Closed-loop three-level charged quantum battery

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Abstract – Quantum batteries are energy storage or extraction devices in a quantum system. Here, we present a closed-loop quantum battery by utilizing a circularly coupled three-state quantum system, and investigate its charging dynamics. The charging performance is greatly improved due to the existence of the third field in the system to form a closed-contour interaction. Through appropriately selecting the third control field, the maximum average power can be increased, even far beyond the ideal maximum power value of non-closed-loop three-level quantum battery. We study the effect of the global driving-field phase on the charging process and find that both the maximum extractable work (“ergotropy”) and the charging power vary periodically under different control fields, with a period of $2\pi$. Possible experimental implementation in nitrogen-vacancy spin is discussed as well.

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Introduction. – Classical batteries on the basis of electrochemical principles are extremely useful to fulfil our daily life needs [1]. Currently, with the ever-increasing demand on the performance of energy storage devices, researchers have tried to exploit quantum phenomena to create a new class of powerful batteries which transcend conventional electrochemistry, i.e., quantum batteries [2]. This entirely new concept was first introduced in [3] and has become a very active research field [4–35]. Quantum batteries are quantum devices that can store or extract energy to perform work [36]. More specifically, a great number of researchers have recently addressed various aspects of quantum batteries, including work extraction [4–9], capacity [10], role of entanglement and many-body interactions [11–15] and environmental effects, etc. [16–19]. The maximum work that can be extracted from the quantum battery compatible with quantum mechanics is called ergotropy [20–23,37]. Generally, the more energy stored, the higher the charging power, the better the battery performance.

Up to now, most researches on quantum cells of quantum batteries have focused on two-level systems [24–32] and spin chains [10–13,33–35]. For example, the collective charging scheme involves the concept of a Dicke quantum battery which consists of $N$ two-level systems, interacting with a photonic mode in a cavity to charge, and resulting in a quantum advantage in the charging power of a factor $\sqrt{N}$ [24]. The spin-spin interactions between one-dimensional spin chain can yield an advantage in charging power, which comes from a mean-field interaction and relies on an intrinsic interaction between quantum batteries [33]. A quantum battery based on a disordered quantum Ising chain is characterized by high extractable work at low entanglement and suppression of energy fluctuation by interaction [34].

A three-level system, where two of the three available transitions are coherently driven, is also an elementary building block of many quantum systems. It is widely used in light storage [38], atomic clock frequency standards [39] and coherent quantum control [40], ranging from ultracold atoms [41], trapped ions [42] to superconducting circuits [43] and nitrogen-vacancy (NV) center [44]. One of the advantages of the three-level system over a two-level one is the additional controllability offered by the coupling field. In recent years, an important research topic in the three-level system is that coherent driving of the third available transition forms a closed-contour interaction (the so-called closed-loop three-level system), which yields fundamentally new phenomena, including phase-controlled coherent population trapping and phase-controlled coherent population dynamics [45]. The closed-loop interactions are used in detection and separation of

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chiral molecules [46], coherent manipulation of a single spin [45] and adiabatic population transfer of a superconducting transmon circuit [47,48]. It is very desirable to take the three-level system as the constituent unit of the quantum battery. Very recently, a three-level system is used to constitute a quantum battery and a stable charging process is realized by employing the stimulated Raman adiabatic passage (STIRAP) technique [21]. The three-level quantum battery allows one to avoid the spontaneous discharging regime. Then a natural and interesting question is what would happen to the performance of a quantum battery if a closed-loop three-level system were applied to the design of a quantum battery.

