Analysis of the $B \to a_1(1260)$ form-factors with light-cone QCD sum rules

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Abstract

In this article, we calculate the $B \to a_1(1260)$ form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$ and $A(q^2)$ with the $B$-meson light-cone QCD sum rules. Those form-factors are basic parameters in studying the exclusive non-leptonic two-body decays $B \to AP$ and semi-leptonic decays $B \to A\ell\nu$, $B \to A\bar{\ell}\ell$. Our numerical results are consistent with the values from the (light-cone) QCD sum rules. The main uncertainty comes from the parameter $\omega_0$ (or $\lambda_B$), which determines the shapes of the two-particle and three-particle light-cone distribution amplitudes of the $B$-meson, it is of great importance to refine this parameter.

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Key Words: $B$ meson, Light-cone QCD sum rules

1 Introduction

The weak $B \to P, V, A$ form-factors with $P = \pi, K$, $V = \rho, K^*$ and $A = a_1, K_1$ final states are basic input parameters in studying the exclusive semi-leptonic decays $B \to P(V,A)\ell\nu$, $B \to P(V,A)\bar{\ell}\ell$ and radiative decays $B \to V(A)\gamma$, they also determine the factorizable amplitudes in the non-leptonic charmless two-body decays $B \to PP(AP, PV, VV)$. Those decays can be used to determine the CKM matrix elements and to test the standard model, however, it is a great challenge to pin down the uncertainties of the form-factors to obtain more precise results. The exclusive semi-leptonic decays $B \to P(V)\ell\nu$, $B \to P(V)\bar{\ell}\ell$ and radiative decays $B \to V\gamma$ and hadronic two-body decays $B \to PP(PV, VV)$ have been studied extensively [1, 2, 3, 4, 5, 6, 7], while the decays $B \to AP, VA$ have been calculated with the QCD factorization approach [8, 9, 10], generalized factorization approach [11, 12], etc. It is more easy to deal with the exclusive semi-leptonic precesses than the non-leptonic precesses, and there have been many works on the relevant form-factors $B \to \pi$, $B \to \rho$ in determining the CKM matrix element $V_{ub}$ [13, 14, 15, 16]. The $B \to a_1(1260)$ form-factors have been studied with the covariant light-front approach [17], ISGW2 quark model [18], quark-meson model [19], QCD sum rules [20], light-cone QCD sum rules [9] and perturbative QCD [21]. However, the values from different theoretical approaches differ greatly from each other.

The BaBar Collaboration and Belle Collaboration have measured the charmless hadronic decays $B^0 \to a_1^\mp \pi^\mp$ [22, 23]. Moreover, the BaBar Collaboration has

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measured the time-dependent CP asymmetries in the decays $B^0 \rightarrow a_1^\pm \pi^\mp$ with $a_1^\pm \rightarrow \pi^\mp \pi^\pm \pi^\mp$, from the measured CP parameters, we can determine the decay rates of $a_1^\pm \pi^-$ and $a_1^\pm \pi^+$ respectively \cite{24}. Recently, the BaBar Collaboration has reported the observation of the decays $B^\pm \rightarrow a_1^0 \pi^\pm$, $B^+ \rightarrow a_1^+ K^0$ and $B^0 \rightarrow a_1^- K^+$ \cite{25, 26}. So it is interesting to re-analyze the $B \rightarrow a_1$ form-factors with the $B$-meson light-cone QCD sum rules \cite{27}.

In Ref.\cite{27}, the authors obtain new sum rules for the $B \rightarrow \pi, K, \rho, K^*$ form-factors from the correlation functions expanded near the light-cone in terms of the $B$-meson distribution amplitudes, and suggest QCD sum rules motivated models for the three-particle $B$-meson light-cone distribution amplitudes, which satisfy the relations given in Ref.\cite{28}. In Ref.\cite{28}, the authors derive exact relations between the two-particle and three-particle $B$-meson light-cone distribution amplitudes from the QCD equations of motion and heavy-quark symmetry. The two-particle $B$-meson light-cone distribution amplitudes have been studied with the QCD sum rules and renormalization group equation \cite{29, 30, 31, 32, 33, 34, 35}. Although the QCD sum rules can’t be used for a direct calculation of the distribution amplitudes, it can provide constraints which have to be implemented within the QCD motivated models (or parameterizations) \cite{32}.

