Robust delay-dependent $H_\infty$ performance for uncertain neutral systems with mixed time delays

Sirada Pinjai$^1$, Jenjira Thipcha$^2$

$^1$Department of Mathematics, Faculty of Science and Agricultural Technology, Rajamangala University of Technology Lanna, Chiang Mai 50300, Thailand
$^2$Mathematics Program, Faculty of Science, Maejo University, Chiang Mai 50290, Thailand

E-mail: sirada@rmutl.ac.th

Abstract. This paper is concerned with the problems of robust stability analysis and $H_\infty$ performance for the uncertain neutral systems with mixed time delays. By constructing an augmented Lyapunov Krasovskii functional with quadruple integral terms, some sufficient criteria are provided in terms of linear matrix inequalities (LMIs). The goal of this paper is to establish some conditions to guarantee the system is robustly stable with a prescribed $H_\infty$ performance level for all uncertainties.

1. Introduction

Nowadays many cases of the field for dynamic systems have been paid attention, such as biological models, epidemic models, transportation systems, aircraft, robotics, neural networks, and so on [2], furthermore the many cases was focused on the effect of delay-dependent stability analysis, since the systems with delays will be instability and poor performance. Therefore various stability and stabilization for dynamical systems with or without state delays have been intensively investigated by many researchers mathematics and control communities [5, 11, 20]. Stability criteria for dynamical systems with time delay are generally divided into two classes: delay-independent one and delay-dependent one. Delay-independent stability criteria tend to be more conservative, especially for small size delay, such criteria do not give any information on the size of a delay. On the other hand, delay-dependent stability criteria are concerned with the size of a delay and usually provide a maximal delay size.

Many problems of stability analysis for neutral time-delay systems have been interested, various improved delay-dependent criteria of the systems have been paid attention. The neutral system is the system that contains time delays on both state space and derivative at state space, the practical model of neutral systems occur as prey-predator model, electrical model. Moreover, the study of robustly stable of the neutral systems with parameter uncertainties aims to guarantee the range of time-delay that made the system still stable and the $H_\infty$ control problem have been concerned. Hence, there are many techniques have been created to improved the method to get the effective results which is guarantee the stability properties for the systems [5, 6, 11, 13, 14, 15, 20, 21, 22].

The cases of $H_\infty$ method have been shown in control theory to integrate controllers succeeding stabilization with guaranteed performance. $H_\infty$ technique has been applied to minimize the cost effects of the external disturbances. It is the objective of $H_\infty$ control to design the controllers
such that the closed-loop system is internally stable and its $H_{\infty}$ norm of the transfer function between the controlled output and the disturbances will not exceed a given level $\gamma$. Moreover, the studies $H_{\infty}$ control systems with interval time-varying delays have been developed so the improvement of the theory of $H_{\infty}$ control have extend the region to study. The problems about delay-dependent robust $H_{\infty}$ for linear system with interval time-varying delay and restricted the derivative of the interval time-varying delay, that mean a fast interval time-varying delay is allowed [6, 17]. The $H_{\infty}$ performance for linear system with nonlinear perturbation have been paid attention moreover, in other hand, [10] showed the time derivative of the Lyapunov-Krasovskii functional produced not only the strictly proper rational functions but also the non-strictly proper rational functions of the time-varying delays with first-order denominators, which was fully handled using reciprocally convex approach [7, 9].

From the many motivations above, this work investigates in robust stability analysis and applying the $H_{\infty}$ performance method for our systems. The parameter uncertainties are bounded in magnitude as some inequality. Based on Lyapunov-Krasovskii theory which is containing in term quadruple integral functional, applying Leibniz-Newton formula, modified version of Jensen’s inequality, Wirtinger-based integral inequality and linear matrix inequality techniques, then the stability criteria and the $H_{\infty}$ performance for neutral system with interval time-varying delays are present in term linear matrix inequality.

Notations: The following notations will be used throughout for this paper. $R^+$ denotes the set of all real non-negative numbers; $R^n$ denotes the n-dimensional space with the vector norm $|| \cdot ||$; $||x||$ denotes the Euclidean vector norm of $x \in R^n$; $R^{n \times r}$ denotes the set of $n \times r$ real matrices; Lebesgue space $L_{2,\infty} = L_2[0, \infty]$ consists of square-integral functions on $[0, \infty]$. $A^T$ denotes the transpose of the matrix $A$; $I$ denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of $A$; $\lambda_{max}(A) = \max \{\Re \lambda : \lambda \in \lambda(A)\}$; $\lambda_{min}(A) = \min \{\Re \lambda : \lambda \in \lambda(A)\}$; $C([-b,0], R^n)$ denotes the space of all continuous vector functions mapping $[-b,0]$ into $R^n$, where $b = \max \{h, r\}$, $h_1, r \in \mathbb{R}^+$; $*$ represents the elements below the main diagonal of a symmetric matrix.

