Saturation and Pion Production in Proton-Nucleus Collisions

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(August 1, 2002)

We study the effects of gluon saturation on pion production in high energy proton-nucleus collisions using the color glass condensate model. At high $p_{\perp}$, we show that the $p_{\perp}$-distribution of gluons behaves as $\sim 1/p_{\perp}^3$ in accordance with both conventional perturbative QCD calculations and experiment. Fragmentation of gluons into pions leads to a rapidity dependent depletion of pions relative to the conventional perturbative QCD predictions. We argue that these clear and systematic differences provide a signal for the onset of gluon saturation which is accessible in upcoming experiments.

PACS numbers: 12.38.Aw, 13.87.Fh

Since the pioneering work of Ref.\textsuperscript{1}, the phenomenon of gluon saturation, occurring at very high gluon density, has been under intense study. One hopes that saturation would naturally tame the rise of the gluon distribution function at small values of Bjorken $x$ and circumvent the subsequent violation of unitarity.

A promising attempt to address the problem of saturation involves replacing the partonic fields by coherent classical fields. The justification is that for large occupation numbers the dynamics are essentially classical. This program was initiated in Ref.\textsuperscript{2} and developed further in e.g. Refs.\textsuperscript{3,4}. Within this approach one is able to show that an intrinsic scale, the saturation scale $Q_{S,A}(x)$, is generated. This scale depends on both Bjorken $x$ and atomic number, $A$, and roughly delineates the dense and dilute regimes. Throughout this work we shall refer to this approach as the color glass condensate (CGC) framework.

The parametric dependence of the saturation scale has been extensively studied. Phenomenological models based on the idea of parton saturation have been successful in describing both the inclusive and diffractive HERA data for $F_2$\textsuperscript{5}. According to this model, the dependence of the saturation scale on Bjorken $x$ is

$$Q_{S,A}^2(x) = (1\text{GeV})^2 \left(\frac{x_0}{x}\right)^{\lambda} A^{1/3}, \quad (1)$$

where $\lambda = 0.25\ldots0.3$ and $x_0 = 3 \cdot 10^{-4}$. For the numerical calculations, we will use $\lambda = 0.3$. Since $Q_{S,A}^2$ can be interpreted as the transverse density of partons, it is enhanced by considering larger nuclei at any given $x$ by a factor of $A^{1/3}$. As a result, one may expect to observe saturation effects better in nuclear collisions at currently available collider energies. Indeed, the present data from BNL–RHIC\textsuperscript{6}\textsuperscript{7} seem to be compatible with saturation based phenomenology\textsuperscript{10–13}. These results, however, are far from being conclusive on account of numerous model dependencies related to our poor knowledge of the details of the spacetime evolution of the produced strongly interacting matter.

The onset of saturation has important qualitative and quantitative consequences for the $p_{\perp}$-distribution of the produced gluons. This was already pointed out in Ref.\textsuperscript{1} in the context of proton–proton collisions where an asymmetry in the saturation scales of the two protons appears away from the mid-rapidity. As one hadron moves deeper into the saturation region, towards smaller $x$, its saturation scale increases, while the saturation scale of the second hadron decreases as is evident from Eq. (1). In proton–nucleus collisions, a similar situation is realized already at mid-rapidity due to the atomic number asymmetry. In particular, one expects three different regions in the transverse momentum distribution of the produced gluons on account of the asymmetries in the saturation momenta, $Q_{S,A} > Q_{S,p}$. There is the region $p_{\perp} < Q_{S,p} < Q_{S,A}$ in which both the proton and the nucleus are saturated. The second possibility is that the nucleus is saturated but the proton is in the linear regime ($Q_{S,p} < p_{\perp} < Q_{S,A}$). The final region is the usual perturbative domain in which both gluon distributions are entirely governed by the linear evolution ($Q_{S,p} < Q_{S,A} < p_{\perp}$). Consequently, proton–nucleus collisions offer the best possibility of experimentally probing the systematics of gluon saturation. The problem of evaluating gluon production in proton–nucleus collisions has been found to be analytically tractable above the smaller saturation scale and in Ref.\textsuperscript{14} this was qualitatively studied in the CGC framework.

