A universally verifiable, software-independent, bare-handed voting protocol

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Abstract

We present a scalable and universally verifiable voting protocol and establish its completeness, soundness and secrecy properties. The protocol supports complete voter-verified accountability. Its correctness is independent of the hardware and software implementation and can be established solely from the output at various stages. The protocol is scalable and efficient.

1 Introduction

India recently concluded the world’s largest election [Wu and Gettleman, 2019] with 543 constituencies and well over 1 million voters per constituency on the average. Complete polling with offline electronic voting machines (EVM) not only ensured efficiency of the polling process and timely announcement of results, but, from several accounts, also ensured that the election was fair [ET-Bureau, 2019, Purkayastha and Sinha, 2019]. Electronic voting perhaps is essential for managing elections of such size and complexity. However, the EVM solution [Election Commission of India, 2019a,b] was not verifiable and had inadequate guarantees [Shukla, 2018, Banerjee and Sharma, 2019], which inevitably generated disquiet during the elections [Vora, 2017, Venkataramakrishnan, 2019].

World-wide concerns with EVMs have resulted in their being discontinued in many countries. After several years of controversy, Netherlands abandoned electronic voting in 2007 [Goldsmith and Ruthrauff, 2007], deciding that the integrity of the democratic process was more important than efficiency. Similar considerations have led to their discontinuation in Germany [NDI, 2019], France [Reuters, 2017], Ireland [O’Halloran and O’Regan, 2010] and several others. Many in the USA have voiced their apprehensions [Mercuri, 2007, Schneier, 2018, Schwartz, 2018] against existing EVMs, and the Defense Advanced Research Project Agency (DARPA) has decided to design and build a secure open source voting system for the future [Zetter, 2019].

Internet voting makes electronic voting more complex. Though countries like Estonia and Switzerland have experimented with internet voting, the concerns with them are many [Rivest, 2001a, Tufekci, 2019]. In a recent report, the national academies in the USA have recommended against internet voting till robust guarantees can be worked out, and have suggested conducting elections with human readable paper ballots [National Academies of Sciences, Engineering and Medicine, 2018].

In this paper, we first identify the concerns that surround electronic voting and then design a protocol that addresses these concerns. We also provide formal proofs of the key characteristics of our protocol.

1.1 Design considerations for electronic voting

The end-to-end (E2E) universally verifiable voting protocols do not rely only on EVMs but try to provide provable guarantees that votes are recorded and tallied correctly [Chaum, 2004, Chaum et al., 2008, Dzieduszycka-Suinat et al., 2015]. In what follows we outline the typical design requirements of such E2E verifiable systems.
A voting system is software-independent if an undetected change or error in its software cannot cause an undetectable change or error in an election outcome [Rivest, 2008]. Software independence is a necessary condition for universal verifiability, because physical verifiability of a hardware-software co-design of a system such as an EVM is almost surely an intractable (at least NP-Hard) problem [Mercuri, 1992]. Thus, the correctness of an E2E verifiable system should not depend on the hardware or software used, and must be established solely from the computed output at various stages. Software-independent systems only rely on the computing platforms to preserve secrecy by not leaking information.

Though universal verifiability implies the correctness of a protocol, some solutions also aim for voter-verified accountability wherein a voter can proactively seek a proof that their vote has been recorded and tallied correctly. Such a proof would also imply that a cast vote is non-repudiable. Voter-verifiable accountability may theoretically be considered redundant in universally verifiable voting protocols, but it adds to voters’ confidence by making the processes more transparent. It is crucial for democracy that not only are elections fair, but that they also appear to be fair.

A voting system must also be free of spurious vote injection, at all times before, during or after polling. Safeguard against spurious vote injection during polling requires strict identity verification of voters against a voter list for all votes, and must rely either on digital authentication or on offline identity verification by a polling officer. In the absence of a de-duplicated digital voter identity system, trust on the latter is unavoidable.

In any polling system voter secrecy must be preserved at all times. Hence, voting systems must never issue a receipt for the cast vote to a voter to ensure that a voter is never able to prove to a coercer or a potential vote buyer who they voted for [Benaloh and Tuinstra, 1994]. Secrecy and receipt-freeness are necessary conditions for coercion-free voting. However, the term receipt-free is somewhat misleading, because it does not prevent from issuing a token receipt to a voter from which no information about who they voted for can be gleaned.

It has also been advocated that a voter should have zero digital computing available at voting time [Chaum, 2004]. The reasons for bare-handed voting are twofold. First, it is unfair to rely on voters to be able to compute cryptographic functions - or even digitally sign - when they may not have the agency or necessary understanding of the process. Second, it is unreasonable to assume that voters can have access to trusted computing platforms that will not leak information [Rivest, 2001a, Adida, 2006]. For example, commodity laptops and handhelds, which a voter may own but not have complete understanding of, certainly cannot be trusted either for correctness of cryptographic computations or for privacy of voting. The secure platform problem [Rivest, 2001b] effectively rules out internet voting [Chaum, 2004, Rivest, 2001a,b, Mercuri, 2007], and bare-handed voting systems must necessarily be polling booth protocols.

Finally, making the vote tally of an EVM or a polling booth - typically of a few thousand voters - public may enable profiling of a locality or a community. Hence, it is essential to aggregate the votes over several polling booths and EVMs leading up to perhaps even an entire constituency before making the tally public. Large aggregations are essential for community privacy.

Any EVM based solution that relies on hardware and software integrity, with or without voter-verified paper audit trails (VVPAT) [Mercuri, 1992, Election Commission of India, 2019a], is not software independent and is hence not universally verifiable. Besides, VVPAT only ensures that the electronic vote count matches that of the paper audit trail, and that by itself provides no guarantee against spurious vote injection or deletion in both. Reliance on ad hoc and unverifiable processes such as in [Election Commission of India, 2019a, Purkayastha and Sinha, 2019] can only result in uncertain technological solutions for electoral democracy.

1.2 Our contribution

In this paper, we present an E2E cryptographic voting protocol with the following main features:

1. It is universally verifiable and software-independent. We achieve universal verifiability through complete voter-verified accountability.

2. It is a direct-recording electronic (DRE) protocol that does not use paper ballots, making it very similar to and suitable for really large elections such as in India [Election Commission of India, 2019b]. The protocol supports voter verifiable paper audit trails (VVPAT).

3. Our protocol can support both digital authentication of voters or offline identity verification. In the latter case the protocol relies on signed certificates for each vote cast from the electoral officers in-charge of polling booths, who work under public scrutiny, to protect against spurious vote injections at the polling booth. However, it provides universally verifiable guarantees against vote injection at all subsequent stages.

4. It relies on signed commitments from the polling officers for various tokens issued by them during polling. However, the procedure is completely auditable and voters can optionally challenge the commitments during voting.
5. A receipt is issued to each voter using which the voter can seek a proof that their vote has been accounted for correctly. However, the vote cannot be determined from the receipt.

6. It is a bare-handed polling booth protocol where a voter interacts with an EVM only using button presses. It however does rely on the voter to be able to do a mental addition of two small numbers to obfuscate the vote. This step can even be replaced with a table lookup.

7. It relies on the EVMs and other storage elements to preserve secrecy by not leaking information. The protocol itself completely guarantees secrecy, including large aggregations to prevent against community profiling.

8. We provide formal proofs of correctness and secrecy of the protocol. The correctness arguments hold even if the authorities collude with each other.

To the best of our knowledge this is the first protocol with all the above guarantees.

The rest of the paper is organised as follows. In Section 2 we review some of the existing E2E protocols. In Section 3 we provide a brief overview of our protocol. In Section 4 we provide some basics of cryptography on which we base our protocol. In Section 5 we present our voting protocol and in Section 6 we discuss its correctness. In Section 7 we discuss some possible simplifications of our protocol and their consequences. In Section 8 we discuss the practicalities of implementation of our protocol and conclude the paper in Section 9.

2 Some E2E verifiable systems

One of the early E2E verifiable voting proposals is the elegant protocol of Fujioka et al. [1993] based on blind signatures [Chaum, 1983] and cryptographic commitments [Pedersen, 1992]. The protocol is universally verifiable, software independent and it also supports complete voter-verified accountability. It is one of the few protocols that display the polled votes in clear text on a bulletin board allowing public verification of the tally. The protocol assures complete voter secrecy from the poll administrators. It is however not bare-handed. Also, each voter has the key of their committed and posted vote, making it not receipt and coercion free.

There are a group of printed ballot paper based bare-handed polling booth protocols [Ryan et al., 2009, Adida and Rivest, 2006, Chaum et al., 2008] that build on the original protocol of Chaum [2004]. They issue specially prepared ballots to voters where the choice of the candidate order is randomised for each voter. The random mappings are kept secret and the voters’ encoded choice, without the candidate names, are displayed after polling on public bulletin boards. Whereas [Ryan et al., 2009] uses an auditable mixnet [Chaum, 1981] for the secret mapping, [Adida and Rivest, 2006] uses homomorphic encryption and [Chaum et al., 2008] uses a simple auditable switchboard to shuffle the candidate order encodings. [Chaum et al., 2008] also reveals one side of the secret shuffles, before or after voting, to enable probabilistic verification by the public. All of them require extensive pre-poll preparation of encoding and printing of ballot papers making large scale deployment difficult.

