Strong enhancement of electron-impact-ionization processes in hot dense plasma by transient spatial localization

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The current standard atomic collision theory cannot explain recent experiments on electron-ion collisional ionization processes in hot dense plasma. We suggest that the use of (distorted) plane waves for incident and scattered electrons is not adequate to describe dissipation during the ionization event. Random collisions with free electrons and ions in plasma cause electron matter waves to lose their phase, which results in partial decoherence of incident and scattered electrons. Such a plasma-induced transient spatial localization of the continuum-electron states significantly modifies the wave functions of continuum electrons, resulting in a strong enhancement of electron-ion collisional ionization of ions in plasma compared with isolated ions. We develop a theoretical formulation to calculate the differential and integral cross sections by incorporating the effects of plasma screening and transient spatial localization. The approach is then used to investigate electron-impact ionization of ions in solid-density magnesium plasma and gives results that are consistent with experiment. The correlation of continuum-electron energies is modified and the ionization cross sections increase several-fold and the rates increase by one order of magnitude. Our findings shed new insight on collisional ionization and three-body recombination and should aid in investigations of the transport properties and nonequilibrium evolution of hot dense plasma.

Electron-impact ionization processes of atoms and ions embedded in hot dense plasma are of interest both for fundamental science and for practical applications in the interiors of stars, white dwarfs, and neutron stars [1, 2], for inertial-confinement fusion [3, 4], and for high-energy-density plasmas [5, 6]. They determine the charge-state distribution and power balance and can also be used to infer the plasma conditions of electron temperature and electron density [7–9]. To date, most atomic data on electron-ion collisional ionization have been obtained for isolated atoms or ions [7, 10–12], where the coupling with the plasma environment is assumed to be trivial and is thus neglected. However, these data are only applicable to rarefied plasmas. If the plasma density is sufficiently high and the coupling cannot be neglected, plasma screening plays a non-negligible role and should be considered when determining the electronic structure. The effect of plasma screening on electron-impact ionization cross sections [13, 14] have also been extensively investigated by using various analytical models [15–18].

Be it for isolated or screened ions, the present standard atomic collision theory describes the incident and scattered electrons in electron-ion collisional ionization as distorted plane waves [19]. In such a treatment, the continuum electrons are completely coherent throughout the entire physical space during the process. In hot dense plasma, however, the continuum electrons collide randomly with the surrounding free electrons and ions, which results in a loss of phase of the electron matter wave and partial decoherence when the electron leaves the parent ion [20]. Partial decoherence of the continuum electrons is a natural many-body effect occurring in dense plasma and differs from plasma screening. The latter supplies an additional Hermitian potential to the physical system, whereas the former is not Hermitian and thus represents a type of dissipation mechanism in hot dense plasma. This work thus investigates how decoherence affects electron-ion collisional ionization.

Elusive signs may be found of anomalous behavior in continuum atomic processes in hot dense plasma [21–26]. In the solar interior, the internal structure predicted by the standard solar model [21] is inconsistent with helioseismic observations, which could be resolved if the Rosseland mean opacity was increased by 10%–20% [22, 23]. Confined micro-explosion experiments using focused femtosecond laser pulses show that ion separation is enhanced in the nonideal plasma [25]. Electron transport in high-plasma-density inductively coupled discharges displays a variety of nonequilibrium characteristics, and the skin depth is anomalous [26]. Only recently has it become feasible to experimentally investigate these processes [27, 28]. Vinko et al. [27] studied the femtosecond collisional ionization rates in solid-density aluminum plasma created by the Linac Coherent Light Source [29], tuned to specific interaction pathways around the absorption edges of ionic charge states. Berg et al. [28] reported the first experimental measurements of ultrafast electron-impact ionization dynamics, for which they used resonant core-hole spectroscopy in a solid-density magnesium plasma. By resonance-pumping the $1s \rightarrow 2p$ transition in highly charged ions within an optically thin plasma, they measured how off-resonance charge states
are populated via collisional processes on femtosecond time scales. These experimental results cannot be explained by the present standard atomic collision theory, even if the plasma screening effect is included.

