Triply charged Higgs bosons at a 100 TeV $pp$ collider

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Abstract In this work, we study the potential of searching for triply charged Higgs boson originating from a complex Higgs quadruplet in the final state with at least three same-sign leptons. A detailed collider analysis of the SM backgrounds and signals is performed at a 100 TeV $pp$ collider for the triply charged Higgs boson mass below 1 TeV and the Higgs quadruplet vacuum expectation value $v_{\Delta}$ ranging from $1.5 \times 10^{-9}$ GeV to 1.3 GeV and the mass splitting $\Delta m$ between the nearby states of the Higgs quadruplet satisfying $|\Delta m| \lesssim 30$ GeV. About 100 fb$^{-1}$ of data are required at most for $5\sigma$ discovery. We also revisit the sensitivity at the Large Hadron Collider (LHC) and find that $5\sigma$ discovery of the triply charged Higgs boson below 1 TeV can be reached for a relatively small $v_{\Delta}$. For example, if $v_{\Delta} = 10^{-6}$ GeV and $\Delta m = 0$, the integrated luminosity of 330 fb$^{-1}$ is needed. But for a relatively large $v_{\Delta}$, i.e., $v_{\Delta} \gtrsim 10^{-3}$ GeV, the triply charged Higgs boson above about 800 GeV cannot be discovered even in the high-luminosity LHC era. For $\Delta m > 0$, the cascade decays are open and the sensitivity can be improved depending on the value of $v_{\Delta}$.

1 Introduction

The discovery of a neutral Higgs boson with mass around 125 GeV [1,2] was a great success of the Standard Model (SM) of particle physics. Nevertheless, the SM is not complete since it does not explain neutrino mass and cannot provide dark matter candidate, et al. Studies of new physics beyond the SM at colliders, such as precision tests of the Higgs boson and direct searches for additional Higgs bosons, can help us to answer these questions, since new physics may be closely connected to the Higg sector at TeV scale.

There are many highly motivated theoretical models, which predict more than one Higgs boson. The two-Higgs-doublet [3], minimal SUSY [4], and multi-Higgs doublet [5,6] models are some of the most studied models. In these models, there are not only new neutral Higgs bosons, but also singly charged Higgs boson. Electrically multi-charged Higgs bosons also exist in other well-motivated models, such as the doubly charged Higgs boson in the Type-II seesaw model [7–15], and even higher (multiple) charged Higgs boson in minimal dark matter models [16,17]. Since the mass splitting in the Higgs multiplet is small compared to the mass of a charged or neutral component itself, the discovery of any of these Higgs bosons can also provide indirect constraints on the other components. To this end, one needs to know how such color singlet multi-charged Higgs bosons can be produced and detected at colliders.

Before going to some detailed discussions, let us briefly discuss the main mechanism of producing multi-charged Higgs bosons and how they can be detected. We will indicate a color singlet higher dimensional Higgs boson $H_n$ transforming under $SU(2)_L \times U(1)_Y$ as $(n, Y)$. We will take values of $Y$ such that the resulting electric charges of the Higgs bosons are zero or integers and write the component fields as $h_n^Q$ with electric charge given by $Q = m + Y_n$. Here $m$ is the third component of isospin $I_n$. The kinetic terms of the Higgs multiplet $H_n$ and the SM Higgs doublet $H$ are given by

$$\mathcal{L}_{\text{kinetic}} = (D^\mu H)^\dagger D_\mu H + (D^\mu H_n)^\dagger D_\mu H_n,$$

$$D_\mu = \partial_\mu + igT^aW^a_\mu + ig'YB_\mu$$

$$= \partial_\mu + i\frac{g}{\sqrt{2}}(T^+W^+_\mu + T^-W^-_\mu)$$
\[ +ie(T_3 + Y)A_\mu + i \frac{g}{c_W} (T_3 - (T_3 + Y)s_W^2) Z_\mu , \]

(1)

where \( T^a \) is the \( SU(2)_L \) generator for \( n \)-dimensional representation with the normalization \( \text{Tr}(T^a T^b) = \delta^{ab}/2 \). \( T^\pm \) are the raising and lowering operators, the operator \( T_3 h Q = mh^Q + Y_h h Q = Y_n h_0^Q \) and \( e = gg'/\sqrt{g^2 + g'^2} \) with \( g' \) and \( g \) being the \( SU(2)_L \) and \( U(1)_Y \) gauge couplings. The abbreviations \( c_W \equiv \cos \theta_W \) and \( s_W \equiv \sin \theta_W \) with \( \theta_W \) being the weak mixing angle are used. The gauge interactions are the main interactions responsible for production of the multi-charged Higgs bosons.

The neutral components of \( H \) and \( H_n \) can be decomposed as \((\nu_H + \nu H^0 + i \nu^H)/\sqrt{2}\) and \((\nu_n + \nu_n^0 + i \nu_n^0)/\sqrt{2}\). If \( H \) and \( H_n \) have non-zero vacuum expectation of values (VEVs) \( v_H / \sqrt{2} \) and \( v_n / \sqrt{2} \), the \( Z \) and \( W^\pm \) will receive masses and the \( SU(2)_L \times U(1)_Y \) will break down to \( U(1)_{\text{em}} \). Certain linear combinations of components in \( H \) and \( H_n \) will become the would-be Goldstone bosons \( G_Z \) and \( G_W^+ \) “eaten” by the \( Z \) and \( W^\pm \) bosons. When discussing detecting physical Higgs bosons, these would-be Goldstone bosons should be separated and counted as longitudinal components of the \( Z \) and \( W^\pm \) bosons. We provide details by expanding Eq. (1) in Appendix A.

After the would-be Goldstone bosons are removed, one can identify the physical degrees of freedom for the Higgs bosons and discuss their production. If there is only one Higgs multiplet \( H_n \), the production of a multi-charged Higgs boson \( h_{Q|\pm|^1} \) at a \( pp \) collider can happen in the following fashions: (1) the Drell-Yan type through the \( s \)-channel exchange of a virtual \( \gamma, Z \) or \( W^\pm \) boson in \( pp \rightarrow \gamma^* \rightarrow h_{Q|\pm|^1} \) or \( pp \rightarrow W^{\pm*} \rightarrow h_{Q|\pm|^1} \); and (2) two vector boson fusion type through the \( pp \) collision to produce pair \( \gamma \gamma, \gamma Z, ZZ, (\gamma, Z) W^\pm \) or \( W^\pm W^\mp \) followed by \( \gamma \gamma, \gamma Z, ZZ, W^\pm W^\mp \rightarrow h_{Q|\pm|^1}h_{Q|\pm|^1} \), or \( (\gamma, Z) W^\pm W^\mp \rightarrow h_{Q|\pm|^1}h_{Q|\pm|^1} \). Here the vector bosons are virtual, but the photons can be almost real at the LHC [18–20].

The multi-charged Higgs boson \( h_{Q|\pm|^1} \) (similarly for \( h_{Q|\pm^-|^1} \)) produced can be detected by the decays \( h_{Q|\pm|^1} \rightarrow h_{Q|\pm^-|^1}W^\mp \rightarrow \cdots \rightarrow h_{Q|\pm^+|^1}W^\pm \cdots + \), where \( Q \geq 2 \), the multi-charged Higgs boson \( h_{Q|\pm^+|^1} \) means a multi-charged Higgs boson with the electric charge being \( \pm |Q| \) with \( |Q| \geq 1 \).

\[ h_{Q|\pm|^1} \text{ means a multi-charged Higgs boson with the electric charge being } \pm |Q| \text{ with } |Q| \geq 1. \]

\[ \uparrow \text{ In some special cases with the VEVs satisfying } v_n = v_H, \rho = 1 \text{ is predicted at tree level for } n = 2 \text{ with } I = 1/2, Y = 1/2 \text{ or } n = 7 \text{ with } I = 3, Y = 2 [24–26]. \]
at the LHC [40], additional b-jet, which is vetoed in our analysis, was also required [40].

When going beyond the LHC, there may be greater chance to discover multi-charged Higgs bosons. In this work we will first concentrate on the study of the discovery potential for the triply charged Higgs boson at a 100 TeV pp collider. After investigating the kinematic distributions at a 100 TeV pp collider and the LHC, we find that it is possible to project the results at a 100 TeV pp collider to the LHC without repeating the collider simulation. Our study shows that 5σ discovery can be reached for a triply charged Higgs boson below 1 TeV with 330 fb−1 of data at the LHC for a relatively small $\sqrt{s} = 10^{−6}$ GeV and with 110 fb−1 of data at a 100 TeV pp collider for the mass splitting $\Delta m = 0$. For $\Delta m > 0$, the sensitivity is further improved due to the cascade decays.

This paper is organized as follows: in Sect. 2, details of the model with a Higgs quadruplet and vector-like triplet leptons are given. In Sect. 3, current (indirect) constraints on this model are discussed. In Sect. 4, we discuss the production and decay of the triply charged Higgs boson, where the cascade decays are systematically studied. In Sect. 5, a detailed collider analysis is performed at a 100 TeV pp collider and the sensitivity at the LHC is revisited. Sect. 6 summarizes our results.

2 A triply charge Higgs boson model

We now provide some information about the triply charged Higgs boson in a complex Higgs quadruplet $\Delta \sim (1, 4, 4, 3/2)$ into the SM to be studied, which is expressed as $\Delta \equiv (\Delta^{+++}, \Delta^{++}, \Delta^{+}, \Delta^0)^T$, the scalar kinetic Lagrangian is shown in Eq. (1) with $\Delta = H_a$. The covariant derivatives

$$D_\mu \Delta = (\partial_\mu - ig T^a W^a_\mu - ig Y^{a*} B_\mu) \Delta, \quad (2)$$

and the Higgs doublet $H \sim (1, 2, 1/2)$ is given by $H = (H^+, H^0)^T$. The hypercharges $Y_\Delta = 3/2, Y_H = 1/2$, and the matrices $T^a$ and $W^a_\mu$ denote the $SU(2)$ generators in the doublet and quadruplet representations, respectively.

