Probing Gauge String Formation in a Superconducting Phase Transition

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Abstract

Superconductors are the only experimentally accessible systems with spontaneously broken gauge symmetries which support topologically nontrivial defects, namely string defects. We propose two experiments whose aim is the observation of the dense network of these strings thought to arise, via the Kibble mechanism, in the course of a spontaneous symmetry breaking phase transition. We suggest ways to estimate the order of magnitude of the density of flux tubes produced in the phase transition. This may provide an experimental check for the theories of the production of topological defects in a spontaneously broken gauge theory, such as those employed in the context of the early Universe.

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1. INTRODUCTION

Formation of topological defects in phase transitions has been of great interest to condensed matter physicists, as well as to particle physicists in the context of the early Universe [1,2]. Production of topological defects in phase transitions is conventionally estimated using either thermal production (with the usual Boltzmann suppression of defect density) [3], or via the Kibble mechanism which dominates over the thermal production in a variety of transitions, and arises due to the formation of a domain structure [4]. Recently, a new mechanism has been proposed for production of defects where defect-antidefect pairs are produced due to energetic oscillations of the magnitude of the order parameter field [5]. Though superconducting transition is dominated by dissipation (which suppresses field oscillations), the presence of gauge fields may make this new mechanism effective in this case also, see ref. [6]. For the purpose of this paper, we will use estimates of string production based on the Kibble mechanism, though actual string distribution may have contributions from this new mechanism as well.

In the Kibble mechanism, the order parameter is taken to be roughly uniform within a correlation region (domain), while varying randomly from one domain to the other. In between any two adjacent domains, the order parameter field is supposed to vary with least gradient (this is usually called the geodesic rule). Consider a superconducting phase transition corresponding to spontaneous breaking of U(1) gauge symmetry and characterized by a complex order parameter. Magnitude of the order parameter gives the degree of superconductivity, while its phase $\theta$ can vary spatially over distances larger than the coherence length. In this case, string defects arise at the junctions of domains if $\theta$ winds non trivially around a loop going through adjacent domains. Simple arguments show [7,8] that the probability of string formation through a triangular domain (in two space dimensions) is equal to 1/4.

It was first suggested by Zurek [9] that one may be able to experimentally test the Kibble mechanism in superfluid $^4$He system. Later, an experimental verification of the Kibble mechanism was carried out in a sample of liquid crystals where strings arise due to formation and coalescence of nematic bubbles in the isotropic phase [8], see also ref. [10]. Subsequently, experimental tests of Kibble mechanism have been carried out in superfluid helium systems [11]. Recently, prediction of correlations in defect-antidefect production via Kibble mechanism has also been experimentally verified in liquid crystals, see ref. [12].

An important limitation of these experimental studies [8,10–12] is that they all correspond to the formation of global defects. Some of the most important types of topological defects in the context of particle theory models of the early Universe are local defects, i.e. those which arise due to spontaneous breaking of a gauged symmetry. Examples of local defects are magnetic monopoles, and many types of cosmic strings. For local defects, the application of Kibble mechanism becomes somewhat non-trivial. For example, it was argued by two of us [13] that the geodesic rule becomes ill defined in the case of gauged topological defects. For global defects, the geodesic rule is well motivated as from energy consideration we expect least gradient of order parameter between two domains. However for gauged defects such energy considerations are absent, as spatial gradient of the order parameter alone, in between two different points in space, does not have any physical meaning. It was shown later in refs. [14,15] that for certain situations, the geodesic rule arises dynamically in gauge theories. Recently, Copeland and Saffin have studied [6] collisions of bubbles in a
first order transition case for Abelian Higgs model, and have shown that geodesic rule can get violated due to field oscillations, (see also ref. [3] in this context).

