Spin and Flavor Contents of the Proton

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Report No: CMU-HEP 95-08, DOE-ER/40682-98, hep-ph/9506397

Abstract

After a brief review of the experimental results obtained in deep inelastic lepton-nucleon scatterings and the Drell-Yan processes and their implications for the spin and flavor contents of the nucleon, we suggest that those features, that are contrary to the expectations of the naive quark model, can all be accounted for in the chiral quark model of Georgi and Manohar. In our formulation of the chiral quark model, the $\eta'$ meson is seen to play an important role.

A few years back, the deep inelastic scattering experiments performed by the EMC \cite{EMC} and NMC \cite{NMC} collaborations at CERN suggested a proton spin and flavor structure that was at variance from the naive quark model expectations. These results have been confirmed and extended by more recent experimental findings: by SMC at CERN, by E142 and E143 at SLAC \cite{SMC}, and by NA51 at CERN \cite{NA51}, respectively. In this paper, we shall discuss a mechanism \cite{Li:1994ay} \cite{Chen:1995na}, in the framework of the chiral quark model of nonperturbative QCD \cite{Georgi:1982as}, which gives a simple and unified account of all such “anomalous” spin and flavor structures of the proton.

1. Structure Function $g_1(x)$ and the Proton Spin

1.1 Polarized lepton-nucleon scatterings and their QCD analysis

Inclusive deep inelastic $lN$ scatterings basically measure the current-current correlation

$$\text{Im} \int e^{iq\xi} d\xi \langle N(p, s) | T \left(J^\mu_{em}(\xi) J^\nu_{em}(0)\right) | N(p, s) \rangle = \frac{i}{M^2} \varepsilon_{\mu\alpha\beta\nu} q^\alpha s^\beta g_1^N(x, Q^2) + ...$$

where $s_\mu$ is the covariant spin vector of the nucleon, $Q^2 \equiv -q^2$ is momentum transfer, and $x = Q^2/2M\nu$ is the usual scaling variable. The spin-dependent structure function $g_1^N(x, Q^2)$ of the nucleon as defined above can be measured by comparing the cross sections of the parallel and anti-parallel longitudinally polarized scatterings. In the operator product expansion of the left-hand-side, one of the leading twist-two terms with the correct quantum number is the dimension-three axial-vector current operator $A^\mu(0) = \sum_{q=u,d,s} e_q^2 \gamma^\mu \gamma_5 q$. This term can be isolated by taking the appropriate moment of the structure function. For the case of the proton target, this leads to the sum rule:

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* Presented by Ling-Fong Li at the Ames Symposium, May 22-24, 1995.
\[
\int_0^1 dx g_1^p(x, Q^2) = \frac{C_{NS}(Q^2)}{36} (3g_A + \Delta s) + \frac{C_S(Q^2)}{9} \Delta \Sigma
\]  

(2)

The factors \(g_A, \Delta s,\) and \(\Delta \Sigma\) are the various flavor non-singlet and singlet combinations of \(\Delta q s\) which are the axial vector current matrix elements between the proton states:

\[
\langle p(s) | \bar{q} \gamma_\mu \gamma_5 q | p(s) \rangle \equiv 2s_\mu \Delta q.
\]  

(3)

\(\Delta \Sigma = \Delta u + \Delta d + \Delta s\) is the singlet combination, while the non-singlet combinations can be related, via \(SU(3)\), to the axial vector couplings measured in the baryon weak decays:

\[
\Delta u - \Delta d = g_A = 1.2573 \pm 0.0028
\]

\[
\Delta u + \Delta d - 2\Delta s = \Delta s = 3F - D = 0.61 \pm 0.038.
\]  

(4)

The Wilson coefficients in Eq.(3) have been calculated in perturbative QCD [8], for the non-singlet case, up to three loops, with an estimate made for the fourth order term:

\[
C_{NS}(Q^2) = 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.5833 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.2153 \left[ \frac{\alpha_s(Q^2)}{\pi} \right]^3 + O(130) \left[ \frac{\alpha_s(Q^2)}{\pi} \right]^4,
\]  

