Abstract
The Minimal Supersymmetric Standard Model (MSSM) is plagued by two major fine-tuning problems: the $\mu$-problem and the proton decay problem. We present a simultaneous solution to both problems within the framework of a $U(1)'$-extended MSSM (UMSSM), without requiring $R$-parity conservation. We identify several classes of phenomenologically viable models and provide specific examples of $U(1)'$ charge assignments. Our models generically contain either lepton number violating or baryon number violating renormalizable interactions, whose coexistence is nevertheless automatically forbidden by the new $U(1)'$ gauge symmetry. The $U(1)'$ symmetry also prohibits the potentially dangerous and often ignored higher-dimensional proton decay operators such as $QQQL$ and $U^cU^cD^cE^c$ which are still allowed by $R$-parity. Thus, under minimal assumptions, we show that once the $\mu$-problem is solved, the proton is sufficiently stable, even in the presence of a minimum set of exotics fields, as required for anomaly cancellation. Our models provide impetus for pursuing the collider phenomenology of $R$-parity violation within the UMSSM framework.

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I. INTRODUCTION

Supersymmetry (SUSY) at the Terascale has been the leading candidate for physics beyond the Standard Model (SM). We do not know the concrete manifestation of supersymmetry at low energies, but the Minimal Supersymmetric Standard Model (MSSM) already incorporates most of the advantages of supersymmetry and has proved to be a useful playground for investigations of the possible SUSY signatures at high energy colliders such as the Tevatron and the Large Hadron Collider (LHC). In spite of its successes, however, the MSSM does not exhaust all possibilities and, given its shortcomings discussed below, it is certainly worth pursuing alternative, more general low-energy supersymmetric theories.

One of the most celebrated successes of low-energy supersymmetry is the resolution of the gauge hierarchy problem of the SM. SUSY protects the Higgs mass and the associated electroweak scale from the dangerous quadratically divergent radiative corrections. However, the MSSM itself suffers from its own fine-tuning problems. First, there is the so-called $\mu$-problem [1], which is associated with the following superpotential coupling of the two MSSM Higgs doublets $H_1$ and $H_2$:

$$W_\mu = \mu H_2 H_1.$$  (1)

Since this coupling is allowed by both supersymmetry and gauge symmetry, there is no natural (i.e. in terms of a symmetry) explanation, at least within the MSSM, as to why the value of the $\mu$ parameter is so much smaller than the fundamental (Planck or string) scale. To fix this problem in a natural way, one has to introduce a symmetry which would prohibit the original $\mu$ term (1). However, in the end this symmetry needs to be broken, since a vanishing $\mu$ term would imply very light charginos, in violation of the LEP search limits [2]. Therefore, a viable model should dynamically generate an effective $\mu$ term. This is typically done by introducing a Higgs singlet $S$ coupling to the MSSM Higgs doublets as

$$W_{\mu_{\text{eff}}} = h S H_2 H_1.$$  (2)

The singlet $S$ is charged under the new symmetry, so that the original $\mu$ term (1) is forbidden. The vacuum expectation value (VEV) of $S$ would then break the symmetry and play the role of an effective $\mu$ parameter. Depending on the type of the new symmetry, the models can be classified into several categories [3]. For instance, when the symmetry is a $\mathbb{Z}_3$ discrete symmetry, one obtains the Next-to-MSSM (NMSSM) [4], when the symmetry is
an Abelian gauge symmetry \( U(1)' \), we have the \( U(1)' \)-extended MSSM (UMSSM) \([5]\), etc. (Other options include the Minimal Nonminimal SSM (MNSSM) \([6]\) and the Essential SSM (ESSM) \([7]\).) In this study we shall work within the UMSSM framework, and we shall use the additional \( U(1)' \) gauge interaction to forbid the original \( \mu \) term \([1]\) while allowing the effective \( \mu \) term \([2]\). We shall completely specify the particle content of the model and will demand that the new \( U(1)' \) gauge symmetry is non-anomalous. An extra \( U(1) \) symmetry is supported by many new physics paradigms including grand unified theories \([8, 9]\), extra dimensions \([10]\), superstrings \([11]\), little Higgs \([12]\), dynamical symmetry breaking \([13]\), and Stueckelberg mechanism \([14]\).

The other fine-tuning problem of the MSSM is related to the existence of lepton number violating (LV) terms

\[
W_{LV} = \mu_i^i H_2 L_i + \lambda_{ijk} L_i L_j E^c_k + \lambda_{ijk}^\prime L_i Q_j D^c_k
\]  

(3)

and baryon number violating (BV) terms

\[
W_{BV} = \lambda_{ijk}'' U^c_i D^c_j D^c_k
\]  

(4)

in the superpotential. Here \( i, j, k \) are generation indices and summation over repeated indices is implied. The couplings \([3]\) and \([4]\) are again allowed by all gauge symmetries and supersymmetry and may even occur in the underlying grand unified theory \([15]\). The presence of both types of such terms would lead to unacceptably rapid proton decay unless certain combinations of couplings are tuned to be extremely small \((\lambda\lambda'' \lesssim 10^{-21}, \lambda\lambda'' \lesssim 10^{-27}[16])\).

The standard practice for dealing with this fine-tuning problem is again to impose a new symmetry, the so-called \( R \)-parity \([17]\), which is the only other new symmetry in the MSSM besides supersymmetry. \( R \)-parity forbids both types of problematic terms \([3]\) and \([4]\) and the proton appears to be safe.

At this point one might question whether it was really necessary to forbid both \([3]\) and \([4]\). Indeed, since proton decay requires both LV and BV interactions, forbidding either of them would be sufficient to stabilize the proton. In this sense, the imposition of \( R \)-parity is far from being the minimalist approach, since it eliminates a large chunk of potentially interesting phenomenology related to the physics of \( R \)-parity violation (RPV) \([18]\). In this study, we shall therefore utilize the \( U(1)' \) gauge symmetry to forbid some, but not all \( R \)-parity violating interactions. More specifically, we shall look for models where the proton
is stable in the presence of either LV interactions (3) or BV interactions (4). We shall find that, without ever demanding it, the LV and BV terms are in fact naturally separated in the sense that the $U(1)'$ symmetry may allow (3) or (4), but not both at the same time. This result, which we shall refer to as “LV-BV separation”, is very general and relies only on the following three simple assumptions:

1. The MSSM Yukawa couplings are allowed by the $U(1)'$ gauge symmetry.

2. The $\mu$-problem is solved as in the UMSSM, namely the $U(1)'$ gauge symmetry forbids the original $\mu$ term (1) while allowing the effective $\mu$ term (2).

3. There are no new exotic $SU(2)$ representations beyond the field content of the MSSM.

The proof of the LV-BV separation is very simple and will be presented in Section II B.

At this point, giving up on $R$-parity may seem like a rather steep price to pay. After all, $R$-parity ensures that the lightest supersymmetric particle is stable and may provide a dark matter candidate. However, it is an under-publicized fact that $R$-parity by itself is not sufficient to stabilize the proton [19, 20, 21, 22]. While $R$-parity does prevent the proton from decaying through the renormalizable operators (3) and (4), it still allows for potentially dangerous dimension five operators such as

$$W_5 = \frac{1}{\Lambda} C_{ijkl}^{L} Q_i Q_j Q_k L_l + \frac{1}{\Lambda} C_{ijkl}^{E} U_i^c U_j^c D_k^c E_j^c + \frac{1}{\Lambda} C_{ijkl}^{N} U_i^c D_j^c D_k^c N_l^c,$$

which violate both lepton number and baryon number. Such operators are generically expected to appear at the cutoff scale $\Lambda$. The problem with $R$-parity is that if, as expected, $\Lambda$ is on the order of the string scale or the Planck scale and the coefficients $C$ are of order one, the proton would still decay too fast [19, 20, 21, 22]. In this sense, $R$-parity does not provide a complete and satisfactory solution to the proton decay problem. The presence of the additional $U(1)'$ symmetry, however, offers new possibilities for dealing with the

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1 In general, our results also hold in the presence of a certain number of additional pairs of Higgs doublets – see Section II B.

2 The lepton number of $N^c$ is given by $-1$ in the presence of an $H_2 L N^c$ term in the superpotential, which will be one of our assumptions later on (Section II A). Strictly speaking, $W_{LV}$ of eq. (4) should also contain right-handed neutrino terms such as $N^c N^c$, $N^c N^c N^c$, and $SN^c N^c$ when a lepton number is assigned to $N^c$.

3 See, for instance Ref. [23], to see how grand unified theories can help with this problem.
dangerous higher dimensional operators \(5\). In fact we shall see that under the same three simple assumptions listed above, not only are the renormalizable LV and BV interactions \(3\) and \(4\) naturally separated, but also the dangerous non-renormalizable operators of the type \(\text{(5)}\) are automatically forbidden. In this sense, in comparison to \(R\)-parity, the \(U(1)\)' gauge symmetry may provide a more attractive alternative solution to the proton decay problem.

Our work is complementary to a number of studies in the literature which have already considered an extra non-anomalous \(U(1)\) gauge symmetry in lieu of \(R\)-parity to address the proton stability problem \[19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30\] \(^4\). The more recent studies have adopted an even more economical approach, where the \(U(1)\)' gauge symmetry is used to simultaneously solve both the \(\mu\)-problem and the proton stability problem \[27, 28, 29, 30\]. In those works the renormalizable \(R\)-parity violating interactions (as well as the non-renormalizable interactions \(\text{(5)}\)) are forbidden by the \(U(1)\)' symmetry \(^5\). The price to pay, however, was to allow for a relatively complicated spectrum, including e.g. \(SU(2)_L\) exotics \[27, 28\], several pairs of Higgs doublets \((N_H)\) \[29\] or several singlet representations \((N_S)\) \[29, 30\]. Even though our motivation here was to allow for either LV or BV interactions, we have also analyzed cases where the \(U(1)\)' symmetry forbids all RPV operators of lowest dimensions. Such examples are presented in appendices \(A\) and \(B\). First in Appendix \(A\) we consider the novel case of \(N_H = 4\), while in Appendix \(B\) we treat the case of \(N_H = 3\) which was previously discussed in Ref. \[29\]. We shall show that in both of those cases the nonlinear \(U(1)\)' anomaly conditions factorize and \(all\) anomaly conditions essentially reduce to linear constraints. Furthermore, the case of \(N_H = 3, N_S = 3\) exhibits an additional simplification: the quadratic and cubic \(U(1)\)' anomaly conditions are not independent, and we find a three-parameter class of anomaly-free solutions which generalize the single model found in Ref. \[29\].

