Evidence for a superfluid density in t–J ladders

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Applying three independent techniques, we give numerical evidence for a finite superfluid density in isotropic hole-doped t–J ladders: We show the existence of anomalous flux quantization, emphasizing the contrasting behaviour to that found in the ‘Luttinger liquid’ regime stabilised at low electron densities; We consider the nature of the low-lying excitation modes, finding the 1-D analog of the superconducting state; And using a density matrix renormalization group approach, we find long range pairing correlations and exponentially decaying spin-spin correlations.

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The behaviour of strongly correlated electrons confined to coupled chains is at present a topic undergoing much investigation; the reasons for this attention are numerous. Firstly, with the behaviour of electrons under t–J or Hubbard type interactions in one-dimension now relatively well understood, the two-chain systems provide an interesting ‘first step’ towards the challenge of two-dimensions. Secondly, the unusual nature of the ground state of the undoped system, in particular the existence of a spin gap, leads to further interest with regards to ‘gapped’ superconducting behaviour. Furthermore, it is believed that compounds such as (VO$_2$)P$_2$O$_5$, SrCu$_2$O$_2$Cl$_2$ may be described by a lattice of coupled chains. Whilst there is considerable literature concerning the possible phases in a t–J ladder, a complete picture is still far from being realised and our aim is to clarify the behaviour in a particular region, specifically considering the nature of the gapped state when the system is doped. Various techniques have been applied previously in this hole doped region, and there is some indication for hole pairing and modified d-wave superconducting correlations.

The t–J Hamiltonian on the 2 × L ladder is defined as,

$$\mathcal{H} = J' \sum_j (S_{j,1} \cdot S_{j,2} - \frac{1}{4} n_{j,1} n_{j,2})$$

$$+ J \sum_{\beta,\gamma} (S_{j,\beta} \cdot S_{j+1,\gamma} - \frac{1}{4} n_{j,\beta} n_{j+1,\gamma})$$

$$-t \sum_{j,\beta,\gamma} P_G(c_{j,\beta,s}^\dagger c_{j+1,\gamma,s} + H.c.)P_G$$

$$-t' \sum_{j,\beta,\gamma} P_G(c_{j,1,\beta,s}^\dagger c_{j,2,\gamma,s} + H.c.)P_G,$$  \hspace{1cm} (1)

where most notations are standard. $\beta (=1,2)$ labels the two legs of the ladder (oriented along the $x$-axis) while $j$ is a rung index ($j=1,\ldots,L$). We shall concentrate on the isotropic case where the intra-ladder (along $x$) couplings $J$ and $t$ are equal to the inter-ladder (along $y$) couplings $J'$ and $t'$.

At half filling, the hamiltonian reduces to the Heisenberg model and the behaviour is generally relatively well understood. A simple interpretation is given by considering the strong coupling limit ($J = 0$): in such a limit, the ground state consists of a singlet on each rung with a spin gap (∼ $J'$) which corresponds to forming a triplet on one of the rungs. With the introduction of interchain coupling $J$, the triplets can propagate and form a coherent band thereby reducing the spin gap. In the isotropic case the gap remains (∼ 0.5$J$) and it is the nature of the state formed on doping such a system that we shall concentrate on.

A possible phase diagram for the isotropic t–J ladder as a function of $J/t$ and doping has been proposed recently, the main features of which we summarize. Away from half filling, the spin gapped region persists, exhibiting hole pairing and, as we will show, possible superconducting correlations. This behaviour is observed for a large region of $J/t$ except for $J/t > \sim 2$, where the system phase separates and also (perhaps) very small $J/t$ where the gap may disappear. As the system is doped further, a Luttinger-like phase is stabilised exhibiting gapless spin and charge excitations. At very small electron densities, an electron paired phase exists.

In this letter we will describe three independent forms of evidence for a finite superfluid density in the spin gapped region of the phase diagram (we will work specifically with an electron density $\langle n \rangle = 0.8$). The first set of results, and the ones we deal in most detail with, are based on the existence of anomalous flux quantization, a feature present in a superconducting state. Secondly, we consider the spin and charge excitation modes which may be used to characterize the state of a system. Finally, we present direct calculations of correlation functions which have been obtained using the density matrix renormalization group method.