2. Preliminaries

Consider the system described by the following state equations of the form

$$
\begin{align*}
\dot{x}(t) &= [C + \Delta C(t)]x(t) + [A + \Delta A(t)]x(t) + [B + \Delta B(t)]x(t - h(t)) + E_\omega \omega(t), \\
\dot{z}(t) &= A_1 x(t) + B_1 x(t - h(t)) + E_1 \omega(t), \\
x(t + t_0) &= \phi(t), \quad \dot{x}(t + t_0) = \psi(t), \quad t \in [-b,0].
\end{align*}
$$

(1)

where $x(t) \in R^n$ is the state variable, $\omega(t) \in R^m$ denotes the disturbance input such that $\omega(t) \in L_{2,\infty}$, $z(t) \in R^q$ is the performance output, $\phi(t), \psi(t)$ are continuously real-valued initial functions on $[-b,0]$. $A, B, C, E_\omega, A_1, B_1, E_1 \omega$ are known real constant matrices with appropriate dimensions. The delay $h(t)$ and neutral delay $\tau(t)$ are time-varying continuous function that satisfies

$$
0 \leq h_1 \leq h(t) \leq h_2, \\
0 \leq \tau(t) \leq \tau, \quad \dot{\tau}(t) \leq \mu
$$

(2)

(3)

where $h_1, h_2, \tau, \mu$ are given real constants. Consider the initial functions $\phi(t), \psi(t) \in C([-b,0], R^n)$ with the norm $||\phi|| = \sup_{t \in [-b,0]} ||\phi(t)||$ and $||\psi|| = \sup_{t \in [-b,0]} ||\psi(t)||$.

The time-varying parameter uncertainties are $\Delta A(t), \Delta B(t), \Delta C(t)$, and $\Delta E_\omega(t)$ and satisfying

$$
[\Delta A(t), \Delta B(t), \Delta C(t), \Delta E_\omega(t)] = LF(t)[G_1, G_2, G_3, G_4],
$$

(4)
Lemma 2 [8] For any positive definite matrix \( \Gamma \in \mathbb{R}^{n \times n} \) and \( F(t) \in \mathbb{R}^{l \times l} \) is an unknown real time-varying function with appropriate dimension and bounded as \( F(t)'F(t) \leq I \).

**Definition 1** The system (1) is robustly exponentially stable, if there exist positive real constants \( k \) and \( N \) such that for each \( \phi(t), \psi(t) \in C([-b,0], \mathbb{R}^{n}) \), the solution \( x(t,\phi,\psi) \) of the system (1) satisfies

\[
\|x(t,\phi,\psi)\| \leq N \max\{\|\phi\|,\|\psi\|\}e^{-kt}, \quad \forall t \in \mathbb{R}^+.
\]

**Definition 2** Given a scalar \( \gamma > 0 \), system (1) is said to be asymptotically stable with the \( H_{\infty} \) performance level \( \gamma \), if it is asymptotically stable and satisfies the \( H_{\infty} \)-norm constraint

\[
\|z(t)\|_2 < \gamma \|\omega(t)\|_2,
\]

for all nonzero \( \omega(t) \in L_2[0,\infty] \) under zero initial condition.

**Lemma 1** [16] (Wirtinger-based Integral Inequality.) For any symmetric constant matrix \( Q \in \mathbb{R}^{n \times n} \) and differentiable signal \( x : [\tau_l, \tau_u] \to \mathbb{R}^n \). The following inequality holds:

\[
\int_{\tau_l}^{\tau_u} x^T(s)Q\dot{x}(s)ds \leq -\frac{1}{\tau_u - \tau_l}\Gamma^T \hat{Q}\Gamma,
\]

where \( \Gamma = [x^T(t - \tau_l), x^T(t - \tau_u), \int_{\tau_l}^{\tau_u} x^T(s)ds] \) and \( \hat{Q} = \left[ \begin{array}{ccc} -4W & -2W & 6W \\ * & -4W & 6W \\ * & * & -12W \end{array} \right] \).

