Neutrino Opacities at High Density and the Protoneutron Star Evolution

S Reddy, J Pons, M Prakash, and J M Lattimer

†Department of Physics & Astronomy
SUNY at Stony Brook, Stony Brook, NY 11794-3800, USA

‡Departament d’Astronomia, Universitat de València
E-46100 Burjassot, València, Spain

Abstract. The early evolution of a protoneutron star depends both on the equation of state and neutrino interactions at high density. We identify the important sources of neutrino opacity and through model calculations show that in-medium effects play an important role in determining the neutrino mean free paths. The effects due to Pauli-blocking and many-body correlations due to strong interactions reduce the neutrino cross sections by large factors compared to the case in which these effects are ignored. We discuss these results in the context of neutrino transport in a protoneutron star.

1. Introduction

To date, calculations of neutrino opacities in dense matter have received relatively little attention compared to other physical inputs such as the equation of state (EOS). The neutrino cross sections and the EOS are intimately related. This relationship is most transparent in the long-wavelength or static limit, in which the response of a system to a weak external probe is completely determined by the ground state thermodynamics (EOS). Thus, in this limit, neutrino opacities consisitent with the EOS can be calculated. However, when the energy and momentum transferred by the neutrinos are large, full consistency is often difficult to achieve in practice. Despite this, many salient features associated with an underlying dense matter model may be incorporated in the calculation of the neutrino opacities. In §2, we describe how this is accomplished for a non-relativistic potential model. The effects of nucleon-nucleon correlations on the neutrino mean free paths are calculated using the random phase approximation (RPA) in §3, where we show that the magnitudes of these many-body effects on both the scattering and absorption reactions are large. The total scattering cross section in a multi-component system including the effects of correlation due to both strong and electromagnetic interactions are presented in §4 using a relativistic formalism. The implications of these results for neutrino transport in a protoneutron star are in §5.

1 E-mail: reddy@nuclear.physics.sunysb.edu.
2. Neutrino Cross-Sections

In the non-relativistic limit for the baryons, the cross-section per unit volume for the two body reaction \( \nu + B_2 \to l + B_4 \) is given by \([4]\)

\[
\frac{\sigma(E_1)}{V} = \frac{G_F^2}{4\pi^2} (V^2 + 3A^2) \int_{-\infty}^{E_1} dq_0 \frac{E_3}{E_1} (1 - f_3(E_3)) \int_{|q_0|}^{2E_1 - q_0} dq q S(q_0, q). \tag{1}
\]

The particle labels are: 1:= incoming neutrino, 2:= incoming baryon, 3:= outgoing lepton (electron or neutrino), and 4:= outgoing baryon. The energy transfer to the baryons is denoted by \( q_0 = E_1 - E_3 \) and the momentum transfer \( q = |\vec{k}_1 - \vec{k}_2| \). Eq. (1) describes both the charged and neutral current reactions with the appropriate substitution of vector and axial vector coupling constants, \( V \) and \( A \), respectively \([4]\). The factor \( (1 - f_3) \) accounts for Pauli blocking of the final state lepton. The response of the system is characterized by the function \( S(q_0, q) \), often called the dynamic form factor. In \( \S 3 \), we consider the modifications required in the presence of spin and isospin dependent forces.

At the mean field level, the function \( S(q_0, q) \) may be evaluated exactly if the single particle dispersion relation is known \([4]\). We illustrate this using a potential model. In this case, if we retain only a quadratic momentum dependence, the single particle spectrum closely resembles that of a free gas and is given by

\[
E_i(p_i) = \frac{p_i^2}{2M_i^*} + U_i, \quad i = n, p. \tag{2}
\]

The single particle potentials \( U_i \) and the effective masses \( M_i^* \) are density dependent. Because the functional dependence of the spectra on the momenta is similar to that of the noninteracting case, it is possible to obtain an analytic expression for the dynamic form factor \( S(q_0, q) \) \([4]\). Explicitly,

\[
S(q_0, q) = \frac{M_2^* M_4^* T}{\pi q} \frac{\xi_- - \xi_+}{1 - \exp(-z)} \tag{3}
\]

where

\[
\begin{align*}
\xi_+ &= \ln \left[ \frac{1 + \exp((e_+ - \mu_2 + U_2)/T)}{1 + \exp((e_+ + q_0 - \mu_4 + U_4)/T)} \right] \\
e_+ &= \frac{2q^2}{2M^* \chi^2} \left[ \left( 1 + \frac{\chi M_4^* c}{q^2} \right) \pm \sqrt{1 + \frac{2\chi M_4^* c}{q^2}} \right],
\end{align*}
\]

with \( \chi = 1 - (M_4^*/M_2^*) \) and \( c = q_0 + U_2 - U_4 - (q^2/2M_4^*) \). The factor \( U_2 - U_4 \) is the potential energy gained in converting a particle of species “2” to a particle of species “4”.