The $B$-meson light-cone distribution amplitudes play an important role in the exclusive $B$-decays, the inverse moment of the two-particle light-cone distribution amplitude $\phi_+(\omega)$ enters many factorization formulas (for example, see Refs.\cite{3, 4}). However, the light-cone distribution amplitudes of the $B$-meson are received relatively little attention comparing with the ones of the light pseudoscalar mesons and vector mesons, our knowledge about the nonperturbative parameters which determine those light-cone distribution amplitudes is limited and an additional application (or estimation) based on QCD is useful.

In this article, we use the $B$-meson light-cone QCD sum rules to study the $B \rightarrow a_1$ form-factors. The semi-leptonic decays $B \rightarrow A l \nu_l$ can be observed at the LHCb, where the $b\bar{b}$ pairs will be copiously produced with the cross section about 500 $\mu$b. We can also study the form-factors with the light-cone QCD sum rules using the light-cone distribution amplitudes of the axial-vector mesons. Recently, the twist-2 and twist-3 light-cone distribution amplitudes of the axial-vector mesons have been calculated with the QCD sum rules \cite{36}.

The $B$-meson light-cone QCD sum rules have given reasonable values for the $B \rightarrow \pi, K, \rho, K^*$ form-factors \cite{27}, so it is interesting to study the $B \rightarrow a_1$ form-factors and cross-check the properties of the $B$-meson light-cone distribution amplitudes. Furthermore, it is necessary to investigate the form-factors with different approaches and compare the predictions of different approaches.

The article is arranged as: in Section 2, we derive the $B \rightarrow a_1(1260)$ form-factors with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and Section 4 is reserved for conclusion.
2  \( B \rightarrow a_1(1260) \) form-factors with light-cone QCD sum rules

In the following, we write down the definitions for the weak form-factors \( V_1(q^2), V_2(q^2), V_3(q^2), V_0(q^2) \) and \( A(q^2) \) \cite{17},

\[
\langle a_1(p)|J_\mu(0)|B(P)\rangle = i \left\{ (M_B - M_a)\epsilon^*_\mu V_1(q^2) - \frac{\epsilon^* \cdot P}{M_B - M_a}(P + p)_\mu V_2(q^2) \right. \\
\left. -2M_a\epsilon^* \cdot \frac{P}{q^2} q_\mu[V_3(q^2) - V_0(q^2)] \right\}, \tag{1}
\]

\[
\langle a_1(p)|J^A_\mu(0)|B(P)\rangle = \frac{1}{M_B - M_a} \epsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu (P + p)_\alpha q_\beta A(q^2), \tag{2}
\]

where

\[
V_5(q^2) = \frac{M_B - M_a}{2M_a} V_1(q^2) - \frac{M_B + M_a}{2M_a} V_2(q^2),
\]

\[
J_\mu(x) = \bar{d}(x)\gamma_\mu b(x),
\]

\[
J^A_\mu(x) = \bar{d}(x)\gamma_\mu \gamma_5 b(x), \tag{3}
\]

