Dynamical Generation of Spacetime Signature by Massive Quantum Fields on a Topologically Non-Trivial Background

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Abstract

The effective potential for a dynamical Wick field (dynamical signature) induced by the quantum effects of massive fields on a topologically non-trivial $D$ dimensional background is considered. It is shown that when the radius of the compactified dimension is very small compared with $\Lambda^{1/2}$ (where $\Lambda$ is a proper-time cutoff), a flat metric with Lorentzian signature is preferred on $\mathbb{R}^4 \times S^1$. When the compactification radius becomes larger a careful analysis of the 1-loop effective potential indicates that a Lorentzian signature is preferred in both $D = 6$ and $D = 4$ and that these results are relatively stable under metrical perturbations.

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1 Introduction

In classical physics, the principle of causality is traditionally required as a basic assumption. According to relativistic notions, it may be expressed in terms of a Lorentzian geometry for a 4-dimensional differentiable spacetime manifold possessing a continuous field of light cones with a preferred time orientation. However, when applying quantum theory to spacetime itself \cite{[1]-[4],[5]-[8],[9]-[11]} a number of intriguing possibilities emerge that force one to re-examine the pre-eminent role of a Lorentzian geometry as the arena for physical phenomena on all scales.

In a quantum description that includes gravitation, it is natural to consider semi-classical 4-geometries that undergo a change of metric signature together with a possible *topological transition* \cite{[12]-[18]}. It has long been conjectured that classical spacetime may have an intricate topology on a microscopic scale or during a Planckian era, and there have recently been attempts to examine the consequences of such a structure on macroscopic physics, including the possibility that the constants of nature may have their origin in the topology of spacetime \cite{[19]}. A number of cosmological models also envisage a primordial geometry that undergoes a signature transition from Euclidean to Lorentzian thereby separating the origin of classical time from the initial quantum creation of the Universe.

The phenomenon of dynamical spacetime topology change may be accompanied by a dynamical signature change of the spacetime metric. Thus, a more general formulation of gravitation should accommodate geometries with degenerate metrics and non-trivial topologies. In the last year or so, there has been considerable activity in the analysis of classical solutions to Einstein’s gravitational field equations that admit degenerate metrics \cite{[20]-[26]}. Their relevance to a number of approaches to the full quantum theory has also been pursued.

The existence of certain manifolds that cannot sustain a global Lorentzian metric implies that signature is inherently dynamical in theories with dynamical topology. The question is how best to model such effects in the language of quantum field theory.

Percacci \cite{[27]} seems to have been the first to offer a formalism in which one can discuss such notions at the level of effective actions. His approach is to dissociate the conventional
geometrical interrelations between the metric tensor components and a field of coframes, and work in close analogy with the Higg’s model in non-Abelian gauge theories. Classical geometry is then regarded as an interpretation of certain expectation values which minimise an effective action. Greensite [28, 29] has developed this idea further by assuming that a particular pattern of signatures arises dynamically as a result of a dynamical phase field that interpolates between signatures. In particular, he assumes that the effective action can become complex, thereby destroying all vestige of a geometrical interpretation for the gravitational degrees of freedom in the theory. However if one adopts Percacci’s viewpoint then this extension might be pursued at the quantum level, since one should only expect a spacetime geometry to emerge in some classical limit. Greensite has argued that, at least for the free scalar field theory interacting with such a dynamical Wick field, the Lorentzian signature of a four dimensional manifold can be predicted as a ground state expectation value. There are a number of issues raised by this claim. In particular, Elizalde et al [30] have examined the dependence of the result on the influence of other topological configurations on the computation. It appears that the minimising signature is rather sensitive to the details of the field system under consideration. Thus the approach offers a means of exploring self-consistent solutions to effective action theories that can correlate the observed properties of classical gravitation with the quantum structure of matter.

In the present paper we discuss the effective potential of a dynamical Wick field induced by the quantum effects of massive quantum fields in $D$-dimensions. In section 2, after a short review of the formalism of refs. [28, 29], we develop the general structure of the 1-loop effective potential for a dynamical Wick field. In section 3 we discuss an expansion for this potential when topological effects are relevant, i.e. when the radius of the compactified dimension is very small compared with $\Lambda^{1/2}$, where $\Lambda$ is a proper-time cutoff. Section 4 is devoted to study the case of large compactification radius, in addition to mass corrections, for $D = 4$ and $D = 6$. An appendix is included, where we outline the application of the same type of calculation to the study of curvature effects on massless fields in $\mathbb{R}^{D-N} \times S^N$ spacetimes, with $S^N$ meaning
the N-dimensional De Sitter space.

2 Induced effective potential

We describe briefly the formalism employed in [28, 29]. Consider the matrix

\[ \eta_{ab} = \text{diag}(e^{i\theta}, 1, 1, \ldots, 1). \] (2.1)

We shall refer to this as a complexified spacetime metric. It has a Euclidean signature for \( \theta = 0 \), and a Lorentzian one for \( \theta = \pm \pi \). It was suggested in [28] that the Wick angle \( \theta \) in (2.1) should be treated as a dynamical degree of freedom. In order to effect a Fourier analysis below we shall require that \( \cos \theta / 2 > 0 \) and we shall restrict our attention to values of \( \theta \in [-\pi, \pi] \).

We compute the 1-loop effective potential \( V(\theta) \) as a function of \( \theta \) under the following assumptions [28, 29]:

Let

\[ Z = \int d\mu(e, \phi, \psi, \bar{\psi}) \exp \left[ -\int d^D x \sqrt{g} L \right], \] (2.2)

with \( e \) standing for the vielbein, \( \phi \) for bosons and \( \psi, \bar{\psi} \) for fermions. \( g_{\mu
u} = e^a_\mu \eta_{ab} e^b_\nu \), with the above \( \theta \)-dependent \( \eta_{ab} \).

