A Channel Allocation Algorithm to Maximize Aggregate Throughputs in DCB WLANs

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Abstract—IEEE 802.11 has evolved from 802.11a/b/g/n to 802.11ac to meet rapidly increasing data rate requirements in WLANs. One important technique adopted in 802.11ac is the channel bonding (CB) scheme that combines multiple 20MHz channels for a single transmission in 5GHz band. In order to effectively access channel after a series of contention operations, 802.11ac specifies two different CB operations: Dynamic Channel Bonding (DCB) and Static Channel Bonding (SCB). This paper proposes an optimal channel allocation algorithm to achieve maximal throughputs in DCB WLANs. Specifically, we first adopt a continuous-time Markov Model (CTMC) model to analyze the equilibrium throughputs. Based on the throughput analysis, we then construct an integer nonlinear programming (INLP) model with the target of maximizing system throughput. By solving the INLP model, we then propose an optimal channel allocation algorithm based on the Branch-and-Bound Method (BBM). It turns out that the maximal throughput performance can be achieved under the channel allocation scheme with the least overlapped channels among WLANs. Simulations show that the proposed algorithm can achieve the maximal system throughput under various network settings. We believe that our analysis on the optimal channel allocation schemes brings new insights into the design and optimization of future WLANs, especially for those adopting channel bonding technique.

I. INTRODUCTION

It is known that the channel bonding (CB) technique has been used in wireless networks to boost data rates. The adoption of the CB technique in WLANs was first introduced in the IEEE 802.11n amendment [1], where two basic 20MHz channels can be aggregated to obtain a 40MHz channel. To support high-speed applications, the IEEE802.11ac amendment [2] further extends the allowable bandwidth in a single transmission from 40MHz to 80MHz and even 160MHz. The design target of 802.11ac is to offer very high throughput (VHT) while keep backward compatibility with the legacy 802.11 specifications [5]. However, the usage of wider channels also makes the channel contention between the neighboring WLANs more complicated, in which the contending node is allowed to dynamically select its transmission channels based on the instantaneous spectrum occupancy status just at the beginning of the transmission. Such a CB technique is usually referred to as Dynamic Channel Bonding (DCB) [4]. This paper makes an attempt to analyze the interactions and dependencies under DCB and seek for the optimal channel allocation strategy to maximize the aggregate throughputs in DCB WLANs.

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There have been several studies on the performance of 802.11ac DCB networks [4]–[9]. By simulations, refs. [4, 5] showed that the channel bonding technique can provide significant throughput gains. An experimental evaluation on different network parameters that affect the performance of the CB in IEEE 802.11 WLANs was presented in [6]. From the perspective of performance analysis, [7] proposed an analytical model based on a decoupling approximation to evaluate the performance of an IEEE 802.11ac WLAN with dynamic bandwidth channel access. Ref. [8] constructed a Continuous-Time Markov Chain (CTMC) to analyze the network performance under the Static Channel Bonding (SCB). Later, ref. [9] extends the CTMC model to analyze the interactions between neighboring WLANs operating under DCB, in which all WLANs are “all-inclusive” in the sense that all WLANs can sense each other. However, although several research efforts have been made to analyze the performance of DCB networks, none of them have been devoted to investigate the impacts of different channel allocation schemes (such as the number of basic channels and the location of the primary channel of each WLAN) on the system performance in DCB networks and this paper attempts to fill this gap.

It is known that WLANs adopts the CSMA/CA protocol for multiple user access at the MAC layer, whose main components are carrier sensing and random backoff to alleviate packet collisions. In DCB networks, the maximum number of basic channels that can be used by a WLAN and the selection of the primary channel are two important parameters that affect the interactions and dependencies among WLANs and lead to different system performance. Observing this, we investigate the optimal channel allocation algorithms to achieve maximal throughputs in DCB WLANs. More specifically, we first adopt the CTMC model proposed in [9] to analyze the throughput performance under different channel allocations. Importantly, we prove that for all-inclusive DCB networks, the optimal
throughput performance is achieved under one of the channel allocation scheme with the least overlapped channels among WLANs. Based on this understanding, we then construct an integer nonlinear programming (INLP) model with the target of maximizing system throughput. By solving the INLP model, we then proposed an optimal channel allocation algorithm based on the BBM to seek for the optimal channel allocation scheme that maximizes the aggregate throughputs of all WLANs. Simulation results validate that the proposed channel allocation algorithm can achieve the maximal network throughput and maintain good fairness among WLANs for “all-inclusive” DCB networks.

We believe that our analysis on channel allocations of “all-inclusive” DCB networks provides new insights into the 802.11ac networks and moves a new step towards the optimization of the DCB networks. For example, by theoretical analysis we show that the too much overlapped channels among WLANs could in fact decrease the aggregate throughputs under current 802.11ac parameter settings\(^1\). Moreover, the proposed channel allocation scheme can achieve optimal throughput performance and is suitable for engineering implementation in practical WLANs.

The remainder of the paper is organized as follows. Section II introduces the background on wide bandwidth operation defined in 802.11ac as well as the channel allocation algorithms in WLANs. Section III describes the system model and introduces the throughput computation under CTMC model. Section IV validates the analytical result with simulation and formulates the channel allocation problem that we are interested in. Section V presents our findings on the optimal channel allocation in “all-inclusive” DCB networks, and constructs an INLP model with the target of maximizing system throughput, then we propose a channel allocation algorithm based on BBM to solve the INLP model in Section VI. Simulations are shown in Section VII. Finally Section VIII concludes this paper.

II. BACKGROUND

A. Channelization and Channel Contention Defined in 802.11ac

802.11ac allows WLANs to use multiple non-overlapping channels\([2], [3]\) in a single transmission. As shown in Fig.1, two adjacent 20MHz channels can form a 40MHz channel, and two adjacent 40MHz channels can form an 80MHz channel. A 160MHz channel can be formed by two adjacent or separated 80MHz channels. We call a 20MHz channel as a basic channel. To support this expanded channelization, each node uses control fields in the beacon to indicate its bandwidth and the selected primary channel\([2]\).

Under CB, a wider bandwidth channel is composed of a primary channel and one or more secondary channels. Each node in the network uses the basic distributed coordination function (DCF) to compete for channel occupancies only on the primary channel\([2], [3]\). When a node has packets to transmit, it first senses its primary channel. Once the primary channel has been sensed idle for a DCF inter-frame space (DIFS) duration, the node starts the backoff procedure by selecting a random value of the backoff counter. The node then starts decreasing the backoff timer linearly with time while sensing the primary channel idle. If the primary channel is sensed busy during the backoff process, the backoff timer is frozen with the remaining time recorded. Upon the primary channel is sensed idle for a DIFS time again, the backoff process resumes with the recorded remaining time.

Different from the case of single-channel WLANs, before the timer expires, the node has to sense its secondary channels for a point coordination function (PCF) inter-frame space (PIFS) period. When the time expires, the node has two options to determine to transmit on which channels: i) under SCB, as shown in Fig. 2(a), only when all the channels (including both the primary and the non-primary channels) are idle, the node starts transmitting using the whole assigned channel. Otherwise, it will initializes a new backoff procedure; ii) under DCB, as shown in Fig. 2(b), even though some of the non-primary channels may be busy during the PIFS, the node begins to transmit using the primary channel and the idle secondary channels that are adjacent to the primary channel, without initializing a new backoff process. It is known that DCB has much better performance than SCB\([4]\), and thus this paper considers the DCB WLAN.

