The role of spin in $NN \rightarrow NN\pi$

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Abstract. Chiral effective field theory (EFT) of Quantum Chromodynamics (QCD) provides a model independent understanding of the nature at low energies. The short range physics in such an EFT is encoded in the so called low energy constants (LECs). These LECs, once determined from one process, can be applied to make predictions for many other processes. In this talk we focus on pion production in nucleon-nucleon collisions which provides access to the $(NN)^2\pi$ LEC that plays an important role connecting different few-nucleon reactions. We demonstrate that this LEC can be extracted from $NN \rightarrow NN\pi$ by studying double polarization observables in the reaction $\vec{n}\vec{p} \rightarrow pp\pi^-$. The corresponding measurement is planned in the near future at the COSY accelerator.

1. Introduction

Chiral perturbation theory (ChPT) is a modern framework to systematically investigate low-energy hadronic reactions [1, 2]. ChPT is based on the most general effective Lagrangian consistent with the symmetries of QCD. At low energies quarks are confined into hadrons which are thus the relevant degrees of freedom in the effective Lagrangian. The general idea of effective field theory is based on clean separation of hadronic scales around chiral limit. This means that hadronic reactions can be studied perturbatively based on the expansion parameter $\chi \sim q/\Lambda_\chi$ with $q$ being some small dynamical scale inherent in the process and $\Lambda_\chi \sim 1\text{ GeV}$ being the chiral symmetry breaking scale. According to the scheme, all long-ranged operators such as one-pion exchange, pion loops etc. are explicitly included in the evaluation of the transition amplitude whereas all short-ranged mechanisms are parameterized by the local contact operators. The framework has been successfully applied to study, in particular, meson-meson [3] and meson-baryon [4, 5] scattering observables as well as nuclear forces [6, 7, 8]. Extended by Weinberg [9] the framework has also been applied in recent years to study different pion reactions on few nucleon systems [10, 11, 12, 13, 14, 15, 16, 17]. In particular, significant progress has been achieved in understanding of the pion production mechanism in $NN \rightarrow NN\pi$ [18, 19, 20, 21, 22]. In this Proceeding we would like to focus on the specific feature of chiral symmetry to connect different reactions at low energies. Specifically, we will discuss how the reaction $NN \rightarrow NN\pi$ can be used to pin down the $(NN)^2\pi$ low-energy constant (LEC) that plays an important role connecting different few-nucleon reactions. Apart from pion production, this LEC contributes to the three-nucleon force [23, 24], to electroweak processes such as $pp \rightarrow de^+\nu$ and triton $\beta$
decay [25, 26, 24] and muon absorption on deuterium \( \mu^−d → n n ν_μ \) [27, 28], as well as to the reactions involving photons \( πd → γNN \) [26, 14] and \( γd → nnπ^+ \) [15, 29], as illustrated in Fig. 1. The connection can be easily understood based on the corresponding interaction Lagrangian in the two-nucleon sector

\[
L_{NN}^{\text{int}} = -2d \left( N^† S u N \right) N^†N
\]

where \( S \) is the nucleon spin vector and the 4-vector \( u \) connects pion production with the vector and axial currents via

\[
f_π u_μ = -\bar{π} \partial_μ \bar{π} - ε_{3ab} V_μ π_a τ_b + f_π A_μ + \ldots
\]

and \( f_π \) is the pion decay constant (\( f_π = 92.4 \) MeV). The strength of this short-ranged operator is parameterized by the LEC \( d \) which is a two-body analogue of the axial constant \( g_A \) [26]. Clearly, to allow for a high accuracy description of the abovementioned processes, this LEC needs to be determined with high accuracy.

In the next section we will briefly explain how one can apply effective field theory to the production process. Sec. 3 will be about peculiarities of p-wave pion production and its connection to the LEC \( d \). In sec. 4 we will discuss what is needed from theory and experiment to perform high-accuracy extraction of the LEC \( d \).

2. How to calculate \( NN → NNπ \) within EFT

To apply the EFT approach to the pion production process the following steps are necessary:

(I) The pion transition (production) operators are perturbative. Thus, they can be calculated systematically using ChPT. At any given order they should consist of all possible irreducible graphs.

