The coordinate coherent states approach revisited

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Abstract

We revisit the coordinate coherent states approach through two different quantization procedures in the quantum field theory on the noncommutative Minkowski plane. The first procedure, which is based on the normal commutation relation between an annihilation and creation operators, deduces that a point mass can be described by a Gaussian function instead of the usual Dirac delta function. However, we argue this specific quantization by adopting the canonical one (based on the canonical commutation relation between a field and its conjugate momentum) and show that a point mass should still be described by the Dirac delta function, which implies that the concept of point particles is still valid when we deal with the noncommutativity by following the coordinate coherent states approach. In order to investigate the dependence on quantization procedures, we apply the two quantization procedures to the Unruh effect and Hawking radiation and find that they give rise to significantly different results. Under the first quantization procedure, the Unruh temperature and Unruh spectrum are not deformed by noncommutativity, but the Hawking temperature is deformed by noncommutativity while the radiation spectrum is untack. However, under the second quantization procedure, the Unruh temperature and Hawking temperature are untack but the both spectra are modified by an effective greybody (deformed) factor.

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1 Introduction

After the seminal work by Seiberg and Witten [1], the noncommutativity of spacetimes was revived and paid more and more attention to henceforth, for instance, it was believed [2, 3, 4] to be an indispensable ingredient for the quantization of gravity. In general there exist two common methods to deal with the noncommutative spacetime. The first one is the “star-product”, which encodes the noncommutativity through replacing the ordinary product between functions by the Moyal-Weyl product [5, 6, 7]. This method has intensively been applied to the construction of field theories and gravity theories on noncommutative spacetimes. For reviews, see refs. [8, 9, 10, 11]. Within this framework one calculates only order by order in noncommutative parameters and then loses the nonlocality of noncommutative theories. Nevertheless, the second method, i.e. the “coordinate coherent states approach” [12, 13, 14] is quite different from the first one in the study of noncommutative quantum mechanics and quantum field theory. The models established in quantum field theory with such an approach are consistent with the Lorentz invariance (restricted in the Euclidean space), unitarity and UV-finiteness. The main idea of this approach is that the physical position of a point is represented by the mean value of its coordinate operators on coherent states. Associated with the quantization procedure based on the normal commutation relation between an annihilation and creation operators, the plane wavefunction which usually represents a “free point particle” gets deformed by a damping factor, and then the Feynman propagator of scalar fields acquires an extra damping factor. This indicates that a point mass can be described by a Gaussian function instead of the usual Dirac delta function, i.e. a point mass is smeared over the width $\sqrt{\theta}$, where $\theta$ is the noncommutative parameter. Black hole solutions with this Gaussian point source, known as “noncommutative inspired black holes”, have been shown to possess some special properties [15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

However, the specific quantization procedure mentioned above is noncanonical, under which, instead of the commutator between a field and its conjugate momentum, the commutator between an annihilation and creation operators is imposed to take the normal form as a basic point. This gives rise to the result that the commutator between a field and its conjugate momentum takes a deformed form containing a damping factor. From the point of view of the canonical quantization of field theory, we argue in this paper the noncanonical quantization procedure by still following the coordinate coherent states approach but taking the canonical quantization procedure, and quite interestingly we have a significantly different result. That is, if we demand the commutator between the field and its conjugate momentum takes the canonical form as the starting point, the Feynman propagator will be the usual form as that in the commutative theory. This indicates that a point mass should still be described by the usual Dirac delta function. As a result, the concept of point particles is still valid if we deal with the spacetime noncommutativity by making use of the coordinate coherent states approach. We note that the commutator between an annihilation and creation operators now gets deformed in the canonical quantization procedure, which still leads to deformed modes
as the representation of the spacetime noncommutativity.

In order to investigate the dependence on quantization procedures, we apply the two quantization procedures to the Unruh effect and Hawking radiation and find that they indeed lead to significantly different results. There have been some literature [25, 26, 27, 28, 29, 30, 31] concerning the issues based on the coordinate coherent states approach associated with the noncanonical quantization procedure. For instance, in ref. [25] the Unruh effect on noncommutative spacetimes was studied by the introduction of the Unruh-DeWitt detector. It was obtained that the positive Wightman-Green function acquires an extra damping factor in momentum space and then the response rate is suppressed by the noncommutativity of spacetime. As a result, it was argued that the Unruh-DeWitt detector registers a temperature which is so greatly suppressed by the noncommutativity that it can be neglected. In addition, based on the quantum field theory in curved spacetime and the Bogoliubov transformation, it was claimed in ref. [26] that the Hawking radiation spectrum is not deformed by the noncommutativity and the Hawking temperature cannot be neglected. We note that the different results caused by the specific treatments in refs. [25, 26] emerge from the noncommutativity of spacetime and such a difference disappears on the ordinary (commutative) spacetime. As to the reasons behind, see ref. [26] for the details. We re-examine the issues by using the coordinate coherent states approach associated with both the noncanonical and canonical quantization procedures and by considering the technique of quantum field theory in curved spacetime and the Bogoliubov transformation. The results are shown to be dependent on the quantization procedures. Under the noncanonical quantization procedure, the Unruh temperature and Unruh spectrum are not deformed by noncommutativity, but the Hawking temperature is deformed by noncommutativity while the radiation spectrum is untack. However, under the canonical quantization procedure, the Unruh temperature and Hawking temperature are untack but the both spectra are modified by an effective greybody (deformed) factor.

