Assessing the adequacy of the gompertz regression model in the presence of right censored data

Nur Niswah Naslina Azid @ Maarof¹,², Jayanthi Arasan¹, Hani Syahida Zulkafli¹ and Mohd Bakri Adam¹

¹Department of Mathematics, Faculty of Science, Universiti Putra Malaysia
²Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Cawangan Kelantan
E-mail: niswah@uitm.edu.my

Abstract. The Gompertz distribution is often used to model human mortality and establish actuarial tables. Cox-Snell residual is the most general practice to evaluate the model adequacy. However, standard statistical procedures are not amenable to handle the censored observations. Therefore, this research investigates the adequacy of the two parameter Gompertz parametric survival model that was extended to incorporate with fixed covariate in the presence of right censored data. Performance of the model is assessed and compared at various combinations of sample sizes and censoring proportions via simulation study. The newly proposed modifications to the Cox-Snell residuals based on the geometric mean and harmonic mean as well as the jackknife means were compared with Cox-Snell and Modified Cox-Snell residuals via simulation studies at different settings. The log-cumulative hazard plot of residuals is obtained by plotting the proposed residuals against the cumulative hazard function to assess the model’s fit. Based on the model adequacy study, we found out that jackknife geometric mean and jackknife harmonic mean outperform CS, MCS, GMCS and HMCS residuals.

1. Introduction
The two parameter Gompertz distribution had been found to be useful in analyzing the lifetime data. This model is one of the commonly used parametric survival function based on laws of mortality. It is a continuous probability distribution which takes non-negative random variables that has exponentially increasing failure rate over time. The Gompertz model was initially proposed by a British actuary, [1] as a model for age-specific human mortality and fitting the actuarial tables. It is well known and widely used in many aspects of biology and demography. The hazard function of this distribution is,

\[ h(t; \gamma; \lambda) = \lambda \exp(\gamma t), \quad t \geq 0, \lambda > 0, \gamma > 0, \]

where \( \lambda \) is known as baseline mortality, and \( \gamma \) is the senescent component. While, \( T \) is the non-negative continuous random variable which represents the individual’s survival time. The survivor function that denotes the probability of an individual survives longer than \( t \) is,

\[ S(t; \gamma; \lambda) = \exp \left[ \frac{\lambda}{\gamma} (1 - e^{\gamma t}) \right], \]

and the probability density function is,
Numerous numbers of researchers have done studies on different characteristics and statistical methodology of the Gompertz distribution. [2] investigated on the properties of the Gompertz model and compared the estimates by using the least-squares and maximum likelihood methods. [3] discussed on the theoretical explanation of Gompertz model in the cases of accretionary growth. [4] developed an exact confidence interval and an exact joint confidence region for the parameters of the Gompertz distribution. Later, [5] explored on the unweighted and weighted least squares estimates for parameters of the Gompertz distribution under complete set of data and first failure censored data. [6] proved that compared with the method of moments, the maximum likelihood estimation has higher accuracy in the parameter estimation of the Gompertz model. [7] studied the performance of the Gompertz model with time-dependent covariates and fixed covariates in the presence of right-censored data, and compared the confidence interval estimates of Wald and Jackknife methods. Based on the coverage probability, [8] discovered on the confidence intervals of the Gompertz model with the time-dependent covariate and fixed covariate in the presence of interval-, right-, left-censored and uncensored data. [9] discussed various properties and methods for estimating the unknown parameters of the Gompertz distribution.

Although there are well known methods for estimating unconditional survival distributions, most interesting survival modeling examines the relationship between survival and one or more predictors known as covariates. Residuals are a widely used tool to assess the adequacy of a model. When modeling survival data, it is not as easy to define a residual as for a general linear model. It is the most general practice to use Cox-Snell residuals to evaluate the model adequacy in survival models [10]. Therefore, newly proposed modified Cox-Snell residuals were applied in the Gompertz model with covariate and right censored data by using harmonic mean, geometric mean as well as jackknife means via simulation study.

This study focuses on the adequacy of the Gompertz distribution with covariate in the presence of right censored data at various censoring proportions and sample sizes. Firstly, an approach of simulating the Gompertz regression model with the right censored data was carried out. Then, several modifications of the Cox-Snell residuals have been proposed. The performance of these methods was analyzed comprehensively at different level of censoring percentiles and sample sizes. The newly proposed modifications to the Cox-Snell residuals based on the geometric mean and harmonic mean as well as the jackknife means were compared with Cox-Snell and Modified Cox-Snell residuals via simulation study by comparing the residual’s intercept, slope, and R-square at different settings.

