Resonant Casimir–Polder forces in planar meta-materials

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Received 8 January 2009
Accepted for publication 12 January 2009
Published 31 July 2009
Online at stacks.iop.org/PhysScr/T135/014019

Abstract
We study the resonant Casimir–Polder potential of an excited atom near a half-space containing magneto-electric meta-material of various kinds on the basis of macroscopic quantum electrodynamics. Analytical results are obtained in the nonretarded and retarded distance regimes and numerical examples are given. We compare our findings with the potential of an excited atom near a left-handed superlens.

PACS numbers: 12.20.−m, 42.50.Wk, 34.35.+a, 42.50.Nn

1. Introduction
In view of the rapid progress made in the fabrication and application of meta-materials [1], which often exhibit left-handed properties [2, 3], a theoretical understanding of the striking optical effects associated with such materials is desirable. Mainly in the past decade, phenomena such as negative refraction [2, 4], superlenses [5, 6], invisibility devices [7, 8] and the reversed Doppler effect [9, 10] has been investigated and measured. Of particular interest to the field of nano-technologies is the research on dispersion forces in the presence of meta-materials with specially tailored magneto-electric properties described by complex \( \varepsilon(\omega) \) and \( \mu(\omega) \). For example, the Casimir force between two magneto-electric plates [11, 12], as well as that between anisotropic meta-materials [13], has recently been addressed where particular attention was drawn to the question of whether attractive behavior can be turned into repulsion, an issue closely related to problem of stiction [14]. Whereas precision measurements of Casimir forces between ordinary materials have already been carried out [15], an experimental confirmation of dispersion interactions in meta-materials remains a prospective task.

We have recently investigated the resonant Casimir–Polder (CP) interaction of an excited atom near a superlens of left-handed material [16], finding that for very weak absorption the CP potential may be enhanced near the focal plane of the lens. In this paper, we study the resonant CP interaction near a magneto-electric half-space (section 2). Endowed with an understanding of the behavior of the potential near a single interface, we then reexamine the superlens geometry in order to clarify whether focal-plane enhancement is a genuine left-handed effect or whether similar behavior can be observed with ordinary magneto-electrics (section 3).

2. Magneto-electric half-space
It has been recognized that the impact of the magneto-electric properties of the meta-material (e.g. left-handedness) on dispersive interactions is much more pronounced for excited systems due to the fact that selected frequencies (i.e. atomic transition frequencies) contribute dominantly to the electromagnetic response. For an excited atom (position \( \mathbf{r}_A \), energy eigenstate \( |n\rangle \), transition frequencies \( \omega_{nk} \), electric-dipole transition matrix elements \( d_{nk} \)) the off-resonant part of the interaction can often be neglected, so that the CP potential may be given as [17, 18]

\[ U_n(\mathbf{r}_A) = -\mu_0 \sum_{k<n} \omega_{nk}^2 d_{nk} \cdot \text{Re} \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega_{nk}) \mathbf{d}_{kn}. \]
where $\Theta$ is the unit step function and $\mathbf{G}^{(1)}$ the scattering Green tensor.

Consider first an atom placed in front of a magneto-electric half-space (at $z_A \leq 0$) of permittivity $\varepsilon_0(\omega)$ and permeability $\mu_0(\omega)$. The associated scattering Green tensor reads (cf e.g. [19])

$$
\mathbf{G}^{(1)}(z_A, z_A, \omega_{nk}) = \frac{i}{8\pi} \int_0^\infty dq \frac{q}{\beta} e^{2i\beta z_A} \left[ \left( r_s - \frac{\beta^2 c^2}{\omega^2_{nk}} r_r \right) (e_x, e_x, e_x) + 2 \frac{q^2 c^2}{\omega^2_{nk}} r_r e_x, e_x, e_x \right],
$$

where

$$
r_s = \frac{\mu \beta - \beta_1}{\mu \beta + \beta_1},
$$

$$
r_r = \frac{\varepsilon \beta - \beta_1}{\varepsilon \beta + \beta_1},
$$

$$
\beta = \sqrt{\frac{\omega^2_{nk}}{c^2} - q^2},
$$

$$
\beta_1 = \sqrt{\frac{e \mu \omega^2_{nk}}{c^2} + q^2}.
$$

[$\varepsilon \equiv \varepsilon_0(\omega_{nk})$ and $\mu \equiv \mu_0(\omega_{nk})$]. The square root of $\beta_1$ has to be chosen such that $\text{Im} \beta_1 > 0$ for a passive medium. Note that when the atom is embedded in a medium, local-field corrections need to be taken into account [20].

