**Time-series image denoising of pressure-sensitive paint data by projected multivariate singular spectrum analysis**

Yuya Ohmichi1 · Kohmi Takahashi2 · Kazuyuki Nakakita1

Received: 21 July 2022 / Revised: 24 September 2022 / Accepted: 16 October 2022 / Published online: 10 November 2022
© The Author(s) 2022

**Abstract**

Time-series data, such as unsteady pressure-sensitive paint (PSP) measurement data, may contain a significant amount of random noise. Thus, in this study, we investigated a noise-reduction method that combines multivariate singular spectrum analysis (MSSA) with low-dimensional data representation. MSSA is a state-space reconstruction technique that utilizes time-delay embedding, and the low-dimensional representation is achieved by projecting data onto the singular value decomposition (SVD) basis. The noise-reduction performance of the proposed method for unsteady PSP data, i.e., the projected MSSA, is compared with that of the truncated SVD method, one of the most employed noise-reduction methods. The result shows that the projected MSSA exhibits better performance in reducing random noise than the truncated SVD method. Additionally, in contrast to that of the truncated SVD method, the performance of the projected MSSA is less sensitive to the truncation rank. The projected MSSA achieves denoising effectively by extracting smooth trajectories in a state space from noisy input data. Expectedly, the projected MSSA will be effective for reducing random noise in not only PSP measurement data, but also various high-dimensional time-series data.

**Graphic abstract**

---

**1 Introduction**

Understanding the nature of physical phenomena via high-dimensional time-series images is a fundamental approach employed in scientific research. In the field of fluid dynamics, the spatiotemporal distribution of pressure and velocity fields in a flow field can be measured using methods such as pressure-sensitive paint (PSP) (Liu et al. 2021) and...
particle-image velocimetry (Westerweel et al. 2013), for example. However, since experimental data usually contain unwanted signals, i.e., noise, it is necessary to properly separate the noise from the signals representing the physical phenomena.

The PSP measurement, which is the subject of this study, is an optical measurement technique that has been applied to a variety of flow fields, from low to hypersonic speeds (Nakakita 2011; Peng and Liu 2019; Running et al. 2019), because of its ability to measure pressure with high temporal and spatial resolution. However, when the pressure fluctuation of the observation target is small, the PSP image is significantly affected by shot noise (Jiao et al. 2019; Varigonda and Narayanaswamy 2021; Liu et al. 2022). Shot noise is noise that is independent of each pixel, and it generates random statistical fluctuations that follow the Poisson distribution. Generally, increasing the light intensity is one method for reducing shot noise. However, it may be impossible to obtain signals with sufficient intensity from the unsteady PSP measurement (called fast PSP) because the exposure time is insufficient. A common technique for reducing shot noise is spatial averaging (Panda 2017; Tang et al. 2021). However, this technique does not sufficiently reduce the noise in some cases, and it reduces the spatial resolution.

Recently, noise-reduction methods based on data-analysis techniques, such as modal decomposition and sparse sensing, have been explored. The singular value decomposition (SVD), also called proper orthogonal decomposition (POD) (Lumley 1967; Berkooz et al. 1993) or principal component analysis (PCA), is often used to PSP images to extract coherent patterns or reduce noise (Pastuhoff et al. 2013; Gordeyev et al. 2014; Peng et al. 2016; Sugioka et al. 2019; Zhou et al. 2022). The SVD is employed with a reduced-rank approximation (truncated SVD) and noise is removed by reconstructing the PSP images using only modes that represent signal components. One of the difficulties with the truncated SVD is extracting only the signal components, which is achieved by setting the appropriate truncation rank. The choice of the appropriate truncation rank is nontrivial, and several criteria based on the cumulative sum of singular values (Pastuhoff et al. 2013), the signal periodicity (Peng et al. 2016), or reference pressure signals (Sugioka et al. 2019) have been used. Recently, Gavish and Donoho (2014) proposed the optimal truncation rank based on asymptotic analysis. However, the appropriate truncation rank is problem-dependent, and it is difficult to determine the truncation rank optimally based on a single criterion. In addition, these methods do not modify the SVD temporal coefficients that represent the temporal behavior of the modes, even though the coefficients are also affected by noise. Modifying the temporal coefficients is expected to achieve further noise reduction and suppress the truncation rank dependence of noise-reduction performance. Wen et al. (2018) proposed a data fusion algorithm that optimizes the temporal coefficients using a limited number of scattered clean data. Noda et al. (2018) employed the coherent output power between PSP and sound-level meter signals to extract pressure-distribution patterns relevant to the target periodic acoustic phenomena. These methods can make effective use of the temporal behavior information of target phenomena to reduce noise but cannot be applied to PSP data alone. Recently, Inoue et al. (2021) proposed a method that combines SVD with sparse sensing that can modify the SVD temporal coefficients using PSP data alone.

