A comparative study of empirical formulas for gamma-ray dose build-up factor in iron and lead materials

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Abstract. To analyse the difference between different empirical formulas for calculating gamma-ray dose build-up factors, we compared the gamma-ray dose build-up factors calculated according to Taylor’s formula and Berger’s formula for iron and lead materials. The results show that the differences between the two formulas are related to energies of gamma-ray, shield thicknesses and types of shielding materials. When the energy of gamma-ray and thickness of shield are determined, the gamma-ray dose build-up factor calculated by Taylor’s formula is higher than that by Berger’s formula for iron material. For lead material, the gamma-ray dose build-up factor calculated by Taylor’s formula is lower than that by Berger’s formula. Generally speaking, the higher the gamma-ray energy and the thicker the shield material, the greater the difference between the gamma-ray dose build-up factors calculated according to the two empirical formulas.

1. Introduction

The main ways of external irradiation protection of gamma-rays are to control the exposure time, to increase the distance between object and the radiation source, or to use shielding [1-2]. Among them, shielding protection is the most widely used method in many fields of nuclear engineering and nuclear technology research [3-4]. In the design of gamma-ray shielding, the build-up factor of gamma-rays is an important factor to be considered because the gamma-rays in practice are mostly wide beams, not narrow beams [5-6].

There are many factors which may influence the gamma-ray build-up factor, including energy of gamma-ray, type of shielding material, geometric size of shielding layer, distance between radiation source and shielding body, etc. [7-9]. The commonly used methods to obtain accurate gamma-ray build-up factor are Monte Carlo method, cross-section method and moment method [10-12]. The numerical calculation method such as Monte Carlo method and moment method are not only complicated but also time-consuming.

To facilitate the practical use, the researchers summarized and presented a variety of different forms of build-up factors, and also tabulated the calculation result of the build-up factors [10]. Even for a same shielding material, the results obtained according to the various build-up factor expressions are different [13-14]. In this paper, for the most commonly used gamma-ray shielding materials, iron and lead, the differences between the gamma-ray dose build-up factors calculated by Taylor’s formula and Berger’s formula are compared and analyzed. The results can provide a reference for the design of gamma-ray shielding.
2. Calculation model and method

As shown in figure 1, in addition to the direct gamma-rays that don’t interact with the shield, scattered photons may also travel through the shield to reach the detection point.

The attenuation law of the isotropic wide beam gamma-rays in shielding materials is as follows.

\[ I = I_0 \exp(-\mu R) \]  

(1)

Where, \( I_0 \) and \( I \) are the gamma-ray intensity at the detection point before and after the shielding body is set, respectively; \( \mu \) is the line attenuation coefficient of the shielding material; \( R \) is the thickness of the shielding body; \( B \) is the build-up factor of gamma-ray.

In this paper, we mainly consider the dose build-up factor. Assuming that the dose produced by scattering gamma-rays in the shielding process is \( \phi_s \), and the dose produced by direct gamma-rays is \( \phi_d \), so the expression of the dose build-up factor \( B \) is

\[ B = 1 + \phi_s / \phi_d \]  

(2)

Both Taylor’s formula and Berger’s formula can give the gamma-ray dose build-up factor of an infinitely large uniform shield. The empirical formula for calculating build-up factor by Taylor’s formula is

\[ B_T(E,\mu R) = A_0 \exp(-A_1\mu R) + (1 - A_0) \exp(-A_2\mu R) \]  

(3)

Among them, \( B_T \) is the build-up factor calculated by Taylor’s formula, \( \mu \) is the line attenuation coefficient of the shielding material, in the unit of cm\(^{-1}\); \( R \) is the thickness of the shield, in the unit of cm. \( A_0, A_1 \) and \( A_2 \) are the gamma-ray energy dependent fitting parameters.

The empirical formula for calculating build-up factor by Berger’s formula is

\[ B_B(E,\mu R) = 1 + B_1\mu R \exp(B_2\mu R) \]  

(4)

Among them, \( B_0 \) is the build-up factor calculated by Berger’s formula; \( B_1 \) and \( B_2 \) are the gamma-ray energy dependent fitting parameters.

