A Precise Determination of $\tan \beta$ from Heavy Charged Higgs Decay

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Abstract

We compute the energy spectrum of charged leptons in the decay $H^+ \rightarrow \bar{b} + (t \rightarrow bl\nu_l)$. The shape of the lepton spectrum obtained, and also the mean lepton energy, are sensitive to the handedness of the intermediate top quark. This sensitivity can be used to precisely determine $\tan \beta$, a fundamental parameter of two Higgs doublet models.

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Precision measurements of electroweak parameters and the strong coupling constant ($\alpha_s$) at LEP [1] are completely consistent with the minimal supersymmetric extension to the standard model (MSSM), with a MSSM mass scale of around 1 TeV [2].

In the MSSM the single standard model (SM) complex scalar doublet, or Higgs doublet, is replaced by two Higgs doublets (for a review see ref. [3]). After spontaneous symmetry breaking the MSSM Higgs sector consists of: two CP-even bosons $h^0$ and $H^0$ ($m_{h^0} < m_{H^0}$); a CP-odd boson $A^0$; and two charged scalars $H^\pm$. The presence of a Higgs sector that includes two doublets is a feature of most supersymmetric models. In models with only Higgs doublets (and singlets) there is no coupling of the charged Higgs to vector boson pairs ($WZ$ or $W\gamma$) at tree level. The most important decay channels of the $H^\pm$ are into fermion pairs or into $W^\pm h^0$. However, $H^\pm$ decays into states involving charginos can, in some regions of the parameter space, be quite large [4]. For a $H^\pm$ with mass ($m_{H^\pm}$) greater than the sum of top and bottom quarks ($m_t + m_b$) the dominant decay process is $H^+ \rightarrow t\bar{b}$. A future linear $e^+e^-$ collider would be an ideal place to search for the decay $H^+ \rightarrow t\bar{b}$. However, in practice, planned machines are limited to beam energies around 500 GeV. Recent studies [5, 6] indicate that it is feasible to also search for a $H^\pm$ (with $m_{H^\pm} > m_t$) at hadron colliders, such as the LHC, over a large region of the available parameter space.

In the present letter we observe that a charged Higgs boson of the Minimal Supersymmetric Model significantly heavier than the top quark offers an opportunity to precisely measure the fundamental parameter of a two Higgs doublet model, namely the ratio of vacuum expectation values of the two doublets, $\tan\beta \equiv v_2/v_1$. The reason for this lies in the structure of the coupling of the charged Higgs boson to $tb$ quarks:

$$
\mathcal{L} = \frac{g V_{tb}}{\sqrt{2} m_W} H^+ t [m_t \cot\beta L + m_b \tan\beta R] b + H.c.,
$$

where $R, L = (1 \pm \gamma_5)/2$, $m_W$ is the mass of the $W$ boson, $g$ is the weak coupling constant, and $V_{tb}$ is the relevant element of the CKM matrix. From this Lagrangian we can see that the ratio of numbers of left and right polarized top quarks produced in the decay $H^+ \rightarrow t\bar{b}$ depends on the coefficients of left and right chiral projection operators $L, R$. At the tree level:

$$
\frac{N_L}{N_R} = \left( \frac{m_b}{m_t} \right)^2 \tan^4\beta,
$$

where $N_L(R)$ are the partial rates of production of left (right) handed top quarks. If $H^+$ is significantly heavier than the top quark, so that in its decay the top is emitted with large velocity, we can determine the handedness of the top quark by analyzing the energy spectrum of the charged lepton produced in the semileptonic decay of $t$ [7]. We see from the equation (2) that the ratio $N_L/N_R$ is very sensitive to the value of $\tan\beta$. However, in practice this means that using this method one will only be able to determine the precise value of $\tan\beta$ over a limited range. For extreme values of $\tan\beta$ the top quark will be produced exclusively in one polarization state, $t_R$ for small $\tan\beta$ and $t_L$ for large. Examination of the lepton spectrum will then be useful to determine the order of magnitude of $\tan\beta$. 

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It is well known \cite{8, 9, 10, 11} that the decay width of the charged Higgs boson is strongly modified by QCD corrections. These corrections will also influence equation \cite{E}. However, the dominant part of corrections can be absorbed by replacing $m_b$ by the running mass of $b$ ($\overline{m}_b$) at the energy scale of the mass of $H^+$. We use the threshold condition $m_b(2m_b) = 4.7 \text{ GeV}$, which gives $\overline{m}_b \approx 3.6 \text{ GeV}$ at the typical mass scale of the $H^+$ of a few hundred GeV \cite{E}. The corresponding effect on the mass of the top quark is of the order of 5%. In our numerical estimates we take $m_t = 150 \text{ GeV}$ and $m_{H^+} \equiv m_{H} = 300 \text{ GeV}$.

