Testing the Zurek-Kibble Causality Bounds with Annular Josephson Tunnel Junctions

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Zurek has provided simple causal bounds for the onset of phase transitions in condensed matter, that mirror those proposed by Kibble for relativistic quantum field theory. In this paper we show how earlier experiments with annular Josephson tunnel Junctions are consistent with this scenario, and suggest how further experiments might confirm it.
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1. INTRODUCTION

As the early universe went through a period of rapid cooling it changed phase several times. A signal of such transitions would be the direct or indirect observation of topological defects (e.g. magnetic monopoles) arising from the inhomogeneities of the ordered phase. Such defects appear because the correlation lengths $\xi(t)$ of the order parameter fields are necessarily finite for transitions implemented in finite time. This is even though, for the continuous transitions that we have in mind, the equilibrium correlation lengths $\xi_{eq}$ would diverge at the critical temperature if there were time enough (the adiabatic approximation).

Kibble observed that this simple causality imposed useful constraints on domain growth and the density of defects at the time of their formation. Unfortunately, because of our lack of knowledge about the details of the transitions in the early universe it is impossible to find reliable predictions for current observations. However, since notions of causality are not specific to the relativistic quantum field theory (QFT) appropriate to the early uni-
verse, Zurek argued that similar causal bounds were valid in condensed matter systems for which direct experiments on defect formation could be performed. These bounds can be understood in several ways. For our purposes the most useful is the following:

As an ordering transition begins to be implemented, from time \( t = 0 \), say, there is a maximum speed \( c(t) \) at which the system can become ordered. This leads to causal horizons, outside of which there are no correlations. Zurek proposed that the earliest time (the ‘causal time’) at which defects can form is when a local causal horizon is large enough to accommodate a single defect at that time. Since the defect size \( \xi(t) = \xi(T(t)) \) depends on temperature \( T \), which depends on time, the causal time \( \bar{t} \) is determined from

\[
\xi(\bar{t}) \approx 2 \int_0^{\bar{t}} dt \ c(t).
\] (1)

This time is not to be confused with the earlier time, that depends on the global structure of the system, when the coherence length becomes smaller than the system. The two successful experiments on defect production in superfluid \(^3\)He give very clear evidence for the primacy of the causal horizon in determining when defects form.

There are other ways to formulate causality, most simply by requiring that \( \xi(t) \) cannot grow faster than \( c(t) \). This can be imposed either before the transition, or after. In simple systems all bounds agree, up to numerical factors approximately unity. To the level at which we are working (better than an order of magnitude, but to a factor of a few), they are indistinguishable.

Whatever the case, although the causal bounds are robust, the extent to which they are saturated depends on the details of the microscopic dynamics. From the microscopic level, causality along the lines above is not explicit, although encoded in the relevant dynamical equations. The picture is rather one of order being established through the growth of the amplitudes of long-wavelength instabilities. The earliest time at which we can identify defects from this viewpoint is when the order parameters have achieved their equilibrium magnitudes. Qualitatively, for simple models this time is also in agreement with the causal time above, showing that the causal bounds are approximately saturated. There is no real surprise in this. The causal time and distance scales \( \bar{t} \) and \( \bar{\xi} = \xi(\bar{t}) \) are just as we would expect from dimensional analysis (in the mean-field approximation) and unstable modes grow exponentially, whereby the dependence of the causal time (and corresponding defect density) on the microscopic parameters is only (square-root) logarithmic. However, before the causal time we now have a picture in which there is a fractal thermal fuzz of potential defects, whose density depends on the scale at which we look. By the causal time some of these have matured
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into the (scale-independent) defects that we see subsequently. This is seen in numerical simulations, and see the article by one of us (RR) in these proceedings, where this is discussed at greater length.

