Investigation of the effect of cylindrical insert devices on laminar convective heat transfer in channel flow by applying the Field Synergy Principle

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Abstract. The Field Synergy Principle is widely applied to the evaluation of the convective heat transfer mechanism. In fact, as highlighted in literature, the evaluation of the synergy between the velocity and the temperature gradient vectors could provide a better insight on the local convective heat transfer mechanism. In this paper, the field synergy approach is adopted to numerically investigate the fluid dynamic and thermal behaviour of a fully developed flow between parallel plates with asymmetric heating, when cylindrical inserts are present. To better evaluate the influence of the inserts on the convective heat transfer mechanism, different values of the insert diameter are considered, for a given pitch value. The numerical results in terms of Nusselt number point out that the convective heat transfer coefficient decreases as the insert diameter increases. The Field Synergy Principle allows to explain the cause of the convective heat transfer reduction identifying the regions in which the heat transfer mechanism is ineffective: the extent of these areas increases as the insert diameter increases.

1. Introduction

The heat transfer enhancement is one of the main scientific and technological challenge of the last decades; therefore, various kinds of technologies have been developed for improving the efficiency of heat transfer equipment. Since the duct shape significantly influences the flow behaviour, the most widespread techniques for enhancing the internal convective heat transfer that do not require external power (i.e. passive techniques) are based on the proper conformation of the duct wall [1].

To assess the performance of passive heat transfer enhancement techniques, many experimental and numerical methodologies have been proposed. While the experimental approach generally enables predicting the average performance of the enhanced heat transfer equipment [2-7], numerical investigations allow to evaluate the velocity and temperature fields in the duct, thus providing a useful tool to better understand the causes of the convective heat transfer augmentation [8-15]. Although the performance of enhanced heat transfer equipment can be properly assessed in terms of Nusselt number, Guo et al. [16] proposed a new method to evaluate the heat transfer enhancement, namely the Field Synergy Principle (FSP), which is based on the interaction between the velocity vector and the temperature gradient.
Their analysis reveals that the investigation of the synergy between the two vectors provides a better insight on the local convective heat transfer mechanism. The results presented in [16] point out that the convective heat transfer enhancement can be achieved by reducing the intersection angle between the velocity and the temperature gradient and by increasing simultaneously the local values of both fields.

Since the feasibility of the FSP has been confirmed in many works [17-26], several analyses of passive heat transfer enhancement techniques have been carried out by adopting this approach. In particular, by numerically investigating the heat transfer mechanism in several geometries (i.e. wavy channels, corrugated ducts, obliquely positioned plate array, circular tube with a coaxially inserted cylindrical bar and parallel plates with inserted square blockages), Tao et al. [17] observed that an enhanced heat transfer surface can reach the best performance for minimum average intersection angle between the velocity vector and the temperature gradient.

The same conclusions were highlighted by He et al. [20] who evaluated the influence of the Reynolds number and the geometric parameters on the dynamic and thermal behaviour of laminar flow in plain plate fin-and-tube heat exchangers.

Guo et al. [24] investigated the heat transfer augmentation in the curved channels by considering different boundary conditions for the thermal problem. Their numerical results confirm that the better synergy between the temperature gradient and the velocity vector corresponds to the higher convective heat transfer rate.

This trend was also confirmed by the analysis of the effect of the irregular boundary conditions on the convective heat transfer mechanism carried out by Zhu et al. [26] who analysed the influence of the irregular boundary conditions on the convective heat transfer coefficient, both in laminar and in turbulent flow regimes.

By applying the FSP to the investigation of heat transfer augmentation in a corrugated tube, Vocale et al. [27] pointed out that this approach allows to identify the zones that do not positively contribute to the convective heat transfer.

More recently, Vocale et al. [28] have presented an analytical investigation on the heat transfer mechanism in non-Newtonian fluids by applying the FSP approach. They highlighted that this approach provides a detailed analysis of the local interaction between temperature and velocity fields, by then offering a promising tool to investigate convective heat transfer in systems where velocity and boundary conditions are totally asymmetric.

