Compact quantum gates on electron-spin qubits assisted by diamond nitrogen-vacancy centers inside cavities

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Constructing compact quantum circuits for universal quantum gates on solid-state systems is crucial for quantum computing. We present some compact quantum circuits for a deterministic solid-state quantum computing, including the CNOT, Toffoli, and Fredkin gates on the diamond nitrogen-vacancy centers confined inside cavities, achieved by some input-output processes of a single photon. Our quantum circuits for these universal quantum gates are simple and economic. Moreover, additional electron qubits are not employed, but only a single-photon medium. These gates have a long coherent time. We discuss the feasibility of these universal solid-state quantum gates, concluding that they are feasible with current technology.

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I. INTRODUCTION

Quantum logic gates are the key elements in quantum computing. It is well known that two-qubit entangling gates can be used to implement any n-qubit quantum computing, assisted by single-qubit gates [1,2]. The family composed of controlled-NOT (CNOT) gates and one-qubit gates is the most popular universal set of quantum gates for quantum computing today [3,13]. The simulation of any two-qubit gate requires at least three CNOT gates and 15 single-qubit rotations [16–20]. Therefore, projects for realizing a CNOT gate in a solid-state system are highly desired for quantum computing in the future.

An optimal unstructured quantum circuit for any multi-qubit gate requires \(4^n - 3n - 1\) CNOT gates [19]. In the domain of a three-qubit case, people pay much attention to Toffoli and Fredkin gates [21]. (Toffoli (Fredkin) gate, Hadamard gates) is a universal set for multi-qubit quantum computing [21,22]. It is usual much more complex and difficult to realize a Toffoli gate or a Fredkin gate with CNOT and one-qubit gates in experiment because it requires at least six CNOT gates [23] to synthesize a Toffoli gate and it requires two CNOT and three controlled-\(\sqrt{\text{NOT}}\) gates [24] to synthesize a Fredkin gate. It is particularly interesting to discuss the physical realization of a Toffoli gate and a Fredkin gate in a simpler way.

Quantum gates on solid-state systems have attracted much attention as they have a good scalability, and it has been demonstrated for superconducting qubits [25–27] and quantum dots [28]. Electron-spin qubits in solid-state systems, in particular, associated with nitrogen-vacancy (NV) defect centers, are particularly attractive.

The negatively charged NV defect center occurs in the diamond lattice consisting of a substitutional \(^{14}\text{N}\) atom and an adjacent vacancy, and is one of the most attracting and promising solid-state candidates for quantum information processing, due to the long room-temperature coherent time (1.8 ms) that can be manipulated and coupled together in a scalable fashion. The procedures have been established for optical initializing, optical preparing, fast microwave or magnetic manipulating, and optical detecting the long-lived spin triplet state associated with NV centers [30–33].

Tremendous theoretical and experimental progress has been made on quantum information processing based on NV centers. The schemes for the quantum entanglement generation between a photon and an NV center [34], and between electrons associated with NV centers [37,41] were proposed. Recently, the schemes for the quantum state transfer between separated NV centers were introduced [42–44]. Multiqubit quantum registers associated with separated NV centers in diamonds have been proposed [37,38,42]. Hyperentanglement purification and concentration of two-photon systems in both the spatial-mode and polarization degrees of freedom were investigated [45] with the assistance of diamond NV centers inside photonic crystal cavities. Yang et al. [46] proposed a scheme for implementing the conditional phase gate between NV centers assisted by a high-Q silica microsphere cavity. As the electron spin of the NV defect center couples to nearby \(^{13}\text{C}\) nuclear spins, a high-fidelity polarization and the detection of the single-electron and nuclear-spin states can be achieved, even under ambient conditions [47–50], which allows quantum information transfer [51,53]. Entanglement generation between an electron-spin qubit and a nuclear-spin qubit [51,52] and between two nuclear spins [53], and the construction of the quantum gate between an electron and a nuclear spin [51].

In 2011, Chen et al. [39] proposed a composite system, i.e., a diamond NV\(^-\) center with six electrons from the nitrogen and three carbons surrounding the vacancy, which is confined in a microtoroidal resonator (MTR) [57].

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with a quantized whispering-gallery mode (WGM). This system allows for an ultrahigh-Q and a small mode volume of WGM microresonators ([58–60]). When the MTR couples to the fiber, the ultrahigh-Q is degraded. The experiments in which a diamond NV center couples to WGMs in a silica microsphere ([61–63]), diamond-GaP microdisk ([64]), or SiN photonic crystal ([65]) have been demonstrated. The photon input-output process of a coupled atom and MTR platform has been demonstrated in experiment ([57]).

It is important to construct compact quantum circuits for universal quantum gates because they reduce not only time but also errors. In this paper, we investigate the possibility of constructing compact universal quantum gates for a deterministic solid-state quantum computing, including the CNOT, Toffoli, and Fredkin gates on the diamond NV centers confined in cavities, by some single-photon input-output processes. The qubits of these deterministic gates are encoded on two of the electron-spin triple ground states associated with the diamond NV centers, and they have a long decoherence time even at the room temperature. Our quantum gates on NV centers are obtained by interacting a photon with the NV centers, detecting the emitting photon medium, and applying some proper feedforward operations on the electron-spin qubits associated with NV centers. Our quantum circuits for these gates are compact and economic. The CNOT and Toffoli gates are particularly appealing as the photon medium only interacts with each electron qubit one time. Compared with the synthesis programs, our schemes are simple. In our proposals, auxiliary electron-spin qubits are not required and only one photon medium is employed, which is different from the quantum gates on moving electrons based on charge detection ([15]) and the photonic quantum gates based on cross-Kerr nonlinearities ([6]). With current technology, these universal solid-state quantum gates are feasible. If the photon loss, the detection inefficiency, and the imperfection of the experiment are negligible, the success probabilities of our gates are 100%.

This article is organized as follows. In Sec. II, we introduce the photon-matter platform based on the diamond NV center coupled to a resonator and the compact quantum circuit for a deterministic CNOT gate on two separated diamond NV centers. Subsequently, the quantum circuits for constructing three-qubit Toffoli and Fredkin gates on three separated diamond NV centers in a deterministic way are given in Secs. III and IV respectively. The fidelities and efficiencies of our proposals are estimated in Sec. V. Finally, we discuss the feasibility of our universal quantum gates and give a summary in Sec. VI.

