Effect of combined pitching and heaving on propulsion of tandem flapping foils

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Flow dynamics across flapping foils in tandem arrangement at Reynolds number of Re = 1100 has been numerically investigated in the present study. The kinematic motion of the foils consists of sinusoidal heaving and pitching motion. The two-dimensional computation has been carried out through the arbitrary Lagrangian-Eulerian (ALE) moving mesh based algorithm with the variational modeling of the incompressible flow equations. The effects of the heave amplitude, pitch amplitude and the flapping frequency on the propulsive performance have been comprehensively studied for a single as well as tandem system of foils at two gaps of 4 and 7 times the downstream foil’s chord length. It is found that the effective angle of attack, effective projected area, type of vortex interaction and the timing of the interaction during the downstroke of the flapping motion influence the propulsive performance. Increasing the heave amplitude of the upstream foil leads to an increase in the average thrust generated by the downstream foil for the gap of 7, while the opposite effect is observed for gap of 4. The propulsion of the downstream foil increases monotonically with the heave amplitude of the downstream foil. Upstream foil’s pitch amplitude is noticed to have a minor effect on the performance of the downstream foil, while an increase in the pitch amplitude of the downstream foil increases the thrust for larger gap between the foils. The study of the propulsive performance in the frequency-pitch amplitude parametric space for the tandem foils has been performed for the first time, where a more complex behavior is observed. The trends in the thrust coefficient and the propulsive efficiency are corroborated by the study of various vortex interaction mechanisms leading to favorable or unfavorable thrust generating conditions. Finally, the three-dimensional flapping dynamics of the tandem configuration is demonstrated, where no spanwise wake structures are noted, indicating the wake interaction to be inherently two-dimensional.

I. INTRODUCTION

Bio-inspiration has been a key factor in the design of underwater vehicles and robots. Recently, there has been an increased interest in learning from nature to improve underwater propulsion performance. The natural propulsion systems utilized by aquatic creatures can out-perform the conventional propulsion devices by as much as 40%1,2. Such propulsion systems have many benefits over screw propellers and other traditional propulsors including the absence of cavitation1,3–5, low acoustic signature6 and excellent manoeuvring performance7, among others.

Based on the mode of swimming, bio-inspired underwater propulsion can be broadly classified into two categories8, viz., finned propulsion and jet propulsion. Finned propulsion systems employ flapping foils to generate thrust while jet propulsion utilize flexible membranes and surfaces to squeeze water through a small space (creating a jet). Several species of fish use multiple fins to generate thrust and consequently, some bio-mimetic marine vehicles9–12 also use multiple foils for propulsion. If the fins/foils are arranged in-line, the configuration is known as a tandem configuration. On the other hand, if the foils are arranged such that they are parallel to one another, the configuration is said to be side-by-side.

In addition to making use of flapping fins/foils, fish often swim together in formations known as “schools”. It has been shown that this collective swimming behavior offers a hydrodynamic advantage to a fish within the formation as a result of the wake created by fish leading the school13. The enhanced hydrodynamic performance corresponds to the tandem configuration of fins/foils and could be beneficial for underwater vehicles and robots. To create robust and reliable tandem flapping foil propulsors, it is essential that the physics behind the relevant flow phenomena is well understood. Therefore, in this study we focus on the flapping dynamics of a tandem foil configuration.

Literature pertaining to flapping foils highlight several governing parameters such as Reynolds number of the flow, foil

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geometry (chord length and thickness), and foil kinematics. In addition, the frequency of flapping is also of great significance. Based on the combinations of above-mentioned parameters, a flapping foil can either produce thrust or extract energy from the surrounding flow. Several studies have been carried out in the energy extraction regime for single as well as tandem flapping foils. In the case of tandem foils, the phase difference in the flapping and the streamwise gap between the foils are also crucial parameters.

Several works have investigated the relationship between the phase difference of tandem flapping foils and the propulsive performance. As studied by Cong et al., the performance of the downstream foil is affected significantly by the phase difference between the upstream and the downstream foil for streamwise gap distances of 0.25-0.75 chord lengths at Re = 200. Studies carried out by Sampath et al. at Re = 36500 have revealed that the downstream foil generates more thrust than the upstream foil when it lags by a quarter cycle, but performs worse if it leads by the same amount. It has also been found that the downstream foil generates maximum thrust when the flapping of both the foils is in-phase at Re = 5000.

Apart from the phase difference, the gap between the tandem foils has significant effect on the flow dynamics and propulsive performance. For synchronized plunging foils, Chen et al. found that at Re = 5000, thrust enhancement is maximum when the streamwise gap between the foils is between 1.5 and 2 chord lengths. Studies on tandem self-propelled flexible flapping plates have also been carried out. At Reynolds number of 100, significant improvements in thrust generation was obtained by reducing the streamwise gap. At Re = 200, the propulsive efficiency was found to be larger when the upstream plate was longer than the downstream plate. A numerical study by Pan and Dong identified that an increase in the streamwise spacing in a school of flapping foils reduced the influence of the lateral neighbors on the performance of the flapping foils. Recently, Joshi and Mysa studied the effect of gap and the chord sizes for the tandem foils on the propulsion at Re = 1100. The mechanism of wake interaction with the downstream foil was identified and studied in detail. A periodic variation in the thrust performance for the downstream foil was observed with the gap between the foils, indicating the crucial effect of the wake interaction. Furthermore, the combined effects of phase difference and the gap between the tandem foils was studied experimentally in the work.

The effect of oscillation frequency has received significant attention in the context of a single flapping foil. Several numerical simulations have examined the effect of flapping frequency on the propulsive performance. Gupta et al. numerically examined the effect of Strouhal number Sr and Reynolds number Re on the propulsive performance of a National Advisory Committee for Aeronautics (NACA) 0012 foil. It was found that the time averaged thrust coefficient exhibits a quadratic relationship with increasing Sr and is greatly enhanced by Re. Gungor et al. investigated the effect of wake asymmetry on the performance of a tandem hydrofoil configuration. Both in-phase and out-of-phase oscillations were considered at high Sr and found to exhibit unsteady behavior with distinct wake characteristics. The influence of Strouhal number and stream-wise gap for a tandem foil configuration has also received some attention in the literature.

The influence of pitch and heave amplitude on propulsion has not been comprehensively investigated for tandem flapping foils. However, some of the works deal with the effect of these parameters on a single foil. The Sr - pitch amplitude parametric space was studied for a single foil and a rich spectrum of wake behavior was observed. In the case of asymmetric foil motion, the mean thrust coefficient was noted to increase with pitch amplitude at Re = 3000 and Re = 10000. The effect of heave amplitude was considered by Floryan et al. where its importance at low reduced frequency was noted.

Majority of the computational research conducted on flapping foils has been two-dimensional and the three-dimensional spanwise as well as end effects have not been taken into consideration. Recent works have focused on finite foils and end effects and studied the effect of aspect ratio of the foil on propulsive performance. However, the conditions under which three-dimensional (3D) flow effects become important are yet to be comprehensively investigated for tandem foils.

To the best of the authors’ knowledge, the effects of heave and pitch amplitudes of the two foils in the tandem configuration on their propulsive performance have not been explored in detail. Although the effect of reduced frequency and Sr have been studied in detail for a single flapping foil, the literature lacks in the study of these parameters for the tandem foil configuration. Moreover, the impact of the collective frequency-pitch amplitude parametric space on tandem foil propulsion can give an insight about possible optimal operational regimes of flapping foil propulsion systems.

In the present study, we numerically investigate the flow dynamics of a tandem flapping foil system at low Reynolds number of 1100. To accomplish this, we employ a moving mesh arbitrary Lagrangian-Eulerian framework for the flapping motion of the foil. The fluid dynamics is modeled with the help of variational finite element method applied to incompressible Navier-Stokes equations. Flow across a single foil is also considered for reference and comparison. The salient and novel contributions from the present work are as follows:

- Effect of heave and pitch amplitudes on the thrust performance of single and tandem flapping foil(s),
- Thrust generation of single and tandem foil(s) in the frequency-pitch amplitude parametric space,
- The variation of the above parameters with the streamwise gap between the tandem foils,
where the fluid velocity is given by $v$.

The three dimensional spanwise behavior of the flow around the tandem foils.

