Measurement of the branching fraction, polarization, and $CP$ asymmetry in $B^0 \rightarrow \rho^+ \rho^-$ decays

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Abstract

We have measured the branching fraction, longitudinal polarization fraction $f_L$, and the CP asymmetry coefficients $A$ and $S$ in $B^0 \rightarrow \rho^+\rho^-$ decays. These results are obtained from a 253 fb$^{-1}$ data sample containing 275 million $B\bar{B}$ pairs collected by the Belle detector running at the KEKB $e^+e^-$ collider. We obtain $B = [24.4 \pm 2.2 \text{(stat)} ^{+3.8}_{-4.1} \text{(syst)}] \times 10^{-6}$, $f_L = 0.951 ^{+0.033}_{-0.039} \text{(stat)} ^{+0.029}_{-0.031} \text{(syst)}$, $A = 0.00 \pm 0.30 \text{(stat)} ^{+0.10}_{-0.09} \text{(syst)}$, and $S = 0.09 \pm 0.42 \text{(stat)} \pm 0.08 \text{(syst)}$. These values are used to determine the CKM phase angle $\phi_2$ via an isospin analysis; the central value and 1σ error are $(87 \pm 17)^\circ$, and $59^\circ < \phi_2 < 115^\circ$ at 90% CL.

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The decay $B^0 \rightarrow \rho^+ \rho^-$ has received much attention in the literature because it allows one to determine the CKM phase angle $\phi_2$\[1\] with relatively little theoretical uncertainty. This is because the penguin contribution to $B^0 \rightarrow \rho^+ \rho^-$ is small, as indicated by the small branching fraction for $B^0 \rightarrow \rho^+ \rho^-$\[2\]. The angle $\phi_2$ is determined by measuring the decay time ($\Delta t$) distribution of $B^0$ and $\bar{B}^0$ decays; the difference between the distributions is fit to the function $A\cos(\Delta m \Delta t) + S\sin(\Delta m \Delta t)$, where $\Delta m$ is the mass difference between the two $B^0$-$\bar{B}^0$ mass eigenstates, and the CP asymmetry coefficients $A$ and $S$ depend on $\phi_2$. To determine $\phi_2$ requires knowledge of the polarization of the $\rho$ mesons, as different polarization amplitudes can give different values of $A$ and $S$ for the same $\phi_2$. For a negligible penguin contribution, the sign of $S$ for the $CP$-odd transversity amplitude $A_\perp$ will be opposite to that for the $CP$-even amplitudes $A_0$ and $A_\parallel$. In this paper we present a measurement of the branching fraction, longitudinal polarization fraction ($f_L$), and coefficients $A$ and $S$ for $B^0 \rightarrow \rho^+ \rho^-$ decays.

The data sample consists of 253 fb$^{-1}$ recorded by the Belle experiment running at the KEKB energy-asymmetric $e^+e^-$ collider\[3\]. The Belle detector\[4\] consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals. These detectors are located within a superconducting solenoid coil providing a 1.5 T magnetic field. An iron flux-return located outside the coil is instrumented to identify muons and detect $K_L^0$ mesons. Two inner detector configurations were used: a 2.0 cm beampipe and a 3-layer silicon vertex detector were used for the first sample of 152 million $B\bar{B}$ pairs, and a 1.5 cm beampipe, a 4-layer silicon detector, and a small-cell inner drift chamber were used for the remaining 123 million $B\bar{B}$ pairs\[5\].

Candidate $B^0 \rightarrow \rho^+ \rho^-$, $\rho^\pm \rightarrow \pi^\pm \pi^0$ decays are selected by requiring two oppositely-charged tracks satisfying $p_T > 0.10$ GeV/$c$, $dr < 2.0$ mm, and $|dz| < 4.0$ cm, where $p_T$ is the momentum transverse to the beam axis, and $dr$ and $dz$ are the radial and longitudinal distances, respectively, between the track and the beam crossing point. The tracks are fit to a common vertex, and the resulting $\chi^2$ is required to be satisfactory. We require that the tracks be identified as pions based on information from the TOF and ACC systems, and from $dE/dx$ measurement in the CDC. This information is combined into a likelihood for a track to be a pion ($L_\pi$) or a kaon ($L_K$). To identify pions, we require that the ratio $L_K/(L_K + L_\pi)$ be less than 0.40. The efficiency of this requirement is approximately 89%, and the kaon misidentification rate is about 10%. Tracks are rejected if they satisfy an electron identification criterion based on information from the ECL system.

