UHECR BOUNDS ON LORENTZ VIOLATION IN THE PHOTON SECTOR

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The aim of this brief review is to present a case study of how astrophysics data can be used to get bounds on Lorentz-violating parameters. For this purpose, a particularly simple Lorentz-violating modification of the Maxwell theory of photons is considered, which maintains gauge invariance, CPT, and renormalization. With a standard spin $\frac{1}{2}$ Dirac particle minimally coupled to this nonstandard photon, the resulting modified-quantum-electrodynamics model involves nineteen dimensionless “deformation parameters.” Ten of these parameters lead to birefringence and are already tightly constrained by astrophysics. New bounds on the remaining nine nonbirefringent parameters have been obtained from the inferred absence of vacuum Cherenkov radiation in ultrahigh-energy-cosmic-ray (UHECR) events. The resulting astrophysics bounds improve considerably upon current laboratory bounds and the implications of this “null experiment” may be profound, both for elementary particle physics and cosmology.

1 Introduction

One of the fundamental questions of physics is the following: does space remain smooth as one probes smaller and smaller distances?

A conservative limit on the typical length scale $\ell$ of any nontrivial small-scale structure of space comes from particle-collider experiments which see no need to change Minkowski spacetime:

$$\text{LEP/Tevatron: } \ell \lesssim 10^{-18} \text{ m} \approx \hbar c/(200 \text{ GeV}) .$$

Yet, astrophysics (or, more specifically, “astroparticle physics”) provides us with very much higher energies. Here, we intend to present a case study of what astrophysics can do, provided the underlying physics is well understood. References will be primarily to research papers.

The proposed case study proceeds in three steps. First, we discuss the phenomenology of a simple photon-propagation model. Second, we obtain bounds on the model parameters from ultrahigh-energy cosmic rays (UHECRs). Third, we consider the theoretical implications of our results. The contribution concludes with a brief outlook.

2 Phenomenology

2.1 Model

Consider the following action for a Lorentz-violating (LV) deformation of quantum electrodynamics (QED):

$$S_{\text{modQED}} = S_{\text{modM}} + S_{\text{standD}},$$

(2)
with a modified-Maxwell term [1, 2],
\[
S_{\text{modM}} = \int_{\mathbb{R}^4} \, d^4 x \left( -\frac{1}{4} \left( \eta^{\mu\rho} \eta^{\nu\sigma} + \kappa^{\mu\nu\rho\sigma} \right) \left( \partial_{\mu} A_{\nu}(x) - \partial_{\nu} A_{\mu}(x) \right) \left( \partial_{\rho} A_{\sigma}(x) - \partial_{\sigma} A_{\rho}(x) \right) \right), \tag{3a}
\]
and the standard Dirac term for a spin-- particle with electric charge \( e \) and mass \( M \),
\[
S_{\text{standD}} = \int_{\mathbb{R}^4} \, d^4 x \, \bar{\psi}(x) \left( \gamma^{\mu} \left( i \partial_{\mu} - e A_{\mu}(x) \right) - M \right) \psi(x). \tag{3b}
\]

Here and in the following, natural units are used with \( c = \hbar = 1 \), but, occasionally, \( c \) or \( \hbar \) are displayed in order to clarify the physical dimension of a particular expression. The fundamental constant \( c \) now corresponds to the maximum attainable velocity of the Dirac particle or, more importantly, to the causal velocity from the underlying Minkowski spacetime with Cartesian coordinates \( (x^\mu) = (x^0, x) = (ct, x^1, x^2, x^3) \) and metric \( g_{\mu\nu}(x) = \eta_{\mu\nu} \equiv \text{diag} \left( +1, -1, -1, -1 \right) \).

The real dimensionless numbers \( \kappa^{\mu\nu\rho\sigma} \) in (3a) are considered to be fixed once and for all, which makes the model Lorentz noninvariant. However, the action (2) is still gauge-invariant, CPT–even, and power-counting renormalizable [3]. Clearly, these properties make the model worthwhile to study theoretically and to verify/falseify experimentally.

In the modified-Maxwell term (3a), \( \kappa^{\mu\nu\rho\sigma} \) is a constant background tensor with the same symmetries as the Riemann curvature tensor and a double trace condition \( \kappa^{\mu\nu}_{\;\;\rho\sigma} = 0 \), so that there are \( 20 - 1 = 19 \) components.

