Meta-Learning with Adaptive Layerwise Metric and Subspace

Yoonho Lee and Seungjin Choi
Department of Computer Science and Engineering
Pohang University of Science and Technology
77 Cheongam-ro, Nam-gu, Pohang 37673, Korea
{einet89, seungjin}@postech.ac.kr
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Abstract

Recent advances in meta-learning demonstrate that deep representations combined with the gradient descent method have sufficient capacity to approximate any learning algorithm. A promising approach is the model-agnostic meta-learning (MAML) which embeds gradient descent into the meta-learner. It optimizes for the initial parameters of the learner to warm-start the gradient descent updates, such that new tasks can be solved using a small number of examples. In this paper we elaborate the gradient-based meta-learning, developing two new schemes. First, we present a feedforward neural network, referred to as $T$-net, where the linear transformation between two adjacent layers is decomposed as $TW$ such that $W$ is learned by task-specific learners and the transformation $T$, which is shared across tasks, is meta-learned to speed up the convergence of gradient updates for task-specific learners. Second, we present $MT$-net where gradient updates in the T-net are guided by a binary mask $M$ that is meta-learned, restricting the updates to be performed in a subspace. Empirical results demonstrate that our method is less sensitive to the choice of initial learning rates than existing meta-learning methods, and achieves the state-of-the-art or comparable performance on few-shot classification and regression tasks.

1 Introduction

Recent deep learning methods seem to demonstrate intelligence, achieving remarkable results on many tasks including image classification [14] or playing games [19]. However, one shortcoming of current deep learning methods is that they require copious amounts of data. As opposed to ‘big data’ applications, many problems of interest are ‘small data’ problems, where a learner must learn with very little data. The main challenge with such problems with little data is that there is insufficient information to make conclusions and thus some inductive bias is required to learn effectively.

Learning to learn, or meta-learning refers to a set of methods in which a model learns about the learning process itself [24, 25, 28]. This line of research typically casts learning as a two-level process with different scopes. The meta-learner uses information gathered from several instances of fast learners to learn a ‘good’ learning rule.

A straightforward approach for applying meta-learning to deep networks is to meta-learn some pattern on the weights of a neural network or its updates; previous work has learned an update rule [21, 16, 2], directly generated weights [8], or learned good initial weights to fine-tune [6]. Another line of research uses recurrent neural networks to encode a learning rule and inserts the small amount of training data as sequential inputs [23, 18, 20, 3, 31]. An approach that has been successful in the domain of few-shot classification is to learn a
distance metric between images and use this metric to find the most similar previously seen image to a new image [13, 30, 26].

Our primary contribution is the MT-net (Figure 1), a neural network architecture and training method that lets a meta-learner learn what and how to learn. In our proposed method, the meta-learner decides the initial parameters of task-specific learners. Unlike previous work, our method additionally learns which weights task-specific learners should update, thereby learning how many degrees of freedom are required to adapt to a new task. Note that in Figure 1, the activation space has 3 degrees of freedom, but task-specific learning (colored lines) happens on a meta-learned subspace (black plane) with 2 degrees of freedom. Additionally, meta-learned parameters (T) alter the geometry of the task-specific learner’s activation space to enable faster task-specific learning.

2 Background

2.1 Problem Setup

We briefly explain the meta-learning problem setup which is applied to few-shot tasks (regression, classification, and reinforcement learning).

The problems of k-shot regression and classification are as follows. In the training phase for a meta-learner, we are given a (possibly infinite) set of tasks {T_1, T_2, T_3, ...}. Each task provides a training set and a test set \( \{D_{T_i,\text{train}}, D_{T_i,\text{test}}\} \). We assume here that the training set \( D_{T_i,\text{train}} \) has k examples, hence the name k-shot learning. A particular task \( T \in \{T_1, T_2, T_3, ...\} \) is assumed to be drawn from the distribution of tasks \( p(T) \).

Given a task \( T \sim p(T) \), the task-specific model \( f_{\theta_T} \) (a feedforward neural network is considered in this paper) parameterized by \( \theta_T \) is trained using the dataset \( D_{T,\text{train}} \) and its corresponding loss \( L_T(\theta_T, D_{T,\text{train}}) \).

Denote by \( \tilde{\theta}_T \) parameters obtained by optimizing \( L_T(\theta_T, D_{T,\text{train}}) \). Then, the meta-learner \( f_\theta \) is updated using the feedback from the collection of losses \( \{L_T(\tilde{\theta}_T, D_{T,\text{test}})\}_{T \sim p(T)} \), where the loss of each task is evaluated using the test data \( D_{T,\text{test}} \). Given a new task \( T_{\text{new}} \) (not considered during meta-training), the meta-learner helps the model \( f_{\theta_{T_{\text{new}}}} \) to quickly adapt to the new task \( T_{\text{new}} \), by warm-starting the gradient updates.

Few-shot reinforcement learning is similar but has one difference: each task \( T \) is a single Markov decision process (MDP) rather than a tuple consisting of train and test sets. In a k-shot reinforcement learning problem, the agent \( f_{\theta_T} \) interacts with the environment for k episodes, after which the parameters are denoted \( \tilde{\theta}_T \). The meta-learner is trained to maximize the expected return of \( \tilde{\theta}_T \) in the MDP \( T \).

2.2 Model-Agnostic Meta-Learning

We briefly review the model-agnostic meta-learning (MAML) [5], emphasizing common characteristics and differences in MAML and our method. MAML is a meta-learning method that applies to any model that resorts to gradient updates for learning. This method is loosely inspired by fine-tuning, and it learns initial parameters of a network such that the network’s loss after a few gradient steps is minimized.

