Polynomial to Linear: Efficient Classification with Conjunctive Features

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Abstract

This paper proposes a method that speeds up a classifier trained with many conjunctive features: combinations of (primitive) features. The key idea is to precompute as partial results the weights of primitive feature vectors that appear frequently in the target NLP task. A trie compactly stores the primitive feature vectors with their weights, and it enables the classifier to find for a given feature vector its longest prefix feature vector whose weight has already been computed. Experimental results for a Japanese dependency parsing task show that our method speeded up the SVM and LLM classifiers of the parsers, which achieved accuracy of 90.84/90.71%, by a factor of 10.7/11.6.

1 Introduction

Deep and accurate text analysis based on discriminative models is not yet efficient enough as a component of real-time applications, and it is inadequate to process Web-scale corpora for knowledge acquisition (Pantel, 2007; Saeger et al., 2009) or semi-supervised learning (McClosky et al., 2006; Spoustová et al., 2009). One of the main reasons for this inefficiency is attributed to the inefficiency of core classifiers trained with many feature combinations (e.g., word n-grams). Hereafter, we refer to features that explicitly represent combinations of features as conjunctive features and the other atomic features as primitive features.

The feature combinations play an essential role in obtaining a classifier with state-of-the-art accuracy for several NLP tasks; recent examples include dependency parsing (Koo et al., 2008), parse re-ranking (McClosky et al., 2006), pronoun resolution (Nguyen and Kim, 2008), and semantic role labeling (Liu and Sarkar, 2007). However, ‘explicit’ feature combinations significantly increase the feature space, which slows down not only training but also testing of the classifier.

Kernel-based methods such as support vector machines (SVMS) consider feature combinations space-efficiently by using a polynomial kernel function (Cortes and Vapnik, 1995). The kernel-based classification is, however, known to be very slow in NLP tasks, so efficient classifiers should sum up the weights of the explicit conjunctive features (Isozaki and Kazawa, 2002; Kudo and Matsumoto, 2003; Goldberg and Elhadad, 2008).

\( \ell_1 \)-regularized log-linear models (\( \ell_1 \)-LLMs), on the other hand, provide sparse solutions, in which weights of irrelevant features are exactly zero, by assuming a Laplacian prior on the weights (Tibshirani, 1996; Kazama and Tsujii, 2003; Goodman, 2004; Gao et al., 2007). However, as Kazama and Tsujii (2005) have reported in a text categorization task and we later confirm in a dependency parsing task, when most features regarded as irrelevant during training \( \ell_1 \)-LLMs appear rarely in the task, we cannot greatly reduce the number of active features in each classification. In the end, when efficiency is a major concern, we must use exhaustive feature selection (Wu et al., 2007; Okanohara and Tsujii, 2009) or even restrict the order of conjunctive features at the expense of accuracy.

In this study, we provide a simple, but effective solution to the inefficiency of classifiers trained with higher-order conjunctive features (or polynomial kernel), by exploiting the Zipfian nature of language data. The key idea is to precompute the weights of primitive feature vectors and use them as partial results to compute the weight of a given feature vector. We use a trie called the feature sequence trie to efficiently find for a given feature vector its longest prefix feature vector whose weight has been computed. The trie is built from feature vectors generated by applying the classifier to actual data in the classification task. The time complexity of the classifier approaches time that
is linear with respect to the number of primitive features when the retrieved feature vector covers most of the features in the input feature vector.

We implemented our algorithm for SVM and LLM classifiers and evaluated the performance of the resulting classifiers in a Japanese dependency parsing task. Experimental results show that it successfully speeded up classifiers trained with higher-order conjunctive features by a factor of 10.

The rest of this paper is organized as follows. Section 2 introduces LLMs and SVMs. Section 3 proposes our classification algorithm. Section 4 presents experimental results. Section 5 concludes with a summary and addresses future directions.

2 Preliminaries

In this paper, we focus on linear classifiers that calculate the probability (or score) by summing up weights of individual features. Examples include not only log-linear models but also support vector machines with kernel expansion (Isozaki and Kazawa, 2002; Kudo and Matsumoto, 2003). Below, we introduce these two classifiers and their ways to consider feature combinations.

In classification-based NLP, the target task is modeled as one or more classification steps. For example in part-of-speech (POS) tagging, each classification decides whether to assign a particular label (POS tag) to a given sample (each word in a given sentence). Each sample is then represented by a feature vector \( x \), whose element \( x_i \) is a value of a feature function \( f_i \) of \( x \).