This paper is organized as follows. In the next section we shall discuss the quantum field theory on the noncommutative Minkowski plane by means of the coordinate coherent states approach associated with the two quantization procedures. This section contains two subsections. In the first subsection, we shall give a detailed review on the noncanonical quantization procedure proposed in refs. [12, 13, 14, 26]. In the second subsection, we shall take a look at the coordinate coherent states approach associated with the canonical quantization procedure. In section 3 we shall discuss the Unruh effect and Hawking radiation by following the two quantization procedures respectively in subsections 3.1 and 3.2. The final section is devoted to the conclusion.
2 Quantum field theory on noncommutative Minkowski plane

In this section, we discuss the quantum field theory on the noncommutative Minkowski plane in terms of the coordinate coherent states approach under the two quantization procedures. In the noncanonical quantization, the commutator between the annihilation and creation operators takes the normal form as the starting point and then the commutator between the field and its conjugate momentum is deformed by noncommutativity. In the canonical quantization, on the contrary, the commutator between the field and its conjugate momentum is set to be normal at the beginning and this leads to the noncommutative deformed commutator between the annihilation and creation operators. Due to the fact that the modes of fields are related closely to the annihilation and creation operators, the noncommutative effects are thus represented by the deformed modes. From the point of view of quantization of field theory, the latter procedure seems to be more reasonable. We note that the two quantization procedures are identical if no noncommutativity is involved in.

2.1 The noncanonical quantization procedure

In this subsection we just review the treatment and its resulting outcomes under the noncanonical quantization procedure proposed in refs. [12, 13, 14, 26]. For simplicity, we consider a two-dimensional noncommutative Minkowski plane with coordinate operators $\hat{x}^0$ and $\hat{x}^1$ satisfying the algebra

$$[\hat{x}^0, \hat{x}^1] = i\theta, \tag{1}$$

where $\theta$ denotes a constant noncommutative parameter. In terms of complex coordinate operators $\hat{z} \equiv \hat{x}^0 + i\hat{x}^1$, $\hat{z}^\dagger \equiv \hat{x}^0 - i\hat{x}^1$, the algebra eq. (1) becomes

$$[\hat{z}, \hat{z}^\dagger] = 2\theta. \tag{2}$$

The coherent states $|z\rangle$, which are defined as the eigenstates of the complex coordinate operator $\hat{z}$: $\hat{z}|z\rangle = z|z\rangle$, have the following normalized form,

$$|z\rangle = e^{-\frac{i\theta}{4\sigma}} e^{\frac{z\hat{z}}{2\theta}} |0\rangle. \tag{3}$$

The physical position of a particle $(x^0, x^1)$ is defined as the expectation values of the coordinate operators $(\hat{x}^0, \hat{x}^1)$ on the coherent states,

$$x^0 \equiv \langle z|\hat{x}^0|z\rangle = \text{Re}z, \quad x^1 \equiv \langle z|\hat{x}^1|z\rangle = \text{Im}z. \tag{4}$$

For the noncommutative version of the plane wave operator $e^{ip_{\mu}\hat{x}^\mu}$, where $\mu = 0, 1$, and the metric is $\eta_{\mu\nu} = \text{diag}(-1, 1)$, its mean value on the coherent states takes the form\(^1\),

$$\langle z|e^{ip_{\mu}\hat{x}^\mu}|z\rangle = e^{-\frac{i}{4}(p_0^2 + p_1^2) + ip_{\mu}x^\mu}, \tag{5}$$

\(^1\)It can be verified that the damping factor $e^{-\frac{i}{4}(p_0^2 + p_1^2)}$ takes the same form in both the Euclidean and
which is interpreted as the wavefunction of a “free point particle” on the noncommutative plane. Note that the plane wavefunction acquires a damping factor $e^{-\frac{\theta}{4}(p_0^2+p_1^2)}$ which will play a central role in quantization.

Now we consider a massless real scalar field $\phi$ propagating on the noncommutative Minkowski plane. In terms of the coordinate coherent states approach [12, 13, 14] the action is undeformed by the spacetime noncommutativity and thus takes the usual form,

$$S = -\frac{1}{2} \int d^2x \, \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi,$$

which is conformally invariant. The field $\phi$ obeys the Klein-Gordon equation,

$$\partial_\mu \partial^\mu \phi = 0.$$ (7)

When quantizing the field, we have to expand the field $\phi(t, x)$ in terms of a set of plane wavefunction modes. According to the above prescriptions, on the noncommutative plane the positive frequency modes are modified as

$$u_p(t, x) = e^{-\theta \omega^2/2} \sqrt{4\pi \omega} e^{-i\omega t + ipx},$$ (8)

where we identify $p_0$ with $\omega = |p|$ and set $p_1 = p$. These modes are orthogonal but have a deformed Klein-Gordon product,

$$(u_p, u_{p'}) \equiv -i \int dx \, (u_p \partial_t u_{p'}^* - u_{p'} \partial_t u_p) = e^{-\theta \omega^2} \delta(p - p').$$ (9)

When expanding the field $\phi(t, x)$, we have to use the deformed modes,

$$\phi(t, x) = \int_{-\infty}^{\infty} dp \, [\hat{a}_p u_p(t, x) + \hat{a}_p^\dagger u_p^*(t, x)],$$ (10)

where the annihilation and creation operators are imposed to satisfy the normal commutation relation,

$$[\hat{a}_p, \hat{a}_{p'}^\dagger] = \delta(p - p').$$ (11)

In this case, the equal-time commutator between the field $\phi$ and its conjugate momentum $\dot{\phi}$ is deformed to be

$$[\phi(t, x), \dot{\phi}(t', x')] = \frac{i}{2\sqrt{\pi \theta}} e^{-\frac{(x-x')^2}{4\theta}}.$$ (12)

