Vortices, monopoles and confinement

L. Del Debbio\textsuperscript{a,1}, A. Di Giacomo\textsuperscript{a,2}, B. Lucini\textsuperscript{b,3}

\textsuperscript{a}Dipartimento di Fisica, Universit\`{a} di Pisa, and INFN Sezione di Pisa, Italy
\textsuperscript{b}Theoretical Physics, University of Oxford, UK

Abstract

We construct the creation operator of a vortex using the methods developed for monopoles. The vacuum expectation value of this operator is interpreted as a disorder parameter describing vortex condensation and is studied numerically on a lattice in $SU(2)$ gauge theory. The result is that vortices behave in the vacuum in a similar way to monopoles. The disorder parameter is different from zero in the confined phase, and vanishes at the deconfining phase transition. We discuss this behaviour in terms of symmetry. Correlation functions of the vortex creation operator at zero temperature are also investigated. A comparison is made with related results by other groups.

Key words: Confinement, vortices, monopoles
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1 Introduction

Recently there has been renewed interest in understanding the role of vortices in the mechanism of colour confinement \cite{1–3}. Vortices were originally introduced in the continuum theory as string-like topological defects \cite{4}, and have been studied both in the continuum and on the lattice \cite{5}. In the notation of Ref. \cite{4}, a vortex creation operator $B(C)$, for a gauge group $SU(N)$, can be associated to each closed oriented curve $C$ and has the following commutation relation with a Wilson loop $W(C')$, bounded by a closed curve $C'$:

$$W(C') B(C) = B(C) W(C') \exp(2\pi i n/N)$$  \hspace{1cm} (1)
where \( n \) is the winding number of \( C' \) around \( C \). In \( SU(2) \) such a vortex contributes a factor \((-1)\) to each Wilson loop with an odd linking number with the vortex line. On a lattice, the world-sheet \( M \) of the vortex can be associated to a surface on the dual lattice. A vortex creation operator is defined by twisting the plaquettes in the action, whose duals belong to that surface.

While creating a vortex is a well defined procedure, detecting vortices in lattice gauge configurations is more difficult. Attempts in this direction stemmed from [6], and results were obtained by several groups [7], indicating that vortices can play a role in confinement. However, it is not clear that this procedure detects the vortices as defined in Eq. 1 (see e.g. [8,9]). The current picture of the role of vortices in colour confinement is based on Eq. 1: at low temperature, vortices disorder the Wilson loop to produce the well-known area law; at high temperature, vortices are suppressed and the Wilson loop obeys a perimeter law.

In three dimensions, a conserved topological quantum number can be associated to a vortex, and a vortex creation operator is defined as a local complex scalar field. The dynamics of this scalar field is described by an effective Lagrangian with a global dual \( Z_N \) symmetry [4]. The breaking of this symmetry is responsible for the different phases of the theory. The scalar field \( \phi(x_0) \) in three dimensions has a non-trivial commutation relations, analogous to those of Eq. 1, with any Wilson loop encircling the point \( x_0 \). Operators that create such topological excitation are known in statistical mechanics as “disorder operators” [10] and the expectation value of a disorder operator is a disorder parameter and is related to the free energy of the associated topological excitation.

In four dimensions the situation is less clear. The vortex corresponds to a string-like topological defect and the dual symmetry does not emerge naturally. In this respect, there is a fundamental difference between the vortex and the monopole picture of colour confinement in terms of symmetry: for the monopoles, a topologically conserved current exists, whose zero component is the generator of the dual symmetry, and a picture based on a mechanism, namely the dual Meissner effect [11], emerges as a consequence of the breaking of the dual symmetry [12]. For the vortices the symmetry pattern is not understood. In close analogy with previous work on abelian-projected monopoles, it is nonetheless possible to define a vortex creation operator obeying the above commutation relation, which can be associated to the free energy of the vortex. Starting from this point of view, in this paper we study the role of vortices across the deconfinement phase transition, using the techniques developed in Refs. [13,14,12] for investigating the condensation of monopoles defined via the abelian projection.
A vortex creation operator $\mu$ is introduced as a disorder operator for the $SU(2)$ lattice gauge theory. When applied to a gauge configuration, $\mu$ creates a vortex associated to a string closing in the $z$ direction via periodic boundary conditions (PBC). As well as for other systems, both in statistical mechanics [10,15,16] and in quantum field theory [17,13,14,12], such an operator can be constructed without any reference to the dual theory. In Sect. 2 we give the explicit construction of this operator. While our study was in progress, other papers appeared which addressed similar problems by using quantities related to the vortex creation operator examined in this work. The explicit relationship between the operator studied here and other quantities in the literature is reviewed in Sect. 3.

