Analysis focused image formation in double exposure speckle photography

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Abstract. This paper presents the theoretical and experimental results of the analysis of the formation of the focused image in a double-exposure speckle photography. Theoretical analysis of the formation of the focused image in double-exposure speckle photography was carried out using the Vander Lugt method. It is shown that as a result of the summation of speckle structures displaced in the space of the photographic plates, moiré fringes are formed in double-exposure speckle photography. Experimental results confirming theoretical studies are presented.

1. Introduction

Double-exposure speckle photography is a simple and universal method for measuring displacements and strains in the plane of diffusely scattering objects. Speckle interferometry, due to the low demands on the stability of the optical system in comparison with holographic interferometry, is becoming widespread for use in industrial environments. These experimental methods have high metrological characteristics, provide the ability to measure the deformed state on the entire surface of the study object, and have a high level of automation [1-4].

The essence of the method is to illuminate a rough surface by coherent light. In this case, each point on the surface is a point source of secondary waves with a different phase. Due to the interference of these secondary waves in space the objective speckle pattern is formed, which is associated with the surface characteristics (static and dynamic deformations) of the investigated object. A speckle pattern formed from a diffusely scattering a rough surface in the image plane of an optical system is called the subjective, which is also associated with the surface characteristics (static and dynamic deformations) of the investigated object. When the rough surface area is moved or deformed, the observed speckle pattern in a space or the image plane will also move accordingly.

In the double-exposure speckle photography the speckle pattern is recorded on one photographic plate in two states of the investigated object – in unloaded and loaded. To measure the displacements of the studied surface coherent optical processing of the photographic plate is performed. As a result interference fringes are formed that are associated with these displacements. One of the simplest coherent optical methods is the Young's method. In this case, a photographic plate with the recoded double-exposure specklegram is illuminated by a narrow laser beam. The distribution of the intensity of light diffracted on the specklegram in the Fraunhofer plane is Young's fringes pattern. This result is interpreted by the well-known two-point experiment of Young. In this case, two point coherent light sources interfere, i.e. two identical speckles. One speckle corresponds to the unloaded state of the
studied object, and the second speckle corresponds to the loaded state of the studied object. The width and inclination of the Young's fringes pattern determine the direction and value of the displacement of the area of the studied surface selected by a narrow laser beam [1-4].

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Theoretical studies of the principles of operation of optical devices have shown that increasing resolution of optical systems, i.e. overcoming is possible on the basis of apodization [6]. Researches the possibility of increasing resolution of optical systems in the presence of aberration based on amplitude apodization was performed in articles [7, 8]. In articles [9-11] analysis the possibility of reducing the lateral size and increasing the longitudinal size of a high-aperture focal system focus using a vortex phase transmission function for different types of input polarisation (including the general vortex polarisation) was performed. In article [12] was shown that the size of speckle grain has an important influence on the quality of speckle pattern.

In article [13-16] was shown that the use of ring apertures in speckle interferometry increases the sensitivity of the method, widens the range of measured displacements, improves the quality of interference fringes when reconstructed by the Young's method, which leads to an increase in the measurement accuracy. This is due to the fact that when using a ring aperture, the size of subjective specks decreases compared to the open aperture. In addition, the use of a ring aperture allow to reduce the influence the aberrations of optical system on the formation of a focused image.

In all early works [1-4, 17] there is no complete analysis of the formation of a focused image in a double-exposure speckle photography, since the main task to obtain the final result by coherent optical processing of the photographic plate for decryption of the recorded information.

Thus, the aim this work is a more detailed theoretical and experimental study of the focused image formation in double-exposure speckle photography with ring aperture and what results can be additionally obtained from this analysis.

2. The calculation of the intensity distribution of the focused image in two-exposure speckle photography

Let us consider the formation of a focused image in the double-exposure speckle photography with diffuse reflection of laser radiation from the surface of an object. On the photographic plate, the sum of the light intensities is recorded when the radiation is reflected from two states of the object: the initial one and offset by the value \( L \) under the action of the load.

The optical recording scheme of the focused image of the investigated object is shown in figure 1.

The investigated object is located in the \( x_1, y_1 \) plane, the lens and ring aperture are in the \( x_2, y_2 \) plane, and the photographic plate is in the \( x_3, y_3 \) plane. The coordinate system is chosen so that the model is displaced during the second exposure only along the \( x \) axis. This choice greatly simplifies the calculation. To describe the passage of a wave through an optical system, we use the method proposed by Vander Lugt [18]. For this purpose, the function is used:

\[
\psi(x, y, \rho) = e^{-ikp\left(x^2 + y^2\right)},
\]  

(1)
where $x$, $y$ are spatial coordinates; $p = 1/d$ ($d$ is the distance over which the wave propagates); $k = 2\pi/\lambda$, ($\lambda$ is the wavelength of the laser radiate on).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image}
\caption{Optical recording scheme for focused speckle photography.}
\end{figure}

The wave that has passed the distance $d$ in space is described by a convolution of complex amplitude and expression: $i \frac{k}{2} D \psi(x, y, D)$, where $D = \frac{f}{d}$.

