Strongly interacting photons in asymmetric quantum well via resonant tunneling

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Abstract: We propose an asymmetric quantum well structure to realize strong interaction between two slow optical pulses. The essential idea is the combination of the advantages of inverted-Y type scheme and resonant tunneling. We analytically demonstrate that giant cross-Kerr nonlinearity can be achieved with vanishing absorptions. Owing to resonant tunneling, the contributions of the probe and signal cross-Kerr nonlinearities to total nonlinear phase shift vary from destructive to constrictive, leading to nonlinear phase shift on order of $\pi$ at low light level. In this structure, the scheme is inherent symmetric for the probe and signal pulses. Consequently, the condition of group velocity matching can be fulfilled with appropriate initial electron distribution.

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1. Introduction

Photons are ideal carriers of quantum information as they do not interact strongly with their environment and can be transmitted over long distances [1, 2]. Realizing efficient nonlinear interactions between single photons is considered a key step towards all-optical quantum computation and quantum information processing. While nonlinear effect whereby one light beam influences another requires large numbers of photons or else photon will be confined in a high-$Q$ cavity. Hence the major obstacle to construct scalable and efficient quantum computation with photonic qubits is the absence of giant cross-Kerr nonlinearity capable of entangling pairs of photons. A promising avenue has been opened by studies of enhanced nonlinear coupling via electromagnetically induced transparency (EIT) [3, 4]. In a four-level $N$-type scheme, it was proposed that the ultrahigh sensitivity of EIT dispersion to the two-photon Raman detuning in the vicinity of an absorption minimum can be used to enhance cross-Kerr nonlinearity between two weak optical fields [4–6]. Subsequently, the proposal for $\pi$ phase shift and high-fidelity implementation of the controlled phase gate has been suggested by trapping the weak signal pulse in a photonic band gap [7–9]. Large cross-Kerr nonlinearity emerges when two optical pulses, a probe and a signal, interact for a sufficiently long time. This happens when their group velocities are both reduced and comparable [10–12]. In order to eliminate the mismatch between the slow group velocity of the probe pulse subject to EIT and that of the nearly free propagating signal pulse, versatile novel symmetric configurations have been suggested theoretically and experimentally to achieve strongly interacting photons and therefore realize the polarization qubit phase gate using an excited-doublet four-level atom [10–12].
Semiconductor heterostructures provide a potential energy well with a size comparable to the de Broglie wavelength, trapping the carriers in discrete energy levels resulting in objects with atom-like optical properties. Different from atomic system, the interaction between semiconductor heterostructures and optical fields is strongly enhanced with merits such as the large electric dipole moments due to the small effective electron mass. Moreover, the intersubband energies and the electron function symmetries can be engineered as desired in accordance with the requirement. These advantages create the opportunities of building opto-electron devices that harness atom physics. Another important motivation of such study comes from the drastic increase in applications because of the wide-spread use of semiconductor components in optoelectronics and quantum information science. As a consequence, there has been a fast growth of research activity aimed at studying the quantum interference effects in semiconductors, for examples, the strong EIT [30], tunneling induced transparency (TIT) [31], ultrafast optical switching with Fano interference [32], slow light [33], etc [34–38]. Nonlinear optical properties in semiconductor heterostructures have also been paid much attention such as ultraslow optical solitons with TIT [39, 40], enhancement of self-Kerr nonlinearity [41, 42], controlled phase shift up to $\pi/4$ in a single-quantum dot coupled to a photonic crystal nanocavity [43], giant cross-Kerr nonlinearity with spin-orbit coupling [44], and so on [45]. Recently, the realization of giant cross-Kerr nonlinear phase shift and the related quantum information processing (QIP) have been investigated in quantum well (QW) structures based on interband and intersubband transitions [46, 47].

In QW structure, resonant tunneling can induce not only transparency but also large cross-Kerr nonlinearity [48]. However, the probe and signal pulses cannot interact for a sufficiently long time. The main hindrance is the mismatch between the group velocities of the probe field that is subject to TIT and its nearly free propagating partner [48]. This fact limits its applications in all-optical switching with single photons and QIP. In the present paper, we suggest an alternative asymmetric QW structure, which combines resonant tunneling and the advantages of inverted-Y-type configuration. The linear optical properties and nonlinear optical responses including the cross-Kerr and self-Kerr nonlinearity are investigated. We analytically demonstrate, owing to resonant tunneling, that the cross-Kerr nonlinearities can be enhanced dramatically. More importantly, resonant tunneling modifies the contributions of cross-Kerr nonlinearities (the probe and signal pulses) to the total nonlinear phase from destructive to constructive. Our numerical calculation shows that nonlinear phase shift on order of $\pi$ can be realized at low light level. For the probe and signal pulses, the structure is an inherent symmetric configuration. Hence the condition of group velocity matching can be easily satisfied by adjusting the initial electron distribution.

