This paper considers closed-string states of type 2b superstring theory in which the whole string is localized at a single point in superspace. Correlation functions of these (scalar and pseudoscalar) states possess an infinite number of position-space singularities inside and on the light-cone as well as a space-like singularity outside the light-cone.
General off-shell string correlation functions are parametrization-dependent expressions of little apparent direct interest. However, the class of Green functions which describe the correlation functions of strings in BRST-invariant point-like position eigenstates have special properties ([1] and references therein). A correlation function with $M$ external point-like states is defined by a sum over world-sheets of arbitrary genus with $M$ boundaries on which the space-time coordinates satisfy fixed Dirichlet conditions, i.e., each boundary (labelled $B$) is fixed at a space-time point $y_B$. These Green functions can be obtained (at least formally) from the partition function of the usual open string theory (in which the boundary conditions are the usual Neumann ones) by a $R \to 1/R$ spatial duality transformation [1].

The Dirichlet boundary correlation functions in critical bosonic string theory exhibit a rich space-time singularity structure [2]. For example the two-boundary correlation function (with boundaries at positions $y_{1}^\mu$ and $y_{2}^\mu$) not only possesses a light-cone singularity (at $y_2^2 \equiv (y_2 - y_1)^2 = 0$), as in conventional point-particle Green functions, but there is an infinite sequence of singularities inside the light-cone ($y_2^2 < 0$) as well as a single space-like singularity ($y_2^2 > 0$). This illustrates a sort of duality between position space and momentum space with the positions of the $y_2^2$ singularities corresponding to the positions of the usual singularities in momentum space (with $\alpha'$ replaced by $1/\alpha'$). The presence of a space-like singularity is a consequence of the exponential degeneracy of massive closed-string states (and should therefore be expected in any string theory) and is closely related to the open-string tachyon state — a familiar feature of bosonic strings.

The following will describe BRST-invariant point-like states of ten-dimensional superstring theory and their correlation functions. The discussion will be given first in the formalism with world-sheet supersymmetry. The space-time properties of the point-like states will become clearer in the manifestly space-time supersymmetric light-cone formalism. It will be seen that the states are not only point-like eigenstates of the position operator but also satisfy (at fixed $\tau$) $\theta^a(\sigma, \tau) \rangle = 0$ or $\partial/\partial \theta^a(\sigma, \tau) \rangle = 0$, which are natural extensions of the point-like conditions to the superspace Grassmann spinor coordinate $\theta^a(\sigma, \tau)$. These conditions define two distinct scalar point-like states, corresponding to off-shell continuations of complex combinations of the two massless scalar states of type 2b supergravity. For certain (‘supersymmetric’) correlation functions the GSO projection leads to a cancellation of the space-like position-space singularity that is present in the individual spin structures that contribute to the correlation functions. This cancellation is a position-space analogue of the cancellation of the momentum-space tachyon in superstring theories. However, the propagator (and the commutator) of closed-string fields in point-like states possesses a space-like singularity that can again be related to an open-string tachyon (in a spin structure that breaks supersymmetry).

To begin, recall some properties of the bosonic theory in the presence of world-sheet boundaries. In the usual theory the space-time coordinates $X^\mu$ satisfy Neumann boundary conditions $\partial_n X^\mu|_B = 0$ (where the subscript $n$ denotes the derivative normal to the boundary, $B$). In this case the theory describes interacting open and closed strings and the boundary represents the trajectory of an end-point of an open string. In the theory with Dirichlet boundary conditions, $\partial_t X^\mu|_B = 0$ (where $t$ denotes the derivative tangential

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3 The theories discussed here have orientable world-sheets so that cross-caps are not included.
to the boundary) each boundary represents a closed string localized at a single (and arbitrary) space-time point, \( y_B \). The partition function in this ‘Dirichlet’ theory describes the position-space correlation functions between these point-like boundary states \([1]\).