To describe the passage of a wave through a spherical lens with a focal length $f$, the complex amplitude of the light incident on the lens must be multiplied by the function $\psi(x, y, f)$. The asterisk denotes the complex conjugate, $F = 1/f$.

We apply the Vander Lugt method for analyzing the recording of double-exposure speckle photography using the optical system shown in figure 1.

At the first exposure, the object is in an unloaded state. The diffuse surface illuminated by laser radiation can be represented as consisting of uniformly located point sources of secondary waves of the same kind, but with a different phase. Then the wave reflected from the object can be written in the following form [19]:

$$E_1(x_1, y_1) = \sum_{n=1}^{N} a e^{i \mu_n} \delta(x_1 - u_n; y_1 - v_n),$$

where $N$ is the number of a point sources; $u_n$, $v_n$ – coordinates of the $n$-th point; $\mu_n$ – phase of the $n$-th point source; and $a$ – the amplitude of the $n$-th point source (with high accuracy, we can assume that the amplitude of all point sources is the equally).

The distribution of amplitudes on the left surface of the lens is determined by the Kirchhoff-Fresnel integral, i.e. convolution $E_1(x_1, y_1) \times i \frac{k}{2} D \psi(x_1, y_1, D)$:

$$E_2(x_2, y_2) = i \frac{k}{2} D \int P_1 E_1(x_1, y_1) \psi(x_2 - x_1; y_2 - y_1; D) dx_1 dy_1.$$

Multiplication by, which describes the transmission of a thin lens, gives the distribution of complex amplitudes on the right surface of the lens:

$$E_3(x_2, y_2) = E_2(x_2, y_2) \psi^*(x_2; y_2; F).$$

Calculating the convolution with the function, we obtain complex amplitudes in the plane of the photographic plate:

$$E_4(x_3, y_3) = i \frac{k}{2} D_2 \int P_2 E_3(x_2, y_2) \psi(x_3 - x_2; y_3 - y_2; D_2) dx_2 dy_2.$$

Using the properties of the function $\psi$ [18] the equation (5), we bring to a more convenient form:

$$E_4(x_3, y_3) = -\frac{k^2}{4} D_1 D_2 \sum_{n=1}^{N} a e^{i \mu_n} \psi(u_n, v_n, D_1) \psi(x_3, y_3, D_2) \times$$

$$\times \int_{P_1} \psi(x_2; y_2; D_1 - F + D_2) e^{i k (D_1 u_n + D_2 x_3)} x_2 e^{i k (D_1 v_n + D_2 y_3)} y_2 dx_2 dy_2.$$
This expression describes the formation of an image of the investigated object. Let’s consider image formation in the limits of the geometric optics. This condition recorded through the parameters of the optical system (figure 1) has the form:

\[
\frac{l}{d_1} + \frac{l}{d_2} = \frac{1}{f},
\]

or, in our symbols, \(D_1 + D_2 = F\). Then \(\psi(x_2; y_2; D_1 - F + D_2) = 1\) and equation (6) takes the form:

\[
E_4(x_3, y_3) = -\frac{k^2}{4} D_1 D_2 \sum_{n=1}^{N} a e^{i \mu n} \psi(u_n, v_n, D_1) \psi(x_3, y_3, D_2) \times
\]

\[
\frac{e^{ik(D_1 u_n + D_2 x_3)} x_2 e^{ik(D_1 v_n + D_2 y_3)} y_2}{P_1} dx_2 dy_2.
\]

The photographic plate is a square detector, i.e. it records the light intensity. Then the distribution of the light intensity in the plane of the photographic plate is written as follows:

\[
I_0 = |E_4(x_3, y_3)|^2 = E_4(x_3, y_3) E_4^*(x_3, y_3) = |A_0|^2 \frac{e^{ikD_1 x_3 (x_2 - x_2')}}{P_2} \frac{e^{ikD_2 y_3 (y_2 - y_2')}}{P_2} dx_2 dy_2.
\]

where \(|A_0|^2 = \left(\frac{k}{2} D_1 D_2\right)^2 \sum_{n=1}^{N} \sum_{m=1}^{N} a^2 e^{i(\mu n - \mu m)} \psi(u_n, v_n, D_1) \psi^*(u_m, v_m, D_1)\).

