A Game-Theoretic Approach for NOMA-ALOHA

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Abstract—Non-orthogonal multiple access (NOMA) can improve the spectral efficiency by exploiting the power domain and successive interference cancellation (SIC), and it can be applied to various transmission schemes including random access that plays a crucial role in the Internet of Things (IoT) to support connectivity for a number of devices with sparse activity. In this paper, we formulate a game when NOMA is applied to ALOHA to decide the transmission probability. We consider a payoff function based on an energy-efficiency metric and drive the mixed strategy Nash equilibrium (NE).

Index Terms—random access, non-orthogonal multiple access, game theory

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) is to exploit the power domain and allows multiple users to share the same radio resource. As in [1], successive interference cancellation (SIC) can be employed in NOMA to effectively decode signals in the presence of interference. Since NOMA can improve the spectral efficiency of multiuser systems, it has been widely investigated to and considered for 5th generation (5G) systems [2–5].

For downlink NOMA, superposition coding and SIC are employed, where the power allocation becomes crucial to guarantee successful decoding with SIC. Beamforming with user clustering is considered in [6], the sum rate optimization is investigated with a minorization-maximization method in [7], and a generalized NOMA beamforming approach is studied in [8] in order to take into account the spatial correlation.

NOMA can also be employed for uplink transmissions. For uplink NOMA, the power allocation to guarantee successful SIC is studied in [9]. In [10, 11], NOMA is applied to ALOHA, which is a random access scheme [12], with different power levels. With NOMA, since more (virtual) channels can be available, the throughput of ALOHA can be improved, which implies that NOMA-ALOHA can support more users or devices in the Internet of Things (IoT) with a limited bandwidth.

In general, random access schemes are suitable for a number of users or nodes with sparse active as signaling overhead to allocate radio resources is not required. Thus, random access is considered for machine-type communications (MTC) in [13], and employed for standards as in [14, 15]. While signaling overhead is low in random access, each node or user has to decide its transmission parameters. In [16–18], it is shown that the notion of game theory [19] becomes useful to locally optimize transmission parameters for random access schemes, since random access can be seen as a noncooperative game.

In this paper, we propose a game-theoretic approach for NOMA-ALOHA proposed in [10] to decide the transmission probability or the mixed strategy for transmissions. In particular, we formulate a NOMA-ALOHA game based on an energy-efficiency metric for the payoff function and derive the mixed strategy Nash equilibrium (NE) for each user’s transmission or access probability.

Notation: Matrices and vectors are denoted by upper- and lower-case boldface letters, respectively. The superscript T denotes the transpose. \( E [\cdot] \) denotes the statistical expectation.

II. SINGLE-CHANNEL TWO-PERSON GAME

In this section, we consider NOMA-ALOHA with single channel in order to demonstrate how a game-theoretic approach can be employed to decide transmission probabilities.

A. Two-Person NOMA-ALOHA Game

Suppose that there are two users\(^\dagger\) for multiple access (or uplink transmissions) with a single channel and a receiver or base station (BS). We assume that NOMA is employed with two different power levels. Thus, the number of transmission strategies for each user is 3 as follows:

\[ S_k = \{ H, L, 0 \}, \quad k = 1, 2, \]

where the subscript \( k \) is the index for users, \( H \) and \( L \) represent the high and low transmit powers, respectively, and 0 represents no transmission. Denote by \( s_k \) the strategy of user \( k \). In addition, \( s_{-k} \) represents the set of the strategies of the users except user \( k \), i.e., \( s_{-k} = \{ s_1, \ldots, s_{k-1}, s_{k+1}, \ldots, s_K \} \), if there are \( K \) users. For two-person games, \( s_{-1} = s_2 \) and \( s_{-2} = s_1 \).

The payoff function of user \( k \) is given by

\[
   u_k(s_k, s_{-k}) = u(s_k, s_{-k}) = R(s_k, s_{-k}) - C(s_k), \quad k = 1, 2, 
\]

where \( R(s, s') \) is the reward function of successful transmission and \( C(s) \) is the cost function of transmission strategy. In particular, we consider the following reward function:

\[
   R(s, s') = \begin{cases} 
   W, & \text{if } s \neq s' \text{ and } s \in \{ H, L \} \\
   0, & \text{o.w.} 
   \end{cases}
\]

\(^\dagger\)Throughout the paper, we assume that users and players are interchangeable.

