Potentials of Processed Palm Kernel Shell Ash (Local Stabilizer) and Model Prediction of CBR and UCS Values of Ntak Clayey Soils in Akwa Ibom State, Nigeria

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Abstract — It is very essential to improve on the study of stabilization, as we investigate the potential of Processed Palm Kernel Shell Ash (PPKSA) as a Local stabilizer in stabilizing clay soil. The ever-increasing cost of construction materials in Nigeria and other developing countries has created the need for improved research into locally and readily available materials and also on how to convert these local materials such as Palm Kernel Shell Ash for use in construction and soil improvement. To achieve this; soil samples were collected from Ntak – Uyo, Akwa Ibom State classified as an A-2-5 soil on AASHTO and CL on UNIFIED SYSTEM of classification, were sieved and passed through sieve No. 36. It was then stabilized with (2-7%) Processed Palm Kernel Shell Ash (PPKSA) by weight of the dry soil. The investigation includes evaluation of the engineering and geotechnical properties of the soil.

The results obtained shows that the increase in PPKSA content at 4.5% increase the Optimum Moisture Content (OMC) by 16.74%, Maximum Dry Density (MDD) by 1.89 gm/cm³, Unconfined Compressive Strength (UCS) by 433.12 kN/m², California Bearing Ratio (CBR) by 55% for uns soaked and 36% for soaked while there was a significant reduction in the value of Liquid Limit (LL) by 30.92% and Plasticity Index (PI) by 10%. The predictive models were developed, and these models showed a good correlation with experimental results in the control tests as they possess a reasonable significant difference and a strong relationship between the measured and predicted values.

The study concluded that PPKSA can be used to improve the properties of soil for construction purposes and 4.5% PPKSA content was observed to yield maximum improvement for OMC, MDD, CBR and UCS values.

Index Terms — Processed Palm Kernel Shell Ash (PPKSA), Mixture Design, Simplex Lattice, Compaction, Consistency, Stabilization, Scheffe’s Models.

I. INTRODUCTION

Road is the key infrastructure of a country. It contributes to the economic, industrial, and cultural development of a country. A country’s economic status depends upon how well served the country is by its roads. The importance of road is comparable to the veins in the human just as veins in the human body maintain health by circulation of blood to different parts of the body.

Similarly, means of transportation keep the health of a nation in good condition by keeping the goods and people moving from one place to another. Thus, road is vital for the all-round development of a nation since every goods and services, need transport facilities both at the production stage as well as distribution stage. In the production stage, road is needed for carrying raw materials such as seeds, manure in the case of food production, sugarcane, cotton, coal, steel etc., in the case of cloth and sugar industry. In the distribution stage transportation is required to transport finished products from farm and factories to the distribution centers. Thus, for the economic, cultural, and social development of a country, an effective and adequate system of transportation is essential in order words there must be well constructed roads linking the cities because a country’s economic status depends so much on how well served the country is by its roads. Hence the rate at which a country’s economic grows is very closely linked to the rate at which the transport sector grows.

Processed Palm Kernel Shell Ash (PPKSA) is the ash produced from burning of palm kernel shell and it is a byproduct of palm oil mills. The ash has a pozzolanic properties that enables it as a partial replacement for cement and also plays an important role in soil stabilization as well as in strength and durability of concrete. The moisture content in kernel shell is low compared to other biomass residues with different sources suggesting values between 11-13%. Palm Kernel Shell contains residues of palm oil which accounts for its slightly higher heating value than average lignocellulosic biomass. When compared to other residues from the industry, it is a good quality biomass fuel with uniform size distribution, easy handling, easy crushing and limited biological activity due to low moisture content. Palm Kernel Shells are the shell fractions left after the nut has been removed after crushing in the palm oil mill. Kernel Shells are fibrous materials and can be easily handled in bulk directly from the product line to end users. Palm Kernel Shell Ash help improve the increasing challenges of scarcity and high cost of construction materials used by the construction industry in Nigeria and Africa, by reducing the volume of cement usage in concrete and other related works.

II. MATERIALS AND METHOD

A. Sample Collection, Preparation, and Identification

Palm Kernel Shell will be obtained from different locations within Akwa Ibom State, Nigeria. It will be dried, incinerated to a certain temperature in a furnace, allowed to cool then pulverized and the shells sieved through sieve no 36, after which it will be processed by adding 1500 g of
gypsum for every 19300 g of additive for experimental study and the specific gravity of the palm kernel shells and the soil sample will be recorded. The soil sample will be collected in bags by method of disturbed sampling at reasonable depth in Uyo, Akwa Ibom State. Preliminary tests for identification of the natural soil, stabilizer and the geotechnical properties of the soil treated with stabilizer will be carried out in accordance with BS 1377: Part 2: 1990: 4.3/5.3, BS 1377: Part 2: 1990: 8.3 and BS 1377: Part 2: 1990: 9.3 and BS 1377: Part 4: 1990: 3.3/3.5 and BS 1377: Part 4: 1990: 3.4/3.6. The determination of chemical properties for the stabilizer and the soil sample were done in accordance with American Society for Testing and Materials (ASTM) 1999 and American Public Health Association (APHA) 20th Edition 1998. The standard proctor energy level was used for compaction test, which was also used in determining the moisture content for California bearing ratio and unconfined compressive strength specimens. The stabilizer was thoroughly mixed with pulverize soil and then with distill water. The results of the combined percentages of the oxides obtained from the selected materials satisfied certain minimum requirement value as specified at the end of the study for local stabilizers and this standard was compared to the recommended ASTM requirement for English stabilizers, example cement. The loss on ignition showed the extent of carbonation in sample mixtures during test and the maximum value was obtained and compared with the maximum value required for pozzolan.

