**Abstract**

Hard processes at the TeV scale exhibit enhanced (double log) EW corrections, that need resummation in view of the high level of precision of Next Linear Colliders. The fact that the weak sector is spontaneously broken causes some peculiarities with respect to unbroken theories like QCD. For observables that are exclusive with respect to $W, Z$ emission, some peculiar technical problems have yet to be solved. Surprisingly, double logarithmic enhancements are present even for inclusive observables like $e^+e^- \rightarrow$ hadrons, leading to violation of the Bloch-Nordsieck theorem. The last effect is particularly important, producing weak effects that in some cases compete in magnitude with the strong ones.
1 Introduction

In last two years, starting from the observation made in [1], it has become clear that the bulk of radiative electroweak (EW) corrections at the TeV scale is given by logarithms of infrared origin, also called Sudakov logarithms [2]. Such logarithms occur because at energies much larger than the EW scale $M_Z \approx M_W \equiv M$, the latter acts as a cutoff for the collinear and infrared (IR) divergences that would be present in the vanishing $M$ limit.

Due to this (double) logarithmic enhancement, EW corrections become pretty big at the TeV scale, producing corrections of the order of 10% to cross sections. Then, resummation becomes necessary and has been addressed for instance in [3, 4, 5] giving rise to some still unresolved controversy. Partly because of this controversy, one can state that Standard Model EW corrections are not under control at the 1% level at the TeV scale [6]. Moreover, this kind of corrections is ubiquitous, being present also in inclusive quantities as has been noticed in [7, 8].

There are various aspects of interest in these issues. First, one of the main goals of TeV scale accelerators like Next Linear Colliders [9] will be to explore possibilities for New Physics with a high level of precision. However, it is clear that no serious limit on, say, anomalous gauge couplings can be given and, quite in general, no signal of New Physics can be established until the aforementioned questions are resolved and established on a firm theoretical ground. Second, in particular papers [7, 8] make it clear that the IR dynamics of a broken theory like the SM electroweak sector is poorly known at the moment and might give rise to surprises; the Block-Nordsieck theorem is violated for instance. Therefore, in my opinion studying and testing this subject can be in itself a reason of interest for NLCs experiments.

In the following I consider processes of the kind $2$ fermions $\rightarrow 2$ fermions $+X$, characterized by a single hard scale, typically the c.m energy $\sqrt{s}$, much greater than the EW symmetry breaking scale $M$. Here $X$ represents emitted soft weak bosons of energy $\omega \ll \sqrt{s}$. All other hard scales are of the same order, namely $|s| \sim |t| \sim |u| \gg M$. Two cases are taken in exam: observables that are inclusive with respect to $W, Z$ emission (Par. 3) and observables that are exclusive with respect to $W, Z$ emission (Par. 2), but still include soft photon radiation up to a given experimental resolution $\lambda$. One can summarize the present situation like this:

- The study of exclusive EW form factors at the TeV scale is a challenging one and results are still controversial; a complete two loop calculation could help in solving some of the open questions.
• inclusive observables are characterized by unsuppressed double logarithmic corrections of IR origin, leading to violation of the BN theorem and to weak corrections that can be of the order of the strong ones.

2 Exclusive observables

Exclusive observables are characterized by 3 energy scales: the c.m. energy $\sqrt{s}$, the symmetry breaking scale $M_W \sim M_Z \equiv M$ and the infrared cutoff $\lambda_{IR}$. One must always be inclusive with respect to radiated photons up to a certain energy/angle resolution $\lambda$, which amounts to making the substitution $\lambda_{IR} \rightarrow \lambda$. A typical example is $e^+e^- \rightarrow \mu^+\mu^- + X$, where $X$ is a soft photon. The presence of 3 scales is a major difference with unbroken theories like QCD where the analogous problem is characterized by a single expansion parameter $\log \frac{\sqrt{s}}{\lambda}$; here, there are two expansion parameters $L \equiv \log \frac{\sqrt{s}}{M}$ and $l \equiv \log \frac{M}{\lambda}$ (see 4)). Since right fermions do not carry non-abelian charges, the interesting case to consider is the one with left (L) fermions on the external legs. One limit in which one already knows what should happen is the SU(2)$\otimes$ U(1) symmetric limit, i.e. $\sqrt{s} \gg M, \lambda$. In this regime the resummed matrix element is given in terms of the Born one by the following expression

