Approximating Instance-Dependent Noise via Instance-Confidence Embedding

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Abstract

Label noise in multiclass classification is a major obstacle to the deployment of learning systems. However, unlike the widely used class-conditional noise (CCN) assumption that the noisy label is independent of the input feature given the true label, label noise in real-world datasets can be aleatory and heavily dependent on individual instances. In this work, we investigate the instance-dependent noise (IDN) model and propose an efficient approximation of IDN to capture the instance-specific label corruption. Concretely, noting the fact that most columns of the IDN transition matrix have only limited influence on the class-posterior estimation, we propose a variational approximation that uses a single-scalar confidence parameter. To cope with the situation where the mapping from the instance to its confidence value could vary significantly for two adjacent instances, we suggest using instance embedding that assigns a trainable parameter to each instance. The resulting instance-confidence embedding (ICE) method not only performs well under label noise but also can effectively detect ambiguous or mislabeled instances. We validate its utility on various image and text classification tasks.

1 Introduction

In modern machine learning, large-scale data has become indispensable [Russakovsky et al., 2015, Wang et al., 2019a]. A prevalent approach to collecting large-scale labeled datasets is to use imperfect sources such as crowdsourcing and web crawling [Fergus et al., 2005, Schroff et al., 2010, Wang et al., 2019a], which is usually less expensive and time-consuming than manual annotation by domain experts. However, such methods inevitably introduce label noise that may lead to overfitting and hurt the generalization of deep models [Arpit et al., 2017, Zhang et al., 2017].

In such situations, it is often beneficial to (i) remove mislabeled data or abstain from using confusing instances [Hara et al., 2019, Thulasidasan et al., 2019]; (ii) increase robustness and reduce harmful influences of noisy labels [Malach and Shalev-Shwartz, 2017, Mirzasoleiman et al., 2020, Liu et al., 2020]; or (iii) explicitly model the transition from the unobservable true label to the noisy observation [Goldberger and Ben-Reuven, 2017, Patrini et al., 2017, Xia et al., 2020]. In this work, we focus on explicit modeling of the label corruption process, which is model-agnostic and data-efficient.

Most existing studies in this direction employ the class-conditional noise (CCN) assumption, i.e., the noisy label is independent of the input feature given the true label [Angluin and Laird, 1988, Natarajan et al., 2013, Patrini et al., 2017]. However, this assumption could be too strong to fit some real-world data well [Xiao et al., 2015, Chen et al., 2021]. More importantly, CCN only captures the general label flipping patterns between classes for all instances. In applications such as data cleansing and human-in-the-loop interaction, instance-specific noise information itself could be of central interest. This urges us to consider not only the class-conditional noise pattern but also the instance-specific noise modeling.
Figure 1: **Graphical representations** of noise models, including the the conditionally independent labels (IND) model, class-conditional noise (CCN) model, instance-dependent noise (IDN) model, and the proposed *instance-confidence embedding* (ICE) approximation of IDN. Here, $X$ is the *input feature*, $Y$ is the *true label*, $\tilde{Y}$ is the *noisy label*, and $C \in [0,1]$ is a scalar *confidence* parameter.

To handle this problem, in this work, we study the *instance-dependent noise* (IDN) model, where the noisy label also depends on the input. Several methods have been reported in the literature, but they either only focus on binary classification under strong assumptions [Menon et al., 2018, Cheng et al., 2020] or are based on domain-specific knowledge [Xia et al., 2020]. In contrast, we propose a simple domain-agnostic approximation method for the multiclass IDN model, referred to as *instance-confidence embedding* (ICE). Concretely, to avoid estimating a noise transition matrix for each instance, we propose a variational approximation that uses a scalar *confidence* parameter (Section 3.2). Then, we suggest to use *instance embedding* that assigns a trainable parameter to each instance because the mapping from the instance to its confidence value could be non-smooth and is usually not required to generalize to unseen examples (Section 3.3). Lastly, we show the effectiveness of the proposed method and its ability to detect ambiguous or mislabeled instances through experiments on various image and text classification tasks (Section 5).

## 2 Problem: Instance-Dependent Noise

In this section, we give a brief overview of learning with *instance-dependent noise* (IDN).

### 2.1 Notation

Consider a $K$-class classification problem, where $X \in \mathcal{X}$ is the *input feature* and $Y \in \{1, \ldots, K\}$ is the unobservable *true label*. We assume that the *clean class-posterior* $p(Y|X)$ comes from a parametric family of distributions:

$$p_\phi(Y|X) := \text{Categorical}(Y|p = f(X; \phi)),$$

(1)

where $p \in \Delta^{K-1}$ is the probability parameter for $Y$ in the $(K-1)$-dimensional probability simplex $\Delta^{K-1}$, and $f : \mathcal{X} \rightarrow \Delta^{K-1}$ is a differentiable function parameterized by $\phi$ that maps the feature $X$ to its corresponding probability parameter $p$. Then, let $\tilde{Y} \in \{1, \ldots, K\}$ be the *noisy label*. The goal is to predict $Y$ from $X$ based on a finite i.i.d. sample of $(X, \tilde{Y})$-pairs.

### 2.2 Dependence

Next, we introduce the dependence structure between $X$, $Y$, and $\tilde{Y}$, which characterize different noise models. The graphical representations of noise models are illustrated in Fig. 1.

In IDN, we assume that the joint distribution of $X$, $Y$, and $\tilde{Y}$ can be factorized as follows:

$$p(X, Y, \tilde{Y}) = p(\tilde{Y}|Y, X)p_\phi(Y|X)p(X).$$

(2)
That is, the noisy label $\tilde{Y}$ depends on both the instance $X$ and the true label $Y$. Then, the noisy class-posterior $p(\tilde{Y}|X)$ can be obtained by marginalizing $p(Y, \tilde{Y}|X)$ over $Y$:

$$p_\phi(\tilde{Y}|X) := \text{Categorical}(\tilde{Y}|q = \sum_{Y=1}^{K} p(\tilde{Y}|Y, X)p_\phi(Y|X)),$$

where $q \in \Delta^{K-1}$ denotes the probability parameter for $\tilde{Y}$.

