Numerical modeling and parallel computations of heat and mass transfer during polymer flooding of non-uniform oil reservoir developing by system of producing and injecting wells

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Abstract. The mathematical and numerical models are developed for computation of interrelated thermal and hydrodynamic processes in the unified oil-producing complex during the polymer flooding of the heterogeneous oil reservoir exploited with a system of arbitrarily located injecting wells and producing wells equipped with submersible multistage electric centrifugal pumps with regulation of their working modes by the surface control stations. The complete differential model includes equations governing non-stationary two-phase three-component filtration in the reservoir and quasi-stationary heat and mass transfer in the wells and working channels of pumps. Special non-linear boundary conditions and dependences simulate the influence of the drossel diameter and the frequency electric current, respectively, on the flow rate and pressure at the wellhead of each producing well and the performance characteristics of all submersible units. The oil field development is also regulated by changes in bottom-hole pressure of each injection well, the concentration of polymer in water solution, its total volume and duration of injection. The problem is numerically solved with the use of the finite difference method and the iterative algorithms with application of technologies of parallel computing. It is shown that parallelization can improve the performance of calculations at several times in comparison with sequential computing.

1. Introduction
The subject of the study is the interrelated heat and mass transfer in the unified oil-producing complex "the oil reservoir – a system of arbitrarily located injecting wells and producing wells equipped with multistage electric submersible pumps (ESP) – the surface control stations (SCS)" during polymer flooding of the heterogeneous reservoir. Oil field development induces very intricate non-steady filtration motion of oil and water phases in the non-uniform porous medium [1–12]. Quasi-stationary thermal and hydrodynamic processes in the multiphase flows in the wells and working channels of ESP are also very complicated, they are accompanied by such factors as the phase transitions at degassing of oil in the pipes and gas dissolution in the channels of pump, compressibility of phases, friction, gravity force, restructuring of gas-liquid flow, inversion of phases, drift motion of disperse components, heat exchange between the flow and rock formation around the well, etc., see [13–22]. Performance characteristics of submersible unit significantly depend on properties of the pumped heterogeneous mixtures [10, 11, 16, 19, 22]. In addition, now the control for the current working modes of ESP are often realized with SCS on the base of analysis of the telemetering data (a feed-
forward) and generation of the needed actions (a feedback) to improve the work conditions of submersible equipment, up to its disable in emergency situations [11]. Therefore, computation of these processes and optimization of the oil recovery can be effectively done on the basis of mathematical and numerical modeling.

Previously, we proposed in [9–11] a computer model to study the features of thermal and hydrodynamic processes in such oil-producing object in the case of water-oil displacement. In this article, we continue our research and generalize mathematical, numerical and algorithmical models [9–11] to more complex practical situations when a water polymer solution of the desired concentration is injected into the porous medium with the purpose to create high-viscous moving fields and redirect the filtration flows. Chemical reagent (polymer) is soluble only in water and can enter into the reservoir both from all or some injection wells with different concentration, its total volume, an initial time and the end moment of injection. The sorption of thickener is irreversible and affects the permeability of porous medium, so the resistance factor is a function of the polymer concentration.

2. Mathematical model

Using [1, 4–6, 8] and generalizing the mathematical model [9–11], the system of conjugated non-linear two-dimensional differential equations governing simultaneous filtration of the polymer-water solution and the oil phase in the porous reservoir averaged over its thickness, one-dimensional quasi-stationary momentum (force-balance) and energy equations for the disperse water-oil-gas flow with discrete phases (gas bubbles, water drops or oil drops) in the \( m \)-th producing well and the \( m \)-th pumping unit, located in this well, can be written in the following form:

a) two-phase three-component filtration in the reservoir \( D_r = \{0 < x < L_x, \ 0 < y < L_y\} \):

\[
\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0; \quad v = v_w + v_o; \quad v_w = -KH_r \frac{k_w^*(S)}{\mu_w(C)} \nabla P; \quad v_o = -KH_r \frac{k_o^*(S)}{\mu_o} \nabla P; \quad (1)
\]

\[
mH_r \frac{\partial S}{\partial t} + \frac{\partial (f v_x)}{\partial x} + \frac{\partial (f v_y)}{\partial y} = 0; \quad mH_r \frac{\partial u}{\partial t} + \frac{\partial (C f v_{w,x})}{\partial x} + \frac{\partial (C f v_{w,y})}{\partial y} = 0; \quad (2)
\]

