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Stabilization of Scale Model Vertical Take-Off and Landing Vehicles without Velocity Measurements

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1. Introduction

Miniature rotorcraft-based Unmanned Aerial Vehicles (UAVs) have received a growing interest in both industrial and academic research. Thanks to their hover and vertical take-off and landing (VTOL) capabilities, they are indeed particularly well suited for many civil missions such as video supervision of road traffic, surveillance of urban districts, victims localization after natural disasters, fire detection or building inspection for maintenance.

Design of guidance navigation and control algorithms for the autonomous flight of small rotorcraft-based UAVs is a challenging research area because of their nonlinear dynamics and their high sensitivity to aerodynamic perturbations. Various control strategies such as backstepping (Bouabdallah & Siegwart, 2005), (Frazzoli et al., 2000), (Mahony & Hamel, 2004), nonlinear model predictive control (Kim et al., 2002), (Bertrand et al., 2007a) or sliding modes (Bouabdallah & Siegwart, 2005) have been successfully applied to stabilization or trajectory tracking of UAV models. Nevertheless most of them require full state knowledge for feedback control design.

For robotic systems it may be useful, for cost or payload reasons, to limit the number of embedded sensors. For a miniature UAV, the nature of the mission itself may also directly impact the choice of the sensors that will be used, and therefore the type of measurements that will be available for the vehicle control.

In constrained environments, for example, the use of a vision based sensor may be preferred to a GPS to estimate the relative position of the vehicle with respect to its environment. In that case, linear velocity measurements may not be available. Another example is the case of a test bench design, where a “ready-to-use” radio controlled vehicle is used along with external sensors that do not require structural modifications of the vehicle. Such external sensors are for example motion capture systems (Kondak et al., 2007), (Kundak & Mettler, 2007), (Valenti et al., 2006), or magnetic field based sensors (Castillo et al., 2004). With such equipments, only the position and the attitude angles of the vehicle can be directly measured.

Nevertheless, knowledge of the vehicle state components (positions, linear velocities, attitude angles and angular velocities) is required for control.

A practical approach may consist in computing the velocities from the position measurements by finite differentiations. This method is used in (Kondak et al., 2007) to...
compute the linear velocity of rotorcraft-based miniature UAVs, and in (Castillo et al., 2004) to compute both linear and angular velocities to control a four-rotors vehicle. However, no theoretical stability guarantee is provided.

One way to theoretically deal with partial state measurement is to define an observer. In (Do et al., 2003) the problem of trajectory tracking for a planar Vertical Take-Off and Landing (VTOL) aircraft with only position and attitude angle measurements is solved by designing a full-order observer. Changes of coordinates are then used to put the system in a triangular form so that a backstepping technique can be used to develop a velocity-independent stabilizing controller.

However, the use of an observer may introduce additional computational burden in the control loop. It is also necessary to prove firstly its own convergence. In addition, compatibility between the frequency of the observer and the frequency of the controller must be checked to ensure the closed loop stability of the complete observer-based controlled system.

Another approach that can be used to avoid computational burden or complexity due to the introduction of an observer is partial state feedback: the controller is designed directly from the available measurements.

Early work on partial state feedback has been done in the context of rigid-link robot manipulators when no velocity measurement is available. In (Burg et al., 1996) and (Burg et al., 1997) the velocity measurement is replaced by a velocity-related signal generated by a linear filter based only on link position measurement. An extension of this work, using a nonlinear filter, can be found in (Dixon et al., 2000). The same method has been applied to solve the problem of attitude tracking of rigid bodies. A velocity-related signal generated by a linear filter is indeed employed in (Wong et al., 2000), where a kinematic representation using modified Rodrigues parameters has been chosen. In (Cotic et al., 2000), a unit-quaternion-based representation is adopted and a nonlinear filter generates a signal replacing the angular velocity measurement in the feedback controller.

First-order dynamic attitude feedback controllers have been proposed in (Arkella, 2001) and (Astolfi & Lovera, 2002) to respectively solve the attitude tracking problem for rigid bodies and spacecrafts with magnetic actuators. The kinematic representations that are used in these two works are respectively based on modified Rodrigues parameters and quaternions. A unit-quaternion representation is also used in (Tayebi, 2007) where a feedback controller depending on an estimation error quaternion is designed to solve the problem of a rigid spacecraft attitude tracking.

Attitude control of rigid bodies without angular velocity measurement is also addressed in (Lizarralde & Wen, 1996) and (Tsiotras, 1998) where a passivity-like property of the system is used to design feedback controllers for kinematic representations respectively based on unit-quaternions and Rodrigues parameters. Both of them use a filtering technique to avoid the use of velocity measurement.

In this chapter, we deal with the problem of position and attitude stabilization of a six degrees of freedom VTOL UAV model when no measurement of the linear velocity nor of the angular velocity is available. Contrary to the previous works, the kinematic representation we use exploits the $SO(3)$ group and its manifold. The method we present is based on the introduction of virtual states in the system dynamics; no observer design is required.
The rest of the chapter is organized as follows. Section 2 introduces the notations and the mathematical identities that will be used in the chapter. Section 3 presents the VTOL UAV model dynamics and the cascaded structure of the controller. The design of the position controller is detailed in Section 4 whereas the attitude controller is presented in Section 5. In Section 6, the closed loop stability of the system is analyzed, and simulations results are provided in Section 7. Concluding remarks are finally given in the last part of this chapter.

