Resonant and nonresonant new phenomena of four-fermion operators for experimental searches

She-Sheng Xue

ICRANeT, Piazzale della Repubblica, 10-65122, Pescara, Physics Department, University of Rome “La Sapienza”, Rome, Italy

In the fermion content and gauge symmetry of the standard model (SM), we study the four-fermion operators in the torsion-free Einstein-Cartan theory. The collider signatures of irrelevant operators are suppressed by the high-energy cutoff (torsion-field mass) $\Lambda$, and cannot be experimentally accessible at TeV scales. Whereas the dynamics of relevant operators accounts for (i) the SM symmetry-breaking in the domain of infrared-stable fixed point with the energy scale $v \approx 239.5$ GeV and (ii) composite Dirac particles restoring the SM symmetry in the domain of ultraviolet-stable fixed point with the energy scale $E \gtrsim 5$ TeV. To search for the resonant phenomena of composite Dirac particles with peculiar kinematic distributions in final states, we discuss possible high-energy processes: multi-jets and dilepton Drell-Yan process in LHC $pp$ collisions, the resonant cross-section in $e^- e^+$ collisions annihilating to hadrons and deep inelastic lepton-hadron $e^- p$ scatterings. To search for the nonresonant phenomena due to the form-factor of Higgs boson, we calculate the variation of Higgs-boson production and decay rate with the CM energy in LHC. We also present the discussions on four-fermion operators in the lepton sector and the mass-squared differences for neutrino oscillations in short baseline experiments.

PACS numbers: 12.60.-i, 12.60.Rc, 11.30.Qc, 11.30.Rd, 12.15.Ff

Introduction. The parity-violating (chiral) gauge symmetries and spontaneous/explicit breaking of these symmetries for the hierarchy of fermion masses have been at the center of a conceptual elaboration that has played a major role in donating to mankind the beauty of the SM for particle physics. The Nambu-Jona-Lasinio (NJL) model [1] of dimension-6 four-fermion operators at high energies and its effective counterpart, the phenomenological model [2] of elementary Higgs boson and its Yukawa-coupling to fermions at low energies, provide an elegant and simple description for the electroweak symmetry breaking and intermediate gauge boson masses. After a great experimental effort for many years, the ATLAS [3] and CMS [4] experiments have recently shown the first observations of a 126 GeV scalar particle in the search for the SM Higgs boson at the LHC. This far-reaching result begins to shed light on this most elusive and fascinating arena of fundamental

*Electronic address: xue@icra.it
In order to accommodate high-dimensional operators of fermion fields in the SM-framework of a well-defined quantum field theory at the high-energy scale $\Lambda$, it is essential and necessary to study: (i) what physics beyond the SM at the scale $\Lambda$ explains the origin of these operators; (ii) which dynamics of these operators undergo in terms of their couplings as functions of running energy scale $\mu$; (iii) associating to these dynamics where infrared (IR) or ultraviolet (UV) stable fixed point of physical couplings locates; (iv) in the domains (scaling regions) of these stable fixed points, which operators become physically relevant and renormalizable following renormalization group (RG) equations, and other irrelevant operators are suppressed by the cutoff at least $O(\Lambda^{-2})$.

The strong technicolor dynamics of extended gauge theories at the TeV scale was invoked [5, 6] to have a natural scheme incorporating the relevant four-fermion operator $G(\bar{\psi}^a_L t^a_{Ra})(\bar{t}^b_R \psi^b_L b)$ of the $\langle \bar{t}t \rangle$-condensate model [7]. On the other hand, these relevant operators can be constructed on the basis of phenomenology of the SM at low-energies. In 1989, several authors [7–9] suggested that the symmetry breakdown of the SM could be a dynamical mechanism of the NJL type that intimately involves the top quark at the high-energy scale $\Lambda$. Since then, many models based on this idea have been studied [10]. The low-energy SM physics was supposed to be achieved by the RG equations in the domain of the IR-stable fixed point with $v \approx 239.5$ GeV [6, 7, 9]. In fact, the $\langle \bar{t}t \rangle$-condensate model was shown [11] to be energetically favorable, the top-quark and composite Higgs-boson masses are correctly obtained by solving RG equations in this IR-domain with the appropriate non-vanishing form-factor of Higgs boson in TeV scales [12, 13].

Inspired by the non-vanishing form-factor of Higgs boson, the formation of composite fermions and restoration of the SM gauge symmetry in strong four-fermion coupling $G$ [14], we preliminarily calculated the $\beta(G)$-function and showed [13] the domain of an UV-stable fixed point at TeV scales, where the particle spectrum is completely different from the SM. This is reminiscent of the asymptotic safety [15] that quantum field theories regularized at UV cutoff $\Lambda$ might have a non-trivial UV-stable fixed point, RG flows are attracted into the UV-stable fixed point with a finite number of physically renormalizable operators. The weak and strong four-fermion coupling $G$ brings us into two distinct domains. This lets us recall the QCD dynamics: asymptotically free quark states in the domain of an UV-stable fixed point and bound hadron states in the domain of a possible IR-stable fixed point.

In this Letter, we proceed a further study on this issue, distinguishing physically relevant four-fermion operators from irrelevant one in the both domains of IR- and UV-stable fixed points, and focusing on the discussion of relevant operators and their resonant and nonresonant new phenomena.
for experimental searches.

**Four-fermion operators from quantum gravity.** A well-defined quantum field theory for the SM Lagrangian requires a natural regularization (cutoff \( \Lambda \)) fully preserving the SM chiral-gauge symmetry. The quantum gravity provides a such regularization of discrete space-time with the minimal length \( \tilde{a} \approx 1.2a_{\text{pl}} \), where the Planck length \( a_{\text{pl}} \sim 10^{-33} \text{ cm} \) and scale \( \Lambda_{\text{pl}} = \pi/a_{\text{pl}} \sim 10^{19} \text{ GeV} \). However, the no-go theorem \[16\] tells us that there is no any consistent way to regularize the SM bilinear fermion Lagrangian to exactly preserve the SM chiral-gauge symmetry. This implies that the natural quantum-gravity regularization for the SM leads us to consider at least four-fermion operators.