It does seem, therefore, that the production of local defects may show qualitatively new features as compared to the case of global defects. Thus it becomes important to investigate whether experimental studies such as in refs. [8][12] can be extended to systems with local defects. We consider this issue in this paper. The only system known to actually occur in nature with local defects is that of flux tubes in type II superconductors. [We note that it is known that certain types of liquid crystals resemble gauge systems [16]. It will be very interesting to study the defect production in this class of liquid crystals.] It has been suggested by Zurek [17] that superconducting transition in a torus geometry may provide a good way to detect domain formation. However, one can not probe the string distribution directly in this manner. In Sect. 2, we describe our proposal for one experiment which is aimed to directly detect open string segments exiting the surface of a superconductor. This provides a possible test of defect formation in two space dimensions. In section 3, we describe another experiment to detect the formation of the full 3-dimensional network of the strings formed inside the sample.

2. DETECTION OF OPEN STRING SEGMENTS

Consider a superconducting phase transition producing a string network as shown in Fig.1a. The arrows on the strings denote the direction of the magnetic field. Top surface of the sample is shown in Fig.1b with a + sign denoting a string exiting the surface while a − sign denoting a string entering the surface. If strings and antistrings form independently then there will be statistical fluctuations leading to an excess, of order $\sqrt{N}$, of one kind of string over the other, where $N \sim A/(4\xi^2)$ is the total number of all strings ($A$ being the surface area). However, as pointed out in ref. [7], the net string number through a surface bounded by a loop of perimeter $L$ is proportional to $\sqrt{L}$. Basically the argument is that the length $L$ of the perimeter consists of $n_L = L/\xi$ segments of size correlation length ($\xi$) each and $\theta$ varies randomly beyond each segment. This, then, is a random walk problem with average step size equal to $\pi/2$ (since the smallest step size for $\theta$ is equal to zero while the largest step size has magnitude $\pi$). The typical value of the net winding (i.e. net increase in $\theta$ divided by $2\pi$) along the perimeter will then fluctuate about zero with typical width given by $n_{ex} \approx \sqrt{n_L}/4$. This suppression in net winding holds for other topological defects as well [18] and arises since the presence of a string in a given domain increases the probability of the formation of an antistring in the adjacent domain due to partial winding of the phase $\theta$ along the boundary common to the two domains. This correlation between the formations of defects and antidefects has been recently experimentally verified for global defects in nematic liquid crystals, see ref. [12].

This excess string number $n_{ex}$ will then lead to a net magnetic flux through the surface with magnitude

$$\phi_{net} \approx \frac{\phi_0}{4} \sqrt{\frac{L}{\xi}}.$$ (1)

Here $\phi_0 = \frac{hc}{2e} (= 2 \times 10^{-7}$ Gauss.cm$^2$) is the flux quantum. Note that it may not be
appropriate to associate a single flux quantum with each vortex since these vortices will be strongly overlapping. However, the excess vortices (or anti-vortices) will be very dilute and \( \phi_{net} \) should be obtainable by assuming that these excess vortices each carry a single flux quantum.

For the domain size, one usually takes the correlation length at the Ginzburg temperature \( T_G \), since for temperatures above \( T_G \) thermal fluctuations can make defects unstable \[1\]. However, large scale structure of string distribution may remain unaffected by such fluctuations, as recently argued by Kibble (see, ref. \[1\]), and by Zurek \[17\]. They have further argued that the appropriate value of \( \xi \) should depend on the rate of phase transition \[4\]. For example, it has been argued in ref. \[17\] that, due to critical slowing down, the order parameter may be frozen above \( T_c \) itself. For a superconducting phase transition, the corresponding frozen out correlation length is given by \[17\],

\[
\xi_f = 10^{-2} \left( \frac{\xi_0}{1000 \text{Å}} \right) \tau_Q^{1/4} \text{ cm.} \tag{2}
\]

Here, \( \xi_0 \) is the zero temperature correlation length and \( \tau_Q \), measured in seconds, is the time scale of the quench (assuming that the relative temperature \( (T_c - T)/T_c \approx t/\tau_Q \), \( t \) being the time). Clearly, \( \xi_f \) can be much larger than the value of \( \xi \) at \( T_G \). In the present paper, we will use \( \xi_f \) to be the appropriate correlation length. Though we mention that the issue of the appropriate correlation length for defect formation is not a settled one. Thus it is important to emphasize that all our conclusions about the experimental proposals in this paper will be straightforwardly applicable if the correlation length happens to be smaller than the value given by Eq.(2). In fact, smaller correlation length will make the experiments much easier.