Note that $p(\tilde{Y}|Y, X)$ plays a central role in IDN. Since both $Y$ and $\tilde{Y}$ are categorical random variables, for a certain instance $x$, $p(\tilde{Y}|Y, X = x)$ can be seen as a $K \times K$ stochastic matrix $T(x)$, whose elements are $T_{ij}(x) := p(\tilde{Y} = j|Y = i, X = x)$ for $i, j \in \{1, \ldots, K\}$. Conventionally, $T(x)$ is called a noise transition matrix [Patrini et al., 2017]. Then, $p(\tilde{Y}|Y, X)$ can be regarded as a matrix-valued function $T : X \rightarrow [0, 1]^{K \times K}$ that maps each instance $x$ to its corresponding IDN transition matrix $T(x)$. Without any restriction, we need $K \times K$ parameters for each instance $x$.

### 2.3 Approach

Owing to its complexity, IDN has only been studied to a limited extent but is of great interest recently. A straightforward method is to jointly estimate the matrix-valued function $T(x)$ as well as the clean class-posterior $p_\theta(Y|X)$ using neural networks [Goldberger and Ben-Reuven, 2017]. However, the estimation error of $T(x)$ could be high, which deteriorates the classification performance. Another direction is to restrict the problem under certain conditions, so that we can provide theoretical guarantees [Menon et al., 2018, Cheng et al., 2020]. However, existing work mainly focused on binary classification.

A promising approach is to approximate IDN using a simpler dependence structure, such as a mixture of noises with different semantic meanings [Xiao et al., 2015] or a weighted combination of noises that depend on parts of the instance [Xia et al., 2020]. In this work, we also suggest that it might be unnecessary to obtain a $K \times K$ matrix for each instance $x$: Note that $p_\phi(Y|x)$ can be seen as a linear combination of columns of $T(x)$ weighted by $p_\theta(Y|x)$; If the maximum value of $p_\phi(Y|x)$ is close to 1, i.e., the label of the instance $x$ is almost deterministic, the estimation of $K - 1$ columns of $T(x)$ has only limited influence on the estimated noisy class-posterior $\hat{p}(\tilde{Y}|x)$. This suggests the possibility of using a relatively simple model to approximate $p(\tilde{Y}|X)$ in real-world applications. In this work, we consider a single-parameter approximation for each instance, which is introduced in Section 3.2 and illustrated in Fig. 2.

Another issue is that existing methods still introduce some level of smoothness w.r.t. $x$ into $T(x)$ [Goldberger and Ben-Reuven, 2017, Xiao et al., 2015, Xia et al., 2020]. In real-world problems, however, we can only access a finite sample of $(X, \tilde{Y})$-pairs that are possibly annotated by non-experts or web crawlers [Fergus et al., 2005]. Thus, the label noise could be aleatory and $T(x)$ could vary significantly for two adjacent instances. Also, the classifier $p_\phi(Y|X)$ is desired but the generalization of $T(x)$ to unseen examples is usually dispensable. This inspires us to use instance embedding instead of neural network approximation, which is discussed in Section 3.3 and demonstrated in Fig. 3.

### 3 Proposed Method

In this section, we present our proposed method, instance-confidence embedding (ICE).

#### 3.1 Variational lower bound

Note that $T(x)$ serves as a linear mapping from $p$ to $q$ (Eq. (3)). Due to the difficulty of estimating the matrix-valued function $T(x)$, we use a simpler function $q_{\theta,\phi}(\tilde{Y}|X)$ parameterized by $\theta$ as a variational approximation to $p_\phi(\tilde{Y}|X)$. The choice of the approximation family is discussed in Section 3.2.

3
Figure 2: An illustration of the transformation (▲ → ▼) from the clean class-posterior $p_\phi(Y|x)$ (the leftmost) to the noisy class-posterior $p_\phi(\tilde{Y}|x)$ (the rightmost). We can see that when the label is almost deterministic (▲ is close to a vertex), the estimation of $K - 1$ columns of the transition matrix $T(x)$ (the two deviated vertices of the dotted triangle) has only limited influence on the estimated noisy class-posterior $\hat{p}(\tilde{Y}|x)$ (△ is still close to ▼). This inspires us to go a step further and use single-parameter approximations (▲ → ▽) $q_{\theta,\phi}(\tilde{Y}|x)$ (Eqs. (8) and (9)).

Then, let us consider the expected log-likelihood as the learning objective, which can be rewritten as

$$\mathbb{E}_{\tilde{Y} \sim p(\tilde{Y}|X)} [\log p(\tilde{Y}|X)] = D_{KL}(p_\phi(\tilde{Y}|X) \parallel q_{\theta,\phi}(\tilde{Y}|X)) + \mathcal{L}(\theta,\phi; X),$$

(4)

where $D_{KL}$ denotes the Kullback-Leibler (KL) divergence, and the second term is

$$\mathcal{L}(\theta,\phi; X) := \mathbb{E}_{\tilde{Y} \sim p(\tilde{Y}|X)} [\log q_{\theta,\phi}(\tilde{Y}|X)].$$

(5)

Since the KL-divergence is always non-negative, this term gives a variational lower bound of the expected log-likelihood. Then, we have the following learning objective to maximize:

$$L(\theta,\phi) := \mathbb{E}_{X \sim p(X)} [\mathcal{L}(\theta,\phi; X)] = \mathbb{E}_{X,\tilde{Y} \sim p(X,\tilde{Y})} [\log q_{\theta,\phi}(\tilde{Y}|X)].$$

(6)

In practice, the expectation can be approximated using the empirical distribution based on a finite i.i.d. sample of $(X, \tilde{Y})$-pairs.

3.2 Variational approximation

Next, we discuss the choice of the variational approximation family of $q_{\theta,\phi}(\tilde{Y}|X)$.

To approximate the effect of multiplying an IDN transition matrix $T(x)$ that requires $K \times K$ parameters for each instance $x$, in this work, we use a simpler transformation from $p$ to $q$, which is not necessarily linear. Compared with estimating a full matrix for each instance without any restriction [Goldberger and Ben-Reuven, 2017], obtaining only an approximation may cause higher approximation error, but on the other hand, it may reduce estimation error and thus improve the classification performance. The high estimation error of complex models might be more harmful, which is empirically validated in Section 5. It is also the case when using CCN as an approximation of IDN to balance this trade-off. The difference is that CCN obtains a complete transition matrix for all instances, but ICE aims to obtain an approximated trend for each instance, which gives useful instance-specific noise information.

