A Fast Deterministic Algorithm for Side Lobe Level Reduction of Open Loop Coplanar Distributed Antenna Arrays in WSNs

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Abstract—Distributed beamforming (DBF) is an efficient technique for reliable communications in wireless sensor networks (WSNs). In DBF based networks, the randomly distributed nodes cooperate to form a randomly distributed antenna array (RAA) which has a main beam directed towards the intended receiver. Due to the nodes randomness, the DBF results in poor pattern characteristics such as high side lobe level (SLL) and pattern asymmetry around the main beam sides. In this paper, a fast deterministic algorithm for SLL reduction of open loop distributed antenna arrays is introduced. Unlike the existing state of the art optimization techniques for SLL reduction, the proposed algorithm provides a fast deterministic solution for energy transmission or the weight of each node without changing its location. Consequently, the exhaustive search burden of the optimization based techniques for the optimum weights is avoided. The simulation results reveal that the proposed algorithm has superior performance to the optimization techniques in terms of execution time, synthesized SLL, and half power beamwidth (HPBW).

1. INTRODUCTION

The traditional antenna arrays consisting of periodic structures such as linear, planar, and circular antenna arrays configurations suffer from scan blindness problem and tight fabrication constraints [1]. Also, the utilization of co-located antennas or traditional arrays in wireless communication systems may lead to significant frequency selective fading, limited transmit power, limited bandwidth, and reduced system capacity. As a promising solution for these critical problems, the distributed antenna networks have been introduced in [2]. In the same context, the distributed Multi-input Multi-output (D-MIMO) has been introduced in [3] for further enhancement of the spectral and energy efficiency of the conventional co-located MIMO (C-MIMO). Wireless Sensor Networks (WSNs) consist of a large number of sensor nodes distributed over a specific area. The nodes are collaborating together for sensing, collecting, and processing information. They have a limited power supply and can’t transmit a signal for a long distance [4]. Distributed beamforming is the key solution for mitigating these problems. In DBF, each sensor node acts as a virtual antenna element to construct a randomly distributed antenna array (RAA). However, the randomness of the distributed nodes creates an array pattern having a high SLL which causes high interference with the unintended receivers located within the same region as well as reducing the received power level at the intended receiver [5,6]. The interference with the unintended receivers limits the system capacity and increases the bit error rate. As antenna arrays with low side lobe levels are required for efficient and reliable communications, many research works are introduced for SLL reduction of distributed antenna arrays. In [7], a node selection based technique for SLL reduction was introduced. It is mainly based on selecting a combination of nodes from the available set of nodes in the WSN and determines the nodes weights according to their locations.
However, it depends on the MAC protocol and one-bit feedback from the unintended receiver which is impossible in some cases. In [8], a modified version of the node selection technique denoted as Bat-Chicken Swarm Optimization (BATCSO) was introduced. It tends to optimize the peak SLL of the array pattern by controlling the nodes transmission energies. Along these lines, a Genetic Algorithm (GA) based technique for SLL minimization was introduced in [9]. It synthesizes the transmission energy of each node without changing the nodes locations. It provides better performance compared to the conventionally distributed beamforming (CDBF). In [10], two Weightless Swarm Algorithm (WSA) and Particle Swarm Optimization (PSO) based techniques were introduced for SLL reduction by adjusting the nodes transmission energy. But, they suffer from increased computational complexity. The WSA based technique provided higher SLL reduction than GA and PSO based techniques which consequently improves the signal to noise and interference ratio (SINR) and the capacity of unintended receivers. Along these lines, PSO and Gravitational Search Algorithm (GSA) based SLL reduction techniques were introduced in [11]. They control both transmission energy and transmission phase of each node without any changes in the nodes locations. In [12], a hybrid meta-heuristic optimization algorithm denoted as (PSOGSA-E) which is a combination between the PSO and GSA-Explore was introduced. It suppresses the SLL by optimizing the weight (amplitude and phase) of each node in the RAA. Also, the Non-dominated Sorting GA with selective distance (NSGA-SD) algorithm was introduced in [13]. It provides a bi-objective optimization formulation for the DBF. It controls the weight (amplitude and phase) of each node to minimize the SLL and at the same time maximizes the directivity of the array pattern. But, it is worth pointing to that all the aforementioned SLL reduction techniques which are based on optimization algorithms are time consuming.

There are several applications such as satellite communications, radar systems, and wireless sensor networks where large arrays sizes are very critical to achieve the desired radiation patterns to fulfill the required systems performances. However, large antenna arrays based systems have large computation burden, complex RF front end chains, and high power consumption. To mitigate these problems, adaptive beamforming making the use of sparse characteristics of large antenna arrays based systems is the key solution. In [14], an efficient $l_0$-norm constrained normalized least-mean-square ($L_0$-CNLMS) adaptive beamforming algorithm for controllable sparse antenna arrays was introduced. It is suitable for sparse antenna arrays of different configurations such as standard hexagonal array (SHA) used in satellite communications, rectangular array (RA) used for C-band based radar systems, triangular array (TA) used for P-band based stealth aircraft and satellite detection systems, and irregular arrays (IA) used for S-band communications. Also, it converges faster and utilizes fewer number of antenna elements compared to state of the art sparsity based adaptive beamforming algorithms. However, for non-sparse arrays, its performance is degraded and provides high SLL compared to the conventional non-sparse beamforming algorithms. In the same context, several sparsity based optimization filtering algorithms can be utilized in the SLL reduction as introduced in [15–17]. These algorithms have proved their effectiveness in the well-defined antenna arrays structures such as linear and planar configurations. However, they have to be modified to be applied to randomly distributed antenna arrays. In RAAs, the randomness of nodes distribution may make the separation distances between some of the array nodes to be small enough to maximize the mutual coupling between the neighboring nodes. Several techniques have been introduced to minimize the coupling effect between the antenna arrays elements as in [18] and [19].

