Causality and Electromagnetic Transmissions Through Materials

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ABSTRACT

There have been several experiments which hint at evidence for superluminal transport of electromagnetic energy through a material slab. On the theoretical side, it has appeared evident that acausal signals are indeed possible in quantum electrodynamics. However, it is unlikely that superluminal signals can be understood on the basis of a purely classical electromagnetic signals passing through a material. The classical and quantum theories represent quite different views, and it is the quantum view which may lead to violations of Einstein causality.

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1. Introduction

The difference between the classical notion of an electromagnetic wave, and the quantum notion of a photon can become easily blurred. The Maxwell wave of the classical electromagnetic theory, when Fourier transformed from space and time $(r, t)$ into momentum space and time $(k, t)$, mathematically produces the Schrödinger equation for the photon[1]. Thus, experiments which would have been completely understood by Maxwell from a classical electromagnetic viewpoint, are sometimes mistakenly held as evidence for a purely quantum mechanical effect.

Our purpose is to discuss whether or not it is possible to propagate electromagnetic information at speeds faster than light speed, especially within the classical electromagnetic theory of continuous media[2]. Care shall be taken to consider the classical electromagnetic theory as different from the quantum electromagnetic theory. The profound difference between classical and quantum electrodynamics is already evident if one merely considers the vacuum propagators[3] of the two pictures (in the Feynman gauge); (i) If in the classical theory one employs the retarded vacuum photon propagator,

$$ D_{\text{retarded}}(r - r', t - t') = \delta(c(t - t') - |r - r'|) \quad (\text{classical}). $$

then the classical electromagnetic signal in the vacuum moves strictly on the light cone. (ii) If in the quantum theory one employs the Feynman propagator,

$$ D_{\text{Feynman}}(r - r', t - t') = \frac{i}{\pi} \left( \frac{1}{|r - r'|^2 - c^2(t - t')^2 + i0^+} \right) \quad (\text{quantum}), $$

then the signal might move on the light cone (forward and backward in time), but the signal might also move off the light cone. The Feynman propagator describes a photon which can move any way it pleases.
In what follows, we discuss classical solutions of the retarded type, similar to Eq.(1), but in the presence of material media. We discuss whether or not causality can be violated in the form of a superluminal signal within the context of locally retarded material response functions. We conclude that causality from the strictly classical electromagnetic viewpoint remains intact. Our mathematical methods of proof avoid all of the many different definitions of signal velocities and “barrier” transit times which fill much of the literature in this subject. For us, the system is causal if and only if the “output” depends only on what the “input” has done in the past. If the “output” depends on what the “input” will do in the future, then the system is acausal.

2. Material Polarization

Consider a material described by the displacement field

\[ \mathbf{D}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + 4\pi \mathbf{P}(\mathbf{r}, t), \]  

(4)

wherein the polarization response of the material is modeled as a linear retarded response to the electric field which is local in space but non-local in time[4],

\[ \mathbf{P}(\mathbf{r}, t) = \int_0^\infty \mathbf{E}(\mathbf{r}, t - s)d\mathcal{F}(s). \]  

(5)

Eq.(5) corresponds to the case of a frequency dependent dielectric response function

\[ \epsilon(\zeta) = 1 + 4\pi \int_0^\infty e^{i\zeta t}d\mathcal{F}(t), \quad (\Im \zeta > 0), \]  

(6)

which is analytic in the upper half frequency plane[5], and obeys the dispersion relation

\[ \epsilon(\zeta) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \Im \epsilon(\omega + i0^+)d\omega}{(\omega^2 - \zeta^2)}, \quad (\Im \zeta > 0). \]  

(7)

3. Electromagnetic Propagation Through a Slab

Now let us consider the propagation of an electromagnetic wave passing through a slab of such a material. For an appropriate component \( V \) of the electromagnetic field, and for (say) normal incidence along the \( z \)-axis, one expects for a material slab or “barrier” that

\[ V(z, t) \to V_{in}(t - (z/c)) + V_{reflected}(t + (z/c)), \quad \text{as} \quad z \to -\infty, \]  

(9)

and that

\[ V(z, t) \to V_{out}(t - (z/c)), \quad \text{as} \quad z \to \infty. \]  

(10)

Eqs.(9) and (10) correspond to an incoming wave \( V_{in} \), a reflected wave \( V_{reflected} \), and a transmitted wave (through the barrier) \( V_{out} \).
The transmission through the barrier is evidently causal in the sense of classical special
relativity if the outgoing wave at time $t$ depends only on how the incoming wave would
have behaved at previous times $t - s$ with $s \geq 0$; Explicitly,

$$V_{\text{out}}(t) = V_{\text{in}}(t) - \int_{0}^{\infty} V_{\text{in}}(t - s)dG(s), \quad \text{(causal response)}.$$ (11)

The causal Eq.(11) will hold true if and only if the transmission amplitude

$$\tau(\zeta) = 1 - \int_{0}^{\infty} e^{i\zeta t}dG(t),$$ (12)

is analytic in the upper half frequency plane, $\Im \zeta > 0$. The point is that Eqs.(9) and
(10) read, respectively, in the frequency domain as

$$V_\omega(z) \to e^{i\omega z/c} + \rho_\omega e^{-i\omega z/c}, \quad \text{as} \quad z \to -\infty,$$ (13)

and

$$V_\omega(z) \to \tau_\omega e^{i\omega z/c}, \quad \text{as} \quad z \to \infty,$$ (14)

where $\rho_\omega$ is the reflection amplitude and $\tau_\omega$ is the transmission amplitude.

