How Recent is Cosmic Acceleration?

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Abstract

Possibly the most peculiar expectation of the standard fine-tuned cosmological paradigm is that cosmic acceleration is to only be a very recent \((z < 1)\) phenomenon, with the universe being required to be decelerating at all higher redshifts. Detailed exploration of the Hubble plot out to \(z = 2\) or so will not only provide an absolute test of this expectation but will also allow for testing of conformal gravity, a non fine-tuned alternate cosmological theory which provides equally good fitting to the current \(z < 1\) Hubble plot data while requiring the universe to be accelerating at higher \(z\) instead. With both standard and conformal gravity being found to be compatible with a very recently reported \(z = 1.7\) data point, additional data points will thus be needed to determine whether the universe is accelerating or decelerating above \(z = 1\).

I. THE HUBBLE PLOT OF STANDARD COSMOLOGY

A variety of recent observational measurements [1–4] have sharply constrained the space available to the standard model cosmological parameters. Specifically, phenomenological data fitting using the standard Einstein-Friedmann cosmological evolution equation

\[
\ddot{R}(t) + k c^2 = \dot{R}(t)(\Omega_M(t) + \Omega_\Lambda(t))
\]

(where \(\Omega_M(t) = 8\pi G \rho_M(t)/3c^2 H^2(t)\) is due to ordinary \(\rho_M(t) \sim 1/R^3(t)\) matter and where \(\Omega_\Lambda(t) = 8\pi G \Lambda/3c^2 H^2(t)\) is due to a cosmological constant \(c\Lambda\)) is found to favor current era values \(\Omega_M(t_0) \simeq 0.3\) and \(\Omega_\Lambda(t_0) \simeq 0.7\), with the current era deceleration parameter \(q_0 = q(t_0) = (n/2 - 1)\Omega_M(t_0) - \Omega_\Lambda(t_0)\) then having to be of order \(-1/2\). While these values are very supportive of the flat \(\Omega_k(t) = -kc^2/\dot{R}^2(t) = 1 - \Omega_M(t) - \Omega_\Lambda(t) = 0\) inflationary universe paradigm [4], they are nonetheless extremely troubling. Specifically, solving Eq. (1) for \(\rho_M(t) = B/\dot{R}^3(t)\) and \(k = 0\) yields \(R(t) = (B/c\Lambda)^{1/3} \sin h^{2/3}(3D^{1/2}t/2)\) where \(D = 8\pi G \Lambda/3c\), so that

\[
\Omega_\Lambda(t) = D/H^2(t) = 1 - \Omega_M(t) = \tanh^2(3D^{1/2}t/2).
\]

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The identification $c|\Lambda| = \sigma T^4_V$ of the cosmological constant with a typical particle physics temperature scale $T_V$ of order $10^{16}$ degrees (viz. $D = 4.7 \times 10^{22}$ sec$^{-2}$) would then yield (for $H_0 = H(t_0) = 65$ km/sec/Mpc) a value for $\Omega_\Lambda(t_0)$ of order $10^{69}$, a value not only overwhelmingly larger than its obtained fitted value, but one not at all compatible with the bounded $\tanh^2(3D^{1/2}t_0/2)$ required by Eq. (2). Or, alternatively, if an $\Omega_M(t_0)$ of order one is taken as a given, an associated $\Omega_\Lambda(t_0)$ of order $10^{69}$ could then only be reconciled with Eq. (2) in the event that $\Omega_k(t_0)$ was of order $-10^{60}$ and thus nowhere near flat.

