Remarks on the compacted six-dimensional model of a particle

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ABSTRACT

Non-homogeneous gauge ground state solutions in a six-dimensional gauge model in the presence of non-zero extended fermionic charge density fluctuations are reviewed and fully reinterpreted.

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1 Introduction

The goal of this paper is to present a simple, self-consistent example of trivially coupled fields in a six-dimensional spacetime with compactified internal dimensions (see Section 2). It is devoted mainly to the existence of the nontrivial topological gauge ground field configuration [1, 2] on the internal space (torus [3]). The basic result of this paper (see Sections 4 and 5) is the appearance of the gauge field as a ground state solution (see Section 3) of coupled Dirac and Maxwell wave equations. Because of vanishing current density fluctuations in the internal space the screening current problem [1, 3] in the internal space is avoided.

2 A short introduction to N-circles compactification

Since the discovery of the cancellation of infinities in superstring models, there has been a renewed interest in higher-dimensional gauge theories. To begin, let us study a (4 + N) dimensional Minkowskian spacetime with one temporal dimension and (3 + N) spatial dimensions, equipped with the metric tensor

\[ g = \text{diag}(+1; 1; \cdots; 1). \]

Now, consider a Poincare invariant model in this spacetime - for example, a scalar model given by action

\[ Z = \int d^{4+N} x \left( \frac{1}{2} \bar{\phi} \partial_0 \phi - \frac{1}{2} \partial^2 \phi - \frac{1}{4!} \partial^4 \phi \right). \] (1)

To construct a classical model, we choose \( N \) supplementary space dimensions as circles of circumference \( L_1, \cdots, L_N \) respectively [3]. This assumption is equivalent to periodic boundary conditions for the field:

\[ (\phi_{3+i} + L_i) = (\phi_{3+i}) : \] (2)
Now, make a Fourier decomposition of the eld:

\[
(x; x_{3+1}) = \frac{1}{(L_1 \cdots L_N)^2} \sum_{n_{i,g}} f_{n_{i,g}} \exp \left( \sum_{i=1}^{N} \frac{x_{3+1} n_i}{L_i} \right); \quad (3)
\]

where the coefficients \( f_{n_{i,g}} \) are elds depending on the \( 4 \) spacetime dimensions \( x \) only. They fill the condition of being "real" \( f_{n_{i,g}} = f_{n_{i,g}}^\dagger \).

Integrating Eq. (1) over the additional coordinates \( x_{3+1} \) from 0 to \( L_i \) one easily obtains the action

\[
S = \iiint d^4x \frac{1}{2} \sum_{n_{i,g}} \left( -\frac{1}{2} m_{n_{i,g}}^2 f_{n_{i,g}} f_{n_{i,g}}^\dagger \right) \left( n_{i,g}^1 + n_{i,g}^2 + n_{i,g}^3 + n_{i,g}^4 \right); \quad (4)
\]

where

\[
= \frac{0}{(L_1 \cdots L_N)} \quad (5)
\]

and

\[
m_{n_{i,g}}^2 = \frac{2}{0} + 4 \sum_{i=1}^{N} \frac{n_{i,g}^2}{L_i^2}; \quad (6)
\]

So, the model with compacted internal dimensions has in finite number of states with an increasing ladder of masses. The formula given by Eq. (6) was obtained, in the case of \( N = 2 \) and \( \theta = 0 \), for particles and solitons in supersymmetric gauge theory.

The dimensional reduction is based on \( L_1 \) ! 0 transition (for fixed). Then only one \( \text{eld}, f_{n_{i,g}} = 0g \), has the finite mass. Therefore, we receive the usual model in 4-dimensional spacetime:

\[
S ! S_R = \iiint d^4x \frac{1}{2} \left( -\frac{1}{2} m^2 \right) \left( n^1 + n^2 + n^3 + n^4 \right); \quad (7)
\]

In the above proposal the dimensional reduction erases any trace of the hidden model of the world.
A little bit complicated example is obtained when we start with the Maxwell eld in \((4 + N)\) dimensions:

\[ S = \int \frac{1}{4} d^{4+N} x F \cdot F \]  

(8)

where

\[ F = \partial A \cdot \partial A ; \]

(9)

Leaving aside the subsequent steps in compactification (analogous to Eqs. (6) - (7)), we suppose that \(A ( = 0; 1; \ldots; N)\) eld does not depend on \(x^{3+i}\). Now, the \(A\) eld is decomposed an \(A\) eld \(( = 0; \ldots; 3)\) and \(i\) scalar elds \((i = 1; \ldots; N)\), so \(A = (A ; i; \ldots; N)\) (see Section 3).

