ON THE MAIN EQUATIONS OF ELECTRODYNAMICS

N. N. CHAUS

ABSTRACT. Instead of a linear system of equations for a free electromagnetic field, we propose a nonlinear system of equations. The classical electrodynamics is preserved. There appeared solutions (the electromagnetic fields) having photon properties. The theory posits the vacuum is a physical medium. The most important problems of relativistic interaction of interpenetrating mediums are studied.

THE PROBLEM

Before a photon was experimentally discovered, the classical electrodynamics was, at the same time, a theory of light. The locality and stability of photons have lead specialists to a perplexity, since solutions of electrodynamics equations do not possess these properties. For this reason, the system of Maxwell’s equations needed to be fixed by replacing it with a nonlinear system. This problem was not solved those days. It was a new science, the quantum electrodynamics, that should have saved the situation and explained the phenomenon of photon. But it did not solve the problem, and it could not solve it, for again, the same linear equations as in the electrodynamics were used. As the result, a photon still remains nowadays an unexplained, even mysterious object, and the theory is similar to the astronomy of Ptolemy, accompanied by obscure philosophical doxies and conjurations.

In this work, the author comes back to the problem of existence of such a system of equations that would “admit” a photon. That is to say that the author believes that the quantum theory is also not flawless.

SOLUTION OF THE PROBLEM

The system we are looking for comes just immediately if we will accurately understand the main principals of electrodynamics and find correct answers to the following questions:

a) what is an electric charge;

b) what is an electric current;

c) does an electromagnetic field act on electric charges and currents?

For the reader to easier accept the author’s answers to these questions, consider a very simple example from mechanics.
Example. Let \( \phi(x) \), \(-\infty < x < \infty \), be a sufficiently smooth function, and \( \phi(x) = 0 \) for \( |x| \geq 1 \). Let us cover the graph of the function \( \phi(x) \) with a sufficiently wide plate, and cut it along the graph of the function \( \phi(x) \). Then let a stretched along the \( x \)-axis homogeneous string be crimped from below and above by the parts of the cut plate, thus making the form of the string repeat the graph of the function \( \phi(x) \). Keeping everything still let us analyze the situation. It is clear that the deflection of the string from the initial condition \( u(x) (= \phi(x)) \) completely determines the state of the string. Nevertheless, let us introduce another useful characteristic of the string. Let us call it a string charge and set the density of the string charge to be \( \sigma(x) = \frac{\partial^2 u}{\partial x^2} \). It is clear that:

1. The density of the force with which the lower or upper part of the plate act on the string is proportional to the density of the string charge (in the theory of linear string). In particular, in the places where \( \sigma(x) = 0 \), the string can be undercut so that it would not touch the string. The form of the string will remain the same after such a procedure.

2. One can consider the force with which the string acts on the plate, and also one can consider the force with which the plate acts on the string. One can as well consider forces acting inside the string, and forces acting inside the plate. But saying that there are forces acting on the string charge in the considered example is absurd. In this example the forces are connected with the charge, these forces and the charge are interconnected, they accompany one another, they cause one another, but no forces act on the string charge. This is impossible.

Answers to questions a) — c). The vacuum is a physical medium. The classical electrodynamics should be regarded first of all as a continuous theory of this medium. The known in the electrodynamics constants \( \varepsilon_0 \) and \( \mu_0 \) are characteristics of this medium. The fields \( \mathbf{E} \) and \( \mathbf{B} \) are also main characteristics of the state of this medium.

According to the classical formula \( \rho = \varepsilon_0 \text{div} \mathbf{E} \), the author claims that the electric charge is only a special characteristic of the state of the vacuum-medium (together with the main characteristics — the vectors \( \mathbf{E} \) and \( \mathbf{B} \)).

According to the classical formula \( \mathbf{j} = \mu_0^{-1}(\text{rot} \mathbf{B} - c^{-2} \dot{\mathbf{E}}) \), the author claims that the electric current is nothing more then another special characteristic of the vacuum-medium (together with the main characteristics \( \mathbf{E} \) and \( \mathbf{B} \), and the characteristic \( \rho \)).

According to the formula \( \mathbf{f} = \rho \mathbf{E} + [\mathbf{j}, \mathbf{B}] \) it is accepted to think that the electromagnetic field exerts a force on currents and charges. The author claims that there is nothing of the sort. There is not a single experiment where one can observe an action of any force on an electric charge. And there is not a single experiment where one could observe an action of any force on electric current. There is not a single physicist who could say about what exactly happens when a force acts on a charge or current. In all experiments we only observe an action of a force on certain physical bodies. And what
acts on them is not the electromagnetic field but the medium, the vacuum. This is similar to hydromechanics where the action is exerted not by some characteristics of the liquid (the distribution of speed, pressure, etc.) but acts the liquid itself, being in a certain state.