In this paper, a fast deterministic DBF algorithm for maximum SLL reduction of open loop distributed antenna arrays is introduced. It determines the transmission energy of each node without inspiring the node location. It saves the computation time and search burden of the best weights required to synthesize the desired pattern, especially for large size RAAs. Also, it does not require feedback from the unintended receivers. The simulation results revealed that the proposed technique outperforms the recent state of the art optimization based SLL reduction techniques which handle the nodes weights. It is suitable for the distributed automotive 77 GHz radar sensors. The existing low power 77 GHz radar sensors suffer from their limited detection range. To extend the radar range, some randomly distributed sensors within a specific area can be grouped to form a RAA to take the advantage of array gain in increasing the radar range. This work is done under the contract between the National Telecom Regulatory Authority (NTRA), Ministry of Communications and Information Technology (MCIT), Egypt and the Electronics Research Institute (ERI), Ministry of scientific research, Egypt start date
2018. The paper is organized as follows. In Section 2, the system model is introduced. The proposed SLL reduction algorithm is presented in Section 3. The simulation results are illustrated in Section 4. Finally, the paper is concluded in Section 5.

2. SYSTEM MODEL

In this section, the geometrical configuration of distributed antenna arrays is introduced. Consider $K$ nodes which are distributed over a circular disk of radius $R$ meters. Each node has polar coordinates $(r_k, \psi_k)$ where $r_k$ is the distance of the $k^{th}$ node from the central point of the cluster, $r_k \in [0, R]$, and $\psi_k$ is the azimuth angle of the $k^{th}$ node with respect to $x$-axis, $\psi_k \in [-\pi, \pi]$. It is assumed that all nodes are isotropic antennas and coplanar with each other. Furthermore, all nodes are perfectly synchronized in phase, time, and frequency. Assume that an intended receiver exists in the proximity of other unintended receivers distributed randomly in space as shown in Fig. 1. The intended receiver has spherical coordinates $(A, \theta_0, \phi_0)$, where $A$ is the distance between the intended receiver and the central point of the RAA, $\theta_0$ the elevation direction, $\theta_0 \in [0, \pi]$, and $\phi_0$ the azimuth direction, $\phi_0 \in [-\pi, \pi]$ of the intended receiver. The spherical coordinates of the $L$ unintended receivers are $(A_l, \theta_l, \phi_l)$, $l = 1, 2, \ldots L$, where $A_l$ is the distance between the unintended receiver and the central point of the RAA, $\theta_l$ the elevation direction, $\theta_l \in [0, \pi]$, and $\phi_l$ the azimuth direction, $\phi_l \in [-\pi, \pi]$ of the unintended receivers. Also, assume that the intended and unintended receivers are located within the same plane as the distributed nodes where $\theta_0 = \theta_l = \frac{\pi}{2}$. Fig. 1 shows the geometrical structure of distributed antenna array [5].

![Figure 1. Geometrical structure of the distributed antenna array.](image)

3. PROPOSED SLL REDUCTION ALGORITHM

In WSNs, to mitigate the high interference with the unintended receivers and increase the received power level at the intended receiver, SLL reduction is the key solution. It significantly improves the capacity and the bit error rate performance of the network. In this section, the proposed algorithm for SLL reduction of RAAs is introduced. The steps of the proposed algorithm are presented as follows:

Consider an ordinary RAA consisting of $K$ nodes distributed over a circular disk of radius $R$ with coordinates $r = [r_1, r_2, \ldots, r_K]$ and $\psi = [\psi_1, \psi_2, \ldots, \psi_K]$ where $r_k$ and $\psi_k$ are the radius and angle of the
The synthesis of an antenna array pattern with a specified null placement has been studied extensively, with a focus on minimizing side lobe levels (SLL). For an antenna array (AA) with a main beam directed towards the receiver, the radiation pattern is given by:

\[ AF(\varphi) = \frac{1}{K} \sum_{k=1}^{K} w_k e^{-j \frac{2\pi}{\lambda} r_k \cos(\varphi - \psi_k)} \]

where \( w_k \) is the transmission weight of the \( k^{th} \) node which is given by:

\[ w_k = \xi_k e^{j \Psi_k} \]

where \( \xi_k \) and \( \Psi_k \) are the \( k^{th} \) node transmission energy amplitude and initial transmission phase, respectively. For a uniformly fed array, \( \xi_k = 1 \) and \( \Psi_k \) is determined by [5] as follows:

\[ \Psi_k = \frac{2\pi}{\lambda} r_k \cos (\varphi_0 - \psi_k) \]

It is required to synthesize an array pattern which has a main beam directed towards the intended receiver with minimum SLL. In this case, the desired array pattern, \( AF_d(\varphi) \), can be defined as follows:

\[ AF_d(\varphi) = \begin{cases} 
0, & -\pi \leq \varphi < \varphi_{NL1} \\
AF(\varphi), & \varphi_{NL1} \leq \varphi \leq \varphi_{NL2} \\
0, & \varphi_{NL2} < \varphi \leq \pi 
\end{cases} \]

where \( \varphi_{NL1} \) and \( \varphi_{NL2} \) are the angles of the first two nulls of the ordinary array pattern \( AF(\varphi) \). For clarification, consider the ordinary array pattern for \( K = 16 \) elements, \( R = 1 \) m, and the main beam is directed at \( \varphi_0 = 0^\circ \). Then, the desired array pattern \( AF_d(\varphi) \) is plotted as shown in Fig. 2.

