Quark-Meson Coupling Model, Nuclear Matter Constraints
and Neutron Star Properties

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Abstract

We explore the equation of state for nuclear matter in the quark-meson coupling model, including full Fock terms. The comparison with phenomenological constraints can be used to restrict the few additional parameters appearing in the Fock terms which are not present at Hartree level. Because the model is based upon the in-medium modification of the quark structure of the bound hadrons, it can be applied without additional parameters to include hyperons and to calculate the equation of state of dense matter in beta-equilibrium. This leads naturally to a study of the properties of neutron stars, including their maximum mass, their radii and density profiles.

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I. INTRODUCTION

Bulk nuclear matter properties have served as an excellent testing ground for models of baryonic many-body systems for many years. This hypothetical medium possesses many similarities with matter in the interior of heavy nuclei, neutron stars and core-collapse supernovae. The relative simplicity of the nuclear matter concept, such as the assumption of a uniform density distribution without surface effects, allows the derivation of several key variables which are generally accepted as necessary conditions that must be satisfied by any successful nuclear model.

The uncertainty in the determination of the forces acting among baryons and their modification by the medium has led to a great variety of models. These traditionally start from a bare nucleon-nucleon interaction, fit to experimental data from nucleon-nucleon scattering and the properties of the deuteron, which serves as input to into a many-body formalism such as the relativistic Dirac–Bruckner–Hartree–Fock (DBHF) approximation and its nonrelativistic counterpart BHF [1, 2], variational methods [3], correlated basis function models [4], self-consistent Greens function (SCGF) models [5, 6], quantum Monte Carlo techniques [7] and chiral effective field theory [8, 9]. An alternative is to develop an effective density dependent baryon-baryon interaction such as the non-relativistic Skyrme or Gogny interaction, or one of the various relativistic effective Lagrangian models and use it directly in a many-body theory.

With the exception of the role of the $\Delta$ excitation in the generation of the three-nucleon force, none of these models consider the internal structure of the nucleon and, in particular, its possible modification in the presence of other hadrons. They depend on a large number of variable parameters which are determined by fitting calculated observables to experimental data. The parameters are often correlated, making it difficult to extract an unambiguous set from such fits, leading to—in principle—an infinite number of such parameter sets [10].

The quark-meson coupling model (QMC model) is unique within the existing literature as it is based upon a very different approach to this problem. Rather than starting with the nucleon-nucleon (NN) force, it begins with the study of a hadron built from quarks immersed in a nuclear medium. The original model, which is employed here, begins with the MIT bag model. One then self-consistently includes the effects of the coupling to the $u$ and $d$ quarks of a scalar-isoscalar meson ($\sigma$) mean field, generated by all the other hadrons in the medium,
on the internal structure of that hadron. As in earlier boson-exchange models, the $\sigma$ is a crude but convenient way to simulate the effects of correlated two-pion exchange between hadrons. While the quarks are also coupled to $\omega$ and $\rho$ mesons, their Lorentz vector nature means that, at least at Hartree level, they simply shift quark energies and do not generate non-trivial, density dependent modifications of the internal structure of the bound hadron.

The QMC model was originally introduced by Guichon [11]. Subsequent development significantly improved the treatment of centre of mass corrections [12], which had generated an unrealistic amount of repulsion in the original model. This development also included a consistent treatment of finite nuclei, including the spin-orbit force [12]. When applied to $\Lambda$ hypernuclei, the model provided a very natural explanation of the very small spin-orbit force observed in those systems [13–15]. In an important, recent development, the inclusion of the density dependence of the “hyperfine” interaction between quarks arising from one-gluon-exchange (OGE) gave a parameter free explanation of the empirical absence of medium mass and heavy $\Sigma$-hypernuclei, while simultaneously yielding a good description of $\Lambda$-hypernuclei [15]. For a review of the many applications of the QMC model we refer to Ref. [29].

A clear connection has also been established between the self-consistent treatment of in-medium hadron structure and the existence of many-body [16] or density dependent [17] effective forces. In particular, in all of the models explored so far involving confined quarks, the self-consistent response to the applied mean scalar field tends to oppose that applied field. This effect can be represented as a “scalar polarisability” which effectively reduces the coupling of the $\sigma$ to an in-medium baryon as the applied scalar field increases. We stress that this scalar polarisability is a calculated property of each hadron and hence introduces no new parameters into the model. Moreover, it is this scalar polarisability which yields the density dependence of the derived Skyrme forces, or equivalently the three-body forces between all combinations of hadrons. That is, the model predicts the existence and strength of the three-body forces between not just nucleons, but nucleons and hyperons and hyperons and other hyperons, without additional parameters.

As an indication of the reliability of these predictions, we note that Dutra et al. [10] critically examined a variety of phenomenological Skyrme models of the effective density dependent nuclear force against the most up-to-date empirical constraints. The Skyrme model SQMC700—derived from the QMC model—was amongst the few percent of the
Skyrme forces studied which satisfied all of these constraints. This, together with the very reasonable description of the empirical properties of hypernuclei, gives us confidence in exploring the consequences of the model for dense matter, including hyperons.

As we have already observed, in a recent development of the QMC model \[15\], the self-consistent inclusion of the gluonic hyperfine interaction led to a successful description of the binding energies of Λ-hypernuclei, as well as the observed absence of medium and heavy mass Σ-hypernuclei, with no additional parameters. We stress that these results were obtained under the minimal assumption (consistent with the OZI rule) the σ, ω and ρ mesons do not couple to strange quarks. Furthermore, the consistency with hypernuclear properties is obtained without the need to introduce additional, much heavier mesons which do couple to the strange quarks \[18\]. This again has the effect of keeping the number of free parameters in the model low. While the model could be supplemented with heavier mesons containing strange quarks \[18\], Occam’s razor suggests that one should not introduce them if they are not needed.

In this paper we present the latest development of the QMC model which extends the earlier study of Stone et al. \[19\]. While that earlier study demonstrated the importance of exchange (Fock) terms in calculations of the EoS of dense baryonic matter in beta-equilibrium, it included only the Dirac vector term in the vector-meson-nucleon vertices. In this work we include the full vertex structure, which one might expect to enhance the pressure at high density, particularly in the case of the ρ meson for which the tensor coupling is much larger than that of the ω. These terms were already included within the QMC model by Krein et al. \[20\] for symmetric nuclear matter. Here we generalize their work by evaluating the full exchange terms for all octet baryons and adding them, in the same way as Stone et al. \[19\], as additional contributions to the energy density. Our investigation complements the important work of Miyatsu et al. who performed a relativistic Hartree–Fock calculation incorporating the tensor interaction \[21\].

In Sec. II we present the basic features of the QMC model used in this work. The application of the model leading to the equation of state (EoS) of dense matter and a description of its parameters is given in Sec. III A. Results obtained for infinite nuclear matter, symmetric and asymmetric as well as beta-equilibrium matter are followed by those for cold neutron stars and their comparison with experimental and observational constraints can be found in Secs. III B - III E. Sensitivity of the EoS and related quantities to variation
of some model parameters is explored in Sec. III F. We then make a comparison between the present work and recent variations of the QMC model studied in Refs. [21, 22] and others. Some discussion and concluding remarks are presented in Sec. IV.

II. THE QMC MODEL

The QMC model is based upon the self-consistent modification of the structure of a baryon embedded in nuclear matter. It is a relativistic mean field model which incorporates the internal quark structure of the baryons, represented as MIT bags containing three quarks in a color-singlet configuration. Interactions occur between quarks in distinct bags via the exchange of mesons coupled locally to the quarks. In the literal interpretation of the bag model, where quarks and gluons exclusively reside within the bag, this coupling would be unnatural. However, the bag may also be understood as an average approximation to a more complex description of confinement in which the surface and size of the bag represent an average description of far more complex dynamical configurations. For example, the QCD lattice simulations of Bissey et al. [23] suggest that the true confinement may be closer to a Y-shaped color string attached to the quarks immersed in a non-perturbative vacuum rather than the effect of the bag boundary. In this case, the quarks in one baryon may approach the quarks of another baryon and interact through vacuum fluctuations, described by meson fields.

Thus, in addition to the usual terms in the Lagrangian density of the MIT bag, the QMC model adds the simplest local couplings of $\sigma$, $\omega$ and $\rho$ mesons to the confined quarks. That is, the couplings are $g^\sigma_q \bar{q}q\sigma$, $g^\omega_q \bar{q}\gamma^\mu q\omega^\mu$ and $g^\rho_q \bar{q}\gamma^\mu \vec{\tau} q\cdot \vec{\rho}^\mu$, respectively [11, 12]. Here $q$ represents the SU(2) doublet of $u$ and $d$ quarks and the coupling of these mesons to the $s$ quark is taken to be zero. These quark-meson couplings describe the interaction between quarks in different hadrons. They act as the source of mean fields in-medium as well as serving to modify the equation of motion of the confined quarks. This leads to a self-consistency problem which is highly non-trivial for the scalar field, whereas the vector couplings in uniform, infinite nuclear matter involve only time components – e.g., $\omega^\mu = \bar{\omega}^\delta^0$ – and so they simply shift energy levels. As a result, the effective strength of the coupling of the scalar meson to a hadron containing light quarks is suppressed as the scalar field increases — or equivalently, as the density increases. Thus, as a result of this self-consistent calculation at the quark
level, one can express the in-medium baryon masses, \( M_B^* \), as functions of the scalar field (as in Ref. [15]) through a calculated, density dependent, scalar coupling, \( g_{\sigma B}(\bar{\sigma}) \).

