COSMIC MACH NUMBER AS A FUNCTION OF OVERDENSITY AND GALAXY AGE

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ABSTRACT

We carry out an extensive study of the cosmic Mach number ($M$) on scales of $R = 5, 10$, and $20\, h^{-1}$ Mpc using a $\Lambda$-dominated flat cold dark matter hydrodynamical simulation. We particularly put emphasis on the environmental dependence of $M$ on overdensity, galaxy mass, and galaxy age. We start by discussing the difference in the resulting $M$ according to different definitions of $M$ and different methods of calculation. The simulated Mach numbers are slightly lower than the linear theory predictions even when a nonlinear power spectrum was used in the calculation, reflecting the nonlinear evolution in the simulation. We find that the observed $M$ is higher than the simulated mean $\langle M \rangle$ by more than $2$ standard deviations, which suggests either that the Local Group is in a relatively low density region or that the true value of $\Omega_m$ is $\sim 0.2$, significantly lower than the simulated value of $0.37$. We show from our simulation that the Mach number is a weakly decreasing function of overdensity. We also investigate the correlations between galaxy age, overdensity, and $M$ for two different samples of galaxies—DWARFs and GIANTS. Older systems cluster in higher density regions with lower $M$, while younger ones tend to reside in lower density regions with larger $M$, as expected from the hierarchical structure formation scenario. However, for DWARFs, the correlation is weakened by the fact that some of the oldest DWARFs are left over in low-density regions during the structure formation history. For giant systems, one expects blue-selected samples to have higher $M$ than red-selected ones. We briefly comment on the effect of the warm dark matter on the expected Mach number.

Subject headings: cosmology: theory - galaxies: formation - large-scale structure of universe - methods: numerical - shock waves

1. INTRODUCTION

The cosmic Mach number $M$ is the ratio of the bulk flow $V$ of the velocity field on some scale $R$ to the velocity dispersion $\sigma$ within the region. It was introduced by Ostriker & Suto (1990, hereafter OS90), who stressed that it is independent of the normalization of the power spectrum and is insensitive to the bias between galaxies and dark matter (DM). Basically, it characterizes the warmth or coldness of the velocity field by measuring the relative strength of the velocities at scales larger and smaller than the patch size $R$, so that it effectively measures the slope of the power spectrum at the scale corresponding to the patch size. OS90 made rough estimates of $M$ using available observational data on three different scales and found that the observed $M$ was higher than the expected values of the standard cold dark matter model (SCDM; $\Omega_m^T = 1$, where $\Omega_m$ is the cosmological matter density divided by the critical density of the universe) in the linear regime by more than a factor of $2$ ($M_{\text{obs}} \approx 1-4$ and $M_{\text{SCDM}} \approx 1$). Subsequently, Suto & Fujita (1990), using $N$-body simulations, argued that the constraint on $M$ derived by OS90 holds at the $90\%$ confidence level and that the distribution of $M$ is close to Maxwellian in linear and mildly nonlinear regimes. Park (1990) has also argued that the biased open CDM models are preferred to the SCDM models using an $N$-body simulation.

The first serious calculation of $M$ using the first generation of large-scale hydrodynamical simulations that include star formation was carried out by Suto, Cen, & Ostriker (1992, hereafter SCO92). Using this type of simulation enables one to examine the velocity field of galaxies and DM independently without an ad hoc assumption of bias between galaxies and DM. They used the patch size of $R = 18$ and $40\, h^{-1}$ Mpc and argued that there was no significant difference in $M$ between galaxies and DM, although the galaxies had somewhat larger $\sigma, V$, and $M$ than did DM. Their best estimate of the mean Mach number derived from SCDM simulations is $\langle M \rangle = 0.6$, lower than the observational estimate of $M_{\text{obs}} \approx 1$.

Strass, Cen, & Ostriker (1993, hereafter S93) made more realistic and direct comparison of observations and models. Accepting the fact that the existing peculiar velocity data do not allow us to compute the ideally defined $M$ as in OS90, they defined a modified Mach number that incorporates the observational errors in measured distances due to the scatter in the Tully-Fisher relation. They constructed a mock catalog of the observations using SCDM hydrodynamical simulations similar to those that were used by SCO92 and calculated their modified $M$ from them. S93 obtained smaller $M$ than did OS90 because they included all velocity components on scales less than the bulk flow into the velocity dispersion, whereas OS90 erased the small-scale dispersion by smoothing. As a consequence, the estimates of S93 on $\sigma$ are large, resulting in a smaller $M$. They found that $95\%$ of the mock catalogs had smaller $M$ than observed and that the Mach number test rejects the SCDM scenario at $94\%$ confidence level.

Since $M$ is defined as $V/\sigma$ on a certain scale $R$, a larger $M$ implies a smaller $\sigma$ if the variation of $V$ is weaker than that of $\sigma$. Observationally, it has been recognized for at least a
decade that the velocity field is very cold outside of clusters (Brown & Peebles 1987; Sandage 1986; Groth, Juszkiewicz, & Ostriker 1989; Burstein 1990; Willick et al. 1997; Willick & Strauss 1998). We ask ourselves in this paper how typical such a cold region of space would be in the entire distribution of the velocity field. We note that van de Weygaert & Hoffman (2000) also address the same question using N-body simulations that simulate the Local Group via constrained initial conditions.

A closely related quantity is the pairwise velocity dispersion \( \sigma_{12} \), which has been much studied due to its cosmological importance in relation to the "cosmic virial theorem." This theorem relates \( \sigma_{12} \) to the two- and three-point correlation functions and \( \Omega_m \). Unfortunately, the determination of \( \sigma_{12} \) is quite unstable, and its value differs significantly from author to author (Mo, Jing, & Bolender 1993; Zurek et al. 1994; Somerville, Davis, & Primack 1997; Guzzo et al. 1997; Strauss, Ostriker, & Cen 1998). This is because \( \sigma_{12} \) is a pair-weighted statistic and is heavily weighted by the objects in the densest regions. Inclusion or exclusion of even \( \sim 10 \) galaxies from the Virgo Cluster can change \( \sigma_{12} \) by \( \sim 100-200 \, \text{km s}^{-1} \) and the correction for the cluster infall also affects the result significantly.

To overcome this problem, alternative statistics have been suggested. Kepner, Summers, & Strauss (1997) proposed the redshift dispersion as a new statistic and suggested a calculation of the dispersion as a function of local overdensity \( \delta \). They analytically showed that \( \sigma_{12} \) is heavily weighted by the densest regions of the sample. In the same spirit, Strauss et al. (1998) defined and calculated a new measure of \( \sigma_{12} \) as a function of \( \delta \) in redshift space using the Optical Redshift Survey (Santiago et al. 1995) and showed that \( \sigma_{12} \) is indeed an increasing function of \( \delta \). As we will show in this paper, the above statement for \( \sigma_{12} \) holds for the velocity dispersion \( \sigma \) as well: it is heavily weighted by the densest regions. Davis, Miller, & White (1997) proposed a single-particle-weighted statistic that measures the one-dimensional rms peculiar velocity dispersion of galaxies and applied the statistic to the subsample of the Optical Redshift Survey and the 1.2 Jy IRAS catalog (Fisher et al. 1995). Baker, Davis, & Lin (2000) applied the same statistic to the Las Campanas Redshift Survey (Shectman et al. 1996) and found that the low \( \Omega_m \) is favored when the results are compared with N-body simulations. We also note the work by Juszkiewicz, Springel, & Durrer (1999), who derived a simple closed-form expression relating the mean pairwise relative velocity \( v_{12} \) to the two-point correlation function of mass. Their results can also be used to estimate \( \Omega_m \).

Since the original work of OS90, many things have changed. The resolution and the accuracy of simulations have increased significantly due to increased computer power and more realistic modeling of galaxy formation. The favored cosmology shifted from SCDM to LCDM (A-dominated flat cold dark matter model), as more and more modern observational data suggest a flat low-\( \Omega_m \) universe with a cosmological constant \( \Lambda \) (e.g., Efstathiou, Sutherland, & Maddox 1990; Ostriker & Steinhardt 1995; Turner & White 1997; Perlmutter et al. 1998; Garnavich et al. 1998; Bahcall, Ostriker, & Steinhardt 1999; Balbi et al. 2000; Lange et al. 2001; Hu et al. 2001). Thus, we are motivated to study this statistic again using a state-of-the-art LCDM hydrodynamical simulation, which allows us to treat baryons and dark matter separately without invoking an ad hoc bias parameter.

