Multiobjective design of porous air bearing using group inching fortification method

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Abstract
This study presents a new approach, group inching fortification (GIF) method, to deal with multiobjective optimization problems found in various mechanical designs. The GIF method is exemplified by a four-factor porous air bearing design. In the GIF method the initial group of designs in Pareto rank 1 is used as the basis to inch up the formation of Pareto solution set, which is fortified over the search process by uniting superior or non-dominated solutions from base-point exploration moves. In this study, a comparison of the GIF method with genetic algorithm (GA) and hyper-cube dividing method (HDM) for the same air bearing design is presented. The results show that the Pareto solution set obtained by the GIF method has more design selections with a wider coverage (breadth). Equally important, the number of objective-function calls required (179, 736, and 3600 for GIF, HDM, and GA, respectively) in the GIF method is significantly reduced. In this study, the GIF method is suggested to be terminated when all the designs are non-dominated by each other, a criterion which is very difficult to achieve by using the GA and HDM. This study proposes an effective design tool which is easy-to-implement for solving the problems with multiple objectives.

Key words: Porous air bearings, Multiobjective optimization, Group inching fortification method, Genetic algorithm, Hyper-cube dividing method, Pareto optimality

1. Introduction
For some mechanical design problems a pre-optimization parametric study is commonly conducted (e.g., Gherca, et al., 2013; Shen, et al., 2014). However, a parametric study does not correlate the design factors well for a given objective in the global point of view. For a single-objective optimization problem (SOOP), including the problems being formulated to have a weighted sum of multiple objectives, a single optimal solution is sought. In recent years, multifactor single-objective optimization has been carried out in many fluid-film lubrication studies (Wang, et al., 2000; Wang and Chang, 2002; Zhu and Bogy, 2002; Zhu and Bogy, 2004; Wang and Chen, 2004; Wang, et al., 2009a; Wang, et al., 2009b; Wang, et al., 2013; Senthil Kumar, et al., 2014) to obtain the optimum set of design variables. The weighted sum approach is also effectively applied to solve a bi-objective problem for the sequencing planning of mixed-model assembly line (Shimizu, et al., 2011). Among these studies, the various versions of dividing-rectangle (DIRECT) algorithm (Zhu and Bogy, 2002; Zhu and Bogy, 2004; Wang, et al., 2009) and the particle swarm optimization method (Wang, et al., 2013) have been demonstrated as practical and effective solvers for SOOOPs.

On the other hand, mechanical design of a component or system usually consists of more than one objective. Even in the case of a design where a single most important objective is desired, the minimization of product cost (a second objective) cannot be ignored in industrial applications. Thus, to effectively solve a problem of two or more objectives is of practical importance. Some studies show the results of recent effort in developing efficient schemes (Wang and Chang, 2004; Hirani and Suh, 2005; Wang, 2005; Bhat and Barrans, 2008; Wang and Cha, 2010; Lu and Xie, 2014) for solving multiobjective optimization problems (MOOPs). To minimize the execution time many of these optimization analyses were carried out by using parallel computing or approximating the computationally intensive function with a surrogate model (Li, et al., 2008; Srirat, et al., 2012).

The solution of an MOOP is usually a family of designs, the so-called Pareto optimality solution set or Pareto front. The strategy of solving MOOPs has been very actively developed, due to the problems’ complexity and practical necessity. A few pioneer works for solving multiobjective optimization problems using evolutionary or genetic
algorithms in engineering applications can be found in the literature (e.g., Deb, 2001; Coello Coello, et al., 2013; Knowles, et al., 2008; Branke, et al., 2008; Li, et al., 2008). Some recent MOOP solvers for tribological designs are adopted from modifying single-objective optimization algorithms, such as the multiobjective genetic algorithm (GA) (Wang and Chang, 2004; Hirani and Suh, 2005; Wang, 2005; Bhat and Barrans, 2008; Chiba, et al., 2014) and hyper-cube dividing method (HDM) (Wang and Cha, 2010).

Instead of using the fitness or merit of objective functions as the reference to advance the search, the multiobjective GA and HDM incorporate the Pareto ranking of the designs as the guide in the optimization process. Therefore, the commonly applied roulette-wheel selection of offspring in the single- or multi-objective GA is based on the fitness of the target function or the Pareto ranking of multiple objectives, respectively. Similarly, the selection of the design space to be divided for searching the potential optimum in the DIRECT algorithm (Zhu and Bogy, 2002; Zhu and Bogy, 2004; Jones, 2001) or HDM (Wang and Cha, 2010) is based on the merit of the objective function or the Pareto ranking, respectively. The difference among the population-based multiobjective optimization methods is that the strategy of generating new designs (points) in the subsequent epochs or generations.

