ON PRECANONICAL QUANTIZATION OF GRAVITY 
IN SPIN CONNECTION VARIABLES

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Abstract

The basics of precanonical quantization and its relation to the functional Schrödinger picture in QFT are briefly outlined. The approach is applied to quantization of Einstein’s gravity in vielbein and spin connection variables and leads to a quantum dynamics described by the covariant Schrödinger equation for the transition amplitudes on the bundle of spin connection coefficients over space-time, that yields a novel quantum description of space-time geometry. A toy model of precanonical quantum cosmology based on the example of flat FLRW universe is considered.

Keywords: quantum gravity, De Donder-Weyl theory, precanonical quantization, tetrad gravity, spin connection, Clifford algebra, FLRW cosmology, quantum cosmology

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1 Introduction

We are accustomed to the notion that fields are infinite dimensional Hamiltonian systems and the fact that time plays a special role in the formalism and interpretation of quantum theory. Both aspects can be seen as inherited from the canonical Hamiltonian treatment which is a basis of canonical quantization both in quantum mechanics and quantum field theory. However, it is much less known that the Hamiltonian formulation can be extended to field theory without explicitly distinguishing the time dimension and without referring to the infinite dimensional configuration or phase space. The examples of such an extension are known in the calculus of variations of multiple integrals, the simplest of them is the De Donder-Weyl (DW) theory (see e.g. [1]).

The DW Hamiltonization of a field theory given by the first order Lagrangian

\[ L = L( y^a, y^\mu_a, x^\nu) \]

is based on the covariant Legendre transformation to the new set of

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variables: polymomenta

\[ p_{a}^{\mu} := \frac{\partial L}{\partial y_{\mu}^{a}} \]

and the DW Hamiltonian function

\[ H(y^{a}, p_{a}^{\mu}, x^{\mu}) := y_{\mu}^{a}(p) p_{a}^{\mu} - L. \]

If the transformation is regular, i.e.

\[ \det \left| \frac{\partial^2 L}{\partial y_{\mu}^{a} \partial y_{\nu}^{b}} \right| \neq 0, \]

the field equations can be cast into the DW covariant Hamiltonian form:

\[ \partial_{\mu} y_{\mu}^{a}(x) = \frac{\partial H}{\partial p_{a}^{\mu}}, \quad \partial_{\mu} p_{a}^{\mu}(x) = -\frac{\partial H}{\partial y_{\mu}^{a}}. \quad (1) \]

Precanonical quantization is aimed at the construction of quantum field theory based on the space-time symmetric version of the Hamiltonian formalism like the DW formulation. It has been demonstrated (see e.g. \([2]\)) that the mathematical structures of the DW formulation (also known as multi- or polysymplectic) are in a sense intermediate between the Lagrangian and the canonical Hamiltonian description, whence the term ”precanonical” proposed by us.

In the previous papers \([3]\), to which I refer for more details, it was argued that precanonical quantization leads to the following representation of the operators of polymomenta:

\[ \hat{p}_{a}^{\nu} = -i\hbar \gamma_{\nu}^{a} \frac{\partial}{\partial y_{\mu}^{a}}, \quad (2) \]

which act on the Clifford-valued (in general) wave functions \( \Psi(y, x) \) on the finite dimensional space of field and space-time variables \( y \) and \( x \). The constant \( \gamma \) naturally appears in precanonical quantization on the dimensional grounds: its dimension in \( n \) space-time dimensions is \( cm^{1-n} \) and it is related to the inverse of a very small "elementary volume" as it appears e.g. in the representation of the \((n-1)\)-forms

\[ \varpi_{\nu} := \partial_{\nu} \bigwedge(dx^{0} \wedge ... \wedge dx^{n-1}) \]

in terms of the Clifford algebra elements:

\[ \gamma_{\nu} = \frac{1}{\varpi} \gamma_{\nu}. \]

