Same initial states attack in Yang et al.’s quantum private comparison protocol and the improvement

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Abstract In Yang et al.’s literatures [J. Phys. A: Math. 42, 055305, 2009; J. Phys. A: Math. 43, 209801, 2010], a quantum private comparison protocol based on Bell states and hash function is proposed, which aims to securely compare the equality of two participants’ information with the help of a dishonest third party (TP). However, this study will point out their protocol cannot resist a special kind of attack, TP’s same initial states attack, which is presented in this paper. That is, the dishonest TP can disturb the comparison result without being detected through preparing the same initial states. Finally, a simple improvement is given to avoid the attack.

Keywords Quantum cryptography · Quantum computation · Quantum private comparison · Same initial states attack

1 Introduction

The principles of quantum mechanics, such as no-cloning theorem, uncertainty principle, and entanglement characteristics, provide some interesting ways for cryptography communication and secure computation. During the past thirty years, quantum communication has developed in a variety of directions, including quantum key distribution (QKD) [1, 2], quantum secret sharing (QSS) [3, 4], quantum direct communication (QDC) [5-7], quantum

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teleportation (QT) [8, 9], etc. On the other hand, secure multi-party computation (SMC) has also been discussed in the quantum mechanism. Many special SMC problems have been solved in quantum setting, for instance, quantum private comparison (QPC) [10-19], quantum protocols for millionaire problem [20, 21], quantum voting [22, 23] and quantum auctions [24, 25].

As an important branch and the foundation of quantum secure multi-party computation (QSMC), QPC has attracted more and more attention. The main goal of QPC is to compare the equality of secret inputs between two participants without disclosing any information about each other’s secret content. The pioneering QPC protocol was proposed by Yang et al. [10] in 2009, and then in 2010, they revised the protocol by removing the step (4) which is in fact unnecessary [26]. Enlightened by Yang et al.’s work, more QPC protocols are proposed subsequently [11-18].

However, while revisiting Yang et al.’s literature [10] and its corrigendum [26], we find there is a loophole undiscovered up to date, and the dishonest TP can prepare the same initial states instead of original random EPR pairs to confuse the two participants’ comparison result (we call it as TP’s same initial states attack). Under this kind of attack, TP is likely to make the participants get the wrong result without being detected, which directly results in the failure of private comparison. What’s more, TP can know the comparison result of the participants, which is contrary to what they claimed: “TP cannot learn any information about the participants’ respective secret inputs and even about the comparison result”.

The structure of this paper is organized as follows. At first, Yang et al.’s protocol is briefly reviewed in Section 2. In Section 3, TP’s same initial states attack is introduced and the loophole is pointed out, and then a simple improvement is given as well. Finally, a brief summary is concluded in Section 4.

2 Review of Yang et al.’s protocol

In Ref. [10], Yang et al. presented a QPC protocol based on Bell states and hash function, which aims to securely compare the equality of two participants’ secret inputs (x, y). It should be noted that the third party (TP) is assumed to be dishonest. However, they find the step (4) is in fact unnecessary, and the protocol security will be ensured by the security check in steps (5) and (6). In order to improve the efficiency of the original protocol, Yang et al. removed the first security check in step (4) in Ref. [26]. Combining the thought of Ref. [10] and its corrigendum [26], the main procedures can be described as follows (also shown in Figure 1).

**Step 1.** Bob, Charlie and TP agree that these four unitary operations $U_i (i = 00, 01, 10, 11)$ represent two-bit information ‘00’, ‘01’, ‘10’ and ‘11’,
respectively.

\[
\begin{align*}
U_{00} &= I = |0\rangle\langle 0| + |1\rangle\langle 1|, \\
U_{01} &= \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|, \\
U_{10} &= \sigma_x = |1\rangle\langle 0| + |0\rangle\langle 1|, \\
U_{11} &= i\sigma_y = |0\rangle\langle 1| - |1\rangle\langle 0|.
\end{align*}
\] (1)

At the same time, Bob and Charlie share a secret hash function \( H \) beforehand, the hash values of \( x, y \) are \( H(x) = (x_{M-1}, x_{M-2}, \ldots, x_0) \), \( H(y) = (y_{M-1}, y_{M-2}, \ldots, y_0) \), respectively, and \( M \) denotes the length of the hash values.