(a) Due to the presence of new small scale, that is the initial nucleon momentum in the center of mass system (cms) \( p ≈ \sqrt{m_πM_N} ≈ 360 \) MeV with \( m_π(M_N) \) being...
the pion (nucleon) mass), the expansion parameter in the production process is different compared to the standard ChPT. It was first argued in Refs. [30, 31], see also a review article [32] that the expansion parameter in $NN \to NN\pi$ should be $\chi \sim p/M_N \approx \sqrt{m_\pi/M_N}$.

(b) An explicit treatment of the $\Delta(1232)$-resonance is mandatory for $NN \to NN\pi$ [33, 34] because the $\Delta$-nucleon mass difference ($M_\Delta - M_N \approx 300$ MeV) is numerically close to the small scale $p$. It turns out that the propagator of the $\Delta$ is suppressed by one order of $\chi$ as compared to the one of the nucleon. Thus, the $\Delta$ starts to contribute at next-to-leading (NLO) order [20].

(II) The transition operators have to be convoluted with non-perturbative $NN$ wave functions. Ideally, the $NN$ wave functions need to be calculated within the same framework, i.e ChPT. However, due to the absence of chiral wave functions in the energy regime above pion threshold realistic phenomenological wave functions are used.

3. p-wave pion production and the role of LEC $d$

As follows from Eqs. (1)-(2) the contact operator supplemented by the LEC $d$ is proportional to the pion derivative. Therefore it should contribute to production of p-wave pions in $NN \to NN\pi$ while connecting S-wave nucleons. Using the scheme sketched in the previous section we have studied p-wave pion production up to next-next-to-leading ($N^2$LO) order in Ref. [20]. The production operator in this case consists of tree level graphs only, as shown in Fig.2. The LEC $d$ contributes at $N^2$LO. In Ref. [20] we have performed a combined analysis of the p-wave pion production amplitudes contributing to $pp \to d\pi^+, pp \to pn\pi^+$ and $pn \to pp\pi^-$ with only one unknown parameter, namely the LEC $d$. We varied the value of the LEC $d$ to achieve qualitatively the best overall description of the differential cross sections and analyzing powers in different channels of $NN \to NN\pi$. The conclusion was that it is possible to qualitatively describe all the data available with the same value of the LEC $d$ which was found to be around $3/(f_\pi^2 M_N)$ for the CCF model [35] of NN interaction. Note that the first extraction of the LEC $d$ from $NN \to NN\pi$ was performed in Ref. [19], although only one channel, $pp \to pn\pi^+$, was used in the analysis. Also in Ref. [36] an attempt to connect $pp \to d\pi^+\nu$ and $pp \to pn\pi^+$ through the LEC $d$ was perfomed. On the other hand, it is pointed out in Ref. [20] that the reaction channels with the isospin one initial state (e.g., $pp \to pn\pi^+$ and $pp \to d\pi^+$) are not very suited for a high-accuracy extraction of the LEC $d$, although can indeed put important constraints on it. For the reaction channels with the proton-proton interaction in the initial state there are two relevant partial waves when producing p-wave pions: $1S_0 \to 3S_1p$ and $1D_2 \to 3S_1p$. The contact term contributes to the former but the latter turns out to be much larger numerically. In addition, for the reaction channel $pp \to pn\pi^+$ the final nucleons may contribute not only in S-
but also in higher partial waves which, of course, makes the extraction of the LEC more difficult. In the current stage of the research these higher partial wave contributions were disregarded.

