Functions for estimating aboveground biomass of birch in Norway

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A suite of regional allometric aboveground biomass functions were derived for Betula pubescens and Betula pendula for Norwegian conditions. The data consisted of 67 trees sampled throughout Norway. A total of 14 component functions were developed for total aboveground, total stem, stemwood, stem bark, live crown, live branch, leaf, and dead branch biomass using combinations of diameter at breast height and height as predictor variables. Application of the derived functions to existing local southern Norwegian mountain birch and regional Swedish biomass datasets indicated an overall good predictive ability of the developed functions. However, the functions produced slight underestimates, suggesting that the respective birch populations had differing biomass allocation patterns. When the developed functions were applied to Norwegian National Forest Inventory data, they produced slightly higher biomass stock and stock change estimates than what is obtained using existing Swedish functions. The higher estimates were evident in the north, central, and western part of Norway, while estimates were similar in southeastern Norway where growing conditions are most similar to Swedish conditions. The analysis indicates that the derived functions are the best available for regional birch biomass stock and stock change estimation in Norway.

Keywords: national biomass; biomass functions; allometry; birch; Kyoto Protocol; mixed-effects

Introduction

Tree biomass stock and stock change estimation are central for forest-based bioenergy feedstock assessments, in studies of the terrestrial carbon cycle, and for reporting under the United Nations Framework Convention on Climate Change (UNFCCC) and the Kyoto Protocol. Currently, Norway uses the regional Swedish functions developed by Marklund (1987, 1988) for reporting Picea abies (L.) Karst (Norway spruce), Pinus sylvestris L. (Scots pine), Betula pubescens Ehrh. (downy birch), and Betula pendula Roth (silver birch) biomass. The practice of geographically extrapolating the functions to Norway likely leads to some unknown error in the biomass estimate resulting from the potentially differing biomass allocation patterns of the same tree species growing in different conditions.

B. pendula and B. pubescens with its high elevation subspecies B. pubescens Ehrh. ssp. czerepanovii (N.I. Orlova) Hämet-Ahti (mountain birch) are the two main birch species in Norway. Downy birch is the most common of the two birch species, comprising over 95% of total birch volume and occurring throughout the country (Norwegian National Forest Inventory [NNFI] 2009). Silver birch occurs at lower elevations and has a more southerly distribution, although the species is also locally present in parts of northern Norway up to a latitude of 66°N and in eastern parts of Finnmark county (NNFI 2009). Birch is the third most common tree genus in Norway, representing 16% of the standing tree volume behind Scots pine (30%) and Norway spruce (45%) (Granhus et al. 2012).

Several local biomass functions have been derived for both birch species in Norway, but they are limited in their applicable geographic range. Functions for southeastern silver birch were developed for stem, branch, and leaf biomass by Korsmo (1995). Mountain birch functions have been derived from birch sampled from the southwest (Kjellvik 1974; Bollandsås et al. 2009) and the southeast (Opdahl 1987; Bollandsås et al. 2009) for various biomass components including aboveground, stem, stemwood, stem bark, total crown, branches, and leaves. The existing local functions do not cover large areas of the birch zone in Norway including low elevation coastal western, mid-elevation southeastern, central, or northern Norway.

Birch is one of the most common trees by growing stock throughout Scandinavia, the Baltic nations, Russia, and locally in northern North America (Global Forest Resources Assessment [GFRA] 2010). Regional allometric birch biomass functions have been developed for Iceland (Snorrason & Einarsson 2006), Sweden (Marklund 1987, 1988), and Finland (Repola 2008). Regional generalized regression (Pastor et al. 1984) functions have been developed for birch from meta-analyses of existing local...
functions for Canada (Lambert et al. 2005; Ung et al. 2008) and the USA (Jenkins et al. 2003). To the authors' knowledge, no regional birch functions currently exist for the Baltic nations or Russia.

The objectives of this study were to: (1) derive regional allometric aboveground biomass functions for birch in Norway; (2) compare the derived functions by applying them to two existing birch biomass datasets; and (3) use NNFI data to compare the birch biomass stock and stock change estimates obtained with the derived functions with estimates from existing local southern Norwegian mountain birch functions and the regional Swedish functions currently used to obtain birch biomass stock and stock change estimates for Norway.

**Materials and methods**

**Site and sample tree selection**

In order to obtain empirical allometric functions for biomass estimation, individual birch trees were destructively sampled. Sample site locations were subjectively selected to represent (to the extent possible) the range of conditions in which birch occurs in Norway. Sample site selection was initially made by dividing Norway into four regions: southeastern, western, central, and northern. For each region, four to five sites were located (Figure 1) and within each site, four trees were sampled with adequate spacing from each other, resulting in a total of 67 sampled trees on 17 sites (one small tree was lost during processing). Within each region, the sample sites were located to represent the regional variability in site, stand, and tree variables (Table 1). The sample trees were selected from vigorous rot-free trees to reflect the full diameter at breast height at 1.3 m (dbh) range present on the site. For each sample tree, a 250 m² ($r = 8.92$ m) plot was established with the sample tree as plot center. Species and dbh were recorded for all trees (other than birch) on the plot with a total height in excess of 50% of the dominant tree height in young

![Figure 1. Birch biomass sampling site locations. Seventeen total sites were selected with five located in the southeast, four in the west, four in central, and four in northern Norway.](image-url)
Table 1. Descriptive data for the current study, 9th NNFI, Marklund (1987, 1988), and Bollandsås et al. (2009) datasets.

| Variable | Mean | Minimum | Maximum | Standard deviation |
|----------|------|---------|---------|--------------------|
| TAG biomass (kg) | 116.3 | 2.9 | 1101.0 | 178.9 |
| dbh (cm) | 15.3 | 4.0 | 45.5 | 8.4 |
| Height (m) | 12.0 | 5.8 | 29.6 | 5.2 |
| Crown height (m) | 3.9 | 0.1 | 15.2 | 3.2 |
| Age (years at breast height) | 50 | 6 | 144 | 36 |
| Basal area (m² ha⁻¹) | 16.0 | 3.0 | 41.8 | 11.7 |
| Stems (number ha⁻¹) | 1449 | 370 | 2770 | 632 |
| Birch site index | 13.9 | 8.4 | 23.7 | 5.1 |
| Elevation (m.a.s.l.) | 202 | 23 | 547 | 185 |
| Plot proportion birch (%) | 84.6 | 13.6 | 100.0 | 21.1 |
| Diameter-to-height-ratio | 1.2 | 0.6 | 2.3 | 0.4 |
| Mean annual temperature (°C) | 3.4 | −0.1 | 6.9 | 2.1 |
| Minimum monthly mean temperature (°C) | −5.8 | −10.0 | 1.0 | 3.4 |
| Maximum monthly mean temperature (°C) | 13.2 | 11.0 | 16.0 | 1.6 |
| Mean annual precipitation (mm) | 1155.2 | 583.4 | 2282.8 | 650.4 |
| Latitude | − | 59°36′ N | 69°09′ N | − |

