Short-Baseline Active-Sterile Neutrino Oscillations?

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Abstract

We suggest the possibility that the anomalies observed in the LSND experiment and the Gallium radioactive source experiments may be due to neutrino oscillations generated by a large squared-mass difference of about $20 - 30 \text{eV}^2$. We consider the simplest 3+1 four-neutrino scheme that can accommodate also the observed solar and atmospheric neutrino oscillations. We show that, in this framework, the disappearance of $\bar{\nu}_e$ and $\bar{\nu}_\mu$ in short-baseline neutrino oscillation experiments is mainly due to active-sterile transitions. The implications of the first MiniBooNE results, appeared after the completion of this paper, are discussed in an addendum.

Neutrino oscillation experiments have shown that neutrinos are massive and mixed particles (see the reviews in Refs. [1, 2, 3, 4, 5, 6, 7, 8]). The observation of $\nu_e \rightarrow \nu_{\mu,\tau}$ oscillations with a squared-mass difference

$$\Delta m^2_{\text{SOL}} \simeq 8 \times 10^{-5} \text{eV}^2$$  \hspace{1cm} (1)

in solar and reactor neutrino experiments and the observation of $\nu_\mu \rightarrow \nu_\tau$ oscillations with a squared-mass difference

$$\Delta m^2_{\text{ATM}} \simeq 2.5 \times 10^{-3} \text{eV}^2$$  \hspace{1cm} (2)

in atmospheric and accelerator neutrino experiments can be accommodated in the minimal framework of three-neutrino mixing, in which the three active flavor neutrinos $\nu_e, \nu_\mu$, and $\nu_\tau$ are superpositions of three massive neutrinos $\nu_1, \nu_2, \text{and } \nu_3$. This three-neutrino mixing framework cannot explain through neutrino oscillations the LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ signal [9, 10, 11, 12], which requires a squared-mass difference

$$\Delta m^2_{\text{LSND}} \gtrsim 10^{-1} \text{eV}^2.$$ \hspace{1cm} (3)
Another anomaly observed in neutrino experiments is the disappearance of $\nu_e$’s in the Gallium radioactive source experiments GALLEX [13, 14] and SAGE [15, 16, 17]. These experiments are tests of solar neutrino detectors in which intense artificial $^{51}$Cr and $^{37}$Ar neutrino sources were placed near or inside the detectors. Both $^{51}$Cr and $^{37}$Ar decay through electron capture ($e^- + ^{51}$Cr $\rightarrow ^{51}$V $+ \nu_e$ and $e^- + ^{37}$Ar $\rightarrow ^{37}$Cl $+ \nu_e$). The energies of the emitted neutrinos are, respectively, $E = 752.73 \pm 0.24$ keV and $E = 813.5 \pm 0.3$ keV [18]. The neutrinos emitted by the artificial sources were detected through the same reaction used for the detection of solar neutrinos [19]:

$$\nu_e + ^{71}$Ga $\rightarrow ^{71}$Ge $+ e^-$, \hspace{1cm} (4)$$

which has the low neutrino energy threshold $E_{th} = 0.233$ MeV. The weighted average value of the ratio $R$ of measured and predicted $^{71}$Ge production rates is [17]

$$R = 0.88 \pm 0.05. \hspace{1cm} (5)$$

In Ref. [17] it has been suggested that this anomaly may be due to an overestimate of the theoretical cross section of the Gallium detection process in Eq. (4). However, a Gallium cross section rescaled by the factor in Eq. (5) leads to a significant deterioration of the fit of solar neutrino data [20].

In this paper we consider the possibility that the anomaly observed in Gallium radioactive source experiments is due to neutrino oscillations$^1$.

