DESIGN OF LPV FAULT-TOLERANT CONTROLLER FOR HYPERSONIC VEHICLE BASED ON STATE OBSERVER

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Abstract. Considering the parameter uncertainty and actuator failure of hypersonic vehicle during maneuvering, this paper proposes a state observer-based hypersonic vehicle fault-tolerant control (FTC) system design method. Because hypersonic vehicles are prone to failure during maneuvering, the state quantity cannot be measured. First, a state observer-based FTC control method is designed for the linear parameter-varying (LPV) model with parameter uncertainty and partial failure of the actuator. Then, the Lyapunov function is used to demonstrate the asymptotic stability of the closed-loop system. The performance index function proved that the system has robust stability under the disturbance condition. Subsequently, the linear matrix inequality (LMI) was used to solve the observer parameters and the corresponding gain matrix in the control system. The simulation results indicated that the designed controller can track the flight command signal stably and has strong robustness, which verified the effectiveness of the design controller.

1. Introduction. Due to the advantages of higher flight speed, rapid global response, and strong penetration ability, hypersonic vehicle has become an important research direction in the aerospace field and has gained the attention of the world’s major military powers. Because the hypersonic vehicle adopts the engine-airframe integrated design and has high flight altitude and flight speed, it is very sensitive to changes in flight conditions and aerodynamic parameters [1]-[5]. In addition, due to the complex flight environment of hypersonic vehicle, any fault in the actuator will greatly damage the stability of the system and cause immeasurable losses [6]-[9]. Therefore, it is very important to study the fault-tolerant control of hypersonic vehicles.
Many experts and scholars at home and abroad have been studying the problem of fault-tolerant control of hypersonic vehicles, and have obtained certain achievements. [10] proposed an improved adaptive fault-tolerant control method. Two different adaptive laws are used to estimate the upper boundary of the external disturbance and the minimum value of the actuator efficiency factor. [11] linearized the nonlinear longitudinal dynamics model of hypersonic vehicle into a linear multivariable control system, and introduced the actuator fault model. In order to achieve an efficient fault-tolerant control, a dynamic sliding surface must be designed and the system must be placed over it. [12] proposed a fast-adaptive terminal sliding mode fault-tolerant controller for the problem of unknown upper bound disturbance and multiple failures of actuators, which ensures that the system stability is quickly restored in a finite time when actuator failures occur. [13] proposed an adaptive backstepping fault-tolerant controller, in which the dynamic surface control strategy was adopted. By introducing two first-order filters, the phenomenon of computation expansion caused by the repeated derivation of virtual control is eliminated, and fault-tolerant control can be implemented by the adaptation law. [14] proposed a fault-tolerant control law based on the neural network and adaptive backstepping method. The radial basis function neural network was used to approximate the nonlinear dynamic model, and a neural network observer was constructed to evaluate the unknown system fault and obtain adaptive online parameters. The update law ensures the stability of the state errors.

However, in comparison with the complex sliding mode method and backstepping method, the LMI-based control method is relatively simple and easy to implement, and is currently the most popular control method in the field of fault-tolerant control. In order to reduce the conservativeness of the controller, different Lyapunov variables are used in different fault states. [15] proposed an iterative LMI algorithm to solve the non-convex optimization problem, and the relevant linear matrix inequalities are derived using the bounded real theorem. The fault-tolerant controller design method is implemented by solving the LMI. [16] proposed a reference model for fault-tolerant tracking control, in which an observer-based fault-tolerant output feedback tracking controller was designed. Further, the Lyapunov theorem was used to prove that the system is asymptotically stable. Due to the strong nonlinearity and strong coupling characteristics of hypersonic vehicles, the design process of nonlinear control methods, such as sliding mode and backstepping, is very complicated. On comparison with these nonlinear control methods, the fault-tolerant control method based on the LPV system can be transformed into an LMI-based fault-tolerant control method, which has the advantages of strong practicability and simple design. However, only a few studies are available on fault-tolerant control based on the LPV system in literature; thus, further studies on this topic are necessary.

Considering these issues, this paper proposes an observer-based LPV fault-tolerant control method and applies it to designing the hypersonic vehicle maneuvering flight control system. First, the height and Mach number are selected as the scheduling variables, and a gridded space is used to obtain the equilibrium state working point of the vehicle. The typical LPV system is obtained using the Jacobian linearization. Then, considering the existence of the parameter uncertainty and actuator partial failure, an LPV model with parameter uncertainty and partial failure of the actuator is established. Due to the special flight environment and integrated design of hypersonic vehicles, the pneumatic sensor is prone to failure during actual
flight, and the states are difficult to measure. Therefore, a fault-tolerant control law based on state observer is introduced. Furthermore, because the failure degree of the actuators is known, the linear matrix inequality conditions are proposed to maintain the controllers stability and robustness. In actuality, the actuators failure degree during maneuvering of the vehicle is often unknown. Therefore, we introduce the inequality constraints, so that the controller has a wide tolerance range. Finally, the corresponding observer and controller gain matrices are obtained using the Matlab LMI toolbox. The simulation results indicated that the proposed control method is efficient.

Notation: Throughout the paper, the notations used are fairly standard. We use $A^{-1}$ and $A^T$ to denote the inverse matrix and transpose of matrix $A$, respectively. The symbol diag $\{A_1, A_2, \ldots, A_n\}$ stands for a block-diagonal matrix whose diagonal elements are $A_1, A_2, \ldots, A_n$. $I_n$ denotes the identity matrix with $n$ dimensions. Finally, for a real symmetric matrix $P$, the notations $P > 0$ ($P \geq 0$) and $P < 0$ ($P \leq 0$) are used to denote their positive definite (positive semi-definite) and negative definite (negative semi-definite), respectively.