The present work is focused on the numerical investigation of the influence of flow insertion devices on the fluid-flow and heat transfer phenomena in parallel plates with asymmetric heating. This geometry is often used in compact heat exchangers or in solar collector. For the purpose of the here presented analysis, cylindrical inserts are considered and their effects are evaluated from the viewpoint of the FSP, to better evaluate the local heat transfer enhancement.

2. Mathematical model

The geometry here investigated is characterized by a parallel plates channel with cylinders inserted along the flow direction, as shown in figure 1(a), being \( L \), \( H \) and \( W \) the channel length, height and width, respectively, \( D_c \) and \( P_c \) the cylinders diameter and pitch, respectively.

Since \( W \gg L \), the analysis has been carried out considering a two-dimensional parallel plates channel, as shown in Figure 1(b).

A Cartesian coordinate system \( x, y \) is introduced, being \( x \) the axial coordinate and \( y \) the perpendicular one. The centre of the cylinders is positioned on the channel axis (i.e. at \( y=H/2 \)), as depicted in figure 1(b).
By assuming that the fluid is Newtonian with constant physical properties, the flow is laminar and steady-state the governing equations in Cartesian coordinates are as follows:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) &= -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\end{align*}
\]

\[
\frac{\rho c_p}{\lambda} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\lambda}{D_h} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right]
\]

where \( u \) and \( v \) are the components of the velocity field, \( \rho, \mu, c_p \) and \( \lambda \) are the fluid density, dynamic viscosity, specific heat at constant pressure and thermal conductivity, respectively, \( p \) is the pressure and \( T \) the fluid temperature.

The governing equations have been solved by considering the following inflow boundary conditions at \( x=0 \):

\[
u = W, \quad v = 0, \quad T = T_w
\]

The boundary conditions for the fluid flow problem have been completed by the no-slip condition on any solid boundary (i.e. at the walls and on the cylinders surface) and by pressure condition at the channel outlet (i.e. \( x=L \)). Regarding the thermal problem, a prescribed and uniform heat flux \( q \) has been applied to the wall at \( y=H \) while the wall at \( y=0 \) has been considered to be adiabatic as well as the cylinders surface. Outflow boundary conditions has been applied at the channel outlet.

The influence of the inserted cylinders on the heat transfer mechanism can be evaluated in terms of the Nusselt number, which on the non-adiabatic wall is defined as follows:

\[
Nu = \frac{hD_h}{\lambda} = \frac{qD_h}{\lambda(T_w - T_b)}
\]

being \( h \) the convective heat transfer coefficient, \( D_h \) the channel hydraulic diameter, \( T_w \) and \( T_b \) the wall and bulk temperature averaged over a single representative module of the channel, respectively.
Although the performance of enhanced heat transfer equipment can be properly assessed by means of the Nusselt number, the Field Synergy Principle approach is a promising tool to investigate convective heat transfer, since it provides a detailed analysis of the local interaction between temperature and velocity fields [2].

In literature, it has been demonstrated that the convective heat transfer rate is proportional to the dot product of the velocity vector and the temperature gradient integrated over the entire domain [2,28].

In the here analyzed model, the dot product between the velocity vector and the temperature gradient reads as follows:

\[
\vec{v} \cdot \nabla T = U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} \tag{6}
\]

The dot product in Eq.(6) can be also defined as a function of the intersection angle between the two vectors:

\[
\vec{v} \cdot \nabla T = \|v\| \|\nabla T\| \cos \beta \tag{7}
\]

where \(\beta\) is the intersection angle between the velocity vector and the temperature gradient.

