A poisson regression approach for modelling spatial autocorrelation between geographically referenced observations

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Abstract

Background: Analytic methods commonly used in epidemiology do not account for spatial correlation between observations. In regression analyses, omission of that autocorrelation can bias parameter estimates and yield incorrect standard error estimates.

Methods: We used age standardised incidence ratios (SIRs) of esophageal cancer (EC) from the Babol cancer registry from 2001 to 2005, and extracted socioeconomic indices from the Statistical Centre of Iran. The following models for SIR were used: (1) Poisson regression with agglomeration-specific nonspatial random effects; (2) Poisson regression with agglomeration-specific spatial random effects. Distance-based and neighbourhood-based autocorrelation structures were used for defining the spatial random effects and a pseudolikelihood approach was applied to estimate model parameters. The Bayesian information criterion (BIC), Akaike's information criterion (AIC) and adjusted pseudo R², were used for model comparison.

Results: A Gaussian semivariogram with an effective range of 225 km best fit spatial autocorrelation in agglomeration-level EC incidence. The Moran's I index was greater than its expected value indicating systematic geographical clustering of EC. The distance-based and neighbourhood-based Poisson regression estimates were generally similar. When residual spatial dependence was modelled, point and interval estimates of covariate effects were different to those obtained from the nonspatial Poisson model.

Conclusions: The spatial pattern evident in the EC SIR and the observation that point estimates and standard errors differed depending on the modelling approach indicate the importance of accounting for residual spatial correlation in analyses of EC incidence in the Caspian region of Iran. Our results also illustrate that spatial smoothing must be applied with care.

Keywords: Cancer incidence, Dietary pattern, Disease mapping, Multilevel generalised linear model, Socio-economic status, Spatial analysis

Background

In general, disease counts in areas that are geographically proximate will display residual spatial dependence; ‘residual’ here acknowledging that dependence of these counts on known risk factors has been taken account of in the analysis model. Spatially correlated observations do not satisfy the independence assumption central to generalized linear model (GLM) theory [1]. However, generalized linear mixed models (GLMMs) can accommodate spatial random effects and provide a flexible means of analysing spatially correlated disease counts [2]. Left unaddressed, residual spatial correlation can bias regression parameter estimates and cause standard errors to be underestimated, leading to confidence intervals that are too narrow and, potentially leading to incorrect inferences regarding exposure-disease associations [3].

The purpose of this paper is to demonstrate for medical statisticians and health researchers how to fit common types of GLMMs with spatially autocorrelated random effects. While Bayesian or frequentist
approaches can be used, we limit attention to frequen-
tist approaches here. Most applications of GLMMs use
approximate likelihood methods to estimate model para-
eters. Rather than try to cover a broad array of models
(without providing sufficient depth for the reader to
understand the logic behind the model), we focus on
Poisson regression with three of the most common
autocorrelation structures: Poisson regression with
agglomeration-specific nonspatial random effects; Pois-
son regression with distance-based agglomeration-speci-
cific spatial random effects; and Poisson regression with
neighbourhood-based agglomeration-specific spatial ran-
dom effects. We aim to compare these three autocorre-
lation structure approaches for Poisson regression
models. In this study these methods are illustrated by
investigating the association between the geographic
pattern of esophageal cancer, EC, incidence and socio-
economic pattern in the Caspian region of Iran.

The structure of this paper is as follows: In the
methods section, the EC study’s setting and data col-
lection are described and exploratory analysis methods
are detailed; then regression models for count data
incorporating different assumptions about spatial cor-
relation are presented. In the results section, the appli-
cation of the different models to the EC data is
described, and finally conclusions from these analyses
are presented.

Methods
Study Population
Residents of Mazandaran and Golestan provinces consti-
tute the study population (see Figure 1). The estimated
midyear population of Mazandaran and Golestan pro-
vinces between 2001 and 2005, stratified for age in five-
year intervals and place of residence, was obtained from
the statistical centre of Iran [4]. These estimates were
projections for 2001 to 2005 based on 1995 census data
using the 2000 geographic boundaries [5,6]. Geographic
coordinates for each agglomeration were also obtained
that approximately reflected the geographical centroid of
each agglomeration [4].

