A calculation method and experiment study of high-pressure common rail injection rate with solenoid injectors

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Abstract
The high-pressure common rail system has been widely used owing to its precise control of fuel injection rate profile, which plays a decisive role in cylinder combustion, atomization, and emission. The fuel injection rate profile of high-pressure common rail system was studied, and a fuel injection rate profile calculation model is proposed. The model treats the injector as a black box. Some measured data are needed to calculate the parameters in the model. The rise and fall of injection rate is regarded as trigonometric function to reduce the complexity and increase the accuracy. The model was verified using two different types of fuel injectors. The model calculation results were evaluated under various data input conditions. The results show that the model has good applicability to different input data and injectors. In addition, because the model building requires a large amount of experimental data, a comprehensive analysis of various input data was also conducted. The injection profile was analyzed from a new perspective and the regularity of injection rate profile was established.

Keywords
High pressure common rail, injection rate profile, injection rate model, injection characteristics, diesel engine

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Introduction

Nowadays, the high-pressure common rail system is widely used in various types of engines because of the flexibility, precise fuel injection timing, and injection rate control, improving the engine power and achieving good economy and emission capabilities.\textsuperscript{1–5} The fuel injection profile describes the variation of mass flow rate with time, affecting the fuel mixing in the cylinder\textsuperscript{6–9} and thus affecting the emission and combustion performance of the engine.\textsuperscript{10–15} Therefore, the fuel injection profile has been studied by various engine research institutes and companies. The research shows that the fuel injection profile significantly affects the main heat release. A slower fuel injection rate results in a higher peak heat release rate and a higher level of engine noise.\textsuperscript{16} The shape of fuel injection profile also significantly affects the atomization and combustion. Different shapes of fuel injection have proven to have a very important impact on ignition delay and combustion duration.\textsuperscript{17} The fuel injection profile also has a very important impact on emission.\textsuperscript{18} The slow rise in the initial phase of fuel injection profile can reduce NOx emission and indicated mean effective pressure (IMEP).\textsuperscript{19} Therefore, the trade-off between NOx fuel consumption and NOx particle emissions can be achieved by appropriate control of fuel injection parameters.\textsuperscript{23} In summary, it is important to study the fuel injection profile of high-pressure common rail system.

Injection rate profile calculation models are important for one-dimensional (1D) and three-dimensional (3D) diesel engine simulation. Accurate injection rate prediction models can determine small changes in injection strategy on system combustion and emission characteristics in 1D simulation calculations about diesel engines. Regarding 3D simulation, especially for the development and validation of spray models, it has been found that a successful CFD run conducted by, for example, CONVERGE,\textsuperscript{20} OpenFoam\textsuperscript{21} and KIVA,\textsuperscript{22} requires an accurate description of the initial and boundary conditions, and detailed information about the injection rate profiles.

Because of such importance of fuel injection profile, many simulation studies have been carried out. These studies focus on hydraulic 1D modeling,\textsuperscript{24} commonly used tools are Simulink or other similar software such as AMESim and GT-SUITE. Solenoid\textsuperscript{25–32} and piezoelectric\textsuperscript{33–35} injectors have been extensively studied. Research institutes often use these models to obtain fuel injection characteristics. Using these models, the movement stroke of injector needle was obtained under different injection pressures.\textsuperscript{29} The movement of needle valve during multiple injections was obtained.\textsuperscript{32} The pressure of some relatively small chambers which are not convenient to measure, such as the pressure of plunger and spool, and the stroke of each valve, can also be obtained by these model. Currently, hydraulic 1D modeling has a high degree of accuracy in many cases. However, to achieve an accurate calculation of fuel injection process, not only structural parameters are needed, but parameters such as the orifice flow coefficient that are difficult to obtain should be measured or estimated, making the modeling and tuning process very complex.\textsuperscript{25}
To reduce time in verification and tuning, Payri et al.\textsuperscript{36} regarded the injector as a black box, ignoring the internal structural parameters of the injector, and the fuel injection rate profile was obtained using the empirical model. The author used eight variables to describe the fuel injection profile, start of injection (SOI), duration of injection (DOI), start slope, end slope, and four parameters describing a curve at the corner. The injection rate profiles can be calculated from simple parameters such as fuel density, injection pressure, energizing time, back pressure, and orifice diameter. Recently, other scholars have used similar methods to study fuel injection.\textsuperscript{24,37} Soriano et al.\textsuperscript{24} simplified the complex injection rate into a trapezoidal form. The injection start time is calculated first, and then the injection end time is calculated based on the energizing time. According to the operating conditions, fuel density, and fuel dynamic viscosity, the slopes of the injection start time and end time are calculated. Finally, a stable maximum injection rate is calculated according to the fuel injection amount. The model needs to use the injection pressure, energizing time, total injection quantity, injection temperature, and nozzle parameters in the calculation. The authors considered the effects of fuel density and viscosity, and then simplified the calculation process of the model, thus reducing the variables in the model calculation. However, the accuracy of fuel injection profile is significantly affected due to the simplification of trapezoid. Some post-processing methods are also needed to obtain relatively reasonable results, making this approach unintuitive. Xu et al.\textsuperscript{37} compared the experimental data with a large number of injection profiles, the fuel injection profile was divided into five stages. The calculation results obtained by the model are almost the same as the experimental results, but it is relatively difficult to obtain the parameters used in the calculation process. Based on the five-stage division, the model calculation process is relatively complex.

