Realization of a coupled-mode heat engine with cavity-mediated nanoresonators

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We report an experimental demonstration of a coupled-mode heat engine in a two-membrane-in-the-middle cavity optomechanical system. The normal mode of the cavity-mediated strongly coupled nanoresonators is used as the working medium, and an Otto cycle is realized by extracting work between two phononic thermal reservoirs. The heat engine performance is characterized in both normal mode and bare mode pictures, which reveals that the correlation of two membranes plays a substantial role during the thermodynamic cycle. Moreover, a straight-twin nanomechanical engine is implemented by engineering the normal modes and operating two cylinders out of phase. Our results demonstrate an essential class of heat engine in cavity optomechanical systems and provide an ideal platform for investigating heat engines of interacting subsystems in small scales with controllability and scalability.

INTRODUCTION

Heat engine, as an essential achievement in thermodynamics, has regained significant attention in the nonequilibrium regime with the developments in nanotechnology and laser cooling. Heat engines at nano/microscales or single-atom levels have been experimentally realized with a single trapped ion (1, 2), nano/microresonators (3–7), nitrogen vacancy centers (8), cold atoms (9, 10), and nuclear spins (11, 12). On the other hand, optomechanics, because of the flexible controllability and extremely low decoherence of mechanical oscillators, has witnessed tremendous achievements in exploring quantum physics in mesoscopic or macroscopic scales (13–18) and ultrasensitive metrology (19–24). Recently, significant progress in fabrication of mechanical devices at micro- and nanoscales has made the optomechanical system as an excellent candidate for studying nonequilibrium thermodynamics (25–28). A hallmark example is to study the stochastic (4–7) and quantum (8–12) heat engines in such a system. Although the optomechanical system has several appealing advantages, e.g., it is a truly mechanical system and has the ability to operate deep in the quantum regime, and several theoretical models for heat engines in optomechanics have been proposed (29–38), the experimental studies of heat engine in cavity optomechanical systems remain elusive so far.

In this work, we demonstrate a coupled-mode stochastic heat engine in a multimode optomechanical system with two nanomechanical membranes inside an optical cavity (39–41). Although the strongly coupled oscillators have been intensively investigated as an important model of stochastic or quantum heat engines in many theoretical works (42–45), the experimental demonstration has not been reported. Here, we extend the proposal (29) and realize a heat engine based on cavity-mediated strongly interacting nanomechanical membranes. The normal mode of two nanomechanical resonators is used as the working medium, and an Otto cycle is implemented by controlling the frequency of membrane and the phononic thermal bath. We have developed a method to analyze the work and efficiency of such a coupled-mode engine in the normal mode and bare mode pictures, incorporating with the technique of single trajectory real-time measurement. The correlation of two membranes plays an important role and performs considerable work in the thermodynamic cycle. Moreover, such a multimode system can be straightforwardly extended to multicylinder heat engines. A straight-twin nanomechanical heat engine is realized by engineering the normal modes and exploiting two normal mode branches alternatively in the same thermodynamic cycle. The realization of such a coupled-mode heat engine with optomechanics extends the nanomechanical heat engines to multipartite systems with high flexibility and provides an opportunity to study more interesting phenomena in nonequilibrium thermodynamics with interacting systems.

RESULTS AND DISCUSSION

Experimental setup

The experimental setup of the coupled-mode nanomechanical heat engine is similar to the one used in (28, 46), as shown in Fig. 1A. Two spatially separated silicon nitride nanomechanical membranes are placed inside a Fabry-Perot cavity. One membrane is in contact with a room temperature thermal bath, and the other is driven by a white noise to obtain the high-temperature thermal bath (28). The motions of membranes are monitored by two weak probe laser fields (not shown in the figure). The vibrational (1,1) modes are used in the experiment, which are nearly degenerate with eigenfrequencies \( \omega_m \approx 2\pi \times 400 \text{kHz} \) (\( m = 1,2 \)). Piezos are used to precisely control the frequencies of membranes (41). The cavity is driven by a red-detuned laser field, which interacts with both membranes simultaneously due to the dynamical backaction, and consequently, two individual eigenmodes (bare modes) of membranes are effectively coupled by the cavity field.