As ten birefringent parameters are already constrained at the \( 10^{-32} \) level [4] by spectro-polarimetric measurements of distant astronomical sources, the model can be restricted to the nonbirefringent sector by making the following Ansatz [5]:
\[
\kappa^{\mu\nu\rho\sigma} = \frac{1}{2} \left( \eta^{\mu\rho} \bar{\kappa}^{\nu\sigma} - \eta^{\mu\sigma} \bar{\kappa}^{\nu\rho} + \eta^{\nu\sigma} \bar{\kappa}^{\rho\mu} - \eta^{\nu\rho} \bar{\kappa}^{\mu\sigma} \right), \tag{4}
\]
for a symmetric and traceless matrix \( \bar{\kappa}^{\mu\nu} \) with \( 10 - 1 = 9 \) components.

Hence, there are nine LV deformation parameters \( \bar{\kappa}^{\mu\nu} \) to investigate. It turns out to be useful to rewrite these parameters as follows:
\[
\left( \bar{\kappa}^{\mu\nu} \right) \equiv \text{diag} \left( 1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \bar{\kappa}^{00} + \left( \delta \bar{\kappa}^{\mu\nu} \right), \quad \delta \bar{\kappa}^{00} = 0, \tag{5}
\]
with a single independent parameter \( \bar{\kappa}^{00} \) for the spatially isotropic part of \( \bar{\kappa}^{\mu\nu} \) and eight independent parameters \( \delta \bar{\kappa}^{\mu\nu} \) for the nonisotropic part. Finally, we can express these parameters in terms of the so-called standard-model-extension (SME) parameters [4]:
\[
\begin{pmatrix}
\bar{\kappa}^{00} \\
\delta \bar{\kappa}^{01} \\
\delta \bar{\kappa}^{02} \\
\delta \bar{\kappa}^{03} \\
\delta \bar{\kappa}^{11} \\
\delta \bar{\kappa}^{12} \\
\delta \bar{\kappa}^{13} \\
\delta \bar{\kappa}^{22} \\
\delta \bar{\kappa}^{23} \\
\end{pmatrix}
= \begin{pmatrix}
\frac{3}{2} \bar{\kappa}_{\text{tr}} \\
-\bar{\kappa}_{\text{O+}}^{(23)} \\
-\bar{\kappa}_{\text{O+}}^{(31)} \\
-\bar{\kappa}_{\text{O+}}^{(12)} \\
-\bar{\kappa}_{\text{e-}}^{(11)} \\
-\bar{\kappa}_{\text{e-}}^{(12)} \\
-\bar{\kappa}_{\text{e-}}^{(13)} \\
-\bar{\kappa}_{\text{e-}}^{(22)} \\
-\bar{\kappa}_{\text{e-}}^{(23)} \\
\end{pmatrix}, \tag{6}
\]
where three parity-odd parameters determine an antisymmetric traceless \( 3 \times 3 \) matrix \((\bar{\kappa}_{\text{O+}})^{mn}\) and five parity-even parameters a symmetric traceless \( 3 \times 3 \) matrix \((\bar{\kappa}_{\text{e-}})^{mn}\).
2.2 Possible spacetime origin

We have already mentioned that the modified-Maxwell model has attractive properties (gauge invariance, CPT invariance, and renormalizability). Still, it would be better if we had a concrete example of how the modified-Maxwell model might arise from an underlying theory.

Precisely this is accomplished by a calculation \cite{6} of the propagation of standard photons and standard Dirac particles over a classical spacetime-foam manifold (Fig. 1–left), which reproduces a restricted, isotropic version of model (2):

\[
2\tilde{\kappa}_{\mu\nu} = -\tilde{\sigma}_2 \tilde{F}, \quad \delta\tilde{\kappa}^{\mu\nu} = 0 ,
\]

in terms of the quadratic coefficient of the modified photon dispersion relation to be given shortly.

