On Soft Generalized Closed Sets in a Soft Topological Space with a Soft Weak Structure

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Abstract

In this work, we introduce the notion of generalized ω-closed set in a soft topological space with a soft weak structure. And some basic properties of this new class are investigated by using the concept of weak structure. Moreover, we study soft ω-T₁²-spaces defined by soft gω-closed sets and study some properties of it by using gω-open sets.

Keywords: Soft set, Soft topology, Soft weak structure, Soft generalized closed set

1. Introduction

As a solution to many problems, scientists have resorted to use approximate solution. In 1999, Molodtsove [1] initiated the concept of soft set theory as a new mathematical tool which is free from the problems mentioned above. Later on Maji et al [2] proposed several operations on soft sets and some basic properties and then Pei and Miao [3] investigated the relationships between soft sets and information systems.

In 2011, Shabir and Naz [4] introduced the notion of soft topological spaces and Min [5] corrected some their results. Zorlutuna et al. [6] continued to study the properties of soft topological spaces by defining the concepts of interior, soft neighborhoods in soft topological spaces. Varo and Aygun [7] presented soft Hausdorff spaces and introduced some new concepts such as convergence of sequences. Levine [8] introduced generalized closed sets in topological spaces.

In 2013, Cagman et al. [9] defined soft topological spaces by modifying the soft set. Also, Roy and Samanta [10] strengthen the definition of the soft topological spaces presented in [9] and they used the base and the subbase to characterize its properties.

In 1997, Csaszar [11] has studied generalized topological notions in collections which are closed under unions. Many other authors [12–15] investigated the properties of the generalized topology. Recently, Császár [16] introduced a new notion called weak structures. Let X be a non-empty set and P(X) be its power set. A structure ω on X is called a weak structure (briefly, W) on X if and only if ϕ ∈ ω [16].

In 2012, Al-Omari and Noiri [17] introduced and studied a kind of sets called generalized ω-closed (briefly, gω-closed) sets in topological space. In this paper, we introduce the notions of soft generalized ω-closed (briefly, soft gω-closed) sets and soft gω-open sets in soft topological spaces. Also, we will investigate some new soft separation axioms. In particular, we study soft ω-T₁²-spaces.
Let $X$ be a non-empty set, $E$ a set of parameters, $P(X)$ denote the power set of $X$ and $A$ be a non-empty subset of $E$.

**Definition 1.1** ([1]). For $A \subseteq E$, a pair $(F, A)$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \rightarrow P(X)$.

For $e \in A$, $F(e)$ may be considered as the set of $e$-approximate elements of the soft set $(F, A)$.

**Definition 1.2** ([5]). A soft set $(F, A)$ over $X$ is said to be:

1. A null soft set denoted by $\emptyset$ if $F(e) = \emptyset$ for all $e \in A$.
2. An absolute soft set denoted by $\hat{X}$ if $F(e) = X$ for all $e \in A$.

**Definition 1.3** ([2][5][18]). For any two soft sets $(F, A)$ and $(G, B)$ defined over a common universe $X$, we have:

1. $(F, A) \subset (G, B)$ iff $A \subseteq B$ and $F(e) \subseteq G(e)$ for all $e \in A$.
2. $(F, A) \equiv (G, B)$ iff $(F, A) \subset (G, B)$ and $(G, B) \subset (F, A)$.
3. $(F, A) \cup (G, B) = (H, C)$ where $C = A \cup B$ and $H(e) = \begin{cases} F(e), & \text{if } e \in A - B; \\ G(e), & \text{if } e \in B - A; \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$

for all $e \in C$.
4. $(F, A) \cap (G, B) = (K, D)$ where $D = A \cap B$ and $K(e) = F(e) \cap G(e)$ for all $e \in C$.
5. $x \in (F, A)$ where $x \in X$ iff $x \in F(e)$ for all $e \in A$ and $x \notin (F, A)$ whenever $x \notin F(e)$ for some $e \in A$.