2. Methodology
In this section, we would like to apply the simulation study to the Gompertz survival time distribution.

2.1. The Gompertz regression model
The effect of covariate on the survival time of the $i^{th}$ can be incorporated to the hazard function by letting parameter $\lambda$ be a function of the covariate,

$$\lambda = \exp(\beta'X).$$

(4)

For data set with a covariate $x_i$ where $i = 1, 2, ..., n$, the hazard function for the $i^{th}$ subject can be expressed as,

$$h(t_i; \gamma; \lambda) = \lambda_i \exp(\gamma t_i),$$

(5)

where

$$\lambda_i = \exp(\beta_0 + \beta_1 x_i).$$

(6)

Subsequently, the hazard function is
\[ h(t_i; x_i; \beta; \gamma) = e^{\beta_0 + \beta_1 x_i + \gamma t_i}, \]

the probability density function is
\[ f(t_i; x_i; \beta; \gamma) = e^{\beta_0 + \beta_1 x_i + \gamma t_i + \frac{e^{\beta_0 + \beta_1 x_i}}{\gamma} (1 - e^{\gamma t_i})}, \]

with the corresponding survivor function given by,
\[ S(t_i; x_i; \beta; \gamma) = e^{\frac{e^{\beta_0 + \beta_1 x_i}}{\gamma} (1 - e^{\gamma t_i})}. \]

### 2.2. The Gompertz regression model in the presence of right censored data

Suppose there is a random sample of size \( n \), let \( t_i \) denotes as failure time of the \( i^{th} \) observation and \( \delta_i \) its censoring indicator, then the general likelihood function of the model with uncensored (observed) and right censored data can be defined. Consequently, the likelihood function for the full sample consisting of uncensored and right censored data for \( i = 1, 2, \ldots, n \), is as follows,

\[
L(\beta; \gamma) = \prod_{i=1}^{n} \left[ f(t_i, x_i, \beta, \gamma) \right]^{\delta_{ri}} S(r_i, x_i, \beta, \gamma)^{1-\delta_{ri}}
\]

\[
= \prod_{i=1}^{n} \left[ \exp(\beta_0 + \beta_1 x_i + \gamma t_i + \frac{e^{\beta_0 + \beta_1 x_i}}{\gamma} (1 - e^{\gamma t_i})) \right]^{\delta_{ri}} \left[ \exp(\frac{e^{\beta_0 + \beta_1 x_i}}{\gamma} (1 - e^{\gamma t_i})) \right]^{1-\delta_{ri}}
\]

and log-likelihood function is,
\[
\ln[L(\beta; \gamma)] = \sum_{i=1}^{n} \delta_{ri} \left[ \beta_0 + \beta_1 x_i + \gamma t_i + \frac{e^{\beta_0 + \beta_1 x_i}}{\gamma} (1 - e^{\gamma t_i}) \right] + \sum_{i=1}^{n} \delta_{ri} \left[ \frac{e^{\beta_0 + \beta_1 x_i}}{\gamma} (1 - e^{\gamma t_i}) \right].
\]

The MLE method estimates the parameters of the model by maximizing the log-likelihood function. A simulation study using 1000 samples each with \( n = 30, 50, 80 \) and 100 was conducted for the Gompertz regression model with both censored and uncensored observations as well as fixed covariate, \( x_i \). The covariate values were simulated independently from the standard normal distribution. The values of \(-5, 0, 3 \) and 0.5 were selected as the parameters of \( \beta_0, \beta_1 \) and \( \gamma \) to mimic the real life survival data. A sequence of random numbers, \( u_i \)'s from the standard uniform distribution on the interval \((0,1)\) was simulated to produce lifetimes \( t_i \) for \( i = 1, 2, \ldots, n \) subjects. The censoring times, \( c_i \) were generated from the exponential distribution where the value \( \mu \) might be adjusted to obtain the desired approximate censoring proportion (cp) for the data with cp = 0\%, 10\%, 30\% and 50\%. The simulated survival time is considered censored if \( t_i > c_i \), and will be replaced by the corresponding censoring time. Censoring proportions means percent of data are right censored. Thus, the survival time \( t_i \) was generated by,
\[
t_i = \log \left[ \frac{\gamma}{\lambda_i} \right] \left[ 1 - \frac{\gamma \log(1 - u_i)}{\lambda_i} \right]^{\lambda_i}
\]
3. Assessing model adequacy

