Dynamic analysis and modeling of three-dimensional crane incorporating payload

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Abstract. This study developed the efficient dynamic models and system dynamics analysis on three-dimensional (3D) crane with a default load. The direction of movement that will be discussed in the 3D crane system is the simultaneous movement on the X axis, up and down in the direction of the Y axis and simultaneously moving in the direction of the Z axis. The dynamic equation of movement of the 3D crane system uses the Lagrange method and non-linear differential equations. System dynamics analysis was performed using Matlab/ Simulink to observe dynamic behaviour in the 3D crane system in the time domain and frequency domain. The response of the 3D crane system includes rail position, trolley and default load response, and default load swing angle. The results showed for the swing angle, α showed a swing angle that was significant at ± 0.09 rad, while the swing angle, β shows a significant angle at ± 0.07 rad. The system oscillates at ± 0.009 rad.

1. Introduction
Crane technology has increasingly used, especially in the fields of transportation and construction. Crane technology occupies an important role in the industry. Crane is used to perform important and challenging tasks such as the construction of bridges, dams, buildings, and high-rise towers. Robot's arms are also very much needed in industry [1] such as cranes that are widely used to transport heavy loads and hazardous materials in shipyards, factories and warehouses.

A crane consists of a mechanical lift, a rope as a lifting and carrying load, a hook and a mechanic supporting trolley-girder and trolley-jib. The mechanical support moves at the suspension point around the crane workspace. Mechanical lifters lift and reduce load loads to avoid obstacles in the movement path and so that load loads move on the target point area [2], [3].

Crane technology can be classified based on the degree of freedom of mechanical movements and suspension points [4]. The mechanical support of crane cranes consists of trolleys that move on rails / bridges. In some cranes, rails / bridges are installed with orthogonal fences in horizontal fields. This arrangement allows one or two suspension points in the horizontal plane. Crane support pole, rail / bridge rotate in a horizontal plane around a fixed vertical axis. This allows the suspension point of the two patterns of movement in the horizontal plane namely displacement and rotation. The suspension point on the crane is fixed at the end of the movement. This suspension point has two rotational movement patterns around two orthogonal axes located at the base of the pole.

Several studies have developed dynamic models on 3D crane systems. The development of modelling and control methods for the 3D crane system has been presented by [5]. The performance of the dynamic crane model to determine the optimal speed in minimizing load swing has been investigated by [6], [7].
Modelling of the crane system is very important to get a precise model. Very fast swing conditions when the 3D crane system carries a load are the problem that often occurs so that it is dangerous for anyone who works with the 3D crane system. Fast motion maneuvers resulting in rope swing are increasing so that they can reduce the performance of the overall 3D crane system [8].

The purpose of this simulation is to get a dynamic model of the 3D crane system based on several approaches. Next is also to analyse the dynamic behaviour of the 3D crane system.

In modelling the crane 3D system there are two methods that are often used, namely the method of mass equation (lumped-mass) or often called the reduction method (reduced method) and the method of expansion (extended method). The most widely used approach is the lumped-mass model. In this model, the power of the rope lift is considered to be sourced from a massless rope. The rope that carries the load is modelled as a pendulum so that a simple representation of mathematical equations is produced which describes the complex dynamics of the load movement. The lumped-mass model can be categorized into two classes, depending on how the external connector is connected to the system, namely by reduced (reduced model) or extended (extended) models [4].

The extended model is a dynamic model of the crane's mechanical support system, load carrier rope and the main part of the crane. The extended model is a unique way to capture a dynamic crane model.

In this simulation, the 3D crane system modelling was developed using the Lagrange equation. Matlab and Simulink are used to simulate and observe system behaviour. The verification process of the results of dynamic modelling on the 3D crane system is done by comparing dynamic models that have been published in several international journals.

2. Method

The method of combining non-linear and linear models to obtain the equation of the load system, trolley, rails, rope length and swing angle. The 3D crane system is developed based on the swing angle along the degree of freedom (DOF). The equation of motion of the 3D crane system obtained using the Lagrange equation. Furthermore, the dynamic equations of motion of the crane system are represented in differential equations. The 3D crane mathematical model is simulated with Matlab and Simulink to observe the dynamic behaviour of system responses. The system response observed is the position of the rail $x(t)$, trolley position $y(t)$, and the angle of load swing $\alpha$ and $\beta$. System responses are analysed in the time domain and frequency domain.