Protocols [Bell et al., 2013] and [Farhi, 2013] are similar but they compute and print the ballots on the spot. [Bell et al., 2013] uses homomorphic encryption for the secret mapping whereas [Farhi, 2013] uses mixnets [Chaum, 1981].

In all the above protocols the voters are allowed to take away a receipt using which they can identify their cast ballot, without the candidate information, on the bulletin board. Hence, voter-verified accountability is only partial in these protocols, but it makes them receipt-free. While all these protocols are universally verifiable because the secret mappings between the candidate names and the ballot encodings can be audited, they assume the secrecy of encoding of ballot forms, and hence trust on the poll administrators is implicit. Trust on the administrators is also implicit for no spurious injection of votes after polling is over and before results are displayed.

The preprepared ballot based protocols also allow auditing authorities to completely open some random ballots before elections to verify the authenticity of the encodings. Voters may also optionally choose to ask for another ballot and keep the originally received one for audit, according to the voter initiated audit-or-cast challenge suggestion of Benaloh [2006] (see also [Benaloh, 2008]). However, relying just on such voter initiated challenge, especially in the polling booth, is problematic because on the one hand there is no way to ensure that sufficient number of voters do actually challenge so that some statistical guarantees can be ensured, and, on the other hand, too many challenges may clog the polling process.

[Neff, 2004] is different in that it does not provide the voter with a paper to submit or destroy, but only a receipt, and yet supports both cast-as-intended and counted-as-cast verifications. It requires a complicated protocol and uses special printers with cuffs, making it somewhat impractical.

[Bohli et al., 2007] is a DRE, bare-handed protocol. It uses a set of pre-committed dummy numbers and freshly generated random numbers to obfuscate the votes. The universal verifiability is established by opening commitments
Our voting protocol involves three authorities - a polling officer in-charge of each polling booth in a constituency, a collection authority responsible for collecting the voting records from the EVMs after verification and further uploading to an election authority, whose responsibility is to finally store the records and tally the votes. The election authority is also responsible for providing proofs that all votes have been accounted for correctly in the tally. Under some mild assumptions, the roles of the collection and election authorities can be merged. Universal verifiability is guaranteed even if the authorities collude. For secrecy, we rely on the authorities and the hardware and software used to not leak information.

We use some key concepts from cryptography, which we explain below along with the protocol overview.

The first concept is that of a digital signature [Diffie and Hellman, 2006] which provides a unique binding of the identity of the signer to a message and confirms the integrity of the message. A digital signature is non-repudiable and the signature and the integrity of the message can be publicly verified by anybody using the public key of the signer.

The second crucial concept that we rely upon is that of a cryptographic commitment [Pedersen, 1992]. A commitment of a chosen value implies that the committer can no longer alter their choice, but this does not require them to reveal their choice in any way. A useful metaphor is to put the choice in a box, lock the box and hand over the box to an adversary or a third party but retain the key. The committer cannot go back on the choice because others have digital copies of the box, but the privacy of their choice is preserved completely because nobody else can open the box or its copies. At a later time the committer may open the box and reveal their choice.

We assume that the polling official in-charge of a polling booth is trusted to ensure that the polling protocol is correctly followed. The polling officer verifies the identity of the voter (Figure 1a), strips the real identity and generates a random identity for the records. The polling officer also generates an obfuscation key privately for a voter before the voter actually votes. The polling officer cryptographically commits these values and affixes their signature to the commitments confirming their correctness (Figure 1b).

We cannot directly obtain a commitment of the vote from the voter because we have assumed that voters cannot digitally compute. So, we follow a protocol witnessed by the polling officer to obtain a confirmation from the voter on their vote in complete secrecy. The key idea is to convert the confirmation to a cryptographic commitment indicating that the vote has been recorded correctly. The EVM prints out a modular sum of the vote and the obfuscation key (Figure 1d). The voter confirms that the sum is correct (Figure 1e), and this according to the protocol implies that the commitment of the vote is correct. The confirmation also independently reaches the polling officer, who records that the protocol is complete. Note that issuing the commitment receipt to the voter does not violate the receipt-free condition, because it is impossible to determine the vote from the commitment alone. Our protocol amplifies the confirmation of a small addition by the voter into a provably correct cryptographic commitment.

The correctness of our protocol crucially hinges on the correctness of the commitments of the random identity and the obfuscation key generated by the polling officer. Since these are generated before - and are hence independent of - the vote, even a malicious polling officer cannot orchestrate a targeted mis-assignment of the vote, but only a random mis-assignment. We optionally provide an audit-or-cast facility for the voter to challenge the polling officer’s commitments prior to voting [Benaloh, 2006]. If a voter decides to challenge, the polling officer must reveal the commitments and provide signed printouts to the voter so that they may verify later, and generate a fresh pair of random identity and obfuscation key and the corresponding commitments. The challenge step is optional, depending on a risk assessment of whether or not the polling officers can be trusted to not attack the voting protocol even after affixing their signature. If the challenge is opted for, then there has to be a sufficient number of challenges, determined according to the hypergeometric
(a) Identity verification.

(b) PO issues a token that contains the random id and the obfuscation key, $u$, and the encoded commitments.

(c) Voter presents the token from the PO and the EVM reads the information. Voter presses the button corresponding to vote $v$.

(d) Voter performs the sum $u + v \mod m$, either mentally or using a table look-up, and checks the receipt generated by the EVM.

(e) Voter acknowledges that the receipt is correct.

Figure 1: Voter experience at the polling booth

Figure 2: After polling is over a voter may get the receipts verified through a trusted verifier who can obtain a zero knowledge proof from the election authority. The election authority must never see the voter.
distribution, to make the risk statistically negligible.

After voting ends, the collection authority reads in the records from each EVM in a constituency after verifying the digital signature of the polling officer and pipes the records to a shuffler. We do not require any cryptographic guarantees of the shuffling process, and a simple random permutation would do. The collection authority is required to maintain a record of the shuffling for later audit. All private data pass encrypted through this stage. The output of the shuffler is passed on to the election authority who decrypts and stores the private information securely, and displays the votes along with the random identifiers, for which a cryptographic commitment has been issued during voting, publicly on a bulletin board in clear text. Anybody can verify the tally from the bulletin board, which allows downloading of the data and verification of a hash signature. The aggregation and permutation at the shuffler removes all traceability of the records to particular EVMs giving perfect anonymisation. Thus the records arrive at the election authority through an anonymous channel.

The final cryptographic construct is a zero knowledge proof that the previously issued commitments correspond to one of the publicly displayed items on the bulletin board, without revealing any information about which [Camenisch et al., 2008]. A zero knowledge proof (ZKP) of a statement is an interactive protocol between a prover and a verifier where the prover convinces the verifier of the validity of the statement without revealing any additional information [Goldwasser et al., 1985]. For voter-verified accountability, a voter anonymously presents the commitments issued to them during voting through any trusted verifier to obtain a ZKP that the entry corresponding to their commitment is indeed on the bulletin board (Figure 2). The prover can be a secure online service of the election authority to which only a set of voting through any trusted verifier to obtain a ZKP that the entry corresponding to their commitment is indeed on the bulletin board (Figure 2). The prover can be a secure online service of the election authority to which only a set of commitments can be uploaded for verification. A trusted verifier can be anybody who can execute the protocol online.

We discuss some possible simplifications and alterations of the protocol in Section 7.

4 Cryptography basics

Throughout this paper $p$ and $q$ denote large primes such that $q$ divides $p - 1$, $G_q$ is a unique cyclic subgroup of $\mathbb{Z}_p^*$ of order $q$, and $g$ and $h$ are generators of $G_q$. There exist standard and efficient procedures to generate such prime pairs, and $G_q = \langle g \rangle = \langle h \rangle$. We assume that $g$ and $h$ are system initialized and publicly known, but the discrete logarithm $\log_q h$ is not known to anybody and is hard to compute. (See supplementary Section A for details.)

4.1 Pedersen commitment

Given a message $\rho \in G_q$ we use the Pedersen commitment scheme [Pedersen, 1992], $C = g^\rho h^r$, where $r \in G_q$ is a secret randomness, to compute a value $C$ that hides $\rho$.

Pedersen commitment is perfectly hiding because $C$ gives no information about $\rho$. The binding property, which demands that given a commitment $C$, it is hard to compute a different pair of message and randomness $(\rho', r')$ with the same commitment, is derived from the hardness of the discrete logarithm problem, because distinct openings $(\rho, r)$ and $(\rho', r')$ of a given commitment $g^\rho h^r = g^{\rho'} h^{r'}$ reveal that $\log_q(h) = (\rho - \rho')/(r' - r) \mod q$.

