Structure functions in deep inelastic scattering from gauge/string 
duality beyond single-hadron final states

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(Dated: July 1, 2014)

Abstract

We study deep inelastic scattering at large ’t Hooft coupling and finite $x$ from gauge/string 
duality beyond single-hadron final states which gives the leading large-$N_c$ contribution. Within 
the supergravity approximation, we calculate the subleading large $N_c$ contribution by introducing 
an extra hadron into the final states. We find the contribution from these double-hadron final 
states will dominate in the Bjorken limit $q^2 \rightarrow \infty$ compared with the single-hadron states. We 
discuss the implications of our results.

PACS numbers: 11.25.Tq, 13.88.+e, 13.60.Hb

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I. INTRODUCTION

The gauge/string correspondence, since first conjectured \[1,3\], has been used widely in studying nonperturbative aspects of QCD. As the first application to deep inelastic scattering (DIS), Polchinski and Strassler \[4\] have employed the correspondence to calculate the structure functions for hadrons at large t’ Hooft coupling and large \(N_c\) limit by introducing an infrared cutoff in the fifth dimension to mimic the confinement. Since then, there have been a lot of further investigations \[5–22\] along the direction. Compared to QCD, these studies show that the hadron structures in the strong coupling limit bear very different features at finite \(x\), while sharing similar features at small \(x\). It turns out that the structure functions in the strong coupling regime are all power suppressed at finite \(x\), implying few partons gaining a finite amount of longitudinal momentum of the target hadron and almost all the partons squeezed into small \(x\) region. However, such conclusion can be arrived at only in the large \(N_c\) limit from the contribution of single-hadron final states. It is valuable to investigate the structure functions beyond this limit, which is just the major concern of the current work.

It is non-trivial task to obtain complete contributions to the structure functions at the subleading large \(N_c\) from gauge/string duality. For simplicity, we will restrict ourselves to the supergravity approximation, considering subleading contributions from the processes with only two scalar hadrons involved in final states. Through the specific calculation and power analysis, we find in the large \(N_c\) expansion, compared with leading contribution, the subleading contribution can be less suppressed in the power expansion of \(1/q\). Thus the subleading contribution will dominate in the Bjorken limit \(q \to \infty\), which implies that large \(N_c\) limit and the Bjorken limit do not commute with each other.

The paper is organized as follows. In Sec.\[II\] we formulate the DIS on a scalar target in the gauge/string correspondence. In Sec.\[III\] we evaluate successively the transition amplitudes, hadronic tensor and structure functions for the DIS process under the supergravity approximation. In Sec.\[IV\] we analyze the power dependence on \(1/q\) for varieties of channels and phase spaces and extract the leading contribution for the structure functions in the Bjorken limit \(q \to \infty\). In Sec. \[V\] we discuss our results and give the summary.
II. DIS FROM THE GAUGE/STRING DUALITY

In the one-photon exchange approximation for DIS, the initial lepton interacts with the hadron target by the exchange of a virtual photon and the hadron absorbs the photon and decays into the final states. The cross section is determined by the hadronic tensor $W^{\mu\nu}$ which is defined as

$$W^{\mu\nu} = \sum_X (2\pi)^4 \delta(p + q - P_X) \langle H|J^{\mu}(0)|X\rangle\langle X|J^{\nu}(0)|H\rangle,$$  \hspace{1cm} (1)

where $J^{\mu}$ is the electromagnetic current, $q^{\mu}$ is the momentum of the virtual photon, $p^{\mu}$ denotes the momentum of the initial hadron $H$ and $P_X$ denotes the total momentum of the final hadron states $X$. For the spinless or spin-averaged hadrons, the hadronic tensor can be decomposed into

$$W^{\mu\nu} = F_1(x, q^2) \left( \eta^{\mu\nu} - \frac{q^{\mu}q^{\nu}}{q^2} \right) + \frac{2x}{q^2} F_2(x, q^2) \left( p^{\mu} + \frac{q^{\mu}}{2x} \right) \left( p^{\nu} + \frac{q^{\nu}}{2x} \right).$$  \hspace{1cm} (2)

All the information for the hadron structure is encoded in the structure functions $F_1(x, q^2)$ and $F_2(x, q^2)$.

In the gauge/string duality, scalar hadrons correspond to normalizable supergravity modes of dilaton and the electromagnetic current corresponds a nonnormalizabel mode of a Kaluza-Klein gauge field at the boundary of the AdS$_5$ space. The mass gap of hadrons can be generated by breaking the conformal invariance through introducing a sharp cut-off $0 \leq z \leq z_0 \equiv 1/\Lambda$. The metric in AdS$_5$ space can be written as

$$ds^2 = \frac{1}{z^2} (\eta_{\mu\nu} dy^\mu dy^\nu + dz^2)$$  \hspace{1cm} (3)

where $\eta_{\mu\nu} = (-, +, +, +)$ is the flat space metric at the boundary. The initial/final dilaton wave function satisfies the Klein-Gordon equation in AdS$_5$ and the corresponding normalizable solution with the boundary condition $\Phi(y, z_0) = 0$ is given by

$$\Phi(y, z) = c_{\kappa,n} e^{ip\cdot y} z^2 J_{\kappa}(M_{\kappa,n} z)$$  \hspace{1cm} (4)

where $\kappa = \Delta - 2$ with $\Delta$ the conformal dimension of the state, $M_{\kappa,n} z_0$ denotes the $n$-th zero point of the Bessel function $J_{\kappa}$ and $c_{\kappa,n}$ is the normalization factor

$$c_{\kappa,n} = \frac{\sqrt{2}}{z_0 |J_{\kappa+1}(M_{\kappa,n} z_0)|}.$$  \hspace{1cm} (5)
In order to calculate the subleading large $N_c$ contribution from the final multiple-hadron states, we need the bulk-to-bulk propagator of dilatons in AdS$_5$ space which is given by

$$G(y, z; y', z') = -\int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (y-y')} \int_{0}^{\infty} d\omega \frac{\omega}{\omega^2 + k^2 - i\epsilon} z^2 J_k(\omega z) z'^2 J_k(\omega' z'),$$

With the consideration of boundary condition, the accurate propagator should take the discrete form

$$G(y, z; y', z') = -\int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (y-y')} \sum_{M, n} \frac{M_{k,n}^2 \epsilon_{k,n}^2}{M_{k,n}^2 + k^2 - i\epsilon} z^2 J_k(M_{k,n} z) z'^2 J_k(M_{k,n} z'),$$

The gauge field corresponding to the current satisfies the Maxwell’s equations in AdS$_5$ space and the nonnormalizable solution with the boundary condition $A_\mu(y, \infty) = n_\mu e^{iqy}$ and the Lorentz-like gauge fixing $\partial_\mu A^\mu + z \partial_z (A_z/z) = 0$ is given by

$$A_\mu = n_\mu e^{iqy} qz K_1(qz), \quad A_z = i n \cdot q e^{iqy} z K_0(qz),$$

where $K_1$ and $K_0$ are both modified Bessel functions and $n_\mu$ is the polarization vector.