The experimental confirmation of the Kibble-Zurek predictions for vortex production in superfluid $^3$He, at better than order-of-magnitude level, reinforced the hope that, if we understood why the bounds were approximately respected there and elsewhere, we might have a better understanding of whether they would be respected in QFT. While $^3$He has an unparalleled richness in its order parameters and the topological defects that can be produced in it, it is a difficult system in which to understand the fundamentals of defect formation. As an independent test of his assumptions Zurek also proposed experiments to measure spontaneous superflow in $^4$He and spontaneous flux generation in superconductors (essentially topological charge density) in much simpler one-dimensional annular geometries, for which predictions are easier to make. In practice, correlation lengths in both these systems are so small at the relevant timescales that it has been difficult to make reasonably 1-dimensional systems, and the experiments have not been performed.

It is with this in mind that, in this paper, we shall examine the spontaneous generations of defects ('fluxons') in annular Josephson tunnel Junctions (JTJs) for which correlations lengths are sufficiently large that effective 1-dimensional systems can be fabricated and the Zurek predictions can be tested. One of us (RM) has conducted such experiments in JTJs in the context of a different experimental programme that was completed before the suggestion was made that the Zurek bounds were relevant in that case.

Although the choice of materials was not optimised for the Zurek scenario the results were very striking. Firstly, defects were, indeed, detected on quenching the system. This is in contrast to their non-observance in some other experiments, on planar superconductors and $^4$He, specifically designed to check the Zurek picture. Secondly, although the circumference of the ring was at least three orders of magnitude larger than a fluxon, their production was rare. A single fluxon only appeared a few percent of the time, two very rarely, and more than two, never. At the time there was no understanding as to why it was so difficult to produce fluxons and no way to estimate the likelihood of their appearance. For the purpose of the experiments this did not matter, since those that were produced were then used for other purposes. We aim to show that these experimental results are compatible with Zurek’s predictions which, to date, provide their only quantitative explanation. However, because of the compound nature of JTJs, causal bounds and their saturation is more complicated.

The possibility that the Zurek-Kibble analysis could be applied to JTJs
was first considered briefly by two of us (EK and RR), and later a more substantial analysis was reported elsewhere. This paper provides an extended and updated presentation of the theory and future experiments.

2. JOSEPHSON TUNNEL JUNCTIONS

An annular JTJ consists of two superimposed annuli of ordinary superconductors of thickness $d_s$, width $\Delta r$, separated by a layer of oxide of thickness $d_{ox}$, with relative dielectric constant $\epsilon_r$. We consider a quench in which the (spatially uniform) temperature $T(t)$ of the JTJ varies in time as

$$\frac{dT}{dt} = -\frac{T_c}{\tau_Q}$$

in the vicinity of $T_c = T(0)$, $\tau_Q$ being the quenching time.

Above the transition, at temperature $T_c$, we have two normal conductors but, after the transition has been completed, the complex order parameters $\Psi_1 = \rho_1 e^{i\phi_1}$ and $\Psi_2 = \rho_2 e^{i\phi_2}$ of the individual superconductors are characterised by the phases $\phi_1$ and $\phi_2$ alone. The Josephson current density at any point $x$ in the annulus is

$$J = J_c(T) \sin \phi(x),$$

where $\phi(x) = \phi_1(x) - \phi_2(x)$, and $J_c(T)$ depends on the nature of the junction. All other things being constant, the larger the resistance of the oxide, the weaker the coupling between the superconductors and the smaller the critical current $J_c(0)$.

2.1. The individual superconductors

Before considering the effect of a quench on a JTJ it is helpful, for purposes of comparison, to consider the effect of the same quench on each of the individual superconductors, in the limit (vanishing critical current $J_c$) in which they are not connected. We also take the limit in which their thickness is smaller than any other scale.

This has been analysed already by Zurek, and we repeat his conclusions. Consider a single superconducting ring with scalar order parameter $\Psi = \rho e^{i\phi}$. We are being simplistic here in our evasion of the gauge properties of the theory. However, for strong second-order transitions it should be
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true - essentially we are looking for a benchmark. In a mean-field approximation the relevant adiabatic correlation length has the behaviour

$$\xi(t) = \xi(T(t)) = \xi_0 \sqrt{\frac{\tau_Q}{t}}$$

(4)
in terms of the fundamental length scale $\xi_0$ (the size of a cold defect, among other things). The maximum speed at which the field can order itself is

$$c(t) = c(T(t)) = C_0 \sqrt{\frac{t}{\tau_Q}},$$

(5)

where $C_0 = \xi_0/\tau_0$ is given in terms of $\tau_0$, the relaxation time for long wavelength modes. [We save the symbol $c_0$ for the speed of light in vacuo.] Note that $c(t)$ shows critical slowing down.