**Step 2.** TP prepares \( n(n > M/2) \) EPR pairs, each of which is randomly one of the four Bell states \( |\Phi^\pm \rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \), \( |\Psi^\pm \rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) \). Later, TP divides these \( n \) EPR pairs into two sequences \( T_B \) and \( T_C \), which are composed of the first and the second particles of each Bell state, respectively. Then TP randomly selects decoy photons \( |0\rangle, |1\rangle, |+\rangle, |-\rangle \) and inserts these decoy photons into \( T_B \) and \( T_C \) at random positions, respectively. Finally, TP sends the two sequences to Bob and Charlie.

**Step 3.** Having received all particles, Bob and Charlie check whether there is any eavesdropper in quantum channel by measuring these inserted decoy photons. If there is no eavesdropper, they continue to the next step; otherwise, they will terminate the protocol.

**Step 4.** Bob and Charlie perform unitary operation on the remaining particles according to \( H(x) \) and \( H(y) \). They perform \( U_B \) and \( U_C \) on the \((k + 1)\)th photon, respectively.

\[
U_B = \begin{cases} 
U_{x_{2k}x_{2k+1}} & \text{if } M \text{ is even} \\
U_{x_{2k-1}x_{2k}} & \text{if } M \text{ is odd}
\end{cases} \quad (2)
\]

\[
U_C = \begin{cases} 
U_{y_{2k}y_{2k+1}} & \text{if } M \text{ is even} \\
U_{y_{2k-1}y_{2k}} & \text{if } M \text{ is odd}
\end{cases} \quad (3)
\]

where \( k = 0, 1, \ldots, \lfloor M/2 \rfloor \) and \( x_{2k}^\prime x_{2k+1}^\prime, x_{2k-1}^\prime x_{2k}^\prime, y_{2k}^\prime y_{2k+1}^\prime, y_{2k-1}^\prime y_{2k}^\prime \) are their hash values’ bits of secret inputs. In order to check whether TP will cheat in the following announcement of his measurement outcomes, Bob and Charlie should make some disarrangement operations as follows, firstly they secretly generate a random number \( l \) by using the QKD method, and then insert the remaining intact EPR photons into the encoded photon sequence at the positions determined by the value of \( l \), respectively. Besides, Bob and Charlie require TP to publish the states of the remaining EPR pairs beforehand. For checking eavesdropping in the Bob-TP and Charlie-TP quantum channels, Bob and Charlie choose some decoy photons randomly from \( \{|0\rangle, |1\rangle, |+\rangle, |-\rangle\} \) and insert them into EPR photon sequences at random positions. Finally, they send the sequences back to TP.

**Step 5.** After received all particles, TP, Bob and Charlie check the security of their quantum channels using the same method as Step 3. If they confirm no eavesdropping, TP takes the Bell-basis measurement on each two correlated photons received from Bob and Charlie, records these measurement outcomes.
and publishes his initial states of EPR pairs except for eavesdropping check. These measurement outcomes are composed of two sets: the set of the sampling EPR pairs’ measurement outcomes $C$, and the set of the encoding EPR pairs’ measurement outcomes $M$. It should be noted that TP cannot know the information about which set each measurement outcome of EPR pair belongs to. Bob and Charlie choose a subset of the positions from one of the two sets randomly and ask TP to publish the measurement outcomes at these positions. For the positions chosen from $C$, if the inconsistency rate between the measurement outcomes and TP’s beforehand announcements is higher than the threshold, Bob and Charlie know that TP is cheating and abort the protocol. Otherwise, they continue to choose a position subset randomly from one of the two sets and ask TP to public the measurement outcomes. For the positions chosen from $M$, Bob and Charlie can deduce the comparison result according to TP’s measurement outcomes and their initial states.

