Magnetic moments of the spin-1/2 singly charmed baryons in covariant baryon chiral perturbation theory

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Abstract

Recent experimental advances have reignited theoretical interests in heavy-flavor hadrons. In this work, we study the magnetic moments of the spin-1/2 singly charmed baryons up to the next-to-leading order in covariant baryon chiral perturbation theory with the extended-on-mass-shell renormalization scheme. The pertinent low energy constants $g_{1-4}$ are fixed with the help of the quark model and the heavy quark spin flavor symmetry, while the remaining $d_2$, $d_3$, $d_5$ and $d_6$ are determined by fitting to the lattice QCD pion-mass dependent data. We study the magnetic moments as a function of $m_\pi^2$ and compare our results with those obtained in the heavy baryon chiral perturbation theory. We find that the loop corrections induced by the anti-triplet states are dominated by the baryon pole diagram. In addition, we predict the magnetic moments of the spin-1/2 singly charmed baryons and compare them with those of other approaches.

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I. INTRODUCTION

In the last two decades, tremendous progress has been made in our understanding of heavy-flavor hadrons, thanks to the experimental discoveries by collaborations such as LHCb, BELLE, and BESIII and the related theoretical studies. In the charmed baryon sector, 24 singly charmed baryons and two doubly charmed baryons are listed in the current version of the review of particle physics [1]. Among them, the newest members include the $\Lambda_c(2860)$ [2], the five $\Omega_c$ states [3], and the $\Xi^{++}_{cc}$ [4]. Inspired by these and other experimental discoveries, there are extensive theoretical and lattice QCD studies on their nature and their decay and production mechanisms (see, e.g., Refs. [5-12] references cited therein).

The magnetic moment of a baryon plays an extremely important role in understanding its internal structure. Historically, the experimental measurement of the magnetic moments of the proton and the neutron revealed that they are not point-like particles. The subsequent studies helped the establishment of the quark model as well the theory of the strong interaction, Quantum Chromo Dynamics. Unlike those of the ground-state baryons, the magnetic moments of the spin-1/2 singly charmed baryons have not been measured experimentally. Nevertheless, they have been studied in a variety of phenomenological models [13-19] as well as QCD sum rules [20]. Lately, they have also been studied in the mean-field approach [21], the self-consistent SU(3) chiral quark-soliton model [22], the heavy baryon chiral perturbation theory (HB ChPT) [23], and lattice QCD simulations [24-27]. In Ref. [23], the low energy constants (LECs) are determined by the quark model and the heavy quark spin flavor symmetry and by fitting to the lattice QCD data extrapolated to the physical point. In this work, we will study the magnetic moments of the spin-1/2 singly charmed baryons up to the next-to-leading order (NLO) in covariant baryon chiral perturbation theory (BChPT) with the extended-on-mass shell (EOMS) renormalization scheme. The unknown LECs will be determined by the quark model and the heavy quark spin flavor symmetry and by directly fitting to the lattice QCD data at unphysical pion masses [24-26]. One notes that many previous studies, such as Refs. [28-29], have shown that the EOMS BChPT can provide a better description of the lattice QCD quark-mass dependent data than its non-relativistic counterpart.

Chiral perturbation theory (ChPT) [30], as a low-energy effective field theory of QCD, is an appropriate framework to study the magnetic moments of hadrons, particularly, their light quark mass dependence. It provides a systematic expansion of physical observables in powers of $(p/\Lambda_{\chi})^n$, where $p$ is a small momentum and $\Lambda_{\chi}$ is the chiral symmetry breaking scale. However, its application to the one-baryon sector encountered a difficulty, i.e., a systematic power counting (PC) is lost due to the large non-vanishing baryon mass $m_0$ in the chiral limit. Over the years, three approaches were proposed to overcome this issue, i.e., the HB [31-32], the infrared (IR) [33], and the EOMS [34] schemes. The IR and the EOMS schemes are the
relativistic formulations of BChPT. A brief summary and comparison of the three different approaches can be found in Ref. [35].

The EOMS scheme is different from the HBChPT, because it retains a series of higher-order terms within the covariant power counting (PC) rule when removing the power-counting-breaking (PCB) terms. In recent years, many physical observables have been successfully studied in this scheme such as the magnetic moments [29,36–40], the masses and sigma terms [28,41–43] of the octet, decuplet and spin-1/2 doubly heavy baryons, the hyperon vector couplings [44,45], the axial vector charges [46], the pion nucleon [47,48] and kaon-nucleon scattering [49]. Thus, inspired by these studies, we would like to study the magnetic moments of the spin-1/2 singly charmed baryons in the EOMS scheme.

This work is organized as follows. In Sec. II, we provide the effective Lagrangians and calculate the relevant Feynman diagrams up to $O(p^3)$. Results and discussions are given in Sec. III, followed by a short summary in Sec. IV.

II. THEORETICAL FORMALISM

The magnetic moments of singly charmed baryons are defined via the matrix elements of the electromagnetic current $J_\mu$ as follows:

$$\langle \psi(p_f) | J_\mu | \psi(p_i) \rangle = \bar{u}(p_f) \left[ \gamma_\mu F_1^B(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_B} F_2^B(q^2) \right] u(p_i),$$

where $\bar{u}(p_f)$ and $u(p_i)$ are the Dirac spinors, $m_B$ is the singly charmed baryon mass, and $F_1^B(q^2)$ and $F_2^B(q^2)$ denote the Dirac and Pauli form factors, respectively. The four-momentum transfer is defined as $q = p_i - p_f$. At $q^2 = 0$, $F_2^B(0)$ is the so-called anomalous magnetic moment, $\kappa_B$, and the magnetic moment is $\mu_B = \kappa_B + Q_B$, where $Q_B$ is the charge of the singly charmed baryon.

