Genetic Bi-objective Optimization Approach to Habitability Score

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Abstract. The search for life outside the Solar System is an endeavour of astronomers all around the world. With hundreds of exoplanets being discovered due to advances in astronomy, there is a need to classify the habitability of these exoplanets. This is typically done using various metrics such as the Earth Similarity Index or the Planetary Habitability Index. In this paper, Genetic Algorithms are used to evaluate the best possible habitability scores using the Cobb-Douglas Habitability Score. Genetic Algorithm is a classic evolutionary algorithm used for solving optimization problems. The working of the algorithm is established through comparison with various benchmark functions and its functionality is extended to Multi-Objective optimization. The Cobb-Douglas Habitability Function is formulated as a bi-objective as well as a single objective optimization problem to find the optimal values to maximize the Cobb-Douglas Habitability Score for a set of promising exoplanets.

Keywords: Exoplanetary Habitability Score · Genetic Algorithm · Astroinformatics · Multi-Objective Optimization · Cobb-Douglas Production Function · Machine Learning

1 Introduction

The search for life has been one of the oldest endeavours of mankind. But only recently have we acquired the capability to take even a step towards this lofty goal. With the first exoplanet discovered in 1991 [3], we have now reached a point where we have discovered over 4000 exoplanets. We have also taken steps in discovering if life exists on these planets through the use of various metrics such as the Earth Similarity Index [15] or the Cobb-Douglas Habitability Score (CDHS) [2]. These metrics take various planetary parameters as inputs and give us an intuitive understanding of the likelihood of life existing on these planets.

The Cobb-Douglas Habitability Production Function (CD-HPF) can quickly give us a score that is representative of the potential of habitability of an exoplanet. It takes in the Radius, Density, Escape Velocity and Mean Surface Temperature of a planet as inputs. All these inputs are in Earth Units(EU) i.e.
the metric measurements of these parameters are divided by Earth’s own measurements. Simply put, the values of any parameter of Earth in Earth Units is 1.

The Cobb-Douglas function was first developed in 1927 [4], seeking to mathematically estimate the relationship between workers, capital and goods produced. In its most standard form for production of a single good with two factors, it is written as

\[ Y = AL^\beta K^\alpha \]

Where, \( Y \) is the total production, \( A \) is total factor productivity, \( L \) and \( K \) being the labour and capital inputs, and \( \alpha, \beta \) being output elasticities of labour and capital respectively. The function itself is highly adaptable and has been utilized for various tasks like revenue models for data centers [13], frameworks for computing scholastic indicators of influence of journals [8] successfully.

The CDHS is calculated in a two-fold manner: by calculating the interior-CDHS using radius and density, and the surface-CDHS, by using escape velocity and surface temperature; the final score is computed by a convex combination of the two scores. Thus the function is formulated as a multi-objective optimization problem of the two scores.

Most optimization functions require the gradient of a function to minimize or maximize it. However, this can prove computationally costly and all functions are not differentiable, and even then, the derivative might not be smooth or continuous. In this paper, we use Genetic Algorithms, a class of gradient-free optimization functions, which are more widely applicable by virtue of them not requiring the derivative of the function to optimize it.

In the book, "On the Origin of Species" by Charles Darwin, he concluded that only those species survived who were successful in adapting to the changing environment and others died. He called this "Natural Selection" which has three main processes; Heredity, Variation and Selection. These involve species receiving properties from their parents, making variations to evolve and then being selected based on their adaptation to the environment for their survival. Along these lines, genetic algorithms [9] were introduced with five phases of process to solve an optimization problem. We create a initial population of randomly generated elements, known as solutions to the problem and then evaluate the correctness of the solutions using a fitness function which tells us how well the solution helps in optimizing the problem. Genetic Algorithms revolve around the twin principles of Exploration and Exploitation. There must be enough variety in the population to ‘explore’ the solution space which is usually vast, and on finding good solutions, the algorithm must ‘exploit’ these solutions and generate incrementally better solutions.

The typical Genetic Algorithm consists of 3 processes: Selection, Crossover and Mutation. In this paper, we use a modified version of a GA that combines the processes of Mutation and Crossover into one. This Proto-Genetic Algorithm is simpler to implement and understand while not compromising on performance.

We evaluate the ‘fitness’ of the population, that is to say we find the value of the function to be optimized using the members of the population, generate
children using one parent and then test their fitness as well, choosing the best for the next generation. It is similar to the biological process of asexual reproduction where the child inherits all the traits from one parent alone. In this case, the child is generated from a Gaussian Distribution (as shown in Figure 2) centered at the parent’s value.

