Integrated photonic qubit quantum computing on a superconducting chip

Lianghui Du, Yong Hu, Zheng-Wei Zhou, Guang-Can Guo and Xingxiang Zhou

Key Laboratory of Quantum Information, University of Science and Technology of China, Chinese Academy of Sciences, Hefei 230026, People’s Republic of China
E-mail: xizhou@ustc.edu.cn

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Abstract. We study a quantum computing system using microwave photons in transmission line resonators on a superconducting chip as qubits. We show that linear optics and other controls necessary for quantum computing can be implemented by coupling to Josephson devices on the same chip. By taking advantage of the strong nonlinearities in Josephson junctions, photonic qubit interactions can be realized. We analyze the gate error rate to demonstrate that our scheme is realistic even for Josephson devices with limited decoherence times. As a conceptually innovative solution based on existing technologies, our scheme provides an integrated and scalable approach to the next key milestone for photonic qubit quantum computing.

Despite the vast potential of quantum computers, no perfect physical implementation has been found for them, as can be seen by examining two representative systems. Josephson device-based superconducting systems are easily integrable and scalable, but are limited by the short decoherence times of Josephson qubits due to coupling to their complex solid-state environment. Photonic qubits, which exhibit strong quantum coherence, suffer from the fact that photons do not interact easily. Also, systems based on conventional bulk optical devices are hard to miniaturize and scale.

1 Author to whom any correspondence should be addressed.
Recognizing the importance of integrated systems for scalable quantum computing, a number of investigators have demonstrated on-chip waveguide-based quantum gates for photonic qubits recently [1]. This is a significant step that may represent the future direction of photonic qubit quantum computing technologies. However, it is a daunting task to achieve a fully integrated photonic qubit quantum computer using conventional technologies, including those developed in the latest experiments. This is because conventional optical devices such as lasers, lenses, optical cavities and photodetectors are bulk devices based on very different technologies and no process exists yet to integrate them on the same chip [2]. Therefore, alternative realistic approaches to fully integrated photonic qubit quantum computing systems are highly valuable.

We combine the strengths of photonic and superconducting systems to realize fully integrated photonic qubit quantum computing. Our physical system is a superconducting chip on which high-$Q$ transmission line resonators (TLRs) and Josephson devices are fabricated. The same system has been used for the study of cavity QED based on Josephson qubits [3]–[5]. However, in our scheme the quantum information is carried by the microwave photon modes in the TLRs and the Josephson junctions play the role of optical devices. For high-$Q$ TLRs, the photons have a long lifetime, which is a major advantage. Easy operation and accurate control are available because Josephson devices can be fabricated with great precision and controlled conveniently by monitoring their electrical signals. A further key advantage is that we can use the strong nonlinearities inherent in Josephson devices to induce interactions between photons. It is shown that high gate fidelities can be achieved even for Josephson devices with limited decoherence times, making their unavoidable noisy environment no longer a limiting factor. Therefore, our scheme is a realistic approach to scalable photonic qubit quantum computing.

We start by considering the two identical TLRs shown in figure 1(a). The TLR mode frequencies are given by $\omega = n\pi/\sqrt{LC}$, $n$ being an integer and $L$ and $C$ the total inductance and capacitance of the TLR. We use the $n = 2$ mode. For $L = 0.5$ nH and $C = 5$ pF, its frequency $\omega_0/2\pi \approx 20$ GHz. The second-quantized voltage and current associated with this mode are $V(x, t) = \sqrt{\hbar\omega_0/C} \cos \frac{2\pi}{l} (\hat{a}(t) + \hat{a}^\dagger(t))$ and $I(x, t) = -i\sqrt{\hbar\omega_0/\pi L} \sin \frac{\pi}{l} (\hat{a}(t) - \hat{a}^\dagger(t))$, where $l$ is the length of the TLR, $x \in [-l/2, l/2]$ the position along the TLR and $\hat{a}(t) = \hat{a} e^{-i\omega_0 t}$ the mode’s annihilation operator.

