Image analysis for velocity profile estimation in A-SOFT hybrid rocket combustor

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Abstract
Altering-intensity Swirling-Oxidizer-Flow-Type (A-SOFT) hybrid rocket engine (HRE) was proposed as a technique to solve problems of current hybrid rockets. It uses axial and tangential oxidizer injections and their mass flow rates are manipulated independently to control the thrust and O/F. The visualization experiment of combustion of gaseous oxygen (GOX) and polymethyl methacrylate (PMMA) of A-SOFT HRE is carried out under combustion pressure of 1 bar. Combustion gas flow in the combustor is captured by high speed cameras whose fps is 30000. In order to capture the nature of the flow field quantitatively, obtaining velocity profiles by image analysis applied to its visualization images is effective. Basic image analysis method is Direct Cross Correlation method and, for high precision and spatial resolution, Correlation Based Correction is applied recursively. Averaged velocity profiles are obtained by averaging the calculation results of 10000 images corresponding to the actual time duration of 0.3 s. Flames with strong white light emission are characteristic of hybrid rockets and regarded as traceable markers for the image analysis. These luminous flames are considered to flow in the boundary layer and velocity of flames in the boundary layer is important to capture the characteristics of combustion flow field. Axial velocity of the luminous flame is proportional to the total mixed mass flow rate for each location x. Tangential velocity is proportional to angular momentum given by tangential GOX injection and inversely proportional to the total mixed mass flow rate for each location x. And how much GOX injected in axial and tangential directions are mixed is speculated by velocity profiles. It is found that mixing of GOX injected in axial and tangential directions occurs at almost the same constant rate regardless of the ratio of GOX mass flow rate injected in axial and tangential directions.

Keywords : Hybrid rocket, Swirling, Combustion, Velocity profile, Image analysis, Direct cross correlation, Correlation based correction

1. Introduction

The hybrid rocket motor has both properties of the liquid rocket motor and the solid rocket motor because its propellants are the combination of solid fuel and liquid oxidizer. In the hybrid rocket motor, the combustion process is called the boundary-layer combustion that with heat transfer from the combustion gas to the fuel grain’s surface, solid fuel on the surface is gasified and added into combustion gas flow and combustion reaction with the gasified solid fuel and the injected oxidizer occurs. There are many advantages offered by the hybrid rocket, such as non-explosive propulsion system, concise structure, controllability of thrust, etc. However there also exist many technical challenges of current hybrid rockets; low fuel regression rate, low combustion stability, O/F-shift, etc.

O/F shift means that O/F cannot be kept optimum due to the change of oxidizer mass flux because fuel regression rate (which refers to regression speed of the fuel grain’s surface by gasification) is nonlinearly dependent on oxidizer mass flux (Marxman, et al., 1964). Oxidizer mass flux variation happens when the fuel port diameter grows with long
time burning or oxidizer mass flow rate changes in throttling. The main problems of O/F shift are that 1) it decreases the regression rate systematically and a lot of fuel residuals remain after mission and 2) when O/F varies much from optimum particularly in deep throttling, time averaged specific impulse decreases less than the optimal one and leads to the engine performance loss. So O/F should be kept optimum by preventing O/F shift.

Alternating-intensity Swirling-Oxidizer-Flow-Type (A-SOFT) hybrid rocket engine (HRE) as shown in Fig.1 has been proposed as a technique to solve problems of current hybrid rockets in our research group (Ozawa, et al., 2015). A-SOFT has two types of oxidizer injection (axial and tangential). Tangential oxidizer injection is known to be able to produce higher regression rate than conventional axial oxidizer injection (Yuasa, et al., 1999). With the constant total flow rate of axial and tangential oxidizer, only changing the swirl intensity (the ratio of mass flow rate of axial and tangential one) controls the fuel regression rate and O/F. A-SOFT HRE has a specific combustion flow field with axial and tangential GOX injection added into it. But there are few cases of combustion experiments of A-SOFT HRE (Ozawa, et al., 2016) and combustion structure remains unclear.