\[
k_w^* = \begin{cases} 0 & \text{if } 0 \leq S \leq S_1; \\ \frac{(S^*-S)/(S^*-S_1)}{S_1 \leq S \leq S^*}; \\ \frac{1}{(S^*-S)/S_1} & \text{if } S \leq S_1; \end{cases}
\]

\[
u = CS + a/m; \quad \mu_w = \mu_w(1 + \alpha_w C) \left[1 + \left(1 - B \cdot K(x,y)\right)\right]a(S,C)/a(S^*,C_{max}); \quad (3)
\]

b) water-oil-gas flow in the \( m \)-th producing well \( D_m = \{0 < z < H_m\} \), \( m = 1, M_p \):

\[
\frac{1}{f_r} \frac{d}{dz} \sum_{l=0,g,w} G_{li} = -\frac{dP}{dz} - \frac{2\tau_f}{r_t} + \rho_g \cdot \cos\phi; \quad (4)
\]

\[
C_{p,g}^l \frac{dT}{dz} = T \phi_{p,g}^l \frac{dP}{dz} - L \frac{G_{pg}}{G_r} \frac{d(C_{p,F})}{dz} + Q + \frac{2}{r_t} (r_{cm} v - q_{cm}); \quad (5)
\]

\[
\beta_g = \frac{G_s}{G_s + G_p \rho_G / \rho_s + G_p \rho_s / \rho_e}; \quad \beta_e = \frac{G_s}{G_e + G_p \rho_e / \rho_s + G_p \rho_s / \rho_e}; \quad G_s = f_q \rho_q \phi_q, \quad \rho = \sum_{l=0,g,w} \rho \phi \nu_l; \quad C_p^l = \sum_{l=0,g,w} \rho \beta_l C_{p,l}; \quad \alpha_p^l = \sum_{l=0,g,w} \alpha_{p,l} \beta_l; \quad x = a, g, w; \quad v_j = C_{p,j}^l v + \nu_{d,j}, \quad j = g, w, or j = g, o; \quad (6)
\]

c) three-phase flow in channels of \( m \)-th electric centrifugal pump \( D_{e,m} = \{0 < \xi \leq 1\} \), \( m = 1, M_p \):
\[
\frac{dP}{d\xi} + \frac{\rho M_E H}{\xi} = \frac{C_p^o}{\rho} \frac{dT}{d\xi} = \left( T\alpha_p^o + \frac{1-\eta}{\eta} \right) \frac{dP}{d\xi} - \frac{L G_w}{1-C_F} \frac{d(C_F)}{d\xi},
\]

where \(M_p\) is the number of the producing wells; \(\tau\) is the time; the sub-indexes “\(w, o, g\)” denote characteristics of water, oil and gas, respectively; \(L_o\) is the length of the filtration region \(D_i\); \(L_y\) is its width; \(H_m\) is the depth of the \(m\)-th producing well; \(M_E\) is the total number of the pump stages.

Equations (1), (2) govern the two-phase isothermal filtration under the Darcy’s law without regard to the capillary effects, gravity, compressibility of the phases and the porous medium [1, 4–6, 8]. Here, \(x\) and \(y\) are spatial coordinates; \(P\) is the pressure; \(S\) is the water saturation; \(C\) is the mass concentration of the thickener; \(a\) is mass amount of thickener that adsorbed in pores; \(v_x, v_y, v_{w,y}\) are the projections of the filtration velocity vectors \(\mathbf{v}, \mathbf{v}_w\) of the water-oil mixture and water to axis \(Ox\) and \(Oy\); \(K(x,y), m(x,y)\) and \(H_r(x,y)\) are the absolute permeability, the dynamic porosity and the thickness of the reservoir; \(\mu_w\) and \(\mu_o\) are the dynamic viscosity of water and oil phase; \(f\) is the fraction of water in the total two-phase flow (the Bacley-Leverett function); \(K_w^*\) and \(K_o^*\) are the water-oil relative permeability; \(n_w\) and \(n_o\) are the power exponents; \(S\) is the irreducible water saturation; \(S^*\) is the limiting water saturation; \(C_{max}\) is the maximum value of the thickener concentration; \(\mu_w^0\) is the viscosity of water without the thickener at \(C = 0\); \(A, B\) and \(\alpha_p\) are empirical coefficients that must be determined throughout the range of values of absolute permeability of the oil reservoir. In this paper we assume that the adsorption \(a = a(S, C)\) is an equilibrium and irreversible, so that this dependence is determined by the Henry sorption isotherm \(a(S, C) = \Gamma SC\) at \(C \in [0, C_{max}]\), where \(\Gamma\) is the Henry coefficient.

