Direct \( N \)-body simulation of the Galactic centre

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ABSTRACT

We study the dynamics and evolution of the nuclear star cluster of the Milky Way galaxy performing a high resolution direct one-million-body simulation. We focus on the interaction of the stellar system with the supermassive black hole. We find that the 3D density distribution of stellar black holes after 5 Gyr is fully consistent with the Bahcall-Wolf slope of -1.75. We obtain the rate of tidal disruption events to be \( 4 \times 10^{-6} \) yr\(^{-1} \) and estimate the number of objects emitting gravitational waves during the accretion onto the supermassive black hole to be \( \sim 230 \) per Gyr with \( \approx 100 \) of them being possible extreme mass ratio inspirals. The examined binary fraction drops by less then a half from the initial value of 5\% with the final value of 3\% inside the inner parsec. We discuss the contribution of binaries with compact objects in the presence of pulsars and of Supernovae Ia rates.

Key words: stellar dynamics – stars: kinematics – black hole – compact binaries – gravitational waves – pulsars

1 INTRODUCTION

The centre of the Milky Way (MW) galaxy hosts the nearest supermassive black hole (SMBH). The proximity of the SMBH allows us to monitor individual stars in its immediate vicinity. Although our line-of-sight to the Galactic Centre (GC) is obscured by dust clouds, the light absorption is sufficiently low in the near-infrared to give us a chance to follow the trajectories of individual stars from ground based telescopes using adaptive optics techniques. Such observations, that are being carried out for more than 25 years (Ghez et al. 2000; Eckart et al. 2017), revealed the presence of a cluster of bright high velocity stars in the inner arcsecond (0.04 pc) of the MW, the so-called S-star cluster. One of these stars has already completed a full orbit (Schödel et al. 2002) and the next pericentre passage is expected to be in 2018. The orbital motion of stars in the S-star cluster is the strongest evidence of the SMBH so far (see for instance Ghez et al. 2000; Eckart et al. 2017 for a review). Next to the S-star cluster, at a distance between 0.04 and 0.5 pc from the SMBH, a disc(s) of even younger and more massive stars was detected (Levin & Beloborodov 2003; Paumard et al. 2006). Despite the presence of the young massive stars, the stellar component of the GC is dominated by older population (see Genzel et al. 2010, for a review). Summarizing the observational results, the GC is comprised of a central SMBH with mass \( M_{\text{SMBH}} \approx 4.3 \times 10^6 M_\odot \) (Gillessen et al. 2017) surrounded by an old nuclear star cluster (NSC) with mass \( M_{\text{NSC}} \approx 2.5 \times 10^7 M_\odot \) (Schödel et al. 2014). The inner part of the NSC features distinct dynamical components such as the S-star cluster and disc(s) of young massive stars.

The SMBH dominates the dynamics of the NSC inside the influence radius \( r_{\text{inf}} \) (a distance where the black hole potential is equal to the potential of the enclosed stellar system, see reviews by Alexander 2005; Alexander 2017). Hence, the formation of a stellar cusp is expected with the density of \( \rho \sim r^7 \) where the power-law index approaches a
value of $\gamma = -1.75$ (Bahcall & Wolf 1976; Bahcall & Wolf 1977). Although initially it seemed that densities of old stellar population in the NSC are flat or even decreasing inside 0.1 pc (e.g. Buchholz et al. 2009), recent studies show agreement with theory and simulations (Gallego-Cano et al. 2018; Schödel et al. 2018; Baumgardt et al. 2018).

If a star gets very close to the SMBH, at the distance where the tidal forces from the SMBH exceed the self gravity of the star, then the star will be tidally disrupted (Hills 1975; Frank & Rees 1976). The tidal disruption can be observed as a bright flare caused by heated stellar debris falling towards the event horizon. The classical solution of the mass fallback rate follows a power-law decay $\dot{M} \sim r^{-3}$ (Rees 1988; Phinney 1989). More than 20 tidal disruption events (TDE) have been observed in other galaxies (Komossa 2015) implying a rate of $\sim 10^{-3} \text{yr}^{-1}\text{gal}^{-1}$ (Stone & Metzger 2016). The proximity of the TDEs to the event horizon of the SMBHs allows to test general relativity in the strong gravity regime.

In the case of compact stellar remnants, such as white dwarfs, neutron stars and black holes, the accretion onto the SMBH will radiate the binding energy in form of low-frequency gravitational waves. As the compact stellar object approaches the last stable orbit, the gravitational radiation becomes more efficient and it can be detected by space-borne interferometers like LISA (Babak et al. 2017). The inspiraling objects can make $\sim 10^6$–$10^7$ orbital revolutions before being swallowed by the SMBH. The analysis of such a signal will allow to obtain information on the space-time geometry and to measure the redshifted mass and spin of the SMBH with high accuracy (Amaro-Seoane et al. 2007, 2015).

