Steady dynamic buckle propagation simulation of confined pipelines under external pressure by user defined elements in ABAQUS

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Abstract. This paper simulates steady dynamic buckle propagation of confined linearly elastic pipelines under external pressure by developing a user-defined-element in ABAQUS allowing the simulation of dynamic propagation through conventional static step. The simulation is separated into two parts: the static indenting step to deform a pipeline locally and the steady dynamic propagation step to obtain the relationship between the deformation profile and propagation speed.

1. Introduction
Subsea pipelines are often buried in soil for thermal insulation. Kyriakides and Lee[1] investigated quasi-static buckle propagation of confined steel pipeline. The study of confined collapse of rings is indeed not new in literature. For example Boot [2] derived a simple elastic buckling criterion for rings (liner pipes) confined by host pipes using the minimum potential principle considering the effect of a gap between the host pipe and liner pipe. Jacobson S [3] derived a semi-analytical result for buckling of confined rings considering material inelasticity. Recently El-Sawy [4] used finite element analysis to conduct the parametric analysis of buckling of confined rings under external pressure and evaluated the results against the Jacobson’s analytical results. Numerical analysis of confined collapse of a ring under uniform external pressure by finite difference method was conducted by Kyriakides and Youn [5]. The main difficulty was about the proper determination of separation point where the confined pipe was separated from the confining rigid circle. Vasilikis and Karamanos [6] studied the structural stability of confined thinned walled cylinder under external pressure through a comprehensive finite element analysis. The main novelty lies in the inclusion of the deformability of confining medium. As noticed by Kyriakides [7] the confined elastic pipeline has a load-displacement curve with “up-down-up” feature while unconfined elastic pipeline has no such feature. So theoretically it is not possible for a linearly elastic pipeline to develop buckle propagation. The quasi-static buckle propagation of confined linearly elastic pipeline has been investigated by Kyriakides [7] and Li and Kyriakides [8, 9]. They divided the problem into two cases: doubly symmetric and symmetric with one axis. However, no researches have been on the dynamic propagation of confined pipelines. This paper presents a simple but effective method to study the effect of buckle propagation velocity on the deformation of long confined linearly elastic tubes. From the above, most researches are directed to quasi-static buckle propagation while there
are some relevant results on dynamic buckle propagation. For example, Netto and Kyriakides [10] conducted experiments on the efficiency of buckle arrestor (see DNV-OS-F101 [11]) using the air as pressurizing medium to keep constant pressure and not without surprise the dynamics improves the efficiency of buckle arrestor due to the reversed ovality in the end. Tassoulas [12] developed an analytical method to include the effect of surrounding fluid on dynamic buckle propagation of pipelines by using the acoustic fluid model. Netto and Kyriakides [13] treated the fluid as acoustic medium in investigating the dynamic buckle propagation of elastically supported beam. Recently Omrani et al. [14] conducted extensive finite element analysis on dynamic buckle propagation of pipelines to obtain the relationship between propagation velocity and geometric parameters such as diameter-to-thickness ratios. Although buckle propagation has attracted attention in literature, the dynamic effect has not been fully understood and the dynamic buckle propagation researches all involve a time-costing transient dynamic analysis. In this paper, we assume the dynamic buckle propagation is in a steady state and develop a user-defined element UEL in ABAQUS to include the inertia forces directly eliminating the necessity of transient dynamic simulation replaced by only some numerical iterations.

2. Problem description and FEM

The subsea pipeline is often buried under the soil for the thermal insulation purpose. Fig.1(a) shows an subsea pipeline confined in a rigid cylinder. The subsea pipeline is externally pressurized by uniform pressure $p$. As an initial attempt on this problem, we assume that the subsea pipeline’s material is linearly elastic and deformation is double-symmetric, i.e., the deformed pipeline is symmetric to the two axes in Fig. 1(b). The radius of thin pipeline is 1m, and the radius of inner circle of rigid cylinder is 1.01m such that the gap between thin pipe and rigid cylinder is 0.01m. The uniform thickness of pipe is 0.01m. The material of subsea pipeline is steel with elastic modulus 206GPa, density is 7700kg/m$^3$ and Poisson’s ratio is 0.3. The rigid confining cylinder is meshed very coarsely by the rigid element in ABAQUS [15]. The model’s boundary conditions are shown in Fig. 1(b). The leading generator is X-symmetric (symmetric about OYZ plane). The left end’s deformation is symmetric about OXY plane (Z-symmetric). Also this structure is Y-symmetric about OZX plane. We assume a hydrostatic condition for the right end for $Z=-L$ boundary (see Fig. 1(c)). The mesh by shell element S4R is generated by assigning the model in Fig.1(b) axially 500 element and circumferentially 30 elements for 1/4 circle uniformly. $L=30$m. The hydrostatic Boundary Condition 1/4 circle of the right end is created by:

1. Firstly create a fixed reference node at $Z=-L$, $X=Y=0$ with all degrees of freedom restrained.
2. Create 30 triangular planes as with surface element SFM [15].
3. Use the *Tie technique in ABAQUS [15] to couple the free nodes of these planes with nodes of S4R shell element on this boundary.
4. Use the shell elements and the SFM elements to create a closed cavity by using the “cavity” technique in ABAQUS. The hydrostatic load would be imposed automatically by prescribing the volume change of this cavity.

![Figure 1. Schematic of confined thin subsea pipeline](image-url)
contact pair between the pipe and the plane OXZ and the contact pair between the pipe and the plane OYZ. All contact pairs have the normal behavior as “hard” and tangential behavior as “frictionless”. The separation of contacting pair is allowed. The buckle propagation initiation needs a local buckle. This buckle is artificially generated by dragging quasi-statically point A in Fig.1(b) downward in –Y direction to the point where Y=0 (no pressure in this phase). The reaction force for this drag during the initiation phase is plotted in Fig.2. The reaction force would be decreasing when the displacement of point A is increasing. Fig.3 gives the deformation sequence in this phase. The point A is kept fixed ever since in the analysis.

![Figure 2. Force deflection curve for the initiation phase](image)

![Figure 3. Displacement contour for the initiation phase correspond to points in Fig.2](image)

A quasi-static buckle propagation should be initiated by prescribing the change of enclosed volume of this pipe (see Yan et al.[16]). Fig. 4 and Fig. 5 show the pressure change and corresponding displacement contours during this phase. \( \Delta V / V_0 \) means the enclosed volume change divided by the initial enclosed volume. The pressure would be constant after some deformations as quasi-static steady propagation state has been reached.

### 3. Dynamic buckle propagation

We investigate the steady dynamic buckle propagation by using the UEL in ABAQUS (see section 4 below for technical details). The leading generator’s Y-direction displacements for different propagating speed is plotted in Fig. 6 (S is the conventional distance measuring Lagrangian coordinate in the undeformed pipe and \( \Delta S \) is the difference of S for two consecutive nodes in each generator). It turns out that the length of transitional part is decreasing as the propagation speed is increasing. The variation of external pressure for different velocities is very small and we conclude that the dynamic pressure is almost independent of the propagation speed.

There are some concerns over the meshing and loading method: the mesh should be fine enough otherwise the steady buckle propagation state would not be obtainable due to the separation of contact introducing severe discontinuity; the Riks step should not be used in this case because the Riks step fails easily if contact separation occurs.
4. Algorithm principles of user defined elements in ABAQUS
We present the algorithm of user defined elements in ABAQUS in simulating the steady state dynamic propagation. Suppose that the buckle is propagating in a constant speed $c$ and there is an observer travelling along with this buckle in speed $c$. As propagation is steady, the observer would always see the identical picture and each material point in each generator repeatedly deforms in the same manner sequentially, i.e., each generator of this cylindrical panel is indeed a streamline in which the material points flow. Each generator during the steady propagation is deformed in a spatial curve. An infinitesimal line element $dS$ ($S$ is the conventional distance-measuring Lagrangian coordinate for each generator, see Fig. 7) is deforming into line element $12$. The predeformed line element $12 \frac{dS}{\rho}$ deforms into a line element $\frac{\partial}{\partial S} \frac{\rho}{\partial S}$ where $\rho$ is the spatial position vector in the deformed configuration. By “steady” property, the velocity can be written as:

$$\vec{v}(\xi) = -\tau \left( \frac{\partial \vec{x}}{\partial S} \right) |\vec{x}| \frac{\partial S}{\partial S} \left| \frac{\rho}{\partial S} = -c \frac{\vec{x}}{\partial S} \right. \right)$$