We illustrate the results of our algorithm on the set of Earth-like exoplanets that is the TRAPPIST system from the exoplanet catalog [12], hosted by the Planetary Habitability Laboratory at the University Of Puerto Rico at Acerbio.

2 Genetic Algorithms

A Genetic Algorithm (GA) is a meta heuristic which is based on the process of natural selection. It is a subset of the class of Evolutionary Algorithms which take cues from biological processes. They are most commonly used in optimization or search problems as they are capable of searching large combinatorial solution spaces to find globally optimal solutions.

Figure 1 indicates the pseudo-code of a typical Genetic Algorithm where a population of solutions are initialized randomly, given the constraints of a specific problem. The fitness of each solution is calculated, which is the value returned by the given function for that solution. Following which the genetic operators of Selection, Crossover and Mutation take place in order to create an incrementally better population. This process is repeated until a termination condition is met, such as a specified number of generations.

```
generate an initial random population
while iteration <= maxiteration
    iteration = iteration + 1
    calculate the fitness of each individual
    select the individuals according to their fitness
    perform crossover with probability p_c
    perform mutation with probability p_m
    population = selected individuals after crossover and mutation
end while
```

Fig. 1: Pseudo-code of a typical Genetic Algorithm

2.1 Proto-Genetic Algorithm

In this paper, we have utilized a simpler version of the Genetic Algorithm. While GA’s typically generate children using traits from both two parents, we have utilized a single-parent reproduction which is both crossover and mutation rolled
into one. The best half of the population is selected and a single child is created for each parent. This child is created using a Gaussian Distribution centered at the parent, thus allowing for a mutation of sorts to occur. Due to the nature of the Gaussian Distribution, a small change is much more likely to happen than a drastic one, which reflects real life as well.

At the end of this process, we have a highly fit population. This algorithm is simpler to understand and implement but gives satisfactory results.

![Fig. 2: The bell-shaped curve of the Gaussian Distribution](image)

3 Implementation

3.1 Single Objective Optimization

Various test functions like Mishra, Rastrigin, Schaffer and others share many similarities. All of them have 2 parameters and are highly multimodal. Thus, these functions serve as suitable benchmarks for GA.

We first initialize 2 sets of values for $x$ and $y$, having populations of 200 values each. Our next step is to run the Genetic Algorithm. Here we choose to run the algorithm for 1000 generations, that is to say the processes of crossover, mutation and recombination take place 1000 times at the end of which we have solutions which are very close to the global minimum. The fitness measure here is of course the value of the function for the parameters $x$ and $y$. After calculating the fitness for each pair we then choose the best pairs, i.e. the ones with the lowest fitness and then use them as the parents of the next generation. Generation of children is done using the Gaussian Distribution, allowing us to vary the children slightly in each generation. This is followed by checking the fitness of each child and
arranging the children and the parents in order of their fitness. This weeds out all the parents who were not good enough and the children who were worse than their parents. Finally, we remake the population choosing the best of both the old and the new generation.

For example, the Rastrigin Function [11], a commonly used benchmark function used to test optimization algorithms due to its highly multimodal nature:

\[ f(x, y) = 20 + x^2 + y^2 - 10(\cos(2\pi x) + \cos(2\pi y)) \]

Fig. 3: The Rastrigin Function

The Rastrigin function in Figure 3 has a global minima of 0.0 with the domain being from -5.12 to 5.12. Thus, our algorithm generates 200 values of \(x\) and \(y\) in the given domain, which is the first generation of the algorithm. They are sorted according to their fitness and new children are generated from the best half of the population. Following this the population is remade by sorting according to fitness again and the second generation is created, with the members being slightly fitter than their parents. Table 1 compares the actual global minima and that obtained using GA for various test functions.

3.2 Constrained Optimization

These functions are also single objective optimization problems, however they are constrained. Whereas the previous batch of functions are only limited by the search domains, these functions have additional constraints. They tend to be more challenging to optimize.

