Neutrinos in the Electron

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We will show that one half of the rest mass of the electron is equal to the sum of the rest masses of electron neutrinos and that the other half of the rest mass of the electron is given by the energy in the sum of electric oscillations. With this composition we can explain the rest mass, the electric charge, the spin and the magnetic moment of the electron.

Introduction

After J.J. Thomson [1] discovered the small corpuscle which soon became known as the electron an enormous amount of theoretical work has been done to explain the existence of the electron. Some of the most distinguished physicists have participated in this effort. Lorentz [2], Poincaré [3], Ehrenfest [4], Einstein [5], Pauli [6], and others showed that it is fairly certain that the electron cannot be explained as a purely electromagnetic particle. In particular it was not clear how the electrical charge could be held together in its small volume because the internal parts of the charge repel each other. Poincaré [7] did not leave it at showing that such an electron could not be stable, but suggested a solution for the problem by introducing what has become known as the Poincaré stresses whose origin however remained unexplained. These studies were concerned with the static properties of the electron, its mass $m(e^\pm)$ and its electric charge $e$. In order to explain the electron with its existing mass and charge it appears to be necessary to add to Maxwell’s equations a non-electromagnetic mass and a non-electromagnetic force which could hold the electric charge together. We shall see what this mass and force is.

The discovery of the spin of the electron by Uhlenbeck and Goudsmit [8] increased the difficulties of the problem in so far as it now had also to be explained how the angular momentum $\hbar/2$ and the magnetic moment $\mu_e$ come about. The spin of a point-like electron seemed to be explained by
Dirac’s [9] equation, however it turned out later [10] that Dirac type equations can be constructed for any value of the spin. Afterwards Schrödinger [11] tried to explain the spin and the magnetic moment of the electron with the so-called Zitterbewegung. Later on many other models of the electron were proposed. On p.74 of his book “The Enigmatic Electron” Mac Gregor [12] lists more than thirty such models. At the end none of these models has been completely successful because the problem developed a seemingly insurmountable difficulty when it was shown through electron-electron scattering experiments that the radius of the electron must be smaller than $10^{-16}$ cm, in other words that the electron appears to be a point particle, at least by three orders of magnitude smaller than the classical electron radius $r_e = e^2/mc^2 = 2.8179 \cdot 10^{-13}$ cm. This, of course, makes it very difficult to explain how a particle can have a finite angular momentum when the radius goes to zero, and how an electric charge can be confined in an infinitesimally small volume. If the elementary electrical charge were contained in a volume with a radius of $O(10^{-16})$ cm the Coulomb self-energy would be orders of magnitude larger than the rest mass of the electron, which is not realistic. The choice is between a massless point charge and a finite size particle with a non-interacting mass to which an elementary electrical charge is attached.

We propose in the following that the non-electromagnetic mass which seems to be necessary in order to explain the mass of the electron consists of neutrinos. This is actually a necessary consequence of our standing wave model [13] of the masses of the mesons and baryons. And we propose that the non-electromagnetic force required to hold the electric charge and the neutrinos in the electron together is the weak nuclear force which, as we have suggested in [13], holds together the masses of the mesons and baryons and also the mass of the muons. Since the range of the weak nuclear force is on the order of $10^{-16}$ cm the neutrinos can only be arranged in a lattice with the weak force extending from each lattice point only to the nearest neighbors. The size of the neutrino lattice in the electron does not at all contradict the results of the scattering experiments, just as the explanation of the mass of the muons with the standing wave model does not contradict the apparent point particle characteristics of the muon, because neutrinos are in a very good approximation non-interacting and therefore are not noticed in scattering experiments with electrons.
1 The mass and charge of the electron

The rest mass of the electron is \( m(e^\pm) = 0.51099892 \pm 4\cdot10^{-8} \text{ MeV}/c^2 \) and the electrostatic charge of the electron is \( e = 4.8032041\cdot10^{-10} \text{ esu} \), as stated in the Review of Particle Physics [14]. Both are known with great accuracy. The objective of a theory of the electron must be the explanation of both values. We will first explain the rest mass of the electron making use of what we have learned from the standing wave model, in particular of what we have learned about the explanation of the mass of the \( \mu^\pm \) mesons in [13]. The muons are leptons, just as the electrons, that means that they interact with other particles exclusively through the electric force. The muons have a mass which is 206.768 times larger than the mass of the electron, but they have the same elementary electric charge as the electron or positron and the same spin. Scattering experiments tell that the \( \mu^\pm \) mesons are point particles with a size \(<10^{-16} \text{ cm}\), just as the electron. In other words, the muons have the same characteristics as the electrons and positrons but for a mass which is about 200 times larger. Consequently the muon is often referred to as a “heavy” electron. If a non-electromagnetic mass is required to explain the mass of the electron then a non-electromagnetic mass 200 times as large as in the electron is required to explain the mass of the muons. These non-electromagnetic masses must be non-interacting, otherwise scattering experiments could not find the size of either the electron or the muon at \( 10^{-16} \text{ cm} \).

