Searching for Mini Extreme Mass Ratio Inspirals with Gravitational-Wave Detectors

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A compact object with a mass $O(1 \sim 1000)M_\odot$, such as a black hole of stellar or primordial origin or a neutron star, and a much lighter exotic compact object with a sub-solar mass could form a non-standard mini extreme mass ratio inspiral (EMRI) and emit gravitational waves within the frequency band of ground-based gravitational-wave detectors. These systems are extremely interesting because detecting them would definitively point to new physics. We study the capability of using LIGO/Virgo to search for mini-EMRIs and find that a large class of exotic compact objects can be probed at current and design sensitivities using a method based on the Hough Transform that tracks quasi power-law signals during the inspiral phase of the mini-EMRI system.

Introduction. The direct detection of binary black hole, neutron star and neutron star/black hole mergers by the LIGO/Virgo collaborations [1–3] has opened the era of gravitational wave (GW) and multi-messenger astronomy. Though these sources were expected, the masses of some of them have surprised us [4], most of all, the existence of $O(100M_\odot)$ black holes, which cannot be explained by traditional stellar evolution models. Such departures from our current understanding of the universe motivate the study of compact objects that differ from the canonical ones predicted by stellar evolution (white dwarfs, neutron stars, and black holes). Though these so-called exotic compact objects (ECOs), e.g. primordial black holes (PBHs) [5], boson stars [6] and quark stars [7], have not been detected, their existences could hint at new physics [8], and help explain puzzling cosmological observations [9, 10]. If ECOs are dark, they would constitute a macroscopic dark matter species, in contrast to other hypothetical dark matter candidates, such as weakly interacting massive particles (WIMPs) [11], or ultralight dark matter such as axions [12] or dark photons [13]. In particular, the low-spinning black holes detected by LIGO/Virgo indicate that they could have primordial origins [8], and therefore that PBHs constitute all or a fraction of dark matter [14].

At the moment, however, only limited efforts have been made to detect ECOs by the GW community. Searches for sub-solar mass black holes (of $O(0.1M_\odot)$) have been carried out [15–17], resulting in constraints on the fraction of dark matter that PBHs could compose. Scalar [18], vector [19, 20], or tensor [21] dark matter clouds around black holes have also been theorized, and constraints on boson/black hole mass pairs have been placed using the non-observation of quasi-monochromatic GW signals across the whole sky [22, 23]. Furthermore, a method has been proposed to search for planetary-mass PBHs, with masses of $O(10^{-7} – 10^{-2})M_\odot$ [24], and constraints have been placed on the existence of such objects in binary systems [25, 26]. However, in order to probe the existence of a variety of ECOs, and cover an extensive portion of the mass parameter space in which ECOs could lie, a more systematic approach is needed. Binary systems composed of lighter ECOs, with masses $\leq O(0.1M_\odot)$, would emit long-lived GWs in the inspiral portion of their lives [24]. Here, we consider a highly asymmetric mass ratio between the two compact objects, inspired by the possibility of future space-based GW interferometers, such as DECIGO [27], LISA [28, 29], Taiji [30–32] and Tianqin [33–35], to detect extreme mass ratio inspirals (EMRIs). In ground-based detectors, ordinary compact objects would be bound to much lighter ECOs, which we call “mini-EMRI” systems. Depending on the mass ratio of the two compact objects, mini-EMRIs could last anywhere from $O(\text{hours – days}) – O(\text{years})$ in the frequency band of ground-based GW detectors, which would allow signal-to-noise ratio to accumulate over time, as is expected for EMRIs in space-based detectors. A stochastic GW background for the kinds of systems considered in this paper could also exist, but is estimated to be very weak [36], motivating the need to consider individual mini-EMRI systems.

The durations of these signals, and the presence of non-stationary noise and gaps in the data, imply computational challenges for traditional matched-filtering algorithms that search for binary neutron star mergers and sub-solar mass and stellar-mass binary black hole mergers [37–39]. Furthermore, mini-EMRIs could last in the detector band for timescales compatible with those expected from continuous waves from isolated neutron stars [40, 41], boson clouds around black holes [23, 42–44], and quasi-monochromatic signals arising from dark matter interactions with GW detectors [45–49]. Hence, methods that have typically been used in these searches can also be applied to detect signals from inspiraling mini-EMRIs.