$(6)$ $(F, E) \equiv (G, E) \equiv (M, E)$ where $M(e) = F(e) - G(e)$ for all $e \in E$.

**Definition 1.4** ([19]). For a soft set $(F, A)$ over $X$, the relative complement of $(F, A)$ (denoted by $(F, A)'$) is defined by $(F, A)' \equiv (F', A)$, where $F' : A \rightarrow P(X) = \{F(e) \subseteq X - F(e) \mid e \in A\}$.

**Definition 1.5** ([4]). Let $\tau$ be the collection of soft sets over $X$, then $\tau$ is called a soft topology on $X$ if $\tau$ satisfies the following axioms:

1. $\emptyset, \hat{X}$ belong to $\tau$.
2. The union of any number of soft sets in $\tau$ belong to $\tau$.
3. The intersection of any two soft sets in $\tau$ belong to $\tau$.

The triple $(X, \tau, E)$ is called a soft topological space over $X$. The member of $\tau$ are said to be soft open set in $X$.

A soft set $(F, E)$ over $X$ is said to be soft closed in $X$ if its relative complement $(F, E)'$ belong to $\tau$.

**Lemma 1.6** ([14]). Let $(F, E)$ be a soft set over $X$ and $x \in X$, then:

1. $x \in (F, E)$ iff $(x, E) \subset (F, E)$
2. if $(x, E) \cap (F, E) = \emptyset$, then $x \notin (F, E)$.

Czarsk defined $i_{\omega}(A)$ as the union of all $\omega$-open subsets of $A$ and $c_{\omega}(A)$ as the intersection of all $\omega$-closed sets containing $A$.

Let $X$ be a non-empty set and $E$ be a set of parameters. A collection $\omega$ of soft sets defined over $X$ with respect to $E$ is called a soft weak structure ([20]) iff $\emptyset \in \omega$. A soft set $(F, A)$ is called $\omega$-open soft set iff $(F, A) \in \omega$ and called $\omega$-closed soft set iff $(F, A)' \in \omega$.

**Definition 1.7** ([19]). In a soft topological spaces $(X, \tau, E)$, a soft set $(F, E)$ over $X$ is called a soft generalized closed set (briefly, soft g-closed) if $C(I(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and $(G, E)$ is soft open in $(X, \tau, E)$.

**Lemma 1.8** ([20]). Let $\omega$ be a soft weak structure defined over $X$ with respect to $E$ and $(F, E), (G, E)$ are two soft sets over $X$, then:

1. $i_{\omega}(F, E) \subset (F, E)$
2. If $(F, E) \subset (G, E)$, then $i_{\omega}(F, E) \subset i_{\omega}(G, E)$ and $c_{\omega}(F, E) \subset c_{\omega}(G, E)$
3. $i_{\omega}(i_{\omega}(F, E)) = i_{\omega}(F, E)$ and $c_{\omega}(c_{\omega}(F, E)) = c_{\omega}(F, E)$.

4. $i_{\omega}(F, E)' = (c_{\omega}(F, E))'$ and $c_{\omega}(F, E)' = (i_{\omega}(F, E))'$.

**Lemma 1.9** ([20]). Let $\omega$ be a soft weak structure defined over $X$ with respect to the parameters set $E$ and $(F, E)$ be a soft set, then:

1. $x \in i_{\omega}(F, E)$ if there exists an $\omega$-open soft set $(G, E)$ such that $x \in (G, E) \subset (F, E)$.
2. $x \in c_{\omega}(F, E)$ if and only if $(G, E) \cap (F, E) = \emptyset$ for all $(G, E) \in \omega$ such that $x \in (G, E)$.
3. If $(F, E) \in \omega$, then $(F, E) = i_{\omega}(F, E)$ and if $(F, E)$ is $\omega$-closed soft set, then $(F, E) = c_{\omega}(F, E)$.
Let \((X, \tau, E)\) be a soft topological space over \(X\). If \((x, E)\) is a soft closed set in \(\tau\) for each \(x \in X\), then \((X, \tau, E)\) is a soft \(T_1\)-space.