Physical bodies on which the vacuum acts, exert a counteraction. They must. In the continuous theory, where we are dealing not with forces but with their densities, it is necessary and sufficient to consider an interaction of interpenetrating mediums. The vacuum can interact simultaneously with two distinct mediums. This is seen from the formulas \( \rho = \rho^+ + \rho^- \), \( j = \rho^+ v^+ + \rho^- v^- \). Existence of such mediums is due to the fact that in nature there exist electrons and protons that “from the birth” sit on vacuum. These particles need not be used in the continuous theory, similarly as in the hydromechanics, the formula \( H_2O \) is not used. The mediums \( M^+ \) and \( M^- \) do not own the charges or currents. They only interact with the vacuum and induce a condition where the characteristics \( \rho \) and \( j \) appear. The charges and currents is not a cause but a consequence. The theory about flows of electrons, coils, magnets, capacitors, etc. is a specific part of the theory of electrodynamics, where we are dealing with technical means to act on vacuum, to control its state. By using recipes of this part of electrodynamics, we prepare a special medium, or the mediums \( M^+ \) and \( M^- \), as to exert a needed action on the vacuum, to get it into a needed state. This is precisely in this part of electrodynamics where the formulas \( j = \rho^+ v^+ + \rho^- v^- \) appear that are not included in the main equations. There are no such formulas in the theory of string; one can touch the string with a finger, it is more difficult to reach the vacuum.

Comparing a string and vacuum, the formulas \( f = -\mu \partial_t^2 u + T \partial_x^2 u \) and \( f = \rho E + [j, B] \) address the same question, — the force of an external action on the medium (the string or vacuum). If the condition of the string, \( u(x, t) \), is such that \( f \equiv 0 \), then this means that the string does not interact with the ambient medium, the string is free and self contained. The condition \( f = 0 \) is necessary for the fields \( E \) and \( B \) describe the free vacuum. But since \( f \), in the general case, is the sum of the forces \( f^+ \) and \( f^- \), the condition \( f = 0 \) does not imply that the vacuum is free. The mediums \( M^+ \) and \( M^- \) could “stretch” the vacuum in opposite directions and yield \( f = 0 \), but still there will be a nontrivial energy exchange between the mediums. This nontriviality can be eliminated by imposing the condition \( (E, j) = 0 \).

On the basis of the preceding discussion the author claims that, if the fields \( E \) and \( B \) and all other possible additional characteristics of the vacuum condition are such that \( \rho E + [j, B] = 0 \) and \( (E, j) = 0 \), then the vacuum is not interconnected with anything, it is free and self contained.

**Nonlinear system of equations.** The system of equations

\[
\begin{align*}
\dot{B} &= -\text{rot} \, E, & \text{div} \, B &= 0, & \rho &= \varepsilon_0 \text{div} \, E, \\
c^{-2} \dot{E} + \mu_0 j &= \text{rot} \, B, & \rho E + [j, B] &= 0, & (E, j) &= 0
\end{align*}
\]
is the system of equations of the state of the free vacuum.

The author believes that system (1) should replace the following system used in physics:

\[ \dot{\mathbf{B}} = -\text{rot} \mathbf{E}, \quad \text{div} \mathbf{B} = 0, \quad \text{div} \mathbf{E} = 0, \quad \dot{\mathbf{E}} = c^2 \text{rot} \mathbf{B}. \quad (2) \]

**First corollaries**

If the field \( \mathbf{E} \) (stationary or not) is sufficiently smooth and vanishing at infinity, and the assembly \( \{ \mathbf{E}; \mathbf{B} \equiv 0 \} \) is a solution of system (1), then \( \mathbf{E} \equiv 0 \). Indeed, system (1) yields, for \( \mathbf{E} \), the representation \( \mathbf{E} = \nabla \varphi \) and \( \Delta \varphi \nabla \varphi = 0 \). Whence \( \Delta \varphi = 0 \) and \( \Delta E_i = 0 \) in \( \mathbb{R}^3 \) if \( |E_i| \to 0 \) at infinity.

In the same elementary way one can deduce that there do not exist stationary or nonstationary spherically symmetric states of the vacuum, i.e. solutions of the form \( \mathbf{E} = \nabla e(r,t), \mathbf{B} = \nabla b(r,t), r^2 = x^2 + y^2 + z^2 \).

Solutions of system (2) satisfy system (1). There exist solutions of system (1) that are not solutions of system (2). Let us show this.

Let \( a_0(x,y,z) \) be a sufficiently smooth function on \( \mathbb{R}^3 \) with compact support. Denote by \( a = a_0(x-c t,y,z) \) and let \( \mathbf{E} = (0,c \partial_y a, c \partial_z a), \mathbf{B} = (0,-\partial_y a, \partial_z a) \). Such \( \mathbf{E} \) and \( \mathbf{B} \) satisfy system (1) but not (2).

Let us stress on the following properties of the solution \( \mathbf{E} \) and \( \mathbf{B} \):

1. The vectors \( \mathbf{E} \) and \( \mathbf{B} \) and the support of the function \( a_0 \) travel with the velocity of light along the \( x \)-axis without change.
2. The vectors \( \mathbf{E} \) and \( \mathbf{B} \) are orthogonal to the direction of the travel.
3. The characteristic \( \rho \) for this solution equals \( \varepsilon_0 c (\partial_y^2 a + \partial_z^2 a) \), and, consequently, the full charge transported by the wave is zero.
4. The total energy \( \mathcal{E} \) and the total momentum \( \mathbf{P} \) of the wave satisfy \( \mathcal{E} = c \mathbf{P} \).
5. It is easy to see that, if the wave meets a similar wave with another direction of the travel, one observes an interaction, since there is no superposition for system (1).

