1. Introduction

The object of this work is to study the global solution to the following boundary value problem for the Moore-Gibson-Thompson equation

$$\alpha u_{ttt} + \beta u_{tt} - c^2 \Delta u - r \Delta u_t + f(u) = 0, \text{ in } \Omega \times (0, +\infty),$$

$$u(x, t) = 0 \text{ on } \partial \Omega,$$

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), u_{tt}(x, 0) = u_2(x), x \in \Omega,$$

where $\Omega$ is a bounded domain in $\mathbb{R}^n (n \geq 1)$ with sufficiently smooth boundary $\partial \Omega$, $u_0(x), u_1(x)$ and $u_2(x)$ are given functions and $f$ is a given nonlinear function. All the parameters $\alpha, \beta, c^2, r$ are assumed to be positive constants.

In recent years, increasing attention has been paid to the well-posedness and asymptotic behavior of the Moore-Gibson-Thompson (MGT) equation, see [1–7]. The MGT model is considered through third-order (in time), strictly hyperbolic partial differential equation as follows

$$\alpha u_{ttt} + \beta u_{tt} - c^2 \Delta u - r \Delta u_t = f(u),$$

it is one of the nonlinear acoustic models describing the propagation of acoustics wave in gases and liquid, it has a wide range of applications in medical and industry. In the physical context of the acoustic waves, $u$ is the velocity potential of the acoustic phenomena, $\alpha$ denotes the thermal relaxation time, $c$ denotes the speed of sound, $\beta$ denotes friction, and $b$ denotes a parameter of diffusivity.

It is often convenient to write MGT equation as an abstract form

$$\alpha uu_{tt} + \beta u_{tt} + c^2 Au + rAu_t = f(u, u_t, u_{tt}),$$

and it has been shown [8,9] that the linear part of Eq. (5) generates a strongly continuous semigroup as long as $r > 0$. In [10], the authors provided a brief overview of well-posedness results, both local and global, pertinent to various configurations of MGT equations. Especially, the authors in [11] considered the following model with nonlinear control feedback

$$\tau uu_{tt} + \alpha \beta uu_t + c^2 Au + bAu_t + \beta u^3_t = 2k u_t^2 + p(u),$$
where the parameter $\beta > 0$, $p(u)$ denotes an active force and the operator $A$ is strictly positive. By semigroup method, it was proved in [11] that (6) with initial data of arbitrary size in $H$ is locally and globally well-posed under the following assumption: $p \in C^1(\mathbb{R})$ and its derivative satisfies $-\delta \leq p'(s) \leq m$ for some positive constants $\delta$ and $m$. Kaltenbacher et al., [12] established the well-posedness by Galerkin approximations and then employ fixed-point arguments for well-posedness of the Jordan-Moore-Gibson-Thompson (JMGT) equation

$$\alpha u_{ttt} + \beta u_{tt} - b\Delta u_t - c^2\Delta u = \frac{B}{c^2}u_t^2 + |\nabla u|^2.$$

More recently, Boulaaaras et al., [13] proved the existence and uniqueness of the weak solution of the Moore-Gibson-Thompson equation with the integral condition by applying the Galerkin method.

In this paper, we extend the results in [11] to Problem (1)-(3) by applying the Galerkin method and compact method. The contents of this paper are organized as follows; In §2, we prepare some materials needed for our proof. Finally, in §3, we give the main result and the proof.

2. Preliminaries

Throughout this paper, the domain $\Omega$ is assumed to be sufficiently smooth to admit integration by parts and second-order elliptic regularity. We use $C$ to denote a universal positive constant that may have different values in different places. $W^{m,2}(\Omega) = H^m(\Omega)$ and $W^{m,2}_0(\Omega) = H^m_0(\Omega)$ denote the well-known Sobolev space. We denote by $||.,||$ the $L^p(\Omega)$ norm and by $||.,||$ the norm in $H^m_0(\Omega)$. In particular, we denote $||.,|| = ||.,||_{L^2}$.