The article is organized in the following manner. First, we briefly discuss the numerical framework in Section II. The next section III discusses the definition of the various parameters utilized in the study. Flapping dynamics of a single foil along with the effects of the kinematic parameters are studied in Section IV. The tandem arrangement of flapping foils is examined in Section V. This is followed by the demonstration of the three-dimensional spanwise effects for tandem foils in Section VI. Finally, the key findings are summarized and the study is concluded in section VII.

II. NUMERICAL FRAMEWORK

In the current study, the flapping dynamics of the foils is modeled using the moving mesh arbitrary Lagrangian-Eulerian (ALE) framework. Discretization of the flow equations is performed using a stabilized Petrov-Galerkin variational formulation, while the foil motion is specified by satisfying the kinematic equilibrium condition or the velocity continuity at the interface between the fluid and the foil. Here, we briefly review the governing equations of the formulation for the sake of completeness.

The flow is modeled with the help of incompressible Navier-Stokes equations written in the ALE framework as

$$
\rho \frac{\partial v}{\partial t} + \rho (v \cdot \nabla) v = \nabla \cdot \tau + \rho g, \quad \nabla \cdot v = 0,
$$

where the fluid velocity is given by $v = (v_x, v_y)$ with its x and y components, and the mesh velocity is denoted as $w$. The body force acting on a fluid element is written as $b$ and the fluid density is given as $\rho$. We consider a Newtonian fluid for which the Cauchy stress tensor can be written as $\tau = -pI + \mu \left(\nabla v + (\nabla v)^T\right)$ in which the fluid pressure is denoted by $p$, fluid dynamic viscosity by $\mu$ and the identity matrix by $I$. In Eq. (1), $\chi$ denotes the ALE reference coordinate system pertaining to the moving mesh coordinates. The governing equations are temporally discretized in the time interval $t \in [t^n, t^{n+1}]$ with the help of the Generalized-$\alpha$ method\textsuperscript{47} while the spatial discretization is carried out by stabilized finite element approximations. Detailed description of the present formulation can be found in the works\textsuperscript{45–50}.

The flapping kinematic motion of the foil is illustrated in Fig. 1. For a single foil as shown in Fig. 1(a), the kinematics consists of a heave component $h(t) = h_0\sin(2\pi ft)$ along with a pitch component $\theta(t) = \theta_0\sin(2\pi ft)$ about a pitching axis located at the leading edge of the foil. Here, $h_0$, $\theta_0$, $f$, and $\phi$ denote the heave amplitude, pitch amplitude, flapping frequency and the phase difference between the heaving and pitching motion, respectively. We also consider the tandem configuration of flapping foils with a gap of $g$ between the foils (Fig. 1(b)). The chord length of the upstream and the downstream foils are denoted by $c_u$ and $c_d$, respectively. The upstream foil’s motion is given as

$$
\theta^u(t) = \theta_0^u\sin(2\pi f^u t), \quad h^u(t) = h_0^u\sin(2\pi f^u t + \phi^u),
$$

where $h_0^u$, $\theta_0^u$, $f^u$ and $\phi^u$ represent the heave amplitude, pitch amplitude, flapping frequency and phase difference between the heaving and pitching motion, respectively, for the upstream foil. Similarly, the motion of the downstream foil is given by

$$
\theta^d(t) = \theta_0^d\sin(2\pi f^d t + \varphi), \quad h^d(t) = h_0^d\sin(2\pi f^d t + \phi^d + \varphi),
$$

where $\varphi$ is the phase difference between the kinematic motions of the upstream and downstream foils and other the symbols have their usual meanings. In the current work, we consider NACA 0015 foils in tandem with $c_u/c_d = 1$, $\varphi = 0^\circ$ and $\phi_0 = \phi^d = 90^\circ$.

The current problem involves an interaction between the fluid and structure in which the structural displacements are imposed on the surface the foil. This means that there is a one-way coupling that is facilitated by matching the structural and fluid velocities at the boundary between the foil and the fluid (kinematic equilibrium condition). The mathematical equation associated with this boundary condition is given as

$$
v_h^f(\varphi(X), t) = v_h^s(X, t), \forall X \in \Gamma^f,
$$

where $\varphi$ represents a one-to-one mapping between the structural position $X$ at time $t = 0$ and its corresponding position at time $t > 0$. $\Gamma^f$ is the fluid-structure interface at $t = 0$.

The nonlinear Navier-Stokes equations are solved by the Newton-Raphson iterative technique. The coupling between the flow equations and the moving mesh framework is carried out in a partitioned iterative manner, the details of which can be found in the works\textsuperscript{27,50}. The above formulation has been verified and validated, consisting of mesh convergence and time convergence studies in the earlier work\textsuperscript{27} for the single and tandem flapping foils and will not be discussed in the present work for brevity.

III. PARAMETERS OF INTEREST

The non-dimensional parameters pertaining to the single flapping foil are the Reynolds number $Re = (\rho U_{\infty} c)/\mu$, non-
dimensional heave amplitude $h_0/c$ and non-dimensional flapping frequency $f^* = (f c)/U_\infty$, where $U_\infty$ is the freestream velocity of the flow. Similarly, for the tandem foils, we consider the characteristic length as the chord of the downstream foil $c_d$. Thus, Reynolds number $Re = (\rho U_\infty c_d)/\mu^*$, non-dimensional heave amplitude of upstream and downstream foils are denoted by $h_0/c_d$ and $h_d/c_d$, non-dimensional flapping frequency of upstream and downstream foils are given by $f^*_u = (f^* c_d)/U_\infty$ and $f^*_d = (f^* c_d)/U_\infty$, respectively, and the gap ratio between the foils is $g/c_d$.

The propulsive performance of the flapping foils is determined by evaluating the integrated values of the hydrodynamic forces on the foil surface. The instantaneous coefficients are given as follows for a single foil:

\[
C_L = \frac{F_y}{\frac{1}{2} \rho U_\infty^3 c_l} = \frac{1}{\frac{1}{2} \rho U_\infty^2 c_l} \int_\Gamma \sigma \cdot n \, d\Gamma, \quad (8)
\]

\[
C_D = \frac{F_x}{\frac{1}{2} \rho U_\infty^3 c_l} = \frac{1}{\frac{1}{2} \rho U_\infty^2 c_l} \int_\Gamma (\sigma \cdot n) \, d\Gamma, \quad (9)
\]

\[
C_T = -\frac{F_z}{\frac{1}{2} \rho U_\infty^3 c_l} = -C_D. \quad (10)
\]

\[
C_P = -\frac{P}{\frac{1}{2} \rho U_\infty^3 c_l} = -F_z v_{\text{heave}} - M_t \omega \frac{1}{\frac{1}{2} \rho U_\infty^2 c_l}. \quad (11)
\]

In the set of equations given above, $C_L$, $C_D$, $C_T$ and $C_P$ are the lift, drag, thrust and power coefficients respectively. $F_x$, $F_y$ and $P$ are the X-component of the force, Y-component of the force and the power supplied to the structure respectively. The moment of the forces acting about the pitching axis of the foil is denoted by $M_t$ while the heave translational velocity of the foil is written as $v_{\text{heave}} = 2\pi h_0 c_l \cos(2\pi f t + \phi_0)$. The angular velocity of the foil in pitching motion is denoted by $\omega$. Here, $c$ and $l$ are the chord and the span of the foil respectively. The variable $X$ represents the time averaged mean value of $X$ over a time period $T$ of the oscillation. The propulsive efficiency of the foil can thus be written as $\eta = C_T/C_P$. Similarly, one can extend the coefficients for the tandem foils.

Next, we discuss the propulsive performance of a single and tandem foil configurations subjected to a freestream flow under the variation of the heave amplitude, pitch amplitude and the flapping frequency. We investigate the different mechanisms of thrust generation to comprehensively understand the flow dynamics of flapping foils for such scenarios.

IV. FLAPPING OF A SINGLE FOIL

A flapping foil generates thrust through the development of a leading edge vortex (LEV) in the propulsion regime. During the downstroke, this LEV is responsible for suction pressure (negative pressure) on the upper surface of the foil, whereas a positive pressure exists at the lower surface. This pressure differential along with the orientation of the flapping foil during the flapping motion leads to a net thrust force. This is depicted in Fig. 2. Therefore, the favorable conditions for generation of thrust during the downstroke of a flapping foil are:\n
(i) suction pressure on the upper surface, and
(ii) positive pressure on the lower surface.