For the pair of identical TLRs in figure 1(a), we introduce a single photon of frequency $\omega_0$ and it being in the left or right TLR denotes the logic 0 or 1 state [6] for a single qubit. This is analogous to the conventional optical cavity mode representation of photonic qubit where the information is encoded by which cavity the photon is in [7]. In comparison to encoding the quantum information with the vacuum and one-photon states, by using this representation we can keep the system energy in the photons only and implement linear or nonlinear photonic qubit quantum computing by manipulating the photons only. For a dilution refrigerator temperature of 40 mK, the thermal photon number in the TLRs is smaller than $10^{-10}$ and thus the 0 or 1 photon state for the TLRs is an excellent approximation. To effect arbitrary transformations on this single qubit, we need to be able to shift the relative energies of the TLRs and transfer photons between them, which implement the functionalities of phase shifters and beam splitters in optics. We realize this by coupling the TLRs capacitively to current biased Josephson junctions (CBJJs), as shown in figure 1(a). Approximating the CBJJs as two-state...
systems whose energy splittings $\Omega_c$ and $\Omega_r$ can be easily adjusted by their bias currents\(^2\), we can write the system Hamiltonian $H = \hbar \omega_0 (\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}) + \frac{1}{2} \hbar \Omega_c \sigma_c^z + \frac{1}{2} \hbar \Omega_r \sigma_r^z + \hbar g_c [(\hat{a} + \hat{b})\sigma_c^+ + (\hat{a}^\dagger + \hat{b}^\dagger)\sigma_c^-] + \hbar g_r (\hat{b} \sigma_r^+ + \hat{b}^\dagger \sigma_r^-)$ [8], where $\sigma_{c,r}^{\pm}$ are the Pauli matrices of the coupling and right CBJJs, $\hat{a}$ and $\hat{b}$ are the annihilation operators for photons in the two TLRs, and the coupling strengths $g_{c,r} = \omega_0 C_{c,r}/\sqrt{2C(C_{c}^{-1} + 2C_{c,r})}$ and $C_{c,r}^{-1}$ are the capacitance of the coupling and right CBJJs.

Since the CBJJ energies can be easily adjusted by tuning the bias current, we can control the interactions between the TLRs and CBJJs. To transfer photons between the TLRs, we adjust the bias currents of the CBJJs to tune $\Omega_r$ far away from $\omega_0$ so the right CBJJ has no effect. We further tune the coupling CBJJ close to resonance with $\omega_0$ and work in the dispersive regime where the magnitude of detuning $\Delta_c = \Omega_c - \omega_0$ is much greater than $g_c$. In this case, assuming the CBJJ was prepared in the ground state, its virtual excitation gives rise to the following effective Hamiltonian for the TLRs [4] in the rotating frame defined by the uncoupled TLR Hamiltonian:

$$H_{\text{eff}} = \frac{\hbar g_c^2}{\Delta_c} (\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}) + \frac{\hbar g_r^2}{\Delta_c} (\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b})$$.

(1)

Since there is only one photon in the system, $\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b} = 1$, the energy shift described in the second term in $H_{\text{eff}}$ is a constant. The first exchange term implements what we call the photon transfer operation. A photon can be transferred between the two TLRs with a rate $g_c^2/2\pi \Delta_c$, which is about 20 MHz for $C_{c} = 0.5 \text{ pF}$, $C_{r} = 23 \text{ fF}$, and $\Delta_c/2\pi = 2 \text{ GHz}$ [9, 10].

\(^2\) Note that even though we use the two-state formalism for the CBJJ for the convenience of keeping only the relevant terms in the system Hamiltonian, our scheme does not rely on treating the CBJJ as a two-state system. The CBJJ will be only virtually excited so potential problems with the two-state approximation such as insufficient anharmonicity do not affect our results at all.

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To shift the relative energies of the TLRs, we tune the coupling CBJJ far off resonance and tune the right CBJJ into the dispersive region. Similarly, an effective Hamiltonian \((\hbar g_c^2/\Delta_c)\hat{b}^\dagger \hat{b}\) results, which gives a relative phase when the photon is in the right TLR.