In this Letter, we undertake the quantitative computation of the produced hadronic spectra based on the framework of Ref.\textsuperscript{14} and compare it to the conventional perturbative QCD (pQCD) calculation which does not include any saturation effects. The main emphasis is on a numerical calculation of the fragmentation of the gluon distributions into pions, and on the study of how saturation effects manifest themselves in the $p_{\perp}$-distributions.
of the measured hadrons. We shall see that although the spectra at the gluonic level are qualitatively very different, this difference is made less striking by the fragmentation of the gluons into pions.

In pQCD, the inclusive cross section for gluon production is given as a convolution of the hard partonic scattering cross section with the gluon distribution functions. The hard cross section behaves as $\sim 1/p_1^4$, but the final result receives additional contributions from the two gluon distributions and from the running of the strong coupling constant. As a result, the slope of the transverse momentum distribution actually falls much more rapidly with a scaling close to $\sim 1/p_1^2$. Any realistic model of particle production must reproduce this scaling at large transverse momenta.

We consider a CGC model obtained by combining two separate limits of the results of Ref. [14]. The gluon production cross section in the perturbative domain is given by

$$\frac{d\sigma^\text{pert}_{g}}{d^2p_1 dy} = \frac{8N_c(N_c^2-1)}{\pi^3} \frac{\alpha_s^3}{p_1^2} \chi_p(x_p, p_1^2) \chi_A(x_A, p_1^2),$$  \hspace{1cm} (2)

where

$$\chi_i(x_i, p_1^2) = \frac{1}{\pi R_s^2} \frac{N_c}{N_c^2-1} \times \left( \int_{x_i}^{z_{1\pi}} dx_i(x, p_1^2) + \frac{C_F}{N_c} \int_{x_i}^{1} dx_i(x, p_1^2) \right),$$

with $i = A, p$. $R_s$ is the transverse radius, $x_{A,p} = p_1 e^{y}, \sqrt{s}$, $C_F = (N_c^2-1)/(2N_c)$ and the nucleus is chosen to have negative beam rapidity. Here $q_i$ and $g_i$ denote the valence quark and gluon distributions in the nucleus or in the proton. One needs to evaluate the color charge functions $\chi_i$ only in the weak field regime, and therefore the use of DGLAP evolved parameterizations is appropriate.

In the saturation domain of the nucleus, the cross section is given by

$$\frac{d\sigma^\text{sat}_{g}}{d^2p_1 dy} = \frac{C(N_c^2-1)}{2\pi^2} \frac{\alpha_s}{p_1^2} \chi_p(x_p, p_1^2) R_s^2 A, \hspace{1cm} (3)$$

The constant $C$ is chosen so that the sub-cross-section is continuous at $Q_{S,A}$. We have neglected the logarithms appearing in the results of Ref. [14] since one expects the overall behavior to be driven by the strong powers of $p_1$ and the constant under the logarithm is in any case difficult to fix. The overall normalization is unimportant as we will discuss later.

From these equations, it is clear that retaining the DGLAP evolved parton distributions for the weak fields and letting the strong coupling run will affect the qualitative results of Ref. [14] which display the expected $\sim 1/p_1^4$ and $\sim 1/p_1^2$ behavior in the perturbative domain and in the saturation domain of the nucleus, respectively. The inclusion of these effects is essential for making the proper contact with conventional result for high $p_1$ mentioned above.

