Does Inflationary Particle Production suggest $\Omega_m < 1$?

Varun Sahni1,* and Salman Habib2,†

1 Inter-University Centre for Astronomy & Astrophysics, Post Bag 4, Pune 411007, India
2 T-8, Theoretical Division, MS B285, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(March 19, 2018)

We study a class of FRW spacetimes with a nonminimally coupled light massive scalar field. Values of the coupling parameter $\xi < 0$ enhance long range power in the vacuum expectation value of the energy-momentum tensor $\langle T_{\mu \nu} \rangle$ and fundamentally alter the nature of inflationary particle production: the energy density of created particles behaves like an effective cosmological constant, leading generically to $\Omega_m < 1$ in clustered matter and providing a possible resolution of the “$\Omega$ problem” for low density cosmological models.

PACS numbers: 04.62.+v, 98.80.Cq

In this Letter we discuss particle creation of light nonminimally coupled scalar fields due to the changing geometry of a spacetime which underwent an early inflationary phase. Nonminimally coupled fields have arisen in diverse cosmological contexts, e.g., density perturbations from strongly coupled scalars [1], the possibility that a very light scalar particle such as the axion may couple nonminimally to gravity [2] and production of primordial magnetic fields from nonminimal electromagnetism [3]. Ultra-light scalars (with $m \sim 10^{-6} e V$) have been discussed in the context of pseudo-Nambu-Goldstone bosons [4].

As shown below, for negative values of the coupling the dominant contribution to both $\langle \phi^2 \rangle$ and $\langle T_{\mu \nu} \rangle$ comes from modes having wavelengths larger than the Hubble radius which makes both quantities formally infrared (IR) divergent. IR divergences cannot be renormalized away, instead a physically motivated IR cutoff has to be invoked [5]. The IR-regularized $\langle T_{\mu \nu} \rangle$ describing particle creation is finite, and behaves like an effective cosmological constant as the Universe expands, consequently the energy density of created particles can dominate the matter density leading to $\Omega_m < 1$ in a flat Universe.

We consider a spatially flat FRW model with expansion factor either de Sitter-like $a \propto e^{Ht}$, or power law $a \propto t^\nu$. Massive free scalar fields satisfy the wave equation

$$[\Box + \xi R + m^2] \Phi = 0 \quad (1)$$

where $R$ is the Ricci scalar and $\xi$ parametrizes the coupling to gravity, $\xi < 0$, $1/6$ corresponding to minimal and conformal coupling respectively. In a spatially flat FRW Universe the field variables separate and $\Phi_k = (2\pi)^{-3/2} \phi_k(\eta) e^{-ik \cdot x}$. The comoving wavenumber $k = 2\pi a/\lambda$ where $\lambda$ is the physical wavelength of scalar field quanta. Defining the conformal field $\chi_k = a \phi_k$ and using $R = 6\ddot{a}/a^3$ (differentiation being with respect to the conformal time $\eta = \int dt/a$), Eq. (1) leads to

$$\ddot{\chi}_k + [k^2 + m^2 a^2 - (1 - 6\xi)\ddot{a}/a] \chi_k = 0 \quad (2)$$

For de Sitter space (2) has the exact solution

$$\chi_k(\eta) = c_1 \sqrt{\eta} H^{(2)}_{\mu}(k\eta) + c_2 \sqrt{\eta} H^{(1)}_{\mu}(k\eta) \quad (3)$$

where $\mu^2 = 9/4 - 12\xi - m^2/H^2$, $c_1 = \sqrt{\pi}/2$, $c_2 = 0$ gives the state associated with the Bunch-Davies vacuum [6].