**Lemma 2** [8] For any positive definite matrix \( Z \in \mathbb{R}^{n \times n} \), scalars \( \tau_l, \tau_u > 0 \), vector function \( w : [\tau_l, \tau_u] \to \mathbb{R}^n \) such that the following integration are well defined, the inequality holds:

\[
-(\tau_u - \tau_l) \int_{-\tau_u}^{l-\tau_u} w^T(s)Zw(s)ds \leq -\left( \int_{t-\tau_u}^{t-\tau_l} w(s) \right)^T Z \left( \int_{t-\tau_u}^{t-\tau_l} w(s)ds \right),
\]

\[
-\frac{(\tau_u^2 - \tau_l^2)}{2} \int_{-\tau_u}^{l-\tau_u} \int_{t}^{t+\theta} w^T(s)Zw(s)dsd\theta
\]

\[
\leq -\left( \int_{-\tau_u}^{l-\tau_u} \int_{t}^{t+\theta} w(s)dsd\theta \right)^T Z \left( \int_{-\tau_u}^{l-\tau_u} \int_{t}^{t+\theta} w(s)dsd\theta \right),
\]

\[
-\frac{(\tau_u^3 - \tau_l^3)}{6} \int_{-\tau_u}^{l-\tau_u} \int_{t}^{t+\theta} \int_{t}^{t} w^T(\theta)Zw(\theta)d\theta duds
\]

\[
\leq -\left( \int_{-\tau_u}^{l-\tau_u} \int_{t}^{t} \int_{t}^{t} w(\theta)d\theta duds \right)^T Z \left( \int_{-\tau_u}^{l-\tau_u} \int_{t}^{t} \int_{t}^{t} w(\theta)d\theta duds \right).
\]

**Lemma 3** [18] For given matrices \( Q = Q^T, H, E \) and \( R = R^T > 0 \) of appropriate dimension, then

\[
Q + HFE + E^T F^T H^T < 0
\]

for all \( F \) satisfies \( F^TF \leq R \), if and only if there exist a positive number \( \epsilon > 0 \), such that

\[
Q + \epsilon^{-1}HH^T + \epsilon E^T RE < 0.
\]
3. Main Result

In this section, we study asymptotically stable for the neutral system (1) and also consider the $H_{\infty}$ performance for its with interval time-varying delays and parameter uncertainties. Some notations for system (1), of several matrix variables are following defined:

\[
\Sigma = \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & 0 & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} & \Sigma_{17} & \Sigma_{18} & \Sigma_{19} & \Sigma_{110} & 0 & 0 \\
* & \Sigma_{22} & 0 & \Sigma_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Sigma_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Sigma_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Sigma_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -Q_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -Q_7 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -Q_9 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Sigma_{99} & \Sigma_{910} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Sigma_{1010} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -R_2 & \end{bmatrix},
\]

where

\[
\begin{align*}
\Sigma_{11} &= 2KP + PA + AT^TP + N_1^TA + ATN_1 + Q_1 + Q_2 - 4Q_3 \\
&\quad - \frac{1}{h_2}Q_2 + h_2Q_5 - h_2^2Q_6 - (h_2 - h_1)^2Q_7 - \frac{1}{4}h_2^4Q_8 - \frac{1}{4}(h_2^2 - h_1^2)^2Q_9, \\
\Sigma_{12} &= PB + N_1^TB + h_2Q_3, \quad \Sigma_{14} = Q_1 + \frac{1}{h_2}Q_4 \quad \Sigma_{15} = h_2Q_6, \\
\Sigma_{16} &= (h_2 - h_1)Q_7, \quad \Sigma_{17} = \frac{1}{2}h_2^2Q_8 + ATN_3, \quad \Sigma_{18} = \frac{1}{2}(h_2^2 - h_1^2)Q_9, \\
\Sigma_{112} &= h_2^2Q_4 + (h_2 - h_1)^2Q_5 + ATP - N_1^T + ATN_2, \quad \Sigma_{110} = PC + N_1^TC, \\
\Sigma_{22} &= -2h_2Q_3, \quad \Sigma_{24} = h_2Q_3, \quad \Sigma_{44} = -Q_2 - 4Q_3 - 4Q_4, \\
\Sigma_{33} &= -4Q_4, \quad \Sigma_{55} = -\frac{1}{h_2}12Q_3 - Q_6, \\
\Sigma_{99} &= h_2^2Q_3 + h_2Q_4 + \frac{1}{4}h_2^2Q_6 + \frac{1}{4}(h_2 - h_1)^2Q_7 + \frac{1}{36}h_2^6Q_6 \\
&\quad + \frac{1}{36}(h_2^3 - h_1^3)^2Q_9 + R_1 + \tau^2R_2 - P - P - N_2 - N_2^T, \\
\Sigma_{910} &= PC + N_2^T C, \quad \Sigma_{1010} = -(1 - \mu)R_1.
\end{align*}
\]