When we were working in the leading large $N_c$ approximation, only single hadron in the final states was needed. The corresponding Witten diagram for the hadronic tensor can be represented in Fig. I in which the dashed cut line denotes the final states. In our present work, we will devote ourselves to calculating the subleading large $N_c$ contribution and analyzing the power dependence of $1/q$. It should be mentioned that the complete subleading contribution of large $N_c$ can come from different origins, however in our paper, we will only restrict ourselves to consider the contribution by introducing an extra hadron into the final states. For further simplicity, we will only consider the spinless hadron and the final states only include two dilatons, in which the gauge propagator and gravity propagator do not contribute. The relevant supergravity interaction is assumed as

$$S = -\int d^5 x \sqrt{-g} \left[ \sum_{i=1}^{3} D^m \Phi_i D_m \Phi_i^* + \sum_{i=1}^{3} \mu_i^2 \Phi_i^* \Phi_i + \lambda \Phi_1 \Phi_2 \Phi_3^* + \lambda \Phi_1^* \Phi_2 \Phi_3 \right]$$

$$= -\int d^5 x \sqrt{-g} \left[ \sum_{i=1}^{3} \partial^m \Phi_i \partial_m \Phi_i^* + \sum_{i=1}^{3} \mu_i^2 \Phi_i^* \Phi_i + A^m A_m \sum_{i=1}^{3} Q_i \Phi_i^* \Phi_i 

+ iA^m \sum_{i=1}^{3} Q_i (\Phi_i \partial_m \Phi_i^* - \Phi_i^* \partial_m \Phi_i) + \lambda \Phi_1 \Phi_2 \Phi_3^* + \lambda \Phi_1^* \Phi_2 \Phi_3 \right].$$

where we have introduced three different dilatons $(i = 1, 2, 3)$ which have different charges $Q_i$ with $Q_1 + Q_2 + Q_3 = 0$ and five-dimensional mass $\mu_i^2 = \Delta_i(\Delta_i - 4)/R^2$ with $\Delta_i$ the conformal dimension of the states and $R$ the AdS radius.
It follows that the subleading contributions of large \( N_c \) with two dilatons in the final states come from the Witten diagrams in Fig. [2] Fig. [3] Fig. [4] and the other six crossed-channel diagrams which have not been displayed here. The Witten diagrams with cutline here actually represent the squared amplitudes: the left to the cut line means the transition amplitude and the right means the complex conjugate. Therefore, in order to calculate the hadronic tensor, we need calculate the transition amplitudes first. From the action given in Eq. (9), it is straightforward to write down all the transition amplitudes corresponding to different channels: \( s \)-channel amplitude,

\[
\mathcal{M}_s = iQ_1 \int d^5xd^5x' \sqrt{-g(x)} \sqrt{-g(x')} \Phi_1(x) A^M(x) \left[ \partial_M G(x, x') \right] \Phi_1^*(x') \Phi_3^*(x') \\
- iQ_1 \int d^5xd^5x' \sqrt{-g(x)} \sqrt{-g(x')} \left[ \partial_M \Phi_1(x) \right] A^M(x) G(x, x') \Phi_2^*(x') \Phi_3^*(x')
\]

\( t \)-channel amplitude,

\[
\mathcal{M}_t = - iQ_2 \int d^5xd^5x' \sqrt{-g(x)} \sqrt{-g(x')} \Phi_1(x) \Phi_3^*(x) \left[ \partial'_M G(x, x') \right] A^M(x') \Phi_2^*(x') \\
+ iQ_2 \int d^5xd^5x' \sqrt{-g(x)} \sqrt{-g(x')} \Phi_1(x) \Phi_3^*(x) G(x, x') A^M(x') \left[ \partial'_M \Phi_2^*(x') \right]
\]

and \( u \)-channel amplitude,

\[
\mathcal{M}_u = - iQ_3 \int d^5xd^5x' \sqrt{-g(x)} \sqrt{-g(x')} \Phi_1(x) \Phi_3^*(x) \left[ \partial'_M G(x, x') \right] A^M(x') \Phi_3^*(x') \\
+ iQ_3 \int d^5xd^5x' \sqrt{-g(x)} \sqrt{-g(x')} \Phi_1(x) \Phi_3^*(x) G(x, x') A^M(x') \left[ \partial'_M \Phi_3^*(x') \right]
\]

where \( x = (y, z) \), \( x' = (y', x') \) and

\[
A^\mu(x) = r^\mu e^{iqy} qz^3 K_1(qz), \quad A^z(x) = i n \cdot q e^{iqy} z^3 K_0(qz), \\
\Phi_1(x) = c_1 z^2 J_{\kappa_1}(M_1 z) e^{ipy}, \quad \Phi_2^*(x) = c_2 z^2 J_{\kappa_2}(M_2 z) e^{-iq'y}, \\
\Phi_3^*(x) = c_3 z^2 J_{\kappa_3}(M_3 z) e^{-iq'y}
\]

The main task of the remaining parts of this work is to calculate the above transition amplitudes, then square them to obtain the hadronic tensor and finally extract the structure functions.
Fig. 1: Leading large-$N_c$ contribution from the single-hadron final states

Fig. 2: $s$-channel contribution from the double-hadron final states
Fig. 3: $t$-channel contribution from the double-hadron final states

Fig. 4: $u$-channel contribution from the double-hadron final states
III. CALCULATION OF THE STRUCTURE FUNCTIONS

