Hyperonic Bulk Viscosity in Neutron Star Mergers

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In this paper we present a computation of bulk viscosity arising from hyperonic processes in matter at densities and temperatures typical of neutron star mergers. To deal with the high temperatures in this environment we go beyond the Fermi surface approximation in our rate calculations and numerically evaluate the full phase space integral. We include processes where quarks move between baryons via meson exchange: these have been largely omitted in previous analyses but provide the dominant contribution to the bulk viscosity. We obtain the dissipation times for harmonic oscillations at the frequencies seen in merger simulations, and find that hyperon bulk viscosity can be highly relevant at densities just below the onset of the first hyperon species and temperatures up to \( T = 5 \text{ MeV} \), with dissipation times as fast as \( \tau_{\text{diss}} \approx 9 \text{ ms} \).

I. INTRODUCTION

The discovery of gravitational waves from a binary neutron star merger in 2017, named GW170817 \cite{Abbott:2017dke}, opened a new window to study dense nuclear and possibly quark matter at high densities and temperatures \cite{Thorne:1987�z, Arts:1989vi, Thorne:1989zc, Prakash:1996jg, Prakash:2000bm, Prakash:2000ug, Prakash:2001ye, Alford:2003yi}. In order to relate the phase structure of dense matter to the astrophysical observations detailed simulations using numerical relativity and relativistic hydrodynamics have to be performed \cite{Bauswein:2018bex, Tout:2019sro, Mroue:2021eop, Rezzolla:2019mgv}, using accurate representations of the relevant material properties. Therefore, it is necessary to improve our understanding of dense matter in merger conditions. Studies of GW170817 \cite{Abbott:2017wbe, Abbott:2017oio} estimate that the central densities of the merging stars were more than two times saturation density (\( n_0 = 0.153 \text{ fm}^{-3} \)). Numerical simulations of the first 20 ms after the initial contact of the stars provide further insight. They suggest that the density reaches several times saturation density and that temperatures can reach tens of MeV \cite{Bauswein:2018bex, Rezzolla:2019mgv}, where some simulations even predict up to \( T \approx 100 \text{ MeV} \) \cite{Mroue:2021eop}. Furthermore, fluid elements undergo strong density oscillations with central frequencies of around 1 kHz \cite{Bauswein:2017vtn, Mroue:2021eop}. This raises the question of which microscopic transport phenomena and dissipation mechanisms are important on the 20 ms neutron star merger timescale. Initial estimates of various transport phenomena in Ref. \cite{Bauswein:2017vtn} showed the potential importance of bulk viscosity in ordinary nuclear matter. Bulk viscosity is a dissipative mechanism, which converts oscillation energy into heat or radiated neutrinos. The magnitude of the bulk viscosity and the equation of state (EOS) of nuclear matter together determine the dissipation time scale on which oscillations are damped. A detailed study in neutrino-transparent matter showed that dissipation times for npeµ\text{-}matter due to direct and modified Urca processes are indeed on a millisecond timescale \cite{Alford:2018ykr, Alford:2019tnc}, whereas in the neutrino-trapped regime, bulk viscosity seems to be negligible \cite{Alford:2019tnc}.

The intriguing prospect for nuclear physics is that other forms of matter might have different bulk viscosity, leading to observable signatures of their presence in the merger. In this paper we focus on hyperonic matter, where several weak, non-leptonic processes can contribute to beta equilibration and hence to bulk viscosity. Although the existence of hyperons in cold, isolated neutron stars is contested (the “hyperon puzzle” \cite{Shapiro:1983du, Shapiro:1998cu}), the higher temperatures and densities reached in the merger render their appearance highly likely. In the past, hyperonic bulk viscosity has been exclusively studied at low (keV range) temperature, often in the context of r-modes \cite{Alford:2006ve, Alford:2007sd, Alford:2009wc, Alford:2009mg, Alford:2010av, Alford:2010gg, Alford:2010us, Alford:2011do, Alford:2011ve, Alford:2011wz, Alford:2013vha, Alford:2015eza, Alford:2015xxb, Alford:2017yjx, Alford:2017qyj, Alford:2017glo, Alford:2018uwa, Alford:2019zse, Alford:2019tnc, Alford:2020shl}. At these temperatures one can use the Fermi surface (FS) approximation since all particles participating in beta equilibration processes are close to their Fermi surfaces. Furthermore, an ultra non-relativistic approach, where the baryon momenta in the matrix element are set to zero, is sometimes adopted \cite{Alford:2006ve, Alford:2009wc} in order to obtain analytic results. In the merger environment, both of these assumptions are invalid and need to be improved on. Additionally, most studies only consider the contact interaction diagram where a W boson is exchanged between baryons. In Refs. \cite{Alford:2010av, Alford:2010us}, it has been shown that, at least at the studied low temperatures, the one meson exchange (OME) contribution, where the W exchange is internal to a hadron, dominates the rates that are relevant to the bulk viscosity. In our treatment of the beta equilibration rate we improve on previous treatments and obtain results that are valid in the merger environment by

(a) taking the OME contributions for all processes into account;
(b) Computing numerically the full twelve dimensional phase space integral instead of using the FS approximation;

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(c) Using a fully relativistic approach, which is particularly important at high densities where the Fermi momenta are largest.

This allows us to calculate the re-equilibration rates for four different strangeness changing, weak decay processes, two of which predominantly occur via OME, not via the contact interaction which is heavily suppressed. We show that all of these rates contribute to the bulk viscosity and have to be taken into account. We find that the re-equilibration rates at high density are generally too fast to lead to a sizeable bulk viscosity and correspondingly short dissipation times. However, at densities below the onset of the lowest lying hyperon, the equilibration rates are sufficiently suppressed by the smaller phase space of the thermal hyperon population to match the external oscillation frequency. This leads to a resonance in the bulk viscosity and dissipation times as short as \( \tau_{\text{diss}} \approx 9 \text{ ms} \). Between saturation density and the hyperon onset, dissipation times below 20 ms can be found up to temperatures of \( T = 6 \text{ MeV} \). Above this temperature, neutrino trapping, which we have neglected in this treatment, would likely become important. In this paper we use natural units, where \( \hbar = c = k_B = 1 \) and the mostly-minus signature of the Minkowski metric, \( g^{\mu\nu} = \text{diag}(1, -1, -1, -1) \).