In neutral current reactions, the initial and final state particles are the same; hence, the strong interaction corrections are due only to \( M_2^* \). For the charged current reactions, modifications due to interactions are twofold. First, the difference in the neutron and proton single particle potentials appears in the response function and also in \( \tilde{\mu} = \mu_n - \mu_p \). Second, the response depends upon the nucleon effective masses.

Results obtained using Eq. (3) are shown in Fig. 1, where electron neutrino absorption (top panels) and scattering (bottom panels) mean free paths are shown in matter containing nucleons and leptons. Various limiting forms are also shown for comparison. The charged current mean free paths are shown as thick solid lines in the bottom panel. These results demonstrate that effects due to kinematics, Pauli blocking, mass, and energy shifts are quantitatively important \([4]\).
3. Effects of Correlations

The random phase approximation (RPA) is particularly suited to investigate the role of particle-hole interactions on the collective response of matter. In this approximation, ring diagrams are summed to all orders. In terms of this more general response, the differential cross-sections are given by

$$\frac{1}{V} \frac{d^3\sigma}{d^2\Omega \, dE_3} = \frac{G_F^2}{\pi} (1 - f_3(E_3)) \, R(q_0, q),$$

(4)

where $R(q_0, q)$ describes the system’s response. For the neutral current reactions

$$R_S(q_0, q) = \left[ c_V^2 (1 + \cos \theta)S_{00}(q_0, q) + c_A^2 (3 - \cos \theta)S_{10}(q_0, q) \right].$$

(5)

$S_{00}$ and $S_{10}$ are the density-density and spin-density response functions. Similarly, the charged current response may be written in terms of the isospin-density and the spin-isospin density response functions as

$$R_A(q_0, q) = \left[ g_V^2 (1 + \cos \theta)S_{01}(q_0, q) + g_A^2 (3 - \cos \theta)S_{11}(q_0, q) \right].$$

(6)

3.1. Neutral Currents

Neutrino scattering on neutrons is the dominant source of scattering opacity. We therefore begin by considering the response in pure neutron matter. For a one component
system, the RPA response functions are given by

\[
S_{ij}(q_0, q) = \left[ \frac{1}{1 - \exp(q_0/T)} \right] \frac{\text{Im} \, \Pi^0(q_0, q)}{\epsilon_{ij}}
\]

\[
\epsilon_{ij} = \left[ 1 - V_{ij} \text{Re} \, \Pi^0(q_0, q) \right]^2 + [V_{ij} \text{Im} \, \Pi^0(q_0, q)]^2.
\] (7)

The dielectric screening function \(\epsilon_{ij}\) is the modification introduced by the RPA to Eq. (1). The zeros of \(\epsilon_{ij}\) correspond to collective excitations, such as zero-sound and spin zero-sound. The potential \(V_{ij}\), which measures the strength of the particle-hole interaction in the medium, is a function of density, temperature, \(q_0\), and \(q\). Both the energy and momentum transferred in the particle-hole channel are of order \(T\). Since the particle-hole interaction is short ranged (\(\sim 1/\text{meson mass}\)), explicitly density dependent interactions play a major role in determining the magnitude of \(V_{ij}\). Momentum dependent interactions introduce additional structure to \(V_{ij}\), which in turn alters the structure of \(S_{ij}\). We will consider these latter effects separately.

For situations in which \(q/k_{Fi} << 1\) and \(T/\mu_i << 1\), the quasi-particle interaction may be obtained using Fermi-liquid theory. Here, the quasi-particle interaction is given by \(V_{ij} = (\delta^2 E/\delta n_i \delta n_j)\), and is usually expressed in terms of the Fermi-liquid parameters. For pure neutron matter, the force in the spin independent channel is given by \(F_0\), while the force in the spin dependent channel is given by \(G_0\). Thus, \(V_{00} = F_0/N_0\) and \(V_{10} = G_0/N_0\), where \(N_0\) is the density of states at the Fermi surface [5]. We employ the results of Bäckmann et al [6] who have calculated \(F_0\) and \(G_0\) in pure neutron matter for the densities of interest here.