The two kinds of boundary condition are formally related by a target-space duality transformation. Recall that closed string theory compactified on a flat torus with radii \( R^\mu (\mu = 0, \ldots , 25) \) is equivalent to the Dirichlet theory on the dual torus with radii \( R'\mu \equiv \alpha'/R^\mu \) and with the integer Kaluza–Klein charges (labelling the discrete momenta) interchanged with the winding numbers. Equivalently, this transformation replaces \( X^\mu \) by the dual coordinates, \( Y^\mu \), defined by \( \partial_\alpha Y^\mu = \epsilon_{\alpha\beta}\partial^\beta X^\mu \). In the presence of world-sheet boundaries the Neumann boundary condition on \( X \) is therefore equivalent to the Dirichlet condition on \( Y \) \([3]\). The Neumann Minkowski-space theory is recovered in the limit \( R \to \infty \) while the Dirichlet Minkowski-space theory is obtained in the limit \( R' \to \infty \) (\( R \to 0 \)).

The usual theory involves a sum over all possible insertions of boundaries with Neumann boundary conditions in world-sheets of arbitrary topology while the theory obtained from this by space-time duality is one with a sum of all possible insertions of Dirichlet boundaries in world-sheets of arbitrary topology. If the sum is taken to include integration over the space-time location of each boundary (so that no momentum flows through it) \([4]\) the result is a string theory with radically different properties from those usually considered.

A world-sheet with one or two boundaries may be parametrized as a cylinder with its axis in the \( \tau \) direction (where the angular coordinate is \( \sigma \)) and a boundary is defined by an end-state, \(|B\rangle \) (as in \([3]\), \([5]\)). When the cylinder ends in a Neumann boundary (on which \( \partial X^\mu /\partial \tau = 0 \)) the end-state satisfies \( (\alpha^\mu_n + \tilde{\alpha}^\mu_{-n})|B\rangle = 0 \) (where the left-moving and right-moving normal modes satisfy \( [\alpha^\mu_n, \alpha^\nu_n] = m\delta_{m+n}\eta^{\mu\nu} = [\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n] \)) so that the total momentum, \( k^\mu \equiv \sqrt{2/\alpha'\alpha^\mu_0} = \sqrt{2/\alpha'\tilde{\alpha}^\mu_0} \), vanishes due to the \( n = 0 \) condition. The conditions on a Dirichlet boundary state localized at \( X^\mu = y^\mu \), are

\[
(\alpha^\mu_n - \tilde{\alpha}^\mu_{-n})|B, y\rangle = 0,
\]

for all \( n \) (where cylinder end-states of the Dirichlet theory are denoted by a hat). A corresponding point-like eigenstate of the total momentum is defined by \( |B, k\rangle \equiv \int d^Dy e^{ik\cdot y}|B, y\rangle/(2\pi)^{D/2} \). A BRST formulation involves ghost coordinates, \( b(\sigma, \tau) \) and \( c(\sigma, \tau) \) and the ghost modes satisfy the boundary conditions

\[
(c_n + \tilde{c}_{-n})|B, k\rangle = 0 = (b_n - \tilde{b}_{-n})|B, k\rangle,
\]

(2)

(where \( \{b_n, c_m\} = \delta_{m+n} = \{\tilde{b}_m, \tilde{c}_m\} \)) as they do in the Neumann theory. These conditions ensure BRST invariance of the state,

\[
Q_{BRST}|B, k\rangle = 0.
\]

(3)

Open superstring theory with Dirichlet conditions.

In type 2b superstring theory (world-sheet) Majorana–Weyl fermionic spinors, \( \psi^\mu \) and \( \tilde{\psi}^\mu \) (of opposite chirality), are introduced with various combinations of boundary conditions.
(spin structures). The sectors containing closed-string scalar bosons which couple to the end-states of the cylinder are the anti-periodic \( A \) sector (the \( NS \otimes NS \) sector) and the periodic \( P \) sector (the \( R \otimes R \) sector). When the end-state of a cylinder is either a Neumann or Dirichlet boundary, these fields satisfy the boundary conditions (using conventions which approximate to those of \([4]\)  

\[
\psi_{n\mu}(\sigma, l) \equiv (\psi^{\mu}(\sigma, l) + i\eta \tilde{\psi}^{\mu}(\sigma, l))/\sqrt{2} = 0,
\]  