Equations (3), (4) are written in a framework of the quasi-stationary approximation [14–19] for the dispersed water-oil-gas flow in the producing well. In such three-phase mixture the discrete components move as the gas bubbles and drops of water (or oil) inside the continuous (oil or water) phase. In these equations \(O\xi\) is the vertical coordinate axis directed upwards the well from its beginning on the reservoir roof; \(P\) and \(T\) are the pressure and the temperature, identical for all phases; \(\rho\) is the mean multiphase mixture density; \(\nu\) is the overall mixture flow velocity; \(\nu_x, \nu_y, \varphi_i, \beta_i\) and \(G_i\) are the density, the actual phase velocity, the volumetric concentration, the consumption content and the mass flow rate (debit) of \(i\)-th phase, averaged over the well pipe cross-section \(f_r\) of radius \(r_i\), \(i = o, g, w\); \(G\) is the overall mixture debit; \(F(P,T)\) is the relative gas factor which is defined as the ratio of the mass of gas released from the oil phase at certain \((P,T)\) to the total amount of the initially dissolved gas; \(C_s\) is the corresponding mass concentration of gas in the oil phase at \(P > P_g\), where \(P_g\) is a saturation pressure; \(L\) is latent heat of gas dissolution into oil; \(\tau_{cm}\) and \(q_{cm}\) are the hydraulic friction and the heat flux density at the internal surface of the producing well; \(Q_{\nu}\) is intensity of the external heat source distributed along the producing well; \(C_F^p\) is the generalized Zuber’s parameter [21, 22] taking into account the nonuniform distribution of the overall velocity \(\nu\) and volumetric concentration \(\varphi_i\) of \(i\)-th dispersed phase in the cross section of the well: \(l = g, w\) or \(l = g, o\) in the mixture with a continuous oil or water phase, respectively; \(\nu_{d,i}\) is the drift velocity of the discrete phase; \(\alpha_{pi}\) and \(C_{pi}\) are the coefficients of
volumetric thermal expansion and volumetric elasticity of \(i\)-th phase; \(g\) is the gravity acceleration; 
\(\phi(z)\) is the angle between the well profile and the axis \(Oz\).

It should be stressed that the systems (3), (4) and (5) are written here only for \(m\)-th producing well \(D_m\) with its own pumping unit \(D_{e,m}\). Obviously, the complete mathematical model of processes in the oil-extracting complex consists of \(M_p\) systems similar to equations (3)–(5), \(m = 1, M_p\), which differ in their geometric parameters, type of ESP unit and its location in the well, the overall and phase debits from the oil reservoir, distribution of thermal and hydrodynamic characteristics along the well, the values of the bottom-hole and wellhead pressure, etc. For convenience, we omit the index \(m\) in notation of functions and parameters in equations (3)–(5).

Equations (5) govern the thermal and hydrodynamic processes in the channels of the multi-stage pump. They were developed in [16, 19] under the assumption that all the phases become highly dispersed in the pump stage and move without slippage, i.e. \(\varphi_i = \beta_i, v_i = \nu_i\), in result of the enormous rotation speed of blades. In the equations (5) \(\xi_i\) is a share of the pump stages; \(H, \eta = g \rho HQ / N\) and \(N\) are the head, the efficiency factor and the power consumption of the pump stage. These characteristics depend on the volumetric flow rate \(Q = G / \rho\) and the effective viscosity \(\mu\) of the three-phase mixture which can significantly decrease in the result of compression of phases and gas dissolution in oil as flow moves along the pump.