II. HYPERONIC MATTER AND BULK VISCOSITY

A. Equation of State

There are many proposed equations of state for nuclear matter with hyperonic degrees of freedom. Depending on the EOS, different hyperons appear at different onset densities [32–39]. Since our analysis requires calculations, including derivatives, of the EOS both in and out of chemical equilibrium with respect to strangeness, we use a simple EOS, that we call “PK1+H”, which can be computed at arbitrary strangeness fraction, rather than using an EOS that is defined via a table of numbers. PK1+H allows stars up to a maximum mass of \( 1.88 M_\odot \), putting it at the edge of compatibility with current constraints (\( M_{\text{max}} \geq 1.928 \pm 0.017 M_\odot \) [40], \( M_{\text{max}} \geq 2.01 \pm 0.04 M_\odot \) [41]). To check that our main conclusions are not specific to the EOS that we used, we computed the peak bulk viscosity using another hyperonic EOS, GM1’B, which has \( M_{\text{max}} = 2.02 M_\odot \) [42]. GM1’B predicts a different order of the onset of the different hyperon species and includes an additional (strange) exchange meson, which leads to a repulsion between the hyperons. However, as we will discuss in Sec. III, in the relevant density and temperature range it predicts a maximum bulk viscosity comparable to PK1+H. This is an indication that our findings concerning the relevance of hyperonic bulk viscosity are valid for any EOS where at least one hyperonic degree of freedom appears at a density that is reachable in mergers.

The PK1+H EOS is based on a relativistic mean field model (RMF) which includes nonlinear mesonic terms which interact with the nucleons and the \( \Lambda \) and \( \Sigma^- \) hyperons, which have the lowest onset densities. We neglect the other hyperons in the baryon octet because they only appear at much higher densities. In PK1+H, the \( \Sigma^- \) hyperon appears first as a function of density due to its contributions to the overall charge neutrality of matter. The nuclear part of the Lagrangian including the Yukawa couplings \( g_{N\sigma}, g_{N\omega}, g_{N\rho} \) between the nucleons and the three mesons follow the conventions in [43], the numerical parameters are chosen according to the PK1 parametrization from Table 1 in Ref. [44]. We extend the PK1 EOS to the hyperonic sector by adding the hyperons to the Langrangian as shown below. The hyperonic coupling constants are chosen in accordance with Ref. [32] in such a way that the model reproduces the hyperon spectrum similar to the one from the DD-ME2 hyperonic EOS investigated in Ref. [35]. All numerical parameters are summarized in App. B. The Lagrangian of the model is

\[
\mathcal{L} = \mathcal{L}_B + \mathcal{L}_m + \mathcal{L}_l, \tag{1a}
\]

\[
\mathcal{L}_B = \sum_i \bar{\psi}_i \left[ i\gamma^\mu \partial_\mu - M_i - g_{\sigma i} \sigma - g_{\omega i} \gamma^\mu \omega_\mu - g_{\rho i} \gamma^\mu \tau \cdot \rho_\mu \right] \psi_i, \tag{1b}
\]

\[
\mathcal{L}_m = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{g_\sigma}{3} \sigma^3 - \frac{g_\omega}{4} \omega^\mu \omega_\mu + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu + \frac{c_3}{4} (\omega^\mu \omega_\mu)^2 + \frac{1}{2} m_\rho^2 \rho^\mu \cdot \rho_\mu - \frac{1}{4} R^{\mu\nu} \cdot R_{\mu\nu}, \tag{1c}
\]

\[
\mathcal{L}_l = \sum_i \bar{\psi}_i \left[ i\gamma^\mu \partial_\mu - m_i \right] \psi_i, \tag{1d}
\]
FIG. 1: Logarithm of the ratios of all baryonic particle densities over the total baryon density at \( T = 2 \) MeV plotted as a function of total baryon density in units of the saturation density. In the used parametrization PK1+H, saturation density is given by \( n_0 = 0.148 \) fm\(^{-3}\). Although the \( \Lambda \)-hyperon is less massive than the \( \Sigma^- \)-hyperon, the order of their onset is reversed because of charge neutrality.

The particle content in or out of chemical equilibrium are then obtained by solving the Euler-Lagrange equations in the mean field approximation.

The resultant particle content for neutral matter in beta equilibrium is shown in Fig. 1. Chemical equilibrium, charge neutrality and baryon number can be expressed as

\[
\begin{align*}
n_B &= n_n + n_p + n_{\Sigma^-} + n_{\Lambda} \quad \text{baryon number,} \\
n_p &= n_e + n_\mu + n_{\Sigma^-} \quad \text{charge neutrality,} \\
\mu_p &= \mu_n - \mu_e \quad \text{chemical equilibrium,} \\
\mu_e &= \mu_\mu, \\
\mu_{\Sigma^-} &= \mu_n + \mu_e, \\
\mu_{\Lambda} &= \mu_n. 
\end{align*}
\]

The resulting dispersion relations for the baryons are given by

\[
E_i = \sqrt{p_i^2 + (M_i^*)^2 + g_{\omega i} \langle \omega_0 \rangle + g_{\rho i} I_{i3} \langle \rho_{03} \rangle},
\]

with the modulus of the three-momentum \( p_i = |p_i| \), the effective baryon mass \( M_i^* = M_i - g_{\sigma i} \langle \sigma \rangle \), where \( \langle \sigma \rangle \) is the vacuum-expectation value (vev) of the \( \sigma \)-meson and \( \langle \omega_0 \rangle \) the vev of the temporal component of the \( \omega \). Only the temporal part of the third isospin-vector component of the \( \rho \) develops a finite expectation value \( I_{i3} \langle \rho_{03} \rangle \), where \( I_{i3} \) denotes the third component of the isospin projection of the \( i \)-th baryon.