Correlation functions of the vortex creation operator at zero and finite temperature yield useful informations on the behaviour of these topological defects in the $SU(N)$ Yang-Mills vacuum. As in previous studies on monopoles, it is convenient to compute

$$\rho = \frac{d}{d\beta} \log \langle \mu \rangle$$

and analogous quantities for $n$-point correlators. Unlike $\mu$, $\rho$ can be determined with good accuracy and provides all the needed information. The vacuum expectation value (vev) of $\mu$ proves to be a disorder parameter for the finite temperature deconfining phase transition, as reported in Sect. 4. Another interesting quantity to be investigated is the correlator $\langle \mu(t_0)\mu(t_0 + t) \rangle$ at zero temperature.

This approach can be generalised to $SU(N)$ gauge group and to vortex loops of generic geometry, also not wrapping through PBC. Work is in progress on these aspects.

2 Vortex creation operator

The vortex creation operator $\mu$ is constructed by the same technique used for the monopole creation operator in previous works [13,14]. The vacuum expectation value of $\mu$ is defined as the ratio of two partition functions:

$$\langle \mu(t_0, x_0, y_0) \rangle = \frac{\tilde{Z}}{Z} = Z^{-1} \int [dU] e^{-\beta \tilde{S}[U]}$$

where $Z$ is the usual partition function with the Wilson action,

$$S[U] = \sum_{x, \mu, \nu} \text{Tr} [1 - P_{\mu\nu}(x)]$$

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and \( \tilde{S} \) is obtained from \( S \) by twisting a line of plaquettes in the 0\( \hat{y} \) plane:

\[
P_{0y}(t_0, x > x_0, y_0, z) \mapsto -P_{0y}(t_0, x > x_0, y_0, z), \forall z
\]  

(5)

The net effect of this transformation is best understood by performing a series of changes of variables in the functional integral as follows. The first change:

\[
U_y(t_0 + 1, x > x_0, y_0, z) \mapsto -U_y(t_0 + 1, x > x_0, y_0, z), \forall z
\]  

(6)

yields

\[
\langle \mu(t_0, x_0, y_0) \rangle = Z^{-1} \int [dU] e^{-\beta \tilde{S}^{(1)}[U]}
\]

(7)

where \( \tilde{S}^{(1)} \) is defined by the following transformation of the Wilson action:

\[
\begin{align*}
P_{0y}(t = t_0 + 1, x > x_0, y_0, z) & \mapsto -P_{0y}(t = t_0 + 1, x > x_0, y_0, z), \forall z \\
P_{0x} & \mapsto P_{0x}, \quad P_{0z} \mapsto P_{0z} \\
P_{xy}(t = t_0 + 1, x_0, y_0, z) & \mapsto -P_{xy}(t = t_0 + 1, x_0, y_0, z), \forall z \\
P_{xy}(t = t_0 + 1, N_s - 1, y_0, z) & \mapsto -P_{xy}(t = t_0 + 1, N_s - 1, y_0, z), \forall z \\
P_{yz} & \mapsto P_{yz}, \quad P_{xz} \mapsto P_{xz}
\end{align*}
\]

Fig. 1. Set of plaquettes changing sign in the hyper-plane \( t_0 + 1 \) after the first change of variables described in Eq. 6. The plaquettes in the plane \( xy \) correspond to the location of the vortex lines at \((x_0, y_0)\), and at \((N_s - 1, y_0)\) due to PBC in the \( x \) direction.

The change of sign in the \( P_{0y} \) plaquettes introduced at \( t = t_0 \) in Eq. 5 has been shifted at \( t = t_0 + 1 \) and two vortices have been created in the \((x, y)\) plane for \( t = t_0 + 1 \) at \((x_0, y_0)\) and \((N_s - 1, y_0)\). It is worthwhile to remark that the second vortex at \((N_s - 1, y_0)\) is due to the fact that we are working in a finite volume with PBC in the \( x \) direction. In an infinite volume, or with free
BC, the vortex at \((x_0, y_0)\) would have been created alone. In the \(z\) direction, the vortices either extend to infinity (for an infinite lattice), or they form a closed loop due to PBC. The set of \(xy\) plaquettes changing sign after this first change of variables is shown in Fig. 1.