As \( \phi_{net} \) is larger for smaller \( \xi_f \), High \( T_c \) superconductors should be excellent for this purpose as they have very small coherence length (with \( \xi_0 \) of the order of 10 Å). [Though, the value of \( \phi_{net} \) may then also depend on the surface chosen due to the anisotropic magnetization in high \( T_c \) materials, see \[19\].] Assuming a very rapid quench, (presumably by taking very thin layer of superconductor and rapidly cooling it with \( \tau_Q \) of order of 1 sec. to about 10 milli seconds) one may get \( \xi_f \approx 0.3 - 1 \mu \text{m} \). With \( \xi_f \approx 0.3 \mu \text{m} \), Eqn.(1) gives a magnetic flux of the order of 50 - 90 \( \phi_0 \) for a square sample with 1 cm sides which should be easily measurable using SQUID. Also, the number of strings and antistrings \( N \) through the sample is about \( 10^8 \). As flux tubes are generally strongly pinned for high \( T_c \) superconductors, one may be able to use STM techniques to directly probe these flux tubes exiting the surface \[20\].

As suppression in \( \phi_{net} \) arises from the correlation between string and antistring formation, it suggests that if we somehow disrupt this correlation then a larger value of \( \phi_{net} \) can be achieved. For example, if a sample consists of \( n_g \) isolated grains, then string-antistring correlation will be absent in-between two different grains. The typical net flux through the whole sample will now be \( \simeq \phi_{net} \sqrt{n_g} \) where \( \phi_{net} \) is typical flux through a single grain. It is important to note that for such a sample one can not just calculate the net increase in the phase \( \theta \) along the perimeter of the whole sample as we did before, since that required the use of the geodesic rule to prescribe \( \theta \) between two domains. However, now for two isolated grains, \( \theta \) is not defined in-between the grains.
For a grain of about 3 µm size (as in ref. [21]), the net flux through its surface will be about 1 - 2 φ₀, assuming ξᵢ ≃ 3 - 1 µm (with the total number of strings through the grain’s surface being about 2 - 25). A square sample with sides equal to 1 cm will contain about 10⁷ grains, leading to φₙₑₜ ≃ 3 - 6 × 10³φ₀. We should mention here that for very small grains flux creeping may be a serious problem. Also, if the size of each grain is comparable to (or smaller than) ξᵢ then one will not expect even a single flux tube to be formed. In fact, this can provide a nice way to determine the value of ξᵢ appropriate for superconducting transition (occurring in a given time scale). By taking samples, consisting of grains of different sizes, one can find the size when flux tube formation abruptly falls to zero. The corresponding size of the grains will be of the order of ξᵢ.

Note that φₙₑₜ as given in Eqn.(1) measures just the difference between strings and antistrings, and hence will not change due to string-antistring annihilations. φₙₑₜ can change due to migration of open segments out of the boundary of the sample and should lead to a fractional correction of the order of 1/√A (for short times after the transition). Here we mention an important point that during the early stages of the formation, vortices having only winding number of the order parameter without any associated magnetic field, may have a faster creeping rate compared to the creeping of flux vortices. The effects of vortex creep can be reduced by taking large sample area to increase the vortex creep time while reducing the phase transition time by taking very thin sample. φₙₑₜ can also change due to strings which enter from the top surface but exit from the side surface (instead of the bottom surface). These strings may then shrink quickly and disappear through the edges of the sample. This, however, will lead to a fractional correction in φₙₑₜ of the order of the ratio rₛ of the area of the side surfaces to the area of the top and bottom surfaces, as strings entering from top will have a probability of rₛ to get out through the side surface.

