Photon vortex generation in quantum level by high-order harmonic synchrotron radiations from spiral moving electrons in magnetic fields

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We explore synchrotron radiations from a spiral moving electron under a uniform magnetic field along $z$-axis using Landau quantization. We found that this process generates a photon vortex with Bessel wave-function as the eigen-state of the $z$-component of the total angular momentum and the photon vortices with large angular momenta are generated by high-order harmonic radiations. We also calculate the decay widths and the energy spectra. Under strong magnetic fields as $10^{13}$ G, which are found in astrophysical objects such as magnetars, photon vortices are predominantly generated.

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Light vortices carrying large angular momenta are interesting for fundamental physics study and applications. One of remarkable features of light vortices is that a single photon could have a vortex wave-function such as Laguerre-Gaussian (LG) and Bessel functions in quantum level. The photon vortex is useful for quantum control of materials such as atoms, molecules, and nuclei and for quantum communications. At present, light vortices are mainly generated from laser with optical devices. Fundamental processes such as high-order harmonic synchrotron radiations from spiral moving electrons under magnetic fields and nonlinear inverse Compton scattering with high intense circularly polarized laser are candidates to produce directly photon vortices because in these processes a single electron emits a photon. Furthermore, they have an advantage that it is possible to generate photon vortices in a wide energy range from eV to GeV. However, these processes have not been studied theoretically well in quantum mechanics.

Synchrotron radiation is one of fundamental phenomena in the universe, which is considered to be one of sources for observed high energy X/γ rays in astronomy. Furthermore, synchrotron radiations generated from relativistic electrons using bending magnets are used as bright light sources on the ground. In 1949, Schwinger calculated high-order harmonic synchrotron radiations from circular or spiral moving electrons, and subsequently he partly took quantum effects into account. In 2017, Katoh et al. calculated the phase structure of high-order harmonic radiations from spiral moving electrons under uniform magnetic fields in classical electro-magnetism and indicated that an \( l \)-th harmonic photon is the photon vortex carrying \( l\hbar \) total angular momentum from the ratio of the total angular momentum to the total energy of the radiation field. This result opens a possibility of a new class of photon vortex sources. However, the wave-function of the radiative photons has not been obtained yet.

In quantum mechanics, the electron orbitals under magnetic fields are in Landau quantization. Landau quantization plays an important role in various phenomena such as terahertz nonlinear optics. Furthermore, theoretical calculations based upon Landau quantization with strong magnetic fields may lead a result different largely from that based upon classical electromagnetism. Thus, to answer the question of whether each single photon emitted from a spiral motion electron under magnetic fields has a wave-function of a vortex, we should calculate its wave-function using Landau quantization. In this letter, we report that the photon generated by high-order harmonic radiations from a spiral moving electron under a uniform magnetic field is the photon vortex with the Bessel wave-function in quantum mechanics. We also present the decay widths from electrons in Landau levels and the energy spectra.
In the present study, we consider a uniform dipole magnetic field along z-direction, \( B = (0, 0, B) \), and that an electron trajectory draws a circle in a plane perpendicular to the z-direction under this magnetic field (see Fig. 1). A moving electron in a magnetic field loses its energy through a transition associated with a photon emission, where its initial and final states are the eigen-states in Landau quantization. We use the natural unit \( \hbar = c = 1 \). The electron wave-function \( \psi(r) \) in this system is obtained from the following Dirac equation:

\[
\{ \alpha \cdot (-i \nabla + eA) + \beta m_e - E \} \psi(r) = 0,
\]

where \( \alpha \) and \( \beta \) are the Dirac matrices, \( A \) is an electro-magnetic vector potential, \( E \) is the electron energy, \( e \) is the elementary charge, and \( m_e \) is the electron mass. We choose the symmetry gauge with the vector potential being \( A = (-y, x, 0)B/2 \). As a solution of this Dirac equation, we can obtain the electron wave-function with Laguerre function as an eigen-state of the z-component of the total angular momentum (zTAM), \( J \), and the z-component of the momentum, \( p_z \). The electron energy in a Landau level is given by

