THE ORIGIN AND EVOLUTION OF HALO BIAS IN LINEAR AND NONLINEAR REGIMES

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ABSTRACT

We present results from a study of bias and its evolution for galaxy-size halos in a large, high-resolution simulation of a low-density, cold dark matter model with a cosmological constant. In addition to the previous studies of the halo two-point correlation function, we consider the evolution of bias estimated using two different statistics: power spectrum $b_p$ and a direct correlation of smoothed halo and matter overdensity fields $b_\rho$. We present accurate estimates of the evolution of the matter power spectrum probed deep into the stable clustering regime $[k \sim (0.1-200) \ h \text{Mpc}^{-1} \text{at} \ z = 0]$ and find that its shape and evolution can be well described, with only a minor modification, by the fitting formula of Peacock & Dodds. The halo power spectrum evolves much slower than the power spectrum of matter and has a different shape which indicates that the bias is time and scale dependent. At $z = 0$, the halo power spectrum is antibiased ($b_p < 1$) with respect to the matter power spectrum at wavenumbers $k \sim (0.15-30) \ h \text{Mpc}^{-1}$ and provides an excellent match to the power spectrum of the Automatic Plate Measuring Facility (APM) galaxies at all probed $k$. In particular, both the halo and matter power spectra show an inflection at $k \approx 0.15 \ h \text{Mpc}^{-1}$, which corresponds to the present-day scale of nonlinearity and nicely matches the inflection observed in the APM power spectrum. We complement the power spectrum analysis with a direct estimate of bias using smoothed halo and matter overdensity fields and show that the evolution observed in the simulation in linear and mildly nonlinear regimes can be well described by the analytical model of Mo & White, if the distinction between formation redshift of halos and observation epoch is introduced into the model. We present arguments and evidence that at higher overdensities the evolution of bias is significantly affected by dynamical friction and tidal stripping operating on the satellite halos in high-density regions of clusters and groups, and we attribute the strong antibias observed in the halo correlation function and power spectrum to these effects. The results of this study show that despite the apparent complexity, the origin and evolution of bias can be understood in terms of the processes that drive the formation and evolution of dark matter halos. These processes conspire to produce a halo distribution quite different from the overall distribution of matter, yet remarkably similar to the observed distribution of galaxies.

Subject headings: cosmology: theory — galaxies: halos — large-scale structure of universe — methods: numerical

1. INTRODUCTION

The distribution of galaxies may, in general, be biased with respect to the overall matter distribution. Therefore, the galaxy density field can be used as a probe of matter distribution only if we fully understand how the galaxy distribution relates to the distribution of matter. Understanding this relationship, the bias, and its evolution are of primary importance for the interpretation of the ever-increasing amount of high-quality galaxy clustering data at low and high redshifts. Although the problem of bias has been studied extensively during the last decade (e.g., Kaiser 1984; Davis et al. 1985; Bardeen et al. 1986; Dekel & Silk 1986; Cole & Kaiser 1989; Babul & White 1991), new data on galaxy clustering at high redshifts (e.g., at $z \leq 1$: Le Fèvre et al. 1996; Shepherd et al. 1997; Carlberg et al. 1997, 1998; Connolly, Szalay, & Brunner 1998; Postman et al. 1998; and at $z \approx 3$: Steidel et al. 1998; Giavalisco et al. 1998; Adelberger et al. 1998) and the anticipation of upcoming accurate measurements of galaxy clustering at $z \approx 0$ from large redshift surveys (e.g., Tegmark et al. 1998 and references therein) have recently generated substantial theoretical progress in modeling galaxy clustering and bias.

In hierarchical structure formation models, galaxies are assumed to form inside dark matter (DM) halos via the energy dissipation by baryons (e.g., White & Rees 1978; Kauffmann, White, & Guiderdoni 1993; Baugh, Cole, & Frenk 1996; Somerville & Primack 1998). Mo & White (1996) showed how bias of DM halos can be predicted analytically in the framework of the extended Press-Schechter theory (Bond et al. 1991; Bower 1991; Lacey & Cole 1993; see § 2). This analytical model is rapidly gaining popularity in theoretical analyses (see, e.g., recent studies by Kauffmann, Nusser, & Steinmetz 1997, Moscardini et al. 1998, and Baugh et al. 1998) which requires it to be tested and its capabilities and limitations evaluated. The model has been tested against numerical simulations by Mo, Jing, & White (1996), Catelan, Matarrese, & Porciani (1998b), Jing (1998), and Porciani, Catelan, & Lacey (1998b), and we present additional tests in this paper. More elaborate analytical models have been developed by Catelan et al. (1998a, 1998b), Porciani et al. (1998a), and Sheth & Lemson (1998).

The effects of nonlinearity of the bias on the observable statistics have been studied by Fry & Gaztaña (1993), Coles (1993), and Mann, Peacock, & Heavens (1997) using heuristic models of local nonlinear bias. Recently, Coles (1993) and Dekel & Lahav (1998) have developed a formalism for studies of galaxy biasing that allows one to account explicitly for the nonlinearity and stochasticity of the bias. They have also analyzed the effects of nonlinearity and stochasticity on results of some of the observational analyses.
Scherrer & Weinberg (1998) have analyzed effects of stochasticity of the local bias on the correlation function and power spectrum and concluded that stochasticity should not affect the shape of these statistics in the linear regime (or, in other words, that in linear regime the bias should be linear). Most recently, Narayanan, Berlind, & Weinberg (1998) studied the effects of nonlinearity, stochasticity, and nonlocality of the bias using heuristic models applied to large N-body simulations. From the observational perspective, Pen (1998) showed how the stochasticity of the galaxy bias can be tested and measured using redshift space distortions, and Tegmark & Bromley (1998) presented evidence that present-day galaxy bias is nonlinear and stochastic based on analysis of galaxy clustering in the Las Campanas Redshift Survey.

The evolution of bias in the linear regime has been recently analyzed by Tegmark & Peebles (1998), who generalized the results of Fry (1996) for the case of stochastic bias in an arbitrary cosmology. Taruya, Koyama, & Soda (1998) and Taruya & Soda (1998) used perturbative analysis to extend the linear analysis of bias evolution into the weakly nonlinear regime. Evolution of halo clustering and bias in the nonlinear regime has been analyzed in several recent numerical studies employing dissipationless simulations (e.g., Brainerd & Villumsen 1994; Bagla 1998; Colin et al. 1999; Ma 1999), simulations that include both dissipationless DM and dissipative baryonic components (e.g., Katz, Hernquist, & Weinberg 1998; Blanton et al. 1998; Cen & Ostriker 1998; Jenkins et al. 1998b), and “hybrid” studies in which dissipationless simulations are complemented with a semianalytical model of galaxy formation (Roukema et al. 1997; Kauffmann et al. 1998a, 1998b; Diaferio et al. 1998; Benson et al. 1998; Baugh et al. 1998; Kolatt et al. 1998). All of these studies, although very diverse in their methods, qualitatively agree on one important result: the galaxy bias is expected to be nonlinear, to depend on the properties of the DM halos and the galaxies they host, and to be a (generally nonmonotonic) function of cosmic time.

In this paper we present results of a detailed analysis of halo clustering evolution in a large high-resolution dissipationless simulation of a representative and fairly successful variant of the cold dark matter (CDM) models: the low-density spatially flat CDM model with the cosmological constant (ΛCDM). The primary goal of this analysis is to identify and study the main processes that drive the evolution of halo bias in linear and nonlinear regimes. Understanding what causes the bias and particular features of its evolution (notably, the ant bias at late stages of evolution in some of the models) is crucial for the interpretation of current observational and theoretical results. We focus therefore on the interpretation of general features of bias evolution observed in our simulations and in studies done by other authors. The main novel feature of this study is inclusion of satellite halos located inside virial radii of more massive isolated halos in the halo catalogs; we refer reader to Colin et al. (1999) for a more detailed description of our approach.

The approach that we have adopted in this project is to consider bias estimated using two different statistics: power spectrum $P(k)$ (Peebles 1980) and direct comparison of the smoothed halo and matter overdensity ($\delta$) fields. This complements several recent studies, including a study by Colin et al. (1999) which used the simulation presented here, of the halo bias defined using the two-point correlation function $\xi(r)$. The bias can be defined using these statistics as follows:

$$b_\delta(r) = \sqrt{\xi(r)/\xi_m(r)},$$  
(1)

$$b_P(k) = \sqrt{P_h(k)/P_m(k)},$$  
(2)

$$b_s = \frac{\delta^R_s}{\delta^0_m},$$  
(3)

where quantities with subscripts $h$ and $m$ correspond to statistics of the halo and matter distributions, respectively, and superscript $R$ indicates the density fields smoothed on a scale $R$. The three estimates of bias given above are, of course, related. In the special case of deterministic local linear bias, $b_\delta = b_P = b_s$. Nevertheless, in the general case of stochastic nonlinear bias, these estimates are complementary to each other, and it is important to consider all three of them to understand fully the manifestations and properties of the bias. All three functions, $b_\delta$, $b_P$, and $b_s$, may depend on a number of (both local and nonlocal) parameters; the most important point to notice, however, is that they are different functions of their parameters, and we use the subscripts to indicate this explicitly. For example, $b_\delta$ and $b_P$ have different scale dependence because $\xi(r)$ and $P(k)$ are related via the Fourier transform, $\xi(r) \propto \int P(k)e^{ik \cdot r}d^3k$, which gives

$$b_\delta(r) = \int \frac{b_P(k)P_m(k)e^{ik \cdot r}d^3k}{P_m(k)e^{ik \cdot r}d^3k}.$$

The bias $b_\delta(r)$ that is scale dependent in a narrow range of $r$ will be scale dependent in a wide range of $k$ (see Coles 1993 for a more detailed discussion). We will show below that the antibias required at small $r$ for the ΛCDM model to be consistent with the $z = 0$ galaxy correlation function is also perfectly consistent with $b_\delta(k) < 1$ at small wavenumbers (down to $k \sim 0.2$ $h$ Mpc$^{-1}$) required for the model to be consistent with the galaxy power spectrum (Gaztañaga & Baugh 1998; Hoyle et al. 1998).

