Big bang nucleosynthesis and entropy evolution in $f(R, T)$ gravity

Snehasish Bhattacharjee$^{1,a}$, P. K. Sahoo$^{2,b}$

1 Department of Astronomy, Osmania University, Hyderabad 500007, India
2 Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad 500078, India

Received: 13 January 2020 / Accepted: 24 March 2020 / Published online: 6 April 2020
© Società Italiana di Fisica and Springer-Verlag GmbH Germany, part of Springer Nature 2020

Abstract The present article is devoted to constraining the model parameter $\chi$ for the $f(R, T) = R + \chi T$ gravity model by employing the constraints coming from big bang nucleosynthesis. We solve the field equations and constrain $\chi$ in the range $-0.14\kappa^2 < \chi < 0.84\kappa^2$ (where $\kappa^2 = \frac{8\pi G}{c^4}$) from the primordial abundances of light elements such as helium-4, deuterium and lithium-7. We found the abundances of helium-4 and deuterium agree with theoretical predictions; however, the lithium problem persists for the $f(R, T)$ gravity model. We also investigate the evolution of entropy for the constrained parameter space of $\chi$ for the radiation and dust universe. We report that entropy is constant when $\chi = 0$ for the radiation-dominated universe, whereas for the dust universe, entropy increases with time. We finally use the constraints to show that $\chi$ has a negligible influence on the cold dark matter annihilation cross section.

1 Introduction

The current accelerated expansion of the universe favors big bang cosmology. The model predicts the abundances of several light elements of the primordial universe with great precision. The elements were produced as a result of nuclear fusion started seconds after the big bang and lasted for some minutes. Additionally, the model predicts inflation which is a super-exponential increase in the volume of the universe for a very short time ($10^{-43}$ s). Inflation has been successful in solving the flatness, horizon and homogeneity problems of the universe [1].

However, many cosmological puzzles exist which hitherto cannot be explained by the standard big bang cosmology such as origin of dark matter and dark energy, cosmological constant problem, cosmic coincidence problem and the exact form of the inflation potential [2–4]. To answer these problems, modifying GR has become a promising alternative, giving rise to a plethora of modified gravity theories.

$f(R, T)$ gravity is a widely studied modified gravity theory introduced in the literature in [5] and is a generalization of $f(R)$ gravity (see [6,7] for a review on modified gravity theories). In this theory, the Ricci scalar $R$ in the action is replaced by a combined function
of $R$ and $T$ where $T$ is the trace of the energy–momentum tensor. $f(R, T)$ gravity has been widely employed in various cosmological scenarios and has yielded interesting results in areas such as dark matter [8], dark energy [9], super-Chandrasekhar white dwarfs [10], massive pulsars [11, 12], wormholes [13–22], gravitational waves [23, 24], baryogenesis [25], bouncing cosmology [26] and in varying speed of light scenarios [27].

In this article, we are interested in constraining the model parameter of $f(R, T)$ gravity theory for the ansatz $f(R, T) = R + \chi T$, where $\chi$ is the model parameter. Constraining $\chi$ can help us to better understand the impact of $\chi$ in cosmological models and also in the above-mentioned astrophysical areas.

Big bang nucleosynthesis can be an excellent way to constrain the model parameters of any modified gravity theory as the abundances of primordial light elements such as deuterium ($^2H$), helium ($^4He$) and lithium ($^7Li$) have been observationally constrained to great accuracy. These abundances are directly related to the Hubble parameter $H$, which ultimately involve the model parameters of any chosen modified gravity theory. This method has been successfully employed to constrain the model parameters of $f(R)$ gravity [28–30], $f(T)$ gravity [31], scalar–tensor gravity models [32] and to test the viabilities of Brans–Dicke cosmology with varying $\Lambda$ [33], higher-dimensional dilaton gravity theory of steady-state cosmological (HDGS) model in the context of string theory [2] and massive gravity theory [34]. The discrepancy between predicted and observed abundances of lithium (‘lithium problem’) is investigated in [35,36] (and in references therein).

The paper is organized as follows: In Sect. 2, we provide an overview of $f(R, T)$ gravity. In Sect. 3, we summarize big bang nucleosynthesis and present a thorough analysis to constrain $\chi$. In Sect. 4, we investigate the evolution of entropy for the radiation- and matter-filled universes for the constrained range of $\chi$. In Sect. 5, we investigate whether $\chi$ influences the cold dark matter annihilation cross section, and Sect. 6 is devoted to discussions and conclusions.