Since the neutrino path in the Gallium radioactive source experiments was of the order of 10 cm, an explanation of the observed disappearance of $\nu_e$’s through neutrino oscillations requires a large squared-mass difference $\Delta m^2_{Ga}$. In fact, requiring an oscillation length $L_{osc}^{Ga} = 4\pi E/|\Delta m^2_{Ga}|$ smaller than about 10 cm, we obtain

$$\Delta m^2_{Ga} \gtrsim 20 \text{eV}^2. \hspace{1cm} (6)$$

Assuming CPT invariance, the survival probability of neutrinos and antineutrinos are equal. It follows that the disappearance of electron neutrinos at the level indicated by Gallium radioactive source experiments appears to be in contradiction with the results of reactor neutrino oscillation experiments (see the review in Ref. [22]), which did not observe any disappearance of electron antineutrinos with an average energy of about 4 MeV at distances between about 10 and 100 m from the reactor source. Let us notice, however, that the oscillation length of reactor neutrinos implied by Eq. (6) is much shorter than 10 m:

$$L_{osc}^{reactors} \lesssim 40 \text{cm}. \hspace{1cm} (7)$$

Hence, in reactor neutrino experiments the oscillations due to $\Delta m^2_{Ga}$ are seen as an energy-independent suppression of the electron antineutrino flux by the factor in Eq. (5). A measurement of such a suppression requires a precise calculation of the absolute electron antineutrino flux produced in a reactor$^2$. Since this calculation is rather difficult, it

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1 The results of the first GALLEX artificial $^{51}$Cr source experiment [13] has been used in Ref. [21] in order to constrain the neutrino mixing parameters.

2 Information on $\bar{\nu}_e$ disappearance which is independent of the absolute flux calculation can be obtained through the measurement of the energy spectrum (assuming it to be known with small uncertainties) or the comparison between rates measured with different source-detector distances. In these cases, reactor neutrino experiments are not sensitive to oscillations generated by a squared-mass difference $\Delta m^2 \gtrsim 2 \text{eV}^2$, as one can see, for example, from Fig. 13a of Ref. [22].
is possible that its systematic uncertainties have been underestimated. Therefore, a $\bar{\nu}_e$ disappearance in reactor neutrino oscillation experiments at the level indicated by Eq. (5) with the oscillation length in Eq. (7) is not excluded with absolute certainty.

In this paper, we consider the possibility that both the LSND and Gallium anomalies are due to neutrino oscillations, through the same large squared-mass difference

$$\Delta m^2_{\text{LSND+Ga}} \gtrsim 20 \text{eV}^2.$$  \hspace{1cm} (8)

We consider, for simplicity, a four-neutrino mixing scheme, in which the three active flavor neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$, and one sterile neutrino $\nu_s$ are superpositions of four massive neutrinos $\nu_1$, $\nu_2$, $\nu_3$, and $\nu_4$. This is the simplest scheme in which there are three independent squared-mass differences which can accommodate the hierarchy

$$\Delta m^2_{\text{SOL}} \ll \Delta m^2_{\text{ATM}} \ll \Delta m^2_{\text{LSND+Ga}}.$$ \hspace{1cm} (9)

Four-neutrino mixing have already been considered in many papers as the explanation of the LSND anomaly (see the reviews in Refs. [3, 4, 6, 8]). Here, we further constrain the allowed values of the large squared-mass difference and the mixing of the electron neutrino by requiring that $\Delta m^2_{\text{LSND+Ga}}$ is responsible of both the LSND and Gallium anomalies.

Since the so-called 2+2 schemes are disfavored by the data [24, 25, 26, 27, 28, 29, 30, 6], we consider a 3+1 scheme, in which there is a group of three neutrino masses which is separated from an isolated neutrino mass by the LSND + Ga mass splitting. In this case, we have

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{31}, \quad \Delta m^2_{\text{ATM}} = |\Delta m^2_{41}| \simeq |\Delta m^2_{32}|,$$

$$\Delta m^2_{\text{LSND+Ga}} = |\Delta m^2_{41}| \simeq |\Delta m^2_{42}| \simeq |\Delta m^2_{43}|,$$ \hspace{1cm} (10, 11)

where $\Delta m^2_{kj} \equiv m_k^2 - m_j^2$. Furthermore, we take into account the upper limit

$$m_\beta < 2.3 \text{eV} \quad (95\% \text{CL}),$$ \hspace{1cm} (12)

obtained in the Mainz [31] and Troitzk [32] tritium $\beta$-decay experiments on the effective electron neutrino mass [33, 34, 35]

$$m_\beta^2 = \sum_{k=1}^4 |U_{ek}|^2 m_k^2.$$ \hspace{1cm} (13)