In this work, we consider a quantum battery for a closed-loop three-level system driven by three laser fields in which the population dynamics depends on the three control fields and associated phases. We design the closed-loop three-level quantum battery model and study the charging dynamics, including charging energy and power. The rest of the paper is organized as follows. In the next section we introduce model and basic concepts of the closed-loop three-level quantum battery. Then we study the dynamic characteristics of quantum batteries in the closed-loop three-level system driven by three laser fields and associated phases. We design the closed-loop three-level system model and study the control fields and associated phases. We design the closed-loop three-level quantum battery. Very recently, a three-level system is take the three-level system as the constituent unit of the battery. To drive the system and promote transitions between different states of the battery and are charged through three laser fields: $\Omega_{12}, \Omega_{23}$ and $\Omega_{13}$. The spaces in the battery show the degree of charge, and the spaces gradually fill up from bare to fully charged batteries. When the three-level system is in the ground state (red), it is equivalent to a bare battery. We regard the intermediate state (yellow) as a partially charged battery. It represents a fully charged battery at the maximum excited state (green).

Our major objective is to analyze the performance of a three-level quantum battery, so that $d = 3$. We sketch our proposal in fig. 1. At the initial moment, the system is prepared in the ground state, representing a depleted battery. To drive the system and promote transitions between the energy levels, one utilizes auxiliary fields, i.e., suddenly switching on a transitional Hamiltonian $H_1$. We seek to inject as much energy into the quantum batteries as possible during the charging time $\tau$. The full Hamiltonian which describes the dynamics of the battery can be written as

$$H(t) = H_0 + \lambda(t)H_1(t),$$

where $\lambda(t)$ is a dimensionless parameter, whose explicit dependence on time $t$ manifests the external control exerted on the system. For the sake of definiteness, we assume $\lambda(t)$ equal to one for $0 < t < \tau$ and zero elsewhere. The character of the Hamiltonian $H_1(t)$ is equivalent to a quantum charger, and $\lambda(t)$ guarantees that it charges the battery only at $t \in [0, \tau]$.

The transitional Hamiltonian $H_1$ reads [45]

$$H_1(t) = \hbar \begin{bmatrix} 0 & \Omega_{12}(t)e^{-i(\omega_{12}t + \phi_1)} & \Omega_{13}(t)e^{-i(\omega_{13}t + \phi_3)} \\ \Omega_{12}(t)e^{i(\omega_{12}t + \phi_1)} & 0 & \Omega_{23}(t)e^{-i(\omega_{23}t + \phi_2)} \\ \Omega_{13}(t)e^{i(\omega_{13}t + \phi_3)} & \Omega_{23}(t)e^{i(\omega_{23}t + \phi_2)} & 0 \end{bmatrix}.$$

Here $\hbar$ is the reduced Planck constant. $\Omega_{12}, \Omega_{23}$ and $\Omega_{13}$ are amplitudes of three driving fields (see fig. 1). $\omega_{12}, \omega_{23}, \omega_{13}$ and $\phi_1, \phi_2, \phi_3$ are frequencies and phases of three driving fields, respectively. We assume they are real and positive.

The Hamiltonian $H_0$ plays a crucial role in how much energy a quantum battery stores. The total energy in the battery at time $t$ is implicitly

$$E(t) = \text{Tr}\{H_0\rho(t)\},$$

with $\rho = \sum_n r_n |r_n\rangle\langle r_n|$ being the density matrix of the system [22]. And the time evolution of the quantum state reads according to the Liouville-von Neumann equation [3]

$$\dot{\rho}(t) = \frac{1}{i\hbar}[H(t), \rho(t)].$$

Indeed, we explore the dynamics of the system charging in a time-dependent interaction picture and the
Hamiltonian $H(t)$ can be written as [49]

$$H_{\text{int}}(t) = \hbar \begin{bmatrix}
\begin{array}{ccc}
0 & \Omega_{12}(t) & \Omega_{13}(t)e^{i\phi} \\
\Omega_{12}(t) & 0 & \Omega_{23}(t) \\
\Omega_{13}(t)e^{-i\phi} & \Omega_{23}(t) & 0
\end{array}
\end{bmatrix},$$

(6)

where we assumed that the driving fields are in resonance. The global driving-field phase reads $\phi = \phi_1 + \phi_2 - \phi_3$, which strongly influences the resulting dynamics [45].