![Figure 2: Generation of thrust as a result of LEV during the downstroke of a flapping foil.](image-url)

Prior to understanding the effects of heave and pitch amplitudes and flapping frequency on the performance of the tandem foil configuration, we discuss their effects for a single isolated foil in this section. We perform two-dimensional computations and give insights about the influence of varying heave and pitch amplitudes and explain the trends with...
the effective area is the straight line connecting the leading edge to the trailing edge. For this scenario, the foil is assumed to be a rigid body translating in the flow direction. To identify the projected effective area of the foil as seen from the upstream direction, the foil is assumed to be a rigid body translating in the flow direction. To identify the projected effective area of the foil as seen from the upstream direction, the foil is assumed to be a rigid body translating in the flow direction.

The temporal variation of the thrust coefficient in a flapping cycle for a single foil with varying $h_0/c$ (fixed $\theta_0 = 30^\circ$) is discussed by plotting the mean thrust coefficient and efficiency in the $(f^*, \theta_0)$ parametric space. This transition from drag-producing to thrust-producing regime can be noticed in the wake signature of the flapping foil, depicted in Fig. 6. At $h_0/c = 0$ (Fig. 6(a)), the clockwise (CW) and the counter-clockwise (CCW) vortices (blue and red in color respectively) are aligned such that they produce a von-Kármán (vK) vortex street, which is drag producing. As heave amplitude is increased to $h_0/c = 0.4$ (Fig. 6(b)), the vortex street resembles a scenario close to the transition to thrust-producing inverted von-Kármán (IvK) vortex street (this transition has been referred as “feathering limit” in the literature). At $h_0/c = 1$ (Fig. 6(c)), we observe the thrust-producing IvK vortex street. Thus, for the parameters considered, an increase in the thrust coefficient with heave amplitude is noticed as a consequence of increase in the effective angle of attack (effective area does not vary with change in heave amplitude). The experimental study conducted by Buren et al. considered a single pitching and heaving foil at $Re = 8000$ where a similar observation was noted. At fixed pitch amplitude and $f^*$, the mean thrust coefficient was found to increase with heave amplitude.

### A. Effect of heave amplitude on propulsion

The temporal variation of the thrust coefficient in a flapping cycle considering $f^* = 0.2$, $\theta_0 = 30^\circ$ and $Re = 1100$ is shown for different heave amplitudes in Fig. 3(a). It can be observed that an increase in the heave amplitude leads to higher thrust generation for the single foil with maximum thrust coefficient noted for $h_0/c = 1$.

To understand this variation, we quantify the effective angle of attack for the foil in Fig. 3(b), given by

$$\alpha_{eff}(t) = \tan^{-1}\left(\frac{-v_{b,heave}^h(t)}{U_\infty}\right) - \theta(t),$$  \hspace{1cm} (12)

where $v_{b,heave}^h(t)$ is the heave translational velocity of the foil. We observe that the effective angle of attack $\alpha_{eff}$ is negative for $h_0/c \in [0, 0.2, 0.4]$ during the downstroke (first half of the flapping cycle) leading to net negative thrust (i.e., drag). As heave amplitude is increased, $\alpha_{eff}$ becomes positive and increases with $h_0/c$ producing net positive thrust. We also quantify the projected effective area of the foil as seen from the upstream direction. For this scenario, the foil is assumed to be a straight line connecting the leading edge to the trailing edge. Based on the prescribed flapping motion, the effective area is evaluated as

$$A_{eff}(t) = |y_{TE} - y_{LE}| = |h(t) + c\sin(\theta(t)) - h(t)|,$$

$$= |c\sin(\theta(t))|,$$  \hspace{1cm} (13)

where $y_{LE} = h(t)$ and $y_{TE} = h(t) + c\sin(\theta(t))$ denote the positions of the leading edge and the trailing edge in the direction perpendicular to the freestream velocity, respectively and $l = 1$ is the span of the foil. As expected, with a fixed $\theta_0 = 30^\circ$, the instantaneous effective area does not vary with the change in heave amplitude (Fig. 3(c)). Therefore, effective angle of attack is the prominent factor for thrust generation in this case.

### B. Effect of pitch amplitude on propulsion

For observing the variation of the propulsive performance with the pitch amplitude, the heave amplitude is fixed at $h_0/c = 1$ along with the Reynolds number $Re = 1100$ and the flapping frequency $f^* = 0.2$. The pitch amplitude is varied between $0^\circ$ and $30^\circ$ with increments of $5^\circ$. The time history

![Figure 3: Temporal variation of the following quantities in a flapping cycle for a single foil with varying $h_0/c$ (fixed $\theta_0 = 30^\circ$): (a) $C_T$, (b) $\alpha_{eff}$, and (c) $A_{eff}$.](image-url)
Figure 4: Temporal variation of the following quantities in a flapping cycle for a single foil with varying $\theta_0$ (fixed $h_0/c = 1$): (a) $C_T$, (b) $\alpha_{eff}$, and (c) $A_{eff}$.

Figure 5: Z-vorticity contours of the wake of the flapping single foil at $t/T = 0.25$, $h_0/c = 1$ and (a) $\theta_0 = 0^\circ$, (b) $\theta_0 = 15^\circ$, and (c) $\theta_0 = 25^\circ$.

Figure 6: Z-vorticity contours of the wake of the flapping single foil at $t/T = 0.25$, $\theta_0 = 30^\circ$ and (a) $h_0/c = 0$, (b) $h_0/c = 0.4$, and (c) $h_0/c = 1$.

The peak thrust coefficient is delayed as the pitch amplitude increases. To comprehend this relationship, the effective angle of attack and the effective area are depicted in Figs. 4(b) and
Figure 7: Variation of the (a) mean thrust coefficient and (b) propulsive efficiency for the single foil at $h_0/c = 1$ with varying $\theta_0$ and $f^*$

Contrary to the observation in the previous sub-section, the effective angle of attack $\alpha_{\text{eff}}$ during the downstroke is positive but the peak of $\alpha_{\text{eff}}$ decreases with increase in the pitch amplitude. This trend of $\alpha_{\text{eff}}$ does not translate to the thrust coefficient plot. This can be attributed to the increase in the effective area projected to the incoming flow with increase in the pitch amplitude (Fig. 4(c)). The increased projected area leads to an increase in net thrust as a consequence of suction pressure on the upper surface of the foil and high pressure on the lower surface of the foil during downstroke. Therefore, in this scenario, the effective projected area is the prominent factor driving the propulsive performance.

The wake signature of the flapping foil is visualized by Z-vorticity contours in Fig. 5. As all the pitch amplitudes produce net thrust, an IvK vortex street is observed for all the cases. Furthermore, with increase in pitch amplitude, the width of the wake increases, leading to more jet-like action for thrust generation. Therefore, for the parameters considered, an increase in the thrust coefficient with pitch amplitude is observed as a consequence of increase in the effective projected area and increased width of the wake. In addition, the observation of the vortex pattern suggests that the LEV is more prominent for lower $\theta_0$ as the effective angle of attack is higher.

C. Propulsive performance in frequency-pitch amplitude parametric space

The effect of flapping frequency is studied for a single flapping foil by plotting the propulsive characteristics on $(f^*, \theta_0)$ parametric space in Fig. 7. Here, $Re = 1100$, $h_0/c = 1$ and both $f^*$ and $\theta_0$ are varied. The computations carried out by varying $(f^*, \theta_0)$ are denoted by the black dot in Fig. 7 while a linear interpolation is utilized to form the contour plot. It can be observed that the mean thrust coefficient increases with an increase in $f^*$ for a fixed $\theta_0$. There is a transition region where drag-producing regime or energy-extraction regime transitions to thrust-generation regime which is demarcated by the feathering limit, pointed out in the work\textsuperscript{15}. In Fig. 7, the upper portion deals with the energy-extraction regime and one can observe the transition to this regime as thrust tends to zero and becomes negative ($f^* = 0.1$ with increasing $\theta_0$). A similar behavior of the thrust coefficient in the Strouhal number-pitch amplitude parametric space was observed in the literature\textsuperscript{52}. In Fig. 7(b), the propulsive efficiency is shown for the cases considered. Note that it is not plotted for the cases where negative thrust is observed. It can be inferred that the highest efficiency that can be obtained is around 40% in the region $(f^*, \theta_0) = (0.2, 30^\circ)$.