(5)

for the singlet-channel, to the second, and estimated to the third, order:

\[
C_S(Q^2) = 1 - \frac{\alpha_s(Q^2)}{\pi} - 1.0959 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - O(6) \left[ \frac{\alpha_s(Q^2)}{\pi} \right]^3.
\]  

(6)

The \(Q^2\)-dependence appearing through the \(\alpha_s(Q^2)\) is important to reconcile the various experimental results taken at different \(Q^2\)s. [10]

Thus the \(g_1(x)\) measurements can be used to evaluate the moment-integral. This, together with the \(SU(3)\) values as given in Eq. (4), allows us to deduce the axial vector matrix element for each quark flavor \(\Delta q\). In the following we display both the original and the more recent results:

| Experiments | \(\int_0^1 dx g_1^p(x, Q^2)\) | \(\Delta s\) | \(\Delta \Sigma\) |
|-------------|-------------------------------|-------------|--------------|
| EMC (1987)  | 0.126 \pm 0.025               | -0.19 \pm 0.06 | 0.12 \pm 0.17 |
| SMC, E142-3 (1994) | 0.136 \pm 0.016 | -0.10 \pm 0.04 | 0.31 \pm 0.11 |

We can thus conclude that the new data do support the original findings of a significant strange quark contribution \(\Delta s \neq 0\) [4] and a much-less-than-unity total-quark term \(\Delta \Sigma < 1\),

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1These authors also conclude that the Bjorken sum rule \(\int dx (g_1^p - g_1^n) = \frac{1}{6} g_A\) is satisfied to within 12%.
although the size of $\Delta s$ has decreased somewhat and $\Delta \Sigma$ is perhaps not as small as originally thought.

### 1.2 Theoretical interpretation - proton spin contents

From above we see that the $g_1$-sum rule measures the matrix element of the axial vector current which corresponds to the non-relativistic spin operator

$$\vec{A} = \bar{q} \gamma^\mu \gamma_5 q = q^\dagger \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} q.$$  

(8)

Thus we can interpret the $\Delta q$ defined in (3) as the quark contribution to the proton spin.

The phenomenological results of $\Delta s \neq 0$ and $\Delta \Sigma < 1$ is contrary to the simple quark model expectation. The $SU(6)$ wave function appropriate for the usual constituent quark model of the proton has the form of

$$|p_\uparrow\rangle = \frac{1}{\sqrt{6}} (2 |U\uparrow U\uparrow D\downarrow\rangle - |U\uparrow U\downarrow D\uparrow\rangle - |U\downarrow U\uparrow D\uparrow\rangle)$$  

(9)

which yields the quark spin contributions of

$$\Delta U = 2 \times \frac{4}{6} = \frac{4}{3}, \quad \Delta D = \frac{1}{6} [4(-1) + 1 + 1] = -\frac{1}{3}, \quad \Delta S = 0, \quad \Delta \Sigma = 1.$$  

(10)

However, we have to distinguish between current quarks which appears in the current operators being probed by lepton-proton scatterings and the constituent quarks which appear in the $SU(6)$ calculation. Presumably, constituent quarks are the current quarks dressed up by some nonperturbative QCD interactions. A simple proposition \[11\] will be that they are proportional to each other $\langle |\bar{q} \gamma_\mu \gamma_5 Q\rangle = Z_q \langle |\bar{q} \gamma_\mu \gamma_5 q\rangle$ with these "renormalization factors" $Z_q's$ still obey the flavor $SU(3)$ symmetry.

$$\langle |\bar{U} U + \bar{D} D - 2 \bar{S} S|\rangle = Z_8 \langle |\bar{u} u + \bar{d} d - 2 \bar{s} s|\rangle$$

$$\langle |\bar{U} U + \bar{D} D + \bar{S} S|\rangle = Z_0 \langle |\bar{u} u + \bar{d} d + \bar{s} s|\rangle$$

Thus, even there is no constituent strange quark in the proton $\langle |\bar{S} S|\rangle = 0$, the proton matrix elements of the strange quark bilinears is still nonzero if the singlet and octect operators renormalize differently

$$\langle |\bar{s} s|\rangle = \frac{1}{3} \left( \frac{1}{Z_0} - \frac{1}{Z_8} \right) \langle |\bar{U} U + \bar{D} D|\rangle.$$  

(11)

This means that the constituent quarks have some structure of their own; the renormalization effects that turns the current quark into a constituent quark surround the constituent quarks with a cloud of quark pairs which may include strange quarks. In Sec. 3 we shall discuss a specific realization of this possibility in the framework of the chiral quark model.