Then, we suggest to use a single-scalar parameter $C \in [0,1]$ for each instance to control this approximation, which is useful for sorting and comparing training examples. This parameter is referred to as the confidence and is obtained via a function $g : \mathcal{X} \rightarrow [0,1]$ parameterized by $\theta$, i.e., $C = g(X; \theta)$. The confidence $C$ plays a central role in our method, where $C = 0$ means that the instance is ambiguous or mislabeled and thus the classifier should not give a confident prediction.
The last piece of our method is the choice of $g : \mathcal{X} \rightarrow [0, 1]$; the function that maps the instance $x$ to its confidence value $C$. It is also possible to use a neural network to approximate this function. However, because we usually only have a limited number of training examples and $g$ could be non-smooth w.r.t. its input $x$, $g$ may not be well approximated by a neural network with similar complexity to the classifier $f$, which is illustrated in an example in Fig. 3. Further, $g$ may be rarely needed after training so its generalization ability is not required in many cases.
Based on these facts, we propose to use instance embedding, i.e., to assign a trainable parameter to each instance \( x \). In other words, the only feature for an instance we use is its index in the training dataset. In this way, \( g \) is expressive and flexible but cannot be used for predicting the confidence of unseen instances. Accordingly, for a training dataset of size \( N \), we need \( N \) parameters for a one-dimensional instance embedding.

This seems to be a high additional computational cost when the dataset size is large, but it is often acceptable, because (i) in modern deep learning, it is common to use over-parameterized models [Nakkiran et al., 2019], and the number of instances is usually not comparable to the number of parameters of the classifier \( f \) (e.g., CIFAR-10 [Krizhevsky, 2009]: \( 5 \times 10^4 \), ResNet-18 [He et al., 2016]: \( \sim 1 \times 10^7 \)); and (ii) the gradient of the instance embedding is sparse and only a small subset of parameters needs to be updated at each iteration.

The idea of associating an entity with a scalar or vector embedding using a simple lookup table with a fixed dictionary size has been widely used in natural language processing [Mikolov et al., 2013, Pennington et al., 2014, Peters et al., 2018, Devlin et al., 2019] due to the discrete nature of tokens, and can be seen recently in contrastive learning [Wu et al., 2018, He et al., 2020] for vision tasks. Instance embedding enables the function to take any possible value on all observed instances but cannot generalize to any unseen token or image.

### 4 Related Work

In this section, we review related problem settings and methods.

**Class-conditional noise (CCN).** Compared with the IDN model, the instance-independent and class-conditional noise (CCN) model has an additional assumption: \( p(\tilde{Y}|Y,X) = p(\tilde{Y}|Y) \), i.e., the noisy label \( \tilde{Y} \) only depends on the true label \( Y \). CCN has been well studied in both binary [Angluin and Laird, 1988, Long and Servedio, 2010, Natarajan et al., 2013, Van Rooyen et al., 2015, Liu and Tao, 2015] and multiclass [Patrini et al., 2017, Xia et al., 2019, Yao et al., 2020] classification. Also, robust loss functions [Ghosh et al., 2017, Zhang and Sabuncu, 2018, Wang et al., 2019b, Charoenphakdee et al., 2019, Ma et al., 2020, Feng et al., 2020, Lyu and Tsang, 2020, Liu and Guo, 2020] have been mainly developed under the CCN setting. In practice, CCN methods can serve as practical approximations of IDN but the assumption could be too strong to fit some real-world data well [Xiao et al., 2015].

**Conditionally independent labels (IND).** The other direction is to assume \( p(\tilde{Y}|Y,X) = p(\tilde{Y}|X) \), i.e., \( Y \) and \( \tilde{Y} \) are two sets of independent labels conditioned on the feature \( X \). This dependence structure is used in the information bottleneck framework [Tishby et al., 1999, Tishby and Zaslavsky, 2015, Alemi et al., 2017, Saxe et al., 2018], where the learning objective is to find a representation \( Y \) that is maximally informative about the observation \( \tilde{Y} \) based on the mutual information. This framework can be adapted for learning from noisy labels if we choose a categorical representation \( Y \). The graphical representations of IND and CCN are given in Fig. 1.

**Label smoothing.** Note that Eq. (8) is similar to the label smoothing (LS) technique [Szegedy et al., 2016, Pereyra et al., 2017, Lukasik et al., 2020], where the empirical distribution is linearly interpolated with a uniform distribution with a fixed mixing parameter. It is also related to the soft/hard bootstrapping loss [Reed et al., 2015], where the observed label is mixed with the predicted probability/predicted label. In contrast, in our method, it is the prediction \( p \) that is “smoothed”, not the label. We elucidate their relations and differences in Appendix B.
Temperature scaling. If we use softmax as the final layer of the neural network for $p_\phi(Y|X)$, the proposed method is closely related to the temperature scaling (TS) technique [Guo et al., 2017]. Concretely, if $p_i \propto \exp\{f_i(X;\phi)\}$ for $i = 1, \ldots, K$, then Eq. (9) becomes

$$q_i \propto \exp\{Cf_i(X;\phi)\},$$

which shows that $C$ is the reciprocal of the temperature. The difference is that the parameter $C$ is instance-dependent in our formulation, rather than being fixed for all instances. Also, TS [Guo et al., 2017] and its extensions [Kull et al., 2019, Rahimi et al., 2020] have been mainly used as post-hoc confidence calibration methods, while our method is used during training.

Sample selection. In a broader sense, the proposed method belongs to a category of methods that treat training examples differently in order to reduce the harmful effects of mislabeled instances. Besides the class-posteriors that our method uses, these methods exploit the training dynamics, loss characteristics, gradient information, or information of data itself from various perspectives. Examples include data cleansing [Liu et al., 2008, Northcutt et al., 2019, Hara et al., 2019] that first removes harmful instances and then (re-)trains the model on the remaining subset; dynamic training sample selection [Malach and Shalev-Shwartz, 2017, Jiang et al., 2018, Han et al., 2018, Wang et al., 2018, Yu et al., 2019, Mirzasoleiman et al., 2020, Wu et al., 2020, Chen et al., 2021] that selects training examples dynamically during training; training techniques [Menon et al., 2020, Liu et al., 2020] that are designed to increase robustness and avoid memorization of noisy labels; learning with rejection or selective classification [El-Yaniv et al., 2010, Thulasidasan et al., 2019, Mozannar and Sontag, 2020] that abstains from using confusing instances while improving the classification performance on accepted instances; and semi-supervised learning [Nguyen et al., 2020, Li et al., 2020] that exploits unlabeled data.

In the same spirit, our proposed method also attempts to detect harmful instances and reduce their influences automatically so as to improve the robustness of the class-posterior estimation. However, unlike explicit sample selection methods, the resulting algorithm is lightweight and has a low computational cost. Also, because the proposed method only affects the class-posterior, it is usually compatible with other training methods. Thus, the proposed method can be used alone or integrated into an existing training pipeline to further improve the performance.