In this letter, we show that a mini-EMRI system formed by an ordinary or exotic compact object and a much lighter ECO can be detected in ground-based GW interferometer data. The current observing run of Advanced LIGO/Virgo can already be used to probe a large region in the parameter space of ECOs, while future detectors will be able to provide even broader coverage. We
also describe a new way to search for these ECOs in the much smaller mass region using the Hough Transform.

**Mini Extreme Mass Ratio Inspirals.** An EMRI system, in its standard definition [29], consists of a supermassive black hole in the galactic center with a mass in the range $10^6 \sim 10^9 M_\odot$, and an inspiraling ordinary compact object such as a black hole of astrophysical origin, a neutron star or a white dwarf, with a typical mass of $1 \sim 10 M_\odot$ [29], and thus a mass ratio $\lesssim 10^{-5}$. Nonstandard EMRIs could have a much lighter ECO [50, 51], or a superheavy boson star replacing the role of the supermassive black hole [52]. Such EMRI systems are expected to form in the inner parsec region of the galaxies due to complicated stellar dynamics under the influence of the gravitational potential of supermassive objects (see, e.g., [53, 54]).

A mini-EMRI system, instead, is defined here to consist of one object (with mass $M$) much lighter than a supermassive black hole, and another one even lighter (with mass $m \ll M$). While $M$ can take any value much smaller than that of the supermassive black hole, we restrict to, as an example, searches for a special class of mini-EMRIs that could be detected by LIGO/Virgo. The maximally achievable GW frequency from such a mini-EMRI system, assuming for simplicity a circular orbit, occurs at the innermost stable circular orbit (ISCO) [55]

$$f_{\text{ISCO}} = 4.4 \text{kHz} \left( \frac{1 M_\odot}{M} \right)^{1/2} \frac{n}{2} g(a),$$

where $a$ is the dimensionless spin of the heavier component, $n$ is the harmonic number, with $n=2$ being the dominant contributor to GW emission, and $g$ is a monotonically increasing function of $a$, normalized such that $g(0) = 1$, with $g(-1) \approx 0.57$ and $g(1) \approx 7.35$. Fig. 1 shows the band of $f_{\text{ISCO}}$ as a function of $M$ by varying also $a$. For the signal to be within the LIGO/Virgo band, we need $M \lesssim O(1000) M_\odot$, and with a mass ratio of $10^{-5}$, which requires $m \lesssim O(10^{-2}) M_\odot$.

An advantage of such mini-EMRI searches is that the GW amplitude and the signal-to-noise ratio generally increase with the chirp mass $M_c \equiv (mM)^{3/5}/(m + M)^{1/5}$; thus, for a given $M_c$, the mini-EMRI search can probe a much smaller $m$ than what a search for comparable sub-solar mass binaries (with both masses being $\bar{m}$) can achieve, as

$$\frac{m}{\bar{m}} \approx 0.8 \left( \frac{\bar{m}}{M} \right)^{2/3} \ll 1.$$  

This implies that with a larger $M$, we can carve into a much deeper portion of the subsolar mass regime. Currently, the most massive black hole detected by LIGO/Virgo was GW190521, in which two merging black holes left behind an intermediate-mass black hole with a mass $142^{+28}_{-16} M_\odot$ [56]. These larger $M$ compact objects could serve as good targets to search for mini-EMRI GW signals, since much lighter ECOs could orbit around them. Furthermore, heavier ECOs would allow us to probe black holes whose mass falls into the traditional intermediate-mass regime. While the heavier mass could be an arbitrary ECO, to focus on the search for sub-solar ECOs, we concentrate here on mini-EMRIs with a heavier compact mass, but allow an arbitrary compactness $C$, the dimensionless mass radius ratio, for its lighter partner, such that the parameters characterizing the system are $m$, $M$, $a$ and $C$.

