1 are

\[
\left. v \frac{d^2 \sigma}{dy \, dz} \right|_{O_{M1}} = \left| c_{M1}^{(8,s)} \right|^2 \frac{2A^2 M_{\text{DM}}^6}{2\pi^3 \Lambda^8} \left[ 1 - y - z + \frac{m_f^2}{4M_{\text{DM}}^2} \right] \left[ y^2 + z^2 - \frac{m_f^2}{2M_{DM}^2} \right],
\]

(3.8)

\[
\left. v \frac{d^2 \sigma}{dy \, dz} \right|_{O_{M2}} = \frac{4}{|c_{M2}^{(8,s)}|^2} \left| v \frac{d^2 \sigma}{dy \, dz} \right|_{O_{M1}},
\]

(3.9)

\[
\left. v \frac{d^2 \sigma}{dy \, dz} \right|_{O_{M3}} = \frac{4}{|c_{M3}^{(8,s)}|^2} \left| v \frac{d^2 \sigma}{dy \, dz} \right|_{O_{M1}},
\]

(3.10)

\[
\left. v \frac{d^2 \sigma}{dy \, dz} \right|_{O_{M4}} = \left| c_{M4}^{(8,s)} \right|^2 \frac{2A^2 M_{DM}^6}{\pi^3 \Lambda^8} \left[ (1 - y - z)(y^2 + z^2) + \frac{m_f^2}{4M_{\text{DM}}^2} \left[ (y + z)^2 - 2(y + z) + 2 \right] \right],
\]

(3.11)

\[
\left. v \frac{d^2 \sigma}{dy \, dz} \right|_{O_{M5}} = \left| c_{M5}^{(8,s)} \right|^2 \left| v \frac{d^2 \sigma}{dy \, dz} \right|_{O_{M4}}.
\]

(3.12)

Notice that in the limit \( m_V/M_{\text{DM}} \to 0 \) the differential cross sections are the same for all operators, up to an overall numerical factor.

In the indirect searches for DM, the observables measured experimentally are the fluxes of cosmic rays, which are directly related to the energy spectra of particles generated by DM annihilations at the production point. By integrating the double-differential cross sections listed above once, one obtains the energy spectra of the SM fermions and of the gauge bosons at production. We consider the distributions of the final fermion energy \( (E_f) \) and of the final gauge boson energy \( (E_V) \), defined as

\[
\frac{dN}{d\ln x} \equiv \frac{1}{\sigma(DMDM \to f\bar{f}V)} \frac{d\sigma(DMDM \to f\bar{f}V)}{d\ln x},
\]

(3.13)

where \( x \equiv E_{f,V}/M_{\text{DM}} \), and shown in Fig. 2.

The distributions originating from the set of operators \( O_{M1}, O_{M2}, O_{M3} \) differ just by an overall factor, which cancels out by normalizing the spectra, and therefore produce the same curves. The same argument applies to the other set \( O_{M4}, O_{M5} \). However, the operator \( O_{M1} \) would instead give a different result with respect to \( O_{M4} \), but the difference is not visible in the fermion energy distribution (left panel of Fig. 2) because of the smallness of \( m_V/M_{\text{DM}} \).

The dimension-6 operator \( O_6 \) mediates the two-body annihilation \( \chi \chi \to f_L \bar{f}_L \) in \( p \)-wave. The processes where a gauge boson is radiated from the final state fermion are still in \( p \)-wave, as discussed in Ref. [3]. Therefore, the energy spectra of fermions and gauge bosons originating from the dimension-6 operator are going to be much less important than those from the dimension-8 operators. The total cross section for the two-body process \( \chi \chi \to f_L \bar{f}_L \) mediated by \( O_M \) is simply

\[
v\sigma(\chi \chi \to f_L \bar{f}_L)|_M = \left| c^{(6,s)} \right|^2 \frac{M_{\text{DM}}^2}{12\pi \Lambda^4} v^2,
\]

(3.14)
Figure 2: Majorana fermion DM. The energy distributions \( dN/d\ln(E/M_{DM}) \) of the final fermion (left panel) and of the final gauge boson (right panel), for the different dimension-8 operators. We set \( c^{(6,p)} = c^{(8,s)}_{M_4} = 1 \) and \( M_{DM} = 1 \) TeV.