where

\[
\omega^{\mu\nu} \equiv \partial^\mu \omega^{\nu} - \partial^\nu \omega^{\mu}, \quad (2a)
\]

\[
\rho^{\mu\nu} \equiv \partial^\mu \rho^{\nu} - \partial^\nu \rho^{\mu} + g_\omega \rho^\mu \times \rho^\nu, \quad (2b)
\]

with bold symbols being vectors in isospin space. The first term \( L_B \) includes the sum over the four baryons (neutron, proton, \( \Lambda \) and \( \Sigma^- \)) with their masses \( M_i \) and their Yukawa interactions with the mesonic fields. We include the scalar \( \sigma \) meson, the vector meson \( \omega^\mu \) and the isovector triplet \( \rho^\mu \), which breaks isospin symmetry, and self-interactions for the scalar and the vector mesons. Note that the Yukawa couplings are different for every baryon-meson interaction. Their values are given in App. B. The leptonic Lagrangian \( L_l \) introduces free electrons and muons, where we assume the electrons to be massless. The particle fractions in or out of chemical equilibrium are then obtained by solving the Euler-Lagrange equations in the mean field approximation.
B. Rate Calculation and Matrix Element

Computations of hyperonic bulk viscosity have been performed using various nucleonic interactions, approximations and EOS in the past, but exclusively for low enough temperatures so that the FS approximation is valid, and often in the context of the \( r \)-mode instability \([24, 26, 27, 30, 31, 45]\). In this work we are interested in mergers where the temperature is high enough to eliminate nucleonic or hyperonic superfluidity and to invalidate the FS approximation.

Hyperonic bulk viscosity arises from beta equilibration of the strangeness fraction, which will be dominated by the fastest strangeness-changing processes. We focus on non-leptonic processes, which are typically faster than leptonic ones \([46]\). The processes we are including in this work all change strangeness by one unit and are mediated by the weak interaction,

\[
\begin{align*}
\text{I: } & \quad n + n \leftrightarrow p + \Sigma^- , \quad (5a) \\
\text{II: } & \quad n + p \leftrightarrow p + \Lambda , \quad (5b) \\
\text{III: } & \quad n + n \leftrightarrow n + \Lambda , \quad (5c) \\
\text{IV: } & \quad \Lambda + \Lambda \leftrightarrow \Lambda + n . \quad (5d)
\end{align*}
\]

In general there are two main contributions to such processes.

(a) “contact interaction”: exchange of a \( W \) boson between the baryons, which at the energy scales relevant to our calculations can be reduced to a contact interaction between the baryons, depicted for process I in Fig. 2(c);

(b) “one meson exchange” (OME): a combined weak-strong channel, depicted for process I in Fig. 2(a) and (b). In this channel, the flavor-changing \( W \)-boson exchange occurs inside one of the incoming baryons, creating an off-shell intermediate state. A strong interaction with the second incoming baryon rearranges the quarks and improves the kinematics of the process. We model that strong interaction as one-meson exchange.

Early work by Jones \([24]\) and Lindblom and Owen \([26]\) only included contact interactions, so they neglected processes III and IV which would require exchange of a \( Z \) boson between the baryons, and such flavor changing neutral currents are highly suppressed by the GIM mechanism \([47]\).

However, there are OME contributions to all four processes in Eq. (5), and at temperatures in the sub-MeV range the OME channel is the dominant contribution. For processes I and II, the OME contribution to the rate is an order of magnitude larger than the contact interaction contribution \([27]\). Process III, in particular, is non-negligible at most densities. This can partially be attributed to the large phase space near the neutron Fermi surface compared to the other baryon species. We calculate the OME contribution to all 4 processes. The rates \( \Gamma_{12\rightarrow 34} \) can be calculated either in the FS approximation for low temperatures, or by computing the full phase space integral:

\[
\Gamma_{12\rightarrow 34} = \frac{1}{S} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^3} \frac{d^3p_4}{(2\pi)^3} \sum_{s} |M_{1234}|^2 (2\pi)^4 \delta(E_1+E_2-E_3-E_4) \delta^3(p_1+p_2-p_3-p_4) \times (6)
\]

\[
f_1(E_1,\mu_1)f_2(E_2,\mu_2)[1-f_3(E_3,\mu_3)][1-f_4(E_4,\mu_4)] ,
\]

with the symmetry factor \( S = 2 \) for all processes with two identical baryons on one side of the reaction, i.e. processes I, III and IV, and \( S = 1 \) for process II. The spin-summed\(^1\), squared matrix element of the process \( \sum_{s} |M_{1234}|^2 \) turning the incoming baryons with labels 1 and 2 into baryons 3 and 4, where the labels stand for the corresponding baryons in Eqs. (5), can be obtained from the Feynman diagrams in panel (a) of Fig. 2 which give the matrix element

\[
M_{1234} = \left[ \bar{u}_3 F^S_{23} u_2 \bar{u}_4 F^W_{14} u_1 D_\epsilon(k_1^2) - \bar{u}_3 F^S_{13} u_1 \bar{u}_4 F^W_{24} u_2 D_\epsilon(k_2^2) \right] , \quad (7)
\]

where the Dirac bispinors are normalized following Refs. \([48]\) and \([49]\) to \( u^\dagger u = 2E^+ \) which leads to the corresponding energy denominators in Eq. (6). When we evaluate \( |M_{1234}|^2 \), the spin summation over Dirac bispinors, which follow equations of motion derived from meson exchange Lagrangians as used here, leads to an expression in terms of the quasi-momentum \( (E^*,-\vec{p}) \) where

\[
E_\epsilon^* = \sqrt{p_\epsilon^2 + (M_\epsilon^*)^2} . \quad (8)
\]

\(^1\) In rate calculations, normally the spin averaged matrix element is used. This would lead to different rates of the reverse processes in cases where the number of incoming particles is not the same as the number of outgoing particles, even in chemical equilibrium.
FIG. 2: Panels (a) and (b) show Feynman and quarkflow diagrams for the OME contribution to process I. The flavor changing weak-interaction vertex connecting the incoming neutron \( n \) with a pion and the \( \Sigma^- \)-hyperon represents a combination of a flavor changing \( W^- \)-boson exchange within the baryon and a quark exchange (modeled via one meson exchange) with the spectator baryon. The strong-interaction vertex connecting the nucleons \( n \) and \( p \) with a pion. For the matrix element in Eq. (7), we have to subtract a second Feynman diagram with the two initial neutrons exchanged. Panel (c) shows the Feynman diagram for the contact interaction contribution, where the two nucleons exchange a charged \( W \)-boson that is integrated out. This is the basis for the matrix element in Eq. (12). All coupling constants can be found in App. B. The remaining diagrams are shown in App. A.

