ON THE BLACK HOLE UNITARITY ISSUE

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Abstract
I discuss features required for preserving unitarity in black hole decay and concepts underlying such a perspective. Unitarity requires that correlations extend on the scale of the horizon. I show, in a toy model inspired by string theories, that such correlations can indeed arise. The model suggests that, after a time of order $4M \ln M$ following the onset of Hawking radiation, quantum effects could maintain throughout the decay a collapsing star within a Planck distance of its Schwarzschild radius. In this way information loss would be avoided. The concept of black hole “complementarity”, which could reconcile these macroscopic departures from classical physics with the equivalence principle, is interpreted in terms of weak values of quantum operators.

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1 Introduction

In view of the successful computation of the quantum degeneracy of some extreme black holes in string theory [1, 2], it may seem likely that further developments will provide the solution to the problem posed by black hole decay and that there is not much to gain from general considerations. However, even if string theory would ultimately lead to a black hole decay consistent with unitarity, the nature of such a decay is presently unclear. It may therefore be of some help to investigate features required for preserving unitarity in black hole decay and concepts underlying such a perspective.

I shall first briefly review the unitarity problem and stress the highly nonlocal correlations which would be needed to avoid information loss. I shall then examine in a toy model how such nonlocal effect can “materialise” the black hole horizon and how such a materialisation suggests that the degrees of freedom of a collapsing star are recovered in the Hawking radiation. I shall then discuss the black hole complementarity issue and its possible quantum significance.

2 The Unitarity Issue

The Bekenstein assumption [3] that a black hole of mass \(M\) has an entropy proportional to the area \(A = 4\pi M^2\) of its event horizon, combined with the Hawking computation of the black hole radiation temperature [4] \(T_H = 1/8\pi M\), yields for this entropy \(S\) the value

\[ S = A/4 + C \]  \hspace{1cm} (1)

where \(C\) is an unknown integration constant. This is an immediate consequence of the identity

\[ dM = T_H dS. \]  \hspace{1cm} (2)

The original derivation of the Hawking radiation [4] was based on conventional local field theory and did not take into account the gravitational backreaction. According to these assumptions, the emitted quanta are only correlated to states formed out of degrees of freedom in the horizons vicinity and no information about the infalling matter is stored in the radiation itself. Complete evaporation of the black hole would then inevitably lead in
the semi-classical approximation to a density matrix out of any initial state, even out of a pure quantum state, and thus to a violation of unitarity [5]. An alternative is however conceivable if, when the black hole reaches the Planck regime where semi-classical considerations become unreliable, evaporation would stop and leave a remnant with very high and possibly infinite degeneracy to which the radiation would remain correlated [6]. If a statistical interpretation of the black hole entropy were available, infinite degeneracy would mean an infinite integration constant in Eq. (1).

It was first suggested by ’t Hooft that the semi-classical approximation breaks down already at macroscopic scales and that unitarity could be maintained without the remnant hypothesis[7]. Information must then be transmitted to the radiation. Causality implies that information about the collapsing star should not be trapped inside the horizon. This means that macroscopic correlations must be present to turn the horizon into a system where information can be deposited. Such “materialisation” of the horizon can only appear at the quantum level and has dramatic implications. Firstly, the black hole entropy is interpretable as a counting of a finite number of states, and thus the integration constant in the area entropy (1) must be zero or finite. Furthermore, any system defined in a finite volume has an energy limited by black hole formation and the number of states in this volume is bounded by the logarithm of the area entropy of the surrounding surface. This is at odds with local field theory which would lead, even in presence of a Planckian cut-off, to an entropy proportional to the volume and hence to an infinite constant of integration in the thermodynamic limit. Thus a theory underlying unitary black hole decay is expected to be genuinely non local. Finally, the materialisation of a horizon, where the classical curvature is vanishingly small in the large $M$ limit, violates the equivalence principle. This motivated the introduction of a complementarity hypothesis according to which an inertial observer who would cross the horizon would not feel its material structure [8].

