Top-Down Nested Supervisory Control of State-Tree Structures Based on State Aggregations *

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Abstract: With a structured state space, state-tree structures (STS) are a powerful framework to model hierarchical finite state machines (HFSM). The boundary consistency property of STS endows them a compact and neatly representation. In this study, by naturally decomposing an STS into a set of STS nests in a top-down nested approach and finding a supervisor for each, the boundary consistency property is extended to the supervisory control of STS. As a consequence, the state spaces for both the system model and optimal supervisor are significantly reduced. Two examples are provided, in which the state space of a large scale HFSM example is reduced from $10^{24}$ to $2 \times 10^{18}$.

Keywords: Discrete-event system, nonblocking supervisory control, state-tree structure, symbolic computation, nested state feedback control.

1. INTRODUCTION

Hierarchical finite state machines (HFSM) (Marchand & Gaudin (2002); Gaudin & Marchand (2004, 2005)) were first developed in Harel (1985, 1987) as finite state machines (FSM) (Wonham & Cai (2013); Ramadge & Wonham (1985)) with multi-levels. As stated in Alur et al. (1999), a superstate in an HFSM can also be other machines. Traditionally, the calculation of the supervisor for an HFSM is obtained by two steps: flattening an HFSM to be an equivalent FSM, and which is followed by using the standard supervisory control of discrete-event systems (DES) (Wonham & Cai (2013); Ramadge & Wonham (1985)). For the purpose of managing the notorious state explosion problem caused by flattening HFSM, state-tree structures (STS) are proposed in Ma & Wonham (2005, 2006) to build HFSM in a compact and natural model. A main feature of STS is that it satisfies boundary consistency: without changing the input/output transitions of a given level, several lower level structures can be “plugged” into its superstates. Binary decision diagram (BDD) (Bryant (1986)) is utilized in the supervisory control of STS as a powerful computational representation of predicates, based on which the state explosion problem faced by the supervisory control of DES is managed.

Several theoretical extensions and applications of STS have been made. The modular supervisory control of an STS is studied in Chao et al. (2013). Supervisor localization based on STS is proposed in Cai & Wonham (2015) to calculate the controller of a controllable event by considering the agent’s neighborhood information only. The research in Jiao et al. (2017) studies the symmetry of STS with parallel components. The supervisory control of STS with partial observation is investigated in Gu et al. (2018) and Gu et al. (2019). In Wang et al. (2019), the nonblocking supervisory control of STS with conditional-preemption matrices is assigned.

In this study, the boundary consistency of STS is extended to the supervisory control of STS. Given an STS with a set of superstates assigned, it is naturally decomposed into a set of STS nests rooted by superstates, which describe the system behavior locally. As a consequence, instead of calculating the optimal behavior of an STS monolithically, a top-down nested approach is presented to calculate the optimal behavior of each STS nest. The main contributions of this study are: 1) based on the superstates, the STS nests describing the system behavior on each hierarchical level are formally defined; 2) the subordination relation of the STS nests is investigated; and 3) the state spaces for both the system model and optimal supervisor are significantly reduced. Two examples are provided in this study. For a large scale example AIP (Ma & Wonham (2005, 2006)), its state space is reduced from $10^{24}$ to $2 \times 10^{18}$.

The rest of this paper is organized as follows. Section 2 presents the STS terminology used throughout the paper.

* This work was supported in part by the Natural Science Foundation of China under Grant No. 61703322, the Alexander von Humboldt Foundation, the Science and Technology Development Fund, MSAR, under Grant No. 0012/2019/A1, and the Fundamental Research Funds for the Central Universities under Grant Nos. XJS200403 and JBF180401. (Corresponding author: Xi Wang.)
The nested structure of STS is studied in Section 3. The nested supervisory control of STS is presented in Section 4. Two case studies are presented in Section 5 to demonstrate the nested supervisory control of STS. Further relevant issues are discussed in Section 6. Finally, conclusions and future work are presented in Section 7.