while the total cross sections of the three-body processes (3.1) mediated by the dimension-8 operators are obtained by integrating over the full phase space

\[
\begin{align*}
\sigma_{M_1} &= |c^{(8,s)}_{M_1}|^2 \frac{A^2 M_{DM}^6}{240\pi^4 \Lambda^8} \left[ 4 - 15 \frac{m_V^2}{M_{DM}^2} + 5 \frac{m_V^4}{M_{DM}^4} \left( -4 + 6 \ln \frac{2M_{DM}}{m_V^3} \right) + \mathcal{O}\left( \frac{m_V^6}{M_{DM}^6} \right) \right], \\
\sigma_{M_2} &= 4 \left| \frac{c^{(8,s)}_{M_2}}{c^{(8,s)}_{M_1}} \right|^2 \sigma_{M_1}, \\
\sigma_{M_3} &= 4 \left| \frac{c^{(8,s)}_{M_3}}{c^{(8,s)}_{M_1}} \right|^2 \sigma_{M_1}, \\
\sigma_{M_4} &= |c^{(8,s)}_{M_4}|^2 \frac{A^2 M_{DM}^6}{120\pi^3 \Lambda^8} \left[ 8 + 15 \frac{m_V^2}{M_{DM}^2} + \frac{m_V^4}{M_{DM}^4} \left( 40 - 60 \ln \frac{2M_{DM}}{m_V^3} \right) + \mathcal{O}\left( \frac{m_V^6}{M_{DM}^6} \right) \right], \\
\sigma_{M_5} &= \left| \frac{c^{(8,s)}_{M_5}}{c^{(8,s)}_{M_4}} \right|^2 \sigma_{M_4}.
\end{align*}
\]  

For simplicity, we have only reported here the leading terms in the expansion in powers of \( m_V/M_{DM} \), but the complete analytical expressions are used in the plots. The sub-leading terms can be of the same order as the contributions from higher-dimensional operators we are neglecting.

The relative importance of the \( s \)-wave three-body process due to dimension-8 operators with respect to the \( p \)-wave two-body annihilation due to the dimenion-6 operator is captured by the ratios of the total cross sections, plotted in Fig. 3. Three-body cross sections can be sizeably larger than the two-body ones. It is evident that the dimension-8 operators dominate the total cross section, provided that the effective operator scale \( \Lambda \) is not too large with respect to the DM mass \( M_{DM} \). It is clear that limiting an EFT analysis for DM annihilations to the dimension-six operators misses the right result, as the cross section receives important contributions from operators of dimension higher than six.

One could have expected this result by an order-of-magnitude estimate of the two-body and
As discussed in the previous section, there is no two-body annihilation \( \phi \to f_L f_L \) in s-wave in the limit \( m_f = 0 \). So the EW bremsstrahlung opens up a three-body annihilation channel which is otherwise absent. The first non-vanishing contribution to the s-wave annihilation cross section

\[ \sigma_{3b}(s) \sim \left| \frac{1}{2} \frac{\Lambda^2}{v} \frac{(8,s)}{(6,p)} \right|^2 \frac{M_{DM}}{A^4} \]
comes from the dimension-8 operators in Table 2 and there is no contribution from lower-dimensional operators to compare with.

For the case of real scalar DM, we computed the double-differential cross sections for the annihilation process (3.1) mediated by the dimension-8 operators $\mathcal{O}_{R1}, \ldots, \mathcal{O}_{R7}$ listed in Table 2

$$
v \frac{d^2\sigma}{dydz} \bigg|_{R1} = |c_{(8,s)}^{(8,s)}| \frac{4A^2 M_{DM}^6}{\pi^3 \Lambda^8} \left( (1 - y - z)(y^2 + z^2) + \frac{m_V^2}{4M_{DM}^2} \left[ (y + z)^2 - 2(y + z) + 2 \right] \right),
$$

$$
v \frac{d^2\sigma}{dydz} \bigg|_{R2} = |c_{(8,s)}^{(8,s)}| \frac{4A^2 M_{DM}^6}{\pi^3 \Lambda^8} \left( (1 - y - z)(y^2 + z^2) - \frac{m_V^2}{4M_{DM}^2} \left[ 3(y + z)^2 - 2(y + z + 3yz) - 2 \right] \right.
\left. - \frac{m_V^4}{8M_{DM}^4} + \frac{m_V^6}{16M_{DM}^6} \right),
$$

