Study on muon anomalous magnetic dipole moment in BLMSSM via the mass insertion approximation

Xi Wang$^{1,2}$*, Shu-Min Zhao$^{1,2}$†, Xin-Xin Long$^{1,2}$, Yi-Tong Wang$^{1,2}$, Tong-Tong Wang$^{1,2}$, Hai-Bin Zhang$^{1,2}$, Tai-Fu Feng$^{1,2,3}$

$^1$ Department of Physics, Hebei University, Baoding 071002, China
$^2$ Key Laboratory of High-precision Computation and Application of Quantum Field Theory of Hebei Province, Baoding 071002, China and
$^3$ Department of Physics, Chongqing University, Chongqing 401331, China

(Dated: November 23, 2022)

Abstract

There are 4.2σ deviations between the updated experimental results of muon anomalous magnetic dipole moment (MDM) and the corresponding theoretical prediction of the Standard Model (SM). We calculate the muon MDM in the framework of the MSSM extension with local gauged baryon and lepton numbers (BLMSSM). In this paper, we discuss how the muon MDM depends on the parameters in the BLMSSM in detail within the mass insertion approximation. Among the many parameters, tan β, $g_L$, $m_{λL}$ and $μ_H$ are more sensitive parameters. Considering the experimental limitations, our best numerical result of $a_{μ}^{BL}$ is around $2.5 \times 10^{-9}$, which can well compensate the departure between the experiment data and SM prediction.

PACS numbers:

Keywords: muon MDM, BLMSSM, mass insertion approximation

* wx_0806@163.com
† zhaosm@hbu.edu.cn
I. INTRODUCTION

In the development of the Standard Model (SM), the muon anomalous magnetic dipole moment (MDM) is an urgent problem to be solved, which indicates that there must be the new physics beyond SM. Therefore, the study of muon MDM has very important practical significance. The muon MDM denoted by \( a_\mu \equiv (g_\mu - 2)/2 \).

The SM contributions to muon MDM have the following parts: 1. the QED loop contributions [1–15]; 2. the electroweak contributions [14, 15]; 3. the hadronic vacuum polarization contributions [1, 4, 16]; 4. the hadronic light-by-light contributions [10–12]. The specific expressions are as follows:

\[
\begin{align*}
{a}_{\mu}^{\text{QED}} &= 116584718.931(104) \times 10^{-11}, \\
{a}_{\mu}^{\text{EW}} &= 153.6(1.0) \times 10^{-11}, \\
{a}_{\mu}^{\text{HVP}} &= 6845(40) \times 10^{-11}, \\
{a}_{\mu}^{\text{HLBL}} &= 92(18) \times 10^{-11}.
\end{align*}
\]

Based on the above, SM prediction of muon anomaly is \( a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11} (0.37 \text{ppm}) \) [1, 8, 17, 18]. New result on the muon MDM is reported by the E989 collaboration at Fermilab [19]: \( a_\mu^{\text{FNAL}} = 116592040(54) \times 10^{-11} (0.46 \text{ppm}) \) and 3.3 standard deviations larger than the SM prediction, which is in great agreement with BNL E821 result [2]. The new averaged experiment value of muon anomaly is \( a_\mu^{\text{exp}} = 116592061(41) \times 10^{-11} (0.35 \text{ppm}) \). Combining all available measurements, we now obtain 4.2\( \sigma \) deviations between the experiment and SM expectation (\( \Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251(59) \times 10^{-11} \)).

Studying muon MDM is very important for exploring new physics. There are some works for the supersymmetric (SUSY) one-loop contributions to muon MDM. The authors [20, 21] obtain the approximate SUSY one-loop contributions by simplification

\[
|a_\mu^{\text{SUSY}}| = 13 \times 10^{-10} \left( \frac{100 \text{GeV}}{M_{\text{SUSY}}} \right)^2 \tan \beta \text{sign}[\mu_H].
\]

Here, \( M_{\text{SUSY}} \) represents the masses of neutralinos, charginos and scalar leptons of the second generation equal. The SUSY contributions can be easily evaluated from Eq. (2). The authors [22] study the muon g-2 in several GUT-scale constrained SUSY models including CMSSM/mSUGRA, pMSSM, CMSSM/mSUGRA extensions and GMSB/AMSB extensions. The numerical result of muon g-2 is researched with the MultiNest technique for the
parameter space \[23, 25\] in the GNMSSM with a singlino-dominated neutralino as a dark matter candidate. The muon g-2 is further studied under \(Z_3\)-NMSSM with LHC analyses in Ref. \[26\]. They study to what extent the g-2 can be explained in anomaly mediation scenarios \[27\]. Even if there is no new particle in this energy range, one can measure the g-2 directly via the channel to a Higgs boson and a monochromatic photon \[28\]. Next, let’s briefly summarize our previous work on muon MDM. We study the corrections from loop diagrams to muon MDM with the mass eigenstate basis in the BLMSSM and B-LMSSM \[29, 31\]. With the effective Lagrangian method \[32-34\], we also research the contributions to muon MDM from loop diagrams under the \(U(1)_X\)-SSM and \(\mu\nu\)SSM.

In the extension of SM, the minimal supersymmetric extension of the standard model (MSSM) \[35\] is one of the most widely studied models. The authors propose the extension of the MSSM with local gauged B and L (BLMSSM) \[36, 37\]. It has two advantages: one is that the broken baryon number (B) can explain asymmetry of matter-antimatter in the universe, and the other is that neutrinos should have tiny mass from the neutrino oscillation experiment. In theory, the tiny mass of neutrino can be induced from the heavy majorana neutrinos by the seesaw mechanism. So, the lepton number (L) should also be broken at some scale.

In this paper, we investigate the BLMSSM contributions to the muon MDM via the mass insertion approximation (MIA). In the process of analysis, we simply give the results of calculating muon MDM by the mass eigenstate basis, and introduce the mass insertion approximation in detail. In comparison, the latter makes it easier and more intuitive to observe sensitive parameters. In the BLMSSM, the one-loop corrections are similar to the MSSM results in analytic form. The difference is that the BLMSSM contributions have the new gaugino \(\lambda_L\) and gauge coupling constant \(g_L\). Under the latest experimental constraints, our results can well compensate for the deviation.

The rest of the paper is organized as follows. In the next section, we briefly summarize the main ingredients of the BLMSSM. In Sec.III, we show analytic forms and degenerate results of the BLMSSM contributions to the muon MDM \((a_{\mu}^{BL})\). In Sec.IV, some numerical results are shown. The last section is devoted to summary.
II. THE BLMSSM

The local gauge group of BLMSSM is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$ \[38, 39\]. Compared with MSSM, BLMSSM includes exotic quarks ($\hat{Q}_4, \hat{U}_4, \hat{D}_4, \hat{Q}_5, \hat{U}_5, \hat{D}_5$) and exotic leptons ($\hat{L}_4, \hat{E}_4, \hat{N}_4^c, \hat{L}_5, \hat{E}_5, \hat{N}_5$), which are used to eliminate B and L anomaly, respectively. The exotic Higgs $\hat{\Phi}_B, \hat{\varphi}_B$ are used to break $B$ spontaneously with nonzero vacuum expectation values (VEVs), and the exotic Higgs $\hat{\Phi}_L, \hat{\varphi}_L$ are used to break $L$ spontaneously with nonzero VEVs. The superfields $\hat{X}$ and $\hat{X}'$ are used to make the exotic quarks unstable. The right-handed neutrinos $N_R^c$ are introduced to provide tiny masses of neutrinos through seesaw mechanism. The Table I specifically displays these additional fields.