2 Overview of $f(R, T)$ gravity

The action in $f(R, T)$ gravity is given by

$$S = \int \sqrt{-g} \left[ \frac{1}{2\kappa^2} f(R, T) + L_m \right] d^4x$$

(1)

where $L_m$ represents the matter Lagrangian and $\kappa^2 = \frac{8\pi G}{c^4}$.

Stress–energy–momentum tensor for the matter fields is given as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(-gL_m)}{\delta g^{\mu\nu}} = g_{\mu\nu} L_m - 2 \frac{\delta L_m}{\delta g^{\mu\nu}}$$

(2)

varying the action (1) with respect to the metric yields

$$f^1_R(R, T) R_{\mu\nu} + \Pi_{\mu\nu} f^1_R(R, T) - \frac{1}{2} g_{\mu\nu} f(R, T) = \kappa^2 T_{\mu\nu} - (T_{\mu\nu} + \Theta_{\mu\nu}) f^1_T(R, T)$$

(3)

where

$$-\nabla_\mu \nabla_\nu + g_{\mu\nu} \Box = \Pi_{\mu\nu}$$

(4)

$$g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g_{\mu\nu}} = \Theta_{\mu\nu}$$

(5)
and \( f_{,R}^1(R, T) \). Upon contraction (3) with \( g^{\mu\nu} \), the trace of the field equations is obtained as

\[
f_{,R}^1(R, T) R - 2f(R, T) + 3\Box f_{,R}^1(R, T) = -\left(\Theta + T\right) f_{,T}^1(R, T) + \kappa^2 T.
\] (6)

We now consider a flat FLRW background metric as

\[
ds^2 = dt^2 - a(t)^2 [dx^2 + dy^2 + dz^2]
\] (7)

where \( a(t) \) denotes the scale factor. For a universe dominated by a perfect fluid, the matter Lagrangian density is given as \( \mathcal{L}_m = -p \). Substituting this to (3) and (6) yields

\[
\frac{1}{f_{,R}^1(R, T)} \left[ -3\dot{R}H f_{,R}^2(R, T) + pf_{,T}^1(R, T) - R f_{,R}^1(R, T) + \frac{1}{2} (f(R, T)) \right]
\]
\[
+ \frac{f_{,T}^1(R, T) + \kappa^2}{f_{,R}^1(R, T)} \rho = 3H^2
\]
\[
- \frac{1}{2} \left( f(R, T) + \dot{R} f_{,R}^1(R, T) + \ddot{R} f_{,R}^2(R, T) - R f_{,R}^1(R, T) \right) - pf_{,T}^1(R, T)
\]
\[
+ 2H \dot{R} f_{,R}^1(R, T) + \kappa^2 f_{,T}^1(R, T) - p = -3H^2 - 2\ddot{H}
\] (8)

where \( H \) denotes the Hubble parameter, overhead dots denote the derivative with respect to time, \( p \) represents the pressure and \( \rho \) represents the density with \( T = \rho - 3p \).

Set \( f(R, T) \) functional form to be

\[
f(R, T) = R + \chi T.
\] (10)

Substituting (10) to (8) and solving for Hubble parameter \( (H_{f(R,T)}) \), we obtain

\[
H_{f(R,T)} = \frac{\epsilon}{t},
\] (11)

where

\[
\epsilon = - \frac{1}{3} \left[ -2\kappa^2 + (\omega - 3) \chi \right] / (\omega + 1) (\chi + \kappa^2).
\] (12)

where \( \omega = p/\rho \) denote the EoS parameter. The scale factor \( a(t) \) takes the form

\[
a \sim t^\epsilon.
\] (13)

The expression of density \( \rho \) reads

\[
\rho = \frac{(2\kappa^2 - \chi (\omega - 3)) \left( \chi (3 + 8\chi - \omega) + 2\kappa^2 (1 + \chi (3 + \omega)) \right)}{3t^2 (\kappa^2 + \chi)^2 (1 + 6\chi + 8\chi^2) (1 + \omega^2)}.
\] (14)

