Response of the Unruh-DeWitt detector in flat spacetime with a compact dimension

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Abstract

In a flat spacetime with one spatial dimension compactified, inertial reference frames are not all equivalent, but there are the preferred ones. This paper investigates nonequivalence of inertial frames in connection with responses of the Unruh-DeWitt detector coupled to a massless scalar field. If the detector moves with an arbitrary constant velocity, it registers no signals. However, within an inertial frame, by instantaneously accelerating the Unruh-DeWitt detector, one can infer the length of the compact dimension and the frame’s moving velocity in the compact direction with respect to the preferred frame from the detector’s response to acceleration.

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I. MOTIVATIONS

The twin paradox is a well-known puzzle in the theory of relativity: one of the two identical twins travels on a high-speed spacecraft away from Earth and then turns around and comes back, while the other stays on Earth. The puzzle appears because each twin apparently sees the other as moving, and therefore time dilation seems to suggest that, paradoxically, the twins should find each other less aged by the time when they meet. As the paradox is resolved within the standard theory of relativity, it turns out the traveling twin is less aged than the earthbound sibling, not the other way around.

There have been various explanations of the twin paradox, all recognizing the crucial fact that the symmetry between the two twins is in fact illusory. The earthbound twin is in the same inertial (rest) frame all the time, while the traveling twin undergoes two different (outbound and inbound) initial frames throughout the journey. The frame switch upon the traveling twin is essentially the reason for the aging difference. The switch of initial frames implies that the traveling twin must experience acceleration at the time of turnaround, which can also be used to account for his slowed aging in terms of gravitational time dilation (although it is often argued that acceleration per se plays no direct role). For more discussions on the twin paradox, see [1] and references therein.

The puzzle strikes again when we consider the twin paradox in a flat spacetime with one spatial dimension compactified. If the traveling twin moves at a constant velocity in the compact direction, his frame remains inertial for the entire journey, yet the topology allows him to meet the earthbound twin after he circumnavigates the compact dimension (see Fig. 1). As the traveling twin undergoes no frame switch at all, the standard explanation of the aging difference no longer works. The resolution to the puzzle lies in the fact that compactifying a spatial dimension breaks the global Lorentz invariance. As a consequence, there is now a class of preferred inertial reference frames, namely, those at rest in the compact direction [2–6].

As inertial frames are not all equivalent now, an observer in principle can experimentally determine the frame’s moving velocity in the compact direction with respect to the preferred frame. This can be done by performing a “global” experiment: sending two light beams in opposite directions along the compact dimension and measuring the arrival time of both signals when they come back. The frame’s moving velocity relative to the preferred frame can be inferred from the time delay between the two arriving signals [2, 3]. On the other hand, a “local” experiment is also possible. For instance, as one spatial dimension is compactified, the form of the electrostatic field of a point charge is deviated from $1/r^2$. Measuring the deviation can also determine the frame’s
moving velocity relative to the preferred frame [7].

It is instructive to look for other kinds of local experiments, as they will teach us to what extent the initial reference frames are inequivalent. Particularly, as the velocity relative to the preferred frame bears an absolute meaning now (and in a sense analogous to acceleration in the Minkowski spacetime), it is suggestive that even the Unruh-DeWitt detector moving at a constant velocity might register signals. Recently, it was shown that, in the Minkowski spacetime, coupled to a massless scalar field in the polymer quantization (which implements some features of the microscopic discreteness in loop quantum gravity) [8], the Unruh-DeWitt detector moving at constant velocity detects radiation [9]. This is essentially because the Lorentz invariance is violated in the UV scale by the microscopic discreteness. In our case of flat spacetime with a compact dimension, as the Lorentz invariance is violated in the IR scale by the large length of the compact dimension, it is curious to know whether the Unruh-DeWitt detector moving at a constant velocity also sees radiation.