Here, we assume a binary feature function \( f_i(x) \in \{0, 1\} \), in which a non-zero value means that particular context data appears in the sample. We say that a feature \( f_i \) is active in sample \( x \) when \( x_i = f_i(x) = 1 \) and \( |x| \) represents the number of active features in \( x \) (\(|x| = |\{i \mid f_i(x) = 1\}|\)).

2.1 Log-Linear Models

The log-linear model (LLM), or also known as maximum-entropy model (Berger et al., 1996), is a linear classifier widely used in the NLP literature. Let the training data of LLMs be \( \{(x_i, y_i)\}_{i=1}^L \), where \( x_i \in \{0, 1\}^n \) is a feature vector and \( y_i \) is a class label associated with \( x_i \). We assume a binary label \( y_i \in \{\pm 1\} \) here to simplify the argument.

The classifier provides conditional probability \( p(y|x) \) for a given feature vector \( x \) and a label \( y \):

\[
p(y|x) = \frac{1}{Z(x)} \exp \sum_i w_{i,y} f_i(x, y),
\]

where \( f_{i,y}(x, y) \) is a feature function that returns a non-zero value when \( f_i(x) = 1 \) and the label is \( y \), \( w_{i,y} \in \mathbb{R} \) is a weight associated with \( f_{i,y} \), and \( Z(x) = \sum_y \exp \sum_i w_{i,y} f_{i,y}(x, y) \) is the partition function. We can consider feature combinations in LLMs by explicitly introducing a new conjunctive feature \( f_{x_i,y}(x, y) \) that is activated when a particular set of features \( F' \subseteq F \) is combined and that \( f_{x_i,y}(x, y) = \bigwedge_{f_{i,y} \in F'} f_{i,y}(x, y) \).

We then introduce an \( \ell_1 \)-regularized LLM (\( \ell_1 \)-LLM), in which the weight vector \( w \) is tuned so as to maximize the logarithm of the a posteriori probability of the training data:

\[
\mathcal{L}(w) = \sum_{i=1}^L \log p(y_i|x_i) - C \|w\|_1.
\]

Hyper-parameter \( C \) thereby controls the degree of over-fitting (solution sparseness). Interested readers may refer to the cited literature (Andrew and Gao, 2007) for the optimization procedures.

2.2 Support Vector Machines

A support vector machine (SVM) is a binary classifier (Cortes and Vapnik, 1995). Training with samples \( \{(x_i, y_i)\}_{i=1}^L \) where \( x_i \in \{0, 1\}^n \) and \( y_i \in \{\pm 1\} \) yields the following decision function:

\[
y(x) = \text{sgn}(g(x) + b) = \sum_{x_j \in SV} y_j \alpha_j \phi(x_j)^T \phi(x),
\]

where \( b \in \mathbb{R} \), \( \phi : \mathbb{R}^n \mapsto \mathbb{R}^H \) and support vectors \( x_j \in SV \) (subset of training samples), each of which is associated with weight \( \alpha_j \in \mathbb{R} \). We hereafter call \( g(x) \) the weight function. Nonlinear mapping function \( \phi \) is chosen to make the training samples linearly separable in \( \mathbb{R}^H \) space. Kernel function \( k(x_j, x) = \phi(x_j)^T \phi(x) \) is then introduced to compute the dot product in \( \mathbb{R}^H \) space without mapping \( x \) to \( \phi(x) \).

To consider combinations of primitive features \( f_j \in F \), we use a polynomial kernel \( k_d(x_j, x) = (x_j^T x + 1)^d \). From Eq. 3, we obtain the weight function for the polynomial kernel as:

\[
g(x) = \sum_{x_j \in SV} y_j \alpha_j (x_j^T x + 1)^d.
\]

Since we assumed that \( x_i \) is a binary value representing whether a (primitive) feature \( f_i \) is active in the sample, the polynomial kernel of degree \( d \) implies a mapping \( \phi_d \) from \( x \) to \( \phi_d(x) \) that has
$H = \sum_{k=0}^{d} \binom{n}{k}$ dimensions. Each dimension represents a (weighted) conjunction of $d$ features in the original sample $x$.\(^1\)

