Abstract

We study how to obtain a sufficiently flat inflaton potential that is natural from the particle physics point of view. Supersymmetry, which is broken during inflation, cannot protect the potential against non-renormalizable operators violating slow-roll. We are therefore led to consider models based on non-linearly realized symmetries. The basic scenario with a single four-dimensional pseudo Nambu Goldstone boson requires the spontaneous breaking scale to be above the Planck scale, which is beyond the range of validity of the field theory description, so that quantum gravity corrections are not under control. A nice way to obtain consistent models with large field values is to consider simple extensions in extra-dimensional setups. We also consider the minimal structures necessary to obtain purely four-dimensional models with spontaneous breaking scale below $M_P$; we show that they require an approximate symmetry that is supplemented by either the little-Higgs mechanism or supersymmetry to give trustworthy scenarios.

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I. INTRODUCTION

Inflation is surely the most compelling paradigm for solving many problems of the standard big bang cosmology $^{1,2,3}$. Besides its theoretical appeal, its basic predictions of a flat Universe with a nearly scale-invariant spectrum of adiabatic perturbations are now experimentally well tested by the Cosmic Microwave Background Radiation (CMBR) anisotropies and the Large Scale Structures (LSS) galaxy surveys.

The basic framework can be realized in models as simple as a single scalar field with a monomial potential. Although such simple toy-models can be attractive, they are tremendously unnatural from the particle physics point of view. In scenarios where the inflaton takes values above the Planck mass ($M_P = (8\pi G)^{-1/2}$) $^4$, the use of a simple potential requires the fine-tuning of an infinite number of non-renormalizable operators, suppressed by powers of $M_P$.

The inflaton potential must be sufficiently flat to allow a slow-rolling phase, but at the same time it must couple to other fields to provide an efficient reheating and, in hybrid models, to trigger the final phase transition. There are only two known candidates for keeping a scalar potential nearly flat and stable under radiative corrections: supersymmetry (SUSY) and non-linearly realized symmetries. The latter mechanism applies both to a pseudo Nambu-Goldstone boson (PNGB) and to the extra components of gauge fields propagating in extra dimensions; both are protected at lowest order by a shift symmetry.

So far most of the attention in inflation model-building has been devoted to supersymmetry, but this symmetry alone cannot naturally provide potentials that are flat enough for inflation, once supergravity effects are included. In the next section (section III), we will describe this problem and review the proposed solutions. None of them is completely compelling.

In section III we turn our attention to the other, much less studied, candidate: the shift symmetry. Even if the Goldstone theorem ready provides flat directions for inflation, it is not trivial to build inflationary models based on PNGBs, essentially because both the potential and its slope vanish in the limit in which the explicit breaking is turned off. The simplest scenario with a single PNGB does not work unless the symmetry breaking scale is higher than the Planck scale, which is presumably outside the range of validity of an effective field theory description. Moreover it is expected that quantum gravity effects will
explicitly break the global symmetry, giving a typical scale for the potential of order \( M_P \), far too big to satisfy the COBE constraint.

We show that these problems are not present in theories with extra dimensions. In particular, the extra components of gauge fields living in extra dimensions provide natural candidates for the inflaton [5].

In the rest of the paper we concentrate on the requirements for building purely 4d models with PNGBs, with a symmetry breaking scale below \( M_P \). This requires more complicated structures such as hybrid inflation models [6]. In section [V] we discuss the necessary ingredients for building natural 4d models. The PNGB potential of the inflaton needs protection from the interactions which are required to end inflation and to reheat the Universe. We present a SUSY model as well as non-SUSY models based on the same recent ideas which were used to build new models of electroweak symmetry breaking. Some of the details are left to the Appendix. We draw our conclusions in section [V].

II. HIGHER DIMENSION OPERATORS AND THE SUGRA \( \eta \) PROBLEM

As noted in the introduction, non-renormalizable operators are clearly very crucial in models of inflation in which the inflaton variation is bigger than the Planck scale, because they are naively more important than lower dimension operators. This makes it very hard to justify any 4d model with a big variation of field values. We will discuss in the next section how this problem can be solved in models in which the 4d effective field theory is the dimensional reduction of an higher dimensional theory.

Non-renormalizable operators are important also in theories where the inflaton variation is much smaller than \( M_P \). This is clear if one considers operators of dimension 6, which can give a mass term

\[
\frac{V}{M_P^2} \phi^2 \sim H^2 \phi^2
\]  

(1)

to the inflaton, spoiling slow-roll.

One would think that supersymmetry can provide flat directions for inflation in a rather natural way; however, it is known that this is not quite true once supergravity corrections are included [7]. Non minimal terms in the Kähler potential can obviously give contributions like (1), but the same kind of corrections are present also with a minimal Kähler potential. The supergravity potential, neglecting for the moment the D-term contribution, can be
expressed as a function of the Kähler potential $K$ and the holomorphic superpotential $W$ as
\[ V = e^{K/M_P^2} \left[ (K^{-1})^i_j L_i L_j - 3\frac{|W|^2}{M_P^2} \right], \] (2)
where $L_i \equiv W_i + K_i \cdot \frac{W}{M_P^2}$. During inflation supersymmetry is broken because the vacuum energy is positive. Taking, at the lowest order, a canonically normalized Kähler potential $K = \phi^* \phi$, the exponential factor in front of $V$ gives a mass to any flat direction of order $V/M_P^2 \sim H^2$. This point is quite clear in the superconformal formalism, where the kinetic term for $\phi$ can be expressed using a superconformal compensator $\Phi$ as
\[ \int d^2 \theta d^2 \bar{\theta} \Phi \Phi^\dagger \phi \phi^\dagger. \] (3)
As $\Phi \Phi^\dagger$ contains the Ricci scalar, we obtain a non-minimal coupling of $\phi$ to gravity which gives the mass correction during inflation.

This effect gives a tilt to the inflaton potential and it is simple to check that its contribution to the slow-roll $\eta$ parameter ($\eta \equiv M_P^2 V''/V$) is exactly 1, while a slow-roll phase requires $\eta \ll 1$. There can be additional contributions in the potential which are of the same order of magnitude and a cancellation is possible. Nevertheless this required cancellation introduces a fine-tuning problem, which is often referred to as the $\eta$-problem. The on-going experiments on the CMBR and on LSS are making the problem increasingly acute. A conservative limit on the spectral index is now $|n - 1| < 0.1$, which turns into a limit for $\eta$: $\eta < 0.05$. Unless a better reason for the cancellation is found, a fine-tuning of at least $1/20$ is required. The situation somewhat resembles the Higgs hierarchy problem: the top Yukawa and the gauge and quartic couplings would drive the Higgs mass towards the scale $\Lambda$ where new physics comes in, but a certain separation is required to account for electroweak precision tests. Here gravity itself drives the inflaton mass towards $H$, but again the two scales must be separated to allow a sufficient amount of inflation.

Several ways to overcome the problem have been proposed. Before reviewing them, we want to stress that none of them is entirely convincing. Most of them rely on assumptions about the fundamental theory, which cannot be justified from the effective low energy point of view. This is not better than assuming a certain cancellation among the various terms in (2). Below we discuss some proposed solutions to the $\eta$-problem. For additional references see \cite{8}.
Superpotential linear in the inflaton. It is easy to verify that in this case the contribution to $\eta$ coming from the exponential factor in (2) is canceled. However, one is left to assume a small quartic term $(\phi\phi^*)^2$ in $K$. Even if the situation might be considered better than in the general case, the fine-tuning problem is still there, as there is no symmetry which can protect the smallness of this term.

**Particular form of the Kähler potential.** If $T$ is a modulus (e.g., a compactification radius) and $\phi$ is the inflaton and the Kähler potential depends only on the combination $\rho \equiv (T + T^* - \phi^*\phi)$ (e.g., $K = -3 \log \rho$), a flat direction is preserved. This could be ensured by a so-called Heisenberg symmetry and it seems to be quite generic in orbifold compactifications of superstrings. The problem is that, during inflation, one also gets a runaway potential in the $\rho$ direction. It is hard to justify why a stabilization mechanism should depend on the $\rho$ variable and not on $T$ itself, as the Heisenberg symmetry is not a symmetry of the full theory. On the other hand, if $T$ is stabilized, corrections to the inflaton potential are reintroduced, giving $\eta \sim 1$. All this kind of solutions relies on particular features of the Kähler potential, which cannot be justified in terms of symmetries of the low energy theory, but must be taken from the UV stringy completion.