The paper is organized as follows. In § 2 we give a brief account of analytical model of the bias developed by Mo & White (1996) and how the epochs of halo formation and observation can be separated in this model (e.g., Moscardini et al. 1998; Catelan et al. 1998a). In § 2 and 4 we describe the numerical simulation and halo identification and selection algorithms used in our analysis. We complement the analysis of the evolution of the halo two-point correlation function based on this simulation and presented by Colin et al. (1999) with the analysis of the evolution of the halo power spectrum and bias $b_P$ in § 5.1. In §§ 5.2 and 5.2.2 we present the analysis of the evolution of $b_\delta$ estimated using smoothed halo and matter density fields at different epochs and identify the processes that drive this evolution. We discuss the results in § 6 and summarize our conclusions and arguments in § 7.

2. ANALYTICAL MODEL OF BIAS

An overdensity field $\delta(x) \equiv [\rho(x) - \bar{\rho}]/\bar{\rho}$ can be characterized in terms of its power spectrum $P(k) = \langle |\tilde{\delta}(k)|^2 \rangle$, where $\tilde{\delta}$ is the Fourier transform of $\delta(x)$. Similarly, if the overdensity field is smoothed on a scale $R$ with a spherically symmetric filter $W(r, R)$, the smoothed field can be charac-
terized by its variance

$$\sigma^2(R) = \frac{1}{(2\pi)^3} \int P(k) \tilde{W}(R)^2 d^3 k,$$

(5)

where $\tilde{W}$ is the Fourier transform of the window function. In the following and throughout the paper we use the real space top-hat filter, $W(r, R) = (4\pi R^3/3)^{-1} \Theta(R - r)$, where $\Theta(R - r)$ is the step function. For this filter the scale $R$ can be interchanged with the mass contained within radius $R$, $M = (4\pi/3)\hat{\rho}R^3$, where $\hat{\rho}$ is the mean density of the universe. The standard Press-Schechter model (PS) (Press & Schechter 1974) assumes that any region of initial comoving size $R$ (or mass $M$) becomes a part of a virialized halo by redshift $z_f$, if its overdensity extrapolated linearly to the present epoch is greater than $\delta_c/D_+(z_f)$, where $D_+(z)$ is the linear growth factor (e.g., Peebles 1980) normalized to unity at the present epoch. The value of $\delta_c$ is motivated by the top-hat collapse model; we use $\delta_c = 1.69$ throughout this work. If the initial overdensity field is Gaussian, the PS model leads to the following expression for the number density of collapsed halos of mass $M$ at redshift $z$:

$$n(M, z, z_f) dM = \frac{1}{\sqrt{2\pi}} \frac{\delta_c(z, z_f)}{M \sigma^2(M, z)} \frac{d\sigma^2(M, z)}{dM} \times \exp \left[ -\frac{\delta_c^2(z, z_f)^2}{2\sigma^2(M, z)} \right] dM,$$

(6)

where $\delta_c(z, z_f) \equiv \delta_c D_+(z)/D_+(z_f)$ and $\sigma(M, z)$ is the variance of the initial density field smoothed on a scale $M$ and extrapolated linearly to the epoch $z$. For the reasons that will be discussed below, we follow Catelan et al. (1998a) in distinguishing the epoch $z_f$ from the ”observation” epoch $z$ ($z < z_f$). The $z_f$ and $z$ dependencies are shown explicitly in the above expression for the mass function. Note, however, that $n(M, z, z_f)$ does not change with $z_f$ for a given power spectrum, the mass function depends only on halo mass $M$ and $z_f$. Equation (6) translates into the commonly used form if we assume $z_f = z$.

In a seminal paper, Mo & White (1996, hereafter MW) showed how the extended Press-Schechter formalism (EPS) (Bond et al. 1991; Bower 1991; Lacey & Cole 1993) can be used to derive the analytical expression for the bias of DM halos. The EPS can be used to derive the expression for the conditional mass function of halos (Bond et al. 1991). Namely, the number density of halos of mass $M$ that collapse at epoch $z_f < z$ in a region of initial Lagrangian radius $R_0$ (mass $M_0$) and the initial overdensity in the growing mode extrapolated linearly to the present $\delta_0$ [$\delta_0(z) = \delta_0 D_+(z)$] is given by

$$n(M, z, z_f) \mid M_0, \delta_0) dM = \frac{1}{\sqrt{2\pi}} \frac{\delta_c(z, z_f) - \delta_0(z)}{M \sigma^2(M, z)} \frac{d\sigma^2(M, z)}{dM} \times \exp \left[ -\frac{[\delta_c(z, z_f) - \delta_0(z)]^2}{2\sigma^2(M, z) - \sigma^2(M_0, z)} \right] dM.$$

(7)

The average overdensity of halos of mass $M$ in spheres of overdensity $\delta_0$ and radius $R_0$ at epoch $z$, $\delta_0(M, z) \mid R_0, \delta_0)$, can be obtained by dividing number densities given by equations (7) and (6) and subtracting unity. The halo bias is then defined as $b \equiv \delta_h/\delta_0$. So far the bias is defined in terms of the Lagrangian radius and linearly extrapolated overdensity. For practical purposes, however, we need the expression for bias in spheres of observed radius $R$ and (generally nonlinear) overdensity $\delta$. MW use a spherical collapse model to relate $(R_0, \delta_0)$ to $(R, \delta)$: $R_0^3 = (1 + \delta)R^3$ and $\delta_0$ are calculated for given $R$, $R_0$, and $\delta$ using the equations of spherical collapse. Having made the translation from $(R, \delta)$ to $(R_0, \delta_0)$, we can calculate the average overdensity of DM halos in spheres of the observed radius $R$ and overdensity $\delta$ as

$$\delta_h(M, z, z_f \mid R, \delta) = (1 + \delta) \frac{n(M, z, z_f) \mid R_0, \delta_0) - 1}{n(M, z, z_f)},$$

(8)

$$b_{NL}(M, z, z_f, \delta) \equiv \delta_h(M, z, z_f \mid R, \delta)/\delta,$$

(9)

where $b_{NL}$ is the halo bias. Note that in general $b_{NL}$ depends on the overdensity of matter $\delta$ and is therefore nonlinear. The quantities $(R, \delta)$ and $(R_0, \delta_0)$ in equation (8) are related by the spherical collapse model. In the limit of linear overdensities and large scales, $\delta_0(z) \ll \delta(z, z_f)$ and $M_0 \gg M$, the bias is given by

$$b(M, z, z_f) = 1 + \frac{v^2 - 1}{\delta(z, z_f)},$$

(10)

where $v \equiv \delta(z, z_f)/\sigma(M, z)$. Equation (10) shows that in this regime the bias is linear (does not depend on $\delta$). Note that the bias of halos with a range of masses $[M_{min}, M_{max}]$ should be computed as a mass function weighted average:

$$b(z, z_f) = \bar{n}^{-1}(z, z_f) \int_{M_{min}}^{M_{max}} b(M, z, z_f)n(M, z, z_f)dM,$$

(11)

where $\bar{n}(z, z_f) = [M_{max} \int n(M, z, z_f) dM]$ and $n(M, z, z_f)$ is given by equation (6).

We use equations (8)-(11) to calculate the bias predicted by this analytical model. The linear overdensity $\delta_0$ is calculated for given $\delta$, $R$, and $R_0 = (1 + \delta)^{1/3}R$ using the equations of the spherical collapse model appropriate for our ΛCDM cosmology (e.g., Lahav et al. 1991; Eke, Cole, & Frenk 1996). The resulting function $\delta_0(\delta)$ is well described by the fitting formula given by MW (eq. [18] in their paper), except for $\delta \sim 20-30$, where deviations reach $\approx 5\%-10\%$.

3. NUMERICAL SIMULATION

We have chosen to study the evolution of the halo bias in a representative variant of the CDM-type models: the low matter density, flat, CDM model with cosmological constant ($\Lambda$CDM), $\Omega_0 = 1 - \Omega_0 = 0.3$, $h = 0.7$, where $\Omega_0$ and $\Omega_0$ are present-day matter and vacuum densities and $h$ is the dimensionless Hubble constant defined as $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$. This model is arguably the most successful model in matching a variety of existing data. Observations of galaxy cluster evolution (Eke et al. 1996), baryon fraction in clusters (Evrard 1997), and high-redshift supernovae (e.g., Perlmutter et al. 1998) strongly favor the value of matter density $\Omega_0 \approx 0.3$, while various observational measurements of the Hubble constant (e.g., Falco et al. 1997; Salas & Cassisi 1998; Madore et al. 1998) tend to converge on the

Note that the bias weighted in this way will give correct value only for the bias estimated from point-to-point comparisons of halo and matter overdensities $\delta_h$. For the spatially averaged bias calculated using correlation functions or power spectra, the weighting will not give a correct value because of the cross terms resulting from squaring the overdensities.
values of $h \approx 0.6$–0.7. We use a normalization of the spectrum of fluctuations that is consistent with both observed cluster abundances (Eke et al. 1996) and the 4 yr COBE data (e.g., Bunn & White 1997): $\sigma_8 = 1$, where $\sigma_8$ is the rms fluctuation in spheres of $8 h^{-1}$ Mpc comoving radius.