The same change of variables can be iterated at successive times, yielding, after \(n\) iterations:

\[
\langle \mu(t_0, x_0, y_0) \rangle = Z^{-1} \int [dU] e^{-\beta \tilde{S}^{(n)}[U]}
\]

and \(\tilde{S}^{(n)}\) is obtained from the Wilson action via:

\[
\begin{align*}
\left\{ \begin{array}{l}
P_{0y}(t = t_0 + n, x > x_0, y_0, z) & \mapsto -P_{0y}(t = t_0 + n, x > x_0, y_0, z), \quad \forall z \\
P_{xy}(t = t_0 + 1, x_0, y_0, z) & \mapsto -P_{xy}(t = t_0 + 1, x_0, y_0, z), \quad \forall z \\
P_{xy}(t = t_0 + 1, N_s - 1, y_0, z) & \mapsto -P_{xy}(t = t_0 + 1, N_s - 1, y_0, z), \quad \forall z \\
\vdots & \\
P_{xy}(t = t_0 + n, x_0, y_0, z) & \mapsto -P_{xy}(t = t_0 + n, x_0, y_0, z), \quad \forall z \\
P_{xy}(t = t_0 + n, N_s - 1, y_0, z) & \mapsto -P_{xy}(t = t_0 + n, N_s - 1, y_0, z), \quad \forall z \\
\end{array} \right. 
\]

Such a configuration corresponds to the propagation of the two vortices, from time \(t = t_0\) to time \(t = t_0 + n\).

Correlations of \(\mu\) operators at different times \(\{t_1, \ldots, t_n\}\) are defined by repeating the transformation in Eq. 5 for each \(t_i\). For a two-point correlation:

\[
\Gamma(t) = \langle \mu(t_0, x_0, y_0) \mu(t_0 + t, x_0, y_0) \rangle \tag{9}
\]

the change in the \(P_{0y}\) plaquettes is reabsorbed when \(n = t\) and the correlator describes the propagation of a pair of vortices from the time \(t_0\) to the time \(t_0 + t\).

We remark here that these prescriptions create vortex lines that are closed in space, due to PBC in the \(z\)-direction, and propagate in time.

At large values of \(t\), \(\Gamma\) decreases exponentially to an asymptotic value, which, by cluster property, is \(\langle \mu \rangle^2\), the square of the disorder parameter:

\[
\Gamma(t) \simeq Ae^{-mt} + \langle \mu \rangle^2 \tag{10}
\]

At \(T = 0\), \(\langle \mu \rangle\) can be extracted from the large-\(t\) value of the correlator, according to Eq. 10.

At finite temperature, there is no propagation in time, and a direct measurement of \(\langle \mu \rangle\) is needed by use of a single operator. In order to have a consistent
implementation of the changes of variables described above, $C^*$ boundary con-
ditions are needed in the time direction, as explained in [14].

Closed vortex lines propagating in time can be created in a field configuration
with the same prescription, but with a different modification of the action. For
instance, using for the modified action:

$$P_{0y}(t_0, x_0 < x < x_1, y_0, z_0 < z < z_1) \mapsto -P_{0y}(t_0, x_0 < x < x_1, y_0, z_0 < z < z_1)$$

(11)

gives a vortex line encircling the rectangle in the $xz$ plane:

$$R = \{(x, y, z) : x_0 < x < x_1, y = y_0, z_0 < z < z_1\}$$

(12)

The same definition is used in a recent publication [21].

3 Comparison with related works

Our disorder parameter is strictly related to observables introduced by other
authors [1–3]. The detailed comparison is as follows.

The paper [2] presents a computation of the free energy of a vortex pair. The
latter is defined as:

$$F = -T \log \frac{Z(\beta, -\beta)}{Z(\beta, \beta)}$$

(13)

where

$$Z(\beta, \beta') = \int [dU] e^{-S(\beta, \beta')}$$

$$S(\beta, \beta') = \frac{1}{2} \left( \beta \sum_{P \not\in M} \text{Tr} P + \beta' \sum_{P \in M} \text{Tr} P \right)$$

and $T = (N_t a)^{-1}$ is the temperature of the system. $M$ indicates a set of pla-
quettles with modified coupling $\beta' = -\beta$, which can be seen as the plaquettes
transversed by the vortex string. The following two cases are examined:

- a vortex solution is placed at $(x_0, y_0, z_0, 0)$ and an anti-vortex at $(x_0, y_0, z_0 + d, 0)$, with a straight string of twisted plaquettes connecting them;
- a single vortex is placed in the middle of the lattice and the string extends
to the boundary in the $z$-direction, where free boundary conditions are used.

