Train Speed Trajectory Optimization using Dynamic Programming with speed modes decomposition

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Abstract. When applying a dynamic programming algorithm to train speed trajectory optimization, there is a problem of too many discrete points which leading to dimension disaster. Different from using uniform discretization of the time and space by previous research, In this paper, a train operation model is proposed based on conditions (speed limit and slope change) and described in the network diagram. Armed with this model, through the pre-planning of the line, the optimization model of train operation is established, and the dynamic programming algorithm is utilized to find the global optimal value of train traction energy consumption. Taking Beijing Yizhuang line as a simulation case, the validity of the proposed model is verified by comparing common optimization algorithms. The results demonstrate that the proposed model can effectively reduce the calculation time of the dynamic programming algorithm by 95.06%, and has better optimization effect. The energy-saving effect reaches 6.0806%.

1. Introduction
With the expansion of the line, the energy consumption of urban rail transit has surged, and energy conservation has become a strategic issue for the travel of urban rail transit. Among them, train traction energy consumption accounts for half of the total energy consumption of the subway system, which is the existing aggregate consumption ratio. It is particularly important to study how to reduce the energy consumption of the subway while ensuring that the subway operates on time.

The hot issue of urban rail transit energy conservation research aims to reduce the traction energy consumption of trains on the premise of ensuring that trains operate according to the target curve. Scholars at home and abroad have done a lot of research and have achieved many results. Its control strategy is roughly divided into two categories [1]: coasting control and general control. In 1980, the University of South Australia's Milroy [2] proposed that when the short-distance train ran on a flat road or a route with a small change in slope, train energy control can consist of three working conditions: maximum traction, coasting and maximum braking. Chang et al. [3] first used the genetic algorithm to find the best unoccupied point, ie the operating point transition point. Wong et al. [4] also applied the genetic algorithm and dynamically assigned the number of unoccupied points to the
chromosome, this strengthens the practical application of the algorithm. Artificial neural networks and genetic algorithms have been applied to the study of idle control of trains [5-6].

General control obtains the best train operation curve by controlling the arrangement and combination of different working conditions (i.e. traction, braking, inertia, cruise). This optimization method can be realized directly because the input of control corresponds to the actual train operation control. The aim is to optimize train operation sequence and minimize energy consumption under time and other physical constraints. Pontryagin maximum principle is a commonly used numerical method for solving continuous or discrete input models and for optimum control systems [7]. Dynamic programming algorithm has been used to solve the optimal train curve with minimum energy consumption [8-10].

In summary, dynamic programming is typically used in most control systems. However, as the solving time and complexity of dynamic programming algorithm increase exponentially with the increase of discrete points, how to reduce the number of separation points directly affects the complexity and time of the algorithm.

2. Modeling context

Train operation is affected by line conditions (slopes, speed limits, etc.), train characteristics (load, motor characteristics, etc.) and processing requirements (safety, punctuality, comfort, etc.).

This chapter takes the total energy consumption of train traction as the optimization goal and establishes the train optimization operation model. In the case of a given train run time, the best energy-saving maneuver sequence (optimal speed-distance speed profile) for the train running between the two stations is found.

2.1. Vehicle motion modeling

According to the known timetable, the inter-station interval time $T_{\text{max}}$ of the train is obtained.

The general equation of motion of a train is called Lomonossoff’s equation, as shown in eqn (1).

\[
(1 + \gamma)M \frac{d^2s}{dt^2} = F - \left( A + B \frac{ds}{dt} + C \frac{d^2s}{dt^2} \right) - Mg \sin(\alpha)
\]

where $F$ is the tractive effort or braking effort if applicable within the adhesion limit; $A$, $B$, and $C$ are Davis constants; $M$ is the mass; $\gamma$ is the rotational inertia constant; $t$ is the dependent element time; $s$ is the instant distance of the train; $\alpha$ is the slope angle.