By a weak solution $u(x,t)$ of Problem (1)-(3) on $\Omega \times [0,T]$ for any $T > 0$, we mean $u \in L^\infty(0,T;H^2(\Omega) \cap H^1_0(\Omega)) \cap W^{1,\infty}(0,T;H^1_0(\Omega)) \cap W^{2,\infty}(0,T;L^2(\Omega))$, $\Delta u_t, u_{tt} \in L^\infty((0,T);H^{-1}(\Omega))$ such that $u(x,0) = u_0(x)$ a.e. in $\Omega$, $u_t(x,0) = u_1(x)$ a.e. in $\Omega$, $u_{tt}(x,0) = u_2(x)$ a.e. in $\Omega$, and

$$\alpha u_{ttt} + \beta u_{tt} - c^2\Delta u - r\Delta u_t + f(u),v = 0$$

for any $v \in H^1_0(\Omega)$, a.e. $t \in [0,T]$.

In this paper, we assume $\alpha, \beta, c^2, r > 0$ and

$$f \in C^1$$

and $|f'(s)| \leq C_1$.

Lemma 1. [14] Let $\Omega \subset \mathbb{R}^n$ be a bounded domain and $w_i$ be a base of $L^2(\Omega)$. Then for any $\epsilon > 0$ there exist a positive constant $N_\epsilon$, such that

$$||u|| \leq \left( \sum_{j=1}^{N_\epsilon} (u, w_j) \right)^{1/2} + \epsilon ||u||_{L^p}$$

for any $u \in W^{1,p}_0(\Omega)$ ($2 \leq p < \infty$), where $N_\epsilon$ is independent on $u$.

Lemma 2. [15] Let $G(z_1, z_2, ..., z_h)$ be the function of the variables $z_1, z_2, ..., z_h$ and suppose that $G$ is continuous differentiable for $k$-times ($k \geq 1$) with respect to every variable. Let $z_i(x,t) \in L^\infty([0,T];H^k(\Omega)) (i = 1, 2, ..., h)$, then the estimation

$$\int_\Omega |D_x^k G(z_1(x,t), z_2(x,t), ..., z_h(x,t))|^2 dx < C(M, k, h) \sum_{i=1}^{h} ||z||_{H^k(\Omega)}$$

holds, where $D_x = \frac{\partial}{\partial x}$, $M = \max_{i=1,2, ..., h} \max_{0 \leq t \leq T, x \in \Omega} |z_i(x,t)|$.

3. Solvability of the problem

In this section, by using Galerkin’s method and compactness method, we shall prove the existence of global solutions of Problem (1)-(3).

Let $\{w_i(x)\}_{i \in N}$ be the eigenfunctions of the following boundary problem

$$-\Delta w = \lambda w, w \in \Omega; w = 0, x \in \partial \Omega,$$
Lemma 3. Assume the relation using the estimate below. Problem (11)-(12) leads to a system of ODEs for unknown functions

\[ \alpha \left( u_{jN}^N, w_j \right) + \beta \left( u_{jN}^N, w_j \right) - c^2 \left( \Delta u_j^N, w_j \right) - \tau \left( \Delta u_j^N, w_j \right) + (f(u_j^N), w_j) = 0, \]

\[ (u_j^N(0), w_j) = (u_0, w_j), (u_j^N(0), w_j) = (u_1, w_j), (u_{jN}^N(0), w_j) = (u_N, w_j) = u_{jN}. \]  

\[ \text{Proof.} \] Problem (11)-(12) leads to a system of ODEs for unknown functions \( T_{jN}(t) \). Based on standard existence theory for ODE, one can obtain functions \( T_{jN}(t) : [0, t_k) \rightarrow \mathbb{R}, j = 1, 2, \ldots, k \), which satisfy approximate Problem (11)-(12) in a maximal interval \( [0, t_k), t_k \in (0, T] \). This solution is then extended to the closed interval \( [0, T] \) by using the estimate below.