In both cases the modification of the action is done in all time-slices.
Using the notation introduced in the previous section:

\[
\frac{Z(\beta, -\beta)}{Z(\beta, \beta)} = Z^{-1} \int[dU] e^{-S'[U]} \tag{14}
\]

and now \(S'\) is obtained from \(S\) by:

\[
\begin{align*}
P_{xy}(t, x_0, y_0, z = z_0 + 1) &\mapsto -P_{xy}(t, x_0, y_0, z = z_0 + 1), \forall t \\
&\vdots \\
P_{xy}(t, x_0, y_0, z = z_0 + d) &\mapsto -P_{xy}(t, x_0, y_0, z = z_0 + d), \forall t
\end{align*}
\]

With a \(t \equiv z\) relabelling of the axes, the correlator \(\Gamma\) defined in this paper by Eq. 9 yields the free energy of \(\textit{two}\) vortex pairs as defined in [2] at distance \(N_s/2\).

At \(T = 0\) we use symmetric lattices \((N_s = N_t)\) and measure the time correlator of two \(\mu\) operators at \(x_0 = y_0 = N_s/2\), which is in fact the correlator of two pairs, when the effect of periodic boundary conditions are taken into account. At non-zero temperature, we use a single vortex operator, which actually introduces two vortices, again due to PBC. The extra vortices that are created by PBC are \(N_s/2\) lattice spacings away from their partners. If \(N_s \gg \xi\), where \(\xi\) is the correlation length, then our disorder parameter \(\langle \mu \rangle\) is the square of \(\exp(-F/T)\).

The authors of Ref. [3] are concerned with the behaviour of the ’t Hooft loop [4] in hot QCD, and relate the dual string tension to the wall tension of \(Z_N\) domain walls. For \(SU(2)\), the lattice definition of the loop operator is given by [18]:

\[
V[C] = \exp \left\{ \beta \sum_{x \in S} (\text{Tr} P_{zt}(x) + \text{h.c.}) \right\} \tag{15}
\]

where \(S\) is the surface in the \(xy\) plane bounded by the curve \(C\). The vev of the operator defined in Eq. 15 is again the ratio of two partition functions: one with a modified action divided by the (standard) Wilson action. The modified action is obtained by changing the sign of the plaquettes in \(S\). The definition given in sect. 2 coincides exactly with Eq. 15, for the curve \(C\) depicted in Fig. 2, after a relabelling of the axes \(y \equiv z\). It is easy to realise in this formulation that our vortex creation operator is precisely a ’t Hooft loop operator: every Wilson loop with non-trivial linking to the curve \(C\) receives a \((-1)\) contribution from the vortex.

Finally, the authors of Ref. [1] compute the free energy of a closed \(SU(2)\) vor-
Fig. 2. The operator $\mu$ introduced in sect. 2 corresponds to $V(C)$ for the curve $C$ depicted here. The box represents a time-slice of the whole lattice.

The operator $\mu$ is defined from the logarithm of the ratio of two partition functions:

$$\exp \left\{ -F(\tau_{\mu\nu}) \right\} = \frac{Z(\tau_{\mu\nu})}{Z} \quad (16)$$

$Z(\tau_{\mu\nu})$ is defined as usual by multiplying a given (co-closed) set $V_{\mu\nu}$ of plaquettes in the action with an element $\tau_{\mu\nu}$ of the center of the gauge group. For $SU(2)$, the only non-trivial possibility is $\tau_{\mu\nu} = -1$. For the set of plaquettes:

$$V_{\mu\nu} = \{ P_{\mu\nu}(t, x, y, z) : (\mu, \nu) = (0, 2), t = t_0, x_0 < x < x_1, y = y_0, z_0 < z < z_1 \}$$

the definition in [1] coincides with the one in Eq. 11.

4 Numerical results

In this paper, we present data only for open vortex lines wrapping in the $z$ direction by PBC. Work is in progress to study closed vortices as done in [21].