The Higgs potential is expressed as [36]

$$V(H, \Delta) = -\mu_H^2 H^+ H + \mu^2 \Delta^3 \Delta + \lambda_1 (H^+ H)^2 + \lambda_2 (\Delta^3 \Delta)^2 + \lambda_3 (H^+ H) (\Delta^3 \Delta) + \lambda_4 (H^+ T^a H) (\Delta T^a \Delta) + (\lambda_5 H^3 \Delta^* + H.C.). \quad (4)$$

The last term is explicitly written as $\lambda_5 H_a H_b H_c \Delta_{abc}^* + H.C.$ with the totally symmetric tensors

$$H_1 = H^+, \quad H_2 = H^0. \quad (5)$$

$$\Delta_{111} = \Delta^{+++}, \quad \Delta_{112} = \frac{1}{\sqrt{3}} \Delta^{++}, \quad \Delta_{122} = \frac{1}{\sqrt{3}} \Delta^{+}, \quad \Delta_{222} = \Delta^0. \quad (6)$$

After the electroweak symmetry breaking, $H^0 \rightarrow (v_H + h^0)/\sqrt{2}$ and $\Delta^0 \rightarrow (\lambda_H + h^0)/\sqrt{2}$. One obtains that VEVs of the fields $H$ and $\Delta$,

$$v_H = \sqrt{\frac{\mu_H^2}{\lambda_1}}, \quad v_\Delta = -\frac{\sqrt{\frac{\mu_\Delta^2}{2m_\Delta^2}}}{\lambda_4} \quad \frac{\lambda_4}{4} v_H^2. \quad (7)$$

Here $m_\Delta$ denotes the mass of the neutral field $h^0$ of the quadruplet $\Delta$.

$$m_\Delta^2 = \mu_\Delta^2 + \frac{1}{8} v_H^2 (4\lambda_3 + 3\lambda_4). \quad (8)$$

We can see from the Higgs potential that the $\lambda_4$ term induces the mass splitting between the nearby states of the Higgs quadruplet. To be more concrete, the mass of the field $\Delta^{n+}$, is given by

$$m_{\Delta^{n+}}^2 = m_\Delta^2 - \frac{n}{4} \frac{\lambda_4}{v_H^2}. \quad (9)$$

The singly charged states $H^\pm$ and $\Delta^\pm$ can mix with each other, thus it is necessary to define the normalized and orthogonal states via

$$\left( \begin{array}{c} G^\pm \\ \phi^\pm \end{array} \right) = 1/v \left( \begin{array}{c} v_H + \sqrt{3} v_\Delta \\ -\sqrt{3} v_\Delta \end{array} \right) \left( \begin{array}{c} H^\pm \\ \Delta^\pm \end{array} \right) \quad (10)$$

with $v \equiv \sqrt{v_H^2 + 3v_\Delta^2} \approx 246$ GeV, where $G^\pm_\Delta$ and $\phi^\pm$ are the would-be Goldstone boson and physical singly charged Higgs boson, respectively. The electroweak parameter is equal to $g_\Delta^2 + 3v_\Delta^2)/(v_H^2 + 9 v_\Delta^2)$ in this model. After removing the Goldstone mode, one obtains interactions of the physical singly charged Higgs boson $\phi^\pm$ to SM fermions and gauge bosons. With the experimental measurement of $\rho$ [23], one obtains $v_\Delta \lesssim 1.3$ GeV at 3σ level. Since the mixing effects are highly suppressed by $v_\Delta/v$, we will not consider them but keep in mind that singly charged Higgs boson can couple to SM leptons even if there are no other fields being introduced. Similarly, neutral Higgs bosons from the doublet and quadruplet can also mix, depending on the parameter $\lambda_5$ in the Higgs potential. For $v_\Delta \ll v$, the mass eigenstates of neutral Higgs bosons are

$$\left( \begin{array}{c} h^0_1 \\ h^0_2 \end{array} \right) = 1/\sqrt{2} v_H^2 + 9v_\Delta^2 v_H / (3v_\Delta - v_\Delta) \left( \begin{array}{c} h^0 \\ \Delta^0 \end{array} \right). \quad (11)$$

Here, $\Delta^{1+} = \Delta^{++}, \Delta^{2+} = \Delta^{++},$ and $\Delta^{3+} = \Delta^{++}$. 

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where \( h_0^0 \) is identified as the discovered Higgs boson with mass of about 125 GeV.

Motivated by the non-zero neutrino masses, we consider the scenario, in which a pair of vector-like triplet leptons \( \Sigma_{L,R} \sim (1, 3, 1) \) with \( \Sigma = (\Sigma^{++}, \Sigma^+, \Sigma^0)^T \) are introduced into the SM [36]. This enables the quadruplet Higgs boson to couple to SM leptons after integrating out the heavy fields. The Yukawa Lagrangian is described as

\[
\mathcal{L}_{\text{Yuk}} = Y_i \bar{L} e^{a_i} \Sigma L_i H^0 + Y_j \bar{e}^{a_j} \Sigma R_j H^0 + \sum_{i=1}^{\Delta} \lambda_i \bar{L}^c e^{a_i} \epsilon^{cd} + \text{H.C.},
\]

where \( L \) is the left-handed lepton doublet, \( Y_i \) and \( Y_j \) are the Yukawa couplings with \( i \) being the generation index. The total symmetric tensors

\[
\Sigma_{11} = \Sigma^{++}, \quad \Sigma_{12} = \frac{1}{\sqrt{2}} \Sigma^+, \quad \Sigma_{22} = \Sigma^0.
\]

Integrating out \( \Sigma_{L,R} \), we obtain the dimension-5 effective operator

\[
\mathcal{L}_{\text{Yuk}}^{\text{eff}} = \frac{m_{\Sigma}}{\Sigma} \left( \frac{v_{\Delta}}{2} - v_{\Delta} \ell_L \ell_L \right) + \frac{\Delta^+}{\sqrt{6}} \ell_L \ell_L \epsilon^{cd} \epsilon^{cd} + \text{H.C.},
\]

where \( m_\Sigma \) is the mass of \( \Sigma \) fields. Assuming that the neutrino mass is generated by the above interaction, we obtain

\[
\mathcal{L}_{\text{Yuk}}^{\text{eff}} \supset \left( \frac{m_{\nu_{ij}}}{v_\Delta} \right)^4 - \frac{\Delta^+}{\sqrt{6}} \ell_L \ell_L + \text{H.C.},
\]

The first term gives rise to neutrino masses in the flavor basis, the second and third terms contribute to the singly charged Higgs boson decaying into leptons, and the fourth term induces the leptonic decay of the doubly charged Higgs boson as we will discuss in detail in Sect. 4.2.

### 3 Constraints

In this section, we will discuss indirect constraints on the model with an extended Higgs quadruplet proposed in Sect. 2 from the decay of Higgs boson into \( \gamma \gamma \), the electroweak precision tests (EWPTs), perturbativity, and low-energy rare processes \( \mu \rightarrow e\gamma \) decay and \( \mu-e \) conversion. It is well known that charged Higgs bosons can contribute at 1-loop level to the decay of \( h_0^0 \rightarrow \gamma \gamma \), which has been measured by the ATLAS and CMS Collaboration and combined in terms of signal strengths \( \mu_{\gamma\gamma}^{\text{ATLAS}} = 1.06 \pm 0.12 \) [41] and \( \mu_{\gamma\gamma}^{\text{CMS}} = 1.20^{+0.17}_{-0.14} \) [42] with the integrated luminosities of 80 fb\(^{-1}\) and 35.9 fb\(^{-1}\), respectively. Due to the larger integrated luminosity, we will take the combined signal strength \( \mu_{\gamma\gamma}^{\text{ATLAS}} \) to constrain the model parameters.

The couplings between \( h_0^0 \) and charged Higgs bosons \( \Delta n^\pm \), i.e., \( h_0^0 \Delta^+ \Delta^0 \), are

\[
\lambda_n = v_H \left( \lambda_3 + \frac{3 - 2n}{4} \lambda_4 \right), \quad n = 1, 2, 3.
\]

From Eq. (9), one sees that \( \lambda_4 \) is fixed by the mass splitting \( \Delta \), which is given by

\[
\lambda_4 = \frac{8m_\Delta}{v_H^2} \Delta m.
\]

The partial width of \( h_0^0 \rightarrow \gamma \gamma \) is thus modified as [43],

\[
\frac{\Gamma(h_0^0 \rightarrow \gamma \gamma)}{\Gamma(h_0^0 \rightarrow \gamma \gamma)_{\text{SM}}} = \frac{\left| N_c Q_i^2 A_{1/2}(\tau_i) + A_1(\tau_W) + \sum_{n=1}^3 \frac{v_{\Delta}^2 \lambda_n Q_n^2}{2m^2_{\Delta_{n^\pm}}} \right|^2}{\left| N_c Q_i^2 A_{1/2}(\tau_i) + A_1(\tau_W) \right|^2}
\]

where \( \tau_i = m_{h_0^0}^2/(4m^2_{1/2}) \), \( \tau_W = m_{h_0^0}^2/(4m^2_{W}) \) and \( \Delta_{n^\pm} = m_{h_0^0}^2/(4m^2_{\Delta_{n^\pm}}) \),

\[
A_{1/2}(\tau_i) = 2 \left[ \tau_i + (\tau_i - 1) f(\tau_i) \right] \tau_i^{-2},
\]

\[
A_1(\tau_W) = -2 \left[ 3 \tau_W^2 + 3 \tau_W + 2(\tau_W - 1) f(\tau_W) \right] \tau_W^{-2},
\]

\[
A_0(\tau_i) = -\left[ \tau_i - f(\tau_i) \right] \tau_i^{-2},
\]

and the function \( f(\tau_i) = \arcsin^2 \sqrt{\tau_i} \) for \( \tau_i < 1 \). In Eq. (18), we have neglected the terms proportional to \( v_\Delta/v_H \).

Following Ref. [39], we also consider the indirect constraints from the EWPTs [44] by considering modification to the oblique parameters [45, 46] and perturbativity, \( \lambda_4 \leq \sqrt{4\pi} \). In Fig. 1, we show the indirect constraints, which are almost independent of \( v_\Delta \), in the plane of \( m_{\Delta_{n^\pm}} \) and \( \Delta m \). For the \( h_0^0 \rightarrow \gamma \gamma \) measurements, we consider the combined signal strength by the ATLAS Collaboration. The pink regions are excluded at 2\( \sigma \) confidence level (C.L.), where two benchmark values of the coupling \( A_3 = 1, 0.1 \) are depicted. The cyan regions are excluded at 2\( \sigma \) C.L. by the perturbativity requirement. The regions between blue curves are however allowed at 2\( \sigma \) C.L. by the EWPTs. Thus there is still large room in the range of \( |\Delta m| \lesssim 30 \text{ GeV} \) satisfying indirect constraints.

Charged Higgs bosons can contribute to other processes or observables at one-loop level [47]. The 90\% C.L. upper
limits on the decay branching ratio of $\mu \rightarrow e \gamma$ and rate of $\mu$-e conversion are [48,49]

$$
\text{Br}(\mu^+ \rightarrow e^+\gamma) \leq 4.2 \times 10^{-13},$$
$$R(\mu N \rightarrow eN) \leq 7 \times 10^{-13} \text{ for Au. (22)}$$

The decay branching ratio of $\mu \rightarrow e \gamma$ [50] and the conversion rate of $\mu$-e [51,52] induced by charged Higgs bosons are given by

$$
\text{Br}(\mu^+ \rightarrow e^+\gamma) = \frac{\alpha (m_{\nu}^2)_{12}^2}{108\pi G_F^2 v^4_\Delta} \left( \frac{1}{m_{\Delta^+}^2 + m_{\Delta^+}^2} \right)^2, \\
\text{Br}(\mu^+ \rightarrow e^-\gamma) = \frac{\alpha (m_{\nu}^2)_{12}^2}{108\pi G_F^2 v^4_\Delta} \left( \frac{1}{m_{\Delta}^2 + m_{\Delta}^2} \right)^2, \\
\text{Br}(\mu^+ \rightarrow e^-\gamma) = \frac{\alpha (m_{\nu}^2)_{12}^2}{108\pi G_F^2 v^4_\Delta} \left( \frac{1}{m_{\Delta^+}^2 + m_{\Delta^+}^2} \right)^2.$$

In Fig. 2, the constraints from $\mu \rightarrow e \gamma$ and $\mu$-e conversion measurements are shown. We obtain the most stringent constraint $v_\Delta \gtrsim 1.5 \times 10^{-9}$ GeV from $\mu^+ \rightarrow e^+\gamma$.