It is known that four-fermion operators of the classical and torsion-free Einstein-Cartan (EC) theory are naturally obtained by integrating over “static” torsion fields at the Planck length,

\[
\mathcal{L}_{\text{EC}}(e, \omega, \psi) = \mathcal{L}_{\text{EC}}(e, \omega) + \bar{\psi} e^\mu \partial_\mu \psi + G J^d J_d,
\]

(1)

where the gravitational Lagrangian \( \mathcal{L}_{\text{EC}} = \mathcal{L}_{\text{EC}}(e, \omega) \), tetrad field \( e_\mu(x) = e_a^\mu(x) \gamma_a \), spin-connection field \( \omega_\mu(x) = \omega_\mu^{ab}(x) \sigma_{ab} \), the covariant derivative \( \mathcal{D}_\mu = \partial_\mu - ig_\omega_\mu \) and the axial current \( J^d = \bar{\psi} \gamma^d \gamma_5 \psi \) of massless fermion fields. The four-fermion coupling \( G \) relates to the gravitation-fermion gauge coupling \( g \) and basic space-time cutoff \( \tilde{a} \). In the regularized and quantized EC theory \[17\] with a basic space-time cutoff, in addition to the leading term \( J^d J_d \) in Eq. (1) there are high-dimensional fermion operators \((d > 6)\), e.g., \( \partial_\sigma J^\mu J^\sigma J_\mu \), which are suppressed at least by \( O(\tilde{a}^4) \).

We consider massless left- and right-handed Dirac fermions \( \psi_L \) and \( \psi_R \) carrying the SM quantum numbers, as well as right-handed Dirac sterile neutrinos \( \nu_R \) and their Majorana counterparts \( \nu_R^c = i\gamma_2(\nu_R)^* \). Analogously to the EC theory \[1\], we obtain a torsion-free, diffeomorphism and local gauge-invariant Lagrangian

\[
\mathcal{L} = \mathcal{L}_{\text{EC}}(e, \omega) + \bar{\psi}_{L,R} e^\mu \mathcal{D}_\mu \psi_{L,R} + \bar{\nu}_R^c e^\mu \mathcal{D}_\mu \nu_R^c + G \left(J^\mu_{L,R} J_{L,R}^\mu + J^\mu_{R,R} J_{R,R}^\mu + 2J^\mu_{L,R} J_{R,L}^\mu \right) + G \left(j^\mu_{L,L} j_{L,L}^\mu + 2J^\mu_{L,L} j_{L,R}^\mu + 2J^\mu_{R,R} j_{R,L}^\mu \right),
\]

(2)

where we omit the gauge interactions in \( \mathcal{D}_\mu \) and fermion flavor indexes of axial currents \( J^\mu_{L,R} \equiv \bar{\psi}_{L,R} \gamma_\mu \gamma_5 \psi_{L,R} \) and \( j^\mu_L \equiv \bar{\nu}_R^c \gamma_\mu \gamma_5 \nu_R^c \). The four-fermion coupling \( G \) is unique for all four-fermion operators and high-dimensional fermion operators \((d > 6)\) are neglected. If torsion fields that couple to fermion fields are not exactly static, propagating a short distance \( \tilde{\ell} \gtrsim \tilde{a} \), characterized by their large masses \( \Lambda \propto \tilde{\ell}^{-1} \), this implies the four-fermion coupling \( G \propto \Lambda^{-2} \).
In this article, we only discuss the relevance of dimension-6 four-fermion operators (2), which can be written as

\[ + \left( \frac{G}{2} \right) (J_L^\mu J_{L,\mu} + J_R^\mu J_{R,\mu} + j_L^\mu j_{L,\mu} + 2J_L^\mu j_{L,\mu}) \] (3)

\[ - G \left( \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L + \bar{\nu}^c_R \psi_R \bar{\nu}_R \psi_R \right), \] (4)

by using the Fierz theorem [19]. Equations (3) and (4) represent repulsive and attractive operators respectively. It will be pointed out below that four-fermion operators (3) cannot be relevant and renormalizable operators of effective dimension-4 in both domains of IR and UV-stable fixed points. We will consider only four-fermion operators (4) preserving the SM gauge symmetry without the flavor-mixing of three fermion families.

**SM gauge symmetric four-fermion operators.** In the quark sector, the four-fermion operators [11]

\[ G \left[ (\bar{\psi}_L^{i a} t_{R a})(\bar{t}^a_R \psi_L^{i b}) + (\bar{\psi}_L^{i a} b_{R a})(\bar{b}^a_R \psi_L^{i b}) \right] + \text{“terms”}, \] (5)

where \(a, b\) and \(i, j\) are the color and flavor indexes of the top and bottom quarks, the quark \(SU_L(2)\) doublet \(\psi_L^{i a} = (t^a_L, b^a_L)\) and singlet \(\psi_R^{a i} = t^a_R, b^a_R\) are the eigenstates of electroweak interaction. The first and second terms in Eq. (5) are respectively the four-fermion operators of top-quark channel and bottom-quark channel, whereas “terms” stands for the first and second quark families that can be obtained by substituting \(t \rightarrow u, c\) and \(b \rightarrow d, s\).

In the lepton sector, we introduce three right-handed sterile neutrinos \(\nu_R^\ell (\ell = e, \mu, \tau)\) that do not carry any SM quantum number. Analogously to Eq. (5), the four-fermion operators in terms of gauge eigenstates are,

\[ G \left[ (\bar{\nu}_L^{i c} \ell_R^c)(\bar{\ell}_R^c \nu_L^{i i}) + (\bar{\nu}_L^{i c} u_{a,R})(\bar{u}_{a,R}^c \nu_L^{i i}) + (\bar{\nu}_L^{i c} d_{a,R})(\bar{d}_{a,R}^c \nu_L^{i i}) \right], \] (6)

preserving all SM gauge symmetries, where the lepton \(SU_L(2)\) doublets \(\ell_R^c = (\nu_R^\ell, \ell_L^c)\), singlets \(\ell_R^c\) and the conjugate fields of sterile neutrinos \(\nu_R^\ell = i\gamma_2(\nu_R^\ell)^*\). Coming from the second term in Eq. (4), the last term in Eq. (6) preserves the symmetry \(U_{\text{lepton}}(1)\) for the lepton-number conservation, although \((\bar{\nu}_R^c \nu_R^\ell)^c\) violates the lepton number of family “\(\ell\)” by two units. Similarly, there are following four-fermion operators

\[ G \left[ (\bar{\nu}_R^c \ell_R^c)(\bar{\ell}_R^c \nu_R^\ell) + (\bar{\nu}_R^c u_{a,R})(\bar{u}_{a,R}^c \nu_R^\ell) + (\bar{\nu}_R^c d_{a,R})(\bar{d}_{a,R}^c \nu_R^\ell) \right], \] (7)

