Development of a generalized physical and mathematical model of working substance processes in an axial turbine stage

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Abstract. Methods of continuum mechanics and equilibrium thermodynamics are used to study a generalized physical and mathematical model of the processes of a working substance (steam, gas) moving in the flow part of an axial turbine stage in the nominal operating mode of the turbine. The mathematical description of the hydrodynamics and thermodynamics of the process is carried out for a one-dimensional model, based on the energy equation in thermomechanical form (the first law of thermodynamics in combination with the Bernoulli equation) for the gas (or steam) flow during its adiabatic (isentropic) expansion in the direction of the axis of the turbine stage. The energy equation is written taking into account both thermodynamic (due to the thermal movement of particles) and hydraulic pressure forces (due to the mechanical action of a continuous medium) of the working substance flow on the walls of the stage flow channel. Numerical calculation of flow rates and enthalpy losses on various elements of the stage (nozzle and working blades) is performed on the example of a model turbine with eight axial-type stages, under specified initial conditions of the process.

1. Introduction
Existing methods of modernization of steam turbine power units are most effective for medium-capacity heat-generating turbines (100–150 MW) and condensing turbines with a wide range of installed capacity (150–1250 MW).

The main part of the STHP that are in operation at the HPS in Kazakhstan was designed 20–40 years ago. It is obvious that the technical re-equipment and commissioning of new turbines requires the development and creation of a more modern profile of thermal turbine units, taking into account the results of scientific research, as well as the accumulated operating experience. The advantage of combined method of production of electric and thermal energy compared to separate, is known to be
associated with the ability to reduce cost per unit of heat for electricity generation due to reduction of losses in cold spring. Therefore, the main direction of improving the efficiency of heating turbines is to reduce heat losses in condensers and increase the share of electricity generated by heat consumption [1,2], which determines the relevance of the research problem. In principle, this goal can be achieved in two ways: 1) the useful use of waste steam heat (for example, for heating network or process water in the condenser) and minimizing heat flows to the condenser cooled by circulating water [2]. The implementation of these technical solutions, in most cases, is focused on improving the reliability of the Central heating system and thermal efficiency, and the efficiency of the turbine unit as a whole, which determines the practical significance of the research problem.

The specifics of the design, working principles and technological scheme STHP suggests the possibility of improving the heat efficiency of turbines of this class of methods [3,4]: 1) reconstruction of flow part of cylinders of high and medium pressure by [5–8]: a) Replacement of rotor blades of the last stages of medium-pressure cylinders on saber blades. As a rule, the design of a medium-power MPC STHP (100 – 160 MW) involves the selection of steam from the last stages of the MPC, for heating network water in boilers; b) Removing the nozzle stages of the HPC and MPC. Part of the stages are all-forged-poured into the turbine shaft; 2) Switching the STHP to partial back pressure mode by disconnecting the condenser from the turbine heat circuit, turning the separation diaphragm between the medium and low-pressure cylinders [9–11].

The solution of the above technical problems cannot be built without the development of universal algorithms for theoretical analysis and numerical (based on computer programs) prediction of optimized parameters (thermodynamic and hydrodynamic) processes of the working substance (steam) on the elements of the flow part of the upgraded turbine stage, which is the Central working body of turbine units (power units), both condensing and heating type, working, in General, to cover both electric and thermal loads of petrolierous, in variable operating modes. The mathematical model of technological processes of this kind will be based on solutions of a nonlinear system of energy equations in thermomechanical form (the first law of thermodynamics in combination with the Bernoulli equation), equations of continuity of mass and energy flows, equations of conservation of momentum of the gas flow (in the special case, steam) and equations of state of the working substance (gas).

To determine the temperature field in a body volume with the surface of the local insulation, heat flows, heat exchange and internal heat sources (different types of heat sources), the corresponding problem is reduced to the construction and minimization (using the methods of regression analysis and the method of minimizing the comparison function, Insert the tag (MFS method) [12]) for the functional of the total energy (thermal and mechanical) of the physical system, including the flow of working medium moving in a closed heat insulated circuit of the technological process in a steam turbine, connected to the primary external energy source (steam boiler) and the cooling system in the turbine exhaust ballast (condensing unit). Using the procedure of minimization of this functional, a splitting system of equations is constructed from the conditional values of temperature, taking into account natural boundary conditions. The solution of this set of theoretical questions determines the scientific and technical significance of the research problem.