This paper investigates the response of the Unruh-DeWitt detector coupled to a massless scalar field in a flat spacetime with one spatial dimension compactified. It turns out the Unruh-DeWitt detector moving at a constant velocity (as in the left of Fig. 1) does not register signals, contrary to the case of the polymer quantization. However, within the inertial frame (as in the right of Fig. 1), the frame’s moving velocity in the compact direction in principle can be determined by instantaneously accelerating the Unruh-DeWitt detector and measuring the instantaneous transition probability. That is, the response of the Unruh-DeWitt detector can be used to discriminate between inertial reference frames with different velocities in the compact direction.
II. THE UNRUH-DEWITT DETECTOR

The Unruh-DeWitt detector [10, 11] is an idealized point-particle detector coupled to a scalar field $\phi$ via a monopole interaction. If the detector moves along a world line $x^\mu(\tau)$, where $\tau$ is the detector’s proper time, the Lagrangian for the monopole interaction is given by $cm(\tau)\phi(x^\mu(\tau))$, where $c$ is a small coupling constant and $m$ is the operator of the detector’s monopole moment.

For a generic trajectory $x^\mu(\tau)$, the detector in general does not remain in its ground state labelled by the energy $E_0$ but can ascend to an excited state with energy $E > E_0$, while at the same time the field $\phi$ makes a transition from the vacuum state $\langle 0 \rangle$ to an excited state $|\psi\rangle$. By first-order perturbation theory, the amplitude for the transition $|0, E_0\rangle \to |\psi, E\rangle$ is given by

$$ic \langle \psi, E | \int_{-\infty}^{\infty} m(\tau)\phi(x^\mu(\tau)) d\tau |0, E_0\rangle,$$

which leads to the transition probability to all possible $E$ and $\psi$ as

$$c^2 \sum_{E} |\langle E | m(0) | E_0 \rangle|^2 \mathcal{F}(E - E_0),$$

where

$$\mathcal{F}(\Delta E) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-i\Delta E(\tau - \tau')} G^+(x(\tau), x(\tau'))$$

is the response function and the remaining factor represents the detector’s selectivity. The Wightman functions $G^\pm$ are defined as

$$G^+(x, x') := \langle 0 | \phi(x)\phi(x') | 0 \rangle,$$

$$G^-(x, x') := \langle 0 | \phi(x')\phi(x) | 0 \rangle.$$

If the detector is in equilibrium with the field $\phi$ along the trajectory, we have

$$G^+(x(\tau), x(\tau')) = G^+(\Delta \tau), \quad \Delta \tau := \tau - \tau'.$$

In this case, the transition probability per unit proper time is given by

$$R = c^2 \sum_{E} |\langle E | m(0) | E_0 \rangle|^2 \int_{-\infty}^{\infty} d(\Delta \tau)e^{-iE\Delta \tau} G^+(\Delta \tau).$$

More details can be found in [12].

1 This paper follows the notations used in [12] as closely as possible.
III. THE WIGHTMAN FUNCTION

Consider a real scalar field $\phi(x) \equiv \phi(t,x)$ in $d$-dimensional spacetime, where events are coordinated as $x^\mu = (x^0, x) = (t, x^1, \ldots, x^{d-1})$. The mode expansion of $\phi(x)$ is given by

$$\phi(t,x) = \sum_k \left( a_k u_k(t,x) + a_k^\dagger u^*_k(t,x) \right).$$

If the spacetime is flat but the $(d-1)$-th spatial direction is compactified with a finite length $L$, the Fourier modes $u_k$ are given by

$$u_k(t,x) = \frac{1}{(2\omega_k(2\pi)^{d-2}L)^{1/2}} e^{i k \cdot x - i \omega_k t},$$

where the frequency associated with $k = (k^1, \ldots, k^{d-1})$ is

$$\omega_k := \sqrt{k^2 + m^2},$$

and the $(d-1)$-th component of $k$ takes only discrete values:

$$k^{d-1} = \frac{2\pi n}{L}, \quad n \in \mathbb{Z}.$$  \hspace{1cm} (3.4)

Let $|0_L\rangle$ be the vacuum state in accordance with the above mode expansion, i.e.,

$$a_k|0_L\rangle = 0, \quad \text{for } \forall k.$$  \hspace{1cm} (3.5)