Multiplying (11) by \( T_{jN}(t) \), summing up the products for \( j = 1, 2, \ldots, N \) and integrating by parts, we get

\[ \alpha \left( u_{jN}^N, u_{jN}^N \right) + \beta \left( u_{jN}^N, u_{jN}^N \right) + c^2 \left( \nabla u_j^N, \nabla u_j^N \right) + r \left( \nabla u_j^N, \nabla u_j^N \right) + (f(u_j^N), u_j^N) = 0. \]

Integrating (14) with respect to \( t \) from 0 to \( t \), we obtain

\[ \alpha \| u_j^N \|^2 + 2 \beta \int_0^t \| u_j^N \|^2 \, dt + r \| \nabla u_j^N \|^2 + 2 \int_0^t (f(u_j^N), u_j^N) \, dt \]

\[ = -c^2 \int_0^t \left( \nabla u_j^N, \nabla u_j^N \right) \, dt + \alpha \| u_j^N(0) \|^2 + r \| \nabla u_j^N(0) \|^2. \]

We observe that

\[ \int_0^t (f(u_j^N), u_j^N) \, dt = (f(u_j^N), u_j^N) |_0^t - \int_0^t \int_\Omega f'(u_j^N)(u_j^N)^2 \, dx \, dt \]

and

\[ \int_0^t \left( \nabla u_j^N, \nabla u_j^N \right) \, dt = (\nabla u_j^N, \nabla u_j^N) |_0^t - \int_0^t \| \nabla u_j^N \|^2 \, dt. \]

Adding \( 2 \left( (u_j^N, u_j^N) + (u_j^N, u_j^N) + (\nabla u_j^N, \nabla u_j^N) \right) \) to both sides of (15) and a substitution of the equalities (16) and (17) in (15) gives

\[ \frac{d}{dt} \left( \| u_j^N \|^2 + \| u^N \|^2 + \| \nabla u_j^N \|^2 + \alpha \| u_j^N \|^2 + 2 \beta \int_0^t \| u_j^N \|^2 \, dt + r \| \nabla u_j^N \|^2 \right) \]

\[ = 2 (u_j^N, u_j^N) + (u_j^N, u_j^N) + (\nabla u_j^N, \nabla u_j^N) + \alpha \| u_j^N(0) \|^2 + r \| \nabla u_j^N(0) \|^2 - 2 (f(u_j^N), u_j^N) |_0^t \]

\[ + 2 \int_0^t \int_\Omega f'(u_j^N)(u_j^N)^2 \, dx \, dt - 2c^2 (\nabla u_j^N, \nabla u_j^N) |_0^t + 2c^2 \int_0^t \| \nabla u_j^N \|^2 \, dt. \]

Then, by Hölder inequality and the fact \( |f(s)| = |\int_0^t f'(s) \, ds| \leq C_1 |s| \) by (A1), we arrive at
\[
\frac{d}{dt} \left( ||u^N||^2 + ||u_t^N||^2 + ||\nabla u^N||^2 \right) + \alpha ||u_t^N||^2 + 2\beta \int_0^t ||u_t^N||^2 d\tau + r ||\nabla u_t^N||^2 \\
\leq 2||u^N|| ||u_t^N|| + 2||u_t^N|| ||u_t^N|| + 2||\nabla u^N|| ||\nabla u_t^N|| + \alpha ||u_t^N(0)||^2 + r ||\nabla u_t^N(0)||^2 \\
+ 2C_1 ||u^N|| ||u_t^N|| + 2C_1 ||u^N(0)|| ||u_t^N(0)|| + 2C_1 \int_0^t ||u_t^N||^2 d\tau \\
+ 2c^2 ||\nabla u^N|| ||\nabla u_t^N|| + 2c^2 ||\nabla u_t^N(0)|| ||\nabla u_t^N(0)|| + 2c^2 \int_0^t ||\nabla u_t^N||^2 d\tau \\
\leq \frac{1}{2} (\alpha ||u_t^N||^2 + r ||\nabla u_t^N||^2) + C_2 (||u^N||^2 + ||u_t^N||^2 + ||\nabla u^N||^2) \\
+ 2C_1 \int_0^t ||u_t^N||^2 d\tau + 2c^2 \int_0^t ||\nabla u_t^N||^2 d\tau + \alpha ||u_t^N(0)||^2 + r ||\nabla u_t^N(0)||^2 \\
+ C_3 ||u^N(0)||^2 + C_4 ||u_t^N(0)||^2 + c^2 ||\nabla u^N(0)||^2 + c^2 ||\nabla u_t^N(0)||^2. 
\]