In this paper, we calculate \( \sigma, V, \) and \( \mathcal{M} \) with patches of size \( R = 5, 10, \) and \( 20 \, \text{h}^{-1} \, \text{Mpc} \) using an LCDM hydrodynamical simulation that includes star formation. The details of the simulation are explained in § 2 and in Appendix A. We correct for the underestimation of the bulk flow due to the limited size of the simulation box using the linear theory of gravitational instability in § 3. In § 4, we describe the method of the calculation of \( V, \sigma, \) and \( \mathcal{M} \). The results of the calculation are presented in § 5, where we discuss the difference in \( \mathcal{M} \) resulting from different methods of calculation. The distribution of \( \mathcal{M} \) is discussed in § 5.2. In §§ 6 and 7, we divide the sample of simulated galaxies by the local overdensity and by their age and study the correlation with \( \mathcal{M} \) under the hierarchical structure formation scenario.

All velocities in this paper are presented in the cosmic microwave background frame, and the velocity dispersion is three-dimensional (i.e., not just the one-dimensional line-of-sight component).

2. THE SIMULATION

The hydrodynamical simulation that we use here is similar to but greatly improved over that of Cen & Ostriker (1992a, 1992b). The adopted cosmological parameters are \( \Omega_m = 0.37, \, \Omega_L = 0.63, \, \Omega_b = 0.049, \, n = 0.95, \, \sigma_8 = 0.8, \) and \( h = 0.7, \) where \( H_0 = 100 \, \text{km s}^{-1} \, \text{Mpc}^{-1} \) and \( n \) is the primordial power spectrum index. The power spectrum includes a 25% tensor-mode contribution to the cosmic microwave background fluctuations on large scales. The present age of the universe with these parameters is 12.7 Gyr. The simulation box has a size of \( L_{\text{box}} = 100 \, \text{h}^{-1} \, \text{Mpc} \) and \( 512^3 \) grid points, so the comoving cell size is \( 200 \, \text{h}^{-1} \, \text{kpc} \). It contains \( 256^3 \) dark matter particles, each weighing \( 5.3 \times 10^8 \, \text{M}_\odot \).

The code is implemented with a star formation recipe summarized in Appendix A. It turns a fraction of the baryonic gas in a cell into a collisionless particle (hereafter "galaxy particle") in a given time step once the following criteria are met simultaneously (see Appendix A): (1) the cell is overdense, (2) the gas is cooling fast, (3) the gas is Jeans unstable, and (4) the gas flow is converging into the cell.

Each galaxy particle has a number of attributes at birth, including position, velocity, formation time, mass, and initial gas metallicity. Upon its formation, the mass of the galaxy particle is determined by \( m_*=c_\ast m_{\text{gas}} \Delta t / t_\ast \), where \( c_\ast \) is the star formation efficiency parameter, which we take to be \( c_\ast = 0.25 \). The term \( \Delta t \) is the current time step in the simulation, and \( t_\ast = \max(t_{\text{dyn}}, 10^7 \, \text{yr}) \). The galaxy particle is placed at the center of the cell after its formation with a velocity equal to the mean velocity of the gas and followed by the particle mesh code thereafter as a collisionless particle in gravitational coupling with DM and gas. Galaxy particles are baryonic galactic subunits with masses ranging from \( 10^3 \) to \( 10^{10} \, \text{M}_\odot \); therefore, a collection of these particles is regarded as a galaxy. Feedback processes such as ionizing UV, supernova energy, and metal ejection are also included self-consistently. Further details of these treatments can be found in Cen & Ostriker (1992a, 1992b). We also refer the interested reader to Cen & Ostriker (1999a, 1999b, 2000), Blanton et al. (1999, 2000), and Nagamine, Cen, & Ostriker (2000, 2001), where various analyses have been performed using the same simulation.

In addition to the above \( L_{\text{box}} = 100 \, \text{h}^{-1} \, \text{Mpc} \) simulation, we have a newly completed \( L_{\text{box}} = 25 \, \text{h}^{-1} \, \text{Mpc} \) simulation with 6 times better spatial resolution and 260 times better...
DM mass resolution. We use this new simulation as a reference by verifying that the same trend found in the $L_{\text{box}} = 100\ h^{-1}\ Mpc$ simulation is seen in the $L_{\text{box}} = 25\ h^{-1}\ Mpc$ simulation as well, although the velocity field in the new one is significantly underestimated as the box size is not large enough.

3. LINEAR THEORY AND DEFINITIONS OF $\mathcal{M}$

Under the linear theory of gravitational instability, the mean square bulk flow and the mean square velocity dispersion in a window of size $R$ can be calculated as follows (e.g., Peebles 1993; OS90):

$$\langle V^2(R) \rangle = \frac{\Omega^2 \Delta H^2}{2\pi^2} \int_0^\infty P(k)W^2(kR)\,dk,$$  

$$\langle \sigma^2(R) \rangle = \frac{\Omega^2 \Delta H^2}{2\pi^2} \int_0^\infty P(k)[1 - W^2(kR)]\,dk,$$

where $P(k)$ is the power spectrum of density fluctuations and $W(kR)$ is the Fourier transform of the window function of size $R$. In this paper, we adopt the top-hat window function $W(x) = 3(\sin x - x \cos x)/x^3$. The effect of the cosmological constant on the term $\Omega^2 \Delta H^2$ is small (Lahav et al. 1991; Martel 1991). Although OS90 include an observational correction term $RV \cdot V/3$ in the integrand, we do not include this term since it makes only a slight difference. The rms cosmic Mach number can be defined in two ways, depending on how one does the averaging (OS90):

$$\langle \mathcal{M}^2(R) \rangle^{1/2} = \left(\frac{\langle V^2(R) \rangle}{\langle \sigma^2(R) \rangle}\right)^{1/2} \text{ or } \left(\frac{\langle V^2(R) \rangle}{\langle \sigma^2(R) \rangle}\right)^{1/2}.$$  

In practice, we can calculate $V(R)$ and $\sigma(R)$ for each patch that we take in the simulation and assign $\mathcal{M}(R) = |V(R)/\sigma(R)|$ to each patch. We can then later take the ensemble average by

$$\langle \mathcal{M}(R) \rangle = \left(\frac{V(R)}{\sigma(R)}\right).$$

This method allows us to observe the distribution of the Mach number before we take the ensemble average. In the next section, we show the differences between the different definitions.

The simulation that we use has a box size of only $L_{\text{box}} = 100\ h^{-1}\ Mpc$ and lacks long-wavelength perturbations beyond this scale. This lack of long-wavelength perturbations results in an underestimate of the bulk flow, as it is determined by the perturbations on scales larger than the patch size $R$. In particular, for the currently popular low $\Omega_m$ models, the peak of the power spectrum lies at scales larger than $200\ h^{-1}\ Mpc$. So a box size larger than $500\ h^{-1}\ Mpc$ is necessary for the correct and direct treatment of the bulk flow accurate to 10% in the simulation on the scale of $20\ h^{-1}\ Mpc$. However, the rms bulk flow can be calculated correctly by the above equation at large enough scales. The solid and the dashed lines in Figure 1 show the predicted rms Mach number calculated from equations (1), (2), and (3) (the latter definition). The solid line is calculated by using the $P(k)$ obtained from the COSMICS package, which was also used to generate the initial conditions of our simulation. The dashed line was calculated with the $P(k)$ that was evolved to nonlinear regime by Peacock & Dodds’s (1996) scheme from the empirical double-power-law linear spectrum. This nonlinear $P(k)$ is known to provide a good fit to the observed optical galaxy power spectrum (Peacock 1999). The two upper lines are calculated with the full $P(k)$ and the bottom two using the cutoff $P(k)$ at $100\ h^{-1}\ Mpc$ to show the effect of the limited simulation box size. The three vertical lines indicate the range of simulated raw values of $\mathcal{M}$ before the correction of the bulk flow for the lack of long-wavelength perturbations. See §§ 3 and 4 for discussion.

![Figure 1](image.png)

**Figure 1.** Cosmic Mach number as a function of scale $R$. The solid (top boundaries of gray regions) and dashed (bottom boundaries of gray regions) lines are the linear theory predictions calculated from the equations in § 3 using the COSMICS $P(k)$ (solid line) and the nonlinear $P(k)$ (dashed line). The nonlinear $P(k)$ was evolved from an empirical double-power-law linear spectrum that is known to provide a good fit to the observed optical galaxy power spectrum (Peacock 1999). The two top lines are calculated using the full $P(k)$ and the bottom two using the cutoff $P(k)$ at $100\ h^{-1}\ Mpc$ to show the effect of the limited simulation box size. The three vertical lines indicate the range of simulated raw values of $\mathcal{M}$ before the correction of the bulk flow for the lack of long-wavelength perturbations. See §§ 3 and 4 for discussion.