The vortex patterns in the wake of the flapping foil are shown in Fig. 8 for various representative values of $(f^*, \theta_0)$. The thrust-producing IvK vortex street is observed for $(f^*, \theta_0) = (0.1, 10^\circ)$ (Fig. 8(a)), while it transitions to the vK vortex street with increase in the pitch amplitude (Figs. 8(b) and (c)). The feathering limit is close to the case $(f^*, \theta_0) = (0.1, 30^\circ)$ where a peculiar vortex pattern can be observed. For $f^* = 0.2$ (Figs. 8(d-f)), the vortex street is IvK with increasing width of the wake as the pitch amplitude increases (discussed in the previous subsection). In Figs. 8(g-h), a similar observation can be made for $f^* = 0.3$.

With this background context about the influence of the parameters such as heave and pitch amplitudes and flapping frequency on the propulsive characteristics of a single isolated flapping foil, we next consider the tandem configuration.
V. FLAPPING FOILS IN TANDEM CONFIGURATION

In the tandem configuration of flapping foils, the downstream foil interacts with the wake of the upstream foil. The effect of the gap between the flapping foils on the propulsive performance of the downstream foil was investigated in detail in Joshi et al. for gap ratios of $g/c_d \in [1, 14]$ for tandem foils of same chord length, i.e., $c_u/c_d = 1$. Various mechanisms of vortex interactions were proposed and categorized into constructive and destructive interactions, which have been noticed in the literature as well.

A constructive interaction occurs when same-signed vortices interact, leading to supply of vorticity on the surface of the downstream foil. On the other hand, a destructive interaction involves the interaction of opposite-signed vortices which drives away the vorticity on the surface of the foil. It is the combination of these interactions on the upper and lower surfaces of the foil which determines the favorable and unfavorable conditions for generation of thrust. Major interactions have been shown in Fig. 9, and the details of which can be found in Joshi et al. To summarize, a constructive interaction (same-signed) on the upper surface (Interaction-1 or I-1) and a destructive one (opposite-signed) on the lower surface (Interaction-4 or I-4) of the foil favors the generation of thrust during the downstroke. On the contrary, a destructive interaction on the upper surface (Interaction-2 or I-2) and a constructive one on the lower surface (Interaction-4 or I-4) leads to unfavorable thrust generating conditions. Furthermore, when an opposite-signed vortex travels in proximity to the upper surface of the foil, it tends to pull the shear layer leading to
Figure 9: Major types of interactions of the downstream foil with the upstream foil’s wake for (a) favorable, and (b) unfavorable, thrust generating conditions \(27\).

Figure 10: Variation of the mean thrust coefficient for the tandem flapping foils at \(h_u^0/c_d = 1, \theta_u^0 = \theta_d^0 = 30^\circ\) with varying upstream foil heave amplitude \(h_u^0/c_d\) for (a) upstream, and (b) downstream foils.

Figure 11: Variation of the propulsive efficiency \(\eta\) for the tandem flapping foils at \(h_u^0/c_d = 1, \theta_u^0 = \theta_d^0 = 30^\circ\) with varying upstream foil heave amplitude \(h_u^0/c_d\) for (a) upstream, and (b) downstream foils.

Enlargement of LEV, which is favorable (Interaction-3 or I-3) (not shown in Fig. 9). The earlier (later) the favorable (unfavorable) condition occurs during the downstroke, the better is the thrust generation for the downstream foil in the tandem arrangement.

Based on the gap ratios and the considered parameters in the work \(27\) \((Re = 1100, c_u/c_d = 1, \theta_u^0 = \theta_d^0 = 30^\circ, f_u^* = f_d^* = 0.2)\).
A. Effect of heave amplitude of upstream foil on propulsion

Here, we discuss the influence of the heave amplitude of the upstream foil on propulsion efficiency of the tandem foils. As mentioned earlier, two gap ratios of \( g/c_d = 4, 7 \) are considered. The downstream heave amplitude is fixed \((h_0^d/c_d = 1)\) and both the upstream and downstream pitch amplitudes are held constant \((\theta^u_0 = \theta^d_0 = 30^\circ)\). As expected, the mean thrust coefficient of the upstream foil follows the trend for the single foil, indicating null interference of the downstream foil on the upstream foil characteristics, as shown in Fig. 10(a). An increase in \( C_T \) is observed with \( h_0^u/c_d \) which has been discussed in detail in the previous section pertaining to a single flapping foil. For the downstream foil, the variation in mean thrust is shown in Fig. 10(b). The mean thrust decreases and increases with \( h_0^u/c_d \) for \( g/c_d = 4 \) and \( g/c_d = 7 \), respectively. The propulsive efficiency is depicted in Fig. 11. The efficiency for the upstream foil is identical to the isolated single foil. It increases sharply as the dimensionless heave amplitude grows from 0.6 to 0.8 and continues to increase at higher amplitudes albeit slowly. For \( h_0^u/c_d = [0, 0.2, 0.4] \), negative thrust or drag is observed for the upstream foil and therefore, the efficiency has not been plotted. For the downstream foil, an overall decrease in efficiency with \( h_0^u/c_d \) is observed for \( g/c_d = 4 \), while it increases gradually for \( g/c_d = 7 \). Note that in Figs. 10(b) and 11(b), the “single” foil depicts the results for an isolated foil with \( h_0^u/c = 1 \) as the heave amplitude of the downstream foil is also unchanged, i.e., \( h_0^d/c_d = 1 \).

The effect of the tandem configuration on the propulsive performance of the downstream foil can be investigated by observing the vortex interaction mechanism of the upstream foil’s wake with the downstream foil. This is shown in the wake signature diagram by visualizing the Z-vorticity contours at different instances for the gap ratios \( g/c_d = 4 \) and \( g/c_d = 7 \) in Figs. 12 and 13 respectively. For \( g/c_d = 4 \), at \( h_0^d/c_d = 0.2 \), the CCW vortex just passes the upper surface of the foil in close proximity at \( t/T = 0.28 \) (I-3) which leads to the LEV getting pulled on the upper surface during the downstroke. This gives the favorable condition for thrust generation. As the heave amplitude is increased to \( h_0^u/c_d = 0.6 - 1 \), the interaction becomes unfavorable as the CCW vortex interacts with the upper and lower surface of the foil very early in the downstroke, leading to premature shedding of the LEV.

At \( g/c_d = 7 \), the variation of the mean thrust coefficient is opposite to that of \( g/c_d = 4 \). Although all the \( h_0^u/c_d \) cases generate positive thrust or are favorable conditions, as the \( h_0^u/c_d \) increases, the interaction of the CW vorticity with the downstream foil occurs earlier (I-1 and I-4) during the downstroke, as can be seen in Fig. 13. For example, this interaction occurs at \( t/T = 0.42 \) (almost the end of downstroke) for \( h_0^u/c_d = 0.2 \) (Fig. 13(a)) but occurs at the start of the downstroke \( t/T = 0.03 \) for \( h_0^u/c_d = 1 \) (Fig. 13(c)). This earlier interaction generates a larger LEV during the prime configuration of the downstroke where the projected surface area of the foil to the freestream is the largest, thus leading to higher thrust.
Figure 13: Z-vorticity contours of the wake of the flapping foils at different time instances of a flapping cycle at $g/c_d = 7$, $h_0^d/c_d = 1$, $\theta_0^u = \theta_0^d = 30^\circ$ and $h_0^u/c_d$: (a) 0.2, (b) 0.6, and (c) 1

B. Effect of heave amplitude of downstream foil on propulsion

Next, we understand the effect of varying the heave amplitude of the downstream foil on its propulsive performance. To accomplish this, we fix the heave amplitude of the upstream foil $h_0^u/c_d = 1$, the pitch amplitudes as $\theta_0^u = \theta_0^d = 30^\circ$ and the non-dimensional flapping frequency is selected as $f^* = 0.2$. The heave amplitude of the downstream foil is varied in the range $h_0^d/c_d \in [0, 1]$.