The numerical simulation of the $\Lambda$CDM model followed the evolution of $256^3 \approx 1.67 \times 10^7$ particles in a periodic $60 h^{-1}$ Mpc box. The particle mass is thus $\approx 1.1 \times 10^8 h^{-1} M_\odot$. The simulation was run using adaptive refinement tree $N$-body code (ART) (Kravtsov, Klypin, & Khokhlov 1997). The ART code reaches high force resolution by refining all high-density regions with an automated refinement algorithm. The refinements are recursive: the refined regions can also be refined, each subsequent refinement having half of the previous level’s cell size. This creates a hierarchy of refinement meshes of different resolution covering regions of interest. The criterion for refinement is local overdensity of particles: in the simulation presented in this paper the code refined an individual cell only if the density of particles (smoothed with the cloud-in-cell scheme; Hockney & Eastwood 1981) was higher than the threshold of $n_{\text{th}} = 5$ particles. Therefore, all regions with overdensity higher than $\delta = n_{\text{th}} 2^{3d/n}$, where $n$ is the average number density of particles in the cube, were refined to the refinement level $L$. For the simulation presented here, $n$ is $\frac{1}{3}$. The peak formal dynamic range reached by the code on the highest refinement level is 32,768, which corresponds to the smallest grid cell of $1.83 h^{-1}$ kpc; the actual force resolution is approximately a factor of 2 lower (see Kravtsov et al. 1997). The simulation that we analyze here has been used by Colín et al. (1999), and we refer the reader to this paper for further numerical details.

4. HALO IDENTIFICATION AND SELECTION

Identification of DM halos in the very high-density environments (e.g., inside groups and clusters) is a challenging problem. The goal of this study is to investigate the halo bias at both low and high matter overdensities, and we therefore need to identify both isolated halos and satellite halos orbiting within the virial radii of larger systems. The problems associated with halo identification within high-density regions are discussed by Klypin et al. (1999, hereafter KGKK). In this study we use a halo-finding algorithm called bound density maxima (BDM). The main idea of the BDM algorithm is to find positions of local maxima in the density field smoothed at a certain scale and to apply physically motivated criteria to test whether the identified site corresponds to a gravitationally bound halo. The detailed description of the algorithm is given by KGKK and Colín et al. (1999). The publicly available version of the BDM algorithm used here is described by Klypin & Holtzman (1997).\textsuperscript{2} The halo catalogs used in the present study were constructed using numerical parameters given by Colín et al. (1999).

To construct a halo catalog we have to define selection criteria based on particular halo properties. Halo mass is usually used to define halo catalogs (e.g., a catalog can be constructed by selecting all halos in a given mass range). However, the mass and radius are very poorly defined for the satellite halos because of tidal stripping which alters a halo’s mass and physical extent (see KGKK). Therefore, we will use maximum circular velocity $V_{\text{max}}$ as a proxy for the halo mass. This allows us to avoid complications related to the mass and radius determination for satellite halos. Moreover, when a halo gets stripped $V_{\text{max}}$ changes less dramatically than the mass and is therefore a better “label” of the halo. For isolated halos, $V_{\text{max}}$ and the halo’s virial mass are directly related. For example, a halo with a density distribution described by the Navarro, Frenk, & White (1996, 1997, hereafter NFW) profile $\rho(r) \propto r^{-1}(1 + r)^{-2}$ ($x \equiv r/R_{\text{vir}}$; $R_{\text{vir}}$ is the scale radius):

$$V^2_{\text{max}} = \frac{GM_{\text{vir}}}{R_{\text{vir}}} \frac{c}{f(c)} \frac{2}{2} f(c) ,$$

where $M_{\text{vir}}$ and $R_{\text{vir}}$ are the virial mass and radius, $f(x) = \ln (1 + x) - x/(1 + x)$, and $c \equiv R_{\text{vir}}/R_{\text{c}}$. While for the satellite halos $V_{\text{max}}$ may not be related to mass in any obvious way, it is still the most physically and observationally motivated halo quantity.

We constructed halo catalogs using thresholds in the maximum circular velocity (i.e., selecting all halos with $V_{\text{max}}$ higher than a specified threshold). The cluster-size halos are not explicitly excluded from the halo catalogs. We assume therefore that the center of each cluster can be associated with a central cluster galaxy. The latter (because of the lack of hydrodynamics and other relevant processes) cannot be identified in our simulations in any other way. We limit the extent of these “galaxies” to the central 150 $h^{-1}$ kpc of the cluster.

The redshift dependency of the relationship between halo mass and $V_{\text{max}}$ is expected to evolve because the concentration factor $c$ (see eq. [12]) is expected to evolve with redshift. We use the prescription of NFW to compute the evolution of $c(M, z)$ and to convert $V_{\text{max}}$ to the virial mass, but note that this prescription has been found to deviate significantly from the results of numerical simulations (Eke, Navarro, & Frenk 1998; Bullock et al. 1999). The values of the virial mass corresponding to the $V_{\text{max}}$ thresholds of 120 and 200 km s$^{-1}$ used in our analysis and calculated using equation (12) and the NFW prescription for $c(M, z)$ are given in Table 1. This table also gives the number of halos at different epochs identified by the halo finder in the simulation box using these thresholds.

5. RESULTS

5.1. Evolution of the Power Spectrum

Recent numerical and semianalytical studies have focused on the bias evolution as determined from the two-point correlation function. However, as new accurate measurements of the power spectrum at both low (e.g., Baugh & Efstathiou 1993, hereafter BE93; Gaztañaga & Baugh 1998, hereafter GB98; Tadros & Efstathiou 1996, hereafter TE96;\textsuperscript{2} See also http://astro.nmsu.edu/~aklypin/pmcode.html.

\begin{table}[ht]
\centering
\caption{Halo Catalogs}
\begin{tabular}{ccccccc}
\hline
$V_{\text{max}}$ & $M_{\text{vir}}$ & $N_{\text{halo}}$ & $M_{\text{vir}}$ & $N_{\text{halo}}$ \\
$\geq 120$ km s$^{-1}$ & $\geq 200$ km s$^{-1}$ & & & & \\
\hline
0.0 & 5.3 & $4.3 \times 10^{11}$ & 4707 & 1.4 & $10^{12}$ & 1027 \\
1.0 & 2.0 & $10^{11}$ & 7867 & 9.0 & $10^{11}$ & 1443 \\
2.0 & 1.1 & $10^{11}$ & 10437 & 5.4 & $10^{11}$ & 1675 \\
3.0 & 8.0 & $10^{10}$ & 9650 & 3.5 & $10^{11}$ & 1636 \\
\hline
\end{tabular}
\end{table}

* Masses are given in $h^{-1} M_\odot$.\textsuperscript{2}

\textsuperscript{2} See also http://astro.nmsu.edu/~aklypin/pmcode.html.
Hoyle et al. 1998) and high (Croft et al. 1998; Weinberg et al. 1998) redshifts become available, it is also useful to examine and compare the evolution of the power spectra of matter, $P_m(k)$, and halo, $P_h(k)$, distributions.

Figure 1 shows the evolution of real space $P_m(k)$ and $P_h(k)$ in our simulation. The halo power spectrum is shown for the halo catalog with the maximum circular velocity threshold of $V_{\text{max}} > 120$ km s$^{-1}$. To estimate the power spectra over a wide range of wavenumbers, we have used the method of Jenkins et al. (1998a). Both $P_m(k)$ and $P_h(k)$ have been obtained by combining a series of spectra, $\{P_i(k)\}$, in overlapping ranges of $k$: $[k_{\text{max}}^{-1}, k_{\text{max}}^{i}]$, where $k_{\text{max}}$ is the maximum wavenumber at which an accurate estimate of the $P(k)$ can be obtained. To compute $P(k)$, the computational cube (of size $L_{\text{box}}$) is divided into $i^3$ subcubes and the particle (or halo) distributions in these subcubes are superposed. The fast Fourier transform (FFT) of the resulting density field, estimated using the cloud-in-cell density assignment scheme (Hockney & Eastwood 1981), gives an accurate estimate of the power spectrum in modes that are periodic on the scale $L_{\text{box}}/i$. We have used the FFTs on a $256^3$ grid, and $i = 2^m$, where $m = 0, \ldots, 6$ and $m = 0, \ldots, 4$ for the DM and halos, respectively. Comparisons with direct $512^3$-grid FFT spectra suggested the use of $k_{\text{max}} = k_{N} = 128i(2\pi/L_{\text{box}})$ is the Nyquist wavenumber for $P(k)$ ($k_{\text{max}}^0 = 2\pi/L_{\text{box}}$). The individual power spectra have been corrected for the shot noise by subtracting the noise spectrum estimated using the same method.