Minkowski signatures. Although the commutation relation between coordinate operators (eq. (1)) preserves the Lorentz boost (rotational) symmetry in the Minkowski (Euclidean) signature, the noncommutative plane wavefunction eq. (5) violates the Lorentz boost invariance. This violation stems from the Lorentz non-invariance of the coherent states (see eq. (3)). We point out that such a Lorentz non-invariance is intrinsic in the coordinate coherent states approach [12, 13, 14]. It is easy to check that the coherent states eq. (3) preserve the Lorentz rotational symmetry in the Euclidean signature but violate the Lorentz boost symmetry in the Minkowski signature. For the details, see the Appendix. The reason that we take the Minkowski signature is that we shall discuss the radiation of black holes by using the technique of quantum field theory in curved spacetime. Therefore, it is natural to choose the Minkowski signature for investigating relative physical phenomena.
Using eqs. (10) and (11) we can derive the Wightman’s positive frequency function

\[ G^+(x^\mu, x'^\mu) \equiv \langle 0 | \phi(x^\mu) \phi(x'^\mu) | 0 \rangle = \int \frac{dp}{4\pi \omega} e^{-\omega^2 - i\omega(t-t') + ip(x-x')}. \] (13)

As usual, the Feynman propagator \( G_F(x^\mu, x'^\mu) \) is defined as

\[ G_F(x^\mu, x'^\mu) \equiv \langle 0 | T \phi(x^\mu) \phi(x'^\mu) | 0 \rangle = \Theta(t - t')G^+(x^\mu, x'^\mu) + \Theta(t' - t)G^+(x'^\mu, x^\mu), \] (14)

where \( \Theta(x) \) is the step function. With eq. (13) and a proper choice of integral contour, the Feynman propagator eq. (14) can be expressed as

\[ G_F(x^\mu, x'^\mu) = i \int \frac{d^2p}{(2\pi)^2} \frac{e^{-\theta(p_0^2 + p_1^2)/2 - ip\mu(x^\mu - x'^\mu)}}{p_0^2 - p_1^2 + i\epsilon}. \] (15)

Therefore, we can read off the Feynman propagator in momentum space,

\[ G_F(p_0, p_1) = \frac{i}{p_0^2 - p_1^2 + i\epsilon} e^{-\theta(p_0^2 + p_1^2)/2}. \] (16)

It is easy to check that it satisfies the following equation,

\[ (\partial_t^2 - \partial_x^2)G_F(x^\mu, x'^\mu) = -\frac{i}{2\pi \theta} e^{-\frac{(t-t')^2 + (x-x')^2}{2\theta}}. \] (17)

Note that the damping factor \( \exp\{-(t-t')^2 + (x-x')^2\}/2\theta \) in eq. (17) originates from the damping factor \( e^{-\theta(p_0^2 + p_1^2)/2} \) in eq. (5), and both are independent of the spacetime signatures. For the Euclidean signature, we only have to replace the temporal coordinate \( t \) in eq. (17) by one spatial coordinate. We have already emphasized this point in footnote 1.

For a massive real scalar field, we can obtain the same result as eqs. (15), (16) and (17) just with the replacement of the suitable mass shell condition [12, 13, 14, 26]. Note that a Gaussian function appears on the right-hand side of eq. (17) instead of the usual Dirac delta function. The corresponding result given in refs. [15, 16, 17, 18, 20, 21, 22, 23, 24] is that a point with mass \( M \) is described by the following Gaussian function rather than the usual Dirac delta function when one considers the noncommutativity abiding by the coordinate coherent states approach under the noncanonical quantization procedure,

\[ \rho_\theta(r) = \frac{M}{(4\pi \theta)^{3/2}} e^{-\frac{r^2}{4\theta}}. \] (18)

\(^2\)Actually, eq. (18) is obtained based on the spacetime with only space-space noncommutativity where \( \theta_0^1 = 0 \). In this situation, the exponential factor appeared in eq. (17) only involves in the spatial coordinates and the Gaussian distribution eq. (18) is thus deduced. See also the previous footnote and the comment under eq. (17).
2.2 The canonical quantization procedure

In this subsection we re-examine the issues mentioned in the previous subsection but follow the canonical quantization procedure. In other words, if we begin with the equal-time canonical commutator between $\phi$ and $\dot{\phi}$, that is, if eq. (12) is replaced by

$$[\phi(t, x), \dot{\phi}(t, x')] = i\delta(x - x'),$$

(19)

the commutation relation between the annihilation operator and creation operator eq. (11) then changes to be

$$[\hat{a}_p, \hat{a}^\dagger_{p'}] = e^{i\theta\omega^2}\delta(p - p').$$

(20)

This keeps the same modes of fields (see eqs. (8) and (10)) as in the noncanonical case. Making a similar analysis to that done in the previous subsection, we can derive the Feynman propagator in momentum space,

$$G_F(p_0, p_1) = \frac{i}{p_0^2 - p_1^2 + i\epsilon}.$$

(21)

It is easy to check that the Feynman propagator now satisfies the usual equation,

$$(\partial_t^2 - \partial_x^2)G_F(x^\mu, x'^\mu) = -i\delta^2(x^\mu - x'^\mu),$$

(22)

which is not deformed by the noncommutativity of spacetime. We note that a similar result has been mentioned in the star-product formalism, which provides a helpful support to our result from a different point of view. That is, in the star-product formalism the noncommutativity does not affect free fields although it affects interactions. For more details, see refs. [32, 8, 9].