**Special example.** Take \( a_0 = (Ay \sin \omega x + Bz \cos \omega x) \chi(x,y,z) \), where \( \chi \) is a sufficiently smooth function equal to zero outside of a compact set \( G_0 \), and \( \chi \equiv 1 \) in a smaller domain \( G_1, G_1 \subset G_0 \). For such \( a_0 \), the solution \( \mathbf{E}, \mathbf{B} \) in the domain \( G_1 \) gives a classical ellipse polarized field.

**Hypothesis.** The photon, considered as a real object, is characterized in the classical electrodynamics by the fields \( \mathbf{E} \) and \( \mathbf{B} \) from the preceding construction. It is a special state of the free vacuum. The diversity of photons is limited by a special class of the function \( a_0 \).

**Some problems**

1. To formulate conditions on the functions \( a_0 \) such that the functions would correspond to real photons. It can happen that the special example of the considered \( a_0 \) serves as a useful remark on this question. Or this is not true. The author does not understand very well how an elliptically polarized wave...
could manage to go through a stationary plate of a polaroid. At this time
the author does not understand this phenomenon at all.

2. The class of functions $a_0$ corresponding to real photons will be fairly
small. It is not clear in principle whether or not it is technologically possible
to produce artificial photons.

3. The majority of problems in the electrical and radio technology can be
solved by using classical theory of electromagnetic waves that dissipate at
infinity. But as far as the real radiation is concerned, a dominating opinion
is that everything consists of photons. System (1) contains both types of
solutions, and, hence, there is a suspicion that this is what indeed happens
in reality.

4. Let us look for solutions of system (1) in the case where there is a sym-
metry axis.

a). Let us consider the fields as follows:

$$
E = \nabla f(u), \quad B = \left( \frac{x}{s} \partial_x g(u) + \frac{y}{s} \partial_y g(u), \frac{y}{s} \partial_z g(u) - \frac{x}{s} h(u), -2 \partial_s g(u) \right),
$$

where $f(\lambda), g(\lambda), h(\lambda)$ are certain functions of one variable, $u = u(s, z)$,
$s = x^2 + y^2$. For such kind of field, we immediately have that $\text{rot} E = 0$,
$\text{div} B = 0, (E, j) = 0$, and

$$
[j, B] = \frac{1}{\mu_0} \left( 4g'(u) \partial_s^2 g(u) + \frac{1}{s} g'(u) \partial_s^2 g(u) + \frac{1}{s} h(u) h'(u) \right) \nabla u.
$$

To obtain this formula, we use that $\partial_x = 2x \partial_s, \partial_y = 2y \partial_s, \partial_x^2 + \partial_y^2 = 4 \partial_s (s \partial_s)$.

As we see, the equation $\rho E + [j, B] = 0$ in system (1) means that there is
the following relation between the functions $f, g, h, u$:

$$
\frac{s}{c^2} \Delta f(u) \cdot f'(u) = 4 s g'(u) \partial_s^2 g(u) + g'(u) \partial_s^2 g(u) + h(u) h'(u). \quad (3)
$$

Formally, each collection of functions $f, g, h, u$ satisfying equation (3) generates
fields $E$ and $B$ that are solutions of system (1).

Let us set in (3) $f(\lambda) = a_0 \lambda, g(\lambda) = a \lambda, h(\lambda) = b \lambda$, where $a_0, a, b$ are
constants. Equation (3) becomes the linear equation for $u$:

$$
a_0^2 \frac{c^2}{s} \Delta u = 4a^2 s \partial_s^2 u + a^2 \partial_s^2 u + b^2 u. \quad (4)
$$

Formally, each solution $u$ of equation (4) generates a solution of system (1),
$E = a_0 \nabla u, B = \left( \frac{ax}{s} \partial_x u + \frac{by}{s} \partial_y u, \frac{ay}{s} \partial_z u - \frac{bx}{s} u, -2a \partial_s u \right)$. In particular, the
function $u_k = e^{k(s + z^2)} \left( s + z^2 \right)^{-2k - 1/2}$ satisfies the equation $s \Delta u_k = 4k^2 u_k$, and,
thus, the corresponding fields $E^{(k)} = \nabla u_k, B^{(k)} = \frac{2k}{cs} (yu_k, -xu_k, 0)$ is a
solution of system (1), however, not in the whole space. All these solutions
$\{E^{(k)}, B^{(k)}\}, k \in \mathbb{R}^1$, have a singularity at the origin, and hence will not
serve as states of the free vacuum. But these solutions could be of interest.
If considered as supplementing the well-known electromagnetic field of a stationary point charge \((k = 0)\).