5 Experiments

In this section, we experimentally verify if the proposed instance-confidence embedding (ICE) method is able to differentiate mislabeled instances from correct ones and consequently improve the classification performance. We also demonstrate that there already exist ambiguous or mislabeled training examples in the original datasets which can be detected by the proposed method. We evaluated on both image classification (Section 5.1) and text classification (Section 5.2).

5.1 Image classification

Datasets. We evaluated our method on six image classification datasets, namely MNIST [LeCun et al., 1998], Fashion-MNIST (FMNIST) [Xiao et al., 2017], and Kuzushiji-MNIST (KMNIST) [Clanuwat et al., 2018] datasets, which contain $28 \times 28$ grayscale images in 10 classes; and SVHN [Netzer et al., 2011], CIFAR-10, and CIFAR-100 [Krizhevsky, 2009] datasets, which contain $32 \times 32$ colour images in 10, 10 and 100 classes, respectively.

Methods. We compared the following eight methods: (1) (CCE) categorical cross-entropy loss; (2) (Bootstrapping) (hard) bootstrapping loss [Reed et al., 2015] that regularizes
ICE-LIN (4) (10 trials are reported. Outperforming methods are highlighted in boldface using one-tailed t-tests with a significance level of 0.05.

|                  | MNIST | FMNIST | KMNIST | SVHN  | CIFAR-10 | CIFAR-100 |
|------------------|-------|--------|--------|-------|----------|-----------|
| CCE              | 94.91 (0.43) | 85.05 (0.52) | 80.40 (1.25) | 71.50 (1.68) | 68.34 (0.82) | 47.09 (0.65) |
| Bootstrapping    | 97.30 (0.28) | 87.24 (0.36) | 84.21 (1.01) | 76.62 (0.97) | 75.97 (0.45) | 49.56 (0.42) |
| Adaptation       | 96.27 (0.41) | 88.30 (0.87) | 82.38 (2.07) | 66.85 (6.45) | 63.95 (5.94) | 31.70 (1.28) |
| Forward          | 95.09 (0.56) | 85.51 (0.45) | 80.76 (1.29) | 74.43 (6.42) | 68.26 (0.62) | 47.92 (0.31) |
| GCE              | 98.31 (0.13) | 88.76 (0.26) | 88.30 (0.60) | 75.03 (0.98) | 80.38 (0.67) | 55.64 (0.40) |
| ICE-LIN          | **98.64 (0.15)** | **89.41 (0.18)** | **89.61 (0.41)** | **77.51 (0.75)** | **82.08 (0.39)** | **55.30 (0.47)** |
| ICE-POW          | **98.60 (0.09)** | **89.29 (0.20)** | 89.21 (0.53) | **79.91 (0.96)** | **82.14 (0.44)** | **54.31 (0.48)** |

Figure 4: **Ridgeline plots** of the confidence $C$ during training (ICE-LIN). The density is estimated via Gaussian kernel density estimation (KDE). The red/blue curves represent the confidence of instances with flipped/original labels, respectively.

the output with the predicted label; (3) **Adaptation** noise adaptation layer [Goldberger and Ben-Reuven, 2017] that estimates a full $K \times K$ transition matrix for each instance; (4) **Forward** forward correction [Patrini et al., 2017] that estimates a transition matrix for all instances; (5) **DAC** deep abstaining classifier [Thulasidasan et al., 2019] that uses abstention for robust learning; (6) **GCE** generalized cross-entropy loss [Zhang and Sabuncu, 2018] as a robust loss; (7) **ICE-LIN** instance-confidence embedding with the linear interpolation (Eq. (8)); and (8) **ICE-POW** the one with the power transformation (Eq. (9)). For a fair comparison, we implemented aforementioned methods using the same network architecture and hyperparameters.

**Models.** For MNIST, FMNIST, and KMNIST, we used a sequential convolutional neural network (CNN) and an Adam optimizer [Kingma and Ba, 2015]. For SVHN, CIFAR-10 and CIFAR-100, we used a residual network model ResNet-18 [He et al., 2016] and a stochastic gradient descent (SGD) optimizer with momentum [Sutskever et al., 2013]. Hyperparameters are provided in Appendix C.

**Improving classification performance.** To verify if the proposed method is able to improve the classification performance under label noise, we constructed semi-synthetic noisy datasets so that the true labels are known. We regarded the original labels as clean labels, although as will be shown in the next experiment, label errors already exist in the original datasets to some extent. Following a common setup [e.g., Reed et al., 2015, Patrini et al., 2017, Thulasidasan et al., 2019], we simply flipped a fraction of labels randomly where the overall noise rate is 50%, i.e., half of instances are mislabeled. We ran 10 trials and reported the means and standard deviations of the test accuracy in Table 1.

We can observe that the proposed method generally outperforms the baseline methods. It is worth noting that estimating a full transition matrix for each instance (Adaptation) may improve the accuracy over CCE when the number of classes is relatively small, but when there are more classes (e.g., CIFAR-100), the performance may drop drastically because it requires an additional $K \times K$ output and the estimation error could be high. On the
Figure 5: The 32 most low-confidence training examples in the MNIST, FMNIST, SVHN and CIFAR-10 datasets, ordered left-right, top-down by increasing confidence. The index in the dataset, original label, and predicted label are annotated above each image.

contrary, the complexity of our single-parameter approximation does not increase as the number of classes increases. Additionally, in the ridgeline plots in Fig. 4, we can observe the separation of instances with flipped/original labels using the learned confidence $C$, which may explain the performance improvement.

Detecting ambiguous/mislabeled instances. Next, we demonstrate that the proposed method can be used for detecting ambiguous or possibly mislabeled instances. We trained the model on the original datasets with the proposed method. A benefit of using a single-parameter approximation is that it naturally derives an order of the training examples. We sorted the training examples via the confidence and showed the 32 most low-confidence ones in in Fig. 5. Results of other datasets are provided in Appendix C.