An important thing to realize here is that for high Tₐ materials vortices are generally strongly pinned. This clearly suppresses both the above effects and should make high Tₐ materials best suited for this experiment.

We mention here the work in ref. [21] where flux trapping was observed in high Tₐ materials with varying external magnetic field. It was observed in ref. [21] that for extremely small applied field the magnetization in the sample was consistent with zero, perhaps suggesting suppression of string formation. The high Tₐ material used in ref. [21] was in the form of a fine powder with grain size of about 3 µm. First, for such small samples, flux creeping (or, more importantly, vortex creeping) may be crucial. This is especially so when we note that for such grains the area of all surfaces may be of similar size which will then lead to shrinking of many strings through the edges. [This shrinking of strings can be avoided by taking grains of very small thickness.] It is also possible that the phase transition in that work may not have been carried out sufficiently fast, so the value of ξᵢ (Eq.(2)) may have been larger than the grain size. In such a situation one will not expect any flux tubes to form.

3. DETECTING STRING LOOPS INSIDE THE SAMPLE

Vortex formation in two space dimensions is quite different from that in three space
dimensions. Even though the underlying mechanism may remain the same, namely the Kibble mechanism, the dynamics of string network in three dimensions has qualitatively new features. In two dimensions, vortex-antivortex annihilate leading to decrease in the number density of vortices in time. In three dimensions, string density decreases due to shrinking of string loops, accompanied by another important feature, usually termed as the intercommuting property of strings. Intercommutation means that when two strings cross, they exchange partners at the crossing point (see Fig.2). We now describe a proposal for a second experiment to detect the formation of full 3 dimensional string network inside the sample, utilizing this intercommutation property of strings.

Numerical simulations show \[7\], that the average string length passing through a (cubic) domain is about 0.88 $\xi$ (for U(1) strings). Thus the density of strings (string length per unit volume) expected is of the order of $\xi^{-2}$. Evolution of the resulting string network happens by shrinking of loops, and by intercommutation of strings. There are various numerical simulations which show that strings (global as well as local) intercommute under most generic conditions \[22\], see also ref. \[23\] for a qualitative discussion. Intercommutativity for global strings has been seen to occur in liquid crystal experiments \[24\]. \[A proposal for experimentally checking intercommutativity of gauge strings by observing the crossing of flux tubes in type II superconductors has been discussed in ref. \[24\].\] Based on the overwhelming evidence for intercommutativity of strings, we assume that flux tubes in type II superconductors also intercommute under most generic conditions. \[Recently, there has been some discussion of cutting and reconnection of vortex lines in the context of high $T_c$ superconductors \[25\].\] Though, as we will explain below, due to presence of impurities high $T_c$ superconductors are not very suitable for this second experiment.

Consider now the experimental setup shown in Fig.3a. A superconducting slab is surrounded by four electromagnets which are suspended so that their deflections (in the plane of all the four magnets) can be measured. Dashed lines show the magnetic field lines. As the temperature is lowered through the transition temperature, the magnetic flux will get expelled out, except in regions where it will form flux tubes going through the sample (assuming that the magnetic field is (slightly) larger than the lower critical field $H_{c1}$). Along with this there will also be some strings produced due to the Kibble mechanism (most or all of which are assumed to be in the form of closed loops for simplicity). Let us first illustrate the basic idea of the experiment by considering a simple, idealized case when only two flux tubes are passing through the sample due to external magnets and a single large string loop is formed due to Kibble mechanism which surrounds the two straight flux tubes, see Fig.3b. If these straight flux tubes were not present, then this string loop would have collapsed all the way. However, when the straight strings are present then the collapsing string loop will have to cross these straight strings. \[Note that, though parallel strings can get entangled due to repulsion, a collapsing loop can not remain entangled.\] As soon as this crossing happens, the strings are going to intercommute (by tilting of the loop, if required) resulting in the flux tubes as shown in Fig.3c. These strings will then shrink down towards the surface of the sample. We see that the final flux tube distribution in Fig.3d is very different from the initial one as shown in Fig.3b. The force between the external magnets and the superconducting sample depends upon the distribution of trapped flux tubes \[26\]. As the flux tube distribution has drastically changed from Fig.3b to Fig.3d, this should lead to different forces on magnets which could be measured.
We mention that the reverse of the process, leading to a distribution of flux tubes as in Fig. 3d evolving back into the one shown in Fig. 3b, is very difficult because the strings shrink due to dissipation and for strings to change from Fig. 3d to Fig. 3b it is necessary that a large loop of string forms somewhere which then expands and touches the outer surfaces of the sample. Of course the original, thermodynamically stable, string configuration may get restored due to strings creeping from outside. However, for intermediate times one will be stuck in the situation where the flux tubes have been broken and diverted in different directions.

