Fresnel coefficients, coherent optical scattering, and planar waveguiding

Alejandro Doval, Carlos Damián Rodríguez-Fernández, Héctor González-Núñez and Raúl de la Fuente

Grupo de Nanomateriais, Fotónica e Materia Branda, Departamento de Física Aplicada, Universidade de Santiago de Compostela, E-15782, Santiago de Compostela, Spain
Grupo Mesturas, Departamento de Física e Ciencias da Terra, Universidade de A Coruña, E-15071, A Coruña, Spain

E-mail: raul.delafuente@usc.es

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Abstract
The Fresnel formalism of transmission and reflection of coherent waves on the abrupt interface between two different optical media is revisited and extended to study optical phenomena concerning guided waves. Using the Fresnel formalism, the propagation of light through different sequences of parallel layers of constant refractive index is analysed, revealing that, in all cases, the solutions corresponding to waveguiding share major features. In the analysis, aiming to connect waveguiding with coherent optical scattering, the possibility of the system having a resonance is examined, and it is shown that guided modes always emerge as singular solutions of the associated coherent scattering configuration. Invariably, the resonance condition that must be satisfied to obtain waveguiding corresponds to the pole of the associated total reflection and transmission coefficients. Even more, the fulfillment of the requirements to meet the resonance condition is a clear indication of the possibility of waveguiding.

1. Introduction

More than two centuries ago, Fresnel analysed the reflection and refraction of light at a single interface between two media with different optical densities using fluid mechanics, and defined two coefficients whose expressions as a function of the angles of incidence and refraction are called Fresnel’s laws of sines and tangents [1, 2]. Subsequently, using electromagnetic theory, the so-called Fresnel coefficients were derived again in order to describe the relationships between the amplitudes of the reflected and transmitted waves and the amplitude of the incident wave (see, for example, [3–7]). Today, Fresnel coefficients are commonly used to describe more general problems involving multiple interfaces. Application examples are the Fabry–Perot interferometer [8, 9], anti-reflection coatings [10, 11] or spectral filters [7, 12].

Moreover, many optical systems consisting of parallel layers allow waveguiding, but they are addressed with a rather different approach. In the standard approach waveguiding is dealt with by considering evanescent waves at the boundaries of a given optical system and applying electromagnetic boundary conditions at each interface [13, 14]. These conditions lead to relations between waves in consecutive layers, and a necessary interferometric constraint to have guided waves.

There exists a vast literature on both the Fresnel formalism of transmission and reflection of coherent waves [3–7] and on planar optical waveguiding in the context of integrated optics [13–17]. However, we are not aware of applications of the Fresnel formalism to address the problems associated to waveguiding. In this paper, we introduce a new approach that connects waveguiding to the propagation of light in a set of parallel layers, and we show interesting physical insights than can be derived from doing so.

A coherent light wave incident in such a system is reflected and refracted at each interface between layers, giving rise to coherent global reflected and transmitted waves at the boundaries of the optical system (see
figure 1). These waves can be said to originate from the coherent scattering of the initial incident wave, and we can therefore describe such processes as coherent optical scattering processes. In our approach, we first begin by analysing a non-bounded coherent optical scattering problem in terms of the Fresnel coefficients. Then, the associated bounded system is established by cancelling the external wave. This gives a resonant condition, which provides singular bounded solutions, which are the modes of the associated guiding system. In addition, the relationships between the different amplitudes are described as a function of the Fresnel coefficients. Considering the explicit expression of these coefficients, the components of the electric field of every mode are obtained.

We believe that this strategy is insightful for understanding the physics underlying optical waveguiding and visualizing its connection with the standard study of reflection and refraction of electromagnetic (EM) waves at a single interface separating two distinct media. Furthermore, solving the waveguiding problem in terms of the Fresnel formalism allows to extract conditions that are common to very different waveguiding systems but often go unnoticed, and even predict whether or not a particular optical system will hold guided solutions.

The manuscript is organized as follows. In section 2, we consider surface problems. First, reflection and refraction of an EM wave at a planar interface is reviewed. Next, we cancel the external field to obtain the resonant guiding condition and we recover the surface plasmon as the unique possible solution. In section 3 we deal with optical phenomena in a thin film sandwiched between two media. In the first place, we address the problem of reflection and refraction in a system composed by three media separated by two parallel planar interfaces. Later, the condition for waveguiding in step index waveguides is established and two different configurations are dealt with: dielectric waveguides and waveguiding in a metallic film surrounded by dielectric media. In section 4 we comment the generalization of our approach to arbitrary optical scattering configurations, including gradient index waveguides. Finally, conclusions are presented in section 5.