It can happen that there are no solutions of equation (3) that are regular and vanishing at infinity for \(s + z^2 \to \infty\). Let us explain this. First of all if such solutions exist, then there would be sufficiently many of them, since, together with a solution \(u(s, z)\), the functions \(u(s, z + \lambda)\), \(\int_0^h u(s, z + \lambda) \varphi(\lambda) \, d\sigma(\lambda)\), and \(L(\partial_z)u(s, z)\) would also be solutions, where \(L(\partial_z)\) is a linear polynomial in \(\partial_z\) with constant coefficients. Let now \(u_1(s, z)\) and \(u_2(s, z)\) be two such solutions, and \(\{E^1, B^1\}\) and \(\{E^2, B^2\}\) — the corresponding electromagnetic fields. Then the superposition of these fields, \(\{E, B\} = \{E^1 + E^2, B^1 + B^2\}\) also satisfies system (1), because \(\{E, B\}\) is generated by the solution \((u_1 + u_2)\) of equation (3). As far as particles are concerned, it is apparent that particles 1 and 2 are noninteracting. After this, let us consider the electromagnetic field \(\{E^2, B^2\}\) which is obtained by a translation of the field \(\{E^1, B^1\}\) in \(\mathbb{R}^3\) but not along the \(z\)-axis. This field will also be a solution of system (3), but it will be generated by a solution \(\tilde{u}_2(x, y, z)\) of a linear equation distinct from equation (3). As a result, the superposition \(\{E^1 + \tilde{E}^2, B^1 + \tilde{B}^2\}\), in general, will not satisfy system (1), and we get that “threaded on an axis” particles do not interact, whereas they become interacting in another position. It seems to the author that such physics is too exotic, and thus we can leave the problem of finding solutions of equation (3) and again look for the functions \(f, g, h, u\) which would satisfy the nonlinear equation (3) and generate reasonable electromagnetic fields.

\(b)\). Let a solution of system (3) have the form: \(E = (\partial_x \Psi, \partial_y \Psi, \partial_z \Psi + \Phi), B = (-\partial_y \Phi, \partial_z \Phi, 0), \Psi = \Psi(s, z, t), \Phi = \Phi(s, z, t), s = x^2 + y^2.\)

One can check that such \(E\) and \(B\) are solutions of system (3) if \(\Psi\) and \(\Phi\) satisfy the following system:

\[
\begin{align*}
\partial_s \Phi [4c^2 \partial_z (s \partial_s \Phi)] - \tilde{\Phi} - \partial_z \tilde{\Psi} &= \partial_z [\Delta \Psi + \partial_z \tilde{\Phi}], \\
\partial_s \Phi [4c^2 \partial_z (s \partial_s \Phi)] + 4s \partial_s \tilde{\Psi} &= \tilde{\Phi} + \partial_z \Psi [\Delta \Psi + \partial_z \tilde{\Phi}].
\end{align*}
\]

Assuming that \(\Phi\) and \(\Psi\) are independent of \(t\) and denoting \(s \partial_s \Phi = g\), we get that the pair \(\Psi, g\) satisfy the system

\[
4c^2 g \partial_s g = s \Delta \Psi \partial_s \Psi, \quad 4c^2 g \partial_z g = s \Delta \Psi \partial_z \Psi.
\]

Formally, each solution \(g, \Psi\) of this system generates a solution of system (3), \(E = \nabla \Psi, B = (-2ys^{-1}g, 2xs^{-1}g, 0)\). For example, such is the pair \(\Psi, g = c^{-1} s \partial_s \Psi\) for an arbitrary but independent of \(z\) function \(\Psi\).

The author does not have a more interesting example of a pair \(g, \Psi\) as well as a regular solution of equation (3).

5. Talking about the interaction of photon solutions as they meet, we should add that there are no known formulas describing interactions between photon solutions and a stationary electromagnetic field. Only superposition of the solution \(E = (0, c \partial_y a, c \partial_z a), B = (0, -\partial_z a, \partial_y a)\) and a stationary field
of the form $\mathbf{E} = (0, ch_2, ch_3), \mathbf{B} = (h_1, -h_3, h_2)$ yields again a solution of system $[\mathbb{I}]$.

**General theory. Necessity.**

Interaction between the vacuum and physical bodies in the continuous theory, when the force density is used, could only be interactions between mediums that interpenetrate each other. In the general case, we will be considering a medium $M$ and a medium $\Phi$ that simultaneously fill a certain part of space and have there, and in other part of the space, the velocities $v^M_\alpha(x, y, z, t)$ and $v^\Phi_\alpha(x, y, z, t)$, respectively. Let there be a force interaction between the mediums. Let $f^M_\alpha$ be the force density with which the medium $M$ acts on the medium $\Phi$, and let $f^\Phi_\alpha$ be the force density with which the medium $\Phi$ acts on $M$. There is no relation $f^M_\alpha = -f^\Phi_\alpha$ in the relativistic theory. It is replaced by a more complicated formula obtained by switching from the densities $f^M_\alpha, f^\Phi_\alpha$ to the corresponding four dimensional force densities. The author did not succeed in obtaining a unique transition, hence in the sequel we give two versions of all main formulas. The reason for this is that, in every point where $f^M_\alpha$ and $f^\Phi_\alpha$ are not equal to zero, there are two velocities $v^M_\alpha$ and $v^\Phi_\alpha$, instead of one, that makes the forth component of the four dimensional force density. It turns out that each one of these velocities is capable to control the forth component of the 4-density of either force.

