Excitation of monochromatic and stable electron acoustic wave by two counter-propagating laser beams

C Z Xiao\textsuperscript{1,2}, Z J Liu\textsuperscript{1,3}, C Y Zheng\textsuperscript{2,4} and X T He\textsuperscript{2,3,4}

1 Key Laboratory for Micro-/Nano-Optoelectronic Devices of Ministry of Education, School of Physics and Electronics, Hunan University, Changsha, 410082, People’s Republic of China
2 HEDPS, Center for Applied Physics and Technology, Peking University, Beijing, 100871, People’s Republic of China
3 Institute of Applied Physics and Computational Mathematics, Beijing, 100084, People’s Republic of China
4 Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai, 200240, People’s Republic of China
5 Author to whom any correspondence should be addressed.

E-mail: xiaocz@hnu.edu.cn

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Abstract

The undamped electron acoustic wave is a newly-observed nonlinear electrostatic plasma wave and has potential applications in ion acceleration, laser amplification and diagnostics due to its unique frequency range. We propose to make the first attempt to excite a monochromatic and stable electron acoustic wave (EAW) by two counter-propagating laser beams. The matching conditions relevant to laser frequencies, plasma density, and electron thermal velocity are derived and the harmonic effects of the EAW are excluded. Single-beam instabilities, including stimulated Raman scattering and stimulated Brillouin scattering, on the excitation process are quantified by an interaction quantity, \( \eta = \gamma_B \tau_B \) where \( \gamma \) is the growth rate of each instability and \( \tau_B \) is the characteristic time of the undamped EAW. The smaller the interaction quantity, the more successfully the monochromatic and stable EAW can be excited. Using one-dimensional Vlasov–Maxwell simulations, we excite EAW wave trains which are amplitude tunable, have a duration of thousands of laser periods, and are monochromatic and stable, by carefully controlling the parameters under the above conditions.

1. Introduction

Electrostatic plasma waves are fundamental research topics in plasma physics due to their significance in inertial confinement fusion (ICF)\textsuperscript{[1]}, laser wakefield acceleration \textsuperscript{[2]} and other fields. Electron plasma waves and ion acoustic waves are the most familiar linear electrostatic waves and they are of great interest. They can not only trigger laser plasma instabilities such as stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS), which have detrimental effects on ICF \textsuperscript{[3]}, but are also often used as a medium to accelerate electrons to relativistic energy due to their advantageous high phase velocities.

Recent observations of nonlinear electrostatic waves enrich the category of electrostatic plasma waves. These waves include nonlinear stable electron acoustic waves (EAWs) \textsuperscript{[4]} and kinetic electrostatic electron nonlinear (KEEN) waves \textsuperscript{[5]}, whose frequencies are well below the electron plasma frequency and belong to nonlinear waves described by the BGK modes \textsuperscript{[6]}. The linear electron acoustic wave is strongly damped because its phase velocity is near the electron thermal velocity which may suffer from strong Landau damping. However, particle trapping helps in diminishing the linear Landau damping rate by modifying the electron distribution function \textsuperscript{[7]}, hence these linear strongly damped waves can propagate nearly undamped in plasma. The observations of undamped EAWs and KEEN waves in laser-produced plasma, particularly in the context of stimulated scattering \textsuperscript{[8–13]}, greatly extend the frequency domain of electrostatic waves. However, these two waves are qualitatively different. The KEEN waves are naturally nonlinear undamped BGK-like waves that contain multiple pronounced phase-locked harmonics. The pronounced phase-locked harmonics here mean that amplitudes of some orders of harmonics are comparable with the fundamental mode \textsuperscript{[5, 11]}, while the
undamped EAWs are considered a predominant fundamental mode where their frequency and wavenumber satisfy the nonlinear dispersion relation [4, 9]. Discussions about EAWs and KEEN waves are focused on their linear and nonlinear natures, the physical mechanism of exciting such waves, and the harmonics that may intrinsically exist [13–16]. In this paper, we use the terminology nonlinear/undamped EAWs’ to distinguish the linear EAW that is strongly Landau damped, and refer to nonlinear waves as those that are trapping dominant with a predominant fundamental mode. Our goal is to excite such monochromatic and stable EAWs and exclude harmonics effects. In our previous paper [17], we found that to amplify one or some of these harmonic modes resonantly, one should diminish the imaginary part of the nonlinear dielectric function and set the driven parameters near the zero value of its real part. This results in particular parameter spaces for the excitation of undamped EAWs, which is the start point of the present work. In addition, other explanations for the nature of EAWs and KEEN waves based on the wave–particle interaction, such as negative mass instability and vortex merging, are also proposed [13, 14, 18].

