Chapter

Nonlinear Giant Magnetostrictive Actuator and Its Application in Active Control

Lihua Yang, Haipeng Zhang, Hailong Jiang, Shuyong Liu, Haiping Wu, Haifeng Li and Ehsan Noroozinejad Farsangi

Abstract

The giant magnetostrictive actuator has great use in vibration control, but the linear model cannot fully describe its dynamic characteristics. In this chapter, based on the domain wall theory and piezomagnetic theory, a hysteresis nonlinear model is established to fully describe the actuator dynamic characteristics. In combination with the regularisation method, a sliding mode controller has been designed, and the giant magnetostrictive actuator is also studied in the application of active control. Experimental results show that the hysteresis nonlinear model proposed in the chapter can fully describe the actuator’s dynamic characteristics in a wider frequency band and the active control also has a much better isolation effect than the passive vibration; it can significantly attenuate the external incentives.

Keywords: giant magnetostrictive actuator, hysteresis nonlinearity, sliding mode control, active control

1. Introduction

Passive vibration isolation has been widely used as an effective isolation method; it can significantly reduce the vibration transmission between mechanical equipment and the base, while the isolation effect is limited in the micro-vibration and low frequencies. Therefore, the active control has been a focus of research at home and abroad [1–4]. With the development of smart materials, intelligent actuators manufactured by those materials including the magneto-rheological actuator, shape memory alloy actuator, giant magnetostrictive actuator (GMA), and others. These actuators have played a huge role in promoting the active control as the executing agency [5, 6].

With the advantages of high-positioning accuracy, fast response, and wide frequency band, among others, the GMA has a wide variety of applications in fields including vibration control and precision positioning [7–10]. Many scholars have studied the linear modelling method of GMA, which is only suitable for describing low-frequency dynamic characteristics [11, 12]. However, the hysteresis nonlinear model based on the domain walls theory can more clearly reveal the coupling relationship among the magnetization process, the stress magnetic machine effect, and stretching amount, and it can more fully describe the GMA’s dynamic characteristics on a wide frequency band [13], which is suitable for the active control
actuator. Zhang established the GMA dynamics equations and studied the active control on the basis of the proportional-integral-derivative (PID) algorithm, the results showed that the GMA has some damping effect, but the system’s adaptive capacity is a little weak [14]. Francesco calculated the actuator’s amplitude-frequency curve and studied the active control in a single freedom degree vibration isolation system. The simulations showed the GMA can significantly reduce the force transmitted to the base [15]. Wang designed a magnetostrictive actuator rod and analysed the system structure vibration on the basis of the linear-quadratic regulator (LQR) algorithm, the results showed the GMA can effectively reduce the structure response of acceleration and displacement [16]. However, these studies mainly focus on the linear actuator model; the isolation frequency band is narrow relatively. Therefore, the application of the GMA hysteresis nonlinear model is urgently needed in the active control.

Currently, some of the more commonly active control strategies are the PID control [17], robust control [18], fuzzy control [19], optimal control [20], adaptive control [21], and sliding mode control [22]. In particular, the sliding mode control has no effect on system parameter perturbation and external disturbance when the system is in sliding mode, so it has much better robustness [23], which is more suitable for the active vibration control algorithm.

Based on the domain wall theory [24], this chapter studied the GMA broadband hysteresis nonlinear model and applied it to the active control of the vibration isolation system. By optimising the design of the hysteresis nonlinear model, the actuator linearity is much better than the linear model on a wider frequency band. The results showed that the GMA hysteresis nonlinear model on a wider frequency band can more fully describe its dynamic characteristics. Compared with passive vibration isolation, the active control has a better isolation effect and wider frequency band isolation, and the sliding mode control strategy also has stronger robustness.

2. Hysteresis nonlinear magnetostrictive actuator model

2.1 Magnetostrictive actuator model

The GMA structure is shown in Figure 1. It is mainly made of the push rod, preload spring, magnetostrictive rod, drive coil, and bias coil, where the preload mechanism is comprised of the preload spring, push rod, and preload screw. The polarised magnetic field of the bias coil can make the magnetostrictive rod deform in a linear manner and can prevent the multiplier phenomenon from affecting system control precision. When an alternating current is passed through the drive coil, the magnetostrictive rod can also generate an alternating magnetic field, which causes a dynamic redistribution of magnetic domains in the magnetostrictive rod. Then the telescopic length can be obtained microscopically to promote a displacement and thrust the output push rod, which achieves energy conversion by transferring electromagnetic energy into mechanical energy.

