Do we understand the incompressibility of neutron-rich matter?

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Abstract

The “breathing mode” of neutron-rich nuclei is our window into the incompressibility of neutron-rich matter. After much confusion on the interpretation of the experimental data, consistency was finally reached between different models that predicted both the distribution of isoscalar monopole strength in finite nuclei and the compression modulus of infinite matter. However, a very recent experiment on the Tin isotopes at the Research Center for Nuclear Physics (RCNP) in Japan has again muddled the waters. Self-consistent models that were successful in reproducing the energy of the giant monopole resonance (GMR) in nuclei with various nucleon asymmetries (such as $^{90}$Zr, $^{144}$Sm, and $^{208}$Pb) overestimate the GMR energies in the Tin isotopes. As important, the discrepancy between theory and experiment appears to grow with neutron excess. This is particularly problematic as models artificially tuned to reproduce the rapid softening of the GMR in the Tin isotopes become inconsistent with the behavior of dilute neutron matter. Thus, we regard the question of “why is Tin so soft?” as an important open problem in nuclear structure.

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The incompressibility coefficient of infinite nuclear matter—also known as the compression modulus—is a fundamental parameter of the equation of state (EOS). The compression modulus controls small density fluctuations around the equilibrium point, thereby providing the first glimpse into the “stiffness” of the EOS. Whereas existing ground-state observables (e.g., masses and charge radii) have accurately constrained the saturation point of symmetric nuclear matter (at a baryon density of $\rho_0 \approx 0.15$ fm$^{-3}$ and a binding energy per nucleon of $\varepsilon_0 \approx -16$ MeV), the extraction of the compression modulus ($K_0$) is significantly more complicated as it requires to probe the response of the nuclear ground state. It is widely accepted that the nuclear compressional modes—particularly the quintessential “breathing mode” or isoscalar Giant Monopole Resonance (GMR)—provide the cleanest, most direct route to the nuclear incompressibility [1, 2].

Earlier attempts at extracting the compression modulus of symmetric nuclear matter relied primarily on the distribution of isoscalar monopole strength in $^{208}$Pb—a heavy, doubly-magic nucleus with a well developed monopole peak [3, 4]. Although such measurements have existed for some time, the field has enjoyed a renaissance due to new and improved experimental facilities and techniques. Indeed, an improved $\alpha$-scattering experiment found the position of the giant monopole resonance in $^{208}$Pb at $E_{\text{GMR}} = 14.17 \pm 0.28$ MeV [5]. As this measurement was combined with the distribution of monopole strength in other nuclei—and compared with microscopic calculations—a value of the incompressibility coefficient in the range of $K_0 = 220-240$ MeV was extracted.

As the experimental program reached a high level of maturity and sophistication, the same strict standards were demanded from the theoretical program. Indeed, calculations of nuclear compressional modes based on consistent Mean-Field plus Random Phase Approximation (RPA) approaches became routine. Moreover, such consistent models—without any recourse to semi-empirical mass formulas—were able to simultaneously predict the incompressibility of infinite nuclear matter as well as the distribution of isoscalar monopole strength in finite nuclei [6, 7]. However, as the experimental story was coming to an end, the theoretical picture remained unclear. On the one hand, nonrelativistic calculations that reproduced the distribution of isoscalar-monopole strength in $^{208}$Pb predicted a nuclear incompressibility coefficient in the $K_0 = 210-230$ MeV range [2, 8, 9]. On the other hand, relativistic models that succeeded in reproducing a large body of nuclear observables—including the GMR in $^{208}$Pb—suggested the significantly larger value of $K_0 \approx 270$ MeV [10, 11].

The solution to this puzzle was originally proposed by Piekarewicz [12, 13] and has since been confirmed by several other groups [14, 15, 16]. The solution is based on the realization that the GMR in $^{208}$Pb does not constrain the compression modulus of symmetric nuclear matter but rather the one of neutron-rich matter. In particular, it was concluded that the compression modulus of a neutron rich system having the same neutron excess as $^{208}$Pb is lower than the compression modulus of symmetric nuclear matter. This could explain how models with significantly different incompressibility coefficients $K_0$ may still reproduce the GMR in $^{208}$Pb [12, 13]. As a result, accurately-calibrated theoretical models were built to reproduce simultaneously the distribution of isoscalar-monopole strength in both $^{90}$Zr and $^{208}$Pb—nuclei with a well developed monopole peak yet significantly different nucleon asymmetries [17, 18]. Since then, the large difference in the predicted value of $K_0$ between nonrelativistic and relativistic models has been reconciled and a “consensus” has been reached that places the value of the incompressibility coefficient of symmetric nuclear
matter at $K_0 = 240 \pm 10$ MeV [15, 16, 17, 19, 20].

