Radiative Neutrino Mass Matrix for Three Active plus One Sterile Species

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Abstract

A simple unifying mass matrix is presented for the three active and one sterile neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$, and $\nu_s$, using an extension of the radiative mechanism proposed some time ago by Zee. The total neutrino-oscillation data are explained by the scheme $\nu_e \leftrightarrow \nu_s$ (solar), $\nu_\mu \leftrightarrow \nu_\tau$ (atmospheric) and $\nu_e \leftrightarrow \nu_\mu$ (LSND). We obtain the interesting approximate relationship $(\Delta m^2)_{\text{atm}} \simeq 2[(\Delta m^2)_{\text{solar}}(\Delta m^2)_{\text{LSND}}]^{1/2}$ which is well satisfied by the data.
Three neutrinos, each associated with a charged lepton (e, µ, τ), are now known. The invisible width of the Z boson, coming from the decay \( Z \rightarrow \nu\bar{\nu} \), is also consistent with exactly three such neutrinos. This means that if there is a fourth neutrino, either it has to be very heavy (with mass greater than \( M_Z/2 \)) or it does not couple to Z. In particular, if it is light, then it must not have any electroweak gauge interactions. Such an object is often referred to as a “sterile” neutrino. The reason that this may be a necessary part of our understanding of particle physics is that there are at present three classes of neutrino experiments\[2, 3, 4\] which show evidence of neutrino oscillations with three very different \( \Delta m^2 \)'s, \textit{i.e.} differences of mass-squares. If all three interpretations are correct, then we need four light neutrinos. (A possible but rather extreme three-neutrino scenario\[3\] is to have large anomalous \( \nu_\tau \)-quark interactions.) It is thus of theoretical interest to find a natural mechanism which explains the masses and mixings of these four neutrinos in the present experimental context.

A specific model for a \( 4 \times 4 \) neutrino mass matrix was proposed already some time ago. The form of this matrix agrees with subsequent purely phenomenological analyses\[7, 8\] of all neutrino-oscillation data. Our present study concerns the possibility that all neutrino masses are zero at tree level, but are generated radiatively at one-loop to match the pattern in [6], using a mechanism first proposed by Zee\[9\]. We extend previous work\[10, 11\] on this topic to include a sterile neutrino\[12\] with the help of an extra U(1) gauge symmetry\[13\]. The resulting mass eigenvalues lead to the approximate relationship

\[
(\Delta m^2)_{\text{atm}} \simeq 2\sqrt{(\Delta m^2)_{\text{solar}}(\Delta m^2)_{\text{LSND}}} \tag{1}
\]

which is well satisfied by the data.

Our model extends the standard electroweak gauge model to include three singlet fermion fields \( \nu_{sL}, N_R, \) and \( S_R \), as well as 3 singlet scalar fields \( \chi^+_1, \chi^+_2, \) and \( \chi^0_2 \). There are also two scalar doublets \( (\phi^+_1, \phi^0_1) \) and \( (\phi^+_2, \phi^0_2) \), where only one is needed in the minimal standard
model. To obtain radiative masses for the three doublet neutrinos, just $(\phi_1^+, \phi_2^0)$ and $\chi_1^+$ are enough\[9, 11\]. The more difficult task is to include the singlet neutrino $\nu_{sL}$ into a $4 \times 4$ radiative mass matrix of the same form. A natural way that this may come about is to have an extra gauge symmetry $U(1)'$ for the fields $\nu_{sL}$, $\chi_2^+$, and $\chi_2^0$ which is broken at a higher ($\sim$ TeV) scale. The axial-vector anomaly, generated by $\nu_{sL}$, is cancelled by $N_R$ which transforms as $\nu_{sL}$ under $U(1)'$. We also add $S_R$ which is trivial under $U(1)'$. A large mass for $N_R$ is then ensured through the Yukawa interaction $\bar{S}_RN_R^C\chi_2^0$ since $\langle \chi_2^0 \rangle >\sim 1$ TeV. The particle content of the model is summarized in Table 1.