2. **Structure and linear optical properties**

Our asymmetric double QW structure is shown in Fig. 1. The growth sequence of the structure from left to right is as follows. A thick $\text{Al}_{0.50}\text{Ga}_{0.50}\text{As}$ barrier is followed by an $\text{Al}_{0.10}\text{Ga}_{0.90}\text{As}$ layer with thickness of 8.8 nm (shallow well). This shallow well is separated from a 6.9 nm GaAs layer (deep well) on the right by a 3.8 nm Al$_{0.50}$Ga$_{0.50}$As potential barrier. Finally, a thin (2.4 nm) $\text{Al}_{0.25}\text{Ga}_{0.75}\text{As}$ barrier separates the deep well from the last $\text{Al}_{0.40}\text{Ga}_{0.60}\text{As}$ thick layer on the right. In this structure, one would observe the ground subbands of the right deep well [1] and the left shallow well [2] with energies 57.2 meV and 123.1 meV, respectively. The eigenenergies of the second excited subband of the left shallow well [5] is 385.9 meV. Two new subbands [3] and [4] with eigenenergies 224.1 meV and 231.4 meV are, respectively, created by mixing the first excited subbands of the shallow ($\text{|se}\rangle$) and deep ($\text{|de}\rangle$) wells by tunneling. Their corresponding wave functions are symmetric and antisymmetric combinations of $\text{|se}\rangle$ and $\text{|de}\rangle$, i.e., $|3\rangle = (\text{|se}\rangle - \text{|de}\rangle)/\sqrt{2}$ and $|4\rangle = (\text{|se}\rangle + \text{|de}\rangle)/\sqrt{2}$. The basic idea is to combine resonant
tunneling with the inherent symmetry of invert-Y-type configuration. To do so, we apply a weak pulsed probe field with frequency $\omega_p$ and a weak pulsed signal field with frequency $\omega_s$ to drive the transitions $|1\rangle \leftrightarrow |3\rangle$, $|1\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, $|2\rangle \leftrightarrow |4\rangle$, respectively. The subbands $|3\rangle$ and $|4\rangle$ are coupled with $|5\rangle$ by a continuous-wave control field with angular frequency $\omega_c$. Thus, an inverted-Y-type configuration with two-fold degenerate middle subbands is realized. Under the dipole and rotating-wave approximations (RWA), this structure is governed by a set of density matrix equations given below,

$$
\begin{align}
\sigma_{21} &= id_2 \sigma_{21} - i\Omega_p (\sigma_{23} + m\sigma_{24}) + i\Omega_s (\sigma_{31} + q\sigma_{41}), \\
\sigma_{31} &= id_3 \sigma_{31} + i\Omega_p (\sigma_{11} - \sigma_{33}) - im\Omega_p \sigma_{34} + i\Omega_s \sigma_{21} + i\Omega_c \sigma_{51}, \\
\sigma_{41} &= id_4 \sigma_{41} + im\Omega_p (\sigma_{11} - \sigma_{44}) - i\Omega_p \sigma_{43} + iq\Omega_s \sigma_{21} + i\Omega_c \sigma_{51}, \\
\sigma_{51} &= id_5 \sigma_{51} - i\Omega_p (\sigma_{33} + m\sigma_{34}) + i\Omega_c (\sigma_{31} + k\sigma_{41}), \\
\sigma_{32} &= id_3 \sigma_{32} + i\Omega_p (\sigma_{32} - \sigma_{33}) + i\Omega_p \sigma_{12} - iq\Omega_s \sigma_{34} + i\Omega_c \sigma_{52}, \\
\sigma_{42} &= id_4 \sigma_{42} + iq\Omega_s (\sigma_{22} - \sigma_{44}) + im\Omega_p \sigma_{12} - i\Omega_p \sigma_{32} + i\Omega_c \sigma_{52}, \\
\sigma_{52} &= id_5 \sigma_{52} - i\Omega_p (\sigma_{33} + q\sigma_{34}) + i\Omega_c (\sigma_{32} + k\sigma_{42}),
\end{align}
$$