We can observe that, surprisingly, in these supposedly clean datasets, a number of instance might be mislabeled. In MNIST and SVHN, we found clearly mislabeled images. There are ambiguous images such as 2-7 and 4-9 pairs in MNIST and shirt/T-shirt/pullover/coat photos in Fashion-MNIST. In CIFAR-10, it is interesting that images in the animal category are more likely to have a low confidence. We conjecture that the spurious correlation between the object and the background color plays an important role. We also found multi-modality issues, e.g., kiwi, owl, and chicken are all labeled as bird but are not visually prototypical birds. This phenomenon suggests the possibility of using the proposed method for diagnosing label issues in large-scale datasets.
Table 2: **Performance (%)** on the GLUE benchmark for natural language understanding. We reported Matthews correlation coefficient on CoLA, F1 score/accuracy on MRPC and QQP, and accuracy otherwise. MNLI-(m/mm) denotes MultiNLI matched/mismatched, respectively.

|         | CoLA  | SST2  | MRPC  | QQP   | MNLI-(m/mm) | QNLI | RTE  | WNLI  |
|---------|-------|-------|-------|-------|-------------|------|------|-------|
| CCE     | 54.76 | 92.55 | 88.04 | 82.35 | 87.80/90.96 | 83.83/84.31 | 90.77 | 66.43 | 50.70 |
| ICE     | 57.83 | 92.20 | 89.54 | 85.05 | 87.85/90.92 | 83.81/84.36 | 91.14 | 63.90 | 56.34 |

Table 3: Selected low-confidence training examples in the CoLA dataset.

| Index | Label      | Guess      | Text                                                |
|-------|------------|------------|-----------------------------------------------------|
| 390   | acceptable | unacceptable | He I often sees Mary.                               |
| 7756  | acceptable | unacceptable | That monkey is ate the banana.                      |
| 8332  | acceptable | unacceptable | I wanted Jimmy for to come with me.                 |
| 2801  | unacceptable | acceptable  | Paula hit the sticks.                               |
| 2479  | unacceptable | acceptable  | Kelly buttered the bread with butter.               |
| 6795  | unacceptable | acceptable  | Henry wanted to possibly marry Fanny.               |

5.2 **Text classification**

We discovered that noisy label issues also exist in text datasets. We conducted similar experiments on the GLUE benchmark [Wang et al., 2019a], which is a collection of datasets for natural language understanding. We trained a BERT-base model pretrained using a masked language modeling (MLM) objective [Devlin et al., 2019] with a default AdamW optimizer [Loshchilov and Hutter, 2017]. The performance in terms of the suggested evaluation metric was reported in Table 2.

We can observe that, except on the RTE dataset, the performance was improved or approximately the same compared with the default CCE method, which shows the benefits of using instance-specifically adjusted confidences. Although, if the dataset is relatively clean, the improvement might be marginal.

We also found mislabeled or ambiguous instances in these datasets. A typical example is the Corpus of Linguistic Acceptability (CoLA) [Warstadt et al., 2019] dataset, which consists of English grammatical acceptability judgments. Six selected low-confidence training examples are given in Table 3. We found that several ungrammatical sentences were mislabeled as acceptable, and some syntactically acceptable sentences were labeled as unacceptable by annotators possibly because they have semantic errors. In this way, we may use the proposed method to probe if the model prediction is consistent with our intent. More results are provided in Appendix C.

6 **Conclusion**

We have introduced a novel variational approximation of the instance-dependent noise (IDN) model, referred to as instance-confidence embedding (ICE). Compared with existing methods based on the class-conditional noise (CCN) assumption, the proposed method is able to capture instance-specific noise information and consequently improve the classification performance. The use of the one-dimensional instance embedding naturally derives an order of training examples which can be used for detecting ambiguous or mislabeled instances. For future directions, it is interesting to explore its combination with other training techniques and its extensions in data cleansing, learning with rejection, or active learning.
Acknowledgement

We thank Gang Niu, Xi Huang, Nontawat Charoenphakdee, Han Bao, Masato Ishii, Yoshihiro Nagano, and Shota Nakajima for helpful discussion. We also would like to thank the Supercomputing Division, Information Technology Center, the University of Tokyo, for providing the Reedbush supercomputer system. YZ was supported by Microsoft Research Asia D-CORE program and Junior Research Associate (JRA) program at RIKEN. MS was supported by JST AIP Acceleration Research Grant Number JPMJCR20U3 and the Institute for AI and Beyond, UTokyo, Japan.

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In this way, we can equip the neural network model with an option of changing the confidence of prediction for individual training examples to mitigate overfitting possibly mislabeled instances.

**A Gradient analysis**

In this section, we provide a basic gradient analysis and visualization for our proposed method.

The gradients of the log-likelihood using Eqs. (8) and (9) are

\[
\frac{\partial}{\partial C} \log(q_i) = \frac{p_i - \frac{1}{K}}{Cp_i + (1-C)\frac{1}{K}} \quad \text{(linear interpolation)}
\]

\[
\frac{\partial}{\partial C} \log(q_i) = \sum_{j=1}^{K} p_j C \log \frac{p_i}{p_j} \quad \text{(power transformation)}
\]

respectively. Their sign boundaries are \( p_i = \frac{1}{K} \) and \( p_i = e^{-H(q,p)} \), respectively, where \( H(\cdot, \cdot) \) denotes the cross-entropy.

We can find that for the linear interpolation (Eq. (8)), when \( p_{y_i} < \frac{1}{K} \) \( \frac{\partial L}{\partial C} < 0 \) and when \( p_{y_i} > \frac{1}{K} \) \( \frac{\partial L}{\partial C} > 0 \). Similarly, for the power transformation (Eq. (9)), when \( p_{y_i} < e^{-H(q,p)} \) \( \frac{\partial L}{\partial C} < 0 \) and when \( p_{y_i} > e^{-H(q,p)} \) \( \frac{\partial L}{\partial C} > 0 \).

The contours of the likelihood for different parameters are plotted in Figs. 6 and 7. Note that the class-posterior \( p \) is obtained from a neural network, so it can be influenced by other instances, especially adjacent instances. On the other hand, the confidence \( C \) is obtained via instance embedding, so it can take any value independently. If the predicted class-posterior \( p_{y_i} \) for an instance \( x \) is low (e.g., because this instance is mislabeled and the majority of adjacent instances are predicted to belong to other classes), then the classifier tends to decrease its confidence value so as not to overfit this possibly mislabeled instance. The gradient magnitude is the largest when \( C \) is high and \( p_{y_i} \) is low (confident wrong prediction), the smallest when both \( C \) and \( p_{y_i} \) are high (confident correct prediction), and in the middle when \( C \) is low (uncertain prediction like a random guess). In this way, we can equip the neural network model with an option of changing the confidence of prediction for individual training examples to mitigate overfitting possibly mislabeled instances.
Figure 8: Illustration of related methods, including the categorical cross-entropy (CCE), label smoothing (LS), soft/hard bootstrapping loss (SB/HB), and the proposed instance-confidence embedding (ICE) with the linear transformation (Eq. (8)).