where quark fields \(u_{a,R}^\ell = (u, c, t)_{a,R}\) and \(d_{a,R}^\ell = (d, s, b)_{a,R}\).
In addition, there are SM gauge-symmetric four-fermion operators that contain quark-lepton interactions [20],

\[ G \left[ (\ell_L^e e_R)(\bar{d}_R^a \psi_{Lia}) + (\ell_L^\nu_R \nu_L^e)(\bar{u}_R^a \psi_{Lia}) \right] + \text{“terms”}, \tag{8} \]

where \( \ell_L = (\nu_e L, e_L) \) and \( \psi_{Lia} = (u_{La}, d_{La}) \) for the first family. The “terms” represent for the second and third families with substitutions: \( e \rightarrow \mu, \tau \), \( \nu_e \rightarrow \nu_\mu, \nu_\tau \), and \( u \rightarrow c, t \) and \( d \rightarrow s, b \). Here we do not consider baryon-number violating operators. It would be interesting to study four-fermion operators in the framework of the \( SU(5) \) or \( SO(10) \) unification theory.

**Relevant and irrelevant four-fermion operators in the IR-domain.** Based on the approach of large \( N_c \)-expansion with a fixed value \( GN_c \), it is shown that in the domain (IR-domain) of IR-stable fixed point \( G_c N_c \), where \( N_c \) is the color number, the top-quark channel of operators [5] undergoes the NJL-dynamics of spontaneous symmetry breaking [4]. As a result, the \( \Lambda^2 \)-divergence (tadpole-diagram) is removed by the mass gap-equation, the top-quark channel of operators [5] becomes physically relevant and renormalizable operators of effective dimension-4. Namely, the effective SM Lagrangian with bilinear top-quark mass term and Yukawa-coupling to the composite Higgs boson \( H \) at low-energy scale \( \mu \) obey the RG equation approaching to the low-energy SM physics characterized by the energy scale \( v \approx 239.5 \text{ GeV} \) [7].

\[
L_{\text{eff}} = L_{\text{kinetic}} + L_{\text{gauge}} + m_t \bar{t}t + \bar{g}_t \bar{t}tH + \bar{Z}_H |D_\mu H|^2 - m_H^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2. \tag{9}
\]

The Dirac part of lepton-sector (6) and (7) does not undergo the NJL-dynamics of spontaneous symmetry breaking because its effective four-fermion coupling \((G_c N_c)/N_c\) is smaller than the critical value \((G_c N_c)\) [11]. Therefore, except the top-quark channel [2], all Dirac fermions are massless and four-fermion operators [1] are irrelevant dimension-6 operators, whose tree-level amplitudes of four-fermion scatterings are suppressed \( O(\Lambda^{-2}) \) in the IR-domain. The hierarchy of fermion masses and Yukawa-couplings is actually due to the explicit symmetry breaking and flavor-mixing, which were preliminarily studied [20, 21] by using the Schwinger-Dyson equation for Dirac fermion self-energy functions and we will present some detailed discussions on this issue in the coming paper [22].

On the other hand, in the domain (UV-domain) of UV-stable fixed point \( G_{\text{crit}} > G_c N_c \), the phase transition takes place from the symmetry-breaking phase to the symmetric phase, the four-fermion operators [4] undergo the dynamics of forming composite fermions, e.g. \([\bar{\psi}_L (\bar{\psi}_R \psi_L) \psi_R]\), preserving all SM gauge symmetries, and the characteristic energy scale is \( E \gtrsim 5 \text{ TeV} \) [13]. In
the UV-domain, four-fermion operators (4) are expected to acquire anomalous dimensions and thus become relevant operators of effective dimension-4, in the sense of being renormalizable and obeying RG equations.

However, in the both IR- and UV-domains, four-fermion operators (3) are irrelevant operators of dimension-6, and thus suppressed $O(\Lambda^{-2})$ for the following reasons. In the IR-domain for low-energy SM physics, the four-fermion operators (3) are not associated with the NJL-dynamics of spontaneous symmetry breaking. In the UV-domain, four-fermion operators (3) are not associated with the dynamics of forming composite fermions, due to the absence of strong coupling limit that is a necessary condition to form three-fermion bound states [14]. As a consequence, the tree-level amplitudes of four-fermion scatterings represented by these irrelevant operators (3) are suppressed $O(\Lambda^{-2})$.

In fact, using dilepton events the ALTAS collaboration [23] has carried out a searched for nonresonant new phenomena originating from contact interactions [18] (the left-left isoscalar model $J^\mu_LJ_{L,\mu}$, which is commonly used as a benchmark for this type of contact interaction searches [24]), and shown no significant deviations from the SM expectation up to $\Lambda = \pi/\tilde{a} > 10$ TeV.

Moreover, it should be mentioned that in the IR domain, even taking into account the loop-level corrections to the tree-level amplitudes of four-fermion scatterings, the four-point vertex functions of irrelevant four-fermion operators in Eqs. (3) and (4) are also suppressed by the cutoff scale $\Lambda$, thus their deviations from the SM are experimentally accessible. However, we recall that in both the IR- and UV-domains, four-fermion operators in Eqs. (3) and (4) have loop-level contributions, via rainbow diagrams of two fermion loops, to the wave-function renormalization (two-point function) of fermion fields [14] and these loop-level contributions to the $\beta(G)$-function are negative [13].

**Nonresonant new phenomena of four-fermion operators.** In the IR-domain with the electroweak breaking scale $v = 239.5$ GeV, the dynamical symmetry breaking of four-fermion operator $G(\bar{\psi}^a_{L,R}t_Ra)(\bar{\psi}^b_{R,L}b)$ of the top-quark channel (5) accounts for the masses of top quark, $W$ and $Z$ bosons as well as a Higgs boson composed by a top-quark pair ($tt$) [7]. It is shown [12, 13] that this mechanism consistently gives rise to the top-quark and Higgs masses of the SM (9) at low energies, provided the appropriate value of non-vanishing form-factor of composite Higgs boson at the high-energy scale $E \gtrsim 5$ TeV.

The finite form-factor of composite Higgs boson is actually the wave-function renormalization of composite Higgs-boson field, which behaves as an elementary particle after performing the wave-function renormalization. However, the non-vanishing form-factor of composite Higgs boson is in fact related to the effective Yukawa-coupling of Higgs boson and top quark, i.e., $\tilde{Z}_H^{-1/2}(\mu) = \tilde{g}_t(\mu)$
of Eq. (9). The effective Yukawa coupling $\bar{g}_t(\mu)$ and quartic coupling $\bar{\lambda}(\mu)$ monotonically decrease with the energy scale $\mu$ increasing in the range $m_\mu < \mu < E \approx 5$ TeV (see Fig. 1). This means that the composite Higgs boson becomes more tightly bound as the energy scale $\mu$ increases.