Within the framework of this scientific article, we will lay the theoretical foundations for further development of a universal physical and mathematical model based on a multidimensional (investigated on a set of variable parameters) functional of the total energy of the working substance (steam, gas) involved in a washed turbine installation (power unit) of any model and any functional purpose.

A model axial turbine with eight stages is accepted as the object of research. The subject of the study is the construction of a generalized thermomechanical energy equation and numerical calculation of the parameters of the working substance flow in this model.
2. Generalized equations of continuity of mass and energy flows of the working substance

The design of the turbine stage, consisting of adjacent rows of nozzle (accelerating) and working (guiding) blades, provides for the following processes. On the nozzle blades, due to the adiabatic expansion of the working substance (gas, steam), the internal energy (enthalpy) of the high-potential flow of the working substance is converted into the kinetic energy of the directed flow movement [9,10]. Further, on the working blades, due to the inelastic impact of the flow on the surface of the blade, according to the law of conservation of momentum, the kinetic energy of the flow of the working substance is converted into the energy of the rotational movement of the rotor mounted on the common shaft of the turbine [9,10].

Methods of continuum mechanics involve a formal division of the total mass \( \mathbf{m} = \int \rho(\mathbf{r}; t) dV \) of the hydrodynamic flow of the working substance (gas) into elementary sections of mass \( \mathbf{d}m = \rho(\mathbf{r}; t) dV = \frac{dV}{\mathbf{u}(\mathbf{r}; t)} \), where \( \mathbf{u}(\mathbf{r}; t) = \rho(\mathbf{r}; t) \) the specific volume of the flow substance \( \rho(\mathbf{r}; t) \) with a density calculated at an arbitrary point with a radius vector \( \mathbf{r}(t) = \{x(t), y(t), z(t)\} \) at time \( t \) from the beginning of the process. At the same time, the elementary section of the flow is a curved cylinder with a point trajectory \( d\mathbf{l} = d\mathbf{r} \) that makes sense of the elementary arc length \( M(x; y; z) \), moving \( \mathbf{c}(\mathbf{r}; t) = \frac{d\mathbf{r}}{dt} \) at a speed whose vector is \( d\mathbf{l} \equiv d\mathbf{r} \) oriented tangentially to the arc perpendicular to the cross section \( S_\perp \), the geometric center of which is located \( M(x; y; z) \) at the point with the radius vector \( \mathbf{r}(t) = \{x(t), y(t), z(t)\} \). It is obvious that the volume differential (elementary flow volume) of the flow channel is equal to the volume of the elementary cylinder \( dV = (dS_\perp \cdot d\mathbf{r}) \). The flow rate of the gas \( G = \frac{d\mathbf{m}}{dt} \) mass through the cross-section of the flow channel \( S_\perp \) is \( \mathbf{j}(\mathbf{r}; t) = \frac{d\mathbf{m}}{dS_\perp dt} \), where the gas current density vector is determined by the expression \( \mathbf{j}(\mathbf{r}; t) = \frac{dG}{dS_\perp} \), and, can be calculated from the identity \( \mathbf{j}(\mathbf{r}; t) = \frac{dG}{dS_\perp} \). From the theorem

\[
\frac{d}{dt} \left[ \int \rho(\mathbf{r}; t) dV \right] = \int \left[ \frac{\partial \rho(\mathbf{r}; t)}{\partial t} + \nabla \cdot (\rho(\mathbf{r}; t) \mathbf{c}) \right] dV
\]

\( \int \nabla \cdot (\rho(\mathbf{r}; t) \mathbf{c}) dV = \int \left( \int (\mathbf{j}(\mathbf{r}; t) \cdot dS_\perp) \right) \), taking into account the identity

\[
\frac{d}{dt} \left[ \int \rho(\mathbf{r}; t) dV \right] = -\int \left( \int \mathbf{j}(\mathbf{r}; t) \cdot dS_\perp \right)
\]

admitting an additional identity in the diffusion approximation, we obtain

\[
\frac{d}{dt} \left[ \int \rho(\mathbf{r}; t) dV \right] = \int \left( \int \mathbf{j}(\mathbf{r}; t) \cdot dS_\perp \right)
\]

Then, finally write down. We find the density of the total mass current.