The Wightman function $G^+_{L}(x, x')$ then takes the form

$$G^+_{L}(x, x') := \langle 0_L | \phi(x) \phi(x') | 0_L \rangle$$

$$= \left( \frac{1}{L} \sum_{k^{d=1}} \right) \int \frac{d^{d-2}k}{(2\pi)^{d-2}} \frac{1}{2\omega_k} e^{i k \cdot (x-x') - i \omega_k (t-t')}$$

$$= \sum_{n=-\infty}^{\infty} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{2\omega_k} e^{i k \cdot (x-x') - i \omega_k (t-t')} e^{-i n L k^{d-1}},$$ \hspace{1cm} (3.6)

where we have used the Poisson summation formula. The Wightman function depends only on the difference of $x$ and $x'$, i.e., $G^+_{L}(x, x') = G^+_{L}(x-x')$; furthermore, as can be seen from (3.6), it is periodic in the $x^{d-1}$ direction, i.e.,

$$G^+_{L}(t' - t, x^1 - x^1', \ldots, x^{d-1} - x^{d-1}' + nL) = G^+_{L}(t' - t, x^1 - x^1', \ldots, x^{d-1} - x^{d-1}'), \quad n \in \mathbb{Z}.$$  \hspace{1cm} (3.7)

\hspace{1cm} $^2$ The Poisson summation formula is

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{k=-\infty}^{\infty} \int_{n=-\infty}^{\infty} dx \, f(x) e^{-2\pi i nx}. $$
Eq. (3.6) can be cast as
\[ G^+_L(x, x') = \sum_{n=-\infty}^{\infty} G^+(t' - t, x^1 - x'^1, \ldots, x^{d-1} - x'^{d-1} - nL), \] (3.8)
where \( G^+(x, x') \equiv G^+_{L \rightarrow \infty}(x, x') \) is the ordinary Wightman function in the Minkowski spacetime.

The Green functions (Wightman function included) are generally very complicated. In the case of a massless \((m = 0)\) scalar field in 4-dimensional spacetime, \( G^+(x, x') \) can be explicitly calculated as (see [12])
\[ G^+(x, x') = -\frac{1}{4\pi^2} \frac{1}{(t - t' - i\epsilon)^2 - |x - x'|^2}, \] (3.9)
where a small (infinitesimal) imaginary number \( i\epsilon, \epsilon > 0, \) is prescribed as a regularization parameter to ensure convergency. The rest part of this paper will focus on this 4-dimensional case with \( x^\mu = (t, x, y, z). \) We will first study the response of the Unruh-DeWitt detector moving at a constant velocity and then the detector moving with a constant acceleration, in the compact and noncompact directions, respectively.

IV. CONSTANT VELOCITY

Consider that the detector moves in the compact \((z)\) direction with a constant velocity \( v = |v| \hat{z}. \)

The trajectory is given by the world line:
\[ t = u^0 \tau, \quad x = y = \text{const}, \quad z = |u| \tau, \] (4.1)
where the 4-velocity \( u^\mu \) is given by
\[ u^\mu = (u^0, u) = \left( \frac{1}{\sqrt{1 - v^2}}, \frac{v}{\sqrt{1 - v^2}} \right). \] (4.2)

Eq. (3.8) with (3.9) now reads as
\[ G^+_L(x(\tau), x(\tau')) \equiv G^+_L(\Delta \tau) = -\frac{1}{4\pi^2} \sum_{n=-\infty}^{\infty} \frac{1}{(u^0 \Delta \tau - i\epsilon)^2 - (|u| \Delta \tau - nL)^2}. \] (4.3)
Each summand has two poles \( \pm nL + i\epsilon/(u^0 \pm |u|) \) if \( \Delta \tau \) is considered to be a complex number.

The transition rate (2.6) can be calculated by a contour integral along an infinite semicircle on the lower-half \( \Delta \tau \) plane. However, as \( u^0 > |u|, \) all poles in (4.3) are on the upper-half plane and hence the contour integral turns out to be zero. This tells that no particles are detected.
Furthermore, although $|0_L\rangle$ is not invariant under the boost in the $z$ direction, it remains invariant under boosts in $x$ and $y$ directions. Therefore, the response of the detector should be the same if we boost it in $x$ and $y$ directions. Consequently, we arrive at the conclusion that no signals are registered if the detector moves with an arbitrary constant velocity.

