Decay $H^+ \rightarrow W^+\gamma$ in a nonlinear $R_{\xi}$-gauge

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A new evaluation of the charged Higgs boson decay $H^+ \rightarrow W^+\gamma$ is presented in the context of the general two-Higgs doublet model. A nonlinear $R_{\xi}$-gauge which considerably simplifies the calculation is introduced and simple expressions are obtained for the fermionic and bosonic contributions. The $H^+ \rightarrow W^+\gamma$ branching ratio is analyzed for several values of the parameters of the model. Although this decay can have a branching fraction as large as $10^{-4}$ in a certain region of the parameter space, it is found that such a region is disfavored by the most recent constraints on $b \rightarrow s\gamma$, $g-2$ of the muon, $Z \rightarrow bb$, and the $\rho$ parameter, along with the exclusions from direct searches at the CERN $e^-e^+$ LEP collider. The possibility of detecting this decay at future colliders is discussed.

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I. INTRODUCTION

One of the most popular extensions of the standard model (SM) is the two-Higgs doublet model (THDM), mainly because of its simplicity and consistency with the minimal realization of supersymmetry (SUSY) [1]. The THDM predicts the existence of five Higgs bosons: two neutral CP-even scalar bosons $h^0$ and $H^0$, one neutral CP-odd scalar boson $A^0$, and a pair of charged scalar bosons $H^{\pm}$. The detection of the charged Higgs boson would be an irrefutable signal of an extended Higgs sector, and its experimental study might help us to elucidate the structure of the Higgs potential and shed light on the group representation of the Higgs fields through the study of its couplings to the gauge boson fields. In models which only comprise singlets and doublets of Higgs bosons, the decays of the charged scalar boson into a gauge boson pair, namely $H^+ \rightarrow W^+\gamma$ [2,3,4] and $H^+ \rightarrow W^+Z$ [5,6], are absent at the tree level, though they may occur at higher orders. While the decay mode $H^+ \rightarrow W^+\gamma$ is forbidden at the tree level due to electromagnetic gauge invariance, the $H^+ \rightarrow W^+Z$ decay can be induced at this order in models including Higgs triplets or more complicated representations [1]. In spite of their suppressed branching ratios, these decay modes are very interesting, which is due to the fact that their experimental study may provide important information concerning the underlying structure of the gauge and scalar sectors. Apart from being rather sensitive to new physics effects [7], these channels have a clear signature and might be at the reach of future particle colliders.

In this work we present an evaluation of the one-loop induced decay $H^+ \rightarrow W^+\gamma$ in the framework of the THDM. This decay was already studied in the context of the minimal supersymmetric standard model (MSSM), which is a restricted case of the general THDM. At the one-loop, the main contributions to this decay arise from the third-generation quarks $t$ and $b$, whereas the bosonic sector contributes through the CP-even Higgs bosons $h^0$ and $H^0$ along with the $W$ boson. The fermionic contribution and the effects of a fourth family were studied in Ref. [2]. The bosonic contribution was studied in the linear $R_{\xi}$-gauge [8], which is not suitable for such a calculation since gives rise to several complications that can be evaded by the use of a nonlinear $R_{\xi}$-gauge [9]. In this work it is reevaluate the $H^+ \rightarrow W^+\gamma$ decay in a nonlinear $R_{\xi}$-gauge, which leads to considerably simplifications due to the fact that some unphysical vertices are removed from the interaction Lagrangian. We will see that such a gauge not only reduces considerably the number of Feynman diagrams but also renders manifestly gauge-invariant ultraviolet-finite amplitudes. Apart from emphasizing the advantages of using the nonlinear $R_{\xi}$-gauge, we will analyze the $H^+ \rightarrow W^+\gamma$ decay in some scenarios which are still consistent with the most recent bounds obtained from electroweak precision measurements. Indeed, it has been argued quite recently [2] that the parameter space of the type-II THDM has become tightly constrained according to the last reported value of the muon anomalous magnetic moment $a_{\mu}$ [10], so we will concentrate essentially on the still-allowed region.

The rest of the paper is organized as follows. A brief review of the model and the nonlinear $R_{\xi}$-gauge used in our calculation are presented in Sec. II. Sec. III is devoted to discuss the most interesting details of the calculation with emphasis on the facilities brought about by the nonlinear $R_{\xi}$-gauge. The discussion and conclusions are presented in Sec. IV and V respectively. Finally, the Feynman rules necessary for the calculation are shown in Appendix A.
II. THE CP-CONSERVING HIGGS POTENTIAL IN THE TWO-HIGGS DOUBLET MODEL