**Theorem 1** For $\|C\| + \|L\| \cdot \|G_3\| < 1$ and given positive scalars $h_1, h_2, \tau, \mu$ and a prescribed $\gamma > 0$, if there exist a scalar $\varepsilon > 0$ and the positive symmetric matrices $P, Q_i, (i = 1, 2, \ldots, 9)$ and any matrices $N_j, j = 1, 2, 3$ with appropriate dimensions that the following LMIs hold

\[
\Omega = \begin{bmatrix}
\Sigma & \Lambda_1 & \Lambda_2 \\
* & -\gamma^2I & E_{1\omega} \\
* & * & -I
\end{bmatrix} < 0,
\]

\[
\begin{bmatrix}
\Omega & \Gamma & \varepsilon\Pi^T \\
* & -\varepsilon I & 0 \\
* & * & -\varepsilon I
\end{bmatrix} < 0,
\]
where

\[
\begin{align*}
\Lambda_1^T &= [E^T_0 P + E^T_1 N_1, 0, 0, 0, 0, E^T_0 N_3, 0, E^T_1 P, 0, 0], \\
\Lambda_2^T &= [A^T, B^T, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], \\
\Gamma^T &= [G^T P + G^T N_1, 0, 0, 0, 0, G^T N_3, 0, G^T P + G^T N_2, 0, 0, 0, 0, 0, 0, 0, 0], \\
\Pi &= [G_1, G_2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
\end{align*}
\]

then the system (1) for time-delay (2) and (3) is robustly stable and satisfies $H_{\infty}$ condition $\|z(t)\|_2 \leq \gamma \|\omega(t)\|_2$ for any non-zero $\omega(t) \in L_2[0, \infty)$.

**Proof 1** Construct a Lyapunov-Krasovskii functional as

\[
V(t) = \sum_{i=1}^{6} V_i(t),
\]

where

\[
\begin{align*}
V_1(t) &= \left[ \begin{array}{c}
\begin{bmatrix} x(t) & \dot{x}(t) \end{bmatrix}
\end{bmatrix}
\end{array} \right]^T \left[ \begin{array}{ccc}
I & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array} \right] \left[ \begin{array}{ccc}
P & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array} \right] \left[ \begin{array}{c}
\begin{bmatrix} x(t) & \dot{x}(t) \end{bmatrix}
\end{bmatrix}
\end{array} \right], \\
V_2(t) &= \int_{t-h_2}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_2}^{t} x^T(s)Q_2x(s)ds, \\
V_3(t) &= h_2 \int_{-h_2}^{t} \int_{t+s}^{t} \dot{x}^T(\theta)Q_3\dot{x}(\theta)d\theta ds + (h_2 - h_1) \int_{t-h_2}^{t} \int_{t+s}^{t} \dot{x}^T(\theta)Q_4\dot{x}(\theta)d\theta ds \\
&+ \int_{t-h_2}^{t} \int_{t+s}^{t} \dot{x}^T(\theta)Q_5x(\theta)d\theta ds, \\
V_4(t) &= \frac{h_2^2}{2} \int_{-h_2}^{t} \int_{t+\omega}^{t} \dot{x}^T(\theta)Q_6\dot{x}(\theta)d\theta d\omega ds + \frac{h_2^2 - h_1^2}{2} \int_{-h_2}^{t} \int_{t+\omega}^{t} \dot{x}^T(\theta)Q_7\dot{x}(\theta)d\theta d\omega ds, \\
V_5(t) &= \frac{h_2^3}{6} \int_{-h_2}^{t} \int_{t+\omega}^{t} \dot{x}^T(\theta)Q_8\dot{x}(\theta)d\theta d\omega d\nu ds + \frac{(h_2^3 - h_1^3)}{6} \int_{-h_2}^{t} \int_{t+\omega}^{t} \dot{x}^T(\theta)Q_9\dot{x}(\theta)d\theta d\omega d\nu ds, \\
V_6(t) &= \int_{t-\tau(t)}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds + \tau \int_{-\tau}^{t} \dot{x}^T(\theta)R_2\dot{x}(\theta)d\theta ds.
\end{align*}
\]
Taking the time derivatives of $V_i(t), i = 1, 2, 3, ..., 6$, along the trajectory of (1) yields

