A Novel VIKOR-TODIM Approach Based on Havrda–Charvat–Tsallis Entropy of Intuitionistic Fuzzy Sets to Evaluate Management Information System

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ABSTRACT
The utilisation of Management information system has grabbed the attention since last decades. The selection of management sources is a difficult assignment for decision-experts. Thus, the present article develops a new technique using entropy measures and a novel extended VIKOR-TODIM model of Intuitionistic Fuzzy Sets. A new parametric entropy measure based on Havrda–Charvat–Tsallis entropy for a probability distribution is proposed and discussed their particular cases. Some properties of this measure has been proved mathematically. A comparison of proposed entropy measure with several existing entropy measures is shown to demonstrate the consistency of the proposed approach. In this paper, we proposed an integrated approach based on VIKOR and TODIM and entropy-weighted method for solving multi-criteria decision-making problems under intuitionistic fuzzy environment and the criteria weights are partial known. Firstly, the problem with multi-criteria decision-making is designed and the steps, principles of the proposed VIKOR-TODIM method are presented. Finally, to verify the applicability and feasibility of the proposed approach, a problem of management information system in a teaching hospital is presented and compared the results with other existing methods. The present work demonstrated, that the proposed VIKOR-TODIM is effective to select and evaluate management information system in any organisation.

1. Introduction
Shannon [1] introduced a measure of uncertainty in information theory and proved various properties for continuous and discrete probability distributions. Zadeh [2] pioneered the theory of Fuzzy sets (FSs) to extend the probability theory, depicted by crisp membership degrees whose range lies between 0 and 1. The theory of FSs play a significant role in decision-making under uncertainty. After this, Zadeh [3] suggested the concept of fuzzy entropy in 1968. Following Zadeh’s work, DeLuca and Termini [4] proposed a probabilistic entropy measure for FSs. They likewise advanced some aphoristic properties for the fuzzy entropy measure, according to which fuzzy entropy can be defined. Yager [5] proposed an entropy measure for FSs based on the distance between an FS and its complementation.
while Higashi and Klar [6] stretched out Yager’s concept to a more general kind of fuzzy complementation. In fuzzy set, negative membership value of a member is complement of its membership degree from one, but practically it is not true. The concept of FSs has undergone further extensions and generalisations [7,8]. Motivated by FSs, Atanassov [9] proposed the notion of intuitionistic fuzzy sets (IFSs) by adding one more element in it and is termed as ‘hesitancy degree’. The prominent concept of intuitionistic fuzzy (IF) set theory is that it assigns to every element a membership function ($\xi \in [0, 1]$) and a non-membership function ($\nu \in [0, 1]$) satisfies a condition that the sum of membership ($\xi$) and non-membership ($\nu$) functions is less than or equal to 1. Due to its flexibility and ability to tackle real-life problems, IFSs have been widely used in many disciplines like as image processing, robotics, algebraic structures, pattern recognition, agriculture areas, irrigation and engineering fields, etc. Therefore, the proposed article focuses under intuitionistic fuzzy theory.

Kaufmann [10] introduced an entropy measure for measuring the degree of a FS by a metric distance between its membership and non-membership function of its closest crisp set. Firstly, Burillo and Bustince [11] introduced the entropy on IFSs can be utilised to assess intuitionism of an IFS. De Luca and Termini’s axioms reformulated by Szmidt and Kacprzyk [12] in 2001 and characterised an entropy measure for IFSs with an alternative approach dependent on geometric translation of IFS and the closest crisp set. Hung and Yang [13] suggested their axiomatic characterisations of entropy on IFSs by utilising the theory of probability. Verma and Sharma [14] presented an exponential entropy measure for IFSs by extending the exponential fuzzy entropy as defined by Pal and Pal [15]. Ye [16] defined two entropy measures for IFSs using triangular function. Ali et al. [17] proposed a graphical method by employing uncertainty index and IF entropy. Garg et al. [18] proposed a generalised IF entropy measure of order $\alpha$ and degree $\beta$ using logarithmic function. Therefore, it has been observed that the entropy measures are playing a dominate role in the decision-making problems. But it has been concluded from the above mentioned studies that all IF entropy measures only consider the derivation of membership and non-membership degree, not consider the effect of hesitancy degree of the IF set. In fact, the degree of uncertainty of an IFS involves both fuzziness and intuitionism. The fuzziness is associated by deviation of membership degree and non-membership degree, while the intuitionism is dominated with hesitancy degree. To enrich the abovementioned shortcomings, some new entropy measures for IFSs are proposed, such as Garg and Kaur [19] proposed an IF entropy measure with two parameters and Joshi and Kumar [20] proposed a parametric entropy under IF environment and applied in multiple attribute decision-making. Therefore, encouraged from the characteristic of fuzziness and intuitionism of IFS, the present paper presents a new parametric intuitionistic fuzzy entropy measure to measure the fuzziness of a set. Recently, many authors have explored the concept of intuitionistic fuzzy entropies from their points of view [17–21] and applied them in different areas like supplier selection, real life situations, medical diagnosis, education, image processing pattern recognition, failure mode.

The multi-criteria decision-making (MCDM) method is an important branch of decision-making. A problem that contains a set of fixed criteria and a set of alternatives is named as multi-criterion decision-making problem. Decision-making theory is characterised as a process of selecting the best alternatives among finite feasible alternatives according to the preference given by various decision-experts. Its theory is a very important branch, which is
mostly used in human activities. Because decision-making problem is frequently produced from a complicated environment, evaluated information is usually fuzzy. Under different environments such as IFS [9], T-spherical FSs [22], interval valued T-spherical FSs [23], q-rung orthopair FSs [24,25], various researchers presented various kinds of algorithms based on aggregation operator and entropy measures to solve the decision-making problems [18,21,23,26–28].

TODIM (an acronym in Portuguese for Interactive and Multi-Criteria Decision Making) is the most implemented technique for decision-making problems which was introduced by Hwang and Yoon [29] especially in economics, medical sciences, social sciences, engineering etc. Many scholars have contributed in fuzzy TODIM models [30,31], intuitionistic fuzzy TODIM models [32,33], the multi-hesitant fuzzy linguistic TODIM approach [34,35] and the intuitionistic linguistic TODIM method [34]. In a MCDM problem, we want to select the foremost reasonable alternative from a finite set of alternatives and the goal is to find the best possible alternative satisfying a specific set of criteria. Frequently, the criterion is thus conflicting and equivalent that it turns out to be very difficult to take a final decision, for example purchasing a cellphone or buying a car etc. are some familiar real world applications of MCDM problems. However, in the solution of MCDM problems criteria weights play a vital role. It is desirable to find an alternative performing closest to an ideal alternative. VIKOR (ViseKriterijumska Optimizacija I Kompromisno Resenje) method initially proposed by Opricovic [36] has been widely used in solving MCDM problems with incommensurable and different criteria. This is because of the way that the VIKOR method provides a compromise solution(s). You and Liu [37] presented an extended VIKOR approach under the IFS environment. Garg et al. [24] presented a VIKOR method using power aggregation operators and applied to solve the decision-making problems. However, TOPSIS (technique for order preference with respect to the similarity to the ideal solution) method, introduced by Hwang and Yoon [29] is a well-known MCDM method. The aim of this method is to choose the best alternative whose distance from its positive ideal solution is shortest. After their invention/existence, numerous attempts have been done by the authors to apply the TOPSIS method under the fuzzy and IFS environment. For instance, Boran et al. [38] presented the TOPSIS method to solve the problem of Personnel selection. Kumar and Garg [39,40] worked on connection number of the set pair analysis (SPA) theory and presented TOPSIS method to solving the expert problems. Arya and Kumar [7] developed TOPSIS method to solve decision problems under bipolar fuzzy environment. Recently, Garg and Kaur [27] developed the distance and weighted distance measures to solve the problems under cubic intuitionistic fuzzy set environment. Apart from this, some authors have proposed many theories and tools to solve MCDM problems, for example, the ELECTRE method [41], PROMETHEE [42], WASPAS method [43], Choquet integral [44] etc., each having its own benefits and downsides.

The objectives of this paper are outlined as follows:

1. Intuitionistic fuzzy numbers (IFNs) demonstrate more and more execution in providing reliable, vague and inexact assessment information.
2. Taking into account that entropy is an important measure and used for calculating weights for solving decision-making issues, this article, in general, aims to define new entropy measure based on Havdral–Charvat–Tsallis entropy in the IFS context to handle complicated problems of choice.
Weights calculated by the entropy method are extraordinary and these objective weights make the decision results more exact and precise when applied to real-life MCDM based on the IFS environment.

The judgement of a perfect alternative in an IFS environment is a laborious MCDM problem. In existing techniques, optimal alternative is characterised by single model which promote to do work in IF environment with combined TODIM and VIKOR method.

The proposed VIKOR-TODIM model overcome the restrictions of existing models.

The main contributions of this article are:

1. The idea of entropy measures is extended to IFNs and properties of this measure are discussed.

2. As the random weights may lead to wrong interpretation to the final ordering of the alternatives, thus computation of weight vector objectively is more important than subjectively. Hence, the presented approach is more suitable than existing approaches.

3. A MCDM problem is discussed using proposed entropy measure and an algorithm is developed with intuitionistic VIKOR-TODIM to tackle complex realistic problems with IF data.

4. Finally, the proposed IF information measure has been studied in MCDM problems for evaluating university teaching hospital to make a newly information managing system and discuss a comparative study to show the effectiveness and rationality of the proposed approach with other existing decision approaches.

The remainder of this communication is classified as follows. In Section 2, we quickly audit some essential concepts about FSs and IFSs. In Section 3, we propose a new entropy measure of IFSs and discuss their properties with the proof of its validity. Section 4 provides an example to examine and conform to the performance of the proposed entropy measure and compare it with the existing entropy measures to demonstrate its validity. In Section 5, we propose a new MCDM method based on the proposed entropy measure and the proposed modified VIKOR-TODIM method. Section 6 describes the analysis of the result associated with modified VIKOR-TODIM approach. A sensitivity analysis and comparative analysis has been described in Sections 6.2 and 6.3, respectively. Finally, the concluding remarks are presented in Section 7.

2. Preliminaries

This section recalls the vital concept and characterisations of FSs and IFSs.