The time evolution of the quantum state is obtained from the equation

$$\dot{\rho}_{\text{int}}(t) = \frac{1}{\hbar}[H_{\text{int}}(t), \rho_{\text{int}}(t)],$$

(7)

with $\rho_{\text{int}}(t) = e^{iH_{\text{int}}t}\rho(t)e^{-iH_{\text{int}}t}$. In our charging protocol, we already assumed that the three external fields mentioned above are in resonance with the levels of the battery. Therefore, the population in each energy level satisfies

$$P_n = \text{Tr}\{\hat{P}_n \rho_{\text{int}}(t)\} = \text{Tr}\{\hat{P}_n \rho(t)\},$$

(8)

where $\hat{P}_n$ represents the projector $\hat{P}_n = |\varepsilon_n\rangle\langle \varepsilon_n|$. Furthermore, $\text{Tr}\{H_0 \rho_{\text{int}}(t)\} = \text{Tr}\{H_0 \rho(t)\}$, the extractable work can be obtained from the difference

$$C(t) = \text{Tr}\{H_0 \rho_{\text{int}}(t)\} - \varepsilon_1.$$  

(9)

Allowing the system to undergo adiabatic dynamics, the evolved state is $|\psi^{\text{ad}}(t)\rangle$, and the ergotropy is

$$C(t) = \langle \psi^{\text{ad}}(t)|H_0|\psi^{\text{ad}}(t)\rangle - \varepsilon_1|\langle H_0|\varepsilon_1\rangle|$$

$$= \langle \psi^{\text{ad}}(t)|H_0|\psi^{\text{ad}}(t)\rangle - \varepsilon_1.$$  

(10)

Notice that if $\varepsilon_1 = 0$ then the ergotropy coincides with the mean energy of $\rho_{\text{int}}$, i.e., $C(t) = E(t)$. For the total charging time $\tau$, the corresponding work is $C(\tau)$,

$$P(\tau) = \frac{C(\tau)}{\tau}.$$  

(11)

In ref. [21], $\Omega_{13} = 0$, the STIRAP protocol can avoid the oscillatory behavior and achieve a stable charging process. Consistently with the previous works [21], we take

$$\Omega_{12}(t) = \Omega_0 f(t), \quad \Omega_{23}(t) = \Omega_0 [1 - f(t)],$$  

(12)

where $\Omega_0$ is a constant and $f(t) = t/\tau$. In what follows, we calculate and analyze the ergotropy and the charging power for different parameters shown in the interaction Hamiltonian $H_{\text{int}}$.

**Dynamics in closed-loop quantum batteries.** — We now analyze the charging process focusing on the closed-loop quantum battery. We first consider a special case, i.e., the phase $\phi = \pi/2$. The corresponding eigenstates of the Hamiltonian (6) are

$$|E_-(t)\rangle = \frac{1}{\sqrt{2}} \left( \frac{\Omega_{12}(t)\Omega_{23}(t)}{\Omega_{13}(t)} - \frac{\Omega_{13}(t)}{\Omega_{12}(t)} \right) |\varepsilon_1\rangle$$

$$- \frac{1}{\sqrt{2}} \left( \frac{\Omega_{23}(t)}{\Omega_{13}(t)} - \frac{\Omega_{13}(t)\Omega_{23}(t)}{\Omega_{12}(t)\Omega_{23}(t)} \right) |\varepsilon_2\rangle + \frac{1}{\sqrt{2}} \frac{\Omega_{13}(t)}{\Omega_{12}(t)} |\varepsilon_3\rangle,$$

(13)

$$|E_0(t)\rangle = \frac{\Omega_{23}(t)}{\Omega_{13}(t)} |\varepsilon_1\rangle + i \frac{\Omega_{13}(t)}{\Omega_{12}(t)} |\varepsilon_2\rangle - \frac{\Omega_{13}(t)}{\Omega_{12}(t)} |\varepsilon_3\rangle,$$

