A phenomenological model of the Resonance peak in High $T_c$ Superconductors

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A notable aspect of high-temperature superconductivity in the copper oxides is the unconventional nature of the underlying paired-electron states. The appearance of a resonance peak, observed in inelastic neutron spectroscopy in the superconducting state of the High $T_c$ cuprates, its apparent linear correlation with the critical superconducting temperature of each of the compounds and its disappearance in the normal state are rather intriguing. It may well be that this peak is the signature of the singlet to triplet excitation, and is an unique characteristic of a d-wave superconductor. We develop a simple criterion for the resonance peak which is based on the concept of twist stiffness and its disappearance at $T=T_c$.

The most notable feature of the unconventional nature of the High $T_c$ cuprates besides its near-neighbor singlet ground state, is its superconducting gap $\Delta$; unlike conventional B.C.S behavior, where $\Delta$, the amplitude of the gap goes up as $T_c$ goes up, the measured $\Delta$ as revealed by angular resolved photoemission spectroscopy goes down as $T_c$ goes up! This had led some authors to postulate that the energy scale governing $T_c$ is phase stiffness of the order parameter or the superfluid density rather than the modulus of $\Delta$ where $\Delta$ is the superconducting condensate density. Inelastic neutron scattering in High $T_c$ cuprate compounds has been of immense help to enhance our understanding of the magnetic aspects underlying physics of High $T_c$. It told us right away without any ambiguity that there are at least two clear signatures of the superconducting ground state, is its superfluid density rather than the modulus of $\Delta$ where $\Delta$ is the superconducting condensate density. Inelastic neutron scattering in High $T_c$ cuprate compounds has been of immense help to enhance our understanding of the magnetic aspects underlying physics of High $T_c$. It told us right away without any ambiguity that there are at least two clear signatures of the superconducting ground state, is its superfluid density rather than the modulus of $\Delta$ where $\Delta$ is the superconducting condensate density. Inelastic neutron scattering in High $T_c$ cuprate compounds has been of immense help to enhance our understanding of the magnetic aspects underlying physics of High $T_c$.

We assume to start with that the superconducting ground state is a d-wave singlet. In order to bring out the underlying symmetry elements of the superconducting and the normal state, let us introduce the well known concept of superconducting phase stiffness (related to charge stiffness), spin stiffness as well as that of twist stiffness which is of particular relevance to near neighbor singlets and is associated with chirality. Each of these three stiffnesses are associated with a distinct symmetry operation and expresses the energy increase of the system as each symmetry operation is applied. Let us consider a spinor on site $i$

$$\psi_i = \begin{pmatrix} c_{i\uparrow}^\dagger \\ c_{i\downarrow}^\dagger \end{pmatrix}$$

(1)

Here the $c_{i\sigma}^\dagger$ are the electron creation operator on site $i$ in a spin state $\uparrow$ and similarly for the other spin $\downarrow$. There are three sets of transformation that we can consider on the spinors, one in the charge sector, one in the spin sector and one in the twist sector.

(a) In the charge sector it is given by

$$\psi_i' = \exp (ie\varphi_i) \psi_i$$

(2)

where $e$ is the electron’s charge causing a rotation by an angle $\varphi_i$, in the electromagnetic gauge space. This is the one parameter transformation of symmetry group $U(1)$. In any superconducting ground state, the $U(1)$ symmetry will be broken signifying blocking of the phase $\varphi$ of the superconducting order parameter and hence a non zero superconducting phase stiffness $D_{\varphi}$.

(b) We can also rotate the spinor in the spin sector by rotating the spin through an angle $\theta_i$ around the spin $\sigma - axis$ so that

$$\psi_i' = \exp \left( i \frac{\sigma \theta_i}{2} \right) \psi_i$$

(3)

where $\sigma$ is the Pauli spin matrix. The group symmetry is $SO(3)$ or $SU(2)$. We note that if the ground state is a superconducting $d-$spin singlet $S = 0$, the ground state energy will be unaffected by rotation of the spin axis whatever the Hamiltonian is and as a result the spin stiffness $D_{\sigma}$, is necessarily zero.
(c) The twist stiffness is best understood by introducing the chirality where we write
\[ \psi'_i = \exp \left( \frac{i \sigma \gamma \theta_j}{2} \right) \psi_i \]
here the chirality operator \( \gamma \) transcribes the fact that the spin rotation \( \theta \) on site \( i \) is exactly equal and opposite to that on the near neighbor site \( j \) whence \( \theta_i - \theta_j = 2 \theta \). This gives
\[ \psi'_i = \exp (\frac{\theta}{2}) \psi_i \]
\[ \psi'_j = \exp (-\frac{\theta}{2}) \psi_j \]
If the site \( i \) and \( j \) belong to sublattice \( A \) and \( B \), then the chiral rotation twists one sublattice around another by a rigid angle \( \theta \). The symmetry of the operation because of the pair representation of the spin operators yields the twist rotation by the singlet with the triplet and hence leads to increase of the ground state energy.