where $d_{21} = \Delta_p - \Delta_s + i\gamma_{21}$, $d_{31} = \Delta_p + i\gamma_{31}$, $d_{41} = \Delta_p - \delta + i\gamma_{41}$, $d_{51} = \Delta_p + \Delta_c - \Delta_s + i\gamma_{51}$, $d_{32} = \Delta_c + i\gamma_{32}$, $d_{42} = \Delta_c - \delta + i\gamma_{42}$, $d_{52} = \Delta_c + i\gamma_{52}$ with $\Delta_p$, $\Delta_s$, and $\Delta_c$ being the detunings of the probe, signal and control fields with the corresponding transitions, and they are defined as $\Delta_p, (s,c) = \omega_p, (s,c) - (\omega_3, (3,5) - \omega_1, (2,3))$. $\delta = \omega_4 - \omega_3 \simeq 7.3$ meV denotes the energy difference between the subbands $|3\rangle$ and $|4\rangle$. Halves of the Rabi frequencies of the probe, signal and control fields are $\Omega_p = \vec{\mu}_{13} \cdot \vec{E}_p / 2\hbar$, $\Omega_s = \vec{\mu}_{23} \cdot \vec{E}_s / 2\hbar$, and $\Omega_c = \vec{\mu}_{33} \cdot \vec{E}_c / 2\hbar$ with $\vec{\mu}_{ij}$ being electric dipole momentum between subbands $|i\rangle$ and $|j\rangle$ ($i,j = 1-5$ and $i \neq j$), while $m = \mu_{41}/\mu_{31} = -0.73$, $q = \mu_{42}/\mu_{32} = 1.2$, and $k = \mu_{54}/\mu_{53} = 2.3$ give the ratios between the relevant subband transition dipole momentum. $E_p$, $E_s$, and $E_c$ are, respectively, the slowly varying electric field amplitudes of the probe, signal and control fields. The half linewidths are, respectively, given by $\gamma_{31} = \gamma_s + \gamma_{31}^{\text{dip}}$, $\gamma_{41} = \gamma_s + \gamma_{41}^{\text{dip}}$, $\gamma_{31} = \gamma_s + \gamma_{32}^{\text{dip}}$, $\gamma_{32} = \gamma_s + \gamma_{32}^{\text{dip}}$, $\gamma_{42} = \gamma_s + \gamma_{42}^{\text{dip}}$, $\gamma_{52} = \gamma_s + \gamma_{52}^{\text{dip}}$. Here $\gamma_s$ ($\gamma_c$, $\gamma_e$) is the electron decay rate of subband $|3\rangle$, $|4\rangle$, $|5\rangle$ and $\gamma_{ij}^{\text{dip}}$ the electron dephasing rates, which are introduced to account not only for intrasubband phonon scattering and electron-electron scattering but also inhomogeneous broadening due to scattering on interface roughness. The dipole transition rate from subband
that the probe and signal propagate with comparable group velocity. To investigate the group 

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of the group velocities of the probe and signal pulses on the initial electron distribution

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\[ \gamma_2 \approx \gamma_{21}^{\text{deph}} \], leading to the strengths of susceptibility associated with cross-Kerr nonlinearity are two orders of higher than those in Ref. [48] (details will be shown in next section). Electron decay rates can be calculated by solving effective mass Schrödinger equation [49]. For temperature up to 10 K and electron density smaller than 10^{12} \text{ cm}^{-2}, \gamma_{i}^{\text{deph}} \text{ can be estimated according to Ref. [31].}

If all electrons remain in ground subband \(|1\rangle\), which means that the signal field drives two virtually empty transitions, the contribution to the susceptibility comes only from higher order. It is hard to achieve group velocity matching since the asymmetry of configuration. Thus, as done in Ref. [44], we assume that electrons distribute not only in subbands \(|1\rangle\) but also in \(|2\rangle\). The symmetric configuration is hence formed. In the presence of the control field, the subbands \(|3\rangle, |4\rangle \text{ and } |5\rangle\) are mixed into three new subbands. The symmetry of scheme ensures that the probe and signal propagate with comparable group velocity. To investigate the group velocities of the probe and signal pulses, following the standard processes [26], we assume \(|\Omega_{p}\rangle, |\Omega_{i}\rangle \ll |\Omega|, |\Delta_{1}, \Delta_{2}, \Delta_{i}, \delta\rangle\text{ and solve the density matrix equations [Eqs. (1)-(7)] in the non-depletion approximation (} \sigma_{11} + \sigma_{22} \approx 1\) together with Maxwell’s equations and expand the linear dispersion relations as Taylor series around their center frequency. The group velocities of the probe and signal pulses are, respectively, given by

\[
\nu_{p}^g = 1/\text{Re}[K_{p}^{(1)}], \quad \nu_{s}^g = 1/\text{Re}[K_{s}^{(1)}],
\]

