Gauged Lepton Number, Dirac Neutrinos, Dark Matter, and Muon $g - 2$

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Abstract

Lepton number is promoted to an $U(1)_L$ gauge symmetry in a simple extension of the standard model. The spontaneous breaking of $U(1)_L$ by three units allows a conserved $Z^L_3$ lepton symmetry to remain, guaranteeing that neutrinos are Dirac fermions, which acquire naturally small masses from a previously proposed mechanism. Dark matter appears as a singlet scalar, with dark symmetry $Z^D_3$ derivable from $Z^L_3$. Muon $g - 2$ may be explained.
Introduction: In the minimal standard model (SM) of quarks and leptons, it is well-known that baryon number \( B \) and lepton number \( L \) appear as accidental global symmetries. It is also well-known that \( B - L \) may be gauged with the addition of a singlet right-handed neutrino \( \nu_R \) per family. If \( B \) and \( L \) are allowed to be different for each family, the condition for an anomaly-free \( U(1)_X \) gauge symmetry is simply [1]

\[
\sum_{i=1}^{3} 3n_i + n'_i = 0, \tag{1}
\]

where \( n_i, n'_i \) are the \( U(1)_X \) values for each quark and lepton family. If \( n_i = 1/3 \) and \( n'_i = -1 \), then \( B - L \) is obtained. Many other choices have been considered, such as \( L_\mu - L_\tau \) [2, 3], \( B - 3L_\tau \) [4], etc.

The separate gauging of \( B \) and \( L \) is possible [5, 6, 7] with the addition of new fermions. Since \( \nu_R \) is also a new addition, its \( U(1)_L \) assignment [8] is not necessarily the same as \( \nu_L \) which must be the same as the charged leptons. This opens up the interesting option that neutrinos are naturally light Dirac fermions [9]. To prevent \( \nu_R \) from having a Majorana mass, \( U(1)_L \) must not be broken by a singlet scalar with \( U(1)_L \) charge double that of \( \nu_R \). In the conventional \( B - L \) extension of the SM, this may be achieved with the breaking of gauge \( B - L \) by three units [10, 11] with a scalar singlet \( \chi^0 \). Here \( \nu_R \) is chosen to transform as \(-2\) under \( U(1)_L \), so it works just as well. To connect \( \nu_L \) with \( \nu_R \), a heavy Higgs doublet \( (\eta^+, \eta^0) \) transforming as \(-3\) under \( U(1)_L \) is added with a large positive mass-squared. The mechanism of Ref. [12] is then applicable. Let the trilinear coupling connecting the SM Higgs doublet \( \Phi = (\phi^+, \phi^0) \) to \( \eta \) and \( \chi^0 \) be \( \mu \chi^0 \Phi \eta \); then the induced vacuum expectation value (VEV) of the heavy \( \eta^0 \) is given by \(-\mu \langle \chi^0 \rangle \langle \phi^0 \rangle / m^2_\eta \), which is very much suppressed. Being proportional to \( \langle \eta^0 \rangle \), the Dirac neutrino masses are thus naturally very small. This mechanism has recently been invoked in a number of scenarios [13, 14, 15, 16].

The \( U(1)_L \) gauge symmetry is broken to \( Z_3^L \), with each particle transforming as \( \omega^L \) with \( \omega^3 = 1 \). As a bonus, some of the new fermions added to render \( U(1)_L \) anomaly-free form a
dark sector [17]. The dark symmetry is derived [18, 19] from lepton symmetry by multiplying with $\omega^{-2j}$, where $j$ is the particle’s intrinsic spin.

The new particles, including the $U(1)_L$ gauge boson $Z_L$, are not easily produced. They may be rather light, of order 100 GeV, and have escaped detection. In particular, the dark sector may explain the discrepancy of the experimental measurement of the muon anomalous magnetic moment compared against the theoretical SM prediction.

**Model**: The relevant particles of this model are listed in Table 1. The SM leptons come in 3 families; the other fermions and scalars are not duplicated. The $U(1)_L$ charge of $\nu_R$ is chosen to allow it to have a naturally small mass because it couples to $\eta$ not $\Phi$.

