The observed shock wave positions and expansion in Cas A can be interpreted in a model of supernova interaction with a freely expanding stellar wind with a mass loss rate of $\sim 3 \times 10^{-5} \ M_\odot \ yr^{-1}$ for a wind velocity of 10 km s$^{-1}$. The wind was probably still being lost at the time of the supernova, which may have been of Type IIIn or IIb. The wind may play a role in the formation of very fast knots observed in Cas A. In this model, the quasi-stationary flocculi (QSFs) represent clumps in the wind, with a density contrast of several $10^3$ compared to the smooth wind. The outer, unshocked clumpy wind is photoionized by radiation from the supernova, and is observed as a patchy HII region around Cas A. This gas has a lower density than the QSFs and is heated by nonradiative shocks driven by the blast wave. Denser clumps have recombined and are observed as HI compact absorption features towards Cas A.

Subject headings: ISM: individual (Cassiopeia A) — supernovae — supernova remnants

1. INTRODUCTION

The supernova remnant Cas A (Cassiopeia A) gives us our best view of the outcome of the explosion of a massive star. Spectral imaging with Chandra at X-ray wavelengths (Hughes et al. 2000) and HST at optical wavelengths (Fesen et al. 2001) has shown the complex structure of the ejected heavy elements. The Chandra image also revealed a central compact X-ray source (Tananbaum 1999), probably a neutron star, and lines of the radioactive isotope $^{44}$Ti have been detected (Iyudin et al. 1994). Despite these many developments, the evolutionary status of Cas A remains uncertain. The most common assumption is that the supernova is interacting with a constant density interstellar medium (Gull 1973b; Gotthelf et al. 2001; DeLaney & Rudnick 2003), or perhaps with a molecular cloud (Keohane, Rudnick, & Anderson 1996). Interaction with a circumstellar shell has also been suggested...
(Chevalier & Liang 1989; Borkowski et al. 1996). The immediate environment of a massive star is expected to be strongly influenced by mass loss, and the pervasive, high-velocity, heavy element ejecta in Cas A indicate that the star underwent strong mass loss before the explosion.

Here, we propose that the supernova is interacting with the slow wind from the progenitor star, with a $\rho_w \propto r^{-2}$ density profile. The resulting model can be compared to the width and expansion of the shocked region ($\S$ 2), giving constraints on the basic parameters. Implications of the model for inhomogeneities in the the wind, for the supernova, and for a surrounding HII region and HI knots are discussed in $\S$ 3.

2. WIND INTERACTION MODEL

The distance to Cas A has been determined from the fast knot expansion to be $3.4^{+0.3}_{-0.1}$ kpc (Reed et al. 1995). Ashworth (1980) claimed an observation of the Cas A supernova by Flamsteed in 1680, but that claim has been controversial (Stephenson & Green 2002). On the basis of very fast knots that show little sign of deceleration, Thorstensen et al. (2001) determined an explosion date of $1671.3 \pm 0.9$. We take an explosion date of $1675 \pm 5$. The outer shock front has been clearly observed in Chandra images to have a radius of $153''$ (Gotthelf et al. 2001), or $7.8 \times 10^{18}$ cm at a distance of 3.4 kpc. The position of the reverse shock front is less clear, but was determined by Gotthelf et al. (2001) from the inner edge to the bright ring of emission at X-ray and radio wavelengths; they found a ratio of the forward shock radius to that of the reverse shock of $r_f/r_r = 1.5$ with a variation of 14% around the remnant.

The youth of Cas A has enabled studies of its expansion from proper motion studies. DeLaney & Rudnick (2003) have recently measured the expansion of the forward shock in X-rays from Chandra observations over 2000–2002 and found it to be in the large range $0.02 - 0.33 \% \, \text{yr}^{-1}$, with a median of $0.21 \% \, \text{yr}^{-1}$. The median corresponds to an expansion parameter $m_f = d \ln r_f / d \ln t$ of 0.68. The bright ring of X-ray emission has previously been found to be expanding at $0.20 \pm 0.01 \% \, \text{yr}^{-1}$ from Einstein and ROSAT observations covering 1979–1996 (Koralesky et al. 1998; Vink et al. 1998), or $m = 0.62 \pm 0.03$. The bright radio ring is approximately co-extensive with the X-ray one. Agüeros & Green (1999) studied the minima in the visibility plane at 151 MHz over the period 1984–1997 to determine a timescale for the bulk ring expansion of $460 \pm 30$ years, or $m = 0.69 \pm 0.05$. While this result is consistent with the X-ray expansion, other radio studies have yielded a slower expansion; Anderson & Rudnick (1995) find an expansion age of $750 - 1300$ years ($m = 0.33 \pm 0.11$). DeLaney & Rudnick (2003) recently examined the motion of the radio ring with an emphasis on angle-
averaged emissivity profiles and found an expansion of $0.07 \pm 0.03 \% \, \text{yr}^{-1}$ ($m = 0.22 \pm 0.09$). DeLaney & Rudnick suggested the difference is due to the more rapid flux drop of the ring compared to the plateau, but this is not definitively established. We regard the current situation on the radio ring expansion to be uncertain; more observations are needed.

