A CLASS OF SELF-GRAVITATING, MAGNETIZED ACCRETION DISKS

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ABSTRACT

The steady state structure of self-gravitating, magnetized accretion disks is studied using a set of self-similar solutions that are appropriate in the outer regions. The disk is assumed to be isothermal, and the magnetic field outside the disk is treated in a phenomenological way. However, the internal field is determined self-consistently. The behavior of the solutions is investigated by changing the input parameters of the model, i.e., mass accretion rate, coefficients of viscosity and resistivity, and magnetic field configuration.

Subject headings: accretion, accretion disks — hydrodynamics

1. INTRODUCTION

One of the key physical ingredients in accretion disks is self-gravity. It plays a significant role in many such systems, ranging from protostellar disks to active galactic nuclei (AGNs). The radial and vertical equations for the disk structure are significantly modified because of the possible impact of the self-gravity, although traditional models of accretion disks ignore self-gravity just for simplicity (e.g., Pringle 1981). Nevertheless, deviations from Keplerian rotation in some AGNs and the flat infrared spectrum of some T Tauri stars can both be described by self-gravitating disk models.

Because of the complexity of the equations and in order to obtain analytical results, authors have studied the effects related to disk self-gravity either in the vertical structure of the disk (e.g., Bardou et al. 1998) or in the radial direction (e.g., Bodo & Curir 1992). In a situation where we lack strong empirical evidence for the detailed mechanisms involved, Bertin (1997) proposed a new class of self-gravitating disks for which efficient cooling mechanisms are assumed to operate so that the disk is self-regulated in a condition of approximate marginal Jeans stability. Thus, the energy equation is replaced by a self-regulation prescription. He showed that in the absence of a central point mass, there is a set of self-similar solutions describing the steady state structure of such self-regulated accretion disks. The self-similar solution corresponds to a flat rotation curve, while the disk has a fixed opening angle. In fact, Bertin’s self-similar solution is a generalization of the self-similar solution for self-gravitating disks (Mestel 1963) dominated by viscosity.

Subsequent analysis confirmed the validity of this simple self-similar solution (Bertin & Lodato 1999). Moreover, such kinds of self-regulated accretion disk can successfully describe the spectral energy distribution of protostellar disks (Lodato & Bertin 2001). Recently, Lodato & Rice (2004) studied the transport associated with gravitational instabilities in a relatively cold disk using numerical simulations. They showed that the disk truly settles into a self-regulated state, in which the axisymmetric stability parameter $Q \approx 1$ and in which transport and energy dissipation are dominated by self-gravity. However, these studies neglected the possible effects of magnetic fields. Many authors tried to construct models for a magnetized disk either in a phenomenological way based on some physical considerations (e.g., Shvartsman 1971; Lovelace et al. 1994; Kaburaki 2000; Shadmehri 2004; Shadmehri & Khajenabi 2005) or by direct numerical simulations (Stone et al. 1999; McKinney & Gammie 2002). Bisnovatyi-Kogan & Lovelace (2000) suggested that recent papers discussing advection-dominated accretion flow (ADAF) as a possible solution for astrophysical accretion should be treated with caution, particularly because of our ignorance surrounding the magnetic field. Models of magnetized accretion disks with externally imposed large-scale vertical magnetic field and anomalous magnetic field due to enhanced turbulent diffusion have also been studied (see, e.g., Campbell 2000; Ogilvie & Livio 2001). These models are restricted to subsonic turbulence in the disk, and the viscosity and magnetic diffusivity are due to hydrodynamic turbulence.

Recently, we presented a set of analytical self-similar solutions for the steady state structure of a magnetized, radiation-dominated disk (Shadmehri & Khajenabi 2005). Although this analysis is just a simple model, one can see some possible effects of the magnetic field on the structure of radiation-dominated disks, at least at a fundamental level. However, we applied some simplifying assumptions concerning the magnetic field both outside and inside the disk. Our approach was based on interesting studies by Lovelace et al. (1987, 1994). We noticed that it is possible to take a similar approach in studying a magnetized, self-gravitating disk.

