Peculiarities of superconducting properties of thin superconductor-normal metal bilayer with large ratio of resistivities

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We demonstrate, both theoretically and experimentally, that thin dirty superconductor-normal metal bilayer with resistivity of normal metal \( \rho_N \) much smaller than normal-state resistivity of superconductor \( \rho_S \) has unique superconducting properties. First of all the normal layer provides the dominant contribution to the diamagnetic response of whole bilayer structure in wide temperature interval below the critical temperature due to proximity induced superconductivity. Secondly, the presence of the normal layer may increase the critical current \( I_c \) in several times (the effect is not connected with enhanced vortex pinning), provides strong temperature dependence of both \( I_c \) and effective magnetic field penetration depth even at temperatures much below the critical one and leads to the diode effect in parallel magnetic field. Besides of general interest we believe that the found results may be useful in construction of different kinds of superconducting detectors.

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I. INTRODUCTION

If superconductor (S) is attached to the normal metal (N) and the SN interface is transparent for electron motion then superconducting electrons penetrate the normal metal on the characteristic length scale \( \xi_N(T) \). It leads to superconducting properties of the normal metal, namely, it can carry non-dissipative current and screen applied magnetic field. The screening effect was observed experimentally in many works on different SN systems in clean and dirty limits using Eilenberger or Usadel equations.

Here we demonstrate that thin low resistive normal (N) layer placed on the superconducting (S) layer with large normal-state resistivity \( \rho_S \) can considerably increase its superconducting properties, namely considerably enhances the diamagnetic response and critical current \( I_c \). We argue that the effect comes from proximity induced superconductivity and locally smaller London penetration depth \( \lambda \) in N-layer - see inset in Fig. 1(a). In this system the critical current enhancement is not connected with enhanced vortex pinning in SN bilayer but it is related with large superconducting current density \( j_s \sim 1/\lambda^2 \) in N-layer. The strong temperature dependence of \( I_c \) even at \( T \ll T_c \), suppression of \( I_c \) in rather weak perpendicular magnetic field, the diode effect in parallel magnetic field and enhanced diamagnetism validates in favor of this interpretation of the found experimental results.

We have to mention that the critical current density \( j_{cN} \) in N-layer of dirty SNS trilayer with thickness of N-layers \( d_N \ll \xi_N(T) = (\hbar D_N/k_BT)^{1/2} \), thickness of S-layer \( d_S \gg \xi_c = (\hbar D_S/k_BT_{c0})^{1/2} \sim \xi(0) \) \( (D_{S(N)} \) is a diffusion coefficient of corresponding layers, \( T_{c0} \) is the critical temperature of superconductor, \( \xi(0) \) is the zero temperature superconducting coherence length) and ratio of resistivities \( \rho_S/\rho_N \gg 1 \) first was analytically calculated in Refs. \cite{4, 10}. Authors predicted that \( j_{cN} \) may exceed depairing current density \( j_{dep} \) of the superconductor and \( j_{cN}(T) \sim 1/\sqrt{T} \) but they did not calculate the critical current of whole structure. Our calculations show that N-layer could lead to enhancement of critical current \( I_c \) of whole bilayer and nontrivial \( I_c(T) \) only at low temperatures \( T \ll T_{c0} \) when thickness of S or N layers exceed several \( \xi_c \) (for bilayer with realistic \( \rho_S/\rho_N \lesssim 200 \) while in wide temperature interval nontrivial \( I_c(T) \) exists when \( d_S, d_N \lesssim 2\xi_c \). Because in Refs. \cite{9, 11} the symmetric SNS system was studied the diode effect in parallel magnetic...
field was absent.

The structure of the paper is following. In Sec. II we present our theoretical results. In Sec. III we show results of the experiment and in Sec. IV we compare our experimental and theoretical results, discuss their relation with other experiments and possible application of such bilayers.