### 2. Anomalous Flavor Structure of the Proton

#### 2.1 Flavor asymmetry measurements in DIS processes
The difference of the proton and neutron (spin-averaged) structure functions $F_2^p(x) - F_2^n(x)$ can be expressed in terms of the quark densities,

$$F_2^p(x) - F_2^n(x) = \frac{x}{3} \left[ (u - d) + (\overline{u} - \overline{d}) \right] = \frac{x}{3} \left[ 2I_s + 2(\overline{u} - \overline{d}) \right]$$

where we have used isospin symmetry in the first equation, and used, in the second equation, the definition $I_s = \frac{1}{2} (u - d) - \frac{1}{2} (\overline{u} - \overline{d})$, with its integral being the third component of the isospin: $\int_0^1 I_s dx = \frac{1}{2}$. The simple assumption that $u = d$ in the quark sea, which is consistent with it being created by the flavor-independent gluon emission, then leads to the Gottfried sum rule [12]

$$I_G \equiv \int_0^1 \frac{dx}{x} \left[ F_2^p(x) - F_2^n(x) \right] = \frac{1}{3} \ . \ (13)$$

Experimentally, NMC found that, with a reasonable extrapolation in the very small-$x$ region, the integral $I_G$ deviated significantly from one third [4].

$$I_G = 0.235 \pm 0.026 = \frac{1}{3} + \frac{2}{3} \int_0^1 (\overline{u} - \overline{d}) \ dx$$

This translates into the statement that, in the proton quark-sea, there are more down-quark pairs as compared to the up-quark pairs.

There were suggestions that the small-$x$ extrapolation could be incorrect and that a more direct confirmation of $u \neq d$ at a fixed $x$ value would be a measurement of the difference of the Drell-Yan process of proton on a proton vs. neutron targets [13]. NA51 collaboration at CERN has performed such an experiment and found that at $x = 0.18$ the $\overline{u}$ density is only about half as much as the $\overline{d}$ density [4]:

$$\frac{\overline{u}}{\overline{d}} = 0.51 \pm 0.04 \pm 0.05 \ . \ (14)$$

Remarks: In this discussion of the anomalous flavor content of the proton, we should also recall the old $\pi N$ sigma-term problem [14]. A simple $SU(3)$ calculation, that is entirely similar to that discussed above in connection with the proton spin, can relate $\sigma_{\pi N}$ to the "fraction of strange quarks in the proton" as:

$$f_s \equiv \frac{2\pi}{3 + 2(\overline{u} + \overline{d} + s)} \simeq \frac{\sigma_{\pi N} - 25 \ MeV}{3\sigma_{\pi N} - 25 \ MeV}$$

where in the second (approximate) equality we have plugged into the $SU(3)$ result the "octect-baryon-mass" value of $M_8 = \frac{1}{3} (2M_N - M_\Xi - M_\Sigma) = M_A - M_\Xi \simeq -200 \ MeV$ and the current-quark-mass ratio of $m_s/m_u,d = 25$. Thus the generally accepted value [15] of $\sigma_{\pi N} = 45 \ MeV$ translates into a surprisingly large strange quark fraction of $f_s \simeq 18\%$.

2.2 Theoretical interpretation - proton flavor contents

The simple picture that quark pairs are produced by the flavor-independent gluons would suggest $\overline{u} = \overline{d}$. From this viewpoint, the results of (14) and (15) are very surprising.