Imposing the causal bound (1) gives a causal time $\bar{t}_s$ for individual superconductors of, approximately,

$$\bar{t}_s = \sqrt{\tau_0 \tau_Q},$$

(6)
at which time the causal horizon is

$$\bar{\xi}_s = \xi(\bar{t}_s) = \xi_0 \sqrt{\frac{\tau_Q}{\bar{t}_s}} = \xi_0 \left(\frac{\tau_Q}{\tau_0}\right)^\frac{1}{4}.$$  

(7)

At these very early times the single length $\bar{\xi}_s$ serves to set both the coherence length in $\rho$, as it achieves its equilibrium value, and in $\varphi$. With each change in $\varphi$ of $2\pi$ will be associated with a unit of flux within the annulus. At the risk of causing some confusion, we could also term these fluxons. On average, the number $N$ of such fluxons will be zero. However, the variance in $N$ will be non-zero. If, as a first guess, we assume that the phase $\varphi$ takes a random walk around the annulus, circumference $C = 2\pi r$,

then for $C \gg \bar{\xi}_s$,

$$\Delta N = \frac{\Delta \varphi}{2\pi} = O\left(\frac{1}{2\pi} \sqrt{\frac{C}{\bar{\xi}_s}}\right).$$

(8)

There are many qualifications to (8), but this is a good starting point.

Numerically, Zurek proposed a system-independent estimate for $\tau_0$ from Gorkov’s equation as $\tau_0 = \pi \hbar /16 k_B T_c \approx 0.15$ ps for $T_c \approx 10$ K. The end result is that, for $\tau_Q$ in seconds and $T_c$ in degrees Kelvin,

$$\bar{t}_s \approx \sqrt{\tau_Q/T_c} \mu s.$$ 

(9)

For $\tau_Q = 1$ s and $T_c = 10$ K, $\bar{t}_s$ is a fraction of a microsecond. Further, with a reasonable choice of $\xi_0 \approx 100$ nm, we have $\bar{\xi}_s \approx 0.1$ mm.
When we first tried to understand whether the Zurek-Kibble bounds had any relevance to the experiments of Ref. 10, two of us made the naive approximation that, up to the formation of fluxons, we could treat the superconductors as independent, with uncorrelated phase angles $\varphi_1$ and $\varphi_2$. In that case, $(\Delta \varphi)^2 = (\Delta \varphi_1)^2 + (\Delta \varphi_2)^2$. For a ring of circumference 0.5 mm, as in the experiments, we would predict $\Delta N = O(1)$ from (8). This is an order of magnitude too large, but there is a certain amount of latitude in all these estimates, and it did not seem totally unreasonable.

However, what made the approximation untenable was that, on keeping the superconductors otherwise identical, but changing the coupling between them, no fluxons were seen at a level at which they would have been seen, had the superconductors behaved independently. We therefore have to turn to coupled junctions, where we find a reason for this.

### 2.2. Josephson tunnel junctions

In order to establish causality bounds for JTJs we need to identify the velocity that establishes the size of the causal horizons and the length that characterises the fluxon size.

The strength of Zurek’s predictions lies in the assumption that, in general, these can be read off from the time and space derivatives alone in the equations of motion in the adiabatic regime. To see how well the bounds are saturated requires further knowledge, but we shall assume that they will be well saturated here, as exemplified by the $^3$He experiments. Further, in this scenario the adiabatic approximation is pushed to its limits, by which we take it to be approximately valid from the causal time onwards.