When a binary star approaches the SMBH it can be disrupted (Hills 1988). As a result, one of the components is captured by the SMBH and the second one is kicked out with high velocity up to several thousand km s$^{-1}$. Therefore, unveiling the origin of hypervelocity stars can provide useful informations on the existence of the Galactic SMBH. We refer to the review provided recently by Brown (2015) for further details. In general, binaries do not dominate the energy budget of the NSC because single stars bound to the SMBH can become very energetic (Trenti et al. 2007). The diverging velocity dispersion profile with decreasing radius from the SMBH implies that a hard binary at outskirts of the NSC will be tidally disrupted and the stellar mass is added to the SMBH. If a star gets inside the tidal disruption happens outside $R_\text{S}$, where $R_\text{S}$ is a fraction of the Schwarzschild radius of the black hole (see Eq. 3), however for compact objects such as stellar mass black holes, neutron stars and white dwarfs the tidal disruption radius is located inside the event horizon and their accretion leads to the emission of low frequency gravitational waves. We note that due to the limit of resolution in the number of particles, we set $r_\text{t}$ larger than in reality. In this simulation we analyse number counts for

2 METHOD

We model the NSC of the Milky Way galaxy using the direct $N$-body fully parallel code NBODY6++GPU (Wang et al. 2015). We approximate the NSC with $N \approx 10^6$ particles. Although this is the largest number of particles ever used in the direct $N$-body methods for the GC so far, the real number of stars is two orders of magnitude higher. The simulation takes into account stellar evolution as well as the formation and evolution of binary stars. We set the number of primordial binaries to be 5% of the total number of particles.

The SMBH is implemented as external point-mass potential with initial mass of 10% of the total stellar mass of the system. We allowed SMBH growth via stars accretion by defining the accretion radius $r_{\text{acc}} = 4.2 \times 10^{-2}$ pc $\approx 10^5 R_\odot$, where $R_\odot$ is the Schwarzschild radius of a $4.0 \times 10^6 M_\odot$ mass black hole. If a star gets inside $r_{\text{acc}}$ then it is considered to be tidally disrupted and the stellar mass is added to the SMBH. For the case of the $4.0 \times 10^6 M_\odot$ mass black hole the tidal disruption happens outside $R_\odot$ and can produce an electromagnetic counterpart (see Eq. 3), however for compact objects such as stellar mass black holes, neutron stars and white dwarfs the tidal disruption radius is located inside the event horizon and their accretion leads to the emission of low frequency gravitational waves. We note that due to the limit of resolution in the number of particles, we set $r_\text{t}$ larger than in reality. In this simulation we analyse number counts for
extreme mass ratio inspirals (accretion of compact objects onto the SMBH) as well as the tidal disruption events.

2.1 Initial conditions

We construct equilibrium $N$-body initial conditions of a Plummer distribution with external point-mass potential generated according to McMillan & Dehnen (2007) with number of single stars $N = 950k$ and number of binaries $N_b = 50k$ (5% of total number of particles). The point-mass potential represents the SMBH with initial mass of 10% of the total stellar mass and it is fixed at the origin. Since this paper focuses on the inner part of the NSC, the effects from bulge, Galactic disc and dark matter halo are ignored.

We assumed a Kroupa (2001) initial mass function (IMF), selecting masses in the range $0.08-100M_\odot$. The initial binaries are paired with mass ratios $f(q) \propto q^{-0.4}$ motivated by observed values of the Scorpius OB2 association (Kouwenhoven et al. 2007), log-uniform distribution in semi-major axis with minimum and maximum values of 0.005 and 50 astronomical units (AU) and thermal eccentricity distribution: $f(e) = 2e$.

The single and binary stars are evolved according to stellar evolution packages from Hurley et al. (2000) and Hurley et al. (2002). For the neutron star velocity kicks we use a Gaussian distribution with 1D velocity dispersion of $\sigma = 265$ km s$^{-1}$ (Hobbs et al. 2005), fallback prescription for black holes (Belczynski et al. 2002) and no natal kick for white dwarfs. The initial parameters are chosen to be as close as possible to those of the DRAGON simulations (Wang et al. 2016).

The NSC is represented by a single stellar population of solar metallicity stars.