As a result of this procedure, the Poincare invariance of the original \((4 + N)\) dimensional theory remains hidden, and only the Poincare group in 4 spacetime dimensions and \(O(N)\) group are effective, with \(O(N)\) singlet-\(A\) and \(O(N)\) vector-\(i\).

3 Introduction to the self-consistent theory of classical elds

In the present paper the language of self-consistent theory of classical elds is used. Hence in order to understand the model in this broader context let us for a moment simplify our considerations taking into account a real scalar eld theory model defined by the following Lagrangian density:

\[ L = \frac{1}{2} \dot{x}^2 (x;t) - \frac{1}{2} r (x;t^2) V ( ) \]  

(10)

where \(-= \partial = \partial t, r = \partial_i \phi = \partial x^i (i\text{ is the versor}.)\)
\( V(\ ) \) is a function of \( \phi \) and the dependence on the coupling constant \( g \) is given by

\[
V(\ ) = \frac{1}{g^2} V(\ ); \quad \phi = g 
\]  
(11)

where \( V \) is an even function independent of \( g \). Depending on the choice of the functional form of \( V \) one can consider various models. One of its realization is illustrated on Figure 1.

The Hamiltonian density derived from the Lagrangian of Eq. (10) is given by

\[
H = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\psi}^2 + V(\ ) \quad (12)
\]

Let \( \phi_0 \) be the scalar timeless field solution of the equation of motion for the field in the ground state of the system given by this Hamiltonian. We use the name of a scalar ground field for the solution \( \phi_0 \).

If we are interested in the Lagrangian density

\[
L = \left( \frac{i}{\hbar} \psi \right) m + J \cdot A - \frac{1}{4} F \cdot F \quad (13)
\]

where \( J = e \) is the electron current density fluctuation and, \( A \) is the total electromagnetic field, four-potential \( A = A^e + A^s \), with the superscripts \( e \) and \( s \) standing for external field and self field (which is adjusted by the radiative reaction to suit the electron current and its fluctuations, see [1]), respectively, then, in the minimum of the corresponding total Hamiltonian, the solution of the equation of motion for \( A^e \) is called electromagnetic ground field. At this point a serious warning has to be given: In the present paper, as in Barut model [1], also, the Dirac-Maxwell coupled equations contain the wavefunction \( \psi(x) \) which does not have the interpretation connected with full charge density distribution,
as in the original linear Schrodinger or Dirac equations, but it is connected with the fluctuations of charge density distribution.

More generally we use the name of a boson ground state for a solution of an equation of motion for a boson field in the ground state of a whole system of fields (fermion, gauge boson, scalar) under consideration. This boson field is the self field (or can be treated like this) when it is coupled to a source-"basic" field. By "basic" field we mean a wave field which is proper for a fermion, a scalar or a heavy boson. This concept of a wave function and the Schrodinger wave equation is dominant in nonrelativistic physics of atoms, molecules and condensed matter. In the relativistic quantum theory this notion had been largely abandoned in favor of the second quantized perturbative Feynman graph approach, although the Dirac wave equation is used approximatively in some problems. What was done by Barut and others was an extension of the Schrodinger's "charge density interpretation" of a wave function to a "fully-edged" relativistic theory. They implemented successfully this "natural interpretation" of a wave function in many specific problems with coupled Dirac and Maxwell equations (for characteristic boundary conditions). But the "natural interpretation" of the wave function could be extended to the Klein-Gordon equation coupled to Einstein field equations, thus being a rival for quantum gravity in its second quantized form. In both cases the second quantization approach is connected with the probabilistic interpretation of quantum theory, whereas the "natural interpretation" together with the self field concept goes in tune with the deterministic interpretation composing a

\footnote{Electron is a classical distribution of charge.}
relativistic, self-consistent field theory.

