D-BRANES, MONOPOLES AND NAHM EQUATIONS

Duiliu-Emanuel Diaconescu

Department of Physics and Astronomy
Rutgers University
Piscataway, NJ 08855-0849
e-mail: duiliu@physics.rutgers.edu

January 3, 2022

Abstract

We study the correspondence between IIb solitonic 1-branes and monopoles in the context of the 3-brane realization of $D = 4$ $N = 4$ super Yang-Mills theory. We show that a bound state of 1-branes stretching between two separated 3-branes exhibits a family of supersymmetric ground states that can be identified with the ADHMN construction of the moduli space of $SU(2)$ monopoles. This identification is supported by the construction of the monopole gauge field as a space-time coupling in the quantum mechanical effective action of a 1-brane used as a probe. The analysis also reveals an intriguing aspect of the 1-brane theory: the transverse oscillations of the 1-branes in the ground states are described by non-commuting matrix valued fields which develop poles at the boundary. Finally, the construction is generalized to $SU(n)$ monopoles with arbitrary $n > 2$. 
1 Introduction

It has been argued recently [11],[12],[31],[33],[35] that the $SL(2, Z)$ duality of the $D = 4, N = 4$ super-symmetric Yang-Mills theory can be viewed as the field-theoretic counterpart of the more fundamental $SL(2, Z)$ duality of type IIb super-string theory [30]. A precise formulation of this correspondence can be achieved in the context of Dirichlet branes of super-string theory, whose existence and basic properties have been first shown in [7],[21],[27],[26]. Especially, $D = 4, N = 4$ super-symmetric Yang-Mills theory with gauge group $SU(2)$ is realized as the low energy effective theory of two parallel type IIb 3-branes. A configuration with separated 3-branes corresponds to a point of spontaneously symmetry breaking on the moduli space of the theory, the scale of the Higgs mechanism being essentially proportional to the distance between the branes. The excitations of this system consist of charged W-bosons which can be identified with fundamental open strings stretching between the 3-branes and of magnetic monopoles which can be similarly identified with solitonic strings also stretching between the branes. More generally, the dyonic states of the world-volume Yang-Mills theory correspond to bound states of solitonic and fundamental strings. This correspondence is compatible with $SL(2, Z)$ transformations.

The present paper gives an explicit construction of the moduli space of world-volume monopoles as a moduli space of super-symmetric ground states of solitonic strings. This construction relies on the remarkable description of solitonic excitations of super-string theory as Dirichlet branes [27]. The relevant formulation of the monopole moduli spaces turns out to be that given by Nahm in [24], usually called ADHMN construction. In this respect our results are the monopole counterpart of the similar constructions carried out in [9],[10],[36],[37] in the framework of instanton moduli spaces.

We first give a detailed analysis for $SU(2)$ gauge group emphasizing the role of boundary conditions. A probe analysis along the lines of [9] proves to be crucial for a proper understanding of the correspondence. In the last section we generalize the construction to $SU(n)$ gauge groups with $n > 2$.

Acknowledgments I am deeply grateful to Michael R. Douglas for introducing me to these problems as well as for constant encouragement and support. Without his patient supervising and enlightening suggestions this work would have never been possible. I thank Roger Bielawski for an enlightening e-mail discussion and for making his work [4] available to me prior to publication.
2 D-Branes and SU(2) Monopoles

2.1 SU(2) Monopoles and Nahm Equations

The data for an SU(2) monopole on $R^3$ consist of a connection $A_\mu(x)$ in the trivial bundle $SU(n) \times R^3$ and a Higgs field $\Phi(x)$ transforming in the adjoint representation of $SU(n)$. The energy of the Yang-Mills-Higgs functional has a lower topological bound which is attained by certain static field configurations, the BPS monopoles. In the limit of vanishing Higgs potential these are defined by Bogomolnii equations:

$$B_i = D_i \Phi, \quad B_i = \frac{1}{2} \epsilon_{ijk} F_{jk}$$

with asymptotic boundary conditions on $\Phi$:

$$\Phi(r) = i \text{diag}(1, -1) - \frac{i}{2r} \text{diag}(k, -k) + O\left(\frac{1}{r^2}\right)$$

The expectation value of $\Phi$ breaks the gauge group $SU(2)$ down to an electromagnetic $U(1)$ in the asymptotic region. Moreover, it obviously determines a map from the two sphere at infinity $S_\infty^2$ into the orbit $SU(2)/U(1) \cong S^2$ which defines in fact a homotopy class in $\pi_2(S^2)$. This class yields a topological invariant of the solution. In this case $\pi_2(S^2) \cong Z$ and the topological invariant is given simply by $k \in Z$, also called magnetic charge.

A remarkable description of the moduli space of monopoles for arbitrary (classical) gauge group has been given by Nahm in [24] and further developed in [8], [15], [18]. We will quote their results only for the moduli space $M(k)$ of $SU(2)$ monopoles with fixed magnetic charge $k > 0$. A complex of Nahm data consists of $su(2)$ valued functions $X^i, i = 1, 2, 3$ on the interval $(-1, 1)$ satisfying Nahm equations:

$$\frac{dX^i}{ds} + \frac{1}{2} \sum_{j,k=1}^3 \epsilon_{ijk} [X^j, X^k] = 0$$

subject to the following boundary conditions:

(i) The $X^i$ are analytic in the interior of the interval with simple poles at $s = \pm 1$:

$$X^i = \frac{T^i}{s-1} + O(1), \quad s \mapsto \pm 1$$
The residues $T^i$ define an irreducible $k$ dimensional representation of $SU(2)$:

$$[T^i, T^j] = \epsilon^{ijk} T^k.$$  

(5)

Then there is a $1-1$ correspondence between $U(k)$ conjugacy classes of Nahm complexes and $SU(2)$ monopoles of charge $k$. This construction can be further refined [15] by imposing a symmetry condition on the functions $X^i$:

$$X^i(s) = X^{i\top}(-s)$$  

(6)

and restricting to $O(k)$ conjugation.

