Construction of the approximant of complete diagram for rock deformation

A B Tsvetkov, L D Pavlova and V N Fryanov
Siberian State Industrial University, 42 Kirova Street, Novkuznetsk, Russia, 654007
E-mail: atsvet@mail.ru

Abstract. The function for approximation of the diagram of rock deformation, intended for nonlinear model of stress-strain state of coal geomassive, is developed, which is realized in the package of problem-oriented software. The computing experiments and the analysis of outcomes of numerical modeling are performed, confirming the correspondence of the calculated results with the measured settlings of underworked seams.

1. Introduction
Intensive technogenic influence on the geomassive leads to formation in the periphery areas of coal seam and in the zones of increase pressure of such geodynamic phenomena, as mine bumps, sudden outbursts of coal, rock and gas, caving and dynamic settlings of roof rocks with release from the goaf of dangerous gases in the form of shock air waves. The pointed phenomena limit the efficiency of the mechanized stopes equipped with expensive machines. As coal seam excavation leads to formation of discharge and bearing pressure zones around goaf, in which the rocks possess various resistance to extending and to compressive forces, then the consideration of a multimodule rock properties, the deformation law of which is defined by nonlinear equations, will allow qualitatively new information to be obtained about geomechanical state in the field of mine workings influence.

To solve this problem authors developed a nonlinear mathematical model of stress-strain state of geomassive [1]. For its defining correlations, constructed by the variable parameter method, it is necessary to elaborate function approximating a complete diagram of deformation.

The aim of the present work is to develop the function, approximating the complete diagram of rock deformation, which is constructed by the basic points, separating fields of elastic, elastoplastic, limiting and out-of-limit deformation.

2. Construction of approximants and computing experiments
The physical equations of the problem of plasticity theory in the process of linearization can be written in the form of formulas [2]:

\[
\begin{align*}
\varepsilon_x &= \frac{1}{E} \left[ \sigma_x - \nu \left( \sigma_y + \sigma_z \right) \right] ; \\
\gamma_{xy} &= \frac{1}{\mu} \tau_{xy} ; \\
\varepsilon_y &= \frac{1}{E} \left[ \sigma_y - \nu \left( \sigma_x + \sigma_z \right) \right] ; \\
\gamma_{yz} &= \frac{1}{\mu} \tau_{yz} ;
\end{align*}
\] (1)
The equations are:

\[ \varepsilon_z = \frac{1}{E} \left[ \sigma_z - v (\sigma_x + \sigma_y) \right] \]

\[ \gamma_{zx} = \frac{1}{\mu} \tau_{zx} \]

which provide recalculation of variable parameters according to dependencies:

\[ E^* = \frac{3E}{2E \psi + 1 - 2\nu} \]
\[ \nu^* = \frac{1}{1 + \frac{1 - 2\nu}{E \psi}} \]
\[ \mu^* = \frac{1}{\frac{1 - 2\nu}{E \psi}} \]

where (\( e_\alpha, e_\beta, e_\gamma, \gamma_{\alpha\beta}, \gamma_{\alpha\gamma} \)) – components of a deformation tensor; (\( \sigma_e, \sigma_\alpha, \sigma_\beta, \sigma_\gamma, \tau_{\alpha\beta}, \tau_{\alpha\gamma}, \tau_{\gamma\alpha} \)) – components of a stresses tensor; \( E \) – modulus of elasticity; \( \nu \) – Poisson’s ratio; \( E^*, \nu^*, \mu^* \) – variable parameters. \( \psi = \frac{3e_\alpha}{2\sigma} \) – dependence of the generalized stresses \( \sigma \) on the generalized deformations \( e \); \( \sigma = \sigma(e) \) – function, defining the complete diagram of rock deformation.

On the basis of research in [3], the typical complete diagram of “stress-strain” has the form as of a diagram in Figure 1. On the complete diagram of deformation the characteristic points are marked: \( (\varepsilon_e, \sigma_e), (\varepsilon_s, \sigma_s), (\varepsilon_o, \sigma_o) \), where \( \sigma_e \) – the stresses corresponding to elastic limit; \( e_e \) – limiting elastic deformations; \( \sigma_s \) – ultimate rock strength at contraction or extension; \( e_s \) – limiting contraction deformations; \( \sigma_o \) – residual stresses; \( e_o \) – residual deformations.