The $\pi^\pm$ candidates are subsequently combined with $\pi^0$ candidates. The latter are reconstructed from pairs of photons having an invariant mass in the range 0.1178–0.1502 GeV, which corresponds to $\pm 3\sigma$ in $\pi^0$ mass resolution. The energy of each photon in the laboratory frame must be $>50$ (90) MeV in the ECL barrel (endcap) region, which subtends the angular range with respect to the beamline of 32°–129° (17°–32° and 129°–150°). In order to reduce combinatorial background, $\pi^0$ candidates are required to have $p > 0.35$ GeV/$c$ in the $e^+e^-$ center-of-mass (CM) frame. To identify $\rho^\pm \rightarrow \pi^\pm \pi^0$ decays, we require that $m_{\pi^\pm \pi^0}$ be in the range 0.62–0.92 GeV/$c^2$; this window corresponds to $\pm 2\sigma$ in the $\pi^\pm \pi^0$ mass distribution. To further reduce combinatorial background, we require that each $\rho$ candidate satisfy $-0.80 < \cos \theta < 0.98$, where $\theta$ is the angle between the direction of the $\pi^0$ and the negative of the $B^0$ momentum in the $\rho^\pm$ rest frame.

To select $B^0 \rightarrow \rho^+ \rho^-$ decays, we calculate the quantities $M_{bc} \equiv \sqrt{E_{beam}^2 - p_B^2}$ and
\[ \Delta E \equiv E_B - E_{\text{beam}}, \] where \( E_B \) and \( p_B \) are the reconstructed energy and momentum of the \( B \) candidate, and \( E_{\text{beam}} \) is the beam energy, all evaluated in the CM frame. The \( \Delta E \) distribution has a long tail on the lower side due to incomplete containment of the electromagnetic shower in the ECL. We define a signal region \( 5.27 \, \text{GeV}/c^2 < M_{bc} < 5.29 \, \text{GeV}/c^2 \) and \(-0.12 \, \text{GeV} < \Delta E < 0.08 \, \text{GeV} \). The fraction of events having multiple candidates in this region is 9.5%. Many of these arise from fake \( \pi^0 \)'s combining with good tracks, and thus we choose the best candidate based on the mass difference \( |m_{\gamma\gamma} - m_{\pi^0}| \). From MC simulation we find that this criterion correctly identifies the \( B^0 \rightarrow \rho^+\rho^- \) decay 87\% (95\%) of the time for longitudinal (transverse) polarization. Signal decays that have one or more daughters incorrectly identified but nonetheless pass all selection requirements are referred to as “self-cross-feed” (SCF) background. Most such decays have the \( \pi^0 \) daughter swapped with a \( \pi^0 \) originating from the rest of the event; this preserves the vertex position and thus the \( \Delta t \) value.

We identify whether a \( B^0 \) or \( \bar{B}^0 \) evolved and decayed to \( \rho^+\rho^- \) by tagging the \( b \) flavor of the non-signal (opposite-side) \( B \) decay in the event. This is done using a tagging algorithm \( [\] that categorizes charged leptons, kaons, and \( \Lambda \)’s found in the event. The algorithm returns two parameters: \( q \), which equals \(+1\) (\(-1\)) when the opposite-side \( B \) is likely to be a \( B^0 \) (\( \bar{B}^0 \)); and \( r \), which represents the quality of the tag as determined from MC simulation and varies from \( r=0 \) for no flavor discrimination to \( r=1 \) for unambiguous flavor assignment.

The dominant background is \( e^+e^- \rightarrow q\bar{q} \) \((q = u, d, s, c)\) continuum production. We discriminate against this using event topology: \( e^+e^- \rightarrow q\bar{q} \) processes in the CM frame tend to be jet-like, while \( e^+e^- \rightarrow B\bar{B} \) events tend to be spherical. To quantify this sphericity, we calculate a set of 16 modified Fox-Wolfram moments and combine them into a Fisher discriminant \( [\] . We calculate a probability density function (PDF) based on this discriminant and multiply it by a PDF for \( \cos\theta_B \), where \( \theta_B \) is the polar angle in the CM frame between the \( B \) direction and the positron beam direction. \( B\bar{B} \) events are produced with a \( 1 - \cos^2\theta_B \) distribution while continuum events are produced uniformly in \( \cos\theta_B \). The PDFs for signal and continuum background are obtained using MC simulation and the data sideband \( 5.21 \, \text{GeV}/c^2 < M_{bc} < 5.26 \, \text{GeV}/c^2 \), respectively. We use the products of the PDFs to calculate a signal likelihood \( \mathcal{L}_s \) and continuum likelihood \( \mathcal{L}_q\bar{q} \) and require that \( \mathcal{R} = \mathcal{L}_s/(\mathcal{L}_s + \mathcal{L}_q\bar{q}) \) be above a threshold. This threshold is determined by optimizing a figure-of-merit \( N_S/\sqrt{N_S + N_B} \), where \( N_S \) (\( N_B \)) is the number of signal (background) events estimated to be in the signal region from MC simulation (extrapolating from an \( M_{bc} \) sideband). For this calculation we assume a \( B^0 \rightarrow \rho^+\rho^- \) branching fraction of \( 25 \times 10^{-6} \). As the tagging parameter \( r \) also discriminates against \( q\bar{q} \) background, we divide the data into six \( r \) intervals (denoted by \( \ell = 1 - 6 \)) and determine the \( \mathcal{R} \) threshold separately for each. The \( \mathcal{R} \) requirement removes about \( 97\% \) of continuum events while retaining about \( 62\% \) of \( B^0 \rightarrow \rho^+\rho^- \) signal. The efficiency of all selection criteria as determined from MC simulation is \((2.19 \pm 0.02)\%\). This value corresponds to \( f_L = 1 \); the efficiency for our measured value of \( f_L \) (which is slightly less than one – see below) is about \( 4\% \) higher, and we include this difference as an additional systematic error in the branching fraction.