2. Hypersonic vehicle model and problem formulation.

2.1. Longitudinal plane motion equation of hypersonic vehicle. The dynamic system model of the longitudinal flight of hypersonic vehicles [17] is the research objective of this paper. In the ground inertial coordinate system, the longitudinal motion equation of a hypersonic vehicle is expressed as follows:

$$
\begin{align*}
\dot{V} &= \frac{T \cos \alpha - D}{m} - \frac{\mu \sin \gamma}{r^2} \\
\dot{\gamma} &= \frac{L + T \sin \alpha}{mV} - \frac{(\mu - V^2 r) \cos \gamma}{V r^2} \\
\dot{h} &= V \sin \gamma \\
\dot{q} &= \frac{M_{yy}}{I_{yy}}
\end{align*}
$$

(1)

where, $V$, $\gamma$, $h$, $\alpha$, $q$ represent speed, flight angle, flight altitude, angle of attack, and pitch rate of the hypersonic vehicle; $T$, $D$, $L$, $M_{yy}$ represent thrust, drag, lift, and pitch moment.

The forces and moment coefficients are given by:

$$
\begin{align*}
L &= \frac{1}{2} \rho V^2 S C_L \\
D &= \frac{1}{2} \rho V^2 S C_D \\
T &= \frac{1}{2} \rho V^2 S C_T \\
r &= h + R_E \\
M_{yy} &= \frac{1}{2} \rho V^2 S \bar{c} [C_M (\alpha) + C_M (\delta_e) + C_M (q)]
\end{align*}
$$

(2)

where, $g$, $R_E$, $\rho$ represent the gravitational constant, earth’s radius, and atmospheric density; $C_L$, $C_D$, $C_T$ represent the lift coefficient, drag coefficient, and thrust coefficient; $C_M (\alpha)$, $C_M (\delta_e)$, $C_M (q)$ represent the angle of attack torque, the elevator moment coefficient, and pitching moment coefficient.

The aerodynamic coefficients and moment coefficients are expressed as follows:

$$
C_L = \alpha (0.493 + \frac{1.91}{M_{\alpha}})
$$
\[ CD = 0.0082(171\alpha^2 + 1.15\alpha + 2.0)(0.0012M_a^2 - 0.054M_a + 1) \]

\[
CT = \begin{cases} 
38 \left[ 1 - 164 (\alpha - \alpha_0)^2 \right] \left( 1 + \frac{17}{M_a^2} \right) (1 + 0.15) \eta, \eta < 1 \\
38 \left[ 1 - 164 (\alpha - \alpha_0)^2 \right] \left( 1 + \frac{17}{M_a^2} \right) (1 + 0.15) \eta, \eta < 1 
\end{cases}
\]

\[ CM(\alpha) = 10^{-4}(0.06 - e^{-\frac{M_a}{30}})(-2\alpha^2 + 120\alpha - 1) \]

\[ CM(q) = \frac{\pi}{2Vq} (-0.025M_a + 1.37) \cdot (-0.0021\alpha^2 + 0.0053\alpha - 0.23) \]

\[ CM(\delta_e) = 0.0292 (\delta_e - \alpha) \]

where, \( \eta \), \( \delta_e \) represent the diffuser area ratio and control surface deflection angle, which are the inputs of the system.

Detailed information on the forces and moment coefficients and other parameters of the vehicle defined in Equations (1)-(3) can be found in [18].

2.2. Establishment of LPV model with parameter uncertainty and partial failure of actuator. Because the main eigenvalues of the hypersonic vehicle Mach number and altitude greatly influence the model [19], we selected the Mach number and altitude as the scheduling variables \( \theta = [Ma, h]^T \). Within the allowable range of the hypersonic vehicle maneuvering parameters, the scheduling variables are space gridded, which are composed of Mach number and height; each grid point is trimmed to obtain the equilibrium state working point of the vehicle. Finally, the Jacobian linearization method is adopted, and a typical LPV model is obtained [20]. The gridding division of the scheduling variables is shown in Figure 1.

![Figure 1. Curve of flight path angle under actuator fault](image)

A typical LPV system is given by:

\[
\dot{x}(t) = A(\theta(t)) x(t) + B(\theta(t)) u(t)
\]

\[ y(t) = Cx(t) \]

where, \( x(t) \) is the state vector, \( u(t) \) is the input vector, \( y(t) \) is the output vector, \( \theta(t) = [\theta_1(t); \theta_2(t); \ldots; \theta_N(t)]^T \) is the time-varying parameter vector, \( A(\theta) \) and \( B(\theta) \) are the explicit matrix functions of the time-varying parameter \( \theta(t) \).
In order to more accurately describe the hypersonic vehicle dynamic model, the uncertain parameter $\Delta A$ and the external disturbance $\omega(t)$ are introduced in Equation (4). Then, the LPV model can be expressed as:

$$\dot{x}(t) = (A_p(\theta) + \Delta A)x(t) + B(\theta)u(t) + B_\omega \omega(t)$$

$$y(t) = Cx(t).$$  \tag{5}$$

**Remark 1.** In order to more accurately represent the dynamic model of the hypersonic vehicle, an external disturbance signal $\omega(t)$ is introduced in the LPV model. In addition, because the hypersonic vehicle adopts the engine-airframe integrated design, the angle of attack $\alpha$ is very sensitive to its own variation $\Delta \alpha$; therefore, $\Delta \alpha$ can be introduced in Equation (3) as the parameter uncertainty term in the angle of attack moment coefficient $C_M(\alpha)$. Then, the expression for $C_M(\alpha)$ is as follows:

$$C_M(\alpha) = 10^{-4}(0.06 - e^{-\frac{M \alpha}{3}})[-2(\alpha + \Delta \alpha)^2 + 120(\alpha + \Delta \alpha) - 1].$$

The expression of $C_M(\alpha)$ directly affects the solution of the state space matrix $A$, and it is considered that the state space matrix $A$ has an uncertainty $\Delta A$.

