Local Spectral Density for a Periodically Driven System of Coupled Quantum States with Strong Imperfection in Unperturbed Energies

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I. INTRODUCTION

The statistical properties of many-body quantum objects attract considerable attention in a broad range of modern physics including condensed matter physics and quantum optics. Of a special interest are the properties specifying the temporal evolution for mesoscopic systems of quantum states coupled by interaction. Extensive investigations of many-body interacting systems (such as nuclei, many-electron atoms, quantum dots, quantum spin glasses and quantum computers) have shown that the state-state interaction can be responsible for the quantum localization effect and the quantum chaos in isolated complex systems [1–9]. As in Random Matrix Theory (RMT) [10,11], the level spacing statistics is well described by the Wigner-Dyson distribution and the individual basis states are spread over the large number of eigenstates. In a sense the interaction leads to dynamic thermalization without coupling to an external thermal bath. On the other hand, to our knowledge, there are no direct theoretical studies for mesoscopic systems driven by an external field.

In the work we apply a RMT approach in order to analyze the localization properties of local spectral density for a generic system of coupled quantum states with strong static imperfection in the unperturbed energy levels. The system is excited by an external periodic field, the temporal profile of which is close to monochromatic one. The shape of local spectral density is shown to be well described by the contour obtained from a relevant model of periodically driven two-states system with irreversible losses to an external thermal bath. The shape width and the inverse participation ratio are determined as functions both of the Rabi frequency and of parameters specifying the localization effect for our system in the absence of external field.

b. This band random matrix with disordered diagonal (BRMDD) is a generic Hamiltonian model for the systems with strong static imperfection in the unperturbed energy levels. The imperfection plays an essential role for the qubit composition and can destroy the operability of quantum computer [9,12,13]. BRMDD-based models were applied to study the electron transport problem [14–16] and the problem of interacting particles in a random potential [17–19]. The results obtained with the help of these models are of an obvious interest also in analyzing such few-freedoms physical objects as the vibrational quasicontinuum of polyatomic molecule [20–22].

The statistical properties of conservative BRMDD-based systems were studied in some detail [23–29]. The shape, the localization length and the inverse participation ratio (IPR) for eigenfunction have been investigated with the help of numerical simulations [23,27,28] and the supersymmetry approach [28,29]. These investigations have exhibited the Lorentzian shape for the local spectral density (LSD) of states under the circumstances when a non-perturbative localization regime is realized. The state-state interaction strength, at which the eigenstates are extended over the whole matrix size \(N\) and the eigenenergy level spacing statistics has the Wigner-Dyson form, have been revealed for the BRMDD with essentially large band (when \(2\hbar + 1 \gg \sqrt{N}\)) [26–29]. The association between ergodic properties of LSD and the matrix parameters have been clarified recently for the BRMDD with arbitrary small band [30]. The lack of ergodicity for LSD is shown in [30] to be identified with an exponential increase in IPR with the strength of state-state interaction.

We study the LSD of states for a generic BRMDD-based quantum system to be excited by an external quasi-monochromatic field. The LSD was introduced in 1955 by Wigner [31,32] and successfully employed in RMT to describe statistically the localization effects for complex quantum systems [10,11] (including the systems represented by band random matrices with reordered leading diagonal [33,34]). In our study this quantity specifies spreading of the energy concentrated initially in an individual basis state \(|g\rangle\), between the quasienergies and gives the quasienergy spectrum of \(|C(\omega)|^2\) where \(C(\omega)\) is the Fourier transform for correlation

\[
C(t) = \langle g|\exp(-i\mathbf{H}(t)/\hbar)|g\rangle. \tag{1}
\]

As for the conservative systems [10,11], the shape of LSD
may be characterized by the width $\Gamma$ of a quasienergy scale, on which the individual state $|g\rangle$ is localized. The number of quasienergies populating this scale is given by the ratio $\Gamma/\Delta_\omega = L$ designated here as a localization length of LSD ($\Delta_\omega$ is the mean quasienergy spacing). Hence the quantity $L$ specifies the greatest possible number of quasienergies, where the basis state $|g\rangle$ can be effectively admixed. The ergodic properties for the system can be identified with the structure of LSD. Th non-ergodic LSD is a strongly fluctuating spiked function and the IPR $\xi$, which gives the actual number of quasienergies involving the state $|g\rangle$, is low in comparison with $L$. In the ergodicity case the LSD is monotonic and the number $\xi$ approaches the value of $L$.

