Computing the Polarimetric and Photometric Variability of Be Stars

K. C. Marr1, C. E. Jones1, and R. J. Halonen2
1 Department of Physics and Astronomy, Western University, London, ON N6A 3K7, Canada
2 School of Arts & Science, Red Deer College, Red Deer, AB T4N 5H5, Canada

Received 2017 August 28; revised 2017 December 6; accepted 2017 December 7; published 2018 January 11

Abstract

We investigate variations in the linear polarization as well as in the $V$-band and $B$-band color–magnitudes for classical Be star disks. We present two models: disks with enhanced disk density and disks that are tilted or warped from the stellar equatorial plane. In both cases, we predict variation in observable properties of the system as the disk rotates. We use a non-LTE radiative transfer code BEDISK (Sigut & Jones) in combination with a Monte Carlo routine that includes multiple scattering (Halonen et al.) to model classical Be star systems. We find that a disk with an enhanced density region that is one order of magnitude denser than the disk’s base density shows as much as ~0.2% variability in the polarization while the polarization position angle varies by ~8°. The $\Delta V$ magnitude for the same system shows variations of up to ~0.4 mag while the $\Delta(B-V)$ color varies by at most ~0.01 mag. We find that disks tilted from the equatorial plane at small angles of ~30° more strongly reflect the values of polarization and color–magnitudes reported in the literature than disks tilted at larger angles. For this model, the linear polarization varies by ~0.3%, the polarization position angle varies by ~60°, the $\Delta V$ magnitude varies up to 0.35 mag, and the $\Delta(B-V)$ color varies by up to 0.1 mag. We find that the enhanced disk density models show ranges of polarization and color–magnitudes that are commensurate with what is reported in the literature for all sizes of the density-enhanced regions. From this, we cannot determine any preference for small or large density-enhanced regions.

Key words: circumstellar matter – polarization – stars: early-type – stars: emission-line, Be

1. Introduction

Classical Be (B-emission) stars are main-sequence, B-type stars for which there is observational evidence for the presence of a geometrically flat decretion disk of ionized gas in Keplerian motion (Rivinius et al. 2013). The evolution of the circumstellar disks of Be stars is thought to be facilitated by nonradial pulsations (Balona & Rozowsky 2016). The defining characteristics of classical Be stars include spectral line emission in the Balmer series, periodic brightness and line profile variations over timescales ranging from minutes to decades (Harmanec 1983), an excess of IR and radio continuum emission, and linearly polarized light (Hall & Mikesell 1950; Behr 1957).

The linear polarization signature of Be stars can vary over time and can account for up to ~2% of the total light emitted. The source of the polarization is electron scattering of the star’s light as it interacts with the disk. Through electron scattering, the light within the disk becomes linearly polarized perpendicular to the plane containing the incident and scattered radiation (Quirrenbach et al. 1997). The polarized light from a Be star provides a wealth of information on the geometry and physical nature (e.g., chemical composition, ionization levels, opacity) of the disk without the need to resolve the source. The strength of the polarization signature is proportional to the number of particles available to scatter the light. However, there is a complex interaction between the scattering and absorption processes in the disk, which imparts a wavelength dependence on the polarization signature. The hydrogen bound–free absorption imprints a sawtooth shape on the polarization as a function of wavelength that weakens as the wavelength increases and rapidly increases at each series limit (see Figure 1 of Halonen & Jones 2013b).

Be stars exhibit variability on many different timescales ranging from hours to decades, a characteristic that suggests the involvement of various astrophysical phenomena. The fastest variations in Be stars, called ultra-rapid variations (Huang 1986), occur on timescales of hours and are associated with B-Cephei type pulsations if recurring (Balona & Rozowsky 1991). The short-term variations occurring over periods of days are primarily driven by non-radial pulsations (Baade 1982; Rivinius et al. 2003), but stellar rotation (Balona 1990, 1995) and localized mass ejections in the inner disk may produce variations of similar length. The intermediate length variations occur over weeks to decades. They are characterized by variability in emission-line strength, often measured by the $V/R\ (\text{violet to red})$ ratio of doubly peaked lines (Hanuschik et al. 1996; Rivinius et al. 2006), and are due to changes in the disk as a result of binarity on shorter terms or to the presence of precessing one-arm density waves on longer terms. The longest timescale for variations measured in Be stars occurs over years to decades. These variations are classified by the loss and return of spectral emission properties and have been linked to the growth and dissipation of the circumstellar disk, for example, $\pi$ Aqr’s (HD 212571) disk dissipation between 1986 and 1996 (Bjorkman et al. 2002; Wisniewski et al. 2010; Draper et al. 2011).