9th Norwegian National Forest Inventory (2005–2009)

| Variable | Mean | Minimum | Maximum | Standard deviation |
|----------|------|---------|---------|--------------------|
| dbh (cm) | 9.7 | 5.0 | 69.1 | 4.6 |
| Height (m) | 7.8 | 1.7 | 30.8 | 3.4 |
| Crown height (m) | 3.8 | 1.0 | 17.4 | 2.2 |
| NNFI birch site index | 8.1 | Unprod. | 23.0 | 4.9 |
| Elevation (m.a.s.l.) | 411 | 1.0 | 1130 | 286 |
| Latitude | − | 57°59′ N | 70°46′ N | − |

Marklund (1987, 1988)

| Variable | Mean | Minimum | Maximum | Standard deviation |
|----------|------|---------|---------|--------------------|
| TAG biomass (kg) | 75.0 | 1.4 | 783.0 | 113.0 |
| dbh (cm) | 12.7 | 3.2 | 36.8 | 6.7 |
| Height (m) | 11.2 | 4.0 | 24.8 | 4.4 |
| Crown height (m) | 4.1 | 0.1 | 14.5 | 2.7 |
| Age (years at breast height) | 46 | 9 | 128 | 28 |
| Basal area (m² ha⁻¹) | 19.0 | 2.4 | 43.0 | 10.0 |
| Elevation (m.a.s.l.) | 233 | 35 | 570 | 121 |
| Diameter-to-height-ratio | 1.1 | 0.5 | 1.9 | 0.3 |
| Mean annual temperature (°C) | 3.0 | −1.9 | 6.5 | 2.6 |
| Minimum monthly mean temperature (°C) | −7.8 | −16.0 | −2.0 | 4.3 |
| Maximum monthly mean temperature (°C) | 14.3 | 11.0 | 16.0 | 1.3 |
| Mean annual precipitation (mm) | 640.3 | 472.7 | 1021.4 | 95.3 |
| Latitude | − | 56°21′ N | 67°35′ N | − |

Bollandsås et al. (2009)

| Variable | Mean | Minimum | Maximum | Standard deviation |
|----------|------|---------|---------|--------------------|
| TAG biomass (kg) | 16.8 | 2.0 | 116.7 | 17.2 |
| dbh (cm) | 7.8 | 2.8 | 21.5 | 3.3 |
| Height (m) | 5.8 | 1.8 | 11.5 | 2.1 |
| Elevation (m.a.s.l.) | 840 | 750 | 950 | 75 |
| Diameter-to-height-ratio | 1.3 | 0.8 | 2.3 | 0.3 |
| Mean annual temperature (°C) | −0.8 | −1.4 | −0.1 | 0.7 |
| Minimum monthly mean temperature (°C) | −9.7 | −12.0 | −5.0 | 4.0 |
| Maximum monthly mean temperature (°C) | 8.7 | 7.0 | 10.0 | 1.5 |
| Mean annual precipitation (mm) | 769.1 | 489.7 | 1239.2 | 409.6 |
| Latitude | − | 60°29′ N | 62°08′ N | − |

Note: TAG, total aboveground biomass; dbh, diameter at breast height (1.3 m); crown height = distance from the ground to the base of the live crown (ignoring one time a single live branch if separated by more than two whorls from the next live branch); basal area = stand basal area; birch site index = the dominant height of the largest tree by dbh at the reference age of 40 years at breast height (Strand 1967); elevation = meters above sea level; plot proportion birch (%) = percentage of birch stems within sample tree plot (r = 8.92 m); diameter-to-height-ratio = dbh (cm)/height (m). Temperature (Tveito et al. 2000) and precipitation (Tveito et al. 1997) data are derived from climate data for all of Norway; normal from 1961 to 1990. NNFI birch site index = the mean height of the 100 largest trees by dbh at the reference age of 40 years at breast height per hectare.

*Area comprised of several sample sites. Abbreviation for unproductive forest.
stands or with a dbh > 5 cm in older stands (Supplementary material, Appendix A). No differentiation between downy and silver birch species was made for the sample trees due to the phenotypic plasticity of identifying traits between the two species that vary with growing conditions and age (Atkinson 1992; Atkinson et al. 1997). All sampled trees were growing on mineral soils with depth between 15 cm and greater than 70 cm.

**Destructive sampling**

The sample trees were felled and cross-cut at stump height after measuring the total tree height and height-to-live crown (Supplementary material, Appendix A) from the stump surface. Branch sampling was then carried out mostly following the methodology of Marklund (1987, 1988) and all weighing was done with tripod-suspended field scales (OCS™, 500 kg, ±0.1 kg for large pieces or UWE™, HS-15K, ±0.01 kg for smaller pieces). First, an estimate of the total number of live branches in the live crown was obtained by: (1) dividing the live crown into three equal lengths, (2) counting the number of live branches present in a centered 1.3 m subsection for each crown part, and (3) summing the branch counts from each section. Second, live sample branches were randomly sampled by deliming the crown starting from the base of the live crown according to: live sample branch = (estimate of the total number of live branches / 5) × (random number between 0 and 1). After each sampled branch was cut and set aside for further processing, the formula was applied again until between five and nine live sample branches were sampled per tree depending on the results of iteratively applying the formula.

The fresh weight (FW) of live sample branches was recorded with leaves and catkins (if present) attached using a portable table-top scale (UWE™, SHC-6C, ±0.2 g). Then, woody branch material, leaves, and catkins (if present) of the live sample branches were separated and packaged for storage. The FW of the live crown was obtained by summing the FW of all live sample branches and remaining live branches with leaves and catkins (if present) attached. All dead branches (if present) were also weighed to obtain the total FW of dead branches and a subjectively selected sample (ca. 500 g) was brought to the lab after FW determination in the field. On the delimbed stem, distance from marked dbh to stump height, and stem diameter starting from 0.5 m below dbh to a 5 cm top was recorded in 0.5 m intervals.