We need to study the decoherence properties of our scheme to analyze its reliability. The photonic qubits have superb coherence and their lifetimes are orders of magnitude longer than that of superconducting qubits. For TLRs fabricated on superconducting chips, a high-quality factor of \(10^6\text{--}10^7\) has been demonstrated \([11]\). For TLR frequencies of tens of GHz, the photon loss rate \(\kappa/2\pi\) can be as low as kHz. In contrast, the CBJJ has a short decoherence time, and we assume its dephasing rate \(\Gamma_2/2\pi \approx 1\) MHz. The CBJJ’s decay rate from the excited state \(\Gamma_1/2\pi\) is of the order of 0.1 MHz.

A major advantage of our scheme is that the relatively lossy CBJJ does not damp the coherence of the photonic qubits very much, since it is only virtually excited. The CBJJ’s decay from the virtually excited state increases the photon’s loss rate by \((g_c/\Delta_c)^2\Gamma_1\), which is not a concern since \((g_c/\Delta_c)^2\) is no greater than \(\kappa\). To study the effect of the CBJJ’s dephasing rate \(\Gamma_2\), we model the dephasing effect as the result of a random fluctuation \(\delta_n\) in the CBJJ’s energy splitting, following the treatment in \([12]\). This introduces an uncertainty in the detuning during, for instance, a photon transfer operation, \(\Delta_c = \Omega_c - \omega_0 \rightarrow \Delta_c + \delta_n\). Here, \(\delta_n\) is the uncertainty in the energy splitting of the CBJJ caused by noise in its Josephson energy and bias current. It is proportional to the first-order derivative of the CBJJ energy with respect to these parameters and it determines the CBJJ’s dephasing rate \([12]\). Due to the presence of the noise, the system will have a random Hamiltonian \(H_{\text{noise}} = -\hbar(g_c/\Delta_c)^2\delta_n(\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger)\) in addition to that in equation (1). Assuming the distribution of \(\delta_n\) is Gaussian, we can estimate the photonic qubit’s decoherence time due to \(H_{\text{noise}}\) by using the free induction decay function \([12]\). Since the free inductor decay function is determined by the spectral density of \(\delta_n\), which in turn is related to the CBJJ’s dephasing rate \(\Gamma_2\), it can be estimated that the system’s dephasing rate is no greater than \(2(g_c^2/\Delta_c^2)\Gamma_2\).

Following the quantum theory of damping, we now calculate the gate error of a photon transfer operation under the influence of cavity loss and CBJJ dephasing using the master equation for the qubit’s density matrix \(\rho\), \(d\rho/dt = -i[H_{\text{eff}}, \rho] + \kappa[\hat{a}\rho\hat{a}^\dagger - \frac{1}{2}\hat{a}^\dagger\hat{a}\rho - \frac{1}{2}\rho\hat{a}^\dagger\hat{a}] + \kappa[\hat{b}\rho\hat{b}^\dagger - \frac{1}{2}\hat{b}^\dagger\hat{b}\rho - \frac{1}{2}\rho\hat{b}^\dagger\hat{b}] + 2(\frac{\pi}{\Omega_c})\Gamma_2(\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger)\rho(\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger) - \rho\). The gate error probability of a single-qubit bit flip is plotted in figure 1(b) as a function of \(\kappa\) and \(\Gamma_2\). The result indicates that, for the already demonstrated \(\Gamma_2/2\pi = 1\) MHz and \(\kappa/2\pi = 10\) kHz \([13, 14]\), the gate error is of the order of \(10^{-3}\).

The manipulations demonstrated so far perform linear optics. Together with the ability to generate and detect photons on the same chip (discussed later), we can perform fully integrated linear photonic qubit quantum computing on a superconducting chip. Alternatively, we can consider interactions between TLR photon qubits. Making photons interact is a major difficulty in conventional optics. However, at microwave frequencies, we can take advantage of the strong nonlinearities in Josephson devices to make photons interact.