Following [28, 29], we assume that the integration measure for scalar fields is given by the real-valued, invariant volume measure (DeWitt measure) in superspace \( d\mu(\phi) = D\phi \sqrt{|G|} \), where \( G \) is the determinant of the scalar field supermetric \( G(x, y) = \sqrt{g} \delta(x - y) \).

From (2.2) and under these conditions, the one-loop potential induced by a free bosonic scalar field of mass \( m_B \) in flat spacetime is given by

\[ -\log \det^{-1/2}[\sqrt{\eta}(\eta^{ab}\partial_a \partial_b - m_B^2)]. \] (2.3)

We work in the representation where the contribution from every free fermion of mass \( m_F \) (neglecting terms proportional to \( \log \det[ \frac{\mathcal{D}_m}{D} ] \)) is \( -\frac{\log \det[D m_F]}{\int d^D x} \), where \( \mathcal{D}_m = \mathcal{D} - m_F \).

\[ \text{the exponential of such terms is a factor which can be absorbed in our integration measure.} \]
Taking into account Dirac conjugacy, one has \( \det[i/\partial - m_F] = \det^{1/2}[\partial^2 - m_F^2] \). Therefore, up to the above mentioned irrelevant terms, this contribution to the one-loop effective potential behaves like (2.3) but with the determinant raised to the power +1/2, instead of −1/2 and \( m_B \) replaced by \( m_F \). After taking the logarithm, this implies a relative sign between boson and fermion contributions. As a result we can write the complete one-loop effective potential \( V(\theta) \) as

\[
V(\theta) = \sum_B V_{m_B}(\theta) - \sum_F V_{m_F}(\theta),
\]

where

\[
V_m(\theta) \equiv \frac{1}{2} \log \det[-\sqrt{-\eta}(\eta^{ab}\partial_a\partial_b - m^2)],
\]

Use of heat-kernel regularization gives [28, 29]

\[
V_m(\theta) = -\frac{1}{2} \int_{\Lambda}^{\infty} ds \int \frac{d^D p}{(2\pi)^D} e^{-s[\alpha p_0^2 + \beta (p^2 + m^2)]},
\]

where \( \Lambda \) is a proper-time cutoff, \( p = \{ p_0, p_1, \ldots, p_{D-1} \} \) and

\[
\alpha \equiv e^{-i\frac{\theta}{2}}, \quad \beta \equiv e^{i\frac{\theta}{2}}.
\]

It is explained in [29] why heat-kernel regularization is to be preferred in this context.

The potential \( V_m(\theta) \) is complex. As discussed in [28, 29], we seek a solution \( \theta = \bar{\theta} \) that ensures:

\[
\text{Im } V(\bar{\theta}) \text{ is stationary,}
\]

\[
\text{Re } V(\bar{\theta}) \text{ is a minimum.}
\]

These conditions (2.8) are essentially *ad hoc* and can only be justified a posteriori.

In order to study topological effects, periodic compactifications of one dimension will be considered (as observed in [30], once \( \mathbb{R}^{D-1} \times S^1 \) has been studied, it is not difficult to extend results to \( \mathbb{R}^{D-n} \times S^n \).) We shall take \( p_1 \), discretized with

\[
p_1^2 = \frac{n^2}{R^2}, \quad n \in \mathbb{Z}.
\]

where \( R \) is some fundamental length. Consequently, in the expression of \( V_m \) we let

\[
\int \frac{dp_1}{2\pi} \to \frac{1}{2\pi R} \sum_{n=-\infty}^{\infty},
\]
and, integrating over the remaining $p$-components, we find

$$V_m(\theta) = -\frac{1}{(4\pi)^{\frac{p+1}{2}} R^\alpha \beta^{\frac{D-2}{2}}} \int_\Lambda^\infty ds \ s^{\frac{D-1}{2} - 1} e^{-s^2 m^2} \theta_3\left(0 \ \frac{s \beta}{\pi R^2}\right), \quad (2.9)$$

where

$$\theta_3(0|z) \equiv \sum_{n=-\infty}^{\infty} e^{-\pi z n^2} \quad (2.10)$$

is a Jacobi theta function.

It is convenient to introduce the dimensionless variables

$$x \equiv \beta \Lambda m^2,$$

$$y \equiv \frac{\beta \Lambda}{R^2}$$

and write

$$V_m(\theta) = -\frac{1}{(4\pi)^{\frac{p+1}{2}} R\Lambda^{\frac{p-1}{2}}} e^{-\frac{\beta\Lambda m^2}{R}} \left[ x^{\frac{D-1}{2}} \Gamma\left(-\frac{D-1}{2}, x\right) + 2 \sum_{n=1}^{\infty} (x + yn^2)^{\frac{D-1}{2}} \Gamma\left(-\frac{D-1}{2}, x + yn^2\right) \right]. \quad (2.12)$$

in terms of the incomplete gamma function

$$\Gamma(a, z) \equiv \int_z^\infty dt \ t^{a-1} e^{-t}. \quad (2.13)$$

It is this expression we analyse as a function of $\theta, x, y, m$. Note our choice of the regularization scheme has made $V$ cutoff dependent.

