Vertical transport in multiple-wide-quantum-well structures with homogeneous and with large-scale disorder nonhomogeneous interfaces.

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Abstract. The time-dependent approach is applied to the description of resonant tunnelling vertical transport accompanied by LO-phonon scattering in asymmetric double-quantum-well systems both with ideal flat and large-scale disorder interfaces. The square-law dependence of dissipative resonant tunnelling rate on the value of coupling matrix element $V$ in the structures with flat interfaces has been found to be radically different from the linear dependence tunnelling rates on $V$ in the structures with nonhomogeneous interfaces. As a result a substantial decrease in the effective tunnelling time in the case of nonhomogeneous interfaces is exhibited in compare with the case of structures with an ideal flat interface, particularly in weakly coupled structures. The physical reasons for the diversity of the functional dependence are discussed.

Resonant tunneling transport of carriers combined with scattering processes in multiple-quantum-well structures has attracted considerable interest to be investigated by theoretically and experimentally. The theoretical description of electron resonant tunneling is complicated by the consideration of elastic and inelastic scattering processes in double-quantum-well systems, due to the reciprocal action of tunneling and relaxation processes. The electronic systems in such a situation cannot reach equilibrium, and consequently resonant states in neighboring quantum wells cannot be considered as stable and should be described in the framework of the time-dependent model. Besides the tunneling and scattering processes must be considered together in the case, a many-particle aspect of the problem should take into consideration. The most advanced method for the calculation of the scattering effects in the electron tunneling is the Keldysh many-particle nonequilibrium Green’s function technique [1], which is rather complicated. Wacker used a less labor-consuming simple single electron Green’s function to consider different elastic and inelastic scattering processes in resonant and nonresonant electron tunneling [2]. But this method is applied only to a weak coupling limit. Gurvitz et. al. formulated a different technique using the Weisskopf-Wigner approximation for the calculation of resonant tunneling of electrons with the LO-phonon scattering in double-quantum-well structures with homogeneous interfaces [3]. This method is simpler and more transparent than the method used by Wacker and is not limited by conditions of weak or strong coupling.

Gurvitz et. al. showed that a dissipative tunneling rate $\tau^{-1}_{tun}$ depends on electron coupling as $V^2$ ($2V$ is the coupling matrix element) in the case of weak coupling and is independent of $V$ in the case of strong coupling [3] structures. However, linear type dependence of $\tau^{-1}_{tun} \sim V$, deduced from the
experiments [4], is rather different. The reasons for the distinctions of functional dependences on $V$ are still unclear.

In this paper we present the results of the study of dissipative resonant carrier tunneling current in quantum-well structures following the approach Gurvitz et al. [3] within the Weisskopf-Wigner approximation, but for quantum-well structures both with ideal flat and large-scale disorder interfaces. We note some new effects in such systems, in particular, that the divergences between the theory [3] and experiment [4] can be explained by the influence of interface roughness.

As an example we have considered an asymmetric double-quantum-well structure, consisting of a narrow and a wide well. There are resonant connected levels $E_1$ in the narrow well and $E_2$ in the wide well ($E_1\approx E_2$). The state $E_2$, as well as $E_1$, is metastable due to combined tunneling with optical phonon emission. The ground state $E_3$ in the wide well differs from the state $E_2$ on value of LO-phonon energy.

At first we shall consider electron transport in double-quantum-well system with ideal flat interface having three energy levels of quantum Schrödinger confinement.

The set of time-dependent Schrödinger equations the time-dependent Schrödinger equations describing evolution electron tunneling associated with LO-phonon scattering in structures with homogeneous interface is

$$
\begin{align*}
\dot{\alpha}(t) &= E_1\alpha(t) + V\beta(t) \\
\dot{\beta}(t) &= V\alpha(t) + E_2\beta(t) + \sum_{q} W\varphi_q \\
\dot{\varphi}_q(t) &= W\beta(t) + E_3\varphi_q(t)
\end{align*}
$$

where $|\alpha(t)|^2$ and $|\beta(t)|^2$ are the probabilities to find the electron at time $t$ in the resonant energy level in the narrow well and in the wide well, respectively. $|\varphi_q(t)|^2$ is the probability of finding the electron in the ground energy level $E_3$ in the wide well and the emitted phonon in the state $|q\rangle$. $V$ is the coupling matrix element of the resonant states, $W$ is the matrix element of the electron-phonon coupling, $\Gamma = 1/T_0$, $T_0$ is an electron- LO-phonon scattering time.