This should have some effects on the rates or cross-sections of the following three dominate processes of Higgs-boson production and decay [3, 4] or other relevant processes. Two-gluon fusion produces a Higgs boson via a top-quark loop, which is proportional to the effective Yukawa coupling $\bar{g}_t(\mu)$. Then, the produced Higgs boson decays to the two-photon state by coupling to a top-quark loop and to the four-lepton state by coupling to two massive $W$-bosons or two massive $Z$-bosons. Due to the $\bar{t}t$-composite nature of Higgs boson, the one-particle-irreducible (1PI) vertexes of Higgs-boson coupling to a top-quark loop, two massive $W$-bosons or two massive $Z$-bosons are proportional to the effective Yukawa coupling $\bar{g}_t(\mu)$. As a result, both the Higgs-boson decaying rate to each of these three channels and total decay rate are proportional to $\bar{g}_t^2(\mu)$, which does not affect on the branching ratio of each Higgs-decay channel. The energy scale $\mu$ is actually the Higgs-boson energy, representing the total energy of final states, e.g., two-photon state and four-lepton states, to which the produced Higgs boson decays.

These discussions imply that the resonant amplitude (number of events) of two-photon invariant mass $m_{\gamma\gamma} \approx 126$ GeV and/or four-lepton invariant mass $m_{4l} \approx 126$ GeV is expected to become smaller as the produced Higgs-boson energy $\mu$ increases, i.e., the energy of final two-photon and/or four-lepton states increases, when the CM energy $\sqrt{s}$ of LHC $pp$ collisions increases with a given luminosity. Suppose that the total decay rate or each channel decay rate of the SM Higgs boson is measured at the Higgs-boson energy $\mu = m_t$ and the SM value of Yukawa coupling $\bar{g}_t^2(m_t) = 2m_t^2/v \approx 1.04$ (see Fig. 1). In this scenario of composite Higgs boson, as the Higgs-boson energy $\mu$ increases to $\mu = 2m_t$, the Yukawa coupling $\bar{g}_t^2(2m_t) \approx 0.98$ (see Fig. 1), the variation of total decay rate or each channel decay rate is expected to be 6% for $\Delta \bar{g}_t^2 \approx 0.06$. Analogously, the variation is expected to be 9% or 11%, at $\mu = 3m_t$, $\bar{g}_t^2(3m_t) \approx 0.95$ or $\mu = 4m_t$, $\bar{g}_t^2(4m_t) \approx 0.93$ (see Fig. 1). This variation may be still too small to be clearly distinguished by the present LHC experiments. Nevertheless, these effects are the nonresonant new signatures of low-energy collider that show the deviations of this scenario from the SM. It will be shown [22] that the induced (1PI) Yukawa couplings $\bar{g}_b(\mu)$, $\bar{g}_\tau(\mu)$ and $\bar{g}_f(\mu)$ of composite Higgs boson to the bottom-quark, tau-lepton and other fermions also weakly decrease with increasing Higgs-boson energy, this implies a slight decrease of number of dilepton events in the Drell-Yan process.

However, the nonresonant new phenomena stemming from four-fermion scattering amplitudes of irrelevant operators of dimension-6 in Eqs. (3) and (4) are suppressed $\mathcal{O}(\Lambda^{-2})$ and hard to be
FIG. 1: Using all experimentally measured SM quantities (including $m_t$ and $m_\mu$) at low energies, we numerically solve the RG equations [13] to uniquely determine the Yukawa coupling $g_t(\mu)$ of composite Higgs boson and top quark and the quartic coupling $\lambda(\mu)$ of composite Higgs field in terms of the Higgs-boson energy scale $\mu > M_z$ up to $E \approx 5$ TeV at which $\lambda(E) = 0$.

identified in high-energy processes of LHC $pp$ collisions (e.g., the Drell-Yan dilepton process, see Ref. [23]), $e^-e^+$ annihilation to hadrons and deep inelastic lepton-hadron $e^- p$ scatterings at TeV scales.

**Composite particles in the UV-domain.** We turn to discuss the composite particle spectra and interacting vertexes in the UV-domain with the energy scale $\mathcal{E} \gtrsim 5$ TeV. As the energy scale $\mu$ and four-fermion coupling $G(\mu)$ increase, composite Dirac fermions are formed at high energies $\mu > \mathcal{E}$ and strong coupling $G > G_{\text{crit}}$. For the top-quark channel, the composite Higgs boson combines with another top quark to form the massive composite Dirac fermions: $SU_L(2)$ doublet $\Psi_D^b = (\psi^b_L, \Psi^b_R)$ and singlet $\Psi^b = (\Psi^b_L, t^b_R)$, where the renormalized composite three-fermion states are:

$$\Psi^b_R = (Z^S_{R})^{-1} (\psi^a_L t^a_R) t^b_R; \quad \Psi^b_L = (Z^S_{L})^{-1} (\psi^a_L t^a_R) \psi^b_L,$$

and the composite bosons ($SU_L(2)$-doublet) are $H^i = [Z^S_{H}]^{-1/2} (\psi^a_L t^a_R)$, where the form-factor $Z^S_{R,L}$ and $[Z^S_{H}]^{-1/2}$ are generalized wave-function renormalization of composite fermion and boson operators. For the bottom-quark channel, the composite particles are represented by Eq. (10) with the replacement $t^a_R \rightarrow b^a_R$, carrying the different quantum numbers of the $U_Y(1)$ gauge group.

The same discussions also apply for the first and second quark families by substituting the $SU_L(2)$ doublet $(t^a_L, b^a_L)$ into $(u^a_L, d^a_L)$ or $(c^a_L, s^a_L)$ and singlet $t^a_R$ into $u^a_R$ or $c^a_R$, as well as singlet $b^a_R$ into $d^a_R$ or $s^a_R$. Some detailed discussions can be found in Ref. [13].