![Figure 2](image.png)

**Figure 2.** The desired array pattern \( AF_d(\varphi) \) for \( K = 16 \) elements, \( R = 1 \) m and the main beam directed at \( \varphi_0 = 0^\circ \).

The synthesized array pattern \( AF_{syn}(\varphi) \) should have the same characteristics as the desired array pattern \( AF_d(\varphi) \) such that:

\[ AF_{syn}(\varphi) = \frac{1}{K} \sum_{k=1}^{K} v_k e^{-j \frac{2\pi}{\lambda} r_k \cos(\varphi - \psi_k)} \approx AF_d(\varphi) \]

where \( v_k \) is the synthesized transmission weight of the \( k^{th} \) node which equals \( \delta_k e^{j \Psi_k} \). \( \delta_k \) is the synthesized transmission energy of the \( k^{th} \) node, while the initial transmission phase \( \Psi_k \) of the \( k^{th} \) node remains fixed as defined in Eq. (3). Substituting \( v_k \) in Eq. (5), the synthesized array pattern is rewritten as

\[ AF_{syn}(\varphi) = \frac{1}{K} \sum_{k=1}^{K} \delta_k e^{j \Psi_k} e^{-j \frac{2\pi}{\lambda} r_k \cos(\varphi - \psi_k)} \approx AF_d(\varphi) \]
To estimate the transmission energy of each node, \( \delta_k \), Eq. (6) is transformed into a matrix form as follows:

\[
\begin{bmatrix} \delta \end{bmatrix}_{1 \times K} \times \begin{bmatrix} S \end{bmatrix}_{K \times N} = \begin{bmatrix} U \end{bmatrix}_{1 \times N}
\]

(7)

or for simplicity Eq. (7) is written as:

\[
\delta S = U
\]

(8)

where \( N \) is the number of samples of the desired pattern \( AF_d(\varphi) \). The number of samples is chosen to be large enough to maintain the pattern smoothness and details. For a given number of samples, \( N \), the sample angles of \( \varphi \in [-\pi, \pi] \) can be calculated by:

\[
\varphi_n = \frac{2n\pi}{N}, \quad n = 1, 2, 3, \ldots N
\]

(9)

The elements of \( [U]_{1 \times N} \) vector are the samples of the desired pattern \( AF_d(\varphi) \) at the sample angle \( \varphi_n \) within the range \( -\pi \leq \varphi \leq \pi \) and can be defined as:

\[
[U]_{1 \times N} = [AF_d(\varphi_1) AF_d(\varphi_2) \ldots AF_d(\varphi_N)]
\]

(10)

\( \delta \) is the \( (1 \times K) \) vector of the synthesized transmission energies of the distributed nodes which is given by

\[
\delta = [\delta_1 \delta_2 \ldots \delta_K]
\]

(11)

\( S \) is a \( K \times N \) matrix whose elements are given by

\[
S_{kn} = \frac{1}{K} e^{j\Psi_k} e^{-j\frac{2\pi}{N} r_k \cos(\psi_n - \psi_k)}, \quad k = 1, 2, \ldots, K \quad \text{and} \quad n = 1, 2, \ldots, N
\]

(12)

The synthesized transmission energy vector can be obtained by solving Eq. (8). As \( S \) is a non-square matrix, both sides of Eq. (8) are multiplied by the Hermitian transpose of the matrix \( S \) which is denoted as \( S^H \). Then Eq. (8) is written as:

\[
\delta S S^H = U S^H
\]

(13)

Let \( R_{SS} = S S^H \) which is a \( K \times K \) square matrix. Then Eq. (13) can be rewritten as:

\[
\delta R_{SS} = U S^H
\]

(14)

Multiplying both sides of Eq. (14) by the inverse of the \( R_{SS} \) matrix, the synthesized transmission energy vector can be calculated by:

\[
\delta = U S^H R_{SS}^{-1}
\]

(15)

where \( R_{SS}^{-1} \) is the inverse of the square matrix \( R_{SS} \).

4. SIMULATION RESULTS

In this section, several simulations are carried out to evaluate and compare the performance of the proposed algorithm with that of the GA based synthesis techniques introduced in [9, 10] and that of the NSGA-SD algorithm introduced in [13]. The GA is utilized to synthesize the antenna array for the maximum SLL reduction by optimizing the transmission energy \( \delta \) which minimizes the following cost function.

\[
CF(\delta) = 20\log_{10}\frac{\max(AF(\varphi_{SL}))}{AF(\varphi_{ML})}
\]

(16)

where \( AF(\varphi_{SL}) \) is the amplitude of the array pattern at the side lobe angle \( \varphi_{SL} \) which is defined as \( \varphi_{SL} \in [(-\pi, \varphi_{NL1}) \cup (\varphi_{NL2}, \pi)] \), while \( AF(\varphi_{ML}) \) is the amplitude of the array pattern at the main lobe angle \( \varphi_{ML} \). Also, the maximum number of iterations \( (I_{ga}) \) of the GA is limited to \( I_{ga} = 100 \) as introduced in [9, 10]. In the simulations, the number of array pattern samples is set to \( N = 1000 \) samples for the proposed algorithm and the other algorithms of comparison introduced in [9], [10], and [13]. The simulations are carried out using MATLAB R2016a on ASUS laptop Intel core i5-5200U. The simulation results are divided into two sections; Section 1 handles the analysis of the synthesized array pattern, and Section 2 handles the analysis of the SLL under the impact of the variations in the number of nodes, \( K \), and the circular area radius, \( R \).
4.1. Synthesized Array Pattern Analysis

In this section, the synthesized array patterns using the proposed algorithm, GA, and NSGA-SD optimization based techniques are compared with the ordinary array pattern in terms of the maximum SLL, HPBW, execution time, and dynamic range ratio (DRR) which is defined as:

$$ DRR = \frac{\text{maximum transmission energy}}{\text{minimum transmission energy}} = \frac{\delta_{\text{max}}}{\delta_{\text{min}}} $$  \hspace{1cm} (17)

Four test cases are considered using a small number of nodes distributed over a small circular area of radius $R$, i.e., $(K = 16$ and $R = 1\text{ m})$, $(K = 8$ and $R = 1\text{ m})$, and using a large number of nodes distributed over a moderate area, i.e., $(K = 32$ and $R = 4\text{ m})$ and $(K = 64$ and $R = 6\text{ m})$.