![FIG. 1: (Color online) Schematic diagram of a diamond NV center coupling to a resonator and the possible Λ-type optical transitions in an NV center. The transition $|−⟩ → |A_2⟩$ is derived by a left-circularly polarized photon (denoted by $|L⟩$ or $S_z = +1$), and $|+⟩ → |A_2⟩$ is derived by a right-circularly polarized photon (denoted by $|R⟩$ or $S_z = −1$). The levels in bold encode the qubits, i.e., $|+⟩ = |m_s = +1⟩$ and $|−⟩ = |m_s = −1⟩$.](image)
magnetic field to mix the ground states. Alternatively, it is possible to find a Λ-type system at zero magnetic field as the inevitable strain in diamond reduces the symmetry and primarily modifies the excited-state structure according to their orbital wave functions. The excited state is separated into two branches, \(|A_1\), \(|A_2\), \(|E_x\), and \(|E_y\), \(|E_2\) at moderate and high strain. Togan et al. demonstrated that the state \(|A_2\) is robust to low strain and magnetic fields due to the stable symmetric properties, and it decays with an equal probability to the ground-state sublevels \(|-\rangle\) through a left circularly polarized radiation \(|L\rangle\) \((S_z = +1)\) and to \(|+\rangle\) through a right circularly polarized radiation \(|R\rangle\) \((S_z = -1)\). That is, the zero phonon line (ZPL) was observed after the optical resonant excitation at 637 nm \((-\rangle \rightarrow |A_2\) driven by a \(L\)-polarized photon and \(|+\rangle \rightarrow |A_2\) driven by a \(R\)-polarized photon). The mutually orthogonal circular polarization will be destroyed by high strain. The preparation and measurement of the electron spin can be realized by exploiting resonant optical excitation techniques. As illustrated in Ref. 39, the electron spin can be polarized by first preparing the electron spin to \(|0\rangle\) by means of optical pumping with a 532-nm light, and then transferring the population to either \(|\pm\rangle\) by means of microwave \(\pi\) pulses. The spin can be a high-fidelity (\(\sim 93.2\%)\) readout and addressed at low temperature \((T=8.6K)\) based on spin-dependent optical transitions. The state \(|A_2\) connects \(|\pm\rangle\), and \(|E_{x,y}\) connects \(|0\rangle\), after spin manipulation by a microwave pulse and resonant excitation transition \(|0\rangle \leftrightarrow |E_{x,y}\rangle\). The presence or absence of fluorescence decay reveals the spin state 38-53.

The Heisenberg equations of the motion for the annihilation operator of the cavity mode \(\hat{a}\) and the lowering operator of the NV center operation \(\sigma_-\) and the input-output relation for the cavity are given by 70

\[
\frac{d\hat{a}}{dt} = -\left[i(\omega_c - \omega_p) + \frac{\kappa}{2}\right]\hat{a}(t) - g\sigma_-(t) - \sqrt{\kappa}\hat{a}_\text{in},
\]

\[
\frac{d\sigma_-}{dt} = -\left[i(\omega_0 - \omega_p) + \frac{\gamma}{2}\right]\sigma_-(t) - g\sigma_z(t)\hat{a}(t) + \sqrt{\kappa}\sigma_z(t)\hat{b}_\text{in}(t),
\]

\[
\hat{a}_\text{out} = \hat{a}_\text{in} + \sqrt{\kappa}\hat{a}(t),
\]

where \(\omega_c\), \(\omega_p\), and \(\omega_0\) are the frequencies of the cavity, the single photon, and the NV center, respectively. \(\hat{a}_\text{in}(t)\) and \(\hat{a}_\text{out}\) are the cavity input and output operators, respectively. \(\sigma_z(t)\) is the inversion operator of the cavity. \(\gamma\) is the decay of the NV center. \(\kappa\) is the damping rate of the cavity. \(g\) is the coupling rate. \(\hat{b}_\text{in}(t)\) is the vacuum input field felt by the NV center with the commutation relation \([\hat{b}_\text{in}(t), \hat{b}_\text{in}^\dagger(t')] = \delta(t - t')\).

In a weak excitation, i.e., taking \(\langle \sigma_z \rangle = -1\), the adiabatic elimination of the cavity mode leads to the reflection coefficient of the NV center confined in the cavity as

\[
r(\omega_p) = \frac{\hat{a}_\text{out}}{\hat{a}_\text{in}} = \left[i(\omega_c - \omega_p) - \frac{\kappa^2}{2}\right][i(\omega_0 - \omega_p) + \frac{\gamma^2}{2}] + g^2.
\]

(2)

The phase shift and the amplitude of the reflected photon are a function of the frequency detuning \(\omega_c - \omega_p\), with \(\omega_c = \omega_p\). For \(\omega_c = \omega_p\), i.e., when the cavity mode resonant with the NV center interacts with the resonant photon pulse, one can obtain 72

\[
r(\omega_p) = -\frac{\kappa^2}{\gamma^2} + g^2,
\]

\[r_0(\omega_p) = -1.
\]

(3)

Here, \(r_0\) is the reflection coefficient of the cold (or the empty) cavity, that is, \(g = 0\) and the cavity is not coupled to the NV center. \(r(\omega_p)\) is the one for the hot cavity, i.e., \(g \neq 0\). Therefore, the change of the input photon is summarized as 39

\[
|\mathbf{R}\rangle + \rightarrow |\mathbf{R}\rangle +,
\]

\[
|\mathbf{L}\rangle - \rightarrow |\mathbf{L}\rangle -,
\]

\[
|\mathbf{R}\rangle - \rightarrow -|\mathbf{R}\rangle -,
\]

\[
|\mathbf{L}\rangle + \rightarrow -|\mathbf{L}\rangle +.
\]

(4)

The effect of the coupling strength \(g/\sqrt{\kappa}\) on the amplitude of the reflected photon and that of the frequency detuning on the phase shift have been discussed in 39. Chen et al. 39 showed that when \(g \geq 5\sqrt{\kappa}\) with \(\omega_c = \omega_0 = \omega_p\),

\[
r(\omega_p) \approx 1, \quad r_0(\omega_p) = -1.
\]

(5)

That is, Eq. 4 becomes

\[
|\mathbf{R}\rangle + \rightarrow |\mathbf{R}\rangle +,
\]

\[
|\mathbf{L}\rangle - \rightarrow |\mathbf{L}\rangle -,
\]

\[
|\mathbf{R}\rangle - \rightarrow -|\mathbf{R}\rangle -,
\]

\[
|\mathbf{L}\rangle + \rightarrow -|\mathbf{L}\rangle +.
\]

(6)

From the Λ-type diamond NV-center optical transition depicted by Fig. 11 one can see that it requires a polarization-degenerate cavity mode. Therefore, it is suitable for not only WGM microresonators 38, 39, 57-73, but also H1 photonic crystals 74, 75, micropillars 76, 78, and fiber-based 79 cavities.