2.2 Scaling

The single and binary IMF in the simulation give a total stellar mass of $M_{\text{tot}} = 6.18 \times 10^4 M_\odot$, while for the real value we use $M_{\text{NSC}} = 4.0 \times 10^5 M_\odot = 10M_{\text{SMBH}}$, thus one particle in the simulation represents a group of 65 stars. For the radial scaling, we measure the influence radius of the SMBH at $t = 0$ to be 0.66 $N$-body units and equate it with the value of the influence radius for the MW which is calculated using the central velocity dispersion taken from Gültekin et al. (2009), $r_{\text{inf}} = GM_{\text{SMBH}}/\sigma^2 = 1.4$ pc. Assuming that half-mass radius $r_{\text{hm}} = 3r_{\text{inf}}$, we can calculate the half-mass relaxation time

$$t_{\text{rel}} = \frac{0.14N}{\ln(0.4N)} \left( \frac{r_{\text{hm}}}{GM_{\text{inf}}} \right)^{1/2},$$

(1)

for the MW NSC in physical units and for the model in $N$-body units and scale the time accordingly. By equating the relaxation time of the model with the relaxation time of the real system we can set the stellar evolution time in correspondence with the dynamical time of the system by

$$\frac{t_{\text{rel}}}{t_{\text{ev}}},$$

(2)

where the prime denotes the modelled system and $t_{\text{ev}}$ can be any stellar evolution time-scale.

Figure 1. Evolution of the Lagrangian radii of the NSC.

Figure 2. Time-evolution of the average stellar mass between shells of Lagrangian radii.

3 GENERAL EVOLUTION OF THE SYSTEM

The NSC overall evolution can be monitored through the time evolution of the Lagrangian radii, which are the radii containing a certain fraction of the total stellar mass. As seen in Fig. 1, the stellar system experiences an initial adjustment to the SMBH potential, explained by the fact that the binaries are not taken into account for the generation of initial conditions. Also, there is a strong mass-loss rate during the first tens of Myr. Overall, the expansion of the NSC is driven by the stellar evolution mass-loss (the magenta line in Fig. 1 clearly shows the expansion), but the Galactic bulge would keep the outer Lagrange radii at roughly a constant value. The small expansion of the inner Lagrange radius (0.1%) is driven by the accretion of stars onto the SMBH.

The time evolution of the average stellar mass, in Lagrangian shells, reveals mass segregation, as shown in Fig. 2. After all the heavy stars lost most of their mass (∼300 Myr), the mass segregation overtakes the time evolution of the average masses in Lagrangian shells. After ∼3 Gyr of evolution, a quasi-steady state is established for the innermost regions. The total number of stars in terms of different stellar evolution components and their properties are described in subsequent sections and summarized in Table 1.
3.1 Density profiles

Typically the 3D stellar density (as well as the surface density) is described by a power law of the form $\rho(r) \propto r^{-\gamma}$, where $r$ is the distance from the SMBH. For the case of equal mass solar type stars the slope becomes $\gamma = -1.75$ inside the influence radius of the black hole (Bahcall & Wolf 1976). For the case of a mass spectrum the dominant component obtains the $-1.75$ slope (Bahcall & Wolf 1977). In Fig. 3 we present 3D stellar density profiles for various stellar types. In order to get a better accuracy, we measured the density profile power-law slopes for 10 snapshots around $t = 5$ Gyr and averaged the results. Due to the low particle number in the inner part, we required at least 3 particles for the calculation of the density. Black holes have the steepest slope of $\gamma = -1.72 \pm 0.04$ while the low mass and high mass main sequence (MS) stars are characterized by a shallower slope, being $\gamma = -0.87 \pm 0.01$ and $\gamma = -0.96 \pm 0.02$, respectively, calculated at 5 Gyr. The slopes are measured inside the SMBH influence radius. White dwarfs have a similar slope ($\gamma = -1.00 \pm 0.02$), but red giants are slightly steeper with $\gamma = -1.22 \pm 0.12$. The slopes are fitted for the density profiles inside the influence radius of the SMBH. Comparison with BAS2018 (see their Fig. 2) yields very similar slopes for the giants, although, for the upper and lower main-sequence stars their simulations show steeper slopes. In principle, the results are consistent with each other since BAS2018 use slightly different definitions for lower and upper main sequence stars, and they show the results at $t = 13$ Gyr. Another point is that BAS2018 have exponentially declining star formation rate, they implement it by adding new stars every Gyr. The power law slope for the stellar mass black holes is remarkably consistent with the analytical prediction of Bahcall & Wolf (1977). The cusp is already formed at $t < 2$ Gyr, less than one NSC half-mass relaxation time. However, as shown by Amaro-Seoane & Preto (2011), the cusp regrowth time is 1/4 of the relaxation time, thus our results are consistent with the assumed initial half-mass relaxation time. Due to stellar mass-loss the influence radius of the SMBH expanded from 1.4 to 2.8 pc and we present the linear fitting for the density slopes for the region $r < 2.8$ pc as well (see columns 4-5 of Table 1). The power-law indices for the influence radius at 5 Gyr are more consistent with the strong mass segregation solution, but are still shallower than the values proposed by Alexander & Hopman (2009) and Preto & Amaro-Seoane (2010).