$$
v \frac{d^2\sigma}{dydz} \bigg|_{R3} = |c_{(8,s)}^{(8,s)}| \frac{4A^2 M_{DM}^8}{\pi^3 \Lambda^8} \left[ \left( 1 - y - z + \frac{m_V^2}{4M_{DM}^2} \right) \left( y^2 + z^2 - \frac{m_V^2}{2M_{DM}^2} \left[ 2(y + z) - 1 \right] + \frac{m_V^4}{4M_{DM}^4} \right) \right],
$$

$$
v \frac{d^2\sigma}{dydz} \bigg|_{R4} = \frac{1}{16} \frac{|c_{(8,s)}^{(8,s)}|^2}{|c_{(8,s)}^{(8,s)}|^2} v \frac{d^2\sigma}{dydz} \bigg|_{R1},
$$

$$
v \frac{d^2\sigma}{dydz} \bigg|_{R5} = |c_{(8,s)}^{(8,s)}| \frac{4A^2 M_{DM}^8}{\pi^3 \Lambda^8} \left[ \left( 1 - y - z + \frac{m_V^2}{4M_{DM}^2} \right) \left( y^2 + z^2 - \frac{m_V^2}{2M_{DM}^2} \right) \right],
$$

$$
v \frac{d^2\sigma}{dydz} \bigg|_{R6} = \frac{|c_{(8,s)}^{(8,s)}|^2}{|c_{(8,s)}^{(8,s)}|^2} v \frac{d^2\sigma}{dydz} \bigg|_{R5},
$$

$$
v \frac{d^2\sigma}{dydz} \bigg|_{R7} = \frac{|c_{(8,s)}^{(8,s)}|^2}{|c_{(8,s)}^{(8,s)}|^2} v \frac{d^2\sigma}{dydz} \bigg|_{R5}.
$$

They are clearly symmetric under the exchange $y \leftrightarrow z$. The energy distributions of the fermion and gauge boson, defined as in Eq. (3.13), are shown in Fig. 4. Some of the distributions (e.g. those due to the operators $\mathcal{O}_{R1}, \mathcal{O}_{R4}$) differ just by an overall factor, which cancels out by normalizing the spectra, and therefore produce the same curves. In the limit $m_V \ll M_{DM}$ the distributions from all different operators become proportional to each other. Therefore, because of the smallness of $m_V/M_{DM}$, the differences in the fermion energy distribution are not visible (left panel of Fig. 4).

For convenience, we also report the results for the total cross sections, obtained after integrating over the kinematical variables
Figure 4: Real scalar DM. The energy distributions $dN/d\ln(E/\Lambda_{DM})$ of the final fermion (left panel) and of the final gauge boson (right panel), for the different dimension-8 operators. We set $c_{R1}^{(8,s)} = 1$ and $\Lambda_{DM} = 1 \, TeV$.