\( V_0(0) = V_3(0) \), and the \( \epsilon_\mu \) is the polarization vector of the axial-vector meson \( a_1(1260) \). We study the weak form-factors \( V_1(q^2), V_2(q^2), V_3(q^2), V_0(q^2) \) and \( A(q^2) \) with the two-point correlation functions \( \Pi^a_\mu(p,q) \),

\[
\Pi^i_\mu(p,q) = i \int d^4x e^{ipx} \langle 0|T \{ J^a_\mu(x) J^i_\mu(0) \}|B(P)\rangle,
\]

\[
J^a_\mu(x) = \bar{u}(x)\gamma_\mu \gamma_5 d(x), \tag{4}
\]

where \( J^i_\mu(x) = J_\mu(x) \) and \( J^A_\mu(x) \) respectively, and the axial-vector current \( J^a_\mu(x) \) interpolates the axial-vector meson \( a_1(1260) \). The correlation functions \( \Pi^i_\mu(p,q) \) can be decomposed as

\[
\Pi^1_\mu(p,q) = \Pi_1 A g_{\mu\nu} + \Pi_2 B q_\mu P_\nu + \Pi_3 C q_\mu q_\nu + \Pi_4 D q_\nu q_\mu + \Pi_5 D P_\mu P_\nu,
\]

\[
\Pi^2_\mu(p,q) = \Pi_2 \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta + \cdots \tag{5}
\]

due to Lorentz covariance. In this article, we derive the sum rules with the tensor structures \( g_{\mu\nu}, q_\mu P_\nu \) and \( \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \) respectively to avoid contaminations from the \( \pi \) meson.

According to the basic assumption of current-hadron duality in the QCD sum rules approach \cite{37,38}, we can insert a complete series of intermediate states with the same quantum numbers as the current operator \( J^a_\mu(x) \) into the correlation functions \( \Pi^a_\mu(p,q) \) to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the meson \( a_1(1260) \), the correlation functions
\( \Pi^i_{\mu\nu}(p, q) \) can be expressed in the following form,

\[
\Pi^1_{\mu\nu}(p, q) = -i f_a M_a (M_B - M_a) V_1(q^2) \frac{g_{\mu\nu}}{M_a^2 - p^2} + \frac{2i f_a M_a V_2(q^2)}{(M_B - M_a)(M_a^2 - p^2)} q_\mu p_\nu + \cdots , \tag{6}
\]

\[
\Pi^2_{\mu\nu}(p, q) = i f_a M_a A(q^2) \frac{\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta + \cdots }{(M_B - M_a)(M_a^2 - p^2)} , \tag{7}
\]

where we have used the standard definition for the decay constant \( f_a \), \( \langle 0 | J^a_\mu(0) | a_1(p) \rangle = f_a M_a \epsilon_\mu \).

In the following, we briefly outline the operator product expansion for the correlation functions \( \Pi^i_{\mu}(p, q) \) in perturbative QCD theory. The calculations are performed at the large space-like momentum region \( p^2 \ll 0 \) and \( 0 \leq q^2 < m_B^2 + m_b p^2 / \Lambda \), where \( M_B = m_b + \Lambda \) in the heavy quark limit. We write down the propagator of a massless quark in the external gluon field in the Fock-Schwinger gauge and the light-cone distribution amplitudes of the \( B \) meson firstly [39],