Huang (1986) suggested that there may be a correlation between the short- and long-term variations in Be stars. The local mass ejection of stellar surface material into the disk is theorized to cause an appreciable increase in the density of local regions, which may lead to variability in the observational characteristics of the stars (Okazaki 1991). Since
this prediction, there have been many studies of one-armed density waves. Notably, Carciofi et al. (2009) were able to confirm the presence of a spiral density arm in the disk of the Be star ζ Tauri (HD 37202). Additionally, they were able to show that the oscillation mode must extend to the outer regions of the disk to properly fit the large amplitude of the V/R variations. This suggests that long period variations could result from mass ejection into the disk. However, Carciofi et al. ’s (2009) model also exhibited inconsistencies with particular observations, such as a large variation in the polarization signature over the V/R cycle, which suggests that the innermost disk may not contain the expected density structure.

Perturbations in the circumstellar disk of Be star systems may also cause long period variations as seen, for example, in the behavior of γ Cas (HD 5394) and 59 Cyg (HD 200120). These two stars are notable as having showed two successive shell events at the same time as a variation in emission-line width. According to Hummel (1998), γ Cas varied in this manner from 1934 to 1940, after which its disk dissipated, while the same variations occurred for 59 Cyg from 1973 June to 1975 April. In the same work, he suggested that these variations are the result of a disk that has been tilted from the equatorial plane of the star. Such a system could form as between one-third and one-half of Be stars are found to be in binary systems, where there is a reasonable chance that the companion star would be misaligned with the equatorial plane of the Be star. Through tidal interactions, the disk can become inclined with respect to the star and align with the companion’s orbital plane. See Martin et al. (2011) for numerical models involving tidal warping and the evolution of misaligned disks.

The variation in the emission-line width and profile can be explained by the change in the orientation of the system on the plane of the sky as the disk rotates while in a tilted state.

In this study, we investigate two cases of Be star disk perturbations, density enhancement (DE) and tilted disk (TD), by studying the linear polarization and the B-band and V-band magnitudes predicted by the models. Through the use of a non-LTE radiative transfer code called BEDISK (Sigut & Jones 2007), the self-consistent radiative equilibrium models of the disk were acquired. From these models, the Stokes parameters were computed using the Monte Carlo routine MCTRACE (Halonen et al. 2013). In Section 2, we discuss the computational method used in creating each model. Section 3 provides the results of the DE model, while Section 4 gives the results of the TD model. Finally, in Section 5, we discuss the results of each model, we compare our predictions to the observations in other works, and we summarize the findings of this report.

2. Computational Modeling

2.1. BEDISK

In order to determine the Stokes parameters for the perturbed disks of Be stars, BEDISK was used to create models of axisymmetric disks. This modeling routine creates a two-dimensional cross-section of the disk with self-consistent thermal and density structures. The disk is axisymmetric around the star’s rotational axis and symmetric above and below the equatorial plane. A synthetic spectrum emulates the radiative output of the central star based on the desired model atmosphere from Kurucz (1993), which is selected through the log(g) and Teff of the star considered. (See Sigut & Jones 2007 for more details.) Multiple studies with BEDISK have shown its utility in modeling spectroscopy (Arcos et al. 2017; Jones et al. 2011), interferometry (Jones et al. 2009, 2017), and also polarimetry and photometry (Halonen & Jones 2013a, 2013b).

For this work, we assume that the disk density distribution falls off in the equatorial plane with an R−4 power law, which was first suggested by Waters (1986) and revisited by Cote & Waters (1987) and Waters et al. (1987). The density grid of the modeled disk is expressed in terms of the cylindrical coordinates corresponding to the radial distance from the star, R, and the height of the disk from the midplane, Z, and is given by

\[ \rho(R, Z) = \rho_0 \left( \frac{R}{R_*} \right)^n e^{-|Z/H|^2}. \]  

Here, ρ0 is the density at the disk’s inner edge at Z = 0, n is the radial density slope, and H is the disk’s scale height. At each radial grid point in the disk, the gas was taken to be in approximate vertical hydrostatic equilibrium. In the literature, values of ρ0 are often within the range of 10−12 g cm−3 to 10−10 g cm−3, while those for n are usually between 2 and 4 (Rivinius et al. 2013).

The BEDISK models for this work were computed using the stellar parameters provided in Table 1.

2.2. MCTRACE

MCTRACE calculates the Stokes parameters and photometric colors by using the thermal solution and level populations from BEDISK as a base model of the circumstellar gas. The routine uses the two-dimensional solution created by BEDISK and interpolates it into a three-dimensional grid to simulate the entire disk. It then performs radiative transfer through a Monte Carlo simulation, following the trajectory of each photon packet until it exits the disk after multiple scattering events. The parameters used for each model in MCTRACE are provided in Table 2. For each simulation, 20 billion photon packets were followed through the disk to ensure appropriate resolution of our results. The routine has been previously used in Halonen et al. (2013) and Halonen & Jones (2013a, 2013b, 2015) to synthesize the polarimetric signatures of classical Be stars.

An effective method for quantifying and interpreting polarization with the Stokes parameters was first presented in Stokes (1852). The linear polarization signature can be quantified using the Stokes Q and U parameters. These parameters represent the differences in the total intensity I...
across two sets of orthogonal axes, with one set rotated 45° from the other.

