Positron acoustic solitary waves in an inhomogeneous multi-component plasma

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Abstract. The theoretical investigations have been made for the propagation of positron acoustic solitary waves (PASWs) in a weakly inhomogeneous plasma composing immobile positive ions, mobile cold positrons, and superthermal hot positrons and electrons. The Korteweg-de Varies (KdV) and modified KdV (mKdV) equations with variable coefficients are derived using the appropriate coordinate transformation and the reductive perturbation method (RPM). The effects of positron concentration, temperature ratios for hot positrons and electrons, hot to cold positrons density ratio, electron to cold positron density ratio, ion to cold positron density and population of hot electrons as well as positrons superthermality on the nonlinear propagation of PASWs are examined to understand the local electrostatic disturbances. It is also found that the presence of superthermal (kappa distributed) hot positrons and hot electrons significantly modify the basic features of PASWs. The critical values for hot positrons and cold positrons also play a vital role in the formation of only compressive PASWs in the plasmas.

1. Introduction
It is well known that the electron–positron–ion (e-p-i) plasmas are existed in astrophysical, space as well as laboratory plasmas, especially in ionosphere [1], auroral acceleration regions [2], solar wind [3], quasar and pulsar magnetosphere [4], active galactic nuclei [4], polar cup of fast rotating neutron stars [5], semiconductor plasmas [6], intense laser fields [7] and so on. However, it is not predominantly easy to the researchers for the production of astrophysical or space like plasmas in the laboratory. One may be studied by finding the basic characteristics of plasmas for understanding the physical issues involved in the plasmas. Further, one may frequently encounter nonlinear collective influences to the plasmas, which cannot appropriately be studied without tedious mathematical techniques. The dispersion or dissipation along with nonlinearities provide several types of consistent structures, such as solitary waves, shock structures, double layer (DL), vortices, etc. which play important roles for understanding the physical phenomena from both the theoretical and experimental point of views. On the other hand, space plasmas with an excess of superthermal electrons or positrons are generally characterized by a long tail in the high energy region, which can be studied by
considering generalized Lorentzian or kappa distributions [8, 9]. The presence of a significant number of superthermal particles follow kappa distribution, but not the Maxwellian one, can significantly change the rate of resonant energy transfer between particles and plasma waves [10]. Besides, C. M. Surko et al. [11] and Abdullah et al. [12] have shown that the cold positrons are now available as new sources in laboratory plasmas, such as semiconductor plasmas and intense laser fields [7]. Therefore, the studies of nonlinear wave propagation in the e-\( p \)-i superthermal plasmas become one of essential aspects in recent years due to its broad applications and potentiality as mentioned earlier. On the other hand, many authors [13-20] have studied the nonlinear positron acoustic waves (PAWs) in different e-\( p \)-i homogenous plasmas considering various types of plasma assumptions. Recently, Alam et al. [20] have studied the positron acoustic (PA) solitary waves (SWs) and double layer (DL) by deriving Korteweg-de Varies (KdV), modified KdV (mKdV) and Gardner equations in an unmagnetized homogeneous plasma system, which is composed of immobile positive ions, mobile cold positrons and superthermal hot electrons as well as positrons. They have found that the kappa distributed hot electrons and positrons significantly modify amplitudes, widths, polarity, and phase speed of PASWs and DLs in the plasmas.

In most of the studies [13-29] have investigated the basic properties of electrostatic waves in the homogeneous plasmas. But, the influence of plasma inhomogeneity on the propagation of electrostatic acoustic waves is another significant problem from various points of view. For instance, one may consistently encounter inhomogeneous plasmas under actual conditions both in the laboratory as well as space plasmas. Inhomogeneity may stem from density gradient or temperature or it could be due to the magnetic field in space and they are more pronounced closer to the edges and the boundaries of the system. The characteristics such as propagation, collision, and reflection of solitons etc., are significantly influenced under such conditions. Only a few authors [30-34] have been investigated that the propagation of ion acoustic solitary waves in an inhomogeneous plasma considering different plasma assumptions. Mowafy and Moslem [33] have studied the propagation of the IAWs in an inhomogeneous plasma comprised warm negative and positive ions with different masses and isothermal electrons. It is best to our knowledge no one investigate the nonlinear propagation of PASWs in an inhomogeneous plasma consisting of immobile positive ions, mobile cold positron, and superthermal hot positrons as well as electrons. Due to the significance of the problems related to the astrophysical, space as well as laboratory plasmas, the effects of plasma parameters on the nonlinear propagation of PASWs in a weakly unmagnetized inhomogeneous plasma consisting of immobile positive ions, mobile cold positrons, superthermal hot positrons as well as electrons are investigated. Thus, the paper is organized as follows: the theoretical model and derivations of KdV and mKdV equations in inhomogeneous plasmas with analytical solutions are presented in Section 2. The results and discussion are described in Section 3. Finally, the conclusion is drawn in section 4.

