Dynamical Electroweak Symmetry Breaking
and Fourth Family

Michio Hashimoto

Theory Center, IPNS, KEK,
1-1 Oho, Tsukuba, Ibaraki 305-0801, JAPAN,
E-mail: michioh@post.kek.jp

We propose a dynamical model with a (2 + 1)-structure of composite Higgs doublets: two nearly degenerate composites of the fourth family quarks $t'$ and $b'$, $\Phi_{t'} \sim \bar{t'}_{\text{R}}(t', b')_{\text{L}}$ and $\Phi_{b'} \sim \bar{b'}_{\text{R}}(t', b')_{\text{L}}$, and a heavier top-Higgs resonance $\Phi_t \sim t_R(t, b)_L$. This model naturally describes both the top quark mass and the electroweak symmetry breaking. Also, a dynamical mechanism providing the quark mass hierarchy can be reflected in the model. The properties of these composites are analyzed in detail.

1. Introduction

Repetition of the generation structure of quarks and leptons is a great mystery in particle physics. Although three generation models have been widely accepted, the basic principle of the standard model (SM) allows the sequential fourth generation (family). Also, the electroweak precision data does not exclude completely existence of the fourth family. Noticeable is that the LHC has a potential for discovering the fourth family quarks at early stage.

If the fourth generation exists, we can naturally consider a scenario that the condensates of the fourth generation quarks $t'$ and $b'$ dynamically trigger the electroweak symmetry breaking (EWSB). The Pagels-Stokar (PS) formula suggests that their contributions to the EWSB should not be small, because the masses of $t'$ and $b'$ should be heavy, $M_{t'} > 311$ GeV and $M_{b'} > 338$ GeV, respectively.

On the other hand, the role of the top quark is rather subtle, i.e., although the contribution of the top is obviously much larger than the other three generation quarks, $b$, $c$ and etc., it is estimated around 10-20% of the EWSB scale.

Recently, utilizing the dynamics considered in Ref. 6, we introduced a new class of models in which the top quark plays just such a role. Its signature is the existence of an additional top-Higgs doublet $\Phi_t$ composed of the quarks and antiquarks of the third family, $\Phi_t \sim t_R(t, b)_L$. In the dynamical EWSB scenario with the fourth family, the top-Higgs $\Phi_t$ is heavier than the fourth generation quark composites.
\( \Phi_t \sim \bar{u}'_R(t', b')_L \) and \( \Phi_{t'} \sim \bar{b}'_R(t', b')_L \). However, in general, \( \Phi_t \) is not necessarily ultraheavy and decoupled from the TeV-scale physics. This leads to a model with three composite Higgs doublets.\(^8\) We explore such a possibility, based on Refs. 7,8.

As for the fourth family leptons, we assume that their masses are around 100 GeV,\(^9\) and thus their contributions to the EWSB are smaller than that of the top quark. For the dynamics with very heavy fourth family leptons, and thereby with a lepton condensation, one needs to incorporate more Higgs doublets, say, a five composite Higgs model. Also, the Majorana condensation of the right-handed neutrinos should be reanalyzed. This possibility will be studied elsewhere.

2. Model

Based on the dynamical model in Ref. 7, we study a Nambu-Jona-Lasinio (NJL) type model with the third and fourth family quarks:\(^8\)

\[
\mathcal{L} = \mathcal{L}_f + \mathcal{L}_g + \mathcal{L}_{NJL},
\]

where \( \mathcal{L}_g \) represents the Lagrangian density for the SM gauge bosons, the fermion kinetic term is

\[
\mathcal{L}_f = \sum_{i=3,4} \bar{\psi}^{(i)}_L i D\psi^{(i)}_L + \sum_{i=3,4} \bar{u}^{(i)}_R i D\psi^{(i)}_R + \sum_{i=3,4} \bar{d}^{(i)}_R i D\psi^{(i)}_R,
\]

and the NJL interactions are described by

\[
\mathcal{L}_{NJL} = G_{t'}(\bar{\psi}^{(4)}_L t'_R)(\bar{\psi}^{(4)}_R \psi^{(4)}_L) + G_{t'}(\bar{\psi}^{(4)}_L \psi^{(4)}_R)(\bar{\psi}^{(4)}_R t'_R) + G_{t'}(\bar{\psi}^{(3)}_L \psi^{(3)}_R)(\bar{\psi}^{(3)}_R t'_R) + G_{t'}(\bar{\psi}^{(3)}_L \psi^{(3)}_R)(\bar{\psi}^{(3)}_R t'_R) + \text{(h.c.)}.
\]

Here \( \psi^{(i)}_L \) denotes the weak doublet quarks of the \( i \)-th family, and \( \bar{u}^{(i)}_R \) and \( \bar{d}^{(i)}_R \) represent the right-handed up- and down-type quarks, respectively.

