Measurements of the fluctuation-induced in-plane magnetoconductivity at high reduced temperatures and magnetic fields in the iron arsenide BaFe$_{2-x}$Ni$_x$As$_2$

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Abstract
The superconducting fluctuations well inside the normal state of Fe-based superconductors were studied through measurements of the in-plane paraconductivity and magnetoconductivity in high quality BaFe$_{2-x}$Ni$_x$As$_2$ crystals with doping levels from the optimal level ($x = 0.10$) up to highly overdoped ($x = 0.20$). These measurements, performed in magnetic fields up to 9 T perpendicular to the $ab$ (Fe) layers, allowed a reliable check of the applicability to iron-based superconductors of Ginzburg–Landau approaches for 3D anisotropic compounds, even at high reduced temperatures and magnetic fields. Our results also allowed us to gain valuable insight into the dependence on the doping level of some central superconducting parameters (coherence lengths and anisotropy factor).

(Some figures may appear in colour only in the online journal)

1. Introduction
As the pairing mechanism of Fe-based superconductors is not yet established, phenomenological descriptions of their superconducting transition are at the forefront of the research in these materials [1]. A powerful tool to probe these descriptions is the superconducting fluctuation effect that appears in the normal state [2–12]. To date, some experimental aspects of these fluctuations have been studied through observables like the magnetization [3–6], the specific heat [7] or the electrical conductivity [8–12]. However, their behavior at high reduced temperatures and magnetic fields (in the short wavelength fluctuation regime) and their onset temperature remain at present unexplored at a quantitative level. The importance of these aspects, which include the possible presence of phase fluctuations well above the superconducting critical temperature ($T_c$) [3–6], is enhanced by comparison with the high-$T_c$ cuprate superconductors (HTSCs), for which the onset and the influence of doping on their superconducting fluctuations are at present some of the most central and debated issues of their phenomenology [13–17].

The central aim of this paper is to present detailed experimental results on the superconducting fluctuations above $T_c$ in iron-based superconductors as a function of the doping level. To achieve this, we have measured in an extended temperature region above the superconducting transition the in-plane fluctuation electric conductivity, $\Delta\sigma_{ab}(T, H)$, in optimally doped and overdoped BaFe$_{2-x}$Ni$_x$As$_2$ crystals ($0.1 \leq x \leq 0.2$), under magnetic fields up to 9 T perpendicular to the...
ab (Fe) layers. We have also used these experimental data to probe the phenomenological descriptions of the superconducting fluctuations based on the Gaussian Ginzburg–Landau (GGL) approach, adapted to the 3D anisotropic nature of these compounds, and also to take into account the short wavelength fluctuation regime [18, 19]. Our analysis allows us to gain valuable insight into the doping dependence of the superconducting parameters. In particular, the anisotropy factor is shown to increase significantly in strongly overdoped samples.

2. Experimental details and results

2.1. Crystal fabrication and characterization

The BaFe\textsubscript{2-\textit{x}}Ni\textsubscript{x}As\textsubscript{2} samples used in this work were plate-like single crystals (typically 5 × 2 × 0.1 mm\textsuperscript{3}) with the c crystallographic axis perpendicular to their largest face. They were cleaved from larger single crystals grown by the self-flux method. Their nominal Ni doping levels were \textit{x} = 0.10, 0.15, 0.18 and 0.20, although the real doping level was found to be a factor of ~0.8 smaller (see [20], where all the details of the growth procedure and characterization may be found). We checked the excellent stoichiometric and structural quality of the crystals studied here by x-ray diffraction. In particular, the (00l) linewidths were found to be slightly larger (\Delta l(20) \sim 0.10\degree, FWHM) than the corresponding instrumental linewidths, \Delta l(20) \sim 0.07\degree. This was attributed to a dispersion in the c-axis lattice parameter \textit{L}_\textit{c},\textsuperscript{5} and was used to roughly estimate \Delta \textit{x} \sim 10^{-2} through the \textit{L}_\textit{c}(\textit{x}) dependence presented in [20]. In turn, the rocking curves for the (008) lines indicated that the dispersion in the c-axis orientation was lower than ~0.05\degree.