To summarize: Depending on the model, the role of a self field can be played by a electromagnetic field \([8, 9, 10, 11, 12, 13, 14, 15]\) or by the gravitational field \([1, 2]\) or by the \(\text{boson field} W^+\). The main law for

arising of these self fields would be taking the lead existence of "basic" fields.

4 The model

Let us consider a six-dimensional field theory of a "fermion" particle in a curved spacetime.

\[
L = \frac{1}{4} F_{MN}^a F^{aMN} + i \, ^M F_M ; \tag{14}
\]

where is the "fermion" wave function of an extended particle coupled to its self field with a field strength tensor \(F_{MN}^a\) given by

\[
F_{MN}^a = \Theta_M W_N^a - \Theta_N W_M^a - \text{gf}_{abc} W_M^b W_N^c ; \tag{15}
\]

The covariant derivative \(r_M\) is

\[
r_M = \Theta_M + \frac{1}{2} T_M^{AB} A_B + \text{ig}W_M ; \tag{16}
\]

where

\[
W_M = W_M^a T_a ; \quad [T_a, T_B] = \text{if}_{abc} T_c ; \tag{17}
\]

and

\[
A_B = \frac{1}{4} A^a_B ; \tag{18}
\]

\(^1Q\) uation mark denotes the fact that half spin of the particle is observed from the outside onl; generally speaking inside a particle the Lorentz symmetry is broken.

\(^2C\) onnected with the charge (matter) fluctuation density (see Section 3).
Indices $M;N;:::$ take the values $0,1,2,3$ and $5,6$. $\Gamma^R_{AB}$ is the spin connection.

$\Gamma^R_{AB}$, where $A$ are matrices in the at six-dimensional Minkowski spacetime. Generator $T_a$ is an element of a Lie algebra $g$.

We impose the geometry of the six-dimensional spacetime to be the topological product of the at four-dimensional Minkowski (external) spacetime $(\cdot)$ (with $; = 0;1;2;3$), and the internal space (with metric $g_{eh}; e;h = 5;6$) which forms a generalized torus. Therefore, the metric tensor can be factorized as

$$g_{MN} = \begin{pmatrix} 0 & 0 \\ 0 & g_{eh} \end{pmatrix}$$

with the two-dimensional internal part

$$g_{eh} = \begin{pmatrix} 0 & 0 \\ 0 & r^2 \end{pmatrix}$$

The ordinary torus can be visualized as a two-dimensional surface embedded in the three-dimensional Euclidean space with coordinates given by

$$w^1 = (R + r \cos \#) \cos \quad w^2 = (R + r \cos \#) \sin \quad w^3 = r \sin \# \quad 2 \; [0;2) \; \# \; 2 \; [0;2]$$

with $x^5 = r, x^6 = \#$, which corresponds to the case with $; = 1$. The case $R = 0$ and $2 \; [0;2) \; \# \; 2 \; [0;2]$ corresponds to a sphere. In general, even for the torus case ($; = 1$), the internal manifold is not at (see Appendix). Only for $; = 0$ ($S^1 \times S^1$ torus) the internal space is obtained.

The Lagrange density function of Eq. (14) leads to the Dirac equation

$$i \Gamma^R_{MN} \psi = 0$$

\footnote{The observationally meaningful case with at two-dimensional internal manifold is discussed in [13].}
with \( r_M \) given by Eq. (16) and \( M^\alpha \) and to the Maxwell equation coupled with the Dirac equation
\[
\frac{1}{\sqrt{g}} D_M \rho - g^{\alpha M} N = j^{\alpha N}
\]
with
\[
(D_M \rho)^a = \delta_M^a - g f_{abc} A_M^b \Gamma^c
\]
where \( \delta_M^a \) are explicitly given in Section 7 and \( j^{\alpha N} \) is the fermionic current density fluctuation
\[
j^{\alpha N} = g \chi N T_a
\]

The fundamental result of this Section is that the gauge field appears as a ground state solution of the coupled wave equations (22)–(23). Hence, according to our notation, it is a ground field.