$$\dot{V}_1(t) = 2x^T(t)Px(t) + 2 \left[ \begin{array}{c} x(t) \\ \dot{x}(t) \end{array} \right] ^T \left[ \begin{array}{cc} P & 0 \\ 0 & N_1^T \end{array} \right] \left[ \begin{array}{c} \dot{x}(t) \\ 0 \end{array} \right],$$

and using Wirtinger-based integral inequality Lemma 1, then

$$\dot{V}_1(t) = 2x^T(t)(A + LF(t)G_1)x(t) + (B + LF(t)G_2)x(t - h(t)) + (C + LF(t)G_3)\dot{x}(t - \tau(t)) + (E\omega + LF(t)G_4)\omega(t) + 2x^T(t)(\dot{x}(t - \tau(t)) + (E\omega + LF(t)G_4)\omega(t) - \dot{x}(t)) + 2 \int_{t-h_2}^{t-h_1} \dot{x}(\theta)\,d\theta \int_{t-h_2}^{t-h_1} N_1^T \left[ (A + LF(t)G_1)x(t) + (B + LF(t)G_2) \right],$$

$$\dot{V}_2(t) = \left[ x^T(t)Q_1x(t) - x^T(t - h_1)Q_1x(t - h_1) + x^T(t)Q_2x(t) - x^T(t - h_2)Q_2x(t - h_2) + V_2(t) \right],$$

$$\dot{V}_3(t) \leq h_2^2 \dot{x}(t)Q_3 \dot{x}(t) - h_2 \int_{t-h_2}^{t-h_1} \dot{x}(t)Q_3 \dot{x}(t) \, ds$$

$$+ h_2 \dot{x}^T(t)Q_4 \dot{x}(t) - \frac{1}{h_2} \int_{t-h_2}^{t-h_1} \dot{x}(t)Q_4 \dot{x}(t) \, ds$$

and using Wirtinger-based integral inequality Lemma 1, then

$$\dot{V}_3(t) \leq h_2^2 \dot{x}(t)Q_3 \dot{x}(t) + (h_2 - h_1) \dot{x}^T(t)Q_4 \dot{x}(t)$$

$$+ \left[ \begin{array}{c} x(t) \\ x(t - h_2) \end{array} \right] ^T \left[ \begin{array}{cc} 1 & \frac{1}{h_2} \int_{t-h_2}^{t-h_1} x(s) \, ds \end{array} \right],$$

$$\dot{V}_3(t) = \left[ \begin{array}{c} x(t) \\ x(t - h_2) \end{array} \right] ^T \left[ \begin{array}{cc} -Q_3 & 6Q_3 \\ -Q_3 & 6Q_3 \end{array} \right] \left[ \begin{array}{c} x(t - h_2) \\ \frac{1}{h_2} \int_{t-h_2}^{t-h_1} x(s) \, ds \end{array} \right],$$

$$\dot{V}_3(t) \leq \frac{h_2^2}{2} \dot{x}^T(t)Q_6 \dot{x}(t) - \frac{h_2^2}{2} \int_{t-h_2}^{t-h_1} \dot{x}^T(t)Q_6 \dot{x}(t) \, ds + \left( \frac{h_2^2 - h_1^2}{2} \right) \dot{x}^T(t)Q_7 \dot{x}(t)$$

$$- \left( \frac{h_2^2 - h_1^2}{2} \right) \int_{t-h_2}^{t-h_1} \dot{x}^T(t)Q_7 \dot{x}(t) \, ds.$$
applying Lemma 2 to estimate integral terms, we obtain

\[ \dot{V}_4(t) \leq \left( \frac{h_2^3}{2} \right)^2 \dot{x}^T(t)Q_6 \dot{x}(t) + \left( \frac{h_2^3 - h_1^3}{6} \right)^2 \dot{x}^T(t)Q_\tau \dot{x}(t) \]

\[ + \left[ \int_{t-h_2}^t \frac{1}{3} h_2^2 x(t) \right] \right] \left[ \begin{array}{cc} -Q_8 & Q_8 \\ Q_8 & -Q_8 \end{array} \right] \left[ \begin{array}{c} \int_{t-h_2}^t \dot{x}^T(\nu) d\nu ds \\ \dot{x}(t) \end{array} \right] \]

\[ + \left[ \int_{t-h_2}^t \frac{1}{3} h_2 h_1 x(t) \right] \right] \left[ \begin{array}{cc} -Q_9 & Q_9 \\ Q_9 & -Q_9 \end{array} \right] \left[ \begin{array}{c} \int_{t-h_2}^t \dot{x}^T(\nu) d\nu ds \\ \dot{x}(t) \end{array} \right] \].