2. Coherent scattering and waveguiding at a single interface

Here, we consider a simple coherent optical scattering problem (figure 2). A monochromatic wave of frequency $\omega$ travelling through a medium with homogeneous refractive index $n_1$ crosses a plane interface and propagates through a second medium of homogeneous refractive index $n_2$. At the boundary, the incoming electromagnetic wave gives rise to two coherent optical waves of the same frequency. They are the reflected wave, coming back into the first medium, and the transmitted wave, which is refracted at the interface at an angle given by Snell’s law. Within the framework of classical electromagnetic theory, the Fresnel Equations describe the relationship between the complex wave amplitudes of reflected, transmitted and incident electromagnetic waves at the boundary between the two optical media. They are determined by two boundary conditions of the EM waves, i.e., the continuity of the tangential or parallel components of both electric and magnetic fields across the boundary surface. For simplicity, we are going to assume that the EM waves upon consideration are plane waves, which are the simplest solution to Maxwell’s equations. Upon this assumption, the electric fields $\vec{E}_j$ of the incident ($j = 0$), reflected ($j = 1$) and transmitted ($j = 2$) waves involved in our boundary problem are:

$$\vec{E}_j = A_j \vec{e}_j e^{i\varphi_j} \quad j = 0, 1, 2$$

$$\varphi_j = \vec{k}_j \cdot \vec{r}$$

where the time dependence $e^{-i\omega t}$ has been omitted. $A_j$ is the complex amplitude of the electric field which vibrates in the direction of the unit vector $\vec{e}_j$ which indicates the positive direction for each wave. On the other hand, $\varphi_j$ is its phase, which depends on the wavevector of light in the material, $\vec{k}_j$, that specifies the direction of propagation of light. These wavevectors are, in general, complex numbers with their real part associated to the phase velocity, and their imaginary part associated with absorption in the medium. The magnitude of the
wavevector (the wavenumber) is related to the frequency of the waves and the refractive index of the medium as follows:

$$k_j = \frac{\omega}{c} n_j \quad j = 1, 2; \quad k_0 = k_i$$

Furthermore, since EM waves are transverse waves, the directions of vibration of the fields and the directions of wave propagation are orthogonal. This means that:

$$\vec{e} \cdot \vec{k}_j = 0$$

Let us consider that the plane of incidence is perpendicular to the $y$-axis, and that the normal vector of the boundary surface is directed along the $z$-axis, as indicated in figure 2. The continuity of the tangential component of the EM fields at a boundary surface should be applied in particular to the phase of the fields. This results in the invariance of the wavevector component tangential to the boundary surface:

$$k_{0x} = k_{kx} = k_{2x} = \beta$$

which defines the wave propagation constant, $\beta$. Meanwhile, the components normal to the boundary surface are defined by equations (2) and (4):

$$k_{0z} = -k_{1z} = \sqrt{(\omega n_1 / c)^2 - \beta^2}$$
$$k_{2z} = \sqrt{(\omega n_2 / c)^2 - \beta^2}$$

where the change of sign takes into account that the incident wave is an ‘incoming’ wave towards the second medium, while the reflected wave is an ‘outgoing’ wave into the first medium.

In order to obtain the relation between wave amplitudes from the boundary conditions, it is convenient to distinguish between two types of waves: Transverse Electric waves (also known as TE or s-polarized waves), in which the electric field vibrates perpendicular to the plane of incidence, and Transverse Magnetic waves (also known as TM or p-polarized waves), in which the vibration perpendicular to the plane of incidence is that of the magnetic field. The boundary conditions imply two relationships between the electric field amplitudes of the three waves as:

$$\begin{align*}
(A_0 + A_1) &= \eta A_2 \\
(A_1 - A_0) k_{1z} &= A_2 k_{2z} / \eta
\end{align*}$$

where $n_{12} = n_2 / n_1$ is the relative refractive index of medium 2 with respect to medium 1. These conditions can be recast in matrix form as follows:

$$M_{12} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} \eta & 1 \\ -k_{1z} / \eta & -k_{2z} / \eta \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = A_0 \begin{pmatrix} 1 \\ -k_{1z} \end{pmatrix}$$