Our first set of results then concern the existence of anomalous flux quantization. The calculation involves threading the double chain ring with a flux $\Phi$ and studying the functional form of the ground state energy with
A distinction should be made between the threaded flux, namely \( E_0(\Phi) \); throughout this letter we will measure the flux \( \Phi \) in units of the flux quantum \( \Phi_0 = \hbar c/e \). In general, \( E_0(\Phi) \) consists of an envelope of a series of parabola, corresponding to the curves of individual many body states \( E_0(\Phi) \), exhibiting a periodicity of one. Byers and Yang have shown that in the thermodynamic limit, \( E_0(\Phi) \) exhibits local minima at quantized values of \( \Phi \), the separation of which is \( 1/n \) where \( n \) is the sum of charges in the basic group; these local minima in \( E_0(\Phi) \) must be separated by a finite energy barrier. Hence, for a superconductor we would expect minima in \( E_0(\Phi) \) at intervals of \( 1/2 \); these minima are related to the existence of supercurrents which are trapped in the metastable states corresponding to the flux minima and are thus unable to decay away. This phenomenon is known as anomalous flux quantization. It should be mentioned that the existence of anomalous flux quantization is an indication of pairing and is not sufficient in itself to imply a superconducting state.

Detailed studies of the attractive Hubbard model on two-dimensional lattices have indicated the presence of anomalous flux quantization, confirming the existence of superconducting correlations in the ground state; in contrast, the repulsive Hubbard model exhibits no anomalous flux quantization.

In addition to the existence of flux quantization, the function \( E(\Phi) \) also gives a quantitative value of the superfluid density, defined in one dimension by

\[
D_s = \frac{\partial^2}{\partial \Phi^2} \left( \lim_{L \to \infty} |LE_0(\Phi)| \right)
\]

A distinction should be made between \( D_s \) (the superfluid density) and \( D \) (the Drude weight). The superfluid density corresponds to the curvature of the envelope of the individual many body states as a function of flux, whilst the Drude weight is obtained from the curvature of a single ground state many body energy level. In general these quantities are different. In the thermodynamic limit no particular applied flux is preferred when calculating \( D_s \) and hence we consider the curvature of the whole \( LE_0(\Phi) \) curve when considering the superfluid density. Note that the existence of superconductivity requires both anomalous flux quantization and a finite superfluid density (and indeed \( D_s \) has no real meaning in the absence of anomalous flux quantization).

Numerically, the application of a flux through the double chain ring is achieved by modifying the kinetic term of the hamiltonian such that

\[
c^{\dagger}_{j, \beta, s} c_{j+\beta, s} \rightarrow c^{\dagger}_{j, \beta, s} c_{j+\beta, s} e^{i \Phi j} \quad \text{(3)}
\]

where \( \Phi \) is the flux through the ring measured in units of \( \Phi_0 \) and \( L \) is the length of either chain. Hence the application of a flux is numerically equivalent to a change in the boundary conditions of the problem; \( \Phi = 0 \) representing periodic boundary conditions and \( \Phi = \frac{\pi}{L} \) representing anti-periodic boundary conditions.

The technique we have employed is exact diagonalisation of finite systems, specifically \( 2 \times 5 \) and \( 2 \times 10 \) double chain rings with intermediate electron densities \( \langle n \rangle = 0.8 \) and \( \langle n \rangle = 0.4 \) corresponding to the regions of the phase diagram where we expect spin gapped or Luttinger liquid behaviour respectively. Note that the electron number is always a multiple of 4 in order to guarantee that antiferromagnetic correlations are not frustrated when one goes around each chain. The modes of the system are characterized firstly by their spin: singlet and triplet excitations correspond to charge and spin modes respectively. It is also useful to consider the parity of the states of the system under a reflection in the symmetry axis of the ladder along the direction of the chains: Even (\( R_s = 1 \)) or odd (\( R_s = -1 \)) excitations correspond to bonding or anti-bonding modes respectively. Finally, it is necessary to consider the momentum, \( k_s = 2 \pi n / L \) in order to determine the dispersion relation of each mode. Implementation of these quantum numbers and symmetries is straightforward using exact diagonalisation methods and the various excitation modes may be obtained by calculating the ground state energy in each symmetry sector.

