Research Article

$H_\infty$ Controller Design for an Observer-Based Modified Repetitive-Control System

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This paper presents a method of designing a state-observer based modified repetitive-control system that provides a given $H_\infty$ level of disturbance attenuation for a class of strictly proper linear plants. Since the time delay in a repetitive controller can be treated as a kind of disturbance, we convert the system design problem into a standard state-feedback $H_\infty$ control problem for a linear time-invariant system. The Lyapunov functional and the singular-value decomposition of the output matrix are used to derive a linear-matrix-inequality (LMI) based design algorithm for the parameters of the feedback controller and the state-observer. A numerical example demonstrates the validity of the method.

1. Introduction

In control engineering practice, many systems exhibit repetitive behavior, such as a robot manipulator, a hard disk drive, and many other servo systems. Repetitive control [1], or RC for short, has proven to be a useful control strategy for a system with a periodic reference input and/or disturbance signal [2–4]. The distinguishing feature of RC is that it contains a pure-delay positive-feedback loop, which is the internal model of a periodic signal. For a given periodic reference input, a repetitive controller gradually reduces the tracking error through repeated learning actions [5], which involves adding the control input of the previous period to that of the present period to regulate the present control input. This theoretically guarantees gradual improvement and finally eliminates any tracking error and provides very precise control, which is a chief characteristic of the human learning process.

From the standpoint of system theory, an RC system (RCS) is a neutral-type delay system. Asymptotic tracking and stabilization of the control system are possible only when the relative degree of the compensated plant is zero [5]. To use RC on a strictly proper plant, that is, the case that most control engineering applications deal with, the repetitive controller has to be modified by the insertion of a low-pass filter into the time-delay feedback line. The resulting system is called a modified RCS (MRCS). Since a modified repetitive controller is just an approximate model of a periodic signal, there exists a steady-state tracking error; that is, in an MRCS, the low-pass filter relaxes the stabilization condition but degrades the tracking precision [6].

RC is similar to iterative learning control (ILC), which is another well-known method that makes use of previous control trials. However, as pointed out by [7–9] and others, there are significant differences between them. First of all, the initial state of a period is different. In an RCS, the state at the beginning of a period is the same as the final state in the previous period. However, in an ILC system (ILCS), the reference trajectory is defined over a finite time interval, and the state of an ILCS is usually reset after a trial. In the literatures of ILC, the initial or boundary conditions, that is, the initial state on each trial and the initial trial profile, are commonly taken to be zero. The difference in initial-condition resetting leads to the different analysis techniques and results.

One problem with an RCS is that the improved disturbance rejection at the periodic frequency and its harmonics is achieved at the expense of degraded system sensitivity at intermediate frequencies. In other words, an RCS cannot reject, and may even amplify, an aperiodic disturbance.
Various solutions have been presented in the literatures addressing this problem. Time-varying and adaptive RC was proposed in [10]. Kim et al. presented a design method of a two-parameter robust RCS that used discrete-time m-synthesis and $H_{\infty}$ control to reject both periodic and aperiodic disturbances [11]. However, the high order of the controller makes it difficult to implement. She et al. devised an $H_{\infty}$ control method to design the parameters of the stabilization controller for a robust MRCS with time-varying uncertainties [12]. But it only considers the case that there is no disturbance input to the controlled plant. In addition, the full states of a plant are needed to design the state-feedback controller. In many practical applications, the complete state of a plant is not available because of running cost and difficulty of installation.

Zhou et al. presented a method based on two-dimensional system theory of designing a robust observer based MRCS [13]. But it only considered the robust stability of the system. To enable RC method to handle a larger class of systems, this paper presents a method of designing an MRCS with a prescribed bound on disturbance attenuation for a class of strictly proper linear systems. First, we present a configuration of an observer based MRCS and formulate the design problem as an equivalent $H_{\infty}$ state-feedback control problem. Then, combining the Lyapunov stability theory and the singular-value decomposition (SVD) of the output matrix, we derive a linear-matrix-inequality (LMI) based sufficient stability condition. The condition can be used directly to design the parameters of the feedback controller and the state-observer. Compared to the one in [13] and other RC methods, the advantage of this method is that it takes into consideration the transient performance of the MRCS. Finally, a numerical example demonstrates the validity of the method.