II. THEORETICAL RESULTS

To calculate superconducting properties of SN bilayer we mainly use Usadel equation for anomalous $F$ and normal $G$ Green functions (equations and details of numerical calculations are present in Appendix A). In Fig. 1(a,b) we show calculated inverse effective magnetic field penetration depth $\Lambda^{-1} = \int dx/\lambda(x)^2$ (in a case of single superconducting layer with $\lambda(x) = \text{const}$, $\Lambda = \lambda^2/d_{S}$ - is the Pearl penetration depth). $\Lambda$ is normalized in units of $\Lambda_0 = \lambda(0)^2/d_{S}$, where $\lambda(0)$ is the London penetration depth of single S-layer at $T = 0$. $\Lambda^{-1}$ describes the screening ability of bilayer and for relatively large ratio $\rho_S/\rho_N$ and temperature not far below from $T_c$ it could considerably exceeds $\Lambda^{-1}$ of single S-layer (see Fig. 1(a)). This effect originates from relation $1/\lambda^2 \sim 1/\rho$ valid in the dirty limit and proximity induced superconductivity in N-layer (see inset in Fig. 1(a)). Besides one can see non-BCS (Bardeen-Cooper-Shreffer) like temperature dependence of $\Lambda^{-1}$ which is consequence of the temperature dependence of superconducting order parameter $\Delta(d_S)$ near the SN interface (see inset in Fig. 1(b)) which controls the strength of induced superconductivity in N-layer. Value of $\Delta(d_S)$ depends not only on the temperature and ratio of resistivities but also on the thickness of N-layer. We consider the situation when $d_N < \xi_N(T_c)$ and even small variation of $d_N$ or temperature leading to small change of ratio $d_N/\xi_N(T)$ strongly influences $\Delta(d_S)$ and $\Lambda^{-1}$ due to large parameter $\rho_S/\rho_N > 1$ in the boundary condition for anomalous Green function $F$ at SN interface: $dF/dx|_{d_S=0} = (\rho_S/\rho_N)dF/dx|_{d_S=0}$ and boundary condition $dF/dx|_{d_S=d_N} = 0$ at outer edge.

In Fig. 2 we present calculated dependence of superconducting current $I_s$ flowing along the bilayer as a function of value $q = \nabla \phi - 2\pi A/\Phi_0$ ($\phi$ is a phase of superconducting order parameter, $A$ is a vector potential, $\Phi_0$ is the magnetic flux quantum) which is proportional to

![FIG. 1: Temperature dependence of inverse effective magnetic field penetration depth $\Lambda^{-1}$ of SN bilayer at different $\rho_S/\rho_N = 200, 100, 10, 0.1$ (a) and different thicknesses of N-layer $d_N/\xi_c = 1/2, 1, 2, 4$ (b). In inset to Fig. 1(a) we show dependence of local $1/\lambda^2$ across the thickness of bilayer. In S-layer $1/\lambda^2$ is strongly suppressed near SN interface due to inverse proximity effect. In inset to Fig. 1(b) we present dependence $\Delta(d_S)(T)$ ($\Delta$ near SN interface) to demonstrate its correlation with $\Lambda^{-1}(T)$.](image1)

![FIG. 2: Dependence of the superconducting current $I_s$ (it is normalized to depairing current of single S-layer) flowing along the bilayer on $q$ which is proportional to supervelocity $v_s \sim q$. Results are present for different thicknesses of N-layer $d_N/\xi_c = 1/2, 1, 2, 4$. Critical current corresponds to maximal possible value of $I_s$ and for bilayers it is reached at $q_{cN} \sim 0.06 - 0.09$ for chosen parameters. In single S-layer maximum in dependence $I_s(q)$ (empty squares) is located at $q_{cN} \simeq 0.88$.](image2)
the superconducting velocity $v_s \sim q$ and those values do not vary across the bilayer (when parallel/perpendicular magnetic field $H_{||/\perp} = 0$). In calculations we assume no vortices and uniform current distribution across the superconductor/bilayer. Note that this dependence may have two maxima in contrast with single superconducting film (see for example [12]). At $q < q_c$ (see Fig. 2) the major part of superconducting current flows via the normal layer (due to locally larger $1/\lambda^2$) while at $q > q_c$ proximity-induced superconductivity in N-layer is suppressed ($\lambda^{-2}$ rapidly decreases with increasing $q > q_c$) and near the second maxima the major part of superconducting current flows via S-layer. For relatively small thickness of the N-layer the maximal (critical) superconducting current can exceed the depairing current of single superconducting film.

Temperature dependence of critical current of bilayer also is non-BCS like (see Fig. 3) and resembles temperature dependence of $\Lambda^{-1}$. Origin of this dependence comes from larger magnitude of proximity induced superconductivity in the N-layer when temperature decreases and corresponding increase of $\Lambda^{-1}$.

So far we present results for fixed thickness of S-layer $d_S = 2\xi_c$. We find that the nontrivial (non BCS like) temperature dependence $I_c(T)$ shifts to lower temperatures with increasing $d_S$ - see Fig. 4(a) and in wide temperature interval ($0 \div T_c$) it exists only for relatively thin bilayers with $d_S \lesssim 2\xi_c$ (for realistic ratio $\rho_S/\rho_N \lesssim 200$). The reason is in increasing contribution of S-layer to the superconducting current when $d_S$ increases while contribution of N-layer stays practically the same. Similar effect occurs with increasing $d_N$ (at fixed $d_S$, $T$ and not very low temperatures) because of weaker induced superconductivity in N-layer. In inset to Fig. 4(a) we present calculated temperature dependent critical (maximal) superconducting current density in N-layer at the outer boundary ($x = d_S + d_N$) and analytical expression for $j_{cN}(T)$ found in Ref. [9] (solid curve is Eq. (28) and dashed line is Eq. (30) from [9]). In inset to Fig. 4(b) we show temperature dependence of $\Delta$ at SN interface ($\Delta(d_S)$).