According to [11], Cox-Snell residuals, \( r_{Ci} \), is commonly used in the analysis of survival data as a model adequacy procedure. In this study, model adequacy was evaluated via graphical plot of residuals. A log-cumulative hazard plot of residuals can be obtained by plotting the Cox-Snell residual against the cumulative hazard function to assess the model’s fit. A well fit model should have an intercept that approaches to 0 and the slope as well as R-square approach to 1. One criticism of Cox-Snell residuals is that they do not account for censored observations, therefore the adjusted Cox-Snell residuals were devised by [12] whereby the standard Cox-Snell residual, \( r_{Ci} \), could be used for uncensored observations and \( r_{Ci} + \Delta \) which \( \Delta = \log(2) = 0.693 \), is used to adjust the residual. The Cox-Snell residuals for the \( i^{th} \) individual, \( i = 1,2,\ldots,n \) is given by,

\[
r_{Ci} = -\log (\hat{S}(t_i))
\]  

(13)

The modified Cox-Snell residuals has been proposed to account for censored data. [12] found that the addition of unity to a Cox-Snell residual for a censored observation inflated the residual to too great an extent. Hence, the median value was calculated for the excess residual. A second version of the modified Cox-Snell residual is,

\[
r_{MCi}^* = \begin{cases} r_{Ci} & \text{for uncensored observations,} \\ r_{Ci} + 0.693 & \text{for censored observations.} \end{cases}
\]

3.1. Modifications of Cox-Snell residuals

In this research, we proposed four modifications to the Cox-Snell residuals as follows,

\[
r_{GMCi}^* = \begin{cases} r_{Ci} & \text{for uncensored observations,} \\ r_{Ci} + G & \text{for censored observations.} \end{cases}
\]

and

\[
r_{HMCi}^* = \begin{cases} r_{Ci} & \text{for uncensored observations,} \\ r_{Ci} + H & \text{for censored observations.} \end{cases}
\]

where \( G \) is the geometric mean of residuals and \( H \) is the harmonic mean of residuals. In the next simulation study, jackknife techniques were used to compare the residual values of the geometric mean and harmonic mean. Hence, there are six type of residuals were applied in the next simulation study which are CS, MCS, GMCS, HMCS, JGMC and JHMCS residuals. Plot of \( \ln \left[ -\ln (\hat{S}(r_{Ci})) \right] \) against \( \ln(r_{Ci}) \) should exhibit a linear line through the origin with a unit gradient if the data fits the model well. Several modifications of the Cox-Snell residuals were used and compare the performance for the right censored and uncensored data.

a) Cox-Snell Residuals, \( r_{Ci} \).
b) Modified Cox-Snell Residuals, \( r_{MCi}^* \).
c) Replace the median with geometric mean of existing data, \( r_{GMCi}^* \).
d) Replace the median with harmonic mean of existing data, \( r_{HMCi}^* \).
e) Replace the median with jackknife geometric mean of existing data, \( r_{jGMCi}^* \).
f) Replace the median with jackknife harmonic mean of existing data, \( r_{jHMCi}^* \).

4. Simulation results and discussion

From table 1 and figure 1 below, the results from the simulation study clearly shows that the range of intercept decrease as the censoring proportion increase. Nevertheless, there are having an opposite trend
as number as sample sizes increases. This result will lead to conclusion that when the percentage of right censored data is decrease, then intercept values approach to 0. The results also applicable for all six residuals and we can conclude that HMCS residual outperforms than other residuals.