The abovementioned analysis shows that it has engineering significance to regard the injection system as a black box and calculate the fuel injection profile, while the existing model is either too complex in the calculation process or too simplified in the calculation results. Based on the current research status, there is a lack of a simple and accurate model for fast calculation of fuel injection rate profile.

The purpose of this study is to propose an extensive form of fuel injection rate profile calculation method considering both accuracy and complexity. The model regards the common rail system as a black box, bypassing the modeling process that requires complex structural parameters, and using few injection rate profiles, fuel injection delay, and injection duration to build an empirical model for calculating fuel injection rate profile in other conditions. The detailed definition and necessity of each parameter are discussed in Section 3. By comparing the injection profile separately, it is found that the rising process is similar to the falling process. This model innovatively treats the rising and falling profiles of injection rate as trigonometric functions, thus ensuring that the calculation process is fast and accurate. The model treats injectors as a black box, thus posing almost no limit to type of injectors. The maximum injection rate, injection delay (or SOI) and DOI are needed to determine the model parameters. The injection rate profile is divided into two curves known as rising and falling profiles. The injection profiles obtained by
experiment are necessary for calculating these two profiles. Two injection profiles are needed at least in different pressures. Injection duration needs plenty of experimental data because of its complexity. To verify the model, two experiments of injectors were carried out, and the calculation accuracy of injection profile under different conditions was evaluated.

**Experimental**

**Experimental setup**

Figure 1 shows the schematic diagram of a fuel injection profile measuring device. The experimental measurement system is mainly composed of several parts. High-pressure common rail system: including high-pressure oil pump, common rail pipe, fuel injector, low-pressure oil pump, and fuel filter. Auxiliary system includes fuel heating system and fuel injection rate test system. Electronic control system includes mainly controls for fuel injection, pump system, and the fuel injection rate test system. Table 1 shows the key equipment and model parameters of test bench. The test bench model is EFS ITV34/C. The fuel injection measuring device is EFS 8420 provided by EFS, which measures the fuel injection amount and fuel injection profile according to the pressure change in the chamber with a configurable back pressure. An oscilloscope was used during the experiment to monitor the current of the solenoid valve of the injector. The injector and common rail system are controlled using an EFS 8427 injector drive box. Two different injectors were used during the experiment to verify the applicability of the model. Injector A is a Delphi DFI2 injector, a six-hole injector with 0.15 mm diameter and 150° injection angle and a balanced valve structure. Injector B is a CYCR-4 injector, which is a seven-hole injector with 0.275 mm diameter and 146° injection angle with an unbalanced valve structure. There are differences in injector fuel line design between balanced valve and unbalanced valve. The balanced valve structure contains a chamber in the fuel line designed to balance the forces exerted by the high-pressure fuel while the needle valve is opening or closing, so that only minimal force is required to open or close the solenoid valve. There is no such design for the unbalanced valve structure, thus, some performance of the injectors will vary. During the experiment, 50 repeated experiments were performed for each working condition, and the rail pressure fluctuation did not exceed 5 MPa in each injection circle. These experiments were conducted at a back pressure of 5 bar due to the measuring device. The injector was inserted into a chamber. The fuel was injected into the chamber, causing pressure changes in the chamber, and then the rate at which the injector injects fuel was calculated from the pressure changes. The rising profile of injection rate is defined as the curve obtained from the detection of injection rate, generating a nonzero value until the injector injection rate reaches its maximum value. The falling profile of injection rate is defined as the curve by which the injection rate has a negative slope and decreases from a maximum value until the injector injection rate reaches zero.
Table 1. Specifications of major experimental apparatus.

| Apparatus                  | Model          | Specification                                                                 |
|----------------------------|----------------|-------------------------------------------------------------------------------|
| Injection system           | Common rail    | $P < 220 \text{ MPa}$, rail volume: $30 \text{ cm}^3$                      |
| Injector drive box         | EFS 8427       |                                                                               |
| Injection qualifier        | EFS 8420       | $0.5\text{–}600 \text{ mm}^3$, $5\text{–}100 \text{ bar}$ back pressure     |
| Injector A                 | Delphi DFI2    | $12 \text{ V}$ balance valve, $6 \times 0.15$ nozzle holes and $150^\circ$ injection angle |
| Injector B                 | CYCR-4         | $4 \text{ mm}$ needle diameter, $7 \times 0.275$ nozzle holes, $146^\circ$ injection angle |

Figure 1. Schematic diagram of experimental measurement system.

Experimental results

The model is derived from the observation of a large number of experiments. The parameters that are important for the calculation of injection profile, such as the fuel injection duration and fuel injection rate curve, are briefly introduced in this section.