In all other parts of the calculation, including the delta distributions in the rate integral Eq. (6), on-shell nucleons are characterized by four-momenta that obey the dispersion relation Eq. (4). Therefore, the meson propagator \( D_\varphi \), defined in Eq. (10), depends on the dispersion relations from Eq. (4) as well, whereas the remaining matrix element is given in terms of the quasi-momentum. For a detailed calculation of spin sums in RMFs see appendix B of Ref. [49].

The weak and strong interaction vertices are given by

\[
F_{ij}^W = G_F m_\pi^2 \left( A_{ij} + B_{ij} \gamma_5 \right), \quad F_{ij}^S = g_{ij} \gamma_5, \tag{9}
\]

with the Fermi constant \( G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2} \), the fifth gamma matrix \( \gamma_5 \), and the strong interaction coupling constants \( g_{ij} \) and weak interaction coupling constants \( A_{ij} \) and \( B_{ij} \), which depend on the baryons in the corresponding vertex and are summarized in App. B. The coupling constants \( A_{ij} \) and \( B_{ij} \) are rendered massless via the insertion of a factor of the pion mass squared, \( m_\pi^2 \), for all processes (whether the exchanged meson is a pion or not). The meson propagator is given by

\[
D_\varphi(k) = \frac{1}{k_0^2 - k^2 - m_\varphi^2}, \tag{10}
\]

where the energy \( k_0 \) and the momentum \( k \) of the meson \( \varphi \), which would be a pion in processes I to III and a kaon in IV, is determined by energy-momentum conservation in the vertices.

The Fermi-Dirac distribution functions

\[
f_i(E_i, \mu_i) = \frac{1}{1 + \exp \left( \frac{E_i - \mu_i}{T} \right)} \tag{11}
\]

account for Pauli blocking and depend on the full dispersion relation of the incoming \((i = 1, 2)\) and outgoing \((i = 3, 4)\) baryons, see Eq. (4), the chemical potentials \( \mu_i \) and the temperature \( T \). Since the effective masses become smaller than the corresponding Fermi momenta at high densities, we treat all baryons as relativistic particles. A non-relativistic treatment leads to nonphysical behavior of the bulk viscosity at medium to high densities (around \( n_B \approx 3n_0 \) [50].

Although they will turn out to be small compared to the OME channel, we also compute the rates for the processes \( n + n \leftrightarrow p + \Sigma^- \) and \( n + p \leftrightarrow p + \Lambda \) in the contact interaction channel. The corresponding matrix elements are derived from the Feynman diagrams in Fig. 2(c), Fig. 8(c) and Fig. 9(c) and are, after spin-summation, given by [25–27, 31]

\[
\sum_s |M_{nnp\Sigma^-}|^2 = 8G_F^2 \sin^2(2\varphi_C)M_n^2 M_{p\Sigma^-} \left( 1 + 3c_A^{np} c_A^{\Sigma^-} \right)^2 \tag{12}
\]

and

\[
\sum_s |M_{npp\Lambda}|^2 = 8G_F^2 \sin^2(2\varphi_C)M_n M_p^2 M_{\Lambda} \left( 1 + 3c_A^{np} c_A^{\Lambda} \right), \tag{13}
\]
All numerical constants can be found in App. B. Since the OME processes provide the dominant contribution to the rates, we only need to make a rough estimate of the subdominant contribution from contact interactions. Following the approach used widely in the literature \cite{24, 25, 30} we simplify the matrix element by applying the ultra non-relativistic approximation, where $E_i = M_i^*$ and the energy denominators in Eq. (6) are replaced with the effective masses $M_i^*$. The contact interaction contribution to the rates can then be computed analytically. We emphasize that this is an extremely crude approximation: in cold hyperonic matter, the ultra non-relativistic approximation underestimates the rates by up to two orders of magnitude. However, even in an improved relativistic treatment, the contact interaction produces rates which are still an order of magnitude slower than the ones derived from the OME process \cite{27, 31}.

The results for the rates in the OME and contact interaction channel are shown in Sec. III. At low temperatures, the Fermi spheres are sharply defined and only particles close to the Fermi surface can participate in the processes given in Eqs. (5). In this case, we can simplify the full phase space integral from Eq. (6) by using the FS approximation: we fix all the momentum magnitudes to their respective Fermi momenta, and split the integral into angular and energy contributions. The FS approximation can be applied to the OME and contact-interaction contributions. For details on the FS approximation see Refs. \cite{51, 52}. For a momentum-independent matrix element, like the contact interaction channel matrix element in the ultra non-relativistic approximation, the rate is

$$\Gamma_{12\rightarrow34} = \frac{T^3|M_{1234}|^2}{(2\pi)^3 3S} \int d\phi d\psi \int_{\pm}^{\pm} ds \frac{k_1^+|M_{1234}|^2(k_1^+)\theta(r_1^2 - 1) + k_1^-|M_{1234}|^2(k_1^-)\theta(r_1^2 - 1)}{\sqrt{p_{F2}^2 - (1 - s^2)p_{F4}^2}},$$

where $p_{Fi}$ is the Fermi momentum of the $i$–th particle in Fig. 2, with momentum transfers $k_1 = p_1 - p_3$ and $k_2 = p_4 - p_2$. Energy-momentum conservation demands that the moduli of the momentum transfer vectors are equal, $k_1 = k_2$. Furthermore, the delta distribution has two zeros, which lead to the two separate contributions to the rate integral with the modulus for $k_1$ (and therefore $k_2$) given by $k_1^\pm = p_{F4} s \pm \sqrt{p_{F4}^2 (s^2 - 1) + p_{F2}^2}$, and $\theta$ being the Heaviside function of $r_\pm = (p_{F1}^2 - k_1^\pm - p_{F3}^2) / (2k_1^\pm p_{F3})$. The angles $\varphi$ and $s = \cos \theta$ are the azimuthal and polar angle between $p_4$ and $k_1$. The integration boundaries for $s \in [-1, 1]$ have to be chosen such that $k_1^\pm$ is real and positive. $|M_{1234}|^2(k_1^+)\theta(r_1^2 - 1)$ is the spin summed, squared matrix element evaluated on the Fermi surface, i.e. $|p_3| = p_{F3}$, $|p_4| = p_{F4}$ and $|k_1| = |k_2| = k_1^\pm$. Energy momentum conservation sets the polar angle $\cos \theta_3 \equiv r$ between $k_1$ and $p_3$ to $r = r_\pm$ defined above. Note that in chemical equilibrium, $\xi = 0$ and $\text{lim}_{\xi \rightarrow 0} I(\xi) = 2\pi^2/3$. The remaining integrals are evaluated numerically.