We have an unbroken discrete $Z_2$ symmetry, namely $L$-parity, to distinguish between two classes of fermions. The leptons now have odd $L$-parity, replacing the usual additive lepton number. This allows the four neutrinos to acquire Majorana masses. However, tree-level neutrino masses are forbidden by the assumed particle content of our model, even after the

| fermions | L-parity | $SU(2)_L \times U(1)_Y$ | $U(1)'$ |
|----------|----------|-------------------------|---------|
| $(\nu_i, l_i)_L$ | - | $(2, -1/2)$ | 0 |
| $l_R$ | - | $(1, -1)$ | 0 |
| $\nu_{sL}$ | - | $(1, 0)$ | 1 |
| $N_R$ | + | $(1, 0)$ | 1 |
| $S_R$ | + | $(1, 0)$ | 0 |

| scalars | L-parity | $SU(2)_L \times U(1)_Y$ | $U(1)'$ |
|---------|----------|-------------------------|---------|
| $(\phi_{1,2}^+, \phi_{1,2}^0)$ | + | $(2, 1/2)$ | 0 |
| $\chi_1^+$ | + | $(1, 1)$ | 0 |
| $\chi_2^+$ | + | $(1, 1)$ | 1 |
| $\chi_2^0$ | + | $(1, 0)$ | 1 |

Table 1: List of fermion and scalar fields in our model.

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spontaneous breaking of the gauge symmetry. Note that $\nu_s$ does not get a Majorana mass because of $U(1)'$; it also does not get a Dirac mass by pairing up with $N_R$ or $S_R$ because of L-parity. More specifically, consider the following interaction Lagrangian density of the fields shown in Table 1.

$$\mathcal{L}_{\text{int}} = \sum_{i,j} f_{ij}(\nu_{iL}l_{jL} - l_{iL}\nu_{jL})\chi_1^+ + \sum_i f'_i\bar{\nu}_{iL}l_iR\chi_2^+ + \sum_i h_i(\bar{\nu}_{iL}\phi_1^+ + \bar{l}_iL\phi_1^0)l_iR$$

$$+ \mu(\phi_1^+\phi_2^0 - \phi_1^0\phi_2^+)\chi_1^- + \mu'\chi_1^+\chi_2^+\chi_2^- + h'N_R S_R \chi_2^0 + \text{h.c.},$$

(2)

where we have used the notation $\psi_i\zeta_j = \bar{\psi}_i^C\zeta_j$ for two fermion fields $\psi$ and $\zeta$. Evidently, $f_{ij}$ is antisymmetric in its generation indices. We have assumed in (2) that $(\phi_2^+, \phi_2^0)$ do not couple to leptons. This is easily achieved by a separate discrete $Z_2$ symmetry which is explicitly broken, but only by soft terms such as $\phi_1^-\phi_2^+ + \phi_2^0\phi_1^0 + \text{h.c.}$ in the Higgs potential, as in the minimal supersymmetric standard model, for example. As shown below, the above interactions induce a radiative neutrino mass matrix for $\nu_e$, $\nu_\mu$, $\nu_\tau$, and $\nu_s$ of the form

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & a & b & d \\ a & 0 & c & e \\ b & c & 0 & f \\ d & e & f & 0 \end{bmatrix},$$

(3)

which generalizes the $3 \times 3$ matrix of the Zee model [9] by including a fourth row and column.

In Fig. 1 we show the one-loop diagram linking $\nu_i$ and $\nu_j$ which contributes to the corresponding entry in $\mathcal{M}_\nu$. This is of course identical to that of Ref. [9] and [11]. Note that $i \neq j$ necessarily, hence only off-diagonal entries can be nonzero. Since $h_i = m_{\nu_i}/\langle \phi_1^0 \rangle$, we obtain

$$a = f_{e\mu}(m_{\mu}^2 - m_{e}^2)\left(\frac{\mu v_2}{v_1}\right)F(m_{\chi_1}^2, m_{\phi_1}^2),$$

