A trivariate meta-analysis of diagnostic studies accounting for prevalence and non-evaluable subjects: re-evaluation of the meta-analysis of coronary CT angiography studies

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Abstract

Background: A recent paper proposed an intent-to-diagnose approach to handle non-evaluable index test results and discussed several alternative approaches, with an application to the meta-analysis of coronary CT angiography diagnostic accuracy studies. However, no simulation studies have been conducted to test the performance of the methods.

Methods: We propose an extended trivariate generalized linear mixed model (TGLMM) to handle non-evaluable index test results. The performance of the intent-to-diagnose approach, the alternative approaches and the extended TGLMM approach is examined by extensive simulation studies. The meta-analysis of coronary CT angiography diagnostic accuracy studies is re-evaluated by the extended TGLMM.

Results: Simulation studies showed that the intent-to-diagnose approach under-estimate sensitivity and specificity. Under the missing at random (MAR) assumption, the TGLMM gives nearly unbiased estimates of test accuracy indices and disease prevalence. After applying the TGLMM approach to re-evaluate the coronary CT angiography meta-analysis, overall median sensitivity is 0.98 (0.967, 0.993), specificity is 0.875 (0.827, 0.923) and disease prevalence is 0.478 (0.379, 0.577).

Conclusions: Under MAR assumption, the intent-to-diagnose approach under-estimate both sensitivity and specificity, while the extended TGLMM gives nearly unbiased estimates of sensitivity, specificity and prevalence. We recommend the extended TGLMM to handle non-evaluable index test subjects.

Keywords: Meta-analysis, Diagnostic test, Non-evaluable subjects

Background

In studies of meta-analysis of diagnostic test comparing an index test with a reference test, non-evaluable test outcome is an important issue that could potentially lead to biased estimates of index test accuracy. Many papers in the literature discussed missing reference test outcome (missing disease status) and how to correct such bias, so called partial verification bias or work up bias [1-4]. However, index test outcomes can be non-evaluable as well, especially for tests yielding dichotomous results. Different situations were discussed where index test result can be non-evaluable: uninterpretable, intermediate and indeterminate [5,6].

For a single study, there are many discussions about how to deal with non-evaluable index test outcomes, such as excluding them [7], grouping them with positive or negative outcomes [5,7], or use $3 \times 2$ table to report them as an extension of the standard $2 \times 2$ table [7]. On the other hand, in meta-analysis, there is little discussion on how to deal with missing index test outcomes [6]. The “classic” $2 \times 2$ table models such as the bivariate linear mixed models [8-13], bivariate generalized linear mixed model (GLMM) [14-16] and TGLMM
Based on the estimates of Se and Sp:

Hence, excluding non-evaluable subjects will have unbi-
and T

test outcome,

false positives. However, no simulation studies have
false negatives and non-evaluable non-diseased subjects
and recommended the conservative intent-to-diagnose
1) leads to overestimation of sensitivity and specificity
excluding the index test non-evaluable subjects (Model

jects are taken as true positives and non-diseased subjects
as false negatives such that
sensitivity and specificity won’t be over-estimated. We
name the other three alternative approaches in Schuetz
et al. [6] as Model 1 (non-evaluable subjects are excluded
from the study), Model 2 (non-evaluable diseased sub-
jects are taken as true positives and non-diseased subjects
are taken as false positives) and Model 3 (non-evaluable
diseased subjects are taken as false negatives and non-
diseased subjects are taken as true negatives). We use
Model 1-3 to denote the above three approaches through-
out the rest of this paper. The authors concluded that
excluding the index test non-evaluable subjects (Model
1) leads to overestimation of sensitivity and specificity
and recommended the conservative intent-to-diagnose
approach by treating non-evaluable diseased subjects as
false negatives and non-evaluable non-diseased subjects
as false positives. However, no simulation studies have
been conducted to evaluate the performance of these
approaches. Moreover, the above conclusions can be mis-
leading.