2. Model equations

A fully ionized, collisionless, unmagnetized four-component plasma system consisting of immobile positive ions, mobile cold positron, and kappa distributed hot positrons, as well as electrons, is considered. At equilibrium, the charge neutrality condition yields \( n_{e0} = n_{pc0} + n_{ph0} + n_{i0} \), where \( n_{i0}, n_{e0} \) and \( n_{pc0}(n_{ph0}) \) are the unperturbed ion number density, electron number density and cold (hot) positron number density, respectively. The concentrations of hot positrons \( (n_{ph}) \) and hot electrons \( (n_e) \) taking the kappa distributed function into account are obtained [20] as

\[
n_{ph} = n_{ph0} \left[ 1 + \frac{e\phi}{k_BT_{ph}(\kappa_p - \frac{3}{2})} \right]^{-\kappa_p^{-\frac{1}{2}}} \quad \text{and} \quad n_e = n_{e0} \left[ 1 - \frac{e\phi}{k_BT_e(\kappa_e - \frac{3}{2})} \right]^{-\kappa_e^{-\frac{1}{2}}},
\]

where \( T_{ph} \) and \( T_e \) are respectively the temperatures of hot positrons and hot electrons, and \( \kappa_p \) and \( \kappa_e \) are respectively the superthermal parameters of hot positrons and hot electrons and \( k_B \) is
the Boltzmann constant. In order to study the nonlinear dynamics of PASWs in the plasma system [20] taking in homogeneity into account, the normalized hydrodynamic fluid equations can be written as

\[ \frac{\partial n_{pc}}{\partial t} + \frac{\partial}{\partial x}(n_{pc}u_{pc}) = 0, \]  

\[ \frac{\partial u_{pc}}{\partial t} + u_{pc} \frac{\partial u_{pc}}{\partial x} = -\frac{\partial \phi}{\partial x}, \]  

\[ \frac{\partial^2 \phi}{\partial x^2} = -n_{pc} - \mu ph \left(1 + \frac{\sigma_1 \phi}{\kappa_p - \frac{3}{2}}\right)^{-\kappa_p + \frac{1}{2}} + \mu_e \left(1 - \frac{\sigma_2 \phi}{\kappa_e - \frac{3}{2}}\right)^{-\kappa_e + \frac{1}{2}} - \mu_t, \]  

where, \( n_{pc} \) is the cold positron number density normalized by its equilibrium value \( n_{pc0} \), \( u_{pc} \) is the cold positron fluid speed normalized by positron acoustic speed \( c_{pc} = (k_B T_{eff}/m_p)^{1/2} \), \( \phi \) is the electrostatic potential normalized by \( k_B T_{eff}/\epsilon \), \( \sigma_1 = T_{eff}/T_{ph} \), \( \sigma_2 = T_{eff}/T_e \), \( \mu ph = n_{ph0}/n_{pc0} \), \( \mu_e = n_{e0}/n_{pc0} \), \( \mu_t = n_{t0}/n_{pc0} \), \( T_{eff} = T_{ph} (\mu e T_{ph} + \mu ph T_e) \) is the effective temperature, \( m_p \) is the positron mass and \( e \) is the magnitude of electron charge. The time and space variables are normalized by the plasma period \( \omega_p^{-1} = (m_p/4\pi n_{pc0} e^2)^{1/2} \) and Debye length \( \lambda_{dp} = (k_B T_{eff}/4\pi n_{pc0} e^2)^{1/2} \), respectively.