To make it less confusing, we preserve the notations $f^M_\alpha$ and $f^\Phi_\alpha$ for the first version, and assume that the corresponding 4-densities $f^M_k, f^\Phi_k$ are of the form $\{f^M_\alpha, \frac{i}{c} f^M_\beta v^M_\beta\}$ and $\{f^\Phi_\alpha, \frac{i}{c} f^\Phi_\beta v^\Phi_\beta\}$. For the second version, $g^M_\alpha$ and $g^\Phi_\alpha$ will denote the force densities exerting by $M$ onto $\Phi$ and $\Phi$ onto $M$, and the corresponding densities $g^M_k$ and $g^\Phi_k$ are of the form $\{g^M_\alpha, \frac{i}{c} g^M_\beta v^M_\beta\}$ and $\{g^\Phi_\alpha, \frac{i}{c} g^\Phi_\beta v^\Phi_\beta\}$.

The formulas connecting $f^M_k$ and $f^\Phi_k$ ($g^M_k$ and $g^\Phi_k$). Denote by $V^M_k$ and $V^\Phi_k$ the 4-velocities (fields) of the mediums $M$ and $\Phi$, i.e. $V^M_\alpha = \gamma^M v^M_\alpha$, $V^\Phi_\alpha = \gamma^\Phi v^\Phi_\alpha$, $\alpha = 1, 2, 3$, $V^M_4 = ic\gamma^M$, $V^\Phi_4 = ic\gamma^\Phi$. Denote by $\tilde{v}_\alpha(x, y, z, t)$ such a velocity in the initial inertial reference frame (IRF) for the new IRF$'$ such that the mediums $M$ and $\Phi$ would have the velocities $(v^M_\alpha)' = -(v^\Phi_\alpha)'$ at a point $(x, y, z, t)'$ in this new IRF$'$. Such $\tilde{v}_\alpha$ is defined by the formula

$$\tilde{v}_\alpha = (\gamma^M + \gamma^\Phi)^{-1}(V^M_\alpha + V^\Phi_\alpha),$$

the corresponding 4-velocity $\tilde{V}_k$ will be

$$(\tilde{\gamma}\tilde{v}_\alpha, ic\tilde{\gamma})$$

with $\tilde{\gamma} = (\gamma^M + \gamma^\Phi)(2 + 2\gamma^M\gamma^\Phi - 2\gamma^M\gamma^\Phi c^{-2}v^M_\alpha v^\Phi_\alpha)^{-1/2}$. It is natural to think that IRF$'$ have the property that observed forces that act and counteract between the mediums at this very point $(x, y, z, t)'$ differ by

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1 Here and in the sequel, the main 4-vector is $(x_1, x_2, x_3, x_4) = (x, y, z, ict)$ and the 4-velocity is $V_k = (\gamma v_\alpha, ic\gamma)$, $\partial_k = \partial/\partial x_k$. 
the sign, i.e.
\[ (f^M_\alpha)' = -(f^\Phi_\alpha)', \quad (g^M_\alpha)' = -(g^\Phi_\alpha)', \quad \alpha = 1, 2, 3, \]
\[ (f^M_4)' = (f^\Phi_4)', \quad (g^M_4)' = (g^\Phi_4)' \]
We should also add that all the densities with and without prime are connected by the Lorenz transformation defined by the velocity \( \bar{v}_\alpha \). All this leads to the following formulas for the initial IRF \( (k, m = 1, \ldots, 4) \):
\[ f^\Phi_k = -f^M_k - \frac{2}{c^2} f^M_i \bar{v}_m \bar{V}_k, \quad f^\Phi_k \bar{V}_k = f^\Phi_k \bar{V}_k, \quad (5) \]
\[ g^\Phi_k = -g^M_k - \frac{2}{c^2} g^M_i \bar{v}_m \bar{V}_k, \quad g^\Phi_k \bar{V}_k = g^\Phi_k \bar{V}_k. \quad (6) \]
A boring derivation of these formulas are left to the reader.

The medium energy-momentum tensors. The interacting mediums \( M \) and \( \Phi \) have some domain \( G = G_M \cap G_\Phi \subset \mathbb{R}^3 \) of their mutual existence as well as regions in \( \mathbb{R}^3 \) where they exist by themselves. This fact suggests that one must define the energy and write its conservation law separately for each medium counting the energy exchange and the energy transformation from one form into another. We thus assume that the variables \( f^M_\alpha \) and \( f^\Phi_\alpha \) (or \( g^M_\alpha \) and \( g^\Phi_\alpha \)) together determine the energy of both the energy of the medium \( M \) and the energy of the medium \( \Phi \), and the conservation laws have the form:
\[ \dot{W}^M + \text{div} S^M = -k_M f^M_\alpha \bar{v}_\alpha + k_\Phi f^\Phi_\alpha \bar{v}_\alpha, \quad (7) \]
\[ \dot{W}^\Phi + \text{div} S^\Phi = -k_\Phi f^\Phi_\alpha \bar{v}_\alpha + k_M f^M_\alpha \bar{v}_\alpha, \quad (8) \]