The scalar sector of the CP-conserving THDM consists of two scalar doublets with hypercharge +1: \( \Phi_1 = (\phi_1^-, \phi_1^{0*}) \) and \( \Phi_2 = (\phi_2^-, \phi_2^{0*}) \). The most general gauge invariant potential can be written as

\[
V(\Phi_1, \Phi_2) = \mu_1^2(\Phi_1^\dagger \Phi_1) + \mu_2^2(\Phi_2^\dagger \Phi_2) - \left( \mu_{12}(\Phi_1^\dagger \Phi_2) + H.c. \right) + \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \left( \lambda_5(\Phi_1^\dagger \Phi_1)^2 + \left( \lambda_6(\Phi_1^\dagger \Phi_1) + \lambda_7(\Phi_2^\dagger \Phi_2) \right)(\Phi_1^\dagger \Phi_2) + H.c. \right)
\]

(1)

It has been customary to impose the discrete symmetry \( \Phi_1 \rightarrow \Phi_1 \) and \( \Phi_2 \rightarrow -\Phi_2 \) in order to avoid dangerous flavor changing neutral current (FCNC) effects. This symmetry is strongly violated by the terms associated with \( \lambda_6 \) and \( \lambda_7 \), but it is softly violated by that associated with \( \mu_{12} \). It is worth mentioning that the last term plays an important role in SUSY models. All of the terms in Eq. (1) are essential to obtain the decoupling limit of the model, in which only one CP-even scalar boson is light. In this work we will assume that this discrete symmetry is softly violated, which means that we will take \( \lambda_6 = \lambda_7 = 0 \) throughout the rest of the presentation.

The scalar potential (1) has to be diagonalized to yield the mass-eigenstates fields. The charged components of the doublets lead to the physical charged Higgs boson and the pseudo-Goldstone boson associated with the \( W \) gauge field as follows

\[
H^\pm = -\phi_1^\pm \sin \beta + \phi_2^\pm \cos \beta,
\]

(2)

\[
G_W^\pm = \phi_1^\pm \cos \beta + \phi_2^\pm \sin \beta,
\]

(3)

with \( \tan \beta = v_2/v_1 \), being \( v_1 \) (\( v_2 \)) the vacuum expectation value (VEV) associated with \( \Phi_1 \) (\( \Phi_2 \)).

On the other hand, the imaginary part of the neutral components \( \phi_{1l}^0 \) define the neutral CP-odd scalar and the pseudo-Goldstone boson associated with the \( Z \) gauge boson. The corresponding rotation is given by

\[
A^0 = -\phi_{1l}^0 \sin \beta + \phi_{2l}^0 \cos \beta,
\]

(4)

\[
G_Z = \phi_{1l}^0 \cos \beta + \phi_{2l}^0 \sin \beta,
\]

(5)

whereas the real part of the neutral components \( \phi_{1R}^0 \) define the CP-even Higgs bosons \( h^0 \) and \( H^0 \):

\[
h^0 = -\phi_{1R}^0 \sin \alpha + \phi_{2R}^0 \cos \alpha,
\]

(6)

\[
H^0 = \phi_{1R}^0 \cos \alpha + \phi_{2R}^0 \sin \alpha,
\]

(7)

where the mixing angle \( \alpha \) is given by

\[
\tan 2\alpha = \frac{2 m_{12}}{m_{11} - m_{22}},
\]

(8)

with the elements of the mass matrix being \( m_{11} = 2v_1^2 \lambda_1 + \tan \beta \mu_{12}^2 \), \( m_{22} = 2v_2^2 \lambda_2 + \cot \beta \mu_{12}^2 \), and \( m_{12} = v_1 v_2 (\lambda_3 + \lambda_4 + \lambda_5) - \mu_{12}^2 \).

The masses of the charged and CP-odd scalar bosons satisfy the following expression

\[
m_{H^\pm}^2 = m_{A^0}^2 + \frac{2m_W^2}{g^2}(\lambda_5 - \lambda_1),
\]

(9)

where \( g \) is the coupling constant associated with the \( SU_L(2) \) group. Obviously \( m_{H^\pm} = m_{A^0} \) when \( \lambda_5 = \lambda_1 \), which is a reflect of the underlying custodial symmetry.

We now would like to comment on the gauge-fixing procedure which was used to simplify our calculation. To this end we introduce the following gauge-fixing functions [8]:

\[
f^{+} = \left( D^{\mu}_\rho + \frac{i g W^\rho}{c_W} Z_\mu \right) W^{+\mu} - i \xi m_W G_W^+, \quad (10)
\]

\[
f^{Z} = \partial_\mu Z^\mu - \xi m_Z G_Z, \quad (11)
\]