**D-term inflation.** In addition to the F-term potential (2), D-term contributions are also present:

$$V_D = \frac{g^2}{2} \text{Re} f^{-1}_{AB} D^A D^B, \quad D^A = K^i (T^A)^i_\phi^j + \xi^A,$$

where $f$ is the gauge kinetic function, $T^A$'s are the gauge group generators and $\xi$ is a Fayet-Iliopoulos (FI) term, which is admissible only for U(1) groups. If the vacuum energy during inflation is dominated by a D-term, the $\eta$-problem is simply not there. One can easily build a hybrid inflation model taking the inflaton to be a neutral superfield $S$, coupled to two charged multiplets $\phi_+$ and $\phi_-$ through a superpotential $W = \lambda S\phi_+\phi_-$. For large values of the scalar component of $S$, $\phi_+$ and $\phi_-$ are stuck at the origin, so that the gauge symmetry is unbroken and the vacuum energy is dominated by the FI term. For smaller values, the negatively charged scalar becomes tachyonic and we go to a vacuum where the U(1) is broken and the vacuum energy is zero. The $S$ direction is classically flat, but it is lifted by quantum corrections as supersymmetry is broken. The potential is generically flat enough to allow slow-roll and no $\eta$-problem seems to be present. However, after a closer look, it seems difficult to get a viable scenario of inflation both with an anomalous U(1) and with a non-anomalous one.
1. Anomalous U(1) with Green-Schwarz mechanism of anomaly cancellation \[11\]. As in this case the non-linear transformation of the dilaton cancels the anomaly, its behavior is clearly crucial: during inflation the dilaton gets a runaway potential and it must be stabilized. The stabilization mechanism generically gives F-term contributions bigger than D-terms \[12\], thus reintroducing the \(\eta\)-problem.

2. An explicit FI term for a non-anomalous U(1) is introduced. Supergravity requires this U(1) to be an R-symmetry \[13\]. This point is often overlooked in the literature about inflation.\(^1\) In the superconformal formalism it is easy to understand why the naive extension of the rigid FI term is not gauge invariant by itself. With a compensator chiral multiplet \(\Phi\), a rigid FI term would be promoted to \([\Phi \bar{\Phi} \xi V]_D\), which is no longer invariant under \(V \to V + \Lambda + \bar{\Lambda}\) since \([\Phi \bar{\Phi} \Lambda]_D\) and \([\Phi \bar{\Phi} \bar{\Lambda}]_D\) are non-zero. To write a gauge-invariant generalization of a FI term, \(\Phi\) must transform under the U(1) symmetry: a term of the form \([\Phi \Phi e^{\xi V}]_D\) will be invariant, provided the compensator undergoes a super-Weyl transformation \(\Phi \to e^{-\xi \Lambda} \Phi\), \(\bar{\Phi} \to e^{-\xi \bar{\Lambda}} \bar{\Phi}\). This implies that the U(1) symmetry must in fact be a gauged version of the U(1)\(_R\) symmetry: the gravitino must be charged.

The framework is now quite constrained: taking the normalization in which the charge of the gravitino is \(-1\), the vacuum energy during inflation is given by

\[
V_0 = \frac{g^2}{2} \xi^2 = \frac{g^2}{2} (2M_P^2)^2 ;
\]

where the FI term is fixed by the gravitino charge. The R-symmetry requires that the superpotential has charge \(+2\): as the scalar component of \(S\) must be neutral, \(\phi_+\) and \(\phi_-\) have charge \(2 + Q\) and \(-Q\) respectively. The classically flat direction \(S\) is lifted by quantum corrections and it gets a potential\(^2\):

\[
V_{1\text{-loop}} = \frac{g^2 \xi^2}{2} \left(1 + \frac{2 + 2Q + Q^2}{8\pi^2} g^2 \log \frac{\lambda^2 |S|^2}{\mu^2}\right),
\]

\(^1\) We thank M. Luty for stressing this point to us.

\(^2\) Note that the effective potential for \(S\) is different from the case \[10\] of a U(1) which is not an R-symmetry, because here the charges of \(\phi_+\) and \(\phi_-\) do not add up to zero and an additional term \(\propto |S|^2 \log(|S|^2/\mu^2)\) is induced. Anyway its contribution is at most comparable to the \(\log |S|^2\) piece, so that neglecting it does not alter the conclusions.
where $\mu$ is the renormalization scale. It is easy to obtain the constraint from the COBE normalization (see e.g. [8]), which is independent of the coupling constant $g$ and requires a huge charge $Q$:

$$\sqrt{\frac{\xi}{Q}} \simeq 10^{16}\text{GeV} \quad \Rightarrow \quad Q \sim 10^6 .$$

(7)

This is technically natural but quite unreasonable, especially because anomaly cancellation gives strong constraints on the spectrum [14, 15]. Even if we allow additional contributions to lift the inflaton potential, the vacuum energy (5) is clearly too big to satisfy the COBE normalization unless $g$ is very small.

In summary, due to the large vacuum energy during inflation, supersymmetry is badly broken in such a way that it lost the power to preserve the flat direction required by inflation. We need some other mechanism to obtain a natural flat potential for inflation.

III. SHIFT SYMMETRIES I: GENERAL DISCUSSION

If we want to explain the lightness of the inflaton from the low energy point of view, we must rely on symmetry arguments. Supersymmetry alone is insufficient, as we explained above, so that one is naturally led to consider approximate bosonic shift symmetries: i.e., the inflaton as a pseudo Nambu-Goldstone Boson. This is certainly not a new idea. Models of inflation based on PNGBs were discussed in [16, 17] and in many subsequent works$^3$. In this paper we want to emphasize that this seems to be the only natural way to keep the potential flat for a slow-roll inflation.

However, it is not straightforward to obtain satisfactory models. The original “natural inflation” model is based on a single PNGB field parametrized by an angular variable $\theta \sim \theta + 2\pi$. In the limit of exact symmetry $\theta$ is a flat direction. With the addition of an explicit breaking term the Lagrangian is of the form

$$L = \frac{1}{2} f^2 (\partial \theta)^2 - V_0 (1 - \cos \theta) ,$$

(8)

where $f$ is the spontaneous breaking scale. The canonically normalized field is $\phi = f \theta$, so the potential is naturally a function of $\phi/f$, which can be flat for large $f$. This scenario is

$^3$ An approximate symmetry has also been used to have light fields, different from the inflaton itself, during inflation. This is important for the curvaton scenario [18, 19] or for moduli fields [20].
however problematic, because the requirements $\epsilon \ll 1$ and $\eta \ll 1$ on the slow-roll parameters

$$
\epsilon \equiv \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 \sim \frac{M_P^2}{f^2}, \quad \eta \equiv \frac{M_P^2 V''}{V} \sim \frac{M_P^2}{f^2}
$$

(9)
gives $f \gg M_P$.

If we interpret $f$ as some symmetry breaking vacuum expectation value (VEV), then this would require that the field theory is valid above $M_P$, which is hard to justify. But the real problem is that we expect that quantum gravity effects, such as the virtual appearance of black holes, will explicitly break the approximate symmetry\footnote{Quantum gravity effects on a PNGB potential are known to be dangerous in the case of the axion. See e.g.\cite{21}.}. These effects, usually suppressed by powers of $f/M_P$, are here unsuppressed\footnote{Naive dimensional analysis suggests, in the limit of strong coupling gravity, that higher dimension operators arising from quantum gravity effects are suppressed by $M_P = (8\pi G)^{-1/2}$ and not by the alternative definition of the Planck mass $G^{-1/2}$.}, so that it is hard to justify why $V_0$ should be smaller than $M_P$, as required by the COBE bound on the overall normalization of density perturbations: $\delta \rho/\rho \sim 10^{-5}$. It is the same problem of higher dimension operators we discussed in the previous section: here the inflaton variation is bigger than $M_P$ so that non-renormalizable operators are important. Quantum gravity effects will induce higher-dimension operators which badly break the symmetry, changing the potential in (8). Therefore a single PNGB in a 4d field theory with the simple potential in (8) cannot provide a satisfactory model of inflation.