(4)

$$b = f_{e\tau}(m_{\tau}^2 - m_{e}^2)\left(\frac{\mu v_2}{v_1}\right)F(m_{\chi_1}^2, m_{\phi_1}^2),$$

(5)

$$c = f_{\mu\tau}(m_{\tau}^2 - m_{\mu}^2)\left(\frac{\mu v_2}{v_1}\right)F(m_{\chi_1}^2, m_{\phi_1}^2),$$

(6)

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where \( v_{1,2} \equiv \langle \phi_{1,2}^0 \rangle \), and the function \( F \) is given by

\[
F(m_1^2, m_2^2) = \frac{1}{16\pi^2 \frac{m_1^2}{m^2_2}} \frac{1}{m_2^2} \ln \frac{m_1^2}{m^2_2}.
\]  

(7)

In Fig. 2 we show the analogous one-loop diagram linking \( \nu_i \) to \( \nu_s \). We find

\[
d = (f_{\tau f} f'_{\tau} m_{\tau} + f_{e\mu} f'_{\mu} m_{\mu}) \left( \frac{m^2_{\chi_1}}{v_1} \right) F(m^2_{\chi_1}, m^2_{\chi_2}), \\
e = (f_{\mu f} f'_{\tau} m_{\tau} + f_{\mu e} f'_{e} m_{e}) \left( \frac{m^2_{\chi_1}}{v_1} \right) F(m^2_{\chi_1}, m^2_{\chi_2}), \\
f = (f_{\tau f} f'_{\mu} m_{\mu} + f_{\tau e} f'_{e} m_{e}) \left( \frac{m^2_{\chi_1}}{v_1} \right) F(m^2_{\chi_1}, m^2_{\chi_2}),
\]

(8)  

(9)  

(10)

where \( u \equiv \langle \chi_2^0 \rangle \). In the following, we will assume that \( f_{e}^c m_e \) is negligible. Moreover, while \( u \) is expected to be large compared to \( v_{1,2} \), that can be compensated by \( m_{\chi_2} \) being larger than \( m_{\chi_1} \) or \( m_{\phi_1} \). Thus \( d, e, f \) need not be larger in magnitude than \( a, b, c \). In any case, we have the important relationship

\[
d = \frac{b e}{c} \left( 1 - \frac{m^2_{\mu}}{m^2_{\tau}} \right) + \frac{f f_{e\mu}}{f_{\tau\mu}},
\]

(11)

where \( m^2_e \) in Eq. (5) has been neglected.

We make the same observation as in Refs. [9] and [11] that \( b \) and \( c \) are likely to be the dominant entries of \( M_{\nu} \) because they are proportional to \( m^2_{\tau} \). This means that \( \nu_\tau \) combines with a linear combination of \( \nu_e \) and \( \nu_\mu \) to form a pseudo-Dirac particle. Let us also assume that \( |f_{e\tau}| << |f_{\mu\tau}| \), so that \( |b| << |c| \). Then the \( 2 \times 2 \) submatrix spanning \( \nu_e \) and \( \nu_s \) is given by

\[
M_{\nu_e \nu_s} = \\
\begin{bmatrix}
-2ab/c & d - be/c - af/c \\
\frac{d}{c} - be/c - af/c & -2ef/c
\end{bmatrix}
\]

(12)

where we have used Eq. (11) and the fact that \( |a/c| << |f_{e\mu}/f_{\mu\tau}| \). Hence

\[
m_{\nu_e} = -2ab/c, \quad m_{\nu_s} = -2ef/c,
\]

(13)
and for $m_{\nu_e} \ll m_{\nu_s}$, the $\nu_e - \nu_s$ mixing is $(e f_{e\mu}/e f_{e\tau} + b m^2_{\mu}/f m^2_{\tau})/2$. This is assumed to be small, so as to satisfy the solar neutrino data. We now have

$$(\Delta m^2)_{\text{solar}} \simeq 4\frac{e^2 f^2}{c^2}. \quad (14)$$

Since $\mathcal{M}_\nu$ has zero trace, it can easily be shown that the leading expressions for its eigenvalues are given by

$$-\frac{2 a b}{c}, \quad c + \frac{a b}{c} + \frac{e f}{c}, \quad - c + \frac{a b}{c} + \frac{e f}{c}, \quad -2 \frac{e f}{c}. \quad (15)$$