**Definition 2.1:** An FS [2] \( M \) in a finite universe of discourse \( \tilde{K} \) is defined as \( M = \{ (l_i, \xi_M(l_i)) : l_i \in \tilde{K} \} \) where \( \xi_M : \tilde{K} \rightarrow [0, 1] \) is membership function. The number \( \xi_M(l_i) \) denotes the degree of membership of \( l_i \in \tilde{K} \) to \( M \).

**Definition 2.2:** An IFS [9] \( M \) in a finite universe of discourse \( \tilde{K} \) is defined as \( M = \{ (l_i, (\xi_M(l_i), \nu_M(l_i))) : l_i \in \tilde{K} \} \) where \( \xi_M, \nu_M : \tilde{K} \rightarrow [0, 1] \) shows the membership and non-membership functions. The numbers \( \xi_M(l_i), \nu_M(l_i) \) denote the degrees of membership.
and non-membership of \( l_i \in \hat{K} \) to \( M \), correspondingly, under the requirement \( 0 \leq \xi_M(l_i) + \nu_M(l_i) \leq 1 \), for all \( l_i \in \hat{K} \).

For each \( l_i \) in \( \hat{K} \), the degree of hesitancy or intuitionistic index is given by \( \phi_M(l_i) = 1 - \xi_M(l_i) - \nu_M(l_i) \). Obviously, when \( \phi_M(l_i) = 0 \), that is \( \nu_M(l_i) = 1 - \xi_M(l_i) \) for all \( l_i \in \hat{K} \), IFS \( M \) alters into an ordinary FS.

For simplicity, IFN is denoted by \( \sigma = (\xi_\sigma, \nu_\sigma) \) which fulfills \( \xi_\sigma, \nu_\sigma \in [0, 1] \) and \( \xi_\sigma + \nu_\sigma \leq 1 \).

### 2.1. Geometrical Representation of IFSs

A convenient geometrical representation of IFS is illustrated in Figure 1 with triangle POQ. In an IFS \( M \), each member \( l_i \) has a membership degree \( \xi_M(l_i) \), negative membership degree \( \nu_M(l_i) \) and a hesitation degree \( \phi_M(l_i) \) with the condition \( \xi_M(l_i) + \nu_M(l_i) + \phi_M(l_i) = 1 \) and \( \xi_M(l_i), \nu_M(l_i), \phi_M(l_i) \in [0, 1] \). In triangle POQ, \( P(1, 0, 0) \) and \( Q(0, 1, 0) \) points are known as crisp elements, for example, any point \( M(1, 0, 0) \) on membership axis denotes fully belongingness / membership degrees with \( \xi = 1 \) and \( \nu, \phi = 0 \). Analogously, point \( Q(0,0,1) \) denotes hesitant part while the segment \( PQ \) considers the elements belong to the FS as \( \xi + \nu = 1 \) and \( \phi = 0 \). For example, consider an IFS \( M = \{(l_1, 0.3, 0), (l_2, 0.4, 0.5)\} \) in Figure 1 in which the point \( (l_1, 0.4, 0) \) represents an element with hesitation parameter 0.7 while, the point \( (l_2, 0.5, 0.3) \) represents a degree of hesitation is equal to 0.1.

**Definition 2.3** ([45]): Let \( \sigma = I(\xi_\sigma, \nu_\sigma) \), \( \sigma_1 = I(\xi_{\sigma_1}, \nu_{\sigma_1}) \) and \( \sigma_2 = I(\xi_{\sigma_2}, \nu_{\sigma_2}) \) be three IFNs. Then the following laws are satisfied with IFNs:

1. \( \sigma_1 \oplus \sigma_2 = (\xi_{\sigma_1} + \xi_{\sigma_2} - \xi_{\sigma_1} \xi_{\sigma_2}, \nu_{\sigma_1} \nu_{\sigma_2}) \);
2. \( \sigma_1 \otimes \sigma_2 = (\xi_{\sigma_1} \xi_{\sigma_2}, \nu_{\sigma_1} + \nu_{\sigma_2} - \xi_{\sigma_1} \nu_{\sigma_2}) \);
3. \( \lambda' \sigma = (\xi_\sigma, 1 - (1 - \nu_\sigma)^{\lambda'}) \) for \( \lambda' > 0 \);
4. \( \lambda' \sigma = (1 - \xi_\sigma, \nu_{\sigma}^{\lambda'}) \) for \( \lambda' > 0 \);

![Figure 1. Geometrical representation of IFS.](image-url)
In order to rank the IFNs, Chen and Tan [46] and Hong and Choi [47] further gave a score and an accuracy function to compare the numeric size of the IFNs:

**Definition 2.4:** Consider an IFN $g = l(\xi_g, v_g)$. Then $H(g)$ and $s(g)$ are called the accuracy and score function of $g$ are defined as:

$$H(g) = \xi_g + v_g, \quad s(g) = \xi_g - v_g,$$

(1)

Based on score functions of IFNs, the ranking laws of IFNs can be illustrated as below:

**Definition 2.5:** Let $g_1 = l(\xi_{g_1}, v_{g_1})$ and $g_2 = l(\xi_{g_2}, v_{g_2})$ be two IFNs. $s(g_i)(i = 1, 2)$ be the score function values of $g_1$ and $g_2$, respectively and $H(g_i)(i = 1, 2)$ be the accuracy degree of $g_1$ and $g_2$, then:

- for $s(g_1) < s(g_2)$, we have $g_1 < g_2$;
- for $s(g_1) = s(g_2)$, we have
  1. if $H(g_1) < H(g_2)$, then $g_1 < g_2$.
  2. if $H(g_1) = H(g_2)$, then $g_1 = g_2$.

**Definition 2.6:** Suppose $g_1 = l(\xi_{g_1}, v_{g_1})$ and $g_2 = l(\xi_{g_2}, v_{g_2})$ be two IFNs. Then the Hamming distance measures of $f_1$ and $f_2$ proposed by Szmidt et al. [48] is computed as follows:

$$d_h(g_1, g_2) = \frac{1}{2} \left[ (|\xi_{g_1} - \xi_{g_2}|) + (|v_{g_1} - v_{g_2}|) + (|\phi_{g_1} - \phi_{g_2}|) \right]$$

(2)

To illustrate the Hamming distance measure, we can use an example as follows: Take any two IFNs like as: $g_1 = (0.6, 0.2)$ and $g_2 = (0.4, 0.5)$, then the hamming degrees of $f_1$ and $f_2$ are

$$\phi_{g_1}(l) = 1 - 0.6 - 0.2 = 0.2, \quad \phi_{g_2}(l) = 1 - 0.4 - 0.5 = 0.1$$

then the Hamming distance between $g_1$ and $g_2$ is:

$$d_H(g_1, g_2) = \frac{1}{2} \left[ |0.6 - 0.4| + |0.2 - 0.5| + |0.2 - 0.1| \right] = 0.3$$

**Definition 2.7 ([9]):** For any two IFSs $M$ and $N$ defined in the universe set $\tilde{K}$ given by

$$M = \{ l_i, \xi_M(l_i), v_M(l_i) | l_i \in \tilde{K} \} \quad \text{and} \quad N = \{ l_i, \xi_N(l_i), v_N(l_i) | l_i \in \tilde{K} \};$$

(3)

then properties and operations on IFSs are briefly defined as follows:

1. $M \subseteq N \Leftrightarrow \forall l_i \in \tilde{K}, \xi_M(l_i) \leq \xi_N(l_i), v_M(l_i) \geq v_N(l_i)$ for $\xi_N(l_i) \leq v_N(l_i)$ OR $\xi_M(l_i) \geq \xi_N(l_i)$, $v_M(l_i) \leq v_N(l_i)$ for $\xi_N(l_i) \geq v_N(l_i)$;
2. $co M = M^c = \{ l_i, v_M(l_i), \xi_M(l_i) | l_i \in \tilde{K} \}$;
3. $M \cap N = \{ \xi_M(l_i) \land \xi_N(l_i) \land v_M(l_i) \lor \xi_N(l_i) | l_i \in \tilde{K} \}$;
4. $M \cup N = \{ \xi_M(l_i) \lor \xi_N(l_i) \land v_M(l_i) \land \xi_N(l_i) | l_i \in \tilde{K} \}$.
**Definition 2.8 ([12]):** A mapping \( \Upsilon_e \) defined on \( IFS(\tilde{K}) \) is an entropy if it holds the following four requirements:

(1) **(IFP1).** \( \Upsilon_e(M) = 0 \iff \Upsilon_e \) is a crisp set, i.e. for all \( l_i \in \tilde{K}, \xi_M(l_i) = 0, \nu_M(l_i) = 1 \); or \( \xi_M(l_i) = 1, \nu_M(l_i) = 0 \).

(2) **(IFP2).** \( \Upsilon_e(M) = 1 \), that is, attains maximum value \( \iff \xi_M(l_i) = \nu_M(l_i) = \phi_M(l_i) = \frac{1}{3} \), for all \( l_i \in \tilde{K} \).

(3) **(IFP3).** \( \Upsilon_e(M) \leq \Upsilon_e(N) \iff M \subseteq N \), i.e. \( \xi_M \leq \xi_N \) and \( \nu_M \leq \nu_N \) for max \( (\xi_N, \nu_N) \leq \frac{1}{3} \) and \( \xi_M \geq \xi_N \) and \( \nu_M \geq \nu_N \) for min \( (\xi_N, \nu_M) \geq \frac{1}{3} \).

(4) **(IFP4).** \( \Upsilon_e(M) = \Upsilon_e(M^c) \) for all \( M \in IFS(\tilde{K}) \).

Throughout this paper, \( FS(\tilde{K}) \) and \( IFS(\tilde{K}) \) will, respectively, denote the set of all fuzzy set and the set of all intuitionistic fuzzy set on set \( \tilde{K} \).

It is realised that Renyi [49], Tsallis [50], Pal and Pal [15], Boekee and Vander Lubbe [51], Taneja [52] measures have been summed up based on the Shannon entropy [1]. Now we can construct a new parametric information measure based on the Havrda–Charvat–Tsallis entropy in the following section.

### 3. A New Parametric Measure for IFSs

#### 3.1. Background

For a finite set of probability distribution defined by \( \Delta_r = \{ \tilde{K} = (l_1, l_2, \ldots, l_r) : l_i \geq 0, \sum_{i=1}^{r} l_i = 1 \}, r \geq 2 \), the entropy given by

\[
V^e(\tilde{K}) = \frac{1}{(\varrho - \varrho^{-1})} \left[ \sum_{i=1}^{r} \left( \frac{\varrho^{l_i} - l_i}{\varrho - 1} \right) \right]
\]  

for some \( \tilde{K} \in \Delta_r \) and \( \varrho > 0 (\neq 1) \).