### 3.2 Stellar mass black holes and other compact objects

Compact objects may play an important role in the evolution of the NSC. Fig. 4 shows the time evolution of compact objects divided by type: carbon-oxygen white dwarfs (COWD), oxygen-neon white dwarfs (ONeWD), neutron stars (NS) and black holes (BH). The COWDs are still forming due to stellar evolution and we see that their number is increasing, while the formation of ONeWDs stopped after ~ 100 Myr, but there are still $\sim 1.4 \times 10^5$ (~ $1.3 \times 10^6$) of them at 2 (5) Gyr. While white dwarfs represent still a noticeable population after 5 Gyr, almost all the neutron stars are ejected, due to the high natal kick received consequently to supernova explosions. We have to note that in a real galactic nucleus the NSs may be still bound to the system under the influence of the potential from the galactic bulge and dark matter halo, but in our simulation all NSs escaped. Fig. 5 shows normalized distribution of velocities for the escaped NSs and BHs. As we can see from the figure, ~ 60% of all the escaped NSs have velocities less than the escape velocity of the bulge at 100 pc, meaning that ~ 60% of them (~ 80000) would be still bound to the GC. Assuming that the Milky Way bulge potential is reasonably represented by a standard Plummer sphere $\Phi = -GM_{\text{tot}}/\sqrt{r^2+b^2}$ with total mass $M_{\text{tot}} = 2.0 \times 10^{10} M_\odot$ (Valenti et al. 2016) and the scale length $b = 350$ pc (Dauphohle & Colin 1995), we found that ~ 60% of escaped NSs have velocities lower than the bulge escape velocity calculated at 100 pc. This suggests that as many as 80000 NSs might be still wandering in the galactic bulge, and possibly can come back to the NSC. For the stellar-mass BHs the situation is different in a way that their kick velocity depends on the fallback factor (Bekczynski et al. 2002). This explains the initial peak in the lower panel of Fig. 4: more than half of the BHs escaped but after that the number of BHs declines slowly and we expect $\sim 2.2 \times 10^4$ (~ $1.8 \times 10^4$) stellar-mass BHs after 2 (5) Gyr. The black line on Fig. 5 shows that ~ 95% of all escaped BHs would be still bound to the system, increasing their total number.

Fig. 6 shows the number of stellar-mass BHs in the inner part of the NSC as a function of time. Since the BHs are the heaviest objects in the NSC, they experience the strongest mass segregation. The number of BHs in the inner 0.5 and 0.3 pc significantly increased. We expect ~ 2000 and ~ 1000 BHs inside central 0.5 and 0.3 pc respectively and ~ 6000 inside the initial influence radius of the SMBH (1.4 pc) at 5 Gyr. Having in mind Fig. 5, we note that the numbers of BHs above have to be treated as lower limits.
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4 THE SUPERMASSIVE BLACK HOLE

Close stellar passages around the SMBH may result either in the star disruption, a phenomenon called tidal disruption event (TDE), or in its gravitational capture. In the latter case, the tight SMBH-star binary evolve mostly through GW emission, forming a so-called extreme mass-ratio inspiral (EMRI).

In this section, we try to quantify the amount of TDEs and EMRIs expected to form over the whole NSC lifetime.

4.1 Tidal disruption events

A star with mass $m$, and radius $r$, can get tidally disrupted if the SMBH tidal forces overcome the star self-gravity. The resulting stellar debris distribute in a disc, feeding the SMBH while emitting X-ray radiation, giving rise to an observable phenomenon called TDE (Hills 1975; Frank & Rees 1976). Equating the gradients of these two competing forces allows us to calculate the tidal disruption radius, which is given by

$$r_t \approx r_s \left( \frac{M_{\text{SMBH}}}{m_s} \right)^{1/3}. \quad (3)$$
In our model, we assumed that a star passing sufficiently close to the SMBH is completely accreted, without any mass left. Since the number of particles used to model the NSC is 65 times smaller than in the real NSC, the number of possible TDE is limited by our low-resolution in the SMBH vicinity. To deal with this problem, we initially set a large tidal radius, \( r_t = 4.2 \times 10^{-4} \) pc, scaling down \textit{a posteriori} to the actual \( r_t \) values our results. In particular, we scale the number of events using the relation obtained from losscone theory, according to which the number of stars accreted through tidal disruption depend on the stellar tidal radius and the total number of stars in the system, \( \frac{N_{\text{acc}} \propto r_t^{-3} \times (N/\ln(0.4N))^4/9}{(Bau�gardt et al. 2004a; Kennedy et al. 2016).} \) Therefore, the number of accreted stars in the real system can be estimated using the above scaling relation