| fermion/scalar   | $SU(2)$ | $U(1)_Y$ | $U(1)_L$ | $Z_3^L$ | $Z_3^D$ |
|------------------|---------|----------|----------|---------|---------|
| $(\nu, e)_L$     | 1       | $-1/2$   | 1        | $\omega$ | 1       |
| $e_R$            | 2       | $-1$     | 1        | $\omega$ | 1       |
| $\nu_R$          | 1       | 0        | $-2$     | $\omega$ | 1       |
| $(N^0, E^-)_L$   | 2       | $-1/2$   | 0        | 1       | $\omega^{-1}$ |
| $(N^0, E^-)_R$   | 2       | $-1/2$   | 0        | 1       | $\omega^{-1}$ |
| $F^-_R$          | 1       | $-1$     | $-3$     | 1       | $\omega^{-1}$ |
| $F^-_L$          | 1       | $-1$     | 0        | 1       | $\omega^{-1}$ |
| $n_{1L}$         | 1       | 0        | $-5$     | $\omega$ | 1       |
| $n_{1R}$         | 1       | 0        | 4        | $\omega$ | 1       |
| $n_{2L}$         | 1       | 0        | $-3$     | 1       | $\omega^{-1}$ |
| $n_{2R}$         | 1       | 0        | $-6$     | 1       | $\omega^{-1}$ |
| $(\phi^+, \phi^0)$ | 2       | $1/2$   | $0$     | 1       | 1       |
| $(\eta^+, \eta^0)$ | 2       | $1/2$   | $-3$   | 1       | 1       |
| $\chi_1^0$       | 1       | 0        | 3        | 1       | 1       |
| $\chi_2^0$       | 1       | 0        | $-9$    | 1       | 1       |
| $\zeta^0$        | 1       | 0        | 1        | $\omega$ | $\omega$ |

Table 1: Fermions and scalars under gauge $U(1)_L$ with resulting $Z_3^L$ and $Z_3^D$. 
This model is free of anomalies:

$$[SU(2)]^2 U(1)_L : 3[(1/2)(1)] + (1/2)[−3 − 0] = 0,$$

$$[U(1)_Y]^2 U(1)_L : 3[2(−1/2)^2 − (−1)^2] + 2(−1/2)^2[−3 − 0] + (−1)^2[0 − (−3)] = 0,$$

$$U(1)_Y[U(1)_L]^2 : 3[2(−1/2) − (−1)] + 2(−1/2)[(−3)^2 − 0] − [0 − (−3)] = 0,$$

$$[U(1)_L]^3 : 3[2 − 1 − (−2)^3] + 2[(−3)^3 − 0] + [0 − (−3)]^3]$$

$$+ [(−5)^3 − (4)^3] + [(−3)^3 − (−6)^3] = 0,$$

$$U(1)_L : 3[2 − 1 − (−2)] + 2[−3 − 0] + [0 − (−3)]$$

$$+ [−5 − 4] + [−3 − (−6)] = 0.$$

The allowed Yukawa terms are:

$$(\bar{\nu}_L \phi^+ + \bar{e}_L \phi^0)e_R, \quad \bar{\nu}_R(\nu_L \eta^0 − e_L \eta^+), \quad \bar{n}_1 LN_1, \quad \bar{n}_1 LN_1 R \eta_1, \quad \bar{n}_1 LN_1 R \eta_1,$$

$$F_L F_R \chi_1, \quad (\bar{N}_R N_L + \bar{E}_R E_L) \chi_1, \quad (\bar{N}_R \phi^+ + \bar{E}_R \phi^0) F_L, \quad (\bar{N}_L \phi^+ + \bar{E}_L \phi^0) F_R,$$

$$n_2 LN_2, \quad \bar{n}_2 LN_2 (N_L \phi^0 − E_R \phi^+), \quad \bar{n}_2 LN_2 (N_L \phi^0 − E_R \phi^+),$$

$$\bar{e}_R F_L \zeta, \quad (\bar{\nu}_L N_R + \bar{e}_L E_R) \zeta, \quad \bar{\nu}_R n_2 L \zeta.$$

After symmetry breaking by $\langle \chi_1 \rangle = u_1$ and $\langle \chi_2 \rangle = u_2$, the terms involving the SM leptons show that the Dirac fermion $n_1$ mixes with one linear combination of $\nu_R$. This means that the three $\nu_L$ of the SM pair up with three linear combinations out of the four right-handed singlets, i.e. the three $\nu_R$ plus $n_1 R$. The remaining combination pairs up with $n_1 L$.

The Dirac fermions $(N,E), F,$ and $n_2$ mix among themselves, but are distinct from the SM leptons and $n_1$. The two sectors are connected only through $\zeta$. The residual symmetry is $Z_3^L$ as shown in Table 1. The dark symmetry $Z_3^D$ is obtained by multiplying each with $\omega^{-2j}$, where $\omega^3 = 1$ and $j$ is the particle’s intrinsic spin. The lightest particle of the dark sector is assumed to be $\zeta$. 