2. Notations and Mathematical Background

Let $SO(3)$ denote the special orthogonal group of $\mathbb{R}^{3\times3}$ and $\mathfrak{so}(3)$ the group of antisymmetric matrices of $\mathbb{R}^{3\times3}$. We define by $(.)_\times$ the operator from $\mathbb{R}^3 \rightarrow \mathfrak{so}(3)$ such that

$$
\forall b \in \mathbb{R}^3, \quad b_\times = \begin{bmatrix} 0 & -b_3 & b_2 \\
 b_3 & 0 & -b_1 \\
 -b_2 & b_1 & 0 \end{bmatrix}
$$

where $b_i$ denotes the $i^{th}$ component of the vector $b$.

Let $V(.)$ be the inverse operator of $(.)_\times$, defined from $\mathfrak{so}(3) \rightarrow \mathbb{R}^3$, such that

$$
\forall b \in \mathbb{R}^3, \quad V(b_\times) = b \quad \forall B \in \mathfrak{so}(3), \quad V(B)_\times = B
$$

For a given vector $b \in \mathbb{R}^3$ and a given matrix $M \in \mathbb{R}^{3\times3}$, let us consider the following notations and identities:

$$
P_a(M) = \frac{M - M^T}{2} \quad P_s(M) = \frac{M + M^T}{2}
$$

$$
\text{tr}(P_a(M)P_s(M)) = 0
$$

$$
\frac{1}{2} \text{tr}(b_\times M) = -b^T V(P_a(M))
$$

The following identity will also be used:

$$
\forall (A_a, B_a) \in \mathfrak{so}(3), \quad \frac{1}{2} \text{tr}(A_a^T B_a) = V(A_a)^T V(B_a)
$$

Denote by $(\gamma_R, n_R)$ the angular-axis coordinates of a given matrix $R \in SO(3)$, and by $I_d$ the identity matrix of $\mathbb{R}^{3\times3}$. One has:

$$
\forall R \in SO(3), \quad \text{tr}(I_d - R) = 2(1 - \cos(\gamma_R))
$$
3. UAV Model and Control Strategy

3.1 VTOL UAV Model

The VTOL UAV is represented by a rigid body of mass \( m \) and of tensor of inertia \( I = \text{diag}(I_1, I_2, I_3) \) with \( I_1, I_2 \) and \( I_3 \) strictly positive. To describe the motion of the UAV, two reference frames are introduced: an inertial reference frame \((\mathcal{I})\) associated with the vector basis \((e_1, e_2, e_3)\) and a body frame \((\mathcal{B})\) attached to the UAV and associated with the vector basis \((b_1, b_2, b_3)\) (see Figure 1).

![Reference frames](image)

Figure 1. Reference frames

The position and the linear velocity of the UAV in \((\mathcal{I})\) are respectively denoted \(\chi = [\chi_x, \chi_y, \chi_z]^T\) and \(\nu = [v_x, v_y, v_z]^T\). The orientation of the UAV is given by the orientation matrix \(R \in SO(3)\) from \((\mathcal{I})\) to \((\mathcal{B})\), usually parameterized by Euler’s pseudo angles \(\psi, \theta, \phi\) (yaw, pitch, roll):

\[
R = \begin{bmatrix}
c\theta & c\phi & -s\phi & s\psi & c\theta s\phi - c\phi s\psi & c\theta c\phi + s\phi s\psi \\
c\theta & s\phi & -c\phi & c\psi & s\theta c\psi + s\phi c\psi & c\theta s\psi - c\phi s\psi \\
-s\theta & s\psi & c\theta & c\phi & -s\phi c\psi & c\phi c\theta
\end{bmatrix}
\]  

(8)

with the trigonometric shorthand notations \(c\alpha = \cos(\alpha)\) and \(s\alpha = \sin(\alpha)\), \(\forall \alpha \in \mathbb{R}\).

Let \(\Omega = [\omega_p, \omega_q, \omega_r]^T\) be the angular velocity of the UAV defined in \((\mathcal{B})\).

The dynamics of a rigid body can be described as:

\[
\begin{align*}
\dot{\chi} &= \nu \\
m\dot{\nu} &= F \\
\dot{R} &= R\Omega \chi \\
I\dot{\Omega} &= -\Omega \times I\Omega + \Gamma
\end{align*}
\]  

(9)

where the inputs are a translational force \(F \in \mathbb{R}^3\) and a control torque \(\Gamma \in \mathbb{R}^3\).
For the VTOL UAV, the translational force \( F \) combines thrust, lift, drag and gravity components. In quasi-stationary flight we can reasonably assume that the aerodynamic forces are always in direction \( e_3^b (= Re_3) \), since the lift force predominates the other components (Hamel & Mahony, 2004). By separating the gravity component \( mge_3 \) from the combined aerodynamic forces, the dynamics of the VTOL UAV are rewritten as:

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= -T Re_3 + mge_3 \\
\dot{\rho} &= R\Omega_x \\
I\dot{\Omega} &= -\Omega\times I\Omega + \Gamma
\end{align*}
\]  

(10)

where the inputs are the scalar \( T \in \mathbb{R} \) representing the magnitude of the external forces applied in direction \( e_3^b \), and the control torque \( \Gamma = [\Gamma_1 \Gamma_2 \Gamma_3]^T \) defined in \((B)\).