The saturation of symmetric nuclear matter [11] is a natural effect of the self-consistent response of the quark wave functions to the mean scalar field, a direct consequence of which is the reduction of the effective \( \sigma N \) coupling as the \( \bar{\sigma} \)-field increases. By analogy with the electric polarisability of an atom, which tends to arrange its internal structure to oppose an applied electric field, this reduction of the \( \sigma N \) coupling is characterised as the scalar polarisability of the nucleon. It is remarkable that the influence of baryon sub-structure, in a mean field approximation, is entirely described in terms of the parameterisation of the effective mass of the baryon through the density dependent scalar coupling derived from the quark model of the baryon and \( g_q^\sigma \). One can therefore replace the explicit description of the internal structure of the baryons by constructing an effective Lagrangian on the hadronic level, with the calculated non-linear \( \sigma \)-baryon couplings given in [15]

\[
M_B^* = M_B - w_{\sigma B} g_{\sigma N} \bar{\sigma} + \frac{d}{2} \tilde{w}_{\sigma B} (g_{\sigma N} \bar{\sigma})^2 ,
\]

(where the weightings \( w_{\sigma B} \) and \( \tilde{w}_{\sigma B} \) simply allow the use of a unique coupling to nucleons) and proceed to solve the relativistic mean field equations in a standard way [24].

The QMC Lagrangian density used in this work is given by a combination of baryon, meson, and lepton components

\[
\mathcal{L} = \sum_B \mathcal{L}_B + \sum_m \mathcal{L}_m + \sum_\ell \mathcal{L}_\ell ,
\]

for the octet of baryons \( B \in \{n,p,\Lambda,\Sigma^-,\Sigma^0,\Sigma^+,\Xi^-,\Xi^0\} \), selected mesons \( m \in \{\sigma,\omega,\rho,\pi\} \), and leptons \( \ell \in \{e^-,\mu^-\} \) with the individual Lagrangian densities

\[
\mathcal{L}_B = \bar{\Psi}_B \left( i\gamma_\mu \partial^\mu - M_B + g_{\sigma B}(\sigma) \sigma - \Gamma^\mu_{\omega B} \omega^\mu - \tilde{\Gamma}^\mu_{\rho B} \cdot \tilde{\rho}^\mu - \tilde{\Gamma}^\mu_{\pi B} \cdot \tilde{\pi} \right) \Psi_B ,
\]

\[
\sum_m \mathcal{L}_m = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \tilde{\Omega}_{\mu\nu} \cdot \tilde{\Omega}^{\mu\nu} + \frac{1}{2} m_\rho^2 \tilde{\rho}_\mu \cdot \tilde{\rho}^\mu + \frac{1}{2} m_\pi^2 \tilde{\pi}_\mu \cdot \tilde{\pi}^\mu ,
\]

for which the vector meson field strength tensors are \( \Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \) and \( \tilde{R}_{\mu\nu} = \partial_\mu \tilde{\rho}_\nu - \partial_\nu \tilde{\rho}_\mu \), and

\[
\mathcal{L}_\ell = \bar{\Psi}_\ell \left( i\gamma_\mu \partial^\mu - m_\ell \right) \Psi_\ell .
\]
In a mean-field description of infinite nuclear matter with uniform density, one can set spatial derivatives of all fields to zero and replace the meson field operators by their expectation values:

\[ \sigma \rightarrow \langle \sigma \rangle \equiv \bar{\sigma} , \]  
\[ \omega_\mu \rightarrow \langle \omega_\mu \rangle = \langle \delta_{\mu0} \omega_\mu \rangle \equiv \bar{\omega} , \]  
\[ \bar{\rho}_\mu \rightarrow \langle \bar{\rho}_\mu \rangle = \langle \delta_{\mu0} \delta_{\alpha3} \rho_{\mu\alpha} \rangle \equiv \bar{\rho} , \]  
\[ \bar{\pi} \rightarrow \langle \bar{\pi} \rangle = 0 . \]  

This is usually called the Hartree mean-field approximation.

The next step is to include the Fock level contributions involving the meson baryon vertices which are expressed as:

\[ \Gamma_{\sigma B} = g_{\sigma B} C_B(\bar{\sigma}) F^\sigma(k^2) \Gamma_1 - \frac{\partial M^*_B}{\partial \bar{\sigma}} F^\sigma(k^2) 1 , \]  
\[ \bar{\Gamma}^\eta_{\eta B} = \epsilon_\eta^\mu \bar{\Gamma}^{\mu \eta}_B = \epsilon_\eta^\mu \left[ g_{\eta B} \gamma_\mu F_1^\eta(k^2) + \frac{i f_{\eta B} \sigma_{\mu\nu}}{2 M^*_B} k^\nu F_2^\eta(k^2) \right] \tau ; \quad \eta \in \{ \omega, \rho \} , \]  
\[ \bar{\Gamma}^\pi_{\pi B} = i \frac{g_A}{2 f_\pi} F^\pi(k^2) \gamma^\mu k^\mu \gamma_5 \bar{\tau} , \]

with the isospin matrix only applicable to isovector mesons.

The \( \sigma, \omega, \rho \) and \( \pi \) form factors are all taken to have the dipole form \( F(k^2) \simeq F(\vec{k}^2) \) with the same cutoff \( \Lambda \). Clearly, these form factors are only of concern for the Fock terms. We make specific note of the two terms which contribute to the vector meson vertices, a vector ‘Dirac’ term and a tensor ‘Pauli’ term.

Through the Euler-Lagrange equations, we obtain from this Lagrangian density a standard system of coupled, non-linear partial differential equations for the meson mean fields [12]. Meson retardation effects are not included and contact terms are subtracted – see the Appendix for details. We note that the mean field approximation becomes progressively more reliable with increasing density. Finally, we note that we have also neglected any modification of the Dirac sea of negative energy states with increasing density (see, however, the discussion of such effects within the NJL model in Ref. [25]).
III. EQUATION OF STATE OF BARYONIC MATTER

A. Formalism

The equation of state relates energy density, pressure, and temperature to baryon number densities $\rho_B$. In this work, we include contributions from the full baryon octet in the limit $T = 0$. The total energy density is given as a sum of the baryonic, mesonic and leptonic contributions

$$\epsilon_{\text{total}} = \epsilon_B + \epsilon_{\sigma\omega\rho} + \epsilon_\pi + \epsilon_\ell.$$

(13)

The non-leptonic energy density can be divided into a direct (Hartree) part, $\epsilon_H = \epsilon_B + \epsilon_{\sigma\omega\rho}$, where

$$\epsilon_B = \frac{2}{(2\pi)^3} \sum_B \int_{|p|<p_F} dp \sqrt{p^2 + M_B^*}^2 ,$$

(14)

$\epsilon_{\sigma\omega\rho} = \sum_{\alpha=\sigma,\omega,\rho} \frac{1}{2} m_\alpha^2 \bar{\alpha}^2$,

(15)

where $\bar{\alpha}$ refers to the mean field value of meson $\alpha$, plus an exchange (Fock) contribution

$$\epsilon_F = \frac{1}{(2\pi)^6} \sum_{m=\sigma,\omega,\rho} \sum_{BB'} C^m_{BB'} \int_{|p|<p_F} \int_{|p'|<p_F} dp dp' \Xi^m_{BB'} ,$$

(16)

The coefficients $C^\sigma_{BB'} = C^\omega_{BB'} = \delta_{BB'}$, $C^\rho_{BB'}$, and $C^\pi_{BB'}$, which arise from symmetry considerations, are given in Ref. [19]. This non-leptonic energy density is then given by

$$\epsilon_{\text{hadronic}} = \epsilon_H + \epsilon_F = \epsilon_B + \epsilon_{\sigma\omega\rho} + \epsilon_F.$$ Note that the pion contributes only at exchange level as parity considerations lead to a vanishing direct level contribution. It nonetheless plays an important role in reducing the incompressibility of nuclear matter [19].

The leptonic energy density is simply

$$\epsilon_\ell = \frac{2}{(2\pi)^3} \sum_\ell \int_{|p|<p_F,\ell} dp \sqrt{p^2 + m_\ell^2} .$$

(17)

The scalar mean field in Eq. [15] is calculated self-consistently as

$$\bar{\sigma} = -\frac{1}{m_\sigma^2} \frac{\partial \epsilon_H}{\partial \bar{\sigma}} - \frac{1}{m_\sigma^2} \frac{\partial \epsilon_F}{\partial \bar{\sigma}}$$

$$= -\frac{2}{m_\sigma^2 (2\pi)^3} \sum_B \int_{|p|<p_F} dp \sqrt{p^2 + M_B^*} \frac{\partial M_B^*}{\partial \bar{\sigma}} - \frac{1}{m_\sigma^2} \frac{\partial \epsilon_F}{\partial \bar{\sigma}} .$$

(18)

(19)
while the vector meson mean fields simply scale with either the total or isovector baryonic density

\[ 
\bar{\omega} = \sum_B \frac{g_{\omega B}}{m_{\omega}^2} \rho_B , \\
\bar{\rho} = \sum_B \frac{g_{\rho B}}{m_{\rho B}^2} I_{3B} \rho_B .
\]

For \( \epsilon_F \), shown in Eq. (16), the integrand has the form

\[ 
\Xi_{BB'}^m = \frac{1}{2} \sum_{s,s'} |\bar{u}_{B'}(p',s')\Gamma_{mB}(p,s)|^2 \Delta_m(k) ,
\]

where \( \Delta_m(k) \) is the Yukawa propagator for meson \( m \) with momentum \( k = p - p' \), and \( u_B \) are the baryon spinors. The integrands are presented in the Appendix.

