Impossibility of distant indirect measurement of the quantum Zeno effect

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Abstract

We critically study the possibility of the quantum Zeno effect for indirect measurements. If the detector is prepared to detect the emitted signal from the core system, and does not reflect this signal back to the core system, then we can prove that the decay probability of the system is not changed by the continuous measurement of the signal and the quantum Zeno effect would never take place. This argument also applies to the quantum Zeno effect for accelerated two-level systems and unstable particle decays.
1 Introduction

Continuous measurement of a quantum system freezes the dynamics. This quantum Zeno effect provides extreme nature of quantum mechanics\cite{1,2}. There are three kinds of causes of the quantum Zeno effect; (1) continuous measurement of the system, (2) the effect of the environment surrounding the system, and (3) the renormalization effect due to the interaction between the system and the detectors. In any case, the freezing of the total Hilbert space into several subspaces\cite{3} is the essence of the mechanism; a strong external disturbance dominates the total Hamiltonian whose eigenspaces form the subspaces. Among the above three causes, the last two can be described by (effective) Hamiltonian and therefore are relatively well understood. However, the first one relies on the phenomenological projection postulate a la von Neumann and needs a more careful investigation. In particular, the consistency between the global nature of the projection postulate and the causality, in the sense that any signal cannot exceed the light velocity $c$, is problematic. This issue will become particularly evident in the indirect quantum Zeno effect.

For a simple example of the distant indirect measurement, suppose the excited state of a two-level system of size $d$ decays to the ground state by emitting light, which is continuously monitored by a detector located at $l(>d/2)$. (See Fig.1.)

![Figure 1: Schematic description of the distant indirect measurement. The excited state of a two-level system of size $d$ decays to the ground state by emitting light, which is continuously monitored by the detector located at $l(>d/2)$.](image)

How and when does the quantum Zeno effect take place? Or does the effect never take place? Considering both the projection postulate and the causality, we may naturally expect that the Zeno effect, provided the decay law is appropriate for it, takes place only after the time $(2l - d)/c$. If this is the case, then what would happen when the two-level system is constantly accelerated and the detectors are set outside the causal boundary such that they can never affect the system? In Fig.2., we show this situation schematically. The trajectory of the two-level system (the solid line) is described by the hyperbolic function which approaches to the “horizon” $ct = x$ asymptotically in the future. The excited state of the two-level system decays to the ground state by emitting light.

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In order to monitor this signal, we prepare many detectors which are aligned so that their trajectories are entirely located above this horizon. The causality principle prevents such devices from affecting the two-level system through physical interaction. (See Fig. 2.) Then one could ask the questions we posed at the beginning of the previous paragraph. The measurement process may be able to make the system wave function change beyond causality because the “wave function” itself has no physical reality. However since the “probability” is a physical reality, the causality may prohibit the Zeno effect from taking place. Therefore, the problem here is the fact that, in this case, the change in “wave function” describes that of “probability”, and thus the possibility of the Zeno effect is very problematic.

Figure 2: Schematic description of the indirect measurement of an accelerated two-level system in the space-time diagram. The trajectory of the two-level system (the solid line) is described by the hyperbolic function which approaches to the “horizon” $ct = x$ asymptotically in the future. All the activated detectors are prepared so that their trajectories are entirely located above this horizon. Causality principle prevents such devices from affecting the system through physical interaction.

This distant indirect measurement is very common in high-energy particle experiments. Especially the issue becomes prominent in the decay experiments of unstable particles. Can we expect that the particle decay law is modified by the continuous monitoring of the emitted photon associated with the decay, by detectors surrounding the unstable particle? Moreover the issue becomes much prominent in macroscopic situations. Is it possible to expect that the life of the Schrödinger’s cat\textsuperscript{1} is elongated by the continuous measurement of the system?

\textsuperscript{1}The Schrödinger’s cat system is the strongly entangled state of the microscopic unstable atom and the macroscopic life.
from outside?

In this paper, we study this consistency issue between the global nature of the von Neumann projection postulate in quantum mechanics and the causality principle in relativity specifying the case of indirect measurements.

This paper is organized as follows. In section two, we specify, in the simplest form, the general feature of the indirect quantum Zeno experiments, and prove, under some conditions, the impossibility of this effect in the indirect distant measurement. Based on this argument, in section three, we study a simple model which demonstrates the situation. Finally, in the last section of this paper, we discuss several other applications of our argument.