**Test case (1):** in this case, consider a $K = 16$ distributed antenna array whose nodes are randomly distributed over a small circular disk area of radius $R = 1\text{ m}$ as shown in Fig. 3. The direction of the intended receiver is at the azimuth angle $\phi_0 = 0^\circ$. The estimated first two null angles of the ordinary pattern are $\phi_{NL1} = -34.56^\circ$ and $\phi_{NL2} = 41.04^\circ$. Consequently, the desired array pattern, $AF_d(\varphi)$ is defined according to Eq. (4) and the number of samples is set to $N = 1000$ samples. The synthesized patterns using the proposed algorithm and the GA based algorithm compared to the ordinary pattern are shown in Fig. 4. The resultant maximum SLL, HPBW, and DRR are listed in Table 1. It is clear that the proposed algorithm provides the lowest SLL and the same HPBW as the ordinary pattern. However, the DRR of the proposed algorithm is slightly greater than that of the GA. Also, it provides about 256.78% reduction in the SLL while the GA provides only 56.60% reduction in the SLL. Using the same number of samples $N = 1000$, the estimated execution time of the proposed algorithm is 0.16163 sec which is much lower than the execution time of the GA based algorithm which equals 320.2466 sec. The polar coordinates $(r_k,\psi_k)$ and nodes energy transmissions $(\delta_k)$ of the synthesized patterns are tabulated in Table 2.

![Figure 3](image.png)

**Figure 3.** The nodes distribution for $K = 16$ and $R = 1\text{ m}$.

**Table 1.** The resultant maximum SLL, HPBW, DRR, and execution time of the proposed algorithm and the GA compared to the ordinary pattern for $K = 16$ and $R = 1\text{ m}$.

| Algorithm          | Maximum SL | HPBW   | DRR         | Execution time   |
|--------------------|------------|--------|-------------|------------------|
| Ordinary Pattern   | $-8.9428$ dB | 28.44° | 1           | –                |
| Proposed Algorithm | $-31.9066$ dB | 28.44° | 19.5920° | 0.16163 sec      |
| GA [9, 10]         | $-14.0047$ dB | 30.96° | 14.8762    | 320.2466 sec      |
Table 2. The polar coordinates \((r_k, \psi_k)\) and nodes energy transmissions \((\delta_k)\) of the synthesized pattern for \(K = 16\) and \(R = 1\) m.

| Polar coordinates | Nodes energy transmissions \((\delta_k)\) |
|-------------------|----------------------------------------|
| \(k\)  | \(r_k\) | \(\psi_k\) | Ordinary | Proposed Algorithm | GA |
| 1 | 0.4842 | -0.914 | 1 | 15.7500\(-0.2129\) | 0.9978 |
| 2 | 0.6779 | 0.6792 | 1 | 29.7940\(-0.6868\) | 0.9542 |
| 3 | 0.8411 | 1.2698 | 1 | 4.6343\(-3.1496\) | 0.2946 |
| 4 | 0.9956 | -1.440 | 1 | 1.8428\(-6.0317\) | 0.2378 |
| 5 | 0.2612 | 0.4538 | 1 | 26.9703\(-5.5416\) | 0.2100 |
| 6 | 0.7952 | -0.219 | 1 | 36.1041\(-8.4292\) | 0.5661 |
| 7 | 0.2640 | -0.660 | 1 | 24.8261\(-10.8984\) | 0.2108 |
| 8 | 0.9916 | 0.4501 | 1 | 7.4936\(-9.4163\) | 0.9451 |
| 9 | 0.8105 | 1.2628 | 1 | 6.5367\(-6.3128\) | 0.4485 |
| 10 | 0.8275 | -1.077 | 1 | 8.3310\(-4.6522\) | 0.0671 |
| 11 | 0.9862 | -1.113 | 1 | 4.8548\(-7.7595\) | 0.4811 |
| 12 | 0.3394 | 0.7940 | 1 | 15.0842\(-4.9390\) | 0.7645 |
| 13 | 0.8955 | 0.1461 | 1 | 24.2017\(-2.9559\) | 0.1962 |
| 14 | 0.7340 | -0.677 | 1 | 14.6938\(-0.9781\) | 0.8216 |
| 15 | 0.3555 | 0.2202 | 1 | 7.1869\(-3.1065\) | 0.1985 |
| 16 | 0.9713 | -0.103 | 1 | 3.3245\(-5.8279\) | 0.4555 |

Test case (2): in this case, consider a \((K = 32\) and \(R = 4\) m) RAA whose main beam is directed at the azimuth angle \(\varphi_0 = 0^\circ\) as shown in Fig. 5. The first two null angles of the ordinary pattern are \(\varphi_{NL1} = -8.68^\circ\) and \(\varphi_{NL2} = 8.28^\circ\). Fig. 6 shows the synthesized patterns using the proposed algorithm and the GA compared to the ordinary pattern, and the resultant maximum SLL, HPBW, and DRR are listed in Table 3. The percentages of SLL reduction are 88.36\% and 42.98\% for the proposed algorithm and the GA based algorithm respectively. The execution time of the proposed algorithm is 0.2523 sec while the execution time of the GA equals 565.0136 sec. Furthermore, the DRR of the proposed algorithm is smaller than that of the GA. The polar coordinates \((r_k, \psi_k)\) and nodes energy transmissions \((\delta_k)\) of the synthesized patterns are listed in Table 4 and Table 5, respectively.

