Standing balance maintenance by virtual suspension model control for legged robot

Junwen Cui, Zhan Li, Yiqun Kuang and Hong Cheng

Abstract

Legged robots demand to keep the balance under required durations when performing standing and locomotion. It may be a challenge for legged robots to maintain balance during standing without actuation from ankle joints and involvement of support polygon plane to estimate the center of mass (CoM). In order to maintain the standing balance of legged robots under such a scenario, we propose a virtual suspension model control (VSMC) method which is concise and can get rid of updating model iterations on sophisticated dynamics. Furthermore, we optimize the range and propose the criteria for the virtual height of the CoM for quickly adjusting parameters and adapting to task characteristics. Simulation experiments on balance keeping are performed for single-leg robot vertical standing, quadruped robot diagonal standing and biped robot parallel standing cases, and the proposed method can achieve promising results within the approximative real setting condition that demonstrate the feasibility and strong anti-interference ability of the VSMC approach.

Keywords

Balance control, virtual suspension model control, virtual height, quadruped robot, biped robot, single-leg robot

Date received: 4 March 2020; accepted: 7 August 2020

Handling Editor: James Baldwin

Introduction

The ability of legged robots to maintain balance is fundamental before performing locomotion tasks. Currently, there are two mainstream manners to maintain balance for legged robots, one is to let legged robots keep immobile standing or low-speed pace state, while another is to configure constant apace leg motion. For the former one, it demands the Centre of Gravity (CoG) to be within the area inside the convex region of the supporting,1–3 and another formulation under normal circumstances for achieving dynamic equilibrium is to require the zero moment point (ZMP) to be within the support boundary.4–7 Usually, biped robots take ankle and hip strategy single or mixed to counteract perturbations or the gravity moment but can not satisfy both translational and rotational of the CoM due to the coupling effects, and also requires the soles with certain area and ankle to react with the ground for ankle strategy.8–10 For the latter one, the most traditional way for constant jumping to keep balance is Raibert’s one-leg hopping and its extension to quadruped and biped robots that they may need touch down with nearly point contact.11,12 The standing balance has the upside of less energy consumption and presents different natural stability ranges according to the formed supporting polygon area. The downside is that it requires additional end structures or multiple support legs. The dynamic movement is just the
opposite, with higher consumption, but simpler end mechanism.

Without ankle and soles, the pivot that calf end contact approximative point or line is one or two underactuated degrees of freedom (DOF) with no torque control that makes the control problem particularly tricky. There are several methods have been put forward before ours. Emura and Arakawa proposed a reaction-wheel-type inverted pendulum method to control quadruped standing with diagonal feet for short instant.\textsuperscript{13} The LQR-based model transformation method proposed by Meng et al. needs restrictions on modeling to guarantee control performance.\textsuperscript{14} Ding et al. built a double inverted pendulum robot model under a pole placement method to balance a biped robot.\textsuperscript{15} Through adding extra devices such as reaction wheel systems, Maurice et al. achieved the underdriven biped robot posture balance in which the initial state is deviation.\textsuperscript{16}

As an intuitive control method, virtual model control (VMC) utilizes virtual components to generate the desired joints torque. The VMC approach reduces the complexity of motion control and has a relatively lower computation consumptions through a discrete high-level controller which can produce fluid motion automatically. Due to these advantages, the adaptive terrain stable walking and attitude stabilization of the biped robot are utilized by Pratt et al.\textsuperscript{17,18} Hu et al. proposed an adaptive VMC method to improve the robustness of controlling biped robot walking by using an adaptive sliding control approach in virtual space which means that the VMC method has the development potential of adaptation.\textsuperscript{19} Zhang et al. applied the modus to quadruped robot going upstairs and uphill and also proved that the quadrupeds could self-stabilize with their method.\textsuperscript{20} Xie et al. proposed to use the decomposed VMC to control the diagonal axis of the torso being effectively when the diagonal foot is touching the ground and applied VMC to constraint the swing foot end as going with planned trajectory.\textsuperscript{21} Desai et al. developed a VMC controller to increase humanoids lateral balancing ability based on the assumption that humans prefer an intuitive task-space control for lateral balancing.\textsuperscript{22}

Havoutis et al. used mass-spring dampers to produce actively compliant behavior for dissipating unexpected perturbation when quadruped robot with trotting gaits, and rarely changed the parameters of the virtual component.\textsuperscript{23} Huang et al. proposed a suspension model applied to a hexapod robot to reduce vibration during walking.\textsuperscript{24} In addition, the VMC method is also appropriate to parallel mechanisms and used for hexapod robot walking with tripod gait proposed by Pratt, Torres et al.\textsuperscript{25,26}