### 3 Strong topological effect: small $R$ ($|y| \gg 1$)

#### 3.1 Small radius and large mass ($|y| \gg 1, |x| \gg 1$)

We note that $|y| \gg 1$ means $\Lambda/R^2 \gg 1$, which amounts to saying that $R$ is small compared with $\Lambda^{1/2}$. Similarly, $|x| \gg 1$ is equivalent to $m \gg \Lambda^{1/2}/\Lambda$. The asymptotic expansion of the incomplete gamma function \((2.13)\) with a large second argument is

$$x^a \Gamma(-a, x) \sim x^{-1} e^{-x} \left[ 1 + \sum_{k=1}^{\infty} \frac{\Gamma(-a)}{\Gamma(-a - k)} \frac{1}{x^k} \right], \ |x| \gg 1. \quad (3.1)$$
Note that, interpreting the quotient of ordinary gamma functions as a finite product, this holds for both non-integer and integer $a$ (here $a = (D - 1)/2$). Inserting this formula into (2.12), one gets

$$V_m(\theta) = -\frac{1}{(4\pi)^{D+1} R A^{D-1}} e^{-\frac{D+1}{4} x} \left[\frac{e^{-x}}{x} \left(1 + O\left(\frac{1}{x}\right)\right) + 2 \sum_{n=1}^{\infty} \frac{e^{-(x+yn^2)}}{x + yn^2} \left(1 + O\left(\frac{1}{x + yn^2}\right)\right)\right],$$

from which we extract the leading term

$$V_m(\theta) \approx -\frac{1}{(4\pi)^{D+1} R A^{D-1} m^2} e^{-\Lambda m^2 \cos \frac{\theta}{2} - i \left[\frac{D - 1}{4} \theta + \Lambda m^2 \sin \frac{\theta}{2}\right]}.$$  

Typical curves representing $\text{Re} V(\theta)$ and $\text{Im} V(\theta)$ have been plotted in Fig. 1. This sort of potential was already studied by the authors in [30], so that the comment made there applies also to the present case. $V_m(\theta)$ drastically changes its nature depending on the specific value of $\Lambda m^2$. When this quantity is small, the real and imaginary parts of $V_m(\theta)$ are approximately sinusoidal, but as it grows, they tend to form a flat plateau around the origin (notice that our assumption in this subsection implies that $\Lambda m^2$ must be large). Since, under these circumstances there is no coincidence between the minima of $\text{Re} V$ and the stationary points of $\text{Im} V$, we cannot speak of any preferred $\theta$.

### 3.2 Small radius and small mass ($|y| \gg 1, |x| \ll 1$)

Now, in order to expand the first term in (2.12), we need a power series for the same gamma function when its second argument, $x$, is small. The expansion is

$$x^a \Gamma(-a, x) = \begin{cases} x^a \Gamma(-a) & a \neq 0, 1, 2, \ldots \\ x^a \frac{(-1)^a}{a!}[\psi(a + 1) - \ln x] - \sum_{n=0}^{\infty} \frac{(-1)^n}{(n - a)n!} x^n, & a = 0, 1, 2, \ldots \end{cases}$$

An equivalent series was employed in [29]. Note, however, that our expression for integer $a$ is handier than the one used in that reference. In fact, we have obtained it from the noninteger-$a$
case by analytic continuation in a, and its particular forms for every value of this parameter coincide with the ones resulting from \[29\]. In any case, the power series obtained will be denoted by

\[ x^a \Gamma(-a, x) = b_a^{(a)} x^a + b_L^{(a)} x^a \ln x + \sum_{n \geq 0} b_n^{(a)} x^n. \]  

(3.5)

Since the values of interest are \( a = (D - 1)/2 \), we list the special forms corresponding to the first integer and half odd integer a’s in Table 1. In particular the fifth and seventh rows will be used in the cases to be studied later.

| a     | power series for \( x^a \Gamma(-a, x) \)                                                                 |
|-------|----------------------------------------------------------------------------------------------------------|
| 0     | \(-\gamma - \ln x + x - \frac{x^2}{4} + \frac{x^3}{18} + O(x^4)\)                                      |
| \(\frac{1}{2}\) | \(2 - 2 \sqrt{\pi} x^{1/2} + 2x - \frac{x^2}{3} + \frac{x^3}{15} + O(x^4)\)                           |
| 1     | \(1 + (\gamma - 1)x + x \ln x - \frac{x^2}{2} + \frac{x^3}{12} + O(x^4)\)                             |
| \(\frac{3}{2}\) | \(2 - 2x + \frac{4}{3} \sqrt{\pi} x^{2/3} - x^2 + \frac{x^3}{9} + O(x^4)\)                          |
| 2     | \(\frac{1}{2} - x + \frac{1}{2} \left(3 - \gamma\right) x^2 - \frac{1}{2} x^2 \ln x + \frac{x^3}{6} + O(x^4)\) |
| \(\frac{5}{2}\) | \(2 - \frac{2}{3} x + x^2 - \frac{8}{15} \sqrt{\pi} x^{5/2} + \frac{x^3}{3} + O(x^4)\)            |
| 3     | \(\frac{1}{3} - \frac{x}{2} + \frac{x^2}{2} - \frac{1}{6} \left(\frac{11}{6} - \gamma\right) x^3 + \frac{1}{6} x^3 \ln x + O(x^4)\) |

Table 1: \( x^a \Gamma(-a, x) \) as a power series in \( x \).

We substitute (3.5) in the first term of (2.12), while the remaining contribution is expressed in terms of the asymptotic expression (3.1)

\[
V_m(\theta) = -\frac{1}{(4\pi)^{\frac{D+1}{2}}} RA R^{\frac{D-1}{2}} e^{-\frac{D-3}{2}\theta} \left[ b_{\frac{D-1}{2}}^{(\frac{D-1}{2})} x^{\frac{D-1}{2}} + b_{\frac{D-1}{2}}^{(\frac{D-1}{L})} x^{\frac{D-1}{2}} \ln x + \sum_{n \geq 0} b_n^{(\frac{D-1}{2})} x^n \right] \\
+ 2 \sum_{n=1}^{\infty} \frac{e^{-(x+yn)^2}}{x+yn^2} \left(1 + O \left(\frac{1}{x+yn^2}\right)\right). \]  

(3.6)
When \( x = 0 \) and \( |y| \to \infty \), the only contribution left is the leading term \( b_0 \) in the (3.5) series, which, for \( D > 1 \) takes on the value

\[
b_0 = \frac{2}{D - 1}.
\]  
(3.7)

Then the potential reads

\[
V_m(\theta) \approx -\frac{2}{(D - 1)(4\pi)^{\frac{D+1}{2}} R \Lambda^{\frac{D-1}{2}}} e^{-i\frac{D-3}{4} \theta}.
\]  
(3.8)

If several fermions and bosons are present, this contribution appears multiplied by \( -\Delta n \equiv -(n_F - n_B) \), where \( n_F \) (\( n_B \)) is number of fermions (bosons). In this limit one recovers the result for massless particles found in [30]. As reported in that reference, the only solution satisfying the requirements (2.8) is \( n_F > n_B \) and \( \theta = \pm \pi, D = 5 \).

