Rapidity and centrality dependence in the percolating colour strings scenario

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Abstract

In AA collisions fusion and percolation of colour strings is studied at fixed rapidity $y$. Distribution of strings in rapidity is obtained from the observed rapidity spectra in pp collisions. For $y$-dependence of multiplicities in Au-Au collisions good agreement is obtained with the existing experimental data. Predictions for LHC energies coincide with the extrapolation of the data. Agreement with the data of the transverse momentum spectra requires introduction of quenching into the model.

1 Introduction

The color string model with fusion [1], [2] and percolation [3]-[6] has produced results on multiplicities of secondaries which are in general agreement with the existing experimental data. The string fusion model predicted a strong reduction of multiplicities both at RHIC and LHC energies. At the time of the ALICE Technical proposal of 95 [7], most of model predictions, (including VENUS [8], HIJING [9], SHAKER [10] and DPM [11]) for LHC energies were more than 4000 charged particles at central rapidity region for central ($b \leq 3$ fm) Pb-Pb collisions and only the prediction of the string fusion model was much lower. Since then many of the models have lowered their predictions introducing several mechanisms, such as the triple pomeron coupling in DPM [12], stronger shadowing in HIJING [13] or other modifications in VENUS [14]. In the parton saturation picture, predictions for the central rapidity density of charged particles per participant for central Pb-Pb at the LHC energy range from around 15 [15] to a lower value around 9 [16]. Assuming that the observed geometrical scaling for the saturation momentum in lepton-hadron scattering is also valid for the nucleus-nucleus scattering, the value around 9 is also obtained [17]. In percolation of strings the obtained values 7.3 [18] and 8.6 [19] are not far from the above ones, as expected, given the similarities between percolation of strings and saturation of partons [20]. In any case, these values lie above 6.4 which is obtained by extrapolation to the LHC energy of the values experimentally found at $\sqrt{s} = 19.4, 62.8, 130$ and 200 GeV [21]. The values obtained in percolation have some uncertainty due to simplifications done in the calculations, mainly related to the dependence of the multiplicity on the energy of a simple string and on the rapidities of fused strings. One of the goals of this paper is to take into account both these dependencies, using as an input the rapidity and energy dependence of multiplicities in pp collisions. We try to answer whether the charge particle density per participant is compatible with the one obtained in the mentioned extrapolation. We assume that strings
occupy different regions in the available rapidity interval. Then at different rapidities one will
see different number of overlapping strings, depending on the rapidities of the string ends. As a
result the string density and the process of percolation become dependent on rapidity together
with all the following predictions for observable quantities. The central part of our derivation
is the calculation of the number of strings at a fixed rapidity which follows from the known
parton distributions in the projectile and target. To this aim we shall use the simple parton
distributions employed in the review [11]. Also, we pay attention to the fragmentation rapidity
region. As an input we take the observed limiting fragmentation scaling in pp collisions. We
then obtain a similar scaling for A-A collisions, in agreement with the experimental data [22].

One of the most interesting features of the RHIC data is the suppression of high transverse
momenta. The nuclear modification factor defined as the ratio between inclusive A-A cross
section normalized to the number of collisions and the inclusive proton-proton cross-section is
found to lie below unity, in disagreement with the perturbative QCD expectations.

In the previous paper [6], in the framework of percolation of strings, a reasonable agreement
with the data was obtained, describing A-A collisions as an exchange of clusters of overlapping
strings. In percolation each cluster behaves like a new string with a larger tension, its value
depending on the number of strings fused into the cluster and the cluster’s transverse area.
Fragmentation of each cluster was assumed to give rise to an exponential distribution in $p_T^2$.
Superposition of different exponential distributions then builds up a power-like distribution
$\sim p_T^{-\kappa}$ with $\kappa$ inversely related to the magnitude of the dispersion in the number of different
clusters. At low density, there is no overlapping of strings, thus no fluctuations and $\kappa$ is large.
As the density increases, so does the string overlapping and more clusters are formed with different
number of strings. So the dispersion increases and $\kappa$ decreases. Finally, at very high density,
above the percolation threshold, there remains a single large cluster of nearly all the strings.
Therefore, there are no fluctuations and $\kappa$ increases again becoming large. In this way, the
suppression of high $p_T$ at large density follows as a result of formation of large clusters of color
strings. In [6] we assumed that the spectrum of a simple string was a single exponential in $p_T^2$.
However, at low string density, as in pp collisions, when fusion of strings is insignificant, the
experimental data clearly show a power-like tail for the $p_T$ distribution.

