The Entropy of the Rotating Charged Black Threebrane from a Brane-Antibrane System

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Abstract

We show that a model based on a D3-brane–anti-D3-brane system at finite temperature, proposed previously as a microscopic description of the non-rotating black threebrane of type IIB supergravity arbitrarily far from extremality, can also successfully reproduce the entropy of the rotating threebrane with arbitrary charge (including the neutral case, which corresponds to the Kerr black hole in seven dimensions). Our results appear to confirm in particular the need for a peculiar condition on the energy of the two gases involved in the model, whose physical interpretation remains to be elucidated.

1 Introduction

The problem of finding, through explicit state-counting, a statistical-mechanical interpretation of the Bekenstein-Hawking formula for the entropy of a black hole has long been regarded as a crucial test for any theory that aims at describing gravity microscopically. In recent years, the two main approaches to quantum gravity, string theory and loop quantum gravity, have given strong indications that they can successfully overcome this test (the original references are [1, 2]; for reviews, see, e.g., [3, 4]). In both approaches, however, there are practical as well as conceptual issues that remain to be grappled with.

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In particular, most of the string-theoretic analyses, going back to the pioneering work [1], have concentrated on black holes (or branes) that are either extremal or near-extremal. The far-from-extremal regime, which includes in particular the neutral (Schwarzschild) black hole, has proven to be more challenging. In the past, it has been modelled in terms of a highly excited fundamental string [5] (as a particular instance of the correspondence principle [6]), and in the matrix theory [7] context [8]. In these two approaches the microscopic state-counting leads to rough agreement with the black hole entropy, a result that is clearly non-trivial, but falls short of providing a fully quantitative account of the black hole thermodynamics.

A third approach, related to matrix theory, makes use of a boost and various dualities to map the far-from-extremal configuration onto a near-extremal one [9], and is capable of producing quantitative agreement. In light of the analysis of the boost given in [10], however, the validity of the mapping procedure is not altogether clear (see for instance [11]).

A fourth approach invokes the gauge/gravity correspondence [12, 13] to relate a specific non-asymptotically-flat black hole to the quantum mechanics of D0-branes. Remarkably, in this context it is possible to extract directly from a mean-field approximation to the strongly-coupled gauge theory not only the form of the non-extremal supergravity entropy, but also geometric properties such as the location of the horizon [14]. Based on the results of this line of work, an effective description for a large class of black holes (including the Schwarzschild case) has been proposed, in terms of a gas of non-interacting quasi-particles [15] (a similar picture has been considered in [16]).

A couple of years ago it was shown that the entropy of the black threebrane of Type IIB supergravity and the twobrane and fivebrane of eleven-dimensional supergravity arbitrarily far from extremality (including in particular the Schwarzschild black hole in seven, nine, and six spacetime dimensions) can be reproduced employing microscopic models based on branes and antibranes [17, 18]. (A different model for the Schwarzschild black hole, based on Euclidean brane-antibranes, has been considered in [22].) Already in [19] it had been noted that the entropy of the non-extremal D1-D5 system could be rewritten in a form suggestive of a model involving branes and antibranes, an observation subsequently generalized to other systems in [20]. The understanding of D-brane–anti-D-brane systems at that time was however insufficient to attempt a direct formulation of such a model. In the past few years this situation has changed completely; starting with [21] we have gained quantitative control over the physics of D-D systems. Indeed, the model proposed in [17] emerged naturally from a study, carried out in the same work, of the properties of D-D systems at finite temperature (other such studies may be found in [23]). The brane-antibrane model is able to successfully account for various properties of the black brane; in particular, the negative specific heat and pressure of the system find a natural explanation in terms of brane-antibrane annihilation.

In this paper we will subject the brane-antibrane model of [17] to an additional non-trivial test, by exploring the possibility of describing with it the rotating black threebrane. The addition of rotation is interesting from the theoretical perspective
because it results in a significant modification of the functional form of the supergravity entropy. It is of course also of interest from the phenomenological viewpoint, because it allows us to study the Kerr black hole, thus bringing us closer to the actual black holes that are believed to exist in our universe. Section 2 contains our calculations, starting in 2.1 with a summary of the model of [17] and an analysis of the neutral rotating threebrane, and ending in 2.2 with the generalization to the case with arbitrary charge. Our conclusions are presented in Section 3.