\[
\langle 0 | T \{ q_i(x_1) \bar{q}_j(x_2) \} | 0 \rangle = i \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x_1-x_2)} \cdot \left\{ \frac{k_i}{k^2} \delta_{ij} - \int_0^1 dv G^{ij}_{\mu\nu}(vx_1 + (1 - v)x_2) \right. \\
\left. \left[ \frac{1}{2} \frac{k_i}{k^2} \sigma^{\mu\nu} - \frac{1}{k^2} v(x_1 - x_2)^\mu x_2^\nu \right] \right\} , \\
\langle 0 | \bar{q}_a(x) h_{\nu\beta}(0) | B(v) \rangle = -\frac{i f_B m_B}{4} \int_0^\infty d\omega e^{-i\omega v \cdot x} \cdot \left\{ (1+ \gamma_5) \left[ \phi_+(\omega) - \frac{\phi_+(\omega) - \phi_-(\omega)}{2v \cdot x} \right] \gamma_5 \right\} \beta_\alpha , \\
\langle 0 | \bar{q}_a(x) G_{\lambda\rho}(ux) h_{\nu\beta}(0) | B(v) \rangle = -\frac{i f_B m_B}{4} \int_0^\infty d\omega \int_0^\infty d\xi e^{-i(\omega + u \xi) v \cdot x} \cdot \left\{ (1+ \gamma_5) \left[ (v_\lambda \gamma_\rho - v_\rho \gamma_\lambda) \left( \Psi_A(\omega, \xi) - \Psi_V(\omega, \xi) \right) \\
-i \sigma_\lambda \Psi_V(\omega, \xi) - \frac{x_\lambda v_\rho - x_\rho v_\lambda}{v \cdot x} X_A(\omega, \xi) \\
+ \frac{x_\lambda v_\rho - x_\rho v_\lambda}{v \cdot x} Y_A(\omega, \xi) \right] \gamma_5 \right\} \beta_\alpha , \tag{8}
\]
where
\[
\phi_+(\omega) = \frac{\omega}{\omega_0^2} e^{-\frac{\omega}{\omega_0}}, \quad \phi_-(\omega) = \frac{1}{\omega_0} e^{-\frac{\omega}{\omega_0}},
\]
\[
\Psi_A(\omega, \xi) = \Psi_V(\omega, \xi) = \frac{\lambda_B^2}{6\omega_0^2} \xi^2 e^{-\frac{\omega + \xi}{\omega_0}},
\]
\[
X_A(\omega, \xi) = \frac{\lambda_B^2}{6\omega_0^2} \xi^2 (2\omega - \xi) e^{-\frac{\omega + \xi}{\omega_0}},
\]
\[
Y_A(\omega, \xi) = -\frac{\lambda_B^2}{24\omega_0^4} \xi (7\omega_0 - 13\omega + 3\xi) e^{-\frac{\omega + \xi}{\omega_0}},
\]
(9)

the \(\omega_0\) and \(\lambda_B^2\) are some parameters of the \(B\)-meson light-cone distribution amplitudes.

Substituting the \(d\) quark propagator and the corresponding \(B\)-meson light-cone distribution amplitudes into the correlation functions \(\Pi^\mu(p, q)\), and completing the integrals over the variables \(x\) and \(k\), finally we obtain the representation at the level of quark-gluon degrees of freedom. In this article, we take the three-particle \(B\)-meson light-cone distribution amplitudes suggested in Ref.\[27\], they obey the powerful constraints derived in Ref.\[28\] and the relations between the matrix elements of the local operators and the moments of the light-cone distribution amplitudes, if the conditions \(\omega_0 = \frac{2}{3} \bar{\Lambda}\) and \(\lambda_B^2 = \lambda_H^2 = \frac{3}{2} \omega_0^2 = \frac{2}{3} \bar{\Lambda}^2\) are satisfied \[29\].

In the region of small \(\omega\), the exponential form of distribution amplitude \(\phi_+(\omega)\) is numerically close to the more elaborated model (or the BIK distribution amplitude (BIK DA)) suggested in Ref.\[32\],
\[
\phi_+(\omega, \mu = 1\text{GeV}) = \frac{4\omega}{\pi\lambda_B(1 + \omega^2)} \left[ \frac{1}{1 + \omega^2} - 2\frac{\sigma_B - 1}{\pi^2} \ln \omega \right],
\]
(10)

where \(\omega_0 = \lambda_B\). The parameters \(\lambda_B\) and \(\sigma_B\) are determined from the heavy quark effective theory QCD sum rules including the radiative and nonperturbative corrections. There are other phenomenological models for the two-particle \(B\)-meson light-cone distribution amplitudes, for example, the \(k_T\) factorization formalism \[10, 41\], in this article, we use the QCD sum rules motivated models.