Let us consider a model that is parameterized by \( \theta \). MAML alternates between the two updates (1) and (2) to determine initial parameters \( \theta \) for task-specific learners to warm-start the gradient descent updates, such that new tasks can be solved using a small number of examples. Each task-specific learner updates its parameters by the gradient update (1), using the loss evaluated with the data \( \{D_{T,\text{train}}\} \). The meta-optimization across tasks is performed such that the parameters \( \theta \) are updated using the loss evaluated with \( \{D_{T,\text{test}}\} \), which is given in (2).
Figure 1: Activation (y) space of an MT-net. Task-specific learners perform gradient descent (∇y) with respect to a task (T_i). The model learns a subspace (black plane) to do gradient descent (colored lines) on. The activation space is more well-behaved from the task-specific learner’s point of view (bottom right), due to a meta-learned transformation matrix (T). Best seen in color.

\[ \tilde{\theta}_T \leftarrow \theta - \alpha \nabla_\theta \mathcal{L}_T (\theta, D_{T,train}) \]  

(1)

\[ \theta \leftarrow \theta - \beta \nabla_\theta \left( \sum_{T \sim p(T)} \mathcal{L}_T (\tilde{\theta}_T, D_{T,test}) \right) \]  

(2)

where \( \alpha > 0 \) and \( \beta > 0 \) are learning rates and the summation in (2) is computed using minibatches of tasks sampled from \( p(T) \).

Intuitively, a well-learned initial parameter \( \tilde{\theta}_T \) is close to some local optimum for every task \( T \sim p(T) \). Furthermore, the update (1) is sensitive to task identity in the sense that \( \tilde{\theta}_{T_1} \) and \( \tilde{\theta}_{T_2} \) have different behaviors for different tasks \( T_1, T_2 \sim p(T) \).

Recent work has shown that gradient-based optimization is a universal learning algorithm [6], in the sense
that any learning algorithm can be approximated up to arbitrary accuracy using some parameterized model and gradient descent. Thus, no generality is lost by only considering gradient-based learners as in (1).

Our method is similar to MAML in that our method also differentiates through gradient update steps to optimize performance after fine-tuning. However, while MAML updates all parameters in $\theta$ to make $\hat{\theta}_T$, our method only alters a (meta-learned) subset of its weights. Furthermore, whereas MAML learns with standard gradient descent, a subset of our method’s parameters effectively ‘warp’ the parameter space of the parameters to be learned during meta-testing to enable faster learning.

Figure 2: A diagram of the adaptation process of a Transformation Network (T-net). Blue values are meta-learned and shared across all tasks. Orange values are different for each task.

3 Meta-Learning Models

We present our two models in this section: Transformation Networks (T-net) and Mask Transformation Networks (MT-net), both of which are trained by gradient-based meta-learning. Both of these methods enable the model to adapt to a new task using a small number of labeled examples.

A T-net (which is a special case of an MT-net) is capable of embedding information about a metric in its activation space; this metric informs each task-specific learner on what gradient direction to take and the step size to take in that direction. An MT-net additionally learns which subset of its weights to update for task-specific learning. Therefore, an MT-net learns to automatically assign one of two roles (task-specific or task-mutual) to each of its weights.

Throughout this section, we assume that the few-shot meta-learning task at hand is regression or classification. Adapting these two models to the few-shot reinforcement learning setting involve simple changes.
Algorithm 1 Transformation Networks (T-net)

Require: \( p(T) \)

Require: \( \alpha, \beta \)

1: randomly initialize \( \theta \)

2: while not done do

3: Sample batch of tasks \( T_i \sim p(T) \)

4: for all \( T_j \) do

5: for \( i = 1, \ldots, L \) do

6: \( \tilde{W}_{i,T_j} = W_i - \alpha \nabla_{W_i} L_{T_j}(\theta_W, \theta_T, D_{T_j,\text{train}}) \)

7: end for

8: \( \tilde{\theta}_{W,T_j} = \{ \tilde{W}_1^{T_j}, \ldots, \tilde{W}_L^{T_j} \} \)

9: end for

10: \( \theta \leftarrow \theta - \beta \nabla_\theta \sum_{T} L_T(\tilde{\theta}_{W,T_j}, \theta_T, D_{T,\text{test}}) \)

11: end while

3.1 T-net

We consider a model \( f_\theta(x) \), parameterized by \( \theta \), that is a \( L \)-layer feedforward neural network

\[
f_\theta(x) = T^L W^L (\sigma (T^{L-1} W^{L-1} (\ldots \sigma (T^1 W^1 x)))) ,
\]

where \( x \in \mathbb{R}^D \) is an input, and \( \sigma(\cdot) \) is a nonlinear activation function. Note that we parameterize connections between two adjacent layers as the product of two matrices \( T^i W^i \). Each \( T^i \) is a square matrix. Model parameters \( \theta \) are a collection of \( W \)'s and \( T \)'s, i.e.,

\[
\theta = \left\{ W_1, \ldots, W_L, T_1, \ldots, T_L \right\} .
\]

Parameters \( \theta_T \), which are shared across task-specific models, are determined by the meta-learner. All task-specific learners have the same initial \( \theta_W \) but update to different values since each uses their corresponding train set \( D_{T,\text{train}} \). Thus we denote such (adjusted) parameters for task \( T \) as \( \tilde{\theta}_{W,T} \). Though they may look similar, \( T \) denotes tasks whereas \( T \) denotes transformation weights. Both task-specific learners \( f_{\theta_T} \) and the meta-learner \( f_\theta \) are parameterized as (3). Note that parameters \( \tilde{\theta}_{W,T} \) are altered by task-specific learners while parameters \( \theta_T \) are determined solely by the meta-learner.