ARXIV: 1801.02351v1 [cs.IT] 8 Jan 2018.
where $W > 0$ is the reward of successful transmission for a user. Due to NOMA, if the user of interest chooses $s = H$, while the other user chooses $s' = L$ or 0, the user of interest can successfully transmit his signal. Thus, the main difference of NOMA-ALOHA from conventional ALOHA is that the BS is able to recover the signals from two users simultaneously as long as one user employs strategy $H$ and the other user adopts strategy $L$ as in (2). Note that in (2), as in conventional ALOHA, if two users choose $(H, H)$ or $(L, L)$, we assume collision and the BS is not able to receive any signal (12).

For the cost function, we can consider the following assignment as an example:

$$C(H) = 2, \quad C(L) = 1, \quad \text{and} \quad C(0) = 0,$$

because strategy $H$ requires a higher transmit power than strategy $L$.

It is noteworthy that the payoff function in (1) can be seen as an energy-efficiency metric, which is widely used in wireless systems, e.g., (20) for power control game. To see that the payoff function in (1) is an energy-efficiency metric, we can consider the logarithm of the ratio of the spectral efficiency or throughput to the transmit power as follows:

$$\frac{\text{Throughput}}{\text{Transmit Power}} = \ln(\text{Throughput}) - \ln(\text{Transmit Power}),$$

where $\ln(\text{Throughput})$ becomes the reward function and $\ln(\text{Transmit Power})$ becomes the cost function in (1).

The two-person NOMA-ALOHA game has a strategic form of triplet: 1) $K = \{1, 2\}$, where $K$ represents the set of users or players; 2) $S_k = S$, where $S_k$ denotes the set of strategies of user $k$; 3) the payoff functions in (1). That is, the two-person NOMA-ALOHA game is given by $G = (K, \{S_k\}_{k=1}^K, \{u_k\}_{k=1}^K)$ with $K = 2$. The resulting game is symmetric, that is, both players have the same set of strategies, and their payoff functions satisfy $u_1(s_1, s_2) = u_2(s_2, s_1)$ for each $s_1, s_2 \in S$ (21). Furthermore, its bimatrix can be found as in Table I.

| TABLE I |
| BIMATRIX OF TWO-PERSON NOMA-ALOHA GAME. |
| $H$ | $L$ | $0$ |
| --- | --- | --- |
| $L$ | $(W - 2, W - 2)$ | $(W - 2, 0)$ |
| $L$ | $(0, W - 2)$ | $(0, W - 1)$ |

It is noteworthy that the two-person NOMA-ALOHA game can be seen as a generalization of a multiple access game in (22) with the notion of NOMA (10). The multiple access game in (22) has two strategies for each user: Transmit (T) and Quite (Q). Strategy T is further divided into $H$ and $L$ in the two-person NOMA-ALOHA game, while strategy Q becomes 0.

**B. Finding NEs**

In this subsection, we find NEs of the two-person NOMA-ALOHA game and show that the NEs depend on the reward of transmission, $W$.

If $W \leq 2$, there exist pure strategy NEs (19), (23), denoted by $\{s^*_k\}$, which are characterized by

$$u_k(s_k^*, s_{-k}^*) \geq u_k(s_k, s_{-k}) \quad \text{for all} \quad s_k \in S_k, \quad k \in K.$$

For example, for $0 \leq W < 1$, from Table I we can see that $(s_1, s_2) = (0, 0)$ is the pure strategy NE. That is, if the reward of successful transmission, $W$, is sufficiently small (compared to the cost of transmissions), the users do not want to transmit signals and non-transmission strategy (i.e., $s_k = 0$) becomes NE. For $1 \leq W \leq 2$, the pure strategy NEs are $(s_1, s_2) = (0, L)$ and $(s_1, s_2) = (L, 0)$. If $W > 2$, there is no pure strategy NE.

In general, we are interested in mixed strategy NEs as randomized strategy can be well employed for random access. In order to find the mixed strategy NEs, the principle of indifference (19) can be used. Since the two-person NOMA-ALOHA game is symmetric, it suffices to find one user’s mixed strategy NE. To this end, let $a$ and $b$ denote the probabilities to choose $H$ and $L$, respectively. Thus, a mixed strategy is represented by $\sigma = (a, b, 1 - a - b)$, where $a + b \leq 1$.