The expression for total energy density is therefore dependent on just the three main adjustable coupling constants, which control the coupling of the mesons to the two lightest quarks, \( g_\sigma^u, g_\omega^u, \) and \( g_\rho^u \) for \( q = u, d \) (\( g_\alpha^s = 0 \) for all mesons \( \alpha \)). In addition, one has the meson masses, the value of the cut-off parameter \( \Lambda \) appearing in the dipole form factors needed to evaluate the Fock terms and finally the radius of the free nucleon. The \( \sigma, \omega, \) and \( \rho \) couplings to the quarks are constrained to reproduce the standard empirical properties of symmetric (\( N=Z \)) nuclear matter; the saturation density \( \rho_0 = 0.16 \text{ fm}^{-3} \), the binding energy per nucleon at saturation of \( E(\rho = \rho_0) = -15.86 \text{ MeV} \) as well as the asymmetry energy coefficient \( a_{\text{asy}} \equiv S_0 \equiv S(\rho_0) = 32.5 \text{ MeV} \) [19] (see also Secs. III C).

The \( \omega, \rho \) and \( \pi \) meson masses are set to their experimental values. The ambiguity in defining the mass of the \( \sigma \) after quantising the classical equations of motion was explained in detail in Ref. [12]. Here it is set to the value that gave the best agreement with experiment for the binding energies of finite nuclei in a previous QMC model calculation [17], which was 700 MeV. This is a common value taken for the sigma meson mass which is generally considered in RMF models to be in the range 400–800 MeV.

The form factor cut-off mass, \( \Lambda \), controls the strength of the Fock terms Eq. (10 - 12). We considered a range of values; \( 0.9 \text{ GeV} \leq \Lambda \leq 1.3 \text{ GeV} \), with the preferred value, as we shall see, being 0.9 GeV. For simplicity we have used the same cutoff for all mesons. Since the pion mass is much lower than that of the other mesons, we have confirmed that using a lower cutoff for the pion does not significantly influence the results. This is not surprising as Fock terms are expected to be more significant at higher density where we have found that the pion does not contribute greatly.
All the other coupling constants in the expression for the total energy density are calculated within the QMC model or determined from symmetry considerations without further need for adjustable parameters. The one exception is $g_{\sigma B}(\bar{\sigma})$, which shows a weak dependence on the free nucleon radius $R_{N}^{\text{free}}$. We checked that changes of order 20% in $R_{N}^{\text{free}}$, consistent with nucleon properties, have no significant effect on the properties of nuclear matter and chose $R_{N}^{\text{free}} = 1.0$ fm.

The baryon-meson coupling constants $g_{\sigma N}(0)$, $g_{\omega B}$, and $g_{\rho B}$ (or equivalently the three quark-meson coupling constants) are determined by solving the MIT bag model equations of motion in medium. Only $g_{\sigma B}$ is density dependent and that dependence is calculated self-consistently according to

$$\frac{\partial}{\partial \bar{\sigma}} \left[ g_{\sigma B}(\bar{\sigma}) \bar{\sigma} \right] = g_{\sigma B}(0) C_{B}(\bar{\sigma}) = - \frac{\partial M_{B}^{*}(\bar{\sigma}, g_{\sigma N}, R_{N}^{\text{free}})}{\partial \bar{\sigma}},$$  \hspace{1cm} (23)

where $M_{B}^{*}$ is calculated in the QMC model using the MIT bag with one gluon exchange for the baryon structure. The couplings $g_{\omega B}$ and $g_{\rho B}$ are expressed in terms of the quark level couplings as:

$$g_{\omega B} = n_{u,d}^{B} g_{\omega}^{q}; \quad g_{\rho B} = g_{\rho N} = g_{\rho}^{q},$$ \hspace{1cm} (24)

where $n_{u,d}^{B}$ is the number of light quarks in baryon $B$.

At densities $\sim 2 - 3 \rho_0$ one expects, simply because the Fermi level of the neutrons rises rapidly, that for matter in beta-equilibrium hyperons must be considered. There is very little experimental data on the $N-Y$ and $Y-Y$ interactions, which makes the traditional approach through phenomenological pair-wise interactions very difficult. There is certainly no hope of determining the relevant three-body forces which are expected to be critical at high density. One of the attractive features of the QMC model is that it predicts all of these forces in terms of the underlying quark-meson couplings, the scalar meson mass and the particular quark model chosen (the MIT bag here). Furthermore, the density dependence of the scalar couplings to each baryon is also determined by the bag model mass parameterisation. The inclusion of this density dependent, in-medium interaction is equivalent in a density independent framework to including the appropriate three-body force between all baryons.

Of course, it is one thing to predict effective hyperon-nucleon forces but another to expect that they are realistic. Nevertheless, to the extent that we can test the QMC model it does indeed pass that test. In particular, the binding energy of a $\Lambda$-hyperon in the 1s-level of
Pb agrees with the experimental value within one MeV and the general agreement with the energy levels of Λ-hypernuclei is quite satisfactory [15]. For a Σ-hyperon in nuclear matter one does have some constraints on the optical potential depth, which is known to be significantly repulsive, and this too is reproduced within the model [15]. This repulsion in the QMC model arises very naturally through the one gluon exchange hyperfine interaction in the bag model which increases with density. It gives a clear physical explanation of the absence of Σ-hypernuclei (apart from one special case). A critical test for the model is its prediction of the binding energies of Ξ-hypernuclei, which will hopefully be tested soon at J-PARC. Until then the density dependent hyperon interactions predicted within the QMC model satisfy all the relevant empirical constraints and this gives us some confidence in applying it to dense matter in beta-equilibrium.

It is well known that the coupling of the ρ meson to a particular baryon has a relatively large Pauli, or tensor, coupling (i.e. $f_{\rho B}$ in Eq. (11)). The value used varies from one model of the nuclear force to another. In the QMC model the prediction of the tensor coupling at zero momentum transfer is unambiguous —it is exactly the anomalous, iso-vector magnetic moment of the baryon in the MIT bag model. Similarly, the tensor coupling of the $\omega$, which in the case of the nucleon is much smaller than for the $\rho$, is determined by the isoscalar magnetic moment. Since the MIT bag model reproduces the experimental values of the magnetic moments quite well, the tensor coupling required within the QMC model is equivalent to using vector meson dominance [27] and in practice we use values for the magnetic moments from the Particle Data Group [28]. Finally and purely as an exercise aimed at exploring the model dependence, we consider two different choices for the ratios of tensor to vector coupling constants $f_{\alpha B}/g_{\alpha B}$; with $\alpha \in \{\rho, \omega\}$. Whereas, as we explained, in the standard QMC calculation we take $f_{\rho N}/g_{\rho N} = 3.70$, we also explore the consequences of arbitrarily setting $f_{\rho N}/g_{\rho N} = 5.68$ in the 'Increased $f_{\rho N}/g_{\rho N}$' scenario. In this scenario we arbitrarily take the ratios of tensor to vector couplings of all baryons from the Nijmegen potentials (Table VII of Ref. [56]).

The only other parameters in the QMC model are those entering the bag model. We refer the reader to Ref. [15] where those parameters were obtained. None of them have been adjusted to any property of nuclear matter, although all calculations involving the QMC model at present rely on the MIT bag model with one gluon exchange and could be in principle improved upon by using a more sophisticated model of quark confinement.
 Nonetheless, with this simple quark-based model, remarkable agreement with a broad range of experimental data has been obtained [29].

Having established the QMC model parameters, in the following section we calculate properties of symmetric (SNM) and pure neutron (PNM) nuclear matter as well as matter in beta-equilibrium (BEM). The latter consists of nucleons and leptons, while matter in generalized beta-equilibrium (GBEM) contains the full baryon octet and leptons. Using the derived EoS, we calculate the properties of cold neutron stars and make a comparison with up-to-date experimental and observational data. We also examine the robustness of those results on the limited number of parameters entering the model.

B. Infinite symmetric and pure neutron nuclear matter

A minimal set of saturation properties of symmetric nuclear matter, the saturation density, the binding energy per particle and the symmetry energy at saturation, were used to fix the quark-meson coupling constants as described in Sec. [III A]. None of those properties is actually an empirical quantity, since they are not measured directly but extracted from experiments or observations in a model dependent way. However, there is a general consensus that all meaningful theories of nuclear matter should reproduce these quantities correctly. Moreover, other properties of both symmetric and pure neutron matter, derived from derivatives of the energy per particle with respect to particle number density, together with their density dependence, can be compared to empirical data to further test the theories. These include the pressure, incompressibility (compression modulus) and the slope of the symmetry energy.