The most studied ECO is the PBH, and binaries of PBHs could form in the early universe, either under the
torques of nearby PBHs or density fluctuations [57–59], in which mini-EMRIs could arise for a PBH population with an extended mass spectrum [60–63]. A PBH binary could also form through capture in a dense PBH halo, and the details of this clustering of PBHs determine the merging rates [64, 65]. Additionally, if a stellar-mass black hole passes by a PBH, it could capture it and form a binary [36]. Finally, PBH binaries could be formed by capture in the galactic center [24]. Despite the many competing explanations for what constitutes an ECO, we propose in this work a way to probe the existence of any sub-solar mass ECO. We are therefore mostly sensitive to the mass $m$ and compactness $C$ of the ECO, but are agnostic to how that ECO formed.

**Gravitational-Wave Signal Properties.** The calculation of the GW signal from a mini-EMRI system is similar to that of two approximately equal-mass objects when they are far away from each other, such that a post-Newtonian treatment is sufficient, which corresponds to the early inspiral stage of the inspiral-merger-ring-down waveform [66, 67]. However, as they approach each other, relativistic effects become significant enough that a fully numerical calculation is necessary, which is a difficult task and still an ongoing effort (see [29, 68, 69] for reviews). While full numerical relativity simulations are advancing toward higher mass ratios (see, e.g., [70, 71]), the extreme mass ratio of this system makes possible a perturbation theory of a different kind, which is based on an expansion in the small mass ratio, in the waveform computation (see, e.g., [69]). We use the fully numerical result obtained for quasi-circular evolution of the binary [72] based on the Teukolsky formalism [73, 74], which is consistent [75] with those obtained based on the numerical or analytical Kludge waveforms [76, 77]. Fig. 2 shows the characteristic strain $h_c(f)$ for the left red vertical line of Fig. 1 as the blue line, as well as for mini-EMRI systems with spins of 0.99 and $-0.99$. We can see that this signal could spend a very long time (relative to detected binary black hole and neutron star mergers) in current and future ground-based GW detectors’ most sensitive frequency bands.

The GW signal can be divided into two stages with different properties: the early inspiral and the late plunging. For the inspiral part, the signal evolves with a frequency “spin-up” rate $\frac{df}{dt}$:

$$\frac{df}{dt} = \frac{96}{5} \pi^{8/3} \left( \frac{GM_c}{c^3} \right)^{5/3} f^{11/3} C_f(a, f),$$

where $G$ is Newton’s gravitational constant and $c$ is the speed of light. The factor $C_f$, as shown in the left panel of Fig. 3, captures relativistic effects and asymptotes to 1 for small $f$, or a large separation. As the ISCO is approached, $C_f$ increases rapidly and diverges corresponding to the second plunging phase. The GW emission, as represented by $h_c$, decreases significantly during this stage, making its detection potentially difficult.

Since $M_c$ is small for subsolar mass searches, the frequency increases slowly during this stage. At the same time, the GW amplitude evolves with a similar relativistic correction factor $C_h$ (as shown in the right panel of Fig. 3):

$$h_0 = \frac{4}{d} \left( \frac{GM_c}{c^2} \right)^{5/3} \left( \frac{\pi f}{c} \right)^{2/3} C_h(a, f).$$

Thus, the signal does not behave like a typical chirp signal that lasts for $O$(seconds), but instead like continuous waves from neutron stars, which LIGO/Virgo are actively searching for using techniques and methodology readily applicable here [78–81]. This means that directed searches [82] for mini-EMRI systems, in which the heavier object could be a known black hole or neutron star, could be performed, as well as all-sky searches [26], in which the heavier mass would be of arbitrary origins.

Compared with previous searches for compact ECOs, allowing a generic compactness $C$ leads to significant changes in the GW signals, since the ECO might be

FIG. 2. The dimensionless characteristic strain $h_c$ for a mini-EMRI with mass ratio $10^{-5}$ at a distance of $d$ from the detector, for $a = -0.99, 0, 0.99$, where the markers denote remaining times to ISCO: 1 minute, 1 hour, 100 hours, 1 year, and 10 years.