The five Feynman diagrams contributing to $\mu_B$ up to $O(p^3)$ are shown in Fig.1. The leading order contribution of $O(p^2)$ is provided by the following Lagrangian:

$$L^{(2)}_{33} = \frac{d_2}{16m_3} \text{Tr}(\bar{B}_3 \sigma^{\mu\nu} F_\mu^+ B_3) + \frac{d_3}{16m_3} \text{Tr}(\bar{B}_3 \sigma^{\mu\nu} B_3) \text{Tr}(F_\mu^+),$$

$$L^{(2)}_{66} = \frac{d_4}{8m_6} \text{Tr}(\bar{B}_6 \sigma^{\mu\nu} F_\mu^+ B_6) + \frac{d_6}{8m_6} \text{Tr}(\bar{B}_6 \sigma^{\mu\nu} B_6) \text{Tr}(F_\mu^+),$$

where the numbers in the superscript are the chiral order, $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$, $F_\mu^+ = [e^\dagger Q h F_{\mu\nu} u + u Q_h F_{\nu\mu} u^\dagger]$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $Q_h = \text{diag}(1,0,0)$ is the charge operator of the charmed baryon, $u = \exp[i\Phi/2F_\phi]$, $\Phi$. 


FIG. 1: Feynman diagrams contributing to the singly charmed baryon magnetic moments up to NLO. Diagram (a) contributes at LO, while the other diagrams contribute at NLO. The solid, dashed, and wiggly lines represent singly charmed baryon, Goldstone bosons, and photons, respectively. The heavy dots denote the $O(p^2)$ vertices.

with the unimodular matrix containing the pseudoscalar nonet, and $F_\phi$ the pseudoscalar decay constant. In the following analysis, we take $F_\pi = 92.4$ MeV, $F_K = 1.22F_\pi$, and $F_\eta = 1.3F_\pi$. In the SU(3) flavor representation, there are three kinds of singly charmed baryons, which are denoted as $B_\bar{3}$, $B_6$, and $B_6^{\mu\mu}$, respectively,

$$B_\bar{3} = \begin{pmatrix} 0 & \Lambda_+^\bar{3} & \Xi_c^+ \\ -\Lambda_+^\bar{3} & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix},$$

$$B_6 = \begin{pmatrix} \Sigma^{++} & \Sigma^+ & \Xi_c^+ \\ \Sigma^+ & \Sigma^0 & \Xi_c^0 \\ \Xi_c^+ & \Xi_c^0 & \Omega_c^0 \end{pmatrix}, \quad B_6^{\mu\mu} = \begin{pmatrix} \Sigma^{++} & \Sigma^+ & \Xi_c^+ \\ \Sigma^+ & \Sigma^0 & \Xi_c^0 \\ \Xi_c^+ & \Xi_c^0 & \Omega_c^0 \end{pmatrix}. \tag{2}$$

The spin of the $B_\bar{3}$ and $B_6$ states is $1/2$ while the spin of the $B_6^{\mu\mu}$ states is $3/2$.

In the numerical analysis, we take the average of the masses for each flavor multiplet, i.e., $m_3 = 2408$ MeV, $m_6 = 2535$ MeV, and $m_6^{\mu\mu} = 2602$ MeV \[1\]. The mass differences are $\delta_1 = m_6 - m_3 = 127$ MeV, $\delta_2 = m_6^{\mu\mu} - m_3 = 194$ MeV, and $\delta_3 = m_6^{\mu\mu} - m_6 = 67$ MeV.

The loop diagrams arising at NLO are determined in terms of the lowest order LECs from
\[ L^{(1)}_B = \frac{1}{2} \text{Tr}[\bar{B}_3(iD - m_3)B_3] + \text{Tr}[\bar{B}_6(iD - m_6)B_6] + \text{Tr}[\bar{B}_6^{\mu\nu}(-g_{\mu\nu}(iD - m_6\gamma_5) + i(\gamma_\mu D_\nu + \gamma_\nu D_\mu)) - \gamma_\mu(iD + m_6\gamma_5)\gamma_\nu B_6^{\mu\nu}], \]
\[ L^{(1)}_{MB} = \frac{g_1}{2} \text{Tr}[\bar{B}_6\gamma_5 B_6] + \frac{g_2}{2} \text{Tr}[\bar{B}_6\gamma_5 B_3 + \text{h.c.}] + \frac{g_3}{2} \text{Tr}[\bar{B}_6^{\mu\nu}u_\mu B_6 + \text{h.c.}] + \frac{g_4}{2} \text{Tr}[\bar{B}_6^{\mu\nu}u_\mu B_3 + \text{h.c.}] + \frac{g_5}{2} \text{Tr}[\bar{B}_6^{\mu\nu}\gamma_5 B_6u^\dagger] + \frac{g_6}{2} \text{Tr}[\bar{B}_3\gamma_5 B_3], \]
\[ L^{(2)}_M = \frac{F_\phi^2}{4} \text{Tr}[\nabla_\mu U(\nabla^\mu U)^\dagger], \]

with

\[ D_\mu B = \partial_\mu B + \Gamma_\mu B + B\Gamma^T_\mu, \]
\[ \Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) - \frac{i}{2}(u^\dagger v_\mu u + u v_\mu u^\dagger) = -ieQ_\mu A_\mu, \]
\[ u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) + (u^\dagger v_\mu u - uv_\mu u^\dagger), \]
\[ U = u^2 = e^{i\phi}, \quad \nabla_\mu U = \partial_\mu U + ieA_\mu[Q_\mu, U], \]  

where \( v_\mu \) stands for the vector source, and the charge matrix for the light quark is \( Q_l = \text{diag}(2/3, -1/3, -1/3) \).

The total spin of the light quarks is 0 for the singly charmed baryon in the \( B_3 \) state. Considering parity and angular momentum conservation, the \( B_3 B_3 \phi \) vertex is forbidden, i.e., \( g_6 = 0 \).

For the \( B_3 \) and \( B_6 \) states, the tree level contributions of the magnetic moments can be easily obtained from Eq. (1), which are:

\[ \kappa_3^{(a,2)} = \alpha_3 d_2 + \beta_3 d_3, \]
\[ \kappa_6^{(a,2)} = \alpha_6 d_5 + \beta_6 d_6. \]  

The values of \( \alpha_3, \beta_3, \alpha_6, \) and \( \beta_6 \) are tabulated in Table III and Table IV. The four LECs \( d_2, d_3, d_5, d_6 \) will be determined by fitting to lattice QCD data.

