Behavior of unsaturated cohesive-frictional soils over a whole range of suction/thermo-controlled stress paths and modes of deformation

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Abstract
The present work documents some of the most recent experimental evidence of the thermo-hydro-mechanical behavior of compacted soils over a whole range of suction- and/or thermo-controlled stress paths and modes of deformation, including data from a series of triaxial, true triaxial, plane strain, ring shear, and resonant column tests conducted on different types of cohesive-frictional soils in the low-to-medium matric suction range under either room temperature or thermally controlled conditions. The work has been accomplished at the Advanced Geomechanics Laboratory of the University of Texas at Arlington, focusing primarily on the following essential features of unsaturated soil behavior: (1) Loading-collapse and apparent tensile strength loci assessed from suction-controlled triaxial and true triaxial testing on clayey sand, (2) Critical state lines from suction-controlled plane strain testing on silty soil, (3) Peak and residual failure envelopes from suction-controlled ring shear testing on clayey soil, (4) Frequency response curves and cyclic stress-strain hysteretic loops from thermo-controlled, constant-water content resonant column testing on clayey soil, and (5) Residual failure envelopes from suction/thermo-controlled ring shear testing on clayey soil. The work is intended to serve as a succinct yet reasonably thorough state-of-the-art paper contribution to PanAm-UNSAT 2021: Third Pan-American Conference on Unsaturated Soils, Rio de Janeiro, Brazil, July 21-25, 2021.

Keywords
Suction-controlled testing
Triaxial testing
True triaxial testing
Plane strain testing
Ring shear testing
Resonant column testing

1. Introduction
Over the last five decades, intensive research efforts undertaken worldwide have defined the threshold of the state-of-the-knowledge of unsaturated soil behavior. The adoption of matric suction and the excess of total stress over air pressure, i.e., net normal stress, as the relevant stress state variables, has facilitated the investigation of essential features of unsaturated soil response, via either the axis-translation or the vapor transfer technique, for a wide range of matric and total suction states. It is the relative success of these techniques that has prompted researchers in the discipline to devote countless hours to fine-tuning myriad details of standardized soil testing devices, and thus keep the focus of their efforts on expanding and upgrading the capabilities of such devices for testing unsaturated soil materials.

The present work documents some of the most recent experimental evidence of the thermo-hydro-mechanical behavior of compacted soils over a whole range of suction- and/or thermo-controlled stress paths and modes of deformation, including data from a series of triaxial, true triaxial, plane strain, ring shear, and resonant column tests conducted on different types of compacted cohesive-frictional soils in the low-to-medium matric suction range under either room temperature or thermally controlled conditions.

The work focuses primarily on the following essential features of unsaturated soil behavior: (1) Comparative analysis of loading-collapse and apparent tensile strength loci assessed from both suction-controlled triaxial and true triaxial testing on clayey sand, (2) Critical state lines from suction-controlled plane strain testing on silty soil, (3) Peak and residual failure envelopes from suction-controlled ring shear testing on clayey soil, and (5) Residual failure envelopes from suction/thermo-controlled ring shear testing on clayey soil.

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shear testing on clayey soil, (4) Frequency response curves and cyclic stress-strain hysteresis loops from thermo-controlled, constant-water content resonant column testing on clayey soil, and (5) Residual failure envelopes from suction/thermo-controlled ring shear testing on clayey soil.

The first section of the work is devoted to the hydro-mechanical response of compacted cohesive-frictional soils under isothermal and suction-controlled conditions via the axis-translation technique. Key modifications made to standardized triaxial, true triaxial, plane strain, and ring shear devices, in order to make them suitable for suction-controlled testing via axis-translation, are summarily described and illustrated.

The second section of the work is devoted to the thermo-hydro-mechanical response of compacted cohesive-frictional soils under non-isothermal and either constant-water content or suction-controlled conditions. Key modifications made to standardized resonant column and ring shear devices, in order to make them suitable for suction/thermo-controlled testing of soils over a whole range of shear strain amplitudes, are also summarily described.