In fact, the simplest possible classical spacetime-foam model has identical “defects” (equal spatial and temporal size \( \tilde{b} \)) embedded with random orientations in Minkowski spacetime (equal spatial and temporal average separation \( \tilde{l} \)). For this type of manifold, modified dispersion relations for protons (\( p \)) and photons (\( \gamma \)) have been obtained in the large-wavelength approximation:

\[
\omega_p^2 \equiv M_p^2 c_p^4 / \hbar^2 + c_p^2 k^2 + O(k^4) ,
\]

\[
\omega_\gamma^2 = \left( 1 + \tilde{\sigma}_2 \tilde{F} \right) c_p^2 k^2 + \left( \tilde{\sigma}_4 \tilde{F} \tilde{b}^2 \right) c_p^2 k^4 + O(k^6) ,
\]

with wave number \( k \equiv |k| \equiv 2\pi/\lambda \), effective defect on/off factors \( \tilde{\sigma}_2 , \tilde{\sigma}_4 \in \{ \pm 1 , 0 \} \), effective defect size \( \tilde{b} \), and effective defect excluded-volume factor,

\[
\tilde{F} \equiv (\tilde{b}/\tilde{l})^4 ,
\]

which is assumed to be less than 1. Remark that the coefficient of the quadratic term in (8a) is simply defined as \( c_p^2 \) and that \( \tilde{F} \) appears in both the quadratic and quartic terms of (8b) [the inclusion of \( \tilde{F} \) in the quartic photon term defines the meaning of the squared length \( \tilde{b}^2 \)].

Calculations of specific space-time-foam models \cite{6} give the effective parameters (with tilde) in terms of the underlying spacetime parameters (with bar):

\[
\tilde{b} = \beta \bar{b}, \quad \tilde{l} = \lambda \bar{l} , \quad \tilde{\sigma}_2 = -1 , \quad \tilde{\sigma}_4 = 1 ,
\]

for positive constants \( \beta \) and \( \lambda \) of order unity. From (9) and (10), the restricted (isotropic) model (7) is defined by the fundamental length scales of the classical space-time-foam model considered, namely, the typical defect size \( \tilde{b} \) and the average separation \( \tilde{l} \). [Note that this classical space-time-foam model, in its simplest form, involves a preferred frame of reference, as the assumed equality of spatial and temporal defect sizes makes clear.] In principle, spacetime defects can also generate CPT–violating terms in the effective photon action \cite{7, 8}, but the corresponding physical effects are expected to be suppressed by at least one power of the fine-structure constant \( \alpha \equiv e^2/4\pi \approx 1/137 \).
2.3 Vacuum Cherenkov radiation

In certain Lorentz-violating photon models, the decay process \( p \rightarrow p\gamma \) is allowed. This process is similar to that of Cherenkov radiation in material media \([9, 10, 11, 12, 13]\) but now occurs already \textit{in vacuo} due to the modified photon propagation and may, therefore, be called “vacuum Cherenkov radiation” \([14, 15, 16, 17]\). For the modified-QED model (2), the decay process \( p \rightarrow p\gamma \) has been studied classically by Altschul \([18]\) and quantum-mechanically (Fig. 1–right) by Kaufhold and the present author \([19]\).

The radiated-energy rate of a point particle with electric charge \( Ze \), mass \( M > 0 \), momentum \( \mathbf{q} \), and ultrarelativistic energy \( E \sim c|\mathbf{q}| \) is given by \([19]\)

\[
\frac{dW_{\text{modQED}}}{dt} \bigg|_{E \gg E_{\text{thresh}}} \sim \frac{Z^2e^2}{4\pi} \frac{\xi(\mathbf{q})}{\mathbf{q}^2} \frac{E^2}{\hbar},
\]

with a direction-dependent coefficient \( \xi \geq 0 \) and a threshold energy \([18, 19]\)

\[
E_{\text{thresh}}^2 = \frac{M^2c^4}{R[2\tilde{\kappa}_{tr} + 2\delta\tilde{\kappa}^{(0)}\tilde{\mathbf{q}}^j + \delta\tilde{\kappa}^{jk}\tilde{q}^j\tilde{q}^k]} + O(M^2c^4),
\]

for LV parameters \(|\tilde{\kappa}^{\mu\nu}| \ll 1\) and ramp function

\[
R[x] \equiv (x + |x|)/2.
\]

Note that (11) becomes infinite in the classical limit \( h \rightarrow 0 \), which traces back to the fact that quantum mechanics provides a frequency cutoff that renders the total radiated energy finite; see Refs. \([11, 12, 13, 19]\) for further discussion and references.

