Minimization of losses in a structure having a negative index of refraction

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Minimization of losses in a structure having a negative index of refraction

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Abstract. A structure consisting of an array of wires cladded with a nonmagnetic dielectric and embedded in a ferrimagnetic host has been calculated to have a negative index of refraction. The structure has moderate losses over a bandwidth of a few GHz. The calculation takes into account the skin effect within the wires and is valid provided the wavelength of electromagnetic waves in the structure is long compared to the radius of the cladded wires. The structure’s electromagnetic response is accurately described by the ferrimagnet’s permeability and a permittivity derived in the long wavelength limit. Losses can be minimized by choosing the pass band to be between 30 and 80% of the plasma frequency and by choosing wires to be of the highest possible conductivity and largest radius compatible with the required plasma frequency.

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1. Introduction

The demonstration by Smith et al [1] of an artificial medium which exhibited a negative index of refraction created a flurry of activity on metamaterials with simultaneously negative permittivity and permeability. The negative phase velocity of electromagnetic waves which these media support differentiates them from substances which exhibit negative refraction simply because they are uniaxial [2]. Some of the novel optical behaviours which arise because the phase velocity is directed opposite to the direction of energy propagation were described by Veselago [3]. However, the main feature of negative refraction due to a negative phase velocity had been described by Shuster [4] a century ago and Mandelshtam [5, 6] investigated the idea of negative refraction more than half a century ago.

One of the reasons for interest in negative phase velocity (NPV) media is that Pendry [7] has shown that one can use such a medium to make a perfect lens. The perfect lens requires the medium to be lossless and have both $\epsilon$ and $\mu$ negative but equal in magnitude to their free space values. Such a lens can image details smaller than the light’s wavelength.

Most NPV media demonstrated thus far possess two separate sets of elements which are separately responsible for $\mu < 0$ and $\epsilon < 0$. The $\mu < 0$ property arises from sets of rings [8] which primarily respond to the magnetic field of the electromagnetic waves within the medium. For waves with a frequency higher than the ring array’s resonance frequency, but lower than the anti-resonance frequency, the $\mu$ of the structure is negative. Arrays of metal posts or wires have a plasma frequency which is much lower [9, 10] than that of the metal forming the wires and can be designed to be comparable to the antiresonance frequency of the ring structure. The wires have $\epsilon < 0$ for frequencies below the plasma frequency. The plasma frequency is essentially the antiresonance frequency of the array of wires whilst the array’s resonant frequency is zero if the wires have no breaks.

The NPV structure considered here is much simpler than the wire/ring array which Smith et al [1] fabricated. The ring array is replaced by a non-conducting ferrimagnetic host medium which is penetrated by a regular array of wires. As pointed out by Pokrovsky and Efros [11], the interaction between the ferrimagnet and the wire array destroys the $\epsilon < 0$ property of the wire array. This interaction can be circumvented by cladding the wires with a non-magnetic material which essentially uncouples the wires from the host ferrimagnet [12, 13] restoring the $\epsilon < 0$ property. Similar considerations hold for the medium Smith et al [1] made in that the posts and rings must be arranged so that the magnetic flux due to current in the posts does not induce current in the rings.

The calculation described below allows the determination of the propagation constants for electromagnetic waves in a two-dimensional photonic crystal composed of long, thin, cladded wires within a ferrimagnetic host without recourse to numerically intensive methods. In particular, finite element methods are not used to solve Maxwell’s equations. These calculations indicate that, provided the ferrite is properly biased by an external magnetic field, there is a range of frequencies for which the structure exhibits a negative index of refraction. The solution to Maxwell’s equations within the structure is also used to elucidate the role the wire radius plays in determining the attenuation experienced by the electromagnetic waves within the structure.