Due to the exponential in its definition, $\mu$ has large fluctuations, which make a direct measurement a challenging task. Instead, as in previous studies [13,12], we define

$$\rho = \frac{d}{d\beta} \log \langle \mu(t_0, x_0, y_0) \rangle = \langle S \rangle_s - \langle \tilde{S} \rangle_{\tilde{s}} \quad (17)$$

Being the difference of the expectation values of two actions, $\rho$ can be easily computed, and proves to yield all the relevant information. At finite temperature, $\rho$ is expected to have a sharp negative peak in the critical region [13,14,12], if $\langle \mu(t_0, x_0, y_0) \rangle$ is related to the deconfinement phase transition.
Fig. 3. Comparison of $\rho$ defined with the vortex and the monopole creation operator. Our results for $\rho$ are displayed in Fig. 3, for several lattice sizes. For comparison, we also report a plot of the corresponding quantity for the vev of a monopole creation operator on a $12^4 \times 4$ lattice [12]. Already at a qualitative level, it is clear that, at weak coupling, $\rho$ is negative, with its absolute value increasing with the volume. This behaviour implies that $\langle \mu \rangle$ vanishes for large $\beta$ in the thermodynamical limit. At low $\beta$, $\rho$ is compatible with 0 for all the volumes considered in this study, which means that $\langle \mu \rangle$ has a non-zero value in the infinite volume limit in the confined phase. The presence of the negative peak connecting the confined and deconfined phases suggests that vortices play a role at the deconfinement transition. This behaviour is in agreement with the results in [1,2]. As in the case of monopoles [12], we can perform a finite size scaling analysis of the data presented above. In neighborhood of the critical coupling $\beta_c$, if $\langle \mu \rangle$ is a disorder parameter for the deconfining phase transition, in the infinite volume limit we have

$$\langle \mu \rangle \propto (\beta_c - \beta)^\delta$$  \hspace{1cm} (18)$$

being $\delta$ the corresponding critical index. In a finite volume, the previous equation is replaced by

$$\langle \mu \rangle = (\beta_c - \beta)^\delta \Phi(N_s/\xi)$$  \hspace{1cm} (19)$$

where $\xi$ is the correlation length and $\Phi$ is a function of the ratio $N_s/\xi$. Since

$$\xi \propto (\beta_c - \beta)^{-\nu}$$  \hspace{1cm} (20)$$
Eq. (19) can be written as

\[ \langle \mu \rangle = L^{-\delta/\nu} \Phi(L^{1/\nu}(\beta_c - \beta)) \]  

(21)

being \( \nu \) the critical index of the correlation length.

Eq. (21) implies

\[ \frac{\rho}{L^{1/\nu}} = f \left( L^{1/\nu}(\beta_c - \beta) \right) \]  

(22)

i.e. the ratio \( \rho/L^{1/\nu} \) is an universal function of the scaling variable

\[ x = L^{1/\nu}(\beta_c - \beta) \]

By guessing that [12]

\[ \frac{\rho}{L^{1/\nu}} = -\frac{\delta}{x} + c \]  

(23)

it is possible to extract from our data the critical exponents \( \delta \) and \( \nu \) and the critical coupling \( \beta_c \). We perform three different fits: a fit with three parameters, a fit to \( \delta \) and \( \nu \) at fixed \( \beta_c \), using the value in [19], and again a two parameter fit to \( \delta \) and \( \nu \) at fixed \( \beta_c \), using the value obtained in the three parameter fit. The error is estimated from the variation in the fitted values when using the different methods described above and when the points corresponding to smaller correlation lengths are excluded. Our results are:
\[ \beta_c = 2.30(1) \]
\[ \delta = 0.5(1) \]
\[ \nu = 0.7(1) \]

The values of \( \beta_c \) and \( \nu \) are in good agreement with the values obtained in [12], while the value of \( \delta \) varies by two standard deviations, when compared to previous results.

Fig. 4 shows how well the scaling relation is obeyed with the values of \( \beta_c \) and \( \nu \) from the above fit.

Assuming that in the weak coupling limit all link variables are close to unity, and remembering that the twisted action is obtained by flipping the sign of \( 2 \times N_t \times L \) plaquettes, a naive prediction would yield:

\[ \rho \simeq -16L + \text{const} \quad (24) \]

This is a justification for the linear dependence used in the ansatz above. However, the actual value of the coefficient \( a \) does not need to be the one in Eq. 24, since the configuration with all links set to the identity is not the true minimum of the modified action which enters the definition of \( \rho \).

To check this behaviour, we have measured \( \rho \) for different volumes and very large values of the coupling. In this region, we expect the data to be independent of \( \beta \). Within the errors the data do lie on a straight line as predicted by Eq. 24. The fact that the slope is negative ensures that the disorder operator vanishes in the weak coupling phase in the thermodynamic limit.