In this paper, we divide the standing balance issue into the overall balance and the attitude balance. We propose a virtual suspension model control (VSMC) method that generates a virtual spring-damper force based on the position discrepancy to drag the body. It requires the cascade joint as a closed-form to obtain the ground reaction force (GRF). We believe that the level of control is critical, first of all, to satisfy the overall posture balance relative to feet, and secondly to fine-tune the body attitude based on a position discrepancy from the end of virtual height (VH) where we make this sequencing process naturally transition. By adopting this approach, the simulation shows that the legged robots can maintain balance on foot in which the support area in line or point and can resume balance promptly after suffering strong unexpected perturbations. To further increase the difficulty, we separately validate the legged robots’ ability to adjust the attitude to maintain balance under dynamic slope disturbances and the CoM distribution outside the support area fixed on the ground.

The main contributions of the paper can be summarized as follows:

1. An intuitionistic virtual suspension model which can effectively maintain the standing balance of a legged robot that without specific support polygon area is proposed.
2. The robot’s attitude can be transformed into the position space by the VH, and the design principle on the friction factor and feasibility is addressed.
3. Simulative experiments on three types of legged robots demonstrate that the proposed method can maintain the standing balance promisingly.

We organize this paper into the following sections to address. In Section 2, we firstly investigate the fundamental single-leg model in legged robots in the $x$-$y$ plane and describe the proposed VSMC formula. Section 3 expounds that the specific VSMC method for three types of legged robot models (quadruped, biped and single-leg) to maintain standing balance. We compared the characteristics of VMC and VSMC in Section 4. Afterwards, we present the simulation results with interpretations in Section 5. Finally, we conclude with some results in Section 6.

**VSMC applied on single-leg**

In this section, the formulation of the VSMC method is introduced and investigated based on the single-leg robot model, which shown in Figure 1. For a cascade leg system, the dynamics can be written as

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau + F_q$$  \hspace{1cm} (1)
where \( M, C, G, \tau, F_q \) respectively denote the inertia matrix, Coriolis force, gravitation force, joint force and virtual joint force. Since \( \dot{\theta} \) and \( \ddot{\theta} \) are both extremely small that we assume \( M(\theta)\dot{\theta} + C(\theta, \dot{\theta})\dot{\theta} \approx 0 \). The main body mass is larger than those of the leg, and the body is already balanced with very low acceleration, we consequently assume the system to be approximately statics for simplifying computation.

The conventional VMC method usually takes joint actuators in a closed-form to produce intuitive virtual forces through virtual components. However, the definition of the virtual components object in VSMC is outside the cascade joint to restrict the CoM movement. The virtual components we define are spring dampers and suspension points with pre-designed parameters. The suspension points can be the desired position of the CoM in the global frame but no specific practical role except for guidance, and the spring dampers are the link to the CoM successively towing.

Virtual components can be composed of three coordinate frames as action frame \( \{B\} \), reaction frame \( \{R\} \), and reference frame \( \{O\} \) proposed by Pratt et al. In this paper, we establish the action frame \( \{B\} \) in the CoM position and the reaction frame \( \{R\} \) in the suspension points. Uniquely, we also newly configure the attitude frame \( \{A\} \) in the hip joints for clear expression.

To maintain a balanced target relative to the ground, the cascade joint can obtain GRF in reaction frame \( \{R\} \) to guide the CoM to ensure the desired position or we can say that contents the overall balance in other words, but this will ignore the attitude control. The original intention is to convert attitude difference to position space, where we propose to use the VH in frame \( \{A\} \) to implement. Therefore, when the CoM deviates from the desired position and attitude, two-position error vectors can be generated. For these two error vectors, it can be regarded as the virtual stretching and shortening of the spring damper of the suspension model, where we design the physical properties, respectively. The sum of the two virtual forces can be abstracted into the action of a set of spring damper. Intuitively, the hanging body guaranteed the position and attitude process in space could be seen as the effect of an object being constrained.

The VH is the distance that the \( \{A\} \) origin extends in the \( y \)-axis direction, and can coincide with the CoM as Figure 1(b). In another case, when the CoM and hip joint plane coincides, an appropriate VH can still be defined as Figure 1(a). In general, the VH is the amount that satisfies the control target within limited conditions, and the proper VH will be discussed later.

