Renormalization-group running of the cosmological constant and the fate of the universe

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For a generic quantum field theory we study the role played by the renormalization-group (RG) running of the cosmological constant (CC) in determining the ultimate fate of the universe. We consider the running of the CC of generic origin (the vacuum energy of quantum fields and the potential energy of classical fields), with the RG scale proportional to the (total energy density)\(^{1/4}\) as the most obvious identification. Starting from the present-era values for cosmological parameters we demonstrate how the running can easily provide a negative cosmological constant, thereby changing the fate of the universe, at the same time rendering compatibility with critical string theory. We also briefly discuss the recent past in our scenario.

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Recent Type Ia supernovae [1] and the CMB data [2] show that our universe is spatially flat (\(\Omega_k = 0\)) and accelerating at present time. Specifically, the experimental situation at present is such that the present energy density consists of approximately 1/3 of ordinary matter and 2/3 of dark energy with a negative pressure. For this dark energy to accelerate the expansion, its equation of state \(w = p/\rho\) must satisfy \(w < -1/3\).

The most promising candidate for this cosmic acceleration and dark energy seems to be a “true” cosmological constant (CC) \(\Lambda\) (with \(\Lambda > 0\)), for which the equation of state is simply \(w = -1\). Another interpretation of dark energy relies on the existence of an ultralight scalar field \(\phi\) (“quintessence”) [3], in which case the dark energy density is the result of the scalar field slowly evolving along its effective potential \(V(\phi)\), with \(w \approx -1\). Some alternate explanations for the present acceleration beyond dark energy include theories in more than four dimensions [4] as well as the Chaplygin gas [5].

If the presently dominating dark energy and the acceleration of the universe are due to the “true” CC or “quintessence”, the expansion will keep accelerating perpetually, rapidly approaching a de Sitter (dS) regime. This implies a fate for open (\(\Omega_k > 0\)) and flat (\(\Omega_k = 0\)) universes in which the scale factor expands exponentially, \(a \sim e^{Ht}\), for an indefinitely long time. The same is destined even for a closed universe, unless \(\Omega_\Lambda\) is too low so that the universe collapses before the CC becomes predominant. More importantly, an eternal expansion and acceleration would exhibit future event horizons, which presents a rather serious challenge to critical string theory [6]. Namely, the finite event horizon implies the existence of regions in space that are inaccessible to light probes, thereby preventing the definition of asymptotic states and therefore the construction of the conventional S-matrix description required for field theory. This implies the impossibility of formulating the S-matrix for strings, which are by definition theories of the on-shell S-matrix, in such backgrounds. A noncritical string theory, characterized by a “graceful exit” from the inflationary phase, has recently been considered in [7].

Nonetheless, there are other models describing dark energy in which the dS phase is transient, thereby bringing back compatibility with string theory. One loophole providing an alternative to a perpetually accelerating universe is to have a potential \(V(\phi)\) which has a minimum at \(\phi_0\), with \(V(\phi_0) = 0\) and \(V''(\phi_0) > 0\). In this case, during the coherent oscillation phase, the energy density decreases as \(a^{-3}\) (\(w = 0\)), leading to a decelerating universe. In another scenario, \(V(\phi)\) has a minimum at \(V(\phi_0) = 0\) and then becomes flat for \(\phi > \phi_0\). After passing the minimum, the epoch of kinetic energy domination sets in where the energy density scales as \(a^{-6}\) (\(w = 1\)), resulting again in a decelerating universe. Nonetheless, the simplest case is to have a negative CC. In this case, no matter the universe is open, flat, or closed, or how extremely tiny the CC is, it collapses eventually, thereby bypassing the troublesome problems with the event horizon. As a way of example, we mention a mechanism to generate a negative CC in [8] (for supergravity theories, see, e.g., [9]).