Hence the mass-squared difference between the two Majorana components of the pseudo-Dirac neutrino with mass $c$ is

$$\Delta m^2 = 4(ab + ef) \simeq 4ef \simeq (\Delta m^2)_{\text{atm}}. \quad (16)$$

Since this is for a $\nu_\mu - \nu_\tau$ mixing of $45^\circ$, we have taken it to explain the atmospheric neutrino data. Finally, the LSND data involve the mixing of $\nu_e$ and $\nu_\mu$, hence

$$(\Delta m^2)_{\text{LSND}} = c^2, \quad (17)$$

with mixing given by $b/c$. Combining Eqs. (14), (16), and (17), we obtain Eq. (1), as claimed.

Current neutrino-oscillation data are consistent with $(\Delta m^2)_{\text{LSND}} \sim 1 \text{ eV}^2$ and $(\Delta m^2)_{\text{solar}} \sim 6 \times 10^{-6} \text{ eV}^2$. In that case, $(\Delta m^2)_{\text{atm}}$ is predicted by Eq. (1) to be about $5 \times 10^{-3} \text{ eV}^2$, which is supported by the most recent data from Super-Kamiokande. In our model, $\nu_\mu$ and $\nu_\tau$ have the same mass $c \simeq 1 \text{ eV}$ and they mix maximally. Let $b \simeq 0.04 \text{ eV}$, then the $\nu_\mu - \nu_e$ mixing parameter $(\sin^2 2\theta)_{\text{LSND}}$ is $4b^2/c^2 \sim 6 \times 10^{-3}$, in good agreement with data. For $\nu_e - \nu_s$ oscillations, we let

$$\frac{f_{e\mu} c}{2 f_{\mu\tau} e} + \frac{b m^2_{\mu}}{2 f m^2_{\tau}} \simeq 0.04, \quad (18)$$

so that $(\sin^2 2\theta)_{\text{solar}}$ is also about $6 \times 10^{-3}$, again in good agreement with data. More specifically, we can let $e \simeq 0.12 \text{ eV}$ and $f \simeq 0.01 \text{ eV}$, then $m_{\nu_s} \sim 2ef/c \simeq 2.4 \times 10^{-3} \text{ eV}$. 

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Furthermore from Eq. (18), $f_{e\mu}/f_{\mu\tau}$ is now about 0.008 and from Eqs. (4) and (6), $a \sim 3 \times 10^{-5}$ eV, hence $m_{\nu_e} \sim 2ab/c \sim 2 \times 10^{-6}$ eV, justifying our assumption that $m_{\nu_e} << m_{\nu_s}$. We have thus a completely successful phenomenological picture of neutrino oscillations.

The model of Ref.[11] differs from ours in that $\nu_s$ is assumed there to acquire a tree-level mass which is just slightly bigger than the radiative mass of $\nu_e$. [This is of course rather ad hoc, but it is necessary to satisfy solar data.] Let us compare its consequences with those of ours. In the former, the parameter $a$ is forced to be large in magnitude because $4ab$ is identified there with $(\Delta m^2)_{\text{atm}}$, resulting in $|f_{e\mu}| \sim |f_{\mu\tau}|$. This condition is subject to severe phenomenological constraints because $f_{e\mu}$ contributes to $\mu$ decay. In fact, in that scenario, $f_{e\mu}^2 \sim f_{\mu\tau}^2 < 7 \times 10^{-4} G_F$. $(\cos^2 \phi M_1^{-2} + \sin^2 \phi M_2^{-2})^{-1}$ where $M_{1,2}$ are the physical charged Higgs masses and $\phi$ is their mixing angle. In our model, because of Eq. (16), $a$ can be and is very small, hence $|f_{e\mu}| << |f_{\mu\tau}|$, so that our $|f_{\mu\tau}|$ is not constrained to be small.