We now discuss, in detail, the realistic case when a large number of strings are formed. Let us first consider the situation when there is no external magnetic field and the sample is cooled to a superconducting state. After waiting for long enough time so that any strings produced during transition have all disappeared (apart from the possibility of pinned vortices), we apply the external magnetic field which will then lead to a group of straight strings (as shown in Fig. 4a). All magnets are chosen to be of equal strength and the distance between a pair of magnets on each side of the sample is chosen to be large so that the two bundles of strings are reasonably confined and far away from each other. Next, one heats the sample back to the normal phase and then again lowers the temperature to superconducting state, now in the presence of the external magnetic field. Along with straight strings due to the external magnetic field, many strings will form due to the Kibble mechanism as well, see Fig. 4b. We assume for simplicity that all Kibble strings are closed loops. As these loops shrink, many of these straight strings (due to the external magnets) will get broken and diverted (similar to the situation in Fig. 3c) resulting in a different distribution of strings, see Figs. 4b and 4c.

Let us now consider the question of the observation of these diverted strings. As we mentioned, recent techniques [20] seem very promising in real time observations of strings even though the density and the randomness of the string network may pose serious problems. Even if the real time observation techniques in ref. [20] can not be used for an initially dense, fast evolving, random network of strings, one may be able to use it to observe the final resulting networks of strings at stages shown in Fig. 4a and Fig. 4c. Comparison of the two will directly give the fraction of strings which have been diverted.

Note that the Kibble estimate of strings will be modified for domains through which flux tubes due to external magnetic field pass. However, for a strongly type II superconductor, with a dilute bundle of strings, the number of domains affected by this will be small. For example, if the penetration depth is 5 times bigger than the correlation length then at most 4% domains will be affected in that region.

There is a simple way to observe diverted strings. This depends on the estimate of how many strings are expected to get diverted. Let us consider the case when the magnetic field $H$ due to the external magnets is only slightly above the lower critical field $H_{c1}$, with $H - H_{c1} << H_{c1}$. The resulting flux tube network is very dilute with the spacing $d$ between the flux tubes (for triangular flux tube lattice) given by [27],

$$d \simeq \lambda \ln \left[ \frac{6\phi_0}{8\pi \lambda^2 (H - H_{c1})} \right]$$

(3)

where $H_{c1}$ can be expressed in terms of the coherence length $\xi$ and the penetration depth $\lambda$ [27],
\[ H_{c1} = \frac{\phi_0}{4\pi \lambda^2} \ln \left( \frac{\lambda}{\xi} \right). \] (4)

\( \xi \) here should be taken to be the equilibrium correlation length. We take the domain size to be again determined by the frozen correlation length \( \xi_f \) as given in Eq.(2). [We again emphasize that qualitative aspects of our results do not depend on this choice. In fact for smaller correlation length, flux tube density will be larger and hence, detection of flux tubes will become easier in these experiments.] The length of strings per unit volume due to Kibble mechanism, \( L_s \), is expected to be, \( L_s \approx 0.88 \times \xi_f^{-2} \) (see ref. [7]), while the string length per unit volume due to external field will be \( L_{ext} \approx d^{-2} \). By varying \( H \) from a value very close to \( H_{c1} \) up to a much larger value, we can explore situations ranging from \( L_{ext} \ll L_s \) to \( L_{ext} \gg L_s \). [For the latter situation, one can use slower phase transition rate to increase \( \xi_f \), leading to lower \( L_s \).]

