The accretion disc dynamo in the solar nebula

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ABSTRACT

The nearest accretion disc to us in space if not time was the protosolar nebula. Remnants of this nebula thus potentially offer unique insight into how discs work. In particular the existence of chondrules, which must have formed in the disc as small molten droplets, requires strong and intermittent heating of disc material. We argue that this places important constraints on the way gravitational energy is released in accretion discs, which are not met by current shearing-box simulations of magnetorotational instability (MRI)-driven dynamos. A deeper understanding of accretion energy release in discs may require a better model for these dynamos.

Key words: accretion, accretion discs.

1 INTRODUCTION

It is now generally agreed that accretion discs are driven mainly by magnetic torques (Shakura & Sunyaev 1973) and that the magnetic fields in such discs are maintained by local dynamo processes. What is not understood, however, despite considerable theoretical efforts, is how such dynamo processes work, and exactly where and in what form the accretion energy is released (King, Pringle & Livio 2007; Blackman & Pessah 2009).

Given the difficulty of this problem, we should look for observational evidence which bears on it. We have unique insight into the remains of one accretion disc – the protosolar nebula. It is therefore worth asking if the present-day Solar system offers evidence which can constrain the way energy is released in accretion discs.

Among the oldest solid objects in the Solar system are chondrules, the round grains present in the majority of meteorites. These must have formed as molten droplets in space before being accreted. The need to heat them sufficiently implies a connection with energy release in the protosolar nebula. We consider here the overall energetics of the heating process. We argue that since most of the meteoritic material has been subject to this heating, the energy source required must be quite widespread and substantial and must therefore involve the major local source of energy, i.e. disc accretion. We find that around 10 per cent of the accretion energy is released in accretion discs.

This paper is organized as follows. In Section 2, we give a brief introduction to chondrules and ideas about their formation. In Section 3, we consider the global energetics of the heating process. A likely mechanism for the provision of transient heating within the solar nebula involves strong shocks (Desch & Connolly 2002; Connolly et al. 2006). In Section 4, we briefly present the model of chondrule formation through shock heating suggested by Desch & Connolly (2002) but argue that their proposed mechanism for generating these shocks by gravitational instabilities within the disc requires disc properties which do not sit easily with our current understanding of disc evolution. In Section 5, we describe the disc properties which appear likely to hold at the time of chondrule formation. The need for significant disc dissipation in the form of shocks, together with evidence for magnetic fields within the nebula at that time (Section 6), leads us to a picture of a disc dynamo (Section 7). This differs markedly from the results of current numerical simulations. Our picture draws on the dynamo model of Baggerly et al. (2009a,b) and involves thin flux ropes and reconnection, analogous to models of flaring on the solar surface. Section 8 is a discussion.

2 CHONDRULES

Most meteorites are chondrites (more than 75 per cent; Sears & Dodd 1988; Hutchinson 2004), and the most abundant constituent of the majority of chondritic meteorite groups is chondrules (Grossman et al. 1988). Chondrules are small particles of silicate material (typically around a millimetre in size; Weissberg, McCoy & Krot 2006) that experienced melting before incorporation into chondritic meteorite parent bodies (Hewins 1996). The properties of chondrules are consistent with formation in the protosolar nebula from primitive material and thus provide an important record of events in this disc. Isotopic dating of chondrules indicates ages of 4.5 Gyr and also suggests that high temperature nebular processes, such as chondrule formation, lasted for about 3–5 Myr (Russell et al. 2006; Scott 2007). This time-scale coincides with the observed lifetime of protostellar discs around solar-type stars (e.g. Hernández et al. 2009). Chondrules formed a few Myr after the earliest Ca–Al-rich inclusions (CAIs; Swindle et al. 1996). These formed at lower temperatures (∼1500 K) and in general have longer cooling times.

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and thus a specific kinetic energy

\[ E_{\text{kin}} = \frac{1}{2} V_\phi^2 = 1.5 \times 10^{12} \left( \frac{M}{M_\odot} \right) \left( \frac{R}{3 \text{ au}} \right) \text{erg g}^{-1}. \]  

(3)

We can think of this as being equivalent to the accretion energy available locally. This is the energy source that we shall need to use to power chondrule formation.

The specific energy required to heat the material from a few hundred kelvin up to \( \sim 1800 \text{ K} \) and then melt it is given by Wasson (1996) as

\[ E_{\text{mel}} = 2.1 \times 10^{10} \text{erg g}^{-1}. \]  

(4)

To form chondrules, it is necessary to heat the protochondrules (or ‘dust balls’) to the melting point. The fraction of chondrule to matrix (the remainder of the chondrite, which has never melted) in chondrites is large, and in addition there is evidence that about 15 per cent of chondrules have undergone more than one melting event. Thus, we shall assume that on average each protochondrule (or ‘dust ball’) is subject to around one melting event.