The authors of the literature [12, 13, 14, 15] deduced from the damped factor in eq. (17) that the distribution of “point mass” can be described by a Gaussian function (eq. (18)). However, as we see now, when the canonical quantization procedure is applied, the damped factor disappears in the Green’s equation (eq. (22)). As a result, we can deduce that the “point mass” is not smeared over the width $\sqrt{\theta}$ and should still be described by the usual Dirac delta function. This is the key point of the present paper and it does not depend on the spacetime signatures. The difference of the two Green functions in eq. (17) and eq. (22) leads to some significant results when we discuss the Unruh effect and Hawking radiation of the noncommutative inspired Schwarzschild black hole as we shall show in section 3. We may give a natural interpretation, i.e. one can still keep the concept of point particles when one takes the average effect of noncommutativity, which seems to be a merit for the canonical procedure. That is, the effect of noncommutativity is diluted when the coordinate coherent states approach together with the canonical quantization is adopted.

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3The characteristic of the coordinate coherent states approach is the close relationship with the average value of various operators on coherent states. See, for instance, eqs. (4) and (5).
3 Unruh effect and Hawking radiation on noncommutative Minkowski plane

In order to investigate the dependence on quantization procedures, we now use the coordinate coherent states approach to discuss the Unruh effect and Hawking radiation on the noncommutative Minkowski plane under the two quantization procedures. As we shall see, the two quantization procedures will give rise to significantly different results. Under the noncanonical procedure, the Unruh temperature and Unruh spectrum are not deformed by noncommutativity, but the Hawking temperature is deformed by noncommutativity while the radiation spectrum is untack. Under the canonical procedure, the Unruh temperature and Hawking temperature are untack but the both spectra are modified by an effective greybody (deformed) factor.

3.1 Under the noncanonical quantization procedure

First we give the general formulae of the Bogoliubov transformation. The left- \((p < 0)\) and right-moving \((p > 0)\) modes do not affect each other in our following discussions and can be considered separately. For simplicity, in the following we only consider the right-moving mode. In the quantization of fields on a general spacetime, there exist more than one complete orthonormal set of modes with which we can expand the field operator \(\phi\). Assume that \(u_\omega(t, x)\) and \(v_\Omega(t, x)\) are two bases of orthonormal modes of the type\(^4\) as eq. (8),

\[
\begin{align*}
(u_\omega, u_{\omega'}) &= C^2_\omega \delta(\omega - \omega'), \\
(v_\Omega, v_{\Omega'}) &= C^2_\Omega \delta(\Omega - \Omega'),
\end{align*}
\]

(23)

where we use \(C_\omega (C_\Omega)\) to denote the damping factor \(C_\omega = e^{-\theta \omega^2/2}\) \((C_\Omega = e^{-\theta \Omega^2/2}\)). As done in ref. [26], we can rewrite the damped modes as \(u_\omega = C_\omega U_\omega\) and \(v_\Omega = C_\Omega V_\Omega\), where \(U_\omega\) and \(V_\Omega\) are the standard modes of the commutative theory.

We expand the scalar field \(\phi(t, x)\) as

\[
\phi(t, x) = \int_0^\infty d\omega \left( \hat{a}_\omega u_\omega + \hat{a}^\dagger_\omega u^*_\omega \right) = \int_0^\infty d\Omega \left( \hat{b}_\Omega v_\Omega + \hat{b}^\dagger_\Omega v^*_\Omega \right),
\]

(24)

where the both sets of operators \(\hat{a}_\omega (\hat{a}^\dagger_\omega)\) and \(\hat{b}_\Omega (\hat{b}^\dagger_\Omega)\) are imposed to satisfy the normal commutation relations,

\[
[\hat{a}_\omega, \hat{a}^\dagger_{\omega'}] = \delta(\omega - \omega'), \quad [\hat{b}_\Omega, \hat{b}^\dagger_{\Omega'}] = \delta(\Omega - \Omega').
\]

(25)

There exists the so-called Bogoliubov transformation which connects the two sets of operators,

\[
\hat{b}_\Omega = \frac{1}{C_\Omega} \int_0^\infty d\omega \ C_\omega (\alpha^*_{\Omega\omega} \hat{a}_\omega - \beta^*_{\Omega\omega} \hat{a}^\dagger_\omega),
\]

(26)

\(^4\)Henceforth, we use energy \(\omega\) or \(\Omega\) rather than momentum \(p\) as subscripts to label different modes.
where $\alpha_\Omega$ and $\beta_\Omega$ are the standard Bogoliubov coefficients given by

$$\begin{align*}
\alpha_\Omega &= (V_\Omega, U_\omega), \\
\beta_\Omega &= -(V_\Omega, U^*_\omega).
\end{align*} \tag{27}$$

We note that the Bogoliubov transformation in ref. [26] is incorrect (see eq. (24) of ref. [26]) although the final result on the Hawking radiation coincides with ours. Substituting eq. (26) into the second equality of eq. (25), we get the following normalization condition

$$\int_0^\infty d\omega \, C^2_\omega (\alpha^*_\omega \alpha_\Omega - \beta^*_\omega \beta_\Omega) = C^2_\Omega \delta(\Omega - \Omega') \tag{28}.$$  

When the field is at the “$a$-vacuum” $|0_a\rangle$ defined as $\hat{a}_\omega |0_a\rangle = 0$, the “$b$-particle” number measured in the $\Omega$th mode $\hat{N}_\Omega = \hat{b}^\dagger_\Omega \hat{b}_\Omega$ is

$$\langle 0_a | \hat{N}_\Omega | 0_a \rangle = \frac{1}{C^2_\Omega} \int_0^\infty d\omega \, C^2_\omega |\beta_\Omega\rangle^2 \tag{29}.$$  

Let us use the above formulae to re-examine the Unruh effect on the noncommutative Minkowski plane. For reviews on the Unruh effect on the commutative spacetime, see refs. [33, 34, 35].