Dynamic mode decomposition (DMD) has also been actively studied as a data-analysis method similar to SVD (Schmid 2010; Taira et al. 2017). With the DMD method, one can simultaneously obtain spatial and temporal patterns by assuming a linear time evolution. Ali et al. (2016) applied DMD to PSP images and extracted pressure-fluctuation patterns as DMD modes. To apply the DMD to noisy data, several algorithms that reduce the effect of noise have been proposed (Hemati et al. 2017; Nonomura et al. 2019; Ohmichi et al. 2022). Ohmichi et al. (2022) applied the DMD algorithm for noisy data to fast PSP images of transonic flow over a swept wing (Ohmichi et al. 2018; Sugioka et al. 2021), and successfully extracted the pressure distribution related to transonic buffet phenomenon.

In fluid dynamics research, the time evolution of SVD temporal coefficients is often analyzed to understand the nature of various phenomena. The phase plot of the coefficients, which represents the relationship between different SVD modes, shows a smooth trajectory (Sieber et al. 2016; Edgington-Mitchell et al. 2018). These researches inspired us to explore a noise-processing method that combines the low-dimensional representation of data on modal bases with multivariate singular spectrum analysis (MSSA), which is a multivariate extension of basic SSA (Broomhead and King 1986; Hassani and Thomakos 2010). Basic SSA is employed for the state-space reconstruction of time-series signals using time-delay embedding, which is often employed for noise processing (Golyandina 2010). By combining MSSA and data representation using the temporal coefficients and extracting smooth trajectories in the state space that represents the temporal behavior of physical phenomena, we expect to achieve effective noise reduction for high-dimensional time-series data. Notably, a similar approach has been proposed by Chhatkuli et al. (2015) in medical-imaging research. They conducted a time-series forecasting for medical imaging using the MSSA combined with PCA coefficients, which are identical to the temporal coefficients of SVD. In this study, we utilized a similar approach to reduce random noise in high-dimensional time-series data.

The objective of this study is to clarify the effectiveness of a noise-reduction method that combines MSSA with low-dimensional data representation via projection onto the
SVD basis. For brevity, this method is referred to as the projected MSSA (PMSSA) in this paper. The projected MSSA is applied to two test cases. One is the pressure field data of the Kármán vortex street behind a square cylinder obtained by a numerical fluid simulation, and the other is the practical unsteady PSP measurement data relevant to a tone trailing-edge (TE) noise of a two-dimensional airfoil. The noise-reduction performance of the projected MSSA is compared with that of the truncated SVD (TSVD), which is one of the most common noise-reduction methods. Additionally, the effects of the truncation rank used in the projected MSSA and the truncated SVD on the noise-reduction performance are discussed.

2 Approach

2.1 Input dataset

The input dataset comprises high-dimensional time-series data, such as time-series spatial-distribution data. We assume that each instantaneous data, denoted as a snapshot in this paper, has $d$ elements ($d$ is the number of pixels of the images) and $j$-th snapshot $x(j)$ can be represented as $x(j) = [x_{1,j}, \ldots, x_{d,j}]^T$, where $j = 1, \ldots, m$ and $m$ is the number of snapshots. Thus, the input dataset can be represented as the matrix $X$ whose columns are $x(j)$.

\[
X = [x(1) \ldots x(m)] = \begin{bmatrix} x_{1,1} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots \\ x_{d,1} & \cdots & x_{d,m} \end{bmatrix}. \tag{1}
\]

That is, $X$ is a $d \times m$ matrix, where the row and column indices correspond to the spatial and temporal distributions, respectively.

2.2 Noise reduction by truncated SVD

The input data matrix $X$ can be factorized by SVD to

\[
X = USV^T = \sum_{i=1}^{r_X} u_i \sigma_i v_i^T, \tag{2}
\]

where $U$ and $V$ are the orthogonal matrices, and $S$ is the diagonal matrix containing the singular values. The left and right singular vectors, $u_i$ and $v_i$, are the $i$-th columns of $U$ and $V$, respectively; $\sigma_i$ is the $i$-th singular value, and $r_X$ is the rank of $X$. The singular values are arranged in descending order. The $u_i$ and $v_i$ represent a spatial mode and its temporal behavior, respectively. If the input dataset $X$ is extremely large, hindering the application of the batch SVD algorithm, online algorithms (Ross et al. 2008; Halko et al. 2011; Ohmuchi 2017) can be applied.

In the noise-reduction method by the truncated SVD, we assume that the noise can be separated as the component corresponding to small singular values ($\sigma_i < \sigma_r$).

\[
X = \sum_{i=1}^{r} u_i \sigma_i v_i^T + \sum_{i=r+1}^{r_X} u_i \sigma_i v_i^T. \tag{3}
\]

The first and second terms in the right-hand side of Eq. 3 represent the signal and noise components, respectively. That is, the noise-reduced data $X_{\text{svd}}$ are obtained as

\[
X_{\text{svd}} = U_r S_r V_r^T = \sum_{i=1}^{r} u_i \sigma_i v_i^T. \tag{4}
\]

The truncation rank $r$ is often selected based on the cumulative sum of singular values or prior information, such as the temporal periodicity.

