Leptogenesis from spin-gravity coupling following inflation

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The energy levels of the left and the right handed neutrinos is split in the background of gravitational waves generated during inflation which, in presence of lepton number violating interactions, gives rise to a net lepton asymmetry at equilibrium. Lepton number violation is achieved by the electroweak sphaleron processes. Sphaleron processes conserve \((B - L)\) so \((B/L)\) violating reactions in the forward and reverse channels do not cancel and (c) departure from thermal equilibrium as the statistical distribution of particles and anti-particles is the same if the Hamiltonian commutes with \(CPT\). In the theory there is CP violation it is possible to generate lepton/baryon asymmetry at thermal equilibrium without requiring \(CP\) violation.

In this paper we show that \(CP\) violation is spontaneous due to the spin-connection couplings of fermions with cosmological gravitational waves. It is well known that inflation \(2\) generates a nearly scale invariant spectrum of gravitational waves \(1\). The spin connection couplings split the energy levels of neutrinos compared to anti-neutrinos and in presence of lepton number violating interaction there is a net asymmetry generated between neutrinos and anti-neutrinos at thermalodynamic equilibrium. Lepton number violation is generated by the dimension five operator introduced by Weinberg \(2\) which also generates the neutrino masses after electro-weak symmetry breaking. The lepton-asymmetry gets frozen in when the lepton-number violating processes decouple. Baryon asymmetry can then be generated from this lepton-asymmetry by the electro-weak sphaleron processes \(2\). Sphaleron processes conserve \((B - L)\) so a lepton asymmetry generated in the GUT era can be converted to baryon asymmetry of the same magnitude \(2\).

The general covariant coupling of spin 1/2 particles to gravity is given by the Lagrangian \(\mathcal{L} = \sqrt{-g} \left( \bar{\psi} \gamma^\mu D_\mu \psi - m \psi \bar{\psi} \right)\), \(D_\mu = \partial_\mu - i \omega_{\mu ab} \sigma^{bc}\) is the covariant derivative, and \(\omega_{\mu ab}\) is the spin-connections \(\omega_{\mu ab} = e_{a\lambda} \left( \partial_\mu e^{\lambda}_c + \Gamma^\lambda_{\mu \kappa} e^\kappa_c e^{\mu}_d \right)\). This Lagrangian is invariant under the local Lorentz transformation of the vierbein \(e^a_\mu (x) \rightarrow A^a_\mu (x) e^b_\mu (x)\) and the spinor fields \(\psi (x) \rightarrow \exp (i \epsilon_{ab} (x) \sigma^{ab}) \psi (x)\). Here \(\sigma^{bc} = \frac{1}{4} \left[ \gamma^b, \gamma^c \right]\) are generators of tangent space Lorentz transformation \((a, b, c\) etc. denote the inertial frame 'flat space' indices and \(\alpha, \beta, \gamma\) etc. are the coordinate frame 'curved space' indices such that \(\epsilon_{a\mu} e^{\alpha}_c = g^{\alpha \beta} \eta^{ab}\) where \(\eta^{ab}\) represents the inertial frame Minkowski metric, and \(g_{\mu \nu}\) is the curved space metric).

The spin-connection term in the Dirac equation is a product of three Dirac matrices which after some algebra can be reduced to a vector \(A^a \gamma_a\) and an axial vector \(iB^a \gamma_a\). The vector term turns out to be anti-hermitian and disappears when the hermitian conjugate part is added to the lagrangian \(\mathcal{L}\). The surviving interaction term which describes the spin-connection coupling of fermions to gravity can be written as a axial-vector

\[
\mathcal{L} = \det (e) \left( i \gamma^a \partial_a - m - \gamma_5 \gamma_d B^d \right) \psi ,
\]

\[
B^d = e^{abcd} e_{\lambda b} \left( \partial_a e^{\lambda}_c + \Gamma^\lambda_{\mu \kappa} e^{\kappa}_c e^{\mu}_d \right) .
\] (1)

In a local inertial frame of the fermion, the effect of a gravitational field appears as an axial-vector interaction term shown in \(\mathcal{L}\). We now calculate the four vector \(B^d\) for a perturbed Robertson-Walker universe.

