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Tensor products of quandles and 1-handles attached to surface-links. (English)

A quandle is a set \( X \) equipped with two binary operations \( \ast \) and \( \tau \) satisfying certain conditions related to Reidemeister moves in knot theory. A symmetric quandle is a pair \( (X, \rho) \) of a quandle \( X \) and a certain involution \( \rho \) on \( X \) called a good involution. For a quandle \( X \), the symmetric double of \( X \) is a symmetric quandle \( (D(X), \rho) \) such that \( D(X) = X \mathbin{\#} X \) with binary operations determined from those of \( X \), and \( \rho \) is a particular good involution.

The main results are as follow. Let \( X \) be a quandle and let \( D(X) \) be the symmetric double of \( X \). The author introduces the notion of the canonical tensor product, or simply the tensor product \( X \otimes X \) of \( X \), and involutions \( \tau \) and \( \rho \) on \( X \otimes X \). The author gives a method of computing \( D(X) \otimes D(X) \) from \( X \otimes X \). The author computes \( X \otimes X \) and the quotient set \( X \otimes X/\langle \tau \rangle \) for every dihedral quandle \( X = \mathbb{R}^n \), and for its symmetric double \( X = D(\mathbb{R}^n) \), the author computes \( X \otimes X/\langle \tau \rangle \), \( X \otimes X/\langle \rho \rangle \) and \( X \otimes X/\langle \tau, \rho \rangle \); the results consist of two types, one when \( n \) is odd, and the other when \( n \) is even. Further, the author discusses 1-handles and chords attached to any surface-link \( F \) that may be disconnected or non-orientable, and considers the knot quandle (when \( F \) is oriented) and the knot symmetric quandle and their tensor products, and shows that the tensor products provide us with complete classifications of 1-handles, and can be used to construct invariants of 1-handles.

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MSC:
57K45 Higher-dimensional knots and links
57K12 Generalized knots (virtual knots, welded knots, quandles, etc.)

Keywords:
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