RF field-attenuation formulae for the multilayer coating model

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Abstract

Formulae that describe the RF electromagnetic field attenuation for the multilayer coating model with a single superconductor layer and a single insulator layer deposited on a bulk superconductor are derived from a rigorous calculation with the Maxwell equations and the London equation.

INTRODUCTION

An idea to enhance the rf breakdown field of superconducting cavities by multilayered nanoscale coating is proposed by A. Gurevich in 2006 [1]. The model consists of alternating layers of superconductor layers (S) and insulator layers (I) deposited on bulk Nb. The S layers are assumed to withstand higher field than the bulk Nb and shield the bulk Nb from the applied rf surface field $B_0$, by which $B_0$ is decreased down to $B_i < B_0$ on the surface of the bulk Nb. Then the cavity with the multilayered structure is thought to withstand a higher field than the Nb cavity, if $B_0$ is smaller than the vortex penetration-field $2\lambda_0$ of the top S layer, and $B_i$ is smaller than 200 mT, which is thought to be the maximum field for the bulk Nb. In order to evaluate the shielded magnetic field $B_s$, the magnetic field-attenuation formulae for the multilayered structure are necessary.

When a magnetic field is applied to a superconductor, the Meissner screening current is induced, which restricts the penetration of the field to a surface layer. This effect is often explained by applying the London equation $d^2B/dx^2 = B/\lambda^2$ to a semi-infinite bulk superconductor in the region $x \geq 0$ with boundary conditions $B(0) = B_0$ and $B(\infty) = 0$, where $\lambda$ is the London penetration depth and $B_0$ is the applied surface field parallel to the superconductor surface. The solution is written as $B(x) = B_0 e^{-x/\lambda}$, which means the penetration of the field is restricted to depth $\lambda$. This solution, however, is just a solution of the London equation for a special case: a solution for semi-infinite bulk superconductor with boundary conditions given above. For different configurations such as a multilayer coating model, the London equation should be solved with appropriate boundary conditions, and solutions are generally different from $B(x) = B_0 e^{-x/\lambda}$. In this paper the RF electromagnetic field-attenuation formulae for the multilayer coating model are derived from a rigorous calculation with the Maxwell equations and the London equation.

\[ \Delta E = \kappa^2 E, \quad \Delta B = \kappa^2 B, \quad (1) \]

where

\[ \kappa^2 = \frac{1}{\lambda^2} \left( 1 - i\mu_0 \sigma_n \omega \lambda^2 - \omega^2 \frac{\sigma_n}{c^2} \lambda^2 \right), \quad (2) \]

$\kappa^2$ is the susceptibility, $\omega$ is the angular frequency, and $\sigma_n$ is a conductivity for the normal conducting carriers. Assuming the typical value of $\lambda \approx 10^{-7} \text{m}$ with $\omega \approx 10^9 \text{Hz}$ for rf applications, the third term of the right-hand side of Eq. (2) becomes $\approx 10^{-12}$. Furthermore, assuming Nb, NbN, MgB$_2$, or Nb$_3$Sn as material of the superconductor, normal conductivities $\sigma_n$ are less than $10^7 \text{S/m}$, and then the second term becomes less than $10^{-3}$. Thus contributions from the second and the third terms to the electromagnetic field distribution are negligible. Then Eq. (1) are reduced to

\[ \Delta E = \frac{1}{\lambda^2} E, \quad \Delta B = \frac{1}{\lambda^2} B, \quad (3) \]

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The solutions that are consistent with Maxwell equations are given by

\[ E_1 = P_1 e^{\frac{x}{\kappa \lambda_1}} + Q_1 e^{-\frac{x}{\kappa \lambda_1}}, \]
\[ B_1 = \frac{1}{i \kappa \lambda_1} (P_1 e^{\frac{x}{\kappa \lambda_1}} - Q_1 e^{-\frac{x}{\kappa \lambda_1}}), \]

where \( E_1 \) and \( B_1 \) are electric and magnetic field in the region I, respectively, \( P_1 \) and \( Q_1 \) are integration constants, and \( k = \omega/c \). In the region II, the Maxwell equations should be solved, where contributions from dielectric losses to the electromagnetic field distribution are neglected. Then the solutions are given by

\[ E_{II} = P_{II} e^{i \sqrt{\kappa} (x - d_s)} + Q_{II} e^{-i \sqrt{\kappa} (x - d_s)}, \]
\[ B_{II} = \frac{\sqrt{\kappa}}{c} (P_{II} e^{i \sqrt{\kappa} (x - d_s)} - Q_{II} e^{-i \sqrt{\kappa} (x - d_s)}). \]

where \( E_{II} \) and \( B_{II} \) are electric and magnetic field in the region II, respectively, and \( P_{II} \) and \( Q_{II} \) are integration constants. In the region III, Eq. (3) should be solved as same as the region I, and the solutions that are consistent with the Maxwell equations are given by