**Figure 1.** The complete diagram of “stress-strain”.

For the analytical representation of diagrams of rock deformation the dependence in the form of rocks association in the form piecewise function is offered:

\[ \sigma(e) = \begin{cases} 
\sigma_e(e), & 0 \leq e < e_e; \\
\sigma_s(e), & e_e \leq e < e_s; \\
\sigma_m(e), & e_s \leq e < e_o; \\
\sigma_o, & e \geq e_o.
\end{cases} \]

where \( \sigma \) – generalized stresses; \( e \) – generalized deformations.

For stresses approximation \( \sigma(e) \) on each section the functions \( \sigma_e(e), \sigma_s(e) \) are elaborated. In the field of elastic deformation the connection between stresses and deformations is defined by the Hooke’s law.
\[ \sigma_e (\varepsilon) = E \cdot \varepsilon, \]  
\hspace{1cm} (4) 

where \( E \) – modulus of elasticity.

The elastoplastic section of the deformation diagram is defined by the parabola, the coefficients of which are calculated from the conditions of conformity with a straight line (4) in a point \((\varepsilon_e, \sigma_e)\) of magnitudes of stresses and the first order derivatives

\[ \sigma_s (\varepsilon) = \sigma_e + \left( \sigma_s - \sigma_e \right) \frac{\varepsilon - \varepsilon_e}{\varepsilon_s - \varepsilon_e} - \frac{(E + E^*)}{2(\varepsilon_s - \varepsilon_e)} \cdot \varepsilon \cdot (\varepsilon - \varepsilon_s), \]  
\hspace{1cm} (5) 

where \( E^* \) – deformation modulus, matching the limiting and out-of-limit diagram sections of rock deformation, which is defined by the dependence

\[ E^* = E - \frac{2(\sigma_s - \sigma_e)}{(\varepsilon_s - \varepsilon_e)}. \]  
\hspace{1cm} (6) 

Taking into account formula (6) the function \( \sigma_s (\varepsilon) \) will be

\[ \sigma_s (\varepsilon) = \sigma_e + \left( \sigma_s - \sigma_e \right) \frac{\varepsilon - \varepsilon_e}{\varepsilon_s - \varepsilon_e} + \frac{(\sigma_s - \sigma_e) - E(\varepsilon_s - \varepsilon_e)}{(\varepsilon_s - \varepsilon_e)^2} \cdot (\varepsilon - \varepsilon_s). \]  
\hspace{1cm} (7) 

The section of out-of-limit deformation is approximated by straight lines and matched with the parabola in the point \((\varepsilon_s, \sigma_s)\):

\[ \sigma_m (\varepsilon) = \sigma_o + \left( \sigma_s(\varepsilon) - \sigma_o \right) \frac{\varepsilon - \varepsilon_o}{\varepsilon_s - \varepsilon_o}, \]  
\hspace{1cm} (8) 

The experimental diagrams of rock deformation are given in [4]. For evaluation of the results reconciliation with the in-situ data, the graphs were compared, and the correlation dependence between them estimated.

In figures 2a, b the graphs of experimental diagrams were compared for sandstones presented in in [4] and approximations of their piecewise functions constructed with the use of dependences (3) – (8).
Figure 2. Approximation of experimental diagrams of sandstones deformation.

From the analysis of the graphs in Figures 2a, b the reconciliation of the results with the in-situ data is observed. The correlation factor is 0.994 and 0.996, respectively.

In Figure 3 the deformation diagrams for coal stretching and coal compression are given, constructed by the characteristic points \((\varepsilon_e, \sigma_e), (\varepsilon_s, \sigma_s), (\varepsilon_o, \sigma_o)\) with the use of formulas (3) – (8).

Figure 3. Deformation diagram of coal extension and contraction.

Approximant (3) is used in nonlinear mathematical model [1] of deformation of rocks in the process of their transition from elastic into elastoplastic, limiting and out-of-limit states.

To carry out calculation experiments the complex of the problem-oriented software was developed [6] which allow the numerical solutions of nonlinear problems of geomechanics to be obtained for evaluation of the stress-strain state of geomassive being under the technogenic influence and with the help of the developed modules to carry out all stages of the computing experiment.

To define quantitative reconciliation of numerical outcomes with the in-situ data computing experiments were performed, for which the mining and geological and technical conditions of seam 22 at the mine “Yubileynaya” in Kuzbass were accepted. The outcomes of numerical solution were compared with the in-situ data of vertical movements recorded during advancement of the
mechanized face from the installation chamber in the direction of the coal massive by rappers of the observant station R1–R7, established in the well at different depths (Figure 4).