Previous studies found that the additional gauge symmetry usually also requires exotic fields for the cancellation of certain anomalies \[27, 28, 29, 30\]. This tends to ruin the successful gauge coupling unification which is a hallmark of supersymmetry \[32\] \(^6\). Here we

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\(^4\) For anomalous \(U(1)\) approaches, see for example Ref. \[31\] and references therein.

\(^5\) Previous studies \[26\] which considered \(R\)-parity violating interactions within the \(U(1)\)' framework did not address the \(\mu\)-problem.

\(^6\) Ref. \[33\] considered an UMSSM with family non-universal charges which was free of exotics. However, in
do not require gauge coupling unification, and follow a bottom-up approach by introducing only the minimal set of exotic fields (three vectorlike pairs of colored triplets $K_i$ and $K_i^c$, see Section II A) required for anomaly cancellation. For simplicity, we will also assume family universal $U(1)'$ charges for all MSSM fields, including the right-handed neutrinos, but will let the exotics have family non-universal charges.

Our paper is organized as follows. In Section II we describe the general properties of our solutions. For this purpose, we shall only need to use the linear constraints on the $U(1)'$ charges following from the Yukawa-type couplings in the superpotential, plus the $U(1)' - [SU(2)_L]^2$ anomaly condition from Section III B. We begin by introducing our formalism and notation in Section II A and proceed to derive some of our main results in the remainder of Section II. In Section II B we explicitly show the LV-BV separation, namely, that the renormalizable LV terms and BV terms can not coexist: if we allow for the LV terms (3) in the superpotential, then the BV terms (4) are automatically forbidden by the $U(1)'$ gauge symmetry, and vice versa. Then in Section II C we extend our discussion to the case of the non-renormalizable RPV terms such as (5) and show that those are absent as well. In Section II D we derive a simple expression for the $U(1)'$ charge of the right-handed neutrino in terms of the $U(1)'$ charges of the other UMSSM fields, and discuss the origin of neutrino masses in our scenario. Finally, in Section II E we present the general solution to the linear constraints discussed in Section II A and then its specific form for the LV case or the BV case alone. In Section III we discuss the remaining constraints on the $U(1)'$ charges arising from the absence of gauge anomalies. We consider the anomaly conditions one at a time and discuss their implications for the model building to follow in the next three sections. In Section IV we present our simplest models ($N_H = 1$) with either LV or BV, but not both, type of interactions. We summarize and conclude in Section V. In Appendix A (Appendix B) we discuss models with $N_H = 4$ ($N_H = 3$) in which both types of RPV terms are forbidden by the $U(1)'$ symmetry. In Appendix C we discuss a special case of $N_H = N_S = 1$ with an altered particle spectrum.

that case one can not write down Yukawa couplings for all fermions at tree level, and in Ref. [33] non-holomorphich terms were introduced in order to radiatively generate the problematic Yukawa couplings.
TABLE I: Chiral fields in the model and their quantum numbers. \( z[F] \) denotes the \( U(1)' \) charge of a field \( F \). In general, we consider \( N_H \) pairs of Higgs doublets \( H_1 \) and \( H_2 \) with identical quantum numbers, and \( N_S \) copies of SM Higgs singlets \( S \).

II. GENERAL PROPERTIES OF THE \( U(1)' \) MODELS

A. Setup and Formalism

In the same spirit as the earlier works \cite{27, 28, 29, 30}, we consider the \( U(1)' \)-extended MSSM where both the \( \mu \) term and the \( R \)-parity violating terms in the superpotential are controlled by the \( U(1)' \) gauge symmetry. In contrast to previous studies along these lines, we shall not forbid all renormalizable RPV terms from the very beginning. Instead, we shall in principle allow for the presence of either LV or BV terms in the superpotential. We will not be particularly concerned whether the RPV terms (3) and (4) arise at the renormalizable level or through a higher dimensional operator. In fact, we shall find examples of both types below. We shall then demonstrate that, as a result of the \( U(1)' \) symmetry, the proton is nevertheless still sufficiently stable, even at the non-renormalizable level. Our result is quite general and relies only on our three simple assumptions listed in the Introduction.

To set up our discussion, in Table I we list the particles of the UMSSM with their corre-
sponding SM quantum numbers and $U(1)'$ charges. The first column lists the corresponding field, and the next two columns give its representation under $SU(3)_C$ and $SU(2)_L$. The last two columns show the hypercharge $y[F]$ and the $U(1)'$ charge $z[F]$ of a field $F$. In addition to the MSSM fields $Q, U^c, D^c, L, E^c, H_1$ and $H_2$, we also include three right-handed neutrinos $N^c$. The Higgs singlet $S$ is introduced in order to generate the effective $\mu$ term (2), and a successful solution to the $\mu$-problem requires that

\[ z[S] = -z[H_1] - z[H_2] \neq 0 . \] (6)

In what follows, we shall make repeated use of this equation which is nothing but the second of our three basic assumptions listed in the Introduction. In general, we shall consider $N_H$ pairs of Higgs doublets $H_1$ and $H_2$ with identical quantum numbers, and $N_S$ SM Higgs singlets of type $S$. The Abelian gauge symmetry $U(1)'$ is assumed to be broken at the TeV scale where all Higgs fields ($S, H_1$ and $H_2$) get VEV’s of that order. An effective $\mu$ term ($\mu_{\text{eff}} = h(S)$) is thus dynamically generated at the TeV scale, completing the solution to the $\mu$-problem. This is very similar to the case of the NMSSM, but having the $U(1)'$ gauge symmetry of the UMSSM has the additional advantage of eliminating the domain wall problem associated with the discrete symmetry of the NMSSM [34]. As we mentioned earlier, a minimum set of vectorlike colored exotics $K_i, K^c_i$ ($i = 1, 2, 3$) is also required for anomaly cancellation (see Section III A). At this point, the hypercharges of the exotics and the $U(1)'$ charges of all fields listed in Table I are yet to be determined.

In the remainder of this Section we shall analyze the main properties of our solutions, based on a limited set of linear constraints for the $U(1)'$ charges. The remaining constraints will be analyzed in Section III. We shall first list the set of relevant equations, and proceed to analyze them in the subsequent subsections.

In addition to (2), we also require that the $U(1)'$ symmetry allows the usual Yukawa couplings in the superpotential

\[ W_{\text{Yukawa}} = y^D_{jk} H_1 Q_j D^c_k + y^U_{jk} H_2 Q_j U^c_k + y^E_{jk} H_1 L_j E^c_k + y^N_{jk} \left( \frac{S}{\Lambda} \right)^a H_2 L_j N^c_k . \] (7)

Here capital letters denote the superfields of the MSSM whose quantum numbers are listed in Table I. Because of the observed smallness of the neutrino masses, we have in general

\[ \text{In addition, quantum gravity effects may violate a global symmetry unless it is a remnant of a gauge symmetry [35].} \]
allowed neutrino Yukawa couplings to arise from a non-renormalizable operator suppressed by some high scale $\Lambda$ \cite{36}. However, in principle we do not exclude the possibility of $a = 0$. We discuss the possible appearance of a Majorana mass term for $N^c$ in Section II.D. The presence of the Yukawa terms (7) leads to the following constraints

\begin{align}
Y_D : & \quad z[H_1] + z[Q] + z[D^c] = 0 \\
Y_U : & \quad z[H_2] + z[Q] + z[U^c] = 0 \\
Y_E : & \quad z[H_1] + z[L] + z[E^c] = 0 \\
Y_N : & \quad z[H_2] + z[L] + z[N^c] + az[S] = 0 .
\end{align}

We supplement these with eq. (6) which we write as

\begin{align}
Y_S : & \quad z[S] + z[H_1] + z[H_2] = 0
\end{align}

and the $U(1)' - [SU(2)_L]^2$ anomaly condition from Section III.B

\begin{align}
A_2 : & \quad 9z[Q] + 3z[L] + N_H(z[H_1] + z[H_2]) + A_2(\text{exotics}) = 0 .
\end{align}

The set of 6 equations (8-13) is the starting point for our analysis in the remainder of this Section. These 6 equations exactly correspond to our three basic assumptions listed in the Introduction: the existence of the Yukawa terms (7) is guaranteed by eqs. (8-11), the solution to the $\mu$-problem is implied by eq. (12) and the absence of $SU(2)_L$ exotics among our particle content in Table I simply means that there is no additional contribution to the $U(1)' - [SU(2)_L]^2$ anomaly and $A_2(\text{exotics}) = 0$ in eq. (13).

B. LV-BV Separation

Starting with eqs. (8-13) and taking the linear combination $6Y_D + 3Y_U - 3Y_E + (N_H - 3)Y_S - A_2$ gives the following constraint among the $U(1)'$ charges

\begin{align}
3(z[U^c] + 2z[D^c]) - 3(2z[L] + z[E^c]) + (N_H - 3)z[S] - A_2(\text{exotics}) = 0 .
\end{align}

We find this equation particularly useful both in illustrating one of our main points, as well as in categorizing the existing $U(1)'$ models in the literature. Each term in eq. (14) corresponds to a particular physical situation:
1. The first term in eq. (14) represents the baryon number violating interactions of eq. (4).
If this term is zero, BV interactions will be present in the model. In order to forbid (4),
one must have \( z[U^c] + 2z[D^c] \neq 0 \), which would require at least one of the remaining
three terms in eq. (14) to be non-vanishing as well.