In the BLMSSM, the superpotential is expressed as \[40\]

$$W_{BLMSSM} = W_{MSSM} + W_B + W_L + W_X,$$

(3)

where, $W_{MSSM}$ is the superpotential of the MSSM. The concrete forms of $W_B, W_L, W_X$ are

$$W_B = \lambda_4 \hat{Q}_4 \hat{Q}_5 \hat{\Phi}_B + \lambda_4 \hat{U}_4 \hat{U}_5 \hat{\varphi}_B + \lambda_4 \hat{D}_4 \hat{D}_5 \hat{\varphi}_B + \mu_B \hat{\Phi}_B \hat{\varphi}_B$$

$$+ Y_{u_4} \hat{Q}_4 \hat{H}_u \hat{U}_4 + Y_{d_4} \hat{Q}_4 \hat{H}_d \hat{D}_4 + Y_{u_5} \hat{Q}_5 \hat{H}_u \hat{U}_5 + Y_{d_5} \hat{Q}_5 \hat{H}_d \hat{D}_5,$$

$$W_L = Y_{e_4} \hat{L}_4 \hat{H}_u \hat{E}_4 + Y_{\nu_4} \hat{L}_4 \hat{H}_u \hat{\nu}_4 + Y_{e_5} \hat{L}_5 \hat{H}_d \hat{E}_5 + Y_{\nu_5} \hat{L}_5 \hat{H}_d \hat{\nu}_5$$

$$+ Y_{\nu_5} \hat{L}_5 \hat{H}_u \hat{\nu}_5 + \lambda_5 \hat{\nu}_e \hat{\nu}_5 \hat{\varphi}_L + \mu_L \hat{\Phi}_L \hat{\varphi}_L,$$

$$W_X = \lambda_1 \hat{Q}_5 \hat{\bar{X}} + \lambda_2 \hat{U}_5 \hat{\bar{X}}' + \lambda_3 \hat{D}_5 \hat{\bar{X}}' + \mu_X \hat{X} \hat{X}'.$$  

(4)

The local gauge symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$ can break down to the electromagnetic symmetry $U(1)_e$, when the $SU(2)_L$ doublets ($H_u, H_d$) and singlets ($\Phi_B, \varphi_B, \Phi_L, \varphi_L$) obtain nonzero VEVs $v_u, v_d$ and $v_B, \bar{v}_B, v_L, \bar{v}_L$ respectively. The $SU(2)_L$ doublets and singlets are shown as

$$H_u = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_u + H^0_u + iP^0_u) \end{pmatrix}, \quad H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + H^0_d + iP^0_d) \end{pmatrix},$$

$$\Phi_B = \frac{1}{\sqrt{2}}(v_B + \Phi^0_B + iP^0_B), \quad \varphi_B = \frac{1}{\sqrt{2}}(\bar{v}_B + \varphi^0_B + i\bar{P}^0_B),$$

$$\Phi_L = \frac{1}{\sqrt{2}}(v_L + \Phi^0_L + iP^0_L), \quad \varphi_L = \frac{1}{\sqrt{2}}(\bar{v}_L + \varphi^0_L + i\bar{P}^0_L).$$  

(5)

The soft breaking terms of BLMSSM are shown as follows \[37, 39\]

$$L_{soft} = L_{soft}^{MSSM} - (m^2_{\nu_e})_{IJ} \bar{N}_I \bar{N}_J - m^2_{Q_4} \bar{Q}_4 \bar{Q}_4 - m^2_{\tilde{U}_4} \bar{\tilde{U}}_4 \bar{\tilde{U}}_4 - m^2_{\tilde{D}_4} \bar{\tilde{D}}_4 \bar{\tilde{D}}_4.$$
TABLE I: The new fields in the BLMSSM.

| Superfields | SU(3)$_C$ | SU(2)$_L$ | U(1)$_Y$ | U(1)$_B$ | U(1)$_L$ |
|-------------|-----------|-----------|----------|----------|----------|
| $\tilde{Q}_4$ | 3 | 2 | $1/6$ | $B_4$ | 0 |
| $\tilde{U}^c_4$ | $\bar{3}$ | 1 | $-2/3$ | $-B_4$ | 0 |
| $\hat{D}^c_4$ | $\bar{3}$ | 1 | $1/3$ | $-B_4$ | 0 |
| $\hat{Q}^c_5$ | $\bar{3}$ | 2 | $-1/6$ | $-(1 + B_4)$ | 0 |
| $\hat{U}_5$ | 3 | 1 | $2/3$ | $1 + B_4$ | 0 |
| $\hat{D}_5$ | 3 | 1 | $-1/3$ | $1 + B_4$ | 0 |
| $\hat{L}_4$ | 1 | 2 | $-1/2$ | 0 | $L_4$ |
| $\hat{E}^c_4$ | 1 | 1 | 1 | 0 | $-L_4$ |
| $\hat{N}^c_4$ | 1 | 1 | 0 | 0 | $-L_4$ |
| $\hat{L}^c_5$ | 1 | 2 | $1/2$ | 0 | $-(3 + L_4)$ |
| $\hat{E}_5$ | 1 | 1 | -1 | 0 | $3 + L_4$ |
| $\hat{N}_5$ | 1 | 1 | 0 | 0 | $3 + L_4$ |
| $\hat{\Phi}_B$ | 1 | 1 | 0 | 1 | 0 |
| $\hat{\varphi}_B$ | 1 | 1 | 0 | -1 | 0 |
| $\hat{\Phi}_L$ | 1 | 1 | 0 | 0 | -2 |
| $\hat{\varphi}_L$ | 1 | 1 | 0 | 0 | 2 |
| $\hat{X}$ | 1 | 1 | 0 | $2/3 + B_4$ | 0 |
| $\hat{X}'$ | 1 | 1 | 0 | $-(2/3 + B_4)$ | 0 |
| $\hat{N}^c_R$ | 1 | 1 | 0 | 0 | -1 |