Applying Laplace transition to $\alpha(t) \rightarrow \tilde{\alpha}(E)$, $\beta(t) \rightarrow \tilde{\beta}(E)$, $\varphi_q(t) \rightarrow \tilde{\varphi}_q(E)$ we can solve the equations set (1) with respect to $\tilde{\alpha}(E)$ and $\tilde{\beta}(E)$. After the inverse Laplace transformation we got as in the Gurvitz case [3] the set of time-dependent Schrödinger equations (1) describing evolution dissipative resonant tunneling in structures with homogeneous interface is reduced to a modeling system [5],

$$
\begin{align*}
\dot{\alpha}(t) &= E_1\alpha(t) + V\beta(t) \\
\dot{\beta}(t) &= V\alpha(t) + (E_2 - i\frac{\Gamma}{2})\beta(t)
\end{align*}
$$

We also found the effective resonant tunneling time as the decision functions,
The received analytical expressions for $\alpha(t)$, $\beta(t)$ and $\tau_{\text{tun.}}$ are suitable for weak ($V<\Gamma/2$), strong ($V>>\Gamma/2$) and intermediate ($V\approx\Gamma/2$) coupling. These rather bulky expressions are submitted in our previous paper [5]. It can be shown nevertheless that the dependence $\tau_{\text{tun.}}$ on $V$ in weakly coupled structures is basically of the square-law type $\tau_{\text{tun.}}^{-1} \sim V^2$.

The kinetics of dissipative resonant tunneling changes in quantum well structures with nonhomogeneous interfaces. Carrier scattering effects on large-scale nonhomogeneous interfaces result in an occurrence of energy level bands $E_{1n}$ and $E_{2n}$ near the spatially located resonant states $E_1$ and $E_2$ the last one playing the determining role. Now we should consider electron transport in multilevel system $E_i$ and $E_{2n}$ (which is denoted below as $E_n$). We assume, what the tunneling is possible only between resonant energy levels, that is between energy levels in a narrow and a wide well concentrated inside a energy interval $2V$ is strictly carried out.

We consider the case when the charge spreading in the interface area is much faster than resonant tunneling and LO-phonon scattering.

Here $\alpha(t)$ and $\beta(t)$ are the spatially located wave function of resonant energy levels in the narrow well and in the wide well, respectively. $\gamma_n(t), \gamma_n'(t)$ is the delocalized wave functions of the nonresonant levels $E_n, E_{2n}$. $W_{n}$ is the coupling matrix element of the states $E_2$ and $E_n$, $\Gamma_n = 1/T_n$, and $T_n$ is the charge spreading time on the interface.

In this case the non-stationary Schrödinger equations (3) are reduced to the modeling system,

$$
\begin{align*}
\dot{\alpha}(t) &= E_i \alpha(t) + V \beta(t) \\
\dot{\beta}(t) &= V \alpha(t) + E_2 \beta(t) + W_{2n} \gamma_n(t) \\
\dot{\gamma}_n(t) &= W_n \beta(t) + E_n \gamma_n'(t) + \sum \gamma_n''(t) \\
\dot{\gamma}_n'(t) &= W_n \gamma_n'(t) + E_n \gamma_n''(t) + W_{2n} \phi_{2n}(t) \\
\vdots \\
\dot{\phi}_{2n}(t) &= W_{2n} \phi_{2n}(t) + E_2 \phi_{2n}(t).
\end{align*}
$$

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\begin{align*}
\dot{\alpha}(t) &= E_i \alpha(t) + V \beta(t) \\
\dot{\beta}(t) &= V \alpha(t) + E_2 \beta(t) + W_{2n} \gamma_n(t) \\
\dot{\gamma}_n(t) &= W_n \beta(t) + (E_n - i \frac{\Gamma_n}{2}) \gamma_n'(t)
\end{align*}
$$
FIGURE 1. Calculated resonant tunneling time in double-quantum-well structures with optical phonon relaxation with (solid lines) and without (dot line) carrier scattering on large-scale interface disorder via resonance splitting $V$ for the different inhomogeneous broadening $\Gamma_n$: $1.5\text{meV}(1)$, $3\text{meV}(2)$, $10\text{meV}(3)$.

We have calculated wave functions $\alpha(t)$, $\beta(t)$, $\gamma_n(t)$ from (4), and found effective resonant tunneling time

$$\tau_{\text{tun.}}^{-1} = \frac{1}{T_0} \sum_n |\gamma_n(t)|^2.$$  

As a result a linear dependence for tunneling effective time on the value of coupling matrix element $V$ has been found, $\tau_{\text{tun.}}^{-1} \sim V$.

This dependence radically differs from square-law dependence which is characteristic for structures with the flat interface. Indeed, in flat interface structures, the tunneling and LO-phonon scattering processes are strongly coupled that leads to square-law dependence, $\tau_{\text{tun.}}^{-1} \sim V^2$. In structures with nonhomogeneous interfaces tunneling and LO-phonon scattering processes are much less coupled because of incorporation of more fast processes determined by charge spreading on the interface. Because of the weak coupling of tunneling and scattering the linear dependence $\tau_{\text{tun.}}^{-1} \sim V$ appeared in the case the same as in a situation without scattering.

As shown in fig.1 the tunneling rate is higher in electron weakly coupled quantum well structures with nonhomogeneous interfaces than in those with an ideal flat interface.

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