Ultra-light fields $m/H \sim 1$ today, corresponds to $m/\H \sim 10^{-60}$ during inflation, hence the field modes can be treated as being effectively massless. Setting $m = 0$ in (3) and specializing to a power law expansion

$$a = (t/t_0)^p \equiv \left( \frac{\eta}{\eta_0} \right)^{(1-2\nu)/2} \quad (4)$$

where $2\nu = (1 - 3p)/(1 - p)$, (the inflationary range $p > 1$ corresponds to $\nu > 3/2$ and for $a \propto e^{Ht}$, $\nu = 3/2$), we find exact solutions of (2) having the form (3) with $\mu^2 - 1/4 = (\nu^2 - 1/4)(1 - 6\xi)$. For $\nu = 1/6$, $\mu = 1/2$ while for $\nu = 0$, $\mu = \nu$. The choices $c_1 = \sqrt{\pi}/2$, $c_2 = 0$ correspond to the adiabatic vacuum state.

To study quantum fluctuations we define the operator

$$\Phi(x) = \int d^3 k \left[ a_k \Phi_k(x, \eta) + a_k^\dagger \Phi_k^*(x, \eta) \right] \quad (5)$$

where $a_k$, $a_k^\dagger$ are annihilation and creation operators $[a_k, a_k^\dagger] = \delta_{kk'}$, defining the vacuum state $a_k|0\rangle = 0 \forall k$. The two-point function

$$\langle \Phi(x)\Phi(x') \rangle_{vac} = \frac{1}{(2\pi)^3} \int d^3 k e^{i(k \cdot (x-x') - \omega \eta)} \phi_k(\eta) \phi_k^*(\eta') \quad (6)$$

is IR divergent over a certain range of $\mu$ values. To see this, substitute Eq. (3) in Eq. (6), using $\phi_k = \chi_k/a$ and the small-argument limit of the Hankel functions

$$H^{(2)}_{\mu}(k\eta) \sim \sum_{k<\lambda} \frac{(kn/2)^{\mu}}{(1 + \mu)} \pm i\frac{\Gamma(\mu)}{\pi} \left( \frac{kn}{2} \right)^{-\mu} \quad (7)$$

The integral controlling the presence of IR divergences is

$$\int dk k^2 k^{-2\mu} c_1^2 - c_2^2 \quad (8)$$

For the adiabatic vacuum state (3), IR divergences arise when $\mu^2 \geq 9/4$. For $\xi = 0$, IR divergences are encountered for $p \geq 2/3$ ($p \neq 1$) [6]. The situation is substantially different for $\xi \neq 0$ (Fig. 1), IR divergences being
present over a wide range of expansion rates: $p < 0$; $p > 1/2 \ (p \neq 1); \ 0 < p < 1/2$. Since there is no particle production when $p = 0$, $1/2$, $\xi = 1/6$, these special cases are free of IR divergences.

FIG. 1. IR divergent regions (shaded) of $\langle \Phi^2 \rangle$: The special cases $p = 0$, $1/2$, $1$ and $1/6 \leq \xi \leq 3/2$ have no IR divergences.

Significantly, $\langle T_{\mu \nu} \rangle$ can also be IR divergent: The (bare) energy density in a spatially flat Universe is

$$
\rho = \langle T_{00} \rangle = \frac{1}{4\pi^2 a^2_0} \int_0^\infty dk k^2 \left( |\phi_k|^2 + k^2 |\phi_k|^2 \right) + \frac{3}{2\pi^2} \int_0^\infty dk k^2 \left[ H |\phi_k|^2 + \frac{\phi_k \dot{\phi}_k + \phi_k \dot{\phi}_k}{a} \right] + \frac{m^2}{4\pi^2} \int_0^\infty dk k^2 |\phi_k|^2. \tag{9}
$$

Restricting attention momentarily to $\xi = 0$, $m = 0$, we conclude that $(T_{00})$ is IR divergent for $|\mu| \geq 5/2$, i.e., $3/4 \leq p \leq 2$ ($p \neq 1$). [We correct a minor error in Ref. [2] who quote $2/3 \leq p \leq 2 \ (p \neq 1).$. This is a much smaller range than for the two-point function which is divergent for $p \geq 2/3 \ (p \neq 1)$. In contrast, the key aspect of nonminimal coupling is that $(T_{00})$ contains terms proportional to $\xi \langle \Phi^2 \rangle$, and consequently (for ultra-light fields) $(T_{00})$ is IR divergent over the same range of parameters as $\langle \Phi^2 \rangle$.