Throughout this paper, $L_2[0, +\infty)$ is the linear space of square integrable functions from $[0, +\infty)$ to $C^\mathbb{R};\|G(\omega)\|_{\infty} := \sup_{\omega \in [0, +\infty]} |G(j\omega)|$ is the $H_{\infty}$ norm of a transfer function $G(s)$, and $^*$ denotes the transpose of a block entry in a matrix.

### 2. Problem Description

Consider the MRCS in Figure 1. The compensated single-input, single-output (SISO) plant is

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t) + B_p w(t),$$

$$y(t) = C_p x_p(t),$$

where $x_p(t) \in \mathbb{R}^n$ is the state of the plant; $u(t), y(t) \in \mathbb{R}$ are the control input and output, respectively; and $w(t) \in L_2[0, +\infty)$ is the disturbance input. Setting $B_p = 0$ adds the disturbance to the system, and setting $B_p = 0$ removes it. $A_p, B_p, C_p$ are real constant matrices. Assume that $(A_p, B_p)$ is controllable and $(C_p, A_p)$ is observable, which are standard for a servo system.

In Figure 1, $r(t)$ is a periodic reference signal with a period of $T$, and

$$e(t) = r(t) - y(t)$$

is the tracking error. The modified repetitive controller is

$$C_{MR}(s) = \frac{1}{1 - q(s) e^{-sT}}.$$
\( q(s) \) is a first-order low-pass filter that ensures the stability of the control system. Without loss of generality, we assume that \( q(s) \) is a first-order filter; that is
\[
q(s) = \frac{\omega_c}{s + \omega_c},
\] (4)
where \( \omega_c \) is the cutoff angular frequency of the filter. The inverse Laplace-transform of (3) yields
\[
v(t) = -\omega_c v(t) + \omega_c v(t - T) + \omega_c e(t) + \dot{e}(t),
\] (5)
where the output, \( v(t) \), of \( C_{MBR}(s) \) is chosen to be the state of the modified repetitive controller.
The following state-observer is used to reproduce the state of the plant
\[
\dot{x}_p(t) = A_p x_p(t) + B_p u(t) + L [y(t) - \ddot{y}(t)]
\] (6)
where \( L \) is the observer gain.
The error between the states of the actual plant and the observer is
\[
x_e(t) = x_p(t) - x_e(t).
\] (7)
Thus, from (1) and (6), we have
\[
x_e(t) = (A_p - LC_p) x_e(t) - B_w w(t).
\] (8)
A linear control law based on the states of the observer and the repetitive controller is
\[
u(t) = K_e v(t) + K_p \ddot{x}_p(t), \quad K_e \in \mathbb{R}, \quad K_p \in \mathbb{R}^{1 \times n}.
\] (9)
where \( K_e \) is the feedback gain of the repetitive controller and \( K_p \) is the reconstructed state-feedback gain.
Since the stability of the system does not depend on an exogenous signal, we set \( r(t) = 0 \). Choose \( x_e(t), x_e(t), \) and \( v(t) \) to be the state variables of the MRCS in Figure 1. Then, from the dynamic equations (1), (5), (8), and (9) yields the following state equation:
\[
x_e(t) = \dot{x}_p(t) + A_1 x_e(t - T) + B u(t) + \ddot{B} w(t),
\] (10)
where
\[
\dot{x}_e(t) = \begin{bmatrix} x_p^T(t) & x_e^T(t) & v^T(t) \end{bmatrix},
\]
\[
A = \begin{bmatrix} A_p & 0 & 0 \\ -\omega_c C_p & A_p - LC_p & 0 \\ 0 & 0 & -\omega_c_c \end{bmatrix},
\] (11)
\[
A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} B_p \\ 0 \\ 0 \end{bmatrix},
\]
\[
\ddot{B} = \begin{bmatrix} B_w \\ -B_c \\ -C_p B_w \end{bmatrix}.
\]
Thus, from (1) and (6), we have
\[
\ddot{x}(t) = A x(t) + B_1 w_1(t) + B_2 u(t),
\]
\[
z_1(t) = C_1 x(t),
\] (12)
where
\[
w_1(t) = \begin{bmatrix} \omega^T(t) & v^T(t - T) \end{bmatrix}^T, \quad y_1(t) = \begin{bmatrix} x_p^T(t) & 0 & v^T(t) \end{bmatrix}^T,
\]
\[
B_1 = \begin{bmatrix} B_w \\ -B_c \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} B_p \\ 0 \\ -C_p B_p \end{bmatrix},
\]
\[
C_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\] (13)
Substituting the control input (9) into (12) yields a representation of the closed-loop system in Figure 1:
\[
x_e(t) = A x_e(t) + A_1 x_e(t - T) + B u(t) + \ddot{B} w(t),
\]
\[
z(t) = C_1 x(t),
\] (14)
where
\[
K = [K_p \quad K_e \quad K_c].
\] (15)
This paper considers the following \( H_{\infty} \) disturbance attenuation problem.
Design suitable control gains, \( K_e \) and \( K_p \), and the state-observer gain, \( L \), such that
\[
\begin{align*}
(1) \text{ the MRCS in Figure 1 is internally stable; and} \\
(2) \text{ the } H_{\infty} \text{ norm from } w_1(t) \text{ to } z(t) \text{ is less than } \gamma; \quad \text{that is,}
\end{align*}
\[
\|G_{wz}(s)\|_{\infty} = \|C_1 [s I - (A + B_2 K)]^{-1} B_1\|_{\infty} < \gamma,
\] (16)
where \( \gamma \) is a positive number.
3. Design of the $H_{\infty}$ Controller