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1 Available at http://arcturus.mit.edu/cosmics.
the nonlinear effects are not completely described by just plugging the nonlinear $P(k)$ into the linear theory equations.

We wish to correct our simulated values of $V$ and $\mathcal{M}$ for the lack of long-wavelength perturbations, but this is not a trivial task (Strauss et al. 1995; Tormen & Bertchinger 1996). We first followed the method of Strauss et al. (1995) and computed the additional contribution to the bulk flow from the long-wavelength perturbations larger than the simulation box size by adding random phase Fourier components in Fourier space using the linear theory equations. In Figure 2, we show the distributions of the simulated bulk flow before and after this process, calculated with the grouped galaxy velocities. The raw simulated bulk flow is shown by the short-dashed histogram. The solid histogram is the one after the addition of the random Fourier components. The dotted histogram is obtained by simply multiplying the raw simulated bulk flow by the numerical factors of 1.2 ($R = 5 \ h^{-1} \ Mpc$), 1.25 ($R = 10 \ h^{-1} \ Mpc$), and 1.4 ($R = 20 \ h^{-1} \ Mpc$). The smooth curves are the “eyeball” fits to the histograms by a Maxwellian distribution. All histograms show a good fit to the Maxwellian distribution except that the raw simulated histogram of the $R = 5 \ h^{-1} \ Mpc$ has a longer tail than the Maxwellian.

We find that the change in the distribution is fairly well approximated by simply multiplying a numerical factor to the raw simulated bulk flow. We also confirm that the distribution does not change very much on the $\mathcal{M}$-$V$ plane when the random Fourier components of the bulk flow are added. Another thing is that the method of Strauss et al. (1995) is explicitly dependent on the normalization of the power spectrum. On the other hand, if we simply take the ratio of the two rms Mach numbers calculated with the full $P(k)$ and the cutoff $P(k)$ (Fig. 1, the two solid or dashed lines) and use this ratio, we can correct the bulk flow being independent of the normalization of the power spectrum, though within the limitation of using the equations of the linear theory.

For these reasons, we choose to correct for the lack of long-wavelength perturbations in the latter manner, as it is sufficient for our purpose. The ratios of the two solid lines [COSMICS $P(k)$ case] in Figure 1 are 1.43 ($R = 5 \ h^{-1} \ Mpc$), 1.56 ($R = 10 \ h^{-1} \ Mpc$), and 1.96 ($R = 20 \ h^{-1} \ Mpc$). For the dashed lines [nonlinear $P(k)$ case], the ratios are slightly smaller: 1.30 ($R = 5 \ h^{-1} \ Mpc$), 1.50 ($R = 10 \ h^{-1} \ Mpc$), and 1.80 ($R = 20 \ h^{-1} \ Mpc$). These factors are larger than the factors obtained by adding the random Fourier components. However, even if these correction factors turned out to be overestimates, our conclusion would be strengthened in that case, because our corrected $\mathcal{M}$ are still well below the observed $\mathcal{M}$. Hereafter, we adopt the correction factors of 1.43, 1.56, and 1.96 for the $R = 5, 10,$ and $20 \ h^{-1} \ Mpc$ cases, respectively.

4. METHOD OF CALCULATION OF $V, \sigma$, AND $\mathcal{M}$

In this section, we describe how we calculate the bulk flow, the velocity dispersion, and the cosmic Mach number from our simulation. We explore various options of calculations to see if they cause any difference in $\mathcal{M}$. We are also interested in the difference in $\mathcal{M}$ of different tracers of the velocity field.

There are many ways one can place the patches in the simulation. One also has to decide whether to use the particle-based ungrouped data set or to apply a grouping algorithm and identify galaxies and dark matter halos. Here, we consider the following cases:

1. Particle based:
   a) Centered on grouped galaxies: use galaxy particles (gal-pt),
   b) Centered on grouped galaxies: use DM particles (dm-galctr-pt),
   c) Centered on grouped DM halos: use DM particles (dm-dmclt-prt);

2. Group based:
   a) Centered on grouped galaxies: use grouped galaxy velocity (gal-gp),
   b) Centered on grouped DM halos: use grouped DM halo velocity (dm-gp).

We first identify galaxies and DM halos in the simulation using the HOP grouping algorithm (Eisenstein & Hut 1998). Using a set of standard parameters [$N_{\text{galn}}, N_{\text{hop}}, N_{\text{merge}}, (\delta_{\text{peak}}, \delta_{\text{saddle}}, \delta_{\text{outer}}) = (64, 16, 4)(240, 200, 80)$, we obtain 8601 galaxies and 9554 DM halos in the simulation box. To select out dynamically stable objects as the centers of the patches, we pick objects that occupy more than two cells in the simulation and those that satisfy the criteria of $M_{\text{group}} \geq 3 \times 10^{16} \ h^{-1} \ M_{\odot} \sigma_{\text{in}}^{3/2}$, where $M_{\text{group}}$ is the mass of the grouped object and $\sigma_{\text{in}}$ is the internal velocity dispersion in units of kilometers per second. This cutoff is motivated by looking at Figure 3, where grouped objects that occupy more than two cells in the simulation are shown. DM halos are not affected by the latter cutoff. We have confirmed that...
the results are robust to this pruning. We are left with 1585 galaxies and 4142 DM halos after this pruning. Changing the grouping parameters certainly affects the number of objects, which in turn affects the estimate of the velocity dispersion. Without grouping, for example, the velocity dispersion would be overestimated, as it would include the internal motions of particle in each object. However, Eisenstein & Hut (1998) showed that the sample is quite stable to the choice of parameters, so this effect is likely to be small. But this is an unavoidable numerical uncertainty, and one should keep this in mind upon reading the results below.

For the particle-based calculation, we calculate the bulk flow \( V \) and the velocity dispersion \( \sigma \) for each top-hat patch in a mass-weighted manner: 
\[
V = \frac{\sum_j m_{p,j} v_{p,j}}{\sum_j m_{p,j}} \quad \text{and} \quad \sigma^2 = \frac{\sum_j m_{p,j} (v_{p,j} - V)^2}{\sum_j m_{p,j}},
\]
where \( v_{p,j} \) is the particle velocity and \( m_{p,j} \) the mass of the jth object. The sum \( \sum_j \) is over all particles in the spherical top-hat patch of a given radius.

For the group-based calculation, we need to calculate the mean velocity of each group, which in turn affects the estimate of the velocity dispersion. We call this velocity \( v_\text{gp} \) the galaxy velocity or the DM halo velocity. We then place spherical top-hat patches of radius \( R = 5, 10, \) and \( 20 \) \( h^{-1} \) Mpc at the centers of the grouped objects and calculate \( V \) and \( \sigma \) for each patch using the objects’ velocity \( v_i \) for galaxies and DM halos separately: 
\[
V = \frac{1}{N} \sum_i v_i \quad \text{and} \quad \sigma^2 = \frac{1}{N(N-1)} \sum_{i,j} (v_i - V)^2,
\]
where \( N \) is the number of objects in the patch and the sum \( \sum_{i,j} \) is over all the objects in the patch. Note that we do not weight by the mass in the group-based calculation to mimic the real observations of galaxies. Periodic boundary conditions are used for all the calculations.

All the calculations are done in real space as it is more straightforward than doing it in Fourier space. We did not smooth the velocity field prior to these calculations. The effect of the smoothing is discussed in OS90, where they noted that the non-zero smoothing length simply increases the theoretical prediction of \( M \) compared with the non-smoothed case. This is obvious because smoothing would erase the velocity dispersion on scales smaller than the smoothing length. Here, our intention is not to erase the small-scale dispersion by the smoothing but rather to observe it as a function of local overdensity.

5. RESULTS

5.1. Mean and rms of \( V, \sigma, \) and \( M \)

From the above calculations, we now have \( V \equiv |V| \) and \( \sigma \) for each patch. We can now calculate the mean and rms Mach number following equations (3) and (4) for both galaxies and DM. We summarize the results in Table 1.

The standard deviation (SD) is indicated to show the typical uncertainty associated with the calculation of the mean in each case, although the error in the mean is not exactly same as the SD. The mean of all trials is shown at the bottom of the table. One immediately sees that \( \langle V^2 \rangle / \langle \sigma^2 \rangle^{1/2} < (\sigma / \langle V \rangle) \langle \sigma^2 \rangle^{1/2} \). If one were to assume a Gaussian distribution for \( M \), the standard deviation of the mean is SD/\( \sqrt{N} \approx 0.04 \), where, for \( R = 10 \) and \( 20 \) \( h^{-1} \) Mpc cases, \( N \) is the number of independent spheres that fit in the simulation box. However, we will show in the next section that, for \( R = 5 \) and \( 10 \) \( h^{-1} \) Mpc, the distribution of \( M \) is not well described by a Gaussian, so SD/\( \sqrt{N} \) is not the correct error in these cases.

