The Higgs mass bound in the SUSY multi-Higgs-doublet model

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Abstract

The upper bound on the mass of the lightest Higgs boson is provided in the supersymmetric standard model with multi-Higgs doublets, up to two-loop order. Relatively large corrections are expected from the experimentally unconstrained extra Yukawa couplings. We calculate it including the two-loop leading-log contributions as a function of the scale $\Lambda$, below which the theory remains perturbative. Even for $\Lambda = 10^4$ GeV, we obtain the upper bound of 140 GeV, which is heavier than that of the MSSM by 10 GeV.

1 Introduction

There is a famous upper bound on the mass of the lightest Higgs boson in the minimal supersymmetric standard model (MSSM), i.e., which is lighter than Z boson at tree level. This comes from the fact that the quartic couplings of the neutral Higgses are determined by only the weak gauge couplings $g$ and $g'$ due to the supersymmetry (SUSY). However, since the top Yukawa coupling is large, this tree-level upper bound receives large corrections. Further, QCD correction, which begins from a two-loop order, is also large because the strong gauge coupling $\alpha_s$ first appears at two-loop order, i.e., two-loop is the lowest order in terms of $\alpha_s$. Consequently, the upper bound can be set about 130 GeV in the MSSM when all the squark masses are at order of 1 TeV.

Here we will provide the upper bound on the mass of the lightest Higgs boson in the supersymmetric standard model with arbitrary number of Higgs doublets, which is the minimal extension of the MSSM in the meaning that its Higgs sector is composed of doublets only. This model has the same upper bound as the MSSM up to one-loop level. So, we must include at least two-loop corrections to the upper bound in order to see the difference from the MSSM. Actually, the two-loop order corrections include the contributions of the extra Yukawa couplings, which are characteristic of the multi-Higgs-doublet model and can be relatively large.

We demand that the theory remains perturbative up to the scale $\Lambda$ in order to set the upper bound on the extra Yukawa couplings, which are unconstrained experimentally. Then the resulting upper bound $m_h$ depends on the scale $\Lambda$, and becomes more stringent as $\Lambda$ is larger. For example, we obtain $m_h = 128$ GeV in the case of $\Lambda = 10^{16}$ GeV, which is almost the same as the upper bound in the MSSM. On the other hand, for $\Lambda = 10^4$ GeV, which is one order above the SUSY-breaking scale $m_S$, the upper bound is lifted up by 10 GeV due to the large extra Yukawa couplings. Throughout this letter, the soft SUSY-breaking squark masses are assumed to be 1 TeV and degenerate for simplicity.

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2 Up to one-loop order

We denote the number of the Higgs doublets as $2n$. Then the superpotential of this model is written as

\[
W = \sum_{i=1}^{n} h_{2i} Q^3 H_{2i} U^3c - \sum_{i=1}^{n} h_{2i-1} Q^3 H_{2i-1} D^3c \\
+ \sum_{i,j=1}^{n} \mu_{2i-1,2j} H_{2i-1} H_{2j} + h.c. + \cdots,
\]

(1)

where $Q^3, U^3c$ and $D^3c$ are the superfields of the left-handed quark doublet in the third generation, right-handed top quark and right-handed bottom quark respectively, and $H_{2i-1}$ and $H_{2i}$ are the superfields of the Higgs doublets. The ellipsis denotes the terms involving quark superfields in the first and second generations and lepton superfields, which are irrelevant to our result.