\[
f^{A} = \partial_\mu A^\mu, \quad (12)
\]
with $D^\mu_\rho$ the electromagnetic covariant derivative and $\xi$ the gauge parameter. Note that $f^+$ is nonlinear and transforms covariantly under the electromagnetic gauge group. This gauge-fixing procedure is suited to remove the unphysical vertices $WG_W\gamma$ and $WG_WZ$, which arise in the Higgs kinetic-energy sector, and also modifies the Yang-Mills sector. One important result is that the expression for the $WW\gamma$ vertex satisfies a QED-like Ward identity, which turns out to be very useful in loop calculations. In particular, in the Feynman-t’Hooft gauge the Feynman rule for the $W^+_\rho(q)W^-_\nu(p)A_\mu(k)$ coupling can be written as

$$\Gamma^{WW\gamma}_{\rho\nu\mu} = -ie (g_{\mu\rho}(p - k + q)\rho + g_{\nu\rho}(q - p)\mu + g_{\mu\nu}(k - p - q)\nu),$$

where all the momenta are incoming. It is easy to see that this expression satisfies a QED-like Ward identity.

The remaining Feynman rules necessary for our calculation do not depend on the gauge fixing procedure. For completeness they are presented in Appendix A. As far as the Yukawa couplings are concerned, we will concentrate on the type-II THDM, in which the up-type quarks couple to the first Higgs doublet whereas the remaining fermions couple to the second one.

### III. The $H^+ \to W^+\gamma$ Decay Width

In the nonlinear $R_\xi$-gauge, the decay $H^+ \to W^+\gamma$ receives contributions from the Feynman diagrams shown in Fig. 1. As far as the fermionic sector is concerned, the main contribution comes from the third-generation quarks, which induce three diagrams, whereas in the bosonic sector there are contributions from the $(H^\pm, \phi^0)$ and $(W^\pm, \phi^0)$ pairs, with $\phi^0 = h^0$ or $H^0$. We would like to emphasize that the nonlinear $R_\xi$-gauge considerably simplifies the calculation of the $H^+ \to W^+\gamma$ decay. First of all, the removal of the unphysical vertex $W^\pm G^\mp_W\gamma$, allows one to get rid of all those diagrams which are displayed in Fig. 2 and do have to be calculated in the linear $R_\xi$-gauge. Even more, in the nonlinear $R_\xi$-gauge the tadpole graphs shown in Fig. 3 vanish. Apart from these simplifications, we will be able to group the Feynman diagrams into subsets which separately yield a manifestly gauge-invariant amplitude free of ultraviolet singularities.

![Feynman diagrams](image)

**FIG. 1:** Feynman diagrams contributing to the $H^+ \to W^+\gamma$ decay in the nonlinear $R_\xi$-gauge. $\phi^0$ stands for $h^0$ and $H^0$, and $X^+$ for $H^+$ and $G^+_W$. Compare with Fig. 1 of Ref. 3.

Once the amplitude for every Feynman diagram shown in Fig. 1 was written down, the Passarino-Veltman method was applied to express it in terms of scalar integrals, which are suitable for numerical evaluation. The full amplitude for the decay $H^+(p) \to W^+(q)\gamma(k)$ can be written as

$$\mathcal{M}(H^+ \to W^+\gamma) = \frac{\alpha^{3/2}}{2\sqrt{\pi m_W s_W}} (G(q \cdot k, g_{\mu\rho} - k_\rho q_\mu) - i H \epsilon_{\mu\omega\alpha\beta}k^\alpha q^\beta) \epsilon^\nu(k)e^\nu(q),$$

which is manifestly gauge invariant. The $H$ function only receives contributions from the quarks:

$$H = \frac{i \lambda_{tb}}{\delta_{(H^\pm,\gamma)}} \left( 3 \delta_{B(3, H^\pm, W^\pm)} + \delta_{(H^\pm, W^\pm)} \left( c^2_{3\beta} C(3, t) + 2 s^2_{3\beta} C(1, b) \right) \right),$$
where we have introduced the shorthand notation $\delta_{a,b} \equiv m_a^2 - m_b^2$, $\delta B_{(a,b,c,d)} = B_0 \left( m_a^2, m_b^2, m_c^2 \right) - B_0 \left( m_b^2, m_a^2, m_c^2 \right)$, and $C_{a,b} \equiv C_0 \left( m_a^2, m_b^2, 0, m_c^2, m_d^2 \right)$, where $B_0$ and $C_0$ stand for Passarino-Veltman scalar integrals. In addition, $c_\beta = \cos \beta$, $s_\beta = \sin \beta$, etc. From the definition of $\delta B_{(a,b,c,d)}$, it is clear that the $H$ function is ultraviolet finite. As for $G$, it can be written as

$$ G = \frac{1}{2 m_W^2 \delta^2_{(H^+, W)}} \left( G_{tb} + \sum_{\phi_0 = H^0, h^0} \left( G_{H^+ \phi_0} + G_{W^+ \phi_0} + G_{W^+ \phi_0} \right) \right), $$

where $G_{AB}$ stands for the contribution of the $(A, B)$ pair:

$$ G_{tb} = \lambda_{tb} \left( m_H^2 \left( 3 \left( \delta_{(t,b)} + c_\beta \delta_{(H^+, W)} \right) + m_W^2 \right) \delta B_{(H^+, W,b, t)} - 3 \delta_{(t,b)} \delta B_{(0,H^+, b,t)} + m_{H^2} \delta_{(H^+, W)} \left( 1 - 2 m_b^2 - c_\beta^2 \delta_{(H^+, W)} \right) C_{(b,t)} + \left( 2 m_t^2 - s_\beta^2 \delta_{(H^+, W)} \right) C_{(t,b)} \right), $$

$$ G_{H^+ h^0} = \lambda_{h^0 H^+} \left( \delta_{(H^+, h^0)} \delta_{(H^+, W)} \delta B_{(0,W,H^+, h^0)} - \left( 2 m_{H^+}^2 \delta_{(H^+, h^0)} + m_{h^0}^2 m_W^2 \right) \delta B_{(H^+, W,H^+, h^0)} - m_{H^+}^2 \delta_{(H^+, W)} \left( 1 + 2 m_{H^+}^2 C_{(H^+, h^0)} \right) \right), $$

When $a = 0$, $m_a$ must be substituted by 0.
\[ G_{\phi^0, h^0} = \lambda_{G_{\phi^0, h^0}} \left( \left( m_W^2 \delta_{(h^0, W)} + m_W^2 (3m_W^2 - 2m_W^2) \right) \delta B_{(H^0, W, h^0)} + \delta_{(H^0, h^0)} \delta B_{(0, W, h^0)} \right) \delta B_{(0, W, h^0)} \right) \]
\[ + m_W^2 \delta_{(H^0, W)} \left( 1 + 2 m_W^2 C_{(W, h^0)} \right), \]

and

\[ G_{\phi^0, h^0} = \lambda_{G_{\phi^0, h^0}} \left( \left( m_W^2 \delta_{(h^0, W)} + m_W^2 (7m_W^2 - 2m_W^2 - 4m_W^2) \right) \delta B_{(H^0, W, h^0)} + \delta_{(H^0, W)} \delta B_{(0, W, h^0)} \right) \delta B_{(0, W, h^0)} \]
\[ + m_W^2 \delta_{(H^0, W)} \left( 1 - 2 \left( 2 \delta_{(H^0, W)} - m_W^2 \right) C_{(m_W^2, m_W^2)} \right), \]

with

\[ \lambda_{H^0, h^0} = \frac{s_2(\alpha - \beta)}{2}, \]

\[ \lambda_{G_{\phi^0, h^0}} = \frac{s_2(\alpha - \beta)}{2}, \]

and

\[ \lambda_{W^0, h^0} = - \frac{m_W^2 s_2(\alpha - \beta)}{2}. \]

Finally, the contribution of the heaviest CP-even scalar boson \( H^0 \) is obtained from that of the lightest one once the substitutions \( m_{h^0} \rightarrow m_{h^0} \) and \( \alpha \rightarrow \alpha - \pi/2 \) are done.

The above expressions are very simple and should be compared to those presented in Refs. [2, 3]. It is also evident that the partial amplitudes induced by the pairs \( (t, b) \), \( (\phi^0, W^\pm) \), \( (\phi^0, H^\pm) \), and \( (\phi^0, G^\pm_W) \) are gauge invariant and ultraviolet finite on their own. Again, this is to be contrasted with the situation arising in the linear \( R_\xi \)-gauge, where showing gauge invariance is somewhat cumbersome, and the cancellation of ultraviolet divergences in the bosonic sector is achieved only after adding up all the Feynman diagrams [3].

Once Eq. (24) is squared and the spins of the final particles are summed over, the decay width can be written as

\[ \Gamma(H^0 \rightarrow W^+ \gamma) = \frac{\alpha^3 \delta_{(H^0, W)}^3}{2\pi^2 s_W m_W^2 m_W^2} \left( |G|^2 + |H|^2 \right). \]

We will evaluate this decay width for some values of the parameters of the model.

IV. NUMERICAL RESULTS AND DISCUSSION

In order to evaluate the \( H^+ \rightarrow W^+ \gamma \) decay, there are six free parameters of the THDM which should be addressed. They are the masses of the four scalar bosons \( m_{H^\pm}, m_{h^0}, m_{\phi^0} \) and \( m_{A^0} \), together with the VEVs ratio \( \tan \beta \) and the mixing angle \( \alpha \). Without losing generality, we will assume that \( \mu_{12} = 0 \). Although the \( H^+ \rightarrow W^+ \gamma \) decay width does not depend on \( m_{\phi^0} \), the respective branching ratio does depend on all of these parameters. Instead of examining the behavior of \( H^+ \rightarrow W^+ \gamma \) for arbitrary values of these parameters and looking for those which enhance its branching ratio, a realistic analysis must take into account the most recent constraints obtained from precision measurements and direct searches at particle colliders. We will thus concentrate on the region of the parameter space which is still consistent with low-energy data.