If we ignore dissipative effects from quasiparticle tunneling and surface losses in the adiabatic regime at temperature $T < T_c$, $\varphi$ satisfies the one-dimensional Sine-Gordon (SG) equation

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{c^2(T)} \frac{\partial^2 \varphi}{\partial t^2} = \frac{1}{\lambda_J(T)^2} \sin \varphi,$$

provided the width $\Delta r$ of the annulus, of radius $\bar{r}$, satisfies $\Delta r \ll \bar{r}$ and $\Delta r \ll \lambda_J(T)$, the Josephson coherence length. In this case $x$ measures the distance along the annulus, with periodic boundary conditions. Unlike the case of superfluid $^4$He and annular (single) superconductors, it is not difficult to implement $\Delta r \ll \lambda_J(T)$, as we shall see.

As with other established models of defect formation, the classical equations are only valid once the transition is complete. We shall not use the SG equation, together with its dissipative and other terms, to study the evolution of defects. In fact, we do not study their evolution at all. However,
Eqs. 10 and 12 are sufficient to enable us, in the spirit of the Zurek-Kibble scenario, to identify $\lambda_J(T)$, diverging at $T_c$, as the equilibrium correlation length $\xi_{eq}$ to be constrained by causality. Further, the Swihart velocity $\varpi(T)$ (with critical slowing down at $T = T_c$), measures the maximum speed at which the order parameter can change\textsuperscript{[17,18]}. We shall see later that, formally, $\varpi(t)$ has a behaviour similar to $c(t)$,

$$c(t) = c(T(t)) = c_0 \sqrt{\frac{t}{\tau_Q}}. \quad (11)$$

Away from the critical temperature dissipative effects are small.

The classical topological defects (the JTJ fluxons) are the "kinks" of the Sine-Gordon theory

$$\varphi_{\pm}(x, t) = 4 \arctan \exp \left[ \pm \frac{x - ut}{\lambda_J(T) \sqrt{1 - (u/c(T))^2}} \right], \quad (12)$$

where $|u| < \varpi(T)$.

In fact, (12) is oversimplified on many counts. For example, the maximum speed of the JTJ fluxon is reduced in the presence of an external magnetic field (which reduces its behaviour to that of a damped pendulum). Nonetheless, at our level of approximation, the Swihart velocity remains a good candidate for determining the causal horizon, after very early times.

At very early times after the beginning of the transition ($t > 0$), before the individual superconductors have the ability to superconduct, the causal bound is, most plausibly, driven dissipatively by $c(t)$ of (5). If it were the case that $C_0$ and $\varpi_0$ were equal, we would be happy to use the formula (13) all the way down to $t = 0$. Although $\varpi_0$ depends on the nature of the junction, rather than just the individual superconductors, for the junctions of Ref. [10], $\varpi_0 \approx 10^7$ m/s for the samples used. This is an order of magnitude larger than $C_0$, as diagnosed earlier. However, we note that the size of the causal horizon at the causal time only depends on the scale of the velocity to its quarter power. To the level at which we are working (better than an order of magnitude, but to a factor of a few), we can take $\bar{c}(t)$ as the velocity in (11).

Our horizon bound for the earliest time $\bar{t}$ that we can see classical JTJ fluxons is when the causal horizon is big enough to accommodate one. That is,

$$\lambda_J(\bar{t}) \approx 2 \int_0^{\bar{t}} dt \bar{c}(t) \quad (13)$$

at $\bar{t} = \bar{t}_J$. This is consistent in that, at time $\bar{t}_J$, classical JTJ fluxons that have been formed will be contracting at the Swihart velocity\textsuperscript{[14]}. To have
R. Monaco, R.J. Rivers and E. Kavoussanaki attempted to produce them before (as would happen with a slower velocity) would make them want to contract faster than causality permits.

We shall give details later, but the end result is that $t_J \geq t_s$, at our level of approximation. We now understand better why our earlier assumption of independent superconductors is not correct. As the superconductors establish themselves, thermal fluctuations lead to the production of phase rotations in each superconductor (as discussed in Section 2.1). However, with the conductors connected, there will be continuous cancellation between them. Flux lines of opposite direction can annihilate in the oxide and disappear from the system to leave only the residual locally unpaired flux lines now classical JTJ fluxons with correlation length $\lambda_J$. They will be reduced in number to our earlier guess, in agreement with experiment.