V. CONSTANT ACCELERATION IN THE COMPACT DIRECTION

Consider that the detector moves in the $z$ direction with a constant acceleration $1/\alpha$. The trajectory is given by the world line:

$$t = \alpha \sinh \frac{\tau}{\alpha}, \quad x = y = \text{const}, \quad z = \alpha \cosh \frac{\tau}{\alpha}.$$  \hfill (5.1)

Eq. (3.8) with (3.9) now reads as

$$G^+_L(x(\tau), x'(\tau')) = -\frac{1}{4\pi^2} \sum_{n=-\infty}^{\infty} \frac{1}{(\alpha \sinh \frac{\tau}{\alpha} - \alpha \sinh \frac{\tau'}{\alpha} - i\epsilon)^2 - (\alpha \cosh \frac{\tau}{\alpha} - \alpha \cosh \frac{\tau'}{\alpha} - nL)^2}.$$  \hfill (5.2)

Consequently (see Appendix), we have

$$G^+_L(x(\tau), x'(\tau')) = -\frac{1}{16\pi^2\alpha^2} \sum_{n=-\infty}^{\infty} \sinh^2 \left( \frac{\Delta \tau}{2\alpha} - \frac{n\epsilon}{2\alpha} \right) + n \left( \frac{\tau}{\alpha} \right) \sinh \frac{\Delta \tau + \tau'}{2\alpha} \sinh \frac{\Delta \tau}{2\alpha} - \frac{n^2}{4} \left( \frac{\tau}{\alpha} \right)^2.$$  \hfill (5.3)

When $L \gg \alpha$, only the summand with $n = 0$ survives and (5.3) reduces to the ordinary result of the Minkowski spacetime:

$$G^+(x(\tau), x'(\tau')) = -\frac{1}{16\pi^2\alpha^2} \sinh^2 \left( \frac{\Delta \tau}{2\alpha} - \frac{\epsilon}{2\alpha} \right) = -\frac{1}{4\pi^2} \sum_{k=-\infty}^{\infty} \frac{1}{(\Delta \tau - i\epsilon + 2\pi i\alpha k)^2},$$  \hfill (5.4)

where we have used the identity

$$\csc^2 \pi x = \frac{1}{\pi^2} \sum_{k=-\infty}^{\infty} \frac{1}{(x-k)^2}.$$  \hfill (5.5)

Taking (5.4) into (2.6) and performing the contour integral, we obtain the transition rate as

$$R = \frac{c^2}{2\pi} \sum_{E} \frac{(E - E_0)(\langle E|m(0)|E_0\rangle)^2}{e^{2\pi(E-E_0)\alpha} - 1}.$$  \hfill (5.6)

The celebrated Planck factor $(e^{2\pi(E-E_0)\alpha} - 1)^{-1}$ indicates that the accelerated detector registers particles of $\phi$ as if it was immersed in a bath of thermal radiation at the temperature

$$T = \frac{1}{2\pi k_B\alpha} \equiv \frac{|\text{acceleration}|}{2\pi k_B}.$$  \hfill (5.7)
For generic cases that $L \gg \alpha$, (5.3) can be cast in a closed form by the identity
\[
\sum_{n=-\infty}^{\infty} \frac{1}{n^2 + an + b} = \frac{\pi \cot \left( \frac{1}{2} \left( \pi a - \pi \sqrt{a^2 - 4b} \right) \right) - \pi \cot \left( \frac{1}{2} \left( \pi \sqrt{a^2 - 4b} + \pi a \right) \right)}{\sqrt{a^2 - 4b}}.
\] (5.8)

Consequently, it turns out that $G_L^+(x(\tau), x(\tau'))$ depends not only on $\Delta \tau \equiv \tau - \tau'$ but also on $\tau + \tau'$, indicating that the detector moving in the $z$ direction with a constant acceleration is not in equilibrium with the field $\phi$.³