Multiplying by the inverse matrix, $M_{12}^{-1}$, we obtain the ratio between the amplitudes of the reflected and transmitted waves and the amplitude of the incident wave:
\[
\begin{pmatrix}
A_1 \\
A_2
\end{pmatrix} = \begin{pmatrix}
\eta_2 & 0 \\
0 & \eta_1
\end{pmatrix} A_0 = A_0 M_{12}\begin{pmatrix}
1 \\
-1
\end{pmatrix}
\]
(8)

being the \(\eta_1\) and \(\eta_2\) parameters the well-known Fresnel Coefficients, which define the ratio between reflected or transmitted wave amplitudes and the amplitude of the incident wave from medium 1 to medium 2. From the expression of matrix \(M_{12}\) they yield [18]:

\[
r_{12} = \frac{\eta k_{1z} + k_{2z}/\eta}{\eta k_{1z} - k_{2z}/\eta}; \quad t_{12} = \frac{2k_{1z}}{\eta k_{1z} - k_{2z}/\eta}
\]
(9)

where \(\eta\) was defined in equation (6) Specifically:

\[
n_{12} = \frac{k_{1z} + k_{2z}}{k_{1z} - k_{2z}}; \quad t_{12} = \frac{2k_{1z}}{k_{1z} - k_{2z}} \quad \text{(TE wave)}
\]

\[
r_{12} = \frac{n_{12}k_{1z} + k_{2z}/n_{12}}{n_{12}k_{1z} - k_{2z}/n_{12}}; \quad t_{12} = \frac{2k_{1z}}{n_{12}k_{1z} - k_{2z}/n_{12}} \quad \text{(TM wave)}
\]
(10)

Furthermore, the Fresnel equations for TM waves can be expressed as a function of the dielectric constants of both media, \(\varepsilon_j = \eta_j^2\), \(j = 1,2\), yielding:

\[
n_{12} = \frac{\varepsilon_2 k_{1z} + \varepsilon_3 k_{2z}}{\varepsilon_2 k_{1z} - \varepsilon_3 k_{2z}}; \quad t_{12} = \frac{2\sqrt{\varepsilon_2 \varepsilon_3} k_{1z}}{\varepsilon_2 k_{1z} - \varepsilon_3 k_{2z}}
\]
(11)

Considering the first relation in equation (6), or directly from the Fresnel formula in equation (9), we obtain the following relation, which will be useful later:

\[
\eta t_{12} - r_{12} = 1
\]
(12)

In transparent media, the Fresnel equations are usually written in terms of the angles of incidence and refraction, but this alternative form is not useful here. Note that in the Fresnel formulas described in equations (9–11), we use the z-component of the reflected wavevector and not that of the incident wavevector, as it is usually done. The reason for proceeding this way will become evident in the next section.

The Fresnel Equations are often used to obtain insightful descriptions of classical optical problems involving dielectric media such as the existence of Brewster’s angle or the phenomenon of total reflection, if \(n_1 > n_2\). However, their validity is not limited to dielectric media, but they also hold in conductor materials. At this stage, we will consider the generation of surface plasmons [19], a problem that is not normally addressed using the Fresnel Equations in which an electromagnetic wave is guided along the interface between dielectric and conducting media.

In order to face the plasmon theory with the tools of the Fresnel Equations, we have to describe plasmons in the very same language of the Fresnel formalism. Hence, the question we want to address is: in view of the results presented in the previous section, can light be guided on a boundary? A waveguide can be described as an optical system that allows the propagation of light in a bounded region without an external source, hence the situation is by far different when considering TM waves. For those waves we obtain:

\[
n_{2z}/n_{12} - k_{1z}/n_{2z} = 0
\]
(13)

which allows \(A_1\) and \(A_2\) to be non-zero. Hence, it means that this simple form of optical guiding corresponds to the pole of the Fresnel coefficients. It must be mentioned that the infiniteness of the reflection Fresnel Coefficient for plasmonic modes was already noted by Cardona [20] for the first time, using the Fresnel law of tangents to express the reflection coefficient. Surprisingly, the same condition for the transmission coefficient was not mentioned in that work.

Equation (13) is equivalent to cancelling the determinant of the matrix \(M_{12}\), that is:

\[
\det[M_{12}] = k_{1z}/\eta - k_{2z}/\eta = 0
\]
(14)

In this situation, equation (8) does not hold anymore since there is no inverse matrix for \(M_{12}\). For TE waves, equation (14) yields \(k_{2z} = k_{1z}\), which is a meaningless solution since the reflected and transmitted waves cannot propagate in the same direction. Hence, surface plasmon waves do not occur for TE polarization. Nevertheless, the situation is by far different when considering TM waves. For those waves we get:

\[
k_{1z}/n_{12} - k_{2z}/n_{2z} = 0
\]
(15)