4 Weak topological effect: large \( R \) (\( |y| \ll 1 \))

Going back to the initial form of \( V_m(\theta) \), we apply the Jacobi theta function identity

\[
\theta_3(0|z) = \frac{1}{\sqrt{z}} \theta_3 \left( 0 \left| \frac{1}{z} \right. \right).
\]  
(4.1)

to eq. (2.9). The resulting integrals will be evaluated by conveniently splitting the integration domains and using

\[
\int_0^\infty ds \, s^{\nu - 1} e^{-as - \frac{b}{s}} = 2 \left( \frac{b}{a} \right)^{\frac{\nu}{2}} K_{\nu}(2\sqrt{ab})
\]  
(4.2)

(\text{where } K_{\nu} \text{ is the modified Bessel function of index } \nu), \text{ together with (3.1) in combination with the exponential power series. We find, for each } n,

\[
\int_{\Lambda}^\infty ds \, s^{-\frac{D}{2} - 1} e^{-\frac{\pi^2 R^2 n^2}{\beta s}} - \beta m^2 s =
\]

\[
2 \left( \frac{\pi R n}{\beta m} \right)^{-\frac{D}{2}} K_{-\frac{D}{2}} \left( 2\pi nmR \right) - \frac{1}{\Lambda^{\frac{D}{2}}} \left( \frac{\pi^2 R^2 n^2}{\beta \Lambda} \right)^{-1} e^{-\frac{\pi^2 R^2 n^2}{\beta \Lambda}} - \beta m^2 \Lambda \left[ 1 + O \left( \frac{\pi^2 R^2 n^2}{\beta \Lambda} \right)^{-1} \right]
\]  
(4.3)
The effective potential becomes

\[ V_m(\theta) = -\frac{\pi^{1/2}}{(4\pi)^{D-1/2}\alpha^{1/2}\beta^{D-1/2}\Lambda^{D/2}} \left[ x^{D/2} \Gamma \left( -\frac{D}{2}, x \right) \right. \]

\[ + 4x^{D/2} \sum_{n=1}^{\infty} \left( \frac{\pi n}{\sqrt{y}} \right)^{-D/2} K_{-D/2} \left( 2\pi n \sqrt{\frac{x}{y}} \right) \]

\[ - 2 \sum_{n=1}^{\infty} \frac{y}{\pi^2 n^2} e^{-\frac{s^2 \pi^2}{n^2}} \left( 1 + O \left( \frac{y}{\pi^2 n^2} \right) \right) \]  

(4.4)

We discuss this for various values of \(|x|\) and \(|y|\).

**4.1 Large radius and large mass** (\(|y| \ll 1, |x| \gg 1\))

We consider the domain where

\[ \left| \frac{x}{y} \right| \gg 1. \]

We need an expression for \(V_m\) asymptotically valid for large values of \(a \equiv \sqrt{x/y}\). To this end, we take the usual asymptotic series of \(K_{\nu}\) and carry out its commutation through the \(n\)-series, as explained in [31]. Thus, we arrive at

\[ \sum_{n=-\infty}^{\infty} \left( \frac{\pi n}{a} \right)^{s-1/2} K_{s-1/2}(2\pi an) = \]

\[ \frac{1}{2} \Gamma \left( s - \frac{1}{2} \right) a^{-2s+1} + \pi^{s-1/2} a^{-s} \sum_{n=0}^{\infty} \frac{\Gamma(s+n)}{\Gamma(s-n)n!} \frac{1}{(4\pi a)^n} \text{Li}_{n+1}(e^{-2\pi a}). \]  

(4.5)

where \(\text{Li}\) stands for the polylogarithm function

\[ \text{Li}_s(z) \equiv \sum_{n=1}^{\infty} \frac{z^n}{n^s}. \]  

(4.6)

(4.5) is valid when \(s > 0\), which is satisfied here since \(s = (D - 1)/2\) and we are taking \(D > 1\). Otherwise, extra terms arising from the commutation procedure can appear. Notice, too, that the \(n\)-series in (4.5) is truncated for positive integer \(s\) (odd \(D\)) due to the pole of \(\Gamma(s - n)\) downstairs.
Using (3.1) for the first term in (4.4) together with the above we have

\[ V_m(\theta) = -\frac{\pi^{1/2}}{(4\pi)^{D+1}} \Lambda^\frac{D}{2} e^{-i\theta - 2\theta} \left\{ \frac{e^{-x}}{x} \left[ 1 + O\left( \frac{1}{x} \right) \right] + y \frac{D}{2} \left( \frac{x}{y} \right)^{D-4} e^{-2\pi \sqrt{x/y}} + O\left( \left( \frac{x}{y} \right)^{D-5} e^{-2\pi \sqrt{x/y}}, \ldots, \left( \frac{x}{y} \right)^{D-1} e^{-4\pi \sqrt{x/y}}, \ldots \right) \right\} \]

\[ -2 \sum_{n=1}^{\infty} \frac{y}{n^2} e^{-\frac{2\pi^2}{y} - x} \left[ 1 + O\left( \frac{y}{\pi^2 n^2} \right) \right]. \]  

(4.7)

The infinite R or topologically trivial limit is \( y = 0 \) and in this limit:

\[ V_m(\theta) \approx -\frac{\sqrt{\pi}}{(4\pi)^{D+1}} \Lambda^\frac{D}{2} m^2 e^{-2\pi \sqrt{x/y}} \left( -\Lambda m^2 \cos \frac{\theta}{2} - i \left[ \frac{D}{4} \theta + \Lambda m^2 \sin \frac{\theta}{2} \right] \right), \]

(4.8)

This is similar to (3.3) and has already been discussed.