The present work does not make any pretenses to absolute originality or even a thoroughly comprehensive literature review, given the extent and importance of the work accomplished by other researchers and scholars on the subject to this date. It is rather intended to serve as a succinct yet reasonably thorough state-of-the-art paper contribution to PanAm-UNSAT 2021: The Third Pan-American Conference on Unsaturated Soils, with particular emphasis on essential features of unsaturated soil behavior that have not been thoroughly addressed in the existing literature, largely due to a lack of suitable testing devices.

2. Hydro-mechanical behavior: recent evidence

2.1 Experimental program and scope

Over the last three decades, several critical-state based constitutive models, incorporating suction as an independent stress state variable, have been postulated for unsaturated soils with varying degrees of success in capturing the true nature of soil response (e.g., Alonso et al., 1990; Toll, 1990; Sivakumar, 1993; Maatouk et al., 1995; Wheeler, 1996; Cui & Delage, 1996; Bolzon et al., 1996; Adams & Wulffsohn, 1997; Rampino et al., 2000; Sivakumar & Wheeler, 2000; Tang & Graham, 2002; Wang et al., 2002; Chiu & Ng, 2003; Thu et al., 2007; Hoyos et al., 2012). Additional experimental evidence for a wider variety of soils, however, particularly for materials that are cohesive-frictional in nature, is still sorely needed to conclusively substantiate their validation.

In this regard, triaxial testing continues to be the most universally used method to characterize unsaturated soil behavior, owing primarily to its versatility in accommodating the required modifications for suction-controlled testing via the axis-translation technique (e.g., Estabragh et al., 2004; Zhang & Li, 2011; Liu & Muraleetharan, 2012; Usmani et al., 2012; Estabragh & Javadi, 2014). In nature, however, soil deposits well above the ground-water table are intrinsically heterogeneous, and hence may be subject to a state of three-dimensional anisotropic tensions and stress gradients. Therefore, accurate predictions of unsaturated soil response require that the constitutive relations be valid for all principal stress paths that are likely to be experienced in the field. It is in this context that a suction-controlled true triaxial (cubical) apparatus becomes of paramount importance (e.g., Hoyos & Macari, 2001; Matsuoka et al., 2002; Reis et al., 2011).

Despite recent advances in suction-controlled triaxial (TX) and true triaxial (TTX) testing of unsaturated soils, there is a glaring lack of experimental substantiation of test results obtained from one technique based on results from the other, which constitutes a first chief motivation under this section of the present work (Hydro-mechanical Behavior). As previously mentioned, a comparative analysis of loading-collapse and apparent tensile strength loci, assessed from both suction-controlled TX and suction-controlled TTX testing on clayey sand, was accomplished. (Results are summarized in the following subsection.)

Suction-controlled TX testing was carried out in a fully automated, double-walled triaxial system, as shown in Figure 1a, featuring the following key items: (1) Base pedestal with 15-bar ceramics, (2) Top cap with coarse porous stones, (3) Inner cell water inlet, (4) Pore water pressure inlet, (5) Differential pressure transducer, (6) Pore air pressure inlet, (7) Flushing inlet, (8) Flushing outlet, (9) Outer cell water outlet, and (10) Soil volume change outlet. A typical cylindrical specimen has a 71.1 mm (2.8 in) diameter and a 142.2 mm (5.6 in) height. A detailed description of the main components and earlier performance verification tests is presented by Patil (2014).

Likewise, suction-controlled TTX testing was accomplished in a servo-controlled cubical cell with similar features, except with the cubical soil sample resting on a bottom wall assembly, as shown in Figure 1b, featuring the following key items: (1) Stainless steel bottom wall, (2) Cubical base pedestal, (3) Full set of symmetrically spaced coarse porous stones, and (4) Full set of symmetrically spaced 5-bar ceramics. Silicon rubber membranes form a pressure seal between each wall assembly and the core reaction frame of the cubical test cell, thus acting as the fluid barrier for distilled water pressurizing the top and four lateral faces of the specimen. A typical cubical specimen measures 76.2 mm (3 in) per side. A detailed description of the main components and earlier performance verification testing is presented by Pérez-Ruiz (2009).