Note that the SVD is identical to the POD method; however, in the POD method, mean subtraction from the input dataset is essential.

2.3 Noise reduction by the projected MSSA

In this study, we examine the noise-reduction performance of the projected MSSA, which is composed of the MSSA and the low-dimensional representation using the SVD basis.

2.3.1 Step 1: Projecting data onto the subspace

Using SVD, the input data $X$ can be projected onto the subspace spanned by the first $r$ singular vectors $U_r$:

\[
\tilde{X} = U_r^T X = S_r V_r^T. \tag{5}
\]

The $\tilde{X}$ represents the low-dimensional representation of the input data and is a $r \times m$ matrix. We denote the $i$-th row and element of $\tilde{X}$ as $\tilde{x}_i$ and $\tilde{x}_{ij}$, respectively.

\[
\tilde{X} = \begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_r \end{bmatrix} = \begin{bmatrix} \tilde{x}_{1,1} & \cdots & \tilde{x}_{1,m} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{r,1} & \cdots & \tilde{x}_{r,m} \end{bmatrix}. \tag{6}
\]

As shown in Eqs. 4 and 5, $\tilde{x}_i$ (denoted as the temporal coefficient hereinafter) represents the temporal change in the expansion coefficients for $u_j$. For example, if $r = r_X$, the $j$-th snapshot can be expressed using $\tilde{x}_{ij}$ as

\[
x(j) = \tilde{x}_{j,1}u_1 + \tilde{x}_{j,2}u_2 + \cdots + \tilde{x}_{j,r}u_r. \tag{7}
\]

Thereafter, the noise reduction is implemented by applying MSSA to $\tilde{X}$. In other words, this algorithm applies MSSA to the SVD-based denoised data via projected low-dimensional representation. The MSSA consists of the following steps.
2.3.2 Step 2: Embedding

The SSA reduces noise by extracting smooth trajectories in a state space. The state-space reconstruction is performed by time-delay embedding as follows. We define the trajectory matrix \( \hat{T}_i \) of \( \tilde{x}_i \) as

\[
\hat{T}_i = \begin{bmatrix}
\tilde{x}_{i,1} & \tilde{x}_{i,2} & \cdots & \tilde{x}_{i,K} \\
\tilde{x}_{i,2} & \tilde{x}_{i,3} & \cdots & \tilde{x}_{i,K+1} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{x}_{i,L} & \tilde{x}_{i,L+1} & \cdots & \tilde{x}_{i,L+m}
\end{bmatrix}.
\]

(8)

Here, \( L \) and \( K \) represent the window length and the window width, respectively, and \( K = m - L + 1 \). Each column of \( \hat{T}_i \) is a partial time series with the length of \( L \) extracted from the time series \( \tilde{x}_i \) and represents a state-space vector converted from \( \tilde{x}_i \) by time-delay embedding. \( \hat{T}_i \) has the same elements along the antidiagonals. The multivariate trajectory matrix \( \hat{T} \) is constructed by stacking each trajectory matrix \( \hat{T}_i \):

\[
\hat{T} = \begin{bmatrix}
\hat{T}_1 \\
\vdots \\
\hat{T}_r
\end{bmatrix}.
\]

(9)

\( \hat{T} \) is a \( rL \times K \) matrix.

2.3.3 Step 3: SVD and grouping

In general SSA procedures, the trajectory matrix undergoes SVD, and the decomposed components are grouped by some criteria. Here, the grouping is performed following the same procedure employed for noise reduction using the truncated SVD method (Sect. 2.2). That is, we decompose the trajectory matrix \( \hat{T} \) by SVD as \( \hat{T} = PDQ^T \), and assume that the noise components can be separated as the components corresponding to small singular values. Then, the noise-reduced trajectory can be extracted as

\[
\hat{\hat{T}} = \begin{bmatrix}
\hat{\hat{T}}_1 \\
\vdots \\
\hat{\hat{T}}_r
\end{bmatrix} = P_{\text{mssa}}D_{\text{mssa}}Q_{\text{mssa}}^T.
\]

(10)

Here, we set the truncation rank \( r_{\text{mssa}} \) to \( r_{\text{mssa}} = r \).

2.3.4 Step 4: Diagonal averaging

Since the trajectory matrix has elements corresponding to temporal coefficients of the same snapshot along the antidiagonals (Eq. 8), the noise-reduced trajectory matrix \( \hat{T}_i \) can be transformed into the temporal coefficient \( \hat{x}_i \) by obtaining the average along the antidiagonals:

\[
\hat{x}_{i,j} = \sum_{(a,b)\in A_j} \frac{(\hat{T}_{i})_{ab}}{n(A_j)},
\]

and

\[
\hat{\hat{x}} = \begin{bmatrix}
\hat{x}_1 \\
\vdots \\
\hat{x}_r
\end{bmatrix} = \begin{bmatrix}
\hat{x}_{1,1} & \cdots & \hat{x}_{1,m} \\
\vdots & \ddots & \vdots \\
\hat{x}_{r,1} & \cdots & \hat{x}_{r,m}
\end{bmatrix}.
\]

(12)

Here, \( (\hat{T}_{i})_{ab} \) is the element in the \( a \)-th row and \( b \)-th column of \( \hat{T}_i \). \( A_j \) represents a set of indices \( (a, b) \), comprising the antidiagonals of \( \hat{T}_i \), i.e., \( A_j = \{(a, b) : a + b - 1 = j, 1 \leq a \leq L, 1 \leq b \leq K \} \). \( n(A_j) \) is the number of elements in \( A_j \).