3.2 Control Strategy

In this chapter, we consider the problem of the vehicle stabilization around a desired position \( \chi^d \) assumed to be constant \( (\dot{\chi}^d = 0) \).

For control design, let us define the position error \( \xi = (\chi - \chi^d) \). The system (10) becomes:

\[
\begin{align*}
\dot{\xi} &= v \\
\dot{v} &= -T Re_3 + mge_3 \\
\dot{\rho} &= R\Omega_x \\
I\dot{\Omega} &= -\Omega\times I\Omega + \Gamma
\end{align*}
\]  

(11)

Designing a controller for the model (11) can be realized by a classical backstepping approach applied to the whole dynamical system. In that case, the input vector \(- (T/m) Re_3 \) must be dynamically extended (Frazzoli et al., 2000), (Mahony et al., 1999). To avoid such a dynamical extension, the singular perturbation theory can be used to split the system dynamics into two reduced order subsystems (Khalil, 2002), (Calise, 1976). This approach leads to a time-scale separation between the translational dynamics (slow time-scale) and the orientation dynamics (fast time-scale). Reduced order controllers can therefore be designed to stabilize the system dynamics (Njaka et al., 1994).

We introduce the scaling parameter \( \varepsilon \in (0,1] \) such that:

\[
\begin{align*}
\dot{\xi} &= v \\
\dot{v} &= -\frac{T}{m} Re_3 + ge_3 \\
\varepsilon\dot{\rho} &= R\Omega_x \\
\varepsilon I\dot{\Omega} &= -\Omega\times I\Omega + \Gamma
\end{align*}
\]  

(12)
For $\varepsilon=1$, we obtain the full order system. Setting $\varepsilon=0$ leads to the slow time-scale reduced-order system, where the orientation dynamics satisfy a quasi steady state condition $\Omega=0$.

For the translational dynamics (12), we will define the full vectorial term $\mathbb{T} \mathbb{e}_3$ as the position control input. We will assign its desired value

$$(\mathbb{T} \mathbb{e}_3)^d = f(\xi, \nu)$$

Assuming that the actuator dynamics can be neglected with respect to the rigid body dynamics of the UAV, the value $\mathbb{T}^d$ is considered to be instantaneously reached by $\mathbb{T}$. Therefore, we have $(\mathbb{T} \mathbb{e}_3)^d = \mathbb{T}^d \mathbb{e}_3$, where $\mathbb{T}^d$ is the desired orientation of the vehicle. The vector $\mathbb{T} \mathbb{e}_3$ will then be split into its magnitude

$$T = \|f(\xi, \nu)\|$$

representing the first control input, and its direction

$$\mathbb{T}^d \mathbb{e}_3 = \frac{1}{T} f(\xi, \nu)$$

representing the desired orientation.

**Remark 1:** The desired orientation $\mathbb{T}^d$ can then be deduced from the given direction $\mathbb{T}^d \mathbb{e}_3 = \frac{1}{T} f(\xi, \nu)$, solving for $(\psi, \theta, \phi)$ for a given specified yaw value $\psi^d$ and using (8) (Hamel, 2002).

Since $\varepsilon \ll 1$ for the considered system, the design of the position controller can be done in the slow time-scale, i.e. for $\mathbb{R} = \mathbb{R}^d$.

For the orientation dynamics (13), we will assign the control torque $\Gamma$ such that the orientation $\mathbb{R}$ of the UAV converges asymptotically to the desired orientation $\mathbb{R}^d$, and such that the angular velocity $\Omega$ converges to $\Omega^d$ defined by:

$$\dot{\mathbb{R}}^d = \mathbb{R}^d \Omega^d$$

Therefore, the assigned control law will be of the form

$$\Gamma = h(\mathbb{R}, \mathbb{R}^d, \Omega, \Omega^d)$$

The design of the attitude controller can be done in the fast time-scale, assuming $\Omega^d = 0$.

Indeed, defining $\Lambda = \mathbb{R} - \mathbb{R}^d$ and using the singular perturbation theory, we get

$$\varepsilon \dot{\Lambda} = \Omega \times \mathbb{R}^d - \varepsilon \mathbb{R}^d \Omega^d$$

(19)
Since $\varepsilon \ll 1$ for the considered system (the translational dynamics are characterized by a slow time-scale with respect to the orientation dynamics), the term $\varepsilon R^d \Omega^d$ can therefore be ignored.

The structure of the controller we will develop is summarized in Figure 2. It is defined as a cascaded combination of the position controller and the attitude controller.

![Figure 2. Cascaded structure of the controller](image)

Remark 2: Note that in the considered case where no velocity measurements are available, the functions $f$ and $h$ defining the control laws will not depend on $v$ nor on $\Omega$.

4. Position Controller

Consider the translational dynamics. According to the above discussion of Section 3.2, we assume, for control design, that $TRe_3 = TR^d e_3$ is the control input of the translational dynamics.