**Particular Cases:**

1. \( V^e(\tilde{K}) = V^{e^{-1}}(\tilde{K}) \).
2. If \( \varrho \to 1 \), (4) recovers the entropy studied by Shannon [1], i.e. \( \lim_{\varrho \to 1} V^e(\tilde{K}) = -\sum_{i=1}^{r} (l_i) \log(l_i) \).
3. If \( l_1 = l_2 = \ldots = l_r = \frac{1}{r} \), \( V^e(\tilde{K}) = \frac{1}{e - 1} \left[ r^{1 - e} - r^{1 - e^{-1}} \right] \).
4. If \( \varrho = 2 \), (4) becomes

\[
V^{e=2}(K) = \frac{2}{3} \left[ \sum_{i=1}^{r} \left( \sqrt{l_i} - l_i^2 \right) \right],
\]

which alters into an entropy for probability distribution \( \tilde{K} = (l_1, l_2, \ldots, l_r) \).
5. Setting \( \varrho^{-1} = \sigma \), (4) becomes Sharma and Taneja entropy [52].

\[
\text{i.e. } V(\varrho, \varrho^{-1} = \sigma)(\tilde{K}) = \frac{1}{\varrho - \sigma} \left[ \sum_{i=1}^{r} \left( \frac{\varrho^{l_i} - l_i^\sigma}{\varrho - \sigma} \right) \right].
\]
Proof: The idea of Renyi [49] entropy from probabilistic setting to fuzzy to develop a new entropy.

Theorem 3.1: The parametric entropy $V^\alpha(\tilde{K})$, $\tilde{K} \in \Delta_r$, fulfills the following properties:

1. Non-negative: $V^\alpha(\tilde{K}) \geq 0$ for all $\alpha > 0 (\neq 1)$.
2. Expandable: $V^\alpha(l_1, l_2, \ldots, l_r, 0) = V^\alpha(l_1, l_2, \ldots, l_r)$.
3. Symmetry: $V^\alpha(l_1, l_2, \ldots, l_r)$ is a symmetric function of $(l_1, l_2, \ldots, l_r)$.
4. Decisive: $V^\alpha(0, 1) = 0 = V^\alpha(1, 0)$.
5. Maximality: $V^\alpha(l_1, l_2, \ldots, l_r) \leq V^\alpha(\frac{1}{r}, \frac{1}{r}, \ldots, \frac{1}{r}) = \frac{1}{e-e^{-1}}[r^{1-e^{-1}} - r^{1-e}]$.
6. Concavity: $V^\alpha(t\tilde{K}_1 + (1-t)\tilde{K}_2) \geq V^\alpha(\tilde{K}_1) + (1-t)V^\alpha(\tilde{K}_2)$.
7. Continuity: $V^\alpha(l_1, l_2, \ldots, l_r)$ is continuous, where $l_i \geq 0$ for all $i = 1, 2, \ldots, r$ and $\alpha > 0$.

Proof: Proof of the above theorem is trivial and easy to calculation.

DeLuca and Termini [4] suggested fuzzy entropy for a fuzzy set $M$ analogously to Shannon entropy as

$$L_{DLT}(M) = -\frac{1}{r} \sum_{i=1}^{r} [\xi_M(l_i) \log_2(\xi_M(l_i)) + (1 - \xi_M(l_i)) \log_2(1 - \xi_M(l_i))],$$  \hspace{1cm} (5)

where $M \in FS(\tilde{K})$ and $l_i \in \tilde{K}$. Analogous to Equation (5), Bhandari and Pal [56] extended the idea of Renyi [49] entropy from probabilistic setting to fuzzy to develop a new entropy.
information measure given by

$$L_{BP}(M) = \frac{1}{r(1-\varrho)} \sum_{i=1}^{r} \log_2[\xi_M(l_i)^\varrho + (1 - \xi_M(l_i))^{1-\varrho}].$$  \hspace{1cm} (6)

The well known idea of Bhandari and Pal [56] was further generalised to IFSs by Hung and Yang [13] to introduce a new intuitionistic fuzzy entropy given by

$$L_{H\&Y}(M) = \frac{1}{r(1-\varrho)} \sum_{i=1}^{r} \log_2[\xi_M(l_i)^\varrho + \nu_M(l_i)^{1-\varrho} + \phi_M(l_i)^{1-\varrho}]; \quad \text{where } \varrho \in (0,1). \hspace{1cm} (7)$$

Keeping these concepts in mind, we present a new information measure for IFSs by extending the idea of Hung and Yang [13] in the next subsection.

### 3.2. Definition

For any $M \in IFSs$, we define

$$V^\varrho(M) = \frac{1}{r(\varrho - \varrho^{-1})} \sum_{i=1}^{r} \left[ \left( \xi_M(l_i)^{\varrho^{-1}} + \nu_M(l_i)^{\varrho^{-1}} + \phi_M(l_i)^{\varrho^{-1}} \right) - \left( \xi_M(l_i)^\varrho + \nu_M(l_i)^{1-\varrho} + \phi_M(l_i)^{1-\varrho} \right) \right]; \hspace{1cm} (8)$$

where $\varrho > 0 (\neq 1)$.

**Limiting Cases:**

1. If $\varrho = 1$, then Equation (8) alters an intuitionistic fuzzy entropy, which was studied by Hung and Yang [13]:

$$H_{HY} = \frac{1}{r} \sum_{i=1}^{r} (\xi_M(l_i) \log_2(\xi_M(l_i)) + \nu_M(l_i) \log_2(\nu_M(l_i)) + \phi_M(l_i) \log_2(\phi_M(l_i))) \hspace{1cm} (9)$$

2. If $\phi_M(l_i) = 0$, then Equation (8) becomes the fuzzy entropy studied by Arya and Kumar [7]:

i.e. $V^\varrho(M) = \frac{1}{r(\varrho - \varrho^{-1})} \sum_{i=1}^{r} \left[ \left( \xi_M(l_i)^{\varrho^{-1}} + \nu_M(l_i)^{\varrho^{-1}} + \phi_M(l_i)^{\varrho^{-1}} \right) - \left( \xi_M(l_i)^\varrho + \nu_M(l_i)^{1-\varrho} + \phi_M(l_i)^{1-\varrho} \right) \right]$.

where $\varrho > 0 (\neq 1)$

Keeping above said in mind, in the next subsection, we will prove the existence of proposed entropy measure by satisfying the requirements in Definition 2.8.

### 3.3. Justification

Before proving the efficiency of the proposed entropy measure, we will investigate a property needed in the proof of justification.
Property 1:
Under the condition IFP4, we have
\[
\left| \xi_M(l_i) - \frac{1}{3} \right| + \left| v_M(l_i) - \frac{1}{3} \right| + \left| \phi_M(l_i) - \frac{1}{3} \right| 
\geq \left| \xi_N(l_i) - \frac{1}{3} \right| + \left| v_N(l_i) - \frac{1}{3} \right| + \left| \phi_N(l_i) - \frac{1}{3} \right| 
\]
(10)
and
\[
\left[ \xi_M(l_i) - \frac{1}{3} \right]^2 + \left[ v_M(l_i) - \frac{1}{3} \right]^2 + \left[ \phi_M(l_i) - \frac{1}{3} \right]^2 
\geq \left[ \xi_N(l_i) - \frac{1}{3} \right]^2 + \left[ v_N(l_i) - \frac{1}{3} \right]^2 + \left[ \phi_N(l_i) - \frac{1}{3} \right]^2 
\]
(11)

Proof: If \( \xi_M(l_i) \leq \xi_N(l_i) \) and \( v_M(l_i) \leq v_N(l_i) \) with \( \max\{\xi_N(l_i), \xi_N(l_i)\} \leq \frac{1}{3} \), then \( \xi_M(l_i) \leq \xi_N(l_i) \leq \frac{1}{3} \) and \( v_M(l_i) \leq v_N(l_i) \leq \frac{1}{3} \) which shows that (10) and (11) hold. Similarly, if \( \xi_M(l_i) \geq \xi_N(l_i) \) and \( v_M(l_i) \geq v_N(l_i) \) with \( \min\{\xi_N(l_i), \xi_N(l_i)\} \geq \frac{1}{3} \), then (10) and (11) hold.

Theorem 3.2: Measure \( V^\circ(M) \) is a valid entropy measure for IFSs.

Proof: IFP1: Let \( M \) is a crisp set having either \( \xi_M(l_i) = 1 \), and \( v_M(l_i) = \phi_M(l_i) = 0 \) or \( v_M(l_i) = 1 \), and \( \xi_M(l_i) = \phi_M(l_i) = 0 \) or \( \phi_M(l_i) = 1 \) and \( \xi_M(l_i) = v_M(l_i) = 0 \). \( (\xi_M(l_i)^e - v_M(l_i)^e + \phi_M(l_i)^e) = 0 \).

Conversely, if \( V^\circ(M) = 0 \), we have
\[
(\xi_M(l_i)^e - v_M(l_i)^e + \phi_M(l_i)^e) = 0. 
\]
Since \( \rho > 0 (\rho \neq 1) \), this is possible only in the following cases:

- either \( \xi_M(l_i) = 1 \), and \( v_M(l_i) = \phi_M(l_i) = 0 \),
- or \( v_M(l_i) = 1 \), and \( \xi_M(l_i) = \phi_M(l_i) = 0 \),
- or \( \phi_M(l_i) = 1 \), and \( \xi_M(l_i) = v_M(l_i) = 0 \).

From the above cases, \( V^\circ(M) = 0 \) if \( M \) is crisp. This proves (IFP1).

