Capture of material by the protosatellite disks of Jupiter and Saturn

V A Kronrod1, A B Makalkin2
1Vernadsky Institute of Geochemistry and Analytical Chemistry (GEOKHI RAS), 19, Kosygin str., Moscow 119991, Russia
2The Schmidt Institute of Physics of the Earth of the Russian Academy of Sciences (IPE RAS), Bolshaya Gruzinskaya str., 10-1, Moscow 123242, Russia

E-mail: va_kronrod@mail.ru

Abstract. The problem of passing planetesimals through the accretion disks of Jupiter and Saturn and capture of their material into the disks is considered taking into account processes of aerodynamic braking, fragmentation, and ablation of the bodies in the disk’s gas medium. We have obtained limitations on planetesimal sizes at which the body remains in the protosatellite disk. Estimates are made for the mass of material captured into the disk for the case of power-law mass (and size) distribution of planetesimals. The narrower size range for the captured bodies and longer duration of satellite formation could yield the low differentiation of Callisto as compared with Ganymede. Only minor planetesimals with radii less than 12 m could be captured into the disk of Saturn in the formation region of Titan. This feature promoted significant lengthening of the process of Titan formation.

1. Introduction
Research missions successfully carried out by the Galileo and Cassini-Huygens orbital space stations made it possible to substantially clarify the data on the morphology of the ice surface of the satellites, their physical characteristics, and information on gravitational, magnetic and thermal fields. On the basis of this factual, analytical and calculated information, modern models of the internal structure of Ganymede, Callisto and Titan [1-5] were built. It can be considered established that Ganymede underwent a process of full differentiation into a metallic Fe-FeS core, a silicate mantle and a powerful ice-water shell [2].

Neighboring Ganymede Callisto has a similar size and density. However, the values of the moments of inertia, based on the gravitational field data, show that the satellite consists of an ice shell, an undifferentiated stone-ice mantle and a silicate core [3]. Titan, as shown in [4, 5], can also have undifferentiated rock-ice mantle. There is reason to believe that the degree of differentiation of ice satellites depends on the accretion processes of the satellite - the accretion time, the mass of planetesimals falling on the growing satellite per unit of time, the distribution of these bodies in size and composition.

The model of an accretionary protosatellite disk of small mass at any given time contains only \(10^{-3}-10^{-2}\) of the total mass of regular satellites in dust particles and small bodies [6-9]. The mass necessary for the formation of satellites is accumulated by them gradually, as the capture of solid particles entering...
the protosatellite disk together with gas from the surrounding area of the protoplanetary disk. Therefore, the conditions for the existence of a small massive accretion disk imply the existence of a constant mass inflow into the disk in the form of dust particles and planetesimals trapped by the gravitational field of the central planet.

This paper discusses the problem of calculating the interaction of planetesimals falling on the surface of an accretion disk from the zone of gravitational influence of the central planet with a disk. It is assumed that the solution of this problem will allow to estimate the mass and composition of the body, falling on the growing ice satellites, and explain the differences in the average density and internal structure of ice satellites in the systems of the giant planets Jupiter and Saturn. The multiparameter problem of braking, destruction and ablation of planetesimals in the gaseous medium of an accretion disk is solved by numerical simulation methods using modified approaches of meteoric physics [10-11]. The equations of motion and ablation of mass as a result of ablation were written in the form [10]. The task of fra planetesimal during the passage of a disk from aerodynamic loads is solved within the framework of the known models of destruction of a meteorite when entering the atmosphere [12, 13].

2. The parameters of the protosatellite disks of Jupiter and Saturn.

The dependence of the disk surface density $\Sigma_g$ on the radial coordinate in the disk $r$ is determined by the structure models of the accretion proto-satellite disks of Jupiter and Saturn [7,8]. For models that are consistent with observational and theoretical data [7], the dependence $\Sigma_g(r)$ can be approximated by a power function

$$\Sigma_g = \Sigma_{20} \left( \frac{r}{20R_p} \right)^{-3/4},$$

(1)

where $R_p$ is the average radius of the central planet, $\Sigma_{20}$ is the surface density of the disk at a distance of 20 radii of the planet. Approximate dependence of gas density in the mid-plane of the disk $\rho_{g0}$ on the radial coordinate:

$$\rho_g = \rho_{20} \left( \frac{r}{20R_p} \right)^{-7/4},$$

(2)

where $\rho_{20} = 6 \times 10^{-9}$ g / cm$^3$ and $2 \times 10^{-9}$ g / cm$^3$ are the gas density in the mid-plane protosatellite disk moves inside the disk according to the equation

$$m \frac{dv}{dy} = \alpha \rho gn s v$$

(3)

Here $m=M/M_1$, $v = V/V_1$; $s = S/S_1$; $\rho gn = \rho_g / \rho_{g0}$; $y = z/h$; $\alpha = c_d c_h \Sigma_g S_1 / 4 M_1 \sin \gamma$.

