Controllable Persistent Atom Current of Bose-Einstein Condensates in an Optical Lattice Ring

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Abstract: In this paper the macroscopic quantum states of Bose-Einstein condensates in optical lattices is studied by solving the periodic Gross-Pitaevskii equation in one-dimensional geometry. It is shown that an exact solution seen to be a travelling wave of excited macroscopic quantum states resultes in a persistent atom current which can be controlled by adjusting of the barrier height of the optical periodic potential. A critical condition to generate the travelling wave is demonstrated and we moreover propose a practical experiment to realize the persistent atom current in a toroidal atom waveguide.

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Dramatic features of fundamental and applied importance for Bose-Einstein condensates (BECs) loaded in a periodic potential have attracted much interest recently where the periodic potential is created by a laser standing wave known as optical lattice. For example the direct observation of an oscillating atomic current in a one-dimensional array of Josephson junctions is realized with atomic BECs trapped in the valleys of the periodic potential and weakly coupled by the interwell barriers.[1,2] The experimental realization of BECs of weakly interacting atoms has provided a route to study atom current in a controllable and tunable environment. The possibility of loading a BEC in a one-dimensional periodic potential

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admits the observation of quantum phase effects on a macroscopic scale such as quantum interference and the study of superfluidity on a local scale. The superfluid array may allow investigation of phenomena so far inaccessible to superconducting Josephson junctions and lays a bridge between the condensate dynamics and the physics of discrete nonlinear media.

Moreover the BEC trapped in an optical lattice exhibits a novel feature that is the quantum phase transition between Mott-insulator and superfluid. As a matter of fact if the corrugated potential is shallow enough that tunnelling establishes phase coherence across the array the atomic gas of bosons in BEC can be kept in the superfluid phase as long as the atom-atom interactions are small comparing with the tunnel coupling. In this regime kinetic energy is dominant in total energy of the boson system. With increase of the potential depth of the optical lattice to a level that suppresses the transport between adjacent wells the system attends an insulator phase above a critical value of the potential depth. In this case the phase coherence is absent and the number of boson atoms in each lattice site becomes the same. The system then possesses a Mott-insulator behavior. Theoretically the equation of motion for the system which realizes the array of weakly coupled condensates satisfies a discrete nonlinear Schrödinger equation (NSE). Various approaches have been proposed to understand theoretically the quantum phase transition and to determine phase diagram as a function of BEC parameters.

The soliton solution of Gross-Pitaevskii (GP) equation (or NSE) describing a continuous BEC in the mean field approximation is also studied. It is well known that the periodic potential in NSE allows to stabilize bright solitons which are not admitted for defocusing nonlinear system. The formation of bright solitons of the NSE has been recently demonstrated for repulsive BECs in optical lattices. The soliton solutions of NSE with periodic potential correspond to localized states of excitations with energies inside the gaps of underlying linear band structure called gap solitons. On other hand a new family of stationary solutions of NSE with periodic potential which is not in the tight-binding regime has been constructed by Bronski, Carr, Deconink and Kutz (BCDK) and is shown to model a quasi-one-dimensional dilute gas BEC trapped in a standing light wave. The BCDK so-
olution is interesting and fruitful of physics. In this paper we following BCDK\cite{12} re-study the exact solution of the NSE for BEC in an optical periodic potential for both repulsive and attractive interatom collisions. It is observed that the solution is seen to be a travelling wave and leads to a persistent atom current. Since the periodic solution in one-dimensional space with spatial period $L$ is equivalent to the solution in a ring of circumstance $L$ the steady current can be realized in an optical lattice ring.

