A multi-contact model to study the dynamic stick-slip and creep in mechanical frictional pair

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Abstract
The original Iwan-type model is always based on the quasi-static assumption, which neglects the dynamic properties of the frictional interface. In this paper, A Iwan-type multi-contact model is established to study dynamic stick-slip and creep in mechanical frictional pair. The individual contacts in this model follow a rupturing-reforming law, which enable the model to describe the relation between the fluctuation of friction force and the behavior of individual contacts. Thermal effect influences the rupturing of individual contacts, a process that contributes to a considerable difference in the prediction of friction force. Creep is examined in two aspects, namely, time dependence of nominal static friction coefficient and fretting response. This model exhibits the logarithm dependence of static friction coefficient on stationary time, a result that the quasi-static Iwan-type model cannot describe. The fretting responses under different parameters are investigated. The proposed model can satisfactorily describe the manner the average rupture threshold of individual contacts influences the maximum friction force and maximum fretting displacement. The energy dissipations under different oscillatory displacements are also evaluated. The relationship between tangential stiffness and average contact rupture threshold is studied under Gaussian and power law distributions of contact rupture thresholds. The paper elucidates that by introducing a new rupturing-forming law of individual contacts, the Iwan-type model can better describe the dynamic properties of the frictional interface.

Keywords: Multi-contact model, Stick-slip, Creep, Hysteresis response, Tangential stiffness

1. Introduction

Stick-slip and creep have been the most prevalent topics in nonlinear motion of mechanical frictional pairs over the past decades. Precise description of the stick-slip phenomenon is an indispensable task in the modeling of sliding friction(Rice and Ruina, 1983; Ruina, 1983). The mechanism of stick-slip in dry friction is mainly caused by the inhomogeneous distribution of shear strengths of junctions over time(Heslot et al., 1994), which is a complex problem involving elastic rupture, shear melting and plastic flow(Filippov et al., 2019).

Based on studies of static friction in various contacts, including metal– metal, rock–rock, and paper–paper contacts, to speak of a truly motionless stick-state rather than a stress-induced creep under external force is not possible(Persson, 1995). Creep is characterized by the transition from partial slip to gross-slip. This transition is caused by the gradual rupturing of individual contacts. The dependence of static friction coefficient on the duration of stationary state is an important property of creep. This dependence can be explained by the fact that the growth of individual contacts in stationary state increases the maximum tangential force and stiffness before gross-slip (Persson et al., 2003). Another example is that when a bolt joint is loaded by a tangential oscillatory force or displacement, the force–displacement phase diagram shows a series of hysteresis loops, a phenomenon that is referred to as “fretting response”. This phenomenon is a repeating process of creep and indispensable in designing a joint interface. Moreover, it can be observed in the low-speed motion of a linear-guide system.

These two phenomena are both important in the prediction of motion of mechanical frictional pairs. Experiments and theories show that they are both caused by the inconsistent rupturing and forming of individual contacts(Heslot et al., 1994; Persson et al., 2003). Thus, how to relate the behaviors of individual contacts with the macro frictional motion is important to simulate these two phenomena.
Using a quasi-static model deducted from contact theory, Eriten et al. (Eriten et al., 2011b) evaluated the tangential stiffness and displacement during the transition from partial slip to gross-slip. This physics-based model is highly reliable in predicting the tribological properties of a quasi-static interface because the required parameters, such as surface roughness, can be obtained from real surfaces. The precise contact stiffness between two mechanical surfaces can also be obtained based on contact theory (Shi and Polycarpou, 2006).

Empirical models, such as the LuGre (Canudas de Wit et al., 1995), Leuven (Swevers et al., 2000), and Iwan models (Segalman, 2005a), can describe both the stick-slip and creep. Although these multi-contact models are empirical in nature (friction force is not conducted from a physics-based contact model), they have been widely adopted in designing control systems because they relate the dynamic stick-slip and creep phenomena with the stick-slip behavior of individual contacts. Among these empirical models, the Iwan-type model (Iwan, 1966, 1967) is probably the most explicit and efficient. Individual contacts in the Iwan-type model are elastically transferred to a series of spring-sliders structures; these structures are called Jenkins elements. When an external force is applied, these elements gradually overcome the local static forces and transition from stick state to slip state, eventually leading to a gross-slip. Segalman (Segalman, 2005b) used the Iwan-type model to investigate the partial-slip phenomenon of lap-type joint. The author reported that the transition from partial slip to gross-slip is abrupt and thus the residual slip is not predicted. Moreover, the stiffness of the system becomes zero after gross-slip in the Iwan-type model, which stands in contrast with the experimental observation by Gaul et al. (Gaul and Lenz, 1997) and the widely observed friction fluctuation after the gross-slip of frictional interface. Song et al. (Song et al., 2003) adjusted the Jenkins elements by adding a residual stiffness that enabled the model to predict the dynamic stick-slip phenomenon after gross-slip. Wang et al. (Wang et al., 2018) established a reducing order of the stick-slip model with exponential distribution of critical sliding force to predict the frictional instability of a bolt joint. The addition of residual stiffness enables the model to simulate the dynamic stick-slip motion after gross-slip, but the model degenerates into a single spring–block system after all the Jenkins elements rupture, and thus the forming and rupturing of individual contacts in dynamic friction cannot be described.