\[ E_{III} = Q_{III} e^{-x \sqrt{\kappa} x_2} - \frac{1}{i \kappa \lambda_2} Q_{III} e^{-x \sqrt{\kappa} x_2}, \]
\[ B_{III} = - \frac{1}{i \kappa \lambda_2} Q_{III} e^{-x \sqrt{\kappa} x_2}. \]

where \( E_{III} \) and \( B_{III} \) are electric and magnetic field in the region III, respectively, and \( Q_{III} \) is an integration constant. The constants \( P_1, Q_1, P_{II}, Q_{II} \) and \( P_{III}, Q_{III} \) are determined from the boundary conditions: continuity conditions of the electric and magnetic field at \( x = d_s \) and \( x = d_s + d_I \). Then, we obtain

\[ P_1 = \frac{Q_{III}}{2} \left(1 - \frac{1}{i \sqrt{\kappa} \lambda_1} \right) e^{-i \sqrt{\kappa} \sqrt{d_s}}, \]
\[ Q_1 = \frac{Q_{III}}{2} \left(1 + \frac{1}{i \sqrt{\kappa} \lambda_1} \right) e^{+i \sqrt{\kappa} \sqrt{d_s}}, \]

from the continuity conditions at \( x = d_s + d_I \), and

\[ P_{II} = \frac{1}{2} e^{i \sqrt{\kappa} \lambda_1} P_{II} e^{\frac{d_s}{\kappa \lambda_1}} + \frac{1}{i \kappa \lambda_1} Q_{II} e^{\frac{d_s}{\kappa \lambda_1}}, \]
\[ Q_{II} = \frac{1}{2} e^{-i \sqrt{\kappa} \lambda_1} P_{II} e^{\frac{d_s}{\kappa \lambda_1}} + \frac{1}{i \kappa \lambda_1} Q_{II} e^{\frac{d_s}{\kappa \lambda_1}}, \]

from the continuity conditions at \( x = d_s \). Substituting Eq. (10) and (11) into Eq. (12) and (13), we obtain

\[ P_1 = \frac{Q_{III}}{2} e^{\frac{d_s}{\kappa \lambda_1}} \left[ \left(1 - \frac{1}{i \lambda_1} \right) e^{i \sqrt{\kappa} \sqrt{d_s}} \right], \]
\[ Q_1 = \frac{Q_{III}}{2} e^{\frac{d_s}{\kappa \lambda_1}} \left[ \left(1 + \frac{1}{i \lambda_1} \right) e^{i \sqrt{\kappa} \sqrt{d_s}} \right]. \]

Now the integration constants \( P_1, Q_1, P_{II} \) and \( Q_{II} \) are expressed in terms of \( Q_{III} \). The constant \( Q_{III} \) can be expressed by the surface magnetic field \( B_0 \), which is given by

\[ B_0 = B_{II} = \frac{1}{i \kappa \lambda_1} (P_1 - Q_1) \]
\[ = \left[ \left( \frac{1}{\lambda_2} \cos \sqrt{\kappa} \sqrt{d_s} \right) \right] \sinh \frac{d_s}{\lambda_1} \times Q_{III}. \]

Then substituting Eq. (14), (15), (10), (11) and (16) into Eq. (5), (7) and (9), the magnetic fields in the region I, II and III are given by

\[ B_1 = \left[ \left( \frac{1}{\lambda_2} \cos \sqrt{\kappa} \sqrt{d_s} \right) \right] \sinh \frac{d_s}{\lambda_1} \times B_0, \]
\[ B_{II} = \left[ \frac{\lambda_1}{\lambda_2} \cos \sqrt{\kappa} \sqrt{d_s} + \frac{\sin \sqrt{\kappa} \sqrt{d_s}}{\sqrt{\kappa} \lambda_2} \right] \sinh \frac{d_s}{\lambda_1} \times B_0, \]

where the denominator \( D \) is given by

\[ D = \left( \frac{1}{\lambda_2} \cos \sqrt{\kappa} \sqrt{d_s} \right) \left( \frac{1}{\lambda_2} \sin \sqrt{\kappa} \sqrt{d_s} \right) \left( \frac{1}{\sqrt{\kappa} \lambda_2} \right) \sinh \frac{d_s}{\lambda_1}. \]

It should be noted that these equations are reduced to the well known expression for the semi-infinite superconductor given by \( B = B_0 e^{-x/\lambda_1} \) when the \( S \) layer and the bulk superconductor are the same material \( \lambda_1 = \lambda_2 \) and the \( I \) layer vanishes \( (d_I \to 0) \).

Assuming an insulator thickness \( d_I \ll 10^{-2} \text{m} \), the above equations can be simplified. Completed results and discussions are seen in Ref. [5].

**SUMMARY**

The formulae that describe the RF field attenuation in the multilayer coating model with a single superconductor layer and a single insulator layer deposited on a bulk superconductor were derived from a rigorous calculation with the Maxwell equations and the London equations. Completed results and discussions are seen in Ref. [5].

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