Simplicial Gravity and Strings

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Abstract

String theory, as a theory containing quantum gravity, is usually thought to require more dimensions of spacetime than the usual 3+1. Here I argue on physical grounds that needing extra dimensions for strings may well be an artefact of forcing a fixed flat background space. I also show that discrete simplicial approaches to gravity in 3+1 dimensions have natural string-like degrees of freedom which are inextricably tied to the dynamical space in which they evolve. In other words, if simplicial approaches to 3+1 dimensional quantum gravity do indeed give consistent theories, they may essentially contain consistent background-independent string theories.

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I. INTRODUCTION

String theories are well known to require unphysical (not 3 space and 1 time) numbers of dimensions in which to propagate. Here I briefly review why this might reasonably be expected on physical grounds. I then show how approaches to simplicial quantum gravity could contain consistent theories of 1-dimensional string-like objects.

In other words, it might be that rather than getting gravity from strings, we could get strings from gravity.

II. WHY SHOULD CLOSED STRING THEORIES CONTAIN GENERAL RELATIVITY?

There are at least two reasons why one might expect string theory to contain gravity. One is that closed strings have modes of vibration which can be interpreted as spin-2 massless particles, and the only consistent coupling of such particles seems to be linearized general relativity [1]. This is an essentially perturbative result, with the hope that the full nonlinear theory can be correctly represented by infinite sums of graviton exchanges (although it does take some strain to imagine getting a black hole out of lots of spin-2 gravitons in flat space)!

The other is to view closed strings as maps from loops with the topology of the circle into some background spacetime manifold $M$. The theory is then really one of the infinite dimensional space of loops on $M$. This loop space is a very complicated object but in the limit of shrinking the loops to points one sees just the geometry of points of $M$ and recovers the usual notions of differential geometry. This would lead one to expect that the geometry of infinite dimensional loop space would naturally contain the finite dimensional geometry of space, and indeed Bowick and Rajeev[2] were able to argue eloquently that string theory could be seen as the Kähler geometry of loop space.

III. THE UNPHYSICAL SPACETIME DIMENSIONALITIES OF STRING THEORY

Let us quickly review why string theory as usually formulated leads to the need for extra spacetime dimensions.
The usual argument is to try to quantize strings propagating in a flat fixed background spacetime $\mathbb{R}^{n+1}$ with $n$ space dimensions and one time. Consistent quantization then requires that the spacetime have certain critical dimensions: $25+1$ if the string is assumed to be bosonic, and $9+1$ if the string is given fermionic degrees of freedom and required to be supersymmetric.

From the loop space point of view, requiring an appropriately defined Ricci tensor for loop space to vanish as an analog of Einstein’s equations leads, for strings in flat spacetime, to require the same unphysical dimensions.

The most common response to these extra dimensions has been to find ways to argue that they are compactified at a scale so small as to render them effectively inaccessible to us, or re-interpret them as additional fields in the theory. I would like to argue, however, that there is a very good physical reason to expect nonsensical predictions for the dimension of a flat nondynamic spacetime in a theory that one hopes would reproduce general relativity.

### IV. WHY SHOULD STRING THEORY REQUIRE UNPHYSICAL NUMBERS OF SPACETIME DIMENSIONS?

The key point is that Einstein’s equations tell us that spacetime is dynamical and is curved by the presence of matter and energy. If a string carries energy it’s hard to see why it would make sense then, given that one wants a theory that will reproduce the results of general relativity, to expect consistent solutions with zero spacetime curvature where the strings are.

Indeed, simply relaxing the requirement that the background be flat, as is done in studies of strings propagating on group manifolds, one can find lower critical dimensions, getting closer to our own 3+1. Even then, the assumption of a completely homogeneous space with no localized curvature where a string is, seems bizarre from a physical point of view.

People working in loop quantum gravity have long argued for the importance of background independence and the development of a relational theory – a view that I personally would agree with – but I think the argument given here of what’s physically inconsistent with the usual approach to string theory is quite robust and independent of any particular philosophical viewpoint: The conditions under which unphysical dimensions are derived are
also those which would require the theory to behave in a way that is directly in conflict with
the physical content of Einstein’s equations.\\[12\\]

Might string theory be viable if one did not insist on the physically unreasonable ansatz of
a flat nondynamical background (perhaps to be relaxed later by some sort of back-reaction)?
Perhaps, in a sense, string theory, in a sense, back-reacts to having natural degrees of freedom
frozen by requiring extra dimensions.

V. STRINGS IN SIMPLICIAL QUANTUM GRAVITY

One way of thinking about a discrete spacetime is to imagine that space (or spacetime) is
made of piecewise flat simplices which fit together to approximate a smooth geometry (for
an excellent review, see \[5\]). Examples include Regge calculus\[6\], in which the dynamical
degrees of freedom are taken to be distances between the points that define the simplices, and
dynamical triangulation\[7\] in which those distances are kept fixed but simplices are added
and subtracted as needed. Such piecewise linear approximations can be used as numerical
approximations for classical general relativity, or used to construct geometries to go into a
path integral (or other) types of quantization.