To get round this problem the standard paradigm then proposes that instead of using such a particle physics based $D$ one should instead, and despite the absence of any currently known justification, fine-tune $D$ down by about 60 orders of magnitude and replace it by the phenomenological $D = 3.1 \times 10^{-36}$ sec$^{-2}$ with the values $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ then ensuing. However, in its turn, such a proposal then engenders a further fine-tuning problem for the standard model since for such a value of $D$ the early universe associated with Eq. (2) would have had to be have been incredibly close to one at the Planck time $t = t_{PL}$, while $\Omega_\Lambda(t = t_{PL})$ would have had to have been as small as $O(10^{-120})$. In fact, given such initial conditions, the universe would then be such that it would decelerate ($q(t) > 0$) continually in all epochs until the cosmological constant finally manages to catch up with the red-shifting matter density, something which for the phenomenologically chosen value for $D$ would occur at the incredibly late $z = 0.67$ when $q(t)$ would at long last finally change sign. While it is very peculiar that such a turn around is to occur just in our particular epoch, nonetheless, independent of one’s views regarding the merits or otherwise of such a proposal, the proposal itself is actually readily amenable to testing, with a modest increase in the range of $z$ (say to $z = 2$) in the $(d_L, z)$ Hubble plot being able to reveal the presence of any possible such turn around. Moreover, such a study would be completely independent of any dynamical assumptions (such as those required for the (complementary) extraction of cosmological parameters from the structure of the cosmic microwave background) and would thus be completely clear cut. Thus in and of itself it would be extremely informative to extend the range of the Hubble plot. However, as we now show, it would be of additional interest since it would allow for a rather unequivocal comparison between standard cosmology and the recently proposed alternate conformal cosmology, a theory where cosmic acceleration is not at all of recent vintage.

II. THE HUBBLE PLOT OF CONFORMAL COSMOLOGY

Given the fine-tuning needs of the standard cosmology, it is of value to explore candidate alternate cosmologies both in and of themselves and also as a (potentially instructive) foil to the standard theory itself. Of such possible alternate theories conformal gravity (viz. gravity based on the fully covariant, locally conformal invariant Weyl action $I_W = -\alpha_g \int d^4x (-g)^{1/2} C^\lambda_{\mu
u\kappa} C^\lambda_{\mu
u\kappa}$ where $C^\lambda_{\mu
u\kappa}$ is the conformal Weyl tensor and where $\alpha_g$ is a purely dimensionless gravitational coupling constant) is immediately suggested since it possesses an explicit symmetry (conformal invariance) which when unbroken would require the cosmological constant to vanish. The cosmology associated with the conformal gravity theory was first presented in [6] where it was shown to possess no flatness problem, to thus release conformal cosmology from the need for the copious amounts of cosmological dark matter required of the standard theory. Subsequently [7,8], the cosmology was shown to also
possess no horizon problem, no universe age problem, and, through negative spatial curvature, to naturally lead to cosmic repulsion. Finally, it was shown that even after the conformal symmetry is spontaneously broken by a $\Lambda$ inducing cosmological phase transition, the cosmology is still able to control the contribution of the induced cosmological constant to cosmic evolution even in the event that $\Lambda$ is in fact as big as particle physics suggests, to thereby provide a completely natural solution to the cosmological constant problem. In the present paper we show that this control actually enables us to provide for a complete and explicit accounting of the recent high z supernovae Hubble plot data without the need for any fine tuning at all.

To explicitly analyze conformal cosmology it is convenient to consider the generic conformal matter action

$$ I_M = -\hbar \int d^4x (g)^{1/2} [S^\mu S_\mu /2 - S^2 R^\mu /12 + \lambda S^4 + \bar{\psi} \gamma^\mu (x)(\partial_\mu + \Gamma_\mu (x))\psi - g S \bar{\psi} \psi] $$

for massless fermions and a conformally coupled order parameter scalar field. For such an action, when the scalar field acquires a non-zero expectation value $S_0$, the entire energy-momentum tensor of the theory is found (for a perfect matter fluid $T^\mu_\nu$ of fermions) to take the form

$$ T^\mu_\nu = T^\mu_\nu_{kin} - \hbar S_0^2 (R^\mu_\nu - g^\mu_\nu R^\alpha_\alpha /2)/6 - g^\mu_\nu \hbar \lambda S_0^4, $$

with the complete solution to the scalar, fermionic and gravitational field equations of motion in a background Robertson-Walker geometry (viz. a geometry in which $C^{\mu_1 \nu_1 \kappa \lambda} = 0$) then reducing to just one relevant equation, viz. $T^\mu_\nu = 0$, a remarkably simple condition which immediately fixes the zero of energy. We thus see that the evolution equation of conformal cosmology looks identical to that of standard gravity save only that the quantity $-\hbar S_0^2/12$ has replaced the familiar $c^2/16\pi G$, so that instead of being attractive the effective cosmological $G_{eff} = -3c^2/4\pi \hbar S_0^2$ is actually negative to thus naturally lead to cosmic repulsion, and instead of being fixed as the standard low energy $G$, the cosmological $G_{eff}$ is instead fixed by the altogether different scale $S_0$ to thus enable it to be altogether smaller than $G$. As we shall see below, it is precisely the replacing of the cosmological $G$ by an altogether smaller $G_{eff}$ that enables conformal gravity to naturally solve the cosmological constant problem.

