NLO QCD corrections to polarised di-boson production in semi-leptonic final states

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ABSTRACT: Understanding the polarisation structure and providing precise predictions for multi-boson processes at the LHC is becoming urgent in the light of the upcoming run-3 and high-luminosity data. The CMS and ATLAS collaborations have already started using polarised predictions to perform template fits of the data, getting access to the polarisation of W and Z bosons. So far, only fully-leptonic decay channels have been considered in this perspective. The natural step forward is the investigation of hadronic decays of electroweak bosons. In this work, we compute NLO QCD corrections to the production and decay of WZ pairs at the LHC in final states with two charged leptons and jets. The calculation relies on the double-pole approximation and the separation of polarised states at the level of Standard Model amplitudes. The presented NLO-accurate results are necessary building blocks for a broad understanding and precise modelling of polarised di-boson production in semi-leptonic decay channels.

KEYWORDS: polarisation, electroweak bosons, NLO QCD, semi-leptonic, LHC
1 Introduction

The pioneering measurements with the run-2 LHC dataset in di-boson inclusive production [1–3] and scattering (VBS) [4] have paved the way towards a refined experimental investigation of the polarisation states of electroweak (EW) bosons produced in multi-boson processes. The most striking difference of the methods adopted in Refs. [1–4] with respect to previous polarisation analyses [5–11] is the usage of polarised-signal templates directly generated with Monte Carlo generators, which is expected to give a more complete picture of the polarisation structure than the simple evaluation of angular coefficients, giving access to interference effects and spin correlations.

The extraction of angular coefficients from unpolarised decay-angle distributions was first proposed in Ref. [12, 13] and has been applied in several theoretical studies on $V + j$ [12–16] and inclusive di-boson production [17–21]. The direct Monte Carlo simulation of processes with intermediate EW polarised bosons was proposed for VBS [22–25] and it is currently available in public codes at leading-order (LO) accuracy matched to parton-shower (PS) in the Standard Model (SM) as well as in the presence of beyond-SM effects. The extension of this approach to higher perturbative orders has been carried out focusing on di-boson inclusive production, reaching next-to-leading-order (NLO) in the QCD coupling [26, 27] and later on NLO in the EW coupling [28–30] and next-to-next-to-leading-order (NNLO) in QCD [31]. Analogous methods have been applied to Higgs decays [32, 33] and to $V + j$ production [34].
A number of works have tackled the polarisation structure of VBS with machine-learning techniques with promising results [35–38].

So far, both experimental measurements [1–11] and phenomenological studies [12–37, 39–43] have focused on leptonic decays of weak bosons, cleaner than the hadronic ones in a hadron-collider environment, but with a smaller branching ratio and affected by reconstruction effects in the presence of neutrinos in the final state. Indeed, the increasing interest in measuring gauge-boson polarisations at the LHC and the lack of statistics in fully-leptonic decay channels has triggered a number of phenomenological studies [44–47] of processes with hadronically decaying bosons. The hadronic decay of W and Z bosons has the great advantage of larger branching ratio w.r.t. the leptonic one, but the disadvantage of much larger backgrounds to deal with in the LHC environment that is dominated by QCD-induced processes. The focus of these polarisation studies has been put especially on the discrimination between longitudinal and transverse bosons in boosted kinematic configurations [44–46]. The driving idea is that the reconstruction of the boosted-fat-jet substructure, making use of either traditional substructure observables like $N$-subjettines [44] and soft-drop [46], or machine-learning [45] techniques, is expected to maximise the information about the polarisation state of the decayed boson, since the jet constituents can be associated to some degree of precision to the decay quarks. Recently it has been proposed to use energy correlators to improve the polarisation discrimination [47]. The existing studies of polarised bosons in the leptonic decay channel suggest that there are LHC observables that enable to discriminate polarisation states without the need to reconstruct individual decay products [3].

So far, no polarisation measurement has been carried out yet with hadronic decays of gauge bosons, although a number of sensitivity studies have been performed for the high-energy [48] and high-luminosity (HL) [49] runs of the LHC, mostly targeting VBS processes.

In the specific case of inclusive di-boson production, the semi-leptonic decay channel has been investigated with 13-TeV LHC data with the aim of searching for new resonances [50–54]. Measuring polarisations in this channel could further constrain new-physics effects [55] and in particular the spin of possible underlying resonances decaying in two gauge bosons. In spite of the high-precision SM predictions available for unpolarised [56–77] and polarised [26–31] di-boson production at the LHC, the fully-leptonic decay channel has always been considered and no tailored study beyond LO (+PS) exists for the semi-leptonic channel in the presence of polarised intermediate bosons. The NLO QCD corrections to the hadronic decay of polarised weak bosons have been known for many years [78, 79] and can be generated easily with the help of any one-loop amplitude provider, but they have not been combined yet with realistic LHC production processes.

In this work we perform a consistent combination of NLO QCD corrections to di-boson production and to the hadronic decay of one of the two bosons in the
double-pole approximation\(^1\) [82–86] and in the presence of polarised and unpolarised intermediate bosons, preserving partial off-shell effects and the complete spin correlations between production and decay. This target is pursued following the strategy proposed at LO in Ref. [22] and later extended to NLO in Refs. [26, 28, 30]. We consider a boosted regime for the weak bosons, where the doubly-longitudinal signal is expected to be sizeable [27], but we do not make use of any jet-substructure-reconstruction technique.

This paper is organised as follows. In Section 2 we describe the technical details of the calculation, the SM input parameters and the kinematic setups that are considered. In Section 3 we present and discuss the integrated and differential predictions for doubly-polarised signals at the LHC with 13.6-TeV centre-of-mass (CM) energy. Our conclusions and outlook are given in Section 4.

2 Details of the calculation

We consider di-boson (ZW\(^+\)) inclusive production at the LHC:

\[
pp \rightarrow Z (\rightarrow \ell^+\ell^-) W^+ (\rightarrow jj) + X, \quad pp \rightarrow Z (\rightarrow \ell^+\ell^-) W^+ (\rightarrow J) + X, \quad (2.1)
\]

where the two semi-leptonic decay channels differ by the number of jets from the decay of the W boson. In the first case (resolved topology) the W\(^+\) boson decays into two light jets, while in the second case (unresolved topology) it decays into a single fat jet. We calculate the NLO QCD corrections to the tree-level EW process, i.e. of perturbative order \(O(\alpha_s\alpha^4)\), working in the double-pole approximation (DPA) [82–86]. The non-resonant contributions and off-shell effects beyond the DPA, as well as the other perturbative orders contributing to the same final state (which cannot embed two weak-boson propagators) are not considered here. In other words, we do not include the irreducible non-resonant and QCD multi-jet backgrounds to the di-boson signal. We also restrict the calculation to ZW\(^+\) production, neglecting the ZW\(^-\) and ZZ resonant processes which also contribute to the same final state with two leptons and a hadronic system. Since the ZW\(^+\) contribution is expected to be larger than ZW\(^-\) and ZZ at the LHC (owing to parton-density and coupling-strength considerations), our analysis, which is not meant to be fully realistic, is expected to give useful insights for future analyses in semi-leptonic final states with run-3 and HL-LHC data.