**Kernel Expansion (SVM-KE)** The time complexity of Eq. 4 is $O(|x| \cdot |SV|)$. This cost is usually high for classifiers used in NLP tasks because they often have many support vectors ($|SV| > 10,000$). Kernel expansion (KE) was proposed by Isozaki and Kazawa (2002) to convert Eq. 4 into the linear sum of the weights in the mapped feature space as in LLM ($p(y|x)$ in Eq. 1):

$$g(x) = w^T x^d = \sum_i w_i x_i^d,$$

where $x^d$ is a binary feature vector whose element $x_i^d$ has a non-zero value when $(\phi_d(x))_i > 0$, $w$ is the weight vector for $x^d$ in the expanded feature space $F^d$ and is precalculated from the support vectors $x_j$ and their weights $\alpha_j$. Interested readers may refer to Kudo and Matsumoto (2003) for the detailed computation for obtaining $w$.

The time complexity of Eq. 5 (and Eq. 1) is $O(|x^d|)$, which is linear with respect to the number of active features in $x^d$ within the expanded feature space $F^d$.

**Heuristic Kernel Expansion (SVM-HKE)** To make the weight vector sparse, Kudo and Matsumoto (2003) proposed a heuristic method that filters out less useful features whose absolute weight values are less than a pre-defined threshold $\sigma$.\(^2\) They reported that increased threshold value $\sigma$ resulted in a dramatically sparse feature space $F^d$, which had the side-effects of accuracy degradation and classifier speed-up.

### 3 Proposed Method

In this section, we propose a method that speeds up a classifier trained with many conjunctive features. Below, we focus on a kernel-based classifier trained with a polynomial kernel of degree $d$ (here,\(^3\))

\[^1\]For example, given an input vector $x = (x_1, x_2)^T$ and a support vector $x' = (x'_1, x'_2)^T$, the 2nd-order polynomial kernel returns $k_2(x', x) = (x'_1 x_1 + x'_2 x_2 + 1)^2 = 3x'_1 x_1 + 3x'_2 x_2 + 2x'_1 x_1 x'_2 x_2 + 1$ (the $x'_1, x'_1 \in \{0, 1\}$). This function thus implies a mapping $\phi_2(x) = (1, \sqrt{3}x_1, \sqrt{3}x_2, \sqrt{2}x_1 x_2)^T$. In the following argument, we ignore the dimension of the constant in the mapped space and assume constant $b$ is set to include it.

\[^2\]Precisely speaking, they set different thresholds to positive ($\alpha_i > 0$) and negative ($\alpha_i < 0$) support vectors, considering the proportion of positive and negative support vectors.

\[^3\]When a feature vector $x$ includes (explicit) conjunctive features $f \in F^d$, we assume weight function $g(y|x') = g(y|x)$, where $x'$ is a projection of $x$ by $\phi_a : F^d \rightarrow F$.

Figure 1: Efficient computation of $g(x)$.

SVMs), but an analogous argument is possible for linear classifiers (e.g., LLMs).\(^3\)

We hereafter represent a binary feature vector $x$ as a set of active features $\{f_i \mid f_i(x) = 1\}$. $x$ can thereby be represented as an element of the power set $2^F$ of the set of features $F$.

#### 3.1 Idea

Let us remember that weight function $g(x)$ in Eq. 5 maps $x \in 2^F$ to $W \in \mathbb{R}$. If we could calculate $W = g(x)$ for all possible $x$ in advance, we could obtain $g(x)$ by simply checking $|x|$ elements, namely, in $O(|x|)$ time. However, because $|\{x | x \in 2^F\}| = 2^{|F|}$ and $|F|$ is likely to be very large (often $|F| > 10,000$ in NLP tasks), this calculation is impractical.

We then compute and store weight $W_{x'} = g(x')$ for $x' \in \mathcal{V}_c(\subset 2^F)$, a certain subset of the possible value space, and compute $g(x)$ for $x \notin \mathcal{V}_c$ by using precalculated weight $W_{x_c}$ for $x_c \subseteq x$ in the following way:

$$g(x) = W_{x_c} + \sum_{f_i \in x^d - x'^d} w_i.$$

Intuitively speaking, starting from partial weight $W_{x_c}$, we add up remaining weights of primitive features $f \in F$ that are not active in $x_c$ but active in $x$ and conjunctive features that combine $f$ and the other active features in $x$.