2.2. Resistivity measurements: determination of the superconducting transition temperatures and transition widths

The in-plane resistivity (along the \textit{ab} layers) was measured in the presence of magnetic fields of up to 9 T perpendicular to the \textit{ab} layers with a Quantum Design physical property measurement system (PPMS) by using four contact layers in an in-line configuration and an excitation current of \sim 1 mA at 23 Hz. The uncertainties in the geometry and dimensions of the crystals, and the finite size of the electrical contacts (stripes typically 0.5 mm wide) led to an uncertainty in the \textit{\rho}_{\textit{ab}} amplitude of ~25\%. An example of the \textit{\rho}_{\textit{ab}}(T) dependence around the transition (corresponding to the crystal with \textit{x} = 0.1) is presented in figure 1(a). The zero-field transition temperature, \textit{T}_\textit{c}, was estimated from the maximum of the corresponding d\textit{\rho}_{\textit{ab}}/dT curve, and the transition width as \Delta \textit{T}_\textit{c} \approx 2(\textit{T}_\textit{c} - \textit{T}_\textit{c}^\text{onset})\textsuperscript{5}, where \textit{T}_\textit{c} is the temperature at which \textit{\rho}_{\textit{ab}} = 0 is attained. In the optimally doped crystal (\textit{x} = 0.1) \textit{T}_\textit{c} = 20.0 K and \Delta \textit{T}_\textit{c} \approx 0.3 K, which are among the best values in the literature for crystals of the same composition [22–25]. This allows investigation of fluctuation effects down to reduced temperatures, \textit{\varepsilon} \equiv \ln(\textit{T}/\textit{T}_\textit{c}), of the order of \Delta \textit{T}_\textit{c}/\textit{T}_\textit{c} \sim 10^{-2}. Overdoped crystals present a slightly wider resistive transition (\Delta \textit{T}_\textit{c} \approx 0.6 K), probably due to the \textit{T}_\textit{c}(\textit{x}) dependence and an \textit{x}-distribution (taking into account that in the overdoped regime |d\textit{T}_\textit{c}/d\textit{x}| \sim 200 K, it may be estimated that \Delta \textit{x} \sim 0.003). Nevertheless, even in these samples, fluctuation effects may be studied in a wide range of temperatures above the zero-field \textit{T}_\textit{c} by just applying a magnetic field of the order of \sim 1 T due to the \textit{T}_\textit{c}(\textit{H}) shift to lower temperatures. A detailed analysis of the effect on the measured fluctuation conductivity of the uncertainty in \textit{T}_\textit{c} is presented in appendix A.

2.3. Determination of the fluctuation contribution to the electric conductivity

An overview of the \textit{\rho}_{\textit{ab}}(T) data up to room temperature is presented in the inset of figure 1(a). The seemingly non-monotonic \textit{x}-dependence at 300 K may be explained in terms of the above mentioned geometrical uncertainties. As expected [20, 26], for \textit{x} > 0.1, the kinks associated with structural (tetragonal–orthorhombic) and magnetic (paramagnetic–antiferromagnetic) transitions are not observed, and for \textit{x} = 0.1 they are irrelevant.

\textsuperscript{5} A similar procedure was previously used to investigate compositional inhomogeneities in non-stoichiometric high-\textit{T}_\textit{c} cuprates, see e.g., Mosquera et al [21].
when compared with the rounding associated with the fluctuations. This is an important experimental advantage for determination of the superconducting contribution to the electric conductivity, \( \Delta \sigma_{ab}(T) \), since \( \rho_{ab,B} \) is the normal state or background contribution. In view of the linear temperature dependence of \( d\rho_{ab}/dT \) (an example for \( x = 0.1 \) is presented in figure 1(b)), the background contributions were estimated by fitting a quadratic polynomial to the measured \( \rho_{ab}(T) \) from \( \sim 4T_c \) down to \( T_{\text{set}} \). The temperature below which fluctuation effects are measurable.

In turn, this was determined as the temperature at which \( d\rho_{ab}/dT \) rises above the extrapolated normal-state behavior beyond the noise level (see figure 1(b)). As shown in figure 1(c), \( T_{\text{set}}(x) \approx 1.5T_c(x) \), an issue that will be analyzed later. An example of the background contribution (for the \( x = 0.1 \) crystal) is shown in figure 1(a).

On the scale of the effect of the superconducting fluctuations, \( \rho_{ab,B}(T) \) is independent of the applied magnetic field. This further experimental advantage for study of the fluctuation-induced in-plane magnetoconductivity is confirmed by the detailed representation of figure 2(a), where it is shown that \( \sigma_{ab}(0) - \sigma_{ab}(H) \) becomes negligible well above \( T_c \). A detailed analysis of the effect on the measured \( \Delta \sigma_{ab}(T) \) of the typical uncertainty in the background contribution is presented in appendix A.

3. Data analysis

3.1. Comparison with the conventional Aslamazov–Larkin approach

An example (for the \( x = 0.1 \) crystal) of the \( \Delta \sigma_{ab}(H) \) dependence at several temperatures just above \( T_c \) is presented in figure 2(b). In the \( H \to 0 \) limit \( \Delta \sigma_{ab} \) tends to a constant value, in qualitative agreement with the conventional Aslamazov–Larkin (AL) result [27],

\[
\Delta \sigma_{ab} = \frac{e^2}{32\hbar \xi_c(0)} e^{-1/2},
\]

where \( e \) is the electron charge, \( \hbar \) is the reduced Planck constant, and \( \xi_c(0) \) is the \( c \)-axis coherence length amplitude. However, \( \Delta \sigma_{ab} \) decreases above a temperature-dependent magnetic field which is close to the scale for the observation of finite-field effects (see below)\(^6\). An example of the \( \Delta \sigma_{ab} \) dependence on \( \epsilon \) (also corresponding to the \( x = 0.1 \) crystal) is shown in figure 3(a). The double-logarithmic scale was chosen to explore in detail the high-\( \epsilon \) region. As may be clearly seen in this figure, the fit of equation (1) to the low-field data (dashed line) is excellent at low reduced temperatures (up to \( \epsilon \approx 0.06 \)), except for \( \epsilon < \Delta T_c/T_c \approx 0.015 \), where \( T_c \) inhomogeneities may play a role (see below).