The covariant analogue of the Schrödinger equation in precanonical quantization takes the form:

\[ i\hbar \gamma_{\nu} \gamma_{\mu} \partial_{\mu} \Psi = \hat{H} \Psi, \quad (3) \]

where \( \hat{H} \) is the DW Hamiltonian operator. For the free scalar field \( y \)

\[ \hat{H} = -\frac{1}{2} \hbar^{2} \gamma^{\nu} \frac{\partial^{2}}{\partial y^{\nu}} y^{2} + \frac{1}{2} \frac{m^{2}}{\hbar^{2}} y^{2}. \quad (4) \]

It corresponds to the harmonic oscillator along the field dimension \( y \) and its spectrum \( \gamma m(N + \frac{1}{2}) \) means that free particles of mass \( m \) correspond to the transitions between nearby eigenstates of DW Hamiltonian operator.
The question arises, how the description in terms of the wave function on the finite dimensional space, $\Psi(y, x)$, is related to standard QFT, e.g. the description in terms of the Schrödinger wave functional $\Psi([y(x)], t)$ on the infinite dimensional space of field configurations $y(x)$, which is derived from canonical quantization? If $\Psi(y, x)$ has a probabilistic meaning of the probability amplitude of observing the value of a field $y$ at the space-time point $x$, then the wave functional $\Psi([y(x)], t)$ should be a composition of amplitudes $\Psi(y, x)$ taken along the surface $\Sigma: y = y(x)$ at the time $t$. Although the idea has been around already since 1998 [4] the exact form of this composition in the case of scalar field theory was established only recently in [5]:

$$\Psi = \text{Tr} \left\{ \prod_x e^{-iy(x)\gamma^\alpha \partial_\alpha y(x)/\kappa} \Psi_{\Sigma}(y(x), x, t) \right\}_{\gamma^0 \kappa \rightarrow \delta(0)}.$$  \hspace{1cm} (5)

Here, starting from the solution of (3) restricted to the surface $\Sigma$ we construct the functional $\Psi$ which, under the map $\gamma^0 \kappa \rightarrow \delta(0)$ (which is actually the inverse of the Clifford-algebraic "quantization map" from the exterior forms to Clifford numbers in the limit of vanishing "elementary volume" $1/\kappa \rightarrow 0$), satisfies the canonical Schrödinger equation in functional derivatives. In this way, the standard QFT based on canonical quantization appears as a singular limit of QFT based on precanonical quantization.

2 DW formulation of vielbein gravity

Our earlier application of precanonical quantization in quantum gravity [6] was based on the DW formulation in metric variables. The resulting formulation is necessarily a hybrid (quantum-classical) theory because a part of the spin connection term in the curved space-time Dirac operator in (3) cannot be expressed in terms of the variables of metric formulation and, therefore, quantized.

Here we explore an alternative approach proceeding from the Lagrangian density written in terms of the vielbein $e^\mu_I$ and torsion-free spin connection variables $\omega_{IJ}^\alpha$, viz.

$$\mathcal{L} = \frac{1}{\kappa_E} e^I e_J^\alpha \left[ \frac{\partial e^\mu_I}{\partial e^\alpha_J} \partial_\alpha \omega_{IJ}^\beta \right] + \frac{1}{\kappa_E} \Lambda e,$$ \hspace{1cm} (6)

where $\kappa_E := 8\pi G$ and $\varepsilon := \det ||e^\mu_\alpha||$.

If we treat the components of vielbeins and spin connections as independent dynamical variables (c.f. [7]), then the corresponding polymomenta

$$p^\alpha_{e^\mu_I} = \frac{\partial \mathcal{L}}{\partial \dot{e}^\alpha_I} \approx 0, \quad p^\alpha_{\omega_{IJ}^\beta} = \frac{\partial \mathcal{L}}{\partial \dot{\omega}_{IJ}^\beta} \approx \frac{1}{\kappa_E} e^I e_J^\alpha$$ \hspace{1cm} (7)