We have carried out simulations of supernova interaction to compare to these observations of the shock dynamics. The explosion of Cas A appears to have been that of a massive star core, even though there was some H near the surface (Fesen & Becker 1991). We thus use the model of Matzner & McKee (1999) for the density distribution resulting from the explosion of a massive star with a radiative envelope. We concentrate on interaction with a stellar wind from the progenitor star with density $\rho_w = A r^{-2}$, although we also briefly consider a constant density environment. For a steady wind, $A = \dot{M}/4\pi v_w$, where $\dot{M}$ is the mass loss rate and $v_w$ is the wind velocity. The outer part of the Matzner & McKee (1999) profile has the form $\rho_{sn} \propto r^{-10.12}$. The self-similar solutions of Chevalier (1982) show that when such a profile interacts with a wind, both reverse and forward shocks expand with $m = 0.88$ and the thickness of the shocked region is $r_f/r_r = 1.26$. The observed shock parameters indicate that the reverse shock wave has propagated in from the power law region.

In order to calculate the further evolution of the shock fronts, we used the VH-1 hydrodynamics code to compute the 1-dimensional evolution of the wind interaction flow. In order to take advantage of the scaling that applies to this problem (Gull 1973a; Truelove & McKee 1999), we used the dimensionless variables $r' = r/R'$, $v' = v/V'$, and $t' = t/T'$, where $R' = M_{ej}/(4\pi A)$, $V' = (2E/M_{ej})^{1/2}$, $T' = R'/V'$, $M_{ej}$ is the ejecta mass, and $E$ is the explosion energy. We have $R' = 3.16 \times 10^{19} M_1 A_{-5}^{-1/2}$ cm, $V' = 3160 E_{51}^{1/2} M_1^{1/2} A_{-5}^{-1/2}$ km s$^{-1}$, and $T' = 3160 E_{51}^{1/2} M_1^{3/2} A_{-5}^{-1}$ yr, where $M_1$ is the ejecta mass in units of $10 M_\odot$, $E_{51}$ is the energy in units of $10^{51}$ ergs, and $A_{-5}$ is $A$ in units of $10^{-5}$ $M_\odot$ yr$^{-1}/(4\pi 10$ km s$^{-1})$. Fig. 1 shows that $r_f/r_r = 1.5$ when $t' = 1.56$ in the scaled variables. At this time, the forward shock has an expansion parameter $m_f = 0.76$ and the reverse shock has $m_r = 0.68$; the forward shock radius is $r_f' = 1.48$. In a computation with constant density ejecta, we found similar values of the $m$ parameters when $r_f/r_r = 1.5$; the sensitivity to the supernova density profile is weak. Using the Matzner & McKee (1999) supernova density profile, we have also carried out a computation for expansion in a uniform (interstellar) medium. In this case, when $r_f/r_r = 1.5$, the forward shock has an expansion parameter $m_f = 0.50$ and the reverse shock has $m_r = 0.27$.

For the forward shock motion, the wind model gives an expansion rate of $0.235 \% \, \text{yr}^{-1}$, while the uniform model gives $0.153 \% \, \text{yr}^{-1}$. The wind value appears to better represent the observations, which have a median value of $0.21 \% \, \text{yr}^{-1}$ (see Fig. 3 of DeLaney & Rudnick 2003). As discussed above, there is ambiguity in the motion of the bright ring, which is
identified as gas that has passed through and is bounded by the reverse shock. The X-ray data and some radio data are consistent with the wind interaction model, while other radio data (expansion of 0.07 % yr$^{-1}$ found by DeLaney & Rudnick 2003) are consistent with a constant density surroundings. In the wind model, the small rate of expansion of compact radio features and parts of the forward shock front may be due to interactions with dense inhomogeneities in the wind like the QSFs (quasi-stationary flocculi; Van den Bergh 1971b). Despite the current small rate of expansion for some segments of the forward shock, the shock is fairly circular overall (Gotthelf et al. 2001), suggesting that the slowing is due to recent interaction with clumps.