FIG. 3: Temperature dependence of the critical current of SN bilayer at different $\rho_S/\rho_N = 0.1, 10, 50, 200$ (dashed curve corresponds to temperature dependence of depairing current of single S-layer). In the inset we present temperature dependence of the critical current of SN bilayer at different thicknesses of N-layer $d_N/\xi_c = 1/2, 1, 2, 4$ and $d_S = 2\xi_c$. For bilayer with $d_N/\xi_c = 4$ dependence $I_c(T)$ has a kink at $T \approx 0.22T_{c0}$ because of dominant contribution of N-layer in $I_c$ at $T \lesssim 0.22T_{c0}$.

FIG. 4: Temperature dependence of $I_c$ (a) and $\Lambda^{-1}$ (b) of SN bilayer with different thicknesses of S-layer. With increasing $d_S$ nontrivial temperature dependence of $I_c$ shifts to lower temperatures where superconducting current mainly flows in N-layer. In inset to figure (a) we show temperature dependence of critical (maximal) superconducting current density in N-layer at outer boundary ($x = d_S + d_N$) and analytical expression for $j_{cN}(T)$ found in Ref. [9] (solid curve is Eq. (28) and dashed line is Eq. (30) from [9]). In inset to figure (b) we show temperature dependence of $\Delta$ at SN interface ($\Delta(d_S)$).
perconducting film. As $\Lambda_0/\Lambda$ decreases (for example with increasing $d_S$) the critical current is mainly determined by S-layer (except at very low $T$) while main contribution to $\Lambda^{-1}$ still comes from N-layer when $q < q_c$.

Due to difference in critical supervelocities of S and N-layers value of critical current in 'positive' ($I^+$) and 'negative' ($I^-$) directions are different in parallel magnetic field ($I^\pm H||$ - see inset in Fig. 5). Indeed, parallel magnetic field either increases $q = \nabla \phi - 2\pi A/\Phi_0$ in N-layer or decreases it depending on direction of the current (which is determined by direction of $\nabla \phi$). In the first case $I_c$ rapidly decreases because $q$ reaches $q_c$ at smaller $\nabla \phi$ while in the second case $I_c$ may even slightly increase at weak magnetic field (note that these fields weakly affect superconductivity in S-layer due to much larger value of $q_c$). Difference in $I_c^\pm$ provides the diode effect (appearance of nonzero average voltage) in the regime with ac current (with zero time-averaged current). In Fig. 5 we showed calculated dependence $I_c^\pm (H||)$ for bilayer with following parameters: $d_N = d_S = 2\xi_c$, $\rho_S/\rho_N = 200$, $T = 0.2T_c$. At large $H||$ the superconductivity in N layer is suppressed and $I_c^+ \approx I_c^-$.

![FIG. 5: Dependence of the critical current of SN bilayer on the parallel magnetic field. Currents flowing in opposite directions ('positive' and 'negative' - see inset) have different critical values when $H|| \neq 0$ which is a consequence of different critical supervelocities in S and N layers.](image)

Note that in symmetric SNS or NSN system the diode effect is absent. It is rather weak in SN bilayer with $\rho_S \lesssim \rho_N$ (comparable with one shown in Fig. 5 at large $H||$) because in this case the main part of superconducting current flows via S-layer. Therefore the presence of noticeable diode effect could be some kind of experimental verification of large/dominant contribution of N-layer in $I_c$ at zero magnetic field.

### III. Experimental Results

To verify theoretical predictions we perform experiments on NbN/Al, NbN/Ag and MoN/Ag bilayers with $50 \lesssim \rho_S/\rho_N \lesssim 400$ and single NbN, MoN films. Using $T_\text{c}(\text{NbN}) = 9$ K and $T_\text{c}(\text{MoN}) = 7.5$ K we estimate $\xi_c = 6.5nm$ for NbN (we take $D_S = 0.5cm^2/s$ from [13] and $\xi_c = 6.4nm$ for MoN (we take $D_S = 0.4cm^2/s$ from [14]). We have to mention, that critical temperature of our NbN and MoN films gradually decreases with decreasing their thickness when $d_S \lesssim 20nm$ (results for relatively thin MoN films are present in Ref. [14]). We relate this effect with presence of 'dead' nonsuperconducting layer with thickness $2 - 3nm$ at the interface with substrate. Proximity effect with this layer could provide decreasing $T_c$ with decreasing $d_S$. Therefore we estimate the effective 'superconducting' thickness of our NbN and MoN in the range $11 - 13nm$ which is close to $2\xi_c$. In our experiment $\Lambda^{-1}$ is measured using two coils technique [16] (via measurements of their mutual inductance $M$ with sample between them) while $I_c$ is extracted from current voltage characteristics of S and SN bridges. To study effect of the thickness of N-layer on superconducting properties of bilayer we change $d_N$ by consequent ion etching (for experimental details see Appendix B).