The situation is changed when we consider theories with extra dimensions\footnote{This is possible if the 4d effective field theory description comes from the dimensional reduction of a higher dimensional theory, it is possible to build models with variation of the inflaton field bigger than $M_P$, while keeping the effects of higher-dimension operators under control. Locality in extra dimensions can in fact prevent large corrections to the inflaton potential from quantum gravity effects. Consider a 5d model with the extra dimension compactified on a circle of radius $R$. The extra component $A_5$ of an abelian gauge field propagating in the bulk cannot have a local potential, due to the higher dimensional gauge invariance; a shift symmetry protects it similarly to what happens to a four-dimensional PNGB. A non-local potential as a function}. If the 4d effective field theory description comes from the dimensional reduction of a higher dimensional theory, it is possible to build models with variation of the inflaton field bigger than $M_P$, while keeping the effects of higher-dimension operators under control. Locality in extra dimensions can in fact prevent large corrections to the inflaton potential from quantum gravity effects.
of the gauge invariant Wilson loop

$$e^{i\theta} = e^{i \oint A_5 dx^5}$$

will however be generated in presence of charged fields in the bulk. At energies below $1/R$, \( \theta \) is a 4d field with a Lagrangian of the form

$$\mathcal{L} = \frac{1}{2 g_{4d}^2 (2\pi R)^2} (\partial \theta)^2 - V(\theta) + \cdots$$

where $g_{4d}^2 = g_5^2 / (2\pi R)$ is the 4D gauge coupling, and the potential $V(\theta)$ is given at one-loop by

$$V(\theta) = -\frac{1}{R^4} \sum_I (-1)^{F_I} \frac{3}{64\pi^6} \sum_{n=1}^{\infty} \frac{\cos(nq\theta)}{n^5},$$

where $F_I = 0(1)$ for massless bosonic (fermionic) fields of charge $q$ coupled to $A_5$. Note that the potential is of the same form as in natural inflation (with small corrections from additional terms in the sum), with the effective decay constant given by

$$f_{\text{eff}} = \frac{1}{2\pi g_{4d} R}.$$  

It is easily seen that $f_{\text{eff}}$ can be bigger than $M_P$ for sufficiently small $g_{4d}$; the slow-roll condition $f_{\text{eff}} \gg M_P$ requires only that

$$2\pi g_{4d} M_P R \ll 1.$$  

The canonically normalized field is $\phi = \theta f_{\text{eff}}$. Due to the higher dimensional nature of the model, the potential \( V(\theta) \) can be trusted even when the 4d field $\phi$ takes values above $M_P$; no dangerous higher-dimension operator can be generated in a local higher-dimensional theory. This conclusion is quite important as it is commonly believed that any inflation model with field values above $M_P$ cannot be justified from a particle physics point of view; we see that this conclusion is valid only if we restrict to purely 4d models. Quantum gravity corrections to the potential \( V(\theta) \) are negligible if the extra dimension is bigger than the Planck length, different from what is expected in a 4d PNGB model. Again locality in the extra space is the key feature; virtual black holes cannot spoil the gauge invariance and do not introduce a local potential for $A_5$, while non-local effects are exponentially suppressed by $\sim e^{-2\pi M_5 R}$, because the typical length scale of quantum gravity effects (the 5d Planck length $M_5^{-1}$) is much smaller than the size of the extra dimension.
It is worthwhile stressing that a variation of the inflaton field during inflation bigger than $M_P$ is required to have a significant and measurable production of gravitational waves. It seems that the only way to get a realistic scenario of this kind is in an extra-dimensional setup.

Another example using extra dimensions is the idea of “brane inflation”. Also this model can be considered based on a PNGB. In fact, the inflaton is the field which describes the distance between two branes. It is massless in the limit in which we neglect the interactions between the two branes, because it is the Goldstone boson of the broken translational invariance. The non-trivial potential generated by the interactions between the two branes has to be very flat when two branes are far apart, again by the locality in extra dimensions. From the 4d point of view the inflaton takes values above the Planck scale, but the extra dimensional completion allows to control higher-dimension operators. Moreover quantum gravity effects are again suppressed by locality, which is really the key ingredient of this type of models.

One could ask whether it is possible to derive a purely 4d theory from the simple 5d model based on the Wilson line by applying the recent idea of deconstructing dimensions, where the Wilson line in the extra dimension corresponds to a 4d PNGB. In this case one replaces the 5d gauge theory by a chain of 4d gauge groups, with the adjacent gauge groups connected by the link fields, which get nonzero VEVs and break the gauge groups down to the diagonal one. There is one linear combination of the Nambu-Goldstone bosons not eaten by the massive gauge fields. It remains light and corresponds to the non-local Wilson line field in the 5d case. However the symmetry breaking scale, $f_{\text{link}} = \sqrt{N} f_{\text{eff}}$, where $f_{\text{link}}$ is the VEV of the link fields and $N$ is the number of the sites, is still required to be bigger than $M_P$.

In the rest of the paper we concentrate on 4d models, by which we mean that there is no (gravitational) extra dimension with size larger than the Planck length and the theory is 4-dimensional all the way up to the 4d $M_P$. In this case we can not use the locality in extra dimensions to protect the flat inflaton potential and it is only sensible to consider $f \ll M_P$. As explained, this is not consistent with slow-roll in a scenario with the simplest PNGB potential, so that one is naturally led to consider models which involve more than one field. With $f \ll M_P$ the corrections to the inflaton potential due to quantum gravity effects can be sufficiently suppressed if the explicit symmetry breaking operators arising
from quantum gravity are prohibited up to a high dimension. Operators of dimension six contributing directly to the inflaton mass are still dangerous because their effect on the mass can be of order $V/M_P^2 \sim H^2$. We would like to emphasize that quantum gravity corrections crucially depend on the UV completion of each model below the Planck scale. There are many ways to suppress quantum gravity effects. Besides locality in the extra space discussed above, additional (discrete or continuous) gauge symmetries in the UV theory can forbid dangerous operators. For example, in a dimensionally deconstructed gauge theory with many sites, the PNGB is the product of many link variables, so that the only gauge invariant operators are of very high dimensions and the Planck scale effects are suppressed by many powers of $f/M_P$.

IV. SHIFT SYMMETRIES II: FIELD VALUES SMALLER THAN $M_P$

To obtain trustworthy 4d inflation models we must require the symmetry breaking scale $f$ and field values smaller than $M_P$ so that the simple potential (8) does not work. A more complicated PNGB potential is needed. In particular, the variation of the potential during inflation and the total height of the potential should be controlled by different terms with different scales. A sharp drop of the potential is therefore needed at some point to end inflation. However, such sharp drop in the potential explicitly breaks the shift symmetry of the inflaton field and may spoil the flatness of the inflaton potential through radiative corrections. We need to examine this point in more detail.

Let us first consider the case of single field inflation. Being a PNGB, its potential is periodic. To have a separation of scales inflation must occur near the maximum so that the potential is sufficiently flat, otherwise we come back to the requirement of natural inflation $f \gg M_P$. Near the maximum (chosen to be at $\psi = 0$) we can expand the potential

$$V(\psi) = V_0 - \frac{m^2}{2} \psi^2 - \frac{\lambda}{4} \psi^4 + \cdots.$$  

(15)

If $V_0 \sim m^2 f^2$, where $f$ is the symmetry breaking scale and is also the maximal variation of $\psi$, as in the case of (8), then again we are stuck with the troublesome relation $f \gg M_P$. Therefore, we must demand that the total potential near the maximum is dominated by the higher order terms, e.g.,

$$m^2 f^2 \ll |\lambda| f^4;$$

(16)
the potential is flattened near the maximum with respect to a conventional PNGB. On the other hand, a quadratically divergent contribution to the mass term will be generated by the quartic coupling,

\[ \Delta m^2 = -\frac{3\lambda \Lambda^2}{16\pi^2}. \]  

Comparing it with (16), we see that the cutoff \( \Lambda \) of (17) must be much smaller than the naively expected value \( 4\pi f \). In other words, there must be other fields with masses much below \( 4\pi f \) which cut off the quadratic divergence. This is possible if their interactions soften the symmetry breaking due to the quartic term. This requires some special structure of the theory which will be discussed in subsection IV A.