After matching with the hadronic representation below the continuum threshold \(s_0\), we obtain the following three sum rules for the weak form-factors \(V_1(q^2)\), \(V_2(q^2)\)
and $A(q^2)$ respectively,

\[
V_1(q^2) = \frac{1}{f_a M_a (M_B - M_a)} e^{\frac{M^2}{M^2}} \left\{ -\frac{1}{2} f_B M_B M^2 \int_0^{\sigma_0} d\sigma \phi_+ (\omega') \frac{d}{d\sigma} e^{-\frac{\sigma}{\bar{\sigma}}} - \frac{f_B M_B}{2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma_{MB}} d\omega \int_{\sigma_{MB} - \omega}^{\infty} \frac{d\xi}{\xi} \frac{\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)}{\bar{\sigma}^2} \frac{d}{d\sigma} e^{-\frac{\sigma}{\bar{\sigma}}} + \frac{f_B M_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma_{MB}} d\omega \int_{\sigma_{MB} - \omega}^{\infty} \frac{d\xi}{\xi} \frac{(1 - 2u) \tilde{X}_A(\omega, \xi)}{\bar{\sigma}^3} e^{-\frac{\sigma}{\bar{\sigma}}} \left[ \tilde{M}_B^2 - 4s M_B^2 \right] - 2 \tilde{M}_B^2 - 2 M_B^2 + 1 \right\}, \tag{11}
\]

\[
V_2(q^2) = \frac{M_B - M_a}{2 f_a M_a} e^{\frac{M_B^2}{M^2}} \left\{ f_B M_B \int_0^{\sigma_0} d\sigma \left[ \frac{1 - 2\sigma}{\bar{\sigma}} + \frac{2 M_B}{M^2} \left[ \bar{\phi}_+(\omega') - \bar{\phi}_-(\omega') \frac{\sigma}{\bar{\sigma}} \right] e^{-\frac{\sigma}{\bar{\sigma}}} + \frac{f_B M_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma_{MB}} d\omega \int_{\sigma_{MB} - \omega}^{\infty} \frac{d\xi}{\xi} \frac{(1 - 2u) \tilde{X}_A(\omega, \xi)}{\bar{\sigma}^2} \frac{d}{d\sigma} e^{-\frac{\sigma}{\bar{\sigma}}} + \frac{f_B M_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma_{MB}} d\omega \int_{\sigma_{MB} - \omega}^{\infty} \frac{d\xi}{\xi} \frac{(1 - 2u) \tilde{Y}_A(\omega, \xi) \frac{\sigma}{\bar{\sigma}}}{\bar{\sigma}} e^{-\frac{\sigma}{\bar{\sigma}}} \right\}, \tag{12}
\]

\[
A(q^2) = \frac{M_B - M_a}{2 f_a M_a} e^{\frac{M_B^2}{M^2}} \left\{ f_B M_B \int_0^{\sigma_0} d\sigma \frac{\phi_+ (\omega')}{\bar{\sigma}} e^{-\frac{\sigma}{\bar{\sigma}}} + \frac{f_B M_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma_{MB}} d\omega \int_{\sigma_{MB} - \omega}^{\infty} \frac{d\xi}{\xi} \frac{[\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)]}{\bar{\sigma}^2} e^{-\frac{\sigma}{\bar{\sigma}}} + \frac{f_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma_{MB}} d\omega \int_{\sigma_{MB} - \omega}^{\infty} \frac{d\xi}{\xi} (1 - 2u) \tilde{X}_A(\omega, \xi) \frac{d}{d\sigma} e^{-\frac{\sigma}{\bar{\sigma}}} \right\} \tag{13}
\]
where