In our work, all the devices work under the resonant condition \(\omega_c = \omega_0 = \omega_p\). In the following, we first consider the case \(g \geq 5\sqrt{\kappa}\), that is, \(r(\omega_p) \approx 1\), and then we discuss the effect of \(g/\sqrt{\kappa}\) on the fidelities and the efficiencies of our universal quantum gates on NV-center systems.

B. Compact quantum circuit for a two-qubit controlled-not gate on an NV-center system

Our quantum circuit for a CNOT gate on two NV centers is shown in Fig. 2. The two NV centers are initially prepared in two arbitrary superpositions of the two
ground states $|+\rangle$ and $|-\rangle$; that is,

$$
|\psi_{cl}^c\rangle = \alpha_c|+\rangle_c + \beta_c|-\rangle_c,
|\psi_{el}^t\rangle = \alpha_t|+\rangle_t + \beta_t|-\rangle_t.
$$

Here $|\alpha_c|^2 + |\beta_c|^2 = |\alpha_t|^2 + |\beta_t|^2 = 1$. The subscripts $c$ and $t$ stand for the control qubit NV$_1$ and the target qubit NV$_2$, respectively. The single-photon medium is initially prepared in the equal superposition of $|R\rangle$ and $|L\rangle$; that is,

$$
|\psi_0\rangle_{ph} = \frac{1}{\sqrt{2}} (|R\rangle + |L\rangle).
$$

Here and after, the subscript $i$ of $|L_i\rangle$ (or $|R_i\rangle$, $i = 1, 2, 3, \ldots$) stands for the spatial mode $i$ from where the $L$-polarized photon (R-polarized photon) emits. After the $|R\rangle$ and the $|L\rangle$ waves arrive at PBS$_2$ simultaneously, the photon emits from spatial mode 4. The specific evolution process of the whole system composed of the input photon and two NV centers can be shown as follows:

$$
\begin{align*}
\text{PBS}_1 & \rightarrow |\psi_1\rangle = \frac{1}{\sqrt{2}}(|R\rangle_1 + |L\rangle_2) \otimes |\psi_{cl}^c\rangle \otimes |\psi_{el}^t\rangle \\
\text{NV}_2 & \rightarrow |\psi_2\rangle = \frac{1}{\sqrt{2}}(|R\rangle_1 (\alpha_c|+\rangle_c + \beta_c|-\rangle_c) \otimes |\psi_{el}^t\rangle \\
& \quad + \frac{1}{\sqrt{2}}(|L\rangle_3 (\alpha_c|+\rangle_c + \beta_c|-\rangle_c) \otimes |\psi_{el}^t\rangle \\
\text{PBS}_2 & \rightarrow |\psi_3\rangle = \frac{1}{\sqrt{2}}(|R\rangle_4 (\alpha_c|+\rangle_c + \beta_c|-\rangle_c) \otimes |\psi_{el}^t\rangle \\
& \quad + \frac{1}{\sqrt{2}}(|L\rangle_4 (\alpha_c|+\rangle_c + \beta_c|-\rangle_c) \otimes |\psi_{el}^t\rangle.
\end{align*}
$$

From Eq. (9), one can see that the balanced Mach-Zehnder (MZ) interferometer composed of PBS$_1$, NV$_1$, and PBS$_2$ completes the operation

$$
PBS_1 \rightarrow \text{NV}_1 \rightarrow PBS_2 = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
$$

in the basis $\{|R\rangle, |R\rangle, |L\rangle, |L\rangle\}$.

Next, the photon passes through a half-wave plate HWP whose optical axes is set at $22.5^\circ$ to complete the Hadamard gate ($H^{ph}$) on the polarization photon,

$$
|R\rangle \xrightarrow{H^{ph}} |F\rangle = \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle),
|L\rangle \xrightarrow{H^{ph}} |S\rangle = \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle).
$$

That is, after an $H^{ph}$, the state of the whole system becomes

$$
\xrightarrow{H^{ph}} |\Psi_4\rangle = (\alpha_c|L\rangle_5|+\rangle_c + \beta_c|R\rangle_5|\rangle_c) \otimes (\alpha_t|+\rangle_t + \beta_t|-\rangle_t).
$$

PBS$_3$ transforms the wave packet $|L\rangle_5$ into $|L\rangle_6$, and transforms $|R\rangle_5$ into $|R\rangle_7$ for interacting with NV$_2$ and then it reaches PBS$_4$ simultaneously with $|L\rangle_6$. Before and after the photon passes though NV$_2$, a Hadamard operation $H^{el}$ is performed on NV$_2$, respectively. According to Eq. (10), one can see that the above operations ($H^{el} \rightarrow PBS_3 \rightarrow NV_2 \rightarrow PBS_4 \rightarrow H^{el}$) complete the transformation as

$$
\rightarrow |\Psi_5\rangle = \alpha_c\alpha_t|L\rangle_9|+\rangle_c|+\rangle_t + \alpha_c\beta_t|L\rangle_9|+\rangle_c|-\rangle_t \\
+ \beta_c\alpha_t|R\rangle_9|-\rangle_c|+\rangle_t + \beta_c\beta_t|R\rangle_9|-\rangle_c|-\rangle_t.
$$

Here Hadamard operation $H^{el}$ completes the following transformations:

$$
|+\rangle \xrightarrow{H^{el}} |\pm\rangle \equiv \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle),
|\rightarrow \xrightarrow{H^{el}} |\mp\rangle \equiv \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle),
$$

FIG. 2: (Color online) Compact quantum circuit for a CNOT gate on two NV centers. HWP is a half-wave plate set at 22.5° to complete the Hadamard operation ($H^{ph}$) on the polarization photon. The polarizing beam splitter PBS$_i$ ($i = 1, 2$) in the basis $\{|R\rangle, |L\rangle\}$ transmits the right-circularly polarized photon $|R\rangle$ and reflects the left-circularly polarized photon $|L\rangle$, respectively. PBS’ represents a PBS which transmits the photon in the state $|F\rangle = (|R\rangle + |L\rangle)/\sqrt{2}$ and reflects the photon in the state $|S\rangle = (|R\rangle - |L\rangle)/\sqrt{2}$, respectively. $D^c$ and $D^e$ are two single-photon detectors.