Substituting the wave functions of the initial or final states in Eq.(13) into the transition amplitudes and integrating out the boundary coordinates \( y \) and \( y' \), we can have

\[
\mathcal{M}_s = Q_1c_1c_2c_3(2\pi)^4\delta^4(p + q - p' - q') n \cdot \left(2p + \frac{1}{x}q\right)
\times \int dzdz'\frac{q}{z'}J_{\kappa_1}(M_1z)K_1(qz)G_s(z, z')J_{\kappa_2}(M_2z')J_{\kappa_3}(M_3z')
- Q_1(2\pi)^4\delta^4(p + q - p' - q') n \cdot q
\times \frac{1}{q} \int dz'z'^2J_{\kappa_1}(M_1z')K_1(qz')J_{\kappa_2}(M_2z')J_{\kappa_3}(M_3z') \tag{14}
\]

\[
\mathcal{M}_t = Q_2c_1c_2c_3(2\pi)^4\delta^4(p + q - p' - q') n \cdot (2q' + \frac{1}{y'}q)
\times \int dzdz'\frac{q}{z}J_{\kappa_1}(M_1z)J_{\kappa_3}(M_3z)G_t(z, z')K_1(qz')J_{\kappa_2}(M_2z')
+ Q_2(2\pi)^4\delta^4(p + q - p' - q') n \cdot q
\times \frac{1}{q} \int dzz^2J_{\kappa_1}(M_1z)K_1(qz)J_{\kappa_2}(M_2z)J_{\kappa_3}(M_3z) \tag{15}
\]

\[
\mathcal{M}_u = Q_3c_1c_2c_3(2\pi)^4\delta^4(p + q - p' - q') n \cdot (2p' + \frac{1}{x'}q)
\times \int dzdz'\frac{q}{z}J_{\kappa_1}(M_1z)J_{\kappa_2}(M_2z)G_u(z, z')K_1(qz')J_{\kappa_3}(M_3z')
+ Q_3(2\pi)^4\delta^4(p + q - p' - q') n \cdot q
\times \frac{1}{q} \int dzz^2J_{\kappa_1}(M_1z)K_1(qz)J_{\kappa_2}(M_2z)J_{\kappa_3}(M_3z) \tag{16}
\]

where we have defined three scalar variables

\[
x = -\frac{q^2}{2p \cdot q}, \quad x' = -\frac{q^2}{2p' \cdot q}, \quad y' = -\frac{q^2}{2q' \cdot q} \tag{17}
\]

and the reduced bulk-to-bulk propagators in the holographic radial coordinate which are given by

\[
G_s(z, z') = -\int_0^\infty d\omega \frac{\omega c_s^2}{\omega^2 + (p + q)^2 - i\epsilon} z^2 J_{\kappa_1}(\omega z) z'^2 J_{\kappa_1}(\omega z'), \tag{18}
\]

\[
G_t(z, z') = -\int_0^\infty d\omega \frac{\omega c_t^2}{\omega^2 + (p' - p)^2 - i\epsilon} z^2 J_{\kappa_2}(\omega z) z'^2 J_{\kappa_2}(\omega z'), \tag{19}
\]

\[
G_u(z, z') = -\int_0^\infty d\omega \frac{\omega c_u^2}{\omega^2 + (p' - q)^2 - i\epsilon} z^2 J_{\kappa_3}(\omega z) z'^2 J_{\kappa_3}(\omega z'), \tag{20}
\]

8
which correspond to $s$-channel, $t$-channel and $u$-channel, respectively. It should be noted that for brevity we will use the integral notation instead of the sum notation in the propagator. However, we have introduced the normalization factors $c_s$, $c_t$ and $c_u$ so that they can be consistent with the cut off in the AdS space. The total transition amplitude is obtained by summing over all the contributions from different channels

$$\mathcal{M} = \mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_t$$

where we have defined

$$C_s = \mathcal{Q}_1 \int dz dz' \frac{q}{z} J_{\kappa_1}(M_1 z) K_1(q z) G_s(z, z') J_{\kappa_3}(M_3 z')$$

$$C_t = \mathcal{Q}_2 \int dz dz' \frac{q}{z} J_{\kappa_1}(M_1 z) J_{\kappa_3}(M_3 z) G_t(z, z') K_1(q z') J_{\kappa_2}(M_2 z')$$

$$C_u = \mathcal{Q}_3 \int dz dz' \frac{q}{z} J_{\kappa_1}(M_1 z) J_{\kappa_2}(M_2 z) G_u(z, z') K_1(q z') J_{\kappa_3}(M_3 z')$$

From the relation between the hadronic tensor and the squared transition amplitude

$$n^\mu n^\nu W_{\mu\nu} = \mathcal{M} \mathcal{M}^*$$

and the definitions of the structure functions in Eq. (2), we can extract the the structure functions in the Bjorken limit $q \to \infty$ with $x$ fixed

$$F_1(x, q^2) = c_s^2 \sum_{M_2} \sum_{M_3} c_2 c_3^2 \int \frac{d^3 p'}{2E_p(2\pi)^3} \frac{d^3 q'}{2E_q(2\pi)^3} (2\pi)^4 \delta^4(p + q - p' - q')$$

$$\times 2q^2 \left\{ [v_u^2 + 4x^2(v_u \cdot v_u)^2] C_u C_u^* + [v_t^2 + 4x^2(v_s \cdot v_t)^2] C_t C_t^* + [v_s \cdot v_t + 12x^2(v_u \cdot v_s) v_s] (C_u C_t^* + C_t C_u^*) \right\}$$

$$F_2(x, q^2) = c_t^2 \sum_{M_2} \sum_{M_3} c_2 c_3^2 \int \frac{d^3 p'}{2E_p(2\pi)^3} \frac{d^3 q'}{2E_q(2\pi)^3} (2\pi)^4 \delta^4(p + q - p' - q')$$

$$\times 4xq^2 \left\{ [v_s^2 + 12x^2 v_s^2] C_s C_s^* + [v_u^2 + 12x^2(v_u \cdot v_s)^2] C_u C_u^* + [v_t^2 + 12x^2(v_t \cdot v_s)^2] C_t C_t^* + [v_s \cdot v_t + 12x^2(v_u \cdot v_s) v_s] (C_s C_t^* + C_t C_s^*) \right\}$$

where we have define three vectors as

$$v_s^\mu = \frac{1}{q} \left( p^\mu + \frac{q^\mu}{2x} \right), \quad v_u^\mu = \frac{1}{q} \left( p^\mu + \frac{q^\mu}{2y'} \right), \quad v_t^\mu = \frac{1}{q} \left( q^\mu + \frac{q^\mu}{2y'} \right)$$
In order to extract the leading contribution in the Bjorken limit \( q \to \infty \) with \( x \) fixed, it is convenient to define the following scaled variables

\[
\hat{p}^\mu = p^\mu / q, \quad \hat{q}^\mu = q^\mu / q, \quad \hat{\omega} = \omega / q, \quad \hat{z} = qz, \quad \hat{z}' = qz'
\]