Panel a of Figure 1 shows the evolution of the matter power spectrum from $z = 5$ to the present. For comparison we also show the nonlinear evolution predicted by the fitting formula of Peacock & Dodds (1996, hereafter PD96): $\Delta^2_{NL}(k_{NL}, z) = f_{NL}[(1 + k^2_{NL}k^2_{NL})/2]^{1/3}$, where $\Delta^2(k) = dP(k, z)/d\ln k$ and $\Delta^2_{NL}(k_{NL}, z) = f_{NL}[(1 + k^2_{NL}k^2_{NL})/2]^{1/3}$, and $n_{NL}$ denote linear and nonlinear quantities, respectively. The analytical expression for $f_{NL}$ depends on five fitting parameters (see § 3.3 in PD96) obtained by fitting the power spectra of the scale-free $N$-body simulations. Although there are some minor difficulties in applying these fitting results to the realistic models with a scale-dependent power-law spectral index (PD96; Smith et al. 1998; Jenkins et al. 1998a), the prediction works remarkably well. We have been able to match the nonlinear spectra in our simulation at all epochs with only a small change in the fitting parameters of PD96. Namely, we have used $n_{eff}(k_{NL}) = d\ln P(k)/d\ln k_{NL}$ [as opposed to an alternative $n_{eff}(k_{NL})$ for the estimate of the spectral index at wavenumber $k_{NL}$] and slightly changed the power index dependence for the fitting parameter $V$. This parameter controls the amplitude of the high-$k$ asymptote; instead of using $V = 11.55 (1 + n_{eff}/3)^{-\eta}$, with a single time-independent value of $\eta = 0.423$ given by PD96, we use $\eta$ varying from 0.7 at $z = 5$ to 0.45 at $z = 0$. The PD96 formula, with their fitting parameters unchanged, fares well at $z < 1$ but underpredicts the amplitude of the asymptote at high $z$. Figure 1a shows that, with this small modification, the PD96 prediction is a success. Nevertheless, if the desired accuracy of the nonlinear spectrum estimate is $\leq 50\%$, the necessity of such (generally time and cosmology dependent) modifications should be kept in mind.

Figure 1b shows that the evolution of $P_h(k)$ is much slower than the evolution of the matter power spectrum. Although the scale of nonlinearity [wavenumber of an

Fig. 1.—Evolution of the matter (panel a) and halo power spectra (panel b). (a) DM power spectra, $P_m(k)$, at different redshifts (solid lines) are compared with the linear spectra (dotted lines) and predictions of the Peacock & Dodds (1996) fitting formula (dashed lines; see text for details). Note that the power spectra in the simulations agree with the analytical predictions at all scales, including highly nonlinear scales at which the “stable clustering” approximation appears to work well. (b) Evolution of the power spectrum for halos with maximum circular velocities greater than 120 km s$^{-1}$ (solid lines) as compared with the linear evolution of the matter power spectrum (dotted lines). Note that the evolution of the halo power spectrum, $P_h(k)$, is much slower than that of the matter spectrum, $P_m(k)$, shown in panel a. At high redshifts the amplitude of $P_h(k)$ exceeds that of $P_m(k)$ by a factor of $\sim 5-10$, while at lower redshifts the differences are smaller. The ratio of amplitudes, $P_h(k)/P_m(k)$, depends on the scale. These differences imply that the halo bias is time and scale dependent.

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3 This modification is, of course, arbitrary, and the same result could possibly be achieved with other changes, for example, by varying the wavenumber at which $n_{eff}$ is computed.
upward inflection of $P_h(k)$ is seen clearly in the halo spectra and approximately matches the corresponding scale in $P_m(k)$ at the same epoch, the shapes of the halo and matter power spectra are quite different: the $P_h(k)$, for example, can be approximated well by a simple power law, while the shape of $P_m(k)$ is more complicated. This difference, together with the difference in the evolution rate, means that the bias of the halo distribution is time and scale dependent. A similar conclusion has been reached by many researchers from comparisons of halo and matter two-point correlation functions (e.g., recently, Bagla 1998; Colin et al. 1999; Kauffman et al. 1998a, 1998b; Katz et al. 1998 and references therein). Note, however, that as we mentioned in § 1, the scale dependence of the bias is different in real and $k$-space (see eq. [4]). Although we will be referring to both $b(h)$ and $b(r)$ (defined in eqs. [1] and [2]) as the bias functions, it should be kept in mind that they have different functional forms.

Figure 2 shows the evolution of the halo bias, $b(k) \equiv [P_h(k)/P_m(k)]^{1/2}$, from $z = 3.0$ to the present epoch for two halo catalogs selected with low- and high-mass thresholds: $V_{\text{max}} > 120 \text{ km s}^{-1}$ and $V_{\text{max}} > 200 \text{ km s}^{-1}$. The bias evolves from a value of $\approx 2-4$ at $z = 3$ to $\approx 0.5-1$ at $z = 0$. At high redshifts, the bias depends on the selection threshold indicating that it is mass dependent; the differences, however, become progressively smaller for lower redshifts. Also, the scale dependence of the bias of the lower mass halos, albeit being redshift dependent and generally $\neq 1$, is significantly weaker than the scale dependence of the higher mass halos. At $z = 1$, for instance, the power spectrum of $V_{\text{max}} > 120 \text{ km s}^{-1}$ halos follows that of the mass almost exactly at all probed wavenumbers. In agreement with analytical prediction of Scherrer & Weinberg (1998), the bias is virtually scale independent at linear scales $k < k_{NL}$, where $k_{NL}$ is the wavenumber where the $P_m(k)$ becomes nonlinear and exceeds the linear prediction ($k_{NL} \approx 0.2-0.4$). Note also that during the evolution the bias at these linear scales is driven to the value of $\approx 1$, as expected in the linear regime (Tegmark & Peebles 1998). This is true for both low- and high-mass catalogs, which indicates that the bias evolution at these scales is driven by the gravitational growth of clustering that tends to erase any initial (mass dependent) differences in the halo and mass distributions (Tegmark & Peebles 1998).

The bias evolution at nonlinear ($k > k_{NL}$) scales is more complicated. The bias evolves to values less than unity at

![Figure 2](image-url)
z < 1, and its scale dependence becomes progressively stronger over a wider range of wavenumbers. We will discuss the possible interpretation of the bias evolution in the nonlinear regime in the following section. However, we would like to point out here that the net result of this evolution is the $P_d(k)$ at $z = 0$ which is significantly antibiased with respect to the overall matter distribution, but which agrees very well with the power spectrum of observed galaxy distribution. Figure 3 compares the $z = 0$ power spectra of halos and matter in our simulation with the power spectrum of galaxies (BE93; GB98; TE96) in the Automatic Plate Measuring Facility (APM) survey. At small scales ($k \geq 2 \text{ h Mpc}^{-1}$), the $P_d(k)$ depends on the catalog's $V_{\text{max}}$ threshold. The galaxy power spectrum, however, lies comfortably in between two likely possibilities of galaxy mass cutoffs. The maximum circular velocities of 120 and 200 km s$^{-1}$ correspond at $z = 0$ to the virial masses of $\approx 3 \times 10^{11} \text{ h}^{-1} M_\odot$ and $\approx 1.5 \times 10^{12} \text{ h}^{-1} M_\odot$, respectively. At larger scales, the power spectra of halos and galaxies agree within the errors. The "inflection" in the galaxy power spectrum (GB98) is reproduced at the correct wavenumber of $k \approx 0.2 \text{ h Mpc}^{-1}$. We interpret the "inflection" wavenumber in the observed spectrum as the scale of nonlinearity, because both halo and matter power spectra in the simulation become nonlinear at higher $k$. The amplitude of the matter power spectrum at these $k$, however, is $\approx 3$--$4$ times higher and does not come anywhere close to matching the observed power spectrum. This shows clearly that it is crucial to consider the distribution of halos, not matter, when comparing model predictions to the observations.

It is not clear whether the power spectrum in the $\Lambda$CDM model considered here can reproduce the amplitude and shape of the turnover observed in the galaxy power spectrum at $k \sim 0.01$--0.06 h Mpc$^{-1}$. Figure 3 shows that the $\Lambda$CDM spectrum does not reproduce the turnover in the spectrum recovered from the APM angular correlation function (BE93; GB98); however, it is in better agreement with the power spectrum derived by TE96 from the Stromlo-APM redshift survey. Also, a recently published power spectrum of the Durham/UKST (United Kingdom Schmidt Telescope) survey (Hoyle et al. 1998) agrees very well with the Stromlo-APM spectrum and with the spectrum of the $\Lambda$CDM model. The Stromlo-APM and Durham/UKST spectra have been computed in redshift space, but the differences from the real space spectrum in the $\Lambda$CDM model at these scales are expected to be $\leq 40\%$ (TE96; Smith et al. 1998). Even with the 40\% lower amplitude, the spectra are in agreement with the model and, within 2 $\sigma$ errors, with the APM power spectrum. The latter indicates either that there are systematic differences between the galaxy surveys or that the cosmological model is incorrect (because it predicts an incorrect redshift-to-real space correction). There is also a possibility that the amplitude and errors of the APM power spectrum are somewhat underestimated at these large scales (Peacock 1997). Unfortunately, at present, statistical and systematic observational errors are too large at these scales to be able to make a decisive conclusion. In any case, considering the whole range of observationally probed wavenumbers, the agreement between the data and the model is much better when we compare observations with $P_d(k)$, as opposed to $P_m(k)$.