Let us define the hadronic energy per particle, \( E = \frac{\epsilon_{\text{hadronic}}}{\rho} \), where \( \rho \) is the total baryonic density and define the following quantities as a function of \( \rho \): The first derivative of \( E \) provides an expression for baryonic pressure

\[
P = \rho^2 \frac{\partial E}{\partial \rho}.
\]  

(25)

The second derivative of \( E \) is the compression modulus or incompressibility

\[
K = 9\rho^2 \left( \frac{\partial^2 E}{\partial \rho^2} \right).
\]  

(26)
The third derivative defines the so-called skewness coefficient (some authors define $K' = -Q$)

$$Q = 27\rho^3 \left( \frac{\partial^3 E}{\partial \rho^3} \right).$$

These quantities can be evaluated at any density and any proton/neutron asymmetry ratio $\beta = (\rho_n - \rho_p)/\rho$ at which the model for the baryonic energy per particle is valid. The particular values at saturation density, $\rho_0$, are indicated with a subscript zero (e.g., $K_0, Q_0$ etc.). In symmetric nuclear matter, $\rho_n = \rho_p = 1/2 \rho$, the values of the incompressibility and skewness at saturation density can be compared to experiment. Obviously, the pressure at saturation density is equal to zero. It is convenient to express the density dependence of the energy per particle in SNM as a Taylor expansion of $E$ about the saturation density in terms of a variable $x = (\rho - \rho_0)/3\rho_0$

$$E_{SNM}(\rho) = E_0 + \frac{1}{2}K_0x^2 + \frac{1}{6}Q_0x^3 + \mathcal{O}(x^4).$$

The value of the incompressibility of infinite nuclear matter at saturation density has been the subject of considerable debate for several decades. It can be extracted either from measurement of energies of giant monopole resonances (GMR) in spherical nuclei or calculated theoretically in non-relativistic and relativistic models, typically involving mean-field plus RPA (see e.g. Refs. [30–32]). The consensus has gravitated to a value of $K_0 = 240 \pm 20$ MeV, as calculated in non-relativistic approaches, although somewhat higher values are predicted in relativistic models. Recent re-analysis of experimental data on GMR energies in nuclei with $56 < A < 208$, in an empirical approach [32] showed that $K_0$ critically depends on properties of the nuclear surface and the most likely values of $K_0$ are between 250 and 315 MeV. The preferred QMC value is 298 MeV, which is in line with these results. There is no rigorous constraint available for the skewness coefficient except for the results of Farine et al. [33]. They obtained a model dependent value $K' = 700 \pm 500$ MeV from an analysis of the nuclear breathing mode, using a selection of Skyrme forces.

In PNM, $\rho_n = \rho$ and $\rho_p = 0$. Although PNM does not exist in nature, it is seen as a first approximation to matter in the outer core of neutron stars at densities higher than $\rho_0$. The density dependence of the energy per particle of PNM is poorly known, except for the fact that PNM does not bind — i.e. the energy per particle is positive at all densities.

At very low densities, below $\sim 0.1 \rho_0$, experiments with cold Fermi atoms have yielded information about strongly interacting fluids, similar to low density matter in neutron star
crusts. Dutra et al. \[10\] studied these constraints in detail. In this work we concentrate on the higher density region, above $\sim 0.1 \rho_0$, as the QMC model may have limited applicability at very low densities. Very recently, Tews et al. \[34\] presented the first complete $N^3$LO calculation of the PNM energy, and Hebeler and Furnstahl \[35\] investigated the energy per particle in PNM at sub-saturation densities using two- and three-nucleon CEFT interactions that were consistently evolved within the framework of the similarity renormalization group. We compare their results with the QMC predictions in Fig. 1. Clearly the QMC prediction for the density dependence of the energy per particle in PNM is very similar to that of Tews et al. \[34\] at sub-saturation density, with a somewhat steeper increase at densities above saturation.

An interesting connection has been made between the pressure in the PNM neutron skin in heavy nuclei and the radius and crust thickness of a cold neutron star \[36\]. Thus a microscopic theoretical calculation of the PNM pressure became of interest, in particular at sub-saturation densities. Tsang et al. (see Fig. 4 and related references in Ref. \[41\]) collected several recent calculations of the PNM pressure as a function of particle number density. We show in Fig. 2 a selection of the models; Bruckner-Hartree-Fock (BHF) with Av18 two-body potential \[37\], Quantum Monte Carlo (QuMoCa) with Av8’ two-body potential \[38\] and CEFT \[9\]. The main uncertainty in these calculations is the strength of three-body forces, which clearly make a significant contribution to the total pressure in these models (compare the left and right panels of Fig. 2; our QMC result is shown in the right panel). The QMC model, which naturally includes three-body forces without additional parameters (see Sec. \[IV\]), indicates a somewhat faster growth of pressure with increasing density than the models with empirical three-body interactions.

Limits for the pressure-density relationship in SNM and PNM in the density region $2 – 5 \rho_0$ have been inferred from a comparison of experimental data on matter flow in energetic heavy ion collisions and predictions of a dynamical transport theory by Danielewicz et al. (see Ref. \[42\] and references therein). The matter created in the collision, lasting $\sim 10^{-23}$s at an incident kinetic energy per nucleon varying from about 0.15 to 10 GeV per nucleon, was modeled as consisting of stable and excited nucleons ($\Delta$ and $N^*$) as well as pions. The basic constraints on this matter are charge symmetry and strangeness conservation (although in this case the strangeness is zero). This is in contrast to matter in cold neutron stars, constrained by charge neutrality and generalized beta-equilibrium, where strangeness...
will not be conserved. The transport theory was extrapolated to cold symmetric and pure neutron matter, with the latter augmented by empirical symmetry pressure [42]. We show in Fig. 3 the pressure versus density for SNM and PNM, as predicted in the QMC model. In both cases our standard QMC model is consistent with the suggested constraints but at the upper end of the preferred range.

C. Asymmetric nuclear matter

Our knowledge of asymmetric nuclear matter is rather limited, mainly because of a still inadequate understanding of the symmetry energy which describes the response of forces acting in a nuclear system with an excess of protons and neutrons. This is an important property of highly asymmetric systems, such as heavy nuclei and the nuclear matter found in neutron stars, and is defined as

\[
S(\rho) = \frac{1}{2} \left. \frac{\partial^2 E}{\partial \beta^2} \right|_{\rho,\beta=0}, \tag{29}
\]

where \(S(\rho)\) is equal to the asymmetry coefficient in the Bethe–Weisacker mass formula in the limit \(A \to \infty\) [26].

The definition of \(S(\rho)\) in Eq. (29) is related but not identical to the commonly used approximation as the difference between the binding energy per baryon in PNM and SNM

\[
S(\rho) = \mathcal{E}(\rho, \beta = 1) - \mathcal{E}(\rho, \beta = 0), \tag{30}
\]

where the binding energy per baryon is

\[
\mathcal{E} = \frac{1}{\rho} \left( \epsilon_{\text{hadronic}} - \sum_B M_B \rho_B \right). \tag{31}
\]

This difference approximation is valid under two assumptions: (i) \(E(\rho, \beta = 0)\) is a minimum energy of the matter at a given density \(\rho\) and thus in the expansion of \(E(\rho, \beta)\) about this value with respect to \(\beta\) the leading non-zero term is the second derivative term and (ii) all the other derivatives in the expansion are negligible [43]. In this work we consider Eq. (30) only to examine the validity of this approximation and to observe the impact of the Fock terms, specifically the tensor contribution, upon the symmetry energy.

The density dependence of the symmetry energy can be expanded about its value at saturation \(S_0\) in terms of the slope \(L\), curvature \(K_{\text{sym}}\) and skewness \(Q_{\text{sym}}\) (all evaluated at
saturation density) as

\[ S = S_0 + L x + \frac{1}{2} K_{\text{sym}} x^2 + \frac{1}{6} Q_{\text{sym}} x^3 + \mathcal{O}(x^4), \]  

(32)

where

\[ L = 3 \rho_0 \left( \frac{\partial S}{\partial \rho} \right)_{\rho=\rho_0}, \]
\[ K_{\text{sym}} = 9 \rho_0^2 \left( \frac{\partial^2 S}{\partial \rho^2} \right)_{\rho=\rho_0}, \]
\[ Q_{\text{sym}} = 27 \rho_0^3 \left( \frac{\partial^3 S}{\partial \rho^3} \right)_{\rho=\rho_0}. \]

(33)

We note that the curvature of the symmetry energy, \( S \), at saturation density in symmetric matter, is called here \( K_{\text{sym}} \), the symmetry incompressibility. It should not be confused with \( K_\tau \), which is the isospin incompressibility, defined in ANM by Eq. (40).