**Single-leg implementation**

Due to the physical characteristics of the suspension model, we describe the CoM as a floating object in the global frame \( \{O\} \). The CoM position vector can be expressed as \( X_B \in \mathbb{R}^2 \). Considering the body attitude, the second position vector generated by the VH in frame \( \{A\} \) is denoted as \( A_H \in \mathbb{R}^2 \). The desired position and attitude vectors of the CoM are denoted as \( X_d \in \mathbb{R}^2 \) and \( A_d \in \mathbb{R}^2 \) respectively. The distance vector \( X_f = X_d - X_B \) is used to describe the CoM position difference between the desired position and the actual one in frame \( \{O\} \) as follows

\[
X_f = X_d - X_B
\]

Where \( X_B \) can be calculated by contacted leg end position \( X_{leg} \in \mathbb{R}^2 \) in frame \( \{B\} \) and the inertial
measurement unit (IMU) when there is no device to monitor the CoM, and the premise is to confirm the leg position \( X_{d, leg} \in \mathbb{R}^3 \) and the CoM position \( X_{init} \in \mathbb{R}^2 \) in frame \( \{O\} \) while initialization and no slipping throughout the process. We can describe as

\[
X_B = X_{d, leg} - R(\theta)X_{leg} + X_{init} \tag{3}
\]

where \( R(\cdot) \in \mathbb{R}^{2 \times 2} \) denotes the rotation matrix with joint angle input \( \theta_d \) or \( \theta \) for the robot body. The CoM attitude difference between \( A_d \) and \( A_H \) can be described by

\[
A_f = A_d - A_H = [R(\theta_d) - R(\theta)]H \tag{4}
\]

where \( H = [0 \ h_v]^T \) denotes a vector with \( h_v \) being the VH.

Virtual forces are generated along both the position and attitude vectors \( X_d \) and \( A_f \). In the VMC, the virtual spring-damper has the stiffness coefficients \( K_x, K_a \) and damping coefficients \( D_x, D_a \),\(^{17} \) and the two virtual forces \( F_x \) and \( F_a \) can be expressed separately by

\[
F_x = K_x X_f + D_x (\dot{X}_d - \dot{X}_B) \\
F_a = K_a A_f + D_a (A_d - A_B) \tag{5}
\]

Thus, the comprehensive virtual vector force is the sum of \( F_x \) and \( F_a \), which can be described as

\[
F_d = F_x + F_a \tag{6}
\]

Forward kinematics\(^{27} \) which describes the relationship between \( n \) joints space and task space can be formulated as

\[
T_n^0(\theta) = \begin{bmatrix}
\alpha & r & \theta \\
0 & 1 & \theta \\
0 & 0 & 1
\end{bmatrix} = T_n^1(\theta)T_n^2(\theta)T_n^3(\theta) \ldots T_n^{n-1}(\theta) \tag{7}
\]

where \( T_n^0(\theta) \in \mathbb{R}^{3 \times 3} \), \( \alpha \in \mathbb{R}^{2 \times 2} \), \( r \in \mathbb{R}^2 \) respectively denote the homogeneous matrix, the orientation matrix and the position vector of the end-effector. \( T_n^1(\theta)T_n^2(\theta)T_n^3(\theta) \ldots T_n^{n-1}(\theta) \) denotes the homogeneous matrix of adjacent joints at their designed joints coordinate.

Due to the task requirements of standing balance, \( \dot{X}_d \) and \( \dot{A}_d \) are both very close to 0 m/s. With reference to the motion of the foot, the virtual force is the desired GRF that needs to be converted into an exert force to the support area, as shown by Figure 2. For the simplified one-leg, we can compute the leg’s Jacobian matrix \( J \in \mathbb{R}^{2 \times 2} \) which can be generated by forwarding kinematics, and the joint force \( F_q \) which is limited as

\[
F_q = - J^T R(\theta) \begin{bmatrix}
F_d \\
0 \\
m \ g
\end{bmatrix} \tag{8}
\]

where \( F_d \) denotes the desired virtual force, \( m \) denotes the mass of the robot, and \( g \) denotes the gravitational acceleration.

The purpose of the robot to keep balance is to minimize the \( X_f \) and \( A_f \) through the VSMC. Nevertheless, this kind of dynamic constraint needs to meet specific physical provisions, such as the fact that the foot does not slide relative to the ground. It requires the adoption of appropriate parameters, particularly for the VH.