or
\[ \dot{W}^M + \text{div} \bar{S}^M = -\kappa_\Phi g^M_\alpha \bar{v}_\alpha + \kappa_M g^\Phi_\alpha \bar{v}_\alpha, \quad (9) \]
\[ \dot{W}^\Phi + \text{div} \bar{S}^\Phi = -\kappa_\Phi g^\Phi_\alpha \bar{v}_\alpha + \kappa_M g^M_\alpha \bar{v}_\alpha. \quad (10) \]

Here \( W^M, W^\Phi \) (or \( \bar{W}^M, \bar{W}^\Phi \)) are energy densities in \( M \) and \( \Phi \), \( S^M, S^\Phi \)
(or \( \bar{S}^M, \bar{S}^\Phi \)) are energy fluxes, if the forces \( f^M_\alpha, f^\Phi_\alpha \) (or \( g^M_\alpha, g^\Phi_\alpha \)) operate. The parameters \( k_M, k_\Phi, \kappa_M, \kappa_\Phi \) control the energy exchange between the mediums, i.e. these parameters make a quantitative characteristic of the pair of mediums. We assume that they are all positive and \( k_M + k_\Phi = \kappa_M + \kappa_\Phi = 1 \). The case where \( k_M = k_\Phi \) \( (\kappa_M = \kappa_\Phi) \) correspond to a symmetric interaction between the mediums \( M \) and \( \Phi \). Note that each of the equations \( (7) - (10) \) is considered in its own domain \( (G_M \text{ or } G_\Phi) \).

Starting with formulas \( (7) - (10) \) and using the 4-vectors \( f_k \) and \( g_k \) we introduce the tensors \( \tau^M_{ik}, \tau^\Phi_{ik}, \bar{\tau}^M_{ik}, \bar{\tau}^\Phi_{ik} \) such that the following equations hold (in the corresponding domains):
\[ \partial_k \tau^M_{ik} = k_M f^M_i - k_\Phi f^\Phi_i, \quad \partial_k \tau^\Phi_{ik} = k_\Phi f^\Phi_i - k_M f^M_i, \quad (11), (12) \]
\[ \partial_k \bar{\tau}^M_{ik} = \kappa_M g^M_i - \kappa_\Phi g^\Phi_i, \quad \partial_k \bar{\tau}^\Phi_{ik} = \kappa_\Phi g^\Phi_i - \kappa_M g^M_i. \quad (13), (14) \]
The signs in these equations are put in such a way that \( \tau^M_{ik}, \tau^\Phi_{ik}, \bar{\tau}^M_{ik}, \bar{\tau}^\Phi_{ik} \)
could serve in equations \( (7) - (10) \) as energy densities, and the vectors \( \iota \tau^{\alpha M}_{4k}, \tau^{\alpha \Phi}_{4k}, \bar{\tau}^{\alpha M}_{4k}, \bar{\tau}^{\alpha \Phi}_{4k} \)
serve as such.


icτ_Φ, ic^2τ_M, ic^2τ_Φ — the energy fluxes. The tensors defined by equations (7) — (14) could be called energy-momentum tensors for the mediums M and Φ. Also the vectors ic^{−1}τ_M, ic^{−1}τ_Φ, ic^{−1}τ_M, ic^{−1}τ_Φ will be impulse densities for the mediums M and Φ.

If one of the parameters k_M, k_Φ, κ_M, κ_Φ equals 0 or 1, then this is a case of a limiting nonsymmetrical interaction between the mediums. In particular, if k_M = 1 and k_Φ = 0, we get from (11), (12) that

\[ \partial_k τ_Φ^{ik} V_i^M = 0, \quad \partial_k τ_Φ^{ik} V_i^Φ = 0. \]

The first of these conditions appears, for example, in relativistic hydrodynamics. There V_i^M is a field of 4-velocities of the liquid, and the tensor τ_Φ^{ik} is the energy-momentum tensor of the liquid itself that interacts with the so-called mass-forces (a one more phantom). The second condition is fundamental in electrodynamics. There τ_Φ^{ik} is the Poynting’s energy-impulse tensor which characterizes the state of the vacuum, V_i^Φ is not a 4-velocity of the vacuum but that of the medium M interacting with the vacuum Φ.

Existence in physics of these two very different formulas on a similar subject lead the author to an understanding that, in the general theory of interacting mediums, there must appear special control parameters k_M, k_Φ, κ_M, κ_Φ. It is clear that the energy states of both mediums could influence to a great extend the process of energy exchange between the mediums, and, hence, the scalars k_M, k_Φ, κ_M, κ_Φ, in general, are not constants but, for example, if a process is considered in a small volume and for a short period of time, could be regarded as such.

