Detection technology of multi-magnetic source in spacecraft based on magnetic field gradient tensor

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Abstract. Multi-magnetic source resolution in spacecraft has been a difficult problem in the field of magnetic survey. Detection technology of magnetic gradient tensor is available to the problem for its high resolution and precision. A spacecraft magnetic source model is established, and a multi-magnetic source model fitting method for spacecraft is presented. The principal invariants of the magnetic field gradient tensor are introduced to ascertain the number and horizontal location of the source, and the Euler equations are combined to complete the multi-magnetic source depth calculation, the best resolution is up to 0.012m, meeting the engineering requirements. The study opens up a new way of multi-magnetic source target detection in spacecraft.

1. Introduction

During the development of spacecraft, in order to make its magnetic control meet the requirements, the whole magnetic moment of spacecraft will be tested and compensated. At present, the spacecraft magnetic testing mostly regards the spacecraft as a whole magnetic field source [1]. Therefore, the testing results are the comprehensive information of several magnetic components in the spacecraft, which can not reflect the real magnetic field source condition inside the spacecraft. With the rapid development of space technology, new magnetic testing requirements not only need to evaluate the overall magnetic moment of the spacecraft, but also need to analyze the main magnetic distribution inside the spacecraft. Therefore, it is very important to study the resolution of magnetic field sources inside the spacecraft.

Equivalent source method is used to fit multi-magnetic field sources in spacecraft abroad, but this method can not really reflect the distribution of magnetic field sources in spacecraft. At present, there is almost no research and application of multi-magnetic field source detection technology in small scale (spacecraft scale) in our country. Some people have proposed that Euler method [3] be used to solve the magnetic field source in spacecraft to eliminate magnetic interference [4-5]. However, the leakage and multi-solution of this method are serious. Similar magnetic detection techniques are mainly used in the fields of ocean [6] and terrestrial magnetic survey [7], which can be used for reference to solve the problem of multiple magnetic field sources in spacecraft.

Firstly, the multi-magnetic field source model of spacecraft is established, and then the fitting method of the model is given. The target of multi-magnetic field source is predicted by the maximum
of the principal invariant of the magnetic field gradient tensor, and the measurement and calculation of multi-magnetic field source in spacecraft are completed by combining Euler inversion method.

2. Modeling of Spacecraft Magnetic Field Source

For a single ideal magnetic field source (i.e. only magnetic moment and particle position information), the Legendre function polynomial expansion of the magnetic field formula can be carried out, and the detailed characteristics of the magnetic field can be analyzed by calculating the terms of the expansion. In most cases, only the magnetic dipole terms in the expansion are retained, while the higher order terms are ignored, because they decay rapidly with distance.

A magnetic dipole term can be expressed as

$$B(r, M) = \frac{\mu_0}{4\pi} \left[ \frac{3(M \cdot r)r - M}{r^5} \right] \approx \frac{4\pi \times 10^{-7} Tm}{r^3}$$

In the formula, $M$ is the magnetic moment of the magnetic field source and $r$ is the displacement vector from the magnetic field source to the detection point.

The total magnetic field of spacecraft obtained by magnetic dipole approximation method is

$$B(r, M) = \sum B_j (r_j, M_j)$$

When the distance from the detection point to the magnetic field source is 2.5 times or more than its own size, the magnetic dipole approximation method[8] can be used to analyze the magnetic field source. The internal magnetic field source of spacecraft is complex, and the magnetic moment, position and shape of magnetic components are different. The magnetic dipole approximation method is an important means to deal with multi-objective magnetic field source problem. In the preliminary magnetic modeling of spacecraft, the shape and size of the magnetic entity are neglected, and only the position of the magnetic source and the parameters of the magnetic moment are considered. Two points should be paid attention to here: one is to neglect the magnetic source with relatively small magnetic moment; the other is to treat the very close magnetic source as an equivalent source. Finally, the spacecraft can be equivalent to a cube containing multiple ideal magnetic field sources. As shown in Figure 1.