Given a task \( T \) sampled from \( p(T) \), \( \tilde{\theta}_{W,T} \) are adjusted by the gradient update

\[
\tilde{\theta}_{W,T} \leftarrow \theta_W - \alpha \nabla_{\theta_W} L_T(\tilde{\theta}_{W,T}, \theta_T, D_{T,\text{train}}) .
\]

Using the task-specific learners \( \tilde{\theta}_{W,T} \), the meta-learner improves itself with the gradient update

\[
\theta \leftarrow \theta - \beta \nabla_\theta \left( \sum_{T \sim p(T)} L_T(\tilde{\theta}_{W,T}, \theta_T, D_{T,\text{test}}) \right) .
\]

\( \alpha > 0 \) and \( \beta > 0 \) are learning rate hyperparameters. We show our full algorithm in Algorithm 1.
Suppose that we are given a new task $T_*$ with the training set $D_{T_*\text{train}}$. The model parameters $\theta_{W,T_*}$ are updated by (4), where the gradient update starts from the initial value $\theta_{W}$ that was determined by the meta-learner.

We now briefly examine the mapping between two adjacent layers, i.e.,

$$y = TWx,$$

where $x$ is the input to the layer, $y$ is the pre-activation ($f(y)$ is the output of the layer). We omit superscripts in $T$ and $W$ since the same type of mapping is applied to each layer. The squared length of a small increment of the pre-activation $dy$ is calculated as

$$\|dy\|^2 = (dWx)\top (T\top T) (dWx).$$  \hspace{1cm} (6)

We see here that the magnitude of $dy$ is determined by the interaction between $dWx$ and a metric matrix $T\top T$. Since a task-specific learner performs gradient descent only on $W$ and not $T$, the change in $y$ resulting from (4) is guided by the meta-learned metric $T\top T$. We provide a further mathematical investigation of this behavior in a later section.

3.2 MT-net

The MT-net is built on the same feedforward model (3) as the T-net:

$$f_\theta(x) = T^L W^L (\sigma (T^{L-1} W^{L-1} (\ldots \sigma (T^1 W^1 x)))),$$  \hspace{1cm} (7)

where the MT-net differs from the T-net is in the binary mask applied to the gradient update to determine which parameters are to be updated. The update rule for task-specific parameters $\tilde{W}_T$ is given by

$$\tilde{W}_T \leftarrow W - \alpha M \odot \nabla W \mathcal{L}(\theta_{W,T_*}, D_{T_*\text{train}}),$$  \hspace{1cm} (8)
Algorithm 2 Mask Transformation Networks (MT-net)

Require: $p(T)$

Require: $\alpha, \beta$

1: randomly initialize $\theta$
2: while not done do
3: Sample batch of tasks $T_i \sim p(T)$
4: for all $T_j$ do
5: for $i = 1, \ldots, L$ do
6: Sample binary mask $M^i$ according to (11)
7: $\tilde{W}^{i,j} = W^{i} - \alpha M^{i} \odot \nabla W, L_T(\theta_W, \theta_T, D_{T_j,t})$
8: end for
9: $\tilde{\theta}_{W,T_j} = \{\tilde{W}^{1,j}, \ldots, \tilde{W}^{L,j}\}$
10: end for
11: $\theta \leftarrow \theta - \beta \nabla_{\theta} \sum_{j} L_T(\tilde{\theta}_{W,T_j}, \theta_T, \theta_\zeta, D_{T,\text{test}})$
12: end while

where $\odot$ is the Hadamard (elementwise) product between matrices of the same dimension and $M$ is a binary ‘gradient mask’ which is sampled each time the task-specific learner encounters a new task. We omit superscripts for layer index since each layer uses the same learning rule (8). Each row of $M$ corresponds to all-ones vector 1 or all-zeros vector 0. We parameterize the probability of row $j$ in $M$ being 1 is using a scalar variable $\zeta_j$. We sample a new $M$ each time we adapt to a task:

$$M = [m_1, \ldots, m_n]^T,$$

$$m_j^T \sim \text{Bern}\left(\frac{\exp (\zeta_j)}{\exp (\zeta_j) + 1}\right) 1^T,$$  \tag{9}

where $\text{Bern}(p)$ denotes the Bernoulli distribution with the parameter $p$ representing the probability that the corresponding binary random variable is one. Each logit $\zeta_j$ acts on a row of a weight matrix $W$, so weights that contribute to the same immediate activation are updated or not updated together.

We backpropagate through the Bernoulli sampling of masks using the Gumbel-Softmax estimator [10]:

$$g_1, g_2 \sim \text{Gumbel}(0, 1),$$  \tag{10}

$$m_j^T \leftarrow \frac{\exp \left( \frac{\zeta_j + g_1}{T} \right)}{\exp \left( \frac{\zeta_j + g_1}{T} \right) + \exp \left( \frac{g_2}{T} \right)} 1^T,$$  \tag{11}

where $T$ is a temperature hyperparameter. This reparameterization allows us to directly backpropagate through the mask, which at the limit of zero temperature, follows the behavior of (9).