Here we study the shape and the ergodic features of LSD and estimate the shape width $\Gamma$ and the IPR, as functions of field-system interaction strength. The functions are analyzed in relation to relevant properties (the mean quasienergy spacing, the width, the localization length and the IPR) specifying the localization effect for our BRMDD-based system in the absence of external field.

II. MODEL DESCRIPTION

We consider a generic system of quantum states, the unperturbed energy levels of which are depicted in the inset of figure 1. The system consists of a single state $|g\rangle$ and $N = 2K + 1$ states $|k\rangle$ ($k = -K, \ldots, K$). The total Hamiltonian of the system is considered as the sum

$$H(t) = H^{(0)}(t) + H^{(1)}(t).$$

Here the time-independent operator $H^{(0)}$ describes a state-state interaction in the absence of field. The time-dependent part $H^{(1)}(t)$ specifies a field-system interaction.

In the basis of unperturbed states the Hamiltonian $H^{(0)}$ is represented in terms of a real symmetric matrix with statistically independent random elements

$$\langle i | H^{(0)} | j \rangle = E^{(0)}_i \delta_{ij} + \langle i | V | j \rangle$$

where the off-diagonal elements $\langle i | V | j \rangle = V_{ij} = V_{ji}$ specify the state-state interaction ($i, j = g, -K, \ldots, K$). In the model we take into account an interaction between the states $|k\rangle$. The corresponding elements $V_{kk'}$ are distributed uniformly in the interval $[-V, V]$ with $\langle V_{kk'} \rangle = 0$ and $\langle V_{kk'}^2 \rangle = V^2/3 = v^2$ if $|k' - k| < b$ or are zero otherwise. The coupling of the state $|g\rangle$ with $|k\rangle$ is ignored and $V_{gg} = 0$. The diagonal elements $E^{(0)}_k$ corresponding to the energy levels of states $|k\rangle$ are uniformly distributed according to the Poisson statistics with the mean spacing $\Delta$ between adjacent levels: $-K\Delta \leq E^{(0)}_k \leq K\Delta$. One of the energy levels (for definiteness, we take the level $E^{(0)}_0$ for the state $|0\rangle$) is located in the midpoint of interval $[-K\Delta, K\Delta]$: $\langle E^{(0)}_k \rangle \approx E^{(0)}_0 = 0$. The state $|g\rangle$ is the lowest one. The energy levels are chosen so that $E^{(0)}_g / E^{(0)}_g = 2K\Delta$. Note that in the basis of states $|k\rangle$ the operator $H^{(0)}$ is represented by a BRMDD.

![FIG. 1. A temporal shape of external periodical field (solid lines, $M = 16$). The fat dashed line gives the envelope $\sin(\omega t)$. The inset shows a diagram of unperturbed energy levels for represented BRMDD-based quantum system.](image)

An external periodical field with the frequency $\omega_f = E^{(0)}_0 - E^{(0)}_g$ stimulates the transitions between the non-perturbed basis states $|g\rangle$ and $|0\rangle$. The field-induced coupling between $|g\rangle$ and the states $|k\rangle$ with $k \neq 0$ is accepted to be negligible. The temporal profile of the field is shown in figure 1 to be a piecewise function with the envelope $\sin(\omega_f t)$. The total number of pieces covering the field period interval $T_f = 2\pi/\omega_f$ is equal to $M$. At large $M$ the field shape is close to monochromatic one. On each of the time pieces $[t_m, t_{m+1}]$ (the integer $m$ gives the number of piece interval) the field is time-invariant and the field-system interaction is represented by

$$H^{(3)}(t_m) = \Omega \sin(2\pi m/M) D.$$  

Here the Rabi frequency $\Omega$ specifies the strength of field-system interaction. The operator $D$ specifies the field-induced coupling between the states:

$$\langle \delta | D | \delta \rangle = \delta_{\delta g} \delta_{gj} + \delta_{\delta j} \delta_{0j}.$$  