\[
E = \sqrt{2eBN_L + p_z^2 + m_e^2},
\]

where \( N_L \) indicates the Landau level number defined as \( N_L = (L + |L|)/2 + n \), \( L \) is the z-component of the orbital angular momentum satisfying a relationship of \( L = J + 1/2 \), and \( n \) is the number of nodes in the amplitude of the wave-function along the radial direction. The obtained wave-function is

\[
\psi(r) = \left\{ \frac{1 + \Sigma_z}{2} G_{L-1}^{n'} \left( \frac{eB}{2} r_T \right) + \frac{1 - \Sigma_z}{2} G_L^n \left( \frac{eB}{2} r_T \right) \right\} \times \frac{E + m_e}{2R_z E} \left[ \chi_s \tilde{\sigma} \frac{\dot{r}_T}{E + m_e} \chi_s^* \right] e^{ip_z z},
\]

\[
G_L^n(r_T) = \sqrt{\frac{n!}{\pi(n + |L|)!}} e^{iL \phi} \sqrt{eBr_T^{|L|} e^{-r^2/2} L_n^{(L)}(r_T^2) e^{iL \phi}},
\]

\[
\tilde{p} = (0, \sqrt{2eBN_L}, p_z),
\]

where the spatial coordinate is written as \( r = (r_T, z) = (r_T \cos \phi, r_T \sin \phi, z) \), \( R_z \) is the size of the system along the z-direction, \( \sigma \equiv (\sigma_x, \sigma_y, \sigma_z) \) is the Pauli matrix, \( \chi_s \) is the two-dimensional Pauli spinor, \( \Sigma_z = \text{diag}(1, -1, 1, -1) \), and \( L_n^{(L)} \) is the associated Laguerre function. Note that \( n' = n \) when \( L \geq 0 \) and \( n' = n - 1 \) when \( L \leq -1 \). The form of \( \chi_s \) is arbitrary when \( N_L \geq 1 \) because there are two degenerate states at fixed \( L \) and \( n \), whereas the state with \( N_L = 0 \),
which is so called the lowest Landau state, is not degenerate and its spinor is taken to be only
\( \chi_s = (0, 1) \).

One of features of the electron wave-function in Eq. (3) is that it has the node number, \( n \), originating from the Laguerre function. For example, the wave-function for \( n = 0 \) has an annual structure without nodes in the front \((xy)-plane\), whereas that for \( n = 1 \) has a double annual stricture. This node number is correlated with the position of the helical motion axis \[32\]. The wave-function for \( n = 0 \) corresponds to the helical motion along the \( z \)-axis when \( L \geq 0 \), whereas the wave-function for \( n \geq 1 \) indicates the helical motion along an axis that is different from the initial axis \[32\]. Thus, we consider various node numbers for the final state and take only \( n = 0 \) for the initial state.

Here, we consider a photon emission from a transition of an electron between two Landau levels. Using the Coulomb gauge \( \nabla \cdot A = 0 \) with the photon as \( A_0 = 0 \), we obtain the wave-function with Bessel function propagating along the \( z \)-direction for the emitted photon as a solution of the Klein-Gordon equation. The photon wave-function is

\[
A_{Lh}(r, t) = \epsilon_h J_L(q_T r)e^{iL\phi}e^{i(q_z - e_q t)},
\]

(4)

where \( L \) is the \( z \)-component of the orbital angular momentum, \( h \) is the helicity, \( q_z \) is the \( z \)-component of the photon momentum, \( e_q \) is the photon energy, \( q_T = \sqrt{e_q^2 - q_z^2} \), \( \epsilon_h = (1, ih, 0)/\sqrt{2} \), and \( J_L \) is the associated Bessel function. This \( A \) does not generally satisfy the gauge condition \( \nabla \cdot A = 0 \) except for \( e_q^2 - |q_z|^2 \ll 1 \) in the para-axial limit. Then, we add a \( z \)-component of \( A_z \) and rewrite \( A \) as

\[
A_{Lh}(r; q_z, k) \propto e^{i(q_z - e_q t)} \left[ q_z J_L(q_T r)e^{iL\phi}, ihq_z J_L(q_T r)e^{iL\phi}, ihq_T J_{L+h}(q_T r)e^{i(L+h)\phi} \right].
\]