Table 3. The resultant maximum SLL, HPBW, DRR, and execution time of the proposed algorithm and the GA compared to the ordinary pattern for \(K = 32\) and \(R = 4\) m.

| Algorithm          | Maximum SL | HPBW | DRR | Execution time |
|--------------------|------------|------|-----|----------------|
| Ordinary Pattern   | -8.9381 dB | 7.56\(^\circ\) | 1   | –              |
| Proposed Algorithm | -16.8359 dB | 7.56\(^\circ\) | 17.9752 | 0.2523 sec |
| GA [6, 7]          | -12.7795 dB | 8.28\(^\circ\) | 187.0206 | 565.0136 sec |

Test case (3): in this case, consider a RAA whose parameters are \((K = 64\) and \(R = 6\) m) as shown in Fig. 7. The direction of the intended receiver is at \(\varphi_0 = 0^\circ\). The estimated first two null angles are \(\varphi_{NL1} = -6.84^\circ\) and \(\varphi_{NL2} = 6.84^\circ\). Fig. 8 shows the synthesized patterns using the proposed algorithm and the GA compared to the ordinary pattern. The resultant maximum SLL, HPBW, and DRR are listed in Table 6. The proposed algorithm and the GA provide SLL reduction about 116.83\% and 34.19\% respectively. The proposed algorithm provides a suitable DRR which is smaller than that of the GA. The execution time of the proposed algorithm is 0.259299 sec which is much lower than the execution time of the GA which equals 1721.023 sec. The polar coordinates \((r_k, \psi_k)\) and nodes energy transmissions \((\delta_k)\) of the synthesized pattern are listed in Table 4 and Table 5, respectively.
transmissions ($\delta_k$) of the synthesized patterns are listed in Table 7, Table 8, and Table 9, respectively.

**Test case (4):** in this case, the proposed algorithm is compared with the NSGA-SD algorithm introduced in [13] for synthesizing a RAA whose parameters are ($K = 8$ and $R = 4m$) as shown in Fig. 9. Fig. 10 shows the synthesized patterns using the proposed algorithm and NSGA-SD compared to the ordinary pattern. The resultant maximum SLL, HPBW, and DRR are listed in Table 10. The simulation results revealed that the proposed algorithm outperforms the NSGA-SD algorithm in terms of maximum SLL reduction and HPBW. It provides SLL reduction of about 123.97% while the NSGA-SD provides a few reduction of about 42.76%. Also, the proposed algorithm provides the same HPBW as the ordinary pattern while the NSGA-SD algorithm provides HPBW which is slightly greater than that of the ordinary pattern. Furthermore, using the same number of samples $N = 1000$ samples, the proposed algorithm provides a very small execution time of 0.09438 sec which is much lower than the execution time of the NSGA-SD algorithm which equals 10 sec. The polar coordinates ($r_k, \psi_k$) and nodes transmission weights ($v_k$) of the synthesized patterns are listed in Table 11.
Figure 6. The synthesized patterns using the proposed algorithm and the GA compared to the ordinary pattern for \( K = 32 \) and \( R = 4 \text{ m} \).

Table 4. The polar coordinates \((r_k, \psi_k)\) and nodes energy transmissions \((\delta_k)\) of the synthesized pattern for \( K = 32 \) and \( R = 4 \text{ m} \).

| Polar coordinates | Nodes energy transmissions \((\delta_k)\) |
|-------------------|-----------------------------------|
| \( k \) | \( r_k \) | \( \psi_k \) | Ordinary | Proposed Algorithm | GA |
|---|---|---|---|---|---|
| 1 | 2.6553 | −0.757 | 1 | 0.9140∠0.8556 | 0.3433 |
| 2 | 3.9234 | 0.3059 | 1 | 0.6666∠−0.3410 | 0.9521 |
| 3 | 2.0722 | −0.591 | 1 | 0.5477∠0.2746 | 0.0053 |
| 4 | 3.5536 | −0.505 | 1 | 0.6914∠−1.1944 | 0.1701 |
| 5 | 3.1862 | 0.6389 | 1 | 0.5505∠1.1972 | 0.0484 |
| 6 | 2.2991 | −0.669 | 1 | 0.1842∠1.3524 | 0.0100 |
| 7 | 3.2846 | −0.296 | 1 | 1.3402∠−0.3347 | 0.7344 |
| 8 | 2.7649 | 1.2815 | 1 | 1.8963∠1.7440 | 0.1082 |
| 9 | 2.2200 | 1.4641 | 1 | 3.3109∠−0.6330 | 0.5389 |
| 10 | 2.1880 | −0.544 | 1 | 0.4172∠−1.3665 | 0.5891 |
| 11 | 3.3880 | 0.5188 | 1 | 1.4029∠0.7994 | 0.4571 |
| 12 | 0.9962 | 0.9142 | 1 | 0.9722∠−2.5973 | 0.3741 |
| 13 | 3.2684 | 0.1062 | 1 | 1.6215∠0.2036 | 0.9758 |
| 14 | 3.6165 | 0.5609 | 1 | 1.0629∠0.9961 | 0.7570 |
| 15 | 2.8659 | −0.499 | 1 | 0.9242∠1.7201 | 0.6325 |
| 16 | 3.5685 | −1.539 | 1 | 0.9499∠−0.9651 | 0.2144 |

4.2. Side Lobe Level Analysis

In this section, the resultant SLL is examined under the impact of the variations in the number of nodes, \( K \) and the circular area radius \( R \).