2.1 Double-pole approximation and polarised-signal definition

Sample diagrams for ZW\(^+\) production that contribute in the DPA at LO and at NLO QCD are shown in Figure 1. In the DPA [82–86] all diagrams without two s-channel boson propagators (one W\(^+\) and one Z boson) are neglected, giving an
amplitude (which is not gauge invariant) that is fully factorised in a production × decay form. In order to recover the EW gauge invariance, the numerator of the doubly-resonant amplitude is evaluated with a modified kinematics obtained via an on-mass-shell projection of the resonant EW bosons, while the denominator (i.e. the two Breit–Wigner distributions from the weak-boson propagators) is evaluated with the original kinematics (for off-shell bosons). The widths of the W and Z bosons are set to zero in the amplitude numerator, while they are kept finite in the denominators of the weak-boson propagators targeted by the DPA.

This technique preserves all spin correlations (the full spin matrix is accounted for) and partial off-shell effects (thanks to the Breit–Wigner modulation and the use of the off-shell phase space). An alternative method that is often used for polarised-signal simulation [25, 31, 34] is the narrow-width approximation [87, 88].

The DPA approach makes it natural to separate polarisation states of intermediate weak bosons at amplitude level [22, 26]. A priori, the polarisation of particles in scattering processes can only be defined for stable external particles, while for intermediate (virtual) particles it is required to perform the sum over all polarisations (physical and unphysical). The factorised structure of double-pole-approximated, doubly-resonant amplitudes enables the splitting of the numerator of each gauge-boson propagator into the sum over physical polarisation states: longitudinal (L), left handed (−) and right handed (+). The contributions of unphysical polarisa-
tion states are exactly cancelled by those of would-be Goldstone bosons on the mass shell. Therefore, replacing the polarisation sum in the propagator numerator with the contribution of a specific polarisation state \( \lambda \) gives a gauge-invariant \( \lambda \)-polarised amplitude, where \( \lambda = 0, \pm \). A convenient choice \([23]\) is to consider the transverse-polarisation state (T) which is defined as the coherent sum of the left- and right-handed states, including also the left–right interference term.

It is essential to recall that the polarisation vectors appearing in the propagator numerator depend on the Lorentz frame where the kinematics is evaluated. This implies that the definition of the polarised signal is reference-frame dependent. The preferred choice for di-boson inclusive production is the di-boson–system CM frame \([1, 20, 27]\), as it allows for a natural interpretation in terms of the corresponding tree-level 2 \( \to \) 2 process (\( g_{\nu \bar{q}_d} \to ZW^+ \)). This is the choice adopted in this work.

As previously stated, the novel aspect of the presented calculation with respect to Ref. \([27]\) is the semi-leptonic decay channel [see Eq. (2.1)]. In the fully-leptonic decay channel, the NLO QCD corrections only enter as initial-state virtual and real radiation. Here we also need to consider the NLO QCD corrections to the \( W^+ \)-boson decay. As can be appreciated in Figure 1, the factorisable virtual corrections to the decay [Figure 1(c)] and to the production sub-process [Figure 1(b)] are accounted for. The real corrections include initial-state-radiation [Figures 1(d)–1(e)] and final-state-radiation [Figure 1(f)] contributions. Both virtual and real non-factorisable NLO QCD corrections to doubly-resonant amplitudes (gluon exchange between production and decay) vanish owing to colour algebra.

The treatment of factorisable real QCD corrections in the DPA is carried out using the general approach introduced in Refs. \([28, 89–91]\) for NLO EW corrections, upon the replacement of the EW coupling \( \alpha \) with a running strong coupling \( \alpha_s \). In order to end up with a final result that is free of infrared (IR) divergences, unresolved real radiation from the production and decay parts of the process need to be managed separately in the DPA and combined consistently within the employed subtraction scheme, in order to have

- the correct matching between subtraction counterterms and the corresponding integrated ones, and
- the cancellation of IR poles between the integrated counterterms and the virtual matrix element.

In particular, gluon radiation from the W-boson decay products and from the WZ-pair-production process are treated separately within the DPA. In our calculation, we employ the dipole subtraction scheme \([92–94]\). For additional technical details, we refer to Ref. \([28]\).
2.2 Monte Carlo tools and input parameters

The presented SM calculation has been performed with two independent in-house multi-channel Monte Carlo codes, MoCaNLO and BBMC. MoCaNLO has been recently employed for NLO EW and QCD corrections to processes with polarised EW bosons in the fully-leptonic decay channel [26–28]. BBMC has been used for NLO corrections to off-shell di-boson production [57–59, 95] and has been modified to enable the treatment of resonances in the DPA. The UV-renormalised tree-level and one-loop SM amplitudes are provided to both codes by RECOLA [96, 97]. The tensor reduction and integration of loop integrals is performed with COLLIER [98].

The calculation is carried out in the SM at NLO QCD accuracy. The five-flavour scheme and no quark-family mixing (unit CKM matrix) are understood. All light quarks and leptons are considered massless. The pole masses \( M_V \) and widths \( \Gamma_V \) of the EW bosons are calculated from the on-shell values \( M_{V}^{\text{OS}}, \Gamma_{V}^{\text{OS}} \) [99],

\[
M_{W}^{\text{OS}} = 80.379 \text{ GeV}, \quad \Gamma_{W}^{\text{OS}} = 2.085 \text{ GeV}, \\
M_{Z}^{\text{OS}} = 91.1876 \text{ GeV}, \quad \Gamma_{Z}^{\text{OS}} = 2.952 \text{ GeV},
\]

according to the relations [100]

\[
M_V = \frac{M_V^{\text{OS}}}{\sqrt{1 + \left(\frac{\Gamma_V^{\text{OS}}}{M_V^{\text{OS}}}\right)^2}}, \quad \Gamma_V = \frac{\Gamma_V^{\text{OS}}}{\sqrt{1 + \left(\frac{\Gamma_V^{\text{OS}}}{M_V^{\text{OS}}}\right)^2}} \tag{2.3}
\]

The EW coupling \( \alpha \) is calculated in the \( G_\mu \) scheme [84], i.e. as a function of the Fermi constant \( G_\mu \) and the weak-boson pole masses:

\[
\alpha = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right), \quad G_\mu = 1.16638 \cdot 10^{-5} \text{ GeV}^{-2}. \tag{2.4}
\]

Since we only consider NLO QCD corrections, the masses of the top quark and of the Higgs boson do not enter the calculation. We use NNPDF31nlo_as_0118 [101, 102] parton-distribution functions (PDFs), provided to the Monte Carlo codes via the LHAPDF interface [103]. Also the running of the strong coupling constant \( \alpha_s \) is evaluated with built-in LHAPDF routines. The dipole formalism [92–94] is used for the subtraction of IR singularities of QCD origin. The \( \overline{\text{MS}} \) factorisation scheme is employed for the treatment of initial-state collinear singularities. The central factorisation and renormalisation scales are both set to the same dynamical value \( \mu_F = \mu_R = \mu_0 \) (defined in Section 2.3), and the QCD-scale uncertainties are estimated with independent 7-point variations of \( \mu_F \) and \( \mu_R \), i.e. via the maxima and minima of the corresponding observables for the scale choices

\[
(\mu_R/\mu_0, \mu_F/\mu_0) = (1/2, 1/2), (1/2, 1), (1, 1/2), (1, 1)(1, 2), (2, 1), (2, 2).
\]
2.3 Kinematic setups and scale definition