For a radiation-dominated universe \( (\omega = 1/3) \), the expression of the Hubble parameter reads

\[
H_{f(R,T)} = \left[ \frac{8\chi/3 + 2\kappa^2}{4(\kappa^2 + \chi)} \right] / t.
\] (15)

In Einstein’s GR, the expression of the Hubble parameter in radiation-dominated universe reads

\[
H = \frac{1}{2t}.
\] (16)
3 Nucleosynthesis in \( f(R, T) \) gravity

In this method, we are interested in finding a suitable value or range of \( \chi \) which can suffice the primordial abundances of light elements. Specifically, we will be studying the ratio of the Hubble parameter in \( f(R, T) \) gravity to the Hubble parameter of standard big bang cosmology for the radiation-dominated universe. The ratio is represented as

\[
Z = \frac{H_f(R,T)}{H_{\text{SBBN}}} \tag{17}
\]

where \( H_f(R,T) \) is given by \( (15) \) and \( H_{\text{SBBN}} \) is given by \( (16) \) and SBBN stands for the standard big bang nucleosynthesis. The primordial abundances of the light elements \( (^2D, ^4He, ^7Li) \) depend on the expansion rate of the universe and on the baryon density \([37,38]\). The baryon density parameter reads

\[
\eta_{10} \equiv 10^{10} \eta_B \equiv 10^{10} \frac{\eta_B}{\eta_y} \tag{18}
\]

where \( \eta_{10} \simeq 6 \) and \( \eta_B \) represents the baryon-to-photon ratio \([39]\).

\( Z \neq 1 \) corresponds to the non-standard expansion factor. This can arise due to GR modification or due to the presence of additional light particles such as neutrinos which would make the ratio to be, \( Z = (1 + \frac{2}{43}(N_\nu - 3))^{1/2} \) [2]. However, we are interested for the case where the value of \( (Z - 1) \) comes from GR modification and hence we shall assume \( N_\nu = 3 \).

3.1 \(^4He\) abundance in \( f(R, T) \) gravity

The first step in producing helium \( (^4He) \) starts with producing \( ^2H \) from a neutron \((n)\) and a proton \((p)\). After that, deuterium is converted into \(^3He\) and tritium \((T)\).

\[
n + p \rightarrow ^2H + \gamma; \quad ^2H + ^2H \rightarrow ^3He + n; \quad ^2H + ^2H \rightarrow ^3H + p. \tag{19}
\]

\(^4He\) is finally produced from the combination of \(^3H\) with \(^2H\) and \(^3He\):

\[
^2H + ^3H \rightarrow ^4He + n; \quad ^2H + ^3He \rightarrow ^4He + p. \tag{20}
\]

The simplest way to ascertain the \(^4He\) abundance is from the numerical best fit given in [40,41]

\[
Y_p = 0.2485 \pm 0.0006 + 0.0016 \left[ (\eta_{10} - 6) + 100 (Z - 1) \right] \tag{21}
\]

For \( Z = 1 \), we recover the SBBN \(^4He\) fraction, which reads \( (Y_p)|_{\text{SBBN}} = 0.2485 \pm 0.0006 \). Observations reveal the \(^4He\) abundance to be \( 0.2449 \pm 0.0040 \) [42]. Thus, we obtain

\[
0.2449 \pm 0.0040 = 0.2485 \pm 0.0006 + 0.0016 [100(Z - 1)] \tag{22}
\]

where we have set \( \eta_{10} = 6 \). This constrains \( Z \) in the range 1.0475 \pm 0.105.

3.2 \(^2H\) abundance in \( f(R, T) \) gravity

Deuterium \(^2H\) is produced from the reaction \( n + p \rightarrow ^2H + \gamma \). Deuterium abundance can be ascertained from the numerical best fit given in [37]

\[
y_{DP} = 2.6(1 \pm 0.06) \left( \frac{6}{\eta_{10} - 6(Z - 1)} \right)^{1.6} \tag{23}
\]
Table 1 The abundances He-4, deuterium and Li-7 for different models

| Models and data/abundances | Yp         | yDp        | yLip       |
|----------------------------|------------|------------|------------|
| SBBN model                 | 0.2485 ± 0.0006 | 2.6 ± 0.16 | 4.82 ± 0.48 |
| f(R,T) Gravity             | 0.2574 ± 0.0006 | 2.8485 ± 0.1715 | 5.2927 ± 0.52925 |
| Observational data         | 0.2449 ± 0.0040 [42] | 2.55 ± 0.03 [42] | 1.6 ± 0.3 [42] |