![Diagram of rappers location](image)

**Figure 4.** Scheme of a rappers location in front of the advancing face, seam 22, mine “Yubileynaya”, Kuzbass.

In Figure 5 the graphs of vertical movements are shown, constructed according to the in-situ data ($V^*(R1)$–$V^*(R7)$) and the results of computing experiments ($V(R1)$–$V(R7)$).

![Graphs of vertical movements](image)

**Figure 5.** The vertical movements of rappers received by the data of mine measurements ($V^*$) and the results of computing experiments ($V$).
3. Results
As a quantitative measure of reconciliation of numerical modelling results with the in-situ data it is offered to use integral criterion of the form

\[
I = \sqrt{\frac{\int_0^L (f^{\text{solve}}(x) - f^{\text{nat}}(x))^2 \, dx}{\int_0^L f^{\text{nat}}(x)^2 \, dx}} ,
\]

where \( L \) – length of the goaf; \( f^{\text{nat}}(x) \) – the function approximating the in-situ data; \( f^{\text{solve}}(x) \) – the function approximating the results of numerical modelling.

For evaluation of the reconciliation of numerical results with in-situ data by criterion (9) the approximant of the following form are constructed:

\[
f^{\text{nat}}(x) = \sum_{i=1}^{N} a_i^{\text{nat}} \left| x - x_i \right| ; \quad f^{\text{solve}}(x) = \begin{cases} \sum_{i=1}^{N} a_i^{\text{e}} \left| x - x_i \right| , & \text{for solution of a problem of the elastic theory} \\ \sum_{i=1}^{N} a_i^{\text{nonlinear}} \left| x - x_i \right| , & \text{for nonlinear solution} \end{cases}
\]

where \( N \) – number of observations; \( a_i^{\text{nat}} \) – coefficients defined from the solution of equations system, constructed with use of mine measurements; \( a_i^{\text{nonlinear}}, a_i^{\text{e}} \) – coefficients defined for solutions of a nonlinear problem and a boundary value problem of the elasticity theory.

According to integral criterion (9) with the use of the constructed dependencies (10) the maximum relative error of numerical modeling is calculated, which for the results of the computing experiments, received by means of the elaborated nonlinear model, amounted 0.21, and for the known model constructed by the the defining proportions of the elastic theory – 0.90. The carried out analysis of results of modelling shows the adequacy of the constructed nonlinear model to the research subject.

4. Conclusions
The function, approximating the complete diagram of rock deformation, is developed. By its means in the nonlinear model of stress-strain state of coal-bearing massive [1], constructed by the method of variable parameters, the value of the intersecting modulus of deformation in equations (1) are algorithmically defined. Application of nonlinear mathematical model of a geomechanical state of the coal massive, including the elaborated piecewise function and the complex of the problem-oriented software developed on its ground, ensure results corresponding to the real data and allows the predictive estimates of stress-strain state of geomassive in the affected area of mine workings to be performed.

The developed software complex can be applied to solve the problems of mathematical modelling, linked with:

- Performance of computing experiments for evaluation of a geomechanical state in the vicinities of goaf;
• Forecast of geomechanical parameters of underground mining technology in the process of
design documentation preparation.

The recommended requirements to hardware and software providing functioning of the program
packages:
• processor – Intel Core i5;
• operative memory – 8 Gb;
• empty space on the hard disk – 10 Gb;
• operating system – Microsoft Windows XP or later versions;
• package of symbolic mathematics – Mathematica 9;
• package for launching software on other computers written in the package Mathematica -
  Wolfram CDF Player Pro.

References
[1] Tsvetkov A B, Pavlova L D and Fryanov V N 2015 GIAB 1 365–370
[2] Pisrenko G S and Mozharovsky N S 1981 The Equations and Boundary Value Problems of the
  Theory of Plasticity and Creep (Kiev: Naukova dumka) p 496.
[3] Petukhov I M and Linkov A M 1983 Mechanics of Pressure Bumps and Outbursts (M.:Nedra) p
  280
[4] Stavrogin A N and Protosenya A G 1979 Plasticity of Rocks (M.: Nedra) p 301
[5] Tsvetkov A B and Pavlova L D 2015 Proc. of the Int. Conf. on High Technologies of
  development and Use of Mineral Resources (Novokuznetsk) pp 121–125
[6] Tsvetkov A B and Pavlova L D 2016 Proc. of the IV All-Russian Conf. on Modelling and the
  Information Technologies in Engineering and Social and Economic Systems (Novokuznetsk:
  SibSIU) pp 175–178