Substituting (2) into (1) gives for the resonant CP potential

$$
U_n(z_A) = -\mu_0 \sum_{k<0} \omega^2_{nk} \left[ \text{Re} \ G^{(1)}_{zz}(z_A, z_A, \omega_{nk}) \right] \left| d^\|_{nk} \right|^2 + \text{Re} \ G^{(1)}_{zz}(z_A, z_A, \omega_{nk}) \left| d^\perp_{nk} \right|^2
$$

(5)

where $[d^\|_{10} = (d_{10})_e, (d_{10})_m, 0]$ and $[d^\perp_{10} = (0, 0, (d_{10})_c)]$. Equation (5) can be further investigated analytically in the limits of short and long atom–interface separations. In the nonretarded regime where $z_A \omega_{nk}/c \ll 1$ we can assume $\beta \simeq \beta_1 \simeq i$ which leads to

$$
r_s = \frac{\mu - 1}{\mu + 1},
$$

$$
r_r = \frac{\varepsilon - 1}{\varepsilon + 1}.
$$

Carrying out the integral in (2) gives

$$
U_n(z_A) = -\sum_{k<0} \frac{\left| d^\|_{nk} \right|^2 + 2 \left| d^\perp_{nk} \right|^2}{32\pi \varepsilon_0(c_A^2 - 1)} \frac{|\varepsilon(\omega_{nk})|^2 - 1}{|\varepsilon(\omega_{nk}) + 1|^2},
$$

(7)

unless the half-space is purely magnetic, in which case

$$
U_n(z_A) = -\sum_{k<0} \frac{\mu_0 \omega^2_{nk} \left| d^\|_{nk} \right|^2}{16\pi z_A} \frac{|\mu(\omega_{nk})|^2 - 1}{|\mu(\omega_{nk}) + 1|^2}.
$$

(8)

This shows that close to the surface, the resonant CP potential is attractive for $|\varepsilon| > 1$ and—in the case of a purely magnetic material—it is attractive for $|\mu| > 1$. In particular, for meta-materials with $|\varepsilon| < 1$ and/or $|\mu| < 1$, a repulsive near-surface behavior of the potential can be expected.

In the retarded regime $z_A \omega_{nk}/c \gg 1$, the main contribution to the integral in equation (2) is due to the stationary-phase point $q = 0$, so the reflection coefficients are approximated by

$$
r_s = -r_p = \sqrt{\mu - 1} - \sqrt{\mu + 1}.
$$

(9)

After substituting (9) into (2), the integral can be carried out and keeping only the leading order in $\varepsilon/Z_A$, the retarded CP potential (5) takes the form

$$
U_n(z_A) = \sum_{k<0} \frac{\mu_0 \omega^2_{nk} \left| d^\|_{nk} \right|^2}{8\pi z_A} \left( \text{Re} \ G^{(1)}_{zz}(z_A, z_A, \omega_{nk}) \right) = \pm \sum_{k<0} \frac{\mu_0 \omega^2_{nk} \left| d^\|_{nk} \right|^2}{8\pi z_A} \cos(2z_A \omega_{nk}/c),
$$

(10)

where the second equality holds for strongly electric and magnetic half-spaces, respectively. It can be seen that the retarded potential is dominated by an oscillating term of decreasing amplitude where purely electric and purely magnetic materials give rise to potentials of different signs.

In figure 1, we show the resonant potential (5) (together with equations (2)–(4)) for purely electric (above) and purely magnetic (below) materials. As expected from the retarded limit, the potential at large distances is governed by decaying oscillations whose phase depends on the signs and electric/magnetic nature of the medium response. The amplitude of these oscillations is largest for large negative $\varepsilon$ or $\mu$. The figure confirms the predicted attraction (repulsion) in the nonretarded regime for $|\varepsilon| > (\|), |\mu| > (\perp)$ and further reveals that a repulsive potential barrier can form for $\varepsilon < -1$ or $\mu < -1$; in the latter case, it is even more pronounced.

