QED Radiative Effects in Diffractive Vector Meson Electro- and Photoproduction at HERA Energies

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Abstract: QED radiative corrections to the cross section of diffractive electro- and photoproduction of vector mesons is calculated at HERA energies. Both semi-analytical and Monte Carlo approaches are discussed and compared.

1 Introduction

The experience with data analysis in inclusive measurements of deep inelastic scattering (DIS) shows that radiative effects are very important and should be taken into account with maximally possible accuracy. Numerical results obtained for the case of the HERA collider [1, 2, 3, 4, 5, 6, 7, 8] have shown that the ratio of the observed and the Born cross sections varies from 0.5 to 2.0, and even larger values are reached at the borders of the kinematical region. The calculation of radiative corrections (RC) for semi-inclusive and exclusive reactions can not be easily reduced to the case of inclusive scattering, but rather requires separate considerations. There are two important approaches which can be applied for this task. The first one is the so-called semi-analytical approach, where covariant formulae for RC are obtained after applying the procedure of covariant cancellation of infrared divergences [9] and integration over the photon phase space. No other approximation than the ultrarelativistic one is used within this approach. Explicit analytical formulae for RC in diffractive vector meson electroproduction were obtained in Ref. [10]. The radiative correction factor has the following form

\[ \eta = \frac{\sigma_{\text{obs}}}{\sigma_0} = \exp(\delta_{\text{inf}})(1 + \delta_{\text{vr}} + \delta_{\text{vac}}) + \frac{\sigma_{\text{hard}}}{\sigma_0}. \]  

Here \( \delta_{\text{vr}} \) and \( \delta_{\text{vac}} \) come from so-called virtual corrections: the vertex function and vacuum polarization by lepton pairs and hadrons, resp. \( \delta_{\text{inf}} \) appears after cancellation of infrared divergences. The exponential results from the summation of soft photon contributions over all orders of perturbation theory. \( \sigma_{\text{hard}} \) is the contribution of hard photon radiation. In the general case it has the form of a three-dimensional integral over the photon phase space. In Ref. [10] the numerical analysis was done for fixed target experiments. In this report we provide numerical results also for deep inelastic scattering at HERA. Moreover, we discuss the case of photoproduction within the semi-analytical approach.

Another possible approach for the calculation of RC in semi-inclusive and exclusive processes is based on using a Monte-Carlo generator for inclusive DIS. In this report we discuss how the
RC can be calculated using information obtained during event generation. Also we show that, under certain assumptions, both methods give the same results. This fact is illustrated for the case of measurements at HERMES.

It should be noted that all numerical results are shown for the electron method of the reconstruction, when the kinematical variables are calculated using the measurement of the final lepton. There are other reconstruction methods (for example, the double angle method using for the case of diffractive vector meson production), which are, in the most of the phase space, less sensitive to these radiative corrections.

2 The code DIFFRAD

The FORTRAN code DIFFRAD was developed to calculate radiative corrections for diffractive vector meson electroproduction and is based on the semi-analytical approach. This code has two versions which are referred to as IDIFFRAD.F and MDIFFRAD.F. The first version allows to calculate the RC to the cross section \(d^3\sigma/dxdQ^2dt\) of diffractive vector meson electroproduction using usual (non Monte-Carlo) methods of numerical integration. The second one exploits the same formulae but using Monte Carlo methods for integration. It allows to integrate not only over the photon kinematic variables, but also over those of the final lepton. Integration over \(Q^2, W^2\) and \(t\) in arbitrary bins can be performed as well as over the full kinematically allowed phase space of the final lepton. Therefore MDIFFRAD.F has more possibilities in comparison to IDIFFRAD.F, but it requires more CPU time.

The code requires input for the model of the virtual photoproduction cross sections \(\sigma_{LT}\) and allows to apply a cut on inelasticity. As input for \(\sigma_{LT}\) we use the model presented originally in Ref. [11] and refined in Ref. [12] (DIPSI). The implementation of other models is possible and straightforward. A cut on the inelasticity \(v (v = p_h^2 - M^2, \text{where } p_h \text{ is the total momentum of unobserved particles and } M \text{ is the proton mass})\) is often suitable to reduce the RC. The value of the cut is related to the experimental resolution on \(v\) and depends on details of the experimental procedure of non-exclusive background subtraction. A detailed discussion of this point can be found in Ref. [10]. Both versions of DIFFRAD allow one to calculate RC with and without this cut. Note that two additional options also exist for the calculation of the inelasticity distribution with and without RC and for the fast evaluation of an approximate calculation of \(\sigma_{hard}\).