It is not easy to excite such nonlinear waves because of their strict excitation conditions. Rosenbluth and Liu [19] proposed a method to excite an electron plasma wave using two parallel laser beams with a frequency difference of \( n\omega_{pe} \), where \( \omega_{pe} \) is the electron plasma frequency and \( n \) is a natural number. Afterwards, it was used for generating relativistic electrons known as the laser beat wave acceleration (LBWA) [20]. This method can also be used for exciting undamped EAWs under certain conditions, as we will show in our paper. Such undamped EAWs may have potential applications due to their special frequency domains, which has not been emphasised before. EAWs compensate the frequency gap between electron plasma waves and ion acoustic waves, which ranges from 0.1\( \omega_{pe} \) to \( \omega_{pe} \), and many physical phenomena may occur in these frequency domains. For example, since the phase velocity of EAWs is not too high (around electron thermal velocity \( v_e \)), ions can be trapped in these waves and accelerated as electrons in the laser wakefield. The consequent monoenergetic ions may be comparable to the conventional ion acceleration mechanism such as target normal shear acceleration (TNSA) [21] and shock wave acceleration [22]. In addition, this nonlinear eigenmode may act as a medium for laser amplification just like that in Raman/Brillouin amplification. The waves may also be used as detectors for plasma diagnostics. To achieve these potential applications, the excitation of such a monochromatic and stable EAW is an underlying problem.

Thus, in this paper we use the method proposed by Rosenbluth and Liu to resonantly excite undamped EAWs using two counter–propagating laser beams with appropriate frequencies, amplitudes, and durations. The excitation conditions based on the nonlinear dispersion relation of the EAW are derived, along with particular parameter spaces in order to exclude the harmonic effects. Our simulation results show that under such conditions, the excited waves have a predominant fundamental component and negligible harmonics, termed the ‘undamped EAWs’. The amplitude of the EAW depends on the products of the two laser amplitudes. Laser plasma instabilities such as SRS and SBS also have a great influence on the excitation process. We find that we can minimize the influences by minimizing the interaction quantities \( \eta = \gamma \eta_0 \), where \( \gamma \) is the growth rate of each instability and \( \eta_0 \) is the characteristic time of the EAW. Both theoretical and one-dimensional (1D) Vlasov–Maxwell simulation results have confirmed our idea of exciting stable and monochromatic EAWs.

The paper is organized as follows. In section 2, the theoretical model for exciting undamped EAWs is depicted where a matching condition relevant to laser and plasma conditions is derived. Section 3 presents 1D Maxwell simulation results considering the effects of both SRS and SBS. In section 4, we summarise the conclusions and discuss the potential future applications.

2. Theoretical model

For simplicity, we consider two parallel propagating laser beams with the form \( E_i = \hat{y} E_i \cos(k_i x - \omega_i t) \), \( E_2 = \hat{y} E_2 \cos(k_2 x - \omega_2 t) \). The frequencies are required to be different, \( \omega_1 \neq \omega_2 \), thus the ponderomotive force due to the beats of the two laser beams \( (\nabla \times \mathbf{B}) \) force is \( F_p = -e \nabla \times \mathbf{B}/c = -e(v_1 B_2 + v_2 B_1)/c \). Assume the lowest order in fluid velocity, i.e., \( v_i = e E_i \sin(k_i x - \omega_i t)/m \omega_i, i = 1, 2 \) and neglect nonresonant terms, and one obtains that,

\[
F_p = \frac{me^2}{2} \Delta k a_i a_j \sin(\Delta k x - \Delta \omega t),
\]

where \( \Delta k = |k_1 - k_2|, \Delta \omega = \omega_1 - \omega_2 \) are the wavenumber and frequency difference. \( m, e, c \) are electron mass, electron charge, and light speed, respectively. \( a_i = e E_i/m \omega_i c \) is the normalized laser amplitude.

The linear longitudinal \( \nabla \times \mathbf{B} \) force acts as an external ponderomotive force on plasma driving the plasma wave with a forced frequency \( \Delta \omega \) and forced wavenumber \( \Delta k \). As is well known, when the frequency difference equals \( \omega_{pe} \) or some integer multiple of \( \omega_{pe} \), the electron plasma wave (Langmuir wave) will be resonantly excited [19], since it is the linear eigenmode in plasmas. Otherwise, plasmas act as a forced oscillation in response to the external ponderomotive force. New eigenmodes emerge as wave–particle interaction diminishes the linear
Landau damping rate when the driven characteristic time is larger than the bounce time \( \tau_b = 2\pi \sqrt{|m/eE|} \), where \( E \) is the plasma wave amplitude, \( k \) is the wavenumber of the plasma wave [23]. Then the nonlinear eigenmode appears in the electrostatic plasma with its frequency lower than \( \omega_{pe} \) and phase velocity of around electron thermal velocity, which is the so-called electron acoustic wave. The dispersion relation of the nonlinear EAW with infinitesimal wave potential \( \phi \) is a slightly modified linear plasma dispersion relation that eliminates the Landau damping rates, which is given by