We consider the low current biased 4-junction SQUID (FJS) device \([15]\) in figure 2(a). The two small identical SQUIDs are coupled inductively to TLR C of length \(l\) at positions \(\pm l/4\). (This does not mean that the FJS must extend to a length of \(l/2\) because the TLR can be laid out in a zigzag fashion.) Since \(l\) is much larger than the dimension of the FJS, we can adopt the long wave approximation and use the TLR current at the SQUIDs’ locations in calculating the SQUIDs’ flux bias. At the two coupling points, the TLR currents are the largest in magnitude and opposite in direction. The main loop is coupled to TLR D at \(l/4\).
Figure 2. (a) The FJS for interacting photons. The junctions are shunted by large capacitances $C_s$ to stabilize the circuit’s phase. (b) Dependence of the fidelity of a controlled phase gate on the ratio between photon transfer rate $\lambda$ and uncertainty in photon energy shift $\omega_s$. Top: comparison between the fidelity calculated by treating the FJS phase as a classical random variable and ignoring the photon loss and FJS dissipation and decoherence (solid line) and the fidelity obtained by solving the density matrix of the entire system and incorporating all decoherence sources (dashed line). Bottom: the gate fidelity without the phase error cancellation procedure. (c) The setup for controlled photon interaction. The circuit in the dashed-line box is the same as that in (a). The TLRs are coupled to CBJJs so that single-bit operations including photon transfer can be performed. The system can be scaled by extending the setup in both ends.

Assuming there is no other external flux bias, the small SQUIDs and main loop are biased by the TLR currents $I_C = \mp i I_{C0}(\hat{c} - \hat{c}^\dagger)$ and $I_D = -i I_{D0}(\hat{d} - \hat{d}^\dagger)$, where $I_{C0} = \sqrt{\hbar \omega_c/L_c}$ and $I_{D0} = \sqrt{\hbar \omega_d/L_d}$ are the zero point current fluctuations in the TLRs, $L_c,d$ the inductance of the TLRs and $\hat{c}$ and $\hat{d}$ the annihilation operators for photons in $C$ and $D$. Since the SQUID phases are constrained by these flux biases, we can work out the system’s Hamiltonian,
\[ H = H_{\text{TLR}} + H_{\text{FJS}} + H_{\text{int}}, \]

where \( H_{\text{TLR}} = \hbar(\omega_c + \omega_s)\hat{c}^{\dagger}\hat{c} + \hbar(\omega_d + \omega_s^d)\hat{d}^{\dagger}\hat{d}, \)

\[ H_{\text{FJS}} = -\frac{\hbar k_B}{2e} \cos \phi, \]

and

\[ H_{\text{int}} = \hbar \omega_{\text{int}} \hat{c}^{\dagger}\hat{d}^{\dagger}\hat{d} + \hbar \omega_{\text{int}} \hat{c} \hat{d}, \]

In these equations, \( E_c = (2e)^2/4(C_j + C_c) \) and \( E_d = \hbar I_c/2e \) are the charging energy and Josephson energy of the junctions, where \( C_j \) and \( I_c \) are the junctions’ capacitance and critical current. \( \phi \) is the average phase of the four junctions determined by the low bias current \( I_b \approx 0 \) of the FJS. The frequency shift \( \omega_s^c = -2E_0^c(\phi^2\chi_c^2 + \chi_c^2\chi_d^2)/\hbar, \)

\( \omega_s^d = -2E_0^d(\phi^2\chi_d^2 + \chi_c^2\chi_d^2)/\hbar, \)

where \( \chi_c = \pi M_c I_c 0/(\pi L_s I_c + \Phi_0), \)

\( \chi_d = \pi M_D I_D 0/(\pi (L_s + L_L) I_s + \Phi_0) \) and \( L_s \) and \( L_L \) are the self-inductances of the small SQUIDs and the circuit loop. To simplify the expressions, we set \( \chi = \chi_c = \chi_d \) and denote the photon frequency shift by \( \omega_s^c = \omega_s^d = \omega_s \). The photon interaction strength \( \omega_{\text{int}} = -4E_0^c\chi^4\cos \phi/\hbar \). In deriving the system Hamiltonian, we have used the rotating wave approximation and also dropped terms involving creation and annihilation of two photons. These terms have no effect since there is not more than one photon in the TLRs.