From Eq.(7) it can be deducted that to properly evaluate the dot product (i.e. the convective heat transfer phenomenon), it has to be considered not only the magnitude of the two vectors but also the value of the intersection angle between them, which can be evaluated as follows:

\[
\cos \beta = \frac{\vec{v} \cdot \nabla T}{\|v\| \|\nabla T\|} = \frac{U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y}}{\sqrt{U^2 + V^2} \sqrt{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2}} \tag{8}
\]

3. Results

Equations (1) to (3), together with their boundary conditions, have been numerically solved by adopting the finite element method, within the COMSOL Multiphysics environment. Both the governing equations have been solved using a direct linear solver and imposing a relative tolerance equal to 1e-06.

To evaluate the effects of inserted devices on the convective heat transfer three values of the cylinders diameter have been considered, namely \(D_c=0.25\) \(H\), \(D_c=0.375\) \(H\) and \(D_c=0.50\) \(H\).

The accuracy of the numerical results have been checked by testing different grid sizes for each value of the cylinders diameter considered in the present study. Due to the interest of the present investigation in the hydrodynamically and thermally fully developed flow, for each considered grid size, the value of Nusselt number in the fully developed region has been evaluated. Representative results of the mesh convergence analysis are shown in figure 2 where the Nusselt number is depicted as a function of the grid elements number \(N\) for \(D_c=0.50\) \(H\), together with the adopted mesh.

It has been observed that for a number of elements higher than 50000 the value of the Nusselt number becomes independent on the grid size, as shown in figure 2. Since this trend has been observed for all values of the cylinders diameter here considered, a mesh characterized by at least 50000 elements has been adopted in all numerical runs.

After having checked the accuracy of the numerical solution, several runs have been performed to evaluate the effects of the cylinders diameter on the dynamic and thermal fluid behaviour.
As above discussed, the here presented analysis is focused on hydrodynamically and thermally fully developed flow, therefore all the results are presented for the fully developed region.

The influence of the inserted cylinders on the flow pattern and temperature field is presented in Figure 3 where the streamlines and the dimensionless temperature contours for two representative values of the cylinders diameter (i.e. the lowest and the highest values considered in the present analysis) are depicted.

Concerning the flow pattern, due to the symmetric conditions and the low value of the Reynolds number (i.e. \( Re = 100 \)), the flow is characterized by a symmetric behaviour with respect to the channel symmetry axis. It has been also observed that for all values of the here considered cylinders diameter there is interference between two adjacent cylinders [8].

As expected, since only the upper wall is subjected to the heat flux, the fluid presents an asymmetric thermal behaviour, as shown in Figure 3(b) where the temperature contours (normalized with respect to the maximum and minimum values) are depicted.

Due to this behaviour, the interaction between the velocity vector and the temperature gradient is not easily predictable and the Field Synergy Principle approach becomes fundamental to understand heat transfer enhancement mechanisms.

By evaluating the influence of the inserted cylinders in terms of Nusselt number it has been observed that \( Nu \) decreases as the cylinders diameter increases. This effect becomes more evident for high values of the cylinders diameter; in fact, for \( D_c = 0.5H \) the heat transfer mechanism is less efficient than in the parallel plates channel without insertion (i.e. the reference case [29]), as shown in Table 1.

The FSP helps to understand this behaviour. In fact, while the Nusselt number definition is limited to the system’s boundary, the Field Synergy Principle conveys information representative of an effect distributed over the whole domain. Therefore, the analysis of the local interaction between the velocity vector and the temperature gradient could suggest suitable geometry modifications, which allow to enhance the convective heat transfer mechanism.

On the left side of Figure 4, the velocity vectors (red arrows) and the temperature gradients (blue arrows) in the non-dimensional form are depicted for all the values of the cylinders diameter here considered. It can be observed that, due to the non-symmetric boundary conditions, the two vectors are almost perpendicular over the whole domain. Smaller intersection angles are observable only in the region close to the adiabatic wall.