The diffusion current density is calculated from Fick's law \( \mathbf{j}_{D_{\text{diff}}}(\mathbf{r}; t) = -\nabla(D_{\text{diff}}(\mathbf{r}; t) \rho(\mathbf{r}; t)) \) [13,14]. The function \( \rho(\mathbf{r}; t) \), under a strict approach, should be constructed from the solution of the kinetic equation describing the mechanism of relaxation transfer of matter particles in the direction of the action of the injection force generated by the pressure (pressure gradient) in the system. The value \( D_{\text{diff}}(\mathbf{r}; t) \) that has the meaning of the diffusion coefficient is a function of temperature \( T \) and is calculated using kinetic theory methods, taking into account all possible interactions of gas molecules during their collisions. The total energy of the hydrodynamic flow, taking into account the processes of heat transfer with the environment (elements of the blade apparatus) can be represented in the form
E = \int E_v(\vec{r}; t) \, dV, \quad \text{where} \quad E_v(\vec{r}; t) = \frac{\partial E}{\partial V} \quad \text{the volume energy density of the flow} \left[ \frac{J}{m^3} \right]. \quad \text{The mechanical power of the flow} \quad N = \frac{dE}{dt} \quad \text{can be written as a surface integral} \quad N = \mathbb{A}(q(\vec{r}; t) \cdot dS), \quad \text{where the mechanical energy flux density vector can be calculated from the identity} \quad \vec{q}(\vec{r}; t) = \frac{dE}{dS} = E_v(\vec{r}; t) \cdot \vec{c}(\vec{r}; t) \left[ \frac{J}{m^2} \cdot s \right].

Entering the specific energy of the flow \quad w(\vec{r}; t) = \frac{dE}{dm} = \frac{E_v(\vec{r}; t)}{\rho(\vec{r}; t)}, \quad \text{taking into account} \quad \vec{q}(\vec{r}; t) = \rho(\vec{r}; t) \cdot \vec{c}(\vec{r}; t), \quad \text{we have} \quad \vec{q}(\vec{r}; t) = w(\vec{r}; t) \cdot \vec{j}(\vec{r}; t).

On the basis of the generalized differential equality
\[ \frac{d}{dt} \left[ \int E_v(\vec{r}; t) \, dV \right] = \left[ \frac{\partial E_v(\vec{r}; t)}{\partial t} + \text{div} \left( E_v(\vec{r}; t) \cdot \vec{c}(\vec{r}; t) \right) \right] \, dV, \quad \text{taking into account the losses of mechanical energy to heat} \quad N_{\text{diff}} = \mathbb{A}(q_{\text{diff}}(\vec{r}; t) \cdot dS), \quad \text{where the heat flux} \quad \vec{q}_{\text{diff}}(\vec{r}; t) = w(\vec{r}; t) \cdot \vec{j}_{\text{diff}}(\vec{r}; t), \quad \text{is the heat flux density, power diffusion flux of energy} \quad \vec{\lambda}_{\text{diff}}(\vec{r}; t) = -\langle \vec{\lambda}^2 \vec{\nabla} (\Omega(\vec{r}; t) \cdot \rho(\vec{r}; t)) \rangle, \quad \langle \vec{\lambda} \rangle = \text{the average energy mean free path length,} \quad \Omega(\vec{r}; t) = \text{averaged energy, the frequency of collisions of gas molecules, we build a generalized differential equation of continuity of energy flow in the system} \left[ 15–17 \right]
\[ \frac{\partial}{\partial t} \left( w(\vec{r}; t) \rho(\vec{r}; t) \right) + \text{div} \left( w(\vec{r}; t) \cdot \left( \rho(\vec{r}; t) \cdot \vec{c}(\vec{r}; t) - \langle \vec{\lambda} \rangle \vec{\nabla} (\Omega(\vec{r}; t) \cdot \rho(\vec{r}; t)) \right) \right) = 0. \quad (1)