We can treat index test non-evaluable subjects as miss-
ing data. Schuetz et al. [6] concluded that sensitivity and
specificity could be over-estimated by excluding non-
evaluable subjects. In fact, under a reasonable general
assumption, missing at random (MAR), excluding non-
evaluable subjects can provide unbiased estimates of
sensitivity (Se) and specificity (Sp). Under MAR assump-
tion, the probability of missing only depend on observed
information, such as patient characteristics and known
t true disease status [18,19]. For example, when diagnosing
extrahepatic cholestasis using percutaneous transhepatic
cholangiography, non-diseased subjects can have unin-
terpretable results more often than diseased patients [5].
A special case of MAR is missing completely at random
(MCAR), where missing is independent of both observed
and unobserved variables [18]. E.g., accidental contamina-
tion of a urine sample such that the test result is discarded.
Under MAR, T and M are independent given disease sta-
tus D, where M = 1, 0 indicates missingness of index
test outcome, D = 1, 0 indicates diseased or non-diseased
and T = 1, 0 represents index test positive or negative.
Hence, excluding non-evaluable subjects will have unbi-
ased estimates of Se and Sp: $Se = Pr(T = 1|D = 1, M = 0) = Pr(T = 1|D = 1)$ and $Sp = Pr(T = 0|D = 0, M = 0) = Pr(T = 0|D = 0)$. Similarly, positive
and negative likelihood ratios (LR+ and LR−) and area
under the curve (AUC) are unbiased too. Under MCAR,
$Pr(M = 1|D = 1) = Pr(M = 1|D = 0)$, and hence disease
prevalence (π) estimate is also unbiased if non-evaluable
subjects are excluded. However, when missing proba-
bilities are not equal between diseased and non-diseased
participants, disease prevalence estimate can be biased if
non-evaluable subjects are excluded, leading overall esti-
mates of positive predictive value (PPV) and negative
predictive value (NPV) biased. PPV and NPV are gener-
ally preferred by clinicians as measurements of how well
a test predicts true disease status because their interpre-
tations are more intuitive: PPV is the probability that a
subject with positive index test result is truly diseased and
NPV is the probability that a subject with negative index
test result is truly non-diseased [19]. However, none of
the approaches discussed in Schuetz et al. [6] can correct
bias in their estimates.

In this article, we propose to extend the TGLMM
approach [17] by treating non-evaluable subjects as miss-
ing data to adjust for potential bias. The TGLMM was
proposed by Chu et al. [17] as an extension of the bivariate
GLMM [9,10,14]. Sensitivities and specificities are found to
be potentially dependent on disease prevalence [20-22].
The TGLMM models disease prevalence together with
sensitivity and specificity to account for potential correla-
tions among them. Moreover, once overall disease preva-
ience is evaluated, other test accuracy indices such as PPV
and NPV can be calculated. By extending the TGLMM to
account for missing data, potential bias in disease preva-
ience estimate can be adjusted and thus, bias in PPV and
NPV estimates can be avoided.

In the rest of this paper, we first present the extended
TGLMM approach in the “Methods” section. Next, in
section “Results”, simulation studies are carried out to
systematically evaluate the performance of the extended
TGLMM, Model 1-3 and the intent-to-diagnose approach
when there are non-evaluable index test subjects. The
meta-analysis of coronary CT angiography studies is re-evaluated by the extended TGLMM approach. The
SAS code for the extended TGLMM is available in the
Appendix. Finally, we conclude the paper with some dis-
cussions in section “Conclusions”.

Methods

Assume there are $i = 1, \ldots, N$ studies in one meta-
alysis data set. We generalize the TGLMM approach
to account for missing index test outcomes by extending
the “classic” $2 \times 2$ table to Table 1. Each cell in Table 1
reports the cell count and cell probability corresponding
to a combination of index test and disease outcomes in
study $i$. Let $n_{tid}$ denote the cell counts in study $i$ with index
test outcome $T = t$ and reference test outcome $D = d$,
where $t = 1, 0, m$ stands for positive, negative and miss-
ing, and $d = 1, 0$ denotes positive and negative. $Se_i$, $Sp_i$
and \( \pi_i \) are sensitivity, specificity and prevalence of study \( i \), respectively. Let \( \omega_{imd} \) denote the missing probability of index test given disease status \( d \) in study \( i \): \( \omega_{imd} = Pr(T = m|D = d) \). The missing probabilities and disease prevalence are incorporated in the cell probabilities in Table 1. Assuming a multinomial distribution, the likelihood for \( \theta_i = (Se_i, Sp_i, \pi_i) \) and \( \omega_i = (\omega_{im1}, \omega_{im0}) \) given data (cell counts) is:

\[
L(\theta_i, \omega_i|Data) \propto (1 - \omega_{im1}) \pi_i Se_i \{(1 - \omega_{im0}) (1 - \pi_i) (1 - Sp_i)\}^{n_{i0}} \left\{ (1 - \omega_{im0}) \pi_i (1 - Se_i) \right\}^{n_{i10}} \{(1 - \omega_{im0}) (1 - \pi_i) Sp_i \}^{n_{i01}} (\pi_i \omega_{im1})^{n_{i11}} \{(1 - \pi_i) \omega_{im0} \}^{n_{i00}}
\]