By using equations (7) — (14) one can obtain a series of other equations eliminating some density from (7) — (14) by using formulas (5), (6). In particular, we have

\[ \partial_k τ_Φ^{ik} V_i^M = -f_i^M - k_Φ \frac{2}{c^2} f_m^Φ V_i^M V_i^Φ, \quad \partial_k τ_Φ^{ik} V_i^Φ = g_i^Φ + \kappa_M \frac{2}{c^2} g_m^Φ V_i^M V_i^Φ. \]

(15)

**A more complicated interaction of mediums.** A more complicated scheme of interactions will be used to consider electromagnetic phenomena. At this point we leave out the question on whether an electron is a free state of the vacuum. Our goal is to obtain analogues of formulas (15) for three mediums M_1, M_2 and Φ which simultaneously occupy the same region in space G = G_1 ∩ G_2 ∩ G_Φ. This means that we have at our disposal the velocities v_1, v_2, v^Φ and force densities f^{12}, f^{21}, f^{1Φ}, f^{2Φ}, f^{Φ1}, f^{Φ2}, where f^{12} is the force density with which the medium M_1 acts on the medium M_2, etc. Passing to 4-densities we again obtain 2 versions of them: f_k^{12}, f_k^{21}, f_k^{1Φ}, f_k^{2Φ}, g_k^{12}, g_k^{21}, g_k^{Φ1}, g_k^{Φ2}, g_k^{Φ1}, g_k^{Φ2}. Also the forces of action and counteraction are connected by formulas similar to (5), (6). For
example,

\[
f_k^{\phi_1} = -f_k^{\phi} - \frac{2}{c^2}f_m^{\phi_1} \Phi\Phi_1 \Phi_1, \quad f_k^{\phi_2} = f_k^{\phi_1} \Phi_1 \Phi_2, \quad (16)
\]

\[
g_k^{\phi_1} = -g_k^{\phi} - \frac{2}{c^2}g_m^{\phi_1} \Phi\Phi_1 \Phi_1, \quad g_k^{\phi_2} = g_k^{\phi_1} \Phi_1 \Phi_2, \quad (17)
\]

where the 4-velocity \( V_k = \dot{V}_k \) is determined by using the well known procedure applied to the pair of velocities \( \mathbf{v}^1, \mathbf{v}^\Phi \).

There is an exchange of energies in the mediums \( M1, M2, \Phi \). Let \( W^1, W^2, W^\Phi \) be energy densities of the mediums \( M1, M2, \Phi \), and \( S^1, S^2, S^\Phi \) be flux of these energies. The the most simple equations that could control the energy exchange between the mediums are the following natural generalizations of (17), (18):

\[
\dot{W}^1 + \text{div} S^1 = -k_1 \Phi f_k^{\phi_1} \Phi_1 + k_2 \Phi f_k^{\phi_2} \Phi_2 - k_2 \Phi_1 f_k^{\phi_2} \Phi_1 + k_2 \Phi_2 f_k^{\phi_2} \Phi_1, \quad (18)
\]

\[
\dot{W}^2 + \text{div} S^2 = -k_2 \Phi f_k^{\phi_2} \Phi_2 + k_2 \Phi f_k^{\phi_2} \Phi_2 - k_2 \Phi_1 f_k^{\phi_2} \Phi_1 + k_2 \Phi_2 f_k^{\phi_2} \Phi_1, \quad (19)
\]

\[
\dot{\Phi} + \text{div} \Phi^\Phi = -k_2 \Phi f_k^{\phi_2} \Phi_2 + k_1 \Phi f_k^{\phi_1} \Phi_1 - k_2 \Phi_2 f_k^{\phi_2} \Phi_1 + k_2 \Phi_2 f_k^{\phi_2} \Phi_1, \quad (20)
\]

and analogous three equations, corresponding to (17), (18) with the forces \( g^\phi \) and constants \( \kappa_{\alpha\beta} \). Similarly to the case of two mediums, all the coefficients now are in pairs in the sense that \( k_{1\Phi} + k_{2\Phi} = 1, k_{2\Phi} + k_{2\Phi} = 1, \kappa_{1\Phi} + k_{1\Phi} = 1, \kappa_{2\Phi} + k_{2\Phi} = 1, k_{12} + k_{21} = 1, k_{12} + k_{21} = 1 \).

The three formulas we gave, (17), (18), (19), and the other three formulas can be used to introduce the tensors \( \tau^{\phi}_{ik}, \tilde{\tau}^{\phi}_{ik}, \tilde{\tau}^{\phi}_{ik} \) relating them to the 4-densities \( f^\alpha_{\beta} \) and \( g^{\alpha}_{\beta} \). As an example we give two such equations that correspond to equations (11) and (14):

\[
\partial_k \tau^{\phi}_{ik} = k_1 \phi f_k^{\phi_1} \Phi_1 - k_2 \Phi_2 f_k^{\phi_2} \Phi_2 - k_2 \Phi_2 f_k^{\phi_2} \Phi_1, \quad (21)
\]

\[
\partial_k \tilde{\tau}^{\phi}_{ik} = \kappa_{1\Phi} g_k^{\phi_1} \Phi_1 - \kappa_{1\Phi} g_k^{\phi_2} \Phi_2 + \kappa_{2\Phi} g_k^{\phi_2} \Phi_2. \quad (22)
\]