In the inertial frame $(t, x)$, the metric takes the usual form,

$$ds^2 = dt^2 - dx^2 = du dv, \tag{30}$$

where $u$ and $v$ are lightcone coordinates defined as

$$u \equiv t - x, \quad v \equiv t + x. \tag{31}$$

The proper set of modes of the type as eq. (8) corresponding to the inertial observer is

$$u_\omega = \frac{e^{-\theta \omega^2/2}}{\sqrt{4\pi \omega}} e^{-i\omega u} = C_\omega U_\omega. \tag{32}$$

In the comoving frame $(\eta, \xi)$ of the Rindler observer with a constant acceleration $a$, the metric has the form

$$ds^2 = e^{2a\xi}(d\eta^2 - d\xi^2) = e^{a(\tilde{v} - \tilde{u})} d\tilde{u} d\tilde{v}, \tag{33}$$

where the lightcone coordinates of the comoving frame are given by

$$\tilde{u} \equiv \eta - \xi, \quad \tilde{v} \equiv \eta + \xi. \tag{34}$$

The proper set of modes of the type as eq. (8) corresponding to the Rindler observer is

$$v_\Omega = \frac{e^{-\theta \Omega^2/2}}{\sqrt{4\pi \Omega}} e^{-i\Omega \tilde{u}} = C_\Omega V_\Omega. \tag{35}$$
The transformation between the inertial frame \((t, x)\) and the comoving frame \((\eta, \xi)\) is given by
\[
\begin{align*}
u &= -\frac{1}{a} e^{-a\bar{u}}, \\
v &= \frac{1}{a} e^{a\bar{u}}.
\end{align*}
\] (36)

We compute the relation between the standard Bogoliubov coefficients \(\alpha_{ij}\) and \(\beta_{ij}\) and obtain the well-known result [36, 37, 34, 35],
\[
|\alpha_{\Omega\omega}|^2 = e^{\frac{2\pi\Omega}{a}}|\beta_{\Omega\omega}|^2.
\] (37)

For \(\Omega = \Omega'\) the normalization condition eq. (28) becomes
\[
\int_0^\infty d\omega \, C_\omega^2 (|\alpha_{\Omega\omega}|^2 - |\beta_{\Omega\omega}|^2) = C_\Omega^2 \delta(0).
\] (38)

From eqs. (29), (37) and (38), we know that when the field is at the “Minkowski-vacuum” \(|0_M\rangle\) defined by the inertial observer \(\hat{a}_\omega|0_M\rangle = 0\), the number of “\(b\)-particles” with energy \(\Omega\) observed by the Rindler observer is
\[
\langle \hat{N}_\Omega \rangle \equiv \langle 0_M|\hat{b}^\dagger_\Omega \hat{b}_\Omega|0_M\rangle = \frac{1}{C_\Omega^2} \int_0^\infty d\omega \, C_\omega^2 |\beta_{\Omega\omega}|^2 = \frac{\delta(0)}{e^{\frac{2\pi\Omega}{a}} - 1},
\] (39)

where \(\delta(0)\) appears due to an infinite volume \((\delta(0) \sim V)\). Therefore, the density of the number of particles takes the form: \(\langle \hat{n}_\Omega \rangle \equiv \frac{\langle \hat{N}_\Omega \rangle}{\delta(0)} = \frac{1}{e^{\frac{2\pi\Omega}{a}} - 1}\). From the spectrum eq. (39) we can read out the Unruh temperature,
\[
T_a = \frac{a}{2\pi}.
\] (40)

This result shows that both the Unruh spectrum and Unruh temperature are not modified by the spacetime noncommutativity.

We now compare our result with that of ref. [25] which was obtained based on the Unruh-DeWitt detector. In ref. [25] a suppressed response rate in fact corresponds [26] to a suppressed energy flux. In our case, the Hamiltonian operator for the scalar field is
\[
\hat{H} = \frac{1}{2} \int d\xi \left( \dot{\phi}^2 + \phi'^2 \right) = \frac{1}{\pi} \int_0^\infty d\Omega \, \Omega e^{-\theta\Omega^2} \left[ \hat{b}^\dagger_\Omega \hat{b}_\Omega + \hat{b}^\dagger \Omega \hat{b}_\Omega \right]
\] (41)

where the dot and prime mean derivatives with respect to the Rindler time \(\eta\) and Rindler space \(\xi\), respectively. Note that eqs. (24), (25) and (35) have been utilized in the derivation of eq. (41). It is obvious that the energy density of the detected particles is suppressed and so is its flux through the detector. Due to the finite density of the number of particles mentioned under eq. (39), the density of zero point energy is also finite although a divergent term related to \(\delta(0)\) appears in eq. (41). A similar case happens again in eq. (48) which is associated with the canonical quantization procedure and will be analyzed in the next subsection. As a
consequence, our results are only agreeable with that of ref. [25] in the aspect of the response rate but not in the aspect of the Unruh spectrum and Unruh temperature.

We turn to the Hawking radiation of the noncommutative inspired Schwarzschild black hole. If the point source of the Gaussian function (eq. (18)) is adopted and the Einstein equation is not modified by the noncommutativity of spacetime, the noncommutative inspired Schwarzschild black hole solution takes the form [15],

$$ds^2 = - \left(1 - \frac{2M_\theta}{r}\right) dt^2 + \left(1 - \frac{2M_\theta}{r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

(42)

where the parameter $M_\theta$ with the dimension of mass satisfies the following formula,

$$M_\theta(r) = \int_0^r \rho_\theta(r') 4\pi r'^2 dr' = \frac{2M}{\sqrt{\pi}} \gamma \left(\frac{3}{2}, \frac{r^2}{4\theta}\right).$$

(43)

The gamma function in the above equation is defined to be $\gamma(a, b) = \int_0^b \frac{dt}{t} t^{a-1} e^{-t}$. To simplify our discussion, we consider a two-dimensional black hole whose metric is same as the time-radial part of the noncommutative inspired Schwarzschild metric.

