A Geometrical Explanation for Repeating Fast Radio Bursts

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ABSTRACT

We propose a geometrical explanation for the periodically and nonperiodically repeating fast radio bursts (FRBs) under neutron star (NS)-companion system. We suggest a critical binary separation, \( r_c \), within which the interaction between the NS and its companion can trigger FRBs. On one hand, for an elliptic orbit with the minimum and maximum binary separations, \( r_{\text{min}} \) and \( r_{\text{max}} \) (satisfying the condition \( r_{\text{min}} < r_c < r_{\text{max}} \)), this binary system could reproduce periodical FRBs. A small separation \( r (\leq r_c) \) of such an elliptic orbit would result in an active period of periodical FRBs. On the other hand, if \( r_{\text{max}} < r_c \), the modulation of orbital motion will not work due to persistent interaction, and this kind of repeating FRBs should be nonperiodic. It is estimated that the critical separation \( r_c \sim 10^{12} \) cm for stellar binary system.

Keywords: binaries: general — neutron stars

1. INTRODUCTION

The origin of fast radio bursts (FRBs) is controversial (see Petroff et al. 2019 for review). From one-off bursts (Lorimer et al. 2007; Thornton et al. 2013) to nonperiodically repeating bursts (Spitler et al. 2016) \(^1\), and to periodically repeating bursts (The CHIME/FRB Collaboration et al. 2020; Chawla et al. 2020), observations continue to refresh the understanding of FRBs. More exhilaratingly, the periodic FRB 180916.J0158+65 (The CHIME/FRB Collaboration et al. 2020) indicates the source of FRBs may be either a neutron star (NS)-companion system or a precessing NS, since the sizes of nonrelativistically moving systems should be under \( \sim 10^7 \) cm as evident from FRB durations. For these three types of bursts with different repeatability, many models associated with NSs are proposed (e.g., Totani 2013; Lyubarsky 2014; Geng & Huang 2015; Dai et al. 2016; Zhang 2017; Kumar et al. 2017; Lyutikov et al. 2020; Yang & Zou 2020; Levin et al. 2020; Gu et al. 2020; Zanazzi & Lai 2020; Beniamini et al. 2020; Ioka & Zhang 2020). Even more, Dai & Zhong (2020) have already applied FRB 180916.J0158+65 to constrain the details on certain FRB model (e.g., the orbital configuration of pulsar-stellar-asteroid belt system). Inspired by the fact that long and short gamma-ray bursts originate from different sources but a similar system of “accretion disc + compact star”, in this Letter, we propose a general and geometrical frame about NS-companion systems to explain both the periodically and nonperiodically repeating FRBs without considering a specific generation mechanism of FRBs.

The remainder of this Letter is organized as follows. The geometry of orbit is illustrated in Section 2. The dynamic parameters are estimated in Section 3. Summary and discussion are presented in Section 4.

2. ORBITAL GEOMETRY

As shown in Figure 1, there is an NS-companion binary with an orbit period \( T \). We assume there is a critical binary separation \( r_c \) (corresponding to the polar angle \( \theta_c \)) under which the interaction between the NS and companion can trigger FRBs. The NS \( N_1 \) is approaching the companion \( N_2 \) from a distance. When the separation \( r \) is small enough, i.e., \( r < r_c < r_{\text{max}} \), the interaction which leads to FRBs occurs until the NS \( N_1 \) passes the critical position and get away from the companion \( N_2 \). Therefore, the duration \( \Delta T \) of the NS crossing the interacting region (shadow region) is just the active period of this periodic FRB. However, if \( r_{\text{max}} < r_c \), this binary are always in an interactive state, the FRB will not be modulated by the orbit period.

\(^1\) Recently, a possible period \( \sim 160 \) day of FRB 121102 is reported (Rajwade et al. 2020).
On the other hand, no rotation period of the NS is reported in these repeating FRBs. This indicates the radio emission of FRBs is not a 'lighthouse', i.e., the radiation is not come from the NS polar cap but from a position in the NS magnetosphere where is always can be seen by observers. Furthermore, the repeatability of FRBs during one orbit period should be mainly determined by the activeness of the companion. Once the condition $r_m < r_c$ satisfied, the repeatability of the nonperiodicly repeating FRBs will not depend on other geometric conditions. Therefore, we will only discuss the geometric conditions about periodically repeating FRBs in the next.

![Schematic diagram of the geometry of our model. $N_1$ is the NS and $N_2$ is the companion. $r_c$ is the critical binary separation (corresponding to the polar angle $\theta_c$) under which the interaction between the NS and companion can trigger FRBs. $r_{max}$ is the maximum binary separation. When the NS $N_1$ moves into the shaded region on the left, the binary begin to interact and FRBs are produced.](image)