2. The second term in eq. (14) represents the lepton number violating interactions of
eq. (3). If this term is zero, LV interactions will be present in the model. In order to
forbid (3), one must have \( 2z[L] + z[E^c] \neq 0 \), which would require at least one of the
remaining three terms in eq. (14) to be non-vanishing as well.

3. The third term in eq. (14) simply counts the number \( N_H \) of Higgs doublet pairs in the
model. This term would vanish only if \( N_H = 3 \), since the solution to the \( \mu \)-problem
requires \( z[S] \neq 0 \) (see eq. (5)).

4. The fourth term \( A_2 \) (exotics) represents the contribution to the \( U(1)' - [SU(2)_L]^2 \)
anomaly from states not listed in Table I. It is a model-builder’s choice whether this
term is present or not.

Eq. (14) allows us to categorize the existing \( U(1)' \) models according to how many and
which of these four terms are non-vanishing. For example, Ref. \( [29] \) forbids all renormalizable
RPV terms, hence the first two terms in eq. (14) are both nonzero. In fact, they cancel each
other, since Ref. \( [29] \) assumes three pairs of Higgs doublets \( (N_H = 3) \) and no \( SU(2)_L \)
exotics, so that the last two terms in eq. (14) are zero. On the other hand, the models of
Refs. \( [27, 28] \) illustrate the case where all four terms in eq. (14) are non-vanishing: those
models also forbid RPV interactions, but contain \( SU(2)_L \) exotics and have \( N_H \neq 3 \).
Finally, the models of Ref. \( [30] \) have \( N_H = 1 \) and no \( SU(2)_L \) exotic representations, so they illustrate
the intermediate case where three terms in eq. (14) are non-vanishing.

According to our third basic assumption (see Introduction), our approach will be to
assume that there are no \( SU(2)_L \) exotic representations so that \( A_2 \) (exotics) = 0, in which
case eq. (14) becomes

\[
3(z[U^c] + 2z[D^c]) - 3(2z[L] + z[E^c]) + (N_H - 3)z[S] = 0 .
\]  

(15)

We shall be mostly interested in cases with \( N_H \neq 3 \), so that the third term in eq. (15) is
nonzero. For simplicity, we shall concentrate on \( N_H = 1 \) in Section IV (the case of \( N_H = 4 \)
is treated in Appendix A). Under those circumstances, eq. (15) reveals that, at least at the
renormalizable level, the LV terms (3) and the BV terms (4) can not coexist (i.e. the first
two terms in eq. (15) can not vanish simultaneously), since we need at least one of them to
cancel the non-vanishing third term proportional to $z[S]$. We refer to this mutual exclusion
as the “LV-BV separation”. The proton is then safe from decaying through renormalizable
RPV interactions, even though $R$-parity is not present in the model. Furthermore, one does
not need both of the first two terms in eq. (15) in order to cancel the third one – only one
of the first two terms will suffice. Therefore we are free to consider models where either
the first or the second term in eq. (15) is zero and the corresponding RPV interactions are
allowed. For example, in the LV case, where $2z[L] + z[E^c] = 0$, eq. (15) gives
$$z[U^c] + 2z[D^c] = \left(1 - \frac{N_H}{3}\right) z[S] \neq 0$$
and the BV interactions (4) are not allowed. Similarly, in the BV case, where $z[U^c] + 2z[D^c] = 0$,
eq 0$$
and the LV terms (3) are not allowed. It is straightforward to see that the LV-BV separation
also holds if the corresponding LV and BV terms arise at the non-renormalizable level – in
that case, there are extra contributions to the right-hand side of eqs. (16) and (17) which
are integer multiples of $z[S]$, so that our argument still applies as long as $N_H = 1$.

C. Higher Dimensional Operators and Proton Decay

As we already mentioned in the Introduction, $R$-parity allows for potentially dangerous
higher-dimensional operators like (5) which may still destabilize the proton. The new $U(1)'$
gauge symmetry can now be used to eliminate those as well [27, 28, 29, 30]. It is interesting
to note that simply by making use of eqs. (8-13), and without specifying the further details
of the model, we can readily compute the $U(1)'$ charge of any such operator and test whether
it is allowed or not. For example, the linear combination $Y_D + 2Y_U + Y_E + (\frac{N_H}{3} - 2)Y_S - \frac{1}{3}A_2$
leads to
$$2z[U^c] + z[D^c] + z[E^c] + \left(\frac{N_H}{3} - 2\right) z[S] = 0,$$
which allows us to determine the $U(1)'$ charge of the $U^c U^c D^c E^c$ operator as
$$U^c U^c D^c E^c : \quad 2z[U^c] + z[D^c] + z[E^c] = \left(2 - \frac{N_H}{3}\right) z[S].$$
Since the solution to the \( \mu \)-problem already implies \( z[S] \neq 0 \) (see eq. (6)), this operator is forbidden, unless one allows for exactly 6 pairs of Higgs doublets in the model. Similarly, the operator \( QQQL \) is also absent, since its charge can be obtained from the linear combination \( \frac{1}{3}A_2 - \frac{N_H}{3}Y_S \):

\[
QQQL : 3z[Q] + z[L] = \frac{N_H}{3}z[S].
\] (20)

Because of eq. (6), again it is clear that the \( U(1)' \) symmetry does not allow this operator, since we already have at least one pair of Higgs doublets as in the MSSM. Finally, one can obtain the \( U(1)' \) charge of the operator \( U^cD^cD^cN^c \) from the combination \( 2Y_D + Y_U + Y_N + (\frac{N_H}{3} - 2)Y_S - \frac{1}{3}A_2 \) as

\[
U^cD^cD^cN^c : z[U^c] + 2z[D^c] + z[N^c] = \left(2 - a - \frac{N_H}{3}\right)z[S].
\] (21)

Since \( a \) is an integer, we see that, in general, as long as the number \( N_H \) of Higgs doublet pairs is not divisible by three, this operator is also forbidden. Even when \( N_H \) is divisible by three, there will be only one special value of the integer \( a \), namely \( a = 2 - \frac{N_H}{3} \), which would allow the existence of this operator. Since \( a \) must be positive, there are only two special cases that one should be worried about: \( (N_H = 3, a = 1) \) and \( (N_H = 6, a = 0) \). The case \( N_H = 6 \) is already disfavored by (19), while in the case \( N_H = 3 \) which we study in Appendix B, we shall consider only the case \( a = 0 \) as in Ref. [29].

To summarize, so far we have shown that in the simplest cases such as \( N_H = 1, 2, 4, \cdots \) the conditions (8-13) are sufficient to rule out the dangerous dimension 5 operators (5) which simultaneously violate baryon and lepton number. This is already an important advantage of our models compared to the usual \( R \)-parity conserving scenario. However, since in our approach we are allowing some of the dimension 4 \( LV \) or \( BV \) interactions, we should also check for potentially dangerous pairs of dimension 4 and dimension 5 operators, which may in general arise from either \( F \)-terms or \( D \)-terms. For the case of the MSSM, the problematic combinations were identified in Ref. [37] as

\[
\{LQD^c, QQQH_1\}, \{U^cD^cD^c, QU^cE^cH_1\}, \{U^cD^cD^c, U^cD^cE^c\}, \{U^cD^cD^c, QU^cL^1\}. \] (22)

Using eqs. (8-13), it is easy to derive the following relations between the \( U(1)' \) charges of
the operators in each pair:

\[
(z[L] + z[Q] + z[D^c]) + (3z[Q] + z[H_1]) = \frac{N_H}{3} z[S],
\]

\[
(z[U^c] + 2z[D^c]) + (z[Q] + z[U^c] + z[E^c] + z[H_1]) = \left(2 - \frac{N_H}{3}\right) z[S],
\]

\[
(z[U^c] + 2z[D^c]) + (z[U^c] - z[D^c] + z[E^c]) = \left(2 - \frac{N_H}{3}\right) z[S],
\]

\[
(z[U^c] + 2z[D^c]) + (z[Q] + z[U^c] - z[L]) = \left(2 - \frac{N_H}{3}\right) z[S].
\]

We see that all of the dangerous pairs of operators are forbidden by the \(U(1)'\) symmetry, due to the condition \([19]\). (The case \(N_H = 6\) would in principle allow the last three pairs, but \(N_H = 6\) was already disfavored by eq. (19) and we shall not be considering it any further.)

So far we have shown that the proton is not destabilized by the potentially dangerous pairs of operators constructed out of MSSM fields only. Since our models have additional fields present \((N^c, S, K, and K^c)\) beyond those of the MSSM, we still need to check that those extra fields do not give rise to dangerous pairs of operators analogous to (22). We systematically checked all relevant combinations of dimension 4 and/or dimension 5 operators involving \(N^c\) and \(S\) in addition to the usual MSSM fields, and verified that all combinations which violate lepton number and baryon number are forbidden by the \(U(1)'\) symmetry when \(z[S] \neq 0\), and \(\frac{N_H}{3}\) is not an integer\(^8\).

It remains to discuss the effect of the colored exotics \(K, K^c\) on proton decay. Since they are heavy, they can not appear among the proton decay products. However, they may still mediate proton decay. It is more difficult to see that the proton is safe from such processes because the \(U(1)'\) charges and hypercharges of the colored exotics are not determined by eqs. \([8,13]\). One possible approach would be to choose the exotic hypercharges so that the lowest dimension operators coupling exotic quarks to the MSSM fields are absent \([25, 29]\).