2. Model

The model consists of a photonic crystal filling a half-space. The elements of the crystal are conducting wires which form a square two-dimensional array. The wires are cladded
Figure 1. A schematic representation of the wire structure and supporting ferrimagnetic host is illustrated in (a). An applied magnetic field $H$ causes the medium’s saturation magnetization $M_s$ to be parallel to the wires. The polarization of the electromagnetic waves is sketched at left with $s$ the direction of Poynting’s vector. A top view of the wire lattice is sketched in (b). The conducting regions are within the circles of radius $r_1$. The region between $r_1$ and $0r_2$ is filled with a nonmagnetic insulator. The region surrounding cladded wires is filled with ferrimagnetic material. The lattice spacing $a$ is much larger than $r_1$ with $r_2$ chosen such that $r_2 \geq \sqrt{r_1 a}$. The boxed region is the unit cell of the lattice.

with a nonmagnetic insulating layer of dielectric. The situation is schematically illustrated in figure 1.

Time- and space-dependent solutions to Maxwell’s equations are sought for plane electromagnetic waves propagating normal to the surface of the crystal. The polarization of the electromagnetic waves is such that the electric field is along the wires (z-direction in figure 1) and the magnetic field is in the plane of the surface (y-direction in figure 1). The orthogonal polarization interacts very little with the wires and ferrimagnet and will not be considered further. The wire radius $r_1$ is chosen to be much smaller than the lattice spacing $a$ and the outer radius of the cladding material $r_2$ is chosen to be near the geometric mean of $a$ and $r_1$.

Electromagnetic waves propagating within the crystal are taken to be at a low enough frequency that the wavelength is much larger than that the radius $r_2$. The lattice constant is not assumed small with respect to the radiation’s wavelength.

3. Ferrimagnetic host

The ferrimagnet surrounding the wire structure provides the negative permeability necessary for this medium to exhibit negative refraction. It is the negative permeability which directly leads to the phase of electromagnetic waves advancing in the direction opposite to the energy flow. Historically, such waves in ordinary magnetic media are known as backward waves.
The permeability $\mu_f$ of the ferrimagnetic host supporting the wires is

$$\frac{\mu_f}{\mu_0} = \frac{(\tilde{H}_0 + M_s)^2 - \left(\frac{\omega}{\mu_0 \gamma}\right)^2}{\tilde{H}_0(\tilde{H}_0 + M_s) - \left(\frac{\omega}{\mu_0 \gamma}\right)^2},$$

where $\tilde{H}_0 = H_0 - i \left(\frac{\omega}{\mu_0 \gamma}\right) \left(\frac{\lambda}{\mu_0 \gamma M_s}\right)$. (1)

This expression for the permeability follows from the torque exerted on the magnetization by the total magnetic field [14] appropriate to the case in which the magnetization is oriented as in figure 1 [12]. Here $\mu_0$ is the permeability of free space, $H_0$ the externally imposed magnetic field parallel to the wires, $M_s$ the ferrimagnet’s magnetization, $\omega$ the angular frequency of the electromagnetic radiation, $\gamma$ the gyromagnetic ratio and $\lambda$ a phenomenological damping parameter [15] describing losses intrinsic to the ferrimagnet. For angular frequencies satisfying the relation

$$\mu_0 \gamma \sqrt{\tilde{H}_0(\tilde{H}_0 + M_s)} < \omega < \mu_0 \gamma (\tilde{H}_0 + M_s),$$

the real part of $\mu < 0$ and the imaginary part of $\mu$ are small except near the lower limit of the frequency range. The lower limit of (2) corresponds to ferromagnetic resonance. Above the resonant frequency the response of the magnetization is out of phase with the driving field, i.e., the susceptibility is negative. At the upper limit of (2) the magnetic field due to the magnetization’s response just equals the driving field and $\mu \approx 0$. This upper limit is the ferromagnetic antiresonance frequency.

The cylindrical holes in the ferrimagnet filled by the cladded wires can scatter electromagnetic waves [16]. Since the radii of the holes are much smaller than the wavelength of any electromagnetic waves considered here, and the scattering is much weaker than the electric dipole scattering from the wires, this magnetic dipole scattering is ignored. Since the holes dilute the structure’s magnetization slightly, the medium’s permeability $\mu_m$ used in the calculations below is

$$\mu_m = \left(1 - \frac{\pi r_h^2}{a^2}\right) \mu_f.$$ (3)

4. Dispersion relation

The objective of the calculation is to determine the dispersion relation for electromagnetic waves propagating transverse to the wire structure with the electric field polarized parallel to the wires. This breaks naturally into three parts. Firstly, the fields radiated by the induced current in a single wire are determined. The induced current is, of course, the wire’s response to an incident electromagnetic plane wave within the ferrimagnet. Secondly, the total electric field at any one wire is found by adding the waves scattered from all the other wires to the incident electromagnetic wave. Thirdly, the resulting sum for the total electric field, essentially a discrete form of an integral equation, is solved for the allowed propagation constants of the structure. The development presented below follows [17] but without the constraint that the wire radius be much smaller than the skin depth within the wire.