Let $q, w \in \mathbb{R}^3$ be two virtual states and let $\delta \in \mathbb{R}^3$ be a virtual control such that:

$$\begin{aligned}
\dot{\xi} &= v \\
\dot{\psi} &= -\frac{T}{m} R^d e_3 + ge_3 \\
\dot{\phi} &= -w \\
\dot{w} &= \delta
\end{aligned} \quad (20)
$$

Lemma 1: Consider the system dynamics (20). Let us define the control vector

$$TR^d e_3 = \frac{m}{k_v} \{k_x \xi + k_1 (\xi - q) + k_2 (\xi - q + w)\} + mge_3 \quad (21)$$

and the virtual control.
\[ \delta = -\frac{1}{k_2} \left\{ k_2 w + k_1 (\xi - q) + k_2 (\xi - q + w) \right\} \]  

(22)

where \( k_X, k_V, k_1 \) and \( k_2 \) are strictly positive gains.

Consider the Lyapunov function candidate

\[ S = \frac{1}{2} k_X \| \xi \|^2 + \frac{1}{2} k_V \| v \|^2 + \frac{1}{2} k_1 \| \xi - q \|^2 + \frac{1}{2} k_2 \| \xi - q + w \|^2 \]

(23)

Define \( k_{\text{min}} = \min(k_X, k_V, k_1, k_2) \) and \( k_{\text{max}} = \max(k_X, k_1, k_2) \).

Then, for any initial conditions \( \xi(0), v(0), \) and \( q(0) = \xi(0) \) and \( w(0) = 0 \) verifying

\[ S(0) < \frac{1}{18} \frac{g^2 k_2 k_{\text{min}}}{k_{\text{max}}} \]

(24)

the control vector (21) along with the virtual control (22) exponentially stabilizes the translational dynamics (20), and the input \( T \) is strictly positive and bounded.

**Proof:**

The time derivative of the Lyapunov function candidate \( S \) is

\[ \dot{S} = k_X \xi^T v + k_V v^T \left( -\frac{T}{m} R^d e_3 + g e_3 \right) + k_1 (\xi - q)^T (v + w) + k_2 (\xi - q + w)^T (v + w + \delta) \]

(25)

which can be expressed as:

\[ \dot{S} = v^T \left[ k_X \xi - k_1 \frac{T}{m} R^d e_3 + k_V v e_3 + k_1 (\xi - q) + k_2 (\xi - q + w) \right] + k_1 w^T (\xi - q) + k_2 (\xi - q + w)^T (w + \delta) \]

(26)

Taking the control vector \( T R^d e_3 \) as defined in (21), we get:

\[ \dot{S} = k_1 w^T (\xi - q) + k_2 (\xi - q + w)^T (w + \delta) \]

(27)

We introduce \( (\xi - q) \) in the first term to obtain:

\[ \dot{S} = k_1 (w + (\xi - q) - (\xi - q))^T (\xi - q) + k_2 (\xi - q + w)^T (w + \delta) \]

(28)

which leads to:

\[ \dot{S} = -k_1 \| \xi - q \|^2 + (\xi - q + w)^T (k_1 (\xi - q) + k_2 (w + \delta)) \]

(29)

Choosing the virtual control \( \delta \) according to (22) makes \( \dot{S} \) become:

\[ \dot{S} = -k_1 \| \xi - q \|^2 - k_2 \| \xi - q + w \|^2 \]

(30)

The application of La Salle’s principle leads to \( \xi \rightarrow q \) and \( w \rightarrow (\xi - q) \), i.e. \( w \rightarrow 0 \). By continuity, we get \( \dot{\xi} \rightarrow q \), that is \( v \rightarrow -w \). Since \( w \rightarrow 0 \), it yields \( v \rightarrow 0 \) and \( \dot{\xi} \rightarrow 0 \), and by
continuity $\dot{v} \to 0$. Combining the second equation of system (20) with (21) and the fact that $\dot{v} \to 0$ leads to:

$$(k_x \xi + k_1 (\xi - q) + k_2 (\xi - q + w)) \to 0$$  \hspace{1cm} (31)

Using $(\xi - q) \to 0$ and $w \to 0$, we finally get $\xi \to 0$ and $q \to 0$. Therefore, the closed-loop system (20) is asymptotically stable, and since it is linear, we can conclude that it is exponentially stable.

It remains to show that the input $T$ is strictly positive and bounded.

From (21), we have:

$$T = \left\| \frac{m}{k_v} \left( k_x \xi + k_1 (\xi - q) + k_2 (\xi - q + w) \right) + mg \right\|$$  \hspace{1cm} (32)

By triangular inequality, we get:

$$T \geq mg - \frac{m}{k_v} k_{\text{max}} \left\{ \|\xi\| + \|\xi - q\| + \|\xi - q + w\| \right\}$$  \hspace{1cm} (33)

$$T \geq mg - \frac{3 m}{k_v} k_{\text{max}} \left\{ \|\xi\|^2 + \|v\|^2 + \|\xi - q\|^2 + \|\xi - q + w\|^2 \right\}^{\frac{1}{2}}$$  \hspace{1cm} (34)

Using the definition of $S$ and $k_{\text{min}}$, we have:

$$\frac{1}{2} k_{\text{min}} \left\{ \|\xi\|^2 + \|\xi - q\|^2 + \|\xi - q + w\|^2 \right\} \leq S$$  \hspace{1cm} (35)

Since $S$ is decreasing, we have:

$$S \leq S(0)$$  \hspace{1cm} (36)

where, taking $q(0) = \xi(0)$ and $w(0) = 0$, the initial value $S(0)$ is defined by:

$$S(0) = \frac{1}{2} k_x \|\xi(0)\|^2 + \frac{1}{2} k_v \|v(0)\|^2$$  \hspace{1cm} (37)

Using (34) along with (35) and (36), we get:

$$T \geq mg - \frac{3 m}{k_v} k_{\text{max}} \sqrt{\frac{2 S(0)}{k_{\text{min}}}}$$  \hspace{1cm} (38)

Using condition (24), we obtain $T > 0$.

