Optimal dividends problem with a terminal value for spectrally positive Lévy processes

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Abstract

In this paper we consider a modified version of the classical optimal dividends problem of de Finetti in which the dividend payments subject to a penalty at ruin. We assume that the risk process is modeled by a general spectrally positive Lévy process before dividends are deducted. Using the fluctuation theory of spectrally positive Lévy processes we give an explicit expression of the value function of a barrier strategy. Subsequently we show that a barrier strategy is the optimal strategy among all admissible ones. Our work is motivated by the recent work of Bayraktar, Kyprianou and Yamazaki (2013).

Key Words: Barrier strategy, Dual model, Optimal dividend strategy, Scale functions, Spectrally positive Lévy process, Stochastic control.
Introduction

In this paper we consider a modified version of the classical optimal dividends problem of de Finetti in which the dividend payments subject to a penalty at ruin. Within this problem we assume that the underlying dynamic of the risk process is modeled by a spectrally positive Lévy process. In recent years, quite a few interesting papers deal with this type of model. For example, Avanzi et al. (2007), Avanzi and Gerber (2008), Bayraktar and Egami (2008), Li and Wu (2009), Ng (2009), Yao, Yang and Wang (2010), Dai, Liu and Luan (2010, 2011), Avanzi, Shen and Wong (2011), Bayraktar, Kyprianou and Yamazaki (2013), Yin and Wen (2013) to name but a few.

We now state the optimal dividends problem considered in this paper. Let \( X = \{X(t)\}_{t \geq 0} \) be a Lévy process without negative jumps defined on a filtered probability space \( (\Omega, \mathcal{F}, \mathcal{F}, P) \), where \( \mathcal{F} = (\mathcal{F}_t)_{t \geq 0} \) is generated by the process \( X \) and satisfies the usual conditions. As the process \( X \) has no negative jumps, its Laplace exponent exists and is given by

\[
\Psi(\theta) = \frac{1}{t} \ln E e^{-\theta X(t)} = c\theta + \frac{1}{2} \sigma^2 \theta^2 + \int_0^\infty (e^{-\theta x} - 1 + \theta x 1_{\{x < 1\}}) \Pi(dx), \tag{1.1}
\]

where \( 1_A \) is the indicator function of a set \( A \), \( c > 0 \), \( \sigma \geq 0 \) and \( \Pi \) is a measure on \((0, \infty)\) satisfying

\[
\int_0^\infty (1 \wedge x^2) \Pi(dx) < \infty.
\]

Denote by \( P_x \) for the law of \( X \) when \( X(0) = x \). Let \( E_x \) be the expectation associated with \( P_x \). For short, we write \( P \) and \( E \) when \( X(0) = 0 \). To avoid trivialities, it is assumed that \( X \) does not have monotone sample paths. In the sequel, we assume that \(-\Psi'(0+) = \mathbb{E}(X(1)) > 0 \) which implies the process \( X \) drifts to \(+\infty\). It is well known that if \( \int_1^\infty y \Pi(dy) < \infty \), then \( \mathbb{E}(X(1)) < \infty \), and \( \mathbb{E}(X(1)) = -c + \int_1^\infty y \Pi(dy) \). Note that \( X \) has paths of bounded variation if and only if

\[
\sigma = 0 \quad \text{and} \quad \int_0^\infty (1 \wedge x) \Pi(dx) < \infty.
\]

In this case, we write (1.1) as

\[
\Psi(\theta) = c_0 \theta + \frac{1}{2} \sigma^2 \theta^2 + \int_0^\infty (e^{-\theta x} - 1) \Pi(dx), \tag{1.2}
\]
with \( c_0 = c + \int_0^1 x \Pi(dx) \) the so-called drift of \( X \). For more details on spectrally positive Lévy processes, the reader is referred to Bertoin (1996), Sato (1999) and Kyprianou (2006).

The process \( X \) is an appropriate model of a company driven by inventions or discoveries, or the cash fund of an investment company before dividends are deduced. Let \( \pi = \{ L^\pi_t : t \geq 0 \} \) be a dividend strategy consisting of a nondecreasing, right-continuous and \( \mathbb{F} \)-adapted process starting at 0, where \( L^\pi_t \) standards for the lump-sums of dividends paid out by the company up until time \( t \). The risk process with initial capital \( x \geq 0 \) and controlled by a dividend strategy \( \pi \) is defined by \( U^\pi = \{ U^\pi_t : t \geq 0 \} \), where

\[
U^\pi_t = X(t) - L^\pi_t, \quad t \geq 0.
\]

The ruin time is then given by

\[
\tau_{\pi} = \inf \{ t > 0 | U^\pi_t = 0 \}.
\]