The general form of perturbations on a flat Robertson-Walker expanding universe can be written as \(\mathcal{L}\)

\[
ds^2 = a(\tau)^2 \left[ (1 + 2 \phi) d\tau^2 - \omega_i dx_i d\tau - ((1 + 2 \psi) \delta_{ij} + h_{ij}) dx^i dx^j \right] ,
\] (2)

where \(\phi\) and \(\psi\) are scalar, \(\omega_i\) are vector and \(h_{ij}\) are the tensor fluctuations of the metric. Of the ten degrees of freedom in the metric perturbations only six are independent and the remaining four can be set to zero by suitable gauge choice. For our application we need only the tensor perturbations and we choose the transverse-traceless (TT) gauge \(h^i_t = 0, \partial^\tau h_{ij} = 0\) for the tensor perturbations. In the TT gauge the perturbed Robertson-Walker metric can be expressed as

\[
ds^2 = a(\tau)^2 \left[ (1 + 2 \phi) d\tau^2 - \omega_i dx^i d\tau - (1 + 2 \psi - h_t) dx_2^2 \right]
\]

\[- (1 + 2 \psi + h_t) dx_2^2 - 2 h_2 dx_1 dx_2 - (1 + 2 \psi) dx_3^2 \] . (3)
An orthogonal set of vierbiens \( e^a_\mu \) for this metric is given by

\[
e^a_\mu = a(\tau) \begin{pmatrix}
1 + \phi & -\omega_1 & -\omega_2 & -\omega_3 \\
0 & -(1 + \psi) + h_+/2 & h_x & 0 \\
0 & 0 & -1(1 + \psi) - h_+/2 & 0 \\
0 & 0 & 0 & -(1 + \psi)
\end{pmatrix}.
\] (4)

Using the vierbiens \( \mathbf{e}^4 \) the expression for the components of the four vector field \( \mathbf{B}^d \) is given by

\[
\mathbf{B}^0 = \partial_0 h_x, \quad \mathbf{B}^i = (\nabla \times \mathbf{\omega})^i + \partial_i h_x \delta^{i3}. \quad (5)
\]

The choice of vierbiens \( \mathbf{e}^4 \) which gives the metric \( \mathbf{g} \) is not unique as one can make a local Lorentz transformation (LLT) \( e^\mu_{\mu'} = \Lambda^\mu_{\mu'} e^\mu_{\mu'} \). Under a LLT the four-vector \( B^d \) transforms as \( B^{d'} = \Lambda^d_{a'} B^a \). The dispersion relations of the left and the right helicity fermions have \( B^d \) dependent terms of the form \( \eta_{mn} B^m B^n \) and \( \eta_{mn} \hat{b}^m B^n \), and therefore the dispersion relations do not change with a transformation of the local inertial frame.

The fermion bilinear term \( \bar{\psi} \gamma_5 \gamma_\alpha \psi \) is odd under CPT transformation. When one treats \( B^a \) as a background field then the interaction term in \( \mathbf{B} \) explicitly violates CPT. When the primordial metric fluctuations become classical, i.e., there is no back-reaction of the microphysics involving the fermions on the metric and \( B^a \) is considered as a fixed external field, then CPT is violated spontaneously.

The gravitational spin connection coupling for the neutrinos at high energy is given by

\[
\mathcal{L} = \det(e)[(i\bar{\nu}_L \gamma^a \partial_a \nu_L + i\bar{\nu}_R \gamma_a \partial_a \nu_R) + m\bar{\nu}_L \nu_R + m^\dagger \nu_R \bar{\nu}_L + B^a(\bar{\nu}_R \gamma_\alpha \nu_R - \bar{\nu}_L \gamma_\alpha \nu_L)],
\]

where \( B^a \) are the parameters of the gravitational waves as defined in \( \mathbf{B} \). If we consider only the Standard Model fermions then the right handed neutrinos carry the opposite Lepton number compared to the left handed neutrinos, \( \nu_R = (\nu_L)^c \) and the mass term in \( \mathbf{B} \) is of the Majorana type (we have suppressed the generation index).

The dispersion relation of left and right helicity neutrinos fields are given by \( \eta^{\alpha\beta}(p_\alpha + \xi B_\alpha)(p_\beta + \xi B_\beta) = m^2 \), where \( \xi = -1 \) for \( \nu_L \) and \( \xi = 1 \) for \( \nu_R \). Keeping terms linear in the perturbations \( B^a \), the free particle energy of the left and right helicity states is

\[
E_{L,R}(p) = p + \frac{m^2}{2p} \mp \left(B_0 - \frac{p \cdot B}{p}\right), \quad (7)
\]

with \( p = |\mathbf{p}| \). In the Standard Model \( \nu_L \) carry lepton number \(+1\) and \( \nu_R \) are assigned lepton number \((-1\)). In the presence of non-zero metric fluctuations, there is a split in energy levels of \( \nu_{L,R} \) given by \( \mathbf{B} \). If there are GUT processes that violate lepton number freely above some decoupling temperature \( T_d \), then the equilibrium value of lepton asymmetry generated for all \( T > T_d \) will be

\[
n(\nu_L) - n(\nu_R) = \frac{g}{2\pi^2} \int d^3 p \left[ \frac{1}{1 + e^{E_L / T}} - \frac{1}{1 + e^{E_R / T}} \right]. \quad (8)
\]