For gamma-rays with different energies, the fitting parameters in empirical formulas (3) and (4) are shown in table 1 and table 2 [14].

| Table 1. Fitting parameters in Taylor’s and Berger’s formula (for iron material) [14]. |
| --- |
| \( E /\text{MeV} \) | Fitting parameters in Taylor’s formula | Fitting parameters in Berger’s formula |
| --- | --- | --- |
| \( A_0 \) | \( A_1 \) | \( A_2 \) | \( B_1 \) | \( B_2 \) |
| 0.5 | 31.379 | -0.0684 | -0.03742 | 0.9081 | 0.0752 |
| 1.0 | 24.957 | -0.0609 | -0.02463 | 0.8214 | 0.0684 |
| 2.0 | 17.622 | -0.0460 | -0.00526 | 0.7020 | 0.0319 |
| 3.0 | 13.218 | -0.0443 | -0.00087 | 0.5323 | 0.0384 |
| 4.0 | 9.624 | -0.0470 | 0.00175 | 0.4366 | 0.0358 |
| 6.0 | 5.867 | -0.0615 | -0.00186 | 0.3721 | 0.0457 |
Table 2. Fitting parameters in Taylor’s and Berger’s formula (for lead material) [14].

| $E$ /MeV | $A_0$ | $A_1$ | $A_2$ | $B_1$ | $B_2$ |
|---------|-------|-------|-------|-------|-------|
| 0.5     | 1.677 | -0.03084 | 0.30941 | 0.2526 | -0.0848 |
| 1.0     | 2.984 | -0.03503 | 0.13486 | 0.3779 | -0.0403 |
| 2.0     | 5.421 | -0.03482 | 0.04379 | 0.3862 | 0.0032 |
| 3.0     | 5.580 | -0.05422 | 0.00611 | 0.3267 | 0.0253 |
| 4.0     | 3.897 | -0.08468 | 0.02383 | 0.2530 | 0.0547 |
| 6.0     | 0.926 | -0.17860 | -0.04635 | 0.1622 | 0.1027 |
| 8.0     | 0.368 | -0.23691 | -0.05684 | 0.1220 | 0.1112 |
| 10.0    | 0.311 | -0.24024 | -0.02783 | 0.0939 | 0.1167 |

3. Simulation results and discussion

As shown in table 3, the mass attenuation coefficients and mean free paths of gamma-rays with different energies in iron and lead are obtained from NIST. Here, the density of iron is $\rho_{Fe} = 7.8 \text{ g/cm}^3$; the density of lead is $\rho_{Pb} = 11.34 \text{ g/cm}^3$; $E_\gamma$ is the energy of gamma-rays; $\mu/\rho$ is the mass attenuation coefficient, and $\lambda$ is the mean free path. In this paper, we consider the difference between the calculated results of the two empirical formulas in the range of 0–20 times of mean free path thickness.

Table 3. The gamma attenuation coefficient and mean free path in iron and lead.

| $E_\gamma$ /MeV | Iron | Lead |
|-----------------|------|------|
|                 | $\mu/\rho_{Fe}/10^{2}\text{cm}^2/\text{g}$ | $\mu/\text{cm}^2$ | $\lambda/\text{cm}$ | $\mu/\rho_{Pb}/10^{2}\text{cm}^2/\text{g}$ | $\mu/\text{cm}^2$ | $\lambda/\text{cm}$ |
| 0.5             | 8.414 | 0.656 | 1.524 | 16.14 | 1.83 | 0.546 |
| 1               | 5.995 | 0.468 | 2.139 | 7.102 | 0.805 | 1.242 |
| 2               | 4.265 | 0.333 | 3.006 | 4.606 | 0.522 | 1.915 |
| 3               | 3.621 | 0.282 | 3.541 | 4.234 | 0.48 | 2.083 |
| 4               | 3.312 | 0.258 | 3.871 | 4.197 | 0.476 | 2.101 |
| 6               | 3.057 | 0.238 | 4.194 | 4.391 | 0.498 | 2.008 |
| 8               | 2.991 | 0.233 | 4.286 | 4.675 | 0.53 | 1.886 |
| 10              | 2.994 | 0.234 | 4.282 | 4.972 | 0.564 | 1.774 |

3.1. Gamma-ray dose build-up factor for iron material

Figure 2 are the curves of the gamma-ray dose build-up factor with the thickness of the iron shield calculated according to the two different empirical formulas. Figure 2(a) shows that when the gamma-ray energy is determined, the gamma-ray dose build-up factor increases with the increase of the thickness of the iron shield, and the growth rate of the gamma-ray dose build-up factor gradually slows down as the thickness of the shield increasing; when the thickness of the iron shield is determined, the higher the gamma-ray energy, the smaller the gamma-ray dose build-up factor. Because the number of fitting parameters in Taylor’s formula is more than that in Berger’s formula, it is generally believed that the fitting result of Taylor’s formula is more accurate than that of Berger’s formula. The relative difference $C$ between the fitting results of Berger’s formula and Taylor’s formula is given as follows.

