A New Phenomenology for the Disordered Mixed Phase

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Abstract

A universal phase diagram for type-II superconductors with weak point pinning disorder is proposed. In this phase diagram, two thermodynamic phase transitions generically separate a “Bragg glass” from the disordered liquid. Translational correlations in the intervening “multi-domain glass” phase are argued to exhibit a significant degree of short-range order. This phase diagram differs significantly from the currently accepted one but provides a more accurate description of experimental data on high and low-T_c materials, simulations and current theoretical understanding.

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Quenched randomness destabilizes the Abrikosov flux-line lattice in a pure type-II superconductor, yielding novel glassy phases \[1\]. In the Bragg glass (BrG) phase \[2\], translational correlations decay asymptotically as power laws. On increasing disorder or temperature, the BrG phase is unstable to a phase in which such correlations decay exponentially \[2\]. At large values of the applied field \(H\), the disordered liquid (DL) evolves smoothly into a glassy state \[3\] when the temperature \(T\) is reduced. This glassy state may be a new thermodynamic phase, the “vortex glass”, with spin-glass-type correlations in the superconducting order parameter, separated from the disordered liquid by a line of phase transitions \[4\]. An alternative picture, attractive in many respects, is that this state resembles a structural glass \[5\], topologically disordered at the largest length scales but lacking long-ranged phase correlations \[6\].

Fig. 1 summarizes the currently popular view of phase behaviour in the disordered mixed phase of type-II superconductors \[2,3,6–9\]. The BrG phase is shown to be unstable to the vortex glass on increasing \(H\); it also melts directly into the DL phase upon increasing \(T\). The continuous vortex-glass to DL transition line meets the BrG-DL transition line at a multicritical point \[9\]. Recent studies \[10\] indicate that the underlying field-driven Bragg glass-vortex glass transition is first-order.

While this phase diagram by construction reflects early experiments on YBCO \[3\] and BSCCO \[9\], many systems, among them the low-\(T_c\) superconductors 2H-NbSe\(_2\) \[11–13\], CeRu\(_2\) \[13–15\] and Ca\(_3\)Rh\(_4\)Sn\(_{13}\) \[16\], the cuprate NCCO \[17\], the bismuthate (K,Ba)BiO\(_3\) \[18\], the borocarbide YNi\(_2\)B\(_2\)C \[15\] and the mercury \[19\] and thallium \[20\] based compounds, behave qualitatively differently. Even for the high-\(T_c\) systems, more recent experiments \[21\] suggest features inconsistent with Fig. 1. That these differences imply qualitatively different types of phase behaviour is one possibility. Alternatively, such differences might simply reflect different limits of an underlying universal phase diagram; this possibility is both theoretically attractive and economical. It would be sensible, however, to require that Fig. 1 be recovered from such a phase diagram in the appropriate limit.

This Letter proposes Fig. 2 as a universal phase diagram for all weakly disordered type-II
superconductors. The term “multi-domain glass” (MG) describes the narrow sliver of glassy phase which we propose always intervenes between BrG and DL phases. This sliver expands into the putative “vortex glass” phase [4] of Fig. 1, as $H$ is increased. Experiments indicate that the MG phase melts via a first-order phase transition on $T$ scans at intermediate $H$; this first-order line is shown to meet a line of continuous transitions at a tricritical point, although more complex topologies are possible. The inset shows the expected phase behaviour at $H \sim H_{c1}$. The predicted reentrance of the glassy phase at low fields was established in Ref. [23]. Recent experiments [15, 24] clearly show the disordered phase enveloping a quasi-lattice phase at small $H$. Fig. 2 incorporates recent proposals for such phase behaviour in low-$T_c$ superconductors [13], derived from an analysis of peak effect phenomena in these systems [15], the anomalous and sharp increase in critical currents $j_c$, seen over a narrow regime below $H_{c2}$, the “peak regime”, as $H$ or $T$ is varied.