3. SYMMETRIC JTJs

After these generalities, we need a specific model. For simplicity we take a symmetric JTJ, in which the electrodes are made of identical materials with common critical temperatures $T_c$ and assume a quench in which the (spatially uniform) temperature $T(t)$ of the JTJ varies in time as in (2).

We reiterate that, in our approximation, the Zurek-Kibble bounds rely on the fact that the Swihart velocity is, numerically at least, the maximum speed at which the order parameter can change at any time. For speeds lower than this, we retain the adiabatic approximation, in which the correlation length of the field $\lambda_J(t)$ is just $\lambda_J(T(t))$, where $\lambda_J(T)$ is the adiabatic (equilibrium) coherence length at temperature $T$.

For a symmetric JTJ

$$\lambda_J(T) = \sqrt{\frac{\hbar}{2e\mu_0 d_e(T)J_c(T)}},$$  \hspace{1cm} (14)$$

where $J_c(T)$ is the critical Josephson current density introduced earlier and $d_e(T)$ is the magnetic thickness. As for the latter, if $\lambda_L(T)$ is the London penetration depth of the two (identical) superconducting sheets,

$$\lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1 - (\frac{T}{T_c})^4}},$$  \hspace{1cm} (15)$$

then

$$d_e(T) = d_{ox} + 2\lambda_L(T) \tanh \frac{d_s}{2\lambda_L(T)}.$$  \hspace{1cm} (16)$$
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Neglecting the barrier thickness \( d_{ox} \ll d_s, \lambda_L \), gives \( d_e = d_s \) close to \( T_c \), i.e. the magnetic thickness equals the film thickness and can be set constant in \( \text{(14)} \).

All the \( T \)-dependence (and \( t \)-dependence) of \( \lambda_J \) resides in \( J_c \), which has the form\(^3\)

\[
J_c(T) = \frac{\pi}{2} \frac{\Delta(T)}{e\rho_N} \tanh \frac{\Delta(T)}{2k_B T}. 
\] (17)

In \( \text{(17)} \), \( \Delta(T) \) in the superconducting gap energy and varies steeply near \( T_c \) as

\[
\Delta(T) \approx 1.8\Delta(0)\sqrt{1 - \frac{T}{T_c}}.
\] (18)

and \( \rho_N \) is the JTJ normal resistance per unit area.

Introducing the dimensionless quantity \( \alpha = 1.6\Delta(0)/k_B T_c \), whose typical value is between 3 and 5, enables us to write \( J_c(T) \) as

\[
J_c(T) \approx \alpha J_c(0)(1 - \frac{T}{T_c}).
\] (19)

Thus, on repeated substitution of \( T(t) \) of \( \text{(2)} \) in the above, we see that in the vicinity of the transition,

\[
\lambda_J(t) = \lambda_J(T(t)) = \xi_0 \sqrt{\frac{\tau_Q}{t}}.
\] (20)

where

\[
\xi_0 = \sqrt{\frac{\hbar}{2\varepsilon\mu_0 d_s \alpha J_c(0)}}.
\] (21)

At the same time, for a finite electrode thickness JTJ, the Swihart velocity takes the form

\[
\tau(T) = c_0 \sqrt{\frac{d_{ox}}{\varepsilon \epsilon d_i(T)}},
\] (22)

where

\[
d_i(T) = d_{ox} + 2\lambda_L(T) \coth(\frac{d_s}{2\lambda_L(T)}).
\] (23)

In the vicinity of the transition it follows, for the quench \( \text{(2)} \), that we recover \( \tau(t) \) of \( \text{(11)} \), \( \tau(t) = c_0 \sqrt{t/\tau_Q} \), where \( c_0 = c_0 \sqrt{d_s d_{ox}/\varepsilon \epsilon \lambda_L^2(0)} \). Inspection of \( \text{[3]} \) shows that

\[
\tilde{t}_J = \sqrt{\tau_0 \tau_Q}
\] (24)
with \( \tau_0 = \xi_0 / c_0 \), in analogy with (7).