\[
N_{\text{acc}} = \left( \frac{r_t}{r_t^{\text{sim}}} \right)^{-3/4} \times \left( \frac{N_{\text{acc}}^{\text{sim}}}{N_{\text{sim}}} \right)^{4/9} \times \left( \frac{1}{\ln(0.4N_{\text{real}}/N_{\text{sim}})} \right)^{4/9} N_{\text{sim}}^{\text{real}}. \tag{5}
\]

For each of the 5 star groups summarized in Table 1, we calculated the corresponding average tidal radius through Eq. 3, and the number of stars passing closer than \( r_t \) in our simulation, namely \( N_{\text{acc}} \). This quantity is then used in Eq. 5 to scale our results to the real NSC.

Table 1 (column 6 and 7) lists the number of tidally disrupted (or accreted) stars per Gyr derived from the total number of objects undergoing one of these events using the relation obtained from loss-cone theory, according to which the number of stars accreted through tidal disruption depend on the stellar tidal radius and the total number of stars in the system, \( \frac{N_{\text{acc}} \propto r_t^{-3} \times (N/\ln(0.4N))^4/9}{(Bau�gardt et al. 2004a; Kennedy et al. 2016).} \) Therefore, the number of accreted stars in the real system can be estimated using the above scaling relation

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For each of the 5 star groups summarized in Table 1, we calculated the corresponding average tidal radius through Eq. 3, and the number of stars passing closer than \( r_t \) in our simulation, namely \( N_{\text{acc}} \). This quantity is then used in Eq. 5 to scale our results to the real NSC.

Table 1 (column 6 and 7) lists the number of tidally disrupted (or accreted) stars per Gyr derived from the total number in 5 Gyr. The majority of TDEs are due to low-mass MS stars, while the SMBH growth is mostly due to MS stars. As shown in Fig. 7, the time evolution of the accreted mass saturates to a nearly constant value in 2.8 Gyr, allowing us to provide an upper limit to the SMBH accreted mass by 5 Gyr as \( \Delta M_{\text{SMBH}} = 4 \times 10^4 M_\odot \) or \( 0.23\% \) of the initial SMBH mass. This implies a mass accretion rate \( M = 2 \times 10^{-4} M_\odot \text{yr}^{-1} \) and a TDE rate \( N_{\text{TDE}} \approx 4 \times 10^{-6} \text{yr}^{-1} \) which is consistent with the observed number of tidal disruption events obtained per MW-like galaxy (Stone & Metzger 2016). We note that the accretion rate is higher in the beginning and is smaller at later stages of evolution.

As stated above, we assume that a star undergoing a TDE in our model is completely disrupted and 100% of its mass is added to the SMBH. However, the tidal disruption radius for a red giant is large while its typical density is generally low, thus a close encounter with the SMBH may lead to an envelope stripping and the core will remain as a white dwarf. These white dwarfs are very hot (~ 10^9 K) and may be observable. Fig. 8 shows possible locations of these objects, the radii at which the red giants were ‘disrupted’. The remnant WDs may also increase the fraction of EMRIs (see next subsection), but these effects will be explored in future work.

### 4.2 Gravitational waves

In general, compact objects (WDs, NSs or BHs) can survive tidal disruption due to their compact sizes and can lead to the formation of an EMRI, a tight binary emitting GWs in the LISA (e.g. Amaro-Seoane et al. 2007) and TianQin (Luo et al. 2016) expected observational bands. In our models, we consider the accretion of a compact object if it gets inside the innermost stable circular orbit (3\( R_S \) for a non-spinning black hole). Beyond this limit, we must distinguish between a \textit{direct plunge} and a possible EMRI. In the following, we refer to the total number of objects undergoing one of these two fates using \( N_{\text{acc}}(r < 3R_S) \). Objects scattering directly into the \( R_S \) are direct plunges, they emit a burst of gravitational radiation, but are difficult to detect even for the GC since they do not spend much time in the LISA band. On the other

