Medium modification of singly heavy baryons in a pion-mean field approach

Nam-Yong Ghim,¹‡⁴ Hyun-Chul Kim,¹,² Ulugbek Yakhshiev,¹,³ and Ghil-Seok Yang⁴

¹Department of Physics, Inha University, Incheon 22212, Republic of Korea
²School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, Korea
³Theoretical Physics Department, National University of Uzbekistan, Tashkent 100174, Uzbekistan
⁴Department of General Education for Human Creativity, Hoseo University, Asan 31499, Republic of Korea

We investigate how the masses of singly heavy baryons undergo changes in nuclear matter. The mass spectrum of the singly heavy baryons was successfully described in a pion-mean field approach even with isospin symmetry breaking, based on which we extend the investigation to the medium modification of the singly heavy baryons. Since all dynamical parameters were determined by explaining the mass spectrum of the SU(3) light and singly heavy baryons in free space, we can directly implement the density-dependent functionals for the dynamical parameters, of which the density dependence was already fixed by reproducing the bulk properties of nuclear matter and medium modification of the SU(3) light baryons. We predict and discuss the density dependence of the masses of the singly heavy baryons.

Keywords: Chiral solitons, singly heavy baryons, medium modification of hadrons

I. INTRODUCTION

Understanding hadrons in nuclear medium have been one of the most important issues in hadronic and nuclear physics, since it is deeply connected to nonperturbative aspects of quantum chromodynamics (QCD): the restoration of chiral symmetry and quark confinement [1–4]. The quark condensate, an order parameter of spontaneous breakdown of chiral symmetry, is known to decrease in nuclear medium, which indicates that chiral symmetry tends to be restored as the nuclear density increases [1]. Experimentally, it has also been observed that the mass of the nucleon undergoes change in nuclear medium with parameters already fixed [5–10]. The singly heavy baryons Λc and Σc in nuclear matter was examined in relativistic mean-field theory [17], the quark-meson coupling model [18–20], and QCD sum rules [21–24] (see also a recent review and references therein [25]). Recently, the SU(3) Skyrme model with a bound-state approach was applied to the masses of the singly heavy baryons [26]. We have first examined baryonic matter including symmetric matter, asymmetric matter, neutron matter, and strange baryonic matter, taking empirical information on the bulk properties of nuclear matter as a guiding principle. By describing the various matters and masses of the SU(3) baryons in nuclear medium, we were able to fix all density-dependent parameters. Thus, we can proceed to study the masses of the singly heavy baryons in nuclear medium with parameters already fixed. The pion mean-field approach, also known as the chiral quark-soliton model (χQSM), was constructed by Witten’s seminal idea [28]: in the large Nc (the number of colors) limit, the nucleon can be regarded as a state of Nc valence quarks bound by the pion mean field generated self-consistently by the presence of the Nc valence quarks. The same idea can be applied to the singly heavy baryons. If we take the limit of the infinitely heavy-quark mass (mQ → ∞), a heavy quark resided in a singly heavy baryon can be decoupled from the Nc − 1 valence quarks inside it. Thus, the heavy quark inside a singly heavy baryon is considered as a mere static color source and the quark dynamics inside it is governed by the light quarks. Since the heavy quark is infinitely heavy, the heavy-quark spin is conserved, which leads to the conservation of the light-quark spin. It is known as the heavy-quark spin symmetry. In this heavy-quark mass limit, the singly heavy baryon is independent of the heavy flavor, which is called the heavy-quark flavor symmetry [29, 31]. In this picture, the singly heavy baryons are represented by a baryon antitriplet (3) and two baryon sextets (6) with spin 1/2 and 3/2. Thus, the singly heavy baryons can be considered as a bound state of the Nc − 1 valence quarks with the single heavy quark detached. The heavy quark is required only for making the singly heavy baryon a color singlet.

Based on this idea, the pion mean-field approach was directly extended to the singly heavy baryons [32]. It has successfully described various properties of the singly heavy baryons in free space [33–41] (see also a recent review [42]). As mentioned previously, using the pion mean-field approach, we were able to describe how the masses of the SU(3) baryons are modified in nuclear
medium [27]. The bulk properties of nuclear matter evaluated from the present approach were in good agreement with empirical and experimental data. We proceed now to describing the masses of the singly heavy baryons with both spin 1/2 and 3/2.

The paper is organized as follows: In the next Section, we briefly review the general formalism for the pion mean-field approach. In Section III we show how to implement the density-dependence into the dynamical parameters. In Section IV we present the numerical results and discuss them. The last Section is devoted to the summary and conclusions of the present work. The explicit expressions for the baryon masses are presented in Appendix A.

II. GENERAL FORMALISM

The pion mean-field approach allows one to describe both the light and singly heavy baryons on an equal footing. Replacing one light quark by a heavy quark with the infinitely heavy mass, we can construct a state for the singly heavy baryon [22]. We first define the normalization of the baryon state in the large $N_c$ limit as

$$\langle B(p', J_3')|B(p, J_3)\rangle = 2M_B\delta_{J_3,J_3'}(2\pi)^3\delta^{(3)}(p' - p),$$