Geographic region
Iran has high rates of EC (esophageal cancer) [7,8].
Strong spatial aggregations in esophageal cancer have
been identified in the two study provinces with a ten-
dency for high rates in eastern and central wards and
low rates in the west [9]. A recent study showed EC was
associated with aggregated risk factors related to socio-
economic status (SES) including income and urbanisa-
tion [10]. The total population of these two provinces is
approximately 4.5 million (1.6 million in Golestan pro-
vince) [4]. The provinces of Iran are subdivided into
wards. There are usually a few cities and rural agglom-
erations in each ward. Rural agglomerations are a

Figure 1 Geographic boundaries of wards (bold polygons), and cities (gray polygons) and rural agglomerations within wards, in Mazandaran and Golestan provinces.
collection of a number of villages. Currently, Mazandaran province has 15 wards, 46 cities and 110 agglomerations and Golestan province has 11 wards, 24 cities and 50 agglomerations. Figure 1 shows geographic boundaries of cities and rural agglomerations within wards in Mazandaran and Golestan provinces.

Data sources
The cases of interest were all EC patients registered between 2001 and 2005 among the study population. The results for both sexes combined are presented in this paper. Data on incident cases of cancer were obtained from the Babol Cancer Registry; issues related to methods, quality and completeness of data collection for this cancer registry are described elsewhere [9,11]. In summary, the major sources of data collection related to cancer in the Babol cancer registry were reports from pathology laboratories, hospitals, and radiology clinics.

For each agglomeration the following socio-economic variables were obtained from the 1995 statistical yearbooks of Mazandaran and Golestan [5,6] and the income and expenses survey in urban and rural areas in 1995 [12,13]: population density (inhabitants per square kilometre), relative level of activity (a synthetic indicator devised by the statistical centre of Iran that is calculated from the number of households, number of telephone lines, number of bank offices, number of commercial licences, electricity consumption, annual construction budget), annual income per family, annual expenditure on food per family, annual expenditure on fruit and vegetables per family, percentage of occupation in the industrial sector, percentage of occupation in the services sector, percentage of occupation in the agricultural sector, percentage of occupation in the construction sector, percentage of male unemployment, and percentage of illiteracy for males and females. In addition to the villages, some agglomerations contain one or more cities; a proportional as likely basis method was used to calculate socio-economic characteristics of these agglomerations.

Factor analysis of socio-economic variables
A factor analysis was performed to synthesize the socio-economic variables into a few independent and uncorrelated factors. A Varimax rotation with Kaiser normalisation was used to facilitate interpretation of the factors. The Anderson-Rubin method was used to create factor scores from the factor solution [14]. The factor scores extracted with this method are uncorrelated with a zero average and variance of one. We attached labels to the factors by interpreting coefficients of the items. This process identified three factors: income, urbanisation and literacy. Scores for these factors for each agglomeration were subsequently used in regression models.

Standardised incidence rates calculation
Adjustment of incidence rates for differences in the age structure of agglomerations was accomplished by age-standardisation. The SIR for an agglomeration was obtained from the ratio of the observed and expected number of cases in that agglomeration. The indirect method of standardisation was used for internal comparisons [15]. Since the population of the region was stable between 2001 and 2005, the 2003 population size was used for computing the incidence rates in age categories of the overall region and the subsequent expected number of cases in each agglomeration. Five-year intervals were used for age categorisation.

Exploratory spatial data analysis
Two methods were used to measure spatial aggregation of the agglomeration SIRs; Moran’s I [16] and semivariograms [17]. Moran’s I was adjusted for agglomeration counts by comparing the observed count in a region with its expected count under the constant risk hypothesis [18].

To calculate the empirical semivariogram, we used studentised residuals (residuals that are each divided by their estimated standard error) obtained from a weighted least squares (WLS) regression. This involved a linear regression model of the form

\[ Z_i = \alpha_0 + \epsilon_i \]  

where \( \alpha_0 \) was an intercept parameter to be estimated, \( \epsilon_i \) was the residual error of the \( i \)th agglomeration, and \( Z_i \) was a Box-Cox transformation of the SIRs. The weights used in parameter estimation by WLS were proportional to the population size at risk within each agglomeration. To obtain a succinct statistical description of the spatial correlation three different parametric models (exponential, Gaussian, and spherical) were fitted to the empirical semivariogram, each of which can be described in terms of nugget, sill and range parameters [17]. The model considered most appropriate was that which minimized the residual sum of squares between the theoretical model and the empirical semivariogram [17].

