Modal analysis of the coupled vibration in an aero-turbine compressor blisk considering both aerodynamic and centrifugal load

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Abstract. To lighten the aircraft engine and improve its propulsion efficiency, many aero engines adopt the blisk structure comprising both the rotor disk and blades. Since the disk becomes thinner, the stiffness of the structure decreased apparently, which causes the occurrence of large amplitude vibration in the blisk. This passage utilizes the finite element analysis (FEA) and computational fluid dynamics (CFD) simulation software to investigate the coupled vibration characteristics between the blades and the disk of the blisk with realistic geometric data, in which the aerodynamic load and centrifugal load are considered. At first, the CFD and finite element models of an aero-turbine compressor blisk are built. Then the pressure load data under some critical rotating speeds from CFD simulation are communicated to FEA software by data interpolating method, and the simulation of modal characteristics of the blisk is conducted in FEA. At last, the natural modal characteristics of the blisk structure are obtained, including the resonance speed Campbell diagram, the natural frequencies and corresponding mode shapes. According to the Campbell diagram, it indicates that the most critical mode is the fourth order mode. It is found that the mode shape of the fourth order in the blisk is the coupling of the disk torsional and blades first bending vibration. It is easy to lead to vibration failure when large amplitude vibration of the fourth mode occurs in the blisk. Also, it could also conclude that the modal characteristics of the coupled vibration in the rotating blisk are very sensitive to the disk stiffness. These results provide basic data for the further research in nonlinear vibration and structure optimization of the blisk, and also a simulating method to investigate coupled vibration between the blades and the disk of the blisk considering both aerodynamic and centrifugal load.

1. Introduction
Nowadays, the blisk structure, a turbomachine component comprising both the rotor disk and blades, becomes more and more common in turbo-machines because of its considerable effectiveness in lightening the weight of turbo engines and boosting the propulsion power. However, when the disk becomes thinner, the stiffness of the structure reduces apparently, which causes the vibration
amplitude to increase rapidly and results in obvious blisk coupled vibration. These coupled vibrations with large vibration amplitude diminish the stability and safety of the blisk dramatically. The thinner disk and blades design, the larger the system produce the deflection, which leads to vibration failure. Thus, it is essential to pay attention to the modal characteristics of the coupled vibration characteristics of blisk. Besides, it is difficult to maintain the blisk because of the integrity feature of the structure. Therefore, it is vital to ensure the dynamic stability of the blisk to avoid the existence of vibration failure in aero engines.

To predict accurately and investigate the dynamic responses and the nonlinear oscillations of the blisk, it needs to discuss the modal characteristics of the realistic blisk. Thus, the realistic mode of the blisk plays a key role in guaranteeing the accuracy of the analysis. Comparing with the analysis on vibration of the blades, modal characteristics of coupled vibration in the blisk are more complicated and more sensitive to lots of parameters. If the modes of the blisk selected are of a large deviation from the realistic vibration characteristics, it is impossible to analyze precisely the dynamic responses and nonlinear oscillations of the blisk. Therefore, it is of vital importance to study the vibration characteristics of the realistic blisk, which is the basis of the investigation on the nonlinear coupled vibration of rotating blisk.

In recent years, considerable attentions have been given to research on natural vibration characteristics of the rotating blisk.