We determine the signal yield in two steps: we first fit the \( M_{bc}-\Delta E \) distribution to obtain the fraction of \( B^0 \rightarrow \rho^+\rho^- + B^0 \rightarrow \rho^+\pi^+\pi^0 \) non-resonant decays (as this fit cannot distinguish between the two modes); we then fit the \( m_{\pi^+\pi^0} \) distribution to determine the non-resonant fraction and hence the \( \rho^+\rho^- \) signal yield.

The first fit is an unbinned maximum likelihood (ML) fit to the two-dimensional \( M_{bc}-\Delta E \) distribution in the wide range \( 5.21 \, \text{GeV}/c^2 < M_{bc} < 5.29 \, \text{GeV}/c^2 \) and \(-0.20 \, \text{GeV} < \Delta E < 20 \, \text{GeV}/c^2 \).
0.30 GeV. The fit includes components for the signal and several backgrounds: continuum events, \( b \rightarrow c \) decays, and \( b \rightarrow u \) decays. The PDFs for signal and \( b \rightarrow u \) background are modeled by smoothed two-dimensional histograms obtained from large MC samples. The PDF for \( b \rightarrow c \) background is the product of a threshold ("ARGUS" \( \tilde{M} \)) function for \( M_{bc} \) and a quadratic polynomial for \( \Delta E \), also obtained from MC simulation. The PDF for continuum background is taken to be an ARGUS function for \( M_{bc} \) and a linear function for \( \Delta E \); the slope of the linear function depends on the tag quality \((r)\) bin \( \ell \), and all seven shape parameters are floated in the fit. The signal PDF is adjusted to account for small differences observed between data and MC for a high-statistics sample of \( B^+ \rightarrow \Upsilon^0 \rho^+, \Upsilon^0 \rightarrow K^+\pi^-\pi^0 \) decays, which, like the signal mode, contain two neutral pions.

The \( b \rightarrow u \) background is dominated by \( B^{(0,+)} \rightarrow \rho \pi, B \rightarrow a_1 \pi \), and \( B \rightarrow a_1 \rho \) decays. As their contributions are small, their normalization is fixed to the yield obtained from MC simulation. For \( B^+ \rightarrow a_1^+\pi^0 \) and \( B \rightarrow a_1 \rho \), the branching fractions are unknown; we therefore estimate them to be \( 3 \times 10^{-6} \) and \( 2 \times 10^{-6} \), respectively, and vary these values by \( 50\% \) and \( 100\% \), respectively, to obtain the systematic error due to these estimates. There are thus nine free parameters in the fit, and the resulting fraction of \( B_{\text{simulation}} \) for \( \Delta E \) tends to zero. The PDFs for continuum and \( b \rightarrow c \) backgrounds are grouped together and taken from the data sideband \( 5.22 \text{ GeV}/c^2 < M_{bc} < 5.26 \text{ GeV}/c^2 \); we use MC simulation to check that the shapes of these backgrounds and their ratio in the sideband region are very close to those in the signal region. We impose the constraint that the fraction of signal + non-resonant events in the \( m_{\pi^+\pi^-\pi^0} \) range \( 0.62-0.92 \text{ GeV}/c^2 \) equals that which we obtained from the \( M_{bc}-\Delta E \) fit; there is then only one free parameter in the fit \((f_{\rho\pi\pi})\). The result of the fit is \( f_{\rho\pi\pi} = (8.9 \pm 6.2)\% \), and thus \( N_{\rho\rho} = 142 \pm 13 \) events. Figure 2 shows the data (two entries per event) and the projection of the fit result. The \( \chi^2 \) of the projections divided by the number of bins is 0.97 for \( M_{bc} \) and 1.02 for \( \Delta E \).