B. Laboratory Analysis

Preliminary tests were performed on six samples with result presented. In preparation of all specimens, the required amount of stabilizers by dry weight of soil was measured and mixed in the dry state before addition of water for any given test.

All tests were performed in accordance with BS1377 (1990) as mentioned above. Specimens for the unconfined compressive strength and the California bearing ratio were prepared at maximum dry density and optimum moisture content using BS compaction energy level.

C. Chemical Analysis

The machines used for the chemical test is called the Atomic Absorption Spectrometer (AAS). The sample is first passed through the digestion process which is done by measuring 100 ml of the sample into a 125 ml beaker, then 0.5 ml of nitric acid (HNO₃) was added, then followed by the addition of 5 ml of hydrochloric acid (HCL) to the beaker. The sample is then heated to a temperature of 90 °C for 2 hours, then allowed to cool after which it is filtered to remove solid particles before taking the sample for testing on the Atomic Absorption Spectrometer. This equipment called Atomic Absorption Spectrometer is used to determine the elements in a substance. Atomic Absorption is the process that occurs when a ground state atom absorbs energy in the form of light of a specified wave length, elevated substance is applied on the chopper and a beam of light from the lamp sent across the chopper which passes through the flame through the monochrometer to the detector before giving out a reading. The different elements have certain percentages to which they react to. For example, at 70% if there be the presence of Nickel in the substance, the machine will give a readout value. Here is a schematic representation for better understanding.

**Lamp - Chopper - Flame - Monochrometer - Detector - Readout**

D. Design and Analysis of Experiments

Literally, an experiment is a test. Researchers perform experiments in virtually all fields of inquiry, usually to discover something about a particular process or systems. More formally, an experiment is a test or series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for the changes that may be observed in the output respond Montgomery et al. [1]. Experimentation plays an important role in product realization activities, which consist of new product design and formulation, manufacturing process development and process improvement. Experimentation is a vital part of the scientific or engineering method. Now there are certain situations where the scientific phenomena are so well understood that useful results including mathematical models can be developed directly by applying these well understood principles Montgomery et al. [1]. However, most problems in science and in engineering require observation of the system at work and experimentation to elucidate information about why and how it works. Well designed experiments can often lead to a model of system performance, such experimentally determined models are called *empirical models*, he posited.
In general, experiments are used to study the performance of processes and systems. The process under consideration can be as a combination of operations, machines, methods, people, materials, and other resources that transforms some inputs (often a material) into an output that has one or more observable response variables. Some of the process variables and materials properties \( x_1, x_2, \ldots, x_p \) are controllable, whereas other variables \( y_1, y_2, \ldots, y_p \) are uncontrollable (although they may be controllable for purposes of a test).

### E. Scheffe’s Model

Interests among researchers have changed from determining which process variables affect the response to determining the region or the important factors that leads to the best possible response Montgomery [1]. The process of obtaining this response is termed response surface methods. Response surface methodology RSM is a collection of mathematical and statistical techniques useful for the analysis and modeling of problems in which a response of interest is influenced by several variables and the objective is to optimize this response Myers et al. [2]. The first step in the RSM is to find approximation for the true functional relationship between the response and a set of independent variables. If the response is well modelled by a linear function of the independent variables, then the approximating function is of the first order model Myers et al. [4]. This is given as:

\[
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon \tag{1}
\]

And in compact form is expressed as:

\[
y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \epsilon \tag{2}
\]

where

\( y = \text{Dependent variables.} \)
\( x = \text{Independent variables.} \)
\( \beta_0 = \text{Model constant.} \)
\( \beta_i(\text{For } i = 1-k) = \text{Independent variables.} \)
\( \epsilon = \text{Random error.} \)

If there is a curvature in the system, then a polynomial of higher degree must be used such as the second order model:

\[
y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{1<j}^{k} \beta_{ij} x_i x_j + \epsilon \tag{3}
\]

where \( \beta_0 \) - Interaction constant between independent components.

The least square method (LSM) estimates the parameters \( \beta_0 \) in the polynomials. The response surface analysis is then performed using the fitted surface. If the fitted surface is an adequate approximation of the true response function then, analysis of the fitted surface will be approximately equal to the analysis of the actual system.