3D crane system with built-in load consists of three components: pendulum, trolley and rail. Figure 1 shows a diagram of the 3D crane system used in this modelling. The built-in load is attached to the pendulum, which is lifted by the rope, lifted and lowered in the direction of the Z-axis. The trolley moves and is parallel to the rail towards the Y-axis. Rail and trolley make horizontal movements towards the X-axis. Trolley movements on rails and lifting weights can be done in two DOFs.

Using the XYZ as a coordinate system, $m_c$ as a mass of trolleys moves on the X-axis and $m_r$ as the mass of the moving rail on the Y-axis. $\alpha$ shows the load angle associated with the vertical direction (Z-axis) and $\beta$ shows the angular load projection along the X-axis direction. $R$ is the length of the rope. In this observation, the system response observed includes the position of rails, trolleys and loads, and load swings. This is so that rails and trolleys can carry loads to the desired location as quickly as possible with a relatively small swing.
The following are some parameters that are determined to get a dynamic model on a 3D crane system.

trolley mass, \( m_c = 1 \) kg
mass load, \( m_p = 0.7 \) kg
rail mass, \( m_r = 1.5 \) kg
Load rope length, \( l = 0.30 \) m
Gravity, \( g = 9.81 \) m/s\(^2\)
input torque, \( F = 1 \) Nm
damping ratio, \( D_x \) and \( D_y = 0 \)

3. Results and Discussion

Based on Figure 1, the default load position can be formulated as follows:

\[
x_p = x_c + R \sin \alpha \cos \beta
\]

(1)

\[
y_p = y_c + R \sin \alpha \sin \beta
\]

(2)

\[
z_p = z_c - R \cos \alpha
\]

(3)

where \( x_p, y_p \) and \( z_p \) shows the position of load loads on the X, Y and Z axes. \( x_c, y_c \) and \( z_c \) shows the position of the trolley on the X, Y and Z axes. If known \( \dot{x}_p, \dot{y}_p, \dot{z}_p, \dot{x}_c, \dot{y}_c \) and \( \dot{z}_c \) is the first derivative form of \( x_p, y_p, z_p, x_c, y_c \) and \( z_c \). The derivative equation can be written as follows:

\[
\dot{x}_p = \dot{x}_c + (\dot{R} \sin \alpha \cos \beta + R \dot{\alpha} \cos \alpha \cos \beta - R \dot{\beta} \sin \alpha \sin \beta)
\]

(4)

\[
\dot{y}_p = \dot{y}_c + (\dot{R} \sin \alpha \sin \beta + R \dot{\alpha} \cos \alpha \sin \beta + R \dot{\beta} \sin \alpha \cos \beta)
\]

(5)

\[
\dot{z}_p = \dot{z}_c - (\dot{R} \cos \alpha - R \dot{\alpha} \sin \alpha)
\]

(6)
The system equation can be written using the following Lagrange equation:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = Q
\]  

(7)

where \( L = T - P \) is Lagrangian, \( T \) is kinetic energy, \( P \) energy potential, \( q = [x_c, y_c, \alpha, \beta]^T \) are general coordinates and torque input \( Q \). In this simulation use two torque inputs, \( f_x \) and \( f_y \). Dynamic equation of the 3D crane system is in coordinate \( x_c, y_c, \alpha \) and \( \beta \).

The equation of the coordinates and Lagrange equations can be written

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_c} \right) - \frac{\partial L}{\partial x} = f_x
\]  

(8)

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}_c} \right) - \frac{\partial L}{\partial y} = f_y
\]  

(9)

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\alpha}} \right) - \frac{\partial L}{\partial \alpha} = 0
\]  

(10)

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\beta}} \right) - \frac{\partial L}{\partial \beta} = 0
\]  

(11)

While kinetic energy in the 3D crane system can be written

\[
T = \frac{1}{2} \left( m_c \dot{x}_c^2 + m_r \dot{y}_c^2 \right) + \frac{1}{2} m_p \left( \dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2 \right)
\]  

(12)

and energy potential

\[
P = m_p \ g \ z_p = - m_p \ g \ R \cos \alpha
\]  

(13)

with \( g \) gravitational constant, \( m_p \) mass of load load. Therefore the Lagrange equation becomes

\[
L = T - P = \frac{1}{2} \left( m_c \dot{x}_c^2 + m_r \dot{y}_c^2 \right) + \frac{1}{2} m_p \left( \dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2 \right) + m_p \ g \ R \cos \alpha
\]  

(14)