Another possibility is that the total vacuum energy during inflation is carried by another field as in hybrid inflation models [6]. The slow-roll field acts as a trigger of the phase transition of the other field. In this case there is a similar worry that the coupling between these fields, which is an explicit breaking contribution, can destroy the flatness of PNGB potential through radiative corrections. We will discuss this case in subsection IV B.

A. A model of single field inflation based on little Higgs theories

From the discussion above we see that a single field inflation with the field value smaller than \( M_P \) requires that the quadratic term is smaller than that naively induced by higher order terms. This is quite similar to what happens for the Higgs potential, so that we can use the same ideas of little Higgs theories [32, 34, 35, 36, 37, 38], recently proposed as a new solution to the hierarchy problem in electroweak symmetry breaking.

Let us first describe the general feature of the little Higgs theories. For specific models we refer the reader to the literature [32, 34, 35, 36, 37, 38]. A little Higgs model is based on a chiral Lagrangian from some spontaneously broken global symmetry. This symmetry is also explicitly broken by two (or more) sets of couplings. Each set of couplings preserves a different subset of the global symmetry under which the little Higgs is an exact Nambu-Goldstone boson. The little Higgs only learns its PNGB nature in the presence of both sets of couplings when the symmetry is completely broken. Therefore, there is no one-loop quadratically divergent contribution to the little Higgs mass. On the other hand, the quartic coupling can be generated at tree-level combining both sets of couplings. The potential for
all little Higgs models has the following form
\[ c_1 g_1^2 f^2 \left| \phi + i \frac{h^2}{f} \right|^2 + c_2 g_2^2 f^2 \left| \phi - i \frac{h^2}{f} \right|^2, \]  
(18)
where \( f \) is the symmetry breaking scale, \( g_1 \) and \( g_2 \) represent the two sets of couplings, \( c_1, c_2 \) are order 1 constants, \( h \) is the little Higgs, and \( \phi \) is a “fat” Higgs which receives large contributions to its mass both from \( g_1 \) and \( g_2 \) alone. The first term preserves a shift symmetry
\[ h \rightarrow h + \epsilon, \quad \phi \rightarrow \phi - \frac{2i \epsilon h}{f}, \]  
(19)
while the second one preserves a different symmetry,
\[ h \rightarrow h + \epsilon, \quad \phi \rightarrow \phi + \frac{2i \epsilon h}{f} ; \]  
(20)
each one forbids a mass term for \( h \). For \( c_1 g_1^2 + c_2 g_2^2 > 0 \), we can integrate out the heavy \( \phi \) field and obtain a quartic coupling for \( h \),
\[ \frac{4c_1 c_2 g_1^2 g_2^2}{c_1 g_1^2 + c_2 g_2^2} |h|^4 . \]  
(21)
In addition, the radiative contribution to \( h \) mass squared is
\[ \frac{c_3 g_1^2 g_2^2}{16\pi^2} f^2, \]  
(22)
with \( c_3 = \mathcal{O}(1) \). The coefficients \( c_1, c_2, c_3 \) can be either positive or negative depending on the model and the types of interactions. In little Higgs theories of electroweak symmetry breaking, one requires \( c_1, c_2 > 0 \) and \( c_3 < 0 \). To obtain a model for inflation we make a different choice, \( c_1 \cdot c_2 < 0, c_3 < 0 \), so that both the squared mass term and the quartic coupling are negative. In terms of the inflaton \( \psi \), which is assumed to be the real part of \( h \), \( \psi = \sqrt{2} \Re(h) \), the potential is
\[ V(\psi) = V_0 - \frac{m^2}{2} \psi^2 - \lambda \frac{\psi^4}{4}, \]  
(23)
where
\[ m^2 = \frac{|c_3| g_1^2 g_2^2}{16\pi^2} f^2, \quad \lambda = \frac{4c_1 c_2 g_1^2 g_2^2}{|c_1 g_1^2 + c_2 g_2^2|}, \quad V_0 \approx \lambda f^4. \]  
(24)
Generically one expects a cutoff of the order of \( 4\pi f \), therefore the quadratic term is suppressed by a loop factor with respect to that naively induced by the quartic coupling. The form of the inflaton potential is similar to the one proposed in Ref. 39, 40 in a gauge mediated SUSY breaking model (though the \( \eta \)-problem was not addressed there). Note that
the real potential is not unbounded from below because $\psi$ is a PNGB; the true minimum occurs at $\psi \sim f$.

The Universe is assumed to start near $\psi = 0$. In the beginning, when the $m^2\psi^2$ dominates
the tilt of the potential, the Universe undergoes slow-roll inflation. Inflation ends after $\psi$ grows and the $\lambda\psi^4$ term becomes dominant. To be specific, we will assume that $c_1 > 0$, $c_2 < 0$, and $|c_2|g_2^2 < c_1g_1^2$, then $\lambda \approx 4|c_2|g_2^2$. During inflation the slow-roll parameter $\eta$ is
given by

$$\eta = \frac{M_P^2V''}{V} \approx -\frac{g_1^2}{64\pi^2} \left| \frac{c_3}{c_2} \right| \frac{M_P^2}{f^2}.$$ (25)

The observational constraint $|\eta| < 1/20$ requires

$$\frac{f}{M_P} > \frac{g_1}{4\pi} \sqrt{\frac{5c_3}{c_2}}.$$ (26)

It can be satisfied with $f < M_P$ if $g_1 \lesssim \mathcal{O}(1)$. One can easily check that with this choice of parameters also the other slow-roll parameter $\epsilon$ is small. The number of e-foldings the Universe expands after the $\lambda\psi^4$ term dominates is roughly given by $|\eta|^{-1}$, so if we assume that the COBE scale occurs when the $m^2\psi^2$ term is still more important, $|\eta|$ can not be too small and should be close to the current limit. In the opposite limit only the quartic term is relevant during observable inflation. The tilt of the spectral index $n - 1 \approx 2\eta$ in this model is predicted to be negative. From the COBE normalization for curvature perturbation, we have

$$5.3 \times 10^{-4} = \frac{V^{3/2}}{M_P^3V} \approx |\eta|^{-3/2} \frac{m_\psi}{\psi} \approx |\eta|^{-3/2}\sqrt{\lambda},$$ (27)

where the last relation is obtained because the COBE scale should be near the point where the $m^2\psi^2$ term and the $\lambda\psi^4$ term are comparable. This requires

$$g_2 \approx 2.7 \times 10^{-3}|c_2|^{3/2} - 1/2.$$ (28)

The very small coupling $g_2$ can be seen as a weak point of this model, though it is natural in the ’t Hooft’s sense [41], because a larger symmetry is recovered in the limit $g_2 \to 0$. Another concern is whether the flat potential is preserved in the presence of quantum gravity effects. One expects that, in addition to the couplings $g_1$ and $g_2$, higher dimensional operators generated by quantum gravity effects may also break the global symmetry explicitly and give rise to a potential for the PNGB. These effects are suppressed by powers of $f/M_P$. Which higher dimensional operators can be generated depends on the specific little Higgs theory and its UV completion as discussed in the previous section.
B. Hybrid inflation models

In this subsection we consider hybrid inflation models with the inflaton being a PNGB\(^6\). In these models the slow-rolling field is protected by an approximate symmetry, while the vacuum energy is dominated by another (waterfall) field. The first field acts like a trigger on the other one: when a critical value is reached we are quickly driven to the true vacuum and inflation ends.

At first sight, however, this introduces another problem: how can the approximate symmetry protect the flatness of the potential without suppressing the coupling between the slow-rolling field and the other one? Another way of phrasing this problem is that since the coupling between the two fields breaks the global symmetry, one may worry that it generates a large potential for the inflaton and spoil slow-roll inflation.