Given the equation of motion $T^\mu_\nu = 0$, the ensuing conformal cosmology evolution equation is then found (on setting $\Lambda = \hbar \lambda S_0^4$) to take a form quite similar to Eq. (4), viz.

$$ \ddot{R}^2 (t) + kc^2 = \ddot{R}^2 (t) (\tilde{\Omega}_M (t) + \tilde{\Omega}_\Lambda (t)) $$

where $\tilde{\Omega}_M (t) = 8\pi G_{eff} \rho_M (t) /3c^2 H^2 (t)$, $\tilde{\Omega}_\Lambda (t) = 8\pi G_{eff} \Lambda /3c H^2 (t)$. Further, unlike the situation in the standard theory where preferred values for the relevant evolution parameters (such as the magnitude and even the sign of $\Lambda$) are only determined by the data fitting itself, in conformal gravity essentially everything is already a priori known. With conformal gravity not needing dark matter to account for non-relativistic issues such as galactic rotation curve systematics, $\rho_M (t_0)$ can be determined directly from luminous matter alone, with galaxy luminosity counts giving a value of it of order $0.01 \times 3c^2 H_0^2 /8\pi G$ or so. Further, with $c\Lambda$ being generated by particle physics vacuum breaking in an otherwise scaleless theory, since such breaking lowers the energy density, $c\Lambda$ must unambiguously be negative, with it thus being typically given by $-\sigma T^4_V$ where $T_V$ is a necessarily particle physics sized scale. Then
with $G_{\text{eff}}$ also being negative, the quantity $\bar{\Omega}_\Lambda(t)$ must thus be positive, just as needed to give cosmic acceleration ($q(t) = (n/2 - 1)\bar{\Omega}_M(t) - \bar{\Omega}_\Lambda(t)$). Similarly, the sign of the spatial 3-curvature $k$ is known from theory \[\text{[11]}\] to be negative, something which has been independently confirmed from a phenomenological study of galactic rotation curves \[\text{[14]}\]. Moreover, since $G_{\text{eff}}$ is negative, the cosmology is singularity free and thus expands from a (negative) quantity $\bar{\Omega}_G$ \[\text{[15]}\].

Consequently in the conformal theory we never need to fine tune in order to make any particular epoch such as our own be an accelerating one, and as we shall show below, the conformal theory not only gives some current era acceleration but in fact gives just the amount needed to account for the currently available Hubble plot data.

Given only that $\Lambda$, $k$ and $G_{\text{eff}}$ are in fact all negative in the conformal theory, the evolution of the theory is then completely determined, with the expansion rate being found \[\text{[11]}\] to be given by

$$R^2 = -k(\beta - 1)/2\alpha - k\beta \sinh^2(\alpha^{1/2}cT)/\alpha, \quad T_{\max}^2/T^2 = 1 + 2\beta \sinh^2(\alpha^{1/2}cT)/(\beta - 1),$$

where $\alpha c^2 = -2\lambda S_0^2 = 8\pi G_{\text{eff}}\Lambda/3c$, $\beta = (1 - 16\Lambda/k^2hc)^{1/2} = (1 + T_V^4/T_{\max}^4)/(1 - T_V^4/T_{\max}^4)$.