2 Entropy Determination

We take from [24] (see also [25]) the rotating black threebrane solution of Type IIB supergravity, with all but one of the rotation parameters set to zero. The metric takes the form

$$ds^2 = \frac{1}{\sqrt{f}}(-h dt^2 + d\vec{x}^2) + \sqrt{f} \left[ \frac{dr^2}{r^4\Delta} \frac{2lr_0^4 \cosh \alpha}{r^4 f} \sin^2 \theta d\theta d\phi + r^2 (\Delta d\theta^2 + \tilde{\Delta} \sin^2 \theta d\phi^2 + \cos^2 \theta d\Omega_3^2) \right],$$

with

$$f = 1 + \frac{r_0^4 \sinh^2 \alpha}{r^4 \Delta},$$
$$\Delta = 1 + \frac{l^2 \cos^2 \theta}{r^2},$$
$$\tilde{\Delta} = 1 + \frac{l^2}{r^2} + \frac{r_0^4 l^2 \sin^2 \theta}{r^6 \Delta f},$$
$$h = 1 - \frac{r_0^4}{r^4 \Delta},$$
$$\tilde{h} = \frac{1}{\Delta} \left(1 + \frac{l^2}{r^2} - \frac{r_0^4}{r^4}\right).$$

This geometry has an ADM mass density

$$m_{SG} \equiv \frac{M_{SG}}{V} = \frac{\pi^3}{\kappa^2 r_0^4} \left( \cosh 2\alpha + \frac{3}{2} \right),$$

and an angular momentum density (conjugate to $\phi$)

$$j_{SG} \equiv \frac{J_{SG}}{V} = \frac{\pi^3}{\kappa^2} lr_0^4 \cosh \alpha.$$

There is an event horizon at $r_H = \sqrt{r_0^4 + l^4/4 - l^2/2}$, whose area corresponds to an entropy density

$$s_{SG} \equiv \frac{A_H/4G_N}{V} = \frac{2\pi^2}{\kappa^2} r_0^4 \sqrt{r_0^4 + l^4/4 - l^2/2} \cosh \alpha.$$
In addition, the solution involves a RR five-form, associated with a charge

\[ Q_{SG} = \frac{\pi^{5/2}}{\kappa} r_0^4 \sinh 2\alpha \]  

(4)

### 2.1 Neutral case

As can be seen from (4), the neutral case corresponds to setting \( \alpha = 0 \). The mass (1) and angular momentum (2) then reduce to

\[ m_{SG} = \frac{5\pi^3}{2\kappa^2} r_0^4, \quad j_{SG} = \frac{\pi^3}{\kappa^2} l_0^4. \]  

(5)

Using these expressions in (3) we can eliminate \( r_0 \) and \( l_0 \) in favor of \( m_{SG} \) and \( j_{SG} \), to obtain the supergravity entropy as a function of the mass and angular momentum of the brane,

\[ s_{SG} = \sqrt{2^{9/4}\pi^{-5/4}\kappa^{1/4} m_{SG}^{5/4} + 4\pi^4 j_{SG}^4} - 2\pi^2 j_{SG}^2. \]  

(6)

For small \( j_{SG} \), this implies

\[ s_{SG} = 2^{9/4} 5^{-3/4} \pi^{1/4} \kappa^{1/2} m_{SG}^{5/4} \left[ 1 - 2^{-9/2} 5^{5/2} \pi^{3/2} \frac{j_{SG}^2}{\kappa m_{SG}^{5/2}} + 2^{-10} 5^{3} \pi^{3} \frac{j_{SG}^4}{\kappa^2 m_{SG}^5} + O(j_{SG}^6) \right]. \]  

(7)

It was shown in [17] that the first term in (7), the entropy for the non-rotating neutral black threebrane, can be reproduced with a field-theoretic model based on a system of \( N \) D3-branes, \( \bar{N} = N \) anti-D3-branes, and two gases of \( \mathcal{N} = 4 \) SYM particles, arising respectively from the massless modes of 3-3 and 3-3 open strings. The total mass density of the system is then

\[ m_{FT} = 2N\tau_3 + e, \]  

