Arbab I. Arbab

Viscous Dark Energy Models with Variable $G$ and $\Lambda$

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Abstract We consider a cosmological model with bulk viscosity ($\eta$) and variable cosmological ($\Lambda \propto \rho^{-\alpha}$, $\alpha =$ const.) and gravitational ($G$) constants. The model exhibits many interesting cosmological features. Inflation proceeds due to the presence of bulk viscosity and dark energy without requiring the equation of state $p = -\rho$. During the inflationary era the energy density ($\rho$) does not remain constant, as in the de-Sitter type. Moreover, the cosmological and gravitational constants increase exponentially with time, whereas the energy density and viscosity decrease exponentially with time. The rate of mass creation during inflation is found to be very huge suggesting that all matter in the universe was created during inflation.

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1 Introduction

The present acceleration of the universe as favored by the supernovae data can be explained by hypothesizing some exotic matter dominated the present universe evolution that breaks the strong energy condition, viz., $p + 3\rho \geq 0$ (Reiss et al., Perlmutter et al.). One variant of this exotic matter is the one violates the null energy condition, viz., $p + \rho > 0$. Such exotic matter can be modelled by a scalar field having a positive energy (dark energy) or negative energy (phantom) (Caldwell et al.). Phantom scalar field can be motivated by
S-brane arising in string theory (Townsend and Wohlfarth). Phantom fields are introduced by bulk viscosity ($\eta$) effects that are equivalent to replacing the pressure $p$ by an effective pressure, viz., $p_{\text{eff}} = p - 3\eta H$, where $H$ is the Hubble constant. Viscous effects in an expanding universe are connected with dissipations that are attributed to creation of energy (matter) in the universe.

Several authors suggested that the bulk viscosity can drive the universe into a period of exponential expansion (inflation) (Murphy, Grøn, Beesham, Arbab). This is really the case, as the effect of bulk viscosity in an expanding universe is to decrease the pressure making the total pressure negative. Inflation can also be induced by higher order corrections (Starobinsky, 1980). In scalar field, inflation stopped by the slow roll-down of the scalar field from the false vacuum to the true vacuum. In this work, we investigate the coupling of gravity with vacuum and viscosity. Provided a certain conspiracy is maintained the evolution of the universe can proceed in an attractive way. We have found that such a recipe is possible and leads to interesting features related to the cosmic evolution. Inflation is triggered by the vacuum energy bulk viscosity cooperation. This work generalizes our recent model of phantom and dark energy models (Arbab, 2007).

2 The Model

Consider the Einstein-Hilbert action with a cosmological constant ($\Lambda$)

$$ S = -\frac{1}{16\pi G} \int d^4x \sqrt{g} (R + 2\Lambda) + S_{\text{matter}} $$

(1)

The variation of the metric with respect to $g_{\mu\nu}$ with $f(R) = R - 2\Lambda$, gives (Amarzguioui et al.)

$$ f'(R)R_{\mu\nu} - \frac{1}{2} f(R)g_{\mu\nu} = -8\pi GT_{\mu\nu}, $$

(2)

where $T_{\mu\nu}$ is the energy momentum tensor of the cosmic fluid. For an ideal fluid one has

$$ T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu}, $$

(3)

where $u_\mu, \rho, p$ are the velocity, density and pressure of the cosmic fluid. Contracting Eq.(2), using Eq.(3) and taking its $00$ components give the equation,

$$ R f'(R) - 2 f(R) + 8\pi G T = 0, $$

(4)

and

$$ f'(R) T_{00} + \frac{1}{2} f(R) + 8\pi G T_{00} = 0, $$

(5)

with $T_{00} = \rho, T = \rho - 3p$ and $T_{ij} = -p$ for $i, j = 1, 2, 3$.