Analytic methods
Three approaches for modelling with agglomeration-specific random effects were considered: (1) a standard Poisson GLMM with independent agglomeration-specific random effects (nonspatial Poisson GLMM); (2) a discrete autocorrelation structure for agglomeration-specific random effects (neighbourhood-based spatial Poisson GLMM); and (3) a continuous autocorrelation structure for agglomeration-specific random effects (distance-based spatial Poisson GLMM).
For all approaches it was assumed that, conditional on random effects, the number of cancer cases in each of the \( N \) agglomerations, \( Y_1, \ldots, Y_N \), were independent Poisson random variables each with mean \( \mu_i \).

The models contained income, urbanisation and literacy. The aim of the analyses was to identify macro scale SES factors that determine the distribution of EC at the agglomeration level.

\[
\log(\mu_i) = \log(E_i) + (X\beta)_i + u_i \tag{2}
\]

where \( X_{\text{SES}} \) is a design matrix for the three socio-economic factors; \( \beta_0 \) is an intercept parameter to be estimated, \( \beta_{\text{SES}} \) is a vector of parameters that describe the socio-economic factor effects on EC and the offset term \( \log(E_i) \) is the logarithm of the expected number of cases for that agglomeration (assumed fixed). Since theoretical \( \text{SIR} = \frac{E_i}{E_i} \), this is a model for observed agglomeration level SIRs and the parameters \( \beta_{\text{SES}} \) describe association between factor scores and (log) SIR with \( \exp(\beta_{\text{SES}}) \) interpretable as relative risk parameters within each agglomeration.

Nonspatial Poisson generalized linear mixed model (nonspatial GLMM)

A nonspatial GLMM for the number of cases can be specified as

\[
\log(\mu_i) = \log(E_i) + (X\beta)_i + \Theta_i \tag{3}
\]

where \( u_i \) is the random effect (one for each agglomeration). These are independent random effects, and as per convention, were assumed to be distributed as \( \text{N}(0, \Sigma) \). The parameter \( \tau^2 \) indicates the variance in the population distribution, and therefore the degree of heterogeneity of agglomerations. These random effects represent the influence of agglomeration \( i \) that is not captured by the observed covariates.

Neighbourhood-based spatial Poisson generalized linear mixed model (neighbourhood-based GLMM)

The neighbourhood-based GLMMs took the following form:

\[
\log(\mu_i) = \log(E_i) + (X\beta)_i + \Theta_i \tag{4}
\]

The vector of random effects \( \Theta_i \) was assumed to have conditional autoregressive structure (CAR) [19,20]. A generalization of the CAR called the intrinsic conditional autoregressive structure (ICAR) was used here, where the conditional distribution of the random effects \( \Theta_i \) is

\[
\Theta_i | \Theta_j \sim \text{N} \left( \frac{\Theta_j}{n_i}, \frac{\sigma^2}{n_i} \right). \tag{5}
\]

\[
\Theta_j = \sum_{i \in \delta_i} \frac{\Theta_j}{n_i}, \quad n_i \text{ is the number of neighbours for agglomeration } i, \text{ and } \delta_i \text{ indicates the neighbourhood of agglomeration } i \text{ [21]. Using the properties of the multivariate normal distribution, Eq. (4) can be specified in a joint formulation, where } Q = \sum_{i=1}^{n} \left(1 - W_{ij}\right) \text{ is the precision matrix as } \Theta \sim \text{MN}(0, (\sigma^{-2}Q)^{-1}). \text{ The diagonal matrix } \Sigma_{\Theta} \text{ has entries } \frac{1}{n_i} \text{ and } W \text{ has entries } W_{ij} = \begin{cases} \frac{1}{n_i} & \text{if } j \in \delta_i \\ 0 & \text{otherwise} \end{cases}.
\]

In this way, the precision matrix has a rather simple form in ICAR, with the number of neighbours \( n_i \) on the diagonal and off-diagonal entries -1, where \( i \) and \( j \) are neighbours and zero otherwise.