Here we shall consider a more general setup, where the hypercharges of the colored exotics in principle may allow couplings to the MSSM fields (see Section [III C]). The proof of proton

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\(^8\) This statement is strictly true in the LV case. In the BV case the only potentially troublesome pair of operators is \(U^c D^c D^c\) and \(N^c N^c N^c S\). The latter has \(U(1)'\) charge \((7 - 3a - N_H)z[S]\) (see eq. (30)) and is in principle allowed for the following three choices: \(\{a, N_H\} = \{0, 7\}, \{1, 4\}, \{2, 1\}\). However, neither of these three options is a viable one: \(N_H = 7\) is incompatible with the \(A_3\) anomaly (see eq. (12) below); \(a = 1, N_H = 4\) is inconsistent with the \(A_4\) anomaly (see Appendix [A]); while \(a = 2\) would imply too small neutrino masses.
stability in that case will be presented in a separate publication \cite{38} where we will discuss
the discrete gauge symmetries \cite{37,39} encoded in our models.

\section{D. Majorana Neutrino Masses}

Recent experiments show that the active neutrinos have masses. There are different
possibilities regarding the origin of neutrino masses: e.g. Dirac neutrino masses may arise
from the SM Higgs mechanism, and their smallness can be naturally explained through a
seesaw mechanism with heavy right-handed Majorana neutrinos \cite{40}. Other possibilities
invoke extra dimensions \cite{41} or higher dimensional operators \cite{42}. Since we allow for a
neutrino Yukawa coupling (see eq. \eqref{eq:Yukawa}), our models can readily accommodate Dirac type
neutrinos. In this subsection we investigate whether in addition to the neutrino Yukawa
coupling, one could write down a Majorana term for the right-handed neutrinos, so that we
can have some kind of a seesaw mechanism as well.

Taking the linear combination $Y_E + Y_N - Y_S$ allows us to express the $U(1)'$ charge of the
right-handed neutrinos $N^c$ as

\[ z[N^c] = -(2z[L] + z[E^c]) + (1 - a)z[S] = \begin{cases} 
(1 - a)z[S] & \text{(LV case)}; \\
(2 - a - \frac{N_H}{3})z[S] & \text{(BV case)}.
\end{cases} \tag{27} \]

We see that in the BV case, lepton number violating terms involving the $N^c$ field (e.g.
$N^c N^c$, $N^c N^c N^c$, and $SN^c N^c$) can not be generated, unless $\frac{N_H}{3}$ is an integer. Therefore
when $N_H \neq 3, 6, \cdots$, the LV-BV separation holds even in the presence of $N^c$ fields with
lepton number $-1$. While the BV case can then have only Dirac neutrino mass terms,
the LV case may in general allow a Majorana neutrino mass term $N^c N^c$ whenever $a = 1$.
However, in the LV case the $SN^c N^c$ term has a $U(1)'$ charge of $(3 - 2a)z[S]$ and is not
allowed. The active neutrinos of the LV case may also get their masses without the RH
neutrinos through $f\tilde{f}$ loops involving the $\lambda$ and $\lambda'$ couplings, or through $\nu\tilde{H}_2^0$ mixing due
to the $\mu'_{\text{eff}}LH_2$ term in eq. \eqref{eq:mu_eff}.

\section{E. General Solution to the Yukawa Constraints and the $A_2$ Anomaly}

In this subsection we present the general solution to the constraints \eqref{eq:constraints} and then
specify its particular form separately for the LV case and the BV case.
Since (8-13) are 6 constraints for 9 variables, we find a three-parameter solution as

\[
\begin{pmatrix}
  z[Q] \\
  z[U^c] \\
  z[D^c] \\
  z[L] \\
  z[E^c] \\
  z[N^c] \\
  z[H_2] \\
  z[H_1] \\
  z[S]
\end{pmatrix} = \frac{\ell}{3} \begin{pmatrix}
  -1 \\
  1 \\
  1 \\
  3 \\
  -3 \\
  -3 \\
  0 \\
  0 \\
  0
\end{pmatrix} + h_1 \begin{pmatrix}
  0 \\
  1 \\
  0 \\
  0 \\
  -1 \\
  1 \\
  -1 \\
  1 \\
  0
\end{pmatrix} + s \frac{9}{9} \begin{pmatrix}
  N_H \\
  9 - N_H \\
  -N_H \\
  0 \\
  9(1 - a) \\
  0 \\
  0 \\
  0 \\
  9
\end{pmatrix},
\]

(28)

where \( \ell, h_1 \) and \( s \) are arbitrary coefficients. The notation for those is suggestive of their interpretation: \( \ell = z[L] \), \( h_1 = z[H_1] \), and \( s = z[S] \).

In the LV case, we have an additional constraint, e.g. \( 2z[L] + z[E^c] = 0 \), which implies the relation \( h_1 = \ell \) and the solution (28) becomes

\[
\begin{pmatrix}
  z[Q] \\
  z[U^c] \\
  z[D^c] \\
  z[L] \\
  z[E^c] \\
  z[N^c] \\
  z[H_2] \\
  z[H_1] \\
  z[S]
\end{pmatrix} = 2\ell \begin{pmatrix}
  -\frac{1}{6} \\
  \frac{2}{3} \\
  -\frac{1}{3} \\
  \frac{1}{2} \\
  -1 \\
  0 \\
  -\frac{1}{2} \\
  \frac{1}{2} \\
  0
\end{pmatrix} + \frac{s}{9} \begin{pmatrix}
  N_H \\
  9 - N_H \\
  -N_H \\
  0 \\
  9(1 - a) \\
  0 \\
  0 \\
  0 \\
  9
\end{pmatrix}.
\]

(29)

Not surprisingly, we recognize in the first column vector on the right-hand side the hypercharge assignments of the UMSSM fields from Table I. Indeed, the constraints (8-12) arise from gauge-invariant operators, so clearly they will be satisfied by the hypercharges of the UMSSM fields. What is more important at this point is the additional remaining degree of freedom represented by the second term in the right-hand side of eq. (29), which will allow us to find nontrivial solutions for the \( U(1)^{I'} \) charges, different from the usual hypercharge.

In the BV case, the corresponding additional constraint \( 2z[U^c] + z[D^c] = 0 \) implies \( h_1 = \).
| Identifier | Anomaly | Equation |
|------------|---------|----------|
| $A_1$      | $U(1)'-[SU(3)_C]^2$ | $\text{tr}[z^a t^b] = \frac{1}{4} \delta^{ab} \sum q \ z = 0$ (color triplet fermions only) |
| $A_2$      | $U(1)'-[SU(2)_L]^2$ | $\text{tr}[z^a t^b] = \frac{1}{2} \delta^{ab} \sum f_L \ z = 0$ (doublet fermions only) |
| $A_3$      | $U(1)'-[U(1)_Y]^2$ | $\text{tr}[z y^2] = \sum f \ z y^2 = 0$ |
| $A_4$      | $U(1)_Y-[U(1)']^2$ | $\text{tr}[y z^2] = \sum f \ y z^2 = 0$ |
| $A_5$      | $[U(1)']^3$ | $\text{tr}[z^3] = \sum f \ z^3 = 0$ |
| $A_6$      | $U(1)'-[\text{gravity}]^2$ | $\text{tr}[z] = \sum f \ z = 0$ |

TABLE II: Anomaly cancellation conditions for the $U(1)'$ charges of the particles in our model listed in Table I. The first column lists a shorthand identifier for each condition, which will be used throughout the text.

$\ell + (1 - \frac{N_H}{3}) s$ and the solution (28) can be written as

$$
\begin{pmatrix}
  z[Q] \\
  z[U^c] \\
  z[D^c] \\
  z[L] \\
  z[E^c] \\
  z[N^c] \\
  z[H_2] \\
  z[H_1] \\
  z[S]
\end{pmatrix} = \left(2\ell - \frac{2}{3} N_H s\right) 
\begin{pmatrix}
  -\frac{1}{6} \\
  \frac{1}{3} \\
  \frac{1}{3} \\
  \frac{1}{2} \\
  -1 \\
  0 \\
  -\frac{1}{2} \\
  \frac{1}{2} \\
  0
\end{pmatrix}
+ \frac{s}{3} 
\begin{pmatrix}
  0 \\
  6 \\
  -3 \\
  N_H \\
  N_H \\
  6 - N_H - 3a \\
  3 \\
  3
\end{pmatrix}.
$$

(30)

Just as in the LV case (29), the usual hypercharges appear as a particular solution to the constraints (8-13), but there is an additional class of solutions with nonzero $z[S]$, so that in general our solutions will be a linear combination of these two classes.

### III. ANOMALIES

Table III summarizes the anomaly cancellation conditions for the $U(1)'$ charges of the fields in our model. In this section, we investigate these anomaly cancellation conditions one by one and discuss their implications for model building.
A. Anomaly $A_1 \left( U(1)' - [SU(3)_C]^2 \right)$

We begin with the mixed $U(1)' - [SU(3)_C]^2$ anomaly which we denote with $A_1$. First we rederive the well known result that the presence of the Yukawa couplings in the superpotential \[ \left( U(1)' - [SU(3)_C]^2 \right) \] requires exotic representations beyond those of the MSSM. Denoting the contribution of such exotics to the $U(1)' - [SU(3)_C]^2$ anomaly by $A_1(\text{exotics})$ we can write $A_1$ as

$$A_1 : 3 \left( 2z[Q] + z[U^c] + z[D^c] \right) + A_1(\text{exotics}) = 0 .$$

The first term is the contribution of the 3 generations of quarks in the MSSM, while the second term is the potential colored exotics contribution. Now taking the linear combination $A_1 - 3Y_U - 3Y_D + 3Y_S$, we get

$$A_1(\text{exotics}) = -3z[S] ,$$

which, in light of eq. (6), shows the need for colored exotic representations [27, 28, 29, 30].

