Dynamical Characteristics of Electromechanical Energy Dissipation due to Translatory Motion: A Minimal Example

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Electromechanical systems cover a broad technological range of application. Typical examples are electrical machines, eddy current brakes, or electrical machines. A most relevant aspect in this process is the dissipation of some part of the energy into heat. Both the conversion process and the dissipation are accompanied by some interesting dynamical characteristics involving strong geometrical and physical non-linearities [1]. Consequently, different vibration issues can arise from this coupling, which have to be understood in order to properly design the considered devices [2–5].

In this work, the classical textbook examples of a simple mechanical oscillator, a magnetically conductive structure and an electric RL-circuit are combined. To this end, in the following the model equations are described and relevant parameters are introduced. After that, the system dynamics are analysed for three different scenarios: first in terms of equilibria, second for a prescribed mechanical motion and third for the full electromechanical coupling. These situations typically occur when considering the attractions of (electro-) magnets, in eddy current brakes and dampers and in lateral rotor oscillations of electric machines. All effects described have been observed experimentally [2,6].

1 Introduction

Electromechanical energy conversion plays an important role in many technological applications. Typical examples are (electro-) magnets of all kinds, eddy current brakes and dampers, or electrical machines. A most relevant aspect in this process is the dissipation of some part of the energy into heat. Both the conversion process and the dissipation are accompanied by some interesting dynamical characteristics involving strong geometrical and physical non-linearities [1]. Consequently, different vibration issues can arise from this coupling, which have to be understood in order to properly design the considered devices [2–5].

In this work, the classical textbook examples of a simple mechanical oscillator, a magnetically conductive structure and an electric RL-circuit are combined. To this end, in the following the model equations are described and relevant parameters are introduced. After that, the system dynamics are analysed for three different scenarios: first in terms of equilibria, second for a prescribed mechanical motion and third for the full electromechanical coupling. These situations typically occur when considering the attractions of (electro-) magnets, in eddy current brakes and dampers and in lateral rotor oscillations of electric machines. All effects described have been observed experimentally [2,6].

2 Modelling

Figure 1 a) depicts the electromechanical oscillator investigated in this work. It consists of a simple mechanical oscillator, a magnetically conductive structure and an electrical circuit. If the oscillator moves, the air-gap varies in length, thus modulating the magnetic field caused by the permanent magnet. This modulation induces voltage into the electrical circuit, where an electrical current flows. Through this process, mechanical energy is transformed to electrical energy and conducted to the electrical circuit. Finally, it is dissipated into heat in the resistance.

From NEWTON’S, AMPÈRE’S and FARADAY’S laws, the following dynamical equations can be derived

\begin{equation}
\eta^2 \ddot{X} + X = f(\tau) + \gamma \Lambda^2, \quad \Lambda = \frac{1}{1 + \kappa - \frac{X}{\gamma}} (I + \kappa - \sigma h_{le}(\Lambda)), \quad \nu \Lambda' + I = 0.
\end{equation}

They are already given in non-dimensional form. The dynamical variables are the specific displacement $X$, the relative flux linkage $\Lambda$ and the specific electric current $I$ (in per unit notation). Here, a non-dimensional time $\tau = \Omega t$ has been introduced, where $\Omega$ is some reference frequency, e.g., related to the base frequency of external forcing $f$. Thus, there are four relevant parameters in this system: The specific mechanical frequency $\eta$ relating the reference frequency $\Omega$ to the eigenfrequency $\omega_0$ of the underlying mechanical system. The parameter $\nu$ comparing the external frequency $\Omega$ to the electrical time constant $T_e$ of the corresponding RL-circuit. The quantity $\kappa$ resembling the specific strength of the magnet compared to the air-gap reluctance. And at last, the parameter $\gamma$ measuring the strength of the magnetic forces compared to the elastic restoring forces. In this sense, $\eta, \nu$ and $\gamma$ are frequency related quantities, while $\kappa$ and $\gamma$ characterise the static magnetic behaviour. The additional parameter $\sigma \ll 1$ relates the iron reluctance to the air-gap reluctance. The function $h_{le}$ represents the magnetisation curve of the involved material. In many cases this physically non-linear influence can be neglected. However, if large oscillations occur, magnetic saturation may have a significant influence on magnetic forces [7], therefore this term is considered here.

3 Results

First scenario: equilibria (e.g., attraction of (electro-) magnets). Figure 1 b) shows the equilibrium displacement $X_0$ as a function of the strength $\kappa$ of the magnet for varying values $\gamma$ of the specific magnetic force. Here, two major dependencies are...
discovered: at the one hand an intuitive trend is observed: the stronger the magnet, the larger the displacement. However, for higher magnetic strength the displacement value saturates. At the other hand, for a weak magnet and weak elastic restoring force compared to the magnetic force, a fold bifurcation occurs. It can be shown, that the upper solution branches are unstable and the lower ones are stable.

\[ \sigma \ll 1 \]

Fig. 1: Dynamical characteristics of electromechanical energy dissipation due to translatory motion. a) Minimal model and relevant parameters, b) Static equilibrium position \( X_0 \) vs. specific strength \( \kappa \) of the magnet for different values \( \gamma \) of the specific magnetic force. c) Forced mechanical motion: equivalent damping force \( f_d \) as a function of the electrical frequency \( \nu \). d) Full electromechanical dynamics: Amplitude of the first harmonic \( \hat{X}_1 \) of the mechanical response in relation to the forcing \( \hat{f} \) as a function of the mechanical frequency \( \eta \).

Second scenario: stationary solutions for prescribed mechanical motion (e.g. eddy current brakes and dampers). In the upper part of figure 1 c), an equivalent specific damping force \( f_d \) is plotted as a function of the electrical frequency \( \nu \) for \( \eta = 0 \) (prescribed mehanical motion, e.g. eddy current brake or damper). The force has been derived by dividing the dissipated electrical power by the mechanical velocity. In this figure, two distinct regions can be identified: For \( \nu < 1 \), the force appears to rise linearly with the frequency, similar to common viscous damping. However, for higher frequencies (\( \nu > 1 \)), the damping effect breaks down. This behaviour can be explained, when considering the induced current \( I \), depicted in the lower part of figure 1 c). This current weakens the magnetic field and as the field mediates the energy conversion, the dissipative effect is reduced by the induced current.

Third scenario: fully coupled electromechanics (e.g. lateral vibrations in electrical machines). As a last point, figure 1 d) shows the fundamental harmonic amplitude \( \hat{X}_1 \) of the displacement in relation to the harmonic forcing amplitude \( \hat{f} \) as a function of the mechanical frequency \( \eta \). For simplicity, the parameters have been set to \( \eta = \nu \), as both are functions of \( \Omega \). The graph corresponding to \( \gamma = 0 \) shows the typical forced response of a pure mechanical oscillator. Increasing \( \gamma \) leads to a reduced resonance frequency and reduced vibration amplitudes. Comparing solutions derived from a linearised model and the full non-linear problem reveals, that a linear analysis is only permissible for very weak forcing. The non-linear solution has been approximated using the harmonic-balance method.

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