We operate with a low bias current \( I_b \approx 0 \) for the FJS so that \( \langle \cos \phi \rangle \) is large and the FJS’s energy splitting is far away from the frequencies of the TLR photons. Thus, the FJS will not be excited by the TLR photons and they hardly get entangled. The FJS then acts as a ‘nonlinear medium’ and equation (2) describes the interaction between photons in \( C \) and \( D \) modulated by the FJS’s phase. For \( I_c = 50 \mu A, L_s \approx 10 \text{ pF} \) and \( M_c \approx 80 \text{ pF} \) [16], the photon interaction strength \( \omega_{\text{int}} \approx 1 \text{ MHz} \), much greater than the photon loss rate. Unfortunately, there are difficulties in using this interaction for quantum computing. First, \( \phi \) has fluctuations in it due to the FJS’s charging energy and thus the interaction strength is not a constant. Also, it is not easy to turn off the interaction. Tuning \( \phi \) close to \( \pi/2 \) requires biasing the FJS close to its critical current, which makes the system unstable. The uncertainty in \( \phi \) grows too.

To have the photons interact only when needed, we use a setup shown in figure 2(c). Here TLRs \( A, B, E, F \) are two qubits with photons in \( A \) and \( F \) representing their logic 0 state. When both qubits are in the 1 state, we can use the photon transfer operation discussed earlier to transfer the photons from \( B \) and \( E \) to the auxiliary TLRs \( C \) and \( D \) whose frequencies are made different from that of the qubit TLRs by \( \omega_s \) to account for their energy shifts. Once the photons are in \( C \) and \( D \) they can interact due to coupling to the FJS. Afterwards, we transfer them back to \( B \) and \( E \).

To stabilize the FJS’s phase, we shunt its junctions with large capacitances \( C_s \) as shown in figure 2(a). At low bias currents, the FJS’s behavior can be very well approximated by that of a harmonic oscillator with Hamiltonian \( H_{\text{FJS}} = \omega_{\text{FJS}}F_{\text{FJS}}(\hat{F}_{\text{FJS}}), \)

\( \omega_{\text{FJS}} \) being the oscillator frequency and \( F_{\text{FJS}} \) its annihilation operator. The distribution of the FJS phase \( \phi \) is given by its ground state wavefunction \( \sqrt{\alpha}/\sqrt{\pi} \exp[-\alpha^2(\phi - \phi_0)^2/2], \)

where \( \alpha = \sqrt{4E_0^c}\cos \phi_0/E_c \) and \( \phi_0 = \langle \phi \rangle = \arcsin(hI_b/8eE_0^c) \). If we choose a total capacitance \( C_j + C_s = 20 \text{ pF} \), the relative uncertainty \( \delta(\omega_{\text{int}})/\omega_{\text{int}} \approx 10^{-4} \) is not a concern for the photon interaction term. However, uncertainties in the photon energy shift terms \( \hbar \omega_s \) can be comparable to \( \hbar \omega_{\text{int}} \) and can cause large errors.

We employ a two-phase technique in the spirit of spin-echo to address this problem. In phase 1, we first perform a photon transfer operation between \( B, C, E, D \) with a speed relatively fast compared to \( \omega_{\text{int}} \) and \( \delta \omega_s = -2E_0^c\chi^2\delta(\phi^2)/\hbar, \) the uncertainty in the photon frequency shift. We then wait for a desired interaction time \( t = \pi/\omega_{\text{int}}, \) after which we perform another photon transfer between \( B, C \) and \( E, D \). In phase 2, we first perform a bit flip on the two qubits.
in other words we perform a photon transfer operation between $A$, $B$ and $E$, $F$. We then repeat phase 1. At the end, we perform a bit flip on the two qubits again. In this process, depending on their initial states, the qubits will acquire the same random phase due to $\delta \omega_s$ in either phase 1 or 2, thus removing the effect of the randomness in $\omega_s$. The end result is a $\pi$ phase shift on the two-qubit states if they are both in 0 or 1 initially. This is equivalent to a controlled phase gate, and it enables universal quantum computing in combination with the single qubit operations.