Taking into account that
\[
||u_t^N(0)||^2 + ||\nabla u_t^N(0)||^2 + ||\nabla u_t^N(0)||^2 \to ||u_2||^2 + ||u_0||^2 + ||\nabla u_1||^2 
\]
and
\[
||u^N(0)||^2 + ||u_t^N(0)||^2 \to ||u_0||^2 + ||u_1||^2 
\]
as \( N \to \infty \), then applying the Gronwall inequality to (19) and then integrating from 0 to \( t \) appears that
\[
||u^N||^2 + ||u_t^N||^2 + ||\nabla u^N||^2 + \alpha ||u_t^N||^2 + r ||\nabla u_t^N||^2 + \int_0^t ||\nabla u_t^N||^2 d\tau + \beta ||\nabla u_t^N||^2 d\tau \leq C. 
\]

Multiplying (11) by \( \lambda_j T_{N}(t) \) summing up the products for \( j = 1, 2, ..., N \), integrating by parts and integrating with respect to \( t \), we get
\[
r||\Delta u^N||^2 + c^2 \int_0^t ||\Delta u^N||^2 d\tau = 2\alpha \int_0^t (u_{tt}^N, \Delta u^N) d\tau + 2\beta \int_0^t (u_t^N, \Delta u^N) d\tau + \int_0^t (f(u^N), \Delta u^N) d\tau + r||\Delta u^N(0)||^2 .
\]

Combining Cauchy inequality, the fact \( ||\Delta u^N(0)||^2 \to ||\Delta u_0||^2 \), and \( |f(s)| \leq C_1 |s| \), and making use of the following inequality
\[
\int_0^t (u_{tt}^N, \Delta u^N) d\tau = (u_{tt}^N, \Delta u^N)_t^t|_0 - \int_0^t (u_{tt}^N, \Delta u^N) d\tau = (u_{tt}^N, \Delta u^N) - (u_{tt}^N, \Delta u^N(0)) + \frac{1}{2} ||\nabla u_t^N||^2 - \frac{1}{2} ||\nabla u_t^N(0)||^2,
\]
we have
\[
r||\Delta u^N||^2 + c^2 \int_0^t ||\Delta u^N||^2 d\tau
\]
\[
\leq 2\alpha ||u_t^N|| ||\Delta u^N|| + 2\alpha ||u_t^N(0)|| ||\Delta u^N(0)|| + \alpha (||\nabla u_t^N||^2 - ||\nabla u_t^N(0)||^2) + 2\beta \int_0^t ||u_t^N|| ||\Delta u^N|| d\tau + \int_0^t ||f(u^N)|| ||\Delta u^N|| d\tau + r||\Delta u^N(0)||^2
\]
\[
\leq \epsilon_1 ||\Delta u^N||^2 + C_6 ||u_t^N||^2 + ||\nabla u_t^N||^2 + C_7 (||u_t^N(0)||^2 + ||\Delta u^N(0)||^2 + ||\nabla u_t^N(0)||^2) + \epsilon_1 \int_0^t ||\Delta u^N||^2 d\tau + C_8 \int_0^t ||u_t^N||^2 d\tau + C_9 \int_0^t ||u^N||^2 d\tau.
\]

Choosing \( \epsilon_1 \) sufficiently small and \( \epsilon_2 \) sufficiently large such that \( \epsilon_2 > 2c^2 \), then it follows from (22) and (20) that
\[ ||\Delta u^N||^2 \leq C_{10} \int_0^t ||\Delta u^N||^2 d\tau + C_{11}. \]  
\hspace{1cm} (23)

Thus, applying Gronwall’s inequality to (23), we deduce
\[ ||\Delta u^N||^2 \leq C. \]  
\hspace{1cm} (24)

Combining (20) and (24), we get
\[ ||u^N||^2 + ||u_t^N||^2 + ||u_{tt}^N||^2 + \int_0^t ||\nabla u^N||^2 d\tau + \beta ||\nabla u^N||^2 d\tau \leq C. \]  
\hspace{1cm} (25)

Furthermore, by (25), we have that (11)-(12) possesses a global solution.