The FORTRAN code DIFFRAD can be received upon request via email from aku@hep.by

3 Electroproduction

Results obtained with the help of DIFFRAD for the three-fold differential cross section are presented in Fig. 1 (\(\sqrt{s}=300 \text{ GeV}\)). Different plots in this figure show that there is no strong \(W\)-dependence of the RC factor \(\eta\). This is due to the fact that \(\sigma_L\) and \(\sigma_T\) are almost flat in the considered kinematical region. Moreover there is no large explicit dependence on \(W\) in the RC factor itself. The quantities \(\delta_{\text{vr}}\) and \(\delta_{\text{vac}}\) in Eq. (1) are practically independent of \(W\) and only a small dependence comes from \(\delta_{\text{inf}}\) and \(\sigma_{\text{hard}}\). The \(Q^2\)-dependence shown in this figure is typical for the case when the inelasticity cut is not applied. If this cut is used then the rise of \(\eta\) when \(Q^2\) goes down would be suppressed. From this figure one can also see that the
Figure 1: RC factor for $\rho(770)$ electroproduction at HERA; $\sqrt{s}=300$ GeV. Symbols from top to bottom correspond to $t = -0.7, -0.5, -0.3, -0.1$ GeV$^2$

The $t$-dependence is rather important. Figures 2 and 3 illustrate this last property. $\eta$ crosses unity for $-t \sim 0.25-0.3$ GeV$^2$ and rises with increasing $|t|$. The large positive correction in this case is a result of the large phase space for photon radiation. The cut applied to the inelasticity (or $E - p_z$) can again reduce the value of the RC factor for large $|t|$. As a consequence of the $t$-dependence of $\eta$, the observed slope parameter also receives large RC. This is illustrated in Fig. 3. The Born cross section has a steeper slope with respect to $|t|$ than the observed cross section. The radiative correction to the slope parameter is negative and $\sim 10\%$.

The model used for exclusive virtual photoproduction of vector mesons is based on the hypothesis of s-channel helicity conservation (SCHC)\footnote{We should note that now there are experimental indications for the violation of this hypothesis \cite{13, 14} as well as theoretical models beyond SCHC \cite{15, 16}. Unfortunately the development of DIFFRAD for using such models is not straightforward, because the calculation of additional terms is required. The calculation for the general case of scalar meson production is considered in \cite{17}.}. In the case of SCHC the Born cross section is independent of the azimuthal angle $\phi_h$ between the scattering and the production plane. However, the observed cross section does depend on $\phi_h$, because the formulae for RC includes this angle. Figure 4 shows an example. Since only $\sigma_{hard}$ has an essential dependence on the azimuth, $\eta$ has a visible $\phi_h$ dependence at large $|t|$, where $\sigma_{hard}$ gives a relatively large
Figure 2: Observable and Born cross sections for exclusive $\rho(770)$ electroproduction as a function of $t$; $\sqrt{s}=300$ GeV; $W=70$ GeV; $Q^2=4$ GeV$^2$.

contribution to the total RC.

We note finally that the RC to the cross section of diffractive vector meson electroproduction is very sensitive to the inelasticity cut. This cut suppresses the contribution of hard photon radiation, which is always positive. Thus the use of harder cuts leads to smaller values of the radiative correction factor. However, in the region of small $|t|$ where radiative corrections are negative ($\eta < 1$) the influence of the cut is not essential or leads to larger absolute values of RC ($\eta$-1). We do not apply the cut for this numerical analysis.

All numerical results shown here were obtained for the case of exclusive $\rho(770)$ production. However there is no essential dependence on the type of the observed vector meson, and all discussed features are very similar for the production of heavier vector mesons.