To distinguish \( B^0 \rightarrow \rho^+\rho^- \) decays from non-resonant \( B^0 \rightarrow \rho^+\pi^+\pi^-\pi^0 \) and \( B^0 \rightarrow \pi^+\pi^-\pi^0 \) decays, we do an unbinned ML fit to the \( \pi^+\pi^-\pi^0 \) mass distribution. For this fit we require \( 5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2 \) and \( -0.12 \text{ GeV} < \Delta E < 0.08 \text{ GeV} \) and fit the \( m_{\pi^+\pi^-\pi^0} \) distribution in the range \( 0.30-1.80 \text{ GeV}/c^2 \). Only one \( \rho \) candidate is required to satisfy \( 0.62 \text{ GeV}/c^2 < m_{\pi^+\pi^-\pi^0} < 0.92 \text{ GeV}/c^2 \); the mass of the other \( \rho \) candidate is then fit. We include additional PDFs for non-resonant \( B \rightarrow \rho\pi\pi \) and \( B \rightarrow \pi\pi\pi \) components; however, the fit result for the latter is negligibly small \((\ll 1\%) \) and consistent with zero, and thus we set this fraction to zero. The PDFs for continuum and \( b \rightarrow c \) backgrounds are grouped together and taken from the data sideband \( 5.22 \text{ GeV}/c^2 < M_{bc} < 5.26 \text{ GeV}/c^2 \); we use MC simulation to check that the shapes of these backgrounds and their ratio in the sideband region are very close to those in the signal region. We impose the constraint that the fraction of signal + non-resonant events in the \( m_{\pi^+\pi^-\pi^0} \) range \( 0.62-0.92 \text{ GeV}/c^2 \) equals that which we obtained from the \( M_{bc}-\Delta E \) fit; there is then only one free parameter in the fit \((f_{\rho\pi\pi})\). The result of the fit is \( f_{\rho\pi\pi} = (8.9 \pm 6.2)\% \), and thus \( N_{\rho\rho} = 142 \pm 13 \) events. Figure 2 shows the data (two entries per event) and the projection of the fit result. The \( \chi^2 \) of the projections divided by the number of bins is 1.35.

The branching fraction is evaluated as

\[
B(B^0 \rightarrow \rho^+\rho^-) = \frac{N_{\rho\rho}}{\varepsilon \cdot \varepsilon_{\text{PID}} \cdot N_{B\overline{B}}},
\]

where \( N_{\rho\rho} \) is the number of \( B^0 \rightarrow \rho^+\rho^- \) candidates \((142 \pm 13)\), \( N_{B\overline{B}} \) is the number of \( B\overline{B} \) pairs produced \((274.8 \pm 2.3) \times 10^6\), \( \varepsilon \) is the geometric acceptance and efficiency of selection criteria obtained from MC simulation \((2.19 \pm 0.02)\%)\), and \( \varepsilon_{\text{PID}} \) is a correction factor for the pion identification requirement to account for small differences between data and MC \((0.969 \pm 0.012)\). Inserting these values into Eq. (1) gives a branching fraction of \((24.4 \pm 2.2) \times 10^{-6} \), where the error given is statistical.

There are several sources of systematic uncertainty; their effect is evaluated by varying the relevant parameter(s) by \( \pm 1\sigma \) and noting the resulting change in the branching fraction.
The different sources and the corresponding changes are: track reconstruction efficiency (1.2% per track); $\pi^0$ reconstruction efficiency (4% per $\pi^0$); the $\pi^\pm$ identification efficiency $\varepsilon_{\text{PID}}$ in Eq. (1) (1.4%); MC statistics used to calculate the acceptance $\varepsilon$ in Eq. (1) (1.0%); the dependence of the acceptance upon $f_L$ (+0.0%, −4.1%); the continuum suppression requirement (13%); and the fraction of signal + non-resonant decays used as a constraint in the $m_{\pi\pi\pi}$ fit. This last error itself has several components: the calibration factors used to correct the $M_{bc}$-$\Delta E$ distribution of MC $B^0 \rightarrow \rho^+\rho^-$ events (to better match the data); the $M_{bc}$-$\Delta E$ shapes used for $b \rightarrow c$ background; the fraction of $b \rightarrow u$ background, which is obtained from MC simulation and includes several estimated branching fractions; and the $\Delta E$ range used in the fit. These components give an overall variation in $f_{\rho\rho} + f_{\rho\pi\pi}$ of (+2.9%, −8.6%). We subsequently redo the $m_{\pi\pi\pi}$ fit, varying $f_{\rho\rho} + f_{\rho\pi\pi}$ by this amount; the
FIG. 2: The $m_{\pi^+\pi^0}$ plus $m_{\pi^-\pi^0}$ distribution (two entries per event) for events satisfying 5.27 GeV/$c^2 < M_{bc} < 5.29$ GeV/$c^2$ and $-0.12$ GeV $< \Delta E < 0.08$ GeV. Superimposed are projections of the ML fit result. The dashed curve represents $\rho^+\rho^-$ signal; the dot-dashed curve represents non-resonant $\rho\pi\pi$ decays; the dotted curve represents $q\bar{q} + (b\to c) + (b\to u)$ backgrounds; and the solid curve shows the overall result.