II. INCOMPRESSIBILITY OF NEUTRON-RICH MATTER

To summarize some of the above findings it is convenient to discuss in detail the incompressibility coefficient of infinite, neutron-rich matter. On very general grounds—indeed, in a model-independent way—the incompressibility coefficient of neutron-rich matter may be written as

$$K_0(\alpha) = K_0 + K_\tau \alpha^2 + O(\alpha^4),$$

where $\alpha \equiv (N - Z)/A$ is the nucleon asymmetry. From this expression it is immediately evident that the GMR in $^{208}\text{Pb}$ (with a neutron excess of $\alpha = 0.21$) should be sensitive to a linear combination of the incompressibility coefficient of symmetric nuclear matter $K_0$ and $K_\tau$—a quantity that determines the evolution of the incompressibility coefficient with neutron excess. Note that $K_\tau$ plays the same role in determining the incompressibility coefficient as the symmetry energy at saturation density (a quantity often denoted by $J$ or $a_4$) plays in determining the energy-per-nucleon of asymmetric matter.

To compute the incompressibility coefficient of neutron-rich matter one proceeds exactly as in the case of symmetric nuclear matter. First, one determines the equilibrium point at a fixed value of $\alpha$ and then extracts $K_0(\alpha)$ from computing the curvature at the minimum. Having done so for various values of $\alpha$, one can extract $K_\tau$ from a simple fit to Eq. (1) [21]. An alternative procedure that is highly accurate and significantly more illuminating starts from the equation of state of neutron-rich matter parametrized in terms of several bulk parameters defined at normal saturation density. Starting from such a parametrization, it becomes a simple exercise to compute the equilibrium point (as a function of $\alpha$) and the corresponding curvature at the minimum. In particular, one obtains the following closed-form expression for $K_\tau$ [21]:

$$K_\tau = K_{\text{sym}} - 6L - \frac{Q_0}{K_0}L,$$  \hspace{1cm} (2)

where $L$ and $K_{\text{sym}}$ represent the slope and curvature of the symmetry energy at saturation density [21]. Although often neglected, note that $K_\tau$ also depends on the third derivative of the EOS of symmetric nuclear matter $Q_0$ (a quantity often referred to as the “skewness” parameter). Quite generally, as the infinite nuclear system becomes neutron rich, the saturation density moves to lower densities, the binding energy weakens, and the nuclear incompressibility softens [21]. It is important to stress the dominant role of the symmetry pressure $L$ on these conclusions and in particular on Eq. (2) (because of the large coefficient in front of it). We note that the symmetry pressure $L$—a quantity that strongly influences the neutron-skin thickness of heavy nuclei—is directly proportional to the pressure of pure neutron matter, namely,

$$P_{nm} = \frac{1}{3} \rho_0 L.$$  \hspace{1cm} (3)

This connection is important as significant theoretical progress has been made in constraining the equation of state of low-density neutron matter. We will draw heavily on this connection in what follows.
III. MEASURING THE BREATHING MODE OF THE TIN ISOTOPES

The realization that the distribution of monopole strength in heavy nuclei is sensitive to the density dependence of the symmetry energy motivated a recent experimental study of the GMR along the isotopic chain in Tin [20, 22]. This important experiment was carried out at the Research Center for Nuclear Physics (RCNP) in Osaka, Japan. Such an experiment probed the incompressibility of asymmetric nuclear matter by measuring the distribution of isoscalar strength in a chain of isotopes ranging from $^{112}\text{Sn}$ (with $\alpha = 0.11$) to $^{124}\text{Sn}$ (with $\alpha = 0.19$). Because of the sensitivity of $K_\tau$ to the pressure of pure neutron matter [see Eqs. (2) and (3)], this experiment represents an attractive hadronic complement to the purely electroweak Parity Radius Experiment (PREx) at the Jefferson Laboratory that aims to measure the neutron radius of $^{208}\text{Pb}$ accurately and model independently via parity-violating electron scattering [23, 24]. We note that such a fundamental measurement will have far-reaching implications in areas as diverse as nuclear structure [25], heavy-ion collisions [26, 27, 28, 29, 30], atomic parity violation [25, 31, 32] and nuclear astrophysics [33, 34, 35, 36, 37, 38, 39, 40].