In this paper, we relax the self-regulation condition of Bertin (1997) simply by assuming an isothermal equation of state. We present a self-similar solution for the steady state structure of such a self-gravitating disk for which the effect of the magnetic field is taken into account. While the magnetic field outside the disk is studied in a phenomenological approach similar to that in Lovelace et al. (1994, hereafter LRN94), the magnetic field inside the disk is obtained self-consistently. We show that magnetic fields may cause significant changes in the typical behaviors of the solutions compared to the nonmagnetized case. The basic equations of the model are described in § 2. Self-similar solutions are obtained in § 3. In spite of the similarity of Bertin’s solutions (1997) to ours, there are differences between his work and the present analysis. A summary of the results are discussed in § 4.

2. GENERAL FORMULATION

The basic equations are integrated over the vertical thickness of the disk. The mass continuity equation becomes

$$-2\pi R \Sigma \varepsilon_R = \dot{M},$$

where $\varepsilon_R$ is the radial velocity and $\Sigma = \int dz \rho \simeq 2h\rho$ is the surface density of the disk. The half-thickness is denoted by $h$. 

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439
where we consider the magnetic field effect on the disk thickness. We see that the disk can be compressed or flattened depending on the field configuration. We also note that since the radial velocity is negative for accretion (i.e., $v_R < 0$), the accretion rate $M$ as an input parameter of our model is positive.

The radial momentum equation reads

$$
\Sigma v_R \frac{dv_R}{dr} = - \frac{P}{R} - \frac{\Sigma v^2}{R} - \frac{GM(R)}{R^2} + \int F^\text{mag}_R dz,
$$

where $P = \int d\hat{z} \rho$ is the integrated disk pressure and $M(R)$ is the mass of the disk within radius $R$. We are neglecting the mass of the central object compared to the disk mass. This is relevant to protostellar disks at the beginning of the accretion phase, during which the mass of the central object is small and self-gravity of the disk plays an important role. In addition, one may argue that our model corresponds to disks at large radii because the effects of the central mass become important in the outer regions of the disk. The term associated with self-gravity of the disk, i.e., $GM(R)/R^2$, is generally applicable to spherical geometry. In fact, the correct term takes on an integral form (see eq. [4] of Bertin 1997). But as we show, our solution describes a disk in which the surface density is inversely proportional to the radius. Interestingly, for such a disk, which resembles a Mestel disk, the correct gravitational force is known, which is exactly equal to our approximate formula (Binney & Tremaine 1987). It implies that we consider the exact gravity force in our equations. Now we can write

$$
\frac{dM(R)}{dr} = 2\pi R \Sigma.
$$

The last term of equation (2) represents the height-integrated radial magnetic force, which can be written as (LRN94)

$$
\int F^\text{mag}_R dz = \frac{1}{2\pi} (B_R B_z)_h - \frac{1}{4\pi R^2} \frac{d}{dr} \left[ h R^2 \left( B^2_{\varphi} - B^2_z \right) \right]
\frac{d}{dr} \left( h \left( B^2_{\varphi} \right) \right) + \frac{1}{4\pi} \frac{d}{dr} \left( h B^2_{\varphi} + B^2_{\varphi} - B^2_z \right)_h,
$$

where $\langle \ldots \rangle \equiv \int_h^a dz \langle \ldots \rangle/(2h)$ and the $h$ subscript denotes that the quantity is evaluated at the upper disk plane, i.e., $z = h$. Similarly, integration over $z$ of the azimuthal equation of motion gives

$$
R \Sigma v_R \frac{d}{dr} \left( R v_{\varphi} \right) = \frac{d}{dr} \left[ R^2 v_{\varphi} \frac{d}{dr} \left( \frac{v_{\varphi}}{R} \right) \right] + \int R^2 F^\text{mag}_{\varphi} dz,
$$

where

$$
\int R^2 F^\text{mag}_{\varphi} dz = \frac{1}{2\pi} \left( R^2 B_R B_{\varphi} \right)_h - \frac{1}{2\pi} \frac{d}{dr} \left( R^2 B_R B_{\varphi} \right)_h
\frac{d}{dr} \left( h R^2 \left( B_{\varphi} \right) \right).
$$

Here the last term of equation (5) represents the height-integrated toroidal component of the magnetic force multiplied by $R^2$.