The method of steepest ascent is also one of the response surface methods Myers et al. [2]. It is a procedure for moving sequentially in the direction of the maximum increase in the response. In the case where minimization is desired, the method is called the steepest descent and the fitted order model given in Equation (2). The challenge which any model developed using polynomials in Equation (1), (2) or (3) is that the developed model will always give an expected response, even when all the components are absent (zero). The limitation is due to the presence of the constant \( \beta_0 \) and random term, \( \epsilon \) in the polynomials. Hence, models developed by the polynomials in Equation (1), (2) and (3) may not be reliable. Therefore, this research work will proffer systematic order to model development using Scheffe’s model. In this approach, the constants in the polynomials are expressed implicitly as functions of the components of the mixture. Thus, the limitation inherent in the ordinary polynomial functions is overcome, leading to a reliable model.

### F. Mixture Experiments

The major interest in any mixture experiments is to model the response and components relationship. Thereon, the major challenge lies on the use of suitable polynomial which will give a realistic prediction of this response component relation. Many products are mixtures of several components. Characteristics of the products such as the strength of steel, concrete, fibre, polymer etc. depend only on the relative proportions of the components in the mixture properties caused by varying the ingredient proportions is the objective of performing mixture experiments Cornell [3]. In mixture experiments, the levels of individual components of the mixture are not independent Myers et al. [2].

For example, if \( x_i, x_2, \ldots, x_p \) denote the proportions of \( p \) components of a mixture, then:

\[
0 \leq x_i \leq 1 \quad i = 1, 2, \ldots, p \tag{4}
\]

and

\[
x_i + x_2 + \cdots + x_p = 1 \quad \text{(i.e., 100%)} \tag{5}
\]

With three components, \( p(i = 1, 2, 3) \), the mixtures space is a triangle with vertices corresponding to formation that are pure blends (mixtures that are 100% of a single component).

1. **Simplex Design**

Simplex is the structural representation (shape) of the line or planes joining the assumed positions of the constituent materials of the mixtures Obam [5]. They are used to study the effects of mixture components on the response variable. A \((q, m)\) simplex lattice design for \( p \) components consist of points defined by the following coordinate setting; the proportions assumed by each component take the \( m+1 \) equally spaced value from 0 to 1.

That is,

\[
x_i = 0, \frac{1}{m}, \frac{2}{m}, \ldots, 1 \quad l = 1, 2, \ldots, q \tag{6}
\]

where

\( Q = \text{Number of components of the mixture.} \)
\( M = \text{Maximum possible number of the components which the mixture can be composed of (Mixture level).} \)

For example, when \( q = 3 \) and \( m = 2 \); then, the possible number of runs is:

\[
(x_1, x_2, x_3) = (0, 0, 0), (1/2, 0, 0), (0, 1/2, 0), (1, 0, 0), (0, 0, 1), (1/2, 1/2, 0), (1/2, 0, 1/2), (0, 1/2, 1/2)
\]

\( = 6 \text{ runs} \)
These can be represented in a simplex lattice as shown in Fig. 2.

For a $(3,3)$ lattice design, the number of runs is

\[ N = \frac{(q+m-1)!}{M!(q-1)!} \]  

where

- $N$ = Number of points in a simplex lattice.
- $Q$ = Number of components in the lattice.
- $M$ = Order of the lattice.

An alternative to Simplex Lattice Design is Simplex Centroid Design Scheffe [6].

Mixture models differ from the usual polynomials employed in response surface work because of the constraint Scheffe [7].

\[ \sum x_i = 1 \]  

The standard forms of the mixture models (Scheffe’s models) that are in widespread use are:

Linear:

\[ E(y) = \sum_{i=1}^{p} \beta_i x_i \]  

(9)

Quadratic:

\[ E(y) = \sum_{i=1}^{p} \beta_i x_i + \sum_{i<j}^{p} \beta_{ij} x_i x_j \]  

(10)

Full cubic:

\[ E(y) = \sum_{i=1}^{p} \beta_i x_i + \sum_{i<j}^{p} \beta_{ij} x_i x_j + \sum_{i<j<k}^{p} \beta_{ijk} x_i x_j x_k \]  

(11)

Special cubic:

\[ E(y) = \sum_{i=1}^{p} \beta_i x_i + \sum_{i<j}^{p} \beta_{ij} x_i x_j + \sum_{i<j<k}^{p} \beta_{ijk} x_i x_j x_k \]  

(12)

where

- $\beta_i$ = Expected response to the pure blend, $x_i = 1$ and $x_j = 0$ when $i \neq j$.
- $E(y)$ = Expected response.

The portion $\sum_{i=1}^{p} \beta_i x_i$ is called the linear blending portion. When curvature arises from nonlinear blending between component pairs, the parameters $\beta_{ij}$ and $\beta_{ijk}$ represent either synergistic or antagonistic blending. Higher order terms are frequently necessary in mixture models because the phenomena studies may be complex and the experimental region is frequently the entire operability region and are therefore large, requiring an elaborate models Montgomery [1].

