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Effect of varying fidelity turbine models on wake loss prediction

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Abstract. Wind farm simulations are widely used in estimating energy yield and for optimal wind farm layout design. In the early design stages of a wind farm, low fidelity wind turbine models are used to estimate the farm power output, often due to incomplete knowledge of the turbine characteristics and the additional complexities. The discrepancies introduced in a wind farm simulation as a result of using low fidelity models can often be overlooked, leading to a misrepresentation of a wind farm’s yield. In this paper, the discrepancies between five levels of fidelity for two turbine designs are quantified, focusing on the produced wake and the downstream flow effects. Two high fidelity aeroelastic turbine models and three low fidelity models are described, where the wake is produced using the Dynamic Wake Meandering (DWM) model for each turbine and model. The results provide insight into the expected uncertainties in wake simulations as a result of changing the turbine fidelity level.

1. Introduction

Reducing the impact of wake interactions in a wind farm is one of the leading challenges in the early stages of wind farm design. In these early design stages, it is in the interest of wind farm developers to establish a near-optimal wind plant layout even when various components of the system are unknown or poorly defined. Wind plant optimisation is an intricate problem consisting of many interconnected components, each with their own set of constraints and complexities. These components range from atmospheric conditions to turbine characteristics, cabling layout, power systems, and others [1, 2]. Errors and uncertainty originating from any one of these models can cascade through the system, causing misrepresentations of the annual energy production and levelized cost of energy of the plant.

Despite the potential errors and model uncertainties, low fidelity wind farm simulations are a useful in providing early estimates of plant output. Wind plant models typically require fast wind field computations due to the large number of iterations required during optimization. For this reason, multi-fidelity modeling techniques play an important role, where the accuracy of high fidelity models and the speed of low fidelity models are leveraged to produce a hybrid model. Three key elements in the wind plant modeling composition are the wind turbine model, the atmospheric conditions, and the coupling of the two in the form of a wake model. Wind turbine models can range in fidelity from steady-state look up tables [3], actuator disks [4], and aeroelastic models [5]. The level of fidelity in these models has an effect on not only the individual turbine’s performance, but also the neighboring turbines due to wake effects.
Numerous models exist for replicating wake effects, ranging from low fidelity models such as Jensen and Bastankah, to high fidelity models such as large eddy simulations and RANS solvers. Lying in the middle of the fidelity spectrum is the dynamic wake meandering (DWM) model, which unifies three engineering models for the most significant characteristics of a wake: the wake deficit, the wake meandering, and the added micro-turbulence \[6\]. Static variations of the DWM model have also been explored in \[7\] and \[8\], bridging the gap between steady and dynamic wake simulations, making the DWM model suitable for both steady-state wind plant optimizations, and unsteady aeroelastic turbine simulations. The model requires the axial induction over the blade span which represents a higher level of detail than other engineering models. To couple a low fidelity wind farm model, for example, using the look up table method, with the DWM model, some assumptions must be made regarding the axial induction profile.

In this paper, five turbine models of varying levels of fidelity are used to generate input for wake simulations, and the flow downstream is analyzed. Sec. 2 describes the implementation of the wake model, and Sec. 3 describes the 5 varying fidelity turbine models. In Sec. 4, the results of the investigation are outlined, in particular, the differences in the axial induction blade profiles, the rotor effective wind speed downstream of each turbine model, and the power output of a downstream turbine in full wake.

2. Wake Model

The Dynamic Wake Meandering (DWM) model is used to model the wakes in this analysis. The DWM model is an engineering model which unifies three components of a turbine wake: the deficit profile, wake meandering, and added rotor turbulence.

