Status of Eikonal Two-Loop Calculations with Massive Quarks

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Abstract

We present results for two-loop diagrams with massive quarks in the eikonal approximation. Explicit expressions are given for the UV poles in dimensional regularization of several of the required integrals.

1 Introduction

The calculation of threshold corrections to hard scattering cross sections beyond leading logarithms requires the calculation of loop diagrams in the eikonal approximation [1]. One-loop calculations have been performed for all $2 \rightarrow 2$ partonic processes in heavy quark [2] and jet [3] production. The soft anomalous dimension matrix $\Gamma_S$ at one-loop allows the resummation of soft-gluon corrections at next-to-leading logarithm (NLL) accuracy [2]. The exponentiation follows from the renormalization group evolution of $\Gamma_S$ and involves the calculation of the ultraviolet (UV) poles in dimensional regularization of one-loop diagrams with eikonal lines. To extend resummation to next-to-next-to-leading logarithms (NNLL) two-loop calculations are required. For massless quark-antiquark scattering the two-loop $\Gamma_S$ was completed in [4]. For heavy quark production, however, the result is not known. In this contribution we present several results for two-loop diagrams involved in the calculation of the two-loop $\Gamma_S$ for massive quarks. In the eikonal approximation the usual Feynman rules are simplified by letting the

\[ p + k \]

\[ p \]

\[ k \rightarrow 0 \]

\[ p_i + k \]

\[ p_i \]

\[ k \]

\[ p_j - k \]

\[ p_j \]

Figure 1: The eikonal approximation (left) and a one-loop diagram (right).

\[1\]Presented at DIS 2008, London, England, April 7-11, 2008
gluon momentum approach zero (left diagram in Fig. 1):

$$
\bar{u}(p) (-ig_s T_F^\alpha) \gamma^\mu \frac{i(y^\mu + k^\mu + m)}{(p+k)^2 - m^2 + i\epsilon} \to \bar{u}(p) g_s T_F^\alpha \gamma^\mu \frac{y^\mu + m}{2p \cdot k + i\epsilon} = \bar{u}(p) g_s T_F^\alpha \frac{v^\mu}{v \cdot k + i\epsilon}
$$

with \( p \propto v \), and \( T_F^\alpha \) the generators of SU(3).

### 2 One-loop and two-loop diagrams

We perform our calculation for eikonal massive quarks in Feynman gauge using dimensional regularization with \( n = 4 - \epsilon \).

We begin with the one-loop diagram in Fig. 1. The momentum integral is given by

$$
I_{1l} = g_s^2 \int \frac{d^n k}{(2\pi)^n} \frac{(-i)g^{\mu\nu} v_i^\mu}{k^2} \frac{(-v_j^\nu)}{v_i \cdot k}
$$

Using Feynman parametrization, followed by integration over \( k \), and after several manipulations, we find

$$
I_{1l} = \frac{\alpha_s}{\pi} \frac{(1 + \epsilon^2/2)}{2} 2\epsilon/2 \pi^{\epsilon/2} \Gamma \left(1 + \frac{\epsilon}{2}\right) (1 + \beta^2) \int_0^1 dx x^{-1+\epsilon}(1-x)^{-1-\epsilon}
\times \left\{ \int_0^1 dz \left[ 4z\beta^2(1-z) + 1 - \beta^2 \right]^{-1} - \frac{\epsilon}{2} \int_0^1 dz \ln \left[ 4z\beta^2(1-z) + 1 - \beta^2 \right] + \mathcal{O}(\epsilon^2) \right\}
$$

where \( \beta = \sqrt{1 - 4m^2/s} \). The integral over \( x \) contains both UV and infrared (IR) singularities. We isolate the UV singularities, \( \int_0^1 dx x^{-1+\epsilon}(1-x)^{-1-\epsilon} = \frac{1}{\epsilon} + \text{IR} \), and find the UV pole and constant terms at one loop:

$$
I_{1l}^{UV} = \frac{\alpha_s}{\pi} \frac{(1 + \beta^2)}{2\beta} \left\{ \frac{1}{\epsilon} \ln \left( \frac{1 - \beta}{1 + \beta} \right) + \frac{1}{2} \left( 4\ln 2 + \ln \pi - \gamma_E - i\pi \right) \ln \left( \frac{1 - \beta}{1 + \beta} \right) + \frac{1}{4} \ln^2(1 + \beta) - \frac{1}{4} \ln^2(1 - \beta) - \frac{1}{2\ln 2} \left( \frac{1 + \beta}{2} \right) + \frac{1}{2\ln 2} \left( \frac{1 - \beta}{2} \right) \right\}
$$