(29)

With these scaled variables, we can rewrite the structure functions as

\[
F_1(x, q^2) = c_1^2 \sum_{M_2} \sum_{M_3} c_2^2 c_3^2 \int \frac{d^3 \hat{p}'}{2E_{p'}(2\pi)^3} \frac{d^3 \hat{q}'}{2E_{q'}(2\pi)^3} (2\pi)^4 \delta^4 (\hat{p} + \hat{q} - \hat{p}' - \hat{q}')
\]

\[
\times \frac{2}{q^6} \left\{ \left[ v_u^2 + 4x^2 (v_s \cdot v_u)^2 \right] \hat{C}_u \hat{C}_u^* + \left[ v_t^2 + 4x^2 (v_s \cdot v_t)^2 \right] \hat{C}_t \hat{C}_t^* + \left[ v_u \cdot v_t + 4x^2 (v_s \cdot v_u)(v_s \cdot v_t) \right] \left( \hat{C}_u \hat{C}_t^* + \hat{C}_t \hat{C}_u^* \right) \right\}
\]

\[
F_2(x, q^2) = c_1^2 \sum_{M_2} \sum_{M_3} c_2^2 c_3^2 \int \frac{d^3 \hat{p}'}{2E_{p'}(2\pi)^3} \frac{d^3 \hat{q}'}{2E_{q'}(2\pi)^3} (2\pi)^4 \delta^4 (\hat{p} + \hat{q} - \hat{p}' - \hat{q}')
\]

\[
\times \frac{4x}{q^6} \left\{ \left[ v_s^2 + 12x^2 v_s^4 \right] \hat{C}_s \hat{C}_s^* + \left[ v_u^2 + 12x^2 (v_s \cdot v_u)^2 \right] \hat{C}_u \hat{C}_u^* + \left[ v_s \cdot v_u + 12x^2 (v_s \cdot v_u) v_s^2 \right] \left( \hat{C}_s \hat{C}_u^* + \hat{C}_u \hat{C}_s^* \right) + \left[ v_u \cdot v_t + 12x^2 (v_t \cdot v_s)(v_u \cdot v_s) \right] \left( \hat{C}_u \hat{C}_t^* + \hat{C}_t \hat{C}_u^* \right) \right\}
\]

(30)

where

\[
\hat{C}_s = Q_1 \int \frac{dz d\hat{z}}{\hat{z}} J_{k_1}(\hat{M}_1 \hat{z}) K_1(\hat{z}) \hat{G}_s(\hat{z}, \hat{z}') J_{k_2}(\hat{M}_2 \hat{z}') J_{k_3}(\hat{M}_3 \hat{z}')
\]

(32)

\[
\hat{C}_u = Q_3 \int \frac{dz d\hat{z}}{\hat{z}} J_{k_1}(\hat{M}_1 \hat{z}) J_{k_2}(\hat{M}_2 \hat{z}) \hat{G}_u(\hat{z}, \hat{z}') K_1(\hat{z}') J_{k_3}(\hat{M}_3 \hat{z}')
\]

(33)

\[
\hat{C}_t = Q_2 \int \frac{dz d\hat{z}}{\hat{z}} J_{k_1}(\hat{M}_1 \hat{z}) J_{k_3}(\hat{M}_3 \hat{z}) \hat{G}_t(\hat{z}, \hat{z}') K_1(\hat{z}') J_{k_2}(\hat{M}_2 \hat{z}')
\]

(34)

with

\[
\hat{G}_s(\hat{z}, \hat{z}') = -\int_0^\infty d\hat{\omega} \frac{\hat{\omega} c_s^2}{\hat{\omega}^2 - \hat{s} - i\epsilon} \hat{z}^2 J_{k_1}(\hat{\omega} \hat{z}) \hat{z}'^2 J_{k_1}(\hat{\omega} \hat{z}')
\]

(35)

\[
\hat{G}_t(\hat{z}, \hat{z}') = -\int_0^\infty d\hat{\omega} \frac{\hat{\omega} c_t^2}{\hat{\omega}^2 - \hat{t} - i\epsilon} \hat{z}^2 J_{k_2}(\hat{\omega} \hat{z}) \hat{z}'^2 J_{k_2}(\hat{\omega} \hat{z}')
\]

(36)

\[
\hat{G}_u(\hat{z}, \hat{z}') = -\int_0^\infty d\hat{\omega} \frac{\hat{\omega} c_u^2}{\hat{\omega}^2 - \hat{u} - i\epsilon} \hat{z}^2 J_{k_3}(\hat{\omega} \hat{z}) \hat{z}'^2 J_{k_3}(\hat{\omega} \hat{z}')
\]

(37)