In this paper, we shall assume that the exotics are $N_K$ vectorlike pairs of chiral fields $K_i$ and $K^c_i$ so that they do not alter the anomaly cancellation conditions among the SM gauge groups. More specifically, we assume that they are triplets and anti-triplets of $SU(3)_C$ with equal and opposite $U(1)_Y$ hypercharges $\pm y[K_i]$ (see Table I). Perhaps most importantly, as already mentioned earlier in the Introduction, we are assuming that the exotics which are needed to cancel the $A_1$ anomaly are $SU(2)_L$ singlets, so that $A_2(\text{exotics}) = 0$. With those assumptions, eq. (31) becomes

$$A'_1 : 3 \left( 2z[Q] + z[U^c] + z[D^c] \right) + \sum_{i=1}^{N_K} \left( z[K_i] + z[K^c_i] \right) = 0 .$$

In order to avoid conflict with experiment, the exotic quarks $K_i$ and $K^c_i$ must be sufficiently heavy [43]. If their masses arise from an ordinary mass term $KK^c$ in the superpotential, then $U(1)'$ invariance implies $A_1(\text{exotics}) = \sum_{i=1}^{N_K} \left( z[K_i] + z[K^c_i] \right) = 0$ and the $\mu$-problem can not be solved because of the conflicting requirements of eqs. (6) and (32). We therefore choose to generate masses for all colored exotics at the TeV scale, through superpotential couplings to the $S$ field:

$$W_{\text{exotics}} = h''_{ij} SK_i K^c_j .$$
Assuming that the couplings in eq. (34) are diagonal, we get the following constraint among the $U(1)'$ charges of the exotics

$$Y_{K_i} : \quad z[S] + z[K_i] + z[K_i^c] = 0 \ . \tag{35}$$

Since $z[S] \neq 0$, this equation reveals that $K$ and $K^c$ do not carry equal and opposite $U(1)'$ charges, even though their hypercharges are equal and opposite ($y[K_i] + y[K_i^c] = 0$).

Now taking the linear combination $A_1' - 3Y_U - 3Y_D + 3Y_S - \sum_{i=1}^{N_K}Y_{K_i}$ gives

$$(3 - N_K)z[S] = 0 \ . \tag{36}$$

Combined with eq. (6), this determines the number of exotic families as

$$N_K = 3 \ . \tag{37}$$

Notice that the $A_1$ anomaly did not impose any constraints on the $U(1)'$ charges themselves, but simply fixed the number of allowed representations in the model. We shall see that the same phenomenon will take place when we consider some of the other anomaly conditions below. In the end, this will leave us with sufficient freedom to find sets of $U(1)'$ charges which will satisfy all of our model requirements.

**B. Anomaly $A_2 (U(1)' - [SU(2)_L]^2)$**

This anomaly condition was already introduced as eq. (13) in Section II A. With our assumption that all exotics in the model are $SU(2)_L$ singlets, it becomes

$$A_2 : \quad 9z[Q] + 3z[L] + N_H (z[H_1] + z[H_2]) = 0 \ . \tag{38}$$

**C. Anomaly $A_3 (U(1)' - [U(1)_Y]^2)$**

In general, the $A_3$ anomaly condition is given by

$$9(2z[Q]y[Q]^2 + z[U^c]y[U^c]^2 + z[D^c]y[D^c]^2) + 3(2z[L]y[L]^2 + z[E^c]y[E^c]^2) + \sum_{i=1}^{N_K} (z[K_i]y[K_i]^2 + z[K_i^c]y[K_i^c]^2) + N_H (2z[H_1]y[H_1]^2 + 2z[H_2]y[H_2]^2) = 0 \tag{39}$$
where \( y[F] \) is the \( U(1)_Y \) hypercharge of a field \( F \) as given in Table II and we have omitted terms involving fields with vanishing hypercharge \((N^c \text{ and } S)\). Substituting the known hypercharges from Table II and using (35), we can rewrite it as

\[
A_3 : \ z[Q] + 8z[U^c] + 2z[D^c] + 3z[L] + 6z[E^c] + N_H (z[H_1] + z[H_2]) - 6z[S] \sum_{i=1}^{N_K} y[K_i]^2 = 0. \quad (40)
\]

Now taking the linear combination \( A_3 + A_2 - 8Y_U - 2Y_D - 6Y_E + (8 - 2N_H)Y_S \) leads to the following simple constraint

\[
\left( 4 - N_H - 3 \sum_{i=1}^{N_K} y[K_i]^2 \right) z[S] = 0. \quad (41)
\]

Because of condition (6), this uniquely reduces to

\[
\sum_{i=1}^{N_K} y[K_i]^2 = \frac{1}{3} (4 - N_H), \quad (42)
\]

where the hypercharges are normalized as in Table II. We see that, just as was the case for \( A_1 \), the anomaly cancellation condition \( A_3 \) did not provide an additional constraint on the \( U(1)' \) charges, but instead only limits the number of Higgs doublet pairs \( N_H \) and the choice for exotic hypercharges \( y[K_i] \). Since the left-hand side of eq. (42) must be positive-definite and \( N_H \) is an integer, there are only four possible choices for the number of Higgs doublet pairs: \( N_H = 1, 2, 3 \) or 4, that we need to consider. The case of \( N_H = 3 \) was already considered in Ref. 29 and we shall revisit it again in Appendix B. We shall also consider the case of \( N_H = 4 \) in Appendix A. Our main interest, however, will be in the minimal case of \( N_H = 1 \), which will be discussed below in Section IV.

Having fixed the number of Higgs doublet pairs \( N_H \), eq. (42) provides a guideline for choosing the hypercharges of the colored exotics. Since the \( A_1 \) anomaly already required \( N_K = 3 \) (see Section III A), it is clear that a family universal choice with rational numbers is only possible for \( N_H = 3 \), with \( y[K_i] = \pm \frac{1}{3} \), or for \( N_H = 4 \), with \( y[K_i] = 0 \). In the case of \( N_H = 1 \) or \( N_H = 2 \), one would have to choose exotic hypercharges in a family non-universal way. In general, there are many possible choices, but here we shall limit ourselves to those where the exotic hypercharges are the same (up to a sign) as the hypercharges of...
the corresponding $SU(2)_L$ singlet, color triplet representations in the MSSM ($U^c$ and $D^c$):

$$N_H = 1 \implies y[K_i] = \left\{ \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{2}{3} \right\},$$

$$N_H = 2 \implies y[K_i] = \left\{ \pm \frac{1}{3}, \pm \frac{1}{3}, \pm \frac{2}{3} \right\},$$

$$N_H = 3 \implies y[K_i] = \left\{ \pm \frac{1}{3}, \pm \frac{1}{3}, \pm \frac{1}{3} \right\}.$$

All three choices (43-45) satisfy the $A_3$ anomaly condition (42). The signs of the exotic hypercharges could be in general chosen arbitrarily. We have limited ourselves to two cases – with the upper signs in eqs. (43-45) the exotics have the wrong quantum numbers to couple to the MSSM quarks and mediate proton decay. In that case, however, the lightest exotic would be stable and may pose problems for cosmology. This could be avoided, e.g. if the reheating temperature is very low, $T_{RH} \lesssim 100$ GeV, which may still be compatible with baryogenesis [44]. On the other hand, choosing the lower signs in eqs. (43-45) allows the exotics to couple to the MSSM quarks, thus avoiding problems with cosmology. Nevertheless, as we already discussed in Section II C, in that case the $U(1)'$ symmetry is sufficient to stabilize the proton. We shall therefore allow for both sets of signs for the exotic hypercharges in eqs. (43-45).

D. Anomaly $A_6$ ($U(1)' - [\text{gravity}]^2$)

The gravitational anomaly $U(1)' - [\text{gravity}]^2$ is given as

$$A_6 : 9(2z[Q] + z[U^c] + z[D^c]) + 3(2z[L] + z[E^c] + z[N^c])
+ 2N_H(z[H_1] + z[H_2]) + N_Sz[S] + 3 \sum_{i=1}^{N_K} (z[K_i] + z[K^c_i]) = 0,$$

where $N_S$ is the number of Higgs singlets $S$ in the model. Taking the linear combination $A_6 - 9Y_U - 9Y_D - 3Y_E - 3Y_N + (12 - 2N_H)Y_S - 3 \sum_{i=1}^{N_K} Y_{K_i}$, we get

$$(N_S - 2N_H - 3a + 3)z[S] = 0 .$$

Because of eq. (6), this implies

$$N_S = 2N_H + 3a - 3 .$$

Once again, the anomaly condition did not constrain the $U(1)'$ charges, but just the number of representations. The simplest possibility appears to be $N_H = 1$, $a = 1$, $N_S = 2$, and this
is the case we shall investigate in Section IV. Another example discussed in Appendix A is $N_H = 4, a = 1$ and $N_S = 8$. Finally, $N_H = 3, a = 0$ and $N_S = 3$ is the case considered in Ref. [29] and below in Appendix B. We see that eq. (48) excludes the minimal (in the sense of total number $N_H + N_S$ of Higgs representations) possibility of $N_H = N_S = 1$ in our current setup. However, this conclusion can be avoided with the addition of extra SM singlet exotic fields. Appendix C provides a specific example of such a model with $N_H = N_S = 1$.

E. The Anomalies $A_4$ ($U(1)_Y - [U(1)']^2$) and $A_5$ ($[U(1)']^3$)

The remaining anomaly conditions $A_4$ and $A_5$ are in general nonlinear equations for the $U(1)'$ charges:

$$A_4 : 9(2y[Q]z[Q]^2 + y[U^c]z[U^c]^2 + y[D^c]z[D^c]^2) + 3(2y[L]z[L]^2 + y[E^c]z[E^c]^2) + 2N_H(y[H_1]z[H_1]^2 + y[H_2]z[H_2]^2) + 3 \sum_{i=1}^{N_K}(y[K_i]z[K_i]^2 + y[K_i^c]z[K_i^c]^2) = 0 ,$$  

$$A_5 : 9(2z[Q]^3 + z[U^c]^3 + z[D^c]^3) + 3(2z[L]^3 + z[E^c]^3 + z[N^c]^3) + 2N_H(z[H_1]^3 + z[H_2]^3) + N_Sz[S]^3 + 3 \sum_{i=1}^{N_K}(z[K_i]^3 + z[K_i^c]^3) = 0 .$$

Because of their nonlinearity, in the past $A_4$ and $A_5$ have typically been the stumbling blocks for finding anomaly-free solutions for the $U(1)'$ charges. Here we shall show, however, that under our previous assumptions [8,13], both of these equations factorize – each one is in fact proportional to $z[S]$ (which according to eq. (5) is nonzero) so effectively we are able to reduce the power of eq. (49) and eq. (50) by one. For example, the $A_4$ anomaly reduces to a linear constraint among the $U(1)'$ charges. The easiest way to see this is to substitute the general solution (28) into eq. (49), which gives

$$\frac{1}{3}s \left\{ (12N_H - 36)h_1 + (7N_H - 18)s - 12\ell - 9 \sum_{i=1}^{N_K}y[K_i](s + 2z[K_i]) \right\} = 0 .$$

Since $s \neq 0$, the expression within the curly brackets must vanish, which allows us to solve e.g. for one of the exotic charges $z[K_i]$ in terms of the other two as well as $s$, $h_1$ and $\ell$.