The triply charged Higgs boson mass and the mass splitting can also be bounded from direct searches for doubly charged Higgs bosons at the LHC. Doubly charged Higgs bosons in these searches are assumed to decay into a pair of same-sign leptons [54,55] or W bosons [56]. It has been shown in Refs. [38,39] that doubly charged Higgs searches can exclude the region of $m_{\Delta^+} \lesssim 400 - 600$ GeV depending on the VEV $v_\Delta$. While it is important to include these constraints for a complete model study, we will concentrate

$$
R(\mu N \rightarrow eN) = (4\pi\alpha)^2 \frac{2G_F^2 m_{\mu}^5}{\Gamma_{cap}} \left( A_R \frac{D}{\sqrt{4\pi\alpha}} + A_L V_p \right)^2, \\
\text{where } \alpha \text{ and } G_F \text{ are the fine-structure constant and Fermion coupling constant, } m_{\nu} \text{ is the neutrino mass matrix in the flavor basis, and } D = 0.189, V_p = 0.0974, \Gamma_{cap} = 13.07 \times 6.58211814 \times 10^{-19} \text{ GeV [53]. Here,}

$$
A_R = -\frac{1}{\sqrt{2}G_F} \frac{(m_{\nu}^2)_{12}}{288\pi^2 v^2_\Delta} \left( \frac{1}{4m_{\Delta^+}^2} + \frac{1}{m_{\Delta}^2} \right), \\
A_L = -\frac{1}{\sqrt{2}G_F} \sum_{i=1,2,3} (m_{\nu}^2)_{i1} (m_{\nu}^2)_{i2} \left[ \frac{1}{6m_{\Delta^+}^2} + \frac{1}{8m_{\Delta^+}^2 f} \left( \frac{m_{\mu}^2}{m_{\Delta^+}^2} \right) - \frac{m_{\mu}^2}{m_{\Delta^+}^2} \right],
$$

with $m_1 = m_e, m_2 = m_\mu, m_3 = m_\tau$, and the function

$$
f(r,s) = \frac{4s + \ln(s)}{r + 4s + \sqrt{r}} \left( 1 - \frac{2s}{r} \right) \sqrt{1 + \frac{4s}{r}} \\
\ln \left( \frac{\sqrt{r + 4s} - \sqrt{r}}{\sqrt{r + 4s} + \sqrt{r}} \right).
$$

In Fig. 1, indirect constraints from $\mu \rightarrow e \gamma$ and perturbativity in the plane of $m_{\Delta^+}$ and $\Delta m$. Blue curves are allowed at $2\sigma$ C.L., while the region between solid and dashed curves correspond to $\Delta m = 1$ and 0.1, respectively.
on the sensitivities to triply charged Higgs boson searches at colliders with the model in Sect. 2 as a benchmark model with the mass region up to 1 TeV.

4 Production and decay of triply charged Higgs boson

4.1 Production cross sections

As mentioned in Sect. 1, triply charged Higgs bosons can be pair produced or associated produced with a doubly charged Higgs boson. In the $s$-channel, they correspond to the Drell-Yan processes through an off-shell photon or $Z$ boson and through a $W$ boson, which are termed “DYZ” and “DYW” processes hereafter, respectively. In the $t$-channel, charged Higgs bosons are produced in conjunction with two additional forward jets at leading order [18] by exchange of $\gamma$, $Z$ and/or $W$ boson. It was found in Refs. [18,20] that the photon fusion (PF) process with collinear initial photons dominates over other contributions involving off-shell photon, $Z$ boson and/or $W$ boson, named as vector boson fusion (VBF) process at the LHC. Following Refs. [18–20], we use an effective photon approximation [57] to describe the PF process, which includes elastic, semi-elastic, and inelastic sub-processes but loses potential tagging forward jets. Since the cross section of PF process is proportional to $Q^4_{\Delta}$ with $Q_{\Delta}$ being the electric charge of $\Delta^{\pm\pm}$, it can even surpass the cross sections of DY processes for the production of triply charged Higgs boson. On the other hand, the VBF process, the cross section of which is expected to increase fairly with the collider energy, can be separated by tagging the forward jets and thus will not be considered in this work.

The studies of triply charged Higgs boson at the LHC with the center-of-mass energy $\sqrt{s} = 13 \sim 14$ TeV can be found in Refs. [37–39]. In the potential era of LHC update, named as High Energy LHC (HE-LHC) [58,59], the collider energy can reach 27 TeV, which increases the LHC mass reach of triply charged Higgs boson. A 100 TeV $pp$ collider such as proton-proton Future Circular Collider (FCC-hh) [60,61] or Super Proton-Proton Collider (SPPC) [62,63] is also designed, which provides new possibilities of discovering triply charged Higgs bosons at $pp$ colliders.

In the left panel of Fig. 3 we show the cross sections of triply charged Higgs production at $\sqrt{s} = 13$, 27 and 100 TeV obtained with MGS-amc@NLO v2.6.5 [64] and NNPDF23_lo_as_0130_qed PDF set [65] for the charged Higgs boson mass range between 300 GeV and 1000 GeV.

4.2 Decays of charged Higgs bosons

To evaluate the significance of the production processes, it is essential to investigate the decays of charged Higgs bosons. Triply charged Higgs boson can decay in cascade into doubly charged Higgs boson or in three-body through an off-shell doubly charged Higgs boson. Therefore, we will first discuss the decay of doubly charged Higgs boson.

One can easily obtain the decay widths of doubly charged Higgs boson into $W^+W^+$ and $e^+\bar{e}^+$ by rescaling those in the Type-II seesaw model [12–15], which are

$$\Gamma(\Delta^{\pm\pm} \rightarrow \ell^+\ell^-) = \frac{m_{\Delta^{\pm\pm}}}{12\pi(1 + \delta_{ij})} |h_{ij}|^2,$$

$$\Gamma(\Delta^{\pm\pm} \rightarrow W^+W^\pm) = \frac{3g^4v^2m_W^3}{64\pi m_W^4} \sqrt{1 - 4\xi_W} - (1 - 4\xi_W + 12\xi_W^2)$$

with $\xi_W \equiv m_W^2/m_{\Delta^{\pm\pm}}^2$, $\ell = e, \mu, \tau$, where we have defined [13]

$$h_{ij} = m_{ij}^2/(\sqrt{2}v_\Delta)$$

with $m_{ij}$ denoting the neutrino mass matrix in the flavor basis and assumed $m_{\Delta^{\pm\pm}} > 2m_W$. Here, we only consider the contribution of a Higgs quadruplet to neutrino mass at tree level; for 1-loop level contribution, one could refer to Refs. [37–39]. In the neutrino mass basis, the diagonal neutrino mass matrix is

$$m_{\nu}^{\text{diag}} = U^T m_{\nu} U,$$

cross section is reduced by at most 5% for $\Delta m = 10$ GeV and 15% for $\Delta m = 30$ GeV.

It is worthy to note that there is a large uncertainty of photon PDF in NNPDF23_lo_as_0130_qed PDF set, which could overestimate the photon-fusion production cross section at the LHC [68,69], but the impact is small at a 100 TeV $pp$ collider since the dominant contribution comes from the DY processes.
There is an update of the mixing sin^2 θ_{23} [70], which slightly changes the neutrino mass matrix m_\nu.

\[ \cos2\theta_{13} \text{ and } \sin2\theta_{13} \] 6

\[ \Gamma(\Delta^{\pm\pm} \to \pm W^\pm) \equiv \frac{g^4 \Delta m^5}{160 \pi^3 m_W^4}. \] (32)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^3 f_{\pi}^2}{8 \pi m_W^4}, \] (33)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^3 f_{\pi}^2}{32 \pi m_W^4}. \] (34)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^5 f_{\pi}^2}{160 \pi^3 m_W^4}. \] (31)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^3 f_{\pi}^2}{8 \pi m_W^4}, \] (33)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^3 f_{\pi}^2}{32 \pi m_W^4}. \] (34)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^5 f_{\pi}^2}{160 \pi^3 m_W^4}. \] (31)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^3 f_{\pi}^2}{8 \pi m_W^4}, \] (33)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^3 f_{\pi}^2}{32 \pi m_W^4}. \] (34)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^5 f_{\pi}^2}{160 \pi^3 m_W^4}. \] (31)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^3 f_{\pi}^2}{8 \pi m_W^4}, \] (33)

\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm\pm} W^\mp) \equiv \frac{g^4 \Delta m^3 f_{\pi}^2}{32 \pi m_W^4}. \] (34)
is larger than 0.1 (0.8) in the range $10^{-5}$ GeV $\lesssim v_\Delta \lesssim 10^{-4}$ GeV ($10^{-6.5}$ GeV $\lesssim v_\Delta \lesssim 10^{-2.5}$ GeV), as shown in the right panel. Given the total width in the left panel, the proper decay length $\lambda_{\Delta^{\pm\pm}} = h c / \Gamma_{\Delta^{\pm\pm}}$ can be easily obtained and is smaller than 0.1 mm for 300 GeV $\leq m_{\Delta^{\pm\pm}} \leq 1000$ GeV, which ensures the validity of prompt search of $\Delta^{\pm\pm}$ at pp colliders.