On the other hand, the reaction \(pn \rightarrow pp\pi^-\) opens a good opportunity for a reliable extraction of the LEC \(d\). First, modern experiments [37, 38, 39, 40] can offer high-accuracy data for different observables in this channel. It is important that the experimental analyses can single out those events that correspond to production of very low energetic protons in their center of mass system. This guarantees that the final \(pp\)-system is purely in S-wave (the \(^1S_0\)-state) which is a precondition for a high-accuracy determination of the LEC \(d\). The initial nucleons in this case appear in the partial waves that are coupled: \(^3S_1 \rightarrow ^1S_0p\) and \(^3D_1 \rightarrow ^1S_0p\). Due to the coupled channel effect the contact term contributes to both partial waves which provides a more rich dependence of the observables on the LEC \(d\). In Fig. 3 we compare the results of our recent calculation [20] with the data for the differential cross section as well as for the analyzing power from TRIUMF [37, 38]. To allow for such a comparison one has to know theoretically both the p-wave pion amplitudes and the s-wave amplitude (in the \(^3P_0 \rightarrow ^1S_0\) partial wave) which can interfere (see the details in Sec.4). The s-wave pion production cross section is small in this channel as follows from the studies of \(pp \rightarrow pp\pi^0\) [42, 43, 44]. It turns out to be impossible to describe the s-wave data within the tree level approximation [45]. Thus, a technically very involved treatment of pion loops at N\(^2\)LO is required which will be completed only in the near future [46]. Therefore, in the current study we parameterize the s-wave amplitude in such a way to reproduce the data for the total cross section very near threshold [44] as well as the shape of the \(pp\)-invariant mass distribution [42], see Ref. [20] for more details. In each panel of Fig. 3 we provide three theoretical curves that correspond to the values of the LEC \(d\) equal to 3 (solid red curve), 0 (dashed black curve) and -3 (dot-dashed blue curve) calculated with the CCF NN model [35]. The comparison with the data reveals that the positive value of the contact term is clearly preferred although for the analyzing power only qualitative agreement can be achieved. On the other hand, there is a much better agreement between the theory and new preliminary data for the analyzing power measured at COSY (for details see talk by S. Dymov given at this conference [39]) as shown in Fig. 4. The new data were measured with much larger statistics than at TRIUMF. Both measurements were performed under very similar kinematical conditions: at the same energy \(T_{lab} = 353\) MeV and with very similar kinematical cuts for the

Figure 3. Results for \(d^2\sigma/d\Omega_x dM_{pp}^2\) (left panel) and \(A_y\) (right panel) for \(pn \rightarrow pp(\ ^1S_0)\pi^-\). Shown are the results for \(d = 3\) (red solid line), \(d = 0\) (black dashed line) and \(d = -3\) (blue dot-dashed line). The data are from TRIUMF [37, 38] (black squares) and from PSI [41] (blue circles).
Figure 4. Analyzing power as a function of $\cos(\theta_\pi)$. The data are from TRIUMF [37, 38] (black squares) and from COSY [39] (violate circles). The theoretical curves are the same as in Fig. 3.

4. What is needed for a high-accuracy extraction of the LEC $d$ from $pn \rightarrow (pp)S\pi^-$?

As follows from the discussion in the previous section one thing that is certainly needed to extract reliably the LEC $d$ from the existing data is the theoretical calculation of pion d-waves [48]. At leading and next-to-leading orders this calculation does not involve any unknown parameters. The high-precision single polarization data in $pp \rightarrow pp\pi^0$ should provide the stringent test of the theory.

However, one should also be able to get direct access to the relevant p-wave amplitudes once the double polarization data are available. In the rest of this section we will focus on this issue assuming the existence of the spin correlation coefficient $A_{xx}$ the measurement of which is in the accepted program at COSY in 2011. As long as the final $pp$-system is purely in S-wave the most general structure of the reaction amplitude can be written as

$$M = A \hat{S}_\hat{p} + B \hat{S}_\hat{k}_\pi$$

In addition, it was pointed out in Ref. [49] that the use of LECs $c_3$ and $c_4$ extracted from $\pi N$ data in Ref. [50] is inconsistent with the power counting in $NN \rightarrow NN\pi$. This will be corrected in Ref. [48].
where $\hat{p}$ is the unit vector of the initial nucleon momentum in the overal center of mass system, $\hat{k}$ is the unit vector of the pion momentum and $\hat{p} \cdot \hat{k} = \cos(\theta_\pi)$. Here $\tilde{S} = \chi_2^T \frac{\omega}{\sqrt{2}} \tilde{\chi}_1$ denotes the normalized spin structure corresponding to the initial spin-triplet state and the $\chi$‘s with corresponding indices stand for the spinors of the initial nucleons. Furthermore, up to and including pion d-waves

$$A = M_{s-P-wave}^3 + M_{p-P-wave}^3 \cos(\theta_\pi) - \frac{1}{3} M_{d-P-wave}^3 \cos^2(\theta_\pi) - \frac{1}{5}$$