At such an approximation the evolution of system for one field period interval is described by a unitary operator

$$U(T_f) = \prod_{m=1}^M \exp (-i[H(t_m)] T_f / M \hbar).$$  

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From diagonalization of the matrix $U(T_f)$ we obtain the eigenfunctions $|\alpha_j\rangle$ and the quasienergies $\omega_j$. The quantity $W_{jg} = |\langle\alpha_j|g\rangle|^2$ gives a probability to find the probing basis state $|g\rangle$ in the eigenstate $|\alpha_j\rangle$.

The described model can be considered as a driven two-state system (the states $|g\rangle$ and $|0\rangle$), the upper state $|0\rangle$ of which is coupled by interaction with a number of others quantum states. The interaction-induced probability distribution of the state $|0\rangle$ over the eigenstates of $H^{(0)}$ corresponds to a quasienergetic distribution of $|0\rangle$ in the absence of field ($\Omega = 0$). Since the operator $H^{(0)}$ is given in terms of a BRMDD the probability distribution of $|0\rangle$ over quasienergy scale is described by the Breit-Wigner shape (that is, by the Lorentzian contour) \[ \rho_0(\omega) = \frac{\Gamma_0}{2\pi \omega^2 + \Gamma_0^2/4} \] of the width $\Gamma_0$. In some sense the state $|0\rangle$ is Lorentzian-broadened. Therefore we associate our BRMDD-based system with a two-state system, the upper state of which is continuously Lorentzian-broadened with the width $\Gamma_0$ because of irreversible losses to an external thermal bath. Here the lower state has no losses and hence is not broadened. This two-state system with irreversible losses (TSSIL) is stimulated by an external monochromatic field. Such a well-known model (see, for instance, \[35,36\]) corresponds to our BRMDD-based system in the limit of infinitely large number $N$ of coupled states $|k\rangle$.

Depending on the ratio between the width $\Gamma_0$ and the Rabi frequency $\Omega$ the TSSIL model gives two shapes of $|C(\omega)|^2$ \[35,36\]. At $2\Omega/\Gamma_0 < 1$ the correlation $C(t)$ is an decaying aperiodical function of time. Then the shape of $|C(\omega)|^2$ is described by a contour

\[ |C(\omega)|^2 = \frac{a_1a_2(a_1 + a_2)}{8\pi} \frac{1}{(\omega^2 + a_1^2/4)(\omega^2 + a_2^2/4)} \] (8)

with coefficients $a_{1,2} = \Gamma_0^2/2 - \Omega^2 \pm \Gamma_0\sqrt{(\Gamma_0^2/4 - \Omega^2)}$. At a significantly weak field ($2\Omega/\Gamma_0 \ll 1$) this shape is close to the Lorentzian contour, the width $\Gamma$ of that is a quadratic function of the Rabi frequency $\Omega$:

\[ \Gamma \approx 2\Omega^2/\Gamma_0. \] (9)

That implies a single exponential decay with time for $C(t)$. At a stronger field ($2\Omega/\Gamma_0 > 1$) the shape of $|C(\omega)|^2$ is given by

\[ |C(\omega)|^2 = \frac{d_2(d_2^2 + d_1^2)}{\pi} \frac{1}{(4\omega^2 + d_2^2 - d_1^2)^2 + 4d_2^2d_1^2} \] (10)

where $d_2^2 = \Omega^2 - \Gamma_0^2/4$ and $d_2 = \Gamma_0/2$. Then the correlation $C(t)$ shows a decaying oscillation. In the limit of a strong field ($2\Omega/\Gamma_0 \gg 1$) the shape width $\Gamma$ is invariant to the Rabi frequency and approximates the magnitude $\Gamma_0/2$.