2.3.5 Step 5: Reconstructing the noise-reduced data

Finally, the noise-reduced time-series data \( X_{\text{mssa}} \) can be obtained in a manner similar to Eq. 7 as

\[
X_{\text{mssa}} = U_{\text{r}} \hat{\hat{x}}.
\]

(13)

3 Numerical experiments using fluid-simulation data

3.1 Input data

First, the noise-reduction performances of TSVD and PMSSA were evaluated for the pressure field of a Kármán vortex street behind a square cylinder obtained by spatial two-dimensional fluid simulation. The Mach number of the flow field was 0.2, and the Reynolds number was 120, defined using the length \( l \) of the side of the square cylinder and the velocity \( U_\infty \) of the uniform flow as reference values. For details on this fluid simulation, please refer to the previous study (Ohmichi et al. 2022). The pressure-coefficient distribution of the wake region, i.e., \( x/l = [0, 10] \) and \( y/l = [-5, 5] \), sampled into equally spaced \( 101 \times 101 \) grid points was used as the input dataset. The center of the cylinder was located at the origin \( (x/l, y/l) = (0, 0) \). The number of snapshots was \( m = 1000 \), and the time interval between the snapshots was \( \Delta t = 0.125l/U_\infty \). The shot noise could be modeled as random normal noise. To investigate the noise-reduction performance, random normal noise with a standard deviation \( \sigma \) was added to each snapshot as the observation noise.

3.2 Results and discussion

Figure 1 shows the noise-reduced images by TSVD and PMSSA. The top row shows the input-pressure distribution...
with noise values of $\sigma = 0$, 0.08, and 0.64. The middle and bottom rows show the results of the noise reduction for the input data with $\sigma = 0.08$ and 0.64, respectively. The window length in PMSSA was set to $L = 90$, and the truncation ranks were set to $r = 5$ and 11. Figure 1 indicates that when the noise was small $\sigma = 0.08$, the differences between TSVD and PMSSA and the different $r$ results were visually small. The result of PMSSA had a slightly smoother distribution at high noise level $\sigma = 0.64$ than that of TSVD.

To quantitatively evaluate the noise-reduction performance, we define the relative error as follows:

$$Relative\ error = \frac{\|X_{\text{rec}} - X_0\|_F}{\|X_0\|_F}. \quad (14)$$

$X_{\text{rec}}$ represents the data denoised by TSVD and PMSSA (i.e., $X_{\text{svd}}$ and $X_{\text{mssa}}$), and $X_0$ is the original data without noise ($\sigma = 0$).

The effect of the window length $L$ on the relative error in PMSSA is shown in Fig. 2. The appropriate value of $L$...
may depend on the nature of the signal and noise; however, Golyandina (2010) reported that \( L = m/2 \) is a good choice in many cases for the basic SSA. Figure 2 shows that \( L = m/2 \) exhibited a small error here as well. However, in the interval of \( 30 \lt L \lt 500 \approx m/2 \), the difference in error was relatively small. The size of the trajectory matrix is \( rL \times (m - L + 1) \); thus, the smaller the \( L \), the lower the computational cost. An empirical guideline for achieving the appropriate \( L \) involves choosing a value in the order of \( \sqrt{m} \). In this problem, we set \( L = 90 \approx 3\sqrt{m} \).

Figure 3 shows the change in error for different noise levels at each truncation rank. For TSVD, the difference in the truncation rank had a significant impact on the error. The error was the smallest at \( r = 5 \) at high noise levels. At low noise levels, the errors were smaller at \( r = 11 \) and \( r = 21 \) than that of \( r = 5 \) because even the signal component was truncated. PMSSA is less sensitive to the truncation rank, and a larger truncation rank than that for TSVD can be applied even at high noise levels. When \( r = 11 \) and 21, small errors were achieved at all noise levels. At sufficiently low noise levels, the errors of TSVD and PMSSA were comparable.

Figure 4 shows the spatial modes \( u_i \) extracted by SVD at each noise level. The first mode \( u_1 \) represents the mean field, while the second and third modes represent the pressure patterns indicating Kármán vortex shedding. The fourth/fifth and sixth/seventh mode pairs represent the second and third harmonics of the Kármán vortex mode, respectively, while the sixth and seventh modes at \( \sigma = 0.64 \) did not exhibit coherent patterns because the noise level was extremely high.