As in T-nets, we denote the collection of altered weights as $\tilde{\theta}_{W,T}$:

$$\tilde{\theta}_{W,T} = \{\tilde{W}^1_T, \ldots, \tilde{W}^L_T\} \tag{12}$$

The meta-learner learns all parameters $\theta$:

$$\theta = \left\{ W^1, \ldots, W^L, T^1, \ldots, T^L, \zeta^1, \ldots, \zeta^L \right\}.$$

$$\left\{ \theta_W, \theta_T, \theta_\zeta \right\}.$$  \tag{13}
As in a T-net, the meta-learner performs stochastic gradient descent on $\mathcal{L}_T(\hat{\theta}_{\mathbf{W}, T}, \theta_T, \theta_{\zeta}, D_{T, \text{test}})$:

$$\theta \leftarrow \theta - \beta \nabla_{\theta} \left( \sum_{T \sim p(T)} \mathcal{L}_T(\hat{\theta}_{\mathbf{W}, T}, \theta_T, \theta_{\zeta}, D_{T, \text{test}}) \right).$$  \hspace{1cm} \text{(14)}

The full algorithm is shown in Algorithm 2.

Note that the binary mask used for task-specific learning ($\mathbf{M}$) is determined by meta-learned parameter weights ($\zeta$). Since the meta-learner optimizes the loss in a task after a gradient step (8), the matrix $\mathbf{M}$ gets assigned a high probability of having value 1 for weights that encode task-specific information. Furthermore, since we update $\mathbf{M}$ along with model parameters $\mathbf{W}$ and $\mathbf{T}$, the meta-learner is expected to learn configurations of $\mathbf{W}$ and $\mathbf{T}$ in which there exists a clear divide between task-specific and task-mutual neurons.

4 Analysis

In this section, we provide further analysis of the update schemes of T-nets and MT-nets.

One uncommon thing we do here is that in a layer $y = \mathbf{A}x$, we focus on the space of $y$ instead of $\mathbf{A}$. This is because when thinking about gradients with respect to a loss function, the two are equivalent. Note that the influence of $\mathbf{A}$ on the loss function $\mathcal{L}_T$ is bottlenecked by $y$. The chain rule shows that $\nabla_{\mathbf{A}} \mathcal{L}_T = (\nabla_y \mathcal{L}_T) x^\top$. Assuming $x$ is fixed, the space of possible $\nabla_{\mathbf{A}} \mathcal{L}_T$ under all loss functions is isomorphic to $\nabla_y \mathcal{L}_T$, which is in turn isomorphic to $\mathbb{R}^n$ (since $\mathbf{x}$ is the dimension of $y$). We take advantage of this fact by learning a full-rank $(n \times n)$ metric in the space of $y$, doing this in the space of $\mathbf{A}$ would be unreasonable for most architectures.

4.1 T-nets Learn a Metric in Activation Space

We consider a layer in a T-net where the pre-activation value $y$ is given by

$$y = \mathbf{T} \mathbf{W} x = \mathbf{A} x,$$  \hspace{1cm} \text{(15)}

where $\mathbf{A} = \mathbf{T} \mathbf{W}$ and $x$ is the input to the layer. The subsequent analysis is layer-independent, so superscripts to denote layers are omitted.

A standard feedforward network resorts to the gradient of a loss function $\mathcal{L}_T$ (which involves a particular task $T \sim p(T)$) with respect to the parameter matrix $\mathbf{A}$, to update model parameters. In such a case, a single gradient step yields

$$y^\text{new} = (\mathbf{A} - \alpha \nabla_{\mathbf{A}} \mathcal{L}_T) x$$
$$= y - \alpha \nabla_{\mathbf{A}} \mathcal{L}_T x.$$  \hspace{1cm} \text{(16)}

In a T-net, the update (4) leads to the following new value of $y$ (with $\mathbf{T}$ fixed):

$$y^\text{new} = \mathbf{T} (\mathbf{T}^{-1} \mathbf{A} - \alpha \nabla_{\mathbf{T}^{-1} \mathbf{A}} \mathcal{L}_T) x$$
$$= y - \alpha (\mathbf{T} \mathbf{T}^\top) \nabla_{\mathbf{A}} \mathcal{L}_T x,$$  \hspace{1cm} \text{(17)}

where $\mathbf{T}$ is determined by the meta-learner. Thus, in the T-net, the incremental change of $y$ is proportional to the negative of the gradient $(\mathbf{T} \mathbf{T}^\top) \nabla_{\mathbf{A}} \mathcal{L}_T$, while the standard feedforward net resorts to a step proportional to the negative of $\nabla_{\mathbf{A}} \mathcal{L}_T$. Task-specific learning in the T-net is guided by a full rank metric in each layer’s activation space, which is determined by the transformation matrix $\mathbf{T}$ of in each layer. This metric $(\mathbf{T} \mathbf{T}^\top)^{-1}$ warps (scaling, rotation, etc.) the activation space of the model so that in this warped space, a single gradient step with respect to the loss of a new task yields parameters that are well suited for that task.
4.2 MT-nets Learn a Subspace with a Metric

We now consider MT-nets and analyze what their update (8) means from the viewpoint of $y = TWx = Ax$.

MT-nets can restrict its task-specific learner to any subspace of its gradient space:

**Proposition 1.** Fix $x$ and $A$. Let $U$ be a $d$-dimensional subspace of $\mathbb{R}^n$ ($d \leq n$). There exist configurations of $T, W,$ and $\zeta$ such that the span of $y_{\text{new}} - y$ is $U$ while satisfying $A = TW$.