When deriving equation (1), the expression of the total heat flux density was used \quad \vec{q}(\vec{r}; t) = \vec{q}_{\text{tr}}(\vec{r}; t) + \vec{q}_{\text{diff}}(\vec{r}; t), \quad \text{in which} \quad \vec{q}_{\text{tr}}(\vec{r}; t) = -\vec{\nabla} (\chi(\vec{r}; t) T(\vec{r}; t)) - \text{Fourier law for thermal conductivity} \quad \vec{q}_{\text{tr}}(\vec{r}; t) = \alpha(\vec{r}; t) \times (T(\vec{r}; t) - T_0) - \text{the Newton-Richman law for heat transfer processes;} \chi(\vec{r}; t), \alpha(\vec{r}; t) - \text{nonlinear coefficients of thermal conductivity and heat transfer;} \quad T_0 - \text{the temperature of the flow channel wall, assumed to be unchanged in the wall thickness. In (1) \langle \vec{c}(\vec{r}; t) \rangle = \text{energy-averaged rate of particle transport in the direction of gas flow expansion.}

The solution of equation (1) will be constructed together with the thermal conductivity equation for the working substance (gas) moving in the heat-insulated flow channel of the axial turbine stage. Also, together with (1), the equation of the working substance flow energy should be studied, which we will construct in a generalized thermostatic mechanical form, taking into account the transformations of all types of flow energy (internal (thermal), kinetic, potential) and additional heat flows (internal (local) and external). The degree of accuracy of the developed model is determined by the rigor of the underlying fundamental equations of theoretical and mathematical physics presented in the paper [18].
3. Generalized equation of the energy of the working substance flow in the flow part of the turbine stage

Based on the methods of continuum mechanics, we present the differential of the total energy of the flow of the working substance (gas) with an elementary mass $\text{dm}(\mathbf{r}; t) = \rho(\mathbf{r}; t)\text{dV}$ moving through the cross section (in the direction of the axis) of the flow part of the model device (apparatus) as [19–23]

$$\text{dE} = \frac{\text{dm} \cdot c^2}{2} + \text{dU} + \text{dA} + \text{dQ}_\text{loc},$$

(2)

Here $\text{dE} = \text{dvC, T} = \frac{\text{dm}}{M} \cdot C, T = c, \text{Tdm}$ is the differential of the internal energy of the gas, defined at an arbitrary temperature $T$; $M$ is the molar mass of gas; $c_v$ is the gas molar heat capacity; $c_p = \frac{C_p}{M}$ is the isochoric specific heat of gas taken constant at a temperature $T$; $a = \rho \text{dV}$ is the elementary work performed by external forces when the gas expands by an amount $\text{dV}$; $\text{dE}_k = \frac{\text{dm} \cdot c^2}{2}$ is the differential of the kinetic energy of the flow, calculated in this cross section, $c$ is the flow rate. In expression (2) $Q_{loc} = -\rho \text{dV}$, the differential of the amount of heat entering the system at this local stage of the process. When calculating the mass $\text{dm}(\mathbf{r}; t) = \rho(\mathbf{r}; t)\text{dV}$ differential, the gas (vapor) density will be assumed to be a spatially homogeneous $\rho(\mathbf{r}; t)$ value within the entire elementary section $\text{dV}$ of the flow at temperature $T$ [24]. Also, assuming the process to be stationary $\rho(t) = \rho = \text{const}$, further investigation of expression (2) is performed under the condition $\text{dm} = \rho \text{dV}$ [25]. The total specific energy of the flow $w = \frac{\text{dE}}{\text{dm}}$ takes the form

$$w = \frac{c^2}{2} + u + a + q_{loc},$$

(3)

In (3) $u = \frac{\text{dU}}{\text{dm}} = c_v, T$ is the specific internal energy, $a = \frac{\text{dA}}{\text{dm}} = \frac{p}{\rho} = p_u$ is the specific work, and $u = \frac{1}{\rho}$ is the specific volume of gas.