Concentrating initially on \( J/t = 0.5 \langle n \rangle = 0.8 \), we show in Figs. (a)(b) all possible spin and charge modes of the \( 2 \times 5 \) (\( 2 \times 10 \)) system, for all possible momenta, as a function of applied flux. In the case of the larger system, we have omitted some of the details of the excited states to simplify the diagram, showing the full spectrum only for \( \Phi < 0.25 \). For both system sizes, the minimum energy function \( E_0(\Phi) \) is formed by charge (spin zero) bonding modes; the excited modes with different quantum numbers move further from the ground state as the system size increases (a result we have checked by finite size scaling techniques) and hence will not interfere with \( E_0(\Phi) \). The existence of minima at intervals of half a flux quantum (i.e. anomalous flux quantization) clearly indicates the existence of pairing.

In order to probe the behaviour of \( E_0(\Phi) \) further, we consider the quantity \( L \left[ E_0(\Phi) - E_0(\Phi = 0) \right] \) as a function of \( \Phi \) for various values of \( J/t \) and \( \langle n \rangle \) (\( L \) is the length of the ladder). Note that the curvature of this function in the thermodynamic limit gives the superfluid density. Fig. (a) shows the contrasting behaviour obtained when keeping \( J/t \) fixed at 1.0 and varying the electron filling, specifically \( \langle n \rangle = 0.4 \) and \( \langle n \rangle = 0.8 \) (both the \( 2 \times 5 \) and \( 2 \times 10 \) results are shown). This plot clearly shows the existence of anomalous flux quantization for a filling of \( \langle n \rangle = 0.8 \) and its absence for \( \langle n \rangle = 0.4 \). The occurrence of the absolute minima at different values of flux \( \Phi = 0 \) and \( \Phi = 1/2 \) for \( \langle n \rangle = 0.8 \) and \( \langle n \rangle = 0.4 \) respectively can be explained by considering the non-interacting Fermi sea for the two fillings; a lower energy state is formed by choosing the flux (and hence boundary conditions) to give a closed shell. Fig. (b) shows an equivalent plot but in this case keeping \( \langle n \rangle \) constant at 0.8 and varying the parameter \( J/t \) from 0.5 to 4.0. In this case anomalous flux quantization is exhibited for \( J/t = 0.5 \), whilst for \( J/t = 4.0 \)
\[ L \left[ E_0(\Phi) - E_0(\Phi = 0) \right] \] appears to scale to a flat function, consistent with the existence of a phase separated region \((D_s = 0)\).

Except for the region believed to be phase separated, the form of the curve \( L \left[ E_0(\Phi) - E_0(\Phi = 0) \right] \) appears to show only relatively small finite size effects and may be easily extrapolated to the thermodynamic limit; hence, by considering the curvature, an accurate value of the superfluid density (and the Drude weight) can be found. In a future publication we analyse the specific values in more detail but for this letter we emphasise that \(D_s\) scales to a finite value in the thermodynamic limit in the regions which are not phase separated.

The second form of evidence for a superfluid density lies in the dispersion behaviour of the low energy spin and charge excitations of the system. The possible phases of a particular model can be characterized by the number of charge and spin modes which are gapless at zero momentum. The one-dimensional analog of a superconductor has one gapless charge mode, a gap to all spin excitations and dominant pairing. The phase diagram for the various possible phases for a chain with a Hubbard (rather than t–J) Hamiltonian has recently been found and interestingly on doping away from half filling, this spin gapped phase with one gapless charge mode is stabilised, exactly the situation we will suggest for the t–J case. In addition, both this article and a recent paper by Nagaosa note that in this gapped phase, in contrast to a Luttinger liquid phase, the power law term in the density-density correlation function at \(2k_f\) is missing.