**Case (1):** In this case, the maximum SLL versus \( K \) over the same disk radius \( R = 1 \text{ m} \) is estimated for an intended receiver located at \( \varphi_0 = 0^\circ \). The resultant maximum SLL using the proposed algorithm,
Table 5. The polar coordinates \((r_k, \psi_k)\) and nodes energy transmissions \((\delta_k)\) of the synthesized pattern for \(K = 32\) and \(R = 4\ m\).

| Polar coordinates | Nodes energy transmissions \((\delta_k)\) |
|-------------------|-----------------------------------------|
| \(k\) | \(r_k\) | \(\psi_k\) | Ordinary | Proposed Algorithm | GA |
| 17  | 3.3281 | 0.8202 | 1 | 1.2163 \(\angle 2.1403\) | 0.1399 |
| 18  | 3.1480 | 0.7378 | 1 | 0.8306 \(\angle 2.5130\) | 0.0588 |
| 19  | 1.9732 | -1.038 | 1 | 0.8169 \(\angle 4.4694\) | 0.0703 |
| 20  | 3.4765 | -1.474 | 1 | 0.8774 \(\angle 6.7142\) | 0.1972 |
| 21  | 2.6898 | -0.509 | 1 | 0.9463 \(\angle 6.4790\) | 0.8445 |
| 22  | 3.7000 | 0.3214 | 1 | 0.4570 \(\angle 6.7499\) | 0.9331 |
| 23  | 1.4750 | 1.3368 | 1 | 2.0058 \(\angle 5.0911\) | 0.5518 |
| 24  | 2.5896 | 0.0592 | 1 | 0.9423 \(\angle 6.0520\) | 0.9999 |
| 25  | 3.7174 | -1.125 | 1 | 0.9521 \(\angle 6.9203\) | 0.2661 |
| 26  | 2.7914 | 1.4638 | 1 | 2.0410 \(\angle 5.2970\) | 0.5470 |
| 27  | 2.0031 | 0.2580 | 1 | 0.9731 \(\angle 6.4173\) | 0.6557 |
| 28  | 3.4931 | -0.676 | 1 | 1.4351 \(\angle 6.0215\) | 0.9976 |
| 29  | 2.1547 | 1.2081 | 1 | 1.1327 \(\angle 7.9586\) | 0.3310 |
| 30  | 1.5932 | -1.387 | 1 | 0.2206 \(\angle 6.6079\) | 0.8130 |
| 31  | 3.9965 | -0.927 | 1 | 0.4402 \(\angle 6.2421\) | 0.7302 |
| 32  | 2.4736 | -0.207 | 1 | 0.9784 \(\angle 6.5731\) | 0.7308 |

Figure 7. The nodes distribution for \(K = 64\) and \(R = 6\ m\).

GA, and the ordinary pattern are shown in Fig. 11. The simulation results revealed that the proposed algorithm outperforms the GA technique as it provides maximum SLL range (from \(-31.9066\ dB\) to \(-40.3119\ dB\)) when \(K\) changes from \((K = 16\) to \(K = 80\)). However, the ordinary pattern and the GA provide maximum SLL range (from \(-8.9428\ dB\) to \(-14.7544\ dB\)) and (from \(-14.0047\ dB\) to \(-32.3507\ dB\)), respectively.

**Case (2):** in this case, for a fixed number of the distributed nodes \(K = 16\), the maximum SLL
Figure 8. The synthesized patterns using the proposed algorithm and the GA compared to the ordinary pattern for $K = 64$ and $R = 6\,m$.

Table 6. The resultant maximum SLL, HPBW, DRR, and execution time of the proposed algorithm and the GA compared to the ordinary pattern for $K = 64$ and $R = 6\,m$.

| Algorithm              | Maximum SL | HPBW | DRR  | Execution time |
|------------------------|------------|------|------|----------------|
| Ordinary Pattern       | $-10.5482\,\text{dB}$ | $5.4^\circ$ | 1     | --             |
| Proposed Algorithm     | $-22.8713\,\text{dB}$ | $5.4^\circ$ | 23.1860 | 0.259299 sec   |
| GA [9, 10]             | $-12.8419\,\text{dB}$ | $5.5^\circ$ | 62.8482 | 1721.023 sec   |

Figure 9. The nodes distribution for $K = 8$ and $R = 4\,m$.

versus disk radius $R$ is estimated for an intended receiver located at $\varphi_0 = 0^\circ$. Fig. 12 shows the resultant maximum SLL using the proposed algorithm, GA, and the ordinary pattern over the disk radius range ($R = 1\,\text{m}$ to $10\,\text{m}$). Also, it is clear that the proposed algorithm provides the highest performance over
the entire range of disk radius. It provides maximum SLL range (from −31.9066 dB to −7.8920 dB) when the disk radius \( R \) changes from \((R = 1\, \text{m} \text{ to } 10\, \text{m})\). However, the ordinary pattern and the GA provide maximum SLL ranges (from −8.9428 dB to −3.9208 dB) and (from −14.0047 dB to −5.5796 dB) respectively.