Based on the Jiles-Atherton theory [25], many researchers explored the GMA characteristics of the nonlinear hysteresis model, and the structure optimisation and parameter configuration can make the theoretical calculations accurately match with experiments in a higher linearity. Generally, the GMA motion can be equivalent to a single freedom degree model, shown in Figure 2. Based on the magnetic domain theory and piezomagnetic theory [26–28], the partial differential equations of the magnetisation process are established. Then the GMA dynamics equation is also deduced with the specific equations as follows:
where $H_e$ is the effective magnetic field strength, $H$ is the sum of the drive magnetic field $H_d = nI$ ($n$ is the number of turns per unit coil length, $I$ is the drive...
current) and bias magnetic field $H_b$, $\lambda$ is the saturation magnetostrictive coefficient, $\sigma = \sigma_0 + \sigma$ is the external stress ($\sigma_0$ is the preload), $\mu_0$ is the vacuum permeability, $M_s$ is the saturation magnetisation, $\lambda$ is the axial magnetostriction strain, $a$ is the shape factor of non-hysteresis magnetisation, $k$ is the irreversible loss coefficient, $\alpha$ is the molecular field parameter of interacting magnetic torque, $\tilde{\alpha} = \alpha + 9\lambda_0/2\mu_0 M_s^2$ is the integrated magnetic domain coefficient, $c$ is the reversible factor, the parameter $\delta \equiv +1$ when $dH > 0$ and parameter $\delta \equiv -1$ when $dH < 0$, $M_{an}$ is the non-hysteresis magnetisation, $M_{irr}$ is the irreversible magnetisation, $M_{rev}$ is reversible magnetisation, and $M$ is the total magnetisation. $G$, $\rho$, $L$, $A$, and $S^H_L$ are, respectively, the damping, density, length and cross-sectional area, and axial compliance coefficient of the magnetostrictive rod; $m_1 = \rho LA/3$, $c_1 = cA/L$, and $k_1 = A/S^H_L$ are, respectively, the equivalent mass, damping, and stiffness coefficient of the magnetostrictive rod; $m_2$, $c_2$, and $k_2$ are, respectively, the equivalent mass, damping, and stiffness coefficient of the end load; and $\mathcal{S}$ is the Laplace transform operator.

2.2 Magnetostrictive actuator model for wide frequency range

When the driving frequency is low, temperature and eddy current can be ignored on the actuator linearity. However, to fully describe the actuator dynamic characteristics on a wide frequency range, it is necessary to consider the hysteresis loss, eddy current losses, additional loss, and stress changes. In this case, magnetisation intensity $M$ can be expressed as follows:

$$\frac{dM}{dt} = \frac{\partial M}{\partial H} \frac{dH}{dt} + \frac{\partial M}{\partial \sigma} \frac{d\sigma}{dt}$$

(9)

where magnetisation change rate is made up of differential susceptibility $\partial M / \partial H$, the magnetic field change rate $dH / dt$, magnetisation stress change rate $\partial M / \partial \sigma$, and stress change rate $d\sigma / dt$. The key is to solve for $\partial M / \partial H$ and $d\sigma / dt$. By Eqs. (2)–(5), the differential susceptibility can be expressed as such:

$$\frac{\partial M}{\partial H} = (1 - c) \frac{M_{an} - M_{irr}}{\delta k - \tilde{\alpha}(M_{an} - M_{irr})} + c \frac{\partial M_{an}}{\partial H}$$

(10)

According to magneto-mechanical coupling model [16], the magnetisation stress change rate can be determined by Eq. (11):

$$\frac{\partial M}{\partial \sigma} = \frac{S^H L}{\xi}(M_{an} - M) + c \frac{\partial M_{an}}{\partial \sigma}$$

(11)

where $\xi$ is the energy coupling parameters of per unit volume and other parameters are the same as above. Substituting Eqs. (10) and (11) into Eq. (9), the magnetisation change rate can be obtained:

$$\frac{dM}{dt} = \left( (1 - c) \frac{M_{an} - M_{irr}}{\delta k - \tilde{\alpha}(M_{an} - M_{irr})} + c \frac{\partial M_{an}}{\partial H} \right) \frac{dH}{dt}$$

$$+ \left( \frac{S^H L}{\xi}(M_{an} - M) + c \frac{\partial M_{an}}{\partial \sigma} \right) \frac{d\sigma}{dt}$$

(12)

In summary, Eqs. (1)–(12) constitute the hysteresis nonlinear model of the giant magnetostrictive actuator, which can describe the displacement and force...
output on a wide frequency range, and then the output displacement and force can be calculated by the input current.

3. Giant magnetostrictive actuator experiment

The experiments mainly include the giant magnetostrictive actuator, NI host controller, capture cards and displays, LabVIEW software, FN15150 power amplifier, MEL laser displacement sensor, and CHINT current transformer. NI equipment and software are used to capture and display the current and displacement signal, the amplifier amplifies NI weak signals to drive actuators, and the laser displacement sensor is used to accurately measure the actuator displacement. The GMA experiment setup is shown in Figure 3. This experiment studies the hysteresis nonlinear model displacement responses to the preload and the bias magnetic field intensity, as shown in Figures 4 and 5. By optimising the actuator preload and bias magnetic field, the GMA can eliminate the doubling phenomenon and work perfectly within a linear range, which can enhance the actuator output linearity.