2.1. Deficit Profile

The wake profile in the DWM model, \(U(r, x)\) at a radial position, \(r\), and a downstream distance, \(x\), is generated by solving the axisymmetric thin-boundary layer approximation of the Navier-Stokes equations in polar coordinates:

\[
\begin{align*}
U \frac{\partial U}{\partial x} + V_r \frac{\partial U}{\partial r} &= \frac{1}{r} \frac{\partial}{\partial r} \left( \nu_T r \frac{\partial U}{\partial r} \right) \\
\frac{1}{r} \frac{\partial}{\partial r} (r V_r) + \frac{\partial U}{\partial x} &= 0
\end{align*}
\]

where \(U\) and \(V_r\) are the longitudinal and radial velocities respectively, and \(\nu_T\) is the eddy viscosity. The boundary condition, \(U(r, 0)\), is determined using the axial induction profiles generated by the particular turbine model. Eq. (1) and (2) are solved numerically using finite difference methods. To take into account the missing pressure term in Eq. (1) and (2), the radial coordinates are pre-expanded using the transformation outlined in \[9\].

The eddy viscosity in (1) varies with downstream distance, and is determined by:

\[
\nu_T = 0.008F_2(x)R_w(x)(1 - U_{min}(x)) + 0.1F_1(x)F_0(I_T)I_T
\]

where \(R_w(x)\) is the wake radius, \(U_{min}(x)\) is the minimum wind speed in the deficit profile for a given downstream distance, and \(I_T\) is the ambient turbulence intensity. \(F_0(x)\), \(F_1(x)\), and \(F_2(x)\) are filter functions calibrated in \[10\].

2.2. Wake Meandering Compensation

The wake meandering is typically implemented dynamically as a string of passive tracer particles which drift through an evolving wind field \[6\]. A static variation of the DWM model is used in
order to represent the meandering in a steady wind farm simulation, which is typically used in wind farm optimization. The static implementation of the wake meandering was proposed by Keck [7] and is further elaborated by Reinwardt [8]. The basic idea is to perform a Gaussian smear on the wake deficit profile, where the Gaussian represents the probability density of the wake center location. The standard deviation of the Gaussian is determined by integrating the power density spectrum of the large scale wind field up to a cutoff frequency. In this case, the Kaimal spectrum is integrated up to a frequency of $U/(2D)$ as defined in [6].

3. Turbine models

The definitions of the five types of varying fidelity turbine models are described in this section. Both the constant induction, and generic models assume that the rotor average thrust coefficient, $\bar{C}_T$, of the turbine is available. Typically, the thrust coefficient is used to parameterize low fidelity wake models, such as the Larsen model, and variations of the Jensen model [11]. Although the DWM wake model requires an axial induction profile rather than a value of $C_T$ to simulate a wake, a conversion is performed to remain consistent with the inputs of other wake models. The five turbine models are used to represent two turbine designs: the DTU 10MW Reference Turbine [12], and its redesign, the IEA 10MW Reference turbine [13]. The IEA 10MW is designed for a higher capacity factor, having more flexible blades and a larger diameter of 200m compared to the DTU 10MW's 180m rotor. Bend-twist-coupling is incorporated in the design, thus the IEA 10MW sees a larger amount of tip torsion in the steady state. The IEA 10MW is representative of state-of-the-art offshore wind turbine designs.

Models 1 (fully flexible) and 2 (stiff)

The stiff and flexible models use the software HAWCStab2 to calculate the axial induction over the rotor over a range of operating wind speeds. HAWCStab2 is a linearized aeroelastic code able to model the nonlinear kinematics of a wind turbine using beam elements [14]. Recent verification of the steady state results from HAWCStab2 against HAWC2 shows excellent agreement [15]. The stiff model results follow from purely aerodynamic calculations. The BEM induction in HAWCStab2 uses the same polynomial for the $\alpha-C_T$ relationship (Eq. (5)) as HAWC2 [16]. This relationship includes a high thrust correction which is especially important for the IEA 10MW turbine below rated. The most important aspect of flexible turbine blades regarding the wake is the blade torsion. Especially turbines that are designed to use bend-twist coupling for load alleviation will see some amount of torsion to feather due to the steady-state flapwise deflection, therefore, reducing the loading towards the tip.