Hamiltonian description

The interaction Hamiltonian of such a composite system of two membranes interacting with a common cavity field is given by

\[
\hat{H}_{\text{int}} = -\hbar \sum_{i=1,2} g_i \hat{a}^\dagger \hat{a} \hat{b}_i^\dagger \hat{b}_i,
\]

where \( \hat{a} \) and \( \hat{b}_i \) are the annihilation operators for cavity and mechanical modes, respectively, \( g_i \) is the optomechanical coupling rate of each membrane \( i \). After adiabatically eliminating the cavity mode, the system can be effectively described by a phonon-phonon coupling Hamiltonian (28, 47),

\[
\hat{H}_{\text{eff}} = \hbar \sum_{i=1,2} \Omega_i \hat{b}^\dagger_i \hat{b}_i + \hbar \Lambda (\hat{b}_2^\dagger \hat{b}_1 + \hat{b}_1^\dagger \hat{b}_2),
\]

where \( \Omega_i = \omega_i - i\gamma/2 - \Lambda \) and \( \Lambda = g_1g_2\chi_{\text{eff}} \) is the...
The expansion can be realized by reducing the normal mode frequency $\omega_\alpha$, which is controlled by $\Delta \omega$. The heat engine is based on an Otto cycle with two “adiabatic” and two isochoric processes. The schematic of the Otto cycle associated with the upper normal mode is shown in Fig. 1B. The energy changes of the four strokes for the upper normal mode are denoted as $W_{1\to2}$, $Q_{2\to3}$, $W_{3\to4}$, and $Q_{4\to1}$, respectively. The total work done by the upper normal mode for an ideal Otto cycle is $W = W_{1\to2} + W_{3\to4} = h(\omega_{\alpha_i} - \omega_{\alpha_e})(N_{\gamma_j} - \langle N_{\gamma_j} \rangle)$, where $\langle N_{\gamma_j} \rangle = k_B T/\hbar \omega_{\gamma_j}$ and $\langle N_{\gamma_j} \rangle = k_B T/\hbar \omega_{\gamma_j}$ are the mean phonon numbers of upper normal mode at status 1 and 3, respectively. The heat that the upper normal mode consumes from the hot bath is $Q_{4\to1} = h \omega_{\alpha_e} \langle (N_{\gamma_j} - \langle N_{\gamma_j} \rangle) \rangle$. The work efficiency of the heat engine for the upper normal mode is defined as $\eta = W/Q_{4\to1}$.

The thermodynamic cycle starts at the status 1 (see Fig. 1D). At $t = 0$, $M_1$ contacts with the high-temperature thermal bath, while $M_2$ contacts with the room temperature thermal bath. The initial frequency detuning is $\Delta \omega_0 \sim 2\pi \times 100$ Hz, and the phonon number of upper normal mode is dominated by the phonon number of bare mode $M_1$, i.e., $\langle B_i^\dagger B_i \rangle \approx \langle B_i^\dagger B_i \rangle$. Subsequently, the high-temperature thermal bath is turned off, the frequency of $M_1$ ($\omega_\alpha$) linearly sweeps from $\Delta \omega_0$ to the final frequency detuning $\Delta \omega_f \sim 2\pi \times 200$ Hz within a time duration of 20 ms, and the frequency of $M_2$ ($\omega_\beta$) keeps constant, i.e., the status 1 to 2, as illustrated in Fig. 1D. Consequently, the upper normal mode frequency changes from $\omega_{\alpha_1} \approx 2\pi \times 200$ Hz to $\omega_{\alpha_2} \approx 0$ Hz. The time sequences of $\omega_\alpha$ and the thermal bath are presented in Fig. 2A. This process corresponds to the expansion stroke of the classical heat engine, with a decrease of the upper normal mode frequency $\omega_\alpha$. This is the key stroke of the cycle that the heat engine performs work to the environment during this process.

