Pentaquarks with a hidden flavour and the hypothesis of the intrinsic charm of the proton.

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Using the experimental data on pentaquarks with a hidden charm, new upper limits on the intrinsic charm in the proton are obtained, possible shapes of heavy quark distributions inside the proton are discussed.

1. INTRODUCTION

The hypothesis of the intrinsic charm in the proton was proposed quite a lot time ago to explain disagreement between early experiments on the charmed particles production and QCD predictions [1]. However, as new data has been accumulated and a calculation technique has been improved, it became clear that the assumption of intrinsic charm could not find any univocal confirmation. Nevertheless, due to the elegance of this model and due to the occasional difficulties in describing charm events, this hypothesis attracts the attention of researchers for decades. Now we expect a new wave of interest to the discussed model because of the recent observation of three resonances in the $J/\psi p$ spectrum by the LHCb experiment [2, 3] (see also the review “Pentaquarks” in the 2019 update of [4]):

\begin{align}
P_c(4312)^+ &\quad (M = 4311.9 \pm 0.7^{+6.8}_{-0.6} \text{ MeV}, \quad \Gamma = 9.8 \pm 2.7^{+3.7}_{-4.5} \text{ MeV}), \\
P_c(4440)^+ &\quad (M = 4440.3 \pm 1.3^{+4.1}_{-4.7} \text{ MeV}, \quad \Gamma = 20.6 \pm 4.9^{+8.7}_{-10.1} \text{ MeV}), \\
P_c(4457)^+ &\quad (M = 4457.3 \pm 0.6^{+4.1}_{-1.7} \text{ MeV}, \quad \Gamma = 6.4 \pm 2.0^{+5.7}_{-1.9} \text{ MeV}).
\end{align}

It is clear that states that strongly decays into $J/\psi$-meson and proton can be interpreted as $uudc\bar{c}$ pentaquarks. Moreover, it is possible that at least one of these states has the same

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quantum numbers as the proton [5], [6], which means that the assumption of mixing such a pentaquark and a proton is quite reasonable. This assumption opens up a new view at the old problem of intrinsic charm: now we can discuss the presence in proton of an admixture of a really existing baryon containing charm valence quarks.

In this work we study the possible distributions of $c$-quarks in the proton and estimate an upper limit on the intrinsic charm contribution within the nonperturbative model describing the mixing of a proton and a pentaquark based on [7]. The results are compared with the prediction of the intrinsic charm model [1].

2. DISTRIBUTIONS OF HEAVY QUARKS IN THE PROTON IN THE FRAMEWORK OF VARIOUS MODELS OF INTRINSIC FLAVOUR

In the article [1], where the hypothesis of intrinsic charm was first discussed, it is assumed that the proton wave function can be represented as $|p⟩ = A_0|udd⟩ + A_1|uudc\bar{c}⟩ + \ldots$, where $|A_1|^2$ — nonzero probability of the existence of intrinsic $c\bar{c}$-pairs. To estimate this probability, the “old” perturbation theory is used:

$$G = |A_1|^2 = \left| \frac{\langle u_1u_2dc\bar{c}\rangle M|u_1u_2d⟩}{E_{u_1u_2dc\bar{c}} - E_{u_1u_2d}} \right|^2. \quad (2)$$

From the above expression in the system of infinite momentum and under the assumption that $\langle u_1u_2dc\bar{c}\rangle M|u_1u_2d⟩ = \text{const}$, one obtains, that

$$G(x_{u_1}, x_{u_2}, x_d, x_c, x_{\bar{c}}) \sim \left(M_p^2 - \sum_{i=u_1, u_2, d, c, \bar{c}} \frac{m_i^2}{x_i}\right)^{-2}, \quad (3)$$

which under assumption of very heavy charmed quarks ($m_c^2 \gg M_p^2, m_u^2, m_d^2$) leads to the following equation:

$$G(x_{u_1}, x_{u_2}, x_d, x_c, x_{\bar{c}}) \sim \frac{x_c^2 x_{\bar{c}}^2}{(x_c + x_{\bar{c}})^2} \delta \left(1 - \sum_{i=u_1, u_2, d, c, \bar{c}} x_i\right). \quad (4)$$

After the integration of (4) one obtains a distribution of single $c$-quark in the proton [1]:

$$G_c(x_c) \sim x_c^2 \left(1 - x_c\right)\left(1 + 10x_c + x_c^2\right) - 6x_c(1 + x_c) \ln \frac{1}{x_c}. \quad (5)$$

The normalization of distribution (5), which obviously determines the probability of finding a charmed quark in a proton, is not theoretically calculated in [1]. In a recent theoretical
study [8] a restriction on the intrinsic charm of 1.93% was obtained using the data of the ALTAS experiment. It should also be noted that there are studies that give greater limitations: in [9] was obtained an upper limit of 1% using the ratio of $\Lambda_{QCD}$ to the difference in the energies of the pentaquark and proton, in [10] was obtained even stricter restriction: $10^{-5}$.

