Conditions for the development of Rayleigh-Taylor instability on the spherical flame front

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Abstract. The instability of the flame front is one of the actual problems of modern fluid and gas mechanics. Depending on the processes determining the growth of inhomogeneities, the hydrodynamic (Darrieus-Landau), diffusive-thermal, and Rayleigh-Taylor instabilities are distinguished for a free spherical flame. The present work is devoted to the conditions for the development of the latter. Rayleigh-Taylor instability arises at the interface of two fluids of different density in the presence of acceleration directed from the lighter to the heavier one. When the flame propagates, combustion products with less density move with acceleration in a denser gas. When the acceleration vector and the density gradient at the flame front are co-directed, instability develops. To investigate the development of instabilities, a series of experiments was carried out. Transparent latex shells were filled with a pre-prepared hydrogen-air mixture. In different series of experiments, the percentage of hydrogen changed. The flame was ignited by a spark discharge with energy of 1 mJ, a spark gap located in the center of the shell. Registration of the flame front propagation was performed using the schlieren method implemented in the shadow device IAB-451 and the high-speed VideoSprint camera. The video was shot at a frame rate of 500 to 1000 fps with an exposure of 500 µs. To automate the processing of images in the Matlab environment, a program is written that converts a set of images of the expanding flame front into a dependence of the mean radius on time. It is found that at the initial stage of propagation there are both acceleration and deceleration of the flame front. Experimentally obtained propagation parameters of the flame front are supplemented by calculations carried out on an analytical model from the literature.

1. Introduction

The propagation of a free flame in the initially quiescent homogeneous combustible mixture is often accompanied by the instability of the front. The mechanisms of flame front instability are well described in the literature. Usually, when the flame propagates, three mechanisms of front instability are distinguished.

Diffusive-thermal instability is caused by different rates of heat transfer from the chemical reaction zone and reagents to the chemical reaction zone. The mathematical model of the diffusive-thermal mechanism of flame front instability was proposed in [1] and further developed in [2]. A detailed study of flame patterns due to thermal diffusion instability in a wide range of combustible mixture parameters is presented in [3].
The hydrodynamic instability or Darrieus-Landau instability is caused by the deflection of the flow passing through the curved flame front. The mathematical theory was first suggested in [4]. According to this model, the flame is unstable over the whole range of parameters of the mixture and the geometry of the front. The solution obtained contradicted the experimental data [5]. Later, the model of the flame front was supplemented by taking into account the finite thickness of the flame front by the means of the phenomenological approach [6] or by detailed consideration of the mass and heat transfer inside the flame front [7, 8]. The introduced corrections led to a theoretical description of the hydrodynamic mechanism of the flame front instability in accordance with the experimental data.

Rayleigh-Taylor instability arises when the interface between light and heavy fluids accelerates toward a heavy fluid [9, 10]. Rayleigh-Taylor mechanism for the interface, through which there is no flow of matter, is described mathematically in [11]. In the case of gas explosions of very large volumes, when the flame thickness can be neglected, the Rayleigh-Taylor instability is analytically investigated in [12]. The authors show two different scenarios for the development of the flame shape. In the first case, the flame remains globally spherical and accelerates with the formation of detonation. In the second case, two large regions of combustion are formed.

This article is devoted to the investigation of the conditions for the appearance of Rayleigh-Taylor instability in gas explosion in volumes up to 30 m³ and comparison of the characteristic sizes and growth rates of the flame front instability by the mechanisms of diffusive-thermal, hydrodynamic and Rayleigh-Taylor ones.

2. Experimental setups

The experiments were carried out at two experimental facilities: a laboratory setup and an explosion chamber 13Ya3 (Moscow Regional Explosion Center for Collective Use of Russian Academy of Sciences).

2.1. Small scale explosions

The laboratory setup (Figure 1) consists of a remote-control panel for creating a gas mixture, a shell filler, a spark igniter, an IAB-451 shadow device and a high-speed VideoSprint camera. The combustible mixture was preliminarily prepared in a cylinder at a pressure of 5 atm. After filling, the mixture was kept in the cylinder for at least 24 hours for complete mixing. In different experiments latex or soap shells were filled with combustible mixture. The hydrogen content in the hydrogen-air mixture was 15, 17, and 20%. The pressure was atmospheric, the volume of the shells was 4 ± 1 l. Mixture was ignited in 5 minutes after filling to ensure the immobility of the mixture at the time of ignition. Initiation was carried out in the center, the energy was 1 mJ.