This study proposes a simple and effective stochastic optimization algorithm, group inching fortification (GIF) method, for solving general MOOPs. In this study, a four-factor two- and three-objective optimum designs of a porous air bearing is conducted. The GIF method is formulated to obtain multiple Pareto solutions and is not based on a method for SOOPs. In the GIF method the initial group of designs in Pareto rank 1 is used as the basis to inch up the formation of Pareto front, which is fortified over the search process by uniting superior or non-dominated solutions from base-point exploration moves. In this study, a comparison of the GIF method with GA (Wang and Chang, 2004/) and HDM (Wang and Cha, 2010) for a two-objective optimization is conducted. A stopping criterion suitable for the GIF method in the optimization process is also proposed, which is important for effective search.

2. The porous air bearing model

In this study, the externally pressurized air bearing pad (Wang and Cha, 2010) has four square porous pads which are flush with the bearing surface (Fig. 1). In the cutaway view of Fig. 1, the pressure at boundary 1 is ambient pressure. The pressure at surface 2 is equal to the supply air pressure and the pressure gradient at surface 3, as well as the other three surfaces of the pad, is zero. The pressure gradient at boundary 4 is also zero due to symmetrically located pads. The pressure generated in the air film is obtained by solving iteratively the compressible-fluid Reynolds equation (Eq. (1)) and the Darcy’s Law (Eq. (2)).

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \left\{ \begin{array}{ll} \frac{12}{h^3} \frac{\partial}{\partial z} \left( \frac{\delta p}{\delta z} \right) & \text{in porous material region} \\ 0 & \text{otherwise} \end{array} \right. 
\]

\[
\frac{\partial^2 \hat{p}}{\partial x^2} + \frac{\partial^2 \hat{p}}{\partial y^2} + \frac{\partial^2 \hat{p}}{\partial z^2} = 0 
\]

where \( p \) and \( \hat{p} \) are the air pressure in the film and porous pad, respectively; \( x, y \) and \( \hat{x}, \hat{y}, \hat{z} \) are the coordinates for the film and porous pad, respectively; \( h \) is the film thickness; and \( k_p \) is the permeability coefficient of the porous pad.

Since the porous pads of the square bearing are symmetrically positioned around the bearing’s geometric center, the computation of pressure distribution is performed in a quarter of the bearing. In the computation, the air-film pressure equals the ambient pressure at the bearing border. The pressure gradient is zero at the bearing centerline interfaced with the other quarters of the bearing. In the air film and porous material interface, the film pressure equals the pressure on the porous pad surface. At the other open end of the porous pad, the pressure on the surface equals the supply air pressure. The pressure gradient of the other four surfaces of the porous pad was set to zero due to zero flow condition. The detail description of the solution procedure can be found in Ref. (Wang and Chang, 2004).
Fig. 1. The schematic of the air bearing with four-porous-pad (Wang and Cha, 2010).

3. Multiobjective optimization

A general minimization problem with multiple objectives can be stated as:

\[
\begin{align*}
\text{minimize} & \quad f_k(x) \quad \text{where} \quad k = 1, 2, \ldots, n, \quad x = [x_1, x_2, \ldots, x_m] \\
\text{subject to} & \quad x \in \Omega
\end{align*}
\]  

(3)

where \(f\) is objective function, \(x\) is design vector of size \(m\), \(m\) is the number of design variables, \(n\) is the number of objectives, and \(\Omega\) defines the domain of \(x\). In this study, the values of design variables are normalized in the range of \([0, 1]\). The Pareto optimality for identifying non-inferior solutions in MOOPs (Deb, 2001; Coello Coello, et al., 2013; Knowles, et al., 2008) can be defined as follows: The \(n\)-factor design vector \(\hat{x} \in \Omega\) is Pareto optimal for the \(m\) objectives if and only if there is no vector \(x \in \Omega\) with the characteristics

\[
\begin{align*}
& f_i(x) \leq f_i(\hat{x}) \quad \text{for} \quad i = 1, 2, \ldots, n \\
& f_i(x) < f_i(\hat{x}) \quad \text{for at least one} \quad i = 1, 2, \ldots, n
\end{align*}
\]  

(4)

(5)

Since the solution \(\hat{x}\) is no worse than \(x\) in all objectives (Eq. (4)) and \(\hat{x}\) is strictly better than \(x\) in at least one objective (Eq. (5)), it can be said that \(\hat{x}\) is non-dominated by \(x\) or \(\hat{x}\) is non-inferior to \(x\).