We consider a BEC confined by a cylindrically symmetric harmonic magnetic trap with an optical periodic potential in the axial direction. The normalized GP equation or more generally the NSE for BEC with the motion in the radial direction essentially frozen\cite{13,14} to the ground state of the harmonic trap is seen as:\cite{15}

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + g |\psi|^2 \psi + V(x)\psi, \quad (1)$$

where the coupling constant of two-atom interaction is given by $g = \frac{2\hbar^2 a}{ml_0^2}$ \cite{16,17} with $l_0 \equiv \sqrt{\hbar/m\omega_0}$ denoting the characteristic length extension of the ground state wave function of harmonic oscillator. $m$ is the mass of the atoms and $a$ is the $s$-wave scattering length. The confinement along the radial direction is so tight that the trap energy with frequency $\omega_0$ along the radial direction is much greater than the mean field interaction energy. The optical periodic potential is written as

$$V(x) = V_0 \sin^2 (k_L x), \quad (2)$$

with $V_0$ denoting the height of potential barrier, where $k_L = 2\pi/\lambda$ is the wave vector of the laser light and $\lambda$ is the wavelength. The corresponding lattice period is $d = \lambda/2$. For the case of weakly coupled condensates the condensate order parameter $\psi$ can be decomposed as a sum of wave functions localized in each well of the periodic potential (tight binding approximation) with the assumption relying on the fact that the height of the interwell barrier is much higher than the chemical potential.\cite{1,2} We, however, do not restrict ourself on the low energy case and look for the global condensate wave function of excitation\cite{12} 

$$\psi(x,t) = \phi(x) \exp(-i\mu t/\hbar),$$

where $\mu$ is the chemical potential. Thus the spatial wave function satisfies the stationary NSE that
With the general form of spatial wave function $\phi(x)$ written as \cite{12,18} $\phi(x) = r(x) \exp \left[ i \varphi(x) \right]$, the eq. (3) is separated as real and imaginary equations from which we obtain the first-order differential equation for the phase $\varphi$

$$\varphi'(x) = \frac{\alpha}{r^2(x)}, \quad (4)$$

and the amplitude $r$

$$\left( r r' \right)^2 = \frac{2a}{l_0^2} r^6 - \frac{2m \mu}{\hbar^2} r^4 + \beta r^2 - \alpha^2 + \frac{2m}{\hbar^2} r^2 \int V(x) \, d(r^2), \quad (5)$$

where parameters $\alpha$ and $\beta$ are constants of integration. An exact solution is found as

$$r^2(x) = A \sin^2(k_L x) + B, \quad (6)$$

with the phase determined from the equation given by

$$\tan[\varphi(x)] = \pm \sqrt{1 + \frac{A}{B} \tan(k_L x)}, \quad (7)$$

where the new coming constants of integration $A$ and $B$ along with $\alpha$ and $\beta$ are to be determined. The constant $B$ is obviously the mean amplitude and acts as the dc offset for the number of the condensed atoms.\cite{12}

Substituting the amplitude formula eq. (6) into the eq. (5) the parameters $\alpha$, $A$ and chemical potential $\mu$ are derived as

$$A = -\frac{m V_0 l_0^2}{2a \hbar^2} = -\frac{V_0}{2a \hbar \omega_0}, \quad (8)$$

and $\alpha^2 = k_L^2 B(B + A)$ with the parameter $\beta$ being eliminated. The constant $B$ can be determined from the normalization condition that $N = \int_0^L \left| \psi(x,t) \right|^2 \, dx$, where $N$ denotes the total number of atoms in the cylindrically symmetric magnetic trap, and the result is seen to be

\[4\]
\[ B = n - \frac{A}{2}. \]  

(9)

\( L \) is the length of optical lattice and the wave function is assumed to satisfy the periodic boundary condition such that \( \psi(x) = \psi(x + L) \) which can be fulfilled if the total length is an integer number of \( d \). Here \( n \) denotes the number of atoms per unit length in the BEC of cigar-like shape. Thus the energy spectrum is obtained as

\[ \mu = E_R + 2an\hbar\omega_0 + \frac{V_0}{2}, \]

(10)

where \( E_R = \frac{\hbar^2 k^2}{2m} \) is called the recoil energy of an atom absorbing one of the lattice phonons.\(^{[1]}\) The second term is the energy due to the interaction between atoms which is proportional to the number density of atoms. It is found that the solution of travelling waves is valid only for a condition that the number density of atoms is greater than a critical value i. e.

\[ n \geq n_c, \quad n_c = \frac{|A|}{2} = \frac{V_0}{4|a|\hbar\omega_0} \]

(11)

for both repulsive and attractive interaction between atoms. The value of chemical potential is then confined by \( \mu \geq E_R + V_0 \) for the repulsive interaction and \( \mu \leq E_R \) for attractive interaction for which the energy spectrum becomes negative when the number density of atoms is greater than a value, \( n > \frac{1}{4|a|\hbar\omega_0}(V_0 + 2E_R) \).