Mixture models which are also known as Scheffe’s models from mere observation are distinct from ordinary polynomials by the absence of the random term and independent constant variables in the models.

2. Relationship Between the Pseudo and Actual Components

In Scheffe’s mixture design, the Pseudo – components, $X_i$ have relationship with actual components, $S_i$. The relationship between $X$ and $S$ as expressed by Scheffe (Scheffe, Experiments with Mixtures [7]) is given as:

\[ X = A \cdot S ; \]
\[ A = \frac{X}{S} \quad \text{or} \quad A = S^{-1} \cdot X ; \]
\[ S = A \cdot A^{-1} = S \cdot B \]  

(13)

where $A^{-1} = B$; $A$ is the of the actual – pseudo proportionality coefficient.

Equation (13) is used to determine actual component of the mixture when the Pseudo components are known, vice versa.

For q components and in keeping with the principle of absolute volume, the sum of the actual component mixture in a given factor space is giving as:

\[ S = \sum_{i=1}^{q} S_i = S_1 + S_2 + S_3 + \ldots + S_{q-1} + 1 \]  

(14)

Dividing Equation (14) by the sum of the actual component mixture, we have:
\[
\frac{s_1}{S} = \frac{s_1 + s_2 + s_3 + \ldots + s_{q-1} + s_q}{S} = \frac{s_1}{S} + \frac{s_2}{S} + \frac{s_3}{S} + \ldots + \frac{s_{q-1}}{S} + \frac{s_q}{S}
\]

where

\[
\frac{s_1}{S} = Z_1; \quad \frac{s_2}{S} = Z_2; \quad \frac{s_3}{S} = Z_3; \quad \frac{s_{q-1}}{S} = Z_{q-1}; \quad \frac{s_q}{S} = S_q
\]

In general form, for any factor space, we have:

\[
Z_i = \frac{s_i}{S} (i = 1, 2, 3 \ldots , q)
\]  

Equation (16) is the proportion of the i\(^{th}\) constituent component of any considered mixture design.

As in a general mixture problem, the measured response is assumed to depend only on the proportions of the ingredients in the mixture, not the amount of the mixture. Therefore, modelling, consequent on experimentation can be based on the actual and pseudo components. Thus, the transformation of the actual components, \(s_i\) into actual ratio components, \(Z_i\) is jettisoned [8].

Then, expressing the actual – pseudo proportionality coefficient expression (Equation (13) in matrix form, we have:

\[
[A] = \frac{[X]}{[S]} = [S]^{-1}[X]
\]  

Oguagamba and Mama [8] developed this expression to mean the inverse or transpose matrix of the actual components corresponding to the pure blend pseudo – components of space points as follows:

\[
[A] = [S]^{-1} = [S]^T
\]

\[
[A] = [a_{11}, a_{12}, a_{13}, \ldots \ldots a_{1q}]^{T} = [s_{11}, s_{12}, s_{13}, \ldots \ldots s_{1q}]^{T}
\]

These derived actual components corresponding to the remaining \(N - q\) lattice points of the pseudo – components mixture proportions are used as other mixture proportion in the experimentation to obtain their corresponding responses.

4. Generalized Scheffe’s Second Degree Mathematical Models

Scheffe’s Models are most times referred to as the mixture models. They differ from the usual regression model due to the correlation between all the components in the mixture designs. Another difference is that the intercept term in the model is not usually included in the regression model [8].

The standard form of the quadratic mixture model according to Scheffe [7] is given as:

Quadratic Model:

\[
E(y) = \sum_{i=1}^{q} \beta_i x_i + \sum_{i<j}^{q} \beta_{ij} x_i x_j
\]

where

\(\beta_i\) = linear blending portion due to the pure blend, \(x_i = 1\) and \(x_j = 0\); \(i \neq j \neq k\).

\(E(y)\) =Expected response.

\(\beta_{ij}\) represents the quadratic nonlinear blending between component pairs, whose parameters may be either synergistic or antagonistic blending.

\(\beta_{ijk}\) represents the full cubic nonlinear blending among component sets of 3, whose parameters may be either synergistic or antagonistic blending.

Oguagamba and Mama [8] gave the generalized Scheffe’s model for the second degree – q variables mixture lattice \(q\) as follows:

\[
E(y) = y_1 x_1 (2x_1 - 1) + y_2 x_2 (2x_2 - 1) + y_3 x_3 (2x_3 - 1) + \ldots + y_{q-1} x_{q-1} (2x_{q-1} - 1) + \ldots + y_q x_q (2x_q - 1) + 4y_{12} x_1 x_2 + 4y_{13} x_1 x_3 + 4y_{23} x_2 x_3 + 4y_{(q-1)q} x_{q-1}
\]

The term, \(y_i\) and \(y_0\) correspond to the mixture response at the respective space points \(i\) and \(ij\) of the actual pure blend, \(S_i\) for \(i = 1, 2, 3, \ldots , q\) (principal points); and derived actual binary blend, \(S_{ij}\) (derived mix ratio) in Equation (27) obtained from the laboratory experiments.