The spin-connection coupling with gravitational waves also splits the energy levels between the charged left and right handed fermions. For example \( E(\nu_R^1) - E(\nu_R^c) = 2(B_0 - \mathbf{p} \cdot \mathbf{B} / |\mathbf{p}|) \). But this does not lead to lepton generation of lepton asymmetry as both \( \nu_L^1 \) and \( \nu_R^c \) carry the same lepton number.

In the ultra-relativistic regime \( p \gg m_\nu \) and assuming that \( B_0 \ll T \), the expression \( \mathbf{B} \) for lepton asymmetry reduces to

\[
\Delta n_L = \frac{g T^3}{6} \left( \frac{B_0}{T} \right). \quad (9)
\]

The dependence on \( \mathbf{B} \) drops out after angular integration in \( \mathbf{B} \) and the lepton asymmetry depends on the tensor perturbations only through \( B^0 \).

To compute the spectrum of gravitational waves \( h(x, \tau) \) during inflation, we express \( h_x \) in terms of the creation-annihilation operator

\[
h(x, \tau) = \frac{\sqrt{16\pi}}{a M_p} \int \frac{d^3 k}{(2\pi)^3} \left( a_k f_k(\tau) + a_k^\dagger f_k^\dagger(\tau) \right) e^{i\mathbf{k} \cdot \mathbf{x}},
\]

where \( \mathbf{k} \) is the comoving wavenumber, \( k = |\mathbf{k}| \), and \( M_p = 1.22 \times 10^{19} \text{GeV} \) is the Planck mass. The mode functions \( f_k(\tau) \) obey the minimally coupled Klein-Gordon equation

\[
f''_k + \left( k^2 - \frac{a''}{a} \right) f_k = 0.
\]

During de Sitter era, the scale factor \( a(\tau) = 1/(H \tau) \) where \( H \) is the Hubble parameter, and Eq. \( \mathbf{B} \) has the solution

\[
f_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \left( 1 - \frac{i}{k\tau} \right), \quad (12)
\]

which matches the positive frequency ”flat space” solutions \( e^{-ik\tau} / \sqrt{2k} \) in the limit of \( k\tau \gg 1 \). The first term of \( \mathbf{B} \) represents the decaying part of \( h \) and can be dropped. The second term of \( \mathbf{B} \) represents the amplitude constant gravitational wave, which survives to

\[...\]
In the radiation era the amplitudes of gravitational waves generated by inflation will be

\[ \langle h(\mathbf{x}, \tau) h(\mathbf{x}, \tau) \rangle_{inf} = \int \frac{dk}{k} \langle |h_k|^2 \rangle_{inf}, \]  

with the spectrum of gravitational waves given by the scale invariant form

\[ \langle |h_k|^2 \rangle_{inf} = \frac{4 H^2}{\pi M_p^2}, \]  

where \( H_k \) are the gravitational waves generated by inflation, \( a_k \) and \( T_k \) are the scale factor and the temperature when the modes of wavenumber \( k \) entered the horizon in the radiation era. The horizon entry of mode \( k \) occurs when

\[ \frac{a_H k}{k} = \frac{a(T) T H_k}{T_k} = 1, \]  

where \( H_k = 1.67 \sqrt{g_s} T_k^2 / M_p \) is the Hubble parameter at the time of horizon crossing of the \( k \) mode (\( g_s \) is the number of relativistic degrees of freedom which for the Standard Model is \( g_s = 106.7 \)). Solving equation (16) for \( T_k \) we get

\[ T_k = \frac{1}{1.67 \sqrt{g_s} a(\tau)} \left( \frac{M_p}{k} \right). \]  

The amplitude of the gravitational waves of mode \( k \) inside the radiation horizon is, using (17) and (15), given by

\[ h_k^{rad} = h_k^{inf} \frac{a(T) T}{a(\tau) T} \left( \frac{1.67 \sqrt{g_s}}{M_p} \right). \]  