The relationship between energizing time and fuel injection duration under different injection pressures is shown in Figure 2. The increase in fuel energizing time
will lead to an increase in the fuel injection duration, and an inflection point appears with the increase in injection pressure. The increase in fuel injection duration is faster before the point. This is related to the needle lift position. When the energizing time is small, the needle valve has not been fully opened. At this time, the energizing time is increased, the needle valve opening becomes longer, and the closing stroke is also increased accordingly, thus causing a rapid increase in duration. When the needle valve has been fully opened, an increase in energizing time only affects the stage of fully opened, thus, the injection duration slowly increases. Under high injection pressures, it is easily lifted up to the maximum stroke for the needle. Therefore, the inflection point appears relatively early, and the low-pressure inflection point appears later. The injection pressure is increased from 60 to 160 MPa, and the inflection point of the duration increase is relatively changed from 1 to 0.7 ms.

The relationship between injection pressure and fuel injection duration under different energizing times is shown in Figure 3. The fuel injection pressure is slightly complex against the fuel injection duration. In the case of a lower energizing time, the injection pressure increases, and the injection duration increases almost linearly. As the injection energizing time of fuel injection increases, the linear increase trend of fuel injection duration is gradually weakened and shows a decreasing trend. As the injection pressure keeps increasing, the decreasing trend becomes obvious. When the injection energizing time is 0.6 ms, the injection pressure changes from 60 to 160 MPa, the injection duration is increased by 120 μs, and when the energizing time is 1.5 ms, the injection duration is reduced by 144 μs. This might be the result

**Figure 2.** Injection duration for different injection energizing times of injector A under different injection pressures.
of fuel pressure on the needle valve: Under low injection energizing time conditions, the needle valve has not been fully opened, as the fuel pressure increases, the needle lift position is increased. This makes the duration longer. In the case of a large injection energizing time, the needle valve is fully opened, as the fuel pressure increases, the opening and closing processes are accelerated, and the duration decreases.

Figure 4(a) shows the injection curve of the needle valve opening process, and Figure 4(b) shows the injection curve of the closing process. The curve starts from the moment the injection rate is above zero, that is, the SOI is ignored in Figure 4. The injection energizing time makes slight difference on injection profiles in opening process. Regardless of the injection pressure, the slope of injection rate in opening process has the characteristics of first increasing and then decreasing with slope always being positive. This phenomenon can be explained by the injection pressure and flow area. At the beginning of injection, the flow area slowly increases due to the low pressure inside the sac of the nozzle. With the increase in pressure, the flow area shows a higher increasing trend, which results in a higher increasing trend in the injection rate. At the ending of opening process, the flow area is almost the maximum and injection pressure keeps decreasing. Thus, the injection rate slowly increases. There is a difference in the maximum fuel injection rate that can be achieved under different pressures, but there is a significant similarity in general. Similarly, in closing process the absolute value of the slope of fuel injection rate first increases and then decreases.

Figure 3. Injection duration for injection pressure of injector A under different energizing times.
Model proposal and validation

Model proposal

The fuel injection profile is shown in the Figure 5. From the physical process of actual injection, the injection profile can be divided into three sections: Injector

Figure 4. Experiment results of rise curve (a) and fall curve (b) of injector A mass injection rate.

Figure 5. Schematic diagram of fuel injection rate profile (rising and falling profile mean the curve between the ending point of SOI, maximum injection rate, and ending point of DOI).
needle valve opening stage, maximum injection rate stage, and needle valve closing stage. These three stages correspond to the rapid rise of injection rate, maintaining a stable injection rate and rapid decline. In short injection duration, the injection rate profiles lack the stage maintaining the maximum injection rate, only the needle valve opening and closing processes. Combined with the actual injection process, several key parameters describing the fuel injection profile should be defined. The injection delay is defined as the time difference between the system’s current signal and the needle valve opening time when the injection rate is above zero. The rising profile of injection rate is defined as the curve of injection rate above zero, the fuel injection profile rapidly rises until the value approaches the maximum injection rate. The stable stage is defined as the stage when the injection rate is the maximum. The falling profile of injection rate is defined as the curve from the last maximum injection rate to the zero value. The fuel injection duration is between the two zero injection rates in opening and closing stages. By accurately describing the above variables, a description of the fuel injection profile can be achieved.

The injection delay (or SOI) determines the starting moment of injection rate profile, which is very important for the calculation of multiple injections. There are mainly five types of delays in a fuel system.\textsuperscript{38} Electronic delay: the delay caused by the electromagnetic force conversion and action of electromagnetic module. Electrohydraulic delay: the delay caused by the interaction between electromagnetic force and hydraulic power. Hydraulic delay: the delay caused by the propagation of pressure waves in the system and mechanical delay caused by spring compression and return. Electronic electromagnetic and mechanical delays can be regarded as the inherent delays of an injection system and do not change with injection pressure and injection energizing time, and thus can be described by constant terms. Electrohydraulic and hydraulic delays involve the hydraulic and structure inside the injector. The pressure is less affected by the injection delay of today’s popular balanced injectors, but has a greater impact on unbalanced injectors, while all can be described equation (1).\textsuperscript{36}$P_r$ is the pressure in common rail, $P_b$ is the pressure of the environment. $a_1$ to $e_1$ are obtained from the fitted curve.