3. Derivation of KdV and mKdV equations with variable coefficients

Let us consider the stretched variables, which is spatially applicable for studying the nonlinear propagation of electrostatic PASWs in an inhomogeneous plasma system [33,35] as

\[ \xi = \varepsilon^{1/2} \left( \int \frac{dx}{V(x)} - \ell \right), X = \varepsilon^{3/2} x, \]  

where \( X \) is the time related coordinates, \( V \) is the speed of wave propagation and \( \varepsilon \) is a small parameter measuring the weakness of dispersion.

Now, expanding the perturbed quantities according to the well-known RPM as

\[ \mathcal{H} = \mathcal{H}^{(0)} + \sum_{i=1}^{\infty} \varepsilon^i \mathcal{H}^{(i)}, \]  

where \( \mathcal{H} = (n_{pc} \quad u_{pc} \quad \phi)' \), \( \mathcal{H}^{(0)} = (n_{pc}^{(0)} \quad u_{pc}^{(0)} \quad \phi^{(0)})' \) and \( \mathcal{H}^{(i)} = (n_{pc}^{(i)} \quad u_{pc}^{(i)} \quad \phi^{(i)})' \), taking only spatial gradients \( \frac{\partial \mathcal{H}^{(0)}}{\partial \xi} = 0 \) and \( \frac{\partial \mathcal{N}}{\partial \xi} = 0 \). Substituting Eqs. (4) and (5) into Eqs. (1)-(3) and collecting the quantities based on the different powers of \( \varepsilon \), one can obtain a set of equations in terms of \( \varepsilon \). The first order of \( \varepsilon \) gives the following relations:

\[ n_{pc}^{(1)} = \frac{n_{pc}^{(0)} \phi^{(1)}}{V - u_{pc}^{(0)}} \quad u_{pc}^{(1)} = \frac{\phi^{(1)}}{V - u_{pc}^{(0)}}, \]  

\[ n_{pc}^{(0)} \left( V - u_{pc}^{(0)} \right)^2 - R_1 + 2R_2 \phi^{(0)} - 3R_3 \{ \phi^{(0)} \}^2 = 0, \]

where,
\[ R_1 = \left\{ \frac{\mu_{ph} \left( \kappa_p - \frac{1}{2} \right) \sigma_1}{\kappa_p - \frac{3}{2}} + \frac{\mu_e \left( \kappa_e - \frac{1}{2} \right) \sigma_2}{\kappa_e - \frac{3}{2}} \right\}, \]

\[ R_2 = \left\{ \frac{\mu_{ph} \left( \kappa_p - \frac{1}{2} \right) \left( \kappa_p + \frac{1}{2} \right) \sigma_1^2}{2 \left( \kappa_p - \frac{3}{2} \right)^2} - \frac{\mu_e \left( \kappa_e - \frac{1}{2} \right) \left( \kappa_e + \frac{1}{2} \right) \sigma_2^2}{2 \left( \kappa_e - \frac{3}{2} \right)^2} \right\}, \]

\[ R_3 = \left\{ \frac{\mu_{ph} \left( \kappa_p - \frac{1}{2} \right) \left( \kappa_p + \frac{1}{2} \right) \sigma_1^3}{6(\kappa_p - 3/2)^3} + \frac{\mu_e \left( \kappa_e - \frac{1}{2} \right) \left( \kappa_e + \frac{1}{2} \right) \left( \kappa_e + \frac{3}{2} \right) \sigma_2^3}{6(\kappa_e - 3/2)^3} \right\}. \]