Note that the gravitational wave spectrum inside the radiation era horizon is no longer scale invariant. The gravitational waves in position space have the correlation function

\[ \langle h(\mathbf{x}, \tau) h(\mathbf{x}, \tau) \rangle^{rad} = \int \frac{dk}{k} \langle (h_k^{rad})^2 \rangle, \]  

and hence for the spin connection \( B^0 \) generated by the inflationary gravitational waves in the radiation era, we get

\[ \langle B^0(\mathbf{x}, \tau) B^0(\mathbf{x}, \tau) \rangle = \int \frac{dk}{k} \left( \frac{k}{a} \right)^2 \langle (h_k^{rad})^2 \rangle = \frac{4}{\pi} \left( \frac{H_I}{M_p} \right)^2 T^2 1.67 \sqrt{g_s} \int_{k_{min}}^{k_{max}} \frac{dk}{k}. \]  

Now we find that the spectrum of spin-connection is scale invariant inside the radiation horizon. This is significant in that the lepton asymmetry generated by this mechanism depends upon the infrared and ultraviolet scales only logarithmically. The scales outside the horizon are blue-tilted which means that there will be a scale dependent anisotropy in the lepton number correlation at two different space-time points \( \Delta L(r) \Delta L(r') \sim A k^n, n > 0 \), where \( \Delta L(r) = L(r) - L_\text{hor} \) is the anisotropic deviation from the mean value. Unlike in the case of CMB, this anisotropy in the lepton number is unlikely to be accessible to experiments. Nucleosynthesis calculations only give us an average value at the time of nucleosynthesis (\( T \sim 1 MeV \)). The maximum value of \( k \) for are those modes which leave the de-Sitter horizon at the end of inflation. If inflation is followed by radiation domination era starting with the reheat temperature \( T_{RH} \) then the maximum value of \( k \) in the radiation era (at temperature \( T \) ) is given by \( k_{max} / a(T) = H_I \left( \frac{T}{T_{RH}} \right) \). The lower limit of \( k \) is \( k_{min} = e^{-N} k_{max} \) which are the modes which left the de-Sitter horizon in the beginning of inflation (\( N \) is the total e-folding of the scale factor during inflation, \( N \approx 55 - 70 \)). The integration over \( k \) then yields just the factor \( \ln(k_{max}/k_{min}) = N \). The r.m.s value of spin connection that determines the lepton asymmetry through equation (16) is \( (B_0)_{rms} = \sqrt{\langle B^0_0 \rangle} \),

\[ (B_0)_{rms} = \frac{2}{\sqrt{\pi}} \left( \frac{H_I}{M_p} \right)^2 T^2 1.67 \sqrt{g_s} \sqrt{N}. \]  

The lepton asymmetry \( \Delta n_L \) as a function of temperature can therefore be expressed as (taking \( g = 3 \) for the three neutrino flavors)

\[ \Delta n_L(T) = \frac{1}{\sqrt{\pi}} (1.67 \sqrt{g_s}) \sqrt{N} \left( \frac{T^4 H_I}{M_p^2} \right). \]  

The lepton number to entropy density \( s = 0.44 g_s T^3 \) is given by

\[ \Delta L \equiv \Delta n_L(T) / s(T) \approx 2.14 \frac{T H_I \sqrt{N}}{M_p^2 \sqrt{g_s}}. \]  

Lepton number asymmetry will be generated as long as the lepton number violating interactions are in thermal equilibrium. Once these reactions decouple at some decoupling temperature \( T_d \), which we shall determine, the \( \Delta n_L(T)/s(T) \) ratio remains fixed for all \( T < T_d \).
To calculate the decoupling temperature of the lepton number violating processes we turn to a specific effective dimension five operator which gives rise to Majorana masses for the neutrinos introduced by Weinberg

\[ \mathcal{L}_W = \frac{C_{\alpha\beta}}{M^2} (\bar{L}_\alpha \phi^* \phi^r L_\beta) + h.c. \]  
where \( L_\alpha = (\nu_\alpha, e^\alpha_L)_L \)

is the left-handed lepton doublet (\( \alpha \) denotes the generation), \( \phi = (\phi^+, \phi^0)^T \) is the Higgs doublet and \( \phi \equiv i\sigma_2 \phi^* = (-\phi^0, -\phi^+)^T \). \( M \) is some large mass scale and \( C_{\alpha\beta} \) are of order unity.