$M$, $V$ is the mass and modulus of the velocity of a moving body in the planetocentric inertial coordinate system, $\rho_g$ is the gas density, $S$ is the mid-section area, $c_d$, $c_h$ are the resistance and heat transfer coefficients, $\rho gn = \rho_g / \rho_{g0}$, $\gamma$ is the angle between tangent to the trajectory and the median plane of the disk. The index “1” is assigned to the parameters at the entrance to the disk, $\varphi$ is the angle at which the body enters the accretion disk, $z_a$ is the height of the uniform atmosphere, $\rho_{g0}$ is the gas density in the equatorial plane of the disk. Equation (3) for calculating the motion of a body in a disk does not take into account the gravitational attraction of the body towards the planet. It can be shown that (3) depends on the direction of the bodies falling relative to the mid-plane of the disk and is valid under restrictions on the radius $R < R_{lim}$ of the body falling on the disk. In the disk of Jupiter (for planetesimals with a substance density of 0.5 g / cm$^3$), $R_{lim}$ reaches values of 250 m and 160 m at distances of Ganymede and Callisto, and 60 m in the area of Titan formation.
3. **Mass loss of planetesimal as a result of ablation.** In the dimensionless form the equation of mass loss can be written in the following form:

\[
\frac{dm}{dy} = -2\alpha \beta \rho g n v^2 s ,
\]

where \( \beta = c_h V_1^2/2\epsilon d H^* \), \( H^* \) is the effective heat of destruction. The parameter \( \beta \) is determined by the ablation coefficient \( \delta_{abl} \) and the speed of falling on the disk \( V_1 \) : \( \beta = \delta_{abl} V_1^2/2 \). Assuming a spherical body with an initial radius of \( R_1 \), we have \( \alpha \approx 0.4 \sum \frac{f}{\rho_m R_1} \). Taking ablation into account for the parameter \( \alpha \), we obtain the relation

\[
\alpha = \frac{1}{2} \int_{v_{\text{min}}}^{v_{\text{max}}} \frac{dv}{v \exp[(\beta/3)(1-v^2)]} \]  

Using the parameter \( \alpha \) calculated according to (5), we obtain the value of the maximum radius of the captured body: \( R_{1\text{max}} \approx 0.4 \sum \frac{f}{\rho_m \alpha} \). In the absence of ablation (ie, with \( \beta = 0 \)), as follows from (5), the parameter \( \alpha \) turns out to be maximum: \( \alpha_{\text{max}} = -0.5 \ln v_{\text{min}} = 0.5 \ln (V_1/\nu_e) \).

4. **Fragmentation of planetesimal by a statistical model.**

According to the statistical theory of strength, large structural defects, leading to the destruction of the body, are much less common for smaller defects. Therefore, the probability of finding a large defect increases with increasing body volume. In [13] the authors examined the magnitudes of the real loads experienced by meteoritic bodies during movement in the atmosphere and compared them with the strength of meteoritic material. According to these estimates, aerodynamic loads never reach the tensile strength. The difference between loads and strength is so great that it is impossible to attribute this effect to simple fluctuations of strength. Therefore, the authors estimate the conditions for the destruction of meteorites from the point of view of the statistical theory of strength, taking into account large-scale factors. The dependence of body strength on volume is expressed by the following relationship:

\[
\sigma_{\text{pr}} = \sigma^* (m^*/M)^{\lambda}, \text{ where } \sigma^* \text{ and } m^* \text{ is the tensile strength and mass of the sample for which the tests were performed, } \sigma_{\text{pr}} \text{ is the tensile strength of the body of the same material but the mass of } M. \text{ Turcotte DL (1986) for ice bodies considers } \lambda = 0.12. \text{ The condition for the destruction of planetesimal: dynamic pressure (velocity pressure) is greater than the stress of destruction } \rho_g V_{rel}^2 \geq \sigma_f, \text{ where } V_{rel} \text{ is the velocity of the body relative to the gas. Hence, the minimum mass of the destroyed body is determined for:}

\[
M_f = m^* \left( \frac{\sigma^*}{\rho_g V_{rel}^2} \right)^{1/\lambda} ,
\]