The incident plane electromagnetic wave is assumed to take the form $E_0^{inc}(x) \hat{z} \exp(-i\omega t)$ where $E_0^{inc}(x)$ is the electric field amplitude at $x$ and $\omega$ is the angular frequency of oscillation.
of the field. For a wave turned on at some time in the distant past the initial transients are absent and the electric fields within the structure are in a steady state. Superposition of the fields radiated by the wires and the incident field has two results consistent with the Ewald–Oseen extinction theorem [18]. Firstly, the scattered waves destructively interfere with the incident wave. Secondly, the superposed waves have a different phase velocity and a different propagation constant $k_m$, than the waves in the ferrimagnet. It is the dispersion relation $\omega$ versus $k_m$ for these waves which we seek.

The fields due to radiation by a single wire are found by solving a simple boundary value problem. For concreteness, consider the wire at the origin of the coordinate system in figure 1(b). Cylindrical coordinates are assumed with the $z$-axis along the wire and $\rho$ the radial distance from the wire’s centreline to the point of interest. An incident electromagnetic wave $E_{\text{inc}}$ can be described by an expansion in Bessel functions. The assumption that the cladded wires are small compared to a wavelength, $r_2 \ll \lambda$, leads to only the zeroth order term being retained. This is equivalent to considering only electric dipole scattering by the wire. In essence, the retained term representing $E_{\text{inc}}$ is the electric field that would exist at the centre of the wire if the wire’s conductivity were zero.

The incident wave gives rise to current within the wire. The incident field $E_{\text{inc}}$ plus the fields radiated by the current yield a net electric field $E_{\text{w}}$ within the wire plus a scattered wave $E_{\text{scat}}$ propagating into the ferrimagnet. These fields are

$$E_{\text{inc}} = E_{\text{inc}}^o J_0(k_0 \rho) \hat{z}, \quad E_{\text{w}} = E_{\text{w}}^o J_0(k_w \rho) \hat{z} \quad \text{and} \quad E_{\text{scat}} = E_{\text{scat}}^o H_{1}^{(1)}(k_f \rho) \hat{z}.$$  

(4)

In (4), $J_0$ is a Bessel function, $H_{1}^{(1)}$ a Hankel function, and the propagation constants are

$$k_0 = \omega \sqrt{\epsilon_0 \mu_0}, \quad k_w = \sqrt{1 + \frac{i \sigma}{\omega \epsilon_0} k_0} \quad \text{and} \quad k_f = \omega \sqrt{\epsilon_f \mu_m}.$$  

(5)

Here $k_w$ and $k_f$ are the propagation constants inside the wire and ferrimagnet, respectively, $\sigma$ is the wire’s conductivity and $\epsilon_f$ the ferrimagnet’s permittivity. The wire is assumed to have the same permittivity and permeability as free space.

The electric field at the surface of the wire at position $(x, y)$, with $x = n_x a$ and $y = n_y a$ and where $n_x \geq 0$, $n_y$ are integers, can be viewed as arising from four categories of sources: the incident wave; the self-inductance of the wire at $(x, y)$; the scattered waves from other wires at position $(x', y')$ with the same depth in the crystal, i.e., $x' = x$ and $y' \neq y$; and the scattered
waves from other wires at different depths, i.e., $x' \neq x$. Translational symmetry in the $y$-direction allows us to set $n_y = 0$. Summing the fields associated with these four sources yields the total electric field at the wire’s surface:

$$E_{tot}(n_x, a, t) = \frac{1 + E_0^{inc}}{E_0^{inc}} E_0^{inc} e^{i(n_x k_f a - \omega t)} + \sum_{n'_x=0}^{\infty} E_0^{scat} S_{dd}(n'_x, n_x, a) E_0^{inc} e^{i(n'_x k_f a - \omega t)}, \quad (7)$$

where $E_0^{inc}$ and $E_0^{ind}$ are evaluated at $r_1$ and $E_0^{scat}$ is evaluated at $r_2$. The sum in (7) excludes $n'_x = n_x$. Also in (7) [19],

$$S_{sd}(\omega, a) = 2 \sum_{n'_x=1}^{\infty} H_0^{(1)}(n'_x k_f a)$$

$$= -1 + \frac{2}{k_f a} - \frac{2i}{\pi} \left[ \ln \left( \frac{k_f a}{4\pi} \right) + \gamma - 2\pi \sum_{m=1}^{\infty} \left( \frac{1}{\sqrt{(2\pi m)^2 - (k_f a)^2}} - \frac{1}{2\pi m} \right) \right] \quad (8)$$

and

$$S_{dd}(n'_x, n_x, \omega, a) = \sum_{n'_x=\infty}^{\infty} H_0^{(1)} \left( \sqrt{(n'_x)^2 + (n'_x - n_x)^2 k_f a} \right)$$

$$= \frac{2}{k_f a} e^{i|n'_x - n_x| k_f a} + \sum_{m=1}^{M} \frac{4 e^{i|n'_x - n_x| \sqrt{(k_f a)^2 - (2\pi m)^2}}}{\sqrt{(2\pi m)^2 - (k_f a)^2}}$$

$$+ \sum_{m=M+1}^{\infty} -\frac{4 e^{i|n'_x - n_x| \sqrt{(2\pi m)^2 - (k_f a)^2}}}{\sqrt{(2\pi m)^2 - (k_f a)^2}}, \quad (9)$$

where $\gamma = 0.57721566 \ldots$ is Euler’s constant. Here $S_{sd}$ and $S_{dd}$ are the sums of Hankel functions at the same depth and different depth, respectively, as the wire under consideration. Directly summing the Hankel functions results in a rapidly converging series provided $k_f a$ has a large, positive imaginary part. However, for $k_f a$ nearly a pure real number, the alternate forms in (8) and (9) converge much more rapidly. In (9) $M$ is chosen such that $2\pi M < |\text{Re}(k_f a)| < 2\pi (M + 1)$. The sum in (9) has been broken into two parts to clearly indicate which branch of the square root to take if $k_f a$ is a pure real number. If $M = 0$ then no terms of the series on the second line of (9) are summed. In the event that $k_f a$ is real, the sum on the last line of equation (9) corresponds to evanescent modes. Note that only the absolute value of $n'_x - n_x$ enters (9) since the fields at $(n_x a, 0)$ depend on waves scattered at earlier, retarded times by wires on either side of the one in question.

Equation (7) is a relation connecting $E_0^{tot}$ to $E_0^{inc}$, the wave which would propagate through the ferrimagnet if the wires were removed. Guided by the approach taken in deriving the Ewald–Oseen extinction theorem, one can assert that the total field can be related to the incident field by [17]

$$E^{tot}(x, t) = \left( \sum_{m=0}^{\infty} f_m e^{i\pi m x} \right) E_0^{inc} e^{i(k_f x - \omega t)}. \quad (10)$$
In (10), \( f_m \) is the relative amplitude of the \( m \)th mode, \( \bar{k}_m \) is an augmentation of \( k_f \) for the \( m \)th mode and \( x = n_x a, y = 0 \). The content of (10) is that the total electric field at each wire within the photonic lattice can be described by the superposition of diffracted waves which travel with a phase velocity different from the free space value. The ansatz embodied in (10) is a generalization of the result found for scattering by atoms within an ordinary medium \([20]\).