Stem disk sampling was performed by dividing the stem into eight (if dbh ≥ 7 cm) or four (if dbh < 7 cm) sections of equal length from the stump surface to the tip. Disk locations were randomly selected within each stem section. A disk was taken at each location and an additional disk was taken at breast height (1.3 m above mean ground level). The disks were approximately 2–4 cm thick when the stem diameter was greater than 7 cm and 20–40 cm when the stem diameter was smaller than 7 cm. The distance from the stem disk to the stump height was measured. Stem disk FW was taken with bark intact and cross-sectional over-bark and under-bark diameters were recorded in two perpendicular directions. Total stem FW was determined by cutting up the remaining stem into sections and weighing them with a field scale (OCS™) attached to a portable tripod and adding the total sample disk FW to the obtained weight.

All sampled materials were placed in paper bags and as soon as logistically possible (typically 0–2 days), placed either in a dry ventilated room (ca. 20°C) or cold dry storage (<0°C) (depending on availability) before being sent to the lab for further processing.

**Lab work and data compilation**

In order to obtain dry weight (DW) of the samples, all sample materials were divided into smaller pieces (except stem disks which were left intact) and placed in paper bags. All samples were placed in a forced-air oven at 103°C for 2–11 days depending on sample size, and dried until minimal daily relative mass loss was achieved. The bark was removed from the stem disks after drying and weighed separately.

Age at breast height was determined from the dbh disk by counting the year rings under a stereo microscope. The stem index was calculated for each sample site from using the height of the tree with the biggest dbh and its age (Strand 1967). Sample site values for mean annual temperature (Tveito et al. 2000), minimum and maximum monthly mean temperature (Tveito et al. 2000), and mean annual precipitation (Tveito et al. 1997) were projected into a Geographic Information System (Quantum GIS 1.8.0-Lisboa) and obtained for the current study, Marklund (1987, 1988) and Bollandsås et al. (2009) datasets.

**Aboveground biomass dataset**

The sampled field and lab data were combined to construct estimates for the following eight biomass components for each tree: total stem, stemwood, stem bark, live crown (live branches, leaves, and catkins if present), live branch, leaf, dead branch, and total above-ground biomass. Various methodologies used in the field and lab phases of the project made it necessary to employ a specific multistage process in order to construct the component biomass estimates for each tree. The important steps of the process are outlined here; a more detailed description is available in Appendix B along with other Supplementary material.

**Total stem biomass** was determined by calculating: (1) the DW to FW ratio for each stem disk and assigning each disk to the appropriate stem section; (2) the volumes...
of each stem section using Smalian’s formula; (3) the total stem volume as the sum of the stem section volumes, with forked tree volumes being calculated in the same way for each forked and single stem; (4) the volume-weighted DW to FW ratio of the stem; and (5) the volume-weighted total stem biomass (Supplementary material, Appendices A and B).

**Stemwood biomass** was determined by calculating: (6) the cross-sectional area of the over-bark and stemwood portions of each sample disk; (7) the proportion of stemwood cross-sectional area of each disk assigned to the corresponding stem section; (8) the proportion of the total stem volume that the stem section represents; (9) the proportion of the stemwood in each stem section; (10) the volume-weighted proportion of stemwood in the stem; and (11) the volume-weighted stemwood biomass (Supplementary material, Appendix B).

**Stem bark biomass** was determined by calculating: (12) the proportion of stem bark for each sample disk assigned to the corresponding section; (13) the proportion of the stem bark in the section; (14) the volume-weighted proportion of stem bark in the tree; and (15) the volume-weighted stem bark biomass. Live crown biomass was determined by calculating: (16) the sum of the DWs of woody branches, leaves, and catkins for each sample branch; (17) the sum of the FWs of the sample branches; (18) the DW to FW ratio of the live sample branches; and (19) the biomass of the live crown. Live branch biomass (woody branch material) was determined by calculating: (20) the sum of the DW of the woody portion of the live sample branches; (21) the sum of the DW of the live sample branches; and (22) the live branch biomass. Leaf biomass was calculated as: (23) the sum of the DW of leaves of the live sample branches; (24) the addition of the DW of leaves and catkins for each sample tree; and (25) the leaf biomass. Dead branch biomass was determined by calculating: (26) the DW to FW ratio of sampled dead branches and (27) the biomass of dead branches. Total aboveground biomass (28) was calculated by adding together the total stem (5); live crown (19); and dead branch biomass (27) (Supplementary material, Appendix B).

### Function development

Single- and two-variable nonlinear mixed-effects (NLME) functions were fit to the component biomass data in order to account for the data’s inherent hierarchical, nonlinear, and heteroseudastic structure (Parresol 1999, 2001). Linearizing log transformations (Baskerville 1972) of the response and/or predictor variables were not performed in order to avoid potential bias problems associated with back transformation to the original scale from linearized function fits (Flewelling & Pienaar 1981; Duan 1983; Taylor 1986; Wirth et al. 2004; Wutzler et al. 2008). All functions were fit and evaluated following the NLME procedures outlined in Pinheiro and Bates (2000) and Robinson and Hamann (2010) with the NLME package (Pinheiro et al. 2012) available in R statistical software (R Core Team 2012). All fixed and random effects function assumptions and best fits were evaluated at each function development stage with a combination of diagnostic plots and lowest Akaike information criterion (AIC) value. The best function across all single- and two-variable functions was selected by the lowest root-mean-square error (RMSE).

Single-variable functions with dbh as the sole predictor were derived for total aboveground (TAGd), total stem (TSd), stemwood (SWd), stem bark (SBd), live crown (LCd), live branch (LBd), leaf (LFd), and dead branch (DBd) biomass. The best function form was initially determined by visually fitting scatterplots (Bates & Watts 1988; Sit & Costello 1994) of each of the biomass components against dbh as a predictor. As previously found by many authors (Parresol 1999; Lamb et al. 2005; Johansson 2007; Wutzler et al. 2008), the power function (Sit & Costello 1994) best represented all component biomass data (Equations (1) and (3)):

\[ Y_{js} = \beta_0 X_{js}^{\beta_1} + \varepsilon_{js}, \]  

where \( Y_{js} \) is the observed biomass of tree \( j \) at site \( s \), \( X_{js} \) is the observed value for tree \( j \) of explanatory variable \( d \) (dbh) at site \( s \), \( \beta_0 \) and \( \beta_1 \) are parameters to be estimated for the fixed effects, \( \alpha_{ds} \) represents the random effects for the variable \( d \) on site \( s \), and \( \varepsilon_{js} \) are the residuals. Sample site-wise random effects \( \alpha_{ds} \) were only assigned to the \( \beta_1 \) parameter for all functions.