2.2. Object Function

In this paper, a model based on condition decomposition is used to calculate the traction energy consumption of trains. According to the description of energy-saving operation of train operation, the objective function is defined as minimizing energy consumption. In order to ensure the punctuality of train operation, time constraints are transformed into penalty functions.

\[
U(v_k, a_k, k) = E_k + \beta |T_k - T_{ek}|
\]

Where $E_k$ is the energy consumption which is calculated based on train trajectory composed of a set of candidate speeds $v_1, v_2, ..., v_n$ at preset positions. $T_k$ is the running time of the train station; $T_{ek}$ is the expected running time of the train in the stage; $\beta$ is the penalty factor in the penalty function to indicate the importance of punctuality in the optimization problem.

2.3. Graph representation

According to the eqn (1) a partial curve composed of the curves of the train under four modes (acceleration, braking, cruising, coasting) can be calculated at two points A and B as shown in fig 2. The intersection of the partial curves (C, D, E, F) is the new preset point of the velocity curve. The main principle of the graph expression is to generate forward and backward curves in four operating
modes for each speed at the preset point. As shown in fig.3, a acceleration curve is first generated from the initial preset position and each speed limit increase. The braking curve is generated in reverse from the end position and the position where each speed limit is reduced. The coasting curve is then generated from the brake curve inversion. Finally, the cruising curve is inserted at a predetermined discrete speed set.

Fig.1 Main principle of the graph representation

3. Dynamic programming

3.1. Introduction

The basic idea of dynamic programming algorithm is to decompose the problem to be solved into several sub-problems, which tend to solve multi-stage decision-making problems. \( v_k \) is the rate variable in the current decision-making stage, and the train in the next decision-making stage \( f_{k+1} \) is only related to the state of the current decision-making stage \( f_k \). All train states start from the initial state, which is the original point of the diagram.

3.2. Genotype generation

According to the state transition equation of the dynamic programming algorithm, the recursive function relation of the optimal index function value is as follows:

\[
\begin{align*}
\begin{cases}
    f_{k+1}(x_{k+1}) = \min \{v_{k+1}(x_{k+1}, u_{k+1}) + f_k(x_k)\} \\
    f_1(x_1) = 0
\end{cases}
\end{align*}
\]

In the formula, \( v_{k+1}(x_{k+1}, u_{k+1}) \) is the stage index value of the \( k + 1 \) stage, namely energy consumption, and \( f_k(x_k) \) is the cumulative optimal index function value from the first to the \( k \) sub-interval.

3.3. Solution Construction

The steps of determining the decision variables in the \( K \) stage to obtain the optimal value of the index function are as follows.

1) Dispersion of operating speed

When the train transits from the current state \( v_k \) to the next state \( v_{k+1} \), it is necessary to judge whether such a speed transition is effective before calculating the optimal index function. According to the corresponding speed limits of each stage, the state range of the next stage \( v_{k+1} \) is calculated by
using the maximum traction force and braking force, so as to determine the set of the state of the next stage \( v_{k+1} = i_{k+1} \Delta v \).

2) Calculating the Value of the Optimal Index Function

Because the stage division is short enough, the acceleration in the stage can be regarded as a constant value. In the stage, the \( E_{i,j} \), \( T_{i,j} \) between any two nodes can be expressed as:

\[
EC_k(i, j) = \frac{1}{2} M (v_{k+1,j}^2 - v_{k,i}^2) + M (w_{0k} + w_{f_k}) s_k
\]

(4)

\[
T_{i,j} = TC_k(i, j) = \frac{2s_k}{v_{k,i} + v_{k+1,j}}
\]

(5)

Where, \( w_{0k} \) is the basic resistance; \( w_{f_k} \) is the additional resistance. \( EC_k(i, j) \) represents the energy consumption of the train when it changes from state \( v_{k,i} \) to state \( v_{k+1,j} \).