Nahm equations can be set in covariant form [8] by introducing a fourth component $X^4$ and defining the covariant derivative:

$$\nabla_s X^i = \frac{dX^i}{ds} + [X^0, X^i]$$  

(7)

Then they prove to be equivalent to self-duality equations for the connection $X^0 ds + X^i dx^i$ on the space $(s, x^i)$:

$$\nabla_s X^i + \frac{1}{2} \sum_{j,k=1}^3 \epsilon_{ijk} [X^j, X^k] = 0$$  

(8)

The covariant Nahm data are invariant under unitary gauge transformation which restrict to the identity transformation at $s = \pm 1$:

$$X^i \mapsto g^{-1} X^i g \quad X^0 \mapsto g^{-1} X^0 g + g^{-1} \frac{dg}{ds}$$  

(9)

According to [8] one can further define complex Nahm data:

$$\alpha = \frac{1}{2}(X^0 + iX^1), \quad \beta = \frac{1}{2}(X^2 + iX^3)$$  

(10)

in terms of which the original equations become a complex equation:

$$\frac{d\beta}{ds} + 2[\alpha, \beta] = 0$$  

(11)

and a real equation:

$$\frac{d}{ds} (\alpha + \alpha^*) + 2([\alpha, \alpha^*] + [\beta, \beta^*]) = 0$$  

(12)
The complex equation is preserved by complex $GL(m,C)$-valued gauge transformations:

\[ \alpha \mapsto g^{-1} \alpha + \frac{1}{2} g^{-1} \frac{d}{ds} g \quad \beta \mapsto g^{-1} \beta g \]  

(13)

while the real equation is preserved only by unitary transformations. The equations (11) and (12) present a striking resemblance with the moment map equations for hyperkahler quotients [10]. Actually, it can be rigorously proven [19] that the moduli space arises in this formulation as a hyperkahler quotient.

Finally we briefly review the construction of the monopole solution in terms of Nahm data [2], [6], [15], [24]. Consider the first order linear differential operator:

\[ \Delta(s) = i \frac{d}{ds} + (T^i + ix^i) \sigma_i \]  

(14)

where $\sigma_i$ are the standard generators of $SU(2)$. Then the equation

\[ \Delta^\dagger v = 0 \]  

(15)

has a unique normalizable solution $v(s)$ with

\[ \int ds v^\dagger v = 1 \]  

(16)

Note that this is a quaternionic $1 \times k$ matrix which in complex notation becomes a complex $2 \times 2k$ matrix, [4], [11].The gauge field and the Higgs field of the monopole are given respectively by:

\[ A_i = \int ds v^\dagger \partial_i v \]  

\[ \Phi = -i \int ds sv^\dagger v \]  

(17)

This concludes our brief discussion of $SU(2)$ monopoles.

### 2.2 Moduli Space of Super-Symmetric Ground States

Consider two parallel Dirichlet type IIb 3-branes in the 123 plane separated by a distance $\mu$ along the $x^9$ axis and a bound state of $k$ 1-branes stretching between them. The existence of such bound states has been proven in [35], while the fact that they can end on 3-branes has been shown in [31],[12],[32].
As stated in [7], [21], [27] and emphasized further in [26], [35] the low energy effective action describing the 1-brane dynamics is the dimensional reduction of the $D = 10$, $N = 1$ super Yang-Mills action to the 1-brane world-sheet. The resulting action has $(8,8)$ super-symmetry and it has been derived in [5]. The field content and its D-brane interpretation can be summarized as follows:

a) bosonic sector

\[ A_\mu \quad \mu = 0, 1 \]
\[ \Phi_{A4} = (X_A + iX_{A+3})/\sqrt{2} \quad A = 1, 2, 3 \]
\[ \Phi^{AB} = \frac{1}{2} \epsilon^{ABCD} \Phi_{CD} = \Phi_{AB}^\ast \quad A, B = 1, \ldots, 4 \]
\[ S = X_7, \quad P = X_8 \]

b) fermionic sector

\[ \chi^A, \quad \tilde{\chi}^A = C_2(\bar{\chi}^A)^T \quad A = 1, \ldots, 4 \]

In the above $A_\mu$ is the two dimensional gauge field, $\Phi_{AB}$, $S$, $P$ are two dimensional scalars in the adjoint representation of $U(m)$ that represent the transverse oscillations of the 1-branes and $\chi^A$ are fermions transforming also in the adjoint of $U(m)$. The indices $A, B, \ldots$ are $SU(4)$ symmetry indices arising in the process of dimensional reduction and the fields $\Phi_{AB}$ form an antisymmetric tensor multiplet of $SU(4)$. We follow the conventions of [5] for the two dimensional Dirac algebra. The lagrangian and super-symmetry transformations read

\[
\mathcal{L} = Tr\{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu P D^\mu P + \frac{1}{2} D_\mu S D^\mu S \\
+ \frac{1}{2} D_\mu \Phi_{AB} D^\mu \Phi^{AB} + i \tilde{\chi} A_\gamma \cdot D \chi^A + g(\bar{\chi} A_5[\chi^A, P] + i \chi_A[\chi^A, S]) \\
- \frac{1}{2} g^2(\bar{\chi}^A[\chi^B, \Phi_{AB}] - \bar{\chi}_A[\chi_B, \Phi^{AB}]) - \frac{1}{4} g^2[\Phi_{AB}, \Phi_{CD}][\Phi^{AB}, \Phi^{CD}] - \frac{1}{2} g^2[S, P]^2 \\
- \frac{1}{2} g^2[S, \Phi_{AB}][S, \Phi^{AB}] - \frac{1}{2} g^2[P, \Phi_{AB}][P, \Phi^{AB}]\} \quad (18)
\]

\[
\delta A_\mu = i(\bar{\alpha} A_\gamma \mu \chi^A - \bar{\chi} A_\gamma \mu \alpha^A) \\
\delta P = \bar{\chi} A_5 \gamma^A - \bar{\alpha} A_5 \chi^A
\]
\[
\delta S = i(\bar{\chi} A \alpha^A - \bar{\alpha} A \chi^A)
\]
\[
\delta \Phi_{AB} = i(\bar{\alpha} B \bar{x}_A - \bar{\alpha} A \bar{x}_B + \epsilon_{ABC} \bar{\alpha} C \chi^D)
\]
\[
\delta \chi^A = \sigma_{\mu \nu} F^{\mu \nu} + i \gamma \cdot D P \gamma_5 \alpha^A - \gamma \cdot DS \alpha^A - \gamma \cdot D \Phi_{AB} \bar{\alpha}_B
\]
\[
+ g(i[P, S] \gamma_5 \alpha^A - i[P, \Phi_{AB}] \gamma_5 \bar{\alpha}_B - [S, \Phi_{AB}] \bar{\alpha}_B + \frac{1}{2}[\Phi_{AB}, \Phi_{BC}] \alpha^C)
\] (19)

Throughout this section we normalize the Higgs field so that \( \mu = 2 \), thus the 3-branes can be taken at points \( \pm 1 \) on the \( x^9 \) axis without loss of generality. Let \( s \times t \in (-1, 1) \times \mathbb{R} \) be world-sheet coordinates. Since the 1-branes are constrained to end on the 3-branes one should obviously impose the boundary conditions

\[
X^\mu(\pm 1, t) = 0, \quad \mu = 4, \ldots, 8
\] (20)