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Muon Anomalous Magnetic Moment: The new interactions which contribute to the muon anomalous magnetic moment are

$$\mathcal{L}_{\text{int}} = g_L Z_L^\alpha \bar{\mu} \gamma^\alpha \mu + [y_E^\mu \bar{\zeta} \mu L + y_F^\mu \bar{\zeta} \mu R + H.c.]$$

with the results [20]:

$$\Delta a_\mu (Z_L) = \frac{g_L^2 m_\mu^2}{12 \pi^2 m_{Z_L}^2} \frac{m_{Z_L}^2}{216 \pi^2 (u_1^2 + 9 u_2^2)},$$

$$\Delta a_\mu (\zeta, E) = \frac{\mu_E^2 m_\mu^2 f (m_E^2 / m_\zeta^2)}{96 \pi^2 m_\zeta^2}, \quad \Delta a_\mu (\zeta, F) = \frac{\mu_F^2 m_\mu^2 f (m_F^2 / m_\zeta^2)}{96 \pi^2 m_\zeta^2},$$

where $m_{Z_L}^2 = 18 g_L^2 u_1^2 + 162 g_L^2 u_2^2$ and

$$f(r) = (r - 1)^{-4} (r^3 - 6 r^2 + 3 r + 2 + 6 r \ln r).$$

As a numerical example, let $m_\zeta = 70$ GeV, $m_E = m_F = 110$ GeV, $y_E^\mu = y_F^\mu = 1.2$, and $u_1 = 92$ GeV with $u_2$ negligibly smaller, then the sum of the above yields $2.51 \times 10^{-9}$, which is the central value of the deviation of the world average experimental value [21] from the theoretical prediction [22] of the SM. The $m_E, m_F, m_\zeta$ values are chosen to avoid constraints from collider data [23] on colorless charged particles decaying to muons and missing energy. The $Z_L$ mass is $g_L$ times 390 GeV. Since $Z_L$ couples only to leptons, the bound on $m_{Z_L}$ is 209 GeV, coming from the highest energy of the LEP II $e^+ e^-$ collider. This means that $g_L > 0.54$. Also, $m_E = y_E u_1$ and $m_F = y_F u_1$, so $y_E = y_F = 1.2$ in this example.

Scalar Sector: The scalar potential consisting of $\Phi, \eta, \chi_{1,2}, \zeta$ is given by

$$V = -\mu_0^2 \Phi^i \Phi + m_1^2 \eta^i \eta - \mu_1^2 \chi_1^i \chi_1 + m_2^2 \chi_2^2 + m_3^2 \zeta^* \zeta$$

$$+ \mu_1 \chi_1^i \Phi^j \eta + f \chi_2 \chi_1^3 + f' \chi_1^3 + H.c.]$$

+ the usual quartic terms.

The $\mu'$ and $f$ terms allow $\langle \eta^0 \rangle = v_1$ and $\langle \chi_2 \rangle = u_2$ to be suppressed from $m_1^2 > 0$ and $m_2^2 > 0$, using the mechanism of Ref. [12], i.e.

$$v_1 \simeq -\mu' u_1 v_0 \frac{1}{m_1^2}, \quad u_2 \simeq -f u_1^3 \frac{1}{m_2^2},$$

(16)
where \( \langle \phi^0 \rangle = v_0 \) and \( \langle \chi_1 \rangle = u_1 \). The \( f' \) term allows \( \zeta \) to transform as \( \omega \) under \( Z_3 \) after \( \chi_1 \) picks up a VEV.

The physical scalars after the spontaneous breaking of \( SU(2)_L \times U(1)_Y \times U(1)_L \) are then the SM Higgs boson \( h \), the corresponding \( H \) from \( U(1)_L \), and \( \zeta \), whereas the \( \eta \) doublet and \( \chi_2 \) singlet are much heavier. Since \( \zeta \) is assumed to be dark matter, its interactions with \( h \) and \( H \) are important considerations, i.e.

\[
-\mathcal{L}_{\text{int}} = \lambda (\sqrt{2}v_0 h + \frac{1}{2} h^2)\zeta^*\zeta + \lambda' (\sqrt{2}u_1 H + \frac{1}{2} H^2)\zeta^*\zeta.
\]

The constraints on \( \lambda \) and \( \lambda' \) will be discussed later.

**Neutrino Sector**: There are 4 Dirac neutrinos in this model. The \( n_{1L} \) singlet pairs up with one linear combination of the three right-handed neutrinos (call it \( \nu_R \)) with mass proportional to \( u_1 \). This \( \nu_R \) will connect to one linear combination of the three left-handed neutrinos (call it \( \nu_L \)). The 2 \( \times \) 2 mass matrix linking \((\nu_L, n_{1L})\) to \((n_{1R}, \nu_R)\) is of the form

\[
\mathcal{M}_{\nu,n} = \begin{pmatrix} 0 & f_1 v_1 \\ f_2 u_2 & f_3 u_1 \end{pmatrix}.
\]

Since \( v_1 \) and \( u_2 \) are suppressed, \( \nu_L \) actually pairs up with \( n_{1R} \) to form a very light Dirac neutrino of mass \((f_1 v_1)(f_2 u_2)/(f_3 u_1)\). This is small compared to the other two Dirac neutrino masses which are proportional to \( v_1 \) alone. Hence this model predicts one nearly massless Dirac neutrino, two of order 1 eV, and one of order 100 GeV, which decays through \( Z_L \) to SM leptons through \( \nu_R - n_{1R} \) mixing.