An example of this computation ($d = 2$) is depicted in Figure 1. We can efficiently compute $g(x)$ for a vector $x$ that has four active features $f_1$, $f_2$, $f_3$, and $f_4$ (and $x^2$ has six conjunctive features) using precalculated weight $W_{\{1,2,3\}}$; we should first check the three features $f_1$, $f_2$, and $f_3$ to retrieve $W_{\{1,2,3\}}$ and next check the remaining four features related to $f_4$, namely $f_4$, $f_1.4$, $f_2.4$, and $f_3.4$, in order to add up the remaining
weights, while the normal computation in Eq. 5 should check the four primitive and six conjunctive features to get the individual weights.

**Expected time complexity** Counting the number of features to be checked in the computation, we obtain the time complexity $f(x,d)$ of Eq. 6 as:

$$f(x,d) = O(|x_c| + |x^d| - |x^d|^2),$$

where $|x^d| = \sum_{k=1}^{d} \binom{|x|}{k}$

(e.g., $|x^2| = \frac{|x|^2 + |x|}{2}$ and $|x^3| = \frac{|x|^3 + 5|x|^2}{6}$). Note that when $|x_c|$ becomes close to $|x|$, this time complexity actually approaches $O(|x|)$.

Thus, to minimize this computational cost, $x_c$ is to be chosen from $\mathcal{V}_c$, as follows:

$$x_c = \arg\min_{x' \in \mathcal{V}_c, x' \subseteq x} (|x'| + |x^d| - |x^d|).$$

(9)

### 3.2 Construction of Feature Sequence Trie

There are two issues with speeding up the classifier by the computation shown in Eq. 6. First, since we can store weights for only a small fraction of possible feature vectors (namely, $|\mathcal{V}_c| \ll 2^{|F|}$), we should choose $\mathcal{V}_c$ so as to maximize its impact on the speed-up. Second, we should quickly find an optimal $x_c$ from $\mathcal{V}_c$ for a given feature vector $x$.

The solution to the first problem is to enumerate partial feature vectors that frequently appear in the target task. Note that typical linguistic features used in NLP tasks usually consist of disjunctive sets of features (e.g., word surface and POS), in which each set is likely to follow Zipf’s law (Zipf, 1949) and correlate with each other. We can expect the distribution of feature vectors, the mixture of Zipf distributions, to be Zipfian. This has been confirmed for word n-grams (Egghe, 2000) and itemset support distribution (Chuang et al., 2008). We can thereby expect that a small set of partial feature vectors commonly appear in the task.

To solve the second problem, we introduce a feature sequence trie (fstrie), which represents a hierarchy of feature vectors, to enable the classifier to efficiently retrieve (sub-)optimal $x_c$ (in Eq. 9) for a given feature vector $x$. We build an fstrie in the following steps:

**Step 1:** Apply the target classifier to actual (raw) data in the task to enumerate possible feature vectors (hereafter, source feature vectors).

An fstrie built from six source feature vectors is shown in Figure 2. In fstries, a path from the root to another node represents a feature vector. An important point here is that the fstrie stores the weights of all prefix feature vectors of the source feature vectors, and the trie structure enables us to retrieve for a given feature vector $x$ the weight of its longest prefix vector $x_c \subseteq x$ in $O(|x_c|)$ time. To handle feature functions in LLMs (Eq. 1), we store partial weight $W_{x_c,y} = \sum_{w} w_{i,y}f_{i,y}(x_c,y)$ for each label $y$ on the node that expresses $x_c$.

Since we sort the features in the source feature vectors according to their frequency, the prefix feature vectors exclude less frequent features in the source feature vectors. Lexical features or finer-grained features (e.g., POS-subcategory) are usually less frequent than coarse-grained features (e.g., POS), so they lie in the latter part of the feature vectors. This sorting helps us to retrieve longer feature vector $x_c$ for input feature vector $x$ that will have diverse infrequent features. It also minimizes the size of fstrie by sharing the common frequent prefix (e.g., $\{f_1, f_2\}$ in Figure 2).

**Pruning nodes from fstrie** We have so far described the way to construct an fstrie from the source feature vectors. However, a naive enumeration of source feature vectors will result in the explosion of the fstrie size, and we want to have a principled way to control the fstrie size rather than reducing the processed data size. Below, we present a method that prunes useless prefix feature vectors (nodes) from the constructed fstrie to maximize its impact on the classifier efficiency.
We adopt a greedy strategy that iteratively prunes a leaf node (one prefix feature vector and its weight) from the fstrie built from all the source feature vectors, according to a certain utility score calculated for each node. In this study, we consider two metrics for each prefix feature vector \( x_c \) to calculate its utility score.