Free electrons in the partially ionized disk scatter photons emitted from the disk and the central star. A non-zero polarization signature is observed when the scattered photons are produced in a region that is asymmetric on the plane of the sky of the observer. As a disk becomes less symmetric, fewer photons are polarized along one of the axes, yielding less cancellation of orthogonal polarization vectors and a greater amount of polarization. These asymmetries occur for both non-axisymmetric disks seen at any inclination and axisymmetric disks seen at angles other than pole-on. In this work, Section 3 considers variations in observables which result in non-axisymmetric disks for the DE model, and Section 4 looks at the variable observables in axisymmetric disks for the TD model.

The relative strength of the polarization can be calculated from the Stokes parameters when added in quadrature. The normalized polarization level are given by \( q = Q/I \) and \( u = U/I \), and the normalized net linear polarization is then determined by

\[
p = \sqrt{(q^2 + u^2)}.
\]

The polarization position angle (PA) is given by

\[
\theta = \frac{1}{2} \arctan\left(\frac{u}{q}\right).
\]

As the Stokes parameters change with the inclination of the star, one may expect the strongest polarization signature to occur at 90°. However, in this position, the disk attenuates the polarization by re-absorbing much of the scattered light before it exits the disk. As such, the maximum polarization signature is actually seen at an inclination of \( i \approx 70° \) (Wood et al. 1996; Halonen & Jones 2013a). In contrast, the weakest polarization signature occurs at 0° when the disk appears spherically symmetric on the plane of the sky, causing the complete cancellation of vibrations from orthogonal directions due to the uniformity of the polarizing planes.

The inclination angle can also affect the photometric variations of a Be star disk, in addition to DEs. As shown in Figure 2 of Rivinius et al. (2013), different wavelengths contribute to the integrated flux of the disk as a function of radial distance from the star. In this work, the \( V \) and \( B \) bands are of primary concern. According to Haubois et al. (2012), 80% of the \( V \)-band flux originates from within 1.8 to 2.5 \( R_\odot \) for the same densities considered in the DE model; the \( B \) band is assumed to be similar.

### 3. Density Enhancement Model

To study the effects of disk warping, we adopted a disk with a DE region of variable size. First, we used BEDISK to create two axisymmetric disks with different densities \( p_0 \) (see Table 2). The base density values were chosen to ensure that any effects introduced by the optically thick DE would be apparent in our observables. We note that an increase in the gas density leads to an increase in the scale height of the disk.

These axisymmetric models were then spliced together, with a sector of the greater density model placed within the lower density model (see Figure 1). The spliced model was then evaluated with MCTRACE to determine the Stokes parameters. MCTRACE provides 16 different phases at which the disk may be viewed, shown in Figure 1. Plotting the observables sequentially at each phase angle is analogous to viewing the system undergoing a complete rotation in the plane of the sky. For this model, we vary the size of the DE sector, \( \omega \), and provide results for sectors with enhanced densities of 45°, 90°, 180°, and 270°.

The observables computed by MCTRACE are collected in 100 Å wavelength bands centered around pre-specified wavelengths. In this work, the observables were analyzed in the \( V \)-band (data averaged from 5250 Å and 5750 Å), \( B \) band (4450 Å), and across the Balmer jump from 3800 Å to 3600 Å.

#### 3.1. Polarization

In the DE model, both the inclination of the system and the angular size of the DE sector must be considered when interpreting the polarization signature. First, the amount of polarization increases as the size of the DE sector increases. In other words, an overall denser disk contains more scattering material, which increases the amount of polarized light observed for disks with asymmetric polarization vectors. However, for optically thick regimes, the polarized level may be diminished through the absorption of the scattered photons. In addition to the size of the DE, the inclination of the system also affects the observed polarization. Figure 2 shows the polarization signatures of the DE model, for different sizes of DEs as described in the figure caption.

In the top panel of Figure 2, the \( V \)-band percent polarization increases as the size of the DE increases. In each case, the polarization is decreased as the enhanced density region is behind the star and then increases as it becomes visible again. For example, when \( \omega = 270° \), the polarization increases from around 0.95% to 1.15% as the lower density region is occulted. The opposite case, but the same effect, is seen when \( \omega = 90° \). When \( \omega = 270° \), we see relatively strong polarization when the lower density region is behind the star, which decreases as the higher density region moves behind the star.
of or behind it (i.e., when the DE region is viewed from the side). When looking through the DE region with the lower density region enveloping it, the scattering into the line of sight yields greater polarization as opposed to when the DE region lies directly in front of the star with respect to the observer.