For convenience, let $B$ denote the payoff matrix for the row user in Table I. Let $[B]_{m,n} = B_{n,m}$. According to the principle of indifference, the row user has the same expected payoff for any pure strategy when the column user employs the mixed strategy NE. Thus, it follows

$$U = a^*B_{1,1} + b^*B_{1,2} + (1 - a^* - b^*)B_{1,3} = a^*B_{2,1} + b^*B_{2,2} + (1 - a^* - b^*)B_{2,3}$$

$$= a^*B_{3,1} + b^*B_{3,2} + (1 - a^* - b^*)B_{3,3},$$

where $U$ is the expected payoff of the row user and $(p^*, q^*, 1 - p^* - q^*)$ is the mixed strategy NE. From (3), we have two equations for two unknown variables, $a^*$ and $b^*$. In addition, since $a^* + b^* \leq 1$, we can find $a^*$ and $b^*$.

Noting that $B_{3,i} = 0$ for $i = 1, 2, 3$ (as the row user does not transmit) from Table I we can see that $U = 0$ if $1 - (a^* + b^*) > 0$ (i.e., the probability of non-transmission or strategy 0 is greater than 0). In this case, we can have closed-form expressions for $a^*$ and $b^*$ from (5) as follows:

$$a^* = \frac{W - 2}{W} \quad \text{and} \quad b^* = \frac{W - 1}{W}.$$  (4)

The above solution is valid when $2 \leq W < 3$ since $a^*, b^* \geq 0$ and $1 - (a^* + b^*) > 0$ are required. If $W = 3$, we can see that $a^* + b^* = 1$, which means that the probability of non-transmission is 0. That is, strategy 0 is not used if the reward of successful transmission, $W$, is sufficiently large. Thus, for $W \geq 3$, (3) is reduced to

$$U = a^*B_{1,1} + b^*B_{1,2} + (1 - a^* - b^*)B_{1,3} = a^*B_{2,1} + b^*B_{2,2} + (1 - a^* - b^*)B_{2,3}.$$  (5)

Then, after some manipulations, we have

$$a^* = \frac{W - 1}{2W} \quad \text{and} \quad b^* = \frac{W + 1}{2W}, \quad W \geq 3. \quad (6)$$

We now consider the case that $W < 2$. If $W < 2$, the reward of successful transmission is so small that high-power
transmission is not desirable. Thus, \( a^* = 0 \) (which is the case that \( W = 2 \) as shown in (4)). Thus, (3) is reduced to

\[
U = b^* B_{2,2} + (1 - b^*) B_{2,3} = 0,
\]

which leads to

\[
b^* = \begin{cases} \frac{W-1}{W}, & 1 \leq W \leq 2 \\ 0, & 0 \leq W < 1. \end{cases}
\]

In Fig. 1 we show the mixed strategy NE, \( \sigma^* = (a^*, b^*, 1 - a^* - b^*) \), for different values of the reward of successful transmission, \( W \). As shown in Fig. 1, we can see that the probability of strategy \( L \) is higher than the probability of strategy \( H \) as strategy \( H \) has a higher cost than strategy \( L \). In addition, as \( W \) increases, the probability of strategy 0 decreases. That is, as the reward of successful transmission increases, the users tend to transmit signals. Note that as \( W \to \infty \), \( a^* = b^* \to \frac{1}{2} \) from (7), i.e., the users always transmit.

When users always transmit in conventional ALOHA, there are collisions with probability (w.p.) 1 and the throughput becomes 0. However, in NOMA-ALOHA, the throughput does not approach 0 although collisions happen thanks to NOMA. In the two-person NOMA-ALOHA, as \( W \to \infty \), the asymptotic throughput approaches 1, because the BS is able to recover the two users’ signals simultaneously as long as \((s_1, s_2) = (H, L) \) or \((L, H)\), i.e.,

\[
\text{Throughput} = 2 \times \Pr((s_1, s_2) = (H, L) \text{ or } (L, H)) = 2 \times \frac{1}{2} = 1, \ W \to \infty.
\]

Fig. 1. The mixed strategy NE, \( \sigma^* = (a^*, b^*, 1 - a^* - b^*) \), for different values of the reward of successful transmission, \( W \).

C. Average Payoff Maximization

In this subsection, we consider a different approach that is not based on noncooperative game.