In the present paper, we expand our previous work [10] on the RG running of the CC in the standard model, to a generic field theory model. We then study the future of the universe in such a generic scenario. Recent works [10, 11, 12] show that even a “true” CC cannot be fixed to any definite constant (including zero), owing to RG running effects. Particle contributions to the RG running of \(\Lambda\) due to vacuum fluctuations of massive fields have been properly derived in [10], with a some-
what unexpected outcome that more massive fields do play a dominant role in the running at any scale. If our chosen value for the RG scale \( \mu \) is always below the lowest mass in the underlying quantum field theory, then the scaling evolution of the CC \( \Lambda(\mu) \) towards \( \mu \to 0 \) is essentially given only by two parameters which depend on the field content (particle masses as well as the total number of massive degrees of freedom including the \( +/- \) sign for bosons/fermions). The same structure of the scaling behavior is also expected for \( \Lambda \) arising from vacuum condensates of scalar fields \([10]\), in which case there is an additional dependence of the parameters on the particle interaction. We show that even if we start with a positive \( \Lambda \) in a flat universe, the scaling behavior may naturally be such as to provide a negative \( \Lambda \) for an indefinitely long time, thereby changing the ultimate fate of the universe.

We have already mentioned the possibility that a flat universe may collapse in the future. In most models of dark energy based on extended supergravity, this occurs in the nearby future, within the next few billion years \([13]\). Although provided with a negative CC for an indefinitely long time, our scenario allows quite a distinctive feature: the universe never collapses, starting to decelerate relatively soon and increasing the scale factor to its maximum value in infinite time.

In the formalism of quantum field theory in curved space-time, the cosmological constant receives divergent contributions originating from zero-point energies of the quantum fields considered. Being one of the parameters in the Lagrangian, the cosmological constant undergoes renormalization (for earlier works on the subject see e.g. \([14]\)). After the appropriate choice of the renormalization scheme which correctly comprises the effects of mass thresholds \([10]\), one can formulate the renormalization-group equation in the form

\[
(4\pi)^2 \mu \frac{\partial \rho_\Lambda}{\partial \mu} = \sum_i \frac{I_i}{2} \frac{\rho_\Lambda}{\mu^2 + m_i^2},
\]

where \( \mu \) is the renormalization scale, \( \rho_\Lambda \) represents the energy density attributed to the cosmological constant, and \( i \) is an index of any massive degree of freedom (corresponding to the quantum field), both fermionic and bosonic. The quantity \( I_i \) acquires the value +1 for bosonic and \( -1 \) for fermionic degrees of freedom. It is clear that more massive fields dominate \([11]\) at any scale and this departure from the Appelquist-Carazzone theorem \([15]\) results from the \( \text{(mass)}^4 \) dimensionality of \( \rho_\Lambda \) \([10]\). The following comment is in order. It is the high dimensionality of the CC energy density that causes the failure of the decoupling theorem: the peculiar mass-dependent renormalization scheme we used in \([10]\) in order to renormalize the zero-point energy agrees with the \text{MS} scheme in the high-energy limit. The result is furthermore supported by the failure of the decoupling theorem in the case of vacuum condensate of the Higgs field \([10]\) where the mass-dependent renormalization scheme is unambiguous \(^1\).

In the approach described above, the particle zero-point energies are taken as a generic contribution to the running of the cosmological constant. The results for the running are obtained at the one-loop level. In the framework we used, quantum field theory of matter (particles) is defined on the curved classical background which is described through the vacuum action

\[
S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + \int d^4x \sqrt{-g} [a_1 R_{\mu\nu\rho\sigma} + a_2 R^2_{\mu\nu} + a_3 R^2 + a_4 \Box R]\]

This equation, however, does not include the possible contributions of quantum gravity. As discussed in \([12]\), one could safely neglect the gravity effects related to all terms in \([23]\) proportional to \( a_i \), and the nonminimal term \( \xi^4 \Phi^4 \Phi \) proportional to \( a_4 \).

Although it is presently well known that there are tremendous difficulties in the formulation of quantum field theory of gravity, it is still possible to renormalize the nonrenormalizable theories in the framework of effective field theories \([17]\). A successful example of such a theory is a chiral perturbation theory as a low-energy nonlinear realization of QCD. Gravity is potentially even a better effective theory, since quantum corrections appear to be rather small, persisting so up to the Planck scale. It is then possible, using the effective field theory to calculate the effects of quantum gravity which emerge from the low-energy part of the theory without knowing the "true" quantum field theory of gravity \([18]\).