Polarizing beam splitter PBS$_1$ splits the input single photon into two wave-packets. The component $|R\rangle$ transmits through PBS$_1$ and then arrives at PBS$_2$ directly, while the component $|L\rangle$ is reflected to spatial model 2 for interacting with NV$_1$, which induces the transformation $|L\rangle_2 (\alpha_c|+\rangle_c + \beta_c|-\rangle_c)$ in NV$_1$ to $|L\rangle_3 (\alpha_c|+\rangle_c + \beta_c|-\rangle_c)$. Here and after, the subscript $i$ of $|L_i\rangle$ (or $|R_i\rangle$, $i = 1, 2, 3, \ldots$) stands for the spatial mode $i$ from where the $L$-polarized photon (R-polarized photon) emits. After the $|R\rangle$ and the $|L\rangle$ waves arrive at PBS$_2$ simultaneously, the photon emits from spatial mode 4. The specific evolution process of the whole system composed of the input photon and two NV centers can be
From Eq. (13), one can see that to complete the CNOT gate on two NV centers, which implements the transformation

\[ |\Psi\rangle_{ct} = |\psi\rangle_c^{\ell} \otimes |\psi\rangle_t^{\ell} = \alpha_c |+\rangle_c \otimes \alpha_t |+\rangle_t + \beta_c (-|\rangle_c \otimes \beta_t |\rangle_t) \]

\[ \to_{\text{CNOT}} \alpha_c |+\rangle_c \otimes \alpha_t |+\rangle_t + \beta_c (-|\rangle_c \otimes \beta_t |\rangle_t) \]

after the photon is detected by the detector \( D^F \) or \( D^S \) in the basis \( \{|F\} = (|R\rangle + |L\rangle)/\sqrt{2}, \{|S\} = (|R\rangle - |L\rangle)/\sqrt{2} \), some proper single-qubit operations shown in Tab. 4 should be performed on the control qubit and the target qubit, respectively. Therefore, the quantum circuit shown in Fig. 4 performs the CNOT gate on two NV centers, which flips the state of the target electron qubit in NV2 if and only if (iff) the control electron qubit in NV1 is in the state \( -| \). This gate works with a success probability of 100% in principle.

### Table I: The feed-forward single unitary operations performed on the control and the target qubits correspond to the outcomes of the medium photon for completing the CNOT gate on the two NV centers with a success probability of 100%. \(-\sigma_z = -|+\rangle \langle +| + |+\rangle \langle -| \). \( I_2 \) is a \( 2 \times 2 \) unit operation which means doing nothing on a qubit.

| photon | control qubit | target qubit |
|--------|---------------|--------------|
| \( D^F \) (\(|F\}) \) | \( I_2 \) | \( I_2 \) |
| \( D^S \) (\(|S\}) \) | \(-\sigma_z \) | \( I_2 \) |

III. SOLID-STATE TOFFOLI GATE ON A THREE-QUBIT NV-CENTER SYSTEM

A Toffoli gate is used to complete a NOT operation on the state of the target qubit when both two control qubits are in the state \(-| \); otherwise, nothing is done on the target qubit. The principle for implementing a Toffoli gate on a three-qubit NV-center system is shown in Fig. 4. Suppose the first control qubit \( c_1 \) in the defect center NV1, the second control qubit \( c_2 \) in the defect center NV2, and the target qubit \( t \) in the defect center NV3 are prepared in three arbitrary superposition electron-spin states as follows:

\[ |\psi\rangle_{c_1} = \alpha_{c_1} |+\rangle_{c_1} + \beta_{c_1} (-|\rangle_{c_1} \]
\[ |\psi\rangle_{c_2} = \alpha_{c_2} |+\rangle_{c_2} + \beta_{c_2} (-|\rangle_{c_2} \]
\[ |\psi\rangle_{t} = \alpha_t |+\rangle_t + \beta_t (-|\rangle_t \]. \]

Here, \( |\alpha_{c_1}|^2 + |\beta_{c_1}|^2 = |\alpha_{c_2}|^2 + |\beta_{c_2}|^2 = |\alpha_t|^2 + |\beta_t|^2 = 1 \).

In order to describe the principle of our Toffoli gate on a three-qubit NV-center system explicitly, we specify the evolution of the system as follows.

An input single-photon medium in the equal polarization superposition state \( |\psi\rangle^{ph} \sim (|R\rangle + |L\rangle)/\sqrt{2} \) passes though a balanced MZ interferometer composed of PBS1, NV1, and PBS2 described by Eq. (10), and then an \( H^{ph} \) (with HWP1) is performed on it. PBS2 transforms \( |R\rangle \) into \( |R\rangle \), and transforms \( |L\rangle \) into \( |L\rangle \). The evolution of the total states induced by the above operations (PBS1 → NV1 → PBS2 → HWP1 → PBS3) can be described as follows:

\[ |\Xi_0\rangle = |\psi\rangle^{ph} \otimes |\psi\rangle^{ph} \otimes |\psi\rangle^{ph} \otimes |\psi\rangle^{ph} \]

\[ |\Xi_1\rangle = \frac{1}{\sqrt{2}} [|R\rangle_1 (\alpha_{c_1} |+\rangle_{c_1} + \beta_{c_1} (-|\rangle_{c_1} \otimes |\psi\rangle_t^{ph} \otimes |\psi\rangle_{t}^{ph} \]

\[ + \frac{1}{\sqrt{2}} [|L\rangle_1 (-\alpha_{c_1} |+\rangle_{c_1} + \beta_{c_1} (-|\rangle_{c_1} \otimes |\psi\rangle_t^{ph} \otimes |\psi\rangle_{t}^{ph} \]

\[ \overset{\text{HWP}_1}{\rightarrow} |\Xi_2\rangle = (\alpha_{c_1} |L\rangle_2 |+\rangle_{c_1} + \beta_{c_1} |R\rangle_2 (-|\rangle_{c_1} \otimes |\psi\rangle_t^{ph} \otimes |\psi\rangle_{t}^{ph} \]

\[ \overset{\text{PBS}_2}{\rightarrow} |\Xi_3\rangle = (\alpha_{c_1} |L\rangle_3 |+\rangle_{c_1} + \beta_{c_1} |R\rangle_3 (-|\rangle_{c_1} \otimes |\psi\rangle_t^{ph} \otimes |\psi\rangle_{t}^{ph} \]. \]