In our current setup, the work is performed by the nanomechanical membranes against the environment, i.e., the piezo and substrate, in the form of stress, which is dissipated by exciting unconfined mechanical modes over the system (7). In practice, it is important to find a way to use the work via a flywheel or battery, for example, a flywheel can be directly attached to the end mirror in an optomechanical piston engine (29), the coupling between the engine and flywheel can be realized via a spin-dependent optical dipole force (2), and the work output can be stored in the quantum coherence and used by attaching an electrical load (37). To harvest the work in our situation, the mechanical oscillators can be designed by integrating an additional oscillator in the system, so that the expansion of the membrane can drive a third mechanical oscillator coherently, which is similar to the method used in (3).

The stroke from status 2 to 3 is implemented in the constant frequencies and corresponds to the isochoric stroke. The upper normal mode experiences a full thermalization to a low phonon population at room temperature in this stroke. The process of status 3 to 4 is the compression process, with $\omega_\alpha$ sweeping back to its initial value ($\Delta \omega_0 \to \Delta \omega_0$). During this process, the environment does work to the heat engine; however, this is negligible because the upper normal mode has been thermalized at the room temperature. In the process of status 4 to 1 (isochoric stroke), the high-temperature thermal bath is switched on, and the upper normal mode is thermalized to a high phonon population state, which returns to the origin of the thermodynamic cycle. The measured phonon numbers of bare modes during the whole thermodynamic cycle are plotted in Fig. 2B, which are averaged more than 250 cycles. A typical single trajectory of the cycle is shown in the inset of Fig. 2B. The stochastic property of the engine is dominated by the thermalization process of status 4 to 1. The fluctuation of the thermal population during the thermalization...
process leads to different initial phonon numbers for the expansion stroke in each cycle. Consequently, the work done in each cycle varies, which obeys an exponential function (see the Supplementary Materials).

To better understand the expansion stroke, the detail of this process is illustrated in Fig. 2C. In the experiment, the mechanical displacements of bare modes, i.e., the amplitudes of the membrane vibrations, are measured in real time. By diagonalizing the effective Hamiltonian, the normal mode operators can be written as $\hat{B}_+ = u_{12} \hat{b}_1 + u_{22} \hat{b}_2$ and $\hat{B}_- = u_{11} \hat{b}_1 + u_{21} \hat{b}_2$, where $\{|u_{11}, u_{12}|, |u_{21}, u_{22}|\}$ is the transform matrix. Thus, the phonon numbers of normal modes can be expressed as $N_+ = u_{12}^2 \hat{b}_1^2 + u_{22}^2 \hat{b}_2^2 + u_{12} u_{22} (\hat{b}_1 \hat{b}_2^* + \hat{b}_2 \hat{b}_1^*)$ and $N_- = u_{11}^2 \hat{b}_1^2 + u_{21}^2 \hat{b}_2^2 + u_{11} u_{21} (\hat{b}_1 \hat{b}_2^* + \hat{b}_2 \hat{b}_1^*)$, which are plotted in Fig. 2C. Figure 2D shows the corresponding theoretical simulation, which agrees very well with the experimental results. In Fig. 2E, the components for the phonon number of upper normal mode are illustrated. It is clearly seen that the population of the bare mode $M_1$ transfers to the mode $M_2$ and their correlation during the process, and the correlation between two bare modes reaches maximum at $\Delta \omega = 0$ (the peak of the green curve in Fig. 2E) appears at $t = 10$ ms. The corresponding coefficient of each component for $N_+$ is displayed in Fig. 2F.

In an ideal Otto cycle, the sweep time should be fast enough to avoid the dissipation of membranes and, at the same time, slow enough to avoid the diabatic transition between two normal modes, to keep the phonon number of the upper normal mode constant. In the real situation, the dissipation cannot be completely ignored during this stroke. Following the previous theoretical proposals, we still adopt the term of adiabatic and exploit a more realistic model, Landau-Zener model (50–53), to characterize the transitions including the dissipation in such a process. The Landau-Zener transition depends on the frequency sweep rate $\alpha$ and the coupling strength $\Lambda$. When the frequency sweep time is slow enough, the phonon population transfers from $M_1$ to $M_2$ adiabatically, i.e., along the upper normal mode branch. When the sweep time is fast, some population can transfer to the lower normal branch, i.e., diabatic. The diabatic transition probability can be approximately expressed as $P_{\text{diab}} = \exp[-2\pi \Lambda^2 / |\alpha|] (50)$. In Fig. 2, $\Lambda = 2\pi \times 40$ Hz and $|\alpha| = 2\pi \times 20$ Hz/ms, which leads to $P_{\text{diab}} \approx 4\%$. Therefore, the transfer is approximately an adiabatic transition, while the phonon number of
the upper normal mode dissipates during the transition. The oscillation in Fig. 2C is the Rabi oscillation in the nonresonant case, i.e., with a frequency detuning.