(1)

It is straightforward to tell from (1) that \( L(\theta_i, \omega_i|Data) \propto L(\theta_i|Data) \times L(\omega_i|Data) \), where the log-likelihood of \( \theta_i \) is:

\[
logL(\theta_i|Data) = n_{i11} \{ \log(\pi_i) + \log(Se_i) \} \\
+ n_{i10} \{ \log(1 - \pi_i) + \log(1 - Sp_i) \} \\
+ n_{i01} \{ \log(\pi_i) + \log(1 - Se_i) \} \\
+ n_{i00} \{ \log(1 - \pi_i) + \log(1 - Sp_i) \} \\
+ n_{im1} \log(\pi_i) + n_{im0} \log(1 - \pi_i)
\]

Let \( \theta = \{\theta_i\} \). Assuming independence among studies conditional on \( \theta_i \), the total log likelihood of \( \theta \) is:

\[
logL(\theta|Data) = \sum_{i=1}^{N} logL(\theta_i|Data)
\]

(2)

Let \( \logit(\pi_i) = \eta + \varepsilon_i \), \( \logit(Se_i) = \alpha + \mu_i \) and \( \logit(\text{Sp}_i) = \beta + \nu_i \), where \( \logit(\cdot) \) is the logit link function such that \( \logit(p) = \log(p/(1 - p)) \), for \( 0 < p < 1 \). \( \eta, \alpha, \beta \) are the fixed effect parameters such that median \( \pi \), \( Se \) and \( Sp \) can be approximated as \( \logit^{-1}(\eta) \), \( \logit^{-1}(\alpha) \) and \( \logit^{-1}(\beta) \), respectively, where \( \logit^{-1}(\cdot) \) is the inverse logit function such that \( \logit^{-1}(x) = 1/(1 + \exp(-x)) \). The random effect vector \( (\varepsilon_i, \mu_i, \nu_i) \) is assumed to be trivariate normally distributed:

\[
(\varepsilon_i, \mu_i, \nu_i)^T \sim MVN(0, \Sigma), \quad \Sigma = \begin{bmatrix} \sigma^2 & \rho_{\varepsilon\mu} \sigma_\varepsilon \sigma_\mu & \rho_{\varepsilon\nu} \sigma_\varepsilon \sigma_\nu \\ \rho_{\mu\varepsilon} \sigma_\mu \sigma_\varepsilon & \sigma^2 & \rho_{\mu\nu} \sigma_\mu \sigma_\nu \\ \rho_{\nu\varepsilon} \sigma_\nu \sigma_\varepsilon & \rho_{\nu\mu} \sigma_\nu \sigma_\mu & \sigma^2 \end{bmatrix}
\]

where the diagonal elements in \( \Sigma \) account for between-study variations of \( \pi \), \( Se \) and \( Sp \) and the off-diagonal elements take care of potential correlations among the three parameters.

Median PPV, NPV, LR+ and LR− and median area under the curve (AUC\(_M\)) can be approximated as [16]:

\[
PPV = \frac{\logit^{-1}(\eta) \logit^{-1}(\alpha)}{\logit^{-1}(\eta) \logit^{-1}(\alpha) + [1 - \logit^{-1}(\eta)] \{1 - \logit^{-1}(\beta)\}},
\]

\[
NPV = \frac{[1 - \logit^{-1}(\eta)] \logit^{-1}(\beta)}{[1 - \logit^{-1}(\eta)] \logit^{-1}(\beta) + [1 - \logit^{-1}(\beta)] \{1 - \logit^{-1}(\alpha)\}},
\]

\[
LR+ = \logit^{-1}(\alpha)/\{1 - \logit^{-1}(\beta)\},
\]

\[
LR− = \{1 - \logit^{-1}(\alpha)\}/\logit^{-1}(\beta),
\]

\[
AUC_{M} = \int_{0}^{1} \logit^{-1}\{ (\alpha - \rho_{\mu\varepsilon} \beta \sigma_{\mu})/\sigma_{\nu} + \rho_{\mu\varepsilon} \sigma_{\mu} / \sigma_{\nu} [\logit(1 - \text{Sp})] \} d\text{Sp}.
\]

The extended TGLMM can be fitted by standard software like SAS NL MIXED procedure, which implements an adaptive Gaussian quadrature to approximate the log-likelihood in (2) integrated on random effects with dual
quasi-Newton optimization techniques. The NLMIXED procedure directly outputs fixed effects estimates \( \hat{\beta} \) and \( \hat{\delta} \) and can provide median prevalence, Se, Sp, PPV, NPV, LR+, LR− estimates and their confidence intervals through the “estimate” statements. Sample SAS code is available in the Appendix.