5.2. Overdensity of Matter versus Overdensity of Halos

5.2.1. Linear and Mildly Nonlinear Overdensities

The evolution of bias, as determined from the power spectra in the previous section, agrees qualitatively with the bias evolution derived from the correlation function analyses (e.g., recently, Bagla 1998; Kauffmann et al. 1998a 1998b; Katz et al. 1998; Colín et al. 1999 and references therein). The bias functions $b_m(k) \equiv [P_m(k)/P_m(k)]^{1/2}$ and $b_d(r) \equiv [\xi_m(r)/\xi_m(r)]^{1/2}$ (see § 5.1), defined using the power spectrum and the correlation function, illustrate the scale dependence of the bias. However, it is difficult to interpret $b_p$ and $b_h$ in terms of the most generic definition of bias: $\delta_b = b_h \delta_m$, where we denote the bias in this definition by $b_h$ to distinguish it explicitly from $b_p$ and $b_d$. The $b_h$ shows how bias depends on the matter density at a fixed scale, the information which cannot be extracted easily from $b_d$ and $b_p$. Therefore, to get a full picture of the bias evolution, we examine the evolution of $b_h$ in our simulation.

To estimate $\delta_b$ and $\delta_m$, we use the top-hat filter (see § 2) of comoving radius $R = 5 \text{ h}^{-1}$ Mpc. The size of the radius is a compromise between halo statistics and the range of probed overdensities. Note that our simulation box contains only 216 independent spheres of this size. This is the maximum number of spheres that can be used to study the scatter of the bias. However, we are primarily interested in the average behavior of $b_h$; therefore, in order to probe the wide range of overdensities, we use a large number (50,000) of

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4 All definitions are, of course, equivalent if the bias is linear. This, however, is not true for the nonlinear bias, which appears to be a generic feature of the CDM models.
spheres, placed randomly throughout the simulation box. To calculate \( \delta_h \), we have used halos with maximum circular velocities of \( V_{\text{max}} > 120 \text{ km s}^{-1} \). Because of limited mass resolution, the halo catalogs are somewhat incomplete for \( V_{\text{max}} \lesssim 100 \text{ km s}^{-1} \) (see Gottlöber, Klypin, & Kravtsov 1998). The catalog with the threshold of 120 km s\(^{-1}\) is complete and contains a large enough number of halos (see Table 1) to provide sufficient statistics.

Figure 4 shows overdensity of DM halos, \( \delta_h \), as a function of matter overdensity \( \delta_m \) at four epochs: \( z = 0, 1, 2, 3 \). The solid curves show the average relation, calculated by averaging \( \delta_h \) of the individual spheres in bins of \( \delta_m \). The actual binned distribution of the \( \delta_m \) and \( \delta_h \) values is shown by gray-scale shades on a two-dimensional grid, where the intensity of gray corresponds to the natural logarithm of the density of points in the grid cell. The figure shows that at all epochs the halo overdensity is tightly correlated with the overdensity of matter. However, the slope of the correlation, the bias, depends on \( \delta_m \) (i.e., the bias is nonlinear) and changes nontrivially with time.

The dashed and dot-dashed lines in each panel of Figure 4 show predictions of the analytical model of bias described in § 2 (namely, eqs. [8] and [9]). To account for the range of halo masses used in our halo catalog, we calculate the mass function weighted bias (eq. [11]) using \( M_{\text{max}} = 10^{13} \, h^{-1} M_\odot \) as an upper limit of integration and redshift dependent \( M_{\text{min}}(z) \) corresponding to our selection threshold of maximum circular velocity of 120 km s\(^{-1}\) (see § 3 and Table 1).

The two model curves correspond to different assumptions about the formation redshift of halos, \( z_f : z_f = z \) (i.e., halos forming at the epoch of observation) for the dot-dashed curve and \( z_f = z + 1 \) for the dashed curve. In the standard PS model, hierarchical buildup of halos is a con-

![Figure 4](image-url)
tinuous process and, therefore, if mass is considered as a defining property of a halo, $z_f = z$. However, this interpretation fails if halos can retain their identity over a finite period of time (e.g., see discussion by Moscardini et al. 1998). For example, if a halo is accreted by a more massive system and orbits in the system’s potential instead of merging instantly, it can be identified at $z < z_f$. The galaxy-conserving model of bias evolution (e.g., Moscardini et al. 1998 and references therein) represents an extreme in which halos are never destroyed after being formed and in which the halo clustering is driven solely by the gravitational pull from the surrounding growing structures.

In reality, we expect the merging to take place and the halos to have individual merging histories and thus individual survival times (Lacey & Cole 1993; Kitayama & Suto 1996). A rigorous treatment of bias should therefore take into account only the halos that survive until the epoch of observation $z$. In practice, this is a difficult task: although the halo formation epoch is well defined, definition of the halo destruction is not trivial. Lacey & Cole (1993) define halo lifetime as the period between formation time of a halo and the time by which this halo is incorporated into a more massive system. This definition differs significantly from how we define the destruction when analyzing the simulations: the halo is destroyed either when it merges with another halo and loses its identity or when the tidal stripping brings the mass bound to the halo below some mass threshold (defined as a selection criterion or by the mass resolution of the simulation). Therefore, for illustration purposes, we will treat $z_f$ as a free parameter, leaving a more rigorous treatment for future work.

Figure 4 shows that different assumptions about $z_f$ lead to significant difference in the behavior of bias predicted by equation (9). Although at $\delta_m \approx 0.5$ the two bias predictions are similar, at higher overdensities they diverge, the $z_f = z + 1$ assumption providing a much better match to the simulation results. Note that $z_f = z$ underestimates the bias in the simulation at $\delta_m \approx 1$. This was noted recently by Jing (1998), who studied mass and scale dependence of bias in the linear regime. For the simulation of the $\Lambda$CDM model used in this paper, Jing finds that halos of mass $10^{11} \, h^{-1} M_\odot$ exhibit bias of $b_1(M = 10^{11} \, h^{-1} M_\odot) \approx 0.5$–0.6, while linear MW bias is (eq. [10]) assuming $z = z_f$ $b_1^L \approx 0.28$. For the same halo mass and $z_f = z + 1 = 1$, equation (10) gives $b_1^L \approx 0.67$, and therefore the finite survival time of halos may naturally explain the bias discrepancy. It is worth noting that for cluster halos, the $b_1$ with $z_f = z$ provides a considerably better approximation which likely reflects late cluster formation ($z \approx z_f$) and/or smaller survival times for cluster-size halos, as is indeed predicted by the extended PS theory (Lacey & Cole 1993).

### 5.2.2. Nonlinear Overdensities

Figure 5 shows the present-day $\delta_h$–$\delta_m$ relation at higher overdensities. Although we use a large number of spheres and oversample the density field, the spheres in independent regions of space are independent and may thus give an idea about the true scatter in the halo bias. In order to study this scatter, we have divided the computational box into eight equal-size ($30 \, h^{-1} \text{ Mpc}$) nonoverlapping subcubes. Different symbols in Figure 5 correspond to spheres in the three subcubes that contain most of the massive clusters in the simulation at $z = 0$. The other five subcubes contain lower density regions and are not shown for clarity.

Figure 5 shows that the differences between $\delta_h$–$\delta_m$ relations in different subcubes are significantly larger than the scatter of each individual relation. Note, however, that even within a single subcube there are indications of real scatter: the subcube represented by the triangle markers shows a dichotomy of at fixed $\delta_h \approx 2$–4. Analysis of the subcubes has shown that differences in the $\delta_h$–$\delta_m$ correlation among the subcubes are real and are caused by the differences in the nonlinear structures they contain.

It appears that the scatter and the differences can be explained by the following two effects. The centers of the spheres with $\delta_m \approx 5$ fall preferentially in close vicinity to the massive group- and cluster-size halos. These cluster halos span a wide range of masses and, most importantly, formation times. The differences in formation times result in different fates of DM halos orbiting inside these clusters. If, for example, a cluster forms and accretes the bulk of its mass and halos early, the halos have time to suffer substantial losses of mass because of tidal stripping and losses of orbital energy from dynamical friction. Dynamical friction may lead to a merging between satellite and central cluster object, thus resulting in a “loss” of the satellite. Tidal stripping may lead to a substantial mass loss and may ultimately (although at a much slower rate) decrease the halo’s maximum circular velocity (see KGKK) bringing the halo below our threshold $V_{\text{max}}$. These effects are enhanced because the virial radius is smaller at earlier epochs and so are the typical pericenters of satellite orbits. If, on the other hand, the cluster forms late and accretes most of its mass fast and at relatively low redshifts, the halos are accreted onto a massive cluster and thus have higher orbital energies.

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5 The survival time is the time between halo formation and destruction.
and orbits with larger pericenters. Moreover, many halos simply do not have enough time to suffer substantial tidal or orbital energy losses.

The differences in formation histories may therefore lead to significant differences in the halo content among clusters. This is illustrated in Figure 6 which shows “bias profiles”: the ratios of the integral overdensities of halos and matter in spheres of increasing radii centered on a cluster. The profiles for five of the clusters from the three subcubes of Figure 5 are shown at three different epochs $z = 0.0, 0.5, 1.0$. Note the large differences between profiles at $z = 1$. While the cluster marked CL1 exhibits strong antibias (i.e., low $\delta_h/\delta_m$), the halos in cluster CL2 are strongly biased. Similar differences are seen in the rest of clusters. Interesting differences can also be observed in the subsequent evolution of $b(r)$. Cluster CL1 shows very mild evolution in $b(r)$ at small ($r \lesssim 1 \, h^{-1} \text{Mpc}$) scales, whereas other clusters show very strong evolution between $z = 1.0$ and $z = 0.5$ and much weaker evolution from $z = 0.5$ until present.