Each of the tensors \( \tau^{\phi}_{ik}, \tilde{\tau}^{\phi}_{ik} \) could be called an energy-momentum tensor of the medium \( \Phi \). Let us replace in (21), (22) \( f_k^{\phi_1} \) and \( f_k^{\phi_2} \) by using formulas (16), and \( g_k^{\phi_1} \) and \( g_k^{\phi_2} \) — according to (17). In addition also assume that the interactions between the medium \( \Phi \) and the mediums \( M1 \) and \( M2 \) are the same: \( k_{1\Phi} = k_{2\Phi} = k_{12} = k_{21} = k_{1\Phi} = k_{2\Phi} = k_{1\Phi} = k_{2\Phi} = k_{M}. \) All this leads to the following generalization of formulas (15):

\[
\partial_k \tau^{\phi}_{ik} = -(f_k^{\phi_1} + f_k^{\phi_2}) + \frac{2}{c^2}(f_m^{\phi_1} \Phi_1 \Phi_1 \Phi_1 + f_m^{\phi_2} \Phi_1 \Phi_1 \Phi_1), \quad (23)
\]

\[
\partial_k \tilde{\tau}^{\phi}_{ik} = (g_k^{\phi_1} + g_k^{\phi_2}) + \kappa_{M} \frac{2}{c^2}(g_m^{\phi_1} \Phi_1 \Phi_1 \Phi_1 + g_m^{\phi_2} \Phi_1 \Phi_1 \Phi_1). \quad (24)
\]

**A look at electrodynamics.** We start with the formulas \( \rho = \rho^+ + \rho^- \), \( \rho^+ \geq 0, \rho^- \leq 0, j = \rho^+ \mathbf{v}^+ + \rho^- \mathbf{v}^- \) which allow to state that the vacuum \( \Phi \) interacts with a medium \( M1 \) that has velocity \( \mathbf{v} = \mathbf{v}^+ \), and with a medium \( M2 \) that has velocity \( \mathbf{v} = \mathbf{v}^- \). We now take the representation of the Lorentz 4-force \( f_k = F_k^+ + F_k^- \), \( F_k^\pm = \{ \rho^\pm \mathbf{E} + \rho^\pm [\mathbf{v}^\pm, \mathbf{B}], \mathbf{E} / c^2 \} \), and
address the question on what the 4-forces $F_k^+$ and $F_k^-$ are and what is their place in the theory of interaction of mediums? Comparing $F_k^+$ and $F_k^-$ with the 4-vectors $f_k^1, f_k^2, g_k^1, g_k^2$, etc. we come to the conclusion that there are two answers to the posed question: a) $F_k^+ = g_k^{1\Phi}, F_k^- = g_k^{2\Phi}$, b) $-F_k^+ = f_k^{1\Phi}, -F_k^- = f_k^{2\Phi}$. The answer a) reiterates the well established notion of $f$, the answer b) is new, and the author sees no reasons why the answer a) is more preferable. One also must retain both formulas (23) and (24) which now become:

$$
\partial_k \tau_{ik}^{\Phi} = (F_i^+ + F_i^-) + k_\Phi 2c^{-2} \left( F_m^+ V_m V_i + F_m^- V_m V_i \right),
$$

$$
\partial_k \tilde{\tau}_{ik}^{\Phi} = (F_i^+ + F_i^-) + \kappa_\Phi 2c^{-2} \left( F_m^+ V_m V_i + F_m^- V_m V_i \right). \quad (25)
$$

Let us compare these two formulas with the equation for the energy-momentum tensor $T_{ik}$ in electrodynamics: $\partial_k T_{ik} = F_i^+ + F_i^-$. Since now all tensors ($T_{ik}$, $\tau_{ik}$, $\tilde{\tau}_{ik}$) are responsible for the distribution and flow of energy, as well as for the momentum of the same medium $\Phi$, one can make 2 different claims:

1. In an interaction with the mediums $M_1$ and $M_2$, the vacuum shows its limit properties by having the characteristics $k_\Phi$ and $\kappa_\Phi$ equal to zero. An equation for EMT of the vacuum is the well known equation $\partial_k T_{ik} = F_i^+ + F_i^-.$

2. The characteristics $k_\Phi$ and $\kappa_M$ of an interaction of the vacuum and mediums $M_1$ and $M_2$ are not zero but sufficiently small, and a right equation for EMT of the vacuum is equation (25) with $k_\Phi \neq 0$.

It is clear that, for $k_\Phi \neq 0$, the tensors $\tau_{ik}$ and $T_{ik}$ will give a different picture of distributions of the flux of energy and momentum in the vacuum. And if a measurement will have a sufficient precision to make formula (25) more preferable, then there will appear the quantity $k_\Phi 2c^{-2} \left( F_m^+ V_m V_i + F_m^- V_m V_i \right)$, and together with this, the mysterious velocity $v^{\Phi}$ included in the structure of $V_i^+$ and $V_i^-$ will also be discovered.

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