The rupturing–reforming mode of individual contacts can express the relationship between the time-variant behavior of the individual contacts and the instability of macro friction force. Barel et al. (Barel et al., 2010) established a multi-contact model with reforming–rupturing dynamic law that considers the role of temperature to describe the frictional instability of nanoscale dry friction. This rupturing–reforming model also shows good predictability in simulating molecular complex friction (Guerra et al., 2016). Braun et al. (Braun and Tosatti, 2009) developed a model of boundary lubrication by considering the solid regions of a lubricant as a series of spring–block structures. The authors found that the macro smooth sliding corresponds to the rupturing–reforming process of contacts between the individual contacts and the upper surface. Kligerman et al. (Kligerman and Varenberg) built a multi-slider model to explore the relationship between macro stick-slip motion and the stick-slip behavior of subcontacts. They reported that the stick-slip motion is highly related to the ratio of the individual contacts area to the whole contact area. However, the contact aging effect is not considered. These modeling studies show that macro-scale frictional nonlinearity is highly related to the dynamic behaviors of individual contacts, and the contacts will reform and rupture during the frictional process.

In this study, a multi-contact modeling approach with a rupturing–reforming mode of individual contacts is established and applied to simulate stick-slip and creep in a mechanical sliding-friction pair. The rupturing and forming of individual contacts during the stick-slip motion are examined. The study of creep is focused on two phenomena, namely, time dependence of nominal static friction force and fretting loop (hysteresis loop). The resulting regulations are compared with those obtained by previous experiments and theories.

2. Modeling method

A macroscopic contact of dry friction between an upper slider and a lower base is decomposed into n individual contacts (Jenkins elements) by reducing-order based on the Iwan-type model (Iwan, 1966, 1967). The individual contacts in this study are in asperity scale. In the original Iwan-type model, a Jenkins element is ruptured when the local external force $k_i l_i$ exceeds a certain rupture threshold $f_{r,i}$, and this element stays in slip state permanently. However, the individual contact reforms after rupturing rather than permanently slip during dynamic friction. Thus, the Jenkins elements herein are modified, as shown in Fig. 1: When the force from the base on an individual contact $f$
exceeds $f_{st}$, the contact ruptures, and the individual force becomes $f_r$. The element has a certain chance to reform after a certain number of time steps in the modified model (Fig. 1(b)), but it permanently becomes 0 in the original Iwan model (Fig. 1(a)). Fig. 1(c) presents the mechanical model of the modified Iwan model. The model consists of $n$ linear spring components with their rupture thresholds. A contact transits from the stick state to the slip state if the force on the contact exceeds a certain threshold. Thereafter, this contact has a certain chance to transit to the stick state. The macro friction force can be calculated as:

$$f_{sl}$$

Fig. 1 (a) The original Iwan model with the static-slip law. In this model, a Jenkins element is in static state when the force on the element is smaller than a certain threshold. However, the element is in slip state permanently once the force becomes larger than the rupture threshold to rupture the element. (b) the Modified Iwan model, where a Jenkins element will have a certain chance to return to the slip state (reforming) after rupturing. In this model, $f_a$ is the force on contact, $k_a$ is the stiffness of spring, and $l_a$ is the elongation of the spring. (c) The mechanical model describes that $n$ Jenkins elements (element $e_1$ to $e_n$) exist, and a single element will rupture-reform-rupture during a frictional process.
\[ F_f = \sum_{i=1}^{n} f_i \]  

\[ f_i = \begin{cases} (k_i l_i, \text{state}(i) = 0) \\ (f_s, \text{state}(i) = 1) \end{cases} \]  

where \( \text{state}(i) = 0 \) and \( \text{state}(i) = 1 \) indicate that contact \( i \) is in a stick or ruptured state, respectively, and \( k_i \) and \( l_i \) are the stiffness and elongation of contact \( i \), respectively. The distribution of \( f_{si} \) can be determined by a Gaussian distribution of mean value \( f_s \) and standard deviation \( \Delta f_s \) before frictional process (Braun and Tosatti, 2009). \( k_i \) is calculated using the following equation:

\[ k_i = k (f_{si(i)} / f_s) \frac{1}{2} \]  

where \( k \) is the mean stiffness of the contact, and when a Jenkins element reattaches, a new \( f_{si} \) determined by the distribution is assigned; \( f_s \) is the average rupture threshold value of friction force on an individual contact. \( f_s \) strongly depends on the pressure between two surfaces. If the force between two surfaces is \( P \), then the average force on the individual contacts is \( F / n \), which means that the mean value of the rupture thresholds is \( f_s = P \mu_s / n \), where \( \mu_s \) is the static friction coefficient. Thus, \( f_s \) and \( F \) should have a positive correlation.

The formation of contacts in dry friction strongly depends on the “age” of the contact, signifying that the process is “time dependent” (Heslot et al., 1994). In describing the probability of a contact rupturing during dry friction, Filippov et al. (Filippov et al., 2004) proposed that the probability of a contact to reform should be calculated as:

\[ p_{on} = \Delta t k_{on} g[(\tau - \tau_0)/\Delta \tau] \]  

where \( k_{on}^0 \) and \( \Delta \tau \) are constant values, \( \tau \) is the time of a contact in a “rupture state”, \( \tau_0 \) is a characteristic time scale, and \( g \) is a stepwise function \( g = 1 \) for \( \tau > \tau_0 \) and \( g = 0 \) for \( \tau \leq \tau_0 \). The properties of \( g \) are controlled by \( \tau_0 \) and \( \Delta \tau \). This equation entails that a ruptured contact will have a certain probability \( \Delta t k_{on}^0 \) to reform in a time step of length \( \Delta \tau \) once the contact has been maintained in ruptured state for a period of time longer than \( \tau_0 \); otherwise the contact has no chance to reform.

When the bond energy \( \Delta E \) is smaller than \( \sigma k_B T \), the contact is a “weak contact” that can be easily ruptured by thermal effect (Dudko et al., 2003; Filippov et al., 2004). In \( \sigma k_B T \), \( \sigma \) is a constant, \( k_B \) is Boltzmann constant and \( T \) is temperature. In the present study, we only consider the thermal effect on the rupturing of weak contacts, which is defined as:

\[ \text{Contact } i \text{ is } \begin{cases} \text{weak contact} & \Delta E_i < \sigma k_B T \\ \text{else} & \Delta E_i > \sigma k_B T \end{cases} \]  

This equation entails that if the elastic potential energy of a contact is sufficiently larger, it becomes a weak contact and has a certain probability to be ruptured by thermal effect. If a weak contact is in a stick state with spring force \( f_i = k_i l_i < f_{si} \), then an elastic barrier \( \Delta E \) must be overcome by further pulling the spring or by thermal excitation as follow:

\[ \Delta E = k_i (f_{si(i)} / k_i)^2 - l_i^2 \]  

Persson (Persson, 1995) proposed that an individual contact has a certain chance to be ruptured by thermal excitation before the contact force reaches its rupture threshold. The rate \( p_e \) for thermal excitation over a barrier of height \( \Delta E \) can be written as follows:

\[ p_e = \gamma \exp(-\delta \Delta E) \]
where $\gamma$ and $\delta$ are constant values. In every time step, a random number $r$ between 0 and 1 is generated to determine whether a weak contact will be ruptured by thermal excitation: If $r < p_e$, then the barrier will be overcome by thermal excitation and contact rupture.

The Iwan-type model can describe the relationship between friction force and individual contacts. However, the elasticity between the driving point and the driven surface should also be considered. In real mechanical frictional pairs, the driving point is the point where external displacement is exerted, whereas the driven surface is the frictional surface driven by elastic components. Systematic elasticity is unavoidable between the driven surface and the driving point. For example, a driving point of a slider-guide frictional pair is on a DC motor, but the elasticity of the ball screw (and other structures) between the driving point and the driven surface means the motion of DC motor does not synchronize with the motion of slider(driven surface). An elastic component with stiffness $K$ representing the elasticity of driving system should be add between the driving point and the driven surface, thus, the dynamic equation of driven surface becomes

$$MA + \varphi V = K(v_d t - D) - F_f$$

where $M$ is the mass of the driven surface, $A$ is the acceleration of the driven surface, $\varphi$ is the damping coefficient, $V$ is the velocity of the driven surface, $v_d$ is the driving velocity, $F_f$ is the macro friction force and $D$ is the displacement of the driven surface. During the simulation, the iteration time step is set as $\Delta t = 0.0001s$ via the Runge-Kutta method. The other default parameters of parameters are listed in Table 1 (Unless otherwise stated, the values of parameters are based on Table.1). The boundary condition are $[D V A]=[0 0 0]$ and $f_i(i=1 \sim n)=0$.