$$\text{SOI} = a_1 + b_1 \sqrt{P_r - P_b} + c_1 P_b + d_1 P_r + e_1 P_r^2$$

The DOI determines the end of injection and is equally important for calculating the injection profile. The injection profile is conducted by at least one rising and falling profiles. SOI and DOI determine the start or end point of injection profile, which are vital for the calculation of injection quantity. Both the injection pressure and energizing time can affect duration. The effect on the duration of injection is nonlinear: Under short injection energizing time conditions, the injection energizing time increases and the time for the needle valve to open and close increases. However only a stable stage increases under a long injection energizing time. The system pressure also affects the needle opening and closing stages. Injection duration can also be affected by the maximum needle lift and internal fuel circuit of injector. Equation (2) is obtained through repeated iterative process. $ET$ is
energizing time and $P_r$ is the pressure in common rail system. $a_2$ to $e_2$ are obtained from curve fitting.

$$DOI = a_2 + b_2 ET^2 + c_2 P_r + d_2 ET \cdot P_r + e_2 ET$$

(2)

The maximum injection rate is the steady value of the injection rate when the needle valve is fully open. In this study, the maximum injection rate was selected based on long energizing time conditions. The maximum fuel injection rate that can be achieved by the injector can be obtained by solving the Bernoulli and mass conservation equation. Equation (3) is based on mass conservation equation. Equation (4) is based on the Bernoulli equation. Thus, the maximum flow rate can be described in equation (5).

$\dot{m}$ is defined as the stationary mass flow that needle lift does constrain anymore. $C_d$ is the flow coefficients of all orifices. $A_0$ is the geometrical cross-section of all the orifices. $\rho_f$ is the fuel density. $u_B$ is Bernoulli’s theoretical velocity defined in equation (4). $a_3$ and $b_3$ are adjusted to the experimental data. Considering that $C_d$ varies with the pressure difference of the nozzle, these equations can be used, not much beyond the fitting scope.

$$\dot{m} = C_d \cdot A_0 \cdot \rho_f \cdot u_B$$

(3)

$$u_B = \sqrt{\frac{2(P_r - P_b)}{\rho_f}}$$

(4)

$$\dot{m} = a_3 \sqrt{P_r - P_b} + b_3$$

(5)

After determining the fuel injection delay, the fuel injection rate profile corresponding to the needle valve opening can be determined. This fuel injection rate profile is affected by fuel’s physical properties and injector’s geometrical parameters. Based on the analysis of the experimental results, the injection rate always increases during the opening stage of injector needle valve, and the slope first increases and then decreases. The injection rate in needle lift process could be fitted by many types of functions. To achieve an accurate calculation of fuel injection profile, after repeated iterations, it is considered that the fuel injection rate can be described by a arc tangent function as shown in equation (6). This function can describe the profile with high accuracy and much less parameters than polynomial or other functions. $f_{\text{rise}}$ is the injection rate upon needle lift. $\dot{m}$ is the stationary mass flow defined in equation (5). $P_r$ is the pressure in common rail, $P_b$ is the back pressure. $t$ is the time from SOI (injection delay). $a_4$ to $f_4$ are obtained by fitting injection rate profile. The format of fitting curve is obtained by repeated iteration and test.

$$f_{\text{rise}} = a_4 + \frac{\dot{m}}{2} \cdot \arctan\left(\left(b_4 \cdot \sqrt{P_r - P_b} + c_4\right) \cdot t^4 + e_4 \cdot \sqrt{P_r - P_b} + f_4\right)$$

(6)

The end time of needle closing can be determined according to the fuel injection delay and duration. The injection rate profile of needle closing stage has similar
characteristics as the opening stage: The slope first decreases and then increases, and the injection rate always decreases. Therefore, the form of arc tangent function is also used in the construction of this fuel injection formula shown in equation (7). \( f_{\text{fall}} \) is the injection rate upon the needle is falling. \( \dot{m} \) is the maximum mass flowrate. \(-t\) is the time before injector stops defined as DOI minus current time. \( a_5 \) to \( f_5 \) are obtained by fitting the injection rate profile.

\[
 f_{\text{fall}} = a_5 + \frac{\dot{m}}{2} \cdot \arctan\left( \frac{b_5 \cdot \sqrt{P_r - P_b} + c_5}{d_5 + e_5 \cdot \sqrt{P_r - P_b} + f_5} \right) \cdot (-t)^{d_5} + e_5 \cdot \sqrt{Pr - Pb} + f_5 \]  

(7)

Because DOI and SOI have been determined, the injection rate profile can be obtained by combining with the rising and falling profiles. Commonly these profiles have one or two intersections of rising profile, falling profile, and the maximum injection rate. The final injection profile can be obtained by combining these profiles. When one intersection is found, the final injection profile is composed of only the rising and falling profiles. It always happened in working conditions with limited energizing time. When two intersections were found, the final injection profile is composed by rising profile, falling profile and the maximum injection rate.

**Model validation**

Table 2 shows the experimental results obtained and used for model approach. To verify the model, the amounts of injection mass and injection rate were compared.

**Injection mass modeling.** The fuel injection quantity can directly affect the engine output power. At the same time, the fuel injection quantity is defined as the integral result of the fuel injection profile that can quantitatively describe the difference between the calculated fuel injection profile and the actual fuel injection profile. In this section, the results are evaluated by comparing the integral value of fuel injection amount under various working conditions.