(1)

where $\xi$ is the spatial distance measuring coordinate (see Fig. 7), $\vec{v}$ is the relative velocity for the moving observer, $c$ is defined as $c = \tau / |\vec{x} / \partial S| \left| \frac{\rho}{\partial S} \right|$ and the subscript “left” means the value for uniform buckled part. The $c$ is the reference propagation velocity and it is related to both buckle propagation speed and the stretching ratio of the buckled uniform part. $|\vec{x} / \partial S|$ means the length of vector $\vec{x} / \partial S$. The absolute velocity could be recovered.
\[
\tilde{v}_{ab} = \tilde{v} + \tilde{c} = -c \frac{\partial \tilde{x}}{\partial t} + \tilde{c}
\]  

where \(\tilde{c}\) is the propagation velocity vector. The acceleration could be obtained by:

\[
\ddot{x} = \ddot{v} = \frac{\partial \tilde{v}}{\partial t} + \tilde{v} \cdot \nabla \tilde{v} = \frac{\partial (c \tilde{x})}{\partial t} \cdot \nabla (-c \frac{\partial \tilde{x}}{\partial t}) = c^2 \frac{\partial^2 \tilde{x}}{\partial t^2} \cdot \nabla (c \frac{\partial \tilde{x}}{\partial t}) = c^2 \frac{\partial^2 \tilde{x}}{\partial t^2} \cdot \frac{\partial^2 \tilde{x}}{\partial S^2} / \partial S / \partial \tilde{x} = c^2 \frac{\partial^2 \tilde{x}}{\partial t^2} \cdot \frac{\partial^2 \tilde{x}}{\partial S^2} = c^2 \frac{\partial^2 \tilde{x}}{\partial t^2} / \partial S^2
\]  

where \(\ddot{x}\) is the acceleration with the dot operator as time derivative, \(t\) is the time, \(\nabla\) is the spatial gradient operator, \(\frac{\partial \tilde{x}}{\partial \tilde{S}}\) is the position vector for the first node of this UEL and so on. The inertia force is imposed on every third node of UEL along each generator by \(F = -Cm\ddot{x}\) where \(m\) is the mass of each shell element S4R, \(C\) is a coefficient related to the position of the UEL. For nodes in generator AC and BD, \(C=1/2\) and otherwise \(C\) is 1. A

Figure 7. Schematic of deformed generator as a streamline

The virtual work principle yields: \(\delta W = \delta U + P \delta V_{\text{enclosed}} - \int \rho \delta \dot{u} dV\) where \(\delta (\cdot)\) means variation of \((\cdot)\), \(U\) is the total strain energy, \(P\) is the external pressure, \(V_{\text{enclosed}}\) is the enclosed volume of pipeline and \(dV\) is the volume element and \(\dot{u}\) is the displacement vector. The inertia force is imposed by D’Alembert principle. The first two terms are automatically imposed by ABAQUS and to impose the third term, we extend the functionality of ABAQUS by developing a user-define-element subroutine UEL [15] to include the inertia force.

Figure 8. User defined element (UEL) definition with nodes along streamline

We use finite difference method and UEL is defined in Fig. 8. The circle means a node with index 1, 2, 3 and so on. Each UEL consists of 5 consectutive nodes. The nodes have been generated during the meshing step and in UEL we only use the existing nodes and don’t introduce additional nodes which fact is essential for ABAQUS to integrate UEL into computation. Supposing that an element is associated to 5 nodes the inertia force is lumped on the third node of this element by (a fourth order central difference scheme):

\[
\ddot{\tilde{x}} = c^2 \frac{\partial \tilde{x}}{\partial t} / \partial S^2 \approx \frac{c^2}{\Delta S^2} \cdot \left(-\frac{1}{12} \tilde{x}_{\text{first}} + \frac{4}{3} \tilde{x}_{\text{second}} - \frac{5}{2} \tilde{x}_{\text{third}} + \frac{4}{3} \tilde{x}_{\text{fourth}} - \frac{1}{12} \tilde{x}_{\text{fifth}} \right)
\]  

where \(\Delta S\) is the distance of two consecutive nodes in one generator and \(\tilde{x}_{\text{pos}}\) is the position vector for the first node of this UEL and so on. The inertia force is imposed on every third node of UEL along each generator by \(F = -Cm\ddot{x}\) where \(m\) is the mass of each shell element S4R, \(C\) is a coefficient related to the position of the UEL. For nodes in generator AC and BD, \(C=1/2\) and otherwise \(C\) is 1. A
Fortran subroutine program has been compiled and two key parameters Residual vector RHS and the associated Jacobian matrix AMATRX are defined (refer to Yan [17] for more details).

5. Conclusions
In this paper, we present a method for studying steady dynamic buckle propagation phenomenon by developing a user-defined-element UEL technique and investigate the effect of dynamics of buckle propagation of confined elastic pipeline without transient analysis. Main conclusions are briefly listed hereinafter:

For long confined pipe, the dynamic effect doesn’t change the pressure significantly. The pressure remains independent of the propagation velocity.

Dynamic effect shortens the transitional part significantly.

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