Taking the time derivative of \( V_5(t) \) yields

\[ \dot{V}_5(t) \leq \left( \frac{h_3^2}{6} \right)^2 \dot{x}^T(t)Q_8 \dot{x}(t) + \left( \frac{h_3^2 - h_1^2}{6} \right)^2 \dot{x}^T(t)Q_9 \dot{x}(t) \]

\[ - \frac{h_3^2}{6} \int_{t-h_2}^t \int_s^t \dot{x}(\omega) Q_8 \dot{x}(\omega) d\omega d\nu ds \]

\[ - \left( \frac{h_3^2 - h_1^2}{6} \right) \int_{t-h_2}^t \int_s^t \dot{x}(\omega) Q_9 \dot{x}(\omega) d\omega d\nu ds, \]

using Lemma 2, we obtain

\[ \dot{V}_5(t) \leq \left( \frac{h_3^2}{6} \right)^2 \dot{x}^T(t)Q_8 \dot{x}(t) + \left( \frac{h_3^2 - h_1^2}{6} \right)^2 \dot{x}^T(t)Q_9 \dot{x}(t) \]

\[ + \left[ \int_{t-h_2}^t \frac{1}{3} h_3^2 \dot{x}(t) \right] \right] \left[ \begin{array}{cc} -Q_8 & Q_8 \\ Q_8 & -Q_8 \end{array} \right] \left[ \begin{array}{c} \int_{t-h_2}^t \dot{x}^T(\nu) d\nu ds \\ \dot{x}(t) \end{array} \right] \]

\[ + \left[ \int_{t-h_2}^t \frac{1}{3} h_3^2 h_1 \dot{x}(t) \right] \right] \left[ \begin{array}{cc} -Q_9 & Q_9 \\ Q_9 & -Q_9 \end{array} \right] \left[ \begin{array}{c} \int_{t-h_2}^t \dot{x}^T(\nu) d\nu ds \\ \dot{x}(t) \end{array} \right] \].

Taking derivative of \( V_6(t) \), it is obtained as

\[ \dot{V}_6(t) \leq \left[ \dot{x}^T(t)R_1 \dot{x}(t) - (1 - \mu)\dot{x}^T(t - \tau(t))R_1 \dot{x}(t - \tau(t)) + \tau^2 \dot{x}^T(t)R_2 \dot{x}(t) \right] \]

\[ - \tau \int_{t-\tau(t)}^t \dot{x}^T(s)R_2 \dot{x}(s) ds - V_6(t), \]

using Jensen’s inequality to estimate integral inequality, then

\[ - \tau \int_{t-\tau(t)}^t \dot{x}^T(s)R_2 \dot{x}(s) ds \leq - \left( \int_{t-\tau(t)}^t \dot{x}^T(s) ds \right) R_2 \left( \int_{t-\tau(t)}^t \dot{x}(s) ds \right) . \tag{8} \]

Hence,

\[ \dot{V}_6(t) \leq \left[ \dot{x}^T(t)R_1 \dot{x}(t) - (1 - \mu)\dot{x}^T(t - \tau(t))R_1 \dot{x}(t - \tau(t)) + \tau^2 \dot{x}^T(t)R_2 \dot{x}(t) \right] \]

\[ - \tau \left( \int_{t-\tau(t)}^t \dot{x}^T(s) ds \right) R_2 \left( \int_{t-\tau(t)}^t \dot{x}(s) ds \right) - V_6(t). \]

