Understanding non-linear hydrodynamic response in heavy-ion collisions via event-plane correlations

Soumya Mohapatra
Department of Physics, Columbia University, New York, NY 10027
E-mail: soumya@cern.ch

Abstract. Recently the ATLAS collaboration measured correlations between pairs and triplets of event-plane angles in Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV and observed interesting centrality dependent patterns. In this work, the origin of these event-plane correlations and the quantitative physical implications of their measured values are discussed. Such correlations can originate due to correlations between the eccentricities, $\epsilon_n$, of different orders in the initial geometry. They can also be generated dynamically during the hydrodynamic expansion of the medium. Comparisons to a Glauber initial geometry model, and to AMPT transport model calculations show that the measured correlations are generated during hydrodynamic expansion of the medium. The measured correlations are shown to unambiguously establish that the higher order flow harmonics, $v_4$-$v_6$, are in fact largely driven by non-linear hydrodynamic response to lower order eccentricities $\epsilon_2$-$\epsilon_3$ of the initial geometry.

1. Introduction
The azimuthal distribution of particles in heavy-ion collisions are often expressed in a Fourier series as:

$$dN/d\phi \propto (1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \Phi_n)))$$

where $\phi$ is the azimuthal angle of the particle momentum, and the harmonics $v_n$ and phases $\Phi_n$ represent the magnitude and direction of the $n^{th}$ order anisotropy. The $v_n$ are commonly termed flow harmonics due to their hydrodynamic origin, while the phases $\Phi_n$ are termed as event-plane angles. The $v_n$ contain information about both the initial geometry as well as the expansion mechanics of the medium produced in heavy-ion collisions. Thus they are important observables and have been studied in detail at RHIC and at the LHC [1, 2]. They have been instrumental, in our understanding of the expansion dynamics of the produced medium. Until recently however, very little attention was paid to studying the event-plane angles $\Phi_n$. The ATLAS collaboration recently measured a large set of correlations involving two or three event-planes [3]. These event-plane correlations were measured in detail as a function of centrality and showed interesting centrality dependent patterns. The event-plane correlations are quite novel measurements as they contain information that can not be accessed using only the flow harmonics $v_n$. As will be shown later, these event-plane correlations give a clear understanding of non-linear correlations between the initial geometry and the final particle distributions.
2. Origin of event-plane correlations

Hydrodynamic calculations show that the lower order harmonics \( v_3 \) and \( v_4 \) and their phases are directly correlated with the magnitude and phase of the second and third-order eccentricities of the initial geometry:

\[
v_n e^{i \Phi_n} \propto e_n e^{i \Phi_n}, \quad n = 2, 3
\]

where, the eccentricities of the initial geometry \( e_n \) and their phases \( \Phi_n \) (called participant-planes) are given in terms of the transverse positions \( r, \phi \) of the participating nucleons relative to their common center of mass as:

\[
\begin{align*}
\epsilon_n & = \sqrt{\langle r^n \cos(n\phi) \rangle^2 + \langle r^n \sin(n\phi) \rangle^2} \\
\Phi_n^* & = \arctan \left( \frac{\langle r^n \sin(n\phi) \rangle}{\langle r^n \cos(n\phi) \rangle} \right) + \pi
\end{align*}
\]

where, the \( \langle \rangle \) implies averaging over all participating nucleons. However, for higher order harmonics the response can be driven by lower order \( \epsilon_n \) as well [4]. For example, \( v_4 \) not only receives a contribution from \( \epsilon_4 \), but also a non-linear contribution from \( \epsilon_2 \). Similarly, \( v_5 \) is not only driven by \( \epsilon_5 \), but also by the product \( \epsilon_2 \epsilon_3 \). Taking into consideration both the linear and leading non-linear components, the total \( v_4 \) and \( v_5 \) can be written as:

\[
\begin{align*}
v_4 e^{i4\Phi_4} & = \alpha_4 \epsilon_4 e^{i4\Phi_4} + \alpha_{2,4} \epsilon_2^2 e^{i4\Phi_2} + ... \\
v_5 e^{i5\Phi_5} & = \alpha_5 \epsilon_5 e^{i5\Phi_5} + \alpha_{2,3,5} \epsilon_2 \epsilon_3 e^{i3\Phi_3} + ...
\end{align*}
\]

where, the coupling constants \( \alpha \) are functions of centrality and the +,... at the end indicates that there can be additional non-linear terms. Using Eq. 2, Eq. 4 can be recast as:

\[
\begin{align*}
v_4 e^{i4\Phi_4} & = \alpha_4 \epsilon_4 e^{i4\Phi_4} + \beta_{2,4} \epsilon_2^2 e^{i4\Phi_2} + ... \\
v_5 e^{i5\Phi_5} & = \alpha_5 \epsilon_5 e^{i5\Phi_5} + \beta_{2,3,5} \epsilon_2 \epsilon_3 e^{i3\Phi_3} + ...
\end{align*}
\]

Equation 4 and its modified form, Eq. 5, explain the origin of event-plane correlations. For example, since \( \epsilon_2 \) drives both \( v_2 \) and \( v_4 \), correlation between the pair of phases \( \Phi_2 \) and \( \Phi_4 \) should be expected. Similarly, since \( \epsilon_2 \) and \( \epsilon_3 \) drive \( v_5 \), correlations are expected between the triplet \( \Phi_2-\Phi_3-\Phi_4 \). These strength of these correlations as a function of centrality can be quantified by the mean values of their cosines, \( \langle \cos(4\Phi_2 - 4\Phi_4) \rangle \) and \( \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle \) averaged over all events in that centrality.