![FIG. 6: Evolution of temperature dependence of $\Lambda^{-1}$ of NbN/Al bilayer during etching and reference NbN film (measured mutual inductance $M$ is normalized to its value in the normal state $M_n$ just above $T_c$). Thickness of Al layer is estimated from measured room temperature resistance per square $R_n$ (shown in the inset). Red dashed curve corresponds to dirty limit theoretical expectation for single superconducting layer: $\Lambda^{-1} \sim \tanh(\Delta(T)/2k_B T)$ [16].](image)
becomes smaller at low temperatures, which qualitatively coincides with theoretical predictions (see Fig. 1(b)).

Note that for thinnest Al layer shape of dependence \( \Lambda^{-1}(T) \) is probably affected by nonuniformity of Al layer along the film (it appears during etching procedure and it is seen from our measurements of resistance per square \( R_s \) in different places of the sample). Not etched bilayer NbN/Al with similar \( d_{NbN} \) and \( d_{Al} = 10 \)nm shows much larger diamagnetic response (see Fig. 12 in Appendix C) and no signs of features in dependence \( \Lambda^{-1}(T) \).

In Fig. 7 we show evolution of \( I_c(T) \) of NbN/Al bridge with width \( w = 4 \)µm during etching procedure (similar results are found for NbN/Ag bridge - see Fig. 13 in Appendix C). Because \( \Delta_0 = \lambda(0)^2/d_N \approx 21 \mu m \) (for estimation we use \( \lambda(0) = 600 \)µm found from dirty limit expression \( \lambda(0) = (\hbar / \pi \sigma t_B / T_c)^{1/2} \) with \( \rho = 300 \)µΩ·cm and \( T_c = 9 \)K) and as it follows from Fig. 12 for NbN/Al bridge \( \Lambda(4.2K) \approx \Delta_0/3 \approx 7 \)µm > \( w \) we conclude that current flows uniformly across the bridge. One can see that even few nanometer Al layer modifies \( I_c(T) \) in comparison with one for NbN bridge and makes it similar to the theoretical expectations. We also find that \( I_c \) of NbN/Al bridge with relatively large \( d_N \) exceeds critical current of NbN bridge in about 4 times at \( T = 4.2 \)K. So large critical current enhancement we explain by low \( I_c \) of our NbN bridge, in comparison with its depairing current. Indeed, using theoretical expression following from the Usadel theory (see for example Eq. (30) in 13), measured \( R_s(T = 10K) = 150\Omega \), \( T_{c} = 9K \) for sample #80 and \( D = 0.5 \)cm\(^2\)/s we find \( I_{dep}(4.2K) = 3.8mA \) which is 10 times larger than the measured critical current at \( T = 4.2K \). The small critical current is probably connected with intrinsic inhomogeneities of our NbN films which allow vortex penetration and motion at supervelocities much lower than depairing supervelocity (which corresponds to maxima in dependence \( I_s(q) \) for ideal S-layer - see Fig. 2).

In bilayer value of critical supervelocities is mainly determined by N-layer (see maxima in dependencies \( I_s(q) \) in Fig. 2) and it is order of magnitude smaller (for \( \rho_S / \rho_N = 200 \)) than in ideal (defectless) superconducting film. Because Al-layer is rather homogenous (this fact follows from its relatively low resistivity and mean path length \( \ell_N \approx d_N \)) we expect that at \( q < q_c \) vortices cannot penetrate our bilayer bridge. As a result the superconducting current of bilayer bridge approaches its maximal possible value (with main contribution from N-layer) which is about half of \( I_{dep} \) of NbN layer at this temperature (according to our calculations - see Fig. 3 for bilayers with close parameters).