Exact tree-level results have been obtained \([20, 21]\) for the restricted modified-QED model (labeled “modQED, isotropic case”) with \( \alpha_0 \equiv (4/3)\tilde{\kappa}^{00} \equiv 2\tilde{\kappa}_{tr} > 0 \) and \( \delta\tilde{\kappa}^{\mu\nu} = 0 \), which is precisely the model derived in the previous subsection. Setting again \( h = c = 1 \), the radiated-energy rate for a spin-\( \frac{1}{2} \) Dirac point particle (charge \( Ze \), mass \( M \), and energy \( E = \sqrt{|\mathbf{q}|^2 + M^2} \) above threshold) is given by

\[
\frac{dW^\text{isotropic case}_{\text{modQED}}}{dt} \bigg|_{E \geq E_{\text{thresh}}} = \frac{Z^2e^2}{4\pi} \frac{1}{3\alpha_0^2 E^2 - M^2} \left( \frac{2 - \alpha_0}{2 + \alpha_0} E - \sqrt{E^2 - M^2} \right)^2 \\
\times \left\{ 2(\alpha_0^2 + 4\alpha_0 + 6)E^2 - (2 + \alpha_0) \left( 3(1 + \alpha_0)M^2 + 2(3 + 2\alpha_0) \sqrt{2 - \alpha_0} E \sqrt{E^2 - M^2} \right) \right\},
\]

The high-energy expansion of (14) for fixed parameters \( \alpha_0 \) and \( M \) reads

\[
\frac{dW^\text{isotropic case}_{\text{modQED}}}{dt} \bigg|_{E \geq E_{\text{thresh}}} = \frac{Z^2e^2}{4\pi} \frac{E^2}{2} \\
\times \left\{ \left( \frac{7}{24} \alpha_0 - \frac{1}{16} \alpha_0^2 + O(\alpha_0^3) \right) + \left( -1 + \frac{1}{48} \alpha_0 - \frac{3}{32} \alpha_0^2 + O(\alpha_0^3) \right) \frac{M^2}{E^2} + O \left( \frac{M^4}{\alpha_0 E^2} \right) \right\},
\]

which displays the quadratic behavior of (11) for a constant coefficient \( \xi = (7/24)\alpha_0 + O(\alpha_0^3) \). From (14), one also obtains the exact tree-level threshold energy:

\[
E^\text{modQED, isotropic case}_{\text{thresh}} = \frac{M}{\sqrt{\alpha_0}} \sqrt{1 + \alpha_0/2},
\]
which reproduces (12) for a small positive value of \( \alpha_0 \equiv 2 \kappa_{\text{tr}} \). Incidentally, the numerical value of \( \alpha_0 \) cannot be too large, as the factors \( \sqrt{2 - \alpha_0} \) in (14) make clear.

As mentioned before, the radiated-energy rates (11) and (14) have been derived for point particles. For realistic particles (protons, nuclei, and photons), the partonic content can be expected to significantly modify the prefactors of the radiation rates but not the energy thresholds (12) and (16), which follow from energy-momentum conservation [model (2) violates Lorentz invariance but maintains spacetime translation invariance]. Note that calculations of standard classical Cherenkov radiation from different types of charge distributions give different radiation rates, depending on the details of the distributions, but unmodified energy thresholds, at least for the case of static charge distributions in the rest frame and a nondispersive medium (cf. Sec. 7.2 of Ref. [13]).

3 UHECR bounds

3.1 Cherenkov threshold condition

The basic idea [14, 15] consists of three steps:

- if vacuum Cherenkov radiation has a threshold energy \( E_{\text{thresh}}(\kappa) \), then UHECRs with \( E_{\text{prim}} > E_{\text{thresh}} \) cannot travel far, as they rapidly radiate away their energy,

- this implies that, if an UHECR of energy \( E_{\text{prim}} \) is detected, its energy must be at or below threshold,

\[
E_{\text{prim}} \leq E_{\text{thresh}}(\kappa),
\]

(17)

- the last inequality gives, using expression (12), an upper bound on the LV parameters [19],

\[
R[2 \kappa_{\text{tr}} + 2 \delta \kappa^0 + \hat{\kappa}^j_{\text{prim}} + \delta \kappa^j_{\text{prim}} \hat{q}^k_{\text{prim}} \hat{q}^k_{\text{prim}}] \leq (M_{\text{prim}}^2 c^4)/E_{\text{prim}}^2,
\]

(18)

with primary energy \( E_{\text{prim}} \), flight direction \( \hat{q}_{\text{prim}} \), and rest mass \( M_{\text{prim}} \) as input.