The parameter space of THDMs as constrained by electroweak precision measurements has been recurrently studied in the literature [4, 11, 12]. Along this line, it is well known that the experimental bound on the inclusive decay \( b \rightarrow s \gamma \) can be translated into a bounded area on the \( m_{H^\pm} \times \tan \beta \) plane. In addition, the muon anomalous magnetic moment \( a_\mu \) is useful to constrain the CP-odd scalar boson mass \( m_{A^0} \), which along with the CP-even scalar boson masses \( m_{h^0} \) and \( m_{\phi^0} \) can be further constrained by the \( Z \rightarrow b\bar{b} \) decay mode \( R_b \). All these constraints, when combined with those obtained from the \( \rho \) parameter and the data from direct searches at the CERN \( e^-e^+ \) LEP collider, can yield
the allowed region in the parameter space of THDMs. Quite recently, the most up-to-date measurements on \( b \to s \gamma \) [10], \( a_\mu \) [11], \( R_b \) [17], and \( \rho \) [18], together with the DELPHI [19] and OPAL [20] excluded regions, were considered to constrain the parameter space of the type-II THDM [9,15]. In particular, the possibility of a very light CP-odd scalar boson was considered in [15], whereas a more general treatment was presented in [1]. From these studies one is lead to conclude that the parameter space of the THDM has become tightly constrained by the most recent measurement of \( a_\mu \) [11]. Below we will follow closely the analysis presented in Ref. [9].

The model independent bound \( m_{H^\pm} \gtrsim 80 \text{ GeV} \) has been derived at LEP for a charged Higgs boson decaying solely into \( \tau \nu \) [21]. On the other hand, within the THDM-II, the latest reported value for the decay \( b \to s \gamma \) [10] yields the very stringent lower bound \( m_{H^\pm} \gtrsim 500 \text{ GeV} \) for intermediate and large values of \( \tan \beta \), though this bound can loosen in SUSY models with conserved or broken \( R \)-parity [22]. As far as \( a_\mu \) is concerned, the latest experimental data from the BNL collaboration [10] require a positive contribution from new physics to bring the theoretical value closer to the experimental one. It turns out that the CP-odd scalar yields a positive contribution to \( a_\mu \), whereas the remaining Higgs bosons give a negative contribution. This means that the deviation between the experimental and theoretical values of \( a_\mu \) can only be explained by the existence of a light CP-odd scalar along with a high value of \( \tan \beta \) and relatively heavy \( H^\pm \), \( h^0 \) and \( H^0 \). When the \( R_b \) constraint [17] is taken into account, a large region of the parameter space of the THDM gets excluded, and only a tiny region survives when the DELPHI [19] and OPAL [20] excluded regions for the scalar boson masses are considered. From this analysis, it was concluded [9] that the most favorable region is that in which \( \tan \beta \) is of the order of 50 or larger, \( m_{H^0} \lesssim 80 \text{ GeV} \), \( m_{h^0} \lesssim 140 \text{ GeV} \), and \( -1/2 \pi \leq \alpha \leq -3/8 \pi \). Even more, the \( \rho \sim 1 \) constraint requires fine-tuned solutions for the masses of the scalar bosons and the mixing angles. In fact, this constraint is satisfied by an almost decoupling CP-even scalar boson \( H^0 \), with a mass of the order of 1 TeV, provided that the remaining parameters meet the previous constraints.

Bearing in mind the preceding discussion, we will analyze the \( H^+ \to W^+ \gamma \) decay for 300 GeV \( \lesssim m_{H^\pm} \lesssim 600 \text{ GeV} \). For the sake of illustration we will assume that the lightest CP-even scalar boson is relatively light and the heaviest one is much heavier indeed, \( i.e. \) we will take \( m_{h^0} = 115 \text{ GeV} \) and \( m_{H^0} = 1 \text{ TeV} \). The former value is consistent with the current LEP bound on the mass of a SM-like Higgs boson [18]. Moreover, we will focus on the region where \( \tan \beta \) is large and \( \sin \alpha \) is close to -1. We have verified that these values lie inside the still-allowed region of the parameter space of the THDM-II.