The “multi-domain” glass is argued to be an equilibrium phase, distinct from the vortex glass, consistent with recent theoretical [3], experimental and simulative understanding. Theoretically, screening destroys vortex-glass-type order in three dimensions [25]; recent experiments [26] question earlier reports of vortex-glass scaling. Simulations of classical directed lines interacting with quenched point impurities see topologically disordered glassy phases in equilibrium in a regime which intervenes between relatively ordered (BrG) and disordered (DL) phases [27]. Arguments for the equilibrium nature of this phase and a brief survey of its glassy attributes in the context of low-$T_c$ materials were provided in Ref. [13].

What symmetry might be broken across the MG-DL transition? A recent study of disordered hard-sphere fluids obtains, within mean-field theory, a one-step replica symmetry breaking glass transition for sufficiently strong disorder; these authors stress the resemblance of the features of the mean-field phase diagram they obtain to Fig. 1 [28]. Should these ideas be applicable to the disordered mixed phase, a discontinuous freezing of the disordered density configuration characterizing the glassy phase would be expected, as against relatively smooth behaviour of the entropy and internal energy across this transition. An alternative possibility for the MG phase, consistent with some of the proposals here and testable via
an analysis of Bitter decoration patterns, is a hexatic glass \cite{29}; the recent discovery \cite{30} of an equilibrium hexatic phase in a dense system of long oriented DNA molecules favours a similar possibility for line vortices.

We argue that translational correlations in the MG phase can be fairly long-ranged (unlike correlations in typical glassy phases), and propose that structure in the MG phase just above the BrG-MG phase boundary is best described as a mosaic of ordered domains, with typical scale \( R_d \gg a \), the inter-line spacing, for weak pinning. \( R_d \) should be \textit{largest} in the interaction-dominated regime but should \textit{decrease} rapidly for much larger or smaller \( H \), reflecting the increased importance of disorder both at high-field and low-field ends \cite{13,13,23,31}. Some diagnostics for the multi-domain character of this phase are the following: The transition between BrG and DL phases can occur via several intermediate stages, as individual domains melt as a consequence of disorder-induced inhomogeneities in the melting temperature \cite{32}. Fluctuations of a domain-like arrangement can yield noise signals of large amplitude associated with relatively \textit{few} fluctuators \cite{1,33}, intermediate structure in the ac susceptibility as a function of \( T \) \cite{13,13}, a stepwise expulsion of vortices on cooling \cite{34} and a host of related features \cite{13,35} of the experimental data which are hard to rationalize in other pictures.

The MG-DL transition is interpreted as the true remnant of the underlying freezing transition in the pure system. The presence of a critical point on this transition line \cite{8} follows from the observation that signals of melting should cease to be seen once \( R_d \) is comparable to correlation lengths in the pure liquid at freezing. This phenomenology also rationalizes \cite{13} a large body of data on anomalies associated with the peak effect phenomenon \cite{11}, including “fracturing” of the flux-line array \cite{12}, the association of thermodynamic melting with the transition into the disordered liquid \cite{36} as well as conjectures regarding the dynamic coexistence of ordered and disordered phases in transport measurements in the peak regime \cite{24}.

A simple conjecture for the scale of such domain sizes suggests \( R_d \sim R_a \), a Larkin length, at least at intermediate \( H \). This length is computed using the coarse-grained free
energy \[ F_{el} = \int dr_{\perp} dz [\frac{\epsilon_{\perp}}{2}(\nabla_{\perp} \cdot \mathbf{u})^2 + \frac{\epsilon_{\perp}}{2}(\nabla_{\perp} \times \mathbf{u})^2 + \frac{\epsilon_{\perp}}{2}(\partial_z \mathbf{u})^2], \]

where \( \mathbf{u}(r_{\perp}, z) \) is the displacement field at location \((r_{\perp}, z)\) and the integral represents the elastic cost of distortions from the ideal crystalline state with vortex lines centred at \( \mathbf{R}_c \), governed by the values of the elastic constants for shear \( (c_{66}) \), bulk \( (c_{11}) \) and tilt \( (c_{44}) \). Quenched random pinning is incorporated by adding \( F_d = \int dr_{\perp} dz V_d(r_{\perp}, z) \rho(r_{\perp}, z) \), to \( F_{el} \), where \( \rho(r_{\perp}, z) = \sum_i \delta^{(2)}(r_{\perp} - R_i - \mathbf{u}(\mathbf{R}_i, z)) \). This term models the interaction with a quenched Gaussian disorder potential \( V_d \), assumed to be correlated over \( \xi \), the coherence length \( i.e. [V_d(x)V_d(x')] = K(x - x') \), with \( K(x - x') \) a function of range \( \xi \). The notation \( x \) denotes \((r_{\perp}, z)\).