2. STS PRELIMINARIES

Relevant preliminaries on the supervisory control of STS are summarized from Ma & Wonham (2005, 2006). An STS is a six-tuple \(G = (ST, \mathcal{H}, \Sigma, \Delta, ST_0, ST_m)\), where \(ST\) is a state-tree organizing the state space of an STS hierarchically; \(\mathcal{H}\) is the set of holons (finite automata); \(\Sigma\) is the finite event set appeared in \(\mathcal{H}; \Delta\) is the global function \(ST \times \Sigma \rightarrow ST\); where \(ST(ST)\) is the set of all sub-state-trees; \(ST_0\) is the initial state-tree; and \(ST_m\) is the set of marker state-trees. All the basic state-trees are denoted by \(B(ST)\), in which each element \(b \in B(ST)\) corresponds to a “flat” system state.

A state-tree consists of three types of states: AND, OR, and SIM, in which AND and OR denote the superstates and SIM represents the simple states. In a state-tree \(ST = (X, x_0, T, \mathcal{E})\), \(X\) is the finite structured state set; \(x_0 \in X\) is the root state; \(T : X \rightarrow \{\text{AND}, \text{OR}, \text{SIM}\}\) is the type function; and \(\mathcal{E} : X \rightarrow 2^X\) is the expansion function. The reflexive and transitive closure of \(\mathcal{E}\) is written as \(\mathcal{E}^+=\). Then \(\mathcal{E}^+(x) := \mathcal{E}^- \setminus \{x\}\) represents the set of all descendants of \(x\). In such a state-tree, \((x, y, t, \mathcal{E})\in X\times X\times X\times \mathcal{E}\), \(x \leq y\) (resp., \(x < y\)) if \(y\in \mathcal{E}^+(x)\) (resp., \(y \in \mathcal{E}^-(x)\)). Let \(x < y\). Define that \(y\) is AND-adjacent to \(x\), i.e., \(x < y\) if \(x < y\) and \(T(x) = \text{AND} \wedge \forall \mathcal{E} \prec x < y \Rightarrow T(y) = \text{AND}\). Based on AND and OR superstates, the state space of an STS can be decomposed into several successive layers in a top-down format, which consist of Cartesian products and disjoint unions, respectively. In a well-formed state-tree, all the leaf states are simple states.

A holon is a five-tuple \(H := (X, \Sigma, \delta, X_0, X_m)\), where \(X\) is the nonempty state set that can be partitioned into a (possibly empty) external state set \(X_E\) and an internal state set \(X_I\), i.e., \(X = X_E \cup X_I\) with \(X_E \cap X_I = \emptyset; \Sigma\) is the event set that can be partitioned into a boundary event set \(\Sigma_E\) and an internal event set \(\Sigma_I\), i.e., \(\Sigma = \Sigma_E \cup \Sigma_I\). \(\Sigma\) can also be partitioned into the sets of controllable and uncontrollable events, i.e., \(\Sigma = \Sigma_c \cup \Sigma_u\); The transition structure \(\delta : X \times \Sigma \rightarrow X\) is a partial function. We write \(\delta(x, \sigma)\) if \(\delta(x, \sigma)\) is defined. \(X_0 \subseteq X\) is the initial state set; and \(X_m \subseteq X\) is the terminal state set. The interval behavior of a holon is assigned to an OR superstate, which describes the local behavior of an STS \(G\) Ma & Wonham (2006). In a holon, \(X_0\) (resp., \(X_m\)) contains the target (resp., source) states of the boundary transitions \(i\in X_E \neq \emptyset, \) i.e., a higher level holon exists.

Intuitively, a predicate \(P\) is defined based on \(B(ST)\), such that \(P : B(ST) \rightarrow \{0, 1\}\). The truth-value (1 resp., 0) represents logical true (resp., false). Formally, \(P(b) = 1\) is represented by \(b \models P\). Propositional logic operators are defined by: 1) \((\neg P)(b) = 1\) if \(P(b) = 0\); 2) \((P_1 \land P_2)(b) = 1\) if \(P_1(b) = 1\) and \(P_2(b) = 1\); and 3) \((P_1 \lor P_2)(b) = 1\) if \(P_1(b) = 1\) or \(P_2(b) = 1\). A predicate \(P\) is identified by a set of basic-state-trees if \(B_P := \{b \in B(ST) \mid P(b) = 1\} \subseteq B(ST)\). Let \(P_0\) and \(P_m\) denote the predicate identified by the initial state-tree \(ST_0\) and the marker state-tree set \(ST_m\), respectively. Then we have \(B_{P_0} := \{b \in B(ST) \mid b \models P_0\}\) and \(B_{P_m} := \{b \in B(ST) \mid b \models P_m\}\). The set of all predicates on \(B(ST)\) is defined by \(\text{Pred}(ST)\). The top and bottom elements of a predicate are true \((\top)\) and false \((\bot)\), respectively. For an STS \(G\) and a given predicate \(P\), via all the basic-state-trees \((T)\) satisfying \(P\), the reachability (sub)predicate \(R(G, P)\) holds on a sequence of \(T\) can be reached from some \(b_0 \models P \land P_m\). Dually, \(CR(G, P)\), namely the coreachability predicate, holds all the \(T\) that can reach some \(b_m \models P \land P_m\) by a sequence of \(T\) satisfying \(P\).

