THE EFFECT OF BANDPASS UNCERTAINTIES ON COMPONENT SEPARATION

SARAH CHURCH,1 LLOYD KNOX,2 AND MARTIN WHITE3

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ABSTRACT

Multifrequency measurements of the microwave sky can be decomposed into maps of distinct physical components such as the cosmic microwave background (CMB) and the Sunyaev-Zeldovich (SZ) effect. Each of the multifrequency measurements is a convolution of the spectrum on the sky with the bandpass of the instrument. Here we analytically calculate the contamination of the component maps that can result from errors in our knowledge of the bandpass shape. We find, for example, that for Planck an unknown 10% ramp across each band results in a CMB map \( \delta T = \delta T_{\text{CMB}} - 4.3 \times 10^{-5} \delta T_{\text{SZ}} \) plus the usual statistical noise. The variance of this contaminant is more than a factor of 100 below the noise variance at all angular scales and even further below the CMB signal variance. This contamination might lead to an error in the velocity of rich clusters inferred from the kinetic SZ effect; however, the error is negligible, \( O(50 \text{ km s}^{-1}) \), if the bandpass is known to 10%. Bandpass errors might be important for future missions measuring the CMB–SZ effect correlation.

Subject headings: cosmic microwave background — cosmology: theory — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

Small-scale anisotropies in the cosmic microwave background (CMB) can arise from a number of sources. In addition to the "primary" anisotropies, generated at the surface of last scattering, secondary anisotropies and foregrounds can contribute to the observed brightness fluctuations. Separating the components from multifrequency observations is an important part of the data reduction and interpretation. Here we consider the effect of uncertainties in the frequency response of the instrument within the observational bands and how that impacts our ability to perform component separation.

2. MODEL OF THE SKY

We assume for simplicity that the intensity in any given direction of the sky is the sum of five components. The first is the CMB itself with specific intensity

\[
I_x = \frac{dB_x}{dv} \propto \frac{x^4 e^x}{(e^x - 1)^2},
\]

where \( B_x \) is a blackbody spectrum and \( x = \hbar \nu/k_B T_{\text{CMB}} \approx \nu/56.84 \text{ GHz} \) is the dimensionless frequency. The kinetic Sunyaev-Zeldovich (SZ) effect (Sunyaev & Zeldovich 1972, 1980; for recent reviews, see Birkinshaw 1999 and Rephaeli 1995), arising from the motion of ionized gas with respect to the rest frame of the CMB, has the same frequency dependence as the CMB signal. The second component is the thermal SZ (tSZ) effect—one of the primary sources of secondary anisotropies in the CMB on small angular scales. Ignoring relativistic corrections, the change in the (thermodynamic) temperature of the CMB resulting from scattering off nonrelativistic electrons is

\[
\frac{\Delta T}{T} = y \left( x \frac{e^x + 1}{e^x - 1} - 4 \right) = -2y \text{ for } x \ll 1,
\]

where the second expression is valid in the Rayleigh-Jeans limit and \( y \) is the Comptonization parameter, which is proportional to the integrated electron pressure along the line of sight.

We also include dust, bremsstrahlung (or free-free) emission, and synchrotron radiation following Knox (1999; see below). Specifically for the dust we assume the spectral dependence of a modified blackbody with an emissivity index of 2 and a temperature of 18 K. For the bremsstrahlung and synchrotron we assume power laws in frequency with indices \( -0.16 \) and \( -0.8 \), respectively (Bennett et al. 1992). Our model does not include a contribution from extragalactic point sources. For the purposes of this analysis, we hold the spectral indices of the components fixed (and known). Although we do not expect this to be true of real astrophysical foregrounds, this is not the uncertainty we are interested in exploring.