At \( O(p^3) \), the loop contributions to the magnetic moments, which come from diagrams (b), (c), (d), and
TABLE I: Coefficients of the tree level contributions of Eq. (5) for the $B_3$ states.

| $\Lambda^+$ | $\Xi^+$ | $\Xi^0$ |
|------------|---------|---------|
| $\alpha_3$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| $\beta_3$  | 1       | 1       | 1 |

(e) in Fig. 1 are written as,

\[
\kappa^{(3)}_{\pm} = \frac{1}{4\pi^2} \left( \sum_{\phi=\pi,K} \frac{g_2^2}{F_\phi^2} \xi^{(3,b)}_{B_3,\phi,\delta_1} H^{(b)}_{B_3}(\delta_1, m_\phi) \right. \\
+ \sum_{\phi=\pi,K} \frac{g_4^2}{F_\phi^2} \xi^{(3,c)}_{B_3,\phi,\delta_2} H^{(c)}_{B_3}(\delta_2, m_\phi) \\
+ \sum_{\phi=\pi,K} \frac{g_2^2}{F_\phi^2} \xi^{(3,d)}_{B_3,\phi,\delta_3} H^{(d)}_{B_3}(\delta_3, m_\phi) \\
+ \sum_{\phi=\pi,K} \frac{g_2^2}{F_\phi^2} \xi^{(3,e)}_{B_3,\phi,\delta_4} H^{(e)}_{B_3}(\delta_4, m_\phi) \\
\left. \right) 
\]

with the coefficients $\xi^{(3;b,c,d,e)}_{B_3,\phi,\delta_i}$ listed in Table III and Table IV. The explicit expressions of the loop functions $H^{(b,c,d,e)}_{B_3}(\delta_i, m_\phi)$ and $H^{(b,c,d,e)}_{B_6}(\delta_i, m_\phi)$ can be found in the Appendix.

Once we obtain the loop functions in the EOMS scheme, we can easily obtain their HB counterparts by performing $1/m_0$ expansions. We have checked that our results agree with those of Ref. [23]. In the following section, for the sake of comparison, we study also the performance of the HBChPT in describing the lattice QCD data of Refs. [24,26]. It should be noted that in the following section, unless otherwise
stated, the HBChPT results refer to the ones obtained in the present work, not those of Ref. [23].

TABLE II: Coefficients of the tree level contributions of Eq. (5) for the $B_6$ states.

|       | $\Sigma^+$ | $\Sigma^+$ | $\Sigma^0$ | $\Xi^+$ | $\Xi^0$ | $\Omega^0$ |
|-------|------------|------------|------------|--------|--------|-----------|
| $\alpha_6$ | 1          | $\frac{1}{2}$ | 0          | $\frac{1}{2}$ | 0      | 0         |
| $\beta_6$  | 1          | 1          | 1          | 1      | 1      | 1         |

TABLE III: Coefficients of the loop contributions of Eq. (6) for the $B_3$ states.

|       | $\Lambda^+$ | $\Xi^+$ | $\Xi^0$ |
|-------|-------------|--------|--------|
| $\xi^{(3,b)}_{B_1\pi,\delta_0}$ | 0       | 1      | -1     |
| $\xi^{(3,b)}_{B_1\pi,\delta_1}$ | 1       | 0      | -1     |
| $\xi^{(3,c)}_{B_1\pi,\delta_2}$ | 0       | 1      | -1     |
| $\xi^{(3,d)}_{B_1\pi,\delta_1}$ | 1       | 5      | 1      |
| $\xi^{(3,d)}_{B_1\pi,\delta_2}$ | 0       | $\frac{3}{2}$ | 0      |
| $\xi^{(3,c)}_{B_1K,\delta_0}$  | 3       | $\frac{1}{4}$ | $\frac{1}{2}$ |
| $\xi^{(3,c)}_{B_1K,\delta_1}$  | $\frac{1}{2}$ | $\frac{5}{2}$ | $\frac{1}{2}$ |
| $\xi^{(3,c)}_{B_1K,\delta_2}$  | 0       | $\frac{3}{4}$ | 0      |

III. RESULTS AND DISCUSSIONS

In this section, we determine the LECs $d_2$, $d_3$, $d_5$ and $d_6$ by fitting to the lattice QCD data of Refs. [24–26], which are collected in Table V for the sake of easy reference. Because of the limited lattice QCD data, the other LECs $g_{1–4}$ are fixed by the quark model and the heavy quark spin flavor symmetry. Their values are $g_1 = 0.98$, $g_2 = -\sqrt{\frac{3}{5}}g_1 = -0.60$, $g_3 = \frac{\sqrt{3}}{2}g_1 = 0.85$, and $g_4 = -\sqrt{3}g_2 = 1.04$ [50 50 51]. In our least-squares fit, the $\chi^2$ as a function of the LECs is defined as

$$\chi^2(C_X) = \sum_{i=1}^{n} \frac{(\mu^\text{th}_i(C_X) - \mu^\text{LQCD}_i)^2}{\sigma_i^2},$$  

where $C_X$ denote all the LECs, $\sigma_i$ correspond to the uncertainty of each lattice QCD datum, $\mu^\text{th}_i(C_X)$ and $\mu^\text{LQCD}_i$ stand for the magnetic moments obtained in the BChPT and those of the lattice QCD in Table V, respectively.
The lattice QCD data are better than that of the HB BChPT in both cases.