Distance-based spatial Poisson generalized linear mixed models (distance-based GLMMs)

Distance-based GLMMs took the following form:

\[
\log(\mu_i) = \log(E_i) + (X\beta)_i + \Phi_i \tag{6}
\]

The vector of random effects was assumed to be MVN \((0, \Sigma_{\Theta}(\theta))\) with parameters that are jointly referred to as \( \Theta = \begin{bmatrix} \tau^2 \\ \nu^2 \\ \phi \end{bmatrix} \). If \( d_{ij} \) denotes the distance between agglomeration centroid \( i \) and \( j \), where counts \( y_i \) and \( y_j \) were observed, then

\[
\Sigma_{\Theta}(\Theta) = \begin{bmatrix} \tau^2 & F \nu^2 \\ F \nu^2 & \nu^4 \\ \phi & \phi \nu^2 \end{bmatrix} \tag{7}
\]

where the matrix \( F \) has elements \( F_{ij} \) given by \( \exp\left(-\frac{-d_{ij}}{\psi}\right) \) for exponential, \( \exp\left(-\frac{-d_{ij}^2}{\psi^2}\right) \) for Gaussian, and \( \exp\left[-\frac{2}{3}\frac{d_{ij}}{\psi} - \frac{1}{2}\frac{d_{ij}^2}{\psi^2}\right] \) for spherical semivariogram models. The unknown parameters in these models, namely, \( \tau^2 \) (the nugget), \( \nu^2 \) (the sill), and \( \phi \) (the range), can be obtained from a variogram of the data.

Parameter Estimation

We used a pseudolikelihood approach to estimate unknown parameters in the GLMMs [22]. This approach first assumed that \( \Delta \), the vector of variance-covariance parameters, was fixed and used maximum likelihood to estimate \( \beta \); and then, from the residuals, \( r \),
revised the estimate of Δ and iterated until convergence. The following steps were carried out:

1. An initial estimate $\hat{\beta}_0$ of β was made by assuming no spatial correlation, that is, with $\Sigma$ as a diagonal matrix.

The deviance residuals $r_0$ were calculated using $\hat{\beta}_0$, and Δ was estimated by using maximum likelihood.

2. A new set of estimates $\hat{\beta}_1$ was found with Δ assumed fixed at the values $\Delta_0$; hence, a new set of deviance residuals $r_1$ was calculated.

3. These estimates were used in a new cycle to redraw Δ and thus derive fresh estimates $\Delta_1$.

Steps 2 and 3 formed an iterative cycle that continued until there was no further change in the estimates.

**Model comparison**

The -2 Log-Likelihood statistic and two commonly used penalized model selection criteria, the Bayesian information criterion (BIC) and Akaike’s information criterion (AIC), were used for model comparison. Adjusted pseudo $R^2$ was also used to compare the different models with

$$R^2 = 1 - \frac{\sum y_i \log(\frac{Y}{\hat{\lambda}_i}) - (y_i - \hat{\lambda}_i)}{\sum y_i \log(Y)}$$

where $y_i$ was the observed count and $\hat{\lambda}_i$ was the model expected count, and adjusted pseudo $R^2$ was given by

$$R^2_{adj} = 1 - \frac{N - 1}{d.f.} (1 - R)^2.$$

The adjustment was for the number of degrees of freedom (d.f. = N-no. of model parameters)[23].

**Cartographic display**

In this study the RR (risk ratio) break points were determined by considering values in the range 0.1 to 10. This corresponds to the range -1 to +1 upon logarithmic transformation. Then this logarithmic scale was divided into 11 equal intervals centred on zero, the break point values were transformed back to the original RR scale, and the five middle intervals were used in the maps. As shown in Figure 2, the middle category was further divided above and below 1. A red-green colour scheme was used for the maps, with shading of red for areas with the highest SIR (> 1.33), followed by orange and yellow for areas with moderately elevated SIR, light and medium green for areas with moderately low SIR, and dark green representing areas with the lowest SIR (< 0.75).

**Software**

SIR calculation was done by Microsoft Excel, exploratory spatial analyses were performed using the SAS VARIOGRAM Procedure[24], factor analyses performed using SPSS 17 and the SAS Glimmix procedure was used to carry out the random effect regression models [25,26].