The bottom panel of Figure 2 shows the variation of PA with phase. The PA vector tracks the location of the DE, allowing for the position of the DE to be known (i.e., where the greatest source of polarization is). In each case, the PA oscillates around 0° as the disk rotates. For this system, the $u$ parameter varies from positive to negative (and is 0 when transitioning between) and dominates over the $q$ parameter, by virtue of how they were defined in MCTRACE. In the case where $\omega = 90^\circ$, the maximum PA is seen when viewing from phase $= 0^\circ$. When the disk is rotated by $90^\circ$, the $u$ parameter becomes negative, and the minimum PA occurs. Rotating by another $90^\circ$ (i.e., phase $= 180^\circ$), $u$ becomes positive once again, although because the DE sits partially behind the star, the maximum PA is reduced. The same effect occurs when the phase $= 270^\circ$, where the minimum PA is also reduced. The disk then rotates back to the starting position. This same behavior is seen when $\omega = 45^\circ$, and in an inverse manner for $\omega = 270^\circ$. When $\omega = 180^\circ$, the $u$ and $q$ parameters all cancel out except for what has been occulted by the star, so the maximum PA occurs at phase $= 0^\circ$ and the minimum at phase $= 180^\circ$.

We used the Anderson–Darling (AD) and Cramér–von Mises (CvM) tests to demonstrate that our results come from different distributions. For the DE model, these tests were applied to the results corresponding to $45^\circ$ and $90^\circ$, since our predictions for these warp sizes have the smallest observed difference. For the data presented in Figure 2, the largest $p$-values acquired were $8.0 \times 10^{-3}$ for the AD test and $5.8 \times 10^{-3}$ for the CvM test.

### 3.2. Color–Magnitudes

The top panel of Figure 3 shows how the $V$-band magnitude changes with phase. Overall, the brightness is greatest when viewing from the low-density region of the disk where the star may be seen. As the disk rotates to the high-density region, the $V$-band magnitude decreases as the disk occults the star. The fall of the final data points in each set corresponds to the system becoming dimmer as the denser material returns into view and occults the star.

As the angular size of the DE sector increases, the system becomes redder as seen in Figure 3. In addition, the system becomes redder as the disk occults the star, although less so for larger DE angular size, making it challenging to discern any change when $\omega = 270^\circ$. When $\omega = 45^\circ$ and $90^\circ$, the effect of the DE going behind the star is also apparent in the data, as the DE moves behind the star, making the system appear bluer.

The magnitude–color loops shown in Figure 4 combine the behaviors shown in both panels of Figure 3 and summarized in the previous two paragraphs. The loops are diagonally oriented, appearing dimmer in the $V$-band while simultaneously bluer when viewing the system from a phase corresponding to a DE region. As the disk rotates and the system is seen from a phase corresponding to a lower density region, the system appears brighter and redder, as these parts of the disk have a smaller scale height and occult less of the star. There is no apparent rotational direction to these loops (clockwise versus counterclockwise), as the scale on which these observables vary is small for this particular model.

The middle panel of Figure 2 shows how the size of the Balmer jump, measured in percent polarization, varies with phase. This was computed by taking the difference in the continuum polarization in 100 Å bands on either side of the limit, centered roughly at 3600 Å and 3800 Å. When $\omega = 45^\circ$ and $90^\circ$, there is an increase in polarization received across the first $\sim 125^\circ$ of phase rotation. This occurs when the DE region is visible with the lower density parts of the disk either in front

![Figure 2. Variation in percent polarization (top panel), Balmer jump polarization (middle panel), and PA (bottom panel) as a function of phase.](image)
The AD and CvM tests were applied to the $\Delta V$ and $\Delta (B - V)$ results in Figures 3 and 4, using the same method used in Section 3.1. The resulting $p$-values from the AD test were virtually 0, which in this case means there is essentially no possibility that the data sets tested originate from the same distribution. The largest $p$-value from the CvM test was found to be $4.4 \times 10^{-16}$.

4. Tilted Disk Model

A TD model was first proposed by Hummel (1998) in an effort to explain the quasi-cyclic variability of emission-line width that occurs over years and decades. His model suggested that a Be star’s disk can be perturbed through an unknown mechanism, resulting in the disk being tilted from the equatorial plane. The formation of a TD is commonly thought to involve tidal interaction between a binary companion and the disk; see, e.g., Papaloizou & Terquem (1995), Martin et al. (2011), and Cyr et al. (2017). Over the course of decades, the disk tilts to align with the companion’s orbital plane, although Cyr et al. (2017) found that the disk tilts such that it is $90^\circ$ from the line of nodes of the companion’s orbital plane. As the tilted disk rotates in the equatorial plane, it appears to an observer that the inclination of the Be star is rapidly changing. In this section, we investigate the effect a TD has on the system’s polarization signature, magnitudes, and colors. These observables were analyzed at the same wavelengths as in the DE model.

BEDISK was used as a starting point to create a reference model (see Tables 1 and 2 for the relevant parameters). MCTRACE was then used to calculate the Stokes parameters and photometric colors at 18 different inclinations and 16 phase angles (each phase angle corresponds to the middle of a disk sector, as shown in Figure 1). To emulate a TD, each phase angle was assigned a different inclination corresponding to the apparent inclination of the Be star that an observer would see in the plane of the sky. This method creates the effect that as the system rotates in the equatorial plane, the Be star’s inclination changes from the maximum value (equivalent to the tilt angle $\omega$) to edge-on with just a $90^\circ$ rotation of the disk, and to the negative of the maximum inclination ($-\omega$) as it rotates another $90^\circ$. Figure 5 illustrates the geometry of the TD model and the relevant variables associated with this work.

