Charmed Baryon Weak Decays with Decuplet Baryon and SU(3) Flavor Symmetry

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(Dated: June 6, 2019)

Abstract

We study the branching ratios and up-down asymmetries in the charmed baryon weak decays of $B_c \to B_D M$ with $B_{c(D)}$ anti-triplet charmed (decuplet) baryon and $M$ pseudo-scalar meson states based on the flavor symmetry of $SU(3)_F$. We propose equal and physical-mass schemes for the hadronic states to deal with the large variations of the decuplet baryon momenta in the decays in order to fit with the current experimental data. We find that our fitting results of $\mathcal{B}(B_c \to B_D M)$ are consistent with the current experimental data in both schemes, while the up-down asymmetries in all decays are found to be sizable, consistent with the current experimental data, but different from zero predicted in the literature. We also examine the processes of $\Xi^0_c \to \Sigma^0 K_S/K_L$ and derive the asymmetry between the $K_L/K_S$ modes being a constant.

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I. INTRODUCTION

There have been many interesting progresses in the study of charmed baryon weak decays due to the recent measurement of the absolute branching fraction for the golden mode $\Lambda_c^+ \to pK^-\pi^+$ by the Belle Collaboration \cite{1} with the new world average of $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+) = (6.23 \pm 0.33)\%$ \cite{2} as well as other $\Lambda_c$ measurements by the BESIII Collaboration \cite{3, 13}. In addition, the absolute branching ratio of $\mathcal{B}(\Xi^0 \to \Xi^-\pi^+) = (1.80 \pm 0.52)\%$ is given by the BELLE collaboration for the first time \cite{14}. Theoretically, the charmed baryon decays are dominated by the nonfactorizable effects, such as those associated with the non-vanishing measured branching ratios for the Cabibbo allowed decays of $\Lambda_c^+ \to \Sigma^0\pi^+$ and $\Lambda_c^+ \to \Sigma^+\pi^0$ \cite{2}, which do not receive any factorizable contributions. Many efforts have been made to understand the nonfactorizable effects with different dynamical QCD models \cite{15–23} as well as the use of the flavor symmetry of $SU(3)_F$ \cite{24–38}, which is believed to be the most reliable and simple way to examine the charmed baryon processes. In particular, it has been recently demonstrated that the results for the charmed baryon decays based on the $SU(3)_F$ approach \cite{26–33, 35–38} are consistent with the experimental data.

However, most of the recent experimental and theoretical activities have been concentrated on the charmed baryon decays with the octet baryon in the final states, whereas there has been a little studies for the decuplet modes. Note that most of the charmed baryon experiments with the decuplet baryon were all done before the millennium. In this work, we will examine the two-body weak decays of $B_c \to B_DM$, where $B_{c(D)}$ and $M$ represent the anti-triplet charmed (decuplet) baryon and octet pseudo-scalar meson states based on $SU(3)_F$. There are two important features for the decays of $B_c \to B_DM$. The first one is that all factorizable amplitudes vanish, resulting in theoretically clean predictions for the non-factorizable contributions of the decays. The other one is that the decays involve only a few $SU(3)_F$ parameters, which are able to be determined by the current experimental data.

On the other hand, the up-down asymmetries of $\alpha$ in $\Lambda_c^+ \to \Xi^0K^+$ and $\Lambda_c^+ \to \Xi^0K^+$ have also been given recently by the BESIII Collaboration with the results of $\alpha(\Lambda_c^+ \to \Xi^0K^+, \Xi^0K^+) = (0.77 \pm 0.78, -1.00 \pm 0.34)$ \cite{11}, respectively, where $\Xi^0$ belongs to the decuplet baryon state with spin-3/2. Although the former experimental result is still consistent with zero, the later one is clearly sizable. This non-vanishing large asymmetry is different from the prediction in the most theoretical calculations in the literature \cite{15, 23, 34}. Recently,
based on the flavor symmetry of $SU(3)_F$, we show that $\alpha(\Lambda_c^+ \to \Xi^0 K^+) = (0.94^{+0.06}_{-0.11})$ \cite{38}, which is consistent with the data, but much larger than zero. In this work, we will particular check the up-down asymmetry in $\Lambda_c^+ \to \Xi^0 K^+$ to see if it agrees with the experimental non-zero result in the $SU(3)_F$ approach.

This paper is organized as the follow. In Sec. II, we present the formalism. We show how the decay amplitudes are related based on $SU(3)_F$. In Sec. III, we provide the numerical results of the decay branching ratios and up-down asymmetries in $B_c \to B_D M$. We conclude our study in Sec. IV.