5. Model Validation and Adequacy

Model validation is carried out in two ways. (a) either by randomly splitting an existing data set into two parts, and using part of the data for model fitting, and part of the data for model validation or (b) using one full data set for model fitting, and finding a second independent data set for model validation. The latter approach is adopted in Scheffe’s model and validation [8].
Therefore, the model Equation (22) is tested for adequacy against the experimental results using a new set of design points. These new set of mixture design proportions (now referred to as control mixture design points) are determined in similar manner the binary mixture proportions are determined in the main experiments. The only new set of parameters introduced is the control Pseudo – components.

Prior to this, a statistical hypothesis for this Scheffe’s model would have been stated earlier. That is, the NULL HYPOTHESIS, $H_0$ and the ALTERNATE HYPOTHESIS, $H_a$. Null hypothesis claims that there is no significant difference between specified Scheffe’s model responses and the experimental responses for any other independent actual component mixtures (such as those the control mixtures). Whereas the alternate hypothesis is against the statement (i.e., there is significant difference between specified Scheffe’s model responses and the experimental responses for any other independent actual component mixtures (such as those the control mixtures)).

These hypotheses are tested at a specified significance level, $\alpha$, which represents the maximum tolerable risk of incorrectly rejecting the null hypothesis, $H_0$. Among tests used to check significant levels of difference between model and experimental responses include Student’s $t$ - Test, Fisher’s $F$ – Test, etc.

6. Student’s $t$ – Test Method

The $t$ - Test (also called Student’s $t$ – Test) compares two “means” and tells if they are different from each other. The $t$ - test also defines how significant the differences are. In other words, it lets reveal if those differences could have happened by chance [8].

Oguahamba and Mama [8] gave an advance and simpler expression for obtaining the $t$ – test variance in experimental response as follows:

$$t = \frac{\sqrt{(N-1)\Sigma(Y_m-Y_e)^2}}{\sqrt{\Sigma(Y_m-Y_e)^2}}$$ (23)

where

$Y_e$ and $Y_m$ are the average experimental and model responses, respectively.

N is total design points in the control experiments, t is the variance from the $t$ - statistics.

The $t$ – value obtained in Equation (23) is compared with the one from the standard statistical table according to Dougherty [9] at enhanced ($\frac{\alpha}{N}$) significant level and degree of freedom, $V_e$. That is, $t(\frac{\alpha}{N})(V_e)$. When the $t$ – value from the standard statistical table, $t(\frac{\alpha}{N})(V_e)$ is greater than those of the $t$ - values obtained in Equation (23), the Null hypothesis is accepted, and the model is adequate. Otherwise, the Null hypothesis is rejected, the Alternate hypothesis is accepted, and the model is not adequate.

7. $F$ – Statistics (Fisher’s) Test Method

This test compares the variance from the model response, $S_e$ with that from the experimental responses. The equation for Fisher’s test is given as:

$$F = \frac{\text{explained variance}}{\text{unexplained variance}} = \frac{S_e^2}{S_m^2}$$ (24)

where,

$$S_e^2 = \frac{\Sigma(Y_e - \bar{y}_e)^2}{N - 1}; \quad S_m^2 = \frac{\Sigma(Y_m - \bar{y}_m)^2}{N}$$ (25)

$S_e^2$ is the greater of $S_e^2$ and $S_m^2$; $S_e^2$ is smaller of the $S_e^2$ and $S_m^2$.

$S_e^2$ and $S_m^2$ are variances from are experimental and model responses.

$Y_e$ and $Y_m$ are experimental and model responses; $\bar{y}_e$ and $\bar{y}_m$ are mean values of experimental and model responses; N is the sample group or total control space points.

$$N - k = V_e (\text{Degree of freedom of design points})$$ (26)

Fisher’s tests the adequacy of the model by comparing the responses of the experimental and model results in the control sample group. The Null Hypothesis is accepted, and Alternative Hypothesis rejected if and only if:

$$\frac{1}{F_{\alpha(V_e,V_2)}} < F < F_{\alpha(V_1,V_2)}$$ (27)

where, Dougherty 9] gave critical values of the $F_{\alpha(V_1,V_2)}$ distribution in which $\alpha$ and N have their usual meaning; $V_1$ and $V_2$ are the number of degrees of freedom defined in Equation (26).

