Acoustic instability in the neutral precursor region of collisionless shocks propagating into partially ionized plasmas

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ABSTRACT
Recent studies about collisionless shocks in partially ionized plasmas showed that some of neutral particles leak into the shock upstream region from the downstream region. In this paper, we perform a linear analysis and show that acoustic waves are unstable in the neutral precursor region. The acoustic instability amplifies fluctuations of magnetic field and density in the upstream region. The fluctuations are indispensable for the diffusive shock acceleration and could be important for the downstream turbulence.

Key words: plasmas – instabilities – supernova remnants – shock waves – acceleration of particles – cosmic rays.

1 INTRODUCTION
Galactic cosmic rays (CRs) are thought to be accelerated by collisionless shocks driven by a supernova explosion (Axford et al. 1977; Krymski 1974; Bell 1978; Blandford & Ostriker 1978). Recent observations of supernova remnants (SNRs) support the CR acceleration in SNRs (Kovama et al. 1993; Ohira et al. 2011). Although the second order acceleration by turbulence is a plausible acceleration mechanism for low energy CRs (Ohira 2013), the diffusive shock acceleration (DSA) is thought to be the most plausible acceleration mechanism. In order to quickly accelerate particles, DSA needs strong magnetic field fluctuations compared to that in interstellar medium. Interestingly, it has been shown that particles accelerated by DSA can amplify the magnetic field fluctuations in the shock upstream region (for a recent review, see Schure et al. 2012). Although there are many studies about the magnetic field amplification in the shock upstream region, saturation of the amplification is still an open problem.

The interstellar medium around SNRs is not always completely ionized. Ions produced by ionization of neutral particles excite several plasma instabilities and amplify the magnetic field fluctuations (Raymond et al. 2003; Ohira et al. 2009b; Ohira & Takahara 2010). In addition, Blasi et al. (2012); Ohira (2012) proposed that some of downstream neutral particles leak into the upstream region and the leaking neutral particles change the structure of shocks propagating into partially ionized plasmas. Ohira (2013) showed the leakage of neutral particles into the upstream region by hybrid particles simulations. Moreover, Ohira (2013b) observed plasma instabilities in the upstream and downstream regions. Interestingly, the observed mode in the upstream region (neutral precursor region) is not any modes expected from kinetic instabilities, but the fast magnetosonic mode. The physical mechanism of the instability that excites the fast magnetosonic mode was not discussed in Ohira (2013b). Therefore, we investigate the excitation mechanism of the fast magnetosonic mode observed in the hybrid simulation of Ohira (2013) in this paper.

We first summarize the simulation results of Ohira (2013) and provide a background plasma condition for a linear analysis (section 2). Then, we perform the linear analysis of the fast magnetosonic mode with ionization of the leaking neutral particles (section 3) and discuss the physical mechanism of the instability and impacts on the magnetic field amplification (section 4).

2 BACKGROUND CONDITION IN THE NEUTRAL PRECURSOR REGION
In this section, we summarize simulation results of Ohira (2013b), providing a background plasma condition for a linear analysis in the neutral precursor region. Ohira (2013b) performed a two-dimensional hybrid simulation in order to investigate a perpendicular collisionless shock propagating into partially ionized plasmas. The simulation solved motion of protons and hydrogen atoms as particles, Maxwell’s equations, collisional ionization and charge exchange. It was shown by the simulation that some of downstream neutral particles leak into the shock upstream region, the leaking
neutral particles are ionized and become pickup ions in the upstream region, the pickup ions are preferentially accelerated, and the upstream flow is decelerated by the pickup ions. In the upstream rest frame, the mean velocity of leaking neutral particles is of the order of the shock velocity. In addition, the fast magnetosonic mode is excited in the neutral precursor region. The wavelength of the observed mode is about the gyro radius of the pickup ions, that is much smaller than the precursor length scale. The propagation direction and the amplitude are the direction of the shock normal and $\delta B/B_0 \approx \delta p/\rho_0 \approx 0.5$, respectively.