**Probability** \( p(x_c) \), which denotes how often the stored weight \( W_{x_c} \) will be used in the target task. The maximum-likelihood estimation provides probability:

\[
p(x_c) = \frac{\sum_{x' \supseteq x_c} n_{x'}}{\sum_{x'} n_{x'}} \tag{10}
\]

where \( n_{x} \in \mathbb{N} \) is the frequency count of a source feature vector \( x \) in the processed data.

**Computation reduction** \( \Delta_d(x_c) \), which denotes how much computation is reduced by \( W_{x_c} \) to calculate a weight of \( x \supseteq x_c \). This can be estimated by counting the number of conjunctive features we additionally have to check when we remove \( x_c \). Since the fstrie stores the weight of a prefix feature vector \( x_{c'} \subset x_c \) such that \( |x_{c'}| = |x_c| - 1 \) (e.g., in Figure 2, \( x_{c'} = \{f_1, f_2\} \) for \( x_c = \{f_1, f_2, f_3\} \)), we can define the computation reduction as:

\[
\Delta_d(x_c) = (|x^d_c| - |x^d_{c'}|) - (|x_c| - |x_{c'}|)
\]

\[
= \sum_{k=2}^{d} \binom{|x_c|}{k} - \sum_{k=2}^{d} \binom{|x_c| - 1}{k}
\]

(\( \because \) Eq. 8).

\[\Delta_2(x_c) = |x_c| - 1 \quad \text{and} \quad \Delta_3(x_c) = \frac{|x_c|^2 - |x_c|}{2} \]

We calculate utility score of each node \( x_c \) in the fstrie as \( u(x_c) = p(x_c) \cdot \Delta_d(x_c) \), which means the expected computation reduction by \( x_c \) in the target task, and prune the lowest-utility-score leaf nodes from the fstrie one by one (Algorithm 1). If several prefix vectors have the same utility score, we eliminate them in numerical descending order.

**Algorithm 1 PRUNE NODES FROM FSTRIE**

**Input:** fstrie \( T \), node_limit \( N \in \mathbb{N} \)

**Output:** fstrie \( T \)

1: \hspace{1em} while \# of nodes in \( T > N \) do
2: \hspace{2em} \( x_c \leftarrow \text{argmin } u(x') \)
3: \hspace{2em} remove \( x_c \), \( T \)
4: \hspace{1em} end while
5: \hspace{1em} return \( T \)

**Algorithm 2 COMPUTE WEIGHT WITH FSTRIE**

**Input:** fstrie \( T \), weight vector \( w \in \mathbb{R}^{|F|} \)

**Output:** weight \( W = g(x) \in \mathbb{R} \)

1: \hspace{1em} \( x \leftarrow \text{sort}(x) \)
2: \hspace{2em} \( \langle x_c, W_{x_c} \rangle \leftarrow \text{prefix_search}(T, x) \)
3: \hspace{2em} \( W \leftarrow W_{x_c} \)
4: \hspace{2em} for all feature \( f_j \in x^d - x_c^d \) do
5: \hspace{3em} \( W \leftarrow W + w_j \)
6: \hspace{2em} end for
7: \hspace{1em} return \( W \)

### 3.3 Classification Algorithm

Our classification algorithm is shown in detail in Algorithm 2. The classifier first sorts the active features in input feature vector \( x \) according to their frequency in the training data. Then, for \( x \), it retrieves the longest common prefix vector \( x_c \) from the fstrie (line 2 in Algorithm 2). It then adds the weights of the remaining features to partial weight \( W_{x_c} \) (line 5 in Algorithm 2).

Note that the remaining features whose weights we sum up (line 4 in Algorithm 2) are primitive and conjunctive features that relate to \( f \in x - x_c \), which appear less frequently than \( f' \in x_c \) in the training data. Thus, when we apply our algorithm to classifiers with the sparse solution (e.g., SVM-KEs or \( \ell_1 \)-LLMs), \( |x^d| - |x^d_c| \) can be much smaller than the theoretical expectation (Eq. 8). We confirmed this in the following experiments.

### 4 Evaluation

We applied our algorithm to SVM-KE, SVM-HKE, and \( \ell_1 \)-LLM classifiers and evaluated the resulting classifiers in a Japanese dependency parsing task. To the best of our knowledge, there are no previous reports of an exact weight calculation faster than linear summation (Eqs. 1 and 5). We also compared our SVM classifier with a classifier called polynomial kernel inverted (PKI: Kudo and Matsumoto (2003)), which uses the polynomial kernel (Eq. 4) and inverted indexing to support vectors.