### C. Bulk Viscosity

In this work we study the energy dissipated when a fluid element of hyperonic matter is subjected to a harmonic small amplitude oscillation of the form

$$n(t) = n_B + \Delta n \sin \omega t,$$

where $\Delta n \ll n_B$, the equilibrium baryon density is $n_B$ and the angular frequency of the external oscillation is $\omega$. The oscillation will push the matter out of beta equilibrium, which causes a difference $\Delta \Gamma$ between the backward and forward rates of the individual processes in Eqs. (5). The chemical imbalance can be quantified by

$$\mu_\Delta \equiv 2\mu_n - \mu_p - \mu_{\Lambda^\pm} = \mu_n - \mu_\Lambda,$$

which is equivalent to the deviation of the strangeness chemical potential from its equilibrium value. To compute the re-equilibration rate of the fluid element we combine the rates of all strangeness changing processes. We assume the

\footnote{In Ref. \cite{52}, the baryons are ordered by the magnitude of their Fermi momenta.}
oscillation amplitude is small enough so that \( \mu_\Delta \ll T \), corresponding to the subthermal regime where we can use the linear approximation,

\[
\Delta \Gamma = \lambda \mu_\Delta .
\]

Bulk viscosity in hyperonic systems has been treated in Refs. [24, 26, 27] and revisited in great detail for a different set of hyperons in Ref. [31]. As noted in Sec. I, these works calculated the equilibration rate assuming low temperatures characteristic of isolated neutron stars. In this work we obtain results that are valid at the densities and temperatures that arise in mergers.

The real part of the bulk viscosity can be expressed as

\[
\zeta = -\frac{n_B \gamma}{\gamma^2 + \omega^2} \left( \frac{\partial P}{\partial x_n} \right) \frac{dx_n}{dn_B} ,
\]

\[
\gamma = \frac{\Delta \Gamma (\delta \mu_\Delta/\delta x_n)}{n_B \mu_\Delta} = \frac{\lambda (\delta \mu_\Delta/\delta x_n)}{n_B} ,
\]

In general, the equilibration rate is a function of temperature and density. The bulk viscosity is maximal when the equilibration rate \( \gamma \) and the external oscillation frequency \( \omega \) coincide. It remains to calculate the variation of the chemical imbalance \( \mu_\Delta \) with respect to the neutron fraction. By taking baryon conservation, charge neutrality and chemical equilibrium with respect to the strong process \( n + \Lambda \leftrightarrow p + \Sigma^- \) into account, we find that

\[
\frac{\delta \mu_\Delta}{n_B \delta x_n} = \alpha_{nn} + \frac{(\beta_n - \beta_\Lambda)(\alpha_{np} - \alpha_{n\Lambda} + \alpha_{n\Sigma} - \alpha_{n\Sigma\Lambda})}{2\beta_\Lambda - \beta_p - \beta_\Sigma} - \alpha_{\Lambda n} - \frac{(2\beta_n - \beta_p - \beta_\Sigma)(\alpha_{n\Lambda} - \alpha_{n\Lambda\Lambda})}{2\beta_\Lambda - \beta_p - \beta_\Sigma} ,
\]

where

\[
\alpha_{ij} = \left( \frac{\partial \mu_i}{\partial n_j} \right)_{n_k \neq x,j} , \quad \text{and} \quad \beta_i = \alpha_{ni} + \alpha_{\Lambda i} - \alpha_{pi} - \alpha_{\Sigma i} .
\]

This has been described in great detail in Refs. [26, 27], whose notation we are largely following here. In order to compute the coefficients \( \alpha_{ij} \) from the equation of state we proceed similarly as for the pressure derivative, evaluating the derivatives numerically by solving the RMF field equations for different values of the various baryon densities \( n_j \) for \( j = n, p, \Lambda, \Sigma \).

### III. RESULTS AND DISCUSSION

#### A. Rates in Fermi Surface Approximation and Full Phase Space Calculation

In Fig. 3 we present our calculation of the rates for the processes I to IV in Eqs. (5) as a function of baryon density at a temperature of \( T = 5 \text{ MeV} \). The left panel shows the four individual rates computed numerically from Eq. (6). After analytical simplifications we carry out the remaining five dimensional integration using the CUBA library [53].

At vanishing temperature, \( T = 0 \), there exists a critical minimal density below which no hyperons are present. The actual value of the critical density highly depends on the choice of equation of state. For the PK1+H EOS that we are using, the onset density for hyperons at \( T = 0 \) is \( n_B \approx 1.85 n_0 \). At non-zero temperature there is a thermal population of hyperons at and below this density. The thermal hyperon population increases with temperature and decreases when the density is lowered further. In this regime the hyperon density is exponentially sensitive to temperature and density, so we observe that the rates span many orders of magnitude. They are much less sensitive at densities above the hyperon threshold. We also observe that the rate of process IV is, especially at low densities, suppressed compared to all other rates. This is because this process involves three hyperons and only one nucleon, and therefore has less phase space available. Furthermore, the strong interaction in this case is mediated by kaon exchange instead of pion exchange, so the interaction is suppressed by the higher mass of the kaon \( m_K \) in the meson propagator, even at high densities where the density of \( \Lambda \) hyperons becomes comparable to the neutron density (see Fig. 1).