In this study, the minimization problem has two objectives (\(n = 2\)), in which \(f_i\) is the negative value of the bearing
stiffness and \( f_2 \) is the amount of air flow to the porous bearing. The four design variables \(( m = 4 )\) and their respective ranges are (1) supply air pressure, 200 to 600 kPa; (2) porous material width, \( L_p \), 5 to 15 mm; (3) porous material to pad edge distance, \( L_e \), 5 to 25 mm; and (4) permeability of porous material, \( k_p \), 2 to \( 10 \times 10^{-15} \) m\(^2\). To generalize the procedure in the developed optimization method the values of design variables are normalized to the range of [0, 1]. In the computer model each side of the square bearing pad has a length of 100 mm, the film thickness of the porous air bearing is 10 \( \mu \)m and the thickness of the porous material is 5 mm.

In an MOOP the main bottleneck in the solution procedure is in calculating the objective functions. The execution time spent on an optimization algorithm is usually trivial. Also, the execution time is affected by many factors in a simulation, such as coding, compiler, and hardware. In this study, therefore, the performance comparison of optimization algorithms is based on the number of objective-function calls required, instead of the execution time of computer programs.

4. Group inching fortification (GIF) method

A multiobjective optimization solver is usually based on a global optimizer for single-objective optimization, which performs global search to ensure that potentially optimums in the design space are not overlooked. On the other hand, some effort is also placed on local search near the current best solution, in the design space as well, for an efficient exploration. The main strategy of the GIF method is to inch up to a better Pareto solution set, based on the initial or current group of rank 1 points, in the criterion space. Thus, the search of the optimal Pareto front can be much more effective with a wider coverage (breadth) solution set in the criterion space. The step by step procedure of the GIF method is described below:

1. Create an initial population of \( m \) members (designs). Normally, the initial points (designs) are randomly generated in the design space.
2. Perform the Pareto analysis to setup the ranking of the designs.
3. If all the designs are in rank 1 (no one design is dominated by any other designs), the search of the Pareto solutions is terminated. Otherwise, the exploration for a better design is conducted, which based on the points in the current group of rank 1.
4. For each point in rank 1 a random exploration move of the point is tested, i.e.

\[
\hat{x}_{i}^{k+1} = \hat{x}_{i}^{k} + r \Delta x, \; i = 1, 2, ..., n_p
\]  

(6)

The \( \hat{x} \) is a design in rank 1, \( n_p \) is the total number of the points in rank 1, \( k \) is the epoch index, \( r \) is a random number having a value within [-0.5, 0.5], and \( \Delta x \) is a fixed exploration step to be specified. To prevent each normalized design variable exceeding the range of [0, 1], the operations of \( \hat{x}_{i}^{k+1} = \text{max}(0, \hat{x}_{i}^{k+1}) \) and \( \hat{x}_{i}^{k+1} = \text{min}(1, \hat{x}_{i}^{k+1}) \) are carried out after Eq. (6) is executed.

5. The movement of a new point (step 4) is rejected if the new design is strictly worse (for minimization, i.e. all the objective functions of the new design are having a respective larger value) than the original design. The movement is allowed when the new design is strictly better (all the objective functions of the new design are having a respective smaller value) than the original design. Otherwise, a point in the lowest ranking group is replaced by the new point (a potentially optimal point). In the latter case, the Pareto front is either increased in coverage (by adding an additional point in the edges of the current Pareto front) or is gaining an additional non-dominated point.

6. Repeat steps (4) and (5) for all the points in rank 1 or until all the points of rank 2 or lower are being replaced. Go to step (2).