If the photon transfer operation between $B$, $C$ and $E$, $D$ were perfect, the controlled phase gate would be exact and the random phase shift in $C$ and $D$ could be eliminated completely. However, since the photons in $C$ and $D$ will interact with the FJS even during the photon transfer, our control phase gate will have errors. This can be seen by examining the system Hamiltonian during the photon transfer (in the rotating frame) $H = \hbar \lambda_{bc} (\hat{b} \hat{c} + \hat{b}^\dagger \hat{c}^\dagger) + \hbar \lambda_{de} (\hat{d} \hat{e} + \hat{d}^\dagger \hat{e}^\dagger) - 2E^0 \chi^2 \phi^2 (\hat{c}^\dagger \hat{c} + \hat{d}^\dagger \hat{d}) - \hbar \omega_{int} \hat{c}^\dagger \hat{e} \hat{c}^\dagger \hat{d}$. The first two terms are used for the photon transfer operation; however, the remaining terms cannot be turned off, making the photon transfer imperfect. Obviously, the fidelity of our controlled phase gate will be improved by making the photon transfer frequency $\lambda_{bc}$ and $\lambda_{de}$ large compared to $\delta \omega_s$ and $\omega_{int}$. Treating the FJS phase $\phi$ as a classical random variable whose distribution is given by the FJS ground state wavefunction, we numerically studied our control phase gate by calculating the evolution of the TLR photon wavefunction under the full Hamiltonian and weighing the final photon states with the distribution of the FJS phase. The gate fidelity is plotted as a solid line in the top in figure 2(b). We set $\lambda = \lambda_{bc} = \lambda_{de}$. For our choice of system parameters, $\lambda / \delta \omega_s \approx 20$, and the gate error is of the order of $10^{-3}$.

As explained earlier, in our design we choose a large energy difference between the FJS and TLR photons so that they do not get entangled. Thanks to this measure, the FJS is not excited and its dissipation and decoherence have little effect on the two-qubit gate fidelities. To quantitatively study the effect of the FJS decoherence and photon loss, we recalculate the two-qubit gate fidelities by solving the density matrix of the entire system and taking into account the photon loss and FJS dissipation and decoherence. We use the master equation $\frac{d \rho}{dt} = -i[H, \rho] + \mathcal{L}\rho$, where $\rho$ is the density matrix of the entire system including the FJS and TLR photons, and $H$ is the system Hamiltonian in each step of the two-phase procedure introduced above. The Liouvillian $\mathcal{L}\rho = \Sigma_i \frac{1}{2} \mathcal{L} [s] \rho + \frac{1}{2} \mathcal{L} [F_{\text{FJS}}] \rho + \frac{1}{2} \Sigma_i \mathcal{L} [ | i \rangle \langle i | ] \rho$ describes the dissipation and decoherence of the FJS and photons. Here, $s$ denotes the TLR photon annihilation operators $a, b, c, d, e$ and $f$, $F_{\text{FJS}}$ is the FJS’s annihilation operator, $| i \rangle$ are the eigenstates of the FJS, $\kappa$, $\Gamma_1$ and $\Gamma_2$ are the photon loss rate and junction dissipation and decoherence rates specified before, and $\mathcal{L} [\hat{O}] \rho = 2 \hat{O} \rho \hat{O}^\dagger - \hat{O}^\dagger \hat{O} \rho - \rho \hat{O}^\dagger \hat{O}$. The two-qubit gate fidelity is obtained by tracing out the FJS degree of freedom in the density matrix when the two-qubit gate is accomplished. The result is plotted as a dashed line at the top of figure 2(b) in comparison to that obtained earlier by treating the FJS phase as a classical random variable. We see that the photon loss and FJS decoherence only slightly degrade the two-qubit gate fidelity. Also plotted at the bottom of figure 2(b) is the fidelity without the two-phase procedure to cancel the phase errors. As can be seen, without this carefully designed procedure the gate error is prohibitively large due to fluctuations in the photon interaction term equation (2) arising from the uncertainties in the FJS phase.

Our microelectronic system is easily scalable, as shown in figure 2(c). We can extend the setup for the control phase gate at both ends to integrate many TLR qubits on the same chip with an FJS between each pair of qubits. This is a 1d architecture with controllable interactions between adjacent qubits that can be scaled to a large number of qubits.

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In order to perform photonic qubit quantum computing, we still need to be able to generate and detect single photons. Photon generation on the superconducting chip has been demonstrated experimentally [17]–[19]. For photon detection [13, 18, 20], we again consider a CBJJ coupled to a TLR of frequency $\omega_0$, as shown in figure 3(a). The CBJJ is prepared in the ground state $|g\rangle$ in the well of its washboard potential. We also make use of an unstable excited state $|e\rangle$, where the CBJJ can tunnel to the voltage state with a large rate $\Gamma_1$. By adjusting the CBJJ’s bias current, we can tune the CBJJ in resonance with $\omega_0$. The CBJJ will then be excited by the TLR photon to $|e\rangle$. When it escapes from $|e\rangle$, a detectable voltage appears across the CBJJ.