![FIG. 4: \( H^+ \to W^+ \gamma \) decay width (solid line) as a function of \( \tan \beta \) for the indicated values of \( \sin \alpha \) and \( m_{H^\pm} \). We have taken \( m_{h^0} = 115 \text{ GeV} \) and \( m_{H^0} = 1 \text{ TeV} \). The partial contributions from the quark (dashed line) and boson (dotted line) sectors are also shown.](image)

It is interesting to examine the effects of both the quark and boson sectors on the decay width \( \Gamma(H^+ \to W^+ \gamma) \). The partial contributions, together with the full contribution, are displayed as a function of \( \tan \beta \) in Fig. 4 whereas in Fig. 5 we show them as a function of \( m_{H^\pm} \). We can observe that, in this range of parameters, the \( H^+ \to W^+ \gamma \) decay is slightly dominated by the bosonic contribution. The fermionic contribution is larger only in the region of the parameter space where \( \sin \alpha \) is very close to -1. This is illustrated in Fig. 9 where we show the partial contributions to the \( H^+ \to W^+ \gamma \) decay as a function of \( \sin \alpha \) for \( m_{H^\pm} = 300 \text{ GeV} \) and two different values of \( \tan \beta \). We see that the bosonic contribution is maximal when \( \sin \alpha = 0 \) and minimal when \( |\sin \alpha| = 1 \), where it is smaller than the fermionic contribution. For this particular set of the parameters, the maximal value of the bosonic contribution is more than one order of magnitude than the respective minimal value. A similar behavior is observed for larger values of \( m_{H^\pm} \) and \( \tan \beta \). Also, from Fig. 4 and Fig. 5 it is interesting to note that the partial contribution from the bosonic sector...
observe that $\beta$ branching ratio is shown in Fig. 7 as a function of $\tan\beta$. The constant lines correspond to the fermion contribution and the remaining to the bosonic one. The values of $\tan\beta$ of tan values of $\tan\beta$ exceed the total contribution in some regions of the parameter space. This stems from the fact that the interference between the partial contributions is destructive. While the fermion contribution depends only on $\tan\beta$ for a given $m_{H^±}$, the bosonic contribution depends on four other unknown parameters. This makes harder the analysis of this decay mode.

The full width of the charged scalar boson is given to a good approximation by the sum of the partial widths of the two-body decay channels induced at the tree-level: $H^+ \to \tau \bar{\nu}, c \bar{s}, t \bar{b}, W^+ A^0$, and $W^+ h^0$. The widths for all these decays can be readily obtained and we refrain from presenting the respective expressions here. In the region of the parameter space that we are considering, the charged scalar boson would decay mainly into $t \bar{b}$. This decay mode is somewhat troublesome as it would suffer from large QCD backgrounds at a hadronic collider [23]. The $H^+ \to W^+ \gamma$ branching ratio is shown in Fig. 7 as a function of $\tan\beta$ for $m_{A^0} = 80$ GeV and two different values of $\sin\alpha$. We can observe that $Br(H^+ \to W^+ \gamma)$ is at most of the order of $6 \times 10^{-6}$, it increases for larger values of $\tan\beta$, but falls slowly as $\sin\alpha$ gets closer to $-1$. On the other hand, the behavior of $Br(H^+ \to W^+ \gamma)$ as a function of $m_{H^±}$ is depicted in Fig. 8. For $\tan\beta = 40$, the branching fraction decreases very slowly for larger $m_{H^±}$, whereas for $\tan\beta = 60$ it has a minimal around $m_{H^±} = 350$ GeV and increases slowly until about $m_{H^±} = 550$ GeV, where there is a maxima.

