Building a non-perturbative quark-gluon vertex from a perturbative one

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Abstract. The quark-gluon vertex describes the electromagnetic and the strong interaction among these particles. The description of this interaction at high precision in both regimes, perturbative and non-perturbative, continues being a matter of interest in the context of QCD and Hadron Physics. There exist very helpful models in the literature that explain perturbative aspects of the theory but they fail describing non-perturbative phenomena, as confinement and dynamic chiral symmetry breaking.

In this work we study the structure of the quark-gluon vertex in a non-perturbative regime examining QCD, checking results with QED, and working in the Schwinger-Dyson formalism.

1. Introduction

Studies of the non-perturbative quark propagator show that the mass function in the infrared limit $M(p^2)|_{p^2 \to 0}$ agrees with the mass of the constituent quarks, and its perturbative limit is well reproduced in the ultraviolet limit[1].

The quark propagator is intimately connected with the gluon propagator and with the quark-gluon vertex through the Schwinger-Dyson equations (SDE), and the Slavnov-Taylor identities. This is why it becomes necessary to study the behaviour of the quark propagator and the vertex of this interaction.

The quark-gluon vertex is already described perturbatively at one-loop[2]. Nevertheless, this structure fails as we study mass generation via dynamical chiral symmetry breaking, which is a non-perturbative phenomenon.

In this work we are going to study the behaviour of the quark-gluon vertex, and we present a guide of the basic ideas needed to build this vertex.

2. Key elements

Even though the quark-gluon vertex is not an observable, it is necessary to preserve its gauge invariance. This is accomplished by using Ward-Slavnov-Taylor identity[3, 4]. This identity relates the vertex with the inverse of the quark propagator and the ghost propagator as follows

$$q^\mu \Gamma_\mu(p,k,q) = G(q^2)[S^{-1}(p)H(p,k,q) - \overline{H}(k,p,q)S^{-1}(k)]$$ (1)

In this equation, $\Gamma_\mu(p,k,q)$ is the quark-gluon vertex, $G(p^2)$ is a scalar function associated with the ghost propagator. Functions $H$ and $\overline{H}$ involve the full quark-quark-ghost-ghost vertex, and...
$S^{-1}(p^2)$ is the inverse quark propagator defined as

$$S^{-1}(p^2) = \frac{i\gamma \cdot p + M(p^2)}{F(p^2)} \quad (2)$$

Because it is the tree level in the perturbative expansion for small coupling, a quark-gluon vertex must have the same transformation properties of the bare vertex $\gamma^\mu$ under charge conjugation and parity operations. Moreover, we need the quark-gluon vertex to be free of kinematic singularities. This implies that the vertex has to have a unique and finite limit when $k^2 \to p^2$.

Finally, the SDEs are an infinite set of coupled equations. Then we need to assure the absence of overlapping divergences of these equations. We achieve this by imposing multiplicative renormalizability, which means that functions $F(p^2)$ and $M(p^2)$ have to be written as a power law in the quark propagator, i.e. $F(p^2) = (p^2)^\nu$, where $\nu$ is the anomalous dimension.

3. Procedure

We can separate the full quark-gluon vertex into two kinematic contributions using the Ward-Slavnov-Taylor identity, similar to the case of QED using Ward-Takahashi identity. Actually, WST identity can be split into two parts, abelian and non-abelian, where the abelian part corresponds to the Ward-Takahashi identity.

Thus, we can write the quark-gluon vertex as

$$\Gamma^\mu = \Gamma^\mu_L + \Gamma^\mu_T \quad (3)$$

where $\Gamma^\mu_L$ and $\Gamma^\mu_T$ are the longitudinal and the transverse part of the vertex, respectively. Moreover, $q_\mu \Gamma^\mu_T = 0$, meaning that the transverse part does not contribute to the WST identity.

Now, we extend the decomposition of the QED vertex to the case of QCD. We can write the longitudinal part of the vertex as follows

$$\Gamma^\mu_L(p, k) = \sum_{i=1}^4 \lambda_i(p^2, k^2)L_i^\mu(p, k) \quad (4)$$

where $\lambda_i = \lambda_i^{(a)} + \lambda_i^{(b)}$, with (a) the abelian contribution and (b) the non-abelian one. For the transverse part we have

$$\Gamma^\mu_T(p, k) = \sum_{i=1}^8 \tau_i(p^2, k^2)T_i^\mu(p, k) \quad (5)$$

Similar to the longitudinal form factor $\lambda_i$, we take $\tau_i = \tau_i^{(a)} + \tau_i^{(b)}$. We use the Ball and Chiu basis for both partitions, except for $T_6^\mu$ which has the opposite sign[5].

For QED, the functions $\lambda_i$ are defined in [5], and we can find perturbative definitions at one-loop for QCD in [2]. Nevertheless, in order to have a non-perturbative description, we need to extend these definitions[6, 7].