Let us remind, that equation (5) is obtained in [1] under the assumption of the very heavy charm quark. This automatically leads to the idea of not only intrinsic charm, but also intrinsic beauty. This is why, in the following sections we will discuss this problem a bit. It is clear that the approach [1] predicts the same shapes of distribution of $c$ and $b$ quarks in a proton. On the contrary, as we will show later, in the model of proton and pentaquark mixing, these distributions must be different.

To obtain distributed quarks in hadrons, there is a nonperturbative Kuti-Weisskopf model [7], which has successfully proven itself in the calculation of structure functions for small $q^2$. According to this model, the probability of detecting $n$ partons in the hadron, of which $m$ are valence, is determined by the expression:

$$G(x_1, \ldots, x_n) \sim \prod_{i=1}^{m} x_i^{1-\alpha_i} \prod_{i=1}^{n} \frac{dx_i}{x_i} \delta \left( 1 - \sum_{i=1}^{n} x_i \right).$$  \hspace{1cm} (6)

In the discussed model the probability for sea parton is proportional to the phase volume $\frac{dx_i}{x_i}$. For valence quarks there is an additional factor $x_i^{1-\alpha_i}$, where the parameter $\alpha_i$ is related to the intersection of the Regge trajectory.

In the described approach, the pentaquark differs from the “ordinary” hadrons only in the presence of five valence quarks. Moreover, the value of the parameter $\alpha_c$ for valence $c$-quarks is known and it is equal to $-2.2$ [11, 12]. As for the remaining values of $\alpha_i$, they are known from the Regge phenomenology: $\alpha_u = \alpha_d = \alpha_q = 1/2$ and $\alpha_s = 0$.

Following [7], one can obtain the following distributions for the $uudc\bar{c}$ pentaquark (see [9]):

$$G^P_q(x_q) \sim x^{-\alpha_q}(1 - x)^{-1+\gamma+2(1-\alpha_q)+2(1-\alpha_c)}$$  \hspace{1cm} (7)

for light valence quarks;

$$G^P_c(x_c) \sim x^{-\alpha_c}(1 - x)^{-1+\gamma+3(1-\alpha_q)+(1-\alpha_c)}$$  \hspace{1cm} (8)

for valence $c$ and $\bar{c}$ quarks. Because quarks are valence, the distributions (7) and (8) are normalized to 1.
Figure 1: (a) Distributions of $c$-quark in the proton according to the model [1] (dashed curve) and in $uudc\bar{c}$ pentaquark according to the model [7] (solid curve) (see also [9]); (b) the analogous distributions for the $b$ quark case.

Let us remind, that according to [7] the distributions of valence quarks in proton have the following shape:

$$G_p^q(x_q) \sim x_q^{-\alpha_q}(1 - x_q)^{-1+\gamma+2(1-\alpha_q)}.$$  \hspace{1cm} (9)

For the proton $\gamma = 3$, and we will use the same value to calculate the distribution of quarks in the pentaquark, as it was done in [9].

Comparing the distributions of charmed quarks in a proton obtained within the intrinsic charm model (5) in a pentaquark (8), it is notable that they have very similar shapes (see Fig.1(a)). However, using the example of the hypothetical $uud\bar{b}b$ pentaquark, it can be shown that this is just a coincidence.

Indeed, the distribution of the valence $b$-quark in such a pentaquark can be obtained from (8) with replacement of parameter $\alpha_c = -2.2$ to $\alpha_b = -8$ [13]:

$$G^p_b(x_b) \sim x^{-\alpha_b}(1 - x)^{-1+\gamma+3(1-\alpha_q)+(1-\alpha_b)},$$  \hspace{1cm} (10)

which significantly changes its shape. Herewith, as we noted earlier, the distribution for the “intrinsic beauty” in the proton is exactly the same as for the intrinsic charm. In view of these circumstances, the difference in the distribution shapes becomes very noticeable (see Fig.1(b)), which means the similarities between (5) and (8) cannot be considered as a regularity.
3. PENTAQUARK ADMIXTURE OF THE PROTON

Despite that the similarity in the distributions of $c$ quarks in the proton within the intrinsic charm model [1] and in the pentaquark within the Kuti-Weisskopf model [7] is a coincidence, it prompts to the idea of a possible mixing of a proton and one of the recently discovered pentaquarks (1). Further, on the assumption that one of these pentaquarks has quantum numbers of a proton, one can estimate probability of such mixing and, therefore, the probability of detection of the pentaquark component in the proton.