What really makes the effective field theory of gravity different from other effective theories is the eventual existence of singularity \([19]\) in the future - which, in the limit of extremely low energy (corresponding to the wavelength probed of the order of the size of the universe), breaks the validity of the theory in the infrared limit. For example, conformal invariance breaking (which is also broken by gravitons!) causes infrared instability in de Sitter space \([20]\). Therefore quantum gravity may lead to strong renormalization effects at large distances owing to the infrared divergences \([21]\).

Recently, Bonanno and Reuter \([22]\) derived RG equations with an infrared cutoff which at a given scale stops the running of \( \Lambda \) in the infrared. The basic framework is the effective average action \( \Gamma_k[g_{\mu\nu}] \), which is a Wilsonian coarse grained free energy. It depends on a momentum

\(^1\) Recently, Gorbar and Shapiro \([17]\) studied the form of decoupling for the CC energy density in quantum field theory in the external gravitational field on a flat background. The results fail to reproduce a correct high-energy behavior as given in the \text{MS} scheme, and, therefore, cannot be conclusive.
scale $k$, and defines an effective field theory appropriate for the scale $k$. An additional hypothesis in the formulation of the RG behavior of $\Lambda$ and $G$ is that, in the limit $k \to 0$, both $\Lambda(k)/k^2$ and $k^2G(k)$ run into an infrared attractive non-Gaussian fixed point.

The scale $k$ plays a role of an infrared cutoff\textsuperscript{22}. Intuitively, an infrared cutoff is necessary since, for example, at the present time $t = t_0$ (at the Big Bang), one has to integrate quantum fluctuations down to the scale $k > \frac{1}{t_0}$; larger scales for which $k < \frac{1}{t_0}$ should obviously be truncated.

However, it is very difficult to imagine the precise physical mechanism which acts as a cutoff. It would depend on the problem under consideration and may include many scales including, e.

It is thus pretty obvious that the Shapiro-Sola choice (5) means a restriction to “soft” graviton momenta, whereas our choice (4) involves typical (or “hard”) graviton momenta.\textsuperscript{2} In turn, this implies that the choice (4) corresponds to CC’s that are nearly uniform in both space and time, thus being compatible with models which treat $\Lambda$ as an (almost) true constant.

Moreover, in the radiation dominated universe, the definition of the renormalization scale (5) coincides with the straightforward association $\mu \sim T$ mentioned above.

Next we discuss our choice of the scale, $\mu = \rho^{1/4}$, and compare it with other possible choices, especially with that proposed by Shapiro and Sola\textsuperscript{12}, $\mu \sim \sqrt{R} \sim H$, where $R$ is the curvature scalar and $H$ denotes the Hubble parameter.

Let us recall that in the decoupling theorem, it is the momentum of particles entering a quantum loop, and the mass of particles inside the loop, that are to be compared (with massless gravitons on external legs in the case of CC). On the other hand, the coupling constants play no role in the decoupling. It has been argued\textsuperscript{12} that the value of the curvature scalar $R$ can be taken as an order-parameter for the gravitational energy, i.e., $\mu \sim \sqrt{R}$. The Einstein equations imply that, at present,

\[ \mu_0 \sim R_0^{1/2} \sim (G\rho_0)^{1/2} \sim H_0, \tag{6} \]

where $G$ denotes the Newton’s constant. According to the choice (5), the relevant scale cannot be less than that of typical momenta of background gravitons, which, at present, are of the order of $10^{-4}$ eV and not of $H_0 \approx 10^{-33}$ eV. We notice that the large separation of the scales (5) and (6) is actually due to a coupling of gravitons to ordinary matter ($\sim GE^2$ with $E$ being a typical graviton energy), inherently present in the expression (6). Indeed, one finds that the ratio of scales (5) and (6) is essentially given by the square root of the graviton coupling constant. It is thus pretty obvious that the Shapiro-Sola choice (5) means a restriction to “soft” graviton momenta, whereas our choice (4) involves typical (or “hard”) graviton momenta.\textsuperscript{2} In turn, this implies that the choice (4) corresponds to CC’s that are nearly uniform in both space and time, thus being compatible with models which treat $\Lambda$ as an (almost) true constant. In this case, $\Lambda$ acts as a source of gravitons with extremely large wavelengths, of the order $H_0^{-1}$. Moreover, with the choice (4), the running of the CC on cosmological scales is essentially negligible. On the other hand, adopting the choice (5), one arrives at a model with the pronounced running of $\Lambda$, comparable with that of ordinary matter. This feature is clearly visible in our figures, see below.