**Assumption 1.** $\Delta A$ is a bounded uncertainty matrix that satisfies $\Delta A = GH(t)$, where $G$ and $J$ are the constant matrices of appropriate dimensions, and $H(t)$ is an unknown time-varying matrix that satisfies $H^T(t)H(t) < I$. $B_\omega$ is the perturbation matrix of an appropriate dimension.

When actuator failures occur, the fault parameter matrix $F$ is introduced to describe the control efficiency of the actuator [21]; then, the control input under the actuator fault condition can be expressed as:

$$u^F(t) = F u(t)$$ \tag{6}$$

where,

$$0 \leq F = \text{diag}\left\{ f_1, f_2 \right\} \leq \bar{F} = \text{diag}\left\{ \bar{f}_1, \bar{f}_2 \right\}.$$ 

$f_i$ represents the control efficiency of the actuator.

Let

$$F_0 = \text{diag}\left\{ f_{01}, f_{02} \right\} = \frac{E + \bar{F}}{2} = \text{diag}\left\{ \frac{\bar{f}_1 + f_1}{2}, \frac{\bar{f}_2 + f_2}{2} \right\}$$ \tag{7}$$

$$\bar{F} = \text{diag}\left\{ \bar{f}_1, \bar{f}_2 \right\} = \frac{E - \bar{F}}{2} = \text{diag}\left\{ \frac{\bar{f}_1 - f_1}{2}, \frac{\bar{f}_2 - f_2}{2} \right\}$$ \tag{8}$$

Also, let

$$\tilde{f}_i = \frac{\bar{f}_i - f_i}{2};$$

then, $F$ can be written as:

$$F = F_0 + \Delta = F_0 + \text{diag}\left\{ \delta_1, \delta_2, \right\},$$

$$\|\delta_i\| \leq \tilde{f}_i.$$ \tag{9}$$

Based on the above analysis of the partial failure of the actuator, Equation (5) can be transformed to:

$$\dot{x}(t) = (A_p(\theta) + \Delta A)x(t) + B(\theta)F u(t) + B_\omega \omega(t)$$

$$y(t) = C x(t).$$ \tag{10}$$
From Equation (10), the LPV model with parameter uncertainty and partial failure of the actuator is yielded.

2.3. Problem formulation. For the LPV model with parameter uncertainty and partial failure of actuator in Equation (10), the corresponding observer and controller are designed [22, 23]. The stability of the hypersonic vehicle is demonstrated by the relevant theoretical derivation, and the system is determined to be stable and robust by the simulation.

However, for hypersonic vehicles, it is more practical to design the corresponding fault-tolerant control law for the maneuvering flight process. Therefore, the aim of this paper is as follows:

Problem 1. By introducing the command signal into the LPV model (10) with uncertain parameters and fault, the corresponding controller and observer are designed, so that the proof and simulation of the command tracking stability and robustness of the hypersonic vehicle are accomplished; that is, by designing the controller, the state error \( \hat{e}(t) \) and output \( y(t) \) are satisfied, which are given as follows:

\[
\hat{e}(t) = r(t) - x(t) < d_i \quad (11)
\]

\[
\|y(t)\|^2 < \gamma^2 \|\omega(t)\|^2 \quad (12)
\]

where, \( d_i \) is an unknown scalar and is only used for stability analysis, and \( \gamma > 0 \) is the given disturbance attenuation level for robustness analysis.

Lemma 1[24]. Let

\[
y(s) = G(s)\omega(s),
\]

and

\[
\|y(s)\|_2 < \|G(s)\|_\infty \|\omega(s)\|_2,
\]

then \( \|G(s)\|_\infty \) can be viewed as an amplifier between the \( H_2 \) norms (energy) of the input \( \omega(t) \) and the output \( y(t) \). Therefore, the disturbance attenuation level \( \gamma \) in (12) is the \( H_\infty \) performance index function.

3. Design of fault-tolerant controller for hypersonic vehicle based on state observer.

3.1. LPV model state observer design. Because the hypersonic vehicle has various flight characteristics, such as higher flight speed, integrated design of the airframe and engine, and high surface temperature of the aircraft, during maneuvering, the pneumatic sensor is prone to failure, resulting in an inability to directly yield some flight states during flight. Therefore, the traditional state feedback control method cannot be used for the fault-tolerant controller design. Hence, this paper proposes the design of a fault-tolerant controller (13) based on state observer, which enables the hypersonic vehicle to effectively track the command signal and maintain stable flight in the event of actuator failure [23], which is given by:

\[
\dot{\hat{x}}(t) = A_c\hat{x}(t) + B(\theta)u(t) + L[y(t) - \hat{y}(t)]
\]

\[
\hat{y}(t) = C\hat{x}(t)
\]

\[
u(t) = -K_x\hat{x}(t) - K_r r(t)
\]

where, \( \hat{x}(t) \) represents the estimator of states, \( \hat{y}(t) \) is the observer output vector, \( K_r, K_x \), and \( L \) are the command signal gain, controller gain, and the observer gain, respectively. \( K_r, K_x, L, \) and \( A_c \) are the parameters that must be designed.
From Equations (10) and (13):

\[
\begin{bmatrix}
\dot{e}(t) \\
\hat{e}(t)
\end{bmatrix} = 
\begin{bmatrix}
A(\theta) + \Delta A - B(\theta) F K_x & -B(\theta) F K_x \\
-A(\theta) - \Delta A + A_c + B(\theta)(F - I) K_x - L C \\
B(\theta)(F - I) K_x & A_c + B(\theta)(F - I) K_x - L C
\end{bmatrix}
\begin{bmatrix}
\dot{e}(t) \\
e(t)
\end{bmatrix} 
+ 
\begin{bmatrix}
B(\theta)F(K_x + K_r) - A(\theta) - \Delta A \\
A(\theta) + \Delta A - A_c - B(\theta) F K_x + B(\theta) K_x + B(\theta) F K_r
\end{bmatrix} r(t) + 
\begin{bmatrix}
-B_w \\
B_w
\end{bmatrix} \omega(t)
\]

(14)

where \( e(t) = x(t) - \hat{x}(t) \) represents the observation error.