#### 4.1 Experimental Settings

A Japanese dependency parser inputs bunsetsu-segmented sentences and outputs the correct head (bunsetsu) for each bunsetsu; here, a bunsetsu is a grammatical unit in Japanese consisting of one or more content words followed by zero or more function words. A parser generates a feature vec-
We used TinySVM\(^2\) and a simple C++ library for maximum entropy classification\(^8\) to train SVMs and \(\ell_1\)-LLMs, respectively. We used Darts-Clone,\(^9\)

\[^{2}\text{http://chasen.org/taku/software/TinySVM/}\]  
\[^{3}\text{http://www-tsujii.is.s.u-tokyo.ac.jp/~tsuruoka/maxent/}\]  
\[^{4}\text{http://nlp.kuee.kyoto-u.ac.jp/nl-resource/corpus-e.html}\]  
\[^{5}\text{http://code.google.com/p/darts-clone/}\]  

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\(d\), \(\omega\), \(\sigma\)

| Model type \(d\), \(\omega\) / \(\sigma\) | Model statistics \(\mathbb{F}^d\) | \(\mathbb{A}^d\) | Dep. acc. | Sent. acc. |
|---------------------------------|------------------|---------|-----------|-----------|
| SVM-KE 1 0                      | 39712            | 27.3    | 88.29     | 46.49     |
| SVM-KE 2 0                      | 1478109          | 380.6   | 90.76     | 53.83     |
| SVM-KE 3 0                      | 26194354         | 3286.7  | 90.93     | 54.43     |
| SVM-KE 3.001                    | 1324765          | 2725.9  | 90.92     | 54.39     |
| SVM-KE 3.002                    | 2514385          | 2238.1  | 90.91     | 54.32     |
| SVM-KE 3.003                    | 793195           | 1855.4  | 90.83     | 54.21     |
| SVM-HEK 4 0                    | 293416102        | 20395.4 | 90.91     | 54.69     |
| SVM-HEK 4 0.0002                | 96522236         | 15282.1 | 90.93     | 54.53     |
| SVM-HEK 4 0.0004                | 19245076         | 11565.0 | 90.96     | 54.64     |
| SVM-HEK 4 0.0006                | 7277592          | 8958.2  | 90.84     | 54.48     |
| \(\ell_1\)-LLM 1 1.0            | 9208             | 26.5    | 88.22     | 46.06     |
| \(\ell_1\)-LLM 2 2.0            | 32575            | 898.8   | 90.62     | 53.46     |
| \(\ell_1\)-LLM 3 3.0            | 129503           | 2088.3  | 90.71     | 54.09     |
| \(\ell_1\)-LLM 4 4.0            | 85419            | 1803.0  | 90.61     | 53.79     |
| \(\ell_1\)-LLM 5 5.0            | 63046            | 1699.5  | 90.59     | 53.55     |

**Table 1:** Feature set used for experiments.

Table 2: Specifications of LLMs and SVMs. The accuracy marked with ‘\(\gg\)’ or ‘\(\geq\)’ was significantly better than the \(d = 2\) counterpart (\(p < 0.01\) or \(0.01 \leq p < 0.05\) by McNemar’s test).

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\[^{9}\text{http://code.google.com/p/darts-clone/}\]  
\[^{10}\text{The parameter } C \text{ of } \ell_1\text{-LLM in Eq. 2 was set to } \frac{\omega}{L} \text{ (referred to in Kazama and Tsujii (2003) as ‘single width’).}\]
Table 3: Parsing results for test corpus: SVM-KE classifiers with dense feature space.