III. RESULTS AND DISCUSSION

From Table 3 the value of OMC increased from 15.36% to 16.74% and shows a slight decrease of 0.48% at 4.5% stabilization while the value for the MDD also increases from 1.61 gm/cm$^3$ to 1.89 gm/cm$^3$ with a slight decrease of 0.19 gm/cm$^3$ at 4.5% addition of stabilizer. The liquid limited and the plasticity index results increases from 50.40% to 30.92% and from 18.40% to 10% respectively with slight changes in value at 4.5% additive. The CBR values for unsoak and soak also increases significantly at varying mix ratios from 32.07% to 55% and from 21% to 36% respectively. The UCS values were also increasing significantly with increased percentage stabilizer added and the value obtained increased from 392.77 kN/m$^2$ to 433.12 kN/m$^2$ with a decrease of 58.3 kN/m$^2$ from 4.5% to 5.5% stabilization.

### TABLE 1: RESULTS OF CHEMICAL ANALYSIS OF PROCESSED PALM KERNEL SHELL ASH

| Oxides     | CaO | SiO$_2$ | Al$_2$O$_3$ | K$_2$O | Na$_2$O | Fe$_2$O$_3$ | MgO | LOI | MnO |
|------------|-----|---------|-------------|--------|---------|-------------|-----|-----|-----|
| Processed Palm Kernel shell Ash (%) | 97.94 | 0.13 | 0.004 | 5.78 | 55.81 | 27.47 | 72.52 | 4.02 | 31.41 |

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A. Scheffe’s Second Degree Model for Ntak-Uyo Clay Subgrade

1. Model formulation for Ntak-Uyo Clay Subgrade

Based on the characteristic’s strength of information of the variation of proportions of the stabilizing additives, the pure mixture proportions in Table 4 were selected based on experience of past and previous knowledge of use of additive in soil stabilization technique. They form the basis for the Scheffe’s model development. The pure blends of their pseudo – components are assumed to correspond to these actual components.

Using Equation (20), the coefficient of the relationship of actual and pseudo components in matrix form is obtained as:

\[
A = S = \begin{bmatrix}
S_{11} & S_{21} & S_{31} \\
S_{12} & S_{22} & S_{32} \\
S_{13} & S_{23} & S_{33}
\end{bmatrix} = \begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}^T = \begin{bmatrix}
2 & 3 & 95 \\
4 & 6 & 90 \\
7 & 8 & 85
\end{bmatrix}
\]

From Equation (6), for second degree space lattice (M=Z), the simplex coordinate, \( x_i \) is given as: \( x_i = \frac{2}{m^2} \), \( i = 1, ..., m \); \( x_{i+1} = \frac{1}{2} \), \( i = 0, 1, 2, ..., 1 \) = 0, 0.5 and 1.

In a three variable and second degree scheffe’s polynomial and using Equation (7), the design space points is obtained as: Design space points, \( N = \frac{(9+m-1)}{m!(q-1)!} \) = 6 runs of experiment. The remaining design space points to make up the six design space points are in the order of the coded variables are given in the Table 5.

As in the control experiment, Oguaghamba and Mama [8] explained that the design space points are made of binary mixture and coded arbitrary but must be constrained to sum up to unity. Design points 7–9 are added up to be used as control points.
Using Equation (20), the actual components of the binary mixture, $S_{N,q}$ is obtained as follows:

\[
\begin{align*}
[S_{N,1}] &= [S_{1,1} S_{2,1} S_{3,1}] X \quad [X_{N,1}] \\
[S_{N,2}] &= [S_{1,2} S_{2,2} S_{3,2}] X \quad [X_{N,2}] \\
[S_{N,3}] &= [S_{1,3} S_{2,3} S_{3,3}] X \quad [X_{N,3}]
\end{align*}
\]

\[
\begin{align*}
[S'_{6,1}] &= [2 4 7] X \quad 0.50 = \begin{bmatrix} 2 \ 3 \ 6 \ 8 \ 95 \ 90 \ 85 \end{bmatrix} \\
[S'_{6,2}] &= [3 6 8] X \quad 0.00 = \begin{bmatrix} 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \end{bmatrix} \\
[S'_{6,3}] &= [95 90 85] X \quad 0.25 = \begin{bmatrix} 95 \ 90 \ 85 \end{bmatrix}
\end{align*}
\]

With these pure and binary mixture proportions in Table 4 and 5, as represented in Table 6, experimental test is conducted, the results are given as in Table 7.

Recall Equation (22), for (3,2) space lattice, $q=3$ and that transforms as follows:

\[
y(x,y,z; q=3) = y_1 x_1 (2x_1 - 1) + y_2 x_2 (2x_2 - 1) + y_3 x_3 (2x_3 - 1) + 4y_1x_1x_2 + 4y_2x_1x_3 + 4y_3x_2x_3
\]

(28)

Similarly, substituting the coefficients, $y_i$ and $y_9$ into Equation (28), we have:

\[
y_{CHR(4)} = 41x_1 (2x_1 - 1) + 43x_2 (2x_2 - 1) + 45x_3 (2x_3 - 1) + 192x_1x_2 + 220x_1x_3 + 188x_2x_3
\]

(29)

\[
y_{CHR(4)} = 25x_1 (2x_1 - 1) + 28x_2 (2x_2 - 1) + 31x_3 (2x_3 - 1) + 132x_1x_2 + 144x_1x_3 + 116x_2x_3
\]