\[
D_{NL}(k, \omega, \phi \to 0) = 1 + \frac{1}{k^2 \lambda_D^2} [1 - 2\xi F(\xi)] = 0,
\]

where \( \xi = \omega/\sqrt{2}k \nu_e \), and \( k \) and \( \omega \) are real without any collisionless damping, \( F(\xi) = e^{-\xi^2} \int_0^\infty e^{\xi^2} d\xi \) is the Dawson function, and \( \lambda_D = \nu_e/\omega_{pe} \) is the Debye length. The lower frequency branch of the dispersion curve (EAW branch) has a sound-wave-like behavior with its phase velocity near \( \nu_e \) [23]. To obtain an analytical dispersion relation for nonlinear EAW, we expand the dispersion relation around \( \xi = 1 \) to the first order,

\[
\omega = 1.314\nu_e k + 1.314 \frac{\nu_e^3}{\omega_{pe}^2} k^3.
\]

The approximated dispersion relation, equation (3), is valid for \( k\lambda_D \) not larger than 0.5, otherwise the frequency has ten percent more negative relative deviation. Since the nonlinear EAW dispersion relation is cut off at \( k\lambda_D = 0.53 \) and a positive frequency shift of the order of \( \omega_b = 2\pi/\tau_b \) exists with finite wave potential [24], this formula can roughly represent the EAW dispersion relation.

Next, we discuss the matching condition for resonantly exciting an undamped EAW. In principle, as long as the driven frequency and wavenumber approach the EAW dispersion relation, a linear EAW can be formed first, then due to the trapping effects, an undamped EAW is finally formed. But some additional effects, such as harmonic effects and laser plasma instabilities, should be ruled out. The KEEN waves, which have pronounced phase-locked harmonics to the main wave, are also undamped and can be excited in broad spectra near the EAW dispersion relation. In our previous study, we found that harmonics can also be resonantly excited when their dispersion relations are satisfied and revealed that large amplitude harmonics (meaning that the amplitude of one harmonic wave or some harmonics are in the order of the fundamental mode, the so-called pronounced harmonics) exist when \( \Delta k\lambda_D \lesssim 0.26 \), since the nonlinear dispersion relation of EAW is bounded in the region \( \Delta k\lambda_D \lesssim 0.53 \). The excited harmonics orders can be estimated by \( 0.53/(\Delta k\lambda_D) \) [17]. Notice that our results are based on the system imposing an electrostatic potential to emulate the ponderomotive force produced in a stimulated scattering system. A recent study on KEEN waves excited in the SRS situation shows that through vortex merging KEEN waves can be excited at \( k\lambda_D > 0.3 \) [13]. The phenomenon may be considered as secondary instability; in our paper we carefully deal with that condition. For this reason, to excite a monochromatic and stable EAW, two laser beams should satisfy the sequential conditions. For co-propagating beams, substituting \( \Delta k = k_1 - k_2 \) and \( \Delta \omega \) into equation (3), we obtain

\[
\omega_1 - \omega_2 = 1.314\nu_e (k_1 - k_2) + 1.314 \frac{\nu_e^3}{\omega_{pe}^2} (k_1 - k_2)^3,
\]

where \( k_1 = \sqrt{\omega_1^2 - \omega_{pe}^2}/c \), \( k_2 = \sqrt{\omega_2^2 - \omega_{pe}^2}/c \). Similarly, for counter-propagating beams, \( \Delta k = k_1 + k_2 \), the matching condition is

\[
\omega_1 - \omega_2 = 1.314\nu_e (k_1 + k_2) + 1.314 \frac{\nu_e^3}{\omega_{pe}^2} (k_1 + k_2)^3.
\]

Involving the condition,

\[
0.36 \omega_{pe} \lesssim \omega_1 - \omega_2 \lesssim \omega_{pe},
\]

harmonic effects can be excluded. These conditions imply rigid relations for laser and plasma parameters. For co-propagating beams, the excited wave has a phase velocity near light speed, which means that the undamped EAW can exist only when \( \nu_e/c \approx 1 \). It is too ultra-relativistic in the laboratory, in the co-propagating configuration to excite undamped EAWs. On the other hand, the matching condition can be satisfied as long as \( (\omega_1 - \omega_2)/(\omega_1 + \omega_2) \) has the order of \( \nu_e/c \) in the counter-propagating configuration. Thus, we present the parameter space of the matching condition for counter-propagating beams, equations (4) and (6), in figure 1(a). The condition of exciting a monochromatic and stable EAW lies between the red-dashed lines. To estimate the parameters conveniently, the projection of the 3D plotting matching condition is presented in figure 1(b), which shows realistic parameters for the excitation.