\begin{align}

v\sigma|_{R1} & = |c_{R1}^{(8,s)}|^2 \left[ \frac{2 A^2 M_{DM}^6}{60 \pi^3 A^8} \right] \left[ 8 + 15 \frac{m_V^2}{M_{DM}^2} + \frac{m_V^4}{M_{DM}^4} \left( 40 - 60 \ln \frac{2M_{DM}}{m_V} \right) + O \left( \frac{m_V^6}{M_{DM}^6} \right) \right], \quad (3.31) \\
v\sigma|_{R2} & = |c_{R2}^{(8,s)}|^2 \left[ \frac{2 A^2 M_{DM}^6}{120 \pi^3 A^8} \right] \left[ 4 + 25 \frac{m_V^2}{M_{DM}^2} - 10 \frac{m_V^4}{M_{DM}^4} \left( 1 + 3 \ln \frac{2M_{DM}}{m_V} \right) + O \left( \frac{m_V^6}{M_{DM}^6} \right) \right], \quad (3.32) \\
v\sigma|_{R3} & = |c_{R3}^{(8,s)}|^2 \left[ \frac{2 A^2 M_{DM}^6}{120 \pi^3 A^8} \right] \left[ 4 - 5 \frac{m_V^2}{M_{DM}^2} + \frac{m_V^4}{M_{DM}^4} \left( 50 - 30 \ln \frac{2M_{DM}}{m_V} \right) + O \left( \frac{m_V^6}{M_{DM}^6} \right) \right], \quad (3.33) \\
v\sigma|_{R4} & = \frac{1}{16} \left| \frac{c_{R2}^{(8,s)}}{c_{R1}^{(8,s)}} \right|^2 v\sigma|_{R1}, \quad (3.34) \\
v\sigma|_{R5} & = |c_{R5}^{(8,s)}|^2 \left[ \frac{2 A^2 M_{DM}^6}{120 \pi^3 A^8} \right] \left[ 4 - 15 \frac{m_V^2}{M_{DM}^2} + 5 \frac{m_V^4}{M_{DM}^4} \left( -4 + 6 \ln \frac{2M_{DM}}{m_V} \right) + O \left( \frac{m_V^6}{M_{DM}^6} \right) \right], \quad (3.35) \\
v\sigma|_{R6} & = \frac{|c_{R6}^{(8,s)}|^2 |c_{R5}^{(8,s)}|^2}{|c_{R1}^{(8,s)}|^2} v\sigma|_{R5}, \quad (3.36) \\
v\sigma|_{R7} & = \frac{|c_{R7}^{(8,s)}|^2 |c_{R5}^{(8,s)}|^2}{|c_{R1}^{(8,s)}|^2} v\sigma|_{R5}. \quad (3.37)
\end{align}

As for the Majorana case, we have only shown the leading terms in the expansion in $m_V/M_{DM}$. The sub-leading terms can be of the same order as the contributions from higher-dimensional operators we are neglecting.

4 Conclusions

We have considered the annihilation of a self-conjugate DM particle – either a Majorana fermion or a real scalar – into light SM fermions. In these cases, the simple annihilation DM DM $\to f_L \bar{f}_L$ is helicity suppressed. The radiation of electroweak gauge bosons lifts the suppression and opens up an $s$-wave channel, independently of the relative velocity of the annihilating DM particles. From the effective operator point of view, we found that this effect is encoded by dimension-8 operators.
The dimension-8 operators, despite suffering from a high mass dimensionality, enjoy no suppression by the relative velocity. We have found all the dimension-8 operators mediating s-wave annihilations of DM into SM fermions and a gauge boson, and for each of them we have computed the differential and total cross sections. These operators encode the so-called “virtual internal bremsstrahlung” effect.

In the Majorana DM case, the two-body annihilation DM DM $\rightarrow f_L \bar{f}_L$ proceeds through a dimension-6 operator and is velocity suppressed (p-wave). We have shown explicitly and quantitatively that the dimension-8 operators actually provide a bigger contribution to the annihilation cross section than the dimension-6 operators, for DM masses well above the weak scale.

For real scalar DM, the s-wave annihilation into massless fermions is forbidden, and becomes allowed in the presence of an extra gauge boson in the final state. We found the dimension-8 operators mediating annihilations into SM fermions and a gauge boson, which switches on an s-wave annihilation process.

In conclusion, our analysis has strengthened the statement that, when dealing with the annihilation of heavy self-conjugate DM today, it would be erroneous to neglect the effects of dimension-8 operators. The results presented in this paper are of interest also when applied to phenomenological analyses, as they provide a model-independent language into which to translate the experimental data, present exclusions and possible future signals.

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A \textit{CP} transformation properties

Under charge conjugation the spinor fields $\psi$ and $\bar{\psi}$ transform as

$$\psi \rightarrow \psi^C = C\psi^T, \quad (A.1)$$
$$\bar{\psi} \rightarrow \bar{\psi}^C = \psi^T C, \quad (A.2)$$

where the charge conjugation matrix $C$ obeys $C^T = C^\dagger = C^{-1} = -C$. Under parity transformation

$$\psi \rightarrow P^{-1} \psi P = i\beta \psi, \quad (A.3)$$
$$\bar{\psi} \rightarrow P^{-1} \bar{\psi} P = -i\bar{\psi} \beta. \quad (A.4)$$