Remark that the Cherenkov threshold condition (18) is only effective for certain combinations of deformation parameters \( \tilde{\kappa}^{\mu \nu} \) and flight directions \( \hat{q} \), namely, if there is a positive argument of the ramp function \( R \) defined by (13). Hence, there must be a sufficient number of UHECR events with primary energies \( E_{\text{prim}}^{(n)} \) and flight directions \( \hat{q}_{\text{prim}}^{(n)} \) in order to bound all deformation parameters (6).

3.2 Bounds on the LV photon parameters \( \tilde{\kappa}^{\mu \nu} \)

The last analysis of Refs. [22, 23] is based on the following 29 selected UHECR events: 27 events from Auger–South [24], 1 event from Fly’s Eye [25], and 1 event from AGASA [26]. The last two events are taken in order to provide a better coverage of the northern celestial hemisphere (later, they can perhaps be replaced by Auger–North events). The relevant data of these 29 events are given in Table 1, where the uncertainties in the energies are of the order of 25\% and those in the pointing directions of the order of 1 deg (see the original references for further discussion).

Adopting a conservative value for the unknown primary mass (see below), it is this relatively

\footnote{The proton is a complicated dynamic object (as is the photon) and “vacuum Cherenkov radiation” is not quite the same as standard Cherenkov radiation in a material medium. For this reason, we have embarked on a parton calculation [21] of the proton radiated-energy rate for the isotropic case (7) with only parameter \( \kappa_\mu \) nonzero and positive. Preliminary results indicate that the proton radiated-energy rate is suppressed for energies \( E \in [E_{\text{thresh}}, f E_{\text{thresh}}] \), with \( E_{\text{thresh}} \) given by the point-particle result (16) and \( f \approx 1.25 \) (based on the results of Table 1 in Ref. [16]).}
Table 1: Selected UHECR events from Auger (2004–2007) [24], Fly’s Eye (1991) [25], and AGASA (1993) [26]. Shown are the arrival time (year and Julian day), the primary energy $E$ [in EeV $\equiv 10^{18}$ eV], and the arrival direction with right ascension $\alpha$ and declination $\delta$ [both in degrees].

| Year | Day | E   | $\alpha$ | $\delta$ | Year | Day | E   | $\alpha$ | $\delta$ |
|------|-----|-----|----------|----------|------|-----|-----|----------|----------|
| 1991 | 288 | 320 | 85.2     | 48.0     | 2006 | 81  | 79  | 201.1    | −55.3    |
| 1993 | 337 | 210 | 18.9     | 21.1     | 2006 | 185 | 83  | 350.0    | 9.6      |
| 2004 | 125 | 70  | 267.1    | −11.4    | 2006 | 296 | 69  | 52.8     | −4.5     |
| 2004 | 142 | 84  | 199.7    | −34.9    | 2006 | 299 | 69  | 200.9    | −45.3    |
| 2004 | 282 | 66  | 208.0    | −60.3    | 2007 | 13  | 148 | 192.7    | −21.0    |
| 2004 | 339 | 83  | 268.5    | −61.0    | 2007 | 51  | 58  | 331.7    | 2.9      |
| 2004 | 343 | 63  | 224.5    | −44.2    | 2007 | 69  | 70  | 200.2    | −43.4    |
| 2005 | 54  | 84  | 199.1    | −48.6    | 2007 | 145 | 78  | 47.7     | −12.8    |
| 2005 | 63  | 71  | 331.2    | −1.2     | 2007 | 186 | 64  | 219.3    | −53.8    |
| 2005 | 81  | 58  | 199.1    | −48.6    | 2007 | 193 | 90  | 325.5    | −33.5    |
| 2005 | 295 | 57  | 332.9    | −38.2    | 2007 | 221 | 71  | 212.7    | −3.3     |
| 2005 | 306 | 59  | 315.3    | −0.3     | 2007 | 234 | 80  | 185.4    | −27.9    |
| 2005 | 306 | 84  | 114.6    | −43.1    | 2007 | 235 | 69  | 105.9    | −22.9    |
| 2006 | 35  | 85  | 53.6     | −7.8     | 2007 | 235 | 69  | 105.9    | −22.9    |

large energy uncertainty which dominates the error budget of the bounds to be presented in this subsection and the next.