Figure 6 shows the experimental results, model calculation results, and relative error between the model results and calculated results under various working conditions. The tolerance band is defined according to 10% of the experimental results.

The calculation results show that the model can predict fuel injection under the majority of working conditions. However, in some working conditions, such as in...
0.8 ms and 100 MPa injection energizing time and injection pressure, the calculated and the experimental values have a relatively large mistake. In terms of overall distribution, the calculation results are relatively accurate when the rail pressure is high, and the fuel energizing time is long.

**Mass flow rate modeling.** The verification data of fuel injection profile was used in the working condition table to calculate the unused data in the model process. The verification result is shown in Figure 7.

Figure 7 shows that the model can reproduce the profile of fuel injection rate. However, under certain conditions (150 MPa, 0.7 ms), the maximum fuel injection rate predicted by the model and the experimental data show some error. Combined with the fuel injection profile obtained by the experiment, this is probably because at higher injection pressures, a short energizing time will produce a relatively high pressure shock caused by the injection process, resulting in measurement errors.

**Application and discussion**

**Modeling with limited data input**

The model building process was introduced above. The injector is regarded as a black box and it is necessary to enter the amounts of experimental data for solving the model parameters. In the use of the model, the equations for each parameter introduced in the previous section should be solved. The SOI, DOI, and the
maximum injection rate were calculated first. Such parameters usually require a large and widely distributed number of operating point data to be fitted to the solution, usually about 15 operating points to achieve a high accuracy for DOI, SOI, and the maximum injection rate. Then, the rising and falling profiles of injection rate will be solved. Usually, two or three injection rate curves at different rail pressures are sufficient for the model. This step is a direct fit to the equation using the experimentally obtained points on the injection rate profile. When using the model, the data involved in solving the model parameters may exist in various forms. This section provides a discussion and comparison of the various possible input data and output results. The various input data are designed as shown in Table 3.

The results in the table are divided into five groups. The first group is the original experimental data. The second group is the calculation results used in the model validation, and this group is used as a control group and as the evaluation standard for results derived from different input data. The parameters of third

Figure 7. Injection rate profile of injector A as a function of time after energizing signal.
Table 3. Different data input for model approach.

| Grouping | Category           | Injection pressure (MPa) | Energizing time (ms) | Injection rate profile                                                                 | Number of cases |
|----------|--------------------|--------------------------|----------------------|---------------------------------------------------------------------------------------|-----------------|
| 1        | Experiment results | 60–160, 10 MPa interval  | 0.6–1.5, 0.1 ms interval | 2 ms interval sampling, entire rising and falling process                           | 110             |
| 2        | Control group      | 60, 80, 100, 120, 140, 160 | 0.6, 0.8, 1.0, 1.2, 1.4 | Same as group 1                                                                        | 30              |
| 3        | A                  | 60, 80, 100, 120, 140, 160 | 0.6, 1.0, 1.4         | Same as group 1                                                                        | 18              |
|          | B                  | 60, 100, 160              | 0.6, 0.8, 1.0, 1.2, 1.4 | Same as group 1                                                                        | 15              |
| 4        | C                  | 80, 140                   | 0.6–1.5, 0.1 ms interval | Same as group 1                                                                        | 20              |
| 5        | D                  | 60–160, 10 MPa interval  | 0.8, 1.2              | Same as group 1                                                                        | 22              |
|          | E                  | 60, 80, 100, 120, 140, 160 | 0.6, 0.8, 1.0, 1.2, 1.4 | 0.0175 ms interval sampling, half rising and falling process                        | 30              |
|          | F                  | 60, 80, 100, 120, 140, 160 | 0.6, 0.8, 1.0, 1.2, 1.4 | 0.035 ms interval sampling, entire rising and falling process                   | 30              |

The working conditions involved in the second group are approached by a small amount of experimental data. The input of variable energizing time (A) and injection pressure input (B) is reduced in the third group as energizing time and injection pressure are the main working conditions. The fourth group is designed for extreme data input, C for two injection pressures and D for two energizing times. Because reproducing of injection rate profile involves the input data of rising and falling curves, it is related to the sampling frequency of test bench. The sampling interval of rising and falling curves used in this paper is 0.0175 ms. Here, the calculation result under the interval of 0.035 ms (F) was tested. In addition, the selection of data range of rising curve is also related to the model solving process. The preset range of injection rate is from greater than zero to the maximum injection rate, and the data selected may not be consistent with the preset. Thus, the case with only a part of the rising curve was selected (E).

Figure 8 shows a comparison between the third group results, the experimental results, and control groups. The working conditions involved in the second group are six injection pressures and five energizing times for a total of 30 operating conditions. Six injection pressures, three energizing times (A), three injection pressures, and five energizing times (B) were used in the test. Figure 8 shows that group one, group two, and group three have high consistency, thus, it can be concluded that
with less data input, the calculation results are still accurate. In fact, more groups of tests with few amount of experimental data input have also been tried. In the case of less and less input of experimental data, the calculated fuel injection profile becomes less accurate. This inaccuracy is mainly because of fuel injection duration. Injection duration has a complex regulation and needs sufficient fitting data.