(1)

where $M_B$ denotes the corresponding baryon mass. A state of the singly heavy baryon is then expressed as

$$\langle B, p| = \lim_{x_4 \to -\infty} \exp(ip_4x_4)N(p),$$

$$\times \int d^3x \exp(ip \cdot x)(-i\Psi^\dagger_1(x, x_4)J_B^1(x, x_4)|0\rangle,$$

$$\langle B, p| = \lim_{y_4 \to -\infty} \exp(-ip'_4y_4)N^*(p'),$$

$$\times \int d^3y \exp(-ip' \cdot y)|0\rangle J_B(y, y_4)\Psi_h(y, y_4),$$

(2)

where $N(p)$/$N^*(p')$ denotes the normalization factor depending on the initial (final) momentum. $J_B(x)$ and $J_B^1(y)$ represent the Ioffe-type current of the $N_c - 1$ valence quarks [16] defined by

$$J_B(x) = \frac{1}{(N_c - 1)!}\delta_{\alpha_1 \cdots \alpha_{N_c - 1}}\Gamma_{f_1 \cdots f_{N_c - 1}}^{f_1 \cdots f_{N_c - 1}}(TT_3Y)(J_JY_H)$$

$$\times \psi_{f_1\alpha_1}(x) \cdots \psi_{f_{N_c - 1}\alpha_{N_c - 1}}(x),$$

$$J_B^1(y) = \frac{1}{(N_c - 1)!}\delta_{\alpha_1 \cdots \alpha_{N_c - 1}}\Gamma_{f_1 \cdots f_{N_c - 1}}^{f_1 \cdots f_{N_c - 1}}(TT_3Y)(J_JY_H)$$

$$\times (-i\psi^\dagger(y)\gamma_4)f_{\alpha_1} \cdots (-i\psi^\dagger(y)\gamma_4)f_{\alpha_{N_c - 1}},$$

(3)

where $f_1 \cdots f_{N_c - 1}$ and $\alpha_1 \cdots \alpha_{N_c - 1}$ designate respectively the spin-isospin and color indices. The matrices $\Gamma_{TT_3Y}(J_JY_H)$ carry the quantum numbers $(TT_3Y)(J_JY_H)$ for the corresponding baryon. $\psi_{f\alpha}(x)$ denotes the light-quark field and $\Psi_h(x)$ stands for the heavy-quark field. In the limit of $m_Q \to \infty$, a singly heavy baryon satisfies the heavy-quark flavor symmetry. Then the heavy-quark field can be written as

$$\Psi_h(x) = \exp(-im_Qv \cdot x)\tilde{\Psi}_h(x),$$

(4)

where $\tilde{\Psi}_h(x)$ is a rescaled heavy-quark field almost on mass-shell. It carries no information on the heavy-quark mass in the leading order approximation in the heavy-quark expansion. $v$ denotes the velocity of the heavy quark [29,31].

We now prove that the normalization factor $N^*(p')N(p)$ is correctly reduced to $2M_B$, which can be computed as

$$\langle B(p', J_3')|B(p, J_3)\rangle = \frac{1}{Z_{eff}}N^*(p')N(p)$$

$$\times \lim_{x_4 \to -\infty} \lim_{y_4 \to -\infty} \exp(-iy_4p'_4 + ix_4p_4)$$

$$\times \int d^3x d^3y \exp(-ip' \cdot y + ip \cdot x)$$

$$\times \int DU\bar{\psi}D\psi^\dagger D\tilde{\Psi}_hD\tilde{\Psi}_h^\dagger J_B(y)$$

$$\times \Psi_h(y)(-i\Psi^\dagger_1(x)\gamma_4)J_B^1(x)\exp\left[\int d^4z\left\{\begin{array}{l}
(\psi^\dagger(z))_f (i\partial + iMU^{\gamma_5} + i\hat{m})_{fg} \psi(z) \\

\Psi^\dagger_1(z)v \cdot \partial \Psi_h(z)\end{array}\right]\right]$$

$$= \frac{1}{Z_{eff}}N^*(p')N(p) \lim_{x_4 \to -\infty} \lim_{y_4 \to -\infty} \exp(-iy_4p'_4 + ix_4p_4)$$

$$\times \int d^3x d^3y \exp(-ip' \cdot y + ip \cdot x)$$

$$\times \langle J_B(y)\Psi_h(y)(-i\Psi^\dagger_1(x)\gamma_4)J_B^1(x)\rangle_0.$$  

(5)

Here, $Z_{eff}$ is the low-energy effective QCD partition function defined as

$$Z_{eff} = \int DU \exp(-S_{eff}).$$

(6)

$S_{eff}$ is called the effective chiral action expressed as

$$S_{eff} = -N_c\operatorname{Tr}\ln[i\partial + iMU^{\gamma_5} + i\hat{m}] \cdot$$

(7)

expresses the vacuum expectation value of the baryon correlation function. $M$ denotes the dynamical quark mass and the $U^{\gamma_5}$ represents the chiral field defined by

$$U^{\gamma_5}(z) = \frac{1 - \gamma_5}{2}U(z) + U^\dagger(z)\frac{1 + \gamma_5}{2}$$

(8)

with

$$U(z) = \exp[i\pi^a(z)\lambda^a].$$

(9)