5. **Fragmentation of planetesimal by the model of Ivanov and Ryzhansky.**

Another approach to the assessment of scale factors is proposed in [12]. According to the model, when the required amount of elastic energy is accumulated, planetesimal falls into two equal parts. The condition of fragmentation for the body of the ball shape is the equality of the elastic energy in the body due to the forces of aerodynamic pressure \( (\rho_g V^4/2E) 4/3\pi R^3 \) of the body’s breaking into two parts
2Y*πR²: ρ₂²l⁴ ≥ 3Y·E/R, where 2Y is the specific (per unit of surface) energy of fragmentation, E is the Young's modulus, R is the radius of the parent body, ρ₂ is the density of the gaseous medium, V is the velocity of the body. To simplify the calculations, it is assumed that the fragments of the destroyed parent body have a spherical shape and, in turn, can collapse while the fracture condition is fulfilled. Estimates of the minimum radius of destruction of the planetesimals:

\[ R_{f,2} = \frac{3\gamma E}{2\rho_g^2 V_{rel}^4}. \]  

(7)

6. Estimates of the mass of the substance captured by the disk.

The distribution of bodies by mass is described by a power law (Safronov, 1969): \( n(M)\,dM = c M^{-q}\,dM \)
where \( n(M)\,dM \) is the number of bodies per unit volume with masses in the interval \((M, M+\,dM)\). For the exponent \( p \), the corresponding radial distribution of bodies is obtained \( p = 3q - 2 \). The theoretical results and the distribution of craters in size give the values of and \( q \approx 11/6 \approx 1.8 \) and \( p=3.5 \). Then the ratio of the mass of the substance evaporated during ablation \( (M_a) \) and therefore trapped in the disk to the total mass of bodies \((M_f)\) with radii \( R_{1,max} < R < R_o \), which, after crossing the disk, left it with a speed \( V_2 \) greater than \( V_c \):

\[ \frac{M_a}{M_f} = \frac{(4-p) R_o}{R_o^{4-p} - R_{1,max}^{4-p} \int_{R_{1,max}}^{R_o} \left(1 - \exp [\beta (v^2 - 1)] \right) R_1^{3-p} \,dR_1}. \]  

(8)

where \( v_2 = V_2 / V_1 \). The value of \( R_o \) in our calculations is given. Finally, let us estimate the ratio of the total trapped mass (consisting of the mass of small bodies trapped in the disk \( M_a \), plus the mass of the substance evaporated during ablation \( (M_a) \), and thus captured in the disk) to the total planetesimals mass \( M_t \) of the substance falling on the disk with a power distribution of sizes from the smallest to bodies with a radius \( R_o \):

\[ \frac{M_c + M_a}{M_t} = \frac{R_{1,max}^{4-p} - R_o^{4-p} \int_{R_{1,max}}^{R_o} \left(1 - \exp [\beta (v^2 - 1)] \right) R_1^{3-p} \,dR_1}{R_o^{4-p} \int_{R_{1,max}}^{R_o} \left(1 - \exp [\beta (v^2 - 1)] \right) R_1^{3-p} \,dR_1}. \]  

(9)

7. Results and discussion.

We simulated passing planetesimals through the circumplanetary disks of Jupiter and Saturn and capture of their material into the disks with consideration of combined processes of aerodynamic braking, fragmentation, and ablation of planetesimals in the disk’s gas medium. Below are the results of simulation for the comet material of the planetesimals. We estimated maximum planetesimal size (radius \( R_{1,max} \)) which the body should have at the entrance to the disk in order to stay in the disk after loosing mass and velocity due to gas drag and ablation. The maximum radius of captured planetesimal \( R_{1,max} \) is obtained as a function of distances from the central planet. Ablation coefficient \( (\sigma_{abl} = 10^{-13} \, \text{cm}^2 \) \) is taken from [11]. For the planetesimals with radii \( R > R_{1,max} \), which were able to escape the disk, the velocities at the exit after crossing the disk should be higher then the escape velocity from the Hill (gravitational) sphere of the planet.