**Proof.** See Appendix B.1. \hfill $\square$

What this proposition means is that by changing $W, T,$ and $\zeta,$ the meta-learned can instruct task-specific learners to only change $y$ within a subspace of the possible values of $y$ (which is $\mathbb{R}^n$), and this can be any subspace of $\mathbb{R}^n$. Note that this construction is only possible because of the transformation $T$; if we only had binary masks $M$, we would only be able to restrict gradients to axis-aligned subspaces.

In addition to learning a subspace that we project gradients onto $(U)$, we are also learning a metric in this subspace. We first provide an intuitive exposition of this idea.

The new value of $y$ after a standard gradient step is, again,

$$y_{\text{new}} = (A - \alpha \nabla_A L_T)x$$

$$= y - \alpha \nabla_A L_T x.$$ (18)

The output of a corresponding layer in an MT-net after one update is

$$y_{\text{new}} = T((T^{-1} A - \alpha M \odot \nabla_{T^{-1} A} L_T)x)$$

$$= y - \alpha T (M \odot (T^T \nabla_A L_T))x.$$ (20)

Since $M$ is an $n \times m$ matrix,

$$M = [m_1, \ldots, m_n]^\top,$$ (22)

where each $m_i$ is a vector of size $m$ which has all one or all zeros as elements. Define $M_T$ as

$$M_T = [m_1^\top, \ldots, m_n^\top]^\top,$$ (23)

where each $m_i^\top$ has the same elements as $m_i$ but is of length $n$, so $M_T$ is a square matrix.

Since $M$ and $M_T$ are binary matrices with same values for columns,

$$y_{\text{new}} = y - \alpha T(M \odot (T^T \nabla_A L_T))x$$

$$= y - \alpha T(M_T \odot T^T) \nabla_A L_T x$$

$$= y - \alpha (T \odot M_T)(M_T \odot T^T) \nabla_A L_T x.$$ (24)

Let’s denote $T_M = M_T \odot T^T$. We see that the update of a task-specific learner in an MT-net performs the update $T_M \nabla_A L_T$. Since $T_M M_T$ is an $n \times n$ matrix that only has nonzero elements in rows and columns where $m$ is 1, by setting appropriate $\zeta$, we can use $T_M M_T$ as a full-rank $d \times d$ metric tensor.

We give a formal statement how MT-nets induce a metric here.

**Proposition 2.** Fix $x, A,$ and a loss function $L_T$. Let $U$ be a $d$-dimensional subspace of $\mathbb{R}^n$, and $g(\cdot, \cdot)$ a metric tensor on $U$. There exist configurations of $T$, $W$, and $\zeta$ such that the vector $y_{\text{new}} - y$ is in the steepest direction of descent on $L_T$ with respect to the metric $du$.

**Proof.** See Appendix B.2. \hfill $\square$

In summary, not only are we learning to project gradients onto arbitrary subspaces of the pre-activation $(y)$ space, but we are also learning a metric in that subspace and thereby learning a low-dimensional linear embedding of the activation space in which local optima with respect to the set of tasks in a given distribution $p(T)$ are easy to reach.
Table 1: Loss on sine wave regression. Networks were meta-trained using 10-shot regression tasks. Reported losses were calculated after adaptation using various numbers of examples.

| Models      | 5-shot     | 10-shot    | 20-shot    |
|-------------|------------|------------|------------|
| MAML[5]     | 1.07 ± 0.11| 0.71 ± 0.07| 0.50 ± 0.05|
| Meta-SGD[17]| 0.88 ± 0.14| 0.53 ± 0.09| 0.35 ± 0.06|
| T-net       | 0.83 ± 0.08| 0.56 ± 0.06| 0.38 ± 0.04|
| MT-net      | **0.76 ± 0.09** | **0.49 ± 0.05** | **0.33 ± 0.04** |

5 Related Work

A successful line of research in few-shot learning uses feedforward neural networks as learners. These approaches learn update rules [21, 16, 2] or directly generate weights [8]. A closely related research direction is to learn initial parameters [5] while fixing the learning rule to gradient descent. Another paper observed further improvements by learning learning rates for each weight in addition to initial parameters [17]. A recent result [6] states that neural networks with gradient descent can approximate any learning algorithm.

Our work is closely related to this line of research. Unlike previous work in this line of research, MT-nets learn how many degrees of freedom the fast learner should have at meta-test time. Additionally, while MT-nets learn update rules, these update rules are directly embedded in the network itself instead of being stored in a separate model.

Distance metric learning [33, 32] is a set of methods to learn a distance between inputs. Recently, a few works have used a neural network to learn a metric between images [13, 30, 26], achieving state-of-the-art performance on few-shot classification benchmarks. Our work is similar to these recent methods, but we learn a metric in feature space instead of input space. Since we perform gradient descent on our learned metric (instead of using the metric to compare pairs of inputs), our method applies to a larger class of problems including regression and reinforcement learning. Additionally, like the distance metric methods [33, 32], we learn a full metric matrix. While those methods needed constrained optimization techniques to enforce that the matrix is a metric, our parameterization allows us to directly learn such a metric with gradient descent.

Another line of research in few-shot learning is to use as a recurrent neural network (RNN) as a learner [23, 20]. Here, the meta-learning algorithm is gradient descent on an RNN, and the learning algorithm is the update of hidden cells. The (meta-learned) weights of the RNN specify a learning strategy, which processes training data and uses the resulting hidden state vector to make decisions about test data. Recent works [3, 31] have successfully adapted this line of reasoning to reinforcement learning. A recent work that uses temporal convolutions for meta-learning [18] is also closely related to this line of research.

6 Experiments

We evaluate the performance of our meta-learning method on various few-shot tasks including regression and classification.