Since \( E_{\text{kin}} \gg E_{\text{mel}} \), it seems that there is plenty of energy available. However, this is not the whole story. Once heated to the melting point, the chondrules must cool. If they simply radiated their energy into space as blackbodies they would cool on a time-scale

\[ t_{\text{rad}} = \frac{E_{\text{mel}} m}{4\pi r^2 \sigma T^4}, \]  

(5)

where a chondrule is assumed to have mass \( m \), radius \( r \) and temperature \( T \), and \( \sigma \) is the Stefan–Boltzmann constant. Thus, a typical cooling time for a chondrule would be (see e.g. the cooling curves given by Wasson 1996)

\[ t_{\text{rad}} = 3.5 \left( \frac{E_{\text{mel}}}{2.1 \times 10^{10} \text{erg g}^{-1}} \right) \left( \frac{r}{1 \text{ mm}} \right) \left( \frac{\rho_{\text{chon}}}{3 \text{ g cm}^{-3}} \right) \times \left( \frac{T}{1800 \text{ K}} \right)^{-4} \text{s}, \]  

(6)

where \( \rho_{\text{chon}} \) is the density of the chondrule.

In practice, in order for the chondrules to have the properties observed, they must cool on a much longer time-scale than this, i.e. \( t_{\text{cool}} \gg t_{\text{rad}} \) (Scott 2007). Estimates of cooling times vary greatly. Levy & Araki (1989) suggest \( t_{\text{cool}} \sim 2 \times 10^3 \) to \( 2 \times 10^4 \text{s} \). Hutchinson (2004) agrees with this at the short end, but indicates that some cooling times must be much longer, in the range \( t_{\text{cool}} \sim 2 \times 10^5 \) to \( 3 \times 10^6 \text{s} \). From the detailed discussion given by Desch & Connolly (2002), it is evident that the cooling histories of chondrules are varied and complicated. Scott, Love & Krot (1996) argue that the disc must have been ‘a maelstrom where temperatures fluctuated through \( \sim 1000 \text{ K} \) and solids experienced “multiple cycles of melting, evaporation, recondensation, crystalization and aggregation”’.

Desch & Connolly (2002) give characteristic cooling times in the range of hours to days. This is in line with the recent review by Connolly et al. (2006), who quote the majority of chondrules as having a cooling time of \( \sim 10^5 \text{s} \). Thus, we adopt a typical cooling time for chondrules of

\[ t_{\text{cool}} = 10^5 \text{s}. \]  

(7)

The fact that \( t_{\text{cool}} \gg t_{\text{rad}} \) has implications for the energetics of the formation process. Not only must the chondrules be heated to melting temperature, but they must also be kept near that temperature for a prolonged period. This implies that not just the protochondrules but also the material surrounding them must be heated to around \( 2000 \text{ K} \) (Wasson 1996). In addition, a sufficient volume of

3 ENERGETICS OF THE FORMATION PROCESS

As remarked above, chondritic meteorites appear to come from the asteroid belt. Accordingly, we assume that \( \text{in situ} \) chondrule formation occurs in a region of the protostellar disc at radii in the range of \( 2-5 \text{ au} \) and so at around a typical radius of

\[ R = 3 \text{ au} = 4.5 \times 10^{13} \text{ cm}. \]

(1)

At this radius, the disc material has circular velocity

\[ V_\phi = \left( \frac{GM}{R} \right)^{1/2} = 1.7 \times 10^6 \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{R}{3 \text{ au}} \right)^{-1/2} \text{cm s}^{-1}. \]

(2)
surrounding material must be heated so that its optical thickness implies a cooling time of hours to days. The specific energy required to heat the surrounding gas to a temperature of \( T = 2000 \text{ K} \) is given approximately by

\[ E_{\text{req}} \approx \frac{(3/2)kT}{2m_H} = 1.2 \times 10^{11} \left( \frac{T}{2000 \text{ K}} \right) \text{ erg g}^{-1}, \quad (8) \]

where we have assumed that the typical gas particle is a hydrogen molecule and have ignored molecular dissociation. This implies, again assuming that on average each element of chondritic material is subject to one melting event, that a fraction \( f \) of around

\[ f \sim \frac{E_{\text{req}}}{E_{\text{kin}}} \approx 0.08 \quad (9) \]

of the available accretion energy must be used to provide the heating process for chondrule formation.