A “power of covariate” variance function (Equation (2)) was used to model the variance structure of the within-site errors for all functions (Pinheiro & Bates 2000).

\[ \text{var(}\varepsilon_{js}\text{)} = g(\nu_{js}, \delta) = |\nu_{js}|^{\delta}, \]  

where \( |\nu_{js}| \) is the absolute value of the variance covariate and \( \delta \) is an unrestricted parameter allowing for cases where variance increases or decreases with \( |\nu_{js}| \) (Pinheiro & Bates 2000). For all component biomass fractions, functions were fit using Equations (1) and (2) with equal variance weights of 1 and a variance covariate given by the fitted values (default value) except in the stem bark function (SBd) where a fixed value of \( \delta = 0.9 \) was used because the NLME with the default value would not converge (i.e. could not be fit). The random effects and variance function (Equation (2)) are implicitly part of, but are not explicitly stated in, the final single-variable function as they only reflect site-level deviations from the fixed effects.

Two-variable functions were derived with dbh and height as predictors for total aboveground (TAGdh), total stem (TSdh), stemwood (SWdh), stem bark (SBdh), live crown (LCdh), and live branch (LBdh) biomass. Prior to
the choice of including height as a second variable, nine other candidate predictor variables were evaluated for inclusion in the single-variable component biomass functions including: age at dbh, crown length (data not shown), average crown width (data not shown), crown height, site index, plot basal area around the sample tree (data not shown), stems per hectare, elevation, and region (data not shown) (Table 1). The candidate variables were assessed by a series of tests: (1) visual assessment of scatterplots of the standardized residuals of the single-variable biomass component functions against predictor values (Bates & Watts 1988); (2) statistical assessment using a Bonferroni corrected two-sided t-test of function standardized residuals stratified into subjective low, medium, and high categories ($\alpha = 0.05$); (3) statistical test for a significant ($\alpha = 0.05$) trend of linear function fits of the function residuals against predictor variable values. Based on these evaluations, height was the preferred second variable.

The form of the function for the two-variable functions was determined in the same way as the single-variable functions except that both dbh and height were used in conjunction as the predictors. The inclusion of height additively, multiplicatively, and with different curve forms (Sit & Costello 1994) was evaluated. NLME function fits were attempted for all tested function forms. The best function form for all two-variable functions is as follows:

$$f(X_jhs) = \beta_0 + \beta_d X_{jds} X_{hjs} + e_{jhs},$$

where $X_{jhs}$ is the observed value of explanatory variable $h$ (height) for tree $j$ at site $s$ and $\beta_h$ is a parameter to be estimated for the fixed effects. As in the single-variable functions, sample site-wise random effects were only assigned to $\beta_d$ and the variance structure of the within-site errors was modeled with Equation (2) using the default values. The final two-variable function (Equation (3)) does not explicitly state the random effects or the variance function as in the single-variable function.

The possibility of developing a set of three-variable functions with each of the remaining nine predictor variables was evaluated, but upon careful consideration of the variable selection tests, the relative importance of the variables to improve individual tree biomass estimation, and the general availability of the variables in inventory data, no such functions were derived.

In order to keep the modeling approach as simple as possible, the nonlinear seemingly unrelated regression process (Parresol 2001) used to force the true additivity of component functions for total aboveground biomass was not performed. Therefore, no across-model contemporaneous correlations (Parresol 2001) are accounted for in any total aboveground estimation presented here. The derived total aboveground biomass combinations ($\text{TAG}_{dh}$, $\text{TAG}_{\text{combination}1}$) = $\text{TS}_{dh} + \text{LC}_{dh} + \text{DB}_{dh}$, and $\text{TAG}_{\text{combination}2}$ = $\text{SW}_{dh} + \text{SB}_{dh} + \text{LB}_{dh} + \text{LF}_{dh} + \text{DB}_{dh}$) had mean predicted differences of 0.9%, 0.6%, and 0.1%, respectively, compared to the same data used to derive the functions.

### Comparing the functions with existing data

In order to test if the derived functions were correctly specified, they were applied to two existing datasets. The first was a local southern Norwegian mountain birch dataset (Bollandsås et al. 2009) and the second a regional Swedish birch dataset inventoried in the mid-1980s (Marklund 1987, 1988). Three function evaluation metrics were calculated: (1) RMSE for the prediction errors; (2) t-test of the mean of the prediction errors; and (3) linear function fit of the prediction errors over predictor variables to check for trends. For the Norwegian function comparison, observed biomass values for total aboveground (stem + total crown) (Supplementary material, Appendix A), stem, and total crown were used; predicted values were calculated using the derived functions for total aboveground ($\text{TAG}_{dh}$), total stem ($\text{TS}_{dh}$), and complete crown (live crown ($\text{LC}_{dh}$) + dead branches ($\text{DB}_{dh}$)) (Supplementary material, Appendix A). Prediction errors were plotted against dbh. For the function comparison against Swedish data, observed stemwood, stem bark, live branch, and dead branch biomass values were compared with the predictions of the derived functions for stemwood ($\text{SW}_{dh}$), stem bark ($\text{SB}_{dh}$), live branch ($\text{LB}_{dh}$), and dead branch ($\text{DB}_{dh}$) biomass. Prediction errors were plotted against dbh, height, age, site index, and elevation (Marklund 1987, 1988). Leaf biomass was not included in the Swedish function comparison as no birch leaf biomass was available from Marklund (1987, 1988).