The curing of IR divergences requires that mode functions be modified in the IR limit. The only freedom to accomplish this in Eq. (8) is to change the behavior of $|c_1 - c_2|$ as $k \to 0$: There are no IR divergences if $\lim_{k \to 0} |c_1 - c_2|^2 \propto O(k^2|\mu|^2) \to 0$ (maintaining $c_2 = 0, c_1 = 1$ at large $k$). One way to determine $c_1$ and $c_2$ is to assume the existence of a pre-inflationary radiation-dominated phase [8], as a result [8], $|c_1 - c_2|^2 \simeq \left[1 + 4\pi^2 C(k\eta_0)^{1-2|\mu|} \right]^{-1}$, where $\eta_0$ marks the onset of inflation. Finiteness in the ultraviolet is achieved by imposing a cutoff at the horizon scale. It then follows that

$$
\langle \Phi^2 \rangle = \bar{C} \eta^{2(\nu - |\mu|)} \int_{\eta_0}^{\eta-1} dk k^{2(1-\nu)} k^{-2|\mu|}, \tag{10}
$$

where $\bar{C} = C\eta_0^{-2\nu}$. The behavior of $\langle \Phi^2 \rangle$ depends crucially upon the value of $\nu - |\mu|$. For $\xi = 0$, $\langle \Phi^2 \rangle \propto C \int_{\eta_0}^{1/\eta} dk k^{2(1-\nu)}$, which gives for exponential inflation ($\nu = 3/2$) the standard result $\langle \Phi^2 \rangle \simeq H^3 \Delta T/(4\pi^2)$ [3]. In the case of power law inflation $\nu > 3/2$, and $\langle \Phi^2 \rangle$ freezes to a large value at late times [4]. With negative values of $\xi, \mu > \nu$, $\langle \Phi^2 \rangle$ grows with time approaching the form:

$$
\langle \Phi^2 \rangle = \frac{C}{\eta - \eta_0} \left( \frac{\eta}{\eta_0} \right)^{2(\nu - |\mu|)} \eta_0^{-2}, \tag{11}
$$

at late times. For $\xi \ll -1/6$ and $\nu^2 \gg 1/4$ we have $|\mu| \approx \nu \sqrt{6|\xi|}$, which substituted in (11) gives

$$
\langle \Phi^2 \rangle \propto a^c, \quad c = \frac{4\nu}{2\nu - 1} (\sqrt{6|\xi|} - 1) > 1 \tag{12}
$$

Thus $\xi < 0$ can greatly accelerate the growth of fluctuations in inflationary models. Even for minimal coupling, $\xi = 0$, IR finite physical quantities, e.g., $\langle \Phi(x)\Phi(x') \rangle$ and $(T_{00})$, remain sensitive to the presence of long range power in the field modes. This is reflected in the growth of scalar fluctuations, generation of density fluctuations, and the quantum creation of gravitational waves.

We now examine particle production in a Universe that inflates and then transits to a matter dominated regime of expansion. To do this, we return to Eq. (2): this equation closely resembles the one dimensional Schrödinger equation, the role of the “potential barrier” $V(x)$ being played by $V(\eta) = -m^2a^2 + (1 - 6\xi)\dot{a}/a$. The form of the barrier is shown in Fig. 2 assuming $\xi < 1/6, m \approx 0$. The process of super-adiabatic amplification of zero-point fluctuations (particle production) can be qualitatively described as follows: the amplitude of modes having wavelengths smaller than the Hubble radius decreases conformally with the expansion of the Universe, whereas that of larger-than Hubble radius modes freezes (if $\xi = 0$) or grows with time ($\xi < 0$). Consequently, modes with $\xi \leq 0$ have their amplitude super-adiabatically amplified on re-entering the Hubble radius after inflation (Fig. 2).

