Remarks on the Gribov horizon and dynamical mass generation in Euclidean Yang-Mills theories

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Abstract

The effect of the dynamical mass generation on the gluon and ghost propagators in Euclidean Yang-Mills theory in the Landau gauge is analysed within Zwanziger’s local formulation of the Gribov horizon.

1 The model

In a series of papers [1, 2], D. Zwanziger has shown that the restriction of the domain of integration in the path integral to the Gribov region \( \Omega = \{ A^a_\mu \mid \partial A^a = 0, \ M^{ab} > 0 \} \), where \( M^{ab} = -\partial_\mu (\partial_\nu \delta^{ab} + g f^{abc} A^c_\mu) \) is the Faddeev-Popov operator, can be implemented by adding to the Yang-Mills action the nonlocal horizon term

\[
S_h = g^2 \gamma^4 \int d^4 x f^{abc} A^b_\mu (M^{-1})^{cd} f^{dec} A^e_\mu .
\] (1.1)

The parameter \( \gamma \) is known as the Gribov parameter [3], and is determined by the horizon condition [1, 2], \( \frac{\delta \Gamma}{\delta \gamma} = 0 \), \( \Gamma \) being the quantum effective action. The nonlocal term (1.1) can be localized by introducing a suitable set of additional fields [1, 2]. The resulting action displays two remarkable properties, namely: locality and multiplicative renormalizability [1, 2, 4]. Moreover, these properties are preserved when the local composite operator

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$A^a_{\mu}A^a_{\mu}$ is introduced in the theory. This enables us to discuss the condensate $\langle A^a_{\mu}A^a_{\mu} \rangle$ and the related dynamical gluon mass $m$ in the presence of the Gribov horizon $\partial \Omega$, within a local renormalizable framework. We give here a sketchy account of this analysis by limiting ourselves to consider the Gribov approximation for the horizon action (1.1), by setting $M^{ab} \approx \delta^{ab}\partial^2$. A more complete and detailed analysis is in preparation [7]. The BRST invariant local action implementing the restriction to the region $\Omega$, and allowing setting $M$, limiting ourselves to consider the Gribov approximation for the horizon action (1.1), by a local renormalizable framework. We give here a sketchy account of this analysis by making use of the algebraic renormalization [8], the action $S$ for the inclusion of the operator $A^a_{\mu}$, is $S = (S_{YM} + S_{gf} + S_h + S_\gamma + S_{mass})$. The term $(S_{YM} + S_{gf})$ is the Yang-Mills action together with the Landau gauge fixing, while $S_h$ is the localized version of the horizon action (1.1) in the Gribov approximation, containing the additional fields $\{\varphi^a_{\mu}, \varphi^a_{\mu}\}$ and $\{\bar{\omega}^a_{\mu}, \omega^a_{\mu}\}$. We have

$$S_{YM} = \frac{1}{4} \int d^4 x F^a_{\mu
u} F^a_{\mu
u}, \quad S_{gf} = s \int d^4 x \bar{c}^a \partial \mu A^a_{\mu}, \quad S_h = -s \int d^4 x \bar{\omega}^a_{\mu} \partial^2 \varphi^a_{\mu},$$

Following [1, 2], the term $S_\gamma$ defines the composite operator $A^a_{\mu}\varphi^a_{\mu}$ and its BRST variation, introduced here through the corresponding sources $J, \lambda$. Finally, $S_{mass}$ accounts for the mass operator $A^a_{\mu}A^a_{\mu}$ and its BRST variation, coupled to the sources $\tau, \eta$.

$$S_\gamma = s \int d^4 x \left( \lambda A^a_{\mu} \varphi^a_{\mu} + JA^a_{\mu}\bar{\omega}^a_{\mu} + \xi \bar{\lambda} J \right), \quad S_{mass} = \frac{1}{2} s \int d^4 x \left( \tau A^a_{\mu}A^a_{\mu} - \zeta \tau \eta \right). \quad (1.2)$$

The parameters $\xi$ and $\zeta$ are needed to account for the divergences arising in the vacuum correlation functions of these composite operators. The nilpotent BRST transformations of the fields and sources are as follows:

$$sA^a_{\mu} = -\left( \partial \mu c^a + g f^{abc} A^b_{\mu} c^c \right), \quad s\varphi^a_{\mu} = \bar{\varphi}^a_{\mu}, \quad s\bar{\lambda} = \bar{J}, \quad s\tau = \eta,$$

$$sc^a = \frac{1}{2} g f^{abc} c^b c^c, \quad s\bar{\varphi}^a_{\mu} = 0, \quad s\bar{J} = 0, \quad s\eta = 0,$$

$$sB^a = B^a, \quad s\varphi^a_{\mu} = \omega^a_{\mu}, \quad s\lambda = 0,$$

$$sB^a = 0, \quad s\omega^a_{\mu} = 0, \quad sJ = \lambda. \quad (1.3)$$

By making use of the algebraic renormalization [8], the action $S = S_{YM} + S_{gf} + S_h + S_\gamma + S_{mass}$ turns out to be multiplicatively renormalizable. In particular, as discussed in [1, 2], the horizon condition is obtained by setting the sources $(J, \bar{J}, \lambda, \bar{\lambda})$ equal to $J = \bar{J} = \gamma^2$, $\lambda = \bar{\lambda} = 0$, and by requiring that $\frac{d\Gamma}{d\gamma} = 0$.

## 2 Gap equation and propagators

Proceeding as in [3], it is not difficult to evaluate the effective action $\Gamma(\gamma)$ at one-loop level, in the presence of the dynamical gluon mass $m$. The horizon condition $\frac{d\Gamma}{d\gamma} = 0$ leads to the following gap equation

$$\frac{3Ng^2}{4} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4 + m^2k^2 + \gamma^4} = 1, \quad (2.4)$$

which generalizes that obtained in [3]. Notice now that the dynamical mass $m$ appears explicitly in eq. (2.4). The gap equation (2.4) can be used to obtain the gluon
and ghost propagators in the tree-level approximation. The gluon propagator is found to be

\[
\langle A^a_{\mu}(q)A^b_{\nu}(-q) \rangle = \delta^{ab} \frac{q^2}{q^4 + m^2 q^2 + \gamma^4 \left( \delta_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right)}.
\]  
(2.5)

For the ghost two point function we have

\[
G(q) = \langle c^a(q)\bar{c}^a(-q) \rangle \sim \frac{1}{q^4}.
\]  
(2.6)

Notice that, according to the \[3, 1, 2\] the gluon propagator is suppressed in the infrared, while the ghost propagator is enhanced.

3 Conclusions

The restriction of the domain of integration to the region \(\Omega\) has been implemented by considering Zwanziger’s local action \[3, 1, 2\] in the Gribov approximation. Expression \(2.4\) is the gap equation defining the Gribov parameter \(\gamma\) in the presence of the dynamical gluon mass \(m\). The resulting gluon propagator is suppressed in the infrared, while the ghost propagator is enhanced. This behavior of the gluon and ghost propagators is in agreement with that found in \[3, 1, 2\]. It has also been found in \[10\] within the Schwinger-Dyson framework. Evidences for a dynamical gluon mass in the Landau gauge within the Schwinger-Dyson formalism have been obtained recently in \[11\]. Finally, lattice simulations have provided confirmations of the infrared suppression of the gluon propagator and of the ghost enhancement, see \[12\] and refs. therein, reporting a gluon mass \(m\) of the order of \(\approx 600\) MeV \[13\].

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