Another significant condition is that the characteristic time of excitation of undamped EAWs should be shorter than that of laser plasma instabilities such as SRS and SBS. In an external driver, the electric field that a particle feels includes both external driver and self-consistent electric field, which is readily obtained
$E = E_0 / \varepsilon (\Delta k, \Delta \omega)$, where $\varepsilon$ is the linear plasma dielectric function and $E_0 = F_p / e$ is the ponderomotive electric field [17]. Substitute equation (1) into the bounce time, and we get the characteristic time of EAWs in counter-propagating laser beams,

$$\tau_B = \frac{2\pi}{\Delta k c} \sqrt{\frac{2\varepsilon (\Delta k, \Delta \omega)}{a_1 a_2}}.$$  

(7)

In order to avoid single-beam laser plasma instabilities, one should consider the following situations. (i) **Density lower than quarter-critical density.** In this case SRS may become more detrimentally unstable than others, thus linear EAW can grow stably when $\gamma_{SRS} \approx a_i \sqrt{\omega_{pe} / \omega_i}$, where $\omega_{pe} \ll \omega_i$, $i = 1, 2$ are assumed [25]. The excitation conditions then become

$$\gamma_{SRS1} \approx a_1 \sqrt{\omega_{pe} / \omega_i} \approx 1,$$

$$\gamma_{SRS2} \approx a_2 \sqrt{\omega_{pe} / \omega_i} \approx 1.$$  

(8)

The inequalities imply that the lower the density, the more successfully the undamped EAW can be excited. It is better to choose equivalent laser amplitudes, i.e., $a_1 = a_2$ due to the reciprocal terms. (ii) **Density higher than quarter-critical but still underdense where Raman scattering is prohibited.** The excitation process should only overcome the Brillouin instability, thus the condition is obtained in the weak coupling limit [25],

$$\gamma_{SBS1} \approx \frac{\pi}{2} \frac{\omega_{pi} \omega_{pe}}{\omega_i} \approx 1,$$

$$\gamma_{SBS2} \approx \frac{\pi}{2} \frac{\omega_{pi} \omega_{pe}}{\omega_i} \approx 1.$$  

(9)

where $\omega_{pi} = \omega_{pe} \sqrt{m_i / M_e}$, $\omega_{pe} \approx \sqrt{Te / M_e}$ are ion oscillation frequency and ion sound velocity, respectively, and $M$, $Te$ denote the ion mass and electron temperature. Also, equivalent laser amplitudes are better for the excitation.

3. One-dimensional Vlasov–Maxwell simulation results

We have performed a series of simulations with wide parameter spaces to excite monochromatic and stable EAWs by using a one-dimensional (1D) Vlasov–Maxwell code [26]. Two counter-propagating laser beams from each side incident into a plasma slab, the lasers propagate in the x-direction and polarize in the y-direction. In order to mitigate the risk of single-beam instabilities, the two normalized amplitudes are kept the same and should not be too large so that the waves act as linear perturbations to the plasma. In other words, the generated longitudinal electric field should satisfy $\Delta k / 2 < \sqrt{eE/k} < \nu_i$, where $\Delta k$ is the width of the phase island and $k$ is its wavenumber. In the first set of simulations, we focus on the excitation process without any laser plasma instabilities to see what an undamped EAW looks like in the two-laser system compared with that in the electrostatic excitation situation [9, 17]. After that, we will discuss the influences of SRS and SBS on the excitation process.
3.1. Resonant excitation of EAW in the counter-propagating laser beams

To avoid laser plasma instabilities (LPIs), fixed ion and a density above quarter-critical are assumed here. According to the matching condition, laser frequencies are chosen as \( \omega \omega_{\text{fp}} = 0.76 \), homogeneous electron density is \( n_0 = 0.36n_c \) where \( n_c \) is the critical density of \( \omega \), and the electron temperature is varied to find the resonant point of excitation of the undamped EAW. The two laser beams have the same amplitudes of \( a_0 = 0.0212 \), raise and down times of \( 60 \omega_0 \) and durations of \( 4500 \omega_0 \). Homogenous plasma fills the simulation box with a length of \( c/60 \), space step of \( c/0.2 \) and 300 space grids in total. The velocity space ranges from \( -c \) to \( c \) with 2048 velocity grids in it, so that the grid step of velocity space ranges from \( v_0/0.0067 \) to \( v_0/0.01 \), and the grid steps of position space are a little smaller than \( D_l \) when varying the electron temperature. The total simulation time is \( 6000 \omega_0 \) with a time step of \( 0.2 \omega_0 \). Boundary conditions are periodic for the distribution functions and open for the laser fields.