Let us finally show that $T$ is bounded.

From equation (32) and by triangular inequality, we also get:

$$T \leq mg + \frac{m}{k_v} k_{\text{max}} \left\{ \|\xi\| + \|\xi - q\| + \|\xi - q + w\| \right\}$$  \hspace{1cm} (39)
\[ T \leq mg + 3 \frac{m}{k_v} k_{max} \left( \frac{1}{\|v\|^2 + \|\xi - q\|^2 + \|\xi - q + w\|^2} \right)^{1/2} \]  

(40)

Using (35) and (36) leads to:

\[ T \leq mg + 3 \frac{m}{k_v} k_{max} \sqrt{\frac{2S(0)}{k_{min}}} \]  

(41)

and, therefore, the input \( T \) is bounded. ■

Lemma 1 ensures that the control vector (21) along with the virtual control (22) exponentially stabilizes the translational dynamics (20) without using any measurement of the linear velocity.

**Remark 3:** From our previous discussion, Section 3.2, equation (32) can be directly used to provide the control input \( T \). Furthermore, since \( T \) is strictly positive, the direction given by

\[
R^d e_3 = \frac{1}{T} \left( mge_3 + \frac{m}{k_v} \{k_1(\xi - q) + k_2(\xi - q + w)\} \right)
\]  

(42)

is well defined and can be used to compute the desired orientation \( R^d \).

Due to the position controller we developed, the closed-loop translational dynamics are exponentially stable for \( R= R^d \). However, since the orientation \( R \) will not converge instantaneously to the desired value \( R^d \), an orientation error term is introduced in the translational dynamics:

\[
m\ddot{v} = -TR^d e_3 + mge_3 - T(R-R^d)e_3
\]  

(43)

Therefore an attitude controller must be designed to allow, at least, asymptotic convergence of \( R \) to \( R^d \).

**5. Attitude Controller**

Consider the orientation dynamics (13) and assume that measurements on the angular velocity \( \Omega \) are not available. Let us introduce \( \tau = \varphi \varepsilon \). In the fast time-scale, the time derivative of a given function \( g \) will be denoted by

\[
\mathring{g} = \frac{d}{d\varepsilon} g = \varepsilon \frac{d}{dt} g
\]  

(44)

Similarly to the translational dynamics, we introduce two virtual states \( Q \in SO(3) \), \( W \in \mathbb{R}^3 \) and a virtual control \( \Delta \in \mathbb{R}^3 \) for the orientation dynamics, such that:
\[\begin{align*}
\dot{\mathbf{R}} &= R \mathbf{\Omega}_x \\
I \mathbf{\Omega} &= -\mathbf{\Omega} \times I \mathbf{\Omega} + \Gamma \\
\dot{\mathbf{Q}} &= -\mathbf{Q} W_x \\
\dot{W} &= \Delta
\end{align*}\] (45)

For a given desired orientation \( \mathbf{R}^d \) we define

\[\begin{align*}
\tilde{\mathbf{R}} &= (\mathbf{R}^d)^T \mathbf{R} \\
\tilde{\mathbf{Q}} &= \mathbf{Q}^T \tilde{\mathbf{R}}
\end{align*}\] (46) (47)

According to the previous discussion of Section 3.2, we assume \( \mathbf{\Omega}^d = 0 \) for control design, \( \text{i.e. } \mathbf{R}^d = 0 \).

Using (46) and (47), we rewrite (45) as:

\[\begin{align*}
\dot{\mathbf{R}} &= \mathbf{\ddot{R}} \mathbf{\Omega}_x \\
I \mathbf{\dot{\Omega}} &= -\mathbf{\dot{\Omega}} \times I \mathbf{\Omega} + \Gamma \\
\dot{\mathbf{Q}} &= W_x \mathbf{\ddot{Q}} + \mathbf{\ddot{Q}} \mathbf{\Omega}_x \\
\dot{W} &= \Delta
\end{align*}\] (48)

**Lemma 2:** Consider the orientation dynamics (48).

Define the control torque

\[\mathbf{\Gamma} = \frac{1}{k_\omega} \left[ -k_r V(P_a(\tilde{\mathbf{R}})) - k_3 V(P_a(\mathbf{\dot{Q}})) + k_4 V(P_a(M)) + k_4 V(P_a(N)) \right]\] (49)

and the virtual control

\[\Delta = -\frac{1}{k_4} V \left( \frac{1}{2} k_3 P_a(\mathbf{\ddot{Q}}) + \frac{1}{2} k_4 (W_x \mathbf{\ddot{Q}} + \mathbf{\ddot{Q}}^T W_x) + \frac{1}{2} k_5 (W_x + P_a(\mathbf{\ddot{Q}})) \right)\] (50)

where

\[M = (W_x + P_a(\mathbf{\ddot{Q}}))^T \mathbf{\dddot{Q}}\] (51)

\[N = \mathbf{\ddot{Q}}^T (W_x + P_a(\mathbf{\ddot{Q}}))^T\] (52)