For instance, in the Dirac representation of the $\gamma$-matrices one has

$$C = i\gamma^2 \gamma^0 = \begin{pmatrix} 0 & -i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}, \quad \beta = \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (A.5)$$
The following identities turn out to be useful
\[ \gamma^0 \gamma^2 \gamma^\mu P_L \gamma^2 \gamma^0 = (\gamma^\mu)^T P_L, \quad (A.6) \]
\[ \gamma^0 \gamma^\mu P_R \gamma^0 = (-1)^\mu \gamma^\mu P_L, \quad (A.7) \]
where
\[ (-1)^\mu = \begin{cases} +1, & \mu = 0, \\ -1, & \mu = 1, 2, 3. \end{cases} \quad (A.8) \]

Using these relations it is easy to prove the following transformation properties under \( CP \) of operators built out of left-handed fermion fields \( f_L \):
\[ CP \left\{ [f_L \gamma^\mu \bar{D}^\nu f_L + \bar{f}_L \bar{D}^\nu \gamma^\mu f_L] \right\} = (-1)(-1)^\mu(-1)^\nu, \quad (A.9) \]
\[ CP \left\{ i \left[ \bar{f}_L \gamma^\mu \bar{D}^\nu f_L - \bar{f}_L \bar{D}^\nu \gamma^\mu f_L \right] \right\} = (-1)^\mu(-1)^\nu, \quad (A.10) \]
\[ CP \left\{ \left[ J_L \gamma^\mu \bar{D}^\nu f_L \right] + \left( J_L \bar{D}^\mu \right) \bar{D} f_L \right\} = (-1)(-1)^\mu, \quad (A.11) \]
\[ CP \left\{ i \left[ J_L \gamma^\mu \bar{D}^\nu f_L - \bar{J}_L \bar{D}^\mu \bar{D} f_L \right] \right\} = (-1)^\mu, \quad (A.12) \]
\[ CP \left\{ \left[ J_L \gamma^\mu \bar{D}^\rho (\bar{D} f_L) + \bar{J}_L \bar{D}^\rho \gamma^\mu f_L \right] \right\} = (-1)(-1)^\mu, \quad (A.13) \]
\[ CP \left\{ i \left[ J_L \gamma^\mu \bar{D}^\rho (\bar{D} f_L) - \bar{J}_L \bar{D}^\rho \gamma^\mu f_L \right] \right\} = (-1)^\mu, \quad (A.14) \]
\[ CP \{ J_L \gamma^\mu \bar{W} \gamma^\nu f_L \} = (-1)^\mu, \quad (A.15) \]
\[ CP \{ J_L \gamma^\mu \bar{W}_L \gamma^\nu f_L \} = (-1)(-1)^\mu. \quad (A.16) \]

If \( \chi \) is a Majorana fermion field, the bilinears \( \bar{\chi} \gamma^5 \chi, \bar{\chi} \gamma^\mu \gamma^5 \chi \) transform under \( CP \) as
\[ CP \{ \bar{\chi} \gamma^5 \chi \} = -1, \quad (A.17) \]
\[ CP \{ \bar{\chi} \gamma^\mu \gamma^5 \chi \} = (-1)(-1)^\mu, \quad (A.18) \]
while the bilinears built out of a real scalar \( \phi \) transform as
\[ CP \{ \phi \partial_\mu \phi \} = (-1)^\mu, \quad (A.19) \]
\[ CP \{ \partial_\mu \phi \partial_\nu \phi \} = (-1)^\mu(-1)^\nu. \quad (A.20) \]

**B  Majorana fermion bilinears**

The standard decomposition of a Majorana fermion field in momentum space is
\[ \chi(x) = \int \frac{d^3k}{(2\pi)^3 \sqrt{2E_k}} \sum_s \left( a_s(k) u_s(k) e^{-ik \cdot x} + a_s^\dagger(k) v_s(k) e^{ik \cdot x} \right), \quad (B.1) \]
from which one can compute the matrix element between the initial state with two DM particles and the vacuum, for the bilinears at small velocities. In the Dirac representation of \( \gamma \)-matrices we obtain
\[ \langle 0| \bar{\chi} O_{\chi} | \chi(p^0, \vec{p}) \chi(p^0, -\vec{p}) \rangle \sim \bar{v}(-\vec{p}) O u(\vec{p}) - \bar{v}(\vec{p}) O u(-\vec{p}). \quad (B.2) \]
where the Majorana nature of the initial particles implies to consider the process with spins and momenta interchanged (and a relative minus sign due to Fermi statistics). Let us first consider the case where the operator $O$ is one of the basis matrices $\Gamma_A = \{1, \gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, \sigma^{\mu\nu}\}$. Noting their behaviour under charge conjugation