B. Channel Allocation Algorithms in WLANs

Channel allocation algorithms in WLANs have attracted much interest from the research community\([10]-[14]\). It is commonly cast as graph-coloring where an edge corresponds to interference between two cells, and the set of available colors corresponds to the set of channels. Because graph-coloring is NP-hard for general graphs, heuristics are used to solve it\([10]-[14]\). However, most of prior works do not take into account the challenges brought by the CB technique. The authors in \([15]\) proposed an algorithm simultaneously consider the channel center frequency and channel bandwidth to increase throughputs gains per-AP. Ref. \([16]\) developed

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\(^1\)This result was also observed by experimental evaluation in \([6]\)
an analytical model to estimate the network throughput under client interferences and proposed a distributed channel assignment algorithm. Another decentralized algorithm was proposed in [17] to select both the channel center frequency and the channel width by sensing the interference that is caused by the other neighboring WLANs. Ref. [13] proposed a practical distributed protocol-compatible channel bonding scheme. However, none of [15]-[18] considered the DCB operation in channel competition.

There are also some investigations on the channel allocation schemes in SCB networks. For example, [19] analyzed the hidden terminal problem and proposed a channel allocation algorithm considering the primary channel selection with a given channel width in SCB networks. Moreover, ref. [8] proposed a solution based on the water-filling concept to find the sub-optimal allocation in SCB networks. However, considering the different channel access options between SCB and DCB, the channel allocation in DCB networks deserves a more careful investigation, and this paper makes such an attempt. It is not difficult to see that the maximum number of channels of each WLAN and the selection of the primary channel are two important parameters that affect the interactions and dependencies among WLANs and lead to different system performance, and a good channel allocation algorithm should specify the settings of both parameters above.

III. SYSTEM MODEL

A. Network Model and Problem Formulation

Similar to [9], we consider a wireless CSMA network with $N$ WLANs, in which all WLANs are within the carrier-sensing range of each other. We assume a WLAN-centric model that all nodes in each WLAN are close to each other. Furthermore, all WLANs are assumed to be saturated, and we assume that when a node in WLANs initializes a transmission, the channel allocated to it will be used until the end of this transmission. That is, the node cannot switch between different channel allocations during a single transmission.

Let $K$ be the number of available basic channels (i.e., number of 20MHz channels), $C$ be all possible combinations of these basic channels (specified by the 802.11ac standard as shown in Fig. 1), and $F$ be the set of all possible channel allocations of the entire network. Define a feasible channel allocation $f = [f_1, f_2, f_3, \ldots, f_N]$ as the vector indicating the channels assigned to all WLANs in the network, where $f_i \in C$ denotes the channels assigned to a single WLAN, $WLAN_i$. For example, $f_i = \{1, 2, 3, 4\}$ denotes that $WLAN_i$ is allocated basic channels from 1 to 4 and the assigned primary channel is channel 2. Let $l_i$ be the number of contiguous basic channels assigned to $WLAN_i$, and the bandwidth assigned to $WLAN_i$ is then $BW_i = 20l_i$ MHz.

B. Throughput Computation using the CTMC Model

This subsection gives a brief introduction to the analytical models built up to compute the link throughputs in CSMA wireless networks. Specifically, we use the CTMC model proposed in [9] for link throughputs computation of a DCB network under a specific channel allocation scheme $f = [f_1, f_2, f_3, \ldots, f_N]$.

Under DCB, even two or more WLANs are assigned overlapped basic channels, they could still be transmitting simultaneously as long as the channels they use in this transmission do not overlap. Also, in a DCB network a WLAN may occupy different numbers of basic channels in different transmissions, which is different from the case of SCB as studied in [8]. The selected channels for transmission of a node in $WLAN_i$ based on the status of the basic channels in $f_i$, which are sensed just before the backoff timer reaches zero. Thus, we define a feasible network state as a set of channels on which WLANs are transmitting simultaneously, and we define the state space, $S$, as the collection of the all feasible states $\mathcal{S}$.

We use an illustrative example of Fig.3 to demonstrate the computation process, where two neighboring WLANs are within the carrier-sensing range, and the channel allocation scheme is $f : f_A = \{1, 2\}, f_B = \{1, 2, 3, 4\}$. The set of feasible network states is $S = \{\emptyset, A_1^3, B_1^3, A_2^2, B_2^2\}$, where $\emptyset$ is the state in which none of the WLANs is transmitting, $A_2^2$, $B_1^3$ is the network state in which only WLAN A or B is transmitting and using channels $\{1, 2\}$ or $\{3, 4\}$ respectively. The top number of $\bullet$ is the first selected channel of $WLAN_i$, and the bottom number is the total number of basic channels used by $WLAN_i$ for the current transmission. The transmission channels selected under DCB is always the largest contiguous subset of these available channels that contains the primary channel. Similarly, $A_1^3B_2^2$ is the network state in which the two WLANs are simultaneously transmitting, using channels $\{1, 2\}$ and $\{3, 4\}$ respectively.

We assume that the backoff timer at each node is in continuous time and has an average duration of $E[B_i]$ seconds,

\[ \begin{align*}
\text{Channels:} & \quad \text{Channels:} \\
1+2(p) & \quad 1+2+3(p)+4 \\
WLAN A & \quad WLAN B
\end{align*} \]

Fig. 3. An illustrating example to explain the CTMC model

Since the interactions of channel competitions under SCB and DCB are different, the feasible states and state transitions are different even under the same network settings.
where $B_i$ is the backoff duration of $WLAN_i$. When a node in $WLAN_i$ has a packet waiting for transmission, the attempt rate is $\lambda_i = E[B_i]^{-1}$. The transmission duration is denoted by $T_i(k_i', \gamma_i, L_i)$, which is determined by the number of basic channels that are bound together in this particular transmission, $k_i'$, the Signal-to-Noise Ratio(SNR) observed by all packet transmissions inside $WLAN_i$, $\gamma_i$, and the payload size $L_i$. Therefore, the packet departure rate from a node in $WLAN_i$ is $\mu_i = E[T_i(k_i', \gamma_i, L_i)]^{-1}$. Then the transition rates between two network states, $s, s' \in S$, are

$$q(s, s') = \begin{cases} \lambda_i & \text{if } s' = s \cup \{WLAN_i\} \\ \mu_i & \text{if } s' = s \setminus \{WLAN_i\} \\ 0 & \text{otherwise.} \end{cases}$$

(1)

Define the activity ratio of $WLAN_i$ as the ratio of the mean packet transmission duration to the mean backoff time. That is,

$$\rho_i(k_i') = \frac{E[T_i(k_i', \gamma_i, L_i)]}{E[B_i]} = \frac{\lambda_i}{\mu_i} \quad (2)$$

It is worthwhile to note that in a DCB network a WLAN may occupy different numbers of basic channels in different transmissions(i.e., $k_i'$ of $WLAN_i$ in different transmissions are different), which is quite different from the case of SCB as studied in [8].

Let $s(t) \in S$ denote the network state at time $t$. If we further assume the backoff and transmission durations are exponentially distributed, $s(t)_{t \geq 0}$ is a continuous-time Markov process on the state space $S$. This Markov process is aperiodic, irreducible and thus positive recurrent, since the state space $S$ is finite. A steady-state solution to the CTMC always exists, denote the stationary probability distribution by $\{\pi_s\}_{s \in S}$.

The steady-state probabilities of the CTMC can be computed by solving the general balance equations, yields

$$\pi_s = \pi_0 \prod_{i \in s} \rho_i(k_i') \quad (3)$$

where $\pi_0$ denotes the probability of the network state where none of the WLANs is activating and $i \in s$ means a node in $WLAN_i$ is transmitting packets in network state $s$. Together with the normalizing condition $\sum_{s \in S} \pi_s = 1$, yields

$$\pi_0 = \frac{1}{\sum_{s \in S} \prod_{i \in s} \rho_i(k_i')} \quad (4)$$

and

$$\pi_s = \frac{\prod_{i \in s} \rho_i(k_i')}{\sum_{s' \in S} \prod_{i \in s} \rho_i(k_i')} \quad s \in S \quad (5)$$

Since the process $s(t)_{t \geq 0}$ is irreducible and positive recurrent on $S$, it follows from the classical Markov chains results that $\pi_s$ is equal to the long-run fraction of the time that the system spends on state $s$.