In this section we construct an LMI-based design algorithm for the $H_{\infty}$ state-feedback controller by employing a Lyapunov functional and the SVD of the output matrix.

Definition 1 (see [14]). The matrix $\Pi$ has full row rank (rank($\Pi$) = $p$). The SVD of $\Pi$ is

$$\Pi = U [S \ 0] V^T,$$

where $S \in \mathbb{R}^{p \times p}$ is a diagonal matrix with positive, diagonal elements in decreasing order; $0 \in \mathbb{R}^{p \times (n-p)}$ is a zero matrix; and $U \in \mathbb{R}^{p \times p}$ and $V \in \mathbb{R}^{n \times n}$ are unitary matrices.

The following lemma presents an equivalent condition for matrix equation

$$\Pi X = X \Pi.$$  

Lemma 2 (see [15]). For a given $\Pi \in \mathbb{R}^{p \times n}$ with rank($\Pi$) = $p$, if $X \in \mathbb{R}^{n \times n}$ is a symmetric matrix, then there exists a matrix, $\Xi \in \mathbb{R}^{p \times p}$, such that $\Pi X = \Xi \Pi$ holds if and only if

$$X = V \text{diag} \{X_{11}, X_{22}\} V^T,$$

where $X_{11} \in \mathbb{R}^{p \times p}$ and $X_{22} \in \mathbb{R}^{(n-p) \times (n-p)}$.

The following two lemmas are also employed in the derivation of the existence condition for the $H_{\infty}$ controller of the system (14).

Lemma 3 (Schur complement [16]). For any real matrix $\Sigma = \Sigma^T$, the following assertions are equivalent:

1. $\Sigma = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} < 0$;
2. $S_{11} < 0$ and $S_{12} - S_{12}^T S_{22}^{-1} S_{12} < 0$;
3. $S_{22} < 0$ and $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 4 (see [17]). For the system

$$\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t), \\
z(t) &= Cx(t) + Du(t),
\end{align*}$$

(20)

the following assertions are equivalent.