Generally, three different methods are used to investigate the natural vibration characteristics of the blisk, such as finite element method, the experiment and numerical calculation. Beirov et al. (1) adopted a 3D finite element model to exploit the natural vibration characteristics of a tuned blisk under the centrifugal load. Wang et al. (2) utilized finite element model to study the vibration behaviors of a blisk with Carbon Fiber Reinforced Polymer/Plastic(CFRP) ring. Wang et al. (3) built a finite element model of the blisk with CFRP based on cyclic symmetry and used orthotropic and isotropic 3D elements to compute the vibration characteristics of the blisk. Xu et al. (4) made use of finite element(FE) software to compute the vibration characteristics of the blades on a high-pressure compressor blisk with cracked blades. Nyssen et al. (5) took advantage of a finite element model to analyze the vibration behaviors of a blisk clamped at the inner surface of the disk. Sun et al. (6) neglected the aerodynamic load and applied finite element method to explore the coupled vibration features of a blisk. Gibert et al. (7) designed the vacuum experiment device and embedded piezoelectric actuators to test the vibration behaviors of a rotating blisk. Hou and Chen (8) simulated experimentally the blisk simulator tested coupled vibration characteristics of the blisk. Nipkau et al. (9) conducted numerical calculation for the lumped parameter model to exploit the vibration features of a mistuned blisk with the effect of aerodynamics. Bai et al. (10) applied an improved hybrid interface substructural component-mode synthesis method to study the vibration behaviors of a mistuned blisk. Dong et al. (11) numerically calculated the vibration features of a blisk by computing a substructure sector based on cyclic symmetry. Mignolet et al. (12) studied the forced vibration response of the blisk with harmonic mistuning and random mistuning. Li et al. (13) used a prestressed component-mode synthesis super-element method to calculate vibration features of the blisk.

Despite broad research on the blisk, the studies on the natural vibration characteristics of the rotating blisk under both centrifugal and aerodynamic load are very scarce. Most of the investigations on natural vibration characteristic of blisks only took centrifugal force into account, but, in fact, the aerodynamic load makes a big effect on the modal characteristics of rotating blisks. Therefore, it is necessary to consider both the effects of centrifugal and aerodynamic loads on the modal characteristics of coupled vibrations in the rotating blisks. Due to the thinner disk, the vibration amplitude increases rapidly, and the natural vibration characteristics of the rotating blisk must be considered.

Compared with other studies of rotating blisks, this paper mainly is focused on the natural vibration characteristics of realistic rotating blisks under both centrifugal and aerodynamic loads. The coupled vibration characteristics of the blisk are investigated by CFD and FEA software with realistic geometric data, in which the effects of the aerodynamic load and the centrifugal load are both considered. The pressure load data are communicated from CFD simulation to FEA software, and
then the modal analysis is computed in FEA. The Campbell diagram is used to study the resonance frequencies of blisks to analyze critical modes. It is observed from results that the fourth mode, which is disk torsional and blades bending vibration under the weak excitation, is the most critical mode with the largest vibration amplitude. And the relative stiffness of the disk is a key parameter to the mode shapes sequence of the rotating blisk.

2. Modal analysis of the rotating blisk and the critical mode
The modal analysis of the rotating blisk is conducted together with CFD and FEA software. The Numeca software is used to mesh flow channels between blades. After the whole mesh model is simulated in the commercial CFD package, ANSYS, CFX software, the steady-state pressure data of the blisk surface is solved. Meanwhile, a 3D finite element model is built in Solidworks software, and the model is later used in the FE solver of commercial FEA package, ANSYS, to analyze the coupled vibration characteristics of the blisk when the pressure data were interpolated from CFX into the finite element model. Finally, the natural frequencies, the corresponding mode shapes and Campbell diagram of the rotating blisk are obtained to find the critical mode.

2.1. Formatting the title Modal analysis of the rotating blisk
According to the realistic geometric parameter, one 3D sector of the blisk subtending an angle $\pi/12$, which includes one blade and one corresponding disk sector, is established in Solidworks software, shown in figure 1. Meanwhile, based on the 3D geometric model of the blade, the flow channel of the blisk is meshed in commercial meshing package, Numeca software, in which the rotating speed is set as 18000 rpm, and the mesh together with the mesh statistics are shown in figure 2 and table 1, respectively. After the establishment of the flow channel mesh, the mesh is inputted to the CFX, where the rotating speed is selected as 18000 rpm, and the inlet and outlet parameters are chosen as the same realistic conditions as shown in table 2. Based on the computation, the pressure data of the blade surface, the pressure data and Mach number data along streamwise are collected finally, as shown in figure 3.