It can be noticed that we need to deal with the integrals over triple Bessel functions, which in general cannot be calculated analytically. However we can choose some special cases, e.g.,
Now let us choose the center-mass frame of the initial dilaton and virtual photon, where

\[ p^\mu = \left( \frac{q}{2\sqrt{x(1-x)}}, \frac{q}{2\sqrt{x(1-x)}}, 0, 0 \right), \quad q^\mu = \left( \frac{(1-2x)q}{2\sqrt{x(1-x)}}, -\frac{q}{2\sqrt{x(1-x)}}, 0, 0 \right) \]
It follows that

\begin{align*}
F_1(x, q^2) & = \sum_{M_2} \sum_{M_3} \frac{c_1^2 c_2^2 c_3^2 |\hat{p}'|}{4\pi q^6} \sqrt{\frac{x}{1-x}} \int d\theta \sin \theta \\
& \times \left\{ [v_u^2 + 4x^2(v_u \cdot v_u)^2] \hat{C}_u \hat{C}_u^* + [v_t^2 + 4x^2(v_t \cdot v_t)^2] \hat{C}_t \hat{C}_t^* \\
& \quad + [v_u \cdot v_t + 4x^2(v_u \cdot v_u)(v_t \cdot v_t)] \left( \hat{C}_u \hat{C}_t^* + \hat{C}_t \hat{C}_u^* \right) \right\} \\
F_2(x, q^2) & = \sum_{M_2} \sum_{M_3} \frac{c_1^2 c_2^2 c_3^2 |\hat{p}'|}{2\pi q^6} \sqrt{\frac{x}{1-x}} \int d\theta \sin \theta \\
& \times \left\{ [v_s^2 + 12x^2 v_s^4] \hat{C}_s \hat{C}_s^* + [v_u^2 + 12x^2(v_u \cdot v_s)^2] \hat{C}_u \hat{C}_u^* + [v_t^2 + 12x^2(v_t \cdot v_s)^2] \hat{C}_t \hat{C}_t^* \\
& \quad + [v_u \cdot v_t + 12x^2(v_u \cdot v_s)(v_t \cdot v_s)] \left( \hat{C}_u \hat{C}_t^* + \hat{C}_t \hat{C}_u^* \right) \right\}
\end{align*}

(45) (46)

IV. POWER ANALYSIS

In order to extract the leading contribution in the Bjorken limit \( q \to \infty \), we need analyze the power dependence of the structure functions on \( 1/q \) in different kinetic ranges. In our work, we will always assume \( \hat{M}_1 \ll 1 \) for the initial hadron. Hence we can classify the kinetic ranges into four different parts according to the masses of the final hadrons: \( \hat{M}_2 \sim 1 \) & \( \hat{M}_3 \ll 1 \), \( \hat{M}_2 \ll 1 \) & \( \hat{M}_3 \sim 1 \), \( \hat{M}_2 \sim 1 \) & \( \hat{M}_3 \ll 1 \) and \( \hat{M}_2 \sim 1 \) & \( \hat{M}_3 \sim 1 \). Now let us deal with them one by one.

A. \( \hat{M}_2 \sim 1 \) & \( \hat{M}_3 \sim 1 \)

In this region, we can reduce the integrals in Eqs. (41), (42) and (43) to

\begin{align*}
\hat{C}_s & \approx -Q_1 \frac{16\hat{M}_1}{\hat{M}_3} \int_{|\hat{M}_3 - \hat{M}_2|}^{\hat{M}_3 + \hat{M}_2} d\hat{\omega} \frac{\hat{\omega} C_s^2}{(\hat{\omega}^2 - \hat{s} - i\epsilon)(\hat{\omega}^2 + 1)^3} \\
& \times \frac{\hat{M}_3^2 + \hat{\omega}^2 - \hat{M}_2^2}{[(\hat{M}_3 + \hat{M}_2)^2 - \hat{\omega}^2]^\frac{3}{2} \left[ \hat{\omega}^2 - (\hat{M}_3 - \hat{M}_2)^2 \right]^{\frac{1}{2}}} \tag{47}
\end{align*}
\[
\hat{C}_t \approx -Q_2 c_t^2 \frac{4\pi \tilde{M}_1 \tilde{M}_3 (\tilde{M}_2^2 + \tilde{M}_3^2 + 1)}{[(\tilde{M}_2 + \tilde{M}_3)^2 + 1]^{\frac{3}{2}} [(\tilde{M}_2 - \tilde{M}_3)^2 + 1]^{\frac{3}{2}} (\tilde{M}_3^2 - \hat{t})^2} 
\]

(48)

\[
\hat{C}_u \approx Q_3 c_u \frac{16\pi \tilde{M}_1 \tilde{M}_3 \hat{u}}{[(\tilde{M}_3 + \tilde{M}_2)^2 + 1]^{\frac{3}{2}} [(\tilde{M}_3 - \tilde{M}_2)^2 + 1]^{\frac{3}{2}} (\tilde{M}_2^2 - \hat{u})^2}.
\]

(49)

The normalization coefficient \(c_1\) for the initial hadron in Eqs. (45) and (46) is always the order of 1 when \(\tilde{M}_1 \ll 1\), while the normalization coefficients \(c_2 \sim q^\frac{1}{2}, c_3 \sim q^\frac{1}{2}\) for the final hadron states by using the asymptotic behavior of the Bessel function. Besides, the sum over \(M_2\) and \(M_3\) contribute \(\sum M_2 \sum M_3 \sim 1\). In order to obtain the final power behavior, we need separate this kinetic region further according to the momentum of the internal propagator, which are given in Tab.I. It should be pointed out that the subscripts in the structure functions denote

| kinetic region | \(c_s, c_u, c_t\) | \(\hat{C}_s, \hat{C}_u, \hat{C}_t\) | phase space | structure functions |
|----------------|------------------|------------------|-------------|------------------|
| \(\hat{t} \sim 1\) \(\hat{u} \sim 1\) | \(c_s \sim q^\frac{1}{2}\) \(c_u \sim q^\frac{1}{2}\) \(c_t \sim q^\frac{1}{2}\) | \(\hat{C}_s \sim 1\) \(\hat{C}_u \sim 1\) \(\hat{C}_t \sim 1\) | \(\int \sin \theta d\theta \sim 1\) | \(F_{ss} \sim \frac{1}{q^3}, F_{uu} \sim \frac{1}{q^3}\) |
| \(\hat{t} \ll 1\) \(\hat{u} \sim 1\) | \(c_s \sim q^\frac{1}{2}\) \(c_u \sim q^\frac{1}{2}\) \(c_t \sim 1\) | \(\hat{C}_s \sim 1\) \(\hat{C}_u \sim 1\) \(\hat{C}_t \sim \frac{1}{q}\) | \(\int \sin \theta d\theta \sim \frac{1}{q}\) | \(F_{ss} \sim \frac{1}{q^4}, F_{uu} \sim \frac{1}{q^4}\) |
| \(\hat{t} \sim 1\) \(\hat{u} \ll 1\) | \(c_s \sim q^\frac{1}{2}\) \(c_u \sim 1\) \(c_t \sim q^\frac{1}{2}\) | \(\hat{C}_s \sim 1\) \(\hat{C}_u \sim \frac{1}{q^2}\) \(\hat{C}_t \sim 1\) | \(\int \sin \theta d\theta \sim \frac{1}{q}\) | \(F_{ss} \sim \frac{1}{q^4}, F_{uu} \sim \frac{1}{q^4}\) |
| \(\hat{t} \ll 1\) \(\hat{u} \ll 1\) | \(c_s \sim q^\frac{1}{2}\) \(c_u \sim 1\) \(c_t \sim q^\frac{1}{2}\) | \(\hat{C}_s \sim 1\) \(\hat{C}_u \sim \frac{1}{q^2}\) \(\hat{C}_t \sim 1\) | \(\int \sin \theta d\theta \sim \frac{1}{q}\) | \(F_{ss} \sim \frac{1}{q^4}, F_{uu} \sim \frac{1}{q^4}\) |