It should be noted also that this article provides only some basic relationships which define the operating parameters of the pump stages and characteristics of multiphase flows in the pipes and in the porous medium. A complete set of special constitutive relations to close the equations is too much large and it can be found in our publications [4–11, 16, 18–20]. The boundary, initial and conjugation conditions for system of differential equations (1)–(5) are analogous to that, which has been formulated in [9, 11]. In addition to these conditions, the initial distribution of the polymer concentration in the region \(D_i\) and the boundary conditions for the second transport equation (2) for the function \(\bar{C}\) at the bottom-hole of each injection well can be written by analogy with [4–6, 8] in the following form:

\[
C(x, z, 0) = 0, \quad x, y \in D_i; \quad C \mid_{(x, y) \in \Gamma_i^z} = \begin{cases} C_i^*, & \tau \in (\tau_i^b, \tau_i^e), \\ 0, & \tau \not\in (\tau_i^b, \tau_i^e), \end{cases}, \quad i = 1, M_I, \tag{6}
\]

where \(M_I\) is the number of the injection wells; \(\Gamma_i^z\) is the boundary of the bottom-hole cylindrical surface of the \(i\)-th injection well; \(\tau_i^b\) and \(\tau_i^e\) are the initial time and the end moment of injection of polymer solution; \(C_i^*\) is its initial concentration in water phase.

An important feature of the model is the ability to simulate the control actions from the earth surface on development of the oil reservoir [9–11]. First of all, this can be done by using nonlinear boundary conditions

\[
P\mid_{z = H_m} = P_{m,wh} = P_{m,in} + 0.5 \varepsilon_{dr} (d_{dr}^{-1} (\rho \nu)^2)\mid_{z = H_m}, \tag{7}
\]

that allow changing the flow velocity \(\nu_{m,wh} = \nu\mid_{z = H_m}\) and pressure \(P_{m,wh}\) at the wellhead \(z = H_m\) of each producing well by varying the diameter \(d_{dr}\) of the regulating drossel. Here \(\varepsilon_{dr}\) is local resistance coefficient of drossel; \(P_{m,in}\) is the pipe line pressure behind the drossel, which can be assumed a constant for \(m\)-th producing well. In addition, the control station allows changing the operation modes of the submersible pumps and their electric motors by following relationships [9]

\[
Q_w = Q^* w / \omega^*, \quad H_w = H^* w (\omega / \omega^*)^3, \quad N_w = N^* w (\omega / \omega^*)^1, \quad N_M = N^* M (\omega / \omega^*)^3. \tag{8}
\]
by varying the frequency $\omega$ of the electric current, where $Q^*_w$, $H^*_w$ and $N^*_w$ are the volumetric flow rate, head and useful capacity of a certain pump stage during its operation on water at the nominal conditions at $\omega^* = 50$ Hz; $N^*_M$ is the nominal consumed power of the motor at $\omega^* = 50$ Hz; $Q^*_w$, $H^*_w$, $N^*_w$ and $N^*_M$ are the similar characteristics of the stage and motor at $\omega \neq \omega^*$. From the other hands, oil field development can be also controlled by vary of pressure $P^*_f$ and polymer concentration $C_i^*$, its total volume and duration $T_i = \tau_i^* - \tau_i^{ib}$ of injection at the bottom-hole of every injecting well.

3. Numerical model

An approximate solution of the system of nonlinear differential equations (1)–(8) is founded by the finite difference method and iterative algorithms. In detail the conservative finite difference scheme, approximating equations (1)–(5) in the grid-points of the discrete domains $D^h_r$, $D^h_m$, $D^h_{em}$ is discussed in [11] at absence of the second transport equation (2) for concentration $C$ for the case of only water-oil displacement in the reservoir. The following is a brief summary of the principal results of [11] and their extension to the case of polymer flooding.

The pressure $P$ is calculated from the quasi-linear elliptic equation $\text{div}[\kappa(S,C)\text{grad}P] = 0$, $\kappa(S,C) = K(x,y) \cdot H_r(x,y)[k^*_w(S)/\mu_w(S,C) + k^*_v(S)/\mu_v]$, that can be obtained from equations (1), (2). This equation is approximated by the five-point difference scheme of the second order. The approximate system of algebraic equations in respect to $P$ is solved by the iterative method [4, 5, 7, 11] of a high rate convergence (3-5 iterations) at every time step.

