Autowave process of plastic flow localization

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Abstract. The localized plasticity development was studied for a wide range of pure metals and alloys in single-crystal and polycrystalline state as well as nonmetallic materials, alkaline halide crystals, ceramics and rocks. Using photographic and digital versions of speckle photography, the localization of plastic flow in the test samples was investigated on specially designed units having high spatial and temporal resolution. According to the analysis given here, the different stages of plastic flow are related both qualitatively and quantitatively.

1. Introduction
The kinetics of plastic flow in solids was studied experimentally from the yield point to failure. The results obtained are generalized in studies of Zuev et al. (2008, 2011, 2013, 2014) and Barannikova et al (2012). In the course of plastic flow development there is a strong tendency towards space-time inhomogeneity, which is a most complex problem one comes up against when dealing with the plastic deformation. The findings presented in this monograph give an answer to this problem. This view finds support in recent studies of Asharia et al. (2008) and Fressengeas et al. (2009).

Thus one of the intriguing features of plastic flow is spontaneous layering of material to cause the emergence of mobile or immobile localization nuclei on the deforming sample. Of particular interest is the localized plasticity pattern observed for the stage of linear work hardening. The work hardening coefficient for the latter stage, \( \theta \equiv E \times d\sigma / d\varepsilon = \text{const} \) (here \( \sigma \) and \( \varepsilon \) are stress and strain, respectively, and \( E \) is Young’s modulus). The space-time periodic plastic flow behavior was examined experimentally. The values obtained are \( \lambda \approx 10^2 \) m and \( T \approx 10^2 \ldots 10^3 \) s. Hence, the propagation rate of localized plasticity nuclei, \( V_{aw} = \lambda / T \approx 10^4 \ldots 10^5 \) m/s. The spontaneous formation of nuclei is illustrated for the stage of linear work hardening in Figure 1. Note that the latter pattern differs from those emergent at the other flow stages.

![Figure 1](image1.png)

Figure 1. A typical example of spontaneous material layering in the deforming sample of polycrystalline Al. Dark fringes correspond to plastic deformation nuclei

2. Localized Plasticity Autowaves
The challenge now is to explain the nature of the localized plasticity phenomenon. It has been long recognized that the deformation involves both elastic and plastic wave processes, the latter waves named after Kolsky (1963). The processes of interest have apparently all the salient features of wave
processes; however, these are involved in the plastic deformation and cannot be grouped with the elastic waves.

Kolsky’s waves are similar in a way to the waves of interest, since they would form in a medium by shock loading. Kolsky’s waves propagate at the rate \(10 \leq V_{aw} \approx (\theta/\rho)^{1/2} \leq 10^2 \text{ m/s} \) (here \(\rho\) is the medium’s density). However, matching of the values \(V_{aw} \sim 0^\circ\); \(V_{aw} \sim 0^\circ\) and \(V_{aw}/V_{aw} = 10^\circ\) reveals that Kolsky’s waves and the waves of interest basically differ. Moreover, the waves of interest have dispersion law \(\omega-k^1\), which is generally employed for addressing self-organization processes in nonlinear media as pointed out by Scott (2003).

It is thus inferred herein that the localized plasticity phenomenon is a specific space-time periodic process occurring in a system far from equilibrium. This appears to be a promising line of observation in view of the fact that the available descriptions of plastic flow dynamics contain no references to such processes occurring spontaneously. The first to discover that the above possibility tends to be overlooked were Glansdorf and Prigogine (1971). They argued that it would be possible to describe adequately the deforming medium’s properties in terms of self-organization in the open system. The deforming medium undergoes self-organization via evolutionary processes termed as autowaves. Mathematically, the autowaves differ radically from the elastic waves in solids. The elastic waves are solutions to hyperbolic differential equations of the type \(\ddot{y} = c \cdot y''\) which have solutions of the form \(y = A \cos(\omega t - kx)\) (here \(c = \omega/k\) is the wave propagation rate). Where we have to deal with longitudinal elastic waves, the wave propagation rate may be expressed in terms of medium’s characteristics, i.e. \(c' = E/\rho\).

The autowaves are solutions to parabolic differential equations of the type \(\dot{y} = \partial(x,y) + D y''\). In order to derive such an equation, the nonlinear function \(\partial(x,y)\) is added to the equation of \(\dot{y} = D y''\), which is used to describe the kinematical viscosity or diffusion of the medium. Apparently, the coefficient \(D\) has the dimension \(L^2 \cdot T^{-1}\). This mathematical ambiguity would hamper the analysis of autowave formation by the plastic flow, using the equation \(\dot{y} = \partial(x,y) + D y''\). The theory of parabolic differential equations holds that disturbance transport rate is formally considered to be infinitely high. However, the propagation rate of disturbance fronts is a finite value. This should be taken into account in the studies of the physics and mechanics of solids plasticity.