However, a large discrepancy is found at high-\( \epsilon \) values: while the AL-theory never vanishes, the experimental \( \Delta \sigma_{ab} \) ends up below the experimental resolution for \( \epsilon \sim 0.3 \) for all the applied magnetic fields. A similar rapid falloff of the fluctuation-induced conductivity was already observed more than 30 years ago in low-\( T_c \) conventional superconductors by Johnson and coworkers [29, 30], who stressed that this type of behavior could not be described in terms of a power law in \( \epsilon \). Thus, our present data provide further evidence of the failure of those proposals that, like the momentum cutoff approach [31] and equivalent microscopic calculations [32][7],

\(^6\) It is worth noting that a negative fluctuation magnetoresistance has been observed in granular metallic films in high fields (of the order of the critical field), see Gerber et al [28]. This effect is not observed here even in the vicinity of \( T_c \), where superconducting and normal domains may coexist as a consequence of \( T_c \) inhomogeneities. On one hand, the magnetic field used in our experiments may not be strong enough to observe such an effect. On the other hand, the indirect contributions to the fluctuation conductivity needed to explain the negative magnetoresistance (see Beloborodov and Efetov [28]) may not be present in non-s wave superconductors, as may be the case for iron pnictides.

\(^7\) As we have already commented in [18], the microscopic approach proposed in this work results as being equivalent to applying a momentum cutoff in the GL-theory (see [31]), since both lead to an asymptotic behavior of the paraconductivity at high reduced temperatures proportional to \( \epsilon \) to the \( -3 \). Such a behavior was seemingly observed in cuprate superconductors by Varlamov and coworkers (see figure 7.6, and the corresponding reference, in that book). Nevertheless, such a result, the only account of the paraconductivity in that book, seems to be an artifact of the data analysis and, in any case, it is in contradiction with the measurements published up to now by other groups in any low- or high-\( T_c \) superconductor (for earlier results see, e.g., [31] and references therein).
The failure of GL-based approaches to explain the high-$T_c$ behavior of superconductors has also been observed in HTSCs [18], and it also appears in the fluctuation diamagnetism of both HTSCs [33, 34] and low-$T_c$ alloys [35, 36]. It has been attributed to the fact that the GL-theory overestimates the contribution of the high-energy fluctuation modes [35]. In fact, although GL approaches are formally valid only in the vicinity of the transition, it was found that the applicability may be extended to the high-$T_c$ region through the introduction of an energy cutoff [18, 19, 36, 34]. Since the energy of the fluctuation modes increases with $H$, the inclusion of such a cutoff is also needed when analyzing the effect of a finite applied magnetic field on the superconducting fluctuations [37]. In the case of 3D anisotropic superconductors, a case well adapted to 122 iron arsenides, the paraconductivity in the presence of an energy cutoff may be easily calculated by using the procedure proposed in the pioneering work by Schmid [38].

These calculations, whose details are presented in appendix B, lead to

$$\Delta \sigma_{\alpha\beta} = \frac{e^2}{32\hbar^2 \pi \bar{\xi}_{\alpha}(0)} \sqrt{\frac{1}{\hbar}} \int_0^\infty \sqrt{\frac{x}{\pi}} \left[ \psi^1(\frac{\epsilon + h}{2\hbar} + x^2) - \psi^1(\frac{\epsilon + h}{2\hbar}) \right] dx,$$

where $\psi^1$ is the first derivative of the digamma function, $h = H/|\phi_0/2\pi \mu_0 \bar{\xi}_{ab}(0)|$ is the reduced magnetic field, $\bar{\xi}_{ab}(0)$ is the in-plane coherence length amplitude, and $c$ is the cutoff constant (expected to be of the order of 0.5) [19]. In the zero magnetic field limit (i.e., for $h \ll \epsilon$) and in the absence of a cutoff ($c \to \infty$), equation (2) reduces to the AL expression, equation (1). It is also worth noting that equation (2) leads to the $\Delta \sigma_{ab}$ vanishing at $\epsilon = c$. In an attempt to check the applicability range of the 3D anisotropic GL approach under an energy cutoff, in what follows we will compare our experimental data with equation (2).