Moreover, Pedersen commitment is additively homomorphic, i.e., if $C_1 = g^{\rho_1} h^{r_1}$ and $C_2 = g^{\rho_2} h^{r_2}$ are commitments of $\rho_1$ and $\rho_2$ respectively, then $C_1 \ast C_2 = g^{\rho_1 + \rho_2} h^{r_1 + r_2}$ is a commitment of $\rho_1 + \rho_2$.

4.2 ZKP of set membership: $C$ is a commitment of some $\rho_i \in \Phi$

We use the scheme proposed by [Camenisch et al., 2008] to provide a zero knowledge proof that a given commitment $C$ corresponds to a message $\rho \in \Phi$ where $\Phi$ is a publicly available set without revealing the message. If $\Phi$ is stored indexed by $C$, then the ZKP of set membership is computationally efficient and requires only $O(1)$ sized proofs [Camenisch et al., 2008]. See the supplementary Section B for a description of the procedure and the associated security properties.

5 The protocol

We assume that a vote is an element from the set $\{0, 1, \ldots, m - 1\}$ where $m$ is a small integer.

Each polling booth, $k$, has two machines, PO$_k$ and EVM$_k$, not connected to each other in any way. The EVM$_k$ is unmanned and standalone, but PO$_k$ is controlled by the polling officer of the booth. PO$_k$ has the digital signature $p_k$ of the polling officer, and $e_k$ is the digital signature of EVM$_k$. Messages signed by the polling officer are denoted by $\sigma$ and messages signed by the EVM are denoted by $\mu$.

We assume that
1. The polling officer (using PO\(_k\)):
   1. Verifies the identity of the voter \(V_i\) (may be non-digitally).
   2. Generates a random identity \(rid_i\) for \(V_i\) (uniformly random from \(\mathbb{Z}_q\), with negligible probability of collision), and a Pedersen commitment [Pedersen, 1992] \(C_{rid_i} = g^{rid_i}h^{r_I_i}\) where \(r_I_i \in \mathbb{Z}_q\) is a secret randomness. (Section 4.1).
   3. Generates a random integer \(u_i \in \mathbb{Z}_q\) (an obfuscation key) and a Pedersen commitment \(C_{u_i} = g^{u_i}h^{r_{u_i}}\) where \(r_{u_i} \in \mathbb{Z}_q\) is a secret randomness.
   4. Signs the commitments \(\sigma_{u_i,k} = \text{sign}_{p_k}(C_{u_i})\) and \(\sigma_{rid_{i,k}} = \text{sign}_{p_k}(C_{rid_i})\).
   5. Prints out the signed commitments \((C_{rid_i}, \sigma_{rid_{i,k}}), (C_{u_i}, \sigma_{u_i,k})\), the random secrets \((r_I_i, r_{u_i}), (rid_i, u_i)\) and \((u_i \mod m)\) (the last in plain text) and hands over to \(V_i\). [At this stage the voter may optionally do an audit-or-cast challenge [Benaloh, 2006] of the commitments and ask for regeneration of \(rid_i\) and \(u_i\) keeping the original printouts for audit.]
   6. Allows \(V_i\) to proceed to the booth of EVM\(_k\).

\(rid_i\) is a secret that uniquely identifies the records for voter \(V_i\) after stripping the real identity. \(u_i\) is a random key generated prior to voting - and is hence independent of the vote - which is used to obfuscate the vote. The polling officer issues signed commitments for both to the voter, confirming their correctness. With signed commitments from the polling officer, the audit-or-cast challenge is an optional feature in our protocol, and is not strictly necessary.

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**Figure 3:** The steps for voter \(V_i\) at the polling officer’s (PO) desk.

1. The polling officer, the collection authority and the election authority are independent and all of them preserve secrecy by not disclosing their data.
2. The polling officer does not interact with the voter after identity verification.
3. PO\(_k\) does not store any information corresponding to the voter after the voter leaves the booth (erases all information from the memory). EVM\(_k\) stores only the information specified in the protocol below.
4. PO\(_k\) and EVM\(_k\) are not simultaneously tampered and controlled to subvert the protocol.

The protocol consists of four stages:

1. **At the polling booth:** The steps for each voter \(V_i\) at the polling booth \(k\) comprise two stages - first at the polling officer’s desk (Figure 3) and then at the EVM (Figure 4). After the polling is over the votes are consolidated as per Figure 5.
2. **Data collection after polling:** See Figure 6.
3. **At the election authority (EA):** See Figure 7.
4. **Post polling verification:** See Figure 8.
1. **V_i → EVM_k**: The printout containing \((C_{rid_i}, C_{ui}, r_{id_i}, r_{ui}, r_{u}, \sigma_{u_{id_i}}, \sigma_{r_{id_i}})\) for scanning. After scanning, the parts containing \((r_{id_i}, u_{id_i}, r_{l_i}, r_{u})\) are destroyed and the remaining are returned to the voter. EVM_k checks all commitments.

2. **V_i → EVM_k** (button press): \(v_i\) (vote).

3. **EVM_k**: Sets \(p_i = r_{id_i} + v_i, w_i = u_i + v_i\) and \(w'_i = w_i \mod m\).

4. **EVM_k**: \(C_{vi} = g^{w'_i} h^{v_i},\) where \(r_{vi} \in \mathbb{Z}_m\) is a secret randomness. \(\mu_{vi} = \text{sign}_{pk}(C_{vi})\).

5. **EVM_k**: \(P_i = (w_i, w'_i, r_{wi} = r_{vi} + r_{u}),\) a proof that \(C_{ui} * C_{vi}\) is a commitment of \(w_i, \mu_{P_{vk}} = \text{sign}_{vk}(P_i)\).

6. **EVM_k → V_i**: Print out \((C_{vi}, \mu_{vi},\) and the proof \((P_i, \mu_{P_{vk}}),\) with \(w'_i\) displayed in clear text within \(P_i\), for the voter.

7. **(optional)** **EVM_k → VVPAT**: Printout of \((r_{id_i}, v_i)\). The voter must be able to check the printout before it is dropped in the VVPAT box.

8. **V_i → EVM_k, PO_k** (button press): an acknowledgement that indeed \(w'_i = (v_i + (u_i \mod m)) \mod m\). The acknowledgement must reach EVM_k and PO_k through independent channels.

9. **PO_k**: \(\text{ack}_k = \"The voter corresponding to } C_{rid_i} \text{ has finished voting\". } \sigma_{ack_{vk}} = \text{sign}_{vk}(\text{ack}_i). \text{Print } (C_{rid_i}, \text{ack}_i, \sigma_{ack_{vk}})\).

10. **(optional)** **EVM_k**: The printout of \((r_{id_i}, v_i)\) drops into the VVPAT box. The voter leaves the booth.

11. **EVM_k**: Compute a hash \(h = H((r_{id_i}, v_i))\) where \(H\) is a publicly known, collision and preimage resistant cryptographic hash function [Rogaway and Shrimpton, 2004]. The first column of the table is the set \(\Psi\) and the second column is the set \(\Phi\).

12. **EVM_k**: \(s_i = \text{enc}_{p_EA}(r_{id_i}, s_{ui}, r_{l_i}, r_{ui}, r_{vi})\), the voting secrets and the secret parameters of the commitments are encrypted with \(P_{EA}\), the public key of the election authority.

13. **EVM_k**: \(m_i = ((C_{rid_i}, C_{ui}, \sigma_{u_{id_i}}, \sigma_{r_{id_i}}), (C_{vi}, \mu_{vi}), (P_i, \mu_{P_{vk}}), (h_i, \mu_{h_{vk}}))\), the signed commitments issued to the voter, and \(h_i\).

14. **EVM_k**: Store record \((s_i, m_i)\) indexed by \(C_{rid_i}\).

Note that the mod \(m\) addition in step 8 involves only small integers. The voter \(V_i\) casts a vote \(v_i\) in the EVM, which is obfuscated by adding with the obfuscation key \(u_i \mod m\). After the voter confirms that \(w'_i\) is correct, a proof for the commitment \(C_{ui} * C_{vi}\) of \(w_i, P_i\), is generated.

**Figure 4**: The steps for voter \(V_i\) at the EVM.

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1. **Polling officer → EVM_k**: The printouts \((C_{rid_i}, \text{ack}_i, \sigma_{ack_{vk}})\) (see step 9 of Figure 4) for all successful voters for scanning.

2. **EVM_k**: adds \((\text{ack}_i, \sigma_{ack_{vk}})\) to the records \((s_i, m_i)\) corresponding to \(C_{rid_i}\). Any remaining records without \((\text{ack}_i, \sigma_{ack_{vk}})\) are discarded but marked for audit. \(N_k\) is the count of valid votes acknowledged by the polling officer. \(\sigma_{N_k} = \text{sign}_{vk}(N_k)\).