A dividend strategy is called admissible if \( L^\pi_t - L^\pi_{t-} \leq U^\pi_t \), for all \( t < \tau_{\pi} \), in other words the lump sum dividend payment is smaller than the size of the available capital. We define the dividend-value function \( V_{\pi} \) by

\[
V_{\pi}(x) = E \left[ \int_0^{\tau_{\pi}} e^{-qt} dL^\pi_t + S e^{-q\tau_{\pi}} | U^\pi_0 = x \right],
\]

where \( q > 0 \) is an interest force for the calculation of the present value and \( S \in \mathbb{R} \) is the terminal value. Let \( \Xi \) be the set of all admissible dividend policies. De Finetti’s dividend problem consists of solving the following stochastic control problem:

\[
V(x) = \sup_{\pi \in \Xi} V_{\pi}(x), \quad (1.3)
\]

and to find an optimal policy \( \pi^* \in \Xi \) that satisfies \( V(x) = V_{\pi^*}(x) \) for all \( x \geq 0 \).

Next, we shall have a review on the related literature. This optimal dividend problem has recently gained a lot of attention in actuarial mathematics for spectrally negative Lévy processes. Avram et al (2007), Loeffen (2008) and Kyprianou et al. (2010) studied the case of \( S = 0 \) for spectrally negative Lévy processes. The case \( S < 0 \) was studied by Thonhauser and Albrecher (2007) for the compound Poisson model and Brownian
motion risk process. The case \( S \in \mathbb{R} \) was studied by Loeffen (2009, 2010) for spectrally negative Lévy processes. It was shown that the optimal dividend strategy is formed by a barrier strategy for this type model under some conditions imposed on the Lévy measure. Moreover, Azcue and Muler (2005) have provided a counter-example for the case \( S = 0 \) shows that a barrier strategy can not be optimal. However, this in contrast with the dividend problem in the case of \( S = 0 \) for spectrally positive Lévy processes considered by Bayraktar, Kyprianou and Yamazaki (2013), which shows that there a barrier strategy always forms the optimal strategy, no matter the form of the jump measure. Motivated by the work of Bayraktar, Kyprianou and Yamazaki (2013), the purpose of this paper is to examine the analogous question for the same model in the case of \( S \neq 0 \).

The rest of the paper is organized as follows. In Section 2 we state some facts about scale functions. In Section 3 we give the main results. Explicit expressions for the expected discounted value of dividend payments are obtained, and it is shown that the optimal dividend strategy is formed by a barrier strategy.

## 2 Scale functions

For an arbitrary spectrally positive Lévy process, the Laplace exponent \( \Psi \) is strictly convex and \( \lim_{\theta \to \infty} \Psi(\theta) = \infty \). Moreover, \( \Psi \) is strictly increasing on \( [\Phi(0), \infty) \), where \( \Phi(0) \) is the largest zero of \( \Psi \). Thus there exists a function \( \Phi : [0, \infty) \to [\Phi(0), \infty) \) defined by \( \Phi(q) = \sup\{\theta \geq 0 : \Psi(\theta) = q\} \) (its right-inverse) and such that \( \Psi(\Phi(q)) = q, q \geq 0 \).

We now recall the definition of the \( q \)-scale function \( W^{(q)} \) and some properties of this function. For each \( q \geq 0 \) there exits a continuous and increasing function \( W^{(q)} : \mathbb{R} \to [0, \infty) \), called the \( q \)-scale function defined in such a way that \( W^{(q)}(x) = 0 \) for all \( x < 0 \) and on \( [0, \infty) \) its Laplace transform is given by

\[
\int_0^\infty e^{-\theta x} W^{(q)}(x) dx = \frac{1}{\Psi(\theta) - q}; \ \theta > \Phi(q).
\] (2.1)
Closely related to $W^{(q)}$ is the scale function $Z^{(q)}$ given by

$$Z^{(q)}(x) = 1 + q \int_0^x W^{(q)}(y) dy, \quad x \in \mathbb{R}.$$ 

We will frequently use the following functions

$$\overline{W}^{(q)}(x) = \int_0^x W^{(q)}(z) dz, \quad \overline{Z}^{(q)}(x) = \int_0^x Z^{(q)}(z) dz, \quad x \in \mathbb{R}.$$ 

Note that

$$Z^{(q)}(x) = 1, \quad \overline{Z}^{(q)}(x) = x, \quad x \leq 0.$$ 

If $X$ has paths of bounded variation then, for all $q \geq 0$, $W^{(q)}|_{(0, \infty)} \in C^1(0, \infty)$ if and only if $\Pi$ has no atoms. In the case that $X$ has paths of unbounded variation, then for all $q \geq 0$, $W^{(q)}|_{(0, \infty)} \in C^1(0, \infty)$. Moreover if $\sigma > 0$ then $C^2(0, \infty)$. Further, if the Lévy measure has a density, then the scale functions are always differentiable (see e.g. Chan, Kyprianou and Savov (2011)).