(14)

$$|E_+(t)\rangle = \frac{1}{\sqrt{2}} \left( \frac{\Omega_{12}(t)\Omega_{23}(t)}{\Omega_{13}(t)} + i \frac{\Omega_{13}(t)}{\Omega_{12}(t)} \right) |\varepsilon_1\rangle$$

$$+ \frac{1}{\sqrt{2}} \left( \frac{\Omega_{23}(t)}{\Omega_{13}(t)} + i \frac{\Omega_{13}(t)\Omega_{23}(t)}{\Omega_{12}(t)\Omega_{23}(t)} \right) |\varepsilon_2\rangle + \frac{1}{\sqrt{2}} \frac{\Omega_{13}(t)}{\Omega_{12}(t)} |\varepsilon_3\rangle,$$

(15)

with the eigenenergies $E_0(t) = 0$ and $E_{\pm}(t) = \pm h\Omega(t)$, where $\Omega_2^2(t) = \Omega_{12}^2(t) + \Omega_{23}^2(t) + \Omega_{13}^2(t)$ and $\Omega_2^2(t) = \Omega_{23}^2(t) + \Omega_{23}^2(t)$.

To achieve an efficient charged state, we employ the STIRAP technique [50,51] and assume the initial state of the system $|\psi(0)\rangle = |E_0(0)\rangle = |\varepsilon_1\rangle$. Therefore, the initial values of the control fields satisfy $\Omega_{12}(0) = \Omega_{13}(0) = 0$ and $\Omega_{23}(0) \neq 0$. When the system undergoes adiabatic dynamics, the evolved state becomes $|\psi^{\text{ad}}(t)\rangle = |E_0(t)\rangle$. Then the ergotropy is

$$C(t) = \frac{\Omega_{23}^2(t)}{\Omega_{13}^2(t)} \varepsilon_1 + \frac{\Omega_{13}^2(t)}{\Omega_{12}^2(t)} \varepsilon_2 + \frac{\Omega_{12}^2(t)}{\Omega_{23}^2(t)} \varepsilon_3 - \varepsilon_1.$$  

(16)

One finds that the ergotropy depends on the final values for control fields $\Omega_{12}(t), \Omega_{23}(t)$ and $\Omega_{13}(t)$ at some cutoff time $\tau_c$ and can arrive at the maximal value $C_{\text{max}} = \varepsilon_3 - \varepsilon_1$ when $\Omega_{13}(\tau_c) = \Omega_{23}(\tau_c) = 0$ and $\Omega_{12}(\tau_c) \neq 0$. In the end we select the suitable control fields, such that the above boundary conditions are satisfied. In the following calculation we take $\varepsilon_1 = 0$, $\varepsilon_2 = h$, $\varepsilon_3 = 1.95h$, respectively. The control field $\Omega_{13}(t)$ is taken as

$$\Omega_{13}(t) = \Omega_0 \sin(\pi t/\tau), \quad \Omega_0(1 - \cos(2\pi t/\tau)),$$

$$\Omega_0(1 - \cos(2\pi t/\tau))^2.$$  

(17)

The dependence of the ergotropy $C(t)$ and the average charging power $P(t)$ on $\tau$ is indicated in fig. 2 for several categories of $\Omega_{13}$. Here $h\tau$ is the dimensionless parameter. In order to analyze the advantages of the closed loop, we also plot the situation of the non-closed loop (solid black line), corresponding to $\Omega_{13} = 0$. Differently from the non-closed loop case, the evolution process of the ergotropy is divided into four windows along the $\tau$ dimension. For a fast evolution, the ergotropy is almost 0 due to being far from the adiabatic limit at these timescales. With increasing $\tau$ the ergotropy grows monotonically to a maximum value, which achieves a fully charged state (corresponding to the maximum charging energy), then it begins to oscillate, and finally reaches and stays at its maximum value for large timescales. We also clearly see that, for different control fields, because of the different

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transfer time of all the population from the initial ground state to the maximally excited state, the minimal time to reach the maximum ergotropy for the first time is different. As a result, for a fast protocol our batteries fail to charge (corresponding to small average power). However, as $\tau$ increases, the average charging power also increases until it reaches a maximum value at some point. Beyond this timescale, the average power will be less than this maximum value.