The trend in the simulated Mach number is as follows: 
\[
M_{\text{dm-pt}} < M_{\text{gal-pt}} < M_{\text{gal-ep}} < M_{\text{dm-ep}},
\]
where the subscripts indicate the different methods of calculation as explained in § 4. The term "dm-pt" refers to both "galctr" and "dmctr" cases of the particle-based DM calculations. For the particle-based calculations, the Mach numbers using different centers and velocity tracers tend to converge on large scales. In the group-based calculation, the difference between \( M_{\text{gal}} \) and \( M_{\text{DM}} \) is apparent. We have confirmed that the same trend is observed in our new \( L_{\text{box}} = 25 \) \( h^{-1} \) Mpc simulation as well when the same calculation is performed with a \( 5 \) \( h^{-1} \) Mpc top-hat patch.

To understand where these differences in \( M \) arise, we summarize the mean and the rms value of \( V \) and \( \sigma \) in Tables 2 and 3. From these two tables and the same calculation with the \( L_{\text{box}} = 25 \) \( h^{-1} \) Mpc on \( R = 5 \) \( h^{-1} \) Mpc, the robust trends that we see on scales \( R \gtrsim 5 \) \( h^{-1} \) Mpc are the following: 
\[
V_{\text{gal-ep}} < V_{\text{particle based}} < V_{\text{dm-ep}} \quad \text{and} \quad \sigma_{\text{group based}} < \sigma_{\text{gal-ep}} < \sigma_{\text{dm-ep}} < \sigma_{\text{dm-gal-ep}},
\]
Differences within \( \sim 20 \) \( \text{km s}^{-1} \) are statistically insignificant, but there are some cases where the difference amounts to \( \sim 60 \) \( \text{km s}^{-1} \), although still within 1 standard deviation. We have also carried out the same calculations on the \( R = 1 \) \( h^{-1} \) Mpc scale and find that the first inequality of the bulk flow shown above does not hold in both \( L_{\text{box}} = 25 \) and 100 \( h^{-1} \) Mpc simulations. Also, for all cases of \( L_{\text{box}} = 100 \) \( h^{-1} \) Mpc, we find \( \sigma_{\text{dm-ep}} < \sigma_{\text{gal-ep}}, \) but the opposite relation is found in our new \( L_{\text{box}} = 25 \) \( h^{-1} \) Mpc on the scale of \( R = 5 \) \( h^{-1} \) Mpc.

The difference in the bulk flow between the particle-based and group-based calculations can be ascribed to the way it was calculated. In the particle-based calculation, we weighted each particle velocity by its particle mass, but in
the cases of the group-based calculations, we put equal weight on each galaxy or DM halo to mimic the real observation. We confirmed that, if we weight by the object’s mass in the group-based calculation, the bulk flows reduced to the same values as the mass-weighted particle-based calculations.

For the velocity dispersion, it is natural to see that $\sigma_{\text{group based}} < \sigma_{\text{particle based}}$, as the internal velocity dispersion increases with the group mass.

### Table 1

| Case                                                                 | $R = 5\ h^{-1}\ \text{Mpc}$ | $R = 10\ h^{-1}\ \text{Mpc}$ | $R = 20\ h^{-1}\ \text{Mpc}$ |
|----------------------------------------------------------------------|-------------------------------|-------------------------------|-------------------------------|
|                                                                      | $\langle V^2 \rangle^{1/2}$   | $\langle V^2 \rangle^{1/2}$   | $\langle V^2 \rangle^{1/2}$   |
|                                                                      | $\langle \sigma \rangle$      | $\langle \sigma \rangle$      | $\langle \sigma \rangle$      |
|                                                                      | SD                            | SD                            | SD                            |
| Particle based:                                                      |                               |                               |                               |
| a) gal-pt ....................... | 1.40                          | 1.92                          | 2.39                          |
|                                                                      | 0.95                          | 1.12                          | 1.33                          |
|                                                                      | 0.45                          | 0.69                          | 0.71                          |
|                                                                      | 0.18                          | 0.18                          | 1585                          |
| b) dm-galctr-pt.............. | 1.14                          | 1.46                          | 1.72                          |
|                                                                      | 0.90                          | 1.06                          | 1.22                          |
|                                                                      | 0.39                          | 0.71                          | 0.82                          |
|                                                                      | 0.17                          | 0.17                          | 1585                          |
| c) dm-dmctr-pt..............  | 1.16                          | 1.76                          | 2.67                          |
|                                                                      | 0.89                          | 1.14                          | 1.33                          |
|                                                                      | 0.43                          | 0.69                          | 0.76                          |
|                                                                      | 0.18                          | 0.18                          | 4142                          |
| Group based:                                                         |                               |                               |                               |
| a) gal-gp ....................... | 1.43                          | 2.07                          | 2.77                          |
|                                                                      | 1.05                          | 1.25                          | 1.47                          |
|                                                                      | 0.50                          | 0.76                          | 0.88                          |
|                                                                      | 0.19                          | 0.19                          | 1585*                         |
| b) dm-gp ....................... | 1.72                          | 2.69                          | 3.62                          |
|                                                                      | 1.29                          | 1.56                          | 1.78                          |
|                                                                      | 0.55                          | 1.00                          | 1.14                          |
|                                                                      | 0.22                          | 0.22                          | 4142*                         |
| Mean of all ................. | 1.48                          | 2.36                          | 3.11                          |
|                                                                      | 1.06                          | 1.29                          | 1.52                          |
|                                                                      | 0.77                          | 0.81                          | 0.89                          |

Notes.—Mean and the rms value of the cosmic Mach number are summarized. SD stands for standard deviation. $N_{\text{patch}}$ is the number of patches that were eligible in each analysis (we rejected those patches that contained only one galaxy). All numbers shown are after the multiplication by the factors of 1.43 ($R = 5\ h^{-1}\ \text{Mpc}$), 1.56 ($R = 10\ h^{-1}\ \text{Mpc}$), and 1.96 ($R = 20\ h^{-1}\ \text{Mpc}$) to correct for the underestimation of the bulk flow due to the limited size of the simulation box (see § 3). All these numbers, the standard deviation of the mean ($\sigma/\sqrt{N}$), is $\leq 0.04$ if one were to assume a Gaussian distribution. However, we show in § 5.2 that the distribution is not well described by a Gaussian. For the $R = 10$ and $20\ h^{-1}\ \text{Mpc}$ cases, the uncertainty is dominated by the cosmic variance, i.e., the number of independent spheres that fit in the simulation box. (See § 4 for discussion.)

### Table 2

| Case                                                                 | $R = 5\ h^{-1}\ \text{Mpc}$ | $R = 10\ h^{-1}\ \text{Mpc}$ | $R = 20\ h^{-1}\ \text{Mpc}$ |
|----------------------------------------------------------------------|-------------------------------|-------------------------------|-------------------------------|
|                                                                      | $\langle V \rangle$           | $\langle V^2 \rangle^{1/2}$   | $\langle V \rangle$           |
|                                                                      | SD                            | SD                            | SD                            |
| Particle based:                                                      |                               |                               |                               |
| a) gal-pt ....................... | 425                           | 480                           | 157                           |
|                                                                      | 384                           | 432                           | 127                           |
|                                                                      | 343                           | 380                           | 84                            |
| b) dm-galctr-pt.............. | 419                           | 469                           | 148                           |
|                                                                      | 393                           | 437                           | 121                           |
|                                                                      | 370                           | 404                           | 82                            |
| c) dm-dmctr-pt..............  | 472                           | 528                           | 165                           |
|                                                                      | 435                           | 480                           | 131                           |
|                                                                      | 394                           | 427                           | 85                            |
| Group based:                                                         |                               |                               |                               |
| a) gal-gp ....................... | 415                           | 470                           | 155                           |
|                                                                      | 368                           | 417                           | 124                           |
|                                                                      | 325                           | 357                           | 76                            |
| b) dm-gp ....................... | 493                           | 551                           | 172                           |
|                                                                      | 451                           | 496                           | 133                           |
|                                                                      | 408                           | 439                           | 83                            |

Notes.—Mean and the rms value of the bulk flow in the simulation is summarized. SD stands for standard deviation. All numbers are in units of kilometers per second. Cases correspond to those in Table 1. All numbers except the SD are after the multiplication by the factors of 1.43 ($R = 5\ h^{-1}\ \text{Mpc}$), 1.56 ($R = 10\ h^{-1}\ \text{Mpc}$), and 1.96 ($R = 20\ h^{-1}\ \text{Mpc}$) to correct for the underestimation due to the limited size of the simulation box (see § 3). Discussion is in § 5.1.