(4)

where \( \eta = \pm 1 \). These boundary conditions on the spinors in the cylinder channel are related to the boundary conditions at the ends of an open string, \( \psi = \pm \tilde{\psi} \), by a global diffeomorphism that rotates the coordinate system through \( \pi/2 \) — leading to the factor of \( i \) in \([4] \) \([5]\). These translate into the conditions on the two possible point-like endstates,

\[
\psi_{k}^{\eta\mu}|B, \eta, k\rangle \equiv (\psi_{k}^{\mu} + i\eta \tilde{\psi}_{k}^{\mu})|B, \eta, k\rangle/\sqrt{2}
\]  

(5)

(where the fermionic modes satisfy the anticommutation relations \( \{\psi_{k}^{\mu}, \psi_{l}^{\nu}\} = \eta^{\mu\nu}\delta_{k+l} \)). The index \( k \) takes half-integral values in the \( A \) sector and integral values in the \( P \) sector. These boundary conditions will ensure local supersymmetry and BRST invariance. The same considerations also determine the boundary conditions for the superconformal ghost coordinates, \( \beta, \tilde{\beta}, \gamma, \tilde{\gamma} \) (satisfying \([\gamma_{k}, \beta_{l}] = \delta_{k+l} = [\tilde{\gamma}_{k}, \tilde{\beta}_{l}] \)).

Including all the modes, the Dirichlet boundary states have the form (with an arbitrary overall constant normalization)

\[
|B, \eta, k\rangle_{A,P} = \exp\left(\sum_{n=1}^{\infty} (\alpha_{-n} \cdot \tilde{\alpha}_{-n}/n - b_{-n} \tilde{c}_{-n} - \tilde{b}_{-n} c_{-n}) \right)

\[
- \frac{i\eta}{\sqrt{2}} \sum_{k \neq 0} (\psi_{-k} \cdot \tilde{\psi}_{-k} + \beta_{-k} \gamma_{-k} + \tilde{\beta}_{-k} \tilde{\gamma}_{-k}) \right) |0, \eta, k\rangle_{A,P}
\]  

(6)

where \( \langle 0, \eta', k'|0, \eta, k \rangle = \delta_{\eta+\eta', 0}\delta^{10}(k + k') \) with \( \langle 0, \eta, k \rangle \equiv |0, -\eta, k\rangle^{\dagger} \) (and the dependence on ghost zero modes has been suppressed). The analogous states in the Neumann theory, \( |B, \eta\rangle \), differ only by the sign of \( \tilde{\alpha}, \tilde{\beta} \) and \( \tilde{\gamma} \) in the exponent (and have zero momentum). In the \( A \) sector the ground state, \( |0, \eta, k\rangle_{A} \), is the tensor product of two negative \( G \)-parity \( NS \) scalar tachyon states. In the \( P \) sector the two ground states, \( |0, \eta, k\rangle_{P} \), are the states annihilated by \( \psi_{0}^{\eta\mu} \equiv \partial/\partial \tilde{\psi}_{0}^{-\eta\mu} \) (\( \eta = \pm 1 \)). In either sector the states satisfy the fermionic world-sheet supersymmetry gauge conditions

\[
F_{k}^{\eta}|B, \eta, k\rangle_{A,P} \equiv (F_{k} + i\eta \tilde{F}_{-k})|B, \eta, k\rangle_{A,P}/\sqrt{2} = 0,
\]  

(7)

(\( F_{k} = \sum_{n} \alpha_{n} \psi_{k+n} + \text{ghost terms} \) and \( \tilde{F}_{k} = \sum_{n} \tilde{\alpha}_{n} \tilde{\psi}_{k-n} + \text{ghost terms} \) which can be iterated to give the Virasoro conditions, \( (L_{n} - \tilde{L}_{-n})|B, \eta, k\rangle_{A,P} = 0 \). The conditions \([4]\) apply for arbitrary momentum, \( k^{\mu} \) (whereas the usual Neumann boundary state satisfies the same conditions only if \( k^{\mu} = 0 \)). Using the usual expression for the BRST charge on a cylindrical world-sheet it is easy to see that \([7]\) leads to \( Q_{BRST}|B, \eta, k\rangle = 0 \).
In order to simplify the following it is convenient to specialize to the light-cone gauge, in which the non-transverse components of the modes of $\psi^\mu$ and $X^\mu$ cancel the ghost modes and only the transverse components of the modes $(\psi_n^i, \tilde{\psi}_n^i, \alpha_n^i$ and $\tilde{\alpha}_n^i$ with $i = 1, \ldots, 8$) survive.