We have already explained the mass of the muons with the standing wave model [13]. According to this model the muons consist of an elementary electric charge and a lattice of neutrinos which, as we know, do not interact with charge or mass. Neutrinos are the only non-interacting matter we know of. In the muon lattice are, according to [13], \((N-1)/4 = N'/4\) muon neutrinos \( \nu_\mu \) (respectively anti-muon neutrinos \( \bar{\nu}_\mu \)), \( N'/4 \) electron neutrinos \( \nu_e \) and the same number of anti-electron neutrinos \( \bar{\nu}_e \), one elementary electric charge and the energy of the lattice oscillations. The letter \( N \) stands for the number of all neutrinos and antineutrinos in the cubic lattice of the \( \pi^\pm \) mesons [13, Eq.(15)]

\[
N = 2.854 \cdot 10^9.
\] (1)

It is, according to [13], a necessary consequence of the decay of the \( \mu^- \) muon \( \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \) that there must be \( N'/4 \) electron neutrinos \( \nu_e \) in the emitted electron, where \( N' = N - 1 \approx N \) [13]. For the mass of the electron neutrinos and anti-electron neutrinos we found in Eq.(34) of [13] that
\( m(\nu_e) = m(\bar{\nu}_e) = 0.365 \text{ milli eV/c}\). 

The sum of the energies in the rest masses of the \( N'/4 \) neutrinos or antineutrinos in the lattice of the electron or positron is then

\[
\sum m(\nu_e)c^2 = N'/4 \cdot m(\nu_e)c^2 = 0.260 43 \text{ MeV} = 0.5096 m(e^\pm)c^2. \tag{3}
\]

To put this in other words, one half of the rest mass of the electron comes from the rest masses of electron neutrinos. The other half of the rest mass of the electron must originate from the energy in the electric charge carried by the electron. From pair production \( \gamma + M \to e^- + e^+ + M \), (M being any nucleus), and from conservation of neutrino numbers follows necessarily that there must also be a neutrino lattice composed of \( N'/4 \) anti-electron neutrinos, which make up the lattice of the positrons, which lattice has, because of Eq.(2), the same rest mass as the neutrino lattice of the electron, as it must be for the antiparticle of the electron.

Fourier analysis dictates that a continuum of high frequencies must be in the electrons or positrons created by pair production in a timespan of \( 10^{-23} \) seconds. We will now determine the energy \( E_\nu(e^\pm) \) contained in the oscillations in the interior of the electron. Since we want to explain the rest mass of the electron we can only consider the frequencies of non-progressive waves, either standing waves or circular waves. The sum of the energies of the lattice oscillations is, in the case of the \( \pi^\pm \) mesons, given by

\[
E_\nu(\pi^\pm) = \frac{\hbar \nu_0 N}{2\pi(e^{\hbar \nu_0/kT} - 1)} \int_{-\pi}^{\pi} \phi d\phi. \tag{4}
\]

This is Eq.(14) combined with Eq.(16) in [13] where they were used to determine the oscillation energy in the \( \pi^0 \) and \( \pi^\pm \) mesons. This equation was introduced by Born and v. Karman [15] in order to explain the internal energy of cubic crystals. In Eq.(4) \( h \) is Planck’s constant, \( \nu_0 = c/2\pi a \) is the reference frequency with the lattice constant \( a = 10^{-16} \) cm, \( N \) is the number of all oscillations, \( \phi = 2\pi a/\lambda \) and \( T \) is the temperature in the lattice, for which we found in [13] the value \( T = 2.38 \cdot 10^{14} \) K. If we apply Eq.(4) to the oscillations in the electron which has \( N'/4 \) electron neutrinos \( \nu_e \) we arrive at \( E_\nu(e^\pm) = 1/4 E_\nu(\pi^\pm) \), which is mistaken because \( E_\nu(\pi^\pm) \approx m(\pi^\pm)c^2/2 \) and...
\( m(\pi^\pm) \approx 273 \, m(e^\pm) \). Eq.(4) must be modified in order to be suitable for the oscillations in the electron. It turns out that we must use

\[
E_{\nu}(e^\pm) = \frac{\hbar \nu_0 N \cdot \alpha_f}{2 \pi (e^{\hbar \nu/T} - 1)} \int_{-\pi}^{\pi} \phi \, d\phi ,
\]

(5)

where \( \alpha_f \) is the fine structure constant. The appearance of \( \alpha_f \) in Eq.(5) indicates that the nature of the oscillations in the electron is different from the oscillations in the \( \pi^0 \) or \( \pi^\pm \) lattices. With \( \alpha_f = e^2 / \hbar c \) and \( \nu_0 = c / 2\pi a \) we have

\[
\hbar \nu_0 \alpha_f = e^2 / a
\]

(6)

that means that the oscillations in the electron are \textit{electric oscillations}.