1. $A$ is stable, and the $H_{\infty}$ norm of the transfer function $G_{wz}(s)$, from $w(t)$ to $z(t)$ satisfies $\|G_{wz}(s)\|_{\infty} < \gamma$.
2. There exists a symmetric matrix $P > 0$ such that

$$\begin{bmatrix}
PA + A^T P & PB \\
* & -I
\end{bmatrix} < 0,$$  

(21)

holds.

So, we have the following theorem.

Theorem 5. For a given positive scalar $\gamma$, if there exist symmetric, positive-definite matrices $X_1$, $X_{11}$, $X_{22}$, and $X_3$, and arbitrary matrices $W_1$, $W_2$, $W_3$, and $W_4$ such that the LMI

$$\begin{bmatrix}
\Theta_{11} & B_w W_2 \\
\Theta_{22} & -W_2^T B_w^T C_p \Theta_3 \Theta_3^T C_p^{-1} B_w \omega_c & B_w \omega_c & 0 & 0 & 0 \\
* & -I & 0 & 0 & 0 \\
* & * & -I & 0 & 0 \\
* & * & * & -I & 0 \\
* & * & * & * & -\gamma^2 I
\end{bmatrix} < 0,$$

(22)
holds, where the SVD of the output matrix $C_p$ is

$$C_p = U [S_{0}] V^T,$$  

(23)

$$X_2 = V \text{ diag } \{X_{11}, X_{22}\}V^T,$$

$$\Theta_{11} = A_p X_1 + X_1 A_p^T + B_p W_1 + W_1^T B_p^T,$$

$$\Theta_{13} = B_p W_3 - W_3^T B_p^T C_p^T - \omega_c X_1^T C_p - X_1 A_p^T C_p^T,$$

$$\Theta_{22} = A_p X_2 + X_2 A_p^T - W_4 C_p - C_p^T W_4^T,$$

$$\Theta_{33} = - C_p B_p W_3 - W_3^T B_p^T C_p^T - 2 \omega_c X_3,$$

(24)

then the closed-loop repetitive-control system (14) is asymptotically stable and provides a prescribed $H_\infty$ disturbance attenuation level, $\gamma$. Furthermore, the parameters in control law (15) are

$$K_p = W_1 X_1^{-1}, \quad K_c = W_3 X_3^{-1},$$  

(25)

and the state-observer gain in (6) is

$$L = W_4 U S X_{11}^{-1} S^{-1} V^T.$$  

(26)

Proof. Applying Lemma 2 to (23), there exists

$$\overline{X}_2 = US X_{11} S^{-1} V^T,$$  

(27)

such that

$$C_p X_2 = \overline{X}_2 C_p,$$  

(28)

Thus, the observer gain $L$ in (26) is equal to

$$L = W_4 \overline{X}_2^{-1}.$$  

(29)

Let

$$P_1 = X_1^{-1}, \quad P_2 = X_2^{-1}, \quad P_3 = X_3^{-1}$$

(30)

$$W_1 = K_p X_1, \quad W_2 = K_p X_2,$$

$$W_3 = K_c X_3, \quad W_4 = L \overline{X}_2^2.$$  

Pre- and postmultiplying the matrix on the left-side of LMI (22) by

$$P = \text{ diag } \{P_1, P_2, P_3, I, I, I, I\},$$  

(31)

yields the following matrix inequality that is equivalent to LMI (22)

$$\begin{bmatrix}
\Lambda_{11} & P_1 B_w K_p & \Lambda_{13} & P_1 B_w & 0 & 0 & 0 \\
* & \Lambda_{22} & -K_p^T B_p^T C_p P_3 & P_2 B_w & 0 & 0 & 0 \\
* & * & \Lambda_{33} & -P_3 C_p B_w & P_3 \omega_c & 0 & 1 \\
* & * & * & -I & 0 & 0 & 0 \\
* & * & * & * & -I & 0 & 0 \\
* & * & * & * & * & -\gamma^2 I & 0 \\
* & * & * & * & * & * & -\gamma^2 I \\
\end{bmatrix} < 0,$$  