### 4 Shock Heating

Desch & Connolly (2002) have proposed a detailed model for the formation of chondrules in terms of the thermal processing of particles in shocks in the accretion disc. Their model takes account of the properties of the shock, including gas-drain heating of particles which are not slowed instantly in the gas shock, radiative processes including the disassociation and recombination of \( \text{H}_2 \), and particle evaporation. Their model can account for the thermal history of chondrules. Their canonical model requires a shock velocity of

\[ V_s = 7 \times 10^3 \text{ cm s}^{-1}. \quad (10) \]

For their assumed ambient disc temperature of \( T = 300 \text{ K} \) and sound speed (assuming a gas of molecular hydrogen) of \( c_s = 1.5 \times 10^5 \text{ cm s}^{-1} \), this corresponds to a Mach number of around \( M \sim 5 \).

However, while this model can successfully account for chondrule formation in the disc, the proposed source of the shocks within the disc does not sit easily with our current understanding of protostellar disc evolution. Desch & Connolly (2002) propose that the shocks are caused by gravitational instabilities within the disc. But this proposal is at odds with their disc properties, which are taken from a disc model by Bell et al. (1997) with an accretion rate of \( \dot{M} = 10^{-8} \text{ M}_\odot \text{ yr}^{-1} \) and a viscous parameter of \( \alpha = 10^{-4} \). The disc density is assumed to be \( \rho = 10^{-4} \text{ g cm}^{-3} \), which for a disc semithickness \( H = (c_s / \rho)^{1/2} \approx 4 \times 10^2 \text{ cm} \) gives a disc surface density of \( \Sigma = 8000 \text{ g cm}^{-2} \). This gives rise to a strongly unstable disc with a Toomre parameter (Toomre 1964) of

\[ Q = \frac{\Omega c_s}{\pi G \Sigma} \approx 3.4 > 1. \quad (11) \]

Such strong gravitational instability is indeed required if this process is to give rise to shocks with velocities \( v_s \approx 0.4V_a \). It is, however, hard to envisage how all the material in such a strongly unstable disc would manage to avoid being strongly shocked a large number of times.

Such a disc has a mass at radii around 3 \( \text{au} \) (say, at radii 2–4 \( \text{au} \)) of

\[ M_{\text{disc}}(3 \text{ au}) \approx \pi R^2 \Sigma \approx 0.025 \text{ M}_\odot. \quad (12) \]

Such a violently unstable disc is likely either to fragment into predominantly gaseous bodies, gathering any chondrules with them, or, if not, give rise to strong gravitational torquing and so a high accretion rate. The accretion rate expected for a disc which is so gravitationally unstable is the same as the one for which the viscous parameter has \( \alpha \approx 0.06 \) (Lodato & Rice 2004; Rice, Lodato & Armitage 2005) which would give (Pringle 1981)

\[ \dot{M} = 3\pi c_s H \Sigma \approx 4 \times 10^{-5} \text{ M}_\odot \text{ yr}^{-1}. \quad (13) \]

Such a high accretion rate would give a mid-plane disc temperature of \( \sim 2000 \text{ K} \) (Bell et al. 1997) and contradicts the original assumption that \( \dot{M} = 10^{-8} \text{ M}_\odot \text{ yr}^{-1} \). It would give a local disc lifetime of around \( M_{\text{disc}} / \dot{M} \approx 600 \text{ yr} \).

Similar considerations of accretion rates and lifetimes apply to the more detailed models presented by Boss & Durisen (2005; see also Boss 2007). These authors stress that early times, when infall on to the disc was high, the disc is most likely massive enough for self-gravity to play a major role in angular momentum redistribution (see e.g. Lin & Pringle 1990). They also note, however, that the local, non-axisymmetric instabilities driven by self-gravity occur at, or around, corotation (see also the discussion in Cossins, Lodato & Clarke 2009). This implies that in general, throughout the bulk of the disc, the relative velocity between the disc gas and the (unstable, but transient) spiral pattern is small. Thus, in general, the short-lived, localized, transient, spirals, which are the typical outcomes of self-gravitational disc instabilities, are not able to drive strong shocks in the gas. In this context, in the numerical simulations of Boss & Durisen (2005), it is only close to (within a few grid cells of) the inner grid boundary that they are able to achieve a shock pattern which is neither close to corotation nor strongly trailing.

Thus while we cannot rule out the possibility that a massive disc and self-gravitational instabilities are able to provide the necessary shocks, we now consider what we regard as more likely properties for the disc in the region of a few \( \text{au} \) at the time of chondrule formation.