\[-m_{\tilde{Q}_5}^2 \tilde{Q}_5^c \tilde{Q}_5^c - m_{\tilde{U}_5}^2 \tilde{U}_5^c \tilde{U}_5^c - m_{\tilde{D}_5}^2 \tilde{D}_5^c \tilde{D}_5^c - m_{\tilde{L}_4}^2 \tilde{L}_4^c \tilde{L}_4^c - m_{\tilde{N}_4}^2 \tilde{N}_4^c \tilde{N}_4^c \]
\[-m_{\tilde{E}_4}^2 \tilde{E}_4^c \tilde{E}_4^c - m_{\tilde{L}_5}^2 \tilde{L}_5^c \tilde{L}_5^c - m_{\tilde{N}_5}^2 \tilde{N}_5^c \tilde{N}_5^c - m_{\tilde{E}_5}^2 \tilde{E}_5^c \tilde{E}_5^c - m_{\varphi_B}^2 \varphi_B^c \varphi_B^c \]
\[-m_{\varphi_L}^2 \varphi_L^c \varphi_L^c - (m_B \lambda_B \lambda_B + m_L \lambda_B \lambda_L + h.c.) \]
\[+ \left\{ A_{u4} Y_{u4} \tilde{Q}_4 H_u \tilde{U}_4^c + A_{d4} Y_{d4} \tilde{Q}_4 H_d \tilde{D}_4^c + A_{u5} Y_{u5} \tilde{Q}_5 H_d \tilde{U}_5 + A_{d5} Y_{d5} \tilde{Q}_5 H_u \tilde{D}_5 + A_{BQ} \lambda_Q \tilde{Q}_4^c \tilde{Q}_4^c \Phi_B + A_{BU} \lambda_B \tilde{U}_5 \tilde{U}_5 \varphi_B + A_{BD} \lambda_D \tilde{D}_4 \tilde{D}_4 \varphi_B + B_{B \mu B} \mu_B \Phi_B \varphi_B + h.c. \right\} \]
\[+ \left\{ A_{e1} Y_{e1} \tilde{L}_4 H_d \tilde{E}_4^c + A_{e4} Y_{e4} \tilde{L}_4 H_u \tilde{N}_4^c + A_{e5} Y_{e5} \tilde{L}_5 H_u \tilde{E}_5 + A_{e6} Y_{e6} \tilde{L}_5 H_d \tilde{N}_5 \right\} \]
\[ + A_N Y_b \bar{H}_u \tilde{N}^e_c + A_N c \tilde{N}^c \tilde{N}^c \varphi_L + B_L \mu_L \Phi_L \varphi_L + h.c. \right) \]
\[ + \left\{ A_1 \lambda_1 \tilde{Q}_5^c X + A_2 \lambda_2 \tilde{U}_5^c \tilde{U}_5^c X' + A_3 \lambda_3 \tilde{D}_5^c \tilde{D}_5^c X' + B_X \mu_X X X' + h.c. \right\}. \]  

(6)

The used mass matrices can be found in the works [41, 42]. The relevant Feynman rules of MSSM for the present computation are collected in Ref. [42]. The Feynman rules for vertices uniquely used in BLMSSM are as follows:

\[
\begin{align*}
\mu_i (\lambda_L) & \rightarrow \lambda_L(\mu_i) + \sqrt{2} g_L P_L \\
\bar{\mu}_i^R & \rightarrow \lambda_L(\mu_i) - \sqrt{2} g_L P_L 
\end{align*}
\]

FIG. 1: Feynman rules for the uniquely vertices in BLMSSM.

### III. THE MUON MDM

The muon MDM can be obtained from the following effective Lagrangian by using the on-shell condition for the external leptons,

\[ \mathcal{L}_{MDM} = \frac{e}{4 m_l} a_l \bar{l} \sigma^{\mu \nu} l \cdot \tilde{F}_{\mu \nu}, \]  

(7)

with \( \sigma^{\mu \nu} = i [\gamma_{\mu}, \gamma_{\nu}] / 2 \). \( e \) and \( l \) denote the electric charge and the lepton fermion, respectively. \( \tilde{F}_{\mu \nu} \) is the electromagnetic field strength, and \( m_l \) is the lepton mass.

The Feynman amplitude can be expressed by these dimension 6 operators with the effective Lagrangian method for the process \( l^I \rightarrow l^I + \gamma \). The dimension 8 operators are suppressed by additional factor \( \frac{m^2_{\text{SU,SY}}}{M_{\text{SU,SY}}} \sim (10^{-7}, 10^{-8}) \), which are neglected safely. Therefore, these dimension 6 operators is enough to use in future calculations. The operators related to muon MDM are \( O_{L,R}^{1,2,3}, 4, 5, 6 \), which is the combination of the Wilson coefficients \( C_{2,3,6}^{L,R} \). Here, \( D_\mu = \partial_\mu + ie A_\mu \) and \( P_{L,R} = \frac{i\epsilon_\mu}{2} \). The specific forms of those dimension 6 operators are

\[
\begin{align*}
O^{L,R}_1 &= \frac{1}{(4\pi)^2} i (iD)^3 P_{L,R}, \\
O^{L,R}_2 &= \frac{eQ_f}{(4\pi)^2} (iD_{\mu l}) \gamma^\mu F \cdot \sigma P_{L,R}, \\
O^{L,R}_3 &= \frac{eQ_f}{(4\pi)^2} i F \cdot \sigma \gamma^\mu P_{L,R} (iD_{\mu l}), \\
O^{L,R}_4 &= \frac{eQ_f}{(4\pi)^2} i (\partial_{\mu} F_{\mu\nu}) \gamma^\nu P_{L,R}, \\
O^{L,R}_5 &= \frac{m_l}{(4\pi)^2} i (iD)^2 P_{L,R}, \\
O^{L,R}_6 &= \frac{eQ_f m_l}{(4\pi)^2} i F \cdot \sigma P_{L,R}. \end{align*} 
\]  

(8)
A. The mass eigenstate basis

![Diagram](image)

FIG. 2: The two one-loop diagrams, written in terms of the mass eigenstate basis for $l^I \rightarrow l^I + \gamma$.

The external photon line has to be attached to the charged internal lines.

The analytical form of one-loop corrections obtained by BLMSSM is similar to that of MSSM. The differences are: 1. the squared mass matrixes of scalar leptons because of new parameters $g_L, \bar{v}_L, v_L$ and so on. 2. right-handed neutrinos and scalar neutrinos are introduced, which leads to the neutrinos and scalar neutrinos are doubled. In the BLMSSM, there are four parts which contribute to muon MDM: 1. scalar charged muon ($\tilde{\mu}$) and neutralino ($\chi^0$) [Fig. 2(a)]; 2. scalar neutrino ($\tilde{\nu}$) and chargino ($\chi^\pm$) [Fig. 2(b)]; 3. neutral Higgs ($H^0$) and muon ($\mu$); 4. charged Higgs ($H^\pm$) and neutrino ($\nu$).

The one-loop Higgs contributions to muon MDM are very small, because it is inhibited by the factor $\frac{m_\mu^2}{m_W^2}$. The mass matrix of neutrinos and the square mass matrix of scalar neutrinos are extended to $6 \times 6$, so that the right-handed neutrino contributions are very small ($10^{-15} \sim 10^{-13}$). From these analysis, the contributions of type 3 and 4 are entirely negligible. Due to the mass of the new vector boson $Z_L$ being greater than 5.1 TeV, the one-loop contributions from $Z_L$-muon are suppressed by the factor $\frac{m_Z^2}{m_{Z_L}^2} \sim 4 \times 10^{-4}$. So, we neglect $Z_L$-muon one-loop contributions.