In our model, we also assume (Shakura & Sunyaev 1973)

$$
\nu = \alpha c_s h,
$$

where $c_s$ is the local sound speed and $\alpha$ is a constant less than unity.

The $z$-component of the equation of motion gives the condition for vertical hydrostatic balance, which can be written as

$$
\frac{\Sigma}{h} v^2_z = \pi G \Sigma^2 + \left( \frac{1}{4\pi} \right) \left( B_R^2 + B_{\varphi}^2 \right) - \frac{h}{4\pi} \frac{d}{dR} \left( B_R^2 \right), \tag{8}
$$

where the term on the left-hand side of equation (8) corresponds to the thermal pressure. Since we neglect the central object, the self-gravity of the disk in the vertical direction leads to the first term on the right-hand side of equation (8). Now we can treat the internal magnetic field using the induction equation. LRN94 showed that the variation of $B_z$ with $z$ within the disk is negligible for even field symmetry. Moreover, $B_R$ and $B_{\varphi}$ are odd functions of $z$ and consequently $\partial B_R/\partial z \approx (B_R)_h/\hbar$ and $\partial B_{\varphi}/\partial z \approx (B_{\varphi})_h/\hbar$. Krasnopolsky & Königl (2002) applied similar configurations to study the time-dependent collapse of magnetized, self-gravitating disks. Thus,

$$
B_R(R, z) = \frac{z}{h} (B_R)_h, \quad B_{\varphi}(R, z) = \frac{z}{h} (B_{\varphi})_h,
$$

and the induction equation reads

$$
-R B_{\varphi} v_R - \frac{\eta R}{h} (B_R)_h + \eta R \frac{d}{dR} \left( B_R^2 \right) = 0, \tag{10}
$$

where the magnetic diffusivity $\eta$ has the same units as the kinematic viscosity. We assume that the magnitude of $\eta$ is comparable to that of the turbulent viscosity $\nu$ (e.g., Bisnovatyi-Kogan & Ruzmaikin 1976; Shadmehri 2004). Exactly in analogy to the $\alpha$ prescription for $\nu$, we are using a similar form for the magnetic diffusivity $\eta$,

$$
\eta = \eta_0 c_s h,
$$

where $\eta_0$ is constant. Note that $\eta$ is not constant and depends on the physical variables of the flow, and in our self-similar solutions, as we show, $\eta$ scales with radius as a power law. This form of scaling for diffusivity has been widely used by many authors (e.g., Lovelace et al. 1987; LRN94; Ogilvie & Livio 2001; Rüdiger & Shalybkov 2002).

While equation (10) describes the transport of a large-scale magnetic field [here $B(R)$], the values of $(B_R)_h$ and $(B_{\varphi})_h$ are determined by the field solutions outside the disk. Instead, we are following the approach of LRN94, in which the external field solutions obey the relations

$$
(B_R)_h = \beta_r B_z, \quad (B_{\varphi})_h = \beta_{\varphi} B_z,
$$

where $\beta_r$ and $\beta_{\varphi}$ are constants of the order of unity ($\beta_{\varphi} < 0$). Thus, one can simply show that $(B_R^2)_h = \beta^2_r B_z^2/3$, $(B_{\varphi}^2)_h = \beta^2_{\varphi} B_z^2/3$, and $(B_R B_{\varphi})_h = \beta_r \beta_{\varphi} B_z^2/3$. Note that these $\beta$-factors are really empirical scalings that now have abundant support from computer simulations of MHD and Poynting outflows from disks. Ustyugova et al. (1999) give a detailed analysis and provided strong evidence for $\beta_r \sim |\beta_{\varphi}| \sim 1$.