3) Establishing the Optimal Index Function Matrix

According to the above deduction, formula (2) can be further expressed as the calculated values of each stage:

\[
U(v_{k,i}, v_{k+1,j}) = EC_k(i, j) + \beta [TC_k(i, j) - T_ek]
\]

(6)

where, \( U(v_{k,i}, v_{k+1,j}) \) is the value of the stage index function representing the state transition from node \( i \) to node \( j \) of stage \( K \) is presented.

4. Case study

Case 1: In order to verify the validity of the model proposed in this paper, simulation verification is carried out with reference to the case in document [11]. The prototype of the train traction system used is based on the Voyager model of the British Railway. The specific data of line information (speed limit and slope) are shown in Figure 3.

![Fig.3. Journey altitude and speed limit profile of the case1](image-url)
Table 1 Characteristics comparison between two algorithms applied on the journey with scheduled time of 2800s

| Algorithms | Mean value (kw·h) | time(s)     |
|------------|-------------------|-------------|
| GA         | 885               | 256.0137909 |
| DP         | 784.6             | 401.0416518 |
| GA-SPD     | 897.3             | 178.6874865 |
| DP-SPD     | 762.7             | 39.0113512  |

Using the algorithm described in this paper, the energy-saving optimization curve of train operation is solved by programming code with MATLAB software. Considering the influence of actual slope and speed limit, combined with train operation strategy, the target speed curve of train can be obtained through dynamic programming algorithm and genetic algorithm respectively in the route of timed 2800s, as shown in Fig. 4-5. The total energy consumption of train operation and the calculation time of the algorithm are shown in Table 1. The results show that the dynamic programming algorithm has better performance in the application of this model, which can further save energy by 2.87138%.

Case 2: The validity of the proposed model and method is verified by simulating the line data and train parameters between stations of Beijing Yizhuang Metro Line. The train parameters are shown in Table 2, and the speed limit and slope conditions are shown in Figure 6. Unlike the case 1, the case 2 is a long distance route. The change of line information is relatively small.

Table 2 Parameters of train

| Name            | Characteristics                        |
|-----------------|----------------------------------------|
| Mass/t          | 280                                    |
| Maximum speed (km/h) | 80                                      |
| Maximum acceleration (m/s²) | 1.45                                   |
| Basic resistance /N | f=2.031+0.0622v+0.00807v²              |
| Traction /N     | F=320-0.4779V                          |
| Braking force /N | B=0.372((17v+100)/(60V+100))           |
Table 3 Characteristics comparison between two algorithms

| Algorithms | Mean value (kw·h) | time(s) |
|------------|-------------------|---------|
| DP         | 21.8473           | 37.097541 |
| DP-SPD     | 20.5950           | 1.831980  |

The train speed curve based on condition decomposition is shown in Fig. 7. The original model and condition decomposition model are solved by dynamic programming, and the results are shown in Fig. 8-9. Table 3 shows that the total energy consumption per unit is reduced by 1.2523 kw·h and the time is reduced by 35.265561 s, which saves energy consumption by 6.0806%.

5. Conclusions
In this paper, the minimum energy consumption of trains is taken as the control objective, and the maximum principle is used to analyze the four control conditions of trains. The train route information (speed limit and gradient) is innovatively regarded as input information. By simulating the four working conditions of the train, the train route is initially planned, and the graph expression model of the train and the number of discrete points are obtained. Based on the optimal control theory, the dynamic programming algorithm is used to solve the complete train speed curve, and the effectiveness of the algorithm is verified by two simulation cases. The experimental results show that:
1. Considering more real line information and condition switching factors, the algorithm can effectively reduce the frequency of condition switching in the short distance operation of trains, which does not allow frequent condition switching.

2. The algorithm can solve the optimal running curve of the train, and can minimize the energy consumption of the whole train operation under the constraints of guaranteeing the punctual and speed-limited operation of the train. Moreover, the computational time of the algorithm is greatly reduced.

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