**Table 7.** The polar coordinates \((r_k, \psi_k)\) for \(K = 64\) and \(R = 6\, \text{m} \).

| \(k\) | \(r_k\) | \(\psi_k\) | \(k\) | \(r_k\) | \(\psi_k\) | \(k\) | \(r_k\) | \(\psi_k\) | \(k\) | \(r_k\) | \(\psi_k\) |
|------|--------|---------|------|--------|---------|------|--------|---------|------|--------|---------|
| 1    | 1.9157 | −0.471  | 17   | 2.5249 | 1.0758  | 33   | 3.5861 | −1.521  | 49   | 1.8737 | −0.303  |
| 2    | 3.6516 | −0.621  | 18   | 4.7466 | 1.1031  | 34   | 4.8725 | 0.7798  | 50   | 3.8408 | 0.7141  |
| 3    | 5.7384 | 0.3249  | 19   | 5.6172 | −0.620  | 35   | 3.7670 | −0.584  | 51   | 2.1731 | 0.7369  |
| 4    | 5.9970 | 0.1321  | 20   | 2.5328 | −0.405  | 36   | 2.5348 | 0.4253  | 52   | 5.2119 | 0.0044  |
| 5    | 2.4775 | 1.1248  | 21   | 4.8209 | −0.149  | 37   | 1.3981 | 0.6896  | 53   | 3.5400 | −1.236  |
| 6    | 2.4928 | 0.6844  | 22   | 5.6417 | −0.614  | 38   | 2.9287 | 0.1857  | 54   | 4.0240 | −0.822  |
| 7    | 1.9263 | 1.4323  | 23   | 3.0031 | 1.1940  | 39   | 5.6978 | 1.3289  | 55   | 2.6788 | 0.7561  |
| 8    | 4.6713 | −0.779  | 24   | 4.5939 | −1.023  | 40   | 5.9471 | 0.1632  | 56   | 5.1492 | 1.3315  |
| 9    | 3.8507 | −0.167  | 25   | 4.1866 | −0.370  | 41   | 4.1747 | −0.036  | 57   | 2.7058 | 0.2669  |
| 10   | 3.0345 | −1.556  | 26   | 5.4961 | 0.8670  | 42   | 2.2828 | 1.5371  | 58   | 5.2587 | −1.116  |
| 11   | 2.6946 | 1.1376  | 27   | 1.3639 | −0.123  | 43   | 4.2919 | −0.111  | 59   | 2.9472 | −1.325  |
| 12   | 2.5094 | 0.1341  | 28   | 5.1596 | −0.379  | 44   | 5.9806 | −0.305  | 60   | 4.7005 | −1.077  |
| 13   | 5.6406 | 0.8122  | 29   | 4.4680 | −1.232  | 45   | 5.2395 | −0.178  | 61   | 3.5281 | 0.6887  |
| 14   | 4.7677 | −0.879  | 30   | 3.9320 | −1.145  | 46   | 5.8972 | −0.078  | 62   | 5.1005 | 1.4072  |
| 15   | 4.7336 | −1.277  | 31   | 5.2981 | 1.4547  | 47   | 5.7858 | −0.924  | 63   | 2.0543 | −0.737  |
| 16   | 4.8680 | 1.1787  | 32   | 3.7789 | 0.8876  | 48   | 2.9178 | −1.199  | 64   | 2.5512 | 0.0031  |

**Table 8.** The nodes energy transmissions \((\delta_k)\) of the synthesized pattern for \(K = 64\) and \(R = 6\, \text{m} \).

| \(k\) | Ordinary | Proposed Algorithm | GA | \(k\) | Ordinary | Proposed Algorithm | GA |
|------|----------|--------------------|----|------|----------|--------------------|----|
| 1    | 7.3130\angle−2.2610 | 0.8607 | 17 | 1    | 32.966\angle−1.7764 | 0.8589 |
| 2    | 34.9493\angle−2.3199 | 0.5446 | 18 | 1    | 1.7948\angle−4.1720 | 0.1068 |
| 3    | 28.0571\angle−0.9256 | 0.2117 | 19 | 1    | 14.6898\angle−5.3173 | 0.7646 |
| 4    | 2.3330\angle1.7749 | 0.0801 | 20 | 1    | 30.3403\angle−7.4535 | 0.4388 |
| 5    | 36.7422\angle2.1666 | 0.5406 | 21 | 1    | 34.3154\angle−8.0161 | 0.4295 |
| 6    | 14.0118\angle4.9403 | 0.8883 | 22 | 1    | 2.1285\angle−7.1076 | 0.0578 |
| 7    | 10.6703\angle2.7589 | 0.7297 | 23 | 1    | 20.6731\angle−4.8484 | 0.1366 |
| 8    | 18.1557\angle−0.4330 | 0.5311 | 24 | 1    | 11.7724\angle−3.8676 | 0.2403 |
| 9    | 31.4089\angle0.0851 | 0.7362 | 25 | 1    | 27.0197\angle−4.5441 | 0.0158 |
| 10   | 5.1443\angle−0.5024 | 0.4748 | 26 | 1    | 3.1625\angle−4.7318 | 0.9937 |
| 11   | 19.5027\angle0.8992 | 0.3001 | 27 | 1    | 4.5873\angle−3.9360 | 0.4950 |
| 12   | 5.1431\angle0.8692 | 0.4856 | 28 | 1    | 36.1245\angle−5.0029 | 0.4771 |
| 13   | 6.6639\angle−0.6234 | 0.5951 | 29 | 1    | 11.5907\angle−6.5602 | 0.6842 |
| 14   | 10.6357\angle−2.6139 | 0.5870 | 30 | 1    | 14.0783\angle−3.5431 | 0.9723 |
| 15   | 5.0176\angle−2.6889 | 0.8266 | 31 | 1    | 1.5847\angle−6.4193 | 0.8291 |
| 16   | 2.4591\angle−4.0906 | 0.5432 | 32 | 1    | 9.1922\angle−3.3820 | 0.9717 |
Table 9. The nodes energy transmissions ($\delta_k$) of the synthesized pattern for $K = 64$ and $R = 6$ m.