Tab. I: \(\tilde{M}_2 \sim 1\) & \(\tilde{M}_3 \sim 1\)

the contribution from different channels, e.g., \(F_{ss}\) means the contribution to the structure functions from the term \(\hat{C}_s \hat{C}_s^*\), \(F_{ut}\) means the contribution from \(\hat{C}_u \hat{C}_t^* + \hat{C}_t \hat{C}_u^*\) and so on. It is easy to show that \(\hat{t} + \hat{u} \sim 1\), which implies that we do not need to consider the case where \(t \ll 1\) and \(u \ll 1\) at the same time.
B. \( \dot{M}_2 \ll 1 \) & \( \dot{M}_3 \sim 1 \)

In this region, we can have

\[
\dot{C}_s \approx Q_1 c_s^2 \frac{16\pi \dot{M}_1 \dot{M}_3 \dot{s}}{\left( \dot{M}_3^2 + 1 \right)^3 \left( \dot{s} - \dot{M}_3^2 \right)^2} \tag{50}
\]

\[
\dot{C}_u \approx -Q_3 c_u^2 \frac{8\pi \dot{M}_3}{\dot{M}_1 \left( \dot{M}_3^2 + 1 \right)^3} \frac{\dot{M}_1^2 - \dot{M}_2^2 + u + \sqrt{\left( \dot{M}_1^2 - \dot{M}_2^2 + u \right)^2 - 4u \dot{M}_1^2}}{\sqrt{\left( \dot{M}_1^2 - \dot{M}_2^2 + u \right)^2 - 4u \dot{M}_1^2}} \tag{51}
\]

\[
\dot{C}_t \approx -Q_2 c_t^2 \frac{4\pi \dot{M}_1 \dot{M}_3}{\left( \dot{M}_3^2 + 1 \right)^2} \frac{\dot{M}_2^2 - t}{\left( \dot{M}_3^2 - t \right)^2} \tag{52}
\]

The normalization coefficients \( c_2 \sim 1 \), \( c_3 \sim q^{\frac{1}{2}} \) and the sum over \( M_2 \) and \( M_3 \) contribute \( \sum_{M_2} \sum_{M_3} \sim q \). The detailed power analysis in different kinetic intervals is given in Tab.II.

| kinetic region | \( \hat{t} \sim 1 \) | \( \hat{u} \sim 1 \) | \( \hat{C}_s, \hat{C}_u, \hat{C}_t \) | phase space | structure functions |
|---------------|-----------------|-----------------|-----------------|-------------|-----------------|
| \( \dot{t} \ll 1 \) | \( \dot{t} \sim 1 \) | \( \dot{u} \sim 1 \) | \( \hat{C}_s \sim 1 \) | \( \hat{C}_u \sim 1 \) | \( \hat{C}_t \sim 1 \) | \( F_{ss} \sim \frac{1}{q}, F_{uu} \sim \frac{1}{q} \) |
|              | \( c_s \sim q^{\frac{1}{2}} \) | \( c_u \sim q^{\frac{1}{2}} \) | \( c_t \sim q^{\frac{1}{2}} \) | \( f \sin \theta d\theta \sim 1 \) | \( F_{tt} \sim \frac{1}{q}, F_{su} \sim \frac{1}{q} \) | \( F_{st} \sim \frac{1}{q}, F_{ut} \sim \frac{1}{q} \) |

| \( \dot{t} \ll 1 \) | \( \dot{t} \sim 1 \) | \( \dot{u} \sim 1 \) | \( \hat{C}_s \sim 1 \) | \( \hat{C}_u \sim 1 \) | \( \hat{C}_t \sim 1 \) | \( F_{ss} \sim \frac{1}{q}, F_{uu} \sim \frac{1}{q} \) |
|              | \( c_s \sim q^{\frac{1}{2}} \) | \( c_u \sim q^{\frac{1}{2}} \) | \( c_t \sim 1 \) | \( f \sin \theta d\theta \sim \frac{1}{q} \) | \( F_{tt} \sim \frac{1}{q}, F_{su} \sim \frac{1}{q} \) | \( F_{st} \sim \frac{1}{q}, F_{ut} \sim \frac{1}{q} \) |

| \( \dot{t} \sim 1 \) | \( \dot{t} \ll 1 \) | \( \dot{u} \sim 1 \) | \( \hat{C}_s \sim 1 \) | \( \hat{C}_u \sim 1 \) | \( \hat{C}_t \sim 1 \) | \( F_{ss} \sim \frac{1}{q}, F_{uu} \sim \frac{1}{q} \) |
|              | \( c_s \sim q^{\frac{1}{2}} \) | \( c_u \sim 1 \) | \( c_t \sim q^{\frac{1}{2}} \) | \( f \sin \theta d\theta \sim \frac{1}{q} \) | \( F_{tt} \sim \frac{1}{q}, F_{su} \sim \frac{1}{q} \) | \( F_{st} \sim \frac{1}{q}, F_{ut} \sim \frac{1}{q} \) |