More details on this one-loop integral are given in [5]. We now continue with the two-loop diagrams (these are the eikonal versions of the diagrams involved in the calculation of the two-loop heavy quark form factor [6]). In Fig. 2, we show a diagram with two gluons exchanged between the massive quarks (left) and the crossed diagram (right). We denote by \( I_1 \) the integral for the first diagram and by \( I_2 \) that for the crossed diagram. We have

$$
I_1 = g_s^4 \int \frac{d^n k_1}{(2\pi)^n} \frac{d^n k_2}{(2\pi)^n} \frac{(-i)g^{\mu\nu} (-i)g^{\rho\sigma}}{k_1^2 k_2^2} \frac{v_i^\mu}{v_i \cdot k_1} \frac{v_j^\rho}{v_j \cdot (k_1 + k_2)} \frac{(-v_j^\nu)}{-v_j \cdot k_1 - v_j \cdot (k_1 + k_2)}
$$

We note that \( I_1 \) is symmetric under \( k_1 \leftrightarrow k_2 \) as is the integral for the crossed diagram, \( I_2 \). Utilizing the properties of these two integrals and the one-loop integral, \( I_{1l} \), we find the relation

$$
I_1 = \frac{1}{2} (I_{1l})^2 - I_2.
$$
Therefore $I_1$ is determined once we calculate $I_2$. For the crossed diagram, we have

$$I_2 = g_s^4 \int \frac{d^n k_1 \, d^n k_2 \, (-i) g^{\mu\nu} (-i) g^{\rho\sigma}}{(2\pi)^n (2\pi)^n} \frac{v_i^\mu v_i^\rho}{\varepsilon} \frac{v_j^\nu v_j^\sigma}{v_i \cdot k_1 \varepsilon} \frac{(-v_j^\nu)}{v_j \cdot (k_1 + k_2)} - \frac{(-v_j^\sigma)}{v_j \cdot k_2}.$$ 

We begin with the $k_2$ integral and after some work find

$$I_2 = -i \frac{\alpha_s^2}{\pi^2} 2^{-4+\epsilon} \pi^{-2+3\epsilon/2} \Gamma \left( 1 - \frac{\epsilon}{2} \right) \Gamma \left( 1 + \epsilon \right) (1 + \beta^2)^2 \int_0^1 dz \times \int_0^1 dy \frac{1}{(1-y)^{-\epsilon}} \left[ \frac{2\beta^2 (1-y)^2 z^2 - 2\beta^2 (1-y) z - \frac{(1-\beta^2)}{2} }{k_1^2 v_i \cdot k_1 ((v_i - v_j) z + v_j) \cdot k_1} \right]^{1+\epsilon}.$$

Now we proceed with the $k_1$ integral and separate the UV and IR poles. After many steps, we find the $1/\epsilon^2$ and $1/\epsilon$ UV poles of $I_2$:

$$I_2^{UV} = -i \frac{\alpha_s^2 (1 + \beta^2)^2}{\pi^2 8 \beta^2} \frac{1}{\epsilon} \left\{ \ln \left( \frac{1 - \beta}{1 + \beta} \right) \left[ 2 \text{Li}_2 \left( \frac{2\beta}{1 + \beta} \right) + 4 \text{Li}_2 \left( \frac{1 - \beta}{1 + \beta} \right) + 2 \text{Li}_2 \left( \frac{- (1 - \beta)}{1 + \beta} \right) - \ln(1 + \beta) \ln(1 - \beta) - \zeta_2 \right] \right. $$

$$\left. - 2 \ln^2 \left( \frac{1 - \beta}{1 + \beta} \right) \ln \left( \frac{1 + \beta}{2} \right) + \frac{1}{3} \ln^3(1 - \beta) - \frac{1}{3} \ln^3(1 + \beta) - \text{Li}_3 \left( \frac{(1 - \beta)^2}{(1 + \beta)^2} \right) + \zeta_3 \right\}.$$

We now proceed with the diagrams in Fig. 3 that involve internal quark and gluon loops. For the quark loop we find

$$I_{ql} = (-1) n_f g_s^4 \int \frac{d^n k \, d^n l \, v_i^\mu \, (-v_j^\rho)}{k^2} (-i) g^{\mu\nu} (-i) g^{\rho\sigma} \frac{(-v_j^\rho)}{k^2} \text{Tr} \left[ -i \gamma^\nu \frac{i l}{l^2} (-i) \gamma^\sigma i \frac{(l - k)}{(l - k)^2} \right].$$