### III. NUMERICAL SIMULATION

The properties of LSD are analyzed for a wide range of parameters $N$, $\nu$, $\Delta$ and $\Omega$ ($3 < N < 1000$, $10^{-3} < \nu/\Delta < 10^2$ and $10^{-2} < \Omega/\Delta < 10^2$). The total number $M$ of time pieces covering the period interval $T_f$ is 32. The shape and the IPR of LSD are determined from averaging over disorder (that is, over many random matrices). The number of disorder realization is more than 200.

#### A. Shape of LSD

We restrict the study by the case of moderate strengths of state-state interaction when the width $\Gamma$ is essentially small compared to the whole quasienergy scale ($\Gamma \ll N\Delta/h$). It implies that the perturbation-induced variations in level density are negligible and the quasienergies $\omega_n$ are homogeneously distributed in the energy band $[-K\Delta, K\Delta]$ with the mean quasienergy spacing $\Delta = \Delta/h$. Then the shape of LSD can be defined as

\[ \rho(\omega) = \frac{1}{\Delta\omega} \frac{\sum_j W_{j\nu} \delta(\omega - \omega_j)}{\sum_j \delta(\omega - \omega_j)}. \] (11)

where $\langle \ldots \rangle$ means the averaging over disorder. We associate the shape $\rho(\omega)$ with contours represented in equations (8) and (10) and determine the coefficients $a_{1,2}$ and $d_{1,2}$ by mean square fitting of equation (8) or (10) to an averaged LSD. The shape width $\Gamma$ is estimated from the obtained coefficients as a twice halfwidth for the right slope of $\rho(\omega)$ in the positive part of quasienergy scale.

The numerical simulation for different magnitudes of $\Delta$, $\Gamma_0$, $\xi_0$ and $\Omega$ shows that the properties of $\rho(\omega)$ are determined by some factors. First of all, that is the mean quasienergy spacing $\Delta$. Another important factor is the width $\Gamma_0$ of shape $\rho_0(\omega)$ for a probability distribution of $|0\rangle$ over quasienergies in the absence of field. We should distinguish two localization regimes to be realized for the distribution. Perturbative localization regime takes place when the state-state interaction is weak and the state $|0\rangle$ is concentrated mainly in an individual quasienergies. For this regime the distribution width $\Gamma_0$ is small in comparison with $\Delta$. On the other hand, the strong state-state coupling results in a broad spreading of $|0\rangle$ over the quasienergy scale. Such a regime (designated here as a non-perturbative one) is characterized by large magnitudes of $\Gamma_0$: $\Gamma_0 > \Delta$. Certainly, the third significant factor characterizing the features of $\rho(\omega)$ is the strength $\Omega$ of field-system interaction. Our study demonstrates that the ergodic properties to be specified in terms of IPR $\xi_0$ for the quasienergetic distribution of $|0\rangle$ have no effect on the shape $\rho(\omega)$. Therefore,
we consider the shape width $\Gamma$ as a function of $\Omega$, $\Gamma_0$ and $\Delta_\omega$.

**FIG. 2.** Shape $\rho(\omega)$ of LSD at $2\Omega/\Gamma_0 = 0.5$ (○), 2(Δ), 4(∇). The lines show shapes obtained from fitting of contours (8) (dashed line) and (10) (solid lines) to $\rho(\omega)$.