The stationary distribution of the example shown in Fig.3 is: $\pi_{A_1^1} = \rho_A(2) \pi_0$, $\pi_{B_1^1} = \rho_B(4) \pi_0$, $\pi_{A_1^1 B_2^2} = \rho_A(2) \rho_B(2) \pi_0$, $\pi_{B_2^2} = \rho_B(2) \pi_0$ with $\pi_0 = (1 + \rho_A(2) + \rho_B(2) + \rho_A(2) \rho_B(2))^{-1}$, where $\rho_A(2)$ means the activity ratio of using two basic channels for current transmission.

It is worth mentioning that it has been proven theoretically that in SCB and DCB networks with continuous-backoff time, the stationary distribution of the Markov chain is insensitive to the distributions of both the backoff and the transmission time [8][9]. Indeed, even if the backoff and transmission time are not exponentially distributed, we can still use the continuous time Markov chain to compute the network throughput.

Based on the steady-state probabilities, we can compute the throughputs of WLANs. The throughput of $WLAN_i$, in bits per second, is then given by

$$Th_i = L_i \left(\sum_{s \in S, s \in s} \pi_s \mu_i \right) (1 - p_e) \quad (6)$$

where $p_e$ is the packet error probability. We assume an ideal channel condition, which does not have communication errors or capture effect, and all the packets have the same length, with the assumption of continuous time backoff we have $p_e = 0$.

Note that any change in channel allocation scheme, $f$, results in a different state space, $S$, as well as the different transitions among them. As can be seen in (4)(5), the normalization constant $\pi_0$ and the stationary distribution $\{\pi_s\}_{s \in S}$ depend on the state space $S$ and the state transitions among them. Thus, different channel allocation schemes will lead to different throughput performances. Its crucial to find an optimal channel allocation scheme that can lead to a maximal network throughput.

IV. CHANNEL ALLOCATION PROBLEM

A. Numerical Analysis

In order to find out the effect of different channel allocation schemes, we analyze the throughput performance of a simple network which is composed of four neighboring WLANs under different channel allocation schemes, as shown in Fig. 4, where each block represents the assigned basic channel and each block with diagonals represents the assigned primary channel. The number of available basic channels is set to $K=4$. Scenario 1 represents the case that all WLANs are allocated the same set of basic channels, we name it as “totally-overlapped”. The case that all WLANs are allocated a set of non-overlapped channels is showed in scenario 2, named as “non-overlapped”. Scenario 3 represent a random case that there are some overlapped channels among WLANs, named as “partially-overlapped”. The position of the primary channel of each WLAN in the network is not overlap in the above three scenarios. Therefore, in scenario 4, the set of basic channels assigned to each WLAN is the same as scenario 3, but with different position of primary channels, named as “partially-primary-overlapped”.

Using the aforementioned throughput computation method, we can get the throughputs of four scenarios, which can be denoted as $Th_{tot}$, $Th_{non}$, $Th_{po}$, $Th_{ppo}$ for scenario “totally-overlapped”, “non-overlapped”, “partially-overlapped”, and “partially-primary-overlapped” respectively.
overlapped”, “non-overlapped”, “partially-overlapped”, and “fully-overlapped” respectively:

\[ Th = \frac{4\lambda L}{(1 + 4\rho(4))} \]
\[ Th_{no} = \frac{4\lambda L}{1 + \rho(1)} \]
\[ Th_{po} = \frac{(6 + 8\rho(2) + 6\rho(1) + 2\rho^2(1) + 4\rho(1)\rho(2))L}{1 + \rho(4) + 3\rho(2) + 2\rho^2(2) + 2\rho(1)\rho(2)} \]
\[ Th_{ppo} = \frac{(3 + 6\rho(2) + 2\rho^2(2) + 2\rho(1)\rho(2))L}{1 + \rho(4) + 3\rho(2) + 2\rho^2(2) + 2\rho(1)\rho(2)} \]

(7)

In addition, we assume all WLANs have same size of the transmitted packet, \( L \) and have same attempt rate, \( \lambda \). We denote the normalized throughput of four scenarios shown in (7) as \( Th' \) by remove \( \lambda L \) in each equation, and with the parameters presented in Section VII, we can get

\[ \rho(1) = \frac{E[T'(1)]}{E[T'(2)]} = 170.2778 \]
\[ \rho(2) = \frac{E[T'(2)]}{E[T'(1)]} = 92.0833 \]
\[ \rho(4) = \frac{E[T'(4)]}{E[T'(1)]} = 64.4444 \]

and

\[ Th'_{to} = 0.0155 \]
\[ Th'_{no} = 0.0234 \]
\[ Th'_{po} = 0.0225 \]
\[ Th'_{ppo} = 0.0184 \]

(9)

Obviously, \( Th'_{no} > Th'_{po} > Th'_{ppo} > Th'_{to} \) will holds, a rough thought come into mind that the less overlapped channels among WLANs in the network, the better throughput performance it can achieve. We also write a simulator to describe the network operations under DCB and the achieved throughput of each WLAN is presented in Fig.5 to give us an insight of the interactions among WLANs. The point denotes the output of CTMC model and the bar denotes the output of average 1000 simulations.

From Fig.5, on the one hand, we can see the throughputs obtained from the CTMC model match well with the simulator results, which validates the correctness of the CTMC model. On the other hand, we can get two interesting findings. In scenario 1, the set of feasible network states is \( S = \{\emptyset, A_1^1, B_1^1, C_1^1, D_1^1\} \), four WLANs are all assigned the same set of basic channels, consequently they compete with all of the others for the channels, and get the same transmission probability for all WLANs in the long term, which results in the same throughput for all of them. In scenario 2, the set of feasible network states is \( S = \{\emptyset, A_1^1, B_1^1, C_1^1, D_1^1, A_2^1B_1^1, A_1^1C_1^1, A_2^1D_1^1, B_1^1C_1^1, B_1^1D_1^1, C_2^1D_1^1, A_2^1C_1^1, A_1^1B_1^1, A_1^1D_1^1, A_1^1C_1^1, B_2^1C_1^1, B_2^1D_1^1, C_3^1D_1^1, A_1^1C_1^1, B_3^1D_1^1\} \), all WLANs are allocated non-overlapped channels, which means they are completely independent of each other, each WLAN can be treated as an independent system. In scenario 3, the set of feasible network states is \( S = \{\emptyset, A_1^1, B_3^1, C_1^1, D_1^1, A_2^1B_3^1, A_1^1C_3^1, A_2^1D_1^1, B_1^1C_1^1, B_1^1D_1^1, C_2^1D_1^1, A_3^1C_1^1, B_3^1D_1^1, A_1^1B_1^1, A_1^1D_1^1, A_2^1C_1^1, B_2^1C_1^1, B_2^1D_1^1, C_3^1D_1^1, A_1^1C_1^1, B_3^1D_1^1\} \), \( WLAN_A \) has been allocated total basic channels, thus it has a partial set of channels that is same with the set of channels allocated to all the other three WLANs respectively. In this case, although \( WLAN_A \) has to compete with all the other three WLANs for the channels, it can also simultaneously transmit with \( WLAN_C \) or \( WLAN_D \) due to the channel access scheme in use, which is DCB as we described in background. Still, \( WLAN_A \) cant transmit with \( WLAN_B \) at the same time, because if the primary channel of \( WLAN_A \) is free (the backoff timer in channel 1 reaches 0), and \( WLAN_A \) has packets to transmit, then whenever \( WLAN_A \) finds channel 2 available, it will integrate channel 1 and 2 as one channel for transmission, or channel 1 to 4 if they are all available, \( WLAN_B \) cannot be
transmitting at the same time. Thus $WLAN_A$ can get the same throughput as $WLAN_B$. A comparison of the throughputs of scenarios 1 to 3 indicates that the channel allocation scheme with less number of overlapped basic channels, has a better throughput performance, which is our first interesting finding. In scenario 4, the sets of basic channels assigned to the four WLANs are the same as scenario 3, the only difference is that they have different allocation of the primary channel. In this case, the set of feasible network states is $S = \{0, A_1, B_1, C_2, D_1, A_2, A_1C_2, A_2D_1, B_1C_2, B_1D_1\}$. $WLAN_A$ has the same primary channel with $WLAN_B$, and $WLAN_C$ has the same primary channel with $WLAN_D$, thus, they can transmit together for the WLANs who have the same position of primary channel. A comparison of the throughputs of scenario 3 and 4 indicates that the channel allocation scheme with non-overlapped primary channel can get a better throughput performance, which is our second interesting finding.