Let us in more detail discuss our advocacy for the scale as given by (5). First of all, we would like to have the correct limit towards the past radiation epoch, where all relevant particle scales are essentially given in terms of the temperature of interacting particle species. Obviously, this is not possible with the choice (6). In addition, in our approach the energy-momentum tensor $T_{\mu\nu}$ is no
longer separately covariantly conserved, but only is the
sum $T_{\mu \nu} + \frac{1}{2} \nabla_{\mu} \nabla_{\nu}$ (see (11) below). This implies that
there exists an interaction between matter and the CC
which causes a continuous transfer of energy from matter
to the CC or vice versa, depending on the sign of the in-
teraction term. We shall see below that in our approach
the CC and matter are always tightly coupled, with a
consequence that the CC actually behaves more like ordi-
nary matter. A similar distinction of scales can be found
only for the CC with a negligible interaction with ordi-
nomogeneity and isotropy, otherwise new scales much larger
than the cosmological one need be introduced. Finally,
there exists an interaction between matter and the CC
which causes a continuous transfer of energy from matter
to the CC actually behaves more like ordi-

mological constant energy density by the first two terms in the expansion (3)

$$\rho_\Lambda = A \mu^2 + B \mu^4,$$

which together with the definition of the cosmological energy scale (5) gives

$$\rho_\Lambda = A \rho^{1/2} + B \rho.$$ (8)

We have shown in the previous paper (10) that also for
the vacuum-condensate contribution to $\Lambda$ one should ex-
pect the same scaling behavior of $\rho_\Lambda$ as in (7). Hence, one
may consider (7) a scaling behavior for generic $\Lambda$, where,
besides the definition in (5), the coefficients $A$ and $B$ now
take on an extra dependence on the particle interaction.
In order to round up the survey of our cosmological model
there remains to identify other components of the total
energy density apart from the cosmological constant en-

ergy density. In a general manner, we express the total
energy density and pressure of the content of the universe
in a form

$$\rho = \rho_\Lambda + \rho_m,$$ (9)

$$p = \rho_\Lambda + p_m,$$

where we have used the symbols $\rho_m$ and $p_m$ to specify
the density and pressure of all other components of the
universe, apart from the dark energy ones. Thus we as-
sume that the running cosmological constant is the only
source of the present acceleration of the universe, i.e.,
the only dark energy component.

In order to couple the cosmological term (being a dy-
namical quantity) with the Einstein field equations

$$G_{\mu \nu} + \Lambda g_{\mu \nu} = -8\pi G T_{\mu \nu},$$ (10)

we completely adopt the prescription outlined in a re-
view paper (26) to move $\Lambda$ to the right-hand side of (10),
interpreting it, at the same time, as a part of the mat-
ter content of the universe, rather than a pure geomet-
rical entity. After doing so $^3$, it is the effective energy-
momentum tensor $T_{\mu \nu} + \frac{1}{2} \nabla_{\mu} \nabla_{\nu}$ that has the perfect fluid

$^3$ If one rather prefers to keep a variable $\Lambda$-term on the left-hand side of (10), then introduction of some additional terms into (10)
is necessary, which are functions of the scalar in a tensor-scalar theory.
form and therefore satisfies energy conservation:
\[ a \frac{d \rho}{da} + 3(\rho_m + p_m) = 0. \]  
(11)

The equation of state obeyed by the cosmological term is then of the familiar form, \( p_A = -\rho_A \), and that obeyed by ordinary matter may have a general form \( p_m = w \rho_m \) (although in our analysis we restrict ourselves to the pressure-free Friedmann models with \( w = 0 \)). Note from \[ \[ \] \] that \( \rho_m \) scales in a familiar way, \( \rho_m \sim a^{-3} \), when \( \Lambda = \text{constant} \) as well as \( w = 0 \).

