Analysis of wind coherence in the longitudinal direction using turbine mounted lidar

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Abstract. The use of lidar for wind turbine control relies on the hypothesis that the wind coherence in the longitudinal direction is sufficient to derive useful information from wind measurements at upstream locations. We examined wind coherence using measurements of longitudinal wind speed obtained using a forward facing pulsed turbine mounted lidar over three sites. The effect of different atmospheric conditions was evaluated. We found that the coherence decay parameter increases together with turbulence intensity, while the parameter defining the coherence at 0Hz increases with integral length-scale. A correction was applied for artificial coherence decrease due to lidar measurement effects. Results display encouraging similarities with a computational study carried out based on Large Eddy Simulation.

1. Introduction

Wind field prediction ahead of wind turbines can provide preview time for pre-emptive control [1]. However, as depicted in Figure 1, a flow disturbance represented as a wave-packet is likely to evolve as it travels toward the turbine when convected by the mean flow. Taylor’s frozen turbulence hypothesis [2], which assumes that characteristics of turbulent fluctuations change little as they travel, may not always hold in practice. First, loss of correlation should increase when eddy characteristic time scale decreases and when spatial separation increases. Second, in atmospheric conditions where coherence is high, a disturbance remains almost unchanged as it travels, while for low coherence the disturbance evolves significantly.

Upwind facing lidar, which allow measuring turbine inflow, have the potential to allow loads reduction and AEP benefits. But one individual raw lidar measurement provides limited information: it samples at a specific range the projection of the wind vector onto a lidar beam pointing in a given direction. Commercially available lidar provide sequential measurements across different beam directions and ranges, which can typically be repeated every second. It is then required to make use of several lidar raw measurements in order to obtain an estimate of wind quantity such as rotor average speed or direction. In this context, wind reconstruction models which account for the travel time needed for a disturbance to reach the turbine need to be applied to raw lidar measurements in order to reconstruct wind quantities relevant for turbine controls [3]. Thus, having more adequate models of wind coherence is important to define adequate lidar measurement configurations [4] and develop lidar assisted turbine controls [5].

Bossanyi [6] proposed a method to simulate unfrozen turbulence with a coherence model suggested by Kristensen [7], and other authors have used Davenport’s model [8]. The potential drawback of both models is that they assume that coherence tends to unity when the temporal frequency $f$ tends to 0.
This, however, is not what observations show. Using LES wind fields, Simley and Pao [9] showed that, differently from the mentioned models, the coherence tends to values lower than 1 when the frequency approaches 0 Hz. For this reason they evaluated the coherence model proposed by [10], which is usually used to describe rotor plane coherence in IEC 61400-1 Edition 3 [11]. With this model, coherence does not tend to 1 as $f$ tends to 0, and the model uses two parameters to tune the coherence decay and its level in the very low frequency range.

Using Large Eddy Simulation (LES), Simley and Pao [9] observed that the parameter which governs the decay of coherence with reduced frequency $f\Delta r/U$, increased linearly with turbulence kinetic energy. This could be explained by the fact that Taylor’s hypothesis is usually valid only in the limit of low turbulence intensity ($TI$). Indeed, the convection time scale across $\Delta r$ is $\tau_{conv} = \Delta r/U$, and the life time scale for an eddy of size $\lambda$ and characteristic speed $\sigma$ is $\tau_{eddy} = \lambda/\sigma$. Thus, writing $\lambda = U/f$ and assuming $TI = \sigma/U$, one can show that the ratio of convection time scale to the eddy life time for a frequency $f$ is $\tau_{conv}/\tau_{eddy} = TI f\Delta r/U$. For a given frequency $f$ and separation $\Delta r$, validity of Taylor’s hypothesis requires that this ratio remains much smaller than one, such that the eddy can be considered frozen as it travels. As a result for a fixed $f$ and $\Delta r$, in the case of high $TI$ the eddy time scale can become smaller than the convection time scale across $\Delta r$, which means that Taylor’s hypothesis is less valid than for lower $TI$. In some sense, this scaling shows that coherence should decrease with frequency and $TI$ in a similar manner. The other model constant in [9], defining the coherence at 0Hz, was found to be proportional to the inverse of the integral length-scale $L$ of turbulence. This result suggests that there is less low frequency coherence loss as a function of separation in the case of large turbulence length-scales.

![Figure 1](image.png)

**Figure 1.** Diagram depicting the departure from frozen turbulence for a longitudinal wind fluctuation $u$, which evolves as it travels towards a wind turbine.