Substitution of (10) into (7) leads to a relation which must be true for all \( n_x \). The sums over \( n_x' \) are simple geometric series. Equating the coefficients of the various terms involving \( n_x \) leads to a set of relations for the \( f_m \)'s as well as a polynomial with roots which yield the \( k_m \)'s. This polynomial can be manipulated into the form

\[
\sum_{m=1}^{N} \frac{1}{\sqrt{(k_f a)^2 - (2\pi m)^2}} \left( \frac{1}{\xi p_m - 1} + \frac{1}{\xi^{-1} - 1} \right) = 0,
\]

where

\[
\xi = e^{i(k_m+k_f)a} = e^{ik_m a} \quad \text{and} \quad p_m = e^{-i\sqrt{(k_f a)^2 - (2\pi m)^2}}.
\]

It is evident from the form of (11) that if \( \xi \) is a root then \( 1/\xi \) is also a root. The sum in (11) is cut off after \( N \) terms. If \( N \) is chosen so that seven evanescent modes are included in the sum then the roots \( \xi_m \) of (11) are accurate to at least 12 decimal places. The \( k_m \) required for the dispersion relation is readily extracted from \( \xi_m \). These \( k_m \) are exact in the limit \( k_m r_2^2 \ll 1 \). A similar solution to Maxwell’s equations for a photonic crystal consisting of a two dimensional array of wires in free space has been given by Belov \([21, 22]\).

5. Approximate permittivity

The dispersion relation implicit in (11) does not make use of the notion of a permittivity \( \epsilon_m \) of the photonic crystal. One can postulate the existence of such a permittivity and, with a suitable averaging of fields over a unit cell of the lattice, solve (7) for the medium’s permittivity. This has been done using the assumption that the current density is uniform throughout the wire or, equivalently, that the wire radius is much smaller than the wire’s skin depth: \( r_1 \ll \delta = \sqrt{2/\mu_0\sigma\omega} \).

One obtains \([13]\)

\[
\epsilon_m(\omega) = \epsilon_f \left( 1 - \frac{\sigma^{\text{eff}}}{\omega \epsilon_f \left( 1 + \frac{\sigma^{\text{eff}}}{2\pi} \left[ \ln \frac{r_2}{r_1} + \mu_f \left( \ln \frac{r_2}{r_1} - \frac{3+\ln 2-\pi/2}{2} \right) \right] \right)} \right),
\]

where

\[
\sigma^{\text{eff}} = \frac{\sigma \pi r_1^2}{a^2}.
\]

Postulating the existence of a permittivity for the structure implies that the wavelengths under consideration are greater than a lattice constant. Thus (13) is only valid in the long wavelength...
limit $k_m a \ll 1$ whereas (11) is valid for $k_m r_2 \ll 1$. As will be seen below, (13) is quite accurate up to $k_m a = \pi$ once (14) is modified.

An approximate relation from the plasma frequency $\omega_p$ can be found from (13) by setting the real part of $\epsilon_m(\omega) = 0$. This yields the plasma frequency

$$\omega_p^2 \simeq \frac{2\pi}{\epsilon_f a^2 \left[ \mu_0 \ln \frac{\omega_p}{r_1} + \mu_f \left( \ln \frac{a}{r_2} - \frac{3 + \ln 2 - \pi/2}{2} \right) \right]}.$$

This approximate relation for the plasma frequency can be compared to the plasma frequency implicit in (11) by noting that in the limit of infinite conductivity there is a sharp transition from propagating to non-propagating waves at the plasma frequency. For finite conductivity the frequency at which $|\text{Re}(k_m)| = |\text{Im}(k_m)|$ can be identified with the plasma frequency.

In order for the permittivity of equation (13) to have a negative real part and a small imaginary part the frequency $\omega$ and the structure parameters must satisfy the constraints

$$\frac{\sigma_{eff}}{\omega \epsilon_f} \gg \frac{2\pi}{\epsilon_f \omega a^2 \left[ \mu_0 \ln \frac{\omega_p}{r_1} + \mu_f \left( \ln \frac{a}{r_2} - \frac{3 + \ln 2 - \pi/2}{2} \right) \right]} > 1.$$

These two constraints are equivalent to demanding that (1) the conductivity be large enough that current in the structure is much larger than the displacement current and that (2) the frequency be only a little lower than the plasma frequency so that the structure does not simply behave as an ordinary metal of conductivity given by (14).