Note that the stopping criterion of the GIF method is defined in step (3). Figure 2 shows the flowchart of the GIF method proposed in this study. To illustrate the above procedure a small size of population for the porous air bearing optimization is conducted. The goal was to maximize the bearing stiffness (i.e., minimizing the negative value of the stiffness) while minimizing the air flow to the bearing. The four design variables are pressure of supply air, width of square porous pad, distance of porous pad edge to bearing edge, and permeability of the porous material. A step by step illustration of the search procedure of the GIF method is shown in Figs. 3a to 3d.
Fig. 2 The flowchart of the GIF method for multiobjective optimization

Fig. 3 A simple two-objective four-variable porous air bearing optimization using the GIF method.

In this study, the stiffness and air mass flow of the bearing are normalized by dividing 400 N/μm and \(5 \times 10^{-5}\) g/min, respectively. Figure 3a shows the initial randomly-selected 10 points in the criterion space. The ranking is obtained by applying the Pareto criterion. To proceed the two-objective optimization analysis the five points of rank 1 in
Fig. 3a conduct the exploration move (Step (4)) and three new rank 1 points (points 4, 5, and 8 in Fig. 3b) are generated. At the same time the point 1 of rank 3, and points 3 and 4 of rank 2 (Fig. 3a) are eliminated (Step (5)). Note that the numbering of a point in rank 1 of an epoch may be different from a previous epoch due to additional points are added. Repeating Step 5, the points 1 and 2 of rank 2 (Fig. 3b) are eliminated due to the exploration move of rank 1 points in Fig. 3b. A Pareto ranking process then follows. The two new non-dominated points are points 3 and 7 of rank 1 in Fig. 3c. The only rank 2 point (Fig. 3c) was the point 6 of rank 1 in Fig. 2h, which is now dominated by point 7 in Fig. 3c. In the last epoch (Fig. 3d) the rank 2 point in Fig. 3c is eliminated and the point 5 is generated. A new sequencing of the points is then processed. The optimization process is terminated at this stage due to all the points are in rank 1.

An application of the GIF method for a two-objective optimization problem of nonconvex type (in criterion space) is also conducted. The goal is to maximize the two functions, \( f_1 \) and \( f_2 \) (Eqs. (7) and (8)), simultaneously. The search of the Pareto front is carried out by using the proposed GIF method. In this case, the population size is 60 and the search step is 10.0. The initial randomly distributed points (designs) as well as the results at epochs 5, 10, and 18 are shown in Figs. 4a to 4d, respectively. The nonconvex criterion space of the problem can be visualized by the randomly selected 600 designs in Fig. 4d. It can be seen that the points move to the Pareto front effectively from initial state to epoch 10 (Figs. 4a to 4c). Additional eight epochs are required to have all the points in the Pareto front for this highly nonlinear nonconvex problem. The final Pareto front determined by the GIF method has more design selections and a wider coverage than those found by using the 600 randomly selected points (Fig. 4d).

\[
\begin{align*}
  f_1 &= 1 + (\phi_1(1, 2) - \phi_1(x_1, x_2))^2 + (\phi_2(1, 2) - \phi_2(x_1, x_2))^2 \quad (7) \\
  f_2 &= (x_1 + 3)^2 + (x_2 + 1)^2 \quad (8)
\end{align*}
\]

in which \( \phi_1(x_1, x_2) = 0.5 \sin(x_1) - 2 \cos(x_1) + \sin(x_2) - 1.5 \cos(x_2) \), \( \phi_2(x_1, x_2) = 1.5 \sin(x_1) - \cos(x_1) + 2 \sin(x_2) - 0.5 \cos(x_2) \), and \(-\pi \leq x_1, x_2 \leq \pi\).

![Fig. 4](image_url)

Fig. 4 An application of the GIF method for a two-objective optimization problem of nonconvex type.

5. Results
Figure 5a shows the initial state of the 60 designs in the criterion space. The search step size is 1.0. These designs are obtained by random selecting the values of the variables in the design space. The ranking of the designs is computed according to the criterion of Pareto optimality. Figures 5b to 5d show the search progress of the GIF method. It can be seen that the lower ranking designs are being replaced and a better Pareto front is being shaped in the search process. Figures 6a to 6d illustrate the search results of the next four epochs after Fig. 5d. Due to the random exploration of the rank 1 points in each epoch, the chance of obtaining a superior or non-dominated design becomes small when most of the points are near the optimal Pareto solutions (for instance, Figs. 6c to 6d). The final result of the Pareto front (step size of 1.0) for the porous air bearing optimization analysis is shown in Fig. 7.