$\pi^a(z)$ are the pseudo-Nambu-Goldstone (pNG) fields and $\lambda^a$ the flavor Gall-Mann matrices. $\hat{m}$ is the mass matrix of current quarks $\hat{m} = \operatorname{diag}(m_u, m_d, m_s)$. We treat the
strange current quark mass $m_s$ perturbatively. The propagators of a light quark in the $\chi$QSM [46] is obtained to be

$$
G(y, x) = \left\langle y \left| \frac{1}{i\partial + iM_U \gamma_5 + m} \right| (y) \right\rangle = \Theta(y_4 - x_4) \sum_{E_n > 0} e^{-E_n(y_4 - x_4)} \psi_n(y) \psi_n^\dagger(x) - \Theta(x_4 - y_4) \sum_{E_n < 0} e^{-E_n(y_4 - x_4)} \psi_n(y) \psi_n^\dagger(x),
$$

where $\Theta(y_4 - x_4)$ is the Heaviside step function. We introduce $\bar{m}$, which is the average mass of the up and down current quarks: $\bar{m} = (m_u + m_d)/2$. It properly generates the Yukawa tail of the pion mean field, when we later solve the equation of motion. We define the one-body Dirac Hamiltonian as

$$
H = \gamma_4 \gamma_5 \partial_4 + \gamma_4 M U\gamma_5 + \gamma_4 \bar{m} \mathbf{1}.
$$

Solving the eigenvalue problem of $H$, we find the energy eigenvalues corresponding to the single-quark eigenstate $H \psi_n(x) = E_n \psi_n(x)$. \hspace{1cm} (12)

We now deal with the heavy-quark propagator in the limit of $m_Q \to \infty$

$$
G_h(y, x) = \left\langle y \left| \frac{1}{i\partial_4} \right| x \right\rangle = \Theta(y_4 - x_4) \delta^{(3)}(y - x).
$$

Using these quark propagators and taking the limit of $y_4 - x_4 = T \to \infty$, we evaluate the baryon correlation function $\langle J_B(y)\Psi_h(y)(-i\Psi^\dagger_h(x)\gamma_4)J_B^\dagger(x) \rangle_0$ as follows \[46,47\]:

$$
\langle J_B(y)\Psi_h(y)(-i\Psi^\dagger_h(x)\gamma_4)J_B^\dagger(x) \rangle_0 \\
\sim \exp[-((N_c - 1)E_{val} + E_{sea} + m_Q)T] \\
= \exp[-M_B T],
$$

which cancels the term $\exp[(-iy_4p_4 + ix_4p_4)] = \exp[M_B T]$ in the large $N_c$ limit. Therefore, we can rewrite the normalization factor becomes $N^*(p)N(p) = 2M_B$. Utilizing this normalization and Eq. (14), we derive the classical mass of the singly heavy baryon \[34\] as

$$
M_B = (N_c - 1)E_{val} + E_{sea} + m_Q.
$$

Before we proceed to compute the mass spectrum of the singly heavy baryons, we want to mention the ordering of the two limits: $N_c \to \infty$ and $m_Q \to \infty$. We first take the limit of $m_Q \to \infty$ and then we carry out $N_c \to \infty$. This ordering is compatible with the present pion mean-field approach. If we had taken the ordering inversely, we would not have detached the heavy quark from the singly heavy baryons.

We restate $S_{eff}$ in Eq. (6) in the following form

$$
S_{eff}(U) = -N_c Tr \ln iD(U),
$$

where the trace operator $Tr$ runs over spacetime and all relevant internal spaces. The $N_c$ stands for the number of colors, and $D(U)$ the one-body Dirac differential operator is defined by

$$
D(U) = \gamma_4 (i\partial - \bar{m} - MU\gamma_5),
$$

where $\partial_4$ is the time derivative in Euclidean space. The mass matrix of the current quarks $\bar{m}$ can be expressed in terms of the Gell-Mann matrices

$$
\bar{m} = m_1 \mathbf{1} + m_3 \lambda_3 + m_8 \lambda_8,
$$

where

$$
m_0 = \frac{m_u + m_d}{3}, \quad m_3 = \frac{m_u - m_d}{2}, \quad m_8 = \frac{m_u + m_d - 2m_s}{2\sqrt{3}}.
$$

$U^{\gamma_5}$ denotes the SU(3) chiral field

$$
U^{\gamma_5} = \exp[i\pi^a \lambda^a \gamma_5] = \frac{1 + \gamma_5}{2} U + \frac{1 - \gamma_5}{2} U^\dagger,
$$

where $\pi^a(r)$ is the pNG field with flavor indices $a = 1, \ldots N_f^2 - 1$. $N_f$ is the number of flavors. Since the hedgehog symmetry constrains the form of the classical pion field as $\pi(x) = \bar{n} \cdot \tau P(r)$, where $P(r)$ is called the profile function of the soliton, we keep only the pion fields $\pi^a$ with $a = 1, 2, 3$. Thus, we have the SU(2) chiral $U$ field as $U_{SU(2)} = \exp(\bar{n} \cdot \tau P(r))$. We now embed the SU(2) soliton into SU(3) by Witten’s ansatz \[48\]

$$
U^{\gamma_5}(x) = \begin{pmatrix} U_{SU(2)}^\gamma(x) \\ 0 \\ 0 \end{pmatrix}.
$$

Since we consider the mean-field approximation, we can carry out the integration over $U$ in Eq. (6) around the saddle point $\delta S_{eff}/\delta \pi^a = 0$. This saddle-point approximation yields the equation of motion that can be solved self-consistently. The solution provides the self-consistent profile function $F_p(r)$, which is just the pion mean field. Compared to the SU(3) light baryons, it is weaker than that produced by the $N_c$ valence quarks.