2 Distant Indirect Measurements

Let us consider a quantum system with unitary evolution. The total system may actually be very complicated however a subspace \( H_Z \) which is relevant to consider the quantum Zeno effect would be almost closed in dynamics. We would like to concentrate on this subspace \( H_Z \). Then one can always construct an experiment of quantum Zeno effect using indirect measurement, as will be explained later in detail.

Let \( \{|e\rangle, \{n\rangle \}_{n=1,2,...} \) denote the complete orthonormal basis vectors in \( H_Z \). In the following we concentrate on the survival probability

\[
s(t) = |\langle e(0)|e(t)\rangle|^2
\]

of a state \(|e\rangle\) in \( H_Z \), where \(|e(t)\rangle\) is the state evolved from \(|e\rangle\) \(|e\rangle = |e(0)\rangle\). Since the evolution of the state vector is closed in this space,

\[
|e(t)\rangle = C(t)|e\rangle + \sum_n C_n(t)|n\rangle,
\]

the survival probability \( s(t) = |C(t)|^2 \) can be obtained without direct measurement of the state \(|e\rangle\). Actually, the measurement of the states \(|n\rangle\) provides us with the probability \( s(t) \) through the unitary relation:

\[
s(t) = 1 - \sum_n |C_n(t)|^2.
\]

This is the indirect measurement of \( s(t) \), which is the subject of our investigation. Due to the time reversal symmetry of the Schrödinger equation, for short time intervals, \( s(t) \) behaves like

\[
s(\Delta t) \sim 1 - \alpha \Delta t^2,
\]

where \( \alpha \) is a constant. Therefore, at time \( t = N\Delta t \) and after \( N \) successive measurements the survival probability of the state \(|e\rangle\) is calculated to be

\[
s_N(t) \sim (1 - \alpha(\Delta t)^2)^{\frac{N}{2}} \sim e^{-\alpha t\Delta t}.
\]
Consequently, for the continuous measurements of the states $|n\rangle$ (in the limit $N \to \infty$, $\Delta t \to 0$ with $t$ fixed) one obtains

$$\lim_{N \to \infty} s_N(t) = 1,$$

and thus, the time evolution of the state $|e\rangle$ freezes. This is the essence of the quantum zeno effect with indirect measurement\(^2\).

For a much elaborate argument of the indirect measurement, let us suppose the subspace $\mathcal{H}_Z$ can be decomposed into mutually complementary subspaces $\mathcal{H}_C$ and $\mathcal{H}_W$ ($\mathcal{H}_Z = \mathcal{H}_C \oplus \mathcal{H}_W$). We call $\mathcal{H}_C$ as the core-zone subspace and $\mathcal{H}_W$ as the wave-zone subspace if they satisfy the following property:

(I) Any vector $|W\rangle$ in $\mathcal{H}_W$ remains in $\mathcal{H}_W$ as it evolves at any advanced time:

$$|W(t > 0)\rangle := U_+(t)|W\rangle \in \mathcal{H}_W,$$

where $U_+(t > 0) = e^{-\frac{\hbar}{i}H}$ is the advanced time evolution operator for the system and $H$ is the Hamiltonian of the system.

If we introduce the projection operators $P_C$, $P_W$, and the unit projection operator $1_Z$ which project a state of $\mathcal{H}_Z$ onto $\mathcal{H}_C$, $\mathcal{H}_W$, and $\mathcal{H}_Z$, respectively, where

$$P_C + P_W = 1_Z,$$

the property (I) can also be expressed as the followings:

(II) For any wave-zone vector $|W\rangle$,

$$P_C|W(t > 0)\rangle = P_C U_+(t)|W\rangle = 0,$$

or,

(III) For $t > 0$,

$$P_C U_+(t) P_W U_+(t)^\dagger = 0.$$

Clearly, the emergence of the subspace $\mathcal{H}_W$ with a nontrivial $\mathcal{H}_C$ is allowed only in the case when $\mathcal{H}_W$ is an open system. In this paper we call $|C\rangle (\in \mathcal{H}_C)$ a core-zone state and $|Z\rangle (\in \mathcal{H}_W)$ a wave-zone state. From any of the properties (I), (II), or (III), we obtain the following useful lemma.