Shortly after the completion of the RCNP experiment a serious discrepancy was revealed: accurately calibrated models that reproduce the GMR in $^{90}\text{Zr}$, $^{144}\text{Sm}$, and $^{208}\text{Pb}$ overestimate the distribution of isoscalar monopole strength in the Tin isotopes [41, 42, 43]. Moreover, the discrepancy between theory and experiment appears to grow with neutron excess $\alpha$, suggesting that the models significantly underestimate the value of $|K_\tau|$. We have colloquially referred to this problem as “why is Tin so soft?”. To illustrate this situation we display in Fig. 1 a comparison between the experimental centroid energies [20, 22] for the neutron-even $^{112}\text{Sn}$-$^{124}\text{Sn}$ isotopes and three theoretical calculations that have been extended up to the doubly magic nucleus $^{132}\text{Sn}$. All theoretical predictions were generated using a consistent RPA approach. That is, the linear response of the system was computed using the same interaction employed to generate the mean-field ground state. A detailed description of this approach may be found in Refs. [44, 45].

The results depicted with the blue triangles were generated using the accurately calibrated FSUGold parametrization [17]. This relativistic model is characterized by a soft behavior for both symmetric nuclear matter and the symmetry energy. (Note that the terms “soft” and “stiff” refer to whether the energy increases slowly or rapidly with density.) Such a soft behavior is reflected in the relatively small values of $K_0$, $L$, and $|K_\tau|$ listed in Table I. Clearly, the model overestimates the experimental data (black squares). Moreover, the discrepancy increases with neutron number: from about $0.2 \text{ MeV}$ for $^{112}\text{Sn}$ to $0.7 \text{ MeV}$ for $^{124}\text{Sn}$. Such a serious discrepancy is particularly troublesome given that the same FSUGold model successfully reproduces the centroid energy of the GMR in $^{90}\text{Zr}$, $^{144}\text{Sm}$, and $^{208}\text{Pb}$, as shown in the inset of Fig. 2. Figure 2 also displays the distribution of isoscalar monopole strength from which the centroid energies depicted in the inset were computed (as the ratio of the first to the zeroth moment). In addition to the four nuclei—$^{90}\text{Zr}$, $^{116}\text{Sn}$, $^{144}\text{Sm}$, and $^{208}\text{Pb}$—of Ref. [5], we display the distribution of monopole strength for the doubly-magic nuclei $^{100}\text{Sn}$ and $^{132}\text{Sn}$. We note that the GMR predictions for all six nuclei fall nicely within the “liquid-drop” inspired curve $E_{\text{GMR}} \simeq 69A^{-1/3}$ [16, 17]. Moreover, these predictions reproduce the experimentally extracted GMR energies [5]—except for the case of $^{116}\text{Sn}$. Figures 1 and 2 capture the essence of the problem of why is Tin so soft?

Also shown in Figure 1 are the predictions from the NL3 parameter set [10, 48]. The NL3 set has been remarkably successful in reproducing a myriad of nuclear ground-state
FIG. 1: (Color online) Comparison between the GMR centroid energies \( (m_1/m_0) \) of all neutron-even \(^{112}\text{Sn} - ^{124}\text{Sn} \) isotopes extracted from experiment \([20,22]\) (black solid squares) and the theoretical predictions from the FSUGold (blue up-triangles), NL3 (green down-triangles), and Hybrid (red diamonds) models. The corresponding dashed lines were obtained from a fit to the centroid energies of the form \( E_{\text{GMR}} = E_0 A^{-\lambda} \) [see Eq. (4)].