The radial emissivity profiles are another difference between the models. Compared to the uniform density case, the shocked ejecta are concentrated into a higher density region surrounded by a region of near constant density wind in the wind case (Fig. 2). The ejecta shell in the wind case is broadened by a factor $\sim 3$ by hydrodynamic instabilities (see Fig. 8 of Chevalier, Blondin, & Emmering 1992), which will be crucial for interpreting the spatial distribution of X-ray emission from Cas A. Radial profiles of radio and Si emission (Fig. 4 of Gotthelf et al. 2001) show an outer plateau of emission, but more detailed investigations of the observations, together with multidimensional hydrodynamic models are needed to provide firm results.

Adopting the wind model, we can apply two known properties of Cas A, its age $t = 320$ yr and outer shock radius $r_f = 7.8 \times 10^{18}$ cm, to determine relations between the 3 model parameters. We find $M_1 = 0.16 M_\odot E_{51}$ and $A_{-5} = 1.3E_{51}$. The uncertainties in the shock positions give an uncertainty in the numerical coefficients of $\sim 50\%$ and additional physical effects that could affect the hydrodynamics, such as cosmic ray pressure or clumpiness, increase the uncertainty. As an example, we take $E_{51} = 2$, leading to an ejecta mass of $3.4 M_\odot$ (mostly heavy element core material) and a mass loss rate of $2.6 \times 10^{-5} M_\odot$ yr$^{-1}$ for a wind velocity of 10 km s$^{-1}$. The current mass of shocked wind material is $6.4 M_\odot$ and the shocked ejecta mass is $1.9 M_\odot$. Including a neutron star mass of $1.4 M_\odot$ brings the core mass to $4.8 M_\odot$, corresponding to a main sequence mass of $\sim 17 M_\odot$.

Another constraint on the models comes from the X-ray luminosity, which is related to the X-ray emitting mass. Mass estimates include $\gtrsim 15 M_\odot$ from Einstein data (Fabian et al. 1980), $\sim 14 M_\odot$ from ASCA data (Vink, Kaastra, & Bleeker 1996), and $10 M_\odot$ from XMM-Newton data (Willingale et al. 2002). The model described above is approximately consistent with these results. The model cannot be expected to yield improved values for the ejecta mass and explosion energy. The main point is that the dynamics and emission suggest that the supernova is interacting with a moderately dense stellar wind, considerably denser than the wind expected from a Wolf-Rayet star, which typically have $\dot{M} \approx 10^{-5} M_\odot$ yr$^{-1}$.
and \( v_w \approx 10^3 \text{ km s}^{-1} \), but consistent with the wind from a red supergiant star.

3. IMPLICATIONS

3.1. Wind Inhomogeneities

The presence of QSFs was one of the reasons that Chevalier & Liang (1989) used to argue for a dense circumstellar shell. The optical emission from QSFs indicates that it is from radiative shock fronts with velocities \( v_q \sim 100 - 200 \text{ km s}^{-1} \). The velocity of the forward shock front is \( v_f = 5800 \text{ km s}^{-1} \), so the preshock density in the QSFs is \( n_q \approx n_0 (v_f/v_q)^2 = 3 \times 10^3 n_0 \), where \( n_0 \) is the smooth wind density; in our model, the density at the shock front is currently \( \sim 1 \text{ H atom cm}^{-3} \). The high density contrast is suggestive of a shell, but the positions of the QSFs are not restricted to the bright emitting shell of Cas A (see Fig. 3 of Lawrence et al. 1995 and Fig. 10 of Fesen 2001). This implies that the QSFs are dense clumps within a smoother wind with the properties given above. The presence of wind inhomogeneities is also indicated by the irregular outline of the forward shock front observed in X-rays (Gotthelf et al. 2001).