We performed most of our experiments by modifying the official code of MAML [5], and we follow their experimental protocol unless specified otherwise.

6.1 Regression

We start with a \(K\)-shot regression problem and compare results to previous meta-learning methods [5, 17]. The details of our regression task are the same as [17]. Each individual task is to regress from the input \(x\) to
We performed this set of experiments to see whether the mask Occam’s razor-like effect: it automatically selects a model of just enough complexity to learn in a set of tasks. We interpret this as the meta-learner of MT-nets having an average of higher-order polynomials to require more parameters to adapt to.

In theory, our method can ‘change’ α. To get the effect of implicitly changing the stepsize from α₀ to α_new, one only needs to replace each $T^i$ with $\sqrt{\frac{\alpha_0}{\alpha_{new}} T^i}$ and each $W^i_0$ with $\sqrt{\frac{\alpha_{new}}{\alpha_0}} W^i_0$. We discuss this in more detail in the supplementary material.

We performed experiments to investigate how robust our method is to suboptimal α. We perform the same sinusoid experiment as in section 6.1, but with various step sizes ($\alpha \in \{10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 1, 10\}$). We evaluate on $K = 10$ training examples, and all other settings are identical to the experiments in section 6.1.

We show losses after adaptation of both MAML and MT-nets in Table 2. We can see that MT-nets are more robust to change in step size. Note that the experiments for $\alpha = 0.01$ in Table 2 is identical to $K = 10$ in Table 1.

### 6.2 Robustness to learning rate change

In theory, our method can ‘change’ α. To get the effect of implicitly changing the stepsize from α₀ to α_new, one only needs to replace each $T^i$ with $\sqrt{\frac{\alpha_0}{\alpha_{new}} T^i}$ and each $W^i_0$ with $\sqrt{\frac{\alpha_{new}}{\alpha_0}} W^i_0$. We discuss this in more detail in the supplementary material.

We performed experiments to investigate how robust our method is to suboptimal α. We perform the same sinusoid experiment as in section 6.1, but with various step sizes ($\alpha \in \{10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 1, 10\}$). We evaluate on $K = 10$ training examples, and all other settings are identical to the experiments in section 6.1.

We show losses after adaptation of both MAML and MT-nets in Table 2. We can see that MT-nets are more robust to change in step size. Note that the experiments for $\alpha = 0.01$ in Table 2 is identical to $K = 10$ in Table 1.

### 6.3 Task Complexity and Changeable Parameters in MT-nets

We performed this set of experiments to see whether the mask M of MT-nets reflect the underlying complexity of the set of tasks it is learning to adapt to.

In each experiment, we randomly generated polynomials of the same order. For a polynomial of order $n$ ($\sum_{i=0}^{n} c_i x^i$), we sampled coefficients $c_0, \ldots, c_n$ uniformly from $[-1, 1]$. We used the same network architecture and hyperparameters as in Section 6.1, and performed 10-shot regression for polynomial orders $n \in \{0, 1, 2, 3\}$. Since the number of free parameters is proportional to the order of the polynomial, we expect higher-order polynomials to require more parameters to adapt to.

We show in Figure 4 the fraction of weights that a task-specific learner changes, which is calculated as the average of $e^{\frac{x^0}{10}}$ over all parameters $l$ in logits. We see that the number of weights that the meta-learner sets to be altered increases as the task gets more complex. We interpret this as the meta-learner of MT-nets having an Occam’s razor-like effect: it automatically selects a model of just enough complexity to learn in a set of tasks.

### Table 2: Loss on 10-shot sine wave regression

| Models   | Learning Rates |
|----------|----------------|
|          | 10  | 1   | 0.1 | 0.01 | 0.001 | 0.0001 |
| MAML[5]  | 171.92 ± 25.04 | 5.81 ± 0.49 | 1.05 ± 0.11 | 0.71 ± 0.07 | 0.82 ± 0.08 | 2.54 ± 0.19 |
| MT-net   | 4.18 ± 0.30   | 0.61 ± 0.07 | 0.54 ± 0.05 | 0.49 ± 0.05 | 0.59 ± 0.06 | 0.72 ± 0.07 |

The output $y$ of a sine function

$$y(x) = A \sin(wx + b)$$

(25)
To compare the performance of MT-nets to prior work in meta-learning, we evaluate our method on few-shot classification on the Omniglot [15] and MiniImageNet [21] datasets.

Our CNN model uses the same architecture as [5, 30]. The model has 4 modules: each has $3 \times 3$ convolutions and 64 filters, followed by batch normalization [9]. On the Omniglot dataset, the convolutions have stride $2 \times 2$; while on the MiniImageNet dataset, the convolutions have no stride, but instead, we use $2 \times 2$ max-pooling after batch normalization.

Results are shown in Table 3. Note that the methods above the middle line are only applicable to few-shot classification and not regression or reinforcement learning. MT-nets achieve the state-of-the-art on classification tasks among methods that apply to any problem setting, while having comparable performance to methods that are specialized for few-shot classification.

7 Discussion

We introduced T-nets and MT-nets. An MT-net is a general neural network architecture that can learn which weights to update along with a layerwise metric to adapt according to. Furthermore, one can transform any feedforward neural network into an MT-net, so any future architectural advances can take advantage of our method. Experiments showed that our method alleviates the need for careful tuning of the learning rate in few-shot learning problems and that the mask $M$ reflects the complexity of the set of tasks it is learning to adapt in. Lastly, our method showed state-of-the-art performance among general methods for few-shot learning and comparable performance to few-shot classification methods.

