Ruling Out Inflation Driven by a Power Law Potential: Kinetic Coupling Does Not Help

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Abstract—We demonstrate that the latest constraints on inflationary observables, namely, the tensor-to-scalar ratio \( r \) and the scalar spectral index \( n_S \) from the Cosmic Background Radiation (CMB) observations are already strong enough to rule out the model of a scalar field with a power law potential even in the presence of kinetic coupling to gravity with a positive coupling constant. The case for a negative coupling constant needs a special treatment.

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1. INTRODUCTION

In the recent years, a plethora of modified gravity theories have been put forward in order to source cosmic inflation, an epoch of accelerated expansion in the early universe [1–10] and to match the inflationary predictions with the observational data [11–30]. Scalar-tensor theories represent a large set of gravitational theories studied so far, and some class of the theories based on other principles, such as quadratic gravity [31], can also be reformulated in the scalar-tensor form. The general class of scalar-tensor theories leading to second-order differential equations of motion (the so-called Horndeski theory [32]) is rather wide, it depends on five functions not fixed by a theory. Hence a thorough analysis of particular cases of the Horndeski theory might be well motivated.

The current CMB bounds on the tensor-to-scalar ratio \( r \) and the scalar spectral index \( n_S \) coming from the latest PLANCK and BICEP/Keck observations [13, 14] are already strong enough to rule out a number of inflationary theories. Keeping in mind the large variety of gravitational theories, ruling out some of them is not less important than constructing new theories. Based on how things stand at the present, the case of a minimally coupled scalar field even with a shallow power-law potential is observationally disfavoured (see, e.g., [33]). In the present paper we show that introduction of a kinetic coupling with a positive coupling constant does not help.

2. BASIC EQUATIONS

We start our analysis from the action

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi} - \frac{1}{2} (g^{\mu\nu} - \kappa G^{\mu\nu}) \phi,_{\mu} \phi,_{\nu} - V(\phi) \right],
\]

where \( R \) is the scalar curvature, \( g_{\mu\nu} \) the metric tensor, \( G^{\mu\nu} \) the Einstein tensor, and \( V(\phi) \) the scalar field potential which we choose in the power-law form \( V = V_0 \phi^\alpha, \kappa \) is the coupling parameter which we consider positive in the present paper. This particular model has been proposed in [36] and studied further in many papers (see, e.g., [37–40]).

The field equations of this theory with the Friedmann–Robertson–Walker metric have the form

\[
3H^2 = 4\pi \dot{\phi}^2 (1 + 9\kappa H^2) + 8\pi V_0 \phi^\alpha, \quad (2)
\]

\[
2\dot{H} + 3H^2 = -4\pi \dot{\phi}^2 (1 - \kappa (2\dot{H} + 3H^2)
+ 4H \ddot{\phi}^{\alpha - 1}) + 8\pi V_0 \phi^\alpha, \quad (3)
\]

\[
(\ddot{\phi} + 3H \dot{\phi} + 3\kappa (H^2 \dot{\phi} + 2HH \dot{\phi})
+ 3H^2 \dot{\phi}) = -V_0 \alpha \phi^{\alpha - 1}. \quad (4)
\]

During inflation, the scalar field is in a slow-roll phase, so we introduce the slow-roll parameters:

\[
e0 = -\dot{H}/H^2,
\]
During slow-roll inflation, we have $\epsilon_0, \epsilon_1, k_0, k_1 \ll 1$.

The field equations (2)–(4) in the slow-roll approximation can be rewritten in the following form:

\[
\frac{3H^2}{8\pi} = V_0 \phi^2, \\
\dot{H} = -4\pi \dot{\phi}^2 - 12\pi K H^2 \dot{\phi}^2, \\
3H \dot{\phi} + \alpha V_0 \phi^{\alpha - 1} + 9H^3 \kappa \dot{\phi} = 0,
\]

or in the form, explicitly resolved with respect to highest-derivative terms:

\[
\dot{\phi} = -\frac{\alpha V_0 \phi^{\alpha - 1}}{2\sqrt{6\pi V_0 \phi^\alpha (1 + 8\pi \kappa V_0 \phi^\alpha)}}, \\
\dot{H} = -\frac{\alpha^2 V_0 \phi^{\alpha - 2}}{6(8\pi \kappa V_0 \phi^\alpha + 1)}, \\
\frac{3H^2}{8\pi} = V_0 \phi^\alpha
\]

We can express the slow-roll parameters (5) using Eqs. (7):

\[
\epsilon_0 = \frac{2}{16\pi \phi^2(1 + 8\pi \kappa V_0 \phi^\alpha)}, \\
\epsilon_1 = \frac{\alpha(1 + 8\pi \kappa V_0 \phi^\alpha + 4\pi \kappa V_0 \phi^\alpha)}{4\pi \phi^2(1 + 8\pi \kappa V_0 \phi^\alpha)^2}.
\]

The condition for the end of inflation is $\epsilon_0(\phi_E) = 1$, implying

\[
\frac{\alpha^2}{16\pi \phi^2_E (1 + 8\pi \kappa V_0 \phi^\alpha_E)} = 1.
\]

The number of e-folds can be expressed as

\[
N = \int_{\phi_I}^{\phi_E} \frac{H}{\dot{\phi}} d\phi = -\frac{4\pi}{\alpha} (\phi_E^2 - \phi_I^2) - \frac{64\pi^2 \kappa V_0}{\alpha(\alpha + 2)} (\phi_E^{\alpha + 2} - \phi_I^{\alpha + 2}).
\]

This expression can be used to find the initial value of the scalar field $\phi_I$. Note that both $\phi_E$ and $\phi_I$ depend only on the product $\kappa V_0$, we will use this fact later.