(8)

with \( \tau_3 = \sqrt{\pi/\kappa} \) the D3-brane tension and \( e \) the total energy (density) available to the gases. The entropy of the system is entirely due to the \( \mathcal{N} = 4 \) SYM gases. In the regime of interest, \( g_s N \gg 1 \) (where the supergravity solution is reliable), SYM is strongly-coupled, and so the entropy of the two gases cannot be determined perturbatively. It is however known [26, 27] via the AdS/CFT correspondence [12],

\[ s_{FT} = 2^{9/4} 3^{-3/4} \pi^{1/2} (e/2)^{3/4} \sqrt{N}, \]  

(9)

where we have taken into account the fact that each gas should carry half of the available energy \( e \). Equation (8) is then used in (9) to eliminate \( e \) in favor of \( N \), and the value of \( N \) (the number of D-\( \bar{D} \) pairs) determined by maximizing \( s_{FT} \) at fixed \( m_{FT} \). This leads to

\[ N = \frac{m_{FT}}{5\tau_3} \iff e = \frac{3}{5} m_{FT}. \]  

(10)
As a consistency check, this gas energy corresponds to a temperature \( T \gg 1/\sqrt{g_sN} \), which as explained in \[17\] is precisely the condition for the D-\(D\) pairs not to annihilate (i.e., for the effective tachyon potential to have a \textit{minimum} at the open string vacuum).

Using (10) and (8) in (9), it was found in \[17\] that the entropy of the field-theoretic model exactly coincides with that of the supergravity solution (the first term in (7)), except for a factor of \( 2^{3/4} \). Given the form of (9), this is equivalent to saying that the supergravity entropy behaves as if each of the gases somehow had access to the \textit{total} energy \( e \), rather than the half, \( e/2 \), that we would naturally assign to it. Remarkably, it was found in \[17\] that this same model could equally well reproduce the entropy of the charged threebrane (where \( N \neq \bar{N} \)), and similar models (based respectively on M2-M2 and M5-M5 pairs)) could account for the entropy of the twobrane and fivebrane solutions of eleven-dimensional supergravity. In all cases there was perfect agreement with the functional form of the entropy, and a numerical discrepancy that could be resolved under the uniform (albeit bizarre) assumption that each of the two gases carries in some sense the total energy \( e \).\(^1\)

It is interesting then to test the microscopic model of \[17\] (and in particular, the peculiar associated condition on the energies) by turning on a small amount \( j_{SG} \) of threebrane angular momentum. In the field-theoretic description this corresponds to considering SYM gases that carry a total R-symmetry charge density \( j_{FT} = j_{SG} \). The entropy of a charged \( \mathcal{N} = 4 \) SYM gas at strong coupling is again predicted by the AdS/CFT correspondence: for a given energy, charge, and gauge group size, it is \[24, 25\]

\[
s(e, j, N) = \frac{2^{5/4}3^{-3/4}\pi N e^{3/4}}{\sqrt{1 + \chi + \sqrt{\chi}}} , \quad \chi = \frac{27\pi^2 j_{FT}^4}{8N^2e^3} . \tag{11}
\]

In our microscopic model there are two gases, one on the D3-branes and the other on the anti-D3-branes. Given that \( N = \bar{N} \), by symmetry we expect each gas to contribute half the total charge, \( j = j_{FT}/2 \). For the energy one would expect a similar split; but, in accordance with the findings of \[17\] summarized above, we instead take the energy of each gas to be the \textit{total} energy \( e \) given by \[8\]. The entropy of our system is thus

\[
s_{FT} = \frac{2^{9/4}3^{-3/4}\pi N e^{3/4}}{\sqrt{1 + \chi + \sqrt{\chi}}} , \quad \chi = \frac{27\pi^2 j_{FT}^4}{2^7N^2e^3} . \tag{12}
\]

For small \( j_{FT} \) this can be expanded into

\[
s_{FT} = 2^{9/4}3^{-3/4}\pi N e^{3/4} \left( 1 - \frac{\sqrt{\chi}}{2} + \frac{\chi}{8} + \ldots \right) , \tag{13}
\]