Therefore, the one-loop new physics contributions to muon MDM are given entirely by the Fig. 2. On the basis of the one-loop self-energy diagrams, we can get the one-loop triangle diagrams by attaching a photon on the internal line in all possible ways. These diagrams have been comprehensively discussed in the BLMSSM with the mass eigenstate basis [29], and the exact results have been derived. We show the general results in the form:

$$a_{\mu}^{BL} = a_{\mu}^{\tilde{\mu}\chi^0} + a_{\mu}^{\tilde{\nu}\chi^\pm},$$

(9)
with

$$a_{\mu}^\chi = - \frac{e^2}{2s_W^2} \sum_{i=1}^{3} \sum_{j=1}^{2} \left[ \text{Re}\left[ (S_1)_{ij}^f (S_2)_{ij}^f \right] \sqrt{x_{ij}^0 x_{ji}^0} \frac{\partial^2 B(x_{ij}^0, x_{ji}^0)}{\partial x_{ij}^0} \right] + \frac{1}{3} \left[ |(S_1)_{ij}^f|^2 + |(S_2)_{ij}^f|^2 \right] x_{ij}^0 x_{ji}^0 \frac{\partial B_1(x_{ij}^0, x_{ji}^0)}{\partial x_{ij}^0},$$

$$a_{\mu}^\nu = \frac{e^2}{2s_W^2} \sum_{j=1}^{2} \sum_{j=1}^{2} \left[ \sqrt{2} m_j \text{Re}\left[ Z_{ij}^1 Z_{j\nu}^2 \right] Z_{ij}^1 Z_{j\nu}^2 \right] + \frac{1}{3} \left[ |Z_{ij}^1 Z_{j\nu}^1|^2 + \frac{m_j^2}{2m_W^2} |Z_{ij}^1 Z_{j\nu}^1|^2 \right] x_{ij}^\nu x_{ji}^\nu \frac{\partial B_1(x_{ij}^\nu, x_{ji}^\nu)}{\partial x_{ij}^\nu}. \quad (10)$$

We define the functions $B(x, y)$, $B_1(x, y)$

$$B(x, y) = \frac{1}{16\pi^2} \frac{(x \ln x + y \ln y)}{y - x}, \quad B_1(x, y) = \left( \frac{\partial}{\partial y} + \frac{y}{2} \frac{\partial^2}{\partial y^2} \right) B(x, y). \quad (12)$$

The couplings $(S_1)_{ij}^f$, $(S_2)_{ij}^f$ are shown as

$$(S_1)_{ij}^f = \frac{1}{c_W} Z_{ij}^{1*} Z_{N}^{2j} Z_{L}^{(1+3)i} Z_{N}^{2j} - \frac{m_j^2}{m_W^2} Z_{L}^{(1+3)i} Z_{N}^{2j},$$

$$(S_2)_{ij}^f = -2 \frac{s_W}{c_W} Z_{L}^{(1+3)i} Z_{N}^{1j} Z_{N}^{2j} - \frac{m_j^2}{m_W^2} Z_{L}^{(1+3)i} Z_{N}^{2j}. \quad (13)$$

The matrices $Z_L$, $Z_N$ diagonalize the mass matrices of scalar lepton and neutralino, respectively. $Z_-$, $Z_+$ are used to diagonalize the chargino mass matrix. The mass squared matrix of scalar neutrino are diagonalized by $Z_{\tilde{e}_{i,j}}$. 

**B. The mass insertion approximation**

Through the above discussion of BLMSSM contributions to muon MDM, we can know that the contributions do not represent an enhancement proportional to $\frac{m}{m_W^2}$, because it is suppressed by the combined rotation matrixes. In fact, they produce an overall enhancement factor $\tan \beta$ [44, 45]. In other words, $|a_{\mu}^{\text{BL}}|$ becomes large as $\tan \beta$ increases. Thus, it’s more convenient to use the mass insertion approximation (MIA) [44, 46-48] to calculate, and the role of parameters can be more clearly displayed. However, the mass eigenstate basis in the previous section is more appropriate for an exact evaluation. Now, we obtain the specific forms of the one-loop contributions by using MIA in the BLMSSM.
a. The one-loop contributions from $\bar{B}-\tilde{\mu}_L-\tilde{\mu}_R$.

\[
a^{BL,(a)}_\mu = 2g_1^2 x_\mu x_{1\mu} \tan \beta [I_1(x_1, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R}) + I_2(x_1, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R}) - J_1(x_1, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R}) - J_2(x_1, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R}) - J_3(x_1, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R})].
\]

with $m_1 = m_B$, $x_i = \frac{m_i^2}{M^2_{SUSY}}$. $m_i$ is the particle mass. The functions $I_1(x, y, z)$, $I_2(x, y, z)$, $J_1(x, y, z)$, $J_2(x, y, z)$ and $J_3(x, y, z)$ are defined as

\[
I_1(x, y, z) = \frac{1}{16\pi^2} \left\{ \frac{(z^2 - xy) \log z}{(x - z)^2(y - z)^2} - \frac{1}{x \log x} \right\};
\]

\[
I_2(x, y, z) = \frac{1}{16\pi^2} \left\{ \frac{1}{(x - y)(y - z)^2} - \frac{x \log x}{(x - y)(x - z)(y - z)^2} \right\};
\]

\[
J_1(x, y, z) = \frac{1}{32\pi^2} \left\{ \frac{x(z - 3y) + z(y + z)}{(x - y)(y - z)^3} - \frac{2x^2 \log x}{(x - y)^3(x - z)^3} \right\};
\]

\[
J_2(x, y, z) = \frac{1}{32\pi^2} \left\{ \frac{x(y - 3z) + y(y + z)}{(x - y)^3(x - z)} - \frac{2x^2 \log x}{(x - y)^3(x - z)^3} \right\};
\]

\[
J_3(x, y, z) = \frac{1}{16\pi^2} \left\{ \frac{x(y + z) - 2y z}{(x - y)(x - z)(y - z)^2} - \frac{x^2 \log x}{(x - y)^2(x - z)^2} \right\}.
\]
b. The one-loop contributions from $\tilde{B} - \tilde{H}^0 - \tilde{\mu}_R$.

$$a_{\mu}^{BL,(b)} = -2g_2^2 x \mu \sqrt{x_1 x_{\mu H}} \tan \beta [J_1(x_1, x_{\mu H}, x_{\tilde{\mu}_R}) - J_1(x_1, x_{\mu H}, x_{\tilde{\mu}_R})].$$  (20)

c. The one-loop contributions from $\tilde{B} - \tilde{H}^0 - \tilde{\mu}_L$.

$$a_{\mu}^{BL,(c)} = g_1^2 x \mu \sqrt{x_1 x_{\mu H}} \tan \beta [J_1(x_1, x_{\mu H}, x_{\tilde{\mu}_L}) - J_1(x_1, x_{\mu H}, x_{\tilde{\mu}_L})].$$  (21)

d. The one-loop contributions from $\tilde{W}^0 - \tilde{H}^0 - \tilde{\nu}_L$.

$$a_{\mu}^{BL,(d)} = -g_2^2 x \mu \sqrt{x_2 x_{\mu H}} \tan \beta [J_1(x_2, x_{\mu H}, x_{\tilde{\nu}_L}) - J_1(x_2, x_{\mu H}, x_{\tilde{\nu}_L})].$$  (22)

here, $m_2 = m_{\tilde{W}^0} = m_{\tilde{\nu}^\pm}$.

e. The one-loop contributions from $\lambda_L - \tilde{\mu}_L - \tilde{\mu}_R$.