In ANM, with asymmetry \( \beta \), the saturation density will be shifted \([37]\) to \( \rho_0(\beta) \sim \rho_0(1 - 3(L/K_0)\beta^2) \) and the density dependence of the energy per particle will become

\[ E(\rho, \beta) = E_0(\beta) + \frac{K_0(\beta)}{2} \left( \frac{\rho - \rho_0(\beta)}{3 \rho_0(\beta)} \right)^2 + \frac{Q_0(\beta)}{6} \left( \frac{\rho - \rho_0(\beta)}{3 \rho_0(\beta)} \right)^3 + \mathcal{O}(\beta^4), \]

(34)

where the expansion coefficients are given as

\[ E_0(\beta) = E_0 + S_0 \beta^2 + \mathcal{O}(\beta^4), \]
\[ K_0(\beta) = K_0 + \left( K_{\text{sym}} - 6 L - \frac{Q_0}{K_0} L \right) \beta^2 + \mathcal{O}(\beta^4), \]
\[ Q_0(\beta) = Q_0 + \left( Q_{\text{sym}} - 9 L \frac{Q_0}{K_0} \right) \beta^2 + \mathcal{O}(\beta^4). \]

(35-37)

The search for constraints on the symmetry energy and its slope, \( L \), has received considerable attention during the last decade. Recently Tsang et al. [41] evaluated constraints from a wide range of experiments. However, as again the symmetry energy is not measured directly but extracted from experimental data in a model dependent way, only limits on the symmetry energy can be established. One of the outcomes of the evaluation was a confirmation of a previously observed correlation between the value of \( S_0 \) and its derivative \( L \) at saturation density. Taking this correlation into account, the constraint centered on \((S_0, L) \sim (32.5, 70)\) MeV, with the uncertainty in \( S_0 \) allowing values \( 30 < S_0 < 35 \) MeV and related values of \( L \) in the range of \( 35 < L < 115 \) MeV (see Fig. 2 in Ref. [41] for more details).
While theoretical predictions of $S_0$ are also more or less confined to the range between 30 to 35 MeV, calculated values of $L$, corresponding to the range of $S_0$, vary widely. For example, the QuMoCa and CEFT models predict very similar low values of $L$, between $\sim 30 - 50$ MeV [41]. The best performing Skyrme forces, selected in Ref. [10], produce values of $L$ clustered around 50 MeV. On the other hand, relativistic mean field models show a much larger spread. The models which satisfied most of the constraints on the properties of nuclear matter, studied by Dutra et al. [39], predicted $L$ in the range $\sim 50 - 70$ MeV. However, frequently used relativistic mean field model parameterizations, e.g. NL3, NL-SH, NLC, TM1 and TM2 predict $L$ values of order $\sim 110 - 120$ MeV [40].

In the QMC model the isospin dependent part of the interaction is mostly controlled by the exchange of the rho meson. For this reason, here and in other works (e.g. [26]) the symmetry energy at saturation $S_0 = 32.5$ MeV is used to fix the rho meson coupling constant. The QMC result for $L$ is 101 MeV (see Table I), which is within the broader limits found by Tsang et al. [41], although outside their preferred range.

We show the density dependence of the symmetry energy $S$ and its slope, $L$, in Fig. [4] and the correlation between $S_0$ and $L$ in Fig. [5]. It can be seen that the linear relationship between $S_0$ and $L$, observed in QuMoCa calculations [44] and CEFT models [41] is also predicted in this work, although at higher values of $L$ and a somewhat different incline. However, this relation depends on whether we use the symmetry energy calculated from the derivative of the energy per particle, as in Eq. (29), or from the approximate expression in Eq. (30) and this effect is not negligible. When the approximate expression is used to evaluate the symmetry energy the linear relationship between $S_0$ and $L$ is shifted to values which are roughly 10 MeV lower.

Another manifestation of isospin asymmetry in nuclear matter can be studied in Giant Monopole Resonance (GMR) experiments [32]. The incompressibility of a finite nucleus is obtained, using sum-rule arguments, from the measured energy $E_{\text{GMR}}$ in spherical nuclei [30] as:

$$K(A, \beta) = M < R^2 > E_{\text{GMR}}^2. \quad (38)$$

Here, $M$ is the nucleon mass and $R$ is the rms matter radius of the nucleus with mass number.
A. $K(A, \beta)$ can be expressed in a form of an expansion in terms of $A^{-1/3}$ and $\beta$ [30]

\[
K(A, \beta) = K_{\text{vol}} + K_{\text{surf}} A^{-1/3} + K_{\text{curv}} A^{-2/3} \\
+ K_\tau \beta^2 + K_{\text{coul}} \frac{Z^2}{A^{1/3}} + \cdots,
\]

where the symmetry related coefficient consists of the volume and surface components [30, 45, 46]

\[
K_\tau = K_{\tau,v} + K_{\tau,s} A^{-1/3},
\]

with $K_{\tau,v}$ ($K_{\tau,s}$) the volume (surface) symmetry incompressibility.

The coefficient $K_{\tau,v}$ can be evaluated using Eq. (36)

\[
K_{\tau,v} = \left( K_{\text{sym}} - 6L - \frac{Q_0}{K_0} L \right).
\]

Stone et al. [32] analyzed all currently available GMR data in nuclei with $56 < A < 208$ and found a limit $-700 \leq K_{\tau,v} \leq -372$ MeV. The QMC result is $K_{\tau,v} = -477$ MeV, which is well inside the experimental limits.

D. Generalised Beta Equilibrium Matter and Neutron Stars

In this section we study cold, asymmetric nuclear matter (ANM) which is expected to exist in the outer core of cold neutron stars.

Dense matter just above the saturation density, when all nuclei are dissolved, forms a system of interacting nucleons and leptons. If this form of matter exists long enough on the time scale of weak interactions, $\tau \approx 10^{-10}$ s, beta-equilibrium (BEM) develops between beta decay $n \rightarrow p + e^- + \bar{\nu}$ and its inverse. When the density increases to about $2 - 3 \rho_0$ and because baryons obey the Pauli principle, it becomes energetically favorable for nucleons at the top the corresponding Fermi sea to convert to other baryons. A generalized beta equilibrium (GBEM) develops with respect to all reactions by weak or strong interactions that lead to the lowest energy state. Only two quantities are conserved in GBEM - the total charge (zero in stars) and total baryon number. Strangeness is conserved only on the time scale of strong interaction, $\tau \approx 10^{-24}$ s, and lepton number is conserved only on the time-scale of tens of seconds, because of the diffusion of neutrinos out of the star [26].

To describe GBEM, it is convenient to use the chemical potentials of the participating particles. It can be shown that there are as many independent chemical potentials as the
number of conserved quantities. Thus we need to choose just two, for example the chemical potentials of the neutron and electron. Chemical potentials of all the other species in GBEM are then expressed via a relation

$$\mu_i = B_i \mu_n - Q_i \mu_e,$$  \hspace{1cm} (42)

where the baryon number, $B$, is 0 or 1 and the charge number, $Q$, is 0 or $\pm 1$. Alternatively (and equivalently), the chemical potentials can be related to Lagrange multipliers (as the degrees of freedom for charge conservation ($\nu$) and baryon number conservation ($\lambda$)) in order to solve the following system of equations

$$0 = \mu_i + B_i \lambda + \nu Q_i,$$  \hspace{1cm} (43)

$$0 = \mu_\ell - \nu,$$  \hspace{1cm} (44)

$$0 = \sum_i B_i \rho_i - \rho,$$  \hspace{1cm} (45)

$$0 = \sum_i B_i \rho_i Q_i + \sum_\ell \rho_\ell Q_\ell,$$  \hspace{1cm} (46)

to obtain the number densities for each particle ($i \in \{n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0\}$ and $\ell \in \{e^-, \mu^-\}$), $\rho_i$, as well as the Lagrange multipliers. At Hartree–Fock level, the following formulas to numerically evaluate the chemical potentials, must be used to ensure we encapsulate the Fock contribution to the energy densities correctly

$$\mu_i = \frac{\partial \epsilon_{\text{total}}}{\partial \rho_i}, \quad \mu_\ell = \frac{\partial \epsilon_\ell}{\partial \rho_\ell} = \sqrt{k^2 + m_\ell^2}.$$  \hspace{1cm} (47)

In Figs. 6 and 7 we show the EoS (with various parameter variations) and the distribution of species in GBEM matter for the preferred scenario in this work. We note that the pressure now involves the total energy density (including leptonic contribution) $\epsilon_{\text{total}} = \epsilon_{\text{hadronic}} + \epsilon_\ell$:

$$P_{\text{total}} = \rho^2 \frac{\partial}{\partial \rho} \left( \frac{\epsilon_{\text{total}}}{\rho} \right) = \sum_i \mu_i \rho_i - \epsilon_{\text{total}}.$$  \hspace{1cm} (48)

The kinks in pressure in Fig. 6 appear at hyperon thresholds. A comparison between calculations for either Hartree alone, Hartree–Fock with only the Dirac piece of the coupling to vector mesons, or the full model highlights the importance of the $\rho N$ tensor coupling at high density. As compared to the EoS of matter in which the hyperons are not included above their natural thresholds and nucleons are assumed to be the only baryons up to densities $\sim 5$–6 $\rho_0$, the pressure in GBEM increases with density more slowly. Nevertheless, the pressure
still increases fast enough to support high mass, cold neutron stars, as will be discussed in
the next section.