### 5 Disc Properties

From timings deduced from isotopic data, Russell et al. (2006) conclude that chondrule formation lasted for around 3–5 Myr. This is also the time-scale on which young solar-type stars appear to lose their protoplanetary discs (Hernández et al. 2009). Thus, it seems reasonable to conclude that chondrules were formed during the final stages of the protoplanetary disc when the accretion rates were low (\( \dot{M} \leq 10^{-7} \text{ M}_\odot \text{ yr}^{-1} \); Ciesla & Charland 2006).

Models of protostellar discs at low steady accretion rates are given by Bell et al. (1997). At the accretion rates we consider, it is likely that heating from the central star provides a non-negligible, and perhaps dominant, contribution in the surface layers, which may cause some changes in the mid-plane (e.g. Chiang et al. 2001). There is also the possibility that such discs contain ‘dead zones’ (Balbus & Hawley 2000; Terquem 2008) where the MRI is unable to operate because of high magnetic diffusivity caused by low ionization. However for the canonical disc parameters we use (see below), Matsumura & Pudritz (2003) argue that the MRI should be fully operational. This is in line with our above argument that the full accretion energy must be accessed to power chondrule formation.

At the time of chondrule formation, we take the disc mass as a few times the minimum mass solar nebula (say, \( \sim 0.06 \text{ M}_\odot \)) and its age as \( \sim 3 \text{ Myr} \). An accretion rate of the order of \( \dot{M} \sim 3 \times 10^{-6} \text{ M}_\odot \text{ yr}^{-1} \) is appropriate. Given this, the models of Bell et al. (1997) suggest the following parameters for the disc in the region of \( R \approx 3 \text{ au} \). The disc (mid-plane) density is

\[ \rho_{\text{disc}} = 10^{-10} \text{ g cm}^{-3} \quad (14) \]

and the mid-plane temperature is

\[ T_{\text{disc}} = 200 \text{ K}. \quad (15) \]

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with disc thickness
\[ H = 0.05R = 2.2 \times 10^{12} \text{ cm} \] (16)
and viscosity parameter
\[ \alpha = 0.01. \] (17)

In this model, most of the disc mass still resides at radii 10–20 au, and it is these radii which control the evolution time-scale of the disc, and so the accretion rate. These parameters are in line with the fully ionized models of Terquem (2008) and with the disc properties assumed by Ida & Lin (2004), in their modelling of planet core formation. For these parameters, the disc sound speed is
\[ c_s = 1.2 \times 10^6 \text{ cm s}^{-1} \] (18)
and the gas pressure is
\[ p_g = 0.83 \text{ dyn cm}^{-2}. \] (19)

6 MAGNETIC FIELDS

Remnant magnetic fields found in chondritic material imply cooling through the Curie temperature of \( \sim 600 \text{ K} \) in the presence of a magnetic field. This offers clear evidence that chondrule formation took place in regions with magnetic fields in the range \( B \sim 1–10 \text{ G} \) (Levy & Sonett 1978; Levy & Araki 1989). This is close to the maximum (equipartition) field strength \( B \) of a magnetic flux tube in an accretion disc – equating the magnetic pressure \( B^2/8\pi \) with the local gas pressure \( p_g \) for the disc parameters given above gives
\[ B_{eq} = 4.6 \text{ G}. \] (20)

We note that to achieve the strength of viscosity \( (\alpha = 0.01) \) assumed above, we only require an average magnetic field \( \langle B \rangle \) within the disc given by (Shakura & Sunyaev 1973)
\[ \alpha \approx \frac{\langle B \rangle^2}{8\pi p_g} \] (21)
and
\[ \langle B \rangle \approx 0.5 \left( \frac{\alpha}{0.01} \right)^{1/2} \text{ G}. \] (22)

This suggests that chondrule formation could have taken place in a disc in which a dynamo mechanism driven by the magnetorotational instability was operating.