(5)

The spin and the orbital angular momentum of the photon are not, in principle, independently conserved and the wave-function is not the eigen-state for the helicity. In the present case, the Bessel wave-functions of \( A(h = +1) \) and \( A(h = -1) \) do not satisfy the orthogonal relation and break the Lorentz invariance. Thus, we take the eigen-state wave-functions with \( A(h = +1) - A(h = -1) \) (state-1) and \( A(h = +1) + A(h = -1) \) (state-2) at fixed \( K = L + h \). They are

\[
A_K^{(1)} = \frac{1}{2e_q} e^{i(q_z - e_q t)} \left[ iq_z \left( \tilde{J}_{K+1} - \tilde{J}_{K-1} \right), q_z \left( \tilde{J}_{K+1} + \tilde{J}_{K-1} \right), 2q_T \tilde{J}_K \right],
\]

\[
A_K^{(2)} = \frac{1}{2} e^{i(q_z - e_q t)} \left[ i \left( \tilde{J}_{K+1} + \tilde{J}_{K-1} \right), \left( \tilde{J}_{K+1} - \tilde{J}_{K-1} \right), 0 \right],
\]

\[
\tilde{J}_M(r_T) = J_M(\sqrt{eB}q_T r_T)e^{iM\phi}.
\]

(6)
The electron state with spiral motion can be described as the eigen-state of zTAM when we choose an appropriate symmetry axis. Photons emitted by electron transitions between two zTAM eigen-states are also at the eigen-states of zTAM, so that a photon with higher zTAM must be a photon vortex. The zTAM of an emitted photon, \( K \), satisfies a relationship of 
\[
K = J_i - J_f,
\]
where \( J_i \) and \( J_f \) are the zTAM of the initial and final electron states, respectively. Because the radiation for \( K = 1 \) has the component of \( J_0 \), of which the amplitude at the center is not zero, it loses the vortex feature. In contrast, radiations for \( K \geq 2 \) have the wave-function of the helical structure in quantum level. This result indicates that the radiation for \( K = 1 \) corresponds to the fundamental radiation obtained in classical electromagnetism \([17]\) and when \( K \geq 2 \) a radiation with \( K \) corresponds to the \( K \)-th harmonic radiation.

An electron in a Landau level could decay to any lower levels, and the energy of the emitted photon depends on the final electron state. To obtain the total energy spectrum of the photons, we calculate the decay widths to individual final states for various \( K \) values. A decay width of an electron can be calculated from the initial and final wave-functions of the electron in Eq. (3) by the imaginary part of the electron self-energy. The electron self-energy with an energy of \( E \) is given by

\[
\Sigma(r_1, r_2, E) = i e^2 \int \frac{dk_0}{2\pi} \gamma_\mu S(r_1, r_2, p_0) \gamma_\nu D^{\mu\nu}(r_1, r_2, E - p_0),
\]

where \( S \) and \( D \) are the electron and photon propagators in the magnetic field, respectively. The decay width of the electron at an initial state \( i \) is obtained as

\[
\Gamma_e(i) = -2 \int dr_1 dr_2 \bar{\psi}_i(r_1) \text{Im}\Sigma(r_1, r_2, E_i) \psi_i(r_2)
\]

\[
= \frac{e^2}{8\pi^2} \sum_{f,K,\alpha} \int dq_z dq_T dq_f \frac{dp_fz}{2\pi} \delta(E_i - E_f - e_q) \left| \int dr \bar{\psi}_f(r) A_K^{(\alpha)*}(r) \psi_i(r) \right|^2,
\]

where \( f \) indicates the final electron state. We rewrite the electron wave-function in Eq. (3) and the photon field in Eq. (6) as

\[
\psi_b(r) = \frac{1}{\sqrt{R_z}} \phi_b(r_T) e^{ip_{bz}z} \quad \text{and} \quad A_K^{(\alpha)}(r) = V_K^{(\alpha)}(r_T) e^{i q_z z},
\]