The condensate atom-current can be evaluated from the usual definition, \( j = \left( \frac{\hbar}{m} \right) \text{Im} \left( \psi^* \frac{\partial \psi}{\partial x} \right) \),\(^{[19]}\) with the exact wave function and the result is

\[ j = \pm \frac{\hbar k_L}{m} \sqrt{\left( n - \frac{A}{2} \right) \left( n + \frac{A}{2} \right)}, \]

(12)

which is a persistent current existing when the critical condition eq. (11) is satisfied. The atom current can be controlled by adjusting of the barrier height of the periodic potential \( V_0 \). The current increases with the decrease of the barrier height and approaches the asymptotic maximum value \( j_{\text{max}} = \pm \frac{\hbar k_L}{m} \) for both the repulsive and attractive interactions when the barrier height becomes vanishingly small. The current as a function of the the barrier height is plotted in Fig. 1.
In ref. [1], a one-dimensional array of Josephson Junction was realized experimentally with optical lattice for an atomic BEC, where the height of interwell barriers is assumed to be much higher than the chemical potential and therefore the condensate atoms are trapped in the potential wells with a weak coupling between adjacent wells by quantum tunneling. While in this paper, an opposite limit is considered that the chemical potential is higher than the barrier height for the repulsive interaction.

The persistent atom current is a critical macroscopic quantum phenomenon and it is certainly of importance to see whether or not the persistent current can be realized experimentally with the recent progress made on the confinement of atoms in the light-induced\cite{20,21} and magnetic-field-induced\cite{22,23} atom waveguides.\cite{24} To this end we adopt the typical experimental data in ref. [1] for the BEC of $^{87}Rb$ atoms confined by a cylindrically symmetric harmonic magnetic trap with the radial frequency of magnetic trap being $\omega_0 = 2\pi \times 92$ Hz in order to have a quantitative evaluation. A blue detuned laser standing wave of the wavelength $\lambda = 795$ nm is superimposed on the axis of the magnetic trap and hence the cylindrical magnetic trap is divided into an array of traps. The scattering length of $^{87}Rb$ atoms is $a = 5.8$ nm. According to ref. [1] the barrier height of the periodic potential $V_0$ can be varied from 0 to $5E_R$. It turns out that the critical value of atom number density to generate the travelling wave evaluated from the right hand side of eq. (11) is $n_c \sim 2 \times 10^8$/cm for the barrier height chosen as $V_0 = E_R$. This value of atom number density is too high to be achieved in the experiment for the Josephson junction arrays with BEC\cite{1} and therefore the persistent current is blocked in this case (i.e. with the barrier height $V_0 = E_R$). To have the critical condition eq. (11) be satisfied the barrier height ought to be suppressed to an order such that $V_0 \sim 0.01E_R$, thus the critical atom number density, which becomes $n_c \sim 2 \times 10^6$/cm, is seen to be in the same order as the experimental value\cite{1} and the persistent current therefore can be generated practically with the same experimental apparatus described in refs. [1,2]. The persistent current may be observed in a relatively higher potential barrier if a higher value of radial frequency $\omega_0$ of magnetic trap can be attained (the critical number density $n_c$ is decreased with increase of the radial frequency $\omega_0$ seen from
We hence propose an experiment to test the persistent atom current of BEC confined in a toroidal magnetic trap with a blue detuned laser standing wave superimposed on the axis of the toroid. The emergence and variation of the atom current can be observed over adjusting of the barrier height.

In conclusion, we observe that the travelling wave of macroscopic quantum states for BEC in the periodic potential results in a persistent current and demonstrate that the atom current can be controlled only by adjusting of the barrier height. A practical experiment to generate the matter wave in a toroidal waveguide is proposed. The concept and proposed experimental realization of the persistent atom current for BEC may be of fundamental and applied importance.