Let us now divide the gauge field \( A_M \) into the ground state component \( a_M (y) (y = f x^5; x^6 g, M \quad m = 5,6) \) in the internal space (so we will not be interested in the excited solutions in the internal space), and \( A_M (x) (x = f x^0; x^1; x^2; x^3 g; M \quad = 0; 1, 2, 3) \) which is the component in the four-dimensional spacetime:
\[
A_M (x; y) = a_m (y) \delta_{0 M} + \delta_{M} (x) ; M = 0; 1; 2; 3.
\]

This paper is devoted to the gauge fields in the internal space. 1 and 2 are devoted to the existence of the gauge fields (ground state solutions) in the four-dimensional spacetime.

Let us suppose that a ground field exists in some Lie algebra direction of the Cartan subalgebra \( g \) only. Then
\[
a_m = a_m^i H_i ; H_i 2 \hbar
\]
\[ a^i_m = a_m n^i; \]  \hspace{1cm} (27)

where \( n^i \) indicates a certain direction in the Cartan subalgebra \( \mathfrak{h} \).

Now, the ground state gauge field \( a^i_m \) is a solution of the Maxwell equations due to the Abelian nature of the Cartan subalgebra. We have

\[ \frac{1}{g_m} \sum_p g_p f^m_n = j^m \]  \hspace{1cm} (28)

with

\[ f^m_n = g^m_n g^{n r} f^r_{pr}; \quad f_{pr} = \theta_{pr} \theta_{ap}; \]  \hspace{1cm} (29)

Let us discuss the simplest model with current density fluctuations \( j^m \) in the internal space equal to zero. Thus, we will avoid the screening current problem (see [1,2]) in the internal space.

There are two types of solutions, i.e. homogeneous and non-homogeneous, with respect to the internal coordinates \( y^n \). The homogeneous solutions

\[ a^C = n; \quad a^C_s = 0 \]  \hspace{1cm} (30)

could not be gauged away due to the non-trivial topological structure of a torus.

There is also a non-homogeneous solution of the Maxwell equation Eq. \( \ref{28} \)

\[ a^C (\#) = a_0 (\# + (r=R) \sin \#); \]  \hspace{1cm} (31)

\[ a^C_s (\#) = 0; \]

This is a generalization of the monopole solution. Indeed, if we change the torus to the sphere \( S^2 \), the monopole solution will be obtained. As we shall see, this ground \( \text{field} \) in the internal space is needed to obtain the chiral zero mode in the four-dimensional spacetime.

\[ \text{This assumption will be represented by the example of Section 6.} \]
5  Six-dimensional Dirac equation

Let us consider the Dirac equation

\[ \gamma^\mu A^\mu \delta_{\alpha} = 0 \]  \hspace{1cm} (32)

with \( r_M \) given by Eq. (14), where \( \delta^{\mu}_{\alpha} \) are defined so that

\[ \delta_{mn} = e^a_m e^b_n \quad \text{and} \quad e^a_m e^b_n = \delta^{ab}_m : \]  \hspace{1cm} (33)

\( A \) are matrices in six-dimensional Minkowski spacetime, defined as follows:

\[ \delta_{mn} = \delta_{1}^{1} \quad \text{and} \quad \delta_{5}^{5} = \delta_{6}^{6} : \]  \hspace{1cm} (34)

The Lorentz SO(2) generator \( 56 \) is

\[ i_3 = 56 = \frac{1}{4} i_5 \quad \text{and} \quad 6 = \frac{1}{2} i_3 : \]  \hspace{1cm} (36)

The Riemannian spin connection on compact internal manifold is \[ 17 \]

\[ \lambda^{ab} (e) = \frac{1}{2} e^c_m e^d_n ( \delta_{mn} + \delta_{km} \delta_{mn} ) \]  \hspace{1cm} (37)

where

\[ \delta_{mn} = e^a_m ( \partial_a e_m + \partial_m e_a ) : \]  \hspace{1cm} (38)

The metric Eqs. (15) - (20) give

\[ \lambda_{112} (e) = \lambda_{121} (e) = \sin \theta : \]  \hspace{1cm} (39)

Connection \( mnr \) is a gravitational analog of field strength tensor \( F_{MN} \) in electromagnetism.