with

\[
K_{p}^{(1)} = \frac{1}{c} + \kappa_{p} \left\{ \frac{[d_{31}d_{41} + d_{31}d_{51} + d_{41}d_{51} - (1 + k^2)\Omega_{c}^2][d_{51}(d_{41} + m^2d_{31}) - (k - m)^2\Omega_{c}^2]}{[d_{31}d_{41}d_{51} - (d_{41} + k^2d_{31})\Omega_{c}^2]^2} - \frac{d_{41} + m^2d_{31} + (1 + k^2)d_{51}}{d_{31}d_{41}d_{51} - (d_{41} + k^2d_{31})\Omega_{c}^2} \right\},
\]

\[
K_{s}^{(1)} = \frac{1}{c} + \kappa_{s} \left\{ \frac{[d_{32}d_{42} + d_{32}d_{52} + d_{42}d_{52} - (1 + k^2)\Omega_{c}^2][d_{52}(d_{42} + m^2d_{32}) - (k - m)^2\Omega_{c}^2]}{[d_{32}d_{42}d_{52} - (d_{42} + k^2d_{32})\Omega_{c}^2]^2} - \frac{d_{42} + m^2d_{32} + (1 + k^2)d_{52}}{d_{32}d_{42}d_{52} - (d_{42} + k^2d_{32})\Omega_{c}^2} \right\},
\]

where \(\kappa_{p} = N\omega_{p}|\mu_{31}|^2|\sigma_{11}^{(0)}/(\hbar\epsilon_{0}c)|, \kappa_{s} = N\omega_{s}|\mu_{32}|^2|\sigma_{22}^{(0)}/(\hbar\epsilon_{0}c)|\) with \(N\) being the electron volume density, \(\sigma_{11}^{(0)}\) and \(\sigma_{22}^{(0)}\) initial electron distribution in subbands \(|1\rangle\) and \(|2\rangle\) with \(\sigma_{11}^{(0)} + \sigma_{22}^{(0)} = 1\). The crucial point about Eqs. (8), (9) and (10) is that the group velocities of the probe and signal pulses depend on the initial electron distribution \(\sigma_{11}^{(0)}\) and \(\sigma_{22}^{(0)}\). Such that it is, in principle, possible to control the group velocities by adjusting the initial electron distribution. This is totally different from Ref. [48]. The physics behind this is clear: the group velocity of the probe (signal) pulse is determined by the coherence \(\sigma_{31}\) and \(\sigma_{41}\) (\(\sigma_{32}\) and \(\sigma_{42}\)). The initial electron distribution governs the evolutions of these coherent terms, and hence the steady state solutions [details can be seen in Eqs. (11)-(15)]. The electron decay rates are \(\gamma_{3} \approx \gamma_{4} = 0.5 \text{ meV}, \gamma_{5} = 0.2 \text{ meV}\) (corresponding intrasubband relaxation time \(T_{1} \sim 10\) ps) [50] and \(\gamma_{i}^{\text{deph}} = \gamma_{4}^{\text{deph}} = \gamma_{5}^{\text{deph}} = 0.2 \text{ meV}\) [31]. We take the Rabi frequency and the detuning of the control field as \(\Omega_{p} = 1.5 \text{ meV}\) and \(\Delta_{p} = -5.3 \text{ meV}\). With \(\Delta_{p} = \Delta_{s} = 3.0 \text{ meV}\) (around the center of their transparency windows) and \(N = 5 \times 10^{-17} \text{ cm}^{-3}\), Fig. 2 illustrates the dependence of the group velocities of the probe and signal pulses on the initial electron distribution \(\sigma_{11}^{(0)}\).

By controlling the initial electron distribution (\(\sigma_{11}^{(0)} \approx 0.141\)), the probe and signal pulses will propagate with comparable and small group velocities (\(\nu_{p}^g = \nu_{s}^g \approx 1.0 \times 10^{6} \text{ m/s}\)). The initial electron distribution can be realized with stimulated Raman adiabatic passage [51].
because of the destructive interference between transition pathways, and can be safely ignored.

In the transparent window, the linear absorptions of the probe and signal fields are very small and signal pulses will propagate with comparable and small group velocities, and the influence of the transparent window are linearly proportional to their detunings. This means that the probe with this set of parameters, the dispersion of the probe and signal pulses around the center of

Fig. 2. (color online) The group velocities of the probe (solid) and the signal (dashed) pulses as functions of $\sigma_{11}^{(0)}$. The parameters are explained in the text.