They [27, 28] state that the collisional ionization and recombination cross sections are considerably larger than the values predicted by the present widely used models. The ionization cross sections and rates may be increased by including plasma screening, but they are not large enough to explain the discrepancies. These experiments provide new insight into ionization in dense plasma; nevertheless, understanding the physics and collisional mechanism behind this enhancement remains a great challenge.

The goal of the present work is to understand the new physics occurring in hot dense plasma and to explain the discrepancies between the present theory and experiments. We point out that the transient spatial localization of continuum electrons (TSLCE) in collisional ionization enhances the cross sections and rates: it is thus an important mechanism modifying the electron-impact ionizations in hot dense plasma. Continuum one-electron-state localization is a natural many-body effect occurring in dense plasma and is different from plasma screening. We develop a theory to investigate how one-electron-state localization affects collisional cross sections and rates in both local thermodynamic equilibrium (LTE) plasmas and non-LTE plasmas. The theory is then applied to analyze electron-impact ionization processes of ions embedded in solid-density magnesium plasmas.

The proposed theory predicts features in the energy differential cross sections compared that are not predicted by existing isolated- and screened-ion models, and these features significantly modify the collisional ionization dynamics. The integral cross sections and rates are thus increased considerably. This theory can be widely applied to studies of hot dense plasma, inertial confinement fusion, astrophysics, and high-energy-density physics. It also deepens our understanding of atomic properties within dense plasmas, which cannot be achieved with the current theories. However, obtaining a unified, complete, and consistent treatment of the anomalous behavior of electron-ion collisional processes based on quantum many-body theory remains a challenge.

RESULTS

New features of energy differential cross sections

To begin our discussion, we first consider collisional processes in LTE plasmas. The electron-impact ionization in hot dense plasma exhibits features that are completely different from those of isolated ions. Figure 1 shows the energy-differential cross section for the ionization pathway \( e + 1s^22s^2S \rightarrow 1s^21S + 2e \) of Mg\(^{2+}\) in a solid-density magnesium plasma at an electron temperature of 150 eV with residual energies of 50, 200, 800, and 1400 eV, respectively. For the solid-density magnesium plasma at 150 eV, the free electron density is about \( 4.54 \times 10^{23} \text{ cm}^{-3} \) based on the ionization balance of the plasma. Three sets of results calculated for an isolated ion, an ion in plasma with screening, and an ion in plasma with screening plus TSLCE are given to compare the effects of plasma screening and of the TSLCE. The inclusion of plasma screening clearly lowers the continuum, which increases the differential cross section. With the added consideration of TSLCE under the given plasma conditions, the differential cross section is further increased by about one order of magnitude with respect to that for an isolated ion.

Looking more into detail, we find that, surprisingly, the differential cross section is modified by electron localization in a dense plasma and thus exhibits new features. The isolated-ion model predicts a "U-shaped" energy dependence for the energy differential cross section that is steeper at high residual electron energies [30–35]. The screened-ion model does not significantly modify this energy dependence, although the differential cross section exhibits some notable differences. If we include the effect of electron localization, however, the U-shaped energy dependence is only approximately maintained at low and high residual energies [see Fig. 1(a) at 50 eV and Fig. 1(d) at 1400 eV]. At an intermediate residual energy, the energy differential cross section differs completely from the isolated- and screened-ion models. At a residual energy of 200 eV [Fig. 1(b)], the differential cross section features two minima at energies for a single ejected electron of \( \sim 40 \) eV and \( \sim 160 \) eV. With an equal sharing of energy for two continuum electrons (100 eV), the differential cross section features a local maximum, which should be a minimum in the isolated- and screened-ion models. At the residual energy of 800 eV [Fig. 1(c)], we find a nearly equal differential cross section except at the two ends with one continuum electron capturing a small fraction of the residual energy and the other capturing the remaining residual energy.

