Optimal design of an IPM motor using Taguchi and Rosenbrock’s methods

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Abstract. Techniques for the design optimization for cogging torque minimization and average torque maximization of a high-speed 2-pole interior permanent magnet (IPM) synchronous motor are presented. It is shown by the finite element method (FEM) and measurement, that combined the Taguchi and Rosenbrock’s methods is a very efficient and effective approach in robust design a high performance motor.

1. Introduction
High performance IPM synchronous motors have been used extensively in recent years for industrial applications [1]-[3]. The maximum speed of such motors is about 6000 rpm with four magnet poles in the rotor. But in some applications, such as the fiber industry or hand dryers, a higher motor speed is required. To keep the same inverter, because of cost, the use of a two-pole motor is necessary [4]. Unfortunately, machines equipped with two magnet poles exhibit a high level of cogging torque. It is well known that cogging torque produces vibration and noise which may be detrimental to the performance of position and speed control systems.

In the literature, several methods for reducing the cogging torque in brushless permanent magnet motors have been proposed [5-9]. However, these techniques increase the complexity of the motor construction and reduce the average output torque. In this paper, technique is proposed in order to minimize the level of cogging torque without any sacrifice of average output torque. These goals can be accomplished using the optimization methods.

Stochastic methods of optimization have gained popularity over the last few years for optimization of engineering problems due to their fast convergence on a global optimum. Among the developed of stochastic methods, the Rosenbrock’s method is robot and flexible [10-12]. However, the major concern of this technique is that it requires a large computation time of numerical filed computation. It can not consider for reducing the cogging torque by means of shaping the magnets either [7]. On the other hand, the Taguchi method is not required to use sophisticated algorithms and additional programming aside from the FEM of electromagnetic field analysis [13, 14]. It also allows many settings of as many as necessary design parameters in optimization computation simultaneously. Hence, effects of several factors on cogging torque and average torque can be investigated.
In this paper, the Taguchi method is used for the determination of the initial optimal values of the design variables for minimum cogging torque and maximum average torque [13, 14], and the Rosenbrock’s method is applied to determine the optimal settings of the design parameters [10-12].

2. Model of Motor
Figure 1 shows a three-phase, two-pole, six-slot IPM motor. The initial design parameters are stator outside diameter = 90 mm, inner diameter = 53 mm, stack length = 65 mm, and rotor outside diameter = 52 mm. The core is made of nonoriented silicon steel, and two sintered NdFeB magnets with a remanence of 1.2 T and a relative recoil permeability of 1.05 are inserted in the rotor. The rotating speed is 12000 rpm.

![Figure 1. Model of IPM motor.](image)

3. Taguchi’s Methodology
3.1. Establish orthogonal array
Taguchi’s method provides a systematic and efficient approach for conducting the numerical experimentation to determine near optimum settings of design parameters. An orthogonal array representing factors and their levels allowable in the design is used to study the parameter space. Considering feasible refinements, Table 1 shows the selected eight factors, A, B, C, D, E, F, G and H affecting the values of cogging torque and average torque. Each factor corresponding to one design variable as shown in Figure 1, and each at three levels except factor A that has two levels. The values with the underline denote the initial design.

| Factor | Definition | Level 1 | Level 2 | Level 3 |
|--------|------------|---------|---------|---------|
| A      | magnetization of magnets | parallel | radial | -- |
| B (mm) | magnetic island width | 2 | 2.5 | 3 |
| C (mm) | air gap length | 0.5 | 0.6 | 0.7 |
| D      | ratio of magnet pitch to pole pitch | 0.778 | 0.806 | 0.833 |
| E (mm) | thickness of magnet | 3 | 3.5 | 4 |
| F (mm) | magnetic bridge width | 1 | 1.2 | 1.4 |
| G (mm) | web width | 1 | 1.2 | 1.4 |
| H (mm) | slot opening | 2.5 | 3 | 3.5 |

3.2. Conduct the experiment
There are 18 experiments required for us to determine the optimum combination of the levels of these factors as shown in Table 2. To obtain the values of cogging and average torques for each case, 2D FEM analysis is conducted. Table 2 tabulates the motor performance indexes. Where $T_c$ is the peak to peak value of cogging torque, and $T_{ave}$ is the value of average torque.
Table 2. Motor performance assessments from Taguchi’s experiments.