The resonant curve of the longitudinal field is plotted in figure 2(a), sampled after the lasers were turned off. A narrow resonant peak which is obtained in the thermal velocity of \( v_e = 0.1267c \) roughly satisfies the matching condition \( (v_e = 0.1335c \text{ in theory, the discrepancy stems from the kinetic frequency shift in a finite wave potential}) \). The resonant excited undamped EAW has the frequency \( \omega_{\text{EAW}} = 0.24\omega_1 \), wavenumber \( k_{\text{EAW}} = 1.267\omega_1/c \), and phase velocity \( v_{\text{pEAW}} = 1.5v_e \), and is a trapping modified, nonlinear monochromatic BGK-like wave. The theoretically predicted ponderomotive electric field plotted with the red dashed line is about \( E_0 = mc^2k_{\text{EAW}}a_1a_2/2e = 2.5 \times 10^{-4}n_0\omega_1c/e \), and is 10 times lower than the longitudinal field at the resonant peak. However, for a near resonant excitation, the amplitude of the excited EAW generally has the order of \( E_0 \) which depends on the products of beam amplitudes. Figure 2(b) shows the excitation process at the resonant point. Initially, the longitudinal field oscillates at an invariable value \( E_0 \approx 4.5 \times 10^{-4}n_0\omega_1c/e \) as a response to the ponderomotive force. Once the nonlinear wave–particle interaction is dominant after several bounce times \( (\tau = 1029\omega_1^{-1}) \), it diminishes the linear Landau damping, i.e. the imaginary part of the linear plasma dielectric function, leading to an exponential increase when the real part of the dielectric function is nearly zero. The undamped EAW is then resonantly excited, with a phase velocity of about 1.5 times the electron thermal velocity. The asymptotic state after the lasers are off belongs to a class of undamped BGK-like modes.
Correspondingly, we present temporal spectra of the longitudinal field in figure 2(c), in a logarithmic scale. When the lasers are on, in addition to the highest peak of the linear EAW and its less significant harmonics, three other components in the ponderomotive force are presented with the frequency of $2\omega_1$, $2\omega_2$, and $\omega_1 + \omega_2$. While turning off the lasers, the system maintains its monochromatic characteristics and stability. The electron distribution function at the end of the simulation also shows the coherent phase space islands in figure 2(d). These hypothetical none-LPI simulations have confirmed that linear effects of the ponderomotive force in the two counter-propagating laser beams have a similar way of exciting undamped EAWs with the electrostatic situations [9, 17]. The resonant excited waves are monochromatic and stable under the conditions. Higher order nonlinearities in the ponderomotive force, such as the convective term in the Vlasov equation, will take place at a later time and ruin the harmony, which is beyond our discussion.

3.2. Influence of SBS

In fact, it is inevitable that laser plasma instabilities occur in the underdense plasmas. These instabilities may make big differences to the excitation of undamped EAWs since linear unstable eigenmodes are easier to trigger than nonlinear modes when their matching conditions are satisfied. Thus, surviving undamped EAWs must compete with these linear modes, in other words, the growth of linear EAWs should be faster than that of the instabilities. First, in this subsection, we focus on Brillouin scattering with the following parameters, one is fixed ion, the other uses mobile ions with $M = 1836\text{m}$ and $T_e = 0.27T_e$. The density, laser frequencies, and electron temperature are the same as in the resonant case in section IIA, i.e., $n_e = 0.36n_c$, $\omega_2/\omega_1 = 0.76$ and $\nu_L = 0.1267c$. The normalized laser amplitudes are chosen as $a_1 = a_2 = 0.03$, and the total simulation time are both $8 \times 10^4\omega_1^{-1}$ but with different laser durations, $7 \times 10^4\omega_1^{-1}$ for fixed ion and $6.5 \times 10^4\omega_1^{-1}$ for mobile ion. Other parameters remain the same as in the resonant case of section IIIA (note that the grid steps of velocity space are $0.007\nu_L$ for the electron distribution function and $0.0147\nu_L$ for the ion distribution function, where $\nu_L = \sqrt{T_e/M}$ is the ion thermal velocity).

First, let us come back to the excitation condition under SBS in the weakly coupling limit, equation (9). It is convenient to introduce a dimensionless interaction quantity $\eta = \gamma\tau_{hi}$, where $\gamma$ is the instability growth rate. By simplification, the interaction quantity of SBS correlates with the following parameters, $\eta_{\text{SBS}} \propto \left| \varepsilon \right|^{1/2} \nu_{hi}^{-1}$. Since $\varepsilon$ decreases when increasing $\Delta k\lambda_D$ and $\Delta \omega/\omega_{pe}$ along the nonlinear EAW dispersion relation curve, larger $\Delta k\lambda_D$ (usually larger than 0.26 and smaller than 0.53) is beneficial for the excitation. Besides, smaller density, and larger ion mass and thermal velocity, are better for mitigating SBS.