The correlator \( B(r_{\perp}, z) = [\langle \mathbf{u}(r_{\perp}, z) - \mathbf{u}(0, 0) \rangle^2] \) where \( \langle \cdot \rangle \) and \( [\cdot] \) denote thermal and disorder averages respectively, has the following properties: For \( r_{\perp}, z \ll R_c, L^b_c \) (the transverse and longitudinal Larkin pinning lengths), correlations behave as in the Larkin “random force” model \( i.e. B(x) \sim x^{4-d} \). At length scales between \( R_c \) and \( R_a \), the Larkin length scale referred to above at which disorder and thermal fluctuation-induced positional fluctuations become of order \( a \), \( B(x) \sim x^{2\xi_c \rho_m} \) with \( \xi_c \rho_m \sim (4-d)/6 \sim 1/6 \). The Larkin length scale can then be estimated via \( R_a \sim R_c (\frac{\pi}{2})^{1/\xi_c \rho_m} \). At still larger length scales, \( B(x) \sim \log(x) \).

We estimate the Larkin pinning lengths \( R_c \) and \( L^b_c \) from \( L^b_c = \frac{2\sqrt{2}c_{66}c_{44}}{n_f^2} \), \( R_c = \frac{\sqrt{2\xi_c \rho_m}^{3/2}}{n_f^{1/2}} \), and \( V_c \sim R_c^2 L_c \). Equating \( BjV_c \) to the energy gain from random pinning \( f(nV_c) \) yields the standard weak-pinning expression \( j_c = \frac{1}{B} f \left( \frac{\mathbf{n}}{V_c} \right)^{1/2} \). If \( R_c, L^b_c > \lambda \), \( j_c \sim \left( \frac{\xi_c}{R_c} \right)^2 j_{dp} \) and \( L^b_c \sim \frac{\lambda}{a} R_c \), with \( j_{dp} \) the depairing current. For 2H-NbSe\(_2\): \( \xi_c \sim 7K, \lambda \sim 700\,\text{Å} \) \((H \parallel c)\), \( \xi \sim 70\,\text{Å} \) and \( a \sim 450\,\text{Å} \) at \( B \sim 1T \). Using \( j_c / j_{dp} \sim 10^{-4} \), we obtain transverse and longitudinal Larkin pinning lengths \( \frac{L^b_c}{a} \sim 15 \) and \( \frac{L^b_c}{a} \sim 24 \).

At fields \( \sim 1T \), \( \left( \frac{\xi_c}{\xi} \right)^{1/\xi_c \rho_m} \sim \left( \frac{\xi_{70}}{70} \right) \sim 7 \times 10^4 \) leading to \( R_d/a \sim R_a/a \leq 10^6 \). Assuming, conservatively, \( R_d \sim 10^4a \) and a longitudinal Larkin length comparable to the size of the sample in the c-axis direction, an estimate for the number of “independent fluctuators” obtained in noise measurements on 2H-NbSe\(_2\) \([33][12]\) can be obtained. For a sample in the shape of a thin platelet of transverse area \( A \sim 1\,\text{mm} \times 1\,\text{mm} \) and with \( A_d \sim R^2_d \), the number
of such fluctuators $N_f \sim \frac{\lambda}{\lambda_d} \sim 10^1 - 10^2$, numbers small enough to yield strong non-Gaussian effects $[33]$. 

Experimentally, behaviour in the mixed phase of low-T$_c$ materials such as 2H-NbSe$_2$, CeRu$_2$, Ca$_3$Rh$_4$Sn$_{13}$, YNi$_2$B$_2$C and several related compounds exhibit a remarkable commonality $[15, 13]$. In these relatively pure materials ($j_c/j_d \sim 10^{-4}$) where thermal fluctuations are substantial (Ginzburg numbers $G_i \sim 10^{-4}$ here, as against typical values for low-T$_c$ systems of about $10^{-8}$), there is now considerable evidence for a two-step transition from a relatively ordered low-T phase into a highly disordered fluid phase $[15, 13]$. This transition occurs via an intermediate and profoundly anomalous regime with glassy properties $[13]$. These experiments indicate that the two transition lines which separate the BrG from the DL phase can always be separately resolved. The data on NCCO $[17]$ and (K,Ba)BiO$_3$ $[18]$ are consistent with Fig. 2 but not with Fig. 1.