Since the classical $U_{cl}$ field is not invariant under translation and rotation, we need to restore these symmetries such that we have the singly heavy baryons with correct quantum numbers. Thus, we perform the zero-mode quantization or the semiclassical quantization for the chiral soliton. A detailed formalism for the zero-mode quantization can be found in Ref. \[48,49\]. Having quantized the soliton, we obtain the collective Hamiltonian as

$$
H_{\text{coll}} = H_{\text{rot}} + H_{\text{sb}},
$$

where the rotational part of the collective Hamiltonian is given as

$$
H_{\text{rot}} = \frac{1}{2T_1} \sum_{j=1}^3 \vec{j}_j^2 + \frac{1}{2T_2} \sum_{p=4}^7 \vec{j}_p^2.
$$
Here $\bar{T}_1$ and $\bar{T}_2$ have forms

$$ T_1 = \eta I_1, \quad T_2 = \eta I_2, \quad (24) $$

where $I_1$ and $I_2$ are the usual moments of inertia. Since we take an “model-independent” approach [50], we do not compute all the dynamical parameters such as $I_1$ and $I_2$ but determine them by using the experimental data on the mass splitting of the baryon octet and decuplet. In the case of the singly heavy baryons, we only know that $\bar{T}_1$ and $\bar{T}_2$ should be smaller than $I_1$ and $I_2$ because the pion mean field from the $N_c - 1$ valence quarks is weaker than that with the $N_c$ ones. Thus, we fit $\eta$ to the masses of the singly heavy baryons in free space [32]. The $J_i$ are the generators of the SU(3) group of which the first three components are the ordinary spin operators. More details can be found in Refs. [51, 52].

In representation $R = (p, q)$, the eigenvalues of $H_{\text{rot}}$ in Eq. (23) are given as

$$ E_{(p, q)}^{\text{rot}} = \left( \frac{1}{2T_1} - \frac{1}{2T_2} \right) J (J + 1) + \frac{p^2 + q^2 + 3(p + q) + pq}{6I_2} - \frac{3}{8T_2} Y'^2, \quad (25) $$

where $Y'$ denotes the right hypercharge. In the case of the SU(3) light baryons, the presence of $N_c$ valence quarks imposes a constraint on the collective Hamiltonian: $Y' = N_c/3$, which selects allowed representations: the octet (8) and decuplet (10). Since the singly heavy baryon consists of the $N_c - 1$ valence quarks, the right hypercharge is constrained to be $Y' = (N_c - 1)/3$ that allows the antitriplet (3) and sextet (6). The center masses for the baryon antitriplet and sextet are then given by

$$ M^Q_{\bar{3}} = M_{\bar{cl}} + \frac{1}{2T_2}, \quad M^Q_6 = M^Q_{\bar{3}} + \frac{1}{I_1}. \quad (26) $$

Note that the center masses are flavor-independent.

To describe the mass splitting in a representation, it is essential to introduce the effects of isospin breaking and explicit flavor SU(3) symmetry breaking. Expanding the effective chiral action to the linear order of $\bar{m}$ and carrying out the quantization, we obtain the symmetry-breaking part of the collective Hamiltonian as

$$ H_{\text{ab}} = (m_d - m_u) \times \left( \frac{\sqrt{3}}{2} \pi D^{(8)}_{38}(R) + \beta \hat{T}_3 + \frac{\gamma}{2} \sum_{i=1}^{3} D^{(8)}_{3i}(R) \hat{J}_i \right) $$

$$ + (m_s - m_d) \times \left( \frac{\alpha}{3} D^{(8)}_{88}(R) + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D^{(8)}_{8i}(R) \hat{J}_i \right), \quad (27) $$

where the first term arises from the isospin symmetry breaking to linear order, and the second term comes from the SU(3) symmetry breaking also to linear order. Once we introduce the isospin symmetry breaking, we need to include the contributions from the electromagnetic (EM) self-energies of the soliton [39, 33]. $D^{(8)}_{ij}$ denote SU(3) Wigner functions. The parameters $\alpha$, $\beta$, and $\gamma$ are expressed as

$$ \bar{\alpha} = \frac{N_c - 1}{N_c}, \quad \alpha = -\frac{2}{3} \frac{\Sigma_{\pi N}}{m_u + m_d} - \beta, $$

$$ \beta = -\frac{K_2}{I_2}, \quad \gamma = \frac{2K_1}{I_1} + 2\beta, \quad (28) $$

where $K_1$ and $K_2$ designate the anomalous moments of inertia. $\Sigma_{\pi N}$ stands for the pion-nucleon sigma term. Note that $\alpha$ should be rescaled by $(N_c - 1)/N_c$, because the singly heavy baryon contains $N_c - 1$ valence quarks, which modify the pion mean field. More discussion of $\bar{\alpha}$, $\beta$, and $\gamma$ can be found in Ref. [32].

In the limit of $m_Q \to \infty$, the spin 1/2 and 3/2 states are degenerate. To remove the degeneracy, we have to introduce the hyperfine chromomagnetic interaction (spin-spin interaction) to order $1/m_Q$

$$ H_{\text{HF}}^{QL_0} = \frac{2}{3} \frac{\kappa}{m_Q M_{L0}} S_L \cdot S_Q = \frac{2}{3} \frac{\kappa}{m_Q} S_L \cdot S_Q, \quad (29) $$

where the $\kappa$ stands for the anomalous chromomagnetic moment. The operator $S_L$ and $S_Q$ designate respectively the spin operators for the soliton and heavy quark. Taking into account the hyperfine mass splitting, the center mass of the sextet in Eq. (26) can be decomposed into those for the spin 1/2 and spin 3/2

$$ M^Q_{61/2} = M^Q_6 - \frac{2}{3} \frac{\kappa}{m_Q}, $$

$$ M^Q_{63/2} = M^Q_6 + \frac{1}{3} \frac{\kappa}{m_Q}. \quad (30) $$

In addition to the EM self-energies of the soliton for the effects of the isospin symmetry breaking, we introduce the EM interaction between the soliton and the heavy quark, which can be formulated in the following expression

$$ H_{\text{Coul}}^{QL_Q} = \alpha_{LQ} Q_L \hat{Q}_Q, \quad (31) $$

where the $\hat{Q}_L$ and $\hat{Q}_Q$ represent charge operators acting on the soliton and heavy quark. The parameter $\alpha_{LQ}$ includes the expectation value of the inverse distance between the soliton and heavy quark, and the fine structure constants. We can fix it by reproducing the existing data on the masses of the singly heavy baryons [32].