$$M^L = \exp\left[-\sum_i \frac{C^F_i}{2} \log^2 \frac{s}{M^2} \right] M^L_0 \quad C^F_i = g'^2 y_i^2 + \frac{g^2}{4}$$

involving the sum of the Casimir $C^F_i$ in the fundamental representation over all external legs $i$. However, if the energy $\sqrt{s}$ is not extremely large, the situation is more complicated, due to the presence of the EW symmetry breaking scale $M$. The main point is that there is a separation of scales (see Fig. 1) such that QED soft effects are present below $M$, while the full EW contributions ($\gamma, Z, W$) has to be taken into account above $M$. As a byproduct of this picture, it has been shown in 4) that taking into account QED soft effects separately as was customary for LEP 11, 12), is not anymore correct at the TeV scale.

As already said, controversial results are present in the literature 3, 4, 5), indicating that symmetry breaking makes life harder for this kind of problem. The problem arises, namely, about the details of the scale separation and about the role of symmetry breaking when the three scales are relatively close to each other. The fact is that when one considers resummation to all orders, one is forced to make a priori

\footnote{Fermions can be taken to be massless as long as $\lambda \gg m_f$; the special case of the top quark, requiring a heavy mass cutoff, was considered in 10), but leads to no important differences at the double log level.}
assumptions for the calculation to be possible. A complete two-loop calculation without any a priori assumption would shed light on this controversy. A sort of “minimal calculation” sufficient for this purpose could be the process $Z' \to f \bar{f}$, considered in \cite{4}.

3 Inclusive observables

The unique, and surprising, feature of EW interactions with respect to nonabelian unbroken theories like QCD, is the violation of the Bloch-Nordsieck (BN) theorem \cite{13}. In abelian theories like QED, although radiative corrections are IR divergent in general, one recovers a finite result by summation over all possible degenerate finale states; this is the essence of the BN theorem. In particular then, if one regularizes IR divergences by introducing a cutoff $\lambda_{IR}$, inclusive observables do not depend at all on this cutoff. In principle, the BN theorem is violated in any nonabelian theory, like QCD for instance, since to recover an IR finite result one should sum also over initial degenerate states \cite{14}, which is however unphysical in general. In QCD one is saved at the bottom line due to color confinement: the initial states are color singlets, and averaging over initial color produces cancellation of the leading IR divergences \cite{15}.

The crucial observation of \cite{7, 8} is that the situation with EW interactions is very different from QCD. In fact the colliders initial states ($e^-, p, \nu...$) carry in general a nonabelian weak isospin charge, and averaging over the initial isospin makes no sense from an experimental point of view. This means that the BN theorem is violated, and that even inclusive observables retain the leading (double log) dependence on the IR cutoff $M$. The effect is quite dramatic for the typical case of $e^+e^- \to$ hadrons: while QCD corrections are perturbative, and therefore almost energy independent, EW effects steadily grow with energy and get as big as the strong ones (see Fig. 2): a sort of early unification!

What happens here is that the nonabelian gauge structure of the theory and the breaking of the gauge symmetry itself conspire to produce a nontrivial result. Symmetry breaking is crucial since, besides providing the physical cutoff $M$, it gives a finite range to weak interactions allowing for asymptotic initial states that are “bare” nonabelian charges like $e^-_L$; the analogous situation in QCD is forbidden.

References

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Figure 1: Separation of scales for EW IR effects at the TeV scale. The process considered is $Z' \rightarrow f \bar{f}$ (see [4] for details).

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Figure 2: Resummed double log EW corrections to $e^+e^- \rightarrow$ hadrons and strong corrections (dashed line) up to 3 loops. The dash-dotted line is for a LL polarized beam, while the continuous line is for an unpolarized beam.