In Fig. 3 we show the dispersion of all the spin and charge modes (for both bonding and antibonding symmetry sectors), for the \(2 \times 10\) ladder with \(\langle n \rangle = 0.8\), \(J/t = 1.0\), corresponding to the region of the phase diagram where we believe the superfluid to exist. There are several obvious features of this diagram. Firstly there is a finite gap to spin excitations; secondly there is (at least) one vanishing charge mode (bonding) as \(k_x \rightarrow 0\) and thirdly, there is no sign of \(2k_f\) charge gapless modes. All of these features point towards a spin liquid with dominant superconducting correlations as previously explained. We should note the possibility of a new charge gapless mode at finite momenta corresponding to the fluctuations of the pair density as indicated by the dip of the bonding charge mode in Figure 3. Some more detailed descriptions of these results (along with the corresponding results from other regions of the phase diagram) are given in a separate publication.

The final results we present are direct numerical calculations of various correlation functions obtained using the density matrix renormalization group approach. This technique allows much larger systems to be considered than is possible with exact diagonalization techniques. We present three types of correlations: Firstly the equal-time rung-rung pair field correlation function \(\langle \Delta_j \Delta_j \rangle\) where \(\Delta_j^+ = (c_{j,1;\uparrow}^+ c_{j,2;\downarrow} - c_{j,1;\downarrow}^+ c_{j,2;\uparrow})\), \(i\) and \(j\) are rung indices and 1 and 2 indicate the chain; this correlation function then creates a singlet pair on rung \(j\) and removes a singlet pair from rung \(i\) and dominant pairing. The phase diagram for the various combinations of various correlation functions obtained using the density matrix renormalization group approach allows much larger systems to be considered than is possible with exact diagonalization techniques.

In summary, we have presented three independent forms of evidence for a superfluid density in hole doped t–J ladders. Firstly we have shown the existence of anomalous flux quantization and a well-converged finite \(D_s\); Secondly we have studied the low lying modes, finding a spin-gapped state with a gapless charge mode and no gapless \(2k_f\) excitations; Finally we have presented direct calculations of correlation functions, showing long range pairing correlations and exponentially decaying spin correlations.

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FIG. 1. Energy as a function of flux (in units of $\Phi_0 = \hbar c/e$) for different system sizes with $J/t = 0.5$, $\langle n \rangle = 0.8$. We show all possible momenta for various quantum numbers: For the charge modes the solid lines correspond to bonding and the dotted lines to anti-bonding, whilst for the spin modes the dashed lines correspond to bonding and the dot-dashed lines to anti-bonding. Figure 1a (1b) corresponds to a system size of $2 \times 5$ ($2 \times 10$); for the larger system size we give only the charge bonding modes and the lowest lying spin anti-bonding mode in full in order to simplify the diagram.

FIG. 2. $L [E_0(\Phi) - E_0(\Phi = 0)]$ where $L$ is the length of the ladder and $E_0(\Phi)$ is the ground state energy with an applied flux $\Phi$. The dashed lines correspond to $2 \times 5$, the solid lines to $2 \times 10$. Fig. 2a shows the results for $\langle n \rangle = 0.4$ and $\langle n \rangle = 0.8$ both with $J/t = 1.0$, whilst fig. 2b shows the results for $J/t = 0.5$ and $J/t = 4.0$ both with $\langle n \rangle = 0.8$.

FIG. 3. Spin and charge excitation modes of a $2 \times 10$ ladder versus momentum $k_x$ (in units of $\pi$); $\langle n \rangle = 0.8$ and $J/t = 1.0$. The quantum numbers associated with the various symbols are shown on the plots.

FIG. 4. Log-log plot of various correlation functions versus $|i - j|$ (real space separation) for a $2 \times 30$ open chain with $\langle n \rangle = 0.8$ and $J/t = 1.0$. The dashed line has a slope $-2$ and the dotted line $-1$. The correlation function are explained in the main text.
a) $2 \times 5$, $J/t=0.5$, $\langle n \rangle=0.8$
b) $2 \times 10^{-J/t=0.5}$ $<n>=0.8$
a) $\langle n \rangle = 0.8$

$J/t = 1.0$

$\langle n \rangle = 0.4$

$J/t = 1.0$
$N = 2 \times 10$

$\langle n \rangle = 0.8$

$J/t = 1$

**Charge**
- $R_x=1$
- $R_x=-1$

**Spin**
- $R_x=1$
- $R_x=-1$
2x30 $t-J$, $t_{\perp}=t=1.0$, $J_{\perp}=J=1.0$

- Pairing
- Density–density
- $S^z-S^z$

$\langle n \rangle = 0.80$