The first transport equation (2) for water saturation $S$ is approximated in the all points $(x_i, y_k)$ of the main grid $D^h_r$ except the centers of wells with the use of points $(x_{i+0.5}, y_{k+0.5})$ of additional grid $\overline{D}^h_r$ by the upward finite-difference scheme of the second order in respect to average integral values $J_{i\kappa}$ of saturation:

$$mH_i h(J_{i\kappa} - J_{i,k})/h = V_{r_{1/2,k}} - V_{r_{1,k+1/2}} + V_{r_{1,k+1/2}} - V_{r_{1,k-1/2}}; \quad J_{i\kappa} = h^{-2} \int_{D_k} S \text{d}x \text{d}y.$$  (9)

where $V_{r_{1/2,k}} = h(fv_x)_{x=2/3}$, $V_{r_{1,k+1/2}} = h(fv_y)_{y+k=1/3}$. $h$ is the time step; step $h = h_x = h_y$ is the step of grid along the axes $Ox$, $Oy$ in the discrete domains $D^h_r$; $D_k$ is the rectangular element of the grid $D^h_r$ with the centers located in the points $(x_{i+0.5}, y_{k+0.5}) \in \overline{D}^h_r$. The values of water saturation $S_{i,1/2,k}$ and $S_{i,1,k/2}$ are defined with application of linear-fractional interpolation through integral values $J_{i\kappa}$ in depending on the flow direction. Let, for example, $V_{r_{1/2,k}} < 0$, i.e. fluid flows through the border $\Gamma_{i+1/2,k}$ from the cell $D_{i\kappa}$ into the cell $D_{i+1,k}$. Then

$$S_{i+1/2,k}^* = \begin{cases} S^{*} \leq J_{i\kappa}, \\ F, \quad F \in \left[ J_{i+1,k}, J_{i\kappa} \right], S_\epsilon + \epsilon_\epsilon \leq J_{i\kappa} < S^{*} - \epsilon^*, \\ J_{i\kappa}, \quad F \notin \left[ J_{i+1,k}, J_{i\kappa} \right], S_\epsilon + \epsilon_\epsilon \leq J_{i\kappa} < S^{*} - \epsilon^*, \\ S_\epsilon, \quad J_{i\kappa} \leq S_\epsilon + \epsilon_\epsilon; \end{cases}$$

(10)

$$F = \begin{cases} 0.5(J_{i+1,k} + J_{i\kappa})/J_{i+1,k}, \quad J_{i+1,k} \geq J_{i\kappa}, \\ 0.5(1 + J_{i\kappa} - (1 - J_{i\kappa})^2/(1 - J_{i+1,k})), \quad J_{i+1,k} < J_{i\kappa}. \end{cases}$$

The finite difference scheme for solving of second equation (2) is similar to (9), (10) and can be written in according to [4, 5] as:
\[ mH \frac{h(U_{i,k}^{n+1} - U_{i,k})}{h_t} = W_{i+1/2,k} - W_{i-1/2,k} + W_{i,k+1/2} - W_{i,k-1/2}; \quad U_{i,k} = h^2 \int_{D_{i,k}} udxdy. \] (11)

where \( W_{i+1/2,k} = h(fC_{i,k}^{n+1/2}) \), \( W_{i,k+1/2} = h(fC_{i,k}^{n+1/2}) \). The grid functions \( C_{i+1/2,k} \) and \( C_{i,k+1/2} \) are determined on the basis of parabolic interpolation through values \( U_{i,k} \) also taking into account the flow direction. So, at \( V_{i+1/2,k} < 0 \),

\[
C_{i+0.5,k} = \begin{cases} 
C_{max}, & \text{if } C_{i,k} \leq C_{i,k}, \\
5/6C_{i,k} + 1/3C_{i+1,k} - 1/6C_{i-1,k}, & \text{if } C_{max} \leq C_{i,k} \leq C_{max}, \\
0, & \text{if } C_{i,k} \leq \varepsilon_1,
\end{cases}
\] (12)

In relationships (10), (12) \( \varepsilon^*, \varepsilon_*, \varepsilon_1, \varepsilon_2 \) and \( \varepsilon_3 \) are the given parameters [4, 5, 7].

It is need also to note that the water saturation \( S \) and concentration \( C \) are multivalued function at the boundaries of the producing and injection wells. This specific feature of the problem solution can be taken into consideration by means of the special method [4], which allows us to determine the values of water saturation in eight directions to the well (in the four directions along the coordinate axes and along the four diagonal directions). In this method the average integral values of the water saturation are determined with the use of the unit cells of the special geometry in the vicinity of the borehole of wells which look as the sectors. For example, the difference equations for the saturation \( S \) along axis \( Ox \) and along the diagonal direction of the first quadrant are written in the following form:

\[
J_{i+1,j}^{n+1} = J_{i+1,j}^{n+1} + \frac{4h_t}{3H \rho_m} \left( \varphi_{i+1,j,n+1} - \varphi_{i+1,j,n} \right); \quad J_{i,j+1}^{n+1} = J_{i,j+1}^{n+1} + \frac{2h_t}{3H \rho_m} \left[ \varphi_{i+1,j,n+1} \right].
\] (13)

The formulas for water saturation and concentration along other directions can be written by analogy with equations (13). To provide the stability of solution equations (9), (11), (13) the time step \( h_t \) of the scheme is determined in accordance with the Courant-Friedrichs-Lewy criterion.