Fig. 1 shows the operational data for the stiff and flexible models of the DTU 10MW and IEA 10MW designs. The power and rotor speed agree very well between the stiff and flexible models because different optimal pitch angles are used for the stiff and flexible models, as can be observed in the lower left plot in Fig. 1. The stiff models generally run less optimally than the flexible models. This leads to higher thrust and lower power, mainly around the shoulder of the power curve where the deflections are large.

Model 3: Constant Induction

Among the three low-fidelity turbine models, the first consist of a constant induction model, where the axial induction over the rotor is assumed to be constant, and equal to:

$$\alpha(x) = \bar{\alpha}$$

where the relationship between $\bar{\alpha}$ and $C_T$ is described as a third-order polynomial as described in [16]:

$$\bar{\alpha} = k_3 C_T^3 + k_2 C_T^2 + k_1 C_T$$

where $k_3 = 0.2460$, $k_2 = 0.0586$, and $k_1 = 0.0883$. 

Figure 1. Operational data for the flexible and stiff models of the DTU 10MW and IEA 10MW wind turbines with rated wind speeds at 11.4m/s and 9.8m/s respectively.

Model 4: Generic
Unlike the constant induction model, the presented generic model accounts for tip and root losses while maintaining the rotor average axial induction. It is based on the formulation of [17] with modifications to ensure a consistent rotor-averaged induction. The advantage of using the generic turbine model in wake simulations is threefold. First, the formulation for axial induction is analytical and continuous, making it compatible with gradient-based methods, as well as being computationally fast. Second, the model requires only a value for rotor-average $C_T$ in order to generate a complete axial induction profile of the rotor. Essentially, the generic model acts as a converter between the simple information typically provided to low fidelity wake models, and the DWM model which requires spatially detailed information. Lastly, the generic model contains two parameters, $a$ and $\delta$, which can be selected to fit a given turbine model if geometric turbine data is available. In this paper, the best fitting parameters with values from literature are compared. The axial induction profile for the generic model is:

$$\alpha(x) = CF(x)G(x)$$

where

$$F(x) = \frac{2}{\pi} \arccos \left( \exp \left( -\frac{N_b}{2} (1 - x) \sqrt{1 + \lambda} \right) \right)$$ (7)

$$G(x) = 1 - \exp \left( -a \left( \frac{x}{\delta} \right)^b \right)$$ (8)

$$C = \frac{\bar{\alpha}}{\left\langle F(x)G(x) \right\rangle}$$ (9)

$F(x)$ is the Prandtl tip loss correction [18], parameterised by the number or turbine blades, $N_b$, and the tip speed ratio of the rotor, $\lambda$. $G(x)$ is the blade root correction, parameterised by $a$, $b$ and $\delta$. As described in [17], $b$ can be described in terms of $a$ using $b = (e^a - 1)/a$. $C$ is the axial induction scaling factor, which is chosen such that the rotor averaged axial induction of $\alpha(x)$ is equal to $\bar{\alpha}$. The area weighted average operator, $\langle \cdot \rangle$, on a radial function, $f(r)$, is:

$$\langle f(r) \rangle = \frac{2}{R^2} \int_0^R \rho f(\rho) d\rho$$ (10)
for a rotor with radius, \( R \).

**Model 5: Optimized Generic**

Whereas the Generic model presented in the previous section can be used given limited information of the turbine, the Optimized Generic (OG) turbine model is a low fidelity turbine model which leverages data from high fidelity wind turbine model simulations in order to produce more accurate wakes. The governing equations to calculate the axial induction profile of the OG turbine are the same as the Generic model (Eq. (7), (8), (9)). However, there are two key differences. First, a rotor-mean axial induction curve is used instead of a \( C_T \) curve as it was found that matching the turbine axial induction is important in producing similar wakes across all fidelity levels of turbine model. Second, the two parameters describing the OG, \( a \) and \( \delta \), are optimized for a given turbine design and site. The optimization aims to minimize the error between the axial induction of the OG turbine model and the high-fidelity axial induction profile over a range of wind speeds, where the error at each wind speed is weighted by a weighting function, \( w(U) \):

\[
\text{minimize} \sum_{i=1}^{N} w(U_i) \left( \langle \alpha_i(r) - \alpha_{\text{ref},i}(r) \rangle^2 \right)
\]

with respect to \( a, \delta \).