(30)

\[
y_{UCS(7)} = 405.6x_1 (2x_1 - 1) + 424.5x_2 (2x_2 - 1) + 426.47x_3 (2x_3 - 1) + 1711.6x_1x_2 + 1732.48x_1x_3 + 1499.3x_2x_3
\]

(31)

---

**TABLE 6: BINARY BLEND PSEUDO AND ACTUAL COMPONENTS FOR SCHEFFE’S (3,2) LATTICE**

| N | Points on Factor Space | Actual Components | Expected Response | Pseudo Components |
|---|------------------------|-------------------|------------------|------------------|
|   |                        | $S_1$  | $S_2$  | $S_3$  | $X_1$  | $X_2$  | $X_3$  |
| 1 | A1                     | 2     | 3     | 95     | 1      | 0      | 0      |
| 2 | A2                     | 4     | 6     | 90     | 43     | 0      | 1      |
| 3 | A3                     | 7     | 8     | 85     | 45     | 0      | 1      |
| 4 | A4                     | 3     | 4.5   | 92.5   | 0.5    | 0      | 0      |
| 5 | A5                     | 4.5   | 5.5   | 90     | 0.5    | 0      | 0.5    |
| 6 | A6                     | 5.5   | 7     | 87.5   | 0      | 0.5    | 0      |
| 7 | A7                     | 5     | 6.25  | 88.75  | 0.25   | 0.25   | 0.5    |
| 8 | A8                     | 4.25  | 5.75  | 90     | 0.25   | 0.5    | 0.25   |
| 9 | A9                     | 3.75  | 5     | 91.25  | 0.5    | 0.25   | 0.25   |

---

**TABLE 7: CALIFORNIA BEARING RATIO AND UNCONFINED COMPRESSIVE STRENGTH VALUES CORRESPONDING TO THE DESIGN SPACE POINTS/LATTICE**

| N | Points on Factor Space | CBR (Unsoak) (%) | CBR (soak) (%) | UCS (kN/m²) | Scheffe’s (3,2) Lattice Coefficients, $y_i$ and $y_9$ |
|---|------------------------|------------------|---------------|-------------|------------------------------------------------------|
| 1 | A1                     | 41               | 25            | 405.66      | $y_1$                                                |
| 2 | A2                     | 43               | 28            | 424.50      | $y_2$                                                |
| 3 | A3                     | 43               | 31            | 424.47      | $y_3$                                                |
| 4 | A4                     | 48               | 33            | 427.90      | $y_{12}$                                            |
| 5 | A5                     | 55               | 36            | 433.12      | $y_{13}$                                            |
| 6 | A6                     | 47               | 29            | 374.83      | $y_{23}$                                            |
| 7 | C1                     | 52.55            | 34.11         | 407.19      |                                                      |
| 8 | C2                     | 50.54            | 33.01         | 405.63      |                                                      |
| 9 | C3                     | 52.23            | 34.35         | 417.89      |                                                      |

---

**TABLE 8: STUDENT T-TEST VARIABLES FOR CBR AND UCS RESPONSE**

| N | CBR (unsoak) (%) | CBR (soak) (%) | UCS (7 days) (kN/m²) |
|---|------------------|---------------|----------------------|
| Y₁ | $Y_{10}$ - $Y_{11}$ | $Y_{10}$ - $Y_{11}$ | $Y_{10}$ - $Y_{11}$ |
| C1 | 52.55            | 52.5           | -0.05                | 0.0025 | 34.11 | 34.13 | 0.02 | 0.0004 | 407.19 | 407.18 | -0.01 | 0.0001 |
| C2 | 50.54            | 50.5           | -0.04                | 0.0016 | 33.01 | 33.00 | -0.01 | 0.0001 | 405.63 | 405.59 | -0.04 | 0.0016 |
| C3 | 52.23            | 52.30          | 0.07                 | 0.0049 | 34.35 | 34.38 | 0.03 | 0.0009 | 417.89 | 417.85 | -0.04 | 0.0016 |
| Sum | 155.32          | 155.20         | -0.02                | 0.0009 | 101.47 | 101.47 | 0.04 | 0.0014 | 1230.71 | 1230.71 | -0.09 | 0.0033 |
| Mean (Y) | 51.77   | 51.78          | 0.00                 | 0.00   | 33.82 | 33.82 | 0.00 | 0.00   | 410.24 | 410.24 | 0.00 | 0.00   |
Recall Equation (23) and by substitution; we have:

\[
t_{CBR(\text{unsoak})} = \frac{\sqrt{(3-1)x(-0.02)}}{\sqrt{3x(0.0099)-(-0.02)^2}} = -0.028284271 \\
0.163095064 = -0.1734 = t_{cal}
\]

Since \( t_{cal} < t_{\alpha/N}(V_e) \); (ie) \(-0.1734 < 2.92\), we accept result [10].