Lemma: If two state vectors $|\Psi_1(t_o)\rangle$ and $|\Psi_2(t_o)\rangle$ in $\mathcal{H}_Z$ satisfy the following relation at time $t_o$,

$$P_C|\Psi_1(t_o)\rangle = P_C|\Psi_2(t_o)\rangle,$$

then this relation always holds at time $t (> t_o)$ in the future:

$$P_C|\Psi_1(t)\rangle = P_C|\Psi_2(t)\rangle.$$

\(^2\text{Koshino and Shimizu}\) investigated a model of the indirect measurements of the Zeno effects and found even an incomplete measurement is sufficient to yield the effect. Our argument is different from theirs in an essential point. See the end of discussion for more detail.
To prove this statement, let us define a core-zone state $|C\rangle$ as

$$|C\rangle := P_C|\Psi_1(t_o)\rangle = P_C|\Psi_2(t_o)\rangle.$$  

Then using two wave-zone states $|W_1\rangle$ and $|W_2\rangle$, the initial states can be expressed as

$$|\Psi_1(t_o)\rangle = |C\rangle + |W_1\rangle,$$

$$|\Psi_2(t_o)\rangle = |C\rangle + |W_2\rangle.$$  

Next, let us evolve the two states until the time $t$.

$$|\Psi_1(t)\rangle = U_+(t-t_o)|C\rangle + U_+(t-t_o)|W_1\rangle,$$

$$|\Psi_2(t)\rangle = U_+(t-t_o)|C\rangle + U_+(t-t_o)|W_2\rangle.$$  

Because $|W_1\rangle$ and $|W_2\rangle$ are wave-zone states and $t-t_o > 0$ holds, they satisfy

$$P_C U_+(t-t_o)|W_1\rangle = P_C U_+(t-t_o)|W_2\rangle = 0.$$  

according to the property (II). Thus by operating $P_C$ in eqns. (15) and (16), we obtain the result mentioned in the lemma:

$$P_C|\Psi_1(t)\rangle = P_C U_+(t-t_o)|C\rangle = P_C|\Psi_2(t)\rangle.$$  

Now let us take a core-zone state $|c\rangle$ as the initial state at $t = 0$. We calculate $s_N(t)$, which is the survival probability of $|c\rangle$ at a later time $t$ after $N$ successive $\mathcal{H}_W$ measurements and compare it with the same quantity without any $\mathcal{H}_W$ measurements $s(t) = |\langle c(0)|c(t)\rangle|^2$. Here, we assume that all the measurements are completely ideal and represented by the von Neumann projection postulate. Let us explain the procedure in more detail. The initial state $|c(0)\rangle$ is set at time $t = 0$. The measurements on $\mathcal{H}_W$ are performed at times $t = t_i$ ($i = 1 \sim N$ and $0 < t_1 < t_2 < \cdots < t_N$). Each measurement makes the wave function shrink in either of the two ways, depending upon whether the observed state of the system belongs to $\mathcal{H}_W$ or not. After the $N$-times measurements, we finally check at time $t = t_{N+1}$ whether the decaying two-level atom is still in the initial excited state $|e(0)\rangle$ or not. Repeating the procedure many times would yield the observational value of the survival probability $s_N(t)$.

From $t = 0$ to $t = t_1$ the state vector evolves by the unitary evolution $U_+(t)$ as

$$|\Psi(t)\rangle = U_+(t)|c(0)\rangle = |c(t)\rangle.$$  

At time $t = t_1$, the first measurement is executed. The probability of finding that the observed state belongs to $\mathcal{H}_W$ is given as

$$p_1 = \langle \Psi(t_1) | P_W | \Psi(t_1) \rangle.$$  

If this case is realized, the state shrinks into a wave-zone state $|\Psi_{w_1}(t_1)\rangle$ defined by

$$|\Psi_{w_1}(t_1)\rangle = \frac{1}{\sqrt{p_1}} P_W |\Psi(t_1)\rangle.$$  

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Because $|\Psi_{[w]}(t_1)| \in \mathcal{H}_W$ and any wave-zone state evolves only within $\mathcal{H}_W$ (1), the system is no longer found in any state of $\mathcal{H}_C$ in the remaining measurements. Hence $|\Psi_{[w]}|$ does not contribute at all to the survival probability of $|e(0)|$, which is a core-zone state, and we can completely ignore this possibility in the calculation of $s_N(t)$. If one does not find any state in $\mathcal{H}_W$, the state shrinks into a core-zone state $|\Psi_{[c]}(t_1)|$ defined by

$$
|\Psi_{[c]}(t_1)| = \frac{1}{\sqrt{1 - p_1}} P_C|\Psi(t_1)|.
$$

(22)