The assumption that the wire is much smaller than the skin depth, used in obtaining (13) and (15), is inconsistent with the constraints in (16). At a frequency for which the structure has $n < 0$ and low losses the wire radius must be comparable to, or larger than, the skin depth.

### 6. Results of calculations

The dispersion relations for the wire array/ferrimagnetic host were calculated for several values of permeability using either equation (7) or equation (13) together with Maxwell’s equations. The results are presented in figure 2. The parameters describing the ferrimagnetic host were chosen to be typical of a moderately lossy ferrite with a large magnetization. The primary result is that, for $\text{Re}(\mu) < 0$, the real part of $k_m a$ is negative. Since the index of refraction is $n = \text{Re}(k_m)/k_0$, the proposed structure exhibits a negative index of refraction.

A second result concerns the role played by the wire’s skin depth in determining $k_m$. The filled lines in figure 2 represent the $k_m$ found from Maxwell’s equations and equation (13) and the dotted lines represent $k_m$ found from equation (11), both for $\mu_f = -\mu_0$. Although these curves are qualitatively similar, they differ in their location of the plasma frequency for the wire structure. This is not surprising since the skin depth in the wire is $\delta = 0.83 \mu m$ at 10 GHz. This is comparable to the wire radius while equation (13) was derived for $r_1 \ll \delta$. As the skin depth becomes smaller than $r_1$, with increasing frequency, the total current flowing in the wire is less than that at lower frequency and the wires are less effective in scattering electromagnetic waves. Indeed, if the conductivity used in equations (13) and (14) is adjusted downward with frequency so that the total current calculated to flow in the wire under the assumption of a uniform current density matches the total current flowing within a skin depth of the wire’s
The structure has \( \mu \) the bandwidth associated with 14 GHz. Losses associated with ferromagnetic resonance are appreciable below 7.5 GHz. The antiresonance frequency. This causes electromagnetic waves to be heavily damped between 12 GHz. The ferrimagnet is so small near the antiresonance that the plasma frequency is lower than the pass band associated with the ferrimagnet are less than those for a medium with \( \mu \) is much smaller than for the case where \( k_{ma} \gg 1 \) and relatively small losses over the band from 7.5 to 12 GHz. In contrast, the wavelength of electromagnetic waves is also larger and the ratio \( \text{Im}(k_{ma})/\text{Re}(k_{ma}) \) is about the same in both cases. The losses are primarily due to Joule heating in the wires surface, then equation (13) provides an accurate representation of the permittivity of the structure for \( k_{ma} < \pi \). In the limit that \( r_1 \gg \delta \), the effective conductivity described by (14) should be replaced by

\[
\sigma_{\text{eff}} = \frac{\sigma 2\pi r_1 \delta}{a^2}.
\]

Four features of the dispersion relation calculated for \( \mu(H_0, \omega) \) of equation (1) stand out in figure 2. Firstly, the bandwidth over which \( n < 0 \) and losses are minimal (small \( \text{Im}(k_{ma}) \)) is much smaller than for the case where \( \mu \) is constant. The antiresonance frequency for the ferrimagnet is 14 GHz and \( \mu(H_0, \omega) \) is negative only below this frequency. The permeability of the ferrimagnet is so small near the antiresonance that the plasma frequency is lower than the antiresonance frequency. This causes electromagnetic waves to be heavily damped between 12 and 14 GHz. Losses associated with ferromagnetic resonance are appreciable below 7.5 GHz. The structure has \( n < 0 \) and relatively small losses over the band from 7.5 to 12 GHz. In contrast, the bandwidth associated with \( \mu = -\mu_0 \) stretches from 2.0 to 16 GHz. Secondly, the losses in the pass band associated with the ferrimagnet are less than those for a medium with \( \mu = -\mu_0 \). However, the wavelength of electromagnetic waves is also larger and the ratio \( \text{Im}(k_{ma})/\text{Re}(k_{ma}) \) is about the same in both cases. The losses are primarily due to Joule heating in the wires.