Physical states in the cylinder are obtained by projection with the usual GSO operator

$$P_{GSO} = (1 + \omega(-1)^F)(1 + \omega(-1)^{\tilde{F}})/4,$$

where $\omega = 1$ in the $A$ sector while $\omega = \pm 1$ in the $P$ sector (the sign ambiguity corresponding to the two possible space-time chiralities) and the operators $(-1)^F \equiv \prod_k (-1)^{F_k}$ and $(-1)^{\tilde{F}} \equiv \prod_k (-1)^{\tilde{F}_k}$ anticommute with all the $\psi_k^i$ and $\tilde{\psi}_k^i$, respectively. The Dirichlet end-states satisfy

$$( -1)^F |B, \eta, k\rangle_A = ( -1)^{\tilde{F}} |B, \eta, k\rangle_A = -|B, -\eta, k\rangle_A, \quad (8)$$

in the $A$ sector and

$$( -1)^F |B, \eta, k\rangle_P = ( -1)^{\tilde{F}} |B, \eta, k\rangle_P = |B, -\eta, k\rangle_P, \quad (9)$$

in the $P$ sector, where the property $(-1)^{F_0} |0, +, k\rangle_P = |0, -, k\rangle_P$ has been used (and $(-1)^{\tilde{F}_0} \equiv \psi_0^1 \ldots \psi_0^8$). [The discussion of these conditions follows closely that given in [6] for the usual Neumann boundary.]

The amplitude coupling $M$ on-shell closed-string states on a world-sheet with a single boundary (the disk diagram) contains intermediate open strings that couple to closed strings (which are singlets of the Chan–Paton symmetry). Thus, in the usual bosonic theory with a $U(n)$ Chan–Paton symmetry the massless gauge-singlet open-string vector state (which is absent for the other symmetries allowed by tree-level unitarity, namely, $SO(n)$ and $Sp(2n)$) mixes with the massless closed-string anti-symmetric tensor state in a dynamical ‘Higgs’ mechanism [7]–[9]. Theories with Dirichlet boundaries are quite different since the open-string vector is not a normal propagating state but is a Lagrange multiplier field which leads to an interesting divergence in the bosonic theory [10]. In the case of the usual type 1 superstring theories the disk diagram does not mix massless open-string and closed-string states, irrespective of the Chan–Paton group. This is a signal that $U(n)$ groups are inconsistent due to a mismatch between the $N = 1$ supersymmetry of the open-string states and the $N = 2$ supersymmetry of the closed-string states [11]. Since the open-string sector of the supersymmetric Dirichlet theory does not contain propagating states the status of this argument needs to be reassessed in that case.

**Boundary correlation functions.**

The correlation function of two Dirichlet boundaries, together with an arbitrary number of closed-string vertex operators, defines off-shell amplitudes of the type 2b theory coupling two currents and $M$ on-shell particles. As a special example consider the two-boundary correlation function with $M = 0$, i.e., the correlation between two point-like states at $y_1^\mu$ and $y_2^\mu$, each of which carries a label $\eta_1$ and $\eta_2$, respectively. The complete Green function, $G_{\eta_1, \eta_2}(y_1 - y_2)$, decomposes into a sum over four spin structures. When $\eta_1 = \eta_2 = \eta$ (the ‘supersymmetric’ case) this sum is defined by

$$G_{\eta, \eta}(y_1 - y_2) = G^{(++)} + G^{(+-)} + G^{(-+)} + G^{(--)}, \quad (10)$$
where the superscripts label the antiperiodic (+) and periodic (−) boundary conditions on the fermions in the σ and τ directions, respectively. The two-dimensional boundary correlation function in the case \( \eta_1 = -\eta_2 = \eta \) defines the ‘propagator’ for the point-like state labelled \( \eta \). The relative signs of the spin structures in this case differ from \( \Box \) in certain crucial respects in a manner determined by \( \Box \) and \( \Box \). There are two useful ways of expressing the Green function, which will now be described.