Triply charged Higgs boson $\Delta^{\pm\pm\pm}$ can decay into $W^i_\ell^i_\ell^j_\ell^j$ and $W^\pm W^\pm W^\pm$ if kinetically allowed. The partial widths of three-body decays through an off-shell $\Delta^{\pm\pm}$ are

$$\Gamma(\Delta^{\pm\pm\pm} \to W^\pm \ell^i_\ell^j_\ell^j) = \frac{g^2 m_{\Delta^{\pm\pm\pm}}^3 |h_{ij}|^2}{768 \pi^3 m_W^2} \int_0^{(m_{\Delta^{\pm\pm\pm}} - m_W)^2} ds F(s),$$

$$\Gamma(\Delta^{\pm\pm\pm} \to W^\pm W^\pm W^\pm) = \frac{3 g^6 v^2 m_{\Delta^{\pm\pm\pm}}^5}{4096 \pi^3 m_W^6} \int_{t_{\min}}^{t_{\max}} dt ds G(s, t).$$

with

$$F(s) = \frac{m_{\Delta^{\pm\pm\pm}}^2}{m_{\Delta^{\pm\pm\pm}}^2} [6(-2 - 2r_s + r_W + 1/r_W(1 - r_s)^2)]$$

$$D(s) = \frac{1}{m_{\Delta^{\pm\pm\pm}}}$$

$$G(s, t) = \frac{1}{m_{\Delta^{\pm\pm\pm}}}$$

$$\left[24r_W(-2 - 2r_s + r_W + 1/r_W(1 - r_s)^2)D(s) \times (2r_W^2 + 1/4(r_s - 2r_W)^2) + 48[(1 - r_s)(1 - r_t) - (1/2r_WR_s + 1/2r_WR_t + 5/2r_W - 3/2r_W)]E(s, t) \times \left[3r_W^2 + 1/4(r_s - 4r_W)(r_t - 4r_W)\right]\right](43)$$

$$t_{\max} = \frac{1}{16}[m_{\Delta^{\pm\pm\pm}}^2 - m_W^2]^2 - (\lambda(m_s, m_W, m_W^2)^1)^2$$

$$t_{\min} = \frac{1}{16}[m_{\Delta^{\pm\pm\pm}}^2 - m_W^2]^2 - (\lambda(m_s, m_W, m_W^2)^1)^2$$

$$E(s, t) = \frac{1}{(r_s - (1 + \Delta m/m_{\Delta^{\pm\pm\pm}})^2)^2 + (1 + \Delta m/m_{\Delta^{\pm\pm\pm}})^2 \Gamma_{\Delta^{\pm\pm\pm}}^2/m_{\Delta^{\pm\pm\pm}}^2},$$

Here, $s, t$ denote the invariant mass of the $W$ boson pair from the decay of $\Delta^{\pm\pm\pm}$ and $r_s \equiv s/m_{\Delta^{\pm\pm\pm}}^2, r_t \equiv t/m_{\Delta^{\pm\pm\pm}}^2, r_W \equiv m_W^2/m_{\Delta^{\pm\pm\pm}}^2$, and $\lambda(x, y, z) \equiv (x - y - z)^2 - 4yz$.

In the limit of $m_W/m_{\Delta^{\pm\pm\pm}} \to 0$, the above integrations over $F(s)$ and $G(s, t)$ are equal to 1. It is noted that the total width $\Gamma_{\Delta^{\pm\pm\pm}} < 0.01$ GeV for $10^{-9}$ GeV $\leq v_\Delta \leq 1$ GeV (see Fig. 4), which has negligible effect on the three-body decay widths.

We can see immediately that different from the decays of $\Delta^{\pm\pm\pm} \to \ell^i_\ell^j_\ell^j, W^\pm W^\pm W^\pm$, the three-body decays of $\Delta^{\pm\pm\pm}$ in Eq. (36) depend on the mass splitting $\Delta m$. To estimate its impact, we introduce

$$\delta \Gamma_{W_{\ell\ell'}} = (\Gamma_{W_{\ell\ell'}} - \Gamma_{W_{\ell\ell'}}^0)/\Gamma_{W_{\ell\ell'}}^0$$

$$\delta \Gamma_{WW\ell} = (\Gamma_{WW\ell} - \Gamma_{WW\ell}^0)/\Gamma_{WW\ell}^0,$$

where $\Gamma_{W_{\ell\ell'}} = \sum_{i,j} \Gamma(\Delta^{\pm\pm\pm} \to W^i_\ell^i_\ell^j_\ell^j)$ and $\Gamma_{WW\ell} = \Gamma(\Delta^{\pm\pm\pm} \to W^\pm W^\pm W^\pm)$ and $\Gamma_{WW\ell}^0$ and $\Gamma_{WW\ell}^0$ are the corresponding values with $|\Delta m| = 0$. In Fig. 5, the values of $\delta \Gamma_{W_{\ell\ell'}}$ and $\delta \Gamma_{WW\ell}$ are shown. We find that both $\delta \Gamma_{W_{\ell\ell'}}$ and $\delta \Gamma_{WW\ell}$ are negligible for $|\Delta m| = 1$ GeV and increase to 10% - 25% for $|\Delta m| = 10$ GeV in the mass range 300 GeV $\leq m_{\Delta^{\pm\pm\pm}} \leq 1000$ GeV.

The interplay between the decays $\Delta^{\pm\pm\pm} \to W^\pm \ell^i_\ell^j_\ell^j$ and $\Delta^{\pm\pm\pm} \to W^\pm W^\pm W^\pm$ is the similar to that for $\Delta^{\pm\pm\pm}$ in two-body decays. Therefore, we need to include the cascade decays of $\Delta^{\pm\pm\pm}$ with $\Delta^{\pm\pm}$ being on shell in the medium $v_\Delta$ region for $\Delta m < 0$ with the widths being approximately given by

$$\Gamma(\Delta^{\pm\pm\pm} \to \Delta^{\pm\pm} W^{\pm\pm}) = -\frac{9g^4 \Delta m^5}{160 \pi^3 m_W^4},$$

$$\Gamma(\Delta^{\pm\pm\pm} \to \Delta^{\pm\pm} \pi^\pm) = -\frac{3g^4 \Delta m^3 f_\pi^2}{32 \pi^3 m_W^4},$$

For $\Delta m > 0$, the cascade decay of $\Delta^{\pm\pm\pm}$ is kinetically forbidden. Hence, the total width of $\Delta^{\pm\pm\pm}$ is expressed as

$$\Gamma_{\Delta^{\pm\pm\pm}} = \Gamma(\Delta^{\pm\pm\pm} \to \ell^i_\ell^j_\ell^j) + \Gamma(\Delta^{\pm\pm\pm} \to W^\pm W^\pm W^\pm) + \theta(\Delta m)$$

$$\left[\Gamma(\Delta^{\pm\pm\pm} \to \Delta^{\pm\pm} W^{\pm\pm}) + \Gamma(\Delta^{\pm\pm\pm} \to \Delta^{\pm\pm} \pi^\pm)\right].$$

$$D(s) = \frac{1}{(r_s - (1 + \Delta m/m_{\Delta^{\pm\pm\pm}})^2)^2 + (1 + \Delta m/m_{\Delta^{\pm\pm\pm}})^2 \Gamma_{\Delta^{\pm\pm\pm}}^2/m_{\Delta^{\pm\pm\pm}}^2},$$

$$E(s, t) = \frac{1}{(r_s - (1 + \Delta m/m_{\Delta^{\pm\pm\pm}})^2)(r_t - (1 + \Delta m/m_{\Delta^{\pm\pm\pm}})^2) + (1 + \Delta m/m_{\Delta^{\pm\pm\pm}})^2 \Gamma_{\Delta^{\pm\pm\pm}}^2/m_{\Delta^{\pm\pm\pm}}^2}.$$
Fig. 4 Left: total decay width of $\Delta^{\pm\pm}$ defined in Eq. (35) as a function of the quadruplet VEV $v_\Delta$ for $m_{\Delta^{\pm\pm}} = 300, 600, 900$ GeV for the NH. The solid, dashed, dotted curves correspond to $\Delta m = 0, -1, -10$ GeV, respectively. Right: decay branching ratios of $\Delta^{\pm\pm}$ in the plane of $\log_{10}(v_\Delta/\text{GeV})$ and $\log_{10}(\Delta m/\text{GeV})$ for $m_{\Delta^{\pm\pm}} = 600$ GeV and the NH. The shaded regions represent the branching ratios in the leptonic decay and gauge boson decay channels for $v_\Delta \lesssim 10^{-4.5}$ GeV and $v_\Delta \gtrsim 10^{-4.5}$ GeV, respectively. The blue curves denote the branching ratio in the cascade decay channel.

Fig. 5 Impact of $\Delta m$ on the partial decay widths of $\Delta^{\pm\pm\pm}$ into $W^\pm \ell^\pm \ell^\pm$ and $W^\pm W^\pm W^\pm$ in the left and right panels, respectively. Benchmark values of $|\Delta m| = 1, 10$ GeV are considered.

Fig. 6 Total decay width (left panel) and proper decay length (right panel) of $\Delta^{\pm\pm\pm}$ defined in Eq. (35) as a function of the quadruplet VEV $v_\Delta$ for $m_{\Delta^{\pm\pm\pm}} = 300, 600, 1000$ GeV with the NH. The solid, dashed, dotted curves correspond to $\Delta m = 0, -1, -10$ GeV, respectively. In the magenta shaded region, $c\tau_{\Delta^{\pm\pm\pm}} \geq 0.1$ mm, $\Delta^{\pm\pm\pm}$ is long-lived.
The total width and proper decay width of $\Delta^{\pm\pm\pm}$ are depicted in Fig. 6. It is interesting to observe that since the three-body decay widths of $\Delta^{\pm\pm\pm}$ are much smaller than the two-body decay widths of $\Delta^{\pm\pm}$, the cascade decay dominates in the medium $v_\Delta$ region for $\Delta m < 0$ even with $\Delta m = -1$ GeV. For $\Delta m > 0$, the cascade decays are not allowed and the dependence of three-body decays on $\Delta m$ is not shown explicitly for simplicity. In the left panel of Fig. 6, the total width can be as small as $10^{-15}$ GeV so that the proper decay length can reach $0.1$ mm $\sim$ $0.1$ m—the region that is inappropriate for prompt search [71], as shown in the right panel. For $\Delta m < 0$, however, the proper decay length is large enough for the prompt search with the contribution of cascade decays added.

It is important to emphasize that although the partial widths of three-body decays and cascade decays depend on the mass splitting $\Delta m$, the decay branching ratios of $\Delta^{\pm\pm\pm}$ are almost independent of $\Delta m$. For $\Delta m > 0$, the $\Delta m$ dependence of the partial widths cancels in the branching ratios of $\Delta^{\pm\pm\pm}$, resulting in a modification smaller than 1.7% for $\Delta m = 10$ GeV. For $\Delta m < 0$, the cancellation is similar in the low and high $v_\Delta$ regions as that for $\Delta m > 0$. In the medium $v_\Delta$ region, the cascade decay dominates over the three-body decays, which ensures that the branching ratio is independent of $\Delta m$.

5 Collider analysis

In this section, we will perform collider studies of triply charged Higgs bosons at $pp$ colliders. Given the couplings of $\Delta^{\pm\pm\pm}$ to charged leptons and $W$ bosons, we have the decay channels: for the DYZ and PF processes $\Delta^{+++} \Delta^{-++} \rightarrow (\ell^+ e^+ W^+)(\ell^- e^- W^-)$, $(\ell^+ e^+ W^+)(W^+ W^-)$, and $(W^+ W^+ W^-)(W^- W^- W^-)$; for the DYW process $\Delta^{\pm\pm\pm} \rightarrow (\ell^+ e^+ W^+)(\ell^- e^- W^-)$, $(\ell^+ e^+ W^+)(W^+ W^-)$, $(W^+ W^+ W^-)(\ell^- e^- W^-)$, and $(W^+ W^+ W^-)(W^+ W^-)$, where $\ell \equiv e, \mu, \tau$. If one or more on-shell $W$ bosons decay into leptons, we can always achieve the inclusive final state with at least three same-sign leptons. We also include the cascade decays for $|\Delta m| \lesssim 30$ GeV. In case of $\Delta m > 0$, $\Delta^{\pm\pm} \rightarrow \Delta^{\pm\pm\pm} W^+$, while in case of $\Delta m < 0$, $\Delta^{\pm\pm\pm} \rightarrow \Delta^{\pm\pm\pm} W^+$ and $\Delta^{\pm\pm} \rightarrow \Delta^{\pm\pm\pm} W^+$. In the latter case, the momenta of leptons and jets from off-shell $W$ bosons are limited by the mass splitting, so that they are unlikely to be detected [54–56]. To this end, we will assume that $\Delta m > 0$ and the efficiency of $\Delta^{\pm\pm} \rightarrow (\Delta^{\pm\pm\pm} \rightarrow X)(W^+ \rightarrow \ell v/jj)$ is the same as that of $\Delta^{\pm\pm\pm} \rightarrow X$.