$$a_{\mu}^{BL,(e)} = -4g_2^2 x \mu \sqrt{x_{\lambda L} x_{\mu H}} \tan \beta [I_1(x_{\lambda L}, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R}) + I_2(x_{\lambda L}, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R}) - J_1(x_{\lambda L}, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R}) - J_2(x_{\lambda L}, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R}) - J_3(x_{\lambda L}, x_{\tilde{\mu}_L}, x_{\tilde{\mu}_R})].$$  (23)

f. The one-loop contributions from $\tilde{W}^\pm - \tilde{H}^\pm - \tilde{\nu}_L$.

$$a_{\mu}^{BL,(f)} = 2g_2^2 x \mu \sqrt{x_2 x_{\mu H}} \tan \beta [J_2(x_2, x_{\mu H}, x_{\tilde{\nu}_L}) + J_4(x_2, x_{\mu H}, x_{\tilde{\nu}_L}) + J_5(x_2, x_{\mu H}, x_{\tilde{\nu}_L})].$$  (24)

We define the functions $J_4(x, y, z)$ and $J_5(x, y, z)$ as

$$J_4(x, y, z) = \frac{1}{16\pi^2} \left\{ \frac{z(x+y) - 2xy}{(x-y)^2(x-z)(y-z)} + \frac{x[x(x+y) - 2yz] \log x}{(x-y)^3(x-z)^2} \right\},$$  (25)

$$J_5(x, y, z) = \frac{1}{32\pi^2} \left\{ \frac{x^2 + x(y+z) - 3yz}{(x-y)^2(x-z)^2} - \frac{2[x^3(y+z) - 3x^2yz + y^2z^2] \log x}{(x-y)^3(x-z)^3} \right\} + \frac{2y^2 \log y}{(x-y)^3(y-z)} + \frac{2z^2 \log z}{(x-z)^3(z-y)}.$$  (26)

The one-loop contributions to muon MDM can be expressed as

$$a_{\mu}^{\tilde{\nu}_L} \simeq a_{\mu}^{BL,(a)} + a_{\mu}^{BL,(b)} + a_{\mu}^{BL,(c)} + a_{\mu}^{BL,(d)} + a_{\mu}^{BL,(e)}.$$  (27)

$$a_{\mu}^{\tilde{\nu}_L^\pm} \simeq a_{\mu}^{BL,(f)}.$$  (28)

We ought to notice that the contributions to muon MDM are related to $\tan \beta$ and $x_i = \frac{m_i^2}{M_{SUSY}^2}$ in the Eqs. (14), (20)–(24). This situation is consistent with MSSM. The contributions
relating with the new gaugino $\lambda_L$ are shown in Eq. (23), which include the new gauge coupling constant $g_L$. Furthermore, we obtain the conclusion that $a^{BL(a)}$, $a^{BL(e)}$ and $a^{BL(f)}$ occupy the dominant position after numerical comparison. When $m_{\lambda L}$ is negative, the sign of $a^{BL(a)}$, $a^{BL(e)}$ and $a^{BL(f)}$ are the same. We can get the reasonable corrections of new physics.

C. Degenerate result

Next, we assume that all the masses of the superparticles are almost degenerate to more clearly know the influence factor on $a^{BL}_{\mu}$. The masses of superparticles $(m_1, m_2, \mu_H, m_{\tilde{\nu}_L}, m_{\tilde{\nu}_R}, m_{\tilde{\mu}_L}, m_{\lambda_L})$ are equal to $M_{SUSY}$

$$|m_1| = |m_2| = |\mu_H| = m_{\tilde{\nu}_L} = m_{\tilde{\nu}_R} = m_{\tilde{\mu}_L} = |m_{\lambda_L}| = M_{SUSY}. \quad (29)$$

The functions can be simplified as

$$I_1(1,1,1) = I_2(1,1,1) = \frac{1}{96\pi^2},$$
$$J_1(1,1,1) = J_2(1,1,1) = J_3(1,1,1) = J_4(1,1,1) = J_5(1,1,1) = \frac{1}{192\pi^2}. \quad (30)$$

The one-loop MSSM results (chargino-sneutrino, neutralino-smuon) in this case are consistent with the results of Ref.[44]

$$a^{MSSM}_{\mu} \simeq \frac{1}{192\pi^2} \frac{m_{\mu}^2}{M_{SUSY}^2} (g_1^2 + 5g_2^2) \tan \beta. \quad (31)$$

In BLMSSM, the one-loop results of muon MDM are given by

$$a^{BL}_{\mu} \simeq \frac{1}{192\pi^2} \frac{m_{\mu}^2}{M_{SUSY}^2} (g_1^2 + 5g_2^2) \tan \beta$$
$$- \frac{1}{48\pi^2} \frac{m_{\mu}^2}{M_{SUSY}^2} g_L^2 \tan \beta \text{sign}[\mu_H m_{\lambda L}]. \quad (32)$$

The corrections beyond MSSM can reach large value, when $\text{sign}[m_1] = \text{sign}[m_2] = \text{sign}[\mu_H] = 1$ and $\text{sign}[m_{\lambda L}] = -1$.

$$a^{BL}_{\mu} \rightarrow \frac{1}{192\pi^2} \frac{m_{\mu}^2}{M_{SUSY}^2} (g_1^2 + 5g_2^2 + 4g_L^2) \tan \beta. \quad (33)$$

According to the above expressions, we study the effect of $M_{SUSY}$, $\tan \beta$ and $g_L$ on the BLMSSM contributions to muon MDM. The results are shown in Fig. 4 First, we plot the
results for $\tan \beta = 50$ in the $g_L$-$M_{SUSY}$ plane. As we can see, if we take a smaller value of $M_{SUSY}$, the $a^{BL}_\mu$ is enhanced in the large $g_L$ region. Next, the upper right figure denotes $\tan \beta$-$M_{SUSY}$ plane when $g_L = 0.45$. The results imply that large $\tan \beta$ and small $M_{SUSY}$ can produce suitable the BLMSSM corrections to compensate the departure. At last, the bottom figure shows the results in the plane of $\tan \beta$ versus $g_L$. When the values of $\tan \beta$ and $g_L$ enlarge, the value of $a^{BL}_\mu$ also increases, but $\tan \beta$ is more sensitive than $g_L$. It shows that $M_{SUSY}$, $\tan \beta$ and $g_L$ are sensitive, and have a direct effect on $a^{BL}_\mu$.

FIG. 4: The effect of $M_{SUSY}$, $\tan \beta$ and $g_L$ on $a^{BL}_\mu$. The upper left figure denotes $g_L$-$M_{SUSY}$ plane with $\tan \beta = 50$. The upper right figure denotes $\tan \beta$-$M_{SUSY}$ plane with $g_L = 0.45$. The bottom figure denotes $\tan \beta$-$g_L$ plane with $M_{SUSY} = 1000$ GeV.
IV. NUMERICAL RESULTS

In this section, we numerically calculate the BLMSSM contributions to muon MDM ($a_{\mu}^{BL}$). Based on the above analysis of the mass insertion approximation, $a_{\mu}^{BL}$ mainly depends on 9 parameters, i.e., $\tan \beta$, $g_L$, $m_1$, $m_2$, $m_{\lambda L}$, $\mu_H$, $m_{\tilde{\nu} L}$, $m_{\tilde{\mu} R}$, $m_{\tilde{\mu} L}$. We take these 9 parameters as free parameters and calculate the BLMSSM contributions to the muon MDM for a given set of parameters, and fix the parameter $M_{SUSY} = 1000$ GeV. Meanwhile, we consider the latest experimental limitations of particles [48–52]. The lightest CP-even Higgs mass $m_{h^0}=125.1$ GeV [53, 54]. The slepton mass is greater than 700 GeV, and the chargino mass is greater than 1100 GeV [46]. Taking $Z_L$ boson mass is greater than 5.1 TeV to satisfy the mass constraint of the $Z_L$ boson from LHC experiments [55].