FIG. 3. The relativistic correction factors for $df/dt$ and $h_0$ as a function of frequency $f$ for an mini-EMRI with parameters specified in the figures, for three choices of spin $a$. 

tidentally disrupted before reaching the ISCO, and correspondingly the GW signal will be cut off at this frequency [51]. The tidal radius can be estimated by equating the gravitational force of the heavier object with the self-gravitating force of the lighter one as \( r_{\text{tidal}} = (m^2 M)^{1/3} / C \), which then translates into another maximal frequency cut-off \( f_{\text{tidal}} \), implying that the actual cut-off frequency will be \( \min(f_{\text{tidal}}, f_{\text{ISCO}}) \). Additionally, the tidal disruption could provide potentially valuable electromagnetic counterpart signals to a GW detection of a mini-EMRI, which could reveal nature of the underlying ECO and help probe new physics.

**Search with the Hough Transform.** We now employ strategies, developed in the context of continuous-wave searches, that attempt to detect GWs from asymmetrically rotating neutron stars [41, 83]. At their cores, these methods assume that the GW frequencies evolve linearly and slowly with time [78, 79, 84]. There also exist so-called “transient” continuous-wave methods [80, 81, 85–88], originally developed to search for remnants of neutron star mergers or supernovae [89], that track rapid frequency evolutions over time. The inspiral portion of mini-EMRI systems would follow a power-law frequency evolutions over time. The inspiral

disturbances, gaps and generally non-stationary noise [78, 79], three problems that would likely occur over the duration of the mini-EMRI signal in GW data.

In this method, we break up the data into chunks of durations \( T_{\text{FFT}} \) much less than the signal duration \( T_{\text{obs}} \). Fast Fourier transform (FFT) each chunk, and combine the power in each chunk incoherently. The length of each FFT is chosen to confine the signal power in one frequency bin during that FFT, which means that it is primarily a function of the frequency change of the inspiraling system over time [24]. The choice of \( T_{\text{FFT}} \) significantly affects the sensitivity, or distance reach, of the search. To choose this quantity, for a range of mass ratios, we calculate how quickly the system inspirals, and use that to determine \( T_{\text{FFT}} \) and \( T_{\text{obs}} \), within a given 10-Hz band. The choice of a 10-Hz analysis band is arbitrary, but in a real search, an upper-frequency cutoff must be selected, since observing the signal until the ISCO would not result in a good sensitivity, as \( T_{\text{FFT}} \) would have to be very small to contain the frequency modulation induced by the large spin-up. Instead, we could optimize the choice of the analysis frequency band, as a function of the mass ratio, as done in the case of equal planetary-mass primordial black hole systems [24], which will be the subject of future work.

Here, we set \( T_{\text{FFT}} = 1/\sqrt{f} \) and evaluate \( f \) at the highest frequency in the 10-Hz band, which will lead to a conservative estimation of the sensitivity. We show in Fig. 4 an example of a mini-EMRI system with \( M = 10M_\odot \), \( a = 0 \), in which the y-axis shows the total observation time of the 10-Hz band and the x-axis shows the mass ratio. For each mass ratio, a set of observation times is obtained by varying the starting frequency from 20Hz to a value near \( f_{\text{ISCO}} \), leading to a region on this plot. The color here denotes the FFT length, whose minimum value is set to be one second, which corresponds to the lower boundary of the colored region. Points with the same starting frequency for different mass ratios fall on a line, as labeled for several in the plot. Typically, lighter-mass systems at lower frequencies will exhibit more linear and slower frequency evolutions over time than those with higher-mass, higher-frequency counterparts [24]. Furthermore, the former systems will inspiral for a lot longer due to having smaller changes in frequency over time than those of the latter systems.

To estimate the sensitivity to mini-EMRI binaries, we employ the method in [24] to calculate the minimum detectable GW amplitude at a given confidence level, as a function of the analysis coherence time, the signal duration, and particular analysis parameters. This minimum amplitude can be translated into a maximum distance reach \( d_{\text{max}} \)