In Fig. 7 the particle content of GBEM matter is displayed. The first hyperon to appear
is the \( \Xi^- \), at 0.42 fm\(^{-3} \), followed by the \( \Xi^0 \) at 0.91 fm\(^{-3} \). Since the latter is above the
maximum density reached in any of our model variations it is largely irrelevant. This particle
content is very different from most other models, which generally find that either the \( \Lambda \) or
\( \Sigma^- \) appears first. The “hyperfine” interaction from one gluon exchange makes the \( \Lambda \) more energetically favourable than the \( \Sigma^- \), providing a source of attraction for the former and repulsion for the latter. This has been shown at the Hartree level in the QMC model to suppress the appearance of \( \Sigma^- \) hyperons in GBEM matter \[47\]. This can also be considered a qualitative explanation for the absence of medium to heavy \( \Sigma \) hypernuclei \[15\]. Even though the \( \Lambda \) feels attraction, it appears that it cannot compete with the attraction generated by
the Fock terms —specifically the tensor part —for the \( \Xi \). We show in Fig. 8 that the \( \Lambda \)
chemical potential approaches the neutron chemical potential, meaning that it is very nearly energetically favourable for it to appear. On the other hand, for the \( \Sigma^- \) we see that at low
density it is more favourable than the \( \Xi^- \), while beyond \( \sim 0.4 \) fm\(^{-3} \) this is no longer so.

\section*{E. Cold neutron stars}

In order to calculate neutron star properties, such as the total gravitational mass, \( M(R) \),
and the baryon number, \( A(R) \), within the stellar radius \( R \), we solve the TOV equations \[48\]
for hydrostatic equilibrium of spherically symmetric (non-rotating) matter. Using the EoS
calculated here, this is self-supported against gravitational collapse.

\begin{align}
M(R) & = \int_0^R 4\pi r^2 \epsilon_{\text{total}} \, dr , \tag{49} \\
\frac{dP_{\text{total}}}{dr} & = -(\epsilon_{\text{total}} + P_{\text{total}}) \frac{(M(r) + 4\pi r^3 P_{\text{total}})}{r^2 (1 - 2M(r)/r)} , \tag{50} \\
\frac{dA}{dr} & = \frac{4\pi r^2 \rho}{\sqrt{1 - 2M(r)/r}} . \tag{51}
\end{align}

In Eqs. (49)–(51) we use units in which \( G = 1 \). The difference between the total gravitational
mass and baryonic mass within a radius \( R \) is defined by \( M(R) - A(R)M_N \).

The EoS of GBEM is not valid in the outer regions (crust) of the star, where nuclei and
nuclear processes become dominant. Following the customary procedure, we introduce a
smooth transition between our EoS in GBEM and the standard low density EoS of Baym, Pethick and Sutherland (BPS) \cite{49} at low density.

The relationship between stellar mass and radius, obtained as the solution of the TOV equations, Eqs. (49–51), is summarized in Table I and depicted in Fig. 9. We find that the predicted maximum mass, \(1.93 \, M_\odot\), lies very close to the constraints set by Demorest et al. \cite{50} of a \((1.97 \pm 0.04) \, M_\odot\) pulsar, as well as the new constraint set by PSR J0348 + 0432 with a mass of \(2.03 \pm 0.03 \, M_\odot\) \cite{51}. The corresponding central density is \(5.52 \, \rho_0\) and the radius \(12.27\) km, somewhat larger than that extracted from recent observations of Type I X-ray bursters (see e.g. Refs. \cite{52, 53}). Extraction of radii from observation is rather complicated and there are still many questions to be addressed. For example, Steiner et al. \cite{52} analyzed observations of six low mass X-ray binaries (emitting X-rays regularly) and their statistical analysis yielded \(R\) in the range \(10–12\) km for masses around \(1.6 M_\odot\). However, the uncertainty in the relation between the extracted photospheric radius and the actual radius of the star remains large. The results of Guillot et al., namely \(R = 9.1^{+1.3}_{-1.5}\) km (90%-confidence), are based on observations of five quiescent low mass X-ray binaries (which emit X-rays only occasionally) under the assumption that the radius is constant for a wide range of masses.

Whilst the observations Refs. \cite{50, 51} provide constraints on high mass neutron stars, the observation of the double pulsar J0737-3039 and its interpretation \cite{54} offers a constraint on the neutron star EoS in a region of central densities \(\sim 2 – 3 \, \rho_0\). The constraint concerns the ratio between the gravitational and baryonic mass of the star. The gravitational mass of pulsar B is measured very precisely to be \(M_g = 1.249 \pm 0.001 M_\odot\) and the baryonic mass depends on the mode of its creation, which can be modeled. If pulsar B was formed from a white dwarf with an O-Ne-Mg core in an electron capture supernova, with no or negligible loss of baryonic mass during the collapse, the newly born pulsar should have the same baryonic mass as the progenitor star. Podsiadlowski et al. \cite{54} estimated the baryonic mass of the pulsar B to be between \(1.366\) and \(1.375 \, M_\odot\). Another simulation of the same process, by Kitaura et al. \cite{55}, gave a value for the baryonic mass of \(1.360 \pm 0.002 \, M_\odot\). We show in Fig. 10 the QMC result, which supports the model of Kitaura et al., accepting some small loss of baryonic mass during the birth of pulsar B.
F. Sensitivity to parameter variation and comparison with other models

Our calculations for the Hartree–Fock QMC model follow similar lines to Refs. [17, 19, 20] in that in each case an approximation is made for the Fock terms. More specifically, in our calculation of the Fock terms we omit energy transfer in the meson propagator (meson retardation effects). We also omit the modification of momenta because of the vector component of the self energy, which has been shown to be small in Refs. [20] and [21]. We include the tensor interaction in the Fock terms, with a common form factor, which has a dipole form. The lowest mass, $\Lambda$, for that cut-off, which should be larger than the masses of the mesons included, is 0.9 GeV. This is taken as our standard value because the incompressibility rises significantly as it is increased. Indeed, as we see in Table I, $K_0$ rises above 312 MeV for $\Lambda$ greater than 1.1 GeV. The effect on neutron star properties of increasing $\Lambda$ to 1.1 GeV is shown in Fig. 9. It leads to a relatively mild increase in the maximum possible mass from $1.93M_\odot$ to $2.07M_\odot$, with a 7% decrease in the corresponding central density.

The contribution to the mean scalar field arising from the Fock terms is incorporated in the case denoted “Fock $\delta \bar{\sigma}$”. As one might expect, given that this is a relatively small correction, we see in Fig. 6 it has a mild effect on the EoS for GBEM. When applied to neutron star properties it increases the maximum mass by a few percent above our standard result, to just below $2M_\odot$.

Our preferred tensor couplings, arising from the underlying MIT bag model, are consistent with Vector Meson Dominance (VDM) and hence our tensor couplings are taken as the experimental magnetic moments. Purely as a test of the effect of a variation in those couplings we also took the set of couplings with a larger tensor coupling for the $\rho$ to the nucleon used in the Nijmegen potentials [56]. These were also used by Miyatsu et al. [21 59]. This variation, denoted “Increased $f_{\rho N}/g_{\rho N}$”, produced an EoS for GBEM which was indistinguishable from our standard result.

The Hartree–Fock calculation in Ref. [22] differs considerably from that presented here, as well as from that in Refs. [21 59 61]. The first and major difference is that the tensor interaction of the baryons is ignored, whereas in Refs. [21 59 61] and in our work it is found to have a very significant effect. A second difference between Ref. [22], our work, and Refs. [21 59 61] is that in their preferred QMC scenario (QMC-HF3) they artificially adjust a parameter, $C$, which is related to the scalar polarisability, to obtain a lower value for the
incompressibility. This represents a dramatic change in the model.

The masses of the baryons in the QMC model are determined by the bag equations and the scalar coupling is calculated directly from the density dependence of the baryon mass in-medium. Thus, changing $C$, or equivalently the scalar polarisability, changes the mass and the density dependent coupling in a manner which is totally inconsistent with the traditional form of the QMC model \[29\]. In this manner the many body interaction is also being changed through the density dependent scalar coupling. Their QMC-HF3 variation gives an incompressibility of $K = 285$ MeV and a very low prediction for the maximum mass of neutron stars, $M = 1.66 \, M_\odot$. In our Dirac-only variation we find a slightly larger value for the incompressibility, $K = 294$ MeV with a maximum stellar mass of $M = 1.79 \, M_\odot$. Other variations were considered in Ref. \[22\] where they do not modify $C$: one where they calculate fully relativistic Fock terms, and another where they make a non-relativistic approximation to the Fock terms. These variations both produce maximum masses of neutron stars of $M = 1.97 \, M_\odot$, very similar to ours.

Refs. \[21, 59–61\] carry out a relativistic calculation in which they treat the Fock and Hartree terms on the same level. More precisely they calculate self-energy contributions arising from both terms and these self energies modify the baryon mass, momentum and energy. They include the tensor interaction, subtract contact terms, and consider two variations of the bag model. In their first paper they used much larger values for the tensor couplings without form factors. In the later paper they include the effect of form factors, ignoring effects of meson retardation (as we do) but with a lower cutoff mass, i.e. $\Lambda = 0.84$ GeV. The latter had the effect of keeping the incompressibility from being too large. Their conclusions are very similar to our own, in that they find that the tensor terms provide a source of attraction and that overall the Fock terms enhance the maximum neutron star mass.