Results
Simulation scenarios

We conduct simulation studies under three missing scenarios to systematically evaluate the performance of the proposed extended TGLMM approach and the approaches discussed in Schuetz et al. [6]: missing probabilities for diseased and non-diseased subjects are same (0.1), or missing probability of diseased group (0.1) is smaller than non-diseased group (0.2), or missing probability of diseased group (0.2) is larger than non-diseased group (0.1). All three scenarios satisfy the MAR assumption, and the first scenario is in fact MCAR [18]. True sensitivity and specificity are 0.7 and 0.9, disease prevalence is 0.25 and variances of Se, Sp and prevalence are 1 on logit scale. These assumptions mimic a diagnostic test with relatively low sensitivity, high specificity and a disease with moderate prevalence. A moderate positive correlation of 0.3 is assumed between Se and \( \pi \), and moderate negative correlations of \(-0.3\) are assumed between Sp and \( \pi \) and between Se and Sp, on logit scales. Such correlation directions were observed in some meta-analysis studies [11,20]. Intuitively, a population with higher prevalence may have more diseased cases with clear disease symptoms, leading to increased sensitivity. Under each setting, 5000 meta-analysis data sets are simulated with 30 studies in each data set. \( \pi_i \), Se\(_i\) and Sp\(_i\) for each study were generated according to the trivariate assumption described in the Methods section. True and false positives, true and false negatives and non-evaluable counts are sampled from the multinomial distribution in Table 1. For each simulated meta-analysis data set, the extended TGLMM, Model 1-3 and the intent-to-diagnose approach are fitted. Bias in percentage, mean standard error (SE) and 95% confidence interval coverage probability (CP) are collected and compared for estimates of sensitivity, specificity, prevalence, PPV, NPV, LR+ and LR−. Bias in percentage is calculated by \( (\hat{\delta} - \delta) \times 100/\delta \), where \( \delta \) is the true value and \( \hat{\delta} \) is the estimator.

Simulation results

Table 2 shows the simulation results under different scenarios. When MCAR \( (\omega_{m1} = \omega_{m0} = 0.1) \), disease prevalence estimates from all five models are nearly unbiased (bias less than 1%). The extended TGLMM and Model 1 both give nearly unbiased estimates (bias less than 1.6%) and nominal coverage probabilities around 93% for Se, Sp, PPV, NPV, LR+ and LR− estimates. Model 2 over-estimates sensitivity and under-estimates specificity; bias of sensitivity estimate is 4.6% and bias of specificity estimate is 11.9%. Estimates of PPV and LR+ are more biased (22.6% bias for PPV and 49.2% bias for LR+). Using Model 3 sensitivities are largely under-estimated (12.6% bias) and specificities are over-estimated (1.1% bias). The intent-to-diagnose approach largely under-estimates both sensitivity and specificity (12.6% and 11.9% bias, respectively). The CPs for some estimates from Model 2 and 3 and the intent-to-diagnose approach can be as low as 0 (e.g., specificity estimates from Model 2), indicating that none of the confidence intervals cover the true values. When missing probability of the diseased group is smaller than the non-diseased group \( (\omega_{m1} = 0.1, \omega_{m0} = 0.2) \), the extended TGLMM and Model 1 both give nearly unbiased estimates (bias around 0.1%) of sensitivity and specificity. However, Model 1 over-estimates disease prevalence (9.6% bias) while the extended TGLMM gives nearly unbiased (bias within 1%) estimate of prevalence. As a consequence, Model 1 gives biased estimates of PPV and NPV (3.1% and 1.3%, respectively), while the extended TGLMM provides nearly unbiased estimates for all parameters (within 2%). Again, under this scenario, the intent-to-diagnose approach largely under-estimates sensitivity, specificity, PPV, NPV and LR+ and over-estimates LR−, with CPs less than 40% and some as low as 0. On the other hand, when \( \omega_{m1} = 0.2 \) and \( \omega_{m0} = 0.1 \), the extended TGLMM and Model 1 again give nearly unbiased estimates (bias around 0.1%) of sensitivity and specificity. Model 1 under-estimates disease prevalence (8.4% bias) while the extended TGLMM provides nearly unbiased estimates. The intent-to-diagnose approach largely under-estimates sensitivity, specificity, PPV, NPV and LR+ and over-estimates LR− and some CPs are as low as 0. When the missing probabilities for diseased and non-diseased subjects are more unbalanced, we expect the estimates from Model 1-3 and the intent-to-diagnose approach to have larger bias and smaller CP. In practice, however, depending on the test performance and missing probabilities, the direction and magnitude of the bias from the four approaches discussed in Schuetz et al. [6] can be different from what we observed in these simulation studies.