$$C^{-1}\bar{\psi}\Gamma_A\psi C = \begin{cases} \bar{\psi}\Gamma_A^T\psi & (\Gamma_A = 1, i\gamma^5, \gamma^5\gamma^\mu) \\ -\bar{\psi}\Gamma_A^T\psi & (\Gamma_A = \gamma^\mu, \sigma^{\mu\nu}) \end{cases},$$

we find that

$$\bar{v}(\vec{p})\Gamma_A u(-\vec{p}) = \begin{cases} -\bar{v}(-\vec{p})\Gamma_A u(\vec{p}) & (\Gamma_A = 1, i\gamma^5, \gamma^5\gamma^\mu) \\ +\bar{v}(-\vec{p})\Gamma_A u(\vec{p}) & (\Gamma_A = \gamma^\mu, \sigma^{\mu\nu}) \end{cases},$$

hence the vector and tensor operators do not contribute to the annihilation of Majorana particles – as it is well known – and it is sufficient to compute the bilinears for scalar, pseudo-scalar and pseudo-vector interactions

$$\bar{u}(\vec{p})\Gamma_A v(-\vec{p}) = \begin{cases} \bar{v}(-\vec{p})(\gamma^0\Gamma_A^\dagger\gamma^0)u(\vec{p}) \end{cases} \dagger,$$

such that, for example

$$\bar{u}(\vec{p})\gamma^0\gamma^5 v(-\vec{p}) = (E + M) \xi^\dagger \eta,$$

$$\bar{u}(\vec{p})\gamma^i\gamma^5 v(-\vec{p}) = 2i(\vec{p} \times \vec{\sigma})^i \xi^\dagger \eta.$$

If the operator $O$ contains one derivative, we have

$$\tilde{\chi}\gamma^5\partial_0 \chi \sim v^0,$$

$$\tilde{\chi}\gamma^5\partial_i \chi \sim v,$$

$$\tilde{\chi}\gamma^5\gamma^0\partial_0 \chi \sim v^0,$$

$$\tilde{\chi}\gamma^5\gamma^0\partial_i \chi \sim v,$$

$$\tilde{\chi}\gamma^5\gamma^i\partial_j \chi = 0.$$

The generalization to an arbitrary number of derivatives is straightforward.
C Useful Identities

By a direct computation we find the following identity involving one covariant derivative
\[
\partial_\nu ( \mathcal{F}_L \gamma^\mu f_L ) = \mathcal{F}_L \gamma^\mu ( \overrightarrow{D}_\nu f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\nu ) \gamma^\mu f_L .
\] (C.1)

A particular case is the contraction \( \mu = \nu \), which gives, using the equations of motion
\[
\partial_\mu ( \mathcal{F}_L \gamma^\mu f_L ) = 0 .
\] (C.2)

For two covariant derivatives we find
\[
\partial_\rho \partial_\nu ( \mathcal{F}_L \gamma^\mu f_L ) = \mathcal{F}_L \gamma^\mu \overrightarrow{D}_\rho ( \overrightarrow{D}_\nu f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\rho ) \gamma^\mu ( \overrightarrow{D}_\nu f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\rho ) \gamma^\mu ( \overrightarrow{D}_\rho f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\rho ) \overrightarrow{D}_\rho \gamma^\mu f_L .
\] (C.3)

A useful relation can be extracted by contracting \( \rho = \mu \)
\[
\partial_\mu \partial_\nu ( \mathcal{F}_L \gamma^\mu f_L ) = \mathcal{F}_L \overrightarrow{D}_\nu ( \overrightarrow{D}_\nu f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\nu ) \overrightarrow{D}_\nu f_L .
\] (C.4)

However, exchanging the partial derivatives on the left hand side, and using Eq. (C.2) we find
\[
\mathcal{F}_L \overrightarrow{D}_\nu ( \overrightarrow{D}_\nu f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\nu ) \overrightarrow{D}_\nu f_L = 0 .
\] (C.5)