Tab. II: \( \dot{M}_2 \ll 1 \) & \( \dot{M}_3 \sim 1 \)
C. $\hat{M}_2 \sim 1$ & $\hat{M}_3 \ll 1$

In this region, we can have

\[
\hat{C}_s \approx Q_1 c_s^2 \frac{16\pi \hat{M}_1 \hat{M}_3 \hat{s}}{\left( \hat{M}_2^2 + 1 \right)^3 \left( \hat{s} - \hat{M}_2^2 \right)^2}
\]  
(53)

\[
\hat{C}_u \approx Q_3 c_u^2 \frac{16\pi \hat{M}_3 \hat{M}_1 u}{\left( \hat{M}_2^2 + 1 \right)^3 \left( \hat{M}_2^2 - u \right)^2}
\]  
(54)

\[
\hat{C}_t = -Q_2 c_t^2 \frac{2\pi}{\hat{M}_1 \hat{M}_3 \left( \hat{M}_2^2 + 1 \right)^2} \frac{\hat{M}_1^2 + \hat{M}_3^2 - t - \sqrt{\left( \hat{M}_1^2 + \hat{M}_3^2 - t \right)^2 - 4\hat{M}_1^2 \hat{M}_3^2}}{\left( \hat{M}_1^2 + \hat{M}_3^2 - t \right)^2 - 4\hat{M}_1^2 \hat{M}_3^2}
\]  
(55)

The normalization coefficients $c_2 \sim q^{\frac{3}{2}}$, $c_3 \sim 1$ and the sum over $M_2$ and $M_3$ contribute $\sum M_2 \sum M_3 \sim q$. The detailed power analysis in different kinetic intervals is given in Tab. III.

| kinetic region | $c_s, c_u, c_t$ | $\hat{C}_s, \hat{C}_u, \hat{C}_t$ | phase space | structure functions |
|----------------|----------------|---------------------------------|-------------|--------------------|
| $\hat{t} \sim 1$ | $c_s \sim q^{\frac{1}{2}}$, $c_u \sim q^{\frac{1}{2}}$, $c_t \sim q^{\frac{1}{2}}$ | $\hat{C}_s \sim \frac{1}{q}$, $\hat{C}_u \sim \frac{1}{q}$, $\hat{C}_t \sim \frac{1}{q}$ | $\int \sin \theta d\theta \sim 1$ | $F_{ss} \sim \frac{1}{q^2}$, $F_{uu} \sim \frac{1}{q^2}$, $F_{tt} \sim \frac{1}{q^2}$, $F_{su} \sim \frac{1}{q^2}$, $F_{st} \sim \frac{1}{q^2}$, $F_{ut} \sim \frac{1}{q^2}$ |
| $\hat{u} \sim 1$ | $c_s \sim q^{\frac{1}{2}}$, $c_u \sim q^{\frac{1}{2}}$, $c_t \sim 1$ | $\hat{C}_s \sim \frac{1}{q}$, $\hat{C}_u \sim \frac{1}{q}$, $\hat{C}_t \sim q^{\frac{1}{2}}$ | $\int \sin \theta d\theta \sim \frac{1}{q}$ | $F_{ss} \sim \frac{1}{q^2}$, $F_{uu} \sim \frac{1}{q^2}$, $F_{tt} \sim \frac{1}{q^2}$, $F_{su} \sim \frac{1}{q^2}$, $F_{st} \sim \frac{1}{q^2}$, $F_{ut} \sim \frac{1}{q^2}$ |
| $\hat{t} \ll 1$ | $c_s \sim q^{\frac{1}{2}}$, $c_u \sim q^{\frac{1}{2}}$, $c_t \sim 1$ | $\hat{C}_s \sim \frac{1}{q}$, $\hat{C}_u \sim \frac{1}{q}$, $\hat{C}_t \sim q^{\frac{1}{2}}$ | $\int \sin \theta d\theta \sim \frac{1}{q}$ | $F_{ss} \sim \frac{1}{q^2}$, $F_{uu} \sim \frac{1}{q^2}$, $F_{tt} \sim \frac{1}{q^2}$, $F_{su} \sim \frac{1}{q^2}$, $F_{st} \sim \frac{1}{q^2}$, $F_{ut} \sim \frac{1}{q^2}$ |
| $\hat{u} \ll 1$ | $c_s \sim q^{\frac{1}{2}}$, $c_u \sim 1$, $c_t \sim q^{\frac{1}{2}}$ | $\hat{C}_s \sim \frac{1}{q}$, $\hat{C}_u \sim q^{\frac{1}{2}}$, $\hat{C}_t \sim \frac{1}{q}$ | $\int \sin \theta d\theta \sim \frac{1}{q}$ | $F_{ss} \sim \frac{1}{q^2}$, $F_{uu} \sim \frac{1}{q^2}$, $F_{tt} \sim \frac{1}{q^2}$, $F_{su} \sim \frac{1}{q^2}$, $F_{st} \sim \frac{1}{q^2}$, $F_{ut} \sim \frac{1}{q^2}$ |

Tab. III: $\hat{M}_2 \sim 1$ & $\hat{M}_3 \ll 1$
D. \( \hat{M}_2 \ll 1 \& \hat{M}_3 \ll 1 \)

In this region, we can have

\[
\hat{C}_s \approx Q_1 c_s^2 \frac{16\pi \hat{M}_1 \hat{M}_3}{\hat{s}}
\]

\[
\hat{C}_u \approx -Q_3 c_u^2 \frac{8\pi \hat{M}_3^2}{M_1} \hat{M}_1^2 - \hat{M}_2^2 + u + \sqrt{\left(\hat{M}_1^2 - \hat{M}_2^2 + u\right)^2 - 4u \hat{M}_1^2}
\]

\[
\hat{C}_t = -Q_2 c_t^2 \frac{2\pi}{M_1 \hat{M}_3} \hat{M}_1^2 + \hat{M}_3^2 - t - \sqrt{\left(\hat{M}_1^2 + \hat{M}_3^2 - t\right)^2 - 4\hat{M}_1^2 \hat{M}_3^2}
\]

the normalization coefficients \( c_2 \sim 1, \ c_3 \sim 1 \), and the sum over \( M_2 \) and \( M_3 \) contribute \( \sum_{M_2} \sum_{M_3} \sim 1 \). The detailed power analysis in different kinetic intervals is given in Tab.IV.