### Table 1 Default parameters used to define the properties of the model

| Symbol | explanation | Value |
|--------|-------------|-------|
| $M$    | Mass of driven surface | 0.1kg |
| $n$    | Number of individual contacts | 100 |
| $f_{si}$ | Rupture threshold of individual contact i | Determined by $f_s$ |
| $f_s$  | Mean value of $f_{si}$ | $3 \times 10^{-3}N$ |
| $\Delta f_s$ | Standard deviation of $f_{si}$ | $3 \times 10^{-4}N$ |
| $k_{on}$ | Constant value | 1 |
| $\tau_0$ | Characteristic time for contact reforming | $3 \times 10^{-4}s$ |
| $\Delta \tau$ | Constant value | 1 |
| $T$    | Temperature | 300K |
| $k_B$  | Boltzmann constant | $1.38 \times 10^{-23}J/K$ |
| $\sigma$ | Constant value | $1.29 \times 10^{-9}/k_B$ |
| $\gamma$ | Constant value | 0.01 |
| $\delta$ | Constant value | 500J$^{-1}$ |
| $\varphi$ | Constant value | 1N.s/m |
| $v_d$  | Velocity of driving point | 0.01m/s |
| $K$    | Stiffness of driving coupling | 4N/m |
| $f_r$  | Residual force of individual contacts | 0 |
| $k$    | Characteristic stiffness of individual contacts | $(20/n)N/m$ |

### 3. Results

#### 3.1 Simulation of Stick-slip

The spring force and friction force over time ($v_d=0.005m/s$) are presented in Fig. 2(a) Two curves show similar
characteristics, a strong stick-slip phenomenon can be observed in the friction force, and the curve of spring force is relatively smooth. The delay between the spring force (the force on the elastic component in real frictional pair) and the real friction force (total internal friction force) is always observed because of elastic coupling between the driven surface and the driving point.

The calculation results of the number of individual contacts ruptured by elastic pulling and thermal excitation ($v_d=0.005\text{m/s}$) are shown in Fig. 3(a). The rate of rupturing due to thermal effect (black curve) is larger than that in the absence of the thermal effect (purple curve). From Eq.(5), we can see that the rupture of weak contact is advanced by considering the thermal effect. The difference between the two curves accumulates over time, and the friction forces under these two conditions are remarkably different (Fig. 3(b)), indicating that the thermal effect on the rupturing of individual contacts cannot be neglected in a long-term sliding friction.

![Fig. 2 (a) Simulated spring force and friction force and (b) enlarged graph of the region in the blue rectangle.](image-url)
Fig. 3 (a) Total number of contacts ruptured by different effects over time. (b) Spring force with and without considering the thermal effect.

Fig. 4 shows the influence of driving velocity and temperature on the ratio of the number of individual contacts ruptured by the thermal effect to the total rupture number \( \frac{N_{\text{thermal}}}{N_{\text{total}}} \), where \( N_{\text{thermal}} \) is the total number of contacts ruptured by thermal effect, and \( N_{\text{total}} \) is the total rupture number. Recorded time is 100s; data interval is 0.001; Every data point is obtained by averaging the values calculated from 20 runs. The scatter and error bars are hidden for a better view, but they are included in the appendix section. The ratio of the contacts ruptured by thermal excitation generally initially increases with velocity within a short range and then decreases.
3.2 Simulation of creep: Time-dependence of static friction force and hysteresis response

3.2.1 Time-dependence of static friction coefficient induced by multi-contact nature

In Amonton friction law, no relative motion exists when the interface is under external force smaller than the maximum external friction. This theory is contrary to creep phenomenon, which shows a small displacement once the surface is under an external force smaller than the “maximum static friction”: $F_s = \mu_s N$, where $N$ is the external load and $\mu_s$ is the static friction coefficient.

In previous studies of static friction force (Persson, 1995; Rice and Ruina, 1983), a spike in friction force higher than the previous history is observed by stopping the driving for a stationary time period $t_s$ then recovering the driving velocity instantaneously. This spike can be taken as the static friction force $F_s$ after a stationary time period $t_s$.

The relationship between static friction coefficient $\mu_s$ and $t_s$ is well described by

$$\mu_s = a + b \ln\left(\frac{t_s}{t_{s0}}\right)$$

where $t_{s0}$ is a constant value. This relationship is adequately proved by experiments and theories (Rice and Ruina, 1983; Ruina, 1983). By contrast, in the quasi-static Iwan-type model, this relation cannot be exhibited because the quasi-static model is time-independent. Herein, the ability of this model to describe this relationship is determined.