Motivated by the above facts, we have built a “Hybrid” model with a low incompressibility coefficient and a stiff symmetry energy \([21]\). However, unlike the NL3 and FSUGold parametrizations, the Hybrid model was not accurately calibrated. Thus, the Hybrid model should be simply regarded as a “test” model that illustrates how surprisingly soft are the experimental GMR energies of the Tin isotopes relative to the theoretical predictions. We observe in Fig. [1] that the Hybrid model yields a significant improvement in the description of the experimental data. Indeed, the predictions of the Hybrid model fall within 0.1 MeV

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TABLE I: Bulk parameters characterizing the behavior of neutron-rich matter around saturation density $\rho_0$. The quantities $\varepsilon_0$, $K_0$, and $Q_0$ represent the binding energy per nucleon, incompressibility coefficient, and third derivative (or “skewness” coefficient) of symmetric nuclear matter at $\rho_0$. Similarly, $J$, $L$, and $K_{\text{sym}}$ represent the energy, slope, and curvature of the symmetry energy at saturation density. All quantities are in MeV, except for $\rho_0$ which is given in fm$^{-3}$ and the pressure of pure neutron matter at saturation density ($P_{\text{nm}}$) which is given in MeV/fm$^3$. A detailed explanation of all these quantities may be found in Ref. [21].

| Model  | $\rho_0$  | $\varepsilon_0$ | $K_0$  | $Q_0$  | $J$     | $L(P_{\text{nm}})$ | $K_{\text{sym}}$ | $K_\tau$ |
|--------|-----------|----------------|--------|--------|--------|---------------------|-----------------|---------|
| FSU    | 0.148     | -16.30         | 230.0  | -523.4 | 32.59  | 60.5(3.0)           | -51.3           | -276.8  |
| NL3    | 0.148     | -16.24         | 271.5  | +204.2 | 37.29  | 118.2(5.8)          | +100.9          | -697.4  |
| Hybrid | 0.148     | -16.24         | 230.0  | -71.5  | 37.30  | 118.6(5.9)          | +110.9          | -563.9  |

FIG. 2: (color online) Distribution of isoscalar monopole strength predicted by the FSUGold model of Ref. [17]. The inset includes a comparison against the experimental centroid energies reported in Ref. [5], with the solid line providing the best fit to the theoretical predictions.

of the experimental data for the full isotopic chain. Note that relative to FSUGold, the improved description provided by the Hybrid model is entirely due to its large negative asymmetric term $K_\tau$, as they shared the same value of $K_0$ (see Eq. (1) and Table I). Indeed, we can capture the $A$ dependence of the $E_{\text{GMR}}$ predicted by all the models with a liquid-drop
inspired formula of the form $E_{\text{GMR}} \simeq E_0 A^{-\lambda}$. We obtain,

$$E_{\text{GMR}} = \begin{cases} 
64.5 A^{-0.29} & \text{for FSU}, \\
102.6 A^{-0.38} & \text{for NL3}, \\
112.7 A^{-0.41} & \text{for Hybrid}.
\end{cases}$$

(4)

The Hybrid model suggests a falloff with $A$ that is significantly faster than the $\lambda = -1/3$ value predicted by a liquid-drop description \[16, 47]. And although the improvement in the case of the Tin isotopes is significant and unquestionable, an important problem remains: the Hybrid model underestimates the GMR centroid energy in $^{208}\text{Pb}$ by almost 1 MeV \[21, 43]. This suggests that the rapid softening with neutron excess predicted by the Hybrid model may be unrealistic.

IV. WHY IS TIN SO SOFT?

So why is Tin so soft and why does it become even softer as the nucleon asymmetry increases? Are we any closer to the answer now than we were then \[20, 22]? Unfortunately not! As we elaborate below, we will assume that the experimental extraction of the GMR energies is without error—although the large discrepancy between the RCNP \[20, 22] and the Texas A&M \[49] results should be resolved.