**Appropriate Virtual Height (VH).** The initial state of the robot is almost stable and meets the prospective position and attitude. When the VH is very approaching to 0 m or too small, the robot can reach the position expectations easily, yet the attitude gradually loses control. When the VH is too large, the generated virtual force will not suffice the limit of the ground friction factor. Therefore, the appropriate VH is necessary, which allows the position and attitude vectors to establish the proper virtual force and maintains the robot’s balance under the limitations.

We regard the CoM position control as a kind of overall attitude control relative to the foot, and the CoM attitude control is relative to the \( \{A\} \) origin. From this point of view, the control level is pronounced. We can attempt to link the CoM and the foot as a rod and link the VH end and the \( \{A\} \) origin as another for an idealized representation. The length of the former is significantly larger than the latter. When the former angle with the ground and the latter attitude angle changes, the endpoint position variation of the former is much larger than the latter within a wide range that we translate into corresponding forces to
verify as shown in Figure 3. Therefore, the CoM position is guaranteed prioritized than the attitude within a wide range. Besides, since the CoM initial state meets the desired location, we can simplify the problem as a prior condition of the discussion as $X_f = 0$ m.

For friction factor limitations, there are several ways to limit this effect as follows.

(i) The virtual spring is with the smaller stiffness;
(ii) The robot restores to the desired attitude is a gradual transition;
(iii) The virtual force of the VH contribution should be less by employing the appropriate VH.

For (i), it is a challenge to reach the fast and stability of the VSMC. The transition method in (ii) has no theoretical basis for determining the proper segmentation. Thus, this paper mainly focuses on (iii) and analyzes the theoretical stability range of criterion for equilibrium positions to manage the applicable VH.

We designate the actual height $h_c$, which is the distance between the origin of frame $\{A\}$ and the CoM position in $y$-axis. According to the design principle in (iii), we discussed the upper limit criterion of the virtual height in specific scenarios as shown in Figure 2.

The cascade leg gains GRF of the suspension model as $-F_d$. Combined with an additional gravity support force $mg$, we can easily get the limit friction and actual friction as

$$F_{lf} = (F_d \sin \alpha + mg)\mu$$
$$F_{af} = F_d \cos \alpha$$

where $\alpha$ represents the angle between $F_d$ and the horizontal line and can be denoted as $\alpha = \frac{\pi}{2}$, and $\theta$ is the CoM pitch angle. $\mu$ denotes the friction coefficient. Considering only the attitude and the initial state indicates $A_f$ is 0 m/s, $F_d$ can be described by the following equation as

$$F_d = 2K_a h_v \sin \frac{\theta}{2}$$

As we can ignore the friction generated by gravity and position difference, the following restriction can be written as

$$F_{lf} \geq F_{af}, \theta \in [0, \pi], h_v \in [0, 0.1]$$

In addition to the friction force, the VH value has a bottom-line requirement that reflected in torque. The $\{A\}$ origin torque $\tau_g$ generated by gravity which is described by the following equation:

$$\tau_g = mgh_c \sin \theta$$

Correspondingly, the torque produced by VH can be described as:

$$\tau_{h_v} = F_d h_v \cos \frac{\theta}{2}$$

Similarly, to reach the goal of standing balance, it demands that the torque of the VH $\tau_{h_v}$ is higher than the gravity torque $\tau_g$ which can be expressed by the inequality as:

$$\tau_{h_v} \geq \tau_g, \theta \in [0, \pi], h_v \in [0, 0.1]$$

We subtract the inequalities and combine the two-part contract can roughly recognize the range in which the VH matches the requirements under the given parameters in Figure 4. Then, it necessitates meeting two primary conditions as

1. Limit friction is higher than actual friction;
2. The VH torque is higher than the gravity torque.

Additionally affirming the preconditions, the proper constants $K$ and $D$ and the CoM position are almost stable. Overall, we can conclude that the smaller VH which generates less virtual forces in attitude that can not maintain the balance or get a fast and steady process. Yet, it holds a large adjustment domain. Otherwise, the larger VH has a strong anti-interference ability, but with a little adjustment domain relatively. Integrated, under the specified parameters, the VH is preferred to be 0.02~0.03 m that would be more suitable to suffice general requirements. The criterion we consider is a preliminary basis for quickly adjusting...
parameters and adapting to task characteristics, which has a high correlation with the attributes of the robot and the parameters of the virtual components.

**Virtual model implementation for three robot models**

Keeping the dynamic balance of legged robots while standing still is a vital subject where we implemented the VSMC. The practical application of this paper is on the problems of quadrupeds standing on diagonal feet, bipeds standing on parallel feet and single-leg robot vertical standing. We demonstrate the effectiveness of our method by building the robots simulating in Webots R2020a, as shown in Figure 5. Robots can be efficiently modeling in Webots which relies on ODE(Open Dynamic Engine) to perform accurate physics simulation, the controller for some robot model given in software can be transferred to real robots directly once tested.