One of the biggest weaknesses of deep networks is that they are very data intensive. By learning what to learn when a new task is encountered, we can get networks with high capacity that can be trained with a small amount of data. We believe that designing effective gradient-based meta-learners will be beneficial not just for
Table 3: Few-shot classification accuracy on (top) held-out Omniglot characters and (bottom) test split of MiniImagenet. ± represents 95% confidence intervals.

| Models                              | 5-way   | 20-way  |
|-------------------------------------|---------|---------|
| Siamese Networks[13]               | 97.3    | 88.2    |
| Matching Networks[30]               | 98.1    | 93.8    |
| Meta Networks[20]                  | 98.95   | 97.0    |
| Prototypical Networks[26]          | 98.8    | 96.0    |
| mAP-SSVM[29]                       | 98.6    | 95.4    |
| Graph Neural Networks[7]            | 99.0    | 97.0    |
| Relation Networks[27]               | 99.6 ± 0.2 | 97.6 ± 0.2 |
| Neural Statistician[4]              | 98.1    | 93.2    |
| Memory mod.[11]                    | 98.4    | 95.0    |
| MAML[5]                             | 98.7 ± 0.4 | 95.8 ± 0.3 |
| Meta-SGD[17]                       | 99.53 ± 0.26 | 95.93 ± 0.38 |
| MT-net                              | 99.5 ± 0.3 | 96.21 ± 0.35 |

| Models                              | 1-shot  | 5-shot  |
|-------------------------------------|---------|---------|
| Matching Networks[30]               | 43.56 ± 0.84 | 55.31 ± 0.73 |
| Meta Networks[20]                  | 49.21 ± 0.96 | -       |
| Prototypical Networks[26]          | 49.42 ± 0.78 | 68.20 ± 0.66 |
| mAP-SSVM[29]                       | 50.32 ± 0.80 | 63.94 ± 0.72 |
| Graph Neural Networks[7]            | 49.8 ± 0.22  | 65.5 ± 0.20 |
| Relation Networks[27]               | 51.38 ± 0.82 | 67.07 ± 0.69 |
| Fine-tune baseline[21]              | 28.86 ± 0.54 | 49.79 ± 0.79 |
| Nearest Neighbor baseline[21]      | 41.08 ± 0.70 | 51.04 ± 0.65 |
| meta-learner LSTM[21]               | 43.44 ± 0.77 | 60.60 ± 0.71 |
| MAML[5]                             | 48.70 ± 1.84 | 63.11 ± 0.92 |
| Meta-SGD[17]                       | 50.47 ± 1.87 | 64.03 ± 0.94 |
| MT-net                              | 50.82 ± 0.69 | -       |
| T-net                               | 51.69 ± 0.70 | 64.08 ± 0.35 |

the few-shot learning setting, but also machine learning problems in general.

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Appendices

A Learning rates as a special case of $T$

In this subsection, we restrict our attention to the special case of $T$ being a constant multiple of the identity matrix $I$; we show that in this case, $T$ corresponds to a learned learning rate.

Consider gradient descent with learning rate $\alpha$ on the following linear equation:

$$\hat{y} = Ax$$

and set the values to $x = x_0$ and $A = A_0$. Naturally, the resulting $\hat{y}$ is $\hat{y}_0 = A_0x_0$. The gradient update with respect to the loss function $L(\hat{y}) = \frac{1}{2}(\hat{y} - y)^2$ is:

$$A_{\text{new}} = A_0 - \alpha \nabla Ax \frac{1}{2}(\hat{y} - y)^2|_{A_0}$$

$$= A_0 - \alpha \nabla y \frac{1}{2}(\hat{y} - y)^2|_{\hat{y}_0, x_0^\top}$$

$$= A_0 - \alpha (\hat{y}_0 - y) x_0^\top$$

(27)

and we see that the new value of $y$ for the same input $x_0$ is

$$\hat{y}_{\text{new}} = A_{\text{new}}x_0 = (A_0 - \alpha (\hat{y}_0 - y) x_0^\top)x_0$$

$$= y_0 - \alpha (\hat{y}_0 - y) x_0^\top x_0$$

(28)

Alternatively consider gradient descent on $A$ in the following equation with learning rate $1$:

$$\hat{y} = \sqrt{\alpha} (A^k x)$$

and set the value of $A^k$ to $A^k_0 = \frac{A_0}{\sqrt{\alpha}}$. We can easily see that given the input $x_0$, this equation produces the same output $\sqrt{\alpha} A^k_0 x_0 = \sqrt{\alpha} \frac{A_0}{\sqrt{\alpha}} x_0 = A_0 x_0$.

The update here is:

$$A_{\text{new}}^k = A_0^k - \nabla A^k \frac{1}{2}(\hat{y} - y)^2|_{A_0^k}$$

$$= A_0^k - \sqrt{\alpha} A^k x_0 \frac{1}{2}(\hat{y} - y)^2|_{A_0^k}$$

$$= A_0^k - \sqrt{\alpha} \nabla y \frac{1}{2}(\hat{y} - y)^2|_{\hat{y}_0^k, x_0^\top}$$

$$= A_0^k - \sqrt{\alpha} (\hat{y}_0 - y) x_0^\top$$

(30)

The new value of $y$ is

$$\hat{y}_{\text{new}} = \sqrt{\alpha} A_{\text{new}}^k x_0$$

$$= \sqrt{\alpha} (A_0^k - \sqrt{\alpha} (\hat{y}_0 - y) x_0^\top) x_0$$

$$= \hat{y}_0 - \alpha (\hat{y}_0 - y) x_0^\top x_0$$

(31)

We see that the resulting output from gradient descent on 29 with learning rate 1 is the same as gradient descent on 26 with learning rate $\alpha$.