3. **EVM_k**: Compute \(H_k = \bigoplus h_i\) and \(\mu_{H_k} = \text{sign}_{vk}(H_k)\) (where \(\bigoplus\) is the bitwise XOR operation).

4. **EVM_k → polling officer**: \((H_k, \mu_{H_k})\) (printout or in electronic form for the polling officer).

5. The polling officer publishes \((H_k, \mu_{H_k}), (N_k, \sigma_{N_k})\) along with the name of the polling booth and the constituency on a bulletin board \(BB_1\).

**Figure 5**: For each polling booth \(k\), after polling is over.

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1. For each EVM_k through a secure and private channel:

   a. Carry out a hardware and software integrity check of the EVM and discard if found problematic.

   b. Collect each \((s_i, m_i, (\text{ack}_i, \sigma_{ack_{vk}}))\).

   c. For each \(i\) verify the authenticity of the signatures \(\sigma_{rid_{ik}}, \sigma_{u_{id_i}}, \mu_{vi}, \mu_{P_{vk}}, \sigma_{ack_{vk}}, \mu_{h_{vk}}\) and replace with \(\sigma_{rid_{i,M}}, \sigma_{u_{id_i,M}}, \mu_{vi,M}, \mu_{P_{vk,M}}, \sigma_{ack_{vk,M}}, \mu_{h_{vk,M}}\), where the replacement signatures for the same items are generated with \(s_{M}\), the digital signature of the collection authority. This anonymises the polling booth. Discard the EVM if any entry is found to be incorrect.

2. Post collection from all EVMs, randomize the records \((s_i, m_i, (\text{ack}_i, \sigma_{ack_{vk}}))\) using a shuffler and output the records as \((s_j, m_j, (\text{ack}_j, \sigma_{ack_{vk}}))\) to the election authority through a secure and private channel. Record the shuffle history for future audit.

**Figure 6**: Data collection after polling.
1. Decrypt each $s_i$ in $\{s_i, m_i, (ack_i, \sigma_{ack,iM})\}$, verify each signature and the computation of each commitment and proof. Verify that no two $rid_i$s are within $m$ of each other and that no two $C_{rid,i}$s collide. Flag for audit if any found.

2. Store each record indexed by $C_i = C_{rid,i} \ast C_{v,i}$ and $C_{rid,i}$.

3. (optional) Publish each $[C_{rid,i}, w_i = u_i + v_i]$ on a bulletin board $BB_2$, sorted by $C_{rid,i}$. Anybody can download, identify their $C_{rid,i}$ from their receipts and verify that their $w_i$ is correctly recorded.

4. Publish rows $[rid_i, v_i, \rho_i = rid_i + v_i, (h_i, \mu_{h,iM})]$ on a bulletin board $BB_3$, sorted by $rid_i$. The first column of the table is the set $\Psi$ and the third column is the set $\Phi$. Anybody can download and verify the signature on $h_i$ using the public key of the collection authority; also $h_i$.

5. Demonstrate that $\bigoplus_k H_k = \bigoplus_i h_i$ and $\sum_k N_k = N$, where $N$ is number of rows in $BB_3$. This provides a guarantee against injection of spurious votes pre-polling, and at the collection and mixing stages. Anybody can download and verify.

6. Tally the votes on $BB_3$ and publish. Anybody can download and verify.

7. (optional) Count the VVPAT records and tally.

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**Figure 7:** At the Election authority (EA).

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1. $V_i \rightarrow$ Trusted Verifier: Receipt from the polling officer containing $(C_{rid,i}, \sigma_{rid,i})$, $(C_{v,i}, \sigma_{u,i})$ and receipt from the EVM containing $(C_{v,i}, \mu_{h,i})$ and the proof $(P_i, \mu_{P,i})$ where $P_i = (w_i, w_i', r_w)$.

2. Trusted Verifier: Check that all signatures match, that $C_{u,i} \ast C_{v,i} = g^{w_i h^{r_w}}$ and $w_i' = w_i \mod m$.

3. Trusted Verifier $\rightarrow$ EA: $(C_{rid,i}, C_{v,i})$ (EA receives a commitment pair from a voter through an anonymous channel for verification, denoted by $\rightarrow$; EA should never see the voter, but only the commitments).

4. EA $\leftrightarrow$ Trusted Verifier: Provide ZKPs that $C_i = C_{rid,i} \ast C_{v,i}$ corresponds to a $rid_i + v_i \in \Phi$ and $C_{rid,i}$ corresponds to a row in $\Psi$ [Camenisch et al., 2008]. (see Section 4.2 of the supplementary material for the protocol).

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**Figure 8:** Voter-verified accountability.
6 Attacks, correctness and secrecy

We provide an informal discussion on correctness and secrecy here. See supplementary Section C for formal proofs.

6.1 Correctness of the commitment of each vote

As per the protocol, $C_{u_i} = g^{u_i} h^* r_i$ is the commitment for $u_i \in \mathbb{Z}_q$ (Figure 3); and $C_{v_i} = g^{v_i} h^* v_i$ is a commitment of $v_i$ (Figure 4). It then follows that $P_i = (u_i, w_i, r_i, v_i = h_{u_i} + r_i v_i)$ issued in step 5 of Figure 4 is an opening of the Pedersen commitment $C_{u_i} * C_{v_i} = g^{v_i u_i} h^* v_i$, of $u_i = u_i + v_i$ with randomness $r_i$.

The voter confirms the sum $w_i = (u_i + v_i) \mod m$ from a printout which the EVM cannot change. This, and the correctness of $C_{u_i} * C_{v_i}$, established by $P_i$, and the correctness of $C_{u_i}$ signed and certified by the polling officer, imply that $C_{v_i}$ is the commitment corresponding to the voter’s intended vote.

Tricking the voter by recording a different $u_i'$ and $v_i'$ such that $w_i = u_i' + v_i'$ will require the PO to issue a wrong commitment $C_{u_i'}$ and the EVM to issue a wrong commitment $C_{v_i'}$ in such a way that their product remains the same. This will require simultaneous tampering and control of PO and EVM. Also, since $u_i$ is committed before $v_i$, this can only cause a random mis-assignment of the vote and not a targeted mis-assignment.

6.2 Voter-verified accountability

Every voter can get the signatures of $C_{rid_i}$, $C_{u_i}$ and $C_{v_i}$, and that $P_i$ is an opening of $C_{u_i} * C_{v_i}$, verified using the printouts obtained from the polling officer and the EVM. They can also present $(C_{rid_i}, C_{v_i})$ to a trusted verifier who can obtain ZKPs from the election authority that the commitments $C_{rid_i} * C_{v_i}$ and $C_{rid_i}$ correspond to some $\rho_i = rid_i + v_i \in \Phi$ and $rid_i \in \Psi$ on $BB_3$.

The $rid_i$’s are sparse in a very large space and the probability that there is another within $rid_i \pm m$ is negligible $O(m/q)$. Collisions of $rid_i$’s are similarly unlikely. This not only ensures that the elements of $\Phi$ and $\Psi$ are unique with overwhelming probability, but that the $rid_i$ and $rid_i + v_i$ come from the same row in $BB_3$ in the ZKPs. Anybody can check if the $C_{rid_i}$’s collide or the $rid_i$’s are too close on the sorted bulletin boards and call for an audit if found.

In addition, for satisfaction, they can verify on $BB_3$ that $w_i$ corresponding to what they have in $P_i$ has been correctly recorded against their $C_{rid_i}$.

A sufficiently large number of voters, as determined by the hypergeometric distribution for a high enough confidence level, must opt for voter-verified accountability.

6.3 Universal verifiability

The tally of votes can be verified by anybody on $BB_3$.

The collision and preimage resistant hash computation, the public display of $H_k$, and the verifications $H = \bigoplus_i h_i = \bigoplus_i H_k$ ensure that any post-polling vote manipulation at the collection or the election authorities is computationally hard for a polynomial-time bounded adversary. Hence, manipulation, if any, must happen before or during polling.

Suppose the EVM is untampered. The EVM checks all commitments issued at the polling officer’s desk, so they have to be correct. By Section 6.1, the EVM records the vote correctly and issues correct receipts, ensuring that no vote tampering is possible.

Even if the EVM is tampered, it cannot introduce fake rows because every row must have the $C_{rid_i}$, $C_{u_i}$, $ack_i$ triples signed by the polling officer, and that $N = \sum_i N_k$. It cannot delete or alter votes because of the commitments issued to the voters (see Sections 6.1 and 6.2).

Hence, the only attack possible is the one discussed in Section 6.1, if the PO and EVM are simultaneously tampered and controlled. The correctness arguments presented above hold even if the authorities collude with each other.

Finally, vote injection by the polling officer by producing fake voters is beyond the scope of this protocol.

6.4 Secrecy and receipt freeness

We assume that the EVMs and POs do not leak information and do not store any information corresponding to any voter.