In order to compute the produced hadronic spectra, we convolute the above inclusive gluon production cross section with the appropriate fragmentation functions. For the sake of simplicity, we do not attempt to account for the Cronin effect [15], nuclear shadowing [16] or nuclear modifications of the fragmentation functions [17]. So, $xg_A(x, Q^2) = Axg_p(x, Q^2)$ in the above equations and similarly for quarks. The generalization to nucleus–nucleus collisions with $A_1 \ll A_2$ is trivial through replacements $p \to A_1, A \to A_2$. We note that the overall normalization of the results is difficult to fix. In Ref. [18] it was found that in order to reproduce the data from hadronic collisions above a cutoff momentum of $p_0 \sim 1 \ldots 3 \text{ GeV}$ using pQCD one needs a $\sqrt{s}$-dependent $K$-factor to account for the higher order corrections. The relative normalization between the pQCD and CGC results, however, is almost fixed since both results should match at high $p_1$. Therefore, we do not attempt to fine-tune the overall normalization, but rather consider a leading order pQCD calculation, including only gluons, without any $K$-factors and point out the differences relative to the CGC calculation which will be normalized at high $p_1$ to the pQCD result. Our central results, the slopes of the $p_1$- and $y$-distributions, are not sensitive to the normalization ambiguities. One should note, however, that the $\sqrt{s}$ growth of the gluon multiplicity in the CGC model is driven by the small $x$-growth of the gluon distributions, and is therefore of the order of $dN/dy \sim \sqrt{s}$. We will focus exclusively on pions since this distribution approximates well the distribution of all hadrons.

To compute the fragmentation of gluons into pions, we assume that the pion is produced collinearly with its parent gluon, $\eta_g = \eta_{g*}$, and carries a fraction $z$ of the parent gluon’s energy. The cross section for the production of pions is

$$\frac{d\sigma^{A\to\pi X}_{p}}{d^2q_1 dy} = J(m_1, y) \int \frac{dz}{z} D^\pi_\perp(z, q_1^2) \frac{d\sigma^{A\to\eta X}_{p}}{d^2p_1 dy}, \hspace{1cm} (4)$$

where

$$p_1 = \frac{q_1}{z} J(m_1, y), \quad y_g = \sinh^{-1}\left( \frac{m_1}{q_1} \sinh y \right),$$

$$J(m_1, y) = \left( 1 - \frac{m_1^2}{m_0^2 \cosh^2 y} \right)^{-1/2}$$

and $m_0^2 = q_1^2 + m_1^2$. The integration over $z$ is limited by the maximal energy the gluon can carry and by the lower bound on $p_1$. In pQCD, this latter scale is a fixed cutoff, while in the CGC calculation it is determined by the saturation scale of the proton,
\( \frac{a m}{\sqrt{s}} \cosh y \leq z \leq \min \left( 1, \frac{q_{\perp}}{p_{\perp,\text{min}}} J(m_{\perp}, y) \right) \), \hspace{1cm} (5)

where \( a = 1, 2 \) for the CGC and pQCD calculations respectively. This difference arises from the underlying parton kinematics. In LO pQCD, one produces two minijets, back-to-back in the transverse plane, carrying at most \( E = \sqrt{s}/2 \). In the CGC calculation, the scattering is a BFKL-type \( 2 \to 1 \) fusion, and the produced gluon can have energy up to \( E = \sqrt{s} \). We use the CTEQ5 parameterization of parton distributions \([19]\) and KKP parameterization of the fragmentation functions \([20]\).

In Fig. 1, we plot the \( p_{\perp} \)-distributions of the produced gluons at different rapidities. The topmost three curves are for \( y = 0 \) and the lower two are for \( y = 3 \). The dashed curve is the CGC result including only gluons in the sources \( \chi_i \). Note that at \( y = 0 \) the slopes of the CGC and pQCD calculations almost match at high \( p_{\perp} \). Below the saturation scale of the nucleus, the two results strongly deviate. In a full computation of the gluon distribution, this sharp bend at \( Q_{S,A}(y) \) will become smooth. The essential point is that there is a marked depletion of gluons for \( p_{\perp} < Q_{S,A}(y) \).

If this spectrum were measured, one should be able to determine whether the parton dynamics in the collision are better described by pQCD or by the CGC model. To obtain a measurable spectrum one needs to fragment each of the distributions in Fig. 1 to hadrons.

In Fig. 2, we show the \( p_{\perp} \)-distribution of the pions produced from the two models. The fragmentation changes the qualitative behavior of the spectrum of the CGC model dramatically with the sharp bend disappearing completely on account of the redistribution of momentum from the gluons to the pions. We note that there are significantly fewer produced pions at lower transverse momentum in the CGC calculation than in the conventional quantum pQCD one and that this deficit is strongly rapidity dependent. We have checked using a smooth fit interpolating between the two limiting forms of the gluon distribution that the qualitative, and to some extent even quantitative, results for the pion spectrum in the CGC model are not sensitive to the sharpness of the knee in the gluon distribution.