![Scheme of experimental setup](image)

Figure 1. Scheme of experimental setup. 1 – LED light source; 2 – optical slit; 3 – diagonal mirror; 4 – spherical mirror; 5 – meniscus; 6 – Foucault knife; 7 – high-speed CCD camera; 8 – ignition power source; 9 – spark plug; 10 – feeding system.

In the experiments, the camera was used at frame rates of 500 fps and 1000 fps. In such modes the frame size was 1280*1000 pixels and 1280*500 pixels.

The main parameters obtained are the position and shape of the flame front as a function of time.
2.2. Large scale explosions

The explosion chamber 13Ya3 (Figure 2) is equipped with a system for preparing a combustible mixture, a fan for stirring the combustible mixture inside the shell, a latex shell fixing device, a flame ignition system with an energy of 1 mJ or 5 J, and an infrared imaging system with an InfraTek 8320 camera.

A combustible mixture with a hydrogen content of 15% was prepared directly in the shell. The hydrogen from the cylinder and the air were fed into the shell through the flowmeters. After filling, the gas in the shell was stirred for 30 minutes, 30 minutes after the fan was turned off, the mixture was ignited by a spark with an energy of 1 mJ or an exploding wire with an energy of 5 J.

![Figure 2. Explosion chamber 13Ya3. 1 – ignition point; 2 – shell with combustible mixture; 3 – InfraTek 8320 camera; 4 – ignition system; 5 – feeding system.](image)

3. Experimental data processing

Using high speed cameras faces a problem of great amount of experimental data. For example, one experiment made with the infrared camera could consist of more than 300 frames. The automation of the processing of experimental data is discussed in the literature quite intensively [13, 14, 15]. Since each experimental work has its own characteristics of obtaining images and the object under study, we developed our own technique for automatic image processing.

The center of the explosion is considered to be known. The picture is binarized in order to separate the flame front from inhomogeneities. The distance between the center and the flame front is measured in pixels which are multiplied on scale of mm per pixel. To make the processing faster a MatLab script was made.

The error obtained by such measurements does not exceed 3%.

4. Experimental results

The results of a series of experiments with a hydrogen-air mixture with 15% hydrogen content are presented in Figure 3. This Figure shows the dependence of average flame radii “R” versus time “t” for a big amount of independent experiments with the unchanged composition, pressure, temperature of the mixture and ignition energy. Each symbol corresponds an individual explosion. Significant dispersion of dependences of average flame radii versus time is observed. Such dispersion can be explained by the probabilistic nature of the origin of the inhomogeneities of the flame front and the statistical regularities of their interaction, which leads to a statistical dispersion within one series of experiments with the same mixture and ignition parameters. At the same time, in all experiments from the series, the growth of inhomogeneities in the flame front is caused by the same mechanisms.
Figure 3. R-t diagrams (radii versus time) of 80 experiments with 15% hydrogen content in the mixture.

It should be noted that Figure 3 shows differences in the flame propagation. This discrepancy is explained by the influence of instabilities, which brings randomness into the process. As it was said, the error is 3%, it does not explain this discrepancy.

Figure 4. Sequence of experimental data processing.

Considering one r-t-diagram (Figure 4a) it can be clearly seen that it is not linear, and flame propagates with acceleration and deceleration. R-t-diagram was approximated with polynomial function of fourth power. First derivative of r-t-diagram is the dependence of flame speed versus time. It shows that there are areas where the speed increases and decreases (Figure 4b). Second derivative of r-t-diagram is the dependence of flame acceleration versus time (Figure 4c).

5. Discussion

According to classical theory Rayleigh-Taylor instability arises when the interface between light and heavy fluids accelerates toward the heavy fluid. In the experiments we can see that lighter combustion products move with acceleration towards unburnt gas thus Rayleigh-Taylor instability can take place.