To date, only two plausible scenarios have been advanced to explain why accurately-calibrated models that reproduce the GMR energies in $^{90}\text{Zr}$, $^{144}\text{Sm}$, and $^{208}\text{Pb}$ fail to do so for the Tin isotopes. One of them is encapsulated in the Hybrid model discussed above \[21] whereas the other one suggests that pairing correlations are responsible for the softening of the monopole response \[50, 51]. As discussed earlier, the Hybrid model is based on an effective interaction that generates a soft EOS for symmetric nuclear matter (a small $K_0$) but a stiff symmetry energy (a large $|K_\tau|$). Any such model should generate soft monopole excitations for symmetric ($N = Z$) nuclei and significantly softer ones for the neutron-rich isotopes (see Fig. 1). Unfortunately, the Hybrid model—and others like it \[43]—can only reproduce the GMR energies in Tin at the expense of significantly underestimating the GMR energy in $^{208}\text{Pb}$. In the case of pairing correlations, the explanation is based on the conjecture that a superfluid—such as the open-shell Tin isotopes—may be easier to compress than a normal fluid \[51]. Whereas the validity of this statement is both interesting and presently unknown, recent Quasiparticle RPA (QRPA) calculations seem to support the conjecture—at least in part \[50, 51]. “At least in part” because although pairing correlations yield an appreciable softening for the lighter isotopes (from $^{112}\text{Sn}$ to $^{120}\text{Sn}$), the discrepancy for the heavier ones ($^{122}\text{Sn}$ and $^{124}\text{Sn}$) remains large \[50, 51]. This indicates that pairing correlations can not account for the observed softening of the GMR energies with nucleon asymmetry. To make matters worse, we now argue that the rapid softening displayed by the experimental GMR energies in the Tin isotopes may be even harder to explain as one incorporates the physics of dilute neutron matter.

V. LOW-DENSITY NEUTRON MATTER

By building on the universal behavior of dilute Fermi gases with an infinite scattering length \[57, 58, 59, 60], significant progress has been made in constraining the equation
FIG. 3: (color online) Equation of state of pure neutron matter as predicted from a variety of microscopic models [52, 53, 54, 55, 56]. Also shown are predictions from the relativistic mean-field models FSUGold [17], NL3 [10, 48], and Hybrid [21].

of state of pure neutron matter. One of the biggest challenges in understanding dilute neutron matter arises from the non-negligible effective range of the neutron-neutron ($nn$) interaction ($r_e = +2.7$ fm) which, although significantly smaller than the scattering length ($|a| = 18.5$ fm), induces important corrections to the universal behavior at relatively low densities ($k_F \approx 1/r_e \lesssim 0.4$ fm$^{-1}$). To date, a host of models using different $nn$ interactions and a variety of many-body techniques have been employed to compute the EOS of dilute neutron matter (see Fig. 3). These models range from the venerated equation of state of Friedman and Pandharipande [52], to those based on modern effective field theory approaches [53, 54], to those using sophisticated “ab-initio” Monte Carlo techniques [55, 56], to name just a few (for a more comprehensive list see Ref. [56]). Remarkably, all these microscopic models are fairly consistent with each other.

Also shown in Fig. 3 are the predictions from three relativistic mean-field models whose parameters have been fitted directly to various properties of finite nuclei. By directly fitting to the experimental data, the parameters of these models encode physics (such as few- and many-body correlations) that goes beyond a simple single-particle picture. In this regard, the underlying parameters of the model may have, at best, a tenuous connection to those appearing in microscopic descriptions of the nucleon-nucleon (NN) interaction. As a result, if the mean-field models are not sufficiently constrained by experimental data, they can predict behavior that is inconsistent with microscopic approaches—and with nature. This is clearly the case for the NL3 and Hybrid models displayed in Fig. 3 and for most of
the older relativistic parametrizations. (Note that the energy of pure neutron matter is to a very good approximation equal to the energy of symmetric nuclear matter plus the symmetry energy). Given that existing ground-state observables do not place stringent constraints on the isovector $NN$ interaction, most relativistic mean-field models predict a stiff symmetry energy, namely, one that increased rapidly with density for $\rho \gtrsim 0.1 \text{ fm}^{-3}$ (see the large values of $L$ and $K_{\text{sym}}$ listed in Table I). In contrast, the FSUGold parametrization incorporates collective modes directly into the fit, including GMR energies for both $^{90}\text{Zr}$ (with $\alpha = 0.11$) and $^{208}\text{Pb}$ (with $\alpha = 0.21$). As a result, a significantly softer symmetry energy ensues. And although there is no a-priori guarantee, it is gratifying to observe that the softening of the symmetry energy displayed by the FSUGold model is consistent with the EOS predicted by the various microscopic approaches (see Fig. 3). We suggest that the equation of state of pure neutron matter provides a powerful constraint that should be routinely and explicitly incorporated into future determinations of energy density functionals.