**Definition 2.1.** A soft topological space \((X, \tau, E)\) is called:

1. Soft \(\omega\)-\(T_0\) if for each \(x, y \in X\) such that \(x \neq y\), there exists a soft \(\omega\)-open set \((F, E)\) such that \(x \in (F, E)\) and \(y \notin (F, E)\) or \(x \notin (F, E)\) and \(y \in (F, E)\).
2. Soft \(\omega\)-\(T_1\) if for each \(x, y \in X\) such that \(x \neq y\), there exists soft \(\omega\)-open sets \((F, E)\) and \((G, E)\) such that \(x \in (F, E), y \notin (F, E), x \notin (G, E)\) and \(y \in (G, E)\).

2. Soft \(\omega\)-Closed Set

**Definition 2.2.** Let \(X\) be a non-empty set and \(E\) be a set of parameters. Let a collection \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\). Then a soft set \((F, E)\) over \(X\) is called a soft generalized \(\omega\)-closed set (briefly, soft \(\omega\)-closed) if \(c_\omega(F, E) \bar{\subset} (G, E)\) whenever \((F, E) \bar{\subset} (G, E)\) and \((G, E)\) is soft open in \((X, \tau, E)\). The complement of a soft \(\omega\)-closed set is called a soft generalized \(\omega\)-open (briefly, soft \(\omega\)-open) set.

**Example 2.2.** Let \(X = \{h_1, h_2, h_3\}, E = \{e_1, e_2\}\) and \(\tau = \{\emptyset, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}\) where

\[
\begin{align*}
F_1(e_1) &= \{h_2\}, & F_1(e_2) &= \{h_1\}; \\
F_2(e_1) &= \{h_2, h_3\}, & F_2(e_2) &= \{h_1, h_2\}; \\
F_3(e_1) &= \{h_1, h_2\}, & F_3(e_2) &= \{h_1, h_3\}; \\
F_4(e_1) &= \{h_1\}, & F_4(e_2) &= \{h_3\}.
\end{align*}
\]

Let \(\omega = \{\emptyset, (F_1, E), (F_3, E)\}\) be a soft weak structure over \(X\) with respect to \(E\). Then \((G, E)\) is a soft \(\omega\)-closed defined by \(G(e_1) = \emptyset; G(e_2) = \{h_2\}\), but \((F_1, E)\) is not soft \(\omega\)-closed.

**Remark 2.3.** Let \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\). Then every soft \(\omega\)-closed set is soft \(g\)-closed. The following example shows that the converse need not be true in general.

**Example 2.4.** In Example 2.2 \((F_2, E)\) is soft \(g\)-closed in \((X, \tau, E)\) but not soft \(\omega\)-closed over \(X\).

**Remark 2.5.** For a soft weak structure \(\omega\) on a soft topological space over \(X\), every \(\omega\)-closed set is a soft \(g\)-closed set. In fact, if \((F, E)\) is a \(\omega\)-closed set \((F, E) \bar{\subset} (G, E)\) and \((G, E)\) is soft open, then \((F, E) = c_\omega(F, E) \bar{\subset} (G, E)\), so that \((F, E)\) is soft \(g\)-closed. The following example shows that the converse need not be true in general.

**Example 2.5.** In Example 2.2 \((F_2, E)\) is soft \(g\)-closed but not soft \(\omega\)-closed.

**Theorem 2.6.** Let \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\). If \((F, E)\) is soft \(g\)-closed in \(X\) and \((F, E) \bar{\subset} (H, E) \bar{\subset} c_\omega(F, E)\), then \((H, E)\) is soft \(g\)-closed.