Since almost all the dynamical parameters have already been fixed in the light baryon sector, and their density dependences have also been set up in the previous work [27], we will proceed directly to the masses of the singly heavy baryons in baryonic matter.
III. SINGLY HEAVY BARYONS IN BARYONIC MATTER

We now recapitulate the formalism with which we have described bulk properties of various baryonic matters, and the masses of the SU(3) light baryons [27]. We introduce three density-dependent free parameters $\lambda$, $\delta$, and $\delta_s$, which are respectively related to the normalized density of infinite nuclear matter, the parameter for isospin asymmetry, and that for the strangeness mixing. They are defined as

$$\lambda = \frac{p}{\rho_0}, \quad \delta = \frac{N - Z}{A}, \quad \delta_s = \frac{N_s}{A},$$

where the $\rho_0$ stands for the normal nuclear matter density, $N$ is the number of neutrons, $Z$ the number of protons, $A$ the baryon number, and $N_s$ the number of baryons with the strangeness $s = |S|$. The strangeness is only an external free parameter, of which the fraction identifies strange matter. We introduce the strangeness-mixing parameter $\chi$, which is defined as $\delta_s = s\chi$ such that we do not need to concern specific strange particles that consist of strange matter. Thus, by taking the nonzero value of $\chi$, we can consider the strange matter.

Following Ref. [27], we have the following density-dependent classical mass, moments of inertia, effects of isospin and SU(3) symmetry breaking:

$$M_{c_l}^* = M_{c_l} f_{c_l}(\lambda, \delta, \delta_1, \delta_2, \delta_3),$$

$$T_1^* = T_1 f_1(\lambda, \delta, \delta_1, \delta_2, \delta_3),$$

$$T_2^* = T_2 f_2(\lambda, \delta, \delta_1, \delta_2, \delta_3),$$

$$E_{iso}^* = (m_d - m_u) \frac{K_{1,2}}{f_0(\lambda, \delta, \delta_1, \delta_2, \delta_3)},$$

$$E_{str}^* = (m_s - \bar{m}) \frac{K_{1,2}}{f_s(\lambda, \delta, \delta_1, \delta_2, \delta_3)},$$

where $f_{c_l}$, $f_{1,2}$, and $f_s$ are given as the functions of the baryon density and other medium variables. They are explicitly written as

$$f_{c_l}(\lambda) = (1 + C_{c_l} \lambda),$$

$$f_{1,2}(\lambda) = (1 + C_{1,2} \lambda),$$

$$f_0(\lambda, \delta) = 1 + C_{num} \frac{\lambda}{1 + C_{den} \lambda},$$

$$f_s(\lambda, \delta_s) = 1 + g_s(\lambda) \delta_s,$$

$$g_s(\lambda) = s g(\lambda),$$

$$g(\lambda) = \left( \frac{K_2}{I_2} + \frac{K_1}{I_1} \right)^{-1},$$

$$E_{iso}^* = 5(M_{c_l}^* - M_{c_d}^* + E_{(1,1)1/2} - E_{(1,1)1/2})$$

$$x = -\frac{3(m_s - m)}{3(M_s - m)}.$$
Table I: Masses of the singly charmed baryons in free space and in different baryonic matters at the normal nuclear matter density $\lambda = 1$. The experimental data are taken from the PDG [66]. In the fifth column, the results in symmetric nuclear matter ($\lambda = 1$) are listed, whereas, in the sixth and seventh columns, those in asymmetric matter ($\delta = 0, \chi = 0$) are respectively given. All the masses are given in unit of MeV.

| Multiplet & spin | Baryon              | Exp. | Free space | Baryonic matter at $\lambda = 1$. |
|------------------|----------------------|------|------------|-----------------------------------|
|                  |                      |      | $\delta = 0, \chi = 0$ | $\delta = 0, \chi = 1$, $\delta = 0,\chi = 0.15$ |
| $3_{1/2}$        | $\Lambda_c$          | 2286.46 ± 0.14 | 2272.84 | 2268.71, 2268.71, 2264.49 |
|                  | $\Xi^+_c$            | 2467.71 ± 0.23 | 2475.20 | 2472.97, 2411.19, 2475.09 |
|                  | $\Xi^0_c$            | 2470.44 ± 0.28 | 2478.18 | 2472.16, 2533.94, 2474.27 |
| $6_{1/2}$        | $\Sigma^+_c$         | 2453.91 ± 0.14 | 2445.67 | 2372.37, 2285.03, 2368.51 |
|                  | $\Sigma^0_c$         | 2452.9 ± 0.4  | 2444.65 | 2370.47, 2370.47, 2366.62 |
|                  | $\Xi^0_c$            | 2453.75 ± 0.14 | 2445.55 | 2370.50, 2457.83, 2366.64 |
|                  | $\Xi^+_c$            | 2578.2 ± 0.5  | 2579.83 | 2506.09, 2462.42, 2508.01 |
|                  | $\Omega_c$           | 2695.2 ± 1.7  | 2715.46 | 2641.28, 2641.28, 2648.99 |
| $6_{3/2}$        | $\Sigma^{++}_{c}$    | 2518.41 ± 0.21 | 2513.77 | 2444.52, 2357.18, 2440.66 |
|                  | $\Sigma^+_{c}$       | 2517.5 ± 2.3  | 2512.75 | 2442.62, 2442.62, 2438.77 |
|                  | $\Sigma^0_{c}$       | 2518.48 ± 0.20 | 2513.65 | 2442.64, 2529.98, 2438.79 |
|                  | $\Xi^+_{c}$          | 2645.10 ± 0.30 | 2647.93 | 2578.23, 2534.57, 2580.16 |
|                  | $\Xi^0_{c}$          | 2646.16 ± 0.25 | 2648.83 | 2578.26, 2621.92, 2580.18 |
|                  | $\Omega^+_{c}$       | 2765.9 ± 2.0  | 2784.52 | 2714.38, 2714.38, 2722.09 |