In this paper we study this point with more attention. Instead of the exponential distribu-
tion we take the standard power-like parameterization for pp collisions [23]. The resulting $p_T$
distribution for central A-A collisions is again found suppressed at high $p_T$ but not enough to
agree with the data. In order to describe the data we need additional suppression which would
physically correspond to the fact that the produced particle, passing through a large cluster and
interacting with the strong chromoelectric field, looses a part of its energy. This result is not
unexpected. In fact a version of HIJING [24], with a string junction and doubling the string
tension to simulate stronger color-fields, is able to explain the difference between baryons and
mesons in the low and mid $p_T$ range but at high $p_T$ some jet quenching mechanism is needed. In
our framework quenching at high $p_T$ may be introduced in a simple phenomenological manner
by taking the average $p_T^2$ of a cluster of $n$ strings to grow with $n$ more slowly than $\sqrt{n}$, as
predicted for a single small cluster in absence of others. Choosing an appropriate $n$ dependence
of the average $p_T^2$ for clusters at a given string density allows to obtain a reasonable agreement
with the experimental data on $p_T$ dependence in A-A collisions.

2 pp collisions

2.1 Multiplicities and numbers of strings

The starting point for the calculation of fusion and percolation of strings in heavy-ion collisions
is the distribution of strings in proton-proton collisions, where effects of fusion and percolation
are very small. Our strategy will be to extensively use the existing experimental data for the multiplicity per unit rapidity $d\mu_{pp}(y)/dy \equiv \mu_{pp}(y)$ in pp collisions to extract the necessary distribution of strings in $y$ from them.

We recall that in the original DPM model without string fusion the multiplicity is given by a sum of contribution from the strings formed in the collision. In particular, in a configuration with $n = 2k$ strings formed, which corresponds to the exchange of $k$ pomerons [11], the multiplicity is given by

$$
\mu_{pp}^{(2k)}(y) = \int_{-Y/2}^{Y/2} \prod_{i=1}^{n} du_i dw_i \rho(u_1, ..., u_n) t(w_1, ..., w_n) \sum_{j=1}^{n} \mu_j(y, u_j, w_j).
$$

We work in the c.m system of colliding protons; $Y$ is the overall rapidity admissible for nearly massless quarks. It is related to the beam rapidity as

$$
Y = Y_{beam} + \ln \frac{m^2}{\mu^2},
$$

where $m$ is the nucleon mass and $\mu$ the quark average transverse mass. The strings are enumerated according to their flavour content, that is according to which quark they are attached. Number 1 corresponds to the quark-diquark (qd) string, number 2 to diquark-quark (dq) string and all the rest correspond to sea quarks, which include $ss$ and $\bar{s}s$ strings. Ends of strings in rapidity are denoted by $u_i$ in the projectile and $w_i$ in the target. Distributions $\rho(u_1, ..., u_n)$ and $t(w_1, ..., w_n)$ give the probability to find the relevant quarks in the projectile and target proton respectively. Note that in the assumed notation $\rho(u_1, ..., u_n)$ gives the probability to find in the proton the valence quark at rapidity $u_1$ the diquark at rapidity $u_2$ and the sea quarks at rapidities $u_3, ..., u_n$. Distribution $t(w_1, ..., w_n)$, on the other hand, gives the probability to find the diquark at rapidity $w_1$, the quark at rapidity $w_2$ and sea quarks at rapidities $w_3, ..., w_n$. Function $\mu_j(y, u, w)$ gives the multiplicity per unit rapidity from the $j$th string at rapidity $y$ provided its ends are at $u$ in the projectile and $w$ in the target. Obviously this probability is zero if the string lies outside rapidity $y$. So $\mu_j(y, u, w)$ has a form

$$
\mu_j(y, u, w) = \rho(y, u, w) \tilde{\mu}_j(y, u, w),
$$

where

$$
\rho(y, u, w) = \theta(u - w - y_0) \theta(u - y) \theta(y - w) + \theta(w - u - y_0) \theta(w - y) \theta(y - u).
$$

Here the two terms correspond to the two possibilities of the higher rapidity end of the string to lie on the projectile or on the target parton. The rapidity interval $y_0$ corresponds to the minimal extension of the string in rapidity. We take $y_0 = 2$.