II. FORMALISM

In order to investigate the two-body decays of the anti-triplet charmed baryon ($B_c$) to decuplet baryon ($B_D$) and octet pseudoscalar meson ($M$) states, we write their representations under the flavor symmetry of $SU(3)_F$ as

$$B_c = (\Xi^0_c, -\Xi^+_c, \Lambda^+_c),$$

$$B_D = \frac{1}{\sqrt{3}} \begin{pmatrix}
\sqrt{3} \Delta^{++} & \Delta^+ & \Sigma^+

\Delta^+ & \Delta^0 & \Sigma^0

\Sigma' & \Sigma^0 & \Xi^0
\end{pmatrix},$$

$$M = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+

\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0

K^- & K^0 & -\frac{2}{\sqrt{6}} \eta
\end{pmatrix}.$$ \hspace{1cm} (1)

Here, we have assumed that the physical meson $\eta$ is solely made of the octet state $[2]$. The effective Hamiltonian associated with $c \to u\bar{d}s$, $c \to u\bar{q}q$ ($q = d, s$) and $c \to u\bar{s}\bar{d}$ transitions is given by \cite{27}

$$\mathcal{H}_{eff} = \sum_{i=+,-} \frac{G_F}{\sqrt{2}} c_i \left( V_{cs} V_{ud} O_{is}^{ds} + V_{cd} V_{ud} O_{is}^{qq} + V_{cd} V_{us} O_{is}^{sd} \right),$$

$$O_{qq}^{i\pm} = \frac{1}{2} \left[ (\bar{u}q_1)_{V-A}(\bar{q}_2 c)_{V-A} \pm (\bar{q}_2 q_1)_{V-A}(\bar{u}c)_{V-A} \right],$$

$$O_{ss}^{i\pm} \equiv O_{dd}^{i\pm} - O_{ss}^{i\pm},$$ \hspace{1cm} (2)

where ($|V_{cs} V_{ud}|, |V_{cd} V_{ud}|, |V_{cd} V_{us}|$) $\approx (1, s_c, s_c^2)$ with $s_c \equiv \sin \theta_c \approx 0.225$ \cite{2}, $c_i$ ($i=+,-$) correspond to the Wilson coefficients, $G_F$ is the Fermi constant, and $O_{qq}^{i\pm}$ with $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$ represent the four-quark operators.
As $O_\pm$ belong to $\overline{15}$ and 6 representations under $SU(3)_F$, respectively, which are symmetric and antisymmetric in flavor and color indices, we can decompose the effective Hamiltonian in the tensor forms of $H(\overline{15})$ and $H(6)$, given by

$$H(\overline{15})_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$H(6)_{ij} = \begin{pmatrix} 0 & -s_c^2 & -s_c \\ -s_c^2 & 0 & 0 \\ -s_c & 0 & 0 \end{pmatrix},$$

respectively, where we have used the convention of $V_{cd} = -V_{us} = s_c$.\[1\]

The most significant feature for $B_c \to B_D M$ is that the decay amplitude is essentially non-factorizable due to the vanishing matrix element of the baryonic transition, i.e. $\langle B_D | \bar{q}_\mu (1 - \gamma_5) c | B_c \rangle = 0$. The reason is that the light quark pair in the anti-triplet charmed baryon state is anti-symmetric, whereas that in the decuplet one is totally symmetric. As a result, we can safely neglect $H(\overline{15})$, which only contributes to the factorizable processes $[15, 39–42]$. In general, the decay amplitude of $B_c \to B_D M$ is given by

$$M = \langle B_D M | H(6) | B_c \rangle = i q_\mu w^{\mu}_{B_D} (P - D \gamma_5) u_{B_c},$$

where $q_\mu$ is the four-momentum of the meson, $w^{\mu}_{B_D}$ is the Rarita-Schwinger spinor vector for the spin-3/2 particle of $B_D$, $P(D)$ corresponds to the $P(D)$-wave amplitude, and $u_{B_c}$ is the spin-1/2 Dirac spinor of $B_c$. By assuming CP invariance, $P$ and $D$ can be taken to be real. Under $SU(3)_F$, the amplitudes associated with $P$ and $D$ are related by

$$P_{(B_c \to B_D M)} = P_0 f_{B_c B_D M}, \quad D_{(B_c \to B_D M)} = D_0 f_{B_c B_D M},$$

respectively, where $P_0(D_0)$ is the real parameter to be determined and $f_{B_c B_D M}$ is the $SU(3)_F$ overlapping factor, defined by

$$f_{B_c B_D M} = (B_D)_{ijk} (B_c)_l H(6)_{ijk} M^l e^{ij0} e^{kmq}.$$ \[6\]

The value of $f_{B_c B_D M}$ in Eq. (6) depends on the specific mode in $B_c \to B_D M$, for example,\[7\]

$$f_{\Lambda^+_c \Delta^+ K^-} = (\Delta^{++})_{111} (\Lambda^+_c)_3 H(6)_{22} (K^-)_3 e^{132} e^{123} = -2.$$  

1 Note that there is a sign difference between our convention and the one in Ref. [2], which will not affect our numerical results.
We will list the values of $f_{B_cB_DM}$ in the next section. We note that the factors of $f_{B_cB_DM}$ with $M$ being a singlet under $SU(3)_F$ vanish so that the corresponding decays with a physical meson $\eta'$ are suppressed. The reason is that $\mathbf{3} \otimes \mathbf{6} \otimes \mathbf{10} \otimes \mathbf{1}$ cannot form a singlet to be invariant under $SU(3)_F$, where $\mathbf{3}, \mathbf{6}, \mathbf{10}$ and $\mathbf{1}$ are the $SU(3)_F$ representations for the anti-triplet charmed baryon, anti-symmetric part of the effective Hamiltonian, decuplet baryon and singlet meson states, respectively. In practice, since $P$ and $D$ share the same overlapping factor, one can alternatively combine these two real parameters into one complex parameter for convenience [26–33, 35–38].