The non-vacuum state of the scalar field at late times ($\eta > |\eta_0|$) is described by a linear superposition of positive and negative frequency solutions

$$
\phi_{\text{out}}(k, \eta) = \alpha \phi_k^+ + \beta \phi_k^- \tag{13}
$$

where $\phi_k^+ = (\sqrt{\pi \eta_0}/2)(\eta/\eta_0)^{\nu} H_{\nu}^{(2,1)}(k\eta)$. [a] $|\nu|, \ p < 1; \ 2\nu = (1 - 3\nu)/(1 - p), -3/2 \leq \nu \leq -1/2 \iff 2/3 \geq p \geq 1/2$ corresponding to matter with an equation of state $0 \leq w \leq 1/3, \ u - 1/4 = (\nu^2 - 1/4)/(1 - 6\xi)$, $\xi < 0$. The transition from inflation to a radiation/matter dominated epoch is marked by $\eta_0, \ H_{\text{ch}}^2 \equiv 1/\eta_0^2$ being the Hubble parameter at reheating. [We assume $m^2 < |\eta| \Rightarrow \eta^2 < 6\xi(\nu^2 - 1/4)/(\nu - 1)^2$ allowing us to treat field modes as being effectively massless.]

The Bogolubov coefficients $\alpha$ and $\beta$ are determined by matching $\phi_k^+(\eta, \eta_0)$, $\phi_k^-(\eta, \eta_0)$ given in (13) and $\phi_{\text{out}}(k, \eta)$. $\phi_{\text{out}}(k, \eta)$ at $\eta = \eta_0$. For modes with $k\eta_0 < 1$ we obtain

$$
\phi_{\text{out}}(k, \eta) = \alpha \phi_k^+ + \beta \phi_k^- \tag{13}
$$

where $\phi_k^+ = (\sqrt{\pi \eta_0}/2)(\eta/\eta_0)^{\nu} H_{\nu}^{(2,1)}(k\eta)$. [a] $|\nu|, \ p < 1; \ 2\nu = (1 - 3\nu)/(1 - p), -3/2 \leq \nu \leq -1/2 \iff 2/3 \geq p \geq 1/2$ corresponding to matter with an equation of state $0 \leq w \leq 1/3, \ u - 1/4 = (\nu^2 - 1/4)/(1 - 6\xi)$, $\xi < 0$. The transition from inflation to a radiation/matter dominated epoch is marked by $\eta_0, \ H_{\text{ch}}^2 \equiv 1/\eta_0^2$ being the Hubble parameter at reheating. [We assume $m^2 < |\eta| \Rightarrow \eta^2 < 6\xi(\nu^2 - 1/4)/(\nu - 1)^2$ allowing us to treat field modes as being effectively massless.]

The Bogolubov coefficients $\alpha$ and $\beta$ are determined by matching $\phi_k^+(\eta, \eta_0)$, $\phi_k^-(\eta, \eta_0)$ given in (13) and $\phi_{\text{out}}(k, \eta)$. $\phi_{\text{out}}(k, \eta)$ at $\eta = \eta_0$. For modes with $k\eta_0 < 1$ we obtain
\[ \alpha + \beta = A \frac{i}{\pi} \Gamma(\mu) \Gamma(1 + \bar{\mu}) \left( \frac{k\eta_0}{2} \right)^{-\mu - \bar{\mu}} \]

\[ + B \frac{\Gamma(1 + \bar{\mu})}{\Gamma(1 + \mu)} \left( \frac{k\eta_0}{2} \right) \mu - \bar{\mu} \]

\[ \alpha - \beta = C \frac{i\pi}{\Gamma(\mu) \Gamma(1 + \mu)} \left( \frac{k\eta_0}{2} \right)^{\mu + \bar{\mu}} \]

\[ + D \frac{\Gamma(\mu)}{\Gamma(1 + \mu)} \left( \frac{k\eta_0}{2} \right) \bar{\mu} - \mu \]

\[ |\alpha|^2 - |\beta|^2 = 1 \]  