The probability of this case is given by $1 - p_1$. Here the projective relation $P^2_C = P_C$ directly yields the following equation:

$$
P_C|\Psi_{[c]}(t_1)| = \frac{1}{\sqrt{1 - p_1}} P_C|\Psi(t_1)|.
$$

(23)

Due to the above lemma, we obtain the following relation at the future time $t = t_2(> t_1)$:

$$
P_C|\Psi_{[c]}(t_2)| = \frac{1}{\sqrt{1 - p_1}} P_C|\Psi(t_2)|.
$$

(24)

Consequently, it is possible to write the probability of finding that the state of the system belongs to $\mathcal{H}_C$ at $t = t_2$ if it belonged to $\mathcal{H}_C$ at $t = t_1$ as follows.

$$
1 - p_2 = 1 - \langle \Psi_{[c]}(t_2) | P_W | \Psi_{[c]}(t_2) \rangle.
$$

(25)

Thus the probability of not finding the system in any state of $\mathcal{H}_W$ in both measurements at $t = t_1$ and $t = t_2$ is given by

$$(1 - p_1) \times (1 - p_2).$$

(26)

If this situation is realized, the state shrinks into the core-zone state defined by

$$
|\Psi_{[cc]}(t_2)| = \frac{1}{\sqrt{1 - p_2}} P_C|\Psi_{[c]}(t_2)|.
$$

(27)

By virtue of the above lemma it is easily shown that

$$
P_C|\Psi_{[cc]}(t_3)| = \frac{1}{\sqrt{1 - p_2}} P_C|\Psi_{[c]}(t_3)|.
$$

(28)

Because the relation $t_3 > t_1$ holds, one can also write

$$
P_C|\Psi_{[c]}(t_3)| = \frac{1}{\sqrt{1 - p_1}} P_C|\Psi(t_3)|.
$$

(29)

We can repeat the same procedure $N$-times. For each measurement procedure, in the similar way in eqns. (22) and (27), we define the shrunk states as

$$
|\Psi_{[c\otimes n]}(t_n)| = \frac{1}{\sqrt{1 - p_n}} P_C|\Psi_{[c\otimes (n-1)]}(t_n)|.
$$

(30)
where \( n = 1 \sim N \) and \( p_n \) is defined by

\[
p_n = \langle \Psi_{c^{\otimes (n-1)}} | P_W | \Psi_{c^{\otimes (n-1)}} \rangle.
\]

Moreover note that by using the above lemma, we can prove, similarly to eqns. (28) and (29), that

\[
P_C | \Psi_{c^{\otimes k}}(t_{N+1}) \rangle = \frac{1}{\sqrt{1 - p_k}} P_C | \Psi_{c^{\otimes (k-1)}}(t_{N+1}) \rangle
\]

for \( k = 1 \sim N \). As the last step at time \( t = t_{N+1} \), we probe directly the \( H_C \) sector and determine the probability of finding the system in the initial state, \( s_N(t) \). The explicit form of \( s_N(t) \) is given by

\[
s_N(t = t_{N+1}) = \left[ \prod_{k=1}^{N} (1 - p_k) \right] \times |\langle e(0) | \Psi_{c^{\otimes N}}(t_{N+1}) \rangle|^2.
\]

The first factor in the right-hand-side is the probability of finding the state is the core-zone state \( |\Psi_{c^{\otimes N}}\rangle \) after the \( N \) measurements. The second factor is the transition probability from \( |\Psi_{c^{\otimes N}}\rangle \) to \( |e(0)\rangle \). It should be reminded here that \( |e(0)\rangle \) is a core-zone state, that is,

\[
|e(0)\rangle = (e(0)|P_C.
\]

Using eqns. (31) and (32), we can rewrite \( s_N \) as

\[
s_N(t = t_{N+1}) = \left[ \prod_{k=1}^{N} (1 - p_k) \right] \times |\langle e(0) | \Psi_{c^{\otimes N}}(t_{N+1}) \rangle|^2
\]

and therefore, we obtain the following theorem:

**Theorem:** If we take a core-zone state \( |e(0)\rangle \) as the initial state at \( t = 0 \), under any of the conditions (I), (II) or (III), \( N \)-times measurements on the wave-zone subspace \( H_W \) does not affect the survival probability of \( |e(0)\rangle \) at all, i.e.:

\[
s_N(t) = s(t).
\]

It should be emphasized that we have not specified the time dependence of \( s(t) \). The theorem is applicable even when the form of \( s(t) \) is different from the exponential form \( \propto e^{-\Gamma t} \). Hence, even if \( s(t) \) obeys the standard early behavior.
like \( s(t) \sim 1 - \alpha t^2 \), the wave-zone measurements do not yield the Zeno effect at all.