3 String Instantons and Non Local Effects.
3.1 Entropy and Temperature.

I shall show that, in a toy model inspired from string theory, non local effect can modify both the black hole temperature and the value of its entropy (1). These changes affect drastically the properties of quantized matter in the black hole background. The model discussed here is not meant to be a realistic one but its interest is to reveal a mechanism by which unitarity could be maintained in black hole evaporation.

The possibility of departing from $A/4$ can be understood in thermodynamic terms. The differential mass formula (9) which generalises (2) for black holes surrounded by static matter can be written as (10)

$$\delta M_{tot} = \frac{\kappa}{2\pi} \frac{\delta A}{4} + \sum_i \partial_{\lambda_i} H_{\text{matter}} \delta \lambda_i$$

(3)

where $M_{tot}$ is the total mass, $\kappa$ the surface gravity of the hole, $A$ the area of the event horizon and the $\lambda_i$ are all the parameters in the matter action. This identity is derived from classical physics and cannot involve the Planck constant. The Bekenstein assumption that the black hole contributes to the entropy amounts to interpret Eq.(3) as the expression of the first principle of thermodynamics: $A/4$ is the entropy up to a multiplicative constant proportional to $\bar{h}^{-1}$ and the temperature must then be proportional to $\bar{h}$. The $\partial_{\lambda_i} H_{\text{matter}}$ are generalised forces.

It is well known that to compute thermal correlation functions and partition functions in field theory in flat Minkowski space-time one can use path integrals in periodic imaginary time. The period $\beta$ is the inverse temperature and can be chosen freely. This method was generalised to compute matter correlation functions in static curved backgrounds. For the Schwarzschild black hole, possibly surrounded by matter, the analytic continuation to imaginary time defines a Euclidean background everywhere except at the analytic continuation of the horizon, namely the 2-sphere at $r = 2M$. Gibbons and Hawking (11) extended the analytic continuation to the gravitational action, restricting the hitherto ill-defined path integral over metrics to a saddle point in the Euclidean section. To constitute such a saddle the Euclidean black hole must be regular given that a singularity at $r = 2M$ would invalidate the solution of the Euclidean Einstein equations. This implies a unique Hawking temperature $T_H$ which, in natural units, is always equal to $\kappa/(2\pi)$. Thus one recovers from Eq.(3) the area entropy (1).
This entropy is not affected by mass surrounding the black hole and would therefore seem to depend only on the black hole mass. But this need not be the case: a different relation between entropy and area arises when a conical singularity is generated in the Euclidean section at \( r = 2M \).

As pointed out by many authors, a conical singularity at \( r = 2M \) modifies the Euclidean periodicity of the black hole and hence its temperature. If this singularity would arise from a source term in the Euclidean Einstein equations, Eq.(3) would remain valid and could be written as

\[
\delta M_{\text{tot}} = T \delta[(1 - \eta) \frac{A}{4}] + T \frac{A}{4} \delta \eta + \sum_i \partial \lambda_i H_{\text{matter}} \delta \lambda_i,
\]

where \( \eta \) is the deficit angle and the temperature \( T \) is related to the Hawking value \( T_H \) by \( T = T_H (1 - \eta)^{-1} \). It follows from Eq.(4) that a new generalised force \( X_\eta \), conjugate to \( \eta \),

\[
X_\eta = T \frac{A}{4}
\]

must appear and that the entropy of the hole would become

\[
S = (1 - \eta) \frac{A}{4},
\]

independent of the surrounding matter. I now show that a deficit angle can be generated by a string instanton and that Eqs.(5) and (6) obtain with \( \eta \) determined by the string tension.