lead to what can be called the primary constraints in DW formalism, i.e. the underlying DW Legendre transformation is singular and there is no unique expression of space-time gradients of fields in terms of the field variables and polymomenta.
Unfortunately, the theory of singular DW Hamiltonian systems is not yet sufficiently developed for the purposes of quantization (c.f. [8]). We, therefore, will be guided by our treatment in [9]. Namely, using the primary constraints we write down an extended DW Hamiltonian density

$$\mathcal{H} = -\frac{1}{\kappa E} e_I^{[\alpha} e_{\beta]} \omega^I_{\alpha} \omega^K_{\beta} \omega^J = -\frac{1}{\kappa E} \Lambda e + \mu \cdot p_e + \lambda \cdot (p_\omega - \frac{1}{\kappa E} e \wedge e) ,$$  \hspace{1cm} (8)

where $\mu$ and $\lambda$ are the Lagrange multipliers (for the sake of brevity we omit, when appropriate, writing the indices explicitly). The DW Hamiltonian equations given by $\mathcal{H}$:

$$\partial_{[\alpha} \omega^I_{\beta]} = \lambda^I_{\alpha \beta}, \quad \partial_{\alpha} p^\alpha_I = -\frac{\partial \mathcal{H}}{\partial e^I_{\alpha}},$$  \hspace{1cm} (9)

$$\partial_{\alpha} e^I_{\beta} = \mu^I_{\alpha \beta}, \quad \partial_{\alpha} p^\alpha_{\omega^I_{\beta}} = -\frac{\partial \mathcal{H}}{\partial \omega^I_{\alpha \beta}},$$  \hspace{1cm} (10)

yield, respectively, the Einstein equations as a consequence of preservation of the constraint $p^\alpha_e \approx 0$ and the covariant constancy condition $\nabla_\beta (e_I^{[\alpha} e_{\beta]} \omega^I_{\alpha}) = 0$, which can be transformed into the expression of spin connection in terms of vielbeins and their derivatives.

Note that eqs. (9), (10) are tantamount to the preservation of $(n-1)$-forms constructed from the constraints (7), see [9]. As the Poisson-Gerstenhaber brackets of those forms\footnote{The Poisson-Gerstenhaber (PG) brackets of forms have been found within the DW Hamiltonian formalism in our earlier work, see e.g. [1b], and their quantization underlies the precanonical quantization program, see [3,4]. An attempt to construct their Dirac-like generalization in singular DW theories is undertaken in [9]. However, a more natural approach, which is still a work in progress, would be a construction of brackets from a proper restriction of the polysymplectic form to the surface of constraints instead of a formal generalization of the Dirac formula.} do not weakly vanish, the constraints in (7) are second class [9]. To cope with this we use our generalization of the Dirac bracket to the singular DW Hamiltonian formalism [9] and notice that the following brackets can be obtained formally without knowing the explicit form of the inverse matrix of PG brackets of constraints:

$$\{[p^\alpha_e \omega_\alpha, e'] \} = 0 ,$$  \hspace{1cm} (11)

$$\{[p^\alpha_e \omega_\alpha, \omega'] \} = \{[p^\alpha_\omega \omega_\alpha, \omega'] \} = \delta^\omega_\alpha,'$$  \hspace{1cm} (12)

$$\{[p^\alpha_e \omega_\alpha, p_\omega] \} = \{[p^\alpha_e \omega_\alpha, \omega] \} = \{[p^\alpha_\omega \omega_\alpha, e] \} = 0.$$  \hspace{1cm} (13)

We assume that these brackets are the fundamental ones whose quantization allows us to construct all other operators of the theory by composition.
3 Quantization

Quantization of formal Dirac brackets (11), (13) and the constraint \( p_e \approx 0 \) allow us to set \( \hat{p}_e = 0 \). It means that our precanonical wave functions will depend only on the spin connection and space-time variables, not on the vielbein variable \( s \): \( \Psi = \Psi(\omega, x) \).