During the second exposure, let the reflecting surface of the object shift by the value \(L\). As mentioned above, we arrange the coordinate system so that the movement occurs only along the \(x\) coordinate. In this case, the reflected wave takes the form:

\[
E_{1L}(x_1, y_1) = \sum_{n=1}^{N} a e^{i \mu n} \delta(x_1 - u_n - L; y_1 - v_n).
\]

Calculating in a similar way the signal passes through the optical system, we obtain the distribution of amplitudes in the plane of the photographic plate when the object is displaced in the following form:

\[
E_{4L}(x_3, y_3) = -\frac{k^2}{4} D_1 D_2 \sum_{n=1}^{N} a e^{i \mu n} \psi(u_n + L, v_n, D_1) \psi(x_3, y_3, D_2) \times
\]

\[
\frac{e^{ik(D_1 u_n + D_2 x_3)} x_2 e^{ik(D_1 v_n + D_2 y_3)} y_2}{P_1} dx_2 dy_2.
\]

Then the intensity will take the form:

\[
I_L = |E_{4L}(x_3, y_3)|^2 = E_{4L}(x_3, y_3) E_{4L}^*(x_3, y_3) = |A_L|^2 \frac{e^{ik(D_1 L + D_2 x_3)} x_2 e^{ik(D_1 v_n + D_2 y_3)} y_2}{P_2} dx_2 dy_2.
\]

where \(|A_L|^2 = \left(\frac{k}{2} D_1 D_2\right)^2 \sum_{n=1}^{N} \sum_{m=1}^{N} a^2 e^{i(\mu n - \mu m)} \psi(u_n + L, v_n, D_1) \psi^*(u_m + L, v_m, D_1)\).

The coefficients and differ by an infinitesimal value due to the fact that the surface structure does not change significantly. Therefore, we can assume that.

The two speckle patterns obtained in the unloaded and loaded state are shifted to \(L\) in the image space and, when superimposed on each other, they represent a complex diffraction grating that carries information about changes that have occurred with the surface of the object.

Since light intensities are recorded on one photographic plate during two exposures, the total intensity will have the form:
\[ I = I_0 + I_L = |A_0|^2 \int \int P_2 P_2 \left\{ e^{ik(D_1 + D_2 x_2)(x_2 - x_2')} + e^{ikD_2 x_3(x_2 - x_2')} \right\} dx_2 dy_2 dx_2' dy_2'. \] (13)

Simplifying the amount in braces, we get:
\[ I = I_0 + I_L = |A_0|^2 \int \int P_2 P_2 \left( e^{ikD_1(u_n x_2 - u_m x_2')} \right) dx_2 dy_2 dx_2' dy_2'. \] (14)

As shown in [1-4], the integrand describes the size of speckles, which depend on the type of aperture used at the input of the optical system. It also follows from expression (14) that the total intensity is modulated by the cosine function. The intensity modulation in the plane of the photographic plate can be characterized as the formation of moiré fringes. These moiré fringes are formed due to, as follows from theoretical calculations, the geometric summation of complex non-periodic speckle structures. From the equation (14) is follows that the period of moiré fringes is associated with the value of the displacement at a particular point of the investigated surface.

3. Experimental observation of moiré fringes

For confirm the theoretical calculations, the experiment to record the focused double-exposure speckle photography based on the optical scheme shown in figure 1 was performed.

The investigated object was a diffusely scattering round flat disk that rotates around the center by a small angular value. The standard lens KALEINAR-3B 2.8/150 to record the focused double-exposure speckle photography was used. The investigated object was located at a double focal distance from the lens, so the focused image on the photographic plate was formed with the magnification of 1. The source of coherent illumination was a laser with a wavelength of 532 nm. To record focused speckle structures, VRP-M photographic plates were used.

At the first exposure the investigated object was in a certain stationary state. The second exposure was performed after the investigated object was rotated by a small angular value around its center. As a result, a complex speckle structure will be recorded on the photographic plate, which is described by the equation (14).

To observe the moiré fringes, the double-exposure photographic plate was illuminated with a white light source. The obtained characteristic photographs of moiré fringes are shown in figure 2. Image moiré fringes obtained with the same double-exposure specklegram under different angles of the illumination. The change in the moiré pattern in figure 2 agrees with the basics of the theory of moiré fringes [20]. From an analysis of the photographs it can be seen that the period of the moiré fringes varies depending on the distance from the center of rotation of the disk.

![Figure 2. Photographs of moiré fringes observed at white light.](image-url)

From an analysis of the photographs it can be seen that the period of the moiré fringes varies depending on the distance from the center of rotation of the disk. This corresponds to the course of the experiment - the displacement increases along the radius of the rotating disk.
The observed moiré fringes make it possible to estimate the distribution of displacements in the plane of the object under study without additional experiments and calculations.

4. Conclusion
In the work using the Vander Lugt method, a theoretical analysis of the formation of a focused image in a double-exposure speckle photograph is carried out and the formation of moiré fringes is shown, which make it possible to estimate the distribution of movements in the plane of the investigated object without additional experiments and calculations. The conducted experimental studies confirm the theoretical calculations.

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