| $k$ | Ordinary | Proposed Algorithm | GA | $k$ | Ordinary | Proposed Algorithm | GA |
|-----|----------|--------------------|----|-----|----------|--------------------|----|
| 33  | 1        | 2.3079$\angle-6.8117$ | 0.1726 | 49  | 1        | 3.9142$\angle-11.3588$ | 0.7557 |
| 34  | 1        | 16.5709$\angle-7.0848$ | 0.9876 | 50  | 1        | 20.7471$\angle-12.0369$ | 0.9212 |
| 35  | 1        | 36.6102$\angle-6.6202$ | 0.7685 | 51  | 1        | 28.2178$\angle-10.1590$ | 0.8000 |
| 36  | 1        | 28.5454$\angle-9.2038$ | 0.7136 | 52  | 1        | 8.9211$\angle-12.0014$ | 0.8768 |
| 37  | 1        | 25.1721$\angle-5.8573$ | 0.9249 | 53  | 1        | 11.8328$\angle-11.4923$ | 0.3159 |
| 38  | 1        | 12.5651$\angle-6.6190$ | 0.9840 | 54  | 1        | 5.1548$\angle-11.3130$ | 0.2939 |
| 39  | 1        | 1.5942$\angle-7.0245$ | 0.2941 | 55  | 1        | 20.9906$\angle-12.3521$ | 0.6820 |
| 40  | 1        | 20.2395$\angle-6.8799$ | 0.9397 | 56  | 1        | 2.2180$\angle-12.3755$ | 0.0664 |
| 41  | 1        | 24.6302$\angle-9.6349$ | 0.3558 | 57  | 1        | 5.0911$\angle-15.6113$ | 0.2043 |
| 42  | 1        | 4.0354$\angle-7.8365$ | 0.4616 | 58  | 1        | 4.2611$\angle-12.8152$ | 0.2024 |
| 43  | 1        | 16.3730$\angle-9.4768$ | 0.0829 | 59  | 1        | 26.4804$\angle-14.6527$ | 0.6462 |
| 44  | 1        | 12.6796$\angle-9.5369$ | 0.4293 | 60  | 1        | 3.0665$\angle-14.3484$ | 0.1310 |
| 45  | 1        | 29.0337$\angle-10.6186$ | 0.4842 | 61  | 1        | 29.6211$\angle-14.6274$ | 0.0178 |
| 46  | 1        | 3.7763$\angle-9.7732$ | 0.5034 | 62  | 1        | 2.4838$\angle-16.8718$ | 0.6771 |
| 47  | 1        | 3.0559$\angle-11.5927$ | 0.5997 | 63  | 1        | 26.0455$\angle-18.5841$ | 0.8941 |
| 48  | 1        | 5.4749$\angle-14.6929$ | 0.7557 | 64  | 1        | 11.4999$\angle-18.4030$ | 0.7483 |

Table 10. The resultant maximum SLL, HPBW, and DRR of the proposed algorithm and the NSGA-SD compared to the ordinary pattern for $K = 8$ and $R = 4$ m.

| Algorithm          | Maximum SL | HPBW | DRR | Execution time |
|--------------------|------------|------|-----|----------------|
| Ordinary Pattern   | $-4.63$ dB | 12.20° | 1   |                |
| Proposed Algorithm | $-10.37$ dB | 12.20° | 3.0685 | 0.09438 sec    |
| NSGA-SD [13]       | $-6.61$ dB | 12.35° | 2.0625 | 10 sec         |

Figure 10. The synthesized patterns using the proposed algorithm and the NSGA-SD compared to the ordinary pattern for $K = 8$ and $R = 4$ m.
Table 11. The polar coordinates \((r_k, \psi_k)\) and nodes transmission weights \((v_k)\) of the synthesized pattern for \(K = 8\) and \(R = 4\, \text{m}\).

| Polar coordinates | Nodes transmissions weight \((v_k)\) | Ordinary | Proposed Algorithm | NSGA – SD |
|-------------------|-------------------------------------|----------|--------------------|-----------|
| \(k\) | \(r_k\) | \(\psi_k\) | \(1\angle0.12\) | \(1.2661\angle−0.6484\) | \(0.53\angle0.18\) |
| 1 | 0 | 0 | 1\angle1.31 | 0.5252\angle0.5315 | 0.94\angle1.10 |
| 2 | 3.2755 | 0.5855 | 1\angle−0.69 | 0.4938\angle−1.158 | 0.31\angle−1.12 |
| 3 | 3.1219 | −0.343 | 1\angle0.10 | 1.5151\angle0.8978 | 0.19\angle0.05 |
| 4 | 0.6868 | 0.8369 | 1\angle−1.73 | 1.1818\angle−1.2798 | 0.76\angle−1.03 |
| 5 | 2.8898 | 1.2064 | 1\angle−2.19 | 0.6784\angle2.2068 | 0.56\angle2.05 |
| 6 | 2.7509 | −1.545 | 1\angle−0.12 | 0.8861\angle0.3869 | 0.97\angle0.01 |
| 7 | 1.7605 | 0.0227 | 1\angle−0.04 | 1.1477\angle−0.5825 | 0.72\angle0.16 |

Figure 11. The maximum SLL versus \(K\) for \(R = 1\, \text{m}\).

Figure 12. The maximum SLL versus \(R\) for \(K = 16\).
5. CONCLUSION

In this paper, a fast deterministic distributed beamforming algorithm is proposed for maximum SLL reduction of RAAs in wireless sensor networks. It controls the energy transmission $\mathbf{d}_k$ or transmission weight ($v_k$) of each node without altering the nodes locations. The simulation results verify the feasibility and effectiveness of the proposed algorithm compared to the recent state of the art GA and NSGA-SD optimization based techniques. It provides the highest SLL reduction while maintaining the same HPBW as the ordinary pattern. Furthermore, it is not time consuming which makes it suitable for adaptive beamforming of distributed random antenna arrays.

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