Mishra’s Bird Function displayed in Figure 4 [10] has a global minima of -106.76 and the domain being from -10 to 0 for \(x\) and -6.5 to 0 for \(y\). The function is given as:
Table 1: Single Objective Optimization Results

| Test Functions  | Global Minimum | Actual Values | GA Values |
|-----------------|----------------|---------------|-----------|
| Easom           | -1             | -0.999        |           |
| Rastrigin       | 0.0            | 0.0003        |           |
| Ackley          | 0.0            | 0.009         |           |
| Beale           | 0.0            | 0.0           |           |
| Goldstein-Price | 3.0            | 3.0001        |           |
| Mishra No.4     | -0.199         | -0.193        |           |
| Cross-in-tray   | -2.06          | -2.06         |           |
| Eggholder       | -959.64        | -959.27       |           |
| Holder table    | -19.208        | -19.208       |           |
| McCormick       | -1.913         | -1.913        |           |
| Schaffer No.4   | 0.292          | 0.292         |           |

\[
f(x,y) = \sin(y) \cdot e^{(1 - \cos(x)^2)} + \cos(x) \cdot e^{(1 - \sin(y)^2)} + (x - y)^2
\]

In addition to minimizing this, the solutions must also not violate the additional constraint which is:

\[(x + 5)^2 + (y + 5)^2 < 25\]

We follow the same procedure as with single objective optimization, albeit making sure the solutions do not violate the constraints along with the upper and lower bounds of the domain. Children generated will be discarded if they do not satisfy the constraints. The standard deviation of the Gaussian Distribution goes on increasing to widen the search range if a large number of solutions are discarded. Table 2 lists different test functions along with their actual and GA obtained global minimum.

Table 2: Constrained Optimization Results

| Test Functions                      | Global Minimum | Actual Values | GA Values |
|-------------------------------------|----------------|---------------|-----------|
| Rosenbrock (with a cubic and a line)| 0.0            | 0.0009        |           |
| Rosenbrock (disk)                   | 0.0            | 0.0           |           |
| Mishra’s Bird                       | -106.76        | -106.76       |           |
| Townsend                            | -2.02          | -2.02         |           |
| Simionescu                          | -0.072         | -0.0719       |           |
3.3 Multi-Objective Optimization

Whereas for single-objective optimization problems there exists a single solution which is the best value, no such solution exists for non-trivial multi-objective problems.

Multi-objective Optimization problems involve minimizing/maximizing more than one function simultaneously. If these functions are competitive i.e. minimizing one maximizes the other, it is not possible to find a single best solution. Instead we get a set of non-dominating solutions known as a Pareto Front (as shown in Figure 6). In the absence of other information, each solution in the Pareto Front is equally valid and no solution can be said to be better than another.

Pareto fronts are based on the idea of dominance. If \( \vec{x}, \vec{y} \) are two solutions, then \( \vec{x} \) is said to dominate \( \vec{y} \) if

\[
   f_i(\vec{x}) \leq f_i(\vec{y}) \quad \forall \, i = 1, 2, 3, \ldots k
\]

In another words, the vector \( \vec{x} \) is said to dominate \( \vec{y} \) if and only if, \( f(x) \leq f(y) \) for every single objective in the multi-objective optimization problem. We say that a vector of decision variables \( \vec{x} \in \mathcal{F} \) is said to be Pareto optimal if no other vector \( \vec{x} \in \mathcal{F} \) exists such that \( f(\vec{y}) \leq f(\vec{x}) \). A multi-objective optimization consists of finding the best Pareto front for a given set of objectives.

There are various algorithms for multi-objective optimization. Indeed, one such algorithm, Particle Swarm Optimization (PSO) has already been used in solving the CD-HPF \cite{17}. PSO have many advantages over GA \cite{6} and hybrid PSO-GA have also been used in problems like intelligent routing \cite{18} to great success.
When solving a multi-objective optimization problem using GA, a different approach must be taken. While in single objective problems we can directly compare function values as fitness and choose the best parents, the same cannot be done when we have multiple objectives to optimize. There are numerous algorithms such as MOGA [7], NSGA [16]. In this paper, we have used one of the most popular multi-objective optimization algorithms, NSGA-II [5].
NSGA-II as illustrated in Figure 5 consists of two new processes in order to assign fitness to solutions. The first is the **non-dominated sort**, where the solutions are sorted into sets of non-dominating solutions i.e. fronts. The second is the **crowded-comparison**, which ensures that solutions which have fewer number of solutions in their vicinity have a higher chance of getting selected. In other words, this algorithm favours non-dominated solutions which are well distributed.

Thus we can assign a fitness to the solutions even with multiple objectives. Following this we use our proto-genetic algorithm to evolve the chosen solutions and continue the process iteratively until we have our population closely resembling the optimal Pareto Front. Figures 7 and 8 compare the actual and obtained pareto fronts of different test functions.