A. One-dimensional graphs

In this subsection, we take $m_1 = 300$ GeV, $m_{\lambda L} = -300$ GeV, $m_{\tilde{\nu} L} = 150$ GeV, $m_{\tilde{\mu} L} = 700$ GeV, $m_{\tilde{\mu} R} = 700$ GeV, and plot the following $a_{\mu}^{BL}$ schematic diagram affected by different parameters. The experimental limitations are denoted by the colored areas, where light green area represents $1\sigma$, light orange area represents $2\sigma$.

In Fig. 5, we plot the results for $m_2 = 1100$ GeV and $\mu_H = 1100$ GeV versus $g_L$. $g_L$ is the coupling constant of $U(1)_L$ gauge, and it is the parameter beyond MSSM. From the analysis by MIA, $g_L$ is an important parameter that appears in Eq. (23). It can be seen that from bottom to top are solid line ($\tan \beta = 30$), dashed line ($\tan \beta = 40$) and dotted line ($\tan \beta = 50$), and the overall trend of the three lines is upward. This conclusion can be seen more intuitively from Eq. (33). The dotted line part is entirely all in the colored areas, the dotted lines part of $0.22 - 0.6$ in the colored areas, the solid line part of $0.4 - 0.6$ in the colored areas. That is to say, $\tan \beta$ is a sensitive parameter and larger $\tan \beta$ leads to larger $a_{\mu}^{BL}$. The value of $a_{\mu}^{BL}$ is around $2.5 \times 10^{-9}$, and it can better meet the experimental limitations.

Similarly, we take $m_2 = 1100$ GeV and $\mu_H = 1100$ GeV, and plot the BLMSSM contributions to muon MDM varying with $\tan \beta$ in Fig. 6. The parameter $\tan \beta$ is ratio of the VEVs of the two Higgs doublets ($\tan \beta = v_u/v_d$). It is included in each one-loop contribution and is proportional. The solid (dashed, dotted) line corresponds to the results with
For better numerical results, we set $\tan\beta = 50$ and $\mu_H = 1100$ GeV. The solid line ($g_L = 0.25$), dashed line ($g_L = 0.45$) and dotted line ($g_L = 0.55$) varying with $m_2$ are shown in Fig. 7. $m_2$ represents the particle mass of $\tilde{W}^0$ or $\tilde{W}^\pm$ ($m_2 = m_{\tilde{W}^0} = m_{\tilde{W}^\pm}$), which directly affects the one-loop contributions from Figs. 3(d), 3(f). The three lines are all decreasing.
functions, when $m_2$ turns large from 1100 GeV to 3000 GeV. The downward trend slowly becomes weak. The reason is that the contributions are proportional to $\sqrt{x_2} = \frac{m_2}{M_{\text{SUSY}}}$, but the effect of $m_2$ on the function is considerable and inversely proportional in Eqs. (22), (24) obtained by MIA. On the whole, the increase of $m_2$ leads to the slow decrease of $a^{\mu}_{BL}$. The dotted and dashed lines are all located in the colored areas. The dotted line can reach $2.9 \times 10^{-9}$, the dashed line can reach $2.4 \times 10^{-9}$ and the solid line can reach $1.7 \times 10^{-9}$.

In addition, supposing the parameters with $g_L = 0.45$ and $m_2 = 1100$ GeV, we study the parameter $\mu_H$ influences on muon MDM in Fig. 8. $\mu_H$ is SUSY invariant Higgs mass, which exists in each contribution of Fig. 3. The solid line, dashed line and dotted line respectively correspond to the results with $\tan \beta = 30$, $\tan \beta = 40$ and $\tan \beta = 50$. The three lines show a slow upward trend. Among the dominant terms, $a^{BL,(a)}_{\mu}$, $a^{BL,(e)}_{\mu}$, and $a^{BL,(f)}_{\mu}$ are proportional to $\sqrt{x_{\mu_H}} = \frac{m_{\mu_H}}{M_{\text{SUSY}}}$, but the function parts are inversely proportional to $\mu_H$ and have a relatively small effect. After synthesis, the effect of $\mu_H$ on muon MDM shows a slowly increasing relationship, when $\mu_H$ increases from 1000 GeV to 2500 GeV. The three curves all are in the colored areas, which means that $a^{BL}_{\mu}$ satisfies the experimental limitations under our assumption. The dotted line is at the top, that is, large $\tan \beta$ value results in larger $a^{BL}_{\mu}$. We can find that the three lines can all reach more than $2.0 \times 10^{-9}$.

FIG. 7: The BLMSSM contributions to muon MDM ($a^{BL}_{\mu}$) versus $m_2$. 

FIG. 8: The BLMSSM contributions to muon MDM ($a^{BL}_{\mu}$) versus $\mu_H$. 

\[ a^{BL}_{\mu} = a^{BL,(a)}_{\mu} + a^{BL,(e)}_{\mu} + a^{BL,(f)}_{\mu} \]
1000
1200
1400
- . / 6
7 8 9 :
2000
2200
2400
; < × 10
× 10
× 10
× 10
μ
(GeV)
H
a
BL
\tan \beta = 30
\tan \beta = 40
\tan \beta = 50
FIG. 8: The BLMSSM contributions to muon MDM ($a_{\mu}^{BL}$) versus $\mu_H$.

**B. Multidimensional scatter plots graphs**

In this subsection, we carry out numerical analysis by scanning free parameters and explore the region to explain the BLMSSM contributions to muon MDM. The random ranges of input parameters are as follows:

$$\tan \beta \supset [1, 50], \quad g_L \supset [0.2, 0.8], \quad m_1 \supset [100, 3000] \text{ GeV},$$

$$m_2 \supset [1000, 3000] \text{ GeV}, \quad \mu_H \supset [1000, 3000] \text{ GeV}, \quad m_{\tilde{\nu}_L} \supset [100, 3000] \text{ GeV},$$

$$m_{\tilde{\mu}_L} \supset [700, 3000] \text{ GeV}, \quad m_{\tilde{\mu}_R} \supset [700, 3000] \text{ GeV}, \quad |m_{\lambda_L}| \supset [200, 5000] \text{ GeV}. \quad (34)$$

Table II shows the results represented by shapes in Figs. 9–11.