\[
\begin{align*}
    s &= M_B^2 \sigma - \frac{\sigma}{\bar{\sigma}} q^2 , \quad \omega' = \sigma M_B , \quad \bar{\sigma} = 1 - \sigma , \\
    \sigma_0 &= \frac{s_0 + M_B^2 - q^2 - \sqrt{(s_0 + M_B^2 - q^2)^2 - 4s_0M_B^2}}{2M_B^2} , \\
    u &= \frac{\sigma M_B - \omega}{\xi} , \quad \bar{M}_B^2 = M_B^2 (1 + \sigma) - \frac{1}{\sigma} q^2 , \\
    \bar{X}_A(\omega, \xi) &= \int_0^\omega d\lambda X_A(\lambda, \xi) , \quad \bar{Y}_A(\omega, \xi) = \int_0^\omega d\lambda Y_A(\lambda, \xi) , \\
    \bar{\phi}_\pm(\omega) &= \int_0^\omega d\lambda \phi_\pm(\lambda) .
\end{align*}
\]  

In Ref. \[31\], Lange and Neubert observe that the evolution effects drive the light-cone distribution amplitude \( \phi_+(\omega) \) toward a linear growth at the origin and generate a radiative tail that falls off slower than \( \frac{1}{\omega} \), even if the initial function has an arbitrarily rapid falloff, which implies the normalization integral of the \( \phi_+(\omega) \) is ultraviolet divergent. In this article, we derive the sum rules without the radiative \( \mathcal{O}(\alpha_s) \) corrections, the ultraviolet behavior of the \( \phi_+(\omega) \) plays no role at the leading order \( \mathcal{O}(1) \). Furthermore, the duality thresholds in the sum rules are well below the region where the effect of the tail becomes noticeable. The nontrivial renormalization of the \( B \)-meson light-cone distribution amplitude is so far known only for the \( \phi_+(\omega) \), we use the light-cone distribution amplitudes of order \( \mathcal{O}(1) \), which satisfy all QCD constraints.

3 Numerical result and discussion

The input parameters are taken as \( \omega_0 = \lambda_B(\mu) = (0.46 \pm 0.11) \text{ GeV}, \mu = 1 \text{ GeV} \) \[32\], \( \lambda_E^2 = (0.11 \pm 0.06) \text{ GeV}^2 \) \[29\], \( M_a = (1.23 \pm 0.06) \text{ GeV}, \ f_a = (0.238 \pm 0.010) \text{ GeV}, \ s_0 = (2.55 \pm 0.15) \text{ GeV}^2 \) \[36\], \( M_B = 5.279 \text{ GeV}, \ f_B = (0.18 \pm 0.02) \text{ GeV} \) \[42\ \[43\].

The Borel parameters in the three sum rules are taken as \( M^2 = (1.1 - 1.5) \text{ GeV}^2 \), in this region, the values of the weak form-factors \( V_1(q^2), V_2(q^2) \) and \( A(q^2) \) are stable enough.

Taking into account all the uncertainties, we obtain the numerical values of the weak form-factors \( V_1(q^2), V_2(q^2) \) and \( A(q^2) \), which are shown in Fig.1, at zero momentum transfer,

\[
\begin{align*}
    V_1(0) &= 0.67_{-0.21}^{+0.33} , \\
    V_2(0) &= 0.31_{-0.11}^{+0.18} , \\
    V_3(0) &= 0.29_{-0.06}^{+0.07} , \\
    V_0(0) &= 0.29_{-0.06}^{+0.07} , \\
    A(0) &= 0.41_{-0.13}^{+0.20} .
\end{align*}
\]  

(15)
Figure 1: The form-factors $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$ with the momentum transfer $q^2$. 
The form-factors can be parameterized in the double-pole form,

$$F_i(q^2) = \frac{F_i(0)}{1 + a_F q^2 / M_B^2 + b_F q^4 / M_B^4},$$  \hspace{1cm} (16)$$

where we use the notation $F_i(q^2)$ to denote the $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$, the $a_F$ and $b_F$ are the corresponding coefficients and their values are presented in Table 3.