Despite the fact that the detector is not in equilibrium with $\phi$, we can still ask what the “instantaneous” transition probability $\mathcal{P}_{\tau_1 \rightarrow \tau_2}$ is if the detector is adiabatically switched on from $\tau_1$ to $\tau_2$ for a short period (i.e., $\tau_1 \lesssim \tau_2$). In this setting, (2.2) with (2.3) should be modified accordingly and the instantaneous transition rate from $\tau_1$ to $\tau_2$ is given by
\[
\mathcal{P}_{\tau_1 \rightarrow \tau_2} = c^2 \sum_{E} |\langle E|m(0)|E_0\rangle|^2 \int_{\tau_1}^{\tau_2} d\tau' \int_{\tau_1 - \tau_2}^{\tau_2 - \tau_1} d(\Delta \tau) \, e^{-iE\Delta \tau} G_L^+(\Delta \tau; \tau'),
\] (5.9)
where from (5.3) we have
\[
G_L^+(\Delta \tau; \tau') = -\frac{1}{16\pi^2 \alpha^2} \sum_{n=-\infty}^{\infty} \frac{1}{\sinh^2 \left( \frac{\Delta \tau}{2\alpha} - \frac{n}{2\alpha} \right) + n \left( \frac{L}{\alpha} \right) \sinh \frac{\Delta \tau + 2\tau'}{2\alpha} \sinh \frac{\Delta \tau}{2\alpha} - \frac{n^2}{4} \left( \frac{L}{\alpha} \right)^2}.
\] (5.10)

Note that the regularization parameter $\epsilon$ in (5.10) is in fact superfluous, as the delimiters of $\int d(\Delta \tau)$ in (5.9) are finite now. If $\alpha \ll \tau_2 - \tau_1$, all the summands in (5.10) vanish for large $\Delta \tau$ and thus the integral $\int_{\tau_1 - \tau_2}^{\tau_2 - \tau_1} d(\Delta \tau)$ in (5.9) can be replaced by $\int_{-\infty}^{\infty} d(\Delta \tau)$ provided $\epsilon$ is kept in (5.10); consequently, we can define the instantaneous transition rate from $\tau_1$ to $\tau_2$ as
\[
\mathcal{R}_{\tau_1 \leq \tau \leq \tau_2} = c^2 \sum_{E} |\langle E|m(0)|E_0\rangle|^2 \int_{-\infty}^{\infty} d(\Delta \tau) \, e^{-iE\Delta \tau} G_L^+(\Delta \tau; \tau'), \quad \text{for } \alpha \ll \tau_2 - \tau_1.
\] (5.11)

Because $|0_L\rangle$ is invariant under arbitrary spacetime translations as well as Lorentz boosts in $x, y$ directions, the fact that $G_L^+(\Delta \tau; \tau')$ depends on $\tau'$ is better understood as that it depends on the $z$-component of the instantaneous 4-velocity $u^z(\tau') := dz(\tau')/d\tau' = \sinh(\tau'/\alpha)$ for $\tau_1 \lesssim \tau' \lesssim \tau_2$. This suggests that, within an initial reference frame moving with a constant velocity (as in the right of Fig. II), one can in principle discern the $z$-component of the frame’s moving velocity (provided $L$ is already known) by instantaneously accelerating the Unruh-DeWitt detector in the $z$ direction and measuring the instantaneous transition probability.

³ It is crucial to know whether the dependence on $\tau + \tau'$ in (5.3) is erased away under the summation over $n$. By the identity (5.8), it is rigourously proven that $G_L^+(x(\tau), x(\tau'))$ is dependent on both $\Delta \tau$ and $\tau + \tau'$. 