For Z = 1&$\eta_{10} = 6$, $y_{Dp}|_{SBBN} = 2.6 \pm 0.16$. Observational constraint on deuterium abundance is $y_{Dp} = 2.55 \pm 0.03$ [42]. Thus, substituting this to 23, we obtain

$$2.55 \pm 0.03 = 2.6(1 \pm 0.06)\left(\frac{6}{\eta_{10} - 6(Z - 1)}\right)^{1.6}.$$ (24)

This constraints Z in the range $Z = 1.062 \pm 0.444$. The constraint on Z of the deuterium abundance partially overlaps with that of the helium abundance. Thus, $\chi$ can be fine-tuned to fit the abundances for both $^2H$ and $^4He$.

3.3 $^7Li$ abundance in $f(R,T)$ gravity

The lithium abundance is puzzling in the sense that the $\eta_{10}$ parameter which precisely fits the abundances of other elements successfully does not fit the observations of $^7Li$ and the ratio of the expected SBBN value of $^7Li$ abundance to the observed one is between 2.4 and 4.3 [2,43]. Thus, neither SBBN nor any modified gravity theory can suffice the low abundance of $^7Li$. This is known as the lithium problem [2].

The numerical best-fit expression for $^7Li$ abundance reads [37]

$$y_{Lip} = 4.82(1 \pm 0.1)\left[\frac{\eta_{10} - 3(Z - 1)}{6}\right]^2.$$ (25)

Observational constraint on lithium abundance is $y_{Lip} = 1.6 \pm 0.3$ [42]. The constraint on Z to fit the $^7Li$ abundance is $Z = 1.960025 \pm 0.076675$ which clearly does not overlap with the deuterium-2 and helium-4 constraints.

3.4 Results

From Table 1, it is clear that $f(R,T)$ gravity yields excellent estimates for the abundances of helium and deuterium which match better to observations than the SBBN model. However, the abundance of lithium is still a problem for both the models (SBBN and $f(R,T)$ gravity). In Fig. 1, we show $\chi$ as a function of Z. For Z in the range $0.9425 \leq Z \leq 1.1525$, the theoretical predictions for the abundances of deuterium and helium agree with observations. This constraints $\chi$ in the range $-0.14k^2 \lesssim \chi \lesssim 0.84k^2$.

4 Entropy evolution in $f(R,T)$ gravity

Baryon-to-entropy ratio is a useful parameter characterizing the over-abundance of matter over anti-matter in the universe. Since the law of conservation of energy momentum is not maintained in $f(R,T)$ gravity, we investigate how this affects adiabaticity [44–46]. In SBBN model, the entropy of the universe is a conserved quantity throughout its evolution and this is
due to the fact that at low energies, baryon number is neither created nor destroyed since there are no decays and consequently the baryon-to-entropy ratio $\eta_S$ is a constant [46]. Equivalently, once the large-scale annihilation processes have concluded, the baryon-to-photon ratio $\eta_B$ is also a constant, and both quantities can be connected easily [46].

From the first law of thermodynamics, we obtain

$$dE + pdV = TdS \quad (26)$$

where $S = s(a^3)$ and $E = \rho(a^3)$ are the entropy and internal energy of the universe, respectively. This gives [46]

$$d(\rho a^3) + pd(a^3) = TdS \rightarrow \frac{T}{a^3} \dot{S} = \dot{\rho} + 4H\rho \quad (27)$$

From statistical mechanics, density $\rho$ is related to temperature $T$ as [47]

$$\rho = \frac{\pi^2}{30} g_{ss} T^4 \quad (28)$$

where $g_{ss} = 107$ is the effective number of relativistic degrees of freedom contributing to the entropy of the universe [46].

