Charged and neutral pion production in the S-matrix approach

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Abstract

The S-matrix approach is used to calculate charged as well as neutral pion production reactions from $NN$ scattering, with the same set of underlying processes and interactions. The chiral perturbation theory (χPT) πN scattering amplitude is used. For the nucleon-nucleon distortions a newly developed realistic potential within the Bonn family of potentials, valid well above the pion production threshold, is considered. In the $\pi^+$ production case, the $NN$ potential, the $\pi N$ relative $p$ waves and the treatment of the exchanged pion energy build up the observed cross-section strength.
A central problem underlying the non-relativistic quantum-mechanical operator for any mechanism of pion production in the framework of time-ordered perturbation theory (TOPT) is that the final- and initial-state interaction diagrams correspond to different off-energy shell extensions of the amplitudes, since energy is not conserved at individual vertices. This problem does not occur in the S-matrix approach [1] which was applied to derive the important Z-diagram operators [2]. Moreover, it was recently shown that this approach with a particular prescription for the energy of the exchanged pion reproduces well the energy dependent re-scattering operator for neutral pion production resulting from TOPT [3].

We report here that a calculation using the S-matrix approach with a \( NN \) potential tuned above pion production and a sufficient number of 3-body channels with \( \pi N \) relative \( p \)-waves, overcomes the problem of insufficient production strength in the \( \pi^+ \) charge channel found in Ref. [4]. Therefore, this work shows that it is possible to have an overall description of all the charge channels from the same underlying physical processes and realistic sub-cluster interactions. As in Ref. [4], and alternatively to the Jülich group [5], we take the \( \chi PT \) \( \pi N \) amplitude [6] to construct the irreducible pion re-scattering term. Together with this contribution, we consider the direct-production term, \( Z \)-diagrams mediated by scalar and vector exchanges, and the explicit \( \Delta \)-isobar excitation.

For the initial state, the nucleon-nucleon interaction is necessarily needed above the pion production threshold. We apply here for the first time an \( NN \) interaction recently developed by the Ohio group, including two boson exchange box contributions, with Delta(1232) and \( N^*(1440) \) intermediate states [7]. As an extension of the family of the Bonn potentials, it describes well the \( NN \) phase-shifts and inelasticities up to 1 GeV.

In the S-matrix prescription, the effective operators are defined only on-energy-shell through the non-relativistic reduction of the corresponding Feynman diagrams where the energy is conserved at each vertex [1]. All nuclear currents and other transition operators are defined to be consistent with an hermitian energy independent \( NN \) potential. For the pion re-scattering diagram (Fig. 1), the S-matrix prescription leads to a single effective operator (both for FSI and ISI diagrams) of the form [8]:

\[
\hat{O}_{rs}^{S} = \frac{f(\Omega)}{(\Omega)^2 - (m_\pi^2 + \vec{q}'^2)},
\]

where, adopting the notation of Ref. [8], \( \vec{q}' \) is the momentum of the exchanged pion and \( f(\Omega) \) is the product of the \( \pi N \) amplitude with the \( \pi NN \) vertex. Whenever the operators
are used in convolution integrals for the distortion of the nucleonic states, the momentum variable is free. The symmetric implementation of the energy conservation condition $\omega_\pi = E_2 - \omega_2 = -(E_1 - \omega_1 - E_\pi)$ at the vertices of Fig. 1 is then used

$$\Omega = \frac{(E_2 - \omega_2)}{2} + \frac{(\omega_1 + E_\pi - E_1)}{2}. \tag{2}$$

This prescription minimizes the deviation between the S-matrix and the TOPT results for the re-scattering diagram [3].

The explicit $\Delta$-isobar propagation contribution is

$$V_{\pi N \Delta} = i \left(\frac{h_A}{2f_\pi}\right)^2 \frac{4g_A}{18f_\pi} \frac{1}{\Delta - E_\pi} \frac{1}{\Omega^2 - \omega_\pi^2} \left[2r_a^{(2)} + 4\left(\pi^{(2)}_{cd} \varepsilon^{abc}\right) (\vec{q}_\pi \cdot \vec{q}) \vec{\sigma}^{(2)} \cdot \vec{q}'\right]$$

Since the $\Delta$ is added explicitly to the $\pi N$ amplitude, the static limit of Eq. (3) is subtracted from the $\chi$PT amplitude [4]. This redefines the $c_3 = -5.29\text{GeV}^{-1}$ parameter to $c'_3 = c_3 - c_3^\Delta$, with $c_3^\Delta \approx -h_A^2/(18m_\pi) = -2.78\text{GeV}^{-1}$. To go beyond threshold, we include $p$-wave pion production and $NN$ partial waves with high relative angular momentum. Details will be given in Ref. [9].