Assuming that the slow-rolling inflaton \(\psi\) and the waterfall field \(\phi\) couple through the interaction

\[
\lambda \psi^2 \phi^2 ,
\]

this will generate a correction to the mass of the inflaton field,

\[
\Delta m^2_\psi \sim \frac{\lambda}{16\pi^2} \Lambda^2 ,
\]

where \(\Lambda\) is the cutoff of the integral. In order for \(\psi\) to act as a switch on the waterfall field, we need

\[
\lambda \psi^2_0 > |m^2_\phi| ,
\]

where \(\psi_0\) is the initial value of \(\psi\). This implies

\[
m^2_\psi > \frac{1}{16\pi^2} \frac{\Lambda^2 |m^2_\phi|}{\psi^2_0} .
\]

We can see that the cutoff \(\Lambda\) can not be too high. The hybrid inflation requires \(m^2_\psi \ll |m^2_\phi|\). It would not work with a naive cutoff \(\Lambda \sim 4\pi f\) expected in strong dynamics, which would yield \(m^2_\psi > |m^2_\phi|\).

Therefore, we need a much lower cutoff for the corrections to \(m^2_\psi\). The only known ways to have such a low cutoff are supersymmetry and little Higgs theories. In these cases, one

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\(^6\) Two-field inflation with a PNGB triggering a first order phase transition has been proposed\(^42\), but the problems we are going to discuss are not addressed.
may (at best) cut off the integral at $\Lambda^2 \sim |m_\phi^2|$, then we have

$$m_\psi^2 > \frac{1}{16\pi^2} \frac{m_\phi^4}{\psi_0^2}. \quad (33)$$

On the other hand, the current constraint on the slow-roll parameter $|\eta| < 0.05$ implies

$$m_\psi^2 < 0.05 \times \frac{V}{M^2_P}. \quad (34)$$

Comparing the above two equations, the requirement $\psi_0 \ll M_P$ implies $m_\phi^4 \ll V$; the waterfall field $\phi$ has to be light compared to the scale of the total vacuum energy it controls.

To get a natural small mass for $\phi$ again requires some symmetry reason. In contrast with the case of the slow-roll field, SUSY can protect the lightness of the waterfall field because we only need $|m_\phi| < V^{1/4}$, not $|m_\phi| \ll H$. Another possibility is that also the waterfall field is a PNGB, protected by a shift symmetry.

From these general arguments, we see that we are led to very specific structures for any natural hybrid 4d models of inflation, if all field values are required to be smaller than $M_P$.

Either we need both SUSY and PNGBs, or we need a little Higgs structure with all relevant fields being PNGBs. We present two examples to demonstrate it explicitly.

1. A SUSY model

The idea of SUSY hybrid inflation with the inflaton as a PNGB was discussed in Ref. [43, 44], in the context of non-Abelian discrete symmetries. To illustrate our point and to clarify the requirements, let us study a very simplified model. Consider the superpotential

$$W = \lambda_0 S (\psi_1^2 + \psi_2^2 - f^2) + \frac{\lambda_1}{2} \psi_1 \phi^2 + \lambda_2 X (\phi^2 - v^2), \quad (35)$$

with

$$\lambda_1^2 f^2 > 2\lambda_2^2 v^2. \quad (36)$$

The first term preserves a U(1) symmetry which is spontaneously broken. We can parametrize the flat directions as follows,

$$\psi_1 + i\psi_2 \equiv \sqrt{2}Q = (f + \sigma) e^{i\chi/f}, \quad \psi_1 - i\psi_2 \equiv \sqrt{2}Q = (f - \sigma) e^{-i\chi/f}, \quad (37)$$

where $\chi$ is the Nambu-Goldstone boson of the broken U(1) symmetry, and $\sigma$ is the other flat modulus due to SUSY. When SUSY is broken, $\sigma$ receives a potential and we assume
that it is stabilized at $\sigma = 0$. We will only consider the field $\chi$, which plays the role of the inflaton. For convenience, in the following we will also use $\psi_1$ and $\psi_2$ to simply represent their values along this direction,

$$\psi_1 = f \cos \left( \frac{\chi}{f} \right), \quad \psi_2 = f \sin \left( \frac{\chi}{f} \right).$$

The U(1) symmetry is also explicitly broken by the coupling $\lambda_1$. For the moment we assume that this is the only explicit breaking effect and that the Kähler potential preserves the U(1) symmetry up to corrections of order $\lambda_1^2/(16\pi^2)$. A potential is generated for $\chi$ due to the $\lambda_1$ coupling. We assume the initial condition of the early Universe to be $\chi \approx 0$; this forces $\phi = 0$ because this field receives a large mass from $\psi_1$. SUSY is broken by $F_X$ and the vacuum energy density is

$$V_0 \approx |F_X|^2 = \lambda_2^2 v^4.$$

There are two kinds of contributions which lift the potential of $\chi$. First, supergravity induces a soft SUSY breaking mass of order $H$ for every scalar ($\psi_1, \psi_2, \phi$). However, because $\chi$ is a PNGB, it only receives a potential due to the presence of the explicit breaking $\lambda_1$. The corresponding contribution is loop-suppressed,

$$m_{\chi}^2 \text{ (SUGRA)} \sim \frac{\lambda_1^2}{16\pi^2} 3H^2.$$

One can see that there is no SUGRA $\eta$-problem if $\lambda_1 \lesssim 1$

In addition to the corrections due to supergravity, there is a direct Yukawa mediated contribution through a $\phi$ loop, arising from the splitting of the spectrum of the $\phi$ supermultiplet due to $F_X$. The potential receives $\chi$ dependence at one loop,

$$V(\chi) \approx V_0 \left( 1 + \frac{\lambda_1^2}{4\pi^2} \ln \frac{\lambda_1 \psi_1}{\mu} \right) = V_0 \left( 1 + \frac{\lambda_1^2}{4\pi^2} \ln \frac{\lambda_1 \cos(\chi/f)}{\mu/f} \right).$$

The derivatives are easy to calculate,

$$V'(\chi) = -V_0 \frac{\lambda_1^2 \sin(\chi/f)}{4\pi^2 f \cos(\chi/f)} = -V_0 \frac{\lambda_1^2 \psi_2}{4\pi^2 f \psi_1},$$

$$V''(\chi) = -V_0 \frac{\lambda_1^2}{4\pi^2 f^2 \cos^2(\chi/f)} = -V_0 \frac{\lambda_1^2}{4\pi^2 \psi_1^2}.$$

We see that $\chi$ is rolling in the right direction ($0 \to \pi f/2$), and eq. \ref{eq:eta} agrees with eqs. \ref{eq:eta}, \ref{eq:eta} with the cutoff $\Lambda^2 \sim |m_\phi^2| = \lambda_2^2 v^2$. The slow roll parameter $\eta$ is now

$$\eta = M_P^2 \frac{V''}{V_0} = -\frac{\lambda_1^2}{4\pi^2} \frac{M_P^2}{\psi_1^2}. $$

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We have
\[ \lambda_2 = 2\pi \sqrt{|\eta|} \frac{\psi_1}{M_P}. \] (45)

Given the current constraint \(|\eta| < 0.05, \psi_1 \ll M_P\) requires \(\lambda_2 \ll 1\), which is equivalent to say that \(\phi\) has to be light. In the limit \(\lambda_2 \to 0\), the potential becomes flat because SUSY breaking vanishes. However, in this model there is no enhanced symmetry in the Lagrangian in the \(\lambda_2 \to 0\) limit. This is technically natural in SUSY theories though because of the non-renormalization theorem. For a non-SUSY theory the smallness of \(\lambda_2\) would be unstable against radiative corrections from the other interactions.