After the leaking neutral particles are ionized, the ionized particles start to gyrate around the upstream magnetic field with a relative velocity between the upstream flow and the leaking neutral particles. Then, ionized particles become hot pickup ions. The pickup ions have the pressure anisotropy, that is, the pressure perpendicular to the magnetic field line is larger than that parallel to the magnetic field line. The pressure anisotropy can excite the slow magnetosonic mode and the Alfvén mode (Raymond et al. 2008), but cannot excite the fast magnetosonic mode observed in the simulation. The Drury instability can excite the fast magnetosonic mode if there are diffusive particles and the pressure gradient of the diffusive particles (Drury & Falle 1986; Chalov 1988). However, the simulation results showed that there is no diffusive particle in the upstream region.

In this paper, we consider magnetohydrodynamic equations with injection of a hot plasma due to collisional ionization of the leaking neutral particles. We consider only collisional ionization because essential effects of charge exchange are the same as that of collisional ionization. We use Cartesian coordinates in which the shock normal is along the $x$ axis. We consider a one-dimensional flow in the $x$ direction and the magnetic field in the $z$ direction, where all values depend on position, $x$, and time, $t$. Then, basic equations are given by

\[
\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x} (\rho u) = \rho_n v_i , \tag{1}
\]

\[
\frac{\partial}{\partial t} (\rho u) + \frac{\partial}{\partial x} \left( \rho u^2 + P + \frac{B^2}{2\pi} \right) = \rho_n u_n v_i , \tag{2}
\]

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 + \frac{P}{\gamma - 1} + \frac{B^2}{2\pi} \right) + \frac{\partial}{\partial x} \left( \frac{1}{2} \rho u^2 + \frac{\gamma}{\gamma - 1} P + \frac{B^2}{4\pi} u \right) = \frac{1}{2} \rho_n u_n^2 v_i , \tag{3}
\]

\[
\frac{\partial}{\partial t} B + \frac{\partial}{\partial x} (Bu) = 0 , \tag{4}
\]

\[
\frac{\partial}{\partial t} \rho_n + \frac{\partial}{\partial x} (\rho_n u_n) = -\rho_n v_i , \tag{5}
\]

where $\rho, \rho_n, u, u_n, P, \gamma$ and $B$ are the density of plasma, the density of leaking neutral particles, the velocity of plasma, the velocity of leaking neutral particles, the pressure, the adiabatic index of plasma, and the magnetic field, respectively. The velocity of the leaking neutral particles, $u_n$, can be assumed to be constant because they do not interact with the electromagnetic field and the momentum change is negligible. The ionization frequency, $\nu_i$, is given by

\[
\nu_i = \frac{\sigma_{\text{rel}}}{m} \left( u_{\text{rel}} \right) , \tag{6}
\]

where $\rho/m, u_{\text{rel}} \equiv |u_n - u|$, and $\sigma_{\text{rel}}(u_{\text{rel}})$ are the number density of ions, the relative velocity between plasma and the leaking neutral particles, and the cross section of collisional ionization, respectively. For simplicity, in equation (3), we assume that leaking neutral particles are cold, but this assumption is not crucial.

Strictly speaking, the mean profiles of plasma density and velocity slightly change in the neutral precursor region but they are almost constant because the leaking flux is only a few percent of the flux of the upstream flow in the shock rest frame (Ohira 2013). However, the pressure significantly changes in the neutral precursor region for high Mach number shocks (Ohira 2012). In addition, we consider perturbations with a length scale smaller than that of the neutral precursor, $\sim u_n \nu_i^{-1}$. Therefore, for simplicity, we assume that background fields are spatially uniform and constant with time except for the plasma pressure, $P$. Then, the background uniform solutions in the upstream rest frame are as follows.