**TABLE II: The meaning of shape style**

| Shape style | Figs. 9[11] | Fig. 11 |
|-------------|-------------|---------|
| ♦           | $0 < a_{\mu}^{BL} < 10^{-9}$ | $0 < a_{\mu}^{BL} < 1.5 \times 10^{-9}$ |
| ▲           | $10^{-9} \leq a_{\mu}^{BL} < 1.5 \times 10^{-9}$ | $1.5 \times 10^{-9} \leq a_{\mu}^{BL} < 2.0 \times 10^{-9}$ |
| ■           | $1.5 \times 10^{-9} \leq a_{\mu}^{BL} < 2.0 \times 10^{-9}$ | $2.0 \times 10^{-9} \leq a_{\mu}^{BL} < 3.0 \times 10^{-9}$ |
| •           | $2.0 \times 10^{-9} \leq a_{\mu}^{BL} < 3.0 \times 10^{-9}$ | \(

To better display sensitive parameters, we show the $a_{\mu}^{BL}$ in the $\tan \beta - g_L$ plane (a) and $\tan \beta - m_{\tilde{\mu}_L}$ plane (b) in Fig. 9. The bounds between ♦, ▲, ■ and • are very obvious in Fig. 9(a). The blue part is displayed in a trapezoid and takes up a lot of space. The results represented by the remaining three colors show a slight radian. The red part is on the upper
right, that is, large $\tan \beta$ and large $g_L$ can bring greater contributions. Similarly, ♦ also occupy a large number of positions in Fig. 9(b) and mainly in the wide area $1 < \tan \beta < 40$ and $700$ GeV $< m_{\tilde{\mu}_L} < 5000$ GeV. ● concentrate in the narrow area $\tan \beta$ $(22, 50)$ and $m_{\tilde{\mu}_L}$ $(700, 1600)$ GeV. $m_{\tilde{\mu}_L}$ is left-handed smuon mass. The function parts of $a_{\mu}^{BL,(a)}$, $a_{\mu}^{BL,(c)}$, $a_{\mu}^{BL,(d)}$, and $a_{\mu}^{BL,(e)}$ are inversely proportional to $m_{\tilde{\mu}_L}$. Therefore, this means that light scalar muon improves the BLMSSM contributions to muon MDM.

We plot $a_{\mu}^{BL}$ in the plane of $m_{\tilde{\nu}_L}$ versus $|m_{\lambda_L}|$ by the left diagram in the Fig. 10 and the right diagram shows the relation between $a_{\mu}^{BL}$, $m_{\tilde{\nu}_L}$ and $g_L$. One can find that the styles of Fig. 10(a) and Fig. 9(b) are similar. In Fig. 10(a), the blue area is the most. In the range $800$ GeV $< |m_{\lambda_L}| < 1200$ GeV, ▲ occupy much space. ■ concentrate in the narrow area $|m_{\lambda_L}| < 800$ GeV and $1500$ GeV $< m_{\tilde{\nu}_L} < 3000$ GeV. ● denoting large contributions to $a_{\mu}^{BL}$ concentrate in the area $|m_{\lambda_L}| < 1000$ GeV and $100$ GeV $< m_{\tilde{\nu}_L} < 1500$ GeV. $m_{\lambda_L}$
expresses the mass of new gaugino $\lambda_L$ beyond MSSM. We take $m_{\lambda L}$ as a negative value in the Eq. (23) and can easily find that this contribution is proportional to $\sqrt{|\lambda_L|} = \frac{m_{\lambda L}}{M_{SUSY}}$. For more convenience, we take $|m_{\lambda L}|$ as the ordinate. These indicate that small $|m_{\lambda L}|$ and small $m_{\tilde{\nu}_L}$ can lead to large corrections. In Fig. 10(b), the layers are distinct, with $\blacklozenge$, $\blacksquare$ and $\blacklozenge$ are arched. In the case of $m_{\tilde{\nu}_L} = 3000$ GeV, we can find these laws. When $g_L < 0.38$, the space is filled with $\blacklozenge$. The red, green and blue parts correspond to $0.38 < g_L < 0.5$, $0.5 < g_L < 0.6$ and $0.6 < g_L < 0.74$, respectively. $m_{\tilde{\nu}_L}$ denotes the mass of the left-handed neutrino, which causes a change in $a_\mu^{BL,(f)}$ by directly affecting the function part of Eq. (24). The final effect is that $a_\mu^{BL}$ is inversely proportional to $m_{\tilde{\nu}_L}$. In the whole, small $m_{\tilde{\nu}_L}$ and large $g_L$ can obviously improve the corrections to $a_\mu^{BL}$.

Furthermore, we show the relationship between $g_L$ and $|m_{\lambda L}|$, $m_{\tilde{\nu}_R}$ and $|m_{\lambda L}|$ in Fig. 11. It is worth noting that the meaning of $\blacklozenge$, $\blacktriangle$ and $\blacklozenge$ in Fig. 11 is inconsistent with that in Figs. 9,10. The specific meanings are shown in Table 11. The left figure and the right figure have obvious stratification and strong regularity. In Fig. 11(a), the diagonal of the space divides the results into two parts. $\blacklozenge$ concentrate in the upper left of the diagonal and $\blacktriangle$ and $\blacklozenge$ mainly distribute at the bottom right of the diagonal. In Fig. 11(b), the whole space is covered. $\blacksquare$ concentrate in the narrow area $m_{\tilde{\nu}_R}$ (700, 1000) GeV and $|m_{\lambda L}|$ (200, 1000) GeV. $\blacktriangle$ occupy much space in the range 700 GeV < $|m_{\lambda L}|$ < 2000 GeV and 1000 GeV < $m_{\tilde{\nu}_R}$ < 1800 GeV. The blue part occupies all the remaining positions. $m_{\tilde{\nu}_R}$ is right-handed smuon mass. Only the function parts of $a_\mu^{BL,(a)}$, $a_\mu^{BL,(b)}$ and $a_\mu^{BL,(c)}$ contain $m_{\tilde{\nu}_R}$ and are inversely proportional to $m_{\tilde{\nu}_R}$. The results imply that large $m_{\tilde{\nu}_R}$ and large $|m_{\lambda L}|$ suppress the BLMSSM contributions to muon MDM. Based on the above description,
we can be more clear about the contribution of the above parameters.

V. DISCUSSION AND CONCLUSION

In the framework of the BLMSSM, we study the one-loop contributions to the muon MDM. During the analysis, the mass insertion approximation is used to more clearly display sensitive parameters. All parameters used can satisfy the latest experimental data. As we mentioned before, there are dominant three parts on which $a_\mu^{BL}$ depends, i.e., $a_\mu^{BL,(a)}$, $a_\mu^{BL,(e)}$ and $a_\mu^{BL,(f)}$. We take $\tan \beta$, $g_L$, $m_1$, $m_2$, $m_{\lambda_L}$, $\mu_H$, $m_{\tilde{\nu}_L}$, $m_{\tilde{\mu}_R}$ and $m_{\tilde{\mu}_L}$ as free parameters. Among them, $\tan \beta$, $g_L$, $m_{\lambda_L}$ and $\mu_H$ are more sensitive parameters. $a_\mu^{BL}$ is an increasing function of $\tan \beta$, $g_L$, $\mu_H$ and decreasing function of $m_2$. Small $|m_{\lambda_L}|$ and small $m_{\tilde{\nu}}$ can improve the BLMSSM contributions to muon MDM. In our used parameter space, the contributions to muon MDM can easily reach its upper bound and even exceed it. Our best numerical result of $a_\mu^{BL}$ is around $2.5 \times 10^{-9}$, which can well compensate the departure between the experiment data and SM prediction.