\[ H_o = -t \sum_{i,j} c^\dagger_{i \sigma} c_{j \sigma} - \frac{3J}{4} \sum_{\nu} b^\dagger_{\nu} b_{\nu} + \frac{J}{4} \sum_{i,\nu} t^1_{i \nu} t^2_{i \nu} - \mu \sum_{i} [b^2_{i} + t^2_{i}] \]

Here \( \mu \) is the chemical potential assumed same over all space. This term is unaffected by twist and we assume that the sum \( \sum_{\nu} [b^2_{\nu} + t^2_{\nu}] \) over \( \nu \) near neighbor pairs which \( = N_{\text{electron}} \) is conserved. In the limit of small twist the Hamiltonian gets modified
\[ H' = H(0) + H(\theta) \]
where the first part is the unperturbed untwisted Hamiltonian. By developing \( H(\theta) \) to second order we obtain for the perturbing term
\[ H(\theta) = \sum_{i,j} \left[ j^\dagger_{ij} \theta - \frac{1}{4} T_{ij} \theta^2 \right] \]
where \( j^\dagger_{ij} \) is the spin current operator and \( T_{ij} \) is the kinetic energy operator. They are given respectively by
\[ j^\dagger = \sum_{ij} \left( e^\dagger_{i \sigma} c_{j \sigma} - H.C. \right) - iJ \sum_{\nu} \left( b^\dagger_{\nu} t_{\nu \alpha} - t^1_{i \nu} b_{\nu} \right) \]
\[ T = -t \sum_{ij} \left( c^\dagger_{i \sigma} c_{j \sigma} + H.C. \right) - \frac{j}{2} \sum_{\nu} \left( b^2_{\nu} + t^2_{\nu} \right) \]
We get ground state energy shift due to twist \( \theta \) as
\[ \Delta E_o = \langle H(\theta) \rangle = \frac{N}{2} D_t \theta^2 \]
\( D_t (\omega = 0) \) is the twist stiffness (in two dimensions it has the dimension of energy). It is formally given by
\[ D_t (\omega) = \frac{1}{N} \left[ -\langle \hat{T} \rangle - \sum_{n \neq 0} \frac{\left( \langle 0 \mid \hat{j}^\sigma \mid n \rangle \right)^2}{\epsilon_n - \epsilon_o - h\omega} - \frac{\left( \langle n \mid \hat{j}^\sigma \mid 0 \rangle \right)^2}{\epsilon_o - \epsilon_n - h\omega} \right] \]
In the absence of the hopping term and of the spin current term, the energy increase per electron is precisely \( J \) which is the bare twist stiffness. The first term of \( D_t (\omega) \)
is the diamagnetic current contribution to stiffness due to the average value of the kinetic energy while the second term reflects second order contribution of "paramagnetic spin current conductivity" $\sigma_j(\omega)$ although $q^p = 0$. The energy levels $\epsilon_q$ are the triplet excited states for a momentum transfer $\pi, \pi$ (which has a gap $E_g$ as measured by inelastic neutron spectroscopy). The spin current in the twisted frame is the response to a "twist vector potential" (engendered by local twist) just as the charge current is response to an electromagnetic vector potential. The linear coefficient of the total response is the corresponding twist stiffness. We can rewrite the expression (14) more conveniently in analogy to the missing Drude weight as

$$D_t \delta(\omega) = D_t^0 \delta(\omega) - \int_0^\infty \sigma_t(\omega)d\omega \tag{15}$$

Here the second term on the right reflects the exhaustion of twist rigidity through incoherent spin excitation where $\sigma_t(\omega)$ is $\sim \Im \chi_{\perp}(\omega)$, the transverse spin susceptibility. From the experimental neutron data, we know that $\Im \chi_{\perp}(\omega)$ is very large at the critical hole concentration $\delta_h^c$ at which $T_c = 0$ while $\Im \chi_{\perp}(\omega)$ monotonically decreases (integrated spectral weight) in the superconducting state as optimum doping $\delta_h^c$ is approached so that we can reasonably conclude that $D_t = 0$ at $\delta_h = \delta_h^c$ while $D_t$ ought to be a maximum at $\delta_h = 0$. In other words $D_t$ is a correct indicator of d-wave superconductivity. The non-zero phase stiffness in conventional $s - wave$ superconductor results from broken $U(1)$ electromagnetic gauge symmetry. The non-zero spin stiffness in a system with long range magnetic order is associated with a broken $SO(3)$ symmetry of the rotational invariance of the spin space and $D_r$ goes to zero at $T = T_N$ when the invariance is restored. What symmetry or symmetries are broken when the phase coherent singlet $d$-wave ground state emerges? We may think of the $d - wave$ superconducting state as a state where $SO(4)$ symmetry is explicitly broken as well as $U(1)$. The normal state is then a state with zero twist stiffness where the broken $SO(4)$ symmetry pertaining to singlet and the three triplets has been restored. If now we accept the premise that at $T_c$, twist stiffness $D_t$ goes to zero, then one makes the simple statement that $kT_c$ is equal to the value of twist stiffness at $T = 0$ (strictly speaking one should use renormalised stiffness due to triplet excitations) and we have