Consequently, the decay width ($\Gamma$) for $B_c \to B_DM$ is given by

$$\Gamma(B_c \to B_DM) = \frac{|\vec{q}|}{8\pi m_{B_c}^2} |\langle \mathbf{M}^2 \rangle| = \zeta \left(1 + \xi^2 r^2\right) P_0^2 f_{B_cB_DM}^2,$$

while the up-down asymmetry ($\alpha$) has the form

$$\alpha(B_c \to B_DM) = \frac{2\xi \text{Re}(PD^*)}{|P|^2 + \xi^2 |D|^2} = \frac{2\xi r}{1 + \xi^2 r^2},$$

where $|\vec{q}|$ represents the absolute value of the three-momentum of the octet pseudoscalar meson $M$ or the decuplet baryon $B_D$ in the CM frame, $m_{B_c}$ is the mass of the charmed baryon $B_c$, $|\langle \mathbf{M}^2 \rangle|$ stands for the spin average squared amplitude, $\zeta = (E_{B_D} + m_{B_D})|\vec{q}|^3 m_{B_c}/(6\pi m_{B_D}^2)$ with $E_{B_D}(m_{B_D})$ representing energy (mass) of $B_D$, $\xi = |\vec{q}|/(E_{B_D} + m_{B_D})$, and $r = D_0/P_0$.

Under the exact flavor symmetry of $SU(3)_F$, one can simply impose the equal-mass ($em$) conditions, given by

$$m_{B_c} \equiv m_{\Lambda_c^+} = m_{\Xi_c^0} = m_{\Xi_c^+};$$

$$m_{B_D} \equiv m_{\Delta^{++}} = m_{\Delta^+} = m_{\Delta^0} = m_{\Sigma^{*0}} = m_{\Sigma^{*+}} = m_{\Xi^0} = m_{\Xi^+} = m_\Omega^-;$$

$$m_M \equiv m_{\pi^+} = m_{\pi^0} = m_{\pi^-} = m_{K^+} = m_{K^0} = m_{\bar{K}^0} = m_{\eta},$$

leading to that both $\xi$ and $\zeta$ are the same for all decays of $B_c \to B_DM$. As a result, $\Gamma(B_c \to B_DM)/f_{B_cB_DM}^2$ and $\alpha(B_c \to B_DM)$ are the same for all modes of $B_c \to B_DM$ when the $em$ conditions are chosen. This $em$ scheme has been widely used in the charmed baryon decays with the octet baryon in the final states based on $SU(3)_F$ as shown in Refs. [26–33, 35–38]. However, it is clear that both parameters of $\xi$ and $\zeta$ for the decuplet modes are quite different since the three-momentum $\vec{q}$ varies largely in different decays around 0.4-0.8 GeV when the physical masses of the baryon and meson states are taken, referred to the physical-mass ($pm$) scheme.
III. NUMERICAL RESULTS

In the \( em \) scheme, from Eq. (9) we see that there is only one combined parameter of \( \bar{r} \equiv \xi r \) for \( \alpha(B_c \to B_D M) \). By using the experimental data of \( \alpha(\Lambda^+_c \to \Xi^0 K^+) = -1.00 \pm 0.34 \) in Ref. [11], we expect that the up-down asymmetry in every decay mode of \( B_c \to B_D M \) should have the same value as

\[
\alpha_{em}(B_c \to B_D M) = -1.00^{+0.34}_{-0},
\]

(11)

where the lower uncertainty of “0” reflects that the physical value of \( \alpha \) cannot be less than \(-1\). From Eqs. (9) and (11), we obtain

\[
\bar{r}_{em} = -1.00^{+0.6}_{-1.6}.
\]

(12)

On the other hand, the decay branching ratios in Eq. (8) also depend on one unknown parameter, defined by

\[
\bar{P}_0 \equiv \sqrt{\zeta (1 + \xi^2 r^2)} P_0,
\]

(13)

which can be determined by only one experimental data point. However, there are four experimental branching ratios as shown in Table I. To obtain the most plausible value for \( \bar{P}_0 \) under the current experimental data, we adopt the minimal \( \chi^2 \) fitting, defined by

\[
\chi^2_{em} = \sum_{ex} \frac{(B_{ex} - B_{em})^2}{\sigma_{ex}^2},
\]

(14)

where \( B_{em} \) is the decay branching ratio generated by \( \bar{P}_0 \) in the \( em \) scheme with the experimental measured lifetime in Ref. [2] and \( B_{ex}(\sigma_{ex}) \) corresponds to the measured branching ratio (uncertainty) in the data. By performing \( \chi^2 \) fit with the minimal value of \( \chi^2_{em} \), we obtain that

\[
(\bar{P}_0)_{em} = (8.7 \pm 0.5) \times 10^{-3} G_F \text{GeV}^2,
\]

\[
\chi^2_{em}/d.o.f. = 1.4,
\]