![Figure 1. 3D model of blisk sector.](image1)

![Figure 2. Mesh of the flow channel of blisk.](image2)

| Table 1. Mesh statistics of the flow channel of blisk. |
|------------------------------------------------------|
| Number of nodes                                      | 712626       |
| Number of elements                                   | 662400       |
| Hexahedra                                            | 662400       |
| Negative cells                                       | 0            |
| Volume[m^3]                                         | 0.000635807  |


Table 2. Boundary conditions of aerodynamic load computation.

| Inlet boundary details | Mass flow rate [kg s\(^{-1}\)] | 0.5 |
|------------------------|---------------------------------|-----|
|                        | Turbulence mode                  | Medium (intensity=5\%) |
|                        | total temperature [K]            | 288 |
| Outlet boundary details| Static pressure [Pa]             | 95000 |

Figure 3. (a) Aerodynamic load distribution of blade and hub surface, (b) Mach number data along streamwise, (c) pressure number data along streamwise.

In the static analysis, the 3D geometric sector model is meshed, the mesh and mesh statistics are as
shown in figure 4 and table 3. The CFD aerodynamic load is applied to the blade surface, and its load transfer summary is listed in table 4. The centrifugal load is also put on the blisk sector. Then the static stress data of the blisk are obtained, as shown in figure 5. Inputting the static stress data into modal analysis, setting solver control parameters and solving this modal analysis, the natural frequencies and corresponding modes of the rotating blisk are attained. The natural frequencies are shown in figure 6, and the corresponding modes are listed in table 5.

![Figure 4. Mesh of blisk sector.](image)

**Table 3.** Mesh statistics of blisk sector.

|        |        |
|--------|--------|
| Nodes  | 180000 |
| Elements | 90090  |
| Element size[m] | 5*e-003 |

**Table 4.** CFD load transfer summary.

| Component   | CFD computed forces from CFD results | Mechanical mapped forces for mechanical surface\(^a\) |
|-------------|--------------------------------------|----------------------------------------------------|
| X-component | -11.400                              | -12.100                                            |
| Y-component | 99.441                               | 98.563                                             |
| Z-component | -120.990                             | -118.660                                           |

\(^a\) 94% of mechanical nodes were mapped to the CFD surface, and the remaining nodes were mapped to the closest node or edge.

![Figure 5. Static strain under centrifugal and aerodynamic load.](image)
As shown in table 5, the first order frequency is 228.64 Hz, and the mode shape is disk bending-blades first bending vibration, which means that the disk vibrates out of plane and the blades vibrate by the first bending without the circumferential phase angle, as shown in figure 7. The second and third order vibrations share the same frequency 303.58 Hz, and the mode shape is disk bending-blades first bending vibration, which the blisk has one rotating nodal diameter, but has different circumferential phase angles because of the cyclic symmetry. The second and third modes are shown in figure 8.

The fourth order frequency is 406.47 Hz, and the mode shape is disk torsional-blades first bending vibration, in which the disk twists about the axial and blades vibrate by first bending, as depicted in figure 9. The fifth and sixth order modes share the same frequency 583.33 Hz, and their mode shapes are as well disk bending-blades first bending vibration, which the blisk has two rotating nodal diameters and different circumferential phase angles, as shown in figure 10. The interesting phenomenon is that all modes share the same modal shape from the seventh to twenty-ninth order, which is disk static-blade first bending vibration with different circumferential phase angles, and every two modes have the same frequency. The seventh to the twenty-ninth modes are shown in figure 11. Moreover, these modal frequencies are very close and concentrated. It can be concluded that blade modes dominate in the vibration of the blisk according to the form of these mode shapes.
Since the overall blisk modes consist of the modes of both the disk and the blades, the blisk modes can be seen as a synthesis of the vibrating forms of the disk and the blades. The disk tends to vibrate out of plane because the disk is thin and the stiffness is small, leading to the out of plane bending vibration of the disk in the first order mode. And in the first 29 modes of the blisk, all the blades vibrate in the motion of first bending. This phenomenon can be explained by the modes of the blades assemblage when the disk is fully clamped. It is found that the first ten order modes of the blades assemblage are first bending vibrations with different circumferential phase angles. So, the first 29 blisk modes all consist of the blades first bending mode with the different circumferential phase angles.