**Results**

**Factor analysis**

Factor analysis identified three factors with eigenvalues greater than 1. Table 1 shows the correlations between socio-economic items and the extracted factors. The three factors account for 53% of total variance in socio-economic variables and individually the factors account for: income: 25%, urbanisation: 15% and literacy: 13%.

**Exploratory analysis**

A total of 1693 new EC cases were diagnosed in 2001-2005 in Mazandaran and Golestan. The observed Moran index was 0.22 which was greater than its expected value -0.0066, indicating systematic clustering in the region. Consistent with Moran’s I, Figure 2(a) showed strong spatial aggregations, with a tendency for high rates in the eastern and central agglomerations and low rates in the west.

A Gaussian semivariogram best fitted the empirical semivariogram as illustrated in Figure 3(a). The effective range of spatial autocorrelation was 225 km. The nugget/sill ratio was 0.28, indicating a moderate degree of spatial autocorrelation (Figure 3(b)).

**Spatial regression**

As illustrated in Figure 3(b), exponential, Gaussian and spherical semivariogram models smoothed out the fluctuations of, and provided a reasonable overall fit to, the empirical semivariogram. Hence we used all three models for $\Sigma_0(\theta)$ and compared the results in Table 2. While the Gaussian model had slightly better fit, the choice of autocorrelation structure had little effect on the results from the distance-based GLMMs. We decided to select the Gaussian distance-based GLMM for further comparisons.

Comparison of the overall fit of nonspatial and spatial regression approaches is provided in Table 3. Based on the likelihood ratio, AIC and BIC the neighbourhood-based autocorrelation structure had the best fit to observed EC counts compared to the competing methods. The Pseudo-$R^2$ diagnostic suggested that nearly 25% of the total variation in EC counts could be explained by the three SES factors. This measure also indicated best fit for neighbourhood-based Poisson regression. Figure 4 shows the scatter plots of the observed SIR against the model predicted SIRs for each of the three models. Large observed SIR are smoothed towards 1 in all three approaches, i.e. for those aggregations model predicted SIRs are closer to 1 and the...
agglomeration is represented in Figure 4 with a point below the line of equality. This smoothing was most marked in the nonspatial model and least marked in the neighbourhood-based model.

Table 4 presents the point and interval estimates for parameters of interest in the nonspatial GLMM, neighbourhood-based GLMM and distance-based GLMM. The two spatial models had RR estimates that were qualitatively similar to the nonspatial GLMM model in finding all three risk factors to be, if anything, protective. However the two spatial GLMMs’ RR estimates were considerably further from the null than their nonspatial counterparts, a point returned to below.

In general, 95% CIs for relative risks in the two spatial GLMMs were wider than the corresponding intervals in the nonspatial GLMM, reflecting the reasonably strong inter-agglomeration correlation being taken into account by the spatial model approaches and corresponding effective reduction in sample size in comparison with the nonspatial model that assumes independence of these agglomerations.

As mentioned above, the neighbourhood-based GLMM and distance-based GLMM point estimates were
systematically smaller compared with the nonspatial GLMM. This attenuation of effect away from the null was especially marked for the income and urbanisation factors.

The neighbourhood-based GLMM and distance-based GLMM RR estimates had comparable conclusions based on inspection of p-values but all three associations were more strongly protective in the neighbourhood-based model. The results for the neighbourhood-based autocorrelation structure indicated that an increase in the score of the income or urbanisation factor was significantly associated with decreasing EC SIR. Model smoothed SIR maps after adjustment for the three factors in Table 3 in a neighbourhood-based GLMM are illustrated in Figure 2(b).

Discussion
In this ecologic study we observed significant associations between agglomeration-specific EC SIR and SES. The geographic pattern of cancer SIR was consistently stronger among eastern regions and among rural agglomerations. However this conclusion depended on the choice of model.