The aim of the present contribution is to realize strong interaction between two weak pulses, we hence define the susceptibility as [12]

$$\chi_p = \frac{N|\mu_1|^2}{\hbar\epsilon_0} \frac{\sigma_{11} + m\sigma_{44}}{\Omega_p} \approx \chi_p^{(1)} + \chi_p^{(3,\text{SPM})} |E_p|^2 + \chi_p^{(3,\text{XPM})} |E_s|^2, \tag{11}$$

$$\chi_s = \frac{N|\mu_3|^2}{\hbar\epsilon_0} \frac{\sigma_{32} + q\sigma_{42} + \sigma_{42}}{\Omega_s} \approx \chi_s^{(1)} + \chi_s^{(3,\text{SPM})} |E_s|^2 + \chi_s^{(3,\text{XPM})} |E_p|^2, \tag{12}$$

where $\chi_p^{(1)}$, $\chi_p^{(3,\text{SPM})}$ and $\chi_p^{(3,\text{XPM})}$ are the linear, self-Kerr, and cross-Kerr susceptibilities of the probe and signal pulses, respectively. By solving the set of density matrix equations [Eqs. (1)–(7)] in steady state in the nondepletion approximation, the first and third order susceptibilities (cross-Kerr nonlinearity $\chi_p^{(3,\text{XPM})}$ and self-Kerr nonlinearity $\chi_p^{(3,\text{SPM})}$) will be considered in the next section. The linear susceptibilities can be written as

$$\chi_p^{(1)} = \frac{N|\mu_3|^2}{\hbar\epsilon_0} \chi_p^{(1)}, \quad \chi_s^{(1)} = \frac{N|\mu_3|^2}{\hbar\epsilon_0} \chi_s^{(1)}, \tag{13}$$

in which $\chi_p^{(1)}$ and $\chi_s^{(1)}$ are given by

$$\chi_p^{(1)} = -\sigma_{11}^{(0)} d_{31} (d_{41} + m^2 d_{31}) - (k - m)^2 \Omega_p^2 \frac{d_{31} d_{41} d_{51} - (d_{41} + k^2 d_{31}) \Omega_p^2}{d_{32} d_{42} d_{52} - (d_{42} + k^2 d_{32}) \Omega_p^2}, \tag{14}$$

$$\chi_s^{(1)} = -\sigma_{22}^{(0)} d_{52} (d_{42} + q^2 d_{32}) - (k - q)^2 \Omega_s^2 \frac{d_{32} d_{42} d_{52} - (d_{42} + k^2 d_{32}) \Omega_p^2}{d_{32} d_{42} d_{52} - (d_{42} + k^2 d_{32}) \Omega_p^2}. \tag{15}$$

Equations (14) and (15) show the symmetry of the QW structure between the probe and signal fields. With the simultaneous exchange of $1 \leftrightarrow 2$ and $m \leftrightarrow q$, the expression of $\chi_{s}^{(1)}$ can be obtained.

The real and imaginary parts of $\chi_p^{(1)}$ (and $\chi_s^{(1)}$), respectively, account for the linear absorption and dispersion of the probe (signal) field. With $\sigma_{11}^{(0)} \approx 0.141$, their evolutions versus their corresponding detunings are shown in Figs. 3(a) and 3(b), respectively. We take $\Delta_p = 3.0$ meV in Fig. 3(a) and $\Delta_p = 2.995$ meV in Fig. 3(b). The other parameters are the same as those in Fig. 2. With this set of parameters, the dispersion of the probe and signal pulses around the center of the transparent window are linearly proportional to their detunings. This means that the probe and signal pulses will propagate with comparable and small group velocities, and the influence of group velocity dispersion can be neglected within the region considered. At the center of the transparency window, the linear absorptions of the probe and signal fields are very small because of the destructive interference between transition pathways, and can be safely ignored.
Fig. 3. (color online) The linear absorption (solid curve) and dispersion (dashed curve) of the probe (a) and the signal fields (b) as functions of their corresponding detunings \( \Delta_p \) and \( \Delta_s \) with \( \sigma_1^{(1)} = 0.27 \). The detunings are chosen as (a) \( \Delta_p = \delta/2 = 3.65 \text{ meV}, \) (b) \( \Delta_p = 3.6505 \text{ meV}. \) The other parameters are the same with those in Fig. 2.