The polling officer issues a perfectly random id $rid_i$ and a random obfuscation $u_i$ to each voter and keeps no record of either. Only the voter sees $u_i$. The voter’s receipt contains $(C_{rid_i}, \sigma_{rid_i}), (C_{u_i}, \sigma_{u_i})$ and $(C_{v_i}, hv_{vk})$ (see steps 5 of Figure 3 and 6 of Figure 4) from which none of $rid_i$, $u_i$ or $v_i$ can be determined. If $u_i$ is secret then the probability of being able to derive $v_i$ from $w_i$ is almost equal for each vote. It is also computationally hard to figure out the secrets
respectively. These records cannot be combined to determine which property is maintained. These records cannot be combined to determine which property is maintained. These records cannot be combined to determine which property is maintained.

makes polling booth level profiling of communities impossible.

secrecy.

step 6 of Figure 4 must also contain steps 5 of Figure 3 and 6 of Figure 4 in a box in the polling booth before leaving the booth. In that case, the printout in the knowledge protocol. If voter-verified accountability is dispensed with, the voter needs to deposit the printouts obtained in the polling booth itself free of any identification with the voter, and can be used for post-facto audit using the same zero

receipts issued to voters can discover who they voted for. If the risks are perceived to be too high in a specific situation, or to a breach in the election authority’s data store. Then, any adversary or coercer who may have harvested commitment keys for the commitments that are used to prove the zero knowledge set memberships ever become public, perhaps due

7 Possible simplifications and alterations of the protocol

Depending upon the level threat perception, there can be several simplifications and alterations to the protocol. We describe some of these below.

The only threat to voter secrecy arises if any store of private data is compromised. This is particularly true if the secret keys for the commitments that are used to prove the zero knowledge set memberships ever become public, perhaps due to a breach in the election authority’s data store. Then, any adversary or coercer who may have harvested commitment receipts issued to voters can discover who they voted for. If the risks are perceived to be too high in a specific situation, or if the confidence in the election authority to maintain their servers securely is low, then voter-verified accountability can be dispensed with altogether. Then, instead of issuing the commitment receipts to the voters, they can be printed and kept in the polling booth itself free of any identification with the voter, and can be used for post-facto audit using the same zero knowledge protocol. If voter-verified accountability is dispensed with, the voter needs to deposit the printouts obtained in steps 5 of Figure 3 and 6 of Figure 4 in a box in the polling booth before leaving the booth. In that case, the printout in step 6 of Figure 4 must also contain \((C_{rid_i},\sigma_{rid_i})\) so that a correspondence between the two printouts may be established later during random audit.

Dropping a VVPAT slip in a box after obtaining an acknowledgement from the voter, with the voter’s choice marked in clear, is a weak form of commitment. It is weak because the commitment is easy to identify and tamper, possibly with a corresponding tampering of the EVM records. However, if the VVPAT slip is considered to be a strong enough commitment with enough physical safeguard of the VVPAT box, then the steps involving the obfuscation key \(u_i\) can be dispensed with altogether. Then, after an acknowledgement from the voter confirming the correctness of the VVPAT printout, the signed commitment of the vote, \((C_{v_i},\mu_{v_i})\), can be issued to the voter as the receipt, with a possible option of cast-or-challenge [Benaloh, 2006]. The rest of the protocol can remain unchanged. This simplifies the protocol considerably.

8 Practicalities of implementation

The ZKP set-membership protocol is based on bilinear maps (see supplementary Section B), which are usually realized only for elliptic curves. All operations in Section 4 are equally valid for cyclic groups of elliptic curves over a finite field instead of modular subgroups of \(\mathbb{Z}_p\). Hence we present our analysis on groups over elliptic curves.

We consider a prototypical high security elliptic curve with a base field size of 1024 bits \((\log p = 1024)\), a group order of 160 bits \((\log q = 160)\) and an embedding size of 2. Note that \(p\) and \(q\) are large primes such that \(q\) divides \(p - 1\) (Section 4). The discrete log problem is believed to be hard for such groups.

There exist fast algorithms for computing the commitments, and the ZKP protocol for set membership is computationally efficient [Camenisch et al., 2008]. All commitments and other parameters are groups elements with 1024 bit representations, and their printouts can easily fit into standard QR codes, the largest of which are around 3 KB.

See Section D of the supplementary information for further details.

9 Conclusions

We have presented a verifiable, direct-recording electronic (DRE) voting protocol whose correctness and secrecy properties can be formally established. The protocol is scalable, efficient and easy to implement.
Though the requirement from the voter is simple, the protocol does rely on several cryptographic constructs to establish its correctness. Hence, building public trust on the protocol may require some special efforts. While banning electronic voting the German Constitutional Court made the following observation [NDI, 2019]:

“The use of voting machines which electronically record the voters’ votes and electronically ascertain the election result only meets the constitutional requirements if the essential steps of the voting and of the ascertainment of the result can be examined reliably and without any specialist knowledge of the subject . . .

Whether the protocol is simple enough to pass the test proposed by the German constitutional court, and is fit for large scale deployment in countries with low digital literacy, has to be carefully evaluated.

Finally, the bigger problem of electoral democracy of creating an accurate de-duplicated voter’s list [National Academies of Sciences, Engineering and Medicine, 2018] that has no spurious entries and does not exclude any rightful voter is still very much an open problem.

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Supporting Information (SI)

A SI: Modular groups

For two integers $a$ and $b$ we write $a = b \mod n$ if $b$ is the remainder when $a$ is divided by $n$. We then say that $a$ and $b$ are congruent modulo $n$. For example, $18 = 4 \mod 7$ and $-18 = 3 \mod 7$. When the context of modulo $n$ is obvious, we write this simply as $a = b$.

The set $\mathbb{Z}_n = \{0, 1 \ldots n-1\}$ is the set of all remainders modulo $n$. $\mathbb{Z}_n$ supports two basic operations, addition and multiplication, in the obvious way. For example, $(11 + 13) = 8 \mod 16$ and $(11 \cdot 13) = 15 \mod 16$.

A group is a set with an operation which is closed, has an identity, is associative, and every element has an inverse. In addition, a group which is commutative is called abelian. An abelian group is called cyclic if there is a special element, called the generator, from which every other element can be obtained either by repeated application of the group operation, or by the use of the inverse operation. If $g$ is a generator of the cyclic group $G$ we often write $G = \langle g \rangle$. If $G$ is multiplicative then every element $h$ of $G$ can be written as $h = g^x$ for some integer $x$.

If $n = p$ is a prime, then for all non-zero $a \in \mathbb{Z}_p$, $ax = 1 \mod p$ has a unique solution and $a$ has a multiplicative inverse. Such a $\mathbb{Z}_p$ is a Field. A multiplicative subgroup of $\mathbb{Z}_p^*$ is a non-empty subset $G$ such that if $a, b \in G$ then $ab \in G$. Thus any subgroup contains 1, and the multiplicative inverse of every element is in the subgroup. If $G$ is a subgroup of $\mathbb{Z}_p^*$ of size $q$, then $q$ divides $p-1$.

Any non-trivial $a \in \mathbb{Z}_p^*$ generates a cyclic group $\{a, a^2, \ldots, a^d = 1\}$, for some $d$. Thus $d$, the order of the group, divides $p-1$, hence $a^{p-1} = 1$. All subgroups of a cyclic group are cyclic. If $G = \langle g \rangle$ is a cyclic group of order $n$, then for each divisor $d$ of $n$ there exists exactly one subgroup of order $d$ and it can be generated by $a^{n/d}$.

Finally, the discrete logarithm problem for a cyclic group - that of determining $x$ from the equation $h = g^x$ is believed to be hard.

B SI: ZKP of set membership

Definition B.1. For an instance of commitment $C$, a proof of set membership with respect to a set $\Phi$ is a zero knowledge proof of knowledge of $(\rho, r)$ such that $C = g^\rho h^r \land \rho \in \Phi$. 

| Common Input: | A Group $G_q = \langle g \rangle = \langle h \rangle$, a commitment $C$, and a set $\Phi$ |
| Prover Input: | $\rho, r$ such that $C = g^\rho h^r$ and $\rho \in \Phi$ |
| $P \leftarrow y, \{A_i\}$ | Verifier picks random $x \in \mathbb{Z}_q$ and sends $y \leftarrow g^x$ and $A_i \leftarrow g^{\frac{x}{p_i}}$ for every $i \in \Phi$. |
| $P \rightarrow V$ | Prover picks random $v \in \mathbb{Z}_p$ and sends $V \leftarrow A_i^v$. |
| Prover and verifier run $\mathsf{PK}\{\langle \rho, r, v \rangle : C = g^{\rho} h^{rV} \land V = g^{\frac{x}{p_i}} \}$ |
| $P \leftarrow a, D$ | Prover picks random $s, t, m \in \mathbb{Z}_q$ and sends $a \leftarrow e(V, g)^{-s} e(g, g)^t$ and $D \leftarrow g^s h^m$. |
| $P \leftarrow e$ | Verifier sends a random challenge $e \in \mathbb{Z}_q$. |
| $P \leftarrow s, r, x_r$ | Prover sends $z_p = s - pr, z_v = t - ve, \text{and} z_r = m - re$. |
| Verifier checks that $D = C^s h^{r \cdot x_r}$ and that $a \leftarrow e(V, y)^c \cdot e(V, g)^{-s} \cdot e(g, g)^t$ |