To better evaluate the influence of the cylinders on the dynamic and thermal fluid behaviour, in Figure 5 the velocity vectors and the temperature gradients for the reference case (i.e. the parallel plates channel without insertion) are reported.
Figure 3. Streamlines and dimensionless temperature contours: (a) \( D_c=0.25 \) \( H \); (b) \( D_c=0.50 \) \( H \).

Table 1. Nusselt number for the here analysed cases.

| \( D_c \) | \( Nu \) |
|----------|--------|
| 0.25 \( H \) | 5.984 |
| 0.375 \( H \) | 5.873 |
| 0.50 \( H \) | 5.407 |
| Reference case | 5.385 |

It has been observed that in the reference case the region in which the two vectors are perpendicular was higher than in cases where the cylinders are inserted in the flow direction. As demonstrated in literature \([16,17]\) the higher convective heat transfer coefficient corresponds to a lower value of the intersection angle between the vectors (i.e. the two vectors have to be aligned), therefore, the insertion of the cylinders in the streamwise direction enhances the convective heat transfer coefficient.

On the right side of Figures 4 and 5 the maps of the dimensionless dot product between the velocity vector and the temperature gradient (normalized with respect to the total thermal power and the hydraulic diameter of the channel) are also reported.
Figure 4. Dimensionless velocity vectors (red arrows), temperature gradients (blue arrows) and dot product:
(a) $D_c=0.25\ H$; (b) $D_c=0.375\ H$; (c) $D_c=0.50\ H$.

The maps in Figure 4 show that the region of the channel in which the dot product assumes negative values increases as the cylinders diameter increases. This result can explain the cause of the reduction in the convective heat transfer coefficient that has been observed by evaluating the performance of the heat transfer device in terms of Nusselt number.
Figure 5. Dimensionless velocity vectors (red arrows), temperature gradients (blue arrows) for the reference case (i.e. parallel plates).

Defining the region in which the dot product assumes positive values as effective heat transfer area \( e \) the other one (i.e. the region in which the dot product assumes negative values) as ineffective heat transfer area it is more clear what happens as the cylinders diameter increases. In Figure 6 the effective (red region) and the ineffective (blue region) heat transfer areas are depicted.

Figure 6. Effective (red regions) and ineffective (blue regions) heat transfer area: (a) \( D_c=0.25 \ H \); (b) \( D_c=0.375 \ H \); (c) \( D_c=0.50 \ H \).
It can be observed that with increasing cylinders diameter the effective heat transfer area decreases and the ineffective one increases, as shown in Table 2. Therefore, as the cylinders diameter increases the overall thermal performance of the whole system decreases.

|       | Effective [%] | Ineffective [%] |
|-------|---------------|-----------------|
| $D_c=0.25 H$ | 76.63         | 23.37           |
| $D_c=0.375 H$ | 67.20         | 32.80           |
| $D_c=0.50 H$ | 57.25         | 42.75           |

4. Conclusions

The dynamic and thermal behaviour of a fully developed flow in a channel bounded by two parallel plate walls heated asymmetrically and with cylindrical inserts has been numerically investigated. The influence of the cylinders on the flow pattern and the heat transfer mechanism has been evaluated by considering different values of the cylinders diameter.

The outcomes in terms of Nusselt number point out that the convective heat transfer coefficient decreases with increasing inserts diameter for a given pitch value. The causes of the convective heat transfer reduction have been explained by adopting the Field Synergy Principle approach, which provides a detailed analysis of the local interaction between the velocity and temperature gradient fields.

The numerical results, in terms of dot product between the two vectors, point out that the extent of the region in which the heat transfer mechanism is effective decreases as the cylinders diameter increases, thus reducing the convective heat transfer coefficient. Therefore, it can be concluded that the Field Synergy Principle approach could be a suitable tool to assess the performance of passive heat transfer enhancement techniques in internal flows, especially when the flows are subjected to asymmetric boundary conditions. Moreover, the analysis of the local interaction between the velocity vector and the temperature gradient could suggest suitable geometry modifications, which allow to enhance the convective heat transfer mechanism.