IFP2: Since \( \xi_M(l_i) + v_M(l_i) + \phi_M(l_i) = 1 \), to obtain the maximim value of IF-entropy \( V^\circ(M) \), we write \( g(\xi_M, v_M, \phi_M) = \xi_M(l_i) + v_M(l_i) + \phi_M(l_i) - 1 \) and taking \( \lambda \) as the Lagrange’s multipliers, we construct the Lagrange’s function as:
\[
G(\xi_M, v_M, \phi_M) = V^\circ(\xi_M, v_M, \phi_M) + \lambda g(\xi_M, v_M, \phi_M). 
\]
(12)
To obtain the maximality of \( V^\circ(M) \), differentiating Equation (12) partially with respect to \( \xi_M, v_M, \phi_M \) and put them equal to zero, we obtain \( \xi_M(l_i) = v_M(l_i) = \phi_M(l_i) = \frac{1}{3} \). It can be easy to calculate that all the first-order derivative becomes zero if and only if \( \xi_M(l_i) = v_M(l_i) = \phi_M(l_i) = \frac{1}{3} \). The stationary point of \( V^\circ(M) \) is \( \xi_M(l_i) = v_M(l_i) = \phi_M(l_i) = \frac{1}{3} \). Now, we prove \( V^\circ(M) \) is a concave function of \( M \in IFS(\tilde{K}) \). For this, we compute Hessian of \( V^\circ(M) \) at the stationary point. 

\[\Box\]
Definition 3.1 (Hessian): The Hessian matrix of a function \( \Omega(\xi_1, \xi_2, \xi_3) \) of three variables is given by

\[
[HEN](\Omega) = \begin{bmatrix}
\frac{\partial^2 \Omega}{\partial \xi_1^2} & \frac{\partial^2 \Omega}{\partial \xi_1 \partial \xi_2} & \frac{\partial^2 \Omega}{\partial \xi_1 \partial \xi_3} \\
\frac{\partial^2 \Omega}{\partial \xi_2 \partial \xi_1} & \frac{\partial^2 \Omega}{\partial \xi_2^2} & \frac{\partial^2 \Omega}{\partial \xi_2 \partial \xi_3} \\
\frac{\partial^2 \Omega}{\partial \xi_3 \partial \xi_1} & \frac{\partial^2 \Omega}{\partial \xi_3 \partial \xi_2} & \frac{\partial^2 \Omega}{\partial \xi_3^2}
\end{bmatrix},
\tag{13}
\]

The function \( \Omega \) is called strictly convex at a point in its domain if \([HEN](\Omega)\) is positive definite and strictly concave if \([HEN](\Omega)\) is negative definite and. The Hessian of \( V^\epsilon(M) \) is given by

\[
[HEN](V^\epsilon(M)) = \frac{p}{r(\rho - \epsilon^{-1})} \begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{bmatrix},
\tag{14}
\]

which is negative definite for all \( \rho > 0(\neq 1) \), where \( p = \rho(\rho - 1)3^{(2-\epsilon)} - \rho^{-1}(\epsilon^{-1} - 1)3^{(2-\epsilon^{-1})} \). Thus, \( V^\epsilon(M) \) is strictly concave function for all \( \rho > 0(\neq 1) \) having its maximum value at stationary point \( \xi_M(l_i) = v_M(l_i) = \phi_M(l_i) = \frac{1}{3} \).

**IFP3:** Since, \( V^\epsilon(M) \) be concave function of \( M \in IFS(\tilde{K}) \), with maximum value at stationary point, then if \( \max\{\xi_M(l_i), v_M(l_i)\} \leq \frac{1}{3} \), then \( \xi_M(l_i) \leq \xi_M(l_i) \) and \( v_M(l_i) \leq v_M(l_i) \) implies \( \phi_M(l_i) \geq \phi_N(l_i) \geq \frac{1}{3} \). Therefore, using property (1), we see that \( V^\epsilon(M) \) agrees with condition IFP3.

Similarly, if \( \min\{\xi_M(l_i), v_M(l_i)\} \geq \frac{1}{3} \), then \( \xi_M(l_i) \leq \xi_N(l_i) \) and \( v_M(l_i) \geq v_N(l_i) \). Therefore, using property (1), we see that \( V^\epsilon(M) \) agrees with condition IFP3.

**IFP4:** For an IFS, \( V^\epsilon(M) = V^\epsilon(M^C) \), which is clear from the definition.

**Theorem 3.3:** For any two \( M, N \in IFS(\tilde{K}) \), such that for all \( l_i \in \tilde{K} \) either \( M \subseteq N \) or \( N \subseteq M \); then,

\[
V^\epsilon(M \cup N) + V^\epsilon(M \cap N) = V^\epsilon(M) + V^\epsilon(N).
\tag{15}
\]

**Proof:** To prove Theorem 3.3, we divide universal set \( \tilde{K} \) into two sets say \( \tilde{K}_1 \) and \( \tilde{K}_2 \) such that

\[
\tilde{K}_1 = \{l_i \in \tilde{K} : M \subseteq N\}, \quad \text{and} \quad \tilde{K}_2 = \{l_i \in \tilde{K} : M \supseteq N\}
\tag{16}
\]

\[
\xi_M(l_i) \leq \xi_N(l_i), \quad v_M(l_i) \geq v_N(l_i) \quad \forall \ l_i \in \tilde{K}_1
\tag{17}
\]

\[
\xi_M(l_i) \geq v_N(l_i), \quad \xi_M(l_i) \leq v_N(l_i) \quad \forall \ l_i \in \tilde{K}_2
\tag{18}
\]

Now, \( V^\epsilon(M \cup N) = \frac{1}{r(\rho - \epsilon^{-1})} \sum_{l_i} \{(\xi_M(l_i)^{\rho^{-1}} + v_M(l_i)^{\rho^{-1}} + \phi_M(l_i)^{\rho^{-1}})

- (\xi_M(l_i)^{\rho} + v_M(l_i)^{\rho} + \phi_M(l_i)^{\rho})\}
\tag{19}
\]

\[
= \frac{1}{r(\rho - \epsilon^{-1})} \sum_{K_1} \{(\xi_N(l_i)^{\rho^{-1}} + v_N(l_i)^{\rho^{-1}} + \phi_N(l_i)^{\rho^{-1}}

- (\xi_N(l_i)^{\rho} + v_N(l_i)^{\rho} + \phi_N(l_i)^{\rho})\}
\]
\[- (\xi_N(l_i)^e + \nu_N(l_i)^e + \phi_N(l_i)^e) \bigg) \bigg] \\
+ \frac{1}{r(Q - Q^{-1})} \sum_{k_2} \left[ \left(\xi_M(l_i)^{e^{-1}} + \nu_M(l_i)^{e^{-1}} + \phi_M(l_i)^{e^{-1}} \right) \\
- (\xi_M(l_i)^e + \nu_M(l_i)^e + \phi_M(l_i)^e) \bigg] \right] \tag{20}

Similarly, we get

\[V^e(M \cap N) = \frac{1}{r(Q - Q^{-1})} \sum_{k_1} \left[ \left(\xi_N(l_i)^{e^{-1}} + \nu_N(l_i)^{e^{-1}} + \phi_N(l_i)^{e^{-1}} \right) \\
- (\xi_N(l_i)^e + \nu_N(l_i)^e + \phi_N(l_i)^e) \bigg] \bigg] \\
+ \frac{1}{r(Q - Q^{-1})} \sum_{k_2} \left[ \left(\xi_M(l_i)^{e^{-1}} + \nu_M(l_i)^{e^{-1}} + \phi_M(l_i)^{e^{-1}} \right) \\
- (\xi_M(l_i)^e + \nu_M(l_i)^e + \phi_M(l_i)^e) \bigg] \right] \tag{21}

Now, adding Equations (20) and (21), we have

\[V^e(M \cup N) + V^e(M \cap N) = V^e(M) + V^e(N) \tag{22}\]

\[\blacksquare\]

**Corollary 3.1:** For any intuitionistic fuzzy set $M$ and $M^c$ (known as complement of $M$), we have

\[V^e(M \cup M^c) + V^e(M \cap M^c) = V^e(M) + V^e(M^c). \tag{23}\]

### 4. Numerical Examples

In this section, we demonstrate the performance of the proposed measure $V^e(M)$ will be validated based on the following examples. To illustrate the effectiveness and performance of the proposed measure for IFSs, some existing fuzzy measures will be adopted for comparison. Therefore, firstly we recall some widely used fuzzy measures for IFSs.

The entropy measure proposed by Zeng and Li [57] is appeared as below:

\[V_{ZL}(M) = 1 - \frac{1}{r} \sum_{i=1}^{r} |\xi_M(l_i) - \nu_M(l_i)|\]

The entropy measure proposed by Burillo and Bustince [11] is appeared as below:

\[V_{BB}(M) = \frac{1}{r} \sum_{i=1}^{r} |1 - \xi_M(l_i) - \nu_M(l_i)|\]

The entropy measure proposed by Szmidt et al. [48] is appeared as below:

\[V_{SK}(M) = \frac{1}{r} \sum_{i=1}^{r} \frac{\min(\xi_M(l_i), \nu_M(l_i)) + \phi_M(l_i)}{\max(\xi_M(l_i), \nu_M(l_i)) + \phi_M(l_i)}\]
The entropy measure proposed by Hung and Yang [13] is appeared as below:

\[ V_{HC}(M) = \frac{1}{r} \sum_{i=1}^{r} \left[ 1 - (\xi_M(l_i))^2 - (v_M(l_i))^2 + (\phi_M(l_i))^2 \right] \]

The entropy measure proposed by Zhang and Jiang’s [58] is appeared as below:

\[ V_{ZJ}(M) = \frac{1}{r} \sum_{i=1}^{r} \frac{\min(\xi_M(l_i), v_M(l_i))}{\max(\xi_M(l_i), v_M(l_i))} \]

The entropy measure proposed by Garg and Kaur [19] is appeared as below:

\[ V_{GK}^{\varrho, \beta}(M) = \frac{2 - \beta}{r(2 - q - \beta)} \times \left[ \sum_{i=1}^{r} \log_2 \left( \frac{\xi_M^{\varrho/\beta}(l_i) + v_M^{\varrho/\beta}(l_i)}{\xi_M(l_i) + v_M(l_i)} \right)^{1-\varrho/\beta} \right] \left( 1 - \xi_M(l_i), v_M(l_i) \right) \]

where \( q > 0, \beta \in [0, 1], q + \beta \neq 2. \)

The entropy measure proposed by Garg [21] is appeared as below:

\[ V_{GS}^{R, S}(M) = \frac{R \times S}{r(R - S)} \sum_{i=1}^{r} \left[ \left( \xi_M^S(l_i) + v_M^S(l_i) + \phi_M^S(l_i) \right)^{\frac{1}{S}} - \left( \xi_M^R(l_i) + v_M^R(l_i) + \phi_M^R(l_i) \right)^{\frac{1}{R}} \right] \]

either \( R > 1, 0 < S < 1 \) or \( 0 < R < 1, S > 1. \)