$$C = (B_\text{B} - B_\text{T}) / B_\text{T}$$

As can be seen from figure 2(b), generally speaking, for iron materials, the results calculated by Berger’s formula are higher than those calculated by Taylor’s formula, and the percentage of relative deviation increases with the increase of shielding material thickness. When the thickness of iron material is less than six times of the mean free path of gamma-ray, the relative deviation is less than 10%. When the gamma-ray energy is 8 MeV or 10 MeV, the results calculated by the two empirical...
formulas are closer. The value of the gamma-ray dose build-up factor directly affects the selection of shielding material thickness in radiation shielding design. For iron material, the gamma-ray dose build-up factor calculated by Berger’s formula is higher than that by Taylor’s formula. Therefore, when iron is used to shield gamma-rays, the thickness of iron required by Taylor’s formula is less than that by Berger’s formula.

\[
\frac{B_f - B_T}{B_T}
\]

\[\text{Thickness of shield, } \mu_R\]  

**Figure 2.** Comparison of gamma-ray dose build-up factors calculated by two formulas for iron.

### 3.2. Gamma-ray dose build-up factor for lead material

Figure 3 is the results of gamma-ray dose build-up factor for lead calculated by the two empirical formulas. As can be seen from figure 3(a), the gamma-ray dose build-up factor for lead shield is generally lower than that for iron shield, which indicates that the gamma-ray shielding performance of lead shield is better than iron. The 0.5 MeV gamma-ray dose build-up factors for lead shield with a thickness of 0-20 times of mean free paths are less than 2. When the lead shield thickness exceeds 8 \(\mu_R\), the gamma-ray dose build-up factor does not increase significantly with the increase of lead shield thickness. When the gamma-ray energy is in the range of 2 MeV to 10 MeV and the lead shield thickness is determined, the dose build-up factors of gamma-rays with different energies are close.

\[
\frac{B_f - B_T}{B_T}
\]

\[\text{Thickness of shield, } \mu_R\]  

**Figure 3.** Comparison of gamma-ray dose build-up factors calculated by two formulas for lead.

**Figure 3(b) is the relative deviation between results of the gamma-ray dose build-up factor calculated by Berger’s formula and Taylor’s formula for lead.** As can be seen from figure 3(b), the dose build-up factor calculated by Berger’s formula is generally smaller than that by Taylor’s formula for lead. When the thickness of lead is less than 8\(\mu_R\), the deviation between the results calculated by the two empirical formulas is less than 5% except for gamma-ray with energy of 4 MeV, and the relative deviation increases with the thickness of lead except for a few energies. As the energies of gamma-ray are between 2 MeV and 10 MeV, the relative deviation between the calculated results by Berger’s formula and Taylor’s formula increases with the increase of gamma-ray energy except for 4
MeV energy. For the shield with a certain thickness, the larger the gamma-ray dose build-up factor, the worse the gamma-ray shielding performance. Therefore, when lead is used to shield gamma-rays, the thickness of lead required by Taylor's formula is higher than that by Berger's formula.

4. Conclusions
Both Taylor’s formula and Berger’s formula can be used to calculate the dose build-up factor of isotropic gamma-ray point source and infinite shield. Because of the different fitting forms and different numbers of fitting parameters, there is a difference between the two formulas for calculating the gamma-ray dose build-up factor for the same situation. The comparative study results show that the build-up factors of gamma-ray dose calculated by Taylor’s formula and Berger’s formula both increase with the increase of shield thickness, and the difference of build-up factor calculated by the two formulas is affected by gamma-ray energy, shield thickness and type of shield material. For iron material, the build-up factor of gamma-ray dose calculated by Berger’s formula is higher than that calculated by Taylor’s formula; and generally speaking, the lower the gamma-ray energy, the greater the difference between the calculated results of the two formulas. For lead material, the gamma-ray dose build-up factor calculated by Taylor’s formula are lower than that calculated by Berger’s formula, and generally speaking, the higher the energy of gamma-rays, the greater the difference between the results calculated by the two empirical formulas. The thicker the shielding material is, the greater the difference between the calculated results of the two empirical formulas.

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