We build properties proximity to real robots and summarize the parameter attributes of three legged robots in Table 1.

The vector sum of the forces of each cascade leg in the three-dimensional space can be used to represent the support force to the body. We distribute the suspension model to each support leg, just like the suspension system of a vehicle. Since then, our proposed method naturally converts the body torque control amount into the task space and no necessity to consider the body torque constraint. Regard the integrated vector force at the CoM as

\[
\mathbf{F} = \frac{1}{n} \sum_{i=0}^{n} \mathbf{F}_i \quad (15)
\]

Here we redefine equation (2) for a multi-legged robot system. \(X_d - X_B\) is denoted as the position difference of CoM in frame \(O\). Due to the hips are distributed around the centroid, the CoM position difference can be expressed as the position error of all the hips as

\[
\begin{bmatrix}
X_{f1} \\
\vdots \\
X_{fn}
\end{bmatrix} = \begin{bmatrix}
X_d - X_B \\
\vdots \\
X_d - X_B
\end{bmatrix} \quad (16)
\]

As in equation (3), \(X_B\) can be calculated by any contact leg and IMU as
with the orientation matrix being defined based on the frame in Figure 6 as

\[ R = R_{roll}(\phi)R_{yaw}(\psi)R_{pitch}(\theta) \]

where \( \phi, \theta, \psi \) denote the desired Euler angles for the input of the rotation matrix, and \( \phi, \theta, \psi \) denote the actual Euler angles for the robot.

The constraint of the VH on attitude indicated by \( A_f \in \mathbb{R}^{n \times 3} \) is decomposed into the position space of each leg. Since the CoM position is the \( \{B\} \) origin, and we can obtain the position of the hip as given in Table 1. Use \( h_v \) replaces the \( y_n \), we redefine equation (4) as the following expression

\[ A_f = \begin{bmatrix} A_{f1} \\ \vdots \\ A_{fn} \end{bmatrix} = [R(\phi_d, \theta_d, \psi_d) - R(\phi, \theta, \psi)] \begin{bmatrix} x_1 \\ y_v \\ z_1 \\ \vdots \\ x_n \\ y_v \\ z_n \end{bmatrix} \]

Therefore, the virtual force \( F_d \in \mathbb{R}^{n \times 3} \) and the joint torque \( F_q = [F_{q1}, F_{q2}, \cdots, F_{qn}]^T \in \mathbb{R}^{3n \times 1} \) can be readily expressed as

\[
\begin{bmatrix}
F_{d1} \\
\vdots \\
F_{dn}
\end{bmatrix} = J^T(\phi, \theta, \psi) \begin{bmatrix}
F_{d1x} + \frac{m g}{n} \\
F_{d1y} + \frac{m g}{n} \\
F_{d1z} \\
F_{dny} + \frac{m g}{n} \\
F_{dnz} \\
\end{bmatrix}
\]

with

\[
\begin{bmatrix}
F_{d1} \\
\vdots \\
F_{dn}
\end{bmatrix} = K_aX_f + D_a(\dot{X}_d - \dot{X}_B) \\
\vdots \\
K_dX_f + D_d(\dot{X}_d - \dot{X}_B) \\
K_dA_f + D_d(\dot{A}_d - \dot{A}_B) \\
\vdots \\
K_dA_f + D_d(\dot{A}_d - \dot{A}_B)
\]

where \( F_d = [F_{d1x}, F_{d1y}, F_{d1z}]^T \) and \( \cdots, F_{dn} = [F_{dnx}, F_{dnz}, F_{dnz}]^T \).
Due to balanced purpose requirements, we set the number of support legs \( n = 2 \) in quadrupeds and bipeds, \( n = 1 \) for the single-leg robot. We can see the virtual suspension spring-dampers exert the effect when the body rolls over as Figure 6, which the CoM and the endpoint of the VH coincide on the \( y \)-axis. That is comparable to a floating object suspended from multiple spring-dampers. This approach converts the attitude deviation into a virtual position deviation, decreases the constraint dimension, and achieves the balanced goal through the comprehensive action of uniformly applied force points that requires no ankle joints and specific polygonal support areas. Besides, It does not require to establish a complicated mathematical model or massive computing resources.

