Mechanisms of power distribution: principles of kinematic and force analysis

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Abstract. The article considers the features of performing kinematic and power analysis of a controlled power mechanism for the transmission of wheeled transport and transport-technological machines. Such mechanisms are currently used mainly on cars in order to improve handling, stability, traction and dynamic properties in severe road conditions. Such mechanisms are not produced in Russia, but theoretical studies are currently being conducted, the results of which will allow determining the main parameters of the mechanism and synthesizing its kinematic scheme. The proposed approaches are based on the methods used in the kinematic and force analysis of planetary gears, but it is taken into account that the mechanism of power distribution during operation has two degrees of freedom. It is indicated that there is a structural and functional similarity between the power distribution mechanism of the car and the turning mechanism of the tracked vehicle. A method is proposed for calculating the gear ratio of the power distribution mechanism according to the condition of matching the minimum possible turning radius provided by the steering trapezoid kinematics and the calculated turning radius.

1. Introduction

The power distribution mechanism (PDM) is used in the transmission of almost every transport and technological machine or wheeled vehicle, since there is a need to distribute power between the drive axles or drive wheels. The most widespread on cars was a simple (symmetrical) conical differential. The possibility of blocking is often provided for center-to-center differentials. The inter-wheel differential is performed lockable, as a rule, on wheeled vehicles of high cross-country ability and special road construction equipment. The lock can be forced (by the driver's decision or by a signal from the control system), in addition, self-locking differentials are used. Lockable and self-locking...
differentials do not meet the modern needs of consumers and do not provide effective control of power flows.

To increase the traction and dynamic properties of a wheeled vehicle, improve controllability and stability of movement, controlled inter-wheeled PDM are used (for example, mass-produced ZF Vector Drive and AYC mechanisms). The design features of this type of PDM are the presence of a gear part in the form of a planetary mechanism, friction controls with a hydraulic or electromechanical drive, and a digital control system.

The gear part performs the functions of a simple (symmetrical) differential, and if necessary, through the use of controls, it is possible to redistribute power between the wheels: more power can be directed to the selected wheel and control the torque and angular speed of the wheel.

Such an PDM is in principle two-threaded in the active state. The power supplied to the PDM is divided – part is redistributed through the differential (or its equivalent), the other flow is directed through the PDM to the selected board and is summed with part of the first flow at the selected output from the PDM. Control over the value of the moment and angular velocity can be carried out, for example, by controlled slipping of the disks of the friction control element.

On tracked vehicles, the analog of the PDM is the turning mechanisms (TM). In this regard, it is advisable to consider in the future the possibility of using technologies used in the production of PDM in relation to TM and, conversely, the problem of rational borrowing of technical solutions and theoretical developments from the practice of designing TM for the development of a new direction for the domestic transport engineering of the design of controlled gear PDM [1,2].

2. Problem statement
The purpose of the work is to form a methodology for kinematic and power analysis of the kinematic scheme of a gear-driven planetary PDM for the transmission of a wheeled vehicle [3-5]. Performing such calculations is necessary when designing a mechanism based on the selected kinematic scheme.

Working with literary sources and analyzing the experience of designing the chassis of transport and traction machines [6-12] allow us to formulate the following tasks.

Using the example of a kinematic scheme of a controlled PDM of the ZF Vector Drive family, consider the features of conducting kinematic and force analysis of such mechanisms.

Methodically generalize the applied approaches to the PDM class, abstracting from a specific scheme.

To consider the possibility of manufacturing and testing controlled PDM based on technologies mastered in the Russian industry.

3. Results of theoretical research
The methods of kinematic and force analysis applied to planetary gearboxes are well developed today and are used in the design of planetary gearboxes (PGB). A significant difference between the PDM and the control panel is the presence of two output links and, in general, the preservation of two degrees of freedom during operation. It will be shown later that for the kinematic and force analysis of the PDM, it is necessary to take into account not only the technical characteristics of the wheeled vehicle, but also the driving conditions.

