Spin Gaps in High Temperature Superconductors

A. J. Millis

AT&T Bell Laboratories, 600 Mountain Ave, Murray Hill NJ 07974

L. B. Ioffe

Physics Department, Rutgers University, Piscataway, NJ 08855

Hartmut Monien

Theoretische Physik, ETH Zurich, CH-8093 Zurich, Switzerland

Abstract

The phenomenology and theory of spin gap effects in high temperature superconductors is summarized. It is argued that the spin gap behavior can only be explained by a model of charge 0 spin 1/2 fermions which become paired into singlets and that there are both theoretical and experimental reasons for believing that the pairing is greatly enhanced in the bilayer structure of the $YBa_2Cu_3O_{6+x}$ system.
The spin dynamics of the high $T_c$ superconductors are anomalous in several ways. The aspect of interest here is shown in Fig. 1, in which the uniform spin susceptibility $\chi_0(T) \equiv \lim_{q \to 0} \chi(q, \omega = 0)$ is plotted for several high $T_c$ materials. Focus first on the data for $YBa_2Cu_4O_8$. This is typical of the "underdoped" members of the yttrium-barium family of high temperature superconductors. Now the optimally doped member of this family, $YBa_2Cu_3O_7$, displays the $\chi(T)$ expected for a fermi liquid: it is large, about 3 states/eV-Cu, and temperature independent. $YBa_2Cu_4O_8$ displays two regimes. For $T > T^* \approx 200K$, $\chi_0(T)/\mu_B^2 \approx A + BT$ with $A = 1.5 \text{states/eV-Cu}$ and $B = 1.6 \times 10^{-3} \text{states/eV-Cu-K}$; for $T < T^*$, $\chi_0(T)$ drops more rapidly. Neither temperature regime is compatible with fermi liquid theory; the low temperature regime is particularly difficult to explain. The oxygen relaxation rate $1/17T_1$, shown in Fig. 2 for several high $T_c$ materials, displays similar anomalies. This is related to the low $\omega$ limit of the imaginary part of the spin susceptibility as discussed in [1]. The data shown in Fig. 2 therefore imply that the anomalous behavior found in $YBa_2Cu_4O_8$ at $\omega = 0$ and $q \to 0$ persists for a range of $q$, at small $\omega$.

In a fermi liquid the spin response is determined by a continuum of particle-hole pair excitations. The non-fermi liquid temperature dependences observed in $YBa_2Cu_4O_8$ imply that in this material the particle-hole continuum has been modified in some essential way. We argue that (i) the data for $T > T^*$ imply that in $YBa_2Cu_4O_8$ a particle-hole continuum of spin excitations still exists, (ii) the behavior at $T < T^*$ can only be explained if the fermions making up the particle-hole continuum are pairing into singlets, (iii) because this singlet pairing is not associated with the onset of superconductivity or superconducting fluctuations, spin-charge separation must occur and (iv) there are both theoretical and experimental reasons for believing that the pairing is greatly enhanced in the bilayer structure of the Y-Ba system. These points are not new; here we aim to clarify the issues and summarize the present theoretical understanding.

We consider $T > T^*$ first. In this regime the copper NMR relaxation rates $^{63}T_1^{-1}$ and $^{63}T_2^{-1}$, both increase by almost a factor of 2 as $T$ is lowered from 400K to 200K. $T_1$
measures \( \lim_{\omega \to 0} \sum_q \chi''(q, \omega)/\omega \) while \( T_2 \) measures \( (\sum_q \chi'(q, \omega = 0)^2)^{1/2} \). The simultaneous increase can to our knowledge only be explained by models which assume proximity to a \( T = 0 \) magnetic critical point or phase. NMR and neutron scattering are not consistent (see e.g.\(^4\)); our view is that there are spin fluctuations not seen by neutron scattering, but the issue is not resolved.