A second chief motivation under this section of the work was the assessment of critical state lines from suction-controlled plane strain (PS) testing on silty soil. (Results are summarized in a subsequent subsection.) It is well known that plane strain analyses may render far more accurate predictions...
for a vast majority of geotechnical infrastructure, including slopes, embankments, tunnels and pavements, given the particular geometries, loading paths and boundary conditions that such geosystems normally undergo. The majority of plane strain (biaxial) devices reported to date, however, only allow for soil testing under fully dry or saturated conditions (e.g., Wood, 1958; Vardoulakis & Graf, 1985; Dresher et al., 1990; Alshibli et al., 2004). In this work, suction-controlled PS testing was carried out in a fully automated, Vardoulakis type of biaxial apparatus, as shown in Figure 1c, featuring the following key items: (1) Bottom base plate; (2) U-shaped base frame, coupled with a Schneeberger type sliding table; (3) Bottom pedestal housing a 5-bar ceramic; and (4) Cuboid specimen. Two 8-mm thick rigid walls, made of Type 304 stainless steel, prevent the specimen from deforming along the intermediate principal axis. A typical cuboid specimen has a 90 mm (3.5 in) height, a 60 mm (2.4 in) width, and a 30 mm (1.2 in) depth. A detailed description of the main components and earlier performance verification is presented by Cruz et al. (2011).

The last chief motivation under this first section of the work was the assessment of peak and residual failure envelopes from suction-controlled ring shear (RS) testing on clayey soil. (Results are also summarized in a subsequent subsection.) Despite the crucial importance of peak and residual shear strength properties of compacted clayey soils, there is very limited experimental evidence of unsaturated soil behavior under large deformations as the soil is being subjected to controlled suction states. In recent years, a few researchers have expanded the capabilities of Bromhead-type RS devices for soil testing under suction-controlled conditions (e.g., Infante-Sedano et al., 2007; Merchán et al., 2011). More comprehensive efforts, however, have yet to be undertaken to generate a thorough set of suction-dependent peak/residual failure envelopes for compacted clayey soil. In this work, suction-controlled RS testing was carried out in a fully automated RS apparatus, as shown in Figure 1d, featuring the following key items: (1) Outer wall of bottom annular platen housing a full set of 5-bar ceramics, (2) Inner wall of bottom annular platen, and (3) Spare set of 5-bar ceramics. A typical ring-shaped specimen has a 152.4 mm (6 in) outer diameter, a 96.5 mm (3.8 in) inner diameter, and an average thickness of 15.0 mm (0.6 in). A detailed description of the main components and performance verification is presented by Velosa (2011) and Yepes (2015).

Figure 1. Key modifications to advanced soil testing devices for implementing axis-translation technique: (a) Triaxial, (b) True triaxial, (c) Plane strain, (d) Ring shear.
2.2 Suction-controlled triaxial (TX) vs. true triaxial (TTX) testing

The Barcelona Basic Model (BBM), introduced by Alonso et al. (1990), and the Oxford Model (OM), by Wheeler & Sivakumar (1995), have become two of the most popular critical state based constitutive frameworks postulated to date for unsaturated soils. The OM formulation is similar to that of the original BBM; however, it postulates that all the essential model parameters are suction-dependent and ought to be experimentally predetermined for a particular matric suction state.

A first distinctive feature of the original BBM is the assumption of a monotonic decrease of the volumetric stiffness parameter $\lambda(s)$ with increasing matric suction, which in turn defines a Loading-Collapse (LC) yield locus in the $p,s$ plane as a full set of preconsolidation pressures for each associated value of suction. Another distinctive feature of the original BBM is the postulation of an Apparent Tensile Strength (ATS) locus as a full set of apparent tensile strength values for each associated value of suction. Both conceptual loci are illustrated in Figure 2, which shows the experimental LC and ATS curves assessed from a series of suction-controlled triaxial TX and TTX testing on compacted SP-SC soil: poorly graded clayey sand.

The LC locus on the positive $p,s$ quadrant was obtained from a series of hydrostatic compression tests conducted on statically compacted samples (cylindrical or cubical) of SP-SC soil under sustained matric suction states of 50, 100, 200, or 350 kPa. In each case, pore-fluid equalization was followed by a ramped consolidation from an initial net mean stress of 50 kPa to a final net mean stress of 600 kPa. The experimental data points in Figure 2 represent values of preconsolidation pressures obtained for each corresponding matric suction using either testing device. Likewise, the ATS locus on the negative $p,s$ quadrant was obtained from a short series of conventional triaxial compression tests, and the corresponding critical state lines, conducted on statically compacted samples (cylindrical or cubical) of the same SP-SC soil and for the same range of sustained matric suction states (50 - 350 kPa).