For a flat Friedmann-Lemaître-Robertson-Walker metric,

$$ ds^2 = dt^2 - a^2(t) \left( dr^2 + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2) \right), $$


one has \( R_{00} = -3 \left( \frac{\dot{a}}{a} \right) \) and \( R = -6 \left( \left( \frac{\dot{a}}{a} \right)^2 + \frac{\ddot{a}}{a} \right) \), where \( a \) is the scale factor, so that Eqs.(4) and (5) yield

\[
3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G\rho + \Lambda, \tag{6}
\]

\[
3 \left( \frac{\ddot{a}}{a} \right) = -4\pi G(\rho + 3p) + \Lambda, \tag{7}
\]

and the energy conservation equation reads,

\[
\dot{\rho} + 3 \left( \frac{\dot{a}}{a} \right) (\rho + p) = 0. \tag{8}
\]

The pressure \( p \) and energy density \( \rho \) of an ideal fluid are related by the equation of state,

\[
p = \omega \rho, \quad \omega = \text{const}. \tag{9}
\]

The Einstein field equation, with time-dependent \( G \) and \( \Lambda \), then yields two independent equations [Eqs.(6) and (7)] having the same form as in the standard model. Hence, we can now allow \( \Lambda \) and \( G \) to vary with time, i.e., \( \Lambda = \Lambda(t) \) and \( G = G(t) \).

The Bianchi identity

\[
(R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}) \; ; \mu = -(8\pi G T^{\mu\nu} + A g^{\mu\nu}) \; ; \mu = 0, \tag{10}
\]

with Eqs.(2) and (3) imply that

\[
G\dot{\rho} + 3(\rho + \rho_0)G\frac{\dot{a}}{a} + \rho\dot{G} + \frac{\dot{A}}{8\pi} = 0, \tag{11}
\]

Bulk viscosity can be introduced in a uniform perfect fluid by replacing the pressure term \( p \) by an effective pressure, \( p_{\text{eff}} \), defined by

\[
p_{\text{eff}} = p - 3\eta H, \tag{12}
\]

where, \( \eta \) is the coefficient of bulk viscosity. This is normally modelled by the relation

\[
\eta = \eta_0 \rho^n, \quad n, \eta_0 = \text{const}. \tag{13}
\]

Applying Eq.(12) into Eq(11) and using Eq.(8), one obtains (Arbab, 1997)

\[
8\pi \dot{G}\rho + \dot{A} = 9\eta(8\pi G)H^2. \tag{14}
\]

We consider here the ansatz

\[
\Lambda = \frac{3\beta}{\rho^\alpha}, \quad \beta, \alpha = \text{const}. \tag{15}
\]

Integrating Eq.(11), using Eq.(12), we obtain

\[
\rho = Aa^{-3(1+\omega)}, \quad A = \text{const}. \tag{16}
\]
Using Eqs.(6) and (15), Eq.(14) reads

\[ \frac{\dot{G}}{G} - \frac{3\alpha \beta}{8\pi G \rho^{(\alpha+1)}} \left( \frac{\dot{\rho}}{\rho} \right) = 3\eta_0 \rho^\alpha (8\pi G \rho + 3\beta \rho^{-\alpha}), \]  

Now consider the following functional dependence of the gravitational constant:

\[ 8\pi G = C \rho^{-(\alpha+1)}, \quad C = \text{const}. \]  

Eqs.(17) and (18) imply that

\[ \dot{\rho} = N \rho^{n-\alpha+1}, \quad N = -\frac{3C\eta_0 (C + 3\beta)}{C(1 + \alpha) + 3\alpha \beta}. \]  

so that

\[ \frac{\dot{a}}{a} = K a^{-3(1+\omega)(n-\alpha)}, \]  

where \( K = -\frac{N A^{(n-\alpha)}}{1+\omega}, \quad \omega \neq -1. \) Substituting this in Eq.(13) using Eq.(14), one gets

\[ a = D t^{1/3(1+\omega)(n-\alpha)}, \]  

where \( D = A^{1/3(1+\omega)} [-N(n-\alpha)]^{1/3(1+\omega)(n-\alpha)}, \quad n \neq \alpha. \) Substituting the above equation in Eq.(16), one finds

\[ \rho = [-N(n-\alpha)]^{-1/(n-\alpha)} t^{-1/(n-\alpha)}, \quad n \neq \alpha \]  

so the Eq.(15) becomes

\[ \Lambda = 3\beta [-N(n-\alpha)]^{\alpha/(n-\alpha)} t^{\alpha/(n-\alpha)}, \quad n \neq \alpha. \]  

Eq.(18) now reads

\[ G = \frac{C}{8\pi} [-N(n-\alpha)]^{(1+\alpha)/(n-\alpha)} t^{(1+\alpha)/(n-\alpha)}, \quad n \neq \alpha. \]  

3 Phantom energy

Consider now the following cases:

3.1 Case (1)

Now let \( n = \frac{2}{\omega} \) where \(-1 < \alpha < 0 \) and \( 1+\omega > 0. \) In this case, Eqs.(21), (22), reduce to

\[ a \propto t^{-2/3\alpha(1+\omega)}, \quad \rho \propto t^{2/\alpha}, \]  

and Eqs. (23), (24) and (13) yield

\[ G \propto t^{-2(1+\alpha)/\alpha}, \quad \Lambda \propto t^{-2}, \quad \eta \propto t, \]  

where \( C > 3\beta > 0, \) i.e., \( G > 0. \) These represent the viscous analogue of the dark energy model (Arbab, 2007). In particular, a viscous cosmological model with \( \Lambda \propto H^2 \) (Arbab, 1997) is equivalent to a viscous dark energy model with \( \Lambda \propto \rho^{-\alpha} \) if \( n_{\text{arb}} = 1 + \frac{\alpha}{2}, \) where \( n_{\text{arb}} \) is the index of viscosity in (Arbab, 1997).
3.2 Case (2)

Now let $\alpha > 0$ and $n < \alpha$. In this case Eq.(19) implies that one has the phantom energy solution, viz., $1 + \omega < 0$. One requires here $C < 0$ so that $G < 0$, and for $\beta > 0$ one has $\Lambda > 0$. This solution is found by (Arbab, 2007) and the above solution represents its viscous analogue. It is clear here that though the energy density increases, gravity (decreases) and viscosity (increase) conspire not to allow the phantom energy density to dominate.

4 Inflationary Solution

We notice from Eq.(20) that when $n = \alpha > 0$, we obtain

$$\frac{\dot{a}}{a} = K = \frac{C\eta_0(C + 3\beta)}{C(1 + \alpha) + 3\alpha\beta(1 + \omega)}.$$  

This implies that

$$a = \Gamma \exp(Kt) , \quad \Gamma = \text{const.},$$

where $K > 0$, for $1 + \omega > 0$. Eqs.(16), yield

$$\rho \propto \exp[-3K(1 + \omega)t] ,$$

so that Eqs. (15) and (18) become

$$\Lambda \propto \exp[3\alpha K(1 + \omega)t] , \quad G \propto \exp[3\alpha K(1 + \omega)(1 + \alpha)t] ,$$

and the bulk viscosity, Eq.(13),

$$\eta \propto \exp[-3\alpha K(1 + \omega)t].$$

Notice, however, during inflation $\omega \neq -1$ as evident from Eq.(24). This is unlike the standard case where inflation requires $\omega = -1$. We need a 60 e-folding for successful inflation. It is also remarkable that the cosmological constant during inflation increases exponentially with time, whereas the energy density decreases exponentially with time. Whatever the initial value of the cosmological constant before inflation, its value at the end of inflation will be enormously large. This may explain why $\Lambda$ was large in the very early universe, as suggested from particle physics considerations. The universe exited from inflation due to the huge growth of the gravitational force that halt the exponential expansion. Thereafter, the universe enters a radiation dominated phase. The large decrease of the bulk viscosity during the inflationary era has allowed the universe to isotropize, and eventually led to the isotropic and homogenous universe, we observe today. In the standard de-Sitter model the inflationary expansion is led by the cosmological constant ($\Lambda$), where the energy density stays constant. We notice from Eqs. (28) and (29) that the mass created (annihilated) during inflation is $M \propto \rho a^3 \propto \exp(-3K\omega t)$. But since $\omega > -1$, one has for $-1 < \omega < 0$, a positive mass creation rate. Hence, one would presume that all matter constituting the universe mass was produced during inflation decelerated inflation.
It is remarkable to notice that inflation is induced by dark energy only. Thus, dark energy played an important role by driving the early universe into an exponential expansion, and the present universe into a accelerated expansion. Hence, the existence of dark energy is very crucial to the evolution of the universe.

5 Concluding Remarks

We have studied in this work the effect of bulk viscosity on the evolution of dark matter and phantom energies. We have shown that non-viscous dark matter models are equivalent to viscous ones. The increasing bulk viscosity and decreasing gravitational constant do not allow the phantom energy density to condensate. During inflationary era the universe isotropizes and the cosmological constant attained a very large value. After inflation the cosmological constant decreases with time quadratically (e.g., for \( n = \frac{2}{3} \)). This evolution provides a viable mechanism for the smallness of the present cosmological constant, i.e., why today the cosmological constant is vanishingly small compared with its initial value!

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