**Proof.** Suppose that \((F, E)\) is soft \(g\)-closed over \(X\) and \((F, E) \bar{\subset} (H, E) \bar{\subset} c_\omega(F, E)\). Let \((H, E) \bar{\subset} (G, E)\) and \((G, E)\) is soft open in \(X\). Since \((F, E) \bar{\subset} (H, E) \bar{\subset} c_\omega(F, E)\), we have \((F, E) \bar{\subset} (G, E)\). Since \((F, E)\) is soft \(g\)-closed, then \(c_\omega(A, E) \bar{\subset} (G, E)\). Since \((H, E) \bar{\subset} c_\omega(F, E)\), we have \(c_\omega(H, E) \bar{\subset} c_\omega(F, E) \bar{\subset} (G, E)\). Therefore \((H, E)\) is soft \(g\)-closed.

The next example shows that the intersection of two soft \(g\)-closed sets is not in general soft \(\omega\)-closed.

**Example 2.8.** Let \(X = \{h_1, h_2, h_3\}, E = \{e_1, e_2\}\) and \(\tau = \{\emptyset, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}\) where

\[
\begin{align*}
F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_2\}; \\
F_2(e_1) &= \{h_3\}, & F_2(e_2) &= \{h_1, h_3\}; \\
F_3(e_1) &= \{h_1, h_2\}, & F_3(e_2) &= \{h_2, h_3\}; \\
F_4(e_1) &= \{h_1, h_3\}, & F_4(e_2) &= \emptyset.
\end{align*}
\]

Let \(\omega = \{\emptyset, (F_2, E)\}\) be a soft weak structure over \(X\) with respect to \(E\). Then \((H, E)\) is a soft \(g\)-closed defined by \((H, E) = H(e_1) = \{h_2\}\). Thus it can be easily checked that \((F_1, E) \bar{\subset} (H, E)\) is not a soft \(g\)-closed set.

**Theorem 2.9.** Let \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\). If \((F, E)\) is a soft \(g\)-closed set, then \(c_\omega(F, E) \bar{\subset} (F, E)\) does not contain any non-empty soft closed set.

**Proof.** Let \((H, E)\) be a soft closed subset of \(X\) such that \((H, E) \bar{\subset} c_\omega(F, E) \bar{\subset} (F, E)\), where \((F, E)\) is soft \(g\)-closed. Since \((H, E)\) is soft open, \((F, E) \bar{\subset} (H, E) \bar{\subset} (F, E)\) is soft \(g\)-closed, \(c_\omega(F, E) \bar{\subset} (H, E) \bar{\subset} (F, E)\) and thus \((H, E) \bar{\subset} c_\omega(F, E) \bar{\subset} (F, E)\). Thus \((H, E) \bar{\subset} c_\omega(F, E) \bar{\subset} (F, E)\) does not contain any non-empty soft closed subset of \(X\), then \((F, E)\) need not be soft \(g\)-closed in general.

**Example 2.10.** In Example 2.2 let \((G, E)\) be a soft set defined by \(G(e_1) = \emptyset, G(e_2) = \{h_2\}\). Then \(c_\omega(G, E) \bar{\subset} (F, E)\) does not contain any non-empty soft closed subset of \(X\), then \((F, E)\) need not be soft \(g\)-closed in general.
\((K, E)\) does not contain any non-empty soft closed set such that \(K(e_1) = \{h_1, h_2\}\), \(K(e_2) = \emptyset\), but \((G, E)\) is not a soft \(g\omega\)-closed set.

**Corollary 2.11.** Let \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\) and \((F, E)\) be a soft \(g\omega\)-closed set. Then \(c_\omega(F, E) = (F, E)\) if and only if \(c_\omega(F, E) - (F, E)\) is soft closed.

**Proof.** Let \((F, E)\) be a soft \(g\omega\)-closed set. If \(c_\omega(F, E) = (F, E)\), then \(c_\omega(F, E) - (F, E) = \emptyset\) and \(c_\omega(F, E) - (F, E)\) is a soft closed set.