(14)

where, \( A = (\bar{\mu} + |\bar{\mu}| + \nu - \mu)/2\bar{\mu}, \)

\( B = (\bar{\mu} + |\bar{\mu}| + \nu + \mu)/2\bar{\mu}, \)

\( C = (\nu + \mu + |\nu| - |\bar{\mu}|)/2\bar{\mu}, \)

\( D = (\bar{\mu} - |\bar{\mu}| + \mu - \nu)/2\bar{\mu}. \)

(Indices \( \nu, \mu \) refer to the inflationary epoch; \( \bar{\mu}, \bar{\nu} \) to the matter/radiation dominated epoch.)

If the field is effectively massless. A radiation dominated phase prior to inflation leads to an effective IR cutoff \( k_{min} \approx \eta_0^{-1} \) in \( \langle T_{00} \rangle \), since IR divergent states cannot arise from IR finite initial conditions \([13]\). In addition, a high frequency cutoff appears due to suppression of particle creation at large \( k \) \([13]\). For particles created during inflation, this cutoff is set by the Hubble parameter at the end of reheating and the commencement of radiation domination: \( k_{max} \approx H_{rh} \approx \eta_0^{-1} \([13,14]\).

The integration limits in \([13]\) are therefore \( \eta_{max} \). For \( m/H \leq 1, \langle T_{00} \rangle \) is dominated by modes larger than the Hubble radius, thus the integration limits are effectively \( \eta_{max} \). For \( m/H \leq 1, \langle T_{00} \rangle \) is dominated by modes larger than the Hubble radius, thus the integration limits are effectively \( \eta_{max} \). For \( m/H \leq 1, \langle T_{00} \rangle \) is dominated by modes larger than the Hubble radius, thus the integration limits are effectively \( \eta_{max} \).

Also a full treatment of the semiclassical Einstein equations \( G_{\mu\nu} = -8\pi G(T_{\mu\nu} + \phi_k^2) \) lies beyond the scope of this work, it is easy to perform a qualitative analysis. For small values \( |\xi| < 1 \), the term \( 1/(4\pi a^2) \int dk k^4 |\phi_k|^2 \) in Eq. \([10]\) is dominated by horizon modes, and is small compared to the remaining terms in \( \langle T_{00} \rangle \), which are dominated by the larger infrared cutoff scale \( \bar{\eta}_0 \) (\( \bar{\eta}_0 \gg \eta \)). These terms, excluding \( m^2 \langle \Phi^2 \rangle \), are of the form \( H^2 \langle \Phi^2 \rangle \) and can be absorbed into the left hand side of the \( (00) \) Einstein equation leading to:

\[ \langle \Phi^2 \rangle \approx \frac{N}{\eta_0^2} \left[ \frac{\eta}{\eta_{MD}} \right] \left( \frac{\eta}{\eta_0} \right)^2 \left[ 1 - \left( \frac{\eta}{\eta_0} \right) \right]^{2\mu - 3} \]

(16)

where \( N = (A^2/8\pi \eta^3)2^\mu \Gamma^2(\mu)/(2\mu - 3) \).