3. PARAMETER ESTIMATION

With neglecting the gravitational radiation, the angular momentum of the NS binary

$$L = m_\mu r^2 \dot{\theta}$$

is constant, where $\theta$ is the polar angle, over dot represents time derivative, $m_\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass, and $m_1$, $m_2$ are the masses of the NS and the companion respectively. The elliptic orbital is

$$r = \frac{p}{1 + e \cos \theta},$$

where

$$p = \frac{L^2}{m_\mu \alpha}$$

with $\alpha = G m_1 m_2$, and $G$ is the gravitation constant, $e$ is the orbital ellipticity. Through Kepler’s Second Law, one can write equation (2) in another form that

$$r = \frac{4 \pi^2 m_\mu a^2 b^2}{\alpha T^2} \cdot \frac{1}{1 + e \cos \theta},$$

where $a$, $b$ are the semi-major axis and semi-minor axis respectively. To match the period $T$ and phase window $\Delta T$, there is

$$\frac{\Delta T}{T} \pi a b = \int_0^{\theta_c} r^2 d\theta.$$  

Integrating equation (5) gives

$$\frac{\Delta T}{T} \pi a b = \left( \frac{4 \pi^2 m_\mu a^2 b^2}{\alpha T^2} \right)^2 \left[ \frac{A + B}{AB \sqrt{AB}} \arctan\left( \sqrt{\frac{B}{A} \tan \frac{\theta}{2}} \right) - \frac{(A - B) \tan \frac{\theta}{2}}{AB \tan^2 \frac{\theta}{2} + A^2} \right]_{\theta_c}^{0}.$$  


with $A = 1 + e$ and $B = 1 - e$. Note that for an elliptic orbit, one has

$$a = \left( \frac{\alpha T^2}{4\pi^2 m_\mu} \right)^{1/3},$$

(7)

and

$$b = a\sqrt{1 - e^2}. \tag{8}$$

Therefore, for the given $m_1$, $m_2$, $T$, and $e$, one can solve $\theta_c$, as well as $r_c$, numerically through equation (6). The NS mass is $m_1 \sim 1.4 \, M_\odot$. But the value of $m_2$ is dependent on a certain physical model, for example, NS-NS binary, NS-white dwarf binary, and even NS-black hole system. Nevertheless, if the physical model is based on the geometric frame in this Letter, the value of $r_c$ derived from the physical model should be consistent with the value calculated under the geometric frame.

For clarity, we take $\Delta T/T = 1/8$, 1/4, 1/2 and $m_1 = m_2 = 1.4 \, M_\odot$. The values of $\theta_c$ and $r_c$ versus $e$ is shown in Figure 2.

![Figure 2](image-url)

**Figure 2.** The values of $\theta_c$ and $r_c$ versus $e$. The top three lines are $\theta_c$ versus $e$. The three lines at the bottom are $r_c$ versus $e$. The solid lines are corresponding to $\Delta T/T = 1/2$. The dashed lines are corresponding to $\Delta T/T = 1/4$. The dot-dashed lines are corresponding to $\Delta T/T = 1/8$

4. SUMMARY AND DISCUSSION

In this letter, we show a general geometrical frame (the NS-companion system with a critical binary separation) to explain the periodically and nonperiodically repeating FRBs. We find the critical binary separation is $r_c \sim 10^{12} \, \text{cm}$ if the source of FRB 180916.J0158+65 is a NS binary.

Under this geometric frame, there is an inevitable question that why we do not find any FRBs in Milky Way since many NS-companion systems (even with a period of few hours) in it. We may have two reasons for the no detection
if this geometric frame is proved to be correct, (i) the emission of FRBs is close to the angular momentum of these binaries, however the angular momentum of these binaries usually tends to be normal to the galactic disc; (ii) the conditions for coherent radiation are hard to be satisfied (suitable magnetic field, position and charge density, etc.).

The critical separation, \( r_c \), is the key parameter. The physical FRB models based on our frame should predict similar values of \( r_c \) to that of Figure 2. Especially, the companion for the periodic and nonperiodic FRBs may be different \(^2\), since the critical separation should be larger in the nonperiodic FRBs, i.e., \( r_c > r_m \). This demands the companions in the nonperiodic-FRB sources to be more energetic, such that the wind of the companion (e.g., Ioka & Zhang 2020) can affect the NSs. Alternatively, one can demand a huge asteroid belt as speculated by Smallwood et al. (2019).

One point that’s really worth mentioning is that if there is another critical radius, \( r'_c \) (e.g., the inner radius of the asteroid belt of the pulsar-asteroid belt impact model (Dai et al. 2016)), an extra blank region will exist in the shaded region of Figure 1. Therefore, at this point, even if a periodic FRB is in the active period, there still exists a halcyon period (also periodic) with shorter duration. This is a very important consequence that could be used to distinguish the pulsar-asteroid belt impact model from others (e.g., Lyutikov et al. 2020; Yang & Zou 2020; Levin et al. 2020; Gu et al. 2020; Zanazzi & Lai 2020; Beniamini et al. 2020; Ioka & Zhang 2020) as the latter have no such prediction.

We suggest to fold the period FRBs at their period just like that of The CHIME/FRB Collaboration et al. (2020). If a double-peaked profile of the number of bursts versus phase is find, one can conclude that the pulsar-asteroid belt impact model is correct. However, if the double-peaked profile is not shown in periodic FRBs even after enough samples have been observed, the pulsar-asteroid belt impact model should be ruled out.

In this Letter, we do not discuss the specific radiation mechanism of radio emission, since it depends on the unknown structure of NS magnetosphere and complicated magnetohydrodynamic processes. For completeness, we recommend the references Dai et al. (2016), Zhang (2017), and Wang et al. (2019). Although the details of radio radiation are unknown, this NS-companion frame can still be tested by finding the double-peaked profile and even by detecting the gravitational waves induced by the orbital inspiral (e.g., LISA (Amaro-Seoane et al. 2017), TianQin (Luo et al. 2016) and Taiji (Ruan et al. 2018)).

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\(^2\) Even applying to the possible longer-period FRB 121102 and shorter-period FRB 180916.J0158+65, since the longer-period FRB also needs a more energetic companion to maintain a larger \( r_c \) (This inference is supported by the very different rotation measures by Michilli et al. 2018 and The CHIME/FRB Collaboration et al. 2020).
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