Substituting all the values, we obtain

$$\dot{S} = \frac{1.86121r^{-\left(\frac{3+2\chi}{2+\chi}\right)} \chi \left(0.15188 + 0.911281\chi + 1.55255\chi^2 + 0.810028\chi^3\right)}{(1 + \chi)^3 \left(0.125 + 0.75\chi + \chi^2\right) \left[\frac{(3+4\chi)(3+14\chi+12\chi^2)}{T^2(1+\chi)^2(1+6\chi+8\chi^2)}\right]^{0.25}}. \quad (29)$$

In Fig. 2, we observe that $\dot{S}$ is positive for $\chi > 0$ and negative for $\chi < 0$ at early times but converges to zero at late times. From Table 2, we further note that $\dot{S} = 0$ for $\chi = 0$ (GR) for the radiation universe. However, Fig. 3 shows that $\dot{S} > 0$ for the dust universe and it is also evident that $\dot{S}$ increases as $\chi$ increases. It is also evident that $\dot{S}$ decreases slowly with time for the dust universe.
Fig. 2 Time evolution of $\dot{S}$ in radiation universe for $-0.14\kappa^2 \lesssim \chi \lesssim 0.84\kappa^2$.

Table 2 Rate of change of entropy ($\dot{S}$) for different models

| Models               | Rate of change of entropy ($\dot{S}$) | Dust universe ($\omega = 0$) |
|----------------------|--------------------------------------|-----------------------------|
| Radiation universe   |                                       |                             |
| $\omega = 1/3$       | $2.01018 \left( \frac{t^{0.28}}{t^2} \right)$ | $1.84325 \left( \frac{t^{1.725}}{t^2} \right)$ |
| $\omega = 0$         | $2.05634 \left( \frac{t^{0.5425}}{t^2} \right)$ | $2.05634 \left( \frac{t^{0.5425}}{t^2} \right)$ |

Fig. 3 Time evolution of $\dot{S}$ in dust universe for $-0.14\kappa^2 \lesssim \chi \lesssim 0.84\kappa^2$.

5 Dark matter annihilation cross section in $f(R, T)$ gravity

Recent cosmological observations have constrained the normalized cold dark matter density in the range [48]

$$0.075 \lesssim \Omega_{\text{cdm}}h^2 \lesssim 0.126.$$  (30)

In this section, we shall assume dark matter to be composed of weakly interacting massive particles (WIMPs). In [28], the authors derived an analytical expression where the WIMP
cross section $\bar{\sigma}$ is written in terms of the relic density of dark matter, its mass $m$ and the power $n$ for the power-law $f(R)$ gravity model of the form $f(R) \sim R^n$. We shall now investigate the role of $\chi$ in dark matter annihilation cross section for a given WIMP mass.

The expression relating to the dark matter relic density, its mass, dark matter annihilation cross section and parameters of a modified gravity model reads [28]

$$\Omega_{\text{cdm}}h^2 = 1.07 \times 10^9 \frac{(\bar{\sigma} + 1)x_f}{(h_*/g_{*s})^{1/2}}M_p \bar{\sigma}$$

where

$$\bar{m} = m + (1 - n)$$

in which $\bar{m} = m$ for GR and $m = 0$ & 1 correspond to s-wave and p-wave polarizations, and for $n = 1$, GR is recovered.

$x_f$ is the freeze-out temperature and given as [28,47]

$$x_f = \ln[0.038(\bar{m} + 1)(g/g_{*s})^{1/2}]M_p m \bar{\sigma} - (\bar{m} + 1) \ln[\ln[0.038(\bar{m} + 1)(g/g_{*s})^{1/2}]M_p m \bar{\sigma}]$$

where $g = 2$ is the spin polarizations of the dark matter particle [28] and $M_p$ is the Planck mass.

In [28], the authors found substantial influence of $n$ in $\bar{\sigma}$ although $n$ had very little deviation from GR ($n - 1 \lesssim 0.00016$). We now modify $\bar{m}$ for our $f(R, T)$ gravity model and check the influence of $\chi$ on $\bar{\sigma}$.

For $\bar{m}$ of the form

$$\bar{m} = m + \chi,$$

we get $\bar{m} = m$ when $\chi = 0$. Now from BBN, $\chi$ is constrained in the range $-0.14\kappa^2 \lesssim \chi \lesssim 0.84\kappa^2$ which is $\mathcal{O}10^{-43}$. Hence, our $f(R, T)$ gravity model produces $\bar{\sigma}$ very close to that predicted from GR. Nonetheless, it would be interesting to do the same analysis with $f(R, T)$ gravity models with a power-law dependence on $T$.