Therefore, the learning rate hyperparameter in gradient descent can be learned using our architecture by setting $T$ to $\sqrt{\alpha} I$ and updating with the equation $\theta \leftarrow W - \nabla W L$ whilst meta-learning the value of $\sqrt{\alpha}$.
B Proofs for Section 4

B.1 MT-nets Learn a Subspace

**Proposition 1.** Fix $x$ and $A$. Let $U$ be a $d$-dimensional subspace of $\mathbb{R}^n$ ($d \leq n$). There exist configurations of $T$, $W$, and $\zeta$ such that the span of $y^{\text{new}} - y$ is $U$ while satisfying $A = TW$.

**Proof.** We show by construction that Proposition 1 is true.

Suppose that $\{v_1, v_2, \ldots, v_n\}$ is a basis of $\mathbb{R}^n$ such that $\{v_1, v_2, \ldots, v_d\}$ is a basis of $U$. Let $T$ be the $n \times n$ matrix $[v_1, v_2, \ldots, v_n]$. $T$ is invertible since it consists of linearly independent columns. Let $W = T^{-1}A$ and let $\zeta_1, \zeta_2, \ldots, \zeta_d \to \infty$ and $\zeta_{d+1}, \ldots, \zeta_n \to -\infty$. The resulting mask $M$ that $\zeta$ generates is a matrix with only ones in the first $d$ rows and zeroes elsewhere.

$$y^{\text{new}} - y = T(W^{\text{new}} - W)x = T(M \odot \nabla_W L_T)x$$

(32)

Since all but the first $d$ rows of $M$ are $0$, $(M \odot \nabla_W L_T)x$ is an $n$-dimensional vector in which nonzero elements can only appear in the first $d$ dimensions. Therefore, the vector $T(M \odot \nabla_W L_T)x$ is a linear combination of $\{v_1, v_2, \ldots, v_d\}$. Thus the span of $y^{\text{new}} - y$ is $U$.

B.2 MT-nets Learn a Metric in their Subspace

**Proposition 2.** Fix $x$, $A$, and a loss function $L_T$. Let $U$ be a $d$-dimensional subspace of $\mathbb{R}^n$, and $g(\cdot, \cdot)$ a metric tensor on $U$. There exist configurations of $T$, $W$, and $\zeta$ such that the vector $y^{\text{new}} - y$ is in the steepest direction of descent on $L_T$ with respect to the metric $du$.

**Proof.** We show Proposition 2 is true by construction as well.

We begin by constructing a representation for the arbitrary metric tensor $g(\cdot, \cdot)$. Let $\{v_1, v_2, \ldots, v_n\}$ be a basis of $\mathbb{R}^n$ such that $\{v_1, v_2, \ldots, v_d\}$ is a basis of $U$. Vectors $u_1, u_2 \in U$ can be expressed as $u_1 = \sum_{i=0}^{d} c_i v_i$ and $u_2 = \sum_{i=0}^{d} c_2 v_i$. We can express any metric tensor $g(\cdot, \cdot)$ using such coefficients $c$:

$$g(u_1, u_2) = \begin{bmatrix} c_{11} & \ldots & c_{1d} \\ \vdots & \ddots & \vdots \\ c_{d1} & \ldots & c_{dd} \end{bmatrix} \begin{bmatrix} g_{11} & \ldots & g_{1d} \\ \vdots & \ddots & \vdots \\ g_{d1} & \ldots & g_{dd} \end{bmatrix} c_{1}$$

(33)

where $G$ is a positive definite matrix. Because of this, there exists an invertible $d \times d$ matrix $H$ such that $G = H^T H$. Note that $g(u_1, u_2) = (H c_1)^T (H c_2)$: the metric $g(\cdot, \cdot)$ is equal to the inner product after multiplying $H$ to given vectors $c$.

Using $H$, we can alternatively parameterize vectors in $U$ as

$$u_1 = \begin{bmatrix} v_1 & \ldots & v_d \end{bmatrix} c_1$$

(34)

$$= V H^{-1} (H c_1).$$

(35)

Here, we are using $H c_1$ as a $d$-dimensional parameterization and the columns of the $n \times d$ matrix $V H^{-1}$ as an alternative basis of $U$.

Let $v^H_1, \ldots, v^H_d$ be the columns of $V H^{-1}$, and set $T = [v^H_1, \ldots, v^H_d, v_{d+1}, \ldots, v_n]$. Since $H$ is invertible, $\{v^H_1, \ldots, v^H_d\}$ is a basis of $U$ and thus $T$ is an invertible matrix. As in Proposition 1, set $W =$
Note that this configuration of $\zeta$ generates a mask $M$ that projects gradients onto the first $d$ rows, which will later be multiplied by the vectors $\{v_1^H, \ldots, v_d^H\}$.

We can express $y$ as $y = Vc_y = VH^{-1}(Hc_y)$, where $c_y$ is again a $d$-dimensional vector. Note that $VH^{-1}$ is constant in the network and change in $W$ only affects $Hc_y$. Since $\nabla_W L_T = (\nabla_{Wx} L_T)x^\top$, the task-specific update (8) is in the direction of steepest descent of $L_T$ in the space of $Hc_y$ (with the Euclidean metric). This is exactly the direction of steepest descent of $L_T$ in $U$ with respect to the metric $g(\cdot, \cdot)$. $\square$