10
A more general case is found when we introduce the torsion. This changes the connection \( !_{mb}(e) \) to

\[
!_{mb} = !_{mb}(e) + K_{mb};
\]

(40)

where \( K_{mb} \) is the contorsion. This will induce only fermion field fluctuation.

The introduction of contorsion \( K_{mb} \) changes the Christoffel connection \( ^k_{mn}(e) \) to an asymmetric quantity

\[
^k_{mn} = ^k_{mn}(e) + K^k_{mn};
\]

(41)

with \( K^k_{mn} = g^{k1}e^a_{m} \epsilon^b_{n} K_{ab} \). This also gives us the torsion

\[
S_{m nk} = K_{m nk} + K_{nm k};
\]

(42)

As we shall see, a non-trivial torsion will be necessary to obtain the chiral zero mode of the fluctuation\(^1\). If we express the \( \gamma^3 \) spinor components as the product

\[
X(x; y) = (x)(y);
\]

(43)

of the bispinor \( (x) \) component and two \( (y) \) spinor components, then the Dirac equation can be expressed as\(^2\)

\[
i \partial + 5 = 0;
\]

(44)

\[
0 \quad r^+
\]

(45)

\[
r \quad 0
\]

with

\[
r_{+} = \frac{1}{\sqrt{r}} e^{i \theta / 2} + \frac{(r-R) \sin \theta}{r (R + R \cos \theta)};\]

(46)

\(^8\) Hence the mass of this fermion particle on the ground state does not change.

\(^9\) From the four-dimensional perspective we are investigating a free Dirac field on the ground state, thus the self-field absence at the right hand side of Eq. (44) is justified and the bispinor \( (x) \) is connected with the "Schrodinger" charge density distribution only.
\[ r = \frac{1}{\rho^2} \frac{(i \theta - \text{ga}_0 \# + (r-R) \sin\# + (K_{112} + \sin\#)=2)}{(R + r \cos\#)} \quad \text{(47)} \]

The chiral zero mode \(( \theta = 0 \) exists only if we link the gauge connection \( a_m \) with the spin connection \( \omega_{112} \) (the Riemannian spin connection \( \omega_{112} \)) and contorsion \( K_{112} \). This will occur if

\[ g \frac{a_0}{R} = \frac{1}{2} \quad \text{(48)} \]

and

\[ \text{ga}_0 \# = \frac{1}{2} K_{112} \quad \text{(49)} \]

Substituting

\[ = e^{im \#} \quad \text{(50)} \]

we obtain

\[ r_+ = \frac{1}{\rho^2} + \frac{m}{(R + r \cos\#)} \quad \text{(51)} \]

\[ r = \frac{1}{\rho^2} \frac{m + \sin\#}{(R + r \cos\#)} \quad \text{(52)} \]

The zero mode \( \psi_0 \) condition

\[ Q \psi_0 = Q \psi_0 = 0 \quad \text{(53)} \]

where

\[ Q = \begin{pmatrix} 0 & r^+ \\ 0 & 0 \end{pmatrix} \quad \text{(54)} \]

\[ Q = \begin{pmatrix} 0 & 0 \\ r & 0 \end{pmatrix} \quad \text{(55)} \]

\[ ^{10} \text{So, we see the importance of the existence of the ground field } a_m \text{.} \]
\[ n_0(\#) = \sum_{0}^{Z} \exp \left( \frac{1}{\#} \frac{m + \sin \#}{(R + \cos \#)} \right) \]  
\hspace{1cm} (56)

For \( m = 0 \), equation Eq. (56) gives

\[ n_0(\#) = 0 \quad (R + \cos \#) \]  
\hspace{1cm} (57)

When we define the internal current density fluctuations as

\[ j^i = \nabla \times n_0; \quad n_0 = n_0^0; \quad i = 5; 6; \]  
\hspace{1cm} (58)

then we see that

\[ j^5 = j^6 = 0 \]  
\hspace{1cm} (59)

which is an important result, and vanishing of the current density fluctuations \( j^i \) in the Maxwell equation (see Eq. (28)) is justified.