Furthermore, we introduce a condition (at \( a(0) = a_0 \)) coming from present observational results on the share of dark energy in the total energy density at the present epoch:
\[ \rho_A(a = a_0) = \Omega^0_\Lambda \rho_0, \]  
(12)

with \( \Omega^0_\Lambda \approx 0.7 \). The condition given above determines the parameter \( A \) in \[ \[ \] \]
\[ A = (\Omega^0_\Lambda - B)\rho_0^{1/2}. \]  
(13)

With substitutions \( u_\rho = \rho/\rho_0 \) and \( y = \ln(a/a_0) \), Eq. (11) acquires the form
\[ \frac{du_\rho}{dy} + 3(1 + w)((1 - B)u_\rho - (\Omega^0_\Lambda - B)u_\rho^{1/2}) = 0, \]  
(14)

while the initial condition is \( u_\rho(y = 0) = 1 \). This equation has the following solution:
\[ u_\rho = \frac{\rho}{\rho_0} = \left[ \frac{\Omega^0_\Lambda - B}{1 - B} + \frac{1 - \Omega^0_\Lambda}{1 - B} \left( \frac{a}{a_0} \right)^{-\frac{1}{2}(1-B)(1+w)} \right]^2. \]  
(15)

The expressions for energy densities \( \rho_A \) and \( \rho_m \) are
\[ \rho_A = \rho_0 \left[ \frac{1}{1 - B} \left( \frac{\Omega^0_\Lambda - B}{1 - B} \right)^2 \right. \]
\[ + \left( 1 + B \right) (\Omega^0_\Lambda - B)(1 - \Omega^0_\Lambda) \left( \frac{a}{a_0} \right)^{-\frac{1}{2}(1-B)(1+w)} \]
\[ + B \left( 1 - \Omega^0_\Lambda \right)^2 \left( \frac{a}{a_0} \right)^{-3(1-B)(1+w)} \],
(16)

\[ \rho_m = \rho_0 \left[ \frac{1}{1 - B} \left( \frac{(\Omega^0_\Lambda - B)(1 - \Omega^0_\Lambda)}{1 - B} \right) \left( \frac{a}{a_0} \right)^{-\frac{1}{2}(1-B)(1+w)} \right. \]
\[ + \left( 1 - \Omega^0_\Lambda \right)^2 \left( \frac{a}{a_0} \right)^{-3(1-B)(1+w)} \].
(17)

In the special case \( B = 1 \), the solution of \[ \[ \] \] has the form
\[ u_\rho = \frac{\rho}{\rho_0} = \left[ 1 - \frac{3 - 2}{2(1 - \Omega^0_\Lambda)}(1 + w) \ln \left( \frac{a}{a_0} \right) \right]^2. \]  
(18)

The expressions for energy densities \( \rho_A \) and \( \rho_m \) in the case \( B = 1 \) are
\[ \rho_A = \rho_0 \left[ \Omega^0_\Lambda - 3 - 2(1 - \Omega^0_\Lambda) \Omega^0_\Lambda (1 + w) \ln \left( \frac{a}{a_0} \right) \right. \]
\[ + \left( \frac{1}{4} - \Omega^0_\Lambda \right)^2 \ln \left( \frac{a}{a_0} \right)^2 \],
(19)

\[ \rho_m = \rho_0(1 - \Omega^0_\Lambda) \left[ 1 - \frac{3}{2} (1 - \Omega^0_\Lambda) (1 + w) \ln \left( \frac{a}{a_0} \right) \right]. \]
(20)