We compare in Fig. 2 the cross sections calculated within the S-matrix approach to those obtained with frequently used approximations. For all the charge channels, the deviation is largest for the static approximation ($\Omega = 0$), which overestimates the cross section by a factor of 2 (close to threshold). The on-shell approximation ($\Omega = E_2 - \omega_2$, difference between the nucleon on-shell energies before and after pion emission) also deviates from the reference result. It may be off 20% for all the charge channels, even close to threshold. For the $\pi^0$ and $\pi^-$ production cases, the same happens for the fixed kinematics approximation ($\Omega = m_\pi/2$). For $pp \rightarrow pn\pi^+$, this approximation underestimates the cross section by a factor of $1.5 - 2.5$ near threshold and $5$ at higher energies. This significant difference arises
from the $\pi N$ relative $p$-waves being very sensitive to the energy prescription. They are particularly important for the $\pi^+$ channel and magnify at higher energies the difference between the S-matrix approach and the fixed-kinematics approximation. The results for $\pi^+$ give insight into the discrepancy with the experimental data by factors 2-5 reported in Ref. [4], where the calculation also used the $\chi$PT $\pi N$ amplitude as here, but included only relative $\pi N$ s-wave states and assumed fixed kinematics.

We compare the cross sections with the Ohio and the Bonn B potentials, and simultaneously illustrate the convergence of the amplitude partial waves with increasing angular momentum $J$, in Figs. [3]--[5]. As expected, the importance of channels with high $J$ increases with increasing laboratory energy $T_{lab}$. For $\pi^0$ production, the $J = 0$ channel alone describes the data and suffices for convergence (Fig. [3]). For $\pi^-$ production (Fig. [4]) however the $J = 1$ channels are needed. For $\pi^+$ production (Fig. [5]), the convergence is slower than in the other two reactions, as the $J = 2$ channels are needed for convergence.

We further verified that the direct-production and re-scattering mechanisms alone are not sufficient to describe the $\pi^0$ production data, and Z-diagrams are decisive, as known. For $\pi^-$ production, the Weinberg-Tomozawa term, which does not contribute to $\pi^0$ production, is important relative to the direct production term. The Z-diagrams have also an important role. For $\pi^+$ production, the Ohio $NN$ model gives a significantly better description of the data. It should be noticed that in contrast to the Bonn B potential, it includes explicit $\Delta$ contributions. For $\pi^+$ production, the $\Delta$ contributions dominate and increase with energy. The general trends obtained here were also found for the Jülich phenomenological model [5]. There all short range mechanisms are included through $\omega$-exchange and adjusted to reproduce the total $pp \to pp\pi^0$ cross section close to threshold. In our calculation no adjustment is made. The parameters for the Z-diagrams and for the $\Delta$ contribution are taken from the $NN$ interaction employed.

We achieved an overall S-matrix description of the cross section both for charged and neutral pion production, from the same underlying processes and interactions. The crucial role of the $\Delta$ in $\pi^+$ production enhances the importance of the $\pi N$ $p$-wave states resulting in a slower convergence and in a higher sensitivity to the fixed kinematics approximation. By taking these channels and simultaneously going beyond the fixed kinematics approach, we obtained a good description of the $pp \to pn\pi^+$ reaction, problematic until now [4], when the $\chi$PT $\pi N$ amplitude is used. The newly developed Ohio interaction tuned above the pion
production threshold furthermore improves the description of the $pp \rightarrow pn\pi^+$ reaction. Our results should be relevant for high precision calculations, as in charge symmetry breaking studies.

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FIG. 2: Ratio between the cross section with different approximations and the S-matrix cross section. The dotted, dashed-dotted and dashed line are, respectively, the on-shell approximation, fixed-kinematics and static approximation.

FIG. 3: Effect of the \(NN\) interaction on the \(pp \rightarrow pp\pi^0\) cross section. Dotted \((J = 0)\), dashed-dotted \((J \leq 1)\), dashed \((J \leq 2)\) and solid \((J \leq 3)\) lines with (without) +’s correspond to the Ohio (Bonn B) potential. The data points are from Ref. [10].
FIG. 4: The same of Fig. 3 but for the $pn \to pp\pi^-$ cross section. The data points are from Ref. [11].
FIG. 5: The same of Fig. 3 but for the $pp \rightarrow pn\pi^+$ cross section. The data points are from Ref. [12].