Multiple forms of residual correlation were potentially present in the cancer SIR data; we considered models addressing 2 types of geographical correlation. A semivariogram plot confirmed the presence of substantial distance-based residual spatial correlation in the EC SIR; agglomerations greater than 225 km apart behaved essentially as independent observations in EC, but at lesser distances, SIR were increasingly correlated as the distance between agglomerations centroids diminished. The global Moran’s I showed the importance of neighbourhood-based residual correlation in

Table 1 Socio-economic loadings from factor analysis (Income, Urbanisation and Literacy)*

| Items                              | Rotated Component Matrix | Components |
|------------------------------------|--------------------------|------------|
|                                    |                          | Income     | Urbanisation | Literacy  |
| Annual income per family           | .846                     | -          | -            |
| Annual expenditure on food per family | .654              | .165       | -            |
| Annual expenditure on fruit and vegetables per family | .455     | .151       | -            |
| Population density                 | .318                     | .285       | -            |
| Relative level of activity         | .321                     | .221       | .533         |
| % of male unemployment             | -.321                    | -.679      | -            |
| % of employment in agriculture      | -.213                    | -.808      | -            |
| % of employment in industry        | .199                     | .341       | -            |
| % of employment in construction    | -.208                    | -          | .470         |
| % of employment in services        | .189                     | .824       | -.198        |
| Female illiteracy                  | -                        | -          | -.642        |
| Male illiteracy                    | -                        | -          | -.669        |
* Loadings less than 0.10 in absolute value are not displayed
Table 2 Comparison of distance-based autocorrelation structures for Poisson GLMM regression using exponential, Gaussian and spherical functions

| Model             | Parameter | Goodness of fit method |
|-------------------|-----------|------------------------|
|                   | Nugget    | Partial sill | Range | -2Log-Likelihood | AIC   | BIC   |
| Exponential       | 0.38      | 3.20         | 427.7 | 380.9            | 390.9 | 392.9 |
| distance-based    |           |             |       |                  |       |       |
| Gaussian          | 0.45      | 1.21         | 224.6 | 373.4            | 370.5 | 362.1 |
| distance-based    |           |             |       |                  |       |       |
| Spherical         | 0.40      | 1.65         | 380.5 | 381.5            | 387.5 | 386.5 |
| distance-based    |           |             |       |                  |       |       |

Figure 3 Empirical semivariogram and theoretical semivariogram values (a), and Gaussian semivariogram fit to the empirical semivariograms points (b).
neighbourhood-based GLMM results than was the non-
spatial Poisson regression.

Ignoring spatial autocorrelation can result in explana-
tory variables apparently being associated with incidence
as a result of overstatement of the degrees of freedom
in the data and consequent underestimation of standard
errors. In our analysis, the standard errors of the regres-
sion coefficients were smaller by as much as 35 percent
in the model that ignored spatial effects compared with
the models that adjusted for spatial effects. As a result,
accounting for spatial autocorrelation increased the
width of confidence intervals for point estimates. We
also observed modest attenuation of SES factor associa-
tions with EC in the direction of the null, i.e. a risk ratio
of 1, in the nonspatial model. In general it is difficult to
predict the direction and nature of this attenuation
when a spatial random effect is added to model [29].
Results of simulation studies suggest that the import-
ance of accounting for spatial autocorrelation will
depend on the similarity in the spatial variation of the
agglomeration-level exposure with the extra-Poisson
variation in the outcome, as well as the strength of that
correlation. If the agglomeration-level exposure varies
on a larger scale over space relative to the spatial struc-
ture of the residuals, then ignoring the spatial autocorre-
lation may lead to underestimation of the exposure
effect. If the spatial structures of exposure and disease
incidence are similar, then the exposure effect may be
confounded by the inclusion of spatial random effects
[30-32].

It is noteworthy that the neighbourhood-based
GLMM is optimal if all agglomerations are of similar

Table 3 Comparison of Poisson regression goodness of fit using nonspatial generalized linear mixed model and
spatial Poisson generalized linear mixed models with either neighbourhood based or distance based autocorrelation
structures

| Model                  | Goodness of fit method |
|------------------------|------------------------|
|                        | -2Log- Likelihood | AIC        | BIC        | Adjusted pseudo R² |
| Nonspatial GLMM        | 413.5                 | 388.5      | 401.5      | 18.7%               |
| Neighbourhood-based GLMM | 353.0                 | 347.1      | 357.0      | 24.7%               |
| Distance-based GLMM    | 373.4                 | 370.5      | 362.1      | 21.2%               |