7 THE DYNAMO PROCESS AND CHONDRULE FORMATION

We have seen that chondrule formation requires the heating of each element of the disc (gas plus protochondrules) up to a temperature of around \( T \approx 2000 \text{ K} \approx 10 T_{\text{disc}}, \) for a time \( \approx t_{\text{cool}} = 10^3 \text{ s}, \) on average once. This must happen during the time when this material is at a radius of the order of 3 au. Once the gas is at this temperature it becomes overpressurized relative to its surroundings, and so will tend to expand, and therefore cool adiabatically, at its now elevated sound speed of \( c_s^* \approx 3.8 \times 10^6 \text{ cm s}^{-1}. \) For this adiabatic cooling to take longer than \( t_{\text{cool}}, \) we require the size of the heated region \( h \) to be such that
\[ h \geq c_s^* \times t_{\text{cool}} \approx 3.8 \times 10^{10} \text{ cm}. \] (23)

We have also noted (Desch & Connolly 2002) that a way of achieving such heating is through shocks within the disc, with shock velocities \( V_s \sim 7 \times 10^5 \text{ cm s}^{-1} \sim 6 c_s. \) However, although the standard shearing-box models of MRI-driven dynamos in accretion discs (see Davis, Stone & Pessah 2009) can produce values of \( \alpha \) of the order of 0.01, they predict velocities within the disc which are for the most part strongly subsonic. In this picture, chondrule formation would thus remain unexplained.

However, there are indications that such models do not give a complete description of dynamo processes in accretion discs (cf. King et al. 2007). One problem is that these models use the magnetohydrodynamics approximation with a standard (isotropic) magnetic diffusivity \( \eta \) and standard (isotropic) Navier–Stokes viscosity \( \nu. \) Fromang & Papaloizou (2007) and Fromang et al. (2007) demonstrate that the results obtained depend on the assumed numerical parameters: the Reynolds number \( R = c_s H/\nu, \) the magnetic Reynolds number \( R_m = c_s H \eta \) and, critically, on the ratio of the two, the magnetic Prandtl number \( P_m = R_m/R = \nu/\eta. \) There are also other physical processes, which can provide complications such as the Hall effect (Balbus & Hawley 2000), and anisotropic transport processes (e.g. Dong & Stone 2009). Schekochihin and co-workers (e.g. Schekochihin et al. 2004, 2005) have also argued that the structure of a magnetic field in a turbulent flow at such high Reynolds numbers can depend critically on the Prandtl number and that this has implications for dynamos under such conditions. In addition, Heitsch et al. (2009) and Zweibel & Heitsch (2008) have discussed the implications for magnetic growth and structure in turbulent media where ambipolar diffusion plays a significant role.

7.1 An alternative dynamo picture

In the light of the above, we speculate how an accretion disc dynamo might produce the intermittent high-energy events which appear to be demanded by chondrule formation. As an example, we consider a new model for a dynamo in a turbulent medium suggested by Baggaley et al. (2009a,b). The extent to which such a model is applicable to cooler accretion discs, such as the solar nebula, is somewhat uncertain, and we introduce it here because it is able to illustrate the kind of properties with regard to energy release that we are looking for to facilitate chondrule formation. In this model, the magnetic field is mainly confined to thin flux ropes which are advected by the flow. Magnetic dissipation only occurs via reconnections of the flux ropes, and so the magnetic dissipation is highly localized. This model can be viewed as an implementation of the limiting regime of an infinitely large magnetic Reynolds number: magnetic dissipation can be safely neglected at all scales, but plays a crucial role through reconnection of field lines in permitting rearrangement of the field topology. Such rearrangements of the field topology result in conversion of magnetic energy into kinetic energy of the fluid, and thence to dissipation as heat. This contrasts with the usual models in which magnetic diffusivity converts magnetic energy to heat directly.

With this in mind, the picture we propose is that the magnetic field in the disc can be thought of as a collection of loops of flux ropes. These loops are continually stretched by the azimuthal shear flow. The stretching increases the magnetic energy associated with the loop and at constant total (gas plus magnetic) pressure decreases the mass density along the field line, and so increases the Alfvén speed along the loop. To maintain an equilibrium distribution of loop properties, the loops must also continually undergo reconnection events. The crucial property of these events is that while each reconnection event in itself releases a negligible amount of energy, it does release the magnetic field, which is then able to reconfigure itself (at the Alfvén speed) and so doing heat the gas. The picture here thus differs fundamentally from that presented by Sonett (1979) and Levy & Araki (1989). They considered magnetic
reconnection in low density regions far from the disc plane (in the
disc corona) and took only the magnetic energy dissipated by the
reconnection events into account. We are assuming here that most
of the accretion energy is released in the bulk of the disc. We should
note, however, that some authors have suggested that a substantial
fraction of the accretion energy might be released in such low density
regions (Tout & Pringle 1992; Uzdensky & Goodman 2008),
arguing that magnetic buoyancy can advect energy efficiently away
from the disc plane. For our picture to work, however, we require
that a significant amount of reconnection occurs close to the disc
plane so that the bulk of the energy release occurs there and disc
gas, along with the chondrule, can be efficiently shock heated.