Feynman and Field [16] have pointed out long ago that this equality would not strictly hold even in perturbative QCD, because the Pauli exclusion principle and the $u - d$ valence-quark asymmetry in the proton would bring about a suppression of the gluonic production.
of $\bar{u}s$ (versus $\bar{d}s$). This mechanism is difficult to implement as the parton picture is intrinsically incoherent and it is difficult to see how this can generate such a large asymmetry as experimentally observed.

*Pion cloud mechanism* [17] is another idea to account for the violation of Gottfried sum rule. The suggestion is that the lepton probe also scatters off the pion cloud surrounding the target proton, and the quark composition of the pion cloud is thought to have more $\bar{d}s$ than $\bar{u}s$. There is supposed to be an excess of $\pi^+$ (hence $\bar{d}$) compared to $\pi^-$, because $p \rightarrow n + \pi^+$, but not a $\pi^-$ if the final states are restricted to the proton and neutron. (Of course the neutral pions have $\bar{d} = \bar{s}$.) However, it is difficult to see why the long distance feature of the pion cloud surrounding the proton should have such a pronounced effect on DIS processes which should probe the *interior* of the proton, and also this effect should be significantly reduced by emissions such as $p \rightarrow \Delta^{++} + \pi^-$, etc.

In the following we shall discuss how a modified version of the Georgi-Manohar *chiral quark model* can accommodate these unexpected proton flavor structures.

### 3. Proton Structure in the Chiral Quark Model

#### 3.1 The SU(3) symmetric chiral quark model

Georgi and Manohar have suggested [7] that the successes of the chiral symmetric description of low energy hadron physics naturally indicates a chiral symmetry breaking scale of $\Lambda_{\chi SB} \simeq 1$ GeV, significantly higher than the QCD confinement scale of $\Lambda_{QCD} \simeq (100-300)$ MeV. This means that even inside a hadron the Nambu-Goldstone bosons are relevant degrees of freedom. Namely, while quarks, gluons and perturbative QCD fully describe strong interactions at distances $\ll \Lambda_{\chi SB}^{-1}$, for longer distances, but still inside the nucleon where the nonperturbative QCD effects are expected to dominate, the physical description may be quite simple when given in terms of Goldstone-boson modes coupled to quarks. In this regime the chiral condensates also supply an extra mass to quarks, giving rise to a constituent quark mass $O(\frac{1}{3}M_N)$. While the QCD coupling is expected to be so strong as to trigger the nonperturbative effects of chiral symmetry breaking, the remanent effective gluonic coupling in such a quasiparticle description may well be so small that they can be neglected.

Eichten, Hinchliffe, and Quigg are the first ones to work out the consequences of this chiral quark model description for the proton flavor and spin structures [4]. It has some similarity to the pion cloud approach mentioned above, but without its conceptual difficulties. A straightforward $SU(3)$ calculation yields the averaged antiquark numbers $\bar{s}$s in the proton:

\[ \bar{u} = \frac{6}{3}a, \quad \bar{d} = \frac{8}{3}a, \quad \bar{s} = \frac{10}{3}a \]  

(17)

where the parameter $a$ has the interpretation as the probability of Goldstone mode emission in $u \rightarrow d + \pi^+$, or its $SU(3)$ equivalences.

In the chiral quark regime, emission of a Goldstone-boson by quarks will flip their helicities and thus reduce their contributions to the proton spin, while the component-quarks of the Goldstone boson will not contribute because they are necessarily unpolarized. Again a simple calculation yields

\[ \Delta u = \frac{4}{3} - \frac{37}{9}a, \quad \Delta d = -\frac{1}{3} - \frac{2}{9}a, \quad \Delta s = -a. \]  

(18)
Thus with \( a \simeq 0.22 \) as required by the NMC data \( (14) \), the result shown above in \( (18) \) gives a negative-valued total-quark spin contribution of \( \Delta \Sigma = 1 - 16a/3 \simeq -0.17 \), which is barely consistent with the 1987 EMC data \( (7) \).

3.2 The chiral quark model with a broken \( U(3) \) symmetry

We have proposed a broken-\( U(3) \) version of the chiral quark model with the inclusion of the \( \eta' \) mode \( (6) \).