8
VI. CONSTANT ACCELERATION IN NONCOMPACT DIRECTIONS

Consider that the detector moves with a constant acceleration $1/\alpha$ in noncompact directions (say, $x$ direction). The trajectory is given by the world line:

$$
t = \alpha \sinh \frac{\tau}{\alpha}, \quad y = z = \text{const}, \quad x = \alpha \cosh \frac{\tau}{\alpha}.
$$

(6.1)

Eq. (3.8) with (3.9) now reads as

$$
G^+_L(x(\tau), x(\tau')) = -\frac{1}{4\pi^2} \sum_{n=-\infty}^{\infty} \frac{1}{(\alpha \sinh \frac{\tau}{\alpha} - \alpha \sinh \frac{\tau'}{\alpha} - i\epsilon)^2 - (\alpha \cosh \frac{\tau}{\alpha} - \alpha \cosh \frac{\tau'}{\alpha})^2 - n^2L^2}
$$

$$
= -\frac{1}{16\pi^2\alpha^2} \sum_{n=-\infty}^{\infty} \frac{1}{\sinh^2 \left( \frac{\Delta \tau}{2\alpha} - \frac{i\epsilon}{2\alpha} \right) - \frac{n^2}{4} \left( \frac{L}{\alpha} \right)^2},
$$

(6.2)

where the similar calculation as shown in Appendix has been repeated. Using the identity

$$
\sum_{n=-\infty}^{\infty} \frac{1}{n^2 - a^2} = -\frac{\pi \cot(\pi a)}{a}
$$

(6.3)

as a special case of (5.8), we can rewrite (6.2) as

$$
G^+_L(\Delta \tau) = -\frac{1}{8\pi\alpha L} \cot \left( \frac{2\pi\alpha}{L} \sinh \left( \frac{\Delta \tau}{2\alpha} - \frac{i\epsilon}{2\alpha} \right) \right)
$$

(6.4)

Note that (6.4) reduces to (3.9) when $L \gg a$.

It is crucial to know whether the dependence on $L$ in $G^+_L(\Delta \tau)$ goes away when $G^+_L(\Delta \tau)$ is integrated in (2.6) to obtain $R$. To know the answer, we compute the derivative of $R$ with respect to $L$:

$$
\partial_L R(L) = e^2 \sum_E |\langle E|m(0)|E_0 \rangle|^2 \int_{-\infty}^{\infty} d(\Delta \tau) e^{-iE\Delta \tau} \partial_L G^+_L(\Delta \tau)
$$

$$
= e^2 \sum_E |\langle E|m(0)|E_0 \rangle|^2 \int_{-\infty}^{\infty} d(\Delta \tau) e^{-iE\Delta \tau} \left( -\frac{1}{L} G^+_L(\Delta \tau) - \frac{\alpha \csc^2 \left( \frac{2\pi\alpha}{L} \sinh \left( \frac{\Delta \tau}{2\alpha} - \frac{i\epsilon}{2\alpha} \right) \right)}{4L^3} \right)
$$

$$
= -\frac{R(L)}{L} - \frac{c^2 \alpha}{4L^3} \sum_E |\langle E|m(0)|E_0 \rangle|^2 \int_{-\infty}^{\infty} d(\Delta \tau) e^{-iE\Delta \tau} \sum_{k=-\infty}^{\infty} \frac{1}{\left( \frac{2\pi\alpha}{L} \sinh \left( \frac{\Delta \tau}{2\alpha} - \frac{i\epsilon}{2\alpha} \right) - \pi k \right)^2},
$$

(6.5)

where we have used the identity (5.5). Considering the formal limit $\alpha \gg L$ with $L$ fixed, we have

$$
\frac{2\pi\alpha}{L} \sinh \left( \frac{\Delta \tau}{2\alpha} - \frac{i\epsilon}{2\alpha} \right) - \pi k \approx \frac{\pi}{L} (\Delta \tau - i\epsilon) - \pi k.
$$

(6.6)
In this limit, the poles of the summands of \( \sum_{k=-\infty}^{\infty} \) in (6.5) are all on the upper-half plane, thus giving rise to zero when integrated over \( \int_{-\infty}^{\infty} d(\Delta \tau) e^{-iE\Delta \tau} \cdots \), as the integral can be calculated along an infinite semicircle on the lower-half \( \Delta \tau \) plane. Therefore, it follows from (6.5) that \( \partial_L R(L) = -R(L)/L \neq 0 \) in the formal limit \( \alpha \gg L \), and thus we have rigorously shown that \( R \) cannot be independent of \( L \).