Finally, as the six-dimensional chirality is defined by \( i^5 \frac{3}{i} \), it is easy to show that the zero mode of the internal fermionic field fluctuation \( n_0 \) is chiral. This means that

\[ 3^0 = +^0 \]  
\hspace{1cm} (60)

has positive internal chirality.

When \( R > 0 \) (\( 2 \leq 0; 2 \); \( 2 \geq 0; 2 \)), the zero mode \( n_0 \) (see Eq. (56)) is exactly the same as for the sphere, and the contorsion \( K_{112} \) vanishes.

Note: It is crucial to distinguish between field wave mechanics and field fluctuation wave mechanics. Because of this distinction our model has given an answer to the existence of internal fermionic field fluctuation and its chirality, but is not able to describe the chiral modes for the global ("Schrödinger") internal fermionic field.
Conclusions and Perspectives

The goal of this paper was to present a model of a hypothetical particle existing in the six-dimensional spacetime with two compacted internal dimensions and with two coupled fields: the fermionic field fluctuation and its self field. Both of them compose the ground state of the system. Hence the coupled Dirac-Maxwell wave equations had to be solved. As we saw, the ground field $a_m$ and the non-trivial torsion $S_{m,n,k}$ on the internal space were necessary to obtain the chiral zero mode of the fluctuation in four-dimensional spacetime as well as on the internal torus.

The question remains: If one fermionic field fluctuation has global chirality (or other quantum numbers) of its own, are the quantum mechanical rules relevant for the description of the way in which the global chirality of these fluctuations constitute the global chirality (or its possible change) of an entire fermionic particle? If partons are fluctuations inside the proton then this question seems to be of extreme importance.

Although the model presented above touches in some respect the problem of the structure of one extended particle, it does not mean that the model is able to describe its complicated subtleties. Nevertheless the language is worth to be used and developed. It is the language of the self-consistent field theory. Its four-dimensional solution leads in the case of the specific electroweak model [1,2,13] to the particle and astroparticle (gamma ray bursts) applications.

I hope that from the time of Barut and others [5,6,7,8,9,10,11,12,13,14,15] who have developed the idea of the self-consistent treatment of source-field effects via the process of the radiation reaction, a new model of a matter particle...
have begun to appear. This model should be seen as a consequence of the previous (although not self-consistent) Schrödinger's ideas which concern the interpretation of the wave function. This interpretation underwent changes while he [Schrödinger] developed his wave mechanics, and "Instead of publishing just one final result, he revealed the whole process of his search, the picture of his long wandering in the darkness" [13]. What lightened the darkness was an electromagnetic interpretation of the wave function of the electron. But, because Barut coupled the Dirac equation to the Maxwell one hence the "Barut" wave function does not have an interpretation connected with the full charge density distribution, as in the original linear Schrödinger equation, but connected with the charge density distribution fluctuations. This devious situation persists till now and we have the "dualism" having not been able to accumulate the description of Schrödinger's states and Barut's fluctuations of these states in one single picture. A leo quantum mechanics together with QED (and QCD, and GSW model) do not offer such a consistent picture. But have the model been formulated we might have said that there is the model of an elementary matter particle.