Hall-probe based magnetization measurements on BSCCO favour a single-step transition into the disordered liquid out of the BrG phase $[37]$. However, recent muon-spin rotation experiments see two transitions $[21]$. The asymmetry of the field distribution, $\alpha = \langle \Delta B^3 \rangle^{1/3} / \langle \Delta B^2 \rangle^{1/2}$, exhibits a first transition in which $\alpha$ jumps from a value of about 1.2 (consistent with a vortex-line crystal) to a value of about 1 (indicating a considerable degree of local translational order) as $T$ is increased. Across a second transition boundary, obtained on further increasing $T$, $\alpha$ drops abruptly to values close to zero, characteristic of the fluid.

Early experiments on YBCO found that melting in relatively clean untwinned crystals was best described as a “complex, two-stage phenomenon” $[39]$. Recent simultaneous measurements of ac susceptibility and magnetization in this material $[36]$ see a sharp peak effect in which the location of the peak in ac susceptibility correlates exactly to the magnetization jump which signals melting. The regime in which $\chi'$ shows non-trivial signatures of the transition is enlarged, compared to the magnetization jump which occurs at a sharply defined temperature, a feature which follows transparently from the discussion here – the
width of the transition region is related to the width of the sliver phase, while the most prominent signatures of melting should generically be obtained across a line, the MG-DL transition line.

When does the phase diagram of Fig. 1 approximate that of Fig 2? The sliver of MG phase is exceedingly narrow at intermediate $H$ and for weak disorder, a consequence of the reduction of effective disorder in an interaction-dominated regime [13]. Thermal smoothening of disorder should reduce the width of this sliver further; thermal fluctuations smear the disorder potential $U_p(T)$ seen by a vortex line, strongly renormalizing the effective disorder if $[< u^2 >] \geq \xi^2$. For a single line pinned by quenched disorder $U_p(T) \sim U_p(0) \exp[-c(T/T_{dp})^3]$ [22] where $U_p(0)$ is the pinning potential per unit length at $T = 0$, $c$ is a numerical constant and $T_{dp}$ is the depinning temperature; $T_{dp} \sim (U_p^2 \xi^3 \epsilon_0 / \gamma^2)^{1/3}$ where $\epsilon_0 = (\Phi_0/4\pi\lambda)^2$, and $\gamma$ is the mass anisotropy. For YBCO, $T_{dp} \sim 20 - 30 K$ [40]. Such a substantial (exponential in the simple limit above) reduction in effective disorder should render the sliver unobservable in many types of experiments, yielding an apparent single melting transition out of the ordered phase.