**Example 4.1:** Let us consider a IFS \( M_1 \) in \( \tilde{K} = \{3, 4, 5, 6, 7\} \) is defined as

\[ M_1 = \{(3, 0.1, 0.8), (4, 0.3, 0.5), (5, 0.5, 0.4), (6, 0.9, 0.0), (7, 1.0, 0.0)\}. \]

Then the modifier for the IFS

\[ M = \{(l, \xi_M(l), v_M(l)) | l \in \tilde{K}\} \]

in \( \tilde{K} \) is given by

\[ M' = \{(l, (\xi_M(l))', (1 - v_M(l))') | l \in \tilde{K}\}. \tag{24} \]

Based on the operations De et al. [59] in Equation (24), we have:

\[ M_1^\frac{1}{3} = \{(3, 0.316, 0.552), (4, 0.547, 0.292), (5, 0.707, 0.225), (6, 0.948, 0.0), (7, 1.0, 0.0)\}, \]

\[ M_2^\frac{1}{3} = \{(3, 0.01, 0.960), (4, 0.090, 0.750), (5, 0.250, 0.640), (6, 0.810, 0.0), (7, 1.0, 0.0)\}, \]

\[ M_3^\frac{1}{3} = \{(3, 0.001, 0.992), (4, 0.027, 0.875), (5, 0.125, 0.784), (6, 0.729, 0.0), (7, 1.0, 0.0)\}, \]

\[ M_4^\frac{1}{3} = \{(3, 0.001, 0.998), (4, 0.008, 0.938), (5, 0.063, 0.870), (6, 0.659, 0.0), (7, 1.0, 0.0)\}. \]

By considering the classification of linguistics variables which are shown in Table 1, we justify the performance of proposed measure.
Table 1. Classification of linguistic variables.

| IFSs | Linguistic variables   |
|------|------------------------|
| $M^1_2$ | Medium Large (ML) |
| $M^1_1$ | Large (L)            |
| $M^2_1$ | Very Large (VL)       |
| $M^3_1$ | Absolutely Large (AL) |
| $M^4_1$ | Very Very Large (VVL) |

Table 2. IFVs with different entropy measures.

| Entropies | $M^1_1$ | $M^1_2$ | $M^2_1$ | $M^3_1$ | $M^4_1$ |
|-----------|---------|---------|---------|---------|---------|
| $V_{ZL}$  | 0.4291  | 0.4400  | 0.2160  | 0.1364  | 0.1082  |
| $V_{BB}$  | 0.0683  | 0.0800  | 0.0760  | 0.0752  | 0.0801  |
| $V_{SK}$  | 0.3571  | 0.4074  | 0.2890  | 0.2603  | 0.2397  |
| $V_{HY}$  | 0.3354  | 0.3281  | 0.2117  | 0.1949  | 0.1457  |
| $V_{G_{K}^{β}}$ | 0.5254  | 0.5168  | 0.4763  | 0.4276  | 0.3146  |
| $V_{G_{S}^{β}}$ | 0.4789  | 0.4354  | 0.3654  | 0.3350  | 0.2711  |
| $V_{G=2}$ | 0.5160  | 0.5110  | 0.4122  | 0.3247  | 0.2801  |

Intuitively, from $M^1_1$ to $M^4_1$, the loss of information hidden in them become less. The entropy conveyed by them increasing. So the following relation holds for good performance:

$$V(M^1_1) > V(M^1_2) > V(M^2_1) > V(M^3_1) > V(M^4_1).$$  \(25\)

To make a comparison, eight entropy measures $V_{ZL}(M_1), V_{BB}(M_1), V_{SK}(M_1), V_{HY}(M_1), V_{ZJ}(M_1), V_{G_{K}^{β}}(M_1), V_{G_{S}^{β}}(M_1)$ and $V_{G=2}(M_1)$ are employed to facilitate analysis. In Table 2, we present the results obtained based on different measures to facilitate comparative analysis.

We can note that IFS $M$ will be assigned more entropy than the IFS $M^1_1$ when entropy measures $V_{ZL}(M_1), V_{BB}(M_1), V_{SK}(M_1)$ are applied. The ranking orders obtained based on these measures are listed below.

$$V_{ZL}(M_1) > V_{ZL}(M^1_1) > V_{ZL}(M^2_1) > V_{ZL}(M^3_1) < V_{ZL}(M^4_1),$$

$$V_{BB}(M_1) > V_{BB}(M^1_1) > V_{BB}(M^2_1) > V_{BB}(M^3_1) < V_{BB}(M^4_1),$$

$$V_{SK}(M_1) > V_{SK}(M^1_1) > V_{SK}(M^2_1)^2 > H_{SK}(M^3_1) > V_{SK}(M^4_1).$$

It is shown that these ranked orders do not satisfy intuitive analysis in Equation (25), while other entropy measures can induce desirable results. In this example, $V_{HY}(M_1), V_{G_{K}^{β}}, V_{G_{S}^{β}}$ and $V_{G=2}(M_1)$ perform well. This illustrates that these entropy measures are not robust enough to distinguish the uncertainty of IFSs with linguistic information.

Example 4.2: Let us consider another IFS $M_2$ defines in $\tilde{K}$. The IFS is defined as:

$$M_2 = \{(3, 0.2, 0.5), (4, 0.3, 0.7), (5, 0.4, 0.3), (6, 0.7, 0.2), (7, 0.8, 0.1)\}.$$
We calculate $M_2^1, M_2^2, M_2^3$ and $M_2^4$. Now we compare only $V_{HY}(M_2), V_{ZJ}(M_2), V_{GK}^{o,\beta}, V_{G}^{R,S}$ and $V_{\varrho(=2)}(M_2)$.

Moreover, the results produced by entropy measure $V_{ZJ}(M_2)$ and $V_{GK}^{o,\beta}$ are not reasonable in this example, which are shown as the equations below.

\[
V_{ZJ}(M_2) < V_{ZJ}(M_2^1) > V_{ZJ}(M_2^2) > V_{ZJ}(M_2^3) > V_{ZJ}(M_2^4),
\]
\[
V_{GK}^{o,\beta}(M_2) > V_{GK}^{o,\beta}(M_2^1) > V_{GK}^{o,\beta}(M_2^2) > V_{GK}^{o,\beta}(M_2^3) > V_{HY}(M_2^3).
\]

Therefore, the entropy measures $V_{ZJ}(M_2), V_{GK}^{o,\beta}(M_2)$ are not suitable for differentiating the information conveyed by IFSs. But $V_{HY}(M_2), V_{G}^{R,S}$ and $V_{\varrho}$ are also satisfy the ranking order in Equation (25). The effectiveness of the proposed fuzzy measure $V_{\varrho(=2)}(M_2), V_{G}^{R,S}$ and $V_{HY}(M_2)$ is indicated by this example once again. Hence, the proposed measure consider one parameter which increases the flexibility due to the setting $\varrho$, whereas $V_{HY}$ does not due to the absence of parameters. Therefore, the proposed measure is more encouraging and has broad scope of practical applications in numerous areas. Moreover, we can see that proposed entropy and Garg and Kaur [19] have the same order preference. It proves that the new entropy formula is feasible, but the presence of parameter in proposed entropy and Garg and Kaur [19] is more convincing.

In the next section, the traditional VIKOR and TODIM methods are integrated to solve MCDM problems in intuitionistic fuzzy environments.

5. Intuitionistic Fuzzy VIKOR-TODIM Approach and Its Application

5.1. Vikor Method

In 1998, Opricovic [36], the notion of the VIKOR technique was developed to compute a compromise solution(s) based on the $L^*_p$-metric and should be as close to be a positive solution and as far from a negative solution. This method offers a compromise programming providing a group utility for the ‘majority’ rule and a less individual regret of the ‘opponent’ with clashing criteria.

Assume that each alternative $\psi_j$ according to each criteria $E_j$ are given as $\gamma_{ij} = 1(1)$, $j = 1(1)$. The multi-criteria measure for compromise ranking is created from the $L^*_p$-metric with an aggregating function by Yu [60]. Advancement of the VIKOR method by Yu [60] is given by

\[
L^*_p = \left\{ \sum_{j=1}^{r} \left( u_j \left( \frac{H^+_j - t_j}{H^+_j - H^-_j} \right) \right)^p \right\}^{1/p}, \quad 1 \leq p \leq \infty, \quad i = 1(1)\; m; \quad (26)
\]

where $H^+_j = \max_i t_{ij}$ and $H^-_j = \min_i t_{ij}$ are the best and worst solutions for each criteria. $u_j$ represents the weight of $j$-th criteria. The measure $L^*_p$, denotes the distance of alternative $\psi_i$ to the ideal solution. Here, $L^*_p(\psi_i = S_i)$ and $L^*_p(\psi_i = R_i)$, has been used to formulate the grading of alternatives $\psi_i$.

Now, an algorithm is exhibited to solve the MCDM problems by using the IFSs and proposed VIKOR-TODIM model based on distance measures with the following steps:
Step 1: Establish the IF decision matrix $P = [\gamma_{ij}]_{m \times r}$ with the help of $\gamma_{ij} = (\xi_{ij}, \nu_{ij})$, where $\gamma_{ij}$ is an IFN.

$$P = [\gamma_{ij}]_{m \times r} = \begin{bmatrix}
\psi_1 & (\xi_{11}, \nu_{11}) & (\xi_{11}, \nu_{12}) & \cdots & (\xi_{1m}, \nu_{1r}) \\
\psi_2 & (\xi_{21}, \nu_{21}) & (\xi_{21}, \nu_{22}) & \cdots & (\xi_{2m}, \nu_{2r}) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\psi_m & (\xi_{m1}, \nu_{m1}) & (\xi_{m1}, \nu_{m2}) & \cdots & (\xi_{mr}, \nu_{mr})
\end{bmatrix}$$

(27)

Step 2: Change the decision matrix $P = (\gamma_{ij})_{m \times r}$ into a normalised IF-decision matrix by:

$$q_{ij} = \begin{cases} 
\gamma_{ij}, & \text{for benefit type criteria} \\
\gamma_{ij}^C, & \text{for cost type criteria}
\end{cases}$$

(28)

where $\gamma_{ij}^C = (\nu_{ij}, \xi_{ij})$ is the complement of $\gamma_{ij}$ [61]. Then, we obtain a new normalised IF-decision matrix $P^N = (q_{ij})_{m \times r}$.