There must be \( N'/2 \) oscillations of the elements of the electric charge in \( e^\pm \), because we deal with non-progressive waves, the superposition of two waves. As we will see later the spin requires that the oscillations are circular. That means that \( 2 \times N'/4 \cong N/2 \) oscillations are in Eq.(5). From Eqs.(4,5) then follows that

\[
E_{\nu}(e^\pm) = \alpha_f / 2 \cdot E_{\nu}(\pi^\pm) .
\]

(7)

\( E_{\nu}(\pi^\pm) \) is the oscillation energy in the \( \pi^\pm \) mesons which can be calculated with Eq.(4). According to Eq.(27) of [13] it is

\[
E_{\nu}(\pi^\pm) = 67.82 \text{ MeV} = 0.486 \, m(\pi^\pm)c^2 \approx m(\pi^\pm)c^2/2 .
\]

(8)

With \( E_{\nu}(\pi^\pm) \approx m(\pi^\pm)c^2/2 = 139.57/2 \text{ MeV} \) and \( \alpha_f = 1/137.036 \) follows from Eq.(7) that

\[
E_{\nu}(e^\pm) = \frac{\alpha_f}{2} \cdot \frac{m(\pi^\pm)c^2}{2} = 0.25462 \text{ MeV} = 0.99657 \, m(e^\pm)c^2/2 .
\]

(9)

We have determined the value of the oscillation energy in \( e^\pm \) from the product of the very accurately known fine structure constant and the very accurately known rest mass of the \( \pi^\pm \) mesons. \textit{One half of the energy in the rest mass of the electron comes from the electric oscillations in the electron.} The other half of the energy in the rest mass of the electron is in the rest masses of the neutrinos in the electron.

We can confirm Eq.(9) using Eq.(5) or Eq.(13) with \( N/2 = 1.427 \cdot 10^9 \), \( e = 4.803 \cdot 10^{-10} \text{ esu} \), \( a = 1 \cdot 10^{-16} \text{ cm} \), \( f(T) = 1/1.305 \cdot 10^{13} \), and with the integral
being \( \pi^2 \) we obtain \( E_\nu(e^\pm) = 0.968 m(e^\pm)c^2/2 \). This calculation involves more parameters than Eq.(9) and is consequently less accurate than Eq.(9).

In a good approximation the oscillation energy of \( e^\pm \) in Eq.(9) is equal to the sum of the energies in the rest masses of the electron neutrinos in the \( e^\pm \) lattice in Eq.(3). Since

\[
m(e^\pm)c^2 = E_\nu(e^\pm) + \sum m(\nu_e)c^2 = E_\nu(e^\pm) + N'/4 \cdot m(\nu_e)c^2,
\]

it follows from Eqs.(3) and (9) that

\[
m(e^\pm)c^2(\text{theor}) = 0.5151 \text{ MeV} = 1.0079 m(e^\pm)c^2(\text{exp}).
\]

The measured rest mass of the electron or positron agrees within the accuracy of the parameters \( N \) and \( m(\nu_e) \) with the theoretically predicted rest masses.

From Eq.(7) follows with \( E_\nu(\pi^\pm) \cong m(\pi^\pm)c^2/2 \) that

\[
2E_\nu(e^\pm) \cong m(e^\pm)c^2 = \alpha_f E_\nu(\pi^\pm) = \alpha_f m(\pi^\pm)c^2/2,
\]

or that

\[
m(e^\pm) \cdot 2/\alpha_f = 274.072 m(e^\pm) \cong m(\pi^\pm),
\]

whereas the actual ratio of the mass of the \( \pi^\pm \) mesons to the mass of the electron is \( m(\pi^\pm)/m(e^\pm) = 273.132 \) or \( 0.9965 \cdot 2/\alpha_f \). We have here recovered the ratio \( m(\pi^\pm)/m(e^\pm) \) which we found with the standing wave model of the \( \pi^\pm \) mesons, Eq.(65) of [13]. This seems to be a necessary condition for the validity of our model of the electron.

