Tensorial depolarization of alkali atoms by isotropic collisions with neutral hydrogen.

(Research Note)

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ABSTRACT

Results. We consider the problem of isotropic collisions between an alkali atom and neutral hydrogen. We calculate the collisional tensorial components of general p and s-states, characterized by their effective principal quantum number $n^*$. It is found that the behaviour of the tensorial components obey simple power laws allowing quick calculations of the depolarizing collisional rates. As application, our results should allow a rigorous treatment of the atomic polarization profiles of the D1-D2 lines of alkali atoms.

Conclusions. Close coupling treatments of atomic collisions are needed to decipher the information encoded in the polarized radiation from the Sun. Important problems remain unresolved like the role of collisions in the Paschen-Back conditions.

Key words. Scattering – Polarization – Atomic processes – Sun: atmosphere – Line: formation

1. Context, notations and numerical results

1.1. Context

Atomic polarization consists in an unbalance of the populations of the Zeeman sublevels pertaining to a given atomic level and in the presence of coherences between the sublevels themselves. The scattering polarization is the observational signature of the atomic polarization (e.g. Landi Degl’Innocenti & Landolfi 2004; Trujillo Bueno 2001). In real plasmas like the solar atmosphere, emitting atoms suffer the effects of wide variety of collisions with nearby abundant particles. Sometimes the information encoded in accurate observations would be inaccessible if the effect of the collisions is misunderstood or ignored.

Scattering polarization is sensitive to radiation, to magnetic and electric fields, to collisions, etc. As the scattering polarization is usually rather small in the problems of practical interest (e.g. scattering polarization of the order of 1% in Fraunhofer lines in the photosphere and chro-
mosphere of the Sun), careful and precise modelling of that polarization is of fundamental im-
portance for learning especially about a weak and even unresolved magnetic field by its Hanle
effect. For this purpose, it is of interest to obtain detailed statistical equilibrium equations (SEE)
including all the tensorial components of the collisional rates.

1.2. Notations

Let us consider the atomic states \((\alpha j)\) of alkali atoms where \(j\) is the total angular momentum
and \(\alpha\) represents the other quantum numbers necessary to define the electronic level. The atomic
states \((\alpha j)\) are quantified by the spherical tensor components \(\rho^k_q(\alpha j)\) of the density matrix, \(k\) is
the tensorial order and \(q\) represents the coherences between the levels (e.g. Fano 1963, Omont
1977, Sahal-Bréchot 1977; Landi Degl’Innocenti & Landolfi 2004). It is useful to note that for
isotropic processes the coupling terms implying \(k \neq k'\) and transfer of coherence \(q\) to \(q'\) are zero,
and that the collisional depolarization and polarization transfer rates are \(q\)-independent. At the
solar photosphere where the second solar spectrum is formed and at the low chromosphere, the
dominant collisions with neutral hydrogen are isotropic, so that the non-diagonal components of
the collisional depolarization matrix are zero.

In the case of collisions of neutral atoms with neutral hydrogen, the inelastic and super-
elastic excitation between two different electronic levels are negligible. Therefore, the index
\(\alpha\) is omitted from now on for the sake of brevity. We apply the general theory developed by
Derouich et al. (2003, 2005) and the associated numerical code to provide, the all non zero
tensorial collisional rates of any \(s\) and \(p\) level. We denote by \(C_k^E\) the collisional rates due to elastic
collisions and by \(D^k(j) = C^0_E(j) - C^k_E(j)\) the usual depolarizing rate; the expression of \(D^k\)
is given for example by equations (7) and (9) of Derouich et al. (2003). \(C^I(j, j_l)\) and \(C^S(j, j_u)\)
are the collisional transfer rates due to inelastic and super-elastic collisions respectively (see
Derouich et al. 2007; Landi Degl’Innocenti & Landolfi 2004). The indices \(l\) and \(u\) denote any
level of energy respectively lower or higher than the energy of the level \((j)\). The contribution of
isotropic collisions in the SEE is given by Equation (5) of Derouich et al. (2007) and Equation
(7.101) of Landi Degl’Innocenti & Landolfi (2004).