In order to decompose the contributions of loop diagrams, we will consider two cases. In case 1, all the allowed intermediate baryons are taken into account, while in case 2, only intermediate baryons of the same type as those of the external baryons are considered. Fitting to the lattice QCD data of Table V and with $g_{1-4}$ fixed, the resulting LECs and $\chi^2$ are listed in Table VI. One notes that the EOMS BChPT descriptions of the lattice QCD data are better than that of the HB BChPT in both cases.
TABLE VI: LECs $d_2, d_3, d_5,$ and $d_6$ determined by fitting to the lattice QCD data, with $g_{1-4}$ fixed. In case 1 all the allowed intermediate baryons in the loop diagrams are taken into account, while in case 2 only intermediate baryons of the same type as those of the external baryons in the loop diagrams are considered.

|      | Case 1        | Case 2        |
|------|---------------|---------------|
|      | EOMS 1 HB 1   | EOMS 2 HB 2   |
| $d_2$ | $-1.25(15)$  | $-1.78(15)$  |
| $d_3$ | 2.20(4) 0.65(4) | 0.49(4) 0.49(4) |
| $d_5$ | 7.83(34) 13.49(34) | 5.08(34) 8.69(34) |
| $d_6$ | $-3.76(5)$ $-4.93(5)$ | $-2.66(5)$ $-3.40(5)$ |
| $g_1$ | 0.98 0.98     | 0.98 0.98     |
| $g_2$ | $-0.60$ 0      | $-0.60$ 0     |
| $g_3$ | 0.85 0.85     | 0.85 0.85     |
| $g_4$ | 1.04 0        | 1.04 0        |
| $\chi^2_{\text{min}}$ | 41.42 131.05 | 15.10 34.35 |

Note that we do not fit to the lattice QCD data obtained at $m_\pi = 700$ MeV, which are probably out of the range of validity of NLO ChPT. Furthermore, as can be seen in Fig. 3, the difference between the lattice QCD value and the ChPT prediction for $\mu_{\Xi_c^0}$ is somehow relatively large. Thus, we do not include the lattice QCD magnetic moment of $\Xi_c^0$ in our fitting as well.

For the sake of comparison with the lattice QCD data, in Fig. 2 we present the predicted magnetic moments of the singly charmed anti-triplet baryons as a function of $m_\pi^2$. It is seen that the EOMS BChPT results are of the same quality as those of the HB BChPT for $\Xi_c^+$ and $\Xi_c^0$. However, surprisingly, the EOMS and HB predictions for $\Lambda_c^+$ in case 1 are very different. From Table VII we note that in the HBChPT the contributions from the intermediate anti-triplet and sextet baryons cancel each other at $O(p^3)$. Thus, at this order, loop corrections are quite small. But in the EOMS scheme, the loop contributions are rather large, especially for $\Lambda_c^+$. In addition, we note that the main contributions of the loop diagrams are from the baryon pole diagram. Therefore, the large difference for the prediction of $\mu_{\Lambda_c^+}$ is caused by the absence of the baryon pole diagram in the HB BChPT at $O(p^3)$.

In Fig. 3, we plot the predicted magnetic moments of the singly charmed sextet baryons as a function of $m_\pi^2$, in comparison with the lattice QCD data. The EOMS BChPT results are in better agreement with the lattice QCD data than those of the HB BChPT. As shown in Tables VII on average the description of the lattice QCD data becomes worse if the intermediate anti-triplet states are included. Therefore, on average the results obtained in case 2 are in better agreement with the lattice QCD data. This has been noted in
Ref. [23] as well. In Tables VII and VIII we decompose the loop contributions mediated by the $\bar{3}$, 6, and $6^*$ states. One can see that the convergence pattern in case 2 is in generally better than that in case 1, with probably the exception of $\Sigma_c^0$. Therefore, we take the predictions obtained in case 2 as our final results.

In Fig. 2 and Fig. 3, we compare the predicted magnetic moments of all the singly charmed baryons at the physical point with those obtained in other approaches. We note that the results of different approaches are rather scattered. However, our results are in better agreement with those of the HBChPT of Ref. [23], though we have chosen different strategies to determine some of the LECs. Clearly, further experimental or lattice QCD studies are needed to pin down their values and to discriminate between different theoretical approaches.

![Graphs showing magnetic moments of singly charmed anti-triplet baryons as a function of $m_c^2$.](image)

**FIG. 2:** Magnetic moments of the singly charmed anti-triplet baryons as a function of $m_c^2$. The solid black nablas represent the corresponding lattice QCD data that are fitted.

**TABLE VII:** Decomposition of the loop contributions to the magnetic moments of singly charmed baryons. The subscript $\bar{3}$, 6, and $6^*$ denote the loop diagrams with the intermediate $\bar{3}$, 6, and $6^*$ states at $O(p^3)$, respectively.