3. METHOD

To understand the effect of an uncertainty in the response of the instrument to a particular signal, let us postulate the following situation. We imagine that the sky is observed at \( N \) frequencies, with measurements \( \theta_i \) \((i = 1, \ldots, N)\). The signal is the sum of \( M \) components with amplitudes \( s_{\alpha} \) \((\alpha = 1, \ldots, M)\) such that

\[
\theta_i = \sum_{\alpha} f_{\alpha} s_{\alpha} + n_i,
\]

where \( n_i \) is the noise in channel \( i \) and equation (4) defines the frequency response matrix \( f_{\alpha} \). We write our linear estimator of \( s_{\alpha} \) as

\[
\hat{s}_{\alpha} = \sum_i W_i \theta_i.
\]
The covariance matrix of the statistical errors for this estimator and the estimator can be written as

$$\mathbf{F} = \sum_i F_{ia}^{-1} f_{ia} N_{ia}^{-1},$$

(6)

where $N_{ia} \equiv \langle n_i n_a \rangle$ and $F_{ia} = \sum_j f_{ij} N_{ij}^{-1} f_{ja}$ is the Fisher matrix for the $s_{ia}$. In the case of diagonal noise, $N_{ia} = \sigma_i^2 \delta_{ia}$, the Fisher matrix simplifies to

$$F_{ia} = \sum_j f_{ia} f_{ja} / \sigma_i^2,$$

(7)

and the estimator can be written as

$$\hat{s}_{ia} = \sum_i F_{ia}^{-1} f_{ia} \delta_{ia}. $$

(8)

The covariance matrix of the statistical errors for this estimator is $\langle (\hat{s}_{ia} - s_{ia}) (\hat{s}_{ia} - s_{ia}) \rangle = F_{ia}. $ $p$

We are interested in the effects on our component separation of a band error that causes the actual bandpass to deviate from the design bandpass by $\delta f_a$. Using equation (8) with the design bandpass, but replacing $\delta_{ia}$ with the right-hand side of equation (4) evaluated with the actual bandpass, and subtracting off the true signal, we find

$$\delta s_a \equiv \langle \hat{s}_{ia} - s_{ia} \rangle = \sum \delta s_{ia} \delta_{ia},$$

(9)

where $M$ is the “component-mixing matrix” given by

$$M_{ia} \equiv \sum_j F_{ij}^{-1} f_{ij} \delta f_{ia} / \sigma_i^2,$$

(10)

and $\delta f_{ia}$ is the difference between the design and actual bandpasses. The bandpass uncertainty then induces a relative rms error on component $a$ from component $\beta$ characterized by

$$M_{ia} \equiv M_{ia} \langle s_{ia} \rangle^{1/2} / \langle s_{ia} \rangle^{1/2},$$

(11)

How is $\delta f_a$ related to the bandpass error? If component $a$ has frequency dependence $g_a(\nu)$, then

$$f_{ia} = \int d\nu g_a(\nu) [h(\nu) + \delta h_a(\nu)],$$

(12)

where $h(\nu) + \delta h_a(\nu)$ is the total bandpass, with the latter term the error. It is easy to see that an error in the amplitude of the bandpass, $\delta h_a(\nu) \propto h_{\alpha}(\nu)$, will have no effect on component separation since it will “calibrate out.” In general, we shall model this calibration process by demanding that

$$f_{i\alpha} = \int d\nu g_a(\nu) [h(\nu) + \delta h_{\alpha}(\nu)] = \delta f(\nu),$$

(13)

where component 0 is the CMB. We normalize $g_a(\nu) = 30$ GHz = 1 for all components $\alpha$. Note that this means $s_{ia}$ is the amplitude of component $\alpha$ at 30 GHz.

Note that the above treatment applies in both pixel space (on scales larger than the beam) and spherical harmonic space. In pixel space, the $\sigma_i$ should all be calculated for the same pixel size. In spherical harmonic space, the $\sigma_i$ (interpreted as errors on the beam-deconvolved maps) are $l$-dependent:

$$\sigma_i(l) = \hat{\sigma}_i(l) \exp \left[ \frac{1}{2} \left( \frac{\hat{\sigma}_i(l)}{2.355} \right)^2 \right].$$

(14)

where $\hat{\sigma}$ and $\sigma$ are the beam and noise defined in Table 1 with $\theta$ converted to radians.