This is a very surprising and counterintuitive result. These qualitative changes in the energy differential cross section are closely related to localization-induced changes in the continuum-electron state, which evidently modifies how the one-electron ionization transition amplitude depends on the continuum-electron energy. For an isolated ion, the one-electron ionization transition amplitude decreases monotonically from the ionization threshold to an energy above that of the continuum electron, and this general behavior produces the U-shaped energy dependence of the energy-differential cross section of the isolated ion. The localization of the continuum-electron state and the broadening of the ionization threshold make the energy dependence of the one-electron ionization transition amplitude more complex: initially it increases and then decreases at higher energies; therefore, the product of the two-electron transition amplitude has not U-shaped energy dependence (when plotted as a function of the total energy of the two electrons) but a much more
FIG. 1: Energy differential cross section of electron-ion collisional ionization. The ionization considered, \(e + 1s^2 2s^2 \rightarrow 1s^2 1S + 2e\) of Mg\(^{9+}\), occurs in a solid-density magnesium plasma at an electron temperature of 150 eV and density of \(4.54 \times 10^{23} \text{ cm}^{-3}\) with a residual energy of (a) 50 eV, (b) 200 eV, (c) 800 eV, and (d) 1400 eV. For clarity, the results obtained with the isolated-ion and screened-ion models are scaled up tenfold and threefold, respectively.

**TSLCE-induced enhancement of integral cross sections**

Figure 2 shows the integral cross sections for the ionization channel of Mg\(^{9+}\) \(e + 1s^2 2s^2 \rightarrow 1s^2 1S + 2e\) in a solid-density magnesium plasma at electron temperatures of 50, 100, 150, 200, and 250 eV. The most striking conclusion drawn from this figure is the significant increase caused by the TSLCE of the integral cross section with increasing electron density and temperature. In fact, the plasma screening enhances the collisional ionization processes, but the increase in the integral cross section due to the latter is much larger than the former, in particular at a higher electron density. Here, we only compare the peak cross section to obtain a quantitative understanding. The peak cross section is calculated to be 0.106 Mb for an isolated ion, but this value is increased by 160% and 195% due to plasma screening at the given lowest (50 eV) and highest (250 eV) temperature. The peak cross section is further increased by 490%, 688%, 980%, 1200%, and 1500% at 50, 100, 150, 200, and 250 eV, respectively. The enhancement of the integral cross section caused by the plasma screening depends only weakly on plasma temperature. However, the TSLCE effect depends much more strongly on electron density and temperature. Moreover, the TSLCE has a more pronounced influence near, yet somewhat above, the ionization threshold, whereas the effect of plasma screening remains roughly similar over a much wider energy range of the incident electron.

The next striking conclusion is the evident ionization potential depression (IPD) caused by plasma screening. The ionization potentials of the 2s electron are predicted to be 198.28, 203.89, 206.99, 209.00, and 210.37 eV at plasma temperatures of 50, 100, 150, 200, and 250 eV, respectively, which is considerably less than our calculated value of 367.22 eV and the experimental value \[36\] of 367.49 eV for an isolated ion. Note that we consider energy broadening for the ionization threshold by including the TSLCE. The result is that the cross section near the ionization threshold behaves differently with and without the energy broadening. Finally, if the TSLCE effect is considered, the integral cross section grows more rapidly from the ionization threshold to a peak value and descends more rapidly. However, with the different treatments of the plasma effects, the predicted location of the peak energy for an isolated ion differs from that for an ion in plasma. When the TSLCE is included, the peak shifts to a lower incident electron energy.

The new features of the differential and integral cross sections of the ions in hot dense plasma are closely related to the TSLCE of electrons involved in electron-impact ionization. Momentum broadening is an important determinant of localization. Figure 3 shows the momentum broadening as a function of the energy of a continuum electron at plasma temperatures of 50, 100, 150, 200, and 250 eV. The momentum broadening increases rapidly from the ionization threshold up to a peak value, and then decreases much more rapidly with increasing continuum-electron energy. This dependence of momentum broadening on energy explains the new features shown in Figs. 1 and 2. The energy differential cross section shows the largest modification compared...
Incident electron energy (eV)

Electron impact ionization cross section (Mb)