### Norwegian birch biomass stock and stock change estimates

To explore whether the new and existing birch biomass functions differ markedly with respect to their predictions of birch biomass stock and stock change from NNFI data in Norway, estimates were calculated with the following function combinations: (1) total aboveground birch biomass ($\text{TAG}_{dh}$) = $\text{SW}_{dh} + \text{SB}_{dh} + \text{LB}_{dh} + \text{DB}_{dh} + \text{LF}_{dh}$ (Tables 2 and 3) (current study); (2) $\text{TAG}_{dh}$ = stemwood ($B-5$) + stem bark ($B-8$) + live branch ($B-11$) + dead branch ($B-16$) (Marklund 1987, 1988) + leaves (where leaf biomass = $B-5 \times (0.0117/0.52)^b$) (“factor currently applied by NNFI for UNFCCC reporting; 5de Wit et al. 2006); (3) $\text{TAG}_{ih}$ = total stem (“Stem”) + total crown (“Tree crown”) biomass (Bollandsås et al. 2009). Marklund’s (1987, 1988) function combination used here is currently used for regional birch biomass estimation in Norway. Calculations were made on data from two consecutive 5-year
### Table 2. Parameter estimates and fit statistics for the single-variable individual tree biomass (kg) functions for birch in Norway.

| Function | TAG$_d$ | TS$_d$ | SW$_d$ | SB$_d^a$ | LC$_d$ | LB$_d$ | LF$_d^a$ | DB$_d^a$ |
|----------|---------|--------|--------|----------|--------|--------|---------|---------|
| $\beta_o$ (std. error) | 0.0982 (0.0105) | 0.0855 (0.0110) | 0.0721 (0.0096) | 0.0137 (0.0028) | 0.0208 (0.0052) | 0.0118 (0.0028) | 0.0078 (0.0025) | 0.0031*(0.0023) |
| $\beta_d$ (std. error) | 2.4100 (0.0448) | 2.3350 (0.0548) | 2.3380 (0.0566) | 2.3109 (0.0780) | 2.4849 (0.1036) | 2.6132 (0.0983) | 2.1953 (0.1395) | 1.7879 (0.2880) |
| N | 67 | 67 | 67 | 67 | 67 | 67 | 67 | 67 |
| $\alpha_{ds}$ (residual) | 0.0414 (0.0956) | 0.0677 (0.1102) | 0.0720 (0.1168) | 0.0547 (0.3127) | 0.1005 (0.2633) | 0.0775 (0.2760) | 0.2417 (0.3167) | 5.1497 [10$^{-6}$] (1.2875) |
| Power ($\delta$) | 1.1094 | 1.0926 | 1.0841 | 0.9$^b$ | 1.0852 | 1.0993 | 1.0221 | 0.9997 |
| AIC | 483.5463 | 458.9375 | 440.3287 | 257.9856 | 407.8178 | 379.0242 | 193.8221 | 75.4916 |
| RMSE | 36.0759 | 45.2900 | 47.6746 | 6.9865 | 19.3049 | 18.7063 | 2.4702 | 0.6532 |

Note: Model form: $Y = \beta_oX_{bg}^d \cdot T A G_{bg}$, total aboveground; TS$_d$, total stem; SW$_d$, stemwood; SB$_d$, stem bark; LC$_d$, live crown; LB$_d$, live branch; LF$_d$, leaf; and DB$_d$, dead branch biomass functions. “XX$_d$” represents the biomass function “XX” fit with only dbh (subscript “d”) as the predictor. $\beta_o$ and $\beta_d$ are parameter estimates for the fixed effects. N = number of observations. $\alpha_{ds}$ = sample site-wise random effects only assigned to the $\beta_d$ parameter. $\delta$ = estimated power value of the variance structure model.

*Best biomass function by RMSE between the single- and two-variable functions in the current study; bFixed power value of the variance structure model.

*Parameter estimate not significant at the $\alpha = 0.05$ level ($p = 0.194$).

### Table 3. Parameter estimates and fit statistics for the two-variable individual tree biomass (kg) functions for birch in Norway.

| Function | TAG$_{dh}$ | TS$_{dh}$ | SW$_{dh}$ | SB$_{dh}$ | LC$_{dh}$ | LB$_{dh}$ |
|----------|-----------|----------|----------|----------|---------|---------|
| $\beta_o$ (std. error) | 0.0521 (0.0080) | 0.0236 (0.0028) | 0.0182 (0.0021) | 0.0053 (0.0016) | 0.0532 (0.0226) | 0.0276 (0.0106) |
| $\beta_d$ (std. error) | 2.1372 (0.0715) | 1.9280 (0.0591) | 1.9083 (0.0590) | 1.9215 (0.1225) | 2.8534 (0.1792) | 3.0047 (0.1774) |
| $\beta_h$ (std. error) | 0.5570 (0.1157) | 0.9780 (0.0914) | 1.0394 (0.0905) | 0.8070 (0.2188) | $-0.7884$ (0.3058) | $-0.7731$ (0.2905) |
| N | 67 | 67 | 67 | 67 | 67 | 67 |
| $\alpha_{ds}$ (residual) | 0.0247 (0.1103) | 0.0159 (0.1217) | 0.0134 (0.1167) | 0.0735 (0.1761) | 0.0873 (0.2758) | 0.0663 (0.2796) |
| Power ($\delta$) | 1.0569 | 0.9939 | 1.0123 | 1.0931 | 1.0557 | 1.0797 |
| AIC | 468.7386 | 403.0430 | 382.2565 | 237.0165 | 403.5541 | 374.3551 |
| RMSE | 29.5963 | 21.9721 | 16.0319 | 10.1248 | 16.9534 | 15.6393 |

Note: Model form: $Y = \beta_oX_{bg}^d \cdot T A G_{bg}$. TAG$_{dh}$, total aboveground; TS$_{dh}$, total stem; SW$_{dh}$, stemwood; SB$_{dh}$, stem bark; LC$_{dh}$, live crown; and LB$_{dh}$, live branch biomass functions. “XX$_{dh}$” represents the biomass function “XX” fit with only dbh (subscript “d”) and height (subscript “h”) as the predictors. $\beta_o$, $\beta_d$, and $\beta_h$ are parameter estimates for the fixed effects. $\alpha_{ds}$ = sample site-wise random effects only assigned to the $\beta_d$ parameter. $\delta$ = estimated power value of the variance structure model.

*Best biomass function by RMSE between the single- and two-variable functions in the current study.
inventory cycles of the NNFI, restricted to: (1) undivided plots on forestry land with birch present and (2) plots inventory cycles of the NNFI, restricted to: (1) undivided by NNFI (see Antón-Fernández & Astrup 2012 and modeled following the standard tariff approach applied measured. The remaining tree heights on the plot were subsample with a target of 10 trees per plot was measured. The remaining tree heights on the plot were modeled following the standard tariff approach applied by NNFI (see Antón-Fernández & Astrup 2012 and references therein). It can be expected that stock and stock change estimates using modeled heights are less variable compared to those using measured heights; however, since the same trees were used for all estimates, all comparisons were equivalent.