The investigation of Simley and Pao [9] is based on simulation and findings have not yet been compared to full scale experiment. If coherence in the longitudinal direction was previously investigated with nacelle lidar [12,13], validating a model such as proposed by [9] would still benefit from longer measurement periods and additional sites or environmental conditions. The present paper aims at providing a statistical description of coherence of the longitudinal wind component based on experimental data. The first part provides a description of the experimental data-set, and lidar measurements. The second part describes the methodology used to assess the coherence model. A correction to account for the lidar measurement effects is proposed. The last part focuses on the comparison of the estimated coherence model parameters to the results obtained by LES [9].

2. **Experimental data-set**

2.1. **sites and lidar setup**

In the present work, we assess three sites where a five beam forward facing pulsed Doppler lidar was mounted on a GE wind turbine, with hub-heights ranging from 70 to 85m and rotor diameters ranging from 100 to 120m. Information on the sites and associated data used for the present analysis is provided in Table 1. The turbine mounted lidar characteristics are described in [14], and the present work is based only on the data from this lidar’s central beam. This beam is aligned facing upwind, and acquires data at 0.8Hz for 10 ranges with 15m increments starting from 50m upstream of the turbine.
The averaging time during which lidar pulses are accumulated for each raw lidar measurement corresponds to 0.25s.

### Table 1. Summary of the sites.

| Type                          | Valid 400s data points |
|-------------------------------|------------------------|
| Site 1 North European coastal flat site | 4424                   |
| Site 2 North American flat forested site | 4562                   |
| Site 3 Complex terrain desert site | 548                    |

### 2.2. Lidar data processing and verification

In the following, the reference wind speed $U$ is defined using the lidar central beam measurement located 110m upstream of the turbine. $U$ is then used to evaluate all wind statistics such as $T_l$, $L$ and power spectral density (PSD). This measurement range represents approximately half of the 2.5 rotor diameter recommended distance to measure free stream wind speed, but in the case of Site 3 it was preferable to use this range. Indeed, for this complex terrain site, significant terrain height variations occur for long measurement ranges. On the other hand, measuring closer than 110m would result in stronger induction effects. To remain consistent, this 110m reference distance was used for all sites.

The dataset is filtered to remove waked wind sectors, and retain only periods of normal wind turbine production above 100kW, with yaw misalignments less than 20°, which removed significant data below 4m/s. Time periods of 400s where lidar availability is lower than 30% for any range are removed to insure sufficient data availability across all ranges from the lidar central beam. Time periods of 400s with 50% overlap were selected in order to maximize statistical convergence required for wind spectrum analysis. This does not allow resolving very low frequencies, but they are not of primary interest for turbine control. Over these 400s samples, statistics such as the average wind value and its standard deviation are still similar to the usual 600s (10min) quantities.

![Figure 2](image-url). Comparison of lidar longitudinal wind speed measured 110m (~1D) upstream to 2.5D met. mast horizontal wind speed on Site 1. [Left] scatter plot of average over 400s. [Right] scatter plot of standard deviation over 400s.

A comparison of the 400s mean and standard deviation of the lidar central beam wind measurement against the same quantities obtained from a met. mast mounted cup was then performed for Site 1 and is provided in Figure 2. The met. mast was equipped with a hub-height cup located at 85m above the ground. It can be observed that the lidar central tends to underestimate the wind speed. This can be due to the relatively close measurement range which results in observable flow induction effects,
especially for lower wind speed. A finer analysis of this effect would require a careful control of measurement height. Another cause could be the presence of yaw misalignments, which reduce lidar projected wind speed compared to cup horizontal wind speed. However, neither correction was implemented, as the correlation of average lidar central beam to the met. mast reference was found to be satisfactory for the purpose of the present study. Finally, the comparison of lidar measured standard deviation to cup standard deviation also indicated that it was suited to determine TI for the purpose of classification of different flow regimes.

For the purpose of spectral analysis, Lidar data processing is decomposed in several steps. First, non-available lidar data points due to blade passage are interpolated from neighbors at the lidar sampling 0.8 Hz rate. The time series are de-trended before performing spectral analysis using 400s windows with 50% overlap. For each site, spectral statistics such PSD and cross-spectra are averaged according to bins of 1m/s for wind speed and 5% for longitudinal turbulence intensity (TI) measured by the same lidar beam. For spatial cross-spectrum, averaging is also performed across pairs with constant spatial separation.