Re-evaluation of the meta-analysis of coronary CT angiography studies

Cardiac CT scans can be used to rule out stenoses, however, are found to be subject to non-evaluable results. Schuetz et al. [6] performed a systematic search for diagnostic accuracy studies of coronary CT angiography. The authors searched Medline, Embase and ISI Web of Science databases for prospective studies using conventional coronary angiography as the gold standard and have patients with non-evaluable CT images. Eventually, 26 studies were...
### Table 2 Simulation results under MAR assumption

| Model | TGLMM | Model 1 | Model 2 | Model 3 | Intent-to-diagnose |
|-------|-------|---------|---------|---------|---------------------|
|       |       |         |         |         |                     |
| Estimate | Bias% | meanSE | CP | Bias% | meanSE | CP | Bias% | meanSE | CP | Bias% | meanSE | CP | Bias% | meanSE | CP |
| ω_{m1} = ω_{m0} = 0.1 | | | | | | | | | | | | | | | |
| Se | -0.3 | 0.041 | 0.94 | -0.3 | 0.041 | 0.94 | 4.6 | 0.036 | 0.81 | -12.6 | 0.037 | 0.33 | -12.6 | 0.036 | 0.33 |
| Sp | -0.1 | 0.017 | 0.93 | -0.1 | 0.017 | 0.93 | -11.9 | 0.018 | 0 | 1.1 | 0.015 | 0.84 | -11.9 | 0.017 | 0 |
| Prev | 0.8 | 0.034 | 0.93 | 0.8 | 0.034 | 0.93 | 0.8 | 0.034 | 0.93 | 0.8 | 0.034 | 0.93 | 0 | 0.034 | 0.93 |
| PPV | -0.1 | 0.046 | 0.94 | -0.3 | 0.046 | 0.94 | -22.6 | 0.047 | 0.08 | -0.9 | 0.046 | 0.94 | -29 | 0.049 | 0.01 |
| NPV | -0.1 | 0.018 | 0.93 | -0.1 | 0.018 | 0.93 | -0.2 | 0.018 | 0.93 | -2.9 | 0.020 | 0.81 | -4.6 | 0.022 | 0.59 |
| LR+ | 1.6 | 1.188 | 0.92 | 1.6 | 1.189 | 0.93 | -49.2 | 0.307 | 0 | -0.5 | 1.160 | 0.92 | -57.6 | 0.271 | 0 |
| LR− | 0.9 | 0.044 | 0.94 | 0.9 | 0.044 | 0.94 | 1.5 | 0.044 | 0.94 | 27.9 | 0.039 | 0.33 | 46.8 | 0.045 | 0.04 |
| ω_{m1} = 0.1, ω_{m0} = 0.2 | | | | | | | | | | | | | | | |
| Se | -0.1 | 0.041 | 0.94 | -0.1 | 0.041 | 0.94 | 4.7 | 0.036 | 0.80 | -12.3 | 0.036 | 0.34 | -12.3 | 0.036 | 0.34 |
| Sp | -0.1 | 0.017 | 0.94 | -0.1 | 0.017 | 0.94 | -22.3 | 0.017 | 0 | 2.2 | 0.014 | 0.62 | -22.3 | 0.017 | 0 |
| Prev | 0.4 | 0.034 | 0.93 | 0.6 | 0.036 | 0.90 | 0.4 | 0.034 | 0.93 | 0.4 | 0.034 | 0.93 | 0.4 | 0.034 | 0.93 |
| PPV | -0.3 | 0.046 | 0.93 | 3.1 | 0.044 | 0.88 | -36 | 0.047 | 0 | 2.7 | 0.044 | 0.89 | -42.1 | 0.047 | 0 |
| NPV | -0.1 | 0.018 | 0.94 | -1.3 | 0.020 | 0.93 | -1.4 | 0.020 | 0.92 | -2.7 | 0.020 | 0.83 | -6.3 | 0.024 | 0.36 |
| LR+ | 1.4 | 1.195 | 0.94 | 1.4 | 1.194 | 0.94 | -65.1 | 0.159 | 0 | 12.3 | 1.312 | 0.95 | -70.8 | 0.147 | 0 |
| LR− | 0.6 | 0.044 | 0.93 | 0.6 | 0.044 | 0.93 | 14.7 | 0.050 | 0.85 | 26.1 | 0.038 | 0.39 | 66.1 | 0.051 | 0 |
| ω_{m1} = 0.2, ω_{m0} = 0.1 | | | | | | | | | | | | | | | |
| Se | -0.1 | 0.023 | 0.93 | -0.1 | 0.023 | 0.93 | 8.7 | 0.018 | 0.12 | -21 | 0.020 | 0 | -21 | 0.019 | 0 |
| Sp | 0 | 0.009 | 0.93 | 0 | 0.009 | 0.93 | -10.6 | 0.009 | 0 | 1.1 | 0.008 | 0.74 | -10.6 | 0.009 | 0 |
| Prev | 0 | 0.018 | 0.93 | -8.4 | 0.017 | 0.72 | 0 | 0.017 | 0.91 | 0 | 0.017 | 0.91 | 0 | 0.0168 | 0.89 |
| PPV | -0.1 | 0.025 | 0.93 | -3.7 | 0.027 | 0.83 | -19.1 | 0.025 | 0 | -4 | 0.026 | 0.8 | -30.6 | 0.025 | 0 |
| NPV | 0 | 0.010 | 0.92 | 1.1 | 0.009 | 0.76 | 1.1 | 0.009 | 0.74 | -4.6 | 0.011 | 0.05 | -6.2 | 0.012 | 0 |
| LR+ | 0.3 | 0.655 | 0.93 | 0.3 | 0.653 | 0.93 | -44.1 | 0.196 | 0 | -11.7 | 0.570 | 0.62 | -59.3 | 0.154 | 0 |
| LR− | 0.3 | 0.025 | 0.93 | 0.3 | 0.025 | 0.93 | -10.8 | 0.022 | 0.62 | 47.4 | 0.021 | 0 | 66.7 | 0.024 | 0 |