The event selection and reconstruction are inspired by the recent CMS analysis presented in Ref. [54] (see Table 1 therein). The clustering of jets is carried out with the anti-$k_T$ algorithm [104] recombining only partons with a rapidity smaller than 5. Two different event topologies are considered: resolved and unresolved. In the resolved topology we ask for:

- at least two jets (clustered with $R_0 = 0.4$) with $p_{T,j} > 30$ GeV, $|y_j| < 2.4$ and $\Delta R_{j\ell^\pm} > 0.4$, the two jets with a pair invariant mass closest to $M_W$ being selected as decay jets,

- the system of the two decay jets with $p_{T,jj} > 200$ GeV and $65$ GeV $< M_{jj} < 105$ GeV;

- two opposite-sign, same-flavour leptons with $p_{T,\ell\ell^\pm} > 40$ GeV, $|y_{\ell^\pm}| < 2.4$, $p_{T,\ell\ell} > 200$ GeV and $76$ GeV $< M_{\ell\ell} < 106$ GeV.

In the unresolved topology we ask for:

- at least one jet (clustered with $R_0 = 0.8$) with $|y_J| < 2.4$, $\Delta R_{J\ell^\pm} > 0.8$, $p_{T,J} > 200$ GeV and $65$ GeV $< M_J < 105$ GeV,

- two opposite-sign, same-flavour leptons with $p_{T,\ell\ell^\pm} > 40$ GeV, $|y_{\ell^\pm}| < 2.4$, $p_{T,\ell\ell} > 200$ GeV and $76$ GeV $< M_{\ell\ell} < 106$ GeV.

By requiring large transverse momenta for the vector bosons, we select a boosted regime. We do not apply any veto on additional jets, as the logarithmically-enhanced soft-boson radiation in real contributions [67, 105, 106] is suppressed thanks to the tight transverse-momentum cuts on the two bosons ($p_{T,\ell\ell} > 200$ GeV and $p_{T,J/jj} > 200$ GeV), avoiding huge QCD $K$-factors. Note, however, that the application of symmetric transverse-momentum cuts on the two bosons leads to very large corrections in transverse-momentum distributions close to the cut, due to the sensitivity to quasi-soft and quasi-collinear QCD initial-state radiation [107–111].

The central renormalisation and factorisation scales are set to,

$$\mu_R = \mu_F = \frac{M_{T,Z} + M_{T,J}}{2}. \quad (2.5)$$

The transverse masses are calculated as,

$$M_{T,Z} = \sqrt{p_{T,\ell\ell}^2 + M_{\ell\ell}^2}, \quad M_{T,J} = \sqrt{p_{T,J}^2 + M_J^2}, \quad (2.6)$$

where $p_{T,J}$ and $M_J$ are, respectively, the transverse momentum and invariant mass of the hadronic system $J$ that is identified as the decay-jet system (resolved topology) or as the hardest-$p_T$ fat jet (unresolved topology).
As previously stated, the polarisation states of intermediate gauge bosons are defined in the CM frame, i.e. the rest frame of the system formed by the two leptons and the decay products of the W boson, which are identified for each contribution in the DPA. This choice enables for qualitative comparisons with phenomenological studies of ZW production in the fully-leptonic decay channel [27, 29].

2.4 Validation

The independent implementation in MoCaNLO and BBMC of the general methods described in Ref. [28] has enabled validation checks at several levels of the calculation.

Renormalisation and phase-space integration Detailed comparisons have been performed both for individual phase-space points for Born-level, virtual and real-radiation contributions, giving excellent agreement both for the QCD-scale and matrix-element evaluation. The UV finiteness of virtual amplitudes in the presence of polarised intermediate bosons has been checked varying by several orders of magnitude the UV-scale regulator as input parameter for RECOLA. Note that both codes make use of RECOLA as amplitude provider. The correct application of selection cuts (after jet clustering) in the two setups described in Section 2.3 has been validated comparing results from MoCaNLO and BBMC for each n-body and (n + 1)-body contribution to the cross-section separately. This has been done both for the unpolarised and the polarised process, finding agreement within Monte Carlo uncertainties in all cases.

Subtraction of IR singularities The correct subtraction of IR singularities of QCD origin has been tested in depth, with both comparisons between the two codes and internal checks in each code separately. The dipole-subtraction kernels and the kinematics used to evaluate them has been tested for a number of individual phase-space points, finding good agreement between the two codes. This is especially relevant for subtraction dipoles associated to decay sub-process (gluon radiation off the W decay), whose kinematics has to undergo first the DPA\(^{3,2}\) on-shell projection and second the final-state–final-state Catani–Seymour mapping [28]. Analogous checks have been performed on integrated subtraction counterterms, finding also excellent agreement for each phase-space point considered. The proper cancellation of IR poles in the sum of virtual matrix elements and I-operators [92] has been successfully checked varying the IR-regularisation scale \(\mu_{\text{IR}}\) up and down by 4 orders of magnitude about the scale defined in Eq. (2.5), finding independence of the result from \(\mu_{\text{IR}}\) within the errors of the Monte Carlo integration. The functioning of the subtraction scheme in the DPA has been also tested by means of the technical parameters \(\{\alpha_{\text{dip}}\}\) that set the integration boundaries for the radiation phase space in each dipole [112]. Varying such parameters between 1 and \(10^{-2}\), has enabled to check that the sum of subtracted-real and integrated-counterterm contributions is independent
of the choice of \(\{\alpha_{\text{dip}}\} \) within integration uncertainties. The independence of the NLO corrections from the unphysical parameters \(\mu_{\text{IR}}\) and \(\{\alpha_{\text{dip}}\}\) represents a strong check given the different treatment of real radiation from the production and decay sub-process matrix elements and subtraction counterterms in the DPA. This further confirms that the methods introduced in Ref. [28] provide NLO predictions that are well under control from the IR-subtraction point of view.

**Treatment of polarisations** The definition of polarisation vectors in the CM reference frame represents a crucial step in the calculation, especially for what concerns real radiation. Detailed checks at the phase-space-point level have been done in the Lorentz frame where polarisations are defined in the case of real corrections to the production and decay sub-processes. A complete comparison between MoCaNLO and BBMC has been performed for each contribution to the cross-section, for all doubly-polarised states, giving excellent agreement within integration errors for both integrated result and differential distributions (bin by bin). Further tests have been carried out comparing the two codes for a different definition of polarisation, *i.e.* in the laboratory frame, finding also agreement.

3 Results

In this section we present numerical results at integrated and differential level for doubly-polarised and unpolarised \(ZW^+\) production at the LHC in the semi-leptonic channel. All results shown have been obtained with MoCaNLO. We have considered the specific case of an electron–positron pair (\(\ell = e\)). The sum over light lepton flavours (\(\ell = e, \mu\)) can be simply obtained upon multiplying all cross-sections by a factor of two.