Since the bracket in (12) coincides with the familiar brackets of polymomenta and field variables in DW Hamiltonian formalism, we can use the curved space-time generalization of the operator representation of polymomenta in precanonical quantization, viz.

\[
\hat{p}_\omega^{[\alpha} = -i\hbar \gamma^{[\alpha} \partial_{\omega^\beta]} ,
\]

where \( \hat{e} \) and \( \hat{\gamma}^{[\alpha} \) are yet unknown operators and \( \vdots \) signifies a potential operator ordering ambiguity.

Now, if we contract the second constraint in (7) with the flat \( \gamma^{IJ} \)-s:

\[
\epsilon \epsilon^{[\alpha} \epsilon^{\beta]} \gamma^{IJ} = \epsilon \gamma^{\alpha\beta} \approx \kappa_E \phi^{\alpha\beta} \gamma^{IJ} ,
\]

and insert the precanonical operator representation of \( \hat{p}_\omega^{[\alpha} \) in eq. (14) into the operator version of the constraint (15), we obtain the operator representation of the curved space-time Dirac matrices and vielbeins, viz.

\[
\hat{\gamma}^{[\alpha} = -i\hbar \gamma E \gamma^{[\alpha} \partial_{\omega^\beta]} , \quad \hat{e}^{[\alpha} = -i\hbar \gamma E \gamma^{\alpha} \partial_{\omega^\beta]} .
\]

This allows us to construct the DW Hamiltonian operator \( \hat{H} \), such that \( \hat{H}|_C =: \hat{c} \hat{H} \), where \( |_C \) denotes the restriction to the surface of constraints (7):

\[
\hat{H} = \hbar^2 \gamma E \gamma^{IJ} : \frac{\partial}{\partial \omega^{[\alpha}_{\omega^\beta]} } \omega^{KL} \gamma^{KM} \gamma^{LM} : - \frac{1}{\kappa_E} \Lambda .
\]

4 Covariant Schrödinger equation for quantum gravity

The precanonical analogue of the Schrödinger equation for quantum gravity, which generalizes eq. (3), will have the form

\[
i\hbar \hat{\nabla} \Psi = \hat{H} \Psi ,
\]

were \( \hat{\nabla} := (\gamma^\mu (\partial_\mu + \omega_\mu))^p \) with \( \omega_\mu := \frac{1}{4} \omega_{\mu,IJ} \gamma^{IJ} \) is the "quantized Dirac operator". Using the operator representation of the curved space gamma-matrices in (16) we obtain:

\[
\hat{\nabla} = -i\hbar \gamma E \gamma^{IJ} : \frac{\partial}{\partial \omega^{[\alpha}_{\omega^\beta]} } \left( \partial_\mu + \frac{1}{4} \omega_{\mu,KL} \gamma^{KL} \right) : .
\]

\[\text{We define } \gamma^\nu := e^\nu_{[\gamma} \gamma^{\gamma]} , \gamma^\nu \gamma^\nu + \gamma^\nu \gamma^\nu = 2\eta^{IJ} ; \eta^{IJ} \text{ is a fiducial flat Minkowskian metric with the signature } + - - - - .\]
Hence, our precanonical counterpart of the Schrödinger equation for quantum gravity takes the form

$$\gamma_{IJ}^{\mu} \frac{\partial}{\partial \omega_{IJ}^{\mu}} \left( \partial_{\mu} + \frac{1}{4} \omega_{\mu KL} \gamma_{KL}^{\mu} - \frac{\partial}{\partial \omega_{\mu L}^{K}} \omega_{\mu [K}^{\mu} \omega_{\beta]}^{L} \right) \Psi + \frac{\Lambda}{\hbar^2 \kappa^2} \Psi = 0. \quad (20)$$

This equation determines the wave function $\Psi(\omega, x)$ or the transition amplitudes $\langle \omega, x | \omega', x' \rangle$ which provide a quantum description of geometry generalizing the classical differential geometry which uses smooth connection fields $\omega(x)$. This description is different from the already existing approaches to the quantum geometry of space-time in quantum geometrodynamics, loop quantum gravity or non-commutative geometry.