The mixed states of proton and pentaquark $X_1|p⟩ + X_2|P_c⟩$ are defined with help of mixing matrix:

$$
\begin{pmatrix}
M_p & V \\
V & M_{P_c}
\end{pmatrix}
\begin{pmatrix}
X_1 \\
X_2
\end{pmatrix}
= \lambda
\begin{pmatrix}
X_1 \\
X_2
\end{pmatrix},
$$

(11)

where $M_p$ and $M_{P_c}$ are masses of proton and pentaquark, and $V$ is operator connected with their interaction.

Assuming the smallness of $V$ with respect to the mass difference $M_{P_c} - M_p$, we can find mixing matrix eigenvalues:

$$
\lambda_1 = M_p + \frac{V^2}{M_{P_c} - M_p},
\lambda_2 = M_{P_c} - \frac{V^2}{M_{P_c} - M_p},
$$

(12)

Herewith the state of the proton with an admixture of pentaquark corresponds to the first eigenvalue

$$
|p⟩' = |p⟩ + \varepsilon|P_c⟩,
$$

(13)

where the small parameter

$$
\varepsilon = \frac{V}{M_{P_c} - M_p},
$$

(14)

and the state of pentaquark with an admixture of proton corresponds to the second eigenvalue

$$
|P_c⟩' = |P_c⟩ + \varepsilon|p⟩.
$$

(15)

The operator $V$ is unknown, but it can be assumed that it is bounded in value by the full width of pentaquark

$$
V \lesssim \Gamma_{P_c}
$$

(16)
and therefore
\[ \varepsilon \lesssim \frac{\Gamma_{P_c}}{M_{P_c} - M_p}. \]  
\( (17) \)

Which of the observed pentaquarks (1) has the quantum numbers of a proton is not exactly known, but most likely this is the state with the smallest mass, since it is the main candidate for the ground state. We will use its width (9.8 MeV) to estimate the value of the admixture:
\[ \varepsilon \lesssim \frac{9.8 \text{ MeV}}{4312 \text{ MeV} - 938 \text{ MeV}} \approx 3 \cdot 10^{-3}. \]  
\( (18) \)

Thus, the probability of the presence of a five-quark state in a proton is \( \varepsilon^2 \lesssim 10^{-5} \).

It is worth to note, that a very similar problem of mixing of light baryons with light pentaquarks \(|uuduu\rangle\) was presented in a recent article [14].

4. SUMMARY

The intrinsic charm model [1] allows to determine the shape of \( c \) quark distribution in a high momentum system, but does not predict the normalization. On the contrary, the model of a proton and a pentaquark mixing allows not only to predict the distribution shape, but also to estimate the admixture of \( c \) quarks in the proton. The shape of \( c \) quark distribution is predicted in the framework of the Kuti-Weisskopf [7] approach applied to the pentaquark, and the admixture value is estimated using the masses and the widths of recently discovered LHCb pentaquarks with hidden charm [2, 3].

It is worth to note, that such different approaches lead to very similar \( c \) quark distributions in a proton. However, most likely, this is a coincidence, as it was demonstrated by example of the \( b \) quark distributions predicted in the framework of the same two models.

As an upper limit of the admixture value \( \varepsilon \) of the \( c \) quark in the proton \(|p'\rangle = |p\rangle + \varepsilon |P_c\rangle\) we used the value of ratio \( \Gamma_{P_c}/(M_{P_c} - M_p) \approx 3 \cdot 10^{-3} \), where \( \Gamma(P_c) \) and \( M_{P_c} \) are the width and the mass of the lowest pentaquark from ones detected by the LHCb experiment \([2, 3]\). This state was chosen due to the fact that, most likely, it can have the quantum numbers of the proton.

Thus, the probability of the transition of the proton to the pentaquark is \( \varepsilon^2 \lesssim 10^{-5} \). Let us note that this restriction is stronger than the restrictions obtained in most other studies (see, for example, \([1, 8, 9]\)).
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