Kinematic and force analysis of the PDM as a gearbox, which can contain simple three-link and complex four- and five-link planetary mechanisms, as well as transmissions with fixed axes, is possible according to the methods developed for the control panel [4].

The kinematic and force analysis of the PDM should be preceded by the choice of the scheme and the choice of the number of gear teeth. In principle, the PDM scheme can be adopted on the basis of the prototype [4, 5, 13], but to select the numbers of gear teeth, an original method must be proposed.

The method of determining the value of the gear ratio of the PDM is based on the fact of the functional and structural similarity between the PDM of a wheeled vehicle and the TM of transport tracked vehicles [10, 14, 15].
In addition, when developing the PDM control system, it is advisable to use the experience of building closed control systems for turning a transport vehicle [12, 16, 17, 18].

As an example, let’s consider the PDM, the scheme of which is shown in figure 1. Let’s consider the kinematics of this mechanism by analogy with the method of analyzing the control panel circuits.

The structure of the considered PDM (see figure 1) includes two onboard planetary mechanisms with stepped and a symmetrical conical differential.

Figure 1. Kinematic diagram of a two-flow PDM. k – differential box (housing); 1 – left half-axis; 2 – right half-axis; 3,4 – planetary gear drivers; T – control element; z – number of teeth.

For the onboard planetary mechanism, the parameter is 

\[ k = \frac{z_2 z_4}{(z_2 z_4)} \]  

[4,14].

With \( z_2 > z_4 \) this parameter \( k > 1 \), which means that when the control is turned on, the corresponding semi-axis will rotate \( k \) times slower. Otherwise, when, the parameter.

The system of kinematics equations for the MRM under consideration is based on the kinematics equations of planetary mechanisms (a kind of the Willis equation) and has the form:

\[
\begin{align*}
2\omega_k &= \omega_1 + \omega_2 \\
\omega_1 &= k\omega_1 + (1-k)\omega_3 \\
\omega_3 &= k\omega_3 + (1-k)\omega_4
\end{align*}
\]

Two additional equations are needed to solve this system. One of them is: \( \omega_k = \text{const} \) (it is assumed \( \omega_k = 1 \) that then the angular velocities will be expressed in fractions of this value). We get the other by imposing a restriction of the form \( \omega_k = 0 \) (or \( \omega_k = 0 \)).

The analysis of the system of equations shows that when the control element of the corresponding board is turned on and at \( k > 1 \), the board becomes lagging, and \( 0 < k < 1 \) at-running.

This mode of turning a wheeled vehicle is similar to the mode of turning a tracked vehicle with a "fixed" (calculated) radius \( R_f \). It is more convenient to consider the relative radius of a fixed (calculated) turn \( p_f = R_f / B \), where \( B \) is the track of the driving axle of a wheeled vehicle.

The relative turning radius can be determined using the gear ratio between the sides (PDM gear ratio):

\[
\rho = \frac{1}{2} \frac{u+1}{u-1}
\]

Here \( u = V_f / V_1 \) – the gear ratio of the PDM; \( \rho = R/B \); \( R \) – the turning radius, \( B \) – the track of the car. If we ignore the change in the rolling radii of the wheels, then we can write: \( u = \omega_2 / \omega_1 \).

It is proposed to proceed from the need to ensure kinematic coordination of the relative radius of rotation of the machine, obtained due to the "power turn" and the minimum relative radius of rotation.
allowed by the kinematics of the steering trapezoid. Let’s put them equal to each other and denote \( \rho_j \).

From the above equations, we can find the relationship between \( \rho_j \) and the parameter \( k \) of the control planetary series.

To control the running board \( \rho_j = \frac{k}{2(1-k)} \); \( k = \frac{2\rho_j}{2\rho_j + 1} \).

To control the lagging board \( \rho_j = \frac{k}{2(k-1)} \); \( k = \frac{2\rho_j}{2\rho_j - 1} \).

We assume \( R_j = 5 \) m and \( B = 1.8 \) m, we get \( \rho_j = 2.78 \) and \( k = 0.847 \) for the first case, and \( k = 1.22 \) for the second.