Step 3: Evaluate the Criteria’s weights: If the weights are completely known, at that point MCDM problem can be solved by aggregating all intuitionistic fuzzy information under different criterions and comparing the final intuitionistic fuzzy values. However in a partial application, the criteria weights are usually partially known or completely unknown. Consequently, the criteria weights must be resolved before solving the MCDM models. The criteria weights can be observationally allocated by decision makers. Nonetheless, this method is subjective and the partial information on the weights vector may not be utilised adequately. Therefore, we can propose another model to compute the weights based on the proposed measure. We can set the total information amount as the objective function of optimisation. By minimising the sum of all information amount, we may construct the following model.

$$\text{Min } T = \sum_{i=1}^{m} \sum_{j=1}^{r} \psi (\xi_{ij}) u_j \\ \text{such that } \begin{cases} 
u \in S \\
\sum_{j=1}^{r} u_j = 1 \\
0 \leq u_j, j = 1, 2, \ldots, r
\end{cases}$$

(29)

where $S$ denotes the set of all incomplete information about criterion weights and $\psi (\xi_{ij})$ is the information measure calculated by proposed measure.

Determining the criteria weights of each criterion $E_j$ based on the weights of criteria $u = (u_1, u_2, \ldots, u_r)^T$ as:

$$u_{js} = \frac{u_j}{u_s}, \quad j, r = 1, 2, \ldots, r;$$

(30)

where $u_j$ is the weight of the criterion $E_j$, $u_s = \max\{u_j / j = 1, 2, \ldots, r\}$ and $0 \leq u_{js} \leq 1$. Note that all criteria have equal importance, any criterion may be chosen.
5.2. TODIM Method

Step 4: The TODIM technique is a discrete multi-criteria technique used for qualitative and quantitative criteria based on Prospect Theory. By Equation (30), we can derive the dominance degree of \( \psi_i \) with respect to each criterion \( E_j \):

\[
Z_j(\psi_i, \psi_t) = \begin{cases} 
\sqrt{\frac{u_{js}d_h(q_{ij}, q_{tij})}{\sum_{j=1}^{n} u_{js}}}, & \text{if } q_{ij} > q_{tij} \\
\text{null}, & \text{if } q_{ij} = q_{tij} \\
\frac{1}{\gamma} \left( \sum_{j=1}^{n} u_{js} \right) d_h(q_{ij}, q_{tij}), & \text{if } q_{ij} < q_{tij}
\end{cases}
\]

where the parameter \( \gamma \) indicates the attenuation factor of the losses, and \( d_h(q_{ij}, q_{tij}) \) is to measure the distance between the IFNs \( q_{ij} \) and \( q_{tij} \) the parameter \( \gamma \) represents the attenuation factor of losses. By definition if \( q_{ij} > q_{tij} \), then \( Z_j(\psi_i, \psi_t) \) signifies a gain; if \( q_{ij} < q_{tij} \), then \( Z_j(\psi_i, \psi_t) \) represents a loss.

Step 5: Constructing the dominance matrix of each alternative \( \psi_i \) with respect to each criterion \( E_j \) is shown below:

\[
Z_j = [Z_j(\psi_i, \psi_t)]_{m \times r} = \begin{bmatrix}
E_1 & E_2 & \ldots & E_r \\
\psi_1 & 0 & \ldots & \sum_{t=1}^{m} Z_j(\psi_1, \psi_{t1}) \\
\psi_2 & Z_j(\psi_2, \psi_1) & 0 & \ldots & Z_j(\psi_2, \psi_{m}) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\psi_m & Z_j(\psi_m, \psi_1) & Z_j(\psi_m, \psi_2) & \ldots & 0
\end{bmatrix}
\]

Step 6: Computing the totally dominance degree of each alternative \( \psi_i \) under the criterion \( E_j \) with respect to another alternatives \( \psi_t, (t_1 = 1, 2, \ldots, m) \) as follows:

\[
\Delta_j(\psi_i) = \sum_{t_1=1}^{m} Z_j(\psi_i, \psi_{t1})
\]

Total dominance results by considering all alternatives are shown as:

\[
[Z_j(\psi_i, \psi_{t1})]_{m \times m} = \begin{bmatrix}
\sum_{t_1=1}^{m} Z_j(\psi_1, \psi_{t1}) & \sum_{t_1=1}^{m} Z_j(\psi_2, \psi_{t1}) & \ldots & \sum_{t_1=1}^{m} Z_j(\psi_r, \psi_{t1}) \\
\sum_{t_1=1}^{m} Z_j(\psi_1, \psi_{t1}) & \sum_{t_1=1}^{m} Z_j(\psi_2, \psi_{t1}) & \ldots & \sum_{t_1=1}^{m} Z_j(\psi_r, \psi_{t1}) \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{t_1=1}^{m} Z_j(\psi_1, \psi_{t1}) & \sum_{t_1=1}^{m} Z_j(\psi_2, \psi_{t1}) & \ldots & \sum_{t_1=1}^{m} Z_j(\psi_r, \psi_{t1})
\end{bmatrix}
\]

The dominance matrix \( t = [t_{ij}]_{m \times r} \) from the set of \( n \) criteria obtained as:

\[
[t_{ij}]_{m \times r} = \begin{bmatrix}
\sum_{t_1=1}^{m} Z_1(\psi_1, \psi_{t1}) & \sum_{t_1=1}^{m} Z_2(\psi_1, \psi_{t1}) & \ldots & \sum_{t_1=1}^{m} Z_r(\psi_1, \psi_{t1}) \\
\sum_{t_1=1}^{m} Z_1(\psi_2, \psi_{t1}) & \sum_{t_1=1}^{m} Z_2(\psi_2, \psi_{t1}) & \ldots & \sum_{t_1=1}^{m} Z_r(\psi_2, \psi_{t1}) \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{t_1=1}^{m} Z_1(\psi_m, \psi_{t1}) & \sum_{t_1=1}^{m} Z_2(\psi_m, \psi_{t1}) & \ldots & \sum_{t_1=1}^{m} Z_r(\psi_m, \psi_{t1})
\end{bmatrix}
\]
The positive ideal solution (PIS) and negative ideal solution (NIS) are calculated by using the dominance matrix. Therefore, to rank the alternatives, it is necessary to determine the ideal solutions. Since, the core idea of VIKOR method is based on the distance measures of alternatives from the ideal solutions and so traditional VIKOR method remains neutral regarding the attitudinal character of experts. In the proposed VIKOR-TODIM model, TODIM method was developed with IFNs to calculate the dominance matrix and then it was integrated with VIKOR method (which aims to obtain compromise solution) to obtain extraordinary ranking order.

Step 7: Acquire PIS and NIS for each criterion. The positive solution is the best value \( H^+ \) and the negative solution is the worst value \( H^- \) as follows:

\[
H^+ = (H^+_1, H^+_2, \ldots, H^+_r)
\]

\[
= \left( \max_{i=1}^{m} \sum_{t=1}^{m} Z_1(\psi_i, \psi_{t1}), \max_{i=1}^{m} \sum_{t=1}^{m} Z_2(\psi_i, \psi_{t1}), \ldots, \max_{i=1}^{m} \sum_{t=1}^{m} Z_r(\psi_i, \psi_{t1}) \right)
\]  \hspace{1cm} (35)

and \( H^- = (H^-_1, H^-_2, \ldots, H^-_r) \)

\[
= \left( \min_{i=1}^{m} \sum_{t=1}^{m} Z_1(\psi_i, \psi_{t1}), \min_{i=1}^{m} \sum_{t=1}^{m} Z_2(\psi_i, \psi_{t1}), \ldots, \min_{i=1}^{m} \sum_{t=1}^{m} Z_r(\psi_i, \psi_{t1}) \right)
\]  \hspace{1cm} (36)

Step 8: Obtain the values \( S^*_i \) and \( R^*_i \) by using the below formulas:

\[
S^*_i = \sum_{1 \leq j \leq r} u_j \frac{d_h(H^+_j, t_{ij})}{d_h(H^+_j, H^-_j)}
\]  \hspace{1cm} (37)

\[
R^*_i = \max_{1 \leq j \leq r} \left( u_j \frac{d_h(H^+_j, t_{ij})}{d_h(H^+_j, H^-_j)} \right)
\]  \hspace{1cm} (38)

where

\[
d_h(H^+_j, t_{ij}) = \max_{1 \leq i \leq n} \sum_{t=1}^{m} Z_j(\psi_i, \psi_{t1}) - \sum_{t=1}^{m} Z_j(\psi_i, \psi_{t1}),
\]

\[
d_h(H^+_j, H^-_j) = \max_{1 \leq i \leq n} \sum_{t=1}^{m} Z_j(\psi_i, \psi_{t1}) - \min_{1 \leq i \leq n} \sum_{t=1}^{m} Z_j(\psi_i, \psi_{t1}).
\]

and \( u_j; j = 1(1)r \) denotes the weight of \( j \)th criteria satisfying \( \sum_{j=1}^{r} u_j = 1 \).

Step 9: Obtain the influence values \( Q^*_i, i = 1(1)m \) with Equation (39) as:

\[
Q^*_i = \tau \frac{S^*_i - S^-}{\bar{S} - S^-} + (1 - \tau) \frac{R^*_i - R^-}{\bar{R} - R^-}.
\]  \hspace{1cm} (39)

Here \( \bar{S} = \max(S_i) \), \( S^- = \min(S_i) \), \( \bar{R} = \max(R_i) \) and \( R^- = \min(R_i) \). The coefficient \( \tau \) and \( 1 - \tau \) are introduced as a weight for ‘maximum group utility’ \( S^*_i \) and individual regrets \( R^*_i \). In general, we set the value to \( \tau = \frac{1}{2} \) and \( 1 - \tau = \frac{1}{2} \) denotes a consensus.
**Step 10:** Determining the rank of the alternatives, sorting by the values of \((S^*_{i})\), \((R^*_{i})\) and \((Q^*_{i})\) in descending order.

**Step 11:** Calculating the compromise solution, the solution satisfying the following two conditions will be the most desirable solution.

**C1:** If \(Q^*(\psi^{(2)}) - Q^*(\psi^{(1)}) \geq \frac{1}{r-1}\), where \(\psi^{(1)}\) and \(\psi^{(2)}\), respectively, stand at initial and second positions in the ranking list of \(Q^*_{i}\) and \(r\) denotes the number of alternatives.