### 6 Discussion

Modified gravity theories are becoming popular owing to the failures of GR in explaining the current acceleration of the universe. In modified gravity theories, the model parameters are fine-tuned to obtain the desired results which sometimes differ significantly from GR. In this work, we investigate the viability of the most widely studied and simplest minimal matter–geometry coupled $f(R, T)$ gravity model of the form $f(R, T) = R + \chi T$ in cosmological models and in many astrophysical areas.

The present manuscript uses the constraints of abundances of light elements such as helium-4, deuterium and lithium-7 to constrain the model parameter $\chi$ to unprecedented accuracy. From the analysis, we report a tight constraint on $\chi$ in the range $-0.14\kappa^2 \lesssim \chi \lesssim 0.84\kappa^2$.

We also study the evolution of entropy for the constrained parameter space of $\chi$ for the radiation and dust universe. We report that entropy ($S$) is constant for $\chi = 0$ for the radiation-dominated universe, whereas for the dust universe, $\dot{S} > 0$ for the allowed range of $\chi$.
We also found that $\chi$ has a negligible influence on dark matter annihilation cross section ($\bar{\sigma}$) and produces $\bar{\sigma}$ very close to that predicted by GR. The constraints on $\chi$ obtained from the present analysis make it clear-cut that the parameter $\chi$ has a negligible influence in cosmological models and in the above-mentioned astrophysical areas. It would certainly be interesting to apply the method to constrain the model parameters for other $f(R, T)$ gravity models and to check their viability in representing the current state of the universe.

As a final note, we add that in [49], the authors reported that the gravitational energy–momentum pseudotensor can also be an important tool in distinguishing and constraining different theories of gravity. Specifically, in [50], the authors reported that the gravitational pseudotensor is useful to identify the dissimilarities in quadrupolar gravitational radiation coming from Einstein’s gravity and $f(R)$ gravity. This idea was further extended to teleparallel gravity in [49]. Since gravitational waves differ substantially from one theory of gravity to another [51,52], detection of the polarization modes of the gravitational radiation can be promising to constrain extended theories of gravity [49].

Acknowledgements SB thanks Biswajit Pandey for constant support and motivation. SB also thanks Suman Sarkar and Biswajit Das for helpful discussions. PKS acknowledges CSIR, New Delhi, India, for financial support to carry out the research project [No.03(1454)/19/EMR-II Dt.02/08/2019]. We are very much grateful to the honorable referee and the editor for the illuminating suggestions that have significantly improved our work in terms of research quality and presentation.