In order to determine the Stokes parameters of the TD, the Stokes parameters of the untilted disk were translated using the relevant inclination for the phase angle being considered. We use Equations (4) and (5) for each corresponding inclination of the wedge, where the untilted polarization $p$ is in the desired wavelength band:

$$u_{\text{tilted}} = -p \cdot \sin(2\omega) \cdot \sin(\text{PA}),$$

$$q_{\text{tilted}} = \sqrt{p^2 - u_{\text{tilted}}^2}.$$

![Figure 5. Geometry of the TD model showing the tilt angle, $\omega$, and the phase angle. The 16 wedges of the equatorial plane, on which the phase angles are centered, are shown in red.](image-url)
Once the tilted Stokes parameters were obtained, the amount of polarized light and the PA could be determined using Equations (2) and (3).

### 4.1. Polarization

The top panel in Figure 6 shows the percent of polarized light as the disk undergoes a full rotation. When the system is not tilted (i.e., $\omega = 0^\circ$), the disk is consistently viewed from edge-on, so no change in polarization is observed. As the tilt angle increases, the polarization minima (when the phase angle $= 0^\circ$ and $180^\circ$) continue to decrease as the symmetry of the system on the plane of the sky increases and further cancellation of orthogonal polarization vectors occurs. As the phase angle increases, the disk rotates to be viewed edge-on at phase angle $= 90^\circ$ and $270^\circ$. At these phases, the percent polarization for a TD is equal to that of an untilted disk (i.e., $\omega = 0^\circ$) as the systems are geometrically equivalent. Noting that $\omega$ is measured from the equatorial plane, and as such is offset from the inclination by $90^\circ$, the largest polarization when the phase angle $= 0^\circ$ is seen when $\omega = 30^\circ$, which is equivalent to viewing an untilted disk with $i = 70^\circ$.

The middle panel of Figure 6 shows how the Balmer jump polarization varies with phase. This was computed in the same manner as the middle panel of Figure 2. As with Figure 2, the similarity between the Balmer jump polarization and the percent polarization at 3800 Å (top panel) for the TD model occurs because the continuum at 3600 Å is dominated by absorption, and the fraction of polarized light at this wavelength is almost zero.

The PA as a function of phase angle for the TD is shown in the bottom panel of Figure 6. The greatest deviations from $0^\circ$ are seen when $\omega = 45^\circ$, beyond which the PA decreases as the disk tilts closer to pole-on.

When considering a TD system, the maximum polarization signature is observed when $\omega \approx 30^\circ$ only when the phase angle is $\approx 0^\circ$ and $180^\circ$. As the disk rotates, the maximum polarization will occur for different $\omega$’s at different phase angles. Figure 7 illustrates this phenomenon. Here we see that the variation of percent polarization becomes more significant for greater $\omega$, with relatively little change for small $\omega$, as expected.

The AD and CvM tests were applied to the polarization signatures in Figure 6, using the same method used in Section 3.1. The largest $p$-values acquired are $2.1 \times 10^{-6}$ using the AD test and $2.1 \times 10^{-3}$ using the CvM test.

### 4.2. Color–Magnitudes

The top panel of Figure 8 shows the change in $\Delta V$ as the disk rotates. As $\omega$ increases, the maximum brightness in the
V-band (occurring at phase angle $\sim 0^\circ$ and $\sim 180^\circ$, when the disk is face-on) also increases. When the disk is edge-on, the minimum brightness is seen, since the disk occults the star, in addition to a reduction of the surface area of the disk.

The $\Delta(B - V)$ color shown in the bottom panel of Figure 8 is inverted from the $\Delta V$, as the B-band light is significantly weaker than the V-band light. As expected, the system becomes redder when the star is along the line of sight, and bluer when it becomes occulted, which may be affected by both the phase angle and the tilt angle.

The color–magnitude loops shown in Figure 9 combine the behaviors shown in both panels of Figure 8, summarized in the previous two paragraphs. Similar to the DE model, we see the diagonal orientation of the loop; however, the dimming in the TD model occurs when the rotation causes the disk to occult the star, also causing the system to appear bluer as the star’s V-band light dominates its B-band light.

We note that the variation in magnitude and color would not be apparent for an axisymmetric system viewed edge-on, as the occultation of the star by the disk would not change over time. As such, the case of $\omega = 0^\circ$ was intentionally not shown in Figure 9, since no sensible loop is observed.

The AD and CvM tests were applied to the $\Delta V$ and $\Delta(B - V)$ results in Figures 8 and 9, using the same method used in Section 3.1. The resulting $p$-values from the AD test were again virtually 0. The largest $p$-value from the CvM test was found to be $4.4 \times 10^{-16}$.