In Fig. 2, the differential cross sections for \(\nu + n \rightarrow \nu + n\) are shown. The effects due to correlations lead to significant reductions at small \(q_0\), since the interaction is repulsive and the excitation of collective modes enhances the response at large \(q_0\). The well defined
zero-sound and spin zero-sound seen at $T = 0$ are damped at finite temperatures. The contribution from the low $q_0$ region dominates the total cross-section due to final state blocking. The presence of a repulsive particle-hole force acts to reduce the neutrino cross sections for neutrino energies of order $T$. 

The composition of charge neutral, $\beta$-equilibrated stellar matter depends on the EOS of dense matter, which contains an admixture of protons and electrons. The correlations due to interactions between the different particle species and electromagnetic correlations also play an important role. In §4, we discuss a relativistic framework to describe the RPA response of a mixture of neutrons, protons, and electrons.

### 3.2. Charged Currents

The charged current reaction is kinematically different from the neutral current reaction, since the energy and momentums transfers are not limited only by the matter temperature. The energy transfer is typically of order $\hat{\mu} = \mu_n - \mu_p$. The charged current probes smaller distances; hence, the $q_0$ and $q$ dependencies of the particle-hole force are likely to play an important role. In symmetric nuclear matter $\hat{\mu} = 0$, and therefore, the situation is very similar to the neutral current case. Since the collective response of laboratory nuclei is well established in nuclear physics, we begin by considering the response of symmetric nuclear matter to a charged current weak probe. Here the strength parameters $V_{01}$ for the spin independent iso-spin channel and the $V_{11}$ for the spin dependent iso-spin channel are required. In nuclear matter, the particle-hole force (retaining only the $l = 0$ terms) is given by

$$F(k_1,k_2) = N_0^{-1} [F_0 + G_0 \sigma_1 \sigma_2 + \tau_1 \tau_2 (F_0' + G_0' (\sigma_1 \sigma_2))].$$

For the charged current reaction, isospin and charge are transferred along the particle-hole channel; hence, only the last two terms in Eq. (8) contribute. The potentials required
to calculate the RPA response in Eq. (7) are given by $V_{10} = 2F_0/N_0$ and $V_{11} = 2G_0/N_0$, where the factor two arises due to isospin considerations. The Fermi-liquid parameters may be calculated from an underlying dense matter model, as for example, a Skyrme model, which successfully describes the low-lying excitations of nuclei [7]. In Fig. 3, the response of symmetric nuclear matter to the charged current probe is shown. Results are for two different parameterizations of the Skyrme force, SGII and SkM*. The excitation of giant-dipole and Gamow-Teller resonances shifts the strength to large $q_0$, and as in the case of scattering, the low $q_0$ response is significantly suppressed.

In asymmetric matter, the energy and momentum transfers are large. Therefore, the momentum dependence of the particle-hole force becomes important. At high momentum transfer, the conservation of the vector current implies that the response function $S_{10}$ is not strongly modified. Further, since its contribution to the differential cross section is roughly three times smaller than the spin-dependent response function $S_{11}$, we focus on the Gamow-Teller part and assume that the Fermi matrix elements are not screened. The iso-vector interaction in the longitudinal channel arises due to $\pi$ exchange, and in the transverse channel due to $\rho$ meson exchange. In addition, to account for the large repulsion observed, Migdal [5] introduced screening in this channel through the parameter $g'$. This form for the particle-hole interaction has been successful in describing a variety of nuclear phenomena. The longitudinal and transverse potentials in the $\pi + \rho + g'$ model are given by [8]

\[
V_L(q_0, q) = \frac{f_\pi^2}{m_\pi^2} \frac{q^2}{q_0^2 - q^2 - m_\pi^2} F_\pi^2(q) + g' \\
V_T(q_0, q) = \frac{f_\rho^2}{m_\rho^2} \frac{q^2 C_\rho}{q_0^2 - q^2 - m_\rho^2} F_\rho^2(q) + g',
\]

where $F_\pi = (\Lambda^2 - m_\pi^2)/(\Lambda^2 - q^2)$ and $F_\rho = (\Lambda_\rho^2 - m_\rho^2)/(\Lambda_\rho^2 - q^2)$ are the $\pi NN$ and $\rho NN$ potentials.