From the following zero equation is for positive symmetric matrices \( P \) with:

\[ 2 \dot{x}^T(t)P([A + LF(t)G_1]x(t) + (B + LF(t)G_2)x(t - h(t)) \]

\[ + (C + LF(t)G_3) \dot{x}(t - r(t)) + (E_\omega + LF(t)A_4)\omega(t) - \dot{x}(t)) = 0. \tag{9} \]
For all time derivatives of $V(t)$ and adding zero equation, we get

$$\dot{V}(t, x_t) \leq \sum_{i=1}^{6} \dot{V}_i(t) + 2\dot{x}^T(t)P \left[ (A + LF(t)G_1)x(t) + (B + LF(t)G_2)x(t - h(t)) + (C + LF(t)G_3)\dot{x}(t - r(t)) + (E\omega + LF(t)G_4)\omega(t) - \dot{\omega}(t) \right] + [A_1x(t) + B_1x(t - h(t))]^T \left[ A_1x(t) + B_1x(t - h(t)) + E_1\omega(t) \right] - \gamma^2\dot{\omega}^T(t)\omega - z^T(t)z(t) + \gamma^2\dot{\omega}^T(t)\omega(t)$$

where

$$\begin{align*}
\mathcal{S}^T &= \begin{bmatrix} A_1 & B_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_1\omega \end{bmatrix}, \\
\xi_1(t) &= \begin{bmatrix} x(t), x(t - h(t)), x(t - h_1), x(t - h_2), \int_t^t x(s)ds, \int_{t-h_2}^{t-h_1} x(s)ds, \\
\int_{t-h_2}^{t} x(s)ds, \int_{t-h_2}^{t-h_1} x(s)ds, \int_{s}^{t} x(s)ds, \int_{s}^{t} x(s)ds \end{bmatrix}.
\end{align*}$$

Then, it should simplify in form

$$\begin{bmatrix} \Sigma & A_1 & A_2 \\ * & -\gamma^2I & E_1\omega \\ * & * & -I \end{bmatrix} + \Gamma F(t)\Pi + \Gamma^T F^T(t)\Pi^T < 0,$$

$$\Omega + \Gamma F(t)\Pi + \Gamma^T F^T(t)\Pi^T < 0. \quad (10)$$

where

$$\begin{align*}
\Gamma^T &= \begin{bmatrix} L^T P + L^T N_1, 0, 0, 0, 0, 0, L^T N_3, 0, L^T P + L^T N_2, 0, 0, 0, 0 \end{bmatrix}, \\
\Pi &= \begin{bmatrix} G_1, G_2, 0, 0, 0, 0, 0, 0, G_3, 0, G_4, 0 \end{bmatrix}.
\end{align*}$$

Using Lemma 3, then we obtain

$$\Omega + \varepsilon \Gamma \Pi^T + \varepsilon^{-1} \Pi \Xi^T < 0, \quad (11)$$

$$\Theta = \begin{bmatrix} \Omega & \Gamma & \varepsilon \Pi^T \\ \Gamma^T & -\varepsilon I & 0 \\ \varepsilon \Pi^T & 0 & -\varepsilon I \end{bmatrix} < 0. \quad (12)$$

Since $\Theta < 0$ is guarantee $\dot{V}(t) < 0$ and following the proof which presenting $H_\infty$ performance of Theorem 1, then we can yield the results

$$\dot{V}(t) \leq -z^T(t)z(t) + \gamma^2\dot{\omega}^T(t)\omega(t).$$

Integrate both sides of (13) from $t_0$ to $t$, yield

$$V(t) - V(t_0) \leq -\int_{t_0}^{t} z^T(s)z(s)ds + \int_{t_0}^{t} \gamma^2\dot{\omega}^T(s)\omega(s)ds$$
Then, letting \( t \to \infty \) and under zero initial condition, we have \( V(t_0) = V(0) = 0 \) and \( V(\infty) = 0 \), that leads to

\[
\int_{t_0}^{t} z^T(s)z(s)ds \leq \int_{t_0}^{t} \gamma^2 \omega^T(s)\omega(s)ds,
\] (13)

therefore \( \|z(t)\|_2 \leq \gamma \|\omega(t)\|_2 \) is satisfied for any non-zero \( \omega(t) \in \mathcal{L}_2[0, \infty) \).

This completes the proof. \( \Box \)

4. Conclusion

A class of delay-dependent on stability and \( H_\infty \) performance for the uncertain neutral systems with time-varying delays is proposed in this paper. Based on construct a suitable augmented Lyapunov-Krasovskii functional which are including the quadruple integral terms. Combining the improved Jensen’s inequality, Wirtinger-based integral inequality, free-weight matrices and zero equation method, then a sufficient conditions for robustly stable and a prescribed performance index for \( H_\infty \) performance are illustrated in terms of Linear Matrix Inequality (LMIs).