Suppose that a mixed strategy is used and the two users have the same mixed strategy, \( \sigma = (a, b, 1 - a - b) \), and use it independently (as no cooperation is assumed). From (1), for a given mixed strategy, the average payoff is given by

\[
\bar{u}(a, b) = \mathbb{E}[u(s_1, s_2)] = \mathbb{E}[R(s_1, s_2)] - \mathbb{E}[C(s_1)]
\]

\[
= W(a(1 - a) + b(1 - b)) - 2a - b.
\]

Since \( \bar{u}(a, b) \) is concave in \( a \) and \( b \), the maximization of the average payoff can be carried out. In Fig. 2 we show the optimal mixed strategy, denoted by \( \hat{\sigma} = (\hat{a}, \hat{b}, 1 - \hat{a} - \hat{b}) \) that maximizes the average payoff for different values of the reward of successful transmission, \( W \). We can see that the optimal mixed strategy that maximizes the average payoff is similar to the mixed strategy NEs in Fig. 1 although both the mixed strategies are not the same for \( W \geq 1 \). For example, for \( 0 \leq W \leq 1 \), the probability of strategy 0 is 1 in both the mixed strategies. In addition, we see that the probability of strategy \( L \) is higher than or equal to that of strategy \( H \), while the two probabilities approaches \( \frac{1}{2} \) as \( W \to \infty \) in both the mixed strategies.

Fig. 2. The mixed strategy that maximizes the average payoff for different values of the reward of successful transmission, \( W \).

Fig. 3 shows the average payoff functions of the two mixed strategies, \( \hat{\sigma} \) and \( \sigma^* \), for different values of \( W \). Clearly, \( \hat{\sigma} \) provides the highest average payoff and higher than that of \( \sigma^* \). However, since \( \hat{\sigma} \) is not a mixed strategy NE, any user who uses a slightly different mixed strategy from \( \hat{\sigma} \) can have a higher average payoff at the cost of the degraded average payoff of the other user who uses \( \hat{\sigma} \).

The ratio of the payoff with \( \hat{\sigma} \) to that with \( \sigma^* \) can be seen as a price of anarchy (PoA) [24], [25]. If both the users trust each other, they can employ \( \hat{\sigma} \). On the other hand, if there is no trust, each user may need to employ \( \sigma^* \), which is NE, and has a worse average payoff than that can be obtained with \( \hat{\sigma} \), i.e., the PoA is less than 1. However, as \( W \to \infty \), from (9), we can see that \( \hat{a} \) and \( \hat{b} \) become \( \frac{1}{2} \), which is the same as the asymptotic mixed strategy NE, \( (a^*, b^*) \), with \( W \to \infty \), and the PoA approaches 1.
III. MULTI-USER NOMA-ALOHA GAME

In this section, we generalize the two-person NOMA-ALOHA game that was introduced in Section II with more users.

Suppose that there are $K \geq 2$ users. Since each user has the same payoff function, the NOMA-ALOHA game is symmetric and the principle of indifference [19] can be employed in order to find the mixed strategy NE.

For convenience, let the user of interest be user $k$. Denote by $U(H)$, $U(L)$, and $U(0)$ the payoff values of user 1 if user 1 chooses $s_k = H$, $L$, and 0, respectively, when the other users have the same mixed strategy, $(a, b, 1-a-b)$. Let $q_k(s)$ be the probability that the other users do not employ strategy $s$. Since each user chooses a strategy independently, we have

$$q_k(H) = \prod_{i \neq k} (1-a) = (1-a)^{K-1}$$
$$q_k(L) = \prod_{i \neq k} (1-b) = (1-b)^{K-1}. \quad (10)$$

From this, we show that

$$U(H) = -C(H) (1 - q_k(H)) + (W - C(H)) q_k(H)$$
$$U(L) = (W - C(L)) q_k(L) - C(L) (1 - q_k(L)), \quad (11)$$

while the payoff of user $k$ becomes $U(0) = 0$ when $s_k = 0$.

Like the analysis in Section II we can see that if $W < 1$, both the probabilities of strategies $H$ and $L$ are to be 0 for NE. Thus, $(a^*, b^*, 1-a^*-b^*) = (0, 0, 1)$. For $1 \leq W < 2$, strategy $H$ cannot be applied for the mixed strategy NE (i.e., $a^* = 0$). In this case, since we need to have

$$U(L) = U(0),$$

the resulting mixed strategy NE becomes

$$(a^*, b^*, 1-a^*-b^*) = \left( 0, 1 - \left( \frac{1}{W} \right)^{\frac{1}{W-1}}, \left( \frac{1}{W} \right)^{\frac{1}{W-1}} \right).$$

The mixed strategy NE for $W \geq 2$ can be shown as follows.