B. Problem Formulation

Based on the numerical analysis, we find that a channel allocation scheme for a network should be designed with more cautious. Thus, we design a throughput maximum model for an “all-inclusive” DCB network. Let $Th_i(f)$ be the equilibrium throughput of $WLAN_i$ under the channel allocation scheme $f$, whose computation process is presented in Section III. The problem of finding the optimal channel allocation $f^*$, to maximize the system throughput for an all-inclusive DCB network, can be formulated as the following optimization problem:

$$\max f \sum_{i=1}^{N} Th_i(f)$$

s.t. $f \in F, f_i \in C$

(10)

For an “all-inclusive” DCB network with $N$ WLANs and $K$ available basic channels, let $|C|$ be the number of possible combinations of basic channels, then the number of all possible channel allocations, $|F| = |C|^N$, grows exponentially with $K$, which makes the searching for the optimal channel allocation in the feasible region is of high complexity. We will propose an optimal channel allocation algorithm in Section VI to reduce the time consumption and efficiently find the optimal solution.

Given the problem we formulated above, we then move forward to analyze the throughput performance of different channel allocation schemes and make efforts to get the optimal solution in “all-inclusive” DCB networks.

V. PERFORMANCE ANALYSIS OF CHANNEL ALLOCATION SCHEMES IN DCB NETWORKS

Consider a DCB network composed of $N$ WLANs that are all in the carrier-sensing range of each other. We also assume that there are four basic channels available (note that we will remove this assumption in Section VII, and our analysis still holds.) in the DCB network, which is labeled from channel (1) to (4). Fig. 6 shows the channel index and all possible combinations of basic channels.

Without loss of generality, we assume that the nodes in all WLANs transmit packets of a fixed size $L$, have a backoff process of equal average durations $E[B] = \lambda^{-1}$, and use the same modulation and coding rate regardless of the number of basic channels selected in the transmission. Therefore, if two WLANs use the same number of basic channels for a transmission, they have equal packet transmission durations.

Under a specific channel allocation $f$, we say that WLANs $i$ and $j$ do not overlap if $f_i \cap f_j = \emptyset$. For WLANs with overlapped basic channels, let $O_X$ be a set of WLANs with overlapped basic channels $X$, where $X$ is the intersection of basic channels of all WLANs in set $O_X$, i.e., $f_i \cap f_j = X \forall i,j \in O_X, i \neq j$. We define the number of overlapped channels of $WLAN_i$ as $O_i$, indicating that $WLAN_i$ has $O_i$ channels that interact with other WLANs in the network. It is easy to see that $O_i \leq K, \forall i$. Three situations listed as following based on the overlapped set belongs in to calculate $O_i$:

- If $WLAN_i$ doesn’t belong to any of the overlapped set, it means that the set of basic channels allocated to $WLAN_i$ doesn’t interact with any other WLANs in the network, then $O_i = 0$;
- If $WLAN_i$ only belongs to one overlapped set, i.e., $i \in O_X$, then $O_i = |X|$, where $|X|$ is the cardinality of set $X$;
- If $WLAN_i$ belongs to more than one overlapped set, i.e., $i \in O_{X1} \cap O_{X2}$, then $O_i$ is the cardinality of the union of basic channels in these overlapped sets, $O_i = |X_1 \cup X_2|$.

Then let $O(f) = \max \{O_i, \forall i\}$ be the number of overlapped channels under channel allocation scheme $f$. After further examination, we find that the throughput performance of “all-inclusive” DCB networks under different channel allocations exhibits specific features (as shown in Proposition 1).

**Proposition 1:** Let $F$ be the collection of channel allocation schemes with minimum $O(f)$. For “all-inclusive” DCB networks, the channel allocation scheme achieves the maximal system throughput exists in $F$, that is $\exists f^*, f^* = \left\{ f : \max f \sum_{i=1}^{N} Th_i(f) \right\}, f^* \in F$.

**Proof:** We separate the analysis into two parts:

1) When the number of WLANs $N$ is no more than the number of available basic channels $K$:

We consider a DCB network with four basic channels, when the number of WLANs $N$ is no more than the available basic channels $K = 4$. For each $N$, we can enumerate all possible channel allocation schemes and write out all expressions of the corresponding achieved throughputs. Then, we can compare these expressions through simple mathematical calculation to
find the maximal throughput, which corresponds to the optimal channel allocation scheme.

To save space, we will not enumerate all possible channel allocation schemes for each and only present our analytical comparisons.

i) \( N = 1 \): There is only one WLAN in the network, it is obvious that the maximal throughput can be obtained by letting the WLAN use all the channels. Thus, the optimal channel allocation scheme is \( f : f_1 = \{ 1 \} \).

ii) \( N = 2 \): For two WLANs that are within the carrier sensing range, different channel allocation schemes correspond to three categories: “totally-overlapped” (i.e., \( f : f_1 = f_2 = \{ 1, 2, 3, 4 \} \)), “partially-overlapped” (i.e., \( f : f_1 = \{ 1, 2, 3, 4 \} , f_2 = \{ 1, 2 \} \)) and “non-overlapped” (i.e., \( f : f_1 = \{ 1, 2 \} , f_2 = \{ 3, 4 \} \)). In addition, we also consider the effect of the position of primary channel in each case. If two WLANs have the same primary channel, they will never transmit packets simultaneously. After careful examination, we find that the optimal channel allocation scheme is \( f : f_1 = \{ 1, 2 \} , f_2 = \{ 3, 4 \} \). That is, there is no overlapped channels between two WLANs, and the primary channel of each WLAN does not overlap either.

iii) \( N = 3 \): In this case, although the total number of channel allocation schemes increases, we can still classify them into three categories and make comparisons. The optimal scheme is \( f : f_1 = \{ 1, 2 \} , f_2 = \{ 3 \} , f_3 = \{ 4 \} \), and there is neither overlapped basic channels nor overlapped primary channels over these three WLANs.

iv) \( N = 4 \): After examination, we find that the optimal channel allocation scheme is \( f : f_1 = \{ 1 \} , f_2 = \{ 2 \} , f_3 = \{ 3 \} , f_4 = \{ 4 \} \).