| Model type | $d$ | PKI classify [ms/sent.] | Baseline Mem. (MB) | Time (ms/sent.) | Proposed w/ fstrie Mem. (MB) | Time (ms/sent.) | Proposed w/ fstrie Mem. (MB) | Time (ms/sent.) | Proposed w/ fstrie Mem. (MB) | Time (ms/sent.) | Speed up |
|------------|-----|--------------------------|--------------------|-----------------|-------------------------------|-----------------|-------------------------------|-----------------|-------------------------------|-----------------|---------|
| SVM-KE 1   | 1   | 13.480                   | 0.2 **0.003** (0.015) | +0.6 0.006 (0.018) | +20.2 0.007 (0.018)          | +662.9 0.016 (0.029) | NA                           |
| SVM-KE 2   | 2   | 10.313                   | 13.5 0.041 (0.054)  | +0.5 **0.020** (0.032) | +18.0 0.021 (0.034)          | +662.4 0.023 (0.036) | 2.1                          |
| SVM-KE 3   | 3   | 10.945                   | 142.2 0.345 (0.361) | +0.5 0.163 (0.178)  | +18.2 0.108 (0.123)          | +667.0 **0.079** (0.093) | 4.4                          |
| SVM-KE 4   | 4   | 12.603                   | 648.0 2.338 (2.363) | +0.5 1.156 (1.178)  | +18.6 0.671 (0.690)          | +675.9 **0.415** (0.432) | 5.6                          |

Figure 3: Average classification time per sentence plotted against size of fstrie: SVM-KE.

Results for SVM-KE with dense feature space

The performances of parsers having SVM-KE classifiers with and without the fstrie are given in Table 3. The ‘speed-up’ column shows the speed-up factor of the most efficient classifier (bold) versus the baseline classifier without fstries. Since each classifier solved a slightly different number of classification steps (112, 853 ± 0.15%), we show the (average) cumulative classification time for a sentence. The Mem. columns show the size of weight vectors for SVM-KE classifiers and the size of fstrie_{BE}, fstrie_{M}, and fstrie_{L}, respectively.

The fstries successfully speeded up SVM-KE classifiers with the dense feature space. The SVM-KE classifiers without fstries were still faster than PKI, but as expected from a large $|x^d|_1$ value, the classifiers with higher conjunctive features were much slower than the classifier with only primitive features by factors of 13 ($d = 2$), 109 ($d = 3$) and 738 ($d = 4$) and the classification time accounted for most of the parsing time.

The average classification time of our classifiers plotted against fstrie size is shown in Figure 3. Surprisingly, we obtained a significant speed-up even with tiny fstrie sizes of < 1 MB. Furthermore, we naively controlled the fstrie size by sim-
Table 4: Parsing results for test corpus: SVM-HKE and \( \ell_1 \)-LLM classifiers with sparse feature space.

| Model | Type | \( d \) | \( \sigma \) | \( \omega \) | Baseline | Proposed w/ \( \ell_1 \)-\( \omega \) | Proposed w/ SVM | Proposed w/ SVM | Proposed w/ SVM | Speed up |
|-------|------|-------|-------|-------|---------|--------------|--------------|--------------|--------------|----------|
|       | Mem. Time (ms/sent.) | Mem. Time (ms/sent.) | Mem. Time (ms/sent.) | Mem. Time (ms/sent.) | Mem. Time (ms/sent.) |
| SVM-HKE | 3 | 0.001 | +0.5 | 0.151 (0.166) | +17.6 | 0.097 (0.111) | +63.0 | 0.070 (0.084) | 5.0 |
| SVM-HKE | 3 | 0.002 | +0.5 | 0.123 (0.137) | +17.0 | 0.074 (0.088) | +61.2 | 0.053 (0.067) | 6.2 |
| SVM-HKE | 3 | 0.003 | +0.4 | 0.102 (0.115) | +14.7 | 0.057 (0.070) | +52.6 | 0.041 (0.054) | 7.8 |
| SVM-HKE | 4 | 0.0002 | +0.5 | 1.022 (1.042) | +17.7 | 0.358 (0.575) | +63.7 | 0.330 (0.346) | 6.8 |
| SVM-HKE | 4 | 0.0004 | +0.5 | 0.816 (0.835) | +16.8 | 0.414 (0.430) | +60.1 | 0.234 (0.249) | 8.7 |
| SVM-HKE | 4 | 0.0006 | +0.4 | 0.646 (0.662) | +15.7 | 0.311 (0.326) | +58.9 | 0.168 (0.183) | 10.7 |
| \( \ell_1 \)-LLM | 1 | 1.0 | +0.8 | 0.006 (0.018) | +25.0 | 0.007 (0.019) | +78.7 | 0.016 (0.029) | NA |
| \( \ell_1 \)-LLM | 2 | 2.0 | +0.6 | 0.016 (0.028) | +20.5 | 0.015 (0.027) | +69.0 | 0.018 (0.034) | 2.9 |
| \( \ell_1 \)-LLM | 3 | 3.0 | +0.5 | 0.091 (0.103) | +17.8 | 0.041 (0.054) | +60.1 | 0.027 (0.040) | 11.6 |
| \( \ell_1 \)-LLM | 4 | 4.0 | +0.5 | 0.082 (0.094) | +16.3 | 0.036 (0.049) | +55.0 | 0.024 (0.037) | 12.4 |
| \( \ell_1 \)-LLM | 5 | 5.0 | +0.5 | 0.076 (0.088) | +15.1 | 0.032 (0.045) | +51.0 | 0.022 (0.035) | 13.3 |

Figure 4: Fstrie reduction: utility score vs. processed sentence reduction for SVM-KE (\( d = 4 \)).