In calculation, we observe the dominating contributions in the three sum rules come from the two-particle $B$-meson light-cone distribution amplitudes, the contributions from the three-particle $B$-meson light-cone distribution amplitudes are of minor importance, about 1%, and can be neglected safely. It is un-expected that the main uncertainty comes from the parameter $\omega_0$ (or $\lambda_B$), which determines the shapes of the two-particle and three-particle light-cone distribution amplitudes of the $B$ meson. From Fig.1, we can see that the uncertainty of the parameter $\lambda_B$ almost saturates the total uncertainties, it is of great importance to refine this parameter. In this article, we take the value from the QCD sum rules in Ref. [32], where the $B$-meson light-cone distribution amplitude $\phi_+$ is parameterized by the matrix element of the bilocal operator at imaginary light-cone separation.

In the region of small $\omega$, the exponential (Gaussian) form of distribution amplitude $\phi_+(\omega)$ is numerically close to the BIK DA suggested in Ref. [32]. In Fig.1, we also present the numerical results with the BIK DA for the central values of the input parameters $\lambda_B$ and $\sigma_B$, the Gaussian distribution amplitude and the BIK DA

| theoretical approaches                           | $V_0(0)$ |
|-------------------------------------------------|----------|
| Covariant light front approach [17]               | 0.13     |
| ISGW2 quark model [18]                           | 1.01     |
| quark-meson model [19]                           | 1.20     |
| QCD sum rules [20]                               | 0.23 $\pm$ 0.05 |
| perturbative QCD [21]                            | $0.34^{+0.07}_{-0.06}$ $^{+0.08}_{-0.08}$ |
| light-cone sum rules [9]                         | 0.30 $\pm$ 0.05 |
| This work (light-cone sum rules)                 | $0.29^{+0.07}_{-0.06}$ |

Table 1: The form-factor $V_0(0)$ from different theoretical approaches. I know the updated value $0.30 \pm 0.05$ from private communication with Prof. H.Y.Cheng, their work is still in progress.

| theoretical approaches                           | $A(0)$ |
|-------------------------------------------------|--------|
| Covariant light front approach [17]               | 0.25   |
| quark-meson model [19]                           | 0.09   |
| QCD sum rules [20]                               | 0.42 $\pm$ 0.06 |
| perturbative QCD [21]                            | $0.26^{+0.07}_{-0.05}$ $^{+0.03}_{-0.03}$ |
| This work (light-cone sum rules)                 | $0.41^{+0.20}_{-0.13}$ |

Table 2: The form-factor $A(0)$ from different theoretical approaches.
lead to almost the same values.

From Table 1, we can see that the values of the $V_0(0)$ from the covariant light-front approach, ISGW2 quark model and quark-meson model differ greatly from the corresponding ones from the (light-cone) QCD sum rules, while the values from the (light-cone) QCD sum rules and perturbative QCD are consistent with each other. From Table 2, we observe that the values of the $A(0)$ from the covariant light-front approach, quark-meson model and perturbative QCD differ greatly from the corresponding ones from the (light-cone) QCD sum rules, while the values of the form-factors from the (light-cone) QCD sum rules are consistent with each other.

## 4 Conclusion

In this article, we calculate the weak form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$ and $A(q^2)$ with the $B$-meson light-cone QCD sum rules. The form-factors are basic parameters in studying the exclusive hadronic two-body decays $B \to AP$ and semi-leptonic decays $B \to A\ell \nu_\ell$, $B \to A\bar{\ell}\nu_\ell$. Our numerical values are consistent with the values from the (light-cone) QCD sum rules. The main uncertainty comes from the parameter $\omega_0$ (or $\lambda_B$), which determines the shapes of the two-particle and three-particle light-cone distribution amplitudes of the $B$ meson, it is of great importance to refine this parameter. However, it is a difficult work, as we cannot extract the values of the basic parameter $\lambda_B$ directly from the experimental data on the semi-leptonic decays $B \to A\ell \nu_\ell$.

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