In the following analysis, \( w(U) \) is chosen to follow the Weibull distribution which is shown to be a good representation of the wind speed probability distribution of a given site [19]. The goal of using a weighting function is to minimize the error in the wake for the most frequent wind speeds. In this analysis, a Weibull distribution conforming to wind class I in [20] is used.

4. Results

4.1. Induction profiles

The radial induction distribution, used as input for the initial wake deficit, is shown in Fig. 2 for the IEA 10MW turbine at different wind speeds. It can be seen that the stiff turbine generally has a higher loading towards the tip than the flexible turbine due to the missing elastic torsion. At high wind speeds, this overprediction of the loading at the outboard part is compensated by an increased pitch angle, which leads to an underprediction of the inboard loading. The simplified generic and OG models do not predict the load redistribution from the outboard part to the inboard part at high wind speed.

Both the generic and OG axial induction profiles present a similar shape, however, due to the parameter optimization, the induction of the OG model towards the root better represents the induction profile of the reference induction, particularly at lower wind speeds. As a result, it was found that the OG model captures the wake evolution of the reference turbine with higher accuracy than the generic model and the constant induction model, which is unable to capture the near wake behavior at all as seen in Fig 3.

4.2. Rotor effective wind speed downstream

To determine the impact of the wake on downstream turbines, the rotor effective wind speed (REWS) over a range of downstream distances using the wakes generated as per Fig. 3 is analyzed. REWS is defined as the rotor area weighted average wind speed. The downstream distance is varied between \( 2D \) and \( 10D \), and the crosswind distance is fixed at zero representing a full wake case. Additionally, the values are represented as a ratio against the highest fidelity model, in order to show the differences between the 5 models. It should also be noted that the
Figure 2. Axial induction profiles for the different turbine models and wind speeds (IEA 10MW turbine design).

axial induction profiles presented are invariant to changes in atmospheric shear as the DWM model considers only the axisymmetric component of the wake. The shear profile is therefore not considered in this study.

Between the DTU 10MW in Fig. 4 and the IEA 10MW Fig. 5, similar trends can be seen. Firstly, the stiff model shows the greatest difference in the wake REWS at rated wind speed and below, albeit, less than 1% difference in most cases. In the near wake region, the constant, generic, and OG turbine models show large discrepancies in REWS. This is expected, as these low fidelity models do not capture many of the details in the induction profile compared to the stiff and flexible model. As the wake propagates downstream, the wakes of the low fidelity model converge to the flexible model value. The OG model shows the fastest convergence compared to the constant and generic models. The generic model closely follows, however it is clear that all three low fidelity models capture the REWS with similar performance compared to the flexible model.

It was found that the downstream REWS due to the low fidelity turbine models is sensitive to the input $C_T$ or $\alpha$ at low wind speeds and tight inter-turbine spacing. Visually represented in Fig. 4 and 5, the variation in REWS can be observed for a $\pm 1\%$ variation in the input $C_T$ or $\bar{\alpha}$, emphasizing the importance of using accurate operating data in wake simulations. This matches a similar observation in Barthelmie et al. (2006) where, in a comparison between wake models, a significant source of uncertainty originated from variations in the initial wind speed profile [21].

4.3. Weibull-Weighted Power

In order to quantify the impact of the observed differences in REWS for the different wake loss models, the wind speed-weighted power, $P_w$, is computed for both the DTU 10MW and IEA 10MW turbine designs in a representative onshore and offshore case. Typically, a high turbulence intensity is associated with a site with terrain and surface effects, such as a forest. On the other hand, an offshore case will present a higher mean wind speed with a lower turbulence intensity, which will prolong in downstream distance the effect of the wake deficit. To provide insight into these two scenarios, the onshore and offshore cases are represented as wind classes IIIB and IC, respectively, as defined in [20].