To the next higher order of \( \varepsilon \) gives the following equations

\[
\frac{2}{(V - u_{pc}^{(0)})^2} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \left( \frac{V n_{pc}^{(0)}}{(V - u_{pc}^{(0)})^2} + \frac{V u_{pc}^{(0)} n_{pc}^{(0)}}{(V - u_{pc}^{(0)})^2} \right) \frac{\partial \phi^{(1)}}{\partial X} + \left( \frac{V}{(V - u_{pc}^{(0)})^2} \right) \phi^{(1)} = \left( V - u_{pc}^{(0)} \right) \frac{\partial u_{pc}^{(2)}}{\partial \xi} - n_{pc}^{(0)} \frac{\partial u_{pc}^{(2)}}{\partial \xi},
\]

\[
\frac{V u_{pc}^{(0)}}{(V - u_{pc}^{(0)})^2} \frac{\partial \phi^{(1)}}{\partial X} + \frac{V}{(V - u_{pc}^{(0)})^2} \phi^{(1)} \frac{\partial u_{pc}^{(2)}}{\partial X} + V \frac{\partial \phi^{(1)}}{\partial X} + \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{1}{(V - u_{pc}^{(0)})^2} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi}
\]

\[
= \left( V - u_{pc}^{(0)} \right) \frac{\partial u_{pc}^{(2)}}{\partial \xi},
\]

\[-n_{pc}^{(2)} + R_1 \phi^{(2)} - R_2 \phi^{(1)} - R_3 \phi^{(0)} \phi^{(1)} = R_2 \phi^{(0)} \phi^{(2)} + R_3 \phi^{(2)} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi}
\]

\[
= \frac{1}{(V - u_{pc}^{(0)})^2} \frac{\partial \phi^{(1)}}{\partial \xi}.
\]

Eliminating \( n_{pc}^{(2)} \), \( n_{pc}^{(2)} \) and \( \phi^{(2)} \) from the above equations with the help of Eq. (6), one can derive the following nonlinear evolution equation with variable coefficients as

\[
\frac{\partial \phi^{(1)}}{\partial X} + \frac{A_3}{A_2} \phi^{(1)} \frac{\partial \phi^{(0)}}{\partial X} + \frac{A_1}{A_2} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial X} + \frac{1}{V^2 A_2} \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0,
\]

(7)

where \( A_1 = R_2 - 2 R_3 \phi^{(0)} + \frac{6 n_{pc}^{(0)}}{(V - u_{pc}^{(0)})^4} \), \( A_2 = \frac{2 V^2 n_{pc}^{(0)}}{(V - u_{pc}^{(0)})^2} \), and \( A_3 = \frac{V n_{pc}^{(0)} (3 V - 3 n_{pc}^{(0)})}{(V - u_{pc}^{(0)})^4 n_{pc}^{(0)}} \).

Eq. (7) is known as the KdV like equation with variable coefficients. Using the variable transform \( \phi^{(1)} = \varphi(\xi, X) e^{-A_3/A_2} \phi^{(0)} \), Eq. (7) can be converted to the following KdV equation:

\[
\frac{\partial \varphi}{\partial X} + L \varphi \frac{\partial \varphi}{\partial \xi} + M \frac{\partial^3 \varphi}{\partial \xi^3} = 0,
\]

(8)
where \( M = (1/V^2 A_2) \) and \( L = (A_1/A_2)e^{(-A_3/A_2)}\phi^{(0)} \). However, the nonlinear coefficient functionally occurs based on the space of the plasma due to the sake of simplicity for mathematical development. The variations are assumed in significantly as compared to the scale length or it is supposed that all parameters are locally constant. Therefore, introducing a transformation \( (\chi = \xi - U_0t) \), the solitary wave solutions of Eq. (8) can be obtained as

\[
\phi = \varphi_0 \text{sech}^2 \left( \frac{X}{W_0} \right),
\]

where \( \varphi_0 = (3U_0/L) \) and \( W_0 = \sqrt{(4M/U_0)} \) are respectively the amplitudes and widths of KdV solitary waves and \( \chi \) is the transformed coordinate with regards to a frame moving with constant velocity \( U_0 \). It is found that the amplitudes of the KdV solitary waves approach to infinity when \( L \to 0 \) for \( n_{pc}^{(0)} = (V - u_{pc}^{(0)})^4 (R_2 - 2R_3\phi^{(0)})/6 \), where the validity of the perturbation technique breaks down. In such case, the structures of PA solitary waves around the critical densities are studied considering the higher order nonlinear evolution equation in weakly inhomogeneous multi-components plasma. To do so, one needs to convert the stretched variables of Eq. (4) as

\[
\xi = \epsilon \left( \int \frac{dx}{V - t} \right), X = \epsilon^3 x.
\]