Compatibility with super-symmetry transformations implies similar boundary conditions for fermions:

\[
\chi^A(\pm 1, t) = 0, \quad A = 1, \ldots, 4
\] (21)

Since the super-symmetry transformations of the spinor fields involve derivatives of the fields \( X^\mu \) a simple consistency check of the vanishing order near the boundary shows that we have in fact to restrict to fields with compact support inside the interval or "bump fields". This restriction does not apply to the transverse fields \( X^i \), with \( i = 1, 2, 3 \) which will be seen to have interesting boundary behavior. Moreover since we are interested in super-symmetric ground states with \( X^\mu \) and \( \chi^A \) vanishing identically this restriction is quite natural. There is a slight subtlety related to this picture: since the world-sheet has nonempty boundary, the total derivative terms in the super-symmetric variation of the lagrangian yield surface terms by integration. However these terms cancel by the above boundary conditions leading to a consistent theory. There are also additional fields arising from the quantization of the fundamental 1\text{--}3 and 3\text{--}1 strings. These constitute quantum mechanical degrees of freedom that couple to the world-sheet boundary and they will play an important role latter.

We can now address the problem of super-symmetric ground states for the 1-brane configuration. These are solutions to:

\[
\delta \chi^A = 0, \quad A = 1, \ldots, 4
\] (22)
with:
\[ \chi^A \equiv 0, \quad X^\mu \equiv 0, \quad \mu = 4, \ldots, 8 \]  
(23)

Fixing the axial gauge \( A_0 = 0 \), which restricts us to static gauge transformations and imposing a reality condition on the super-symmetry parameters:
\[ \tilde{\alpha}^A_{\varphi} = \epsilon_{\varphi \gamma} \alpha^A_\gamma \]  
(24)

we find a family of ground states that break half of the original \((8, 8)\) super-symmetries given by:
\[ D_1 \Phi^{AB} + \frac{g}{2} [\Phi^{AC}, \Phi_{CB}] = 0. \]  
(25)

Setting \( g = \sqrt{2} \), the equations can be rewritten as:
\[ D_1 X^i + \frac{1}{2} \epsilon^{ijk} [X^j, X^k] = 0 \]  
(26)

which are formally identical to covariant Nahm equations (8). This is positive evidence for the identification of monopoles with 1-strings but it is by no means sufficient. There are two main problems that have to be answered at this stage.

(i) So far we have proceeded formally, ignoring the boundary conditions on the fields \( X^i(s) \) which are in fact an essential ingredient of Nahm construction. Consequently, it is vital that we understand these boundary conditions in D-brane context.

(ii) The second puzzle is related to the role of the usual super-symmetric ground states. It can be seen easily that if the reality conditions (24) are absent, the 1-brane system has a “trivial” family of ground states:
\[ X^i = \text{diag}(i\lambda^i_1, \ldots, i\lambda^i_m) \]  
(27)

which admit a physical interpretation in terms of positions of the 1-branes \([35]\). Thus one would be tempted to conclude that these are the “real” ground states of the system while those derived above are rather unphysical. As we show shortly these two problems are in fact closely related.

Note that the boundary conditions (20), (21) require that \( \delta \chi^A = 0 \) be identically satisfied in a neighborhood of the boundary, thus the fields \( X^i \) should behave near boundary as general local solutions of Nahm equations. One can easily check that the ansatz below:
\[ X^i = \frac{T^i}{s}, \quad [T^i, T^j] = i \epsilon^{ijk} T^k \]  
(28)
always constitutes such a solution, thus the most general boundary conditions allowed by consistency requirement are:

\[ X^i = \frac{T^i}{s + 1} + O(1), s \mapsto \pm 1 \]  
(29)

with

\[ [T^i, T^j] = i\epsilon^{ijk}T^k \]  
(30)

defining an \( k \) dimensional \( SU(2) \) representation. These are not yet Nahm boundary conditions as the latter also require that the \( SU(2) \) representation be irreducible. It is this aspect that will provide a hint on the solution to the second problem as well.

Suppose that the representation is reducible, decomposing as:

\[ k = k_1 \oplus k_2 \oplus \ldots \oplus k_q \]  
(31)

Then the residues \( T^i \) may be set in block diagonal form:

\[ T^i = \text{diag}(T^i_1, \ldots, T^i_q) \]  
(32)

Using a similar block diagonal ansatz for the fields \( X^i(s) \):

\[ X^i = \text{diag}(X^i_1, \ldots, X^i_q) \]  
(33)

we find that the Nahm equations split in \( q \) groups

\[ \frac{dX^i_a}{ds} + \frac{1}{2}\epsilon^{ijk}[X^{ja}, X^{kb}] = 0 \quad a = 1, \ldots, q \]  
(34)

In D-brane terms this means that the original bound state of \( k \) branes splits in \( q \) sub-bound states infinitely far apart. Each of the above equations determines a ground state for each group taken separately. The extreme limit is the case when the representation defined by the residues splits in a sum of one dimensional representations. In this case the residues vanish and the solutions to Nahm equations following from the ansatz are exactly the “trivial” ground states (27).

Collecting all the facts, we have shown that the bound state of \( k \) 1-branes exhibits a family of ground states which may be formally identified with the monopole moduli space in Nahm formulation. This identification is achieved if one imposes Nahm boundary conditions on the transverse fields \( X^i(s) \).
The usual flat directions of the world-sheet potential appear as degenerate Nahm solutions corresponding to the limit of infinitely separated 1-branes. However, it is not clear why Nahm boundary conditions are the right ones when the D-branes are close by. Their physical interpretation is obscured by the poles of the transverse fields at the boundary and by the fact that in general the matrix valued “coordinate” fields cannot be simultaneously diagonalized as they do not commute. One could argue that the usual ground states should be the “real” ones for any D-brane configuration. It appears that these questions can be clarified by a probe argument.

2.3 Probe Analysis

The main idea is to use a D-brane as a probe in order to construct the monopole gauge field as a coupling on the D-brane world-sheet. This technique has been applied first in [9] for ADHM construction of instantons. In the present case the analysis will be somewhat different due to the particularities of the model.

Perform a T-duality transformation of the previous configuration along the 4567 directions. The result is a system of two parallel type IIb 7-branes in the 1, ..., 7 plane and \( k \) parallel 5-branes in the 4, ..., 7, 9 plane stretching between them. The 5-branes intersect the 7-brane along four dimensional subspaces with three transverse coordinates \( X^1, X^2, X^3 \) within the 7-brane. The probe is a 1-brane along the 9-th axis also ending on the 7-branes. Note that this is exactly the configuration considered in [11] where it is argued that the 5-brane is the monopole of the 7-brane world-volume. The difference is that in this case the branes wrap the \( T^4 \) of T-duality and not \( K_3 \), thus the 5-brane will be identified with a monopole of \( N=4 \) rather than \( N = 2 \) SYM theory.