3.1 Integrated Results

In Table 1 we present integrated cross-sections for different polarisation states in the two setups introduced in Section 2.3. Before analysing the QCD corrections, we focus on the LO picture that is already interesting. The contribution of the LL polarisation state is rather large (\(\approx 35\%)\) besides a sizeable TT contribution (\(\approx 50\%, \) dominated by left–right configurations), while the contribution of the mixed polarisation states is at the 10\% level. The suppression of the mixed modes results from the transverse momentum cuts on the produced vector bosons of 200 GeV and the unitarity suppression of the corresponding cross-sections with the square of the energy of the longitudinal vector bosons [113, 114]. At variance with ZZ production, the LL signal is not suppressed owing to the triple-gauge-boson coupling that contributes to LO diagrams. At high energies, the two longitudinal bosons behave in fact like would-be Goldstone bosons...
cross-sections with the ones described in Section 2.3 for unpolarised and doubly-polarised ZW production. Polarisations are defined in the di-boson CM frame. Numerical errors (in parentheses) and QCD-scale uncertainties from 7-point scale variations (in percentages) are shown. The fractions (in percentage) are computed as ratios of polarised interaction contributions over the LO gluon-induced contributions.

Table 1. Integrated cross-sections (in fb) in the resolved and unresolved fiducial setups described in Section 2.3 for unpolarised and doubly-polarised ZW^+ production in the semi-leptonic decay channel. Polarisations are defined in the di-boson CM frame. Numerical errors (in parentheses) and QCD-scale uncertainties from 7-point scale variations (in percentages) are shown. The fractions (in percentage) are computed as ratios of polarised cross-sections over the unpolarised one. K-factors are defined as ratios of the NLO QCD cross-sections with (K_{NLO}) and without (K_{NLO}^{(no g)}) gluon-induced contributions over the LO ones.

[115, 116] that result from an s-channel W^+ boson carrying the whole partonic energy (which exceeds 400 GeV in our setups). While in the resolved topologies the mixed polarisation states have similar cross-sections, in the unresolved topology the TL cross-section is 15% larger than the LT one. This is due to the fact that the two quarks from the decay of a transverse W boson are preferably produced in and opposite to the direction of the W boson in the W-boson CM frame. Thus, they are roughly back to back in the laboratory frame and less likely recombined to a fat jet. Therefore, the two-light-jet requirement is fulfilled more easily than the requirement of a single fat jet with a mass close to M_W. The decay jets of a longitudinal W boson, on the other hand, are preferably perpendicular to its direction and consequently more collinear in the laboratory frame and more likely to be recombined to a fat jet.
The overall picture at NLO QCD shows a smaller difference between the results in the two setups, especially in the case of LT and TL polarisations. The NLO QCD corrections are very large for polarisation states with at least one transverse boson, with a size that is comparable to the one for the LO cross-section. On the contrary, the corrections are small for the purely longitudinal state. The LL state receives negative corrections (about $-5\%$) in the resolved topologies, while in the unresolved setup the corrections become positive (10%). In the resolved topology the TT and LT states receive comparable corrections at the $+100\%$ level, while a different behaviour between the two is found in the unresolved setup. The largest corrections in both setups (about $+200\%$) characterise the TL polarisation state. In general (both for polarised and unpolarised states), these big corrections are caused by hard QCD radiation in partonic processes with initial-state gluons [105], which are enhanced by the large gluon luminosity in the proton. In fact, omitting the gluon-induced channels, the $K$-factors are much smaller (see column labelled $K_{\text{NLO}}^{(\text{no g})}$ in Table 1).

For the LL polarisation state, not only the gluon-emission real corrections are small but also the gluon-induced ones (with quark emission) do not give large contributions, as already seen in inclusive ZW calculations with leptonic decays [27, 29]. This can be understood as follows. As demonstrated in Ref. [106] the dominant NLO QCD corrections arise for the production of a vector-boson–quark pair in gluon–quark scattering with subsequent radiation of a soft vector boson from the quark (see also Figure 2). However, the sub-process $gq \rightarrow Vq'$ is suppressed for longitudinal high-energy vector bosons, as can seen via the Goldstone-boson equivalence theorem and the absence of LO diagrams for the corresponding process with the vector boson replaced by a would-be Goldstone boson (for massless quarks).

The striking difference (in both setups) between TL and LT at NLO QCD results from hard QCD radiation in gluon-induced partonic channels which enhances the mixed state with a longitudinal W boson much more than the one with a transverse W boson. This is a consequence of the unitarity cancellations for mixed polarisation states with high transverse momenta of the longitudinal vector bosons, which also holds in the presence of additional real QCD radiation. Owing to this suppression of high-energy longitudinal bosons, kinematic configurations are preferred, where the transverse boson recoils against the longitudinal one and the additional jet, favouring (in the gluon-induced processes) configurations with a rather collimated system of the longitudinal boson and the radiation jet. We have checked numerically that the diagrams in Figures 2(a) and 2(b) give the leading contribution to the LT and TL state, respectively. In the TL polarisation mode, all jets are thus produced relatively close in phase space, resulting in a less efficient decay-jet identification. In addition, the transverse $Z$ boson absorbs the entire hadronic recoil in the final state, allowing for a softer longitudinal boson and therefore a less severe unitarity suppression. For the LT mode the leptonically decaying longitudinal $Z$ boson needs a
Figure 2. Leading QCD-radiation contributions in the $q_0 g$ partonic channel for longitudinal-transverse and transverse-longitudinal $ZW^+$ production at the LHC in the semi-leptonic decay channel. Particles carrying colour charge are highlighted in red.

transverse momentum above 200 GeV, which causes a stronger unitarity cancellation. These effects are further confirmed at differential level, as shown in Section 3.2. Up to these differences, the mixed states give a contribution of about 14% to the total cross-section at NLO QCD.

The QCD correction to the TT state is similar in the two setups, giving a NLO cross-section that represents 65% of the total. It is worth recalling that the $ZW$ TT state is characterised by an approximate amplitude zero at tree level [114], which is spoiled by QCD radiation from higher orders (already at NLO).

The interference effects, obtained subtracting the sum of polarised cross-sections from the unpolarised one, contribute $+3\%$ at LO and are almost negligible ($< 0.5\%$) at NLO QCD, with irrelevant differences between the two setups. This effect is more sizeable in differential results.

As expected in general for purely EW processes, the QCD-scale uncertainties at LO are small since they only come from factorisation-scale variations. In addition, the requirement of the boosted regime reduces them roughly by a factor of 4 w.r.t. inclusive calculations [27, 29]. We have checked numerically that the same effect is also found in the fully-leptonic channel for the WZ process. At NLO QCD the renormalisation-scale dependence of the strong coupling and the sizeable real corrections render the scale uncertainties much larger than at LO, ranging between 7\% and 11\% for polarisation modes with at least one transverse boson. A different behaviour is found for the LL mode, for which the real corrections are small and therefore the NLO QCD scale uncertainty is of the same order of magnitude as at LO (1\%). The difference in scale uncertainties between the two setups is motivated by the small differences in the applied kinematic selections. Owing to the EW character of the LO $ZW$ process, a truly NLO QCD scale dependence of the polarised cross-section can only be obtained upon including NNLO QCD corrections, which is possible with the tools developed in Ref. [31]. Notice that this would also provide more reliable scale uncertainties.