resulting change in $N_{\rho\rho}$ is $(+2.0\%, -6.4\%)$. Combining this in quadrature with the other systematic errors gives an overall systematic error of $(+16\%, -17\%)$. Thus the final result for the branching fraction is

$$B(B^0 \to \rho^+\rho^-) = \left[ 24.4 \pm 2.2 \text{ (stat)} ^{+3.8}_{-4.1} \text{ (syst)} \right] \times 10^{-6}. \quad (2)$$

To determine the polarization of $B^0 \to \rho^+\rho^-$ decays, we do an unbinned ML fit to the helicity angle distribution $F(\cos \theta_1, \cos \theta_2)$, where $\theta_1$ ($\theta_2$) is the angle between the direction of the $\pi^0$ from the $\rho^+$ ($\rho^-$) and the negative of the $B^0$ momentum in the $\rho^+$ ($\rho^-$) rest frame. For a longitudinal polarization fraction $f_L$, this distribution is

$$\frac{9}{4} \left[ f_L \cos^2 \theta_1 \cos^2 \theta_2 + \frac{1}{4}(1 - f_L) \sin^2 \theta_1 \sin^2 \theta_2 \right]. \quad (3)$$

In the fit, this PDF is multiplied by a two-dimensional acceptance function for $(\cos \theta_1, \cos \theta_2)$ determined from MC simulation. The acceptance is modeled as the product $A(\cos \theta_1) \cdot A(\cos \theta_2)$, where $A$ is a polynomial function.

We fit events satisfying $5.27$ GeV/$c^2 < M_{bc} < 5.29$ GeV/$c^2$, $-0.12$ GeV $< \Delta E < 0.08$ GeV, and $0.62$ GeV/$c^2 < m_{\pi^+\pi^0} < 0.92$ GeV/$c^2$. The likelihood function includes PDFs for signal, $\rho\pi\pi$ non-resonant decays, and continuum, $b\to c$, and $b\to u$ backgrounds. The PDF for non-resonant decays is taken to be constant, and the PDF for $b\to u$ background is taken from MC
FIG. 3: The $\cos \theta_1$ plus $\cos \theta_2$ distribution (two entries per event) for events satisfying $5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2$, $-0.12 \text{ GeV} < \Delta E < 0.08 \text{ GeV}$, and $0.62 \text{ GeV}/c^2 < m_{\pi^{\pm}\pi^0} < 0.92 \text{ GeV}/c^2$. Superimposed are projections of the ML fit result. The dashed curve represents $\rho^+\rho^-$ signal; the dotted curve represents non-resonant $\rho\pi\pi$ decays and $q\bar{q} + (b \rightarrow c) + (b \rightarrow u)$ backgrounds, and the solid curve shows the overall result.

Simulation. The PDFs for continuum and $b \rightarrow c$ backgrounds are combined and determined from the data sideband $5.21 \text{ GeV}/c^2 < M_{bc} < 5.26 \text{ GeV}/c^2$, $-0.12 \text{ GeV} < \Delta E < 0.12 \text{ GeV}$; we use MC simulation to check that the shapes of these backgrounds and their ratio in the sideband region are very close those in the signal region. The fraction of signal + non-resonant decays is taken from the previous $M_{bc}-\Delta E$ fit; the component $f_{\rho\pi\pi}$ is taken from the previous $m_{\pi^{\pm}\pi^0}$ fit. The fraction of $b \rightarrow u$ background is small and is taken from MC simulation. Since $f_{(q\bar{q} + b \rightarrow c)} = 1 - f_{\rho\rho} - f_{\rho\pi\pi} - f_{b \rightarrow u}$, there is only one free parameter in the fit ($f_L$). The result of the fit is $f_L = 0.951^{+0.033}_{-0.039}$, where the error given is statistical. Figure 3 shows the data and projections of the fit result. The $\chi^2$ of the fit projections divided by the number of bins is 0.83 for $\cos \theta_1$ and 1.05 for $\cos \theta_2$.