Before and after the photon emitting from spatial mode 6 (5) passes through a balanced MZ interferometer composed of PBS5, NV2, and PBS6 (PBS4, NV2, and PBS6), an \( H^{ph} \) is performed on it, respectively. These processes (HWP3 → PBS5 → NV2 → HWP5 and HWP2 → PBS4 → NV2 → PBS6 → HWP6) complete the transformation \( |\Xi_3\rangle \rightarrow |\Xi_4\rangle \). Here:

\[ |\Xi_4\rangle = \alpha_{c_1} |+\rangle_{c_1} (\alpha_{c_2} |R\rangle_9 |+\rangle_{c_2} + \beta_{c_2} |L\rangle_9 (-|\rangle_{c_2} \otimes |\psi\rangle_t^{ph} \otimes |\psi\rangle_{t}^{ph} \]

\[ + \beta_{c_1} (-|\rangle_{c_1} (\alpha_{c_2} |R\rangle_9 |+\rangle_{c_2} - \beta_{c_2} |L\rangle_9 (-|\rangle_{c_2} \otimes |\psi\rangle_t^{ph} \otimes |\psi\rangle_{t}^{ph} \]. \]

The transformation of PBS3 → NV2 → PBS7 can be described by Eq. (16), and PBS4 → NV2 → PBS6 can be written as

\[ \text{PBS}_4 \rightarrow \text{NV}_2 \rightarrow \text{PBS}_6 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \].

in the basis \( \{|R\rangle, |L\rangle\} \). When the photon emits from spatial mode 9, before it reaches the 50:50 BS directly. When the photon emits from spatial mode 9, before it reaches the 50:50 BS, it passes through a balanced MZ interferometer composed of PBS7, NV3, and PBS9 described by Eq. (16), and an \( H^{ph} \) is performed on the defect NV3 before and after the photon transmits through it, respectively. The above operations \( (H^{ph} \rightarrow \text{PBS}_8 \rightarrow \text{NV}_3 \rightarrow \text{PBS}_9 \rightarrow H^{ph}) \) complete the transformation as

\[ \rightarrow |\Xi_5\rangle = \alpha_{c_1} \alpha_{c_2} |R\rangle_{11} |+\rangle_{c_1} |+\rangle_{c_2} (\alpha_t |+\rangle_t + \beta_t (-|\rangle_t) \]

\[ + \alpha_{c_1} \beta_{c_2} |L\rangle_{11} |+\rangle_{c_1} (-|\rangle_{c_1} \otimes |\psi\rangle_t^{ph} \otimes |\psi\rangle_{t}^{ph} \]

\[ + \beta_{c_1} \alpha_{c_2} |R\rangle_{11} (-|\rangle_{c_1} \otimes |\psi\rangle_t^{ph} \otimes |\psi\rangle_{t}^{ph} \]

\[ + \beta_{c_1} \beta_{c_2} |L\rangle_{11} (-|\rangle_{c_1} \otimes |\psi\rangle_t^{ph} \otimes |\psi\rangle_{t}^{ph} \]. \]
FIG. 3: (Color online) Compact quantum circuit for deterministically implementing a Toffoli gate on a quantum system composed of three NV centers.

Next, the wave packet emitting from spatial 11 interferes with the wave packet emitting from spatial 10 at the BS, which implements the transformations

|\[R\]\rangle_{11} \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|\[R\]\rangle_{12} + |\[R\]\rangle_{13}),

|\[L\]\rangle_{11} \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|\[L\]\rangle_{12} + |\[L\]\rangle_{13}),

|\[R\]\rangle_{10} \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|\[R\]\rangle_{12} - |\[R\]\rangle_{13}),

|\[L\]\rangle_{10} \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|\[L\]\rangle_{12} - |\[L\]\rangle_{13}). \hspace{1cm} (21)

\[\Xi\] will be transformed into the state

|\[\Xi\]\rangle_5 = \frac{|F\rangle_{12}}{2} \bigg[ a_{c_1} a_{c_2} \langle c_1 | + \rangle c_2 (\alpha_t | + \rangle t + \beta_t | - \rangle t) + a_{c_1} \beta_{c_2} + \langle c_1 | - \rangle c_2 (\alpha_t | + \rangle t + \beta_t | - \rangle t) \\
+ \beta_{c_1} \beta_{c_2} | - \rangle c_2 (\alpha_t | + \rangle t + \beta_t | - \rangle t) - \beta_{t} \beta_{t} | - \rangle c_2 (\alpha_t | - \rangle t + \beta_t | + \rangle t) \\
+ \langle S\rangle_{12} | c_1 | + \rangle c_2 (\alpha_t | + \rangle t + \beta_t | - \rangle t) - \alpha_{c_1} \beta_{c_2} | - \rangle c_2 (\alpha_t | - \rangle t + \beta_t | + \rangle t) \\
+ \beta_{t} \beta_{t} | - \rangle c_2 (\alpha_t | - \rangle t + \beta_t | + \rangle t) \bigg]. \hspace{1cm} (22)

The photon medium is measured in the basis \{|F\rangle, |S\rangle\} by the detector \(D^f\) or \(D^s\). Following with the feedforward operations performed on the NV centers, shown in Table [II], we accomplish the construction of the Toffoli gate on the three NV centers in a deterministic way. That is, the state of the system composed of the three
defect NV$_1$, NV$_2$, and NV$_3$ becomes
\[
|\Xi\rangle_{\text{Toffoli}} = \alpha_{c_1}\alpha_{c_2}|+\rangle_{c_1}|+\rangle_{c_2}(\alpha_1|+\rangle_t + \beta_1|-\rangle_t) \\
+ \alpha_{c_1}\beta_{c_2}|+\rangle_{c_1}|-\rangle_{c_2}(\alpha_1|+\rangle_t + \beta_1|-\rangle_t) \\
+ \beta_{c_1}\alpha_{c_2}|+\rangle_{c_1}|+\rangle_{c_2}(\alpha_2|+\rangle_t + \beta_2|-\rangle_t) \\
+ \beta_{c_1}\beta_{c_2}|+\rangle_{c_1}|+\rangle_{c_2}(\alpha_2|+\rangle_t + \beta_2|--\rangle_t). \tag{23}
\]

From the processes above, one can see that the setup shown in Fig. 3 completes the transformation,
\[
|\Xi\rangle_{c_1, c_2, t} = |\psi\rangle_{c_1}^e \otimes |\psi\rangle_{c_2}^e \otimes |\psi\rangle_t^e. \tag{24}
\]

That is, the setup shown in Fig. 3 realizes exactly the Toffoli gate on the three-qubit NV-center system, which flips the state of the target qubit iff both the two control qubits are in the state $|--\rangle$.