The maximum stellar masses in their first paper are considerably larger than those in their second paper, almost certainly because the inclusion of the form factor decreases the effect of the Fock term at high density. They consider two variations of the QMC model: one with, and one without the pion contribution in the bag (CQMC) which tends to give a slightly stiffer EoS, because of its effect on the baryon masses. For QMC they obtain $M = 1.86 \, M_\odot$, $R = 11.2$ km, and for CQMC $M = 1.93 \, M_\odot$, $R = 11.5$ km for the maximum stellar mass solutions. Despite the differences in how we handle the Fock terms and their use of larger tensor couplings, and more phenomenological hyperon couplings, we are led to
the same conclusions about the importance of the tensor contribution. We also find a very similar particle content, where the \( \Xi^- \) is the first hyperon to appear.

IV. DISCUSSION

In order to treat the equation of state of matter at the densities typical of neutron stars one must treat the motion of the baryons relativistically. The quark-meson coupling (QMC) model not only does that but it self-consistently treats the in-medium changes in baryon structure induced by the large scalar mean fields generated in such matter. As we have explained, those changes, which may be represented by the corresponding scalar polarisabilities, lead naturally to predictions for the three-body forces between not just the nucleons but the nucleons and hyperons as well as hyperons, without additional parameters. This widely used approach has been extended here to include the effect of Fock terms arising from the tensor (or Pauli) couplings of the baryons to vector mesons, especially the \( \rho \).

The results for a comprehensive set of nuclear matter properties, including \( K_0 \), \( L \), \( K_{\text{sym}} \), \( Q_0 \) and \( K_{\tau,v} \) have been studied in detail. While the incompressibility is increased by this addition and tends to lie at the top end of the acceptable range, it serves as a useful constraint on the additional mass parameter, \( \Lambda \), associated with the form factor that appears at the meson-baryon vertices (the latter only being needed once the Fock terms are computed). The pronounced variation of the nuclear matter observables with this parameter (which must lie above the masses of the exchanged mesons included in the theory) is illustrated in Table I. Increasing \( \Lambda \) beyond 0.9 GeV raises the incompressibility; with the case denoted \( \Lambda = 1.1 \) GeV already very close to the limit \( K < 315 \) MeV.

The symmetry energy is greatly enhanced and significant curvature is introduced into this quantity by the Fock terms, specifically the tensor component of the rho meson. At saturation density we find in all cases that the isospin incompressibility is within accepted constraint limits and while the slope of the symmetry energy is large, it does lie within the broad limits reported by Tsang et al. [41].

It is interesting to note that there is a satisfying level of consistency between theoretical predictions of \( \text{N}^3\text{LO} \) chiral effective field theory and the QMC model results studied here for densities of PNM up to and around nuclear matter density. Above saturation density a slightly higher energy per particle as a function of density is found here. It is also found
that the natural incorporation of many body forces in the QMC model tends to produce a stiffer PNM EoS above saturation density than other models including 3-body forces.

Even at densities above three times nuclear matter density, the nucleon Fock terms are found to contribute significantly to the EoS and the corresponding attraction is what is responsible for the increased pressure and larger maximum stellar masses. This can be seen in Fig. 9 where there is a clear transition from a Hartree QMC calculation to a Hartree–Fock calculation with no tensor interaction (Dirac-only; no Pauli term), to our standard calculation (Dirac and Pauli terms). In these three variations, and those with increasing form factor mass, $\Lambda$, the maximum stellar mass increases because of the increased pressure coming from the Fock terms. This increased pressure arises mainly from the $\rho$ meson contribution. As we have already noted and can be seen in Table I and Fig. 9, the value of $\Lambda$ cannot be varied far. Indeed, for $\Lambda = 1.1$ GeV, the incompressibility is already as high as it can be. We see that the maximum neutron star mass, for the case of nuclear matter in beta-equilibrium where hyperons must appear, lies in the range $1.93$ to $2.07M_\odot$ for $\Lambda \in (0.9, 1.1)$ GeV.

The EoS and the maximum masses of the corresponding neutron stars are insensitive to the use of the larger $\rho$ tensor couplings used, for example, by Miyatsu et al. [21]. Similarly, modest variations in the radius of the free nucleon have only very minor effects on these quantities. Finally, we note that the correction ($\delta \bar{\sigma}$) to the scalar mean field arising from the Fock terms decreases the incompressibility by 13 MeV, yet other observables remain largely unaltered by this addition. This plus the dependence of the incompressibility and maximum mass on $\Lambda$ leads us to the conclusion that the maximum mass allowed in the model lies in the range $1.9 - 2.1M_\odot$.

Turning to the particle content of dense matter in the neutron stars generated in this work, in comparison with the results of Stone et al. [19], who did not include the tensor couplings in the Fock terms, we find that the threshold density for $\Xi^-$ is lowered, while those of the $\Lambda$ and $\Xi^0$ are raised. Over the range of parameter variation allowed by the nuclear matter constraints there is a greater splitting between the thresholds of the $\Xi$ baryons than that found by Stone et al. [19]. We find that $\Lambda$ production is not energetically favoured at the densities considered here, in agreement with Ref. [21].

We stress that the QMC model does not predict the appearance of $\Sigma$ hyperons at any density where the model can be considered realistic. This is in contrast to a number of other relativistic models which do predict the $\Sigma$ threshold to occur, even prior to that of the
We note that Schaffner-Bielich [57] considered a phenomenological modification of the $\Sigma$ potential with additional repulsion, which significantly raised its threshold density. In the case of the QMC model the physical explanation of the absence of $\Sigma$-hyperons is very natural, with the mean scalar field enhancing the repulsive hyperfine force for the in-medium $\Sigma$ (recall that the hyperfine splitting, which arises from one-gluon-exchange, determines the free $\Sigma$–$\Lambda$ mass splitting in the MIT bag model).

Purely for comparison purposes, we also include a nucleon-only scenario, in which hyperons are artificially excluded. In this case the EoS is increasingly stiffer at densities above $0.4 \, \text{fm}^{-3}$, leading to a large maximum stellar mass of $2.26 \, M_{\odot}$, consistent with many other nucleon-only models.

It is worth remarking that upon inclusion of the tensor coupling, the proton fraction increases more rapidly as a function of total baryon density. This is likely to increase the probability of the direct URCA cooling process in proto-neutron stars. As a further consequence, the maximum electron chemical potential is increased in this case, which may well influence the production of $\pi^-$ and $\bar{K}$ condensates. Changes to the $\Lambda$ threshold (it occurs at higher density with lower maximum species fraction) reduce the possibility of H-dibaryon production as constrained by beta-equilibrium of the chemical potentials.

In summary, taking into account the full tensor structure of the vector-meson-baryon couplings in a Hartree–Fock treatment of the QMC model results in increased pressure at high density – largely because of the $\rho N$ tensor coupling – while maintaining reasonable values of the incompressibility at saturation density. The conceptual separation between the incompressibility at saturation density and the slope of the symmetry energy or ‘stiffness’ at higher densities is critical. It is the latter that leads to neutron stars with maximum masses ranging from $1.9 \, M_{\odot}$ to $2.1 \, M_{\odot}$, even when allowance is made for the appearance of hyperons. This suggests that hyperons are very likely to play a vital role as constituents of neutron stars with central densities above three times nuclear matter density.

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Appendix

The integrands take the following form for $B = B'$

$$\Xi_B^\sigma = \frac{1}{2} \left( \frac{g_{\sigma B} C_B(\bar{\sigma}) F^\sigma(k^2))}{E^*(p')E^*(p)} \right)^2 \left\{ M_B^{*2} + E^*(p')E^*(p) - p' \cdot p \right\} \Delta_\sigma(k).$$

(52)

Here for the vector meson integrands we denote $\eta = \omega, \rho$

$$\Xi_B^{\eta V} = \left( \frac{g_{\eta B} F_1^\eta(k^2)}{E^*(p')E^*(p)} \right)^2 \left\{ 2M_B^{*2} - E^*(p')E^*(p) + p' \cdot p \right\} \Delta_\eta(k)$$

(53)

$$\Xi_B^{\eta T} = \left( \frac{g_{\eta B} F_2^\eta(k^2)}{E^*(p')E^*(p)} \right)^2 \left\{ \left( \frac{3M_B^{*2} + 3E^*(p')E^*(p)}{E^*(p')E^*(p)} \right) - 3p' \cdot p \right\} \Delta_\eta(k)$$

(54)

$$\Xi_B^{\pi T} = \left( \frac{g_{\pi B} \kappa_{\pi B} F_2^\pi(k^2)}{E^*(p')E^*(p)} \right)^2 \left\{ \left( \frac{5M_B^{*2} - E^*(p')E^*(p) + p' \cdot p}{4M_B^{*2}} \right) \right\} \Delta_\pi(k)$$

(55)

and for the pion

$$\Xi_B^\pi = - \left( \frac{g_{\pi B} F_2^\pi(k^2)}{E^*(p')E^*(p)} \right)^2 \left\{ M_B^{*2} - E^*(p')E^*(p) + p' \cdot p \right\} \Delta_\pi(k).$$

(56)

where $E^*(\vec{p}) = \sqrt{\vec{p}^2 + M_B^{*2}}$. In the above integrands we expand the terms in the braces multiplied by the propagator to isolate the momentum independent pieces and multiply these contact terms by the variable $\xi$ which we use to investigate the consequences of contact subtraction. We emphasize here the importance of subtraction of the momentum independent piece, which when transformed to configuration space corresponds to a delta function. In this manner our subtraction is implemented by the variable $\xi$, such that $\delta(\vec{r}) \mapsto \xi \times \delta(\vec{r})$.