**Lemma 1.** Let

$$W^* = \left( 1 + 2 \frac{1}{W} \right)^{K-1}. \quad (12)$$

For $2 \leq W < W^*$, we have

$$(a^*, b^*) = \left( 1 - \left( 1 - \frac{2}{W} \right)^{\frac{1}{W-1}}, 1 - \left( \frac{1}{W} \right)^{\frac{1}{W-1}} \right). \quad (13)$$

For $W \geq W^*$, $b^*$ is the solution of

$$Wb^{K-1} = W(1-b)^{K-1} + 1,$$

while $a^* = 1-b^*$.

**Proof:** Suppose that the probability that a user employs strategy 0 is not zero, while $a, b > 0$. Then, by the principle of indifference, we need to have $U(H) = U(L) = U(0) = 0$. From (11), it can be shown that

$$Wq_k(H) - 2 = Wq_k(L) - 1 = 0.$$

Thus, we have

$$a^* = 1 - \left( \frac{2}{W} \right)^{\frac{1}{W-1}} \quad \text{and} \quad b^* = 1 - \left( \frac{1}{W} \right)^{\frac{1}{W-1}}, \quad (15)$$

which is given in (13).

However, if $W$ is sufficiently large (i.e., for a sufficiently large reward of successful transmission), the probability that a user employs strategy 0 becomes zero or $a^* + b^* = 1$. The corresponding $W$ is the solution of $1 = a^* + b^*$ or

$$1 = \left( \frac{2}{W} \right)^{\frac{1}{W-1}} + \left( \frac{1}{W} \right)^{\frac{1}{W-1}}, \quad (16)$$

which is $W^*$ in (12). In this case (i.e., $W \geq W^*$), we only need to have $U(H) = U(L)$ with $a^* + b^* = 1$. Thus, we have

$$Wq_k(H) - 2 = Wq_k(L) - 1 \quad \text{and} \quad a^* + b^* = 1,$$

which can also be expressed as (14) in terms of $b^*$ only. Clearly, the solution of (14) is $b^*$, and $a^*$ becomes $1-b^*$, which completes the proof.

Fig. 4 shows the mixed strategy NE, $\sigma^* = (a^*, b^*, 1-a^*-b^*)$, for different values of the reward of successful transmission, $W$, when $K = 5$ with $W^* = 22.969$. Note that $W^* = 3$ when $K = 2$ according to (12), and $W^*$ can be seen as the threshold value of the reward of successful transmission to set the probability of $s_k = 0$ to 0. Clearly, from (12), $W^*$ increases with $K$. That is, a higher reward of successful transmission is required to force users to keep transmitting as $K$ increases.

Fig. 5 shows the mixed strategy NE, $\sigma^* = (a^*, b^*, 1-a^*-b^*)$, for different numbers of users when $W = 10$. For a fixed reward of successful transmissions, as the number of users increases, the probability of transmissions (either $H$ or $L$) decreases, while the probability of non-transmissions increases. This behavior results from the increase of the probability of collision as $K$ increases.
In this paper, we formulated a multiple access game for ALOHA with (power-domain) NOMA, where the payoff function is based on energy efficiency. The mixed strategy NE has been derived using the principle of indifference to decide transmission parameters, i.e., the probability of transmissions. It was shown that the probability of transmissions can approach 1 as the reward of successful transmission increases. In this case, unlike conventional ALOHA, we showed that the throughput does not approach 0, although there is packet collision because the power levels of users can be different thanks to NOMA.

**IV. CONCLUDING REMARKS**

In this paper, we formulated a multiple access game for ALOHA with (power-domain) NOMA, where the payoff function is based on energy efficiency. The mixed strategy NE has been derived using the principle of indifference to decide transmission parameters, i.e., the probability of transmissions. It was shown that the probability of transmissions can approach 1 as the reward of successful transmission increases. In this case, unlike conventional ALOHA, we showed that the throughput does not approach 0, although there is packet collision because the power levels of users can be different thanks to NOMA.

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