Since the three active flavor neutrinos must have large mixings with $\nu_1$, $\nu_2$, and $\nu_3$ in order to accommodate the observed oscillations due to $\Delta m^2_{\text{SOL}}$ and $\Delta m^2_{\text{ATM}}$, the only scheme allowed is the one in which $\nu_1$, $\nu_2$, and $\nu_3$ are light, with masses

$$m_1, m_2, m_3 \lesssim 2.3 \text{eV},$$ \hspace{1cm} (14)

and $\nu_4$ is heavy, with mass

$$m_4 \simeq \sqrt{\Delta m^2_{\text{LSND+Ga}}} \gtrsim 4.5 \text{eV}.$$ \hspace{1cm} (15)

In four-neutrino schemes, the average $\nu_e$ survival probability in the Gallium experiments is given by

$$\langle P_{\nu_e \rightarrow \nu_e} \rangle = 1 - \frac{1}{2} \sin^2 2\vartheta_{\text{Ga}},$$ \hspace{1cm} (16)
where $\vartheta_{\text{Ga}}$ is an effective mixing angle. Interpreting $R$ in Eq. (5) as $\langle P_{\nu_e \rightarrow \nu_e} \rangle$, we obtain

$$\sin^2 2\vartheta_{\text{Ga}} = 0.24 \pm 0.10. \quad (17)$$

In the 3+1 mixing schemes (see the review in Ref. [3]), the survival and transition probabilities in short-baseline neutrino oscillation experiments have the two-neutrino mixing forms (for $\alpha, \beta = e, \mu, \tau, s$)

$$P_{\nu_{\alpha} \rightarrow \nu_{\alpha}} = P_{\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\alpha}} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2 \left( \frac{\Delta m^2_{41} L}{4E} \right), \quad (18)$$

$$P_{\nu_{\alpha} \rightarrow \nu_{\beta}} = P_{\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\beta}} = P_{\bar{\nu}_{\beta} \rightarrow \bar{\nu}_{\alpha}} = \sin^2 2\vartheta_{\alpha\beta} \sin^2 \left( \frac{\Delta m^2_{41} L}{4E} \right) \quad (\alpha \neq \beta), \quad (19)$$

where $L$ is the source–detector distance and the effective mixing angles are given by

$$\sin^2 2\vartheta_{\alpha\alpha} = 4 |U_{\alpha 4}|^2 \left( 1 - |U_{\alpha 4}|^2 \right), \quad (20)$$

$$\sin^2 2\vartheta_{\alpha\beta} = \sin^2 2\vartheta_{\beta\alpha} = 4 |U_{\alpha 4}|^2 |U_{\beta 4}|^2 \quad (\alpha \neq \beta). \quad (21)$$

Therefore, we have

$$\sin^2 2\vartheta_{\text{Ga}} = \sin^2 2\vartheta_{ee} = 4 |U_{e 4}|^2 \left( 1 - |U_{e 4}|^2 \right). \quad (22)$$

Taking into account that $|U_{e 4}|^2$ is small, in order to accommodate the observed oscillations due to $\Delta m^2_{\text{SOL}}$ and $\Delta m^2_{\text{ATM}}$, we obtain, from Eqs. (17) and (22),

$$|U_{e 4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{\text{Ga}} \simeq 0.06 \pm 0.03. \quad (23)$$

In spite of the relatively heavy mass of $\nu_4$ in Eq. (15), the mixing of $\nu_e$ with $\nu_4$ is not a problem for the bound in Eq. (12) on the effective electron neutrino mass in $\beta$-decay experiments. In fact, the contribution of $\nu_4$ to $m_\beta$ is

$$m_\beta(\nu_4) = |U_{e 4}| m_4 \simeq 1.1 \pm 0.3 \text{eV} \left( m_4 \frac{4.5 \text{eV}}{4.5 \text{eV}} \right). \quad (24)$$