Analogously, we present in this letter for the $e_R$-channel of quark-lepton interactions [8], the massive composite Dirac fermions: $SU_L(2)$ doublet $\Psi_D^\ell = (\ell^i_L, \Psi^\ell_R)$ and singlet $\Psi^\ell = (\Psi^\ell_L, e^\ell_R)$,
where the renormalized composite three-fermion states are:

$$\Psi_R^i = (Z_R^S)^{-1}(\bar{d}_R^a \psi_L^i)e_R; \quad \Psi_L = (Z_L^S)^{-1}(\bar{\psi}_L^i d_R^a)\ell_i L,$$

and the composite bosons ($SU_L(2)$ doublet) are $H^i = [Z_H^S]^{-1/2}(\bar{\psi}_L^i u_R^a)$. For the $\nu_R$-channel, the composite particles are represented by Eq. (11) with the replacements $e_R \rightarrow \nu_R$, $d_R \rightarrow u_R$,

$$\Psi_R^i = (Z_R^S)^{-1}(\bar{u}_R^a \psi_L^i)\nu_R; \quad \Psi_L = (Z_L^S)^{-1}(\bar{\psi}_L^i u_R^a)\ell_i L,$$

and the composite bosons ($SU_L(2)$ doublet) are $H^i = [Z_H^S]^{-1/2}(\bar{\psi}_L^i u_R^a)$, carrying different quantum numbers of the $U_Y(1)$ gauge group. The composite particles from the second and third families can be obtained by substitutions: $e \rightarrow \mu, \tau, \nu^e \rightarrow \nu^\mu, \nu^\tau$, and $u \rightarrow c, t$ and $d \rightarrow s, b$.

In addition, the composite three-fermion states formed by the first term ($\ell_R$-channel) of Eq. (6) are:

$$\Psi_R^i = Z_R^S(\bar{\ell}_R^L \ell_R)\ell_i L; \quad \Psi_L = Z_L^S(\bar{\ell}_L^L \ell_R)\ell_i L,$$

the composite Dirac fermions are $SU_L(2)$ doublet $\Psi_D^i = (\ell_L^i, \Psi_R^i)$ and singlet $\Psi_D = (\Psi_L, \ell_R)$, as well as the composite bosons ($SU_L(2)$ doublet) $H^i = [Z_H^S]^{1/2}(\bar{\ell}_L^L \ell_R)$. The composite particles formed by the second term ($\nu_R$-channel) of Eq. (6) are obtained by the replacement $\ell_R \rightarrow \nu_R$, and they carry no charge of the $U_Y(1)$ gauge group. These composite particles have the same mass and form factor as their counterparts of Eq. (10) in the quark sector, but different quantum numbers of the SM gauge symmetries.

In the UV-domain, all four-fermion operators (4) are expected to become relevant operators of effective dimension-4 by the formation of composite particles. The propagators of these composite particles have poles and residues that respectively represent their masses and form-factors. As long as their form-factors are finite, these composite particles behave as elementary particles with the mass-shell conditions

$$E_{\text{com}} = \sqrt{p^2 + \mathcal{M}^2} \approx \mathcal{M}, \quad \text{for} \quad p \ll \mathcal{M} \approx \mathcal{E}_\xi$$

where the characteristic energy scale $\mathcal{E}_\xi$ sets in the UV-domain

$$\mathcal{E}_{\text{thre}} \lesssim \mathcal{E}_\xi \ll \Lambda, \quad \mathcal{E}_{\text{thre}} \gtrsim 5 \text{ TeV}.$$  

When the energy scale $\mu$ decreases to the energy threshold $\mathcal{E}_{\text{thre}}$ and the four-fermion coupling $G(\mu)$ decreases to the critical value $G_{\text{crit}}(\mathcal{E}_{\text{thre}})$, all three-fermion and two-fermion bound states decay to their constituents of elementary particles of the SM. Needless to say, these composite particles
should play important roles in the early universe and phase transition to the epoch of electroweak symmetry breaking.

Compared with the SM in the IR-domain of the symmetry breaking phase, the spectrum of massive composite particles in the UV-domain of the symmetric phase has the same quantum numbers of SM chiral gauge symmetries, coupling to gauge bosons $\gamma, W^\pm, Z^0$ and gluon, but the spectrum and interacting vertex are vector-like, fully preserving the parity-symmetry. In the UV-domain of the symmetry phase, there is only the spectrum of composite fermions, as example the top quark channel (10). SM elementary fermions and Higgs boson become the constitutes of composite fermions. It would be helpful to explain this scenario by analogy with the QCD of elementary quarks in high-energies and only composite hadrons in low energies. The $\beta_{\text{QCD}}$-function has an opposite sign to the $\beta$-function of this scenario [13]. Similarly to that the composite Higgs behaves as an elementary particle in the IR-domain of the SM, these composite fermions behave as elementary particles, provided their wave-function renormalizations are finite above the threshold energy (15). Otherwise, composite particles dissolve (decay) to their constitutes of SM elementary particles. In Ref. [13], we discussed the resonant new phenomena (multi-jets events) of four-fermion operators (5) and (10) with peculiar kinematic distributions in final states, that are expected to be observed in LHC $pp$ collisions.

**Resonant new phenomena in high-energy processes.** In this section, we discuss that the resonant new phenomena of these composite particles produced in high-energy processes: (i) the Drell-Yan process in LHC $pp$ collisions, (ii) $e^-e^+$ annihilation to hadrons and (iii) deep inelastic lepton-hadron $e^-p$ scatterings.

(i) The Drell-Yan process: in addition to multi-jets produced in LHC $pp$ collisions [13], the pair of lepton and anti-lepton are produced by the annihilation of quark and anti-quark via SM gauge bosons. The final states of lepton and anti-lepton (dilepton) are measured in terms of their invariant mass $m_{\ell\ell}$. In Ref. [23], the analysis up to $m_{\ell\ell} \approx 1$ TeV has been made, showing no deviation from the SM attributed to the four-fermion operators (contact interactions) $J^\mu_L J_{L,\mu}$ in Eq. (3), which are in fact irrelevant and suppressed $O(\Lambda^{-2})$ as previously discussed.

Instead, the relevant four-fermion operators (5) form massive composite fermions of Eqs. (11) and (14), i.e., the $e$-channel $[\bar{e}_L, (\bar{d}_R^a d_{La}) e_R]$ and/or the $\mu$-channel $[\mu_L, (\bar{s}_R^a s_{La}) \mu_R]$, which then decay to lepton and anti-lepton as final states. This implies the resonant new phenomena to appear in the dilepton invariant mass $m_{\ell\ell} \approx \mathcal{M} \gtrsim 5$ TeV. Due to their origin from a very massive composite particle “at rest”, the kinematic distribution of dilepton final states is expected to be two leptons moving apart in opposite directions, each carrying energy-momentum about $\mathcal{M}/4$. In the channels
involving neutrino or sterile neutrinos, the four-fermion operators (S) form the composite fermions of Eqs. (11,12) and (13), which then decay to final states containing neutrinos $\nu_L$ and/or sterile neutrinos $\nu_R$. These neutrinos carry away some missing transverse energy momenta, which is an important signal (or trigger) for a number of new phenomena.