After many steps (see also [5]) we extract the UV poles

$$I_{ql}^{UV} = -n_f \frac{\alpha_s^2 (1 + \beta^2)}{\pi^2 6 \beta} \left\{ \frac{1}{\epsilon^2} \ln \left( \frac{1 - \beta}{1 + \beta} \right) + \frac{1}{\epsilon} \left[ - \text{Li}_2 \left( \frac{1 + \beta}{2} \right) + \text{Li}_2 \left( \frac{1 - \beta}{2} \right) \right. $$

$$\left. + \frac{1}{2} \ln^2(1 + \beta) - \frac{1}{2} \ln^2(1 - \beta) + \frac{5}{6} + 4 \ln 2 + \ln \pi - \gamma_E - i\pi \right] \ln \left( \frac{1 - \beta}{1 + \beta} \right) \right\}.$$
For the gluon-loop integral, we have

\[
I_{gl} = \frac{1}{2} g_s^4 \int \frac{d^n k}{(2\pi)^n} \frac{d^n l}{(2\pi)^n} \frac{v_i}{v_j \cdot k} \frac{(-v^\nu)}{k^2} \frac{(-i)g^{\mu\nu}}{(k-l)^2} \frac{(-i)g^{\rho\sigma}}{k^2} \times \left[ g^{\rho\sigma}(k+l)^\sigma + g^{\rho\sigma}(k-2l)^\sigma + g^{\rho\sigma}(-2k+l)^\rho \right] \times \left[ g^{\rho\nu}(l+k)^\sigma + g^{\rho\nu}(-2k+l)^\rho + g^{\rho\nu}(k-2l)^\nu \right].
\]

We calculate the UV poles and find

\[
I_{UV}^{gl} = -\frac{19}{96} \frac{\alpha_s^2 (1 + \beta^2)}{\beta} \left\{ \frac{1}{\epsilon^2} \ln \left( \frac{1 - \beta}{1 + \beta} \right) + \frac{1}{\epsilon} \left[ -\text{Li}_2 \left( \frac{1 + \beta}{2} \right) + \text{Li}_2 \left( \frac{1 - \beta}{2} \right) \right] + \frac{1}{2} \ln^2(1 + \beta) - \frac{1}{2} \ln^2(1 - \beta) + \left( \frac{58}{57} + 4 \ln 2 + \frac{\ln \pi - \gamma_E - i\pi}{2} \right) \ln \left( \frac{1 - \beta}{1 + \beta} \right) \right\}. 
\]

We also have to add a diagram to those in Fig. 3 involving a ghost loop. The corresponding integral is

\[
I_{gh} = (-1) g_s^4 \int \frac{d^n k}{(2\pi)^n} \frac{d^n l}{(2\pi)^n} \frac{v_i}{v_j \cdot k} \frac{(-v^\rho)}{l^2} \frac{(-i)g^{\mu\rho}}{k^2} \frac{(-i)g^{\nu\sigma}}{k^2} \times \left[ g^{\mu\sigma}(l+k)^\sigma + g^{\mu\sigma}(-2k+l)^\rho + g^{\mu\rho}(k-2l)^\nu \right].
\]

and a calculation of its UV poles gives

\[
I_{UV}^{gh} = -\frac{\alpha_s^2 (1 + \beta^2)}{\pi^2} \left\{ \frac{1}{\epsilon^2} \ln \left( \frac{1 - \beta}{1 + \beta} \right) + \frac{1}{\epsilon} \left[ -\text{Li}_2 \left( \frac{1 + \beta}{2} \right) + \text{Li}_2 \left( \frac{1 - \beta}{2} \right) \right] + \frac{1}{2} \ln^2(1 + \beta) - \frac{1}{2} \ln^2(1 - \beta) + \left( \frac{4}{3} + 4 \ln 2 + \frac{\ln \pi - \gamma_E - i\pi}{2} \right) \ln \left( \frac{1 - \beta}{1 + \beta} \right) \right\}. 
\]

We also note that the integral for another diagram involving an internal gluon loop with a four-gluon vertex vanishes.

There are additional diagrams not discussed here, also including self-energies and counterterms. The color factors for all diagrams have been calculated and must be accounted for in the final result.
Acknowledgements

The work of N.K. was supported by the National Science Foundation under Grant No. PHY 0555372.

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