Quantization of different open string sectors yields:

(i) Open 1−5 strings have DD boundary conditions in 1238 directions, DN boundary conditions in 4567 directions and NN boundary conditions in 09 directions. Quantization of the \( NS \) sector yields four bosonic states forming

\[1\] I thank M.R. Douglas for the idea of this analysis.

\[2\] As pointed out by M.R. Douglas, the 1-brane cannot end on the 7-brane since the \( R − R \) 2-form does not couple to the 7-brane gauge field, so flux conservation would be violated. However, the intersection point can be regarded for our purposes as an ending point for the 1-brane fields. It is in this sense that we will be using this terminology throughout the paper.
an $SO(4) \cong SU(2) \times SU(2)$ multiplet. GSO projection selects $SO(4)$ chirality leaving a complex doublet. Quantization of the $R$ sector yields chiral Weyl fermions $\xi_-$ and $\xi_+$ with an internal symmetry $SO(4)$ group. GSO projection relates world-sheet and internal chirality [9], leaving two $SU(2)$ doublets $\xi_-^A$, $\xi_+^Y$. The fields are charged under both the 1-brane $U(1)$ gauge field and the 5-brane $U(k)$ gauge field, transforming as $(k, -1) \oplus (\bar{k}, 1)$. Moreover one expects a mass-term for these fields proportional to $|X^i - Y^i|$, where $X^i, Y^i, i = 1, 2, 3$ describe transverse oscillations of the five and one branes respectively.

(ii) Open 1–7 have DN boundary conditions along 1, 2, 3, 7, 9 directions, NN boundary conditions along 0, 8 directions and DD boundary conditions along the 9-th direction. There is one chiral fermion mode arising from the $R$ sector for each connected component of the intersection between the 1-brane and the two branes. Thus we are left with two chiral fermions which constitute quantum mechanical degrees of freedom located at different 1-brane endpoints. These fields are expected two play an important role in the discussion conditions for the bulk 1–5 theory.

To derive the latter, we start as in [9] with five and nine branes and then do dimensional reduction. More precisely, we start with $k$ 9-branes and a 5-brane in the 012389 plane and then take dimensional reduction to the 09 plane. The effective theory of the 5-brane is $D = 6$ (0, 2) super Yang-Mills coupled to charged multiplets consisting of a Weyl fermion $\xi$ and a doublet of complex scalars. The fermion kinetic term is

$$L_{\text{kin}} = \int d^6x \ i\bar{\xi} \Gamma \cdot (\partial + A + iB)\xi$$

where $A$ is the $U(k)$ gauge field induced from the 9-branes and $B$ is the Abelian gauge field of the 5-brane. The dimensional reduction is performed in [5]. The six dimensional gauge fields yield two dimensional gauge fields and four scalars representing oscillations of the 1-brane within the five brane. Since the 1-brane is constrained to end on the 7-branes one of these oscillations is frozen. Recall that $X^i, Y^i, i = 1, 2, 3$ denote the oscillations of the five and one brane respectively. The six dimensional Weyl fermion yields precisely the chiral complex fermions derived earlier from string quantization. The relevant part of the two dimensional lagrangian for fermions is:

$$L^{(2)} = \int d^2x \ \{i\bar{\xi}_- D_+ \xi_- + i\bar{\xi}_+ D_- \xi_+\}$$

$$- \int d^2x \ \{i\bar{\xi}_+ (X^i + iY^i)\sigma_i \xi_- + i\bar{\xi}_- (X^i + iY^i)\sigma_i \xi_+\}$$

11
Here $\sigma^i$ denote the standard SU(2) generators acting on the internal symmetry indices of the fermions. Note that because of GSO projection the SU(2) index of left handed fermions is different from that of right handed fermions, thus the $\sigma^i$ carry mixed indices, $\sigma^i_{AY}$, and should not be thought of as generators of any of the SU(2) groups in question. In the following we will simply forget this subtlety and treat both $A$ and $Y$ on equal footing as only one index with values 1, 2. Then we can define:

$$\psi = \frac{1}{2}(\xi_- + i\xi_+), \quad \chi = \frac{1}{2}(\xi_- - i\xi_+)$$

(37)

and rewrite the lagrangian in the form:

$$\mathcal{L}^{(2)} = \int d^2x \left\{ i\bar{\chi}D_0\chi + i\bar{\psi}D_0\psi + i\bar{\chi}D_1\psi + i\bar{\psi}D_1\chi \right\}$$

$$- \int d^2x \left\{ \bar{\chi}(X^i + iY^i)\sigma_i\psi - \bar{\psi}(X^i + iY^i)\sigma_i\chi \right\}$$

(38)

Treating the terms without time derivatives as generalized mass terms we derive a quantum mechanical effective action for fermions in the spirit of [37]. Since the gauge fields will play no role in the following we can gauge them to zero. The mass-less modes for the fermions are determined by the equations:

$$(\partial_1 + (iX^i - Y^i)\sigma_i)\psi = 0$$

(39)

$$(\partial_1 - (iX^i - Y^i)\sigma_i)\chi = 0$$

(40)

The first equation is identical to the equation (15) appearing in the construction of the monopole gauge field while the second equation is it’s dual. We will assume that the first equation has $p$ normalizable solutions $v^\alpha$ analytic and finite near the boundary while the second equation has none. This is the case if the fields $X^i$ satisfy either Nahm or standard boundary conditions, but $p$ is different in each case. Consider then the following low energy ansatz for the fermionic fields:

$$\psi(t, s) = \psi^\alpha(t)v^\alpha(s), \quad \chi(t, s) = 0$$

(41)

where $\psi^\alpha(t)$ are slowly varying functions of time. Take similarly $Y^i \equiv Y^i(t)$ to be slowly varying functions of time, with no spatial dependence. Then the above lagrangian reduces to:

$$\mathcal{L}^{(2)} = \int dt \bar{\psi}^\alpha(\delta_{\alpha\beta}\partial_0 + \partial_0Y^i)\left\{ \int ds \left. v^\alpha L\frac{\partial v^\beta}{\partial Y^i} \right\} \right)\psi^\beta$$

(42)
We see that the probe moves in an $SU(p)$ space-time external background gauge field $\mathcal{A}_j$ given by the Nahm construction (17) where $p$ is the number of $1-5$ independent zero modes. At this stage both Nahm and standard boundary conditions seem to lead to a consistent theory, the probe moving in a well defined monopole gauge field. However one expects only one consistent theory and this is where the $1-7$ boundary fields come into play.

2.4 Boundary Conditions

A proper analysis of the boundary $1-7$ fields and their couplings with the bulk $1-5$ theory seems a rather difficult task, well beyond the limits of this paper. We will not provide a complete solution here but we propose a simple physical mechanism powerful enough to select the boundary conditions.