The \( \Delta L = 2 \) interactions that result from the operator \( \mathcal{L}_W \) are

\[ \nu_L + \phi^0 \leftrightarrow \nu_R + \phi^0, \quad \nu_R + \phi^{0*} \leftrightarrow \nu_L + \phi^{0*}. \]  

(23)

In the absence of the gravitational waves the forward reactions would equal the backward reactions and no net lepton number would be generated. In the presence of a background gravitational waves the energy levels of the left and right helicity neutrinos are no longer degenerate and this leads to a difference in the number density of left and right handed neutrinos of the magnitude given by equation (29) at thermal equilibrium. This process continues till the interactions (28) decouple. The decoupling temperature is estimated as follows. The cross section for the interaction \( \nu_{L\alpha} + \phi^0 \leftrightarrow \nu_{R\beta} + \phi^0 \) is \( \sigma = \frac{|C_{\alpha\beta}|^2 \pi}{M^4} \), and interaction rate \( \Gamma = \langle \sigma v \rangle \sigma \) of the \( \Delta L = 2 \) interactions is \( \Gamma = \frac{0.122 |C_{\alpha\beta}|^2 T^5}{M^4} \). In the electroweak era, when the Higgs field in \( \mathcal{L}_W \) acquires a vev, \( \langle \phi \rangle = (0, v)^T \) (where \( v = 174 \text{ GeV} \)), this operator gives rise to a neutrino mass matrix \( m_{\alpha\beta} = \frac{\epsilon_C C_{\alpha\beta}}{M} \). We can therefore substitute the couplings \( C_{\alpha\beta} \) in terms of the light left handed Majorana neutrino mass. At the decoupling temperature the interaction rate \( \Gamma(T) \) falls below the expansion rate \( H(T) = 1.7 \sqrt{g_*} T^3 / M_p \). The decoupling temperature is obtained from equation \( \Gamma(T_d) = H(T_d) \) and turns out to be

\[ T_d = 13.68 \pi \sqrt{g_*} \frac{v^4}{m^2 \pi M_p}, \]  

(24)

where \( m \) is the mass of the heaviest neutrino. A lower bound on the mass of the heaviest neutrino is given by atmospheric neutrino experiments \( \Delta \text{atm} = 2.5 \times 10^{-3} eV^2 \) which means that the decoupling temperature has an upper bound given by \( T_d = 1.3 \times 10^{13} (\Delta \text{atm} / m^2) \text{ GeV} \). Substituting the expression (22) for \( T \) in (24), we finally obtain the formula for lepton number

\[ L = 92.0 \left( \frac{v^4 H_I}{m^2 M_p^2} \right) \sqrt{N} \]

\[ = 7.4 \times 10^{-11} \frac{H_I}{4 \times 10^{14} \text{GeV}} \frac{2.5 \times 10^{-3} eV^2}{m^2} \sqrt{N} \]  

(25)

As first pointed out in \[ \text{[7]} \] electroweak sphalerons at the temperatures \( T \sim 10^3 \text{GeV} \) violate \( B + L \) maximally and conserve \( B - L \). Therefore a lepton asymmetry generated at an earlier epoch gets converted to baryon asymmetry of the same magnitude by the electroweak sphalerons.

The input parameters needed for generating the correct magnitude of baryogenesis (\( \eta \sim 10^{-10} \)) are the amplitude of the higgs \( \sim 10^{-6} \) (or equivalently the curvature during inflation \( H_I \sim 10^{14} \text{GeV} \) or the scale of inflation is the GUT scale, \( V^{1/4} \sim 10^{10} \text{GeV} \) which is allowed by CMB \[ \text{[12, 13]} \], neutrino Majorana mass in the atmospheric neutrino scale \( m^2_{\nu} \sim 10^{-3} eV^2 \) \[ \text{[11]} \] and duration of inflation \( H_I t = N \sim 100 \) needed to solve the horizon and entropy problems in the standard inflation paradigm. All these parameters are well within experimentally acceptable limits.

To summarize the mechanism of baryogenesis we propose arises in the standard Einstein’s gravity where spontaneous \( CPT \) violation is caused by gravitational waves, and the \( h_x \) gravitational wave modes which give non-zero spin connection are produced in generic inflation scenarios (in contrast to models \[ \text{[14]} \] where baryon asymmetry is created through a gravitational Chern-Simons term, which can be generated if specific \( CP \) violating terms are introduced in the inflaton potential which can give rise to birefringent circularly polarized gravitational waves).