The integral length-scale is defined as:

$$L = U \int_0^\infty \langle u(t)u(t+\tau) \rangle d\tau$$ (1)

Here, $u$ is the wind speed fluctuation from the central beam longitudinal, measured at a given range. The symbol $\langle . \rangle$ represents the ensemble average over a given bin. A computation of the auto-correlation of the Lidar longitudinal wind speed from an inverse Fourier transform of the PSD is employed as a first step to estimate the integral length-scale. $L$ is then determined by integration of the auto-correlation until the first zero-crossing, consistently with [9]. The agreement with length-scales estimated with the met. mast cup on Site 1 is shown in Figure 3.

![Figure 3](image-url)  
**Figure 3.** Comparison between the integral length-scale $L$ estimated from eq. (1) using the lidar central beam and the hub-height met. mast mounted cup available on Site 1.

### 2.3. Wind sites

The objective of this section is to build a basic understanding of the flow conditions across the sites. A bulk Richardson number ($Ri$) was used to evaluate atmospheric stability in Site 1 [15]. Figure 4 (left) represents for this site the dependency of $L$ to $Ri$. It illustrates that positive $Ri$, which characterizes stably stratified atmospheric conditions, tends to be correlated to smaller length-scales. This is expected, as stable atmospheric conditions tend to suppress large scales of turbulence [15]. This also suggests that smaller length-scales are likely to occur when turbulence is low, which is confirmed across all sites Figure 4 (right). For a given turbulence level, it can finally be observed that the length-scale increases with wind speed, an observation which has been previously reported by other authors [16]. Overall, the range of observed length-scales, from 40 to 150m, and TI, from 5 to 20% values, illustrates the varied nature of the wind conditions over these 3 sites.
3. Methodology

3.1. Longitudinal coherence

Coherence $\gamma$ of longitudinal wind speed fluctuations separated by a distance $\Delta r$ along the longitudinal direction is presently defined as a function of frequency by eqs. (2) and (3), where $\Delta r = |r_i - r_j|$.

$$S(r_i, r_j, f) = \langle \hat{u}(r_i, f) \hat{u}(r_j, f) \rangle$$

(2)

$$\gamma(\Delta r, f) = \frac{s(r_i r_j, f) s(r_i r_j, f)}{||s(r_i r_i, f)|| ||s(r_j r_j, f)||}$$

(3)

In the above, $\langle \hat{u}(r_i, f) \hat{u}(r_j, f) \rangle$ represents the Fourier transform of a time series, $\langle \hat{u}(r_i, f) \hat{u}(r_j, f) \rangle$ is the complex conjugate, $r_i$ is a given range along the central beam. Coherence is then determined according to eq. (3) for separations of 30, 60 and 90m. Ensemble average is extended to $r_i$ and $r_j$ pairs with identical separation.

3.2. Coherence model

Figure 5 shows two examples of coherence for Site 1 at 8 m/s. A general observation is that coherence decreases either when $f$ or $\Delta r$ increases. One can notice also that an increase in $TI$ also results in a faster coherence drop as a function of frequency. This could be explained by the reduced validity of Taylor’s hypothesis for higher $TI$, as detailed in the introduction. In addition, for higher $TI$ we observe an increase in low frequency coherence as $f$ tends to 0. This can be explained as the increase of $TI$ is associated to an increase of $L$, consistently with the observation reported in section 2.3.

In order compare the coherence model trends found by Simley and Pao using LES [9] with those resulting from full field measurements, the coherence model indicated in equation (4) was fitted to the dataset introduced previously:

$$\gamma(\Delta r, f) = \exp \left( -a \sqrt{\left( \frac{f \Delta r}{U} \right)^2 + (b \Delta r)^2} \right)$$

(4)

In the above equation, parameter $a$ controls the decay of coherency as a function of reduced frequency, while $b$ governs the level of low frequency coherency as a function of spatial separation.

As in [9], $a$ and $b$ are determined by a fit across all separation distances simultaneously, while weighting different frequencies by the energy in the measured spectrum. The fitting error is plotted in
Figure 6 as function of the number of 400s samples in the bin. In the following, to maintain a good compromise between available data and sufficient statistical convergence, only bins which contain more than 30 independent 400s samples were considered.