5. Discussion and Summary

As mentioned in Section 2, the variability in the polarized light from a Be star may be used to study the geometry of the circumstellar material as the disk evolves. Values of percent polarization are between 0% to 1.5%, with only $\sim 5\%$ of Be stars lying above this range (Yudin 2001). Wisniewski et al. (2010) showed that through the long-term process of disk growth and dissipation (on the order of decades), some stars can show a large degree of variation in polarization, with $\pi$ Aqr and 60 Cyg (HD 200310) varying by up to 1% or greater. These two stars in particular are interesting as both are thought to have binary companions and have recorded episodes of disk loss. Another Be star of interest is Pleione (HD 23862), which,
according to Tanaka et al. (2007), has a double disk in which an old and tilted disk is dissipating as a new disk is forming in the equatorial plane. In another study, Hirata (2007) presented an analysis on Pleione’s TD and showed that the star’s polarization has varied by as much as $\sim0.5\%$ over time. Draper et al. (2014) reported on the variable polarization of $\gamma$ Cas and showed that its V-band polarization had varied by as much as 0.3%. Be stars more often exhibit smaller variations in percent polarization, rather than larger ones, as larger variations require more substantial changes to the disk as the observer sees it.

In the DE model, we find that the variability of the percent polarization, as seen in Figure 2, is comparatively small, where the greatest change in the V-band is $\sim0.2\%$, the same maximum variability reported by Huang et al. (1989), and across the Balmer jump we see up to $\sim0.5\%$ variation. In contrast, Figure 6 shows that the TD model also showed most values of V-band and Balmer jump polarization to be within $\sim0.5\%$, although they varied significantly more within that range than the DE model did. The greatest variation in V-band polarization seen in the TD model is $\sim1\%$, when the tilt angle $\omega = 70^\circ$, which is similar to the values reported by Wisniewski et al. (2010) for disk growth and dissipation. These cases likely produce similar variability as they are both examples of significant changes to the disk. We see that for TDs with tilt angles less than 45°, the variation remains $\lesssim0.5\%$, similar to the values reported by Hirata (2007).

The PA was found to be variable in four Be stars, o And (HD 217675), $\gamma$ Cas, 88 Her (HD 162732), and $\kappa$ Dra (HD 109387), as reported by Vince et al. (1995), in which the variations were as large as 30°. $\gamma$ Cas, in particular, was observed to have PA variations up to 10° (excluding the data point from 1976 as the authors labelled an outlier), in addition to showing percent polarization of up to 1%, similar to Figure 2’s $\omega = 90^\circ$ DE, which varies by over $\sim8^\circ$. The other three stars in their work show greater PA variability than $\gamma$ Cas, of $\sim50^\circ$, 33°, and 20° for o And, 88 Her, and $\kappa$ Dra, respectively, which are similar to the large PA variations seen in the TD model. The variability of polarization for $\gamma$ Cas, which Draper et al. (2014) reported on, showed PA variations by as much as 10°. Significantly larger variations in PA may occur, such as that in Hirata (2007), where he shows that Pleione’s PA has changed by $\sim80^\circ$ over the course of $\sim20$ years. The PA seen from the DE model is consistent with the values reported by Hirata, as shown in Figure 2’s bottom panel. As with the percent polarization, the PA in the TD model changes much more significantly than in the DE model. The bottom panel of Figure 6 shows that the largest possible variation in PA is $\sim80^\circ$ when the tilt angle is 45°, which is consistent with Hirata’s (2007) data on Pleione.

As previously mentioned, when comparing the TD and DE models, it is best to compare the DE model to the $\omega = 30^\circ$ TD model. This is because the DE model is inclined at $i = 70^\circ$ while the TD model’s tilt angle is measured from the equatorial plane, making $\omega = 30^\circ$ in the TD model equivalent to $i = 70^\circ$ in the DE model.

Both models show variability of $\Delta V$ and $\Delta (B - V)$ within one magnitude, which is consistent with the photometry reported by Telting et al. (1998) for X Per (HD 24534), a Be star observed to have a tilted disk. The models are also consistent with the photometry of a group of 500 galactic Be stars reported by Labadie-Bartz et al. (2017). Unlike the polarization signatures, the variations of the color and magnitude are similar when comparing the $\omega = 30^\circ$ TD model with the DE model. The largest variations we see for the DE model in $\Delta V$ are $\sim0.4$ mag and in $\Delta (B - V)$ are $\sim0.01$ mag, where all sizes of the DE show variations of approximately the same amount. For the TD model, the variations in the color–magnitudes get larger as the tilt angle increases, as more of the star is visible. For the largest adopted tilt angle of 70°, we see variations in $\Delta V$ of $\sim1$ mag, while the $\Delta (B - V)$ varies $\sim1.2$ mag.

Harmanec (1983) reports on the variability of Be stars, stating that when the V band becomes brighter, the $B - V$ index is expected to become redder and the $U - B$ index is expected to become bluer, which he attributes to a positive correlation between H I emission strength and stellar luminosity, which exists for long-term variations. Conversely, he also describes an inverse correlation between H I emission strength and the stellar luminosity, such that as the V band becomes dimmer, both the $B - V$ and $U - B$ indices become redder. In both the DE and TD models, we see evidence of the positive correlation described by Harmanec (1983), as the loops that result from the rotation of the disk in Figures 4 and 9 are all positively sloped.