![Figure 1. The multi-dipole model of a spacecraft](image1.png)

![Figure 2. Magnetic field distribution upon spacecraft model.](image2.png)

3. Magnetic Fitting Method for Spacecraft

After establishing the multi-magnetic field source model of spacecraft, the approximation fitting method is often used to infer the position and magnetic moment information of each magnetic field source.
3.1. Measurement and calculation

(1) Setting up the reference coordinate system and measuring range of the spacecraft.
(2) Setting up the measuring plane (surface) and measuring mode of spacecraft magnetic field.
(3) The magnetic field detector is arranged on the measuring plane.
(4) Calculate and retrieve the position and magnetic moment information of multiple equivalent magnetic field sources inside the spacecraft.

3.2. Fitting optimization

(1) Setting the initial values of the number, position and magnetic moment parameters of the magnetic field source, and establishing the simulation model.
(2) The magnetic field data information of the simulation model in the measurement plane.
(3) Comparing the simulated data with the real measured data to find out the difference between them.
(4) Modify the parameters of the simulation model according to certain arithmetic program.
(5) Repeat steps (2) and (3) until the difference between the simulated and measured values satisfies certain tolerance conditions.

In the approximation fitting method, K Mehelm et al. proposed a simple and effective algorithm. In recent years, many studies have introduced artificial intelligence algorithms (such as genetic algorithm, neural network algorithm, etc.) into the problem of multi-magnetic source location. However, in the application of these methods, there is a difficult problem, that is, the selection of initial values: it may lead to tedious calculation or deviation from the real situation. Therefore, the magnetic field gradient tensor is introduced to improve the selection process of initial fitting values.

4. Magnetic gradient tensor

If the magnetic induction intensity $B = (B_x, B_y, B_z)$ at a certain point in space, then the total tensor of the magnetic field at that point is

$$
G = \begin{bmatrix}
\frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\
\frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\
\frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z}
\end{bmatrix} = \begin{bmatrix}
g_{xx} & g_{xy} & g_{xz} \\
g_{yx} & g_{yy} & g_{yz} \\
g_{zx} & g_{zy} & g_{zz}
\end{bmatrix}
$$

(3)

According to Maxwell’s equations of electromagnetic field, for the passive region in magnetic field, the following relations are established,

$$
\nabla \cdot B = 0 \Rightarrow \begin{cases}
B_{sx} + B_{sy} + B_{sz} = 0 \\
B_{sx} = B_{sx}, B_{sy} = B_{sy}, B_{sz} = B_{sz}
\end{cases}
$$

(4)

In this way, $G$ is a symmetric tensor with only five independent components, which facilitates the actual measurement. In practical working conditions, $G$ is usually obtained by measuring magnetic induction intensity $B = (B_x, B_y, B_z)$ and using differential calculation [9]. Self-made magnetic field gradient tensor junction based on the above measurement principle Structure [10] is shown in Figure 3.

The gradient tensor of magnetic field is a second-order tensor obtained by calculating the divergence of three components of magnetic field vector. It has three invariants,
\[
\begin{aligned}
I_0 &= \text{trace} \mathbf{G} = B_{xx} + B_{yy} + B_{zz} \\
I_1 &= B_{xx}B_{yy} + B_{yy}B_{zz} + B_{zz}B_{xx} - \left( B_{x}^{2} + B_{y}^{2} + B_{z}^{2} \right) \\
I_2 &= \det \mathbf{G} = B_{xx} \left( B_{yy}B_{zz} - B_{xy}^{2} \right) + B_{yy} \left( B_{zz}B_{xx} - B_{yx}^{2} \right) + B_{zz} \left( B_{xy}B_{yx} - B_{xx}B_{yy} \right)
\end{aligned}
\]  

(5)

Assuming that 25 magnetic dipoles are distributed at different positions in the detection plane, the distribution of the \( I_1 \) field of the magnetic field generated by them in the detection plane can be obtained, as shown in the following figure 4.