Making the second assumption of Zurek and Kibble that \( \lambda_J(t_J) \) also characterises kink separation at this formation time, we find

\[
\lambda_J = \lambda_J(t_J) = \xi_0 \sqrt{\frac{T_Q}{t_J}} = \xi_0 (\frac{T_Q}{\tau_0})^{\frac{1}{4}},
\]

(25)

identical in form to (7) for superconductors. The quarter-power dependence of \( \lambda_J \) on \( c_0 \) (for given \( \xi_0 \)) that we alluded to earlier is apparent. Moreover, if we keep \( c_0 \) fixed, then \( \lambda_J \propto \xi_0^{3/4} \). Inserting reasonable values of \( \xi_0 \geq 10 \mu m \) and \( c_0 = 10^7 \) m/s gives \( \tau_0 \geq 1 \) ps. Thus, if \( \tau_Q = 1 \) s we have \( t_J = 1 \) µs. We see that \( \tau_s < t_J \) by up to a factor of a few.

Whatever, we have a practical problem with symmetric JTJs in this parameter range, in that substituting these values in \( \lambda_J \) of (25) gives \( \lambda_J \approx 10 \) mm. This \( \lambda_J \) which, in Zurek’s picture, should characterise fluxon separation at the quench, would require very large annuli in order to see fluxons.

Fortunately, such concerns are theoretical since the fabrication of JTJs typically yields non-symmetric devices with more acceptable properties.

### 4. NON-SYMMETRIC JTJs

Suppose the two superconductors, 1 and 2, now have different critical temperatures \( T_{c_2} > T_{c_1} \). Fluxons only appear at temperatures \( T < T_{c_1} \), from which we measure our time \( t \). At this time the individual superconducting gap energies are

\[
\Delta_2(T_{c_1}) \simeq 1.8\Delta_2(0) \sqrt{1 - \frac{T_{c_1}}{T_{c_2}}}
\]

and \( \Delta_1(T) \) vanishes at \( t = 0 \) as

\[
\Delta_1(T_{c_1}) \simeq 1.8\Delta_1(0) \sqrt{\frac{t}{\tau_Q}}.
\]

The critical Josephson current density \( J'_c(T(t)) \) for a non-symmetric JTJ is now

\[
J'_c(T) = \frac{\pi \Delta_1(T)\Delta_2(T)}{\beta(T) e \rho N} \sum_{l=-\infty}^{\infty} \left\{ \left[ \omega_l^2 + \Delta_1^2(T) \right] \left[ \omega_l^2 + \Delta_2^2(T) \right] \right\}^{-1/2},
\]

(28)

where \( \beta = 1/k_BT \) and the \( \omega_l = (2l + 1)\pi/\beta \) are the fermionic Matsubara frequencies. Near \( t = 0 \) (\( T = T_{c_1} \))

\[
J'_c(T(t)) \simeq \alpha' J'_c(0) \sqrt{1 - \frac{T_{c_1}}{T_{c_2}}} \sqrt{\frac{t}{\tau_Q}}
\]

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instead of (28), where

\[ J'_c(0) = \frac{\pi \Delta_1(0) \Delta_2(0)}{[\Delta_1(0) + \Delta_2(0)] e \rho N}, \quad \alpha' = \frac{[\Delta_1(0) + \Delta_2(0)]}{k_B T_{c1}}, \]  

(30)

provided \( \Delta_2(T_{c1}) \ll 2\pi k_B T_{c1} \). This is the case here.

If we now construct \( \lambda_J(T(t)) \) we find

\[ \lambda_J(t) = \lambda_J(T(t)) \approx \bar{\xi}_0 (1 - \frac{T_{c1}}{T_{c2}})^{-1/4} (\frac{\tau_Q}{\tau})^{1/4}, \]  