It is interesting to compare these features with the GM1’B EOS which we have noted above is less convenient to deal with than PK1+H but is more consistent with phenomenological constraints. In GM1’B the zero-temperature hyperon onset involves the \( \Lambda \) rather than the \( \Sigma^- \), and occurs at a higher density, \( n_B = 2.39 n_0 \). However, the rates for processes II, III, and IV for the GM1’B EOS show very similar behavior to PK1+H, just shifted to slightly higher
FIG. 3: *Left panel:* Individual rates of all four strangeness-changing processes defined in Eqs. (5) at $T = 5$ MeV as a function of baryon density normalized to saturation density $n_0$ in the OME channel. Below the hyperon onset at $n_B \approx 1.85 n_0$, the rates drop quickly to zero as the thermal population of hyperons becomes highly suppressed. *Right panel:* Sum of all rates at $T = 5$ MeV (solid blue line) in comparison to the FS approximation (black, dashed line labeled “FS OME”) and the FS approximated rate of the contact interaction (black, dotted line) in the non-relativistic limit presented in Eqs. (12) and (13). Note that restricting to contact interactions in the ultra non-relativistic approximation means ignoring processes III and IV, which underestimates the total rate by a factor of $10^3$. Rates in the FS approximation are only defined above the hyperon density threshold.

FIG. 4: Comparison of the rate for process III (the dominant process at low densities) for the PK1+H EOS, which we use in in this work, and the GM1’B EOS (see Sec. II) at $T = 5$ MeV. The rates show very similar behavior, with GM1’B shifted slightly because it has a higher zero-temperature onset density for hyperons.

densities. Process I, since it involves the $\Sigma^-$, only occurs at higher densities. A direct comparison of the rates for process III, which is the dominant process at low densities, is shown in Fig. 4 for a temperature of $T = 5$ MeV. For our purposes, the important point is that both EOSes show the same pattern in their strangeless equilibration rate. We therefore expect that our results for damping times for PK1+H are representative of hyperonic EOSes, up to a shift in the density where the bulk viscosity achieves its maximum value.

The right panel in Fig. 3 shows the total rate $\Gamma = 2\Gamma_I + \Gamma_{II} + \Gamma_{III} + \Gamma_{IV}$ at $T = 5$ MeV as a function of baryon density (solid blue line). For comparison, we show the total rate in the FS approximation in the OME channel (labeled OME FS approx) and for the sum of process I and II in the contact interaction channel. In the contact interaction channel, the ultra non-relativistic approximation has been performed, see paragraph below Eq. (13) for more details.
FIG. 5: Re-equilibration rate $\gamma$ defined in Eq. (20) for PK1+H as function of baryon density at temperatures of $T = 3, 4, 5, 10, 25, 50$ MeV, where the lowest lying, blue curve corresponds to $T = 3$ MeV and faster rates correspond to higher temperatures. All rates are obtained by evaluating the full phase space integral for the OME matrix elements. The black, dashed line marks the optimal equilibration rate for maximal bulk viscosity where it matches the external oscillation, $\gamma = \omega$. Above the hyperon onset at $n_B \approx 1.85 n_0$ (marked by the thin, black, dashed, vertical line) even at low temperatures the rates are too fast to match the external oscillation. Below the onset, for temperatures below $T = 5$ MeV, $\gamma$ can match $\omega$, leading to a maximal bulk viscosity at a “resonant” density that drops as the temperature rises.

All rates in the FS approximation are only computed above the hyperon threshold, since the Fermi momenta are not properly defined below that density. At high densities, where the temperature becomes negligible compared to the Fermi momenta of the participating particles, the FS approximation works well for the OME contribution. However, it completely fails below the hyperon onset, leaving out the parameter space with the highest bulk viscosity at temperatures above $T > 2$ MeV. Contrary to what one might expect, the FS approximation in the OME channel gives a faster rate than the full phase space integral, although the latter receives contributions from the thermally blurred Fermi surface and not only from particles exactly on their FS. However, further approximations in the energy integral tend to overestimate the rate, see App. B of Ref. [20] for more information.

B. Re-equilibration Rates $\gamma$

From Eq. (20) we can compute the internal re-equilibration rate $\gamma$, shown in Fig. 5. At densities below the onset of the lowest lying hyperon $\Sigma^-$, indicated by a vertical dashed line, the re-equilibration rate spans many orders of magnitude. As with the hyperon creation/annihilation rates $\Gamma$, this is because of the exponential sensitivity of the hyperon population to temperature variation. The horizontal black dashed line shows where the equilibration rate matches the external frequency $\omega$, which is where the bulk viscosity reaches its resonant maximum. In all our calculations we assume an external oscillation frequency of $\omega = 2\pi$ kHz which is typical for the high-amplitude density oscillations that occur immediately after the merger [17]. Above the hyperon threshold, even at temperatures as low as $T = 2$ MeV the equilibration rate is far above the external oscillation frequency. With rising temperature, the equilibration rate increases further, following approximately a $T^3$ law as predicted by the FS approximation. However, below the hyperon onset, the restricted phase space of the thermal hyperonic population leads to significantly slower rates. In the investigated density regime, starting from saturation density (below which nuclear matter might not be uniform [54]), we observe that the equilibration rates intersect with the horizontal line of maximum bulk viscosity up to a temperature of $T = 6$ MeV. Above 6 MeV, the thermal population becomes significant enough to yield relatively fast rates even at low densities. We therefore expect a significant bulk viscosity and correspondingly short dissipation times only below the hyperon onset, and only at temperatures below about 6 MeV.

Above the hyperon threshold, the equilibration rate is not so sensitive to the density. Depending on the temperature, it either rises slowly or falls slowly. This is because, even though the hyperon creation/annihilation rates (Fig. 3) increase with density due to the increased phase space, the density dependence of the susceptibilities in the definition of the equilibration rate Eq. (20) have an opposite density dependence.
C. Bulk Viscosity and Dissipation Times

In Fig. 6 we present the bulk viscosity from Eq. (19). We calculated the OME contribution to the equilibration rate by numerically evaluating the full phase space integral Eq. (6). In both plots we show the bulk viscosity as a function of baryon density at temperatures ranging from \( T = 2 \) MeV to \( T = 50 \) MeV. In the left panel, we explore the baryon density range \( n_B \in [1.4n_0, 5n_0] \), where the vertical dashed line indicates the density at which hyperons would first appear when the temperature is zero. At densities above the threshold we observe a rather low bulk viscosity, which further decreases with temperature. The largest bulk viscosity is therefore obtained at the lowest shown temperature, \( T = 2 \) MeV. This is because in this density range the equilibration rate is always too fast (faster than the typical density oscillation frequency \( \sim 1 \) kHz) so to increase the bulk viscosity one must decrease the equilibration rate, e.g., by reducing the temperature or density.