The Pareto solution sets of using step size of 0.5 and 0.75 are also shown in Fig. 7. The numbers of function call required for the step size of 0.5, 0.75, and 1.0 are 189, 201, and 179, respectively. It is noted that the final Pareto front of step size of 1.0 is resemble to some result obtained in an earlier epoch, such as epoch 3 (Fig. 5d). This demonstrates the marked efficiency of the GIF method proposed in this study. A test of using population of 120 is also conducted (Fig. 8). The number of required objective-function calls is increased significantly (the corresponding numbers are 470, 481, and 498 for step size 0.5, 0.75, and 1.0, respectively), as any population based algorithm, when comparing with the cases of using population of 60.

![Graphs showing search progress of GIF method](image-url)

Fig. 5 The first four search steps (epochs 0 to 3) of the porous air bearing optimization analysis using the GIF method.
Fig. 6 The epochs 4 to 7 of the porous air bearing optimization analysis using the GIF method.

Fig. 7 The effect of step size of exploration on the search efficiency and the performance for the porous air bearing optimization analysis using the GIF method (population size is 60).
Fig. 8 The effect of step size of exploration on the search efficiency and the performance for the porous air bearing optimization analysis using the GIF method (population size is 120).

Figure 9 shows the effect of limiting the distance (in criterion space) of the newly generated rank 1 points for the porous air bearing optimization analysis. The population size is 60. The minimum allowable distance in the criterion space is $\sqrt{2}/46 \approx 0.0307$, in which the number of rank 1 points is 46 and an estimation of the overall length of the Pareto front is $\sqrt{2}$. In this case, the number of the required function calls is much higher than the cases without limiting the distribution distance (e.g., Fig. 7). Since the replacement of the lower ranking points has a higher rejection rate (meeting the allowable distance) some of the points are not yet in rank 1 in the 15th epoch (Fig. 9).

Fig. 9 The result of limiting the newly generated points of rank 1 in distribution distance along the Pareto front (in criterion space) for the porous air bearing optimization analysis.
A test of the population size ranges from 30 to 120 is also performed and the result is shown in Fig. 10. It can be seen that the number of the required function calls is increased significantly when the size of population is large. Nevertheless, a simulation with a large population will have a better Pareto solution set. The expense is a higher computational cost. In this study, the result obtained by using a population size of 30 is not acceptable due mainly to a small size of population, thus, the search is terminated before the points of rank 1 gaining a better Pareto solution set.

Finally, the Pareto curves of rank 1 obtained by the GA, HDM, and GIF method for the two-objective four-factor MOOP are compared. In the GA (Wang and Chang, 2004), each variable is decoded in 12 binary bits; the selection scheme of reproduction is roulette wheel selection; single-point crossover is applied; the bit mutation probability is 1.0%; the population size is 60 and the simulation evolves to 60 generations. In the HDM (Wang and Cha (16)), the selection of the hyper-cubes to be divided is based on (1) the size of the hyper-cube and (2) the Pareto rank of the hyper-cube in the same size group. Only one hyper-cube is selected in each size group, which has the lowest Pareto ranking.

It can be seen that in Fig. 11 the resultant Pareto front obtained by the GIF method (population size = 60, step size = 1.0) is considerably better than the GA and HDM when bearing stiffness is large. If in the final decision of a design the weighting of bearing stiffness is much larger than the amount of air flow, the GIF method can provide a much better solution. It is also found that the number of objective-function calls required (179, 736, and 3600 for GIF, HDM, and GA, respectively) in the GIF method is significantly reduced.

Fig. 10 The effect of population size on the search efficiency and Pareto front.
Fig. 11 The Pareto fronts obtained by the GIF method, HDM and GA for the porous air bearing optimization analysis.

In addition to maximize the bearing stiffness while minimizing the amount of supply air (two-objective), the GIF method can be applied to maximize the bearing load capacity simultaneously. The resultant min-max solutions (Deb, 2001) of the three-objective optimization, as well as the two-objective GIF case of Fig. 11, are shown in Table 1. The step size is 1.0 for the both cases and the numbers of function call required for the two- and three-objective optimizations are 179 and 135, respectively. The number of function calls required for the three-objective optimization is less than that of two-objective case. This is due to the designs points are easier to reach non-dominated solutions in the criterion space of three-dimensional (Pareto surface) than two-dimensional (Pareto curve). The solution can be improved in the three-objective case if more exploration moves are allowed. Note that the load capacity is not the targeted objective of the two-objective optimization case, which is displayed for comparison with the three-objective optimization test. At the expense of reduced stiffness and increased supply air, as expected, the load capacity is increased as compared with the two-objective case when the load capacity is one of the optimization objectives.