We estimated the ratio \( (M_o^a=M_c/M_t) \) of the mass of solid material, lost by the falling bodies through ablation and thus captured by the disk \( (M_a) \), to the total mass of the falling bodies \( (M_f) \) with \( R > R_{1,max} \) (Fig.1). This ratio depends on the maximum value in the mass distribution of falling bodies \( M_M \) which is an input parameter of the model. Another input parameter is the exponent \( q=1.8 \) in the power-law mass distribution for planetesimals [14]. In the results presented here we adopt for the largest body (with mass \( M_M \)) the radius \( R_M = 1000 \, \text{m} \).
We also estimated the ratio $M_{o,ca}=(M_c+M_a)/M_f$ (Fig. 2). Here $(M_c+M_a)$ is the whole mass captured (in the unit volume) in the disk. This value includes the total mass of small bodies with initial $R<R_{1,max}$, which are entirely captured in the disk. The value $(M_c+M_a)$ also includes the mentioned above mass $M_a$ which comes to the disk from the larger bodies through their ablation. The parameter $M_t$ is the total mass containing in all the falling bodies (in unit volume) with power-law mass distribution and with size range from zero to $R_{max}$. It is seen from Fig. 1 that the ratio $M_{o,ca}=M_a/M_f$ is very small both in disks of Jupiter and Saturn due to small mass loss by ablation $M_a$ (at adopted value of ablation coefficient). At the same time, the parameter $M_{o,ca}$ (in Fig. 2), reaches 27% at the distances of Ganymede and 17% in the region of Callisto. For Titan the value $M_{o,ca}$ is about 11% (Fig. 2). Thus, there is a significant difference in the mass of material captured in the formation regions of Ganymede, Callisto, and Titan. Note that the presented calculations of values $M_{o,ca}$ and $M_{o,ca}$ do not take into account the possible fragmentation of planetesimals. The inclusion of fragmentation would increase the captured masses.

Without regard for fragmentation the bodies with initial radius $R<R_{1,max}=30$ m are captured in the circum-Jovian disk at Callisto distance and $R<R_{1,max}=100$ m at Ganymede distance (Fig. 3). When fragmentation is considered [12, 13], the planetesimals with radius $R>100$ m should fragment in the formation region of Callisto (Fig. 4). In the Ganymede region all bodies with $R>0.3$ m do fragment. Assuming fragmentation into many small pieces, we obtain that in the region of Ganymede formation the bodies with radii $R<0.3$ m and $R>100$ m are captured. In the region of Callisto according to the model adopted, the disk captures the planetesimals of any size. In the disk of Saturn at the distance of Titan the bodies with radii $R<R_{1,max}=12$ m and $R>6$ km are being captured and fragmented, correspondingly.

**Figure 1.** The ratio of mass lost through ablation and captured in the disk ($M_o$) to the total mass of the falling bodies ($M_t$) with $R>R_{1,max}$ and $R<R_M$ at different radial distances $r$ from the central planet (in units of planetary equatorial radius $R_p$).

**Figure 2.** The ratio of the captured mass (consisting of the mass of small bodies captured in the disk $M_c$ plus the mass of material lost through ablation $M_a$) to the total mass of the falling bodies $M_f$ at different radial distances $r$ from the central planet.
8. Conclusion
Our research shows that a significant masses of protosatellite material falling on the circumplanetary disks of Jupiter and Saturn are captured in the disks. At the same time the masses captured in the formation region of different moons are very different. Our results also show that the narrow range of sizes of captured bodies, as well as greater duration of satellite formation, due to more remote location could provide a low differentiation of Callisto compared with Ganymede. In the disk of Saturn in the Titan formation region the bodies with radii $R < 12$ m were captured, and fragmentation, apparently, for bodies’ capture is irrelevant. This could assist in significant elongation of accretion of Titan and formation of non-differentiated rock–ice mantle.

Acknowledgments: The studies were obtained with partial financial support from the Russian Foundation for Basic Research (project No. 18-05-00685), the program of the Presidium of the Russian Academy of Sciences No. 17.

References
[1] Kuskov O L, Dorofeeva V A, Kronrod V A, Makalkin A B Jupiter and Saturn Systems: Formation, Composition, and Internal Structure of Large Satellites (Moscow: Izd. LKI, 2009)
[2] Kuskov O L, Kronrod V A 2001 Icarus 151 204–227
[3] Kuskov O L, Kronrod V A 2005 Icarus 177 550–569
[4] Dunaeva A N, Kronrod V A, Kuskov O L 2014 Doklady Earth Sciences 454, Part 1 89–93.
[5] Dunaeva A N, Kronrod V A, Kuskov O L 2016 Geochemistry International 54, No.1 27–47.
[6] Makalkin A B, Dorofeeva V A, Ruskol E L 1999 Solar Syst. Res. 33. No 6 456–463
[7] Makalkin A B, Dorofeeva V A 2014 Solar Syst. Res. 48 No 1 62-78
[8] Canup R M, Ward W R 2002 Astron. J. 124 3404-3423
[9] Sasaki T, Stewart G R, Ida S 2010 Astrophys. J. 714 1052–1064
[10] Gritsevich M, Koschny D 2011 Icarus 212 877–884
[11] Ceplecha A Z, Revelle D O 2005 Meteoritics & Planetary Science 40 No1 35–54
[12] Ivanov A G, Ryzhansky V A 2004 Doklady Akademii Nau No. 6 759-763
[13] Tsvetkov V I, Skripnik A Ya 1991 Astron. vestnik 25 No. 3 364-370.
[14] Safronov V S 1991 Icarus 94 No 2 260–271