The ratio $R = T_{dp}^{low} T_m^{high} / T_{dp}^{high} T_m^{low}$ measures the relative importance of thermal fluctuations in low and high $T_c$ materials; $T_m$ is the melting temperature in the pure case and we have approximated $T_m \sim T_c$. Using the following values: $NbSe_2 : U_p = 10K/\AA, \xi = 70\AA, \lambda = 700\AA, \gamma = 5$ and $YBCO : U_p = 10K/\AA, \xi = 20\AA, \lambda = 1400\AA, \gamma = 50$ we obtain $R \sim 10^3$, suggestive of the relative importance of this effect to the high-$T_c$ materials vis. a vis. its irrelevance in low-$T_c$ systems except very close to $H_{c2}$. Experimentally, the nominal irreversibility line for $H \sim 3 - 7T$ in YBCO lies well below the melting line in weakly disordered samples [40]. Thus, thermal melting occurs in a nearly reversible regime. Local Hall probe-based susceptibility measurements on YBCO [11] find that $j_c$ is actually finite below $T_m$ but extremely small ($\sim 0.4A/cm^2$). These experiments see a peak effect close to and below $T_m(H)$, a feature inaccessible in usual SQUID based magnetization experiments. Other experiments at somewhat smaller $H$ see a very clear and sharp peak effect [36], with properties similar to those in the low-$T_c$ materials [13]. These results illustrate that sig-
nals of a two-step transition in high-T\textsubscript{c} materials may be very hard to access, particularly if discontinuities in \( j_c \) or magnetization across the first transition are small. In contrast, experiments on the low-T\textsubscript{c} materials discussed in Refs. [13,15] indicate an irreversibility line located in the fluid phase and provide clear evidence of a two-step transition. Anomalously small values of \( j_c \) translate to anomalously large values of \( R_d \). The magnetization discontinuity across the BrG-MG phase boundary should scale roughly as the density of unbound dislocations \( \rho_d \) in the MG phase for small \( \rho_d \sim 1/R_a^2 \), yielding \( \Delta M \sim \Delta M_0 (a/R_a)^2 \) where \( \Delta M_0 \) is the magnetization jump in the pure system at \( T_m \). Even if \( R_a/a \sim 30 \), the corresponding induction jump \( \Delta B \sim \Delta M \) is of order \( 10^{-3} M_0 \) or within noise levels in a typical experiment [33,32]. Thus, for sufficiently weak disorder, the sample may behave substantially as a single domain in the intervening MG phase, implying the absence of signals of the BrG-MG transition in a magnetization experiment and thus the phenomenology of Fig. 1.

The apparent vanishing of the sliver rationalizes the putative multicritical point [30] of Fig. 1. While the sliver may not be resolvable in some classes of experiments, it may be apparent in others, particularly those which probe local correlations. However, in the thermodynamic limit, provided other, unrelated direct instabilities to the liquid do not intervene, continuity suggests two transitions as in Fig. 2. In essence, the phenomenology of Fig. 1 assumes by fiat such an instability. There appears to be little justification for this assumption; we suggest that it need not hold. For more disordered superconductors, whether high-T\textsubscript{c} or low-T\textsubscript{c}, a two-step transition should generically be seen.

Many recent simulations, for example Ref. [27], see an intermediate field MG phase with translational correlations comparable to system sizes, although the dynamics changes abruptly across the BrG-MG transition. Interestingly, Ref. [27] concludes that the existence of a sliver of MG phase always preempts a direct BrG-DL transition cannot be ruled out, as in the recent simulations of Sugano et. al. [38].

In conclusion, it is suggested that the BrG phase in all disordered type-II superconductors generically transforms first into an intermediate glassy state on heating, rather than directly
into a liquid; the proposal of a generic two-step transition out of the disordered liquid state should be experimentally testable in thermodynamic measurements. More details will appear elsewhere [35].

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REFERENCES

[1] G. Blatter et. al., Rev. Mod. Phys., 66, 1125 (1994).

[2] T. Giamarchi and P. Le Doussal, Phys. Rev. Lett 72, 1530 (1994); Phys. Rev. B, 52, 1242 (1995); ibid., 55, 6577 (1997).

[3] P.L. Gammel, D.A. Huse and D.J. Bishop, in Spin Glasses and Random Fields, ed. A.P. Young, (World Scientific, Singapore, 1998); T. Giamarchi and P. le Doussal ibid.

[4] D. S. Fisher, M. P. A. Fisher and D. A. Huse, Phys. Rev. B 43 130 (1991).

[5] G. I. Menon, C. Dasgupta and T.V. Ramakrishnan, Phys. Rev. B, 60 7607 (1999).

[6] T. Natterman and S. Scheidl, Advances in Physics, 49, 607 (2000).

[7] V. Vinokur et. al. Physica C, 295, 209, (1998); D. Ertaz and D. R. Nelson, Physica C 272 79 (1996).

[8] J. Kierfeld and V.M. Vinokur, Phys. Rev. B, 61, R14928, (2000).

[9] B. Khaykovich et. al. Phys. Rev. B., 56 R517, (1997).

[10] M.B. Gaifullin, et. al., Phys. Rev. Lett, 84, 2945, (2000); C.J. van der Beek et. al., ibid., 84, 4196, (2000).

[11] M. J. Higgins and S. Bhattacharya, Physica C 257 232 (1996).