This equation can be rearranged to be expressed as a function of the dielectric constant:

\[-\varepsilon_2 k_{1z} + \varepsilon_3 k_{2z} = 0
\]
(16)

which corresponds to the well-known plasmon resonance condition [19] (note that this equation is usually written in terms of the normal component of the incident wavevector: \(k_{1z} = -k_{2z}\), but we are considering
Combining equation (16) and equation (5) together, and working out the resulting expression, the formula of the mode propagation constant is obtained:

\[
\beta = \frac{\omega}{c} \sqrt{\varepsilon_1 \varepsilon_2} = \frac{\omega}{c} \sqrt{n_1 n_2}
\]

(17)
as well as the relationship between the transverse and tangential components of the wave vectors:

\[
\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} k_{2z} = -\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} k_{1z} = \beta
\]

(18)
The simplest solution of equations (16) and (17) corresponds to real electric permittivities of opposite signs. For example, considering medium 2 with negative permittivity we would have:

\[-\varepsilon_2 > \varepsilon_1 > 0\]

(19)which means that \(\beta\) would be real and, \(k_{1z}\) and \(k_{2z}\), pure imaginary. The optical scattering problem associated with this situation corresponds to the incidence of a wave from a transparent dielectric medium to a conductive medium. Its frequency must be lower than the plasma frequency of the conductor, and the imaginary part of the permittivity must approach zero. In terms of the refractive indices of the media, this situation is written as:

\[n_j = n_j' + i n_j''\quad j = 1, 2\]

\[n_1' = \sqrt{\varepsilon_1}, \quad n_1'' = 0\]

\[n_2' = 0, \quad n_2'' = -i \sqrt{\varepsilon_2} = \sqrt{-\varepsilon_2}\]

(20)where we take \(n_2'' > 0\) to take account for absorption, and no amplification, in the conductive medium. Henceforth, substituting these expressions in equations (17) and (18), \(k_{1z}\) and \(k_{2z}\) can be rewritten as:

\[k_{2z} = i \frac{n_2'' \omega}{\gamma c}, \quad k_{1z} = -i \frac{n_1'' \omega}{\gamma c}, \quad \gamma = \sqrt{-n_1^{12} + n_2^{12}} = \sqrt{-\varepsilon_1 - \varepsilon_2} > 0\]

(21)Since the \(z\)-components of the wavevectors are purely imaginary, the wave is evanescent at both sides of the interface, as seen in figure 3. Precisely, when extracting the imaginary unit from the square root, we have chosen signs in such a way that the wave amplitude does not grow but decays into both media. This electromagnetic wave is strongly coupled to the charge motion at the metal surface (the surface plasmon) and, as a whole, they give rise to a surface plasmon polariton (SPP).

On the other hand, taking into account the definitions of the TM Fresnel coefficients in equation (10), the ratio between reflected and transmitted amplitudes is:

\[
\frac{A_1}{A_2} = \frac{n_1}{n_2} = \frac{\varepsilon_2 k_{2z} + \varepsilon_1 k_{1z}}{2 \sqrt{\varepsilon_2 k_{1z}}} = \sqrt{\varepsilon_2} = n_{1z}
\]

(22)where we have used equation (16). These results can also be obtained directly from equation (6) for TM waves for the particular case \(A_0 = 0\).

Finally, using equations (20) and (21) together with the orthogonality condition between the electric field and the wavevector, equation (3), the following expressions for the electric field components are obtained:
where $A$ is an arbitrary constant. In these expressions we have used that $E = E_1$ for $z < 0$ and $E = E_2$ for $z \geq 0$.

To top it off, a small amendment. In this section we continue denominating wave 1 as the reflected wave and wave 2 as the transmitted wave. Since there is no incoming wave, this denomination lacks its sense. Indeed, we have just one wave propagating surrounding the interface. It can be thought as the combination of two waves: wave 1, propagating in medium 1, and wave 2, propagating in medium 2.

3. Coherent optical scattering and waveguiding in thin films

In this section, we address a second problem involving coherent scattering: the propagation of light in a system composed of three media, each one with its own refractive index, and planar parallel interfaces as separation boundaries between them, as shown in figure 4. Light impinges in the system from medium 1, afterwards, one part is reflected backwards, another part goes back and forth in medium 2, which has a characteristic thickness $d$, and a third part leaves the system through medium 3.