In terms of the parameters $T_{\max}$ and $T_V$ we thus obtain

$$\bar{\Omega}_\Lambda(t) = (1 - T^2/T_{\max}^2)^{-1}(1 + T^2T_{\max}^4/T_V^4)^{-1}, \quad \bar{\Omega}_M(t) = -(T^4/T_V^4)\bar{\Omega}_\Lambda(t)$$

at any $T(t)$ without any approximation at all. From Eq. \[\text{[11]}\] we thus immediately see that simply because $T_{\max}$ is overwhelmingly larger than the current temperature $T(t_0)$, i.e. simply because the universe is as old as it is, it automatically follows, without any fine-tuning at all, that the current era $\bar{\Omega}_\Lambda(t_0)$ has to lie somewhere between zero and one today no matter how big (or small) $T_V$ might actually be, with conformal gravity thus having total control over the contribution of the cosmological constant to cosmic evolution. Conformal gravity thus solves the cosmological constant problem by quenching $\bar{\Omega}_\Lambda(t_0)$ rather than by quenching $\Lambda$ itself (essentially by having a $G_{\text{eff}}$ which is altogether smaller than the standard $G$), and with it being the quantity $\bar{\Omega}_\Lambda(t_0)$ which is the one which is actually measured in cosmology, it is only its quenching which is actually needed. With conformal gravity thus being able to naturally accommodate a large $\Lambda$ we are now actually free to allow $T_V$ to be as large as particle physics suggests. Then, for such a large $T_V/T(t_0)$ we see that the quantity $\bar{\Omega}_M(t_0)$ has to be completely negligible today \[\text{[11]}\] so that $q_0$ must thus, without any fine-tuning at all, necessarily lie between zero and minus one today notwithstanding that $T_V$ is huge. Moreover, noting that

$$\tanh^2(\alpha^{1/2}cT) = (1 - T^2/T_{\max}^2)/T_{\max}^2T^2/T_V^4 + 1,$$

we immediately see that the current era $\bar{\Omega}_\Lambda(t_0)$ is given by the completely bounded $\tanh^2(\alpha^{1/2}cT_0)$ (so that the current era curvature contribution to cosmic expansion is then given as $\Omega_k(t_0) = \text{sech}^2(\alpha^{1/2}cT_0)$), with the current deceleration parameter being given by the nicely bounded $q_0 = -\tanh^2(\alpha^{1/2}cT_0)$. The essence of the conformal gravity approach then is not to change the matter and energy content of the universe but rather only their effect on cosmic evolution, with the cosmological constant itself no longer needing to be quenched.
While completely foreign to standard gravity, a universe in which \( \rho_M(t) \) makes a completely negligible contribution to current era cosmic evolution is, as we now show, nonetheless fully compatible with the currently available \( z < 1 \) Hubble plot data. Specifically, through use of type Ia supernovae the authors of [12] were able to measure the dependence of luminosity distance \( d_L \) on redshift out to \( z = 1 \). To fit their data we thus need to determine the dependence of \( d_L \) on \( z \) in the conformal theory, something we can readily do now that we have obtained the expansion factor \( R(t) \). Thus, in the conformal theory we find first that the Hubble parameter is given as

\[
H(t) = \alpha^{1/2}c(1 - T^2(t)/T_{\text{max}}^2)/\tanh(\alpha^{1/2}ct)
\]

with its current \( (T_{\text{max}} \gg T(t_0)) \) value being found to obey \(-q_0 = \tanh^2(\alpha^{1/2}ct_0) = \alpha^2/H_0^2\), with the current age of the universe then being given by \( t_0 = \text{arctanh}((-q_0)^{1/2})/((-q_0)^{1/2}H_0) \), viz. by \( t_0 \geq 1/H_0 \). For temperatures well below \( T_{\text{max}} \) and for the naturally achievable \( T_V \ll T_{\text{max}} \) case of most practical interest to conformal gravity (viz. a case where \( T_{\text{max}}^2 T^2(t_0)/T_V^3 \) can be of order one) we may set \( R(t) = (-k/\alpha)^{1/2}\sinh(\alpha^{1/2}ct) \), so that for geodesics \( \int_{t_0}^{t_1} cdt/R(t) = \int_{t_0}^{t_1} dr/[1 - k \alpha^2 r^2]^{1/2} \) we obtain

\[
(-k)^{1/2}r_1 = \coth(\alpha^{1/2}ct_0)/\sinh(\alpha^{1/2}ct_1) - \coth(\alpha^{1/2}ct_1)/\sinh(\alpha^{1/2}ct_0).
\]