All other relative radii, larger than \( \rho_j \), can be obtained by power control of the driving wheels with controlled slipping in the brakes of planetary mechanisms.

Let us consider separately both cases of rotation with radii \( \rho_j < \rho < \infty \).

We introduce the parameter \( s \)-the sliding of the discs of the included brake relative to the PDM housing. For control on the running board \( s = \omega_4 \), and on the lagging board \( s = \omega_3 \).

The parameter value will change within the range of \( 0 < s < 1 \) when the control is applied.

The value \( s = 0 \) will correspond to the full activation of the control brake, and \( s = 1 \) – the off state of the control element in question.

Then the values of the intermediate relative radii of rotation \( p \) will be determined by the following dependencies.

For running board control:

\[
\rho = \frac{k}{2(1-k)(1-s)}
\]

(2)

For control on the lagging board:

\[
\rho = \frac{k}{2(k-1)(1-s)} .
\]

(3)

The sliding in the off brake will vary within \( 1 < s < 2 \).

Let’s move on to the question of the power analysis of the PDM.

We introduce the notation:

- \( M_{d1} \) and \( M_{d2} \) – moments on the left and right driving wheels;
- \( M_{d1} \) and \( M_{d2} \) – moments on the left and right half-axes before summing with the moment from the planetary series;
- \( M_{d1} \) and \( M_{d2} \) – moments on the left and right gears of planetary mechanisms connected to the differential housing;
- \( M_{s1} \) and \( M_{s2} \) – moments on the left and right gears of planetary mechanisms connected to the semi-axes;
- \( M_{t1} \) and \( M_{t2} \) – moments on the left and right brake mechanisms; - the input moment on the differential housing;
- \( M_z \) – total torque on the differential housing;
- \( f \), \( \varphi \), \( Z \), \( r \) – coefficients of rolling resistance and traction, the normal load on the wheel and the rolling radius.

To determine the values of the 12 entered variables, you will need to create 12 equations.

We neglect the friction in the gears and supports of the differential links.

It follows from the PDM equilibrium equation that \( M_{d1} = M_{d2} \), and \( M_z + M_{s1} + M_{s2} = 0 \).
We get two more pairs of equations from the equilibrium condition of the planetary series:

\[
\begin{align*}
M_p &= -kM_k \\
M_r &= -(1-k)M_k
\end{align*}
\]

Let’s add the coupling equations:

\[
\begin{align*}
M_{s1} &= M_{p1} + M_{d1} \\
M_{s2} &= M_{p2} + M_{d2} \\
M_z &= M_{s} + M_{k1} + M_{k2}
\end{align*}
\]

It is necessary to define the resulting system with three more equations.

When applying control to one (for example, the right) side, the mechanism of the opposite (left) side is not controlled, therefore \( M_{r1} = 0 \).

The remaining two equations can be obtained by combining the values of the four initial parameters \( M_{s1}, M_{s2}, M_{r2}, M_z \).

This approach allows us to consider various options for the task. However, in order to obtain realistic solutions, one should not lose sight of the estimates of the values of these variables.

For an unguided \( |M_s| = fZr \), it should be assumed for a controlled side \( |M_s| < qZr \). The maximum possible moment \( M_z \) for this driving mode is determined from the traction calculation, and the value \( M_r \) is limited by the design of the brake control element.

Kinematic and force analysis were performed for the mechanism, the scheme of which is shown in figure 1 for the control strategies for the running and lagging board. Links 1 and 2 are connected to the driving wheels; 3 and 4 are connected to the controls (brakes); link k is the driving one.

When calculating the internal efficiency for the differential was assumed to be equal to 0.98; for planetary rows with stepped satellites – equal to 0.97.

The use of the control on the running board turned out to be more profitable from the point of view of energy efficiency – a greater efficiency was realized. In this case, the kinematic parameter value of 0.847 is assumed for a series with stepped satellites. When driving on a lagging board, the value 1.22 is assumed.