As a second example, we have considered a genuinely magneto-electric half-space with different signs for $\varepsilon$ and $\mu_0$ (see figure 2). The strongest oscillations are seen in the case of a meta-material with $\varepsilon < 0$ and $\mu_0 > 0$ (or vice versa), where the oscillations have opposite signs in the two cases, as predicted from equation (10). The oscillation amplitude is very weak for a left-handed material or an ordinary one with $\varepsilon_0, \mu_0 > 0$.

3. Perfect lens geometry

Consider an excited atom placed next to a meta-material superlens (i.e. a plate of thickness $d$ with $\varepsilon = \varepsilon_0 = -1$) bounded by a perfectly conducting mirror on the far side (this arrangement was first suggested in [21]). The reflection coefficients of the associated Green tensor (2) are given by

$$
r_s = -r_p = \frac{\mu + \beta_1 - (\mu + \beta_1) e^{2i\beta_1 d}}{\mu + \beta_1 - (\mu + \beta_1) e^{2i\beta_1 d}},
$$

(11)
Figure 1. Resonant CP potential of a two-level atom in front of an electric (above) and magnetic (below) meta-material half-space for different strengths of the electric/magnetic properties. The atomic dipole moment is oriented parallel to the surface and we have assumed $\text{Im} \epsilon = \text{Im} \mu = 10^{-3}$.

We have discussed the associated resonant potential (5) in [16]. As shown, for a perfect, nonabsorbing lens, the reflection coefficients reduce to $r_s = -r_p = -e^{-2i\beta_d}$ and the potential (1) takes the form

$$U(z_A) = -\sum_{k,n} \frac{\mu_0 \alpha_k^3}{4\pi c^2 \tilde{z}} \left[ \cos(\tilde{z}) + \tilde{z} \sin(\tilde{z}) - \tilde{z}^2 \cos(\tilde{z}) \right] |d_{nk}^1|^2 + 2 \left[ \cos(\tilde{z}) + \tilde{z} \sin(\tilde{z}) \right] |d_{nk}^2|^2$$

We have calculated the CP potential (5) (together with equations (2), (4), (11) and (12)) for a two-level atom near a weakly absorbing superlens and compared it with comparable right-handed meta-material slabs, with the results being displayed in figure 3. It is seen that the superlens...
Figure 2. Resonant CP potential of a two-level atom in front of a magneto-electric meta-material half-space. Assumptions of figure 1 apply.

Figure 3. Resonant CP potential of an excited two-level atom in front of a meta-material slab of thickness $d = 5c/\omega_{10}$ with a perfect mirror at its far end. The atomic dipole moment is oriented perpendicular to the surface and we have assumed $\text{Im} \, \epsilon = \text{Im} \, \mu = 10^{-4}$.

gives rise to enhanced attraction which sets in around the focal plane, while the right-handed materials do not give rise to such a behavior. Instead, the highly transparent material $\text{Re} \, \epsilon = \text{Re} \, \mu = 1$ leads to a weak oscillating potential which is entirely due to the mirror while the materials with different signs of $\text{Re} \, \epsilon$ and $\text{Re} \, \mu$ lead to near-surface potential barriers. As seen in section 2, the latter behavior is due to the negative $\text{Re} \, \epsilon$ and $\text{Re} \, \mu$, respectively.

4. Summary

In this paper, we have studied the resonant CP potential of an excited atom in front of meta-material half-space. We have shown that for long distances the potential exhibits attenuated oscillations, while close to the surface the potential becomes attractive or repulsive, depending on whether the absolute values of permittivity and permeability are larger or smaller than unity.

Furthermore, we have reconsidered the potential of an excited atom placed near a weakly absorbing left-handed planar superlens with a perfect mirror at the far end. A comparison with comparable right-handed meta-materials has shown that an enhanced superlens attraction away from the surface is genuinely due to the left-handed properties of the lens, while the potentials of right-handed materials closely resemble those of respective half-spaces without the additional mirror.
Acknowledgments

The work was supported by Deutsche Forschungsgemeinschaft. We acknowledge funding from the Alexander von Humboldt Foundation (HTD and SYB) and the Vietnamese Basic Research Program (HTD).

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