In the first way the process described by the Green function is viewed as the evolution of a closed string from an initial boundary state at \( y_1 \) to the final state at \( y_2 \), thereby defining a cylindrical world-sheet of length \( l \). In this case, \( G_{\eta_1, \eta_2}(y_1 - y_2) = G^A_{\eta_1, \eta_2}(y_1 - y_2) + G^P_{\eta_1, \eta_2}(y_1 - y_2) \), where

\[
G^A_{\eta_1, \eta_2}(y_1 - y_2) = \int_0^\infty dl \ A(B, \eta_1, y_1)(1 + (-1)^F) e^{-(L_0 + \bar{L}_0 - 1)l} |B, \eta_2, y_2\rangle_A
= \eta_1 \eta_2 \left(G^{(++)} + G^{(+-)}\right)
\]

(note that the GSO projection simplifies since a factor of \((-1)^F\) is equivalent to \((-1)^F\) and \((-1)^{F+\bar{F}}\) is equivalent to 1). The evolution operator involves the closed-string hamiltonian, \( L_0 + \bar{L}_0 - 1 \), where the Virasoro generators include all the bosonic and fermionic coordinates of the \( A \) sector. The first spin structure in \( \Box \) can be expressed in the form

\[
G^{(++)} = \int_0^\infty dl \ A(B, \eta, y_1) e^{-(L_0 + \bar{L}_0 - 1)l} |B, \eta, y_2\rangle_A
= \frac{1}{2(\alpha')^5} \int_0^\infty \frac{dl}{l^5} e^{-y^2 / 2\alpha' l - l} \prod_{n=1} \frac{(1 + q^{2n+1})^8}{(1 - q^{2n})^8}
= \frac{\pi}{2(\pi \alpha')^5} \int_0^\infty \frac{dl'}{l'} e^{-l' (y^2 / 4\pi^2 \alpha' - 1/2)} \prod_{n=1} \frac{(1 + w^{n+1/2})^8}{(1 - w^n)^8},
\]

where \( y^\mu = y_1^\mu - y_2^\mu, \ w = e^{-l'}, \ q = e^{-l} \). The last line is obtained by a standard modular transformation \( l' = 2\pi^2 / l \) from the one before. Formally (i.e., ignoring the \( l' = 0 \) end-point divergence), this expression has an infinite number of logarithmic singularities in \( (y_1 - y_2)^2 \) of the form \( \ln((y_1 - y_2)^2 / 4\pi^2 \alpha' + m/2 - 1/2) \), where \( m \geq 0 \) (which is similar to the bosonic Dirichlet theory). The existence of a singularity outside the light-cone is correlated with the fact that this spin structure is not consistent by itself. This singularity will soon be shown to cancel in the case \( \eta_1 = \eta_2 \) when the contribution of \( G^{(-+)} \) (the parity-conserving spin structure in \( G^P \)) is added. Furthermore, the \( l' = 0 \) (\( l = \infty \)) divergence will cancel with a similar term arising from \( G^{(+-)} \) (the second spin structure in \( G^A_{\eta_1, \eta_2} \), which is given by \( \Box \) with an extra factor of \((-1)^F\)) for either \( \eta_1 = \pm \eta_2 \).

The expression for the \( P \)-sector correlation function, \( G^P \), is similar to \( \Box \) with the crucial difference that the correlation function with \( \eta_1 = -\eta_2 \) has the same sign as the one with \( \eta_1 = \eta_2 \) (which follows by use of \( \Box \) and \( \Box \)) so that \( G^P_{\eta_1, \eta_2} = G^{++} + G^{--} \). Furthermore, integration over fermion zero modes causes \( G^{--} \) to vanish (as in the type 1 theory).
The second way of expressing the Green function is to represent the cylindrical world-sheet as an annulus, or a loop formed by an open string with end-points fixed at \( y_1 \) on one boundary and \( y_2 \) on the other. This may be evaluated as a trace over open-string states circulating around the loop. As is usual with open-string loop amplitudes, the resulting expression should be related to the earlier one by a modular transformation in which the evolution time \( l \) along the cylinder (in (12)) transforms into where \( l' = \frac{2\pi^2}{l} \), the evolution parameter for the open strings circulating around the annulus. A string with fixed end-points has the mode expansion,

\[
Y(\sigma, \tau) = y_1 + (y_2 - y_1)^{\sigma} + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n \sin n\sigma e^{in\tau}
\]  

(13)