In order to clarify the nature of combustion in A-SOFT HRE, the visualization experiment of combustion is effective and applying image analysis to it can estimate velocity profile of combustion gas in the combustor. One of the popular image analysis methods is PIV (Particle Image Velocimetry) that tracks movement of tracer particles and measures flow field. However as for many hybrid rocket motors, PIV is not suitable because the fuel grain has carbon atoms so the burning with intense light emission may occur and make tracer particles invisible. In A-SOFT HRE, some part of combustion gas has strong white light emission including the burned residue of carbon atoms. The residue emitting light does not disappear shortly but flows for a while in the combustor. So in order to estimate velocity profile of the combustion gas in A-SOFT HRE, combustion gas with strong light itself can be regarded as the traceable marker instead of tracer particles and image analysis can be applied to them. These flames are considered to include the burned residue of carbon atoms shortly after gasification and be pressed near the fuel grain’s surface due to swirling flow so their velocity profile represents the flow near the fuel grain’s surface, in the boundary layer. The velocity analysis of flames in the boundary layer is important to capture the characteristics of the combustion flow field.

Fig.1 The concept of Alternating-number Swirling-Oxidizer-Flow-Type (A-SOFT) hybrid rocket engine.

2. Experiments

2.1 Experimental Setup

The visualization experiment of combustion in the A-SOFT test motor was conducted (Obata, et al., 2017). This research uses the visualization images captured by this experiment. Fig. 2 shows the overall view of experimental setup. Propellants are the combinations of gas oxygen as oxidizer and PMMA as fuel. Gas oxygen is fed to the test motor through two lines for axial and tangential injection. Two high speed cameras are set vertical (high speed camera1) and parallel (high speed camera2) to the direction of combustion flow.

Fig.3 shows the schematic of the hybrid rocket motor. The fuel grain length and a diameter are 150 mm and 40 mm, respectively. Radial and swirl injectors for axial and tangential GOX injection are arranged side by side on the inlet side of the combustor. For visualization, PMMA has a role not only as fuel but also as a transparent combustion chamber without covered by metal as common engines. Through the quartz glass set at the front of the combustor, the inside of the combustor can be seen from the inlet. In order to view the combustion flow in the combustor from the front side clearly, the motor does not have a convergent nozzle that would prevent recognizing the thickness of the flame with the intake of nozzle. Considering no convergent nozzle, static pressure in the combustor is nearly equal to the ambient pressure (0.1 MPa).

Fig.4 shows the schematic of radial and swirl injectors. Both have 8 holes with the same diameter. The only difference is direction of the injector holes; radial or tangential.
Frame rate of high speed cameras is 30000 fps. Visualization images are captured for about 2.5 seconds from 3 seconds after opening the main valve. Ignition is done with the nichrome wire supplying a small amount of GOX (0.11 g/s) until flame propagated to the leading edge of the grain.

2.2 Experimental conditions

The experiment is carried out in 4 experimental conditions and the important point is that all 4 conditions have the same total GOX mass flow rate (6.3 g/s). This research focuses on the impact on the combustion flow caused by the change of the swirl intensity, the ratio of axial and tangential GOX mass flow rate without changing the total.
Experimental conditions are shown in table 1.

| Case  | 1  | 2  | 3  | 4  |
|-------|----|----|----|----|
| GOX supply pressure [MPa] |    | 0.5|    |    |
| Total GOX mass flow rate [g/s] |    | 6.3|    |    |
| Tangential GOX mass flow ratio (Tangential ratio) [-] | 0.41 | 0.58 | 0.76 | 1.00 |
| Burning time [s] | 6.2 | 6.2 | 6.4 | 6.3 |

2.3 Experiment results

By experiments, the visualization images of combustion flow in the A-SOFT HRE are obtained and Fig.5 shows examples of images. Tangential ratio described in Fig.5 means the ratio of tangential GOX mass flow rate to total GOX mass flow rate. Due to images from side view, as tangential ratio increases, the disturbance of flow is reduced and the combustion flow has helical structure more clearly. Luminous flames with strong light emission flowing near the fuel grain’s surface are confirmed on images from front view. They are considered to include the burned residue of carbon atoms emitting intense light and the residue with light does not disappear shortly but flows for a while. These luminous flames can be subjects of image analysis described in the next chapter.

| Condition | Visualization image from the side view | From the front view |
|-----------|---------------------------------------|---------------------|
| Case 1 (Tangential ratio = 0.41) | ![Visualization image](image1) | ![From the front view](image2) |
| Case 2 (Tangential ratio = 0.58) | ![Visualization image](image3) | ![From the front view](image4) |
| Case 3 (Tangential ratio = 0.76) | ![Visualization image](image5) | ![From the front view](image6) |
| Case 4 (Tangential ratio = 1.00) | ![Visualization image](image7) | ![From the front view](image8) |

Fig.5 Visualization images of the combustion flow viewed from front and side views
3. Image analysis method
3.1 Setting of image analysis