### Table 3

| Case                                                                 | $R = 5\ h^{-1}\ \text{Mpc}$ | $R = 10\ h^{-1}\ \text{Mpc}$ | $R = 20\ h^{-1}\ \text{Mpc}$ |
|----------------------------------------------------------------------|-------------------------------|-------------------------------|-------------------------------|
|                                                                      | $\langle \sigma \rangle$      | $\langle \sigma^2 \rangle^{1/2}$ | $\langle \sigma \rangle$      |
|                                                                      | SD                            | SD                            | SD                            |
| Particle based:                                                      |                               |                               |                               |
| a) gal-pt ....................... | 289                           | 342                           | 182                           |
|                                                                      | 404                           | 454                           | 208                           |
|                                                                      | 521                           | 553                           | 184                           |
| b) dm-galctr-pt.............. | 356                           | 410                           | 204                           |
|                                                                      | 432                           | 483                           | 215                           |
|                                                                      | 533                           | 571                           | 207                           |
| c) dm-dmctr-pt..............  | 369                           | 455                           | 267                           |
|                                                                      | 465                           | 538                           | 271                           |
|                                                                      | 572                           | 620                           | 239                           |
| Group based:                                                         |                               |                               |                               |
| a) gal-gp ....................... | 274                           | 329                           | 182                           |
|                                                                      | 349                           | 398                           | 190                           |
|                                                                      | 434                           | 466                           | 171                           |
| b) dm-gp ....................... | 263                           | 322                           | 186                           |
|                                                                      | 339                           | 385                           | 183                           |
|                                                                      | 415                           | 441                           | 149                           |

Notes.—Mean and the rms value of the velocity dispersion $\sigma$ in the simulation are summarized. SD stands for standard deviation. All numbers are in units of kilometers per second. Cases correspond to those in Table 1. See § 5.1 for discussion.
is erased by the grouping. Also, we expect to see $\sigma_{\text{gal-pt}} < \sigma_{\text{dm-pt}}$, as galaxy particles have formed out of sticky gaseous material compared to collisionless DM particles.

So we regard the following relations as the most robust trends observed in our simulations: (1) $V_{\text{dm-gp}} > V_{\text{gal-gp}}$, and $V_{\text{dm-gp}}$ is always larger than any other case (only for non-mass-weighted calculations); (2) $\sigma_{\text{group based}} < \sigma_{\text{particle based}}$; and (3) $\mathcal{M}_{\text{group based}} > \mathcal{M}_{\text{particle based}}$.

To summarize, our calculations show that the different methods of calculation result in different values of bulk flow and velocity dispersion, hence different Mach numbers as well. We find that the grouping affects the resulting Mach number. However, the differences in the simulated $\mathcal{M}$ are smaller than the discrepancies between the simulated and the observed $\mathcal{M}$, so they are not significant enough to change the arguments to follow.

5.2. Distribution of $V$, $\sigma$, and $\mathcal{M}$

One would like to understand how the observed $\mathcal{M}$ compares with the distribution of the simulated $\mathcal{M}$ and how it arises from the distribution of $V$ and $\sigma$.

Theoretically, bulk flow is expected to follow a Maxwellian distribution. In Figure 2, we have already shown that the simulated bulk flow can be described by a Maxwellian distribution fairly well. The distribution of velocity dispersion is nontrivial. In Figure 4, we show the distribution of the simulated $\sigma$ of the grouped galaxies ("gal-gp" case). The three vertical dashed lines in each panel are the median, the mean, and the rms values of the distribution. The solid curves are the eyeball fits to a Maxwellian distribution. For the $R = 5\ h^{-1}\ Mpc$ case, it is fitted to the Maxwellian relatively well except the longer tail at large values of $\sigma$. For the $R = 10$ and $20\ h^{-1}\ Mpc$ cases, the distribution is not well characterized by the Maxwellian. The simulated distribution has a steeper cutoff at low values.

In Figure 5, we show the distribution of the simulated $\mathcal{M}$ of the grouped galaxies ("gal-gp" case). The smooth solid curves show the eyeball fits to a Maxwellian distribution. For the $R = 5$ and $10\ h^{-1}\ Mpc$ cases, the simulated Mach number distribution has a longer tail than does the Maxwellian distribution. At the scale of $R = 20\ h^{-1}\ Mpc$, the distribution is well fitted by the Maxwellian distribution. For all other methods of calculation listed in Table 1, we find the same qualitative behavior. We note that Suto & Fujita (1990) have argued that the Mach number is distributed slightly broader than a Maxwellian, consistent with our result. The three vertical dotted lines in each panel are, from left to right, $\langle V^2/\sigma^2 \rangle^{1/2}$, $\langle \mathcal{M} \rangle$, and $\langle V^2/\sigma^2 \rangle$ as summarized in Table 1. Because of the long tail in the distribution for the $R = 5$ and $10\ h^{-1}\ Mpc$ cases, the rms Mach number and the mean $\langle \mathcal{M} \rangle$ do not reflect the peak of the distribution well. The dashed lines on the right show the observed $\mathcal{M}$, which will be summarized in the next section. The observed $\mathcal{M}$ is higher than the mean $\langle \mathcal{M} \rangle$ by more than 2 standard deviations at the 92%, 94%, and 71% confidence levels for the $R = 5, 10$, and $20\ h^{-1}\ Mpc$ cases, respectively.

How does this unusually high $\mathcal{M}$ arise from the distribution of $V$ and $\sigma$? In Figure 6, we show the number density distribution of the simulated top-hat patches on the $\mathcal{M}$-$\sigma$ and $\mathcal{M}$-$V$ planes for the patch sizes of $R = 5, 10$, and $20\ h^{-1}\ Mpc$ (group-based calculations). Contours are of the number density distribution of the simulated sample on an equally spaced logarithmic scale. Overall, as the patch size $R$ increases, the bulk flow decreases and the velocity disper-

![Fig. 4](image_url)

**Fig. 4.** Distribution of the velocity dispersion of the simulated galaxies ("gal-gp" case) for $R = 5, 10$, and $20\ h^{-1}\ Mpc$, from top to bottom. The smooth solid curves show the eyeball fits to the Maxwellian distribution. The three vertical dotted lines in each panel are, from left to right, the median, the mean, and the rms value of the distribution. For $R = 5\ h^{-1}\ Mpc$, the simulated distribution has a longer tail than the Maxwellian. For $R = 10$ and $20\ h^{-1}\ Mpc$, the distribution has a sharper cutoff at low values and is not well described by the Maxwellian.

![Fig. 5](image_url)

**Fig. 5.** Mach number distribution of the simulated galaxies ("gal-gp") for $R = 5, 10$, and $20\ h^{-1}\ Mpc$ from top to bottom. This is after the bulk flow correction for the lack of long-wavelength perturbations. The smooth solid curves show the eyeball fits to the Maxwellian distribution. The three vertical dotted lines in each panel are, from left to right, $\langle V^2/\sigma^2 \rangle^{1/2}$, $\langle \mathcal{M} \rangle$, and $\langle V^2/\sigma^2 \rangle$, as summarized in Table 1. The observed $\mathcal{M}$ (dashed line) is higher than the simulated mean $\langle \mathcal{M} \rangle$ by more than 2 standard deviations at 92%, 94%, and 71% confidence levels for the $R = 5, 10$, and $20\ h^{-1}\ Mpc$ cases, respectively. See § 5.2 for discussion.
sion increases as we already saw in Figures 2 and 4. This is what we naively expect in the Friedman universe: \( V \) is a monotonically decreasing function of \( R \) approaching zero as the largest irregularities are smoothed over, and \( \sigma \) grows monotonically, saturating at the scale where \( V \) has leveled off, at the same value that \( \sigma \) had on small scales (OS90). One can also see that the distribution of \( M \) shifts down as the patch size \( R \) increases. This can be seen more clearly in Figures 1 and 5.