### 3.2.1. Optimally doped sample.

A first check may be carried out through measurements performed with $h = 0$, because in this case equation (2) depends only on $\xi_c(0)$ and $c$. In the optimally doped sample the fit of equation (2) to the $\Delta \sigma_{ab}(\epsilon, h = 0)$ data is excellent above $\epsilon = 0.02$, including the $\Delta \sigma_{ab}$ vanishing at high $\epsilon$-values (see figure 3(a)). This analysis leads to $\bar{\xi}_c(0) = 0.8$ nm and $c = 0.39$. By using these values, equation (2) is then fitted to the data obtained with $h > 0$ with $\xi_{ab}(0)$ as the only free parameter. The fit quality is also excellent up to the largest field used in the experiments, leading to $\xi_{ab}(0) = 2.3$ nm. For comparison, the conventional AL approach (equation (1)) evaluated with the same $\bar{\xi}_{ab}(0)$ and $\xi_c(0)$ values is represented as a dotted line in figure 3(a).

As expected, the adequacy of equation (2) extends to the $\Delta \sigma_{ab}(H)$ representation of figure 2(b), where the solid lines were evaluated by using in equation (2) the above $\xi_{ab}(0)$, $\xi_c(0)$ and $c$ values. The dashed line in this figure is the crossover to the region at which finite-field effects are expected to be relevant, and was evaluated by using $h = \epsilon$ in equation (2)$^8$.

It is worth mentioning that the in-plane magnetococonductivity, $\sigma_{ab}(0) - \sigma_{ab}(H)$, in figure 2(a) is also in excellent agreement with $\Delta \sigma_{ab}(0) - \Delta \sigma_{ab}(H)$ as evaluated from equation (2) by using the same $\bar{\xi}_{ab}(0)$, $\xi_c(0)$ and $c$ values (solid lines). As the $\sigma_{ab}(0) - \sigma_{ab}(H)$ data do not depend on a background subtraction, this fact further validates the procedure used to determine the background contribution to obtain $\Delta \sigma_{ab}$ in figures 2(b) and 3.

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8 See, e.g., [35]. This criterion for the presence of finite-field effects may be more intuitively written as $H = \phi_0/(2\pi \xi_{ab}(T))$. This magnetic field scale is sometimes referred to as a ghost critical field, see e.g., Kapitulnik et al [39].
3.2.2. Overdoped samples. An example of the $\Delta \sigma_{ab}(\varepsilon)/H$ dependence corresponding to an overdoped crystal ($x = 0.15$) is shown in figure 3(b). The upturns observed in the low-$H$ isofields (indicated with arrows) are associated with the above mentioned $T_c$ inhomogeneities inherent to non-optimally doped samples. It is expected that a magnetic field of the order of $H=\mu_0 H \gtrsim 1$ T will shift $T_c(H)$ so that $\Delta \sigma_{ab}$ would be unaffected by $T_c$ inhomogeneities down to $\varepsilon = 0$. Then, we have fitted equation (2) to the experimental data for $\mu_0 H \gtrsim 3$ T with $\xi_{ab}(0), \xi_c(0),$ and $c$ as free parameters. The agreement is excellent in the entire $\varepsilon$ range, and even extends to the lowest magnetic fields for $\varepsilon$-values well above $\Delta T_c/T_c$. Similar results are obtained in the other overdoped crystals studied. The $\varepsilon$-dependence of the resulting $\xi_{ab}(0), \xi_c(0),$ and $c$ will be analyzed in the next section.

We have checked that the $T_c$-inhomogeneity model proposed in [40], which is based on Bruggeman’s effective-medium theory [41], accounts for the upturn in the $H$ proposed in [40], which is based on Bruggeman’s effective-

A direct consequence of the excellent agreement of the GL-theory to our data is the absence of appreciable indirect contributions to the fluctuation-induced in-plane conductivity and magnetconductivity of Fe-based superconductors, including the Maki–Thompson (MT) and the so-called density of states (DOS) contributions. These indirect contributions have been found to be negligible also in HTSCs [50], and in both systems this could be attributed to the non s-wave pairing [51].

Finally, it has recently been claimed that conventional GL approaches for the fluctuation effects above $T_c$ are not applicable at low-field amplitudes (typically below 1 T) in SmFeAsO$_{1-\delta}$F$_{\delta}$, due to the presence of phase fluctuations [6]. The relevance of phase fluctuations has also been proposed for a member of the less anisotropic 122 family, Ba$_{1-x}$K$_x$Fe$_2$As$_2$ [3]. However, our present results show that the anomalous increase of fluctuation effects at low field amplitudes may be explained in the framework of GL approaches by taking into account the effect of $T_c$ inhomogeneities. This could suggest that the effect of phase fluctuations in these materials is less important than previously estimated.