![Fig. 7: Actual Pareto Fronts of Test Functions](image-url)
4 Cobb-Douglas Habitability Function

The Cobb-Douglas Habitability Function is given as follows:

\[ Y = R^\alpha . D^\beta . V^\delta . T^\gamma \]

It can also be formulated as a bi-objective optimization problem for easy visualization and understanding. The Cobb-Douglas Habitability Score \( Y \) is divided into two components, CDHS-interior \( Y_i \) and CDHS-surface \( Y_s \). The CDHS is estimated by maximizing both \( Y_i \) and \( Y_s \) which are defined as follows:

\[ Y_i = R^\alpha . D^\beta \]
\[ Y_s = V^\delta . T^\gamma \]

These functions are subject to the constraints:

\[ \alpha + \beta \leq 1 \]
\[ \delta + \gamma \leq 1 \]
\[ 0 < \alpha, \beta, \delta, \gamma < 1 \]

Where \( \alpha, \beta, \gamma, \delta \) are the elasticities of the planetary parameters Radius, Density, Escape Velocity and Mean Surface Temperature. The quality of this model is well noted. \[13\]

Thus we have a bi-objective optimization problem where we have to optimize \( CDHS_i \) and \( CDHS_s \) simultaneously.

\[ \max f(\vec{x}) = [Y_i, Y_s] \]

However, since \( V_e = \sqrt{\frac{2GM}{R}} \), we know that increasing surface score is not possible without compromising on interior score and vice versa. Thus, as shown in \[2\], we use the following relationships:

\[ V_e = \frac{\delta}{\alpha} \frac{W_R}{W_{V_e}} R \]

Where \( W_R \) and \( W_{V_e} \) are weights of \( R \) and \( V_e \) respectively. Rearranging the equation we get:

\[ \delta = \alpha \frac{V_e}{R} C \]

where,

\[ C = \frac{W_{V_e}}{W_R} \]

In order to bring out the trade-off between the two components of the Cobb-Douglas Habitability Score, we calculate \( \delta \) from the other parameters, optimizing the variables \( \alpha, \beta, \gamma \) and \( C \).

We apply the aforementioned proto-genetic algorithm modified with NSGA-II on a set of planets from the exoplanet catalog hosted by the Planetary Habitability Laboratory at the University Of Puerto Rico, the TRAPPIST system. The results are shown in Table 3 with illustrations in Figure 9.

## 5 Results

After testing on multiple exoplanets in the catalog, we found promising results similar to that of past approaches \[13\].

The Pareto fronts also show a trend where increase in one score is compensated for by decrease in the other. These complementary scores bring out the trade-off between \( Y_i \) and \( Y_s \).

The final score is calculated as the weighted linear combination of interior and surface score where the weights sum up to 1.

\[ Y = w_iY_i + w_sY_s \]
Table 3: Comparison of CDHS using GA with past approaches

| Exoplanets  | CDHS(2018) | CDHS(GA) |
|-------------|------------|----------|
| TRAPPIST-1 b | 1.0410     | 1.3753   |
| TRAPPIST-1 c | 1.1589     | 1.2073   |
| TRAPPIST-1 d | 0.8870     | 1.0146   |
| TRAPPIST-1 e | 0.9093     | 0.9990   |
| TRAPPIST-1 f | 0.9826     | 1.0389   |
| TRAPPIST-1 h | 0.8025     | 0.9973   |
| Proxima Cen b | 1.08297    | 1.11909  |

\[ w_i + w_s = 1 \]

We set the weights \( w_i \) and \( w_s \) as 0.5 i.e. equal weights. Thus the calculated CDHS is the mean of the surface score and the interior score. With different weight pairs we get a range of habitability scores for each planet instead of a hard score, making the model more robust than other metrics.

In order to ensure the results are consistent, the CD-HPF was also solved as a single objective optimization problem i.e. its original form:

\[ Y = R^\alpha . D^\beta . V^\delta _e . T^\gamma _s \]

Similar to other single-objective optimization problems, we generated populations of \( \alpha, \beta, \gamma \) and \( \delta \) and evolved them with the proto-genetic algorithm. The results were similar and establish the veracity of the multi-objective optimization approach as listed in Table 4.