3. POWER SPECTRUM

For the theory under consideration, the expressions for the tensor-to-scalar ratio $r$, the scalar spectral index $n_S$ and the amplitude of scalar power spectrum $P_\zeta$ have been derived in [11, 12] to be

\[
r = 16\epsilon_0 = \frac{\alpha^2}{\pi \phi_I^2 (1 + 8\pi \kappa V_0 \phi_I^\alpha)}, \\
n_S = 1 - 2\epsilon_0 - \epsilon_1 = 1 - \frac{\alpha^2}{8\pi \phi_I^2 (1 + 8\pi \kappa V_0 \phi_I^\alpha)} - \frac{\alpha(1 + 8\pi \kappa V_0 \phi^\alpha + 4\pi \kappa V_0 \phi^\alpha)}{4\pi \phi_I^2 (1 + 8\pi \kappa V_0 \phi_I^\alpha)^2}.
\]

Note that $r$ and $n_S$ do not explicitly depend on $k_0$ and $k_1$, it is known that these slow-roll parameters only appear in second-order terms [11]. The role of nonminimal coupling in the slow-roll approximation is to modify the equations for standard slow-roll parameters $\epsilon_0$ and $\epsilon_1$. The expression for the amplitude of the scalar power spectrum in the case of small slow-roll parameters $\epsilon_0, \epsilon_1, k_0, k_1$ takes the following form (taking into account only the leading order term)

\[
P_\zeta \approx \frac{H^2}{8\pi^2} \frac{1}{\epsilon_0} = \frac{16\pi \phi_I^2 (1 + 8\pi \kappa V_0 \phi_I^\alpha)}{3\alpha^2}.
\]

It is important to note that the expressions for $r$ and $n_S$ depend only on the product $\kappa V_0$, but do not depend on them separately. This is the reason why we can test this theory (and actually rule it out) using PLANCK+BICEP/Keck results [14] by changing the values of $\kappa V_0$. On the contrary, the scalar power spectrum amplitude depends on $\kappa$ and $V_0$ separately (13), so it can be easily satisfied by changing these parameters separately.

We use Eqs. (11) and (12) for plotting the dependence $(r, n_S)$ (Fig. 1).

As we see, adding a nonminimal kinetic coupling makes the results better but still not good enough to pass the latest test results. An interesting feature of these results is the fact that as $\kappa V_0 \rightarrow +\infty$, the curve $(r, n_S)$ ends on a line corresponding to the case $\kappa V_0 = 0$ (of course, for some different $\alpha$). Analytic expressions for the case $\kappa V_0 \rightarrow +\infty$ can be written in the form

\[
\lim_{\kappa V_0 \rightarrow \infty} r = \frac{16\alpha}{2N(\alpha + 2) + \alpha}, \\
\lim_{\kappa V_0 \rightarrow \infty} n_S = 1 - \frac{4(\alpha + 1)}{2N(\alpha + 2) + \alpha}.
\]

We can now explicitly express the $r$ dependence of $n_S - 1$:

\[
n_S - 1 = -\frac{(2N - 1)r + 16}{16N}.
\]

Analytic expressions for $r$ and $n_S$ in the case $\kappa V_0 = 0$ (a minimally coupled scalar field) take the form

\[
r = \frac{4\alpha}{\alpha + 2/N}, \\
n_S = 1 - \frac{\alpha + 2}{2N + \alpha/2},
\]

which leads to

\[
n_S - 1 = -\frac{(2N - 1)r + 16}{16N}.
\]
418 AVDEEV, TOPORENSKY

Fig. 1. In this figure, graphs are plotted in the range of the potential power $\alpha = 0.01-2.4$, for (a) $N = 50$, (b) $N = 60$, (c) $N = 70$, each plot is built in the range of values $\kappa V_0 = -0.001-1000$, tan and gray areas describe the $68\%$ and $95\%$ confidence levels of Planck results (TT, TE, EE + lowE + lensing + BK15 + BAO) [14], dashed red lines describe the results for a theory with minimal kinetic coupling.

As can be seen from Eqs. (15) and (17), the line for the limit $\kappa V_0 \rightarrow \infty$ coincides with the line for a minimally coupled scalar field. This fact tells us that increasing the $\kappa V_0$ parameter to high values cannot improve the result for a minimally coupled scalar field, which is already ruled out. In the range of intermediate $\kappa V_0$, the $(r, n_s)$ curve deviates from the line of a minimally coupled field, however, this deviation is not large enough to match the current observational data. This means that the model with a nonminimally kinetically coupled scalar field solely can be already ruled out for any power-law potential.

4. CONCLUSION

In this paper, we have considered the scalar-tensor theory with non-minimal kinetic coupling in the case of a positive coupling constant $\kappa$ and a power-law scalar field potential. Using the latest observational data, it was shown that in this case this theory can be ruled out.

Note that the dynamics of negative $\kappa$ is much richer and includes, for example, such an exotic possibility as realizing inflation without a scalar field potential [34]. This regime requires large $\dot{\phi}$ [35] and hence cannot be described by the usual slow-roll approximations. We leave the case of negative $\kappa$ for a future work.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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