4.2 Large radius and small mass \((|y| \ll 1, |x| \ll 1)\)

Since these conditions are not enough to fix the magnitude of \(|x|/|y| = (mR)^2\) we shall suppose

\[ \left| \frac{x}{y} \right| \ll 1, \]  

(4.9)

The opposite case has already been discussed in the previous subsection.

Fortunately there is an identity linking our Bessel function series and a particular zeta function, namely

\[ E(s; a) \equiv \sum_{n=-\infty}^{\infty} (n^2 + a^2)^{-s} = \frac{2\sqrt{\pi}}{\Gamma(s)} \sum_{n=-\infty}^{\infty} \left( \frac{\pi n}{a} \right)^{s-1/2} K_{s-1/2}(2\pi an), \]

(4.10)

(for its derivation, see e.g. [31]) which will be helpful in the evaluation of the potential (4.4) for small values of \( a \equiv \sqrt{x/y} \). It must also be born in mind that in the end we take \( s = -(D - 1)/2, D = 2, 3, \ldots \). In fact, for our purpose it will be sufficient to know of an expansion.
of $E(s; a)$ in powers of $a$, with due consideration of these values of $s$. A similar problem was addressed in [32] (see also [33]), and here we just quote the (adequately modified) result:

$$\Gamma(s)E(s; a) =$$

$$\begin{cases}
\Gamma(s)a^{-2s} - \sqrt{\pi} \Gamma\left(s - \frac{1}{2}\right) a^{1-2s} \Theta\left(\frac{1}{2} - s\right) \\
+ 2 \sum_{n=0}^{\infty} (-1)^n \frac{\Gamma(s + n)}{n!} \zeta(2n + 2s) a^{2n}, & s \neq \frac{1}{2} - m,
\end{cases}$$

$$\begin{cases}
2 \sum_{n=0}^{-s-1} (-1)^n \frac{\Gamma(s + n)}{n!} \zeta(2n + 2s) a^{2n}, & s = -m,
\end{cases}$$

$$\Gamma(s)a^{-2s} + \frac{(-1)^{1/2-s}\sqrt{\pi} a^{1-2s}}{\left(\frac{1}{2} - s\right)!} \left[ \psi\left(\frac{1}{2}\right) + \psi\left(\frac{1}{2} - s\right) - \ln a^2 + 2\gamma \right]$$

$$+ 2 \sum_{n=0}^{\infty} (-1)^n \frac{\Gamma(s + n)}{n!} \zeta(2n + 2s) a^{2n}, & s = \frac{1}{2} - m,$$

where $m = 0, 1, 2, \ldots$. The coefficients of these series are to be understood with the aid of the Riemann zeta function reflexion formula

$$\Gamma\left(\frac{z}{2}\right) \zeta(z) = \pi^{z-1/2}\Gamma\left(\frac{1-z}{2}\right) \zeta(1-z). \quad (4.12)$$

In any of the above cases, the result found will be represented by the series

$$\Gamma(s)E(s; a) = c_{-2s}^{(-s)} a^{-2s} + c_{1-2s}^{(-s)} a^{1-2s} + c_{(L)}^{(-s)} a^{1-2s} \ln a^2 + \sum_{n \geq 0} c_n^{(-s)} a^{2n}. \quad (4.13)$$

Special values when $s$ is half an odd negative integer or a negative integer $s$ are listed in Table [4].
We now replace the Bessel series in (2.12) with the appropriate form above, getting

\[
V_m(\theta) = -\pi^\frac{3}{2} e^{-i\frac{D-2}{4}\theta} \left\{ \frac{b_0(\frac{D}{2})}{4\pi} x^\frac{D}{2} + \frac{b_1(\frac{D}{2})}{4\pi} x^\frac{D}{2} \ln x + \sum_{n=0} b_n(\frac{D}{2}) x^n \right. 
\]

\[
+ \frac{y^\frac{D}{2}}{\sqrt{\pi}} \left[ c_{D-1}(\frac{D}{2}) \left( \frac{x}{y} \right)^\frac{D-1}{2} + c_{D-2}(\frac{D}{2}) \left( \frac{x}{y} \right)^\frac{D-2}{2} + c_{D}(\frac{D}{2}) \left( \frac{x}{y} \right)^\frac{D}{2} \ln \left( \frac{x}{y} \right) \right] 
\]

\[
+ \sum_{n=0} c_n(\frac{D}{2}) \left( \frac{x}{y} \right)^n 
\]

\[
-2 \sum_{n=1}^\infty \frac{y}{\pi^2 n^2} e^{-x^2 y} \left[ 1 + O \left( \frac{y}{\pi^2 n^2} \right) \right] \left( \frac{y}{\pi^2 n^2} \right) \right\\} 
\]

The simplest case to be considered is the topologically trivial limit \( y = 0 \) when the main contribution comes from the leading term \( b_0 \) given by (3.7),

\[
V_m(\theta) \approx -\frac{2\sqrt{\pi}}{(D-1)(4\pi)^\frac{D+1}{2} \Lambda^\frac{D}{2}} e^{-i\frac{D-2}{4}\theta}. 
\]

When several fermions and bosons exist, (4.15) is multiplied by \( -(n_F - n_B) \). As may be expected, this coincides with the induced potential corresponding to massless fields in a no-
ncompatified spacetime, and has been considered in [28, 29]. Ruling out the special case $n_F = n_B$ or $D = 2$ (which make $V$ independent of $\theta$), the only solution which satisfies (2.8) is $n_F > n_B$ and $\theta = \pm \pi, D = 4$. Thus, under these conditions, a Lorentzian signature is singled out by the dynamics only in $D = 4$.