Using Eqs. (5) and (10) into Eqs. (1)-(3), one can obtain the same values of \( n_{pc}^{(1)}, u_{pc}^{(1)} \) and the dispersion relation as mentioned in Eq. (6). To the next higher order of \( \epsilon \) gives

\[
- (V - u_{pc}^{(0)}) \frac{\partial n_{pc}^{(2)}}{\partial \xi} + n_{pc}^{(0)} \frac{\partial u_{pc}^{(2)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} (n_{pc}^{(1)} u_{pc}^{(1)}) = 0
\]

\[
- (V - u_{pc}^{(0)}) \frac{\partial u_{pc}^{(2)}}{\partial \xi} + u_{pc}^{(1)} \frac{\partial u_{pc}^{(1)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} = 0
\]

\[-n_{pc}^{(2)} + R_1\phi^{(2)} - 2R_2\phi^{(0)}\phi^{(2)} + 3R_3\phi^{(0)}^2 \phi^{(2)} + \left[ -R_2 + R_3\phi^{(0)} \right] \phi^{(1)}^2 = 0.\]

Solving Eqs. (11)-(13) with the help of Eq. (6), one can find the following relations

\[
n_{pc}^{(2)} = \frac{3n_{pc}^{(0)} \phi^{(1)}^2}{2 (V - u_{pc}^{(0)})^3} + n_{pc}^{(0)} \phi^{(2)} (V - u_{pc}^{(0)})^2, u_{pc}^{(2)} = \frac{\phi^{(1)}^2}{2 (V - u_{pc}^{(0)})^3} + \frac{\phi^{(2)} (V - u_{pc}^{(0)})}{2},
\]

\[
\left( \frac{n_{pc}^{(0)}}{(V - u_{pc}^{(0)})^2} - R_1 + 2R_2\phi^{(0)} - 3R_3\phi^{(0)}^2 \right) \phi^{(2)}
\]

\[+ \frac{1}{2} \left( R_2 - 2R_3\phi^{(0)} + \frac{6n_{pc}^{(0)}}{(V - u_{pc}^{(0)})^4} \right) \phi^{(1)}^2 = 0.
\]

Finally, the following equations can be obtained considering the next higher order of \( \epsilon \):
waves are generated as well as the amplitudes of the PASWs decrease. It is also found that the KdV of the parameters, as a result of the amplitudes of the PASWs decrease. The driving force, due to the inertia equation increases due to the enhancement of densities and \( \sigma \)
electrostatic PASWs are discussed considering the typical ranges of the remaining parameter constants. It is seen from Figs. 1 that the amplitudes of PASWs are decreasing with increasing positron concentration, and as a result, the solitary waves are generated as well as the amplitudes of the PASWs decrease. It is also found that the KdV results and Discussion

The physical issues concerned in the weakly inhomogeneous plasma (composing immobile positive ions, mobile cold positrons, and superthermal hot positrons as well as electrons) have been investigated by driving the KdV and mKdV equations with variable coefficients. The parametric effects on the electrostatic PASWs are discussed considering the typical ranges \( \mu_e = 0.1 - 0.6 \), \( \sigma_1 = 1 - 6 \), \( \sigma_2 = 0.1 - 0.9 \), \( \kappa_e = 1.8 - 2.2 \) and \( \kappa_p = 3 - 100 \) of the plasma parameters, which are consistent with space and laboratory plasmas [1-3, 7, 8].