Although a full treatment of the semiclassical Einstein equations \( G_{\mu\nu} = -8\pi G(T_{\mu\nu} + \phi_k^2) \) lies beyond the scope of this work, it is easy to perform a qualitative analysis. For small values \( |\xi| < 1 \), the term \( 1/(4\pi a^2) \int dk k^4 |\phi_k|^2 \) in Eq. \([10]\) is dominated by horizon modes, and is small compared to the remaining terms in \( \langle T_{00} \rangle \), which are dominated by the larger infrared cutoff scale \( \bar{\eta}_0 \) (\( \bar{\eta}_0 \gg \eta \)). These terms, excluding \( m^2 \langle \Phi^2 \rangle \), are of the form \( H^2 \langle \Phi^2 \rangle \) and can be absorbed into the left hand side of the \( (00) \) Einstein equation leading to:

\[ 3H^2 \approx 8\pi G \left[ \frac{H_{rh}}{m_{pl}} + \frac{1}{2} m^2 \langle \Phi^2 \rangle \right] \]

(17)

where \( \bar{G} \approx G/(1 + 8\pi G|\xi| \langle \Phi^2 \rangle) \). For ultra-light fields \( |\xi| < 0.1 \) ensures that \( \langle \Phi^2 \rangle \) grows at a slow rate, satisfying constraints on the time variation of \( \bar{G} \) \([12]\). Consequently, \( 8\pi G \langle \Phi^2 \rangle \approx N \left( \frac{H_{rh}}{m_{pl}} \right)^2 (1 + z_{MD})^{2\mu - 3} \left( \frac{z_{rh}}{z_{MD}} \right)^{2\mu - 3} \]

(18)

where \( \zeta = \eta_0/\eta = \exp \int_{t_{in}}^{t_{rh}} H dt \), \( t_{in} \) is the beginning of inflation, and \( t_{rh} \) is the time when a mode entering the horizon today, left the Hubble radius during inflation. In general \( \zeta \) can be very large, \( \log \zeta \gg 1 \). Substituting \( H_{rh}/m_{pl} \approx 10^{-5} \), \( T_{rh} \approx 10^{14} \text{ GeV} \), \( T_{MD} \approx (1 + z_{MD}) \times 2.3 \times 10^{-13} \text{ GeV} \), \( 1 + z_{MD} \approx 23.19 \Omega_m h^2 \), we find that \( 8\pi G \langle \Phi^2 \rangle \) can also be very large (since \( G \leq G \) this holds for \( 8\pi G \langle \Phi^2 \rangle \) as well). For large \( 8\pi G \langle \Phi^2 \rangle \gg 1 \), \( G \approx 1/(8\pi \xi \langle \Phi^2 \rangle) \) and

\[ \Lambda_{eff} \approx 8\pi G \langle T_{00} \rangle \approx m^2/2|\xi| \]

\[ \Omega_0 \equiv \Lambda_{eff}/3H^2 \approx \frac{1}{8\pi \xi}(m/H)^2 \]

(19)

We thus find that the energy density of created particles behaves like an effective cosmological constant. Furthermore, for ultralight fields \( m/H \approx 1 \), \( \Omega_0 \) can be a significant fraction of the total density of the Universe at
late times. (Recent observations of high redshift supernovae suggest $\Omega_\Lambda > 0$, which could be quite large if the Universe is flat [13].)

At first glance it may appear strange that the equation of state of created nonminimal scalars is $p \equiv -\rho$ and not the familiar $p = \rho/3$ expected for relativistic particles. This is because the dominant contribution to $\langle T_{00} \rangle$ comes from wavelengths larger than the horizon size, as a result $\langle T_{\mu\nu} \rangle \approx \frac{1}{2} g_{\mu\nu} n^2 \langle \Phi^2 \rangle$ at late times. In a similar context it is well known that $\langle T_{00} \rangle$ for gravitational waves (and minimally coupled massless scalars) created during inflation is dominated by wavelengths of order the horizon size, because of which the equation of state of relic gravitational waves is identical to that of classical matter driving the expansion, and in a matter-dominated Universe, is $p = 0$ [14]. (For wavelengths larger than the Hubble radius the distinction between “real” particles and vacuum polarization becomes ambiguous.)