The definition of the cosmological energy scale requires that \( u_\rho^{1/2} \geq 0 \). This requirement for values \( B > \Omega^0_\Lambda \) leads to the existence of the maximal value of the scale factor. In general, the expression for the largest value of the scale factor \( a_{\text{max}} \) is
\[ a_{\text{max}} = a_0 \left( \frac{B - \Omega^0_\Lambda}{1 - \Omega^0_\Lambda} \right)^{-\frac{1}{3(1 - \Omega^0_\Lambda)(1 + w)}}, \]  
(21)

while for the special case \( B = 1 \), we find
\[ a_{\text{max}} = a_0 e^{\frac{2}{3(1 - \Omega^0_\Lambda)(1 + w)}}. \]  
(22)

\[ \text{FIG. 1: The dependence of the energy density components of the universe (in units of } \rho_0 \text{) on the scale parameter } a \text{ for the parameter values } B = 0.6, \Omega^0_\Lambda = 0.7, \text{ and } w = 0. \text{ For large values of the scale factor, } \rho_m \text{ tends to zero, whereas } \rho_A \text{ approaches } \rho \text{ and tends to a constant nonzero value. The hierarchy } \rho_m < \rho_A < \rho \text{ is maintained for all values of the scale factor.} \]
Finally, we can incorporate the laws of change of the total energy density with the scale factor into the Friedmann equation for the flat universe ($k = 0$):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho.$$  \hfill (23)

Solving the Friedmann equation, we obtain the law of the evolution of the scale factor

$$a = a_0 \left[ \frac{1 - B}{\Omega^0_\Lambda - B} e^{\frac{2}{3} H_0(t,t_0)(\Omega^0_\Lambda - B)(1+w)} - \frac{1 - \Omega^0_\Lambda}{\Omega^0_\Lambda - B} \right].$$  \hfill (24)

For $B = \Omega^0_\Lambda$, the $a(t)$ law has the form
tends to interesting consequences for the event horizon for some finite interval of time starting now). The logical energy scale generally acceptable (and not only is nonnegative which makes our definition of the cosmological energy scale generally acceptable (and not only for some finite interval of time starting)

\[ a = a_0 \left[ 1 + \frac{3}{2} H_0 (t - t_0) (1 + w) (1 - \Omega_\Lambda^0) \right]^{\frac{2}{3(1 - \Omega_\Lambda^0) (1 + w)}}, \]

while for \( B = 1 \), we obtain

\[ a = a_0 e^{\frac{3}{2} H_0 (1 - \Omega_\Lambda^0) (1 + w) (t - t_0)}. \]

An important feature of the evolution for \( B > \Omega_\Lambda^0 \) is that, although there exists a maximal value of the scale factor, it takes the universe infinitely long to reach it. Moreover, for any finite time the total energy density \( \rho \) is nonnegative which makes our definition of the cosmological energy scale generally acceptable (and not only for some finite interval of time starting now).

The cosmological model described above also has interesting consequences for the event horizon \( d_E(t_0) = a_0 \int_{t_0}^{\infty} \frac{dt'}{a(t')} \).

In the case \( B = \Omega_\Lambda^0 \), we can give the expression for the event horizon in the closed form

\[ d_E(t_0) = \frac{2}{2 - 3(1 - \Omega_\Lambda^0) (1 + w)} \frac{1}{H_0}. \]

Numerical integration for the values of the parameter \( B \leq \Omega_\Lambda^0 \) gives the finite values for the event horizon as depicted in Fig. 6. On the other hand, the event horizon for the parameter values \( B > \Omega_\Lambda^0 \) diverges. This can be easily seen from the fact that in this parameter range the scale factor is bounded from below and from above:

\[ a_0 < a(t) < a_{\text{max}}, \]

which leads to

\[ a_0 \int_{t_0}^{\infty} \frac{dt'}{a(t')} > d_E(t_0) > a_0 \int_{t_0}^{\infty} \frac{dt'}{a_{\text{max}}}. \]

As both the upper and the lower bound in the above inequality diverge, the event horizon is infinite in this parameter range.

Let us now discuss the main features of our scenario in a more detail. As shown in our figures 1-6, several different outcomes concerning the fate of the universe as well as the scale factor evolution are possible, depending, in essence, on how the parameter \( B \) is related to \( \Omega_\Lambda^0 \).