3.2. Actuator failure design of fault-tolerant controller under known conditions. It is assumed that the hypersonic vehicle occurs actuator failure during maneuvering and that the degree of fault is known; \( F \) is a known actuator fault parameter matrix. Then, the following theorem is established:

**Theorem 1:** For any given positive scalar \( \gamma > 0 \) and known actuator fault matrix \( F \), if the positive definite symmetry matrices \( \bar{P}_1, \bar{P}_2, \bar{P}_3 \), real matrices \( \bar{A}_c, \bar{P}, \bar{K}_x, \bar{K}_r, L \) and a positive scalar \( \varepsilon > 0 \) exist, the following condition is established:

\[
CP_2 = \bar{P}C
\]

(15)

where,

\[
N_{11} = A(\theta) \bar{P}_1 + \bar{P}_1 A^T(\theta) - B(\theta) F \bar{K}_x - \bar{K}_x^T F B^T(\theta)
\]

\[
N_{12} = -\bar{P}_1 A^T(\theta) + \bar{P}_1 A_c^T + \bar{K}_x^T (F - I) B^T(\theta) - B(\theta) F \bar{K}_{2,x}
\]

\[
N_{13} = B(\theta) F (K_x + K_r) - A(\theta) \bar{P}_3 = B(\theta) F \bar{K}_{3,x} + B(\theta) F \bar{K}_r - A(\theta) \bar{P}_3
\]

\[
N_{21} = -A(\theta) \bar{P}_1 + A_c \bar{P}_1 + B(\theta)(F - I) \bar{K}_x - \bar{K}_{2,x}^T F B^T(\theta)
\]

\[
N_{22} = A_c \bar{P}_2 + \bar{P}_2 A_c^T + B(\theta)(F - I) \bar{K}_{2,x} + \bar{K}_{2,x}^T (F - I) B^T(\theta) - C^T L^T - L C
\]

\[
N_{23} = A(\theta) \bar{P}_3 - A_c \bar{P}_3 - B(\theta)(F - I) \bar{K}_{3,x} - B(\theta)(F - I) \bar{K}_r
\]

\[
N_{31} = \bar{K}_{3,x}^T F B^T(\theta) + \bar{K}_r^T F B^T(\theta) - \bar{P}_3 A^T(\theta)
\]

\[
N_{32} = \bar{P}_3 A^T(\theta) - \bar{P}_3 A_c^T + \bar{K}_{3,x}^T (F - I) B^T(\theta) - \bar{K}_{3,x}^T (F - I) B^T(\theta)
\]

\[
K_x = \bar{K}_x \bar{P}_1^{-1} = \bar{K}_{2,x} \bar{P}_2^{-1} = \bar{K}_{3,x} \bar{P}_3^{-1}, K_r = \bar{K}_r \bar{P}_3^{-1}
\]

\[
A_c = \bar{A}_c \bar{P}_1^{-1} = \bar{A}_{2,c} \bar{P}_2^{-1} = \bar{A}_{3,c} \bar{P}_3^{-1}, L = LP^{-1}
\]

Then, based on the observer-based output feedback control law (13), the LPV system (10) remains robust and stable under the disturbance attenuation level \( \gamma \).
Proof. To demonstrate the stability of the system, the Lyapunov function is constructed as follows:

\[ V(t) = \tilde{e}^T(t) P_1 \tilde{e}(t) + \tilde{e}^T(t) P_2 \tilde{e}(t) \]  

(17)

The derivatives for \( V(t) \) can be obtained as follows:

\[
\dot{V}(t) = \tilde{e}^T(t) P_1 \dot{\tilde{e}}(t) + \tilde{e}^T(t) P_1 \dot{\tilde{e}}(t) \\
+ \tilde{e}^T(t) P_2 \dot{\tilde{e}}(t) + \tilde{e}^T(t) P_2 \dot{\tilde{e}}(t)
\]

\[
= \begin{bmatrix} \dot{\tilde{e}}(t) & e(t) & r(t) & \omega(t) \end{bmatrix}^T \begin{bmatrix} M_{11} + P_1 \Delta A + \Delta A^T P_1 & M_{12} - \Delta A^T P_2 & M_{13} & P_1 \Delta A \\
M_{21} - P_2 \Delta A & M_{22} & M_{23} + P_2 \Delta A & 0 \\
M_{31} - \Delta A^T P_1 & M_{32} + \Delta A^T P_2 & 0 & 0 \\
- \Delta_r^T P_1 & 0 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix} \dot{\tilde{e}}(t) \\
e(t) \\
r(t) \\
\omega(t) \end{bmatrix}
\]