4. Conclusions

We have presented detailed measurements of the fluctuation-induced in-plane conductivity and magnetconductivity in a series of high quality BaFe$_{2-\delta}$Ni$_x$As$_2$ single crystals covering doping levels from the optimal one ($x = 0.1$) up to highly overdoped ($x = 0.2$). The sharp superconducting transition allowed us to investigate fluctuation effects in almost all of the accessible reduced temperature window above $T_c$. In turn, the smooth temperature and field dependences of the normal-state

9 Note that some works report the observation of $\xi_{\text{onset}}$ values different from 0.5 in non-optimally doped HTSCs and amorphous low-$T_c$ superconductors. See, e.g., [16, 48]. However, these differences in the $\xi_{\text{onset}}$ values could be related to the strong $T_c$ dependence on the stoichiometry in these materials, and the unavoidable presence of stoichiometric inhomogeneities, see [21, 49].
in-plane resistivity permitted a reliable estimation of the temperature onset for the fluctuation effects. As an example of the usefulness of these data to probe the different approaches for the superconducting fluctuations around $T_c$ in iron pnictides, we have also presented here a detailed comparison with a version of the phenomenological Gaussian Ginzburg–Landau approach that includes an energy cutoff, which reduces to the well-known Aslamazov–Larkin result at low reduced temperatures and magnetic fields. Our data are in good agreement with this approach in all the studied temperature and field regions, suggesting the absence, even in the very low magnetic field regime, of local superconducting order associated with phase fluctuations [3, 6], at present a debated aspect of HTSCs [13–17]. Other contributions to the fluctuation-induced in-plane magnetoconductivity (Maki–Thompson, Zeeman, and DOS) are found to be negligible, as is also the case in HTSCs [50]. Finally, we have also shown that when analyzing the low-field regime, it is crucial to take into account the $T_c$-inhomogeneities associated with chemical disorder, which are always present to some extent even in the best crystals, due to the unavoidable random distribution of doping ions [21]. It would be interesting to extend our present analysis to the critical region by using the LLL scaling proposed in [2], and also to check our present results in other families of Fe-based superconductors, in particular in the more anisotropic 1111 pnictides, where the fluctuation dimensionality is at present a debated issue [7–12].

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*Note added in proof.* After this paper had been submitted, we became aware of a work about superconducting fluctuations in transport properties of LiFeAs [54]. In contrast with our present results, and in spite of the three-dimensional character of this compound, it is claimed that the fluctuation conductivity agrees with the two-dimensional Aslamazov–Larkin expression, and also that the onset temperature is as high as $\sim 2.5 T_c$.

**Appendix A. The uncertainty in $\Delta \sigma_{ab}(\epsilon, H)$ associated with the uncertainties in the transition temperature and the normal-state contribution**

Here, we present some representative examples of how the uncertainty in the transition temperature and the normal-state contribution affect the determination of the fluctuation contribution to the in-plane electrical conductivity.

In our work $T_c$ is estimated as the temperature at which the slope of the $\rho_{ab}(T)$ curve is maximum. In turn, the $T_c$ uncertainty is estimated as $\Delta T_c = 2(T_c - T_{c^-})$, where $T_{c^-}$ is the temperature at which $\rho_{ab}$ vanishes. In figure A.1(a) we present the detail of the zero-field resistive transition of the optimally doped crystal (with $x = 0.1$) showing the locations of $T_c$, $T_{c^-}$, and $T_c^+ = T_c + \Delta T_c/2$ (the latter represents an upper limit for the $T_c$ value). In figure A.1(b) we show the $\Delta \sigma_{ab}(\epsilon)$ dependence on the reduced temperature, $\epsilon = \ln(T/T_c)$, as obtained by using those three representative $T_c$ values. As may be clearly seen, $\Delta \sigma_{ab}$ is almost unaffected by the uncertainty in $T_c$ above $\epsilon = 0.03$. The line is the best fit of equation (2) above this $\epsilon$ value to the solid data points. In the presence of a finite magnetic field $\Delta \sigma_{ab}$ presents a smoother behavior at $T_c$ (its divergence is shifted to lower temperatures) and $\Delta \sigma_{ab}(\epsilon)$ is less affected by the uncertainty in $T_c$. This is illustrated in figure A.1(c), where the result corresponding to $\mu_0 H = 5$ T is presented. The solid line in this figure is the best fit of equation (2) to the solid data points.

The same analysis is presented in figure A.2 for one of the overdoped samples ($x = 0.15$). In this case, as may be seen in the detail of the resistive transition presented in figure A.2(a), $\Delta T_c/T_c$ is larger than in the optimally doped crystal. As commented above, this may be attributed to the random distribution of Ni dopants and to the $T_c$ dependence.

![Figure A.1](image-url)
on the doping level. This effect is also present in doped cuprates (see, e.g., [21]) and may be intrinsic to non-optimally doped superconductors. The $\Delta \sigma_{ab}$ dependence on the reduced temperature, as obtained by using the three representative $T_c$ values shown in figure A.2(a), is presented in figure A.2(b). The circles and rhombuses were obtained with $\mu_0 H = 1$ T and 5 T, respectively. The effect on $\Delta \sigma_{ab}$ of changing $T_c$ to $T_c^-$ or $T_c^+$ may still be accounted for by equation (2) by just changing the $\xi_c(0)$ value (which affects the $\Delta \sigma_{ab}$ amplitude) by $\pm 20\%$.

