Influence of load level during strengthening of reinforced concrete beams on their reliability

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Abstract. The main purpose of article is to study the influence of load level on the reliability of non-damaged reinforced concrete beams with rectangular cross-section during strengthening by adding stretched steel reinforcement. On the basis of real samples testing of the existing and advanced methods of reliability estimation (depending on the variant of random parameter of the load level at the strengthening moment), adapted to the national norms of reinforced concrete structure design, the recommended value of failure-free probability $P(\beta)$ – within ranges $0.999624-0.999758$ and $0.999606-0.999775$ (for two of considered variants, respectively). The comparative analysis of obtained results is conducted depending on the additional steel bar diameter ($Ø10$ mm, $Ø12$ mm, $Ø14$ mm) and load level at the strengthening moment ($0.0\times M_{ult,0}$; $0.3\times M_{ult,0}$; $0.5\times M_{ult,0}$; $0.75\times M_{ult,0}$, where $M_{ult,0}$ is bearing capacity of non-strengthened beam). Practical utility of the study results could be associated with the possibility of usage both existing as well as developed methodology to evaluate reliability for design of bended reinforced concrete elements, strengthened with additional stretched steel rebar subjected to load (simulation of the actual element performance during strengthening), especially in the case of proving reliability levels of its design.

1. Introduction
Nowadays, probabilistic methods of building structure design are becoming more common as they make it possible to assign their reliability guaranteed level at the design stage. Moreover, considering aspects of high level subjectivity while choosing random design parameters of construction bearing capacity reserve, as well as lack of sufficient normative calculation instructions (multiple random parameters have rather wide statistical variations) the topicality of the problem of approach development for their reliability evaluation could be confirmed, namely, in the aspects of reliability. In addition, taking in the account the rapidly increasing reconstruction of existing buildings and structures, the design of elements, which need to be strengthened should be accompanied by quantitative evaluating of their reliability – identification of failure-free probability. Simultaneously, nowadays the problem of effect of load level (as the random factor) on reliability of reinforced concrete structures in the case of their strengthening is not fully investigated and requires further research and analysis.

Recently, numerous research is devoted to the stress-strain state analysis of bended and compressed-bended elements, strengthened while being subjected to load [1-13], as well as to performance of...
reinforced concrete elements in the aggressive environment [14] and performance of RC elements with new specific constructions or different new types of concrete [15-17]. However, despite the originality of obtained experimental data, the issue of estimation of reliability of structures (especially, those, strengthened while being subjected to load) in these works is not considered. On the other hand, reliability of reinforced concrete structures (as the separate field of building construction design) has become the issue of scientific research rather short time ago- at the end of XX century. It is important to note, that most of the studies are devoted to the failure-free probability evaluation (as the part of reliability) for such elements and could be associated with beams and slabs, reinforced with the use of composite reinforcement. Among the others, in some works [18] the reliability analysis of bended elements (beams and slabs) of bridges, reinforced by composite tapes of various cross-sections is conducted. Moreover, the methodology is proposed to calculate safety factors such constructions [19]. However, despite the parameters for reliability estimation [18, 20] obtained in these works and recommended safety factors [19], they do not fully take into account the impact of load level at the moment of analyzed structures strengthening (as the random parameter it is not considered at all) on their reliability. In recent papers [21,22] study of influence of different cross-section composite tapes is proposed. Composite tapes were fixed during the application of the load on the reinforced concrete beams, which did not have sufficient shear strength. The advantage of such work [21] is the structure reliability analysis conducted (according to simplified probability algorithm, based on Monte-Carlo principle) for rather wide range of rated load at which the strengthening took place (from exploitative to maximum value). The preference of the work [22] could be characterized as possibility to evaluate the shear strength reliability on the basis of experimental data statistic analysis for more than 250 beams, strengthened during the load application. Anyway, it could be assumed, that regarding such works [21, 22] certain remark should be made:

1) beams with insufficient shear strength are considered (sidelong sections in most cases do not need additional reinforcement during the reconstruction;
2) rather great diversity among values of safety factors is noted [22];
3) as random parameters for bearing capacity reserve are taken only those, associated with strength (load level at the moment of strengthening was considered as predetermined value). In the paper [23] was proposed the methodology for evaluating reliability of reinforced concrete beams and slabs, strengthened with composite tape during the load application. In contrast to other papers [21, 22], this research was associated to elements (beams and slabs) with deficient shear strength (the normal section of the construction was considered), which makes it rather topical. However, complicated mathematical apparatus for reliability evaluation, based on the preliminary research statistic data and adopted to USA design regulations does not allow to fully use this methodology in the design practice. In addition, the safety factors calculation was conducted only for one load level at the strengthening moment which does not allow to provide the graph of dependency of β- indexes on this parameter- in contrast to another paper [24].