As was shown in Ref. 7, the diagonal parts of the NJL interactions, \( G_{t'} \), \( G_{t'} \) and \( G_t \), can be generated from the topcolor interactions.\(^10\) Following the dynamical model in Ref. 7, we assume that the coupling constants \( G_{t'} \) and \( G_{t'} \) are supercritical and mainly responsible for the EWSB, while the four-top coupling \( G_t \) is also strong, but subcritical.\(^7\) The mixing term \( G_{t'} t \) can be generated by a flavor-changing-neutral (FCN) interaction between \( t' \) and \( t \).\(^7\) On the other hand, \( G_{t'} t \) may be connected with topcolor instantons.\(^10\) In this case, in order to produce the four-fermion type operator, an appropriate dynamical model should be chosen. Although we keep the \( G_{t'} t \) term in a general discussion, it will be ignored in the numerical analysis. Since these four-fermion mixing terms provide the off-diagonal mass terms of the composite Higgs fields in low energy, at least two of them are required so as to evade (pseudo) Nambu-Goldstone (NG) bosons.
3. (2 + 1)-Higgs doublets

3.1. Low energy effective model

In low energy, the model introduced in the previous section yields an approximate (2 + 1)-structure in the sector of the Higgs quartic couplings. Indeed, in the bubble approximation, the composite $\Phi_{t'}(b')$ couples only to $\psi_L^{(3)} \equiv q_L = (t', b')_L$ and $t'_R(b'_R)$, while the top-Higgs $\Phi_t$ couples only to $\psi_L^{(3)} \equiv q_L = (t, b)_L$ and $t_R$. This leads to such a (2 + 1)-structure. (See Figs. 1 and 2.) Although the electroweak (EW) gauge interactions violate this structure, the breaking effects are suppressed, because the yukawa couplings are much larger than the EW gauge ones.

Let us study the low energy effective model.

It is convenient to introduce auxiliary fields at the NJL scale. In low energy these composite Higgs fields develop kinetic terms and hence acquire the dynamical degrees of freedom. The Lagrangian of the low energy model is then

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_g + \mathcal{L}_s + \mathcal{L}_y,$$

with

$$\mathcal{L}_s = |D_\mu \Phi_{t'}|^2 + |D_\mu \Phi_{t'}|^2 + |D_\mu \Phi_t|^2 - V,$$  \hspace{1cm} (5)

and

$$- \mathcal{L}_y = y_t \bar{\psi}_L^{(4)} t'_R \tilde{\Phi}_{t'} + y_t \bar{\psi}_L^{(4)} t'_R \Phi_{t'} + y_t \bar{\psi}_L^{(3)} t_R \Phi_t + \text{h.c.},$$

where $V$ represents the Higgs potential and $\Phi_{t', b', t}$ are the renormalized composite Higgs fields ($\Phi_{t'} = -i\tau_2 \Phi_{t'}$). Taking into account the renormalization group (RG) improved analysis, we study the following Higgs potential:  

$$V = V_2 + V_4,$$  \hspace{1cm} (7)
The Higgs mass terms are connected with the inverse of the four-fermion couplings. While $M_{Φ^b}^2$ and $M_{Φ^t}^2$ are negative, the mass square $M_{Φ_t}^2$ is positive, which reflects a subcritical dynamics of the $t$ quark. Note that the top-Higgs $Φ_t$ acquires a vacuum expectation value (VEV) due to its mixing with $Φ^b_t$. On the other hand, the quartic couplings $λ_1$–$5$, $t$ are induced in low energy as schematically shown in Figs. 1 and 2, and hence these values are dynamically determined.

The structure of the mass term part $V_2$ is general. On the other hand, the $V_4$ part is presented as the sum of the potential for the two Higgs doublets $Φ^b_t$ and $Φ^t_t$, and that for the one doublet $Φ_t$, i.e., it reflects the $(2 + 1)$-structure of the present model. In passing, when we consider general Higgs quartic couplings for the three Higgs, there appear 45 real parameters.

3.2. Mass spectra of the quarks and the Higgs bosons

Let us analyze the mass spectra of the quarks and the Higgs bosons.