For simplicity I shall describe here only the case of a pure black hole of mass \( M \). The general case is treated elsewhere [13]. In presence of a Nambu-Goto string the Euclidean action is

\[
I = -\frac{1}{16\pi} \int_M \sqrt{g} R + \frac{1}{8\pi} \int_{\partial M} \sqrt{h} K
\]

\[
-\frac{1}{8\pi} \int_{(\partial M)_\infty} \sqrt{h_0} K_0 + \mu \int d^2 \sigma \sqrt{\gamma}.
\]

Here \( \mu \) is the string tension and \( \gamma \) the determinant of the induced metric on the world sheet. The latter is taken to have the topology of a 2-sphere. The variation of this action with respect to the metric gives the Einstein equations and the variations with respect to the string coordinates in \( \gamma \) give rise to the stationary area condition for the string.
The Einstein equations still admit ordinary black hole solutions corresponding to zero string area. The Euclidean space is regular at \( r = 2M \) and the \( t \)-periodicity is the inverse Hawking temperature. However there exists a non-trivial solution to the string equations of motion in Euclidean space when the string wraps around the Euclidean continuation of the horizon, a sphere at \( r = 2M \). This solution has a curvature singularity at \( r = 2M \). Expressing the curvature in the trace of Einstein equations as the product of the horizon times a two dimensional curvature and using the Gauss-Bonnet theorem for disc topology tell us that there is a conical singularity with deficit angle \( 2\pi \eta \) such that

\[
\eta = 4\mu. \tag{8}
\]

This deficit angle is the sole effect of the string instanton. It raised the temperature from \( \beta_H^{-1} \) to \( \beta^{-1} = \beta_H^{-1}/(1 - 4\mu) \).

I now evaluate the free energy of the black hole. The contribution of the string term to the action (7) exactly cancels the contribution of the Einstein term. The only contributions comes from the boundary terms and one gets

\[
F(\beta, \mu) = \beta^{-1} I_{\text{saddle}} = \frac{M}{2} = \frac{\beta}{16\pi(1 - 4\mu)}. \tag{9}
\]

From Eq. (8) and the thermodynamic relations \( S = \beta^2 (\partial F/\partial \beta)_{\mu}, \ X_{4\mu} = (\partial F/\partial 4\mu)_{\beta} \), one recovers Eqs. (5) and (6) with \( \eta = 4\mu \).

For a black hole of given mass \( M \), the increase of the Hawking temperature due to the conical singularity is accompanied by a decrease of the entropy in such a way that the product

\[
TS = \frac{M}{2} \tag{10}
\]

remains constant. This result, which is consistent with the fact mentioned before that the classical differential mass formula (5) defines the entropy only up to a multiplicative constant, can be understood in simple terms. Hawking quanta carry away a mass proportional to the temperature but the entropy contained in the radiation is proportional to the number of emitted quanta. Hence the entropy stored in a black hole of mass \( M \) must decrease as the temperature increases.

In the above analysis I considered the entropy associated to an instanton corresponding to a string wrapped once around the horizon. Clearly such a configuration is thermodynamically meaningful only if it describes a
metastable state at the scale of the black hole lifetime. Otherwise the instanton, as well as instanton configurations with higher winding number would contribute only an irrelevant exponentially small correction to the leading entropy term $A/4$. I shall assume temporarily that such a metastability does indeed occur and investigate its consequences at a quantum level. This will allow me to exhibit a mechanism which would lead to a unitary evolution of the collapsing black hole and which could well be realised by any phenomenon implying non local correlations on the scale of the horizon.

3.2 Towards unitarity.

The string instanton at $r = 2M$ in Euclidean space does not alter the classical Lorentzian black hole background which remains regular on the horizon. However dramatic effects occur at the quantum level. To illustrate these let us consider the approximation consisting of retaining only the $s$-wave component of a free scalar field propagating on the Schwarzschild geometry and disregarding the residual relativistic potential barrier. This amounts to take a 2-dimensional scalar field propagating on the radial subspace of the 4-geometry. The metric is

$$ds^2 = -(1 - \frac{2M}{r})dudv \quad u = t - r^*, \quad v = t + r^*$$

where $r^*$ is the tortoise coordinate

$$dr^* = dr(1 - 2M/r)^{-1}.$$  \hspace{1cm} (12)

One can compute the expectation value of the energy-momentum tensor of the scalar field using the trace anomaly \[14\] to get

$$4\pi r^2 T_{uu} = \frac{1}{12\pi} \left[ -\frac{M}{2r^3} (1 - \frac{2M}{r}) - \frac{M^2}{4r^4} \right] + t_u(u).$$  \hspace{1cm} (13)