Quite remarkably, simplicial spaces also arise naturally in loop quantum gravity\[4\], ef-fectively as duals to spin networks – one dimensional graphs with edges labelled by $SU(2)$
representations, and with interwiners at the vertices. The edges can be interpreted as quanta
of area and the vertices as quanta of space, and a quite direct connection can be made with
a picture of space as built up from polyhedra\[8\] glued together. For earlier work connecting
spin networks and quantum gravity, see also reference \[9\].

The point I want to make here is that simplicial spaces, however they arise, naturally
contain things one can interpret as lower-dimensional objects which are intrinsically coupled
to a dynamical background. The idea is easy to understand: In two dimensions one can
approximate a 2-manifold with flat triangles, joined in such a way that the curvature is only
a points where the sums of the angles fail to add to $2\pi$. In three dimensions the curvature
is similarly distributional, but now along 1-dimensional lines. With the usual association
in general relativity of curvature and matter, one could think of these as analogs of string-
theoretic strings moving in a background which is flat everywhere else. I propose that one
calls any 1-dimensional distributional curvature a “simplicial string” or “S-string”.

It is interesting to note that the appearance of 1-dimensional objects is actually implied by starting with a 3+1 dimensional spacetime, so there is a certain naturalness to the idea. If one assumed a D+1 dimensional simplicial spacetime one would have been led to distribution curvature on D-2 dimensional subspaces - sheets, or volumes, or hypervolumes instead of 1-dimensional singular objects.

It is interesting to note the striking degree of similarity between the strings of the usual string theories and S-strings, as well as their differences. Both are 1-dimensional objects in a space which is flat where the strings are not. However, for regular strings, space is still flat where the strings are, while the space where S-strings are, is, by definition, where curvature is localized! The space in which string theories place the strings must be selected by hand and is static. In contrast, S-strings are intrinsically part of a dynamical space and inseparable from it.

S-strings need not be closed, and indeed could form complex and branching networks, dynamically evolving and changing in topology as the associated simplicial 3-space evolves. They thus represent a much richer and more complicated structure than the simple closed loops in flat spacetime of string theory. The implications of this in terms of what else might be hidden in simplicial quantum gravity have yet to be investigated, but there is clearly much richness and unexplored structure present.

Note that while string theory postulates 1-dimensional strings and then tries to derive the dimensionality of space, here we take the physical dimensionality of space as given and deduce, with the simplicial assumption\(^\text{[13]}\), the existence of 1-dimensional objects!

Einstein had lamented the form of the equation \( R_{\mu\nu} = 8\pi GT_{\mu\nu} \) as ugly, with the left side completely geometrical and the right side essentially put in by hand. Here we see the possibility of a picture in which geometry is primary, with stringy matter at the smallest scales being an aspect of space itself – string are where the curvature is concentrated. Similarly, the fact that the highly nonlinear Einstein equations allow one to determine how matter moves in a gravitational field (the geodesic equation) so that “matter tells space how to curve and space tells matter how to move” \(^\text{[10]}\) finds an analog here in that S-strings are both part of the geometry of space and what one might want to think of as matter, with their evolutions inextricably linked together.
VI. CONCLUSIONS

I have argued that the appearance of unphysical spacetime dimensions found in theories of strings which are supposed to contain gravity might very well be expected on physical grounds, and not necessarily a reason to rule out fundamental 1-dimensional objects.

String theory over the last several years has been realized to be a theory containing a variety of extended objects, perhaps most notably D-branes on which strings can end. In a similar fashion, I have argued that any approach to simplicial quantum gravity contains 1-dimensional degrees of freedom – S-strings – which appear naturally together with the geometry and as an intrinsic part of it. S-strings, though genuinely one-dimensional objects which live in a 3-dimensional space, are not put in “by hand”, but, rather, emerge naturally. They must co-evolve with space and indeed have no meaning independent of it. There is no need to make assumptions about some pre-existing background spacetime, much less that it be flat. If simplicial quantum gravity is indeed present in a consistent theory of quantum spacetime, it would effectively unify “space” and “matter” degrees of freedom in a completely natural and relational way.

It is also interesting to see that apparently very different approaches to quantum gravity – simplicial quantum gravity, loop quantum gravity, and string theory - might have more points in the common than is usually recognized.

VII. ACKNOWLEDGEMENTS

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Quite amazingly, the extra dimensions are often compactified as flat tori, with zero curvature, or Ricci-flat ones selected essentially by hand in the hopes of finding some sort of phenomenology that has some sort of resemblance to the real world. Disallowing dynamics, or even any Ricci curvature at all (!) for the spaces of the hidden dimensions is as strange as doing it for the big ones, from the point of view of general relativity.

Which, as noted earlier, arises naturally in loop quantum gravity.