|       | EOMS 1 | HB 1 | LQCD [24] |
|-------|--------|------|-----------|
|       | $O(p^3)$ | $O(p^3)_{\bar{3}}$ | $O(p^3)_6$ | $O(p^3)_0$ | $\mu_{\text{tot}}$ | $O(p^3)$ | $O(p^3)_{\bar{3}}$ | $O(p^3)_6$ | $O(p^3)_0$ | $\mu_{\text{tot}}$ | $\mu_{\text{tot}}$ |
| $B_{\bar{3}}$ | $\mu_{\bar{3}^2}$ | 1.005 | $\cdots$ | 0.035 | $-1.272$ | $-0.232$ | 0.191 | $\cdots$ | $-0.263$ | 0.280 | 0.208 | $\cdots$
|       | $\mu_{\Xi_c^0}$ | 1.005 | $\cdots$ | 0.141 | $-0.913$ | 0.233 | 0.191 | $\cdots$ | $-0.169$ | 0.215 | 0.237 | $\cdots$
|       | $\mu_{\Xi_c^+$} | 0.859 | $\cdots$ | 0.330 | $-0.996$ | 0.193 | 0.253 | $\cdots$ | 0.432 | $-0.495$ | 0.190 | $\cdots$
| $B_3$  | $\mu_{\Sigma_c^0}$ | 2.251 | $-0.293$ | $-0.444$ | 0.090 | 1.604 | 3.916 | $-0.319$ | $-0.988$ | 0.288 | 2.897 | 1.499(202)
|       | $\mu_{\Sigma_c^+$} | 0.428 | $-0.192$ | $-0.094$ | $-0.042$ | 0.100 | 1.044 | $-0.243$ | $-0.349$ | 0.091 | 0.543 | $\cdots$
| $B_6$  | $\mu_{\Sigma_c^0}$ | $-1.394$ | $-0.090$ | 0.256 | $-0.175$ | $-1.403$ | $-1.828$ | $-0.168$ | 0.290 | $-0.106$ | $-1.812$ | $-0.875(103)$
|       | $\mu_{\Sigma_c^+$} | 0.428 | 0.067 | 0.112 | $-0.048$ | 0.559 | 1.044 | 0.084 | $-0.145$ | 0.053 | 1.036 | $\cdots$
|       | $\mu_{\Xi_c^0}$ | $-1.394$ | 0.135 | 0.380 | $-0.198$ | $-1.077$ | $-1.828$ | 0.159 | 0.494 | $-0.144$ | $-1.319$ | $\cdots$
|       | $\mu_{\Xi_c^+$} | $-1.394$ | 0.361 | 0.505 | $-0.220$ | $-0.748$ | $-1.828$ | 0.486 | 0.698 | $-0.182$ | $-0.826$ | $-0.667(96)$
Motivated by the recent experimental progress on heavy flavor hadrons, we have studied the magnetic moments of the singly charmed baryons in the covariant baryon chiral perturbation theory (BChPT) up to
FIG. 4: Magnetic moments of the anti-triplet baryons obtained in different approaches. The light-blue bands represent the result obtained in the present work. The others are taken from Ref. [13] (N. Barik et al., 83), Ref. [14] (B. Julia-Diaz et al., 04), Ref. [15] (S. Kumar et al., 05), Ref. [16] (A. Faessler et al., 06), Ref. [17] (B. Patel et al., 08), Ref. [18] (N. Sharma et al., 10), Ref. [19] (A. Bernotas et al., 12), and Ref. [23] (HB ChPT, 18).
FIG. 5: Same as Fig. 4, but for the sextet baryons. Additional data are taken from Ref. [20] (S.-L Zhu et al., 97), Ref. [21] (G.-S Yang et al., 18), Ref. [22] (J. Y. Kim et al., 18), Ref. [24] (LQCD, 14), and Ref. [27] (LQCD, 15).

the next-to-leading order. Using the quark model and the heavy quark spin flavor symmetry to fix some of
the low energy constants, we determined the rest by fitting to the lattice QCD data. We compared our results with those of the heavy baryon (HB) ChPT and found that on average the lattice QCD quark mass dependent data can be better described by the covariant BChPT, consistent with previous studies. In addition, we found that the baryon pole diagram, which is absent in the HB ChPT, can play an important role in certain cases.

Compared with the results of other approaches, our predicted magnetic moments for the anti-triplets are relatively small. The same is true for the $\Sigma^{++}_c$, $\Sigma^+_c$, and $\Xi^{'+}_c$. On the other hand, our results for $\Sigma^+_c$, $\Xi^{0}_c$, and $\Omega^{0}_c$ are relatively large (small in absolute value). It is not clear how to understand such a pattern at present. We hope that future lattice QCD or experimental studies can help us gain more insight into these important quantities and better understand the singly charmed baryons.

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VI. APPENDIX

The pertinent loop functions, with the PCB terms removed, are given here.

[1] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, 030001 (2018).
\[ H_{B_h}^{(d)}(\delta_1, m_\phi) = H_{B_h}^{(b)}(m_3, \delta_1, m_\phi), \quad H_{B_h}^{(d)}(\delta_1, m_\phi) = H_{B_h}^{(d)}(m_3, \delta_1, m_\phi). \]

\[ H_{B_h}^{(c)}(\delta_2, m_\phi) = \begin{cases} H_{\delta<mc}^{(c)}(m_3, \delta_2, m_\phi), & (\delta_2 < m_\phi) \\ H_{\delta>mc}^{(c)}(m_3, \delta_2, m_\phi), & (\delta_2 > m_\phi) \end{cases} \]

\[ H_{B_h}^{(c)}(\delta_3, m_\phi) = \begin{cases} H_{\delta<mc}^{(c)}(m_6, \delta_3, m_\phi), & (\delta_3 < m_\phi) \\ H_{\delta>mc}^{(c)}(m_6, \delta_3, m_\phi), & (\delta_3 < m_\phi) \end{cases} \]

(8)

\[ H^{(b)}(m_B, 0, m_\phi) = -\frac{1}{4\pi^2} \left[ 2m_B^2 + \frac{m_\phi^2}{m_B^2} (2m_\phi^2 - m_B^2) \log \left( \frac{m_\phi^2}{m_B^2} \right) + \frac{2m_B(m_B^4 - 4m_\phi^2m_B^2 + 2m_\phi^2)}{m_B^2 \sqrt{4m_B^2 - m_\phi^2}} \arccos \left( \frac{m_\phi}{2m_B} \right) \right]. \]  

(9)

\[ H^{(b)}(m_B, \delta, m_\phi) = \frac{m_B}{8\pi^2} \int_0^1 \, dx \int_0^{1-x} \frac{x^3 m_B^3 + \delta x^2 m_B^2}{x(m_B + \delta)^2 + (x - 1)(m_B^2 - m_\phi^2)} 
+ \left[ (4x^2 + 4x - 2)m_B + \delta(3x - 1) \right] \log \left( \frac{x(m_B + \delta)^2 + (x - 1)(m_B^2 - m_\phi^2)}{\mu^2} \right) 
- 2(2x^2 + 2x - 1)m_B \log \left( \frac{x^2 m_B^2}{\mu^2} \right) - x^2 m_B - \delta + 4\delta x, \quad (0 < \delta < m_\phi) \]  

(10)

\[ H^{(d)}(m_B, 0, m_\phi) = -\frac{1}{4\pi^2} \left[ 2m_B^2 + \frac{m_\phi^2}{m_B^2} (2m_\phi^2 - m_B^2) \log \left( \frac{m_\phi^2}{m_B^2} \right) + \frac{2m_B(m_B^4 - 3m_\phi^2) - m_\phi^4}{m_B^2 \sqrt{4m_B^2 - m_\phi^2}} \arccos \left( \frac{m_\phi}{2m_B} \right) \right]. \]  