\(k_r, k_\omega, k_3, k_4\) and \(k_5\) are strictly positive gains with...
Consider the control Lyapunov function candidate

\[
\mathcal{L} = \frac{1}{2} k_r \text{tr}(I_d - \hat{R}) + \frac{1}{2} k_\omega \Omega^T I \Omega + \frac{1}{2} k_3 \text{tr}(I_d - \hat{Q}) + \frac{1}{2} k_4 \text{tr}\left\{(W_X + P_a(\hat{Q}))^T (W_X + P_a(\hat{Q}))\right\}
\] (54)

Then, for any initial condition \(\hat{R}(0), \Omega(0)\), with \(Q(0) = \hat{R}(0)\) and \(W(0) = 0\), such that

\[
\mathcal{L}(0) < 2k_r
\] (55)

the control torque (49) along with the virtual control (50) asymptotically stabilizes the orientation dynamics (48).

**Proof:**

Consider the Lyapunov function candidate \(\mathcal{L}\) defined by (54). It's time derivative along the trajectories of (48) is given by:

\[
\dot{\mathcal{L}} = -\frac{1}{2} k_r \text{tr}(\hat{R} \Omega) + k_\omega \Omega^T (-\Omega X I + \hat{\Gamma}) - \frac{1}{2} k_3 \text{tr}(\hat{Q}) + k_4 \text{tr}\left\{(W_X + P_a(\hat{Q}))^T (\Delta_X + P_a(\hat{Q}))\right\}
\] (56)

where

\[
P_a(\hat{Q}) = \frac{1}{2} \left\{W_X \Omega + \hat{Q} \Omega + \hat{Q}^T W_X + \Omega \Omega^T\right\}
\] (57)

Using (57) and the fact that \(\Omega^T (\Omega X I) = 0\), it yields:

\[
\dot{\mathcal{L}} = -\frac{1}{2} k_r \text{tr}(\hat{R} \Omega) + k_\omega \Omega^T \Omega - \frac{1}{2} k_3 \text{tr}(\hat{Q}) + k_4 \text{tr}\left\{(W_X + P_a(\hat{Q}))^T \left(\Delta_X + \frac{1}{2} \left\{W_X \Omega + \hat{Q} \Omega + \hat{Q}^T W_X + \Omega \Omega^T\right\}\right)\right\}
\] (58)

Using identity (4) we get:

\[
\dot{\mathcal{L}} = -\frac{1}{2} k_r \text{tr}(\Omega X P_a(\hat{R})) + k_\omega \Omega^T \Omega - \frac{1}{2} k_3 \text{tr}(W_X P_a(\hat{Q})) - \frac{1}{2} k_3 \text{tr}(\Omega X P_a(\hat{Q})) + k_4 \text{tr}\left\{(W_X + P_a(\hat{Q}))^T \left(\Delta_X + \frac{1}{2} \left\{W_X \Omega + \hat{Q} \Omega + \hat{Q}^T W_X + \Omega \Omega^T\right\}\right)\right\}
\] (59)

Also using (6) we obtain:

\[
\dot{\mathcal{L}} = \Omega^T \left\{k_r V(P_a(\hat{R}))+k_\omega \Gamma + k_3 V(P_a(\hat{Q}))\right\} - \frac{1}{2} k_3 \text{tr}(W_X P_a(\hat{Q})) + k_4 \text{tr}\left\{(W_X + P_a(\hat{Q}))^T \hat{Q} \Omega X\right\}
\] (60)

\[
+ \frac{1}{2} k_4 \text{tr}\left\{(W_X + P_a(\hat{Q}))^T \Omega \times \hat{Q}^T\right\} + k_4 \text{tr}\left\{(W_X + P_a(\hat{Q}))^T (\Delta_X + \frac{1}{2} W_X \Omega + \hat{Q}^T W_X)\right\}
\]

Using (51) we have:
\[
\frac{1}{2} k_4 \text{tr}\left\{W_x + P_a(\tilde{Q})^T \Omega_{x} \right\} = \frac{1}{2} k_4 \text{tr}\{M \Omega_{x}\} = -k_4 \Omega^T V(P_a(M))
\]

In the same way, we use (52) to get:
\[
\frac{1}{2} k_4 \text{tr}\left\{(W_x + P_a(\tilde{Q}))^T \Omega_{x} \right\} = -k_4 \Omega^T V(P_a(N))
\]

Therefore, the time derivative of \( \mathcal{L} \) can be simplified:
\[
\dot{\mathcal{L}} = \Omega^T \left\{ k_r V(P_a(\tilde{R})) + k_4 \Gamma + k_3 V(P_a(\tilde{Q})) - k_4 V(P_a(M)) - k_4 V(P_a(N)) \right\} - \frac{1}{2} k_3 \text{tr}(W_x P_a(\tilde{Q})) + k_4 \text{tr}\left\{(W_x + P_a(\tilde{Q}))^T (\Delta_x + \frac{1}{2}(W_x \tilde{Q} + \tilde{Q}^T W_x)) \right\}
\]