So what is the connection between the largely model-independent equation of state of dilute neutron matter and the incompressibility coefficient of neutron-rich matter? To appreciate this connection we first combine Eqs. (1) and (2) to write the incompressibility coefficient of asymmetric matter as

$$K_0(\alpha) = K_0 + \left( K_{\text{sym}} - 6L - \frac{Q_0}{K_0}L \right) \alpha^2 + O(\alpha^4) .$$

Then, as the energy of pure neutron matter is to an excellent approximation equal to the sum of the energy of symmetric matter plus the symmetry energy, the EOS of pure neutron matter around saturation density may be expressed in terms of a conveniently defined dimensionless parameter $x = (\rho - \rho_0)/3\rho_0$ and the same bulk parameters appearing in Eq. (5). That is,

$$E_{nm}/N = (\varepsilon_0 + J) + Lx + \frac{1}{2}(K_0 + K_{\text{sym}})x^2 + \frac{1}{6}(Q_0 + Q_{\text{sym}})x^3 + \ldots$$

Thus, the evolution of the incompressibility coefficient with nucleon asymmetry may be parametrized in terms of two bulk parameters of the EOS of symmetric nuclear matter ($K_0$ and $Q_0$) and the slope and curvature of either the symmetry energy ($L$ and $K_{\text{sym}}$) or of pure neutron matter ($L_{nm} \equiv L$ and $K_{nm} = K_0 + K_{\text{sym}}$). That is,

$$K_\tau = K_{\text{sym}} - 6L - \frac{Q_0}{K_0}L = K_{nm} - 6L_{nm} - \frac{K_0^2 + Q_0L_{nm}}{K_0} .$$

This establishes a strong correlation between $K_\tau$ and the density dependence of the EOS of pure neutron matter (through $L_{nm}$ and $K_{nm}$). In most accurately-calibrated models the dominant contribution to $K_\tau$ comes from the slope of the symmetry energy $L = L_{nm}$ \cite{61, 62}. Indeed, in the three relativistic mean-field models considered here the dominant term ($6L$) accounts for at least 75% of the value of $K_\tau$. Given this fact, we believe that values as large as $|K_\tau| \approx 550 \text{ MeV}$—as seem to be suggested by experiment \cite{20, 22}—are inconsistent with the behavior of dilute neutron matter.

VI. CONCLUSIONS

The present contribution centered around the recently measured distribution of isoscalar monopole strength in the Tin isotopes \cite{20, 22}. This critical experiment suggests a significant
softening of the GMR energies that is unexplained by existing theoretical models. Before the publication of the experimental data in 2007 [20, 22], there was strong evidence in support of a value of the incompressibility coefficient of symmetric nuclear matter around $K_0 = 240 \pm 10$ MeV. However, the measurement on the Tin isotopes has forced us to pause and re-examine our models. Particularly confusing is the fact that some of these accurately-calibrated models are successful in reproducing the GMR energies in $^{90}$Zr, $^{144}$Sm, and $^{208}$Pb.

So why is Tin so soft and why does it become even softer with an increase in the neutron excess? One possible explanation relies on the open-shell structure of the Tin isotopes and its assumed superfluid character [50, 51]. Although this approach has met with some success, the impact of pairing correlation on the heavy Tin isotopes ($^{122}$Sn to $^{124}$Sn) is modest so the discrepancy remains. Another approach—encapsulated in the Hybrid model discussed above and introduced in Ref. [21]—adopts a small value for the incompressibility coefficient of symmetric matter and a large value for the slope of the symmetry energy. This “soft-stiff” combination is fairly successful in describing the rapid softening of the GMR energies in the Tin isotopes. However, the same model significantly underestimates the GMR energy in $^{208}$Pb. Moreover, the EOS of neutron matter generated by the Hybrid model—and essential to reproduce the rapid softening of the GMR in the Tin isotopes—is inconsistent with microscopic models that based their predictions in the universality of dilute Fermi gases.

In conclusion, the distribution of isoscalar monopole strength in the Tin isotopes remains an important open problem in nuclear structure. As one attempts to solve this difficult problem, one must remember that the challenge is not solely to describe the distribution of monopole strength in the Tin isotopes, but rather, to do so while simultaneously describing a host of ground-state observables, collective modes, and the equation of state of low-density neutron matter.

Acknowledgments

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