Acknowledgments

This work is supported by National Natural Science Foundation of China (NNSFC) (No. 12075074), Natural Science Foundation of Hebei Province (A2020201002, A202201022, A202201017), Natural Science Foundation of Hebei Education Department (QN2022173), Post-graduate’s Innovation Fund Project of Hebei University (HBU2022ss028, HBU2023SS043), the youth top-notch talent support program of the Hebei Province.

[1] T. Aoyama, N. Asmussen, M. Benayoun, et al., Phys. Rep. 887 (2020) 1.
[2] G.W. Bennett, et al., Phys. Rev. D 73 (2006) 072003.
[3] A. Keshavarzi, D. Nomura, T. Teubner, Phys. Rev. D 97 (2018) 114025.
[4] G. Colangelo, M. Hoferichter, P. Stoffer, J. High Energy Phys. 02 (2019) 006.
[5] M. Hoferichter, B.L. Hoid, B. Kubis, J. High Energy Phys. 08 (2019) 137.
[6] M. Davier, A. Hoecker, B. Malaescu, et al., Eur. Phys. J. C. 80 (2020) 241.
[7] A. Keshavarzi, D. Nomura, T. Teubner, Phys. Rev. D 101 (2020) 014029.
[8] T. Blum, P.A. Boyle, V. Gulpers, et al., Phys. Rev. Lett. 121 (2018) 022003.
[9] T. Aoyama, M. Hayakawa, T. Kinoshita, et al., Phys. Rev. Lett. 109 (2012) 111808.
[10] G. Colangelo, F. Hagelstein, M. Hoferichter, et al., J. High Energy Phys. 03 (2020) 101.
[11] G. Eichmann, C.S. Fischer, R. Williams, Phys. Rev. D 101 (2020) 054015.
[12] T. Blum, N. Christ, M. Hayakawa, et al., Phys. Rev. Lett. 124 (2020) 132002.
[13] T. Aoyama, T. Kinoshita, M. Nio, Atoms 7 (2019) 28.
[14] A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67 (2003) 073006.
[15] C. Gnendiger, D. Stockinger, H.S. Kim, Phys. Rev. D 88 (2013) 053005.
[16] M.T. Hansen, A. Patella, J. High Energy Phys. 10 (2020) 029.
[17] H. Davoudiasl, W.J. Marciano, Phys. Rev. D 98 (2018) 075011.
[18] K. Hagiwara, A. Keshavarzi, A.D. Martin, et al., Nucl. Part. Phys. Proc. 287-288 (2017) 33-38.
[19] Muon g-2 Collaboration, Phys. Rev. Lett. 126 (2021) 141801.
[20] S. Heinemeyer, D. Stöckinger, G. Weiglein, Nucl. Phys. B 690 (2004) 62.
[21] S. Heinemeyer, D. Stöckinger, G. Weiglein, Nucl. Phys. B 699 (2004) 103.
[22] F. Wang, L. Wu, Y. Xiao, et al., Nucl. Phys. B. 970 (2021) 115486 [arXiv:2104.03262].
[23] J.J. Cao, J.W. Lian, Y.S. Pan, et al., J. High Energy Phys. 09 (2021) 175 [arXiv:2104.03284].
[24] J.J. Cao, X.L. Jia, L. Meng, et al., [arXiv:2210.08769].
[25] J.J. Cao, J.W. Lian, Y.S. Pan, et al., J. High Energy Phys. 03 (2022) 203 [arXiv:2201.11490].
[26] J.J. Cao, F. Li, J.W. Lian, et al., Sci.China Phys.Mech.Astron. 65 (2022) 9, 291012 [arXiv:2204.04710].
[27] W. Yin, J. High Energy Phys. 06 (2021) 029 [arXiv:2104.03259].
[28] W. Yin, M. Yamaguchi, Phys.Rev.D 106 (2022) 3, 033007 [arXiv:2012.03928].
[29] S.M. Zhao, T.F. Feng, H.B. Zhang, et al., J. High Energy Phys. 11 (2014) 119 [arXiv: 1405.7561].
[30] J.L. Yang, H.B. Zhang, C.X. Liu, et al., J. High Energy Phys. 08 (2021) 086.
[31] J.L. Yang, T.F. Feng, Y.L. Yan, et al., Phys. Rev. D. 99 (2019) 015002.
[32] L.H. Su, S.M. Zhao, X.X. Dong, et al., Eur. Phys. J. C 81 (2021) 433.
[33] S.M. Zhao, L.H. Su, and X.X. Dong, et al., J. High Energy Phys. 03 (2022) 101.
[34] C.X. Liu, H.B. Zhang, J.L. Yang, et al., J. High Energy Phys. 04 (2020) 002.
[35] J. Rosiek, Phys. Rev. D 41 (1990) 3464.
[36] P.F. Perez and M.B. Wise, J. High Energy Phys. 08 (2011) 068.
[37] P.F. Perez and M.B. Wise, Phys. Rev. D 82 (2010) 011901.
[38] P.F. Perez, Phys. Lett. B 711 (2012) 353 [arXiv:1201.1501].
[39] J.M. Arnold, P.F. Perez, B. Fornal, et al., Phys. Rev. D 85 (2012) 115024 [arXiv:1204.4458].
[40] T.F. Feng, S.M. Zhao, H.B. Zhang, et al., Nucl. Phys. B 871 (2013) 223.
[41] S.M. Zhao, T.F. Feng, X.J. Zhan, et al., J. High Energy Phys. 07 (2015) 124 [arXiv:1411.4210].
[42] E. Arganda, M.J. Herrero, R. Morales, et al., J. High Energy Phys. 03 (2016) 055.
[43] T.F. Feng, L. Sun, X.Y. Yang, Nucl. Phys. B 800 (2008) 221-252.
[44] T. Moroی, Phys. Rev. D 53 (1996) 6565-6575.
[45] D. Stockinger, J. Phys. G 34 (2007) R45-R92.
[46] P. Athron, C. Balazs, D.H.J. Jacob, et al., J. High Energy Phys. 09 (2021) 080 [arXiv:2104.03691].
[47] E. Arganda, M.J. Herrero, R. Morales, et al., J. High Energy Phys. 03 (2016) 055 [arXiv:1510.04685].
[48] M. Endo, K. Hamaguchi, S. Iwamoto, et al., J. High Energy Phys. 07 (2021) 075.
[49] M. Chakraborti, L. Roszkowski and S. Trojanowski, J. High Energy Phys. 05 (2021) 252 [arXiv:2104.04558].
[50] P. Cox, C.C. Han, and T.T. Yanagida, Phys. Rev. D. 104 (2021) 075035 [arXiv:2104.03290].
[51] M.V. Beekveld, W. Beenakker, M. Schutten, et al., SciPost Phys. 11 (2021) 3, 049 [arXiv:2104.03245].
[52] M. Chakraborti, S. Heinemeyer and I. Saha, Eur. Phys. J. C. 81 (2021) 12, 1114 [arXiv:2104.03287].
[53] CMS Collaboration, Phys. Lett. B 716 (2012) 30.
[54] ATLAS Collaboration, Phys. Lett. B 716 (2012) 1.
[55] ATLAS Collaboration, Phys. Lett. B 796 (2019) 68 [arXiv:1903.06248].