![FIG. 7: Evolution of temperature dependence of critical current of NbN/Al bridge during etching procedure. For comparison we also show temperature dependence of critical current of NbN bridge. Thicknesses of Al layers are estimated from room temperature \( R_s \) (shown in the inset).](image)

![FIG. 8: Dependence of critical current (flowing in opposite directions) of NbN/Al (\( d_{NbN} = 19 \)nm, \( d_{Al} = 10 \)nm) and NbN (\( d_{NbN} = 19 \)nm) bridges on perpendicular magnetic field \( H_\perp \). For NbN bridge \( I_c \) does not depend on direction of the current and \( H_\perp \). In the inset we show dependence \( I_c^\pm \) of the same NbN/Al bridge on the parallel magnetic field.](image)
lieve that it is connected with the presence of perpendicular component of the magnetic field which strongly suppresses proximity induced superconductivity in N-layer and $I_c^\perp$. Note, that in the experiment $I_c^\perp \neq I_c^\parallel$ in the perpendicular magnetic field too (most probably it is connected with different quality of edges of the bridge [19]) but relative difference is smaller than in the parallel magnetic field.

IV. DISCUSSION

We present results on superconducting properties of thin dirty SN bilayer with thickness of S layer $d_S$ about several $\xi_N$, thickness of N layer $d_N \ll \xi_N(T_{c0})$ and large ratio of residual resistivities $\rho_S/\rho_N \gg 1$. We show that such a bilayer has unique superconducting properties. First of all the screening ability of the bilayer is determined mainly by the proximity induced superconductivity in the normal layer, where locally the London penetration depth $\lambda$ is smaller. Secondly, at some conditions, the presence of normal layer may considerably increase the critical current (the effect is not connected with enhanced vortex pinning) because the largest part of superconducting current flows via the normal layer where superconducting current density $j_s \sim 1/\lambda^2$. Besides the temperature dependence of critical current and effective magnetic field penetration depth have unusual, non-BCS like temperature dependence. We argue that these properties are consequences of small thickness of N-layer and large ratio of resistivities $\rho_S/\rho_N \gg 1$.

![FIG. 9: Dependence of local $1/\lambda^2$ on the coordinate across the bilayer ($d_S = 2\xi_c$, $d_N = \xi_c$, $T = 0.2T_{c0}$) calculated in Usadel and Eilenberger (see for example Eq. (1) in [23]) models for different ratios of resistivities (mean path lengths).](image)

We believe that our results could be used as an alternative explanation for the several times enhancement of $I_c$ found in NbN/CuNi bilayers in comparison with single NbN film [20, 21]. The thickness of NbN layer was 8 nm, while the thickness of CuNi layer varied from 3 up to 6 nm. It is know that residual resistivity of CuNi strongly depends on Ni concentration and varies in the range $\rho_{\text{CuNi}} \approx 2 - 49 \mu \Omega \cdot \text{cm}$ [22] and, hence, potentially the ratio $\rho_{\text{NbN}}/\rho_{\text{CuNi}}$ could reach 100 (with $\rho_{\text{NbN}} = 200 \mu \Omega \cdot \text{cm}$). Unfortunately due to absence of dependence $I_c(T)$ and actual values of resistivity of used CuNi and NbN layers we cannot make a solid statement about the role of proximity effect in that experiments.

![FIG. 10: Temperature dependence of $\Lambda^{-1}$ for bilayer with $d_S = d_N = 2\xi_c$, $\rho_S/\rho_N = 200$ and finite barrier between S and N layers (strength of the barrier is governed by parameter $\gamma$ - see Eq. A6). In the inset we present temperature dependence of $\Lambda^{-1}(d_S)$ at different $\gamma$. Result are obtained using Usadel model.](image)
with very short mean path length $\ell_S$ in NbN and MoN materials which is in the range of 1A if one uses relation $D_S = v_F \ell_S / 3$ with typical value of Fermi velocity $v_F = 2 \cdot 10^6 cm^2/s$. It questions about boundary conditions for quasiclassical Green functions at SN interface between highly disordered superconductor and relatively clean metal and how it affects both direct and inverse proximity effects in N and S layers.

Due to strong dependence of $I_c$ on temperature such a SN bilayer has unclear perspectives from point of view applications where one needs large critical current. Besides effect of current enhancement could be noticeable only in relatively thin structures and when host superconductor has critical current much smaller than depairing current. From another side steep temperature dependence of $I_c$ and $\Lambda^{-1}$ could be utilized in different kinds of superconducting detectors of electromagnetic radiation \[24\] \[25\], particles \[26\] or dark matter \[27\] based on temperature dependent $I_c(T)$ and/or $\Lambda^{-1}(T)$. For example in superconducting single photon detector (SSPD) absorbed photon or particle locally heats by $\delta T$ the superconducting strip biased below its critical current $I < I_c(T)$ and the superconductor transits to the resistive state when $I > I_c(T + \delta T)$ \[24\]. In the kinetic inductance detector (KID) such a heating leads to change of kinetic inductance $L \sim \Lambda^{-1}(T)$ and resonance frequency of corresponding inductance-capacity circuit \[25\]. It is clear that the steeper the temperature dependence of $I_c$, $\Lambda^{-1}$ the larger will be their change at fixed $\delta T$ (it is determined by energy of absorbed particle or photon) and sensitivity of detector should subsequently increase.