250 eV, $4.71 \times 10^{23}$ cm$^{-3}$

200 eV, $4.62 \times 10^{23}$ cm$^{-3}$

150 eV, $4.54 \times 10^{23}$ cm$^{-3}$

100 eV, $4.37 \times 10^{23}$ cm$^{-3}$

50 eV, $3.80 \times 10^{23}$ cm$^{-3}$

250 eV, Screening

50 eV, Screening

Isolated atom

FIG. 2: Enhanced collisional ionization integral cross section. The electron-impact ionization $e + 1s^2 2s^2 2S \rightarrow 1s^2 1S + 2e$ of Mg$^{9+}$ occurs in solid-density magnesium plasma at electron temperatures and densities of 250 eV and $4.71 \times 10^{23}$ cm$^{-3}$ (black solid line), 200 eV and $4.62 \times 10^{23}$ cm$^{-3}$ (red dotted line), 150 eV and $4.54 \times 10^{23}$ cm$^{-3}$ (green dashed line), 100 eV and $4.37 \times 10^{23}$ cm$^{-3}$ (blue dot-dashed line), and 50 eV and $3.80 \times 10^{23}$ cm$^{-3}$ (violet dot-dot-dashed line). The integral cross sections obtained by the screened-ion model are weakly temperature dependent and, thus, for clarity, we only show the results at the highest (250 eV) and lowest (50 eV) temperatures.

with the isolated and screened ion at a residual energy of 200 eV [Fig. 1(b)], which is a natural result of the TSLCE because the momentum broadening is large near this energy. However, the TSLCE exerts much less effect at the residual energy of 50 eV [Fig. 1(a)] and 1400 eV [Fig. 1(d)] because of the smaller momentum broadening. Similarly, the pronounced enhancement of the integral cross section in a definite energy range near the threshold shown in Fig. 2 also results from momentum broadening.

Greatly increased collisional ionization rates in hot dense plasmas

Figure 4 shows how the TSLCE affects the rates for electron-impact ionization of $e + 1s^2 2s^2 2S \rightarrow 1s^2 1S + 2e$ of Mg$^{9+}$ ion in solid-density magnesium plasma. The results indicate a significant enhancement of the electron-ion collisional ionization rates in hot dense plasma due to plasma screening and the TSLCE. At a lower plasma temperature, plasma screening plays a more important role than the TSLCE in this enhancement. The ratio of the rate obtained by considering plasma screening to that obtained by using the isolated-ion cross section is far greater than the ratio of the rate obtained by using the cross sections of the screened ion without the TSLCE to that obtained with the TSLCE. At a higher plasma temperature, however, this ratio decreases and the TSLCE plays an increasingly important role in enhancing the rates. Compared with the results obtained using the isolated-ion cross section, the rates increase by 64.0, 10.8, 5.5, 3.8, and 3.1 times due to plasma screening and by 177.6, 33.6, 22.1, 19.6, and 19.5 times due to both plasma screening and the TSLCE at plasma temperatures of 50, 100, 150, 200, and 250 eV, respectively.

The larger electron-ion collisional ionization rates in hot dense plasma are due to two factors: the increase of the integral cross section and the shift of the ionization threshold. At a lower temperature, the latter plays a more important role, so the IPD causes a greater enhancement of the electron-ion collisional ionization rates. The statistical weight from the energy distribution of the free electrons in the plasma is greater for the cross section with an IPD than without. As a result, the electron-ion collisional ionization rate increases more at lower temperature but the effect becomes much less profound with increasing temperature. With increasing temperature, the enhancement of the integral cross section caused by the TSLCE plays a more important role in increasing the rates. Based on Fig. 4, the slope of the black solid line (which includes the TSLCE effect) increases with increasing temperature, whereas the slope of the red dashed line (which only takes into account plasma screening) decreases.