(15)

where \( d.o.f. \) represents “degree of freedom.” The small value of \( \chi^2_{em}/d.o.f. \) for the fit in Eq. (15) indicates that the \( em \) scheme is good to explain the current experimental data. Our results for the decay branching ratios of \( B_c \to B_D M \) in the \( em \) scheme are summarized in Tables I-IV. In the tables, we also show the \( SU(3)_F \) overlapping factors of \( f_{B_c B_D M} \) for the decays of \( B_c \to B_D M \).
TABLE I. Our results of the up-down asymmetries of $\alpha_{pm}$ and branching ratios of $B_{pm}$ and $B_{em}$ for the Cabibbo allowed modes of $B_c \to B_D M$ based on $SU(3)_F$ along with the experimental data \cite{2, 11, 13, 14} as well as the theoretical calculations in the literature \cite{15, 16, 34}.

| Channel | $f_{B_c B_D M}$ | $\alpha_{pm}$ (our result) | $10^3 B_{pm}$ | $10^2 B_{em}$ | $10^3 B$ | $10^3 B$ | $10^3 B$ | $10^3 B_{ex}(\alpha_{ex})$ |
|---------|-----------------|------------------|-----------|-----------|-----|-----|-----|----------------------|
| $\Lambda_c^+ \to $ $\Delta^{++} K^-$ | $-2$ | $-0.86^{+0.44}_{-0.14}$ | $15.3 \pm 2.4$ | $12.4 \pm 1.0$ | $9.5$ | $27.0$ | $7.0 \pm 4.0$ | $10.7 \pm 2.5$ \cite{2} |
| $\Lambda_c^+ \to $ $\Delta^+ \bar{K}^0$ | $-2\sqrt{3}$ | $-0.86^{+0.44}_{-0.14}$ | $5.1 \pm 0.8$ | $4.1 \pm 0.3$ | $3.1$ | $9.0$ | $2.3 \pm 1.3$ | |
| $\Lambda_c^+ \to $ $\Sigma'^+ \pi^0$ | $\sqrt{2}$ | $0.91^{+0.45}_{-0.10}$ | $2.2 \pm 0.4$ | $2.1 \pm 0.2$ | $2.1$ | $5.0$ | $4.6 \pm 1.8$ | |
| $\Lambda_c^+ \to $ $\Sigma'^+ \eta$ | $\sqrt{2}$ | $0.97^{+0.43}_{-0.03}$ | $3.1 \pm 0.6$ | $6.2 \pm 0.5$ | - | $10.4$ | $2.1 \pm 1.1$ | $9.1 \pm 2.0$ \cite{13} |
| $\Lambda_c^+ \to $ $\Sigma'^0 \pi^+$ | $\sqrt{2}$ | $0.90^{+0.45}_{-0.10}$ | $2.2 \pm 0.4$ | $2.1 \pm 0.2$ | $2.1$ | $5.0$ | $4.6 \pm 1.8$ | |
| $\Lambda_c^+ \to $ $\Xi'^0 K^+$ | $2\sqrt{3}$ | $1.00^{+0.34}_{-0.00}$ | $1.0 \pm 0.2$ | $4.1 \pm 0.3$ | $0.7$ | $5.0$ | $2.3 \pm 0.9$ | $5.02 \pm 1.04$ \cite{11} |

\footnotesize{\textsuperscript{a} The data has not been included in the data fitting.}
\footnotesize{\textsuperscript{b} The experimental decay branching ratios of $\Xi'^0_c$ are measured relative to $B(\Xi'^+\to \Xi^- 2\pi^+)$.

We now discuss in the $pm$ scheme. From the data of $\alpha_{ex}(\Lambda_c^+ \to \Xi'^0 K^+)$, we find that

$$r_{pm} = -6.6^{+1.1}_{-1.0 \pm 7}. \quad (16)$$

With the value in Eq. (16), our predictions for $\alpha_{pm}(B_c \to B_D M)$ are shown in Tables \cite{11, 15}.}
To valuate the decay branching ratios, we have to refit the data, found to be

\[
(P_0, D_0)_{pm} = (3.2 \pm 0.4, -5.1 \pm 2.5) \times 10^{-2} G_F \text{GeV}, \quad R_{P_0D_0} = 0.70,
\]

\[
\chi^2_{pm}/d.o.f. = 11,
\]

(17)

where \(R_{P_0D_0}\) stands for the correlation between the two fitted parameters of \(P_0\) and \(D_0\) and the data point of \(\alpha_{ex}(\Lambda_c^+ \to \Xi^0K^+)^{0.70}\) has also been included in the fit. Our results of \(B(B_c \to B_D M)\) are listed in Tables I-IV. We note that unlike the cases in the \(em\) scheme, \(\zeta\) and \(\xi\) vary from \((0.02 \sim 0.11)\) GeV\(^3\) and \((0.2 \sim 0.3)\) for the different modes of \(B_c \to B_D M\) in the \(pm\) scheme, respectively, resulting in the main differences for the two schemes. In the \(pm\) scheme, it is clear that the \(SU(3)_F\) flavor symmetry is broken by the mass difference in the kinematic part, but still kept in the \(P(D)\)-wave amplitude. In contrast, \(SU(3)_F\) is exact both kinematically and dynamically in the \(em\) scheme. The larger value of \(\chi^2_{pm}\) compared to that of \(\chi^2_{em}\) may result from the improper handling of the \(P(D)\)-wave amplitude. The \(SU(3)_F\) breaking effect in the amplitude would compensate that from the kinematic part. However, such effect can be considered within the \(SU(3)_F\) approach only when more experimental data points are available in the future.