### 4. RESULTS

As an example, let us consider simple shifts in the bandpass, which we shall model as a top hat of width $\Delta \nu$ about the central frequency as given in Table 1. To gain intuition, let us restrict ourselves to a subspace consisting only of CMB and tSZ effect signals with a 1 GHz shift in the 217 GHz channel to higher frequency. In this case (at angular scales larger than the largest beam),

$$M_{ia} = \begin{pmatrix} 0.0 & -0.00391 \\ 0.0 & 0.00522 \end{pmatrix},$$

(15)

which indicates that the CMB channel does not “contaminate” any of the signals but there is leakage from the tSZ channel. That $M_{\beta\alpha} = 0$ is a direct result of our calibration procedure, which enforces $\delta f_{i\alpha} = 0$. Note that for other calibration sources typically used by ground-based experiments, e.g., planets, the CMB can, via bandpass errors, contaminate other components.

A more realistic scenario is that both the central frequency and bandwidth of the instrument are quite well determined but that the amplitude of the response as a function of frequency is somewhat uncertain. This arises, for example, if a linear “ramp” is introduced across the true spectral response by the calibration process. This can occur for both high electron mobility transistors (M. Seiffert 2002, private communication) and bolometers (P. Ade 2002, private communication). As a simple model of this effect, we introduce, into our otherwise top-hat bandpasses, such a ramp that causes a change in amplitude of 10% across the band. This ramp leaves the bandwidth unchanged and moves the central frequency of each band by 1% or less for the bandwidths shown in Table 1. A 10% ramp is selected because (1) it is quite

### TABLE 1

| Number | Frequency (GHz) | $\Delta \nu$ (arcmin) | Noise ($\mu$K) |
|--------|-----------------|---------------------|--------------|
| 1       | 30              | 0.2                 | 33           |
| 2       | 44              | 0.2                 | 24           |
| 3       | 70              | 0.2                 | 14           |
| 4       | 100             | 0.2                 | 10           |
| 5       | 100             | 0.3                 | 9.2          |
| 6       | 143             | 0.3                 | 7.1          |
| 7       | 217             | 0.3                 | 5.0          |
| 8       | 353             | 0.3                 | 5.0          |
| 9       | 545             | 0.3                 | 400         |

Note: The noise (thermodynamic temperature fluctuation in a square pixel of side “beam”) is quoted assuming a 12 month integration. We do not use the 850 GHz channel of Planck here since our focus is on components at lower frequency.
representative of the level of uncertainty of real calibration techniques and (2) it illustrates that while a 1% error in the location of the center of a 20%–30% band seems quite small, the effect on the component mixing is noticeable, as shown in Table 2 for our five-component model.

The elements of the mixing matrix in Table 2 tell us, for example, that our CMB map, $\mathbf{s}_M$, will have a contribution from dust of $-0.25s_3$, where $s_3$ is the dust amplitude at 30 GHz. Fortunately, the dust amplitude is very low at 30 GHz!

We have also investigated the effect of a 10% ramp in each of the channels individually. For the CMB/tSZ mixing, the 217 GHz channel is the most sensitive, as expected. Ramps in the low-frequency channels give rise to mixings down by a factor of approximately 100 compared to ramps in the 100–353 GHz channels, showing these errors are less important for CMB/tSZ mixing (again as expected). From now on, we will consider only the case where all channels have a 10% ramp.

The importance of this contamination is easier to read from the normalized mixing matrix, $\mathcal{M}_{\text{eq}}$. To calculate $\mathcal{M}_{\text{eq}}$, we need to know the rms sky fluctuations of the various components. We use the model described in Knox (1999) and references therein, most notably Bouchet & Gispert (1999). The exception is our SZ power spectrum, which we take from White, Hernquist, & Springel (2002) and extend to $|l| < 400$ by assuming $C_l = C_{\text{lim}}$, appropriate for the Poisson-dominated regime (see Fig. 1).