3. NESTED STRUCTURE OF STS

An STS is a framework to model hierarchical DES in a compact form. Given an STS with a set of superstates assigned, we can naturally decompose it into a set of STS nests rooted by superstates, which describe the system behavior locally.

3.1 STS Nests

The state aggregation of each superstate is defined below.

**Definition 1. [State Aggregation]** Let \(ST = (X, x_0, T, \mathcal{E})\) be a state-tree. The state aggregation \(X_A : X \rightarrow 2^X\) in a state-tree is defined by

\[X_A(x) := \begin{cases} \{z \mid z \in \mathcal{E}(x)\}, & \text{if } T(x) = \text{OR} \\ \{z \mid z \in X_A(y)\}, & \text{if } T(x) = \text{AND} \end{cases},\]

In the case that \(X_A(y) \subset X_A(x)\), the calculation of state-aggregation \(X_A(y)\) is discarded. The remains partition the structured state set of an STS.

**Definition 2. [STS Nest]** An STS nest \(G^x\) rooted by a superstate \(x\) describes the local behavior in \(x\), which is represented by a six-tuple \(G^x = (ST^x, \mathcal{H}_A(x), \Sigma_A(x), A^x, ST_0^x, ST_m^x)\), where

- \(ST^x\) is a state-tree with a root \(x\) and terminated at the state aggregation \(X_A(x)\). In an STS \(G\), given a state-tree \(ST^x\) rooted by \(x\), \(ST^x\) is obtained by removing all the descendants of \(y\) with respect to \(X_A(x) \subset X_A(y)\);
- \(\mathcal{H}_A(x)\) is the holon aggregation of superstate \(x\). Formally, \(\mathcal{H}_A(x) := \{H^x | X^x_0 \subseteq X_A(x)\}\);
- \(\Sigma_A(x)\) is the event aggregation of superstate \(x\). Formally, \(\Sigma_A(x) := \{\sigma | \sigma \in \Sigma_j^y, H^y \in \mathcal{H}_A(x)\}\);
- \(A^x\) is the nested transition structure of \(G^x\), which will be defined later;
- \(ST_0^x\) is the initial-state-tree of \(G^x\) with respect to \(\mathcal{V}(ST_0^x) = \{z | z \in X^x_0, H^y \in \mathcal{H}_A(x)\}\); and
- \(ST_m^x\) is the marker-state-tree set of \(G^x\) with respect to \(\mathcal{V}(ST_m^x) = \{z | z \in X^x_m, H^y \in \mathcal{H}_A(x)\}\).

By starting from the root state of an STS \(G\), it is naturally decomposed into a set of STS nests by Algorithm 1.

**Definition 3. [Subordination]** STS nests \(G^x\) is subordinate to \(G^y\) if \(y \in X_A(x)\). Formally, \(G^x < G^y\).

**Example.**

We take the transfer line (Wonham & Cai (2013); Ma & Wonham (2005, 2006)) shown in Fig. 1 as an example.
Algorithm 1 STS nest calculation

Input: An STS $G$ with root state $x_0$.
Output: A set $D$ of STS nests $G$.