Once some body postulated the view of a particle as a definite matter identity

\footnote{It is said that in quantum field theory a matter particle is treated as a set of aggregated quanta. The strict point-likeness of mentioned quanta is supposed to be the corner-stone of the theory.}

\footnote{In the beginning, Schrödinger could not see the connection with Heisenberg's method, but within a short time he established the equivalence between the two (see also approaches. Some suggested that there is also a connection between the self-eld approach and quantum field theory. Because the self-eld picture is in line with the classical point of view, where there are no in finite energy density zero-point fluctuations, and the vacuum-eld is identically equal to zero, this states ent sounds strange. But the 1951 paper of Callen and Welton on the fluctuation dissipation theorem showed that there is an intimate connection between vacuum fluctuations and the processes of radiation reaction. The existence of one implies the existence of the other. Finally, to quote Milonni [14]: "It seems s ... that the generalization of these ideas ... may lead us to view the vacuum-eld more as a fum alibi or subterfuge than a "real" physical thing." (see also [2]).}
(for example, a charge distribution described by a wave function) which has a self-field of its own coupled to it. Just here the self-consistent theory with the "natural interpretation" of a wave function finds its important application: a "basic" field (the cornerstone of a particle) extends everywhere in the space accessible for a wave function determined by its equation of motion and at the same time a self-field coupled to this "basic" field propagates according to its field equations (Maxwell's for the electromagnetic self-field, Einstein (or a better one) (for the gravitational self-field) with the velocity of light. Now, this is the main goal of the one particle physics which has to be achieved.

Hence from the mathematical point of view, a stable elementary particle is a self-consistent time and space dependent solution of field equations involved in its description, which moves in the spacetime in the nonspreading way. Then, what does remain? The solution of the initial-value problem for coupled equation of motion and field equations. Some "simple" stationary, self-consistent, non-perturbative case of the coupled Klein-Gordon-Einstein equations is discussed in [13]. However the inclusion of the Einstein equation to the system of equations procures big analytical complications, so in view of the outward similarity of the Brans-Dicke model to the discussed one, even the perturbative calculations of the kind of the Brans-Dicke scalar-tensor theory could be useful.

Finally, we could do all self-consistent calculations in the six-dimensional space. At this point the model presented before as homogeneous in its space-time structure seems to be an oversimplification of the matter particle nature. We can imagine that our six-dimensional world could be compacted in a non-

\[^{13}\] In my opinion the gravitation has to be obligatory incorporated.
homogeneous manner [13]. In this picture the realm assless scalar "basic" eld ' (dilaton) forms a kind of ground eld (but not the self eld). At this point the model which is discussed in [13, 1] raises a new issue of the analysis. The solutions presented in [13] are parametrized by the parameter $A$ which has similar dynamical consequences as the mass $M = \frac{A c^2}{2G}$ | its existence would be perceived by an observer in the same way as invisible mass which could be the extended "center" of a particle.

7 Appendix: Metric tensor in the model

The metric Eq. (20)

$$g_{\alpha\beta} = \begin{pmatrix} R + r \cos \theta & 0 \\ 0 & \frac{1}{r^2} \end{pmatrix}$$

(61)

gives

$$e_{\alpha}^a = \begin{pmatrix} (R + r \cos \theta) & 0 \\ 0 & \frac{1}{r} \end{pmatrix}$$

(62)

and

$$e_{\beta}^m = \begin{pmatrix} (R + r \cos \theta)^{-1} & 0 \\ 0 & \frac{1}{r^2} \end{pmatrix}$$

(63)

The Christoffel symbols are equal to

$$\Gamma^1_{12} = \frac{r \sin \theta}{R + r \cos \theta}; \quad \Gamma^2_{11} = \frac{R + r \cos \theta}{r} \sin \theta.$$  

(64)

The Riemann tensor $R_{\alpha\beta}$ components are different from zero

$$R_{11} = -\frac{\cos \theta (R + r \cos \theta)}{r}; \quad R_{22} = \frac{r \cos \theta}{R + r \cos \theta}$$

(65)

which makes the curvature scalar $R$ equal to

$$R = \frac{2 \cos \theta}{r(R + r \cos \theta)}.$$  

(66)

$14G$ is the gravitational constant.
Let us notice that only if $R > 0$ we get the flat space. On the other hand, if $R < 0$ and $R > 1$, we obtain $2 = r$, similar as for the sphere.

Acknowledgments

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Figure captions

Figure 1

The potential $V(\phi)$ for a toy model in scalar field theory.
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