The key observation is that the $1-5$ zero modes determined by Nahm equations are present even in the limit of large separation between the $1$-brane and the $5$-brane. The only mechanism that can make this fact possible is to think of these modes as corresponding to $1-5$ strings gliding and touching the $7$-branes (see fig.1).

![Figure 1: The D-brane configuration for the probe analysis](image)

1: $1-7$ tensionless strings
2: $1-5$ strings that become tensionless by touching the $7$-branes

In this limit, the $1-5$ strings can be regarded as $1-7$ strings as well, thus there should be a $1-1$ correspondence between $1-5$ and $1-7$ zero modes. According to our analysis, there are only two $1-7$ zero modes located at the two endpoints of the $1$-brane, thus there should be only two zero modes in the $1-5$ theory for any value of the $5$-brane multiplicity $k$ i.e. $p = 2$. This is all we need since that happens precisely when the ground state fields $X^i$ satisfy Nahm boundary conditions! It is easy to check that
standard boundary conditions give $p = 2k$ independent solution leading thus to an inconsistency for any $k > 1$. The exception $k = 1$ is not a flaw in the argument as in that case they coincide [24].

A different aspect of this problem is revealed by the particular form of the would be monopole gauge field $A_j$. Trivial boundary condition lead to an $SU(2k)$ gauge field, in fact a linear superposition of embedded BPS monopoles. The probe analysis shows that this is not physical since one would need the Chan-Paton factors of $2k$ 7-branes to construct it. Actually we have re-derived in the D-brane framework an well known result in the monopole theory. A linear superposition of $k > 1$ BPS monopoles is not an exact $SU(2)$ monopole solution of charge $k$ [20], but it can be an exact solution when embedded along different roots in a gauge group of higher rank [20], [33], [34].

This shows that the natural ground states for the 1-brane configurations considered so far are precisely those defined by Nahm equations, which appear already in covariant form in this context. Defining complex Nahm data, it is straightforward to rewrite them as moment map equations for an $U(k)$ hyperkahler quotient, as expected in theories with extended super-symmetry.

The irreducibility condition in the definition of Nahm data corresponds to the fact that the $k$ 1-branes form a true bound state. Relaxing this condition, we obtain a degenerate configuration corresponding to infinitely separated sub-bound states as in the end of (2.2). This represents a point on the boundary of the monopole moduli space [1]. The usual flat directions of the potential, while inconsistent when the branes are at finite distance, are recovered in the limit of infinitely separated 1-branes. Equivalently these are points on the boundary of the moduli space parameterizing infinitely separated BPS monopoles.

Finally, there is an intriguing aspect of this picture, namely the Nahm ground states do not admit a clear coordinate interpretation as in [35]. This is related to the fact that there is no general way to define the centers of the hypothetical elementary magnetic charges composing a multi-monopole solution [3],[4]. This can be done only for well separated monopoles [1],[3],[4], that is in the asymptotic region of the moduli space when one can show that the Higgs field has exactly $k$ essential zeros (counted with multiplicity). The positions of the zeros can be regarded as centers of magnetic charge. However, this description breaks at a general point on the moduli space [17].

I thank R. Bielawski for clarifying these points to me.

---

\(3\) I thank R. Bielawski for clarifying these points to me.
3 D-Branes and $SU(n)$ Monopoles

We generalize the results of the previous section to $SU(n)$ monopoles. The Nahm construction for $SU(2)$ monopoles can be generalized to arbitrary gauge group as follows.

3.1 $SU(n)$ Monopoles and Nahm Equations

The ADHM construction of arbitrary monopole moduli spaces has been first discussed by Nahm, [23] and further developed in [18]. The brief presentation in this subsection follows closely [18]. The asymptotic conditions for the Higgs field generalize to:

$$\Phi(r) = i\text{diag}(\mu_1, \ldots, \mu_n) - \frac{i}{2r}\text{diag}(k_1, \ldots, k_n) + O\left(\frac{1}{r^2}\right) \quad (43)$$

where $\mu_1 \ldots \mu_n$ and $k_1 \ldots k_n$ satisfy:

$$\sum_{a=1}^{n} \mu_a = \sum_{a=1}^{n} k_a = 0, \quad \mu_1 < \mu_2 < \ldots < \mu_n \quad (44)$$

The gauge group $SU(n)$ is asymptotically broken to a maximal torus $T = U(1) \times \ldots \times U(1)$ and the solutions are topologically classified by homotopy classes $[\Phi_\infty] \in \pi_2(G/T)$. According to [16], [22] these can be represented by $r$-tuples of integers $(m_1 \ldots m_r)$, where $r$ is the rank of the group $G$, $r = n - 1$ for $SU(n)$. The magnetic charges $(m_1 \ldots m_{n-1})$ are given in terms of the asymptotic data by:

$$m_1 = k_1, \quad m_2 = k_1 + k_2, \quad m_{n-1} = k_1 + \ldots + k_{n-1} \quad (45)$$

The Nahm data are defined as analytic $u(m_a)$ valued functions $X^i_a$ defined on each interval $(\mu_a, \mu_a + 1)$ such that they solve Nahm equations:

$$\frac{dX^i_a}{ds} + \frac{1}{2} \sum_{j,k=1}^{3} \epsilon_{ijk}[X^j_a, X^k_a] = 0 \quad (46)$$

subject to the boundary and matching conditions:

(i) Let $t = s - \mu_a$. Then at a point $\mu_a$ with $m_{a-1} < m_a$, $X^i_{a-1}$ is analytic and has finite nonzero limit $C^i_a$ as $t \to 0^-$ and $X^i_a$ has a block form expansion near $t = 0$

$$\left( \begin{array}{cc} C^i_a + O(t) & O(t^\gamma) \\ O(t^\gamma) & \frac{T^a}{t} + O(1) \end{array} \right) \quad (47)$$
where $\gamma = \frac{m_a - m_{a-1}}{2}$ and $X^i_a$ define an irreducible representation of $SU(2)$.

(ii) If $m_{a-1} > m_a$ the boundary conditions are the same with the roles of $m_a$ and $m_{a-1}$ inversed.

(iii) If $m_{a-1} = m_a$, $X^i_a$ and $X^{i-1}_a$ are both analytic near $t = 0$ with finite limits $C^i_a$ and $C^{i-1}_a$ required to satisfy a certain regularity condition which will not be made explicit here since it will play no role in the following. A stronger version of that condition is simple continuity $C^i_a = C^{i-1}_a$ and it restricts us in this case to embedded $U(n-1)$-monopoles [18].