Three scenarios are studied: equal or unequal missing probabilities for the diseased and non-diseased groups. Bias in percentage (Bias%), mean standard error (meanSE) and 95% confidence interval coverage probability (CP) are summarized for estimates of sensitivity (Se), specificity (Sp), prevalence (Prev), positive predictive value (PPV), negative predictive value (NPV), positive likelihood ratio (LR+) and negative likelihood ratio (LR−) from different models. "TGLMM" stands for the extended TGLMM. Model 1 excludes non-evaluable subjects, Model 2 takes non-evaluable subjects as index test positives, Model 3 takes non-evaluable subjects as index test negatives and the intent-to-diagnose approach takes non-evaluable subjects as false positives and false negatives.
included that reports cell counts in a $3 \times 2$ table as Table 1. The authors mentioned that the $3 \times 2$ table can be extended to a $3 \times 3$ table for non-evaluable results of the gold standard, however such cases were rare (0.1%) in this systematic review. We re-evaluate the 26 studies by the extended TGLMM and compare to the estimates following the four approaches discussed in Schuetz et al. [6].

The fitted median estimates and 95% confidence intervals are reported in Table 3. The extended TGLMM accounting for missing subjects gives median sensitivity, specificity, LR+, LR− and AUC estimates close to the estimates when non-evaluable subjects are excluded as in Model 1. The median disease prevalence estimated from the extended TGLMM is slightly lower than the estimate from Model 1. Model 2 gives significantly lower specificity estimate and Model 3 gives lower sensitivity estimate. The intent-to-diagnose approach provides lower estimates for sensitivity, specificity and AUC as it is the most conservative approach. Figure 1 presents the estimated PPV and NPV with 95% confidence bands versus prevalence, based on the overall sensitivity and specificity estimates from the extended TGLMM and the intent-to-diagnose approach. Figure 1 shows that as disease prevalence changes, PPV and NPV estimates from the latter approach are not ever included in the 95% confidence band of the estimates from the extended TGLMM, which suggests potential underestimation of PPV and NPV.