Now, to estimate the fraction of diverted strings, one approach can be to note that at the time of string formation, the number \( N_d \) of loops with sizes greater than \( d \) (for \( d \) comparable with the sample size) is of order one (from the results of numerical simulation in ref. [7]). This will suggest that the fraction of diverted strings will be of order \( 1/N_{ext} \) where \( N_{ext} \) is the number of external strings due to one pair of magnet. Clearly this fraction will be very small. [Still these diverted strings may be observable by Lorentz microscopy [20], as we mentioned earlier.] However, we now argue that the fraction of diverted strings can be much large, even if \( N_d \) is very small. The point is that string loops at the time of formation do not remain unchanged during the evolution and coarsening of string network which happens by shrinking and intercommutation of strings. First consider the case when external magnetic field is chosen to be sufficiently close to \( H_{c1} \), so that \( L_{ext} \ll L_s \). Let us concentrate on the strings due to the external field. As the strings enter the sample from outside, they will very quickly start intercommuting with shrinking strings in the highly dense string network and, within few random walk steps, the memory of the original direction of entrance (or exit) will be lost. This suggests that as the string network thins out due to the shrinking of strings, a given string will either follow its original straight path or will be diverted to the other magnet, both the possibilities being equally probable (as all the magnets are chosen to have the same strength). Thus in the limit \( L_{ext} \ll L_s \), about 50% of the straight strings will be diverted as shown in Fig.4c.

This is a macroscopic effect. Straight strings going through the sample result in a force on the magnets [20]. If 50% of those strings get diverted, the force due to strings on the magnets will change significantly affecting deflections of the magnets (compared to their positions in Fig.4a). Observation of such a deflection will signal the presence of strings produced in the phase transition with density far in excess of \( L_{ext} \). We can now increase the external magnetic field \( H \) which leads to an increase in \( L_{ext} \). When \( H \) has been increased sufficiently above \( H_{c1} \) so that \( L_{ext} \gg L_s \) then a very small fraction of straight strings should be diverted, leading to negligible deflection of the magnets. The deflection of magnets will therefore increase with increasing value of \( \frac{L_s}{L_{ext}} \) and will saturate at some maximum value when \( L_s > L_{ext} \), corresponding to the situation when about 50% of strings get diverted. The value of \( L_{ext} \) for which the deflection almost saturates will give a direct measure of the order of magnitude of the density of strings produced in the Kibble mechanism. We note here that it is not important to know what the actual change in the force on magnets is when
the strings get diverted: The important thing is that the change in force will increase with decreasing value of external magnetic field and will approach saturation when $L_{\text{ext}}$ and $L_s$ are roughly of the same order of magnitude. We should point out here that the presence of open string segments may change the number of strings which ultimately get diverted. It is not difficult to estimate the number of strings which are diverted depending on the density of open strings, though its effect may not be too significant. This is because, $\phi_{\text{net}}$ through a surface of area $A$ is proportional to $A^{1/4}$, and it is $\phi_{\text{net}}$ which can affect number of diverted strings (as strings-antistrings ending on a surface will eventually join due to attractive force, and then will get pulled inside the sample).

It is important to realize that this experiment requires a very clean sample. Thus high $T_c$ superconductors are not suitable for this experiment. (Also, for high $T_c$ materials, mean field results like in Eqn.(3) may be suspect \[28\] near $H_{c1}$. Though, it will only affect the value of $H$ at which saturation in deflections of magnets may happen, without changing any results in this section.) Vortex pinning can affect the results of this second experiment in two important ways. First, with pinning centers present, the magnetization of the superconductor shows irreversible behavior. Thus the number of flux tubes present in the sample for external fields applied after the superconducting transition will be different than the number for the case where the transition is carried out in the presence of the external field. For this experiment we have assumed the two numbers to be the same. The second problem will be that with pinning, we can not assume that the strings which have intercommuted will shrink away to the surface. Even after intercommuting, the strings may remain pinned inside the sample. It seems very difficult to make any estimate of how much impurity can be tolerated for the success of this experiment. If sample is pure enough that the number of pinned strings is much smaller than the number of diverted strings then pinning should not significantly affect the results of the experiment.