Figure 3. Self-made magnetic field gradient tensor detection structure

Figure 4. Multi-source magnetic field gradient tensor principal invariant \( I_1 \) plane distribution

Each "spot" in Figure 4 represents a magnetic field source [11] below the corresponding position. This method of finding the horizontal position of the target magnetic field source by detecting the maximum of the main invariant is often used in large area magnetic anomaly detection [12].

After obtaining the horizontal position \((x, y)\) of the magnetic source, the depth \(z\) of the magnetic source can be obtained by substituting equation (6), so that the position information of the magnetic source can be obtained.

\[
\begin{bmatrix}
B_{xx} & B_{xy} & B_{xz} \\
B_{yx} & B_{yy} & B_{yz} \\
B_{zx} & B_{zy} & B_{zz}
\end{bmatrix} \begin{bmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{bmatrix} = -3 \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}
\]

(6)

In the formula: \((x_0, y_0, z_0)\) is the location of the magnetic field source, \((x, y, z)\) is the location of the detection point.

5. Simulation calculation

In this paper, the maximum of the principal invariant \((I_1)\) of the magnetic field gradient tensor is used to determine the number and horizontal position of the target magnetic field source, and then the horizontal position is substituted for equation (6) to obtain the buried depth of the magnetic source. The size of the spacecraft model is \(3 \text{ m} \times 3 \text{ m} \times 3 \text{ m}\), and four magnetic field sources are preset inside. The initial setting value (digital number) and the simulated calculation value (letter number) of the magnetic field source of the model are shown in Table 1.

| Number | \( x/\text{m} \) | \( y/\text{m} \) | \( z/\text{m} \) | \( r/\text{m} \) | Range difference /\text{m} |
|--------|----------------|----------------|----------------|---------------|--------------------------|
| 1      | 2.3282         | 0.7663         | 0.5780         | 2.5183        | 0.0138                   |
| a      | 2.3111         | 0.7514         | 0.6055         | 2.5045        | 0.0954                   |
| 2      | 0.6928         | 1.1342         | 0.5744         | 1.4479        |                          |
b   0.6700  1.0158  0.5903  1.3525
3   1.1262  0.7196  0.5066  1.4294
8   1.2318  0.7451  0.5865  1.5545  0.1251
4   2.0707  2.1641  0.7849  3.0964
4   2.0875  2.1595  0.8014  3.1086  0.0122

The simulation results are shown in Figure 5. The red dot represents the position of the magnetic field source, the arrow represents the magnetic moment, and the blue dot represents the position of the magnetic field gradient tensor.

Figure 5. Schematic diagram of simulation results of multi-magnetic field sources in spacecraft model

By comparison, it can be seen that the horizontal position of the magnetic source detected by the principal invariant of the magnetic field gradient tensor has a high accuracy. The simulation results are very close to the expected values, and the minimum resolution of the spatial distance can reach 0.012 m.

6. Conclusion

Resolution of multi-magnetic field sources has always been a difficult problem in the field of magnetic measurement. The small-scale resolution mainly depends on the "guess" algorithm of approximation fitting. Using the gradient tensor algorithm, we can quickly determine the number, location and magnetic moment of magnetic field sources. In this paper, the maximum of the principal invariant of the gradient tensor is used to determine the number and horizontal position of the target magnetic field sources. In order to increase accuracy, 2~3 planes can be measured and counted, and then the buried depth of magnetic dipole can be inverted by using Euler equation. The location information of multiple magnetic field sources in space can be measured and inverted accurately at the same time. The inversion calculation of magnetic field gradient tensor can obtain more accurate and reliable results for the case of small-scale multi-magnetic field sources interfering with each other.

This multi-target resolution method can judge the magnetic field distribution inside the spacecraft, detect and inverse the positions of multiple magnetic field sources accurately, improve the pertinence and accuracy of spacecraft magnetic testing and magnetic compensation, and is an ideal method for future spacecraft magnetic testing.

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