5 Defining the Hilbert space

In order to specify the Hilbert space we first assume that our wave functions are vanishing at large $\omega$-s. Physically it means that the regions of very large curvature $R = d\omega + \omega \land \omega$ are avoided by the wave function. The scalar product should have the form: $\langle \Phi | \Psi \rangle := \int [d\omega] \bar{\Psi} \Psi$, where $[d\omega]$ is a Misner-like [10] covariant measure on the space of spin connection coefficients, which we found to have the form

$$[d\omega] = \epsilon^{-(n-1)} \prod_{\mu I} d\omega_{IJ}^{\mu}. \quad (21)$$

However, since $\epsilon := \det |e^{I}_{\alpha}|$ and $e^{I}_{\alpha}$ form the inverse matrix of $e^{I}_{\alpha}$, which are themselves differential operators according to (16), this measure is operator-valued with

$$\hat{\epsilon}^{-1} = \frac{1}{n!} \epsilon^{I_1 \ldots I_n} e^{\mu_1 \ldots \mu_n}_{I_1} \cdots e^{\mu_n}_{I_n}. \quad (22)$$

Hence, our scalar product has the form

$$\langle \Phi | \Psi \rangle := \int \bar{\Psi} [d\omega] \Psi. \quad (23)$$

In order to ensure that the DW Hamiltonian operator $\hat{H}$ is self-adjoint with respect to this scalar product the expectation values are defined as follows,

$$\langle \hat{H} \rangle := \int \bar{\Psi} \left[ [d\omega] \hat{H} \right]_{W} \Psi, \quad (24)$$

where the subscript $W$ denotes the Weyl ordering.

There is a remaining local coordinate freedom in spin connection coefficients which should be fixed. A possible choice could be the De Donder-Fock condition, i.e. the choice of harmonic coordinates on the average:

$$\partial_{\mu} \langle \Psi(\omega, x) | e^{g}^{\mu \nu} | \Psi(\omega, x) \rangle = 0, \quad (25)$$
where
\[
\bar{g}^{\mu\nu} = -\hbar^2 \kappa^2 \gamma^I J \eta^{KL} \frac{\partial^2}{\partial \omega^I \partial \omega^J}
\] (26)
is the metric operator obtained from (16), and the operator ordering in (25) is fixed by the Weyl ordering prescription in (24). Note that in the present formulation the coordinate condition is imposed on the wave functions \(\Psi(\omega, x)\) rather than on the metric or vielbein fields.

6 Precanonical quantum cosmology, a toy model

Let us consider \(n = 4, k = 0\) FLRW metric with a harmonic time coordinate \(\tau\)
\[
ds^2 = a(\tau)^6 d\tau^2 - a(\tau)^2 d\mathbf{x}^2 = \eta_{IJ} e_I^\mu e_J^\nu dx^\mu dx^\nu.
\] (27)

Then \(e_0^0 = a^3 \delta^0_0\) and \(e_J^\nu = a \delta_J^\nu\) for \(J = 1, 2, 3\), and the non-vanishing components of spin connection are \(\omega_0^0 = -\omega_1^0 = \dot{a}/2a^3 = \omega\), where \(i = I = 1, 2, 3\).

In this simple case there is only the \(\Lambda\)-term which remains in the DW Hamiltonian operator, eq. (17), and eq. (20) takes the form
\[
\left( 2 \sum_{i=1}^{3} \alpha^I \partial_i + 3 \omega \partial_\omega + \lambda \right) \Psi = 0,
\] (28)
where \(\alpha^I := \gamma^0 I\) and \(\lambda := \frac{3}{2} + \Lambda/(\hbar \kappa E)^2\), if the Weyl ordering is used. Note that the correct value of \(\Lambda\) can be obtained from the constant of order unity which results from the operator ordering, provided \(\kappa \sim 10^{-3}\) GeV.