Figure 9: Set membership protocol for set $\Phi$

The ZKP protocol requires bilinear groups and associated hardness assumptions. Let $\mathcal{G}$ take a security parameter $k$ written in unary as input and output a description of a bilinear group $(p, G_q, G_T, e) \leftarrow \mathcal{G}(1^k)$ such that

1. $p$ is a $k$-bit prime.
2. $G_q, G_T$ are cyclic groups of order $q$. Let $G_q^* = G_q \setminus \{1\}$ and let $g \in G_q^*$.
3. $e : G_q \times G_q \rightarrow G_T$ is a bilinear map (pairing) such that $\forall a, b : e(g^a, g^b) = e(g, g)^{ab}$.
4. If $g$ generates $G_q$ then $e(g, g)$ generates $G_T$.
5. Membership in $G_q, G_T$ can be efficiently decided, group operations and the pairing $e$ are efficiently computable, generators are efficiently sampleable, and the descriptions of the groups and group elements each have size $O(k)$ bits.

The ZKP protocol relies on the Boneh-Boyen short signature scheme [Boneh and Boyen, 2004]. The secret key of the signer is $x \rightarrow Z_{q^2}$, the corresponding public key is $y = g^x$. The signature on a message $\rho$ is $m \rightarrow g^{\frac{1}{x+\rho}}$; verification is done by checking that $e(m, y \cdot g^\rho) = e(g, g)$. Suppose the $|\Phi|$-Strong Diffie Hellman assumption ($|\Phi|$-SDH) holds in $(G_q, G_T)$, then the basic Boneh-Boyen signature scheme is $|\Phi|$-secure against an existential forgery under a weak chosen message attack [Boneh and Boyen, 2004].

A honest-verifier zero knowledge proof under a strong Diffie-Hellman assumption associated with the above pairing generator ($\mathcal{G}$) is given by the set membership protocol in Figure 9 [Camenisch et al., 2008].

Note that if $\Phi$ is stored indexed by $C$, then the ZKP of set membership is computationally efficient and requires only $O(1)$ sized proofs [Camenisch et al., 2008]. Also, the first communication in Figure 9 from each verifier to the prover needs to happen only once.

C SI: Proofs of correctness of commitments, universal verifiability and secrecy

C.1 Correctness of the commitment of each vote

Theorem 1 (Completeness). Let $u \in Z_q$ denote a random number given by the PO to the voter alongwith its commitment $C_u = g^{uh^r}$. Let $v \in Z_m$ denote the voter’s vote and let $C_v = g^{v^r}$ denote its commitment. If $w = u + v$, $w' = w \mod m$ and $r_w = r_u + r_v$ then $w' = (u \mod m + v) \mod m$ and $C_u C_v = g^{(w')^r}$.

Proof. The statement $w' = (u \mod m + v) \mod m$ follows directly from the properties of modular arithmetic. The statement $C_u C_v = g^{(w')^r}$ follows trivially by the law of exponents. We have $C_u C_v = g^{uh^r} \cdot g^{v^r} = g^{u+v^r} h^{r+rv} = g^{w'} h^{r_w}$.

Theorem 2 (Soundness). Let $u \in Z_q$ denote a random number given by the PO to the voter alongwith its commitment $C_u = g^{uh^r}$. Let $v \in Z_m$ denote the voter’s vote, where $m < q$. If $C_u, C_v = g^{w^r}$ where $w \mod m = w' = (u \mod m + v) \mod m$ and $r_w \in Z_q$ is a random number, then computing $(\hat{v}, r_v)$ such that $C_v = g^{\hat{v}^r}$ and $\hat{v} \in Z_m \backslash \{v\}$ is computationally hard.

Proof. Suppose $C_v = g^{\hat{v}^r}$ where $\hat{v} \in Z_m \backslash \{v\}$ (each member of the group generated by generators $g$ and $h$ can be expressed in this form). Since $C_u, C_v = g^{uh^r}$, we have $g^{u+v^r h^{r+rv}} = g^{w^r}$ by the law of exponents. Let $u' = u \mod m$ and $u'' = u \mod m$. It is given that $w' = w \mod m$. Let $w'' = w \mod m$. We thus have

$$g^{u''m+u'+\hat{v}h^{r_u+r_v}} = g^{w''m+w'h^{r_w}} \tag{1}$$

By the computational binding property of Pedersen commitments, Equation 1 implies that a polynomial-time adversary is computationally bound to satisfy the following equations:

$$u''m + u' + \hat{v} = w''m + w' \tag{2}$$

First, notice that $r_v = r_w - r_u$. We now focus on proving $\hat{v} = v$. We have $w' = (u + v) \mod m$.

Case 1: $u' + v < m$. In this case, $w' = u' + v$. Thus, we have

$$u''m + u' + \hat{v} = w''m + u' + v \tag{3}$$

Since $v < m$ and $\hat{v} < m$, we must have $u'' = w''$ and $\hat{v} = v$ from the above equation.

Case 2: $m \leq u' + v < 2m - 1$. In this case, $w' = u' + v - m$. Thus, we have

$$u''m + u' + \hat{v} = w''m + u' + v - m \tag{4}$$

Again, since $v < m$ and $\hat{v} < m$, we must have $u'' = w'' - 1$ and $\hat{v} = v$ from the above equation.
C.2 Voter-verified accountability

**Theorem 3** (Voter-verified accountability). Let rid\(_i\) and C\(_{rid}\) be the purported random number in \(\mathbb{Z}_q\) and its commitment issued to voter \(i\) at the polling booth, and \(v_i \in \mathbb{Z}_m\) be her intended vote. If voter \(i\) performs the additional verification of Figure 8, then with overwhelming probability, a row containing \(\langle rid_i, v_i \rangle\) exists on \(BB_3\).

**Proof.** By Theorem 2, assuming polynomial-time adversary, we have \(C_{v_i} = g^v h^{r v_i}\). Given the commitment \(C_i = C_{rid_i} * C_{v_i}\), by the soundness of the ZKP of set membership on set \(\Phi\), an element \(rid_i + v_i\) exists in the third column of \(BB_3\).

Also, the ZKP of set-membership for commitment \(C_{rid_i}\) on set \(\Psi\) ensures that an element \(rid_i\) exists on the first column of \(BB_3\). Since \(BB_3\) is verified for no collision up to \(rid_i + m\) and \(v_i \in \mathbb{Z}_m\), this implies that the row containing \(rid_i\) is also the row containing \(rid_i + v_i\). The probability of such collisions for honestly generated random numbers is given by the solution to the birthday problem as approximately \(1 - \exp(-\frac{q(m-1)}{2})\), where \(N\) is the number of rows in \(BB_3\), which is negligible for large \(q/m\).

Finally, the verification of the invariant property that the third column of \(BB_3\) ensures that the second column of the row containing \(rid_i\) is \(v_i\). \(\square\)

C.3 Universal verifiability

**Theorem 4** (No vote injection/deletion/tampering post polling). Let \(rid_i \in \mathbb{Z}_q\) be a random number issued to voter \(i\) at the polling booth, \(v_i \in \mathbb{Z}_m\) be her vote as intended and \(h_i = \mathcal{H}((rid_i, v_i))\) be the hash of her vote, where \(\mathcal{H}\) is a preimage resistant and collision resistant hash function. Let \(V_k\) denote the votes cast in polling booth \(k\) and \(V\) denotes the votes for which a row \(\langle (rid_i, v_i), rid_i + v_i, (h_i, \mu_{h_i, \lambda_i}) \rangle\) exists on \(BB_3\). Let \(H_k = \bigoplus_{i \in V_k} h_i\) be the hash computed by EVM\(_k\) and \(H = \bigoplus_{i \in V} h_i\), where \(\bigoplus\) denotes the bitwise-XOR operation. If \(\bigoplus_{k} H_k = H\) and \(\langle rid_i, v_i \rangle\) is unique for each row, then

1. injecting votes after the polling booth is computationally hard.
2. probability that any votes were deleted after the polling booth is negligible.