Deviations of this result due to mass corrections have been analysed in ref. [29] in which all dimensions were considered non-compact. It is therefore of interest to study the modifications induced by compactification of various dimensions. This requires the inclusion of $y$ contributions in (4.14).

In order to be able to handle (order by order) the resulting combined series, we will have to make a further assumption on the relative size of $|x|$ and $|y|$. A simple hypothesis consistent with (4.9) is

$$O(|x|) = O(|y|^2).$$

(4.16)

With this hypothesis we next consider the ensuing modifications on the most interesting cases found in [29].

4.2.1 $D = 4$

From (4.14), and using Tables 1 and 2, we read off the induced effective potential for a boson, up to orders of $|x|^3$ or equivalent:

$$V_m(\theta) = -\frac{1}{2(4\pi)^2 \Lambda^2} \left\{ \frac{1}{2} e^{-i\frac{\theta}{2}} - |x| + \left[ \frac{|y|^2}{45} - \frac{1}{3} |x||y| + \frac{4}{3} |x|^{3/2}|y|^{1/2} + \frac{1}{2} \left( \frac{3}{2} - \gamma \right) |x|^2 - \frac{|x|^2}{2} \left( \ln |x| + i\frac{\theta}{2} \right) \right] 
+ \frac{1}{2} |x|^2 \left( \psi \left( \frac{1}{2} \right) + 1 + \gamma - \ln \frac{|x|}{|y|} \right) - \frac{\zeta(3) |x|^3}{6 |y|} + \frac{\zeta(5) |x|^4}{80 |y|^2} e^{i\frac{\theta}{2}} 
+ \frac{|x|^3}{6} e^{i\theta} + O(|x|^{7/2}, \ldots) \right\}$$

(4.17)

Imposing the stationarity condition

$$\left. \frac{\partial}{\partial \theta} \text{Im } V_m(\theta) \right|_{\theta = \bar{\theta}} = 0$$

(4.18)
on (4.17), we find
\[
1 - \frac{2}{45}|y|^2 + \frac{2}{3}|x||y| - \frac{8}{3}|x|^{3/2}|y|^{1/2} + |x|^2 \left( -\frac{3}{2} - \psi \left( \frac{1}{2} \right) + \ln \frac{|x|^2}{y} \right) + \frac{\zeta(3)}{3} \frac{|x|^3}{|y|} + \frac{\zeta(5)}{40} \frac{|x|^4}{|y|^2} \cos \frac{\bar{\theta}}{2} \\
- \frac{|x|^2}{2} \bar{\theta} \sin \frac{\bar{\theta}}{2} + \frac{2}{3}|x|^3 \cos \bar{\theta} + O(|x|^{7/2}, \ldots) = 0. \tag{4.19}
\]

We assume a solution around \( \theta = \pm \pi \) of the type
\[\bar{\theta} = \pm \pi + 2\varepsilon, \tag{4.20}\]
and find the approximate solution
\[\bar{\theta} \approx \pm \pi \left[ 1 - |x|^2 \left( 1 + \frac{2}{45}|y|^2 \right) \right]. \tag{4.21}\]

The results so far apply to the case where only one boson is present. We will now generalise them to include bosons of mass \( m_B \) and fermions of mass \( m_F \). For that purpose we introduce the notations
\[
\begin{align*}
\Delta n &= n_F - n_B, \\
\Delta |x|^n &= \sum_F |x_F|^n - \sum_B |x_B|^n, \quad \left( |x_F| \equiv \Lambda m_F \right), \\
\Delta(|x|^n \ln |x|) &= \sum_F |x_F|^n \ln |x_F| - \sum_B |x_B|^n \ln |x_B|.
\end{align*}
\]

In order to generalise the previous expressions, it is enough to make the following replacements
\[
\begin{align*}
V(x, y) &\rightarrow \sum_B V(x_B, y) - \sum_F V(x_F, y) \\
y-independent terms &\rightarrow y-independent terms \cdot (-\Delta n) \\
|x|^n &\rightarrow -\Delta |x|^n \\
|x|^n \ln |x| &\rightarrow -\Delta(|x|^n \ln |x|).
\end{align*}
\]

With these changes the solution becomes
\[\bar{\theta} \approx \pm \pi \left[ 1 - \frac{\Delta |x|^2}{\Delta n} \left( 1 + \frac{2}{45}|y|^2 \right) \right]. \tag{4.24}\]
The \( y \)-independent correction is the one found in \[29\], while the \(|y|^2\)-term provides a first measure of the deviation due to a large-\( R \) effect. Making also the hypothesis
\[
\frac{\Delta |x|^2}{\Delta n} < 0,
\]
we find that the approximate \( \bar{\theta} \) falls outside of the domain considered, i.e.
\[
\bar{\theta} \not\in [-\pi, \pi].
\]

Now, we turn our attention to the real part of \eqref{eq:4.17}, which, up to the same order as \( \text{Im} \ V_m \), reads
\[
\text{Re} \ V_m(\theta) = -\frac{1}{2(4\pi)^2 \Lambda^2} \left\{ \left[ \frac{1}{2} + \frac{|y|^2}{45} - \frac{1}{3} |x| |y| + \frac{4}{3} |x|^{3/2} |y|^{1/2} + \frac{1}{2} \left( \frac{3}{2} - \gamma \right) |x|^2 - \frac{|x|^2}{2} \ln |x| \ight.ight.
\]
\[
\left. + \frac{1}{2} |x|^2 \left( \psi \left( \frac{1}{2} \right) + 1 + \gamma - \ln \left| \frac{x}{y} \right| \right) - \frac{\zeta(3) |x|^3}{6 |y|^3} + \frac{\zeta(5) |x|^4}{80 |y|^4} \right\} \cos \frac{\theta}{2}
\]
\[
- \frac{|x|^2}{4} \theta \cos \frac{\theta}{2} + \frac{|x|^3}{6} \sin \theta + O(|x|^{7/2}, \ldots).
\]
\[
\text{(4.26)}
\]
Its leading term is proportional to \( \cos \theta/2 \) so generalising to include all masses we have
\[
\text{Re} \ V(\theta) = \frac{1}{2(4\pi)^2 \Lambda^2} \left[ \Delta n \left( \frac{1}{2} + \frac{|y|^2}{45} \right) \cos \frac{\theta}{2} + \Delta |x| + O(\Delta |x|^{3/2}) \right].
\]
\[
\text{(4.27)}
\]
Notice that, if \( n_F > n_B \), one has \( \Delta n > 0 \) and the leading term is for \( \theta \in [-\pi, \pi] \). Obviously, when restricted to this interval, this curve has two absolute minima at \( \theta = \pm \pi \). The first correction is \( \theta \)-independent, and leaves the location of these minima unchanged.