Note that the number of the physical Higgs bosons in our model are three for the CP even Higgs bosons ($H_1$, $H_2$ and $H_3$ with the masses $M_{H_2} \leq M_{H_1} \leq M_{H_3}$), two for the CP odd Higgs ($A_1$ and $A_2$ with the masses $M_{A_2} \leq M_{A_1}$), and four for the charged ones ($H^+_1$ and $H^+_2$ with the masses $M_{H^+_1} \leq M_{H^+_2}$). It turns out that the heavy Higgs bosons, $H^+_2$, $A_2$, and $H_3$, consist mainly of the components of the top-Higgs $Φ_t$. 
Fig. 3. $M_{t'}$ and $M_{b'}$. The bold and dashed curves are for $\Lambda^{(3)}/\Lambda^{(4)} = 1, 2$, respectively. The dotted lines correspond to the lower bounds for the masses of $t'$ and $b'$ at 95% C.L., $M_{t'} > 311$ GeV and $M_{b'} > 338$ GeV, respectively.

The VEV's $v_{t', b', t}$ for the Higgs fields $\Phi_{t', b', t}$ are approximately determined by

$$\begin{align*}
\left[2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)\cot^2 \beta_4\right] v_{t'}^2 &\simeq -M_{\Phi_{t'}}^2 - M_{\Phi_{t'}} M_{\Phi_{b'}} \cot \beta_4, \\
\left[2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)\tan^2 \beta_4\right] v_{b'}^2 &\simeq -M_{\Phi_{b'}}^2 - M_{\Phi_{t'}} M_{\Phi_{b'}} \tan \beta_4, \\
v_t &\simeq -\frac{M_{\Phi_{t'}} M_{\Phi_{b'}}}{M_{\Phi_{t'}}^2} v_{t'} + -\frac{M_{\Phi_{t'}} M_{\Phi_{t'}}}{M_{\Phi_{t'}}^2} v_{b'},
\end{align*}$$

(10) (11) (12)

where we defined the ratios of the VEV's by $\tan \beta_4 \equiv v_{t'}/v_{b'}$. Note that the relation $v^2 = v_{t'}^2 + v_{b'}^2 + v_t^2$ holds, where $v \simeq 246$ GeV.

On the other hand, the masses of the CP odd and charged Higgs bosons are approximately given by

$$\begin{align*}
M_{A_1}^2 &\approx -2M_{\Phi_{t'}}^2 (1 - \tan^2 \beta_{34}), \\
M_{A_2}^2 &\approx M_{\phi_{t'}}^2 (1 + 2\tan^2 \beta_{34}) + M_{\phi_{b'}}^2 \tan^2 \beta_{34}, \\
M_{H_1^+}^2 &\approx M_{A_1}^2 + 2(m_{t'}^2 + m_{b'}^2)(1 - \tan^2 \beta_{34}), \\
M_{H_2^+}^2 &\approx M_{A_2}^2 + 2(m_{t'}^2 + m_{b'}^2) \tan^2 \beta_{34},
\end{align*}$$

(13) (14) (15) (16)

where we took $\tan \beta_4 = 1$ and defined $\tan \beta_{34} \equiv v_t/\sqrt{v_{t'}^2 + v_{b'}^2}$. The mass formulae for $H_{1,2,3}$ are quite complicated because of the $3 \times 3$ matrices.

In order to calculate the mass spectra more precisely, we employ the RGE's with the compositeness conditions:

$$y_{t'}^2(\mu = \Lambda^{(4)}) = \infty, \quad y_{b'}^2(\mu = \Lambda^{(4)}) = \infty, \quad y_t^2(\mu = \Lambda^{(3)}) = \infty,$$

(17)
for the yukawa couplings, and
\[
\frac{\lambda_1}{y_{b'}}_{\mu=\Lambda^{(4)}} = \frac{\lambda_2}{y_{t'}}_{\mu=\Lambda^{(4)}} = \frac{\lambda_3}{y_{b'b'}}_{\mu=\Lambda^{(4)}} = \frac{\lambda_4}{y_{b't'}}_{\mu=\Lambda^{(4)}} = 0, \frac{\lambda_t}{y_t}_{\mu=\Lambda^{(3)}} = 0, \tag{18}
\]
for the Higgs quartic couplings, where \(\Lambda^{(4)}\) and \(\Lambda^{(3)}\) denote the composite scales for the fourth generation quarks and the top, respectively. The RGE’s are given in Ref. 8. For consistency with the \((2 + 1)\)-Higgs structure, we ignore the one-loop effects of the EW interactions. As for the Higgs loop effects, although they are of the \(1/N\)-subleading order, they are numerically relevant and hence incorporated.