The string density $dN_{pp}^{(2k)}(y)/dy \equiv N_{pp}^{(2k)}(y)$ at a given rapidity is given by an expression similar to (1) but without $\tilde{\mu}$:

$$
N_{pp}^{(2k)}(y) = \int_{-Y/2}^{Y/2} \prod_{i=1}^{n} du_i dw_i \rho(u_1, ..., u_n) t(w_1, ..., w_n) \sum_{j=1}^{n} \rho(y, u_j, w_j).
$$

To analyse fusion probabilities in nuclear collisions we need to know the latter quantity.

In principle, knowledge of the distributions $\rho(u_1, ..., u_n)$ and $t(w_1, ..., w_n)$ and of string luminosities $\tilde{\mu}_j(y, u, w)$ allows to calculate both $\mu_{pp}^{(2k)}(y)$ and $N_{pp}^{(2k)}(y)$. This was done in the extensive calculations within the original DPM model [11]. However these input quantities are in fact poorly known and our idea is to directly relate $\mu_{pp}^{(2k)}(y)$ and $N_{pp}^{(2k)}(y)$ using the experimental data.
for the former. To do this we assume that string luminosities are approximately \( y \)-independent and the same for all type of strings:

\[
\tilde{\mu}_j(y, u, w) = \mu_0. \tag{6}
\]

This approximation has been widely used in analytical studies of string fusion. It can be justified for relatively long strings far from their ends, when particle production can be well described by the Schwinger mechanism of pair creation in a strong field. With strings of finite dimension it may be considered as a sort of averaging over their length and rapidity of emission. With this approximation we obtain a simple and direct relation between the multiplicity and number of strings per unit rapidity:

\[
\mu_{pp}^n(y) = \mu_0 N_{pp}^n(y). \tag{7}
\]

In fact this relation for the central region was extensively used in earlier studies of string percolation.

Relation (7) allows to find only the total number of strings per unit rapidity from the experimental data on multiplicities. However we need something more. In nucleus-nucleus collisions separately enter multiplicities coming from the valence strings and sea strings. Obviously one cannot find each of them from the experimental data. So we choose to calculate the contribution of valence strings from the theoretical formulas (5) and (7) and then, subtracting this contribution from the experimental multiplicities, find the contribution from sea strings. This procedure can be justified by the fact that distributions of valence quarks are much better known and less dependent on the overall energy than the sea contribution.

### 2.2 Total and sea strings from the experimental data

We calculate the number of quark-diquark strings from (5) as

\[
N_{n}^{qd}(y) = \int_{-Y/2}^{Y/2} du dw q_n^{(p)}(u) d_n^{(t)}(w) \rho(y, u, w)
= \int_{-Y/2}^{Y/2} du \int_{-Y/2}^{w_1} dw \left( q_n^{(p)} (u) d_n^{(t)} (w) + d_n^{(t)} (u) q_n^{(p)} (w) \right), \quad w_1 = \min \{ y, u - y_0 \}. \tag{8}
\]

Here \( q_n^{(p)} (u) \) and \( d_n^{(t)} (w) \) are inclusive probabilities to find a valence quark in the projectile and a diquark in the target at rapidities \( u \) and \( w \) respectively in a configuration with \( n \) strings. The second term in (8) corresponds to inverse strings whose upper ends lie on the target diquark. In our symmetric case the number of diquark-quark strings is obviously the same, so that the total number of valence strings is just twice the expression (8).