In accordance with the discussion for the Unruh effect above, we can deduce the Hawking radiation of the two-dimensional noncommutative inspired Schwarzschild black hole. At present, the Minkowski vacuum is replaced by the so-called “Kruskal vacuum” defined by the inertial observer located at the event horizon and the Rindler observer is replaced by the static observer at infinity [35]. Making the similar computation to that done for the Unruh effect, we can easily obtain the spectrum of Hawking radiation,

$$\langle \hat{\mathcal{N}}_{\Omega} \rangle \equiv \langle 0_K | \hat{b}_{\Omega}^+ \hat{b}_{\Omega} | 0_K \rangle = \frac{\delta(0)}{e^{2\pi \Omega k} - 1},$$

(44)

Note that the spectrum is not modified by the noncommutativity of spacetime but it is related to a modified temperature that involves in the noncommutativity,

$$T_H = \frac{\kappa}{2\pi M} = \frac{1}{8\pi M \left[1 - \frac{2M}{\sqrt{\pi} \theta} e^{-\frac{M^2}{8\theta}} + O(\frac{1}{\sqrt{\theta} e^{-\frac{M^2}{8\theta}}}) \right]},$$

(45)

where in the second equality we have calculated the temperature to order $O(\frac{1}{\sqrt{\theta} e^{-\frac{M^2}{8\theta}}})$. Incidentally, this deformed Hawking temperature coincides with the result that was obtained in terms of the Parikh-Wilczek’s tunneling method [38] in refs. [27, 28, 29, 30, 31].

3.2 Under the canonical quantization procedure

Now we reconsider the Unruh effect and Hawking radiation following the similar analysis to that in the previous subsection but adopting the canonical quantization procedure. The Bogoliubov transformation eq. (26) is still valid while the normalization condition eq. (28) changes to be

$$\int_0^\infty d\omega \left(\alpha_{\Omega}^* \alpha_{\Omega'} - \beta_{\Omega}^* \beta_{\Omega'}\right) = \delta(\Omega - \Omega').$$

(46)
The “b-particle” number measured in the $\Omega$th mode $\hat{N}_\Omega = \hat{b}_\Omega^\dagger \hat{b}_\Omega$ at the “$a$-vacuum” is

$$\langle 0_a | \hat{N}_\Omega | 0_a \rangle = \frac{1}{C^2_\Omega} \int_0^\infty d\omega |\beta_{\Omega\omega}|^2 = \frac{\delta(0)}{e^{\frac{\theta}{a}} - 1} e^{6\Omega^2}.$$  \hspace{1cm} (47)

We note that the Unruh spectrum is modified by an effective greybody factor $\Gamma(\Omega) \equiv e^{\theta\Omega^2}$ which is related to the noncommutativity of spacetime but the Unruh temperature is not altered. Moreover, due to eq. (21) or eq. (22) and the reason given below eq. (22), we obtain an interesting result that the noncommutative inspired Schwarzschild solution goes back to that of the ordinary one in the commutative theory. Therefore, the Hawking radiation spectrum is also modified by the effective greybody factor but the Hawking temperature is not altered.

Now let us take a look at the Hamiltonian operator which can be derived in the same way as that of deriving eq. (41). The Hamiltonian operator corresponding to the canonical quantization procedure takes the form,

$$\hat{H} = \frac{2}{\pi} \int_0^\infty \partial \hat{N}_\Omega \left[ e^{-\theta\Omega^2} \hat{N}_\Omega + \frac{1}{2} \delta(0) \right].$$  \hspace{1cm} (48)

Substituting eq. (47) into eq. (48), we can see that the Hamiltonian is same as the ordinary (commutative) one, which is consistent with the ordinary Feynman propagator (eq. (21)). That is, the response rate will not be deformed by the noncommutativity if we start with the Unruh-DeWitt detector by using the ordinary propagator.

The difference between the two propagators eq. (16) and eq. (21) emerges from the difference between the two Hamiltonians eq. (41) and eq. (48). What we can confirm now is that it is just the spacetime noncommutativity that gives rise to the difference because such a difference disappears when the noncommutative parameter tends to zero. In fact, the noncanonical quantization procedure is identical with the canonical one in the quantum field theory on the ordinary (commutative) spacetime. Although we argue the noncanonical quantization procedure by adopting the canonical one in the re-examination of the Unruh effect and Hawking radiation, to single out the preferable one needs further research in both the theoretical and experimental aspects. At present we may say that the canonical procedure seems to be more reasonable within the framework of quantization of field theory, and that it is probably interesting to reveal that the noncommutativity makes the two quantization procedures produce such a difference. Incidentally, the spacetime noncommutativity usually gives rise to the violation of properties that hold on the ordinary (commutative) spacetime, such as the breaking of self-duality in the noncommutative chiral bosons [39].

## 4 Conclusion

In this paper, we revisit the coordinate coherent states approach in the quantum field theory on the noncommutative Minkowski plane by adopting two quantization procedures. We
argue the noncanonical quantization procedure by proposing the canonical one and find that
the Feynman propagator remains the same form as that in the commutative theory. This
indicates that the concept of point particles is still valid, which is different from the result
associated with the noncanonical quantization procedure. We can give a natural explanation
that the effect of noncommutatitivity is diluted if the noncommutativity is dealt with by the
coordinate coherent states approach associated with the canonical quantization procedure.
In order to see further differences caused by the different quantization procedures, we take
the Unruh effect and Hawking radiation as our examples. In the following we give a summary
of the concrete results.