### Table II: The operations performed on the control and the target qubits correspond to the measurement outcomes of the medium photon for completing the Toffoli gate on the three NV centers with a success probability of 100%.

| Photon          | Qubit $c_1$ | Qubit $c_2$ | Qubit $t$ |
|-----------------|------------|------------|------------|
| $D_5^c$ $(|F\rangle_{12})$ | $I_2$  | $I_2$  | $I_2$  |
| $D_4^c$ $(|S\rangle_{12})$ | $I_2$  | $\sigma_z$ | $I_2$  |
| $D_4^c$ $(|F\rangle_{13})$ | $-\sigma_z$ | $I_2$  | $I_2$  |
| $D_8^c$ $(|S\rangle_{13})$ | $-\sigma_z$ | $\sigma_z$ | $I_2$  |

### IV. SOLID-STATE FREDKIN GATE ON A THREE-QUBIT NV-CENTER SYSTEM

A Fredkin gate is used to exchange the states of the two target qubits iff the control qubit is in the state $|--\rangle$. Our quantum circuit for implementing a Fredkin gate on a three-qubit NV-center system in a deterministic way is shown in Fig. 4. The control qubit $c$ encoded on NV center “NV$_1$”, the first target qubit $t_1$ encoded on NV center “NV$_2$”, and the second target qubit $t_2$ encoded on NV center “NV$_3$” are initially prepared in three arbitrary states
\[
|\psi\rangle_{c_1}^e = \alpha_c|+\rangle_c + \beta_c|--\rangle_c, \\
|\psi\rangle_{t_1}^e = \alpha_t|+\rangle_t + \beta_t|--\rangle_t, \\
|\psi\rangle_{t_2}^e = \alpha_t|+\rangle_t + \beta_t|--\rangle_t. \tag{25}
\]

Here $|\alpha_c|^2 + |\beta_c|^2 = |\alpha_t|^2 + |\beta_t|^2 = 1$. The photon medium $p$ is prepared in the equal superposition state
\[
|\psi\rangle_{p}^h = \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle). \tag{26}
\]

That is, the initial state of the quantum system, composed of the three electrons $c$, $t_1$, and $t_2$, and a single photon $p$, can be written as
\[
|\Pi_0\rangle = |\psi\rangle_{p}^h \otimes |\psi\rangle_{c}^e \otimes |\psi\rangle_{t_1}^e \otimes |\psi\rangle_{t_2}^e. \tag{27}
\]

In the following, let us discuss the construction of the solid-state Fredkin gate on a three-qubit NV-center system step by step.

First, a photon medium is injected into the input port $in$ and it passes through a balanced MZ interferometer composed of PBS$_1$, NV$_1$, and PBS$_2$, and then an $H^p$ is performed on it (i.e., let it pass through HWP$_1$). PBS$_3$ transmits the $R$-polarized photon to spatial model 3, and reflects the $L$-polarized photon to spatial model 4. Based on the argument as made in Sec. III one can see that the state of the whole system composed of a single photon medium and three NV centers then becomes
\[
|\Pi_1\rangle = (\alpha_c|L\rangle_4|+\rangle_c + \beta_c|R\rangle_3|--\rangle_c) \otimes |\psi\rangle_{t_1}^e \otimes |\psi\rangle_{t_2}^e. \tag{28}
\]

Before and after the photon emitting from spatial model 6 (5) passes through a balanced MZ interferometer composed of PBS$_5$, NV$_2$, NV$_3$, and PBS$_7$ (PBS$_4$, NV$_2$, NV$_3$, and PBS$_6$), an $H^p$ is performed on it, respectively. The state of the complicated system after these operations (HW$P_3 \rightarrow$ PBS$_5 \rightarrow$ NV$_2 \rightarrow$ NV$_3 \rightarrow$ PBS$_7 \rightarrow$ HW$P_5$ and HW$P_2 \rightarrow$ PBS$_4 \rightarrow$ NV$_2 \rightarrow$ NV$_3 \rightarrow$ PBS$_6 \rightarrow$ HW$P_4$) becomes
\[
\rightarrow |\Pi_2\rangle = \alpha_c\alpha_t_1\alpha_t_2|L\rangle_4|+\rangle_c + \beta_c|R\rangle_3|--\rangle_c \otimes \alpha_t_1|+\rangle_t_1 + \beta_t|--\rangle_t_2 + \alpha_t_1|+\rangle_t_1 + \beta_t|--\rangle_t_2 + \alpha_t_1|+\rangle_t_1 + \beta_t|--\rangle_t_2 + \alpha_t_1|+\rangle_t_1 + \beta_t|--\rangle_t_2 + \alpha_t_1|+\rangle_t_1 + \beta_t|--\rangle_t_2. \tag{29}
\]

Here the balanced MZ interferometer composed of PBS$_5$, NV$_2$, NV$_3$, and PBS$_7$ (PBS$_4$, NV$_2$, NV$_3$, and PBS$_6$) completes the unitary operation
\[
PBS_{5(4)} \rightarrow NV_2 \rightarrow NV_3 \rightarrow PBS_{7(6)} = \begin{pmatrix}
0 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1 \\
0 & 0 & 0
\end{pmatrix} \otimes I_5, \tag{30}
\]
in the basis \{\{R\rangle|+\rangle, |R\rangle|+\rangle, |R\rangle|--\rangle, |R\rangle|--\rangle, |L\rangle|+\rangle, |L\rangle|+\rangle, |L\rangle|--\rangle, |L\rangle|--\rangle\}.\]

Next, when the photon emits from spatial mode 10, it reaches the 50:50 BS directly. When the photon emits from spatial mode 9, before it reaches the BS, it passes through a balanced MZ interferometer composed
of PBS\textsubscript{8}, PBS\textsubscript{9}, NV\textsubscript{2} and NV\textsubscript{3}, which completes the oper-

tation

PBS\textsubscript{8} → NV\textsubscript{3} → NV\textsubscript{2} → PBS\textsubscript{9} = \begin{pmatrix}
I_5 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}, \quad (31)