In Table I for the Cabibbo allowed modes of \(B_c \to B_D M\) based on \(SU(3)_F\), we also show the experimental data \([2, 11, 13, 14]\) as well as the theoretical calculations in the literature \([15, 16, 34]\). In particular, Xu and Kamal in Ref. \([15]\) consider the baryon pole term as the nonfactorizable amplitude, Korner and Kramer in Ref. \([16]\) take account of the heavy quark symmetry and covariant quark model for the baryon wave function, and Sharma and Verma in Ref. \([34]\) study the branching ratios with \(SU(3)_F\) based on the old experimental data. Note that our result of \(B_{em}(\Lambda_c^+ \to \Xi^0K^+) = (4.1 \pm 0.3) \times 10^{-3}\) is smaller than, but still consistent with, the current experimental value of \((5.02 \pm 1.04) \times 10^{-3}\). However, it fits well with the previous experimental result of \(B(\Lambda_c^+ \to \Xi^0K^+) = (4.0 \pm 1.0) \times 10^{-3}\) as shown in Table 1 of Ref. \([11]\). However, our result of \(B_{pm}(\Lambda_c^+ \to \Xi^0K^+) = (1.0 \pm 0.2) \times 10^{-3}\) is inconsistent with the data. It is interesting to point out that the up-down asymmetries for all decays are expected to be zero by theoretical studies in Refs. \([15, 16, 34]\) due to the vanishing D-wave amplitudes, which are different from our nonzero results and inconsistent with the current experimental result of \(\alpha_{ex}(\Lambda_c^+ \to \Xi^0K^+) = -1.00 \pm 0.34 [11]\). We recommend to measure \(\alpha(\Lambda_c^+ \to \Delta^{++}K^-)\) in the future experiment as this decay channel has the largest decay branching rate, which will be a clean justification of the \(SU(3)_F\)
TABLE II. Results for the Cabibbo suppressed decays of $B_c \to B_D M$ with $SU(3)_F$.

| Channel                  | $s_c^{-1} f_{B_c B_D M}$ | $\alpha_{pm}$ | $10^4 B_{pm}$ | $10^4 B_{em}$ |
|--------------------------|--------------------------|---------------|---------------|---------------|
| $\Lambda_c^+ \to \Delta^{++} \pi^-$ | 2                         | $-0.81^{+0.43}_{-0.19}$ | 12.5 ± 2.0    | 6.6 ± 0.6     |
| $\Lambda_c^+ \to \Delta^+ \pi^0$     | $\frac{2\sqrt{6}}{3}$   | $-0.81^{+0.43}_{-0.19}$ | 8.3 ± 1.3     | 4.4 ± 0.4     |
| $\Lambda_c^+ \to \Delta^0 \pi^+$     | $\frac{2\sqrt{3}}{3}$   | $-0.81^{+0.43}_{-0.19}$ | 4.2 ± 0.7     | 2.2 ± 0.2     |
| $\Lambda_c^+ \to \Sigma'^+ K^0$      | $-\frac{2\sqrt{3}}{3}$  | $-0.95^{+0.44}_{-0.05}$ | 1.3 ± 0.2     | 2.2 ± 0.2     |
| $\Lambda_c^+ \to \Sigma^0 K^+$       | $\frac{3}{\sqrt{3}}$    | $-0.95^{+0.44}_{-0.05}$ | 0.7 ± 0.1     | 1.1 ± 0.1     |
| $\Xi_c^0 \to \Delta^+ K^-$           | $\frac{2\sqrt{3}}{3}$   | $-0.79^{+0.43}_{-0.21}$ | 3.0 ± 0.5     | 1.2 ± 0.1     |
| $\Xi_c^0 \to \Delta^0 K^0$           | $\frac{2\sqrt{3}}{3}$   | $-0.79^{+0.43}_{-0.21}$ | 3.0 ± 0.5     | 1.2 ± 0.1     |
| $\Xi_c^0 \to \Sigma'^+ \pi^-$        | $\frac{2\sqrt{3}}{3}$   | $-0.84^{+0.44}_{-0.16}$ | 2.5 ± 0.4     | 1.2 ± 0.1     |
| $\Xi_c^0 \to \Sigma^0 \pi^0$         | $-\sqrt{3}$              | $-0.84^{+0.44}_{-0.16}$ | 5.6 ± 0.9     | 2.8 ± 0.2     |
| $\Xi_c^0 \to \Sigma^0 \eta$          | -1                       | $-0.89^{+0.45}_{-0.11}$ | 1.1 ± 0.2     | 0.9 ± 0.1     |
| $\Xi_c^0 \to \Sigma^- \pi^+$         | $-\frac{4\sqrt{3}}{3}$  | $-0.84^{+0.44}_{-0.16}$ | 9.9 ± 1.6     | 4.9 ± 0.4     |
| $\Xi_c^0 \to \Xi^0 K^0$              | $\frac{2\sqrt{3}}{3}$   | $-0.96^{+0.43}_{-0.04}$ | 0.9 ± 0.2     | 1.2 ± 0.1     |
| $\Xi_c^0 \to \Xi^- K^+$              | $-\frac{4\sqrt{3}}{3}$  | $-0.96^{+0.43}_{-0.04}$ | 3.6 ± 0.6     | 4.9 ± 0.4     |
| $\Xi_c^+ \to \Delta^{++} K^-$        | 2                        | $-0.79^{+0.43}_{-0.21}$ | 35.0 ± 5.7    | 14.6 ± 1.2    |
| $\Xi_c^+ \to \Delta^+ K^0$           | $\frac{2\sqrt{3}}{3}$   | $-0.79^{+0.43}_{-0.21}$ | 11.7 ± 1.9    | 4.9 ± 0.4     |
| $\Xi_c^+ \to \Sigma'^+ \pi^0$        | $-\sqrt{3}$              | $-0.84^{+0.44}_{-0.16}$ | 4.8 ± 0.8     | 2.4 ± 0.2     |
| $\Xi_c^+ \to \Sigma'^+ \eta$         | $-\sqrt{2}$              | $-0.89^{+0.45}_{-0.11}$ | 8.7 ± 1.4     | 7.3 ± 0.6     |
| $\Xi_c^+ \to \Sigma'^0 \pi^+$        | $-\frac{\sqrt{3}}{3}$   | $-0.84^{+0.44}_{-0.16}$ | 4.8 ± 0.8     | 2.4 ± 0.2     |
| $\Xi_c^+ \to \Xi'^0 K^+$              | $-\frac{2\sqrt{3}}{3}$  | $-0.96^{+0.43}_{-0.04}$ | 3.5 ± 0.6     | 4.9 ± 0.4     |