The final number of valence strings at given \( y \) is obtained after averaging over the number of formed strings:

\[
N_v(y) = 2 \sum_{k=1}^{\infty} \omega_k N_{(n)}^{qd}(y) = 2 \langle N_{(n)}^{qd}(y) \rangle. \tag{9}
\]

Here \( \omega_k \) is the probability for the exchange of \( k \) pomerons, given by [11]

\[
\omega_k = \frac{\sigma_k}{\sum_{l=1}^{\infty} \sigma_l}, \tag{10}
\]

where \( \sigma_k \) is the cross-section for \( k \) inelastic collisions. It is standardly taken in the K.A.Ter-Martirosyan model [25]

\[
\sigma_n(s) = 2\pi \int_0^\infty db \left( \frac{2\chi}{n!} \right)^n, \tag{11}
\]
where the eikonal $\chi$ corresponds to the single pomeron exchange

$$\chi(s, b) = C(s)e^{-b^2/b_0^2(s)}, \quad (12)$$

with

$$b_0^2(s) = 4R_N^2 + 4\alpha' \left( \ln s - \frac{\pi}{2} \right), \quad C(s) = \frac{g^2}{b_0^2(s)} \left( se^{-i\pi/2} \right)^{\alpha - 1}, \quad (13)$$

$\alpha$ and $\alpha'$ are the pomeron intercept and slope, $g$ is its coupling to the proton and $R_N$ the proton radius. Some improvement of these $\sigma_k$ to include the triple pomeron interaction and diffractive states may be found in [11].

To calculate the number of valence strings per unit rapidity we have to know the inclusive distributions of quarks and diquarks. Following [11] we choose the exclusive distribution $p(u_1, u_2, ... u_n)$ for a projectile in a factorized form

$$p_n(u_1, u_2, ... u_n) = c_n\delta(1-\sum_{i=1}^n x_i)\prod_{i=1}^n x_i^{\mu_i}, \quad (14)$$

where for the quark $\mu_1 = 1/2$ and for the diquark $\mu_2 = 5/2$ For the sea quarks and antiquarks we take $\mu = 1/|\ln x_c|$, where in accordance with [11] $x_c = m_c/\sqrt{s}$ with $m_c = 0.1$ GeV is a cutoff at small $x$. Scaling variables are related to rapidities as

$$x = e^{Y/2+u}, \quad (15)$$

Note that the distributions $p_n(u_1, u_2, ... u_n)$ are defined and normalized in the interval $0 < x < 1$, that is for $-\infty < u < Y/2$. The actual strings are formed only in the part of this interval with $u > -Y/2$. This circumstance is inessential for valence quarks whose distributions rapidly vanish towards small values of $x$. For the distributions in the target one has only to invert the rapidities $u \to -w$ in (15). Integration over the scaling variables of unobserved partons gives the desired inclusive distributions. For the valence quark we find

$$q_p^{(n)}(x) = c_v x^{1/2(1-x)^{3/2+(n-2)\mu}}, \quad c_v = \frac{\Gamma(3+(n-2)\mu)}{\sqrt{\pi}\Gamma(5/2+(n-2)\mu)}. \quad (16)$$

For the diquark

$$d_p^{(n)}(x) = c_d x^{5/2(1-x)^{-1/2+(n-2)\mu}}, \quad c_d = \frac{4\Gamma(3+(n-2)\mu)}{3\sqrt{\pi}\Gamma(1/2+(n-2)\mu)}. \quad (17)$$

After the averaged valence string number is found according to (9) we have to transform it into the valence multiplicity using (7). The value of $\mu_0$ can be found from the observed plateau height assuming that at $y = 0$ all strings contribute. Their average number can be found from (9) as $N^{pp} = 2(k)$. As a result we find values of $\mu_0$ slowly rising with energy and visibly saturating at TeV energies. In Fig. 1 we show these values extracted from the data [23] together with their extrapolation to the LHC energies in the assumption that the plateau in the pp multiplicity distribution rises linearly with $\ln s$. The obtained multiplicities from valence strings vanish in the fragmentation region too slowly as compared to the experimental data, and at $y-Y_{beam} > 0$ become greater than the latter. This is obviously related to our assumption of a constant string luminosity throughout the string length, whereas it should go to zero at its ends. Put in other words, in our approach the total energy is conserved in its division between different strings due to the $\delta$-function in (14) but it is not conserved inside each separate string, since near the string end its luminosity should vanish. To cure this defect in a simple manner we just assume that as soon as the calculated valence contribution becomes larger that the data we substitute
the former by the latter, assuming that in this deep fragmentation region sea strings do not contribute at all.