Notice that the initial NJL model contains six four-fermion couplings: The EWSB scale and the pole (\(\overline{\text{MS}}\)) mass of the top quark are fixed to \(v = 246\) GeV and \(M_t = 171.2\) GeV (\(m_t = 161.8\) GeV), respectively. We further convert the NJL couplings into more physical quantities, \(M_{A_1}, M_{A_2}\) and \(\tan \beta_4\). As for \(M_{b't'}^2\), we fix \(M_{b't'}^2 = 0\). Numerically, it is consistent with \(G_{b't'} \approx 0\). For the numerical calculations, we use the QCD coupling constant \(\alpha_3(M_Z) = 0.1176\).\(^9\)

The results are illustrated in Figs. 3 and 4. The masses of \(t'\) and \(b'\) are essentially determined by the value of \(\Lambda^{(4)}\), where we converted the \(\overline{\text{MS}}\)-masses \(m_{t'}\) and \(m_{b'}\) to the on-shell ones, \(M_{t'(b')} = m_{t'(b')}[1 + 4\alpha_s/(3\pi)]\). As is seen in Fig. 3, their dependence on \(\Lambda^{(3)}/\Lambda^{(4)} (= 1 – 2)\) is mild. When we vary \(\tan \beta_4\) in the interval \(0.9 – 1.1\), the variations of \(M_{t'}\) and \(M_{b'}\) are up to 10\% (see Fig. 3). The Higgs masses are relatively sensitive to the value of \(\Lambda^{(4)}\) (see Fig. 4), while their sensitivity to \(\Lambda^{(3)}/\Lambda^{(4)} (= 1 – 2)\) is low. The Higgs mass dependence on \(\tan \beta_4\) is also mild, at most 5\% for \(\tan \beta_4 = 0.9 – 1.1\), and \(\Lambda^{(4)} = 2 – 10\) TeV.

Since at the compositeness scale the yukawa couplings go to infinity, there could in principle be uncontrollable nonperturbative effects. By relaxing the compositeness...
conditions, we estimated such “nonperturbative” effects around 10%. Since the loop
effects of the EW interactions are expected to be much smaller, the uncertainties
of the “nonperturbative” effects are dominant.

The $2\sigma$-bound of $R_b$ yields $M_{A_2} \geq 0.70, 0.58, 0.50$ TeV for $\Lambda^{(4)} = 2, 5, 10$ TeV.
Following the $(S,T)$ analysis a la LEP EWWG, we found that our model is within
the 95% C.L. contour of the $(S,T)$ constraint, when the fourth family lepton mass
difference is $M_{\nu'} - M_{\nu''} \sim 150$ GeV.3

We can introduce the CKM structure in our model.7,8 Since the mixing between
the fourth family and the others is suppressed, $|V_{t\nu'}| \sim |V_{us}|m_c/m_t \sim O(10^{-3})$ and
$|V_{t\nu''}| \sim |V_{tb}| \sim m_c/m_t \sim O(10^{-2})$, the contributions of the $t'$-loop to the $B^0 - \bar{B}^0$
mixing, $b \to s\gamma$ and $Z \to \bar{b}b$ are negligible. Note also that the effects of the charged
Higgs bosons are suppressed, because their masses are relatively heavy. As for the
tree FCNC and FCCC, they are highly suppressed in the first and second families,
because of the assumption that the top-Higgs is responsible for the top mass and
does not couple to the other quarks, in the spirit of the $(2 + 1)$-Higgs structure.8

4. Summary

We have studied the $(2+1)$ composite Higgs doublet model. It describes rather nat-
urally both the top quark mass and the EWSB. We can incorporate the dynamical
mechanism for the quark mass hierarchy and the CKM structure into the model.7,8

It would be interesting to embed the present model into an extra dimensional one.12

The signature of the model is clear, i.e., as shown in Fig. 4, the masses of the
four resonances are nearly degenerate and also the heavier top-Higgs bosons appear.

A noticeable feature is that due to the $t'$ and $b'$ contributions, the gluon
fusion production of $H_1$ is considerably enhanced. For example, for $\Lambda^{(4)} = 3$ TeV,
$\Lambda^{(3)}/\Lambda^{(4)} = 1.5$, $\tan\beta_A = 1$, $M_{A_1} = 0.50$ TeV, and $M_{A_2} = 0.80$ TeV, we obtain
$M_{H_1} = M_{H_2} = 0.33$ TeV and $M_{H_3} = 0.49$ TeV. In this case, $\sigma_{gg-H_1} Br(H_1 \to ZZ)$
is enhanced by 5, where the relative $H_1 ZZ$ and $H_1 t\bar{t}$ couplings to the SM values
are 0.86 and 2.0, respectively. Similarly, the CP odd Higgs production via the gluon
fusion process should be enhanced. Also, the multiple Higgs bosons can be observed
as $t\bar{t}$ resonances at the LHC. Detailed analysis will be performed elsewhere.

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