Table 1 Comparison of the min-max solutions of the two- and three-objective optimizations using the GIF method

| No. of objectives | Supply pressure ($10^3$ N/m$^2$) | Porous pad | Bearing | Supply air ($10^3$ g/min) | Bearing load capacity (N) |
|-------------------|-----------------------------------|------------|---------|---------------------------|--------------------------|
|                   | Width (mm) | Location (mm) | Permeability ($10^{-15}$ m$^2$) | stiffness (N/mm) | |
| 2                 | 477.5      | 7.66        | 25.0   | 10.0          | 313.7 | 0.0213 | (2488) |
| 3                 | 445.9      | 15.0        | 25.0   | 10.0          | 286.5 | 0.0443 | 2688 |

Conclusions

A new multiobjective optimization method is developed and tested in a study for a porous air bearing design. The performance of the GIF method is compared with two multiobjective tribological studies. A three-objective bearing...
design is also conducted by using the GIF method. The potential of the GIF method is demonstrated and the characteristics of the method are detailed in this study. The GIF method is a straightforward algorithm and very few operating parameters are required to adjust. The GIF method is terminated when all the designs are in rank 1 or in Pareto front. This stopping criterion is very difficult to be implemented by the GA and HDM, which can maximize the search efficiency without taking the risk of under or over iteration in the optimization process. Also, this stopping criterion does not require a priori knowledge of the criterion (or objective-function) space.

The comparison of various methods presented in this study is based on the coverage of the Pareto front as well as the number of objective-function computations required. A fair comparison for various optimization methods is always a challenging task. The difficulty comes from the fact that the Pareto solutions are not the same for various methods, in terms of min-max solution and the shape of Pareto front. Other than the number of the objective-function calculations required, the result of the GIF optimization for the porous air bearing design has much wider coverage of Pareto front. Also, at the end of the simulations (Fig. 11) all of the designs (60 for both the GIF method and GA) of the GIF method are of Pareto rank 1. While the result of the GA has 52 designs in Pareto rank 1. Therefore, to reach a similar Pareto front coverage the GA will require more computational time and some parameters tuning. The HDM uses a different approach, in which 221 design points are obtained (Fig. 11), but with a narrower Pareto front coverage.

In this study, the Pareto front in the ninth epoch of the GIF method has broader solution selections (Fig. 11) than those obtained by the GA and HDM. Therefore, if a larger bearing stiffness is preferred in a final decision, the solution obtained by the GIF method can provide a better design than by the other two methods. If no preference is set, however, the most popular criterion to select one best compromise Pareto solution is the min-max method from the non-dominated solutions.

A uniform distribution of Pareto solutions in criterion space is possible (Fig. 9) by using the GIF method. However, the expense is a higher computational cost. It is also shown that a small size of population in using GIF method may not be able to provide a reasonably good Pareto solution set (Fig. 10, a population size of 30). Nevertheless, due to the effective search lead by the rank 1 points the GIF method can sustain a large amount of population (Fig. 8, a population size of 120). Alternatively, the search for the optimal Pareto front may pass beyond the stopping condition (all point are in rank 1) as indicated in the three-objective optimization case. This can be accomplished by allowing the search to continue, until a pre-specified number of epochs is reached. In this case, only an absolutely better design is allowed to substitute a poor one. This requires an extended computational resource, however.