On the one hand, we re-examine the Unruh effect and Hawking radiation on the non-
commutative Minkowski plane following the coordinate coherent states approach associated
with the noncanonical quantization procedure. Based on the Bogoliubov transformation, we
show that the Unruh spectrum (eq. (39)) and Unruh temperature (eq. (40)) keep unchanged
although the general formulae of "b-particle" number measured in the Ω\(^{th}\) mode (eq. (29))
is deformed by the noncommutativity. In addition, the Hawking radiation spectrum is not
deformed but is related to a deformed temperature which involves in the noncommutativity.

On the other hand, if we adopt the canonical quantization procedure, i.e. we begin with
the equal-time canonical commutator between \(\phi\) and \(\dot{\phi}\), both the Unruh spectrum and Hawk-
ing radiation spectrum are deformed by an effective greybody factor \(\Gamma(\Omega) = e^{\Theta \Omega^2}\) although
we do not take into account for the latter the effect of backscattering of the noncommutative
inspired Schwarzschild black hole. When the radiation energy is low (\(\Omega \ll \frac{1}{\sqrt{\Theta}}\)), the greybody
factor can be neglected and the Unruh spectrum and Hawking radiation spectrum will not
be modified by the noncommutative parameter \(\Theta\). However, when the radiation energy is
high enough (\(\Omega \sim \frac{1}{\sqrt{\Theta}}\)), the greybody factor will play an important role. It is believed that
the noncommutative length \(\sqrt{\Theta}\) is of order of the Planck scale \(l_P\), so only when the energy of
radiation approaches the Planck energy \(\Omega_P \sim \frac{1}{l_P}\) can the noncommutative effect be manifest.
The Feynman propagator (eq. (21)) remains the usual form and a point mass should still
be described by the usual Dirac delta function rather than a Gaussian function, which is
contrary to the statement given in refs. [12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24].

The above results can be summarized concisely by the following table:
Quantities | Noncanonical quantization | Canonical quantization
--- | --- | ---
Commutators | $[\phi(t, x), \dot{\phi}(t, x')] = \frac{i}{2\sqrt{\pi\theta}}e^{-\frac{(x-x')^2}{4\theta}}$ | $[\phi(t, x), \dot{\phi}(t, x')] = i\delta(x - x')$
 | $[\hat{a}_p, \hat{a}^\dagger_{p'}] = \delta(p - p')$ | $[\hat{a}_p, \hat{a}^\dagger_{p'}] = e^{i\theta\omega^2}\delta(p - p')$
$G_F(p_0, p_1)$ | $\frac{i}{p_0^2 - p_1^2 + i\epsilon} e^{-\theta(p_0^2 + p_1^2)/2}$ | $\frac{i}{p_0^2 - p_1^2 + i\epsilon}$
$\langle \hat{N}_\Omega \rangle$ | $\frac{\delta(0)}{e^{\frac{\Omega}{4\pi\epsilon} - 1}}$ | $\frac{\delta(0)}{e^{\frac{\Omega}{4\pi\epsilon} - 1}} e^{\theta\Omega^2}$
$\hat{H}$ | $\frac{2}{\pi} \int_0^\infty d\Omega \ \Omega e^{-\theta\Omega^2} \left[ \hat{N}_\Omega + \frac{1}{2}\delta(0) \right]$ | $\frac{2}{\pi} \int_0^\infty d\Omega \ \Omega \left[ e^{-\theta\Omega^2} \hat{N}_\Omega + \frac{1}{2}\delta(0) \right]$  

Recall that the greybody factor for the ordinary Schwarzschild black hole arises from the backscattering of radiations of the gravitational field and has the following limiting property,

$$\Gamma(\omega) \to \begin{cases} 1, & \omega \gg \frac{1}{M}, \\ \frac{A}{4\pi}\omega^2, & \omega \ll \frac{1}{M}, \end{cases} \quad (49)$$

where $A$ is the horizon area of the black hole. We can see that our effective greybody factor has a quite different behavior compared with the ordinary greybody factor (eq. (49)). If we take into account both the effects of noncommutativity and of backscattering, the total effective greybody factor may be very different from the ordinary one (eq. (49)). We shall leave it to our further considerations.

Finally, we make some comments. Under the circumstance that neither the noncanonical nor the canonical quantization procedure can be excluded by the present research, our results reveal that the noncommutativity of spacetime leads to some effects that depend on the quantization procedures in the coordinate coherent states approach. In fact, it is already well-known that quantization schemes are ambiguous on noncommutative spacetimes in the star-product formalism. It has been found [40] that in the perturbative formulation of quantum field theory on the noncommutative Minkowski space, the Feynman and Dyson perturbation expansions, which are equivalent in commutative quantum field theory, lead to different results in noncommutative quantum field theory. For instance, the unitarity is violated in the former, while it seems to be preserved in the latter. In addition, the analogy with these models exists in the twist-deformed version of noncommutative field theory [41, 42, 43, 44, 45, 46]. There one has to choose either undeformed or deformed commutation relations for the field oscillators, and the different choices correspond to different implications, e.g. the former choice gives some interesting outcomes, such as the cancellation of the UV/IR mixing, the preservation of the Lorentz invariance, and the violation of the Pauli exclusion principle. When we compare our case with that of the star-product formalism, we may say that the choice of either noncanonical or canonical quantization procedure is analogous to that of either undeformed or deformed commutation relations. We can see that the similar ambiguity of choosing the noncanonical or canonical quantization procedure
on noncommutative spacetimes also exists in the coordinate coherent states approach, and
that the different choices also lead to different implications in the Unruh effect and Hawking
radiation as have been demonstrated in the present paper. The last comment is that we
restrict our discussion to the two-dimensional case. Since a higher dimensional case can be
reduced to a two-dimensional one [47], our conclusion is expected to be valid in an arbitrary
dimensional spacetime.