The thermodynamic cycle of the Otto cycle is shown in Fig. 3A. The phonon number of the upper normal mode for four strokes is plotted as a function of the upper normal mode frequency $\omega_u$, with a frequency sweep time of 15 ms, which is more close to an ideal Otto cycle compared to a longer sweep time. The cycle starts from the expansion stroke (the top right corner in Fig. 3A), which corresponds to $t = 0$ in Fig. 2B. The phonon number decays are determined by the rates of $\gamma_1$ and $\gamma_2$ (the red dots in Fig. 3A). Then, the cycle follows the processes of thermalization of the normal mode to the low phonon population (the green dots), adiabatic compression (the blue dots), and thermalization to the high phonon population state (the orange dots) in the counterclockwise direction as illustrated in Fig. 3A.

Since the phonon number is not conserved in the expansion stroke in Fig. 3A, in contrast to an ideal Otto cycle, the work performed in the expansion stroke should be an integral $W_{1\rightarrow2} = h \int^{\omega_+}_{\omega_-} N_i(\omega) \, d\omega$. Consequently, the total work $W$ done by the upper normal mode is estimated straightforwardly with the area enclosed by the cycle (the product of two axis units is the energy divided by $h$), which gives $W = 3 \times 10^{-21}$ J according to Fig. 3A and a power of $\sim 3.75 \times 10^{-21}$ W. Because of a relatively fast sweep rate compared to the situation shown in Fig. 2, i.e., $|\alpha| = 2\pi \times 27 \text{ Hz/ms}$, the diabatic transition probability becomes 9%. The diabatic transition can be visualized by the gray area in Fig. 3A, where the red squares present the total phonon numbers of the upper and lower normal modes. The difference between the red squares and the red dots are due to the diabatic transition from the upper normal mode to the lower normal mode. The red circles are the phonon number of upper normal mode without including the correlations (ignoring the cross terms in $N_i$). Therefore, the light blue area in Fig. 3A can be understood as the work done by the correlations of the bare modes.

In general, the injected heat has an integral expression similar to the work calculation. In the current setup, the frequency does not change during the thermalization process, which is shown by the orange dots in Fig. 3A; therefore, $Q_{i\rightarrow i-1} = \hbar \omega_u (\langle N_i \rangle - \langle N_{i-1} \rangle) \approx \hbar \omega_u (\langle N_i \rangle_i)$. Since the coupling strength $\lambda = 2\pi \times 40$ Hz is much smaller than the mechanical mode frequency $\omega_m \approx 2\pi \times 400$ kHz, the local approach can be adopted (37). The work efficiency is limited by the difference of the initial and final normal mode frequency in stroke 1 (29). The work efficiency $\eta_{\text{ideal}}$ is approximately equal to $(\omega_i - \omega_{i-1})/\omega_m \approx 0.05\%$ for an ideal Otto cycle (with negligible small decays and the Otto cycle has a rectangular shape) in the current setup, where $\omega_i$ and $\omega_{i-1}$ (y axis in Fig. 1D) are the initial and final frequencies of the upper normal mode with a frequency detuning $\Delta \omega_i$ (x axis in Fig. 1D). A larger value of $\omega_i - \omega_{i-1}$ could be chosen to increase the work efficiency. A significant enhancement of the efficiency requires a much stronger deformation of the membrane, for example, by using a much larger voltage applied on the piezo or using a more efficient piezo actuator. There is no limiting factor, since the frequency difference $\omega_i - \omega_{i-1}$ can be as large as $\omega_m$ in principle.