With general soft SUSY-breaking terms, the tree-level Higgs potential of this model is given by

\[
V^{(0)}(\phi_1^0, \ldots, \phi_{2n}^0, \phi_1^0*, \ldots, \phi_{2n}^0*) = \frac{g^2 + g'^2}{8}(\phi_d^0 \phi_d^0 - \phi_u^0 \phi_u^0)^2 \\
+ \phi_d^0 M_d^2 \phi_d^0 + \phi_u^0 M_u^2 \phi_u^0 - (v_d^0 M_d^2 v_u^0 + h.c.),
\]

(2)

where

\[
\phi_d^0 = \phi_1^0, \phi_3^0, \ldots, \phi_{2n-1}^0 \quad , \quad \phi_u^0 = \phi_2^0, \phi_4^0, \ldots, \phi_{2n}^0,
\]

(3)

are the neutral components of the Higgs doublets, which couple to the down-type quarks and to the up-type quarks respectively. The matrices $M_u^2$ and $M_d^2$ are $n \times n$ Hermitian matrices and $M_u^2$ is generally an $n \times n$ complex matrices. The couplings $g$ and $g'$ are the gauge couplings of $SU(2)_L$ and $U(1)_Y$ respectively. We concentrate our attention only on the neutral Higgs fields since only neutral fields may take VEVs after the breakdown of the electroweak symmetry.

Now we can rotate the Higgs fields so that

\[
\langle \phi_1^0 \rangle = \frac{v_d}{\sqrt{2}}, \quad \langle \phi_2^0 \rangle = \frac{v_u}{\sqrt{2}}, \quad \langle \phi_3^0 \rangle = \cdots = \langle \phi_{2n}^0 \rangle = 0, \quad (v_d, v_u : \text{real}),
\]

(4)

where prime denotes a quantity after the field transformation.

Just like in the MSSM case, we regard a diagonal element of the mass squared matrix, $\langle \phi | M^2 | \phi \rangle$, as the desired upper bound. Here the matrix $M^2$ is the mass squared matrix of the neutral Higgses, and the field $\phi$ is the real part of the field $\phi^0$ defined by

\[
(\phi^0, \chi^0) = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} (\phi_1^0, \phi_2^0), \quad (\tan \beta = \frac{v_u}{v_d})
\]

(5)

and

\[
\phi^0 = \frac{1}{\sqrt{2}}(\phi + i \chi).
\]

(6)

Then we can restrict the effective potential to $\phi_1^0$ and $\phi_2^0$ directions.

\[
V^{(0)}(\phi_1^0, \phi_2^0, \phi_1^0*, \phi_2^0*) = \frac{g^2 + g'^2}{8}(|\phi_1^0|^2 - |\phi_2^0|^2)^2 \\
+ m_1^2 |\phi_1^0|^2 + m_2^2 |\phi_2^0|^2 - (m_{12}^2 \phi_1^0 \phi_2^0 + h.c.).
\]

(7)

\textsuperscript{1}The number of the Higgs doublets must be even because of the anomaly cancellation.
This is the same form as the Higgs potential of the MSSM. Therefore it is a trivial result that the desired tree-level upper bound $m_h$ is the same one as that of the MSSM, i.e., $m_h \approx m_Z \cos \beta$ \cite{12}.

Next we consider the one-loop radiative corrections to this tree-level upper bound. The one-loop corrections to the effective potential mainly come from the loops of quarks and squarks in the third generation because of the large sizes of their Yukawa couplings. In the $\overline{\text{MS}}$ scheme, this is given by

$$V^{(1\text{loop})}(\phi^0_u, \phi^0_d, \phi^0_u^\dagger, \phi^0_d^\dagger) = \frac{3}{32\pi^2} \left[ \sum_{q=t,\bar{t},b,\bar{b}} m_q^2 \ln \left( \frac{m_q^2}{\mu^2} - \frac{3}{2} \right) - 2 \sum_{q=t,b} m_q^4 \ln \left( \frac{m_q^2}{\mu^2} - \frac{3}{2} \right) \right],$$

where $m_q$ and $m_q$ are Higgs-field-dependent masses of the squarks and quarks respectively, and defined by

$$\begin{align*}
  m_{t_1}^2 &= m_1^2 + m_2^2 + |t h_t A_t \phi^0_u|, \\
  m_{t_2}^2 &= m_1^2 + m_2^2 + |t h_\beta A_t \phi^0_d|,
\end{align*}$$

and

$$\begin{align*}
  m_{b_1}^2 &= m_1^2 + m_2^2 + |t h_b A_b \phi^0_u|, \\
  m_{b_2}^2 &= m_1^2 + m_2^2 + |t h_\beta A_b \phi^0_d|,
\end{align*}$$