Choosing \( \Gamma \) according to (49) leads to:
\[
\dot{\mathcal{L}} = -\frac{1}{2} k_3 \text{tr}(W_x P_a(\tilde{Q}))+ k_4 \text{tr}\left\{(W_x + P_a(\tilde{Q}))^T (\Delta_x + \frac{1}{2}(W_x \tilde{Q} + \tilde{Q}^T W_x)) \right\}
\]

As \( W_x^T = -W_x \), and introducing \( P_a(\tilde{Q}) \) in the first term:
\[
\dot{\mathcal{L}} = -\frac{1}{2} k_3 \text{tr}\left\{P_a(\tilde{Q})^T P_a(\tilde{Q}) \right\} + k_4 \text{tr}\left\{(W_x + P_a(\tilde{Q}))^T (\Delta_x + \frac{1}{2}(W_x \tilde{Q} + \tilde{Q}^T W_x)) \right\}
\]

Taking \( \Delta \) as defined in (50), one has:
\[
\dot{\mathcal{L}} = -\frac{1}{2} k_3 \text{tr}\left\{P_a(\tilde{Q})^T P_a(\tilde{Q}) \right\} - \frac{1}{2} k_5 \text{tr}\left\{(W_x + P_a(\tilde{Q}))^T (W_x + P_a(\tilde{Q})) \right\}
\]

Using again identity (6), we finally have:
\[
\dot{\mathcal{L}} = -k_3 \left\|V(P_a(\tilde{Q}))\right\|^2 - k_5 \left\|V(P_a(\tilde{Q}))\right\|^2
\]

ensuring that \( \mathcal{L} \) is strictly decreasing until \( P_a(\tilde{Q}) \rightarrow 0 \) and \( W_x \rightarrow -P_a(\tilde{Q}) \), i.e. \( W_x \rightarrow 0 \).

Denote by \((\gamma_{\tilde{Q}}, n_{\tilde{Q}})\) the angle-axis coordinates of \( \tilde{Q} \). Using (7), one has:
\[
k_3 (1 - \cos(\gamma_{\tilde{Q}})) = \frac{1}{2} k_3 \text{tr}(I_d - \tilde{Q}) \leq \mathcal{L}
\]

Since \( \mathcal{L} \) is decreasing, we have \( \mathcal{L} \leq \mathcal{L}(0) \). Using (55) it yields:
\[
k_3 (1 - \cos(\gamma_{\tilde{Q}})) \leq \mathcal{L} \leq \mathcal{L}(0) < 2k_r
\]
Using (53), we get:

\[ 1 - \cos(\gamma_{\tilde{Q}}) \leq 2 \frac{k_r}{k_3} \]  

(71)

From \( P_d(\tilde{Q}) \to 0 \), we have \( \gamma_{\tilde{Q}} = 0 \) or \( \gamma_{\tilde{Q}} = \pm \pi \). The second possibility is excluded by (71). Therefore we have \( \tilde{Q} \to I_d \). By (47), it yields \( \tilde{R} \to Q \). By continuity and using La Salle’s principle, we get \( \tilde{R} \to \tilde{Q} \). Using the first equation of (48) and the third equation of (45), one has \( \tilde{R} \Omega_x \to Q W_x \). Since \( \tilde{R} \) is orthogonal, we get \( \Omega_x \to -\tilde{R}^T Q W_x \). Using \( W_x \to 0 \) it yields \( \Omega_x \to 0 \) and then \( \Omega \to 0 \). Therefore, using the first equation of (48), we get \( \tilde{R} \to 0 \). By continuity \( \Omega \to 0 \) and then, by the second equation of system (48), \( \Gamma \to 0 \). Knowing that \( P_d(\tilde{Q}) \) and \( W_x \) converge to zero, one can ensure that, respectively from (51) and (52), \( M \) and \( N \) converge to zero. Combining the above discussion with the fact that \( \Gamma \to 0 \), equation (49) ensures that \( P_d(\tilde{R}) \to 0 \).

Similarly to the previous analysis on \( \tilde{Q} \), let us denote \( (\gamma_{\tilde{R}}, n_{\tilde{R}}) \) the angle-axis coordinates of \( \tilde{R} \). One has:

\[ k_r (1 - \cos(\gamma_{\tilde{R}})) = \frac{1}{2} k_r \operatorname{tr}(I_d - \tilde{R}) \leq \mathcal{L} \leq \mathcal{L}(0) < 2 k_r \]  

(72)

It yields

\[ 1 - \cos(\gamma_{\tilde{R}}) < 2 \]  

(73)

From \( P_d(\tilde{R}) \to 0 \), we have \( \gamma_{\tilde{R}} = 0 \) or \( \gamma_{\tilde{R}} = \pm \pi \). The second possibility is excluded by (73).

Therefore, we finally have \( \tilde{R} \to I_d \) and \( \tilde{R} \to R^d \). ■

Lemma 2 ensures that the control (49) along with the virtual control (50) asymptotically stabilizes the orientation dynamics (48) without using any measurement of the angular velocity.

**Remark 4:** The time-scale parameter \( \varepsilon > 0 \) is chosen such that the deviation \( \tilde{R} \) converges to \( I_d \) faster than the translational dynamics (\( \varepsilon \ll 1 \)).