With thus obtained valence contribution to the multiplicities we find the sea contribution just as the difference between the total and valence one. Dividing it by \( \mu_0 \) we find the number of sea strings per unit rapidity in pp collisions \( N^{s,pp}(y) \). In fact we need not exactly this number but the one in the assumption that all strings are of the sea type, which is obtained from it by rescaling

\[
N^{s}(y) = \frac{N^{pp}}{N^{pp} - 2} N^{s,pp}(y).
\]

This quantity is shown in Fig. 2 for different values of the collision energy \( \sqrt{s} \).

\section{hA and AA collisions}

Generalization to hA and AA collisions is straightforward and follows [11]. At fixed impact parameter \( b \) one introduces the average numbers of participants \( 2 \nu_A(b) \) and collisions \( \nu(b) \). Then the number of strings in AA collisions at given \( b \) and \( y \) is

\[
N_{AA}(b, y) = \nu_{par}(b) N^{pp}(y) + \left( \nu_{col}(b) - \nu_{par}(b) \right) N^{s}(y).
\]

Here \( N^{pp}(y) \) is obtained from the observed multiplicity in pp collisions according to (7) and \( N^{s}(y) \) is given by (18).

One can easily further generalize (19) to collisions of different nuclei (see [11]).

In the Glauber approach the numbers \( \nu_{par} \) and \( \nu_{col} \) for AA collisions are obtained as follows

\[
\nu_{par}(b) = A \frac{\int d^2b' T_A(b') \left( 1 - e^{-A \sigma T_A(b - b')} \right)}{1 - e^{-A^2 \sigma T_{AA}(b)}}, \quad \nu_{col}(b) = A^2 \frac{\sigma T_{AA}(b)}{1 - e^{-A^2 \sigma T_{AA}(b)}}.
\]

Figure 1: String luminosity as a function of energy
Figure 2: Sea string contribution to multiplicities per nucleon as a function of rapidity. Curves from bottom to top correspond to energies 19.4, 62.8, 130, 200 and 6000 GeV.

where $\sigma$ is the total pp-cross-section, $T_A(b)$ is the nuclear profile function normalized to unity and

$$T_{AA}(b) = \int d^2b'T_A(b')T_A(b - b').$$

For hA collision, as mentioned, $\nu_{\text{par}} = 1$ and

$$\nu_{\text{col}}(b) = A\frac{\sigma T_A(b)}{1 - e^{-A\sigma T_A(b)}}.$$  

Note that for AA collisions in the above formulas the denominator is written in the so-called optical approximation [26]. As is well-known, it works reasonably well except close to the nucleus boundary, where the collision numbers obtained from (20) may be quite deceptive.

It is customary to take the profile function $T_A(b)$ generated by the Woods-Saxon nuclear density. However for our purpose it is more convenient to assume the nucleus to have a well-defined radius, which allows to determine the interaction area in the transverse plane as just the area of the overlap. For this reason we take the nucleus as a sphere of radius $R_A = A^{1/3} \cdot 1.2$ fm, which gives

$$T_A(b) = \theta(R_A - b) \frac{2\sqrt{R_A^2 - b^2}}{V_A},$$

where $V_A$ is the nuclear volume. With this choice the most peripheral collisions occur at $b = 2R_A$ with $\nu_{\text{col}} = \nu_{\text{par}} = 1$.