It is interesting to note that the charging performance of the closed-loop battery is greatly improved, and the maximum average charging power is even far beyond the ideal maximum average power value of the non-closed-loop three-level quantum battery [21]. In more detail, the value is close to 4 times that for the original non-closed-loop battery with $\Omega_{13} = \Omega_0 \sin(\pi t/\tau)$. Further study shows that the maximum average charging power can be increased by increasing the index $n$ of the control field. Furthermore, compared with the ergotropy of the non-closed-loop system at its maximum average charging power, the closed-loop batteries charged by three laser fields can store more ergotropy, i.e., the existence of the control field $\Omega_{13}$ can greatly accelerate the charging process and improve the maximum average charging power. Therefore, for an optimized $\Omega_{13}$, the system can realize highly efficient and stable charging process as long as we immediately turn off the $H_1$ after the moment of reaching the maximum average charging power.

So far we have only considered the case of the total driving phase $\phi = \pi/2$. To further demonstrate the effect of the phase on the charging process, we calculate the maximum average charging power and the charging energy at different phases. Figure 3 shows the maximum average charging power and the corresponding energy as a function of $\phi$ under the different control field $\Omega_{13}$. No matter what the control field is, the maximum charging power and energy reach their maximum value at $\phi = \pi/2$ and have the same period $2\pi$. As the index $n$ increases, the maximum average charging power and the corresponding energy increase and the amplitude of oscillation decreases. For a clearer and more comprehensive understanding of the effect of the phase on the charging energy and the average power.
charging power, in fig. 4, we display the charging energy and the average charging power as a function of both phase $\phi$ and charging time $\tau$ for different $\Omega_{13}$. (a) and (b): $\Omega_{13} = \Omega_0 \sin(\pi t/\tau)$. (c) and (d): $\Omega_{13} = \Omega_0 (1 - \cos(2\pi t/\tau))$. (e) and (f): $\Omega_{13} = \Omega_0 (1 - \cos(2\pi t/\tau))^2$.

**Possible implementation in nitrogen-vacancy spin.** – There are several physical systems to implement the closed-loop three-level quantum battery such as trapped ion systems or superconducting circuit systems [21,47,48]. Here we briefly describe a scheme which coherently drives the NV spin using a combination of time-varying magnetic and strain fields to implement a three-level quantum battery. The negatively charged NV center in the diamond lattice forms an $S = 1$ spin system. Under an appropriate rotating frame and the resonant case, the dynamics of the NV spin are described by the Hamiltonian (6) [45,52]. Conveniently, the initialization of the system can be realized by means of optical spin pumping under green laser excitation. Even at room temperatures, the spin of the NV can also be initialized easily. This character makes it become a platform for quantum information processing [44]. The three eigenstates of the spin operator $\hat{S}_z$ are $| - 1 \rangle$, $| 0 \rangle$ and $| + 1 \rangle$, which correspond to $| \varepsilon_1 \rangle$, $| \varepsilon_2 \rangle$ and $| \varepsilon_3 \rangle$ in our battery, respectively. Thus, one can prepare the system corresponding to a bare quantum battery in $| \varepsilon_1 \rangle$ at first, and then utilize a time-varying strain field to drive the $| - 1 \rangle \leftrightarrow | + 1 \rangle$ transition and microwave magnetic fields to drive the $| 0 \rangle \leftrightarrow | \pm 1 \rangle$ transitions. The NV spin can be optically read out by virtue of its spin-dependent fluorescence. At last, the ergotropy and charging power of the three-level quantum battery can be obtained by uncomplicated calculation.

**Conclusions.** – We have introduced the concept of a “closed-loop three-level quantum battery”, which is a three-level system driven by three available transitions forming a closed-contour interaction. We show that the performance of the quantum battery can be greatly improved by choosing the appropriate third driving field. The closed-contour interaction makes the maximum average charging power be greatly increased, even far beyond the most ideal maximum power value of the non-closed-loop three-level quantum battery. In addition, the charging energy and power can reach the peak value at phase $\phi = \pi/2$ and vary with the phase with a period of $2\pi$. Finally, we have briefly described the scheme of realizing the closed-loop three-level quantum battery by a nitrogen-vacancy spin system.

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