Next, we examined the reproducibility of the time-series change. Figure 5 shows the frequency distribution for the pressure at the point, \((x/l, y/l) = (3.0, 0.5)\), behind the square cylinder. For each truncation rank, the frequency distributions for different noise levels are compared. Figure 6 shows the comparison in the time domain. From these figures, we can see that PMSSA was superior to TSVD in removing the noise components under all conditions. Figure 5 shows that TSVD could extract periodic signals, although the noise remained at frequencies different from those of the periodic signal components. Conversely, PMSSA reduced noise over a wide frequency range while retaining the periodic signal.

Fig. 3 Effect of the truncation rank \( r \) on the relative error in TSVD and PMSSA

Fig. 4 Spatial modes \( u_i \) extracted by SVD at different noise levels
representing a Kármán vortex shedding phenomenon. This noise-reduction performance of PMSSA was not significantly affected by the truncation rank whereas the remaining noise increased with the truncation rank for TSVD. Figure 5a and b shows that at low noise levels ($\sigma = 0.08$) and with an overly small truncation rank ($r = 5$), even the signal components, which appeared as harmonic peaks at $r = 11$, were truncated. At high noise levels, this harmonic peak was not captured even at $r = 11$ because the amplitude of the harmonic signal was smaller than the noise floor. This is a limitation of SVD-based methods: since both PMSSA and TSVD reconstruct the signal using SVD modes, if the SVD does not capture a target phenomenon, these methods cannot reproduce the phenomenon.

Finally, we compared the temporal coefficients $\hat{x}_i$ and $\hat{x}_j$ (Eqs. 6 and 12). As explained in Sect. 2.3, these are the results of the noise reduction by TSVD and PMSSA, expressed as the expansion coefficient for $u_i$. Figure 7a shows the time history of the coefficient for $u_1$, which is the mean mode. Evidently, the coefficient for the mean mode fluctuated significantly in the TSVD, while the fluctuation was suppressed in the PMSSA. Figures 7b and c show the phase plots for the coefficients of $u_2$, $u_3$, and $u_4$, respectively. These pairs represent the Kármán vortex and its second-harmonic oscillation, respectively. The figures clearly show that while PMSSA was able to extract smooth trajectories representing the relationship between the modes, the TSVD result retained a significant amount of noise in the trajectories. These results indicate that PMSSA was superior in extracting the smooth trajectories of physical signals from noise-contaminated signals.

4 Demonstration using PSP data

4.1 Input data

As a demonstration using practical experimental data, we applied the PMSSA and TSVD to PSP data containing large noise. The analyzed data were the time-series measurement data of the wing surface pressure distribution for the tone TE noise phenomenon on the NACA0012 airfoil (Nakakita...
The pressure distribution was measured using anodized aluminum PSP, which is one of the fast PSPs. The low-turbulence calibration wind tunnel at JAXA Chofu Aerospace Center was employed for this experiment. Figure 8 shows the experimental model and the measured pressure-distribution image. The main flow velocity was 28.0 m/s, the angle of attack was $-1.5^\circ$, and the pressure measurement area was a circular region on the pressure side of the wing (indicated by an arrow in Fig. 8a). The sampling rate of the pressure-distribution image was 10 kHz, and the signals below 300 Hz were removed by a frequency filter to eliminate the dominant low-frequency noise, which was not random noise. A $2 \times 2$ spatial averaging was performed, and the number of pixels after the spatial averaging was ...
The number of snapshots was $m = 10000$. Figure 8b shows the pressure-distribution image after applying the frequency filter and spatial averaging. Since the shot noise was large compared to the physical signal, coherent patterns could not be observed visually. However, Nakakita (2011, 2013) succeeded in extracting a coherent pressure pattern at approximately 918 Hz relevant to the tone TE noise by cross-spectrum analysis from this data (shown in Fig. 8c). In the present analysis, we apply PMSSA to this data to demonstrate its effectiveness in reducing shot noise. For details on this experiment, please refer to Nakakita (2011, 2013).

### 4.2 Results and discussion

First, we consider the results of the mode decomposition by SVD. Figure 9 shows the spatial mode $u_i$, obtained by SVD on the upper row and the frequency distribution of the temporal coefficient $\tilde{x}_i$ corresponding to each mode on the lower row. The most dominant mode, $u_1$, had a noisy spatial distribution with a slightly coherent structure. The frequency with the largest amplitude was 916 Hz, which corresponds to the fundamental frequency of the TE noise; however, the amplitude was similar in all frequency ranges, indicating that this mode behaved almost randomly in time. Conversely, the second and third modes ($u_2$ and $u_3$) exhibited a clear wavy pattern, and the frequency distribution indicated that they displayed periodic oscillations at 916 Hz. The fourth mode ($u_4$) exhibited a similar wavy pattern with a peak frequency of 916 Hz, although different spatial patterns from the second and third modes were present. In the frequency distribution of this mode, the amplitude of $O(100)$ Hz was slightly higher, and the different patterns shown in the spatial distribution could be considered to have $O(100)$ Hz fluctuations. Although not shown in the figure, the fifth and subsequent modes did not have a clear peak frequency.

