Determination of nonuniform stress field and rheological properties by caliper log measurements

AA Skulkin*, LA Nazarova**, and EV Rubtsova***

Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia

E-mail: *chuptt@yandex.ru; **larisa@misd.ru**; rubth@misd.ru

Abstract. The authors formulate and solve the problem on finding nonuniform stress field and effective viscosity in the plane oriented orthogonally to a borehole axis drilled in rocks with rheological properties by the data of caliper logging using a viscoelastic model. From the numerical research with artificial input data, the effective viscosity range is determined, and the testing time such that the problem has the unique solution is found.

1. Introduction

Knowing stress state of rock mass is important in civic construction, mineral mining and oil industry. A modern specialist in rock mechanics should possess information on properties and stresses of rock mass in order to select and justify a safe and efficient mining technology, to plan horizontal and inclined drilling paths, as well as to construct and operate waterworks, surface and underground atomic stations and other engineering objects contacting adjacent rock mass.

To this effect, there is a whole set of direct [1] and indirect [2] methods. The indirect methods are as a rule implemented through empirical correlation of physical field parameters within geomechanical models of test objects. When measurements are taken in salt or in rock masses with increased rheological properties, difficulties may arise, which is connected with long recording, or with requirement to provide identity of all measurement conditions [3–5]. This paper proposes the method to estimate horizontal external stresses, shear modulus and viscosity of rocks by the data of caliper measurements in a vertical hole based on the inverse problem solving.

2. Statement of the problem and research results

Let in a rock mass with rheological properties, at the time \( t = 0 \) a borehole with radius \( r_0 \) be drilled in an arbitrary direction. It is assumed that the principal stress field components \((p, q)\) in the orthogonal plane to the hole axis different, the plane strain condition [6] is fulfilled and in each hole section \( z = \text{const} \) deformation of the well bore zone is described by the system composed of [7–9]:

— the equations of equilibrium

\[
\sigma_{ij,j} = 0;
\]  

(1)

— the equations of state for the viscoelastic medium

\[
\sigma = K\varepsilon + V\varepsilon', \quad \tau = \mu\gamma + S\gamma';
\]  

(2)

— the Cauchy equations
\[ \varepsilon_{ij} = 0.5(u_{ij,j} + u_{ij,i}), \quad \varepsilon_{\theta\theta} = u / r. \] (3)

where \( \sigma_{ij} \) and \( \varepsilon_{ij} \) are the components of the stress and strain tensors; \( i, j = r, \theta \), \( (r, \theta) \) are the cylindrical coordinates, and the bar means the time derivative; \( K \) and \( \mu \) are the moduli of triaxial compression and shear; \( u_i \) are the displacements; \( \sigma \) is the average stress; \( \gamma \) is the principal shear; \( V \) and \( S \) are the empirical constants assumed as the bulk and shear viscosities, respectively.

For (1)–(3) the boundary conditions are formulated:

\[ \sigma_{rr}(r_0, t) = \sigma_{r\theta}(r_0, t) = 0, \quad \sigma_{rr}(r, t) \rightarrow \frac{1}{2}(p + q) + \frac{1}{2}(p - q)\cos 2\theta, \]

\[ \sigma_{r\theta} \rightarrow \frac{1}{2}(p - q)\sin 2\theta \quad \text{as} \quad r \rightarrow \infty. \] (4)

At \( t = 0 \) the displacement and stresses are zero. The system (1)–(4) is solved by the method of operational calculus [10], and the stresses:

\[ \sigma_{rr} = \frac{p + q}{2}(1 - \xi^{-2}) + \frac{p - q}{2}(1 - 4\xi^{-2} + 3\xi^{-4})\cos 2\theta, \]

\[ \sigma_{r\theta} = \frac{p + q}{2}(1 + \xi^{-2}) - \frac{p - q}{2}(1 + 3\xi^{-4})\cos 2\theta \] (5)

are analogous to the stresses in the Kirsch equation [11] \( (\xi = r / r_0) \), while the displacements at the hole boundary (from caliper logging measurements) have the form:

\[ u(r_0, t) / r_0 = U_V(t) + U_S(t), \] (6)

where

\[ U_V(t) = \frac{r_0}{K} \left( (p + q) + 2(1 - \nu^2)(p - q)\cos(2\theta + \alpha))(1 - e^{-Kt/V}) \right), \]

\[ U_S(t) = \frac{r_0}{2\mu(1 + \nu)} \left( (p + q) + 2(1 - \nu^2)(p - q)\cos(2\theta + \alpha))(1 - e^{-\mu t / S}) \right), \]

where \( \alpha \) is the angle between the hole caliper blades and principal stress.

The lab tests of rheological materials and the numerical experiments (including rocks [3, 6, 12]) show that viscous deformation is associated with shearing mechanisms \( (V \) is much less than \( S \), and at small times it is possible to neglect \( U_V(t) \) in (6). At certain times \( t_n(n = 1, \ldots, N) \) the hole diameter \( D_n \) is measured by four pairs of the hole caliper blades [13].

The inverse boundary problem is formulated as: to find the external stress field components \( (p \) and \( q) \) as well as the mechanical properties \( \mu \) and \( S \) of rocks and the unknown angle \( \alpha \) between the blades and principal stress by \( D_n \).

It follows from (6) that \( U_S(t, \alpha, p, q, \mu, S) = U_S(t, \alpha, ap, aq, a\mu, aS) \) for any \( \alpha \); thus, by \( D_n \) it is impossible to determine unambiguously the wanted parameters—additional information is required.

With this end in view, using the hydraulic fracturing stress measurement technique [14, 15], it is possible to find the values of \( p \) and \( q \) in the orthogonal plane relative to the hole axis.
By the caliper log data, calculate $e_n = 1 - D_n / D$ (where $D$ is the design diameter of the hole, which corresponds to the left-hand side of (6)). We introduce the objective function:

$$\Phi(\mu, S) = \sqrt{\frac{N}{\sum_{n=1}^{N} \left(e_n - U_S(t_n, \mu, \alpha, S)\right)^2}{\sum_{n=1}^{N} e_n}}$$

and analyze its structure. We assign $p = 10$ MPa, $q = 8$ MPa, $\mu = 0.7$ GPa, $S_0 = 10^{15}$ Pa·s typical of salt rocks [3] and synthesize the input data:

$$e_n = [1 + \delta \psi(t_n)]U_S(t, p, \alpha, q, \mu_0, S_0),$$

where $\delta$ is the relative error; $\psi$ is an arbitrary value uniformly distributed over the interval $[-1,1]$. Figure 1 demonstrates the contour line of $\Phi$ at $\delta = 0.2$ and $t_n = 1, 2, \ldots, 50$ days.

![Figure 1. Contour lines of the objective function $\Phi$.](image)

Evidently, the objective function is unimodal, and the inverse problem is solvable. The wanted values $\mu$ and $S$ occur in the equivalence domain $W$, the size of which depends on the level of noise in the input data. The function minimum search can be carried out using the gradient descent method [16].

3. Conclusion
The authors have proposed the method to determine nonuniform stress field in the orthogonal plane relative to the hole axis in rock mass having rheological properties based on the inverse problem solution using the data obtained by the caliper logging and hydraulic fracturing stress measurement.

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