The same orientation and looping behavior have been observed in the works of de Wit et al. (2006) and Jones et al. (2013). de Wit et al. (2006) studied the photometric variability in a sample of Be stars in the Small Magellanic Cloud and found that $\sim90%$ of the the stars had color–magnitude loops that rotated in a clockwise sense, with the loops of the remainder stars rotating counterclockwise. They found that an outflowing disk and variable mass loss could recreate the rotational behavior of the loops. Jones et al. (2013) showed that the positive correlation behavior described by Harmanec (1983) exists for $\delta$ Sco (HD 143275) for disk growth and dissipation events. They also show that $\delta$ Sco’s loops rotate in a clockwise fashion and vary less in both color and magnitude than the TD model with a tilt angle of 30° as shown in the top panel of Figure 9.

The same behavior again has been reproduced in models of Be stars, such as those in the work of Haubois et al. (2012) and Granada et al. (2018). The disk growth and dissipation models reported in Haubois et al. (2012), with $r_0 = 3 \times 10^{-11}$ g cm$^{-3}$, which is an approximate average of the two densities adopted here for the DE model, produce similar loops of equivalent scale to those presented in this work. In terms of average disk density, Haubois et al.’s (2012) system is most comparable to the $\omega = 45^\circ$ DE model. When viewing at $i = 70^\circ$, Haubois et al.’s (2012) model shows modest variations centered around $\Delta V = 0$ and $\Delta (B - V) = 0$, similar to what is seen in the $\omega = 45^\circ$ DE model when the enhancement is behind the star. Once the DE region becomes visible again, the brightness decreases as the DE region begins to once again occult the star. There is a small difference in $\Delta V$ brightness between our model and Haubois et al.’s (2012), which may be explained by the greater average disk density adopted in our model.

Granada et al. (2018) present theoretical color–magnitude loops of Be stars in infrared wavelengths. It is important to note that the primary source of the infrared wavelengths is the disk as opposed to the V-band wavelengths generated primarily by the star as seen in Figure 2 of Rivinius et al. (2013). Granada et al. (2018) show that for Be stars with small and intermediate inclination angles, the stars with stable and developed disks have $n \lesssim 3.5$ and intermediate values of base density $r_0$, although for larger inclination angles, the IR excess diminishes as expected, as less surface area of the disk is visible. In comparison with our TD models, where a large tilt angle is
analogous to a small inclination and vice versa, we see that as the tilt angle increases, the variation of the V-band magnitude increases. So, as the IR excess becomes smaller in Granada et al.’s (2018) work, an equivalent change in a Be star in our work predicts a smaller variation in the V-band magnitude.

The hydrodynamic SPH simulations in Cyr et al. (2017) showed that when the orbital plane of a binary companion is misaligned from the equatorial plane of the Be star (misalignment angle), the greatest tilting effect is seen at 45°, and the effect of tilting the disk diminishes from greater misalignment angles. Their results also showed that the tilt angle never becomes as large as the misalignment angle and is generally less than half the misalignment angle. This in part may explain why the tilt angle \( \approx 30° \) TD model is consistent with observations presented in the literature.

In the DE model, all of the ranges of variability of the polarization and color–magnitudes reported in this work are equally present in the literature. Hence, based on the models presented here, we cannot place bounds on the size of the DE region.

In summary, by using polarimetry, important properties of the disks may be investigated, such as the geometry, chemistry, density, and disk evolution, without the need to resolve the disk. In this work, we created two models with different disk shapes and used the multiwavelength polarimetric and photometric signatures to analyze their disk behavior. In the future, we plan to merge the DE and TD models to create a tilted disk with a DE region and to expand our code to predict other key observables.

C.E.J. wishes to acknowledge support though the Natural Sciences and Engineering Research Council of Canada. The authors would like to thank Professor Roge Mamon from the Department of Statistical and Actuarial Sciences at The University of Western Ontario for helpful discussions related to the statistical analysis of our results.