(18)

where,

\[
M_{11} = P_1 A(\theta) + A^T(\theta) P_1 - P_1 B(\theta) F K_x - K_x^T F B^T(\theta) P_1 \\
M_{12} = -A^T(\theta) P_2 + A_c^T P_2 + K_x^T (F - I) B^T(\theta) P_2 - P_1 B(\theta) F K_x \\
M_{13} = P_1 B(\theta) F (K_x + K_r) - P_1 A(\theta) \\
M_{21} = -P_2 A(\theta) + P_2 A_c + P_2 B(\theta) (F - I) K_x - K_x^T F B^T(\theta) P_1 \\
M_{22} = P_2 A_c + A_c^T P_2 + P_2 B(\theta) (F - I) K_x + K_x^T (F - I) B^T(\theta) P_2 \\
- C^T L^T P_2 - P_2 L C \\
M_{23} = P_2 A(\theta) - P_2 A_c - P_2 B(\theta) (F - I) K_x - P_2 B(\theta) (F - I) K_r \\
M_{31} = (K_x^T + K_r^T) F B^T(\theta) P_1 - A^T(\theta) P_1 \\
M_{32} = A^T(\theta) P_2 - A_c^T P_2 - K_x^T (F - I) B^T(\theta) P_2 - K_x^T (F - I) B^T(\theta) P_2
\]

Using the Young inequality the following equation can be obtained:

\[
\begin{bmatrix}
-\Delta_r^T P_1 & \Delta A^T P_2 & -P_1 \Delta A & 0 \\
-P_2 \Delta A & 0 & P_2 \Delta A & 0 \\
-\Delta A^T P_1 & \Delta A^T P_2 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
In order to demonstrate the robustness of the design control law, the $\mathcal{H}_\infty$ performance index function is introduced \cite{22}:

$$J(t) = y^T(t) y(t) - \gamma \omega^T(t) \omega(t)$$

Because $x(t) = \dot{x}(t) + e(t)$, $x(t) = r(t) - \dot{\hat{e}}(t)$, the following equation can be obtained:

$$\dot{V}(t) + J(t) = \dot{V}(t) + x^T(t) C^T C x(t) - \gamma^2 \omega^T(t) \omega(t)$$

$$= \dot{V}(t) + \begin{bmatrix} \dot{\hat{e}}(t) \\ e(t) \\ r(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C^T C & -C^T C & 0 \\ 0 & -C^T C & C^T C & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \dot{\hat{e}}(t) \\ e(t) \\ r(t) \\ \omega(t) \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\hat{e}}(t) \\ e(t) \\ r(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} M_{11} + P_1 \Delta A + \Delta A^T P_1 & M_{12} - \Delta A^T P_2 \\ M_{21} - P_2 \Delta A & M_{22} + C^T C \\ M_{31} - \Delta A^T P_1 & M_{32} + \Delta A^T P_2 - C^T C \\ -B_w^T P_1 & -B_w^T P_2 \end{bmatrix} \begin{bmatrix} \dot{\hat{e}}(t) \\ e(t) \\ r(t) \\ \omega(t) \end{bmatrix}$$

$$+ \begin{bmatrix} P_1 \Delta A + \Delta A^T P_1 & -\Delta A^T P_2 & -P_1 \Delta A & 0 \\ -P_2 \Delta A & 0 & P_2 \Delta A & 0 \\ -\Delta A^T P & \Delta A^T P_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\hat{e}}(t) \\ e(t) \\ r(t) \\ \omega(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C^T C & -C^T C & 0 \\ 0 & -C^T C & C^T C & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\hat{e}}(t) \\ e(t) \\ r(t) \\ \omega(t) \end{bmatrix}$$

$$\leq \varepsilon$$
where,

\[
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & 0 \\
M_{21} & M_{22} & M_{23} & 0 \\
M_{31} & M_{32} & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
+ \begin{bmatrix}
P_1 \Delta A + \Delta A^T P_1 & -\Delta A^T P_2 & -P_1 \Delta A & 0 \\
-P_2 \Delta A & 0 & P_2 \Delta A & 0 \\
-\Delta A^T P & \Delta A^T P_2 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & C^T C & -C^T C & 0 \\
0 & -C^T C & C^T C & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & -P_1 B_w \\
0 & 0 & 0 & P_2 B_w \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & 0 \\
M_{21} & M_{22} & M_{23} & 0 \\
M_{31} & M_{32} & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
+ \varepsilon \begin{bmatrix}
P_1 G & -P_2 G & 0 \\
-P_2 G & 0 & 0 \\
0 & 0 & G^T P_1 & -G^T P_2 & 0 & 0
\end{bmatrix}
\]

\[
+ \left[ \begin{array}{c}
-J^T \\
0 \\
-J^T
\end{array} \right] \begin{bmatrix}
J & 0 & -J & 0 \\
0 & C^T & -C^T & 0
\end{bmatrix} + \begin{bmatrix}
0 \\
C^T \\
-C^T
\end{bmatrix}
\]

\[
+ \gamma^2 \begin{bmatrix}
-P_1 B_w \\
P_2 B_w \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
-B_w^T P_1 & B_w^T P_2 & 0 & 0
\end{bmatrix}
\]

(22)

Let

\[
U = \begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & 0
\end{bmatrix}
+ \varepsilon \begin{bmatrix}
P_1 G & -P_2 G & 0 \\
-P_2 G & 0 & 0 \\
0 & 0 & G^T P_1 & -G^T P_2 & 0 & 0
\end{bmatrix}
\]

\[
+ \frac{1}{\varepsilon} \begin{bmatrix}
-J^T \\
0 \\
-J^T
\end{bmatrix} \begin{bmatrix}
J & 0 & -J & 0 \\
0 & C^T & -C^T & 0
\end{bmatrix} + \gamma^2 \begin{bmatrix}
-P_1 B_w \\
P_2 B_w \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
-B_w^T P_1 & B_w^T P_2 & 0 & 0
\end{bmatrix}
\]

(23)