approach. In addition, the authors in Ref. [34] use $SU(3)_F$ without neglecting $H(15)$ but treated the D-wave amplitude being zero. Nonetheless, they still arrive the conclusion that $H(15)$ is negligible comparing to $H(6)$. However, our results are somewhat different from those in Ref. [34].

There are some common features between our results and those in Refs. [15, 16, 34]. The most important one is that the vanishing amplitudes in the Cabibbo allowed decays of
TABLE III. Results for the Double Cabibbo suppressed decays of $B_c \to B_{D}M$ with $SU(3)_F$.

| channel       | $s_{c}^{-2}f_{B_c B_{D}M}$ | $\alpha_{pm}$ | $10^5B_{pm}$ | $10^5B_{em}$ |
|---------------|-----------------------------|---------------|--------------|--------------|
| $\Xi_c^0 \to \Delta^+\pi^-$ | $\frac{2\sqrt{3}}{3}$ | $-0.75^{+0.42}_{-0.25}$ | $2.2 \pm 0.4$ | $0.7 \pm 0.1$ |
| $\Xi_c^0 \to \Delta^0\pi^0$ | $-\frac{2\sqrt{6}}{3}$ | $-0.75^{+0.42}_{-0.25}$ | $4.3 \pm 0.7$ | $1.3 \pm 0.1$ |
| $\Xi_c^0 \to \Delta^-\pi^+$ | $-2$ | $-0.75^{+0.42}_{-0.25}$ | $6.5 \pm 1.1$ | $2.0 \pm 0.2$ |
| $\Xi_c^0 \to \Sigma^0K^0$ | $\frac{\sqrt{6}}{3}$ | $-0.88^{+0.45}_{-0.12}$ | $0.4 \pm 0.1$ | $0.3 \pm 0.0$ |
| $\Xi_c^0 \to \Sigma^-K^+$ | $-\frac{2\sqrt{3}}{3}$ | $-0.88^{+0.45}_{-0.12}$ | $0.9 \pm 0.1$ | $0.7 \pm 0.1$ |
| $\Xi_c^+ \to \Delta^{++}\pi^-$ | $2$ | $-0.76^{+0.42}_{-0.24}$ | $25.5 \pm 4.4$ | $7.8 \pm 0.7$ |
| $\Xi_c^+ \to \Delta^+\pi^0$ | $-\frac{2\sqrt{6}}{3}$ | $-0.76^{+0.42}_{-0.24}$ | $17.0 \pm 2.9$ | $5.2 \pm 0.4$ |
| $\Xi_c^+ \to \Delta^0\pi^+$ | $-\frac{2\sqrt{3}}{3}$ | $-0.76^{+0.42}_{-0.24}$ | $8.5 \pm 1.5$ | $2.6 \pm 0.2$ |
| $\Xi_c^+ \to \Sigma^+K^0$ | $\frac{2\sqrt{3}}{3}$ | $-0.88^{+0.45}_{-0.12}$ | $3.5 \pm 0.6$ | $2.6 \pm 0.2$ |
| $\Xi_c^+ \to \Sigma^0K^+$ | $-\frac{\sqrt{6}}{3}$ | $-0.88^{+0.45}_{-0.12}$ | $1.7 \pm 0.3$ | $1.3 \pm 0.1$ |