References

1. For a review, see e.g., A. Linde, Lect. Notes Phys. 738, 1–54 (2008)
2. S. Boran, E.O. Kahya, Adv. High Energy Phys. 2014, 282675 (2014). arXiv:1310.6145
3. P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. 75, 559606 (2003)
4. S. Perlmutter et al., Astrophys. J. 517, 565–586 (1999)
5. T. Harko et al., Phys. Rev. D 84, 024020 (2011)
6. S. Capozziello, M.D. Laurentis, Phys. Rept. 509, 167 (2011)
7. S. Capozziello, Int. J. Mod. Phys. D 11, 483 (2002)
8. R. Zaregonbadi et al., Phys. Rev. D 94, 084052 (2016)
9. G. Sun, Y.-C. Huang, Int. J. Mod. Phys. D 25, 1650038 (2016)
10. F. Rocha et al. arXiv:1911.08894 (2019)
11. S.I. dos Santos, G.A. Carvalho, P.H.R.S. Moraes, C.H. Lenzi, M. Malheiro, Eur. Phys. J. Plus 134, 398 (2019)
12. P.H.R.S. Moraes, J.D.V. Arbañil, M. Malheiro, J. Cosm. Astrop. Phys. 06, 005 (2016)
13. P.H.R.S. Moraes, P.K. Sahoo, Eur. Phys. J. C 79, 677 (2019)
14. E. Elizalde, M. Khurshudyan, Phys. Rev. D 99, 024051 (2019)
15. P.H.R.S. Moraes, W. de Paula, R.A.C. Correa, Int. J. Mod. Phys. D 28, 1950098 (2019)
16. E. Elizalde, M. Khurshudyan, Phys. Rev. D 98, 123525 (2018)
17. P.H.R.S. Moraes, P.K. Sahoo, Phys. Rev. D 97, 024007 (2018)
18. P.K. Sahoo, P.H.R.S. Moraes, P. Sahoo, Eur. Phys. J. C 78, 46 (2018)
19. P.K. Sahoo, P.H.R.S. Moraes, P. Sahoo, G. Ribeiro, Int. J. Mod. Phys. D 27, 1950004 (2018)
20. P.H.R.S. Moraes, P.K. Sahoo, Phys. Rev. D 96, 044038 (2017)
21. P.H.R.S. Moraes, R.A.C. Correa, R.V. Lobato, J. Cosm. Astrop. Phys. 07, 029 (2017)
22. T. Azizi, Int. J. Theor. Phys. 52, 3486 (2013)
23. M. Sharif, A. Siddiqi, Gen. Rel. Grav. 51, 74 (2019)
24. M.E.S. Alves, P.H.R.S. Moraes, J.C.N. de Araujo, M. Malheiro, Phys. Rev. D 94, 024032 (2016)
25. P.K. Sahoo, S. Bhattacharjee, Int. J. Theor. Phys. (2020) https://doi.org/10.1007/s10773-020-04414-3. arXiv:1907.13460
26. P. Sahoo et al., Mod. Phys. Lett. A (2020). https://doi.org/10.1142/S0217732320500959. arXiv:1907.08682
27. S. Bhattacharjee, P.K. Sahoo, Eur. Phys. J. Plus 135, 86 (2020). [arXiv:2001.06569]
28. J.U. Kang, G. Panotopoulos, Phys. Lett. B 677, 6 (2009)
29. R.P.L. Azevedo, P.P. Avelion, Phys. Rev. D 98, 064045 (2018)
30. M. Kusakabe et al., Phys. Rev. D 91, 104023 (2015)
31. S. Capozziello, G. Lambiase, E.N. Saridakis, Eur. Phys. J. C 77, 576 (2017)
32. A. Coe, K.A. Olive, Phys. Rev. D 73, 083525 (2006)
33. R. Nakamura, M. Hashimoto, S. Gamow, K. Arai, A&A 448, 23 (2006)
34. G. Lambiase, JCAP 10, 028 (2012)
35. J. Larena, J.M. Alami, A. Serna, Astrophys. J. 658, 1 (2007)
36. T.R. Makki, M.F.E. Eid, Mod. Phys. Lett. A 34(24), 1950194 (2019)
37. G. Steigman, Adv. High Energy Phys. 2012, 268321 (2012)
38. V. Simha, G. Steigman, JCAP 06, 016 (2008)
39. WMAP Collaboration (E. Komatsu et al.), Astrophys. J. Suppl. 192, 18 (2011)
40. J.P. Kneller, G. Steigman, New J. Phys. 6, 117 (2004)
41. G. Steigman, Annu. Rev. Nucl. Part. Sci. 57, 463 (2007)
42. B.D. Fields et al., JCAP 03, 010 (2020)
43. B.D. Fields, Annu. Rev. Nucl. Part. Sci. 61, 47–68 (2011)
44. T. Harko, Phys. Rev. D 90, 044067 (2014)
45. I. Prigogine, J. Geheniau, E. Gunzig, P. Nardone, Proc. Natl. Acad. Sci. USA 85, 7428 (1988)
46. M.P.L.P. Ramos, J. Paramos, Phys. Rev. D 96, 104024 (2017)
47. E.W. Kolb, M.S. Turner, The Early Universe (Addison-Wesley Publishing Company, Redwood City, 1989)
48. D.N. Spergel et al. WMAP Collaboration, Astrophys. J. Suppl. 170, 377 (2007)
49. S. Capozziello, M. Capriolo, M. Transirico, Int. J. Geom. Meth. Mod. Phys 15, 1850164 (2018)
50. M. De Laurentis, S. Capozziello, Astropart. Phys. 35, 257 (2011)
51. K. Bamba, S. Capozziello, M. De Laurentis, S. Nojiri, D. Sez-Gmez, Phys. Lett. B 727, 194 (2013)
52. H. Abedi, S. Capozziello, Eur. Phys. J. C 78, 474 (2018)