4 Multiplicities and $p_T$ distributions

In our previous studies of string fusion we always stressed that it can only occur in the common rapidity interval. However our attention was mostly centered on the central rapidity region where all (or nearly all) strings contribute, so that the requirement of common rapidity interval
was of no relevance and strings could be considered as of practically infinite length in rapidity. Now we study the fusion process in more detail. At a fixed rapidity \( y \) only strings which pass through this rapidity can fuse. Formulas of the previous sections allow to find the original number of strings \( N(b, y) \) stretched between the projectile and target at fixed rapidity layer \( y \) and impact parameter \( b \). According to the percolation colour strings scenario these strings in fact fuse into strings with higher colour. The intensity of fusion is determined by the dimensionless percolation parameter \( \eta \) proportional to the string density in the interaction area

\[
\eta(b, y) = \frac{N(b, y)s_0}{S(b)},
\]

(24)

where \( s_0 = \pi r_0^2 \) is the transverse area of the string and \( S(b) \) is the interaction area, that is, the overlap area in case of AB collisions. Obviously in our case the percolation parameter depends both on \( b \) and \( y \). At \( \eta \sim 1.2 \div 1.3 \) fusion of strings leads to their percolation and formation of macroscopic string clusters. This phenomenon will take part only in restricted intervals of \( b \) and \( y \), predominantly at central collisions and rapidities, where the effects of string fusion and percolation will be most noticeable.

Considering the case of AA collisions at reasonably high energies we shall assume the total number of strings high enough to allow use of the thermodynamic limit, in which the total areas of \( n \)-fold fused strings \( S_n \) become distributed according to the Poisson law with \( \langle n \rangle = \eta(b, y) \):

\[
S_n(b, y) = S(b)e^{-\eta(b, y)}\frac{\eta^n(b, y)}{n!}.
\]

(25)

Due to averaging of the direction of colour, the \( n \)-fold fused string emits the number of particles which is only \( \sqrt{n} \) times greater than the simple string. So the total production rate at fixed \( b \) and \( y \) will be given by

\[
\mu(b, y) = \mu_0 \frac{S(b)}{s_0}e^{-\eta(b, y)}\sum_{n} \sqrt{n} \frac{\eta^n(b, y)}{n!}.
\]

(26)

where \( \mu_0 \) is the production rate from the single string, which, as stated above, we assume to be independent of \( y \) but dependent on energy.

As to the \( p_T \) distribution, we use a slightly generalized model introduced and discussed in [27], in which the normalized probability \( w_n(p) \) to find a particle with transverse momentum \( p \) emitted from the \( n \)-fold fused string is given by

\[
w_n(p) = \frac{(\kappa_n - 1)(\kappa_n - 2)}{2\pi p_n^2} \left( \frac{p_n}{p + p_n} \right)^{\kappa_n}.
\]

(27)

Here for \( n = 1 \) the parameters are determined by the experimental data on pp collisions:

\[
p_1 = 2 \text{ GeV}/c, \quad \kappa_1 = 19.7 - 0.86 \ln E_{cm}
\]

(28)

and \( E_{cm} \) is the c.m. energy in GeV. With \( n > 1 \) from the string fusion scenario it follows that the average transverse momentum squared of the particles emitted from the \( n \)-fold fused string is \( n^{1/2} \) greater than for a single string. This gives a relation between \( p_n \) and \( \kappa_n \)

\[
p_n^2 = \frac{n^{1/2}}{p_1^2} \frac{(\kappa_n - 3)(\kappa_n - 4)}{(k_1 - 3)(k_1 - 4)}.
\]

(29)

So the distribution from \( n \)-fold string is fully determined by the \( n \)-dependence of \( \kappa_n \). In [27] the simplest choice of \( n \)-independent \( \kappa_n \) was used. However this simple choice does not allow to obtain the \( p_T \) dependence in agreement with the data at RHIC. To improve our description, we introduce corrections to the original string picture which correspond to non-linear phenomena in string clustering and influence both \( \kappa_n \) and the behaviour of \( p_n \).
In fact the value of $\kappa$ controls the difference of the distribution from the purely exponential one, passing into the latter at very large $\kappa$. For a single string a finite $\kappa_1$ may be thought of as a result of fluctuations in the string tension (or equivalently its transverse area) [28]. One may expect this fluctuations to grow as many string fuse and so the value of $\kappa_n$ should fall with $n$. However there is another effect acting in the opposite direction. As many strings fuse into clusters, multiple interactions of emitted particle inside the clusters should lead to thermalization of the particle spectra making it closer to an exponential. Thus eventually at large $n$ parameter $\kappa_n$ should grow to large values. Naturally we cannot determine the exact form of the $n$ dependence of $\kappa_n$ from purely theoretical reasoning. We can only think that the change from fall to growth should occur in the vicinity of the percolation threshold and that in any case $\kappa$ cannot be smaller than 4 to have a convergent $<p^2>$. In practice we take $\kappa_n$ as