What we therefore require for chondrule formation is that some
reconnection events occur in regions with sufficiently high Alfvén
speeds that shock velocities of $V_s \approx 6c_s$ can be generated. For
this to occur, we require that a majority of flux tubes have field
strengths of the order of $B_{\text{eq}}$ and loop mass densities around
40 times lower than the mean disc density before reconnection
occurs. Unfortunately the ‘fluctuation dynamo’ model of Baggaley
et al. (2009a,b) is currently computed only in an incompressible
medium. Further consideration of the model will be required before
it is possible to establish whether or not such high-energy reconnec-
tion events are likely in an accretion disc.

7.2 Implications for magnetic field structure
and internal disc properties

In this picture we suppose that from time to time a reconnection
event occurs which causes a sufficiently large and rapid adjust-
ment of field topology that a region of disc gas of size $h$ is subject
to shock heating, with velocity $V_s \approx 7$ km s$^{-1}$. We need this heated
region to stay sufficiently hot for a time $t_{\text{cool}} \approx 10^5$ s. We can use
this information to deduce requirements for the properties of the
magnetic loops.

7.2.1 Size of heated regions

We expect the cooling time-scale $\tau$ for a region of size $h$ and
temperature $T$ to be given by

$$\tau \sim \frac{\text{heat content}}{\text{heat loss rate}}.$$  

We expect that the heat content is simply

$$\text{heat content} \propto \rho T h^3,$$

where $\rho$ is the gas density. If the region is optically thick, so that
heat loss is mainly by radiative transfer, then we expect

$$\text{heat loss rate} \propto \frac{T^4}{\rho h^2} \times h^2,$$

where $\kappa$ is the opacity. Thus, we expect

$$\tau \propto \frac{\rho^2 h^2}{T^3}. $$  

For an accretion disc in thermal equilibrium, the relevant length-
scale is the disc thickness $H$ and the cooling time-scale is the thermal
time-scale $t_{\text{th}}$, given by (Pringle 1981)

$$t_{\text{th}} \approx \frac{1}{\alpha \Omega} \approx 2.6 \times 10^7 \left(\frac{\alpha}{0.01}\right)^{-1} \text{s}. $$

Using this, we can obtain a very rough estimate of the size re-
quired for the shock-heated regions. To keep things simple we as-
sume that the opacity $\kappa$ is constant (e.g. Bell et al. 1997), although

if some chondritic material is vaporized and/or H$_2$ is significantly
dissociated, this might not be the case. Then from the computations
of Desch & Connolly (2002), we note that once the gas has relaxed
behind the shock, the temperature is $T \approx 10^3$ K, and so a factor of
$\sim 50$ above its pre-shock value, and similarly that the density $\rho$ has
increased by a factor of the order of $\sim 10$. Thus using equation (27)
we find

$$\frac{h^2}{H^2} \sim \frac{t_{\text{cool}}}{t_{\text{th}}} \times (50)^3 \times (10^{-2}),$$

which gives

$$h \sim 0.2H \sim 4 \times 10^{11} \text{ cm}. $$

7.2.2 Heating events and loops

We have seen that a single heating event is likely to have a size of
around $h \sim 4 \times 10^{11}$ cm. Evidently, this must be roughly the size
of the reconnecting loops. This means that the rate per unit volume
at which energy is dissipated locally by the shock is approximately

$$q_{\text{shock}} \sim \rho V^2 \times \left(\frac{V_s}{h}\right).$$

We can contrast this with an accretion disc in local thermal equi-
librium, where the much lower average volume rate at which energy
is dissipated within the disc is approximately

$$q^+ = \frac{\rho c_s^2}{t_{\text{th}}},$$

We can now use this to estimate the probability $p$ that a random
disc element is hot at any one time. The average rate at which shocks
heat the disc is

$$\langle q_c \rangle \approx p \times q_{\text{shock}}.$$  

We have argued earlier (equation 9) that to heat on average each
element of disc gas up to a temperature of around $T \approx 2000$ K once
every viscous time-scale requires about $f \approx 0.08$ of the available
accretion energy. Thus, we require that

$$\langle q_c \rangle \approx f \times q^+.$$  

and hence that

$$p \approx f \left(\frac{c_s^2}{V^2}\right) \frac{h/V_s}{t_{\text{th}}} \approx 3.3 \times 10^{-7}.$$