The above argument leading to the theorem can be summarized as follows. The statement (I) means "wave-zone states evolve within \( \mathcal{H}_W \) in the future":

\[
[P_W, U_+] P_W = 0
\]

Since \( P_C + P_W = 1 \), this leads to \( [P_C, U_+] P_W = 0 \), which is equivalent to \( P_C U_+(t) P_W = 0 \). (These are other forms of (II)(III).) The last relation yields the above lemma \( P_C [P_C, U_+] = 0 \), which claims "Any part of a state which finally evolves to a state in \( \mathcal{H}_C \) has been evolving within \( \mathcal{H}_C \) entirely". The repeated use of the lemma in this form yields

\[
P_C U_+ (\Delta t_n) P_C U_+ (\Delta t_{n-1}) \cdots P_C U_+ (\Delta t_1) = P_C \prod_{j=1}^{n} U_+ (\Delta t_j),
\]

which clearly shows this statement.

By inserting the normalization factor after each projection in eqn. (35),

\[
\frac{1}{\sqrt{1 - p_n}} P_C U_+ (\Delta t_n) \frac{1}{\sqrt{1 - p_{n-1}}} P_C U_+ (\Delta t_{n-1}) \cdots \frac{1}{\sqrt{1 - p_1}} P_C U_+ (\Delta t_1)
\]

one obtains the resultant state after successive negative-result measurement on \( \mathcal{H}_W \), where \( 1 - p_{j} \) is the probability that the state is not in \( \mathcal{H}_W \) at the time \( t_{j} \). When we calculate the survival probability \( s_N(t) \), the above normalization factor totally cancels with these probabilities. Thus the final survival probability is not affected by the repeated measurements; this is the theorem expressed above in eqn. (34). This cancellation of the factors \( \sqrt{1 - p_j} \) simply reflects the fact that only a single intermediate state contributes in the calculation of the survival probability. Since there is no interference between different intermediate states in this case, it is clear that the measurement cannot affect the result. This claim applies not only to the survival probability with the initial condition \( |e(0)\rangle \), but also any process which finally results in a state in \( \mathcal{H}_C \), according to eqn. (35).

In the above arguments for the Hamiltonian system, the boundary/initial condition, which reflects the wave-zone property (I), has been essential. This one-sided property would actually be associated with the most distant indirect measurements of quantum processes. In the next section, we study a typical model of indirect measurement which falls into this category.

### 3 A Model

We have developed the general formalism which leads to the theorem of eqn. (34) in the previous section. Now we would like to study a simple example which possesses a wave-zone subspace and the theorem is applicable rigorously. Let us suppose a one-dimensional space in which \( x \) denotes its spatial coordinate, and set a two-level atom system of size \( d \) in the region \([-d/2, d/2]\). To express the
upper and lower energy level states, we introduce a fermionic pair of annihilation and creation operators $a$ and $a^\dagger$:

\[
\{a, a^\dagger\} = 1,
\]

\[
\{a^\dagger, a^\dagger\} = \{a, a\} = 0.
\]

We also introduce a massless spinor field

\[
\Phi(x) = (\Phi_R(x), \Phi_L(x))^T
\]

and quantize it in the fermionic way:

\[
\{\Phi_h(x), \Phi_h'(x')\} = \delta_{hh'} \delta(x - x'),
\]

\[
\{\Phi_h^\dagger(x), \Phi_h^\dagger(x')\} = \{\Phi_h(x), \Phi_h'(x')\} = 0,
\]

where $h(= R, L)$ is the helicity of the field excitations. As in the Coleman-Hepp model, using the annihilation operator, the vacuum state $|\text{vac}\rangle$ can be introduced as

\[
a|\text{vac}\rangle = 0,
\]

\[
\Phi_h(x)|\text{vac}\rangle = 0.
\]

Then the excited state of the two-level atom is defined by

\[
|e\rangle = a^\dagger|\text{vac}\rangle.
\]

For the spinor field, we concentrate on the two particle states in which only one R-helicity and one L-helicity particles exist. The state in which a R-helicity particle stays at the position $x = x_R$ and a L-helicity particle at $x = x_L$ is denoted by

\[
|x_R, x_L\rangle = \Phi_h^\dagger(x_R)\Phi_h^\dagger(x_L)|\text{vac}\rangle.
\]