[1] F.S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, and M. Inguscio, Science 293 (2001) 843.

[2] S. Burger, F.S. Cataliotti, C. Fort, F. Minardi, M. Inguscio, M.L. Chiofalo, and M.P. Tosi, Phys. Rev. Lett. 86 (2001) 4447.

[3] M. Greiner, O. Mandel, T. Esslinger, T.W. Hänisch, and I. Bloch, Nature 415 (2002) 39.

[4] M.P.A. Fisher, P.B. Weichman, G. Grinstein, and D.S. Fisher, Phys. Rev. B40 (1989) 546.

[5] D. van Oosten, P. van der Straten, and H.T.C. Stoof, Phys. Rev. A63 (2001) 053601.

[6] Jun-Jun Liang, J.-Q. Liang, and W.-M. Liu, Phys. Rev. A68 (2003) 043605.

[7] S. Pötting, O. Zobay, P. Meystre, and E.M. Wright, J. Mod. Opt. 47 (2000) 2653.

[8] V.V. Konotop and M. Salerno, Phys. Rev. A65 (2002) 021602.

[9] A. Smerzi, A. Trombetton, P.G. Kevrekidis, and A.R. Bishop, Phys. Rev. Lett. 89 (2002)
170402.

[10] W.M. Liu, B. Wu, and Q. Niu, Phys. Rev. Lett. **84** (2000) 2294; W.M. Liu, W.B. Fan, W.M. Zheng, J.Q. Liang, and S.T. Chui, Phys. Rev. Lett. **88** (2002) 170408.

[11] W.D. Li, X.J. Zhou, Y.Q. Wang, J.Q. Liang, and W.M. Liu, Phys. Rev. **A64** (2001) 015602; Z.W. Xie, W. Zhang, S.T. Chui, and W.M. Liu, Phys. Rev. **A69** (2004) 053609.

[12] J.C. Bronski, L.D. Carr, B. Deconinck, and J.N. Kutz, Phys. Rev. Lett. **86** (2001) 1402.

[13] A. Görlitz, J.M. Vogels, A.E. Leanhardt, C. Raman, T.L. Gustavson, J.R. Abo-Shaeer, A.P. Chikatur, S. Gupta, S. Inouye, T.P. Rosenband, D.E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **87** (2001) 130402.

[14] M. Greiner, I. Bloch, O. Mandel, T.W. Hänsch, and T. Esslinger, Phys. Rev. Lett. **87** (2001) 160405.

[15] F. Dalfovo, S. Giorgini, L.P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71** (1999) 463.

[16] M. Olshanii, Phys. Rev. Lett. **81** (1998) 938.

[17] D.S. Petrov, G.V. Shlyapuikov, and J.T.M. Walraven, Phys. Rev. Lett. **85** (2000) 3745.

[18] L.D. Carr, C.W. Clark, and W.P. Reinhardt, Phys. Rev. **A62** (2000) 063610.

[19] D.I. Choi and Q. Niu, Phys. Rev. Lett. **82** (1999) 2022.

[20] M.A. Ol'shanii, Y.B. Ovchinnikov, and V.S. Letokhov, Opt. Commun. **98** (1993) 77.

[21] M.J. Renn, D. Montgomery, O. Vдовин, D.Z. Anderson, C.E. Wieman, and E.A. Cornell, Phys. Rev. Lett. **75** (1995) 3253.

[22] J. Denschlag, D. Cassettari, and J. Schmiedmayer, Phys. Rev. Lett. **82** (1999) 2014.

[23] A.E. Leanhardt, A.P. Chikkatur, D. Kielpinski, Y. Shin, T.L. Gustavson, W. Ketterle, and D.E. Pritchard, Phys. Rev. Lett. **89** (2002) 040401.
Figure Caption:

**Fig. 1** The persistent atom-current $j$ (in unit $\frac{\hbar k L}{m}$) as a function of the barrier height (in unit $4|a|\hbar\omega_0$) of the optical lattice.