Note that index $a$ runs from 0 to $n$ with the conventions $m_0 = m_n = 0$. Then there is a $1-1$ correspondence between $U(m_a)$ conjugacy classes of Nahm data and $SU(n)$ monopoles with magnetic charges $(m_1, \ldots, m_{n-1})$. Repeating the steps of the $SU(2)$ construction on each interval one can define complex Nahm data and represent the moduli space as a hyperkahler quotient. Taking into account the boundary conditions for $X^i$ the complex gauge transformations are properly defined [18] as $(n-1)$-tuples $g = (g_1, \ldots, g_{n-1})$ of smooth maps $g : (\mu_{a-1}, \mu_a)$ with finite analytic limits at the boundary such that:

i) If $m_a > m_{a-1}$, $g_a$ preserves the block form (56), the derivatives of the off diagonal blocks are of order $O(\gamma)$ and the limit of the $m_a \times m_a$ diagonal block equals the limit of $g_{a-1}$.

ii) If $m_a < m_{a-1}$ the same boundary conditions are satisfied with roles of $m_a$ and $m_{a-1}$ inversed.

iii) If $m_a = m_{a-1}$ the limits of $g_a$ and $g_{a-1}$ coincide.

3.2 D-Brane Configurations and Moduli Spaces

The construction of an $SU(n)$ monopole with asymptotic conditions (52), (53) requires a configuration of $n$ parallel 3-branes located at points of coordinates:

$$X^0_a = \mu_a, \quad a = 1, \ldots, n$$

(48)

along the $x^0$ axis. This configuration corresponds to a point of maximal symmetry breaking of the effective $U(n)$ gauge theory on the 3-brane world-volume. The gauge group may be factored as $U(n) \cong SU(n) \times U(1)$ where the Abelian factor describes the center of mass dynamics of the 3-branes and the $SU(n)$ factor describes the relative dynamics of the branes. The subsequent construction will thus yield $SU(n)$ monopoles rather than $U(n)$ monopoles.
Figure 2: The D-brane configuration for a spherically symmetric $SU(5)$ monopole

The key is to construct a configuration of 1-branes with endpoints on the $n$ parallel 3-branes so that the net magnetic charge induced on the $a$-th brane be equal to $k_a$ and that in the interval $(\mu_a, \mu_{a+1})$ there be exactly $m_a$ 1-branes stretching along the $x^9$ axis. The best way to illustrate how this works is to present some particular cases which will make the general rules clear. The details for spherically symmetric $SU(5)$ and $SU(6)$ monopoles are presented in figures one and two respectively.

Figure 3: The D-brane configuration for a spherically symmetric $SU(6)$ monopole

Standard Gauss law arguments show that a 1-brane ending on a pair of 3-branes induces a monopole on one of the branes and an anti-monopole on the other. Moreover a 1-brane threading through the core of a 3-brane
does not induce any charge in the 3-brane world-volume. Consequently, the oscillations of the 1-brane are not constrained by charge conservation, the world-sheet fields being continuous.

As in the case of SU(2) monopoles, boundary and matching conditions play a crucial role in the construction. In fact, a part of these are already implicit in the previous paragraph. For a systematic approach, let the $u(m_a)$ valued world-sheet fields $X_a^i$ describe the transverse oscillations of the 1-branes stretching between the $a$-th and the $(a+1)$-th 3-brane. Charge conservation yields $m_a = m_{a-1} + k_a$, where $k_a$ is the net magnetic charge on the $a$-th 3-brane, equal to the net number of 1-branes that end on that brane. The Chan-Paton degrees of freedom of the lower configuration can be split as:

$$C^{m_a} = C^{m_{a-1}} \oplus C^{k_a}$$

and the matrix valued fields $X^\mu$, $X^i$ can be set in block form:

$$X^\mu_a = \begin{pmatrix} X^\mu_{a,11} & X^\mu_{a,12} \\ X^\mu_{a,21} & X^\mu_{a,22} \end{pmatrix} \quad X^i_a = \begin{pmatrix} X^i_{a,11} & X^i_{a,12} \\ X^i_{a,21} & X^i_{a,22} \end{pmatrix}$$

The fermionic fields can be split similarly:

$$X^A_a = \begin{pmatrix} X^A_{a,11} & X^A_{a,12} \\ X^A_{a,21} & X^A_{a,22} \end{pmatrix}$$

The boundary conditions for the fields $X^\mu$, $\mu = 4, \ldots, 8$ are:

(i) $X^\mu_{a-1,11}$, $X^\mu_{a,11}$ must be analytic in a neighborhood of $t = 0$ with finite limits as $t \rightarrow 0$ and these limits must coincide.

(ii) $X^\mu_{a,12}$, $X^\mu_{a,21}$, $X^\mu_{a,22}$ must be “bump fields” that is compactly supported away from $t = 0$.

Note that the condition (i) above holds for the fields $X^i$ as well. The matching conditions for fermions are identical. The surface terms arising from the super-symmetry variation of the upper and lower lagrangian cancel each other leading again to a consistent theory.

Imposing the reality condition (24) on each interval we find a family of super-symmetric ground states which are solutions to covariant Nahm equations:

$$D_1 X^i_a + \frac{1}{2} \epsilon^{ijk} [X^j_a, X^k_a] = 0$$

We fix again the gauge $A_1 = 0$ reducing to standard Nahm equations. In order to analyze the local behavior of the solutions, rewrite these equations
in terms of matrix blocks:

\[
\frac{dX_{11}^i}{ds} + \frac{1}{2} \epsilon^{ijk} ([X_{11}^j, X_{11}^k] + X_{12}^j X_{21}^k - X_{12}^j X_{21}^k) = 0
\]

\[
\frac{dX_{12}^i}{ds} + \frac{1}{2} \epsilon^{ijk} (X_{11}^j X_{12}^k - X_{11}^j X_{12}^k + X_{12}^j X_{22}^k - X_{12}^j X_{22}^k) = 0
\]

\[
\frac{dX_{21}^i}{ds} + \frac{1}{2} \epsilon^{ijk} (X_{21}^j X_{11}^k - X_{21}^j X_{11}^k + X_{22}^j X_{21}^k - X_{22}^j X_{21}^k) = 0
\]

\[
\frac{dX_{22}^i}{ds} + \frac{1}{2} \epsilon^{ijk} ([X_{22}^j, X_{22}^k] + X_{21}^j X_{12}^k - X_{21}^j X_{12}^k) = 0
\]

where the interval index \(a\) has been suppressed for simplicity. The matching conditions for fermions imply by consistency with the unbroken symmetries that:

(i) \(X_{11}^i, X_{a-1}^i\) must be analytic in a neighborhood of \(t = 0\), with finite and equal limits as \(t \to 0^+\) and \(t \to 0^-\) respectively.