The variation in the mean thrust coefficient and the propulsive efficiency for the downstream foil are shown in Fig. 14. Note that as the upstream heave amplitude is fixed, the propulsive performance of the upstream foil will be identical to an isolated single foil at $h_0/c = 1$, $\theta_0 = 30^\circ$ and $f^* = 0.2$. It is observed that the mean thrust decreases with decrease in the heave amplitude of the downstream foil for both the gap ratios, as shown in Fig. 14(a), similar to the variation for a single foil with decreasing $h_0/c$. At $g/c_d = 4$, we do not observe a positive thrust coefficient for the downstream foil for the heave amplitudes considered. On the other hand, for $g/c_d = 7$, the thrust coefficient is negative for $h_0^d/c_d = 0.2$. The propulsive efficiency first increases slightly when $h_0^d/c_d$ is decreased from 1 to 0.8, and then decreases monotonically for $g/c_d = 7$.

The vortex interaction in this scenario can be visualized in Figs. 15 and 16 for $g/c_d = 4$ and $g/c_d = 7$, respectively. In Section IV A, it was observed that for a single foil, at lower $h_0/c$, the wake is drag producing vK vortex street and as the heave amplitude increases, the propulsive performance increases with the wake resembling IvK vortex street. A similar observation can be inferred for the tandem configuration when the heave amplitude of the downstream foil is increased. For gap ratio of $g/c_d = 4$, the individual wake of the downstream foil is inherently vK vortex street at $h_0^d/c_d = 0$ (Fig. 15(a)). The incoming IvK wake of the upstream foil interacts with the downstream foil in such a manner that the opposite-signed vortices of the vK and IvK vortex streets pair up. This pairing can also be observed for $h_0^d/c_d = 0.4$ (Fig. 15(b)). With increase in $h_0^d/c_d$, the CCW vortex of the upstream foil’s
wake interacts with the downstream foil at the early of the downstroke resulting in I-2 where the CW vortex shear layer on the upper surface of the foil is prematurely shed (seen at $t/T = 0.28$ in Fig. 15(c)). Thus, for all the cases at $g/c_d = 4$, the thrust is negative (unfavorable condition) and it is inherently unfavorable as a result of vK street for lower $h_{d0}/c_d$.

For higher gap ratio of $g/c_d = 7$, the vK vortex street for the downstream foil is observed at $h_{d0}/c_d = 0$ (Fig. 16(a)) which leads to the negative thrust. With increase in the heave amplitude of the downstream foil, favorable condition for thrust generation occurs (Figs. 16(b) and (c)). The CW vortex from the wake of the upstream foil interacts with the upper and lower surface of the downstream foil. This results in supply of vorticity to the upper surface (I-1) and the vorticity on the lower surface is driven away (I-4). A pressure differential is created based on the suction pressure on the upper surface and high pressure on the lower surface of the foil during downstroke, leading to thrust. These interactions (I-1 and I-4) occur at the early stages of the downstroke as $h_{d0}/c_d$ increases, thus extending the favorable condition in the duration where projected surface area of the foil is maximum, giving higher average thrust.

C. Effect of pitch amplitude of upstream foil on propulsion

In this subsection, we examine the effect of the pitch amplitude of the upstream foil on the thrust and propulsive efficiency of the tandem configuration. Once again, two gap ratios ($g/c_d = 4, 7$) are considered while the downstream pitch amplitude is fixed at $\theta_{d0} = 30^\circ$. Here, the heave amplitudes of both the upstream and downstream foil are held constant ($h_{u0}/c_d = h_{d0}/c_d = 1$).

The mean thrust coefficient of the upstream foil for both the gap ratios (Fig. 17(a)) exhibits a similar behavior as that of the single foil indicating that the downstream foil does not influence the flow around the upstream foil. As the pitch amplitude ($\theta_{u0}$) is increased to $30^\circ$ from zero, the mean thrust coefficient grows larger, peaking around $\theta_{u0} = 25^\circ$ before dropping slightly at $\theta_{u0} = 30^\circ$. The trend in $C_T$ with $\theta_{u0}$ can be
explained in terms of the behavior of the single foil, explained in the previous section. The variation in the mean thrust coefficient for the downstream foil with $\theta_0^d$ is shown in Fig. 17(b). For $g/c_d = 4$, the thrust generated by the downstream foil increases steadily with pitch amplitude. On the other hand, at $g/c_d = 7$, the thrust generated by the downstream foil exhibits a more complex behavior; it decreases gradually until the region $\theta_0^u \in [20^\circ, 25^\circ]$ before starting to increase again. The thrust coefficient for a single foil at identical parameters ($h_0/c = 1$ and $\theta_0 = 30^\circ$) is also shown in the figure for comparison. We can conclude that the pitch amplitude of the upstream foil has a minor effect on the propulsive performance of the downstream foil.

The propulsive efficiency of the upstream foil increases monotonically with upstream pitch amplitude and little to no variation is observed as the gap ratio is changed, as shown in Fig. 18(a). The trends for both $g/c_d = 4$ and $g/c_d = 7$ are identical to that of the single foil, as expected. On the other hand, the variation in the efficiency of the downstream foil is irregular for $g/c_d = 7$ (Fig. 18(b)) as a slight variation between 0.36 and 0.415 is observed. Note that as the average thrust coefficient is negative for all the $g/c_d = 4$ cases, they are not shown in the efficiency plot.

To elucidate the variation of the thrust of the downstream foil, we turn our attention to the Z-vorticity contours presented in Figs. 19 and 20 for $g/c_d = 4$ and 7, respectively. It can be observed that as $\theta_0^u$ increases, the wake of the upstream foil is wider, identical to the observation for a single foil in Section IV B. For $g/c_d = 4$, the thrust coefficient is negative for all the cases as a consequence of unfavorable condition. However, there is a slight increase in the mean thrust with $\theta_0^u$. This can be explained by observing the increase in the delay of the shedding of the premature LEV by the CCW vortex as $\theta_0^u$ increases (as a consequence of the wider wake). The shedding
occurs around \( t/T = 0.15, 0.18 \) and \( 0.22 \) for \( \theta_0^u = 5^\circ, 15^\circ \) and \( 30^\circ \), respectively (Fig. 19). As the interaction gets delayed in the downstroke, the average thrust coefficient increases although slightly. For higher gap ratio of \( g/c_d = 7 \), we observe a slight dip in the mean \( C_T \) till \( \theta_0^u = 20^\circ \) and then an increase till \( 30^\circ \). At \( \theta_0^u = 5^\circ \) (Fig. 20(a)), the stronger CCW vorticity from the upstream foil’s wake is in close proximity to the downstream foil at \( t/T = 0.28 \) (around mid-downstroke). This leads to the favorable condition (I-3) which pulls the shear layer on the leading edge of the downstream foil, forming LEV. As the angle is increased to \( \theta_0^u = 15^\circ \) in Fig. 20(b), the interaction I-3 is weaker (due to the weaker CCW vortex) and delayed to \( t/T = 0.34 \). Furthermore, the stronger CCW vortex is far away from the upper surface due to the wider wake of the upstream foil at higher \( \theta_0^u \). This delay in the interaction leads to a slightly lower mean thrust compared to \( 5^\circ \) and is the reason for the decreasing trend in \( C_T \). For \( \theta_0^u = 25^\circ \), the favorable condition for thrust generation occurs via I-1 and I-4 during the start of the downstroke (Fig. 20(c)) which results in increasing trend for the average thrust.

Figure 19: Z-vorticity contours of the wake of the flapping foils at different time instances of a flapping cycle at \( g/c_d = 4, h_0^u/c_d = h_0^d/c_d = 1, \theta_0^u = 5^\circ \) and \( \theta_0^u = 15^\circ \) and \( \theta_0^u = 30^\circ \): (a) 5\(^\circ\), (b) 15\(^\circ\), and (c) 30\(^\circ\)

Figure 20: Z-vorticity contours of the wake of the flapping foils at different time instances of a flapping cycle at \( g/c_d = 7, h_0^u/c_d = h_0^d/c_d = 1, \theta_0^u = 30^\circ \) and \( \theta_0^u \): (a) 5\(^\circ\), (b) 15\(^\circ\), and (c) 25\(^\circ\)

D. Effect of pitch amplitude of downstream foil on propulsion

Having studied the effect of the pitch amplitude of the upstream foil, we now focus on the pitch amplitude of the downstream foil. The heave amplitudes of the upstream and downstream foils are fixed (\( h_0^u/c_d = h_0^d/c_d = 1 \)) and the upstream pitch amplitude is \( \theta_0^u = 30^\circ \) for this scenario.