$$\kappa_n = \kappa_1 + a(n-1) + b(n-1)^2$$

(30)

and try to adjust $a$ and $b$ to get a better agreement with the experimental data. In fact the results are not very sensitive to the choice of $a$ and $b$ provided they are taken to have a behaviour of $\kappa_n$ in agreement with the above general theoretical observations. However the generalization (30) is not sufficient to bring our predictions in agreement with the observed quenching of the ratios $R_{AA}$ at RHIC. To this aim we have to introduce some quenching also in the string picture at large values of $\eta$. It corresponds to the fact that passing through a large cluster volume and interacting with the strong chromoelectric field the produced particles loose a part of their energy [29]. On our phenomenological level it would correspond to the behaviour of the average transverse momentum squared as

$$<p^2>_n = n^{\alpha_n} <p^2>_1$$

(31)

with the exponent $\alpha_n$ less than 1/2 and diminishing with $n$. Similarly to (30) we parameterize

$$\alpha_n = \frac{1}{2} + c(n-1) + d(n-1)^2.$$  

(32)

The comparison with the experimental data determines the optimal fit for the parameters $a$, $b$, $c$ and $d$. The resulting values for $\kappa_n$ and $\alpha_n$ are shown in Figs. 3 and 4.

Averaging with the distribution in $n$ we find the final distribution in $p$ from the fusing strings at given $b$ and $y$ as

$$w(p, b, y) = a(b, y) \sum_{n=1}^{\infty} w_n(p) \frac{\eta^n(b, \nu)}{n!}, \quad a^{-1}(b, y) = e^\eta(b, y) - 1$$

(33)

(the change in the normalization is due to the restriction $n \geq 1$).

5 Numerical results

We studied Au-Au collisions at energies 19.4, 62.8, 130, 200 and 6000 GeV corresponding to the existing experimental data and expected at LHC. Using our results on the string numbers in pp collisions we calculated their numbers in nucleus-nucleus collisions at a given $y$. Knowing these numbers and also numbers of participants and collisions we then determined values of the percolation parameter $\eta(y, b)$ at different rapidities and impact parameters. We have taken the transverse radius of the single string 0.3 fm. In Fig. 5 and 6 we illustrate values of $\eta(y, b)$ as a function of $y$ for central collisions and as a function of $b$ at mid-rapidity. As one can observe, at RHIC and LHC energies these values are quite large, far beyond the percolation threshold. As a result one finds a very substantial reduction in multiplicity calculated according to Eq.
Figure 3: Parameter \( \kappa_n \) in the distribution (27) for the \( n \)-fold fused string. Curves from top to bottom correspond to energies 19.4, 62.8, 130, 200 and 6000 GeV. The curves end at maximal \( n \) reached at these energies (26) with luminosities determined from pp collisions. In Fig. 7 and 8 we show multiplicities as a function of \( y \) for central collisions and function of \( b \) at midrapidity. They agree rather well with the existing data both in form and absolute values. In Fig. 8 a rather sharp change is seen in the periphery of the nuclei, between \( b/2R_A = 0.8 \) and unity. The RHIC data do not exhibit such a sharp saturation. This behaviour is a direct consequence of using in our calculations the nuclear profile function (23) corresponding to the step function for the nuclear density. A more realistic profile function would lead to a smoother transition from the periphery to center, although requiring a more complicated definition of the interaction area.

Our prediction for the plateau at LHC energy (divided by \( \nu_{\text{par}} \)) is around 9 for the sum of charged and neutral particles (that is 6 for charged), which is lower than derived in other publications [13]-[17]. The plateau height for charged particles in central Au-Au collisions as a function of energy is illustrated in Fig. 9. Of course our results are directly related to the chosen string radius and go upward if it is lowered. However then we loose the agreement with the existing data.

Note that our calculations also reproduce quite well the behaviour in the fragmentation region (limiting fragmentation). For this the dependence of the string luminosity on energy proved to be quite important. Without it the nice linear dependence of multiplicities in the fragmentation region is spoiled and the line is widened into a band.