With very few fuel injection energizing times (D) and injection pressures (C) as input working conditions, the calculation results are shown in Figure 9. In this comparison, the parameters of C group input model are 80 and 140 MPa, and the fuel injection energizing time of D group input to model is 0.8, 1.2 ms. The calculated results include 70, 150 MPa, 0.7 and 1.5 ms, which is beyond the input data. The calculation results show that the output of C is almost the same as the control group, and the output of D obviously has an error in the calculation of injection duration, which exists under low pressure and high pressure conditions. This phenomenon is also caused by the complex regulation of injection duration. There is an error in the fuel injection duration calculated by inputting only two injection

Figure 8. Comparison between experimental results and modeled results using groups A and B as data input.
energizing time points under each injection pressure. It can be concluded that the model is sensitive to injection duration.

Owing to different experimental setups, a difference is observed in the sampling frequency used to calculate the rise and fall curves. This was verified from the curve calculation with a small sampling frequency (F) and partial curve selection (E). Figure 10 shows the results calculated by E, and F. The calculation results of group F are almost identical to the control group. Group E can better predict the fuel injection rate under low pressure conditions. As the pressure increases, the prediction error begins to increase. The error is mainly reflected in the prediction of the maximum fuel injection rate. The results show that in the selection of input data, it is very important to completely select the rising and falling range, and the sampling frequency has no obvious influence on the model calculation results.

Figure 9. Comparison between experimental results and modeled results using group C and D as data input.
To verify that the model is also suitable for additional injectors, Injector B CYCR-4 was selected to perform the verification. CYCR-4 is an unbalanced injector that differs from the DFI2 injector used above. There are differences in the capabilities of these two injectors due to mechanical design. Table 4 shows the operating conditions for approach model and the comparison conditions.

Figure 11 shows a comparison between the model calculation results of injector B and the experimental results, and the calculated data in the figure agree well with the experimental data. It can be concluded that the method proposed in this study is applicable to different types of injectors. The results were obtained by using the control group as the input.

To verify the adaptability of this model, the results in limited data input were analyzed in Figure 12. Group A represents limited data input in both injection pressure and energizing time. Only two injection pressures were selected as the data input in Group B. Only two energizing times were selected in Group C. Group B
Table 4. Different data input for CYCR-4 injector.

| Grouping | Category | Injection pressure (MPa) | Energizing time (ms) | Injection rate profile | Number of cases |
|----------|----------|--------------------------|----------------------|-----------------------|-----------------|
| 1        | Experiment results | 60–160, 10 MPa interval | 1.0–1.5, 0.1 ms interval | 2 ms interval sampling, entire rising and falling process | 66              |
| 2        | Control group | 60, 80, 100, 120, 140, 160 | 1.0, 1.2, 1.4 | Same as group 1 | 18              |
| 3        | A | 60, 100, 160 | 1.0, 1.2, 1.4 | Same as group 1 | 9               |
|          | B | 80, 140 | 1.0–1.5, 0.1 ms interval | Same as group 1 | 12               |
|          | C | 60, 80, 100, 120, 140, 160 | 1.2, 1.4 | Same as group 1 | 12               |
| 4        | D | 60, 80, 100, 120, 140, 160 | 1.0, 1.2, 1.4 | 0.0175 ms interval sampling, half rising and falling process | 18             |
|          | E | 60, 80, 100, 120, 140, 160 | 1.0, 1.2, 1.4 | 0.035 ms interval sampling, entire rising and falling process | 18             |

Figure 11. Comparison of injection rate profiles for injector B between experimental and model results.
and C represent two types of data input in extreme cases. The calculated injection rates have a relatively large error at a low injection pressure, nearly 10%. The results are much better at a high injection pressure. The results from Group A is better than Group B and C with less cases, that is, the model needs enough input to calculate the injection duration or other characteristic parameters. The results obtained form Group B is better than Group C means that the calculation of injection duration needs more data compared to other parameters.

The selected data in the rising and falling curves also affect the calculation results. Only half of the curve is selected in Group D. A larger simple gap is set in Group E (Figure 13). The results obviously deviate from the experimental results. It can be concluded that the complete curve is vital for the model. The model can roughly reflect the trend of injection rate in this situation. Overall, it can be concluded that the model can be used in different types of injectors.

Figure 12. Comparison between experimental results and modeled results using Group A, B and C as data input.
Conclusion

Fuel injection is a very important process in the operation of engines. It determines the mixing, combustion, and discharge of fuel and oxidant in the cylinder. The fuel injection rate profile directly reflects the fuel injection process and plays an important role in the combustion and atomization simulation of the engine. This study presents an empirical and fast calculation method for the fuel injection rate profile of common rail injectors. The model regards the injector as a black box. Some test data were used as the input for calculating the parameters and fuel injection profile can be calculated. The calculation results show that the model has high calculation accuracy for each injector. Specific studies and conclusions are listed below.