To close the equations of our model, we can write the self-regulation condition as has been done by Bertin (1997). But one should naturally expect another form of self-regulation prescription in the magnetized case. In fact, self-regulation results from a competition of dissipation and instabilities. Indeed, the instability
of a magnetized disk is different from that of an unmagnetized disk. Effects of the magnetic field on linear gravitational instabilities in two-dimensional differentially rotating disks have been investigated in detail by Elmegreen (1987, 1994), Gammie (1996), and Fan & Lou (1997). Axisymmetric and nonaxisymmetric perturbations show significantly different behaviors. While axisymmetric instability in a thin disk borrows its physical grounds from Toomre (1964) and requires \( Q < 1 \), the literature has lacked an explicit theoretical evaluation of the critical \( Q \) for nonaxisymmetric gravitational runaways. In order to avoid such difficulties one can consider the axisymmetric stability. The presence of the magnetic fields modifies the Toomre criterion for a hydrodynamic disk in such a way that magnetized disks are unstable to axisymmetric perturbation if \( Q_M = Q(1 + 1/\beta) < 1 \), where \( \beta \) is the ratio of the thermal pressure to the magnetic pressure (e.g., Shu 1992).

There is a fundamental point concerning the self-regulation prescription. When the disk mass becomes large enough to induce global instability there is nothing like self-regulation taking place. We see numerical simulations that describe a massive unstable disk (e.g., Bonnell 1994; Matsumoto & Hanawa 2003). However, considering existing numerical simulations, we think that it is not possible simply to say that all massive disks fragment because, to our knowledge, not only do current numerical simulations suffer from their own limitations, but they also do not generally give a fully consistent picture of global fragmentation. This situation becomes more complicated when we consider magnetic fields. It seems that there is a complicated interaction between gravitational instability and MHD turbulence that influences disk structure, but MHD turbulence reduces the strength of the gravitational instability (Fromang 2005).

Almost all numerical simulations show that global instabilities are highly dependent on the thermal state of the disk (Gammie 2001; Johnson & Gammie 2003). However, the thermal state of the disk has been neglected by some authors, so their approach is not a complete analysis. Simulations by Matsumoto & Hanawa (2003) that do not include detailed thermal evolution predict fragmentation in an early phase. But all these fragments are on tight orbits and are likely to merge due to disk accretion. Matzner & Levin (2005) argue analytically that viscous heating and stellar irradiation quench fragmentation and conclude that numerical simulations lead to fragmentation, which do not account for irradiation and are unrealistic. On the other hand, not necessarily all numerical simulations of massive disks lead to fragmentation. For example, Pickett et al. (2000) failed to obtain fragmentation in similar thermodynamics conditions. Thus, whether or not massive disks fragment due to global instability remains controversial. At this stage, what we can say is that even if a massive disk with no star in the center does not fragment, it does not mean that self-regulation is occurring in the disk. In order to avoid such difficulties, we relax the self-regulation condition and replace it by an isothermal assumption, i.e.,

\[
P = \Sigma c_s^2,
\]

where \( c_s \) is the constant sound speed. We hope our simple approach will be able to illustrate some possible effects of the magnetic fields on the structure of self-gravitating disks.

### 3. SELF-SIMILAR SOLUTIONS

Equations (1), (2), (3), (5), (8), (10), and (13) constitute the basic equations of our model for the steady state structure of a self-regulated magnetized disk. We can solve these equations numerically using appropriate boundary conditions. However, before doing such an analysis it would be illustrative to study the typical behavior of the solutions by applying semianalytical methods, e.g., self-similarity. Indeed, any self-similar solution contains part of the behavior of the system, in particular far from the boundaries. However, the main goal of our analysis is just to illustrate the possible effects of the magnetic field on the steady state structure of a self-gravitating disk. For this propose, we think that self-similar solutions are very useful.

It is then straightforward to find a self-similar solution with the radial dependence

\[
\frac{\Sigma(R)}{\Sigma_0} = a \left( \frac{R}{R_0} \right)^{-1},
\]

\[
\frac{v_r(R)}{V_0} = b,
\]

\[
\frac{v_\phi(R)}{V_0} = -c,
\]

\[
\frac{P(R)}{P_0} = d \left( \frac{R}{R_0} \right)^{-1},
\]

\[
\frac{B_z(R)}{B_0} = e \left( \frac{R}{R_0} \right)^{-1},
\]

\[
\frac{h(R)}{R_0} = f \left( \frac{R}{R_0} \right),
\]

\[
\frac{M(R)}{M_0} = q \left( \frac{R}{R_0} \right),
\]

where \( a, b, c, d, e, f, \) and \( q \) are numerical constants that can be obtained from the equations (see below) and \( R_0, \Sigma_0, P_0, V_0, \) and \( B_0 \) are convenient units that reduce the equations into nondimensional forms. We are assuming \( V_0 = c_s = (P_0/\Sigma_0)^{1/2} \) and \( M_0 = R_0^2 \Sigma_0 \).