When \( N \leq K \), we have \( F = \{ f : O(f) = 0 \} \). In the first three cases, there is more than one channel allocation scheme with \( O(f) = 0 \). Summarizing these four scenarios above, we can see that the channel allocation scheme achieves the maximum network throughput always exists in \( F \), so that Proposition 1 stands when \( N \leq K \).

Based on Proposition 1 when the number of basic channels are sufficient enough to afford each WLAN non-overlapped channels, we can assign a set of contiguous non-overlapped basic channels to each WLAN, in order to get the channel allocation scheme with minimum \( O(f) \), that is \( O(f) = 0 \) under this situation. With a litter abuse of notion, we let \( k_i \) denote the number of non-overlapped channels allocated to WLAN \( i \). Then the channel allocation problem can be rewritten as an INLP problem:

\[
\begin{align*}
\max \quad h(k_1, \cdots, k_N) &= \sum_{i=1}^{N} Th_i = \sum_{i=1}^{N} \frac{A}{1 + \rho(k_i)} & (11a) \\
\text{s.t.} \quad \sum_{i=1}^{N} k_i &\leq K \quad (11b) \\
k_i &= 2^j, j = 0, 1, 2, 3 & (11c)
\end{align*}
\]

where \( A = \lambda L \), which is a constant in our assumptions. The objective function \( h(k_1, \cdots, k_N) \) in (11.a) represents the aggregate network throughput under the allocating scheme \( K = [k_1, \cdots, k_N] \). (11.b) means that total number of channels used by WLANs must not exceed total number of basic channels available. (11.c) is an accompaniment brought by the IEEE 802.11ac standard, the number of channels used by each WLAN is an exponential function with base number of two, which is also a positive integer, and the number of channels that can be integrated together for a transmission must not exceed 8.

\( \rho(k_i) \) in (11.a) means the activity ratio of WLAN \( i \) using \( k_i \) basic channels for current transmission and \( \rho(k_i) \) is a discrete function of \( k_i \). In order to solve this channel allocation optimization problem, we first use the order of CFTOOL in MATLAB, under the parameters presented in Section VII, to fit a continuous curve for \( \rho(k_i) \), denoted as \( \rho'(k_i) \). Then, we relax \( k_i \) to a continuous variable. Thus the optimization problem can change into a non-integer nonlinear programming (NINLP) problem:

\[
\begin{align*}
\max \quad h'(k_1, \cdots, k_N) &= \sum_{i=1}^{N} Th_i = \sum_{i=1}^{N} \frac{A}{1 + \rho'(k_i)} & (12a) \\
\text{s.t.} \quad \sum_{i=1}^{N} k_i &\leq K \quad (12b)
\end{align*}
\]

where \( \rho'(k_i) = \frac{b}{(k_i)^a} = \frac{168.2}{(k_i)^{89/144}} \).

We can comply with the standard solution of Lagrange Multiplier Approach to solve problem (12). Introducing a Lagrange multiplier \( \gamma \) to establish a Lagrange function \( H(k_1, \cdots, k_N, \gamma) \) first, which is

\[
H(k_1, \cdots, k_N, \gamma) = \sum_{i=1}^{N} \frac{A}{1 + \rho'(k_i)} + \gamma (k_1 + \cdots + k_N - K) \quad (13)
\]

then we take the derivatives of \( H(k_1, \cdots, k_N, \gamma) \) for all unknown quantities in the function, and make them equal to zero to get extreme points, which can be represented as:

\[
\begin{align*}
\frac{\partial H}{\partial k_1} &= \frac{Ank_1^{-a-1}}{[1+\rho'(k_1)]^2} + \gamma = 0 \\
\vdots \quad & \vdots \\
\frac{\partial H}{\partial k_N} &= \frac{Ank_N^{-a-1}}{[1+\rho'(k_N)]^2} + \gamma = 0 \\
\frac{\partial H}{\partial \gamma} &= k_1 + \cdots + k_N - K = 0 \quad (14)
\end{align*}
\]

By solving (14), we can get an allocating scheme, \( K = [k, \cdots, k] \), where \( k = K/N \), \( k \) can be interpreted as the mean number of non-overlapped channels that allocated to each WLAN, which might be a non-integer.

As for (12.b), the constraint is linear, consequently, we need to prove that the objective function is a concave function. To this end, we need prove the odd-order partial derivation of
As last, we use the BBM to solve (11). Problem (12) is the optimal solution of (11). Otherwise, chose a non-integer variable randomly, \( k_i, i \in \{1, \ldots, N\} \), we can get two branches by adding the constrains \( k_i \leq \frac{k_j}{\tilde{k}} \) and \( k_i \geq \frac{k_j}{\tilde{k}} \) to (12) respectively (if \( 2^m \leq k_i \leq 2^n, m, n = 0, 1, 2, 3, \) then \( \frac{k_i}{\tilde{k}} = 2^m, \frac{k_j}{\tilde{k}} = 2^n \). Setting the upper bound as \( h'(\tilde{k}, \cdots, \tilde{k}) \) and the lower bound as \( h(1, \cdots, 1) \). For every variable, \( k_i \), we can get two branches, owing to the objective function is a concave function, using Lagrange Multiplier Approach we can get the optimal solution of each branch. If the optimal solution is a feasible integer solution then stop branching under this branch and we update the lower bound into the objective value with this optimal solution, otherwise, substituting them to the objective function and comparing target values, cutting the branch with smaller value, updating the upper bound into the bigger value. Then keep branching under the remaining branch. Each branch corresponding to a sub-problem, by solving a series of relaxation problems of sub-problems, until we get a feasible integer solution, then updating the lower bound in order to make sure that the lower bound is the biggest objective value among all the feasible solutions. When this iteration end, we take the allocating scheme corresponding to the present lower bound as the optimal feasible solution of (11).

After obtaining the optimal allocating scheme, \( K^* = [k_1, \cdots, k_N] \), for the corresponding optimal channel allocation scheme, we set \( f_i = \{1 + \sum_{j<i} \min(k_j, 8), \cdots, \sum_{j<i} \min(k_j, 8)\} \) for all \( i \in \{1, 2, \ldots, N\} \), and set the first channel in \( f_i \) as the primary channel of WLAN \( i \), i.e., \( K^* = [2, 1] \) and \( f : f_1 = \{1, 2\}, f_2 = \{3\} \).

2) When the number of WLANs \( N \) is larger than the number of available channels \( K \):

If the network has more than 4 WLANs and we do not have enough channels to allocate each WLAN non-overlapped channels, there must be overlapped channels between WLANs. An interesting issue is that how these WLANs overlap can get the optimal throughput. In this case, we use the term Spectrum Efficiency (SE) to measure the expected network throughput per frequency spectrum, that is

\[
\eta(f) = \frac{T_{\theta X}(f)}{BW_{\omega X}(f)}
\]

where \( T_{\theta X}(f) \) and \( BW_{\omega X}(f) \) is the achieved throughput and the total bandwidth been used of an overlapped set, \( O_X \), under channel allocation scheme \( f \) respectively.