Conversely, let \(c_\omega(F, E) - (F, E)\) be a soft closed set. Since \((F, E)\) is soft \(g\omega\)-closed, then by Theorem 2.9, \(c_\omega(F, E) - (F, E)\) does not contain any non-empty soft closed set. Since \(c_\omega(F, E) - (F, E)\) is a soft closed subset of itself, \(c_\omega(F, E) - (F, E) = \emptyset\) and hence \(c_\omega(F, E) = (F, E)\). \(\square\)

**Theorem 2.12.** Let \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\). Then \((H, E)\) is soft \(g\omega\)-open if and only if \((F, E)\tilde{c}(\omega)(H, E)\) whenever \((F, E)\tilde{c}(H, E)\) and \((F, E)\) is soft closed.

**Proof.** Let \((H, E)\) be a soft \(g\omega\)-open set and \((F, E)\tilde{c}(H, E)\), where \((F, E)\) is soft closed. Then \((H, E)'\) is a soft \(g\omega\)-closed set contained in a soft open set \((F, E)'\). Hence \(c_\omega(H, E)'\tilde{c}(F, E)'\), i.e. \((i_\omega(H, E))'\tilde{c}(F, E)'\). So \((F, E)\tilde{c}(i_\omega(H, E))\).

Conversely, Suppose that \((F, E)\tilde{c}(H, E)\) for any soft closed set \((F, E)\) whenever \((F, E)\tilde{c}(H, E)\). Let \((H, E)'\tilde{c}(G, E)\), where \((G, E)\) is a soft open set. Then \((G, E)'\tilde{c}(H, E)\) and \((G, E)'\) is soft closed. By assumption, \((G, E)'\tilde{c}(i_\omega(H, E))\) and hence \(c_\omega(H, E) = (i_\omega(H, E))'\tilde{c}(G, E)\). Therefore, \((H, E)'\) is soft \(g\omega\)-closed and hence \((H, E)\) is soft \(g\omega\)-open. \(\square\)

**Theorem 2.13.** Let \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\). Then the following are equivalent:

1. For every soft open \((G, E)\) of \(X\), \(c_\omega(G, E)\tilde{c}(G, E)\).
2. Every soft subset of \(X\) is soft \(g\omega\)-closed.

**Proof.** (1)\(\implies\) (2) Let \((F, E)\) be any soft subset of \(X\) where \((F, E)\tilde{c}(G, E)\) and \((G, E)\) is any soft open set. Then by (1), \(c_\omega(G, E)\tilde{c}(G, E)\) and hence \(c_\omega(F, E)\tilde{c}(G, E)\).

Thus \((F, E)\) is soft \(g\omega\)-closed.

(2)\(\implies\) (1) Let \((G, E)\) be any soft open set. Then by (2), \((G, E)\) is soft \(g\omega\)-closed and hence \(c_\omega(G, E)\tilde{c}(G, E)\). \(\square\)

**Theorem 2.14.** Let \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\). If a soft set \((H, E)\) is soft \(g\omega\)-open and if for a soft open set \((G, E)\), \(i_\omega(H, E)\tilde{c}(H, E)'\tilde{c}(G, E)\), then \((G, E)' = \emptyset\).

**Proof.** Let \((G, E)\) be a soft open set and \(i_\omega(H, E)\tilde{c}(H, E)'\tilde{c}(G, E)\) for a soft \(g\omega\)-open set \((H, E)\). Then \((G, E)'\tilde{c}(i_\omega(H, E))'\tilde{c}(H, E)\). This implies \((G, E)'\tilde{c}(c_\omega(H, E) - (H, E))\). Since \((H, E)'\) is soft \(g\omega\)-closed, by Theorem 2.9, \((G, E)' = \emptyset\). \(\square\)

**Theorem 2.15.** Let \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\). If a soft set \((H, E)\) is soft \(g\omega\)-open and \(i_\omega(H, E)\tilde{c}(K, E)\tilde{c}(H, E)\), then \((K, E)\) is soft \(g\omega\)-open.