4. CONCLUSIONS

We conclude by emphasizing the importance of experimentally checking theories of formation of gauge defects. As we mentioned earlier, gauge defects play an extremely important role in particle theory models of the early Universe. Constraints on various theories of Grand Unification arising from over abundance of magnetic monopoles are well known. Many models of structure formation are based on production and evolution of gauge cosmic strings. Due to non-trivial issues relating to the validity of the geodesic rule in gauge theories, it becomes important to test defect formation for gauge defects, just as experimental tests have been done for the formation of global defects in liquid crystals and in superfluid helium. The only system known in nature, with spontaneously broken gauge symmetry, with a vacuum manifold nontrivial enough to lead to topological strings is superconductor. The proposals for the two experiments in this paper, to detect string formation in superconductors, thus provide a way to check some of the important conceptual issues relating to theories of gauge defect formation.

We point out some potential problems in the second experimental set up. After the strings get diverted, more strings may eventually creep in from the surrounding areas to make the local string density consistent with the value of the external field, although, once a string
configuration like in Fig.4c has been achieved (due to intercommutativity), there may be a large activation energy required for strings to creep in (something like the energy required to create a large string loop). It is important, therefore, that the two magnets be far apart so that the two bundles of strings in Fig.4a are far away from each other. The observations of strings (either by using Lorentz microscopy or by the deflection of magnets) will have to be done before this creeping becomes significant. Also as we mentioned earlier, the sample need to be extremely clean as otherwise pinned vortices can make any interpretation of the results difficult. From this discussion it is evident that the first experiment should provide a relatively clean way to check the string formation and a high $T_c$ material can be used for this purpose. Vortex pinning not only does not cause any problem for this experiment, it actually helps by reducing the errors caused by flux creep. The second experiment, although requiring very clean sample, provides a way to probe the full 3-dimensional network of gauge strings. Understanding properties of such an evolving string distribution are of crucial importance for cosmic string models of structure formation in the Universe.