1. A comprehensive experimental study and analysis of energizing time and injection pressure was performed. The variation in fuel injection duration, maximum fuel injection rate, and fuel injection rising and falling curve with
injection pressure and fuel injection energizing time was obtained. In the working conditions of a lower energizing time, the injection duration increases with the increase in injection pressure. In the case of a slightly higher energizing time, the injection duration decreases with the increase in injection pressure. The injection duration increases with the increase in injection energizing time. The injection duration under different rail pressures shows a different growth trend with the increase in injection energizing time. The higher the injection pressure, the earlier the fuel injection duration shows a slowdown in growth. The rising and falling curves of the injection rate are almost the same under the same pressure. The slope of the injection rate at different pressures becomes larger and shows a similar growth regulation.

2. Based on the observation of experimental data, a calculation model of fuel rate profile was established, and the model was verified. The fuel injection profile model is divided into two sections, the rising section and the falling section. By cooperating with the calculation method of fuel injection duration, the maximum fuel injection rate, and fuel injection delay, the profile can be reproduced and calculated.

3. The application characteristics of fuel injection rate model are discussed. The model may involve various types of data input during use. Based on this consideration, this study designed three different possible data input forms. The model is consistent with most data input forms. If the input data accurately calculate the fuel injection duration, the injection profile can be precise. In addition, the adaptability of the model under different injectors is discussed. The results show that the injectors with different structures and working principle are also suitable for the model.

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References
1. Wang SY. Modern diesel electronically controlled fuel injection technology. Beijing, China: Mechanical Industry Press, 2013.
2. Yamaki Y, Mori K and Hiroshi K. Application of common rail fuel injection system to a heavy duty diesel engine. SAE paper 942294, 2002.

3. Philippe G and Mourad H. Performance development of the first European heavy duty diesel engine equipped with full electronic high injection pressure common rail system. SAE paper 2000-01-1821, 2000.

4. Lei S, Ji CW, Wang SF, et al. Impacts of dimethyl ether enrichment and various injection strategies on combustion and emissions of direct injection gasoline engines in the lean-burn condition. *Fuel* 2019; 254: 115636.

5. Zhong WJ, Tamilselvan P, Li ZL, et al. Combustion and emission characteristics of gasoline/hydrogenated catalytic biodiesel blends in gasoline compression ignition engines under different loads of double injection strategies. *Appl Energy* 2019; 251: 113296.

6. Agarwal AK, Som S, Shukla PC, et al. In-nozzle flow and spray characteristics for mineral diesel, Karanja, and Jatropha biodiesels. *Appl Energy* 2015; 156: 138–148.

7. Agarwal AK, Dhar A, Gupta JG, et al. Effect of fuel injection pressure and injection timing on spray characteristics and particulate size–number distribution in a biodiesel fuelled common rail direct injection diesel engine. *Appl Energy* 2014; 130: 212–221.

8. Jiang C, Xu H, Srivastava D, et al. Effect of fuel injector deposit on spray characteristics, gaseous emissions and particulate matter in a gasoline direct injection engine. *Appl Energy* 2017; 203: 390–402.

9. Chen C and Tang ZG. Investigation of the spray formation and breakup process in an open-end swirl injector. *Sci Prog* 2020; 103(3): 0036850420946168.

10. Mohan B, Yang W, Yu W, et al. Numerical investigation on the effects of injection rate shaping on combustion and emission characteristics of biodiesel fueled CI engine. *Appl Energy* 2015; 160: 737–745.

11. Macian V, Payri R, Ruiz S, et al. Experimental study of the relationship between injection rate shape and diesel ignition using a novel piezoactuated direct-acting injector. *Appl Energy* 2014; 118: 100–113.

12. Payri R, Salvador FJ, Gimeno J, et al. Influence of injector technology on injection and combustion development – part 1: hydraulic characterization. *Appl Energy* 2011; 88: 1068–1074.

13. Haozhong Huang, Zhongjiu Li, Wenwen Teng, et al. Influence of n-butanol-diesel-PODE3-4 fuels coupled pilot injection strategy on combustion and emission characteristics of diesel engine. *Fuel* 2019; 236: 313–324.

14. Yu XM, Guo ZZ, Sun P, et al. Investigation of combustion and emissions of an SI engine with ethanol port injection and gasoline direct injection under lean burn conditions. *Energy* 2019; 189: 116231.

15. Kazim AH, Khan MB, Nazir R, et al. Effects of oxyhydrogen gas induction on the performance of a small-capacity diesel engine. *Sci Prog* 2020; 103(2): 0036850420921685.

16. Kashdan JT, Anselmi P and Walter B. Advanced injection strategies for controlling lowtemperature diesel combustion and emissions. SAE paper 2009-01-1962, 2009.

17. Ferrari A and Mittica A. Response of different injector typologies to dwell time variations and a hydraulic analysis of closely-coupled and continuous rate shaping injection schedules. *Appl Energy* 2016; 169: 899–911.