Here $t_u(u)$ is defined by boundary conditions. For a radiation flux at a temperature $T$, one has

$$\lim_{r \to \infty} 4\pi r^2 T_{uu} = t_u = (\pi/12)T^2.$$  \hspace{1cm} (14)
Inserting this value into Eq. (13) one sees that if \( T = T_H \), \( T_{uu} \) vanishes on the horizon as \((1 - 2M/r)^2\). This means that in the Kruskal inertial frame defined by

\[
\begin{align*}
  dU &= \exp\left(\frac{-u}{4M}\right) du \\
  dV &= \exp\left(\frac{v}{4M}\right) dv,
\end{align*}
\]

the energy momentum tensor is regular on the horizon. If the temperature is increased above \( T_H \), one gets

\[
4\pi r^2 T_{UU} = \frac{1}{48\pi} \left(\frac{T}{T_H}\right)^2 - 1 \frac{1}{U^2}.
\]

Thus, the string instanton induces, in an inertial frame, a positive energy singularity on the horizon.

Let us suppose that the above analysis of an eternal black hole can be extended to the radiation emitted by a black hole originating from a collapse. In this case the past horizon is absent but the behaviour on the future horizon is unaltered. Hence, in absence of backreaction, the outgoing flux is still given by Eqs.(13), (14) and (17) at asymptotic Schwarzschild times.

To estimate the backreaction, I shall compute the amount of radiation energy stemming from the region just outside the star. In absence of backreaction, a collapsing shell moves at asymptotic times along a trajectory of constant \( v = v_0 \) in the coordinate system defined by Eqs.(11) and (12) which remain valid outside the shell. The latter can be taken as a mimic of the star surface. Outside the star, energy is conserved and the energy flux measured at asymptotic times as \( r \to \infty \) (or \( v \to \infty \)) stems entirely from the negative energy flux \( T_{vv} \) flowing towards the horizon. No radiation flux \( T_{uu} \) is emitted from the vicinity of the star. This is why the latter is not affected by the radiated quanta and why no information about it is carried by the radiation. When the temperature is increased, the vicinity of the star, contributes to the radiation an energy flux which from Eqs.(13) and (14) is

\[
4\pi r^2 T_{uu} = \frac{\alpha}{M^2}, \quad \alpha = M^2 \frac{\pi}{12} (T^2 - T_H^2) = \frac{\mu(1 - 2\mu)}{96\pi(1 - 4\mu)^2}.
\]

It follows from Eq.(18) that, if \( \alpha \) is of \( O(1) \), the radiation stemming from the region just outside the star would tend to infinity as \( u \to \infty \). Hence one
expects that quantum gravity must deeply affect the star and its neighborhood. I shall label these as the quantum star. Even without the knowledge of the equation of state of the quantum star one may use mass conservation to estimate its average coordinate distance \( y \equiv r(t) - 2M(t) \) to its Schwarzschild radius. In view of Eq.(18), the mass \( M(t) \) of the quantum star decreases in time as \( dM/dt \approx -\alpha/M^2 \). As a first approximation I keep \( v = v_0 = \text{constant} \) and thus, from Eqs.(11) and (12), one gets when \( y \ll 2M \)

\[
\frac{dy}{dt} \approx -\frac{y}{2M} + \frac{\alpha}{M^2}.
\]

Equation (19) is similar to a result obtained by Itzhaki [15] in a different but related context. We see that in a time comparable to \( 4M \ln M \) after the onset of the Hawking radiation which occurs when \( y \) is still of order \( M \), the quantum star reaches a coordinate distance \( y = O(1/M) \) of its Schwarzschild radius. The proper distance \( d \) to the horizon in the Schwarzschild time is

\[
d = \int_{2M}^{r} \frac{1}{\sqrt{1 - \frac{2M}{x}}} dx \approx \sqrt{8My} = O(1).
\]

Thus the quantum star remains at a Planckian distance (and at a Planckian temperature) of its Schwarzschild radius up to a time of order \( M^3 \) where it has completely evaporated. After the time \( 4M \ln M \), the nature of the Hawking radiation changes: thermal quanta are now emitted from the burning quantum star and information about its structure gets transmitted to the radiation. These considerations corroborate the results of a previous analysis based on the assumption that quantum gravity should tame the transplanckian frequencies of blueshifted Hawking quanta stemming from the neighbourhood of the horizon after such a time span [16].