The obtained results (in both setups) understand a hard cut on the transverse
momentum of both bosons, which acts like a veto on additional QCD jets. Although this prevents huge QCD $K$-factors, the symmetric character of such selections ($p_{T,\text{jj}}/J > 200 \text{ GeV}$, $p_{T,\ell\ell} > 200 \text{ GeV}$) induces large unphysical NLO corrections in kinematic regions close to the cut [111]. These can be avoided by omitting the cut $p_{T,\text{jj}}/J > 200 \text{ GeV}$. The corresponding results for the resolved setup described in Section 2.3 without this cut are shown in Table 2. While the LO picture is identical to the original setup, the omission of the $p_{T,\text{jj}}/J$ cut induces huge QCD corrections owing to the fact that the large transverse momentum of the leptonic system is now absorbed by the entire hadronic system, including both decay and extra real radiation jets. This leads to much larger $K$-factors for all polarisation states with at least one transverse boson, while the LL contribution does not change so much, confirming the arguments given above. The QCD corrections and, in particular, the gluon-induced contributions are huge for the TL state, giving a TL component that amounts at almost 19% of the total unpolarised result (compared to 5% at LO). As expected, the TL contribution is much larger than the LT one at NLO QCD, because the Z boson is boosted ($p_{T,\ell\ell} > 200 \text{ GeV}$) while the hadronically decaying W boson is not necessarily boosted, therefore the unitarity suppression is strong for the LT state but much smaller for the TL state. Note that the strong increase of the mixed polarisation states is again dominantly due to the gluon-induced channels, as can be seen by inspecting the $K$-factors $K_{\text{NLO}}^{(\text{no g})}$ without these contributions in Table 2. Despite the small QCD corrections, the LL fraction is diminished (13.6%) w.r.t. the

| state | $\sigma_{\text{LO}}$ [fb] | $f_{\text{LO}}$ [%] | $\sigma_{\text{NLO}}$ [fb] | $f_{\text{NLO}}$ [%] | $K_{\text{NLO}}$ | $K_{\text{NLO}}^{(\text{no g})}$ |
|-------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|
| resolved (no minimum $p_{T,\text{jj}}$ cut), $Z(e^+e^-)W^+(-j)$ | | | | | | |
| unpol. | 1.8564(1)$^{+1.2\%}_{-1.4\%}$ | 100 | 5.5388(8)$^{+10.6\%}_{-8.6\%}$ | 100 | 2.984 | 1.371 |
| $Z_LW_L^+$ | 0.64605(3)$^{+0.2\%}_{-0.6\%}$ | 34.8 | 0.7525(4)$^{+1.5\%}_{-1.2\%}$ | 13.6 | 1.165 | 1.194 |
| $Z_LW_T^+$ | 0.08687(1)$^{+0.2\%}_{-0.6\%}$ | 4.7 | 0.3057(1)$^{+11.4\%}_{-9.2\%}$ | 5.5 | 3.519 | 1.462 |
| $Z_TW_L^+$ | 0.08710(1)$^{+0.1\%}_{-0.6\%}$ | 4.7 | 1.0486(1)$^{+14.6\%}_{-11.9\%}$ | 18.9 | 12.04 | 2.408 |
| $Z_TW_T^+$ | 0.97677(7)$^{-2.2\%}_{-2.2\%}$ | 52.6 | 3.5506(9)$^{+11.8\%}_{-9.6\%}$ | 64.1 | 3.635 | 1.424 |
| interf. | 0.0595(1) | 3.2 | -0.119(2) | -2.1 | - | - |

Table 2. Integrated cross-sections (in fb) in the resolved setup described in Section 2.3 without the minimum $p_{T,\text{jj}}$ cut of 200 GeV. Polarisations are defined in the di-boson CM frame. Numerical errors (in parentheses) and QCD-scale uncertainties from 7-point scale variations (in percentages) are shown. The fractions (in percentage) are computed as ratios of polarised cross-sections over the unpolarised one. $K$-factors are defined as ratios of the NLO QCD cross-sections with ($K_{\text{NLO}}$) and without ($K_{\text{NLO}}^{(\text{no g})}$) gluon-induced contributions over the LO ones.
default setups (20%), but remains sizeable compared to the 6% found in inclusive fiducial setups [27, 29].

3.2 Differential Results

In order to understand the relative importance of the various polarisation states and the differences between the two setups at LO and at NLO QCD, it is essential to analyse differential distributions, which are presented in this section. Unless otherwise stated, the two default setups described in Section 2.3 are understood.

We start the discussion on differential results with the angular observable that is directly related to the polarisation state of an EW boson. The polar decay angle of the lepton $e^\pm$ is defined as the angular separation between the lepton direction in the rest frame of the leptonically-decaying boson ($\vec{p}_{e^\pm}^*$) and the direction of the same boson calculated in the reconstructed di-boson CM frame ($\vec{p}_{e^+e^-}^{CM}$),

$$\cos \theta_{e^\pm}^{*,CM} = \frac{\vec{p}_{e^\pm}^* \cdot \vec{p}_{e^+e^-}^{CM}}{|\vec{p}_{e^\pm}^*||\vec{p}_{e^+e^-}^{CM}|}. \tag{3.1}$$

The differential cross-section with respect to $\cos \theta_{e^\pm}^{*,CM}$ is depicted in Figure 3. The polar decay angle $\theta_{e^\pm}^{*,CM}$ is very well suited for the discrimination between polarisation states of the $Z$ boson, while it cannot give access to the polarisation state of the other boson, therefore very similar shapes are expected for LL and LT modes, as well as for the TL and TT ones. The slight differences are a consequence of the (small) interference and spin-correlation effects. As expected from results in the fully-leptonic decay channel [27, 29], a longitudinally polarised $Z$ boson gives leptons produced mostly around $\theta_{e^\pm}^{*,CM} = \pi/2$, while a transverse one populates more regions around $\cos \theta_{e^\pm}^{*,CM} = \pm 0.6$. In both cases the collinear and anti-collinear configurations are suppressed by the transverse-momentum cut on the charged leptons. The NLO QCD corrections distort the polarised and unpolarised shapes only in less-populated regions. In the rest of the spectrum they give rather flat enhancements to the LL and LT distributions, while for the TT and TL states the $K$-factors mildly diminish towards positive $\cos \theta_{e^\pm}^{*,CM}$. In the regions where the TT and TL distributions feature a sharp drop, the interferences reach the 9% level at LO, while they almost disappear at NLO QCD. Overall, the leptonic decay of the $Z$ boson is only indirectly affected by QCD effects. The results for this observables are basically identical in the two setups.