We have thus shown that the rest mass of the electron can be explained by the sum of the rest masses of the electron neutrinos in a cubic lattice with \( N'/4 \) electron neutrinos \( \nu_e \) and the mass in the sum of the energy of \( N/2 \) electric oscillations in the lattice, Eq.(9). The one oscillation added to the \( 2 \times N'/4 \) oscillations is the oscillation at the center of the lattice, Fig.(1). From this model follows, since it deals with a cubic neutrino lattice, that the electron is not a point particle, which is unlikely to begin with, because at a true point the self-energy would be infinite. However, since neutrinos are non-interacting their presence will not be detected in electron-electron scattering experiments.

The rest mass of the muon has been explained similarly with an oscillating lattice of muon and electron neutrinos [13]. We found that \( m(\mu^\pm)/m(e^\pm) \) is \( \cong 3/2\alpha_f = 205.55 \), nearly equal to the actual mass ratio 206.768, in agreement
with what Nambu [16] found empirically. The heavy weight of the muon is primarily a consequence of the heavy weight of the N'/4 muon neutrinos in the muon lattice. The mass of the muon neutrino is related to the mass of the electron neutrino through $m(\nu_e) = \alpha_f m(\nu_\mu)$, Eq.(39) of [13].

In order to confirm the validity of our preceding explanation of the mass of the electron we must show that the sum of the charges of the electric oscillations in the interior of the electron is equal to the elementary electric charge of the electron. We recall that Fourier analysis requires that, after pair production, there must be a continuum of frequencies in the electron and positron. With $h\nu_0 \alpha_f = e^2/a$ from Eq.(6) follows from Eq.(5) that the oscillation energy in $e^\pm$ the sum of $2 \times (N'/4 + 1) \cong N/2$ electric oscillations

$$E_{\nu}(e^\pm) = \frac{N}{2} \cdot \frac{e^2}{a} \cdot \frac{f(T)}{2\pi} \int_{-\pi}^{\pi} \phi \, d\phi,$$

with $f(T) = 1/(e^{h\nu/kT} - 1) = 1/1.305 \cdot 10^{13}$ from p.17 in [13]. Inserting the values for N, f(T) and a we find that $E_{\nu}(e^\pm) = 0.968 m(e^\pm)c^2/2$. The discrepancy between $m(e^\pm)c^2/2$ and $E_{\nu}(e^\pm)$ so calculated must originate from the uncertainty of the parameters N, f(T) and a in Eq.(13). We note that it follows from the factor $e^2/a$ in Eq.(13) that the oscillation energy is the same for electrons and positrons, as it must be.
We replace the integral divided by 2π in Eq.(13), which has the value \( \pi/2 \), by the sum \( \sum \phi_k \Delta \phi \), where \( k \) is an integer number with the maximal value \( k_m = (N/4)^{1/3} \). \( \phi_k \) is equal to \( k\pi/k_m \) and we have

\[
\sum \phi_k \Delta \phi = \sum_{k=1}^{k_m} \frac{k\pi}{k_m} \cdot 1 = \frac{k_m(k_m + 1)\pi}{2k_m^2} \approx \frac{\pi}{2},
\]

as it must be. The energy in the individual electric oscillation with index \( k \) is then

\[
\Delta E_{\nu}(k) = \phi_k \Delta \phi = \frac{k\pi}{k_m^2}.
\] (14)

Suppose that the energy of the electric oscillations is correctly described by the self-energy of an electrical charge

\[
U = \frac{1}{2} \cdot \frac{e^2}{r}.
\] (15)

The self-energy of the elementary electrical charge is normally used to determine the mass of the electron from its charge, here we use Eq.(15) the other way around, we determine the charge from the energy in the oscillations.

The charge of the electron is contained in the electric oscillations. That means that the electric charge is not concentrated in a point but is distributed over \( N/4 = O(10^9) \) charge elements \( Q_k \). The charge elements are distributed in a cubic lattice and the resulting electric field is cubic, not spherical. For distances large as compared to the sidelength of the cube, (which is \( O(10^{-13}) \) cm), say at the first Bohr radius which is on the order of \( 10^{-8} \) cm, the deviation of the cubic field from the spherical field will be reduced by about \( 10^{-10} \). The charge in all electric oscillations is

\[
Q = \sum_k Q_k.
\] (16)

Setting the radius \( r \) in the formula for the self-energy equal to \( 2 \, a \) we find, with Eqs.(13,14,15), that the charge in the individual electric oscillations is

\[
Q_k = \pm \sqrt{2\pi N \cdot e^2 f(T)/k_m^2} \cdot \sqrt{k}.
\] (17)

and with \( k_m = 1/2 \cdot (N/4)^{1/3} = 447 \) and

\[
\sum_{k=1}^{k_m} \sqrt{k} = 6310.8
\]
follows, after we have doubled the sum over $\sqrt{k}$, because for each index $k$
there is a second oscillation on the negative axis of $\phi$, that

$$Q = \sum Q_k = \pm 5.027 \cdot 10^{-10} \text{ esu},$$

(18)

whereas the elementary electrical charge is $e = \pm 4.803 \cdot 10^{-10} \text{ esu}$. That
means that our theoretical charge of the electron is 1.047 times the elementary electrical charge. Within the uncertainty of the parameters the theoretical charge of the electron agrees with the experimental charge $e$. We have confirmed that it follows from our explanation of the mass of the electron that the electron has, within a 5% error, the correct electrical charge.