(31)

where \( \bar{\xi}_0 \) is as in (21), and we have used the fact that \( J'_c(0) \simeq J_c(0) \) and \( \alpha' \simeq \alpha \). The crucial difference between \( \lambda_J(t) \) of (31) and \( \lambda_J(t) \) of (20) for the symmetric case lies in the different critical index. \( \lambda_J(t) \) of (31) is very insensitive to the time at which fluxons form. For the critical behaviour of (29) to be valid, our earlier condition now becomes \( 1 - \frac{T_{c1}}{T_{c2}} \gg O(\tau J/\tau Q) = O(10^{-6}) \), which is always the case. The critical time \( \bar{t}_J \), determined from (24) as before, now gives

\[ \bar{t}_J = \bar{\tau}^{4/7} \tau_Q^{3/7} (1 - \frac{T_{c1}}{T_{c2}})^{-1/7} \]  

(32)

rather than (24). For a typical value \( (1 - T_{c1}/T_{c2}) = 0.02 \) and the same values of \( \bar{\tau}_0 \) and \( \tau_Q \) as used previously, we find \( \bar{t}_J \approx 0.24 \mu s \). In turn, this gives

\[ \lambda_J \lambda_{\bar{t}_J} = \lambda_J(\bar{t}_J) \simeq \bar{\xi}_0 (1 - \frac{T_{c1}}{T_{c2}})^{-1/4} (\frac{\tau_Q}{\bar{\tau}_0})^{1/7} \simeq 1.4 \text{ mm}. \]  

(33)

This is an order of magnitude smaller than in the symmetric case, but still an order of magnitude larger than the corresponding lengths in annuli of single superconductors.

However, we note that \( \bar{t}_J \) of (32) is now slightly smaller than \( \bar{t}_s \) of (4). For all our caveats about the order of magnitude nature of the bounds, if we replace \( \bar{t}_J \) by \( \bar{t}_s \), \( \lambda_J \) of (33) is reduced slightly to 1.1 mm. The difference is negligible, given the crudity of the bounds.

In summary, for annular JTJs of this type we expect \( \lambda_J \sim 1 \text{ mm} \), rather than 10 mm or 100 \( \mu \text{m} \), and we would expect to start to see fluxons once the circumference \( C \sim 1 \text{ mm} \). This is, indeed, what happens.

5. PAST AND FUTURE EXPERIMENTS

As we said earlier, the original experiments performed by one of us (RM) were devised with other aims in mind. The intention was to produce
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fluxons for further experiments and the density and frequency with which they were produced was secondary. The experiments that have been performed, and which we have in mind, are simple in principle. As we cool an annular JTJ uniformly from above its critical temperature $T_c$, domains in $\varphi$ will form whose boundaries are the fluxons. The mean flux is zero.

The relevant observational quantity here is the variance in the flux or, equivalently, the variance $\Delta N$ in the number of fluxons minus antifluxons. This stability gives us a distinct advantage over experiments in vortex production in superfluid $^4$He in which what is measured is a (decaying) density of vortices plus antivortices. In fact the null result of Ref.12 for finding vortices in $^4$He can be attributed to uncertainty over the decay rate of vortex tangles.

If the circumference of the annulus is $C = 2\pi \bar{r}$, and the mean $\varphi$ correlation length at the time $T_J$ of the fluxon formation is $\bar{\lambda}_J$ then, in parallel with our earlier observations, we would expect, for $C \gg \bar{\lambda}_J$,

$$\Delta N = \frac{\Delta \varphi}{2\pi} = O\left(\frac{1}{2\pi} \sqrt{\frac{C}{\lambda_J}}\right), \quad (34)$$

as a result of a random walk in phase around a ring. It is the prediction that we would like to test for a variety of junctions and temperature quenches, with different $\lambda_J$. Unfortunately, for the experiments to date it has not been possible to have $C \gg \bar{\lambda}_J$.

For the experiments of Ref.10 non-symmetric annular Nb/Al-AlO_x/Nb JTJs were fabricated with a whole-wafer process in which the junctions are formed in the window of the SiO insulating layer between the base and the counter electrodes. See Ref.10 for details. The ring-shaped junctions had a mean radius $\bar{r} \cong 80 \mu m$, width $\Delta r = 4 \mu m$ and a geometry in which both the base and top electrode have a hole concentric to the ring. The junction temperature was raised above its critical value ($T_c \simeq 9.2 K$) by means of a heating resistor placed close to the (unbiased) sample; then the temperature was lowered by letting the sample cool down by exchanging heat with the liquid helium bath through some helium gas. By changing the helium gas pressure it was possible to vary the thermal constant and hence the quench time $\tau_Q$ in the range $10^{-2}$-s-1s. The samples were measured in a very well electrically and magnetically shielded environment. The number of fluxons trapped during the transition was determined simply by feeding a proper current to the junction and measuring its voltage.