Concretely, the interaction quantities are $\eta = 0$ for fixed ion as an asymptotic state and $\eta_1 = 7\eta_{\text{SBS}} \tau_{hi} = 1.32$, $\eta_2 = 7\eta_{\text{SBS}} \tau_{hi} = 1.99$ for mobile ion. Here we present their corresponding nonlinear evolution of longitudinal fields in figure 3. Figures 3(a) and (d) are the longitudinal field evolution at $x = 2000c/\omega_1$ and its temporal spectrum for fixed ion, respectively. When enlarging the system, more linear EAWs are generated in the simulation box and behave in collective motion compared with what is observed in figure 2. As is seen in figure 3(a), no such obvious resonant process exists in the whole simulation; the highest amplitude is about $2 \times 10^{-3}m_{\omega_1}c/e$, which is a little bit higher than the initial excited wave amplitude, $E \approx 1.3 \times 10^{-3}m_{\omega_1}c/e$, where the ponderomotive electric field is about $E_0 = 5.7 \times 10^{-3}m_{\omega_1}c/e$. It is observed in the simulation that the phenomenon results from longitudinal field energy transfer on a large scale of tens of wavelengths, which prevents the amplitudes from increasing. The amplitude inhomogeneity results in different kinetic nonlinear frequency shifts of EAWs due to particle trapping [27–29], as shown in our temporal spectrum where the initial resonant peak, $\Delta \omega = 0.24\omega_1$, downshifts to several lower frequencies. This effect also results in different EAW phase velocities, thus adjacent phase space islands merge; it is a common nonlinear phenomenon in the evolution of trapping structures in the phase space [30]. Though the spectrum is broadened at a later time, we have not observed pronounced harmonics, i.e., at integer multiples of $\Delta \omega$, and after lasers are off (red dashed line), the spectra of $E_y$ are coincident with that in the laser pulse, indicating that monochromatic and stable EAWs are finally excited.

When SBS is taken into account, the interaction between undamped EAW and ion acoustic wave (IAW) decides which mode is dominant in the electrostatic field. Generally, the longitudinal field evolution contains both modes, thus we filter out fields of EAW and IAW from the longitudinal field by frequency as shown in figures 3(b)(e) and figures 3(c)(f), respectively. In the frequency domain figure 3(e), the excited undamped EAW...
frequency, $\omega_{\text{IAW}} \approx 0.235\omega_1$ is a little bit smaller than the the initial laser frequency difference, which is also due to the kinetic nonlinear frequency shift. While in figure 3(b), the IAW signal is 2 times weaker than the undamped EAW signal and has a center frequency of $6 \times 10^{-3}\omega_1$. Harmonic effects are still of less significance in this case. The evolution of EAW and IAW reveal the interaction process as shown in figures 3(b) and (c), respectively. Since the interaction quantities are near unity, excited linear IAWs grow faster than SBS to a steady state in the first $2 \times 10^4\omega_1^{-1}$ as shown in figure 3(b). Initial increase of IAW is observed at $t = 1 \times 10^4\omega_1^{-1}$ with the linear IAW frequency $\omega_{\text{IAW}} = k_c = 4 \times 10^{-3}\omega_1$, which is however suppressed by undamped EAWs. As IAW evolves in its nonlinear regime, plenty of laser beams are scattered by IAWs which leads to decreases in laser amplitudes and subsequently suppresses the excitation of undamped EAWs. We pick up a moment in the this evolution stage at $t = 4 \times 10^4\omega_1^{-1}$ and present electron and ion distribution functions in figure 4(a) and 4(c). It is strange that backward SBS of the negative propagating laser beam ($-x$ direction) is stimulated first (figure 4(c)), while in the positive direction IAW has not been formed yet at that time. In the electron distribution function (figure 4(a)), phase space vortex are so small that few electrons are trapped in it, while electrons have been observed to be trapped in IAWs with low phase velocity (not shown). At later time $t = 6 \times 10^4\omega_1^{-1}$, undamped EAWs are reexcited with weaker amplitude, broader spectrum than the initial excitation and then die out when lasers are off, since the nonlinear behaviors of IAWs propagating on both side and frequency shifting to $6 \times 10^{-3}\omega_1$ are dominant. Finally, in this case the monochromatic and stable EAWs last for a duration of $4 \times 10^4\omega_1^{-1}$ (about six thousands of laser periods of $\omega_1$).

In order to further suppress SBS, we performed another case with smaller interaction quantities. The density decreases to 0.28$n_i$, the frequencies are chosen as $\omega_2/\omega_1 = 0.75$, and the thermal velocity remains the same at $v_t = 0.1267c$, which satisfies the matching condition. The corresponding interaction quantities decrease to $\eta_e = 0.94$ and $\eta_i = 1.37$ due to the decrease in density and the increase in $\Delta k \lambda_0$ from 0.27 to 0.33. It is found that the duration of the undamped EAW is further increased to about $5.5 \times 10^4\omega_1^{-1}$. As a consequence, SBS has a negative impact on the excitation of EAW in both the linear and the nonlinear stages, and the more SBS we can suppress, the closer we can get to the fixed ion case, as shown in figure 3(a).