Table 1. Range of intercept for various residual values.

| n   | Censoring proportion (%) | CS min  | CS max  | MCS min  | MCS max  | GMCS min | GMCS max  | HMCS min | HMCS max  | JGMCS min | JGMCS max  | JHMCS min | JHMCS max  |
|-----|--------------------------|---------|---------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 30  | 0                        | -0.4418 | 0.7074  | -0.4418  | 0.7074   | -0.4418  | 0.7074   | -0.4418  | 0.7074   | -0.4418  | 0.7074   | -0.4418  | 0.7074   |
|     | 10                       | -0.5075 | 0.7060  | -0.4630  | 0.7010   | -0.4967  | 0.7010   | -0.4286  | 0.7010   | -0.5470  | 0.7060   | -0.5470  | 0.7060   |
|     | 30                       | -0.6698 | 0.4176  | -0.7965  | 0.1520   | -0.9295  | 0.2910   | -0.6826  | 0.4012   | -0.8840  | 0.1164   | -0.8833  | 0.1167   |
|     | 50                       | -0.9904 | 0.3390  | -1.6125  | -1.3089  | 0.0732   | -0.9541  | 0.3346   | -1.5457  | -0.2724  | -1.5470  | -0.2711  |
| 50  | 0                        | -0.3229 | 0.3258  | -0.3229  | 0.3258   | -0.3229  | 0.3258   | -0.3229  | 0.3258   | -0.3229  | 0.3258   | -0.3229  | 0.3258   |
|     | 10                       | -0.3811 | 0.3750  | -0.4010  | 0.2683   | -0.4036  | 0.3205   | -0.3831  | 0.3547   | -0.3932  | 0.2955   | -0.3932  | 0.2955   |
|     | 30                       | -0.4786 | 0.3439  | -0.6881  | -0.0032  | -0.5697  | 0.2297   | -0.4187  | 0.3352   | -0.7046  | 0.0438   | -0.7045  | 0.0439   |
|     | 50                       | -0.7155 | 0.4313  | -1.3256  | -0.3092  | -0.9783  | 0.2045   | -0.7328  | 0.4872   | -1.3284  | -0.1203  | -1.3282  | -0.1200  |
| 80  | 0                        | -0.3636 | 0.2742  | -0.3636  | 0.2742   | -0.3636  | 0.2742   | -0.3636  | 0.2742   | -0.3636  | 0.2742   | -0.3636  | 0.2742   |
|     | 10                       | -0.4561 | 0.2708  | -0.5124  | 0.1878   | -0.5157  | 0.2168   | -0.4644  | 0.2454   | -0.5365  | 0.1833   | -0.5365  | 0.1833   |
|     | 30                       | -0.6663 | 0.1709  | -0.9838  | -0.1443  | -0.8450  | 0.0309   | -0.6586  | 0.1571   | -1.0554  | -0.1378  | -1.0554  | -0.1378  |
|     | 50                       | -0.9487 | -0.0061 | -1.4077  | -0.4841  | -1.0746  | -0.2012  | -0.8848  | 0.0126   | -1.4461  | -0.4262  | -1.4460  | -0.4261  |
| 100 | 0                        | -0.2159 | 0.2431  | -0.2159  | 0.2431   | -0.2159  | 0.2431   | -0.2159  | 0.2431   | -0.2159  | 0.2431   | -0.2159  | 0.2431   |
|     | 10                       | -0.2246 | 0.2375  | -0.2717  | 0.2224   | -0.2767  | 0.2043   | -0.2509  | 0.2389   | -0.2747  | 0.2212   | -0.2747  | 0.2212   |
|     | 30                       | -0.3495 | 0.2852  | -0.5426  | 0.0114   | -0.4481  | 0.1574   | -0.4112  | 0.2890   | -0.7175  | -0.0250  | -0.7174  | -0.0250  |
|     | 50                       | -0.6776 | 0.1858  | -1.2115  | -0.2876  | -0.8877  | -0.0045  | -0.6826  | 0.1963   | -1.2509  | -0.2767  | -1.2509  | -0.2767  |

Figure 1. Comparison of residual values for estimated values of intercept.
The findings from Table 2 and Figure 2 demonstrate that the range of slope increase as the censoring proportion increase. The performance of the six residuals show that GMCS residual was found to perform best when sample sizes are large based on the range of slope values.