The maximum shock velocity in the QSFs may be determined by the cooling time in the postshock region. A similar situation may be present in the remnant of SN 1987A, and Pun et al. (2002) estimate the postshock cooling time as \( t_{cool} \approx 2.2 (2 \times 10^4 \text{ amu cm}^{-3}/\rho_q) (v_q/250 \text{ km s}^{-1})^{3.8} \text{ yr} \) over the shock velocity range 100 - 600 km s\(^{-1}\). Converting from \( \rho_q \) to the ambient preshock density, \( \rho_0 \), the ram pressure relation above gives \( t_{cool} \propto v_q^{5.8} \). Substituting the current conditions for the blast wave yields \( v_q = 280 (t_{cool}/100 \text{ yr})^{0.17} \text{ km s}^{-1} \). For higher velocities, the shocks are nonradiative, which explains why this is approximately the upper limit of the shock wave velocities in the QSFs. There may be faster shock waves moving into lower density inhomogeneities, which are not visible optically.

The origin of high contrast knots in the circumstellar wind is not clear, but they have probably been observed on other objects. In SNe II In (Type II In supernovae), there is moderately narrow line emission, probably from shocked clumps, as well as broad line emission; for example, SN 1988Z showed broad H\( \alpha \) with velocities to 20,000 km s\(^{-1}\) as well as a narrower 2000 km s\(^{-1}\) component (Stathakis & Sadler 1991). Radiative shocks are present at higher velocities than in Cas A because of the higher ram pressure at early times in the supernova evolution. SN 1995N is another SN II In with an intermediate width H\( \alpha \) component as well as narrow lines that appear to be from clumps in the preshock circumstellar medium (Fransson et al. 2002).
3.2. The Supernova

The presupernova star apparently had little H at the time of the explosion, but did have some (Fesen & Becker 1991; Fesen 2001). This implies that the supernova may have been more closely related to Type IIn and IIb supernovae than to Type Ib and Ic supernovae, which are probably the explosion of Wolf-Rayet stars, and that the dense circumstellar wind around Cas A may have extended down to close to the stellar surface. The Type IIn SN 1995N showed evidence for interaction with a dense, H-rich wind, but also for emission from fast, O-rich ejecta near the reverse shock, showing that the explosion occurred with little H at the surface of the star (Fransson et al. 2002).

The Type IIb SN 1993J also had little H at the time of the explosion and expanded into a dense wind. Radio and X-ray observations imply a mass loss rate of $\sim 4 \times 10^{-5} \, M_\odot \, yr^{-1}$ for a wind velocity of 10 km s$^{-1}$ (Fransson, Lundqvist, & Chevalier 1996), comparable to our estimate for Cas A. In addition, the main sequence mass of the progenitor star is estimated at $13 - 16 \, M_\odot$ (Woosley et al. 1994), close to our estimate for Cas A. Houck & Fransson (1996) estimated that the bulk of the H/He envelope mass of SN 1993J lies between $8500 - 10,000 \, km \, s^{-1}$; although most of the knots in Cas A do not show H lines, Fesen & Becker (1991) found a H knot with a velocity $\sim 9000 \, km \, s^{-1}$. He enrichment is found in both the QSFs (Chevalier & Kirshner 1978) and in the H envelope of SN 1993J (Houck & Fransson 1996). A possible problem is that some models for SN 1993J require a massive star companion, which survives the supernova (Woosley et al. 1994); there is no evidence for a massive star near the explosion center of Cas A (Thorstensen et al. 2001). However, it may be possible for a star to undergo this evolution as a single star.

If the dense wind initially extended in to close to the stellar surface, it can help to explain a puzzling feature of the fastest knots. Fesen & Becker (1991) find N and H-rich knots moving at $\sim 10,000 \, km \, s^{-1}$, which is surprising because the outer supernova ejecta are expected to be shocked to a high temperature and have a long radiative lifetime. These knots must have crossed the reverse shock early in the life of the supernova remnant. In the dense, slow wind, the early evolution is given by a self-similar solution, with $r_f/t = 30,000 E_5^{0.44} M_1^{-0.32} A_5^{-0.12} (t/day)^{-0.12} \, km \, s^{-1}$. For the typical parameters, ejecta moving at 10,000 km s$^{-1}$ crosses the reverse shock at an age of $\sim 25$ years, when the preshock density is $2 \times 10^{-21} \, g \, cm^{-3}$. The smooth, H-rich ejecta are not radiative at this time, but a moderate degree of clumping can lead to radiative cooling and knot formation. The absence of a dense wind would lead to hotter, lower density ejecta and would make it difficult to produce cool ejecta knots at this early time.
3.3. The Outer Wind