When there are overlapped channels among WLANs, we have \( \mathcal{F} = \{f : O(f) = 1\} \), the ways they can overlap with each other will directly influence the interactions of WLANs. Hence, its important to find an efficient way of overlapping to maximize the throughput per spectrum frequency. We want to prove that for a set of WLANs with overlapped channels, \( O_X \), the channel allocation scheme with \( |X| = 1 \) can get the optimal SE for WLANs in this set. The following analysis is based on the mathematical induction.

i) We first consider there are two WLANs in this set, i.e., \( O_X = \{i, j\}, |O_X| = 2 \). There are three ways that WLAN and

Although the acquisitions of primary channels of WLAN \( i \) and \( j \) may be different, we can get the same set of feasible network states, we classify them as the same type (i.e., \( f : f_i = \{1, 2\}, f_j = \{1, 2, 3, 4\} \) and \( f : f_i = \{1, 2\}, f_j = \{1, 2, 3, 4\} \).
ii) We next assume our conclusion stands when there are $M$ WLANs overlap with each other, $|O_X| = M$, which means there is a scheme with $|\mathcal{X}| = 1$ can get the optimal SE of WLANs in this set, $O_X$. iii) Finally we should prove that our conclusion works when there are $M+1$ WLANs overlap with each other, $|O_X| = M+1$, which means there exists a scheme with $|\mathcal{X}| = 1$ can get the optimal SE of WLANs in this set, $O_X$. We label the newly added WLAN as $WLAN_m$. We can treat the original $M$ WLANs as a group, which is using the same basic channel and primary channel for transmission. $f : f_i = \{1\}, \forall i \in M$, according to ii) the optimal SE for this group can be obtained. We use the same manner as i) to characterize the ways of overlapping between group $M$ and $WLAN_m$. We treat them as two WLANs with overlapped channels, i.e., $O_X = \{M, m\}$, $|O_X| = 2$, then after enumerate and compare all possible overlapping allocation schemes we can obtain there is a scheme with $|\mathcal{X}| = 1$ has the optimal SE of WLANs in this set, $O_X$. According to the above analysis, we can arrive at the conclusion that for an overlapped set, $O_X$, there is a channel allocation scheme with $|\mathcal{X}| = 1$ can get the optimal SE of WLANs in this set. Therefore, when the number of WLANs is larger than the number of available channels, we could divide $N$ WLANs into $K$ groups and each group is assigned an independent basic channel. Each group of WLANs is a set of WLANs with one overlapped channel, i.e., $O_X, |\mathcal{X}| = 1$. Thus the optimal SE of WLANs in each set can be obtained under this channel allocation scheme, and the optimal throughput performance will be obtained with some adjustments of the number of WLANs falls into each group. So that Proposition 1 stands when $N > K$.

Based on Proposition 1 when the number of WLANs is greater than the number of available basic channels, we can convert the channel allocation optimization problem into an INLP problem:

\[
\begin{align*}
\text{max} & \quad g(n_1, \ldots, n_K) = \sum_{k=1}^{K} \frac{A_{n_k}}{1 + B_{n_k}} \\
\text{s.t.} & \quad \sum_{k=1}^{K} n_k = N \\
& \quad n_k \geq 1, n_k \in \mathbb{N}^+ 
\end{align*}
\]

where $n_k, k \in \{1, \ldots, K\}$ represents the number of WLANs that is allocated channel $k$. $A = \lambda L$ and $B = \rho (1)$, they are both constant in our assumptions. The objective function $g(n_1, \ldots, n_K)$ in (18.a) represents the aggregate network throughput under the grouping scheme $\mathcal{N} = \{n_1, \ldots, n_K\}$. (18.b) means that the sum of $n_k$ must equal to the total number of WLANs $N$, and (18.c) means $n_k$ must be a positive integer.

To solve this WLAN grouping optimization problem, we first relax this integer nonlinear programming (INLP) problem into a non-integer nonlinear programming (NINLP) problem. Then, we can get the optimal solution of this NINLP using Lagrange Multiplier Approach. Introducing a Lagrange multiplier $\xi$ to establish a Lagrange function $G(n_1, \ldots, n_K, \xi)$, which is

\[
G(n_1, \ldots, n_K, \xi) = \sum_{k=1}^{K} \frac{B_{n_k}}{1 + A_{n_k}} + \xi (n_1 + \cdots + n_K - N)
\]

then we take the derivatives of $G(n_1, n_2, n_3, n_4, \xi)$ for each unknown quantities in the function, and make them equal to

\[
\begin{align*}
\frac{\partial G}{\partial n_k} &= \frac{B_{n_k}}{(1 + A_{n_k})^2} - \xi \\
\text{for } k = 1, \ldots, K \\
\end{align*}
\]

Table I: An Example of the Channel Allocation Algorithm

| The number of overlapped channels | Possible channel allocation scheme | SE |
|----------------------------------|----------------------------------|----|
| $|\mathcal{X}| = 1$             | $f_1 : f_i = \{1\}$, $f_j = \{1\}$ | $\eta(f_1) = \frac{2ML}{20(1 + 2\rho(1))} = \frac{ML}{10(1 + 2\rho(1))}$ |
| $f_2 : f_i = \{1\}$, $f_j = \{1, 2\}$ | $\eta(f_2) = \frac{2ML}{40(1 + \rho(1) + \rho(2))} = \frac{ML}{20(1 + \rho(1) + \rho(2))}$ |
| $f_3 : f_i = \{1\}$, $f_j = \{1, 2\}$ | $\eta(f_3) = \frac{3ML + 2\rho(1)\lambda L}{40(1 + 2\rho(1) + \rho(2) + 2\rho(1) + \rho(2))}$ |
| $f_4 : f_i = \{1\}$, $f_j = \{1, 23, 4\}$ | $\eta(f_4) = \frac{3ML + 2\rho(1)\lambda L}{80(1 + \rho(1) + \rho(2) + 4\rho(4))}$ |
| $f_5 : f_i = \{1\}$, $f_j = \{1, 2, 3, 4\}$ | $\eta(f_5) = \frac{3ML + 2\rho(1)\lambda L}{80(1 + \rho(1) + \rho(2) + 4\rho(4) + 2\rho(1) + \rho(2))}$ |
| $f_6 : f_i = \{1\}$, $f_j = \{1, 2, 3, 4\}$ | $\eta(f_6) = \frac{3ML + 2\rho(1)\lambda L}{80(1 + \rho(1) + \rho(2) + 4\rho(4) + 2\rho(1) + \rho(2))}$ |
| $|\mathcal{X}| = 2$             | $f_7 : f_i = \{1, 2\}$, $f_j = \{1, 2\}$ | $\eta(f_7) = \frac{2ML}{40(1 + 2\rho(2))} = \frac{ML}{20(1 + 2\rho(2))}$ |
| $f_8 : f_i = \{1, 2\}$, $f_j = \{1, 2, 3, 4\}$ | $\eta(f_8) = \frac{2ML}{80(1 + \rho(2) + \rho(4))}$ |
| $f_9 : f_i = \{1, 2\}$, $f_j = \{1, 2, 3, 4\}$ | $\eta(f_9) = \frac{3ML + 2\rho(2)\lambda L}{80(1 + \rho(2) + \rho(4) + 2\rho(1) + \rho(2))}$ |
| $|\mathcal{X}| = 4$             | $f_{10} : f_i = \{1, 2, 3, 4\}$, $f_j = \{1, 2, 3, 4\}$ | $\eta(f_{10}) = \frac{3ML + 2\rho(2)\lambda L}{80(1 + \rho(2) + \rho(4) + 2\rho(1) + \rho(2))}$ |

Convert the channel allocation optimization problem into an INLP problem:

\[
\begin{align*}
\text{max} \quad g(n_1, \ldots, n_K) &= \sum_{k=1}^{K} \frac{A_{n_k}}{1 + B_{n_k}} \\
\text{s.t.} \quad \sum_{k=1}^{K} n_k &= N \\
& \quad n_k \geq 1, n_k \in \mathbb{N}^+ 
\end{align*}
\]
zero to get extreme points, which can be represented as

\[
\frac{\partial g}{\partial n_1} = -\frac{2AB}{1 + Bn_k}, \quad \forall k \in \{1, \cdots, K\} \quad (20)
\]

\[
\frac{\partial^2 g}{\partial^2 n_k} = 0, \quad \forall i, j \in \{1, \cdots, K\}, \quad i \neq j \quad (22)
\]

By solving (20) we can get a group scheme \(N = \{n, \cdots, \bar{n}\}\), where \(\bar{n} = N/k, \bar{n}\) can be interpreted as the mean number of WLANs that be allocated the same single basic channel, which might be a non-integer.