Before and after the photon interacts with NV\textsubscript{3} and NV\textsubscript{2}, an $H^c$ is performed on NV\textsubscript{3} and NV\textsubscript{2}, respectively. These operations ($H^c → PBS\textsubscript{8} → NV\textsubscript{3} → NV\textsubscript{2} → PBS\textsubscript{9} → H^c$) complete the transformation $|\Pi_2\rangle → |\Pi_3\rangle$. Here

$$
\begin{align*}
|\Pi_3\rangle &= \alpha_c\alpha_{t_1}\alpha_{t_2}\ket{L_{10}}|+\rangle_c|+\rangle_{t_1}|+\rangle_{t_2} \\
&\quad -\alpha_c\alpha_{t_1}\beta_{t_2}\ket{R_{10}}|+\rangle_c|\rangle_{t_1}|\rangle_{t_2} \\
&\quad -\alpha_c\beta_{t_1}\alpha_{t_2}\ket{L_{10}}|+\rangle_c|\rangle_{t_1}|+\rangle_{t_2} \\
&\quad +\alpha_c\beta_{t_1}\beta_{t_2}\ket{L_{11}}|\rangle_c|\rangle_{t_1}|+\rangle_{t_2} \\
&\quad +\beta_c\alpha_{t_1}\alpha_{t_2}\ket{R_{11}}|\rangle_c|+\rangle_{t_1}|+\rangle_{t_2} \\
&\quad -\beta_c\alpha_{t_1}\beta_{t_2}\ket{L_{11}}|\rangle_c|\rangle_{t_1}|+\rangle_{t_2} \\
&\quad -\beta_c\beta_{t_1}\alpha_{t_2}\ket{L_{11}}|\rangle_c|\rangle_{t_1}|+\rangle_{t_2} \\
&\quad +\beta_c\beta_{t_1}\beta_{t_2}\ket{R_{11}}|\rangle_c|\rangle_{t_1}|+\rangle_{t_2}. \quad (32)
\end{align*}
$$

The 50:50 BS, described by Eq. (21), transforms $|\Pi_3\rangle$ into

$$
|\Pi_4\rangle = \frac{|F_{13}|}{2} \left[ \alpha_c|+\rangle_c(\alpha_{t_1}|+\rangle_{t_1} - \beta_{t_1}|\rangle_{t_1}) \\
\quad \times (\alpha_{t_2}|+\rangle_{t_2} - \beta_{t_2}|\rangle_{t_2}) + \beta_c|\rangle_{t_2}(\alpha_{t_2}|+\rangle_{t_1} - \beta_{t_1}|\rangle_{t_1}) \\
\right] + \frac{|S_{13}|}{2} \left[ -\alpha_c|+\rangle_c(\alpha_{t_1}|+\rangle_{t_1} + \beta_{t_1}|\rangle_{t_1}) + \beta_c|\rangle_{t_2}(\alpha_{t_2}|+\rangle_{t_1} + \beta_{t_1}|\rangle_{t_1}) \\
\right] + \frac{|F_{12}|}{2} \left[ -\alpha_c|+\rangle_c(\alpha_{t_1}|+\rangle_{t_1} - \beta_{t_1}|\rangle_{t_1}) + \beta_c|\rangle_{t_2}(\alpha_{t_2}|+\rangle_{t_1} + \beta_{t_1}|\rangle_{t_1}) \\
\right] + \frac{|S_{12}|}{2} \left[ -\alpha_c|+\rangle_c(\alpha_{t_1}|+\rangle_{t_1} + \beta_{t_1}|\rangle_{t_1}) + \beta_c|\rangle_{t_2}(\alpha_{t_2}|+\rangle_{t_1} + \beta_{t_1}|\rangle_{t_1}) \\
\right]. \quad (33)
$$

Third, by detecting the single-photon medium in the basis \{F, S\} and following with the feedforward single-qubit unitary operations shown in Table III, one can see that the state of the system composed of NV\textsubscript{1}, NV\textsubscript{2}, and NV\textsubscript{3} becomes

$$
|\Pi_{\text{Fredkin}}\rangle = \alpha_c|+\rangle_c(\alpha_{t_1}|+\rangle_{t_1} + \beta_{t_1}|\rangle_{t_1}) \\
\times (\alpha_{t_2}|+\rangle_{t_2} + \beta_{t_2}|\rangle_{t_2}) + \beta_c|\rangle_{t_2}(\alpha_{t_2}|+\rangle_{t_1} + \beta_{t_1}|\rangle_{t_1}) \\
\times (\alpha_{t_1}|+\rangle_{t_1} + \beta_{t_1}|\rangle_{t_1}). \quad (34)
$$

Comparing Eq. (23) with Eq. (33), one can see that the quantum circuit shown in Fig. 4 implements a Fredkin gate on the three NV centers with the success probability of 100% in principle, which swaps the states of two target qubits iff the control qubit is in the state $|\rangle$. 

FIG. 4: (Color online) Schematic setup for deterministically implementing a Fredkin gate on three NV centers.
TABLE III: The operations performed on the control and the target qubits correspond to the measurement outcomes of the medium photon for completing the Fredkin gate on the three NV centers with a success probability of 100%.

| photon | qubit c | qubit t1 | qubit t2 |
|--------|---------|----------|----------|
| \(D_1^F(\{F\}_{12})\) | \(I_2\) | \(\sigma_z\) | \(\sigma_z\) |
| \(D_2^S(\{S\}_{12})\) | \(-\sigma_z\) | \(I_2\) | \(I_2\) |
| \(D_2^F(\{F\}_{13})\) | \(-\sigma_z\) | \(\sigma_z\) | \(\sigma_z\) |
| \(D_2^S(\{S\}_{13})\) | \(I_2\) | \(I_2\) | \(I_2\) |

V. Fidelities and Efficiencies of Our Universal Quantum Gates

Let us estimate the fidelities and the efficiencies of our universal solid-state quantum gates discussed above, defining the fidelity as

\[ F = \left| \langle \psi_{\text{real}} | \psi_{\text{ideal}} \rangle \right|^2. \]

Here, \(\psi_{\text{ideal}}\) is the target state of the NV-center-cavity system encoded for the quantum gate in the ideal case \(g \geq 5\sqrt{\kappa\gamma}\), and \(\psi_{\text{real}}\) is the target state of a realistic NV-center-cavity system. Defining the efficiency as the yield of the photons, that is, \(\eta = n_{\text{output}}/n_{\text{input}}\). Here, \(n_{\text{input}}\) is the number of the input photon, whereas \(n_{\text{output}}\) is the number of the output photon. The gates are realized by the input-output processes of the photon medium, which means that the reflection coefficient of the NV-cavity system determines the fidelities and the efficiencies of our universal quantum gates.

![Figure 6](image-url) (Color online) The efficiencies of the CNOT (solid line, red), Toffoli (the dash-dotted line, blue), and Fredkin (dotted line, black) gates vs \(g/\sqrt{\kappa\gamma}\) gates vs \(g/\sqrt{\kappa\gamma}\). Here, \(g/\sqrt{\kappa\gamma} \geq 0.5\).