Systems of first order differential equations (3), (4) and (5) are solved with implicit Euler's schemas. However, their realization is significantly complicated because of a large number of special, usually non-linear constitutive relations. For example, performance characteristics (head \( H \), power \( N \), efficiency \( \eta \)) of pump stages during pumping of the non-uniform mixtures are computed with the modified procedure [9, 16, 19] based on nonlinear semi-empirical dependencies [23]. Other similar features of equations also require the use of iterative procedures in calculations. Moreover, the number of the producing wells equipped with ESP-units on the certain oil field can reach several hundreds, so the general number of grid points in all the wells may be much more than the number of grid points in the reservoir. This results to that the calculating times of filtration and heat mass transfer in the all wells become comparable.

In realization of the complete numerical model the linear pressure behind the drossel at the wellhead of each well is determined by its given value \( P_m^{lin} \), which can be much less than an unknown pressure \( P_m^{wh} \) before him, so as the values \( \rho_m^{wh}, w_m^{wh}, \xi_m \) of density, flow velocity and local resistance coefficient of the regulating drossel in the relationship (7) are unknown and should be determined in solving system (1)–(8). Therefore, this system could be iteratively solved in such a way that the nonlinear boundary conditions (7) at \( z = H_m \) have been simultaneously satisfied to the specified accuracy for every producing well, \( m = 1, M_p \).
The developed numerical model is implemented in the program package “PolymerFlood” with using of technologies of parallel computing [24, 25] and simultaneous visualization of the results of computation. This package interacts with the special program “SCS” which simulates an operation of the surface control stations for regulation of work of equipment in the producing wells. PolymerFlood sends telemetry data and values of the current operating parameters of the submersible pump unit to SCS (a direct relation). In turn, this program analyses incoming data and generates the desired control parameters (ω and etc.) for the pump unit. These parameters are sent to PolymerFlood (a feedback). Additionally, there is the interactive interface for changing the parameters $C^i_0$, $\tau^b_0$, $\tau^e_0$, $P^{bh}_i$ of polymer flooding at a given $i$-th injection well during computation, $i = 1, M_I$.

Parallelizing of calculations is effective in the following cases: 1) computation of two-phase three-component filtration in heterogeneous reservoir using grids which can contain tens and hundreds thousand mesh points; 2) calculation of thermal and hydrodynamic processes in the producing wells with electric pumps at the grids with a large number of nodes; 3) realization of the direct relation and feedback between PolymerFlood and SCS packages for all the wells and control stations; 4) visualization of the two-dimensional computed and initial characteristics in the filtration area on display when the number of pixels reaches several millions.

4. The results of numerical experiments

Computational experiments were carried out and analyzed for the concrete oil-extracting complex. Their results shown that parallelization can improve the performance of the calculations at several times in comparison with sequential computing. In particular, it is shown that in depending on the type of CPU and GPU the time of parallel computations at the central processor in solving filtration problem reduces by 2-3 times, calculation of processes in the system of producing wells accelerates proportional to the number of processors and visualization with the use of graphic coprocessors – at about 4-7 times.

5. Conclusion

The complete mathematical model to study interrelated heat and mass transfer in the unified oil-producing complex during the polymer flooding of the heterogeneous oil reservoir exploited with a system of injection wells and producing wells equipped with submersible electric centrifugal pumps is proposed. An important feature of the model is the ability to simulate the control actions from the earth surface on the oil reservoir development. Numerical methods are developed for solving the system of corresponding non-linear differential equations. The numerical and algorithmical models are implemented in the program package. The analysis of results of computational experiments has also shown that on-line creation of the moving polymer fields due to injection of thickener in the desired wells significantly increases the uniformity of oil displacement in the heterogeneous reservoir and improves their basic exploitation parameters by redirecting the filtration flows.

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