Let us examine the other constraints. We will assume that the Yukawa mediated contribution, eq. (41), dominates over the supergravity contributions, eq. (40). The number of e-folds of slow-roll inflation after a given epoch is
\[ N(\chi) \approx \int_{\chi_{\text{end}}}^{\chi} M_P^{-2} \frac{V}{V'} d\chi = \int_{\psi_{\text{end}}}^{\psi(\chi)} M_P^{-2} \frac{d\psi_1}{\lambda_2^2 M_P^2} = \frac{4\pi^2}{\lambda_2^2 M_P^2} \left( \frac{\psi_1^2}{2} - \frac{\psi_{\text{end}}^2}{2} \right) = \frac{1}{2|\eta(\chi)|} - \frac{1}{2|\eta_{\text{end}}|}, \] (46)

where the subscript “end” represents the end of inflation. Generally it is dominated by the first term. We obtain a prediction for the deviation of the spectral index from 1
\[ n - 1 \approx 2 \eta_{\text{COBE}} \approx -\frac{1}{N_{\text{COBE}}}, \] (47)

where the subscript “COBE” denotes the values corresponding to the scale of COBE measurement, and \(N_{\text{COBE}}\) is typically 40 – 60.

From the constraint on curvature perturbation measured by COBE,
\[ 5.3 \times 10^{-4} = \frac{V^{3/2}}{M_P^3 |V'|} \approx \frac{1}{M_P^3} \frac{\lambda_2^2 v^6}{\lambda_2^2 v^4 \frac{\lambda_1^2}{4\pi^2} \frac{\psi_2}{f\psi_1}} = \frac{2\pi v^2 f}{M_P^2} |\eta|^{-1/2} = \frac{2\pi v^2}{M_P^2 \sin(\chi/f)} |\eta|^{-1/2}. \] (48)

we obtain
\[ \frac{v^2}{M_P^2} \approx 8 \times 10^{-6} \sqrt{\frac{50}{N_{\text{COBE}}}} \sin \left( \frac{\chi_{\text{COBE}}}{f} \right) \Rightarrow \frac{v}{M_P} \approx 3 \times 10^{-3} \left( \frac{50}{N_{\text{COBE}}} \right)^{1/4} \sin^{1/2} \left( \frac{\chi_{\text{COBE}}}{f} \right). \] (49)

In this simple model we did not address why there is an approximate U(1) global symmetry, broken only by \(\lambda_1\). We left out many other possible terms which do not respect the U(1) symmetry. While it is technically natural in SUSY theories, it is not very well motivated. In addition, explicit breaking terms can also arise from quantum gravity effects. Our main point here is to demonstrate how an approximate shift-symmetry can protect a
flat direction from SUGRA corrections during inflation and the subtleties involved when one needs to couple the PNGB to other fields for the end of inflation. One can extend the model in such a way that a non-Abelian discrete symmetry gives rise to the approximate global symmetry as in [43, 44], and suppresses any dangerous symmetry breaking terms. Another natural way to obtain an accidental global symmetry is to exploit the locality in (deconstructed) extra dimensions, which we will study next. In particular, one can find natural hybrid models even without SUSY.

2. A 6d hybrid model and its 4d deconstruction

In this subsection we present a non-SUSY hybrid inflation model. As we argued earlier, to get such a model in 4d without SUSY, we need the little Higgs structure with both the inflaton field and the waterfall field being PNGBs. Little Higgs theories were first motivated from deconstructing extra-dimensional theories, where the PNGBs correspond to the extra components of gauge fields in extra dimensions [32, 34]. We will first discuss a 6d model where the extra components of the gauge field, \( A_5, A_6 \), play the roles of the inflaton and waterfall fields. It provides simple physics intuition and a clear picture of inflation dynamics. Later we will show that it can safely be deconstructed to purely 4d models.

We consider an SU(2) gauge theory in 4 ordinary infinite dimensions and 2 extra compact dimensions. We assume that the \( x_5 \) direction is compactified on a circle with radius \( R_5 \), and the \( x_6 \) direction is compactified on an \( S^1/Z_2 \) orbifold with radius \( R_6 \) (Fig. 1). Furthermore we assume that the orbifold projection breaks the SU(2) gauge symmetry down to U(1) at the orbifold fixed points \( x_6 = 0, \pi R_6 \). The assumed parities of the various gauge components under the \( Z_2 \) projection are shown in Table I. We see that in the 4d picture, the \( T^3 \)
TABLE I: The $Z_2$ parities of various gauge components.

|       | $A^a_{\mu}$ | $A^a_5$ | $A^a_6$ |
|-------|-------------|---------|---------|
| $T^{1,2}$ | $-$ | $-$ | $+$ |
| $T^3$    | $+$ | $+$ | $-$ |

Component of $A_5$ and $T^{1,2}$ components of $A_6$ have zero modes. They have a tree level potential from the commutator term in $F_{56}^2$,

$$\frac{1}{2} g^2 (A^3_5)^2 \left[ (A^1_6)^2 + (A^2_6)^2 \right],$$

(50)

where $g$ is the 4d gauge coupling. For simplicity, we used $A_5$, $A_6$ to represent the zero modes and we will omit the generator indices in the rest of the discussion. Note that the full theory is periodic under the transformations $A_5 \rightarrow A_5 + 1/(g R_5)$, $A_6 \rightarrow A_6 + 1/(g R_6)$, where the Kaluza-Klein modes simply shift by one unit.

At tree level, there are flat directions along $A_5$ (with $A_6 = 0$), and $A_6$ (with $A_5 = 0$). As discussed in section III, $A_{5,6}$ can not have local mass terms by gauge invariance. They can only get non-local contributions from Wilson lines and these contributions are indeed generated by radiative corrections, lifting the flat directions. One can imagine that at the origin ($A_5 = A_6 = 0$) the radiative corrections generate a positive squared mass for one direction, say $A_6$, and a (larger) negative mass squared for the other ($A_5$). This can be achieved if there are charged fermions living at the orbifold fixed lines, $x_6 = 0$ or $\pi R_6$. In this case $A_6$ can play the role of the slow-roll field and $A_5$ can be the waterfall field. The Universe with the initial condition $A_6 \neq 0$, $A_5 = 0$ will slowly roll to the origin until the squared mass of $A_5$ turns negative and $A_5$ jumps down to the true vacuum. To satisfy the slow-roll condition during inflation and to have a sufficiently fast waterfall process it is required that

$$m^2_{A_6} \ll H^2 \ll \left| m^2_{A_5} \right|,$$

(51)

which implies $R_6 \gg R_5$.

Similarly to the case of one extra dimension in sec. III, the potential can be computed for a given particle content, and is well known in the literature. A more detailed discussion of the parameters required for slow-roll inflation and all the other observational constraints can be found in the Appendix. The most important feature of this model is again that the locality in extra dimensions protects the flat potential for the inflaton (and also the mass of
the waterfall field) while still allowing significant couplings which trigger the waterfall phase transition and reheat the Universe after inflation.

From the constraints on the parameters discussed in the Appendix, one can check that the effective decay constants or symmetry breaking scales $1/(2\pi gR_5)$, $1/(2\pi gR_6)$ can be smaller than $M_P$ in this model. Therefore, a valid 4d model can easily be obtained by deconstruction. In fact, there is more freedom in the 4d deconstructed theories, since the various couplings are not required to be related as in the 6d theory by the higher dimensional gauge symmetry. The hierarchy between the scales of the inflaton field and the waterfall field can either come from the symmetry breaking scales or the couplings.