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Appendix

Here we give the proof that the coherent states eq. (3) preserve the Lorentz rotational sym-
metry in the Euclidean signature but violate the Lorentz boost symmetry in the Minkowski
signature.

For the Euclidean signature that corresponds to \( SO(2) \) group, the coordinate operators
\( \hat{x}^0 \) and \( \hat{x}^1 \) transform under the rotation as

\[
\begin{pmatrix}
\hat{x}'^0 \\
\hat{x}'^1
\end{pmatrix} = \begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
\hat{x}^0 \\
\hat{x}^1
\end{pmatrix},
\]

where \( \alpha \) is the rotational angle. The constant noncommutative parameter \( \theta \) is invariant under
the rotation. Then we can see eq. (1) is invariant under the rotation,

\[
[\hat{x}^0, \hat{x}^1] = (\cos^2 \alpha + \sin^2 \alpha)[\hat{x}^0, \hat{x}^1] = [\hat{x}^0, \hat{x}^1] = i\theta.
\]

The complex coordinate operators \( \hat{z}, \hat{z}^\dagger \) transform under the rotation as

\[
\hat{z}' = e^{-i\alpha} \hat{z}, \quad \hat{z}'\dagger = e^{i\alpha} \hat{z}^\dagger.
\]

Note that \( \hat{z}, \hat{z}^\dagger \) are not mixed up under the rotation, so we can define universal states for
different coordinate operators: (i) A universal ground state can be defined for different co-
dordinate operators, \( \hat{z}|0\rangle = 0 \), thus the ground state is invariant under the rotation; (ii) The
coherent states \( |z\rangle \) defined by \( \hat{z}|z\rangle = z|z\rangle \) (eq. (3)) are universal for different coordinate op-
erators with the eigenvalues \( z \) transforming as \( z' = e^{-i\alpha}z \). As a result, the coherent states
are also invariant under the rotation, which can be proved directly,

\[
|z'\rangle = e^{-\frac{i\alpha}{i\theta} \hat{z}^\dagger} e^{\hat{z}'} e^{\frac{i\alpha}{i\theta} \hat{z}^\dagger}|0\rangle = e^{-\frac{i\alpha}{i\theta} \hat{z}^\dagger} e^{\frac{i\alpha}{i\theta} \hat{z}^\dagger}|0\rangle = |z\rangle.
\]
While for the Minkowski signature that corresponds to $SO(1,1)$ group, the coordinate operators $\hat{x}^0$ and $\hat{x}^1$ transform under the Lorentz boost transformation as

$$\begin{pmatrix} \hat{x}'^0 \\ \hat{x}'^1 \end{pmatrix} = \begin{pmatrix} \cosh \alpha & -\sinh \alpha \\ -\sinh \alpha & \cosh \alpha \end{pmatrix} \begin{pmatrix} \hat{x}^0 \\ \hat{x}^1 \end{pmatrix},$$  \hspace{1cm} (54)$$

where $\alpha$ is the boost parameter. We can see that eq. (1) is also invariant under the Lorentz boost transformation,

$$[\hat{x}'^0, \hat{x}'^1] = (\cosh^2 \alpha - \sinh^2 \alpha)[\hat{x}^0, \hat{x}^1] = [\hat{x}^0, \hat{x}^1] = i\theta.$$ \hspace{1cm} (55)$$

However, the complex coordinate operators $\hat{z}, \hat{z}^\dagger$ now mix up under the Lorentz boost transformation,

$$\hat{z}' = \hat{z} \cosh \alpha - i\hat{z}^\dagger \sinh \alpha, \hspace{1cm} \hat{z}^\dagger' = \hat{z}^\dagger \cosh \alpha + i\hat{z} \sinh \alpha.$$ \hspace{1cm} (56)$$

It is analogous to the Bogoliubov transformation. Therefore, we cannot define a universal ground state. That is, the ground state for the coordinate operators $\hat{z}$ defined by $\hat{z}|0\rangle = 0$ is no longer the ground state for the transformed coordinate operators $\hat{z}'$ anymore, i.e., $\hat{z}'|0\rangle = (\hat{z} \cosh \alpha - i\hat{z}^\dagger \sinh \alpha)|0\rangle \neq 0$. Similarly, we cannot expect that the coherent states eq. (3) are invariant under the Lorentz boost transformation. We now verify this outcome by an apagogical proof, that is, if we assume that the coherent states $|z\rangle$ (eq. (3)) are invariant under the Lorentz boost transformation, i.e., $|z\rangle = |z'\rangle$, we then have

$$\hat{z}'|z'\rangle = \hat{z}'|z\rangle = (\hat{z} \cosh \alpha - i\hat{z}^\dagger \sinh \alpha)|z\rangle = (z \cosh \alpha - i\sinh \alpha \hat{z}^\dagger)|z\rangle.$$ \hspace{1cm} (57)$$

To preserve the property $\hat{z}'|z\rangle \sim |z\rangle$, the last term in the above equation should have the form $\hat{z}^\dagger|z\rangle \sim |z\rangle$, that is to say, $|z\rangle$ are also the eigenstates of $\hat{z}^\dagger$. However, it is impossible because one cannot find common eigenstates for two operators $\hat{z}, \hat{z}^\dagger$ which are noncommutative.

We can also understand the above result from a more intuitive and physical point of view. It is believed that the minimal length exists due to the noncommutativity of spacetimes. The minimal length is invariant under the Lorentz rotational transformation in the Euclidean signature but not invariant under the Lorentz boost transformation in the Minkowski signature.

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