The 29 primary energies and flight directions from Table 1 (flight directions being the opposite of arrival directions) give 29 inequalities for the deformation parameters from the Cherenkov threshold condition (18), where we set $M_{\text{prim}} = 56 \text{ GeV}/c^2$. [A significant fraction of these primaries may well be protons [24], but we prefer to take a conservative value for $M_{\text{prim}}$. The used value of $56 \text{ GeV}/c^2$ is even larger than the mass of an iron nucleus $^{56}\text{Fe}$.] The 29 inequalities then give the following two-σ Cherenkov bounds [23] on the nine isolated SME parameters (6) of the nonbirefringent modified-Maxwell model (2)–(4):

$$|(\tilde{\kappa}_{o+})^{(ij)}| < 2 \times 10^{-18}, \quad (19a)$$

$$|(\tilde{\kappa}_{e-})^{(kl)}| < 4 \times 10^{-18}, \quad (19b)$$

$$\tilde{\kappa}_{\text{tr}} < 1.4 \times 10^{-19}, \quad (19c)$$

for the Sun-centered celestial equatorial coordinate system in Cartesian coordinates. A single UHECR event suffices for bound (19c) and the 148 EeV Auger event from Table 1 has been used, which has a reliable energy calibration.

Based on a 212 EeV Auger event [27] and setting $M_{\text{prim}} = 52 \text{ GeV}/c^2$, a new result [21] gives the following two-sided bound at the two-σ level on a single universal isotropic parameter: $-2 \times 10^{-19} < \tilde{\kappa}^{\text{univ}}_{\text{tr}} < 6 \times 10^{-20}$, where the lower bound arises solely from partonic effects. Here, “universal” means that the same LV parameter $\tilde{\kappa}_{\text{tr}}$ applies to all gauge bosons of the Standard Model, as might be expected from a spacetime-foam model as discussed in Sec. 2.2.

It is important to realize that the Cherenkov bounds (19abc) only depend on the measured energies and flight directions of the charged cosmic-ray primaries at the top of the Earth atmosphere. As noted in Refs. [15, 16], the travel length from vacuum Cherenkov radiation (if operative) would be of the order of meters rather than megaparsecs. Hence, we only need to be sure of having observed a charged primary traveling over a distance of a kilometer, say, in order to apply the Cherenkov threshold condition (18). [The previous discussion applies, strictly speaking, only to event energies above $1.25 E_{\text{thresh}}$ (as mentioned in Ftn. a), whereas cosmic rays with energies barely above $E_{\text{thresh}}$ would still have to travel over astronomical distances.]
For comparison, the current laboratory bounds are as follows (with selected references):

- direct bounds [28, 29] on the three parity-odd nonisotropic parameters in $\tilde{\kappa}_{o+}$ at the $10^{-12}$ level;
- direct bounds [28, 29] on the five parity-even nonisotropic parameters in $\tilde{\kappa}_{e-}$ at the $10^{-14}$ to $10^{-16}$ levels;
- direct bound [30, 31] on the single parity-even isotropic parameter $\tilde{\kappa}_{tr}$ at the $10^{-7}$ level;
- indirect bound [32] on $\tilde{\kappa}_{tr}$ at the $10^{-8}$ level from the measured value [33, 34] of the electron anomalous magnetic moment $a_e \equiv (g_e - 2)/2$.

Interestingly, the UHECR Cherenkov bounds (19) are the strongest where the laboratory bounds are the weakest. A case in point is the isotropic parameter $\tilde{\kappa}_{tr}$, which is difficult to constrain by laboratory experiments (having no sidereal variations) but easy to constrain by a single UHECR event (the flight direction being irrelevant).