**Theorem 1.** Assume (8) holds, \( u_0 \in H^2(\Omega) \cap H_0^1(\Omega), u_1 \in H_0^1(\Omega), \) and \( u_2 \in L^2(\Omega), \) then for any \( T > 0, \) Problem (1)-(3) possesses a unique global solution.

**Proof.** For any \( v \in H_0^1(\Omega), \) it follows that
\[ a(|u_{tt}^N, v|) \leq (\beta ||u_t^N|| + \epsilon^2 ||\Delta u^N|| + ||\nabla u^N|| + C_1 ||u^N||)||v||_{H_0^1}. \]  
\hspace{1cm} (26)

Thus, using Lemma 3, it follows that
\[ ||u_{tt}^N||_{H^{-1}(\Omega)} \leq M. \]  
\hspace{1cm} (27)

Similarly, we have
\[ ||\Delta u_t^N||_{H^{-1}(\Omega)} \leq M. \]  
\hspace{1cm} (28)

From Lemma 3, (27) and (28), there exist a subsequence of \( \{u^N\}, \) still denoted by \( \{u^N\}, \) and a function \( u, \xi, \eta, \) such that
\[ u^N \to u \text{ weak * in } L^\infty(0,T,H^2(\Omega) \cap H_0^1(\Omega)), \]  
\hspace{1cm} (29)
\[ u_{tt}^N \to u_{tt} \text{ weak * in } L^\infty(0,T,H_0^1(\Omega)), \]  
\hspace{1cm} (30)
\[ u_{ttt}^N \to u_{ttt} \text{ weak * in } L^\infty(0,T,L^2(\Omega)), \]  
\hspace{1cm} (31)
\[ u_{tttt}^N \to u_{tttt} \text{ weak * in } L^\infty(0,T,H^{-1}(\Omega)), \]  
\hspace{1cm} (32)
\[ f(u^N) \to \xi \text{ weak * in } L^\infty(0,T,H^{-1}(\Omega)), \]  
\hspace{1cm} (33)
\[ \Delta u_t^N \to \eta \text{ weak * in } L^\infty(0,T,H^{-1}(\Omega)). \]  
\hspace{1cm} (34)

and for any \( t \in [0,T] \)
\[ u^N \to u \text{ weakly in } H^2(\Omega) \cap H_0^1(\Omega), \]  
\hspace{1cm} (35)
\[ u_{tt}^N \to u_{tt} \text{ weakly in } H_0^1(\Omega), \]  
\hspace{1cm} (36)
\[ u_{ttt}^N \to u_{ttt} \text{ weakly in } L^2(\Omega), \]  
\hspace{1cm} (37)
\[ u_{tttt}^N \to u_{tttt} \text{ weakly in } H^{-1}(\Omega), \]  
\hspace{1cm} (38)
\[ f(u^N) \to \xi \text{ weak * in } H^{-1}(\Omega)), \]  
\hspace{1cm} (39)
\[ \Delta u_t^N \to \eta \text{ weak * in } H^{-1}(\Omega)). \]  
\hspace{1cm} (40)