Multiplying both sides of the equation by

\[
\Xi = \begin{bmatrix}
P_1 & 0 & 0 \\
0 & P_2 & 0 \\
0 & 0 & P_3
\end{bmatrix}
= \begin{bmatrix}
P_1^{-1} & 0 & 0 \\
0 & P_2^{-1} & 0 \\
0 & 0 & P_3^{-1}
\end{bmatrix} > 0
\]

yields:

\[
\Xi U \Xi = \begin{bmatrix}
N_{11} & N_{12} & N_{13} \\
N_{21} & N_{22} & N_{23} \\
N_{31} & N_{32} & 0
\end{bmatrix}
+ \gamma^2 \begin{bmatrix}
-B_w & 0 \\
B_w & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
-B_w^T & B_w^T & 0
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
P_1 J^T \\
0 \\
-P_3 J^T
\end{bmatrix}
\begin{bmatrix}
0 & C \hat{P}_2 & -C \hat{P}_3 \\
0 & 0 & 0
\end{bmatrix}
+ \frac{1}{\varepsilon} \begin{bmatrix}
P_1 J^T \\
0 \\
-P_3 J^T
\end{bmatrix}
\begin{bmatrix}
J \hat{P}_2 & 0 & -J \hat{P}_3
\end{bmatrix}
\]

(24)
As seen from Equation (23), the partial monomial in the matrix element contains two unknown variables, which cannot be solved using the LMI toolbox. Using the related transformation in Equation (3.2), the monomial in each element of the matrix can be transformed into a monomial with only one unknown to be solved, so that the constraints (15) and (16) can be solved using the LMI toolbox; in addition, the unknown gain matrices are solved.

Using Schur’s complement, Equation (21) can be transformed into:

$$
\Xi U \Xi = \begin{bmatrix}
N_{11} & N_{12} & N_{13} & 0 & -B_w & \varepsilon G & \hat{P}_1 J^T \\
N_{21} & N_{22} & N_{23} & P_2 C^T & B_w & -\varepsilon G & 0 \\
0 & C P_2 & -C \hat{P}_3 & -I & 0 & 0 & 0 \\
-\varepsilon G^T & B_w^T & 0 & 0 & -\varepsilon^2 I & 0 & 0 \\
J P_1 & 0 & -J \hat{P}_3 & 0 & 0 & 0 & -\varepsilon I
\end{bmatrix}
$$

Combining Equations (22) and (25) yields:

$$
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & 0 & 0 & 0 & 0 \\
M_{21} & M_{22} & M_{23} & 0 & 0 & 0 & 0 \\
M_{31} & M_{32} & 0 & 0 & 0 & 0 & 0
\end{bmatrix} + \begin{bmatrix}
P_1 \Delta A + \Delta A^T P_1 & -\Delta A^T P_2 & -P_1 \Delta A & 0 \\
-P_2 \Delta A & 0 & P_2 \Delta A & 0 \\
-\Delta A^T P & \Delta A^T P_2 & 0 & 0
\end{bmatrix}
$$

$$
< \begin{bmatrix}
N_{11} & N_{12} & N_{13} & 0 & -B_w & \varepsilon G & \hat{P}_1 J^T \\
N_{21} & N_{22} & N_{23} & P_2 C^T & B_w & -\varepsilon G & 0 \\
0 & C P_2 & -C \hat{P}_3 & -I & 0 & 0 & 0 \\
-\varepsilon G^T & B_w^T & 0 & 0 & -\varepsilon^2 I & 0 & 0 \\
J P_1 & 0 & -J \hat{P}_3 & 0 & 0 & 0 & -\varepsilon I
\end{bmatrix}
$$

Therefore,

$$
\dot{V}(t) + J(t) < 0 \tag{27}
$$

Integrating Equation (27) from 0 to $+\infty$:

$$
\lim_{t \to +\infty} \left( V(t) - V(0) + \|y(t)\|_2^2 - \gamma^2 \|\omega(t)\|_2^2 \right) < 0 \tag{28}
$$

with the initial conditions of $V(0) = 0$ and $\lim_{t \to +\infty} V(t) > 0$.

Then,

$$
\|y(t)\|_2^2 - \gamma^2 \|\omega(t)\|_2^2 < 0 \tag{29}
$$

From the property of the $H_\infty$ performance index function in Lemma 1 [24], when the equation (29) is established, the hypersonic vehicle control system (10) is robust and stable with the disturbance attenuation level $\gamma$. The proof is finished.
Remark 2 The initial condition of 0 used in the equation above does not refer to the initial condition at time \( t = 0 \), but to the facts that the hypersonic vehicle does not develop an actuator failure and does not contain uncertain parameters, and that the state of the command signal for stable flight can be effectively tracked when \( c(0) = \tilde{c}(0) = 0 \). Because the Lyapunov function is \( V(t) = \tilde{c}^T(t) P_1 \tilde{c}(t) + e^T(t) P_2 e(t) \), we can obtain \( \lim_{t \to +\infty} V(t) > 0 \) with \( V(0) = 0 \).

3.3. Fault-tolerant controller design with unknown actuator failure. Theorem 1 proposes a fault-tolerant control law for hypersonic vehicles when the actuator failure is known. However, for actual maneuvering conditions, the failure of the actuator and the degree of failure are often unknown. Therefore, the design of a fault-tolerant control system under unknown conditions of actuator failure is considered.