Each element of the charge distribution is surrounded in the horizontal plane by four electron neutrinos as in Fig.(1), and in vertical direction by an electron neutrino above and also below the element. The electron neutrinos hold the charge elements in place. We must assume that the charge elements are bound to the neutrinos by the weak nuclear force. The weak nuclear force plays here a role similar to its role in holding, for example, the $\pi^\pm$ or $\mu^\pm$ lattice together. It is not possible, in the absence of a definitive explanation of the neutrinos, to give a theoretical explanation for the electro-weak interaction between the electric oscillations and the neutrinos. However, the presence of the range $a$ of the weak nuclear force in $e^2/a$ is a sign that the weak force is involved in the electric oscillations. The attraction of the charge elements by the neutrinos overcomes the Coulomb repulsion of the charge elements. The weak nuclear force is the missing non-electromagnetic force or the Poincaré stress which holds the elementary electric charge together. The same considerations apply for the positive electric charge of the positron, only that then the electric oscillations are all of the positive sign and that they are bound to anti-electron neutrinos.

Finally we learn that Eq.(13) precludes the possibility that the charge of the electron sits only on its surface. The number $N$ in Eq.(13) would then be on the order of $10^6$, whereas $N$ must be on the order of $10^9$ so that $E_\nu(e^\pm)$ can be $m(e^\pm)c^2/2$ as is necessary. In other words, the charge of the electron must be distributed throughout the interior of the electron, as we assumed.

Summing up: The rest mass of the electron and positron originates from the sum of the rest masses of $N^'/4$ electron neutrinos or anti-electron neutrinos in cubic lattices plus the mass in the energy of $N'/2$ electric oscillations in the neutrino lattices. That means that neither the electron nor the positron are point particles. The electric oscillations are attached to the neutrinos
by the weak nuclear force. The sum of the charge elements of the electric oscillations accounts for the elementary charge of the electron, respectively positron.

2 The spin and magnetic moment of the electron

The model of the electron we have proposed in the preceding chapter has, in order to be valid, to pass a crucial test; the model has to explain satisfactorily the spin and the magnetic moment of the electron. When Uhlenbeck and Goudsmit [8] (U&G) discovered the existence of the spin of the electron they also proposed that the electron has a magnetic moment with a value equal to Bohr’s magnetic moment \( \mu_B = e\hbar/2m_e c \). Bohr’s magnetic moment results from the motion of an electron on a circular orbit around a proton. The magnetic moment of the electron postulated by U&G has been confirmed experimentally, but has been corrected by about 0.11% for the so-called anomalous magnetic moment. If one tries to explain the magnetic moment of the electron with an electric charge moving on a circular orbit around the particle center, analogous to the magnetic moment of hydrogen, one ends up with velocities larger than the velocity of light, which cannot be, as already noted by U&G. It remains to be explained how the magnetic moment of the electron comes about.

We will have to explain the spin of the electron first. The spin, or the intrinsic angular momentum of a particle is, of course, the sum of the angular momentum vectors of all components of the particle. In the electron these are the neutrinos and the electric oscillations. Each neutrino has spin 1/2 and in order for the electron to have \( s = 1/2 \) all, or all but one, of the spin vectors of the neutrinos in their lattice must cancel. If the neutrinos are in a simple cubic lattice as in Fig.(1) and the center particle of the lattice is not a neutrino, as in Fig.(1), the spin vectors of all neutrinos in the lattice cancel, \( \Sigma \hat{\mathbf{j}}(\mathbf{n}_i) = 0 \), provided that the spin vectors of the electron neutrinos of the lattice point in opposite direction at their mirror points in the lattice. Otherwise the spin vectors of the neutrinos would add up and make a very large angular momentum. We follow here the procedure we used in [17] to explain the spin of the muons. The spin vectors of all electron neutrinos in the electron cancel just as the spin vectors of all muon and electron neutrinos.
in the muons cancel because there is a neutrino vacancy at the center of their lattices, (Fig.(1) of [17]).