(ii) Deep inelastic lepton-hadron scatterings (DIS): the deep inelastic electron $e(k)$ and hadron $N(p)$ scattering $e(k) + N(p) \rightarrow e(k') + X(p_n)$ occurs via exchange of neutral gauge bosons in the SM, where the energy transfer $q = k - k'$ and $X(p_n)$ represents some hadronic final states with total four-momentum $p_n$. If the energy transfer $q > M > \sim 5$ TeV, the composite Dirac fermion $[e_L, (\bar{d}_R^a d_{La}) e_R]$ formed by the relevant four-fermion operator (8). This implies the appearance of a resonant cross-section $\sigma_{ep}(q)$ whose peak locates at $q \approx M$ the mass of the composite Dirac fermion formed intermediately. In addition, the kinematic distribution of final states $e(k')$ and $X(p_n)$ is expected to be deviated from the SM result. In the channels involving neutrinos $\nu_L$ and $\nu_R$ of Eq. (S), the composite Dirac fermions $[e_L, (\bar{u}_R^a u_{La}) \nu_R^c]$ and $[\nu_L^c, (\bar{d}_R^a d_{La}) e_R]$ form and decay in the electron $e(k)$ and hadron $N(p)$ scattering process, the kinematic distribution of final states $e(k')$ and $X(p_n)$ is expected to be affected by the missing energy-momenta carried away by neutrinos.

In addition, the polarized electron-deuteron deep inelastic (DIS) experiment [24, 25] measured the right-left asymmetry,

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

(16)

where $\sigma_{R,L}(q)$ is the cross-section for the deep-inelastic scattering of a right- or left-handed electron $e_{R,L} + N \rightarrow e + X$. For low energy-momentum transfer $q^2 \ll \mathcal{E}$, $A \neq 0$ for the violation of parity symmetry of the SM. As high energy-momentum transfer $q$ approaches $\mathcal{E}$ and $q > \mathcal{E}$, $A \rightarrow 0$ for the restoration of parity symmetry [12].

(iii) $e^+ e^-$ collisions annihilating to hadrons: the process and cross-section of electron-positron annihilation to hadrons have been important to study the SM. As the CM energy $\sqrt{s}$ (invariant mass) of electron-positron collider increases, the bound states of quarkonia/mesons have been discovered as narrow resonances in the cross-section, the resonance of $Z$-production has been also observed for the CM energy $\sqrt{s} > \sim 100$ GeV [24]. The narrow resonance for (tt) quarkium will appear in final states if $\sqrt{s} > 346$ GeV. On the basis of relevant four-fermion operators (6) and (S), we expect the resonances of composite Dirac particles, e.g., $[\bar{e}_L^c (\bar{e}_L e_R^c) e_R]$, $[\bar{\nu}_R^c, (\bar{\nu}_R^c e_L^c e_R^c)]$, $[\bar{e}_L^c (\bar{d}_R^a d_{La}^a) e_R]$, $\cdots$, to appear in the cross-section of $e^+ e^-$ annihilation, if the CM energy of the electron-positron collider reaches the energy threshold $\mathcal{E}_{\text{thre}}$, i.e., $\sqrt{s} > \mathcal{E}_{\text{thre}} > 5$ TeV. However, this energy scale does not seems to be reached by both $e^+ e^-$ colliders and DIS.
experiments in near future.

In addition, two high-energy photons from the LHC pp collision can produce two electron-positron pairs fusing into a composite Dirac particle $\ell_i$ with $\ell_i^L = e_L$ and $\ell_i^R = e_R$, which can be identified by observing the resonance in the invariant mass $M_{e^+e^-}$ of final states of two electron-positron pairs. In the CM frame, electron and positron of each pair move apart in the opposite direction and the energy-momentum of each particle is about one-fourth of the invariant mass. The cross-section for these channels can be estimated to be $4\pi\alpha^2/M^2$. The massive composite particle $(13,14)$ that comprises electron $\ell_i^L = e_L$ and sterile neutrino $\ell_i^R = \nu_R$ is expected to be identified by observing the resonance with final states of electron and positron oppositely moving apart with energy being one-half of the invariant mass $M$, and the rest carried away by sterile neutrino and anti-neutrino oppositely moving apart. Compared with the multi-jets resonant phenomena due to four-quark operators $(10)$ in LHC pp collisions $[13]$, the probabilities of the dilepton resonant phenomena due to four-lepton operators $(13)$ in pp collisions are smaller, because of the factor $\alpha^2$.

In general, what can be said are following. If the accessible CM energy $\sqrt{s} > M$, the cross section for the allowed inelastic processes forming massive composite Dirac particles will be geometrical in magnitude, of order $\sigma_{\text{com}} \sim 4\pi/M^2$ in the CM frame where massive composite Dirac particles are approximately at rest. Decays of these massive composite particles to their constituents leading to unconventional events of multi-jets, jets-dilepton and multi-leptons states with peculiar kinematic distributions in the CM frame. As a result, these unconventional events will qualitatively depart from the SM, and completely dominate over the SM processes, for which cross sections go roughly as $\pi\alpha_{\text{gauge}}^2/(\sqrt{s})^2$. Thus the SM background is expected to be more or less zero. On the other hand, if the accessible CM energy $\sqrt{s} < M$, then departures from the SM will be quantitative rather than qualitative, as described in the previous section.

In currently scheduled LHC (pp-collision) runs for next 20 years, the integrated luminosity will go from 10 fb$^{-1}$ up to 10$^3$ fb$^{-1}$ and the CM energy $\sqrt{s}$ from 7 TeV up to 14 TeV, then the number of events of composite Dirac particles created in quark-quark or quark-lepton (Drell-Yan) processes can be estimated by $\sigma_{\text{com}} \times 10^{1-3}\text{fb}^{-1} \sim 4\pi \times 10^{5-7}$ for the $(u,d)$ family, $\sim 4\pi \times 10^{3-5}$ for the $(c,s)$ and $(t,b)$ families, assuming $M \sim 5$ TeV.