respectively. Because the electron spin is not a good quantum number in the relativistic framework, we make the average of the strength in Eq. (8) for the initial spin and the summation for the final spin. We finally obtain the decay width of

\[
\frac{d\Gamma_{if}^{(a)}}{dp_fz} = \frac{d\Gamma_{if}^{(a)}}{dq_z} = \frac{e^2}{8\pi^2} \left| \int dr_T \bar{\phi}_f(r_T) V_K^{(a)*}(r_T) \phi_i(r_T) \right|^2.
\]
To discuss clearly the quantum effect, we calculate numerically the decay widths for electrons under strong magnetic fields such as $10^{12}$ G and $10^{13}$ G, which were found in astrophysical objects as discussed later. Figure 2 shows the amplitudes of the initial and final electrons ($n_i = n_f = 0$) and the radiative photon as a function of the radius. The initial and final electron orbitals have annual structures and their amplitudes at the center are zero. The diameter of the electron motion decreases after a photon emission, because of the loss of the total angular momentum and energy. For the radiation for $K = 1$, the wave-function has the peak at the center, whereas the radiation for $K = 2$ has a helical structure and its amplitude at the center is zero.

Figure 3 shows the decay widths of electrons as a function of $K$ using Eq. (10). The results are multiplied with the gamma factor along the $z$-direction, $\gamma_z = E_i/\sqrt{2eBL_i + m_e^2}$, so that the results are independent of the $z$-component of the initial electron momentum of $p_{iz}$ when $|p_{iz}| \gg m_e$. In the low $K$ region, the decay widths to the final electron states with $n_f = 0$ dominate. For $n_f = 0$, the decay width for the radiation with $K = 1$ is the largest and the decay width decreases as $K$ increases. The decay width depends also on the strength of magnetic fields. As the strength becomes stronger, the decay widths for all modes increase and the fraction of the radiations with $K \geq 2$ increases. Furthermore, the fraction of the decay widths with $K \geq 2$ increases with increasing the initial electron angular momentum, which is proportional to the diameter of the electron spiral motion. As stated previously, a state with $n_f \geq 1$ corresponds to an electron recoil of which the spiral motion axis is different from the initial axis. In Fig. 3(a,c,d) the decay widths to the states with $n_f \geq 1$ have the peaks at relatively high $K$ values. This indicates that, when an emitted photon brings a relatively large angular momentum, the electron is recoiled.

The circular polarization is correlated with the helicity. The wave-function of an emitted photon is the eigen-state for the state-1 or state-2 [recall, Eq. (6)]. The result that the state-1 dominates for all modes means that the fractions of states of $h = \pm 1$ are approximately identical and hence the degree of the circular polarization is low. This is because the momentum of the photon is not parallel to the vortex propagating axis. As the angle between the momentum and the propagating axis decreases, the fraction of the state-2 increases. If both axes are parallel, the component with $h = -1$ are canceled out and $h = +1$ photons dominate, corresponding to circular polarization.

In Fig. 4 we present the momentum distribution of radiative photons, which are integrated over all the radiation angle. The possible momentum of an emitted photon with $K$ is limited by

$$\frac{-K}{2L_i + m_e^2/eB} (E_i - p_{iz}) \leq q_z \leq \frac{K}{2L_i + m_e^2/eB} (E_i + p_{iz}).$$

(11)
The average energy of photons increases as $K$ increases and the maximum (minimum) energy is proportional to $K$. Thus, at relatively high energies, photon vortices with large angular momenta are generated. We note that there is a discontinuity at $q_z = 4.1$ MeV/c for the total spectrum. This originates from the fact that only the radiation with $K = 1$ has a component of the constant decay width against the energy, which originates from $J_0(0) = 1$ with $q_T = 0$. When a magnetic field strength is weak and the Landau level density of states is high, this discontinuity shifts to zero energy.