18. Tay KL, Yang W, Zhao F, et al. Effects of triangular and ramp injection rate-shapes on the performance and emissions of a kerosene-diesel fueled direct injection compression ignition engine: a numerical study. *Appl Therm Eng* 2017; 110: 1401–1410.
19. Boggavarapu P and Singh S. Computational study of injection rate-shaping for emissions control in diesel engines. SAE technical paper 2011-26-0081, 2011.
20. Richards K, Senecal P and Pomraning E. *CONVERGE 2.1.0 theory manual*. Middleton, WI: Convergent Science Inc., 2013.
21. Jasak H. OpenFOAM: open source CFD in research and industry. *Int J Nav Archit Ocean Eng* 2009; 1: 89–94.
22. Amsden A; LANL. A block-structured KIVA program for engines with vertical or canted valves. Los Alamos, NM: Los Alamos National Laboratory (LANL), 1999.
23. Tanabe K, Kohketsu S and Nakayama S. Effect of fuel injection rate control on reduction of emissions and fuel consumption in a heavy duty DI diesel engine. SAE technical paper 2005-01-0907, 2005.
24. Soriano JA, Mata C, Armas O, et al. A zero-dimensional model to simulate injection rate from first generation common rail diesel injectors under thermodynamic diagnosis. *Energy* 2018; 158: 845–858.
25. Salvador F, Marti-Aldaravi P, Carreres M, et al. An investigation on the dynamic behaviour at different temperatures of a solenoid operated common-rail ballistic injector by means of a one-dimensional model. SAE technical paper 2014-01-1089, 2014.
26. Piano A, Millo F, Postrioti L, et al. Numerical and experimental assessment of a solenoid common-rail injector operation with advanced injection strategies. *SAE Int J Engines* 2016; 9(1): 565–575.
27. Piano A, Boccardo G, Millo F, et al. Experimental and numerical assessment of multi-event injection strategies in a solenoid common-rail injector. *SAE Int J Engines* 2017; 10(4): 2129–2140.
28. Payri R, Climent H, Salvador FJ, et al. Diesel injection system modelling. Methodology and application for a first-generation common rail system. *Proc Inst Mech Eng D J Automob Eng* 2004; 218: 81–91.
29. Seykens XLJ, Somers LMT and Baert RSG. Modelling of common rail fuel injection system and influence of fluid properties on injection process. In: *Proceedings of VAFSEP2004*, Dublin, Ireland, 6–9 July 2004.
30. Chung NH, Oh BG and Sunwoo MH. Modelling and injection rate estimation of common-rail injectors for direct-injection diesel engines. *Proc Inst Mech Eng D J Automob Eng* 2008; 222: 1089–1001.
31. Payri R, Salvador FJ, Marti-Aldaravi P, et al. Using onedimensional modeling to analyse the influence of the use of biodiesels on the dynamic behavior of solenoid-operated injectors in common rail systems: detailed injection system model. *Energy Convers Manag* 2012; 54: 90–99.
32. Marcic S, Marcic M and Praunseis Z. Mathematical model for the injector of a common rail fuel-injection system. *Engineering* 2015; 7: 307–321.
33. Salvador FJ, Gimeno J, De la Morena J, et al. Using one-dimensional modeling to analyze the influence of the use of biodiesels on the dynamic behavior of solenoid-operated injectors in common rail systems: results of the simulations and discussion. *Energy Convers Manag* 2012; 54: 122–132.
34. Salvador FJ, Plazas AH, Gimeno J, et al. Complete modelling of a piezo actuator last-generation injector for diesel injection systems. *Int J Engine Res* 2014; 15(1): 3–19.
35. Plamondon E and Seers P. Development of a simplified dynamic model for a piezoelectric injector using multiple injection strategies with biodiesel/dieselfuel blends. *Appl Energy* 2014; 131: 411–424.
36. Payri R, Gimeno J, Novella R, et al. On the rate of injection modeling applied to direct injection compression ignition engines. *Int J Engine Res* 2016; 17(10): 1015–1030.
37. Xu LL, Bai XS and Jia M. Experimental and modeling study of liquid fuel injection and combustion in diesel engines with a common rail injection system. *Appl Energy* 2018; 230: 287–304.
38. Wang P. *Research on pressure fluctuation and delay characteristics for high pressure fuel supply system*. Beijing, China: Beijing Institute of Technology, 2016.
39. Payri R, Salvador FJ, Gimeno J, et al. Flow regime effects on non-cavitating injection nozzles over spray behavior. *Int J Heat Fluid Flow* 2011; 32(1): 273–284.

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**Appendix**

**Nomenclature**

| Symbol | Description                        |
|--------|-----------------------------------|
| IMEP   | Indicated mean effective pressure  |
| $P$    | Pressure or injection pressure     |
| SOI    | Start of injection or injection delay |
| DOI    | Duration of injection              |
| ET     | Energizing time                    |
| $P_r$  | Pressure of common rail            |
| $P_b$  | Back pressure                      |
| $t$    | Time after rising edge of energizing signal |
\( \dot{m} \quad \text{Maximum flow rate} \\
C_d \quad \text{Maximum nozzle discharge coefficient} \\
A_0 \quad \text{Geometrical cross section of all the orifices} \\
\rho_f \quad \text{Liquid density} \\
u_B \quad \text{Bernoulli’s theoretical velocity} \\
f_{\text{rise}} \quad \text{Rising profile of injection rate} \\
f_{\text{fall}} \quad \text{Falling profile of injection rate}