Simley and Pao [9] present the evolution of a as a function of $\sigma/U = \sqrt{\frac{TKE}{U}}$. Here $TKE = \frac{1}{2}(\sigma_u^2 + \sigma_v^2 + \sigma_w^2)$ denotes turbulent kinetic energy derived from the 3 wind components. Our lidar setup does not allow to directly measuring the vertical and transverse flow standard deviation $\sigma_v$ and $\sigma_w$, but only the longitudinal component $\sigma_u$, which is also used to define $TI$. In order to compare our results to [9], we derived three expressions for $\sigma$ based on typical anisotropy relationships between $\sigma_v$ or $\sigma_w$ and $\sigma_u$. The first assumption is recommended by the IEC [11] and representative of neutral atmospheric conditions ($\sigma_v = 0.8\sigma_u$ and $\sigma_w = 0.5\sigma_u$), the second of unstable atmospheric conditions ($\sigma_v = 0.8\sigma_u$ and $\sigma_w = 0.9\sigma_u$) and the third of stable atmospheric conditions ($\sigma_v = 0.6\sigma_u$ and $\sigma_w = 0.3\sigma_u$). Using a spectral model, it could be possible to use the other four beams of the lidar to evaluate from the measurements the fluctuations of $v$ and $w$, but this falls outside the scope of the present work.

**Figure 5.** Coherence as a function of frequency at site 1 for $U=8$ m/s for 30m, 60m and 90m separations. Curves are fitted according to eq. 4. [Left] $TI=5\%$ bin. [Right] $TI=15\%$ bin.

**Figure 6.** Convergence analysis from the fit of equation (4) to the measured coherence as a function of the number of samples available for the considered atmospheric condition.

### 3.3. Lidar measurement effects

Lidar measurement process has specificities which have been extensively described in previous studies [17] and probe length averaging is one of the important effects relevant for the present study. The effect can be expressed as the convolution of the beam projected wind field with a measurement weighting function, which will presently be considered to be Gaussian. The characteristic length of
this function can be defined as its half-width, and is also called probe length. For the present pulsed lidar, the probe length specification is 30m. It is easy to understand that if the probe length is much smaller than the integral length-scale (the characteristic length of the wind signal) then the convolution will have little effect on the measurement. On the other hand, if the probe length is comparable or larger than the integral length-scale, then the spatial convolution will cause significant filtering of higher spatial frequencies. Visual comparison of the lidar and cup variance normalized spectrum allows illustrating the effect, and selected examples are shown in Figure 7. Here, temporal frequency \( f \) is converted to spatial wavenumber \( k = \frac{2\pi f}{U} \) to facilitate comparison of different wind speeds. As could be expected, low spatial frequencies are in good agreement but wave numbers above 0.1 rad/m begin to suffer from probe length filtering. Interestingly, the characteristic wave-number for the probe length is close to 0.2 rad/m. This effect is specifically visible for \( TI=5\% \) and \( TI=15\% \) for the 6m/s case, and this low wind speed sensitivity can be explained by the fact that \( L \) remains moderate or small in this case (from 40 to 80m) according to Figure 4 (right). On the other hand, for 12m/s, \( L \) varies with more amplitude and is close to 50m for \( TI=5\% \) while it can reach up to 140m in for \( TI=15\% \). This explains that filtering still affects the \( TI=5\% \) case, while it is difficult to identify an effect for \( TI=15\% \).

![Figure 7](image_url)

**Figure 7.** Comparison of lidar longitudinal wind speed to met. mast horizontal wind speed variance normalized PSD as a function of wave number \( k \) (rad.m\(^{-1}\)) for Site 1. [Top, Left] 6m/s and \( TI=5\% \) bin. [Top, Right] 6m/s and \( TI=15\% \) bin. [Bottom, Left] 12m/s and \( TI=5\% \) bin. [Bottom, Right] 12m/s and \( TI=15\% \) bin.

According to eqs. (2) and (3), measured coherence should decrease with increasing measurement signal to noise ratio. Thus, assuming a constant lidar measurement noise can lead to different bias as function of atmospheric conditions. For instance, when turbulence intensity is low (also correlated to smaller length-scales) the ratio of wind measurement variance over measurement noise variance becomes smaller, which can artificially and prematurely reduce measured coherence. This and the
above observations raises the importance of investigating the impact of the lidar measurement process on the estimation of coherence. A lidar measurement model was implemented to support the development of a correction function which could cancel the potential artefacts due to the lidar measurement principle. Details on the model are provided in the next section.