**Neutrino sector.** In the UV-domain, the four-neutrino operator ($\nu_R^{\ell c}$-channel) of the last term of Eq. (6) forms composite three-fermion states (self-conjugate Majorana states)

$$\Psi_R^\ell = Z_R^S(\nu_R^{\ell c}\nu_R^{\ell'})\nu_R^\ell; \quad \Psi_R^{\ell c} = Z_L^S(\bar{\nu}_R^{\ell'}\nu_R^\ell)\nu_R^{\ell c}, \quad (17)$$

and composite Majorana fermions $\Psi_M^\ell = (\nu_M^{\ell c}, \Psi_R^\ell)$, as well as the composite bosons $H_M = \ldots$
These composite particles comprising only neutrinos are hard to be produced in ground laboratories, except in the early universe.

However, in the IR-domain for the SM, the last term of Eq. (15) can generate a mass term of Majorana type, because the family number $N_i = 3$ ($i = 1, 2, 3$ that are different from the $SU_L(2)$ index) plays the same role as the color number $N_c$ of the top-quark in the $\langle \bar{t}t \rangle$-condensate. In the same way similar to the gauge and mass eigenstates of neutrinos $\nu_L^\ell = U^\ell_i \nu_i^L$, we use the $3 \times 3$ unitary PMNS matrix $U$ for neutrino mixings to define the flavor eigenstates of sterile neutrinos by

$$\nu^\ell_R = U^\ell_i \nu^i_R,$$  \hspace{1cm} (18)

Analogously to the generation of top-quark mass $m_t$, the dynamical symmetry breaking of the $U_{\text{lepton}}(1)$-symmetry generates the Majorana mass of right-handed neutrinos

$$m^M = -G \sum_i (\bar{\nu}^\ell_R \nu^i_R),$$  \hspace{1cm} (19)

together with a sterile massless Goldstone boson, i.e. the bound state $(\bar{\nu}^\ell_R \gamma_5 \nu^i_R)$, and a sterile massive scalar particle, i.e. the bound state $(\bar{\nu}^\ell_R \nu^i_R)$, carrying two units of the lepton number of the family “$i$”. Note that the family index “$i$” is summed over as the color index “$a$” in the $\langle \bar{t}t \rangle$-condensate. In the IR-domain, the sterile neutrino mass $m^M$ and sterile scalar particle mass $m_H^M$ satisfy the mass-shell conditions,

$$m^M = \tilde{g}_t(m^M) v_{\text{sterile}}/\sqrt{2}, \hspace{1cm} (m_H^M)^2/2 = \tilde{\lambda}(m_H^M) v_{\text{sterile}}^2,$$  \hspace{1cm} (20)

where $\tilde{g}_t(\mu^2)$ and $\tilde{\lambda}(\mu^2)$ obey the same RG equations (absence of gauge interactions) and boundary conditions of Eqs. (7,8,9) and (10) in Ref. \cite{13}. However, unlike the electroweak scale $v$ determined by the gauge-boson masses $M_W$ and $M_Z$ experimentally measured, the scale $v_{\text{sterile}}$ is unknown and needs to be determined by the sterile neutrino mass $m^M$. The ratio is approximately estimated

$$\frac{m^M}{m_H^M} = \frac{\tilde{g}_t(m^M)}{2(\tilde{\lambda}(m_H^M))^{1/2}} \approx \frac{m_t}{m_H} = \frac{\tilde{g}_t(m_t)}{2(\tilde{\lambda}(m_H))^{1/2}} = 1.37.$$  \hspace{1cm} (21)

All sterile particles and gauge-singlet (neutral) states of massive composite Dirac particles (TeV-scales) discussed here can be possible candidates of warm and cold dark matter \cite{26}, and they can decay into SM particles via relevant four-fermion operators in Eqs. (5-8).

As a result, in the IR-domain, we write the following bilinear Dirac and Majorana mass terms of neutrinos in terms of their mass eigenstates $\nu_L^\ell$ and $\nu_R^i$,

$$m^D_i \bar{\nu}_L^i \nu_R^i + m^M_i \bar{\nu}_R^i \nu_R^i + \text{h.c.}$$  \hspace{1cm} (22)
We expect the Majorana masses to be approximately equal (degenerate), \( m^M_i \approx m^M \), for three right-handed neutrino \( \nu^i_R \), since there is not any preferential \( i \)-th component of the condensate \( \langle \bar{\nu}^j_R \nu^i_R \rangle \). Whereas, due to the origin from the explicit symmetry breaking terms related to family flavor mixings, the Dirac masses \( m^D_i \) have the structure of hierarchy \([20][22]\). Moreover we assume that \( m^M_i \gg m^D_i \).

Following the usual approach \([27]\), diagonalizing the \( 2 \times 2 \) mixing matrix \((22)\) in terms of the neutrino and sterile neutrino mass eigenstate “\( i \)”, we obtains the mixing angle \( 2\theta_i = \tan^{-1}(m^D_i/m^M_i) \) and two mass eigenvalues

\[
M^\pm_i = \frac{1}{2} \left\{ m^M_i \pm \left[ (m^M_i)^2 + (m^D_i)^2 \right]^{1/2} \right\}, \quad i = 1, 2, 3, 
\]

(23)
corresponding to two mass eigenstates: three heavy sterile Majorana neutrinos \( (\nu^1_R + \nu^2_R) \) of approximately degenerate mass-spectra \( M^+_i \approx m^M_i \approx m^M \), and three light gauge Majorana neutrinos \( (\nu^1_L + \nu^2_L) \) of hierarchic mass-spectra \( M^-_i \approx (m^D_i)^2/4m^M_i \). The mixing angle \( 2\theta_i \approx (m^D_i/m^M_i) \ll 1 \) and mass-squared difference \( (M^+_i)^2 - (M^-_i)^2 \approx (m^M_i)^2 \), indicating that the mixing of gauge and sterile Majorana neutrinos is very small. Therefore the oscillation between gauge and sterile Majorana neutrinos can be negligible.