ORCID iDs

K. C. Marr @ https://orcid.org/0000-0003-0989-2941
C. E. Jones @ https://orcid.org/0000-0001-9900-1000

References

Arcos, C., Jones, C. E., Sigut, T. A. A., et al. 2017, ApJ, 842, 48A
Baade, D. 1982, A&A, 105, 65
Balona, L. A. 1990, MNRAS, 245, 92
Balona, L. A. 1995, MNRAS, 277, 1547
Balona, L. A., & Rozowsky, J. 1991, MNRAS, 251P, 66B
Behr, A. 1957, VeGoe, 7, 175
Björkman, K. S., Miroshnichenko, A. S., McDavid, D., et al. 2002, ApJ, 573, 812
Caballero-García, M. D., Camero-Arranz, A., Arabaci, Özbek., et al. 2016, A&A, 589A, 9C
Carciofi, A. C., Okazaki, A. T., Le Bouquin, J.-B., et al. 2009, A&A, 504, 915
Cote, J., & Waters, L. B. F. M. 1987, A&A, 176, 93
Cox, A. N. 2000, Allen’s Astrophysical Quantities (4th ed.; New York: Springer)
Cyr, I. H., Jones, C. E., Panoglou, D., et al. 2017, arXiv:1706.07029
de Wit, W. J., Lamers, H. J. G. L. M., & Marquette, J. B. 2006, A&A, 456, 1027
Draper, Z. H., Wisniewski, J. P., Björkman, K. S., et al. 2011, in IAU Symp. 272, Active OB Stars: Structure, Evolution, Mass Loss and Critical Limits, ed. C. Neiner et al. (Cambridge: Cambridge Univ. Press), 388
Draper, Z. H., Wisniewski, J. P., Björkman, K. S., et al. 2014, ApJ, 786, 120
Granada, A., Jones, C. E., Sigut, T. A. A., et al. 2018, AJ, 155, 50
Hall, J. S., & Mikesel, A. H. 1950, PUSNO, 17, 1
Halonen, R. J., & Jones, C. E. 2013a, ApJ, 765, 17
Halonen, R. J., & Jones, C. E. 2013b, ApJS, 208, 3
Halonen, R. J., & Jones, C. E. 2015, in Proc. IAU Symp. 307, New Windows on Massive Stars: Asterosisomology, Interferometry, and Spectropolarimetry, ed. G. Meynet et al. (Cambridge: Cambridge Univ. Press), 377
Halonen, R. J., MacKay, F. E., & Jones, C. E. 2013, ApJS, 204, 11
Hamuschik, R. W., Hummel, W., & Sutorius, E. 1996, A&A, 116, 309
Harmance, P. 1983, HvdOB, 7, 55H
Haubois, X., Carciofi, A. C., Rivinius, Th., et al. 2012, ApJ, 756, 156
Hirata, R. 2007, in ASP Conf. Ser. 361, Active OB-Stars: Laboratories for Stellar and Circumstellar Physics, ed. A. T. Okazaki, S. P. Owocki, & S. Stefl (San Francisco, CA: ASP), 267
Huang, L. 1986, A&AS, 112, 124
Huang, L., Hsu, J. C., & Guo, Z. H. 1989, A&AS, 78, 431H
Hummel, W. 1998, A&A, 330, 243
Jones, C. E., Molak, A., & Sigut, T. A. A. 2009, MNRAS, 392, 383
Jones, C. E., Sigut, T. A. A., Grzenia, B., et al. 2017, arXiv:1704.08733
Jones, C. E., Tycner, C., Silaj, J., et al. 2011, IAUS, 272, 398J
Jones, C. E., Wiegent, P. A., Tycner, C., et al. 2013, AJ, 145, 142
Kee, N., Owocki, S., Townsend, R., et al. 2014, arXiv:1412.8511
Kurucz, R. F. 1993, Kurucz CD-ROM No. 13 (Cambridge, MA: Smithsonian Astrophysical Observatory)
Labadie-Bartz, J., Pepper, J., & Virginia McSwain, M. 2017, arXiv:1609.08449v2
Lee, U., Saio, H., & Osaki, Y. 1991, MNRAS, 250, 432
Martin, R. G., Pringle, J. E., & Tout, C. A. 2011, MNRAS, 416, 2827
Okazaki, A. T. 1991, PASJ, 43, 75
Papaloizou, J. C. B., & Terquem, C. 1995, MNRAS, 274, 987
Quirrenbach, A., Bjorkman, K. S., Bjorkman, J. E., et al. 1997, ApJ, 479, 477
Rivinius, T., Baade, D., Steffl, S., et al. 1998, A&A, 335, 125
Rivinius, T., Baade, D., & Steffl, S. 2003, A&A, 411, 229R
Rivinius, T., Carciofi, A., & Martayan, C. 2013, A&ARVS, 21, 69R
Rivinius, T., Steffl, S., & Baade, D. 2006, A&A, 459, 137R
Sigut, T. A. A., & Jones, C. E. 2007, ApJ, 668, 481
Slettebak, A. 1982, ApJS, 50, 55
Stokes, G. G. 1852, TCApS, 9, 399
Tanaka, K., Sadakane, K., & Narusawa, S. 2007, PASJ, 59L, 3ST
Telting, J. H., Waters, L. B. F. M., Roche, P., et al. 1998, MNRAS, 296, 785
Vince, L., Arsenijević, J., Markovic-Krstjanić, S., et al. 1995, IAPP, 59, 32V
Waters, L. B. F. M. 1986, A&A, 162, 121
Waters, L. B. F. M., Cote, J., & Lamers, H. J. G. L. M. 1987, A&A, 185, 206
Wisniewski, J. P., Draper, Z. H., & Björkman, K. S. 2010, ApJ, 709, 1306
Wood, K., Björkman, J. E., Whitney, B. A., & Code, A. D. 1996, ApJ, 461, 828
Yudin, R. V. 2001, A&A, 368, 912