Comparison with experiments and other theoretical results

We now test the proposed theory by comparing its results with recent experimental and theoretical results. The experiment of Berg et al. [28] used an x-ray free-electron laser to produce a solid-density magnesium
FIG. 3: Momentum broadening of the continuum electrons. The continuum electrons are ejected from collisional ionization, \( e + 1s^2 2s^2 2S \rightarrow 1s^2 \, ^1S + 2e \) of Mg\(^{9+}\), occurring in a solid-density magnesium plasma at plasma temperatures of 250 eV (black solid line), 200 eV (red dotted line), 150 eV (green dashed line), 100 eV (blue dot-dashed line), and 50 eV (violet dot-dot-dashed line). Atomic units are used for the half widths at half maximum in the momentum distributions.

FIG. 4: Electron-ion collisional ionization rates. The rates are given as a function of plasma temperature for the solid-density magnesium plasma.

plasma that strongly deviates from LTE and has a considerable density of core-hole states. Because the laser pulse is short (60 fs) and electron-ion collisional ionization rates are very fast, the free electrons in the plasma reach only an approximate equilibrium distribution, whereas the ions remain cold during the entire x-ray interaction.

These results refer to LTE plasma; here, we extend our approach to non-LTE plasma. To simplify the treatment, we assume a two-temperature plasma model with a definite electron density and temperature and a (different) definite ion temperature. The electron temperature and density are assumed to be 75 eV and \( 3.0 \times 10^{23} \) cm\(^{-3}\), which are the same physical conditions considered by Berg et al. [28]. Figure 5 compares our calculated integral cross section with the results of experiment [28] and with other theoretical results for the collisional ionization \( e + 1s 2s^2 2p^2 \, ^2P \rightarrow 1s 2s^2 2p \, ^3P + 2e \) of Mg\(^{7+}\). These other theoretical results were obtained by Berg et al. [28] using different methods, including those of Lotz [37] and the revision by Burgess and Chidichimo (BC) [38], the scaled hydrogenic approximation [39] and its analytic fitting formula by Clark, Abdallah, and Mann [40], binary encounters [41–44], and the distorted-wave approximation [45–48].

To explain their experimental results [28], Berg et al. proposed a parameterized expression for the collisional cross section (referred to by them as the “BCF with IPD” model), which aims to describe the available experimental data in the low-density regime and provide a scaling method to capture the lowering continuum. Thus, the
BCF model (with and without the IPD effect) obtains a consistent result (the same profile and trend) both in the low- and high-density regimes. Figure 5 shows that the predicted enhancement in the cross section obtained by including both screening and the TSLCE is consistent with the experimental result [28], although the very-near-threshold behavior differs. The results of the isolated-ion model under-estimate the cross sections by about one order of magnitude. Even considering plasma screening, the cross section remains 2.0 to 5.0 times less than the experimental data. The results in Fig. 5 indicate that the TSLCE is at the origin of the enhanced collisional cross sections and rates observed experimentally [28].

DISCUSSION

The TSLCE plays an important role in determining the differential and integral cross sections of electron-impact ionization. This localization that occurs in hot dense plasma differs from that of plasma screening. Although both TSLCE and plasma screening originate from many-body effects in plasma, their physical nature and origin differ. The former arises from the dephasing of the continuum electrons induced by multiple collisions with plasma particles, and the latter arises from Coulomb screening between particles and their environment. Thus, plasma screening affects both bound and continuum states, whereas TSLCE affects only continuum states. The new features in differential cross sections are attributed to electron localization (Fig. 1) and indicate that the collisional dynamics and the electron correlation characteristics of electron-impact ionization differ completely from that of isolated ions. Figure 5 shows that TSLCE also modifies the energy dependence of the integral cross sections; in particular, near but somewhat above the ionization threshold, where the collisional cross section increases less steeply than that of the BCF model. We conclude that TSLCE is a significant physical mechanism that enhances the cross sections and rates of ion ionization in hot dense plasma, as observed in experiments [27, 28].

The electron-ion collisional ionization dynamics plays an important role in determining the ionization balance and therefore affects the physical properties of plasmas, such as the equation of state and opacity [49]. Recent experimental findings indicate that the current state-of-the-art theories strongly under-estimate the collisional ionization rates in hot dense plasma. This means that current state-of-the-art models need to be improved to provide more accurate results. The plasma theory (for example, the collision-radiative model) thus needs to be reconstructed to include the effect of TSLCE.