The influence of \( \theta_0^d \) on the mean thrust coefficient of the downstream foil is shown in Fig. 21(a). For the reference case of an isolated (single) foil, the mean thrust coefficient increases until \( \theta_0^d = 25^\circ \) after which it gradually settles at \( C_T \approx 0.8 \). In comparison, for \( g/c_d = 4 \), \( C_T \) increases and then decreases with \( \theta_0^d \). The peak value of \( C_T \) occurs at \( \theta_0^d = 10^\circ \) and it is noted that \( C_T \) for all the cases is very small. On the other hand, for \( g/c_d = 7 \) the mean thrust coefficient consistently for \( \theta_0^d \in [0^\circ, 30^\circ] \) with a maximum value close to 1.6. At low values of \( \theta_0^d \), the behavior of the mean thrust is observed to be similar to that of a single foil.
Figure 21: Variation of the (a) mean thrust coefficient and (b) propulsive efficiency for the downstream foil at $h_{u0}/c_d = h_{d0}/c_d = 1$, $\theta_{u0} = 30^\circ$ with varying downstream foil pitch amplitude $\theta_{d0}$.

The variation in propulsive efficiency is shown in Fig. 21(b). Here, the isolated foil exhibits an almost linear increase in $\eta$ with $\theta_{d0}$. For $g/c_d = 4$, the propulsive efficiency increases with downstream pitch amplitude before peaking at $\theta_{d0} = 20^\circ$ and then decreasing again. When the gap ratio is increased to $g/c_d = 7$, the behavior of $\eta$ closely matches that of the isolated foil case.

Let us further analyse the flow dynamics to comprehend the variation in the propulsive performance. As observed for a single isolated foil, the mean thrust coefficient increases with $\theta_0$ for $f^* = 0.2$ as the projected surface area increases (LEV is more prominent for low $\theta_0$ as effective angle of attack is higher). Therefore, if the upstream foil would not be present, one would observe an increase in $C_T$. In the case of tandem configuration, the behavior of the downstream foil is influenced by the interaction of it with the wake of the upstream foil, leading to either favorable or unfavorable conditions for propulsion. For $g/c_d = 4$, there seems to be a competing effect between the increasing projected surface area of the downstream foil and the effect of the unfavorable condition. For all the cases of $\theta_{d0}$, there is an unfavorable condition caused due to the interaction of CCW vortex of the upstream foil’s wake with the leading edge of the downstream foil (I-2 and I-5) resulting in premature LEV shedding. For $\theta_{d0} = 0^\circ$ (Fig. 22(a)), a secondary LEV is formed during the downstroke as a result of the large effective angle of attack due to pure heaving motion. As $\theta_{d0}$ increases, the projected surface area of the downstream foil also increases leading to an increase in $C_T$ at $\theta_{d0} = 15^\circ$ (Fig. 22(c)). However with further increase in $\theta_{d0}$, there is no generation of secondary LEV and premature shedding of LEV is more dominant leading to decrease in average thrust (Fig. 22(c)). Therefore, at lower $\theta_{d0}$, the downstream foil behaves similar to an isolated foil (increase in mean $C_T$ with $\theta_{d0}$; although slightly due to already existing unfavorable condition), while for higher $\theta_{d0}$, the unfavorable condition is more prominent and reduces the propulsive performance of the downstream foil.

Contrary to low gap ratio, we observe an increase in mean thrust for $g/c_d = 7$. The favorable thrust generating condition as a consequence of I-1 and I-4 can be observed for $\theta_{d0} = 5^\circ$.
C. Stream heave amplitudes are fixed (the parameter space is utilized. The upstream and downstream performance in frequency-pitch amplitude at the same frequency and pitch amplitude. We present the results for the two gap ratios $g/c_d = 4, 7$.

in Fig. 23(a). A comparison of the vorticity contours at the quarter time period for $\theta_0^d = 15^\circ$ and $25^\circ$ is depicted in Fig. 23(b-c). The favorable condition occurs for all the $\theta_0^d$ cases. However, the projected surface area increases with $\theta_0^d$ leading to an increase in overall $\overline{C_F}$, observed in Fig. 21(a).

E. Propulsive performance in frequency-pitch amplitude parametric space

To assess the effects of flapping frequency on mean thrust coefficient and propulsive efficiency, the frequency-pitch amplitude parametric space is utilized. The upstream and downstream heave amplitudes are fixed ($h_0^u/c_d = h_0^d/c_d = 1$) and both $f_u^* = f_d^*$ and $\theta_0^u = \theta_0^d$ are varied, i.e., both the foils flap at the same frequency and pitch amplitude. We present the results for the two gap ratios $g/c_d = 4, 7$.

1. Gap ratio $g/c_d = 4$

For the small gap ratio of $g/c_d = 4$, the contours for mean thrust coefficient and propulsive efficiency are shown in Figs. 24 and 25 for the upstream and downstream foils, respectively. Here, the black dots represent the cases considered for computation and the region in between is linearly interpolated to obtain the contours. Note that $\eta$ is shown only for the cases in which the thrust is positive. In Fig. 24, we observe that the variation of $\overline{C_F}$ and $\eta$ of the upstream foil follow almost identical contours as that of a single foil (Fig. 7). This substantiates the idea that the downstream foil does not affect the performance of the upstream foil in any way and therefore the behavior of the upstream foil can be understood by examining the trend of a single flapping foil, discussed in detail in the previous section. In the case of the downstream foil (Fig. 25), the behavior is more complex as a result of its interaction with the wake of the upstream foil. The maximum $\overline{C_F}$ observed is 2.0 for $(f_d^*, \theta_0^d) = (0.3, 40^\circ)$. The thrust coefficient also peaks to a value of 0.43 at $(f_d^*, \theta_0^d) = (0.15, 20^\circ)$. The propulsive efficiency for the downstream foil reaches the large values of 37% and 33% at the above mentioned $(f_d^*, \theta_0^d)$ values, respectively.

We analyze the vortex contours to comprehend the effect of the combination of flapping frequency and pitch amplitude on the tandem configuration at $g/c_d = 4$. To accomplish this, we consider the representative cases as $(f_d^*, \theta_0^d) \in [(0.05, 20^\circ), (0.1, 20^\circ), (0.15, 20^\circ), (0.2, 20^\circ), (0.25, 20^\circ), (0.3, 20^\circ)]$ for which the Z-vorticity contours at different time instances and the temporal variation of the thrust coefficient in a time period are shown in Fig. 26. Note that the representative cases correspond to varying flapping frequency but constant pitch amplitude of $20^\circ$ for both the foils. One particular observation from the contour plots is that as the flapping frequency increases, the frequency of vortices shed in the wake of the upstream foil also increases. This has significant effect on the propulsive performance of the downstream foil.

At $f_u^* = f_d^* = 0.05$ (Fig. 26(a)), the thrust of the downstream foil hovers around the zero mark and the vortex contours represent the case in close proximity to the feathering limit. It can be clearly observed that there is no prominent vortex shedding or formation of LEV on the foils. As the frequency is increased to 0.1 (Fig. 26(b)), some positive thrust is observed around the quarter-period of the downstream foil. This is a consequence of favorable condition for thrust being formed when the CW vortex interacts with the downstream foil at $t/T = 0.22$. This interaction supplies vorticity to the upper surface of the foil (I-1) and leads to a suction pressure during that period. The developed LEV is clearly observed at $t/T = 0.4$. With further increase in the flapping frequency to 0.15 (Fig. 26(c)), a more prominent increase in thrust is observed for the parameter $(f_d^*, \theta_0^d) = (0.15, 20^\circ)$, although it is delayed compared to $f_d^* = 0.1$ as a result of the delayed interaction which can be seen in the contour plots. A favorable condition on the lower surface of the downstream foil is ob-
Figure 24: Variation of the (a) mean thrust coefficient and (b) propulsive efficiency for the upstream foil in tandem configuration at $h_0^u/c_d = h_0^d/c_d = 1$ and $g/c_d = 4$ with varying $\theta_0^u$ and $f_u^*$

Figure 25: Variation of the (a) mean thrust coefficient and (b) propulsive efficiency for the downstream foil in tandem configuration at $h_0^u/c_d = h_0^d/c_d = 1$ and $g/c_d = 4$ with varying $\theta_0^d$ and $f_d^*$

served at $t/T = 0.3$ (I-4) which drives the CCW vorticity away leading to high pressure region on the lower surface. The LEV developed can be observed at $t/T = 0.45$.