### 4.2.2 \( D = 6 \)

For \( D = 6 \), eq. \eqref{eq:4.14} and the information in Tables 1 and 2 give
\[
V_m(\theta) = -\frac{1}{2(4\pi)^3 \Lambda^3} \left\{ \frac{1}{3} e^{-i\theta} - \frac{1}{2} |x| e^{-i\frac{\theta}{2}} + \frac{|x|^2}{2} \right.
\]
\[
\left. + \left[ -\frac{2}{945} |y|^3 - \frac{1}{45} |x| |y|^2 + \frac{1}{6} |x|^2 |y| + \frac{4}{15} |x|^{5/2} |y|^{1/2} \right.ight.
\]
\[
\left. + \frac{1}{6} |x|^3 \left( \frac{10}{3} + \ln \frac{|x|}{|y|} - \psi \left( \frac{1}{2} \right) + i \frac{\theta}{2} \right) \right] e^{-i\frac{\theta}{2}}
\]
\[
+ O(|x|^{7/2}, \ldots) \right\},
\]
\[
\text{(4.28)}
\]
and the stationarity condition on its imaginary part reads

\[-\frac{4}{3} \cos \bar{\theta} + \left[ |x| - \frac{4}{945} |y|^3 + \frac{2}{45} |x| |y|^2 + \frac{1}{3} |x|^2 |y| + \frac{8}{15} |x|^{5/2} |y|^{1/2} + \frac{1}{3} |x|^3 \left( -\frac{17}{6} - \psi \left( \frac{1}{2} \right) + \ln \frac{|x|^2}{|y|} \right) \right] \cos \frac{\bar{\theta}}{2} - \frac{|x|^3}{6} \bar{\theta} \sin \frac{\bar{\theta}}{2} + O(|x|^7/2, \ldots) = 0. \tag{4.29} \]

We now make the replacements (4.23) on \( V_m \). Further, following [29] we suppose

\[ \Delta n = 0, \tag{4.30} \]

and realize that condition (4.29) becomes

\[-\left[ \Delta |x| + \frac{2}{45} \Delta |x| |y|^2 + \frac{1}{3} \Delta |x|^2 |y| + \frac{8}{15} \Delta |x|^{5/2} |y|^{1/2} \right. \]

\[ + \frac{1}{3} \Delta |x|^3 \left( -\frac{17}{6} - \psi \left( \frac{1}{2} \right) - \ln |y| \right) + \frac{2}{3} \Delta (|x|^3 \ln |x|) \] \cos \frac{\bar{\theta}}{2} \]

\[ + \frac{\Delta |x|^3}{6} \bar{\theta} \sin \frac{\bar{\theta}}{2} + O(\Delta |x|^{7/2}, \ldots) = 0. \tag{4.31} \]

Solving as before, we find an approximate solution to be

\[ \bar{\theta} \approx \pm \pi \left[ 1 - \frac{1}{3} \frac{\Delta |x|^3}{\Delta |x|} \left( 1 - \frac{2}{45} |y|^2 \right) \right], \tag{4.32} \]

where, again, the \( y \)-dependent part provides the first large \( R \) correction to the result in [29].

One might think that this correction vanishes for \( |y| = \sqrt{45}/2 \), but this value lies outside the present small-\( |y| \) approximation. So, taking \( |y| < \sqrt{45}/2 \), everything depends on \( \Delta |x|^3/\Delta |x| \).

Assuming

\[ \frac{\Delta |x|^3}{\Delta |x|} < 0, \tag{4.33} \]

we are lead to the same situation as in the previous case, namely, that the stationary points
are outside of the domain $[-\pi, \pi]$. The real part of (4.28) is
\[
\text{Re } V_m(\theta) = -\frac{1}{2(4\pi)^3 \Lambda^3} \left\{ \frac{1}{3} \cos \theta + \frac{|x|^2}{2} \right. \\
+ \left[ -\frac{1}{2} |x| - \frac{2}{945} |y|^3 + \frac{1}{45} |x||y|^2 + \frac{1}{6} |x|^2 |y| + \frac{5}{15} |x|^{5/2} |y|^{1/2} \\
+ \frac{1}{6} |x|^3 \left( -\frac{10}{3} - \psi \left(\frac{1}{2}\right) + \ln \frac{|x|^2}{|y|} \right) \right. \cos \frac{\theta}{2} \\
\left. \left. + \frac{|x|^3}{12} \theta \sin \frac{\theta}{2} + O(|x|^{7/2}, \ldots) \right\}.
\]
(4.34)

Generalizing by means of (4.23), and recalling the assumption (4.30), we find its first nonvanishing terms to be
\[
\text{Re } V(\theta) = -\frac{1}{2(4\pi)^3 \Lambda^3} \left[ \frac{1}{2} \Delta |x| \cos \frac{\theta}{2} + \frac{1}{2} \Delta |x|^2 + O(\Delta |x|^2 |y|, \Delta |x|^3, \ldots) \right].
\]
(4.35)

If
\[
\Delta |x| > 0,
\]
(4.36)
we get the same as in the $D = 4$ case, \textit{i.e.} the leading term is a nonnegative sinusoidal, vanishing at $\theta = \pm \pi$, multiplied by a positive constant, and the next-to-leading correction is $\theta$-independent. Therefore, the conclusion is the same. Within this approximation, the minima of $\text{Re } V$ on $[-\pi, \pi]$ are at $\theta = \pm \pi$.