Here $h_t$ and $h_b$ are the row vectors of the Yukawa couplings defined by

$$h_t \equiv \begin{pmatrix} h_2, h_4, \ldots, h_{2n} \end{pmatrix}, \quad h_b \equiv \begin{pmatrix} h_1, h_3, \ldots, h_{2n-1} \end{pmatrix},$$

and $A_t = \text{diag}(A_2, A_4, \ldots, A_{2n})$, $A_b = \text{diag}(A_1, A_3, \ldots, A_{2n-1})$ are the soft SUSY-breaking $A$-parameter matrices, and $\mu$ is the $\mu$-parameter matrix whose $(i,j)$-component is $\mu_{2i-1,2j}$. Here we assume the soft SUSY-breaking squark masses to be degenerate and take the common mass $m$. We also neglect the small $D$-term contributions for simplicity, but including these contributions does not change the following discussion.

We should calculate the second derivative of $V^{(1)} \equiv V^{(0)} + V^{(1\text{loop})}$ in terms of $\phi$ to obtain the upper bound $m_h$. In the basis \cite{3}, the effective potential restricted to the directions of $\phi^0_u$ and $\phi^0_d$ again becomes the same form as that of the MSSM. Hence we can easily see that the upper bound $m_h$ is also the same as the one in the MSSM case and conclude that $m_h \approx 140 \sim 150$ GeV, which is realized in the case of the large left-right stop mixing.

Thus $m_h$ is the same as the one in the MSSM up to one-loop order. At two-loop order, however, the extra Yukawa couplings first contribute to $m_h$ as well as the strong gauge coupling $\alpha_s$. Then since two-loop order is the lowest order in terms of these couplings and one-loop corrections are the same order as the tree-level value, the two-loop corrections might be relatively large. In fact, the QCD correction lowers the upper bound by more than 10 GeV in the MSSM due to the large size of $\alpha_s$. Since the extra Yukawa couplings are unconstrained experimentally, we will set their upper bounds by requiring that the theory remains perturbative up to some scale $\Lambda$. The maximal values of the extra Yukawa couplings under this requirement might be large, so that the two-loop corrections to $m_h$ due to these couplings are as large as the QCD correction. So in this letter, the upper bound $m_h$ is calculated using the RGE improved effective potential approach in the two-loop leading-log approximation in order to see how different the upper bound $m_h$ is from that of the MSSM.
3 Upper bound formula

For simplicity, we assume that the squarks are degenerate and their masses are 1 TeV throughout this letter and neglect $O(g^4, g^2 g'^2, g'^4)$ contributions. We can obtain the upper bound formula in a quite similar way to Ref. [13], in which the analytic expression of the lightest Higgs mass is provided in two-loop leading-log approximation in the MSSM.

\[
m^2_t = \frac{m_Z^2}{\pi^2} 2 \beta \left\{ 1 - \frac{3m_t^2}{2\pi^2 v^2} \left[ 1 - \frac{8 \cos^2 \beta}{\cos 2\beta} - \left( 1 + \frac{m_t^2}{m_t^2} \tan^2 \beta \right) \ln \frac{m_s}{m_t} \right] \right\} + \frac{3m_t^4}{\pi^2 v^2} \left[ 1 - \frac{X_t^2}{12m_S^2} \right] - \frac{m_Z^2}{8\cos 2\beta} \ln \frac{m_S}{m_t} \\
+ \frac{X_t^2}{8\pi^2} \left[ (\beta_m(m_t) - 6m_t^2) \ln \frac{m_t}{m_S} \right] + \frac{m_Z^2}{v^2 \cos 2\beta} \left( 6 \cos^2 \beta - \frac{3m_t^2}{m_t^2} \tan^2 \beta \right) - \frac{3\beta}{2} \frac{X_t^2 m_t}{m_S} \ln \frac{m_t}{m_S} \\
+ (\beta_m(m_t) - 12m_t^2) \left( \ln \frac{m_s}{m_t} \right)^2 \right\} \tag{15}
\]