**Figure 3.**
The giant magnetostrictive actuator experiment.

**Figure 4.**
Curves of displacement changes with the preload.
Figure 4 shows that the actuator displacement increases with the preload, but when the preload is greater than 7 MPa, experimental curves began to appear asymmetrical, so the preload cannot be greater than 7 MPa. Figure 5 shows that without the paranoid magnetic field, actuator output is butterfly-shaped with a strong nonlinearity, but when the paranoid magnetic field increases, the output linearity also gradually increases. Finally, $\sigma_0 = 6.7 \text{MPa}$ and $H_b = 14 \text{kA/m}$ are selected in the study which can give the actuators better linearity.

4. Application for GMA in active control

4.1 Active control model of vibration isolation system

The active control model of the vibration isolation system is shown in Figure 6, where $M_1, K_1, C_1, M_2, K_2,$ and $C_2$ are, respectively, the mass, stiffness, and damping of the upper isolated equipment and the middle vibration isolation platform. $x, y, \dot{x}, \dot{y}, \ddot{x},$ and $\ddot{y}$ are, respectively, displacement, velocity, and acceleration as above. $f$ is the active control force of the GMA, and $p$ is the external disturbance. The dynamics equations of the isolation system can be expressed as Eq. (13):

![Active control model of vibration isolation system](image)
4.2 Sliding mode control design

Based on full state feedback, the sliding mode can make the system reach the sliding mode surface and achieve sliding mode movement in a jump way. Therefore, it is essential for dynamic characteristics to rationally design the sliding mode surface, assuming the system switching function as Eq. (4) [29, 30].

\[ S(t) = \Theta X \]  

where \( S \) is the switching function, \( \Theta \) is a 1 \times 4 dimension switching matrix, and \( X = [x, y, \dot{x}, \dot{y}]^T \) are the state variables. The non-singular state transition matrix \( \Gamma \in R^{4 \times 4} \) is taken to regulate Eq. (16), and the coordinate transformation is the following:

\[ Z(t) = \Gamma X(t) \]  

where \( \Gamma = \begin{bmatrix} 1 & -\dot{B}_1 \dot{B}_2 \\ 0 & I_1 \end{bmatrix}, B = \begin{bmatrix} \dot{B}_1 \\ \dot{B}_2 \end{bmatrix}, \dot{B}_1 \in R^{3 \times 1}, \text{and} \dot{B}_2 \in R^{1 \times 1}; \text{substituting} \]  

Eq. (17) into Eqs. (15) and (16), it gets the system canonical form and the switching surface as Eq. (18):

\[ \dot{Z}(t) = \bar{A}Z(t) + \bar{B}U(t) \]

\[ S(t) = \Theta \bar{Z} \]  

where \( \bar{A} = \Gamma \Theta \Gamma^{-1}, \bar{B} = \begin{bmatrix} 0 & \dot{B}_2 \end{bmatrix}^T, \text{and} \Theta = \Theta \Gamma^{-1}; \text{let} Z(t) = \begin{bmatrix} Z_1(t) \\ Z_2(t) \end{bmatrix}, \]  

\[ \bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \text{and} \Theta = \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix}^T, \text{where} Z_1(t) \in R^{3 \times 1}, Z_2(t) \in R^{1 \times 1}, \bar{A}_{11} \in R^{3 \times 3}, \bar{A}_{12} \in R^{3 \times 1}, \bar{A}_{21} \in R^{1 \times 3}, \text{and} \bar{A}_{22} \in R^{1 \times 1}; \text{then} \]  

Eq. (18) can be decomposed as Eq. (19):
\[
\dot{Z}_1(t) = \overline{A}_{11}Z_1(t) + \overline{A}_{12}Z_2(t) \\
S = \overline{\Theta}_1Z_1(t) + \overline{\Theta}_2Z_2(t)
\]  
(19)

Let \( S = 0 \) and \( \overline{\Theta}_2 = I \); in Eq. (19), it gets:

\[
\begin{align*}
Z_2(t) &= -\overline{\Theta}_1Z_1(t) \\
\dot{Z}_1(t) &= (\overline{A}_{11} - \overline{A}_{12} \overline{\Theta}_1)Z_1(t)
\end{align*}
\]  
(20)

where the matrix \( \overline{\Theta}_1 \) can be designed by the optimal control method or pole assignment method, and then the sliding surface \( S(t) \) can also be determined. Finally, the saturation function of exponential reaching law is used as Eq. (21):

\[
\dot{S}(t) = -\beta S(t) - \xi \text{sat}(S(t)) \\
\xi > 0, \beta > 0
\]  
(21)

where \( \alpha > 0 \) and \( 1 > \beta > 0 \); with the combination of Eqs. (15), (16), and (21), it can get active control force as Eq. (22) by ignoring the external disturbances:

\[
f = -(\overline{\Theta}B)^{-1}[\overline{\Theta}AX(t) + \beta S(t) + \xi \text{sat}(S(t))]\]
(22)