We have argued that each chondrule heating event should take
place on a scale of $\sim h \approx 4 \times 10^{11}$ cm. If we divide the disc locally
(say 2–4 au) into $N_{\text{box}}$ boxes, each of size $\sim h^3$, then

$$N_{\text{box}} \sim \pi R^2 \times 2H/h^3 \sim 1.4 \times 10^7.$$

Therefore, the number of these boxes which is ‘hot’ at any one
time is

$$N_{\text{hot}} \approx p \times N_{\text{box}} \approx 4.6 \times 10^{-2}.$$

Since the orbital period at this radius is $P = 1.6 \times 10^8$ s, each box is
therefore heated for a fraction $f_{\text{hot}} \approx (h/V_s)/P \approx 3.6 \times 10^{-3}$ of an
orbit. Similarly, the average time between these large flare events is
given by $t_{\text{flare}} \sim P/N_{\text{hot}} \approx 100$ yr.

Assuming that the reconnection rate for loops is roughly the same
as the rate at which they are stretched by the disc shear flow, so that
on average each loop reconnects once per orbit, then the number of
reconnections per orbit which give rise to major flares, and so
chondrule heating, is

$$N_{\text{req}} \sim N_{\text{hot}}/f_{\text{hot}} \sim 13.$$
7.2.3 Loop strength distribution and relation to the dynamo

We have argued that only about \( f \approx 0.08 \) (equation 9) of the energy is released in heating events which are powerful enough to cause chondrule formation. If we make the simplifying assumption that magnetic loops are all approximately of the same physical size \((\sim h)\) but have varying magnetic field strengths, then this implies that most loops have only weak field strengths. This ties in with the argument (equation 20) that to obtain a viscosity parameter of \( \alpha \approx 0.01 \), we only require an average disc field of the order of \((B) \approx 0.5 \) G.

If all the loops are of the same size, then the total number of loops would be

\[
N_{\text{loops}} \sim N_{\text{box}} \sim 1.4 \times 10^5. \tag{39}
\]

If, in a steady state, each loop reconnects once per dynamical time \((\approx \text{orbital period})\) then over this time there are a total of \( N_{\text{loops}} \sim 1.4 \times 10^5 \) reconnections, of which \( N_{\text{eq}} \sim 13 \) give enhanced heating, and so chondrule formation. These strong flares provide a fraction \( f \approx 0.08 \) of the total power of the disc, and so the energy released in a weak reconnection event is \( \sim N_{\text{eq}} / f N_{\text{loops}} \sim 1.3 \times 10^{-3} \) times that released in a strong one. The frequency distribution \( f(E) \) for events which release an energy \( E \) (cf. Charbonneau et al. 2001) is usually assumed to take a power-law form

\[
f(E) \propto E^{-s}, \quad s > 0. \tag{40}
\]

Then our above reasoning implies that \( s \approx 2.3 \). This is similar to what is found for the energy release in solar flares, for which \( s \) is generally in the range \( 1.4 \lesssim s \lesssim 2.6 \) (Charbonneau et al. 2001). Note that \( s = 2 \) implies equal energy release at all scales so that \( s \approx 2.3 \) implies that energy release occurs mainly (but marginally so) at small scales. For their incompressible loop dynamo model, Baggaley et al. (2009b) find \( s \approx 3 \) so that small scales strongly dominate.

8 CONCLUSIONS

We have argued that the need for strong intermittent heating during the formation of chondrules in the protosolar nebula places important constraints on the way gravitational energy is released in accretion discs. Following Desch & Connolly (2002), we suggest that this requires shock heating and propose that the shocks occurring as magnetic loops in the disc dynamo are released by reconnection events. In contrast, current shearing-box simulations of MRI-driven accretion disc dynamos predict largely subsonic velocity fields, and so leave the formation of chondrules unexplained.

We have proposed that accretion disc dynamos may be much more inhomogeneous, in space and in time, than current simulations indicate. It would not be surprising if the input physics to current numerical simulation was incomplete, and we have briefly discussed reasons why this might be so. We speculate that a better understanding of accretion disc energy release may require consideration of more complicated physical processes than are currently implemented in numerical codes.