\[
\rho = \rho_0 , \quad \rho_n = \rho_{n,0} , \tag{7}
\]

\[
u = 0 , \quad u_n = u_{n,0} , \tag{8}
\]

\[
B = B_0 , \tag{9}
\]

\[
\nu_{i,0} = \frac{\rho_0}{m} u_{n,0} \sigma_{\text{rel}}(u_{n,0}) , \tag{10}
\]

\[
P(t) = P_0 + \frac{\gamma - 1}{2} \rho_{n,0} u_{n,0}^2 \nu_{i,0} t . \tag{11}
\]

The phase velocity of the fast magnetosonic mode is given by

\[
u_{\text{ph}}(t) = \sqrt{\frac{\gamma P(t)}{\rho_0} + \frac{B_0^2}{4\pi \rho_0}} = u_{\text{ph},0} \sqrt{1 + \frac{t}{t_c}} , \tag{12}
\]

where $u_{\text{ph},0} = u_{\text{ph}}(0)$ and $t_c$ is given by

\[
t_c = \frac{2}{\gamma (\gamma - 1) \xi} \left( \frac{u_{\text{ph},0}}{u_{n,0}} \right)^2 \nu_{i,0}^{-1} , \tag{13}
\]

where $\xi = \rho_{n,0}/\rho_0$ is the density ratio and less than about 0.1, that depends on the shock velocity and the electron temperature in the shock downstream region. The pressure and the phase velocity can be assumed to be constant for $t < t_c$. Note that $t_c$ is much smaller than the ionization time scale, $\nu_{i,0}^{-1}$; for high Mach number shocks because the velocity of leaking neutral particles, $u_{n,0}$, is of the order of the shock velocity in the upstream rest frame.

### 3 LINEAR ANALYSIS

In this section, we perform a linear analysis of equations (4)-(11) by using the background solutions. We do not consider perturbations of the leaking neutral particles. They are advected with the velocity of $u_{n,0}$. On the other hand, perturbations of plasma propagate with the phase velocity smaller than $u_{n,0}$, so that we can approximate that the perturbations of plasma are decoupled from perturbations of the leaking neutral particles. Then, the linearized perturbation equations are given by
where the perturbation is assumed to be $\rho_0$.

Furthermore, the above equation does not explicitly contain $t < t_c$ for $u_{n,0} = 10$ and $\gamma = 5/3$. The red solid line shows the growth rate of the most unstable mode for $d \log \nu_1 / d \log u_{n,0} = 1$ and $-1$, respectively.

The above equations (14)–(17) can be reduced to

$$0 = \left\{ \frac{d}{dt} \partial_t \delta \rho + \rho_0 \frac{\partial}{\partial x} \delta u = \rho_0 \left( \frac{d \nu_0}{d \rho} \delta \rho - \frac{d \nu_0}{d u_{n,0}} \delta u \right) \right\} ,$$

$$\partial_t \delta B + B_0 \frac{\partial}{\partial x} \delta u = 0 .$$

The perturbation is assumed to be $\delta u \propto \exp(ikx)$. Note that $-1 \leq d \log \nu_0 / d \log u_{n,0} \leq 1$ for the shock velocity of the order of 2000 km/s (Heng & McCray 2007). Furthermore, the above equation does not explicitly contain the magnetic field and it contributes only the phase velocity. Therefore, the magnetic field is not directly important to the evolution of the perturbation.

### 3.1 For $t < t_c$

For $t < t_c$, the phase velocity, $u_{ph}(t)$, can be assumed to be constant, $u_{ph,0}$, so that the perturbation can be assumed to be $\delta u \propto \exp(ikx)$. Then, the dispersion relation of equation (18) is given by

$$0 = \omega^3 - u_{ph,0}^2 k^2 \omega .$$

Three solutions of the dispersion relation ($\omega = \omega(k)$) can be analytically obtained by using the Cardano’s method. Then, we find that the fast magnetoacoustic modes propagating parallel and antiparallel to the leaking neutral particles are unstable, and the mode propagating parallel to the leaking neutral particle is the most unstable mode.

