Turbulent Combustion in Type Ia Supernova Models

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Abstract
We review the astrophysical modeling of type Ia supernova explosions and describe numerical methods to implement numerical simulations of these events. Some results of such simulations are discussed.

1.1 Astrophysical and numerical models
Type Ia supernovae (SNe Ia) are among the brightest and most energetic explosions observed in the Universe. For a short time they can outshine an entire galaxy consisting of some hundred billions of stars. Assuming that SNe Ia originate from a single stellar object, only two sources of explosion energy come into consideration: the gravitational binding energy of the star and its nuclear energy. Since for the particular class of SNe Ia no compact object is found in the remnant, they are usually associated with thermonuclear explosions of white dwarf (WD) stars consisting of carbon and oxygen. The currently favored astrophysical model assumes it to be part of a binary constellation and to accrete matter from the companion until it comes close to the limiting Chandrasekhar mass. At this stage, the central density of the WD reaches values at which nuclear burning of carbon towards heavier elements ignites. After a simmering phase of several hundreds of years, a thermonuclear runaway in a tiny region close to the center leads to the formation of a thermonuclear flame.

The astrophysical interest in SNe Ia is – among other things – founded on their relevance for cosmology. On the basis of an empirical calibration relating their peak luminosities with the shapes of their lightcurves they are a suitable tool to determine cosmological distances. The geometrical survey of the Universe performed in this way led to one of the greatest surprises of modern astrophysics pointing to the fact that the Universe is predominantly made of a so far unknown “dark energy” component. SNe Ia distance measurements may in the future possibly contribute to the determination of the equation of state of this dark energy. However, the empirical calibration applied here urgently calls for a theoretical explanation and ongoing SN Ia cosmology projects crucially depend on increasing the accuracy of the measurements by getting a handle on the systematic errors. This is achievable only on the basis of a better understanding of the mechanism of SN Ia explosions.

To this end, we attempt to model SN Ia explosions from “first principles” in conjunction with detailed comparison with observations of nearby objects. The goal is to construct numerical models as parameter-free as possible.

Such a SN Ia explosion model has to describe the propagation of the thermonuclear flame from the WD’s center outwards. Hydrodynamics in principle allows for two distinct modes here. One is the so-called deflagration mode, in which the subsonic flame is mediated by the thermal conduction of the degenerate electrons, and the other is a supersonic detonation in which the flame is driven by sound waves.
A prompt detonation has been ruled out as a valid model for SNe Ia, since the entire star is incinerated with sound speed here. Therefore the material has no time to pre-expand and is burned at high densities where the nuclear reactions terminate in iron group elements. This is in disagreement with observations showing that intermediate mass elements need to be produced as well. Hence, the flame must start out subsonically in the deflagration mode. However, a laminar deflagration flame is much too slow to release sufficient energy to explode the star. The main issue of SN Ia models is thus to identify mechanisms to accelerate the flame propagation.

This is the point where turbulence comes into play. The interaction of the flame with turbulent motions defines burning in SNe Ia as a problem of turbulent combustion. The flame propagating from the center of the star outwards produces an inverse density stratification in the gravitational field of the WD leaving light and hot nuclear ashes behind while the fuel in front of it is dense and cold. The resulting Rayleigh-Taylor instability leads to the formation of burning bubbles that buoyantly rise into the fuel. The shear flows at the interfaces of these bubbles are characterized by a Reynolds number of about $10^{14}$ and the Kelvin-Helmholtz instability generates turbulent eddies. These decay in a turbulent energy cascade and the flame interacts with eddies on a wide range of scales. In this way, the flame becomes corrugated and its surface area is enlarged. This enhances the net burning rate and accelerates the flame propagation. A later transition of the flame propagation mode is still hypothetical and not further discussed here.