Then, with \( \sinh(\alpha^{1/2}ct) = (-q_0)^{1/2}/(1 + q_0)^{1/2}(1 + z) \) where \( z = R(t_0)/R(t_1) - 1 \), we find that we can express the general luminosity distance \( d_L = r_1 R(t_0)(1 + z) \) entirely in terms of the current era \( H_0 \) and \( q_0 \) according to the very compact relation

\[
H_0 d_L/c = -(1 + z)^2 \left\{ 1 - [1 + q_0 - q_0/(1 + z)^2]^{1/2} \right\} /q_0
\]

Conformal gravity fits to the luminosity distance can thus be parametrized via the one parameter \( q_0 \), a parameter which must lie somewhere between zero and minus one, with \( d_L \) thus having to lie somewhere between \( d_L(q_0 = 0) = cH_0^{-1}(z + z^2/2) \) and \( d_L(q_0 = -1) = cH_0^{-1}(z + z^2) \) at temperatures well below \( T_{\text{max}} \). Given Eq. (11) we turn now to a data analysis.

### III. FITS TO THE HUBBLE PLOT

For the fitting we shall follow [12] and fit 38 of their 42 data points together with 16 of the 18 earlier lower \( z \) points of [17], for a total of 54 data points with reported effective blue apparent magnitude \( m_i \) and uncertainty \( \sigma_i \). (While we thus leave out 6 questionable data points for the fitting, nonetheless, for completeness we still include them in the figures.) For the fitting we calculate the apparent magnitude \( m \) of each supernova at redshift \( z \) via \( m = 25 + M + 5\log_{10} d_L \) (\( d_L \) in Mpc) where \( M \) is their assumed common absolute magnitude, and minimize \( \chi^2(q_0, M) = \sum(m - m_i)^2/\sigma_i^2 \) as a function of the two parameters \( q_0 \) and \( M \). In all of our fits \( M \) (as determined using \( H_0 = 65 \text{ km/sec/Mpc} \)) is found to be in agreement with the analyses of [12], with our best fit \( \chi^2 = 58.62 \) being obtained for \( q_0 = -0.37, M = -19.37 \). We display this fit as the upper curve in Fig. (1) where we also present as the lower curve the corresponding \( \Omega_M(t_0) = 0.3, \Omega_\Lambda(t_0) = 0.7 \) standard model fit, a fit which gives \( \chi^2 = 57.74 \) (and \( M = -19.37 \)) for the same 54 points. As we thus see, in the detected region the best fits of the two models are completely indistinguishable, only in fact departing
from each other at the highest available redshifts. Moreover, the conformal cosmology fits turn out to be extremely insensitive to the actual value of $q_0$, with other typical $q_0, M$ fits being $\chi^2(0, -19.29) = 61.49, \chi^2(-0.25, -19.34) = 58.96, \chi^2(-0.5, -19.40) = 59.11, \chi^2(-0.75, -19.46) = 63.59, and \chi^2(-1.00, -19.54) = 75.79$. The data will thus support conformal cosmologies with a fairly broad range of negative values of $q_0$ beginning at $q_0 = 0$. In fact such high quality $q_0 = 0$ fits (viz. fits with $R(t) = (-k)^{1/2}ct$) have already been reported earlier. The authors of [2], having noted in passing that such fits were actually as good as their best standard model fits, and with the authors of [20] having presented such $q_0 = 0$ fits in an exploration of generic power law expansion rate cosmologies. What is new here is that we derive such $q_0 = 0$ fits within the framework of a well-defined cosmological model while also extending them to non-zero $q_0$ (i.e. to non-zero $\Lambda$).