In this image analysis, luminous flames are regarded as markers which visualize flow of the combustion gas and velocity profile of combustion gas is estimated by tracking flames on the successive images. A basic image data is composed of a lot of pixels and each pixel has a different luminance value. Because luminous flames emit strong light, whether a pixel on image data contains luminous flame or not can be judged by the magnitude of luminance which it has (the threshold of the luminance value is 210). The way flames are identified is shown in Fig.6 and luminous flames are coloured in blue without the right side with burn marks. At the same time, x, y axis which are used in the discussion are set in Fig.6. Note that this research uses only the visualization images captured by high speed camera1 which gets the side view of the combustor since the influence of perspective is large.

![Fig. 6 Luminous flames coloured in blue in the side view](image)

3.2 Direct Cross Correlation method

Direct cross correlation method is selected as the basic method of this image analysis (Adrian, et al., 1992). This method assumes that in two successive images (frame = n, n+1), a certain area (frame = n) moved to the most similar area (frame = n+1) and the degree of similarity is decided by the cross-correlation function $R(x_1, y_1, x, y)$ in Eq. (1). $(x_1, y_1)$ and $(x, y)$ are the central coordinates of the region1 (frame = n) and any region (frame = n+1) respectively the shape of which is a square. In order to get the average displacement in the region1 $(x_1, y_1)$ (frame = n), first calculate the cross-correlation function $R$ to each region $(x, y)$ (frame = n+1) which indicates the similarity to the region1 $(x_1, y_1)$. When the region2 $(x_2, y_2)$ has the highest value of the cross-correlation function $R$ in the search area, the region1 $(x_1, y_1)$ is assumed to move to the region2 $(x_2, y_2)$ and the average displacement in the region1 $(x_1, y_1)$ is obtained as $(x_2 - x_1, y_2 - y_1)$.

$$R(x_1, y_1, x, y) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} (f_{i,j} - f_m)(g_{i,j} - g_m)}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} (f_{i,j} - f_m)^2 \sum_{i=1}^{N} \sum_{j=1}^{N} (g_{i,j} - g_m)^2}}$$

$f$: luminance of each pixel $(i, j)$ in region1 $(x_1, y_1)$ (size = N x N, frame = n),
$g$: luminance of each pixel $(i, j)$ in a region $(x, y)$ (size = N x N, frame = n+1) in the search area,
$f_m, g_m$: average luminance

However, this method has a serious problem. When the size of region1 $(x_1, y_1)$ is set smaller to raise the spatial resolution, the region1 has fewer pixels and less information of luminance. It is more difficult to distinguish between the true region which the region1 moves to and error regions where the region1 does not actually move but the value of the cross-correlation function $R$ is high. The correlation plane which shows the value of cross-correlation function $R$ for each coordinate $(x,y)$ has the true displacement peak and more noise peaks (in Fig. 7). To solve this problem, correlation based correction (Hart, 2000) is applied recursively and then the displacement of the small region1 can be
obtained.

![Correlation plane with true peak and noise peaks](image)

**Fig.7** Correlation plane which has the true peak and some noise peaks (Hart, 2000)

### 3.3 Correlation Based Correction (CBC)

The first step is to eliminate random peaks in the correlation plane and get the displacement correctly. In order to do that, correlation based correction is the suitable method. About correlation based correction, it is assumed that the correlation planes of the adjacent regions have nearly equal true displacement peaks but noise peaks randomly. So multiplying the values in two correlation planes of two adjacent regions for each relative coordinate cancels noise peaks but leave the true displacement peak. And overlapped region’s displacement is obtained (in Fig.8).

![Correlation anomalies](image)

**Fig. 8** Elimination of correlation anomalies by multiplying the correlation tables from adjacent regions. Correlation values that do not appear in both tables are eliminated allowing tracer particle displacement to be resolved (Hart, 2000)

The second step is that in order to raise the spatial solution, CBC is used recursively with the previous calculation result obtained by CBC. It means that after getting the average displacement in the region1 (N x N pixels) by CBC, resize the region1 (N x N → 0.5 N x 0.5 N pixels) and make the new correlation plane of the new region1 in the surrounding area of the region2 where it is assumed that the region1 moves by the previous calculation of CBC. And then by CBC again, the average displacement in the new region1 (0.5 N x 0.5 N pixels) is obtained. Do this method repeatedly until the region1’s size gets to 2 x 2 pixels which is the smallest applicable size by CBC. Finally, the displacement of the region1 (2 x 2 pixels) is obtained for high precision and spatial resolution by using CBC recursively. 2 pixel corresponds to 0.4 mm. And this research starts with N=32.