Fig. 5 shows a sample of spring force and friction force over time during this process. The corresponding first highest spike after resuming of velocity is denoted by a red circle (driving velocity $v_d=0.05m/s$, stop time length of driving $t_d = 80s$, and stationary time period $t_s = 50s$). An evident static time period after the stopping of driving is observed, wherein the friction and spring forces remain the same. The friction force rapidly increases after restarting during which an obvious “Amonton-style” maximum static friction force can be observed. In this section, the maximum static friction force is considered as the maximum of the spring force between $t = 80s + t_s$ and $t = 80s + t_s + 10s$. 

Fig. 4 Relationship between the ratio of the number of contacts ruptured by the thermal effect and velocity under different temperatures.
Fig. 5 A sample of friction coefficient over time with stationary duration (starting from $t=80s$, with a duration of 50s).

The red circle denote the first spike after the stationary period. The cyan rectangle represents the visible stationary period.

Fig. 6 shows the calculated $F_s - t_s$ relationship and the corresponding fitting curve (Every data point is calculated by averaging 100 simulation results, error bars are hidden but can be viewed in appendix section). The curve can be fitted well by logarithmic equation. In this model, this relationship is enabled by the frame of the Iwan-type model, which restricts the total number of individual contacts. The forming rate of individual contacts decreases toward 0 during stationary time, eventually leading to logarithm-dependence of $F_s$ on $t_s$.

Fig. 6 Data on the calculated $F_s - t_s$ relationship data and logarithm fitting curves. The lines with different kinds of symbols denote the calculated results of static friction force under different temperatures. The dash lines denote represent logarithm fitting curves.
3.2.1 Fretting response under oscillatory external force

Fig. 7 depicts a typical hysteresis loop under the quasi-static assumption. The tangential stiffness $k_t$ and the maximum displacement of driven surface $D_{max}$ are evaluated under different external forces, maximum displacement of driving point, and $f_s$. $k_t$ and $D_{max}$ are calculated as

$$k_t = \frac{y_B - y_A}{x_B - x_A} \quad (10)$$

$$D_{max} = x_D \quad (11)$$

In the previous studies about the Iwan-type model, the hysteresis loop is calculated under the assumption that the whole process is quasi-static. This assumption fails to exhibit the dynamic properties of the frictional interface and the time-evolution of hysteresis response cannot be described. In this section, we aim to simulate the dynamic process of frictional interface under oscillatory external force. To simulate the oscillatory external force from a real mechanical system, we use the scheme of external force from the experiment of Eriten et al. (Eriten et al., 2012). The displacement of the driving point is sinusoidal between a linear loading process and an unloading process. The motion of the driving point is:

$$\Delta(t) = \begin{cases} 
\Delta_1 t/t_1 = v_m t, & 0 \leq t < t_1 \\
\Delta_1 + \Delta_{max} \sin(2\pi f_s (t - t_1)) , & t_1 \leq t \\
(\Delta(t = t_2) - (t-t_2)/(t_3-t_2), & t_2 \leq t \leq t_3
\end{cases} \quad (12)$$

where $\Delta_1$, $t_1$ and $v_m$ are the linear loading displacement, duration and velocity; $\Delta_1 + \Delta_{max}$ is the absolute value of the maximum displacement of the driving point; $f_s$ is the frequency of the sinusoidal displacement; $t_1$ and $t_2$ are the displacement of the driving point at the end of oscillatory motion and the corresponding $t$. The value of external pressure $P$ cannot be directly determined. Thus, we study the relationship between $P$ and other variables by adjusting $f_s$, and we assume that $\Delta f_s = 1/3 f_s$. In ref (Eriten et al., 2012), $\Delta_1 = \Delta_{max}$. By contrast, in this paper we set them as two different values to examine the influence of these parameters. The parameters we use in this section are dimensionless, and the default values are: $n=1000$, $K=40$, $f_s=0.01$, $\Delta_1 = 0.25$, $t_1 = 10$, $\Delta_{max} = 0.1$, $2\pi f_s = 0.1$, $t_2 = t_1 + 6\pi$, $t_3 = t_2 + t_1$, and $v_m = 0.03$.

![Fig. 7 A typical hysteresis loop under the quasi-static assumption. The portion between $F(x_A,y_A)$ and $B(x_B,y_B)$ can be regarded as the linear response region where the displacement increases without the individual contacts (or a...](image-url)
small portion of individual contacts) rupturing. The $F$-$D$ relation can be considered linear. The portion between $B(x_B, y_B)$ and $C(x_C, y_C)$ is the transition stage from the linear response region to gross-slip where the individual contacts gradually rupture and finally leads to the gross-slip between $C(x_C, y_C)$ and $D(x_D, y_D)$.