We will now see whether the electric oscillations in the electron contribute to its angular momentum. As we said in context with Eq.(7) there must be two times as many electric oscillations in the electron lattice than there are neutrinos. The oscillation pairs can either be the two oscillations in a standing wave or they can be two circular oscillations. Both the standing waves and the circular oscillations are non-progressive and can be part of the rest mass of a particle. We will now assume that the electric oscillations are circular. Circular oscillations have an angular momentum \( \vec{j} = m \vec{r} \times \vec{v} \).

And, as in the case of the spin vectors of the neutrinos, all or all but one of the O(10^9) angular momentum vectors of the electric oscillations must cancel in order for the electron to have spin 1/2. As in [13] we will describe the superposition of the two circular oscillations by

\[
\begin{align*}
  x(t) &= \exp[i\omega t] + \exp[-i(\omega t + \pi)], \\
  y(t) &= \exp[i(\omega t + \pi/2)] + \exp[-i(\omega t + 3\pi/2)],
\end{align*}
\]

that means by the superposition of a circular oscillation with the frequency \( \omega \) and a second circular oscillation with the frequency \( -\omega \). The latter oscillation is shifted in phase by \( \pi \). Negative frequencies are permitted solutions of the equations of motion in a cubic lattice, Eqs.(7,13) of [13]. As is well-known oscillating electric charges should emit radiation. However, this rule does already not hold in the hydrogen atom, so we will assume that the rule does not hold in the electron either.

In circular oscillations the kinetic energy is always equal to the potential energy and the sum of both is the total energy. From

\[
E_{pot} + E_{kin} = 2E_{kin} = E_{tot}
\]

follows with \( E_{kin} = \Theta \omega^2/2 \) and \( E_{tot} = \hbar \omega \) that \( 2E_{kin} = \Theta \omega^2 = \hbar \omega \). \( \Theta \) is the moment of inertia. When we superpose the two circular oscillations with \( \omega \) and \( -\omega \) of Eqs.(19,20) we have

\[
2 \times 2E_{kin} = 2 \Theta \omega^2 = \hbar \omega
\]

from which follows that the angular momentum is

\[
j = \Theta \omega = \hbar/2.
\]
That means that each of the $O(10^9)$ pairs of superposed circular oscillations has an angular momentum $\hbar/2$.

The circulation of the oscillation pairs in Eqs.(19,20) is opposite for all $\omega$ of opposite sign. It follows from the equation for the displacements $u_n$ of the lattice points

$$u_n = Ae^{i(\omega t + n\phi)},$$

(Eq.(5) in [13]) that the velocities of the lattice points are given by

$$v_n = \dot{u}_n = i\omega_n u_n.$$  

The sign of $\omega_n$ changes with the sign of $\phi$ because the frequencies are given by Eq.(13) of [13], that means by

$$\omega_n = \pm \omega_0 [\phi_n + \phi_0].$$

Consequently the circulation of the electric oscillations is opposite to the circulation at the mirror points in the lattice and the angular momentum vectors cancel, but for the angular momentum vector of the electric oscillation at the center of the lattice. The center circular oscillation has, as all other electric oscillations, the angular momentum $\hbar/2$ as Eq.(23) says. The angular momentum of the entire electron lattice is therefore

$$j(e^\pm) = \sum j(n_i) + \sum j(el_i) = j(el_0) = \hbar/2,$$

as it must be for spin $s = 1/2$. The explanation of the spin of the electron given here follows the explanation of the spin of the baryons in [13], as well as the explanation of the absence of spin in the mesons. A valid explanation of the spin must be applicable to all particles, in particular to the electron, the prototype of a particle with spin.

We will now turn to the magnetic moment of the electron which is known with extraordinary accuracy, $\mu(e^\pm) = 1.001159652187 \mu_B$, according to the Review of Particle Physics [14], with $\mu_B$ being the Bohr magneton. The decimals after 1.00 $\mu_B$ are caused by the anomalous magnetic moment which we will not consider. As is well-known the magnetic dipole moment of a particle with spin is, in Gaussian units, given by

$$\vec{\mu} = g \frac{e\hbar}{2mc} \vec{s},$$

where $g$ is the dimensionless Landé factor, $m$ the rest mass of the particle and $\vec{s}$ the spin vector. The $g$-factor has been introduced in order to bring the
magnetic moment of the electron into agreement with the experimental facts. As U&G postulated and as has been confirmed experimentally the g-factor of the electron is 2. With the spin \( s = \frac{1}{2} \) of the electron the magnetic dipole moment of the electron is then

\[
\mu(e^\pm) = \frac{e\hbar}{2m(e^\pm)c},
\]

(29)
or one Bohr magneton in agreement with the experiments, neglecting the anomalous moment. For a structureless point particle Dirac [9] has explained why \( g = 2 \) for the electron. However we consider here an electron with a finite size and which is at rest, which means that the velocity of the center of mass is zero. When it is at rest the electron has still its magnetic moment. Dirac’s theory does therefore not apply here.