**Proof.** Suppose there exist some fake votes in \(BB_3\) that do not exist in any \(V_k\). Let’s denote such votes by \(V_{fake}\). Further, suppose that some votes were deleted, i.e., they were counted in some \(V_k\) but their corresponding row does not appear in \(BB_3\). Let’s denote such votes by \(V_{del}\). We thus have:

\[
V = \left( \bigcup_k (V_k \cup V_{fake}) \right) \setminus V_{del} \tag{5}
\]

Since \(\bigoplus_{k} H_k = H\) and \(H_k = \bigoplus_{i \in V_k} h_i\), we have:

\[
\bigoplus_{k} \mathcal{H}((rid_i, v_i)) = \bigoplus_{i \in V} \mathcal{H}((rid_i, v_i)) \tag{6}
\]

By Equations 5 and 6 and the properties of XOR, we have:

\[
\left( \bigoplus_{k} \bigoplus_{i \in V_k} \mathcal{H}((rid_i, v_i)) \right) \oplus \left( \bigoplus_{i \in V_{del}} \mathcal{H}((rid_i, v_i)) \right) = \\
\left( \bigoplus_{k} \bigoplus_{i \in V_k} \mathcal{H}((rid_i, v_i)) \right) \oplus \left( \bigoplus_{i \in V_{fake}} \mathcal{H}((rid_i, v_i)) \right) \tag{7}
\]

\[
\Rightarrow \bigoplus_{i \in V_{del}} \mathcal{H}((rid_i, v_i)) = \bigoplus_{i \in V_{fake}} \mathcal{H}((rid_i, v_i))
\]

**Lemma 4.1.** \(V_{fake} = \emptyset\).
Proof. Suppose there exist some \( k \) fake votes \((k \geq 1)\). Consider the last fake vote \( i_0 \in V_{fake} \). For Equation 7 to hold true, we must have

\[
\mathcal{H}((\text{rid}_{i_0}, v_{i_0})) = \bigoplus_{i \in V_{fake} \setminus \{i_0\}} \mathcal{H}((\text{rid}_i, v_i)) \oplus \bigoplus_{i \in V_{del}} \mathcal{H}((\text{rid}_i, v_i))
\]

(8)

Since \((\text{rid}_i, v_i)\) are all unique, the preimage resistance property of the hash function \( \mathcal{H} \) implies that finding a new preimage \((\text{rid}_{i_0}, v_{i_0})\) such that its hash matches the RHS is computationally hard. This means that injecting fake votes after the polling booth is computationally hard.

\( \square \)

Lemma 4.2. \( V_{del} = \phi \).

Proof. Suppose some \( k \) votes \((k \geq 1)\) are deleted. Consider the last such deleted vote \( i_0 \in V_{del} \). For Equation 7 to hold true, we must have

\[
\mathcal{H}((\text{rid}_{i_0}, v_{i_0})) = \bigoplus_{i \in V_{del} \setminus \{i_0\}} \mathcal{H}((\text{rid}_i, v_i))
\]

(9)

since from the previous lemma, we know that \( V_{fake} = \phi \).

From the collision-resistance property of the hash function \( \mathcal{H} \), the probability that the hash of a given \((\text{rid}_{i_0}, v_{i_0})\) for random \( \text{rid}_{i_0} \) is equal to the RHS is negligible. This means that the probability that any \( k \) votes \((k \geq 1)\) were deleted after polling is negligible.

\( \square \)

\( \square \)

**Theorem 5** (Universal verifiability). Assuming that \( PO_k \) and \( EVM_k \) are not simultaneously tampered, the voting protocol presented in this paper is universally verifiable.

Proof. First, note that anybody can tally the total number of votes for each candidate on \( BB_3 \). By Theorem 4, nobody can create fake votes, change existing votes or delete any vote post polling (i.e. after \( BB_1 \) is populated with correct hashes and counts). We now show that this is not possible before or during polling also.

**Case 1: EVMs are untampered.** If \( EVM_k \) is untampered, it would produce correct signed hash \( H_k \) corresponding to the voters’ intended votes, which cannot be forged. Although this is enough to ensure universal verifiability by Theorem 4, we show that any vote manipulation during polling is also caught by voter verification. Since \( EVM_k \) generates correct commitment \( C_v, w \) and \( r_w \), an incorrect commitment \( C_{v'} \) (not corresponding to the \( u \) given to the voter) by the PO would fail the check \( C_{v'} \checkmark \equiv g^w h^r \) by the voter. An incorrect commitment \( C_{rid'} \) (not corresponding to the \( rid \) recorded against the voter) generated by the PO would also fail the ZKP of set-membership on the set \( \Psi \) as no collision in \( rid \) is allowed on \( BB_3 \).

**Case 2: Some EVM is tampered.** In this case, we can’t rely on the hashes produced by the EVMs. The only guard in this case is the voter verification since voter intent can only be verified by the voter. A vote which is not cast as intended, i.e. one with an incorrect \( C_{v'} \) would fail the \( C_u, C_{v'} \equiv g^w h^r \) check by the voter by Theorem 2, unless \( PO_k \) is also tampered to issue a \( C_{v'} \) that is not a commitment for the \( u \) issued to the voter, which we assume does not happen. Assuming that \( C_u \) is independent of the vote (a reasonable assumption given that \( C_u \) is given before the vote is cast), even such a simultaneous tampering can only cause a random mis-assignment instead of a targeted mis-assignment.

A vote which is not counted as cast (either deleted or counted such that the vote doesn’t match the \( C_v \) issued to the voter) would fail the ZKP on set \( \Phi \) by Theorem 3.

Finally, a tampered \( EVM_k \) cannot have any fake votes without a simultaneous tampering of \( PO_k \) signing fake \( N_k \). Note that we trust the identity verification process at the PO desk and thus assume that the polling officer doesn’t allow fake voters to cast vote.
C.4 Secrecy and receipt freeness

**Definition C.1** (Ciphertext indistinguishability\(^1\)). An encryption scheme \( E : M \times K \rightarrow C \) is said to possess ciphertext indistinguishability if for each polynomially-bounded adversary \( A \), the advantage of \( A \) (defined below) is negligible in the following indistinguishability game between a challenger and the adversary.

- **Challenger** chooses a key \( k \) randomly from the key space \( K \).
- **Adversary** sends \( m_0, m_1 \in M \) to the challenger.
- **Challenger** chooses \( b \in \{0, 1\} \) and sends \( c = E(m_b, k) \) to the adversary.
- **Adversary** outputs \( b' \in \{0, 1\} \) as its guess of \( b \).

The advantage of the adversary \( A \) in the above game is given by:

\[
Adv_{CI}[A, E] = \left| \Pr_{k \in K'}[b = 1 \land b' = 1] - \Pr_{k \in K}[b = 0 \land b' = 1] \right|
\]  

(10)

**Theorem 6** (Vote secrecy). Let \( u \in R \mathbb{Z}_q \) denote a random number given by the PO to the voter alongwith its commitment \( C_u = g^uh^w \). Let \( v \in \mathbb{Z}_m \) denote the voter’s vote and let \( C_v = g^vh^w \) denote its commitment. Let \( P = (w, w', r_w) \) where \( w = u + v, w' = w \mod m \) and \( r_w = r_u + r_v \). Given that \( m \ll q \), the tuple \( R(v, u) := (C_u, C_v, P) \) possesses ciphertext indistinguishability.

**Proof.** By the perfect hiding property of Pedersen commitments, we know that components \( C_u \) and \( C_v \) are perfectly hiding. Perfect hiding is a stronger property than ciphertext indistinguishability, therefore \( C_u \) and \( C_v \) are both ciphertext indistinguishable.

To prove \( P \) is ciphertext indistinguishable, first note that since \( r_w \) is a function of random numbers independent of \( v \), \( r_w \) cannot possibly leak any information about \( v \) and is thus ciphertext indistinguishable. Below we prove that \( w \) and \( w' \) are ciphertext indistinguishable.

The indistinguishability game for \( w \) proceeds as follows:

- **Challenger** chooses a random \( u \in \mathbb{Z}_q \).
- **Adversary** sends votes \( v_0, v_1 \in \mathbb{Z}_m \) to the challenger.
- **Challenger** chooses \( b \in \{0, 1\} \) and sends \( w_b = u + v_b \) to the adversary.
- **Adversary** outputs \( b' = A(w_b) \in \{0, 1\} \) as its guess of \( b \).

Let the event \( b = 0 \land b' = 1 \) be denoted by \( A_0 \) and the event \( b = 1 \land b' = 1 \) be denoted by \( A_1 \). We have to show that for each polynomially-bounded adversary \( A \), \( Adv_{CI}[A, w] \) is negligible. More precisely, we show that:

\[ Adv_{CI}[A, w] = \left| \Pr[A_0] - \Pr[A_1] \right| = O(m/q) \]

which is negligible since \( m \ll q \).