Fig. 1 shows the growth rates of the most unstable mode for $u_{n,0}/u_{ph,0} = 10$ and $\gamma = 5/3$. The red, green, and blue lines show the growth rates for $d \log \nu_1 / d \log u_{n,0} = 1, 0,$ and $-1$, respectively. The growth rates increase with the wavenumber. It should be noted that the dispersion relation is not valid for a length scale smaller than the gyroradius of pickup ions and for a time scale smaller than the gyroperiod of pickup ions because the fluid approximation is not valid for the pickup ions.

Fig. 2 is the same as Fig. 1 but for $u_{n,0}/u_{ph,0} = 100$. The red solid line shows the growth rate of the most unstable mode for $d \log \nu_1 / d \log u_{n,0} = 1$. The black dotted and dashed lines show the asymptotic solution of the growth rate for $k < k_c$ and $k > k_c$, respectively.

$$\text{Im}[\omega] = \frac{\gamma + 1}{2 \gamma t_c} \left\{ \begin{array}{ll} \frac{k^3}{2} & (\text{for } k < k_c) \\ \frac{1}{2} & (\text{for } k > k_c) \end{array} \right.$$
where the characteristic wavenumber $k_c$ is given by

$$k_c = \frac{\gamma + 1}{\gamma \nu_i u_{ph,0}} = \frac{\xi (\gamma^2 - 1)}{2} \left( \frac{u_{n,0}}{u_{ph,0}} \right)^3 \left( \frac{\nu_i}{\Omega_c} \right)^{-\frac{3}{2}} \Omega_c^{-1},$$

(21)

where $\Omega_c$ and $r_g = u_{n,0}/\Omega_c$ are the cyclotron frequency and the gyroradius of pickup ions, respectively. $\nu_i/\Omega_c \approx 10^{-5}$ and $u_{n,0}/u_{ph,0} \approx 10^2$ for young SNRs in the typical interstellar medium, and $\xi \lesssim 0.1$, so that the length scale of $k_c$ is comparable to or larger than the gyroradius of pickup ions, $r_g$, for young SNRs.

The shortest growth time scale is of the order of $t_c$. However, equation (16) is valid for $t < t_c$, so that the exponential growth cannot be expected for a long time. We calculate a long time evolution in the next subsection.

### 3.2 For $t > t_c$

In order to understand a long time evolution of perturbations, we solve equation (15) by numerical simulations. Fig. 3 shows the numerical results for $ku_{ph,0} t_c = 1$, $u_{n,0}/u_{ph,0} = 100$, and $\gamma = 5/3$. The red, green, and blue lines show the results for $d \log \nu_i / d \log u_{n,0} = 1$, 0, and $-1$, respectively. We adopt a normal fast magnetosonic mode propagating parallel to the leaking neutral particles as an initial perturbation, that is, $\delta u \propto \exp[i(kx - ku_{ph,0} t)]$. The transit time in the neutral precursor region is about the ionization time scale, $\nu_i^{-1}$, that is about $10^3 t_c$ for $u_{n,0}/u_{ph,0} = 100$.

The amplitudes slowly increase with time. Therefore, the fast magnetosonic mode is still unstable even when $t > t_c$. Three lines are almost the same in the early phase, but in the late phase, the amplitude for the case of $d \log \nu_i / d \log u_{n,0} = -1$ grows slightly faster than that for the other cases. These behaviors can be understood from the dispersion relations shown in Figs. 1 and 2. The dispersion relation does not depend on the velocity dependence of the ionization frequency, $d \log \nu_i / d \log u_{n,0}$ for a large $u_{n,0}/u_{ph}$, but it depends on $d \log \nu_i / d \log u_{n,0}$ for a small $u_{n,0}/u_{ph}$. The phase velocity, $u_{ph}(t)$, increases with time for $t > t_c$. Therefore, the evolution of the amplitude depends on $d \log \nu_i / d \log u_{n,0}$ in the late phase.