1. Calculate $X_A(x_0)$ and $G^{x_0}$;
2. Put state $y \in X_A(x_0)$ w.r.t. $T(y) \neq SIM$ in a set $C$;
3. while $C \neq \emptyset$ do
   4. Choose an element $y$ from $C$;
   5. $C := C \setminus \{y\}$;
   6. Calculate $X_A(y)$ and $G^y$;
   7. Put state $z \in X_A(y)$ w.r.t. $T(z) \neq SIM$ in $C$;
   8. Put $G^y$ into $D$;
9. end
10. return $D$;

Suppose that the capacities of the buffers $B1$ and $B2$ are both one. The system behavior of machine $M1$ (resp., $M2$) is described by two holons, in which the operations in superstate $M1_1$ (resp., $M2_1$) are depicted in the low level holons. The corresponding state-tree and holons are shown in Figs. 2 and 3, respectively. The events denoted by odd (resp., even) numbers are controllable (resp., uncontrollable).

Fig. 1. Transfer line.

Fig. 2. State-tree of a transfer line after plug in.

Fig. 3. Holons of a transfer line after plug in.

For the STS shown in Figs. 2 and 3, we have $G^{TL} < G^{M11}$ and $G^{TL} < G^{M21}$. The holons are divided into three holon families shown in Figs. 4 and 5. The system behaviors in $G^{M11}$ and $G^{M21}$ are not considered while synthesizing the supervisor for the top level $G^{TL}$.

3.2 Nested Transition Structure in STS

Generally, given an STS $G$, suppose that $G^y$ is subordinate to $G^x$, i.e., $G^y < G^x$. We have holon aggregations $H_A(x)$ and $H_A(y)$ as two holon families. In $H_A(x)$, superstate $y$ is replaced by a simple state with the same name. In order to integrate the system behavior of $G^y$ into $G^x$, we require that

- while the system arrives state $y$ in $G^x$, the system enters $ST_{0y}^y$ automatically;
- after entering $G^y$, the process in $G^x$ is paused temporarily; and
- while the system is ready to leave $G^y$, the events defined at state $y$ in $G^x$ are eligible to occur.

Let $\sigma$ in $\Sigma$ be an event in an STS $G$. In Ma & Wonham (2005, 2006), the largest eligible state-tree and largest next state-tree of $\sigma$, denoted by $Elig_{G}(\sigma)$ and $Next_{G}(\sigma)$, are proposed to build its forward and backward transitions, respectively. Let $\sigma \in \Sigma_A(x)$. Similarly, the largest nested eligible state-tree and largest nested next state-tree in $G^x$, denoted by $Elig_{G^x}(\sigma)$ and $Next_{G^x}(\sigma)$, respectively, are obtained. As a consequence, the nested forward/backward transition structures denoted by $\Delta^f/\Gamma^b$ are built.

Similarly, at a state-transition $T \in ST(S{T}^x)$, the forward (resp., backward) transition relation $\Delta^f$ (resp., $\Gamma^b$) corresponding to event $\sigma$ is defined based on replace source $G^x,\sigma$ (resp., replace target $G^x,\sigma$) operations on $T \land Elig_{G^x}(\sigma)$ (resp., $T \land Next_{G^x}(\sigma)$). According to Ma & Wonham (2005, 2006), function replace source $G^x,\sigma$ (resp., replace target $G^x,\sigma$) replaces the source (resp., target) states of event $\sigma$ appeared in $T$ by the corresponding target states simultaneously.

Example.

For the STS shown in Figs. 4 and 5, while the system arrives state $M1_1$ in $G^{TL}$, the system enters $ST_{0y}^{M11}$ automatically and the behavior in $G^{TL}$ is paused. After the system arrives $ST_{1y}^{M11}$ in $G^{M11}$, event 2 in $G^{TL}$ is paused. After the system arrives state $M1_1$ in $G^{TL}$, the system enters $ST_{0y}^{M11}$ automatically and the behavior in $G^{TL}$ is paused.
eligible to occur. In Fig. 4, at the active state set \( \{M_{20}, B_{11}\} \), we have \( \Delta T_L(M_{20}; B_{11}; 3) = \{M_{21}, B_{10}\} \).

3.3 Predicate Representation

The transition relation structures and the state space of all the STS nests are encoded in predicates by function \( \Theta : ST(S_T^x) \rightarrow \text{Pred}(ST^x) \) defined in Ma & Wonham (2005, 2006).\(^2\) Let \( G^x = (S_T^x, \mathcal{H}_A(x), \Sigma_A(x), \Delta^x, T_{S_T^x}, T_{R_m}^x) \) be an STS nest. A predicate \( P^y \) defined on \( \text{B}(ST^x) \) is a function \( P : \text{B}(ST^x) \rightarrow (0, 1) \). As a consequence, \( G^x \) can be rewritten as \( G^x = (S_T^x, \mathcal{H}_A(x), \Sigma_A(x), \Delta^x, P_0^x, P_m^x) \), in which \( P_0^x \) and \( P_m^x \) are the initial predicate and marker predicate, respectively.