Therefore, the bound in Eq. (12) implies

$$m_4 \lesssim 10 \text{eV}. \quad (25)$$

Taking into account also Eq. (8), we obtain the allowed range

$$20 \text{eV}^2 \lesssim \Delta m^2_{\text{LSND+Ga}} \lesssim 100 \text{eV}^2. \quad (26)$$

Let us now consider the LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ signal, which has been observed with the probability [12]

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} = (2.64 \pm 0.67 \pm 0.45) \times 10^{-3}. \quad (27)$$

Since we are considering large values of $\Delta m^2_{\text{LSND+Ga}}$ in the interval in Eq. (26), the transition probability measured in the LSND experiment is the averaged probability

$$\langle P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} \rangle = \frac{1}{2} \sin^2 2\vartheta_{\text{LSND}}, \quad (28)$$
with the effective mixing angle given by (see Eq. (21))

$$\sin^2 2\vartheta_{\text{LSND}} = \sin^2 2\vartheta_{e\mu} = 4 |U_{e4}|^2 |U_{\mu4}|^2.$$  \hspace{1cm} (29)

Thus, from Eq. (27), we obtain

$$\sin^2 2\vartheta_{\text{LSND}} \simeq (5.3 \pm 1.6) \times 10^{-3}. $$  \hspace{1cm} (30)

Short-baseline ($^{-}\nu_{\mu}$) to ($^{-}\nu_{e}$) oscillations generated by a large squared-mass difference have been recently searched, with negative results, in the CCFR [36], KARMEN [37], NuTeV [38], and NOMAD [39] experiments. From Fig. 8 of Ref. [39], one can see that the range of $\sin^2 2\vartheta_{\text{LSND}}$ in Eq. (30) is compatible with the results of the CCFR, KARMEN, and NuTeV experiments if the allowed interval of $\Delta m^2_{\text{LSND+Ga}}$ in Eq. (26) is restricted to

$$20 \text{ eV}^2 \lesssim \Delta m^2_{\text{LSND+Ga}} \lesssim 30 \text{ eV}^2. $$  \hspace{1cm} (31)

In fact, although a combined analysis of all the relevant neutrino oscillations data yields a poor goodness of fit [41, 42, 43, 44, 29, 30, 6], if the fit is accepted, there is an allowed region in the $\sin^2 2\vartheta_{\text{LSND}}$ range in Eq. (30) and the determination of $|U_{e4}|^2$ in Eq. (23) from the Gallium anomaly allow us to determine the allowed range of $|U_{\mu4}|^2$: from Eq. (29),

$$|U_{\mu4}|^2 = \sin^2 2\vartheta_{\text{LSND}} \simeq 0.02 \pm 0.01. $$  \hspace{1cm} (32)

This small value of $|U_{\mu4}|^2$ implies that the effective mixing angle in short-baseline $\nu_{\mu}$ disappearance experiments is given by

$$\sin^2 2\vartheta_{\mu\mu} \simeq 4 |U_{\mu4}|^2 \simeq 0.08 \pm 0.04. $$  \hspace{1cm} (33)

This value of $\sin^2 2\vartheta_{\mu\mu}$ is compatible with the exclusion curves of the CDHSW [47] and CCFR $\nu_{\mu} \to \nu_{\mu}$ oscillation experiments for $\Delta m^2_{\text{LSND+Ga}}$ in the interval in Eq. (31). It is interesting to notice that the results of the CDHSW $\nu_{\mu}$ disappearance experiment favor a $\Delta m^2_{\text{LSND+Ga}}$ in the range in Eq. (31), as remarked at the end of the appendix of Ref. [40].