\(^1\)In \[17\] it was noted that, to reproduce the functional form of the charged threebrane entropy, one had to assume in any case that both gases have the \textit{same} energy. There then remained the numerical \( 2^{3/4} \) discrepancy. Clearly we do not need the equality of energies as an additional assumption if we interpret the supergravity expression for the entropy as implying that each gas has access to the total energy.
an expansion of the same form as the supergravity expression (7), which seems encouraging. To perform a detailed comparison, we first need to fix the number of D-\(\overline{D}\) pairs as in [17], by maximizing the entropy with respect to \(N\). At this order, the equilibrium value of \(N\) is found to be the same as in the non-rotating case, Eq. (10). Using this and (8) in (13) we obtain

\[
s_{FT} = 2^{9/4} 5^{-5/4} \pi^{1/4} \kappa^{1/2} m_{FT}^{5/4} \left[ 1 - 2^{-9/2} 5^{5/2} \pi^{3/2} \frac{j_{FT}^2}{k m_{FT}^{5/2}} + 2^{-10} 5^{\frac{5}{2}} \pi^{\frac{3}{2}} \frac{j_{FT}^4}{k^2 m_{FT}^{5}} + O(j_{FT}^6) \right],
\]

which correctly reproduces the supergravity result (7)

Having found agreement between supergravity and the microscopic model up to order \(j^4\), one is encouraged to determine whether the agreement extends to higher order, perhaps even to the entire expression (6). So we maximize (12) with respect to \(N\); remarkably, the equilibrium value of \(N\) is again (10). Plugging this back into (12), we obtain a microscopic entropy

\[
s_{FT} = \frac{2^{45} 5^{-5/4} \pi^{1/4} \kappa m_{FT}^{5/2}}{\sqrt{2^7 \kappa^2 m_{FT}^{5} + 5^5 \pi^3 j_{FT}^4 + 5^5/2 \pi^3/2 j_{FT}^2}}.
\]

At first sight this appears to differ from the supergravity result (6), but upon multiplying the numerator and denominator by \(\sqrt{2^7 \kappa^2 m_{FT}^{5} + 5^5 \pi^3 j_{FT}^4 - 5^5/2 \pi^3/2 j_{FT}^2}\), we obtain complete agreement between the two expressions!

### 2.2 Charged case

Now we extend our analysis to the charged case, \(\alpha \neq 0\). We could in principle use again (1), (2) and (4) in (3) to obtain an expression for the supergravity entropy as a function of the mass, angular momentum and charge of the threebrane. It is however easier to keep \(\alpha, r_0\) and \(l\) in the analysis and attempt to reproduce (3) directly.

In the microscopic model we now have

\[
Q_{FT} = N - \bar{N} \neq 0,
\]

and as before, the actual values of \(N\) and \(\bar{N}\) should be determined by maximizing the entropy. Given our experience with the neutral case, it is natural to expect that the equilibrium values of \(N, \bar{N}\) will again coincide with those found in [17] for the non-rotating system. We thus postulate (and later check) that, at equilibrium,

\[
N = \frac{\pi^{5/2}}{2\kappa} r_0^4 e^{2\alpha}, \quad \bar{N} = \frac{\pi^{5/2}}{2\kappa} r_0^4 e^{-2\alpha},
\]

which is indeed the most natural way to split the supergravity expression (4) into two parts. Comparing the mass of the microscopic system,

\[
m_{FT} = (N + \bar{N}) \tau_3 + e,
\]
against the supergravity expression \(1\), we then infer that (as in the non-rotating case) the energy of the gases should be identified with

\[
e = \frac{3\pi^3}{2\kappa^2} r_0^4 .
\]

As before, we will assume that this total energy is somehow available to each of the two gases. The final ingredient is the R-charge density \(j_{FT} = j_{SG}\), which should be split in some way between the two gases:

\[
j_{FT} = j + \bar{j} .
\]

Given the form of \(2\), it is natural to propose that, at equilibrium,

\[
j = \frac{\pi^3}{2\kappa^2} lr_0^4 e^\alpha , \quad \bar{j} = \frac{\pi^3}{2\kappa^2} lr_0^4 e^{-\alpha} .
\]

Based on the AdS/CFT prediction \(11\) for the entropy of a strongly-coupled charged SYM gas, the entropy of our microscopic system is

\[
s_{FT} = \frac{2^{5/4} \sqrt{\frac{\pi N}{3}} e^{3/4}}{\sqrt{\sqrt{1 + \chi} + \sqrt{\chi}}} + \frac{2^{5/4} 3^{-3/4} \sqrt{\pi N e^{3/4}}}{\sqrt{\sqrt{1 + \bar{\chi}} + \sqrt{\bar{\chi}}}} ,
\]

where

\[
\chi = \frac{27\pi^2}{8N^2 e^3} , \quad \bar{\chi} = \frac{27\pi^2}{8N^2 e^3} .
\]