Since \( f \in C^1 \) and \( ||f(u^N)|| \leq C||u^N|| \leq C, \) for any \( v \in H_0^1(\Omega) \) and any \( t \in [0,T], \) we have
\[ (\Delta u_t^N, v) = - (\nabla u_t^N, \nabla v) \to - (\nabla u_t, \nabla v) = (\Delta u_t, v), \]  
\hspace{1cm} (41)
\[ f(u^N) \to f(u) \]  
\hspace{1cm} (42)
as $N \to \infty$. Then we get $\xi = f(u), \eta = \nabla u_t$, combining this with (35)-(40), we have

$$u \in L^\infty((0, T); H^2(\Omega) \cap H^1_0(\Omega)) \cap W^{1, \infty}((0, T); H^2_0(\Omega)) \cap W^{2, \infty}((0, T); L^2(\Omega)),$$

$$\Delta u_t, u_{tt} \in L^\infty((0, T); H^{-1}(\Omega)).$$

By using Lemma 3 and (27), we observe that

$$|(u^N, w_j)| + \sum_{k=1}^3 |(u^N_k, w_j)| \leq M,$$  \hspace{1cm} (43)

where $u^N = \frac{2}{\bar{t}}u^N_{t\bar{t}}$. Then, by Ascoli-Arcela theorem, we can select from $\{u^N\}$ a subsequence, still denoted by $\{u^N\}$, such that as $N \to \infty$, the subsequence

$$(u^N, w_j) \to (u, w_j), \quad (u^N_k, w_j) \to (u_k, w_j), \quad k = 1, 2, 3, \quad j = 1, 2, \ldots $$  \hspace{1cm} (44)

In particular, we take $t = 0$ and we note that $\{w_j(x)\}_{j \in N}$ are an orthogonal basis of $L^2(\Omega)$, we know that

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad u_{tt}(x, 0) = u_2(x) \text{ a.e. in } \Omega.$$  \hspace{1cm} (45)

By (29)-(34), (44) and Lemma 2.1, we have

$$u^N \to u, \quad u^N \to u_t \text{ in } C([0, T], L^2(\Omega)).$$  \hspace{1cm} (46)

Thanks to (29)-(42), letting $N \to \infty$ in (11), leads to

$$a(u_{tt}, v) + \beta(u_{tt}, v) - c^2(\Delta u, v) - r(\Delta u_t, v) + (f(u), v) = 0$$  \hspace{1cm} (47)

for any $v \in H^1_0(\Omega)$. Altogether, we conclude that $u$ is a solution of the initial boundary Problem (1)-(3).

Now, suppose that there exist two different solutions $u_1, u_2$ for Problem (1)-(3), then the difference $w = u_1 - u_2$ satisfies

$$\alpha w_{tt} + \beta w_t - c^2 \Delta w - r \Delta w_t + f(u_1) - f(u_2) = 0, \quad \text{in } \Omega \times (0, +\infty),$$

$$w(x, t) = 0 \text{ on } \partial\Omega, \quad \text{for } x \in \Omega,$$

$$w(x, 0) = 0, w_t(x, 0) = 0, \quad \text{for } x \in \Omega,$$  \hspace{1cm} (50)

Integrating (48) for $t$ from 0 to $t$, we have

$$\alpha w_t + \beta w_t - r \Delta w = \int_0^t (c^2 \Delta w + f(u_2) - f(u_1)) \, d\tau.$$  \hspace{1cm} (51)

Multiplying the Eq. (51) by $w_t$, integrating over $\Omega$, adding up $(w, w_t)$, we obtain

$$\frac{1}{2} \alpha ||w_t||^2 + r ||\nabla w||^2 + ||w||^2 \beta ||w_t||^2 = 2 \int_0^t (c^2 \Delta w + f(u_2) - f(u_1)) w_t \, d\tau$$

$$= 2c^2 (||\nabla w||^2 - ||\nabla w_0||^2) + 2 \int_0^t \theta \omega w_0 \, dx \, d\tau$$

$$\leq C (||\nabla w||^2 + ||w||^2),$$  \hspace{1cm} (52)

where we have used mean value theorem and $|\theta| \leq 1$. By applying Gronwall inequality, we deduce that

$$\alpha ||w_t||^2 + r ||\nabla w||^2 + ||w||^2 = 0.$$  \hspace{1cm} (53)

This implies that $w = 0$ for all $t \in [0, T]$. Thus the uniqueness is proved. \hspace{1cm} $\Box$

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