It is worthwhile to mention that the $H^+ \to W^+ \gamma$ branching ratio may be dramatically enhanced, mainly by the fermionic contribution, around the threshold $m_{H^±} \sim m_t + m_b$. This is shown in Fig. 9 for some illustrative values of $\tan\beta$ and $\sin\alpha$. It can be observed that $Br(H^+ \to W^+ \gamma)$ can be larger than $10^{-4}$ when $\tan\beta$ is small regardless of the value of $\sin\alpha$, and up to $10^{-4}$ when a large value of $\tan\beta$ combines with $\sin\alpha \sim 0$. In the latter case the bosonic contribution is also responsible for the enhancement. However, when a large value of $\tan\beta$ combines with $\sin\alpha \sim 1$, there is large cancellations between the two contributions, which results in a significant suppression of
$Br(H^+ \rightarrow W^+\gamma)$. Unfortunately the region of the parameter space in which such an enhancement shows up is not favored by the constraints from electroweak precision measurements. Although a fourth-fermion family may improve the $H^+ \rightarrow W^+\gamma$ decay rate [2], the enhancement would depend on the splitting between the masses of the fourth-generation quarks $m_u' - m_d'$, which cannot be too large due to the $\rho \sim 1$ restriction. In this respect, it seems that the $H^\pm \rightarrow W^\pm Z$ decay is more promising since it is sensitive to nondecoupling effects when there is a large splitting between the CP-odd and the charged scalar boson masses [6]. As discussed above, this scenario is still consistent with the constraints on the parameter space of the THDM-II. Furthermore, the $H^\pm \rightarrow W^\pm Z$ decay can arise at the tree-level in Higgs triplet models [1].
Let us now discuss shortly on the possibility of detecting the $H^+ \to W^+\gamma$ decay at future colliders. Single and pair production of heavy charged Higgs bosons at both hadronic and linear $e^+e^-$ colliders have long received considerable attention. A charged scalar boson heavier than 180 GeV could be detected at a hadronic collider via several processes, such as $gb \to H^-t$, $c\bar{s} \to H^+\tau$, and $c\bar{b} \to H^+t$. The one-loop processes $gg \to W^+H^\mp$ and $gq \to W^\mpH^\mp$ give no viable signature at the CERN LHC collider. While the $gb \to H^-t$ rate is out of the reach of the Fermilab Tevatron, it could be detected at the LHC for $m_{H^\pm}$ up to 1 TeV. This process is appropriate to look for a charged Higgs boson in the low- and high-tan$\beta$ regime. Though the $gb \to H^-t$ process followed by the decay $H^- \to b\bar{t}$ suffers from irreducible background, the $H^- \to \tau\bar{\nu}$ mode is background free due to the distinctive $\tau$ polarization. The cross section for $gb \to H^-t$ increases for large tan$\beta$ but falls when $m_{H^\pm}$ increases. At the LHC, the cross section for $pp \to H^+\bar{b}b$ can be as large as $10^4$ fb for $m_{H^\pm}$ of the order of a few hundreds of GeV and tan$\beta$ around 50. With an integrated luminosity of 100 fb, this rate is enough to detect a charged scalar boson decaying as $H^\pm \to \pi\nu$, but it is not viable to detect the $H^+ \to W^+\gamma$ mode unless its decay width is dramatically enhanced. There is some chance to detect the $H^+ \to W^+\gamma$ decay if the constraints on the parameter space can be evaded in some way and the charged scalar boson mass is about 180 GeV. In this case the branching ratio might increase up to $10^{-4}$, whereas the cross section for $pp \to H^+\bar{b}b$ can reach the $10^4$ fb level.

In a linear collider (LC) the situation is less promising. If the mass of the charged scalar boson happen to be less than 180 GeV, the linear collider (LC) would also work in the mode $H^+ \to W^+\gamma$ and $\gamma \to \gamma\gamma$. The viability of these modes depends strongly on the strength of the Yukawa couplings of the fermions to the charged scalar boson. It turns out that the rates for these processes can be up to two orders of magnitude larger than the one-loop amplitudes. The advantages of using such a gauge were emphasized. In particular, there are 21 Feynman diagrams in the nonlinear $R_\xi$-gauge, whereas the linear one induces about one hundred. An analysis of the behavior of the $H^+ \to W^+\gamma$ decay width as a function of the parameters of the model was presented, but we focused essentially on the region of the parameter space still allowed by the constraints obtained from the latest reported values of $b \to s\gamma$, $R_b$, $a_\mu$ and the $\rho$ parameter, which favor a charged scalar boson heavier than the $t$ quark, a large value of tan$\beta$, and $-\pi/2 \leq \alpha \leq -3\pi/8$. In this still-allowed region, which was not considered in the previous analyses of the $H^+ \to W^+\gamma$ decay, the respective branching ratio receives contribution of the same order of magnitude from both the bosonic and fermionic sectors, and it can be as large as $10^{-6}$, which seems to be beyond the reach of the LHC.

A study of the decay $H^+ \to W^+\gamma$ was presented in the context of the type-II THDM. A nonlinear $R_\xi$-gauge covariant under the electromagnetic gauge group was used to obtain the one-loop amplitudes. The advantages of using such a gauge were emphasized. In particular, there are 21 Feynman diagrams in the nonlinear $R_\xi$-gauge, whereas the linear one induces about one hundred. An analysis of the behavior of the $H^+ \to W^+\gamma$ decay width as a function of the parameters of the model was presented, but we focused essentially on the region of the parameter space still allowed by the constraints obtained from the latest reported values of $b \to s\gamma$, $R_b$, $a_\mu$ and the $\rho$ parameter, which favor a charged scalar boson heavier than the $t$ quark, a large value of tan$\beta$, and $-\pi/2 \leq \alpha \leq -3\pi/8$. In this still-allowed region, which was not considered in the previous analyses of the $H^+ \to W^+\gamma$ decay, the respective branching ratio receives contribution of the same order of magnitude from both the bosonic and fermionic sectors, and it can be as large as $10^{-6}$, which seems to be beyond the reach of the LHC.