According to the above description, it is not difficult to find Yang et al.’s protocol has the following features:

1. The protocol is based on EPR pairs and hash function. The secret inputs are encrypted by one-way hash function firstly, and then encoded into EPR pairs by using local unitary operations.
2. TP is dishonest, which is different from most of the other QPC protocols [11-18] that assume TP is semi-honest. In addition, TP is demanded to have no knowledge of two participants’ private information and even the comparison result.
3. The whole ERP pairs sequence is composed of two sets: the encoding set ($M$ is the corresponding measurement outcomes) and the sampling set ($C$ is the measurement outcomes). The first set is used to compare the two participants’ secret inputs, while the other set is applied to check whether TP is honest. In addition, In order to prevent TP from knowing the positions of the sample set and further cheating the participants without being detected, the disarrangement operations on the EPR photons are indispensable in the process of Yang et al.’s QPC protocol.
In the protocol, quantum superdense coding [27] method is utilized to conceal private information. Because TP needs to publish both the initial states and the final measurement outcomes of EPR pairs, the participants can deduce the unitary operations that the counterparty used (i.e., the corresponding hash value), but they cannot know the actual bit value of the secret inputs due to the characteristic of one-way hash function. It is ostensible that the scheme is secure, but the security is mainly based on the classic cryptography, i.e., the one-way hash function, instead of the quantum cryptography mechanism. Moreover, since TP is assumed to be dishonest, so he/she can make any attacks. That is to say, TP is likely to launch the same initial states attack to make the participants get wrong comparison result without being detected, and more details will be described in Section 3.

3 TP’s same initial states attack and the improvement

3.1 TP’s same initial states attack on Yang et al.’s protocol

Suppose TP launches the same initial states attack on Yang et al.’s protocol, the detailed process is as follows: TP firstly prepares the EPR pairs sequence in the same Bell state (e.g., $|\Phi^+\rangle$) in Step 2, and the $|\Phi^+\rangle$ sequence is divided into two sequences, $T_B$ and $T_C$, which will be sent to Bob and Charlie, respectively. Secondly, TP will maliciously publish that the final measurement outcomes are the same as their initial states in Step 5. It is obvious that the disarrangement operations in Step 4 will lose the original effect under this special kind of attack. Since all the EPR pairs are the same, Bob and Charlie cannot detect TP’s cheating when he/she announces his/her measurement outcomes. And there are two cases as follows:

1. If the relation of Bob’s and Charlie’s secret inputs is $x \neq y$, but TP maliciously publishes that the measurement outcomes (including $C$ and $M$) are the same as the initial states he/she prepared (i.e., $|\Phi^+\rangle$) in Step 2. Since the measurement outcomes of $C$ are the same as the initial states, Bob and Charlie cannot perceive TP’s cheating behavior. Hence, on the basis of the relation between TP’s measurement outcomes and their initial states, they will mistakenly believe that their secret messages satisfy $x = y$, which means they get a wrong comparison result. What’s more, TP will know the correct comparison result because of knowing that the measurement outcomes are different from the initial states.

2. If $x = y$, and TP still publishes that the measurement outcomes are the same as the initial states, in this case, Bob and Charlie can gain the correct comparison result (i.e., $x = y$), but the result is also known by TP.

As discussed above, if TP launches the same initial states attack on Yang et al.’s protocol, whatever $x = y$ or not, he/she will always know the correct comparison result. What’s more, if $x \neq y$, Bob and Charlie will get wrong comparison result without being detected.
3.2 The improvement of Yang et al.’s protocol

To avoid TP’s same initial states attack, we can add a random unitary operation procedure to fix the loophole of Yang et al.’s protocol. The detailed process is as follows: In Step 4, before Bob and Charlie perform the disarrangement operations, they need to randomly choose one of four unitary operations $U_i$ (see Equation 1) to perform on the sampling EPR pair photons, respectively. That means, the final states of these sampling EPR pairs are not always the same as their initial states. In addition, in order to check whether TP is cheating or not, Bob and Charlie are required to publish the unitary operations on the sampling photons before TP announces the measurement outcomes in Step 5.

In the improved protocol, even if TP tries to launch the same initial states attack, the cheating will be detected because that the inconsistent rate among the random unitary operations, TP’s announcement of $C$ and their initial EPR pair states exceeds a rational threshold. In other words, the improved protocol is also secure against the other conventional attacks presented in Ref. [10].

4 Conclusion

In this paper, by revisiting and analysing Yang et al.’s QPC protocol, we firstly point out there is a security loophole in it. More specifically, TP may launch the same initial states attack to confuse the two participants’ comparison results, what’s more, the result will be revealed to TP. In order to fix the loophole, a simple improvement is proposed by adding a random unitary operation procedure before the disarrangement operations. Analysis shows the improved protocol can resist TP’s same initial states attack.

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