**3.3. Influence of SRS**

In this subsection, we mainly focus on the influence of SRS on the EAW excitation. For simplicity, ions keep immobile to exclude the effects of Brillouin scattering for its small growth rate. Two cases are presented here; the higher density one has the following parameters: $n_e = 0.16n_i$, $\omega_2/\omega_1 = 0.72$, and $v_t = 0.1c$, while for the lower density one $n_e = 0.01n_i$, $\omega_3/\omega_1 = 0.94$, and $v_t = 0.02c$, for comparison. Simulation setups are the same as for the SBS cases, except the total simulation times/durations are $2 \times 10^4\omega_1^{-1}/1.6 \times 10^4\omega_1^{-1}$ for the higher density case and $1 \times 10^4\omega_1^{-1}/0.8 \times 10^4\omega_1^{-1}$ for the lower density case. The grid step of velocity space is 0.0098$v_t$ in the

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**Figure 3.** (a) Evolution of longitudinal field at $x = 2000c/\omega_1$ for fixed ion and (d) corresponding temporal spectrum. (b) and (c) are evolution of undamped EAW and IAW fields for mobile ion by filtering of $E_z(x = 2000c/\omega_1)$. (e) and (f) are corresponding temporal spectra for undamped EAW and IAW, respectively. The red dashed line indicates the time lasers are off. Electric field are normalized by $m\omega_1/e$ and spectra are in arbitrary units (all scales as $\times 10^{-4}$).
Due to the decrease in electron density, the interaction quantities decrease to high density case and monochromatic waves. Long time simulations have also been performed and their main physics will be summarised below.

For this reason, we lower the density to low density case and 0.049\(n_e\) in the low density case. Since we are concerned about the electron instabilities, the timescale for the simulations are enough to reach a nonlinear stage. Long time simulations have also been performed and their main physics will be summarised below.

From equation (8), it is straightforward that the interaction quantity for SRS is relevant to the linear electron density and the plasma dielectric function, \(\eta \propto |\epsilon|^{1/2}n_e^{1/4}\), so that the primary method to mitigate effects of SRS is decreasing the electron density. For the high density case, \(\eta_1 = \gamma_{\text{SRS1}}\tau_B = 2.44\) and \(\eta_2 = \gamma_{\text{SRS2}}\tau_B = 2.88\). The growth of linear EAW is thus slower than the SRS instability. Following the procedures of the SBS cases, we filter out fields of the EAW and Langmuir wave (LW) from the longitudinal field by frequency at \(\pm 2000 n_0 e / \omega_L\), as shown in figures 5(a) and (e), and 5(b) and (f), respectively. In figures 5(a) and (b), it is observed that the monochromatic and stable EAWs last for less than 1000\(\omega_L^{-1}\), instead by strong LWs generated by SRS then. The wave amplitude of initial linear EAWs is about \(4 \times 10^{-5} m_0 e / \omega_L\) compared with 0.03\(m_0 e / \omega_L\) for LWs. Under these parameters, SRS is driven to a highly nonlinear stage, so that no EAWs can compete with LWs. From detailed diagnostics, we find that these LWs are generated from forward Raman scattering, since at such a high temperature a LW from backward SRS is strongly damped \(k_{\text{LW}} \lambda_D = 0.32\), and also the phase space vortex of the undamped EAW \((v_p = 0.18c)\) has squeezed the LW from backward SRS \((v_p = 0.36c)\). Figures 6(a) and 6(b) are typical electron distribution functions when both undamped EAW and LW are excited. The forward SRS generated LW from both beams (figure 6(a)) and EAW (figure 6(b)) are plotted in different colorbars (logarithmic for LW and linear for EAW) to express the multiscale structures. In figure 6(a) we plot the electron distribution function at \(t = 9600 \omega_L^{-1}\) and find that both beams suffer from forward SRS, thus generating a high phase velocity LW. At the same time, excited undamped EAWs in figure 6(b) merge in the phase space due to nonlinear frequency shifts in the spectrum, as shown in figure 5(e). This phenomenon is much like that discovered by Albrecht-Marc et al [30], who considered KEEN waves generated by vortex merging of nonlinear EAWs. The excited waves have broad spectra with an initial frequency of \(\omega_{\text{EAW}} = 0.28 \omega_L\) and a wavenumber of \(k_{\text{EAW}} = 1.52 \omega_L / c\), however, strong LWs make the spectra impure, so they cannot be considered as monochromatic waves.