Note added: After the submission of this manuscript, the muon $g - 2$ collaboration announced a new result for the anomalous magnetic moment of the negative muon, which is based on the data collected during the year 2001. The new result, $a^\exp_\mu = 11659214(8)(3) \times 10^{-10} (0.7$ ppm), is consistent with previous measurements of the anomaly for the negative and positive muon. The new world average for the muon anomaly is thus $a^\exp_\mu = 11659208(6) \times$
$10^{-10}$ (0.5 ppm), which is to be contrasted with the previous one \cite{10}: $a^\mu_{\text{exp}} = 11659203(8) \times 10^{-10}$ (0.7 ppm). This new results confirms the fact that THDMs are very disfavored by the muon anomaly. However, we would like to point out that the scenario that we are considering is still consistent with the new result. So, the analysis presented in this work is not affected significantly and our conclusions remain unchanged.

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APPENDIX A: FEYNMAN RULES FOR THE DECAY $H^+ \rightarrow W^+ \gamma$

In this appendix we present the Feynman rules which do not depend on the gauge fixing procedure described in Sec. \cite{11}. The $H^\mp h^0$ vertex arise from the Higgs potential. In terms of the coupling constants $\lambda_i$ the Feynman rule for this vertex is given by

$$g_{H^\mp h^0} = g_{A^0 A^0 h^0} - \frac{2i m_W}{g} (\lambda_5 - \lambda_4)s_{\beta - \alpha}, \quad (A1)$$

where

$$g_{A^0 A^0 h^0} = \frac{2i m_W}{g} \left( 2\lambda_1 s_\beta^2 c_\alpha - 2\lambda_2 c_\beta^2 s_\beta c_\alpha - (\lambda_3 + \lambda_4 + \lambda_5) (s_\beta^3 c_\alpha - c_\beta^3 s_\alpha) + 2\lambda_5 s_{\beta - \alpha} \right). \quad (A2)$$

The coupling constants $\lambda_i$ can be expressed in turn in terms of the mixing angles and the Higgs boson masses as follows:

$$\lambda_1 = \frac{g^2}{8m_W^2} \left( \left( \frac{s_\beta}{c_\beta} \right)^2 m_{h^0}^2 + \left( \frac{c_\alpha}{s_\beta} \right)^2 m_{H^0}^2 - \frac{s_\beta}{c_\beta} \mu_{12}^2 \right), \quad (A3)$$

$$\lambda_2 = \frac{g^2}{8m_W^2} \left( \left( \frac{c_\alpha}{s_\beta} \right)^2 m_{h^0}^2 + \left( \frac{s_\alpha}{c_\beta} \right)^2 m_{H^0}^2 - \frac{c_\beta}{s_\beta} \mu_{12}^2 \right), \quad (A4)$$

$$\lambda_3 = \frac{g^2}{4m_W^2} \left( 2m_{H^+}^2 s_\beta^2 + \frac{s_\alpha}{s_\beta} (m_{H^0}^2 - m_{h^0}^2) - \frac{\mu_{12}^2}{s_\beta c_\beta} \right), \quad (A5)$$

$$\lambda_4 = \frac{g^2}{4m_W^2} \left( m_{A^0}^2 - 2m_{H^+}^2 + \frac{\mu_{12}^2}{s_\beta c_\beta} \right), \quad (A6)$$

$$\lambda_5 = \frac{g^2}{4m_W^2} \left( \frac{\mu_{12}^2}{s_\beta c_\beta} - m_{A^0}^2 \right). \quad (A7)$$

After some algebra we are left with

$$g_{H^\pm h^0} = \frac{ig}{2m_W} \left( 2s_{\alpha - \beta} m_{H^\pm}^2 + \frac{c_\alpha + \beta}{c_\beta^2 s_\beta^2} \mu_{12}^2 + \left( s_\alpha s_\beta t_\beta - \frac{c_\beta c_\alpha}{t_\beta} \right) m_{h^0}^2 \right). \quad (A8)$$

The Higgs kinetic-energy term induces the $H^\pm G_W^{\mp} h^0$ vertex:

$$g_{H^\pm G_W^{\mp} h^0} = \frac{ig}{2m_W} (m_{H^\pm}^2 - m_{h^0}^2) c_{\beta - \alpha}. \quad (A9)$$

The Feynman rules involving the heavy CP-even scalar boson $H^0$ can be obtained from above after the replacements $m_{h^0} \rightarrow m_{H^0}$ and $\alpha \rightarrow \pi/2$. As for the couplings of the fermions to the $W$ gauge boson and the charged scalar boson, they are \cite{11}: 

\[ g_{H^{-}\bar{t}b} = \frac{ig}{2\sqrt{2}m_W}(v + a\gamma^5), \]  \hspace{1cm} (A10)

\[ g_{W^{-}\bar{t}b} = \frac{ig}{2\sqrt{2}m_W}(v + a\gamma^5)\gamma^\mu, \]  \hspace{1cm} (A11)

with \( v = m_t \cot\beta + m_b \tan\beta \) and \( a = m_b \tan\beta - m_t \cot\beta \).

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