where $\beta_m$ is defined as

\[
Q \frac{\partial (h'_3 \sin \beta)}{\partial Q} = \frac{h'_3 \sin \beta}{16\pi^2} \beta_{m_t}
\]

and $\beta_m(m_t)$ stands for $\beta_m$ defined at the scale $Q = m_t$. The renormalization scale is set to be $m_t$, and $m_Z = m_Z(m_t)$, $m_H = m_H(m_t)$. The mass parameter $m_t$ is the on-shell running mass $m_t(M_t)$, where $M_t$ is the pole mass, and the relation between $m_t$ and $M_t$ in the $\overline{\text{MS}}$ scheme is

\[
m_t = \frac{M_t}{1 + 4\alpha_s(M_t)/3\pi} \tag{17}
\]

The parameter $X_t$ is the mixing parameter of the left-right stop mixing, which is defined by $X_t = |M'_2 + M'_1 \cot \beta|$, and $m_S = \sqrt{m^2 + m_t^2}$ can be regarded as the SUSY-breaking scale.

The factor $m_Z^2 \cos^2 2\beta$ in the first line of (15) corresponds to the tree-level contribution and the remaining factor of the first line represents the anomalous dimension of the lightest Higgs field between the SUSY-breaking scale $m_S$ and the renormalization scale $m_t$. The second line of the formula is the part of the one-loop correction, and the rest is the two-loop leading-log contributions.

It is clear from the above expression that the upper bound $m_h$ gives the absolute upper bound when $\tan \beta$ is large and $\beta_m(m_t)$ has its maximal value. The $\beta$-function $\beta_m$ is expressed by using the step functions as

\[
\beta_m = \beta_{h'_3} + \cos^2 \beta (\gamma^{(1)}_\phi - \gamma^{(1)}_{\phi'}) \\
= -\frac{4}{3} g^2 (6 - \theta_{\tilde{G}\tilde{Q}} - \theta_{\tilde{G}\tilde{G}}) + 3h'_3 \sin^2 \beta \\
+ \frac{1}{2} \sum_{i=1}^n |h'_{2i}|^2 (3\theta_{\tilde{\phi}_{2i}} + 2\theta_{\tilde{\phi}_{2i}\tilde{Q}} + \theta_{\tilde{\phi}_{2i}\tilde{G}}) \\
+ \frac{1}{2} \sum_{i=1}^n |h'_{2i-1}|^2 (\theta_{\tilde{\phi}_{2i-1}} + \theta_{\tilde{\phi}_{2i-1}\tilde{G}}) + \cdots \tag{18}
\]

where $g_s$ is the strong gauge coupling and the ellipsis denotes the terms involving $g, g', \cdots$, which can be neglected because they are irrelevant within our approximations. A step function $\theta_P$ is defined as one above the mass of the particle $P$ and
zero below it, and \( \theta_{AB} \equiv \theta_A \theta_B \). (\( G \) denotes gluino, \( \tilde{Q}, \tilde{U} \) and \( \tilde{D} \) are the squarks in the third generation, \( \phi'_1 \) are Higgses and \( \phi'_i \) are Higgsinos.) Now since we consider below the scale \( Q = m(= m_Q = m_U = m_D) \), the maximal value of \( \beta_{m_1} \) is

\[
\beta_{m_1} = -8g_s^2 + 3h_2^2 \sin^2 \beta + \frac{3}{2} \sum_{i=1}^{n} |h_{2i}'|^2 + \frac{1}{2} \sum_{i=1}^{n} |h_{2i-1}'|^2,
\]