Also in (3), a specific local amount of heat is introduced $q_{loc} = \frac{\text{dQ}_{loc}}{\text{dm}}$. Flow pressure in the system $p = p_r + p_{HD} + p_{HDSt}$ [26], where $p_r$ is the pressure due to thermodynamic forces associated with the impact of chaotically moving gas particles on the walls of the flow channel; $p_{HD}$ is pressure due to the action of hydrodynamic forces associated with the mechanical action of the flow on the walls of the flow channel. The pressure forces of a hydrostatic column $p_{HDSt} = \rho g z$ are determined by the distance $z$ from the axis of the given cross-section to the horizontal surface (taken as the zero level). Then, by virtue of $a = p_u = (p_r + p_{HD} + p_{HDSt})u$, we represent the specific work of the gas as the sum of the technical $a_{HD} = p_{HD}v$ and thermodynamic $a_r = p_r v$ specific work. Applying the Clapeyron-Mendeleev $p_r v = rT$ equation of state, according to $u + a = (c_r + r)T + p_{HD}v + gz = c_p T + p_{HD}v + gz$ [27–31] and, calculating the specific enthalpy of a gas $h_r = \frac{\text{dH}}{\text{dm}} = \frac{\text{dU} + \text{dA} + gz}{\text{dm}} = u + a_r$, in the form $h_r = c_p T$ we get
\[ u + a = h + p + p R S \] [32–35]. Here \( \frac{R}{M} \) is the universal gas constant. The Isobaric specific heat of a gas satisfies the Mayer equation \( c_p = c_v + R \). Then, the expression, in the generalized case, takes the form of an energy equation in thermomechanical form

\[
w = \frac{c^2}{2} + h_r + p \frac{R}{\rho} + p R S + q_{loc}.
\]  

(4)

Since thermodynamic forces make the main contribution to the gas pressure for the processes of the working substance in the flow channel of the turbine stage \( h_r >> p \frac{R}{\rho} \), equation (4) is simplified

\[
w = \frac{c^2}{2} + h_r + q_{loc}.
\]  

(5)

We apply expression (5) to the input cross-section (I - I) and output cross-section (II-II) of the device. Specific local amounts of heat are \( q_{loc1} = \frac{dQ_{loc1}}{d\rho} \), \( q_{loc2} = \frac{dQ_{loc2}}{d\rho} \) calculated in the same way. Determining the increment of the specific energy of the flow in the flow channel of the device (in the space between the sections (I - I), (II-II)) as the flow of heat energy into the system

\[
\Delta q = w_2 - w_1 = \Delta \left( \frac{c^2}{2} + h_r + q_{loc}\right)
\]  

[36–42], we will proceed to the differential of the total amount of heat received by the gas at this stage of the process \( dq = \frac{d c^2}{2} + h_r + q_{loc}\). This expression is differential and holds for an arbitrary elementary cross-section. Near this section (on the set of points of the continuum measure), the gas state can be considered as equilibrium.

4. Adiabatic expansion of the working substance in the axial turbine stage

For the case of an adiabatic gas (or water vapor) process in a flow channel, when \( q_{loc} = 0 \), \( dq = 0 \) and

\[
d \left( \frac{c^2}{2} + h_r \right) = 0
\]  

the generalized energy equation in thermomechanical form (4) is reduced to a form

\[
w = \frac{c^2}{2} + h_r
\]  

convenient for practical calculations of the parameters of the working substance in the flow channel of an axial turbine stage. It is obvious that during isentropic \( (ds = 0) \) expansion of the \( (v_2 > u_1) \) gas, when the pressure \( (p_{f2} < p_{f1}; T_2 < T_1) \) and \( \Delta h_r = h_{r2} - h_{r1} < 0 \) temperature of the gas monotonically...


\[ \Delta \left( \frac{c^2}{2} + h_t \right) = 0 \]

decrease and, accordingly \( c_2 > c_1 \), the velocity of the directed flow movement (along the stage axis) increases \( \left( c_2 > c_1 \right) \).

In this case, the drop in enthalpy on the nozzle apparatus \( \Delta h = h_{nt} - h_{it} = \frac{c_{nt}^2 - c_{it}^2}{2} \) determines the theoretical flow rate at the nozzle outlet \( c_{nt} = \sqrt{c_{it}^2 + 2h_c} \), where \( h_{nt}, h_{it} \) respectively, the actual enthalpy of the flow at the inlet and the theoretical enthalpy at the nozzle outlet. The actual flow rate at the outlet of the nozzle array is reduced in comparison with the theoretical one \( c_1 < c_{nt} \) and is determined using the velocity loss coefficient \( \varphi = \frac{c_{nt}}{c_1} < 1 \), which takes values \( \varphi \approx 0.98 \div 0.99 \).