Now let us write the Hamiltonian of the total system, which is composed of three terms:

\[
H = H_{\text{atom}} + H_\Phi + H_{\text{int}}.
\]

The first term $H_{\text{atom}}$ is the Hamiltonian of the free motion of the two-level atom and is given by

\[
H_{\text{atom}} = \hbar \omega a^\dagger a,
\]

where the energy of the excited state is set to be $\hbar \omega$. The second term $H_\Phi$ is the free Hamiltonian of the massless spinor field and is defined by

\[
H_\Phi = -i\hbar c \int_{-\infty}^{\infty} \Phi^\dagger \sigma_3 \partial_x \Phi dx
\]

\[
= -i\hbar c \int_{-\infty}^{\infty} \left[ \Phi_R^\dagger \partial_x \Phi_R - \Phi_L^\dagger \partial_x \Phi_L \right] dx,
\]

where $\sigma_3$ is the third component of the Pauli matrix. If no interaction term is added, the field Hamiltonian yields right-moving particles for $h = R$ and
left-moving particles for $h = L$ with the light velocity. The third term $H_{int}$ in eqn.(46) expresses the interaction between the two-level atom and the spinor field and is given by

$$H_{int} = \hbar \int_{-d/2}^{d/2} dx \int_{-d/2}^{d/2} dx' \left[ g(x, x') \Phi_R^\dagger(x) \Phi_L^\dagger(x') a + g(x, x')^* a^\dagger \Phi_L(x') \Phi_R(x) \right]. \quad (49)$$

The interaction induced by the coupling $g(x, x')$ is supposed to take place only in the restricted spatial region defined by $x, x' \in (-d/2, d/2)$ and the excited state of the atom decays into two particle states with different helicities. Note that even after adding the interaction term in eqn.(49), $|\text{vac}\rangle$ is still stable.

In this model, the subspace, whose complete basis is given by $\{ |e\rangle, |x_{R}, x_{L}\rangle \}$ with $x_{R} \geq -d/2$ and $x_{L} \leq d/2$, can be identified as a subspace $\mathcal{H}_{Z}$, because the evolution in this space is closed:

$$|\Psi(t)\rangle = C(t) |e\rangle + \int_{-d/2}^{d/2} dx_{R} \int_{-d/2}^{d/2} dx_{L} F(x_{R}, x_{L}; t) |x_{R}, x_{L}\rangle \quad (50)$$

for an arbitrary vector $|\Psi\rangle(\in \mathcal{H}_{Z})$. From the Schrödinger equation, the amplitudes obey the following equations:

$$i \partial_{t} C = \omega C(t) + \int_{-d/2}^{d/2} dx_{R} \int_{-d/2}^{d/2} dx_{L} g(x_{R}, x_{L})^* F(x_{R}, x_{L}; t), \quad (51)$$

$$\left( \partial_{t} + c \partial_{x_{R}} - c \partial_{x_{L}} \right) F(x_{R}, x_{L}; t) = -ig(x_{R}, x_{L}) C(t). \quad (52)$$

It is possible to integrate eqn.(52) formally and the result can be expressed as

$$F(x_{R}, x_{L}; t) = F_{o}(x_{R} - ct, x_{L} + ct)$$

$$-i \int_{0}^{t} g(x_{R} - ct + c\tau, x_{L} + ct + c\tau) C(\tau) d\tau, \quad (53)$$

where $F_{o}(x_{R}, x_{L})$ is the initial amplitude of $F$ at $t = 0$.

When $x_{R} > d/2$ or $x_{L} < -d/2$, by taking the initial condition as

$$C(0) = 0, \quad (54)$$

$$F_{o}(x_{R}, x_{L}) = \delta(x_{R} - x'_{R}) \delta(x_{L} - x'_{L}), \quad (55)$$

the following relation arises from eqn.(51) and eqn.(53):

$$U_{+}(t)|x_{R}, x_{L}\rangle = |x_{R} + ct, x_{L} - ct\rangle. \quad (56)$$

This means that the right- and left-moving particles propagate freely after leaving the interaction region. It is worth stressing that even if only one of the two conditions $x_{R} > d/2$ or $x_{L} < -d/2$ holds, the evolution in eqn.(56) is still realized. This is because the interaction is activated only when both particles simultaneously stay in the region $(-d/2, d/2)$. 