For the design of the robot feet, as in the real world, we set the feet shape to be spherical, and ensure the continuous contact process at varying contact angles. However, it also leads to an obstacle. The contact point between the foot and the ground is changing with the attitude adjustment, and the impact will be specifically explained and displayed in the simulation experiment.

**Comparison between VMC and VSMC**

VMC can be regarded as a generalized VMC. Different tasks may result in a different definition of VMC for constraining single or multiple rigid bodies. VSMC embodies the body constraint and the key point is to transform the attitude constraint through the VH so that the final constraint force is directly the GRF. Besides, the design of VSMC implies the rules of stability of balancing that is the position is always adjusted before the attitude is satisfied. The natural properties of VSMC lead to a stabilizing effect, but most VMC designs have no stabilizing characteristics since they are intuitional. In Figure 7, a conventional virtual mechanical component to constrain the six degrees of freedom of the rigid body is shown. This conventional VMC case uses linear spring-dampers to constrain the position in the world frame, torsion spring-dampers to constrain the attitude that obtains wrenches in the body frame to allocate to the contact points. The specific process is realized in the source code as an attachment to this paper. The simulation results of this conventional VMC paradigm will be shown in the ensuing section.

**Simulation experiment results**

We experimentally built three robot systems with the same leg system and parameters, and three controllers with similar structures by Python 3.7. The control frequency is \( 0.5 \) kHz in the Webots default simulation environment, all sensors property are set in a reasonable range, and the specific experimental parameters are given in Table 2. To comprehensively show the balanced nature of this method, we examined the robots in the following three ways.

1. The robots are unexpectedly perturbed.
2. The robots stand on a slope and we disturb the slope.
3. The CoM is distributed outside the support polygon to resume balance.

The VSMC sets parameters which are mainly physical parameters of virtual components based on task features. The hardware determines \( h_v \), and the task specifies the standing height, we merely decide the appropriate \( h_v \), \( K \) and \( D \).

Before examining our method, we firstly compare it with the conventional VMC method to illustrate the feasibility and initial advantages of ours. We let the three robots stand on the flat ground with diagonal feet or single foot and give slight noises, the robot would gradually overturn if the system is unstable. We can see that the balance maintaining time for the three robots are respectively in Table 3, which indicates that the direct VMC constraints do not exhibit the balance characteristics and the robots fall over when around 1s, but the VMSC can always balance which is the preliminary requirement of the further challenge.

**Quadrupeds diagonal standing**

We set the quadruped robot to initially stand on four feet to ensure the position of the CoM, followed by two legs are raised, and the others are supporting legs with torque control by VSMC. The coronal plane for quadrupeds is more arduous to accommodate unexpected perturbation where we implement a controllable pendulum ball to strike the body and record the energy of the
To maintain balance, it is necessary to satisfy two requirements. For this perturbation, there is no apparent shift in the offset that maintains no fall under the given parameter. In the $x-z$ plane, we offset distances along the $x$ and $z$ axis respectively, these values are approaching to the limit, given as Table 4. The quadruped robot is finally balanced in a particular attitude as shown in Figure 11.

In the process of standing from quadruped to biped, it is likewise necessary to satisfy two restrictions in the swing.

**Bipeds parallel standing**

Similarly, we performed the three experiments on the biped robot. We implement perturbation to the sagittal plane to maximize the effect, record the impact energy and detect states such as Figure 12(a). Regular and periodic dynamic slope application results such as Figure 12(b). Also taking snapshots of both procedures are separately exhibited in Figures 9 and 10. The effective direction for the centroid shift is along the $x$ axis, as shown in Table 4.

Since the perturbation is mainly along the $x$-axis direction, the bipeds attitude influence is primarily the pitch angle. Notice that the pitch angle deviation is severe and not intuitive to meet the requirements of friction limit in Figure 4, which is because $X_f$ and $A_f$ counteract each other to some extent. This natural process reflects the physical characteristics of the suspension model. The dynamic slope makes the robot rhythmic. When the initial yaw angle is $0$ rad, the robot roll and yaw are well maintained except the pitch. For the CoM offset, we examine that the bipeds can tolerate CoM offset $-0.007$ m along the $x$-axis and its stable attitude are shown as Figure 11(c), the offset along the $z$-axis cannot deviate from the support line in actual that is not attempted.