(32)

Choose a Lyapunov functional candidate to be

$$V(t) = x^T(t) P x(t),$$  

(35)

where

$$P = \text{ diag } \{P_1, P_2, P_3\}.$$  

(36)

Along the time trajectory of (34)

$$\frac{dV(t)}{dt} = 2 x^T(t) P \dot{x}(t) = \xi^T(t) \overline{X}_2 \xi(t),$$  

(37)
where
\[
\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-T) \end{bmatrix}^T,
\]
and \( \Lambda_{11}, \Lambda_{12}, \Lambda_{22}, \Lambda_{33} \) are defined in (33).

From (37), \( \Lambda < 0 \) implies that, for any \( \xi(t) \neq 0, dV(t)/dt < 0 \). Also, \( \Lambda < 0 \) is equivalent to the matrix inequality
\[
\begin{bmatrix}
\Lambda_{11} & \Lambda_{12} & \Lambda_{13} & 0 \\
* & \Lambda_{22} & -K_p^TP_3 & P_3 \\
* & * & \Lambda_{33} & 0 \\
* & * & * & 0 \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix} < 0.
\]

Furthermore, since \( B_w = 0 \), applying Sylvester Criterion [18] and Schur-complement Lemma 3 to (32) yields \( \Lambda < 0 \). So, if LMI (22) holds, then MRCS (34) is asymptotically stable; that is, the closed-loop system (14) is asymptotically stable when \( B_w = 0 \).

Next, we consider the case \( B_w \neq 0 \). Note that closed-loop MRCS (14) is asymptotically stable.

Applying Lemma 4 to (14), it follows that if the matrix inequality
\[
\Omega = \begin{bmatrix}
P(A + B_2K) + (A + B_2K)^TPB_1C_1^T & * & -I & 0 \\
* & \omega_c \omega_c & 0 & 0 \\
* & * & -I & 0 \\
* & * & * & -\gamma^2I \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix} < 0
\]
holds, then the system (14) satisfies the disturbance attenuation performance \( \|G_{wz}(s)\|_{\infty} < \gamma \).

Obviously, from the coefficient matrices and controller parameters in (14), the matrix inequality (40) is equal to (32). Also, LMI (32) is equivalent to (22). So, if LMI (22) holds, then \( \Omega < 0 \). \( \Box \)

Remark 6. Theorem 5 provides an LMI-based sufficient condition for the closed-loop repetitive-control system (14) with a prescribed \( H_\infty \) disturbance-attenuation level \( \gamma \). The condition can be used directly to design the parameters of the controller and the state-observer in Figure 1.

In addition, from Theorem 5, we obtain a sufficient condition for an \( H_\infty \) disturbance-attenuation level for the system (1) under the state-feedback-based control law:
\[
u(k) = K_c x(t) + K_p x_p(k).
\]

The representation of the corresponding closed-loop system in Figure 1 is
\[
\dot{x}(t) = (\bar{A} + \bar{B}_2 \bar{K}) x(t) + \bar{B}_1 w_1(t),
\]
\[
\ddot{x}(t) = \bar{C}_1 x(t),
\]
where
\[
\bar{A} = \begin{bmatrix}
\bar{A}_{11} & \bar{A}_{12} & \bar{A}_{13} & 0 \\
* & \bar{A}_{22} & -K_p^TP_3 & P_3 \\
* & * & \bar{A}_{33} & 0 \\
* & * & * & 0 \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix},
\]
\[
\bar{B}_1 = \begin{bmatrix}
B_{p} & 0 \\
-\omega_c C_p - C_p A_p & -\omega_c \\
0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\bar{B}_2 = \begin{bmatrix}
B_w \\
-\omega_c B_p \\
0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\bar{C}_1 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\bar{K} = \begin{bmatrix}
K_p & K_c \\
\end{bmatrix}.
\]

4. Numerical Example

In this section, we apply our method to the position control of a chuck-workpiece system with a three-jaw chuck (Figure 4, [19]).