Once \( B \) is picked, the parameter \( A \) is given by (13). Our treatment of \( A \) and \( B \) as free parameters means that our analysis is, on the whole, model independent (i.e., we do not restrict ourselves to a given particle theory). Let us mention, for instance, that considering zero-point energies only, we have \( B = \frac{1}{(1 + w)^2} \) in the standard model (treating neutrinos as massless).

First, consider case \( B < \Omega_\Lambda^0 \), as depicted in fig. 4. We see that \( \rho_\Lambda \) approaches a positive constant \( \rho_0 \left( \frac{\Omega_\Lambda^0 - B}{1 - B} \right)^2 \) when \( a \to \infty \), whereas \( \rho_m \) approaches zero in the same limit. This means that the universe once entering the dS regime, stays there for an indefinitely long time, see fig. 5. In this case, the running of \( \rho_\Lambda \) plays no essential role, since it is the asymptotic behavior of \( \rho_\Lambda \) and \( \rho_m \) that matters in determining the ultimate fate. Of course, the
The event horizon is finite, see fig. [7]. Next, consider the limiting case \( B = \Omega^0 \) depicted in fig 2. Now both \( \rho_{\Lambda} \) and \( \rho_{m} \) tend to zero when \( a \to \infty \). The hierarchy \( \rho_{m} < \rho_{\Lambda} \) preserved in the whole running is crucial for the ultimate fate, which turns out to be the same as in the preceding case, see fig. [6]. The event horizon is also finite, see fig. [7]. Now we consider the most interesting case \( B > \Omega^0 \) as depicted in figs. [6, 7]. In all these cases, the universe left the deS phase relatively soon, when \( a \sim a_0 e^{0.5 - 1.5} \) (depending on \( B \)), then starting to decelerate. The reason for this is the change in hierarchy from \( \rho_{\Lambda} > \rho_{m} \) to \( \rho_{m} > \rho_{\Lambda} \). Soon afterwards, when \( a \sim a_0 e^{1 - 2.5} \) (depending on \( B \)), \( \rho_{\Lambda} \) becomes negative, staying such forever. We recall that for the “true” \( \Lambda \) \((\Lambda < 0)\), the universe recollapses unavoidably. The reason why we have a different fate here is that the established hierarchy, \( \rho_{m} > |\rho_{\Lambda}| \), is preserved also asymptotically, where both components tend to zero energy density. Stated differently, since \(|\rho_{\Lambda}| \) goes faster to zero than \( \rho_{m} \) when \( a \to a_{\max} \), the former component never has a chance to start dominating, a feature crucial for having recollapse within a finite time interval. Finally, we need to explain why \( a_{\max} < \infty \) in this case, see fig. [6].

Stating formally, the recollapse here occurs at infinity (i.e., for \( t = \infty \)), when \( |\rho_{\Lambda}| \) finally reaches \( \rho_{m} \). Since any recollapse (starting whenever) requires \( a_{\max} < \infty \), we have \( a(t = \infty) = a_{\max} < \infty \), as seen from fig. [6] and fig. [6]. The matter dominance in the asymptotic regime, helped with the influence of negative \( \Lambda \), makes the event horizon diverge, thus alleviating the problems with strings, see fig. [4].

The results on the evolution of the universe presented in this paper refer to the case of a flat universe \((k=0)\). For this case, it is possible to obtain analytical expressions for the time evolution of the scale factor \( a(t) \). In the cases of open \((k=-1)\) or closed \((k=+1)\) universes, the curvature effects may become important in the considerations of the asymptotic evolution of the scale factor. In these cases, however, it is not possible to maintain the same level of analytical tractability.