From Eq. (1), the function $H$ can be expressed as

$$H(p, k, q) = \chi_0(p^2, k^2, q^2)I + \chi_1(p^2, k^2, q^2)\gamma \cdot p + \chi_2(p^2, k^2, q^2)\gamma \cdot k + \chi_3(p^2, k^2, q^2)\sigma^{\mu\nu}p_\mu k_\nu \quad (6)$$
If we perform a perturbative expansion for the $\chi_i$, it follows that $\chi_0$ is the dominant term[2]. So, we can choose the longitudinal part of the quark-gluon vertex as

$$\Gamma^\mu_L(p,k) = G(q^2)\chi_0(p^2, k^2, q^2)\Gamma^\mu_{BC}(p,k)$$  (7)

where $G(q^2)$ and $\chi_0(p^2, k^2, q^2)$ are defined perturbatively in [2].

On the other hand, we need to analyse the behaviour of the transverse part in order to make a practical ansatz. There are basics requirements for $\tau_i$ coefficients that must be fulfilled. For instance, these coefficients have to be free of kinematics singularities. In general, $\tau_i$ need to satisfy the following:

- All $\tau_i$ must be symmetric under the exchange of $k^2 \leftrightarrow p^2$, except for $\tau_6$ which must be antisymmetric under this exchange.
- Since the quark propagator is expected to be multiplicative renormalizable, then the quark-gluon vertex has to be a function of $F(p^2)$ and $M(p^2)$, because the longitudinal part contains them both. So, $\tau_i = \tau_i(F(p^2), M(p^2))$.
- Multiplicative renormalizability also implies that in the asymptotic limit $k^2 \gg p^2$, the transverse vertex must be unique.

About this last point, we perform the asymptotic limit over the perturbative definitions of $\tau_i$, given in [2]. For the abelian ($a$) and the non-abelian ($b$) coefficients we find [8]

$$k^4 \tau_{a1}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{4\pi}\left(3 + \xi\right)\ln\left(\frac{p^2}{k^2}\right)$$

$$k^4 \tau_{a2}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{12\pi}\left(1 - 2\xi\right)\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{a3}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{12\pi}\left(1 + \xi\right)\ln\left(\frac{p^2}{k^2}\right)$$

$$k^6 \tau_{a4}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{12\pi}\left(\xi - 1\right)\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{a5}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{24\pi}\left(\xi - 1\right)\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{a6}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{12\pi}\left(1 - \frac{\xi}{2}\right)\ln\left(\frac{p^2}{k^2}\right)$$

$$k^4 \tau_{a7}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{12\pi}\left(1 - \xi\right)\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{a8}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{4\pi}\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{b1}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{12\pi}\left[12 + 7\xi - \xi^2\right]\ln\left(\frac{p^2}{k^2}\right)$$

$$k^4 \tau_{b2}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{8\pi}{48\pi}\left[12 + 7\xi - \xi^2\right]\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{b3}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{8\pi}{16\pi}\left[12 + 7\xi - \xi^2\right]\ln\left(\frac{p^2}{k^2}\right)$$

$$k^6 \tau_{b4}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{8\pi}{16\pi}\left[12 + 7\xi - \xi^2\right]\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{b5}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{8\pi}{16\pi}\left[12 + 7\xi - \xi^2\right]\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{b6}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{8\pi}{16\pi}\left[12 + 7\xi - \xi^2\right]\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{b7}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{8\pi}{16\pi}\left[12 + 7\xi - \xi^2\right]\ln\left(\frac{p^2}{k^2}\right)$$

$$k^2 \tau_{b8}^4 = \left(C_F - \frac{1}{2}C_A\right)\frac{8\pi}{16\pi}\left[12 + 7\xi - \xi^2\right]\ln\left(\frac{p^2}{k^2}\right)$$

where $\xi$ is the gauge parameter, $C_F$ and $C_A$ are the color factors of the eigenvalues of the square Casimir operators in the fundamental and the adjoint representation, respectively.

In the asymptotic limit, the transverse quark-gluon vertex only has contributions of $T_3^\mu = -T_6^\mu$, i.e., $\Gamma^\mu \sim (\tau_3 - \tau_6)T_6^\mu$ when $k^2 \gg p^2 \gg m^2$. Thus, in this limit

$$\tau_3 - \tau_6 = -\frac{\alpha_s}{64\pi^2}\left[C_A(1 + \xi) - 8\xi C_F\right]$$  (8)
In Fig. (1) we compare $\tau_3 - \tau_6$ obtained using Eq. (8) with the definitions from [2]. If we take the QED case, where $C_F = 1$ and $C_A = 0$, we obtain the corresponding limit [9].

A non-perturbative proposal for the quark-gluon vertex has to be gauge invariant [10]. Work in this direction will be reported elsewhere [11].

Figure 1. In this plot, the asymptotic approx. is obtained using eq. (8), while Davydychev, et. al. result is produced using the full definition in [2]. The plotting parameters are $\alpha_s = 0.118$ for the coupling, $m = 5\text{MeV}$ for the quark mass and we computed in Landau gauge $\xi = 0$.

4. Conclusions

In this work we presented the basic requirements that a non-perturbative ansatz for the quark-gluon vertex must satisfy, and they agree with the literature of the area.

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