| Experiment | Factor level combination | T_c (Nm) | T_ave (Nm) |
|------------|--------------------------|----------|------------|
| 1          | A1-B1-C1-D1-E1-F1-G1-H1  | 0.663    | 2.137      |
| 2          | A1-B1-C2-D2-E2-F2-G2-H2  | 0.619    | 2.215      |
| 3          | A1-B1-C3-D3-E3-F3-G3-H3  | 0.525    | 2.327      |
| 4          | A1-B2-C1-D1-E2-F2-G2-H2  | 0.503    | 2.161      |
| 5          | A1-B2-C2-D3-E3-F3-G1-H1  | 0.455    | 2.231      |
| 6          | A1-B2-C3-D3-E1-F1-G2-H2  | 0.199    | 2.028      |
| 7          | A1-B3-C1-D2-E1-F3-G2-H3  | 0.183    | 2.004      |
| 8          | A1-B3-C2-D3-E2-F1-G3-H1  | 0.270    | 2.013      |
| 9          | A1-B3-C3-D1-E3-F2-G1-H2  | 0.300    | 2.105      |
| 10         | A2-B1-C1-D3-E3-F2-G2-H1  | 0.658    | 2.603      |
| 11         | A2-B1-C2-D1-E1-F3-G3-H2  | 0.165    | 2.351      |
| 12         | A2-B1-C3-D2-E2-F1-G1-H3  | 0.791    | 2.454      |
| 13         | A2-B2-C1-D2-E3-F1-G3-H2  | 0.852    | 2.440      |
| 14         | A2-B2-C2-D3-E1-F2-G1-H3  | 1.063    | 2.371      |
| 15         | A2-B2-C3-D1-E2-F3-G2-H1  | 0.526    | 2.319      |
| 16         | A2-B3-C1-D3-E2-F3-G1-H2  | 1.151    | 2.367      |
| 17         | A2-B3-C2-D1-E3-F1-G2-H3  | 1.138    | 2.362      |
| 18         | A2-B3-C3-D2-E1-F2-G3-H1  | 0.850    | 2.236      |

3.3. Analysis of results
The means of all results can be calculated as

\[ m = \frac{1}{18} \sum_{i=1}^{18} T_i \]  

The means are T_c = 0.6062 Nm and T_ave = 2.2624 Nm.

3.4. Calculate average effect
The peak to peak value of cogging torque of setting factor C at level 3 are calculated as

\[ m_c(T_c) = \frac{1}{6}(T_c(3)+T_c(6)+T_c(9)+T_c(12)+T_c(15)+T_c(18)) \]

where the factor C is set to level 3 only in experiments 3, 6, 9, 12, 15 and 18. The peak to peak values of cogging torque for all levels of factors can be obtained in a similar way. Figures 2 and 3 illustrate the main factor effects on cogging torque and average torque, respectively.

![Figure 2. Main factor effects on cogging torque.](image)

![Figure 3. Main factor effects on average torque.](image)
3.5 Analysis of variance

Analysis of variance (ANOVA) provides a measure of confidence. To conduct ANOVA, the sum of squares (SS) is calculated first, and it can be calculated as,

$$SSF_A = n \sum_{i=1}^{n} (m_i - m)^2$$  \hspace{1cm} (3)

where \( n \) denotes the number of levels for each factor. The \( SSF_B \sim SSF_H \) can be obtained in the same way. These results are summarized in Table 3.

| Factor | SSF   | Factor effects(%) |
|--------|-------|-------------------|
| \( T_c \) | 1.326 | 78.619            |
| \( T_{avg} \) | 15.452 | 99.427            |

### Table 3. Effects of various factors on motor performance indexes.

| Factor | SSF | Factor effects(%) |
|--------|-----|-------------------|
| \( T_c \) | 1.326 | 78.619 |
| \( T_{avg} \) | 15.452 | 99.427 |