The changes in the rate of evolution may seem counter-intuitive; indeed, the time period corresponding to the $z = 1.0$–0.5 interval is approximately twice as short ($\approx 2.5$ Gyr) as the period between $z = 0.5$ and $z = 0$ ($\approx 5$ Gyr). We could thus expect more significant changes at $z < 0.5$ because of the more prolonged effects of tidal stripping and dynamical friction. However, as we have noted above, as the mass of a cluster grows with time, the halos are accreted on higher pericenter, higher energy orbits and thus are not as likely to approach the dense central region or spend a substantial amount of time there.

To illustrate that this indeed is the case, we have analyzed the dynamical evolution of halos identified within the virial radius of clusters at different moments in time. Figure 7 shows the evolution of five halos randomly selected within the virial radius of cluster CL1 at $z = 3$ (there are a total of 10 halos within the virial radius). At all epochs, CL1 is the most massive cluster in the simulation box; at $z = 3$ its mass was $1.5 \times 10^{13} \, h^{-1} \, M_\odot$. The panels in each of the horizontal rows in Figure 7 show the evolution of particles that are bound to the halos at $z = 3$. The orbits of four halos are contained within the cluster virial radius, $\approx 150 \, h^{-1} \, \text{kpc}$, at $z = 3$ (shown as a circle in each panel), where all of these halos merge with the central $D_{100}$ size object by $z = 1$ (i.e., after $\approx 3.5$ Gyr). The halo on the most eccentric orbit (bottom row) survives until $z \approx 0.5$ and gets tidally destroyed after that.

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**Fig. 6.** Bias profiles (ratio of halo to matter overdensities inside a sphere of radius $r$ centered on the cluster center) of five of the rich ($M_{\text{vir}} \approx 10^{14} \, h^{-1} \, M_\odot$) clusters from the three subcubes shown in Fig. 5 at three different epochs $z = 0.0, 0.5, 1.0$. The vertical lines indicate the virial radii of clusters at $z = 0$; $M_{\text{vir}}$ and $Z_{1/2}$ indicate the $z = 0$ virial mass and redshift at which cluster had half of this mass, respectively. Note the large differences between profiles at $z = 1$. While the cluster marked CL1, the most massive cluster in the simulation, exhibits strong antibias (i.e., low $\delta_h/\delta_m$), the halos in cluster CL2 are strongly biased. Similar differences are seen in the rest of clusters. Interesting differences can also be observed in the subsequent evolution of $b(r)$. Cluster CL1 shows very mild evolution in $b(r)$ at small ($r \lesssim 1 \, h^{-1} \text{Mpc}$) scales, whereas other clusters show very strong evolution between $z = 1$ and $z = 0.5$ and much weaker evolution from $z = 0.5$ until present.
Fig. 7.—Leftmost column of panels shows particles bound to the five halos identified within the virial radius of the cluster CL1 shown in Fig. 6 at $z = 3$ [$R_{\text{vir}} \approx 120 \ h^{-1} \text{ kpc}$ (proper) and $M_{\text{vir}} \approx 1.5 \times 10^{13} \ h^{-1} \ M_{\odot}$ at $z = 3$]. The rest of the columns show positions of the same particles at later moments. Four out of five halos merge with the central cluster halo by $z \approx 1$. In all panels particles are plotted in proper coordinates; the circles in all panels have the same radius equal to the virial radius of the cluster at $z = 3$.

For comparison, Figure 8 shows the evolution of 10 halos randomly selected within the virial radius of the same cluster CL1 at $z = 0.5$. As before, most of the halos stay within the $z = 0.5$ virial radius ($\approx 1.2 \ h^{-1} \text{ Mpc}$). However, unlike the $z = 3$ halos, most of them (eight out of 10) survive until $z = 0$ (i.e., during the period of $\approx 5$ Gyr). Although some halos suffer substantial mass loss in their outer regions, the dense halo cores can be identified at $z = 0$. Note that two halos on low-pericenter orbits do merge with the central object.

Finally, we have also visually examined the fate of the halos identified in cluster CL5 of Figure 6 at $z = 1$ and $z = 0.5$. More than half of the $z = 1$ halos merge with the central object by $z = 0.5$, while most of the $z = 0.5$ halos survive until $z = 0$. The difference is caused by both the lower typical orbit pericenters at higher redshifts and the higher efficiency of the dynamical friction due to a smaller mass of cluster. It explains the evolution of the bias profile shown in Figure 6.

To illustrate the relative efficiency of dynamical friction at different epochs of cluster evolution, we present the evolution of the dynamical friction timescale in a cluster. At a given epoch, the dynamical friction time, $t_{\text{fric}}$, can be estimated using Chandrasekhar's formula (Binney & Tremaine 1987), assuming the cluster mass and density distribution and the mass of the satellite. In the right column of Figure 9 we present estimates of $t_{\text{fric}}$ for halos with maximum circular velocity $V_{\text{max}} = 120 \text{ km s}^{-1}$ for clusters of different final masses. The mass accretion histories of clusters of similar present-day mass exhibit a spread around an average, typical for this mass, evolution track. To account for this spread, we have used both the average mass evolution tracks and two individual evolution tracks representative of the early and late forming tails of the population (these
tracks represent $\approx 2\sigma$ deviations from the average mass evolution track. The cluster mass evolution tracks used here (Avila-Reese & Firmani 1997) have been generated using the Monte Carlo method of Lacey & Cole (1993). For each epoch, we compute $t_{\text{fric}}$ using equations (8)–(10) of KGKK assuming the NFW density distribution [with an appropriate $c(M, z)$; see §4] for both the cluster and satellite at a distance $R_{\text{vir}}/2$ from the cluster center (where $R_{\text{vir}}$ is the virial radius of the cluster at this epoch) and accounting explicitly for the mass loss due to the tidal stripping.

The left column of panels in Figure 9 represents the dynamical friction time in a different, easier to interpret, way. It shows the “merging redshift,” defined as the redshift corresponding to the time $t + t_{\text{fric}}$, where $t$ is the current epoch (redshift $z$). In the sense of the dynamical friction time, $z_{\text{merge}}$ can be interpreted as a redshift at which most of the halos present in clusters at redshift $z$ will merge into a central object. This interpretation assumes that the mass of the cluster would not change, which is not correct. The actual value should therefore be considered as an upper

Fig. 8.—Same as in Fig. 7, but for 10 halos randomly selected from the halos identified within the virial radius of the cluster at $z = 0.5 \left[ R_{\text{vir}} \approx 1.1 h^{-1} \text{ Mpc (proper)} \text{ and } M_{\text{vir}} \approx 5.8 \times 10^{14} h^{-1} M_\odot \right]$. The first and third columns from the left show the positions of the particles bound to the halos at $z = 0.5$, while the second and the fourth panels show the positions of the same particles at $z = 0$. Note that despite substantial mass losses, eight out of 10 of these halos can be identified at $z = 0$ as distinct dense clumps of particles. The circles in all panels have the same radius equal to the virial radius of the cluster at $z = 0.5$. 
Figure 9.—Estimates of the dynamical friction timescale as a function of time for clusters of different final \((z = 0)\) mass \(M_0\). The right column presents estimates of \(t_{\text{fric}}\) for satellite halos of maximum circular velocity \(V_{\text{max}} = 120 \text{ km s}^{-1}\). The three curves correspond to the average evolution (solid curves), early formation (dashed curves), and late formation (dot-dashed curves) computed using Monte Carlo realizations of mass accretion histories (see § 5.3 for details). The dotted curve shows the age of the universe in the \(\Lambda CDM\) model studied here as a function of redshift. The left column shows the corresponding evolution of "merging redshift" defined as a redshift corresponding to \(t(z) = t_{\text{fric}}(z)\), where \(t(z)\) is the age of the universe at redshift \(z\) and \(t_{\text{fric}}\) is the corresponding dynamical friction timescale (both quantities are shown in the right panel). The curve marking has the same meaning as in the right panel. Negative values \(z_{\text{merge}} < 0\) mean that the dynamical friction time is larger than the time span between redshift \(z\) and the present.

The results and arguments presented in the previous section suggest that we can identify major processes that drive the evolution of the halo bias. In the linear (\(\delta_m \ll 1\) the overdensities quoted here and below are for a density field smoothed with the top-hat filter of radius \(R_{\text{TH}} = 5 \text{ h}^{-1} \text{ Mpc}\), and in mildly nonlinear regimes (\(\delta_m \lesssim 3\)) the analytical model of bias developed by Mo & White (1996; see § 2) is in good agreement with the results of our simulation, if the formation epoch of halos \(z_f\) is distinguished from the epoch of observation \(z\) (or, in other words, if halos are assumed to retain their identity during a finite interval of time after their formation). This model reproduces the non-linearity and time evolution of the bias observed in the simulation well (see Fig. 4). The evolution of the bias in linear and quasi-linear regimes is, therefore, driven primarily by the halo collapse and merging rates specific for a given cosmological model.