The regional biomass estimates were further grouped into Norwegian regions by Norwegian county groups according to: southeast = Oppland, Buskerud, Vestfold, Hedmark, Oslo, Akershus, Østfold, Telemark, Aust-Agder, and Vest-Agder; west = Møre og Romsdal, Sogn og Fjordane, Hordaland, and Rogaland; central = Nord-Trøndelag and Sør Trøndelag; and north = Finnmark, Troms, and Nordland counties. Calculated estimates were also grouped by site productivity classes which are grouped Norwegian site indices for birch according to: unproductive = potential yield < 1 m³ ha⁻¹ yr⁻¹; low = height at 40 years of age (H40) 6–8 m; medium = H40 11–14 m; high = H40 17–23 m (Strand 1967). Calculated estimates were finally grouped by forest types which were defined as: birch dominant = plots with ≥ 70% composition of birch; other deciduous = plots with ≥ 70% composition of deciduous trees in total (birch < 70%); mixed forest = other mixed stand types; conifer dominant = plots with ≥ 70% composition of pine or spruce or mixed conifer stands with <10% birch or other deciduous trees; poor stocked = poorly stocked stands under regeneration or mature stands with a basal area of maximally 3–5 m² ha⁻¹ depending on site index class. Tree composition percentages are based on crown cover for sapling to commercial size trees (Supplementary material, Appendix A) and on volume in commercial sized trees.

Results

Component birch biomass functions

A summary of selected sample tree, stand, site characteristics, and climate data is presented in Table 1. The sampled trees in the current study were representative of the dbh and height ranges of the birch trees recorded in the 9th NNFI (2005–2009) in Norway. The sampled dbh range was 4.0–45.5 cm and the NNFI range was 5.0–69.1 cm with only 38 trees with diameters larger than the largest tree in the study. The sampled height range was 5.8–29.6 m and the NNFI range was 1.7–30.8 m with only one tree taller than the tallest tree in the study.

Separate functions were derived using the predictors dbh and height for total aboveground (TAGd, TAGdh), total stem (TSd, TSdh), stemwood (SWd, SWdh), stem bark (SBd, SBdh), live crown (LCd, LCdh), live branch (LBd, LBdh), leaf (LFd), and dead branch (DBd) birch biomass (Tables 2 and 3). There were no trends in the Pearson residuals across the range of the predicted response for any of the functions (including those not shown), with the possible exception of DBd (Figure 2H), indicating that the individual function fits were good (Figure 2).

Incorporating height into the single-variable functions reduced function error (RMSE) markedly for TAG (18.0%), TS (55.4%), SW (66.4%), LC (12.2%), and LB (16.4%) biomass, but did not improve SB (−44.9%) biomass (Tables 2 and 3). No convergent functions were found by including height with the LFd and DBd biomass functions. The best suite of aboveground birch biomass component functions in the current study by RMSE was: TAGdh, TSdh, SWdh, SBd, LCdh, LBdh, LFd, and DBd (Tables 2 and 3; Figure 2).

Comparing the functions with existing data

Applying the derived total aboveground function (TAGdh) to Norwegian total aboveground mountain birch data inventoried by Bollandsås et al. (2009), resulted in an underestimation of the measured biomass by 5.3 kg (p = 0.0083). The derived total aboveground biomass function also showed a trend to especially underestimate trees with a small dbh (Figure 3A). A weak trend (p = 0.0072) to overestimate biomass by increasing dbh was indicated; however, this trend was solely due to two large influential observations (Figure 3A). The total stem biomass function was also found to underestimate by 3.8 kg (p < 0.0001) across the range of dbh (data not shown). A trend was found to overestimate total crown biomass in trees with dbh greater than 6 cm (p = 0.0001) (Figure 3B). The derived complete crown (LCdh + DBd) biomass functions were also found to overestimate by 9.7 kg (p = 0.0036) across the range of dbh.

The derived functions were also applied to a regional dataset inventoried in Sweden (Marklund 1987, 1988). The derived functions for different aboveground components significantly underestimated the measured biomass by: SWd (9.6 kg), SBd (5.9 kg), LBd (11.4 kg), and DBd (1.3 kg). For each of the functions (SBd, LBd, and DBd) except SWd, a weak but significant trend to underestimate by increasing dbh was found (Figure 4).

When the prediction errors were fit with height as the independent variable, the derived functions showed a significant trend to underestimate Swedish birch biomass for SWd, SBd, LBd, and DBd (data not shown) with
increasing dbh. With crown height as the independent variable, the prediction errors showed a similar significant trend for the functions SWdh and SBd (data not shown). No trends were found with respect to age at breast height, site index, or elevation.

Norwegian birch biomass stock and stock change estimates
The derived functions predicted 2.2% and 14.3% higher total birch biomass stock estimates (86.3 million tons) for NNF9 than when using the Marklund (1987, 1988) and Bollandsás et al. (2009) functions, respectively (Figure 5A). The derived functions also predicted 0.53 and 2.04 million tons higher biomass stock change (6.6 million tons) than when using the corresponding functions (Figure 5A). The stock estimate was more sensitive to biomass function errors than the stock change estimates. The relative differences between the stock and stock change predictions were similar and consistent for most comparisons (Figure 5), so further descriptions

Figure 2. Pearson residuals for the best models by RMSE: total aboveground (TAGdh) (A), total stem (TSdh) (B), stemwood (SWdh) (C), stem bark (SBd) (D), live crown (LCdh) (E), live branch (LBdh) (F), leaf (LFd) (G), and dead branch biomass (DBd) (H).
of the comparisons will be in terms of the stock estimates alone.

When the different function estimates were stratified by region, site productivity, and forest type, several trends became evident. The derived function predictions differed markedly compared to those obtained with the functions of Bollandsás across all stratifications (Figure 5A–C). Regional trends revealed that the derived function predictions were higher than those obtained with the functions of Marklund in the west (5.0%), less so in central and northern Norway, and nearly the same in the southeast (0.2%) (Figure 5A). The derived functions predicted much higher in the southeast and west and less so in central and northern Norway compared to Bollandsás (Figure 5A). Compared to Marklund, the derived functions predicted higher on unproductive sites (10.4%), less so on low productive sites, nearly the same on medium sites, and lower (−3.1%) on highly productive sites (Figure 5B). Conversely, the derived functions predicted lower than Bollandsás on unproductive sites (−6.0%), but increasingly higher on low, medium, and high site productivity classes (Figure 5B). Grouping by forest type showed that the derived functions predicted higher (ca. 3.5%) than...
Marklund in birch dominant, other deciduous, and poor stocked forest types, but nearly the same in mixed and conifer dominant types (Figure 5C). Compared to Bollandsås, the derived functions predicted 11.2–21.8% higher depending on the forest type, with the least difference in the birch dominant type (Figure 5C).
Discussion

The presented results suggest that the derived functions are the best available for regional birch biomass stock and stock change estimation in Norway. The derived functions provide a good fit to the data with no visible trends in the residuals for the most important aboveground biomass components (Figure 2). Predictions obtained with the new functions showed good predictive ability for Norwegian mountain birch and regional Swedish birch biomass data. However, the observed underestimation patterns suggest that the biomass allocation pattern of the respective birch populations were different (Figures 3 and 4). The derived functions mostly estimated higher birch biomass stock and stock change throughout Norway, across different regions, site productivities, and forest types (Figure 5) than did existing Norwegian mountain birch or Swedish functions.