To clearly see the effect of string fusion in Fig 10 we show the multiplicities at \( b = 0 \) without fusion. Their values are several times greater than with fusion and do not agree with the experimental data at all.

Passing to the \( p_T \) distributions in Fig. 11 we show the ratios \( R_{AA}(p_T) \) for central collisions at midrapidity. The curve for 200 GeV served to determine our parameters \( a, b, c \) and \( d \) in (30) and (32). Our predictions for the LHC energy show a behaviour similar to the RHIC energy with a still more pronounced quenching effect. In the fragmentation region we prefer to show the ratios \( R_{AA}^{\text{par}} \) with normalization respective to the number of participants, since in this region
the multiplicities are roughly proportional to $\nu_{\text{par}}$ due to low string densities. Fig. 12 shows that these ratios are close to unity and may only fall a little below unity at LHC energies.

6 Conclusions

In the framework of percolation of strings we have obtained a strong reduction of multiplicities at LHC, much larger than the rest of models but in agreement with the extrapolation from the SPS and RHIC experimental data. Due to similarities between percolation of strings and saturation of partons, it would be interesting to explore the possibility for further reduction of multiplicities in the saturation approach. In order to describe the energy dependence of multiplicities in AA collisions we need a rather large transverse size of the elementary string 0.3 fm. This enhances the interaction of strings and so cluster formation, which leads to stronger reduction of multiplicities. In our calculations we have used the standard optical approximation to compute the numbers of participants and collisions. This approximation enhances the number of collisions for peripheral collisions in comparison with Monte-Carlo evaluations and leads to some uncertainties also for central collisions. For this reason, our results must be regarded to have an uncertainty in the range of 10%-15%. So the string transverse size can be lower if the number of strings is in fact lower.

Taking limiting fragmentation scaling for pp collisions as an input, we have found the same behaviour for AA collisions, which is confirmed experimentally up to the RHIC energies. Our calculations predict that limiting fragmentation scaling also remains approximately valid at the LHC energy (with a 5% suppression compared with to SPS or RHIC, see Fig. 7).

We have been able to describe reasonably well the high transverse momentum spectrum at different energies ranging from SPS to RHIC. A large suppression is predicted for LHC. In order to obtain such an agreement, in addition to the usual effects of string clustering, such as reduction of the effective number of independent color sources and suppression of transverse momentum fluctuations, we need a shift of the $p_T$ spectrum due to energy loss. Considering
string fusion as an initial state effect (before particle production), a final state effect is needed to account for the observed suppression, similarly to jet quenching in the QCD picture.

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Figure 6: Percolation parameter $\eta$ in Au-Au collisions at midrapidity as a function of impact parameter. Curves from bottom to top correspond to energies 19.4, 62.8, 130, 200 and 6000 GeV.

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Figure 7:_multiplicities (charged + neutals) per 1/2 number of participants in central Au-Au
    collisions as a function of rapidity. Curves from bottom to top correspond to energies 19.4, 62.8,
    130, 200 and 6000 GeV.
Figure 8: Multiplicities (charged plus neutrals) per 1/2 number of participants in Au-Au collisions at midrapidity as a function of impact parameter. Curves from bottom to top correspond to energies 19.4, 62.8, 130, 200 and 6000 GeV.

Figure 9: The plateau height for charged particles per 1/2 number of participants in Au-Au collisions at midrapidity as a function of energy.
Figure 10: Multiplicities (charged plus neutrals) per 1/2 number of participants in central Au-Au collisions at midrapidity without fusion. Curves from bottom to top correspond to energies 19.4, 62.8, 130, 200 and 6000 GeV.

Figure 11: Nuclear factor $R_{AA}$ for central Au-Au collisions at midrapidity. Curves from top to bottom correspond to energies 19.4, 62.8, 130, 200 and 6000 GeV.
Figure 12: Nuclear factor $R_{AA}^{\text{par}}$ for central Au-Au collisions in the fragmentation region $y = 0.1Y$. Curves from top to bottom correspond to energies 19.4, 62.8, 130, 200 and 6000 GeV.