$$B = M_{p-P-wave}^3 + M_{p-P-wave}^3 \cos(\theta_\pi)$$

where the superscript $3L_J$ in the amplitudes $M_{1-L_J}$ symbolizes the partial wave of the initial nucleons whereas the subscript 1-wave corresponds to the partial wave of the pion in the overall cms. One can deduce from this expression that all observables that involve terms $|A|^2$ and $A^*B$ should necessarily contain the interference of s- and d-waves in addition to the relevant s- and p-wave terms. On the other hand, the term $|B|^2$ depends solely on the p-wave amplitudes up to the interference of p- and d-waves. Unfortunately, most of the observables, such as differential cross section $\frac{d\sigma}{d\Omega}$, analyzing power $A_y$, spin-correlation coefficient $A_{xx}$ depend indeed either on $A^*B$ or on $|A|^2$ (or even on both). Fortunately, the differential cross section and the spin-correlation coefficient $A_{xx}$ depend on the same combination of $A^*B$ and $|A|^2$ which disappears if one measures $(1 - A_{xx}) \frac{d\sigma}{dp}$. Thus, this key observable depends solely on $|B|^2$ as pointed out in the accepted proposal of the ANKE collaboration at COSY [40]

$$1 - A_{xx} \frac{d\sigma}{dp} = 2|B|^2 \sin^2 \theta_\pi = 2|M_{p-P-wave}^3| - \frac{1}{3} M_{p-P-wave}^3 \cos^2 \theta_\pi$$

where in the last equality the interference between p- and d-waves was neglected. In Fig. 5 we demonstrate the dependence of $(1 - A_{xx}) \frac{d\sigma}{dp}(\theta = 90^\circ)$ on the LEC d based on Eq. (5). Assuming that the data have the magnitude indicated by the red curve one obtains two clearly separated solutions with positive and negative contact terms. Imposing constraints from the differential cross section and the analyzing power in this channel as well as from the charged pion production channels one should find out the unique solution.

5. Summary

In this Proceeding we discussed how one can extract the low-energy constant that accompanies the short-range operator in few nucleon sector. This LEC contributes to many low-energy reactions and is thus important. We demonstrate that this LEC can be extracted with high-accuracy by studying pion production in nucleon-nucleon collisions. In particular, we emphasize that the reaction channel $pn \rightarrow (pp)_{S-wave}$ with final protons in the $S$-wave is most suited for the analysis. We present the results of recent ChPT calculations of p-wave pion production that is sensitive to this LEC and discuss what is still missing for a high-accuracy determination of the LEC. In particular, we stress that the double-polarization measurement of this reaction planned at COSY in 2011 is of special importance for it would allow one to single out the relevant p-wave amplitudes from the rest. In addition, a parameter-free ChPT calculation of pion d-waves, which is ongoing, would allow one to extract the LEC from the existing observables and thus would provide a good cross check of the results.

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Figure 5. Dependence of the double polarization observable \( \frac{d\sigma}{d\Omega} (1 - A_{xx}) (\theta = 90^\circ) \) on the LEC \( d \) calculated within ChPT (black curve). The arrows indicate solutions for the LEC \( d \) from the intersection of the theory curve with imaginary experimental data (red line).