Let us now consider the experimental bounds on $\nu_{\mu} \to \nu_{\tau}$ and $\nu_{e} \to \nu_{\tau}$ transitions obtained in short-baseline experiments (CHORUS [49], NOMAD [39] and CCFR [50, 51]). From Fig. 14 of Ref. [52], one can see that, for $\Delta m^2_{\text{LSND+Ga}}$ in the range in Eq. (31), the effective mixing angles

$$\sin^2 2\vartheta_{e\tau} = 4 |U_{e4}|^2 |U_{\tau4}|^2, \quad \sin^2 2\vartheta_{\mu\tau} = 4 |U_{\mu4}|^2 |U_{\tau4}|^2 $$  \hspace{1cm} (34)

\textsuperscript{3} The fit can be improved by introducing a second sterile neutrino [40], in a so-called 3+2 mixing scheme. However, it seems to us that it is highly unlikely that the two large squared-mass differences happen to have just the right values in the small regions which are not excluded by the neutrino oscillation data.
are bounded by
\[ \sin^2 2\vartheta_{e\tau} \lesssim 1 \times 10^{-1}, \quad \sin^2 2\vartheta_{\mu\tau} \lesssim 2 \times 10^{-3}. \] (35)
Taking into account the allowed ranges of \( |U_{e4}|^2 \) and \( |U_{\mu4}|^2 \) in Eqs. (23) and (32), the limit on \( \sin^2 2\vartheta_{e\tau} \) does not give a significant bound, whereas the limit on \( \sin^2 2\vartheta_{\mu\tau} \) yields
\[ |U_{\tau4}|^2 = \frac{\sin^2 2\vartheta_{\mu\tau}}{4 |U_{\mu4}|^2} \simeq \frac{\sin^2 2\vartheta_{\mu\tau} \sin^2 2\vartheta_{Ga}}{4 \sin^2 2\vartheta_{LSND}} \lesssim 0.05. \] (36)
Therefore, also \( |U_{\tau4}|^2 \) is constrained to be small. It follows that
\[ |U_{s4}|^2 = 1 - (|U_{e4}|^2 + |U_{\mu4}|^2 + |U_{\tau4}|^2) \gtrsim 0.8, \] (37)
and the \( \nu_e \) disappearance indicated by Gallium radioactive source experiments is mainly due to \( \nu_e \rightarrow \nu_s \) transitions with an effective mixing angle given by
\[ \sin^2 2\vartheta_{es} = 4 |U_{s4}|^2 |U_{s4}|^2 \simeq 0.2 \pm 0.1. \] (38)
These transitions are compatible with the CCFR bound on \( \nu_e \rightarrow \nu_s \) transitions (Fig. 4 of Ref. [51]) for the effective squared mass difference \( \Delta m^2_{LSND+Ga} \) confined in the range in Eq. (31).
The \( \nu_e \rightarrow \nu_s \) transitions due to \( \Delta m^2_{LSND+Ga} \) affect also solar neutrino experiments. Since the mixing of \( \nu_s \) with \( \nu_1, \nu_2, \) and \( \nu_3 \) is small, in practice solar neutrino experiments should observe an average probability of disappearance of electron neutrinos into sterile neutrinos of the same value as the ratio \( R \) in Eq. (3) measured the Gallium radioactive source experiments:
\[ \langle P_{\nu_e \rightarrow \nu_s} \rangle \simeq \frac{1}{2} \sin^2 2\vartheta_{es} \simeq 0.10 \pm 0.05. \] (39)
It is interesting to notice that a comparison of the SNO Neutral-Current (NC) data with the Standard Solar Model (SSM) prediction is compatible with \( \nu_e \rightarrow \nu_s \) transitions at the level indicated in Eq. (39), although no evidence can be claimed, because of the large theoretical uncertainty of the SSM prediction. In fact, the equivalent flux of \( ^8B \) electron neutrinos measured in SNO through the NC reaction \( \nu + d \rightarrow p + n + \nu \), which is equally sensitive to \( \nu_e, \nu_\mu, \) and \( \nu_\tau \), is [53]
\[ \Phi^{SNO}_{NC} = (4.94 \pm 0.21^{+0.38}_{-0.34}) \times 10^6 \text{ cm}^{-2} \text{s}^{-1}. \] (40)
This value can be compared with the BS05(GS98) [51] and TC04 [53] SSM values
\[ \Phi^{BS05}_S = (5.69 \pm 0.98) \times 10^6 \text{ cm}^{-2} \text{s}^{-1}, \] (41)
\[ \Phi^{TC04}_S = (5.31 \pm 0.6) \times 10^6 \text{ cm}^{-2} \text{s}^{-1}, \] (42)
leading to
\[ \langle P_{\nu_e \rightarrow \nu_s} \rangle_{SNO+BS05} = 1 - \frac{\Phi^{SNO}_{NC}}{\Phi^{BS05}_S} = 0.13^{+0.15}_{-0.17}, \] (43)
\footnote{The flux in Eq. (40) has been measured in the phase II of the SNO experiment (also called “salt phase”), in which about 2 tons of NaCl have been added to the heavy water in order to improve the efficiency and precision of the NC measurement [53].}
\[
\langle P_{\nu e \rightarrow \nu s}\rangle_{\text{SNO+TC04}} = 1 - \frac{\Phi_{\text{SNO}}^{\text{NC}}}{\Phi_{\text{TC04}}^{\text{B}}} = 0.07 \pm 0.13 .
\] (44)