Three classes of models have been proposed: “fermi liquid” models with overdamped spin excitations, “\( z = 1 \)” models with undamped or weakly damped spin waves, and “spin liquid” models. In fermi liquid models, the particle-hole continuum is modified by the presence of overdamped spin excitations. The spin excitations are overdamped because decay of a spin excitation into a particle-hole pair is allowed. There are two sub-cases — either the magnetic wavevector is inside the particle-hole continuum, or it is a “\( 2p_F \)” wavevector of the fermi surface. The critical phenomena of these cases have been worked out\(^5,6\), and the comparison to data is not favorable, as first pointed out in ref.\(^8\). The principle difficulty concerns the \( Cu \) \( T_2 \) rate, which is predicted to vary with temperature as \( (T_1/T)^{-1/2} \), if the ordering wavevector is inside the continuum, or more weakly, if \( Q = 2p_F \), whereas the data suggest \( T_2^{-1} \propto 1/T_1T \)\(^8\).

The \( z = 1 \) models take as their starting point the magnetic insulating parent compounds, which are antiferromagnets with spin waves of dispersion \( \omega = c[|\vec{k} - \vec{Q}|] \) \( (\vec{Q} = (\pi, \pi) \) is the ordering vector and in \( La_2CuO_4 \) \( hc \sim 0.75eV - \hat{A} \). It is assumed that the doping which converts these materials to superconductors also changes the dispersion to \( \omega^2 = c^2[(\vec{k} - \vec{Q})^2 + \Delta^2] \). The critical properties associated with opening the gap have been extensively studied\(^5\). For \( T > \Delta > 0 \), the copper NMR \( T_1 \) and \( T_2 \) rates are predicted to obey \( T_2/T_1T \sim \) constant \( \sim c \) in agreement with data\(^3\). However, the very restrictive kinematics of spin waves implies that e.g. the oxygen relaxation rate in the spin-wave-only model is given by \( 1/17T_1T \sim T^2 \)\(^3\). It is therefore argued\(^8\) that the model must be supplemented by a particle hole continuum of fermions which couple only weakly to the spin waves but contribute to the small \( q \) response, so that \( \chi_0(T) = \chi_{\text{fermion}} + \chi_{\text{spinwave}} \). However, a quantitative comparison of the theory to \( YBa_2Cu_4O_8 \)\(^3\) found that the observed \( T_2/T_1T \) ratio implied \( c \approx 0.35 eV - \hat{A} \),
implying $d\chi_{\text{spinwave}}/dT = 1 \times 10^{-2} \text{states/ev} - Cu - K$, six times greater than the observed $d\chi/dT$.

A third alternative is the “spin liquid” picture. This is based on a gauge theory formalism which is one implementation of Anderson’s spin-change separation hypothesis\[10\]. The spin response is given by a particle-hole continuum of spin $1/2$ charge 0 fermionic ”spinons” which interact via a gauge field. It has been shown that the gauge field interaction may lead to divergences in the “$2p_F$” susceptibilities of the spinons even if the interaction is not tuned to a critical value, and parameters exist for which the physical consequences are roughly consistent with available data\[11\].

We now turn to point (ii). For $T < T^* \approx 200K$, $\chi_0(T)$ drops below the $\chi_s = A + BT$ characterizing the higher $T$ behavior. For $100K < T < 200K$, $\chi_0(T)$ is roughly linear in $T$ and extrapolates to a slightly negative value at $T = 0$. In other words, in this regime one should think of the $\chi_0(T)$ as being due to thermal excitations above a ground state with a vanishing spin susceptibility. In order to have $\chi_0(T) = 0$ at $T = 0$ in a spin rotation invariant system one must have a ground state which is a singlet with a gap to spin excitations. We have already argued that there is a particle-hole continuum of spin excitations in $YBa_2Cu_3O_8$. The only known method of opening a gap in a particle-hole continuum is to pair the fermions into singlets, as occurs in a conventional superconductor. Note in particular that the effect of actual or incipient antiferromagnet order on the spin susceptibility is known\[6\] not to produce a gap; instead it leads in two spatial dimensions to $\chi_0(T) = \chi_0 + DT$, with $\chi_0 > 0$.

The pairing is unlikely to be due to conventional superconductivity for two reasons: the scale, $T^* \approx 200K$, is much larger than the largest superconducting $T_c$, observed in any member of the Y-Ba family, and the observed resistivity is very different from the “paraconductivity” observed in materials with strong superconducting fluctuations. In particular, $YBa_2Cu_3O_{6.7}$ exhibits spin gap effects very similar to those found in $YBa_2Cu_3O_8$ but its resistivity has upward curvature for $80K < T < 150K$ and is rather flat between $100K$ and $T_c = 60K\[12\]$. If strong superconducting fluctuations were present, the resistivity would drop
rapidly in this regime. Thus we believe that theories based on formation of conventional Cooper pairs via an attractive interaction\textsuperscript{13} are unlikely to be be relevant. Rather, the correct theory must involve spin-charge separation in some form.