Table 4 Comparison of Poisson regression results using nonspatial GLMM and spatial GLMMs with neighbourhood
based or distance based autocorrelation structure

| Factor     | RR  | SE* | 95% CI      | P-value | RR  | SE* | 95% CI      | P-value | RR  | SE* | 95% CI      | P-value |
|------------|-----|-----|-------------|---------|-----|-----|-------------|---------|-----|-----|-------------|---------|
|            | lower | upper |         |         | lower | upper |         |         | lower | upper |         |         |
| Literacy   | 1.00 | 0.11  | 0.90  | 1.11   | 0.99  | 0.88  | 0.21  | 0.58  | 1.33  | 0.21  | 0.93  | 0.25  | 0.57  | 1.52  | 0.39  |
| Income     | 0.83 | 0.22  | 0.67  | 1.03   | 0.20  | 0.67  | 0.29  | 0.50  | 0.89  | 0.04  | 0.74  | 0.23  | 0.59  | 0.93  | 0.06  |
| Urbanisation | 0.78 | 0.19  | 0.65  | 0.94   | 0.08  | 0.55  | 0.35  | 0.39  | 0.77  | 0.05  | 0.70  | 0.22  | 0.56  | 0.87  | 0.05  |

* Standard error of RR

Figure 4 Scatter plots of observed SIRs (horizontal axis) against model predicted SIRs (vertical axis).
size and arranged in a regular pattern and the distance-based GLMM is optimal if all inhabitants of each agglomeration live at their agglomeration’s centroid and the measured rate thus refers to this specific location. There have been attempts to model Poisson kriging accounting for size and shape of administrative units as well as population density [33,34].

Bayesian methods have been widely used for the analysis of spatially correlated count data using software such as WinBUGS and MLwiN. WinBUGS was developed purely for Bayesian analysis, while MLwiN also allows likelihood approaches to GLMMs. Bayesian models have to specify prior distributions for all parameters and require computationally intensive MCMC sampling. Likelihood and Bayesian approaches for spatial Poisson regression have been compared for several case studies with similar results found [35].

Valid inference in spatial regression requires acknowledgment of residual spatial dependence, since regression coefficients of interest will often be sensitive to the form of the dependence assumed, as was observed in this study. The exploratory data analysis showed significant geographic autocorrelation suggesting that the nonspatial GLMM, which ignored this type of correlation, was inappropriate. In the nonspatial GLMM presented here, the agglomeration-specific random effects, assumed to be independent, introduced extra-Poisson variation in EC counts but ignored between-agglomeration spatial correlation. In contrast, the spatial GLMMs explicitly defined spatially-structured random effects, accounting for between-agglomeration spatial correlation but not addressing extra-Poisson variation, which would have required a second set of agglomeration-specific random effects. While we have illustrated the use of SAS, one widely available statistical software package, implementations of these methods also exist in R and WinBugs [25,26].

Conclusions
Our results illustrate the importance of accounting for residual spatial correlation in analyses of ecologic studies of disease incidence.

GLMMs are accessible, flexible methods that enable epidemiologists to account for residual spatial correlation. With careful definition of correlation structure and careful application of spatial smoothing, spatial GLMMs are a valuable tool to account explicitly for the effects of residual spatial correlation during the regression modelling process.

List of abbreviations used

AIC: Akaike’s information criterion; BIC: Bayesian information criterion; CAR: Conditional autoregressive; Distance-based GLMM: Distance based spatial Poisson generalized linear mixed model; EC: Esophageal cancer; GLM: Generalized linear model; GLMM: Generalized linear mixed model; ICAR: Intrinsic conditional autoregressive; Neighbourhood-based GLMM: Neighbourhood based spatial Poisson generalized linear mixed model; Nonspatial GLMM: Nonspatial Poisson generalized linear mixed model; RR: Risk ratio; SES: Socio-economic status; SIR: Standardised incidence ratio; WLS: Weighted least squares

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Authors’ contributions
MM was responsible for the data collection process and issues related to data quality. MM performed the statistical analysis. MM and RW wrote the first draft of the manuscript to which all authors subsequently contributed. All authors read and revised the manuscript for important intellectual content and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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