Here, the normalized efficiency $\eta_N$ is used to focus on the effects of diabatic transition and decays, which is defined as the ratio of the actual work efficiency to $\eta_{\text{ideal}}$, i.e., $\eta_N = W/(\eta_{\text{ideal}} Q_{i\rightarrow i-1})$. $\eta_N$ as a function of the frequency sweep time is presented in Fig. 3B. $\eta_N$ decreases with the increase of the sweep time, since the phonon number damping is dominant (the red diamonds and red curve). $\eta_N$ without including the correlations (the cross terms in $N_i$) is shown as the blue diamonds and blue curve. When the sweep time is relatively small, the diabatic transfer occurs. This issue could be solved by using shortcuts to adiabaticity, which allows the fast control of heat engine without decreasing the efficiency (54, 55).

The contribution of the lower normal mode is much smaller than the upper normal mode. This is because the upper and lower normal modes are respectively dominated by the bare modes $M_1$ and $M_2$ at $\Delta \omega_i \sim 2\pi \times 200$ Hz, and consequently, the upper normal mode is thermalized to a high phonon population, and the lower normal mode is thermalized to a low phonon population, with a ratio of $\sim 60$. Therefore, the work efficiencies for the cases of the upper normal mode and two normal modes are similar. While the situation is different at a relatively small sweep time, the efficiency of two normal modes could have a significantly greater value owing to the diabatic transition. The thermodynamic cycle of the lower normal mode and more details can be found in the Supplementary Materials.

Previously, we have demonstrated a single-cylinder optomechanical heat engine. In practice, a multicylinder heat engine is preferred, because of the fact that not only more work can be performed in each cycle, but also the work can be extracted more smoothly. Here, we

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**Fig. 3. Thermodynamic cycle and work efficiency.** (A) The thermodynamic cycle of engine with the upper normal mode. The status numbers are marked according to Fig. 1D. The red dots represent the phonon number of the upper normal mode, the red squares present the total phonon numbers of the upper and lower normal modes, and the red circles are the phonon number of upper normal mode by ignoring the correlations (the cross terms in $N_i$). The fluctuations of phonon numbers follow the Boltzmann distribution, which are not shown in the figure. (B) The normalized efficiency as a function of the frequency sweep time. The red and blue diamonds are the experimental data with and without the contribution of correlations. The red and blue curves are the corresponding theoretical simulations. The error bars are the SDs.
realize a straight-twin nanomechanical heat engine in the same setup but engineering the normal modes in a different way, which allows two normal modes to operate out of phase as two cylinders. The schematic of Otto cycle for the straight-twin engine is shown in Fig. 4A, with both normal modes included (the notation in Fig. 4A is defined in the same way as in Fig. 1B). The normal mode branches are modified to be \( \omega_{\pm} = \pm \sqrt{\omega_0^2 + \Delta \omega^2 / 4} \), as shown in Fig. 4B. In contrast to the situation in Fig. 1D, where \( \omega_1 \) changes and \( \omega_2 \) keeps constant, here, both frequencies of membranes change at the same sweep rate but with opposite directions, and then one can observe a noise spectrum as shown in Fig. 4B. By following the control sequence shown in Fig. 4C, the phonon numbers of \( M_1 \) and \( M_2 \) during a cycle are measured (Fig. 4D). To realize positive work output for both normal modes, it requires the sweep frequency separation range to be offset toward the right from the anticrossing, e.g., \( \Delta \omega \) sweeps from \( 2\pi \times 720 \) Hz to \(-2\pi \times 180 \) Hz, and then sweeps back, as shown in Fig. 4B. Similar to the case in a single-cylinder heat engine, one can obtain the thermodynamic diagrams of both normal modes in 1 cycle, as shown in Fig. 4 (E and F). The top right corner in Fig. 4E and the left bottom corner in Fig. 4F correspond to the initial time of a cycle, i.e., \( t = 0 \). The red, green, blue, and orange dots represent the four strokes in sequence. The work done by the two normal modes in one thermodynamic cycle are \( 2.6 \times 10^{-21} \) J and \( 0.3 \times 10^{-21} \) J, respectively. The difference of the work performed by two normal modes can be reduced by using the eigenmodes of membranes with larger quality factors.