4. NESTED SUPERVISING CONTROL OF STS

In Ma & Wonham (2005, 2006), given a predicate \( P \) with respect to an STS \( G \), by the state feedback control (SDFC), the supremal element of weakly controllable and coreachable behavior of \( G \) (i.e., optimal behavior) \( C = \text{supC}^2 \mathcal{P}(P) \) of \( G \) w.r.t. \( P \) is calculated. Instead of calculating the optimal supervisor of an STS monolithically, a top-down nested approach is presented in this section to calculate the optimal behavior of each STS nest.

4.1 Nested Supervisory Control

Suppose that \( G^x \) and \( G^y \) are two STS nests in an STS \( G \) w.r.t. \( G^x \prec G^y \). By predefining specifications for each STS nest, we obtain predicates \( P^x \) and \( P^y \), respectively. A top-down supervisor synthesis procedure is given in Algorithm 2 to calculate the optimal behavior of all the STS nests. The calculation of the optimal behavior w.r.t. \( P^y \) depends on the result for \( P^x \). Finally, the global optimal behavior of \( G \) is obtained.

In Algorithm 2, Lines 1–3 calculate the supervisor for the STS nest \( G^x \) on the top level. The control function of each controllable event on the top level is calculated in Line 3. Lines 4–21 define a recursive function \( \text{Supcon}() \) that calculates the supervisors of other STS nests. Suppose that \( G^x \prec G^y \). Let holon \( H^y \in \mathcal{H}_A(x) \) and \( H^x \in \mathcal{H}_A(y) \).

Algorithm 2 Nested Supervisory Control of STS

**Input:** A set of Nested STS \( G \) with predefined predicates.

**Output:** Control functions for controllable events.

1. Compute \( C^y = \text{supC}^2 \mathcal{P}(P^y) \) with \( x = x_0 \);
2. \( N_{\text{good}} := \Theta(\text{NextG}(\sigma)) \);
3. \( f_x := \Gamma(N_{\text{good}}) \) for all \( \sigma \in \Sigma_A(x) \cap \Sigma_c \);
4. **start** \( \text{Supcon}(P^y); \)
5. **for** each \( G^y \) w.r.t. \( G^x \prec G^y \);
6. \( (\sigma \in \Sigma_A(x) \cap \Sigma_c) \Rightarrow f_x \wedge P_{m}^y \wedge \text{Elig} \Gamma(\sigma) \Rightarrow B; \)
7. **if** \( B \neq \perp \)
8. \( P_{o}^y = P_{o}^y \wedge B; \)
9. \( P_{m}^y = P_{m}^y \wedge C^y; \)
10. \( C^y = \text{supC}^2 \mathcal{P}(P^y); \)
11. \( N_{\text{good}} := \Theta(\text{NextG}(\sigma)) \wedge C^y \) for all \( \sigma \in \Sigma_A(y) \cap \Sigma_c; \)
12. **endif**
13. **if** \( C^y = \perp \) or \( P_{m}^y \wedge P_{o}^y \wedge \neg R(C^y) \neq \perp; \)
14. **pause;** //The structure of \( G^y \) needs remodel.
15. **else**
16. **for** each \( G^z \) w.r.t. \( G^y \prec G^z \);
17. \( \text{Supcon}(P^z); \)
18. **endfor**
19. **endif**
20. **endfor**
21. **end**
22. **return** \( f_x \) for all \( \sigma \in \Sigma_c \);

with \( \sigma \in \Sigma^T \cap \Sigma_B \cap \Sigma_c \). In case that \( \sigma \) is disabled in \( G^x \), then the basic-state-trees in \( G^y \) containing the corresponding terminal states in \( H^y \) are defined as an illegal state in \( G^y \), which is guaranteed by Lines 6–8. For each controllable event \( \sigma \in \Sigma_A(y) \), Lines 9–11 calculate the control functions that contain all the (redundant) illegal behaviors of \( G^y \) to provide the supremal permissive behavior for event \( \sigma \). Line 13 checks the supremal behavior \( C^y \) for \( G^y \). In case that \( C^y \) is empty or some basic-state-trees in \( G^z \) containing the terminal states in \( G^y \) are not reachable, which shows that the system model in \( G^y \) is problematic and the users should remodel it. Line 17 invokes function \( \text{Supcon}() \) recursively.