The SM backgrounds are those with at least three same-sign charged leptons in the final states. In previous studies, the backgrounds $t\bar{t}W [37–39,72,73]$, $t\bar{t}Z [37,73]$, $t\bar{t}t\bar{t} [37,72,73]$, $t\bar{t}b\bar{b} [37,72]$, $t\bar{t}h [73]$, $WWZ [73]$, $WZZ [73]$ and $ZZZ [73]$ were considered. In Ref. [39], the backgrounds $WZ$ and $ZZ$ were discussed with charge misidentification of leptons taken into account.

In our study, we consider the backgrounds with at least two same-sign leptons at parton level and the third same-sign lepton could come from heavy-flavor hadron decays or charge misidentification. Besides, the $t\bar{t}h$ background is also taken into account, since its cross section is huge. The set of backgrounds can be read off from the experimental searches for final states with same-sign leptons or multiple leptons [40,74,75], which are classified into $t\bar{t}$ production in association with a boson $(t\bar{t}W, t\bar{t}Z/\gamma^*, t\bar{t}h$ with $h$ being the SM Higgs boson), multi-top production $(t\bar{t}t\bar{t}, t\bar{t}t\bar{t}, t\bar{t}t\bar{t})$, multiboson production $(WZ, Z/\gamma^*WW, WW, WZ, WWZ, ZZZ, W\gamma^*, W\gamma^*)$ and rare processes $(t\bar{t}bb, tWZ, t\bar{t}Zq)$ with $q$ denoting one of quarks except $t/\bar{t}$.

The comments on the backgrounds are made as follows. Backgrounds with an off-shell photon, such as $\gamma^*$, are not generated since their contributions are expected to be reduced significantly after imposing the lower cuts on the invariant mass of opposite-sign same-flavor leptons in Cut-3 (see the definition below) as compared to the corresponding backgrounds with an on-shell $Z$ boson. The backgrounds $t\bar{t}h, h \rightarrow b\bar{b}, WW^*$ are not considered since their cross sections are much smaller than those of $t\bar{t}bb, t\bar{t}W$. For the background $t\bar{t}Z$, we only consider the decay $Z \rightarrow \ell^{+}\ell^{-}$ and neglect $Z \rightarrow q\bar{q}$ since the latter cross section is much smaller as compared to $t\bar{t}jj$ and $t\bar{t}bb$. The tri-top production $t\bar{t}t\bar{t}$ [76] with a much smaller cross section than that of $t\bar{t}W$ [77] can be neglected.

The charge misidentification probability is about $10^{-5} \sim 10^{-3}$ for electrons ($e_\nu$) due to bremsstrahlung interactions with the inner detector material and negligible for muons at the 13 TeV LHC [74,75,78,79]. At a 100 TeV $pp$ collider, we assume a conservative and uniform rate $e_\nu = 10^{-3}$ [80]. The charge-misidentified backgrounds are obtained by reweighting the background by the charge misidentification probabilities [78], see Table 1. Backgrounds with a non-prompt lepton may fake the signal, which originates from hadron decays or in photon conversions as well as hadrons misidentified as leptons. It is shown at the 13 TeV LHC that the non-prompt leptons mainly come from heavy-flavor hadron decays in events containing top quark, $W$ boson or $Z$ boson [40]. Besides, the probability of jet faking lepton can also be reduced with the cut on missing energy [72,73,75,79,81], i.e., Cut-5 below. Therefore, we will only consider non-prompt leptons from heavy-flavor hadron decays at $pp$ colliders in this study.

We generate parton-level signal and background events at $\sqrt{s} = 100$ TeV using $\text{MG5\_aMC@NLO}$ v2.6.5 [64], which

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7 Non-prompt leptons from jet faking can be distinguished from the prompt leptons in $W/Z$ decays with dedicated isolation variables [78].
are passed to \texttt{Pythia8} [82] for possible sequential decays, parton shower and hadronization. The default factorization and renormalization scales are used. The backgrounds $WZ$ and $tt$ are matched up to two additional jets [31], $t\bar{t}t\bar{t}$, $t\bar{t}bb$ and $t\bar{t}Zq$ are generated without additional partons for simplicity, while the other backgrounds are matched to additional one jet.

The next-leading-order QCD overall $K$-factors of the background processes are available at the LHC colliding energy $\sqrt{s} = 14$ TeV ranging from 1.2 to 2.0 [83–94]. As an estimate, we apply these $K$-factors to the corresponding processes at $\sqrt{s} = 100$ TeV [80]. The detector response is simulated using \texttt{Delphes} [95] with the built-in baseline FCC-hh detector configuration. The probability of one $b$ quark to be identified as $b$-jet is $[1 - p_T/(20 \text{ TeV})] \cdot 85\%$ and the mis-tagging efficiencies for light-flavor quarks and $c$-quark wrongly identified as $b$-jets are $[1 - p_T/(20 \text{ TeV})] \cdot 1\%$ and $[1 - p_T/(20 \text{ TeV})] \cdot 5\%$ in the central region ($|\eta| < 2.5$) [96].

In order to identify objects, we impose the following criteria [80,97]

$$ p_T{\ell}/j_\mu > 20 \text{ GeV}, \quad p_T{j}/j_b > 30 \text{ GeV}, \quad |\eta_{\ell}/\mu/j_b| < 6, $$

where $j$ and $b$ denote the light-flavor jets and $b$-tagged jet, respectively. The lepton candidates are isolated within a cone of radius 0.3, and the jet candidates are clustered with the anti-$k_T$ algorithm [98] and a radius parameter of 0.4 implemented in the \texttt{FastJet} package [99].

Events are then selected with a series of cuts. It is demanded the angular separation between any two reconstructed objects satisfies\footnote{\Delta \eta and $\Delta \phi$ denote the pseudo-rapidity and azimuthal angle difference between any two reconstructed objects.} $\Delta R \equiv \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} > 0.3$ [97] (Cut-1), which can help to reject leptons from the decay of a $b$-hadron or $c$-hadron [100]. Three or more charged leptons are required with the $p_T$ of the leading, sub-leading and sub-sub-leading leptons larger than 50 GeV, 35 GeV and 25 GeV, respectively and at least two of them have the same charge (Cut-2), where $\ell_i$. To reduce backgrounds from Drell-Yan processes and $Z$ boson decays, events with opposite-sign same-flavor lepton pairs or same-sign electron pairs with the invariant mass below 12 GeV or within the mass window of 15 GeV around the $Z$ boson mass are rejected [74,75] (Cut-3). For the signal processes, the final states can be $\ell^+\ell^+\ell^-\ell^- (\ell^-jj)E_T^{\text{miss}}$, $\ell^+\ell^+\ell^-jj (\ell^-jj)E_T^{\text{miss}}$, $\ell^+\ell^+\ell^-jjj (\ell^-jjj)E_T^{\text{miss}}$, and the charge-conjugated ones. Therefore, we further impose the following selection cuts:

- exactly three same-sign leptons are required (Cut-4);
- missing transverse momentum $E_T^{\text{miss}} > 50$ GeV (Cut-5);
- $b$-tagged jets are vetoed (Cut-6);

It is noted that experimental search for signals in final state with SS3L signature and at least one $b$-tagged jet has been performed [40], which is typically different from our context. Cuts on objects other than the three same-sign leptons can also be imposed. For example, one can require the sum of the residual lepton number and jet number to be larger than 2.

In this paper, we however only consider Cut-1 to Cut-6 for an easier comparison with previous studies.

It is straightforward to obtain the cut flow of cross sections after the selection cuts. In Fig. 7, cross sections of the background processes after Cut-3 to Cut-6 are depicted, where the left endpoints of the gray, orange, blue and red bands correspond to the cross sections after Cut-3 to Cut-6, respectively.
Therefore, the length of each colored band characterizes the cut efficiency of an individual cut. The total cross section of backgrounds $\sim 0.34 \text{ fb}$ after selection cuts is dominated by $WZ, t\bar{t}W, t\bar{t}, t\bar{t}Z$, while the backgrounds $t\bar{t}t\bar{t}$ and $t\bar{t}bb$ are less important [72]. We can see that the background $ZZ$ becomes negligible after imposing selection cut on the missing transverse momentum.

To evaluate the signal significance, we use [101]

$$Z = \sqrt{2 \left( n_s + n_b \log \frac{n_s + n_b}{n_b} - n_s \right)}, \quad (49)$$

where $n_s$ and $n_b$ denote the numbers of signal and background events after selection cuts. This formula is valid even for $n_b \ll n_s$ [101]. The discovery prospects of individual signal process are depicted in Fig. 8. The solid, dashed and dotted curves correspond to the required integrated luminosities to reach $5\sigma$ discovery for $\Delta m = 0, 1 \text{ GeV}$ and $10 \text{ GeV}$, respectively. Doubly and triply charged Higgs bosons decay into on-shell leptons and gauge bosons in the low and high $v_\Delta$ regions, respectively, alongside with a smooth transition in the medium $v_\Delta$ region due to the cascade decay of $\Delta^{\pm \pm}$. It is apparent that a larger integrated luminosity is required for $v_\Delta \gtrsim 10^{-3} \text{ GeV}$ than for $v_\Delta \lesssim 10^{-5} \text{ GeV}$ since in the latter case $\Delta^{\pm \pm(\pm)}$ mainly decays into $W$ bosons and the signal cross section is dissipated by the decays of $W$ bosons. We can see that with the integrated luminosity of about $0.1 \text{ fb}^{-1}$, the triply charged Higgs boson with mass below $1000 \text{ GeV}$ can be discovered. Besides, the required integrated luminosity to reach $5\sigma$ in the region of $v_\Delta \lesssim 10^{-5} \text{ GeV}$ for the IH is smaller than that for the NH. This is because the coupling of $\Delta^{\pm \pm}$ to the electron pair for the IH is larger. For $v_\Delta \gtrsim 10^{-3} \text{ GeV}$, the sensitivities for the NH and IH are the same since $\Delta^{\pm \pm(\pm)}$ mainly decays into $W^\pm W^\pm W^\pm$, which is independent of the neutrino mass hierarchy. Below, we will concentrate on the sensitivities for the NH.