The generation of light vortices in the universe has been also discussed [33–36]. For example, it is suggested that photon vortices could be created around rotating black holes [36]. We pointed out a possibility [37] that Hermite-Gaussian wave $\gamma$ rays are generated by high-order harmonic radiations in $\gamma$-ray bursts (GRBs), based upon the previous study [18]. The present result clearly shows that photon vortices could be generated in various astrophysical environments with strong magnetic fields in the universe. Furthermore, photon vortices dominate, in principle, under strong magnetic fields as $10^{13}$ G, which was found in astrophysical objects such as magnetars [38]. The magnetars are magnetized neutron stars that are sources for soft $\gamma$ repeaters and anomalous X-ray pulsars [38] and central engines of GRBs [39]. The observation of high linearly polarized light from a GRB indicates a magnetized baryonic jet with large-scale uniform strong magnetic fields [40]. Even if a magnetic field is inhomogeneous, it is considered to be locally uniformed compared with electron spiral motion diameters. Thus, it is expected that photon vortices are predominantly generated in strong magnetic fields. The fact that polarized X/$\gamma$ rays were observed in detectors onboard satellites and interplanetary space explorer [40–43] suggests that photon vortices could be also observed near the earth. A method to measure LG light at optical wavelengths from astrophysical objects has been proposed [35]. To measure the LG $\gamma$-rays with energies higher than several ten keV, we have proposed the use of Compton scattering [6]. Because the Bessel function has a helical structure similar to the LG function, the measurements of Bessel wave photons are probably possible in similar manners.

In the present study, we have obtained the result that the $k$-th harmonic photon from a spiral moving electron under a uniform magnetic field has the Bessel wave-function as the eigen-state of zTAM of $k$. This is consistent with the previous result in classical electromagnetism [17]. Furthermore, the present calculation is improved from Ref. [17] at following points. First, we have obtained the wave-function for the radiative photon vortices and the decay widths using Landau quantization. Second, the loss of the total angular momentum of the initial electron is taken into account to obtain more exact result. Third, the present calculation can be applied to
strong magnetic fields up to $10^{13}$ G as well as weak fields. Fourth, we consider the recoil of an electron and thereby obtain the final electron states, of which the spiral motion axis shifts from the initial axis. Fifth, we have found that the discontinuity in the energy spectrum for the emitted photons, and this appears near zero energy for weak magnetic fields. The present study shows a fundamental process to generate directly photon vortices, and it is expected to develop photon vortex sources based upon this study in near future. It is also shown that photon vortices are predominantly produced under strong magnetic fields, which are found in astrophysical objects as magnetars.

![Diagram of electron motion](image)

**FIG. 1.** Coordinate in the present study. We assume the uniform dipole magnetic field along the $z$-axis. The electron is in the spiral moving along the $z$-axis.

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$L_i = 10, \rho_z = 50\text{MeV at } B = 10^{13}\text{G}$

![Graph showing squared amplitudes of initial and final electrons and emitted photon](image)

FIG. 2. The squared amplitudes of the initial and final electrons, $|\psi_i|^2$ (solid lines) and $|\psi_f|^2$ (dashed lines), and that of the emitted photon, $|A_K^{(1)}|^2$ (dotted lines), versus $\sqrt{eB \, r_T}$ [fm].

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FIG. 3. Decay widths of electrons with $n_i = 0$ as a function of zTAM of an emitted photon when the final electron Landau number is fixed. The initial orbital angular momenta are $L_i = 10 \ (J_i = 10 - 1/2$) for (a,b) and $L_i = 100 \ (J_i = 100 - 1/2)$ for (c,d). The magnetic field strengths are $B = 10^{13} \text{G}$ for (a,c) and $B = 10^{12} \text{G}$ for (b,d). The dashed and dotted lines represent the results of the photon field at the state-1 and the state-2, respectively, and the solid line indicates a summation of the two contributions.

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FIG. 4. $d\Gamma_e/rdq_z$ with $n_i = 0$, $L_i = 10$, and $p_{iz} = 50$ MeV at the magnetic field strength $B = 10^{13}$G as a function of the $z$-component of the photon momentum, $q_z$. 

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