(11)

\[ H^{(d)}(m_B, \delta, m_\phi) = \frac{(2m_B + \delta)^2}{16\pi^2 m_B^2} \left\{ 2 \left[ m_\phi^2 \left( m_\phi^2 - 2\delta^2 \right) + 3\delta m_B (m_\phi - \delta) \left( \delta + m_\phi \right) - (m_\phi^2 - \delta^2) \right] \log \left( \frac{m_\phi}{m_B + \delta} \right) 
- 2 \left[ \frac{(m_\phi - \delta)(\delta + m_\phi)}{(2m_B + \delta - m_\phi)(2m_B + \delta + m_\phi)} \right] \left[ m_\phi^2 \left( 8\delta^2 - 3m_\phi^2 \right) + 5\delta m_B (\delta^2 - m_\phi^2) + 4\delta m_B^3 (m_\phi^2 - \delta^2) \right] 
+ \left( m_\phi^2 - \delta^2 \right)^2 \tan^{-1} \left( \frac{-2\delta m_B - 2m_B^2 - \delta^2 + m_\phi^2}{\sqrt{(m_\phi - \delta)(\delta + m_\phi)(2m_B + \delta - m_\phi)(2m_B + \delta + m_\phi)}} \right) 
+ 2 \left[ m_\phi^2 \left( 8\delta^2 - 3m_\phi^2 \right) + 5\delta m_B (\delta^2 - m_\phi^2) + 4\delta m_B^3 (m_\phi^2 - \delta^2) \right] \right\} \right] 
\cdot \frac{(m_\phi - \delta)(\delta + m_\phi)}{\sqrt{(2m_B + \delta - m_\phi)(2m_B + \delta + m_\phi)}} 
\cdot \tan^{-1} \left( \frac{m_\phi^2 - \delta (2m_B + \delta)}{\sqrt{(m_\phi - \delta)(\delta + m_\phi)(2m_B + \delta - m_\phi)(2m_B + \delta + m_\phi)}} \right) 
+ \frac{m_B^2 - 2\delta m_B + m_B^2 + 2(m_\phi - \delta)(\delta + m_\phi)}{\mu^2} \right], \]  