### 4. Result and Discussion

#### 4.1 The example of calculation result on the image

Fig. 9 shows the example of displacement vectors obtained by the calculation described above as arrows on the image (Tangential ratio = 1.00). Note that the subjects of calculation are only luminous flames. As long as arrows of displacement vectors are checked on the image, these directions are well obtained.
However, when the displacements [pixel/frame] of luminous flames are converted to velocity vectors [m/s], there is the serious problem. Because the combustor is made of PMMA and the shape of the combustor is cylindrical, there is the effect of distortion and it is difficult to change the image data captured by the high speed camera to actual scale. It is considered that only the center line of the combustor (as the red dotted line in Fig. 9) is not affected by distortion due to curvature of the combustor and unit conversion [pixel/frame → m/s] can be performed correctly using the distance from the camera to the grain’s surface near which luminous flames occur and are pressed due to swirling flow. Incidentally, the luminous flames on both front and back side are seen on images shown in Fig. 10. But they can be distinguished because flames on front side flow to the upper right and flames on back side flow to the lower right on the image due to swirling flow. So they are judged by whether the displacement in y direction obtained by calculation is positive or not. Flames on front side are nearer and clearer than on back side. So only take calculation results of flames on front side and remove flames on back side.

Finally averaging the calculation results from many visualization images gets the average velocity profile of luminous flames which represent the flow near the fuel grain’s surface. This averaging process is needed because from only two successive images, displacements of a few locations at the center line are obtained as shown in Fig. 9 and effect of flame fluctuation is too large to be unsuitable for discussion. So using and averaging the displacements of many frames covers nearly all x’s displacements and influence of flame fluctuation is eliminated. In the image analysis, 10000 images (actual time duration = 0.3 s) from the end of the burning time of each experiment when combustion is considered stable are used. Considering that the regression rate is about 0.2 mm/s and the initial grain diameter is 40 mm, the fuel regression is about $0.2 \text{ mm/s} \times 0.3 \text{ s} = 0.06 \text{ mm}$ so there is almost no change in the shape of the fuel grain during 0.3 s.

### 4.2 Calculation results

At first, assuming that luminous flames at the center line of the combustor applied to image analysis flow just above the fuel grain’s surface, the distance from any target flame to the camera in the camera’s sight direction is almost the same so magnification of unit conversion can be the same for any flame. Compared to the distance from the camera to the combustor, the position errors of flames from the grain surface are small enough to ignore so this assumption can be applicable.

Magnification of unit conversion [pixel → mm] of side view’s images is 150/804 [mm/pixel] because the length of the combustor which is 150 mm corresponds to 804 pixel on the image data. Considering frame rate of the high speed camera which is 30,000 fps, magnification of unit conversion [pixel/frame → m/s] is $150/804 \times 30000/1000 [(\text{m/s}) / (\text{pixel/frame})]$. Based on the above, the calculation results are described in the next section. The plot interval in the x direction is 0.001.
4.2.1 Axial velocity profile in x direction

L is the length of the combustor which is 150 mm and the value of x is made dimensionless (to x/L). Axial velocity means velocity in the x direction and its profiles are shown in Fig.11.

![Axial velocity profiles of luminous flames in the x direction](image)

Fig.11 Axial velocity profiles of luminous flames in the x direction

In Fig.11, it is confirmed that axial velocity increase as x/L increases because gasified fuel is added to the combustion gas and total mass flow rate increases as it approaches the outlet of the combustor. The characteristic point is that the velocity profile of case 4 has an oscillation. Because case 4 has the most tangential GOX mass flow rate and a strong helical flow field is created in the combustor. As shown in Fig.12, high velocity and low velocity are calculated alternatively at the center line of the combustor where image analysis is applied. Another point in Fig.11 is that the more the tangential ratio is, the larger the axial velocity profile is. It is discussed in a later chapter.

![Helical flow field in the combustor (case 4)](image)

Fig. 12 Helical flow field in the combustor (case 4)
4.2.2 Tangential velocity profile in y direction

Tangential velocity means velocity in the y direction and its profiles are shown in Fig.13.