3. Strongly interacting photons with resonant tunneling

The explicit forms of the probe and signal third order susceptibilities associated with self-Kerr nonlinearity \( \chi_{p,s}^{(3,\text{SPM})} \) and cross-Kerr nonlinearity \( \chi_{p,s}^{(3,\text{XPM})} \) are, respectively, given by

\[
\chi_{p}^{(3,\text{SPM})} = \chi_{s}^{(3,\text{SPM})} = 0,
\]
\[
\chi_{p}^{(3,\text{XPM})} = \frac{N|\mu_{13}|^2|\mu_{23}|^2}{4\hbar^4 E_0} \chi_{p}^{(3,\text{XPM})},
\]
\[
\chi_{s}^{(3,\text{XPM})} = \frac{N|\mu_{13}|^2|\mu_{23}|^2}{4\hbar^4 E_0} \chi_{s}^{(3,\text{XPM})},
\]

where \( \chi_{p}^{(3,\text{XPM})} \) and \( \chi_{s}^{(3,\text{XPM})} \) can be simplified as

\[
\chi_{p}^{(3,\text{XPM})} = -\frac{T_{p1}}{2\pi} [T_{p2} + (T_{p3} - T_{p4}) \Omega_c^2 + T_{p5} \Omega_c^4],
\]
\[
\chi_{s}^{(3,\text{XPM})} = -\frac{T_{s1}}{2\pi} [T_{s2} + (T_{s3} - T_{s4}) \Omega_c^2 + T_{s5} \Omega_c^4],
\]

with

\[
T_{p1} = d_{s1}(d_{14} + mqd_{31}) + (m-k)(k-q)\Omega_c^2,
\]
\[
T_{p2} = d_{s1}d_{15}[\sigma_{11}d_{12}d_{23}(d_{14} + mqd_{31}) - \sigma_{22}d_{31}d_{41}(d_{24} + mqd_{32})],
\]
\[
T_{p3} = d_{s2}[\sigma_{21}(d_{14} + k^2d_{31})(d_{24} + mqd_{32}) + (m-k)(k-q)\sigma_{11}(d_{23}d_{41})],
\]
\[
T_{p4} = d_{s1}[\sigma_{11}(d_{14} + k^2d_{23})(d_{14} + mqd_{31}) + (m-k)(k-q)\sigma_{22}d_{31}d_{41}],
\]
\[
T_{p5} = (k-m)(k-q)[\sigma_{11}(d_{24} + k^2d_{23}) - \sigma_{22}(d_{14} + k^2d_{31})],
\]
\[
T_{s1} = d_{s2}(d_{14} + mqd_{32}) + (m-k)(k-q)\Omega_c^2,
\]
\[
T_{s2} = d_{s2}d_{15}[\sigma_{11}d_{12}d_{23}(d_{14} + mqd_{31}) - \sigma_{22}d_{31}d_{41}(d_{24} + mqd_{32})],
\]
\[
T_{s3} = d_{s1}[\sigma_{11}(d_{14} + k^2d_{31})(d_{24} + mqd_{32}) + (m-k)(k-q)\sigma_{22}d_{31}d_{41}],
\]
\[
T_{s4} = d_{s2}[\sigma_{21}(d_{14} + k^2d_{23})(d_{24} + mqd_{32}) + (m-k)(k-q)\sigma_{11}d_{31}d_{42}],
\]
\[
T_{s5} = (k-m)(k-q)[\sigma_{11}(d_{14} + k^2d_{32}) - \sigma_{22}(d_{14} + k^2d_{31})],
\]
\[
\mathcal{Z} = d_{s1}[d_{23}d_{24}d_{25} - (d_{24} + k^2d_{23})\Omega_c^2][d_{31}d_{41}d_{51} - (d_{41} + k^2d_{31})\Omega_c^2].
\]

It should be noted that the structure yields zero contributions to self-Kerr nonlinearity of the probe and signal pulses [see Eq. (16)], which are not desirable if one is interested in the
Fig. 4. (color online) (a) Re[$\chi_p^{(3)}$] with (solid) and without (longdashed) the control field and Im[$\chi_p^{(3)}$] (dashed) versus $\Delta_p$; (b) Re[$\chi_s^{(3)}$] with (solid) and without (longdashed) the control field and Im[$\chi_s^{(3)}$] (dashed) versus $\Delta_s$; (c) Re[$\chi_p^{(3)}$] (solid) and Im[$\chi_p^{(3)}$] (dashed) versus $\Delta_p$ without tunneling by setting $m = q = k = 0$; (d) Re[$\chi_s^{(3)}$] (solid) and Im[$\chi_s^{(3)}$] (dashed) versus $\Delta_s$ without tunneling. The parameters are the same with those in Fig. 3.