Eq. (C.3) shows that the action of a partial derivative is equivalent to the action of the corresponding covariant derivative on each single term. This relation is crucial to relate operators containing quadratic \((\overrightarrow{\phi}^2)\) and derivative \((\overrightarrow{\phi} \partial_\mu \phi)\) terms of the scalar field. Notice that this kind of relation remains true also for differences of SM operators instead of sums. By direct computation we find
\[
\partial_\rho \left[ \mathcal{F}_L \gamma^\mu ( \overrightarrow{D}_\nu f_L ) - ( \mathcal{F}_L \overrightarrow{D}_\nu ) \gamma^\mu f_L \right] =
\mathcal{F}_L \gamma^\mu \overrightarrow{D}_\rho ( \overrightarrow{D}_\nu f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\rho ) \gamma^\mu ( \overrightarrow{D}_\nu f_L ) - ( \mathcal{F}_L \overrightarrow{D}_\rho ) \gamma^\mu ( \overrightarrow{D}_\rho f_L ) - ( \mathcal{F}_L \overrightarrow{D}_\rho ) \overrightarrow{D}_\rho \gamma^\mu f_L .
\] (C.6)

Similar relations hold for three covariant derivatives
\[
\partial_\sigma \partial_\rho \partial_\nu ( \mathcal{F}_L \gamma^\mu f_L ) =
\mathcal{F}_L \gamma^\mu \overrightarrow{D}_\sigma \overrightarrow{D}_\rho ( \overrightarrow{D}_\nu f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\sigma ) \gamma^\mu \overrightarrow{D}_\rho ( \overrightarrow{D}_\nu f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\sigma ) \gamma^\mu \overrightarrow{D}_\sigma ( \overrightarrow{D}_\nu f_L )
+ ( \mathcal{F}_L \overrightarrow{D}_\sigma ) \gamma^\mu ( \overrightarrow{D}_\rho f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\sigma ) \overrightarrow{D}_\rho \gamma^\mu ( \overrightarrow{D}_\sigma f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\sigma ) \overrightarrow{D}_\rho \overrightarrow{D}_\sigma \gamma^\mu f_L .
\] (C.7)

This cumbersome identity simplifies contracting \( \rho = \mu \); using the same argument previously exploited we find
\[
\partial_\sigma \partial_\mu \partial_\nu ( \mathcal{F}_L \gamma^\mu f_L ) =
\mathcal{F}_L \overrightarrow{D}_\sigma \overrightarrow{D}_\mu ( \overrightarrow{D}_\nu f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\sigma ) \gamma^\mu ( \overrightarrow{D}_\nu f_L )
+ ( \mathcal{F}_L \overrightarrow{D}_\sigma ) \overrightarrow{D}_\sigma \gamma^\mu ( \overrightarrow{D}_\rho f_L ) + ( \mathcal{F}_L \overrightarrow{D}_\sigma ) \overrightarrow{D}_\rho \overrightarrow{D}_\sigma \gamma^\mu f_L ,
\] (C.8)

which is useful to find dependencies among seemingly independent operators.
References

[1] P. Ciafaloni, D. Comelli, A. Riotto, F. Sala, A. Strumia and A. Urbano, JCAP 1103, 019 (2011) [arXiv:1009.0224].

[2] N. F. Bell, J. B. Dent, T. D. Jacques and T. J. Weiler, Phys. Rev. D 83, 013001 (2011) [arXiv:1009.2584]; N. F. Bell, J. B. Dent, T. D. Jacques and T. J. Weiler, Phys. Rev. D 84, 103517 (2011) [arXiv:1101.3357].

[3] P. Ciafaloni, M. Cirelli, D. Comelli, A. De Simone, A. Riotto and A. Urbano, JCAP 1106, 018 (2011) [arXiv:1104.2996].

[4] N. F. Bell, J. B. Dent, A. J. Galea, T. D. Jacques, L. M. Krauss and T. J. Weiler, Phys. Lett. B 706, 6 (2011) [arXiv:1104.3823].

[5] K. Cheung, P. -Y. Tseng and T. -C. Yuan, JCAP 1106, 023 (2011) [arXiv:1104.5329].

[6] M. Garny, A. Ibarra and S. Vogl, JCAP 1107, 028 (2011) [arXiv:1105.5367].

[7] P. Ciafaloni, M. Cirelli, D. Comelli, A. De Simone, A. Riotto, A. Urbano, JCAP 1110, 034 (2011) [arXiv:1107.4453].