These correlations can be quite large in certain circumstances. For example viscous effects are known to damp out the effects higher moments (or higher \( \epsilon_n \) in the initial geometry. In such a scenario, the linear terms in Eq. 4 would be much smaller than the non-linear terms leading to strong correlations between the event-plane angles.

It should be pointed out that non-linear hydrodynamic response in not the only mechanism by which the event-plane correlations can be generated. Such correlations can exist even in the linear response limit, i.e. when only the first terms on the right hand side in Eq. 4 are present. This can happen if there are correlations between the different \( \Phi_n^* \) in the initial geometry. These initial-geometry correlations can then be simply transferred to correlations between the \( \Phi_n \). However, as it will be shown, the correlations between the participant-plane angles calculated in initial geometry models are very different than the measured correlations between the event-planes. However, hydrodynamic and transport model calculations starting with the same initial geometry models reproduce the measured correlations quite well. Thus the measured correlations themselves can be used to determine their origin.
3. ATLAS measurements

Figure 1 shows the dependence of the eight two-plane correlators on the number of participating nucleons, \(N_{\text{part}}\), as measured by ATLAS. The correlators are calculated using the traditional event-plane method (shown as open symbols) as well as the scalar-product (SP) method (shown as solid symbols). The SP method is similar to the EP method but explicitly takes into account effects of event-by-event flow fluctuations, by applying weights to each event that depend on the magnitude of the observed \(v_n\) in that event [5]. On the other hand, in the EP method, each event is given equal weight. For this reason the EP and SP methods are also referred to as unweighted and weighted methods respectively.

Also shown for comparison in Fig.1 are the corresponding correlations between the participant-plane angles \(\Phi_n^*\) calculated in a Glauber geometry model. These model calculations are also done using procedures that mimic the EP and SP methods. Figure 2 shows comparison of the measured correlators to AMPT transport model calculations [5] for the six non-zero correlations.

The first three panels of Fig.1 show the first three moments of the \(\Phi_2 - \Phi_4\) correlation: 
\[
\langle \cos(4j(\Phi_2 - \Phi_4)) \rangle \quad \text{for } j=1-3.
\]
It is seen that the correlations are quite small in more central events (large \(N_{\text{part}}\)), but increase to fairly large values in mid-central and peripheral collisions. That all three moments are large, implies that the \(\Phi_4\) plane is strongly aligned with the \(\Phi_2\) plane. Comparing with the correlations between the participant planes \(\Phi_n^*\), it is seen that the measured correlators are very different than the correlations in the initial geometry. However the AMPT model calculations shown on Fig. 2 that start from a Glauber-like initial geometry model, but include final-state interactions through parton and hadron transport models reproduce the measured correlations quite well. This leads to the conclusion that the \(\Phi_2 - \Phi_4\) correlations are generated during collective expansion due to non-linear hydrodynamic response. Further, the large magnitude of the measured correlator in mid-central and peripheral events indicates that a larger fraction of the \(v_4\) is in fact generated by the second order moment \(v_2\), rather than the fourth order moment \(v_4\), of the initial geometry.

The fourth panel of Fig.1 shows the \(\langle \cos(6(\Phi_2 - \Phi_4)) \rangle\) correlator. The value of this correlator is much smaller than the \(\Phi_2 - \Phi_4\) correlators but still is significant. Due to the small magnitude of this correlator, it is not possible to draw strong physics conclusion from it. But it is interesting to note that even here, the AMPT results are in much better agreement with the measured correlator than the Glauber model calculation of the corresponding participant-plane correlations.

The fifth and sixth panels of Fig.1 show the \(\langle \cos(6(\Phi_2 - \Phi_6))\rangle\) and \(\langle \cos(6(\Phi_3 - \Phi_6))\rangle\) correlators respectively. It is interesting to note that while the \(\Phi_2\) and \(\Phi_3\) are weakly correlated with one another, they are both strongly correlated with \(\Phi_6\). In both cases the measured correlators are very different than the Glauber model calculations, but are recovered in AMPT calculations (Fig.2), clearly showing that these are also generated during the medium expansion. The structure of the correlators can be understood by writing \(v_6\) as:

\[
v_6 e^{i6\Phi_6} = \alpha_6 \epsilon_6 e^{i6\Phi_6^*} + \alpha_{2,6} \epsilon_2 e^{i6\Phi_2^*} + \alpha_{3,6} \epsilon_3 e^{i6\Phi_3^*} + \ldots
\]

\[
= \alpha_6 \epsilon_6 e^{i6\Phi_6^*} + \beta_{2,6} \epsilon_2 e^{i6\Phi_2^*} + \beta_{3,6} \epsilon_3 e^{i6\Phi_3^*} + \ldots
\]