In conclusion, we have realized a coupled-mode stochastic heat engine with nanomechanical resonators. The heat engine is based on a multimode optomechanical system, and the normal mode of two coupled nanomechanical membranes is used as the working medium. Both single- and double-cylinder heat engines are experimentally investigated by properly engineering the normal modes of the coupled system. It is a truly multiple-cylinder heat engine scheme and is possible to generalize to a heat engine array with sophisticated engineering and fabrication. Future work includes investigating the finite-time quantum thermodynamics and enhancement of the output power and efficiency in such a novel platform for the heat engines at nanoscales.

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**Fig. 4. Straight-twin nanomechanical heat engine.** (A) Schematic of the Otto cycle for a straight-twin engine with both normal modes. (B) The experimental measurement and the theoretical simulation of the normal mode frequencies for a straight-twin engine. (C) The frequency detuning and the high-temperature thermal bath connected to \( M_1 \) as a function of time during a cycle. (D) The measured average phonon numbers of \( M_1 \) and \( M_2 \) during a cycle. (E and F) The thermodynamic diagrams of the upper and lower normal mode branches, respectively. The status numbers are marked according to (B). The fluctuations of phonon numbers follow the Boltzmann distribution, which are not shown in the figures.
MATERIALS AND METHODS

Nanomechanical membrane

The nanomechanical membrane used in the experiment is a commercial stochiometric silicon nitride membrane (Norcada), which is deposited on a silicon wafer by low-pressure chemical vapor deposition. It has a large tensile stress of ~1 GPa, a high mechanical quality factor (~10^7 at low temperature), and ultralow optical absorption in the infrared range. The membrane has a thickness of 50 nm and a 1 mm by 1 mm size. Each membrane is attached to two ring piezo actuators (Noliac) for the purpose of independent membrane frequency and position control.

SUPPLEMENTARY MATERIALS

Supplementary material for this article is available at https://science.org/doi/10.1126/sciadv.abt7740.

REFERENCES AND NOTES

1. J. Roßnagel, S. T. Dawkins, K. N. Tolazzi, O. Abah, E. Lutz, F. Schmidt-Kaler, K. Singer, Sci. Adv. 7, eabl7740 (2021).
2. D. von Lindenfels, O. Gräb, C. T. Schmiegelow, V. Kaushal, J. Schulz, M. T. Mitchison, Nat. Photonics 11, 735–739 (2017).
3. M. Serra-Garcia, A. Foehr, M. Molerón, J. Lydon, C. Chong, C. Daraio, Mechanical

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6 of 7

Sheng et al., Sci. Adv. 7, eabl7740 (2021) 8 December 2021
53. O. V. Ivakhnenko, S. N. Shevchenko, F. Nori, Simulating quantum dynamical phenomena using classical oscillators: Landau-Zener-Stückelberg-Majorana interferometry, latching modulation, and motional averaging. *Sci. Rep.* **8**, 12218 (2018).

54. A. del Campo, J. Goold, M. Paternostro, More bang for your buck: Super-adiabatic quantum engines. *Sci. Rep.* **8**, 12218 (2018).

55. A. del Campo, J. Goold, M. Paternostro, More bang for your buck: Super-adiabatic quantum engines. *Sci. Rep.* **8**, 12218 (2018).

56. S. Alipour, A. Chenu, A. T. Rezakhani, A. del Campo, Shortcuts to adiabaticity in driven open quantum systems: Balanced gain and loss and non-markovian evolution. *Quantum* **4**, 336 (2020).

57. F. Bemani, A. Motazedifard, R. Roknizadeh, M. H. Naderi, D. Vitali, Synchronization dynamics of two nanomechanical membranes within a Fabry-Perot cavity. *Phys. Rev. A* **96**, 023805 (2017).

58. S. A. Biehs, G. Agarwal, Dynamical quantum theory of heat transfer between plasmonic nanosystems. *J. Opt. Soc. Am. B* **30**, 700–707 (2013).

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