On the working grid, the drop in enthalpy \( \Delta h_p = h_{tp} - h_{2t} = \frac{w_{2t}^2 - w_{1t}^2}{2} \) mainly affects the increase in the relative flow rate \( \dot{W} = \dot{c} - \dot{u} \), which takes a value (at the exit from the grid) \( w_{2t} = \sqrt{w_{1t}^2 + 2h_p} \). The actual relative flow rate at the inlet of the working grid \( \dot{W}_1 \) is related to the actual absolute flow rate at the outlet of the nozzle grid \( \dot{c}_1 \) by the equality \( \dot{W}_1 = \dot{c}_1 + \dot{u} \), where \( \dot{u} \) is the circumferential velocity of the point on the rotor rim. Absolute flow rate at the outlet of the working grid \( \dot{c}_2 = \dot{w}_2 + \dot{u} \).

Coefficient of speed loss on the working grid \( \psi = \frac{w_{2t}}{w_{1t}} < 1 \).

Available stage heat transfer \( \Delta h_t = h_t - h_{it} = \frac{c_{nt}^2 - c_{it}^2}{2} \). The loss of enthalpy in the nozzle and the working grid are computed from \( \Delta h_c = h_{tc} - h_{it} = \frac{c_{nt}^2 - c_{it}^2}{2} \). Total enthalpy losses in the stage \( \Delta h_p = \Delta h_c + \Delta h_t = \frac{c_{nt}^2 - c_{it}^2}{2} \). Next, we find the relative blade efficiency of the stage \( \eta_{bl} = \frac{E_{nt} - \Delta h_p}{E_{nt}} = \frac{c_{nt}^2 + w_{2t}^2 - w_{1t}^2}{2} - \left( c_{nt}^2 - c_{it}^2 \right) \frac{w_{1t}^2 + w_{2t}^2}{2}. \) (6)

5. Nonlinear hydrodynamic phenomena at the stages of the MPC of a heating turbine

Based on the results of numerical calculations performed for the stage No. 23 (MPC) of the T heat-generating turbine-110-120-130 LMP, in the partial back pressure mode, in the flow channel of the reconstructed part of the average pressure, fundamentally new thermal and hydrodynamic properties of steam are shown in Figure 1
Figure 1. Dependence of the relative internal efficiency $\eta_{oi}$ of the turbine T-110-120-130 LMP, from the pressure $P^H_T$ in the lower heating selection (article No. 23 (MPC)).

In contrast to the existing (linear) hydrodynamic model of the working blades, non-linear hydrodynamic effects are observed at stage 23 (lower heating steam extraction with MPC), which are manifested in the presence of the calculated maximum of the function $\eta_{oi}(P^H_T)$. According to the set initial values of the steam parameters ($12.7$ MPa; $555^0C$): $\eta_{oi(max)} = 0.758$ at $(P^H_T)_{max} = 0.07$ MPa.

Linear model of hydrodynamic flow, for a T turbine-110-120-130 LMP [43-46], gives a monotonically decreasing function of the relative internal efficiency of the sampling pressure in the range of 0–0.14 MPa [47–50].

6. Conclusions
1. The theoretical foundations of a generalized physical and mathematical model of the processes of the working substance (steam, gas) in the flow part of the axial turbine stage are Developed.
2. The Mathematical description of hydrodynamic and thermodynamic processes is carried out for a one-dimensional model based on the nonlinear equation of continuity of the energy flow (1), which is solved together with the energy equation in thermomechanical form (4) for the gas (or steam) flow during its adiabatic expansion.
3. From the numerical study of the model equations, it is established that the introduction of saber-shaped blades into the flow part of the stage leads to an abnormal dependence of the relative internal efficiency of the stage on the pressure in the steam turbine heating selection, which is explained by nonlinear hydrodynamic phenomena caused by the geometry of the blades.

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