Two other types of localized electron states that occur in electron collisions with atoms or ions are similar to TSLCE: the shape-resonance state and the Feshbach-resonance state [50]. In a shape-resonance state, the amplitude of the continuum-electron wave function inside the centrifugal potential barrier is strongly enhanced. In a Feshbach resonance, the coupling between the open and closed channels induces wave-function mixing between the bound electrons and the continuum electrons of the open channel, resulting in a strongly enhanced amplitude of the continuum-electron wave function inside the atomic or ionic sphere. Both the shape-resonance state and the Feshbach-type resonance state increase the collisional cross section many-fold, or even by many orders of magnitude, over the nearby nonresonance states. However, the difference is that, for the shape-resonance states and the Feshbach-resonance states, the large increase in cross sections only occurs very near the resonance energy, whereas the TSLCE effect increases the cross section considerably over a much wider range of collision energies and increases the collisional-ionization rate much more effectively.

To summarize, electron-ion collisional ionization processes in hot dense plasma are strongly enhanced by TSLCE compared with those in rarified plasma. The dephasing and decoherence of the incident and scattered electrons significantly modifies the wave functions of the continuum states and thus increases the electron-ion collisional-ionization cross sections and rates. We develop a theory to account for the TSLCE effect and use it to investigate electron-ion collisional-ionization processes in solid-density magnesium plasma.

The collision dynamics is completely modified at the residual energy where the continuum electrons have a large momentum broadening and a two-peak structure appears in the energy differential cross section. The results of the proposed theory are consistent with the results of recent experiments. TSLCE influences not only the basic atomic processes but also the physical properties: equation of state, opacity, electric conduction, and heat conduction. The results provide insight into the continuum atomic processes and their reverse processes (such as collisional ionization and three-body recombination, which is discussed in detail in this work) in the dense-plasma regime. The proposed concept and theoretical formalism should find wide applications in a variety of disciplines, such as astrophysics, high-energy-density physics, inertial confinement fusion, and atomic and molecular physics. Further theoretical developments based on quantum many-body theory are required to obtain a unified and more natural and consistent description.

METHODS

Plasma screening potential

For an isolated ion of nuclear number \(Z\) with \(N\) electrons (\(N = Z\) refers to an atom), the atomic properties and wave functions are determined by solving the Dirac equation using a consistent field method [51, 52]. In hot dense plasma, which is characterized by electron temperature \(T\) and density \(n_e\), both the bound and con-
tinum electrons of the ion feel a plasma screening potential $V_{scr}(r)$ that originates from the interaction with surrounding electrons and ions in the plasma [53, 54]:

$$V_{scr}(r) = 4\pi \left[ \frac{1}{r} \int_0^r \rho(r_1) dr_1 - \frac{3}{2} \frac{3}{\pi} \rho(r) \right]^{1/3},$$

(1)

where $R_0 = (\frac{3}{4\pi\rho})^{1/3}$ defines the radius of the ion sphere. The free-electron density distribution $\rho(r)$ follows Fermi–Dirac statistics, which is obtained from the ionization equilibrium equation of the plasma. In the theoretical treatment, we calculate the Kohn–Sham potential by using the finite-temperature exchange-correlation functionals of Dharma-Wardana and Taylor [55], although other choices for functionals are possible [56].

### Localized wave functions of continuum electrons

The energy and momentum of the continuum electrons in hot dense plasma have a range of uncertainty because of the randomness of collisions and the resulting energy and momentum that is exchanged with other electrons and ions. As a result, the radial wave function of the continuum electrons (with central momentum $k_0$) is a superposition of normalized wave function $P_{k_\kappa}(r)$ associated with energy within the uncertainty

$$u_{k_\kappa}(r) = \frac{1}{A} \int_0^\infty f(k, k_0) P_{k_\kappa}(r) dk,$$

(2)

where $\kappa$ is the relativistic angular quantum number, $A$ is a renormalization constant, and $f(k, k_0)$ is the expansion coefficient of $P_{k_\kappa}(r)$ in momentum space. The normalized wave function $P_{k_\kappa}(r)$ is determined by solving the Dirac equation with the plasma screening potential and has a definite energy and momentum $k$. In dense plasmas, the dominant momentum broadening has contributions from elastic and inelastic collisions with free electrons and ions in the plasma and therefore can be taken as a Lorentzian distribution

$$f(k, k_0) = \frac{\Delta k/\pi}{(k-k_0)^2 + \Delta k^2},$$

where $\Delta k$ being the half width at half maximum of the momentum broadening. The localized wave function can be normalized to unity over the physical space domain [20].