**Proof.** We have \((H, E)'\tilde{c}(K, E)'\tilde{c}(i_\omega(H, E))'\). Since \((H, E)'\) is soft \(g\omega\)-closed, from Theorem 2.7, it follows that \((K, E)'\) is soft \(g\omega\)-closed, and hence \((K, E)\) is soft \(g\omega\)-open. \(\square\)

3. Separation Axioms

In this section, we introduce the new separation axiom, namely soft \(\omega-T_{\frac{1}{2}}\)-space in soft topological space with a soft weak structure \(\omega\).

**Definition 3.1.** A soft topological space \((X, \tau, E)\) with a soft weak structure \(\omega\) is called a soft \(\omega-T_{\frac{1}{2}}\)-space if for every soft \(g\omega\)-closed set \((F, E)\) of \(X\), \(c_\omega(F, E) = (F, E)\).

**Theorem 3.2.** Let \(\omega\) be a soft weak structure on \(X\). A soft topological space \((X, \tau, E)\) is soft \(\omega-T_{\frac{1}{2}}\) if \((x, E)\) is soft \(g\omega\)-closed set for all \(x \in X\).

**Proof.** Let \(x, y \in X\) such that \(x \neq y\). Then \((x, E)'\) and \((y, E)'\) are soft \(g\omega\)-open sets such \(y \in (x, E)'\), \(x \notin (x, E)'\) and \(y \notin (y, E)'\), \(x \in (y, E)'\). Hence \(X\) is soft \(\omega-T_{\frac{1}{2}}\). \(\square\)

**Corollary 3.3.** If the union of soft \(\omega\)-open sets is soft \(\omega\)-open set, then the converse of Theorem 3.2, is true.

**Theorem 3.4.** Let \(\omega\) be a soft weak structure on a soft topological space \((X, \tau, E)\) with respect to \(E\). Then the following statements are equivalent:

1. \(X\) is a soft \(\omega-T_{\frac{1}{2}}\)-space.
2. Every soft subset of \(X\) is soft \(\omega-T_{\frac{1}{2}}\)-space.

**Proof.** (1)\(\implies\) (2). Suppose \((x, E)\) is not a soft closed subset for some \(x \in X\). Then \((x, E)'\) is not soft open, so if there is, \(x \in X\) is the only soft open set containing \((x, E)'\). Therefore \((x, E)'\) is soft \(g\omega\)-closed. Since \(X\) is soft \(\omega-T_{\frac{1}{2}}\)-space, \(c_\omega(x, E)' = (i_\omega(x, E))' = (x, E)'\) and thus \((x, E) = i_\omega(x, E)\).
Let \( (F, E) \) be a soft \( g\omega \)-closed subset of \( X \) and \( x \in c_\omega(F, E) \). We show that \( x \in (F, E) \). If \( (x, E) \) is soft closed and \( x \notin (F, E) \), then \( x \in (c_\omega(F, E) - (F, E)) \). Then \( (x, E) \subset (F, E) \) and hence \( (F, E) \subset (x, E) \). Since \( (F, E) \) is a soft \( g\omega \)-closed set and \( (x, E) \) is a soft open, \( c_\omega(F, E) \subset (x, E) \) and hence \( (x, E) \subset (c_\omega(F, E), E) \). Therefore, \( (x, E) \subset c_\omega(F, E) \) and \( (c_\omega(F, E), E) \). This is a contradiction. Therefore, \( x \in (F, E) \). If \( (x, E) = i_\omega(x, E) \), since \( x \in c_\omega(F, E) \), then for every soft \( \omega \)-open set \( (G, E) \) such that \( x \in (G, E) \), we have \( (G, E) \cap (F, E) \neq \emptyset \). By assumption \( (x, E) = i_\omega(x, E) \), we have \( (x, E) \) be an \( \omega \)-open set and \( (x, E) \cap (F, E) \neq \emptyset \). Hence \( x \in (F, E) \). Therefore, in both cases we have \( x \in (F, E) \). Therefore, \( c_\omega(F, E) = (F, E) \) and hence \( X \) is a soft \( \omega-T_2 \)-space.