4 Photoproduction

In photoproduction interactions, i.e. very small $Q^2$ (quasi-real photon exchanged), the final positron is scattered at very small angle and escapes detection in the main detector through the beam pipe. In this case QED radiative corrections can be calculated by integration of the analytical formulae over the phase space of the final positron. A numerical analysis shows that radiative corrections in this case are smaller than in the case of electroproduction. The results are shown in Fig. 5. Note that small values of RC in this case reflect the statement of the Kinoshita-Lee-Nauenberg theorem [18, 19]. For our case it says that if we integrate over the phase space of a particle which radiates, then all leading logarithms are mutually cancelled (see Ref. [20], for example).
Figure 4: RC factor for exclusive $\rho(770)$ electroproduction as a function of $\phi_h$; $\sqrt{s}=300$ GeV; $W=70$ GeV.

Figure 5: RC factor for exclusive $\rho(770)$ photoproduction as a function of $t$; $\sqrt{s}=300$ GeV.

Figure 6: Possible scheme of Monte Carlo calculation of the RC factor.

Figure 7: RC factor for HERMES from DIFFRAD (semi-analytic approach, full line) and from RADGEN (Monte Carlo approach, points with error bars); $\sqrt{s}=7.18$ GeV; $Q^2=1.5$ GeV$^2$; $W^2=20$ GeV$^2$. 
5 Monte Carlo approach

The Monte Carlo approach considered here is based on a generator for inclusive processes with radiative effects included. Here we focus on the case of HERMES and use RADGEN \[21\] as such a generator. This generator was developed for the fixed target experiments while for HERA experiments HERACLES \[22\] which includes electroweak effects should be used.

Two Monte Carlo samples with and without radiative effects have to be prepared. The procedure is sketched in Fig. 6. Firstly, the kinematical variables $\nu$ and $Q^2$ defining the momentum of the final lepton are generated in accordance with the Born cross section. In the case of generation of the sample without RC these variables are used as an input to generate the momentum of the vector meson. For the generation of the sample with RC, a Monte Carlo generator including real photon emission has to be used. Three main tasks are performed by this generator:

1. simulation of the appropriate scattering channel (non-radiative; elastic, quasielastic or inelastic radiative tails) in accordance with their contribution to the total observed cross section;

2. calculation of the inclusive RC factor $\eta^{inc}$ for the given values of $\nu$ and $Q^2$. Since for both samples we generate the same number of events this factor has to be used as a weight to obtain the observed cross section as a weighted sum of events\[2\];

3. generation of the kinematical variables $\nu_{true}$ and $Q^2_{true}$ which are then used as input for generating the momentum of the vector meson. Note that $\nu_{true}$ and $Q^2_{true}$ differ from $\nu$, $Q^2$ for events which contain a radiated photon.

The events are collected in predefined bins. If the numbers of events in a given bin for the samples with and without radiative corrections are $n_1$ and $n_2$, resp., the RC factor for vector meson electroproduction $\eta_\rho$ can be estimated as

$$\eta_\rho = \frac{n_1 < \eta^{inc} >}{n_2}$$  \hspace{1cm} (2)

where $\eta^{inc}$ is the RC factor for the inclusive process. We note that due to photon radiation $\nu_{true} \leq \nu$. Thus the exclusive production of a vector meson with a given value for its energy (which is related to $|t|$) is less probable for a radiative event than for a non-radiative event with the same meson energy. As a result $n_1 < n_2$ and the typical value of the RC factor for vector meson production (for semi-inclusive processes in general) is smaller than for the inclusive case.

Figure 7 presents results for the HERMES experiment obtained with in the two approaches. From this picture we can conclude that the Monte Carlo approach correctly reproduces the RC factor for vector meson electroproduction. The inelasticity cut was not applied for these calculations. We note that the parameter $\Delta$ which separates soft from hard photon radiation and which is always necessary in the Monte Carlo approach (see discussion in Ref. \[21\]) should be chosen smaller than its default value $\Delta = 100$MeV. This value is close to the threshold energy

\[2\] Another algorithm without calculation of weights is possible. In this case both samples with and without RC are generated in accordance with the observed and the Born cross section, resp. Both approaches are equivalent, and everything discussed below is valid for this case as well.
for observing photons in the HERMES detector. In the case of vector meson electroproduction the maximal photon energy is smaller (especially for small $|t|$) in comparison with the inclusive case. So the parameter $\Delta$ has to be reduced also. For the present calculation $\Delta = 5$ MeV was chosen. For smaller values of $\Delta$ we do not see an essential dependence on $\Delta$.

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