Fig. 8 shows the spring force, driving displacement and displacement (the displacement of driven surface) over time in a fretting process. Obvious inconsistencies are observed between the driving displacement and the displacement because of the systematic stiffness $K$, which causes the delay between the driving motion and the frictional motion (Heslot et al., 1994). The whole region is divided into three sub-regions by two dash lines, namely, the linear loading, oscillatory loading and linear unloading regions from left to right. We consider the “spring force” as the “tangential force $F$” to depict the tangential force-displacement phase diagram (hysteresis loop). The $F$-$D$ hysteresis loops corresponding to Fig. 8 are shown in Fig. 9. The first hysteresis loop is more irregular because the individual contacts are all in stick state at $t=0$, causing strong fluctuations in tangential force and energy dissipation during partial slip. By contrast, the second and third loops are stable. For better comparison, we choose the second and third loops to evaluate the tangential stiffness, hysteresis displacement and energy dissipation during the hysteresis response. The tangential stiffness is measured as the slope of the purple line, which fits the left side of the “irregular parallelogram”.

Fig. 8 Friction and external forces over time: The red, black and green curves denote the driving displacement, displacement of the driven surface and the force on the spring, respectively.

Fig. 9 Corresponding hysteresis loops of Fig. 8. The black, cyan, green and red curves denote the parts of phase diagram in linear loading and unloading, first loop, second loop and third loop, respectively.
Fig. 10 shows the $F$-$D$ hysteresis loops under the conditions of $t_1=5,10,15$s ($f_s = 0.003$, $t_2 = t_1 + 2\pi$). The hysteresis loop moves to the right by increasing the linear loading and unloading durations. Fig. 11 displays the hysteresis loop under the condition of $f_s = 0.02$, $t_2 = t_1 + 72\pi$. The hysteresis loop decays into a line representing a reciprocating motion without energy dissipation caused by contact rupturing, and the displacement $D$-velocity $D'$ phase graph shows that the trajectory converges toward a torus (limited cycle) (Fig. 12). The position of the driven surface approaches and vibrates around a point where the oscillatory driving displacement does not induce energy dissipation by contact rupturing.

Fig. 10 $F$-$D$ hysteresis loops under different durations of monotonic linear loading and unloading process (recorded time: $t_1 \leq t \leq t_1 + 2\pi$). The red, green and blue curves denote the hysteresis loop under the condition of $t_1=5,10,15$s. The parts of the curves in the linear loading and unloading process are hidden.

Fig. 11 $F$-$D$ hysteresis loops under the condition of $f_s = 0.02$, $t_2 = t_1 + 144\pi$, loop $X$ means the hysteresis loop during $t = (X - 1)/f_{os} ~ X/f_{os}$.
Fig. 12 D-D’ phase diagram corresponding to Fig. 10. The black, green and red curves denote the phase diagram during the loops 2, 4, 36.

The Poincaré phase diagram corresponding to Fig. 12 is displayed in Fig. 13 to determine numerically if the motion has the characteristic of limited circle. The phase points before loop 72 is scattered across the plane, but the points after loop 72 converge to a point, indicating that the motion has the characteristic of a limited cycle (single period motion).

Fig. 13 Poincaré phase diagram corresponding to Fig. 12, where the phase points converge to a point (blue points to a black point) after about loop 72.

The relationship between energy dissipation and maximum oscillatory displacement under different $f_s$ is shown in Fig. 14 ($t_1 = 0, t_2 = 6\pi$). The energy dissipation $W$ is calculated as the mean value of the amounts of energy dissipated during the second and third loops. Each data point is calculated by averaging the results from 10 runs (The length of the error bars (standard deviations) are shown in Fig. 15). The curve of energy dissipation as it changes with maximum oscillatory displacement can be divided into two parts: a nonlinear part where only partial-slip happens and a linear part where gross-slip occurs. The first part of the curve shows that $W$ exponentially increases with maximum oscillatory displacement $\Delta_{\text{max}}$. By contrast, the second part illustrates that $W$ almost linearly increases with $\Delta_{\text{max}}$. Similar results are observed in ref (Eriten et al., 2011b; Wang et al., 2018). The relationship between the standard deviation of $W$ and $\Delta_{\text{max}}$ under the condition of $f_s = 0.01, 0.15, 0.02$ is shown in Fig. 15. The curves demonstrate that during the transition from the first part to the second part, the standard deviation initially increases sharply then
decreases. The reason for this phenomenon is that the motion is very sensitive to the randomness of rupturing of individual contacts during the transition, indicating that even a minute perturbation during the rupturing process may result in a considerable difference as a consequence.