To obtain a useful simple parameterization of the pion spectrum, we consider the form

\[
\frac{d\sigma}{dy dp_{\perp}^2} = \frac{C(y)}{(p_{\perp}^2 + \mu(y)^2)^b},
\]

where the emergence of a rapidity dependent scale, \( \mu(y) \), is a natural consequence of the rapidity dependence of the saturation scale. The rapidity dependence of the effective slope \( b(y) \) is expected since the slope of the gluon distribution already depends on rapidity. We find that \( \mu(y)^2 = 0.50 e^{\lambda y} \), where \( \lambda = 0.3 \). The values of the fit parameters \( C(y) \) and \( b(y) \) at various rapidities are collected in Table I. The inclusion of the quarks plays a role for the overall normalization and the slopes are left almost unchanged as can be seen already from Fig. 2.

Finally, we consider the rapidity distributions at fixed \( p_{\perp} \). One cannot approach the beam rapidity of the nucleus in this framework, since at such large negative rapidities the saturation scales of the nucleus and the proton become comparable and the underlying model breaks.
down. Near midrapidity, the CGC model gluon spectrum in perturbative domain is characterized by constant behavior in rapidity, as is the case for pQCD spectrum. In the saturation domain, the CGC gluon spectrum has a nonzero slope also at midrapidity. The CGC model gluon spectra are shown in Fig. 3 with dotted curves for p$_{\perp}$ = 3 and 1.4 GeV, and the corresponding pion spectrum by solid lines for p$_{\perp}$ = 0.4, 0.6 0.8 1.0 and 1.7 GeV. The pion spectrum from pQCD, shown by dashed lines for 0.4 and 1.0 GeV has the flat behavior at midrapidity over the whole p$_{\perp}$ range. We observe that the small p$_{\perp}$ pions might reveal the characteristic form carried by the gluon spectrum in the saturation domain. This result is easy to understand, since the pion spectrum at scale q$_{\perp}$ effectively probes the gluon spectrum at some higher scale q$_{\perp}$/z. For the perturbative distribution one finds that z $\sim$ 0.5…0.6. This is clearly seen by comparing pions at p$_{\perp}$ = 1.7 GeV and gluons at p$_{\perp}$ = 3 GeV. As the slope of the gluon distribution decreases, z moves towards smaller values as well. This causes the gluons in saturation domain to fragment mainly to much smaller values of p$_{\perp}$ than the perturbative gluons would. The situation improves at LHC energies where the saturation domain extends already at midrapidity to $\sim$ 2 GeV. At large rapidities it is difficult to distinguish between saturation and perturbative spectra, and presumably different dynamics which are not contained in either of these models become dominant towards the fragmentation regions.

We have investigated the quantitative behavior of the p$_{\perp}$- and y-distributions of gluons and pions in collisions of nuclei with a large atomic number asymmetry. We have shown that within the color glass condensate framework one indeed reproduces the behavior of the conventional pQCD at high p$_{\perp}$ which significantly deviates from commonly cited expectation of 1/p$_{\perp}^4$ in the literature on saturation. The fragmentation of gluons into pions was shown to significantly change the qualitative shape of the spectrum. However, a clear difference from the spectrum obtained from the conventional pQCD calculation in the form of a rapidity dependent pion deficit can still be extracted by measuring the rapidity dependence of the p$_{\perp}$-distributions. The rapidity dependence of the slopes were found to be different at small p$_{\perp}$, and a change in the form of the rapidity distribution of pions as a function of p$_{\perp}$ could be used to determine the onset of saturation.

**Acknowledgements:** We would like to thank A. Dumitru, K.J. Eskola, K. Kajantie, J. Jalilian–Marian, L. McLerran and W. Vogelsang for useful discussions. J.T.L. thanks the Nuclear Theory Group at Brookhaven National Laboratory, where part of this work was done, for hospitality.

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