3.4. Lidar measurement model

The model applied to simulate lidar measurements within a synthetic wind generated from an IEC Kaimal spectrum [12] with varying length-scale, $T_l$, and convection speed, assumes frozen wind propagation and no yaw misalignment. First, a wind time trace is generated by the method of random phases from the Kaimal spectrum, to which lidar measurement effects are applied in temporal space. The list of measurement effects, together the implementation approach, is provided in Table 2.

| Specification | Effect                        | Modelling approach                                                                 | Value used        |
|---------------|-------------------------------|-------------------------------------------------------------------------------------|-------------------|
| Probe length  | Spatial averaging             | A Gaussian spatial weighting function is convoluted with the initial simulated wind time series. | 30m half-width    |
| Acquisition duration | Temporal averaging as the flow patterns travel | A rectangular temporal weighting function is convoluted with previously spatially averaged wind time series. | 0.25s width      |
| Sampling rate | Aliasing and limits the access to high frequencies | The convoluted time series is subsampled to mimic lidar beam scanning sequence. | 0.8Hz             |
| Measurement noise | Adds loss of coherence | Gaussian white noise is added to the sub-sampled signal. | 0.25m/s           |

As the initial signal for the simulation is frozen, it should have had unity coherency, had it been perfectly measured in the field. Our method leads us to estimate the loss of coherence due to measurement process by evaluating the coherence of the final synthetic signal. The inverse of this frequency dependent function, evaluated for each set of separation (30, 60 and 90m) and combination of $(U,T_l,L_l)$ is used as a multiplier to correct the experimentally measured coherence, prior to fitting the coherence model. To prevent excessive propagation of noise, this operation is performed only for the frequencies where the simulated coherency used for the correction is greater than 0.2. Also, corrected coherency is not allowed to be greater than 1.

4. Results

In this section, we assess the effects of $T_l$ and length-scale on the fitted $(a,b)$ parameters of the eq. (4) coherence model and compare the field data results to the results obtained LES [9]. Figure 8 (top) provides this comparison, without the use of the coherency correction for measurement effects based on the lidar model. When $a$ is represented as a function of $T_l$, one can observe an important scatter combined with a tendency for over-estimating the simulation results, especially for low $T_l$. Coherence decay seems to be higher for $T_l=5\%$ compared to $T_l=15\%$, which is not consistent with our analysis for the validity of Taylor’s hypothesis. The magnitude for parameter $a$ tends to be closer to the findings of [9] for higher $T_l$. In contrast, the evolution of parameter $b$ as a function of integral length-scale follows rather well the behavior predicted by the simulation.
We observe that measured results obtained from LES. Trends showing the decrease of coherence with reduced frequency and from Taylor’s hypothesis analysis is recovered, even though important scatter remains for low TI. However, the agreement with the LES for b and L degrades for lower integral scales, and suffers in general from higher scatter. We observe that measured b is now higher for lower length-scales, which could be due to the coherency correction procedure which caps above unity coherency after the correction process. As suggested by a referee, a more complete analysis of the correction including a parametric study of effects of the lidar model on the coherence correction would be valuable. Also, an analysis of how to best perform corrections based on simulated but unfrozen wind data would be useful in order to better understand potential a and b bias when estimated from measurements.

5. Conclusion

Longitudinal wind coherence in the longitudinal direction was assessed using a turbine mounted forward-facing lidar measuring across different ranges, using data from three sites. Sensitivity of coherence to turbulence intensity and integral length-scales was evaluated and compared to previous results from LES simulation [9]. A method to compensate for lidar measurement effect was developed and applied to the dataset which was analysed. Using this correction function improved the agreement with the results obtained from LES. Trends showing the decrease of coherence with reduced frequency

Figure 8. [Top, Left] Coherence model parameter a as a function of longitudinal turbulence intensity, colored by integral length-scale L. [Top, Right] Coherence model parameter b as a function of integral length-scale L, colored by TI. The area of each data point is proportional to the number of samples (see Figure 4). All wind speeds bins with more than 30 independent 400s samples are represented. [Bottom] Dependency of a and b with use of corrected coherence data according to lidar measurement effects.
for increasing turbulence intensity, as predicted by Taylor’s hypothesis, were observed. In addition, observations confirmed a non-unity low frequency coherence value which further reduces for shorter integral length-scales.

For future studies, it would be beneficial to consider measurement systems allowing longer spatial separation and higher repeating frequency of the same central beam by disabling measurements along the other beams which are not needed for such an investigation. In addition, more development of the coherence correction obtained from the lidar measurement model would be beneficial.

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