In the 4d picture, the inflaton and the waterfall fields are PNGBs, whose masses are protected by many approximate symmetries. Because in the limit in which one of them is restored the PNGB is exactly massless, its mass can be quite small even in the presence of a large coupling to additional fields. Let us start from two sites with an SU(2) symmetry on each site, with four link fields $X_i$, $i = 1, 2, 3, 4$ (see fig. 2), which transform as fundamentals under both SU(2)'s: the VEVs of the $X_i$ break SU(2) × SU(2) to the diagonal subgroup. To reproduced the orbifold projection we gauge the full SU(2) group on the first site but only the U(1) subgroup corresponding to the $T^3$ generator on the second site. We add to the potential two plaquette operators:

$$V = -\kappa_1 f^4 \text{Tr}(X_1 X_2^\dagger X_3 X_4^\dagger) - \kappa_2 f^4 \text{Tr}(X_2 X_3^\dagger X_4 X_1^\dagger).$$

(52)

Gauge fixing $X_1 = \mathbb{1}$, it is easy to find that the classically flat directions can be parameterized by $X_2$ and $X_4$, ($X_3 = X_2 X_4$), with the additional constraint $X_2 X_4 = X_4 X_2$, coming from the second plaquette [35,37]. In this way we have reproduced the commutator potential between $A_5$ and $A_6$. As only $T^3$ is gauged in the second site, we can add plaquette operators which include the projection $\Omega \equiv \text{diag}(1,-1)$ on this site to get rid of useless light states.
The operator
\[ -\kappa_3 f^4 \text{Tr}(X_1 \Omega X_2^\dagger X_3 \Omega X_4^\dagger) , \] (53)
forces \( X_4 \) to commute with \( \Omega \) so that only the neutral component survives, and
\[ -\kappa_4 f^4 \text{Tr}(X_1 \Omega X_3^\dagger X_4 \Omega X_2^\dagger) , \] (54)
keeps only the charged components of \( X_2 \). It is easy to verify that there are no one-loop quadratically divergent contributions which lift the flat directions \[35, 37\]: each of the plaquette interactions and gauge couplings respect a subgroup of the SU(2)^8 global symmetry of the link fields, leaving the PNGBs exactly massless. Only the combination of two sources of explicit breaking lift the flat directions, so that we have only logarithmic divergences.

As in the 6d model the charged scalars from \( X_2 \) take a positive mass squared, from gauge loops: \( m_2^2 \sim g^4/(16\pi^2)f^2 \). Instead we want that the neutral component of \( X_4 \) receives a tachyonic contribution to the potential: this can be done by introducing fermions in the theory, coupled to \( X_4 \). To avoid the presence of quadratic divergences, fermions must be introduced in a “delocalized way”, similarly to what one does to control the top loop corrections to the Higgs mass in little Higgs models. We get a contribution: \( m_4^2 \sim -\lambda^4/(16\pi^2)f^2 \), where \( \lambda \) is the analogue of the top Yukawa coupling. The required hierarchy can be obtained if \( \lambda \gg g \). Inflation can evolve as in the 6d model: we start away from the origin along the \( X_2 \) direction and we roll towards the origin, because of the positive mass contribution. For sufficiently big values of \( X_2 \), \( X_4 \) is stuck at the origin because of the commutator potential. When a critical point is reached, the potential of \( X_4 \) becomes unstable and the waterfall process starts, ending inflation.

The model is quite similar to the 6d one, except that there are differences due to couplings and volume factors, and to the fact that now gravity is 4-dimensional. As gravity is now 4-dimensional, one needs to worry about the corrections of the potential due to explicit symmetry breaking operators generated by quantum gravity effects (which were exponentially suppressed in “real” extra dimensions). For the simplest 2-site model, there are gauge invariant operators involving only two links. If one imagines that the links come from bilinear fermion condensates, they correspond to dimension-6 operators which are still dangerous as discussed before. These operators can be eliminated by additional (continuous
or discrete) gauge symmetries\(^7\). A deconstruction with more sites (gauge groups) so that the gauge invariant operators require more links is also sufficient to suppress quantum gravity effects \([33]\). As there is more freedom in the 4d deconstructed model, changing the model parameters we can have a very fast waterfall process along \(X_4\) as in the original hybrid inflation \([6]\) or a quite slow one as in the SUSY inspired “supernatural models” \([45]\).

In the above discussion we assumed that \(A_6 (X_2)\) is the slow-roll field and \(A_5 (X_4)\) is the waterfall field. One can also consider the opposite case where the roles of the two fields are reversed. In that case one requires \(m_{A_5}^2 > 0, m_{A_6}^2 < 0, \) and \(R_5 \gg R_6\). Cosmic strings will be generated at the end of inflation because \(U(1)\) is broken; this process can have interesting but complicated consequences \([46]\).

V. CONCLUSIONS

From the point of view of cosmology, inflation is definitely the most attractive scenario describing the very early Universe. On the other hand, from the particle physics point of view, the required inflaton potential is extremely unnatural and seems to need a lot of fine tuning. In this paper we examined in detail the physics ideas which may be used to naturally obtain a viable inflaton potential. We emphasize that SUSY, although being a popular paradigm for inflation models, can not adequately preserve the flat potential for inflaton by itself. The only natural way to obtain a flat potential for inflation is to incorporate some approximate shift symmetry. The examples are PNGBs and extra components of gauge fields living in extra dimensions. In purely 4d theories, the simplest model based on PNGBs has the difficulties that we need to extrapolate the field theory beyond its regime of validity as the symmetry breaking scale has to be greater than the Planck scale. To avoid this problem we need either extra dimensions or more complicated models in pure 4 dimensions as we discussed in sections \([\text{III}]\) and \([\text{IV}]\).

With extra dimensions, locality in the extra space allows to get a trustworthy potential even if the variation of the inflaton field is bigger than \(M_P\), with exponentially suppressed quantum gravity corrections \([3]\).

\(^7\) For example, if in the UV theory \(X_1, X_2, X_3, X_4\) carry additional gauge charges 1, 3, 7, 5, respectively, the lowest dimensional gauge invariant operators which break the global symmetry are the plaquette operators in \([42]\). Note that \(\text{Tr} X_i X_i^\dagger\) does not break the global symmetry.
On the other hand, purely 4d models require more sophisticated structures. The inflaton, besides being a PNGB, must have a potential with further protection from the potentially dangerous explicit symmetry breaking interactions which are required to end inflation. We conclude that supersymmetry or little Higgs structure is necessary for such protection. Discrete or continuous gauge symmetries are required in the UV completion of the 4d models below the Planck scale to control quantum gravity effects.

A generic prediction of the 4d models is that the contribution to the density perturbations from gravitational waves is unobservably small, because the field values are smaller than $M_P$. This has to be contrasted with extra-dimensional setups, where a significant production of gravitational waves is possible.

In hybrid models density perturbations produced during the final phase transition can give interesting phenomenological signatures. The prediction of the spectral index depend on the individual models. However, as it is quite difficult to preserve a flat direction during inflation, it seems quite generic that the spectral index $n$ should deviate from unity considerably. In our models the slow-roll parameters are small because of loop-factor suppressions, so that we do not expect them to be utterly small. Anyway, the same conclusion holds if the $\eta$-problem is solved just by a certain amount of fine-tuning. Therefore, a small deviation from scale-invariance seems to be a smoking gun for the inflationary paradigm itself. In some sense the inflaton mass cannot be too separated from the Hubble scale during inflation for the same reason we do not expect the Higgs mass to be very far from the scale of new physics (whatever it is), in which the SM is embedded.

Note added: As this work was completed ref. [47] appeared, where hybrid models based on PNGBs are discussed.

APPENDIX A: MORE DETAILS ABOUT THE 6 DIMENSIONAL HYBRID MODEL AND ITS DECONSTRUCTION

In this Appendix we present more detailed discussion of the 6d hybrid model and its 4d deconstruction in sec. IV B 2.

For the 6d model, the potential for the extra components of the gauge field consists of a
sum of cosine functions which are periodic in $A_5 \to A_5 + 1/(gR_5)$ and $A_6 \to A_6 + 1/(gR_6)$. Here we will just expand it around the origin to obtain the mass terms for $A_5$ and $A_6$ at the origin. To satisfy eq. (51) we have to assume that $R_5 \ll R_6$. The gauge loops give positive contributions to the squared mass for both $A_5$ and $A_6$. For $R_5 \ll R_6$ they are given by

$$m_{A_5}^2\text{(gauge)} \approx \frac{2g^2\zeta(4) R_6}{\pi^5} \frac{1}{R_5^2},$$  \hspace{1cm} (A1)$$

$$m_{A_6}^2\text{(gauge)} \approx \frac{3g^2\zeta(3)}{4\pi^4} \frac{1}{R_6^2}. \hspace{1cm} (A2)$$