Using \(17\), \(19\) and \(21\), one finds that the ratios \(23\) simplify to

\[
\chi = \frac{l^4}{4r_0^4} = \bar{\chi} ,
\]

and the two terms in \(22\) then combine into

\[
s_{FT} = \frac{2\pi^4 \kappa^{-2} r_0^6 \cosh \alpha}{\sqrt{r_0^4 + l^4/4 + l^2/2}}.
\]

Amazingly, multiplying the numerator and denominator by \(\sqrt{r_0^4 + l^4/4 + l^2/2}\), we obtain perfect agreement with the supergravity expression \(3\)! And, perhaps even more remarkably, the assumed values \(17\) and \(21\) can be verified to be precisely the ones that maximize the entropy \(22\) for a fixed charge \(16\) and mass \(18\).

### 3 Conclusions

In this paper we have shown that the D3-D3 model of \[17\], originally proposed as a microscopic description of the non-rotating black threebrane, can also successfully
account for the entropy of the rotating threebrane arbitrarily far from extremality (including in particular the neutral case, which corresponds to a Kerr black hole in seven spacetime dimensions). We regard this as a highly non-trivial test of the model.

For maximal conceptual clarity, it is worth commenting here on the role played by the AdS/CFT correspondence in our calculations. In the microscopic model, the entropy arises from two strongly-coupled $\mathcal{N} = 4$ SYM gases. AdS/CFT is invoked as a tool to determine this entropy, and the result, after a maximization procedure, is then compared to the supergravity entropy. Since the AdS/CFT prediction for the SYM entropy is deduced from supergravity, our calculation might at first glance appear to be circular. The point to be emphasized, however, is that AdS/CFT uses only the near-extremal threebrane, whereas our analysis extends arbitrarily far away from extremality. In addition, the number of D3-branes in the model is not determined uniquely by the charge of the system, but is instead fixed thermodynamically. At the very least, then, the agreement found here and in [17] is a rather non-trivial property of the supergravity formulae, implying a relation between the behavior of the threebrane entropy in two completely different regimes. Moreover, this property appears to have some degree of universality, for it applies not only to the (rotating or not) Type IIB threebrane, but also to the twobrane and fivebrane of eleven-dimensional supergravity [17], and perhaps to other systems as well [19] [20]. Even from this perspective, then, the agreement would call for an explanation.

A particularly significant aspect of our results relates to what in [17] was regarded as a numerical discrepancy. As explained in Section 2.1 of the present paper, in [17] it was found that the entropy $s_{\text{FT}}$ of the D-D model was in perfect agreement with the functional form of the supergravity entropy $s_{\text{SG}}$; but, under the physically self-evident condition that the total energy $e$ available to the two SYM gases be split between them, it was a factor of $2^{3/4}$ too small, $s_{\text{SG}} = 2^{3/4}s_{\text{FT}}$. Since $s_{\text{FT}} \propto e^{3/4}$, one way to phrase this disagreement was that the supergravity entropy behaved as if each gas somehow had access to the total energy $e$. In the rotating case analyzed in this paper, the entropy of the model depends on the energy $e$ of the gases not just through a simple factor of $e^{3/4}$, but through the rather complicated function seen in (22). It is quite remarkable then that, to reproduce the supergravity entropy, one again requires nothing more and nothing less than the assumption that each gas carries the total energy $e$. Needless to say, the most important pending task in connection with the brane-antibrane model analyzed in [17] and the present paper is to find a physically reasonable explanation for this rather bizarre result.

As noted already in [17], it would also be interesting to test the model by computing quantities other than the entropy. Beyond that, there remain of course the ever-present questions about the precise way in which the geometric and causal properties of the black brane are encoded in the field-theoretic description.

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2 The numerical discrepancies found in [17] for the twobrane and fivebrane cases were also of the form $s_{\text{SG}} = 2p/(p+1)s_{\text{FT}}$, with $p$ the dimension of the brane, and thus lent themselves to exactly the same interpretation.
Note Added: After this paper was submitted to the arXiv preprint server, we learned that overlapping results have been independently obtained in [28].

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