phase shift of the probe (signal) field that is induced by the signal (probe) field. The role of resonant tunneling can be seen from the expressions of $T_{\alpha\beta}$ ($\alpha = p, s, \beta = 1, 3, 4, 5$). In the QW structure under consideration, the symmetric and asymmetric wave functions of subbands $|3\rangle$ and $|4\rangle$ lead to $m \neq q \neq k$, which indicates that the resonant tunneling can modify the optical nonlinearity such as cross-Kerr effect. In Figs. 4(a) and 4(b), we illustrate the evolutions of $\chi_p^{(3)}$ and $\chi_s^{(3)}$ as functions of their corresponding detunings with (solid) and without (longdashed) the control field. All parameters are the same as those in Fig. 3. Within the transparency window, both the strengths of cross-Kerr nonlinearities and nonlinear absorptions of the probe and signal pulses are enhanced dramatically. Fortunately, the probe and signal nonlinear absorption peaks (dashed) are very sharp (Im[$\chi_p^{(3), \text{XPM}}$] accounts for the nonlinear absorption), and the real parts of the two cross-Kerr susceptibilities decay much more slowly than their corresponding nonlinear absorptions. We also notice that in the present QW structure, $\gamma_2$ is dominantly determined by the electron dephasing rate of a long-lived ground-subband coherence, smaller $\gamma_2$ can be attained by decreasing the temperature. Hence, the present QW structure yields the strengths of cross-Kerr nonlinearities that are two orders of magnitude higher than that in Ref. [48]. More importantly, positions of the nonlinear absorption peaks can be controlled by adjusting the detunings $\Delta_p$ and $\Delta_s$, which can be seen from Eqs. (19) and (20). For certain detunings, for example $\Delta_p = 2.9995$ meV and $\Delta_s = 3.0$ meV, we have Re[$\chi_p^{(3), \text{XPM}}$] $\simeq -419.02$ meV$^{-3}$, Re[$\chi_s^{(3), \text{XPM}}$] $\simeq -417.12$ meV$^{-3}$, and the two negative cross-Kerr nonlinearities are of the same order of magnitudes. Therefore, giant cross-Kerr nonlinearities are realized, while the nonlinear absorptions are negligibly small during the interaction time (Im[$\chi_p^{(3), \text{XPM}}$] $\simeq -10.1$ meV$^{-3}$ and Im[$\chi_s^{(3), \text{XPM}}$] $\simeq 8.7$ meV$^{-3}$). As shown in Figs. 4(a) and 4(b), under this set of parameters, the strengths of cross-Kerr nonlinearities can be enhanced with the presence of the control field.

Figures 4(c) and 4(d) illustrate the evolutions of the real (solid) and imaginary (dashed) parts...
of $\chi_p^{(3,\text{XPM})}$ and $\chi_s^{(3,\text{XPM})}$ versus their corresponding detunings with $m = q = k = 0$. In this case, the subband [4] is decoupled, and the system can hence be described as an inverted-Y-type configuration. In order to see the role of resonant tunneling more clearly, we choose $\Delta_c = 0$ and $\Omega_c \approx 2.65$ meV (similar transparency windows as those with tunneling). The other parameters are the same as those in Figs. 4(a) and 4(b). Within the transparency windows, the enhancement of cross-Kerr nonlinearities can still be achieved, while $\chi_p^{(3,\text{XPM})}$ values are two orders of magnitude smaller than those with tunneling under the same conditions. With $\Delta_p = 0.5$ µeV and $\Delta_s = 0$ meV, we have $\text{Re}[\chi_{s}^{(3,\text{XPM})}] \approx -5.77$ meV$^{-3}$, $\text{Re}[\chi_{s}^{(3,\text{XPM})}] \approx 5.77$ meV$^{-3}$ [please see Figs. 4(c)-4(d)]. Thus we can conclude that resonant tunneling can induce giant enhancement of cross-Kerr nonlinearity. Furthermore, the two cross-Kerr nonlinearities (one positive and one negative) exhibit destructive effect on total nonlinear phase shift (details will be shown later).

A significant interaction between photons is a very essential requirement for implementation of all-optical switching and controlled phase gate between two optical qubits. We thus turn our attention to the feasibility of achieving $\pi$ nonlinear phase shift at low light level. For Gaussian probe and signal pulses of time durations $\tau_{p,s}$, and with peak Rabi frequencies $\Omega_{p,s}^0$, solving the propagation equations gives the nonlinear cross-phase shift $\phi_{p,s}^n$ [12]

$$\phi_{p}^n = \frac{2\alpha_p l \hbar^2 |\Omega_{p}^0|^2}{c |\mu_{23}|^2} \text{erf}(\zeta_p) - \text{Re}[\chi_{p}^{(3,\text{XPM})}],$$  
\[(21)\]

$$\phi_{s}^n = \frac{2\alpha_s l \hbar^2 |\Omega_{s}^0|^2}{c |\mu_{13}|^2} \text{erf}(\zeta_s) - \text{Re}[\chi_{s}^{(3,\text{XPM})}],$$  
\[(22)\]

where $\zeta_p = [(1 - v_p^2)/v_p^2] \sqrt{2l}/(v_p^2 \tau_p)$, and $\zeta_s$ can be obtained from $\zeta_p$ upon interchanging $p \leftrightarrow s$. $l$ is the length of the QW structure, and erf($\zeta$) represents the error function.