Figure 2. Scaled propagation constants \( k_{ma} \) versus frequency \( f \) are shown for \( \mu = -\mu_0 \) (filled and dotted lines) and for \( \mu(H, \omega) \) (\( \bullet \), \( \text{Re}(k_{ma}); +, \text{Im}(k_{ma}) \)). The dotted line was calculated from equation (11); and the filled line from Maxwell’s equations and equations (13) and (14). The \( k_{ma} \) shown for \( \mu(H, \omega) \) were calculated from equations (1), (3) and (11). Other parameters used in the calculations are \( a = 5.0 \, \text{mm}, r_1 = 1.0 \, \mu\text{m}, r_2 = 75 \, \mu\text{m} \) and \( \sigma = 3.65 \times 10^7 \, \Omega^{-1} \, \text{m}^{-1} \). Parameters used in equation (1) for \( \mu(H, \omega) \) are \( H_0 = 8.0 \times 10^2 \, \text{A m}^{-1}, M_s = 4.0 \times 10^5 \, \text{A m}^{-1} \), \( \lambda = 1.0 \times 10^6 \, \text{s}^{-1} \) and \( \gamma \) is the gyromagnetic ratio for a free electron. Also, \( -\text{Re}(k_{ma}) = \text{Im}(k_{ma}) \) at 21.2 GHz for the filled line and at 17.5 GHz for the dotted line.
and not due to dissipation within the ferrimagnetic host. Thirdly, there is little dispersion and the group velocity is nearly constant in the structure’s pass band. Fourthly, Re\((k_m a)\) for \(\mu(H_0, \omega)\) approaches the value of Im\((k_m a)\) for \(\mu = -\mu_0\) at 30 GHz. This occurs because \(\mu(H_0, \omega)\) is positive above the antiresonance frequency, approaching \(\mu_0\) and \(\epsilon_m > 0\) above the plasma frequency. An electromagnetic wave has \(n > 0\) in this case and propagates with negligible loss.

The parameters of the model were investigated with the aim of reducing the losses due to joule heating within the wires. These losses are, of course, smallest when the conductivity of the wires is largest. The result of calculating Im\((k_m) / \text{Re}(k_m)\) (about half the loss tangent) from (11) for a range of parameters is that the losses depend on the plasma frequency, the conductivity and the wire radius. This can be understood by using the permittivity of (13) along with \(\mu_f\) and solving Maxwell’s equations. One obtains

\[
-\frac{\text{Im}(k_m)}{\text{Re}(k_m)} = \frac{1}{2} \left( \frac{\omega_p}{(\omega_p - \omega)} \left( \frac{\omega^2 \sigma_{\text{eff}}}{2\pi} \right) \left[ \mu_0 \ln \frac{r_2}{r_1} + \mu_f \left( \ln \frac{a}{r_2} - \frac{3 + \ln 2}{2} \right) \right] \right).
\]

(18)

Losses represented by (18) are minimized when \(\omega_p / \omega\) is as large as possible but consistent with the constraint (16). This is in agreement with calculations based on (11). For \(0.3 \omega_p < \omega < 0.8 \omega_p\) the loss tangent is near its minimum. Other calculations based on (11) carried out with \(\omega\) and \(\omega_p\) constant while \(r_2\) was adjusted so that the logarithmic terms in (18) were fixed showed that the loss tangent is proportional to \(\sigma^{-0.5}\) and \(r_1^{-1}\). This implies that \(\sigma_{\text{eff}} \propto r_1 \sqrt{\sigma}\). This is consistent with (17) but not (14).

7. Conclusions

Calculations indicate that an array of wires cladded in a nonmagnetic dielectric and embedded in a ferrimagnet exhibits a negative index of refraction. Over the frequency range for which \(n < 0\), the electromagnetic response of such a structure can be adequately described by the ferrimagnet’s permeability and a permittivity which correctly accounts for the net current in each wire, regardless of how that current is distributed within that wire.

Typical losses associated with the wires are larger than the dissipation within the ferrimagnet provided the ferrimagnet is not biased near the ferromagnetic resonance. Losses calculated for the wire structure are minimized by (1) having the operating frequency at about half the plasma frequency, (2) using wires of the highest possible conductivity and (3) having the wire radius as large as possible consistent with maintaining an adequately high plasma frequency.

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