---

9 The gravitational anomaly $A_6$ was not taken into account in Ref. [30], which allowed building a model with $N_H = N_S = 1$.

10 The factorization of the $A_4$ and $A_5$ anomalies has been previously noticed in Ref. [30] for the specific case of $N_H = 1, a = 0$ and a particular set of exotics.
Similarly, substituting the general solution (28) into eq. (50), and using eq. (48), we get

\[ s \left\{ -3 \left[ (3a + 4N_H - 12)h_1^2 - 6ah_1\ell + (3a - 4)\ell^2 \right] \\
+ \left[ 3(3a^2 - 6a - 4N_H + 12)h_1 - (9a^2 - 18a + 2N_H)\ell \right] s \\
- \frac{1}{3} \left[ 9a^3 - 27a^2 + 18a - N_H^2 + 9N_H \right] s^2 - 9 \sum_{i=1}^{N_K} z[K_i] (s + z[K_i]) \right\} = 0 . \tag{52} \]

Once again, since \( s \neq 0 \), the expression within the curly brackets must vanish, which translates into only a quadratic constraint on the \( U(1)' \) charges.

As we shall see later in Appendix B, a further drastic simplification of the above formulas (51) and (52) occurs for the case of \( N_H = 3, a = 0 \) and \( y[K_i] = \pm \frac{1}{3} \), when the cubic anomaly completely factorizes, and effectively reduces to a linear constraint. Furthermore, this linear constraint turns out to be equivalent to the constraint implied by eq. (51), so that in effect the cubic anomaly condition is automatically satisfied and in that case does not constrain the \( U(1)' \) charges at all.

This completes our discussion of the anomaly cancellation conditions involving the new \( U(1)' \). To recapitulate, in Section II we first considered the effect of the 6 constraints (8-13) on the \( U(1)' \) charges of the 9 non-exotic fields in our model (see Table I). This resulted in the general three-parameter solution given by eq. (28). Then in Section III we studied the remaining\(^{11}\) 5 anomaly cancellation conditions \( A_1, A_3, A_4, A_5 \) and \( A_6 \), which involved 3 additional variables – the \( U(1)' \) charges \( z[K_i] \) of the exotic fields \( K_i \). We found that only 2 out of these 5 new conditions actually restrict the values of the \( U(1)' \) charges, so that there is still a lot of freedom remaining in the actual \( U(1)' \) charge assignments. In the following we shall demonstrate this explicitly by presenting specific examples of anomaly-free charge assignments which satisfy all of the model-building constraints considered so far. In Section IV we shall find, as anticipated, that there exist solutions which allow for either LV or BV, but not both. Nevertheless, the proton will be stable in such models, as already discussed in Section II C and the \( \mu \)-problem will be solved by eq. (6).

\(^{11}\) Recall that \( A_2 \) was already accounted for in Section II.
IV. MODELS WITH LEPTON OR BARYON NUMBER VIOLATION

In this section we shall concentrate on the simplest case of \( N_H = 1 \). In addition to the usual MSSM fields, the model also contains \( N_S = 2 \) Higgs singlets \( S_i \) and \( N_K = 3 \) vectorlike pairs \((K_i, K^c_i)\) of exotic quarks introduced to cancel the \( A_1 \) anomaly (see Section III A). The \( R \)-parity conserving part of the superpotential is given by the combination of eqs. (2), (7) and (34):

\[
W_{RPC} = y^{D}_{jk} H_1 Q_j D^c_k + y^{U}_{jk} H_2 Q_j U^c_k + y^E_{jk} H_1 L_j E^c_k + y^{N}_{ijk} S_i Λ H_2 L_j N^c_k
+ h_i S_i H_2 H_1 + h''_{ijk} S_i K_j K^c_k.
\]

(53)

Recall that with \( N_H = 1 \) and \( N_S = 2 \), the \( A_6 \) anomaly condition (48) demands \( a = 1 \), so that the neutrino Yukawa couplings arise from a non-renormalizable operator as shown. We assume diagonal couplings of the exotics to \( S \) (i.e. \( z[K_i^c] = -z[K_i] - z[S] \)) but off-diagonal terms may also exist if two or more exotic quarks have identical \( U(1)' \) charges. As discussed in the Introduction, the \( μ \)-problem is solved through an effective \( μ \) term \( μ_{\text{eff}} = h_1 \langle S_1 \rangle + h_2 \langle S_2 \rangle \) by requiring \( z[S] \neq 0 \). This forbids not only the original \( μ \) term, but also mass terms for the exotics \((KK^c)\) and Higgs singlet self-couplings \( S, S^2 \) and \( S^3 \).

The \( R \)-parity violating part of the renormalizable superpotential of the UMSSM is

\[
W_{RPV} = W_{LV} + W_{BV},
\]

(54)

where

\[
W_{LV} = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + h_{ij} S_i H_2 L_j,
\]

(55)

\[
W_{BV} = \lambda''_{ijk} U^c_i D^c_j D^c_k.
\]

(56)

It is easy to see that the \( U(1)' \) symmetry either simultaneously allows all three terms \\{\( LLE^c, LQD^c, SH_2L \)\}, in which case \( z[L] = z[H_1] \), or simultaneously forbids all three. In the \( LV \) case, therefore, we shall expect to have all three terms appearing in eq. (55) present.

A comment is in order regarding the possibility of a bare \( LV \mu'H_2L_i \) term in the superpotential. Such a term is dangerous because it will reintroduce a hierarchy problem (\( \mu' \)-problem) of the type we originally intended to avoid. Indeed, the general solution (28) in principle allows for this term. However, it is easy to see that in both the \( LV \) case and the \( BV \) case we are interested in, this term is absent and the \( \mu' \)-problem is solved in exactly
the same way as the $\mu$-problem. For example, in the LV case the $U(1)'$ charge of $H_2L_i$ from eq. (29) is $z[H_2L_i] = z[S]$ which is not vanishing because of condition (6). In the BV case, from eq. (30) we get $z[H_2L_i] = (N_H - 6)z[S]/3$. Since the case of $N_H = 6$ was already discarded (see Section II C), the $H_2L_i$ is again forbidden by the $U(1)'$ symmetry. An effective $\mu'$ term will be nevertheless generated from the $S_iH_2L_j$ term in $W_{LV}$, once the $U(1)'$ symmetry is broken by the VEV of $S$ at the TeV scale.

The $U(1)'$ symmetry is broken when $S$ gets a VEV $\langle S \rangle$ at the TeV scale. This generates the corresponding effective bilinear terms in the superpotential with coefficients

$$
\mu_{\text{eff}} \equiv h_i \langle S_i \rangle, \quad \mu'_{i,\text{eff}} \equiv h'_{ji} \langle S_j \rangle, \quad m_{K,ij} \equiv h''_{kij} \langle S_k \rangle. \tag{57}
$$

With the natural size of the couplings \(\{h, h', h''\} \sim 1\), the effective $\mu$ and $\mu'$ parameters as well as the masses of the exotic quarks $m_K$ are all of order a TeV. With the effective bilinear terms, the superpotential of the UMSSM becomes similar to that of the MSSM. First, the model predicts a new gauge boson, $Z'$, near the $U(1)'$ symmetry breaking scale:

$$
M_{Z'}^2 = g_{Z'}^2 \left( z[H_1]^2 v_1^2 + z[H_2]^2 v_2^2 + z[S]^2 v_{s1}^2 + z[S]^2 v_{s2}^2 \right). \tag{58}
$$

Here, $g_{Z'}$ is the $U(1)'$ gauge coupling constant, $v_i = \sqrt{2} \langle H_i \rangle$ (with $v_1^2 + v_2^2 \sim 246^2 \text{ GeV}^2$), and $v_{si} = \sqrt{2} \langle S_i \rangle$. The direct constraint on the mass of the $Z'$ comes from searches at the Tevatron in the dilepton channel ($Z' \rightarrow \ell^+ \ell^-$). The typical bound is $M_{Z'} > 600 \sim 900 \text{ GeV}$, depending on the $U(1)'$ charges of the quarks and leptons [45]. The VEV’s of the Higgs doublets will also induce mixing between the $Z$ and $Z'$ gauge bosons. If the $Z'$ is sufficiently heavy, this mixing is quite small, in accordance with the experimental constraints from LEP (per mil level) [46]. The supersymmetric partners of the $Z'$ and $S$ ($Z'$-ino and singlino) become extra components of the neutralinos. The $S$ field gives one physical CP-even Higgs state, while the corresponding CP-odd Goldstone boson gets absorbed as the longitudinal component of the $Z'$ gauge boson. For recent studies on phenomenology of the UMSSM, see Ref. [47].