We note also that the form of Eq. (3) for the neutrino mass matrix with $c$ as the dominant entry is not sufficient by itself to have the correct $\nu_e - \nu_s$ submatrix needed to explain the solar data. Without Eq. (11), which is an automatic consequence of our model, that submatrix would have dominant off-diagonal terms, i.e.

$$M_{\nu_e \nu_s} \sim \begin{pmatrix} 0 & d \\ d & 0 \end{pmatrix},$$

which would make $\nu_e$ and $\nu_s$ pseudo-Dirac partners with the requisite mixing of 45° in conflict with solar neutrino data.

A third point concerns the fermion singlets $N_R$ and $S_R$. They have even L-parity, which is unbroken in our model, hence they do not mix into the lepton sector. Both of them are massive, because the terms $S_R S_R$ and $N_R S_R \chi_2^0 + h.c.$ are allowed in the Lagrangian density. The scale of $U(1)'$-breaking, i.e. $\langle \chi_2^0 \rangle$ can be taken beyond 1 TeV, thereby pushing up these masses. It is to be noted that the off-diagonal terms in the $Z-Z'$ mass matrix are prohibited due to the absence of appropriate Higgs fields in the present model. We also assume that
the kinetic mixing between the $U(1)_Y$ and $U(1)'$ gauge bosons is negligible. Hence our $Z'$ couples at the tree level only to $\nu_s L, N_R, \chi_2^+$ and $\chi_2^0$. Thus present experimental bounds [14] on a possible $Z'$ with standard-model-like couplings do not apply. However, because $\nu_s$ mixes with $\nu_e$ radiatively, $Z'$ develops a small coupling to $\nu_e$. To avoid any possible conflict with nucleosynthesis or current electroweak phenomenology, we assume $M_Z \sim 1 \text{ TeV}$ or greater, which is of course natural since we already assumed $\langle \chi_2^0 \rangle \sim 1 \text{ TeV}$ or greater.

The charged scalar $\chi_1^+$ contributes to the standard-model effective charged-current interaction due to the presence of the $f_{ij}(\nu_i L l_j L - l_i L \nu_j L)\chi_1^+$ term in the Lagrangian density. The corresponding effects on processes, such as electron-neutrino scattering, are experimentally severely constrained. They give rise to the constraint $f_{e\mu}^2/M^2 < 0.036 G_F$ [11], where $M$ is the mass of the charged scalar mediating the process. Since we have $|f_{e\mu}| << |f_{\mu\tau}|$, this is no problem for us. The proposed hierarchical relation $|f_{e\mu}| << |f_{\mu\tau}|$ is also consistent with the constraint from the branching ratio of the decay $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ being $(17.35 \pm 0.10)\%$ [14] since the latter only requires $f_{\mu\tau}^2/M^2 < 0.13 G_F$.

In summary, we have demonstrated that the present results of solar, atmospheric as well as LSND experiments can be explained with three electroweak-active neutrinos and a sterile one with a minimal extension of the standard $SU(2)_L \times U(1)_Y$ electroweak gauge model. The extra $U(1)'$ gauge and $Z_2$ discrete symmetries are needed to avoid tree-level Majorana or Dirac mass terms. All neutrino masses are radiatively generated in one loop by an extension of the Zee model. Our proposal results in an interesting relationship $(\Delta m^2)_{\text{atm}} \simeq 2[(\Delta m^2)_{\text{solar}}(\Delta m^2)_{\text{LSND}}]^{1/2}$ which is well satisfied by the present experimental data and will be critically tested with more accurate data forthcoming in the near future.

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FIG. 1. One loop radiative $\nu_i - \nu_j$ ($i, j = e, \mu, \tau$) mass due to charged Higgs exchange.

FIG. 2. One loop radiative $\nu_i - \nu_s$ ($i = e, \mu, \tau$) mass due to charged Higgs exchange.