Results strongly suggest the existence of a non-linear ATS locus, independent of the testing method: triaxial or true triaxial. The apparent tensile strength is indeed expected to reach a constant value (plateau) at significantly higher suction states ($s \to \infty$), in a similar fashion as the original BBM formulation for preconsolidation pressures on the LC locus (Sheng et al. 2008), as shown in Figure 2. Even more suggestive, however, is the virtually identical response of SP-SC soil from suction-controlled TX and TTX testing, regardless of the level of induced matric suction and despite potential inconsistencies in the boundary conditions imposed on the test samples. It is worth mentioning that the Silastic Type J-RTV cubical membranes proved to yield best possible performance, especially in terms of minimal stress concentration at their corners (e.g., Sture & Desai, 1979; Janoo, 1986). These results serve as a contribution to what it might become a preliminary basis for the substantiation of test results obtained from one method, when implementing the axis-translation technique, based on results from the other.

2.3 Suction-controlled plane strain (PS) testing

A full set of critical state lines (CSLs) obtained from suction-controlled PS testing on compacted ML soil (low compressibility silt) is shown in Figure 3. The material is an artificially mixed soil, made of predominantly silty sand and kaolin clay. A typical specimen, as shown in Figure 1c, was prepared by uniaxial consolidation of a slurry mixture made of 75% fine sand and 25% kaolin. The CSLs were obtained from a series of conventional triaxial compression tests carried out on identically prepared cuboid specimens of ML soil under sustained matric suction states of 50, 75, or 100 kPa, and initial values of net mean stress of 75, 100, and 200 kPa.
Figure 3 shows the effect of increasing net normal stress on the position and slope of the CSLs of ML soil as projected onto the deviatoric stress vs. matric suction plane. Although not definitively conclusive, given the limited set of experimental datapoints, results clearly suggest an increasingly nonlinear CSL with increasing net normal stress, even under suction-controlled PS conditions. This can be directly attributed to the already (and thoroughly) demonstrated correspondence between the nonlinear nature of shear strength envelopes for most unsaturated soils, with increasing matric suction, and their respective soil-water characteristic curve (e.g., Vanapalli et al., 1996; Lu & Likos, 2004).

Within the regime of relatively low suction, and prior to the air-entry pressure, the soil pores remain essentially saturated, and the shear strength envelope is reasonably linear. As the soil becomes unsaturated, the reduction in the pore-water volume within this regime effectively reduces the contribution of matric suction toward shear strength. This effect becomes more readily apparent as the net normal stress increases, as effectively shown in Figure 3. It is worth noting that the air-entry value of the ML soil used in this work is approximately 75 kPa, beyond which the nonlinearity of the CSLs becomes increasingly manifest.

### 2.4 Suction-controlled ring shear (RS) testing

Peak and residual failure envelopes obtained from suction-controlled RS testing on compacted CL soil (low plasticity clay) are shown in Figure 4. The soil consists of 18% sand, 50% silt, and 32% clay. A typical RS specimen was prepared by statically compacting the loose soil-water mixture directly into the bottom annular platen of the RS apparatus, as featured in Figure 1d, via an upper annular platen. The upper annular platen itself features a full set of coarse porous stones for simultaneous pore-air pressure control in the RS specimen, thus allowing for implementation of the axis-translation technique.

The assessment of peak and residual shear strength of compacted CL soil was accomplished under constant matric suction states ranging from 25 to 200 kPa, for a net normal stress of 100 kPa, which was attained via the upper annular platen during the in-place static compaction process. Once the soil attained at least 90% consolidation, a suction-controlled shearing was carried out at an average rotational speed of 0.023°/min, corresponding to an equivalent horizontal displacement rate of 0.025 mm/min. Shearing was terminated when it was readily apparent that a residual stress had been reached. This shearing rate is slightly lower than that reported in previous works where higher total suction values (up to 100 MPa) were induced via relative-humidity based techniques (e.g., Infante-Sedano et al., 2007).