In Table 3, we assume $s_n$ are the amplitudes of an $l = 500$ spherical harmonic, so that $(s_n^2) = C_l^*$. We find that all the elements are quite small; the largest contribution to the CMB map comes from the SZ effect, and the rms of this contaminant is 0.05% of the CMB signal rms. The Galactic contaminants of the CMB map, $M_{\text{gal}}$, for $\beta = 2, 3, 4$ greatly increase with increasing angular scale. Even so, they are very small at all angular scales; at $l = 2$, $M_{\text{gal}} = -0.006, 0.0002$, and $2 \times 10^{-3}$ for dust, synchrotron, and bremsstrahlung, respectively.

In Figure 1, we show how the SZ contamination of the CMB map affects the CMB power spectrum and the CMB-SZ cross-correlation power spectrum. Denoting the CMB contaminant from SZ as $a_{\ell m}^{\text{CMB/SZ}}$, we have

$$C^{CMB(SZ)}_l \equiv \langle a_{\ell m}^{CMB/SZ} a_{\ell m}^{\text{SZ}}^* \rangle = M_{\text{gal}}(l)C^{\text{SZ-SZ}}_l. \tag{16}$$

The $l$ dependence of the mixing matrix arises from the $l$ dependence of the noise in the beam-deconvolved maps but is quite mild: $M(l, l)$ monotonically decreases from $-4 \times 10^{-3}$ to its high $l$ asymptote of $-5.3 \times 10^{-3}$. In general, the cross-correlations are the most affected since they are first order in the component-mixing matrix. Fortunately, for the 10% ramp error, the bandpass error–induced cross-correlation is well below the level of the Planck noise.

Our results are specific to the conservative component separation procedure we have assumed. Other methods can reduce the statistical noise by including assumptions about the statistical properties of the various components (Tegmark & Efstathiou 1996; Bouchet, Gispert, & Puget 1995; Hobson et al. 1998). The results will also differ in detail if one adopts a more realistic (and more complicated) model of the foregrounds.

We do not, however, expect the changes in modeling and component separation methods to lead to qualitatively different conclusions. Let us consider the method of Wiener filtering. Component separation by use of the Wiener filter exploits our assumed knowledge of the variance of each component. The Wiener filter allows us to, for example, put more weight on the 353 GHz channel at high $l$ if we assume less contamination is less at small scales. Similar arguments apply to other Galactic contaminants. A Wiener filter at high $l$ thus approximates the weighting in our two-component model. Since our two-component model mixing matrix is qualitatively similar to our five-component model mixing matrix, we expect a Wiener filtering procedure to lead to a similar mixing matrix. Further evidence of the robustness of our results comes from the dependence of the mixing matrix on $l$ (and therefore on the number of channels included) described above.

The bandpass uncertainties can lead to systematic errors in the velocities of clusters inferred from the kinetic SZ effect. The mixing between tSZ and primary CMB on a cluster of optical depth $\tau$ and temperature $T_e$ induces an error

$$\delta T \approx -8 \times 10^{-3} \tau \frac{kT_e}{m_e c^2} \tag{17}$$

in the CMB signal. If the contamination was all erro-
neously attributed to the kinetic SZ effect from the moving cluster, it would bias the inferred velocity by $v = 8 \times 10^{-3} (kT/m_e c^2) c \approx 50 \text{ km s}^{-1}$ for a rich cluster. This bias is negligible compared to other sources of uncertainty for individual cluster velocities (Haehnelt & Tegmark 1996; Nagai, Kravtsov, & Kosowsky 2002; Holder 2002) but is comparable to errors that might be achievable by Planck on bulk flows in $10^6 h^{-3} \text{ Mpc}^3$ volumes (Aghanim, Gorski, & Puget 2001). This systematic contaminant of the bulk flows will appear as a $T_e$-weighted monopole. Such a pattern is not expected cosmologically, would be evidence of bandpass errors, and could be removed from the data with negligible residuals. Note that while we have ignored relativistic corrections to the SZ distortion for the purposes of estimating the magnitude of the effect, they will be important in the actual analysis of the data (Diego, Hansen, & Silk 2002).

5. CONCLUSIONS

We can conclude that bandpass errors at the $O(10\%)$ level are acceptably small for Planck. More sensitive experiments, though, may have more stringent requirements on the quality of the bandpass measurements because an overall scaling of the sensitivity of each channel leaves the component-mixing matrix unchanged.

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