Using the condition of deformation flow continuity introduced by Hill (1998), equation of the type \(\dot{y} = \partial(x,y) + D y''\) can be derived for the plastic flow. Really, thus we obtain

\[
\dot{\varepsilon} = f(\varepsilon, \sigma) + D_\varepsilon \varepsilon'' \tag{1}
\]

where \(f(\varepsilon, \sigma)\) is a nonlinear function generally called ‘point kinetics for strain’. Scott (2003) proposed the theory of autowave processes, which holds that in order to address the deforming medium, another equation is required. For this purpose, one can write an equation in \(\sigma\), which has the same form as (1), i.e.

\[
\dot{\sigma} = g(\varepsilon, \sigma) + D_\sigma \sigma'' \tag{2}
\]

equation (2) apparently corresponds to the additivity condition \(\dot{\sigma} = \dot{\sigma}_{\varepsilon} + \dot{\sigma}_{\varepsilon}e\). The first term in the right side of (2), so-called ‘point kinetics’ for stress, describes the relaxation rate of elastic stress and the second term, the relaxation rate of viscous stresses.

3. Elastic-plastic Strain Invariant

The phase autowaves of localized plastic deformation, which form at the stage of linear work hardening can be described as a system of equidistant localized plasticity nuclei, which travel synchronously at a constant velocity over the tensile sample (see Figure 1). The study was made for a range of pure metals and alloys in single-crystal or polycrystalline state, which had FCC, BCC or HCP lattice. All the diagrams \(\sigma(\varepsilon)\) obtained for the test samples show up a linear work hardening stage (see Table 1).
Table 1. Numerical data for evaluation of the ratio \( \frac{2\lambda V_{aw}}{\chi V} \approx 0.99 \approx 1 \)

| Cu  | Mg  | Cd  | Zn  | Al  | In  | Zr  | Ti  | Pb  | Sn  | V   | Nb  | γ-Fe | α-Fe | Ni  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|-----|
| 1.5 | 0.25 | 0.55 | 0.6 | 2.2 | 2.3 | 0.6 | 0.6 | 3.2 | 1.3 | 0.9 | 0.7 | 1.1  | 0.7  | 0.7 |

We matched the products \( \lambda \cdot V_{aw} \) and \( \chi \cdot V \) (here \( \chi \) is interplanar spacing of crystal lattice and \( V \) is transverse ultrasound wave rate). These values are quantitative characteristics of the processes involved in the elastic and plastic deformation, which are assumed to characterize, respectively, the autowaves propagating in the deforming solid and the processes involved in crystal lattice straining. Numerical analysis suggests that \( 0.25 \approx \frac{2\lambda V_{aw}}{\chi V} \leq 2.30 \approx \frac{\gamma_{Fe}}{\chi V} \), the mean being \( \langle 2\lambda V_{aw}/\chi V \rangle \approx 1.15 \approx 1 \). Hence,

\[
\lambda \cdot V_{aw} \approx \frac{1}{2} \chi \cdot V
\]

This result testifies that the processes involved in the elastic and plastic deformation are closely related. Hence, equation (3) is termed ‘elastic-plastic strain invariant’.

According to Madelung (1996), \( V = \chi \omega_0 \) (here \( \omega_0 \) is the Debye frequency). Hence,

\[
\lambda V_{aw} \approx \frac{1}{2} \left( \chi V \right) \approx \frac{V^2}{2\omega_0} \approx \frac{G}{2} \left( \frac{\partial^2 W}{\partial \omega^2} \right) \approx \frac{1}{2} \left( \frac{\partial^2 W}{\partial \omega^2} \right) \approx \frac{1}{2} \left( \frac{\partial^2 W}{\partial \omega^2} \right)
\]

where \( \nu \ll \chi \) is the atomic displacement near interparticle potential minimum and the elastic modulus from (4) is expressed in terms of this potential as \( G \approx \left( \frac{\partial^2 W}{\partial \omega^2} \right) \chi^2 \). In this case, the value \( \xi = \left( \omega_0 \chi \right) \cdot \rho = \rho \cdot V \) is the specific acoustic resistance of the medium, which might be related according Nettel (2009) to crystal lattice perturbation due to external actions. The interparticle potential from (4) can be given as

\[
W(\nu) \approx \frac{1}{2} \left( \frac{\partial^2 W}{\partial \omega^2} \right) \cdot \nu^2 + \frac{1}{6} \left( \frac{\partial^2 W}{\partial \omega^2} \right) \cdot \nu^3 = \frac{1}{2} f_2 \cdot \nu^2 - \frac{1}{3} f_3 \cdot \nu^3
\]

where \( f_2 = \frac{\partial^2 W}{\partial \omega^2} \) is a quasi-elastic coupling coefficient and \( f_3 = 0.5 \cdot \frac{\partial^2 W}{\partial \omega^2} \) is an anharmonicity coefficient. With the proviso that \( \frac{1}{2} f_3 \cdot \nu^2 \gg \left| \frac{1}{3} f_3 \cdot \nu^3 \right| \), equation (3) assumes the form

\[
\lambda V_{aw} \approx \frac{1}{\xi} \approx \frac{f_2}{f_3} \frac{1}{V \cdot \rho} = Z \approx 10^4 \text{ m}^2/\text{s}
\]

where \( Z \) is apparently a criterion of plasticity.