One can see that, although the uncertainties are large, the tendency of the ratios in Eqs. (43) and (44) is towards an agreement with the average probability of \(\nu_e \rightarrow \nu_s\) transitions in Eq. (39).

The disappearance of \(\nu_e\) due to \(\nu_e \rightarrow \nu_s\) transitions could affect the search for \(\nu_{\mu} \rightarrow \nu_e\) transitions in the MiniBooNE\(^5\) experiment [56, 57, 58], which has been designed to check the LSND anomaly. This is due to the fact that the MiniBooNE \(\nu_{\mu}\) beam has a natural \(\nu_e\) contamination of about \(5 \times 10^{-3}\). Since the MiniBooNE detector is located at a distance of 541 m from the target and the energy spectrum of the \(\nu_{\mu}\) beam ranges from about 0.2 GeV to about 3 GeV, with a peak at about 0.6 GeV, it is convenient to write the oscillation length due to \(\Delta m^{2}_{\text{LSND+Ga}}\) as

\[
L_{\text{osc}} \simeq 120 \text{ m} \left( \frac{E}{\text{GeV}} \right) \left( \frac{\Delta m^{2}_{\text{LSND+Ga}}}{20 \text{ eV}^2} \right)^{-1} .
\] (45)

Hence, a \(\Delta m^{2}_{\text{LSND+Ga}}\) in the range in Eq. (31) implies that the oscillation length is much shorter than the MiniBooNE source-detector distance and the flavor transitions are practically constant over the energy spectrum. The effect on the \(\nu_{e}\) spectrum at the detector is the superposition of two opposite and competitive contributions: a \(\nu_{e}\) disappearance due to \(\nu_{e} \rightarrow \nu_s\) oscillations with a relatively large mixing (see Eq. (38)) and a \(\nu_{e}\) appearance due to \(\nu_{\mu} \rightarrow \nu_e\) with a relative small mixing (see Eq. (30)). Since the natural contamination of \(\nu_{e}\) in the \(\nu_{\mu}\) beam is at the percent level, the two opposite effects on the \(\nu_{e}\) spectrum are competitive.

The hypothesis of \(\nu_{\mu} \rightarrow \nu_e\) transitions driven by \(\Delta m^{2}_{\text{LSND+Ga}}\) may soon be tested at the T2K beam line (starting from 2009) with the near off-axis detector located at a distance of 280 m from the target. The neutrino energy in T2K is about the same as in MiniBooNE. With a systematic error on the electron neutrino flux \(\sigma(\text{syst}) \sim 5\%\) the 90\% C.L. sensitivity to \(\sin^2 2\theta_{\mu e}\) is about \(3 \times 10^{-3}\).