(ii) \(X_{12}^i, X_{21}^i\) must be analytic neighborhood of \(t = 0\), vanishing to some order \(\gamma\) as \(t \to 0\).

(iii) \(X_{22}^i\) must be identically equal near \(t = 0\) to a general local solution to Nahm equations:

\[
X_{22}^i = \frac{T_i}{t} + O(1) \quad (54)
\]

where \(T_i\) define an irreducible representation of \(SU(2)\).

The vanishing order \(\gamma\) can be determined from representation theoretic consistency considerations similar to those of [13], pg. 625. The result is

\[
\gamma = \frac{k_a - 1}{2} \quad (55)
\]

being identical to that in (56). Finally, if \(m_{a-1} = m_a\) the above conditions reduce simply to:

\[
X_{a-1}^i = X_a^i, \quad t = 0 \quad (56)
\]

which is a stronger condition than that of [13] for monopole moduli spaces and it corresponds to an embedded \(U(n-1)\) monopole. This appears quite naturally in the D-brane picture since \(k_a = 0\) implies that there are no magnetic charges induced on the \(a\)-th 3-brane, thus it can be removed from the configuration with no effect on the charge distribution. The presence
of the poles in the boundary conditions of the transverse fields has been analyzed in detail in the previous section.

In conclusion, the D-brane-monopoles correspondence can be generalized to $SU(n)$ gauge groups with arbitrary $n$. We remark that the moduli space of super-symmetric ground states are naturally described by covariant Nahm equations. The formulation in terms of hyperkähler quotients is again straightforward. One has to rearrange the Nahm data in complex form and to define appropriate gauge transformations. Since we have shown that the boundary conditions for the fields are precisely Nahm conditions, it is clear that the gauge transformations are identically to those presented in (3.1) for monopoles. A distinct feature of the above generalization is that the D-brane configurations admit a natural interpretation in terms of BPS monopoles embedded along different roots. This will be pursued next.

### 3.3 Embedded BPS Monopoles in D-Brane Picture

Embedding of elementary monopole solutions in arbitrary gauge groups has been thoroughly studied in [20], [33], [34]. Following this line, one can choose an orthonormal basis of the $su(n)$ Cartan sub-algebra such that the asymptotic value of the Higgs field takes the form:

$$\Phi = \mu \cdot H - \frac{1}{r} k \cdot H + O\left(\frac{1}{r^2}\right)$$  \hspace{1cm} (57)

In the case of maximal symmetry breaking, the simple roots $\beta^a, a = 1, \ldots, n-1$ can be uniquely chosen so that:

$$\mu \cdot \beta^a > 0$$  \hspace{1cm} (58)

The simple roots can be described conveniently as vectors in an $n$-dimensional space with basis $\{e_i\}$ lying in the hyper-plane perpendicular to $\sum_{i=1}^n e_i$:

$$\beta^a = e_a - e_{a+1}$$  \hspace{1cm} (59)

An arbitrary root $\alpha$ defines an $SU(2)$ subgroup with generators:

$$t_1 = (2\alpha^2)^{-1/2}(E_\alpha + E_{-\alpha})$$

$$t_2 = -i(2\alpha^2)^{-1/2}(E_\alpha - E_{-\alpha})$$

$$t_3 = \alpha^* \cdot H$$  \hspace{1cm} (60)
where

$$\alpha^* = \frac{\alpha}{\alpha^2}$$ (61)

is the dual of $\alpha$. Then one can embed a fundamental $SU(2)$ monopole in $SU(n)$ along any root $\alpha$ in a natural way [20], [33], [34]. If $\alpha \equiv \beta^a$ is a simple root the resulting solution has magnetic charges:

$$m_{ab} = \delta_{ab}$$ (62)

These are called fundamental $SU(n)$ monopoles. If the root $\alpha$ is not simple the solution can be regarded as a superposition at the same point in space of fundamental solutions oriented along different simple roots. The magnetic charges of such a solution are given by:

$$\alpha^* = \sum_{a=1}^{k} m_a \beta^a$$ (63)

Note that in the case of non-simple roots the superposition described above is still an exact solution. This is not true for $SU(2)$ gauge group, leading to the contradiction discussed in section 2.4. Approximate solutions with higher charges can be obtained by embedding many well separated fundamental monopoles along the same simple root, similarly to the construction of the asymptotic region of the moduli space of $SU(2)$ monopoles.

Returning to D-brane configurations, note that there is a 1−1 correspondence between the set of vectors $\{e_i\}$ and the set of $n$ parallel 3-branes and that each pair of consecutive 3-branes ($a$, $a+1$) determines uniquely a simple root. Then, taking into account the results of the previous section, a 1-brane stretching between these two 3-branes can be identified with the fundamental $SU(2)$ monopole embedded along the corresponding simple root. Many widely separated 1-branes in the same position can be identified with an asymptotic superposition of fundamental monopoles. A 1-brane stretching between two non-consecutive 3-branes ($a$, $b$), $a < b-1$ can be identified with an $SU(2)$ monopole embedded along the root:

$$\alpha = (e_a - e_{a+1}) + \ldots + (e_{b-1} - e_b)$$ (64)

that is to a superposition of fundamental monopoles corresponding to $\beta_a$, $\ldots$, $\beta_{b-1}$ at the same point in space. This agrees with the representation of
the \((a, b)\) 1-brane as a collection of 1-branes \((a, a - 1), \ldots , (b - 1, b)\) with endpoints identified. Note also that since

\[
\beta^*_a = \frac{1}{2} \beta_a, \quad \alpha^*_a = \frac{1}{2} \alpha
\]

the magnetic charges of this solution are:

\[
m_1 = 0, \ldots , m_{a-1} = 0, \ m_a = 1, \ldots , m_{b-1} = 1, \ m_b = 0 \ldots , m_{n-1} = 0 (66)
\]

in perfect agreement with those of the D-brane configuration. Many widely separated \((a, b)\) 1-branes would yield an asymptotic multi-monopole configuration as above. This picture suggests that an arbitrary bound state of 1-branes can be interpreted similarly as a generic \(SU(2)\) monopole embedded in \(SU(n)\) yielding an exact \(SU(n)\) solution. It is not clear if this holds true.

4 Conclusion

We have shown that the identification of the 3-brane world-volume monopoles with the D-strings of the type IIb theory can be made explicit in Nahm formalism. The detailed study of this correspondence has revealed several new aspects of D-brane physics. Perhaps the most intriguing of all is the coordinate interpretation of the Nahm super-symmetric ground states. Consistency arguments have shown that the transverse fields \(X^i\) describing the positions of the 1-brane endpoints within the 3-brane develop poles at the boundary! Moreover while they are smooth on the interior, they do not commute, thus a direct coordinate interpretation as in [35] is missing. This fact is related to the fact that monopoles are massive objects which generally do not have a well defined location in space. Since the 1-brane endpoints may be considered point-like particles, the poles appear as an attempt at a reconciliation between these two aspects. This provides further evidence that D-branes are more than simple geometrical objects, their behavior in certain circumstances contradicting standard geometrical interpretation. The consequences, as well as the extent of this phenomenon remain rather mysterious.