Figs. 1(a)-(d) show the electrostatic PA KdV solitary waves for different values of \( n_{pc}^0 \), \( \mu_e \), \( \kappa_e \) and \( \sigma_1 \) considering the remaining parameter constants. It is seen from Figs. 1 that the amplitudes of PASWs are decreasing with the increase of \( n_{pc}^0 \). The nonlinear term \( (L) \) of KdV equation increases due to the enhancement of densities and spectral index kappa of electrons parameters, as a result of the amplitudes of the PASWs decrease. The driving force, due to the inertia of the positron, also decreases with increasing positron concentration, and as a result, the solitary waves are generated as well as the amplitudes of the PASWs decrease. It is also found that the KdV...
equation supports both compressive for $L < 0$ and rarefactive for $L > 0$ solitons depending on the plasma parameters in the inhomogeneous plasmas. Besides, the KdV equation does not support the PASWs in the weakly inhomogeneous plasmas at the critical unperturbed densities $n_{pc}^{(0)} = \left(V - u_{pc}^{(0)}\right)^4 / 6$. In such case, the mKdV equation with variable coefficients is derived considering further higher order nonlinearity terms to investigate the PASWs around the critical densities. Figs. 2(a)-(d) display the effect of $n_{pc}^{0}$, $\kappa_e$, $\kappa_p$ and $\sigma_1$ along with $\chi$ on the electrostatic PA mKdV solitary waves considering the remaining parameter constants. It is seen from Figs. 2 that the amplitudes of mKdV PASWs are increasing with the increase of $n_{pc}^{0}$, $\kappa_e$ and $\kappa_p$, and decreasing with the increase of $\sigma_1$. It is interesting to note that the mKdV equation supports only the compressive solitons depending on the inhomogeneous plasma conditions. The results obtained in this manuscript are in good agreements with the findings of [20] in homogeneous plasmas. Thus, the parametric investigations on the considered inhomogeneous plasmas may be useful for understanding the physical properties of nonlinear electrostatic PA solitary waves in space plasmas [1-5] and laboratory plasmas [6, 7].

**Figure 1.** Effects of (a) $n_{pc}^{(0)}$ ($\kappa_e = 1.8$, $\mu_e = 0.2$, $\sigma_1 = 1$, $\mu_{ph} = 0.08$, $\kappa_p = 3$, $\sigma_2 = 0.5$, $u_{pc}^{(0)} = 0.6$, $V = 1.6$, $\phi^{(0)} = 0.1$, $U_0 = 0.02$) (b) $\mu_e$ with the same values of (a) but $\mu_{ph} = 0.14$ and $n_{pc}^{(0)} = 0.1$, (c) $\kappa_e$ ($\mu_{ph} = 0.14$, $\mu_e = 0.6$, $\kappa_p = 3$, $\sigma_1 = 1$, $\sigma_2 = 0.5$, $u_{pc}^{(0)} = 0.6$, $n_{pc}^{(0)} = 0.7$, $V = 1.6$, $\phi^{(0)} = 0.5$, $U_0 = 0.02$) and (d) $\sigma_1$ with the same values of (c) but $\kappa_e = 2.1$ and $\kappa_p = 5$ on the electrostatic PA KdV solitary waves in weakly inhomogeneous plasma.
Figure 2. Effects of (a) \( n_{pc}^{(0)} \) (\( \kappa_e = 1.8, \kappa_p = 3, \mu_e = 0.2, \sigma_1 = 1, \mu_{ph} = 0.08, \sigma_2 = 0.5, u_{pc}^{(0)} = 0.6, V = 1.6, \phi^{(0)} = 0.1, U_0 = 0.02 \)) (b)\( \kappa_e \) with the same values of (a) but \( n_{pc}^{(0)} = 0.5, \phi^{(0)} = 0.5, \) and \( \mu_e = 0.45, \) (c)\( \kappa_p \) taking the same values of (a) but \( \mu_{ph} = 0.12, n_{pc}^{(0)} = 0.5 \) and \( \mu_e = 0.45, \) and (d) \( \sigma_1 \) with the same values of (b) but \( n_{pc}^{(0)} = 0.71, \kappa_e = 1.8 \) and \( \mu_e = 0.5 \) along with \( \chi \) on the electrostatic PA mKdV solitary waves in weakly inhomogeneous plasmas.

5. Conclusions
The weakly unmagnetized inhomogeneous plasma system composing of immobile positive ions, mobile cold positrons, and superthermal hot positrons, as well as electrons, is considered. The KdV and mKdV equations with variable coefficients are derived using the RPM for investigating the nonlinear propagation of PASWs in the plasmas. It is found that the KdV equation admits both compressive and rarefactive solitons, but only compressive solitons are found from the mKdV equations in inhomogeneous plasmas. The contributions of density ratios, temperature ratios and superthermality parameters to the inhomogeneously solitons propagation characteristics are described. It can be concluded that the energy absorption appears by the solitons without changing the shape and velocity, thus the energy absorption enhances (suppresses) smoothly with the enhancement of plasma parameters for the rarefactive (compressive) PASWs in the plasma system. Furthermore, the system critically changes the polarity, which makes the energy absorption have maximum change near the
critical values. The results obtained in this investigation might be useful for further verification in laboratory plasmas.