A early photograph of Cas A taken by Minkowski, reproduced in van den Bergh (1971a), gives evidence for a patchy HII region surrounding Cas A, extending out 7′ from the remnant (see also Fig. 10 of Fesen 2001). Spectral observations of the nebulosity on the E side of Cas A by Fesen, Becker, & Blair (1987) showed it to be a low-ionization HII region or shock-heated gas; we advocate the HII region interpretation here. Peimbert & van den Bergh (1971) estimated an intrinsic emission measure $\sim 2500$ pc cm$^{-6}$, leading to an electron density $n_e \approx 15$ cm$^{-3}$ for a radius of 5.7 pc. The corresponding mass is several $100\ M_\odot$, which is too high for the material to be stellar mass loss. However, the emission is patchy, and we suggest that the emitting gas is in clumps with $n_e \sim 300$ cm$^{-3}$, reducing the mass by a factor $\sim 20$. The recombination time for the gas is $\sim 300$ years, so it is possible that it was ionized at the time of the supernova shock breakout; the current X-ray luminosity of Cas A is not sufficient to provide the ionization. As with the photoionized gas around SN 1987A (Lundqvist & Fransson 1996), the dominant density component that is observed is the densest one that has not already cooled and recombined. Peimbert & van den Bergh (1971) estimated that an energy in ionizing radiation of $1 \times 10^{50}$ ergs was needed to ionize the gas. This is larger than expected during shock breakout. With our reduced mass, the energy requirement drops to $\sim 5 \times 10^{48}$ ergs, which can be attained at shock breakout for an extended star with a low mass envelope (Matzner & McKee 1999).

In this scenario, the patchy HII region represents mass lost from the progenitor during a red supergiant phase. At 10 km s$^{-1}$, the wind can reach 6.9 pc (7′ at 3.4 kpc) in $7 \times 10^5$ yr. This is approximately the expected age of the red supergiant phase for a 15 – 20 $M_\odot$ mass star. The extended red supergiant wind is apparently also observed around SN 1987A, although by a dust echo in this case (Chevalier & Emmering 1989). The wind ends in a patchy shell of radius 4.5 pc. A termination shell might also be present around Cas A.

Knots in the preshock wind may have also been detected as HI compact absorption features towards Cas A (Reynoso et al. 1997); the knots have sizes < 0.1 pc and show spatial substructure. Their densities are presumably $\gtrsim 300$ cm$^{-3}$ so that the H can recombine. The knots have radial velocities in the range $-10$ to $-17$ km s$^{-1}$ relative to the systemic velocity of Cas A (Reynoso et al. 1997). In the present model, these velocities represent the velocity of the presupernova wind and are consistent with the wind of a red supergiant star.

In summary, we have shown that the dynamical properties of Cas A are consistent with interaction with the dense wind from a red supergiant progenitor star. The wind interaction supports a Type IIb or IIIn supernova designation, which is also indicated by the presence of high velocity hydrogen (Fesen & Becker 1991). The basic emission features of the supernova remnant, fast ejecta (fast moving knots), intermediate velocity shocks
(quasi-stationary flocculi), and narrow line emission (surrounding ionized clumps), have also been identified as emission features in Type II\textit{n} supernovae. The ability to spatially resolve the complex interaction in Cas A may provide a useful guide to the interpretation of the distant supernova emission. Finally, we note that some of the points of view advocated here, including interaction with a red supergiant wind and the possible importance of wind interaction for the formation of fast knots, have recently been discussed by Laming & Hwang (2003).

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Fig. 1.— The ratio of forward shock to reverse shock radius, \( r_f'/r_r' \), and the deceleration parameters for the forward shock, \( m_f \), and reverse shock, \( m_r \), as a function of scaled time. The hydrodynamic model is for a exploded star with a radiative envelope running into a wind with a \( \rho_w \propto r^{-2} \) density profile.
Fig. 2.— The density profile labeled ‘wind’ is the same model as in Fig. 1 shown when $r'_f/r'_r = 1.5$. The ‘ISM’ model has the same supernova model, but is running into a constant density medium. The density and radius are scaled to the values at the outer shock front. The value of $r'_f/r'_r$ is chosen to be close to that observed in Cas A.