Similar to the TSSIL model, at a weak field ($2\Omega/\Gamma_0 < 1$) the shape $\rho(\omega)$ is shown in Figure 2 to be well described by contour (8). The obtained values of $\Gamma$ satisfy the requirement $\Omega^2/\Gamma_0 \leq \Gamma \leq 2\Omega$. The type of $\Omega$-dependence for the width $\Gamma$ is specified by relation between $\Delta_\omega$, $\Gamma_0$ and $\Omega$. Our study demonstrates that the low values of $\Gamma_0$ or $\Omega$ give the linear low for the dependence. At $\Gamma_0 < \Delta_\omega$ the width $\Gamma$ is seen from the inset of Figure 3 to be a linear $\Gamma_0$-independent function of $\Omega$:

$$\Gamma \approx 2\Omega.$$  \hspace{1cm} (12)

This dependence can be considered as a manifestation of the perturbative regime for distribution $\rho_0(\omega)$. At $\Gamma_0 > \Delta_\omega$ (when a non-perturbative localization regime for $\rho_0(\omega)$ is realized) the increase of $\Omega$ results in the transition of $\Omega$-dependence for $\Gamma$ from the linear law to the quadratic one. We associate such a behavior of the width $\Gamma$ with the magnitude of localization length $L$ for LSD. Figure 3 shows that a linear function of $\Omega$

$$\Gamma \approx A\Omega\sqrt{\Delta_\omega/\Gamma_0}$$  \hspace{1cm} (13)

takes place when $L \ll 1$. The coefficient $A = 0.7$ is obtained from mean square fitting of (13) to calculated values of $\Gamma$. On the other hand, at $L \gg 1$ the $\Omega$-dependence for the quantity $\Gamma$ is close to the quadratic law (9) predicted by the TSSIL model. As for distribution $\rho_0(\omega)$, the transition from the linear to quadratic $\Omega$-dependence of $\Gamma$ can be associated with crossover of the shape $\rho(\omega)$ from the perturbative to non-perturbative localization regime.

**FIG. 3.** The localization length $L$ as a function of $\Omega/\sqrt{\Delta_\omega \Gamma_0}$ at a weak field ($2\Omega/\Gamma_0 < 1$) for $\Gamma_0/\Delta_\omega = 0.9$ (squares), 2.5 (△), 20 (stars) and 225 (full circles). The dashed line gives approximation (13) obtained for the points with $\Gamma_0/\Delta_\omega > 1$. The solid line represents law (9) predicted by the TSSIL model. The inset shows the ratio $\Gamma/\Gamma_0$ as a function of $2\Omega/\Gamma_0$ at $2\Omega/\Gamma_0 < 1$ for $\Gamma_0/\Delta_\omega = 0.06$ (+), 0.2 (○), 2.5 (△), 20 (stars) and 226 (full circles). The dashed line corresponds to dependence (12). The solid line represents law (9) predicted by the TSSIL model.

**FIG. 4.** The ratio $\Gamma/\Gamma_0$ as a function of $2\Omega/\Gamma_0$ at a strong field ($2\Omega/\Gamma_0 > 1$) for $\Gamma_0/\Delta_\omega > 1$ (full circles) and $\Gamma_0/\Delta_\omega < 1$ (○). The solid line shows a relevant dependence predicted by the TSSIL model.
At a strong field \((2Ω/Γ_0 > 1)\) the shape \(ρ(ω)\) is shown in Figure 2 to be well fitted by contour (10) obtained from the TSSIL model. As for the case of weak field, the behavior of Ω-dependence for the shape width Γ is determined by the localization regime realized for shape \(ρ_0(ω)\). Figure 4 demonstrates that at \(Γ_0 > Δω\) the Ω-dependence of Γ is in good agreement with a relevant curve obtained from the TSSIL model. At high field amplitudes (when \(2Ω/Γ_0 ∝ 1\)) the width Γ approximates the value \(Γ_0/2\). The minor discrepancy between Γ and the curve predicted by the TSSIL model can be explained by a divergence of the temporal profile of field from the monochromatic shape. At the perturbative regime of \(ρ_0(ω)\) the Ω-dependence of Γ is proportional to the TSSIL curve. Our analysis shows that for any magnitude of Ω the calculated values of Γ are higher then ones from the TSSIL model by a factor of 1.4. In the limit of high field amplitudes the width Γ is close to the magnitude \(Γ_0/1.4\).