![Fig.13 Tangential velocity profiles of the luminous flames in the y direction](image)

In Fig.13, it is confirmed that tangential velocity decreases as x/L increases because angular momentum given by tangential GOX injection does not increase but GOX injected in the tangential direction is mixed with gasified fuel and GOX injected in the axial one and its mixed mass flow rate increases. And the more the tangential ratio is, the larger the tangential velocity profile is. In case 4, the profile has an oscillation because of a strong helical flow field like the axial velocity profile.

4.2.3 Angle of velocity vector

Using the axial velocity and the tangential velocity obtained above, the angle of velocity vector is calculated by arctangent (tangential velocity/axial velocity) and the results are shown in Fig. 14. Regardless of tangential GOX mass flow rate, the angle of velocity vector is about 60° near the inlet of the combustor for all cases, which may be related to how to set the tangential injectors in the combustor. But as tangential ratio is larger, the reduction rate of angle is smaller. As for the relative low tangential ratio (case 1 and 2), at first their angles sharply decrease from about 60° and low angle values are kept at x/L exceeding 0.5. The notable point is that the profile of case 4 does not have an oscillation unlike the axial and tangential profile. So both high velocity regions and low velocity regions in a strong helical flow field have the similar angle profile.

![Fig. 14 Angle profiles of velocity vector of the luminous flames](image)
4.4 Discussion
4.4.1 Fuel Regression Rate

For the further discussion about the relationship between velocity of luminous flame and mass flow rate of combustion gas, local fuel regression rate is measured and how much fuel mass flow rate is added to the combustion flow is obtained.

First, Table 2 shows the average fuel regression rate obtained from the difference in grain weight before and after the combustion test. From Table 2, it can be seen that the averaged fuel regression rate increases as the tangential ratio increases.

Table 2 Averaged fuel regression rate

| case | 1 | 2 | 3 | 4 |
|-------|---|---|---|---|
| Tangential ratio | 0.41 | 0.58 | 0.76 | 1.00 |
| Total GOX mass flow rate | 6.3 |
| Averaged fuel regression rate | 0.11 | 0.14 | 0.17 | 0.25 |

Next, a method of measuring local fuel regression rate is described below. After the experiments the head-end of the fuel grains is sealed and vertically put below the laser distance meter. A small amount of water is poured inside the grain and a piece of colored plastic is floated on it. The height of the water surface is gotten by measuring the distance from the laser distance meter to the piece of colored plastic. By repeating this, the local fuel regression rate can be obtained by Eq. (2) which uses volume of poured water at once and the change of height of water surface. Incidentally R is 20 mm and burning time t is shown in Table 1. Its measuring devices and results are shown in Fig. 15 and 16.

\[
\dot{r} = \frac{1}{t} \left( \frac{\Delta V}{\pi \Delta h} - R \right) \quad (2)
\]

where
\( \Delta V = \) volume of poured water at once (mm\(^3\))
\( \Delta h = \) change of the height of the water surface (mm)
\( R = \) radius of the fuel grain (mm)
\( t = \) burning time (s)

Also as preparation for the later discussion, integrated value of fuel mass flow rate added into combustion gas for each x is calculated by using Eq. (3) and results are shown in Fig.17. Incidentally, the material of fuel grain is PMMA so \( \rho_f \) is 1.19 x 10\(^{-9}\) g/mm\(^3\).

\[
m_f \left( \frac{x}{L} \right) = \rho_f \int_0^x \dot{r} 2 \pi R \, dx \quad (3)
\]

where \( \rho_f = \) density of solid fuel (g/mm\(^3\))
4.4.2 The relationship between velocity of the luminous flame and mass flow rate of combustion gas

First, GOX injected in the axial direction gradually mixes with GOX injected in the tangential direction so this mixed mass flow rate of GOX injected in the axial direction with the tangential is simply assumed as Eq. (4).

\[
\dot{m}_{oa}^{mixed} \left( \frac{x}{L} \right) = \begin{cases} 
  c \left( \frac{x}{L} \right) + d & \quad (\dot{m}_{oa} + d < \dot{m}_{oa}) \\
  \dot{m}_{oa} & \quad (\text{otherwise}) 
\end{cases}
\]

(4)

where \( \dot{m}_{oa} \) = GOX mass flow rate injected in the axial direction (g/s)
\( c \) and \( d \) = constant parameters (g/s)

And the axial velocity of the luminous flame is considered to be proportional to the sum of the integrated fuel mass flow rate, GOX mass flow rate injected in the tangential direction and mixed mass flow rate of GOX injected in
the axial direction with the tangential. Modeling is performed as Eq. (5)

\[ u_x \left( \frac{x}{L} \right) = a \left( \dot{m}_{ot} + \dot{m}_f \left( \frac{x}{L} \right) + \dot{m}_{oa\text{mixed}} \left( \frac{x}{L} \right) \right) \]  