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References

Bhat, N. and Barrans, S.M., Design and test of a Pareto optimal flat pad aerostatic bearing, Tribology International, Vol. 41 (2008), pp. 181-188.
Branke, J., Deb, K., Miettinnen, K. and Slowinski, R., Multiobjective Optimization: Interactive and Evolutionary Approaches (2008), Springer: New York.
Chiba, K., Kanazaki, M., Nakamiya, M., Kitagawa, K. and Shimada, T., Diversity of design knowledge for launch vehicle in view of fuels on hybrid rocket engine, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 8, No. 3 (2014), Paper No. 14-00001, pp. 1-14.
Coello Coello, C. A., Van Veldhuizen, D. A. and Lamont, G. B., Evolutionary Algorithms for Solving Multiobjective Problems (2013), 2nd Ed., Springer: New York.
Deb, K., Multiobjective Optimization Using Evolutionary Algorithms (2001), John Wiley & Sons: West Sussex.
Gherca, A. R., Maspeyrot, P., Hajjam, M. and Fatu, A., Influence of texture geometry on the hydrodynamic performance of parallel bearings, Tribology Transactions, Vol. 56 (2013), pp. 321-332.
Hirani, H. and Suh, N.P., Journal bearing design using multiobjective genetic algorithm and axiomatic design approaches, Tribology International, Vol. 38 (2005), pp. 481-491.
Jones, D. R., Encyclopedia of Optimization (2001), Kluwer Academic: MA.
Knowles, J., Corne, D. and Deb, K., Multiobjective Problem Solving from Nature: from Concepts to Applications

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(2008), Springer: New York.
Li, M., Li, G and Azarm, S., A Kriging metamodel assisted multi-objective genetic algorithm for design optimization, Transactions of ASME, Journal of Mechanical Design, Vol. 130, No. 3 (2008), pp. 031401-10.
Lu, X. and Xie, X., Multi-objective structural optimization of hub unit bearing using response surface methodology and genetic algorithm, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 8, No. 3 (2014), Paper No. 14-00019, pp. 1-10.
Senthil Kumar, P., Manisekar, K. and Narayanasamy, R., Experimental and prediction of abrasive wear behavior of sintered Cu-SiC composites containing graphite by using artificial neural networks, Tribology Transactions, Vol. 57 (2014), pp. 455-471.
Shen, F., Chen, C.-L. and Liu, Z.-M., Effect of pocket geometry on the performance of a circular thrust pad hydrostatic bearing in machine tools, Tribology Transactions, Vol. 57 (2014), pp. 700-714.
Shimizu, Y., Waki, T. and Yoo, J.-K., Multi-objective optimization on a sequencing planning of mixed-model assembly line, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 5, No. 4 (2011), pp. 274-283.
Srirat, J., Kitayama, S. and Yamazaki, K., Simultaneous optimization of variable blank holder force trajectory and tools motion in deep drawing via sequential approximate optimization, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 6, No. 7 (2012), pp. 1081-1092.
Wang, N., A parallel computing application of the genetic algorithm for lubrication optimization, Tribology Letters, Vol. 18 (2005), pp. 105-112.
Wang, N. and Cha, K.-C., Multiobjective optimization of air bearings using hypercube-dividing method, Tribology International, Vol. 43 (2010), pp. 1631-1638.
Wang, N. and Chang, Y.-Z., A hybrid search algorithm for porous air bearings optimization, Tribology Transactions, Vol. 45 (2002), pp. 477-477.
Wang, N. and Chang Y.-Z., Application of the genetic algorithm to the multiobjective optimization of air bearings,” Tribology Letters, Vol. 17, No. 2 (2004), pp. 119-128.
Wang, N. and Chen, L.-W., A divide-and-conquer parallel computing scheme for the optimization analysis of tribological systems, Tribology Transactions, Vol. 47 (2004), pp. 313-320.
Wang, N., Ho, C.-L. and Cha, K.-C., Engineering optimum design of fluid-film lubricated bearings, Tribology Transactions, Vol. 43 (2000), pp. 377-386.
Wang, N., Huang, H.-C. and Hsu, C.-R., Parallel optimum design of foil bearing using particle swarm optimization method, Tribology Transactions, Vol. 56 (2013), pp. 453-460.
Wang, N., Tsai, C.-M. and Cha, K.-C., Optimum design of externally pressurized air bearing using cluster OpenMP, Tribology International, Vol. 42 (2009a), pp. 1180-1186.
Wang, N., Tsai, C.-M. and Cha, K.-C., A study of parallel efficiency of modified DIRECT algorithm applied to thermohydrodynamic lubrication, Journal of Mechanics, Vol. 25 (2009b), pp. 143-150.
Zhu, H. and Bogy, D. B., DIRECT algorithm and its application to slider air-bearing surface optimization, IEEE Transactions on Magnetics, Vol. 38, No. 5 (2002), pp. 2168-2170.
Zhu, H. and Bogy, D. B., Hard disc drive air bearing design: modified DIRECT algorithm and its application to slider air bearing surface optimization, Tribology International, Vol. 37 (2004), pp. 193-201.