The scenario under consideration implies also short-baseline \(\nu_{\mu} \rightarrow \nu_s\) oscillations generated by \(\Delta m^{2}_{\text{LSND+Ga}}\) with the effective mixing angle

\[
\sin^2 2\theta_{\mu s} = 4 |U_{\mu 4}|^2 |U_{s 4}|^2 \simeq 0.08 \pm 0.04 .
\] (46)

Since this value of \(\sin^2 2\theta_{\mu s}\) practically coincides with the value of \(\sin^2 2\theta_{\mu \mu}\) in Eq. (33) and is much larger than the values of \(\sin^2 2\theta_{\mu e} = \sin^2 2\theta_{\text{SND}}\) in Eq. (30) and \(\sin^2 2\theta_{\mu \tau}\) in Eq. (35), the \(\nu_{\mu} \rightarrow \nu_s\) channel is the dominant cause of short-baseline \((\bar{\nu})_{\mu}\) disappearance.

Optimal future experiments which could observe the large disappearance of \((\nu)_{e}\) and \((\bar{\nu})_{\mu}\) due to active–sterile transitions and the \((\bar{\nu})_{\mu} \rightleftharpoons (\nu)_{e}\) transitions due to \(\Delta m^{2}_{\text{LSND+Ga}}\) are: Beta-Beam experiments [59] with a pure \(\nu_{e}\) beam from nuclear decay (see the reviews in Refs. [60, 61]); Neutrino Factory experiments with a beam composed of \(\nu_e\) and \(\bar{\nu}_{\mu}\), from \(\mu^+\) decay, or \(\bar{\nu}_{\tau}\) and \(\nu_{\mu}\), from \(\mu^-\) decay (see the review in Ref. [62, 60]); experiments with a \(\bar{\nu}_e\) beam produced in recoless nuclear decay and detected in recoless nuclear antineutrino capture [63].

\(^5\) The implications of the first MiniBooNE results, appeared after the completion of this paper, are discussed in the addendum at page 8.
In conclusion, in this paper we have suggested the possibility that the anomalies observed in the Gallium radioactive source experiments and the LSND experiment may be due to neutrino oscillations generated by the same large squared-mass difference $\Delta m^2_{LSND+Ga}$. We have shown that, in the framework of the simplest 3+1 four-neutrino scheme that can accommodate also the $(\nu_e \rightarrow \nu_{\mu,\tau})$ oscillations observed in solar and reactor experiments and the $(\nu_\mu \rightarrow \nu_e)$ oscillations observed in atmospheric and accelerator experiments, the short-baseline disappearances of $\nu_e$ and $\nu_\mu$ are due mainly to $\nu_e \rightarrow \nu_s$ and $\nu_\mu \rightarrow \nu_s$ transitions, respectively. We have noticed that in the MiniBooNE experiment flavor transitions are effectively energy-independent and the disappearance of $\nu_e$ due to $\nu_e \rightarrow \nu_s$ transitions could affect the search for $\nu_\mu \rightarrow \nu_e$ transitions, because of the natural $\nu_e$ contamination of the beam. Finally, we have remarked that the scenario under consideration could be tested in future experiments with pure $\nu_e$ and $\nu_\mu$ beams, as Beta-Beam and Neutrino Factory experiments.

**Addendum: First MiniBooNE Results**

After the completion of this paper, the MiniBooNE collaboration released their first results concerning the search for $\nu_\mu \rightarrow \nu_e$ transitions generated by $\Delta m^2_{LSND}$ \[64\]. Since no significant excess of quasi-elastic charged-current $\nu_e$ events was observed above the calculated background for reconstructed neutrino energy $E_{QE} > 475$ MeV, the two-neutrino $\nu_\mu \rightarrow \nu_e$ transitions generated by $\Delta m^2_{LSND}$ are disfavored by the MiniBooNE data at 98% C.L. \[64\].