with the experimental data. From the fifth column to the last one, we list the results for the medium-modified values for the masses of the singly charmed baryons. In the column, their results in symmetric nuclear matter are listed at the normal nuclear matter density. As expected from the previous work [27] The masses of the singly charmed baryons consistently decrease in nuclear matter. We now consider the mass modification in asymmetric nuclear matter with $\delta = 1$ set. Then, as shown in the sixth column, we find a very interesting aspect in the change of the $\Xi_c$ masses. In the asymmetric nuclear matter, the proton and neutron undergo changes in a different manner: the proton mass starts to decrease as $\delta$ increases, whereas the neutron mass gets enhanced with larger values of $\delta$. The down quarks outnumber the up quarks in asymmetric nuclear matter. If one puts a down quark in it, the Pauli exclusion principle brings about the repulsion between the down quarks. Thus, competition between up and down quarks will govern how the mass of a singly charged quark is modified in asymmetric nuclear matter. It explains why the mass of $\Xi^0_c$ increases as $\delta$ increases whereas $\Xi^+_c$ behaves oppositely in asymmetric nuclear matter. A similar propensity can also be observed in the baryon sextet, though it is not as prominent as in the baryon antitriplet. In the last column, we examine how the masses of the singly charmed baryons experience the medium modification in strange matter with $\chi = 0.15$. As discussed above, now the number of the strange quarks increases and hence a singly charmed baryon containing the strange quark may decrease less than the nonstrange ones. We observe this feature in the last column of Table I. We will later discuss the density dependences of the antitriplet and sextet masses quantitatively.

For completeness, we list the results for the mass modification of the singly bottom baryons in Table I. Except for the spin-spin interaction that is proportional to $1/m_Q$, we respect in the current work the heavy-quark flavor symmetry. Thus, the changes of the masses of the singly bottom baryons are in conformity with those of the charmed baryons.

Figure 1: Shifts of the center masses for the singly heavy baryons. The solid line curve draws the mass shift of the baryon antitriplet. The dashed and dotted ones depict respectively the mass shifts for the baryon sextet with spin 1/2 and spin 3/2. The results are given in unit of MeV.
Table II: Masses of the singly bottom baryons in free space and in different baryonic matters at the normal nuclear matter density $\lambda = 1$. The experimental data are taken from the PDG [66]. In the fifth column, the results in symmetric nuclear matter ($\lambda = 1$) are listed, whereas, in the sixth and seventh columns, those in asymmetric nuclear matter ($\delta = 1$) and strange matter ($\chi = 0.15$) are respectively given. All the masses are given in unit of MeV.

| Multiplet & spin | Baryon | Exp. Free space | Baryonic matter at $\lambda = 1.$ |
|------------------|--------|-----------------|----------------------------------|
|                  |        |                 | $\delta = 0$, $\chi = 0$ $\delta = 1$, $\chi = 0$ $\delta = 0$, $\chi = 0.15$ |
| $3_{1/2}$        | $\Lambda_b$ | 5619.60 ± 0.17  | 5595.17  | 5595.17  | 5590.95  |
|                  | $\Xi_b^0$  | 5791.9 ± 0.5    | 5798.05  | 5736.27  | 5800.17  |
|                  | $\Xi_b$    | 5797.0 ± 0.6    | 5800.00  | 5861.78  | 5802.11  |
| $6_{1/2}$        | $\Sigma_b^+$ | 5810.56 ± 0.25  | 5729.83  | 5642.49  | 5725.98  |
|                  | $\Sigma_b^0$ | –              | 5730.69  | 5730.69  | 5726.84  |
|                  | $\Sigma_b^-$ | 5815.64 ± 0.18  | 5733.48  | 5820.81  | 5729.62  |
|                  | $\Xi_b^0$  | –              | 5936.78  | 5864.93  | 5866.85  |
|                  | $\Xi_b^-$  | 5935.02 ± 0.05  | 5940.44  | 5867.71  | 5911.38  |
|                  | $\Omega_b$ | 6046.1 ± 1.7    | 6074.74  | 6002.46  | 6010.16  |
| $6_{3/2}$        | $\Sigma_b^{++}$ | 5830.32 ± 0.27  | 5751.34  | 5664.00  | 5747.48  |
|                  | $\Sigma_b^{0+}$ | –             | 5752.20  | 5752.20  | 5748.35  |
|                  | $\Sigma_b^{-+}$ | 5834.74 ± 0.30  | 5754.98  | 5842.32  | 5751.13  |
|                  | $\Xi_b^{++}$ | –              | 5952.3  | 5886.43  | 5888.36  |
|                  | $\Xi_b^{0+}$ | 5955.33 ± 0.13  | 5960.74  | 5889.22  | 5932.88  |
|                  | $\Xi_b^{-+}$ | –              | 6095.04  | 6023.96  | 6031.67  |