Figure 4 shows the effect of matric suction on both peak and residual failure envelopes of CL soil as projected onto the shear stress vs. matric suction plane. The patterns are strikingly similar to those observed in the experimental CSLs obtained for compacted ML soil from suction-controlled PS testing (Figure 3), namely: (1) Both envelopes remain essentially linear within a range of relatively low values of suction, and (2) The nonlinearity of either envelope becoming more manifest with increasing suction beyond a threshold value. It is also worth noting that the air-entry value of the CL soil used in this work is approximately 100 kPa.

As has been demonstrated from extensive suction-controlled direct shear and triaxial testing on a wide variety of unsaturated soils, within the regime of relatively low suction, and prior to the air-entry pressure, the soil pores remain essentially saturated, the shear strength envelope is reasonably linear, and the beta angle ($\phi^\beta$) with respect to suction remains effectively equal to the friction angle ($\phi'$), as it is readily observed in Figure 4. Results also suggest a more pronounced (and detrimental) effect of increasing matric suction on the residual shear strength of compacted clay soil, as compared to that on its peak shear strength.

### 3. Thermo-hydro-mechanical behavior: recent evidence

#### 3.1 Experimental program and scope

A considerable portion of the increasingly ubiquitous geothermal infrastructure across the world is located in traditionally earthquake prone areas while being supported by soil deposits well above the phreatic surface. To date, however, there is hardly any comprehensive study at the laboratory scale that had focused on a thorough characterization of strength-stiffness properties of unsaturated cohesive-frictional soils over a relatively large range of shear strain amplitudes (0.001% to 0.1%) and under simultaneous thermal conditioning of the pore fluids, which constitutes a first chief motivation under this second section of the work (Thermo-hydro-mechanical Behavior).
Cyclic triaxial testing has proved to be a potentially suitable approach to studying the effect of elevated temperatures on unsaturated soil response under repeated loading, particularly the resilient modulus $M_R$ (e.g., Ng & Zhou, 2014). The non-destructive resonant column (RC) technique, however, has proved to be sufficiently reliable for similar purposes, particularly the assessment of shear-wave velocity, shear modulus, and material damping, and is relatively inexpensive compared to most cyclic triaxial or cyclic simple shear apparatuses. Furthermore, most conventional devices are not suitable to capture small-strain behavior adequately and hence vastly underestimate soil stiffness.

In this work, an existing RC apparatus has been upgraded in order to investigate the dynamic response of compacted clayey soil via thermo-controlled, constant-water content RC testing, with particular emphasis on confinement/moisture/thermal effects on frequency response curves and cyclic shear stress vs. shear strain hysteresis loops. (Results are summarized in the following subsection.) The upgraded RC apparatus, as shown in Figure 5a, features the following key items: (1) Bottom base plate, (2) Base pedestal with optional 5-bar ceramics, (3) Omega Type K thermocouple, and (4) Cyclic torque actuator with Model SR-DF-FO-250 fiber-optics proximitor probe. A Type HTC-250 digital convection heater, featuring an internal fan, was also adapted to the main cell of the RC device for thermal conditioning of the soil sample, ranging from 20 to 60 degrees Celsius. A typical cylindrical specimen has a 70 mm (2.75 in) diameter and a 150 mm (5.9 in) height. A detailed description of the main components and performance verification and calibration testing is presented by Davoodi-Bilesavar (2020).

One last chief motivation under this section of the work was the assessment of residual failure envelopes from suction/thermo-controlled ring shear (RS) testing on clayey soil (results are summarized in a subsequent subsection). Drilled shaft foundations above ground water table which are also used as part of ground-source heat pump systems, hence referred to as energy foundations, have proved to be a promising technology for increasing the energy efficiency of heating-cooling systems. Thermally induced changes in shear strength of surrounding soils may have a significant impact on some of their key design parameters, including side shear resistance, and therefore the overall performance of these foundations (e.g., Cekerevac & Laloui, 2004; Uchaipichat & Khalili, 2009; Alsherif & McCartney, 2016; Cheng et al., 2017). To date, however, there is hardly any comprehensive study at the laboratory scale that has focused on a thorough characterization of shear strength behavior of soils, from peak to residual, over a wider range of shear deformations and simultaneous thermal conditioning of the pore-fluids.