At densities below the hyperon onset density, the equilibration rate is slow enough to reach resonance with the density oscillation, so we see the prominent peak where, for a given temperature, the equilibration rate \( \gamma \) matches \( \omega \). The peak value of bulk viscosity is determined by the susceptibilities (Eq. (19)), whose gradual variation with temperature causes the peak value to drop as the temperature rises.

In the left panel, the bulk viscosity is plotted for temperatures of \( T = 2, 5, 8, 10, 20, 30, 40, 50 \) MeV. For temperatures above \( T = 5 \) to 6 MeV, we expect the bulk viscosity to peak at even lower densities than we investigate in this work. The right panel of Fig. 6 focuses on the density regime below the zero-temperature onset of \( \Sigma^- \) hyperons and temperatures from 2 to 5.5 MeV in 0.5 MeV increments. We can clearly observe how the peak of the bulk viscosity moves to lower densities with increasing temperatures. Additionally, the maximum bulk viscosity (i.e. the height of the peak) decreases with rising temperature as well. Every curve is cut off at a density where the hyperon fraction becomes too small to allow for a reliable numerical computation of the equilibration rate.

For oscillations in mergers, one important measure of the importance of bulk viscosity is the dissipation time \( \tau_{\text{diss}} \) which quantifies how fast a density oscillation of a fluid element in the merger is damped. Following Refs. [19, 55, 56],

\[
\tau_{\text{diss}} = \frac{\varepsilon}{d\varepsilon/dt} = \frac{\kappa_S}{\omega^2 \xi},
\]

where \( \varepsilon \) is the energy carried by an oscillation in baryon density with frequency \( \omega \) and amplitude \( \Delta n \),

\[
\varepsilon = \kappa_S^{-1} \left( \frac{\Delta n}{n_B} \right)^2, \quad \text{where} \quad \kappa_S^{-1} = n_B \frac{\partial P}{\partial n_B},
\]
FIG. 7: **Left panel:** Contour plot of dissipation time in the plane of baryon density and temperature, focused on the region of highest bulk viscosity, i.e. below the hyperon onset density and at temperatures below 6 MeV. The diagonal “valley” of fastest dissipation tracks the position of the bulk viscosity peaks in the right panel of Fig. 6. At lower densities, the equilibration rates are too slow, whereas at higher densities they are too fast for a sizable bulk viscosity and short dissipation times. In the grey area at low $T$ and $n_B$, the hyperon fractions are too small for a reliable calculation. **Right panel:** The minimal dissipation times as a function of temperature are plotted. Each dot corresponds to a peak of the bulk viscosity in the right panel of Fig. 6, where the corresponding density can be read off. Dissipation times as low as 9 ms are found.

and $\kappa_S$ is the incompressibility.

The numerical results are presented in Fig. 7. In the left panel, the dissipation time is plotted as a contour plot in the plane of baryon density and temperature. We focus on the parameter regime of low temperatures and densities in which the bulk viscosity is maximal. The shortest dissipation times we find are as low as $\tau_{\text{diss}} \approx 9$ ms, which makes hyperon bulk viscosity potentially important in neutron star mergers, whose characteristic timescale is around 10 to 20 ms. The peaks of the bulk viscosity in the right panel of Fig. 6 directly translate to the diagonally oriented central “valley” where dissipation times are below 16 ms. For lower densities, a higher temperature is required to render the hyperon population large enough, leading to the diagonal structure of the contours. Moving out of the valley from the innermost contour to higher or lower densities leads to a rapid increase of the dissipation time because the bulk viscosity drops rapidly as we move off resonance. When we move to lower densities, the re-equilibration rate (Fig. 5) becomes slower than the external oscillations. While the re-equilibration rate perfectly matches $\omega$ in the innermost contour, moving to higher densities will lead to faster rates and a mismatch on the other side of the resonance. In the grey shaded area at low densities and temperatures, the hyperon fraction is so small that a reliable numerical calculation of the rates is not possible.

In the right panel we plot the shortest dissipation time (obtained by varying the density) as a function of temperature. Thus every dot in the plot corresponds to a different temperature and density. For a given temperature one can read off the corresponding density by locating the peak of the bulk viscosity with that temperature in the right panel of Fig. 6. Dissipation times as low as $\tau_{\text{diss}} \approx 9$ ms are observed at $T = 2$ MeV, but even at temperatures as high as $T = 6$ MeV, dissipation times that are shorter than 20 ms, and hence potentially relevant for the dynamics of mergers, are found.

**IV. CONCLUSIONS**

In this paper we have presented a calculation of hyperonic bulk viscosity and the resultant dissipation time for density oscillations in the range of densities and temperatures that are expected to exist in binary neutron star mergers. For this purpose, we used the PK1+H EOS, whose maximum neutron star mass is at the edge of compatibility with observations, but we checked that comparable results would be obtained for the GM1'B EOS: for both these EOS the hyperonic bulk viscosity reaches similar peak values at a density approximately $0.3 n_0$ to $0.4 n_0$ below the zero
temperature hyperon onset.

We calculated hyperonic equilibration rates by evaluating the one meson exchange contribution, which, as first discussed in Ref. [27], is the dominant channel in all of the studied parameter space.