B Linear interpolation

Linear interpolation between some properties of an instance and other value is a widely used technique for regularization in machine learning, such as the bootstrapping loss Reed et al. [2015] and the label smoothing technique [Szegedy et al., 2016, Pereyra et al., 2017, Lukasik et al., 2020]. In this section, we briefly summarize related techniques and compare their differences.

Concretely, let \( p \) be the predicted probability vector for \( Y \) (Eq. (1)), \( y \) be the one-hot vector for the observed label, \( \hat{y} = \arg \max_{p} \) is the one-hot vector for the predicted label, \( u \) be the uniform probability vector \( (u_i = 1/K) \) for \( i \in \{1, \ldots, K\} \).

Here, \( p, y, \hat{y}, u \in \Delta^{K-1} \) are all in the probability simplex. Let \( C \in [0, 1] \) be a scalar linear interpolation parameter.

Then, as also shown in Fig. 8, the learning objectives are equivalent to the following KL-divergences:

\[
\begin{align*}
D_{KL}(y \parallel p), & \quad \text{(categorical cross-entropy)} \\
D_{KL}(Cy + (1 - C)u \parallel p), & \quad \text{(label smoothing)} \\
D_{KL}(Cy + (1 - C)p \parallel p), & \quad \text{(soft bootstrapping loss)} \\
D_{KL}(Cy + (1 - C)\hat{y} \parallel p), & \quad \text{(hard bootstrapping loss)} \\
D_{KL}(y \parallel Cp + (1 - C)u). & \quad \text{(instance-confidence embedding)}
\end{align*}
\]

We can see that the label smoothing and the bootstrapping loss methods smooth the target, but the proposed ICE method smooths the prediction. Note that it is impossible to let \( C \) be an instance-dependent parameter in other methods, because when \( C = 0 \), the supervision signal \( Y \) can be completely lost.

Another technique using linear interpolation is mixup [Zhang et al., 2018], which also interpolates the input features \( X \) between two instances. Therefore its characteristics could be more different than the methods mentioned above.
Figure 9: **Ridgeline plots** of the confidence $C$ during training. The density is estimated via Gaussian kernel density estimation (KDE). The red/blue curves represent the confidence of instances with flipped/original labels, respectively.

## C Experiments

### C.1 Image classification

**Data.** We used the MNIST, Fashion-MNIST, Kuzushiji-MNIST, SVHN, CIFAR-10, and CIFAR-100 datasets. The MNIST, Fashion-MNIST, Kuzushiji-MNIST datasets contain $28 \times 28$ grayscale images in 10 classes. The size of the training set is 60000 and the size of the test set is 10000. The SVHN dataset contains $32 \times 32$ colour images in 10 classes. The size of the training set is 73257 and the size of the test set is 26032. The CIFAR-10 and CIFAR-100 datasets contain $32 \times 32$ colour images in 10 classes and in 100 classes, respectively. The size of the training set is 50000 and the size of the test set is 10000. We used 20% of the training sets for validation. We added synthetic label noise into the training and validation sets. The test sets were not modified.

**Models.** For MNIST, Fashion-MNIST, and Kuzushiji-MNIST, we used a sequential convolutional neural network with the following structure: \texttt{Conv2d} (channel=32) ×2, \texttt{Conv2d} (channel=64) ×2, \texttt{MaxPool2d} (size=2), \texttt{Linear} (dim=128), \texttt{Dropout} (p=0.5), \texttt{Linear} (dim=10). The kernel size of convolutional layers is 3, and rectified linear unit (ReLU) is applied after the convolutional layers and linear layers except the last one. For SVHN, CIFAR-10 and CIFAR-100, we used a ResNet-18 model [He et al., 2016]. To ensure that $C \in [0, 1]$, we simply apply the sigmoid function that maps $\mathbb{R}$ to $[0, 1]$ to the embedding.

**Optimization.** For MNIST, Fashion-MNIST, and Kuzushiji-MNIST, we used an Adam optimizer [Kingma and Ba, 2015] with batch size of 512 and learning rate of $1 \times 10^{-3}$. The model was trained for 2000 iterations (17.07 epochs) and the learning rate decayed exponentially to $1 \times 10^{-4}$. For CIFAR-10 and CIFAR-100, we used a stochastic gradient descent (SGD) optimizer with batch size of 512, momentum of 0.9, and weight decay of $1 \times 10^{-4}$. The learning rate increased from 0 to 0.1 linearly for 400 iterations and decreased to 0 linearly for 3600 iterations (4000 iterations/40.96 epochs in total). For SVHN, the setting was the same as CIFAR-10 except the model was trained for 1000 iterations.

**Results.** The ridgeline plots of the confidence $C$ during training are given in Fig. 9. Low-confidence training examples are given in Figs. 10 and 11, which are partially presented in Fig. 5.

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1. MNIST [LeCun et al., 1998] http://yann.lecun.com/exdb/mnist/
2. Fashion-MNIST [Xiao et al., 2017] https://github.com/zalandoresearch/fashion-mnist
3. Kuzushiji-MNIST [Clanuwat et al., 2018] http://codh.rois.ac.jp/kmnist/
4. SVHN [Netzer et al., 2011] http://ufldl.stanford.edu/housenumbers
5. CIFAR-10, CIFAR-100 [Krizhevsky, 2009] https://www.cs.toronto.edu/~kriz/cifar.html
C.2 Text classification

We implemented the BERT-base model [Devlin et al., 2019] using PyTorch [Paszke et al., 2019] and HuggingFace’s transformers [Wolf et al., 2020] libraries. We used a pretrained model\(^6\) and an AdamW optimizer [Loshchilov and Hutter, 2017]. The batch size was 32 and the weight decay was 0.01, otherwise we used the default hyperparameters. The model was trained on 4 NVIDIA Tesla P100 GPUs in parallel with the mixed precision training option (fp16) enabled. For the CoLA, MRPC, RTE, and WNLI datasets, the model was trained for 5 epochs and otherwise 3 epochs. Low-confidence training examples are given in Tables 4 to 11.