Fig. 14 Relationships between energy dissipation and maximum oscillatory displacement under different $f_s$. The black, red, magenta, cyan and green curves denote the relationship between energy dissipation and maximum oscillatory displacement under the condition of $f_s=0.01$, 0.15, 0.02, 0.025 and 0.03, respectively.

Fig. 15 Standard deviation corresponding to Fig. 13. The black, red, and magenta lines denote the relationship between the standard deviation of energy dissipation and maximum oscillatory displacement under the conditions of $f_s=0.01$, 0.15, and 0.02, respectively.
Fig. 16 Chronological order of rupture number. The blue, green and red lines denote the rupture numbers over time in three different processes under the same parameter setting ($f_s=0.02$).

Fig. 17 exhibits the hysteresis loops under different $f_s$. As $f_s$ increases, the maximum tangential force increases, whereas the maximum tangential displacement decreases. These two phenomena are consistent with the conclusions in ref (Eriten et al., 2012; Eriten et al., 2011a). The tangential stiffness remains almost constant under different $f_s$. These results are consistent with those obtained by experiments on smooth steel-steel and aluminum-aluminum interface. However, the prediction fails under several experimental conditions such as rough aluminum-aluminum interface, where the tangential stiffness shows an evident increase when the external pressure increases.

Fig. 17 F-D hysteresis loop under different $f_s$ (recorded time: $t_1 + 2\pi \leq t \leq t_1 + 6\pi$). The blue, green and red curves denote the hysteresis loops under the condition of $f_s=0.015$, 0.01 and 0.003, respectively. Only the second and third loops are presented.

Using Gaussian distribution law to generate the rupture thresholds of individual contacts fails to predict the obvious positive relationship between the average rupture threshold (or static friction force of individual contacts) and the tangential stiffness of several experimental results. However, if we use the power law distribution:
\[
\phi(f_{si}^*) = \begin{cases} 
(f_{si}^*)^{-\chi}, & f_{si\min}^* < f_{si}^* \\
0, & \text{else}
\end{cases}
\]  

(13)

where \( f_{si}^* = f_{si}/b + c \), in which \( b \) and \( c \) are constant values. The distribution law we use herein has a lower bound \( f_{si\min}^* > 0 \) because the power distribution diverges at \( f_{si}^* = 0 \). \( \chi \) is the power exponent with \( \chi > 2 \), and \( f_{si\max}^* \) is the maximum value of \( f_{si}^* \), the mean value of \( f_{si}^* \) can be calculated as:

\[
u_w^* = \frac{f_{si\min}^{2-\chi}}{\chi - 2}
\]  

(14)

where \( u_w^* = f_S/b + c \).

\( b \) and \( c \) can be calculated with the following equation:

\[
\int_{f_{si\min}}^{+\infty} \phi(f_{si}^*)d(f_{si}^*) = 1
\]  

(15)

Fig.18 shows the hysteresis loops under the power law distribution of \( f_{si}^* (\chi = 3, b = 2, c = 2, \Delta_1 = 0, t_1 = 0) \). The model can predict the positive regulation between tangential stiffness and contact stiffness under power law distribution \( \chi = 3 \). When \( \chi > 2 \), similar results are also obtained. The mean value of the rupture thresholds of the individual contacts is the same as that used in Fig.16. However, the energy dissipations of the cycles produced under the power law distribution of \( f_{si} \) are considerably smaller than the simulations under Gaussian distribution of \( f_{si} \) because the individual contacts with a large rupture threshold are easier to generate. The disparate results obtained by the different schemes of \( f_{si} \) demonstrate that choosing the distribution law of threshold values of individual contact forces is important in establishing of the Iwan-type multi-contact model, which should be decided by the application object. For example, in predicting a three-dimensional lap joint interface, the rupture thresholds of individual contacts can be considered as the same to simplify the calculation(Song et al., 2003). For a bolt joint with obvious surface roughness effect, using the power law distribution leads to good agreement with experimental results (Wang et al., 2018).
A contact pair between two rough surfaces is simulated via the finite element method in ANSYS-Workbench (ANSYS Inc, Canonsburg, PA, USA). The surface roughness of the upper and lower surfaces is generated as $Ra=0.8$ ($Ra$ is the profile arithmetic average error) to simulate dry friction under the strong influence of surface roughness. The properties of the upper and lower surfaces are based on a commercial stainless steel and a bearing steel (listed in table 2, 3). The lower surface is fixed, and the upper surface is loaded by constant force $P$ on the top of the upper surface and driven at a constant velocity of 0.471m/s in the X direction during the frictional process.