Next, we present an example (corresponding to the $x = 0.1$ crystal) of the typical uncertainty in $\Delta \sigma_{ab}$ associated with the determination of the background contribution. In figure A.3(a) we present the detail of the temperature dependence of the in-plane resistivity in the normal state up to $\sim 4T_c$. The lines are the background contributions as determined by fitting a degree-two polynomial in different temperature intervals between the onset temperature for the fluctuation effects ($T_{onset}$) and $4T_c$. The robustness of the background contribution to changes in the fitting region is a consequence of the smooth behavior of the normal-state in-plane resistivity in a wide temperature region above $T_{onset}$ (it varies by about 1% from $T_c$ up to $\sim 1.5T_c$). The same background contributions are presented in the $d\rho_{ab}/dT$ versus $T$ representation in figure A.3(b). The well defined linear behavior of $d\rho_{ab}/dT$ up to $\sim 4T_c$ justifies the use of a quadratic form to determine $\rho_{ab,B}(T)$. This last figure also illustrates that $T_{onset}$ (estimated as the temperature at which $d\rho_{ab}/dT$ rises above the extrapolated normal-state behavior beyond the noise level) is almost independent of changes in the background fitting region. In figure A.3(c) we present the $\varepsilon$-dependence of $\Delta \sigma_{ab}$ as obtained by using the background contributions in figure A.3(a). The solid lines correspond to equation (2) evaluated with $\xi_c(0) = 0.8$ nm and the values indicated for the cutoff constant. As is clearly shown, the uncertainty in the background leads to an uncertainty in the cutoff constant below $\pm 10\%$. Given the relation $c = \ln(T_{onset}/T_c)$, this leads to an uncertainty in $T_{onset}$ that remains below $\pm 4\%$. 

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Figure A.2. (a) Detail of the resistive transition corresponding to the $x = 0.15$ crystal, indicating the upper, lower, and midpoint $T_c$ values. The effects of the $T_c$ choice on the resulting $\Delta \sigma_{ab}(\varepsilon)$ are presented in (b) for $\mu_0 H = 1$ and 5 T. The solid (dashed) lines are fits of equation (2) to the data obtained with $T^-_c$ ($T^+_c$). The difference in the resulting $\xi_c(0)$ is about $\pm 20\%$.

Figure A.3. (a) Detail of the normal-state in-plane resistivity for the $x = 0.1$ crystal, showing the robustness of the background contribution to changes in the temperature interval used to determine it. The same backgrounds are presented in (b) in the $d\rho_{ab}/dT$ versus $T$ representation. The effects of the background choice on the resulting $\Delta \sigma_{ab}(\varepsilon)$ are presented in (c), where the lines are fits of equation (2) to the different data sets.
Appendix B. The 3D anisotropic GGL approach for \( \Delta \sigma_{ab} \) in the finite-field regime

Our starting point is the model proposed by Schmid (adapted for a 3D anisotropic superconductor), based on a combination of the standard GL-expression for the thermally averaged current density of the superconducting condensate,

\[
J = \frac{\hbar e}{mi^*} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) - \frac{4e^2}{m^* c} A(t) (\Psi^* \Psi), \tag{B.1}
\]

with the generalized Langevin equation of the order parameter \[38],

\[
- \frac{\pi \hbar a_0}{8k_B T_0} \left( \frac{\partial}{\partial t} + \frac{2ie}{\hbar c} \Phi(r) \right) \Psi(r, t) = \left[ a_0 \epsilon + \frac{1}{2m^*} \left( \frac{\hbar}{c} \nabla - \frac{2c}{A(t)} \right)^2 \right] \Psi(r, t) + G(r, t). \tag{B.2}
\]

In these equations, \( m^* \) is the mass of the Cooper pairs, \( c \) is the speed of light, \( \Phi \) is the electrical potential, \( k_B \) is the Boltzmann constant, \( a_0 = \hbar^2/2m^* \xi^2(0) \) is the Ginzburg–Landau normalization constant (here \( \xi(0) \) is the GL coherence length amplitude), and \( A \) is the vector potential. Note also that equation (B.2) almost coincides with the conventional time-dependent Ginzburg–Landau equation of the order parameter. The only difference is the presence of a random force, \( G(r, t) \), which must be completely uncorrelated in space and time. The latter implies that \( G(r, t) \) will verify

\[
(G^*(r, t)G(r', t')) = a \delta(r - r') \delta(t - t'), \tag{B.3}
\]

where \( a \) is a normalization constant that may be determined by just taking into account that in the stationary limit equation (B.2) has to reproduce the equilibrium thermal average of the squared order parameter \[38]. This directly leads to \( a = \pi \hbar a_0/4 \).