The nonlinear phase shift acquired by the probe and signal pulses propagating through the QW structure can be controlled by the signal and probe pulses intensity. In Figs. 5(a) and 5(b), we plot the probe nonlinear phase shift $\phi_{p}^n$, the signal nonlinear phase shift $\phi_{s}^n$, and the total nonlinear phase shift $\phi_{p}^n = \phi_{p}^n + \phi_{s}^n$ (for the applications of giant cross-Kerr nonlinearity in QIP, the total nonlinear phase shift is important [11]) as functions of $\Omega_{p}^0 = \Omega_{s}^0 = \Omega$ with (a) and without (b) resonant tunneling. The length of QW structure is taken as $l = 1.0$ mm. With resonant tunneling, $\text{Re}[\chi_{p}^{(3,\text{XPM})}] \cdot \text{Re}[\chi_{s}^{(3,\text{XPM})}] > 0$ leads to $\phi_{p}^n > 0$, $\phi_{s}^n > 0$, which indicates the constructive effect of the probe and signal nonlinear phase shift on the total nonlinear phase shift [see Fig. 5(a)]. $\phi_{p}^n = \pi$ can be achieved with $\Omega_{p} = \Omega_{s} = \Omega \approx 6.22 \times 10^{-3}$ meV. The probe and signal pulses can have a mean amplitude of about one photon when these beams are
focused or propagate in a tightly confined waveguide. With these parameters, the corresponding intensities of the probe and signal pulses are, respectively, given by $I_p \approx 4.35 \text{ mW cm}^{-2}$ and $I_s \approx 6.94 \text{ mW cm}^{-2}$. We remark that the intensities of a single probe and signal photons per 1.0 ns on the area of 1 $\mu$m$^2$ are $I_p \approx 2.73 \text{ mW cm}^{-2}$ and $I_s \approx 2.25 \text{ mW cm}^{-2}$, respectively. The numerical results indicate that our semiconductor QW structure can indeed realize a $\pi$-nonlinear phase shift at low light level. In Fig. 5(a), we also illustrate the positive effect of the control field on $\phi^n$. Without resonant tunneling, $\text{Re}[^3\chi^{(3,XPM)}] \cdot \text{Re}[^3\chi^{(3,XPM)}] < 0$ exhibits the destructive effect on total nonlinear phase shift [see Fig. 5(b)]. In this case, $\phi^n = \pi$ can be obtained with $\Omega_p = \Omega_s = \Omega \approx 6.5 \times 10^{-3} \text{ meV} (I_p \approx 0.48 \text{ W cm}^{-2} \text{ and } I_s \approx 0.76 \text{ W cm}^{-2})$, i.e., more than one hundred of photons.

4. Conclusion

To begin with, we should point out that in the literature several semiconductor QW structures have been suggested to investigate the giant cross-Kerr nonlinearity and the possibility of quantum phase gate [46–48]. In asymmetrical N-type [46] or ladder [47] configurations, the probe pulse propagates with slow light because of EIT, while the signal pulse possesses a nonzero Kerr nonlinearity only. The group velocity matching can only be satisfied by controlling the signal detuning when the signal field is continuous wave. However, this is not desirable for photonic controlled phase gate. This is a consequence of the asymmetrical configurations. In the present study, the influence of group velocity dispersion can be safely ignored. In addition, the QW structure supports zero contributions to self-Kerr nonlinearities. That is to say, it is possible that the probe and signal wave packets propagate in QW structure with group velocity matching and higher stability. Our calculation is based on one-dimensional model. The influence of transverse degrees of freedom on the cross-Kerr nonlinearity and the phases will be considered in our ongoing work.

In conclusion, we have designed a double semiconductor asymmetric QW structure to achieve strong interaction between photons. This structure combines the resonant tunneling with the advantages of inverted-Y type scheme. By virtue of resonant tunneling, not only can the strength of cross-Kerr nonlinearities be enhanced dramatically with vanishing linear and nonlinear absorptions simultaneously, but also the effect of cross-Kerr nonlinearities of the probe and signal pulses on the total nonlinear phase shift can be changed from destructive to constructive. Our numerical findings confirm that it is possible to achieve nonlinear phase shift on order of $\pi$ at low light level. Due to the symmetry of the scheme with respect to the probe and signal pulses, their group velocities could be equal by modulating the initial distribution of electron. We believe that the present study is useful for guiding experimental realization of electroptically modulated devices and facilitating more practical applications in solid quantum information processing.

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