Our motivation is two fold: It is clear that the above description, without the \( \eta' \) mode, of \( \pi/d = 0.75 \), \( \Delta s \simeq -0.22 \), and \( \Delta \Sigma \simeq -0.17 \) as given in Eqs.\( (17) \) and \( (18) \) is not adequate to account for the more precise nature of the new data collected in the last two years, as shown in \( (17) \) and \( (7) \), etc. On the theoretical side, we are motivated to include \( \eta' \) because, in the leading \( 1/N_c \) approximation (the planar diagrams), there are nine Goldstone bosons with an \( U(3) \) symmetry. However we also know that if we stop at this order, some essential physics would have been missed: At the planar-diagram level there is no axial anomaly and \( \eta' \) would have been a bona fide Goldstone boson; and the unbroken \( U(3) \) symmetry would also lead \( (3) \) to the phenomenologically unsatisfactory feature of a flavor-independent sea, \( \pi = d = \bar{\pi} \), which clearly violates the results in \( (14) \) and \( (15) \). Thus it will be better to include the \( 1/N_c \) corrections (non-planar diagrams) which break the \( U(3) \) symmetry. The broken-\( U(3) \) is implemented by taking differently-renormalized octet and singlet Yukawa couplings \( g_0/g_8 \equiv \varsigma \neq 1 \).

A simple two-parameter \( (a \) and \( \varsigma \) \) calculation yields the following results \( (3) \):

| Item | \( \chi_{QM} \) | \( a = 0.1 \), \( \varsigma = -1.2 \) | Experimental value |
|------|----------------|-------------------------------|-------------------|
| \( I_G \) | \( \frac{1}{3} - \frac{4}{9}(1 - \varsigma)a \) | 0.236 | 0.235 \( \pm 0.026 \) |
| \( \pi/d \) | \( \frac{6+2\varsigma+\varsigma^2}{8+\varsigma^2} \) | 0.53 | 0.51 \( \pm 0.09 \) |
| \( f_s \) | \( \frac{(10-2\varsigma+\varsigma^2)a}{(9/2)+3(8+\varsigma^2)a} \) | 0.19 | 0.18 \( \pm 0.03 \) |
| \( \Delta u \) | \( \frac{4}{3} - \frac{1}{3}(37 + 8\varsigma^2)a \) | 0.79 | 0.83 \( \pm 0.05 \) |
| \( \Delta d \) | \( -\frac{1}{3} - \frac{2}{9}(1 - \varsigma^2)a \) | -0.32 | -0.42 \( \pm 0.05 \) |
| \( \Delta s \) | \( -a \) | -0.10 | -0.10 \( \pm 0.05 \) |
| \( \Delta \Sigma \) | \( 1 - \frac{2}{3}(8 + \varsigma^2)a \) | 0.37 | 0.31 \( \pm 0.11 \) |

Here \( I_G \) stands for the Gottfried integral of Eq.\( (13) \), and \( f_s \) is the strange quark fraction in the proton as defined in Eq.\( (16) \). The ”experimental values” for \( \Delta q s \) are taken from the phenomenological analysis as given in Ref.\( (10) \).

Our choice of the Goldstone-mode emission-probability \( a = 1.0 \) and the coupling ratio \( \varsigma = -1.2 \) is for illustrative purpose only, to show that a simple calculation in this model can reproduce the principal features of the observed proton spin and flavor contents. (Given the crudity of the model, a detailed ”best-fit” analysis is not warranted at this stage.) It is
gratifying that the parameter $a$ turns out to be fairly small. This means that the constituent quarks are not surrounded by some complicated quark-sea; and it makes this chiral quark picture more self-consistent. The negative value for the singlet-coupling $g_0 \simeq -1.2 g_8$, which is basically required by the $\pi/\eta$ ratio as determined by the NA51 measurement, is, in our opinion, the more intriguing result of this model calculation.

This work is supported at CMU by the US Department of Energy (DE-FG02-91ER-40682), and at UM-St Louis by the National Science Foundation (PHY-9207026).

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