Another advantage of SN bilayer in comparison with highly disordered superconductors is connected with high uniformity of N-layer. Our results show that proximity induced superconductivity in N-layer is weakly sensitive to local inhomogeneities of host superconductor (it follows from our measurements of $I_c(H)$ for SN bilayer - see Fig. 8) and critical current of bilayer approaches to its maximal possible value. Note, that in superconductors with large $\rho$ the value of critical current is dictated by the weakest place in the sample and usually it is smaller than the depairing current in two or more times \[28\].

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### Appendix A: Model

To calculate superconducting properties of SN bilayer we use Usadel equation for anomalous $F = \sin \Theta$ and normal $G = \cos \Theta$ Green functions

$$hD \frac{\partial^2 \Theta}{\partial x^2} - \left(2h\omega_n + \frac{D}{\hbar}q^2 \cos \Theta \right) \sin \Theta + 2\Delta \cos \Theta = 0,$$  \hspace{1cm} (A1)

where $D$ is a diffusion coefficient ($D = D_S$ in superconducting layer and $D = D_N$ in the normal one), $\omega_n = \pi T(2n + 1)$ is a Matsubara frequency, $q = \nabla \varphi - (2\pi / \Phi_0)A(x)$ ($\varphi$ is a phase of the order parameter, $A$ is a vector potential) takes into account nonzero velocity of superconducting condensate $v_s \sim q$ in direction parallel to layers ($y$ direction in our case), $\Delta(x)$ is a magnitude of superconducting order parameter which has to be found in the superconducting layer with help of self-consistency equation

$$\Delta \ln \left(\frac{T}{|T_c|}\right) + 2\pi k_B T \sum_{\omega_n \geq 0} \left( \frac{\Delta}{h\omega_n} - \sin \Theta_s \right) = 0 \text{ \hspace{1cm} (A2)}$$

and we assume that in the normal layer $\Delta = 0$ because of zero BCS coupling constant. $T_{c0}$ in Eq.(A2) is the critical temperature of superconductor with no N-layer.

We consider thin bilayer with thickness of superconducting layer $d_S \ll \lambda$ ($\lambda$ is the London penetration depth) and thickness of normal layer $d_N \ll$ less than characteristic penetration depth of magnetic field in N layer. Therefore we may neglect corrections to $A(x)$ which comes from screening effect and choose following form for $A(x) = H_{||} x$ ($H_{||}$ is a parallel magnetic field).

The inverse effective magnetic field penetration depth by definition is

$$\Lambda^{-1} = \frac{16\pi^2}{\hbar c^2} \int_0^{d_s + d_n} \frac{1}{\rho} \sum_{\omega_n \geq 0} \sin^2 \Theta dx$$  \hspace{1cm} (A3)

with $\rho = \rho_S$ in S layer and $\rho = \rho_N$ in N layer. In the absence of normal layer $\Lambda = \lambda^2 / d_S$ - so called Pearl penetration depth.

To find the sheet critical current (critical current per unit of width of the bilayer) we use the following expression for sheet superconducting current

$$J = \int_0^{d_s + d_n} \frac{2\pi k_B T}{e \rho} q \sum_{\omega_n \geq 0} \sin^2 \Theta dx$$  \hspace{1cm} (A4)

The sheet critical current is defined as maximal sheet superconducting current. For superconducting film without N-layer $J_e = j_{dep} d_s$, where $j_{dep}$ is the depairing current.

At SN interface $x = d_s$ we use Kupriyanov-Lukichev boundary conditions \[24\]

$$D_S \frac{d\Theta}{dx} \bigg|_{x=d_s-0} = D_n \frac{d\Theta}{dx} \bigg|_{x=d_s+0}$$ \hspace{1cm} (A5)
\[
\gamma \xi_c \frac{d\Theta}{dx} \bigg|_{x=d_s+0} = \sin(\Theta(d_s + 0) - \Theta(d_s - 0)) \tag{A6}
\]
and boundary condition with vacuum at \( x = 0, d_s + d_n \): \( d\Theta/dx = 0 \). Eq. (A6) leads to jump of \( \Theta \) on SN boundary in presence of the barrier, which is controlled by parameter \( \gamma = R_{SNSN}/(\sigma_N \xi_c) \) \( (R_{SN} \) is the resistance of SN interface, \( A_{SN} \) is its area and \( \xi_c = \sqrt{\hbar D_S/k_B T_{c0}} \). Usually we choose \( \gamma = 0 \) which leads to continuity of \( \Theta \):
\( \Theta(d_s + 0) = \Theta(d_s - 0) \).