[8] A. Hryczuk and R. Iengo, JHEP 1201, 163 (2012) [Erratum-ibid. 1206, 137 (2012)] [arXiv:1111.2916].

[9] V. Barger, W. -Y. Keung and D. Marfatia, Phys. Lett. B 707, 385 (2012) [arXiv:1111.4523].

[10] M. Garny, A. Ibarra and S. Vogl, JCAP 1204, 033 (2012) [arXiv:1112.5155].

[11] A. De Simone, J. Phys. Conf. Ser. 375, 012046 (2012) [arXiv:1201.1443].

[12] L. Bergstrom, Phys. Lett. B 225, 372 (1989).

[13] T. Bringmann, L. Bergstrom and J. Edsjo, JHEP 0801, 049 (2008) [arXiv:0710.3169]; L. Bergstrom, T. Bringmann and J. Edsjo, Phys. Rev. D 78, 103520 (2008) [arXiv:0808.3725].

[14] V. Berezinsky, M. Kachelriess and S. Ostapchenko, Phys. Rev. Lett. 89, 171802 (2002) [hep-ph/0205218]; C. Barbot and M. Drees, Phys. Lett. B 533, 107 (2002) [hep-ph/0202072]; C. Barbot and M. Drees, Astropart. Phys. 20, 5 (2003) [hep-ph/0211406]; M. Kachelriess and P. D. Serpico, Phys. Rev. D 76, 063516 (2007) [arXiv:0707.0209]; N. F. Bell, J. B. Dent, T. D. Jacques and T. J. Weiler, Phys. Rev. D 78, 083540 (2008) [arXiv:0805.3423]; J. B. Dent, R. J. Scherrer and T. J. Weiler, Phys. Rev. D 78, 063509 (2008) [arXiv:0806.0370]; V. Barger, Y. Gao, W. Y. Keung, D. Marfatia, Phys. Rev. D80, 063537 (2009) [arXiv:0906.3009]; J. F. Fortin, J. Shelton, S. Thomas and Y. Zhao, arXiv:0908.2258; M. Kachelriess, P. D. Serpico and M. A. Solberg, Phys. Rev. D 80, 123533 (2009) [arXiv:0911.0001].

[15] K. Cheung, P. Y. Tseng and T. C. Yuan, JCAP 1106, 023 (2011) [arXiv:1104.5329].

[16] J. -F. Fortin, T. M. P. Tait, Phys. Rev. D 85, 063506 (2012) [arXiv:1103.3289].

[17] K. Cheung, P. Y. Tseng and T. C. Yuan, JCAP 1101 (2011) 004 [arXiv:1011.2310].
[18] J. Goodman, M. Ibe, A. Rajaraman, W. Shepherd, T. M. P. Tait, H. -B. Yu, Nucl. Phys. B844, 55-68 (2011). [arXiv:1009.0008].

[19] P. Ciafaloni, D. Comelli, A. De Simone, A. Riotto and A. Urbano, JCAP 1206, 016 (2012) [arXiv:1202.0692].

[20] A. Rajaraman, T. M. P. Tait and D. Whiteson, JCAP 1209, 003 (2012) [arXiv:1205.4723]; A. Rajaraman, T. M. P. Tait and A. M. Wijangco, [arXiv:1211.7061].

[21] R. C. Cotta, J. L. Hewett, M. P. Le and T. G. Rizzo, [arXiv:1210.0525].

[22] V. Barger, Y. Gao, W. Y. Keung and D. Marfatia, Phys. Rev. D 80, 063537 (2009) [arXiv:0906.3009].

[23] O. Adriani et al. [PAMELA Collaboration], Phys. Rev. Lett. 105, 121101 (2010) [arXiv:1007.0821].

[24] K. Nakamura et al. [Particle Data Group Collaboration], J. Phys. G G 37, 075021 (2010).

[25] T. Bringmann and P. Salati, Phys. Rev. D 75, 083006 (2007) [astro-ph/0612514].

[26] M. Cirelli, R. Franceschini and A. Strumia, Nucl. Phys. B 800, 204 (2008) [arXiv:0802.3378].

[27] M. Cirelli, G. Corcella, A. Hektor, G. Hutsi, M. Kadastik, P. Panci, M. Raidal and F. Sala et al., JCAP 1103, 051 (2011) [arXiv:1012.4515].