Interplay between halo formation and the merging rate in high-density regions [affecting \(n(M, z, z_f|R_0, \delta_0)\) in eq. (8)]...
and halo formation and the merging rate in the field [affecting \( n(M, z, z_p) \)] leads to the decrease of the bias with time for halos in a given mass range. This is simply because halos of a given mass collapse earlier in high-density regions than they do in the field; the high-redshift objects therefore represent rare events in the density field and are initially strongly clustered because of modulation by large-scale modes. The number of halos in high-density regions is then decreased because of merging, while the number density of halos in the field may still increase (or level off depending on mass range considered) at lower redshifts. The ratio is thus a decreasing function of time. In the regions of negative overdensity \( \langle \delta_m \rangle \leq -0.5 \), the evolution is reverse: formation of halos of a given mass in these regions is delayed, and at early epochs their number density is below the average (see Fig. 4). The halos in underdense regions are thus antibiased at early epochs. Figure 4 shows that bias at \( \langle \delta_m \rangle \leq -0.5 \) increases during evolution as the collapse threshold is decreasing function of time. In the regions of negative overdensity \( \langle \delta_m \rangle \leq -0.5 \), the evolution is reverse: formation of halos of a given mass in these regions is delayed, and at early epochs their number density is below the average (see Fig. 4). The halos in underdense regions are thus antibiased at early epochs. Figure 4 shows that bias at \( \langle \delta_m \rangle \leq -0.5 \) increases during evolution as the collapse threshold is reaching lower and lower overdensities and more and more halos are being formed in these underdense regions (Fig. 5 shows that \( b = 1 \) for \( -1 \leq \langle \delta_m \rangle \leq 2 \) at \( z = 0 \)). The prediction of the analytical model at these overdensities matches our numerical results nicely.

In high-density regions \( \langle \delta_m \rangle \gtrsim 3 \), the evolution and amplitude of bias appears to be significantly affected by dynamical friction.\(^6\) Indeed, the process results in halo mergers with the central cluster halo and thereby reduces the ratio of halo to matter overdensities. The efficiency of dynamical friction depends sensitively on the mass of satellite halos, properties of halo orbits, and mass of the cluster. The latter evolves rapidly with time and switches dynamical friction on at some epoch \( z \sim 0.5 \)–1.0 for the \( \Lambda \)CDM model studied here). The mass accretion history is a stochastic process, and some scatter in the mass evolution is expected for clusters that have the same mass at \( z = 0 \) (Lacey & Cole 1993): some clusters accrete most of the mass early on, while others accrete most of their mass at later redshifts. Therefore, for a given observation epoch \( z \), different clusters may be affected by the dynamical friction to a different degree. For example, a cluster which has accreted most of its mass (and halos) just prior to \( z \) will be less affected than a cluster of the same mass that accreted most of its halos earlier because dynamical friction had more time to operate in this cluster. The situation is even more complicated, because clusters, even those in which dynamical friction becomes inefficient because of mass increase, may accrete smaller clusters and groups which, in turn, have different evolution histories and therefore different ratios of halo to matter overdensities.

Dynamical friction is not the only process that affects halo counts in clusters and groups. Tidal fields of clusters strip the outer parts of satellite halos which may result in substantial mass loss for medium- and high-mass clusters (up to a factor of 5–20 depending on the parameters of the halo orbit and the period of time the halo spends in the cluster; see, e.g., KGKK). The maximum circular velocity, \( V_{\text{max}} \), of the halo changes only mildly, even in the case of severe mass loss (which, as a reminder, is the reason we use it for the halo selection). Nevertheless, for some halos \( V_{\text{max}} \) may decrease below the selection threshold, in which case these halos will “drop out” of the catalogs. This process likely contributes to the mild evolution of bias seen in Figure 6 at \( z \leq 0.5 \). The efficiency of tidal stripping is lower for lower mass systems; therefore, we expect it to be important only in relatively massive \( (M_{\text{vir}} \gtrsim 10^{14} \, h^{-1} \, M_\odot) \) clusters. While this effect may seem to depend on the particular halo selection procedure used in our analysis, similar effects may arise for other selection procedures.\(^7\) It is clear, for example, that this effect would be even more severe had we chosen to select halos using their bound mass. Observationally, galaxy catalogs are usually selected using a fixed luminosity limit in a given wavelength band. If the luminosity of galaxies in this band evolves differently in clusters than in the field (which is strongly suggested by a variety of observations), the uniform selection criterion is bound to select somewhat different galaxy populations in high- and low-density regions.

Different evolution histories of different systems may result in a different locally evaluated bias. If, for example, matter and halo density fields are smoothed at some sufficiently large (larger than typical cluster size) scale, regions of a similar matter overdensity may correspond to different halo overdensities, because the latter depends on the evolution history of systems encompassed within the smoothing scale. We can expect, therefore, that bias evaluated at a finite smoothing scale will exhibit some scatter in different regions of space. Note that this scatter arises not from the “stochasticity” of the halo formation, but from the fact that the same matter overdensity may correspond to different evolution histories and, thus, of different halo content. On the other hand, the differences in the evolution histories result from the modulation by large-scale modes in the density distribution. The bias is thus also modulated by the large-scale modes and is therefore nonlocal. Note that this “stochasticity” should decrease as one smooths density fields on progressively larger scales because effects of the modulation by large-scale modes are smaller on larger scales. In general, if the relation between halo and matter density fields is nonlinear, the bias estimated from a density field smoothed at any particular scale will be nonlinear and will exhibit some scatter (Dekel & Lahav 1998). In our simulation, differences in the bias between different regions of the computational volume seen in Figure 5 are caused by different numbers and formation histories of clusters and groups in these regions. The regions that exhibit weaker bias (i.e., stronger antibias) are the regions that contain clusters with earlier formation epochs. The Coma-size cluster that already had a mass of \( \approx 1.3 \times 10^{13} \, h^{-1} \, M_\odot \) at \( z = 3 \) exhibits the highest matter overdensity and the strongest antibias.

Clusters and groups of galaxies contribute a significant fraction of the galaxy clustering signal at small, \( r \lesssim 1–2 \, h^{-1} \, \text{Mpc} \), scales and significantly affect clustering amplitude at larger scales. The antibias arising because of dynamical processes in groups and clusters can therefore explain the antibias seen in comparisons of halo and matter two-point

\(^6\) It is clear that dynamical friction is important for halo evolution; indeed, binary halo mergers are also due to dynamical friction. However, here we discuss dynamical friction that operates on satellites orbiting inside a more massive system (a group or a cluster), which leads to the decay of their orbits, and, ultimately, to a merger with the central cluster object.

\(^7\) Note that we are bound to use some selection procedure because in most cases we are interested in studying the clustering of a particular (selected) class of objects: halos of a given type, galaxies of a given luminosity or color, etc.
correlation functions and power spectra. Our results then imply that an understanding of the evolution of small-scale galaxy clustering requires a deeper understanding of the evolution of galaxies in groups and clusters.

The major caveat in the interpretation of these dissipationless results is a correspondence between DM halos and observable galaxies. We note, however, that the definition of DM halo in this study is significantly different from a conventional definition, which eliminates many problems of halo-galaxy mapping related to the overmerging problem (see KGGK and Colin et al. 1999 for discussion). In the framework of the hierarchical structure formation modeled here, it seems likely that in every sufficiently massive \((M \geq 10^{11} \, h^{-1} \, M_\odot)\) gravitationally bound halo, at some epoch baryons will cool, form stars, and produce an object resembling a galaxy (e.g., White & Rees 1978; Kauffmann et al. 1997; Yepes et al. 1997). This is indeed a cornerstone assumption of the semianalytical models of galaxy formation (e.g., Kauffmann et al. 1993; Baugh et al. 1996; Somerville & Primack 1998). Therefore, even though we cannot predict detailed properties of galaxies hosted by DM halos (which would require inclusion of a substantial amount of additional physics in the simulations), it is likely that the overall halo distribution should be representative of the expected distribution of the overall galaxy population. If this is not true, the excellent agreement between clustering of galaxy-size halos in our simulation and observed galaxy clustering would actually indicate that something is wrong with the model.

Antibias, similar in amplitude and scale dependence to that observed in our simulation, has been found in other recent numerical studies that employ either nonadiabatic hydrodynamics (Jenkins et al. 1998b) or semianalytic recipes (Kauffmann et al. 1998a, 1998b; Diaferio et al. 1998; Benson et al. 1998) to model galaxy formation and the evolution of the two-point correlation function. These studies are very different in their modeling techniques, and the agreement indicates that antibias is real and is not a numerical artifact of a particular simulation. Antibias was traditionally an unfavored proposition because it is easier to envision a biased rather than antibiased galaxy formation. However, as we have argued above, the antibias may arise naturally during the dynamical evolution of the halo population, even though halo formation is biased. Indeed, all of the numerical studies mentioned above are similar to the present study in explicit modeling of the orbital evolution of halos in groups and clusters. Diaferio et al. (1998) present bias profiles \(b(r)\) of the clusters in their simulation that are in good qualitative agreement with the profiles shown in Figure 6, indicating that similar dynamical processes are probably causing the antibias in their study.