For a numerical implementation of the deflagration SN Ia model, the scale down to which the flame interacts with turbulent motions has to be considered. This is the so-called Gibson scale, at which turbulent velocity fluctuations of the cascade reach values comparable with the laminar flame speed. At the beginning of the explosion (the WD star has a radius of about 2000 km and ignites inside the first $\sim 100$ km), the Gibson scale is of the order of $10^4$ cm. The flame width, however, is less than a millimeter. Due to this huge scale separation, turbulent eddies interact with the flame only in a kinematic way but leave the internal flame structure unaffected. Thus, burning proceeds in the so-called flamelet regime of turbulent combustion for most parts of the explosion process. With three-dimensional simulations on scales of the WD star, it is possible to reach resolutions down to less than a kilometer. Of course, these simulations need to take into account effects of turbulence on smaller (unresolved) scales, which is implemented via a sub-grid scale model (cf. the contribution of W. Schmidt et al.). Complementary small-scale simulations are provided to test the assumptions of flame propagation around and below the Gibson scale.

One has to keep in mind, however, that the explosion process takes place on an expanding background. Due to the energy release, the WD expands. With lower fuel densities, the flame structure broadens and the laminar flame speed decreases [1]. Therefore the Gibson scale becomes smaller and eventually, in the very late phases of the explosion, turbulent eddies may be capable of penetrating the flame structure so that the distributed burning regime is entered.

The numerical implementation of the outlined SN Ia model on scales of the WD star follows [2] in a large eddy simulation (LES) approach. The resolved hydrodynamics is described by the PROMETHEUS implementation [3] of a higher-order Godunov scheme. Turbulence on unresolved scales is taken into account with a sub-grid scale model. Seen from scales of the WD, the flame appears as a sharp discontinuity separating the fuel from the ashes. Its propagation is modeled via the level set method [4], where the flame velocity is set by the physics of the flamelet regime. Here, flame propagation completely decouples from the microphysics
of the burning and is determined by the turbulent velocity fluctuations on the grid scale which are known from the sub-grid scale model. The nuclear reactions are implemented in the simplified approach of [5].

1.2 Results

Numerical simulations on the basis of the outlined model have been shown to lead to explosions of the WD star. A flame ignited near the center of the star (cf. top left panel of Fig. 1) develops the typical “mushroom”-like features due to buoyancy instabilities (cf. top right panel of Fig. 1). It becomes increasingly wrinkled and the generated turbulence accelerates the flame propagation. In this way, the flame incinerates considerable fractions of the material (cf. bottom left panel of Fig. 1) and the energy release is sufficient to gravitationally unbind the WD star. A snapshot of the density structure of the remnant after the burning has ceased is shown in the bottom right panel of Fig. 1, where the imprints of turbulent burning are clearly visible. The most vigorously exploding model so far released about $7 \times 10^{51}$ erg of energy [7]. Another important global quantity to assess the explosion process is the mass of produced $^{56}$Ni, because its radioactive decay powers the visible event. In the mentioned simulation, $0.4 M_\odot$ of $^{56}$Ni were obtained. Both values are within the range of expectation.
from observations, albeit on the low side. First synthetic light curves have been derived from explosion simulations [8] and compare well with observations. However, current deflagration models of SNe Ia seem to have difficulties reproducing observed spectra. A spectrum of the late (“nebular”) phase at day 350 after explosion was recently derived [9] from a very simple simulation. Although reproducing the broad iron lines of observed spectra well, it showed strong indication of unburnt material at low velocities which is not seen in the observations. Both features, however, share a common origin. The rising bubbles filled with ashes distribute iron group elements over a wide range in velocity space and thus give rise to broad iron lines. At the same time, downdrafts in between these bubbles transport unburnt material towards the center producing the strong oxygen and carbon lines which are absent in the observations.

This problem may in part be attributed to the simplicity of the underlying explosion simulation. It was performed on only one octant of the star with rather low resolution and the flame was ignited in a very artificial shape. Nonetheless, it seems likely that physical ingredients are still missing in the explosion model. In particular, burning at late phases was ignored as yet. Fuel consumption was ceased when the flame reached densities of unburnt material below $10^7 \text{g cm}^{-3}$, because the distributed burning regime is expected to be entered here. A recent approach [10], however, modeled the transition between the turbulent burning regimes by assuming flamelet scaling for the flame propagation velocity above this density threshold and by applying Damköhler’s limit for the thin reaction zone regime [11] below. The result strongly supports the conjecture that an implementation of burning at low densities may help to cure current problems of the deflagration SN Ia model.

References

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