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**Table 1. Properties of different stellar types at \( t = 5 \) Gyr.**

| Stellar type         | \( N_{\text{acc}} \) | \( \frac{<m>}{(M_\odot)} \) | \( \gamma(r < 1.4 \text{pc}) \) | \( \gamma(r < 2.8 \text{pc}) \) | \( N_{\text{acc}}(r < 3R_S) \) |
|----------------------|---------------------|----------------|----------------|----------------|----------------|
| Low-mass main sequence | \( 5.0 \times 10^7 \) | 0.25            | \(-0.87 \pm 0.01 \) | \(-1.11 \pm 0.03 \) | 3278          |
| Main sequence         | \( 4.3 \times 10^6 \) | 0.92            | \(-0.96 \pm 0.02 \) | \(-1.21 \pm 0.05 \) | 658           |
| Red giant             | \( 1.5 \times 10^3 \) | 1.24            | \(-1.22 \pm 0.12 \) | \(-1.34 \pm 0.15 \) | 39            |
| White dwarf           | \( 1.6 \times 10^6 \) | 0.71            | \(-1.00 \pm 0.02 \) | \(-1.23 \pm 0.03 \) | 138           |
| Black hole            | \( 1.8 \times 10^4 \) | 10.05           | \(-1.72 \pm 0.04 \) | \(-1.98 \pm 0.07 \) | 2             |
| All stars             | \( 5.8 \times 10^7 \) | 0.33            | \(-1.02 \pm 0.02 \) | \(-1.23 \pm 0.03 \) | 4120          |

**Notes.** Column 1 is the name of the stellar type, columns 2 and 3 show the total number of stars at 5 Gyr and the average stellar mass in solar masses, columns 4 and 5 represent the 3D density power-law indices inside 1.4 and 2.8 pc respectively, column 6 shows the number accretion rate per Gyr derived over the period of 5 Gyr and column 7 shows the accretion rate per Gyr for black holes and white dwarfs that accrete inside 3\( R_S \).
hand, compact objects that fall into the region between 1 and 3 $R_\odot$ may become an EMRI.

The last column of Table 1 provides the accretion rates of compact objects in our simulation. Our results suggest that $\gtrsim 100$ WDs are expected to undergo a direct plunge, while the remaining are most likely captured in a possible EMRI. However, we must stress that also NSs can contribute to the total EMRI number. We note that here we do not follow the accreted object after it is gone inside 3 $R_\odot$. The late evolution of an EMRI is determined by GW emission, leading eventually to the SMBH-compact remnant coalescence over the merging time due to the gravitational radiation given by (Peters 1964)

$$t_{GW} \approx \frac{268}{425} \frac{5}{256} \frac{c^5}{G} \frac{d}{m_1 m_2 (m_1 + m_2)} (1 - e^2)^{7/2},$$

where $t_{GW} \ll (1 - e)t_{rc}$.

Note that Fig. 9 is not scaled to the real number of stars in the GC, one point represents more than one star. More precise condition for a compact object to be a EMRI is given by (Amaro-Seoane et al. 2007):

$$t_{GW} \ll (1 - e)t_{rc},$$

where $t_{rc}$ is the local relaxation time, $t_{GW}$ is the merging time due to gravitational radiation.

We can conclude that white dwarfs are the main sources of EMRIs in the GC with a minimum rate of about 100 events per Gyr, we found few black holes and no neutron stars.

5 BINARIES

Although, the GC environment is very extreme, some binaries have been detected there (Muno et al. 2005b; Pfuhl et al. 2014). In this section we analyse the number of GC binaries obtained from our simulation. The upper curve in Fig. 10 shows the total number of binaries as function of time. We start the simulation with 5% of binaries and roughly half of them survive till $t = 5$ Gyr of evolution. The two lower curves in the same figure show the number of binaries inside 1 and 0.1 pc, respectively, meaning that after 5 Gyr we expect 100-1000 of them inside 0.1 pc and $\approx 5.0 \times 10^3$ in the inner parsec. These binaries are characterized by an average total mass of 1.0 and $0.69 M_\odot$ and a binary fraction of $\sim 2\%$ and $\sim 2.5\%$ respectively. The initial distribution of the binary semi-major axis (SMA) was assumed log-uniform between $a = 0.005$ and 50 AU. The SMA defines the binary binding energy ($E_b \propto 1/a$), and allows to determine whether a binary is ‘hard’ or ‘soft’ (Heggie 1975). A sizeable number of ‘soft’ binaries are quickly destroyed because of the repeated interactions with the surrounding dense environment, leading to a strong decrease of the number of binary systems having initial SMA values larger than 1 AU (compare blue and green lines in Fig. 11).