If we substitute the above self-similar solutions in the main equations of the model, the following system of dimensionless algebraic equations are obtained, which are to be solved for \( a, b, c, d, e, f, \) and \( q \):

\[
ac = \dot{m},
\]

\[
-(ab^2 + d) = -2a^2 + \left[ 2b + \frac{2}{3}f \left( 3 + \beta_r^2 - \beta_\phi^2 \right) \right] c^2,
\]

\[
ab \left( \alpha f \sqrt{\frac{d}{a}} \right) = \beta_\phi \left( 2 - \frac{4}{3}f \beta_r \right) c^2,
\]

\[
\frac{d}{f} = a^2 + \left( f \beta_r + \beta_r^3 + \beta_\phi^3 \right) c^2,
\]

\[
c - \eta_0 \beta_r \sqrt{\frac{d}{a}} - \eta_0 \sqrt{\frac{d}{a}} f = 0,
\]

\[
d = a,
\]

where \( \dot{m}, \alpha, \eta_0, \beta_r, \) and \( \beta_\phi \) are the input parameters, and \( \dot{m} = M/(2\pi R_0 \Sigma_0 V_0) \) is the nondimensional mass accretion rate. The radial dependences of physical quantities both in the magnetized case and in the unmagnetized case are similar. After some algebraic manipulations, we can find a complicated algebraic equation for \( f \), and clearly only the real root is a physical solution. Having
the real root of this equation, the other physical quantities are determined using the above equations.

Now we explore the parameter space of the input parameters and their effects on the solutions. We restrict our study to values around unity for $\beta_r$ and $\beta_z$ (Ustyugova et al. 1999). Since we find that the sensitivity of the solutions on the parameter $\beta_z$ is not very strong, we plot all the physical variables as a function of $\beta_r$ for a fixed $\beta_z$. However, we comment about possible effects of variations of $\beta_z$ in Figure 3.

Figure 1 shows various physical variables as functions of $\beta_r$ with $\alpha = 0.1$, $\eta_0 = 0.1$, and $\beta_z = -0.8$, and the mass accretion rate is indicated on the plots. This figure helps us to see the dependence of the physical variables on the variations of the mass accretion rate $\dot{m}$. The top plots show the surface density (left) and the opening angle (right) for $\dot{m} = 0.5, 1.0,$ and $2.0$. We see that as the parameter of the radial component of the magnetic field at the surface of the disk increases, the surface density decreases for a fixed mass accretion rate, but that the surface density increases with increasing $\dot{m}$. In addition, as the mass accretion rate increases, the disk becomes thinner. However, the disk thickness increases with $\beta_r$ for a fixed accretion rate. The middle plots show typical behaviors of the radial (left) and the rotational (right) velocities.
While the ratio of the radial velocity to the sound speed increases with increasing $\beta_r$, the ratio of the rotational velocity to the sound speed decreases when $\dot{m}$ is kept constant. However, increasing the mass accretion rate causes the radial velocity to decrease. But for the rotational velocity, we see completely different behavior, i.e., the ratio $v_r/c_s$ significantly increases with the accretion rate $\dot{m}$.