(5)

where \( \dot{m}_{ot} \) = GOX mass flow rate injected in the tangential direction (g/s)
\[ a = \text{constant parameter (m/g)} \]

As to the tangential velocity of the flame, it is assumed to be proportional to the angular momentum given by the tangential GOX injection and inversely proportional to the total mass flow rate. Its modeling is performed as Eq. (6).

\[ u_\theta \left( \frac{x}{L} \right) = b \left( \dot{m}_{ot}^2 \left( \dot{m}_{ot} \right)^{-1} + \dot{m}_f \left( \frac{x}{L} \right) + \dot{m}_{oa\text{mixed}} \left( \frac{x}{L} \right) \right) \]  

(6)

where \( b = \text{constant parameter (m/g)} \)

So that the difference between axial and tangential velocity profiles obtained by image analysis and modeling is, the smallest parameters \( a, b, c \) and \( d \) are obtained by the least squares method. Each parameter is shown in Table 3 and the axial and tangential velocity distributions for image analysis and modeling are shown in Fig. 18. Also the distribution of \( \dot{m}_{oa\text{mixed}} \left( \frac{x}{L} \right) \) is shown in Fig. 19.

| Case  | 1   | 2   | 3   | 4   |
|-------|-----|-----|-----|-----|
| Tangential ratio [-] | 0.41 | 0.58 | 0.76 | 1.00 |
| Tangential GOX mass flow rate [g/s] | 2.58 | 3.65 | 4.79 | 6.30 |
| Axial GOX mass flow rate [g/s] | 3.72 | 2.65 | 1.51 | 0.00 |
| Parameter a [m/g] | 0.62 | 0.65 | 0.61 | 0.69 |
| Parameter b [m/g] | 0.93 | 1.08 | 1.01 | 1.52 |
| Parameter c [g/s] | 6.78 | 6.73 | 5.99 | -   |
| Parameter d [g/s] | 0.00 | 0.00 | 0.00 | -   |

(a) case 4 (Tangential ratio = 1.00)

Fig. 18 Axial and tangential velocity profiles of each case obtained by image analysis and modeling
Fig. 18 Axial and tangential velocity profiles of each case obtained by image analysis and modeling.
From Fig. 18, the velocity profiles obtained by image analysis and modeling show good agreement. So, the axial velocity of the luminous flame is proportional to the sum of the integrated fuel mass flow rate, GOX injected in the tangential direction and mixed mass flow rate of GOX injected in the axial direction with the tangential. The tangential velocity of the luminous flame is proportional to angular momentum given by tangential GOX injection and inversely proportional to the total mixed mass flow rate.

Moreover, by using the velocity of the luminous flame obtained by image analysis, it gets possible to show how much GOX injected in the axial and tangential directions are mixed in Fig. 19. From parameter c in Table 3 and Fig. 19, it is found that mixing of GOX injected in the axial and tangential directions occurs at almost the same constant rate regardless of the ratio of GOX mass flow rate injected in the axial and tangential directions.

In Table 3, parameter a shows a relatively close value in every case but as for parameter b, in case 4, it is about 1.5 times larger than the other cases. It is because in cases other than case 4, decrease of angular momentum due to viscous force occurs when mixing GOX injected in the axial and tangential direction and parameter b which is related to angular momentum gets smaller than case 4 which has the only tangential GOX injection.

Since $d = 0.00$ in any case, it can be seen that mixing of GOX does not happen at all shortly after injection but gradually occurs in the combustor.

### 5. Conclusion

Flames with strong white light emission are characteristic of hybrid rockets and the image analysis which regards these luminous flames as traceable markers is done and gets their averaged axial and tangential velocity profiles.

The axial velocity of the luminous flame is proportional to the sum of the integrated fuel mass flow rate, GOX injected in the tangential direction and mixed mass flow rate of GOX injected in the axial direction with the tangential. The tangential velocity of the luminous flame is proportional to angular momentum given by tangential GOX injection and inversely proportional to the total mixed mass flow rate.

And how much GOX injected in the axial and tangential directions are mixed is speculated by velocity profiles. It is found that mixing of GOX injected in the axial and tangential directions occurs at almost the same constant rate regardless of the ratio of GOX mass flow rate injected in the axial and tangential directions.
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