(which is the \( R \to 0 \) limit of the more general expression in [1]) and the bosonic part of the open-string evolution operator for such an open string is given by

\[
L_{0,\alpha}^y = \frac{(y_1 - y_2)^2}{4\pi^2\alpha'} + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n
\]

(14)

(where \( y = y_2 - y_1 \)). This is to be added to the usual contributions from the fermionic modes and the ghosts, \( L_{0}^\psi \) and \( L_{0}^{\text{ghosts}} \) to give the total \( L_0^y \). The propagator, \( 1/(L_0^y - a) \), describing the evolution of the world-sheet of an open string with end-points fixed at \( y_1 \) and \( y_2 \) has poles in \((y_1 - y_2)^2\) (with \( a = 0 \) in the \( R \) sector and \( a = 1/2 \) in the \( NS \) sector). These space-time poles in the open-string propagator are the origin of the singularities seen in the last line of (12). More generally, the world-sheet for a correlation function of an arbitrary number of Dirichlet boundaries can be constructed by sewing together three-string vertices with propagators representing the evolution of open strings with fixed endpoints. The space-time singularity structure of these amplitudes is then simply obtained from the singularities of these propagators.

From this viewpoint the expression (12) (the first contribution to \( G_{\eta_1,\eta_2}^A \) in (11)) is obtained from the spin structure in which the fermionic field is anti-periodic (NS) between the open-string endpoints, as well as being anti-periodic around the loop,

\[
\hat{G}^{++} = \frac{\pi}{2(\pi\alpha')^5} \int_0^{\infty} \frac{dl'}{l'} \text{Tr} \left( e^{-(L_0^y - 1/2)l'} \right),
\]

(15)

where the hat indicates that the spin structure is labelled in the frame of the annulus. The \( P \)-sector parity-conserving term \( G^{(-+)} \), which transforms into \( \hat{G}^{(+-)} \) under the modular transformation, arises from a trace over the other spin structure in the NS sector of the open string,

\[
\hat{G}^{(+-)} = \frac{\pi}{2(\pi\alpha')^5} \int_0^{\infty} \frac{dl'}{l'} \text{Tr} \left( (-1)^{F_{\text{open}}} e^{-(L_0^y - 1/2)l'} \right)
\]

(16)

(which includes a factor \(-1\) for terms in the trace with an odd number of fermionic open-string oscillators). When \( \eta_1 = \eta_2 \) the GSO projection gives the sum of (15) and (16), which projects onto the even-moded Fock space states of the \( NS \) sector of the fixed-
endpoint open-string circulating in the loop. The singularity outside the light-cone (at 
\((y_1 - y_2)^2 = 2\pi^2\alpha'\)) cancels in this sum and the leading singularity is on the light-cone 
\(((y_1 - y_2)^2 = 0)\). This closely parallels the cancellation of the tachyonic open-string pole 
in the usual GSO projection. When \(\eta_2 = -\eta_1\) the relative sign of \(G^{++}\) and \(G^{-+}\) (\(i.e.,\) 
of \(\hat{G}^{++}\) and \(\hat{G}^{-+}\)) in the sum changes and the GSO projection no longer eliminates the 
space-like singularity – open-string states of half-integer mode number are not eliminated.

Similarly, the second term in (11), \(G(+-)\), is proportional to the trace over the open-
string states satisfying periodic (R) boundary conditions, \(\hat{G}^{(-+)} \sim \int dl' \text{Tr}(\exp(-L_0^y l')/l')\). 
The Fourier transform of the above expressions with respect to \(y^\mu\) are simple to 
evaluate, resulting in the momentum-space Green function. The sum of the two spin 
structures that contribute to \(\tilde{G}^A(k)\) (where the tilde indicates the Fourier transform) is

given by

\[
\eta_1\eta_2\tilde{G}_{\eta_1,\eta_2}^A = \tilde{G}^{(++)} + \tilde{G}^{(+-)}
\]

\[
= \frac{1}{2} \int_0^\infty dl e^{-(\alpha'k^2-2)l/2} \left( \prod_{n=1}^\infty \frac{(1+q^{2n-1})^8}{(1-q^{2n})^8} - \prod_{n=1}^\infty \frac{(1-q^{2n-1})^8}{(1-q^{2n})^8} \right).
\]