Two different samples were produced, both with $1 - T_{c1}/T_{c2} \simeq 0.02$. For sample B, we estimated $\xi_0 = 6.5 \mu m$, $\tau_0 = 10^7 m/s$ and $\tau_0 = 0.65 ps$. Although $T_J \simeq T_s$, if we take (18) at face value, we find $\bar{\lambda}_J \simeq 1 mm$ (with experimental uncertainty of up to 50%). On the other hand, if we estimate...
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$\lambda_J$ at $\lambda_J(\bar{t}_s)$ we find $\lambda_J \simeq 0.7$ mm, identical at our level of approximation.

With $C \sim \lambda_J$ we cannot use (34) as it stands. However, we would expect to see a fluxon several percent of the time. In practice (invariably single) fluxons formed once every 10-20 times.

In Sample A, having a similar circumference and similar superconductors coupled differently, whereby $\lambda_J$ was 2.5 times larger, there was much less fluxon trapping, with no reliable fluxons seen in several quenches (enough to have seen something in the former case). This demonstrates that the superconductors cannot be treated individually for fluxon production, as our earlier discussion confirms. We are imprecise in the quantitative details since the aim was not to count the frequency with which defects occurred, but just to have defects at all. Nonetheless, it is apparent that, in order to have $\lambda_J \ll C$ for annular JTJs of a size comparable to the one above, we must find ways to reduce $\lambda_J$ further, as well as possibly increase $C$.

The quench time $\tau_Q$ is difficult to establish accurately. Fortunately, the small power of $\tau_Q/\bar{\tau}_0$ in (33) makes the uncertainty largely irrelevant. However, it has the consequence that a huge reduction in the quench time is necessary to have any observable effect on $\lambda_J$. Similarly, we can gain little from increasing the asymmetry of the JTJ, even if that is easily possible experimentally. In consequence, the only realistic way to reduce $\lambda_J$ is by reducing $\bar{\tau}_S$. Primarily, we need to increase $J_c(0)$ to the largest possible value which does not degrade the barrier quality, $J_c \simeq 10^4 A/cm^2$. However, for such currents we shall have $\bar{t}_s > \bar{t}_J$. The simplest assumption is that it will be $\bar{t}_s$ that will set the time-scale at which fluxons are formed. New experiments with such values are under active consideration by us at this time.

We consider this an important test of the Zurek-Kibble programme for establishing common constraints in the production of defects in condensed matter physics and relativistic quantum physics, given that the spontaneous creation of flux has not been seen in high $T_c$ planar superconductors at the level suggested by the Zurek-Kibble bound (in fact not at all), and that vortices are not observed in $^4He$ quenches. This is even though a separate experiment supports the phase separation of the Kibble mechanism.

We conclude with one final concern. Although the absence of fluxons in our Sample A, with larger $\lambda_J/C$, suggests that the observed fluxons are not artefacts of the heating and cooling environment, we might be concerned at the effects of temperature inhomogeneities. In general, moving phase boundaries leave less (or no) defects in their wake.

The critical slowing down of $\bar{c}(t)$ and $c(t)$ weakens the effect of any large scale inhomogeneities in temperature, even though it is modified slightly for idealised non-symmetric JTJs. It is difficult to put quantitative limits on the
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permissible inhomogeneities but, with empirically comparable $\xi_0$, $\tau_0$, and $\tau_Q$, the situation is no better or worse for JTJs than for any other superconducting system undergoing a mechanical quench. A possible check of these ideas could be performed by cooling with a temperature gradient across the ring circumference, although this would cause its own experimental problems. We postpone such considerations to the far future.

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