Another interesting aspect, not emphasized in the text, is the hidden correspondence between instanton/monopole reciprocity, [6], and D-branes. This fact has been first noted in [10] where it is argued that instanton reciprocity can be regarded as a T-duality transformation which interchanges
outer and inner quivers. Since the 3-brane and the 1-brane can also be interchanged by a T-duality transformation, the above results show that this correspondence extends to monopole moduli space. Note also that in the case of instantons, reciprocity transforms the self-duality equations in pure algebraic equations which describe the moduli space of a Dirichlet $p$-brane embedded in a $p + 4$ brane. In the present case reciprocity transforms the self-duality equations in a system of ordinary differential equation, corresponding to the fact that the 1-brane is transverse to the 3-brane. Finally it is interesting to note that performing a T-duality transformation along the 1-brane we end up with a system of type IIa 0-branes embedded in a 4-brane whose moduli space should describe instantons. It appears that the T-duality interchanges monopoles and instantons! Although not very clear at the present stage this line of development might lead to new insights in the interplay between D-branes and moduli spaces of solitonic objects.
References

[1] M.F. Atiyah and N.J. Hitchin The Geometry and Dynamics of Magnetic Monopoles, Princeton Univ. Press, Princeton (1988)

[2] M.C. Bowan, E. Corrigan, P. Goddard, A. Puaca and A. Soper Construction of Spherically Symmetric Monopoles Using the Atiyah-Drinfeld-Hitchin-Nahm Formalism, Phys. Rev. D28, 3100 (1983)

[3] R. Bielawski, Monopoles, Particles and Rational Functions, McMaster preprint (1996), to appear in Ann. Glob. Anal. Geom.

[4] R. Bielawski, Asymptotic Behaviour of SU(2) Monopole Metrics, J.Reine.Angew.Math. 468, 139 (1995)

[5] L. Brink, J. Schwarz and J. Scherk, Supersymmetric Yang-Mills Theories, Nucl. Phys. B121, 77 (1977)

[6] E. Corrigan and P. Goddard, Construction of Instanton and Monopole Solutions and Reciprocity, Ann. Phys. 154, 253 (1984)

[7] J. Dai, R.G. Leigh, J. Polchinski, New Connections Between String Theories, Mod. Phys. Lett. A4, 2073 (1989)

[8] S.K. Donaldson, Nahm Equations and the Classification of Monopoles, Commun. Math. Phys. 96, 387 (1985)

[9] M.R. Douglas, Gauge Fields and D-Branes, hep-th/9604198

[10] M.R. Douglas and G. Moore, D-Branes, Quivers and ALE Instantons, hep-th/9603167

[11] M.R. Douglas and M. Li, D-brane Realization of N=2 Super Yang-Mills Theory in Four Dimensions, hep-th/9604041

[12] M.B. Green and M. Gutperle, Comments on D-Branes, hep-th/9604091

[13] J.A. Harvey, Magnetic Monopoles, Duality and Supersymmetry, hep-th/9603086

[14] N.J. Hitchin, Hyperkahler Quotients, Asterisque 206, 137 (1992)
[15] N.J. Hitchin, *On the Construction of Monopoles*, Commun. Math. Phys. 89, 145 (1983)

[16] P.A. Horvathy and J.H. Rawnsley, *Topological Charges in Monopoles Theories*, Commun. Math. Phys. 96, 497 (1984)

[17] C.J. Houghton and P.M. Sutcliffe, *Monopole Scattering With a Twist*, Nucl. Phys. B 464, 59 (1996)

[18] J. Hurtubise, *The Classification of Monopoles for the Classical Groups*, Commun. Math. Phys. 120, 613 (1989)

[19] J. Hurtubise, *Monopoles and Rational Maps: A Note on a Theorem of Donaldson*, Commun. Math. Phys. 100, 191 (1985)

[20] K. Lee, E.J. Weinberg and P. Yi, *The Moduli Space of Many Monopoles for Arbitrary Gauge Groups*, hep-th/9602167

[21] R.G. Leigh *Dirac-Born-Infeld Action From Dirichlet Sigma Model*, Mod. Phys. Lett. A4, 2767 (1989)

[22] M.K. Murray *Non-Abelian Magnetic Monopoles*, Commun. Math. Phys. 96, 539 (1984)

[23] W. Nahm, *The Construction of All Self-Dual Multimonopoles by the ADHM Method*, “Monopoles in Quantum Field Theory”, Craigie et al. (eds), World Scientific, Singapore (1982)

[24] W. Nahm, *A Simple Formalism for The BPS Monopole*, Phys. Letters 90B, 413 (1980)

[25] H. Nakajima, *Monopoles and Nahm Equations*, Einstein Metrics and Yang-Mills connections (Sanda, 1990), 193, Lecture Notes in Pure and Applied Mathematics, 145, Dekker, New York (1993)

[26] J. Polchinski, S. Chauduri, C.V. Johnson, *Notes on D-Branes*, hep-th/9602052

[27] J. Polchinski, *Dirichlet Branes and Ramond-Ramond Charges*, Phys. Rev. Lett. 75, 4724 (1995)
[28] J. Polchinski and E. Witten, *Evidence for Heterotic-Type I String Duality*, Nucl. Phys. B460, 525 (1996)

[29] C. Schimdhuber, *D-Brane Actions*, hep-th/9601003

[30] J.H. Schwartz, *An SL(2, Z) Multiplet of Type IIB Superstrings*, hep-th/9508143

[31] A. Strominger, *Open P-Branes*, hep-th/9512059

[32] A.A. Tseytlin, *Selfduality of Born-Infeld Action and Dirichlet 3-brane of Type IIB Superstring Theory*, hep-th/9602064

[33] E.J. Weinberg, *Fundamental Monopoles and Multimonopole Solutions for Arbitrary Simple Gauge Groups*, Nucl. Phys. B167, 500 (1980)

[34] E.J. Weinberg, *Fundamental Monopoles in Theories with Arbitrary Symmetry Breaking*, Nucl. Phys. B203, 500 (1982)

[35] E. Witten, *Bound States of Strings and p-Branes*, hep-th/9510135

[36] E. Witten, *Small Instantons in String Theory*, hep-th/9511030

[37] E. Witten, *Sigma Models and The ADHM Construction of Instantons*, hep-th/9410052