Although an easy and reliable method, our scheme may fail to detect a photon. The TLR photon may decay before being detected by the CBJJ. The CBJJ’s intra-well decay from $|e\rangle$ to $|g\rangle$ and its finite decoherence time are concerns, too. To study the influence of the photon loss rate and the CBJJ’s intra-well decay and decoherence on the efficiency of our photon detector, we model it as a three-state system, shown in figure 3(a), where $|f\rangle$ represents the voltage state. We use the master equation $d\rho/dt = -i[H, \rho] + \mathcal{L}\rho$. Here, $\rho$ is the density matrix of the system, the system Hamiltonian $H = \delta \hat{a}^{\dagger} \hat{a} + g_{td}(\hat{a}^{\dagger} \sigma_{ge} + \hat{a} \sigma_{eg})$ and the detuning $\delta = \omega_0 - \mu$ where $\mu$ is the frequency difference between $|e\rangle$ and $|g\rangle$. The Liouvillian $\mathcal{L}\rho = \frac{\Gamma}{2} \mathcal{L}[\hat{a}]\rho + \frac{\Gamma}{2} \mathcal{L}[\sigma_{ef}]\rho + \frac{\kappa}{2} \mathcal{L}[\sigma_{eg}]\rho + \frac{\kappa}{2} \sum_{i=g,e} \mathcal{L}[|i\rangle\langle i|]\rho$, where $\mathcal{L}[\hat{O}]\rho \equiv 2\hat{O}\rho \hat{O}^{\dagger} - \hat{O}^{\dagger}\hat{O}\rho - \rho \hat{O}^{\dagger}\hat{O}$. $\kappa$ is the decay rate of the photon in the TLR; $\gamma_T$ and $\gamma_\phi$ are the intra-well decay rate and dephasing rate of the CBJJ; $\sigma_{ij} = |i\rangle\langle j|$ for $i, j = g, e, f$.

Assuming initially there is a photon in the TLR and the CBJJ is in $|g\rangle$, we plot the detecting efficiency (the probability that the CBJJ ends up in $|f\rangle$) in figure 3(b) as a function of $\Gamma/\kappa$. As can be seen, the efficiency is high even for moderately large escaping rate $\Gamma$. For

![Figure 3](http://www.njp.org/)

**Figure 3.** (a) The photon detection scheme. (b) The dependence of detector efficiency $(1 - 10^{-P})$ on the ratio between the escape rate $\Gamma$ and the photon loss rate $\kappa$. The coupling strength $g_{td}/2\pi = 100$ MHz, photon loss rate $\kappa/2\pi = 10$ kHz and CBJJ decay and dephasing rates $\gamma_T/2\pi = 100$ kHz and $\gamma_\phi/2\pi = 1$ MHz.
Figure 4. The photonic quantum computer on a chip.

$\Gamma/2\pi = 20$ MHz [21] and $\kappa/2\pi = 10$ kHz, the detection efficiency is above 99%. Also, it is demonstrated in our simulation that the influence of $\gamma$ is minor and thus the detecting CBJJ does not need to have long decoherence times.

In summary, we have shown that, by using TLR microwave photons as qubits and Josephson junctions as optical devices, a superconducting chip provides an ideal implementation for fully integrated linear or nonlinear photonic qubit quantum computing. In our scheme, everything needed for universal photonic qubit quantum computing, including the photon generator, single- and two-qubit gates and the photon detector, is based on Josephson devices and can be fabricated on the same microchip, as shown in figure 4. Thanks to our careful design, high gate fidelities can be achieved and thus our scheme is a realistic approach. Since our system is based on existing mature technologies, fast experimental progress can be expected to implement integrated photonic qubit quantum computing. The novel idea of using on-chip microwave photons as qubits also opens up the possibility of investigating many interesting quantum optics effects in an integrated system.

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