In this work, suction/thermo-controlled RS testing was carried out in an upgraded version of the suction-controlled RS apparatus illustrated in Figure 1d, thus featuring all of the essential items for implementing the axis-translation technique. The upgraded version, however, as shown in Figure 5b, features the additional key items: (1) Thermal band, wrapping...
bottom annular platen; (2) Heating lead wire, connecting to a convection heater; and (3) Clayey soil specimen failed under elevated temperature. A Type HTC-250 digital convection heater, featuring an internal fan, was also adapted to the main cell of the RS device for thermal conditioning of the soil, ranging from 20 to 60°C. The upper annular platen, which seats right on top of the specimen, features an Omega Type K thermocouple for direct measurements of soil temperature induced by the featured thermal band around the bottom annular platen. The typical ring-shaped specimen featured the same dimensions as those illustrated in Figure 1d, namely, a 152.4 mm (6 in) outer diameter, a 96.5 mm (3.8 in) inner diameter, and an average thickness of 15.0 mm (0.6 in). A detailed description of its main components and performance verification is presented by Yepes (2015).

3.2 Thermo-controlled resonant column (RC) testing

Typical frequency response curves (FRCs) obtained from thermo-controlled RC testing on a sample of CL soil compacted at optimum moisture content (w = 13.6%) are shown in Figure 6. The soil consists of 20% sand, 28% silt, and 52% clay, thus classifying as low plasticity clay as per the USCS. Likewise, typical FRCs obtained from a CL soil sample compacted on wet side of optimum (w = 17%) are shown in Figure 7. In both cases, the test sample was subjected to an identical multistage stress/thermal history prior to RC testing, summarized as follows.

The sample was first allowed to consolidate for at least 24 hrs under a 5 psi (34.5 kPa) confinement, after which the convection heater was set to room temperature (25°C) and thermal conditioning of the soil allowed for at least 3 hours. A first FRC was generated by sweeping the input-torque frequency range, typically between 100-280 Hz, with a 0.1 kN-m magnitude cyclic torque. The temperature was then gradually increase to 60°C, under constant 5 psi (34.5 kPa) confinement, and thermal conditioning allowed for another 3 hours, after which a new FRC was generated. The convection heater was then shut off and the soil allowed to cool down back to room temperature (25°C).

After soil cooling, and given the non-destructive nature of the RC technique for cyclic torques of 0.1 kN-m magnitude or less, the confinement was raised to 20 psi (138 kPa) and the sample subjected to the same thermal history prior to further RC testing. After the sample was taken apart, the moisture content was measured at its middle, top, and bottom to assess the amount of water loss during cyclic thermal loading. No significant water loss was observed in any of the test samples (less than 1% loss on average), thus rendering all trials as constant-water content RC tests.

The RC test essentially involves a soil column with fixed-free end conditions that is excited to vibrate in one of its natural modes. The FRCs shown in Figure 6a were generated upon thermal conditioning of the test soil to a temperature of 25°C. Results corroborate the critical influence that the level of confinement has been observed to have over a soil column under resonance, with a significant rightward shift of the FRC under higher confinement of 20 psi (138 kPa), which can be directly attributed to the ensuing increase in rigidity (stiffness) of the soil skeleton, and hence the resonant frequency. Conversely (and consequently), the maximum shear strain amplitude induced at resonance by the 0.1 kN-m torque is observed to decrease with increasing confining pressure.

Likewise, the FRCs shown in Figure 6b were generated upon thermal conditioning of the test soil to a temperature of 60°C. Results corroborate the general behavioral trends noted for 25°C, namely, a significant rightward shift of the FRC under higher confinement, and thus lower shear strain amplitude at resonance. However, RC testing under 60°C is observed to yield lower values of resonant frequencies (Hz) and concomitantly higher values of shear strain amplitudes (%) at resonance, as compared to those generated from RC testing under 25°C. This is strongly indicative of an increasingly detrimental effect of rising temperatures on soil stiffness, a behavioral trend that appears to substantiate findings from previous work related to thermal effects on shear strength properties of compacted clayey soils (e.g., Cekerevac & Laloui, 2004; Alsherif & McCartney, 2016).