The initial values of $W^{(q)}$ and its derivative can be derived from (2.1):

$$W^{(\delta)}(0+) = \begin{cases} \frac{1}{c_0}, & \text{if } X \text{ has paths of bounded variation}, \\ 0, & \text{otherwise}, \end{cases}$$

and

$$W^{(\delta)'}(0+) = \begin{cases} \frac{2}{\sigma^2}, & \text{if } \sigma \neq 0, \\ \frac{q + \Pi(0, \infty)}{c_0}, & \text{if } X \text{ is compound Poisson} \\ \infty, & \text{if } \sigma = 0 \text{ and } \Pi(0, \infty) = \infty. \end{cases}$$

The functions $W^{(q)}(x)$ and $Z^{(q)}(x)$ play a key role in the solution of two-sided exit problem. The following results can be extracted directly out of existing literature. See for example Korolyuk et al. (1976), Bertoin (1997), Avram, Kyprianou and Pistorius (2004), Kuznetsov, Kyprianou and Victor Rivero (2012) in a somewhat different form. Define the first passage times for $a < b$, with the convention $\inf \emptyset = \infty$,

$$T^+_b = \inf\{t \geq 0 : X(t) > b\}, \quad T^-_a = \inf\{t \geq 0 : X(t) < a\}, \quad \tau_{ab} = T^-_a \wedge T^+_b.$$
Then we have for \( x, y \in (a, b), q \geq 0, z \geq b, \)

\[
E_x(e^{-qT^a} \mathbf{1}_{\{T^a < T^b_x\}}) = \frac{W'(q)(b - x)}{W(q)(b - a)}, \tag{2.2}
\]

\[
E_x(e^{-qT^b_x} \mathbf{1}_{\{T^a < T^b\}}) = Z'(q)(b - x) - W'(q)(b - x) \frac{Z(q)(b - a)}{W(q)(b - a)}, \tag{2.3}
\]

\[
E_x(e^{-q_{ab}} \mathbf{1}_{\{X(\tau_{ab}) = b\}}) = \frac{\sigma^2}{2} \left( W'(q)(b - x) - W'(q)(b - x) \frac{W'(q')(b - a)}{W(q)(b - a)} \right), \tag{2.4}
\]

\[
E_x(e^{-q_{ab}} \mathbf{1}_{\{X(\tau_{ab}^-) \leq d, X(\tau_{ab}) \leq dz\}) = u(q)(x, y) \Pi(dz - y)dy, \tag{2.5}
\]

where

\[
u(q)(x, y) = W'(q)(b - x) \frac{W(q)(y - a)}{W(q)(b - a)} - W'(q)(y - x).
\]

The identities (2.2) and (2.3) together with the strong Markov property imply that

\[
e^{-q(t \wedge \tau_{ab})} W'(q)(b - X(t \wedge \tau_{ab})), \quad e^{-q(t \wedge \tau_{ab})} Z'(q)(b - X(t \wedge \tau_{ab}))
\]

and

\[
e^{-q(t \wedge \tau_{ab})} \left( Z'(q)(b - X(t \wedge \tau_{ab})) - W'(q)(b - X(t \wedge \tau_{ab})) \frac{Z(q)(b - a)}{W(q)(b - a)} \right)
\]

are martingales.

### 3 Main results

Denoted by \( \pi_b = \{L^b_t, t \leq \tau^b\} \) the constant barrier strategy at level \( b \) and let \( U_b = \{U_b(t) : t \geq 0\} \) be the corresponding risk process, where \( U_b(t) = X(t) - D_b(t), \) with \( L^b_{0-} = 0 \) and \( L^b_{t} = (b \vee \sup_{0 \leq s \leq t} X(s)) - b. \) Note that \( U_b(t) \) is a spectrally positive Lévy process reflected at \( b, \pi_b \in \Xi \) and \( L^b_0 = x - b \) if \( X(0) = x > b. \) Denote by \( V_b(x) \) the expected discounted value of dividend payments, that is,

\[
V_b(x) = E_x \left[ \int_0^{T^b} e^{-q_t} dL^b_t + S e^{-q_{T^b}} \right], 0 \leq x \leq b,
\]

where \( T^b = \inf\{t > 0 : U_b(t) = 0\} \) with \( T^b = \infty \) if \( U_b(t) > 0 \) for all \( t \geq 0. \) Here \( q > 0 \) is the discount factor and \( S \in \mathbb{R} \) is the terminal value.
Denote by $\Gamma$ the extended generator of the process $X$, which acts on $C^2$ function $g$ defined by

$$A g(x) = \frac{1}{2} \sigma^2 g''(x) - cg'(x) + \int_0^\infty [g(x + y) - g(x) - g'(x)y1_{|y|<1}]\Pi(dy).$$  \hfill (3.1)