Suppose that \( G^x \prec G^y \). As shown in Fig. 6, in each STS nest \( G^y \), the system behavior is recorded in an agent \( G^y_{\text{traker}} \). With respect to the specification for \( G^y \), according to the optimal behavior \( C^y \) of \( G^y \) and the current status (a basic state-tree b) provided by \( G^y_{\text{traker}} \), a set of decision makers \( f_{\sigma_i} \), with \( \sigma_i \in \Sigma_c \cap \Sigma_A(y) \) and \( i = 1, 2, \ldots, n \), makes the decisions applying \( b \) as the argument. If \( f_{\sigma_i}(b) = 1 \), then \( \sigma_i \) is allowed to occur. Otherwise, it is disabled.

4.2 Boundary Consistency of Supervisory Control

As stated in Section 1, a well-formed STS satisfies boundary consistency. In this study, this feature is extended to the supervisory control of STS.

**Property 1.** We say that an STS satisfies the boundary consistency of supervisory control if it satisfies: the low level closed-loop (under control) STS nests can be “plugged” into the states of a high level STS nest without changing its control functions (control logics).

11329
Fig. 6. Nested STS control diagram.

As the two-level predicate depicted in Fig. 7, suppose that event $\sigma$ is controllable and it is disabled at basic-state-tree $2$ on the top level. Within predicate $2$ on the lower level, in order to avoid blocking the system at basic-state-tree $c$, it is considered as a new illegal basic-state-tree. Line 8 in Algorithm 2 guarantees that Property 1 is satisfied. Finally, after calculating the supervisor for the lower-level events $1$, $3$, and $5$ are identical, as given in Section 5.1 under GT. However, after plugging the (nonblocking) 

Finally, after calculating the supervisor for the lower-level event $2$ in Algorithm 2 guarantees that Property 1 is satisfied.

5. CASE STUDIES

Two case studies are presented in this section to demonstrate the nested supervisory control of STS.

5.1 Transfer Line

For the transfer line studied in Section 3.1, the nonblocking supervisory control functions are:

- In $G^{TL}$:
  - event $1$ is enabled at: $\{B_{10}, B_{20}, M_{20}, TU_{0}\}$,
  - event $3$ is enabled at: $\{B_{11}\}$, and
  - event $5$ is enabled at: $\{B_{21}\}$;
- In $G^{M1}$: events $11$ and $13$ are always enabled; and
- In $G^{M2}$: events $21$ and $23$ are always enabled.

The transfer line satisfies Property 1. The system behavior of its STS model under nested supervisory control is shown in Fig. 8. By projecting out the low-level behavior shown in Fig. 8 in the dashed line boxes, we obtain a diagram identical with the optimal behavior of the top level.

Fig. 7. Two level predicates.

Fig. 8. Optimal behavior of Transfer Line.

5.2 AIP Example

The diagram of the AIP studied in Ma & Wonham (2005, 2006) is depicted in Fig. 9. AIP has five conveyor loops: one central loop communicates with four external loop by four transfer units. Linked to the external loops are three assembly stations and an I/O station. The primary DES model of AIP studied in Brandin (1994) is the synchronous product of 100 automata with a state space up to $10^{24}$. If the developed nested SFBC, we obtain 36 different STS nests on three hierarchical levels. As a consequence, the total state space of all the $36$ STS nests are around $2 \times 10^{18}$. The computation is finished in several seconds on a personal computer with 2.40 GHz Intel CPU and 8G RAM. The BDD nodes of the local control functions for several important controllable events are listed in Table 1 to compare between the AIP studied in Ma & Wonham (2005, 2006) (under MW) and this study, in which the BDD size $0$ represents that the corresponding event is allowed to occur when it is eligible.