Combing the specific evolutions of the CNOT, Toffoli, and Fredkin gates and the input-output relations of the NV-cavity system in the realistic case given by Eq. (35), one can see that the fidelities of those gates can be calculated as

\[ F_{\text{CNOT}} = \frac{(2 + |r| + |r|^2)^2}{2(5 - 2|r| + 2|r|^2 + 2|r|^3 + |r|^4)}, \]

\[ F_{\text{Toffoli}} = \frac{(3 + |r|)^4}{16(3 + |r|^2)^2}, \]

\[ F_{\text{Fredkin}} = \frac{\zeta_{\text{Fredkin}}}{\xi_{\text{Fredkin}}}. \tag{35} \]

with

\[ \zeta_{\text{Fredkin}} = (29 + 19|r| + 8|r|^2 + 4|r|^3 + 3|r|^4 + |r|^5)^2, \]

\[ \xi_{\text{Fredkin}} = 8[237 - 10|r| + 165|r|^2 - 8|r|^3 + 66|r|^4 - 12|r|^5 + 26|r|^6 + |r|^7(3 + |r|)
\times (8 + 3|r| + |r|^2)]. \tag{36} \]

The efficiencies of those gates can be calculated as

\[ \eta_{\text{CNOT}} = \left[ \frac{3 + |r|^2}{4} \right]^2, \]

\[ \eta_{\text{Toffoli}} = \frac{(3 + |r|^2)^2(7 + |r|^2)}{128}, \tag{37} \]

\[ \eta_{\text{Fredkin}} = \frac{(3 + |r|^2)[4 + (1 + |r|^2)^2][12 + (1 + |r|^2)^2]}{512}. \]

For the diamond NV centers, the photoluminescence is partially unpolarized, and the emission with ZPL is only 4% of the total emission. \(\gamma_{\text{ZPL}}\) with zero phonon line is only 4% of \(\gamma_{\text{total}} = 2\pi \times 15 \text{ MHz}\) \cite{36,66}. \(Q = c/\lambda\), where \(c\) is the speed of light and \(\lambda = 637 \text{ nm}\) is the transition wavelength. The WGM cavities with microrotoidal form have attracted much attention \cite{80}. Ref. \cite{80} shows that the polymer-coated microroid is feasible and robust in experiments. For the diamond NV center in a MTR with WGM mode system, Ref. \cite{36} shows that
when \( g / \sqrt{\kappa \gamma} \geq 3 \) with \( \omega_c = \omega_p = \omega_0 \), \( r(\omega_p) \sim 0.95 \); when \( g / \sqrt{\kappa \gamma} \geq 5 \) with \( \omega_c = \omega_p = \omega_0 \), \( Q \sim 10^5 \) (corresponding to \( \kappa \sim 1 \) GHz) or \( Q \sim 10^4 \) (corresponding to \( \kappa \sim 10 \) GHz), \( r(\omega_p) \sim 1 \).

Figures 5 and 6 show the fidelities and the efficiencies of our universal quantum gates as a function of \( g / \sqrt{\kappa \gamma} \) with \( g / \sqrt{\omega} = g / \sqrt{\omega_0} \) and \( g / \sqrt{\kappa \gamma} \geq 1/2 \). Our results show that the fidelities and the efficiencies of our quantum gates increase with \( g / \sqrt{\kappa \gamma} \). When \( g / \sqrt{\kappa \gamma} = 5 \), the fidelities of the gates are unity with \( \eta_{\text{CNOT}} = 98.05\% \), \( \eta_{\text{Toffoli}} = 97.57\% \), and \( \eta_{\text{Fredkin}} = 96.15\% \).

VI. DISCUSSION AND SUMMARY

Universal quantum gates in solid-state systems are much more attractive as they have a good scalability. Many schemes have been proposed for realizing universal quantum gates on solid-state systems. Based on superconductor, Romero et al. [29] and Stojanovic et al. [56] proposed some interesting schemes for realizing controlled-phase and Toffoli gates in nanosecond time scale, respectively. Liang and Li [81] proposed a scheme for realizing a conditional gate between a quantum-dot qubit and a photonic qubit. In 2010, the quantum circuit for realizing a CNOT between a quantum-dot qubit and a polarized photon qubit was designed by Bonato et al. [13].

Based on appealing diamond NV-center qubits, Yang et al. [46] proposed a scheme for realizing a conditional phase gate between two centers assisted by high-Q silica microsphere cavity, and the control and the target qubits are encoded on different energy levels. Jelezko et al. [50] designed a quantum circuit for realizing controlled-ROT gate between an electron and a nuclear spin qubits in a NV center.

The schemes we proposed for constructing the two-qubit CNOT, and three-qubit Toffoli and Fredkin gates on diamond NV centers inside resonators have some interesting features. (1) The quantum circuits are compact. Especially the schemes for CNOT and Toffoli gates, in which the photon medium only interacts with each qubit one time. The complexity of our schemes for Toffoli and Fredkin gates beats its synthesis procedure. The optimal synthesis of a Toffoli gate requires six CNOT gates. A Fredkin gate can be decomposed into six specific gates, i.e., two CNOT and three controlled-\( \sqrt{\text{NOT}} \) gates. (2) Our schemes are economic. Auxiliary electron qubits are employed in Refs. [13, 82], but they are not required in our schemes. Furthermore, only one single-photon medium is employed in our proposals. (3) The static electron qubits employed in our proposals are more robust than the moving qubits in Ref. [13]. (4) Different from Refs. [13, 50, 51] (the hybrid qubits are employed), all the qubits in our proposals are encoded on the spins of the electrons associated with NV centers, which means our quantum gates are scalable. Unfortunately, identical NV centers are required in our proposals, although the identical NV centers are challenge with current techniques, the energy levels of different NV centers can be adjusted by external magnetic fields. (5) They have a long coherence time in NV centers even at the room temperature. (6) Different from Ref. [46], all of the qubits in our proposals are encoded on the identical energy levels. (7) Our proposals are robust against low strain and magnetic fields, due to the special auxiliary energy level we employed. (8) Compared with an atom-cavity system, the time scale for manipulating an NV center is much shorter than that seen with an atom. Also it is difficult to trapped an atom in the cavity. Although high fidelities and efficiencies can be achieved in our schemes, only 4% of the emitted photon emitting from the NV centers are coherent emissions within the narrowband ZPL at 637 nm due to the particular characteristic of the NV centers.

In summary, we have designed the compact quantum circuits for implementing some deterministic universal quantum gates on NV centers, including the CNOT, Toffoli, and Fredkin gates, by means of the interaction between an NV-cavity-assisted qubit and a single-photon medium in a scalable fashion. The quantum gates are constructed by some input-output processes of a single photon medium, the measurements on the polarizations of the photon medium, and feedforward operations. As these quantum gates have a long coherence time even at the room temperature and they are universal, intrinsically deterministic, and scalable, they provide a different way for quantum computing in solid-state quantum systems.

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