\[
d_{\text{max}} = 0.995 \left( \frac{GM}{c^2} \right)^{5/3} \left( \frac{\pi}{c} \right)^{2/3} T_{\text{FFT}} \frac{1}{\sqrt{T_{\text{obs}}}} \left( \sum \frac{F_i^2}{S_i(f_i)} \right)^{1/2} \left( \frac{p_0(1-p_0)}{Np_1} \right)^{1/4} \sqrt{\frac{\theta_{\text{thr}}}{(CR_{\text{thr}} - \sqrt{2erfc^{-1}(2\Gamma)})).
\]
where $F_i = f_i^{2/3} C_h(a, f_i)$; $i$ is an index that runs over the chunks; $N$ is the number of FFTs in $T_{\text{obs}}$; $p_0$ is the probability of selecting a noise peak above a threshold; $p_1 = e^{-\theta_{\text{thr}}} - 2e^{-2\theta_{\text{thr}}} + e^{-3\theta_{\text{thr}}}$, $\theta_{\text{thr}} = 2.5$ is a threshold on equalized power spectra in the time/frequency map; $\Gamma = 0.95$ is the confidence level; $C R_{\text{thr}} = 5$ is the threshold of the critical ratio in selecting candidates in the frequency-Hough map; and $S_n$ is an estimation of the noise power spectral density of the detector.

**Results.** The hypothetical distance reach and minimum detectable amplitude of a search for mini-EMRI systems are shown in the left panel of Fig. 5 on the y-axis and in color, respectively, as a function of the mass ratio for a system with $M = 10M_\odot$, $a = 0$ two compact objects. The starting frequency of the 10-Hz band is varied from 20Hz to a frequency close to $f_{\text{ISCO}}$. A fixed starting frequency results in a line in this plot and varying it leads to the region shown here. For a fixed mass ratio, increasing the starting frequency from 20Hz leads firstly to an increasing distance reach before dropping to lower values, which is different than in Fig.4, and leads to the mixing of the blue and cyan colors. Systems with less extreme mass ratios could be seen further away, at $O(10\text{Mpc})$, while those with more extreme mass ratios could only be detected $O(\text{kpc-Mpc})$ away.

The result of allowing the lighter one, with mass $m$, of the mini-EMRI system to be non-compact, with compactness $C$, is shown in the right panel of 5. Regions on the plane $(m, C)$ that could be detected are denoted by the green region with the contours for several distances shown. The red dashed line denotes where the tidal radius coincides with the ISCO radius, while for the region to the left (right) of this line the tidal radius is larger (smaller) than the ISCO radius. As the tidal radius becomes larger and cuts into the 10Hz band, the GW signal available to build up the sensitivity is gradually lost, which then leads to the boundary of the green region.

**Discussion.** In this letter, we propose an innovative idea to search current ground-based GW data for mini-EMRI systems. A detection of such a source would imply a huge paradigm shift in the way that we understand compact objects. The prospects of seeing such a system within our galaxy in the current detector era are promising, based on the sensitivity estimation presented here, and even more possible when Einstein Telescope and Cosmic Explorer come online. We also propose a way to search for such systems with the Hough Transform, that would track the quasi-power-law nature of the inspiraling mini-EMRI system and provide an estimate of the chirp mass. Our work paves the way for searches for mini-EMRI systems, and demonstrates the effectiveness of using traditional continuous-wave methods to probe the existence of ECOs.

**Acknowledgements.** This material is based upon work supported by NSF’s LIGO Laboratory which is a major facility fully funded by the National Science Foundation. This research has used data obtained from the Gravitational Wave Open Science Center, a service of LIGO Laboratory, the LIGO Scientific Collaboration and the Virgo Collaboration. LIGO Laboratory and Advanced
LIGO are funded by the United States National Science Foundation (NSF) as well as the Science and Technology Facilities Council (STFC) of the United Kingdom, the Max-Planck-Society (MPS), and the State of Niedersachsen/Germany for support of the construction of Advanced LIGO and construction and operation of the GEO600 detector. Additional support for Advanced LIGO was provided by the Australian Research Council. Virgo is funded, through the European Gravitational Observatory (EGO), by the French Centre National de Recherche Scientifique (CNRS), the Italian Istituto Nazionale della Fisica Nucleare (INFN) and the Dutch Nikhef, with contributions by institutions from Belgium, Germany, Greece, Hungary, Ireland, Japan, Monaco, Poland, Portugal, and Spain. We thank Juan Calderon Bustillo, Carlos Lousto, Cristiano Palomba, Kuver Sinha, Yue Zhao, and the LIGO/Virgo/KAGRA continuous-wave group for helpful comments and discussions. HG is supported by the U.S. Department of Energy under Award No. DESC0009959. ALM is a beneficiary of a FSR Incoming Post-doctoral Fellowship.

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