Our self-similar solution generally corresponds to $\beta > 1$, where $\beta$ is the ratio of thermal pressure to magnetic pressure at the surface of the disk. The bottom left plot of Figure 1 shows this behavior, although $\beta$ decreases with $\beta_r$ for a fixed $\dot{m}$. However, if the accretion rate increases, the ratio of the thermal pressure to the magnetic pressure $\beta$ increases, which implies that the effect of the magnetic field on the structure becomes weaker. We have already discussed the Toomre parameter $Q$ and a modified version of this parameter, i.e., $Q_M$, because of the magnetic field. Figure 1 (bottom right) shows variation of both $Q$ (solid lines) and $Q_M$ (dotted lines) as function of $\beta_r$. Generally, we have $Q_M > Q$ because of the dynamical effects of the magnetic field, and as $\beta_r$ increases these parameters increase to values closer to unity or even higher values. It implies a more stable disk according to the Toomre criteria as $\beta_r$ increases. In addition, the Toomre parameter rapidly increases when the mass accretion rate decreases.

The $\eta_0$-dependence of the solutions is shown in Figure 2. In fact, this figure is similar to Figure 1 except for changing the
input parameter \( \eta_0 \) to different values, 0.05, 0.1, and 0.2. The other input parameters are the same as in Figure 1. The surface density \( \Sigma \), the rotational velocity \( v_R \), and the other factors decrease with increasing \( \eta_0 \). But the radial velocity \( v_R \) increases with increasing \( \eta_0 \). We see that as \( \eta_0 \) tends toward larger values, the disk thickness increases. In addition, for a fixed \( \eta_0 \) the disk thickness increases as \( \beta_r \) increases; however, the ratio \( h/R \) is more sensitive to the variation of \( \beta_r \) for higher values of \( \eta_0 \). Interestingly, as the magnetic diffusivity coefficient \( \eta_0 \) increases, the Toomre parameters \( Q \) and \( Q_M \) significantly increase to values higher than unity.

The \( \alpha \)-dependence of the solutions is not strong as long as \( \beta_r \approx |\beta_\phi| \approx 1 \). But we found that the ratio \( \beta \) and the Toomre parameter change because of viscosity variations. Figure 3 (top left) shows \( \beta \) as a function of \( \beta_\phi \). Clearly, as the viscosity parameter \( \alpha \) increases, the ratio \( \beta \) increases. In addition, the Toomre parameter increases with the viscosity parameter. We found that \( \beta_\phi \)-dependence of the solutions is weak for \( |\beta_\phi| \approx 1 \). However, there are some changes in the ratio \( \beta \) as \( \beta_\phi \) varies. Figure 3 (bottom left) shows the variation of \( \beta \) as a function of \( \beta_\phi \) for \( \beta_\phi = -0.8, -1.0, \) and \( -1.2 \). Evidently, this ratio increases with \( |\beta_\phi| \). In addition, by increasing \( |\beta_\phi| \) the Toomre parameter increases, although it is not very significant.

Bertin (1997) showed that the opening angle of a self-regulated nonmagnetized disk depends only on \( Q \), independent of the viscosity coefficient and the mass accretion rate. He argued that significantly large values of \( Q \) would be undesirable because they conflict with the thin disk approximation. But the disk thickness in our magnetized case depends on the mass accretion rate and the other input parameters as long as the \( \beta \)-factors are of the order of unity. For larger values of \( \beta_r \) or \( \beta_\phi \), the disk thickness may change because of the magnetic fields.

4. CONCLUSIONS

We have obtained a self-similar solution for a self-gravitating, magnetized, viscous disk. The solution has constant rotational and accretion velocities, independent of the radial distance. Since we are considering an isothermal magnetized disk, the sound speed is constant.

Our goal is to study the possible effects of the magnetic fields on the steady state structure of a self-gravitating, magnetized disk, at least at the physical level. Although we have made some simplifying assumptions in order to treat the problem analytically, our self-similar solution shows that magnetic fields can really change the typical behavior of the physical quantities of a self-gravitating disk. Not only the surface density of the disk changes, but also the rotational and radial velocities significantly change because of the magnetic fields. It means that any realistic model for a self-gravitating disk should consider the possible effects of the magnetic fields. Of course, our self-similar solutions are too simple to make any comparison with observations. However, we think that one could relax the self-similarity assumption and solve the equations of the model numerically.
In doing so, our self-similar solutions can greatly facilitate testing and interpretation of results. Then we can calculate the spectral energy distribution of such a self-gravitating magnetized disk.

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