Our analysis indicates that, at a local level, the Universe may be filled with a quasiclassical, stochastic, nearly homogeneous nonminimal scalar field having a Gaussian probability distribution with dispersion $\langle \Phi^2 \rangle$. (This description agrees with the general stochastic approach to inflationary cosmology [9,17].) Due to cosmic variance, the local value of $\Phi^2$ will generally not equal $\langle \Phi^2 \rangle$, remarkably this does not affect the value of $\Omega_\phi$ since $\langle \Phi^2 \rangle$ does not depend upon $\Phi$ (provided $\Phi$ is large).

To summarize, the inflationary production of light nonminimal particles ($\xi < 0$) leads to an energy density which mimics an effective cosmological constant implying $\Omega_m < 1$ in clustered matter in a critical density Universe. Hence, a low density Universe can exist without an “$\Omega$ problem” making inflation compatible with observations indicating a low value for $\Omega_m$ [8,13]. Since nonminimal scalars naturally arise in the low energy limit of string theory [20], our results may apply to a wider class of models than the ones considered here.

It is a pleasure to thank Yuri Shtanov and Alexei Starobinsky for stimulating discussions.

---

* Electronic address: varun@iucaa.ernet.in
† Electronic address: habib@lanl.gov

[1] R. Fakir and W.G. Unruh, Phys. Rev. D 41, 1783 (1990); D. Salopek, J.R. Bond, and J.M. Bardeen, ibid 40, 1753 (1989); E.W. Kolb, D.S. Salopek, and M.S. Turner, ibid 42, 3925 (1990); R. Fakir, S. Habib, and W. Unruh, Ap. J. 394, 396 (1992); R. Fakir and S. Habib, Mod. Phys. Lett. A 8, 2827 (1993).
[2] M.S. Turner and L.M. Widrow, Phys. Rev. D 37, 3428 (1988).
[3] R. Opher and U.F. Wichoski, Phys. Rev. Lett. 78, 787 (1997).
[4] J. Frieman, C.T. Hill, A. Stebbins and I. Waga, Phys. Rev. Lett. 75, 2077 (1995).
[5] J.A. Lima and L. Parker, Phys. Rev. D 16, 245 (1977).
[6] T.S. Bunch and P.C.W. Davies, Proc. R. Soc. A360, 117 (1978).
[7] A. Vilenkin and L.H. Ford, Phys. Rev. D 26, 1231 (1982).
[8] V. Sahni, Class. Quantum Grav. 5, L113 (1988).
[9] A.D. Linde, Particle Physics and Inflationary Cosmology (Harwood Acad., 1990).
[10] C. Pathmanyake and L.H. Ford, Phys. Rev. D 37, 2099 (1988).
[11] T. Souradeep and V. Sahni, Mod. Phys. Lett. A, 7, 3541 (1992).
[12] C.M. Will, in 300 Years of Gravitation (Cambridge University Press, Cambridge, 1987), edited by S. Hawking and W. Israel, p. 80.
[13] S.J. Perlmutter et al., Nature 391, 51 (1998); A.G. Riess et al., astro-ph/9805201 (1998).
[14] S.A. Fulling, M. Sweeney, and R.M. Wald, Commun. Math. Phys. 63, 257 (1978); S.A. Fulling, F. Narcowich, and R.M. Wald, Ann. Phys. 136, 243 (1981).
[15] N.D. Birrell and P.C.W. Davies, Quantum Fields in Curved Space (Cambridge University Press, Cambridge, 1982).
[16] B. Allen, Phys. Rev. D 37, 2078 (1988); V. Sahni, ibid 42, 453 (1990).
[17] A.A. Starobinsky, in Current Trends in Field Theory, Quantum Gravity and Strings, edited by H.J. de Vega and N. Sanchez (Springer, Heidelberg, 1986).
[18] P. Coles and G.F.R. Ellis, Nature 370, 609 (1994).
[19] J.P. Ostriker and P.J. Steinhardt, Nature 377, 600 (1995).
[20] T. Damour and A. Vilenkin, Phys. Rev. D 53, 2981 (1996).