\[
t_{CBR(\text{soak})} = \frac{\sqrt{(3-1)(x-0.04)}}{\sqrt{3x(0.0015)-(0.04)^2}} = 0.056568542 \\
0.050990195 = 1.1094 = t_{cal}
\]

Since \( t_{cal} < t_{\alpha/N}(V_e) \); (ie) 1.1094 < 2.92 we accept result [10].

\[
t_{UCS(7\text{days})} = \frac{\sqrt{(3-1)x(-0.09)}}{\sqrt{3x(0.0033)-(0.09)^2}} = -0.12727922 \\
0.042426406 = \sqrt{0.0000} = -3.0000
\]

Since \( t_{cal} < t_{\alpha/N}(V_e) \); (ie) \(-3.0000 < 2.92\), we accept result [10].

| TABLE 9: FISHER’S F-TEST VARIABLES FOR CBR RESPONSE |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| N               | \( Y_e \)       | \( Y_m \)       | \( Y_{\bar{e}} \) | \( Y_{\bar{m}} \) |
| C_1             | 52.55           | 52.5            | 0.78            | 0.6084          | 0.74            | 0.5476          |
| C_2             | 50.54           | 50.5            | 1.23            | 1.5129          | 1.26            | 1.5876          |
| C_3             | 52.23           | 52.3            | 0.46            | 0.2116          | 0.54            | 0.2916          |
| Sum             | 155.32          | 155.3           | 2.3329          | 2.4268          |                |                |
| Mean \((\bar{Y})\) | 51.77           | 51.76           |                |                |                |                |
| CBR (unsoak) (%) |                |                |                |                |                |                |
| C_1             | 34.11           | 34.13           | 0.29            | 0.0841          | 0.29            | 0.0841          |
| C_2             | 33.01           | 33.0            | -0.81           | 0.6561          | -0.84           | 0.7056          |
| C_3             | 34.35           | 34.38           | 0.53            | 0.2809          | 0.54            | 0.2916          |
| Sum             | 101.47          | 101.51          | 1.0211          | 1.0813          |                |                |
| Mean \((\bar{Y})\) | 33.82           | 33.84           |                |                |                |                |
| CBR (soak) (%)  |                |                |                |                |                |                |

| TABLE 10: FISHER’S F-TEST VARIABLES FOR UCS RESPONSE |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| N               | \( Y_e \)       | \( Y_m \)       | \( Y_{\bar{e}} \) | \( Y_{\bar{m}} \) |
| UCS (KN/m²)     | \( Y_e \)       | \( Y_m \)       | \( Y_{\bar{e}} \) | \( Y_{\bar{m}} \) |
| C_1             | 407.19          | 407.18          | -3.05           | 9.3025          | -3.03           | 9.1809          |
| C_2             | 405.63          | 405.59          | -4.61           | 21.2521         | -4.62           | 21.3444         |
| C_3             | 417.89          | 417.85          | 7.65            | 58.5225         | 7.64            | 58.3696         |
| Sum             | 1230.71         | 1230.62         | 89.0771         | 88.8949         |                |                |
| Mean \((\bar{Y})\) | 410.24          | 410.21          |                |                |                |                |

Recall Equation (24), considering Table 9.

\[
S_1^2 = S^2_e = \frac{2.3329}{3-1} = 1.16645
\]

\[
S_2^2 = S^2_m = \frac{2.4260}{3-1} = 1.2134
\]

Therefore, \( F_{CBR(\text{unsoak})} = \frac{1.16645}{1.2134} = 0.9613 \).

From the statistical tables according to [9] at 5% level of significance, \( \alpha \); at 2 degree of freedom, \( (V_e) \) (see Equation (27)).
IV. CONCLUSION

The following conclusions were drawn from the study. The combined percentage of SiO$_2$, Al$_2$O$_3$, Fe$_2$O$_3$, MgO and CaO of the Processed Palm Kernel Shell Ash is above 58% which satisfies Samuel Assam minimum requirement value of 58% specified for local stabilizers and as such can be used as a pozzolanic material. The loss in ignition value obtained was less than 7% maximum value required for local stabilizer. It means that most of the sample were absorbed by the system and the sample contains very little carbon.

The clay sample used in this experimental study contains elite minerals and was classified as A-2-5 using the AASHTO system of classification while the UNIFIED system of classification was CL (showing an inorganic clay of low to medium plasticity, gravelly clays, silty clays). Although at its natural state the soil is not suitable for use as a filled material.

Processed Palm Kernel Shell Ash (PPKSA) has reflected a significant increase in natural CBR and UCS values of Ntak – Subgrade, Scheffe’s second degree mixture models was developed to determine the mixture proportion ratio and unconfined compressive strength and California bearing ratio responses of the soil.

Models developed correspond with experimental results to a reasonable degree of accuracy and could fit and be successfully used in predicting the soil – PPKSA properties in the absence of experimental data for soil as they satisfy the significance level of differences with standard statistical requirement.
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