As we just noted, current Hubble plot data do not allow us to resolve between standard and alternate gravity theories. However, since the standard theory is a decelerating one above $z = 1$ while the conformal theory continues to accelerate, continuation of the Hubble plot beyond $z = 1$ will actually enable us to discriminate between the various options. Thus in Fig. (2) we plot the higher $z$ expectations. The highest curve in the figure is the conformal gravity fit for $q_0 = -0.37$, the middle curve in the figure is the conformal gravity fit for $q_0 = 0$, and the lowest curve in the figure is the $\Omega_M(t_0) = 0.3, \Omega_\Lambda(t_0) = 0.7$ standard model expectation. As we see, the curves start to depart from each other fairly rapidly once we go above $z = 1$, with the three cases respectively yielding $m = 27.17, m = 27.04$ and $m = 26.75$ at $z = 2$, a difference of at least 0.3 magnitudes between standard gravity and the conformal alternative. (At $z = 5$ the respective magnitudes are $m = 30.40, m = 30.25$ and $m = 29.14$.) Since conformal gravity handles the cosmological constant problem, the primary problem troubling the standard theory, so readily (its other successes and its own difficulties are discussed in [8,10]) it would thus appear to merit further consideration, with only a modest extension of the Hubble plot readily enabling us to discriminate between standard gravity and its conformal alternative while potentially even being definitive for both. The author is indebted to Drs. G. V. Dunne, K. Horne, D. Lohiya, R. Plaga and B. E. Schaefer for helpful comments. This work has been supported in part by the Department of Energy under grant No. DE-FG02-92ER40716.00.

**IV. ADDED NOTE**

Since this paper was first released it was announced [22] that new analysis of SN 1997ff now puts this supernova at $z = 1.7^{+0.1}_{-0.15}$ and establishes a lower bound on its luminosity large enough to definitively exclude the two most commonly considered non-cosmological explanations (dust extinction and intrinsic luminosity evolution) of the supernovae Hubble plot data. Interestingly, this lower bound turns out to be very close not only to the values ($m = 26.52, m = 26.64$) expected of $q_0 = 0$ and $q_0 = -0.37$ conformal gravity at $z = 1.7$ but also to the nearby $m = 26.32$ expectation of an $\Omega_M(t_0) = 0.3, \Omega_\Lambda(t_0) = 0.7$ standard model at that $z$, thus making further exploration of the $z > 1$ region potentially very significant. In their analysis the authors of [22] present their SN 1997ff data in the form of confidence contours in distance modulus ($m - M$) versus redshift space. To illustrate their data we have augmented Fig. (2) by adding in the 68% and 95% confidence region values for the apparent magnitude $m$ (as obtained from the quoted $m - M$ by adding $M = -19.37$) at
redshifts $z = 1.65, z = 1.7$ and $z = 1.75$, and have presented an enlarged view of the relevant region in Fig. (3). (In these figures the two inner horizontal bars on the vertical data points represent the extent of the 68% confidence region at each of the chosen redshifts while the two outer bars represent the 95% confidence one.) To estimate the effect of SN 1997ff on the $\hat{\chi}^2(q_0, M)$ that we obtained for our fits to the $z < 1$ data we have calculated the change in $\Delta \hat{\chi}^2(q_0, M)$ generated by an effective additional data point with the 68% confidence region value of $m = 25.83 \pm 0.5$ at redshift $z = 1.7^{+0.1}_{-0.15}$, to find typical values $\Delta \hat{\chi}^2(0, -19.28) = 1.60$ and $\Delta \hat{\chi}^2(-0.33, -19.35) = 2.08$ to be compared with $\Delta \hat{\chi}^2(-19.36) = 0.83$ for the standard model.