The presence of a homogeneous electrical field, \( E \), applied at the instant \( t = 0 \) may be taken into account in this formalism by applying

\[
A(t) = \begin{cases} -\epsilon Et & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases} \tag{B.4}
\]

and \( \Phi(r) = 0 \) to equations (B.1) and (B.2). After a standard Fourier-like expansion of the order parameter this leads to

\[
J = V^{-1} \sum_p \frac{2e}{m^*} (\hbar p + 2eEt)|\Psi_p|^2, \tag{B.5}
\]

and, respectively,

\[
\frac{\partial \Psi_p(t)}{\partial t} = - \frac{8k_B T_0}{\pi \hbar a_0} \left[ a_0 \epsilon + \frac{(\hbar p + 2eEt)^2}{2m^*} \right] \Psi_p(t) + G_p(t), \tag{B.6}
\]

where \( \Psi_p(t) \) is the Fourier component of the order parameter corresponding to the wavevector \( p \equiv (p_x, p_y, p_z) \) and \( G_p(t) \) represents the random force in momentum space which, according to equation (B.3), will verify

\[
(G_p^*(t)G_p(t')) = a \delta(p - p') \delta(t - t'). \tag{B.7}
\]

The thermally averaged current density at an arbitrarily high electrical field may be now obtained by introducing in equation (B.5) the \(|\Psi_p|^2\)-expression resulting from equation (B.6). The latter is a differential equation with solution

\[
\Psi_p(t) = \frac{8k_B T_0}{\pi \hbar a_0} \int_{-\infty}^{t} dt' G_p(t) \exp \left\{ - \frac{8k_B T_0}{\pi \hbar a_0} \int_{-\infty}^{t} dt'' \left[ a_0 \epsilon + \frac{(\hbar p + 2eEt'')^2}{2m^*} \right] \right\}. \tag{B.8}
\]

Then, by using equations (B.7) and (B.8), we obtain

\[
(|\Psi_p|^2) = \frac{16k_B^2 T_0^2}{\pi \hbar a_0} \int_{-\infty}^{t} dt' \exp \left\{ - \frac{16k_B T_0}{\pi \hbar a_0} \int_{-\infty}^{t} dt'' \left[ a_0 \epsilon + \frac{(\hbar p + 2eEt'')^2}{2m^*} \right] \right\}. \tag{B.9}
\]

and, subsequently, the supercurrent density at high applied electrical fields will be given by

\[
J = \frac{32e^2 E^2}{\pi \hbar m^* a_0 V} \sum_p (\hbar p + 2eEt) \int_{-\infty}^{t} dt' \exp \left\{ - \frac{16k_B T_0}{\pi \hbar a_0} \int_{-\infty}^{t} dt'' \left[ a_0 \epsilon + \frac{(\hbar p + 2eEt'')^2}{2m^*} \right] \right\}. \tag{B.10}
\]

As addressed in [38], to solve the time integrations involved in equation (B.10) it is convenient first to introduce the factor \((\hbar p + 2eEt)\) inside the integral over \( t' \) and, then, to apply the following changes of variables:

\[
\hbar p = \hbar k - eE(t + t'), \tag{B.11}
\]

\[
t' = u + t, \tag{B.12}
\]

\[
t'' = \frac{1}{2}(u - u') + t. \tag{B.13}
\]

Equation (B.10) is then transformed into

\[
J = \frac{32e^2 E^2}{\pi \hbar m^* a_0 V} \sum_k \int_{-\infty}^{0} du (\hbar k - eEu) \exp \left\{ - \frac{8k_B T_0}{\pi \hbar a_0} \int_{-\infty}^{u} du' \left[ a_0 \epsilon + \frac{(\hbar k - eEu')^2}{2m^*} \right] \right\}, \tag{B.14}
\]

an expression that on solving the integration over \( u' \) and introducing the dimensionless variable \( x = \frac{\hbar k - eEu}{\delta^2 e^2} \epsilon u \) reads as

\[
J = - \frac{e^2 E^2}{4\hbar e^2 V} \sum_k \int_{-\infty}^{0} dx x \exp \left[ x + \frac{\epsilon^2 (\hbar k - eEu)^2}{4e^2 V} \right], \tag{B.15}
\]

where \( E^* = \frac{16k_B T_0}{\pi \hbar a_0} \epsilon^{3/2} \) is the temperature-dependent electrical field characteristic of each material which governs the non-Ohmic regime of the fluctuation-induced supercurrent that appears for \( E \gtrsim E^* \). Note also that in this last expression we have already omitted the \( \hbar k \)-term that appears in the integral over \( u \) in equation (B.12). The reason is that this term is odd and, thus, the contributions corresponding to \( k - -k \) cancel each other when carrying out the sum over the momentum spectrum.
In what follows we will restrict ourselves to applied electrical fields verifying $E \ll E^*$. The last term in the exponential of equation (B.13) can then be suppressed, and the current density exhibits the linear dependence with the electrical field characteristic of the Ohmic regime. By using $J = \Delta \sigma \mathbf{E}$ and performing the trivial integral over $x$, we find the following expression for the fluctuation-induced conductivity as a sum over the modes of the spectrum of the fluctuations:

$$\Delta \sigma = \frac{\epsilon^2}{4\hbar c^2 V} \sum_k \frac{1}{[1 + \xi^2(\epsilon)(k^2)^2]^2},$$

(B.14)

that on transforming the $k$-summations into $k$-integrals through

$$\sum_k \to V \int \frac{dk_x}{2\pi} \int \frac{dk_y}{2\pi} \int \frac{dk_z}{2\pi},$$