[12] S. S. Banerjee et. al. Physica C 308 25 (1998); Phys. Rev. B 59 6043 (1999); Phys. Rev. B, 58, 995 (1998).

[13] S.S. Banerjee et. al. Physica C, 355/1-2, 39, (2001).

[14] K. Tenya et. al., J. Phys. Soc. Jpn., 68, 224, (1999).

[15] S. S. Banerjee et. al., J. Phys. Soc. Jpn Suppl. 69 262 (2000).

[16] S. Sarkar et. al. Phys. Rev. B, 61, 12394 (2000).
[17] D. Giller et. al. Phys. Rev. Lett, 79, 2542, (1997).

[18] T. Klein et. al., J. Low. Temp. Phys., 117, 1353, (1999).

[19] A. Wisnewski et. al. in Studies of High Temperature Superconductors Vol. 31, ed. A.V. Narlikar, (Nova Science Publishers, NY, 2000).

[20] V. Hardy et. al. Physica C, 232, (1994) 347.

[21] T. Blasius et. al., Phys. Rev. Lett, 82, 4296, (1999).

[22] D. R. Nelson, Phys. Rev. Lett, 60 1973 (1988);

[23] K. Ghosh et. al., Phys. Rev. Lett, 76 4600 (1996).

[24] Y. Paltiel et. al., Phys. Rev. Lett, 85, 3712, (2000).

[25] H.S. Bokil and A.P. Young, Phys. Rev. Lett, 74, 3021, (1995).

[26] D.R. Strachan et. al., Phys. Rev. Lett, 87, 067007, (2001); B. Brown, Phys. Rev. B, 61, 3267, (2000).

[27] A. van Otterlo, R.T. Scalettar and G.T. Zimanyi, Phys. Rev. Lett, 81, 1497, (1998).

[28] F. Thalmann, C. Dasgupta and D. Feinberg, Europhys. Lett, 50 54, (2000). This possibility was pointed out to the author by C. Dasgupta.

[29] M.C. Marchetti and D.R. Nelson, Phys. Rev. B, 41, 1910, (1990).

[30] H.H. Strey et. al. Phys. Rev. Lett, 84, 3105, (2000).

[31] The increased importance of disorder at low fields should imply a weakening of the first-order behaviour of the MG-DL transition line of Fig. 2 close to $H_{c1}$, mirroring the behaviour at high-fields. This would suggest another tricritical point at the low-field end of the MG-DL transition line, as seen in some recent experiments.

[32] A. Soibel et. al. Nature, 406, 282, (2000)
[33] R. Merithew et al. Phys. Rev. Lett. 77 3197 (1996).

[34] S. Ooi, T. Shibauchi and T. Tamegai, Physica B, 284-288, 775, (2000).

[35] G.I. Menon, Phase Transitions, (to appear); G.I. Menon, unpublished.

[36] T. Ishida, K. Okuda and H. Asaoka, Phys. Rev. B, 56, 5128, (1997).

[37] E. Zeldov et al., Nature, 365, 375 (1995).

[38] R. Sugano et al. Physica B, 284-288, 803, (2000).

[39] G. d’Anna et al. Europhys. Lett, 25, 225, (1994); ibid. 25, 539, (1994).

[40] T. Nishizaki and N. Kobayashi, Supercon. Sci. Tech., 13, 1, (2000);

[41] B. Billon et al., Phys. Rev. B, 55, R14753, (1997).
FIGURES

FIG. 1. The current view of the phase diagram of disordered type-II superconductors \cite{3,6–8}. In addition to Meissner and normal phases, this phase diagram subdivides the mixed phase into three phases – the Bragg glass (BrG), the vortex glass (VG) and the disordered liquid (DL); these are described in the text. Note that the Bragg Glass phase melts directly into the liquid at intermediate field values.

FIG. 2. Proposed universal phase diagram for disordered type-II superconductors. The Meissner, normal, BrG and DL phases are as shown in Fig. 1. The MG phase (see text) intrudes between BrG and DL phases everywhere in the phase diagram. The inset expands the boxed region of the main figure, illustrating the reentrance of the BrG-MG phase boundary. Solid lines indicate discontinuous transitions.