Acknowledgments

This work was supported by the Department of Mathematics, Faculty of Science and Agricultural Technology, Rajamangala University of Technology Lanna, Chiang Mai and Research and Development Institute (RDI) of Rajamangala University of Technology Lanna, Thailand.

References

[1] Boyd S, Ghaoui El L, Feron E and Balakrishnan V 1994 Linear matrix inequalities in system and control theory: studies in applied mathematics vol.15 SIAM Philadelphia, PA
[2] Gu K, Kharitonov V L and Chen J 2003 Stability of time-delay system Birkhäuser: Berlin
[3] Balasubramaniam P and Vembarasan V 2011 Robust stability of uncertain fuzzy BAM neural networks of neutral-type with Markovian jumping parameters and impulses Comput. and Math. with Appl. 62 1838
[4] Balasubramaniam P, Krishnasamy R and Rakkiyappan R 2011 Delay-interval-dependent robust stability results for uncertain stochastic systems with Markovian jumping parameters Nonlinear Anal.: Hybrid Systems 5 681
[5] Chen W H and Zheng W X 2007 Delay-dependent robust stabilization for uncertain neutral systems with distributed delays Automatica 43 95
[6] Jiang X and Han Q L 2005 On \( H_\infty \) control for linear systems with interval time-varying delay Automatica 41 2099
[7] Kwon O M, Park M J, Park Ju H, Lee S M and Cha E J 2013 Analysis on robust \( H_\infty \) performance and stability for linear systems with interval time-varying state delays via some new augmented Lyapunov-Krasovskii functional Appl. Math. and Comput. 108
[8] Kwon O M, Park J H, Lee SM and Cha E J 2014 New augmented Lyapunov-Krasovskii functional approach to stability analysis of neural networks with time-varying delays Nonlinear Dyn. 76 221
[9] Kwon O M, Park M J, Park Ju H, Lee S M and Cha E J 2014 On stability analysis for neural networks with interval time-varying delays via some new augmented Lyapunov-Krasovskii functional. Commun. Nonlinear Sci. Numer. Simulat. 19 3184
[10] Lee W I, Lee S Y and Park P G 2014 Improved criteria on robust stability and \( H_\infty \) performance for linear systems with interval time-varying delays via new triple integral functionals Appl. Math. and Comput. 243 570
[11] Mohajerpoor R, Shanmugam L, Abdil H, Rakkiyappan R, Nahavandi S and Park J H 2017 Improved delay-dependent stability criteria for neutral systems with mixed interval time-varying delays and nonlinear disturbances J. of The Frankl. Inst. 354 1169
[12] Peng C and Tian Y C 2009 Delay-dependent robust \( H_\infty \) control for uncertain systems with time-varying delay. Information Science, 179 3187
[13] Qiu F and Cui B T 2010 Improved exponential stability criteria for uncertain neutral system with nonlinear parameter perturbations International J. of Automation and Computing 7 413
[14] Raja R, Zhu Q, Senthilraj S and Samidurai R 2015 Improved stability analysis of uncertain neural type neural networks with leakage delays and impulsive effects Appl. Math. Comput. 266 1050
[15] Samidurai R, Rajavel S, Zhu Q and Raja R 2016 Robust passivity analysis for neutral-type neural networks with mixed and leakage delays Neurocomputing 175 635
[16] Seuret A and Gouaisbaut F 2013 Wirtinger-based integral inequality: Application to time-delay systems Automatica 49(9) 2860
[17] Thanh N T and Phat V N 2013 $H_\infty$ control for nonlinear systems with interval non-differentiable time-varying delay European J. of Control 19 190
[18] Wang Y, Xie L, and Souza de C E 1992 Robust control of a class of uncertain nonlinear systems. Systems Control Lett. 19 139
[19] Wang F X, Liu X G, Tang M L and Hou M Z 2018 Improved integral inequality for stability analysis of delayed neural networks Neurocomputing 273 178
[20] Zhang W A and Yu L 2007 Delay-dependent robust stability of neutral systems with mixed delays and nonlinear perturbations Acta Automatica Sinica 33 863
[21] Zhang D and Yu L 2010 $H_\infty$ output tracking control for neutral systems with time-varying delay and nonlinear perturbations. Commun. Nonlinear Sci. Number Simulat. 15 3284
[22] Zhu X and Yang G 2008 Delay-dependent stability criteria for systems with differentiable time delays Acta Automatica Sinica 37 765