The one non-vanishing spin structure in the \(P\) sector has the momentum-space form,

\[
\tilde{G}^{(++)}(k) = -8 \int_0^\infty dl e^{-\alpha'k^2l/2} \prod_{n=1}^\infty \frac{(1+q^{2n})^8}{(1-q^{2n})^8}.
\]

For \(\eta_1 = \eta_2\) these expressions may formally be interpreted as the \(R \rightarrow 0\) limit of the terms in the cosmological constant of the Neumann theory in a toroidal space-time. In that case the fact that the sum of (15) and the first term in (17) is not exponentially divergent as \(q \rightarrow 1\) is another expression of the fact that the corresponding sum of position-space terms has no singularity outside the light-cone. The complete Green function (the sum of (17) and (15)) then vanishes for all \(k\) (equivalently, its Fourier transform vanishes for all \(y\)) as a result of the Jacobi\(\ \text{aequatio incessat satia abstrusa},\) which arises here as a statement of the equality of the number of scalar states coupling to the boundary in the \(A\) sector and in the \(P\) sector at every mass level. For \(\eta_1 = -\eta_2\) the \(P\) sector reinforces the \(A\) sector and there is no cancellation. In that case the correlation function possesses contains the space-like singularity at \(y^2 = 2\pi^2\alpha'\) that is manifest in \(G^{(++)}\) (equivalently, in (15)).

**Space-time supersymmetry**

The point-like states can also be described in a manifestly supersymmetric formalism in the light-cone gauge. The two scalar states of the previous sections are given by,

\[
|B, \eta, k\rangle_{lc} = \exp \sum_{n=1}^\infty \left( \alpha^i_{-n} \tilde{\alpha}^i_{-n}/n - i\eta S^a_{-n} \tilde{S}^a_{-n} \right) |0, \eta, k\rangle_{lc},
\]

where the ground states are tensor products of light-cone ground states in each Fock space,

\[
|0, \eta, k\rangle_{lc} = (|k\rangle|i\rangle|\tilde{a}\rangle - i\eta |k\rangle|\tilde{a}\rangle)/4,
\]
\[\eta = \pm, \text{ and } S_n^a, \tilde{S}_n^a \text{ are the modes of the } SO(8) \text{ transverse space-time spinors} \text{ (the indices } a \text{ and } \hat{a} \text{ label the two inequivalent } SO(8) \text{ spinors while } i \text{ labels the vector). These ground states are the two complex scalar ground states of the type 2b theory. The states (13) satisfy}
\]

\[S_n^a |B, k, \eta\rangle_{lc} \equiv \frac{1}{\sqrt{2}} (S_n^a + i\eta \tilde{S}_n^a) |B, k, \eta\rangle_{lc} = 0 \quad (21)\]

(for all \(n\)) in addition to the point-like condition on the bosonic coordinates. The ground states (20) are again normalized so that \(i_c \langle 0, \eta', k' | 0, \eta, k \rangle_{lc} = \delta_{\eta + \eta', 0} \delta^{10}(k + k') \) (using \(\langle i | j \rangle = \delta_{i,j}, \langle a | b \rangle = \delta_{a,b}\), where \(i_c \langle 0, \eta, k \rangle = ([0, -\eta, k])^\dagger\). They are related by \(|0, \eta, k\rangle_{lc} = S_0^{n1} S_0^{n2} \ldots S_0^{n8} |0, -\eta, k\rangle_{lc}. \) The end-state (19) is not an eigenstate of the twist operator, \(\Omega\), that interchanges the left and right-moving Fock spaces. Expanding the exponential in a power series leads to an infinite series of Fock-space states that includes states of the form \((S^i \tilde{S}^\dagger)^{2n+1} |i\rangle \langle i|\) and \((S^i \tilde{S}^\dagger)^{2n} |\dot{a}\rangle \langle \dot{a}|\), which are antisymmetric under \(\Omega\), in addition to the symmetric states.