The gray strips in Figure 6 are the “best-guess” ranges of \( V \) and \( \sigma \) based on observations. We take \( V = 500-700 \) km s\(^{-1}\) and \( \sigma = 100-160 \) km s\(^{-1}\) for \( R = 5 \) h\(^{-1}\) Mpc, \( V = 350-550 \) km s\(^{-1}\) and \( \sigma = 150-250 \) km s\(^{-1}\) for \( R = 10 \) h\(^{-1}\) Mpc, and \( V = 350-550 \) km s\(^{-1}\) and \( \sigma = 250-350 \) km s\(^{-1}\) for \( R = 20 \) h\(^{-1}\) Mpc based on the observed range of values summarized at the end of the next section. These ranges correspond to \( M = 4.6 \pm 1.3 \) (\( R = 5 \) h\(^{-1}\) Mpc), \( 2.3 \pm 0.8 \) (\( R = 10 \) h\(^{-1}\) Mpc), and \( 1.5 \pm 0.4 \) (\( R = 20 \) h\(^{-1}\) Mpc). The tilted strips naturally arise from the definition of the Mach number once we fix the value of either \( V \) or \( \sigma : M \propto 1/\sigma \) or \( \propto V \). Note that the overlapping region of the two strips is off the peak of the entire distribution, as we already saw in Figure 5. This offset is mainly caused by the observed low-velocity dispersion.

In Figure 6, the abscissa and the ordinate are not independent of each other because of the definition of the Mach number. To show the independent quantities on both axes, we show the bulk flow against the velocity dispersion of the simulated sample of grouped galaxies and DM halos for the cases of \( R = 5 \) and \( 10 \) h\(^{-1}\) Mpc in Figure 7. There is a slight hint of positive correlation between the two quantities, but otherwise, they seem to be decoupled. The gray strips are the same as in Figure 6.

We further discuss the implication of the observed high Mach number of the Local Group in the next section by turning our eye to the local overdensity.

6. \( V, \sigma, \) AND \( \mathcal{M} \) AS FUNCTIONS OF OVERDENSITY

In this section, we study the correlations between \( V, \sigma, \mathcal{M}, \) and local overdensity \( \delta \). We calculate \( \delta \) at all sampling points using spherical top-hat patches of the same sizes that we used in calculating \( V, \sigma, \) and \( \mathcal{M} \). For DM particles, we simply add the mass of all the particles in the patch and divide by the total mass in the simulation box to obtain the local mass overdensity \( \delta_{\text{DM}} \). For galaxies, we use the updated isochrone synthesis model GISSEL99 (see Bruzual & Charlot 1993) to obtain the absolute luminosity in \( V \) band and calculate the luminosity overdensity \( \delta_{\text{LV}} \) in the same manner as the mass overdensity. The GISSEL99 model takes the metallicity variation into account. Comparison of this simulation with various observations in terms of “light” is done by Nagamine et al. (2000) in detail. The use of luminosity overdensity is not absolutely necessary here, and one should get the same conclusions as presented in this paper even if one uses the mass overdensity of galaxies, since both overdensities roughly follow each other. We could in principle incorporate dust extinction by using a simple model, but that is a minor detail that would not change our conclusions in a qualitative manner.

In Figure 8, we show \( V \) and \( \sigma \) as functions of \( \delta \) on scales of \( R = 5, 10, \) and \( 20 \) h\(^{-1}\) Mpc, respectively (group-based
calculation). The contour levels are the same as before. Again, the gray strips indicate the same best-guess range based on the observations, as already described in the previous section. An important feature to note here is that \( \sigma \) and \( \delta \) are strongly correlated with each other, while \( V \) and \( \sigma \) are not. Velocity dispersion is an increasing function of overdensity. This correlation between \( \delta \) and \( \sigma \) is similar to that seen in the case of \( \sigma_{12} \), as described in § 1. In the case of \( \sigma \), it is weighted by the pairs always taken relative to the center-of-mass velocity (bulk flow \( V \)) of the patch, whereas in the case of \( \sigma_{12} \), one takes all possible pairs in the patch. The solid line running through the contour in the \( \sigma-\delta \) plot indicates the median of the sample in each bin of overdensity. Willick & Strauss (1998) studied the small-scale velocity dispersion in the observed data under the assumption of a linear relation between \( \sigma \) and \( \delta \), but our calculation predicts a shallower power-law dependence of \( \sigma \propto \delta^{0.3-0.5} \) on all scales, with the power index being larger at larger \( \delta \).

We then plot \( \mathcal{M} \) against \( \delta \) in Figure 9 on scales of \( R = 5, 10, \) and \( 20 \h^{-1} \text{Mpc} \) (group-based calculation). The contour levels are the same as before. The cosmic Mach number is a weakly decreasing function of overdensity. This correlation between \( \mathcal{M} \) and \( \delta \) originates from that between \( \delta \) and \( \sigma \). Roughly speaking, low overdensity suggests low \( \sigma \) and large \( \mathcal{M} \). Therefore, the observed high \( \mathcal{M} \) of the Local Group compared to the mean suggests that the Local Group is likely to be located in a relatively low overdensity region if our model is correct. We note that van de Weygaert & Hoffman (2000) reach a similar conclusion by simulating the Local Group using constrained initial conditions. However, it is also important to note that a given \( \mathcal{M} \) does not correspond to a single value of \( \delta \) due to both the weakness of the correlation and the significant scatter around the median, which is indicated by the solid line.

The dotted vertical line in the \( R = 10 \h^{-1} \text{Mpc} \) panel in Figure 9 indicates that \( 1 + \delta_{\text{IRAS}} = 1.2 \), which is the observed IRAS galaxy number overdensity at the Local Group (Strauss & Willick 1995; it was calculated with an \( R = 5 \h^{-1} \text{Mpc} \) Gaussian window, which corresponds to an \( R = 5 \sqrt{3} = 11.2 \h^{-1} \text{Mpc} \) top-hat window). It shows that the Local Group is off the peak of the distribution for galaxies, supporting our statement. The fact that the IRAS survey samples only star-forming galaxies that tend to reside in low-density regions is not so important here since it is only an issue in the center of clusters.

To illustrate the above point more clearly, we divide the simulated galaxy sample into quartiles of local overdensity and calculate \( \langle \mathcal{M} \rangle \) for each quartile. In Figure 10, the three crosses at each scale are the mean of each quartile of the grouped galaxies (top: first quartile, lowest \( \delta \); bottom: fourth quartile, highest \( \delta \); middle: total sample). Note that the galaxies in low-density regions have higher \( \mathcal{M} \). We will
500 TF spiral galaxies by Tully & Pierce (2000) finds \( V \approx 400 \pm 100 \) km s\(^{-1}\). Taking \( \sigma = 300 \pm 50 \) km s\(^{-1}\) as a typical value, one obtains \( \mathcal{M}(R = 30 \ h^{-1} \text{ Mpc}) = 1.3 \pm 0.4 \) (open circle), exactly the same as the previous estimate by OS90 on the same scale. The IRAS Point Source Catalog Redshift Survey (PSCz) gives \( V = 475 \pm 75 \) km s\(^{-1}\) (Saunders et al. 2001) using linear theory. Again, assuming \( \sigma = 300 \pm 50 \) km s\(^{-1}\) yields \( \mathcal{M}(R = 20 \ h^{-1} \text{ Mpc}) = 1.6 \pm 0.4 \) (open triangle). The Mach numbers from these new surveys seem to confirm that the observed \( \mathcal{M} \) is larger than the SCDM prediction, as originally pointed out by OS90. Bulk flows from other surveys on scales larger than \( R = 30 \ h^{-1} \text{ Mpc} \) are summarized in Dekel (2000).

Although we made new estimates of the Mach number on scales \( R \gtrsim 20 \ h^{-1} \text{ Mpc} \), these numbers should be regarded as tentative since the observed bulk flow on large scales still seems uncertain in the literature (see Courteau et al. 2000; Dekel 2000). But if these estimates are correct, we consider that the high observed \( \mathcal{M} \) reflects the fact that the Local Group is located in a relatively low density region, as we argued earlier in this section.

Another possibility to resolve the discrepancy between the simulated \( \langle \mathcal{M} \rangle \) and the observed \( \mathcal{M} \) is that the real universe has a lower mass density than the simulated value of \( \Omega_m = 0.37 \). We find in Figure 10 that the \( \Omega_m = 0.2 \) line fits all the observational estimates very well. If indeed \( \Omega_m = 0.2 \), the observed low-velocity dispersion of galaxies and the high Mach number would be typical in such universes.

One might wonder if our result would be significantly altered were the power spectrum to be steepened by one of the various mechanisms being proposed to solve the putative problems of the CDM paradigm on small scales (e.g., Dalcanton & Hogan 2000). We explored one typical such variant, the warm dark matter proposal, and found that for a particle mass in the permitted range \( (\gtrsim 1 \text{ keV}; \text{see Narayanan et al. 2000; Bode, Ostriker, & Turok 2000}) \) the effect on the expected Mach number is negligible because the turndown in the power spectrum occurs at such a high wavenumber as to be unimportant on patch sizes greater than \( 1 \ h^{-1} \text{ Mpc} \).