\(^6\) bert-base-cased: https://huggingface.co/bert-base-cased
Figure 10: The 32 most low-confidence training examples in the MNIST, FMNIST, and KMNIST datasets, ordered left-right, top-down by increasing confidence.
Figure 11: The 32 most low-confidence training examples in the SVHN, CIFAR-10 and CIFAR-100 datasets, ordered left-right, top-down by increasing confidence.
| Index | Text |
|-------|------|
| 390   | label: acceptable  
guess: unacceptable  
**sentence:** He I often sees Mary. |
| 5766  | label: acceptable  
guess: unacceptable  
**sentence:** Heidi believes any description of herself. |
| 2801  | label: unacceptable  
guess: acceptable  
**sentence:** Paula hit the sticks. |
| 1522  | label: unacceptable  
guess: acceptable  
**sentence:** That the sun is out was obvious. |
| 8332  | label: acceptable  
guess: unacceptable  
**sentence:** I wanted Jimmy for to come with me. |
| 300   | label: acceptable  
guess: unacceptable  
**sentence:** They failed to tell me which problem the sooner I solve, the quicker the folks up at corporate headquarters. |
| 7813  | label: acceptable  
guess: unacceptable  
**sentence:** I went to the shop for to get bread. |
| 5904  | label: acceptable  
guess: unacceptable  
**sentence:** It hailed. |
| 4159  | label: unacceptable  
guess: acceptable  
**sentence:** Fifteen years represent a long period of his life. |
| 2479  | label: unacceptable  
guess: acceptable  
**sentence:** Kelly buttered the bread with butter. |
| 3846  | label: acceptable  
guess: acceptable  
**sentence:** They parted the best of friends. |
| 7371  | label: unacceptable  
guess: acceptable  
**sentence:** The hiker will reach the top of the mountain for an hour. |
| 430   | label: unacceptable  
guess: acceptable  
**sentence:** It’s probable in general that he understands what’s going on. |
| 6795  | label: unacceptable  
guess: acceptable  
**sentence:** Henry wanted to possibly marry Fanny. |
| 1115  | label: acceptable  
guess: acceptable  
**sentence:** He attributed to a short circuit the fire which. |
| 4155  | label: unacceptable  
guess: unacceptable  
**sentence:** Two drops sanitize anything in your house. |
| 1367  | label: acceptable  
guess: acceptable  
**sentence:** We elected president the boy’s guardian’s employer. |
| 7756  | label: acceptable  
guess: unacceptable  
**sentence:** That monkey is ate the banana |
| 4445  | label: unacceptable  
guess: acceptable  
**sentence:** George has went to America. |
| 4015  | label: acceptable  
guess: unacceptable  
**sentence:** He seems intelligent to study medicine. |
Table 5: The Stanford Sentiment Treebank (SST2)

| Index | Text                                                                 |
|-------|----------------------------------------------------------------------|
| 50155 | label:positive guess:positive                                        |
|       | sentence:a thirteen-year-old ’s book report                          |
| 58416 | label:negative guess:negative                                        |
|       | sentence:blues                                                       |
| 59724 | label:negative guess:positive                                        |
|       | sentence:‘ synthetic ’ is the best description of this well-meaning,  |
|       | beautifully produced film that sacrifices its promise for a high-     |
|       | powered star pedigree.                                               |
| 24696 | label:positive guess:negative                                        |
|       | sentence:lamer instincts                                             |
| 34494 | label:positive guess:positive                                        |
|       | sentence:had released the outtakes theatrically and used the film as |
|       | a bonus feature on the dvd                                           |
| 54555 | label:negative guess:negative                                        |
|       | sentence:pretentious, fascinating, ludicrous, provocative and vainglorious |
| 29155 | label:positive guess:negative                                        |
|       | sentence:he can be forgiven for frequently pandering to fans of the  |
|       | gross-out comedy                                                     |
| 44610 | label:negative guess:positive                                        |
|       | sentence:below                                                       |
| 66148 | label:negative guess:positive                                        |
|       | sentence:’ s cliche to call the film ‘ refreshing                    |
| 11869 | label:positive guess:negative                                        |
|       | sentence:go unnoticed and underappreciated                           |
| 55848 | label:negative guess:positive                                        |
|       | sentence:the film is an earnest try at beachcombing verismo, but it  |
|       | would be even more indistinct than it is were it not for the striking,|
|       | quietly vulnerable personality of ms. ambrose.                      |
| 57359 | label:negative guess:negative                                        |
|       | sentence:( ferrera )                                                 |
| 42232 | label:positive guess:negative                                        |
|       | sentence:forgive any shoddy product as long as there ’ s a little    |
|       | girl-on-girl action                                                  |
| 15783 | label:positive guess:negative                                        |
|       | sentence:have finally aged past his prime ...                        |
| 57186 | label:negative guess:positive                                        |
|       | sentence:hollywood war-movie stuff                                  |
| 52071 | label:positive guess:positive                                        |
|       | sentence:the gags                                                    |
| 1896  | label:positive guess:negative                                        |
|       | sentence:missing from the girls ’ big-screen blowout                 |
| 64779 | label:positive guess:negative                                        |
|       | sentence:growing strain                                              |
| 3940  | label:positive guess:negative                                        |
|       | sentence:you to bite your tongue to keep from laughing at the        |
|       | ridiculous dialog or the oh-so convenient plot twists                |
Table 6: Microsoft Research Paraphrase Corpus (MRPC)

| Index | Text |
|-------|------|
| 799   | **label:** equivalent  **guess:** not equivalent  
sentence1: We need a certifiable pay as you go budget by mid-July or schools wont open in September , Strayhorn said .  
sentence2: Texas lawmakers must close a $ 185.9 million budget gap by the middle of July or the schools wont open in September , Comptroller Carole Keeton Strayhorn said Thursday . |
| 469   | **label:** not equivalent  **guess:** equivalent  
sentence1: It ’s also a strategic win for Overture , given that Knight Ridder had the option of signing on Google ’s services .  
sentence2: It ’s also a strategic win for Overture , given that Knight Ridder had been using Google ’s advertising services . |
| 1037  | **label:** equivalent  **guess:** not equivalent  
sentence1: The broader Standard & Poor ’s 500 Index < .SPX > edged down 9 points , or 0.98 percent , to 921 .  
sentence2: The Standard & Poor ’s 500 Index shed 5.20 , or 0.6 percent , to 924.42 as of 9 : 33 a.m. in New York . |
| 1178  | **label:** equivalent  **guess:** not equivalent  
sentence1: Sens. John Kerry and Bob Graham declined invitations to speak .  
sentence2: The no-shows were Sens. John Kerry of Massachusetts and Bob Graham of Florida . |
| 1753  | **label:** equivalent  **guess:** not equivalent  
sentence1: The Dow Jones industrial average closed down 18.06 , or 0.2 per cent , at 9266.51 .  
sentence2: The blue-chip Dow Jones industrial average < .DJI > slipped 44.32 points , or 0.48 percent , to 9,240.25 . |