The typical temperature in mergers is in the MeV range, which is high enough to invalidate the FS approximation. This approximation was used in all previous studies of hyperonic bulk viscosity since they were concerned with temperatures in the keV range. We therefore numerically evaluate the full phase space integral for the rates instead of relying on the FS approximation. This allows us to study the behavior of the system at densities below the zero-temperature hyperon onset, where there is only a thermal population of hyperons and the Fermi surface is not well defined. We find that the rates above the hyperon onset are too fast to match an external frequency oscillation of $\omega \sim 2\pi$ kHz. However, in the thermally occupied density range the rates are suppressed by the smaller phase space. The bulk viscosity therefore peaks at densities just below the zero-temperature hyperon threshold. At those densities, for temperatures in the MeV range, we find that density oscillations can be damped on time scales as short as 9 ms, which is fast enough to affect the evolution of the merger. It therefore seems worthwhile to incorporate hyperonic bulk viscosity, combined with its nuclear counterpart computed in Ref. [19, 21], in future numerical general relativity simulations of neutron star mergers. One could then look for signatures of the presence of hyperons in future observations of gravitational waves from neutron star mergers.

In future work, the influence of large amplitude oscillations and magnetic fields on the hyperon bulk viscosity could be studied. Different equations of state for the hyperonic matter will predict slightly different values for the dissipation time and will affect the exact temperatures and densities where bulk viscosity becomes maximal and therefore dissipation times become minimal. However, we expect that the resonant peak of hyperonic bulk viscosity will always occur just below the zero-temperature hyperon onset, since the re-equilibration rate must cross through the resonance as we go from very low densities with negligible hyperon population to densities above the onset where the rate is much faster than the typical 1 kHz density oscillation frequency.

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Appendix A: Feynman and Quarkflow Diagrams

In this appendix we present the Feynman diagrams and the corresponding quark flow diagrams for three of the four strangeness changing processes we take into account, see Eqs. (5). Process I is depicted in the main part of this publication, see Fig. 2. For the computation of the matrix element in Eq. (7), a second Feynman diagram with the initial baryons exchanged has to be subtracted. Only for process II this leads to a nontrivial change, since in all other cases the initial particles are identical. In these trivial cases, we do not draw the second Feynman and quark flow diagram.

For process I and II, we additionally show the diagrams for the same process in the contact interaction channel, where the baryons directly exchange a charged $W$-boson. These diagrams are the basis for the matrix elements in Eq. (12) and Eq. (13).

FIG. 8: Feynman- and quarkflow diagram for process II, $n + p \rightarrow p + \Lambda$, in the OME channel (panels (a) and (b)) and the contact interaction channel (panel (c)).
FIG. 9: Feynman- and quarkflow diagram for process II, $n + p \rightarrow p + \Lambda$, in the OME channel (panels (a) and (b)) and the contact interaction channel (panel c), both with the initial nucleons exchanged.

FIG. 10: Feynman- and quarkflow diagram for process III, $n + n \rightarrow n + \Lambda$ in the OME channel. The corresponding contact interaction channel would be mediated by neutral $Z$-boson exchange and is therefore suppressed by the GIM mechanism. For the calculation of the OME matrix element, a diagram with the two incoming neutrons exchanged has to be subtracted from the depicted one.

FIG. 11: Feynman- and quarkflow diagram for process IV, $\Lambda + \Lambda \rightarrow \Lambda + n$ in the OME channel. The corresponding contact interaction channel is suppressed due to the GIM mechanism. For the calculation of the OME matrix element, a diagram with the two incoming neutrons exchanged has to be subtracted from the depicted one.
Appendix B: Numerical Parameters and Coupling Constants

In this appendix we collect all numerical parameters and coupling constants from the EOS and the Feynman diagrams in Fig. 2 and App. A.

| $M_{\pi}$ | $M_{\rho}$ | $m_{\sigma}$ | $m_{\rho}$ | $g_{\sigma N}$ | $g_{\omega N}$ | $g_{\rho N}$ | $g_2 [fm^{-1}]$ | $g_1$ | $c_4$ |
|---|---|---|---|---|---|---|---|---|---|
| 930.5731 | 938.2796 | 514.0891 | 784.254 | 763 | 10.3222 | 13.0131 | 4.5297 | -8.1688 | -9.9976 | 55.636 |

| $m_e$ | $m_{\mu}$ | $m_{\pi}$ | $M_A$ | $M_{\Sigma^-}$ | $g_{\sigma A}$ | $g_{\omega A}$ | $g_{\rho A}$ | $g_{\sigma \Sigma^-}$ | $g_{\rho \Sigma^-}$ | $g_{\omega \Sigma^-}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 106 | 134.976 | 497.611 | 1115 | 1197 | 0.642 | $g_{\sigma N}$ | 0.453 | $g_{\sigma N}$ | 0.66 | $g_{\omega N}$ | 0.66 | $g_{\omega N}$ | 0 | $-2 g_{\rho N}$ |

**TABLE I**: Numerical parameters for the nuclear part and the hyperonic extension of the PK1+H equation of state. The nuclear EOS and all parameters are taken from Ref. [44]. The meson-nucleon Yukawa couplings are identical for neutron and proton, i.e. $g_{\sigma N} \equiv g_{\sigma n} = g_{\sigma p}$ etc.. All masses are given in MeV.

| Vertex | $g_{ij}$ | $A_{ij}$ | $B_{ij}$ |
|---|---|---|---|
| $pp\pi$ | 13.3 | - | - |
| $np\pi$ | 13.3$\sqrt{2}$ | - | - |
| $nn\pi$ | $-13.3$ | - | - |
| $\Lambda n\pi$ | - | $-1.07$ | $-7.19$ |
| $\Lambda p\pi$ | - | 1.46 | 9.95 |
| $\Sigma^- n\pi$ | - | 1.93 | $-0.63$ |
| $\Lambda nK$ | $-14.1$ | - | - |
| $\Lambda KK$ | - | 0.67 | $-12.72$ |

**TABLE II**: Coupling constants for the matrix element in the OME channel taken from Refs. [27, 31]. The kaon couplings were originally published in Refs. [57, 58]. The vertices are defined in Eq. (9).

| $c_A^{np}$ | $c_A^{p\Lambda}$ | $c_A^{\Lambda \Sigma^-}$ | $\sin^2(2\theta_c)$ |
|---|---|---|---|
| -1.26 | -0.72 | 0.34 | 0.18742 |

**TABLE III**: Coupling constants for the matrix element in the contact interaction channel taken from Refs. [26, 27]
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