7. Correlations Between Galaxy Age, Overdensity, and Mach Number

7.1. General Expectations

Under the standard picture of hierarchical structure formation, larger systems form from mergers of small objects. Therefore, one naively expects that the DWARF galaxies that exist in the present-day universe will be the “leftovers” in the low-density regions and the GIANTS will be located in high-density regions where DWARFs gathered to form GIANTS. (We denote DWARFs and GIANTS in capital letters because we will symbolically divide our galaxy sample in the simulation into two subsamples by their stellar mass.) However, DWARFs that are about to merge into larger systems could exist in high-density regions as well.

Now, let us define the formation time of a system in the simulation by the mass-weighted mean of the formation time of the consisting galaxy particles. Larger systems are the assembly of smaller systems that formed earlier, so for GIANTS, the larger the system is, the older the formation time would be. (Note that we are using the terms “young” and “old” relative to the present, i.e., young \( \equiv 0.2 \), but note that they have adopted a modified definition of \( \mathcal{M} \); \( \mathcal{M}(R = 14 \ h^{-1} \text{ Mpc}) = 1.03 \) from 206 galaxies in the infrared Tully-Fisher (TF) spiral galaxy catalog of Aaronson et al. (1982), and \( \mathcal{M}(R = 25 \ h^{-1} \text{ Mpc}) = 0.57 \) from 385 galaxies in the \( D_s-\sigma \) elliptical galaxy catalog of Faber et al. (1989). A more recent sample is the surface brightness fluctuation (SBF) survey of 300 elliptical galaxies by Tonry et al. (2000). They find \( V \approx 300 \pm 150 \) km s\(^{-1}\) and \( \sigma = 312 \pm 24 \) km s\(^{-1}\) at the scale of \( R = 30 \ h^{-1} \text{ Mpc} \), which yields \( \mathcal{M}_{\text{SBF}}(R = 30 \ h^{-1} \text{ Mpc}) = 0.96 \pm 0.5 \) (open pentagon). The recent survey of
smaller $z_{\text{form}}$) DWARFs do not follow this trend, because some of the smallest DWARFs formed at very high redshift will remain as they are without merging into larger systems. Therefore, they are the oldest population by definition despite the fact that they are the smallest systems. This countereffect dilutes the correlation between age and local overdensity for the DWARF population. Systems in high-density regions have larger $\sigma$, hence smaller $M$, and vice versa. We summarize the above points in Figure 11. The three left boxes represent the DWARF galaxies divided in terms of the local overdensity of the region they live in. DWARFs live in both low- and high-overdensity regions (VOIDS and CLUSTERS), while GIANTS live in moderate to high overdensity regions (the two right boxes). We denote the intermediate overdensity region as FILAMENTS. The correlations with $\delta$, galaxy age, mass, $\sigma$, and $\mathcal{M}$ are indicated by the arrows in the figure.

7.2. Do We See the Effect in the Simulation?

To see the above effect in the simulation, we divide the simulated galaxy sample in the simulation into DWARFs and GIANTS at the median mass of $M_{\text{galaxy}} = 10^{10} \, h^{-1} \, M_\odot$, as shown in Figure 12. Note that the galaxies shown in this figure were taken from the $z = 0$ output of the simulation; therefore, GIANTS that formed at late times certainly include galaxy particles that formed very early on. We also divide each sample into quartiles by their formation time. The formation time of each galaxy is calculated as defined above and is converted to redshift ($z_{\text{form}}$). The boundary redshift of the quartiles are shown as the horizontal dashed lines in Figure 12, which are $z_{\text{boundary}} = 2.44, 3.53$, and 4.36 for DWARFs and 0.68, 082, and 1.05 for GIANTS. One sees from this figure that the trend is not as clear-cut as we naively expected above, though the basic line was correct. The formation time of DWARFs ranges widely, and the heavier DWARFs tend to be younger. Very small DWARFs form at very high redshift ($z_{\text{form}} \gtrsim 4$), and the moderate-size DWARFs continue to form through $z \sim 1$. GIANTS mainly form at moderate redshifts ($1 < z_{\text{form}} < 2$), when the global star formation rate is most active in the simulation (Nagamine et al. 2000). One sees that the very massive GIANTS have a slight positive slope as we expected above. But near the boundary of DWARF and GIANT, there are some less massive GIANTS that are older than the heavier GIANTS as well.

Now we discuss the correlation between overdensity, galaxy age, and the Mach number. In Figure 13, we show the formation time of galaxies as a function of DM mass overdensity $\delta_{\text{DM}}$ calculated with a top-hat patch of $R = 5 \, h^{-1} \, \text{Mpc}$. One sees that DWARFs exist in all environments with a weak positive correlation between $z_{\text{form}}$ and $\delta_{\text{DM}}$ and that some older (i.e., larger $z_{\text{form}}$) GIANTS tend to be in high-density regions more than less massive ones, as suggested in Figure 11. The horizontal dashed lines indicate the boundaries of the quartiles in terms of $z_{\text{form}}$. See §7 for discussion.

![Figure 12](image1.png)

**Fig. 12.** Mean formation time of the simulated galaxies at $z = 0$ (converted to redshift) vs. stellar mass of galaxies. Mean formation time of each galaxy was calculated by taking the mass-weighted average of the formation time of consisting galaxy particles. All galaxies in the simulation box are shown. The vertical line at $M_{\text{gal}} = 10^{10} \, h^{-1} \, M_\odot$ divides the sample into DWARFs and GIANTS. The horizontal dashed lines are the boundaries of the quartiles in terms of $z_{\text{form}}$. See §7 for discussion.

**Table 4**

| QUARTILE | DWARF | GIANT |
|----------|-------|-------|
|          | $\delta_{\text{el}}$ | $\delta_{\text{LV}}$ | $\delta_{\text{DM}}$ | $\delta_{\text{el}}$ | $\delta_{\text{LV}}$ | $\delta_{\text{DM}}$ |
| Young    | 3.45  | 3.21  | 2.41  | 0.81  | 1.37  | 0.43  |
| Second   | 4.73  | 4.13  | 3.44  | 1.48  | 1.94  | 0.75  |
| Third    | 4.69  | 4.11  | 3.30  | 2.32  | 2.53  | 1.25  |
| Old      | 4.91  | 4.16  | 3.67  | 5.73  | 4.85  | 4.00  |

**Notes.** Shown are the mean of the local overdensity for each quartile of the galaxy sample divided in terms of its age. Overdensity was calculated with a top-hat $R = 5 \, h^{-1} \, \text{Mpc}$ filter. Both $\delta_{\text{el}}$ and $\delta_{\text{DM}}$ were calculated in terms of their mass, and $\delta_{\text{LV}}$ is the luminosity overdensity calculated with absolute $V$-band luminosity. See §7 for discussion.
DWARF and GIANTS are less clustered and seem to well on scales of 1 of the oldest populations follow the power law close to the oldest GIANTS. The two correlation functions DWARFs are clustered second most strongly, but very oldest GIANTS are clustered most strongly, and the oldest Poisson error bars are shown together. As one expects, the GIANTS populations in the left panel of Figure 15. The oldest and the youngest quartiles of the DWARF and clearly, we show the two-point correlation function of the axies appear just by the ñlaments. GIANTS. Notice also the projection e†ect that some gal-

DWARFs and old GIANTS or young DWARFs and young But it is a little difficult to see the di†erence between the old galaxies is indicated by the solid points. One can clearly see that the older population is more clustered than the younger population for both DWARFs and GIANTS. Some old DWARF galaxies reside in low-density regions as well but it is a little difficult to see the difference between the old DWARFs and old GIANTS or young DWARFs and young GIANTS. Notice also the projection effect that some galaxies appear just by the ñilaments.

To see the di†erence in the clustering property more clearly, we show the two-point correlation function of the oldest and the youngest quartiles of the DWARF and GIANT populations in the left panel of Figure 15. The Poisson error bars are shown together. As one expects, the oldest GIANTS are clustered most strongly, and the oldest DWARFs are clustered second most strongly, but very close to the oldest GIANTS. The two correlation functions of the oldest populations follow the power law \( \xi = (5/r)^{1.8} \) well on scales of 1 \( h^{-1} \) Mpc < \( R < 10 h^{-1} \) Mpc. The youngest DWARFs and GIANTS are less clustered and seem to be consistent with each other on scales of 3 \( h^{-1} \) Mpc < \( R < 10 h^{-1} \) Mpc. The youngest GIANTS seem to have a weaker signal than the youngest DWARFs on scales less than 2 \( h^{-1} \) Mpc, but it is not clear if this is real or a numerical artifact.

In the right panel of Figure 15, we show the cumulative number fraction distribution of different galaxy populations as functions of local mass overdensity (calculated with a top-hat \( R = 1 h^{-1} \) Mpc window). One sees that older galaxies tend to reside in higher density regions than younger galaxies, consistent with the correlation function shown in the left panel. GIANTS prefer higher density regions than DWARFs.