Theorem 3.1. Let $S = 0$. Assume that $V_b(x)$ is bounded and twice continuously differentiable on $(0, b)$ with a bounded first derivative. Then $V_b(x)$ satisfies the following integro-differential equation

$$\mathcal{A} V_b(x) = qV_b(x), \quad 0 < x < b,$$

**Proof** Similar to the case of jump-diffusion (cf. Yin, Shen and Wen (2013)), applying Itô’s formula for semimartingales one has

$$E_x \left[ e^{-qt\wedge T_b} V_b(U_b(t \wedge T_b)) \right] = V_b(x)$$

$$+ E_x \int_0^{t \wedge T_b} e^{-qs} [(\mathcal{A} - q)V_b(U_b(s))] ds - E_x \left[ \int_0^{t \wedge T_b} e^{-qt} dL^b_t \right].$$

Letting $t \to \infty$ and note that $V_b(0) = 0$ we have

$$V_b(x) = E_x \left[ \int_0^{T_b} e^{-qt} dL^b_t \right].$$

This ends the proof.

Lemma 3.1. For $b, q \geq 0$ and $0 \leq x \leq b$, we have

$$E_x \left[ e^{-qT_b} \right] = \frac{Z^{(q)}(b - x)}{Z^{(q)}(b)}.$$  \hfill (3.2)

**Proof** Let $Y_b(t) = b - U_b(t)$, then $Y_b$ is a reflected Lévy process with initial value $b - x$. Define $T_b = \inf\{t > 0 : Y_b(t) \geq b\}$, then

$$E \left[ e^{-qT_b} | U_b(0) = x \right] = E \left[ e^{-qT_b} | Y_b(0) = b - x \right] = \frac{Z^{(q)}(b - x)}{Z^{(q)}(b)},$$

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where in the last step we have used the result of Proposition 2 (i) in Pistorius (2004), see also Theorem 2.8 (i) in Kuznetsov et al (2012). This ends the proof.

The following result due to Bayraktar, Kyprianou and Yamazaki (2013), here we give a different proof.

**Lemma 3.2.** For \( b, q \geq 0 \) and \( 0 \leq x \leq b \), define
\[
V(x, b) = E_x \left[ \int_0^{T_b} e^{-qt} dL_t^b \right],
\]
then we have
\[
V(x, b) = \frac{Z^{(q)}(b)}{Z^{(q)}(b - x)} Z^{(q)}(b - x) - \frac{Z^{(q)}(b - x)}{q} \left( \frac{Z^{(q)}(b - x)}{Z^{(q)}(b)} - 1 \right). \tag{3.3}
\]

**Proof** By the law of total probability and the strong Markov property as in Yin et al (2013), we have
\[
V(b, x) = h_1(x)V(b, b) + h_2(x), \tag{3.4}
\]
where
\[
h_1(x) = E_x \left( e^{-qT_b^+} 1_{\{T_b^+ < T_0^-\}} \right),
\]
and
\[
h_2(x) = E_x \left( e^{-qT_b^+} (X(T_b^+) - b) 1_{\{T_b^+ < T_0^-\}} \right).
\]
By (2.3),
\[
h_1(x) = Z^{(q)}(b - x) - W^{(q)}(b - x) \frac{Z^{(q)}(b)}{W^{(q)}(b)}. \tag{3.5}
\]
By (2.5),
\[
E_x \left( e^{-qT_b^+} X(T_b^+) 1_{\{T_b^+ < T_0^-\}} \right) = \int_{y=0}^{b} \int_{z=b}^{\infty} z u^{(q)}(x, y) \Pi(dz - y) dy
\]
\[
\quad \equiv I_1(x) - I_2(x), \tag{3.6}
\]
where
\[
I_1(x) = \int_{y=0}^{b} \int_{z=b}^{\infty} \frac{W^{(q)}(b - x)}{W^{(q)}(b)} W^{(q)}(y) z \Pi(dz - y) dy
\]
\[
= \frac{bZ^{(q)}(b)}{W^{(q)}(b)} W^{(q)}(b - x) + bcW^{(q)}(b - x)
\]
\[
+ \frac{W^{(q)}(b - x)}{W^{(q)}(b)} \left( \frac{Z^{(q)}(b)}{q} - \Psi'(0+) \right) \frac{Z^{(q)}(b)}{Z^{(q)}(b)} + \frac{\Psi'(0+)}{q}. \tag{3.7}
\]
\[ I_2(x) = \int_{y=0}^{b} \int_{z=b}^{\infty} W^{(q)}(y-x) z \Pi (dz - y) dy \]
\[ = -bZ^{(q)}(b-x) + bW^{(q)}(b-x) \]
\[ + Z^{(q)}(b-x) - \Psi'(0+) \frac{Z^{(q)}(b-x)}{q}, \quad (3.8) \]