Figure 2: Mass shifts of singly charmed baryons in symmetric nuclear matter ($\delta = 1$, $\chi = 0$). In the upper left panel, the $\lambda$ dependences of the baryon antitriplet are drawn. In the right upper panel, those of the baryon sextet with spin 1/2 are depicted, whereas in the lower panel, those of the baryon sextet with spin 3/2 are shown. The results are given in unit of MeV.
for $M^{\Sigma}_{6}$ are given in Eqs. (26) and (30), as functions of $\lambda$. Note that the center $\Delta M^{\Sigma}_{6}$ decreases as $\lambda$ increases till $\lambda \approx 1.2$, and then gets enhanced. On the other hand, $\Delta M^{6/1}_{\Sigma} (\Delta M^{6/3}_{\Sigma})$ is diminished rapidly till $\lambda$ reaches around 2.2 (2.5) and then starts to increase. It implies that when the nucleons inside nuclear matter get more closely packed the repulsion overcomes the attractive interaction in the presence of the singly charged baryons. The difference between the density dependences of the antitriplet and sextet can be understood as follows: the density dependences of $\tilde{T}_{1}$ and $\tilde{T}_{2}$ are different each other. While $\tilde{T}_{1}$ increases as $\lambda$ increases, $\tilde{T}_{2}$ is lessened with the $\lambda$ grown. $M_{c1}$ decreases linearly as the nuclear density increases. When $\lambda$ reaches around 1.2, the second term $1/2 \tilde{T}_{2}$ overtakes $M_{c1}$, so that $M^{\Sigma}_{6}$ starts to increase. However, the second term for $M^{6/1}_{6}$ and $M^{6/3}_{6}$ in (26) is suppressed as $\lambda$ increases. Thus, $M^{6/1}_{6}$ and $M^{6/3}_{6}$ follow the behavior of $M_{c1}$. When $\lambda$ further increases, the term with $\Delta^{*}$ comes into play. In Fig. 2, we draw the mass shifts of the charmed baryon antitriplet and sextet, $\Delta M_{B_{c}}$, in symmetric nuclear matter. The $\lambda$ dependences of $\Delta M_{B_{c}}$ follow those of the center masses shown in Fig. 1. This is natural, because the effects of the flavor SU(3) symmetry breaking, which causes the mass splitting in the representations, are changed only in strange matter. This is the reason why the mass shift in each representation is degenerate.

In Fig. 3 we depict the mass shifts of the singly charged baryons in asymmetric neutron matter with $\delta = 1$ and $\chi = 0$. The neutral and positively-charged baryons generally show rather different behaviors as $\lambda$ increases. The charmed baryons in the antitriplet exhibit the difference prominently. While $\Xi^{0}_{c}$ increases rather rapidly as $\lambda$ increases, $\Xi_{c}^{+}$ decreases until $\lambda$ reaches around $\lambda = 2.0$ in asymmetric nuclear matter. It indicates that the effects of isospin symmetry breaking stand out in neutron matter ($\delta = 1$). This has profound physical implications. The density-dependent function $f_{0}(\lambda, 1, 0)$ in Eq. (40) increases as $\delta$ grows. It contributes to the $\pi$, $\beta$, and $\gamma$ in Eq. (28), so that $d_{3}$ and $d_{6}$ in Eq. (A8) become $\delta$-dependent. The terms containing $d_{3}$ and $d_{6}$ in mass formulae in Eqs. (A1)–(A7) are proportional to the third component of the isospin operator, $T_{3}$, which brings about the isospin symmetry breaking. Thus, the differences between the neutral and positively charged baryons demonstrated in Fig. 3 arise from these terms. As explained above, the underlying physics in these differences comes from the Pauli exclusion principle.

Figure 4 illustrates how the masses of the singly charged baryons are shifted as $\lambda$ increases. Interestingly, the mass shifts of the singly charged baryon show general tendency: They first start to decrease as $\lambda$ increases, and then increases when $\lambda$ gets to some specific values. However, those of $\Omega_{c}^{+}$ and $\Omega_{c}^{-}$ monotonically fall off as $\lambda$ increases. Inspecting Eqs. (A1)–(A7), we find that the terms with $D_{3}$ in the antitriplet and $D_{6}$ in the sextet cause respectively the mass splittings in the corresponding representations. We also observe that the $\lambda$ dependences of the baryon sextet with spin $1/2$ are almost the same as those with spin $3/2$. Note that the sextet baryons with spin $1/2$ and $3/2$ are degenerate before we introduce the hyperfine interaction in Eq. (30). Though the parameter $\Delta^{*}$ in Eq. (45) is also density-dependent, its effect is marginal. The prefactor in the $D_{6}$ term of the $\Omega_{c}$ ($\Omega_{c}^{+}$) baryons is $-4/3$, whereas the those of $\Sigma_{c}$ ($\Sigma_{c}^{+}$) and $\Xi_{c}^{+}$ ($\Xi_{c}^{+}$) are respectively $+2/3$ and $-1/3$. This leads to the different $\lambda$ dependences of the sextet baryons as shown in Fig. 4.