In the framework of the 3+1 four-neutrino scheme considered in this paper, the absence of a signal due to $\nu_\mu \rightarrow \nu_e$ appearance may be, at least partially, explained by a suppression of the background due to $\nu_e \rightarrow \nu_s$ and $\nu_\mu \rightarrow \nu_s$ transitions, as remarked after Eq. (45). In fact, the estimated number of $\nu_e$ events is

$$N_{\nu_e} = P_{\nu_e \rightarrow \nu_e} N^B_{\nu_e} + P_{\nu_\mu \rightarrow \nu_e} N^B_{\nu_\mu} + P_{\nu_\mu \rightarrow \nu_\mu} N_{\nu_\mu},$$

(47)

where $N^B_{\nu_e}$ and $N^B_{\nu_\mu}$ are, respectively, the estimated numbers of $\nu_e$-induced and $\nu_\mu$-induced background events, and $N_{\nu_\mu}$ is the estimated number of $N_{\nu_\mu}$ in the case of full $\nu_\mu \rightarrow \nu_e$ transmutation. In short-baseline experiments $P_{\nu_\mu \rightarrow \nu_e} \simeq 1 - P_{\nu_e \rightarrow \nu_s}$, as remarked after Eq. (37), and $P_{\nu_\mu \rightarrow \nu_\mu} \simeq 1 - P_{\nu_e \rightarrow \nu_s}$, as remarked after Eq. (46). Moreover, the oscillation probabilities are practically constant in the MiniBooNE energy spectrum, as explained after Eq. (45).

From Table I of Ref. \[64\], adding the uncertainties in quadrature, we obtain

$$N^B_{\nu_e} = 229 \pm 32.5 \quad \text{and} \quad N^B_{\nu_\mu} = 129 \pm 17.0,$$

(48)

for $E_{QE}^\nu$ in the range $475 \text{ MeV} < E_{QE}^\nu < 1250 \text{ GeV}$. From the public information kindly given by the MiniBooNE collaboration on the Web\[6\], we obtain

$$N_{\nu_\mu} = 62851.2 \pm 250.7.$$

(49)

\[6\] http://www-boone.fnal.gov/for_physics/aps107datarelease/
From Eqs. (5), (33) and (27), we have

\[ P_{\nu_e \rightarrow \nu_e} = R = 0.88 \pm 0.05, \]
\[ P_{\nu_\mu \rightarrow \nu_\mu} = 1 - \frac{1}{2} \sin^2 2\vartheta_{\mu\mu} = 0.96 \pm 0.02, \]
\[ P_{\nu_\mu \rightarrow \nu_e} = P_{\nu_\mu \rightarrow \nu_e}^{\text{LSND}} = (2.64 \pm 0.81) \times 10^{-3}. \]

(50)

(51)

(52)

Then, we obtain

\[ P_{\nu_e \rightarrow \nu_e} N_{\nu_e}^B = 201.5 \pm 30.8, \quad P_{\nu_\mu \rightarrow \nu_\mu} N_{\nu_\mu}^B = 123.8 \pm 16.5, \quad P_{\nu_\mu \rightarrow \nu_e} N_{\nu_\mu} = 165.9 \pm 50.9. \]

(53)

Comparing with \( N_{\nu_e}^B \) in Eq. (48), one can see that the estimated amount of \( \nu_e \)-induced background is reduced by about 28 events as an effect of \( \nu_e \rightarrow \nu_\mu \) transitions. This reduction can compensate only partially the larger appearance signal due to \( \nu_\mu \rightarrow \nu_e \) transitions.

The estimated and measured numbers of \( \nu_e \) events are, respectively,

\[ N_{\nu_e} = 491.3 \pm 61.7 \quad \text{and} \quad N_{\nu_e}^{\text{MiniBooNE}} = 380 \pm 19.5, \]

(54)

Hence, the 3+1 four-neutrino scheme considered in this paper is compatible with the results of the MiniBooNE experiment within 1.7 standard deviations. Although our scheme is clearly not favored by the MiniBooNE data, we think that further measurements are necessary in order to assess its viability.

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