For comparison, the friction force-displacement curves produced by the proposed method are also presented. A simplified scheme can be used to determine the parameters used in the proposed method:

$$f_s = P\mu s/n$$

(16)

where $\mu s$ is the macro static friction between two surfaces. According to commercial handbooks, $\mu s = 0.15$. The number of individual contact $n$ is determined as $n = 600$. Thus, $f_s$ can be determined. The distribution of $f_{si}$ follows the power law distribution with $\chi = 3$. Every curve is obtained by averaging the results from 300 runs to eliminate the influence of randomness.

Figs. 19 and 20 present the model and the friction force, respectively, during the transition from the stick state to the slip state. Tangential stiffness and maximum friction force increase with external pressure. This result is consistent with that obtained by the multi-contact model in the present study (Using the power law distribution with $\chi = 3$ of $f_{si}$) and with that achieved by previous experiments of two rough surfaces in frictional contact. However, for two smooth surfaces in frictional contact, the hysteresis response shows that the tangential stiffness remains almost the same with the increase of external load both in a previous FEM simulation (Wang et al., 2018) and experiment (Eriten et al., 2011a). Thus, the relationship between external load and tangential stiffness in the frictional behavior of two smooth surfaces is better described by this model under the Gaussian distribution of $f_{si}$. However, the proposed method only provides a coarse comparison because the roles of surface geometry and asperity deformation are neglected. The results also show obvious quantitative deviation between the FEM simulation and the proposed method. This deviation reveals that the simple spring structure of Jenkins elements cannot fully simulate the complex interaction between asperities. We aim to introduce the influence of surface geometry into this dynamic Iwan-type model in a follow-up study.

Table 2 Properties determined to simulate the upper surface.

| Property                   | Value                  |
|----------------------------|------------------------|
| Density                    | 7750Kg/m³              |
| Young’s Modulus            | 1.93×10¹¹ GPa          |
| Poisson’s ratio            | 0.31                   |
| Yield strength             | 2.1×10⁸ Pa             |
| Tangential modulus         | 1.8×10⁹ Pa             |
| Size                       | 0.31mm×0.31mm          |

Table 3 Properties determined to simulate the lower surface.

| Property                   | Value                  |
|----------------------------|------------------------|
| Density                    | 7850Kg/m³              |
| Young’s Modulus            | 2×10¹¹ GPa             |
| Poisson’s ratio            | 0.3                    |
| Yield strength             | 2.5×10⁸ Pa             |
| Tangential modulus         | 1.45×10⁹ Pa            |
| Size                       | 0.62mm×0.31mm          |
Fig. 19 FEM model. The upper surface is first compressed with a constant external pressure on the top, and then move at a constant velocity in the X-direction. The lower surface is fixed during the frictional process.

Fig. 20 Transitions from partial-slip to gross-slip under different external loads. The black, red, blue, and magenta lines denote the relationship between friction force and displacement under the external loads of 3, 4, 5, and 6 N produced by the FE method, respectively. The heavy black, red, blue, and magenta lines depict the relationship between friction force and displacement under the external loads of 3, 4, 5, and 6 N produced by the proposed method, respectively.

4. Conclusions

In this paper, dynamic stick-slip and creep are satisfactorily described without introducing residual stiffness by establishing a dynamic Iwan-type model with rupturing-forming law of individual contacts.

Stick-slip is accompanied by the periodic behavior of concentrated ruptures of individual contacts, and the thermal
effect considerably advances and increases the rate of ruptures of individual contacts. The model exhibits good prediction of logarithm-dependence of nominal static friction force on static duration time, which is a phenomenon the quasi-static Iwan-type model cannot describe. The hysteresis loops under different linear loading periods show that the length of linear loading period changes the position of the loop in the coordinate system. The displacement $D$-velocity $D'$ phase diagram shows a limited cycle after a certain amount of fretting loops, indicating that the fretting response will gradually decay into a motion without the rupturing of individual contacts. The relation between the energy dissipation and the maximum oscillatory displacement has two parts: a nonlinear part and a linear part. The standard deviation substantially increases and then decreases during the transition between these parts. The maximum friction force and the maximum tangential displacement during hysteresis response increases and decreases, respectively, with the averaged rupture threshold of the individual contacts. The positive relationship between tangential stiffness and external pressure of two rough surfaces in frictional contact is better described under the power law distribution of rupture thresholds of individual contacts $f_{sl}$ compared with previous experimental results and FEM comparison.

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**Appendix**

The error bars of Fig.4 and 6 are hidden for better view, but the length of the error bars is shown in this section:

![Fig. 21 Lengths of the error bars corresponding to Fig.4.](image-url)
Fig. 22 Lengths of the error bars corresponding to Fig. 6.