In Figure 4 the differential cross-section in the cosine of the scattering angle is shown. This angle is defined as

$$\cos \theta_{\text{scatt}} = \frac{\vec{p}_{e^+e^-}^{CM,z}}{\sqrt{\vec{p}_{e^+e^-}^{CM} \cdot \vec{p}_{e^+e^-}^{CM}}} \tag{3.2}$$

where $\vec{p}_{e^+e^-}^{CM}$ is the three momentum of the electron–positron pair in the reconstructed di-boson CM frame and $\vec{p}_{e^+e^-}^{CM,z}$ is its component in the $z$ direction. This observable
Figure 3. Distribution in the cosine of the polar decay angle of the positron in semi-leptonic ZW^+ production at the LHC. The definition of this angle is given in Eq. (3.1). Results for the unpolarised and doubly-polarised process are shown in the resolved (left) and unresolved (right) setups described in Section 2.3. From top down: NLO QCD differential cross-sections, ratios of NLO QCD cross-sections over the LO ones, normalised LO (dashed) and NLO QCD (solid) shapes (unit integral), interference contributions relative to the unpolarised cross-section.

is directly related to the production level, while it is expected to be rather decay agnostic, up to small effects due to the decay-product reconstruction. In Figure 4, manifest differences can be seen for various polarisation modes. At large cosθ_scatt the cross-section is dominated by the TT polarised state, while around zero the LL and the TT states give almost the same contribution to the unpolarised result. The QCD corrections sizeably distort the shapes of the LT, TL and TT distributions, with a particularly significant effect in the TT case. The marked shape change for the TT state is determined by the steeply increasing K-factor at low cosθ_scatt. This is caused by real corrections spoiling the approximate amplitude-zero effect which is present at LO in the quark–antiquark annihilation into WZ [55, 114]. Very large K-factors are also found for the LT and TL polarisation modes in forward-scattering regions, where the LO signals are suppressed by unitarity cancellations due to the large
transverse-momentum cuts applied on the two bosons, requiring very high scattering energy when combined with forward/backward scattering angles. All $K$-factors around $\cos \theta_{\text{scatt}} = 1$ become huge, because real radiation leaves room to configurations where the boson trajectory is closer to the beam direction, without being cut away by the rapidity selections or suppressed. Interferences at NLO QCD are almost vanishing in the most populated region, while they become non-negligible in forward-scattering configurations. The discrimination power of this angular variable amongst the polarisation states is marked. However, using this observable is well motivated only for SM studies and measurements, since the model independence of the polarised shapes is not given. In fact, at variance with decay angles, the distribution shapes in the scattering angle could vary sizeably depending on the production dynamics, i.e. it is very sensitive to new-physics effects at production level.

In Figure 5 the differential cross-section in the rapidity of the hadronic system $J$ [defined after Eq. (2.6)] is considered. In the case where both bosons are transversely polarised the NLO QCD corrections significantly change the shape of the distribution. This is caused by the large contributions from real emission via gluon-induced
processes, which mostly fill the central region, while at LO the distributions is almost flat for $|y_J| < 1.8$. This is an indirect effect of the approximate amplitude zero at LO, as already observed in Figure 4 for the distributions in the scattering angle. In the resolved setup, large flat corrections are found for the TL and LT contributions, with mild non-flat effects just around $|y_J| = 2$. In the unresolved setup, the mixed states are characterised by less flat $K$-factors, giving slightly more sizable shape changes than in the resolved case. In both setups, the LL distribution only receives small corrections reflecting the result at integrated level. The interferences at NLO QCD are very small in the whole accessible spectrum, and negligible in the unresolved setup. Overall, the different shapes in the two setups are due to the sharp cut $|y_J| < 2.4$ in unresolved topologies, which is replaced in the resolved ones by rapidity cuts on single jets that suppress the region $|y_J| \gtrsim 2.4$. This causes the steeper fall off at the edges of the distribution of the resolved setup compared to the unresolved setup.

In Figure 6 the distribution in the rapidity of the electron–positron pair is presented. This observable is strongly correlated to the rapidity of the hadronic system $J$ discussed in Figure 5. In fact, the Z-boson momentum absorbs the recoil of the en-
Figure 6. Rapidity distribution of the electron–positron pair in semi-leptonic ZW\(^+\) production at the LHC. Same structure as Figure 3.

tire hadronic system, including the W boson and additional QCD radiation. Very small shape modifications are found comparing LO and NLO QCD distributions. In fact, a non-flat behaviour of the K-factors is only found in the largest-rapidity regions that are not forbidden by selection cuts, with a mild increase towards \(|y_{e^+e^-}| = 2.4\) for the TL state, a mild decrease in the same region for the LT and TT states. Up to different overall normalisations, the corrections behave very similarly in the two setups. The shapes for different polarisation states are very close to each other. Only in the central region the TT component is slightly more peaked than the others. Interference contributions are basically independent of \(y_{e^+e^-}\).

The distribution shown in Figure 7 concerns the absolute value of the rapidity difference between the positron and the hadronic system J. Similarly to the scattering angle, this observable is well suited to discriminate amongst different polarisation states, thanks to the marked shape differences. Concerning model dependence, the same caveats apply as for the distribution in the scattering angle. The LL, the TL, and the LT polarisation states have a maximum at \(|\Delta y_{e^+J}| = 0\), while the maximum of the TT state is shifted to \(|\Delta y_{e^+J}| \approx 0.65\). The LO TT shape is heavily distorted by QCD corrections, again due to real radiation that fills kinematic regions that are

– 18 –
(a) Resolved topology

(b) Unresolved topology

Figure 7. Distribution in the rapidity separation between the positron and the hadronic system J in semi-leptonic $Z W^+$ production at the LHC. The identification of the hadronic system J is described in Section 2.3. Same structure as Figure 3.

suppressed at LO due to the approximate amplitude zero ($|\Delta y_{e+J}| \approx 0$). The LT and TL polarisation states receive large and increasing corrections from real radiation in gluon-induced channels for $|\Delta y_{e+J}| \gtrsim 2.5$, where the LO is extremely suppressed. In general, at large rapidity separation all signals are suppressed by the rapidity cuts. Interferences are very small through the whole spectrum at NLO QCD. Apart from different total cross-sections, the two kinematic setups give almost identical results for all polarisation states, both in terms of normalised shapes and in terms of $K$-factor behaviours.

Although the angular observables are the most promising ones in terms of the discrimination power amongst polarisation states, it is important to complement their investigation with the study of energy-dependent observables. In Figure 8 we show differential results in the invariant mass of the hadronic system J. While the distributions feature, as expected, the Breit–Wigner shape of the W-boson resonance, this observable is heavily affected by the reconstruction of the hadronic decay and therefore subject to contamination from QCD radiation. In fact, the NLO QCD real corrections introduce an ambiguity in the determination of the $W^+$-boson decay jets.
The jets that happen to be part of the reconstructed hadronic system J without being actual decay products of the W\(^+\) boson create a background that does not follow the Breit–Wigner modulation. This effect is particularly manifest in the TL distribution. When the recombined W boson becomes off-shell, the TL curve does not fall off like the others (and in particular the LT one, which is almost identical at LO), but rather results in a much flatter behaviour. The striking difference between the TL and LT states, beyond the different overall normalisation that has already been motivated in Section 3.1, is due to the interplay between the reconstruction of the hadronic system and the suppression of longitudinal bosons in mixed polarisation states in the high-energy regime. In particular, both states are unitarity suppressed at LO (as can be easily seen upon replacement of the longitudinal boson with the corresponding would-be Goldstone boson), while at NLO QCD there is an enhancement due to the opening of the gluon-(anti)quark channel that gives a sizeable real correction. As discussed in Section 3.1, for the LT polarisation state the bremsstrahlung parton is produced preferably opposite to the direction of the hadronically decaying W boson. Therefore little ambiguity is left for the assignment of decay jets to the transverse W boson.
In the TL case, on the other hand, the additional parton is produced close to the longitudinal, hadronically decaying W boson. This deteriorates the reconstruction of the W decay products and results in a distorted shape with a Breit–Wigner peak (events with correct assignment, lower in shape compared to the LT state) on top of a sizeable flat background (events with wrong assignment). This feature originates from the hard $p_T$ cut that is applied to the hadronically decaying W boson, vetoing effectively jets with soft and moderate transverse momentum.