**C2:** The alternative \(\psi^{(1)}\) should also be ranked first in the list of \(S^*_{i}\) or \(R^*_{i}\). The compromise solution \(Q^*_{i}\) is more stable in a decision making process and calculated with ‘voting by majority rule’ \((\tau > 0.5)\), or ‘by conseness’ \((\tau = 0.5)\), or ‘with veto’ \((\tau < 0.5)\). If the conditions C1 and C2 are not simultaneously satisfied, then we seek the compromise solution as given below:

(a) (Acceptable advantage): If only C2 is not satisfied then \(\psi^{(1)}\) and \(\psi^{(2)}\) are the compromise solutions.

(b) (Acceptable stability): If C1 is not satisfied, then we explore the utmost value \(M\) by the following equation:

\[
Q^*(\psi^{(M)}) - Q^*(\psi^{(1)}) < \frac{1}{r-1}
\]

(40)

Hence, it shows the alternatives \(\psi^{(i)}\) \((i=1(1)M)\) are the real compromised solution(s).

**6. Numerical Example**

This section presents a problem of evaluating university teaching hospital (Chandigarh, India) has been studied intends to construct a ‘Management Information System’ (MIS) in order to enhance better services and work productivity. Practical use of the proposed approach involves for illustrating the applicability of the proposed VIKOR-TODIM method. Assume that there are five possible alternatives as: \(\psi_1, \psi_2, \psi_3, \psi_4\) and \(\psi_5\) have been considered as alternatives. To perform the evaluation, a commission composed of five decision experts (say) \(DE_1, DE_2, DE_3, DE_4\) and \(DE_5\) have been established and five evaluation criteria which include Functionality \((E_1)\), Reliability \((E_2)\), Maintenance \((E_3)\), Customer Satisfaction \((E_4)\), and Price \((E_5)\) are considered. Then, the evaluation by the relevant experts of the alternatives \(\psi_i\) regarding each given criterion is presented by an IFN. Intuitionistic fuzzy decision matrices with their evaluation results are presented in Table 3.

Now we put forward the proposed measure to prioritise these information management:

**Table 3. IFVs with different entropy measures.**

| Entropies      | \(M_2^1\)   | \(M_2\)    | \(M_2^2\)   | \(M_2^3\)   | \(M_2^4\)   |
|----------------|--------------|-------------|--------------|--------------|--------------|
| \(V_{WY}\)    | 0.3627       | 0.3620      | 0.2830       | 0.2423       | 0.2185       |
| \(V_{ZJ}\)    | 0.2119       | 0.2486      | 0.2253       | 0.1769       | 0.1478       |
| \(V_{GK}\)    | 0.4650       | 0.4876      | 0.3241       | 0.2847       | 0.2417       |
| \(V_{GR}\)    | 0.4352       | 0.4157      | 0.3476       | 0.3316       | 0.3017       |
| \(V_{GR}^{\beta}\) | 0.4168       | 0.4024      | 0.3945       | 0.3274       | 0.2912       |
6.1. Decision Analysis with Proposed Method

Step 1. We construct an Intuitionistic fuzzy decision matrix $\hat{D}$ which contains five criteria and five alternatives and is showed as below:

| Decision value | $E_1$    | $E_2$    | $E_3$    | $E_4$    | $E_5$    |
|---------------|----------|----------|----------|----------|----------|
| $\psi_1$     | (0.60,0.10) | (0.20,0.70) | (0.50,0.40) | (0.50,0.20) | (0.20,0.80) |
| $\psi_2$     | (0.30,0.50) | (0.70,0.10) | (0.20,0.80) | (0.60,0.40) | (0.50,0.30) |
| $\psi_3$     | (0.30,0.20) | (0.40,0.30) | (0.40,0.20) | (0.50,0.30) | (0.40,0.20) |
| $\psi_4$     | (0.10,0.40) | (0.10,0.50) | (0.40,0.60) | (0.60,0.30) | (0.70,0.20) |
| $\psi_5$     | (0.20,0.30) | (0.30,0.60) | (0.40,0.50) | (0.40,0.20) | (0.10,0.50) |

Step 2. We assume that each criterion is the cost criterion, the normalised decision matrix will become the following one:

| Decision value | $E_1$    | $E_2$    | $E_3$    | $E_4$    | $E_5$    |
|---------------|----------|----------|----------|----------|----------|
| $\psi_1$     | (0.10,0.60) | (0.70,0.20) | (0.40,0.50) | (0.20,0.50) | (0.80,0.20) |
| $\psi_2$     | (0.50,0.30) | (0.10,0.70) | (0.80,0.20) | (0.40,0.60) | (0.30,0.50) |
| $\psi_3$     | (0.20,0.30) | (0.30,0.40) | (0.20,0.40) | (0.30,0.50) | (0.20,0.40) |
| $\psi_4$     | (0.40,0.10) | (0.50,0.10) | (0.60,0.40) | (0.30,0.60) | (0.20,0.70) |
| $\psi_5$     | (0.30,0.20) | (0.60,0.30) | (0.50,0.40) | (0.20,0.40) | (0.50,0.10) |

Step 3. The partial information on criterion weights is listed in set $T$: The entropy values for each criterion are given as below:

$$\tilde{K}_1 = \sum_{i=1}^{5} \xi_{1j} = \sum_{i=1}^{5} V^e(r_{1j}) = 0.102; \quad \tilde{K}_2 = \sum_{i=1}^{5} \xi_{2j} = \sum_{i=1}^{5} V^e(r_{2j}) = 0.055$$

$$\tilde{K}_3 = \sum_{i=1}^{5} \xi_{3j} = \sum_{i=1}^{5} V^e(r_{3j}) = 0.013; \quad \tilde{K}_4 = \sum_{i=1}^{5} \xi_{4j} = \sum_{i=1}^{5} V^e(r_{4j}) = 0.013$$

$$\tilde{K}_5 = \sum_{i=1}^{5} \xi_{5j} = \sum_{i=1}^{5} V^e(r_{5j}) = 0.093.$$ 

The optimal model to determine the attribute weights can be constructed as; $\min T = 0.102u_1 + 0.055u_2 + 0.013u_3 + 0.013u_4 + 0.093u_5$ such that:

$$\begin{align*}
\sum_{j=1}^{5} u_j &= 1 \\
u_j &\geq 0, \quad j = 1, 2, \ldots, 5.
\end{align*}$$

Corresponding to Equation (29), we have

$$u = (0.2271, 0.2615, 0.1826, 0.1808, 0.148)^T.$$ 

Step 4. We compute the dominance of each alternative $\psi_i$ over the alternative $\psi_{t_1}$ with respect to the given criterion $(i, t_1 = 1, 2, 3, 4, 5)$. Assume the value of $\tau = 2.5$. From the set
of criteria and the decision matrix, we can construct five dominance matrices $Z_1 - Z_5$ as follows:

$$Z_1 = \begin{bmatrix} E_1 & E_2 & E_3 & E_4 & E_5 \\ 0.0000 & 0.5310 & -0.4598 & -0.5936 & -0.5310 \\ 0.3013 & 0.0000 & 0.2610 & -0.4598 & 0.2610 \\ 0.2610 & -0.4598 & 0.0000 & -0.4598 & -0.2655 \\ 0.3369 & 0.2610 & 0.2131 & 0.0000 & 0.1507 \\ 0.3013 & -0.4598 & 0.1507 & -0.2655 & 0.0000 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} E_1 & E_2 & E_3 & E_4 & e_5 \\ 0.0000 & 0.3960 & 0.3234 & 0.2800 & 0.1617 \\ -0.6060 & 0.0000 & -0.4285 & -0.6060 & -0.5532 \\ -0.4948 & -0.2800 & 0.0000 & -0.4285 & -0.4285 \\ -0.4285 & 0.3960 & 0.2800 & 0.0000 & 0.2800 \\ -0.2474 & 0.3615 & 0.2800 & -0.4285 & 0.0000 \end{bmatrix}$$

$$Z_3 = \begin{bmatrix} E_1 & E_2 & E_3 & E_4 & E_5 \\ 0.0000 & -0.5920 & 0.2340 & -0.4186 & -0.2960 \\ 0.2703 & 0.0000 & 0.3310 & 0.1911 & 0.2340 \\ -0.5127 & -0.7251 & 0.0000 & -0.5920 & -0.5127 \\ 0.1911 & -0.4186 & 0.2703 & 0.0000 & 0.1351 \\ 0.1351 & -0.5127 & 0.2340 & -0.2960 & 0.0000 \end{bmatrix}$$

$$Z_4 = \begin{bmatrix} E_1 & E_2 & E_3 & E_4 & E_5 \\ 0.0000 & -0.5151 & -0.2974 & 0.0000 & -0.2974 \\ 0.2330 & 0.0000 & 0.0000 & 0.1345 & 0.0000 \\ 0.1345 & 0.0000 & 0.0000 & 0.1345 & 0.0000 \\ 0.0000 & -0.2974 & -0.2974 & 0.0000 & -0.5151 \\ 0.1345 & 0.0000 & 0.0000 & 0.2330 & 0.0000 \end{bmatrix}$$

$$Z_5 = \begin{bmatrix} E_1 & E_2 & E_3 & E_4 & E_5 \\ 0.0000 & 0.2721 & 0.2981 & 0.2981 & 0.2434 \\ -0.7350 & 0.0000 & 0.0000 & 0.1721 & -0.6574 \\ -0.8051 & 0.0000 & 0.0000 & 0.2108 & -0.5693 \\ -0.8051 & -0.4649 & -0.5693 & 0.0000 & -0.8051 \\ -0.6574 & 0.2434 & 0.2108 & 0.2981 & 0.0000 \end{bmatrix}$$

**Step 5 and 6.** With the help of Equation (34), computing the total dominance of each alternative $\psi_i$ with respect to the alternatives $\psi_{t1}$. Following the consequences of Step 4, the dominance matrix is therefore created as below:

$$\begin{bmatrix} \psi_1 & \psi_2 & \psi_3 & \psi_4 & \psi_5 \\ \psi_1 & -2.1154 & 0.3635 & -0.9241 & 0.9617 & -0.2733 \\ \psi_2 & 1.1611 & -2.1937 & -1.6318 & 0.5275 & -0.0344 \\ \psi_3 & -1.0726 & 1.0264 & -0.3425 & 0.1779 & -0.4396 \\ \psi_4 & -1.1099 & 0.3675 & 0.2690 & -1.1099 & 0.3675 \\ \psi_5 & 1.1117 & -1.2203 & -1.1636 & -2.6444 & 0.0949 \end{bmatrix}$$
Step 7. With the help of Equations (35) and (36), the PIS $H^+$ and the NIS $H^-$ are acquiring as follows:

$$H^+ = (1.1611, 1.0264, 0.2690, 0.9617, 0.3675);$$
$$H^- = (-2.1154, -2.1937, -2.3425, -2.6444, -0.4396).$$

Steps 8. In this step, we calculate $S^*_i$ and $R^*_i$ as below:

$$S^*_1 = 0.6787, \quad S^*_2 = 0.5326, \quad S^*_3 = 0.6581, \quad S^*_4 = 0.2712, \quad S^*_5 = 0.5900;$$
$$R^*_1 = 0.2270, \quad R^*_2 = 0.2270, \quad R^*_3 = 0.2270, \quad R^*_4 = 0.1573, \quad R^*_5 = 0.2270.$$

Step 9. Let the value of $\tau$ be 0.5. Compute the values of $Q^*_i = 1(1)5$,

$$Q^*_1 = 1.0000, \quad Q^*_2 = 0.8207, \quad Q^*_3 = 0.9747, \quad Q^*_4 = 0.0000, \quad Q^*_5 = 0.8912.$$

Step 10. Sorting the values $S^*, R^*$ and $Q^*$ on the basic of ascending order, we can compute ranking of alternatives. Table 4 shows the results.