The fact that \( G^+(x(\tau), x(\tau')) \) depends only on \( \Delta \tau := \tau - \tau' \) indicates that the accelerating detector is in equilibrium with the field \( \phi \). However, the form of the transition rate \( R \) is very complicated and it is unlikely that the equilibrium is thermal. The independence of \( \tau + \tau' \) in \( G^+(x(\tau), x(\tau')) \) also means that the response of the detector cannot know about the frame’s moving velocity in \( x, y \) directions. Furthermore, as \( \partial_L R(L) \neq 0 \), one can in principle infer the length \( L \) by measuring \( R \) in relation to \( 1/\alpha \).

VII. SUMMARY AND DISCUSSION

If the Unruh-DeWitt detector moves with an arbitrary constant velocity, it registers no signals. In this sense, the local Lorentz invariance is still preserved while the global Lorentz invariance is violated in the flat spacetime with a compact dimension. This is in contrast to the case of the field in the polymer quantization as studied in [9].

If the detector moves with an constant acceleration in the compact (\( z \)) direction, it registers particles but is not in equilibrium with the field \( \phi \). Within an inertial frame, by instantaneously accelerating the detector in \( z \) direction, the \( z \)-component of the frame’s moving velocity can be inferred from the instantaneous transition probability (provided \( L \) is known).

If the detector moves with an constant acceleration in noncompact (\( x, y \)) directions, it registers particles and is in equilibrium with the field \( \phi \), but the equilibrium is very likely non-thermal. Within an inertial frame, by instantaneously accelerating the detector in \( x, y \) directions and measuring the transition rate in relation to acceleration, one can infer the length \( L \). The frame’s moving velocity in the noncompact directions, however, remain unknowable.

Therefore, even though we cannot discriminate between the Unruh-DeWitt detectors moving along different inertial world lines (see the left of Fig. 1), the response of the Unruh-DeWitt detector can be used to infer the length \( L \) and to discriminate between inertial reference frames with different velocities in the compact direction (see the right of Fig. 1). However, whether we can definitely assert that inertial frames are discriminable by “local” experiments is a matter of interpretation. After all, the vacuum state \( |0_L \rangle \) is a global concept and an accelerating Unruh-DeWitt detector knows
about $|0_L\rangle$ only if the walls of the moving inertial reference frame are transparent to the field $\phi$.\footnote{In the same regard, the experiment by measuring the deviation of the electrostatic field of a point charge (as studied in \cite{2}) is not to be viewed as completely local either, since the deviation relies on the fact that the electric field stretches out to the entire universe and the local electric field is deformed anyway if the charge source is screened by the frame walls.}

It should be noted that the Unruh-DeWitt detector in flat spacetime with a compact dimension as studied in this paper is equivalent to that in Minkowski space in the presence of two parallel Casimir plates which impose periodic boundary conditions as one of the cases studied in \cite{13}. However, as the work of \cite{13} focused only on the response of stationary detectors with static boundaries, acceleration in the direction perpendicular to the Casimir plates (which is equivalent to acceleration in the compact direction in this paper) was not considered.\footnote{It is obvious that the response of the detector accelerating in the direction perpendicular to the Casimir plates is not stationary (i.e., not in equilibrium with the field) if Dirichlet or Neumann boundary conditions are imposed on the plates. However, whether it is stationary or not is not obvious if periodic boundary conditions are imposed.} In this paper, we obtain a rigorous formula for the response of the Unruh-DeWitt detector accelerating in the compact direction. By virtue of the close investigation, we arrive at the conclusion that the response is not in equilibrium with the field $\phi$ and furthermore the frame’s moving velocity can be inferred from it, thus bringing out the connection between responses of the Unruh-DeWitt detector and nonequivalence of inertial frames.

Finally, it should be stressed that measurement of responses of the Unruh-DeWitt detector is completely out of reach of current technology, let alone to distinguish the difference. Nevertheless, it is conceptually important to understand (non)equivalence of inertial reference frames in light of responses of the Unruh-DeWitt detector.