The 67 birch trees were sampled from throughout much of the birch zone in Norway, covering areas not previously represented by the existing local Norwegian birch functions including: low-elevation western coastal, mid-elevation southeastern, central, and northern Norway (Figure 1). The majority of the most prevalent conditions in which birch occurs in Norway (Table 1) were also represented in the sample; however, unproductive forest, high elevation birch in southern Norway (>700 meters above sea level), and birch growing on peatlands were not included. There are considerable land areas in northern Norway and at high elevations throughout Norway with unproductive birch forests; while relatively little birch is found on peatlands (NNFI 2009). Therefore, special care should be taken when applying the derived functions to these forest types.

Even though the sample did not include unproductive forests, the sample did include individual trees from very low productive areas with high similarity to much of Norway’s unproductive forests. The sample contained 12 trees from >540 m.a.s.l. from the west and southeast as well as 12 trees from >180 m.a.s.l. in the north. Environmental conditions on these sample sites are approximately similar to the conditions found on birch-dominated unproductive forest in the north and at high elevations throughout Norway. Some indication of the expected performance of the derived functions on unproductive sites is indicated in the stock and stock change comparison, where the estimate was intermediate to the Bollandsås et al. (2009) and Marklund (1987, 1988) estimates (Figure 5B). Although it is likely that the derived functions will underestimate southern mountain birch biomass when applied in those conditions, they produce less of an underestimate than the Marklund functions on these sites. It is also important that the data material used for developing the southern Norwegian mountain birch functions (Bollandsås et al. 2009) is limited to three sample areas and does not encompass unproductive birch elsewhere in Norway (Bollandsås et al. 2009).

Geographically, extrapolating allometric biomass functions is common practice in regions where no functions exist. Available evidence suggests that this practice can have varying effects on the estimation of component birch biomass. In a widely distributed genus such as birch, growing conditions can range from similar to dissimilar in different regions likely resulting in increasingly different biomass allocation patterns where conditions are most dissimilar. This hypothesis has circumstantial support from comparative Nordic allometric birch biomass studies which have reported both differing and similar birch biomass component estimates. Bollandsås et al. (2009) found various significant differences from measured Norwegian mountain birch biomass compared with predicted values for stem, total crown, and total aboveground biomass using a suite of local and regional birch biomass functions from Norway (Opdahl 1987; Korso 1995) and Sweden (Marklund 1987, 1988; Bylund & Nordell 2001; Claesson et al. 2001; Dahlberg et al. 2004). Bollandsås et al. (2009) reported no significant difference for total aboveground or stem biomass compared to the regional Icelandic functions of Snorrason and Einarsson (2006). Repola (2008) reported differing biomass predictions for birch live branch biomass with increasing dbh in a comparison of his and Marklund’s (1987, 1988) functions applied to Finnish NFI data, but relatively similar stem biomass estimates. In the current study, NNFI total aboveground biomass stock estimates were the same as Marklund’s (1987, 1988) estimate in southeastern Norway where conditions are most similar to Swedish conditions, but increasingly different in northern, central, and western Norway (Figure 5A) where conditions are most dissimilar. Component birch biomass was also significantly different than southern Norwegian mountain birch (Figure 3) and Swedish birch (Figure 4).

The derived functions’ underestimate of measured Swedish biomass, but higher estimate on NNFI data is likely caused by the large proportion of low and unproductive birch forests in Norway compared to the data sampled by Marklund. For forest with relatively high productivity, the Marklund functions estimate slightly higher biomass than the derived functions while the opposite is the case for low and unproductive forests (Figure 5B).

Conclusions

The results indicate that geographic extrapolation of birch biomass functions can lead to divergent biomass estimates. Circumstantial evidence from this and other comparative birch biomass studies from the Nordic countries suggest that it is due to differences in biomass allocation patterns in
trees growing in different conditions. If the estimated differences presented here are representative of the actual error that would result, then the continued application of Marklund’s functions for stock and stock change estimation for carbon accounting and bioenergy stock predictions may result in an underestimation of birch biomass throughout Norway, in the west, in central, and in the north (Figure 5A). The comparison of the derived functions applied to existing biomass and NNFI data indicates that the functions are likely the best choice for estimating regional birch biomass stock and stock change in Norway.

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Supplemental data

Supplemental data for this article can be accessed at 10.1080/02827581.2014.951389.

References

Antón-Fernández C, Astrup R. 2012. Empirical harvest models and their use in regional business-as-usual scenarios of timber supply and carbon stock development. Scand J For Res. 27:379–392.

Atkinson MD. 1992. Betula pendula Roth (B. verrucosa Ehrh.) and B. pubescens Ehrh. Biological flora of the British Isles. J Ecol. 80:837–870.

Atkinson MD, Jervis AP, Sangha RS. 1997. Discrimination between Betula pendula, Betula pubescens, and their hybrids using near-infrared reflectance spectroscopy. Can J For Res. 27:1896–1900.

Baskerville GL. 1972. Use of logarithmic regression in the estimation of plant biomass. Can J For. 2:49–53.

Bates DM, Watts, DG. 1988. Nonlinear regression analysis and its applications. New York (NY): Wiley.

Bollandás OM, Rekstad I, Naesset E, Rasberg I. 2009. Models for predicting above-ground biomass of Betula pubescens spp. czerepanovii in mountain areas of southern Norway. Scand J For Res. 24:318–332.

Bylund H, Nordell KO. 2001. Biomass proportion, production and leaf nitrogen distribution in a polycorinic mountain birch stand (Betula pubescens spp. czerepanovii) in northern Sweden. In: Wielgolaski FE, editor. Nordic mountain birch ecosystems. Man and the biosphere series. Vol. 27. New York (NY): Parthenon; p. 115–126.