**B. Inverse participation ratio of LSD**

We investigate the IPR \(ξ\) under the circumstances of localized regime for the probability distribution of basis state \(|0⟩\) over the eigenstates of \(H^{(0)}\). It implies that the state \(|0⟩\) is spread over a sufficiently large number of the eigenstates and \(N ≫ ξ_0 ≫ 1\). Here the quantity \(ξ_0\) is the IPR for the probability distribution of \(|0⟩\) and gives the actual number of eigenstates involving the state \(|0⟩\) [30]. In the study the main attention is paid to the case of \(ξ ≫ 1\). We accept the IPR as \(ξ = (⟨∑_{j}|W_{jg}|^2⟩)^{-1}\) and obtain this quantity from numerical simulation. Here \(⟨⟩\) means the averaging over disorder. The IPR \(ξ\) is considered as a function of Ω, ξ_0 and Γ_0. Results of the study are represented in Figure 5.

Our study demonstrates that at fixed Ω and Γ_0 the IPR \(ξ\) varies proportionally with the parameter ξ_0. This fact testifies that the change in ξ with the Rabi parameter is due to transformations in the shape of LSD. For instance, in the limit of strong field \((2Ω/Γ_0 ≫ 1)\) the width Γ of the shape \(ρ(ω)\) is Ω-independent and \(Γ ≈ Γ_0/2\). As a result, the quantity ξ is also invariant to Ω and approximates the magnitude of ξ_0 (see Figure 5). In other words, the IPR of LSD is equal to the IPR for the distribution of \(|0⟩\) over the eigenstates of \(H^{(0)}\) (here we should take into account the presence of two peaks of LSD in the negative and positive parts of quasienergy spectrum). As indicated earlier, at a weaker field \((2Ω/Γ_0 ≪ 1)\) the variation in Ω give rise to essential shape transformations both in the shape width and in the contour form. The analysis of calculated data shows that the behavior of IPR for this parameter region can be empirically approximated by the law:

\[ ξ ≈ Bξ_0(Ω/Γ_0)^β, \]

where the coefficients \(B = 1.54\) and \(β = 1.18\) are obtained from mean square fitting of (14) to calculated values of ξ.

**FIG. 5.** Dependence of the IPR \(ξ\) on the ratio \(2Ω/Γ_0\) at \(Γ_0/Δω = 40\) (\(ξ_0 = 30\) (+), 40(stars)) and \(Γ_0/Δω = 100\) (\(ξ_0 = 39\) (C), 100 (full circles)). The dashed line shows approximation (14).

Notice that the observed Ω-dependence of IPR is close to the linear law and differs significantly from the expected quadratic dependence to be realized in the case of the Lorentzian shape for LSD. Probably, this difference is attributed to the fact that we are not far enough in the asymptotic regime of weak field implying \(2Ω/Γ_0 ≪ 1\). In our simulation we have \(2Ω/Γ_0 ≽ 0.1\) and the shape of LSD appers to can not be well reduced to the Lorentzian contour. Unfortunately, the numerical simulation of localized regime for LSD with \(2Ω/Γ_0 < 0.1\) requires too large matrix sizes and much computational efforts being beyond our numerical abilities.

**IV. CONCLUSION**

We have analyzed localization and ergodic properties of LSD for a periodically excited generic system of coupled quantum states with strong imperfection in the unperturbed energies. These properties have been demonstrated to be determined essentially by a state-state interaction resulting in the localization effect for the system in the absence of external field. If the interaction is so strong that the conservative system exhibits this effect the shape of LSD can be obtained from a relevant model of driven two-state system with irreversible losses. In a sense the state-state interaction may be recognized as the
losses acting till the mesoscopic effect becomes evident. The time scale and the rate of a correlation decay caused by such losses are specified in terms of the shape width and the IPR of LSD.

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