1.3. Numerical results

The present work provides new complementary numerical results indispensable for rigorous
analysis of the polarization profiles. Derouich et al. (2003) determined solely the depolarization
rates \(D^{k=2}\) of P-states. In fact, Table 4 of Derouich et al. (2003) gives the alignment \((k = 2)\)
depolarization rates as a function of \(n^*\) for the temperatures of 5000 K and 6000 K. The collisional
rates \(C^k_E, C^k_I\) and \(C^k_S\) were not given. This paper attempts to fill this lacuna by providing
\(C^k_E, C^k_I\) and \(C^k_S\) as a power function of \(n^*\) and \(T\). In addition, compact analytical power laws of
the \(C^k_E\) rates associated to S-states are derived from the collisional method presented in Derouich
et al. (2005).
Grids of collisional rates are computed for the effective principal quantum number \( n^* \) of the s and p-states in the interval \([1.5, 3]\). We adopt a step size of 0.1 and we obtain laws behaviours with \( n^* \) which are, with correlation coefficients \( R > 0.9 \), fit by:

## Table 1. Effective quantum number \( n^* \) of the alkali atoms.

|             | Li I | Na I | K I | Rb I | Cs I |
|-------------|------|------|-----|------|------|
| \( n^*(2S) \) | 1.588| 1.627| 1.770| 1.804| 1.869|
| \( n^*(2P) \) | 1.960| 2.117| 2.234| 2.285| 2.329|

Only the excitation collisional transfer rates \( C_{0I}^{0}(3/2, 1/2) \) and \( C_{1I}^{1}(3/2, 1/2) \) are given. However, it is straightforward to retrieve the values of the deexcitation collisional rates \( C_{0S}^{0}(1/2, 3/2) \) and \( C_{1S}^{1}(1/2, 3/2) \) by applying the detailed balance relation:

\[
C_{kS}^{j}(j, j_u) = \frac{2j + 1}{2j_u + 1} \exp \left( \frac{E_{j_u} - E_{j}}{k_BT} \right) C_{kI}^{j}(j_u, j)
\]

with \( E_{j} \) being the energy of the level \((j)\) and \( k_B \) the Boltzmann constant.

We notice that all the collisional rates are given in \( s^{-1} \), \( n_H \) is the neutral hydrogen density in \( cm^{-3} \) and the temperature \( T \) is in Kelvins. Table 1 gives the effective principal quantum number...
n* of the s and p-states of the alkali atoms Li I, Na I, K I, Rb I and Cs I. In particular, since
\( n^* = 1.627 \) for ground state of Na I, the destruction of orientation rate \((k = 1)\) is

\[
D^1(\frac{1}{2}) = C_E^0(\frac{1}{2}) - C_E^1(\frac{1}{2})
\]

\[
= \left[ 1.838 \times n^*^{3.675} - 0.958 \times n^*^{4.011} \right] \times 10^{-9} n_H \left( \frac{T}{5000} \right)^{0.416}
\]

\[
= 4.246 \times 10^{-9} n_H \times \left( \frac{T}{5000} \right)^{0.416}
\]

which is very close to the fully quantum rate given in the erratum of Kerkeni et al. (2000) (At
\( T = 5000 K \), they found a rate of \( 4.32 \times 10^{-9} n_H \) s\(^{-1} \)).

2. Hyperfine structure

In typical solar conditions, the hyperfine splitting is usually much lower than the inverse of
the typical time duration of a collision and therefore one can assume that the nuclear spin is
conserved during the collision\(^1\). This is the frozen nuclear spin approximation implying that
the hyperfine collisional rates are given as a linear combination of the fine rates \( C_E^b, C_I^b \) and \( C_S^b \)
given in this paper (e.g. Nienhuis 1976 and Omont 1977).

3. Conclusion

Tensorial collisional components are given in this work and could be implemented in the numerical
simulations of the scattering polarization in a way similar to radiative rates. Hyperfine and
fine collisional rates may be derived from simple power laws provided in this work.

Our results are valid for collisions without external magnetic field or for a sufficiently
weak field. In fact, under solar conditions, to the best of the author’s knowledge, there have
been no study which includes magnetic fields explicitly in the calculations of collisional rates.
Depolarizing collisional rates commonly encountered in the literature should be used in the Hanle
effect regime but are not valid in the Paschen-Back effect regime. It remains a challenge to take
into account the effect of the magnetic fields in the calculation of the collisional coefficients

\(^1\) It is important, however, not to confuse this condition with the fact that the SEE must be solved for the
hyperfine levels when the inverse of the lifetime of the level is lower than the hyperfine splitting, i.e. the
hyperfine levels are separated.
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