The only part of Eq.(28) that can be changed in order to explain the g-factor of an electron with structure is the ratio \( e/m \) which deals with the spatial distribution of charge and mass. In the classical electron models the mass originates from the charge. However that is not necessarily always so. If part of the mass of the electron is non-electrodynamic and the non-electrodynamic part of the mass does not contribute to the magnetic moment of the electron, which to all that we know is true for neutrinos, then the ratio \( e/m \) in Eq.(28) is not \( e/m(e^\pm) \) in the case of the electron. The elementary charge \( e \) certainly remains unchanged, but \( e/m \) depends on what fraction of the mass is of electrodynamic origin and what fraction of \( m(e^\pm) \) is non-electrodynamic, just as the mass of a current loop does not contribute to the magnetic moment of the loop. From the very accurately known values of \( \alpha_f, m(\pi^\pm)c^2 \) and \( m(e^\pm)c^2 \) and from Eq.(9) for the energy in the electric oscillations in the electron \( E_\nu(e^\pm) = \alpha_f/2 \cdot m(\pi^\pm)c^2/2 = 0.996570 m(e^\pm)c^2/2 \) follows that very nearly one half of the mass of the electron is of electric origin, whereas the other half of \( m(e^\pm) \) is made of neutrinos which do not contribute to the magnetic moment. That means that in the electron the mass in \( e/m \) is practically \( m(e^\pm)/2 \). The magnetic moment of the electron is then

\[
\vec{\mu}_e = g \frac{e\hbar}{2m(e^\pm)/2 \cdot c} \vec{s},
\]

(30)
and with \( s = \frac{1}{2} \) we have \( \mu(e^\pm) = g e\hbar/2m(e^\pm)c \). Because of Eq.(29) the g-factor must be equal to one and is unnecessary. In other words, if the electron is composed of the neutrino lattice and the electric oscillations as we have suggested, then the electron has the correct magnetic moment \( \mu_e = e\hbar/2m(e^\pm)c \), if exactly 1/2 of the electron mass consists of neutrinos.
The preceding explanation of the magnetic moment of the electron has to pass a critical test, namely it has to be shown that the same considerations lead to a correct explanation of the magnetic moment of the muon \( \mu = \frac{e\hbar}{2m(\mu^\pm)c} \), which is about 1/200th of the magnetic moment of the electron but is known with nearly the same accuracy as \( \mu_e \). Both magnetic moments are related through the equation

\[
\frac{\mu_\mu}{\mu_e} = \frac{m(e^\pm)}{m(\mu^\pm)} = \frac{1}{206.768},
\]

as follows from Eq.(28) applied to the electron and muon. This equation agrees with the experimental results to the sixth decimal. The muon has, as the electron, an anomalous magnetic moment of about 0.11% \( \mu_\mu \), which is too small to be considered here.

In the standing wave model [13] the muons consist of a lattice of \( N'/4 \) muon neutrinos \( \nu_\mu \), respectively anti-muon neutrinos \( \bar{\nu}_\mu \), of \( N'/4 \) electron neutrinos and the same number of anti-electron neutrinos plus an elementary electric charge. For the explanation of the magnetic moment of the muon we follow the same reasoning we have used for the explanation of the magnetic moment of the electron. We say that \( m(\mu^\pm) \) consists of two parts, one part which causes the magnetic moment and another part which does not contribute to the magnetic moment. The part of \( m(\mu^\pm) \) which causes the magnetic moment must contain circular electric oscillations without which there would be no magnetic moment. It becomes immediately clear from the small mass of the electron neutrinos and from Eq.(5) for the energy of the electric oscillations in the electron that \( \Sigma m(\nu_e) \) and \( E(\nu_e(e^\pm)) \) are too small, as compared to the energy in the rest masses of all neutrinos in the muons, to make up \( m(\mu^\pm)/2 \). However, the oscillations in the \( \mu^\pm \) mesons do not follow Eq.(5) for the oscillation energy in the electron, but rather Eq.(4) for the oscillation energy in the muons. Both differ by the factor \( \alpha_f \) in Eq.(5). But even when the oscillation energy in the muons as given by Eq.(4) is considered, the energy of the electric oscillations in the muons would be only \( E(\nu(\mu^\pm))/4 = 16.955 \text{ MeV}, \) if the electric oscillations are attached to \( N/4 \) electron neutrinos, as is the case in the electron.