Therefore, the assumption could be made regarding the appropriority to develop the issue of quantitative reliability estimation (especially, for failure-free probability) for bended reinforced concrete elements, strengthened exactly during the load application (which could model the actual conditions of their work during the real time process). In addition, it is important to consider the load level at the moment of structure strengthening (as the random factor) influence on the probability of these structures’ reliable work.

2. Purpose of the article
The purpose of article is to conduct research on influence of the load level on the failure-free probability while strengthening rectangular beams with additional stretched steel bars during reconstruction. The study is conducted on the basis of tested samples [24] and further developed methodology for reliability evaluation (depending on variant of random load level parameter during strengthening), adopted to the national design code for reinforced concrete structures.
3. The method of reliability evaluation of RC beams, strengthened under an action of load

As the range of statistic diversity of random factors for building constructions is within 5...25 %, reliability evaluation for experimental beams’ samples after strengthening [25] will be conducted with the use of statistic linearization method. Moreover, previously proposed methodology [24] was used and developed. In order to identify the reliability factors was used the existing method for reliability estimation for new designed structures [25], well-known probability theory thesis [25] and recommendations for such thesis application for building constructions [23].

According to previously developed [24] method for reliability estimation, the final equation for calculation the random value of maximum bending moment value \( \bar{M}_{ult} \), which could be perceived by beam after its strengthening (preservation the condition \( x \leq xR \)), will reach the following shape:

\[
\bar{M}_{ult} = f(E_x, E_{s,aver}, f_y, E_{s,add}, y_{mid,add}, \delta, \bar{d}) = (A_s + A_s,add)\hat{E}_{s,add} - 0.5\lambda \varepsilon_c \bar{d} (0.5\lambda + 1) + \\
\varepsilon_c \pm \sqrt{\varepsilon_c^2 + \left( \frac{3.2 \varepsilon_c^3 \bar{d} E_{s,aver}}{A_s,add E_{s,aver}} \right)} + \\
\varepsilon_c \pm \sqrt{\varepsilon_c^2 + \left( \frac{3.2 \varepsilon_c^3 \bar{d} E_{s,aver}}{A_s,add E_{s,aver}} \right)}
\]

where \( X_i(x_i), Y_j(y_j) \) – are respectively random \([X_i(x_i)]\) and determined \([Y_i(y_j)]\) parameters for bearing capacity reserve determination and load level on the structure (all random and determined parameters are given in the work [24]).

Mathematical assumptions of random arguments \( X_i(x_i) \) are substituted in previously obtained equation (1), next the function \( \bar{M}_{ult} = f[X_i(x_i),...,X_n(x_n)] \) is linearized by estimation the coefficients \( D_{X_i(x_i)} [25] \):

\[
D_{X_i(x_i)} = \frac{\partial \bar{M}_{ult}}{\partial X_i(x_i)}.
\] (2)

The equation for definition the standard of maximum bending moment \( \bar{M}_{ult} \) could be presented in the following form [18]:

\[
\bar{M}_{ult} = \sqrt{\sum_{i=1}^{n} D_{X_i(x_i)} \times \hat{X}_i(x_i)},
\] (3)

where \( \hat{X}_i(x_i) \) – standard of random parameter.

The failure-free probability \( P(\beta) \), could be calculated by following equation:

\[
P(\beta) = 0.5 + \Phi(\beta),
\] (4)

where \( \beta = (\bar{M}_{ult} - M_{ult})/M_{ult} \) – is so-called safety characteristic according to [25] or the reliability index according to [19]; \( M_{ult} \) – calculated bearing capacity of normal beam cross-section after strengthening; \( \Phi(\beta) \) – the error function (Laplas function).

4. The results of theoretical research of influence of load level during strengthening of RC beams on the probability of their reliable work

The major difference from the existing evaluation methods for probability of structures’ reliable work definition in this studies is consideration of the load level impact in calculations as the random parameter for construction during strengthening. Such coefficients were defined through the experiment in recent
study [21]. The way to consider these coefficients in this methodology for evaluation of reliable
construction work probability is the main subject of theoretical investigation. Two possible approaches
to evaluate such impact were analyzed.

Variant 1. The value of standards \( (\hat{\gamma}^{add}, \hat{\gamma}^{mid}) \) and their variation coefficients \( (\hat{\gamma}^{add}, \hat{\gamma}^{mid}) \), given
in [33] were taken for all testing conditions (for each load level and additional rebar diameter) separately,
which means that each figure in Table 1 corresponds to separate standard number. Simultaneously were
calculated parameters value for strength and deformability of materials, cross- section geometry, load
levels during strengthening and corresponding coefficients \( \hat{\gamma}^{add}, \hat{\gamma}^{mid} \), as well as bearing capacity
\( M_{ult} \) of strengthened beam given in research one [20]. Calculated numerical values of statistic
characteristic (mathematical assumptions, standards) for above-cited parameters, calculated on the basis
of corresponding source data are also given in [20]. Probability of reliable work of strengthened beams,
obtained from the assumption that the impact of load level is different for different strengthening
conditions (additional rebar diameter) is given in Table 1 and graphically in Figure 1.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{№} & \text{Additional rebar} & \text{Load level at the moment of strengthening} & \text{Probability} \times 10^6 \\
\hline
1 & \text{Ø10 mm} & 0.0 \times M_{ult,0} & 0.999675 & 0.999675 & 0.999675 & 0.999675 \\
2 & \text{Ø12 mm} & 0.3 \times M_{ult,0} & 0.999698 & 0.999698 & 0.999698 & 0.999698 \\
3 & \text{Ø14 mm} & 0.5 \times M_{ult,0} & 0.999709 & 0.999709 & 0.999709 & 0.999709 \\
4 & \text{Ø14 mm} & 0.75 \times M_{ult,0} & 0.999675 & 0.999675 & 0.999675 & 0.999675 \\
\hline
\end{array}
\]