3.6 Design Optimization from Taguchi method

It is noted in Figure 2 that the factor-level combination (A1, B1, C3, D1, E1, F3, G3, H2) contributes to minimization of cogging torque, and it can be seen in Figure 3 that the factor-level combination (A2, B1, C1, D3, E3, F2, G1, H3) contributes to maximization of average torque. Therefore, the element B1 is immediately selected to constitute the elements of the optimum design for minimum cogging torque and maximum average torque. It is shown in Table 3 that factor A has large effect on average torque (99.427%) to cogging torque (78.619%), and factors from C to H have large effect on cogging torque to average torque. Therefore, the best combination of design parameters for minimum cogging torque and maximum average torque is determined to be (A2, B1, C3, D1, E1, F3, G3, H2). Based on this combination of design parameters, 2-D FEM analysis was used to determine the performance of the optimized motor. Results show that cogging torque reduces from the initial design of 1.343 Nm to Taguchi method of 0.241 Nm, and average torque increases from the initial design of 2.307 Nm to Taguchi method of 2.384 Nm.

4. Rosenbrook’s method and results

The Rosenbrook’s method of rotating coordinate is used to search the optimal value. The coordinate system is rotated at any stage for finding the minimum. The details of the steps to reach the solutions are described in [11, 12]. In this section, the optimization using Rosenbrook’s method is carried out so that the cogging torque becomes minimal under the constraint that the average torque is not less than 2.4 Nm. Using the initial design variables from the Taguchi method, the constraints of design variables B, C, D, E, F, G and H are defined as follows:

\[
\begin{align*}
2.0 & \leq B \leq 2.5; \quad 0.6 \leq C \leq 0.7 \\
0.778 & \leq D \leq 0.8; \quad 3 \leq E \leq 3.5 \\
1.4 & \leq F \leq 1.5; \quad 1.4 \leq G \leq 1.5 \\
3 & \leq H \leq 3.3
\end{align*}
\]  \hspace{1cm} (4)

The performance of the motor was obtained using 2-D FEM analysis. Table 4 compares the data of the motor between the initial design, Taguchi, Rosenbrook’s methods and measurement. Figures 4 and 5 compare the results of cogging torque and average torque. It is seen that the values of predicted and measured of cogging torque and average torque are good agreement with each other. Also, it can be seen that cogging torque reduces from the initial design of 1.343 Nm and Taguchi method of 0.241 Nm to Rosenbrook’s method of 0.206 Nm, and average torque increases from the initial design of 2.307 Nm and Taguchi method of 2.384 Nm to Rosenbrook’s method of 2.432 Nm.

| Factor | A   | B   | C   | D   | E   | F   | G   | H   |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|
| \( T_c \) | 1.326 | 0.009 | 0.029 | 0.015 | 0.138 | 0.050 | 0.078 | 0.041 |
| \( T_{avg} \) | 15.452 | 0.042 | 0.003 | 0.003 | 0.036 | 0.003 | 0.001 | 0.001 |

4
Table 4. Comparison results.

|                | A | B(mm) | C(mm) | D | E(mm) | F(mm) | G(mm) | H(mm) | Tc (Nm) | Tave (Nm) |
|----------------|---|-------|-------|---|-------|-------|-------|-------|---------|-----------|
| Initial radial | 3 | 0.5   | 0.833 | 3 | 1     | 1     | 3     | 1.343 | 2.307   |
| Taguchi radial | 2 | 0.7   | 0.778 | 3 | 1.4   | 1.4   | 3     | 0.241 | 2.384   |
| Rosenbrock radial | 2.212 | 0.652 | 0.793 | 3.152 | 1.412 | 1.423 | 3.123 | 0.206 | 2.432   |
| Test           |   |       |       |   |       |       |       |       | 0.210   | 2.410     |

Figure 4. Comparison of cogging torques.  
Figure 5. Comparison of average torques.

5. Conclusion
This paper applied the combination of the Taguchi and Rosenbrock’s methods to the design optimization for cogging torque minimization and average torque maximization of a high-speed 2-pole IPM synchronous motor. It was shown that the technique presented in this paper is effective for obtaining the design parameters having low levels of cogging torque. The technique would link the existing FEM packages to complete an iterative design loop.

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6. References
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