The removal of the contact terms is a common procedure due to the fact that these contact terms represent very short range, effectively zero range correlations between the baryons, which is not consistent in this model which treats the baryons as clusters of quarks and not
as point-like objects. We give this explicitly for the Vector-Vector piece of the vector mesons

\[
\frac{2M_B^* p^2 - E^*(p')E^*(p) + \vec{p}' \cdot \vec{p}}{\vec{k}^2 + m_\eta^2} = \frac{2M_B^* p^2 - p' \cdot p}{\vec{k}^2 + m_\eta^2} \\
\geq \frac{M_B^* p^2 - \frac{\xi^2}{2}}{\vec{k}^2 + m_\eta^2} \\
= \frac{M_B^* p^2}{\vec{k}^2 + m_\eta^2} - \frac{1}{2} \frac{\vec{k}^2}{\vec{k}^2 + m_\eta^2} \\
= \frac{M_B^* p^2 + m_\eta^2}{\vec{k}^2 + m_\eta^2} - \frac{1}{2} \xi
\]

(57)

the remaining subtractions follow in the same manner.
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FIG. 1: (Color online) Pure neutron matter energy per particle as a function of density as obtained in the present work, in comparison with complete CEFT at N^3LO order – for more details of the latter, see Ref. [34].
FIG. 2: (Color online) Density dependence of pressure in PNM as predicted in BHF, DBHF, QuMoCa and CEFT without (with) three-body forces left (right) panel. The QMC model prediction is shown in the right panel. For more details see the text and ref. [41].
FIG. 3: (Color online) (Top) Pressure in SNM as a function of density as predicted in QMC model. The shaded area is taken from Ref. [42]. (Bottom) Pressure in PNM as a function of density as predicted in the QMC model. The upper and lower shaded areas correspond to two different estimates of the contribution of the symmetry pressure to the total pressure. For more detail see Ref. [42]
FIG. 4: (Color online) (Top) Symmetry energy $S$ as a function of baryon number density, as calculated in this work. (Bottom) Slope $L$ of the symmetry energy, as a function of baryon number density $L(\rho) = 3\rho \left( \frac{\partial S}{\partial \rho} \right)$. 
FIG. 5: (Color online) The correlation between the slope and magnitude of the symmetry energy $S_0$. The difference in results obtained from calculation of $S_0$, using either Eq. (29) or Eq. (30) (the latter denoted “App.”) are illustrated. Constraints on the slope $L$ and the symmetry energy $S_0$ at saturation density from different experiments are overlayed. The experimental methods are labeled next to the boxes with the estimated uncertainties. See Ref. [41] for more details.
FIG. 6: (Color online) GBEM equation of state. Kinks occur at significant hyperon threshold densities. The divergence between the standard QMC parameterization and the ‘Hartree Only’ and “Dirac Only” scenarios highlights the importance of the $\rho N$ tensor coupling in Hartree–Fock at high density. The “Nucleon only” BEM EoS is added for a comparison.
FIG. 7: (Color online) Species fraction as a function of baryon number density in GBEM, for the standard scenario EoS shown in Fig. 6.
FIG. 8: (Color online) (Top) Neutral baryon chemical potentials as a function of baryon number density for the standard scenario. (Bottom) Negative charge baryon chemical potentials as a function of baryon number density for the standard scenario.
FIG. 9: Gravitational Mass versus radius relationship for various scenarios described in the text. The black dots represent maximum mass stars and the coloured bars represent observed pulsar constraints.
FIG. 10: Gravitational mass versus baryonic mass. The boxes are constraints from simulations (Yellow) by Kitaura et al. [55] and (Orange) by Podsiadlowski et al. [54], which are explained in the text.
TABLE I: Coupling constants determined for the QMC model in our standard case (for which \( \Lambda = 0.9 \) GeV, and \( R_N^{\text{free}} = 1.0 \) fm) and variations in which differences from that standard parameter set are indicated in column 1. Also shown are the saturation incompressibility, \( K_0 \); stellar radius; maximum stellar mass and corresponding central density (units \( \rho_0 = 0.16 \) fm\(^{-3} \)). In the 'Increased \( f_{\rho N}/g_{\rho N} \)' scenario we arbitrarily take the ratios of tensor to vector couplings of all baryons from the Nijmegen potentials (Table VII of Ref. [56]), where there is a larger value of \( f_{\rho N}/g_{\rho N} = 5.7 \).

| Model               | \( g_{\sigma N} \) | \( g_{\omega N} \) | \( g_{\rho} \) | \( K_0 \) (MeV) | \( L \) (MeV) | \( R \) (km) | \( M_{\text{max}} \) (\( M_\odot \)) | \( \rho_{c_{\text{max}}} \) (\( \rho_0 \)) |
|---------------------|--------------------|--------------------|-------------|----------------|-------------|-------------|-----------------------------|----------------|
| Standard            | 10.42              | 11.02              | 4.55        | 298            | 101         | 12.27       | 1.93                        | 5.52                        |
| \( \Lambda = 1.0 \) | 10.74              | 11.66              | 4.68        | 305            | 106         | 12.45       | 2.00                        | 5.32                        |
| \( \Lambda = 1.1 \) | 11.10              | 12.33              | 4.84        | 312            | 111         | 12.64       | 2.07                        | 5.12                        |
| \( \Lambda = 1.2 \) | 11.49              | 13.00              | 5.03        | 319            | 117         | 12.83       | 2.14                        | 4.92                        |
| \( \Lambda = 1.3 \) | 11.93              | 13.85              | 5.24        | 329            | 124         | 13.02       | 2.23                        | 4.74                        |
| \( R = 0.8 \)       | 11.20              | 12.01              | 4.52        | 300            | 110         | 12.41       | 1.98                        | 5.38                        |
| Fock \( \delta \bar{\sigma} \) | 10.91              | 11.58              | 4.52        | 285            | 109         | 12.29       | 1.98                        | 5.5                          |
| Increased \( f_{\rho N}/g_{\rho N} \) | 10.55              | 11.09              | 3.36        | 299            | 101         | 12.19       | 1.93                        | 5.62                        |
| Dirac Only          | 10.12              | 9.25               | 7.83        | 294            | 85          | 12.47       | 1.78                        | 5.2                          |
| Hartree Only        | 10.25              | 7.95               | 8.40        | 283            | 88          | 11.85       | 1.54                        | 6.0                          |
| Nucleon Only        | 10.42              | 11.02              | 4.55        | 298            | 101         | 11.64       | 2.26                        | 5.82                        |
TABLE II: Additional nuclear matter properties determined for our standard case (for which $\Lambda = 0.9$ GeV, and $R_{N}^{\text{free}} = 1.0$ fm) and the effect of subsequent variations in which differences from the standard parameter set are indicated in column 1. These properties are calculated at saturation. The tabulated quantities at saturation are the symmetry energy $S_0$, slope $L$, curvature $K_{\text{sym}}$, incompressibility $K_0$, skewness coefficient $Q_0$ and volume component of isospin incompressibility $K_{\tau,v}$. (The symbol $^\dagger$ indicates that this parameter is a constraint on the parameters of the QMC model.) All entries are in MeV. In the ‘Increased $f_{\rho N}/g_{\rho N}$’ scenario we arbitrarily take the ratios of tensor to vector couplings of all baryons from the Nijmegen potentials (Table VII of Ref. [56]), where there is a larger value of $f_{\rho N}/g_{\rho N} = 5.7$.

| Model                      | $^\dagger S_0$ | $L$ | $K_{\text{sym}}$ | $K_0$ | $Q_0$ | $K_{\tau,v}$ |
|----------------------------|----------------|-----|------------------|-------|-------|--------------|
| Standard                   | 32.5           | 101 | 66               | -189  | -477  |
| $\Lambda = 1.0$           | 32.5           | 106 | 94               | -141  | -492  |
| $\Lambda = 1.1$           | 32.5           | 111 | 128              | -85   | -509  |
| $\Lambda = 1.2$           | 32.5           | 117 | 166              | -19   | -530  |
| $\Lambda = 1.3$           | 32.5           | 124 | 211              | 64    | -560  |
| $R = 0.8$                  | 32.5           | 110 | 120              | -142  | -485  |
| Fock $\delta\sigma$       | 32.5           | 109 | 136              | 285   | -430  |
| Increased $f_{\rho N}/g_{\rho N}$ | 32.5       | 101 | 68               | 299   | -187  | -475         |
| Dirac Only                 | 32.5           | 85  | 2                | 294   | -298  | -424         |
| Hartree Only               | 32.5           | 88  | -17              | 283   | -455  | -404         |
| App. Standard             | 32.5           | 93  | 43               | 298   | -206  | -451         |
| App. Standard             | 30.0           | 83  | 34               | 299   | -224  | -402         |
| Standard                   | 30.0           | 91  | 58               | 298   | -206  | -426         |