Also, we compute the modal characteristics of the blisk with a thicker disk, which means a bigger stiffness of the disk. Compared with the modal characteristics of the thin blisk, it is found that there are great differences in the mode shapes sequence and the frequencies, as shown in table 6.
Table 5. Frequencies and corresponding mode shapes of the coupled vibration in the blisk.

| Overall Mode Order | Frequency [Hz] | Mode Shape |
|--------------------|----------------|------------|
| 1                  | 228.6          | Disk bending-blades first bending; zero nodal diameter |
| 2                  | 303.58         | Disk bending-blades first bending; one nodal diameter |
| 3                  | 303.58         | same mode shape; different circumferential phase angles |
| 4                  | 406.47         | Disk torsional-blades first bending                   |
| 5                  | 583.33         | Disk bending-blades first bending; two nodal diameter |
| 6                  | 583.33         | same mode shape; different phase angles               |
| 7                  | 631.05         |                                                       |
| 8                  | 631.05         |                                                       |
| 9                  | 640.29         |                                                       |
| 10                 | 640.29         |                                                       |
| 11                 | 640.41         |                                                       |
| 12                 | 640.41         |                                                       |
| 13                 | 640.47         |                                                       |
| 14                 | 640.47         |                                                       |
| 15                 | 640.59         |                                                       |
| 16                 | 640.59         |                                                       |
| 17                 | 640.70         | Disk static-blades first bending; different phase angles |
| 18                 | 640.70         | same mode shape; different phase angles               |
| 19                 | 640.80         |                                                       |
| 20                 | 640.80         |                                                       |
| 21                 | 640.88         |                                                       |
| 22                 | 640.88         |                                                       |
| 23                 | 640.93         |                                                       |
| 24                 | 640.93         |                                                       |
| 25                 | 640.96         |                                                       |
| 26                 | 640.96         |                                                       |
| 27                 | 640.97         |                                                       |
| 28                 | 647.83         |                                                       |
| 29                 | 647.83         |                                                       |

For the blisk with a bigger stiffness, the first and second order mode shapes are disk bending-blades first bending vibrations with no nodal diameter, and the third order mode shape is disk bending-blades first bending vibration with one rotating nodal diameter, which is different in mode shapes sequence from the thin blisk. Therefore, it could conclude that the relative stiffness of the disk is an important parameter to the modal characteristics of the rotating blisk. Thus, the coupled vibration characteristics between the disk and blades are very important to investigate.
Table 6. Differences of frequencies and mode shape sequence between the thin blisk and the thick blisk.

| Overall mode order | Original disk thickness | Increased disk thickness |
|--------------------|-------------------------|--------------------------|
|                    | f [Hz]                  | Mode shape               | f [Hz]                  | Mode shape               |
| 1                  | 228.64                  | Disk bending-blades      | 436.17                  | Disk bending-blades      |
|                    |                         | first bending            |                         | first bending            |
|                    |                         | zero nodal diameter      |                         | one nodal diameter       |
| 2                  | 303.58                  | Disk bending-blades      | 436.17                  | Disk bending-blades      |
|                    |                         | first bending            |                         | first bending            |
|                    |                         | one nodal diameter       |                         | same mode shape          |
|                    |                         | same mode shape          |                         | different phase angles   |
| 3                  | 303.58                  | Disk static-blades       | 503.00                  | Disk bending-blades      |
|                    |                         | first bending            |                         | first bending            |
|                    |                         | zero nodal diameter      |                         | different phase angles   |
| 4                  | 406.47                  | Disk torsional-blades    | 550.78                  | Disk torsional-blades    |
|                    |                         | first bending            |                         | first bending            |
| 5                  | 583.33                  | Disk bending-blades      | 638.18                  | Disk bending-blades      |
|                    |                         | first bending            |                         | first bending            |
|                    |                         | two nodal diameters      |                         | two nodal diameters      |
| 6                  | 583.33                  | Disk static-blades       | 638.18                  | Disk static-blades       |
|                    |                         | same mode shape          |                         | first bending            |
|                    |                         | different phase angles   |                         | same mode shape          |
| 7                  | 631.05                  | Disk static-blades       | 640.90                  | Disk static-blades       |
|                    |                         | first bending            |                         | first bending            |
| 8                  | 631.05                  | Disk static-blades       | 640.90                  | Disk static-blades       |
| 9                  | 640.29                  | Disk static-blades       | 641.74                  | Disk static-blades       |
| 10                 | 640.29                  | Disk static-blades       | 641.74                  | Disk static-blades       |