Based on the above discussion of the SVD mode, the truncation ranks for TSVD and PMSSA were set to $r = 4$. 

![Fig. 9](image) Spatial modes $u_i$ extracted by SVD (upper row), and the frequency distributions of their temporal coefficients $\tilde{x}_i$ (lower row)

---

*Fig. 8 a NACA0012 airfoil model and measurement area for the PSP measurement. b Instantaneous pressure-distribution image contaminated by shot noise. c Coherence map of the 918 Hz signal taken by Nakakita (2013)*
to capture the 916 Hz fluctuation pattern. In PMSSA, the results at \( r = 11 \) were also compared to examine the effect of the truncation rank. The window length was set to \( L = 3\sqrt{m} = 300 \).

Figure 10 shows the pressure-distribution image after the noise removal by the TSVD and PMSSA methods. In the image before denoising (Fig. 8b), shot noise appeared in high levels, and coherent patterns were not observed. Contrarily, after denoising by TSVD and PMSSA, in the image shown in Fig. 10, wavy patterns corresponding to the periodic fluctuation of 916 Hz appeared. However, in the TSVD results, the pressure values of the entire region fluctuated significantly, which is attributed to the random-noise variation of \( u_1 \) shown in Fig. 9. In PMSSA, only the temporal change in the wavy pattern was clearly extracted. It was also observed that PMSSA was less sensitive to the truncation rank \( r \), as it successfully removed the noise even when \( r = 11 \), which included an excessive number of modes to extract the 916 Hz oscillation.

In Fig. 11, the comparison of the pressure fluctuations at the position indicated by the arrow in Fig. 8b is shown. The result of the pressure measurement by the Kulite pressure transducer is also shown for comparison. From the frequency distribution shown in Fig. 11a, it is

---

**Fig. 10** Reconstructed (i.e., denoised) pressure-distribution images obtained by TSVD \((r = 4)\), PMSSA \((r = 4)\), and PMSSA \((r = 11)\)

**Fig. 11** Comparisons of the denoised pressure signals and pressure transducer signal in the a frequency and b time domain
evident that the peak amplitudes at 916 Hz for both methods of TSVD and PMSSA were comparable to that by the pressure transducer. However, for the TSVD, the noise levels in other frequency ranges were high. PMSSA could significantly reduce the noise in both $r = 4$ and 11. From the comparison of the pressure fluctuations in the time domain in Fig. 11b, it was confirmed that the denoised signal with PMSSA exhibited less noise than that with TSVD, and a smooth temporal signal was extracted. As shown in Fig. 11a, the second-harmonic peak frequency of approximately 1840 Hz was captured by the pressure transducer, but not by TSVD and PMSSA. This is because the amplitude of the pressure fluctuation at 1840 Hz was smaller than that of the shot noise generated in the PSP data, and the mode decomposition by SVD could not extract this phenomenon.

Figure 12 shows a comparison of the temporal coefficients ($\tilde{x}_i$ and $\hat{x}_i$) obtained by the TSVD and PMSSA methods. The time variation of the temporal coefficient for the first mode ($u_1$) in Fig. 12a clearly shows that the PMSSA reduced the large random fluctuations remaining in the TSVD result. In addition, in Fig. 12b, the phase plot represented by the coefficients for the second and third modes ($u_2$ and $u_3$) clearly shows that the PMSSA extracted a smoother trajectory than TSVD. In other words, PMSSA can identify the modes behaving randomly and modes representing physical phenomena via modifying their trajectories. The effective denoising is achieved by reconstructing PSP data based on the identified physically meaningful SVD modes and their modified trajectories.

5 Conclusion

A numerical method for reducing random noise from high-dimensional time-series data was investigated for denoising unsteady PSP data. Unsteady PSP data are significantly affected by random noise generated by shot noise. Here, we demonstrated the effectiveness of a noise-reduction method, i.e., the projected MSSA. The projected MSSA comprises a combination of MSSA and low-dimensional data representation. For the low-dimensional representation, we projected the data onto the subspace spanned by the SVD basis. The pressure distributions for Kármán vortex shedding obtained by numerical fluid simulations and PSP data on the NACA0012 airfoil relevant to the tone TE noise were analyzed. The results clearly showed several advantages of the projected MSSA. The projected MSSA exhibited better noise-reduction performance than the conventional truncated SVD method in both analyses. In addition, the projected MSSA was less sensitive to the truncation rank than the truncated SVD and exhibited better performance in a wide range of truncation rank values, since the projected MSSA can remove random variations from the SVD temporal coefficients. Furthermore, the projected MSSA does not require prior information such as noise amplitude or pressure tap data as reference signals to identify noise components. These advantages are achieved by extracting smooth trajectories of temporal coefficients in a state space by using time-delay embedding. It should be noted, however, that since the projected MSSA uses the spatial modes from SVD for low-dimensional projection, signals not captured by SVD modes cannot be recovered. The projected MSSA can be combined with mode decomposition methods other than SVD, and the
development of mode decomposition methods with better detection limits is desired.