V. SUMMARY AND OUTLOOK

In the present work, we aimed at investigating the mass shifts of the singly heavy baryons within a pion mean-field approach ($\chi$QSM) in various nuclear matters. In the limit of the infinite heavy-quark mass, the dynamics in a singly heavy baryon is governed by the light quarks whereas the heavy remains as the mere static color source with the heavy quark spin-flavor symmetry satisfied. The light quarks, which yields the right hypercharge $Y' = 2/3$, select the proper representations of the singly heavy baryons. This allows one to describe the light and singly heavy baryons on an equal footing. Since all the density-dependent variables had been determined in describing the bulk properties of nuclear matter and the mass shifts of the baryon octet and decuplet, we were able to evaluate those of the baryon antitriplet and sextet without fitting the parameters. Then, we first computed the medium-modified masses of the singly charged baryons in symmetric nuclear matter. The center masses of the baryon antitriplet and sextet govern the density dependences of the singly charged baryon masses. In the case of asymmetric nuclear matter, the neutral and positively-charged baryons reveal different density dependences: The neutral baryons tend to increase as the nuclear density increases, whereas the positively charged ones decrease as the nuclear density grows. We explained the reason and discussed its physical implications. As a result, the effects of isospin symmetry breaking are more strengthened as the density increases in asymmetric nuclear matter. We also presented the mass shifts of the singly charged baryons in strange matter.

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Appendix A: Expressions for the masses of the singly heavy baryons

The masses of the antitriplet baryon are expressed as

\[ M_{\Lambda_c} = M_{c1} + E_{(0,1)}^{\text{rot}} + m_Q + \frac{2}{3} D_3 + \frac{1}{4} c_8, \]
\[ M_{\Xi_c} = M_{c1} + E_{(0,1)}^{\text{rot}} + m_Q - \frac{1}{3} D_3 + d_3 T_3 + \frac{3}{4} \left( T_3 + \frac{1}{6} \right) c_8 - \hat{Q}_q \alpha_{\text{LQ}} T_3, \quad (A1) \]

where the \( E_{(0,1)}^{\text{rot}} \) can be obtained from Eq (25). The masses of spin-1/2 sextet baryon are given by following expression: The masses of

\[ M_{\Sigma_{c1}} = M_{c1} + E_{(2,0)}^{\text{rot}} + m_Q - \frac{2}{3} \frac{\kappa}{m_Q} D_6 + d_6 T_3 + \frac{2}{3} \left[ T_3 + \frac{1}{3} \right] c_8 \]
\[ + \frac{1}{9} \left( T_3^2 + \frac{1}{5} T_3 - \frac{3}{5} \right) c_{27} + \hat{Q}_q \alpha_{\text{LQ}} T_3, \quad (A2) \]

with the \( E_{(2,0)}^{\text{rot}} \) can be obtained from Eq (25). The masses of spin-3/2 baryon sextet mass can be written as following expression:

\[ M_{\Sigma_{c3}} = M_{c1} + E_{(2,0)}^{\text{rot}} + m_Q - \frac{1}{3} \frac{\kappa}{m_Q} D_6 + d_6 T_3 + \frac{2}{3} \left[ T_3 + \frac{1}{3} \right] c_8 \]
\[ + \frac{1}{9} \left( T_3^2 + \frac{1}{5} T_3 - \frac{3}{5} \right) c_{27} + \hat{Q}_q \alpha_{\text{LQ}} T_3, \quad (A5) \]
Figure 4: Mass shifts of singly charmed baryons in strange matter ($\delta = 0$, $\chi = 0.15$). In the upper left panel, the $\lambda$ dependences of the baryon antitriplet, i.e., $\Lambda_c$ and $\Xi_c$ are drawn in solid curve and dashed one, respectively. In the right upper panel, those of the baryon sextet with spin 1/2 are depicted, whereas in the lower panel, those of the baryon sextet with spin 3/2 are shown. The results are given in unit of MeV.

$$M_{\Xi_Q} = M_{\text{cl}} + E_{(2,0)}^{\text{rot}} + m_Q - \frac{1}{3} m_Q$$
$$- \frac{1}{3} D_6 + d_6 T_3 + \frac{3}{10} \left( T_3 - \frac{1}{6} \right) c_8$$
$$- \frac{2}{45} \left( T_3^2 + 2 T_3 + \frac{1}{4} \right) c_{27}$$
$$+ \hat{Q}_Q^\alpha L_Q T_3, \quad \text{(A6)}$$

$$M_{\Omega_Q} = M_{\text{cl}} + E_{(2,0)}^{\text{rot}} + m_Q - \frac{1}{3} m_Q$$
$$- \frac{4}{3} D_6 + \frac{1}{5} c_8 - \frac{1}{45} c_{27}. \quad \text{(A7)}$$

Here $d_{3,6}$ and $D_{3,6}$ are defined as

$$D_3 = (m_s - \hat{m}) \left( \frac{3}{8} \bar{\tau} + \beta \right),$$

$$D_6 = (m_s - \hat{m}) \left( \frac{3}{20} \bar{\tau} + \beta - \frac{3}{10} \bar{\gamma} \right),$$

$$d_3 = (m_d - m_u) \left( \frac{3}{8} \bar{\tau} + \beta \right),$$

$$d_6 = (m_d - m_u) \left( \frac{3}{20} \bar{\tau} + \beta - \frac{3}{10} \bar{\gamma} \right). \quad \text{(A8)}$$

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