**Theorem 2:** For any given positive scalar \( \gamma > 0 \) and unknown actuator fault parameter matrix \( F \), if there are positive definite symmetric matrices \( P_1, P_2, P_3, \) real matrices \( A_c, P, \bar{K}_x, \bar{K}_r, L, \) diagonal matrices \( S \), and a positive scalar \( \varepsilon > 0 \), the following conditions are established:

\[
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{13} & CP_2 = \bar{P}C \\
Q_{21} & Q_{22} & Q_{23} & 0 \\
Q_{31} & Q_{32} & 0 & -B_w \\
0 & CP_2 & -C \bar{P}_3 & -\varepsilon G \\
-\varepsilon G^T & -\varepsilon G^T & 0 & 0 \\
J \bar{P}_1 & 0 & 0 & 0 \\
\bar{K}_x & \bar{K}_{2,x} & -\bar{K}_{3,x} - \bar{K}_r & 0 \\
\end{bmatrix}
\begin{bmatrix}
\varepsilon G \\
P_1 J^T \\
\bar{K}_r^T \\
\bar{K}_{2,x}^T \\
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 C^T \\
P_3 C^T \\
P_3 \\
\end{bmatrix}
\begin{bmatrix}
\tilde{c} \\
-\varepsilon I \\
-\gamma^2 I \\
\bar{K}_x \\
\end{bmatrix}
\leq 0
\]

where,

\[
Q_{11} = A(\theta) \bar{P}_1 + \bar{P}_1 A^T(\theta) - B(\theta) F_0 \bar{K}_x - \bar{K}_3^T F_0 B^T(\theta) + B(\theta) S B^T(\theta)
\]

\[
Q_{12} = -\bar{P}_1 A^T(\theta) + A_c^T + \bar{K}_3^T (F_0 - I) B^T(\theta) - B(\theta) F_0 \bar{K}_{2,x} - B(\theta) S B^T(\theta)
\]

\[
Q_{13} = B(\theta) F_0 \bar{K}_x + B(\theta) F_0 \bar{K}_x - A(\theta) \bar{P}_3
\]

\[
Q_{21} = -A(\theta) \bar{P}_1 + \bar{A}_c + B(\theta) (F_0 - I) \bar{K}_x - \bar{K}_{2,x}^T F_0 B^T(\theta) - B(\theta) S B^T(\theta)
\]

\[
Q_{22} = \bar{A}_{2,c} + \bar{A}_{2,c}^T + B(\theta) (F_0 - I) \bar{K}_{2,x} + \bar{K}_{2,x}^T (F_0 - I) B^T(\theta)
\]

\[
- C^T L^T - LC + B(\theta) S B^T(\theta)
\]

\[
Q_{23} = A(\theta) \bar{P}_3 - \bar{A}_{3,c} - B(\theta) (F_0 - I) \bar{K}_3 - B(\theta) (F_0 - I) \bar{K}_r
\]

\[
Q_{31} = \bar{K}_3^T F_0 B^T(\theta) + \bar{K}_3^T F_0 B^T(\theta) - \bar{P}_3 A^T(\theta)
\]

\[
Q_{32} = \bar{P}_3 A^T(\theta) - \bar{A}_{3,c}^T - \bar{K}_3^T (F_0 - I) B^T(\theta) - \bar{K}_r^T (F_0 - I) B^T(\theta)
\]

\[
K_x = K_x \bar{P}_1^{-1} = \bar{K}_{2,x} \bar{P}_2^{-1} = \bar{K}_{3,x} \bar{P}_3^{-1}, \bar{K}_r = \bar{K}_r \bar{P}_3^{-1}
\]

\[
A_C = \bar{A}_c \bar{P}_1^{-1} = \bar{A}_{2,c} \bar{P}_2^{-1} = \bar{A}_{3,c} \bar{P}_3^{-1}, L = \bar{L} \bar{P}_3^{-1}
\]

Then, with the observer-based output feedback control law (13), the LPV system (10) remains robust and stable under the disturbance attenuation level \( \gamma \).

**Proof.** When the actuator fault parameter matrix \( F \) is unknown, combining (9) and (16) results in:
where,

\[
Z_{11} = A(\theta) \bar{P}_1 + \bar{P}_1 A^T(\theta) - B(\theta) F_0 \bar{K}_x - \bar{K}_x^T F_0 B^T(\theta)
\]

\[
Z_{12} = -\bar{P}_1 A^T(\theta) + \bar{P}_1 A_c^T + \bar{K}_x^T (F_0 - I) B^T(\theta) - B(\theta) F_0 \bar{K}_{2, x}
\]

\[
Z_{13} = B(\theta) F_0 \bar{K}_{3, x} + B(\theta) F_0 \bar{K}_r - A(\theta) \bar{P}_3
\]

\[
Z_{21} = -A(\theta) \bar{P}_1 + A_c \bar{P}_3 + B(\theta) (F_0 - I) \bar{K}_x - \bar{K}_{2, x}^T F_0 B^T(\theta)
\]

\[
Z_{22} = A_c \bar{P}_2 + \bar{P}_2 A_c^T + B(\theta) (F_0 - I) \bar{K}_{2, x} + \bar{K}_{2, x}^T (F_0 - I) B^T(\theta) - C^T L^T - LC
\]

\[
Z_{23} = A(\theta) \bar{P}_3 - A_c \bar{P}_3 - B(\theta) (F_0 - I) \bar{K}_{3, x} - B(\theta) (F_0 - I) \bar{K}_r
\]

\[
Z_{31} = \bar{K}_{3, x}^T F_0 B^T(\theta) + \bar{K}_r^T F_0 B^T(\theta) - \bar{P}_3 A^T(\theta)
\]

\[
Z_{32} = \bar{P}_3 A^T(\theta) - \bar{P}_3 A_c^T - \bar{K}_{3, x}^T (F_0 - I) B^T(\theta) - \bar{K}_r^T (F_0 - I) B^T(\theta)
\]

Let

\[
P = \begin{bmatrix}
  -B \\
  B \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  K_x^T \\
  K_{2, x}^T \\
  -K_{3, x}^T \\
  -K_r^T \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  -B \\
  B \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  \Delta \\
  \Delta \\
  \Delta \\
  \Delta
\end{bmatrix}
\]