Acknowledgements The authors thank Hiroki Iwamoto for his technical support regarding the processing of the PSP data.

Author Contributions YO contributed to conceptualization, data curation, funding acquisition, investigation, methodology, software, validation, visualization, writing—original draft, and writing—review and editing; KT contributed to investigation, software, validation, writing—original draft, and writing—review and editing; KN contributed to investigation, resources, and writing—review and editing.

Funding This work was supported in part by the Japan Society for the Promotion of Science (JSPS) KAKENHI (Grant No. 20K14958).

Availability of data and materials The data that support the findings of this study are not publicly available. Data are however available from the authors upon reasonable request and with permission of Japan Aerospace Exploration Agency.

Declarations

Conflict of interest The authors have no competing interests to disclose.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

Ali MY, Pandey A, Gregory JW (2016) Dynamic mode decomposition of fast pressure sensitive paint data. Sensors (Basel, Switzerland) 16(6):862. https://doi.org/10.3390/s16060862

Berkooz G, Holmes P, Lumley JL (1993) The proper orthogonal decomposition in the analysis of turbulent flows. Annu Rev Fluid Mech 25(1):539–575. https://doi.org/10.1146/annurev.fl.25.010193.002543

Broomhead DS, King GP (1986) Extracting qualitative dynamics from experimental data. Physica D 20(2):217–236. https://doi.org/10.1016/0167-2789(86)90031-X

Chhatkuli RB, Demachi K, Miyamoto N et al. (2015) Dynamic image prediction using principal component and multi-channel singular spectral analysis: a feasibility study. Open J Med Imag 05(03):133–142. https://doi.org/10.4236/ojmi.2015.53017

Edgington-Mitchell DM, Weightman JL, Honnery DR (2018) Sound production by shock leakage in supersonic jet screech. In: et al. 2018 AIAA/CEAS aeroacoustics conference. AIAA 2018–3147. https://doi.org/10.2514/6.2018-3147

Gavish M, Donoho DL (2014) The optimal hard threshold for singular values is $4/\sqrt{3}$. IEEE Trans Inf Theory 60(8):5040–5053. https://doi.org/10.1109/TIT.2014.2323359

Golyandina N (2010) On the choice of parameters in singular spectrum analysis and related subspace-based methods. Statis Interface 3(3):259–279. https://doi.org/10.4310/SII.2010.v3.n3.a2

Gordeyev S, De Luca N, Jumper EL et al. (2014) Comparison of unsteady pressure fields on turrets with different surface features using pressure-sensitive paint. Exp Fluids 55(1):1661. https://doi.org/10.1007/s00348-013-1661-9

Halko N, Martinsson PG, Tropp JA (2011) Finding structure with randomness: probabilistic algorithms for constructing approximate matrix decompositions. SIAM Reviews 53:217–288. https://doi.org/10.1137/090771806

Hassani H, Thomakos D (2010) A review on singular spectrum analysis for economic and financial time series. Statis Interface 3:377–397. https://doi.org/10.4310/SII.2010.v3.n3.a11

Hemati MS, Rowley CW, Deem EA et al. (2017) De-biasing the dynamic mode decomposition for applied Koopman spectral analysis of noisy datasets. Theoret Comput Fluid Dyn 31(4):349–368. https://doi.org/10.1007/s00162-017-0432-2

Inoue T, Matsuda Y, Ikami T et al. (2021) Data-driven approach for noise reduction in pressure-sensitive paint data based on modal expansion and time-series data at optimally placed points. Phys Fluids 33(7):077105. https://doi.org/10.1063/5.0049071

Jiao L, Chen Y, Wen X et al. (2019) Resolving vortex-induced pressure fluctuations on a cylinder in rotor wake using fast-responding pressure-sensitive paint. Phys Fluids 31(5):055106. https://doi.org/10.1063/1.5092944

Liu T, Sullivan J, Asai K et al. (2021) Pressure and temperature sensitive paints. Springer International Publishing, Cham, Switzerland. https://doi.org/10.1007/978-3-030-68056-5

Liu X, Qin C, Tang Y et al. (2022) Resolving dynamic features of kilohertz pressure fluctuations using fast-responding pressure-sensitive paint: Measurement of inclined jet impingement. Exp Fluids 63(4):72. https://doi.org/10.1007/s00162-022-04319-4

Lumley JL (1967) The Structure of Inhomogeneous Turbulent Flows. In: Yaglom AM, Taratski VI (eds) Atmospheric turbulence and radio propagation. Nauka, Moscow, pp 166–178

Nakakita K (2011) Unsteady pressure measurement on NACA0012 model using global low-speed unsteady PSP technique. In: 41st AIAA fluid dynamics conference and exhibit. AIAA pp 2011-3901, https://doi.org/10.2514/6.2011-3901

Nakakita K (2013) Phase delay and correlation area detection of unsteady pressure field using unsteady PSP measurement. In: AIAA ground testing conference. AIAA pp 2013-3124, https://doi.org/10.2514/6.2013-3124