The strategy we follow for addressing the system is that used for studying, for instance, a Fabry–Perot interferometer [21], and can be generalized to model an arbitrary multilayer with flat boundaries [22]. We denote as $A_{2+}$ the amplitude of the wave travelling through medium 2 towards medium 3, and we denote as $A_{2-}$ the wave propagating through medium 2 towards medium 1. At the plane $z = 0$, there are two ‘incoming’ waves, $(0)$ and $(2-)$, and two ‘outcoming’ waves, $(1)$ and $(2+)$. As in the previous section, we consider monochromatic plane solutions to Maxwell equations in each medium, so we write the electric fields as in equation (1):

$$E_j = A_j e^{i\omega t - k_j z}$$

where we considered again the invariance of the wavevector component tangential to the interfaces. To obtain the relationship between the amplitudes of these waves we first look at the first interface. From the definition of the Fresnel Coefficients and the application of the superposition principle, we obtain:

$$A_{2+} = f_{21}A_0 + f_{21}A_{2-}$$
$$A_1 = n_2A_0 + t_{21}A_{2-}$$

where the subindices 'jk' mean that the wave is incident from medium j towards medium k. Note that in this expression we must use the Fresnel equations for TE or TM waves, as appropriate. The first equality in equation (24) means that the wave of amplitude $A_{2+}$ results from the superposition of the transmitted part of the incident wave and the reflection of the wave denoted $(2-)$. Similarly, the reflected wave, $A_1$, results from reflection...
of the incident wave and transmission of the wave (2-). Meanwhile, the relation between the waves that get trapped in medium 2 can be found by performing one roundtrip through this medium beginning and ending at $z = 0$, see figure 5:

$$A_{2-}e^{ik_z} = r_{23}A_{2+}e^{ik_z} + \frac{\Delta \tilde{r}}{k_{2+z}}$$

or simplifying:

$$A_{2-} = r_{23}A_{2+}e^{ik_z} = 2k_{2+z}d$$

$$k_{2+z} = \sqrt{\left(\omega n_2/\mu\right)^2 - \beta^2}$$

Being $k_{2+z}$, $k_{2-z}$ the normal components of the wavevector of the waves traveling within medium 2. The relationship between the transmitted wave and the others can be extracted on a similar way. Also looking at figure 4, the amplitude of the wave transmitted at the observation plane $z = d$ can be written as:

$$A_3e^{i\beta(x-\Delta x)}e^{ik_zd} = t_{23}A_{2+}e^{i\beta(x-\Delta x)}e^{ik_zd} \Longrightarrow A_3 = t_{23}A_{2+}e^{-ik_zd\delta d_0/2}$$

The relationships between the different amplitudes are summarized in equations (25), (27) and (28). These relationships can be rewritten in terms of the amplitude of the incident wave, $A_0$:

$$A_{2+} = \frac{n_2}{1 - r_{21}r_{23}}A_0$$

$$A_1 = n_{23}A_0 = \left[ n_2 + \frac{t_{23}t_{13}e^{i\phi}}{1 - r_{21}r_{23}} \right]A_0 = \frac{-r_{21} + r_{23}e^{i\phi}}{1 - r_{21}r_{23}}A_0$$

$$A_{2-} = \frac{t_{23}t_{12}e^{i\phi}}{1 - r_{21}r_{23}}A_0$$

$$A_3 = n_{23}A_0 = \frac{n_2t_{23}e^{-ik_zd/2}}{1 - r_{21}r_{23}e^{i\phi/2}}A_0$$

where the Stokes relations [9] were used to simplify the expressions:

$$r_{ik} = -r_{ki}$$

$$t_{ik}t_{kj} = 1 - r_{ik}^2$$

In equation (29), the parameters $n_{23}$, $t_{23}$ correspond to the total reflection and transmission coefficients, respectively. It should be noted that the denominator in all these relationships is the same. With respect to the directions of the electric field components, we have $\hat{e}_1 = \hat{y}$ for all waves in the TE case. For TM waves we must consider the orthogonality condition equation (3) to calculate the unit vectors $\hat{e}_2$ in the plane of incidence.