Equations (A1,A2) are solved numerically by using iteration procedure. For initial distribution \( \Delta(x) = \text{const} \) we solve Eq. (A1) for Matsubara frequencies ranging from \( n=0 \) up to \( n=100 \). In numerical procedure we use Newton method combined with tridiagonal matrix algorithm. Found solution \( \Theta(x) \) is inserted to Eq. (A2) to find \( \Delta(x) \) and then iterations repeat until the relative change in \( \Delta(x) \) between two iterations does not exceed \( 10^{-8} \). Length is normalized in units of \( \xi_c \), energy is in units of \( k_B T_{c0} \), current is in units of deparing current of single S-layer with the thickness \( d_s \), magnetic field is in units of \( H_0 = \Phi_0/2\pi \xi_c^2 \) and effective magnetic field penetration depth is in units of \( \Lambda_0 = \lambda^2(T = 0)/d_s \). Usual step grid in S and N layers is \( \delta x = 0.02 \xi_c \).

To decrease the number of free parameters we suggest that the density of states in S and N layers are the same and ratio of resistivities is equal to inverse ratio of diffusion constants or mean path lengths \( \rho_S/\rho_N = D_N/D_S = \xi_N/\xi_S \).

### Appendix B: Experimental details

The bilayers NbN/Ag and NbN/Al were prepared on Al_{2}O_{3} 10 × 10 mm\(^2\) substrates in a magnetron vacuum machine (Alcatel SCM-600) with a load-lock chamber. The thin films were fabricated in a single deposition run at the substrates at ambient temperatures. In total three targets were used: pure niobium (99.99\%) as a superconducting material, Al (99.99\%) and Ag (99.99\%) as the normal metals. The design of the deposition machine allows growth of the entire structure in one cycle without disrupting the vacuum. This results in high-quality structures with clean interfaces and strong proximity effect. NbN films were deposited by reactive dc-magnetron sputtering in the Ar (99.999\%) and N2 (99.999\%) gases mixed at the total pressure of \( 7 \times 10^{-3} \) mbar with a residual pressure in the chamber of about 1.5 \( \times 10^{-7} \) mbar. The deposition rate of the NbN layers was 1.3 nm/s. The Ag and Al films were deposited by rf-magnetron sputtering in the Ar at the pressure of \( 2 \times 10^{-2} \) mbar. The deposition rates of the Ag and Al layers were from 1 to 3 nm/s.

MoN/Ag was fabricated by DC-magnetron sputtering on HV system AJA ATC-2200 at room temperature. All samples were fabricated on a silicon substrate (KDB-10) in one vacuum cycle and covered by Si (10 nm) to protect of oxidation a top layer. MoN film was deposited from metallic Mo target in N\(_2\) atmosphere, see [14]. The base vacuum in the main chamber was about \( 2 \times 10^{-8} \) mbar. The working pressure was maintained at a level of 2.6 \( \times 10^{-8} \) mbar. The rate of deposition of layers was about 3-5 nm/min.

Measurements of \( \Lambda^{-1} \) were performed by using standard two-coil measurement technique of mutual inducance (see for example Ref. [15]). The diameter of coils (2 mm) and their height (4 mm) is smaller than the typical lateral size of measured film (10 \( \times \) 10 or 7 \( \times \) 7 mm), separation between coils is \( \approx 1 \) mm, while thickness of the bilayer \( d_s + d_n < 100 \) nm is smaller than London penetration depth. At these parameters mutual inductance \( M \sim \Lambda \) [15] except temperatures close to \( T_c \) where \( \Lambda \) diverges.

The bridges made of bilayer films were fabricated by standard lift-off lithography. Because of low substrate temperature we were able to deposit the bilayers on the photoresist without its apparent degradation. Widths of the bridges range from 3 up to 5 microns, while their length is fixed to 10 microns. Due to low thickness of studied bilayer bridges (\( d_{SN} = 12 - 19 \) nm, \( d_{MN} = 19 \) nm, \( d_{AI} = 2 - 25 \) nm, \( d_{Ag} = 2 - 30 \) nm) current density distribution across the sample is expected to be uniform because \( w < \Lambda \). Transport measurements were performed by four-probe method. During measurements we change the current from large negative up to large positive values and in this way we were able to find both the critical current \( I_c \) (at this current bridge switches from the superconducting state to the resistive one) and retrapping current \( I_r \) (when the bridge switches back from the resistive to the superconducting state).