For this reason, we lower the density to \(n_e = 0.01 n_0\), and the corresponding parameters are \(\omega_2 / \omega_1 = 0.94\), and \(v \approx 0.02 c\), which satisfy the matching condition. Other settings are the same as in the high density case. Due to the decrease in electron density, the interaction quantities decrease to \(\eta_1 = 1.39, \eta_2 = 1.43\). Evolutions of EAW and LW fields are presented in figures 5(c) and (d) which show that the undamped EAW is dominant in
Figure 5. (a) and (b) are the evolution of undamped EAW and LW fields by filtering of $E_\alpha(x = 2000c/\omega_1)$ and corresponding temporal spectra (e) and (f) for the high density case: $n_e = 0.16n_i$, $\omega_2/\omega_1 = 0.72$, $v_\parallel = 0.1c$. (c) and (d) are the evolution of undamped EAW and LW fields by filtering of $E_\alpha(x = 2000c/\omega_1)$ and corresponding temporal spectra (g) and (h) for the low density case: $n_e = 0.01n_i$, $\omega_2/\omega_1 = 0.94$, $v_\parallel = 0.02c$. Ions are fixed in these two cases. Electric field is normalized by $m_\alpha c/e$ and spectra are in arbitrary units (all scales as $10^{-4}$).

Figure 6. Electron distribution functions in different scales ((a) and (b) in logarithmic scale and (c) and (d) in linear scale) to express the multiscale structures of LWs and undamped EAWs. (a) and (b) are sampled at $t = 9600s/c$ for the high density case, and (c) and (d) are sampled at $t = 7500s/c$ for the low density case. (b) and (d) are blowups, but in different scales, of the white-dashed boxes in (a) and (c), respectively.
the first place. The excited EAWs have a monochromatic frequency of $\omega_{EAW} = 0.06\omega_c$, a wavenumber of $k_{EAW} = 1.93\omega/c$, and a corresponding phase velocity of $v_p = 1.55v_c$ as shown in the spectrum in figure 5(g).

The monochromatic and stable EAWs last for nearly 6000$\omega_i^{-1}$ (about 955 laser periods, $T_L$) with the amplitude in the order of $E_0 \approx 8.7 \times 10^{-10}mv_{EAW}/e$. However, at $t \approx 6000\omega_i^{-1}$, LWs are still excited by strong backward SRS, and grow to an amplitude four times that of the EAW amplitude and saturate. In figures 6(c) and (d), we present the interaction of undamped EAWs and LWs in the phase space at $t = 7500\omega_i^{-1}$ when LW is excited. At early time, the linear EAW suppresses the LW in the phase space due to its predominant amplitude, however, LWs generated from both laser beams grow violently and gradually squeeze EAWs in the phase space, even distorting the whole Maxwellian distribution to a highly nonlinear stage, as shown in figure 6(d). At last, both components remain in the wave form after the lasers are turned off. In the lower density case, although strong LW excited in the later time, the duration of undamped EAWs indeed increases when decreasing the interaction quantities, which shows that the mitigation of SRS is good for the excitation of monochromatic and stable EAWs. Further studies of long time simulations to $8 \times 10^4\omega_i^{-1}$ have revealed that once LWs are excited from the instabilities the influences of SRS are ignorable and this then lasts for the whole simulation. Thus better mitigation strategies should be proposed for the excitation.

4. Conclusion and discussion

The motivation for this paper stems from potential applications of undamped EAW such as acceleration, amplification and diagnostics etc., due to its unique characteristic of frequencies and phase velocities that we have never come across before. How to achieve such a monochromatic and stable EAW is a primitive step for the applications. Here we propose a method to excite the wave using two counter-propagating laser beams with a frequency difference and wavenumber difference near the nonlinear EAW dispersion relation. We derive an analytical matching condition and, in addition, we exclude the harmonic effects of EAW and overcome the single-beam laser plasma instability. The strategies proposed in this paper for mitigating LPs are to decrease the interaction quantity, $\eta = \gamma \gamma_0$, where $\gamma$ is growth rate of instabilities and $\gamma_0$ is the characteristic time of EAW.

Next, we present Vlasov–Maxwell simulations to demonstrate the excitation process. Three sets of simulations are performed, the first sets without any instabilities indicate that when the matching condition is well satisfied, stable and monochromatic, EAW can be excited easily. In the second and third sets, we investigate the influences of SBS and SRS, respectively. It is found that both IAW and LW have negative impacts on the excitation process. Ways to mitigate these instabilities include decreasing density or dielectric function, increasing ion mass or electron thermal velocity for suppressing SBS, and decreasing density or dielectric function for suppressing SRS. Although obtaining perfect pure and continuous undamped EAWs is still difficult, we can now excite EAW wave trains that are amplitude tunable, have a duration of thousands of laser periods, and are monochromatic and stable, by carefully controlling the parameters theoretically.

Experiments on the excitation of undamped EAWs may be mostly restricted by the laser and plasma conditions. A frequency multiplication technique could be used for excitation. For example, the frequency difference of a quadruple frequency laser and a triple frequency laser is $4\omega_c / 3\omega_c$ proportional to the product of two normalized laser amplitudes, its effects would be enhanced tremendously by relativistic effects are considered. Since the linear ponderomotive force is proportional to the product of two normalized laser amplitudes, its effects would be enhanced tremendously by relativistic lasers being introduced, and this will be studied in the future.

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