Thus the would-be horizon in absence of backreaction may disappear and mass conservation alone indicates that the incipient black hole may evaporate before a true horizon can form. Such a history of a collapsing star in a topologically trivial background devoid of horizon and singularity would clearly be consistent with unitarity. The above analysis indicates that this is likely to be the case if, as exemplified by the string instanton, non local correlation’s on the scale of the horizon induce at the quantum level a non vanishing positive energy density \( T_{uu} = O(1/M^2) \) when \( R \rightarrow 2M \).
4 The Complementarity Issue

We just saw that unitarity could be maintained in black hole evaporation if the energy density $T_{uu}$ remains finite when $R \to 2M$. The halting mechanism resulting from the non vanishing $T_{uu}$ would also solve, as mentioned above, the well known problem posed by the existence of transplanckian frequencies in an external observer frame. However, it apparently contradicts the equivalence principle as the small classical gravitational field at the horizon appears inconsistent with the huge acceleration required to bring the collapse to a halt.

This clash could be resolved in quantum gravity where the metric field $g_{\mu\nu}(x)$ is promoted to a quantum operator. Indeed, to the extend that a description in terms of matter evolving in a geometrical background could be maintained, the metric $g_{\mu\nu}(x)$ need not be identified with the expectation value of the corresponding Heisenberg operator $\hat{g}_{\mu\nu}(x)$ in a “in” quantum state $|i\rangle$. Rather, the background geometry could be determined, as anticipated by ’t Hooft [17], from both the “in” state $|i\rangle$ and the “out” state. This would lead to different causal histories of the black hole as reconstructed by observers crossing the horizon at different times but would reduce, in accordance with the equivalence principle, to the classical description of the collapse for the proper history of the star as recorded by an observer comoving with it [16].

To understand this point in qualitative terms, consider a detector sensitive only to cisplanckian effects. I call such a detector an observer. Let us first confine the motion of this observer within the space-time outside the event horizon of a sufficiently massive collapsing star. It will necessarily encounter radiation. The radiation recorded by such observers can be encoded in some “out”-state. Thus in the space-time available to “external” observers, there exist outgoing states describing a particular set of detectable quanta covering the whole history of the evaporating black hole. This information about a particular decay mode can be added to the characterisation of the system by the Schrödinger state of the star before collapse, or equivalently by the corresponding Heisenberg state $|i\rangle$. More precisely we could specify that the system is likely to be found at sufficiently late times in a state characterised by some typical distribution of Hawking quanta. It may seem at first sight that this added information about the future detection of Hawking radiation is irrelevant for the analysis of the energy momentum tensor and of the metric
at intermediate times. This could be an incorrect conclusion for reasons I shall now explain.

The expectation value $\langle i|\hat{A}(t)|i \rangle$ of a Heisenberg operator $\hat{A}(t)$ in the normalised quantum state $|i \rangle$ is often expressed as

$$\langle i|\hat{A}(t)|i \rangle = \sum_\alpha P_\alpha A_\alpha P_\alpha = |\langle \alpha|i \rangle|^2 \tag{21}$$

where the eigenvectors $|\alpha \rangle$ relative to eigenvalues $A_\alpha$ form a complete set of orthonormal states. One interprets then $\langle i|\hat{A}(t)|i \rangle$ as the average over the probability distribution $P_\alpha$ of finding the value $A_\alpha$ if exact measurements of a complete set of commuting observables containing $\hat{A}(t)$ are performed at time $t$ on a quantum system "pre-selected" to be in the initial Schrödinger state $|t_1, i \rangle = U(t_1, t_0)|i \rangle$ at time $t_1$. $U(t_1, t_0)$ is the evolution operator to the time $t_1$ from the time $t_0$ where the Schrödinger state is identified with the Heisenberg one.