The Hessian matrix of (18.a) is

\[
H = \begin{bmatrix}
\frac{\partial g^2}{\partial n_1^2} & \frac{\partial g^2}{\partial n_1 n_2} & \cdots & \frac{\partial g^2}{\partial n_1 n_K} \\
\frac{\partial g^2}{\partial n_2 n_1} & \frac{\partial g^2}{\partial n_2 n_2} & \cdots & \frac{\partial g^2}{\partial n_2 n_K} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial g^2}{\partial n_K n_1} & \frac{\partial g^2}{\partial n_K n_2} & \cdots & \frac{\partial g^2}{\partial n_K n_K}
\end{bmatrix}
\]

the elements of \(H\) are given as

\[
\frac{\partial g^2}{\partial^2 n_k} = -\frac{2AB}{1 + Bn_k}, \quad \forall k \in \{1, \cdots, K\} \quad (21)
\]

(1) Solve the relaxation problem of the convex integer nonlinear programming (NINLP) problem (i.e., (12) of (11)), obtain an optimal solution. Substitute the optimal solution to the objective function, set the objective value as the upper bound and set the objective value of a random feasible integer solution as the lower bound.

(2) If this optimal solution is a feasible solution, output this optimal solution; otherwise, go to next step.

(3) Choose a non-integer variable randomly, add two constraints (i.e., \(k_i \leq \bar{k}\) and \(k_j \geq \bar{k}\)) to the relaxation problem to construct two new relaxation problems, that is two branches.

(4) Use Lagrange Multiplier Approach to get the optimal solution of each branch, substitute the optimal solution of each branch to the objective function, obtain the objective value.

(5) If one of the optimal solution is a feasible solution, update its objective value as the new lower bound. What's more, if this objective value greater than or equal to the objective value of the other branch, then cut the other branch and output this optimal solution; otherwise, go to next step.

(6) If there is an objective value of a non-integer solution greater than the objective value of the other branch, update its objective value as the new upper bound, then repeat steps 3-5 to the branch with this non-integer solution until the optimal solution is obtained.

(7) According to the optimal allocating scheme and grouping scheme, obtain the optimal channel allocation scheme.

Due to the convexity in (11.a) and (18.a), this channel allocation algorithm based on the Branch and Bound Method yields a solution that in general is the global optimal solution of (11) and (18). The computations are simple because the equal or less than \(O(N)\) or \(O(K)\) complexity involved in this computation scales only with the size of the number of WLANs, \(N\), or the number of available basic channels, \(K\), and is independent of the size of \(C\) which grows exponentially with \(K\).

To better illustrate the proposed algorithm, we give the procedures of network settings with \(N = 3, K = 7\) in Table II. The initial lower bound is set as the objective value of allocating scheme \(K = \{1, 1, 1\}\), that is \(h(1, 1, 1)\), the initial upper bound is set as the objective value of allocating scheme \(K = \{7/3, 7/3, 7/3\}\), that is \(h^*(7/3, 7/3, 7/3)\). There are two branches in each iteration, we have to judge the feasibility.
Table III: An Example of the Greedy-based Algorithm

| Example | Feasibility | Objective Value |
|---------|-------------|-----------------|
| 1st     | Yes         | 186.8310        |
| 2nd     | Yes         | 239.1467        |
| 3rd     | Yes         | 287.5422        |
| 4th     | No          | /               |
| 5th     | Yes         | 339.8579        |
| Result  | Yes         | 339.8579        |

of each branch, that is to see whether it is a feasible integer solution (satisfy the constrains in (11) or (18)). What more, we update the lower bound and upper bound according to our algorithm and eventually find out the biggest lower bound.

Table III shows an example of the Greedy-based algorithm, each variable attempts to maximize their own interest at each iteration. We set the initial state as $K = \{1, 1, 1\}$, in each iteration, we choose a variable in an ascending order, and double the number of channels assigned to the variable until the number of available basic channels allows it. The final objective values are shown in the last row of two Tables, the proposed algorithm shows better objective value than the Greedy-based algorithm. An interesting question comes into mind, in the Greedy-based algorithm, the whole number of available basic channels come in handy, why achieves a lower objective value? We will explain this phenomenon in the next part.

The procedures to find a grouping scheme of the proposed algorithm and the Greedy-based algorithm are similar to the above. Only in Greedy-based algorithm, we increase the number of WLANs in each group one value in each iteration until the number of WLANs allows it. For example, in the network settings with $N = 7, K = 3$, the grouping scheme obtained by the proposed algorithm is $N = \{2, 2, 3\}$ and the one obtained by the Greedy-based algorithm is $N = \{5, 1, 1\}$, we have $g(2, 2, 3) > g(5, 1, 1)$.

B. Case of Interest

We consider a scene that there are 3 neighboring WLANs and 7 basic channels available. Our allocating scheme obtained by Channel Allocation Algorithm is $K = \{2, 2, 2\}$ and the corresponding channel allocation scheme is $f : f_A = \{1, 2\}, f_B = \{3, 4\}, f_C = \{5, 6\}$, under this scheme, there is only 6 channels are used for transmission. However, the allocating scheme obtained by the Greedy-based algorithm is $K = \{4, 2, 1\}$ and the corresponding channel allocation scheme is $f : f_A = \{1, 2, 3, 4\}, f_B = \{5, 6\}, f_C = \{?\}$, there are 7 channels are used for transmission.

We use the simulation parameters present in Section VII to compare two solutions in terms of achieved throughput in each WLAN, the network-wide throughput and JFI. What more, we use the term Gain to represent the rate of throughput increase of each WLAN between two solutions. For example, the number of channels allocated to $WLAN_A$ is 2 in our solution, and is 4 in the Greedy-based solution, the Gain of $WLAN_A$ is $[Th_A (4) - Th_A (2)] / Th_A (2)$, in the same way, the Gain of $WLAN_C$ is $[Th_C (2) - Th_C (1)] / Th (1)$, the number in brackets represent the number of allocated non-overlapped channels $k_i$ for $WLAN_i$.

From Table IV, we can see although the scheme obtained by the Channel Allocation Algorithm do not use the total number of available basic channels, but it gets a better network-wide throughput and JFI than Greedy-based Algorithm. Through the Gain of each WLAN we can get the reason of this phenomenon, that is the Gain obtained by WLAN A is less than the Gain obtained by WLAN C. From the insight of engineering practice, the duration of some headers and preambles is not affected by the channel width in 802.11ac network. Therefore, doubling the number of a WLANs allocated basic channels, the transmission rate of this WLAN cant boost up to twice that. Indeed, the more number of basic channels is, the less Gain can get by doubling it. What more, from the point of view of JFI, allocating each WLAN approximately equal number of non-overlapped basic channels can guarantee a good fairness among network.

VII. Simulation Results

This section evaluates the performance of our proposed algorithm through MATLAB simulations. The wireless networking environment is configured as a bulk of WLANs which are all in the carrier-sensing range of each other. Simulation parameters are presented in Table V, given by amendment 802.11ac to keep the error probability $p_e$ below 10%. Using these parameters we can calculate the packet transmission duration $T(k')$, for each channel number in use $k'$, as shown in Table VI, $M, R$ is the modulation and the coding rate respectively, $\varepsilon (k')$ is the number of data subcarriers when $k'$ basic channels are used.