We shall now present explicit examples where the $U(1)'$ symmetry allows for either $W_{LV}$ or $W_{BV}$, but not both at the same time. For simplicity, we assume the MSSM chiral fields ($Q, U^c, D^c, L, E^c, N^c$) to have family universal $U(1)'$ charges\footnote{Family non-universal $U(1)'$ charges in the SM quark sector may induce dangerous flavor changing neutral currents [48]. (On the other hand, such a flavor changing $Z'$ may provide an explanation of the discrepancies in rare $B$ decays [49].)}, but we allow family non-
universal $U(1)'$ charges for the exotic quarks ($K_i, K^c_i$). The hypercharges of the exotic quarks may be family non-universal as well. In general, it is possible that there may be additional SM singlet fields which belong to the hidden sector, yet are charged under $U(1)'$ and thus contribute to the $A_5$ and $A_6$ anomalies. However, our primary intention was simply to demonstrate that an anomaly-free $U(1)'$ can be used and is sufficient to achieve all of our goals outlined in the Introduction. Therefore, for concreteness and for simplicity, we shall assume only the field content listed in Table I.

In Table III we show several examples of anomaly-free charge assignments (up to an arbitrary normalization factor) for $N_H = 1$, $N_S = 2$, $a = 1$ and $y[K_i] = \{\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\}$. We have classified our examples in two groups: the first five columns are LV models which allow for LV, but not BV terms in the superpotential, while the remaining six columns are BV models which allow for BV, but not LV terms in the superpotential. In LV models I-IV the LV terms appear already at the renormalizable level as in eq. (55). In model V the terms of eq. (55) appear at the non-renormalizable level ($SLLE^c$, $SLQD^c$ and $S^2H_2L$) and in addition there are renormalizable LV terms involving exotics, e.g. $NK_1K^c_1$ and $EK_2K^c_1$. Similarly, BV models I-III already allow renormalizable BV couplings as in eq. (56), while BV models IV-VI allow only non-renormalizable BV operators such as $QQD^c\dagger$, $QQQH_1$ and $H_1H_2U^cD^cD^c$.

A few comments are in order. First, each example in Table III in fact corresponds to a whole family of solutions. This is because hypercharge itself also satisfies all of our requirements, including the absence of mixed anomalies with $U(1)'$. Therefore, each one of our solutions can be “rotated” by hypercharge in an arbitrary normalization. More specifically, if $z_0[F_i]$ is any particular solution from Table III then a family of anomaly-free $U(1)'$ charges is generated by the linear combination

$$z[F_i] = \alpha z_0[F_i] + \beta y[F_i], \quad (59)$$

where $y[F_i]$ are the hypercharge assignments of our fields $F_i$ from Table I and $\alpha$ and $\beta$ are arbitrary coefficients. Therefore, the numerical values for the $U(1)'$ charges in our models are subject to fixing the convention for eq. (59). In Table III we only listed examples which are not equivalent in the sense of eq. (59).

In spite of the freedom provided by eq. (59), the numerical values of the $U(1)'$ charges are important for phenomenology, as they determine the couplings of the particles in our
|     | LV          | BV          |
|-----|-------------|-------------|
|     | I  II  III  IV  V | I  II  III  IV  V |
| $z[Q]$ | 1  3  3  3  4 | 1  3  15  0  0  0 |
| $z[U^c]$ | 8  24  24  24  5 | 2  6  30  3  9  9 |
| $z[D^c]$ | -1  -3  -3  -3  -4 | -1  -3  -15  0  0  0 |
| $z[L]$ | 0  0  0  0  -9 | -2  -6  -30  1  3  3 |
| $z[E^c]$ | 0  0  0  0  9 | 2  6  30  -1  -3  -3 |
| $z[N^c]$ | 0  0  0  0  9 | 2  6  30  -1  -3  -3 |
| $z[H_2]$ | -9  -27  -27  -27  -9 | -3  -9  -45  -3  -9  -9 |
| $z[H_1]$ | 0  0  0  0  0 | 0  0  0  0  0  0  0 |
| $z[S]$ | 9  27  27  27  9 | 3  9  45  3  9  9 |
| $z[K_1]$ | -5  -13  -23  -25  -5 | -1  -7  -17  -3  -7  -5 |
| $z[K_2]$ | -2  -4  -8  -7  -5 | -1  -4  -20  0  -1  1 |
| $z[K_3]$ | 1  2  1  -1  -5 | -1  -4  -11  0  2  1 |
| $z[K'_1]$ | -4  -14  -4  -2  -4 | -2  -2  -28  0  -2  -4 |
| $z[K'_2]$ | -7  -23  -19  -20  -4 | -2  -5  -25  -3  -8  -10 |
| $z[K'_3]$ | -10  -29  -28  -26  -4 | -2  -5  -34  -3  -11  -10 |

TABLE III: Examples of anomaly-free $U(1)'$ charge assignments for $N_H = 1, N_S = 2, a = 1$ and $y[K_i] = \{\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\}$. These $U(1)'$ charges can be scaled by an arbitrary normalization factor, as well as rotated by hypercharge (see text for details).

model to the $Z'$. For instance, our LV examples I-IV in Table III are completely leptophobic, as they have $z[L] = z[E^c] = z[N^c] = 0$. Under those circumstances, the standard collider bounds on the $Z'$ mass are degraded, and a very light $Z'$ can be allowed. However, this is not a general property of our LV models, since the hypercharge “rotation” (59) could generate nonzero $U(1)'$ charges for $L, E^c$ and $H_1$. On the other hand, $z[N^c] = 0$ is a general property of LV models I-IV in this particular case ($a = 1$), as already anticipated by eq. (27). Similarly, the vanishing entries for the $U(1)'$ charges of $Q, D^c$ and $H_1$ in our BV models, can also be rotated away from zero using eq. (59).

As we mentioned in Section III C, we also consider the case where the exotic hypercharges

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have the opposite sign: $y[K_i] = \{-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\}$. The actual solutions for the $U(1)'$ charges that we find in that case are given simply by those of Table III with the replacement $z[K_i] \leftrightarrow z[K^c_i]$. In general, this choice of $y[K_i]$ appears dangerous, since hypercharge alone would then allow for LV and BV couplings involving exotic fields. However, we find that due to the general phenomenon of LV-BV separation discussed in Section III.B, the $U(1)'$ symmetry is still sufficient to prevent the simultaneous appearance of LV and BV couplings in the superpotential, and in all but one case (namely, BV-IV with opposite exotic hypercharge) the proton turns out to be stable [38].

V. CONCLUSIONS

In this paper, we constructed a $U(1)'$-extended MSSM without $R$-parity, where the extra non-anomalous $U(1)$ gauge symmetry plays the dual role of solving the $\mu$-problem and controlling the $R$-parity violating terms [34]. The $U(1)'$ gauge symmetry provides a solid theoretical framework for discussing the phenomenology of $R$-parity violation. The most important implication of our models is the LV-BV separation: when the lepton number violating terms [3] are allowed by the $U(1)'$ symmetry, the baryon number violating terms [4] in the superpotential are automatically forbidden, and vice versa. Within our approach, the dangerous dimension 5 operators such as $QQQL$ or $U^c U^c D^c E^c$, which are allowed by $R$-parity and could still destabilize the proton, are also eliminated. This presents a very minimal solution to the proton decay problem which is alternative to $R$-parity. We showed that the LV-BV separation holds under very general circumstances. Perhaps the most stringent and least motivated was our assumption that there are no exotic $SU(2)_L$ representations. While one can not judge the validity of this assumption without knowledge of the fundamental theory at high energies, it is certainly consistent with the principle of “Occam’s razor”.

While in our LV and BV examples the corresponding RPV couplings are allowed by the symmetries, the size of those couplings is still undetermined. The experimental upper bounds on the individual RPV couplings range from $10^{-3}$ for $\lambda$ to $10^{-7}$ for $\lambda''$. We do not consider such small values particularly fine-tuned, especially when compared to the Yukawa couplings of the first generation fermions in the SM. In fact such small RPV couplings may naturally originate from higher-dimensional operators, without modifying the analysis and
the conclusions of our paper [38].

An interesting feature of our setup is that all LV terms \((\lambda L LE^c, \lambda' L Q D^c, \mu'_{\text{eff}} H_2 L)\) must co-exist, as long as one of them is allowed. This is phenomenologically interesting since, for instance, the observation of a sneutrino resonance in an \(s\)-channel at hadron colliders such as the Tevatron and the LHC requires both \(\lambda\) and \(\lambda'\) couplings. Besides the relation among the \(R\)-parity violating terms, our models also provide a connection between the phenomenology of \(R\)-parity violation and \(U(1)'\) extensions of the MSSM. In this sense, a potential discovery of a \(Z'\) resonance at the Tevatron or LHC would motivate searches for \(R\)-parity violating SUSY signatures, and vice versa.

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APPENDIX A: MODELS WITH $N_H = 4$

In this Appendix we briefly consider the case of $N_H = 4$. Again we shall choose $a = 1$, which fixes $N_S = 8$ in accordance with eq. (48). The exotic hypercharges are uniquely determined from eq. (42) to be $y[K_i] = 0$. For simplicity, in this Appendix we shall assume that the exotic quarks also have the same $U(1)'$ charges as well: $z[K_1] = z[K_2] = z[K_3] \equiv k$. With those choices, the quadratic and cubic anomaly conditions (51) and (52) can be rewritten as

$$\frac{2}{3} s (6h_1 + 5s - 6\ell) = 0 \tag{A1}$$

$$-\frac{1}{12} s \left( (6h_1 + 5s - 6\ell)(42h_1 + 7s + 6\ell) + 9 (s + 6k) (5s + 6k) \right) = 0 \tag{A2}$$

Notice that taking into account eq. (A1) eliminates the first term in the curly brackets in eq. (A2) and the $A_5$ anomaly condition completely factorizes:

$$-\frac{3}{4} s (s + 6k) (5s + 6k) = 0 \tag{A3}$$

This allows us to obtain explicitly a family of anomaly-free solutions for the $U(1)'$ charges of the fields in Table II. It turns out that all of these solutions forbid both the LV and BV terms, something which could not have been expected on the basis of eq. (15) alone. Indeed, the $A_4$ anomaly constraint (A1) is inconsistent with the individual constraints for the LV case ($h_1 = \ell$) and the BV case ($h_1 = \ell + (1 - \frac{N_H}{3})s$) which were derived earlier in Section II.E. In either case, compatibility with eq. (A1) demands $s = 0$, which is not allowed by the condition (6).