References

[1] Webb R L 1994 Principles of Enhanced Heat Transfer (New York: Taylor & Francis)
[2] Rainieri S and Pagliarini G 2002 Int. J. Heat Mass Tran. 45 4525–36
[3] Rainieri S, Bozzoli F and Pagliarini G 2012 Int. J. Heat Mass Tran. 55 498–504
[4] Garcia A, Solano J P, Vicente P G and Viedma A 2012 App. Therm. Eng. 35 196-201
[5] Rainieri S, Bozzoli F, Cattani L and Pagliarini G 2013 Int. J. Heat Mass Tran. 59 353–62
[6] Bozzoli F, Cattani L and Rainieri S 2016 Int. J. Heat Mass Tran. 101 76–90
[7] Harleß A, Franz E and Breuer M 2017 Int. J. Heat Mass Tran. 107 1076–84
[8] Kundu D, Haji-Sheikh A and Lou D Y S 1991 Numer. Heat Tr. A-App. 19 345-60
[9] Rainieri S, Bozzoli F, Schiavi L and Pagliarini G 2011 Int. J. Numer. Method H. 21 559–71
[10] Rainieri S, Bozzoli F, Mordacci M and Pagliarini G 2012 Heat Tran. Eng. 33 1120–29
[11] Zachár A 2010 Int. J. Heat Mass Tran. 53 3928–39
[12] Zheng N, Liu W, Liu Z, Liu P and Shan F 2015 App. Therm. Eng. 90 232-41
[13] Vocale P, Mocerino A, Bozzoli F and Rainieri S 2016 J. Phys. Conf. Ser. 745 032072
[14] Pagliarini G, Vocale P, Mocerino A and Rainieri S 2017 J. Phys. Conf. Ser. 796 012014
[15] Lin Z M, Wang L B, Lin M, Dang W and Zhang Y H 2017 App. Therm. Eng. 115 644-58
[16] Guo Z Y, Li D Y and Wang B X 1998 Int. J. Heat Mass Tran. 41 2221–25
[17] Tao W Q, Guo Z Y and Wang B X 2002 *Int. J. Heat Mass Tran.* **45** 3849–56
[18] Guo Z Y, Tao W Q and Shah R K 2005 *Int. J. Heat Mass Tran.* **48** 1797–1807
[19] Tao W Q, He Y L, Wang Q W, Qu Z G and Song F Q 2002 *Int. J. Heat Mass Tran.* **45** 4871–79
[20] He Y L, Tao W Q, Song F Q, Zang W 2005 *Int. J. Heat and Fluid Flow* **26** 459–473
[21] Ma L D, Li Z Y and Tao W Q 2007 *Int. Comm. Heat Mass Tran.* **34** 269–276
[22] Tao Y B, He Y L, Wu Z G and Tao W Q 2007 *Int. J. Heat and Fluid Flow* **28** 1531–44
[23] Wei L, Zhichun L, Tingzhen M and Zengyuan G 2009 *Int. J. Heat Mass Tran.* **52** 4669–72
[24] Guo J, Xu M and Cheng L 2011 *Int. J. Heat Mass Tran.* **54** 4148–51
[25] Hamid M O A, Zhang B and Yang L 2014 *Energy* **76** 241–253
[26] Zhu X W and Zhao J Q 2016 *Int. J. Heat Mass Tran.* **100** 347–54
[27] Vocale P, Mocerino A, Bozzoli F and Rainieri S 2017 *Proc. of 7th International Symposium on Advances in Computational Heat Transfer, CHT-17* (Napoli, 2017)
[28] Vocale P, Mocerino A, Bozzoli F and Rainieri S *Advances in Mathematical Physics* (in press)
[29] Bejan A 1993 *Heat Transfer* (New York: John Wiley & Sons, Inc.)