Constitutive modelling of plastic deformation and damage in anisotropic high-purity titanium and validation using ex-situ and in-situ tomography data

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Abstract. A study on plastic deformation and damage in titanium was conducted. The X-ray tomography data reveal that damage distribution and evolution in titanium is markedly different than for a FCC material. Theoretically, it is shown that only by modelling both the anisotropy and the tension-compression asymmetry in plastic behaviour it is possible to realistically predict titanium behaviour. For a smooth specimen under uniaxial tension, the model predicts that damage initiates at the centre of the specimen, and is diffuse; the level of damage close to failure being very low. In contrast, for a notched specimen under the same loading it is predicted that damage initiates at the outer surface of the specimen, and grows towards the centre of the specimen, which corroborates with XCMT data.

1. Introduction
Titanium materials that are predominantly hexagonally close-packed are known to display at room temperature plastic anisotropy and a strong tension–compression asymmetry. Although titanium materials are textured, modeling of their damage and failure characteristics is generally done using evolution laws for the rate of void growth or Gurson-type models. However, these models cannot realistically describe damage in Ti materials because the core hypothesis of such models is that the plastic behavior of the matrix is governed by the isotropic von Mises criterion. In this paper, we present an experimental study on plastic deformation and damage of a polycrystalline Ti material, as well as modelling of the observed behaviour using the Stewart and Cazacu model [1].

2. Plasticity-Damage couplings in polycrystalline Titanium
According to this model, the plastic strain rate is defined with respect to the stress potential \( \varphi \) given by:

\[
\varphi(\sigma, f) = \hat{m}^2 \sum_{i} \left( |\xi_i| \hat{k}_i \right)^2 \frac{1}{\bar{\sigma}_i^2} + \bar{\sigma}_i^2 \left( 2f \cosh \left( \frac{3\sigma_m}{h\bar{\sigma}_i} \right) - \left( 1 + f^2 \right) \right).
\]
In equation (1) k is a parameter describing the tension-compression asymmetry of the matrix; m depends on the anisotropy coefficients and k, $\hat{m}$ is the porosity, and $\tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\sigma}_3$ are the principal values of the transformed stress tensor $\tilde{\sigma} = L : \sigma'$, where $\sigma'$ is the deviator of the Cauchy stress tensor and $L$ is a fourth-order symmetric tensor describing the orthotropy of the fully-dense material. The parameter $\alpha$ which controls damage evolution depends on both the anisotropy coefficients and the sign of the mean stress, $\sigma_m$. Hardening of the matrix (fully-dense material) is due to the effective plastic strain, $\tilde{\varepsilon}^p$. The rate of change of the porosity is given by the evolution of existing voids and the nucleation of new ones. Evolution of existing voids is obtained from mass conservation while nucleation of new ones is due to both the plastic strain and to the mean stress. In the absence of voids ($f = 0$), the potential (1), reduces to a clear physical meaning, being related to the plastic properties of the material. Specifically, all the parameters are expressible in terms of the anisotropy coefficients (components of the tensor $L$) and/or the strength-differential parameter k. For the identification of these parameters we conducted mechanical tests in uniaxial tension on flat specimens with long axis along the rolling direction (RD) and two other in-plane directions, at 45° and 90° (TD) with respect to RD, respectively and in uniaxial compression tests (cylindrical specimens same directions). The evolution of the anisotropy in r-values with accumulated plastic deformation and the evolving difference in hardening rates between tension and compression loadings observed from the test results are indicative of texture evolution. To describe these effects, the anisotropy coefficients and k are considered to evolve with accumulated plastic deformation. Next, the capabilities of the model to predict plastic deformation and damage in the Ti material for loadings that were not used for identification are demonstrated. Key results are presented in the following. For more details, the reader is referred to [3].