The factorized constraints (A1) and (A2) can now be solved rather easily and the general
solution (28) can be written as

\[
\begin{pmatrix}
  z[Q] \\
z[U^c] \\
z[D^c] \\
z[L] \\
z[E^c] \\
z[N^c] \\
z[H_2] \\
z[H_1] \\
z[S] \\
z[K] \\
z[K']
\end{pmatrix} = -2\ell \begin{pmatrix}
  \frac{1}{6} \\
-\frac{2}{3} \\
\frac{1}{3} \\
-\frac{1}{2} \\
1 \\
0 \\
\frac{1}{2} \\
-\frac{1}{2} \\
0 \\
0 \\
0
\end{pmatrix} + \frac{s}{18} \begin{pmatrix}
8 \\
-5 \\
7 \\
0 \\
15 \\
-15 \\
-3 \\
-15 \\
18 \\
18\rho \\
-18(1+\rho)
\end{pmatrix}
\]  

(A4)

where \( \rho = -\frac{1}{6} \) (\( \rho = -\frac{5}{6} \)) in the case of \( s + 6k = 0 \) (\( 5s + 6k = 0 \)). As expected, we obtain a two-parameter family of solutions – one parameter (\( \ell \)) corresponds to the usual hypercharge assignments while the second parameter (\( s \)) gives the nontrivial part of the \( U(1)' \) solution.

**APPENDIX B: MODELS WITH \( N_H = 3 \)**

In this Appendix we shall consider \( U(1)' \) models with \( N_H = 3 \), \( N_S = 3 \) and \( a = 0 \), as in Ref. [29]. As we already saw in Section II B, in that case one should either simultaneously allow or simultaneously forbid the LV and BV terms (see eq. (15)). Furthermore, \( N_H = 3 \) allows for family-universal hypercharges of the exotic quarks (see eqs. (42) and (45)). We shall consider two possible values for the exotic hypercharges: \( y[K_1] = +\frac{1}{3} \) and \( y[K_1] = -\frac{1}{3} \). For simplicity, in this Appendix we shall again assume universal \( U(1)' \) charges for the exotic quarks: \( z[K_1] = z[K_2] = z[K_3] = k \). The \( A_4 \) anomaly (51) can then be written as

\[
A_4 : \begin{cases}
-2s (3k + 2\ell + s) = 0, & \text{for } y[K_1] = \frac{1}{3}; \\
2s (3k - 2\ell + 2s) = 0, & \text{for } y[K_1] = -\frac{1}{3};
\end{cases}
\]  

(B1)

while the \( A_5 \) anomaly condition is independent of \( y[K_1] \) and reads

\[
A_5 : -3s (3k + 2\ell + s) (3k - 2\ell + 2s) = 0.
\]  

(B2)

We can see that \( A_5 \) completely factorizes into linear polynomials which already appear in the expression for \( A_4 \). Therefore, \( A_5 \) does not provide an additional restriction on the
$U(1)'$ charges, i.e. $A_5$ will be automatically satisfied for any choice of $U(1)'$ charges which is consistent with $A_4$. Since $A_4$ is already a linear relation, this allows us to derive a three-parameter class of solutions which generalize the single model found in Ref. [29]. For $y[K_i] = +\frac{1}{3}$, from eqs. (28) and (B1) we find the general solution

$$
\begin{pmatrix}
z[Q] \\
z[U^c] \\
z[D^c] \\
z[L] \\
z[E^c] \\
z[N^c] \\
z[H_2] \\
z[H_1] \\
z[S] \\
z[K] \\
z[K^c]
\end{pmatrix} = \ell \begin{pmatrix}
-1 \\
-1 \\
1 \\
1 \\
-1 \\
-3 \\
2 \\
0 \\
-2 \\
0 \\
2
\end{pmatrix} + h_1 \begin{pmatrix}
0 \\
1 \\
-1 \\
0 \\
0 \\
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix} + k \begin{pmatrix}
-1 \\
-2 \\
1 \\
1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
$$

(B3)

in terms of the $U(1)'$ charges $\ell \equiv z[L], h_1 \equiv z[H_1]$ and $k \equiv z[K_i]$. This solution is anomaly-free and satisfies all of the constraints discussed in Sections II and III. As a special case, it also contains the solution found in Ref. [29], which we recover by imposing $8\ell = -7h_1 = -7k$. For example, $\ell = \frac{7}{12}$, $h_1 = k = -\frac{2}{3}$, gives

$$
z[Q, U^c, D^c, L, E^c, N^c, H_2, H_1, S, K, K^c] = \left\{ \frac{1}{12}, \frac{1}{12}, \frac{7}{12}, \frac{7}{12}, \frac{1}{12}, -\frac{5}{12}, -\frac{1}{6}, -\frac{2}{3}, \frac{5}{6}, -\frac{2}{3}, -\frac{1}{6} \right\}
$$

(B4)

which is exactly the charge assignment in the model of Ref. [29]. In addition to our requirements listed in Sections II and III, Ref. [29] demanded the presence of a Majorana mass term $SN^cN^c$ in the superpotential. This would imply the constraint $8\ell - 2h_1 + 9k = 0$, 

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which still leaves us with a two-parameter class of solutions

\[
\begin{pmatrix}
  z[Q] \\
  z[U^c] \\
  z[D^c] \\
  z[L] \\
  z[E^c] \\
  z[N^c] \\
  z[H_2] \\
  z[H_1] \\
  z[S] \\
  z[K] \\
  z[K^c]
\end{pmatrix}
= \ell
\begin{pmatrix}
  0 \\
  1 \\
  0 \\
  1 \\
  1 \\
  1 \\
  0 \\
  0 \\
  0 \\
  2 \\
  4
\end{pmatrix}
+ \frac{k}{2}
\begin{pmatrix}
  0 \\
  1 \\
  1 \\
  0 \\
  1 \\
  1 \\
  0 \\
  0 \\
  2 \\
  4
\end{pmatrix}
\]

(B5)

as a generalization of eq. (B4).

For completeness, we shall also consider the other possible sign of the exotic hypercharges: \( y[K^c] = -\frac{1}{3} \), since in that case \( A_5 \) is also automatically satisfied due to its factorization (B2), which makes it easy to obtain another class of solutions satisfying all Yukawa constraints and all anomaly cancellation conditions. Putting together eqs. (B1) and (28), we find

\[
\begin{pmatrix}
  z[Q] \\
  z[U^c] \\
  z[D^c] \\
  z[L] \\
  z[E^c] \\
  z[N^c] \\
  z[H_2] \\
  z[H_1] \\
  z[S] \\
  z[K] \\
  z[K^c]
\end{pmatrix}
= \ell
\begin{pmatrix}
  0 \\
  1 \\
  0 \\
  1 \\
  1 \\
  1 \\
  0 \\
  0 \\
  0 \\
  2 \\
  4
\end{pmatrix}
+ h_1
\begin{pmatrix}
  0 \\
  1 \\
  0 \\
  1 \\
  1 \\
  1 \\
  0 \\
  0 \\
  0 \\
  1 \\
  3
\end{pmatrix}
+ \frac{k}{2}
\begin{pmatrix}
  0 \\
  1 \\
  0 \\
  1 \\
  1 \\
  1 \\
  0 \\
  0 \\
  0 \\
  2 \\
  4
\end{pmatrix}
\]

(B6)

Unfortunately, this class of models does not solve the proton decay problem: as can be seen from eq. (B6), the \( U(1)' \) symmetry still allows \( R \)-parity violating couplings involving exotic fields, e.g. \( U^c D^c K^c \) and \( L Q K^c \).
APPENDIX C: MODELS WITH $N_H = N_S = 1$

We have already seen that the $A_6$ anomaly condition \[ \text{(18)} \] restricts the number of Higgs representations $N_H$ and $N_S$. As we mentioned in Section [1111] the minimal case of $N_H = 1$, $N_S = 1$ is not allowed within the model we have discussed so far. However, the constraint \[ \text{(18)} \] varies with the particle spectrum, and here we provide an example with a slightly altered spectrum which can allow $N_H = 1$, $N_S = 1$. We simply add another SM singlet field $X$ with superpotential

$$W_X = \xi SXX,$$  \tag{C1}

so that the $U(1)'$ charge of $X$ is given by $z[X] = -\frac{1}{2} z[S]$. The general solution \[ \text{(28)} \] is then rewritten as

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_2] \\ z[H_1] \\ z[S] \\ z[X] \end{pmatrix} = \frac{\ell}{3} + h_1 -1 + \frac{s}{9} \begin{pmatrix} N_H \\ 9-N_H \\ -N_H \\ 0 \\ 0 \\ 9(1-a) \\ -9 \\ 0 \\ 9 \\ -9/2 \end{pmatrix} \tag{C2}$$

with no additional free parameters.

The new $X$ particles will modify the anomaly conditions $A_6$ ($U(1)' - \text{[gravity]}^2$) and $A_5$ ($[U(1)']^3$) which get additional contributions of $N_X z[X]$ and $N_X z[X]^3$, respectively. Then eq. \[ \text{(18)} \] is modified as

$$N_S = 2N_H + 3a - 3 + \frac{1}{2}N_X$$  \tag{C3}

where $N_X$ is the number of families of the $X$ fields. $N_H = 1$, $N_S = 1$ is now allowed with $a = 0$, $N_X = 4$. As an existence proof, we provide an example of an anomaly-free LV model of this category with $y[K_i] = \{\frac{1}{3}, -\frac{2}{3}, -\frac{4}{3}\}$:

$$z \{Q, U^c, D^c, L, E^c, N^c, H_2, H_1, S, K_1, K_2, K_3, K_1^c, K_2^c, K_3^c, X \} = \{4, 8, 2, -6, 12, 18, -12, -6, 18, -6, -3, -15, -12, -15, -3, -9\}. \tag{C4}$$
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