(B.15)

may be rewritten as

$$\Delta \sigma = \frac{\epsilon^2}{32\pi^2 \hbar} \int \frac{dk_x}{\epsilon + \xi^2(\epsilon)(k^2)^2}. $$

(B.16)

Equation (B.16) has been derived in the framework of a general model for a 3D isotropic superconductor. However, in view of the ratio between the superconducting coherence length amplitudes in BaFe$_2$-xNi$_x$As$_2$ superconductors, the 3D anisotropic scenario seems to be more appropriate for these compounds. In this last dimensional case, the scale variation of the order parameter in each spatial direction is determined by the corresponding superconducting coherence length. Thus, considering that the $x$ and $y$ directions lie on the $ab$-plane and that $z$ corresponds to the crystallographic $c$-axis, equation (B.16) can be adapted to anisotropic 3D superconductors by applying $\xi^2(\epsilon)(k^2)^2 \to \xi_{ab}^2(\epsilon)(k^2)^2 + \xi_{cd}^2(\epsilon)(k^2)^2 + \xi_{xy}^2(\epsilon)(k^2)^2$ (here $\xi_{ab}(0)$ is the in-plane superconducting coherence length amplitude). Using polar coordinates for the $xy$-plane this leads to

$$\Delta \sigma_{ab} = \frac{\epsilon^2}{16\pi \hbar} \int \frac{dk_x}{\epsilon + \xi_{ab}(0)^2 + \xi_{xy}(0)^2 + \xi_{cd}(0)^2}.$$  

(B.17)

If an external magnetic field $H$ is applied parallel to the $c$-direction, the in-plane spectrum of the fluctuations becomes equivalent to that of a charged particle in a magnetic field [53]. Thus, $k_x$ in equation (B.17) must be replaced by $\epsilon k_x/e B_0(n + 1)$, where $\mu_0$ is the vacuum magnetic permeability and $n = 0, 1, \ldots$ is the Landau-level index. As a consequence, the integral with respect to $k_x$ is transformed into a sum over $n$ through $\frac{1}{2\pi} \int k_x dk_x \to \sum_n$. Besides, we must also include as a multiplier to equation (B.17) the so-called Landau degeneracy factor given by $\sum_{n \neq 0} = \frac{\epsilon k_x}{\pi e B_0}$ (here $\phi_0$ is the magnetic flux quantum). The resulting expression for $\Delta \sigma_{ab}$ is

$$\Delta \sigma_{ab} = \frac{\epsilon^2}{16\pi \hbar} \int \frac{dk_x}{\epsilon + h(2n + 1)} + \xi_{ab}^2(0) k_x^2$$

(B.18)

where $h = H/H_c(0)$ is the reduced magnetic field and $H_c(0) = \phi_0/2\pi \mu_0 \xi_{ab}(0)^2$ is the upper critical magnetic field perpendicular to the $ab$-planes, linearly extrapolated to $T = 0$ K.

To take into account the limits imposed by the uncertainty principle on the shrinkage of the superconducting wavefunction when $\epsilon$ or $h$ increases, an energy cutoff must be applied to equation (B.18) [19]. This restricts the sum over $n$ and the integration over $k_x$ through $n_{\text{max}} = \frac{\epsilon}{\hbar - c - \epsilon}/\xi_{ab}(0)$ (here $c$ is a cutoff constant expected to be of the order of $0.5$ [19, 34], leading to

$$\Delta \sigma_{ab} = \frac{\epsilon^2}{32\hbar \pi \xi_{ab}(0)} \int \frac{\sqrt{2}}{\hbar} \int \sqrt{\frac{\epsilon + h}{\hbar}} dx$$

$$\times \left[ \psi^2 \left( \frac{\epsilon + h + 2h}{2h} + x^2 \right) - \psi^2 \left( \frac{\epsilon + h}{2h} + x^2 \right) \right].$$

(B.19)

In the zero magnetic field limit, i.e., for $h \ll \epsilon, c$, this equation is transformed into

$$\Delta \sigma_{ab} = \frac{\epsilon^2}{16\hbar \pi \xi_{ab}(0)}$$

$$\times \left( \arctan \sqrt{\frac{c}{\epsilon}} - \arctan \sqrt{\frac{c}{\epsilon}} \right),$$

(B.20)

which corresponds to the paraconductivity under an energy cutoff. At low reduced temperatures and magnetic fields, for $h, \epsilon, c$, the cutoff effects become unimportant [19, 34]. Accordingly, in this regime equations (B.19) and (B.20) reduce to the $c$-independent expression

$$\Delta \sigma_{ab} = \frac{\epsilon^2}{32\hbar \pi \xi_{ab}(0)} \int \frac{\sqrt{2}}{\hbar} \int \infty dx \psi^2 \left( \frac{\epsilon + h}{2h} + x^2 \right),$$

(B.21)

and, respectively, the AL paraconductivity in a 3D anisotropic superconductor (equation (1)) [27]. Note finally that equations (B.19) and (B.20) lead to the $\Delta \sigma_{ab}$ vanishing at $\epsilon = c$.

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