Fig. 4 shows the numerical results for $d \log \nu_i / d \log u_{n,0} = -1$, $u_{n,0}/u_{ph,0} = 100$, and $\gamma = 5/3$. The grey, blue, and magenta lines show the numerical results for $ku_{ph,0} t_c = 10$, 1, and 0.1, respectively. The evolution hardly depends on the wavenumber for $k > k_c \approx (u_{ph,0} t_c)^{-1}$, but the growth rates increase with the increasing the wavenumber for $k < k_c$. These behaviors are also the same as that of the dispersion relations shown in Figs. 1 and 2. Therefore, for the young SNRs, the mode with $k = k_c \approx r_g^{-1}$ has the maximum growth rate even when $t > t_c$. We again note that the numerical solutions are not valid for a wavelength smaller than the gyroradius of pickup ions because the fluid approximation is not valid for the pickup ions.

### 4 DISCUSSION

First, we discuss the physical mechanism of the acoustic instability. For $t < t_c$, as mentioned in the section 3.1, the driving force is the energy injection due to ionization of leaking neutral particles. The first term of the right hand side of equation (16) means that leaking neutral particles are more ionized in denser regions and their kinetic energy is more injected in the denser regions but the mass injection due to ionization is negligible. Then, the pressure gradient between low density regions and high density regions becomes large compared with that before injection of ionized particles. Therefore, the amplitude of the acoustic wave grows with oscillation.

For $t > t_c$, the phase velocity, $u_{ph}(t)$, becomes comparable to the velocity of leaking neutral particles, $u_{n,0}$, so that all the terms in equation (15) become comparable. Therefore, in addition to the energy injection due to ionization, the mass and momentum injection due to ionization also contributes the acoustic instability. Furthermore, not only the density perturbation but also the velocity perturbation (the

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**Figure 3.** Time evolution of the velocity perturbation at $x = 0$ for $ku_{ph,0} t_c = 1$, $u_{n,0}/u_{ph,0} = 100$, and $\gamma = 5/3$. The red, green, and blue lines show the numerical results for $d \log \nu_i / d \log u_{n,0} = 1$, 0, and $-1$, respectively.

**Figure 4.** Time evolution of the velocity perturbation at $x = 0$ for $d \log \nu_i / d \log u_{n,0} = -1$, $u_{n,0}/u_{ph,0} = 100$, and $\gamma = 5/3$. The grey, blue, and magenta lines show the numerical results for $ku_{ph,0} t_c = 10$, 1, and 0.1, respectively.
second terms of the right hand side of equations (14)-(16) contributes the acoustic instability.

The acoustic instability could be important for other instabilities (e.g. the Drury instability and parametric instabilities) and the magnetic field amplification in the CR precursor (Beresnyak et al. 2009; Drury & Downes 2012) and downstream regions (Giacalone & Jokipii 2007; Inoue et al. 2007; Guo et al. 2012) as a seed of perturbations with a small length scale.

We did not take into account gradient of background velocity and density in the neutral precursor region. Ohira (2013b) showed that the gradient is very small for the shock velocity of about 3000 km/s. For slower shock velocity, it is expected that more neutral particles leak into the upstream region and the gradient in the neutral precursor region becomes larger (Blasi et al. 2012; Ohira 2012). In this case, a wave-action analysis like Drury & Falle (1986) is needed to properly analyze the acoustic instability. Furthermore, we did not take into account kinetic effects of pickup ions in this paper. These issues will be addressed in future work.

5 SUMMARY

In this paper, we have showed that the fast magnetosonic mode is unstable in the neutral precursor region where neutral particles are leaking from the downstream region. The acoustic instability has already been observed in a hybrid simulation for collisionless shocks propagating into partially ionized plasmas (Ohira 2013b). In this paper, we provided the physical mechanism and linear analysis. The instability could be important for other instabilities as a seed of perturbations with a small length scale.

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