We have checked that without the hard $p_T$ requirement on the hadronic system J the shape of the TL distribution is closer to the others, as can be appreciated in Figure 9(a) where we consider the resolved setup as in Figure 8(a) but without the $p_{T,j1} > 200$ GeV cut. The assignment of decay jets to the W boson, although not perfect, behaves much better for the TL mode, further confirming the reasoning above and in particular the correlation between the flat background from misreconstruction and the high-$p_{T,j1}$ cut. The absence of a hard transverse-momentum
cut on the hadronic decay system J leads to abundant real QCD radiation filling the pure radiative region below 100 GeV for the leading decay jet, which is excluded at LO by the required $p_T$ of the Z boson, as can be observed in the distribution in the $p_T$ of the hardest jet from the W-boson decay shown in Figure 9(b). The distribution for the TL mode exhibits a clear peak around 50 GeV, while a hard cut $p_{T,\text{jj}}>200$ GeV would remove all events with $p_{T,j_1}<100$ GeV. The same effect is present also for the LT and TT polarised states, but much less pronounced. The interference contribution is a the level of 10% for $p_{T,\text{J}}<100$ GeV.

Coming back to the results in the default setups shown in Figure 8, the differences in the TL distribution between the resolved and unresolved topology are due to the effects of the clustering algorithms on the reconstruction procedure. In particular, the flat background from mis-reconstruction of the W-boson decay jets is larger (giving a normalised shape with a lower peak) in the unresolved setup, because the larger recombination radius causes more initial-state QCD radiation fall in the decay-jet system. Looking at Figure 8, the TT and LT distributions feature very similar shapes when going far from the on-shell regime, since the QCD effects do not depend much on the polarisation mode of the leptonic Z boson, apart from the different normalisation determined by the unitarity suppression of the longitudinal polarisation. The LL state is characterised by QCD $K$-factors that are below one at the peak, while they are monotonically increasing in off-shell regimes. In the resolved setup, the corrections increase faster for $M_j<M_W$, resulting from events where one of the decay jets is missed in the reconstruction of the W boson, while in the unresolved one the corrections are larger for $M_j>M_W$, indicating an initial-state-radiation jet be clustered together with one of the decay jets. This is a direct consequence of the more inclusive jet clustering in the unresolved setup. For the TT, LT, and LL states the omission of the $p_{T,\text{jj}}$ cut does not change the general picture, apart from the different normalisation.

In Figure 10 the differential cross-section is presented with respect to the invariant mass of the hadronic system J and the positron. Large differences amongst various polarisation states occur. The low-invariant-mass regime ($M_{e+J}<200$ GeV) is dominated by real-emission contributions. In fact, the additional QCD jet allows for the Z and $W^+$ bosons to be produced with a lower boson-pair invariant mass. The larger jet-recombination radius in the unresolved setup leads to higher $M_{e+J}$ thus suppressing the contributions in the low-invariant-mass region. At large $M_{e+J}$ the TT distribution falls off slower than all the others. In the resolved setup the LL distribution becomes lower than the TL one around 900 GeV, while in the unresolved setup the LL signal remains larger than TL, with similar suppressions at high energy. The much stronger suppression of the LL state in the resolved setup leads to a signal that is almost one order of magnitude smaller than in the unresolved case at 1 TeV. This is due to the fact that for the LL state the high partonic energy is shared between the two bosons (effect of additional radiation is small for the LL
Figure 10. Invariant-mass distributions of the system formed by the hadronic system $J$ and the positron in semi-leptonic $ZW^+$ production at the LHC. The identification of the hadronic system $J$ is described in Section 2.3. Same structure as Figure 3.

state) resulting in LO-like configurations with two collimated quarks that are easily clustered together. This clearly disfavours the resolved topologies where two jets are required. Rather large negative interferences are present in the radiation-driven soft part of the spectrum at NLO ($\approx -10\%$). Shape-wise the LL distribution features a pronounced peak around 400 GeV, while the other polarisation states, and especially the TT one, are more spread over the shown spectrum.

Similar features are found in transverse-momentum distributions for the positron, which are shown in Figure 11. The behaviour in the low transverse-momentum region strongly depends on the polarisation of the Z boson (of which the positron is a decay product) and affects the normalised shapes. For a transverse (longitudinal) $Z$ boson the shape has a local minimum (maximum) around $p_{T,e^+} \approx 130$ GeV. This is due explained looking at Figure 3(a): a transverse polarisation gives a positively-charged lepton that goes more frequently in the same or opposite direction w.r.t. the $Z$ boson, therefore sharing in a non-democratic way the boson energy (two peaks at $p_{T,e^+} \approx 20$ GeV and $p_{T,e^+} \approx 200$ GeV), while a longitudinal one gives leptons that are preferably orthogonal to the boson trajectory and share democratically the boson
Figure 11. Transverse-momentum distributions of the positron in semi-leptonic $ZW^+$ production at the LHC. Same structure as Figure 3.

energy (single peak at $p_{T,e^+} \approx 130$ GeV). The marked shape differences and the small interference effects for $p_{T,e^+} \lesssim 200$ GeV, namely in the most-populated region, makes this transverse-momentum observable suitable for the discrimination of the Z-boson polarisation state. Another interesting aspect of the results is the slower fall-off of the distributions in the unresolved setup compared to the resolved setup, which is particularly significant for the double-longitudinal signal. This is the same effect as observed in Figure 10 for the invariant mass of the positron–jet system, which is highly correlated to the transverse momentum of the positron. For $p_{T,e^+} \gtrsim 300$ GeV, all $K$-factors increase faster in the resolved topology compared to the unresolved one, owing to a different LO suppression. Requiring at least two jets in the final state results in high-$p_T$ events being cut away at LO, as the almost collinear decay quarks are often clustered into a single jet. In the unresolved topology such events are not discarded (at least one fat jet is required). Since the two vector bosons are produced with opposite transverse momenta, the same effects are found in the high-energy tails of the transverse-momentum distributions for the hadronic system $J$, the Z boson, and the charged leptons.

In the resolved topology it is possible to distinguish the two jets that come from
Figure 12. Distributions in the leading-jet decay angle (left) and in the subleading-jet transverse momentum (right) in semi-leptonic ZW\(^+\) production at the LHC. The identification of the leading and subleading jet is discussed in Section 2.3 and the decay-angle definition is given in Eq. (3.3). Results for the unpolarised and doubly-polarised process are shown in the resolved setup described in Section 2.2. The panels of the subfigures have the same structure as in Figure 3.