$\Xi_c^+ \to \Sigma^+K^0$ and $\Xi_c^+ \to \Sigma^0\pi^+$. It is clear that the current experimental data of $B(\Xi_c^+ \to \Sigma^+K^0)/B(\Xi_c^+ \to \Xi^-2\pi^+) = (1.0 \pm 0.5)$ and $B(\Xi_c^+ \to \Xi^0\pi^+)/B(\Xi_c^+ \to \Xi^-2\pi^+) < 0.1$ are insufficient to rule out this feature yet. It is interesting to note that the decay branching ratios given in the various theoretical calculations may not obey the flavor symmetry of $SU(3)_F$ in general, but they all preserve the isospin symmetry. In particular, the isospin relations in the Cabibbo allowed decays can be summarized as follows:

$$B(\Lambda_c^+ \to \Delta^{++}K^-) = 3B(\Lambda_c^+ \to \Delta^+\bar{K}^0), \quad B(\Lambda_c^+ \to \Sigma'^+\pi^0) = B(\Lambda_c^+ \to \Sigma^0\pi^+)$$

$$B(\Xi_c^0 \to \Sigma'^+K^-) = 2B(\Xi_c^0 \to \Sigma^0K^0), \quad B(\Xi_c^0 \to \Xi^0\pi^0) = \frac{1}{2}B(\Xi_c^0 \to \Xi'^-\pi^+). \quad (18)$$

Similar relations in the singly and doubly-Cabibbo suppressed decays are also expected.

Finally, we explore the decay processes of $\Xi_c^0 \to \Sigma^0K_S/K_L$, which contain both Cabibbo allowed and doubly-suppressed contributions as shown in Table [IV], resulting in an asymmetry due to the interference between the two contributions. Explicitly, the $K_S - K_L$ asymmetry is found to be

$$R \equiv \frac{\Gamma(\Xi_c^0 \to \Sigma^0K_S^0) - \Gamma(\Xi_c^0 \to \Sigma^0K_L^0)}{\Gamma(\Xi_c^0 \to \Sigma^0K_S^0) + \Gamma(\Xi_c^0 \to \Sigma^0K_L^0)} = \frac{(1 - s_c^2)^2 - (1 + s_c^2)^2}{(1 - s_c^2)^2 + (1 + s_c^2)^2} = -0.106, \quad (19)$$
TABLE IV. Results for $\Xi^0_c \to \Sigma^0 K_S/K_L$ with $SU(3)_F$.

| channel | $f_{B_cB_D M}$ | $\alpha_{pm}$ | $10^3 B_{pm}$ | $10^3 B_{em}$ |
|---------|----------------|---------------|----------------|----------------|
| $\Xi^0_c \to \Sigma^0 K_S$ | $-\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3} s_c^2$ | $-0.88_{-0.12}^{+0.45}$ | $0.70 \pm 0.11$ | $0.52 \pm 0.04$ |
| $\Xi^0_c \to \Sigma^0 K_L$ | $\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3} s_c^2$ | $-0.88_{-0.12}^{+0.45}$ | $0.87 \pm 0.14$ | $0.64 \pm 0.05$ |

which is independent of the fitting. As a consequence, the asymmetry in Eq. (19) provides a clean prediction in the $SU(3)_F$ approach for the charmed baryon decays, which can be tested by the experiments in BELLE and BESIII.

IV. CONCLUSIONS

We have studied the decay branching ratios and up-down asymmetries in the charmed baryon weak decays of $B_c \to B_D M$ based on the flavor symmetry of $SU(3)_F$. It is interesting to emphasize that these $B_c$ decays with the decuplet spin-3/2 baryon receive only non-factorizable contributions. We have shown that our fitting results for $B(B_c \to B_D M)$ are consistent with the current experimental data in both $pm$ and $em$ schemes. In particular, the $em$ scheme leads to a much smaller number for the $\chi^2$ fit than the $pm$ one, resulting in that the predicted values of $B(B_c \to B_D M)$ in the $em$ scheme contain much less uncertainties than those in the $pm$ one. To reduce the large uncertainties in the $pm$ scheme, the $SU(3)_F$ breaking effect should be included in the amplitude as well when more precision measurements of $B(B_c \to B_D M)$ are available. We have demonstrated that the isospin relations for the decay branching ratios in Eq. (18) are scheme- and model-independent. It is also interesting to note that the vanishing rates for the Cabibbo allowed decays of $\Xi^+_c \to \Sigma^{'+ K^0}$ and $\Xi^+_c \to \Xi^0 \pi^+$ have not been supported by the experimental data yet.