The growth rates and wavelengths of maximum growth rate for different mechanisms can be calculated according to the theoretical formulas presented in the Table 1.
Table 1. Theoretical formulas for calculating the instability size and growth speed. \( \chi \) – thermal diffusivity, \( \theta \) – thermal expansion coefficient, \( S_L \) – laminar burning velocity, \( g \) – acceleration, \( \nu_k \) – kinematic viscosity, \( Z_e \) – Zeldovich number, \( L_e \) – Lewis number, \( A_t \) – Atwood number.

| Characteristic wavelength | Growth rate |
|---------------------------|-------------|
| Diffusive-thermal instability [2] | \( \lambda_{TD} = \frac{8\pi \chi}{S_L \sqrt{2Ze(1 - L_e) - 1}} \) | \( \omega_{TD} = \frac{4\pi^2}{\lambda_{TD}^2} \left( \frac{Z_e}{2} \left( 1 - L_e \right) - 1 \right) \) |
| Darrius-Landau instability [8] | \( \lambda_{DL} = \frac{4\pi \chi}{S_L} \left( 1 + \frac{\theta + 1}{(\theta - 1)^2} \theta \ln \theta \right) \) | \( \omega_{DL} = \frac{\pi S_L}{\lambda_{DL}} \left( \frac{\theta + 1 - \frac{1}{\theta}}{\theta} \right) \) |
| Rayleigh-Taylor instability [16] | \( \lambda_{RT} = 4\pi^2 \left( \frac{\nu_k^2}{g \times A_t} \right) \) | \( \omega_{RT} = \left( \frac{(g \times A_t)^{4/3}}{2\nu_k^{2/3}} \right) \sqrt{\frac{2}{\nu_k}} \) |

The inhomogeneities of the flame front initially grow rapidly according to the diffusive-thermal mechanism (Table 2). Growth rate of the inhomogeneity due to the diffusive-thermal instability of the flame front decreases in proportion to the square of the inhomogeneity size. Further growth of the inhomogeneity is due to the hydrodynamic (Darrieus-Landay) mechanism, whose growth rate decreases linearly with the size of the inhomogeneity. The appearance and growth of inhomogeneities on the flame front lead to its acceleration, which creates the conditions for the development of Rayleigh-Taylor instability. Under the experimental conditions, growth rate of the inhomogeneity by the Rayleigh-Taylor mechanism is sufficiently small. This mechanism affects the development of inhomogeneities in the flame front, when they increase to large enough dimensions (Table 3).

Table 2. Theoretically calculated values of the instability size and growth speed.

| %H2 | \( \lambda_{TD} \), m | \( \omega_{TD} \), s\(^{-1} \) | \( \lambda_{DL} \), m | \( \omega_{DL} \), s\(^{-1} \) | \( \lambda_{RT} \), m | \( \omega_{RT} \), s\(^{-1} \) |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|
| 15% | 4.5*10\(^{-4}\) | 1155 | 2*10\(^{-3}\) | 1123 | 2.5*10\(^{-2}\) | 1.02 |
| 17% | 4*10\(^{-4}\) | 945 | 1.6*10\(^{-3}\) | 1749 | 9.5*10\(^{-3}\) | 3.7 |
| 20% | 3.4*10\(^{-4}\) | 355 | 1.2*10\(^{-3}\) | 3193 | 2*10\(^{-3}\) | 23.4 |

The condition for the transition of one type of instability to another is the equality of their growth rates. In other words, inhomogeneity of this size has equal growth rates by two mechanisms.

Table 3. The size of the inhomogeneities at equal growth speed.

| \( \lambda(15\%H_2) \), m | \( \lambda(20\%H_2) \), m |
|-----------------|-----------------|
| \( \omega_{TD} = \omega_{DL} \) | 1.2*10\(^{-4}\) | 1.1*10\(^{-5}\) |
| \( \omega_{DL} = \omega_{RT} \) | 2.14 | 0.16 |

Table 3 shows the values of the wavelengths of the instabilities at equal growth rates. It can be seen that the transition from the diffusive-thermal instability of to the Darrieus-Landau occurs at its small
dimensions, when the Darrieus-Landau transition to Rayleigh-Taylor occurs at a large instability size. The Rayleigh-Taylor instability can indeed be observed if the combustion proceeds in a large volume.

6. Conclusions
The scales of inhomogeneities at the flame front in the lean hydrogen-air mixtures have been determined experimentally and theoretically.

The condition for the appearance of various types of instabilities are shown.

Parameters in the combustion process that can lead to the development of Rayleigh-Taylor instability are determined.

Rayleigh-Taylor instability in lean hydrogen-air flame arises when a linear size of inhomogeneities to be not less than 0.16 m in 20% and 2.14 m in 15% hydrogen-air mixture.

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