Finally, let us look at the correlation between overdensity and the Mach number. The mean Mach numbers of young and old galaxies are summarized in Table 5. Here, “young” denotes the first two younger quartiles in galaxy age, and “old” denotes the two older quartiles. In all cases of GIANTS and of \( R = 5 h^{-1} \) Mpc DWARFs, the older sample has smaller \( M_M \) as expected. On larger scales (\( R \geq 5 h^{-1} \) Mpc), the patch starts to sample more DWARFs in low-density regions, and the naively expected trend turns over in the opposite direction for the DWARFs.

8. CONCLUSIONS

We have studied the bulk flow, the velocity dispersion, and the cosmic Mach number on scales of \( R = 5, 10, \) and 20 \( h^{-1} \) Mpc using an LCDM hydrodynamical simulation, putting emphasis on the environmental effects of the local overdensity and the correlation with galaxy age and size. Different methods of calculation and different definitions of \( M_M \) were tried out to see the differences in the results. We
found that $(\langle V^2 \rangle / \langle \sigma^2 \rangle)^{1/2} < \langle \dot{M} \rangle < (\langle V^2 / \sigma^2 \rangle)^{1/2}$ (Table 1) and that the different methods of calculation result in different values of bulk flow and velocity dispersion, hence different Mach numbers as well. We found that the grouping procedure affects the resulting Mach number significantly. However, the difference in $\dot{M}$ due to different methods of calculation is smaller than the discrepancy between the simulated and the observed $\dot{M}$; therefore, it is not significant enough to change our following conclusions.

We showed the distribution of the bulk flow, the velocity dispersion, and the Mach number in the simulation (Figs. 2, 4, and 5). The bulk flows are fitted by a Maxwellian distribution well except that the uncorrected $R = 5 \ h^{-1} \ Mpc$ case has a longer tail. The velocity dispersion is not well fitted by a Maxwellian; it has a longer tail for the $R = 5 \ h^{-1} \ Mpc$ case and a sharper cutoff at low values of $\sigma$ for the $R = 10$ and $20 \ h^{-1} \ Mpc$ cases. As a result, the Mach number is relatively well fitted by a Maxwellian, but with a longer tail for the $R = 5$ and $10 \ h^{-1} \ Mpc$ cases.

We discussed the theoretical predictions of $\dot{M}$ in § 3, including the scale and $\Omega_m$ dependence of $\dot{M}$. The range of the simulated Mach numbers falls just below the theoretical prediction (Fig. 1), reflecting the nonlinear evolution in the simulation, which cannot be fully taken into account by simply plugging the nonlinear power spectrum into the linear equations. We also discussed in § 3 how we corrected the simulated bulk flow for the lack of long-wavelength perturbations beyond the simulation box size.

The first main conclusion of this paper is that the observed velocity configuration of the Local Group is not the most typical one if the adopted LCDM cosmology is correct. Our calculations show that the observed Mach numbers are higher than the simulated mean by more than 2 standard deviations at high confidence levels (Fig. 5) and that the observed velocity configuration is off the peak of the number density distribution in the $\dot{M}$-$\sigma$ plane (§ 5.2; Fig. 6). This discrepancy is mainly due to the low observed velocity dispersion, while the observed bulk flow is not that uncommon.

Second, we showed that the cosmic Mach number is a weakly decreasing function of overdensity (Fig. 9). The correlation originates from that between overdensity and the velocity dispersion (Fig. 8). This is a similar situation to that of the pairwise velocity dispersion. Roughly speaking, a high Mach number suggests a low-density environment. It is important to take this overdensity dependence of $\dot{M}$ into account.

### Table 5: Galaxy Age Dependence of Mach Number

| Age   | $R = 5 \ h^{-1} \ Mpc$ | $R = 10 \ h^{-1} \ Mpc$ | $R = 20 \ h^{-1} \ Mpc$ |
|-------|------------------------|--------------------------|--------------------------|
|       | Median | Mean  | SDOM | Median | Mean  | SDOM | Median | Mean  | SDOM |
| DWARF:|        |       |      |        |       |      |        |       |      |
| Young | 1.63   | 3.00  | 0.14 | 1.11   | 1.44  | 0.06 | 0.74   | 0.78  | 0.04 |
| Old   | 1.53   | 3.00  | 0.15 | 1.19   | 1.54  | 0.06 | 0.82   | 0.88  | 0.04 |
| GIANT :|        |       |      |        |       |      |        |       |      |
| Young | 2.13   | 2.86  | 0.06 | 1.50   | 1.83  | 0.06 | 0.98   | 1.04  | 0.04 |
| Old   | 1.70   | 2.45  | 0.07 | 1.20   | 1.53  | 0.06 | 0.90   | 0.92  | 0.04 |

**NOTES.**—Shown are the mean, the median and the standard deviation of the mean (SDOM; $\sigma / \sqrt{N}$) of the Mach number for different populations and scales. For $R = 10$ and $20 \ h^{-1} \ Mpc$ cases, SDOM is limited by the independent number of spheres that fit in the simulation box. The values above are after the correction for the underestimation of the bulk flow due to the lack of long-wavelength perturbations in the simulation. See § 7 for discussion.
account in any analysis of cosmic Mach number or velocity dispersion, as is the case for the pairwise velocity dispersion.

Third, a few new observational estimates of $\mathcal{M}$ were made in this paper on scales of $R = 20$ and $30 \, h^{-1} \, \text{Mpc}$ (Fig. 10). They are much higher than the SCDM prediction, confirming the conclusions of earlier studies by other authors. Combined with our second point, the observed local high $\mathcal{M}$ is simply a reflection of the fact that the Local Group is in a relatively low overdensity region, as we know from the IRAS survey. Another possibility that resolves the discrepancy between the simulated and the observed $\mathcal{M}$ is that our universe has a much lower mass density than the simulated value of $\Omega_m = 0.37$. If so, the observed low $\mathcal{M}$ and high $\mathcal{M}$ would be typical in such universes. As we showed in Figure 10, the observed Mach numbers are in good agreement with the linear theory prediction with $\Omega_m = 0.2$. This may be interpreted as the local value of $\Omega_m$ being closer to 0.2 than the simulated value of 0.37. We also explored the possibility of the warm dark matter proposal and found that for a particle mass in the permitted range ($\gtrsim 1 \, \text{keV}$; see Narayanan et al. 2000; Bode et al. 2000) the effect on the expected Mach number is negligible on scales greater than $1 \, h^{-1} \, \text{Mpc}$.

Fourth, we studied the correlation between galaxy mass, galaxy age, local overdensity, and the Mach number. Our major points are summarized in Figure 11. Using the simulation, we showed that the older (redder) systems are strongly clustered in higher density regions with smaller $\mathcal{M}$ while younger (bluer) systems tend to reside in lower density regions with larger $\mathcal{M}$ (Figs. 13, 14, and 15 and Table 5), as expected from the hierarchical structure formation scenario. We divided the galaxy sample into DWARFs and GIANTS in the simulation and found that the GIANTS follow this expected trend while DWARFs deviate from this trend on large scales ($R > 5 \, h^{-1} \, \text{Mpc}$) due to the presence of the old DWARFs in low-density regions that did not merge into larger systems. The two-point correlation functions and the cumulative number fraction distributions of different populations were presented.

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APPENDIX A

GALAXY PARTICLE FORMATION CRITERIA IN THE SIMULATION

The criteria for galaxy particle formation in each cell of the simulation are

$$1 + \delta_{\text{tot}} > 5.5, \quad (A1)$$

$$m_{\text{gas}} > m_j \equiv G^{-3/2} \rho_b^{1/2} C_{\text{crit}} \left(1 + \frac{1}{\delta_d} \frac{\bar{\rho}_d}{\bar{\rho}_b} \right)^{-3/2}, \quad (A2)$$

$$t_{\text{cool}} < t_{\text{dyn}} \equiv \sqrt{\frac{3\pi}{32G\bar{\rho}_{\text{tot}}}}, \quad (A3)$$

$$\mathbf{v} \cdot \mathbf{n} < 0, \quad (A4)$$

where the subscripts $b$, $d$, and “tot” refer to baryons, collisionless dark matter, and the total mass, respectively. $C_{\text{crit}}$ in the definition of the Jeans mass is the isothermal sound speed. The cooling time is defined as $t_{\text{cool}} = n_e k_B T/\Lambda$, where $\Lambda$ is the cooling rate per unit volume in units of ergs s$^{-1}$ cm$^{-3}$. Other symbols have their usual meanings.

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