The W-boson decay, up to potentially relevant reconstruction effects. In Figure 12 we consider two observables that depend on the kinematics of individual jets (those labelled as decay jets, sorted according to their transverse momentum). The polar decay angle of the leading decay jet \( j_1 \) in Figure 12(a) is defined similarly to the one of the charged lepton in Eq. (3.1): it is the angular separation between the leading-jet direction in the rest frame of the hadronic system \( J (\vec{p}_{j_1}) \) and the direction of \( J \) calculated in the reconstructed di-boson CM frame (\( \vec{p}_{j_1}^{\text{CM}} \)),

\[
\cos \theta_{j_1}^{\text{CM}} = \frac{\vec{p}_{j_1}^{\text{CM}} \cdot \vec{p}_{j_2}^{\text{CM}}}{|\vec{p}_{j_1}^{\text{CM}}| |\vec{p}_{j_2}^{\text{CM}}|}.
\] (3.3)

Note that this observable is sensitive to the reconstruction procedure to identify the hadronic system \( J \) and is not selecting the up-like or down-like jet (which would be unphysical) but rather the leading-\( p_T \) one. At variance with the leptonic decay
angle in the Z-boson rest frame, the hardest jet can originate from an up-type or a
down-type quark, strongly distorting the description of the boson decay. Strikingly,
all distributions are non vanishing only between \(\cos \theta_{j_1}^{*,\text{CM}} \approx -0.4\) and \(\cos \theta_{j_1}^{*,\text{CM}} = 1\),
clearly favouring the positive region of the spectrum. In fact, the hardest jet is mostly
produced in the same direction of the decayed boson, of which it takes the largest
fraction of transverse momentum. The analogous distributions for the softest jet show
the opposite behaviour, populating mostly the negative region of the spectrum. The
LO shape for the TL state follows the one of the LL state. This is expected as this
angular variable, up to the sorting of jets in the transverse momentum instead of the
electric charge, is directly related to the polarisation state of the W boson. At NLO
QCD this is no longer the case, with the TL shape following the LT and TT ones. This
dramatic change in the TL shape, driven by large and non-flat real QCD corrections
in gluon-induced channels, is due to the combination of the bad reconstruction of the
W boson, discussed already for Figures 8 and 9, the suppression of the LO signal,
and the choice of sorting the decay jets in \(p_T\). The interference effects at NLO QCD
are practically negligible in the most populated region, while they increase up to
more than 10% towards the endpoints of the spectrum.

In Figure 12(b) we show the differential cross-section with respect to the trans-
verse momentum of the softest jet from the W-boson decay. As expected, all dis-
tributions decrease very fast already at moderate transverse momentum. For the
LL polarisation state (both at LO and at NLO) and the TL one (just at LO), the
distributions are peaked around 100 GeV, namely half of the minimum transverse
momentum required by the selections for the W boson. This behaviour is under-
stood as the longitudinal W boson, mostly produced at large scattering angles (see
Figure 4), favours configurations where the two decay jets are orthogonal to the
boson direction (in its rest frame) and therefore typically share half of the boson
transverse momentum. At NLO, the LL shape does not deviate from the LO one
as the QCD corrections are very small, while the TL shape becomes peaked around
zero, following closely the shape of the TT and LT distributions. This strong mod-
ification of the TL distribution is due to large effects of mis-reconstruction induced
by real QCD radiation. In particular, the gluon-induced contributions that dom-
inate the NLO QCD corrections to the TL state, shown in Figure 2(b), prefer a
boosted, transversely polarised Z boson whose recoil is absorbed by the system of a
hard QCD parton and a soft pair of quarks from the longitudinal-W-boson decay.
After clustering and reconstruction, this topology results in one hard jet and one
soft jet, mimicking the jet pattern characteristic for the hadronic decay of a trans-
verse W boson (TT and LT modes). The TT and LT distributions show very similar
shapes, dominated by the soft region \((p_{T,j_1} < 100 \text{ GeV})\), while for \(p_{T,j_1} > 100 \text{ GeV}\)
the QCD corrections enhance the LT signal. In fact, for transverse W bosons, one
decay jet is preferably emitted opposite to the direction of the boson, resulting in a
small transverse momentum of this subleading decay jet.
4 Conclusion

In this work we have presented NLO QCD corrections to vector-boson-pair inclusive production in the semi-leptonic decay channel. The results focus on the WZ process in final states with two charged leptons and jets, but can be easily extended to ZZ production with the same final state, as well as to processes with a charged lepton, missing transverse momentum and jets (WZ and $W^+W^-$). The building blocks at NLO QCD are exactly the same as those used for this calculation.

Although we have neglected a number of effects, including the overlap with other production mechanisms, the non-resonant background and other sources of corrections (NLO EW, matching to parton shower), the presented calculation represents a crucial step towards precise predictions for di-boson processes in semi-leptonic decay channels. For the first time, we have combined in the double-pole approximation the QCD corrections to the production of two bosons and to the hadronic decay of one of the two, separating doubly-polarised signals at the level of tree-level and one-loop Standard Model amplitudes.

We have considered a boosted regime, where the longitudinal signals give a more sizeable contribution than in inclusive setups. We have applied two different jet selections: a first one with two light jets (resolved) and a second one with a single fat jet (unresolved). Between the two setups moderate differences show up at the level of distribution shapes and more marked deviations are revealed for QCD $K$-factors for the various polarisation states.

The reconstruction of the hadronic decay of the W boson is found to behave very differently for the various polarisation states, distorting angular and energy-dependent distributions and enhancing otherwise suppressed contributions. The largest impact is found when the W boson is longitudinally polarised and the leptonically decaying Z boson is transversely polarised, as a combination of unitarity cancellations and a hard cut on the transverse momentum of the longitudinal bosons. For this polarisation mode, the sizeable QCD corrections and their interplay with the reconstruction procedure causes distributions for decay observables of the longitudinal W boson to mimic those of transverse bosons.

Strikingly, a number of observables turn out to be highly sensitive to polarisation-state discrimination. Many of these observables are inclusive in the hadronic decay structure, i.e. they do not rely on jet-substructure techniques. This suggests that extracting polarisation information from the data is not uniquely related to the reconstruction of the sub-jets from the hadronic decay.

The dramatic change of (doubly-)polarised distributions when going from LO to NLO QCD makes it essential to include at least NLO QCD corrections in any polarisation study or data analysis in the considered decay channel. This statement, however, applies also for fully-leptonic channels, as shown in previous works [26–31]. Including NLO corrections is especially important for multi-boson processes.
that are characterised by a LO suppression in some kinematic configurations. The inclusion of NNLO QCD corrections, though definitely desirable and now feasible for di-boson processes [31], is not expected to give as dramatic shape distortions to the NLO distributions, as those given by the NLO corrections to the LO shapes. With specific regards to the hadronic decays of EW bosons, it will be especially relevant to match NLO QCD (or NNLO QCD) calculations to parton showers and hadronisation, enabling a realistic comparison against LHC data.

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