At this point, we consider the possibility of waveguiding in a three-layer system. Therefore, we impose the condition for the existence of guided waves, i.e., no light incident on the system, $A_0 = 0$. Going back to equations (25) and (27), we obtain:

$$A_{2+} = r_{21}A_{2-} = r_{21}t_{23}A_{2+}e^{i\phi} \Longrightarrow A_{2+}(1 - r_{21}r_{23}e^{i\phi}) = 0$$

Hence, either we have the trivial solution with all the amplitudes cancelling out, or else:

$$1 - r_{21}r_{23}e^{i\phi} = 0$$
which is the resonance condition (or eigenvalue equation) for the existence of modes in this three-media configuration. This condition is nothing more than a consistency requirement: the wave trapped in medium 2 must reproduce itself (in amplitude and phase) after completing one round trip, except for an increase in the phase \(2\beta\Delta x = 2d\beta^2/k_{1z}^2\) in the tangential direction, \(x\). Furthermore, the resonance condition can be equally extracted just by cancelling out the common denominators in equation (26). In particular, this yield poles for the total reflection and transmission coefficients:

\[
n_{123}, t_{123} \rightarrow \infty
\]

The propagation constant of every guided mode can be determined by solving this resonance condition. Besides, according to equations (25) and (28), the relationships between amplitudes in this circumstance are:

\[
A_{2+} = r_{21}A_{2-} \\
A_t = t_{21}A_{2+} \\
A_{3+} = t_{23}A_{2+}e^{-ik_{2+}d}e^{i\phi}/2
\]

These relationships allow to derive the expression of the EM fields. They could also be obtained from equation (29), but with a little more calculation. Note again that since there is no external field \(A_0\), the notions of wave reflected in the first medium and wave transmitted to the third medium lose their sense.

As a first example of application of equation (32) are (34), let us assume that the system considered in this section is only made of transparent dielectric materials, which means real refractive indices. In this situation, the wavenumbers \(k_i = n_i\omega/c\ (j = 1, 2, 3)\) are real quantities, the resonance condition equation (32) can be decomposed into two different constraints:

\[
l|r_{21}r_{32}| = 1 \implies l|r_{21}| = |r_{32}| = 1 \\
k_{2+}d - \delta_{21} - \delta_{23} = m\pi, m \in \mathbb{Z}
\]

where we put \(r_{jk} = l r_{jk} e^{-i2\phi}\) and \(\phi = 2k_{2+}d, \text{real}\). The first relation implies that there is total reflection at the two boundaries, while the second one is a constructive interference condition in medium 2, that means the phase acquired by the wave on a complete roundtrip is a multiple of \(2\pi\). The total reflection condition implies that \(n_2 > n_1, n_3\) and that the angle of propagation of waves in medium 2 is greater than the critical angle for the surrounding boundaries: \(\theta_2 > \theta_{21} (j = 1, 3)\). Moreover, under total reflection conditions, the waves in mediums 1 and 3 are evanescent waves \((k_{21}, k_{23} \text{ imaginary})\), so the propagation constants of the modes verify:

\((n_2, n_3)\omega/c < \beta_m < n_3\omega/c\). Additionally, the specific values of \(\beta_m\) and their number are obtained from the constructive interference condition. Each solution corresponding to an integer value of \(m\) (the mode order) is a mode of the dielectric waveguide.

To complete this example, we give the expressions of the electric field components for a symmetrical guide, i.e., \(n_3 = n_t\). They result from equations (3), (34) and (35) with \(t_{23} = t_{21}\) and \(\delta_{23} = \delta_{21}\). In this case the reflection coefficient reads \(r_{21} = e^{-i2\phi}\), and, from equation (12): \(t_{21} = 1 + r_{21}\). Furthermore, considering first TE modes we have \(\delta_{21} = \tan^{-1}(k_{21}/k_{2+})\), and we can rewrite the second expression in equation (35) as:

\[
\tan (k_{2+}d/2 - m\pi/2) = k_{21}/k_{2+}
\]

which corresponds to the eigenvalue equation for TE modes. Applying equation (34) and considering the Fresnel formulas for TE waves, the expression of the electric field in each medium is obtained. After some calculus, it yields:

\[
\bar{E}_1(x, z) = A_1\left(e^{-ik_{2+}z/2} + (-1)^m e^{ik_{2+}z/2}e^{ik_{21}z}e^{i\phi}\right) z \leq 0 \\
\bar{E}_2(x, z) = A_2\left(e^{ik_{2+}z-iz/2} + (-1)^m e^{-ik_{2+}z+iz/2}e^{i\phi}\right) 0 \leq z \leq d \\
\bar{E}_3(x, z) = A_3\left(e^{ik_{2+}z/2} + (-1)^m e^{-ik_{2+}z/2}e^{i\phi}\right) z \geq d
\]

The amplitudes of the modes for \(m = 0\) and \(m = 1\) are shown in figure 6. Note that equation (36) ensures the continuity of the electric field \(z\)-derivative at \(z = 0\) and \(z = d\). This set of modes can be split in two subsets: symmetric modes with cosine amplitudes in the core layer, and antisymmetric modes with sine amplitudes in the core layer. They correspond to even and odd values of the integer \(m\), respectively.