References
[1] Bremer J, Hoffmann P, Manson A H, Meek C E, Ruster R and Singer W 1996 Ann. Geophys. 14 1317
[2] Franz J, Kintner P and Pickett J 1998 Geophys. Res. Lett. 25 1277
[3] Pierrard V and Lemaire J 1996 J. Geophys. Res. 101 7923
[4] Michel F C 1982 Rev. Mod. Phys. 54 1
[5] Miller H R and Wiita P J 1987 Active Galactic Nuclei (Berlin: Springer) p. 202
[6] Shukla P K, Rao N N, Yu M Y and Tsintsadze N L 1986 Phys. Rep. 138 1
[7] Berezhiani V, Tskhakaya D D and Shukla P K 1992 Phys. Rev. A 46 6608
[8] Vasyliunas V M 1968 J. Geophys. Res. 73 2839
[9] Baluku T K and Hellberg M A 2008 Phys. Plasmas 15 123705
[10] El-Tantawy S A, El-Bedwehy N A and Moslem W M 2011 Phys. Plasmas 18 052113
[11] Surko C M, Leventhal M and Passner A 1989 Phys. Rev. Lett. 62 901
[12] Abdullah K, Haarsma L and Gabrielse G 1995 Phys. Scr. T59 337
[13] Shah M G, Hossen M R, Sultana S and Mamun A A 2015 Chin. Phys. Lett. 32 085203
[14] Uddin M J, Alam M S and Mamun A A 2015 Phys. Plasmas 22 062111
[15] Rahman M M, Alam M S and Mamun A A 2015 Brazilian J. Phys. 45 314
[16] Rahman M M, Alam M S and Mamun A A 2015 Astrophys. Space Sci. 357 36
[17] Shah M G, Hossen M R and Mamun A A 2015 Brazilian J. Phys. 45 219
[18] Uddin M J, Alam M S and Mamun A A 2015 Phys. Plasmas 22 022111
[19] Rahman M M, Alam M S and Mamun A A 2014 J. Korean Physical Society 64 1828
[20] Alam M S, Uddin M J, Masud M M and Mamun A A 2014 Chaos 24 033130
[21] Hafez M G and Talukder M R 2015 Astrophys. Space Sci. 359 27
[22] Hafez M G, Talukder M R and Sakkthivel R 2016 Indian J. Phys. 90 603
[23] Hafez M G, Talukder M R and Ali M H 2017 Plasma Phys. Reports 43499
[24] Hafez M G, Talukder M R and Ali M H 2016 Phys. Plasmas 23 012902
[25] Hafez M G, Talukder M R and Ali M H 2016 Astrophy. Space Sci. 361 154
[26] Hafez M G, Roy N C, Talukder M R and Ali M H 2016 Astrophys. Space Sci. 361 312
[27] Hafez M G, Roy N C, Talukder M R and Ali M H 2016 Phys. Plasmas 23 082904
[28] Alam M S, Hafez M G, Talukder M R and Ali M H 2017 Phys. Plasmas 24 072901
[29] Akter T, Deeba F and Kamal-Al-Hassan M 2016 IEEE Transactions Plasma Sci. 44 1449
[30] El-Wakil S A, Zahran M A, El-Shewy E K and Mowafy A E 2006 Phys. Scr. 74 503
[31] Mowafy A E, El-Shewy E K, Moslem W M and Zahran M A 2008 Phys. Plasmas 15 708
[32] Attia M T, Zahran M A, El-Shewy E K and Mowafy A E 2010 Z. Naturforsch. A 65 91
[33] Mowafy A E and Moslem W M 2012 J. King Saud Univ. Sci. 24 343
[34] Gill T S, Kaur H and Saini N S 2006 Pramana – J. Phys. 66 6
[35] Asano N 1974 Suppl. Prog. Theor. Phys. 55 52