$$kT_c = D_t(T = 0, \omega = 0) \tag{16}$$

The expression relating spin stiffness to some characteristic frequency (which we shall baptise resonance frequency $\omega_r$) can be written as

$$D_t(T = 0, \omega = 0) = \chi_{\perp}(T) h^2 \omega_r^2(T) \tag{17}$$

The resonance frequency $\omega_r$ is a small amplitude harmonic twist oscillation or rigid precession of sublattice $A$ with respect to sublattice $B$. Here $\chi_{\perp}(T)$ is the transverse spin flip magnetic susceptibility, which has its largest value at $Q = \pi, \pi$. The transverse static susceptibility $\chi_{\perp}(T)$ in the High $T_c$ cuprates (as measured by N.M.R. $1/T_2$ spin - spin relaxation rate) can be parametrised as

$$\chi_{\perp}(T) = \frac{A}{k(T + T_c)} \tag{18}$$

where $A$ is a phenomenological constant. That this form of the static susceptibility in the normal state at $T \gg T_c$ is appropriate can be checked from the imaginary part of susceptibility

$$\Im \chi_{\perp}(\omega, T) \sim \frac{\omega}{T} \tag{19}$$

which is of a form universally observed for small $\omega$. This behavior in the normal state probably points to proximity to a quantum critical point for spin excitation. In a temperature range above $T_c$, the spin correlations have a rapid decay in space but a slow decay in time due to a large density of $S = 1$ excited states. Real part of the dynamical susceptibility $\chi_{\perp}(q, \omega)$ would not show a narrow peak around a specific ordering vector but $Im \chi_{\perp}(T, \omega)$ will exhibit considerable weight at low frequency. Using expression (17) and (18), we obtain

$$h\omega_r = akT_c \tag{20}$$

This is our central result. It corroborates a posteri-
The expression (23) is plotted in figure (1), with neutron and ARPES data superimposed. The proportionality constant a measured from fig 1 gives the number 0.42 mev/°K. If we are at the critical hole doping concentration 0p at which both Tc1, the Neel temperature & Tc, the superconducting critical temperature are both zero, then we must have Dp = 0 and Dt = 0, signifying no long range magnetic order and no long range superfluid order; it is a quantum critical point. It is well known that Zn doping destroys Tc. It is seen by neutrons that doping with Zn introduces large low energy spin fluctuations(integrated spectral weight increase), the spin gap Eg(π,π) rapidly goes to zero), that will drive Dp to zero suppressing ωc and killing superconductivity. The normal state can be defined as a spin liquid (by definition has no sublattice magnetisation) where we have considerable low energy spin excitation. We also require that translational invariance be unbroken for the system to qualify as a liquid. Thus it describes a gapless spin liquid more in conformity with the original suggestion of the long range RVB liquid22. In its loss of twist stiffness the spin liquid behaves like any conventional liquid loosing shear rigidity at the melting transition. The concept of twist stiffness is based on infinitesimally small twist as is customary in these definitions; beyond T ≥ Tc, the restored dynamical SO(4) symmetry implies b ⇐⇒ ra pair fluctuation in the spin liquid phase costing no energy around the untwisted singlet. If this symmetry persists for all twist angles then we will be in the frustrated “Henley limit”23 of infinite classical spin degeneracy where one sublattice A will twist freely around the other sublattice B and the two sublattices are totally decoupled. Or else the system may develop a region where Dt(T > Tc) may become negative for large twist angles generating large singlet-triplet excitations and hence may go spontaneously to a distorted or twisted ground state24. Although twist stiffness and superconducting phase stiffness are different at T = 0, their simultaneous disappearance at T = Tc is indicated by the Arpes results23 of the hump and dip structure in the electronic spectral weight and point to strong coupling of triplet and phase fluctuation as Tc is approached.

Several theoretical models exist25 that explain the resonance peak. Our objective in this paper has been relatively simple: can we understand the resonance peak without a detailed model and does it have some predictive ability as to the underlying symmetry nature of the normal and superconducting state? I think the arguments given in this paper will throw some new light on these issues.

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