To make $m_{A_5}^2$ negative, we can introduce fermions extended along the $x_5$ direction but localized in the $x_6$ direction, so that they only contribute to the mass of $A_5$. A natural choice is to have charged fermions living at the orbifold fixed lines, $x_6 = 0$ or $\pi R_6$, which preserves only the U(1) gauge symmetry (corresponding to $A^3$). Their contribution to $m_{A_5}^2$ is

$$m_{A_5}^2\text{(fermion)} \approx -\frac{3g^2\zeta(3)}{4\pi^4} \frac{1}{R_5^2} \sum_i 2Q_i^2,$$  \hspace{1cm} (A3)$$

where $Q_i$ is the U(1) charge of the fermion $i$. From the constraint (which will be discussed later) on the large density perturbations generated at the beginning of waterfall, $R_6/R_5$ is required to be $\gtrsim 20$. We can see that for somewhat large $Q_i$ or many fermions, $m_{A_5}^2$ can become negative. The exact values of $m_{A_5}^2$, $m_{A_6}^2$ depends on the field content. Nevertheless, they have to be cut off by $1/R_5^2$ and $1/R_6^2$ as they should vanish in the $R_{5,6} \to \infty$ limit. Below we will simply parametrize them by

$$m_{A_5}^2 = -\frac{c_5 g^2}{\pi^4} \frac{1}{R_5^2},$$  \hspace{1cm} (A4)$$

$$m_{A_6}^2 = \frac{c_6 g^2}{\pi^4} \frac{1}{R_6^2}, $$  \hspace{1cm} (A5)$$

where $c_5$ and $c_6$ are constants of order 1 or larger. Note that we require that the positive squared mass for $A_5$ from the tree-level potential eq. (50) to be larger than the negative radiative contribution in the beginning of inflation, i.e.,

$$|m_{A_5}^2| = \frac{c_5 g^2}{\pi^4 R_5^2} < g^2 A_6^2 \text{(initial)} \lesssim \frac{1}{4R_6^2} \Rightarrow R_5 > \frac{2\sqrt{c_5} g}{\pi^2} R_6. \hspace{1cm} (A6)$$

\textsuperscript{8} In fact, this contribution is localized at the orbifold fixed lines, which can mix KK modes in the $x_6$ direction. However, as we see later, this term is required to be smaller than the tree-level KK masses $n^2/R_n^2$, so we can treat it as a small perturbation without re-diagonalizing the mass eigenstates.
We can see that eqs. (A5), (A6) are consistent with eqs. (32), (33) as \( \Lambda^2 \sim 1/R_6^2 > |m_{A_5}^2| \).

During inflation, the vacuum energy is dominated by the \( A_5 \) potential,

\[
V \sim \frac{|m_{A_5}^2|}{g^2(2\pi R_5)^2} \sim \frac{c_5}{4\pi^6} \frac{1}{R_5^4},
\]

while the slope is determined by the \( A_6 \) potential,

\[
V' \sim \frac{m_{A_6}^2}{g(2\pi R_6)} \sim \frac{c_6 g}{2\pi^5} \frac{1}{R_6^3}, \quad V'' \sim m_{A_6}^2.
\]

There are several constraints for the parameters \( g, R_5, R_6 \). For the slow roll condition, we require

\[
\eta = M_P^2 \frac{V''}{V} \approx g^2(2\pi R_5)^2 M_P^2 \frac{c_6}{c_5} \frac{R_5^2}{R_6^2} \ll 1.
\]

From the COBE measurement of curvature perturbations, it is required that

\[
5.3 \times 10^{-4} = \frac{V^{3/2}}{M_P^3 V} \approx \eta^{-3/2} \frac{1/2}{\pi} g^2.
\]

Finally, as discussed in Ref. [45], in hybrid inflation models, large density perturbations will be generated during the period when both fields are light. A rough condition for a fast enough end of inflation, so that the large density perturbations do not fall inside the observable window is

\[
\frac{m_{A_5}}{H} \frac{m_{A_6}}{H} \approx \eta \sqrt{\frac{c_5}{c_6}} \frac{R_6}{R_5} \gtrsim 1.
\]

For some sample numbers, if we assume \( c_5, c_6 \sim 1 \), all constraints can be satisfied with

\[
\frac{R_6}{R_5} \sim 100, \quad g \sim 2 \times 10^{-3}, \quad 2\pi R_5 M_P \sim 10^4, \quad \eta \sim 0.03.
\]

One can check that the effective decay constants or symmetry breaking scales \( 1/(2\pi g R_5) \), \( 1/(2\pi g R_6) \) can be smaller than \( M_P \) in this model. Therefore, in this case it is possible to obtain valid 4d models by dimensionally deconstructing this model.

For 4d models there is more freedom in the choice of parameters as couplings are not related by higher dimensional gauge symmetry. For simplicity of discussion, we assume \( \kappa \sim g^2 \) for the couplings of the plaquette operators (52), where \( g \) is the SU(2) gauge coupling: in this way the commutator potential is \( \sim g^2 \) as happens in the 6d model. We also assume that the VEVs for \( X_i \)'s are equal \( \sim f \), and the required hierarchy between the scales of the slow-roll field and the waterfall field is generated by the couplings to fermions.
The charged scalars from $X_2$ have a positive mass squared from gauge loops: $m_2^2 \sim g^4/(16\pi^2)f^2$ (9). The neutral scalar of $X_4$, on the other hand, receives a negative contribution to its mass squared from couplings to fermions, $m_4^2 \sim -\lambda^4/(16\pi^2)f^2$, where $\lambda$ is the analogue of the top Yukawa coupling$^{10}$. As emphasized, the fermion couplings should be introduced in a “delocalized way,” i.e., preserving enough global symmetries, to avoid the quadratic divergences. The required hierarchy can be obtained if $\lambda \gg g$.

Inflation evolves as in the 6d model: we start away from the origin along the $X_2$ direction and we roll towards the origin. For sufficiently big values of $X_2$, $X_4$ is stuck at the origin because of the commutator potential. For this to happen, the positive contribution from the commutator potential must be able to overcome the negative one from the fermion couplings:

$$g^2 f^2 > \frac{\lambda^4}{16\pi^2} f^2 .$$

(A13)

The vacuum energy during the slow-roll is approximately constant and given by $V_0 \sim \frac{\lambda^4}{16\pi^2} f^4$. Before we get to the origin the mass along $X_4$ becomes negative and we end up to the true minimum with restored gauge symmetry. We can easily estimate the slow-roll parameters:

$$\epsilon \equiv \frac{M_P^2}{2} \left( \frac{V'}{V_0} \right)^2 \simeq \frac{M_P^2}{f^2} \left( \frac{g^4}{\lambda^4} \right)^2 ,$$

(A14)

$$\eta \equiv M_P^2 \frac{V''}{V_0} \simeq \frac{M_P^2 g^4}{f^2 \lambda^4} .$$

(A15)

To have slow-roll we must assume that

$$f \gg M_P \frac{g^2}{\lambda^2} .$$

(A16)

Note that, assuming a certain hierarchy between the coupling constant $g \ll \lambda$, we have slow roll, even if $f$ is smaller than the Planck scale, in constrast to what happens in natural inflation. This is possible in a hybrid model because the vacuum energy depends on $\lambda$,

\footnote{With the assumption $\kappa \sim g^2$, the combined contribution of the plaquette operators is comparable to the gauge term.}

\footnote{These 1-loop contributions are slightly enhanced with respect to the explicit UV operator, which can be estimated from the two loop quadratic divergence through naive dimensional analysis, by the logarithmic factor $\log(\Lambda^2/(gf)^2) \simeq 2 \log(4\pi/g)$. We will see in the following that a viable model of inflation requires a small $g$ so that this enhancement is quite consistent, while it is rather small in models of EWSB, where $g \sim 1$.}
while the slow-roll potential is lifted by $g$. Let us now look at the COBE normalization for the large scale perturbations CMBR

$$\left( \frac{V_0}{\epsilon} \right)^{1/4} \approx 0.027 \cdot M_P ; \quad (A17)$$

it gives the constraint

$$\frac{\lambda^3}{\sqrt{4\pi} \cdot g^2} \left( \frac{f}{M_P} \right)^{3/2} = \frac{g \cdot \eta^{-3/4}}{\sqrt{4\pi}} \approx 0.027 . \quad (A18)$$

Using the experimental limit $\eta < 1/20$, we get a rather strong constraint on $g$: $g \lesssim 0.01$.

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