5. Experimental research

In this chapter, compared with passive vibration isolation, the active control experiment is studied to analyse the isolation effect on a double-layer vibration isolation system. The external disturbance is the force of shaker JZK-40, the hardware control system is designed by LabVIEW real time, and the isolation system experiment setup is shown in Figure 7.

When the external stimulus is applied to the system, the controllers can capture the acceleration signal of the upper and middle layers; then the signal is conveyed to a controller by an A/D converter. At the same time, D/A signals are given out by NI control calculations, passing through amplifier YE2706A, and the output finally is transmitted to GMA for carrying out the active control. The vibration isolation system control block diagram is shown in Figure 8.

In low frequencies, taking the middle platform displacement as the evaluation index, the isolation effect is compared between passive vibration isolation and active control. Experimental parameters are as follows:

\[
\begin{align*}
n &= 1200, \, H_b = 10kA/m, \, a = 7102A/m, \, \ddot{a} = -0.01, \, M_s = 7.65 \times 10^5A/m, \\
\lambda &= 1.005 \times 10^{-6}, \, k = 3283A/m, \, b = 0.18, \, S^H = 1.3 \times 10^{-11}, \, d = 1.0 \times 10^{-8}m^2/N, \\
\xi &= 7600Pa, \, C = 3000kNs/m^2, \, \rho = 9250kg/m^3, \, L = 8.6 \times 10^{-4}m, \, A = 78.5 \times 10^{-6}m^2, \\
M_1 &= 155.42kg, \, C_1 = 302.7Ns/m, \, K_1 = 57016.2N/m, \, M_2 = 22.5kg, \, C_2 = 290.544Ns/m, \, \text{and} \, K_2 = 190000N/m.
\end{align*}
\]  
(23)

The system's natural frequency (\( f_1 = 2.8Hz, \ f_2 = 15.6Hz \)) can be calculated according to the above parameters, and the external disturbances were taken as single, multi-frequency, and random signals. The experimental results are in Figures 9–13 and in Table 1.

Figures 9–12 show that active control can effectively suppress the vibration generated by external incentives with a significant isolation effect and speed response. Table 1 shows that, compared with the passive vibration isolation,
active control makes the middle raft displacement decrease by 81.08, 78.21, 81.13, and 55.34% in a single frequency, multi-frequency, and random excitation, respectively. Therefore, the active control has obvious advantages and a good isolation effect.

Figure 13 shows that the natural frequencies corresponding to the two peaks of the passive isolation system are fully consistent with the calculations. Moreover, the active control can change the system mode to eliminate the first- and second-order formants to achieve the purpose of isolation. It also shows the active control is poor when the excitation frequency is less than 2 Hz, and then the isolation effect increases with escalating excitation frequency. However, there are always cross-points between active control and passive vibration isolation in the amplitude-frequency curve, which also shows the active control is more suitable for low- and middle-frequency vibration control. It can compensate for the lack of passive vibration isolation and effectively inhibit the transmission of vibration isolation and broaden isolation frequency band, all of which are of great significance for the study of active vibration control in engineering applications.
Figure 9.
The time history of middle-layer displacement with $f_1$ excitation.

Figure 10.
The time history of middle-layer displacement with $f_2$ excitation.

Figure 11.
The time history of middle-layer displacement with $\sqrt{2f_1} + \sqrt{2f_2} + 50$ excitation.
6. Conclusions

In this study, GMA linear model cannot fully describe the dynamic behaviour. Based on the magnetic domain theory, piezomagnetic theory, and with the consideration of the magnetic hysteresis, eddy current, and alternating stress, the GMA...
A hysteresis nonlinear model is established, which can describe the actuator dynamic characteristics in a broadband segment. The GMA is then used in active control with the sliding mode control theory. The experimental results show that the hysteresis nonlinear model can fully describe the actuator dynamic characteristics in a broadband segment; the active control is better than passive isolation in single, multi-frequency, and random excitation; and it can effectively broaden the isolation frequency band and improve the response speed and suppress vibration transmission. But it also has some isolation range, which means the effect will deteriorate beyond the effective frequency isolation band.

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**Conflict of interest**

The authors declare that there are no conflicts of interest regarding the publication of this chapter.

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