Figure 5: Average classification time per sentence plotted against size of fstrie: SVM-KE (\( d = 3 \)).

Figure 6: Average classification time per sentence plotted against size of fstrie: \( \ell_1 \)-LLM (\( d = 3 \)).

Results for SVM-HKE and \( \ell_1 \)-LLM classifiers with sparse feature space

The performances of parsers having SVM-HKE and \( \ell_1 \)-LLM classifiers with and without the fstrie are given in Table 4. The fstries successfully speeded up the SVM-HKE and \( \ell_1 \)-LLM classifiers by factors of 10.7 (SVM-HKE, \( d = 4, \sigma = 0.0006 \)) and 11.6 (\( \ell_1 \)-LLM, \( d = 3, \omega = 3.0 \)). We obtained more speed-up when we used fstries for classifiers with more sparse feature space \( \mathcal{F}^d \) (Figures 5 and 6). The parsing speed with \( d = 3 \) models are now comparable to the parsing speed with \( d = 2 \) models.

Without fstries, little speed-up of SVM-HKE classifiers versus the SVM-KE classifiers (in Table 3) was obtained due to the mild reduction in the average number of active features \( |x^d| \) in the classification. This result conforms to the results reported in (Kudo and Matsumoto, 2003).

The parsing speed reached 14,937 sentences per second with accuracy of 90.91% (SVM-HKE, \( d = 3, \sigma = 0.002 \)). We used this parser to process 1,005,918 sentences (5,934,184 bunsetsus) randomly extracted from Japanese weblog feeds.
updated in November 2008, to see how much the impact of fstries lessens when the test data and the data processed to build fstries mismatch. The parsing time was 156.4 sec. without fstrieL, while it was just 35.9 sec. with fstrieL. The speed-up factor of 4.4 on weblog feeds was slightly worse than that on news articles ($0.346/0.067 = 5.2$) but still evident. This implies that sorting features in building fstries yielded prefix features vectors that commonly appear in this task, by excluding domain-specific features such as lexical features.

In summary, our algorithm successfully minimized the efficiency gap among classifiers with different degrees of feature combinations and made accurate classifiers trained with higher-order feature combinations practical.

5 Conclusion and Future Work

Our simple method speeds up a classifier trained with many conjunctive features by using precalculated weights of (partial) feature vectors stored in a feature sequence trie (fstrie). We experimentally demonstrated that it speeded up SVM and LLM classifiers for a Japanese dependency parsing task by a factor of 10. We also confirmed that the sparse feature space provided by $\ell_1$-LLMs and SVM-HKEs contributed much to size reduction of the fstrie required to achieve the same speed-up. The implementations of the proposed algorithm for LLMs and SVMs (with a polynomial kernel) and the Japanese dependency parser will be available at http://www.tkl.iis.u-tokyo.ac.jp/~ynaga/.

We plan to apply our method to wider range of classifiers used in various NLP tasks. To speed up classifiers used in a real-time application, we can build fstries incrementally by using feature vectors generated from user inputs. When we run our classifiers on resource-tight environments such as cell-phones, we can use a random feature mixing technique (Ganchev and Dredze, 2008) or a memory-efficient trie implementation based on a succinct data structure (Jacobson, 1989; Delpratt et al., 2006) to reduce required memory usage.

We will combine our method with other techniques that provide sparse solutions, for example, kernel methods on a budget (Dekel and Singer, 2007; Dekel et al., 2008; Orabona et al., 2008) or kernel approximation (surveyed in Kashima et al. (2009)). It is also easy to combine our method with SVMs with partial kernel expansion (Goldberg and Elhadad, 2008), which will yield slower but more space-efficient classifiers. We will in the future consider an issue of speeding up decoding with structured models (Lafferty et al., 2001; Miyao and Tsujii, 2002; Sutton et al., 2004).

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