2.2. Campbell diagram and the critical mode

Resonance Campbell diagram is a tool to identify the critical rotating speeds of turbine engines in rotor dynamics. A Campbell diagram plot represents a system's frequency response spectrum as a function of its spin speed. The point at which the frequency line intersects with the line of engine excitation line identifies the critical rotating speed of turbine engines. Figure 12 is an example of the Campbell diagram. In figure 12, there are two intersecting points shown as point A and point B, which are the points where two mode frequency lines intersect with the frequency line of engine excitation $\omega = \Omega/60$, respectively.

![Campbell diagram](image.png)

Figure 12. an example of Campbell diagram.
The aerodynamic load is calculated, and the modal analysis of the blisk is investigated when the rotating speed is selected as 0 rpm, 2000rpm, 4000 rpm, 6000rpm, 8000rpm, 10000rpm, 12000rpm, 14000rpm, 16000rpm, 20000rpm, respectively. The resonance Campbell diagram of coupled vibration for the rotating blisk is obtained, as shown in figure 13. From the Campbell diagram, the resonance point could be found and evaluated. It can be seen that there are eight crossing points totally which represent eight resonance points. Judged by how close the resonance rotating speed approaches the operating rotating speed, which is 14500rpm, the red crossing point is the critical resonance point when the frequency line of engine excitation \( k=2 \) and the fourth mode frequency line intersect. Thus, the corresponding fourth mode is the critical mode and the rotating speed 13853 rpm is the critical rotating speed, which is very close to the steady-state operating rotating speed of the aero engine. According to the Campbell diagram, the fourth order mode shape, disk torsional-blade first bending vibration as shown in figure 14, is selected to establish the dynamical model.

![Figure 13. The resonant Campbell diagram of rotating blisk coupled vibration.](image)

![Figure 14. The critical mode of blisk coupled vibration.](image)

**Conclusions**

In this paper, the coupled vibration characteristics between the blades and the disk of the rotating blisk with realistic geometric data are investigated utilizing the CFD and FEA software, in which the effects
of aerodynamic load and centrifugal load are both considered. The modal characteristics of the coupled vibration in the blisk are obtained, which includes the frequencies, corresponding mode shapes and the Campbell diagram. With the Campbell Diagram, it could be found that the fourth mode, disk torsional vibration and blades first bending vibration, is the most critical mode, on which could be based to build the mechanical model of the rotating blisk. Also, the modal characteristics of the blisk with a thicker disk are also investigated, and it is found that the relative stiffness of the disk is a key parameter to the mode shapes sequence of the rotating blisk.

The studies have led to the following conclusions:
(i) The fourth mode, disk torsional vibration and blades first bending vibration, is the most critical mode of the coupled vibration in this specific rotating blisk considering both aerodynamic and centrifugal load.
(ii) The modal characteristics of the coupled vibration in the rotating blisk are very sensitive to the disk stiffness.
(iii) By means of CFD and FEA software, there is a simulating method to calculate the coupled vibration characteristics between the blades and the disk of blisks which consider both aerodynamic and centrifugal load.

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