Assume that the parameters of plant (1) are
\[
A_p = \begin{bmatrix}
0 & 1 \\
-1 & -1 \\
\end{bmatrix}, 
B_p = B_w = \begin{bmatrix}
0.5 \\
0 \\
\end{bmatrix},
C_p = \begin{bmatrix}
1 & 0 \\
\end{bmatrix}.
\]
We consider the problem of tracking the reference input
\[ r(t) = \sin(2\pi t), \] (48)
and rejecting the aperiodic disturbance
\[ w = \begin{cases} 
0, & t < 5 \\
\sin(2\pi t) + 5 \sin(\pi t) + 5 \sin(3\pi t) + 5(\tan(t-10) - \tan(t-9)), & 5 \leq t \leq 15 \\
0, & t > 15, \end{cases} \] (49)
below the level \( \gamma = 0.2 \).

Thus, the repetition period is \( T = 1 \).

Carrying out the design procedure in Theorem 5, yields the corresponding control gains
\[ K_e = 4.0882 \times 10^4, \quad K_p = [-63.5735 \quad -1.9221], \] (50)
and the state-observer gain
\[ L = [0.5003 \quad -0.3630]^T. \] (51)

Simulation results in Figure 5 show that the system is asymptotically stable. After it enters the steady state in the third period for the reference input, the disturbance is added to the system. And the largest steady-state peak-to-peak relative tracking error is as small as 0.0893%.

To better assess the disturbance attenuation performance of the observer based MRCS, we design an \( H_\infty \) output-feedback controller [20] (Figure 6) for the plant (47), where the PI controller in Figure 6 is
\[ K_p = 650 + \frac{1400}{s}. \] (52)

Let
\[ w_l(t) = [v^T(t-T) \quad w^T(t)]^T, \quad z(t) = e(t). \] (53)
The \( H_\infty \) disturbance rejection problem was formulated as follows.

Find an \( H_\infty \) output controller \( K_\infty(s) \) such that the MRCS in Figure 6 is internally stable and provides the disturbance attenuation level \( \gamma \); that is, \( \|G_{w_z}(s)\|_\infty < \gamma \).

The problem of designing \( K_\infty(s) \) is shown in Figure 7, and the resulting controller is
\[ K_\infty(s) = \frac{541667.3035(s + 1)(s + 2.154)}{(s + 8.157 \times 10^4)(s + 2.154)(s + 1)}. \] (54)

Choose the index
\[ I_e = \sup_{t \geq t_0} |e(t)| \] (55)
to evaluate the steady-state tracking performance, where \( t_0 \) is the setting time of the control system.

Figure 8 shows the tracking error of the system in Figure 6 for the reference input (48) and the aperiodic disturbance (49). We find that \( I_e = 0.0098 \) for the observer based \( H_\infty \) in Figure 1, but \( I_{E_{\text{appr}}} = 0.022 \) for the \( H_\infty \) MRCS in Figure 6. Moreover, the former has better transient performance.

5. Conclusion

In a conventional repetitive-control system, the tracking performance may be degraded by aperiodic disturbance even if it contains an internal model of a periodic signal,
which theoretically guarantees asymptotic tracking for the periodic reference input. This paper describes a method of designing an observer based repetitive-control system with a prescribed $H_{\infty}$ disturbance attenuation level for a class of strictly proper linear plants. An equivalent system is established to convert the design problem to a static state-feedback $H_{\infty}$ control problem. The Lyapunov stability theory and the SVD of the output matrix are used to derive an LMI-based design algorithm for the parameters of the feedback controller and the state-observer. The validity of the method was demonstrated using the position control of a noncircular cutting workpiece system. The simulation results show that the designed MRCS is asymptotically stable and exhibits satisfactory disturbance-attenuation performance.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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