Figure 6. Frequency response curves (FRCs) from thermo-controlled RC testing on CL soil compacted at optimum moisture (w = 13.6%): (a) Temperature = 25°C, (b) Temperature = 60°C.
It is worth restating that all the FRCs shown in Figure 6 correspond to CL soil compacted at optimum gravimetric moisture content ($w = 13.6\%$), which corresponds to a matric suction of 400 kPa, as per the soil-water characteristic curve (Davoodi-Bilesavar, 2020). The intent was to induce an initial suction state beyond the air-entry value of the soil (60 kPa) and well into the drying loop of the SWCC. On the other hand, all the FRCs shown in Figure 7 correspond to CL soil compacted on wet side of optimum ($w = 17\%$), which now corresponds to a lower matric suction of 40 kPa. In this latter case, the intent was to induce an initial matric suction just below the air-entry value of the soil, and the resulting FRCs virtually replicate the same trends observed in Figure 6 for CL soil compacted at optimum moisture.

The set of FRCs shown in Figure 7, however, appears to suggest a far less detrimental effect of rising temperatures on soil stiffness, that is, for clayey soils compacted at relatively high moisture contents, and hence initial matric suctions below their air-entry value. This is substantiated by the minimal decrease observed in resonant frequencies, and thus minimal increase in shear strain amplitudes, from 25 to 60°C, which is also in conformity with findings from previous work related to thermal effects on shear strength behavior.

The influence of the initial (compaction induced) matric suction can be readily assessed by comparing the main RC test outputs in Figures 6 and 7, respectively. For instance, for 5 psi (34.5 kPa) confinement and 25°C temperature, the output resonant frequency experiences a rather drastic decrease, from 208 to 128 Hz, as the compaction induced suction decreases almost tenfold from 400 kPa ($w = 13.6\%$) to 40 kPa ($w = 17\%$). Likewise, for 60°C temperature, the output resonant frequency decreases from 190 to 126 Hz for the same difference in compaction induced matric suctions. The same trends can be observed for higher confinements (20 psi) and also in terms of both resonant frequencies and shear strain amplitudes.

The main focus of this subsection of the work has been on small-strain stiffness of compacted CL soil. It is well known, however, that the cyclic behavior of soils is nonlinear and hysteretic; consequently, the shear modulus and damping are heavily shear strain dependent. It is hence worth mentioning that the applied 0.1 kN-m torque proved to yield shear strain amplitudes far below a threshold limit (0.01%), thus rendering the CL soil response from all RC testing as low-amplitude or purely linear (a more thorough assessment of shear moduli, damping ratios, and threshold strain amplitudes is far beyond the intended scope of the present work).

Figures 8a and 8b show the cyclic hysteretic stress-strain loops obtained from two additional, identically prepared specimens of CL soil under temperatures of 25 and 60°C, respectively. The loops were both generated under a 5 psi (34.5 kPa) confinement, with the specimens compacted on wet side of optimum ($w = 17\%$), yielding initial suctions of 40 kPa and 42 kPa (verified via filter paper), respectively. In both cases, the frequency of the applied torque was set to the same value corresponding to resonance in Figures 7a and 7b, respectively.

It can be readily observed that the range of maximum shear strain amplitudes generated in the loops (0.002% - 0.003%) is reasonably close to that observed in the corresponding FRCs (Figure 7). Moreover, a slightly degraded secant modulus (slope of hysteretic loop) is observed with increasing temperature, which is consistent with the general trends emerging from Figures 6 and 7. On the other hand, equivalent viscous damping can be evaluated from the area enclosed by the hysteretic loops (e.g., Hardin & Drnevich, 1972; Prakash, 1981; Kramer, 1996). The results hence confirm the inverse proportionality that exist between shear modulus and damping, even under thermally controlled conditions, that is, larger areas enclosed by the loops, and larger shear strain amplitudes induced by the same cyclic shear stress, with increasing temperature.
3.3 Suction/thermo-controlled ring shear (RS) testing

The residual failure envelope obtained from suction/thermo-controlled RS testing on three identically prepared samples of compacted CL soil (low plasticity clay) is shown in Figure 9. The soil is virtually identical to the test material used for generating the peak and residual failure envelopes shown in Figure 4 via suction-controlled RS testing under isothermal conditions. In this case, however, each RS sample was subjected to a multistage stress/suction/thermal history prior to shearing, summarized as follows.