More information can be gained if the system is also "post-selected" to be found at a later time $t_2$ in a given Schrödinger state $|t_2, f \rangle$. One may then express expectation values $\langle i|\hat{A}(t)|i \rangle$ as an average of weak values defined for $t_1 < t < t_2$ by

$$A^\text{weak}_f \equiv \frac{\langle f|\hat{A}(t)|i \rangle}{\langle f|i \rangle} \tag{22}$$

where $|f \rangle = U(t_0, t_2)|t_2, f \rangle$. One gets

$$\langle i|\hat{A}(t)|i \rangle = \sum_f P_f A^\text{weak}_f \quad P_f = |\langle f|i \rangle|^2. \tag{23}$$

Eq. (23) suggests that weak values represent measurable quantities for a pre- and post-selected system. This is indeed the case if a measurement of $\hat{A}(t)$ is performed on the system with sufficient quantum uncertainty to avoid disrupting the evolution of the system. Such "weak" measurements yield not only the real part of $A^\text{weak}_f$ but also its imaginary part and reconstruct in this way the available history between $t_1$ and $t_2$ for a system pre-selected at $t_1$ and post-selected at $t_2$. In the limit $\hbar \to 0$, the weak value is purely real.

Generally, the information gained by post-selection and weak values is relevant only if one post-selects a state describing rare events for the pre-selected state considered, that is if $P_f$ is located in the tail of the distribution probability. However, the situation is different when one considers the
Hawking emission process in the classical background of a collapsing star. There, in absence of backreaction, the energy-momentum tensor of the radiation can be computed exactly in some simplified models. It then appears that post-selected states defined on a space-like surface $\sigma$ arbitrarly close to the union of the event horizon and of the future light-like infinity may yield weak values of the energy-momentum tensor operator $\hat{T}_{\mu\nu}(x)$ very different from its average value. While the latter remains smooth on the scale of the Schwartzschild radius, the former may exhibit in that region oscillations of unbounded amplitudes \[19, 20\]. These features, which are a consequence of the unbounded blueshift experienced in the vicinity of the horizon by the vacuum fluctuations generating the Hawking quanta, persist in generic post-selected states detectable by external observers. The energy content of these fluctuations show up in the weak values of $\hat{T}_{\mu\nu}(x)$ but are averaged out in expectation values. However observers who do cross the horizon detect different post-selected states. These yield weak values of $\hat{T}_{\mu\nu}(x)$ which are smooth as the observer approaches the horizon. In particular free falling observers cannot detect Hawking quanta and their weak values coincide with the expectation values \[16\]. This suggests that, when backreaction is taken onto account, there is no reason to identify for all observers, the relevant metric background to the expectation values of the metric operators $\hat{g}_{\mu\nu}(x)$.

Transplanckian effects and unitarity considerations pose problems only to the external observer who can ultimately detect Hawking quanta encoded in some final state $|f\rangle$. Both problems would be cured by the halting mechanism as the latter would tame the transplanckian fluctuations of the weak value in $\hat{T}_{\mu\nu}(x)$. This raises the possibility that the halting mechanism is a property of the weak value of the metric relative to the final state $|f\rangle$. In other words, halting would occur only in the description of the reconstructed history encoded in the background defined by the weak value of the metric

$$g^{\text{rec}}_{\mu\nu}(x) = \frac{\langle f|\hat{g}_{\mu\nu}(x)|i\rangle}{\langle f|i\rangle}. \quad (24)$$

The inertial observer, which cannot detect Hawking quanta, would observe the proper history encoded in the expectation value

$$g^{\text{proper}}_{\mu\nu}(x) = \langle i|\hat{g}_{\mu\nu}(x)|i\rangle. \quad (25)$$

The above discussion indicates that black hole complementarity \[8\] can fit into the framework of conventional quantum physics. It could reconcile
non local correlations on the horizon scale required for the unitarity of the reconstructed history with the equivalence principle as applied to the proper history. While the correlations necessary for ensuring unitarity appear hardly available in the context of conventional local field theory, there are some indications that the developments of string theories point towards a unitary description of black hole decay. But a full non perturbative formulation is clearly required to settle the issues.

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