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[1] Weinberg S 1979 Physica A 96 327.
[2] Gasser J and Leutwyler H 1984 Ann. Phys. 158 142.
[3] Colangelo G, Gasser J and Leutwyler H 2001 Nucl. Phys. B 603 125.
[4] Bernard V and Mei\ss ner U G 2007 Ann. Rev. Nucl. Part. Sci. 57 33.
[5] Bernard V 2008 Prog. Part. Nucl. Phys. 60, 82.
[6] Bedaque P F and van Kolck U 2002 Ann. Rev. Nucl. Part. Sci. 52 339;
[7] Epelbaum E 2006 Prog. Part. Nucl. Phys. 57 654;
[8] Epelbaum E , Hammer H W and Mei\ss ner U G 2009 Rev. Mod. Phys. 81, 1773.
[9] Weinberg S 1992 Phys. Lett. B 295 114.
[10] Gasser J, Lyubovitskij V E and Rusetsky A 2008 Phys. Rept. 456 167.
[11] Mei\ss ner U G, Raha U and Rusetsky A 2005 Eur. Phys. J. C 41 213; 2006 Phys. Lett. B 639 478.
[12] Baru V , Hanhart C, Hoferichter M, Kubis B, Nogga A and Phillips D R 2011 Physics Letters B 694, 473.
[13] Liebig S, Baru V , B"allout F, Hanhart C and Nogga A, Preprint arXiv:1003.3826 [nucl-th].
[14] G"ardestig A and Phillips D R 2006 Phys. Rev. C 73 014002.
[15] Lensky V, Baru V , Haidenbauer J, Hanhart C, Kudryavtsev A E and Mei\ss ner U G 2005 Eur. Phys. J. A 26 107.
[16] Krebs H, Bernard V and Mei\ss ner U G 2004 Eur. Phys. J. A 22 503.
[17] Liu L, Baru V , Haidenbauer J and Hanhart C, Preprint arXiv:1007.1382 [nucl-th], accepted for publication in Eur. Phys. J. A.
[18] Lensky V, Baru V , Haidenbauer J, Hanhart C, Kudryavtsev A E and Mei\ss ner U G 2006 Eur. Phys. J. A 27 37.
[19] Hanhart C, van Kolck U, and Miller G A 2000 Phys. Rev. Lett. 85 2905.
[20] Baru V , Epelbaum E , Haidenbauer J, Hanhart C, Kudryavtsev A E, Lensky V and Mei\ss ner U G 2009 Phys. Rev. C80 044003.
[21] van Kolck U, Niskanen J A and Miller G A 2000 Phys. Lett. B493 65.
[22] Filin A, Baru V , Epelbaum E , Haidenbauer J, Hanhart C, Kudryavtsev A and Mei\ss ner U G 2009 Phys. Lett. B 681 423.
[23] Epelbaum E , Nogga A, Gl"ockle W, Kamada H, Mei\ss ner U G and Witala H 2002 Phys. Rev. C 66 064001.
[24] Gazit D, Quaglioli S and Navratil P 2009 Phys. Rev. Lett. 103 102502.
[25] Park T S et al. 2003 Phys. Rev. C67 055206.
[26] G"ardestig A and Phillips D R 2006 Phys. Rev. Lett. 96 232301.
[27] Andreev V A et al. [MuSun Collaboration], *Preprint* arXiv:1004.1754 [nucl-ex].
[28] Myhrer F 2010 *AIP Conf. Proc.* 1261 43 (*Preprint* arXiv:1004.2892 [nucl-th]).
[29] Lensky V, Baru V, Epelbaum E, Hanhart C, Haidenbauer J, Kudryavtsev A E and Meißner U G 2007 *Eur. Phys. J.* A 33 339.
[30] Cohen T D, Friar J L, Miller G A and van Kolck U 1996 *Phys. Rev.* C 53 2661.
[31] Hanhart C and Kaiser N 2002 *Phys. Rev.* C 66 054005.
[32] Hanhart C 2004 *Phys. Rept.* 397 155.
[33] Baru V, Haidenbauer J, Hanhart C, Kudryavtsev A E, Lensky V and Meißner U G 2008 *Phys. Lett.* B 659 184.
[34] da Rocha C, Miller G A and van Kolck U 2000 *Phys. Rev.* C 61 034613.
[35] Haidenbauer J, Holinde K and Johnson M B 1993 *Phys. Rev.* C 48 2190.
[36] Nakamura S X 2008 *Phys. Rev.* C 77 054001.
[37] Hahn H et al. 1999 *Phys. Rev. Lett.* 82 2258.
[38] Duncan F et al. 1998 *Phys. Rev. Lett.* 80 4390.
[39] Dymov S, proceedings of the 19th International Spin Physics Symposium (SPIN 2010) September 27 October 2, 2010, Jülich, Germany
[40] Dymov S et al., (ANKE collaboration), “Measurement of the spin correlation parameters of the quasi free $\vec{n}\vec{p} \rightarrow (pp)s\pi^-$ reaction with polarised beam and target at ANKE”, accepted ANKE proposal 205 for COSY accelerator
[41] Daum M et al. 2002 *Eur. Phys. J.* C 25 55.
[42] Bilger R et al. 2001 *Nucl. Phys. A* 693 633.
[43] Meyer H O et al. 1999 *Phys. Rev. Lett.* 83 5439; 2001 *Phys. Rev.* C 63 064002.
[44] Abd El-Samad S et al. [COSY-TOF Collaboration] 2003 *Eur. Phys. J.* A17 595.
[45] Park B Y et al. 1996 *Phys. Rev.* C 53 1519.
[46] Filin A et al., in preparation
[47] Tsirkov D, proceedings of the 19th International Spin Physics Symposium (SPIN 2010) September 27 October 2, 2010, Jülich, Germany
[48] Baru V et al., in preparation
[49] Long B and Lensky V, *Preprint* arXiv:1010.2738 [hep-ph].
[50] Krebs H, Epelbaum E and Meißner U G 2007 *Eur. Phys. J.* A 32 127.