On the other hand, the number of systems with smaller SMA increase in time, thus implying a growing number of ‘hard’ binaries. Fig. 11 compares the SMA distribution at a time $t = 2$ Gyr in our simulation (the peaked brown line) with the SMA distribution obtained evolving all the binaries in isolation. This comparison shows the effect of the dense environment on the binary stellar evolution. Thus, the systems with small separations are getting higher in number and their orbits shrink. As opposite to this, the standalone binary evolution code results show that the number of binary systems with smaller separations will decrease (some of them will merge and some of them will get wider orbits after the supernovae explosions). Step-filled histograms on Fig. 11 show that the low-separation binaries are dominated by low-mass MS stars and some WDs. Fig. 12 shows the distribution of the binding energies of the binaries and their distances to the SMBH at 100 Myr, 1 Gyr and 5 Gyr. As we can see, the number of binaries with binding energies below $10^{-8}$ N-body units decreases with time, especially in the central part.

The total mass of a binary system is typically twice larger than that of a single star, thus implying that most of the binaries will be subjected to mass segregation. While mass segregation brings the binaries to the centre, the soft
ones are being destroyed and hard ones survive, but even a very hard binary can be tidally disrupted by the SMBH. Fig. 13 illustrates how the binary fraction changes with the distance from the SMBH for initial moment (blue curve), 100 Myr (green), 1 Gyr (red) and 5 Gyr (cyan). Initially, the binaries were distributed uniformly but already after 100 Myrs the central binary fraction \( r < 1 \) pc dropped from 5% to \( \sim 2.5 \% \). Comparison of the red and cyan lines yields that the total number of binaries drops but in general the shape of the curve is established.

Our simulation suggests that the NSC contain a substantial number of white dwarf binaries (Fig. 14). These binary systems are of particular interest since they can give rise to supernovae Ia events or, in some cases, they can even form a millisecond pulsar through matter accretion from a companion star onto a highly spinning massive WD. Double degenerate WD - WD binaries can be the progenitor of supernovae Ia explosions, provided that their total mass exceeds the Chandrasekhar limit, while they can also lead to the formation of neutron stars. On the other hand, binary systems containing a NS are almost absent in the system (Fig. 15). These types of binaries are possible progenitors of millisecond pulsars (MSPs), which are thought to be recycled neutron stars spun up by matter accretion from a stellar companion, according to the standard scenario. After the natal kick the neutron star binaries become very wide (if they survive the supernova explosion) and eventually are ionized. We find that 1000 and \( \approx 3000 \) of double and single degenerate pairs are expected to populate the central parsec of the MW galactic nucleus.

We note that the estimation of number of MSPs in the GC is still to be analysed in more detail. We aim to start several new runs taking into account the MW bulge as an external potential and investigate how many neutron stars would be bound to the NSC. We expect that the bulge will
about half of the initial binaries survived until 5 Gyr of the evolution. Most of the binaries are destroyed due to dynamical interactions with single stars. The increasing number of white dwarf binaries implies a high supernovae Ia rate.

We note that due to stellar evolution and stellar accretion, the influence radius of the SMBH grows from 1.4 pc at $t = 0$ to $\sim 2.8$ pc at $t = 5$ Gyr. Thus, fitting the stellar density profiles inside $r = 2.8$ pc gives steeper slopes which are more consistent with the strong mass segregation solution discussed by Alexander & Hopman (2009) and Preto & Amaro-Seoane (2010). They claimed that in the case when the number of lower mass objects (stars with masses up to $1M_\odot$) is much higher than that of heavy objects (stellar black holes with masses of $10M_\odot$) the heavy objects obtain a power-law density slope $\gamma$ of $-11/4$ while the light ones have $\gamma = -3/2$. They parametrized the solution by $\Delta = N_h M_\odot^2/(N_c M_i^2)\Delta_c/(3 + M_h/M_i)$, where $N_c$ and $M_i$ denote the numbers and masses of light and heavy objects. In our simulation the value of $\Delta$ approaches zero ($\Delta \sim 5 \times 10^{-8}$) meaning that we are in the strong mass segregation regime, but the power-law density slopes are shallower in our simulation.

The absence of neutron stars in the NSC after 200 Myr of evolution is due to high velocity kicks at the moment of formation of a neutron star. This lets them escape from the SMBH. As we have seen from Fig. 11, the NSC is completely dominated by binaries with small separations at 5 Gyr, meaning that it is likely to expect hypervelocity stars with velocities above $1000 \text{ km s}^{-1}$. Our results suggest the majority of the ejected objects are low-mass MS stars or, more rarely, WDs.

Since the accretion radius in our simulation exceeds $r_h$ for most of the remained binaries, we leave the detailed analysis of the 3-body interactions that potentially involve them and the SMBH to a forthcoming work.