**Remark 5:** The condition (55) is not very conservative. Choosing \( W(0) = 0 \) and \( Q(0) = \tilde{R}(0) \), the condition (55) can be simplified:

\[ \frac{k_r}{k_0} \geq \frac{E_{\tilde{Q}}(0)}{(2 - E_r(0))} \]  

(74)
with $E_T(0)=\frac{1}{2} \text{tr}(I_d - \tilde{R}(0))$ and $E_{\omega}(0) = \frac{1}{2} \Omega(0)^T I \Omega(0)$.

6. Stability Analysis

We consider the full dynamics of the system along with virtual states and with the orientation error term in the translational dynamics:

\[
\begin{align*}
    \dot{\xi} &= v \\
    \dot{v} &= -\frac{T}{m} R^d e_3 + g e_2 - \frac{T}{m} (R - R^d) e_3 \\
    \dot{q} &= -w \\
    \dot{w} &= \delta \\
    \tilde{R} &= \tilde{R} \Omega_x \\
    \Omega &= -\Omega \times \Omega + \Gamma \\
    \tilde{Q} &= W \Omega + \tilde{Q} \Omega_x \\
    \dot{W} &= \Delta
\end{align*}
\]

with $\tilde{R}$ and $\tilde{Q}$ respectively defined by (46) and (47).

**Proposition 1:** Consider the system dynamics (75). Under the conditions (24), (53) and (55), the control laws (21) and (49), along with the virtual controls (22) and (50), asymptotically stabilize the system (75).

**Sketch of the proof:**

By Lemma 2, under the conditions (53) and (55), the closed loop orientation dynamics are asymptotically stable when (49) and (50) are respectively used as control and virtual control. By Lemma 1, under the condition (24), the input $T$ is bounded. Therefore, the orientation error term $\frac{T}{m} (R - R^d) e_3$ asymptotically converges to zero.

Since, from Lemma 1, the control of the translational dynamics is exponentially stabilizing for $R \equiv R^d$, we can use (Khalil, 2002) to conclude that the control of the translational dynamics is asymptotically stabilizing in presence of the orientation error term.

Therefore, the system (75) is asymptotically stable when the control laws (21) and (49) are used along with the virtual control laws (22) and (50).

By introducing virtual states, we have been able to design stabilizing controllers for the position and attitude of the VTOL UAV model using no measurement of the linear velocity $v$ nor of the angular velocity $\Omega$.

**Remark 6:** In the case where only the linear velocity $v$ of the vehicle is not measured, a detailed proof using the singular perturbation theory can be found in (Bertrand et al., 2008).

**Remark 7:** Control laws for trajectory tracking, in the case where the linear and angular velocities are not measured, have been proposed in (Bertrand et al., 2007b).
7. Simulation Results

The VTOL UAV is described by the following parameters: \( m = 2.5 \, \text{kg} \), \( l_1 = l_2 = 0.13 \, \text{kg.m}^2 \), and \( l_3 = 0.16 \, \text{kg.m}^2 \). The gravitational acceleration is \( g = 9.81 \, \text{m.s}^{-2} \).

Simulation results are provided for stabilization at hover around the desired position \( \chi^d = [3 \, -1 \, 1]^T (\text{m}) \), starting from the initial condition \( \chi_0 = [5 \, -3 \, 4]^T (\text{m}) \), \( [\psi_0 \, \theta_0 \, \phi_0] = [0 \, -8 \, -10]^T (\text{deg}) \), \( v_0 = 0 \) and \( \Omega_0 = 0 \). The desired yaw \( \psi^d \) was chosen to be equal to zero.

The values of the gains are: \( k_x = 0.2 \), \( k_v = 3.0 \), \( k_1 = 0.8 \), \( k_2 = 0.8 \), \( k_r = 0.74 \), \( k_\omega = 3.3 \), \( k_3 = 12 \), \( k_4 = 0.25 \), \( k_5 = 6.1 \).

Figure 3 presents the coordinates of the position error \( \xi = [x \, y \, z]^T \) and attitude angles. Stabilization of the UAV model is achieved from the given initial condition with satisfying behaviour performances. The input \( T \) and the components of the control torque \( \Gamma \) are plotted in Figure 4.

The evolution of the angular deviation terms \( \dot{\phi} = \phi - \phi^d \), \( \ddot{\theta} = \theta - \theta^d \) and \( \dot{\psi} = \psi - \psi^d \) are presented in Figure 5. It can be verified that these terms converge faster than the closed loop translation, hence validating the time scale separation approach used for the design of the controllers.

![Figure 3. Position error and attitude angles](www.intechopen.com)
Figure 4. Control inputs

Figure 5. Angular deviation terms
8. Conclusion

In this chapter, we have presented a method to design guidance and control laws for the stabilization of a scale-model VTOL UAV when no measurements of the linear velocity nor of the angular velocity are available.

Motivated by the cascade structure of the model and by the singular perturbation approach, the controller is designed in two steps by considering a time-scale separation between translational and orientation dynamics. The position controller computes the magnitude of external forces, considered as a control input for the translational dynamics, and the desired orientation of the UAV. The attitude controller delivers the control torque ensuring asymptotic convergence of the actual orientation to the desired one.

By the proposed approach, these two feedback controllers have been designed by introducing virtual states in the system dynamics, and without using any observer. It is also worth noticing that this work is based on a kinematic representation exploiting the $SO(3)$ group and its manifold.

Elements for stability analysis have been given and simulation results have been provided to illustrate the good performances of the proposed approach.

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