Finally, one would like to know if the above scenario is capable of solving the well-known problems that one encounters when explaining dark energy with a “true” CC. The first problem is to ensure a phase of inflation, an epoch when vacuum energy dominated other forms of energy density, characterized by \( \rho_{vac} \gg \rho_{\Lambda}^0 \). Let us also mention the so-called coincidence problem, which requires an extremely large hierarchy between \( \rho_R \) and \( \rho_{\Lambda} \) at the epoch just after the inflation (contrary to the expectation from equipartition), in order to correctly reproduce \( \rho_{\Lambda} \) today, with \( \Lambda = constant \). Since both of the problems are addressed by “quintessence”, one may wish to know whether the above scenario could mimic some of quintessence models. This amounts to study the running \( \rho_{\Lambda} \) backward into a distant past. Whereas at extremely low energies we could keep only the first two terms in the expansion of \( \rho_{\Lambda} \) and perform the analysis without referring to any particular model, this is no longer possible at high energies. Now we need all peculiarities of the underlying particle theory: particle masses, all relevant interactions, symmetry breaking properties, etc. For instance, the description of the running of the cosmological constant energy density at high scales \( \mu \) requires the knowledge of the number and individual masses of the heaviest degrees of freedom (and not just a small number of specific combinations of these parameters like coefficients \( A \) and \( B \)). Such complete information is currently not available even for the simplest theories, like the standard model. There is an additional technical complication in the region of high scales. Namely, one has to take into account the contributions coming from mass scales differing many orders of magnitude, which hinders a straightforward numerical treatment. In addition, higher-loop contributions together with the running of masses and couplings, all of which we have obviously disregarded in (1), may no longer be irrelevant at high energies. We feel, though, that it would certainly be of importance to address the coincidence problem even in a toy model, in view of the obvious connection between \( \Lambda \) and \( \rho_{m} \) as expressed in (8). Since our examples show a clear tendency for \( \rho_{\Lambda} \) to grow faster than \( \rho_{m} \) for \( t < t_0 \), one may object that there is no room in our scenario to explain the discovery of the supernova SN 1997ff, which is believed to signal a switch from cosmological acceleration to deceleration [27]. Once again we must stress that this potential drawback could result from our ignorance of the full theory. We recall [10] that the smallness of the parameter \( A \) requires a fine-tuned relation among particle masses of the underlying theory. One may therefore imagine that the running of masses, although believed to be weak at such energies and therefore ignored in (1), could easily change the sign of \( A \), thereby depleting \( \rho_{\Lambda} \) at higher energies.

As a last step, it would be very useful to compare the effects originating from the matter sector (treated in this paper) with quantum gravity effects on the running of the cosmological constant. Firstly, we could interpret the assumption \( \rho_{\Lambda}(0) = 0 \) as an effective inclusion of the effects of quantum gravity in our model since within our model there is no mechanism or principle that would dictate vanishing of \( \rho_{\Lambda} \) at \( \mu = 0 \). This choice certainly represents a rather crude approximation of the effects of quantum gravity, but, in our view, at the same time single out the most important of these effects. Secondly, in the absence of complete understanding of quantum gravity effects, especially in the infrared region of the scale \( \mu \), it is not possible to say which type of effect (matter sector or quantum gravity) would dominate the running of the cosmological constant energy density at infrared scales. Still, our findings of significant (and possibly sign-changing) running of the cosmological constant energy density coming from the matter sector make reasonable the assumption that quantum gravity effects (apart from those setting \( \rho_{\Lambda}(0) = 0 \)) do not significantly change the running of the CC and the respective consequences on the evolution of the universe. We see our model as complementary to quantum gravity approaches, especially to...
those which draw conclusions from the pure gravity theories assuming that the matter sector does not alter the pattern from the pure gravity theory. Our results show that one cannot a priori expect the effects of the matter sector to play only a secondary role. We expect that only a full solution of quantum effects in gravity along with a careful treatment of contributions coming from the matter sector will resolve the problem of the running of the cosmological constant unambiguously.

In conclusion, we have studied the renormalization-group running of the cosmological constant at scales well below the lowest mass in a generic particle theory. Having specified the renormalization-group scale in cosmological settings, we considered the ultimate fate of the universe in such a scenario. As its main feature, our scenario naturally provides the possibility that the universe endowed with a positive cosmological constant nowadays, necessarily exits a de Sitter regime a few Hubble times afterwards. More surprisingly, although such a universe exhibits the maximum value of the scale factor, it never recollapses. Although the running into past times cannot be provided in a model-independent manner, the relation between $\rho_\Lambda$ and $\rho_m$ implied in (8) indicates that, treated in a specific particle physics model, our approach might also contribute to the resolution of the coincidence problem.

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