For this measurement there are eight sources of systematic error; their effect is evaluated by varying the relevant parameter(s) by $\pm 1\sigma$ and noting the resulting change in $f_L$. The different sources and the corresponding changes are: the signal + non-resonant fraction ($+0.013$, $-0.012$); the non-resonant fraction ($+0.021$, $-0.020$); the pion identification efficiency, which has a small effect upon the acceptance ($+0.000$, $-0.004$); misreconstructed $B^0 \rightarrow \rho^+\rho^-$ decays ($+0.005$, $-0.000$); the continuum suppression requirement ($\pm 0.013$); possible interference with an $L = 0 \pi^{\pm}\pi^0$ system produced in non-resonant $\rho^{\pm}\pi^{\mp}\pi^0$ decays ($\pm 0.005$); a very small bias in the fitting procedure measured from a large toy MC sample ($+0.000$, $-0.005$); and uncertainty in the continuum $+(b \rightarrow c)$ background shape ($+0.004$, $-0.014$). This last uncertainty is evaluated by taking the background shape from
alternative $M_{bc}$ and $\Delta E$ sidebands. Combining all systematic errors in quadrature gives an overall systematic error of $(+0.029, -0.031)$. Thus the final result for the fraction of longitudinally polarized decays is

$$f_L = 0.951^{+0.033}_{-0.039} \text{(stat)} ^{+0.029}_{-0.031} \text{(syst)}. \quad (4)$$

To fit for the $CP$ asymmetry coefficients $A$ and $S$, we divide the data into $q = +1$ and $q = -1$ tagged decays and do an unbinned ML fit to the respective $\Delta t$ distributions. At KEKB, the $\Upsilon(4S)$ is produced with a Lorentz boost of $\beta\gamma = 0.425$ nearly along the electron beamline ($z$); since the $B^0$ and $\bar{B}^0$ mesons are approximately at rest in the $e^+e^-$ CM frame, $\Delta t$ is determined from the displacement in $z$ between the $f_{CP}$ and $f_{\text{tag}}$ decay vertices: $\Delta t \approx (z_{CP} - z_{\text{tag}})/\beta\gamma c$.

The likelihood function for event $i$ is

$$L_i = f_{\rho\rho}^{(i)} \mathcal{P}(\Delta t)_{\rho\rho} + f_{\text{SCF}}^{(i)} \mathcal{P}(\Delta t)_{\text{SCF}} + f_{\rho\pi\pi}^{(i)} \mathcal{P}(\Delta t)_{\rho\pi\pi} + f_{b\rightarrow c}^{(i)} \mathcal{P}(\Delta t)_{b\rightarrow c} + f_{b\rightarrow u}^{(i)} \mathcal{P}(\Delta t)_{b\rightarrow u} + f_{q\bar{q}}^{(i)} \mathcal{P}(\Delta t)_{q\bar{q}}, \quad (5)$$

where the event weights $f^{(i)}$ are functions of $M_{bc}$ and $\Delta E$ and are normalized to the fractions of events obtained from the previous $M_{bc}$-$\Delta E$ and $m_{\pi^{\pm}\rho^0}$ fits. The PDFs $\mathcal{P}(\Delta t)$ for $b \rightarrow c$ and $b \rightarrow u$ backgrounds are determined from MC simulation, and the PDF for continuum $q\bar{q}$ background is determined from an $M_{bc}$ sideband. We include an additional PDF for SCF background in which a $\pi^\pm$ daughter is swapped with a track from the rest of the event; this function is determined from MC simulation and is found to be exponential with an effective lifetime of about 1 ps. The weighting function $f_{\text{SCF}}$ is also determined from MC simulation; its normalization is 5.7% of all $\rho^+\rho^-$ candidates.

The PDF $\mathcal{P}_{\rho\rho}(\Delta t)$ is given by

$$\int_{-\infty}^{\infty} e^{-|\Delta t'|/\tau_{B^0}} \left\{ 1 - q \Delta \omega_{\ell(i)} + q(1 - 2\omega_{\ell(i)}) \times \left[ A \cos(\Delta m \Delta t') + S \sin(\Delta m \Delta t') \right] \right\} R(\Delta t^{(i)}, \Delta t') \, d\Delta t',$$  

where $R(\Delta t, \Delta t')$ is a resolution function determined from data, $\omega_{\ell}$ is the mistag probability for the $\ell$th bin of the tagging parameter $r$, and $\Delta \omega_{\ell}$ is a possible difference in $\omega_{\ell}$ between $q = +1$ and $q = -1$ tags. The PDF $\mathcal{P}_{\rho\pi\pi}$ is taken to be exponential with $\tau = \tau_B$ and is smeared by the same resolution function $R$. We determine $A$ and $S$ by maximizing the log-likelihood $\sum_i \log L_i$, where $i$ runs over the 656 candidate events satisfying $5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2$, $-0.12 \text{ GeV} < \Delta E < 0.08 \text{ GeV}$, and $0.62 \text{ GeV}/c^2 < m_{\pi^{\pm}\rho^0} < 0.92 \text{ GeV}/c^2$. The results of the fit are $A = 0.00 \pm 0.30$ and $S = 0.09 \pm 0.42$, where the errors given are statistical. These values are consistent with the no-$CP$-violation case $A = S = 0$, and the errors are consistent with expectations based on MC studies. Figure 4 shows the data and projections of the fit result for $0.0 < r < 0.5$ and $0.5 < r < 1.0$ subsamples; the $\chi^2$ of the projections divided by the number of bins is 0.98 and 0.97, respectively, which is satisfactory.