The expression for TM modes is worked out in a similar way, excluding that the electric field is contained in the \(xz\) plane. At each layer, the orthogonality condition, equation (3), determines the direction of vibration of the electric field:

\[
\bar{e}_1 = (k_{21}x - \beta z)/k_1, \bar{e}_{2+} = (k_{2+}x - \beta z)/k_{2+}, \bar{e}_3 = (-k_{21}x - \beta z)/k_1 \\
k_j = \sqrt{k_{21}^2 + \beta^2} \quad j = 1, 2, 3, 2, 3
\]

where \(k_{2+} = k_{2-} \equiv k_3\) and the sign convention is consistent with the expression of the Fresnel coefficients in equation (10). Note that since \(k_{21}\) is imaginary, \(\bar{e}_1\) and \(\bar{e}_3\) are complex unitary vectors.
Besides, for TM modes, $\delta_{21} = \tan^{-1}(n_2^2 |k_{1z}|/n_1^2 k_{2z}^2)$ and $r_{21} = (1 + r_{21})/n_{21} = (1 + r_{21})n_2/n_1$. So, the eigenvalue equation for TM modes is:

$$\tan \left( k_{2z} d/2 - m\pi/2 \right) = n_2^2 |k_{1z}|/n_1^2 k_{2z}^2 \quad (39)$$

and the expression of the electric field is:

$$\vec{E}_1(x, z) = A n_2 \vec{e}_1(e^{-ik_{2z}d/2} \pm (-1)^m e^{ik_{1z}z} e^{i\phi_0} \quad z \leq 0$$

$$\vec{E}_2(x, z) = A n_1 \vec{e}_2(e^{ik_{2z}d/2} \pm (-1)^m e^{-ik_{1z}z} e^{i\phi_0} \quad 0 \leq z \leq d$$

$$\vec{E}_3(x, z) = A n_2 \vec{e}_1(e^{ik_{2z}d/2} \pm (-1)^m e^{-ik_{1z}z} e^{i\phi_0} \quad z \geq d \quad (40)$$

Here, equation (39) ensures continuity at the interfaces of the tangential components of the electric field (the x-components). Like TE modes, the TM modes can be divided into symmetric and antisymmetric.

As a second example of thin film waveguiding, we take media 1 and 3 as transparent dielectric materials, while medium 2 is an absorbing metallic layer. As the refractive index of medium 2 is complex ($n_2 = n_2^r + i n_2^i$), so is the transverse component of the wave vector in this medium:

$$k_{2z} = \pm \sqrt{\left(\frac{\omega}{c}\right)^2 - \beta^2}$$

$$= \pm \sqrt{(n_2^r)^2 - n_2^r n_2^i(\omega/c)^2 - \beta^2 - 2n_2^i n_2^r(\omega/c)^2 i} = k_{2z}^r + k_{2z}^i \quad (41)$$

and the phase in equation (32) is also complex:

$$\phi = \phi^r + i\phi^i = 2d(k_{2z}^r + k_{2z}^i i) \quad (42)$$

Hence, we decompose the resonance condition into two components, as we did in equation (35), obtaining:

$$\left| r_{21} \right| r_{23} \left| e^{-2ik_{2z}d} = 1 \right.$$
A particularly simple and ideal case of metallic waveguiding is the one which satisfies the condition \( n_2' = 0 \).
In this case, the permittivity \( \varepsilon_2 = \varepsilon_2'' = -n_2'^2 < 0 \), just as in plasmonic waveguides, and \( k_{3+z} \) is imaginary \((k_{3+z} < 0)\). For simplicity we will address the symmetric case: \( n_1 = n_3 \).

From the expression of the Fresnel coefficients, it is noted that there is no solution of equation (43) for TE waves because \( \left| r_{21} \right| \) always.
While for TM waves we have \( \left| r_{21} \right| > 1 \) if the normal component of the wavevector of the surrounding media is imaginary. That is \( k_{1z} = i k_{1z}'' \), \( k_{3z} = i k_{3z}'' \), so the wave is evanescent in the dielectric media \((/\beta > n_1\omega/c)\), as expected. We choose \( k_{1z}'' = -k_{1z}' > 0 \), so the wave vanishes at \( \pm\infty \).