The “leverage factor” for the UHECR Cherenkov bounds is, according to (18), given by $(E_{prim}/M_{prim}c^2)^2$ and is of the order of $10^{18}$ for $M_{prim} \sim 50$ GeV/c$^2$ and $E_{prim} \sim 50$ EeV. With further information on selected UHECR events (e.g., the shower-maximum atmospheric depth $X_{max}$ used already in Ref. [22]), it may be possible to reduce $M_{prim}$ by a factor 10 to $M_{prim} \sim 5$ GeV/c$^2$ and, with more and more events becoming available, it may be possible to select on a typical energy $E_{prim} \sim 150$ EeV, thereby increasing the Cherenkov leverage factor by a factor $10^3$ to a value of the order of $10^{21}$. For completeness, the leverage factor for the astrophysics bounds [4] on the birefringent parameters is given by $L/\lambda$, which is of the order of $10^{32}$ for a source distance $L \sim 10^{26}$ m and typical wavelength $\lambda \sim 10^{-6}$ m. Both leverage factors make clear what the power of astrophysics can be, provided the physics is well understood.

3.3 Bounds on the LV photon parameter $\tilde{b}^2$

From the 148 EeV Auger event in Table 1, we also get a one-$\sigma$ bound [6] on the general coefficient of the quartic photon term in (8b):

$$|\tilde{\sigma}_4 \tilde{F} \tilde{b}^2| < (2 \times 10^{-35} \text{ m})^2,$$

based on the parton analysis of Ref. [16] but rescaled to $M_{prim} = 56$ GeV/c$^2$ and $E_{prim} = 148$ EeV. In fact, the right-hand side of (20) is of order $(\hbar c^3 M_{prim}/E_{prim}^2)^2$. Note that the above bound is two-sided, whereas genuine Cherenkov radiation would only give a one-sided bound (the phase velocity of the electromagnetic wave must be less than the maximum attainable velocity of the charged particle). The reason for having a two-sided bound (20) is that, in addition to Cherenkov radiation, another type of process can occur. The relevant process is proton break-up $p \rightarrow p e^+ e^-$ (that is, pair-production by a virtual gauge boson), which then gives the “other” side of the bound [16, 21].

It may be of interest to mention that the non-observation of primary photons by the Pierre Auger Observatory [27, 35] has also been used to obtain (modulo some assumptions) a tight one-sided bound [36] on the quartic term of a modified photon dispersion relation.

In the analysis of Refs. [16, 36], it is taken for granted that the modified photon dispersion relation corresponds to a consistent theory with, for example, microcausality and unitarity (cf. the discussion in Refs. [37, 38]). This may very well be the case, provided that the cubic term is absent from the modified dispersion relation, leaving a quartic term as the first higher-order term to be considered (cf. the discussion in Refs. [6, 39]). But it is also clear that a real understanding of potential Lorentz-violating effects can only come from considering a complete theory, even if it is only an effective theory such as the one discussed in Sec. 2.2.
As remarked in an earlier review [40], bound (20) disagrees by many orders of magnitude with a possible “quantum-gravity” effect [41] in a gamma-ray flare from Mkn 501 as observed by the MAGIC telescope [42]. Most likely, the observed time dispersion has an astrophysical origin. Still, this type of measurement suggests how, in principle, astrophysical data could give more than just upper-bounds on the possible small-scale structure of space.

The length scale on the right-hand side of (20) is extraordinarily small, but this inequality becomes less dramatic if the dimensionless number \( \bar{F} \) on the left-hand side is also small. Now this is precisely what has been found in the model calculation leading up to (8), where the quartic photon coefficient has an extra reduction factor \( \bar{F} \) which can be interpreted as the defect excluded-volume factor (9) entering the quadratic photon coefficient. Taking a value \( \bar{F} = 1.5 \times 10^{-19} \), just consistent with the previous result (19c), bound (20) becomes

\[
\bar{b}^2 < (5 \times 10^{-26} \text{ m})^2,
\]

which remains small compared to the current laboratory bound (1).

4 Theoretical implications

In the previous section, we have established two types of bounds on Lorentz violation in the photon sector. First, a combined one–\( \sigma \) bound [4, 23] was obtained on the nineteen Lorentz-violating deformation parameters of the modified-Maxwell model (2):

\[
|\kappa^{\mu\nu\rho\sigma}| < 3 \times 10^{-18},
\]

where, for the sake of argument, the “one-sided” Cherenkov bound on the isotropic parameter \( \bar{\kappa}_{\text{tr}} \) has also been made “two-sided” [as mentioned in the paragraph starting a few lines under (19c), there is a new two-sided bound at the \( 10^{-19} \) level].