Increasing the frequency leads to more frequent vortex shedding. An adverse effect on the propulsion can be observed at $f_d^* = 0.2$ where the CCW vortex interacts with the down- stream foil before the beginning of the downstroke leading to an unfavorable condition (I-2 and I-5), shown in Fig. 26(d). This interaction results in premature shedding of the LEV during the first quarter of the flapping period. Near the end of the downstroke, a favorable condition occurs at $t/T = 0.42$ during which a slight increase in the thrust is observed. However, this interaction occurs too late during the downstroke to have a significant effect on propulsion. At $f_u^* = f_d^* = 0.25$ (Fig. 26(e)), the unfavorable condition occurs at $t/T = 0.1$ where the CCW vortex in the upstream foil’s wake interferes with the downstream foil’s leading edge. This interaction produces an LEV too early in the downstroke which gives higher thrust around $t/T \approx 0.12$. As the flapping progresses, the LEV is shed around $t/T = 0.3$ resulting in lower thrust values during the downstroke. With further increase in the frequency to 0.3, a large instantaneous thrust is noticed at approximately $t/T = 0.14$ (delayed compared to $f_d^* = 0.25$). This high thrust is a consequence of a similar interaction which produces LEV
Figure 26: Time history of the thrust coefficient for the downstream foil and the Z-vorticity contours of the wake of the flapping foils at different time instances of a flapping cycle at $g/c_d = 4$, $h_0/c_d = h_0'/c_d = 1$ in the parametric space $(f_0^*, \theta_0^*) = (f_0^*, \theta_0^*)$: (a) $(0.05, 20^\circ)$, (b) $(0.1, 20^\circ)$, (c) $(0.15, 20^\circ)$, (d) $(0.2, 20^\circ)$, (e) $(0.25, 20^\circ)$ and (f) $(0.3, 20^\circ)$
Next, we discuss the influence of the upstream foil’s wake on the downstream foil at higher gap ratio of $g/c_d = 7$. The propulsive performance of the upstream foil is similar to the gap ratio $g/c_d = 4$ and has not been discussed to avoid redundancy. The contours of mean thrust and efficiency for the downstream foil are shown in Fig. 27. A maximum propulsive performance of the upstream foil is similar to the results for both the tandem foils. At $f^u = f^d = 0.1$ (Fig. 28(a)), the thrust coefficient exhibits mild variation about zero, indicating the case closer to the feathering limit. However, a slightly negative mean thrust for both the foils (Figs. 24 and 27 for upstream and downstream foils respectively) is a consequence of vK vortex street observed in the contour plots. As $f^u$ and $f^d$ are increased to 0.15, the change in $C_T$ is more complex and we observe a dip in the thrust around $t/T = 0.25$ and 0.75. The vorticity contours (Fig. 28(b)) illustrate the interaction of CCW vortex with the leading edge of the downstream foil at $t/T = 0.2$ which leads to an unfavorable condition (I-2). The result of this interaction is that the vorticity on the upper surface of the foil is driven away leading to premature LEV shedding and consequently a loss in $C_T$ at quarter of time period. As the time increases to $t/T = 0.33$, the interaction of vortices changes to I-5 where the CCW vortex interacts with the lower surface of the downstream foil. This creates suction pressure on the lower surface which reduces thrust at the end of downstroke ($t/T = 0.5$).

As the frequency is further increased to $f^u = f^d = 0.2$ (Fig. 28(c)), higher $C_T$ near quarter-period ($t/T = 0.31$) is observed, compared to lower frequency cases. At $t/T = 0.04$, a CW vortex interacts with the leading edge of the downstream foil. The CW vortex supplies vorticity to the upper surface but drives away vorticity on the lower surface. These interactions (I-1 and I-4) produce favourable conditions for thrust generation. The occurrence of favorable condition earlier during the downstream strengthens the LEV on the downstream foil, leading to enhancement of thrust. At $t/T = 0.40$, the LEV on the downstream foil is fully developed and will eventually shed due to the CCW vortex interaction at the start of upstroke.

As shown in Fig. 28(d), at $f^u = f^d = 0.25$, the thrust coefficient $C_T$ drops sharply after $t/T = 0$.

### Figure 27: Variation of the (a) mean thrust coefficient and (b) propulsive efficiency for the downstream foil in tandem configuration at $h_0^u/c_d = h_0^d/c_d = 1$ and $g/c_d = 7$ with varying $\theta_0^d$ and $f^d$.

early in the downstroke which is shed at the quarter of the flapping period (Fig. 26(b)) after which thrust decreases. Therefore, the downstream foil is not subjected to the favorable condition for thrust generation during its optimal range of time period (around quarter of $T$) to utilize the large projected area for thrust generation.

2. **Gap ratio $g/c_d = 7$**

At $f^u = f^d = 0.1$ (Fig. 28(a)), the thrust coefficient exhibits mild variation about zero, indicating the case closer to the feathering limit. However, a slightly negative mean thrust for both the foils (Figs. 24 and 27 for upstream and downstream foils respectively) is a consequence of vK vortex street observed in the contour plots. As $f^u$ and $f^d$ are increased to 0.15, the change in $C_T$ is more complex and we observe a dip in the thrust around $t/T = 0.25$ and 0.75. The vorticity contours (Fig. 28(b)) illustrate the interaction of CCW vortex with the leading edge of the downstream foil at $t/T = 0.2$ which leads to an unfavorable condition (I-2). The result of this interaction is that the vorticity on the upper surface of the foil is driven away leading to premature LEV shedding and consequently a loss in $C_T$ at quarter of time period. As the time increases to $t/T = 0.33$, the interaction of vortices changes to I-5 where the CCW vortex interacts with the lower surface of the downstream foil. This creates suction pressure on the lower surface which reduces thrust at the end of downstroke ($t/T = 0.5$).

As the frequency is further increased to $f^u = f^d = 0.2$ (Fig. 28(c)), higher $C_T$ near quarter-period ($t/T = 0.31$) is observed, compared to lower frequency cases. At $t/T = 0.04$, a CW vortex interacts with the leading edge of the downstream foil. The CW vortex supplies vorticity to the upper surface but drives away vorticity on the lower surface. These interactions (I-1 and I-4) produce favourable conditions for thrust generation. The occurrence of favorable condition earlier during the downstroke strengthens the LEV on the downstream foil, leading to enhancement of thrust. At $t/T = 0.40$, the LEV on the downstream foil is fully developed and will eventually shed due to the CCW vortex interaction at the start of upstroke.

As shown in Fig. 28(d), at $f^u = f^d = 0.25$, the thrust coefficient $C_T$ drops sharply after $t/T = 0$. This is a consequence of an unfavorable condition produced due to the interaction of the CCW vortex (I-2) before the start of the downstroke. This vortex drives away the vorticity from the upper surface of the foil. Therefore, a negative thrust is generated during the initial period of the flapping cycle, in contrast to $f^d = 0.2$ where a fa-
Figure 28: Time history of the thrust coefficient for the downstream foil and the Z-vorticity contours of the wake of the flapping foils at different time instances of a flapping cycle at \( g/c_d = 7 \), \( h_u/c_d = h_d/c_d = 1 \) in the parametric space \((f_u^*, \theta_u^*), (f_d^*, \theta_d^*)\): (a) \((0.1, 30^\circ)\), (b) \((0.15, 30^\circ)\), (c) \((0.2, 30^\circ)\), (d) \((0.25, 30^\circ)\) and (e) \((0.3, 30^\circ)\).