A. Model Validation

In this part, we investigate correctness of the CTMC model. Fig. 7 shows the throughput performance of the illustrating example of Fig. 3 to the Backoff Contention Window ($CW$) duration. The relationship between $\lambda$ and $CW$ is given by $\lambda = \frac{2}{CW T_{min}}$. We use a continuous time backoff in our simulator, which results in a zero probability of collision. Each point of the simulation is the result of average 1000 simulations. From Fig. 7, we can see the analytical and simulated results shows a very good match.

B. Analysis Validation

We set the number of basic channels to be $K = 4$ to validate our previous analysis in Section V. The number of WLANs increases from 1 to 10. When the number of WLANs increases from 1 to 4, we use our proposed algorithm to solve problem (11) to find an optimal allocating scheme and the corresponding channel allocation scheme. When the number of WLANs increases from 5 to 10, we use our proposed algorithm to solve problem (18) to get an optimal group
scheme and the corresponding channel allocation scheme. We also investigate two random selection schemes for comparison, in the first one we select a random channel combination for a fixed bandwidth (see Fig. 8(a)), in the other one we select a channel combination with channel width randomly select from 20MHz to \(BW_{\text{max}}\) (see Fig. 8(b)). Note that the effect of randomly selecting \(BW\) increases the opportunities that different WLANs interact with each other because more combinations are feasible. In order to reduce randomness, the output of random selection schemes is an average value of 1000 simulations. From Fig. 8, we can see our proposed algorithm can always get the same throughput performance as the optimal solution, which obtained by exhaustively search in the feasible region. Moreover, the throughput achieved by our proposed algorithm is always better than random selection schemes. It is worthwhile to mention that when the number of WLANs increases from 1 to 4, the throughput achieved by the Greedy-based Algorithm is same as our proposed algorithm, however, there is a slightly drop compare to the proposed algorithm when the number of WLANs increases from 5 to 10. This is because in Greedy-based Algorithm, when \(N > K\), every group wants to recruit more WLANs to boost its SE, and the grouping scheme obtained by our proposed algorithm tends to uniformly allocate the number of WLANs in each group. In the objective function of (18), \(B=p(1) \gg 1\), we have \(\frac{A_{\text{k}}}{1+B_{\text{k}}} \approx \frac{A_{\text{k}}}{B_{\text{k}}}\), thus when \(N > K\), the optimization of grouping scheme can only get a slightly throughput increment. But it can get a better fairness among WLANs, we will present it in the next part.

C. Effectiveness of Channel Allocation Algorithm

We consider a more practical situation of the IEEE 802.11ac WLAN. Obviously, there are more than 4 available basic channels in 5GHz band, as we can see from Fig. 1. Hence, we set \(K = 17\) and increase the number of WLANs from 1 to

**TABLE IV**

Comparisons Between the Proposed Algorithm and the Greedy-based Algorithm

| Parameters              | Aggregate Throughput(Mbps) | JFI |
|-------------------------|-----------------------------|-----|
| Allocating scheme \(\{k_A, k_B, k_C\}\) | WLAN A | WLAN B | WLAN C | SUM |
| Channel Allocation Algorithm | \(2, 2, 2\) | 114.5927 | 114.5927 | 114.5927 | 343.7780 | 1 |
| Greedy-based Algorithm | \(4, 2, 1\) | 162.9881 | 114.5927 | 62.2770 | 339.9578 | 0.8836 |
| Gain | / | 0.4223 | 0 | 0.4565 | / | / |

**TABLE V**

Parameters Values Based on IEEE 802.11ac

| Parameter                  | Notation | Value |
|----------------------------|----------|-------|
| Packet length              | \(L_d\)  | 12000bits |
| Number of aggregated packets | \(K_A\) | 64 packets |
| Contention window          | \(CW\)   | 16 slots |
| Slot duration              | \(T_{\text{slot}}\) | 9\mu s |
| Average backoff duration   | \(E(B)\) | \(CW\frac{T_{\text{slot}}}{2}\) |

**TABLE VI**

Transmission Duration for Different Channel Number in Use

| \(k'\) | \(\varepsilon (k')\) | \(M\) | \(R\) | \(T (k')\) |
|--------|----------------------|------|------|-----------|
| 1      | 52                   | 6 bits(64-QAM) | 5/6 | 12.26ms  |
| 2      | 108                  | 6 bits(64-QAM) | 3/4 | 6.63ms   |
| 3      | 234                  | 4 bits(16-QAM) | 3/4 | 4.64ms   |
| 4      | 468                  | 4 bits(16-QAM) | 1/2 | 3.52ms   |

Fig. 7. Throughput comparison between analysis and simulation

Fig. 8. Comparison of throughput performance when \(K = 4\)
20. We present the system performance comparison in terms of aggregate throughput, JFI channel and utilization. Channel utilization is computed as the fraction of basic channels that are occupied by one or more WLANs versus the total number of basic channels, i.e.,

$$\Gamma (f) = \frac{1}{K} \sum_{k=1}^{K} I (k)$$

(23)

where the I (k) equals to 1 if the basic channel k is found occupied by one or more WLANs. Moreover, the JFI with respect to the throughput is

$$J = \frac{\left( \sum_{i=1}^{N} T h_i \right)^2}{N \sum_{i=1}^{N} T h_i^2}$$

(24)

When the number of WLANs increases from 1 to 10, we compare the proposed algorithm with optimal (exhaustive) scheme, the Greedy-based algorithm and the randomly selected scheme. From Fig. 5 we can see when the number of WLANs increases, the proposed algorithm can always get the same results as the optimal throughput performance. Although the Greedy-based algorithm has a good channel utilization performance, owing to every WLAN tends to make use of the remaining available basic channels, the performance of the proposed algorithm always better than other schemes in throughput and JFI.

In addition, we investigate when the number of WLANs increase from 10 to 20, since the exhaustive search is not feasible for a large number of WLANs, it has been excluded from the evaluation, and since we set K = 17, the channel utilizations of the proposed algorithm and the Greedy-based algorithm are all 1 when the number of WLANs increases, it means under larger size networks, the network tends to make use of all available channels, thus we dont exhibit it here. From Fig. 11, we can see the proposed algorithm always get a better throughput performance and JFI when he number of WLANs increase from 10 to 20, and the Greedy-based algorithm can get a similar throughput as the proposed algorithm but with a poorer JFI when the number of WLANs gets bigger. When the number of WLANs increases from 17 to 20, JFI of the Greedy-based algorithm presents a downward trend, in the same time, through the optimization of grouping scheme, the proposed algorithm can maintain a better JFI and without throughput drop.

VIII. Conclusion

This paper adopted a CTMC model to analyze the equilibrium throughputs of “all-inclusive” DCB WLANs and examined the throughput performance under different channel allocation schemes. We first proved that the maximal throughput performance can be achieved under one of the channel allocation scheme with the least overlapped channels. Based on this understanding, we then construct an INLP model in order to maximize the system throughput, finally we proposed a channel allocation algorithm based on the Branch and Bound Method to solve the INLP model. Extensive evaluations were conducted and showed that the proposed algorithm can always get the optimal throughput performance and outperforms the random channel selection schemes and the Greedy-based algorithm. We hope that our analysis on the optimal channel allocation schemes brings new insights into the design and optimization of future WLANs. As a future work, we will investigate the performance of a not-all-inclusive DCB network in which not all the WLANs are within the carrier-sensing range of each other and find efficient channel allocation algorithms to optimize system performance.

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Fig. 9. Performance comparison of different algorithms as the number of WLANs increases ($BW$ fixed)

Fig. 10. Performance comparison of different algorithms as the number of WLANs increases ($BW$ random)

Fig. 11. Throughput and JFI performance in larger size networks

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