For this measurement there are twelve sources of systematic error; their effect is evaluated by varying the relevant parameter(s) by $\pm 1\sigma$ and noting the resulting change in $A$ and $S$. The different sources and the corresponding changes are listed in Table I. The error due to wrong-tag fractions is evaluated by varying $\omega_{\ell}$ and $\Delta \omega_{\ell}$ values. The error due to $b \rightarrow c$ and continuum $\Delta t$ distributions is evaluated by varying the $\sim 40$ parameters used to describe
FIG. 4: The $\Delta t$ distribution for events satisfying $5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2$, $-0.12 \text{ GeV} < \Delta E < 0.08 \text{ GeV}$, and $0.62 \text{ GeV}/c^2 < m_{\pi^\pm \pi^0} < 0.92 \text{ GeV}/c^2$. Superimposed are projections of the ML fit result. (a) $q = +1$ tags; (b) $q = -1$ tags; (c) raw $CP$ asymmetry for $0.0 < r < 0.5$; (d) raw $CP$ asymmetry for $0.5 < r < 1.0$. For (a) and (b), the hatched distribution represents signal events and the dashed curve represents background.

The error due to the component fractions is evaluated by varying the fractions obtained from the $M_{bc}$-$\Delta E$ fit. The error due to self-cross-feed (SCF) events is evaluated by varying both the SCF fraction (by 50%) and the effective lifetime used for SCF events (by 2$\sigma$). The effect of a $CP$ asymmetry in $b \to c$ and $q\bar{q}$ backgrounds is evaluated by adding such asymmetries to the $b \to c$ and $q\bar{q}$ $\Delta t$ distributions; the sizes of these asymmetries are taken to be those obtained from fitting an $M_{bc}$ sideband in the data (these values are small and consistent with zero). The error due to a possible fitting bias is evaluated using a full MC simulation, and the error due to tag-side interference is evaluated using a toy MC simulation. The error due to the transversity amplitudes $A_\perp$ and $A_\parallel$ (which may have
different values of $A$ and $S$) is evaluated in two steps: first setting $f_L$ equal to its central value and varying the values of $(A, S)$ for $A_\perp$ and $A_\parallel$ over the range $-1$ to $+1$; second, using common values of $(A, S)$ for all transversity amplitudes (except for an extra minus sign for $A_\perp$) and varying $f_L$. Combining all systematic errors in quadrature gives overall systematic errors of $(+0.10, -0.09)$ for $A$ and $\pm 0.08$ for $S$. Thus the final result is

$$A = 0.00 \pm 0.30 \text{(stat)}^{+0.10}_{-0.09} \text{(syst)} \quad (7)$$

$$S = 0.09 \pm 0.42 \text{(stat)} \pm 0.08 \text{(syst)}. \quad (8)$$

TABLE I: Systematic errors for the $CP$ asymmetry coefficients $A$ and $S$. 

| Type                        | $\delta A \times 10^{-2}$ | $\delta S \times 10^{-2}$ |
|-----------------------------|-----------------------------|-----------------------------|
| Wrong tag fractions         | $+\sigma$ 0.5 0.6 0.8 0.8  | $-\sigma$ 0.1 0.1 0.9 0.9  |
| Parameters $\Delta m, \tau_{\rho\rho}$ | $+\sigma$ 1.3 1.3 1.3 1.3  | $-\sigma$ 1.6 1.5 2.3 2.5  |
| Resolution function         | $+\sigma$ 2.1 2.6 5.0 4.5  | $-\sigma$ 0.0 0.0 0.6 0.6  |
| Background $\Delta t$       | $+\sigma$ 0.0 2.0 0.0 4.3  | $-\sigma$ 0.0 0.2 0.6 0.0  |
| Component fractions         | $+\sigma$ 5.2 4.5 6.0 4.7  | $-\sigma$ 4.1 2.8 0.4 0.4  |
| $\rho\pi\pi$ non-resonant fraction | $+\sigma$ 0.0 0.0 0.6 0.6  | $-\sigma$ 0.0 0.0 0.2 0.2  |
| Self-cross-feed (SCF) fraction | $+\sigma$ 0.0 2.0 0.0 4.3  | $-\sigma$ 0.0 0.2 0.6 0.0  |
| Background asymmetry        | $+\sigma$ 7.2 5.5 0.3 0.3  | $-\sigma$ 7.2 5.5 0.3 0.3  |
| Possible fitting bias        | $+\sigma$ 7.2 5.5 0.3 0.3  | $-\sigma$ 7.2 5.5 0.3 0.3  |
| Vertexing                   | $+\sigma$ 7.2 5.5 0.3 0.3  | $-\sigma$ 7.2 5.5 0.3 0.3  |
| $f_\perp, f_\parallel$ (transversely-polarized components) | $+\sigma$ 5.2 4.5 6.0 4.7  | $-\sigma$ 4.1 2.8 0.4 0.4  |
| **Total**                   | $+\sigma$ 10.2 8.6 8.4 8.4  | $-\sigma$ 10.2 8.6 8.4 8.4  |