(12)
\[
H_{\delta m, \phi} \frac{1}{m_B} \left( \frac{\delta + m_B}{m_B} \right)^2 \left\{ -30 \log \left( \frac{\delta + m_B}{m_B} \right) m_B^6 \right. & - 6 \left( \frac{m_B}{\delta + m_B} + 5 \right)^2 m_B^2 \\
+ 5 \left\{ \tan^{-1} \left( \frac{\delta + 2m_B \delta - m_B^2}{\sqrt{(\delta + 2m_B)^2 - m_B^2}} \right) - \tan^{-1} \left( \frac{\delta + 2m_B \delta + 2m_B^2 - m_B^2}{\sqrt{(\delta + 2m_B)^2 - m_B^2}} \right) \right\} \\
\left. \cdot \sqrt{(\delta + 2m_B)^2 - m_B^2} (m_B^2 - \delta^2) - 4 \log \left( \frac{\delta + m_B}{m_B} \right) \left( 5\delta + 11m_B \right) \right\} m_B^6 \\
+ 3 \left\{ -4 \log \left( \frac{\delta + m_B}{m_B} \right) \left( 15\delta^2 + 66m_B\delta + 79m_B^2 \right) \delta^2 + 2 \tan^{-1} \left( \frac{\delta^2 + 2m_B \delta - m_B^2}{\sqrt{(\delta + 2m_B)^2 - m_B^2}} \right) \right\} \\
\left. \cdot \sqrt{(\delta + 2m_B)^2 - m_B^2} (m_B^2 - \delta^2) \right\} \right\} m_B^6 \\
+ 2 \left\{ 30 \log (\delta + m_B) (12 \delta + m_B)^5 m_B^5 - \left( 45\delta^4 + 204m_B\delta^3 + 354m_B^2\delta^2 + 267m_B^3\delta + 92m_B^4 \right) m_B^2 \\
+ 3 \left\{ \tan^{-1} \left( \frac{\delta^2 + 2m_B \delta + 2m_B^2 - m_B^2}{\sqrt{(\delta + 2m_B)^2 - m_B^2}} \right) - \tan^{-1} \left( \frac{\delta^2 + 2m_B \delta - m_B^2}{\sqrt{(\delta + 2m_B)^2 - m_B^2}} \right) \right\} \\
\left. \cdot \sqrt{(\delta + 2m_B)^2 - m_B^2} (m_B^2 - \delta^2) \right\} (15\delta^4 + 68m_B\delta^3 + 118m_B^2\delta^2 + 91m_B^3\delta + 29m_B^4) \\
+ 3 \left\{ -2 \log \left( \frac{1}{\mu^2} \right) (3\delta + 2m_B) m_B^5 - 6 \log (m_B) (22\delta + 3m_B) m_B^5 \right\} \\
\left. - \delta^2 \log \left( \frac{m_B}{\delta + m_B} \right) (38\delta + 195m_B) m_B^2 + \delta \log \left( \frac{\delta + m_B}{m_B} \right) \right\} m_B^2 \\
\left. \cdot \left( 20\delta^3 + 132m_B\delta^2 + 316m_B^2\delta^2 + 295m_B^3\delta + 360m_B^4 \right) \right\} m_B^2 \\
\left[ -72 \log (m_B) m_B^6 (2 \delta + m_B) + 60 \log (\delta + m_B) m_B^6 (2 \delta + m_B) \right) \right\} (\delta + 2m_B) \\
+ 6 \log \left( \frac{1}{\mu^2} \right) m_B^5 (4 \delta + 5m_B) + 6 \log (\delta + m_B) m_B^5 (5 \delta + 26m_B) \right\} m_B^5 \\
\left. - 6 \delta^5 \log \left( \frac{\delta + m_B}{m_B} \right) (20\delta^3 + 144m_B\delta^2 + 158m_B^2\delta + 293m_B^3) \right\} m_B^5 \\
\left. + 6 \tan^{-1} \left( \frac{\delta^2 + 2m_B \delta - m_B^2}{\sqrt{((\delta + 2m_B)^2 - m_B^2)} (m_B^2 - \delta^2)} \right) - \tan^{-1} \left( \frac{\delta^2 + 2m_B \delta + 2m_B^2 - m_B^2}{\sqrt{((\delta + 2m_B)^2 - m_B^2)} (m_B^2 - \delta^2)} \right) \right\} \\
\left. \cdot \sqrt{(\delta + 2m_B)^2 - m_B^2} (m_B^2 - \delta^2) \right\} (\delta + 2m_B)^2 \left( 5\delta^4 + 14m_B\delta^3 + 14m_B^2\delta^2 + 3m_B^3\delta - 3m_B^4 \right) \\
\left. + 6 \left\{ 30 \delta^2 + 104m_B\delta^2 + 525m_B^2\delta^2 + 618m_B^3\delta^3 + 250m_B^4\delta^2 - 110m_B^5\delta - 86m_B^6 \right\} \right\} m_B^2 \\
\left. - \frac{m_B^2 (30 \log \frac{m_B^2}{\mu^2}) - 43}{432\pi^2} \right\}.
\]
\[
H^{(c)}_{\delta>m_\phi} (m_B, \delta, m_\phi)
= \frac{1}{864 \pi^2 m_B^5 (\delta + m_B)^2} \left\{ -30 \log \left( \frac{\delta + m_B}{m_\phi} \right) m_\phi^8 - 6 \left( 38 \log \left( \frac{m_\phi}{\delta + m_B} \right) + 5 \right) m_B^2 \\
+ 5 \coth^{-1} \left( \frac{m_B^2 + \delta (\delta + 2m_B)}{\sqrt{(\delta^2 - m_\phi^2) ((\delta + 2m_B)^2 - m_\phi^2)}} \right) \sqrt{(\delta^2 - m_\phi^2) ((\delta + 2m_B)^2 - m_\phi^2)} \\
- 4 \delta \log \left( \frac{\delta + m_B}{m_\phi} \right) (5 \delta + 11m_B) \right\} m_\phi^6 \\
+ 3 \left( -4 \log \left( \frac{\delta + m_B}{m_\phi} \right) (15 \delta^2 + 66m_B\delta + 79m_B^2) \delta^2 + 2 \coth^{-1} \left( \frac{m_\phi^2 + \delta (\delta + 2m_B)}{\sqrt{(\delta^2 - m_\phi^2) ((\delta + 2m_B)^2 - m_\phi^2)}} \right) \\
\cdot \sqrt{(\delta^2 - m_\phi^2) ((\delta + 2m_B)^2 - m_\phi^2)} (15 \delta^2 + 34m_B\delta + 28m_B^2) + m_B^2 (30 \delta^2 + 68m_B\delta + 61m_B^2) \right\} m_\phi^4 \\
+ 2 \log \left( \frac{m_\phi}{\delta + m_B} \right) (76 \delta^2 + 195m_B\delta + 75m_B^2) \right\} m_B^2 \\
+ 2 \left[ 30 \log (\delta + m_B) m_B^6 - (45 \delta^4 + 204m_B\delta^3 + 354m_B^2\delta^2 + 267m_B^3\delta + 92m_B^4) m_B^2 \\
- 3 \coth^{-1} \left( \frac{m_B^2 + \delta (\delta + 2m_B)}{\sqrt{(\delta^2 - m_\phi^2) ((\delta + 2m_B)^2 - m_\phi^2)}} \right) (15 \delta^4 + 68m_B\delta^3 + 118m_B^2\delta^2 + 91m_B^3\delta + 29m_B^4) \\
\cdot \sqrt{(\delta^2 - m_\phi^2) ((\delta + 2m_B)^2 - m_\phi^2)} + 3 \left( 20 \log \left( \frac{\delta + m_B}{m_\phi} \right) \delta^5 + 132 \log \left( \frac{\delta + m_B}{m_\phi} \right) m_B\delta^5 \right. \\
+ 316 \log \left( \frac{\delta + m_B}{m_\phi} \right) m_B^2 \delta^4 + 295 \log \left( \frac{\delta + m_B}{m_\phi} \right) m_B^3 \delta^3 - \log \left( \frac{m_\phi}{\delta + m_B} \right) m_B^2 (38 \delta + 195m_B) \delta^3 \\
+ 360 \log \left( \frac{\delta + m_B}{m_\phi} \right) m_B^2 \delta^2 - 6 \log \left( \frac{\delta + m_B}{\mu^2} \right) m_B^2 \delta + 132 \log \left( \frac{\delta + m_B}{m_\phi} \right) m_B^2 \delta \\
- 4 \log \left( \frac{1}{\mu^2} \right) m_B^6 - 18 \log \left( m_\phi m_B^5 \right) \right\} m_\phi^2 \\
+ 60 \delta \log (\delta + m_B) m_B^6 (2\delta + 5m_B) + 6 \coth^{-1} \left( \frac{m_\phi^2 + \delta (\delta + 2m_B)}{\sqrt{(\delta^2 - m_\phi^2) ((\delta + 2m_B)^2 - m_\phi^2)}} \right) \\
\sqrt{(\delta^2 - m_\phi^2) ((\delta + 2m_B)^2 - m_\phi^2)} (30 \delta^6 + 204m_B\delta^5 + 525m_B^2\delta^4 + 618m_B^3\delta^3 + 250m_B^4\delta^2 + 110m_B^5\delta + 86m_B^6) \\
+ m_B^2 (30 \delta^6 + 204m_B\delta^5 + 525m_B^2\delta^4 + 618m_B^3\delta^3 + 250m_B^4\delta^2 + 110m_B^5\delta + 86m_B^6) \\
+ 6 \left[ 10 \log \left( \frac{\delta + m_B}{\mu^2} \right) m_B^8 - 12 \delta \log (\delta + m_B) m_B^6 + 6 \log \left( \frac{1}{\mu^2} \right) (4\delta + 13m_B) m_B^6 \\
+ 5 \delta \log \left( \frac{\delta + m_B}{\mu^2} \right) (59 \delta + 26m_B) m_B^6 - \delta^5 \log \left( \frac{\delta + m_B}{m_\phi} \right) (5 \delta^3 + 44m_B\delta^2 + 158m_B^2\delta + 295m_B^3) \\
\right\} \right\} m_B^2 \left( 30 \log \left( \frac{m_B^5}{\mu^2} \right) - 43 \right). \tag{14}
\]
$$H_{\delta,m_B}(m_B, \delta, m_\phi) = \left( \frac{80 \log \left( \frac{m_B^2}{\mu^2} \right) - 3}{432 \pi^2} \right) m_B^5 + \frac{1}{432 \pi^2 (\delta + m_B)^4} \left\{ -2 \log \left( \frac{\delta + m_B}{m_\phi} \right) (9 \delta^2 + 22m_B \delta + 16m_B^2) \right\} m_\phi^5$$