3. Comparison between F.E. prediction and XCMT measurements of porosity evolution

For the smooth specimen, the XCMT scan was done ex-situ, following uniaxial tension along RD up to an axial displacement of 5.52 mm (i.e. very close to failure). In figure 2(a) are shown the reconstructed 2-D views in the (TD, TT) plane, and in the (RD, TD) plane of the Ti specimen, respectively. For comparison purposes, in figure 2(b) are shown the respective views extracted from an XCMT scan of a copper specimen, which was taken at the same axial displacement. It is very important to note that for the same axial displacement, the copper material is much damaged while titanium material shows very little damage. While a large hole/crack is seen in the middle of the copper specimen, almost no damage is observed in the Ti specimen. It means that for uniaxial tensile loading the rate of damage growth is slower in Ti than in Cu. To extract the experimental void volume fraction, a clustering technique was applied in order to identify 3-D groups of connected voxels, and thus to identify individually each pore within the matrix. A total of 2385 pores have been detected in the total volume of the specimen, which resulted into an average porosity over the specimen volume of 0.052%. However, the average porosity in the minimal cross-section is of 0.25%. Figure 2 shows a comparison between the F.E. cross-sections and isocontours of void volume fraction according to the Stewart and Cazacu [1] model and XCMT data for the same axial displacement. The F.E. predictions of the geometry of the specimen are also superposed on the different cut views It is worth noting that the model also predicts that damage is diffuse, in the minimal cross-section the maximum void volume fraction predicted being of 0.22% (see figure 2a). Likewise, for the RD-TD view (figure 2(b)) most of the voids observed by XCMT are inside the region of maximum void volume fraction predicted by the model.
Figure 1. Post-test XCMT scans of smooth specimens subjected to uniaxial tension: (a) Ti specimen; (b) Cu specimen.

Figure 2. Comparison between F.E. cross-sections and isocontours of porosity of a smooth Ti specimen subjected to uniaxial tension along RD, according to [1]: (a) cross-section; (b) (RD-TD) section.

Note that the model correctly predicts the section geometry in all the planes. Next, we assess the predictive capabilities of the model in terms of capturing the influence of the stress triaxiality on damage and its evolution. For this purpose, in-situ XCMT in uniaxial tension was conducted on a notched axisymmetric RD specimen. The notch radius was of 0.51 mm while the specimen cross-section radius was of 1.27 mm.

Figure 3. In-situ XCMT scans of Ti showing the (RD-TD) view of the notched specimen corresponding to displacement of 0.24 mm, 0.73 mm 1.02 mm, 1.2 mm, respectively.
Scans were taken at notch displacement of 0.24 mm, 0.73 mm, 1.02 mm and 1.20 mm, respectively. As an example, in Figure 3 are shown the cross-section in the (RD, TD) corresponding to each displacement. Comparison between F.E. predictions at 1.2 mm axial displacement and model are shown in Figure 4 (b). Since according to the model, damage is driven by the plastic deformation, and the plastic anisotropy is correctly described, the model correctly predicts the location of the zones of maximum damage for the notched specimen. The importance of accounting for tension-compression asymmetry in the plastic behaviour of Ti is clearly demonstrated by comparing the predicted isocontours of the porosity in the (RD-TD) section with an anisotropic model with no tension-compression symmetry (k=0). Note that if the tension-compression asymmetry in plastic deformation is accounted for, the local distribution of the damage is totally different.

![Image](image.png)

**Figure 4.** Comparison between F.E. cross-sections and isocontours of porosity of a smooth Ti specimen subjected to uniaxial tension along RD, according to [2]: (a) cross-section; (b) (RD-TD) section.

### 4. Conclusions

The ex-situ and in-situ X-ray tomography measurements conducted show that damage distribution and evolution in this material is markedly different than in a typical FCC material such as copper. Stewart and Cazacu [1] anisotropic elastic/plastic damage model predicts correctly the anisotropy in plastic deformation, and its strong influence on damage distribution and damage accumulation. Specifically, for a smooth axisymmetric specimen subject to uniaxial tension, damage initiates at the center of the specimen, and is diffuse; the level of damage close to failure being very low. On the other hand, for a notched specimen subject to the same loading the model predicts that damage initiates at the outer surface of the specimen, and further grows from the outer surface to the center of the specimen, which corroborates with the in-situ tomography data.

### References

[1] Stewart J B and Cazacu O 2011 *Int. J. Solids Struct.* **48** 357.

[2] Cazacu O, Plunkett B and Barlat F 2006 *Int. J. Plasticity* **22** 1171.

[3] Revil-Baudard B, Cazacu O, Flater P, Chandola N and Alves J L 2016 *J. Mech. Phys. Solids* **88** 100.