Substituting (3.7) and (3.8) into (3.6) we get
\[ E_x \left( e^{-qT^+_b} X(T^+_b) 1_{(T^+_b < T^-_0)} \right) = \frac{W^{(q)}(b-x)}{W^{(q)}(b)} \left( Z^{(q)}(b) - \Psi'(0+) \overline{W}^{(q)}(b) - bZ^{(q)}(b) \right) \]
\[ - Z^{(q)}(b-x) + \Psi'(0+) \overline{W}^{(q)}(b-x) + bZ^{(q)}(b-x), \]
from which and (3.5) we arrive at
\[ h_2(x) = \frac{W^{(q)}(b-x)}{W^{(q)}(b)} \left( Z^{(q)}(b) - \Psi'(0+) \overline{W}^{(q)}(b) \right) \]
\[ - Z^{(q)}(b-x) + \Psi'(0+) \overline{W}^{(q)}(b-x). \quad (3.9) \]

Substituting (3.5) and (3.9) into (3.4) and using the boundary condition \( V'(b, b) = 1 \), we get
\[ V(b, b) = \frac{Z^{(q)}(b)}{Z^{(q)}(b)} + \frac{\Psi'(0+)}{qZ^{(q)}(b)} - \frac{\Psi'(0+)}{q}, \]
and the result follows.

From Lemmas 3.1 and 3.2 we have

**Theorem 3.2.** The expected discounted value of dividend payments of the barrier strategy at level \( b \geq 0 \) is given by
\[ V_b(x) = \begin{cases} \Lambda(b)Z^{(q)}(b-x) - Z^{(q)}(b-x) - \frac{\Psi'(0+)}{q}, & \text{if } 0 \leq x \leq b, \\ x - b + \Lambda(b) - \frac{\Psi'(0+)}{q}, & \text{if } x > b, \end{cases} \quad (3.10) \]

where
\[ \Lambda(b) = \left( Z^{(q)}(b) + \frac{\Psi'(0+)}{q} + S \right) \frac{1}{Z^{(q)}(b)}. \]

By differentiating (3.10), we obtain that
\[ V'_b(x) = \begin{cases} -q\Lambda(b)W^{(q)}(b-x) + Z^{(q)}(b-x), & \text{if } 0 < x \leq b, \\ 1, & \text{if } x > b. \end{cases} \quad (3.11) \]
It follows that $V_b'(b) = 1$ if and only if $\Lambda(b) = 0$, or, equivalently $\bar{Z}^{(q)}(b) = -\frac{\Psi'(0^+)}{q} - S$. We denote our candidate barrier level by

$$b^* = \begin{cases} 
(Z^{(q)})^{-1}(-\frac{\Psi'(0^+)}{q} - S), & \text{if } -\frac{\Psi'(0^+)}{q} - S > 0, \\
0, & \text{if } -\frac{\Psi'(0^+)}{q} - S \leq 0.
\end{cases}$$

(3.12)

The expected discounted value of dividend payments of the barrier strategy at level $b^*$ is given by

$$V_{b^*}(x) = \begin{cases} 
-\bar{Z}^{(q)}(b^* - x) - \frac{\Psi'(0^+)}{q}, & \text{if } -\frac{\Psi'(0^+)}{q} - S > 0, \\
x + S, & \text{if } -\frac{\Psi'(0^+)}{q} - S \leq 0,
\end{cases}$$

(3.13)

for any $x \geq 0$.

**Remark 3.1.** Letting $S \to 0$ in (3.12) and (3.13), the results deduced to (2.12) and (2.14) in Bayraktar, Kyprianou and Yamazaki (2013).

From the result Theorem 2.1 in Bayraktar, Kyprianou and Yamazaki (2013), we have

**Theorem 3.3.** Consider the stochastic control problem (1.3). Then the barrier strategy at $b^*$ is an optimal strategy for the control problem and $V(x) = V(x, b^*)$ as defined in (3.13).

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