Using Young inequality, the following equation can be obtained:

\[
P < \begin{bmatrix}
  -B \\
  B \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  K_x^T \\
  K_{2, x}^T \\
  -K_{3, x}^T \\
  -K_r^T \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  \bar{K}_x^T \\
  \bar{K}_{2, x}^T \\
  -\bar{K}_{3, x}^T \\
  -\bar{K}_r^T \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\]

\[
S \bar{P}_2 - C \bar{P}_3 - I - \gamma^2 I
\]

\[
S \bar{P}_3 - -\varepsilon I
\]

\[
\varepsilon G^T - \varepsilon G^T
\]
Using the proof form Theorem 1, we can yield:

$$
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{13} & 0 & -B_w & \varepsilon G & P_1J^T & K^T_{x, x} \\
Q_{21} & Q_{22} & Q_{23} & P_2C^T & B_w & -\varepsilon G & 0 & K^T_{x, 2, x} \\
Q_{31} & Q_{32} & 0 & -P_3C^T & 0 & 0 & -P_3J^T & -K^T_{x, 3, x} - K^T_{r, x} \\
0 & C\bar{P}_2 & -C\bar{P}_3 & -I & 0 & 0 & 0 & 0 \\
-B_w^T & \varepsilon G^T & 0 & 0 & -\gamma^2 I & 0 & 0 & 0 \\
-J\bar{P}_1 & 0 & -J\bar{P}_3 & 0 & 0 & -\varepsilon I & 0 & 0 \\
\bar{K}_x & \bar{K}_{2,x} & -\bar{K}_{3,x} - \bar{K}_r & 0 & 0 & 0 & 0 & -S\hat{F}^2 \\
\end{bmatrix} < 0
$$

(35)

The proof is finished.

**Remark 3:** For the inequality constraints (16) of Theorem 1 and (31) of Theorem 2, the Matlab LMI toolbox can be used to obtain the results. For the equation constraint conditions of Equations (15) and (30), they can be transformed into the corresponding LMI conditions [25]-[27] as follows:

$$
\begin{bmatrix}
\beta I & C\hat{P}_2 - \hat{P}C \\
P_2C^T & C^T\hat{P}^T & \beta I
\end{bmatrix} > 0
$$

(36)

In addition, they can satisfy the minimum value of $\beta$; then, the inequalities can be solved using the Matlab LMI toolbox [28]-[30].

4. **Numerical simulation.** For the dynamic model of hypersonic vehicle given by Equations (1) and (2), the design structure of the state observer-based hypersonic vehicle LPV fault-tolerant controller designed in this paper is shown in Figure 2.

![Figure 2. Structure diagram of control system](image)

It is assumed that the hypersonic vehicle starts maneuvering at the $t = 20$ s, and changes from the equilibrium states $V_0 = 4200$ m/s, $\alpha_0 = 0.978$ deg, $\gamma_0 = 0$ deg, $q_0 = 0$ deg/s, $h_0 = 27$ km to the equilibrium states $V = 4525$ m/s, $\alpha = 0.978$ deg, $\gamma = 0$ deg, $q_0 = 0$ deg/s, $h = 30$ km. When there are parameter uncertainty and actuator failure, it is assumed that the uncertainty range of the angle of attack in the uncertain parameter $C_M(\alpha)$ is $|\Delta\alpha| < 0.1$. It is assumed that the upper
DESIGN OF LPV FAULT-TOLERANT CONTROLLER FOR HYPERSONIC VEHICLE

and lower limits of the actuator failure are $f_i = 1$, $f_i = 0.2$, respectively. Let the external disturbance signal be $\omega(t) = 10 \sin(2000t)$, and the disturbance matrix be $B_\omega = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 & 0 \end{bmatrix}^T$. It is known from actual scenario that the fifth state quantity $q$ of the hypersonic vehicle cannot be directly observed; therefore, $C = [I_4 \ 0]$. If the traditional state feedback control law is adopted, the flight states of the hypersonic vehicle are shown as Figures 3-6.

It can be seen from Figures 3-6 that the control system is divergent in the presence of parameter uncertainties and actuator failures. The traditional state feedback control law cannot maintain vehicle stability and requires further fault-tolerant controller design.

Let the disturbance attenuation level be $\gamma = 1.2$ in the $H_\infty$ performance index function. By solving Equations (30) and (31) using the Matlab LMI toolbox, the unknown parameter matrices in the designed control system can be obtained as follows:
The simulation result is shown in Figures 7-12. Figures 7-10 show the command signals tracking using the state observer-based fault-tolerant control method in the presence of parameter uncertainty, actuator failure, and external disturbances. Figures 11 and Figure 12 show the changes in the control inputs. It can be seen from the simulation results that the maneuvering states can stably track the command signal. Even in the presence of external disturbances, the control system can remain stable and robust. Therefore, the effectiveness of the state observer-based LPV fault-tolerant control system proposed and designed in this paper is proved.

5. Conclusion. In this paper, a typical hypersonic vehicle model was studied, and a state observer-based hypersonic vehicle LPV fault-tolerant controller was designed. The altitude and Mach number were selected as the scheduling variables and space gridded to obtain the equilibrium state working points. The Jacobian
linearization was used at the equilibrium points. Along with the scenario of occurrence of partial failure of the actuator during maneuvering of the hypersonic vehicle, the LPV model with uncertain parameters and actuator failure under external disturbance was obtained. Because the pneumatic sensor is prone to failure during flight, the actual state is difficult to measure; therefore, a fault-tolerant control law based on the state observer was introduced. Considering Lyapunov’s theorem and $H_\infty$ performance index function, the inequality constraints satisfying the stability and robustness of the system were derived and the corresponding gain matrix was solved using Matlab LMI. The preliminary simulation verification using Simulink indicated that the hypersonic vehicle can still track the command signal stably and that the system is robust even in the event of partial failure of the actuator.

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