**Dark Sector**: The dark fermions \( N, E, F \) all decay to \( \zeta^* \) and a lepton. They are produced only through electroweak interactions. As noted earlier, for \( m_E = m_F = 110 \text{ GeV} \), their decay to muons with the large missing energy of \( m_\zeta = 70 \text{ GeV} \) is allowed by the present collider data [23]. The interactions of \( \zeta \) with the SM particles are shown in Eqs. (10),(17), and through the \( Z_L \) gauge boson. In underground direct-search experiments using nuclear recoil, only the Higgs exchange is applicable.
The spin-independent cross section for elastic scattering off a xenon nucleus is

$$\sigma_0 = \frac{1}{\pi} \left( \frac{m_\zeta m_{Xe}}{m_\zeta + m_{Xe}} \right)^2 \left| \frac{54 f_p + 77 f_n}{131} \right|^2,$$  \hfill (19)

where \[24]\]

$$f_p \frac{m_p}{m_\zeta} = \left[ 0.075 + \frac{2}{27} (1 - 0.075) \right] \frac{\lambda}{m_h^2 m_\zeta},$$ \hfill (20)

$$f_n \frac{m_n}{m_\zeta} = \left[ 0.078 + \frac{2}{27} (1 - 0.078) \right] \frac{\lambda}{m_h^2 m_\zeta}. \hfill (21)$$

For $m_\zeta = 70$ GeV, $m_h = 125$ GeV, and $m_{Xe} = 122.3$ GeV, the upper limit on $\sigma_0$ is \[25\] $7 \times 10^{-47}$ cm$^2$, thereby yielding $\lambda < 1.36 \times 10^{-4}$.

As for relic abundance, the cross sections of $\zeta\zeta^*$ to lepton pairs through $Z_L$ or Eq. (10) are helicity-suppressed. Assuming $H$ to be lighter than $\zeta$, the $\zeta\zeta^* \to HH$ amplitudes are shown in Fig. 1. The sum of these diagrams (with a factor of 2 in the last one) for $\zeta\zeta^*$ annihilation at rest is

$$\lambda_{eff} = \lambda' \left[ 1 + \frac{3m_H^2}{4m_\zeta^2 - m_H^2} - \frac{4\lambda' u_1^2}{2m_\zeta^2 - m_H^2} \right] = \lambda'(1.675 - 5.461 \lambda'), \hfill (22)$$

where $m_\zeta = 70$ GeV, $m_H = 60$ GeV, and $u_1 = 92$ GeV have been used. The annihilation cross section times relative velocity is

$$\sigma_{ann} \times v_{rel} = \frac{\lambda_{eff}^2 \sqrt{m_\zeta^2 - m_H^2}}{64\pi m_\zeta^3}.$$ \hfill (23)
Setting this equal to the typical value of $3 \times 10^{-26}$ cm$^3$/s for the correct relic abundance, $\lambda_{\text{eff}} = 0.07$ is obtained, for which $\lambda' = 0.05$ is a solution. Once produced, $H$ decays through its very small mixing with $h$ to SM particles.

**Concluding Remarks** : Three seemingly unrelated fundamental issues in particle and astroparticle physics are shown to be connected in a proposed theoretical framework of gauged lepton number $U(1)_L$. The three topics are (1) naturally small Dirac neutrino masses, (2) dark matter, and (3) the muon anomalous magnetic moment. Assigning charges to leptons under $U(1)_L$, together with the addition of new fermions to render the theory anomaly-free as shown in Table 1, it is discovered that the breaking of gauge $U(1)_L$ results in a conserved $Z_3^L$ symmetry. A dark sector also automatically appears, with dark $Z_3^D$ symmetry obtained by multiplying with $\omega^{-2j}$ for each particle, where $\omega^3 = 1$ and $j$ is the particle’s intrinsic spin.

This breaking of $U(1)_L$ by 3 units is also the key to having naturally small Dirac neutrino masses using the mechanism of Ref. [12]. New particles transforming under $U(1)_L$ are not easily produced. They may be below 100 GeV and have escaped detection. Hence they are also suitable for explaining the discrepancy of the measured muon anomalous magnetic moment compared against the SM theoretical prediction. Here the contributions come mainly from the dark sector, with a smaller share from the $Z_L$ gauge boson. To summarize, the numerical values used in this study are $m_\zeta = 70$ GeV for the scalar dark-matter candidate, $m_H = 60$ GeV for the Higgs boson associated with $U(1)_L$ which is broken by $\langle \chi_1 \rangle = u_1 = 92$ GeV, and $m_E = m_F = 110$ GeV for the charged fermions in the dark sector.

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