**Discussions**

Adequate reporting of the missing outcomes in study reports is essential to apply the discussed models. As shown in the simulation studies, different missing scenarios can have different impact on how estimates are biased and more importantly, missing mechanism can indicate whether the MAR assumption holds. When the MAR assumption is violated, i.e., the probability of non-evaluation depends on unobserved index test outcomes, the direction and magnitude of bias are hard to predict. Few sensitivity analysis methods using pattern mixture models and selection models are available for this scenario [23,24]. These approaches can be explored in further research. On the other hand, number of non-evaluable results need to be known in order to apply the proposed methods. However, a recent study shows that they are not consistently or adequately reported in published studies [25].

A reviewer has pointed out that as long as number of non-evaluable subjects are known, disease prevalence can be estimated unbiasedly through an univariate meta-analysis. Consequently, together with unbiased sensitivity and specificity estimates, PPV and NPV estimates are unbiased too. This approach is a simpler method than the proposed extended TGLMM to estimate prevalence, however, can be less efficient by ignoring the potential correlation between prevalence, sensitivity and specificity, which may result in wider confidence intervals.

For an individual patient, different approaches of treating a missing result can have different impact. For example, if index test results are missing due to the same reason of returning a negative result (and thus is MNAR), then treating such patients as disease negatives can yield unbiased estimate of prevalence for a study, and also won’t affect the patients’ diagnosis. On the contrary, if index test missing patients are treated as positives for reasons such as suspicious of serious disease like cancer [26],

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**Table 3** Median estimates and 95% CI (in brackets) for parameter estimates using different methods

| Method         | Sensitivity | Specificity | Prevalence | PPV          |
|----------------|-------------|-------------|------------|--------------|
| TGLMM          | 98.0 (96.7, 99.3) | 87.5 (82.7, 92.3) | 47.8 (37.9, 57.7) | 87.8 (83.3, 92.3) |
| Model 1        | 98.0 (96.7, 99.3) | 87.4 (82.5, 92.3) | 49.3 (38.9, 59.7) | 88.4 (84.9, 92.7) |
| Model 2        | 98.1 (96.9, 99.3) | 75.9 (69.3, 82.5) | 47.8 (37.9, 57.8) | 78.9 (71.9, 85.9) |
| Model 3        | 91.7 (88.1, 95.4) | 89 (85.4, 92.7) | 47.8 (37.9, 57.7) | 88.4 (84.1, 92.7) |
| Intent-to-diagnose | 91.7 (88.1, 95.3) | 76.2 (69.7, 82.6) | 47.9 (37.9, 57.9) | 78 (70.2, 85.7) |

| Method         | NPV          | LR+         | LR−         | AUC          |
|----------------|--------------|-------------|-------------|--------------|
| TGLMM          | 97.9 (96.4, 99.5) | 7.8 (4.8, 10.9) | 0.02 (0.01, 0.04) | 0.99 (0.96,1) |
| Model 1        | 97.8 (96.1, 99.4) | 7.8 (4.8, 10.9) | 0.02 (0.01, 0.04) | 0.99 (0.96,1) |
| Model 2        | 97.8 (96.2, 99.4) | 4.1 (2.9, 5.2) | 0.02 (0.01, 0.04) | 0.98 (0.97,1) |
| Model 3        | 92.1 (88.4, 95.8) | 8.4 (5.5, 11.3) | 0.09 (0.05, 0.14) | 0.96 (0.93,0.99) |
| Intent-to-diagnose | 90.9 (86.4, 95.5) | 3.8 (2.7, 5.0) | 0.11 (0.06, 0.16) | 0.93 (0.89,0.96) |

*TGLMM* stands for the extended TGLMM. Model 1 excludes non-evaluable subjects, Model 2 takes non-evaluable subjects as index test positives, Model 3 takes non-evaluable subjects as index test negatives and the intent-to-diagnose approach takes non-evaluable subjects as false positives and false negatives. Positive predictive value (PPV), negative predictive value (NPV), positive likelihood ratio (LR+), negative likelihood ratio (LR−) and area under the curve (AUC) are summarized.
it may result in over-estimation of disease prevalence and unnecessary medical cost for the patient. For another example, if index test is repeatable and repeated for subjects with non-evaluable results, then it is appropriate to ignore missing results.

**Conclusions**

In this paper, we propose an extended TGLMM approach to handle non-evaluable index test subjects in meta-analysis of diagnostic tests. The extended TGLMM is compared to an intent-to-diagnose approach and three alternative approaches proposed by Schuetz et al. [6] through simulation studies and re-evaluation of the meta-analysis of coronary CT angiography studies.