Table 1. BDD size of Controller functions for AIP

| Event                                      | MW | This study |
|--------------------------------------------|----|------------|
| AS$_i$ _repaired $(i = 1, 2)$              | 0  | 0          |
| AS$_i$ _stop_close $(i = 1, 2)$            | 1  | 9          |
| AS$_i$ _stop_open $(i = 1, 2)$             | 16 | 9          |
| AS$_i$ _gate_open $(i = 1, 2)$             | 0  | 0          |
| AS$_i$ _read $(i = 1, 2)$                  | 0  | 0          |
| AS$_i$ _pickup3                            | 15 | 0          |
| AS$_i$ _pickup4                            | 15 | 0          |
| AS$_i$ _gate_open                          | 0  | 0          |
| AS$_3$ _read                               | 2  | 2          |
| L$_i$ _gate_open                           | 95 | 44         |
| CL $TU_{i}$ _gate_open $(i = 1, 2)$        | 70 | 37         |
| CL $TU_{i}$ _stop_close                    | 28 | 20         |
| CL $TU_{i}$ _stop_close                    | 36 | 19         |
| TU$_i$ _Drw2L$(i = 1, 2)$                  | 54 | 0          |

6. DISCUSSIONS

For the one-level transfer line depicted in Fig. 4, by following the approach proposed in Ma & Wonham (2005, 2006) (under MW) and this study, the control functions for events $1$, $3$, and $5$ are identical, as given in Section 5.1 under $G^{TL}$. However, after plugging the (nonblocking)
holons shown in Fig. 5, their results are different. The BDD nodes of the control functions are listed in Table 2, which shows that Property 1 is not satisfied in the supervisory control of STS proposed in Ma & Wonham (2005, 2006).

Table 2. BDD nodes of controllers

| Event | MW    | This study |
|-------|-------|------------|
| 1     | 0     | 4          |
| 3     | 1     | 1          |
| 5     | 5     | 1          |
| 11    | 4     | 0          |
| 13    | 4     | 0          |
| 21    | 2     | 0          |
| 23    | 2     | 0          |

More precisely, by following Ma & Wonham (2005, 2006), control functions $f_{11}$ and $f_{13}$ make decisions based on the system behavior in some high level holons. They require that events 11 and 13 should occur if

- buffers $B_1$ and $B_2$ and test unit $TU$ are empty, or
- buffer $B_1$ is empty, buffer $B_2$ is occupied, machine $M_2$ is at the initial state, and test unit $TU$ is empty.

As a consequence, the control functions calculated based on the approach proposed in Ma & Wonham (2005, 2006) may contain redundant control logics. This is caused by the redundant calculation of the synchronous product of $G_1^{M_1}$ and $G_2^{M_2}$. By following Ma & Wonham (2005) and Ma & Wonham (2006), the closed-loop behavior of the STS contains 56 basic state-trees and 126 transitions. However, as shown in Fig. 8, according to the nested approach presented in this study, we obtain the closed-loop behavior of the STS contains 12 basic state-trees and 15 transitions.

7. CONCLUSION

Based on the AND and OR superstates of an STS, we formally decompose it into a set of STS nests that describes its system behavior on each hierarchical level. The subordination relation among different STS nests is also defined. Suppose that an STS nest $G_1^x$ is subordinated to another STS nest $G_2^y$. In $G_1^x$, the complex internal behavior of $G_2^y$ is represented by a simple state $y$. Instead of calculating the optimal supervisor of an STS monolithically, a top-down nested approach is presented in this study to calculate the optimal behavior of each STS nest. By avoiding the redundant calculation of the synchronous product in independent STS nests, the state spaces of both the system model and the supervisor are reduced significantly. Finally, two case studies are presented in Section 5 to demonstrate the nested supervisory control of STS. For the STS model of the AIP studied in Ma & Wonham (2005, 2006); Brandin (1994), it was originally with a state space up to $10^{18}$. In this study, it is decomposed into 36 different STS nests on three hierarchical levels. As a result, the total state space of all the 36 STS nests is reduced to around $2 \times 10^{18}$. The supervisors for each STS nest satisfies the boundary consistency of supervisory control, i.e., the low level closed-loop (under control) STS nests can be “plugged” into the states of a high level STS nest without changing its control functions (control logics). In our future work, we will work on the nested supervisory control of state-tree structures with partial observations.

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