which is realized in the limit that all the Higgs bosons are lighter than the top quark. Here we denote the word “extra” as the meaning of giving no contributions of the “extra” Yukawa couplings corresponding to the “extra” Higgses. The absolute upper bound formula (15) is smaller than the maximal value (19) by the constant to saturate \( m_h \). In this case the scale at which \( \beta_{m_1} \) is estimated is not above all of the Higgs mass eigenvalues and thus \( \beta_{m_1} |m_i| \) that should be used in the upper bound formula (13) is smaller than the maximal value (19) by the contributions of the “extra” Yukawa couplings corresponding to the “extra” Higgses that are decoupled. Here we denote the word “extra” as the meaning of giving no masses to the quarks. For example, the extra Yukawa couplings mean \( h_{2s}' \) and \( h_{2s-1}' \) (\( i \geq 2 \)). The absolute upper bound is thus realized when all Higgses that do not mix with the lightest one are light and the extra Yukawa couplings that correspond to heavy extra Higgses are small enough so that \( \beta_{m_1} \) is close to (19) if the fixed values of \( h''_2 \) and \( h''_1 \) are given. Thus, from now on, we will restrict the parameter space to this region.

Unlike the MSSM case, there is no upper bound on \( h''_2 \) and \( h''_1 \) due to the existence of unconstrained coupling constants \( h_{2i}' \) and \( h_{2i-1}' \) (\( i \geq 2 \)). Then we put the requirement:

"The theory remains perturbative up to some scale \( \Lambda \)."

By substituting (19) for \( \beta_{m_1} |m_i| \) in the formula (13) and setting \( \tan \beta \) large, the desired upper bound is obtained as

\[
m_h^2 = \frac{m_Z^2}{m_t^2} \left( 1 - \frac{3m_t^2}{2\pi^2 v^2} \ln \frac{m_S}{m_t} \right) \\
+ \frac{3m_t^4}{\pi^2 v^2} \left[ \frac{1}{2} \frac{X_2}{m_S} (1 - \frac{X_2^2}{12m_S}) + \frac{m_Z^2}{8m_t^2} X_7^2 \right] \ln \frac{m_S}{m_t} \\
+ \frac{1}{8\pi^2} \left[ \left( \frac{3}{2} h_{2\text{max}}'' + \frac{1}{2} h_{1\text{max}}'' \right) \frac{X_2^2}{m_S} (1 - \frac{X_2^2}{12m_S}) \ln \frac{m_S}{m_t} \\
- \frac{9m_t^2}{2\pi^2} \frac{X_7^2}{m_S} \ln \frac{m_S}{m_t} \\
+ (\frac{3}{2} h_{2\text{max}}'' + \frac{1}{2} h_{1\text{max}}'') \ln \frac{m_S}{m_t} \right],
\]

where \( h_{2\text{max}}'' \) and \( h_{1\text{max}}'' \) are the values of \( h''_2 \) and \( h''_1 \) that maximize the combination \( \frac{3}{2} h_{2\text{max}}'' + \frac{1}{2} h_{1\text{max}}'' \) under the above requirement.
4 Maximal values of \( h_2'' \) and \( h_1'' \)

Now we will compute \( h_2''_{\text{max}} \) and \( h_1''_{\text{max}} \). First, we should notice that the RGEs of \( h_2'' \) and \( h_1'' \) are the same as that of \( h_1 \) and \( h_0 \) in the MSSM because \( h_2'' \) and \( h_1'' \) are the sole pair of the Yukawa couplings that couple to the up-type quark multiplets and the down-type quark multiplets respectively and the gauge interactions do not mix \( \phi_1 \) and \( \phi_2 \) with the other Higgses. Thus, we can use the formulae in \[14\], which are expressed by the step functions just like \[13\]. Here we will use the RGEs that is gained by setting all \( \theta \)'s to one above the SUSY-breaking scale \( m_S \) and all \( \theta \)'s except \( \theta_Q \), \( \theta_C \) and \( \theta_D \) to one below the scale \( m_S \) at the RGE formulae in \[14\]. Of course the correct RGEs below \( m_S \) depend on the mass spectrum of the SUSY particles, but this difference is thought to be small and negligible. Further we will also neglect the \( \tau \) Yukawa coupling \( h_\tau'' \) for simplicity. Neglecting \( h_\tau'' \) does not lower the upper bound \( m_h \) because the coefficient of \( h_\tau'' \) in the RGE of \( h_\tau' \) is positive.