Resistivity of the samples was measured either using van Der Pauw method and/or transport measurements. In this way we find \( \rho_{MoN} = 260 - 380 \mu\Omega \cdot \) cm depending on the sample and \( \rho_{MoN} = 200 \mu\Omega \cdot \) cm at \( T = 10K \). For both materials room temperature \( \rho \) is smaller than at \( T = 10K \) which is common feature of highly disordered metallic films. Our thickest Al film (\( d_{Al} = 70 \) nm) has \( \rho_{Al} = 2.9 \mu\Omega \cdot \) cm while 90 nm thick Ag film has \( \rho_{Ag} = 1.5 \mu\Omega \cdot \) cm (both at room temperature) which are close to literature data [30]. Residual resistance for these and thinner films is 1-4 times smaller and gradually increases with decreasing \( d_N \) [30] which gives us \( \rho_S/\rho_N \lesssim 400 \) depending on the thickness of the normal layer and the pair of S,N materials.

To study effect of the thickness of normal layer on superconducting properties of bilayer we make chips with side \( 10 \) mm \( \times \) 10 mm. The central part of chips with size \( 6 \mu \text{m} \times 6 \mu \text{m} \) is used for measurements of mutual inductance, while on the edge of the chip we fabricate bridges. The thickness of Al and Ag layers is changed by gradual ion etching. The ion etching of Al and Ag layers was performed in a Plasmalab 80 plus (Oxford Instruments) equipped with capacitive (HF) and inductive (ICP) plasma sources. The etching process was performed in Ar at the pressure of 5 mbar, HF power
100 W and ICP power 400 W. The Ag etching rate was 30 nm/min and Al etching rate 1 nm/min.

We also did special experiment and found dependence of resistance per square $R_s$ of normal layer on its thickness at room temperature during etching procedure combined with simultaneous measurement of the resistance of the etched sample. We use these results to estimate thicknesses of the Al and Ag layers in bilayer by measuring their room temperature $R_s$.

**Appendix C: Experimental results for NbN/Ag and MoN/Ag bilayers**

In Fig. 11 we present experimental $\Lambda^{-1}(T)$ for NbN/Ag bilayer after two consequent etching of N-layer. In the inset to Fig. 11 we show results for MoN/Ag bilayer and in Fig. 12 for NbN/Al bilayer without etching. Qualitatively these results coincide with ones for NbN/Al bilayers and theoretical calculations in the Usadel model. Quantitatively in the experiment $\Lambda^{-1}$ is larger than the theory predicts.

![Fig. 11](image1.png)

**FIG. 11:** Evolution of temperature dependence of $\Lambda^{-1}$ of NbN/Ag bilayer after consequent etching and reference NbN film (mutual inductance is normalized to its value in the normal state just above $T_c$). In the inset we present $\Lambda^{-1}(T)$ for MoN/Ag bilayer ($R_s = 0.73 \Omega$ at room temperature) with $d_{MoN} = 19 \text{nm}$, $d_{Ag} = 30 \text{nm}$ and single MoN layer ($R_s = 120 \Omega$) with $d_{MoN} = 19 \text{nm}$.

![Fig. 12](image2.png)

**FIG. 12:** Temperature dependence of $\Lambda^{-1}$ of NbN/Al bilayer (sample #98, $d_{NbN} = 17 \text{nm}$, $d_{Al} \approx 10 \text{nm}$, room temperature $R_s = 8.2 \Omega$) and NbN film (sample #80, $d_{NbN} = 17 \text{nm}$, room temperature $R_s = 135 \Omega$).

In Fig. 13 we show temperature dependence of critical $I_c$ and retrapping $I_r$ currents of NbN/Ag bridge with width $w = 5 \mu m$ during consequent etching (in the inset results for NbN/Al bridge with $w = 4 \mu m$ are present). The presence on the normal layer not only considerably increases the critical current and changes its temperature dependence in comparison with superconducting bridge but it also wipes out the hysteresis of current-voltage characteristics (making $I_r = I_c$) for bilayers with relatively large $d_N$. The last effect was observed earlier for shunted MgGe superconducting bridge with low shunt resistance [31]. The dependence $I_c(T)$ looks more ‘noisy’ for NbN/Ag bridges than for NbN/Al ones. The origin of the 'noise' comes from stochastic nature of switching to the resistive state [31] and it is clear seen in our experiment via repeated measurements of current-voltage characteristics at fixed temperature.

![Fig. 13](image3.png)

**FIG. 13:** Temperature dependence of critical (solid curves) and retrapping (dashed curves) currents of NbN/Ag and NbN/Al bridges (see inset) with different thicknesses of Al and Ag layers. In the same figures we show $I_c(T)$ and $I_r(T)$ of reference NbN bridges.

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