A favorable condition is observed at the initiation of downstroke. At \( t/T = 0.18 \), a weak favorable condition is generated by the interaction of the CW vortex with the lower surface of the foil (I-4) resulting in increased pressure and thus, an increase in thrust. The favorable condition, along with increased effective projected area of the foil at the quarter-period contributes to the rise in \( C_T \). Beyond \( t/T = 0.25 \), the effective area of the downstream foil begins to reduce. However, at \( t/T = 0.32 \), a CW vortex shed from the upstream foil reaches the lower surface of the downstream foil (I-4) and causes the thrust to peak at \( t/T = 0.37 \). The effective projected area of the foil then sharply reduces and the foil approaches a transition to
upstroke leading to a drop in thrust. In this case, although the maximum thrust noticed during flapping motion is the highest of all the cases shown in Fig. 28, the mean thrust is lower due to the unfavorable interaction occurring before the start of the downstroke.

The thrust performance of the downstream foil at \( f_\ast = f_\ast^d = 0.30 \) is depicted in Fig. 28(e). We notice here that with the increase in flapping frequency, the vortex shedding is more frequent. In this case, the unfavorable condition due to interaction of a large CCW vortex with the downstream foil (I-5) occurs at \( t/T = 0 \). This indicates that the suction pressure is large on the lower surface. The suction pressure persists up until \( t/T \approx 0.18 \) beyond which the CC vortex is shed from the lower surface of the foil. The effective projected area beyond this time is close to the maximum value and this increases \( C_T \). At the quarter period, the effective area is maximum and this brings the thrust coefficient above zero. At \( t/T = 0.32 \), a CW vortex arrives at the downstream foil and further enhances the thrust (I-1 and I-4). A peak in \( C_T \) is observed at \( t/T = 0.40 \) due to the development of a large LEV which is favorable for thrust generation. However, this interaction occurs late in the downstroke and as a result, the maximum thrust achieved by the foil is reduced.

Therefore, the type of vortex interaction from the wake of the upstream foil with the downstream foil; and the time and duration of interaction in the flapping cycle determines the propulsive performance of the downstream foil. From an operational perspective, the tandem foils generate maximum thrust where interactions leading to favorable conditions occur at the downstroke. Moreover, the propulsion efficiency is determined by the amount of thrust generated and the power input to the system. Optimal regimes where the thrust is higher and the power input is lower leads to higher efficiency of the propulsion system.

VI. THREE-DIMENSIONAL TANDEM FLAPPING FOILS

As a demonstration of the present numerical formulation and its scalability, we perform a numerical computation considering the three-dimensional effects of the tandem flapping foils. The parameters selected are \( Re = 1100 \), \( g/c_d = 7 \), \( h_0^d/c_d = h_0^d/c_d = 1 \), \( \theta_0^d = \theta_0^d = 30^\circ \), \( \theta_0^d = \theta_0^d = 90^\circ \) and \( \varphi = 0^\circ \). The computational domain with the boundary conditions is depicted in Fig. 29. A freestream velocity in the X-direction is imposed on the inlet boundary which is \( 15c_d \) from the upstream foil, while a stress-free condition is satisfied at the outlet boundary (\( 20c_d \) from the downstream foil). The slip or no-penetration condition is satisfied at the top and bottom boundaries which are equidistant (\( 20c_d \)) from the center of the foils. A no-slip condition is satisfied at the surface of the two foils. The two-dimensional mesh created for the computations presented in the previous sections is extruded in the third dimension with a span size of \( 5c_d \) with \( \Delta z/c_d = 0.125 \) as the resolution in the Z-direction. The boundaries perpendicular to the Z-axis representing the total span of the foils are considered to be periodic boundaries. The discretization consists of around 4.1 million grid points consisting of approximately 4 million eight-node hexahedral elements.

The time history of the thrust coefficient for the upstream and the downstream foils is shown in Fig. 30(a). The temporal variation of the thrust coefficient for the two-dimensional case has also been plotted. It can be observed that for the parameters considered, there is no difference between the propulsive response of the tandem configuration of flapping foils in three-dimensions as compared to the two-dimensional results. This is further corroborated by the visualization of the vortex structures in Fig. 30(b-d). The three-dimensional vortex structures are depicted by the iso-surfaces of Q-criterion at 0.25 colored by the freestream velocity. The two-dimensional Z-vorticity is also shown at three different spans of \( z/c_d \in [0, 2.5, 5] \). It can be concluded that there are no three-dimensional spanwise effects at \( Re = 1100 \) for the tandem flapping foils, suggesting the flow to be inherently two-dimensional. Further study needs to be carried out at high Reynolds numbers including turbulence effects to understand the three-dimensionality of the flow patterns.

VII. CONCLUSIONS

The present study numerically investigates the influence of heave \( (h_0^d/c_d \text{ and } h_0^d/c_d) \) and pitch amplitudes \( (\theta_0^d \text{ and } \theta_0^d) \) and the flapping frequency \( (f_\ast \text{ and } f_\ast^d) \) on the propulsion of a single and tandem foil system. The combination of these kinematic parameters have been explored for the first time and their effects on the performance of NACA 0015 foils in tandem have been studied. The two-dimensional variational formulation has been extended to three-dimensions to demonstrate the three-dimensional flow effects due to flapping at low Reynolds number for tandem foils.

Insights about the propulsive performance of the foils can be gained by comprehending the wake interaction mechanisms with the foil. In the single/tandem configuration, the effective angle of attack of the foil, its effective projected area, type of vortex interaction and the timing of the interaction influence the propulsion of the flapping foil. The salient findings from the investigation are summarized as follows:

Single foil:

- The heave amplitude \( h_0/c \) is directly proportional to the thrust generated during flapping. An increase in the effective maximum angle of attack from negative (at zero amplitude) to positive (at maximum amplitude) during downstroke is the reason for the relationship (effective projected area being constant). The negative to positive thrust transition is reflected in the von-Kármán to inverted von-Kármán vortex street transition.
Figure 29: Schematic of the three-dimensional computational domain for a uniform flow across tandem flapping foils at $Re = 1100$.

Figure 30: Three-dimensional flapping at $g/c_d = 7$, $h_0^u/c_d = h_0^d/c_d = 1$, $\theta_0^u = \theta_0^d = 30^\circ$, $f_u^* = f_d^* = 0.2$ and $Re = 1100$: (a) comparison of the thrust coefficient in a flapping cycle for 2D and 3D computations, and iso-surfaces of $Q$-criterion colored by streamwise velocity (and $Z$-vorticity at various layers along the span) at $t/T$: (b) 0, (c) 0.2, and (d) 0.4.
An increase in the pitch amplitude $\theta_0$ also increases the maximum thrust generated by the single foil. Although the effective angle of attack decreases, an increase in the effective projected area is the reason for this trend. The vortex street is wider for large pitch amplitudes.

Various types of wake signature are observed at different values of $(f_u, \theta_0)$ parametric space. Largest mean thrust is noted for $(0.3, 30^\circ)$, while $(0.2, 30^\circ)$ gives the highest efficiency of 40%.

Tandem foils:

- The upstream foil behaves as the single isolated foil with no interference of the downstream foil for all cases.
- Increasing the heave amplitude of the upstream foil $h_0^u/c_d$ results in increase and decrease in the mean thrust for gap ratios of 7 and 4, respectively.
- The average thrust performance increases monotonically with increasing the heave amplitude of the downstream foil $h_0^d/c_d$.
- The pitch amplitude of the upstream foil $\theta_0^u$ has very minor effect on the propulsion of the downstream foil.
- An increase in the pitch amplitude of the downstream foil $\theta_0^d$ leads to an increase in mean propulsive force for higher gap ratio of 4, while the force slightly increases and then decreases for gap of 7.
- The change in the $(f_u, \theta_0^u) = (f_d, \theta_0^d)$ parametric space has significant difference in the thrust performance for the downstream foil in the tandem configuration. At gap ratio of 4, $(0.3, 40^\circ)$ provides a mean thrust of 2 with 37% efficiency. However, at higher gap ratio of 7, an efficiency of 41.6% is observed for $(0.2, 30^\circ)$.
- The behavior of the propulsive performance of the tandem system of foils can be explained by examining the vortex interaction patterns and linking it with favorable and unfavorable conditions for thrust generation provided in the literature.
- At the low Reynolds number of 1100, the tandem flapping foils do not have any noticeable three-dimensional effects along the spanwise direction.

DATA AVAILABILITY

The data that supports the findings of this study are available within the article.

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