Thus the RGEs we are using are

\[
Q \frac{\partial h_2''}{\partial Q} = \frac{h_2''}{16\pi^2} \left( -\frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2 + h_0'' + h_1'' \right),
\]

\[
Q \frac{\partial h_1''}{\partial Q} = \frac{h_1''}{16\pi^2} \left( -\frac{16}{3} g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^2 + h_0'' + h_1'' \right),
\]

above \( m_S \) and

\[
Q \frac{\partial h_2''}{\partial Q} = \frac{h_2''}{16\pi^2} \left( -8 g_3^2 - \frac{3}{4} g_2^2 - \frac{11}{20} g_1^2 + \frac{9}{2} h_0'' + \frac{1}{2} h_1'' \right),
\]

\[
Q \frac{\partial h_1''}{\partial Q} = \frac{h_1''}{16\pi^2} \left( -8 g_3^2 - \frac{3}{4} g_2^2 + \frac{1}{20} g_1^2 + \frac{1}{2} h_0'' + \frac{9}{2} h_1'' \right),
\]

below \( m_S \). Here \( g_3 = g_s \), \( g_2 = g \) and \( g_1 = \sqrt{5/3} g' \).

Next we will consider the gauge couplings. By extending the RGEs in \[14\] in the same way as the above Yukawa coupling case and solving the RGEs, we obtain above the scale \( m_S \),

\[
g_3^2 = \frac{1}{8\pi^2} \ln(Q/M_3),
\]

\[
g_2^2 = \frac{1}{8\pi^2} \ln(Q/M_2),
\]

\[
g_1^2 = \frac{1}{8\pi^2} \ln(Q/M_1),
\]

and below \( m_S \),

\[
g_3^2 = \frac{1}{8\pi^2} \ln(Q/M_3'),
\]

\[
g_2^2 = \begin{cases} 
\frac{1}{8\pi^2} \ln(Q/M_2') & \text{for } n \neq 2 \\
\text{constant} & \text{for } n = 2
\end{cases},
\]

\[
g_1^2 = \frac{1}{8\pi^2} \ln(Q/M_1'),
\]

where \( M_i \) and \( M_i' \) \( (i = 1, 2, 3) \) are decided by the initial conditions:

\[
\alpha_3(m_Z) = 0.12, \quad \alpha_2(m_Z) = 0.034, \quad \alpha_1(m_Z) = 0.017,
\]

and the boundary conditions at \( Q = m_S \).
Using (22)-(31), \( h_{2 \text{max}}'' \) and \( h_{1 \text{max}}'' \) can be computed.

As the criterion that the theory remains perturbative up to \( \Lambda \), we adopt the condition whether one of the dimensionless coupling constants (gauge or Yukawa) saturates the scale \( \Lambda \). A particular coupling \( \lambda \) is said to “saturate” a scale \( \Lambda \) if

\[
\frac{\lambda^2(Q^2)}{4\pi} \leq 1
\]

(33)

for \( Q \leq \Lambda \), and the equality in (33) holds for \( Q = \Lambda \). One can see from the RGEs (26)-(31) and the conditions (32) that \( g_2 \) is the first gauge coupling to saturate for every scale \( \Lambda \). Thus one cannot set \( \Lambda \) larger than the scale \( \Lambda_n \) determined by the saturation of \( g_2 \) for a given \( n \). However, this constraint does not restrict the allowed region of \( \Lambda \) so strictly. For example, \( \Lambda_2 = 10^{42.2} \) GeV, \( \Lambda_5 = 10^{18.0} \) GeV, \( \Lambda_6 = 10^{15.4} \) GeV and \( \Lambda_{10} = 10^{10.0} \) GeV.