Table 4: CDHS obtained using Multi-Objective and Single Objective Optimization

| Exoplanets  | CDHS(Multi-Objective) | CDHS(Single Objective) |
|-------------|-----------------------|------------------------|
| TRAPPIST-1 b | 1.3753                | 1.3684                 |
| TRAPPIST-1 c | 1.2073                | 1.2065                 |
| TRAPPIST-1 d | 1.0146                | 1.0138                 |
| TRAPPIST-1 e | 0.9990                | 0.9972                 |
| TRAPPIST-1 f | 1.0389                | 1.0343                 |
| TRAPPIST-1 h | 0.9973                | 0.9929                 |
| Proxima Cen b | 1.11909               | 1.1158                 |
6 Conclusion

In this paper, we have used a Proto-Genetic algorithm along with NSGA-II to calculate the best habitability scores for different exoplanets using the Cobb-Douglas Habitability Production Function. The optimizing capability of the proto-genetic algorithm was well established by testing on numerous benchmark functions of the single objective, constrained single-objective and multi-objective optimization types. Finally the algorithm was applied in calculating the habitability scores of promising exoplanets from the TRAPPIST system. The results were further verified using the single-objective optimization approach as well, establishing the merit of a genetic bi-objective optimization approach to habitability scores.

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References

1. Back, T.: Evolutionary algorithms in theory and practice: evolution strategies, evolutionary programming, genetic algorithms. Oxford university press (1996)
2. Bora, K., Saha, S., Agrawal, S., Safonova, M., Routh, S., Narasimhamurthy, A.: Cd-hpf: New habitability score via data analytic modeling. Astronomy and Computing 17, 129–143 (2016)
3. Cochran, W.D., Hatzes, A.P., Hancock, T.J.: Constraints on the companion object to hd 114762. The Astrophysical Journal 380, L35–L38 (1991)
4. Coma, C.W., Douglas, P.H.: A theory of production. In: Proceedings of the Fortyieth Annual Meeting of the American Economic Association. vol. 139, p. 165 (1928)
5. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A fast and elitist multiobjective genetic algorithm: Nsga-ii. IEEE transactions on evolutionary computation 6(2), 182–197 (2002)
6. Eberhart, R.C., Shi, Y.: Comparison between genetic algorithms and particle swarm optimization. In: International conference on evolutionary programming, pp. 611–616. Springer (1998)
7. Fonseca, C.M., Fleming, P.J., et al.: Genetic algorithms for multiobjective optimization: Formulation discussion and generalization. In: Icga. vol. 93, pp. 416–423. Citeseer (1993)
8. Ginde, G., Saha, S., Mathur, A., Venkatagiri, S., Vadakkepat, S., Narasimhamurthy, A., Sagar, B.D.: Scientobase: a framework and model for computing scholastic indicators of non-local influence of journals via native data acquisition algorithms. Scientometrics 108(3), 1479–1529 (2016)
9. Holland, J.H., et al.: Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence. MIT press (1992)
10. Mishra, S.K.: Some new test functions for global optimization and performance of repulsive particle swarm method. Available at SSRN 926132 (2006)
11. Mühlenbein, H., Schomisch, M., Born, J.: The parallel genetic algorithm as function optimizer. Parallel computing 17(6-7), 619–632 (1991)
12. Planetary Habitability Laboratory: Exoplanet catalog (2019), http://phl.upr.edu/projects/habitable-exoplanets-catalog/data/database
13. Saha, S., Basak, S., Safonova, M., Bora, K., Agrawal, S., Sarkar, P., Murthy, J.: Theoretical validation of potential habitability via analytical and boosted tree methods: An optimistic study on recently discovered exoplanets. Astronomy and computing 23, 141–150 (2018)
14. Saha, S., Sarkar, J., Dwivedi, A., Dwivedi, N., Narasimhamurthy, A.M., Roy, R.: A novel revenue optimization model to address the operation and maintenance cost of a data center. Journal of Cloud Computing 5(1), 1 (2016)
15. Schulze-Makuch, D., Méndez, A., Faurén, A.G., Von Paris, P., Turse, C., Boyer, G., Davila, A.F., António, M.R.d.S., Catling, D., Irwin, L.N.: A two-tiered approach to assessing the habitability of exoplanets. Astrobiology 11(10), 1041–1052 (2011)
16. Srinivas, N., Deb, K.: Multiobjective optimization using nondominated sorting in genetic algorithms. Evolutionary computation 2(3), 221–248 (1994)

17. Theophilus, A., Saha, S., Basak, S., Murthy, J.: A novel exoplanetary habitability score via particle swarm optimization of ces production functions. In: 2018 IEEE Symposium Series on Computational Intelligence (SSCI). pp. 2139–2147. IEEE (2018)

18. V. Sarasvathi1, N. Ch. S. N. Iyengar2, S.S.: Qos guaranteed intelligent routing using hybrid pso-ga in wireless mesh networks. CYBERNETICS AND INFORMATION TECHNOLOGIES 15(1), 69–83 (2015)