We turn to discuss the neutrino flavor oscillations in the usual framework. The mass-squared difference of neutrino mass eigenstates \( (i, j = 1, 2, 3) \),

\[
\Delta M^2_{ij} \equiv (M^+_i)^2 - (M^-_i)^2 \approx \frac{1}{4}(2\Delta m_{ij}^{2M} + \Delta m_{ij}^{2D}), 
\]

(24)
where \( \Delta m_{ij}^{2M,D} \equiv (m^{M,D}_i)^2 - (m^{M,D}_j)^2 \) [see Eq. \((22)\)]. Since \( \Delta m_{ij}^{2D} \gg \Delta m_{ij}^{2M} \), the neutrino mass-squared difference \( \Delta m_{ij}^{2M} \) accounts for neutrino flavor oscillations with \( E_\nu/L \sim \Delta m_{12}^{2M} \approx 5 \times 10^{-3}\text{eV}^2 \) in the long-baseline experiments, where \( E_\nu \) and \( L \) respectively are neutrino energy and travel distance from a source to a detector. Whereas the neutrino mass-squared difference \( \Delta m_{ij}^{2D} \) may accounts for neutrino oscillations \( E_\nu/L \sim \Delta m_{ij}^{2D} \gg 10^{-3}\text{eV}^2 \) in short baseline experiments \([28]\).

**Some remarks.** The multitude of seemingly arbitrary parameters required to specify the SM shows the incompleteness of the SM, which is mainly manifested by our ignorance of (i) the relevant operators and dynamics that underlie the spontaneous/explicit breaking of the SM chiral-gauge symmetries, (ii) the global symmetries and mixings of puzzling replication of quark and lepton families. The relevant four-fermion operators \([4]\) potentially give the theoretical description of the SM in the IR-domain with \( v \approx 239.5 \text{ GeV} \), provided the UV-domain with \( E \gtrsim 5 \text{ GeV} \), where the resonant and nonresonant new phenomena are distinct from the SM. We advocate that it is deserved
to theoretically study the particle spectrum and symmetry of the strong-coupling theory in the domain of UV-stable fixed point by non-perturbative numerical simulations, meanwhile experimentally search for resonant and nonresonant new phenomena of relevant four-fermion operators.

Acknowledgment. Author is grateful to Prof. Zhiqing Zhang for discussions on the LHC physics, and Prof. Hagen Kleinert for discussions on the IR- and UV-stable fixed points of quantum field theories.

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.
[2] F. Englert, R. Brout, Phys. Rev. Lett. 13 (1964) 321;
P. W. Higgs, Phys. Lett. 12 (1964) 132; Phys. Rev. Lett. 13 (1964) 508; Phys. Rev. 145 (1966) 1156;
G. S. Guralnik, C. R. Hagen, T. W. B. Kibble, Phys. Rev. Lett. 13 (1964) 585; and T. W. B. Kibble, Phys. Rev. 155 (1967) 1554.
[3] ATLAS Collaboration, Phys. Lett. B 716 (2012) 1, and http://atlas.ch/.
[4] CMS Collaboration, Phys. Lett. B 716 (2012) 30-61.
[5] C. T. Hill, Phys. Lett. B266 (1991) 491 and *ibid* B345 (1995) 483; Phys. Rev. D87 (2013) 065002.
[6] C. T. Hill, Phys. Rev. D24, 691 (1990); C. T. Hill, C. N. Leung, S. Rao, Nucl. Phys. B262, 517, (1985); J. Bagger, S. Dimopoulos, E. Masso, Phys. Rev. Lett. 55 920 (1985).
[7] W. A. Bardeen, C. T. Hill and M. Lindner, *Phys. Rev.* D41 (1990) 1647.
[8] Y. Nambu, in Proceedings of the 1989 Workshop on Dynamical Symmetry Breaking, edited by T. Muta and K. Yamawaki (Nagoya University, Nagoya, Japan, 1990); V.A. Miranski, M. Tanabashi and K. Yamawaki, *Mod. Phys. Lett.* A4 (1989) 1043; *Phys. Lett.* B221 (1989) 117. See also the SU(3)-extension of their work in Chapter 26 of the textbook [http://klnt.de/b6](http://klnt.de/b6).
[9] W. J. Marciano, Phys. Rev. Lett. 62, (1989) 2793.
[10] G. Cvetic, Rev. Mod. Phys. 71 (1999) 513-574; C. T. Hill, E. H. Simmons, Phys. Rept. 381 (2003) 235-402; *Erratum-ibid.* 390 (2004) 553-554.
[11] S.-S. Xue, Phys. Lett. B721 (2013) 347.
[12] S.-S. Xue, Phys. Lett. B727 (2013) 308.
[13] S.-S. Xue, Phys. Lett. B737 (2014) 172.
[14] S.-S. Xue, Nucl. Phys. B486 (1997) 282, *ibid* B580 (2000) 365, Phys. Rev. D 61 (2000) 054502.
[15] S. Weinberg, in Understanding the Fundamental Constituents of Matter, edited by A. Zichichi (Plenum Press, New York, 1977).
[16] H.B. Nielsen and M. Ninomiya, Nucl. Phys. B185 (1981) 20, *ibid* B193 (1981) 173, *Phys. Lett.* B105 (1981) 219, *Int. J. Mod. Phys.* A6 (1991) 2913.
[17] S.-S. Xue, Phys. Rev. D82, 064039 (2010), Phys. Lett. B682 (2009) 300, *ibid* B665 (2008) 54, B711
[18] E. Eichten, K. D. Lane, and M. E. Peskin, Phys. Rev. Lett. 50, 811 (1983);
     E. Eichten, I. Hinchliffe, K. D. Lane, and C. Quigg, Rev. Mod. Phys. 56, 579 (1984).
[19] C. Itzykson and J-B. Zuber “Quantum Field Theory” page 161, McGraw-Hill, Inc. ISBN 0-07-032071-3.
[20] S.-S. Xue, Modern Physics Letters A, Vol. 14 (1999) 2701.
[21] S.-S. Xue, Phys. Lett. B398 (1997) 177.
[22] S.-S. Xue, “Fermion masses and spontaneous/explicit symmetry-breaking” in preparation.
[23] G. Aad et al. (ATLAS Collaboration), Phys. Rev. D 87, 015010 (2013).
[24] J. Beringer et al. (Particle Data Group Collaboration), Phys. Rev. D 86, 010001 (2012).
[25] C.Y. Prescott et al., Phys. Lett. B84, 524 (1979).
[26] S.-S. Xue, J. Phys. G, Nucl. Part. Phys. 29 (2003) 2381, references therein.
[27] T. P. Cheng and L. F. Li, “Gauge theory of elementary particle physics”, Oxford University Press
     Inc. New York, 1984, ISBN 978-019-851961-4.
[28] A. A. Aguilar-Arevalo et al. [MiniBooNE Collaboration], Phys. Rev. Lett. 110 (2013) 161801.