6 SUMMARY AND DISCUSSION

We performed a high resolution direct $N$-body simulation of the GC starting with $\sim 10^6$ particles with 5% of initial binaries taking into account single and binary stellar evolution. This is the largest simulation of this kind so far. We showed that the stellar component forms a cusp with the highest power-law index for the stellar mass black holes $\gamma = -1.72$. Then we demonstrated how mass segregation occurs by analysing average masses between Lagrangian shells. When the stars happen to come very close to the SMBH they are disrupted with a total rate of $\sim 4 \times 10^3$ stars per Gyr. The number of accretion events for compact objects is $\sim 230$ per Gyr with roughly 100 of them being possible EMRIs. About half of the initial binaries survived until 5 Gyr of the evolution. Most of the binaries are destroyed due to dynamical interactions involving a binary star and the SMBH can result in the binary break up, with one component being captured by the SMBH and the other ejected away with velocities up to $1000 \text{ km s}^{-1}$ (Hills 1988). This mechanism is one of the possible scenarios that can explain the observed population of hypervelocity stars (Brown 2015). Indeed, if a binary with mass $1 M_\odot$ and semimajor axis of 0.1 AU approaches the MW SMBH as close as its disruption radius $r_h \approx 10^{-6}$ pc, then it may lead to the formation of a hypervelocity star with $v \approx 1370 \text{ km s}^{-1}$ (Eq. 2 in Brown 2015).

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system. In case of a binary, if the latter survives the supernova explosion, the binary gets a very wide orbit and eventually is destroyed by interaction with single stars in the dense stellar environment. Thus, postulating that all neutron stars have velocity kicks with 1D dispersion of 265 km s$^{-1}$, makes the formation of a close binary with a neutron star unlikely. Therefore, the standard scenario for formation of millisecond pulsars (MSP) fails due to a lack of neutron stars. But in reality MSPs are observed even in globular clusters (e.g. Manchester et al. 1991) where the escape velocity is much smaller than for a NSC. It means that MSP can form in an alternative scenario, for example from the accretion induced collapse of a white dwarf (e.g. Hurley et al. 2010; Taani et al. 2012; Tauris et al. 2013; Freire & Tauris 2014). If a MSP is detected in the close vicinity of a SMBH it can be used to test general relativity in the strong regime (e.g. Psaltis et al. 2016). Moreover, the spatial distribution of MSPs in the NSC can give hints on the formation scenario of the NSC (Arca-Sedda et al. 2017; Abbate et al. 2018).

In this simulation we constructed the initial conditions assuming the in-situ formation of the NSC, but its formation is likely due to star cluster inspiral, at least in part, as firstly suggested by Tremaine (1976) and Capuzzo-Dolcetta (1993), although a fraction is likely due to in-situ star formation (King 2003, 2005). Antonini et al. (2012) provided the first self-consistent simulation tailored to reproduce the MW observational properties. Later on, Arca-Sedda et al. (2015) showed that the formation of a NSC around an SMBH weighing a few $10^6 M_\odot$ is extremely rapid, lasting 0.1-1 Gyr, thus implying that the contributing clusters still are “dynamically young” when arrive to the Galactic centre. Moreover, Arca-Sedda et al. (2015) presented the first simulations to model self consistently a whole galactic nucleus and 11 star clusters using, for the whole system, more than $10^6$ particles. More recently, Tsatsi et al. (2017) pointed out that the MW NSC rotation can be reproduced by the “star cluster inspiral” scenario. Taking into account these facts, our follow-up simulations may be started with the initial stellar distribution according to the “star cluster inspiral” scenario with some initial rotation.

We note that the rate of tidal disruption events may be enhanced in the presence of an accretion disc (Just et al. 2012; Kennedy et al. 2016). The same is true for the gravitational waves: the drag force of the accretion disc may bring compact object close to the SMBH resulting in the enhancement of the EMRI rates detectable by LISA, moreover, the gaseous disc may significantly reduce the semi-major axis of stellar binary black holes boosting their merging time (Bartos et al. 2017; Stone et al. 2017; McKernan et al. 2017). In case of neutron star or white dwarf binaries this mechanism may lead to an enhanced rate of supernovae Ia explosions and gamma-ray bursts. Stellar binaries may merge in the close vicinity of the SMBH due to “eccentric Kozai-Lidov” mechanism (Stephan et al. 2016). The Kozai-Lidov oscillations can be studied via the direct N-body modelling with one-to-one particle resolution or by approximating outer stars as a smooth potential. Panamarev et al. (2018) showed that the interaction of stars with the accretion disc may lead to formation of a nuclear stellar disc in the inner part of the galactic nucleus. Such stellar discs may serve as environment for dynamical formation of compact binaries.

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