### Differential and integral cross sections of electron-impact ionization

The energy differential cross section for electron-impact ionization reads

$$\frac{d\sigma_{ef}(\epsilon_0, \epsilon)}{d\epsilon} = \rho(\epsilon_0) \rho(\epsilon) \rho(\epsilon_0 - \epsilon) \frac{2\pi}{k_i^2 g_i} \sum_{J_T} \sum_{p<q} (2J_T + 1)$$

$$\times \left| \langle \psi_i \kappa_i, J_{T}\rangle M_T \sum_{p<q} \frac{1}{r_{pq}} |\psi_f \kappa_1 \kappa_2, J_{T}\rangle M_T \right|^2,$$

(3)

where $\rho(\epsilon)$ is the density of states of the corresponding continuum electron [20], $I$ is the ionization potential, $g_i$ is the statistical weight of the initial state, $k_i$ is the kinematic momentum of the incident electron, $J_T$ is the total angular momentum when the target state is coupled to the continuum orbital, $M_T$ is the projection of the total angular momentum, $\kappa_i$, $\kappa_1$, and $\kappa_2$ are the relativistic angular quantum numbers of the incident and scattered...
electrons, and $\psi_i$ and $\psi_f$ are the wave functions of the initial and final states, respectively. The density of states is included here to denote the re-normalization formalism of the continuum electron [20]. The integral ionization cross section is obtained by integrating the differential cross section over the energy $\epsilon$ of one scattered electron.

Collisonal-ionization rates

The rate of electron-impact ionization from level $i$ to $j$ may be expressed as [37]

$$R_{ij} = n_e \int_{\sqrt{2I/m_e}}^{\infty} f(v) \sigma_{ij} dv,$$  

(4)

where $I$ is the ionization potential from $i$ to $j$, and $f(v)$ is the velocity distribution function of the free electrons in the plasma.

The data that support the figures in this paper and other findings of our study are available upon request from the corresponding author.