$$+ 2 \left[ 9 \left( \frac{\log \left( \frac{m_\phi}{\delta + m_B} \right) + 1}{\delta^2 + 22m_B \delta + 16m_B^2} \right) m_B^5 + \left( \tan^{-1} \left( \frac{\delta^2 + 2m_B \delta + 2m_{B_0}^2 - m_\phi^2}{\sqrt{((\delta + 2m_B)^2 - m_\phi^2)(m_\phi^2 - \delta^2)}} \right) - \tan^{-1} \left( \frac{\delta^2 + 2m_B \delta - m_\phi^2}{\sqrt{((\delta + 2m_B)^2 - m_\phi^2)(m_\phi^2 - \delta^2)}} \right) \right) \right.$$
\[
H^{(c)}_{d,m_B}(m_B,\delta,m_\phi) = \left(60 \log \left(\frac{m_D^2}{m_B^2}\right) - 3\right) m_B^2 + \frac{1}{432 \pi^2} \left[-2 \log \left(\frac{\delta + m_B}{m_\phi}\right) (9\delta^2 + 22m_B\delta + 16m_\phi^2) m_\phi^2 \right.
\]
\[
+ 4 \frac{\log \left(\frac{m_\phi}{\delta + m_B}\right) (9\delta + 40m_B) \delta^3 - \coth^{-1} \left(\frac{\delta^2 + 2m_B\delta + m_\phi^2}{\sqrt{(\delta^2 - m_\phi^2)} \left((\delta + 2m_B)^2 - m_\phi^2\right)}\right) (9\delta^2 + 22m_B\delta + 16m_\phi^2) m_\phi^2 \right]
\]
\[
\cdot \sqrt{(\delta^2 - m_\phi^2)} \left((\delta + 2m_B)^2 - m_\phi^2\right) + 4 \log \left(\frac{\delta + m_B}{m_\phi}\right) (71\delta^2 + 62m_B\delta + 25m_\phi^2) m_\phi^2
\]
\[
+ \left[54\delta^4 + 420m_B^2\delta^4 + 421m_B^2\delta^2 + 16 \log \left(\frac{m_\phi}{\delta + m_B}\right) (117\delta^2 + 137m_B\delta + 135m_\phi^2) \delta^2
\]
\[
+ 330m_B^2\delta + 84m_B^2 - 24 \log \left(\frac{m_\phi^2}{m_B^2}\right) m_B^3 (3\delta + 5m_B) m_B^2
\]
\[
+ 2 \coth^{-1} \left(\frac{\delta^2 + 2m_B\delta + m_\phi^2}{\sqrt{(\delta^2 - m_\phi^2)} \left((\delta + 2m_B)^2 - m_\phi^2\right)}\right) \sqrt{(\delta^2 - m_\phi^2)} \left((\delta + 2m_B)^2 - m_\phi^2\right) (\delta + 2m_B)
\]
\[
\cdot (27\delta^4 + 120m_B^2\delta^3 + 206m_B^2\delta^2 + 172m_B^2\delta + 68m_B^4)
\]
\[
+ 4 \log \left(\frac{1}{m_B}\right) \left(11m_B^5 + 41\delta m_B^7 + 9\delta^3 m_B^5\right) + 20 \log (\delta + m_B) (2m_B^8 + 2\delta m_B^7 - 85\delta^3 m_B^5)
\]
\[
+ 4 \left(-\log \left(\frac{\delta + m_B}{m_\phi}\right) m_B^2 (271\delta^3 + 257m_B\delta + 620m_B^2) \delta^4
\]
\[
+ 36 \frac{\log \left(\frac{m_B^2}{m_\phi^2}\right) m_B^6 + 4 \log \left(\frac{m_\phi}{\delta + m_B}\right) (9\delta^6 + 76m_B\delta^5 + 274m_B^2\delta^3 + 135m_B^4) \delta^2
\]
\[
+ \log \left(m_\phi\right) (12m_B^8 + 72\delta m_B^7 + 443\delta^3 m_B^5)) \right) m_B^2
\]
\[
+ 2 \log (\delta + m_B) (23\delta + 4m_B) m_B^9
\]
\[
+ 2 \delta \coth^{-1} \left(\frac{\delta^2 + 2m_B\delta + m_\phi^2}{\sqrt{(\delta^2 - m_\phi^2)} \left((\delta + 2m_B)^2 - m_\phi^2\right)}\right) \sqrt{(\delta^2 - m_\phi^2)} \left((\delta + 2m_B)^2 - m_\phi^2\right) (\delta + 2m_B)
\]
\[
\cdot (m_B^2)^3 (9\delta^2 + 22m_B\delta + 24m_B^2\delta^2 + 20m_B^3\delta + 6m_B^4)
\]
\[
+ 2 \delta \log \left(\frac{m_B^2}{m_\phi}\right) (9\delta^5 + 94m_B\delta^7 + 1028m_B^2\delta^5 + 1720m_B^3\delta^3 + 1600m_B^4\delta^2 + 1480m_B^5\delta + 1090m_B^6)
\]
\[
(16)
\]
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