From the above result, we can introduce in $\mathcal{H}_Z$ a wave-zone subspace $\mathcal{H}_W$ for the excited state $|e\rangle$, which is defined using the projection operator onto the subspace $P_W$:

$$P_W = P_+ + P_- + P_{+-},$$ \hspace{1cm} (57)

Then the core-zone subspace $\mathcal{H}_C$ is defined using the projection operator:

$$P_C := 1 - P_W = |e\rangle\langle e| + \int_{-d/2}^{d/2} dx_L \int_{-d/2}^{d/2} dx_L |x_R, x_L\rangle\langle x_R, x_L|.$$ \hspace{1cm} (61)

By construction, we have

$$P_C |e\rangle = |e\rangle.$$}

Now, the states are complete in $\mathcal{H}_Z$ as seen in eqn. (50), and the property (I) holds due to eqn. (56), and therefore, everything is in place to apply the theorem of the previous section. According to this theorem, one can claim that the measurements of the $\Phi$ particles in the outside regions $(-\infty, -d/2] \cup [d/2, \infty)$ do not affect the survival probability of the excited state of the atom at all, hence, neither Zeno nor anti-Zeno effects take place.

On the other hand, if one observes the $\Phi$ particles not only in the outside region but also in the inside interaction region $(-d/2, d/2)$, then the development of $s_N(t)$ is expected to be modified by such measurements. Clearly, when the continuous measurements of the $\Phi$ particles are performed in the whole region $(-\infty, \infty)$, the quantum Zeno effect will most strongly take place; $s_N \to 1$.

### 4 Discussions and Comments

The starting point of our argument has been the question of how the causality and the projection postulate in quantum mechanics can be reconciled with each other in the indirect quantum Zeno effects. Recognizing that the indirect distant measurement often possesses the wave-zone property (I), we have studied this one-sided property, which led to the lemma of eqn. (11). By calculating the survival probability of a system under indirect measurement, we have found the probability is not affected by the measurement at all. Further by using a simple model, we have demonstrated the applicability of the general argument.

Some additional comments on our investigation are in order.
The first one is related to the reflection wave of the detector. One may suppose models in which the apparatus creates reflectional waves of the \( \Phi \) field in the process of measurement. The reflected \( \Phi \) wave returns to the core-zone region and begins to affect the evolution of the \( \mathcal{H}_C \) sector after the arrival time. Thus, the detector Hamiltonian comes to violate the structure of the original wave-zone property (I); the \( \Phi \) particle states outside the atom are no longer exact wave-zone states. Therefore our theorem is not exactly applicable in such cases. However, the contribution of the reflectional wave would be subleading in the coupling expansion and can be negligible in small coupling cases as commented below again.

The second comment is related to the completeness of the measurement. In the above arguments, we have treated the ideal measurement just for convenience of discussion. However our argument can be applicable for more realistic incomplete measurements. For example, as in the work of Koshino and Shimizu [4], we can set the measurement apparatus outside the atom \((x \in [x_-, x_+])\), which measures the \( \Phi \) field in the above model with the following interaction:

\[
\mathcal{H}_{\text{detector}} = \hbar \int_{-\infty}^{\infty} \Omega(k) b_1^\dagger(k) b(k) dk \\
+ \hbar \int_{x_-}^{x_+} dx \int_{-\infty}^{\infty} dk \lambda_R(x, k) \left( b_1^\dagger(k) \Phi_R(x) + \Phi_R^\dagger(x) b(k) \right) \\
+ \hbar \int_{x_-}^{x_+} dx \int_{-\infty}^{\infty} dk \lambda_L(x, k) \left( b_1^\dagger(k) \Phi_L(x) + \Phi_L^\dagger(x) b(k) \right),
\]

where \( x_+ > x_- > d/2 \), and \( b(k) \) (\( b_1^\dagger(k) \)) is the annihilation (creation) operator of the detector excitations:

\[ [b(k), b_1^\dagger(k')] = \delta(k - k'), \]

and \( \lambda_h(x, k) \) is a real coupling function which controls the incompleteness of the detector. The third term of the Hamiltonian in eqn. (62) generates a reflectional left-moving wave and the system may be in two-left-moving-particle states. The time evolution in this case can be expressed in a closed form as

\[
|\Psi(t)\rangle = C(t)|e\rangle + \int_{-d/2}^{d/2} dx_R \int_{-\infty}^{\infty} dx_L F(x_R, x_L; t)|x_R, x_L\rangle \\
+ \int_{d/2}^{d/2} dx_L \int_{-\infty}^{x_+} dx_R \int_{-\infty}^{\infty} dx_R^2 G(x_R^1, x_R^2; t)|x_R^1, x_R^2, x_L\rangle \\
+ \int_{d/2}^{d/2} dx_L \int_{-\infty}^{\infty} dx_R \int_{-\infty}^{\infty} dk D_k(x_L; t)|b_k^\dagger, x_L\rangle
\]