The RS specimen was prepared by statically compacting the loose soil-water mixture directly into the bottom annular platen of the RS apparatus via the upper annular platen. The sample was first allowed to consolidate under a vertical stress of 100 kPa. Once the soil attained at least 90% consolidation, the pore-air pressure was gradually increased to a target value and the pore-fluids allowed to come under equilibrium (equalization stage) under a matric suction state of 50 kPa via the axis-translation technique. The convection heater was then set to a constant temperature of either 20, 30, or 40°C, and thermal conditioning of the soil allowed for at least 3 hours via the thermal band wrapped around the bottom annular platen, as illustrated in Figure 5b. The soil temperature was thoroughly verified by the Omega Type K thermocouple adapted to the upper annular platen.

Upon complete stress/suction/thermal conditioning of the soil, and for procedural consistency purposes, each suction/thermo-controlled RS testing was conducted at the same average rotational speed of 0.023°/min, with the shearing phase terminated when it was readily apparent that a residual stress had been reached. The convection heater was finally shut off and the soil allowed to cool down back to room temperature (approximately 25°C).

For one particular set of experimental variables, namely, net normal stress of 100 kPa, matric suction of 50 kPa, and soil temperature of 25°C, the resulting residual failure envelope, as shown in Figure 9, yields a value of residual shear strength of approximately 35 kPa, which is reasonably close to that generated from suction-controlled RS testing on same CL soil, as shown in Figure 4. This serves as further evidence of the potential suitability of the upgraded RS device for thermo-controlled testing. More importantly, however, the resulting envelope appears to support the notion of a “threshold” temperature beyond which the residual shear strength of compacted clayey soil experiences an increasingly detrimental effect with increasing temperature, very much like the effect of increasing matric suction on a would-be “residual” beta angle (ϕ′), as shown in Figure 4.

4. Concluding remarks

As mentioned earlier as the present work was being introduced in its intent and scope, the main objective was to synthesize some of the most recent experimental evidence of thermo-hydro-mechanical behavior of cohesive-frictional soils over a whole range of suction- and/or thermo-controlled stress paths and modes of deformation, with particular
emphasis on essential features of unsaturated soil behavior that are yet to be thoroughly investigated in the literature.

The virtually identical response of SP-SC soil from suction-controlled TX and TTX testing, in terms of both LC and ATS loci; the nonlinearity of CSLs of ML soil with increasing matric suction, even under suction-controlled PS conditions; the similar patterns observed in peak and residual failure envelopes of CL soil from suction-controlled RS testing; the increasingly detrimental effect of rising temperatures on CL soil stiffness via thermo-controlled RC testing; the inverse proportionality observed between shear modulus and damping under thermally controlled conditions, as evidenced by cyclic hysteretic loops; and finally, the evidence of a “threshold” temperature beyond which the residual shear strength of CL soil decreases progressively with increasing temperature stand as the most enlightening features of unsaturated soil behavior that the present work has gained further insights on.

In a far more general sense, however, the results from the relatively short series of triaxial, true triaxial, plane strain, ring shear, and resonant column tests accomplished in this work underscore the crucial importance of keeping a research focus on expanding and upgrading the capabilities of standardized testing techniques for a thorough and proper characterization of the engineering response of unsaturated soil materials, as well as the significant amount of room still available for further investigative efforts.

The present experimental effort would have been ideally accomplished on the same type of soil in order to facilitate the analysis and comparison of behavioral trends assessed from the different testing methods. However, each individual soil type, and the corresponding experimental program, was investigated under the auspices of different sponsors and programs based on proposals pursuing a wide range of research ideas in the fascinating subject of unsaturated soil mechanics.

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Authors’ contributions

Laureano R. Hoyos: conceptualization and original draft preparation. Roya Davoodi-Bilesavar: resonant column testing. Ujwalkumar D. Patil: triaxial testing. Jairo E. Yepes: ring shear testing. Diego D. Pérez-Ruiz: true triaxial testing. José A. Cruz: plane strain testing.

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