Next, we argue that although anomalous temperature dependences are present, there is no strong evidence for singlet pairing in $La_{2-x}Sr_xCuO_4$, where interplane coupling is believed to be weak. Spin susceptibility data obtained as in\textsuperscript{14} but using more reliable data of\textsuperscript{15} are shown in Fig. 1; although there is a pronounced downturn in the susceptibility for $x = 0.08$ and $x = 0.14$ it is also clear that $\lim_{T \to 0} \chi_s(T) > 0$ in these materials. Similarly, $T_1 T^{-1}$, plotted in Fig. 2 at $x = 0.14$, shows no sign of a downturn. Finally, the Cu relaxation rates $1/T_1 T$ in $La_{2-x}Sr_xCuO_4$ are monotonic in $T^4$, in contrast to the Cu relaxation rate in $YBa_2Cu_4O_8$ which exhibits a pronounced downturn beginning almost a factor of two above $T_c$. We conclude that the bilayer structure of $YBa_2Cu_4O_8$ is important. This conclusion is not universally accepted; for an alternative point of view see\textsuperscript{8}.

There is in fact evidence that the two planes in a bilayer are magnetically coupled. Neutron scattering experiments have only detected fluctuations in which the spins on one plane are perfectly anticorrelated with spins on the other\textsuperscript{16}. An NMR $T_2$ "interplane relaxation" experiment in which Cu spins on one plane are pumped and spins on the adjacent plane are probed has been proposed by Monien and Rice\textsuperscript{17} and performed by Stern et. al.\textsuperscript{18}. The interplane rate is found to be large (of the order of the in-plane $T_2$) and rapidly temperature dependent.

Theories of the spin gap have been discussed by many authors\textsuperscript{19}–\textsuperscript{23}. Because one must deal with "spinons", the effects of gauge interactions must be included. These lead to a large inelastic lifetime and thus strong pairbreaking which in general suppresses\textsuperscript{22} the spinon-pairing instabilities found\textsuperscript{24} in one-plane models (for an exception, see\textsuperscript{21}, where in one model a very weak instability is found). However, in two-plane models the enhanced in-plane magnetic susceptibility leads\textsuperscript{21,22} to a large enhancement of the between-planes pairing originally proposed in\textsuperscript{20}. In fact, the pairing kernel has essentially the same singularity as the theoretical expression for the "interplane relaxation" $T_2$ discussed above, so this experiment
may be regarded as evidence that the singular interaction required by the interplane pairing
theories exists and has the correct order of magnitude.

To conclude: a magnetic susceptibility which decreases as the temperature is decreased
is observed in underdoped high temperature superconductors and is difficult to explain
within fermi liquid theory. We have argued that one should distinguish two regimes: a
$\chi_0(T) \approx A + BT$ regime which occurs in many materials and must be understood in terms of
antiferromagnetism and a particle-hole continuum, and a "spin-gap" regime in which $\chi$ drops
rapidly to zero, which occurs only in the yttrium-barium family and must be understood in
terms of pairing of chargeless fermionic "spinons".

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See, e.g. Fig 3 of B. Batlogg, p. 37 in ref [1]. This figure presents data for YBa$_2$Cu$_3$O$_{6.7}$ which has spin gap behavior similar to YBa$_2$Cu$_4$O$_8$. The resistivity flattening is present in only the former material, which has more impurity scattering. The sensitivity of $\rho$ to impurities shows that there is no paraconductivity.

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FIGURES

FIG. 1. Spin susceptibilities for $YBa_2Cu_3O_8$ (filled squares) and $La_{2-x}Sr_xCuO_4$ ($x=0.08$, open circles, $x=0.14$ open triangles, $x=0.18$, open squares). The solid line is the slope predicted by the quantum critical regime of the $z=1$ theory with the spin wave velocity appropriate to $La_2CuO_4$.

FIG. 2. Oxygen relaxation rates from ref [1] (Y-Ba) and [4] (La-Sr).