**Theorem 3.5.** If the union of soft \( \omega \)-open sets is soft \( \omega \)-open set in a soft weak structure \( \omega \) on a soft topological space, then every soft \( \omega-T_2 \)-space is a soft \( \omega-T_2 \)-space.

**Proof.** Suppose that \( (X, \tau, E) \) is a soft \( T_1 \)-space and the union of soft \( \omega \)-open sets is soft \( \omega \)-open set. It suffices to show that a set which is not soft \( \omega \)-closed also is not soft \( g\omega \)-closed set. Let \( (F, E) \) is not soft \( \omega \)-closed. Let \( x \in c_\omega(F, E) - (F, E) \). Then \( (x, E) \subset c_\omega(F, E) - (F, E) \) and \( (x, E) \) is a nonempty soft \( \omega \)-closed set in \( X \) by Corollary 3.3. Hence, by Theorem 2.9, \( (F, E) \) is not soft \( g\omega \)-closed.

The next example shows that the converse of the above theorem is not true in general.

**Example 3.6.** Let \( X = \{h_1, h_2, h_3\} \), \( E = \{e_1, e_2\} \) and \( \tau = \{\emptyset, \bar{X}, (F_1, E), (F_2, E)\} \) where

\[
F_1(e_1) = \{h_1\}, \quad F_1(e_2) = \{h_1\};
\]
\[
F_2(e_1) = \{h_1, h_2\}, \quad F_2(e_2) = X;
\]

Let \( \omega = \{\emptyset, (F_1, E), (F_2, E)\} \) be an \( \omega \) over \( X \) with respect to \( E \). Then \( (X, \tau, E) \) is soft \( \omega-T_2 \)-space but not soft \( \omega-T_3 \).

**Theorem 3.7.** Let \( \omega \) be a soft weak structure on a soft topological space \( (X, \tau, E) \). If \( X \) is soft \( \omega-T_0 \), then for each \( x, y \in X \) such that \( x \neq y \), we have \( c_\omega(x, E) \neq c_\omega(y, E) \).

**Proof.** Let \( x \) be a soft \( \omega-T_0 \) and \( x, y \in X \) such that \( x \neq y \). Then there exists soft \( \omega \)-open set \( (F, E) \) such that \( x \in (F, E) \) and \( y \notin (F, E) \). Therefore \( (F, E) \) is soft \( \omega \)-closed set such that \( x \notin (F, E) \) and \( y \in (F, E) \). Since \( c_\omega(y, E) \) is the intersection of all soft \( \omega \)-closed subsets that contain \( y \), then \( c_\omega(y, E) \subset (F, E) \) and hence \( x \notin c_\omega(y, E) \). Thus \( c_\omega(x, E) \neq c_\omega(y, E) \).

**Corollary 3.8.** If \( c_\omega(x, E) \) is a soft \( \omega \)-closed set for each \( x \in X \), and if for each distinct \( x, y \in X \), \( c_\omega(x, E) \neq c_\omega(y, E) \), then \( X \) is soft \( \omega-T_0 \).

**Proof.** For each distinct \( x, y \in X \), since \( c_\omega(x, E) \neq c_\omega(y, E) \), there exists some \( z \in X \) such that \( z \in c_\omega(x, E) \) and \( z \notin c_\omega(y, E) \). If \( x \in c_\omega(y, E) \), then \( c_\omega(x, E) \subset c_\omega(y, E) \) which is a contradiction since \( z \notin c_\omega(y, E) \). Thus \( c_\omega(y, E) \) is soft \( \omega \)-open set such that \( x \in (c_\omega(y, E)) \) and \( y \notin (c_\omega(y, E)) \).

Hence \( X \) is soft \( \omega-T_0 \).

**Conflict of Interest**

No potential conflict of interest relevant to this article was reported.

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