| kinetic region | \( c_s, c_u, c_t \) | \( \hat{C}_s, \hat{C}_u, \hat{C}_t \) | phase space | structure functions |
|----------------|---------------------|---------------------|--------------|---------------------|
| \( \hat{t} \sim 1 \) \( \hat{u} \sim 1 \) | \( c_s \sim q^2 \) \( c_u \sim q^2 \) \( c_t \sim q^2 \) | \( \hat{C}_s \sim \frac{1}{q} \) \( \hat{C}_u \sim \frac{1}{q} \) \( \hat{C}_t \sim \frac{1}{q} \) | \( \int \sin \theta d\theta \sim 1 \) | \( F_{ss} \sim \frac{1}{q^2}, F_{uu} \sim \frac{1}{q^2} \) |
| \( \hat{t} \ll 1 \) \( \hat{u} \sim 1 \) | \( c_s \sim q^2 \) \( c_u \sim q^2 \) \( c_t \sim 1 \) | \( \hat{C}_s \sim \frac{1}{q} \) \( \hat{C}_u \sim \frac{1}{q} \) \( \hat{C}_t \sim q^2 \) | \( \int \sin \theta d\theta \sim \frac{1}{q} \) | \( F_{ss} \sim \frac{1}{q^2}, F_{uu} \sim \frac{1}{q^2} \) |
| \( t \sim 1 \) \( u \ll 1 \) | \( c_s \sim q^2 \) \( c_u \sim 1 \) \( c_t \sim q^2 \) | \( \hat{C}_s \sim \frac{1}{q} \) \( \hat{C}_u \sim \frac{1}{q} \) \( \hat{C}_t \sim \frac{1}{q} \) | \( \int \sin \theta d\theta \sim \frac{1}{q} \) | \( F_{ss} \sim \frac{1}{q^2}, F_{uu} \sim \frac{1}{q^2} \) |

Tab. IV: \( \hat{M}_2 \ll 1 \& \hat{M}_3 \ll 1 \)

E. The final dominated contribution

From the above analysis, we find that the dominated contribution is from the \( t \)-channel where \( \hat{M}_1 \ll 1 \) \( \hat{M}_3 \ll 1 \), \( \hat{M}_2 \sim 1 \) with \( \hat{t} \ll 1 \) and \( \hat{u} \sim 1 \). Hence, the leading contribution is
given by

\[ F_1(x, q^2) \approx \left( \frac{\Lambda}{q} \right)^2 f_1(x), \quad F_2(x, q^2) \approx \left( \frac{\Lambda}{q} \right)^2 f_2(x) \]  

(59)

where we have extracted the power dependence and lump all the others into the functions \( f_1(x) \) and \( f_2(x) \) which are independent on \( q \) or at most dependent on \( q \) by \( \ln q \). Since what we are most interested in is the power dependence in our present work, we will not present their specific forms of \( f_1(x) \) and \( f_2(x) \).

V. DISCUSSION AND CONCLUSION

Now let us compare the above results in Eq. (59) from the subleading large-\( N_c \) contribution with those obtained before from the leading large-\( N_c \) contribution which is given by

\[ F_1(x, q^2) = 0 \quad F_2(x, q^2) \approx \left( \frac{\Lambda}{q} \right)^{2\kappa_1+2} f(x) \]  

(60)

Set \( \kappa_1 = 1 \) to be consistent with our present specific case, we can have

\[ F_1(x, q^2) = 0 \quad F_2(x, q^2) \approx \left( \frac{\Lambda}{q} \right)^4 f(x) \]  

(61)

Therefore, we can noticed two significant differences between them. Firstly, for the leading large-\( N_c \) contribution, the structure function \( F_1(x, q^2) \) always vanishes while it will obtain nonzero contribution at the subleading large-\( N_c \) order. As we all know, \( F_1(x, q^2) \) is proportional to the Casimir of the scattered hadron under the Lorentz transformation, so it is natural that \( F_1(x, q^2) \) vanishes when the virtual photon hits the original scalar target hadron directly at the leading large-\( N_c \) order. However at the subleading large-\( N_c \) order, the scalar target hadron can split into two scalar hadrons, each hadron can have orbital angular momentum and can lead to non-vanishing \( F_1(x, q^2) \) when they are hit by the virtual photon. Such arguments can be verified by Eq. (26), in which only \( t \)-channel and \( u \)-channel contribute to \( F_1(x, q^2) \) and \( s \)-channel in which the target hadron interacts directly with the virtual photon does not contribute at all. Secondly, the subleading large-\( N_c \) contribution from the double-hadron final states is power less suppressed than the leading large-\( N_c \) one. The power dependence of the structure function is the same as that of the hadron 2 from the leading large-\( N_c \) contribution. This conclusion makes sense, because in the dominant contribution as discussed above, the incoming hadron 1 splits into two hadrons 2 and 3, hadron 2
has the minimum twist $\kappa_2 = 0$ which propagates to the boundary of AdS $z = 0$ and interacts with the current. In $\hat{t} \ll 1$, we can regard hadron 2 as an almost on-shell hadron, hence the final power dependence should be controlled by the twist of hadron 2. Our calculation and analysis verify this argument, which was originally proposed in Ref. [4]. The result that the subleading contribution in $N_c$ will dominate in the Bjorken limit $q^2 \to \infty$ implies that large $N_c$ limit and the Bjorken limit do not commute with each other. Such conclusion can lead to very important consequence in DIS from gauge/gravity duality. When we are calculating within supergravity, we a priori make the large $N_c$ limit first followed by the Bjorken limit, while when we are analyzing the process from OPE, we actually a priori make the Bjorken limit first followed by the large $N_c$ limit. If the large $N_c$ limit and the Bjorken limit do not commute with each other any more, these mutual comparison and analysis would lose valuable meaning. There’s no doubt that we need further investigation along this direction. We will postpone these work for the future.

Acknowledgments

J.H.G. was supported in part by the Major State Basic Research Development Program in China (Grant No. 2014CB845406), the National Natural Science Foundation of China under the Grant No. 11105137 and CCNU-QLPL Innovation Fund (QLPL2014P01).

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