Table 7: Quora Question Pairs (QQP)

| Index | Text |
|-------|------|
| 216515| **label:** duplicate  **guess:** duplicate  
question1: Why does Quora censor opinions and answers?  
question2: Does Quora censor questions and answers, and should they? |
| 343656| **label:** not duplicate  **guess:** duplicate  
question1: Could India’s surgical strike in POK be an elaborate hoax or play?  
question2: Did India really conduct a surgical strike on Pakistan? |
| 266594| **label:** not duplicate  **guess:** not duplicate  
question1: Why is financial literacy generally not taught in American high schools?  
question2: Why isn’t financial literacy taught in today’s public schools? |
| 251996| **label:** not duplicate  **guess:** not duplicate  
question1: What is life like in communist countries?  
question2: What would life in a legitimate Communist country be like? |
| 7963  | **label:** not duplicate  **guess:** duplicate  
question1: What is the best option for Indian politics and politicians?  
question2: What are the options of Indian politics and politicians? |
Table 8: MultiNLI (MNLI)

| Index | Text |
|-------|------|
| 218290 | **label:** entailment **guess:** neutral  
premise: The ruins of the huge abbey of Jumiyges are perhaps the most the white-granite shells of two churches, the Romanesque Notre-Dame and the smaller Gothic Saint-Pierre.  
hypothesis: Notre-Dame is a larger church than Gothic Saint-Pierre. |
| 39431 | **label:** neutral **guess:** entailment  
premise: Unless you feel really safe in French metropolitan traffic, keep your cycling ‘you can rent a bike at many railway stations’ for the villages and country roads.  
hypothesis: You should not cycle in the French metropolitan area. |
| 27574 | **label:** contradiction **guess:** neutral  
premise: I don’t think so.  
hypothesis: I have no real idea. |
| 258544 | **label:** neutral **guess:** neutral  
premise: A set of stone doors in the wall slid to the side to reveal a screen on which various torture scenes began to appear.  
hypothesis: The doors hid a television screen. |
| 320518 | **label:** contradiction **guess:** neutral  
premise: None seems comfortable with the notion of removing Clinton for sex-related misdeeds.  
hypothesis: People don’t want Clinton touching sex related ordeals |

Table 9: Question NLI (QNLI)

| Index | Text |
|-------|------|
| 1659 | **label:** not entailment **guess:** not entailment  
question: What caused Latin America’s right-wing authorities to support coup o’etats?  
sentence: This was further fueled by Cuban and United States intervention which led to a political polarization. |
| 5876 | **label:** not entailment **guess:** not entailment  
question: What antenna type is a portion of the half wave dipole?  
sentence: The monopole antenna is essentially one half of the half-wave dipole, a single 1/4-wavelength element with the other side connected to ground or an equivalent ground plane (or counterpoise). |
| 5829 | **label:** entailment **guess:** entailment  
question: How are Toxicara canis infections spread?  
sentence: Toxocara canis (dog roundworm)eggs in dog feces can cause toxocariasis. |
| 77419 | **label:** entailment **guess:** entailment  
question: Why did Madrid cede the territory to the US  
sentence: Florida had become a burden to Spain, which could not afford to send settlers or garrisons. |
| 9576 | **label:** not entailment **guess:** not entailment  
question: What has no distinction between the categories of voiced, voiceless, aspirated and unaspirated?  
sentence: Some of the Dravidian languages, such as Telugu, Tamil, Malayalam, and Kannada, have a distinction between voiced and voiceless, aspirated and unaspirated only in loanwords from Indo-Aryan languages. |
Table 10: Recognizing Textual Entailment (RTE)

| Index | Text |
|-------|------|
| 2429  | **label:**not entailment **guess:**entailment  
**sentence1:**Bogota, 4 May 88 - The dissemination of a document questioning Colombia's oil policy, is reportedly the aim of the publicity stunt carried out by the pro-Castro Army Of National Liberation, which kidnapped several honorary consuls, newsmen, and political leaders.  
**sentence2:**Several honorary consuls were kidnapped on 4 May 88. |
| 2463  | **label:**not entailment **guess:**entailment  
**sentence1:**The official religion is Theravada Buddhism, which is also practiced in neighboring Laos, Thailand, Burma and Sri Lanka.  
**sentence2:**The official religion of Thailand is Theravada Buddhism. |
| 1361  | **label:**entailment **guess:**not entailment  
**sentence1:**The Catering JLC formulates pay and conditions proposals of workers in the industry which, if approved by the Labour Court, legally binds employers to pay certain wage rates and provide conditions of employment. However, the QSFA now contends that the JLC has no right to make such a legally binding provision, as Section 15 of the Constitution states that the sole and exclusive power to make laws is vested in the Oireachtas, and no other authority has power to make laws for the State. It also argued that the existence of the minimum wage and 25 other pieces of legislation protecting employees’ rights means that there is no need for JLCs. The chairman of the QSFA, John Grace, warned that the situation would lead to job losses and closures.  
**sentence2:**John Grace works for QSFA. |

Table 11: Winograd NLI (WNLI)

| Index | Text |
|-------|------|
| 266   | **label:**entailment **guess:**not entailment  
**sentence1:**Susan knew that Ann's son had been in a car accident, so she told her about it.  
**sentence2:**Susan told her about it. |
| 478   | **label:**entailment **guess:**not entailment  
**sentence1:**Joe paid the detective after he delivered the final report on the case.  
**sentence2:**The detective delivered the final report on the case. |
| 294   | **label:**entailment **guess:**not entailment  
**sentence1:**Dan had to stop Bill from toying with the injured bird. He is very compassionate.  
**sentence2:**Dan is very compassionate. |
| 586   | **label:**entailment **guess:**not entailment  
**sentence1:**Dan took the rear seat while Bill claimed the front because his "Dibs!" was slow.  
**sentence2:**Dan took the rear seat while Bill claimed the front because Dan’s "Dibs!" was slow. |
| 243   | **label:**entailment **guess:**not entailment  
**sentence1:**Mark was close to Mr. Singer’s heels. He heard him calling for the captain, promising him, in the jargon everyone talked that night, that not one thing should be damaged on the ship except only the ammunition, but the captain and all his crew had best stay in the cabin until the work was over.  
**sentence2:**He heard Mr. Singer calling for the captain. |