Let \( S = \mathbb{Z} \cap [m, q) \) denote the range of integers from \( m \) to \( q \). This represents a safe range of values for \( w_b \) such that if \( w_b \in S \), then for each \( v_b \in \mathbb{Z}_m \), we have a \( u \in \mathbb{Z}_q \) that satisfies \( w_b = u + v_b \). We can write:

\[
\Pr[A_b] = \Pr[A_b \land w_b \in S] + \Pr[A_b \land w_b \not\in S]
\]

(12)

\(^1\)More precisely, this is the definition of ciphertext indistinguishability under a chosen plaintext attack, where the adversary chooses plaintexts of her choice and tries to guess which plaintext a given ciphertext encrypts.
Theorem 7 (Receipt freeness). Assuming that none of the entities Polling Officer (PO), Collection Authority (CA), Election Authority (EA) and Trusted Verifier (TV) collude with one another and EVM behaves honestly, none of PO, CA or TV can obtain any information on a voter’s receipt. Therefore, tuple \( R(v, u) \) is ciphertext indistinguishable.}

**Proof.** Consider the receipts \( R_1 = ((C_{rid_i}, \sigma_{rid_i}), (C_{ui}, \sigma_{ui})) \) and \( R_2 = ((C_{vid}, \mu_{vid}), (P_i, \mu_{P_i})) \) given to voter \( i \) at polling booth \( k \) by the PO and the EVM respectively. \( C_{rid_i} \) does not leak any information about \( rid_i \) because of the perfect hiding property of Pedersen commitments. Also, \( (C_{ui}, C_{vid}, P_i) \) do not leak any information about \( v_i \) by Theorem 6. Finally, none of the signatures leak any information about \( (rid_i, v_i) \), except the polling booth \( k \) where the vote was cast. Since the rows \( (rid_j, v_j) \) displayed at \( BB_3 \) are randomly shuffled by the CA, the row number in \( BB_3 \) cannot leak any information about the polling booth \( k \) and therefore cannot be associated with the voter’s receipt. Thus, no information about \( (rid_i, v_i) \) can be obtained from only \( R_1 \) and \( R_2 \).

Below we show that none of PO, CA or TV can obtain any information about \( (rid_i, v_i) \) from \( R_1 \) and \( R_2 \), even with access to the additional information that they possess.

First, note that we assume the PO destroys all information about a voter, specifically \( rid_i \) and \( u_i \), and therefore does not possess any additional information.

The CA knows \( (s_i, m_i, h_i) \) for each vote. However, \( s_i \) is encrypted with EA’s public key, \( m_i \) only contains signed commitments that possess perfect hiding property, and \( h_i \) is hiding because of its preimage resistance property. Thus, the CA does not obtain any information about \( (rid_i, v_i) \).

Finally, the trusted verifier does not obtain any information about \( (rid_i, v_i) \) because of the zero-knowledge property of the set-membership proof.\[ ☐\]
D SI: Practicalities of implementation

The set-membership protocol of supplementary Section 4.2 is based on bilinear maps, which are usually realized only for elliptic curves. All operations in Section 4 are equally valid for cyclic groups of elliptic curves over a finite field instead of modular groups of $\mathbb{Z}_p$. Hence we present our analysis on groups over elliptic curves.

We consider a prototypical high security elliptic curve with a base field size of 1024 bits ($\log p = 1024$), a group order of 160 bits ($\log q = 160$) and an embedding size of 2. Note that $p$ and $q$ are large primes such that $q$ divides $p - 1$ (Section 4).

The security of the Pedersen commitments in our protocol is derived from the hardness of the discrete log problem in the chosen group. A discrete log problem that requires $O(\sqrt{q}) \equiv 2^{160/2} = 2^{80}$ exponentiations is considered safe from the best known attacks on elliptic curves. The security of a well chosen such curve and that of a 2048 bit finite field are roughly equivalent [Lynn, 2013].

The security of the set membership protocol is derived from the hardness of the $|\Phi|$-SDH problem in the chosen group [Boneh and Boyen, 2004, Camenisch et al., 2008], where $|\Phi|$ is the total size of the electorate in a constituency. Brown and Gallant [2004] and Cheon [2006] have shown that in groups of order $q$, the $|\Phi|$-SDH problem can be solved with $O(\sqrt{q}/d + \sqrt{d})$ exponentiations, for any divisor $d \leq |\Phi|$ of $q - 1$. Assuming the typical size of a constituency to be $|\Phi| \approx 10^6 \approx 2^{20}$, and considering the best case scenario from an adversary’s point of view, the $|\Phi|$-SDH problem can be solved in $O(\sqrt{q}/|\Phi|) \equiv 2^{(160-20)/2} = 2^{70}$ exponentiations. This can be considered adequately safe, and breaking either the Pedersen commitment or the set membership protocol will be practically impossible for the chosen curve.

Each element of the group is a point on the elliptic curve and requires $O(\log p)$ space to represent both its coordinates. An operation in this group requires a constant number of field multiplications and additions, thereby requiring at most $O(\log^2 p)$ bit operations. The exponentiation operation - also called the scalar multiplication operation in case of elliptic curves - requires $O(\log q)$ group operations using the standard double-and-add algorithm, requiring a maximum of $O(\log q \log^2 p)$ bit operations.

The polling booth protocol of Figures 3 and 4 requires computation of a constant number of Pedersen commitments, and each Pedersen commitment requires 2 exponentiations and a group operation, amounting to $O(\log q \log^2 p)$ bit operations. In addition, the polling booth protocol requires a constant number of cryptographic signatures and public key encryptions for which standard fast algorithms are known.

The polling officer needs to print out the signed commitments $(C_{rid_i}, \sigma_{rid_i}, u_i)$ for the voter, $C_{rid_i}$ and $C_{u_i}$ are group elements, each of $\log p = 1024$ bits. For RSA signatures the sizes are of the order of the modulus, 1024 bits. Thus each signed pair is of 2048 bits and can be printed as two adjacent QR codes in a piece of paper. $\text{rid}_i$ is also of 1024 bits and can be printed in a QR code. $u_i \mod m$ is a small integer that needs to be printed in clear text. $\text{rid}_i$ and $u_i$ need to be printed on the same paper that needs to be destroyed before the voter leaves the polling booth.

The EVM needs to print out $(C_{vi}, \mu_{v_{i+k}})$ for the voter, which, likewise, is 2048 bit in a QR code. On the same paper the EVM also needs to print $(P_i, \mu_{u_{i+k}})$ in another 2048 bit QR code with $w'_i$, a small integer, printed in clear text.

The EVM also needs to print $(\text{rid}_i, v_i, u_i \mod m, w'_i)$ in VVPAT. $\text{rid}_i$ is a 1024 bit integer, which can be printed in a QR code. $v_i, u_i \mod m$ and $w'_i$ are small integers that need to be printed in clear text in the same VVPAT slip.

After polling is over, computation of the global hash $H_k$ of EVM$_k$ (Figure 5) requires $O(n)$ group operations, where $n$ is the size of the electorate per EVM (typically $n < 2000$), amounting to $O(n \log^2 p)$ bit operations. Each hash $H_k$ is an element of the group, requiring $O(\log p)$ space.

The data collection stage of the protocol (Figure 6) requires collecting $O(K)$ hashes, where $K$ is the number of polling booths per constituency (typically $K \approx 500$). The size of BB$_1$ is therefore $O(K \log p)$. This is followed by $O(K)$ number of signature verifications and replacement of signatures, for which standard fast algorithms are known. Finally, randomizing the records requires computing a random permutation of $|\Phi|$ indices.

The sizes of BB$_2$ and BB$_3$ (Figure 7) are $O(N \log p)$, where $N = |\Phi|$ is the size of the electorate per constituency, and checking $\bigoplus_i H_k = \bigoplus_i h_i$ requires $O(N)$ XOR operations, amounting to $O(N \log^2 p)$ bit operations.

Finally, the ZKP of set-membership (supplementary Section B) requires a one-time computation and communication of $O(N)$ Boneh-Boyen signatures per verifier. Computation of a Boneh-Boyen signature requires $O(\log^2 q)$ operations for calculating $1/(x + i)$ in a group of order $q$, and $O(\log q \log^2 p)$ operations for the exponentiation. Thus, it imposes a one-time cost of $O(N \log q \log^2 p + N \log^2 q)$ bit operations per verifier per constituency. Given a particular voter’s commitment $C_i$, though, the set membership protocol requires a constant number of bilinear map evaluations, and a constant number of exponentiations. Verifying the correctness of a commitment also requires a constant number of exponentiations and a group operation.

We do not attempt to provide complexity bounds for evaluating bilinear maps as it depends on various domain parameters of the elliptic curve and the bilinear pairing selected. Rather, we provide empirical results on the time required to
evaluate these bilinear maps for the particular curve we select.

We use the PBC library [Lynn, 2013] for estimating the time required for Pedersen commitments and evaluating the bilinear maps - the two most crucial primitives in our protocol. We choose the Type a1 elliptic curve that is provided by PBC with $\log p = 1024$, $\log q = 160$ and an embedding size of 2. Using this curve, the time required to generate the group generators $g$ and $h$ are 6.7 $ms$ on the average, a Pedersen commitment takes 28 $ms$ on average and a bilinear map evaluation takes 22 $ms$ on average on commodity hardware, thereby confirming the practicality of our protocol under a reasonable security guarantee.

Of course, the above estimates are only indicative. For a practical implementation an elliptic curve of sufficient security strength will have to be chosen more thoughtfully.