The results presented in Fig. 6 correspond to a single downstream turbine in full wake, where the
Figure 3. Wake profile slices using varying fidelity turbine models as boundary conditions ($U_\infty = 10 \text{ms}^{-1}$).

Figure 4. Difference in REWS compared to the flexible turbine model at varying downstream distances (full wake, DTU 10MW turbine design). Error bars represent the uncertainty due to a 1% change in $C_T$ for the stiff and generic models, and a 1% change in $\bar{\alpha}$ in for the OG model.

Figure 5. Difference in REWS compared to the flexible turbine model at varying downstream distances (full wake, IEA 10MW turbine design). Error bars represent the uncertainty due to a 1% change in $C_T$ for the stiff and generic models, and a 1% change in $\bar{\alpha}$ in for the OG model.
Figure 6. Model Comparison of Weighted Production as function of downstream distance, turbine model and site in full wake condition. Turbine power output, $P_w$, is normalised with the power output of the Flexible model, $P_{w,ref}$.

Power output is averaged over all operating wind speeds using a Weibull distribution-weighted average power, $P_w$. The power is computed based on the steady power equation using the air density, $1.22 \text{ kg/m}^3$, the swept area of $25446.9\text{ m}^2$ and $31415.9\text{ m}^2$ for the DTU 10MW and IEA 10MW respectively, the power coefficient for each turbine (Fig. 1) and the REWS from Section 4.2. It should be noted that only full wake situations are considered, and that the wind direction probability distribution is not included in the calculation of $P_w$. For wind speeds lower than cut-in ($4\text{ m/s}$), the power is interpolated using the power coefficient at cut-in to avoid discontinuities in $P_w$ in Fig. 6.

Fig. 6 presents the $P_w$ results of the DTU 10MW and the IEA 10MW turbines for the Onshore and Offshore case. There are two important factors that needs to be taken into account when interpreting these results: (1) the effect of turbulence intensity on the wake recovery and (2) the weighting effect of a Weibull distribution. For the first factor, a higher turbulence intensity leads to faster wake recovery, which explains a slightly faster convergence for the onshore case. The second effect modifies the weight of the power region when computing the $P_w$, therefore, the contribution of the wake effects. For the forest site, with a lower mean wind speed, the Weibull distribution will shift the probability function towards the partial load region, below rated wind speed, emphasizing the wake effects. The contrary effect is found for the offshore case.

For the same wind class, Fig. 6 shows that the IEA 10MW turbine presents higher errors in $P_w$ compared to the DTU 10MW. This suggests that the lower fidelity models do not capture $P_w$ for the flexible turbine model as accurately. The highest difference is found in the constant model, where an over-prediction of up to 5% is found on the IEA 10MW onshore case at a downstream distance of $2D$. 

5. Conclusion

In this paper, the wake profile produced by five different fidelity levels of wind turbine model is investigated. Two high fidelity turbine models, (flexible, and stiff), as well as three low fidelity models (constant induction, generic, and optimized generic (OG)) were used to describe two turbine designs, the DTU 10MW and the IEA 10MW. It was found that the greatest discrepancies between fidelity levels occurs at low wind speeds and at close inter-turbine spacings. For realistic onshore distances, 3D to 5D, the errors range for the power output of a downstream turbine in full wake is within 1% for the DTU 10MW and 2% for the IEA 10MW. When comparing realistic offshore distances, 7D to 9D, the errors range within 0.4% for both turbines. The OG turbine model, which leverages characteristics from the high fidelity model in its definition, shows the best representation of the turbine among the low fidelity models. However, at realistic downstream distances, the benefit is small compared to the other low fidelity models. Furthermore, given that the presented investigation considers only full wake cases, the discrepancies between fidelity levels are expected to be even lower when adjusted for wind farm layout and wind climate. Although these discrepancies are small, they assume accurate knowledge of the thrust and induction curves of the turbine. A small error in either of these can lead to significant discrepancies in the wake profile and in the power output of downstream turbines.

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