![Fig. 5. \( \rho \) vs. \( L \) for large values of \( \beta \). The solid line corresponds to a linear fit.](image-url)
The behaviour of \( \rho \) at low \( \beta \)'s is displayed in Fig. 6: the value at low \( \beta \) remains bounded from below and consistent with zero for increasing volumes, which guarantees that \( \rho \) does not vanish in the thermodynamic limit above the phase transition.

![Fig. 6. \( \rho \) vs. \( \beta \) for different spatial volumes.](image)

Another quantity that can give information on the behaviour of center vortices in the vacuum of SU(2) is the correlation function of two \( \mu \) operators as a function of distance at zero temperature. When relabelling the axes, as discussed in the previous Section, this quantity is identical to the free energy recently discussed in [2].

Again, our numerical computation is performed in terms of \( \rho_2 \), now defined as

\[
\rho_2(t) = \frac{d}{d\beta} \log \langle \mu(t_0, x_0, y_0) \mu(t_0 + t, x_0, y_0) \rangle
\]

which is the difference between two actions. Since the usual Wilson contribution is \( t \)-independent, the whole dependence on \( t \) is brought by \( \tilde{S} \).

In Fig. 7 we display the behaviour of \( \tilde{S} \) on a 16\(^4\) lattice at \( \beta = 2.5 \), where the system is confined and scaling is supposed to work.

The \( t \) dependence of the correlation is fitted using two different fitting functions, both depending on three parameters \((a, b, m)\):

\[
\rho_2(t) = a e^{-mt} + b
\]

\[
\rho_2(t) = ac^{-mt} + b
\]
Fig. 7. The modified action $\tilde{S}$ per unit of volume as a function of the distance between the vortex and the antivortex on a lattice $16^4$ at $\beta = 2.5$.

these two fits will be named respectively fit I and fit II in what follows. The functional dependence in fit I describes a Yukawa potential between vortices, as studied in Ref. [2]. In both cases, $b$ is the asymptotic value of $\rho_2$ and is expected to be different from zero if vortices are condensed. Both ansatz fit the data relatively well, despite the fact that the masses obtained are quite different. The values of $\chi^2$ give an indication that fit II could be a better description of the data, although it is impossible to draw any robust conclusion given that it is very difficult to disentangle the logarithmic correction in $t$ which differentiate fit I from fit II. The outcome of the fits is summarised in Table 1 and the two fitting curves are superimposed on the data in Fig. 7. The value of the constant $b$ is determined quite accurately due to the plateau in the data points at large $t$. It is interesting to remark that $a$ turns out to be negative in both cases up to 90% CL, while the error on the fitted mass is very large. This is mainly due to the fact that large variations in $m$ can be reabsorbed by variations in $a$ in the range where data points are available.

| fit type | $a$      | $b$      | $m$ | $\chi^2$/d.o.f. |
|----------|----------|----------|-----|-----------------|
| fit I    | -0.00018 | 0.34923  | 0.650| 2.6             |
| fit II   | -0.00035 | 0.34923  | 1.302| 1.8             |

Table 1  
Fit results for the correlation of vortex operators

We notice from fit II that the correlation length is around 1.3 lattice spacings. This justifies the view of the pair of vortices (the one at the center of the lattice and the one at the border produced by PBC) as independent.
5 Discussion and future outlooks

In previous works [12], we produced evidence of dual superconductivity of the QCD vacuum, supporting the mechanism of Refs. [11]. A peculiar feature of this phenomenon is that monopoles defined by different abelian projections all condense [12]. This possibility was suggested in Ref. [22] and lattice data support it, showing that confinement is related to condensation of magnetic charges defined by a few abelian projections.

If all or a large class of abelian projections are equivalent, there are infinitely many physically equivalent disorder symmetries. We already observed that this fact is not inconsistent, but suggests that maybe a more fundamental dual symmetry pattern exists, which can manifest itself as condensation of all these magnetic charges [23].

Now it is found that also vortices show condensation. We were not able to associate a dual conserved topological quantity to vortices in 3+1 dimensions, contrary to what happens in 2+1 dimensions. However we consider what is observed here an important information on the way to understand the features of the dual description.

We are extending the analysis to the vortices of the $SU(3)$ gauge theory and to some aspects of closed vortex lines. We are also trying to understand what happens in the presence of quarks. Preliminary data on condensation of monopoles, as defined by the abelian projections, show that the presence of quarks does not affect the dual superconductor picture. This is in agreement with the idea that the gross features of the QCD vacuum, including the mechanism of confinement, are determined already at $N_c = \infty$ [24].

We are now trying to understand how to extend to full QCD the analysis of vortices.

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