(6)

The \(\langle \cos(6(\Phi_2 - \Phi_6))\rangle\) correlator is zero in most central events and increases in magnitude for mid-central and peripheral events. This behavior is similar to the \(\langle \cos(4(\Phi_2 - \Phi_4))\rangle\) correlator and can be understood as as the contribution from the second term on the right hand side of Eq. 6. As the \(v_6\) increases from central to mid-central events, the contribution of this term increases and leads to stronger \(\Phi_2 - \Phi_6\) correlations. On the other hand, the \(\langle \cos(6(\Phi_3 - \Phi_6))\rangle\) correlator is non-zero in most central events and decreases in magnitude for mid-central and peripheral events. In most central events, as the \(v_n\) are small, the \(v_3^2\) term is larger than the \(v_3^2\)
term, leading to strong $\Phi_3 - \Phi_5$ correlation. With decreasing $N_{\text{part}}$, the $v_2$ rapidly increases, but $v_3$ changes only weakly. This leads to a relative decrease in the contribution from the $v_3$ term leading to a decrease in the $\Phi_3 - \Phi_5$ correlation.

The last two measured correlators shown in Fig. 1 are $\langle \cos(12(\Phi_3 - \Phi_4)) \rangle$ and $\langle \cos(10(\Phi_2 - \Phi_5)) \rangle$. The measured correlators are consistent with zero, and also consistent with the Glauber model calculations over the measured $N_{\text{part}}$ range. These correlations are not expected to have any non-linear components, as it is not possible to write equations of the form given in Eq. 5 that only couple $v_3-v_4$ or $v_5-v_2$.

![Figure 1](image)

**Figure 1.** The set of two-plane correlators measured by ATLAS. The data points correspond to measurements via the scalar-product method (solid symbols) and the event-plane method (open symbols). The error bars and bands represent the statistical and systematic uncertainties respectively. The correlations between the participant-plane angles $\Phi^*_n$ calculated in a Glauber model are shown by the continuous curves for the weighted case, and by the dashed curves for the unweighted case. Figure taken from [3].

Figure 3 shows the three-plane correlators measured by ATLAS. Also shown for comparison are the corresponding participant-plane correlators calculated in a Glauber model. It is seen that the four correlators that are significant in strength are very different than the Glauber model calculations. Figure 4 compares the four non-zero three-plane correlators to the AMPT calculations, which reproduce the correlators remarkably well. These results clearly show that like the two-plane correlations, these are also generated during hydrodynamic expansion. In particular the large value of the $\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$ is quite informative. For $N_{\text{part}} < 330$ this correlator is well above 0.5, indicating that in this range a larger part of $v_5$ in fact is generated from non linear response to $\epsilon_2\epsilon_3$ rather than from $\epsilon_5$. 


Figure 2. Comparison of six of the two-plane correlators measured by ATLAS to AMPT calculations. Figure taken from [3].

Figure 3. The set of three-plane correlators measured by ATLAS. Comparisons are shown to the corresponding correlations between the participant-plane angles calculated in a Glauber model. Figure taken from [3].
Figure 4. Comparison of four of the two-plane correlators measured by ATLAS to AMPT calculations. Figure taken from [3].

4. Summary
The results presented here clearly demonstrate the ability of the event-plane correlations to elucidate the effects of non-linear hydrodynamic response in heavy ion collisions. Most of the measured correlators are very different than the corresponding correlators between participant-planes of the initial geometry evaluated in Glauber model calculations. However they are quite well recovered in AMPT calculations, indicating that they are the results of collective dynamics, and generated during the expansion of the medium. These correlations show that significant fraction of $v_4$, $v_5$ and $v_6$ are in fact generated by the lower order moments $\epsilon_2$ and $\epsilon_3$ of the initial geometry. It should be pointed out that these correlations are a completely new observable and upon further study may reveal additional interesting insights into the collective dynamics in heavy ion collisions.

The event-plane correlations can also be used to constrain other medium properties such as the viscosity to entropy density ratio $\eta/s$. This is because the viscous effects affect the linear and non-linear terms differently. Similarly these correlations can be used to constrain initial geometry models as well, as they are sensitive to the eccentricities in the initial geometry. This has been demonstrated by Qui and Heinz [6].

References
[1] Aad G et al. (ATLAS Collaboration) 2012 Phys.Rev. C86 014907 (Preprint 1203.3087)
[2] Aad G et al. (ATLAS Collaboration) 2013 JHEP 1311 183 (Preprint 1305.2942)
[3] Aad G et al. (ATLAS Collaboration) 2014 (Preprint 1403.0489)
[4] Gardim F G, Grassi F, Luzum M and Ollitrault J Y 2012 Phys.Rev. C85 024908 (Preprint 1111.6538)
[5] Bhalerao R S, Ollitrault J Y and Pal S 2013 Phys.Rev. C88 024909 (Preprint 1307.0980)
[6] Qiu Z and Heinz U 2012 Phys.Lett. B717 261–265 (Preprint 1208.1200)