Figure 4. Neutrino absorption cross sections in asymmetric matter.
form factors. Numerical values used are $C_\rho = 2, g' = 0.7, \Lambda = 1.4 \text{ GeV}$, and $\Lambda_\rho = 2 \text{ GeV}$. The RPA response function then takes the form

$$S_{11}(q_0, q) = \left[\frac{1}{1 - \exp(q_0/T)}\right] \text{Im} \, \Pi^0(q_0, q) \left(\frac{1}{3\epsilon_L} + \frac{2}{3\epsilon_T}\right) \epsilon_{L,T} = \left[1 - 2V_{L,T} \text{Re} \, \Pi^0(q_0, q)\right]^2 + \left[2V_{L,T} \text{Im} \, \Pi^0(q_0, q)\right]^2$$

(10)

The cross sections in charge neutral stellar matter with a fixed lepton fraction $Y_L = 0.4$ are shown in Fig. 4. The RPA screening reduces the cross sections by about a factor of two. The inclusion of virtual $\Delta$-hole excitations reduces the cross sections further. Unlike in the case of ordinary $\beta$ decay, where the $\Delta$-hole contribution is important, the role of these excitations is marginal here due to the large energy and momentum transfers.

4. Relativistic Treatment

![Figure 5. Neutrino scattering cross sections in a relativistic approach.](image)

At high density the baryons become increasingly relativistic and thus a relativistic description may be more appropriate. Such a description also allows us to treat the baryons and the electrons on an equal footing. The basic formalism and some illustrative results may be found in [3, 4]. Here we present results for a system of neutrons, protons, and electrons at finite temperature. The ground state is described by a field-theoretical model at the mean field level, in which isoscalar $\sigma, \omega$ and isovector $\rho$ meson exchanges modify the in-medium baryon propagators. We calculate the Hartree response by accounting for these modifications. Correlations are incorporated through the relativistic random phase approximation (RRPA), where the particle-hole interaction is mediated by the $\sigma$ and $\omega$ mesons between the baryons, and electromagnetic interactions between protons and electrons are mediated by photons. Fig. 5 shows the results for different
temperatures at a baryon density of 0.32 fm$^{-3}$. Compared to the non-relativistic models, the screening effects are small, the dominant suppression arising mainly due to density dependent nucleon effective masses.

5. Discussion

We have highlighted the influence of correlations and collective phenomena on the neutrino opacities in dense matter. Our findings here indicate that the neutrino cross sections are significantly reduced, and the average energy transfer in neutrino-nucleon interactions is increased due to the presence correlations in the medium. Several improvements are necessary before we can assess the influence of these results on the macrophyscial evolution of a protoneutron star. Among the most important of these are calculations that provide (1) the dynamic form factor, (2) the particle-hole and particle-particle interactions, (3) the renormalization of the axial charge, and, (4) the means to assess the role of multi-pair excitations, in charge neutral, beta-equilibrated dense matter at finite temperature. While investigations along these directions are in progress (see also [3]), some general trends may be anticipated. The many-body effects studied here, including the improvements listed above, suggest considerable reductions in the opacities compared to the free gas estimates often employed in many applications. In the particular instance of the early evolution of a protoneutron star, the suggested modifications imply shorter time scales over which the deleptonization and cooling occur. Since mean free paths $\lambda$ are much less than the stellar radius $R$, evolution is via diffusion. To order of magnitude, therefore, the timescale $\tau \propto 1/\lambda$. However, there are important feedbacks between these evolutionary consequences and the underlying EOS, in particular the specific heat of multicomponent matter, which have to be studied before firm conclusions may be drawn.

Acknowledgments

We thank G. E. Brown for useful discussions. This work was supported in part by the U. S. Department of Energy under contract DOE/DE-FG02-88ER-40388, and the NASA grant NAG 52863.

References

[1] Sawyer R F 1975 Phys. Rev. D11 2740; 1989 Phys. Rev. C40 865.
[2] Iwamoto N and Pethick C J 1982 Phys. Rev. D25 313.
[3] Horowitz C J and Wehrberger K 1991 Nucl. Phys. A531 665; 1991 Phys. Rev. Lett. 66 272; 1992 Phys. Lett. B226 236.
[4] Reddy S, Prakash M, and Lattimer J M 1997 Phys. Rev. D submitted.
[5] Migdal A B 1962, Theory of Finite Fermi Systems and Applications to Atomic Nuclei (New York: Interscience).
[6] Bäckmann S O and Källman C.- G 1973 Phys. Lett. B43 263.
[7] Bertsch G and Tsai S F 1975 Phys. Rep. 18 125.
[8] Oset E, Toki H, and Weise W 1982 Phys. Rep. 83 280.
[9] Burrows A and Sawyer R F 1989 Phys. Rev. D submitted.