We can therefore expect that simulations affected by the overmerging problem and in which a recipe for splitting or weighting the overmerged halos is used may overestimate the clustering amplitude at small scales and not show (or show a weaker) antibias. The amount of antibias should also depend on the box size because small-size \((30–50 \, h^{-1} \, \text{Mpc})\) boxes are unlikely to contain clusters in the high-mass tail of the mass function. Such a trend is indeed observed; the comparison of bias in the \(\Lambda\)CDM simulations of 30 and \(60 \, h^{-1} \, \text{Mpc}\) boxes presented by KGKK and Colin et al. (1999) shows that antibias is stronger in the \(60 \, h^{-1} \, \text{Mpc}\) simulation. Finally and most importantly, the sensitivity of the small-scale bias amplitude to the abundance and evolution histories of halos indicates that we can expect some differences between cosmological models. The models that form clusters at systematically later epochs and/or in lesser abundance should exhibit a weaker antibias (or even absence of antibias) than models that form clusters earlier and in larger numbers. Thus, for example, a \(\Lambda\)CDM model appears to result in a higher value of \(z = 0\) bias than the \(\Lambda\)CDM model (Kaufman et al. 1998a, 1998b; Colin et al. 1999; Diaferio et al. 1998). A more systematic large-box study is needed, however, to test and quantify such dependence on cosmology.

We would like to emphasize that the antibias (in the amount predicted by the numerical studies) is actually needed to reconcile the otherwise very successful \(\Lambda\)CDM model with observed galaxy clustering at \(z = 0\) (e.g., Klypin, Primack, & Holtzman 1996; Cole et al. 1997; Jenkins et al. 1998a). Note that this requirement applies to the overall galaxy population represented in large galaxy surveys such as the CfA and APM; subpopulations of galaxies may exhibit significantly different clustering properties, as is indicated by the differences between clustering of spiral and elliptical galaxies (e.g., Guzzo et al. 1997), infrared- and optically selected galaxies (e.g., Hoyle et al. 1998), etc. In the simulation presented here, certain subsamples of halos are clustered more strongly than the overall population. For example, as shown by Gottlöber et al. (1998), the population of halos that lose mass at \(z < 1\) (the halos that are accreted and being tidally stripped by clusters) are actually biased at \(z = 0\) with respect to the matter distribution, as opposed to the strongly antibiased distribution of the entire halo population. The simulation used in our analysis was also used by Kolatt et al. (1998), who showed that interpretation of the high clustering amplitude of the high-redshift galaxies is not unique. The amplitude can be reproduced equally well when these galaxies are assumed to be located in high-mass halos or are assumed to have smaller masses but undergo a starburst due to a collision/merger with another galaxy. Recent studies by Blanton et al. (1998), Cen & Ostriker (1998), Kaufmann et al. (1998a, 1998b), and Diaferio et al. (1998) demonstrate the existence of age, luminosity, and color segregation of clustering in their models. Some differences in clustering properties of certain sub-samples from the overall population can therefore be expected.

Another observational requirement for the \(\Lambda\)CDM model is a scale-dependent bias (e.g., Jenkins et al. 1998a): the shape of the observed galaxy correlation function is rather different from the shape of the nonlinear matter correlation function; this scale dependency is also reproduced nicely in the simulations. Recent studies of galaxy power spectrum by Gaztañaga & Baugh (1998) and Hoyle et al. (1998) stress that the antibias is required at rather low wavenumbers. However, as we have noted above (see § 1) and have illustrated in Figure 3, the amount of small-scale antibias observed in the galaxy correlation function (Colin et al. 1999) is sufficient to produce the required antibias in the power spectrum at low wavenumbers. Indeed, the power spectrum of the halo distribution in our simulation matches nicely the power spectrum of the APM galaxies at all probed wavenumbers. The matter power spectrum, on the other hand, is different from the galaxy power spectrum at a very high \((> 5–10 \, \sigma)\) significance level. The existence of nonlinear and scale-dependent bias of the galaxy distribution may affect other analyses that either assume there is no bias
or that the bias is linear. These include estimates of the matter density \( \Omega \) from peculiar velocities and redshift distortions (Dekel & Lahav 1998; Pen 1998) and from the observed mass-to-light ratios in galaxy groups and clusters (Diaferio et al. 1998), estimates of the baryon density in the universe (e.g., Goldberg & Strauss 1998; Meiksin, White, & Peacock 1998), estimates of the cosmological parameters based on the joint analysis of galaxy redshift surveys, cosmic microwave background anisotropies, and high-redshift supernovae (e.g., Eisenstein, Hu, & Tegmark 1998), etc. In this respect, the lesson of the present analysis is that any use of the galaxy density field as a cosmological probe requires very careful testing and evaluation.

7. CONCLUSIONS

We presented results from a study of the origin and evolution of bias of the DM halo distribution in a large, high-resolution simulation using a low matter density, flat, CDM model with cosmological constant. The following conclusions can be drawn from the results presented in this paper.

1. The evolution of the power spectrum of the halo distribution is significantly slower than the evolution of the matter power spectrum at all (both linear and nonlinear) scales. The halo and matter power spectra also have significantly different shapes. The differences in shape and rate of evolution imply time- and scale-dependent bias of the halo distribution which is in qualitative agreement with the results of the correlation function analyses. We stress, however, that the scale dependence of the bias determined from the power spectrum \( b_P(k) \equiv [P_h(k)/P_m(k)]^{1/2} \) is different from the scale dependence of \( b_D(r) \equiv [\xi_h(r)/\xi_m(r)]^{1/2} \) because the two statistics are related via the Fourier transform (see §1). Put simply, \( b_P(k) \) cannot be obtained from \( b_D(r) \) by a naive substitution of variable \( r \propto 1/k \). At \( z = 0 \) the halo power spectrum in our simulation matches the observed power spectrum of the APM galaxies well.

2. Despite the differences in shape, the power spectra of both matter and halos exhibit a distinct “inflection point” at approximately the same wavenumber, corresponding to the scale of nonlinearity (i.e., the scale at which the power spectra begin to deviate significantly from the linear power spectrum). The inflection scale is \( \approx 0.15-0.2 h \) Mpc\(^{-1} \) and coincides with the inflection observed in the power spectrum of APM galaxies (Gaztañaga & Baugh 1998); therefore, we interpret the inflection in the APM power spectrum as the present-day scale of nonlinearity \( k_{NL} \). In the simulation analyzed here, \( k_{NL} \) evolves with time from \( \approx 1 h \) Mpc\(^{-1} \) at \( z = 5 \) to \( \approx 0.15-0.2 h \) Mpc\(^{-1} \) at \( z = 0 \). We should note that the distinct inflection point can be seen only in the real space power spectrum; the nonlinear amplitude of the redshift space power spectrum is strongly suppressed and the inflection in the power spectrum of both matter and halos is smoothed out (see Figs. 3 and 4 in Gottlöber et al. 1998).

3. The analytic fitting formula of Peacock & Dodds (1996), with only minor tuning, provides an excellent match to both the shape and evolution rate of the matter power spectrum in our simulation. The latter probes deep into the nonlinear regime, down to wavenumbers of \( \approx 200 h \) Mpc\(^{-1} \) (at \( z = 0 \)); we find that clustering of DM in the highly nonlinear regime in the simulation is approximately stationary (stable clustering).

4. In addition to \( b_P \), we examine the evolution of bias defined using smoothed halo and matter overdensities (\( \delta_h \) and \( \delta_m \)) as \( b_D = \delta_h / \delta_m \). In agreement with results from the correlation function and power spectrum analyses, we find that \( b_D \) is nonlinear (i.e., depends on \( \delta_m \)) and time dependent. If we modify the model and assume that halos can retain their identity for a finite period of time after their formation and distinguish between formation and observation epochs \( z_f \) and \( z_o \), the analytic model of Mo & White (1996) can describe both the nonlinearity and evolution of \( b_D \) at linear and quasi-linear overdensities (\( \delta_m \lesssim 5-7 \); here and below the overdensities are smoothed with the top-hat smoothing radius of \( 5 h^{-1} \) Mpc). The original model \( (z = z_f) \) significantly underestimates the bias of galaxy-size halos at \( \delta_m \gtrsim 1 \).

5. We argue that at nonlinear overdensities the bias evolution is significantly affected by dynamical friction and tidal stripping of halos in groups and clusters. Both processes tend to reduce the number density of cluster halos of a given mass range, thereby reducing the bias and resulting in antibias at \( z \lesssim 0.5 \) at \( \delta_m \gtrsim 5 \) in the LCDM model studied here. The effect of dynamical friction depends sensitively on the cluster formation history, which introduces a certain degree of scatter into the bias \( b_D \).

In summary, the evolution of the bias of galaxy-size halos observed in the simulation in linear and quasi-linear regimes can probably be fully described using the extended PS theory. In other words, the evolution of bias in this regime results from an interplay between halo formation and merging rates in different regions and in the field. In the nonlinear regime, the halo bias evolution appears to be driven by the dynamical processes inside clusters and groups. Thus, despite the apparent complexity, we believe that the origin and evolution of bias can be understood in terms of the processes that drive the formation and evolution of DM halos and galaxies that they host: collapse from the density peaks, merging, tidal stripping, and morphological transformation in the high-density regions. Our results show that these processes may conspire to produce a halo distribution quite different from the overall distribution of matter, yet remarkably similar to the observed distribution of galaxies. This result implies that detailed modeling of the small-scale galaxy clustering requires a good understanding of galaxy evolution in clusters. We would like to emphasize, therefore, the importance of further efforts in modeling galaxy evolution in both clusters and in the field.

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