Second, restricting to the isotropic model (7)–(10) with a possible spacetime-foam origin, one–\( \sigma \) Cherenkov-type bounds [6, 16, 21] have been obtained for combinations of the effective defect size \( \bar{b} \) and separation \( \bar{l} \):

\[
\bar{F} \equiv (\bar{b}/\bar{l})^4 < 1.5 \times 10^{-19},
\]

\[
\bar{b} < 5 \times 10^{-26} \text{ m} \approx \hbar c/(4 \times 10^9 \text{ GeV}),
\]

where the particular parameters \( \bar{b} \) and \( \bar{l} \) are really defined by the modified dispersion relations (8ab). For simplicity, we focus our discussion on this last case with a single quadratic photon coefficient \( \bar{F} \) and a single quartic photon coefficient \( \bar{F} \bar{b}^2 \), both of which can be interpreted in terms of a simple spacetime-foam-like structure.

Bound (23b) is remarkable compared to what can be achieved with particle accelerators on Earth (recall that the proton beam energy of the Large Hadron Collider is \( 7 \times 10^3 \text{ GeV} \)). As it stands, bound (23b) may be easily satisfied by a quantum-gravity theory with length scale \( l_{\text{Planck}} \approx \sqrt{\hbar G/c^3} \approx 1.6 \times 10^{-35} \text{ m} \approx \hbar c/(1.2 \times 10^{19} \text{ GeV}) \). But for TeV–gravity models [43, 44] it would be hard to understand why the effective length scale \( \bar{b} \) for photon propagation in the three-dimensional world would be reduced by a numerical factor of the order of \( 10^6 \) compared to the nonperturbative gravity scale \( L_{\text{grav}} = \hbar c/E_{\text{grav}} \approx \hbar c/(4 \text{ TeV}) \). In fact, the quartic coefficient of the photon dispersion relation would need an additional factor of the order of \( 10^{-12} \) or less. A priori, there is no reason to expect the quartic photon coefficient to be extraordinarily small; see Ref. [6] for a heuristic discussion based on so-called Bethe holes [45].

Bound (23a) is even more interesting as it implies that a single-scale \( (\bar{b} \sim \bar{l}) \) classical spacetime foam is ruled out altogether. This result holds, in fact, for arbitrarily small values of the defect size \( \bar{b} \), as long as a classical spacetime makes sense. In the context of effective theories
with Lorentz invariance violated at an ultraviolet scale $\Lambda$, a similar result holds that strong LV effects can be expected at low energies [46], which have not been seen experimentally.

The conclusion is, therefore, that Lorentz invariance remains valid down towards smaller and smaller distances, which answers in part the question posed at the beginning of the Introduction. This conclusion would hold down to distances at which the classical–quantum transition occurs, which may happen for distances of the order of $l_{\text{Planck}} \approx 10^{-35}$ m or perhaps for distances given by an entirely new fundamental length scale [47].

5 Outlook

Astrophysics data (in particular, results from UHECRs) show that a hypothetical quantum spacetime foam must have “crystalized” to a classical spacetime manifold which is remarkably smooth, as quantified by the defect excluded-volume factor $(\bar{b} / \bar{l})^4 \lesssim 10^{-19} \ll 1$ and Lorentz-violating parameters $|\kappa^{\mu\nu\rho\sigma}| \lesssim 10^{-18} \ll 1$. The result applies to any theory of quantum spacetime, be it Matrix–theory [48, 49] or loop quantum gravity [50].

The outcome is like having a “null experiment” and there is an analogy with the well-known Michelson–Morley experiment [51]: theory foresees physical effects which are not found by experiment.

This suggests the need for radically new concepts, similar in depth to the “relativity of simultaneity” introduced by Einstein [52]. For our problem, a first small step may be the realization that precisely Lorentz invariance is crucial at the high-energy frontier and that a new type of conserved relativistic “charge” can play an important role for the flatness of spacetime by resolving the so-called cosmological constant problem [53, 54, 55]. It remains to be seen if this is a step in the right direction, but experiment has, at least, provided theory with a base camp for the long climb towards the quantum origin of spacetime.

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