While the conformal gravity $\Delta \hat{\chi}^2$ values are actually less than those generated by some of the outlier points in the $z < 1$ data, it is important to note that as well as possessing statistical errors the SN 1997ff data happen to also possess an explicitly identified systematic error. Specifically SN 1997ff just happens to be lensed by two foreground galaxies at $z = 0.56$ both of which lie very close to the line of sight, with the authors of estimating that this would cause SN 1997ff to appear 0.2 magnitudes or so brighter than it actually is. Dimming the quoted distance modulus data by this amount would serve to move conformal gravity into the 68% confidence region with the modified $\Delta \hat{\chi}^2(q_0, M)$ then becoming $\Delta \hat{\chi}^2(0, -19.28) = 0.82$ and $\Delta \hat{\chi}^2(-0.33, -19.35) = 1.16$ to be compared with a modified $\Delta \hat{\chi}^2(-19.36) = 0.29$ for the standard model. Thus we see that at present it is not yet possible to discriminate between standard gravity and the conformal alternative, with both of the theories being compatible with the SN 1997ff data. In fact, given that there is a systematic lensing effect (and as yet only one $z > 1$ data point of course), we see that since the dimmed data straddle the coasting ($q(t) = 0$ at all times) universe (equivalent to $q_0 = 0$ conformal gravity), at the present time one is not in fact able to determine whether the universe is actually accelerating or decelerating above $z = 1$ with further data (such as those anticipated in the upcoming SNAP satellite mission) thus being needed to resolve the issue and identify any specific trend in the $z > 1$ region that might exist.
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[15] Included in this class of q(t) < 0 universes are the coasting ones in which q(t) = 0^{-}.
[16] Ω_{M}(t_{0}) is suppressed by G_{eff} being small, and not by ρ_{M}(t_{0}) itself being small, with G_{eff} being made smaller the larger rather than the smaller S_{0} gets to be, to thus enable the cΛ/ρ_{M}(t_{0}) = Ω_{Λ}(t_{0})/Ω_{M}(t_{0}) ratio to be as large as particle physics suggests while not leading to any 60 order of magnitude conflict with observation.
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[18] Since redshift position uncertainties Δz_{i} are also reported in the supernovae data, for completeness we have also minimized χ^{2}(q_{0}, M) = (m - m_{i})^{2}/(σ_{i}^{2} + σ_{z_{i}}^{2}) where σ_{z_{i}}^{2} = [5log_{10}d_{L}(q_{0}, z_{i} + Δz_{i}) - 5log_{10}d_{L}(q_{0}, z_{i})]^{2}/2 + [5log_{10}d_{L}(q_{0}, z_{i} - Δz_{i}) - 5log_{10}d_{L}(q_{0}, z_{i})]^{2}/2 is the effect of the redshift uncertainty, with it being the fitting procedure itself which is used to normalize σ_{z_{i}} to the apparent magnitude. With this prescription we then find that our best conformal gravity fit is now given as χ^{2}(-0.33, -19.35) = 54.13 for the 54 points, to be compared with the corresponding best Ω_{M}(t_{0}) = 0.3, Ω_{Λ}(t_{0}) = 0.7 standard model fit of χ^{2}(-19.36) = 53.27. Moreover, the conformal cosmology fits continue to be insensitive to the actual value of q_{0}, with other typical q_{0}, M fits then being χ^{2}(0, -19.28) = 56.00, χ^{2}(-0.25, -19.33) = 54.26, χ^{2}(-0.5, -19.39) = 54.78, χ^{2}(-0.75, -19.47) = 59.01, and χ^{2}(-1.00, -19.55) = 69.88.
[19] While such R(t) = (-k)^{1/2}ct fits would have to be associated with an Ω_{M}(t_{0}) = 0, Ω_{Λ}(t_{0}) = 0 empty universe in the standard model, in the conformal theory they would instead be associated with a current era Ω_{M}(t_{0}) = 0, Ω_{Λ}(t_{0}) = 0 universe,viz. one with its normal fill of matter but which is now in a curvature dominated phase.
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[21] With $k$ explicitly being negative in conformal gravity, determining its predictions for the temperature dependence of the cosmic microwave background is thus also paramount for the theory.

[22] A. G. Riess et. al., astro-ph/0104455, Astrophys. J. (in press).

[23] Even in the standard model fits to the data below $z = 1$ some 10 of the 54 fitted points are each individually found to contribute an amount in excess of 2 units to the total $\hat{\chi}^2(-19.36) = 53.27$ (the largest single individual contribution goes as high as 7.10), with all of these particular points thus lying quite a bit away from the expected standard model value. With there thus being a lot of scatter in the Hubble plot data one can only extract limited information from any given single data point.

[24] In fact it is only the assumption of the a priori validity of standard gravity which leads to the favoring of deceleration above $z = 1$. 
FIG. 1. The $q_0 = -0.37$ conformal gravity fit (upper curve) and the $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ standard model fit (lower curve) to the $z < 1$ supernovae Hubble plot data.
FIG. 2. Hubble plot expectations for $q_0 = -0.37$ (highest curve) and $q_0 = 0$ (middle curve) conformal gravity and for $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ standard gravity (lowest curve).
FIG. 3. The $q_0 = -0.37$ (highest curve) and $q_0 = 0$ (middle curve) conformal gravity fits and the $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ standard model fit (lowest curve) to the $z < 2$ Hubble plot data.