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REFERENCES

Baggaley A. W., Barenghi C. F., Shukurov A., Subramanian K., 2009a, Phys. Rev. E, 80, 5301
Baggaley A. W., Barenghi C. F., Shukurov A., Subramanian K., 2009b, preprint (arXiv:0910.5392v2)
Balbus S. A., Hawley J. F., 2000, Space Sci. Rev., 92, 39
Bell K. R., Cassen P. M., Klahr H. H., Henning T., 1997, ApJ, 486, 372
Blackman E. G., Pessah M. E., 2009, ApJ, 710, L113
Boss A. P., 1996, in Hewins R. H., Jones R. H., Scott E. R. D., eds, Chondrules and the Protoplanetary Disk. Cambridge Univ. Press, Cambridge, p. 257
Boss A. P., 2007, ApJ, 660, 1707
Boss A. P., Durisen R. H., 2005, ApJ, 621, L173
Charbonneau P., Mcintosh S. W., Liu H.-L., Bogdan T. J., 2001, Sol. Phys., 203, 321
Chiang E. I., Joung M. K., Creech-Eakman M. J., Qi C., Kessler J. E., Blake G. A., van Dishoeck E. F., 2001, ApJ, 547, 1077
Ciesla F. J., Charney S. B., 2006, in Lauretta D. S., McSween H. Y., eds, Meteorites and the Early Solar System II. Univ. of Arizona Press, Tucson, p. 209
Clarke C. J., Pringle J. E., 1988, MNRAS, 235, 365
Connolly H. C., Desch S. J., Ash R. D., Jones R. H., 2006, in Lauretta D. S., McSween H. Y., eds, Meteorites and the Early Solar System II. Univ. of Arizona Press, Tucson, p. 383
Cossins P., Lodato G., Clarke C. J., 2009, MNRAS, 393, 1157
Davis A. M., MacPherson G. J., 1996, in Hewins R. H., Jones R. H., Scott E. R. D., eds, Chondrules and the Protoplanetary Disk. Cambridge Univ. Press, Cambridge, p. 71
Davis S. W., Stone J. M., Pessah M. E., 2009, preprint (arXiv:0909.1570)
Desch S. J., Connolly H. C., 2002, Meteorite Planet. Sci., 37, 183
Dong R., Stone J. M., 2009, ApJ, 704, 1309
Fromang S., Papaloizou J., 2007, A&A, 476, 1113
Fromang S., Papaloizou J., Lesur G., Heinemann T., 2007, A&A, 476, 1123
Grossman J. N., Rubin A. E., Nagahara H., King E. A., 1988, in Kerridge J. F., Matthews M. S., eds, Meteorites and the Early Solar System. Univ. of Arizona Press, Tucson, p. 619
Heitsch F., Zweibel E., Slyz A. D., Devriendt J. E. G., 2009, ApJ, in press
Hernández J., Calvet N., Hartmann L., Muzerolle J., Gutermuth R., Stauffer J., 2009, ApJ, 707, 705
Hewins R. H., 1988, in Kerridge J. F., Matthews M. S., eds, Meteorites and the Early Solar System. Univ. of Arizona Press, Tucson, p. 660
Hewins R. H., 1996, in Hewins R. H., Jones R. H., Scott E. R. D., eds, Chondrules and the Protoplanetary Disk. Cambridge Univ. Press, Cambridge, p. 1
Hutchinson R., 2004, Meteorites: a Petrologic, Chemical and Isotopic Synthesis. Cambridge Univ. Press, Cambridge
Ida S., Lin D. N. C., 2004, ApJ, 604, 388
King A. R., Pringle J. E., Livio M., 2007, MNRAS, 376, 1740
Lin D. N. C., Pringle J. E., 1990, ApJ, 538, 515
Lodato G., Rice W. K. M., 2004, MNRAS, 351, 630
Levy E. H., Araki S., 1989, Icarus, 81, 74
Levy E. H., Sonett C. F., 1978, in Gehrels T., ed., Protostars and Protoplanets. Univ. of Arizona Press, Tucson, p. 516
Matsumura S., Pudritz R. E., 2003, ApJ, 598, 645
Morbidelli A., Levison H. F., Tsiganis K., Gomes R., 2005, Nat, 435, 462
Morfill G., 1988, Icarus, 53, 41
Pringle J. E., 1981, ARA&A, 19, 137
Rice W. K. M., Lodato G., Armitage P. J., 2005, MNRAS, 364, 56
Russell S. S., Hartmann L., Cuzzi J., Krot A. N., Gounelle M., 2006, in Lauretta D. S., McSween H. Y., eds, Meteorites and the Early Solar System II. Univ. of Arizona Press, Tucson, p. 233
Schekochihin A. A., Cowley S. C., Taylor S. F., Moran J. L., McWilliams J. C., 2004, ApJ, 612, 276
Schekochihin A. A., Haugen N. E. L., Brandenburg A., Cowley S. C., Moran J. L., McWilliams J. C., 2005, ApJ, 625, L115
Scott E. R. D., 2007, Annu. Rev. Earth Planet. Sci., 35, 577