It appears to be necessary to consider the case that the electric oscillations in the \( \mu^\pm \) mesons are attached to all neutrinos of the electron neutrino type in the \( \mu^\pm \) lattice, regardless whether they are electron neutrinos or anti-electron neutrinos. That would mean that the electric charge is distributed
uniformly in the $\mu^\pm$ lattice. There are, as has been shown in the paragraph below Eq.(31) of [13], $3/4N$ neutrinos of the electron neutrino type in the muons, of which $N/4$ neutrinos originate from the charge $e^\pm$ carried by $\mu^\pm$. If the electric oscillations are attached to $3/4N$ electron neutrinos, regardless of their type, then the energy in all electric oscilllations or the energy in the electric charge is, with Eq.(8) and $E_\nu(\mu^\pm) = E_\nu(\pi^\pm) = 67.82$ MeV from Eq.(31) in [13], as well as with $m(\mu^\pm)c^2 = 105.6583$ MeV, given by

$$3/4 \cdot E_\nu(\mu^\pm) = 3/4 \cdot 67.82 \text{ MeV} = 50.865 \text{ MeV}$$

In other words, the energy in the electric oscillations or the electric charge makes up, in a good approximation, $1/2$ of the mass of the muons. The other half of the rest mass of the muons consists of the sum of the rest masses of the neutrinos in the muon lattice plus the oscillation energy of the muon neutrinos, neither of which contributes to the magnetic moment. It is

$$1/4E_\nu(\mu^\pm) + N/4m(\nu_\mu)c^2 + 3/4Nm(\nu_e)c^2 = 53.347 \text{ MeV} = 0.50490 m(\mu^\pm)c^2.$$  \hspace{1cm} (33)

The theoretical total energy in the rest mass of the muons is then $E(m(\mu^\pm)) = 0.9863 m(\mu^\pm)c^2(\text{exp})$.

In simple terms, if $E_\nu(\pi^\pm) = E_\nu(\mu^\pm) = 1/2m(\pi^\pm)$, not $0.486 m(\pi^\pm)$ as in Eq.(27) of [13], then it follows from $3/4E_\nu(\mu^\pm) = 3/8m(\pi^\pm)$ and from the neutral part of the muon mass in Eq.(33), which is likewise $\approx3/8m(\pi^\pm)$, that the rest mass of the muons is $m(\mu^\pm) \approx 3/8m(\pi^\pm) + 3/8m(\pi^\pm) = 3/4\cdot m(\pi^\pm)$, as it must be in a first approximation, whereas the actual $m(\mu^\pm)$ is $1.00937\cdot3/4\cdot m(\pi^\pm)$. That means that in a good approximation the charged part of the rest mass of the muons is $1/2$ of the mass of the muons.

If the charged part of the muon mass as expressed by Eq.(32) makes up $1/2$ of the mass of the muons and if the other part of the muon mass does not contribute to the magnetic moment, then the magnetic moment of the muon is given by

$$\bar{\mu}_\mu = \frac{e\hbar}{2m(\mu^\pm)/2 \cdot c} \cdot \vec{s}.$$  \hspace{1cm} (34)

With $s = 1/2$ we have $\mu_\mu = e\hbar/2m(\mu^\pm)c$ as it must be, without the artificial g-factor.
Conclusions

One hundred years of sophisticated theoretical work have made it abundantly clear that the electron is not a purely electromagnetic particle. There must be something else in the electron but electric charge. It is equally clear from the most advanced scattering experiments that the “something else” in the electron must be non-interacting, otherwise it could not be that we find that the radius of the electron must be smaller than $10^{-16}$ cm. The only non-interacting matter we know of with certainty are the neutrinos. So it seems to be natural to ask whether neutrinos are not part of the electron. Actually we have not introduced the neutrinos in an axiomatic manner but rather as a consequence of our standing wave model of the stable mesons, baryons and $\mu$-mesons. It follows necessarily from this model that after the decay of the $\mu^-$ meson there must be electron neutrinos in the emitted electron, and that they make up one half of the mass of the electron. The other half of the energy in the electron originates from the energy of electric oscillations. The theoretical rest mass of the electron agrees, within 1% accuracy, with the experimental value of $m(e^\pm)$. We have learned that the charge of the electron is not concentrated in a single point, but rather is distributed over $O(10^9)$ elements which are held together with the neutrinos by the weak nuclear force. The sum of the charges in the electric oscillations is, within the accuracy of the parameters, equal to the elementary electrical charge of the electron. From the explanation of the mass and charge of the electron follows, as it must be, the correct spin and magnetic moment of the electron, the other two fundamental features of the electron. With a cubic lattice of anti-electron neutrinos we also arrive with the same considerations as above at the correct mass, charge, spin and magnetic moment of the positron.

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