We use these values along with the measured branching fraction for $B^0 \rightarrow \rho^+\rho^-$ and previously-measured branching fractions for $B^+ \rightarrow \rho^+\rho^0$ $^{[10]}$ and $B^0 \rightarrow \rho^0\rho^0$ $^{[2]}$ to constrain the angle $\phi_2$. We use the isospin relations of Ref. $^{[11]}$ (originally applied to the $B \rightarrow \pi\pi$ system), neglecting a possible $I=1$ contribution to the $B^0 \rightarrow \rho^+\rho^-$ amplitude $^{[12]}$. We first fit the measurements to obtain a minimum $\chi^2$; this is denoted $\chi^2_{\text{min}}$. We then scan values of $\phi_2$ from $0^\circ$–$180^\circ$ and for each value calculate the difference $\Delta \chi^2 \equiv \chi^2(\phi_2) - \chi^2_{\text{min}}$. We insert $\Delta \chi^2$ into the cumulative distribution function for the $\chi^2$ distribution for one degree of freedom to obtain the confidence level (CL) for the $\phi_2$ value. The curve 1–CL for all $\phi_2$ values is plotted in Fig. 5. From this curve we obtain a central value and $1\sigma$ error $\phi_2 = (87 \pm 17)^\circ$; the 90% CL interval around the central value is $59^\circ < \phi_2 < 115^\circ$.

In summary, we have measured the branching fraction, longitudinal polarization fraction $f_L$, and $CP$ asymmetry coefficients $A$ and $S$ for $B^0 \rightarrow \rho^+\rho^-$ decays. Our results are $B = 2.4 \pm 2.2^{+3.8}_{-4.1} \times 10^{-6}$, $f_L = 0.951^{+0.033}_{-0.039}^{+0.039}_{-0.031}$, $A = 0.00 \pm 0.30^{+0.10}_{-0.09}$, and $S = 0.09 \pm 0.42 \pm 0.08$, where the first error listed is statistical and the second error listed is systematic. These
FIG. 5: Results of fitting the branching fractions $B(B^0 \to \rho^+ \rho^-)$, $B(B^+ \to \rho^+ \rho^0)$, $B(B^0 \to \rho^0 \rho^0)$, and the $\rho^+ \rho^-$ asymmetry parameters $A$ and $S$ for the angle $\phi_2$, using the isospin relations of Ref. [11]. The vertical axis is one minus the confidence level (see text). The horizontal line at $1 - CL = 0.317$ corresponds to a 68.3% CL interval for $\phi_2$, and the horizontal line at $1 - CL = 0.10$ corresponds to a 90% CL interval.

results are consistent with previously-published measurements [13]. The result for $f_L$ is consistent with the theoretical expectation based on QCD factorization [14], and the results for $A$ and $S$ are consistent with the case of no $CP$ violation. From an isospin analysis we obtain the constraint $59^\circ < \phi_2 < 115^\circ$ at 90% CL.

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The naming convention $\alpha (= \phi_2)$, $\beta (= \phi_1)$, and $\gamma (= \phi_3)$ is also used in the literature.

B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 94, 131801 (2005).

S. Kurokawa and E. Kikutani, Nucl. Instr. and Meth. A 499, 1 (2003), and other papers included in this volume.

A. Abashian et al. (Belle Collaboration), Nucl. Instr. and Meth. A 479, 117 (2002).

Y. Ushiroda (Belle SVD2 Group), Nucl. Instr. and Meth. A 511, 6 (2003).

H. Kakuno et al., Nucl. Instr. and Meth. A 533, 516 (2004).

S. H. Lee et al. (Belle Collaboration), Phys. Rev. Lett. 91, 261801 (2003).

H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. B 241, 278 (1990).

O. Long et al., Phys. Rev. D 68, 034010 (2003).

S. Eidelman et al. (PDG), Phys. Lett. B 592, 1 (2004).

M. Gronau and D. London, Phys. Rev. Lett. 65, 3381 (1990).

A. Falk et al., Phys. Rev. D 69, 011502(R) (2004).

B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 93, 231801 (2004).

A. Kagan, Phys. Lett. B 601, 151 (2004).