To illustrate the dependence of $5\sigma$ contours on the mass splitting $\Delta m$, we show the $5\sigma$ discovery reach in the plane of $\log_{10}(L/\text{fb}^{-1})$ and $\Delta m$ in Fig. 10 with two benchmark values $v_\Delta = 10^{-6} \text{ GeV}$ and $5 \times 10^{-5} \text{ GeV}$, which ensure $\text{Br}(\Delta^{\pm \pm} \rightarrow \ell^\pm \ell^\mp W^\pm W^\mp) = 1$ and $\text{Br}(\Delta^{\pm \pm} \rightarrow W^\pm W^\pm W^\mp) = 1$ for $\Delta m = 0$, respectively. Moreover, from the right panel of Fig. 6, the proper decay lengths for $\Delta^{\pm \pm} = 10^{-6} \text{ GeV}$ and $5 \times 10^{-5} \text{ GeV}$ are both larger than 0.1 mm, which ensures the validity of prompt search. We can see that the integrated luminosities to reach $5\sigma$ discovery decreases with $\Delta m$ for $0 < \Delta m \lesssim 10 \text{ GeV}$ (15 GeV) for $v_\Delta = 10^{-6} \text{ GeV}$ ($5 \times 10^{-5} \text{ GeV}$) as shown in Fig. 10.

Finally, the sensitivities at the FCC-hh and the LHC are compared. The latter one has been investigated in Refs. [38,39] with the above benchmark values of $v_\Delta$ being chosen. Different from the significance formula in Eq. (49), they used $n_s/\sqrt{n_s + n_b}$ to quantify the significance and found that at $5\sigma$ level $m_{\Delta^{+++}} \lesssim 950 \text{ GeV}$ can be reached at the LHC with the integrated luminosity of $3 \text{ ab}^{-1}$ for $v_\Delta = 10^{-6} \text{ GeV}$, while it is reduced to $m_{\Delta^{+++}} \lesssim 600 \text{ GeV}$ for $v_\Delta = 5 \times 10^{-5} \text{ GeV}$. However, it is known [101] that $n_s/\sqrt{n_s + n_b}$ is a good approximation of the significance $Z$ in Eq. (49) only if $n_s \ll n_b$. In our case, $n_s$ and $n_b$ can be comparable. As a result, we find that $n_s/\sqrt{n_s + n_b}$ underestimates the significance by several times. As a result, the integrated luminosities required to reach $Z = 5$ can be smaller than that with $n_s/\sqrt{n_s + n_b} = 5$ by one or two orders. To be more concrete, in Table III of Ref. [39], the signal cross section with $(m_{\Delta^{+++}}, \Delta m, v_\Delta) = (400 \text{ GeV}, 0, 10^{-6} \text{ GeV})$ for the NH is $1.19 \times 10^{-3} \text{ pb}$ and the total background cross section is $1.21 \times 10^{-3} \text{ pb}$. From the left panel of Fig. 22 of Ref. [39], we can infer that the signal cross section with $(m_{\Delta^{+++}}, \Delta m, v_\Delta) = (600 \text{ GeV}, 0, 10^{-6} \text{ GeV})$ for the NH is $2.66 \times 10^{-4} \text{ pb}$. The corresponding integrated luminosities required to satisfy $Z = 5$ and $n_s/\sqrt{n_s + n_b} = 5$ are $3.2 \text{ fb}^{-1}$ and $21.2 \text{ fb}^{-1}$ for $m_{\Delta^{+++}} = 400 \text{ GeV}$, and are $23.7 \text{ fb}^{-1}$ and $3 \text{ ab}^{-1}$ for $m_{\Delta^{+++}} = 600 \text{ GeV}$.
Fig. 8  5σ discovery prospects of searching for triply charged Higgs boson at a 100 TeV $pp$ collider with SS3L signature in the DYZ, PF and DYW processes. The benchmark scenarios with $\Delta m = 0, 1, 10$ GeV and $m_{A^{\pm\pm\pm}} = 300, 600, 1000$ GeV for the NH are depicted in the plane of $\log_{10}(v_{\Delta}/\text{GeV})$ and $\log_{10}(L/\text{fb}^{-1})$.

Fig. 9  5σ discovery prospects of searching for triply charged Higgs boson at a 100 TeV $pp$ collider with SS3L signature. The benchmark scenarios with $\Delta m = 0, 1, 10$ GeV and $m_{A^{\pm\pm\pm}} = 300, 600, 1000$ GeV for the NH and IH are depicted in the plane of $\log_{10}(v_{\Delta}/\text{GeV})$ and $\log_{10}(L/\text{fb}^{-1})$.

Fig. 10  5σ discovery reach in the plane of $\log_{10}(L/\text{fb}^{-1})$ and $\Delta m$ with two benchmark values of $v_{\Delta} = 10^{-6}$ GeV and $5 \times 10^{-3}$ GeV in the left and right panels, respectively. The 5σ contours are labelled by the mass $m_{A^{\pm\pm\pm}}$ in units of GeV.
Table 2 Cross sections (in units of pb) of the backgrounds at 100/13 TeV before cuts and after cuts, which are denoted as $\sigma_0(100/13 \text{ TeV})$ and $\sigma_{\text{cut}}(100/13 \text{ TeV})$, respectively. The $K$-factors for each process are listed in the second column. The notation of “$a \times 10^b$” stands for $a \times 10^b$

| background | $K$-factor | $\sigma_0(100 \text{ TeV})$ | $\sigma_{\text{cut}}(100 \text{ TeV})$ | $\sigma_0(13 \text{ TeV})$ | $\sigma_{\text{cut}}(13 \text{ TeV})$ |
|------------|-----------|-----------------|-----------------|-----------------|-----------------|
| ttjj       | 1.5       | 1.07E+03        | 3.15E-05        | 6.58E+01        | 1.94E-06        |
| ttbb       | 1.77      | 1.07E+02        | 4.86E-06        | 1.21E+00        | 5.49E-08        |
| tttt       | 1.21      | 2.39E-01        | 1.36E-05        | 7.20E-04        | 4.09E-08        |
| ttwj       | 1.28      | 8.41E-01        | 9.04E-05        | 2.23E-02        | 2.40E-06        |
| ttzz       | 1.35      | 1.05E+00        | 1.22E-05        | 1.41E-02        | 1.65E-07        |
| twzj       | 1.45      | 7.06E-01        | 1.55E-05        | 1.44E-02        | 3.16E-07        |
| tzzj       | 1.1       | 4.88E-01        | 6.15E-07        | 1.13E-02        | 1.43E-08        |
| wwwj       | 1.74      | 1.55E-01        | 1.20E-05        | 7.15E-03        | 5.33E-07        |
| wwwz       | 1.98      | 7.27E-02        | 2.87E-06        | 2.88E-03        | 1.14E-07        |
| wwwj       | 1.96      | 3.28E-02        | 1.73E-05        | 1.17E-03        | 6.20E-07        |
| wwwj       | 1.58      | 2.02E-03        | 3.93E-08        | 1.18E-04        | 2.30E-09        |
| wzjj       | 1.83      | 7.23E+00        | 1.34E-04        | 4.94E-01        | 9.14E-06        |
| zzzj       | 1.47      | 5.19E-01        | 1.23E-07        | 4.60E-02        | 1.09E-08        |

Fig. 11 The normalized distributions with an arbitrary unit (A.U.) of $p_T$ and $E_{\text{miss}}^T$. Upper left: $p_T$ for leptons in the DYZ process for the NH and $m_{\Delta^{+++}} = 600 \text{ GeV}$; upper right: $p_T$ for leptons in the background WZ process; lower left: $p_T$ for leading-$p_T$ b-jet in the backgrounds $ttW$ and $tt$ processes; lower right: $E_{\text{miss}}^T$ in the DYZ process for the NH and $m_{\Delta^{+++}} = 600 \text{ GeV}$ and in the background WZ process. Leptons ordered by $p_T$ are denoted by 1st, 2nd and 3rd ones

Ref. [39] present the discovery prospects of $\sqrt{s}/\sqrt{n_s + n_b}=5$ with the integrated luminosities of 100 fb$^{-1}$ and 3 ab$^{-1}$, which are unable to be converted into the discovery prospects of $Z = 5$ with varying $\Delta m$ and $m_{\Delta^{+++}}$. On the other hand, we find that it is feasible to obtain the sensitivity at the LHC by projecting the result at the FCC-hh, which is obtained by the delicate detector simulation shown above. To verify this point, we compare the kinematic distributions of the signals and backgrounds at the 13 TeV LHC and FCC-hh in Figs. 11 and 12. Figure 11 displays the distributions of $p_T$ for leptons and leading-$p_T$ b-jet and $E_{\text{miss}}^T$. One can see that these distributions at the 13 TeV LHC and FCC-hh are close to each other. The most notable difference at these two colliders comes from the rapidity distributions [80], which are
Fig. 12 The normalized distributions of the rapidity \(|\eta|\) in the DYZ process for the NH and \(m_{\Delta^{+++}} = 300, 600\) and 1000 GeV and backgrounds. \(|\eta|\) for the leading \(p_T\) lepton and \(b\)-jet for the left and right panels, respectively.

shown in Fig. 12. The leptons and \(b\)-jets tend to have a larger rapidity at the FCC-hh than that at the LHC. The cut efficiencies mainly depend on the \(p_T\) and \(\eta\) of leptons and \(E_T^{\text{miss}}\). Therefore, if we impose the same cuts\(^9\) at the LHC as that at the FCC-hh, the cut efficiencies at these two colliders are expected to be roughly the same. This enables us to evaluate the signal significance at the LHC without repeating the collider simulation.

In the last two columns of Table 2, we show the cross sections of background processes without any cut and after all cuts at the 13 TeV LHC as a recast of cross sections at the FCC-hh assuming the same cut efficiencies, which are denoted as \(\sigma_0\) (13 TeV) and \(\sigma_{\text{cut}}\) (13 TeV), respectively. We obtain that the dominant backgrounds at the LHC are \(WZ\), \(t\bar{t}\) and \(Z\). The cross section of \(WZ\) is 9.14 \times 10^{-6} \text{pb} after cuts, which is slightly smaller than that in Ref. [39] probably owing to a larger \(E_T^{\text{miss}}\) cut that we have imposed. The cross section of background \(ZZ\) we obtain is smaller than that in Ref. [39] since we have further rejected events with lepton invariant mass below 12 GeV as in Cut-2. The cross section of background \(t\bar{t}W\) we obtain is larger since we have considered both leptonic and hadronic decays of top quark. The signal cross section with \((m_{\Delta^{+++}}, \Delta m, v_\Delta) = (400 \text{GeV}, 0, 10^{-6} \text{GeV})\) for the NH is 2.2 \times 10^{-3} \text{pb}, which is about 2 times of that in Ref. [39]. This is because we have multiplied a \(K\)-factor of 1.25 for the DY cross section. Besides, the PF process has also been included in our analysis, the cross section of which for \(m_{\Delta^{+++}} = 400\) GeV is comparable to that of the DYW or DYZ cross section, see Fig. 3.