5 Self-energy contribution

The upper bound \( m_h^2 \) discussed so far is \( \partial^2 V^{(1)} / \partial \phi^2 \). Since the effective potential \( V \) is a sum of one-particle-irreducible (1PI) Feynman diagrams with zero external momenta, \( m_h^2 \) is written as

\[
m_h^2 = m_{h \text{tree}}^2 + \Pi(0),
\]

(34)

where \( \Pi(\rho^2) \) is the self-energy of the Higgs boson.

On the other hand, physical mass \( M_h \) is defined as a pole of the propagator, that is,

\[
M_h^2 = m_{h \text{tree}}^2 + \Pi(M_h^2).
\]

(35)

Then from (34) and (35), the relation between \( m_h \) and \( M_h \) is

\[
M_h^2 = m_h^2 + \Pi(M_h^2) - \Pi(0).
\]

(36)

More precisely,

\[
M_h^2 = m_h^2 + \text{Re}\Pi(M_h^2) - \text{Re}\Pi(0).
\]

(37)

The imaginary part of \( \Pi(M_h^2) - \Pi(0) \) contributes to the decay width of the Higgs boson, and if we suppose \( M_h < 2M_W \), where \( M_W \) is the physical mass of the W boson, the Higgs is stable at tree level and (36) is correct.

This correction \( \Delta \Pi(M_h^2) \equiv \Pi(M_h^2) - \Pi(0) \) mainly comes from one-loop electroweak interaction effects.

This correction is larger than the MSSM case due to the existence of the extra Yukawa couplings, but still small and lift up the upper bound \( m_h \) by 2 GeV at most.

6 Results

The results including the self-energy contribution is plotted in Fig.1 in the case of \( n = 2 \). Since we are interested in the absolute upper bound on the mass of the lightest Higgs, only large \( \tan \beta \) case (\( \tan \beta = 40 \)) is plotted. The lines are the case of \( \Lambda = 10^{10}, 10^{12}, 10^8, 10^4 \) GeV from bottom to top. The common soft SUSY-breaking stop mass parameter \( m \) is set to be 1 TeV in all the cases. We can see from this figure that setting the saturation scale \( \Lambda \) larger, the upper bound \( m_h \) becomes more stringent.

Because the RGEs (22)-(25) have \( n \)-dependence only through the terms involving the weak gauge couplings \( g \) and \( g' \), the \( n \)-dependence of \( m_h \) is quite small. In fact,
since the $n$-dependence of the result becomes larger as $\Lambda$ is larger and $g$ becomes no longer small at $10^{15}$ GeV for $n = 6$ ($\Lambda_6 = 10^{15.4}$ GeV), we can see the most enhanced $n$-dependence of $m_h$ when we compare the two cases: $n = 2$ and $n = 6$ for $\Lambda = 10^{15}$ GeV. This situation is plotted in Fig.2.

From this figure we can see that the $n$-dependence of $m_h$ can be neglected.

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Figure 1: The upper bound $m_h$ in the case of $n = 2$. The lines correspond to the case of $\Lambda = 10^{16}, 10^{12}, 10^8, 10^4$ GeV from bottom to top. The parameters, $\tan \beta$ and the common soft SUSY-breaking stop mass $m$, are set to be 40 and 1 TeV respectively.

Figure 2: The comparison of the two cases: $n = 2$ and $n = 6$ for $\Lambda = 10^{15}$ GeV. In both cases we set $\tan \beta = 40$ and $m = 1$ TeV.