**Single-leg standing**

The single-legged robot is relatively unique, and it cannot actively maintain the yaw angle due to leg structure and point contact. The leg is bent which makes the structure asymmetric, the disturbance in the $z$-axis direction will cause the yaw angle to deflect and...
subsequently overturn. Consequently, the perturbation, dynamic slope and centroid shift experiments are performed in a uniaxial course. We apply perturbation mainly along with $x$-axis and accept results such as Figure 13(a). Active hills only deliver periodic action events in the $z$-axis and get results such as Figure 13(b). We take a snapshot of both procedures are separately exhibited in Figures 9 and 10. The effects of shifting the CoM along the $x$-axis near the limit are shown in Table 4. According to consequent, the single-legged robot experiment is similar to the biped robot. Because it cannot guarantee the yaw angle and the asymmetry of the structure exacerbates the yaw deflection when the roll

Figure 8. Unexpected perturbation and dynamic slope disturbance examination of the quadrupeds. (a) After the quadrupeds receive the perturbation of the pendulum ball, 0.8686 kgm/s (starting from the blue dotted line), the CoM position and attitude are recovering, and the expected decomposition forces of the corresponding right front leg and left rear leg are responding. (b) The platform $x$ and $z$ axes increase progressively with sine and cosine, respectively. The period is 2.51 s, and the maximum angle is 0.1 rad.

Figure 9. Snapshots of quadrupeds, bipeds, and single-leg robots while suffering unintended perturbation.

Figure 10. Snapshots of quadrupeds, bipeds, and single-leg robots while suffering dynamic slope disturbance.
is tilted, the single-legged robot is exceptionally vulnerable. We attempt to regularize the two angle disturbances of the floating platform similar to the experiment on quadrupeds and bipeds which causes the support point to move along the spherical arc surface, and the yaw angle gradually oscillates. At the same time, it also creates a continuously enhanced centrifugal result and eventually overturns. Of course, utilizing the VSMC to stand still on the plane is entirely satisfactory. Consequently, we barely make a relatively small offset of the CoM along the $x$-axis. Because the constant swing in the balancing process causes the deflection of the yaw and roll, which eventually causes the slip to fail to meet the friction limit, it can be seen from the steady-state results in Figure 11(d) and Table 4 that the single-legged robot barely maintained the yaw and roll angle not too large.

**Conclusion**

This paper addresses that the virtual suspension model control successfully solves the problem that the robots with the support line or point maintain stand balance. We re-integrate the attitude control amount into the
task space for execution by the virtual height variable, which is mainly manifested in the force control of the leg system in closed-form, and ankle joints are not required for restraint attitude. Inspired by the physical characteristics of the suspended object of the spring damper, we use the reaction effect of the force exerted by the legs to approach. For the virtual height variable, we propose a suitable range of criteria to facilitate faster adjustment of parameters and adaptation to task characteristics. To this end, we construct three types of legged robots without ankle joints with the same three-degree leg system and performed three experiments on them, unexpected perturbation, dynamic slope, and centroid shift. Experimental results show that our method can resist perturbation and slope disturbance when the robot is standing statically, and it can tolerate a certain amount of centroid shift. Different from Raibert’s need to keep balance by non-stop hopping, our method is exhilarating to fill the gap of robots standing balance in our conditions. We believe this will improve the balance of motion of multi-legged or single-legged robots.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work is supported by the National Natural Science Foundation of China under Grant U1613223 and 61603078 and the Fundamental Funds for the Central Universities.

Reference code: https://github.com/JameScottX/Virtual-suspension

ORCID iDs
Junwen Cui https://orcid.org/0000-0001-6577-9121
Zhan Li https://orcid.org/0000-0002-3928-1642

Supplemental material
Supplemental material for this article is available online.