Finally, we summarize constraints and discovery prospects in the plane of \(m_{\Delta^{+++}}\) and \(\Delta m\) with the benchmark values of \(v_\Delta = 10^{-6} \text{GeV}\) and \(5 \times 10^{-3} \text{GeV}\) for the NH in Fig. 13. The 5\(\sigma\) contours correspond to the integrated luminosities required to satisfy \(\mathcal{Z} = 5\). At the 13 TeV LHC, the regions of \(\Delta m \geq 0\) and 300 GeV \(\leq m_{\Delta^{+++}} \leq 1000\) GeV can be discovered with the integrated luminosity of 330 fb\(^{-1}\) for \(v_\Delta = 10^{-6} \text{GeV}\), while the region of \(m_{\Delta^{+++}} > 800\) GeV for \(v_\Delta = 5 \times 10^{-3} \text{GeV}\) is unable to be discovered even with the integrated luminosity of 3 ab\(^{-1}\). At the FCC-hh, the regions of \(\Delta m \geq 0\) and \(m_{\Delta^{+++}} \leq 400\) GeV for \(v_\Delta = 10^{-6} \text{GeV}\) and \(v_\Delta = 5 \times 10^{-3} \text{GeV}\) can be discovered with the integrated luminosities of 3.3 fb\(^{-1}\) and 110 fb\(^{-1}\), respectively. Our studies promote the possibility of discovering new physics in searches for final states with at least three same-sign leptons without \(b\)-jet at the LHC and future 100 TeV \(pp\) colliders.

6 Conclusions

In this work, we have studied the potential of searching for triply charged Higgs bosons at the LHC and a 100 TeV \(pp\) collider. We first discuss the methodology of producing and detecting a multi-charged Higgs boson at \(pp\) colliders. While the singly and doubly charged Higgs bosons have been discussed thoroughly, the triply charged Higgs boson has not been paid much attention. The details of a specifically non-trivial model with a Higgs quadruplet and a pair of vector-like triplet leptons are given. The indirect constraints on this model are subsequently discussed, which indicate that the magnitude of mass splitting \(\Delta m\) between the nearby states of the Higgs quadruplet is restricted to be smaller than 30 GeV while the quadruplet VEV \(v_\Delta\) larger (smaller) than 1.5 \times 10^{-9} \text{GeV} (1.3 \text{GeV}) is allowed.

We then discuss the production cross section and decay branching ratio of the triply charged Higgs boson. With the increase of collider energy, the production cross section becomes larger significantly. This motivates us to study the sensitivity of searching for a triply charged Higgs bosons at a 100 TeV \(pp\) collider. Triply charged Higgs boson can...
decay into $W^\pm W^\mp W^\mp$ or $\ell^\pm \ell^\pm W$ through an off-shell doubly charged Higgs boson with the decay branching ratios being nearly independent of the mass splitting $\Delta m$. The cascade decays are open if $\Delta m \neq 0$ and can be dominant in the medium region of $v_\Delta$.

Thanks to the high charge, three same-sign leptons can be produced in the decays of triply charged Higgs boson. In previous studies with SS3L signature at the LHC, only part of SM backgrounds were considered. We consider a complete set of backgrounds, simulate them at a 100 TeV pp collider by taking the FCC-hh as an example and perform a detailed collider analysis with at least three same-sign leptons in the final state being selected, which is inclusive for the signal processes with one or two $\Delta^{\pm\pm\pm}$ and the decays $\Delta^{\pm\pm\pm} \rightarrow \ell^\pm \ell^\pm (W^\pm)$ and $W^\pm W^\pm (W^\pm)$. The cascade decays giving rise to the SS3L signature for $\Delta m > 0$ are also properly included. Signal events are generated according to their dependence on the mass splitting $\Delta m$ and the quadruplet VEV $v_\Delta$, so that we can obtain the discovery significance as a function of $v_\Delta$.

For a comparison, we choose two benchmark values of $v_\Delta$, for which prompt searches are valid. We find that previous studies at the LHC underestimated the significance by several times. We revisit the sensitivity at the LHC by projecting that at the FCC-hh since the differential distributions at these two colliders are close except the rapidity distributions. Our study shows that 5$\sigma$ discovery can be reached for a triply charged Higgs boson below 1 TeV with 330 fb$^{-1}$ of data at the LHC for a relatively small $v_\Delta = 10^{-6}$ GeV and with 110 fb$^{-1}$ of data at a 100 TeV pp collider in the whole region.
$1.5 \times 10^{-9}$ GeV $\lesssim v_\Delta \lesssim 1.3$ GeV for the mass splitting $\Delta m = 0$. For $\Delta m > 0$, the sensitivity is further improved due to the cascade decays.

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**Appendix**

**Appendix A: The $Z$, $W^\pm$ masses and the would-be Goldstone modes**

In this Appendix, we will give more details of the model with a SM Higgs doublet $H$ and a Higgs multiplet $H_n$ in Sect. 1 by expanding Eq. (1).

The VEV of $H_n$ will modify the $W$ and $Z$ boson masses compared to the model with just $H$ to have

$$m_W^2 = \frac{g^2}{4}(v_H^2 + 2(I_n(I_n + 1) - Y_n^2)v_n^2) ,$$

$$m_Z^2 = \frac{g^2}{4c_W^2}(v_H^2 + 4Y_n^2v_n^2) . \tag{A1}$$

where $I_n$ is the isospin of the $SU(2)_L$ of a $n$-th rank Higgs representation. Therefore, the $\rho$ parameter is expressed as

$$\rho = \frac{v_H^2 + 2(I_n(I_n + 1) - Y_n^2)v_n^2}{v_H^2 + 4Y_n^2v_n^2} . \tag{A2}$$

Experimentally, the $\rho$ parameter is determined to be very close to unity, $\rho = 1.00039 \pm 0.00019$ [23]. In the usual Higgs representation, the VEV $v_n$ is constrained to be small compared with the doublet VEV $v_H$. The new Higgs boson couplings to SM fermions are small proportional to $v_n/v_H$. If $n$ is larger than 3, $H_n$ does not couple to SM fermions directly for a Lagrangian that is renormalizable.

The Goldstone bosons and charged Higgs fields of $H_n$ are

$$G_Z = \frac{v_HI_0 + 2Y_nv_nI_n}{\sqrt{v_H^2 + 4Y_n^2v_n^2}} ,$$

$$G^+_W = \frac{v_Hh^+ + v_nv\sqrt{2(I_n(I_n + 1) - Y_n^2)}\phi^+}{\sqrt{v_H^2 + v_n^22(I_n(I_n + 1) - Y_n^2)}} . \tag{A3}$$

where $h^+$ denotes the singly charged field from the doublet representation $H$.

After removing the Goldstone bosons, one can obtain the physical pseudoscalar $A^0$ and singly charged Higgs bosons $h^+_i$ as given by

$$A^0 = \frac{2Y_nv_nI_0 - v_HI_0}{\sqrt{v_H^2 + 4Y_n^2v_n^2}} , \tag{A4}$$

and

$$h^+_1 = \frac{v_n\sqrt{2(I_n(I_n + 1) - Y_n^2)}h^+ - v_H\phi^+}{\sqrt{v_H^2 + v_n^22(I_n(I_n + 1) - Y_n^2)}} ,$$

$$h^+_2 = \frac{\sqrt{2(I_n + Y_n + 1)(I_n - Y_n)h^+_2} + \sqrt{2(I_n - Y_n + 1)(I_n + Y_n)h^+_2}}{\sqrt{2(I_n(I_n + 1) - Y_n^2)}} . \tag{A5}$$

with $\phi^+$ given by

$$\phi^+ = \frac{\sqrt{2(I_n - Y_n + 1)(I_n + Y_n)h^+_2} - \sqrt{2(I_n + Y_n + 1)(I_n - Y_n)h^+_2}}{\sqrt{2(I_n(I_n + 1) - Y_n^2)}} \tag{A6}$$

Note that $h^+_i$ may or may not be mass eigenstates depending on the details of Higgs potential. For simplicity, we will assume that they are mass eigenstates.

It is convenient to write the two real neutral components $h^0$ and $h^0_n$ as

$$h^0_1 = \frac{v_Hh^0 + 2Y_nv_nh^0_n}{\sqrt{v_H^2 + 4Y_n^2v_n^2}} , \quad h^0_2 = \frac{2Y_nv_nh^0 - v_Hh^0_n}{\sqrt{v_H^2 + 4Y_n^2v_n^2}} \tag{A7}$$

In general, $h^0$, $h^+$, $h^0_n$ and $h^0_n$ are not mass eigenstates. From Eqs. (A3), (A4), (A5) and (A7), the mass eigenstates can be written as the following basis transformations. For real neutral fields $h_\alpha$ ($h_1 \equiv h^0$, $h_2 \equiv h^0_n$)

$$h_\alpha = \sum_{\alpha, \beta=1}^3(N_R)_{\alpha\beta}h^0_{\beta} \tag{A8}$$

where $N_R$ denotes the $2 \times 2$ orthogonal matrix and $h^0_{\alpha}$ are the mass eigenstates ($h^0_{1(2)} \equiv h^{0}_{1(2)}$). For imaginary neutral
### Table 3 Feynman Rules. All momenta flow into the vertex