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REFERENCES

[1] *Formation and interactions of topological defects*, Edited by, A.C. Davis and R. Brandenberger, Proceedings of NATO Advanced Study Institute, 1994, (Plenum, New York).
[2] A. Vilenkin and E.P.S. Shellard, *Cosmic strings and other topological defects*, (Cambridge University Press, Cambridge, 1994); A.J. Gill, Contemp. Phys. 39, 13 (1998).
[3] F. A. Bais and S. Rudaz, Nucl. Phys. B170, 507 (1980); F. Liu, M. Mondello and N. Goldenfeld, Phys. Rev. Lett. 66, 3071 (1991).
[4] T.W.B. Kibble, J. Phys. A9, 1387 (1976).
[5] S. Digal and A.M. Srivastava, Phys. Rev. Lett, 76, 583 (1996); S. Digal, S. Sengupta, and A.M. Srivastava, Phys. Rev. D55, 3824 (1997); *ibid* D56, 2035 (1997).
[6] E.J. Copeland and P.M. Saffin, Phys. Rev. D54, 6088 (1996).
[7] T. Vachaspati and A. Vilenkin, Phys. Rev. D30, 2036 (1984).
[8] M.J. Bowick, L. Chandar, E.A. Schiff and A.M. Srivastava, Science 263, 943 (1994).
[9] W.H. Zurek, Nature 317, 505 (1985). See also, Acta Phys. Pol. B24, 1301 (1993).
[10] I. Chuang, R. Durrer, N. Turok and B. Yurke, Science 251, 1336 (1991).
[11] P.C. Hendry, N.S. Lawson, R.A.M. Lee, P.V.E. McClintock, and C.D.H. Williams, J. Low. Temp. Phys. 93, 1059 (1993); G.E. Volovik, Czech. J. Phys. 46, 3048 (1996) Suppl. S6.
[12] S. Digal, R. Ray, and A.M. Srivastava, *hep-ph/9805502*.
[13] S. Rudaz and A.M. Srivastava, Mod. Phys. Lett. A8, 1443 (1993).
[14] M. Hindmarsh, A.C. Davis and R.H. Brandenberger, Phys. Rev. D49 (1994) 1944; see also, R. H. Brandenberger and A.C. Davis, Phys. Lett. B332, 305 (1994).
[15] T.W.B. Kibble and A. Vilenkin, Phys. Rev. D52, 679 (1995); see also, J. Borrill, T.W.B. Kibble, T. Vachaspati and A. Vilenkin, Phys. Rev. D52, 1934 (1995).
[16] P.G. de Gennes, “The Physics of Liquid Crystals” (Clarendon Press, Oxford, 1974).
[17] W.H. Zurek, Phys. Rep. 276, 177 (1996).
[18] A.M. Srivastava, Phys. Rev. D43, 1047 (1991).
[19] S. Kolesnik, T. Skoskiewicz, J. Igalson and Z. Korczak, Phys. Rev. B45, 10158 (1992).
[20] K. Harada, T. Matsuda, J. Bonevich, M. Igarashi, S. Kondo, G. Pozzi, U. Kawabe and A. Tonomura, Nature 360, 51 (1992); K. Harada, T. Matsuda, H. Kasai, J. E. Bonevich, T. Yoshida, U. Kawabe and A. Tonomura, Phys. Rev. Lett. 71, 3371 (1993).
[21] T. J. Jackson, M. N. Keene, W. F. Vinen and P. Gilberd, Physica B 165&166, 1437 (1990).
[22] E.P.S. Shellard, Nucl. Phys. B283, 624 (1987); K.J.M. Moriarty, E. Myers and C. Rebbi, Phys. Lett. B207, 411 (1988).
[23] C. Rosenzweig and A.M. Srivastava, Phys. Rev. D43, 4029 (1991).
[24] A.M. Srivastava, Phys. Lett. A194, 141 (1994).
[25] C. Carraro and D.S. Fisher, Phys. Rev. B51, 534 (1995); M.A. Moore and N.K. Wilkin, Phys. Rev. B50, 10294 (1994).
[26] R. J. Adler and W. W. Anderson, J. Appl. Phys. 68, 695 (1990).
[27] A.L. Fetter and P.C. Hohenberg in “Superconductivity”, Vol. 2, Edited by R.D. Parks, (Marcel Dekker, Inc., New York, 1969).
[28] D.R. Nelson and H.S. Seung, Phys. Rev. B39, 9153 (1989).
FIGURE CAPTIONS

Fig.1: (a) Formation of a string network in the phase transition in a thin slab of superconductor. [The random network we show in these figures is only an illustration and is not exactly what one actually gets in a simulation.] (b) Distribution of string ends at the top surface of the sample. + and − denote strings exiting and entering the surface respectively.

Fig.2: Intercommutation of strings.

Fig.3: A superconducting slab is shown in (a) surrounded by four electromagnets which are suspended so that their deflections can be measured. Dotted lines show the magnetic field lines. (b) shows the idealized case when only a single large string loop is formed through the Kibble mechanism, which surrounds only two straight flux tubes. (c) shows that strings have intercommuted as the loop collapses and crosses the straight strings. The final flux tube distribution is shown in (d) which is very different from the initial one as shown in (b).

Fig.4: Bundles of strings due to the external field in the superconducting state obtained by switching on the external field long after the superconducting transition is completed (so that any strings produced during the transition have disappeared). (b) shows the network of strings expected after cooling the sample to superconducting state in the presence of the magnetic field of the external magnets. (c) shows the diverted strings due to string intercommutation.
FIG. 1.
FIG. 3.
FIG. 4.