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Appendix: Some calculation details

Here we present the detailed derivation from (5.2) to (5.3). The similar calculation is also applied in (6.2).

The denominator of each summand in (5.2) can be recast as

\[
\left( \alpha \sinh \frac{\tau}{\alpha} - \alpha \sinh \frac{\tau'}{\alpha} - i\epsilon \right)^2 - \left( \alpha \cosh \frac{\tau}{\alpha} - \alpha \cosh \frac{\tau'}{\alpha} - nL \right)^2
\]

\[
= -\alpha^2 \left( \cosh^2 \frac{\tau}{\alpha} - \sinh^2 \frac{\tau}{\alpha} + \cosh^2 \frac{\tau'}{\alpha} - \sinh^2 \frac{\tau'}{\alpha} \right) + 2\alpha^2 \left( \cosh \frac{\tau}{\alpha} \cosh \frac{\tau'}{\alpha} - \sinh \frac{\tau}{\alpha} \sinh \frac{\tau'}{\alpha} \right)
\]

\[
- 2i\alpha \left( \sinh \frac{\tau}{\alpha} - \sinh \frac{\tau'}{\alpha} \right) + 2nL \alpha \left( \cosh \frac{\tau}{\alpha} - \cosh \frac{\tau'}{\alpha} \right) - n^2 L^2 + O(\epsilon^2)
\]

\[
= -2\alpha^2 + 2\alpha^2 \cosh \frac{\tau - \tau'}{2\alpha} - 4i\alpha \cosh \frac{\tau + \tau'}{2\alpha} \sinh \frac{\tau - \tau'}{2\alpha} + 4nL\alpha \sinh \frac{\tau + \tau'}{2\alpha} \sinh \frac{\tau - \tau'}{2\alpha}
\]

\[- n^2 L^2 + O(\epsilon^2)
\]

\[
= 4\alpha^2 \sinh^2 \frac{\tau - \tau'}{2\alpha} - 4i\alpha \sinh \frac{\tau - \tau'}{2\alpha} \left( \cosh \frac{\tau - \tau'}{2\alpha} \cosh \frac{\tau'}{2\alpha} - \sinh \frac{\tau - \tau'}{2\alpha} \sinh \frac{\tau'}{2\alpha} \right)
\]

\[+
4nL\alpha \sinh \frac{\tau + \tau'}{2\alpha} \sinh \frac{\tau - \tau'}{2\alpha} - n^2 L^2 + O(\epsilon^2)
\]

(A.1a)

\[=
4\alpha^2 \sinh^2 \frac{\Delta\tau}{2\alpha} \left(1 + O(\epsilon)\right) - 4i\alpha \sinh \frac{\Delta\tau}{2\alpha} \cosh \frac{\Delta\tau}{2\alpha}
\]

\[+
4nL\alpha \sinh \frac{\tau + \tau'}{2\alpha} \sinh \frac{\Delta\tau}{2\alpha} - n^2 L^2 + O(\epsilon^2)
\]

(A.1b)

\[
\approx
4\alpha^2 \sinh^2 \left( \frac{\Delta\tau}{2\alpha} - \frac{i\epsilon}{2\alpha} \right) + 4nL\alpha \sinh \frac{\tau + \tau'}{2\alpha} \sinh \frac{\Delta\tau}{2\alpha} - n^2 L^2,
\]

(A.1c)

where from (A.1a) to (A.1b) we have absorbed the positive factor \(\cosh(\tau'/\alpha)\) into \(\epsilon\) and from (A.1b) to (A.1c) we have used

\[
sinh \left( \frac{\Delta\tau}{2\alpha} - \frac{i\epsilon}{2\alpha} \right) = \sinh \frac{\Delta\tau}{2\alpha} \cosh \frac{i\epsilon}{2\alpha} - \cosh \frac{\Delta\tau}{2\alpha} \sinh \frac{i\epsilon}{2\alpha}
\]

\[=
\sinh \frac{\Delta\tau}{2\alpha} - \frac{i\epsilon}{2\alpha} \cosh \frac{\Delta\tau}{2\alpha} + O(\epsilon^2).
\]

(A.2)

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