| Vertices                                                                 | Coefficients                                                                 |
|-------------------------------------------------------------------------|------------------------------------------------------------------------------|
| $W^\pm h_{\mu_1}^{[0]} h_{\mu_2}^{[m+n]}$                              | $i \frac{g}{2}(N_R)_{1a} S_{1\beta} - \sqrt{(I_n - Y_n)(I_n + Y_n + 1)} (N_R)_{2a} S_{2\beta}$ |
| $W^\pm A^0 h_{\mu_1}^{[m+n]}$                                           | $+ \sqrt{(I_n + Y_n)(I_n - Y_n + 1)} (N_R)_{2a} S_{3\beta} (P_2 - P_1)_{\mu}$ |
|                                                                          | $- \frac{g}{2} (N_R)_{12} S_{1\beta} - \sqrt{(I_n - Y_n)(I_n + Y_n + 1)} (N_R)_{12} S_{2\beta}$ |
|                                                                          | $+ \sqrt{(I_n + Y_n)(I_n - Y_n + 1)} (N_R)_{12} S_{3\beta} (P_2 - P_1)_{\mu}$ |
| $W^\pm h_{\mu_1}^{[m+n]} h_+^{[2]}$                                     | $i \frac{g}{2} \sqrt{(I_n - Y_n + 2)(I_n + Y_n - 1)} S_{2\beta} (P_2 - P_1)_{\mu}$ |
| $W^\pm h_{\mu_1}^{[m+n]} h_-^{[2]}$                                     | $i \frac{g}{2} \sqrt{(I_n - Y_n + 2)(I_n + Y_n - 1)} S_{2\beta} (P_2 - P_1)_{\mu}$ |
| $W^\pm h_{\mu}^{[1]} h_+^{[1]*}, (Q \geq 2)$                           | $i \frac{g}{2} \sqrt{(I_n - Y_n + 2)(I_n + Y_n - 1)} S_{2\beta} (P_2 - P_1)_{\mu}$ |
| $W^\pm h_{\mu}^{[Q]} h_+^{[Q+1]}, (Q \geq 2)$                          | $i \frac{g}{2} \sqrt{(I_n - Y_n + 2)(I_n + Y_n - 1)} S_{2\beta} (P_2 - P_1)_{\mu}$ |
| $Z_\mu Z_\nu h_{\mu_1}^{[0]}$                                           | $\frac{g}{4\sqrt{2}} ((N_R)_{1a} v_{\mu} + 4 Y_3^2 (N_R)_{2a} v_{\mu}) g_{\nu\mu}$ |
| $Z_\mu Z_\nu h_{\mu_1}^{[0]} h_{\nu_1}^{[0]}$                          | $\frac{g}{4\sqrt{2}} ((N_R)_{1a} v_{\mu} + 4 Y_3^2 (N_R)_{2a} v_{\mu}) g_{\nu\mu}$ |
| $A_\mu A^\nu h_{\mu_1}^{[m+n]} h_{\nu_1}^{[m+n]}$                      | $((3_{1a})^2 + (S_{2a})^2 + (S_{3a})^2) g_{\mu\nu}$ |
| $Z_\mu Z_\nu h_{\mu_1}^{[m+n]} h_{\nu_1}^{[m+n]}$                      | $\frac{g}{4\sqrt{2}} ((S_{1a}^2 - \frac{1}{2}) (S_{1a})^2 + (S_{2a}^2) (S_{2a})^2 + (S_{3a}^2) (S_{3a})^2) g_{\mu\nu}$ |
| $Z_\mu Z_\nu h_{\mu_1}^{[Q]} h_{\nu_1}^{[Q]}$, $Q \geq 2$              | $Q^2 c^2 g_{\mu\nu}$ |
| $Z_\mu h_{\mu_1}^{[m+n]} h_{\mu_2}^{[0]}$                              | $\frac{g}{4\sqrt{2}} ((Q c_{2w}^2 + (N_R)_{1a} v_{\mu} + 4 Y_3^2 (N_R)_{2a} v_{\mu}) g_{\nu\mu}$ |
| $Z_\mu A_\nu h_{\mu_1}^{[m+n]}$                                         | $-i \frac{g}{2\sqrt{2}} ((N_R)_{1a} v_{\mu} + 4 Y_3^2 (N_R)_{2a} v_{\nu}) g_{\mu\nu}$ |
| $A_\mu A_\nu h_{\mu_1}^{[m+n]}$                                         | $-i \frac{g}{2\sqrt{2}} ((N_R)_{1a} v_{\nu} + 4 Y_3^2 (N_R)_{2a} v_{\mu}) g_{\mu\nu}$ |
| $A_\mu h_{\mu_1}^{[m+n]} h_+^{[2]}$                                     | $(i_{12} S_{1\beta} + 2 Y_3 (N_R)_{2a} S_{1\beta} (P_2 - P_1)_{\mu}$ |
| $A_\mu h_{\mu_1}^{[Q]} h_+^{[Q+1]}$, $Q \geq 2$                        | $(i_{12} S_{1\beta} + 2 Y_3 (N_R)_{2a} S_{1\beta} (P_2 - P_1)_{\mu}$ |
| $Z_\mu h_{\mu_1}^{[Q]} h_+^{[Q+1]}$, $Q \geq 2$                        | $(i_{12} S_{1\beta} + 2 Y_3 (N_R)_{2a} S_{1\beta} (P_2 - P_1)_{\mu}$ |
| $Z_\mu h_{\mu_1}^{[Q]} h_+^{[Q+1]}$, $Q \geq 2$                        | $(i_{12} S_{1\beta} + 2 Y_3 (N_R)_{2a} S_{1\beta} (P_2 - P_1)_{\mu}$ |

Fields $I_\alpha$ ($I_1 \equiv I^0, I_2 \equiv I^0_n$)

$$I_\alpha = \sum_{\alpha, \beta=1}^{3} (N_I)_{\alpha \beta} I_\beta^m , \quad \text{(A9)}$$

where $N_I$ denotes the $2 \times 2$ orthogonal matrix and $I_\mu^m$ are the mass eigenstates. $I_\mu^m \equiv G_Z$ is the would-be Goldstone boson and $I_\mu_n^m \equiv A^0$. For singlet charged fields $H_\alpha^+ (H_1^+ \equiv h^+, H_2^+ \equiv h^+_n, H_3^+ \equiv h^+_n)$\(^{11}\)

$$H_\alpha^+ = \sum_{\alpha, \beta=1}^{3} S_{\alpha \beta} H_\beta^{m+n} , \quad \text{(A10)}$$

where $S$ denotes the $3 \times 3$ orthogonal matrix and $H_\beta^{m+n}$ denotes the mass eigenstates. $H_1^{m+n} \equiv G^+_W$ is the would-be Goldstone boson and the physical Higgs bosons $H_{2(3)}^{m+n} \equiv h_{1(2)}^+$.

**Appendix B: Feynman rules in the general Higgs representation**

The production and detection additional Higgs boson depend on their couplings to photon, $W^\pm$ and $Z$ bosons. Using Eq. (1), we have the following interaction terms relevant to $A, W^\pm$ and $Z$ fields ($g_2 \equiv g$ is the $SU(2)_L$ gauge coupling),

$$\mathcal{L}_{\text{int}}^W = i \frac{g_2}{\sqrt{2}} \left[ \sqrt{(I_n + m)(I_n - m + 1)} \bar{h}_n^{Q} h_n^{Q+1} - \sqrt{(I_n - m)(I_n + m + 1)} \bar{h}_n^{Q+1} h_n^{Q+1} \right]$$

---

\(^{11}\) In general, $h_n^{Q+1} \neq h_n^{Q}$. The equality only holds for real representations.
### Table 4 Feynman Rules (continued)

| Vertices | Coefficients |
|----------|--------------|
| $W_\mu^{-}W_\nu^{+}h_a^0h_a^{0}$ | $\frac{g}{3}((N_R)^2_{1a} + (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |
| $W_\mu^{-}W_\nu^{+}h_a^{0}$ | $\frac{g}{3}((N_R)_{1a}v_{I_a} + (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b}g_{\mu\nu}$ |
| $W_\mu^{-}W_\nu^{+}A_0^0$ | $\frac{g}{3}((N_a)^2_{12} + (I_a(I_a + 1) - Y^2_{\alpha})(N_a)^{2a}_{2b})g_{\mu\nu}$ |
| $W_\mu^{-}W_\nu^{+}h_a^{0}h_a^{0}H_{\mu
u}^{\pm}$ | $\frac{g}{3}((S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 + Y_a)^{2})S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 - Y_a)^{2})S_{\mu\nu}^{2})g_{\mu\nu}$ |
| $W_\mu^{-}W_\nu^{+}h_a^{0}h_a^{0}H_{\mu
u}^{\pm}$ | $\frac{g}{3}((S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 + Y_a)^{2})S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 - Y_a)^{2})S_{\mu\nu}^{2})g_{\mu\nu}$ |
| $W_\mu^{-}W_\nu^{+}Q_{\pm}^{\pm}h_{\mu
u}^{Q}$ | $\frac{g}{3}(2(\mu + Q) + (Q + 1))g_{\mu\nu}$ |
| $W_\mu^{-}W_\nu^{+}h_a^{0}h_a^{0}H_{\mu
u}^{\pm}$ | $\frac{g}{3}((S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 + Y_a)^{2})S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 - Y_a)^{2})S_{\mu\nu}^{2})g_{\mu\nu}$ |
| $W_\mu^{-}W_\nu^{+}h_a^{0}h_a^{0}H_{\mu
u}^{\pm}$ | $\frac{g}{3}((S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 + Y_a)^{2})S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 - Y_a)^{2})S_{\mu\nu}^{2})g_{\mu\nu}$ |
| $W_\mu^{-}W_\nu^{+}h_a^{0}h_a^{0}H_{\mu
u}^{\pm}$ | $\frac{g}{3}((S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 + Y_a)^{2})S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 - Y_a)^{2})S_{\mu\nu}^{2})g_{\mu\nu}$ |
| $W_\mu^{-}W_\nu^{+}h_a^{0}h_a^{0}H_{\mu
u}^{\pm}$ | $\frac{g}{3}((S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 + Y_a)^{2})S_{\mu\nu}^{2} + (I_a(I_a + 1) - (1 - Y_a)^{2})S_{\mu\nu}^{2})g_{\mu\nu}$ |

### Table 5 Feynman Rules (continued). It is noted that there is no coupling of physical singly charged Higgs boson to $\gamma W^\pm$. After removing the would-be Goldstone boson, the interaction of $A_\mu W^\mu W^\mu H^m_{\mu
u}$ becomes vanishing.

| Vertices | Coefficients |
|----------|--------------|
| $A_\mu W^\mu h_a^0 H_{\mu\nu}^m$ | $\frac{g}{3}((N_R)^{2a}_{2b} - (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |
| $A_\mu W^\mu A^0 H_{\mu\nu}^m$ | $\frac{g}{3}((N_R)^{2a}_{2b} - (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |
| $Z_\mu W^\mu H_{\mu\nu}^m$ | $\frac{g}{3}((N_R)^{2a}_{2b} - (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |
| $Z_\mu W^\mu h_a^0 H_{\mu\nu}^m$ | $\frac{g}{3}((N_R)^{2a}_{2b} - (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |
| $Z_\mu W^\mu A^0 H_{\mu\nu}^m$ | $\frac{g}{3}((N_R)^{2a}_{2b} - (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |
| $A_\mu W^\mu h^2_a H_{\mu\nu}^m$ | $\frac{g}{3}((N_R)^{2a}_{2b} - (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |
| $A_\mu W^\mu h_a^{2a} H_{\mu\nu}^m$ | $\frac{g}{3}((N_R)^{2a}_{2b} - (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |
| $Z_\mu W^\mu h_a^{2a} H_{\mu\nu}^m$ | $\frac{g}{3}((N_R)^{2a}_{2b} - (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |
| $A_\mu W^\mu h_a^{2a} H_{\mu\nu}^{Q+1}$ | $\frac{g}{3}((N_R)^{2a}_{2b} - (I_a(I_a + 1) - Y^2_{\alpha})(N_R)^{2a}_{2b})g_{\mu\nu}$ |

$(Q \geq 2$ for positive $Q$ & $Q \leq -3$ for negative $Q)$
Substituting the physical components defined in Appendix A into Eq. (B1), one can get the Feynman rules of Higgs-Gauge couplings. We list the tables of Feynman rules in the following. Note that we have removed the would-be Goldstone bosons $G_Z$ and $G_W$ after the electroweak symmetry breaking, thus $\alpha = 2$, 3 for the singly charged Higgs field $H_u^{\pm}$.

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