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[1] N. Booth, A. P. L. Robinson, P. Hakel, R. J. Clarke, R. J. Dance, D. Doria, L. A. Gizzo, G. Gregori, P. Koester, and L. Labate, Nat. Commun. 6, 8742 (2015).
[2] S. J. Rose, Plasma Phys. Controlled Fusion 47, B735 (2005).
[3] S. X. Hu, B. Militzer, V. N. Goncharov, and S. Skupsky, Phys. Rev. Lett. 104, 235003 (2010).
[4] L. Caillabet, B. Canaud, G. Salin, S. Mazeve, and P. Loubye, Phys. Rev. Lett. 107, 115004 (2011).
[5] S. H. Glenzer and R. Redmer, Rev. Mod. Phys. 81, 1625 (2009).
[6] R. P. Drake, Phys. Plasmas 16, 055501 (2009).
[7] A. Müller, Adv. At. Mol. Opt. Phys. 55, 293 (2008).
[8] E. Landi and M. Landini, Astron. Astrophys. 347, 401 (1999).
[9] P. Bryans, E. Landi, and D. W. Savin, Astrophys. J. 691, 1540 (2009).
[10] T. R. Kallman and P. Palmeri, Rev. Mod. Phys. 79, 79 (2007).
[11] M. A. Lennon, K. L. Bell, H. B. Gilbody, J. G. Hughes, A. E. Kingston, M. J. Murray, and F. J. Smith, J. Phys. Chem. Ref. Data 17, 1285 (1988).
[12] E. Kallne and L. A. Jones, J. Phys. B: At. Mol. Opt. Phys. 10, 3637 (1977).
[13] R. K. Janev, S. B. Zhang, and J. G. Wang, Matter and Radiation at Extremes 1, 237 (2016).
[14] M. C. Zammit, D. V. Fursa, and I. Bray, Phys. Rev. A 82, 052705 (2010).
[15] P. Debye and E. Hückel, Phys. Z. 24, 185 (1923).
[16] B. F. Rozenhayn, Phys. Rev. A 5, 1137 (1972).
[17] J. C. Stewart and K. D. Pyatt, Jr., Astrophys. J. 144, 1203 (1966).
[18] G. Eckert and W. Kröll, Phys. Fluids 6, 62 (1963).
[19] B. H. Branden, The Benjamin/Cummings publishing company, INC., (1983).
[20] P. F. Liu, C. Gao, Y. Hou, J. L. Zeng, and J. M. Yuan, Communications Physics 1, 95 (2018).
[21] J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. 60, 297 (1988).
[22] J. N. Bahcall, A. M. Serenelli and M. Pinsoneault, Astrophys. J. 614, 464 (2004).
[23] S. Basu and H. M. Antia, Phys. Rep. 457, 217 (2008).
[24] L. G. Christophorou and J. K. Olthoff, Applied Surface Science 192, 309 (2002).
[25] E. G. Gamaly, L. Rapp, V. Roppo, S. J. Riedel, and A. V. Rode, New J. Phys. 15, 025018 (2013).
[26] Alex V. Vasekov and Mark J. Kushner, Phys. Rev. E 66, 066411 (2002).
[27] S. M. Vinko et al., Nat. Commun. 6, 6397 (2015).
[28] Q. Y. van den Berg et al., Phys. Rev. Lett. 120, 055002 (2018).
[29] LCLS Website. Available at http://lcls.slac.stanford.edu/, (2014).
[30] H. Ehrhardt, K. Jung, G. Knoth, and P. Schlemmer, Z. Phys. 203, 1269 (1983).
[31] R. Biswas, and C. Sinha, Phys. Rev. A 54, 2944 (1996).
[32] T. Kai, Phys. Rev. A 81, 023201 (2010).
[33] A. Bandyoodhyay, K. Roy, S. Mandal, and N. C. Sil, Phys. Rev. A 51, 2151 (1995).
[34] M. Guerra, P. Amaro, J. Machado, and J. P. Santos, J. Phys. B: At. Mol. Opt. Phys. 48, 185202 (2015).
[35] P. F. Liu, J. L. Zeng, and J. M. Yuan, J. Phys. B: At. Mol. Opt. Phys. 51, 075202 (2018).
[36] A. Kramida, Yu. Ralchenko, J. Reader, and NIST ASD Team (2018). Available at: https://physics.nist.gov/asa1 [2018, November 19].
[45] G. Csanak, H. S. Taylor, and R. Yaris, Phys. Rev. A 3, 1322 (1971).
[46] S. M. Younger, Phys. Rev. A 22, 1425 (1980).
[47] J. B. Mann, At. Data Nucl. Data Tables 29, 407 (1983).
[48] D. H. Sampson, H. L. Zhang, and C. J. Fontes, Phys. Rep. 477, 111 (2009).
[49] J. E. Bailey et al., Nature (London) 517, 56 (2015).
[50] H. Friedrich, Theoretical atomic physics, Springer-Verlag Berlin Heidelberg (2006).
[51] M. F. Gu, Can. J. Phys. 86, 675 (2008).
[52] P. Jönsson, X. He, C. F. Fischerb, and I. P. Grantc, Comput. Phys. Commun. 177, 597 (2007).
[53] Sang-Kil Son, R. Thiele, Z. Jurek, B. Ziaja, and R. Santra, Phys. Rev. X 4, 031004 (2014).
[54] S. X. Hu, Phys. Rev. Lett. 119, 065001 (2017).
[55] M. W. C. Dharma-Wardana and R. Taylor, J Phys. C: Solid state phys. 14, 629 (1981).
[56] S. Ichimaru, H. Iyetomi and S. Tanaka, Phys. Rep. 149, 91 (1987).