Claesson S, Sahlén K, Lundmark T. 2001. Functions for biomass estimation of young Pinus sylvestris, Picea abies, and Betula spp. from stands in northern Sweden with high stand densities. Scand J For Res. 16:138–146.

Dahlberg U, Berge TW, Petersson H, Venecatasuny BP. 2004. Modelling biomass leaf area index in a sub-arctic Scandinavian mountain area. Scand J For Res. 19:60–71.

de Wit HA, Palosuo T, Hylen G, Liski J. 2006. A carbon budget of forest biomass and soils in southeast Norway calculated using a widely applicable method. For Ecol Manag. 225:15–26.

Duan N. 1983. Smearing estimate: a nonparametric retransformation method. J Am Stat Assoc. 78:605–610.

Flewelling JW, Pienaar LV. 1981. Multiplicative regression with lognormal errors. For Sci. 27:281–289.

Global Forest Resources Assessment (GFRA). 2010. Country Reports. Rome (Italy): Food and Agriculture Organization of the United Nations, Forestry Department (FRA2010/155). Reports for Belarus, Canada, Estonia, Finland, Iceland, Latvia, Lithuania, Norway, Russia, Sweden, and United States of America. Available from: http://www.fao.org/forestry/fra/67090/en/

Granhus A, Hylen G, Nilsen J-E. 2012. Statistikk over skogforhold og skogressurser i Norge registrert i perioden 2005–2009 [Statistics of forest conditions and forest resources in Norway registered in 2005–2009]. Ås (Norway): Ressursoversikt fra Skog og landskap [Overview of resources from forest and landscape].

Jenkins JC, Chojnacky DC, Heath LS, Birdsey RA. 2003. National-scale biomass estimators for United States tree species. For Sci. 49:12–35.

Johansson T. 2007. Biomass production and allometric above- and below-ground relations for young birch stands planted at four spacings on abandoned farmland. Forestry. 80:41–52.

Kjelvik S. 1974. Primærproduksjon i en subalpin bjørkstog på Maurset i Øvre-Eidfjord, Hordaland [Primary production in a subalpine birch forest in Maurset in Øvre-Eidfjord municipality, Hordaland county] [dissertation]. Ås: Norwegian Agricultural University. Norwegian with English summary.

Korsmo H. 1995. Weight equations for determining biomass fractions of young hardwoods from natural regenerated stands. Scand J For Res. 10:333–346.

Lambert MC, Ung C-H, Raulier F. 2005. Canadian national tree aboveground biomass equations. Can J For Res. 35:1996–2018.

Marklund LG. 1987. Biomass functions for Norway spruce (Picea abies (L.) Karst.) in Sweden (Rapport 43). Umeå: Department of Forest Survey, Swedish University of Agricultural Sciences.

Marklund LG. 1988. Biomassfunktörer för tall, grön och björk I Sverige [Biomass functions for pine, spruce and birch in Sweden (Report 45)]. Umeå: Department of Forest Survey, Swedish University of Agricultural Sciences. Swedish with summary in English.

Norwegian National Forest Inventory (NNFI). 2009. Norwegian National Forest Inventory: 9th Inventory Cycle. Ås (Norway): Norwegian Forest and Landscape Institute.

Opdahl H. 1987. Bjørkesskog til energiformål I Nord-Østerdalen [Birch forests for energy purposes in the north of Østerdalen]. Norsk Skogbruk. 2:28–30. Norwegian.

Parresol BR. 1999. Assessing tree and stand biomass: a review with examples and critical comparisons. For Sci. 45:573–593.

Parresol BR. 2001. Additivity of nonlinear biomass equations. Can J For Res. 31:865–878.

Pastor J, Aber JD, Melillo JM. 1984. Biomass prediction using generalized allometric regressions for some northeast tree species. For Ecol Manage. 7:265–274.

Pinheiro JC, Bates DM. 2000. Mixed-effects models in S and S-PLUS. New York (NY): Springer Verlag.

Pinheiro J, Bates D, DebRoy S, Sarkar D, The R Development Core Team. 2012. nlme: linear and nonlinear mixed effects models. R package version 3. 1–104.
Repola J. 2008. Biomass equations for birch in Finland. Silva Fennica. 42:605–624.

Robinson AP, Hamann JD. 2010. Forest analytics with R: an Introduction. New York (NY): Springer.

R Core Team. R: A language and environment for statistical computing [Internet]. 2012. R Foundation for Statistical Computing, Vienna (Austria). Available from: http://www.R-project.org/

Sit V, Poulin-Costello M. 1994. Catalog of curves for curve fitting (Biometrics Information handbook series, No. 4). Victoria (BC): British Columbia Ministry of Forests.

Snorrason A, Einarsson SF. 2006. Single-tree biomass and stem volume functions for eleven tree species used in Icelandic forestry. Icel Agric Sci. 19:15–24.

Strand L. 1967. Høydekurver for bjørk [Height curves for birch]. Meddelelser fra det Norske Skogforsøksvesen. 22, Kapittel IX, side 291. Calculator. Available from: http://www.skogoglandskap.no/kalkulator/bonitering_og_produksjonsevne/bonitering_og_produksjonsevne/nyبونيتات_کالمکاتور/calculator_mode=True

Taylor J. 1986. The retransformed mean after a fitted power transformation. J Am Stat Assoc. 81:114–118.

Tveito OE, Førland EJ, Dahlström B, Elomaa E, Frich P, Hanssen-Bauer I, Jónsson T, Madsen H, Peráli J, Rissanen P, Vedin H. 1997. Nordic precipitation maps. Oslo: Det Norske Meteorologiske Institutt. (Report no. 22/97 Klima, pp. 22).

Tveito OE, Førland E, Heino R, Hanssen-Bauer I, Alexanderson H, Dahlström B, Drebs A, Kern-Hansen C, Jónsson T, Vaarby Laursen E, Westman Y. 2000. Nordic temperature maps. Oslo: Det Norske Meteorologiske Institutt (Report no. 09/00 Klima, pp. 54).

Ung C-H, Bernier P, Guo X-J. 2008. Canadian national biomass equations: new parameter estimates that include British Columbia data. Can J For Res. 38:1123–1132.

Wirth C, Schumacher J, Schulte ED. 2004. Generic biomass functions for Norway spruce in Central Europe—a meta-analysis approach toward prediction and uncertainty estimation. Tree Physiol. 24:121–139.

Wutzler T, Wirth C, Schumacher J. 2008. Generic biomass functions for Common beech (Fagus sylvatica) in Central Europe: predictions and components of uncertainty. Can J For Res. 38:1661–1675.