References
1. Song SM and Waldron KJ. Machines that walk. Cambridge: The MIT Press, 1989.
2. McGhee RB and Frank AA. On the stability properties of quadruped creeping gaits. Math Biosci 1968; 3: 331–351.
3. Shih CL. The dynamics and control of a biped walking robot with seven degrees of freedom. J Dyn Syst Meas Control 1996; 118(4): 683–690.
4. Vukobratovic M and Borovac B. Zero-moment point — thirty five years of its life. Int J Hum Robot 2004; 1(1): 157–173.
5. Firmani F and Park EJ. Theoretical analysis of the state of balance in bipedal walking. J Biomech Eng 2013; 135(4): 041003.
6. Vukobratovic M and Stepanenko J. On the stability of anthropomorphic systems. Math Biosci 1972; 15(1–2): 1–37.
7. Troy JJ. Dynamic balance and walking control of biped mechanisms. Doctor of Philosophy, Iowa State University, 1995.
8. Winter DA. Human balance and posture control during standing and walking. *Gait Posture* 1995; 3(4): 193–214.
9. Goswami A. Postural stability of biped robots and the foot-rotation indicator (FRI) point. *Int J Robot Res* 1999; 18(6): 523–533.
10. Nenchev DN and Nishio A. Ankle and hip strategies for balance recovery of a biped subjected to an impact. *Robotica* 2008; 26(5): 643–653.
11. Raibert M. *Legged robots that balance*. Cambridge, MA: MIT Press, 2000.
12. Raibert MH, Brown HB Jr, Chepponis M, et al. *Dynamically stable legged locomotion*. Technical report, Massachusetts Inst of Tech Cambridge Artificial Intelligence Lab, 1989.
13. Emura T and Arakawa A. Attitude control of a quadruped robot during two legs supporting. In: 5th International conference on advanced robotics’ robots in unstructured environments, Pisa, Italy, 19–22 June 1991, pp.711–716. IEEE.
14. Meng J, Li Y and Li B. A dynamic balancing approach for a quadruped robot supported by diagonal legs. *Int J Adv Robot Syst* 2015; 12(10): 142.
15. Ding B, Plummer A and Iravani P. Investigating balancing control of a standing bipedal robot with point foot contact. *IFAC-PapersOnLine* 2016; 49(21): 403–408.
16. Maurice CM, Goodwine B and Schmiedeler JP. Postural balance of an underactuated biped robot with a reaction wheel. In: *International design engineering technical conferences and computers and information in engineering conference*, Vol. 58172 Ohio, USA, 6–9 August 2017, p.V05AT08A057. ASME.
17. Pratt J, Dilworth P and Pratt G. Virtual model control of a bipedal walking robot. In: *Proceedings of international conference on robotics and automation*, Albuquerque, NM, 25–25 April 1997, 193–198. IEEE.
18. Pratt J, Chew CM, Torres A, et al. Virtual model control: an intuitive approach for bipedal locomotion. *Int J Robot Res* 2001; 20(2): 129–143.
19. Hu JJ, Pratt JE, Chew CM, et al. Virtual model based adaptive dynamic control of a biped walking robot. *Int J Artif Intell T* 1999; 8(3): 337–348.
20. Zhang G, Rong X, Hui C, et al. Torso motion control and toe trajectory generation of a trotting quadruped robot based on virtual model control. *Adv Robot* 2016; 30(4): 284–297.
21. Xie H, Ahmadi M, Shang J, et al. An intuitive approach for quadruped robot trotting based on virtual model control. *Proc Inst Mech Eng I J Syst Contr Eng* 2015; 229(4): 342–355.
22. Desai R, Geyer H and Hodgins JK. Virtual model control for dynamic lateral balance. In: *2014 IEEE-RAS international conference on humanoid robots*, Madrid, Spain, 18–20 November 2014, pp.856–861. IEEE.
23. Havoutis I, Semini C, Buchli J, et al. Quadrupedal trotting with active compliance. In: *2013 IEEE international conference on mechatronics (ICM)*, Vicenza, Italy, 27 February–1 March 2013, pp.610–616. IEEE.
24. Huang Q. Sliding mode control based on virtual suspension model for controlling posture and vibration of six-legged walking robot. In: *2006 IEEE international conference on robotics and biomimetics*, Kunming, China, 17–20 December 2006, pp.642–647. IEEE.
25. Pratt J, Torres A, Dilworth P, et al. Virtual actuator control. In: *Proceedings of IEEE/RSJ international conference on intelligent robots and systems IROS ’96*, Osaka, Japan, 8 November 1996, pp.1219–1226. IEEE.
26. Torres A. Virtual model control of a hexapod walking robot. Technical report, AITR-1582, 1996, http://hdl.handle.net/1721.1/7083
27. Li Z, Li C, Li S, et al. A fault-tolerant method for motion planning of industrial redundant manipulator. In: *IEEE transactions on industrial informatics*, 02 December 2019. IEEE.
28. Michel O. Cyberbotics Ltd. Webots™: professional mobile robot simulation. *Int J Adv Robot Syst* 2004; 1(1): 5.
29. Havoutis I, Semini C and Caldwell DG. Virtual model control for quadrupedal trunk stabilization. In: *Dynamic walking conference*, Pittsburgh, Pennsylvania, 9–13 June 2013.
30. Wang J, Wilson DA, Xu W, et al. Active suspension control to improve vehicle ride and steady-state handling. In: *Proceedings of the 44th IEEE conference on decision and control*, Seville, Spain, 15 December 2005, pp.1982–1987. IEEE.