For the up-down asymmetries, we have found that they are sizable, which are different from the prediction of zero due to the vanishing D-wave contributions in the literature. In particular, we have obtained that $\alpha(B_c \to B_D M) = -1.00_{-0.34}^{+0.00}$ for all decay modes in the $em$ scheme, while they range from $-1$ to $-0.42$ at $1\sigma$ level in the $pm$ scheme, consistent with the current only available data of $\alpha_{ex}(\Lambda^+_c \to \Xi^{0} K^0) = -1.00 \pm 0.34$ [11] for the up-down asymmetry. To justify the $SU(3)_F$ approach, we have proposed to search for
\( \alpha(\Lambda_c^+ \to \Delta^{++}K^-) \), which is predicted to be \(-0.86^{+0.44}_{-0.14} \), in the future experiments, as the decay has the largest branching rate among \( B_c \to BDM \).

In addition, we have examined the processes of \( \Xi_c^0 \to \Sigma^0K_S/K_L \), which contain both Cabibbo allowed and doubly-suppressed contributions. We have predicted the \( K_L - K_S \) asymmetry of \( R(\Xi_c^0 \to \Sigma^0K_S/K_L) \) is \(-0.106 \), which depends on neither model/scheme nor the data fitting. Clearly, this asymmetry is a clean result in the \( SU(3)_F \) approach, which should be tested by the experiments.

ACKNOWLEDGMENTS

This work was supported in part by National Center for Theoretical Sciences and MoST (MoST-104-2112-M-007-003-MY3 and MoST-107-2119-M-007-013-MY3).

[1] S. B. Yang et al. [Belle Collaboration], Phys. Rev. Lett. 117, 011801 (2016).
[2] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, 030001 (2018).
[3] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 115, 221805 (2015).
[4] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 116, 052001 (2016).
[5] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 117, 232002 (2016);
[6] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 118, 112001 (2017).
[7] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 95, 111102 (2017).
[8] M. Ablikim et al. [BESIII Collaboration], Phys. Lett. B 767, 42 (2017).
[9] M. Ablikim et al. [BESIII Collaboration], Phys. Lett. B 772, 388 (2017).
[10] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 121, 062003 (2018).
[11] M. Ablikim et al. [BESIII Collaboration], Phys. Lett. B 783, 200 (2018).
[12] M. Ablikim et al. [BESIII Collaboration], arXiv:1811.08028 [hep-ex].
[13] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 99, no. 3, 032010 (2019).
[14] Y. B. Li et al. [Belle Collaboration], Phys. Rev. Lett. 122, 082001 (2019).
[15] Q. P. Xu and A. N. Kamal, Phys. Rev. D 46, 3836 (1992).
[16] J. G. Korner and M. Kramer, Z. Phys. C 55, 659 (1992).
[17] Q. P. Xu and A. N. Kamal, Phys. Rev. D 46, 270 (1992).
[18] H. Y. Cheng and B. Tseng, Phys. Rev. D 46, 1042 (1992) Erratum: [Phys. Rev. D 55, 1697 (1997)].
[19] H. Y. Cheng and B. Tseng, Phys. Rev. D 48, 4188 (1993).
[20] T. Uppal, R. C. Verma and M. P. Khanna, Phys. Rev. D 49, 3417 (1994).
[21] P. Zenczykowski, Phys. Rev. D 50, 5787 (1994).
[22] M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij and A. G. Rusetsky, Phys. Rev. D 57, 5632 (1998).
[23] K.K. Sharma and R.C. Verma, Eur. Phys. J. C 7, 217 (1999).
[24] M.J. Savage and R.P. Springer, Phys. Rev. D 42, 1527 (1990).
[25] M.J. Savage, Phys. Lett. B 257, 414 (1991).
[26] C.D. Lu, W. Wang and F.S. Yu, Phys. Rev. D 93, 056008 (2016).
[27] C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, JHEP 1711, 147 (2017).
[28] C. Q. Geng, Y. K. Hsiao, Y. H. Lin and L. L. Liu, Phys. Lett. B 776, 265 (2018).
[29] C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Phys. Rev. D 97, 073006 (2018).
[30] C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Eur. Phys. J. C 78, 593 (2018).
[31] C. Q. Geng, C. W. Liu and T. H. Tsai, Phys. Lett. B 790, 225 (2019).
[32] C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Phys. Rev. D 99, 073003 (2019).
[33] Y. K. Hsiao, Y. Yao and H. J. Zhao, Phys. Lett. B 792, 35 (2019).
[34] K. K. Sharma and R. C. Verma, Phys. Rev. D 55, 7067 (1997).
[35] D. Wang, P. F. Guo, W. H. Long and F. S. Yu, JHEP 1803, 066 (2018).
[36] C. Q. Geng, C. W. Liu, T. H. Tsai and S. W. Yeh, Phys. Lett. B 792, 214 (2019).
[37] H. J. Zhao, Y. K. Hsiao and Y. Yao, arXiv:1811.07265 [hep-ph].
[38] C. Q. Geng, C. W. Liu and T. H. Tsai, arXiv:1902.06189 [hep-ph].
[39] J. G. Körner, Nucl. Phys. B 25 (1971) 282
[40] J. C. Pati and C. H. Woo, Phys. Rev. D 3, 2920 (1971).
[41] Y. Kohara, Phys. Rev. D 44, 2799 (1991).
[42] L. L. Chau, H. Y. Cheng and B. Tseng, Phys. Rev. D 54, 2132 (1996).