The fraction of the electromagnetic wave intensity at frequency $\omega$ which passes through
the material slab is given by $P(\omega) = |\tau_\omega|^2$, which leads to the following:

**Central Theorem:** If the transmission amplitude is the boundary value $\tau_\omega = \tau(\omega + i0^+)$
of a function $\tau(\zeta)$ of complex frequency $\zeta$ analytic in the upper half plane, i.e. if

$$P(\omega) = \lim_{\sigma \to 0^+} |\tau(\omega + i\sigma)|^2,$$ (15)

then the transmission through the slab is causal in the sense of Eq.(11).

It is thus not required to perform a detailed Fourier transform from the frequency
domain to the time domain in order to prove causality or acausality. All that is required is
an explicit demonstration that the transmission amplitude $\tau(\omega + i0^+)$ is indeed the boundary
value of function $\tau(\zeta)$ analytic for $\Im \zeta > 0$. The rest follows from the mathematical
nature of the Fourier transform according to general theorems proved by Titchmarsh[6].

To see what is involved, consider a slab of material of thickness $L$ with a dielectric
response function $\epsilon(\zeta)$ which obeys Eq.(7). If $\omega \Im \epsilon(\omega + i0^+) \geq 0$, then the complex index
of refraction $\eta(\zeta)$ obeys a similar dispersion relation with $\omega \Im \eta(\omega + i0^+) \geq 0$,

$$\eta(\zeta) = \sqrt{\epsilon(\zeta)} = 1 + \frac{2}{\pi} \int_{0}^{\infty} \frac{\omega \Im \eta(\omega + i0^+)d\omega}{(\omega^2 - \zeta^2)}, \quad (\Im \zeta > 0).$$ (16)

The transmission amplitude obeys

$$\tau(\zeta) = \frac{2\eta(\zeta)e^{i\zeta L/c}}{(1 + \eta(\zeta)^2)\cos(\zeta\eta(\zeta)L/c) + 2i\eta(\zeta)\sin(\zeta\eta(\zeta)L/c)}.$$ (17)
Since Eqs.(16) and (17) imply that $\tau(\zeta)$ is analytic in the upper half complex frequency plane, then Eq.(11) holds true and the transmission through the slab is strictly causal in the classical electrodynamic theory. The outgoing wave depends on what the incoming wave would have been doing in the past if the slab were not present. Note that the inequality $\omega \text{Im} \epsilon(\omega + i0^+) \geq 0$ is essential for the classical proof of causality.

4. Conclusions

There is a very large literature on superluminal group velocities[7] and delay times[8] for electromagnetic transmission through slabs of materials. Much of this work uses many different definitions of velocities and delay times. Some work hints at the notion that there is a superluminal transmission of an electromagnetic signal. More often, the discussion exhibits a disclaimer which asserts that Einstein causality is not violated. For example, in one mathematical model[9] of microscopic objects with inverted populations, the complex index of refraction is given by

$$\eta_{Chiao}(\zeta) = \sqrt{1 - \left(\frac{|f|\omega_p^2}{\Omega^2 - 2i\gamma - \zeta^2}\right)}.$$  

(18)

If $\Omega^2 < |f|\omega_p^2$, then there is a “cut” for both $\eta$ and $\tau$ in the upper half frequency plane. For such a case, in accordance Eq.(17) and (18), as a mathematical model one finds a non-analytic $\tau(\zeta)$ for $\text{Im}\zeta > 0$ and this model does indeed violate Einstein Causality! No disclaimer needs to be be made for this result. However, the sum rule

$$\frac{2}{\pi} \int_0^{\infty} \omega \text{Im} \epsilon(\omega + i0^+)d\omega = \omega_p^2 > 0,$$

(19)

holds true even if $\omega \text{Im} \epsilon(\omega + i0^+) < 0$ for a limited bandwidth. Since the model in Eq.(18) obeys the inequality $\omega \text{Im} \epsilon(\omega + i0^+) < 0$ for all $\omega$, the model violates the sum rule Eq.(19) and is not entirely physical.

There is presently the possibility of building mathematical models with negative noise temperatures in finite bandwidth which genuinely violate Einstein causality. Some examples model “inverted populations” and the microscopic mechanism for the causality violation is to be found in quantum mechanics. An atom or molecule in an excited state can scatter a photon in the following manner: Firstly (in time), the excited atom may emit the outgoing photon, and secondly (in time) the “virtual” ground state atom may absorb the incoming photon. The total scattering is elastic and energy conserving. This is only true over times long with respect to the the duration of the scattering event.

Typical of definitions often employed for time delay in the literature (when an electromagnetic signal passes through a material slab) is that

$$t_{delay}(\omega) = \left(\frac{d\theta(\omega)}{d\omega}\right),$$

(20)

where

$$\tau_\omega = \sqrt{P(\omega)e^{i\theta(\omega)}}.$$  

(21)
The criteria of a negative delay time $t_{\text{delay}}(\omega) < 0$ or of group velocity $v = (d\omega/dk) > c$
are not reliable for studying acausality in experiments[10-12]. Such criteria often amount to little more than approximate Fourier transformations. Our central point is that criteria such as Eq.(11), take place strictly in the time domain. On this more strict basis, acausality is theoretically possible in mathematical models but has not yet been definitively observed.

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