From Table 5, we conclude that the most desirable alternative is $\psi_4$ corresponding to different values of $\tau$. From this analysis, it has been observed that the optimal alternative is remain same which shows that the proposed approach is more consistent and can be effectively used to solve the decision-making problems in a more efficient ways (Figure 2).

Step 11. From Table 5, it is obvious that alternatives $\psi_4$ and $\psi_2$ stands at the initial and second positions in the ranking list of $Q^*_i$. However, applying the condition C1, $Q^*(\psi^{(4)}) - Q^*(\psi^{(2)}) = 0.0000 - 0.8207 = -0.8207 < \frac{1}{5-1} = 0.25$. It implies that C1 is not satisfied. Then, we search for compromise solution as follows:

$$Q^*(\psi^{(5)}) - Q^*(\psi^{(2)}) = 0.8912 - 0 = 0.8912 > \frac{1}{5-1} = 0.25.$$ Thus, $\psi_2$ and $\psi_4$ are taken as compromised solutions.

### Table 4. The ranking $C^*$, $D^*$ and $Q^*$ and the compromise measure.

| $\psi_1$ | $\psi_2$ | $\psi_3$ | $\psi_4$ | $\psi_5$ | Ranking | Compromise solutions |
|---|---|---|---|---|---|---|
| $S^*$ | 0.6787 | 0.5326 | 0.6581 | 0.2712 | 0.5900 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_4 |
| $R^*$ | 0.2270 | 0.2270 | 0.2270 | 0.1573 | 0.2270 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_4, \psi_2 |
| $Q^*$ | 1.0000 | 0.8207 | 0.9747 | 0.0000 | 0.8912 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_4, \psi_2 |

### Table 5. The $S^*_i, R^*_i, Q^*_i$ values obtained by the weight $\tau$ changes and the compromise solution.

| $\tau$ | $S^*$ | $R^*$ | $Q^*(\tau)$ | $\psi_1$ | $\psi_2$ | $\psi_3$ | $\psi_4$ | $\psi_5$ | Ranking | Compromise solutions |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.6787 | 0.5326 | 0.6581 | 0.2712 | 0.5900 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_4 |
| 0.1 | 0.2270 | 0.2270 | 0.2270 | 0.1573 | 0.2270 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_4, \psi_2 |
| 0.2 | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_4 |
| 0.3 | 0.1000 | 0.9283 | 0.9899 | 0.0000 | 0.9565 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_4 |
| 0.4 | 0.3000 | 0.8924 | 0.9848 | 0.0000 | 0.9347 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_4 |
| 0.5 | 0.5000 | 0.8207 | 0.9747 | 0.0000 | 0.8912 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_2, \psi_4 |
| 0.6 | 0.7000 | 0.7849 | 0.9697 | 0.0000 | 0.8694 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_2, \psi_4 |
| 0.7 | 0.9000 | 0.7490 | 0.9646 | 0.0000 | 0.8476 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_2, \psi_4 |
| 0.8 | 1.0000 | 0.7132 | 0.9596 | 0.0000 | 0.8259 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_2, \psi_4 |
| 0.9 | 1.0000 | 0.6773 | 0.9545 | 0.0000 | 0.8041 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_2, \psi_4 |
| 1 | 1.0000 | 0.6415 | 0.9494 | 0.0000 | 0.7823 | $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$ | \psi_2, \psi_4 |
6.2. Sensitivity Analysis

During the decision analysis of proposed approach, the parameter \( \tau \) plays a significant role for the assurance of \( Q^* \), see (39), which demonstrates the weight age of the strategy of the ‘utmost group utility’. Specifically, when \( \tau \) is 0, then the ranking is as \( \psi_4 = \psi_2 = \psi_5 = \psi_3 = \psi_1 = \psi_4 \). If \( 0.1 \leq \tau \leq 1 \), then the ranking order is as follows: \( \psi_4 > \psi_2 > \psi_5 > \psi_3 > \psi_1 \) where \( > \) means ‘preferred to’. In what pursues, we examine the impact of the parameter \( \tau \) to the estimation of \( Q^* \) in detail. The results are appeared in Table 5. From this analysis, we conclude that the decision-maker can plan to choose the values of parameter \( \varrho \) and therefore, their respective alternatives according to his goal. Therefore, the proposed measures give various choices to the decision-maker to reach the target. It can be seen that the change of \( \tau \) has no impact on the ranking order.

6.3. Comparative Analysis and Further Discussion

To verify the usefulness of the proposed MCDM method, the above example was solved by using the methods in the existing literature as given by the researchers with same attribute weights information and results are depicted in Table 6.

Table 6. Comparison of the alternatives by different methods under IF environment.

| Methods          | Environment | Model      | Ranking                  |
|------------------|-------------|------------|--------------------------|
| Boran et al. [38] | IFSs        | TOPSIS     | \( \psi_4 > \psi_3 > \psi_5 = \psi_2 > \psi_1 \) |
| Gomes and Rangel [62] | IFSs        | TODIM      | \( \psi_4 > \psi_5 > \psi_3 > \psi_2 > \psi_1 \) |
| You and Liu [37]  | IFSs        | VIKOR      | \( \psi_4 > \psi_1 > \psi_2 > \psi_3 > \psi_5 \) |
| Proposed method   | IFSs        | VIKOR-TODIM| \( \psi_4 > \psi_2 > \psi_5 > \psi_3 > \psi_1 \) |
The ranking of alternatives show that the first choice $\psi_4$ as the most suitable alternative in above methods. In our proposed method $\psi_4$ is best choice but ranking order does not matter for other alternatives. Now, a very common query that arises in mind is that how the proposed method is superior to other methods for comparison. The weight assigned to each criteria can vary much ultimately obtained reflected in result of a method. This example proves that the proposed decision-making approach is competent to getting reasonable results. Thus, the priority of the new information measure is also verified.

(1) Gomes and Rangel [62] approach does not consider the entropy information, which produces various results for different operators. In the proposed method, there is no risk of loss information because it is useful to consider the entropy information of the criteria as well as the discrimination between the alternatives.

(2) Boran et al. [38] method only considers the distances from the ideal solution and from the negative ideal solution, without considering their relative importance. The final results yielded by the intuitionistic fuzzy TOPSIS are not always the closet to the ideal solution.

(3) In comparison with You and Liu [37], the proposed method can effectively handle the MCDM problems by taking decision makers behaviours into consideration. The approach given by You and Liu [37] fails to get sensible decision reliability.

(4) The results obtained by the proposed method found to be more accurate as it takes DMs’ bounded rationality into account as well as an aggregating function of all criteria with an advantage rate. As a result, the combined VIKOR-TODIM method can make full use of their merits in the complicated decision-making problems. The advantages of VIKOR and TODIM are all fully utilised. Hence, it can efficiently address the circumstance where DMs have limited rationality and some conflicting criteria. The modified VIKOR-TODIM method is more generalised and suitable to solve the real-life problem more accurately than the existing ones.

![Figure 3](image-url)  
*Figure 3. Statistics of Comparison rankings.*
**A Graphical Analysis:** For a quick analysis, the results of the above discussion are displayed in Figure 3. The bar chart in Figure 3 has illustrated that all results from three methods are not exactly the same. In the present case, some parts of the outcomes are totally different.

The functional utility of proposed technique is altogether clarified dealing with the issue of the board information framework in an educating clinic.

**7. Conclusions**

In this study, when there is an absence of data and partial complete information and decision experts are in an intuitionistic fuzzy environment, then we proposed a new intuitionistic information measure for IFSs. The largest contribution of the proposed work is that it integrated the advantages of IFNs, entropy information of the criteria, TODIM and VIKOR to well address the decision-making problem. Firstly, we have developed an entropy-weight method to determine the weights of criterion’s under intuitionistic fuzzy environment with the weights of the criteria are completely unknown. An advantage of the proposed method is it deduces the subjectivity influence and remains the original decision information sufficiently. Secondly, we have stretched out the VIKOR-TODIM method to deal with become an appropriate MCDM solution in which the importance of criteria are given as linguistics described by IFNs and the preference rating of alternatives. The functional utility of the proposed IF-approach is thoroughly explained dealing with the problem of management information system in a teaching hospital. The entropy-based VIKOR-TODIM method coordinated with IFS has excellent opportunities for success in multi-criteria decision-making because it implies relatively objective, intuitive and vague information of decision experts. The proposed VIKOR-TODIM approach has less loss of information and delivers more correct, more general and precise results and can be utilised to tackle the uncertainties in decision-making problems under intuitionistic fuzzy environment.

In the future, authors can expand the proposed method by applying various MCDM techniques (e.g. PROMETHEE, LINMAP (Linear Programming Method for Multidimensional Analysis of Preference), ELECTRE or TOPSIS) to select best criteria. Furthermore, we should also be considered that the method could be found applications on some other problems like as strategic project evaluation, supplier selection, factory location, marketing management, renewable energy technologies, etc. The proposed IF-entropy measure can be extended to the concept of the symmetric or parametric divergence measure for Intuitionistic fuzzy sets, Picture fuzzy sets, Pythagorean fuzzy sets.

**Disclosure statement**

No potential conflict of interest was reported by the author(s).

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