**Table 2. Range of slope for various residual values.**

| n    | censoring proportion (%) | CS min  | CS max  | MCS min  | MCS max  | GMCS min  | GMCS max  | HMCS min  | HMCS max  | JGMCS min  | JGMCS max  | JHMCS min  | JHMCS max  |
|------|--------------------------|---------|---------|----------|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 30   | 0                        | 0.4518  | 1.1671  | 0.4518   | 1.1671   | 0.4518    | 1.1671    | 0.4518    | 1.1671    | 0.4518    | 1.1671    | 0.4518    | 1.1671    |
|      | 10                       | 0.5004  | 1.1857  | 0.4125   | 1.1982   | 0.4060    | 1.1937    | 0.4797    | 1.2142    | 0.3965    | 1.1424    | 0.3965    | 1.1425    |
|      | 30                       | 0.5195  | 1.1286  | 0.3880   | 1.0927   | 0.4860    | 1.2133    | 0.5413    | 1.2576    | 0.3835    | 1.0648    | 0.3835    | 1.0649    |
|      | 50                       | 0.4962  | 1.3726  | 0.3310   | 1.5035   | 0.4995    | 1.5920    | 0.5439    | 1.3265    | 0.3701    | 1.4596    | 0.3701    | 1.4651    |
| 50   | 0                        | 0.5102  | 1.1759  | 0.5102   | 1.1759   | 0.5102    | 1.1759    | 0.5102    | 1.1759    | 0.5102    | 1.1759    | 0.5102    | 1.1759    |
|      | 10                       | 0.5306  | 1.1811  | 0.4826   | 1.1793   | 0.4864    | 1.1823    | 0.5300    | 1.2341    | 0.4834    | 1.1659    | 0.4834    | 1.1659    |
|      | 30                       | 0.5876  | 1.2282  | 0.4690   | 1.2008   | 0.5977    | 1.2129    | 0.5787    | 1.3205    | 0.4612    | 1.1433    | 0.4612    | 1.1433    |
|      | 50                       | 0.5468  | 1.3283  | 0.4198   | 1.3921   | 0.5140    | 1.4592    | 0.5528    | 1.3843    | 0.4099    | 1.3872    | 0.4099    | 1.3872    |
| 80   | 0                        | 0.6704  | 1.2662  | 0.6704   | 1.2662   | 0.6704    | 1.2662    | 0.6704    | 1.2662    | 0.6704    | 1.2662    | 0.6704    | 1.2662    |
|      | 10                       | 0.6694  | 1.3654  | 0.6605   | 1.3636   | 0.6621    | 1.3565    | 0.6746    | 1.3859    | 0.6561    | 1.3372    | 0.6561    | 1.3372    |
|      | 30                       | 0.6169  | 1.3387  | 0.6048   | 1.4363   | 0.6610    | 1.4442    | 0.6488    | 1.3769    | 0.5878    | 1.3518    | 0.5878    | 1.3518    |
|      | 50                       | 0.5852  | 1.4539  | 0.5488   | 1.6216   | 0.6672    | 1.6125    | 0.6554    | 1.4737    | 0.5422    | 1.5930    | 0.5423    | 1.5930    |
| 100  | 0                        | 0.6736  | 1.1872  | 0.6736   | 1.1872   | 0.6736    | 1.1872    | 0.6736    | 1.1872    | 0.6736    | 1.1872    | 0.6736    | 1.1872    |
|      | 10                       | 0.6842  | 1.2279  | 0.6613   | 1.2027   | 0.6640    | 1.2085    | 0.6873    | 1.2046    | 0.6562    | 1.1980    | 0.6562    | 1.1980    |
|      | 30                       | 0.6413  | 1.3726  | 0.6306   | 1.2044   | 0.6600    | 1.2683    | 0.6690    | 1.3475    | 0.6151    | 1.1249    | 0.6151    | 1.1249    |
|      | 50                       | 0.6317  | 1.3764  | 0.5700   | 1.3368   | 0.6886    | 1.4997    | 0.6472    | 1.4000    | 0.5755    | 1.4274    | 0.5755    | 1.4274    |

**Figure 2.** Comparison of residual values for estimated values of slope.
The residuals perform well in model diagnosis when sample size is large and censoring proportion is small. Overall, we can conclude that the proposed modification of the Cox Snell residual using jackknife harmonic mean and jackknife geometric mean outperform than the other residuals.