In order to explicitly evaluate the energy distribution of leptons in the decay chain $H^+ \rightarrow \bar{b} + (t \rightarrow b + (W^+ \rightarrow \bar{\ell}_l \nu_l))$ we adopt the narrow width approximation for both intermediate particles $t$ and $W^+$. The total width of this decay is then

$$\Gamma(H^+ \rightarrow \bar{b} \ell_l \nu_l) = \frac{\Gamma(H^+ \rightarrow t\bar{b})\Gamma(t \rightarrow bW^+)\Gamma(W^+ \rightarrow \bar{\ell}_l \nu_l)}{\Gamma_t \Gamma_W} = \frac{G_F^3}{128 \sqrt{2\pi^3}} \frac{m_W^3}{\Gamma_t \Gamma_W m_t m_H^3} \left(\frac{m_t^2}{m_H} \cot^2 \beta + \overline{m}_b \tan^2 \beta\right) \times \left(\frac{m_H^2}{m_H^2 - m_t^2}\right)^2 \left(\frac{m_t^2}{m_W^2} - \frac{m_t^2}{m_W^2}\right)^2 \left(2m_W^2 + m_t^2\right),$$

(3)

where we have neglected mass of $b$ everywhere except in the coupling. It can be seen that the total width of $H^+$ depends on $\tan \beta$. However it does not distinguish between the two values: $\tan^2 \beta = X$ and $\tan^2 \beta = \overline{m}_b / (m_H X)$. Much more information can be gained from the partial decay width $d\Gamma/dx$, with $x \equiv 2E_l / m_H$ denoting the scaled energy of the lepton in the rest frame of the $H^+$. We introduce the following dimensionless variables

$$u = \frac{m_t^2}{m_H^2}, \quad y = \frac{m_W^2}{m_H^2}, \quad a = \frac{m_b^2}{m_H^2} \tan^2 \beta, \quad b = \frac{m_t^2}{m_H^2} \cot^2 \beta.$$  

(4)

The kinematic limits of lepton energy are expressed by

$$y \leq x \leq 1.$$  

(5)

In order to compute the energy distribution we use

$$d\Gamma = \frac{1}{2m_H} \frac{1}{(2\pi)^8} |\mathcal{M}|^2 dR_4(H; b, q, l, \nu),$$

(6)

where $\mathcal{M}$ is the matrix element and $R_4(H; b, q, l, \nu)$ is the 4 body phase space with letters denoting 4-momenta of the corresponding particles, and since we have to distinguish between the two $b$ quarks, we assign the letter $b$ to the one produced in the primary decay $H^+ \rightarrow t\bar{b}$ and the letter $q$ to the product of the top quark decay. We decompose the 4 body phase space in a manner similar to the calculation of QCD corrections to the lepton energy spectrum in semileptonic top quark decays \cite{12}

$$dR_4(H; b, q, l, \nu) = dR_3(H; P, l, \nu) dz dR_2(P; b, q),$$

(7)
where \( P \equiv b + q \) is the 4-momentum of the two \( b \) quarks, and \( z \) is the square of their invariant mass, \( z \equiv P^2/m_H^2 \). In the narrow width approximation the phase space integration is simplified by replacing the propagators of \( t \) and \( W \) by delta functions, and we end up with the following formula for the energy distribution of leptons

\[
\frac{d\Gamma}{dx} = \frac{3G_F^3}{64\sqrt{2}\pi^3 m_t m_W^7} f(x) \tag{8}
\]

where the dimensionless function \( f(x) \) defined by

\[
f(x) = \begin{cases} 
2ux(a - b) [x - y - u \ln(x/y)] + (a - bu)(x - y)(2u - x - y) & \text{if } y \leq x \leq \min(u, y/u) \\
2u^2x(a - b)(1 - x + \ln x) + u^2(a - bu)(1 - x)^2 & \text{if } \max(u, y/u) \leq x \leq 1
\end{cases} \tag{9}
\]

For values of \( x \) between \( u \) and \( y/u \) we have to distinguish two cases:

\[
f(x) = \begin{cases} 
2ux(a - b) [u - y + u \ln(y/u)] + (a - bu)(y - u)^2 & \text{if } u < y/u \\
2ux(a - b) [x(1 - u) + u \ln u] + x(a - bu)(1 - u)(2u - x - xu) & \text{if } u > y/u
\end{cases} \tag{10}
\]

We use these formulae to compute the mean scaled energy of leptons, \( \bar{x} \). The result is the same in both mass cases:

\[
\bar{x} = \frac{a(1 + 2u) + b(2 + u) u^2 + 2uy + 3y^2}{a + b} \frac{1}{6u(u + 2y)}. \tag{11}
\]

We see from this formula that if mass of the top quark approaches mass of the decaying Higgs boson \( (u \to 1) \) the mean energy loses sensitivity to values of \( a \) and \( b \). Therefore the determination of \( \tan\beta \) using this method will be better if the charged Higgs boson is significantly heavier than the top quark, i.e. when the top produced in its decay has high velocity.

In order to illustrate the sensitivity of the lepton energy distribution to the value of \( \tan\beta \) we show in fig. 1 plots of \( f(x) \) for two extreme cases, \( \tan\beta = 0.5 \) (solid line) and for \( \tan\beta = 83.33 \) (dashed) (these two values correspond to the same total decay rate). We obtain the characteristic spectra of leptons from totally polarized right and left handed top quarks [7]. In figure 2 we plot the dependence of the mean energy of leptons on the value of \( \tan\beta \). Provided the charged Higgs is significantly heavier than the top quark, we obtain good sensitivity to the precise value of \( \tan\beta \) in the range \( 2 \leq \tan\beta \leq 10 \). Outside of this range, the spectrum of leptons can only serve to determine whether \( \tan\beta \) is small or large.

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Figure captions

Figure 1: Distribution of lepton energy in the decay $H^+ \rightarrow b\bar{b}l\nu$, according to equations (9-10), calculated for $m_H = 300$ GeV, $m_W = 80.22$ GeV, for two values of $\tan \beta$.

Figure 2: Sensitivity of the mean scaled energy of leptons (see eq. [11]) to the value of $\tan \beta$, shown for three values of mass of $H^+$. 
This figure "fig1-1.png" is available in "png" format from:

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