where

\[ |x_R^1, x_R^2\rangle = \Phi_L^\dagger(x^1) \Phi_L^\dagger(x^2)|\text{vac}\rangle, \]

and

\[ |b_k^\dagger, x_L\rangle = b_k^\dagger \Phi_L^\dagger(x_L)|\text{vac}\rangle. \]
Note that the interaction between the atom and the two left-moving particles does not exist in the model. Therefore, the following relation is always satisfied provided $x^1_L, x^2_L < x_-$:

$$U_+(t)|x^1_L, x^2_L\rangle = |x^1_L - ct, x^2_L - ct\rangle.$$  

A crucial point is that the evolution of $C(t)$ and $F(x_R, x_L)$ for $x_R, x_L \in (-d/2, d/2)$ in this case is exactly the same as in eqn. (51) and eqn. (52). Consequently, both the couplings $\lambda_R$ and $\lambda_L$ give no contribution to the survival probability of the atom's excited state $s$ defined by

$$s(t, \lambda_h) = \langle e| \exp \left[ -\frac{i}{\hbar} t (H + H_{\text{detector}}) \right]|e\rangle^2.$$  

(64)

$$\frac{\delta s(t, \lambda_h(x, k))}{\delta \lambda_h(x, k)} = 0.$$  

(65)

This implies that no quantum Zeno effect is observed even in this incomplete measurement model with a reflectional wave.

Our argument will also be applicable for the decay experiments of unstable particles with long life times. In the early study of the Zeno effect, it has been argued that the unstable particle cannot decay when it is monitored continuously. In general in this experiment, the light (or any signal) emitted from the decayed particle is detected by the high sensitive measuring device at distance, which is a typical case of the indirect measurement. In the past analyses \[5\], the focus has been on the early time behavior of the survival probability $s(t)$. Especially it is pointed out that the Zeno effect analysis inevitably requires the nonperturbative behavior of $s(t)$ in the small-Q S-wave decay which is still controversial due to the nonperturbative difficulty. However, independent from their analysis, our theorem does not need any explicit time dependence of $s(t)$ and can be quite useful. In the real three-dimensional situation, most of the reflectional wave is expanded in space and only a tiny part of it can go back to the decaying area. Further, this tiny part has to reproduce the inverse-decay process followed by another decay process in order to make the decay probability change. This process is at higher order in small coupling constant and therefore makes the effect negligibly small. Hence the property (I) would hold with fairly high precision. As a result, this type of continuous measurement yields only negligible Zeno suppression of the unstable particle decay, irrespective of the decay law’s detail.

For the quantum Zeno effect in the constantly accelerated system, the confrontation between the causality and the projection postulate has become prominent, as explained in the introduction. According to our argument however, the situation has become very clear and the paradox is settled. Since this case is the typical indirect measurement with the property (I) which is guaranteed by the one-sided property of the causality horizon, the measurement does not affect the decay probability at all. The same argument would also apply to the evaporation of Black Holes due to the Hawking radiation.
When the size of the total system $H_Z$ becomes macroscopic, the property (I) tends to hold more naturally. For example in the macroscopic Schrödinger’s cat system, the external measurement of the cat state would permit the property (I), and the quantum Zeno effect does not take place; the continuous measurement does not change the destiny of the cat.

In the context of indirect measurement of the quantum Zeno effects, the work by Koshino and Shimizu [4] is important. By investigating a model of indirect measurements of Zeno and anti-Zeno effects in spontaneous decay of a two-level atom, they found that the quantum Zeno effect really takes place even if the detector of a photon excitation emitted from the atom does not cover the full solid angle. We cannot however apply our argument to their model because their detector is not spatially localized and spreads over the space; the emitted photon continues to propagate in the atom region until it is observed and therefore our property (I) does not hold. On the other hand in our paper, we explicitly utilized the local property of the (anti-)Zeno effect which was not considered in [4]. Actually the model we have used describes the case in which the emitted excitations get out of the spatial region of the decaying two-level atom at a finite time and propagate freely before the detection. Restricted in such a situation we have proved that the survival probability of the decaying state is not affected at all by the successive indirect measurements.

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