Simple mathematical reasoning shows that the insertion of plasticity criterion is justified for the case of deformation initiated by a chaotic dislocation arrangement. Let \( \rho_{md} \) be mobile dislocation density. Then the average distance between dislocations is approximately equal to dislocation path and is given as \( \langle l \rangle = \frac{\rho_{md}^{\nu/2}}{2\pi} \). According to Friedel (1964), \( \sigma \approx \frac{Gb}{2\pi \rho_{md}^{\nu/2}} \); hence, \( \rho_{md}^{\nu/2} = \langle l \rangle = \frac{Gb}{2\pi \sigma} \approx \sigma^{-1} \). The velocity of quasi-viscous dislocation motion, \( V_{aw} = (b/B) \cdot \sigma \) (here \( B \) is a coefficient of viscous drag of dislocations by the phonon and electron gases in the crystal). Then we can write

\[
l \cdot V_{aw} \approx \text{const} = \frac{Gb}{2\pi \cdot B} = Z
\]

The modulus \( G \) and the coefficient \( B \) are conventionally employed in dislocation motion descriptions. Following Al’chits and Indenbom (1986), we use the values \( G \approx 40 \text{ GPa} \) and \( B \approx 10^3 \text{ Pa} \cdot \text{s} \) and thus obtain \( Z \approx 10^7 \text{ m}^2/\text{s} \), which is close to the product \( l/2 \chi \cdot V \), calculated for studied materials. The above suggests that we have established a reliable quantitative criterion, which might be useful for analysis of the interaction of elastic and plastic deformation on the macro- and micro-scale levels. In its
universal form the above criterion is also appropriate for the description of elastic and dislocation deformation. The same criterion can apparently be used to address autowave processes as well; hence, it is taken to be a more general form of the elastic-plastic strain invariant:

\[ \lambda \cdot V_{\text{ave}} = l \cdot V_{\text{disl}} = \frac{1}{2} \chi \cdot V = Z \quad (8) \]

Thus equation (8) applies to both localized plasticity autowaves and plastic deformation via dislocation glide; it might be used for description of lattice straining due to elastic wave propagation. Hence, equation (8) can be regarded as a more general version of invariant (3).

4. Conclusions
1. The plastic flow and failure are regarded here as individual processes, which involve formation and evolution of autowaves in the deforming medium. The plastic deformation is shown to exhibit an intermittent behavior with a changeover in the flow stages from a steady-state flow to the onset of necking and failure.
2. The parabola exponent \( n \) is considered an indication of the loss of plastic flow stability; the value \( n \) may vary significantly and reverse sign for each nucleus observed at the pre-failure stage. Hence, the deformation behavior and capability for work hardening of individual material volumes might vary significantly.
3. The examination of localized deformation patterns suggests that the location and time of future fracture can be determined long before a macroscopic neck forms in the test sample.
4. On the base experimental evidence a new method can be developed for predicting the place and time of material failure.

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References
[1] Alshits V I and Indenbom V L 1986 Mechanisms of dislocation drag Dislocations in Solids 7 ed F R N Nabarro (Amsterdam: Elsevier) pp 43-111
[2] Asharia A, Beaudoin A and Miller R 2008 Mathematics and Mechanics of Solids 13 292-315
[3] Fressengeas C, Beaudoin A, Entemeyer D, Lebedkina T, Lebyodkin M and Taupin V 2009 Physical Review B 79 014108-10
[4] Friedel J 1964 Dislocations (Oxford: Pergamon Press)
[5] Glansdorf P and Prigogine I 1971 Thermodynamic Theory of Structure, Stability and Fluctuations (London: Wiley-Interscience)
[6] Hill R 1998 The Mathematical Theory of Plasticity (Oxford: University Press)
[7] Kolsky H 1963 Stress Waves in Solids (New York: Dover)
[8] Madelung O 1996 Introduction to Solid-State Theory (Berlin: Springer)
[9] Nettel S 2009 Wave Physics (Berlin: Springer)
[10] Scott A 2003 Nonlinear Sciences. Emergence and Dynamics of Coherent Structures (Oxford: University Press)
[11] Zuev L B and Barannikova S A 2011 Sol. Stat. Phen. 172-174 1279-83
[12] Zuev L B and Barannikova S A 2014 Int. J. Mech. Sci. 88 1-8
[13] Barannikova S A, Ponomareva A V, Zuev L B, Vekilov Yu Kh and Abrikosov I A 2012 Sol. Stat. Commun. 152 784-7
[14] Zuev L B, Danilov V I and Barannikova S A 2008 Physics of macrolocalization of plastic flow (Novosibirsk: Nauka Publishing) (In Russian)
[15] Zuev L B, Barannikova S A, Maslova A A 2019 Materials Research 22 e20180694:1-12