Table 3. R-square for various residual values.

| n     | censoring proportion (%) | CS    | MCS   | GMCS  | HMCS  | JGMCS | JHMCS |
|-------|--------------------------|-------|-------|-------|-------|-------|-------|
| 30    | 0                        | 0.6978| 0.6978| 0.6978| 0.6978| 0.9778| 0.9778|
|       | 10                       | 0.6179| 0.6153| 0.5769| 0.6294| 0.9738| 0.9738|
|       | 30                       | 0.7395| 0.5690| 0.4832| 0.7199| 0.9167| 0.9168|
|       | 50                       | 0.7359| 0.5241| 0.4826| 0.7404| 0.8920| 0.8877|
| 50    | 0                        | 0.7679| 0.7679| 0.7686| 0.7679| 0.9683| 0.9683|
|       | 20                       | 0.8111| 0.7424| 0.6878| 0.8083| 0.9687| 0.9687|
|       | 30                       | 0.8018| 0.6956| 0.6742| 0.8178| 0.9582| 0.9583|
|       | 50                       | 0.7956| 0.5830| 0.5852| 0.7923| 0.9070| 0.9072|
| 80    | 0                        | 0.7929| 0.7929| 0.7929| 0.7929| 0.9899| 0.9899|
|       | 10                       | 0.7912| 0.7738| 0.7733| 0.7865| 0.9887| 0.9887|
|       | 30                       | 0.7878| 0.7065| 0.7066| 0.7937| 0.9452| 0.9452|
|       | 50                       | 0.7389| 0.6440| 0.6217| 0.7614| 0.9185| 0.9186|
| 100   | 0                        | 0.9069| 0.9069| 0.9069| 0.9069| 0.9871| 0.9871|
|       | 10                       | 0.9000| 0.9016| 0.8885| 0.9000| 0.9829| 0.9829|
|       | 30                       | 0.9118| 0.8640| 0.8595| 0.9086| 0.9660| 0.9660|
|       | 50                       | 0.8768| 0.7520| 0.7700| 0.8792| 0.8737| 0.8737|

Figure 3. Comparison of residual values for R-square.
5. Conclusion

Based on the model adequacy study, we found that the jackknife geometric mean and jackknife harmonic mean residuals outperform than CS, MCS, GMCS and HMCS residuals. When the censoring proportions increase, range of simulated slope, intercept and R-square of the residuals will increase. We can conclude that higher number of sample sizes make intercept closer to 0 and slope closer to 1. While for the R-square values becomes wider as the censoring proportion increases. Based on results, we can see that proposed modification of the Cox-Snell residual using jackknife geometric mean and jackknife harmonic mean perform better than the other residuals.

References

We would like to thank the Putra Grant, Vot 9595300, Universiti Putra Malaysia for supporting this research project.

References

[1] Gompertz B 1825 On the nature of the function expressive of the law of human mortality and on a new mode of determining the value of life contingencies Philosophical Transactions of the Royal Society of London 115 pp 513-583
[2] Garg M L, Rao B R and Redmond C K 1970 Maximum-likelihood estimation of the parameters of the Gompertz survival function Journal of the Royal Statistical Society Series C (Applied Statistics) 19(2) pp 152-159
[3] Makany R 1991 A theoretical basis for Gompertz’s curve Biometrical Journal 33(1) pp 121–128
[4] Chen Z 1997 Parameter estimation of the Gompertz population Biometrical Journal 39(1) pp 117-124
[5] Wu J W, Hung WL and Tsai C H 2004 Estimation of the parameters of the Gompertz distribution using the least squares method Applied Mathematics and Computation 158(1) pp 133–147
[6] Lenart A 2012 The moments of the Gompertz distribution and maximum likelihood estimation of its parameters Scandinavian Actuarial Journal 2014(3) pp 255–277
[7] Kiani K, Arasan J and Midi H 2012 Interval estimations for parameters of gompertz model with time dependent covariate and right censored data Sains Malaysiana 41(4) pp 471-480
[8] Kiani K and Arasan J 2013 Gompertz model with time-dependent covariate in the presence of interval-, right- and left-censored data. Journal of Statistical Computation and Simulation 83(8) pp 1472-1490
[9] Dey S, Moala F A and Kumar D 2018 Statistical properties and different methods of estimation of Gompertz distribution with application Journal of Statistics and Management Systems 21(5) pp 0839–876
[10] Cox D R and Snell E J 1968 A general definition of residuals Journal of the Royal Statistical Society Series B (Methodological) 30(2) pp 248-275
[11] Collet D 2003 Modelling Survival Data in Medical Research Chapman and Hall/Crc; 2nd edition
[12] Crowley J and Hu M 1977 Covariance Analysis of Heart Transplant Survival Data Journal of the American Statistical Association 72(357) pp 27-36