By defining the magnitude \( R = -k_{2+z}'' n_1^2/(n_2'^2 k_{1z}) > 0 \), the \( r_{21} \) coefficient is written as:

\[
r_{21} = \frac{R+1}{R-1} \tag{44}
\]

With this expression and considering that \( k_{2+z}' = 0 \), equation (43) transforms to:

\[
\left( \frac{R-1}{R+1} \right)^2 = e^{-2k_{1z}'dz}
\]

\[
\delta_{21} = \delta_{31} = \frac{m\pi}{2} \tag{45}
\]
and taking the square root:

\[
\frac{R-1}{R+1} = \pm e^{-k_{1z}'dz} \tag{46}
\]

Therefore, we have two different solutions, one corresponding to \( \delta_{21} = 0 \), \( R > 1 \), and the other to \( \delta_{21} = \frac{\pi}{2}, 0 < R < 1 \). They yield two possible values for the propagation constant, \( \beta \), one corresponding to the symmetric or even propagation mode and the other to the anti-symmetric or odd propagation mode, see figure 7. When \( k_{1z}''d \gg 1, e^{-k_{1z}'dz} \rightarrow 0, R \rightarrow 1 \) and \( r_{21} \rightarrow \infty \), each one of these modes is characterized by two independent waves which share the same propagation constant and correspond to two distant Surface Plasmon Polaritons (SPPs) propagating at the boundaries \( z = 0 \) and \( z = d \). For the symmetric solution the SPP waves propagate in phase, while in the anti-symmetric case they travel with opposite phases.

Finally, we note that equation (45) can also be written in a more usual way \([23]\) as:

\[
R^l = \tanh (k_{1z}''d/2), l = -1, 1 \tag{47}
\]

As in the preceding sections, we write the electric field components of the modes. They are:

\[
\vec{E}_1(x, z) = A_{21} e^{ik_{1z}'dz/2} + (-1)^m e^{-ik_{1z}'dz/2} e^{ik_{1z}dz} \tag{48}
\]

\[
\vec{E}_2(x, z) = A_{11} (e^{ik_{1z}'dz} + e^{-ik_{1z}'dz}) + (-1)^m (e^{ik_{1z}'dz} + e^{-ik_{1z}'dz}) e^{ik_{1z}dz} \tag{45}
\]

\[
\vec{E}_3(x, z) = A_{21} e^{ik_{1z}'dz/2} - (-1)^m e^{-ik_{1z}'dz/2} e^{ik_{1z}dz} \tag{48}
\]

4. Comment on the general problem

We can consider a general process of coherent light scattering as a black box in which an incident wave is divided into a reflected and transmitted wave. In linear optics, the amplitudes of these waves are proportional to the amplitude of the incident wave. This defines de reflection and transmission coefficients, \( r, t \) as:

\[
r = \frac{A_r}{A_i}, t = \frac{A_t}{A_i} \tag{49}
\]
In this situation, optical waveguiding can be analysed by considering the poles of these coefficients. This means that setting both denominators equal to zero identifies the different configurations that support light guiding. However, to solve the guiding process, one must first solve for the fields through the concerned optical system, relating the amplitudes at each boundary by means of the corresponding Fresnel coefficients, and determining the expression of and . Note that this formalism can also be applied to gradient index waveguides, in which the refractive index varies continuously along a given direction. In this case, the coefficients and are defined for the ratio between amplitudes at .

5. Conclusions

In this paper we unify the treatment of coherent scattering and optical guiding processes, using the Fresnel model of light propagation through a surface separating two media of different refractive index. The distinction in the treatment of the two processes is clear: the scattering process is induced by an external signal that generates an affected and a transmitted wave in the system under study; while in the guiding phenomenon there is no such external wave, but a wave that propagates confined in several media. Proceeding in this way, we can switch from one phenomenon to the other by simply cancelling the external wave. It is in this step when the relations between the waves participating in the process are simplified and a certain resonance condition is obtained. On the one hand, this condition provides the propagation constants of the guided modes, and, on the other hand, it is used to specify the relations between the waves defining a mode. It has been proved that the resonance condition corresponds to the common poles of the reflection and transmission coefficients of the studied system. It has also been shown that the obtained condition can be particularized for different configurations. For example, when studying the propagation of light through three media, one can consider either fully dielectric guides or a metallic film between dielectrics. In addition, the resonance condition associated with a given number of media allows us to identify all possible associated guiding configurations.

In our opinion, this way of treating two different types of processes (coherent optical scattering and waveguiding) following a common line allows us to dig in the physical insight of this processes. Besides, this way of proceeding also shows the shared characteristics of different guiding configurations.

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Data availability statement

No new data were created or analysed in this study.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

ORCID iDs

Raúl de la Fuente @ https://orcid.org/0000-0001-7990-6910

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