5 Closing Remarks

Regarding signature as a dynamical quantity, we have examined its influence on the one-loop effective potential. Using Grensite’s criteria for a stable potential we have analysed the dependence of a preferred signature on both the inclusion of massive fields and a compactified background. We have argued that a flat metric with a Lorentzian signature on $\mathbb{R}^{D-1} \times S^1$ may arise in a number of distinct ways.

We extend the solutions found previously in $D = 4$ and $D = 6$ by examining their stability under the influence of compactification and massive fields. For suitably large masses the nature of potential as a function of the compactification radius ((3.3) and (4.8)) indicates...
that the preferred signature is insensitive to compactification. If the requirements on the
solution are relaxed slightly by accepting approximate coincidence of minima and stationary
points, we get similar results in 4- and 6-dimensional universes with small $m$ and close to
the topologically trivial limit $R \rightarrow \infty$. Assuming that $\Lambda$ and $m^2 R^4$ are comparable ($\Lambda^2 / R^4 \rightarrow \infty$)
deviations from the previous cases have been evaluated in first approximation. Eqs. (4.21)
and (4.32) provide a first measure of the separation from Greensite’s stationary points ($\bar{\theta}$)
for small $m > 0$ due to weak compactification effects in $R^3 \times S^1$ and $R^5 \times S^1$, respectively.
The value of $\bar{\theta} = (\pm \pi)$ found in ref. [29] receives now a multiplicative correction of the form
$(1 + \text{constant} |y|^2)$, where $|y|^2 = \Lambda^2 / R^4$. The radius-independent parts are in agreement with
ref. [29], which dealt with topologically trivial spaces.

In view of these results one may argue that, for our present universe with $D = 4$, 1-loop
quantum fluctuations away from a flat metric with a Lorentzian signature are largely suppressed
according to the hypotheses that we have adopted. Clearly this result may be modified by the
inclusion of gravity.

A Appendix: curvature effects in De Sitter spaces

The same sort of analysis can be easily applied to the study of curvature effects in the presence
of the dynamical Wick angle. As an example, we consider the potential for a massless scalar
field in an $R^{D-N} \times S^N$ spacetime, where $S^N$ is an $N$ dimensional De Sitter space of radius $a$.
As usual $\xi$ will denote the coupling with the scalar curvature $R$, so that our operator is now
$\Box + \xi R$. After integrating over the continuous momentum components, the effective potential reads

$$
V(\theta) = -\frac{[f(\theta)]^{\frac{D-N}{2}}}{2(4\pi)^{\frac{D-N}{2}} \text{Vol}(S^N)} \sum_{n=0}^{\infty} (\bar{\lambda}_n y)^{-\frac{D-N}{2}} \Gamma \left( -\frac{D-N}{2}, \bar{\lambda}_n y \right),
$$

(A.1)

where $\bar{\lambda}_n$ and $d_n$ are the dimensionless eigenvalues and their degeneracies

$$
\begin{align*}
\lambda_n &= \frac{1}{a^2} \bar{\lambda}_n = \frac{1}{a^2} [n^2 + n(N-1) + \xi Ra^2], \\
d_n &= (2n + N - 1) \frac{(n + N - 2)!}{n!(N-1)!},
\end{align*}
$$

(A.2)
and

\[ f(\theta) = e^{i\frac{\theta}{2}}, \]
\[ y = f(\theta) \frac{\Lambda}{\sqrt{\sigma}}. \]  

(A.3)

First, we look at the large-\(|y|\) limit, which implies small radius or large curvature. If \(\xi \neq 0\), the leading term of the resulting expansion has the form

\[ V(\theta) \approx -\frac{1}{2(4\pi)^{D-N} \text{Vol}(S^N) \xi \Lambda} e^{-i\frac{D-N}{4}\theta - \xi \Lambda \left(\frac{\cos \theta}{2} + i \frac{\sin \theta}{2}\right)}. \]  

(A.4)

Since we suppose \(R\) to be large, this gives a plateau-like potential, which indicates the above commented lack of preference for any signature. On the other hand, carefully repeating the calculation for \(\xi = 0\) one obtains

\[ V(\theta) \approx -\frac{1}{(D-N)(4\pi \Lambda)^{D-N} \text{Vol}(S^N)} e^{-i\frac{D-N-2}{4}\theta}. \]  

(A.5)

In the light of the previous discussions, it is clear that this produces Lorentzian signature when \(D = N + 4\), i.e. for spacetimes of the type \(\mathbb{R}^4 \times S^N\), which includes the Kaluza-Klein-style solution for \(N = 1\).

Of course, taking the opposite case, or small-\(|y|\), brings nothing new. This corresponds to the flat-space limit and, once more, gives a main contribution to \(V(\theta)\) of the form constant \(\cdot e^{-i\frac{D-2}{4}\theta}\), which reproduces Greensite’s original set-up, with its ensuing Lorentzian solution for \(D = 4\).

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Figure Caption

**Figure 1.** Changing nature of a (rescaled) potential of the type $v(\theta) = e^{-B \cos \frac{\theta}{2} + i \left( A\theta + B \sin \frac{\theta}{2} \right)}$, for $-\pi \leq \theta \leq \pi$. Curves drawn in solid and dashed line represent Re $v(\theta)$ and Im $v(\theta)$, respectively. The plots shown correspond to fixed $A = 0.75$ and to different values of $B$: (a) $B = 1$, (b) $B = 10$. In (b) the formation of a wide plateau around the origin is already noticeable. The width of this plateau can be seen to increase as $B$ grows. Changes in the value of $A$ do not produce significant alterations on this varying behaviour.
This figure "fig1-1.png" is available in "png" format from:

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