In this simulation use long values \( R, \ddot{R} = \dot{R} = 0 \) so that the completion of equations (8), (9), (10) and (11) are

\[
(m_c + m_p) \ddot{x}_c + m_p R \ddot{c} \cos \alpha \cos \beta - m_p R \dot{\beta} \sin \alpha \sin \beta - m_p R \alpha \dot{\alpha}^2 \sin \alpha \cos \beta
- m_p R \dot{\beta}^2 \sin \alpha \cos \beta - 2 m_p R \dot{c} \ddot{\beta} \cos \alpha \sin \beta = f_x - D_x \dot{x}_c
\]  

\[
(m_r + m_p) \ddot{y}_c + m_p R \dot{\beta} \cos \beta - m_p R \dot{\beta}^2 \sin \beta = f_y - D_y \dot{y}_c
\]  

(15)
with $D_x$ and $D_y$ is the viscous coefficient next to the crane on the X and Y axes. With the method of elimination and substitution of equations (15), (16), (17), (18), the equation of the dynamic model of nonlinear 3D crane systems can be formed in the following differential matrix equations:

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) = u$$

(19)

In order to obtain a model system that can be used to create a control system, a linearization process is carried out. The linearization step is to take a small swing angle is $\alpha \approx 0$ and $\beta \approx 0$, so obtained:

$$\sin \beta \approx \beta$$

$$\sin \alpha \approx \alpha$$

$$\cos \alpha \approx 1$$

$$\cos \beta \approx 1$$

(20)

So the matrix (19) can be simplified to become

$$M(q)\ddot{q} = \begin{bmatrix} (m_c + m_p) & 0 & m_p R & -m_p Ra\beta \\ 0 & (m_r + m_p) & m_p R\beta & m_p R a \\ m_p R & m_p R\beta & m_p R^2 & 0 \\ -m_p Ra\beta & m_p Ra & 0 & m_p R^2 a^2 \end{bmatrix} \begin{bmatrix} \ddot{x}_c \\ \ddot{y}_c \\ \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix}$$

(21)

$$V_m(q, \dot{q})\dot{q} = \begin{bmatrix} D_x & 0 & 0 & 0 \\ 0 & D_y & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix}$$

$$G = \begin{bmatrix} 0 \\ 0 \end{bmatrix} m_p g Ra$$

and $u = [f_x \ f_y \ 0 \ 0]^T$

Next, the differential equation in Equation (19) can be resolved to get the position of the rail and trolley and the swing angle $\alpha$ and $\beta$. The dynamic model of the 3D crane system was also designed by Lee [5], but the authors made several different approaches.

The next step, the derived mathematical model is then simulated with Matlab / Simulink to observe the dynamic behaviour of the 3D crane system. Simulink is a platform for multi-domain simulation
and dynamic system based model design. Simulink provides a set of interactive graphics and library sets, so you can see a variety of time systems.

In this simulation, the input signal bang with an amplitude of ± 1 Nm is used as the input torque. Figure 2 shows the bang input torque for the motor which drives the trolley and rail. Bang torque input has a positive and negative period to allow the crane system to move and then slow down and eventually stop at the target position.

![Figure 2. Input bang](image)

Figure 3 shows the response of the 3D crane system with bang input torque in the time domain. With input torque, the rail moves as far as 0.55 m and the trolley moves as far as 0.36 m. While the rail response with settling time of 1.93 s and 3.2% overshoot. The trolley with settling time 1.81 s and overshoot 0.21%. This response shows the position of the rail swinging and oscillating at the targeted location.

Based on figure 3 it appears that the load swing occurs significantly during and after the movement of the 3D crane system. As expected with no attenuation, the load will continue to oscillate. The swing angle, α shows a significant and continuous swing angle ± 0.09 rad.

On the other hand, swing angle, β shows the initial swing angle which is significant ± 0.07 rad. Then the system oscillates at a fixed angle of ± 0.009 rad. Evaluation of load swing is carried out using the integral squared error (ISE) method, by comparing the angles of α and β. In this case, ISE for α is obtained at 6.70 and ISE β is 0.75. Figure 4 shows the frequency response of the angles of the charge swing α and β. Data shows that the dominant swing frequency for α is 1.07 Hz and β is 1.17 Hz.
4. Conclusion
System dynamics analysis and 3D crane system modelling have been presented. Dynamic models have been derived using the Lagrange equation and simulated with Matlab/Simulink. System behaviour has been observed and discussed in the time domain and frequency domain. The results show that load swings occur significantly during and after the movement of the 3D crane system. By obtaining valid system dynamics and models, it is expected to be used to design an accurate and accurate control system on the 3D crane system.
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