Table 1 shows the results of kinematic calculation – the angular velocities of the main links and couplings in fractions of the angular velocity of the leading link. The latter is assumed to be constant and equal to one. Mode 1 corresponds to the control strategy for the running board, and mode 2 corresponds to the lagging board.

| Mode number | The number of the main link | 1  |
|-------------|---------------------------|----|
|             |                           | 2  |
| 1           |                           | 3  |
| 1           |                           | 4  |
| 2           |                           | 2.000 | 0.000 |
| 2           |                           | 2.000 | 2.000 |

Table 2 shows the main results of the force analysis – the values of the moments on the first link (in a simple planetary mechanism, this corresponds to the solar gear) in fractions of the moment of the leading link. The numbering of the planetary mechanisms is taken from left to right according to the scheme in figure 1. It should be understood that this calculation refers only to the additional torque applied to the running board when the corresponding control element is turned on.

Negative values of the moment on the elements of the planetary row of the lagging side in mode 2 indicate that the power from this row will be redirected from it through the differential box (housing) to the running side.

The relative moment on the included controls 3 and 4 is - 0.153.
Table 2. Relative moment of the first links.

| Mode number | The number of the planetary mechanism |
|-------------|--------------------------------------|
| 1           | 0.000  0.000  1.000                  |
| 2           | -0.694 -0.847 0.000                  |

Thus, regardless of the kinematic scheme of the PDM, two main stages can be distinguished when conducting kinematic and force analysis.

The imposition of external links and the formation of the basic equations of kinematics and force balance in accordance with the traditional principles of the analysis of planetary gears.

The imposition of additional links based on the consideration of the operation of the engine of the machine in specific operating conditions.

The joint solution of the obtained equations, the result of which makes it possible to determine the angular velocities and torques at all the main links of the PDM in the considered operating modes.

In the production of PDM belonging to this family, it is advisable to use technologies and materials tested during the serial production of transport tracked vehicles. In this industry (see works [14, 19, 20, 21].

To control the mechanism, as an alternative to an electromechanical drive, the technology of using which is poorly developed, it is proposed to use a closed electronic-hydraulic system. The hydraulic drive of the friction clutches in such a system operates in the mode of low-frequency pulse-width modulation of the control pressure [17, 22, 23, 24].

Tests of a prototype of the mechanism as part of the transmission can be carried out using the test stand used at Peter the Great St. Petersburg Polytechnic University [25].

4. Practical implications and prospects

The proposed approach to the construction of the equations of kinematic and force analysis of the PDM and the principles of determining the values of unknown quantities can be used in the design of the gear part of the controlled gear PDM and used as the basis of the mathematical model of the gear part of the PDM when working out control algorithms in the Matlab Simulink environment.

The application of the considered mechanism in the transmissions of wheeled transport and transport-technological will lead to an improvement in the quality of traffic control and an increase in transport productivity [1, 26, 27]. In the future, the proposed mechanism can be used as part of the transmission of an unmanned tractor in order to simplify the algorithms for controlling the machine when working in autonomous mode [28].

The mechanism of the ZF Vector Drive family can replace the symmetrical differential in the transmission of a wheeled tractor and get the operational advantages expected when implementing competing solutions [1,27].

Of further interest is the use of the proposed mechanism in the design of the chassis of new tracked and wheeled vehicles and the modernization of existing models of ground vehicles for various purposes [18, 29, 30].

5. Conclusions

For the controlled PDM of the ZF Vector Drive family, angular velocities and moment values at the main links are obtained, thus, the tasks of kinematic and force analysis are completed.

Regardless of the specific kinematic scheme of the controlled inter-wheeled planetary PDM, kinematic and force analysis include three characteristic stages (drawing up the basic equations of kinematics and force balance, superimposing the main connections; drawing up additional equations that take into account the features of the propulsion system and give additional connections; joint solution of the resulting system of equations).
The considered mechanism can be manufactured using technologies approved at domestic enterprises.

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