Comment: \( M_{ult,0} \) – bearing capacity of non-strengthened beam.

\[
\begin{align*}
\text{Figure. 1.} & \quad \text{Dependence graph of the “load level – failure-free probability } P(\beta) \text{” (var. 1).} \\
\text{Table 1.} & \quad \text{The failure-free probability } P(\beta) \text{ (var. 1).}
\end{align*}
\]

Variant 2. In this variant the standards value \( (\hat{\gamma}^{add}, \hat{\gamma}^{mid}) \) and corresponding variation coefficients
\( (\hat{\gamma}^{add}, \hat{\gamma}^{mid}) \) could be taken equal and averaged for all testing conditions, which does not depend on
load levels and additional rebar diameters. Therefore, following averaged values for coefficients
\( \hat{\gamma}^{add}, \hat{\gamma}^{mid} \),
\( \gamma_{\text{mid}} \) standards are obtained with use of table 4 [18]: \( \gamma_{s, \text{dis}} = 0.005; \) \( \gamma_{s, \text{inc}} = 0.002 \). Thus, taking into account the change in bearing capacity values for beams’ normal cross-sections after strengthening \( M_{\text{ult}} \) (in most cases towards its decrease) and standard \( \hat{M}_{\text{ult}} \) (mostly towards increasing), which leads to changes (in most cases towards increasing) of reliability indexes \( \beta \) values and corresponding values for the failure-free probability \( P(\beta) \). Probability of strengthened beams’ failure-free work, obtained through assumption that load level influence is the same for different strengthening conditions (additional rebar diameter) given in Table 2 and graphically in Figure 2.

![Graph](image.png)

**Figure 2.** Dependence graph of the “load level – failure-free probability \( P(\beta) \)” (var. 2).

**Table 2.** Probability of reliable work \( P(\beta) \) (var. 2).

| №  | Additional rebar | Load level at the moment of strengthening |
|----|------------------|----------------------------------------|
|    |                  | 0.0 \( \times M_{\text{ult},0} \) | 0.3 \( \times M_{\text{ult},0} \) | 0.5 \( \times M_{\text{ult},0} \) | 0.75 \( \times M_{\text{ult},0} \) |
| 1  | Ø10 mm           | 0.999694                              | 0.999675                              | 0.999651                              | 0.999606                              |
| 2  | Ø12 mm           | 0.999748                              | 0.999720                              | 0.999689                              | 0.999638                              |
| 3  | Ø14 mm           | 0.999775                              | 0.999748                              | 0.999694                              | 0.999641                              |

Comment: \( \text{Mult,0} \) – bearing capacity of non-strengthened beam.

As could be seen from the results of theoretical investigations, probability of reliable work of tested samples, obtained with consideration of two different variants of load level impact also vary. Namely, at load levels less than 0.3 failure-free probability \( P(\beta) \) could be obtained with the use of variant 1. At the load levels which exceed 0.5 greater probability of failure-free work \( P(\beta) \) could be obtained considering the calculation variant 2. Taking in the account all the above cited results in order to reach higher reliability for strengthening reinforced concrete beams with additional rebar it could be recommended to use both variants of load level impact evaluation.

Specifically, in order to obtain higher level of reliable work probability 2 variant should be used for beams strengthened at low load levels (0.3 and lower). Also for beams, strengthened at high load levels (0.5 and higher) it could be recommended to use variant 1 to evaluate the influence of load level. Such an approach will give an opportunity to design strengthened structures with higher failure-free probability.
Practical utility of article results could be associated with possibility to use both variants for evaluating the load level influence in the methodology of estimating the reliability in design of bended reinforced concrete elements, strengthened by additional stretched rebar with simultaneous load application.

5. Conclusion
In this article is conducted the theoretical analysis of load level influence while strengthening of non-damaged reinforced concrete beams with rectangular cross-section with additional stretched rebar on their failure-free probability $P(\beta)$. On the basis of testing of developed methodology for estimating the probability of reliable work, adopted to national codes for reinforced concrete structures design two variants for evaluation the load level influence are proposed. Such an approach makes it possible to design structures strengthened by additional rebar with higher reliability.

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