In summary, by simulation studies we showed that under MAR assumption, excluding index test non-evaluable subjects (Model 1) will not lead to biased estimates of sensitivity, specificity, LR+, LR− and AUC. Thus in practice, researchers can be confident to apply Model 1 when there is a belief in the MAR assumption. However, when disease prevalence or PPV and NPV are of interest, excluding non-evaluable subjects could lead to biased estimates of these parameters. Under this situation, the extended TGLMM accounting for missingness should be preferred. Even though the extended TGLMM is more theoretically complex than the widely used bivariate random effects model, it is easy to program use SAS NLMIXED procedure. Sample SAS code with an application to the meta-analysis of coronary CT angiography studies is provided in the Appendix. Model 2, Model 3 and the intent-to-diagnose approach all largely under- or over-estimate sensitivity and specificity, so that they should not be recommended when MAR assumption is not seriously violated.

**Claims**

Ethical approvals and informed consents are not applicable to this paper.
Appendix: SAS code of the extended TGLMM approach: meta-analysis of coronary CT angiography studies

/*Read in data for the coronary CT angiography studies*/ data observe;
input sid year ntp1 nfp1 nfn1 ntn1 nm1 nm0;
datalines;
1 2005 25 4 0 19 2 0
2 2008 57 3 2 79 0 9
... ...
26 2008 32 6 4 6 10 0
; run;

/*Macro to compute AUC*/
%macro auclogit;
estimate "AUC" 0
%do sp=1 %to 1000;
+0.001/(1+exp(-alpha0-RhoSeSp*exp(sigse)/
exp(sigsp)*
(log(1-0.001*&sp+0.0005)
-log(0.001*&sp-0.0005)+beta0)) )
%end;
%mend auclogit;

/*TGLMM using NLMIXED procedure*/
proc nlmixed data=observe fd df=1000
cov corr ecorr gtol=1e-10;
parms beta0=2 alpha0=1.5 eta0=0.2
sigse=0.4 sigsp=0.3 sigpi=0.5
fZ=0 fZ1=0 fZ2=0;
lsei = alpha0 + muse 
lspi = beta0 + musp 
lppi = eta0 + mupi 
Sei=1/(1+exp(-lsei));
Spi=1/(1+exp(-lspi));
ppli=1/(1+exp(-lppi));
RhoSeSp= (exp(2*fZ)-1)/(exp(2*fZ)+1);
RhoSePi= (exp(2*fZ1)-1)/(exp(2*fZ1)+1);
RhoSpPi= (exp(2*fZ2)-1)/(exp(2*fZ2)+1);

/* The log likelihood accounting for
missing cell counts by nm1*log(ppi)
and nm0*log(1-ppi)*/
logL= ntp1 * (log(ppi) + log(Sei)) +
nfp1 * (log(1-ppi) + log(1-Sei)) +
nfn1 * (log(ppi) + log(1-Sei)) +
nln1 * (log(1-ppi) + log(Spi)) + ntn1 *
(log(1-ppi) + log(Spi)) + nm1 * log(ppi) +
nm0 * log(1-ppi);
model ntp1 - general(logL);
random muse musp mupi - normal([0, 0, 0],
[exp(2*sigse),
RhoSeSp*exp(sigse+sigsp), exp(2*sigsp),
RhoSePi*exp(sigse+sigpi),
RhoSpPi*exp(sigsp+sigpi), exp(2*sigpi)])
subject=sid;
%auclogit;

/*estimate statements in proc NLMIXED
can give point estimates of functions
of parameters as well as variances
from delta method*/
estimate "Se" 1/(1+exp(-alpha0));
estimate "Sp" 1/(1+exp(-beta0));
estimate "Prev" 1/(1-exp(-eta0));
estimate "PPV" exp(eta0+alpha0)*
(1+exp(beta0))/(exp(eta0+alpha0)*
(1+exp(beta0))+(1+exp(alpha0)));
estimate "NPV" exp(eta0)*
(1+exp(alpha0))
/(1-exp(beta0));
estimate "LR+" exp(alpha0)/(1+exp(alpha0))
/(1-exp(beta0)+exp(eta0)*
(1-exp(beta0)));
estimate "LR-" (1-exp(alpha0)/
(1-exp(alpha0))/
(1-exp(beta0))/
(1-exp(beta0)));
or;

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All the authors contributed substantively to the study and approved the
manuscript submitted for review. XM and HC conceived of the idea of the
study, XM was responsible for data analysis. FS, XM and HC all contributed in
drafting and revising the manuscript. All authors read and approved the final
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