By separating variables \(\Psi := u(x) f(\omega)\) we obtain the equation on \(u\):
\[
2 \sum_{i=1}^{3} \alpha^I \partial_i u = iqu,
\]
where the imaginary unit comes from the anti-hermicity of \(\partial_i\), and the equation on the wave function in \(\omega\)-space:
\[
(iq \partial_\omega + 3 \omega \partial_\omega + \lambda) f = 0,
\]
whose solution \(f \sim (iq + 3 \omega)^{-\lambda}\) yields the probability density (similar to t-distribution)
\[
\rho(\omega) := f f \sim (9 \omega^2 + q^2)^{-\lambda}.
\] (29)

At \(\lambda > 1/2\), which is required by \(L^2[(-\infty, \infty), d\omega] = d\omega\) normalizability in \(\omega\)-space, this distribution has a bell-like shape centered at the zero expansion rate \(\dot{a} = 0\). The most probable expansion rate can be shifted by accepting complex values of \(q\), and the inclusion of minimally coupled matter fields changes \(\lambda\).

Although our toy model bears some similarity with the minisuperspace models, its origin and the content are different. It is obtained from the full quantum Schrödinger
equation (20) when the field $\omega$ is one-component, rather than via quantization of a reduced mechanical model deduced under the assumption of spatial homogeneity. In fact, the naive assumption of spatial homogeneity of the wave function: $\partial_i \Psi = 0$, or $q = 0$, would not be compatible with its normalizability in $\omega$-space. Instead, our model implies a quantum gravitational structure of space at the scales $\sim \Re \frac{1}{q}$ and $\sim \Im \frac{1}{q}$ given by $u(x)$.

7 Discussion

This paper presents rather a prototype of the theory of quantum gravity which results from a brute force implementation of precanonical quantization in general relativity. Our goal was to highlight the potential of the approach in constructing a mathematically well-defined, nonperturbative, covariant and background independent formulation of quantum gravity and to discuss some features of the resulting theory.

A difficulty we faced is related to the second class constraints in the DW Hamiltonian formulation whose complete analysis is not well understood. In fact, in addition to the brackets in (11)–(13), one can also calculate e.g. $\{e^{\alpha I}_{\varpi} \omega^{J}_{\nu}, \omega^{L}_{\mu}\}^D$ which, however, is hardly possible to quantize directly using the Dirac’s rule, because it explicitly depends on the generalized inverse of the matrix of PG brackets of constraints. Because of this we proceeded from quantization of brackets in (11)–(13) alone, hoping that the results can be compatible with the right hand side of the commutators similar to $[e^{\alpha I}_{I} \varpi^{\alpha}, \omega^{L}_{J}]$ when $e^{\alpha I}_{I} \varpi^{\alpha}$ is treated as a composite operator. However, the validity of this workaround and formal calculation of Dirac brackets (11)–(13) needs further scrutiny.

The immediate consequences of our approach are that (i) the quantum dynamics is described in the space of spin connection coefficients $\omega$, (ii) the metric becomes a composite operator, eq. (26), (iii) the quantum description of geometry is achieved in terms of the transition amplitudes $\langle \omega, x | \omega', x' \rangle$ which obey the covariant Schrödinger equation, eq. (20).

We also noticed that the correct value of the cosmological constant can be obtained from the dimensionless number of order unity, which appears from the ordering of operators in eq. (20), if the parameter $\varpi$ of precanonical quantization corresponds to the energy scale of roughly $10^2 MeV$, which is a rather unexpected fact to be understood.

One of the motivations to consider the vielbein formulation as a starting point of precanonical quantization was a desire to understand if the hybrid formulation of precanonical quantum gravity in [6], with its ”bootstrap condition” and the space-time emerging from the self-consistency requirement, is indeed a feature of precanonical quantum gravity or a reflection of limitations of the metric formulation. Whereas the present consideration seems to suggest the latter, the certain answer requires clarity regarding the treatment of the second class constraints in DW formulation of vielbein gravity.
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