Noda T, Nakakita K, Wakahara M et al. (2018) Detection of small-amplitude periodic surface pressure fluctuation by pressure-sensitive paint measurements using frequency-domain methods. Exp Fluids 59(6):94. https://doi.org/10.1007/s00348-018-2550-z

Nomura T, Shibata H, Takaki R (2019) Extended-Kalman-filter-based dynamic mode decomposition for simultaneous system identification and denoising. PLOS ONE 14(2):e0209836. https://doi.org/10.1371/journal.pone.0209836

Ohmichi Y (2017) Preconditioned dynamic mode decomposition and mode selection algorithms for large datasets using incremental proper orthogonal decomposition. AIP Advances 7(7):075318. https://doi.org/10.1063/1.4996024

Ohmichi Y, Ishida T, Hashimoto A (2018) Modal decomposition analysis of three-dimensional transonic buffet phenomenon on a swept wing. AIAA J 56(10):3938–3950. https://doi.org/10.2514/1.1056855

Ohmichi Y, Sugioka Y, Nakakita K (2022) Stable dynamic mode decomposition algorithm for noisy pressure-sensitive-paint measurement data. AIAA J 60(3):1–6. https://doi.org/10.2514/1.J061086
Panda J (2017) Experimental verification of buffet calculation procedure using unsteady pressure-sensitive paint. J Aircr 54(5):1791–1801. https://doi.org/10.2514/1.C033917
Pastuhoff M, Yorita D, Asai K et al. (2013) Enhancing the signal-to-noise ratio of pressure sensitive paint data by singular value decomposition. Meas Sci Technol 24(7):075301. https://doi.org/10.1088/0957-0233/24/7/075301
Peng D, Liu Y (2019) Fast pressure-sensitive paint for understanding complex flows: from regular to harsh environments. Exp Fluids 61(1):8. https://doi.org/10.1007/s00348-019-2839-6
Peng D, Wang S, Liu Y (2016) Fast PSP measurements of wall-pressure fluctuation in low-speed flows: improvements using proper orthogonal decomposition. Exp Fluids 57(4):45. https://doi.org/10.1007/s00348-016-2130-z
Ross DA, Lim J, Lin RS et al. (2008) Incremental learning for robust visual tracking. Int J Comput Vision 77(1–3):125–141. https://doi.org/10.1007/s11263-007-0075-7
Running CL, Sakaue H, Juliano TJ (2019) Hypersonic boundary-layer separation detection with pressure-sensitive paint for a cone at high angle of attack. Exp Fluids 60(1):23. https://doi.org/10.1007/s00348-018-2665-2
Schmid PJ (2010) Dynamic mode decomposition of numerical and experimental data. J Fluid Mech 656:5–28. https://doi.org/10.1017/S0022112010001217
Sieber M, Paschereit CO, Oberleithner K (2016) Spectral proper orthogonal decomposition. J Fluid Mech 792:798–828. https://doi.org/10.1017/jfm.2016.103
Sugioka Y, Hiura K, Chen L et al. (2019) Unsteady pressure-sensitive-paint (PSP) measurement in low-speed flow: characteristic mode decomposition and noise floor analysis. Exp Fluids 60(7):108. https://doi.org/10.1007/s00348-019-2755-9
Sugioka Y, Nakakita K, Koike S et al. (2021) Characteristic unsteady pressure field on a civil aircraft wing related to the onset of transonic buffet. Exp Fluids 62(1):20. https://doi.org/10.1007/s00348-020-03118-y
Taira K, Brunton SL, Dawson STM et al. (2017) Modal analysis of fluid flows: an overview. AIAA J 55(12):4013–4041. https://doi.org/10.2514/1.J056060
Tang L, Hand L, Murakami D, et al. (2021) Unsteady pressure-sensitive-paint shot noise reduction. In: AIAA AVIATION 2021 FORUM. AIAA pp 2021-2579. https://doi.org/10.2514/6.2021-2579
Varigonda SV, Narayanaswamy V (2021) Methodology to image the panel surface pressure power spectra in weakly coupled fluid/structure interactions. Exp Fluids 62(11):237. https://doi.org/10.1007/s00348-021-03314-4
Wen X, Liu Y, Li Z et al. (2018) Data mining of a clean signal from highly noisy data based on compressed data fusion: a fast-responding pressure-sensitive paint application. Phys Fluids 30(9):097103. https://doi.org/10.1063/1.5046681
Westerweel J, Elsinga GE, Adrian RJ (2013) Particle image velocimetry for complex and turbulent flows. Annu Rev Fluid Mech 45(1):409–436. https://doi.org/10.1146/annurev-fluid-120710-101204
Zhou L, Wang S, Song J et al. (2022) Study of internal time-resolved flow dynamics of a subsonic fluidic oscillator using fast pressure sensitive paint. Exp Fluids 63(1):17. https://doi.org/10.1007/s00348-021-03370-w

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.