The states (19) satisfy the 16-component supersymmetry conditions

\[Q_n^a |B, \eta, k\rangle_{lc} = 0 = Q_\dot{a}^\dot{a} |B, \eta, k\rangle_{lc}, \quad (22)\]

where \(Q_n = (Q + i\eta \tilde{Q}) / \sqrt{2}\) and

\[Q^a = \sqrt{2k^+} S_n^a, \quad Q^\dot{a} = \sqrt{\frac{2}{k^+}} \gamma^i_{\dot{a}a} \sum_{-\infty}^{\infty} S_{n-i}^a \alpha_{n}^i, \quad (23)\]

are the two \(SO(8)\) components of a 16-component space-time supercharge (with analogous definitions for \(\tilde{Q}^a\) and \(\tilde{Q}^\dot{a}\)). These supercharges satisfy the usual superalgebra which includes the relations \(Q^a \dot{Q}^\dot{a} = Q^{\dot{a}} \dot{Q}^a = P^- \tilde{P}^-\) and \(\{Q^a, Q^{\dot{b}}\} = \delta^{\dot{a}b}(P^- + \tilde{P}^-)\), where \(P^- \equiv H\) is the light-cone gauge hamiltonian. The supersymmetry conditions, (22), differ from those of the type 1 superstring by the presence of important factors of \(i\) in the definition of \(Q_n\) (arising from the replacement \(k^+ \to -k^+\) in the definition of \(\tilde{Q}\) in passing from the Neumann to the Dirichlet theory).

In a light-cone superspace formulation of the type 2b theory [12] a \(SO(8)\) Grassmann spinor coordinate can be identified,

\[\theta^a(\sigma, \tau) = \frac{1}{\sqrt{2k^+}} \left( S^a(\sigma, \tau) + i\tilde{S}^a(\sigma, \tau) \right), \quad (24)\]

together with a conjugate spinor momentum,

\[\lambda^a(\sigma, \tau) = \sqrt{k^+ / 2} \left( S^a(\sigma, \tau) - i\tilde{S}^a(\sigma, \tau) \right) \equiv \partial / \partial \theta^a(\sigma, \tau). \quad (25)\]

The components of the supercharges \(Q^{\pm a}\) can be identified with the zero modes, \(Q^a = \sqrt{2k^+} \theta^a_0, Q^- = \sqrt{2k^+} \lambda^0_0\), while the supercharges \(Q^{\pm \dot{a}}\) have simple representations as bilinears in \(\theta^a, \partial / \partial \theta^a\) and \(\partial_\mu Y^\mu\) [12]. The two states, (19), satisfy

\[\theta^a(\sigma, \tau) |B, k, +\rangle_{lc} = 0, \quad \lambda^a(\sigma, \tau) |B, k, -\rangle_{lc} = 0. \quad (26)\]
Either of these conditions is an obvious superspace extension of the Dirichlet condition, \([13]\).

A string light-cone superfield containing these states can be written as an expansion in component string fields,

\[
\Phi[Y^i(\sigma), \theta^a(\sigma), k^+] = \phi[Y^i(\sigma), k^+] + \cdots + \phi^{a_1 a_2 \cdots a_m}[Y^i(\sigma), k^+] \theta^{a_1} \theta^{a_2} \cdots \theta^{a_m} + \cdots, \tag{27}
\]

which is the string generalization of the light-cone superfield of ten-dimensional type 2b supergravity \([13]\). The states (26) are those at the top and bottom ends of the infinite-dimensional supermultiplet (annihilated by \(Q^\pm a\)). Upon imposing the point-like condition (so that the component string fields are proportional to \(\delta(Y^i(\sigma) - y^i)\)) these top and bottom states are also annihilated by \(Q^+ a\) and \(Q^- a\), respectively (these are the non-linearly realized \(SO(8)\) light-cone supercharges) so that these states are the top and bottom states of a covariant physical supermultiplet. They therefore have the appropriate quantum numbers to couple to the complex massless scalar (or its complex conjugate) of type 2b supergravity, as well as an infinite sequence of massive scalar states.

Scattering amplitudes can be calculated simply using the off-shell states \([19]\) in the Dirichlet theory. For example, the cylinder amplitude with two Dirichlet boundaries (with momentum \(p_1\) and \(p_2\)) and \(M\) massless closed-string ground states with momenta \(k_r\) (satisfying momentum conservation, \(p_1 + p_2 + \sum_r k_r = 0\)) attached at points \(\rho_r = (\sigma_r, \tau_r)\) to the interior is proportional to (setting \(\alpha' = 2\) for simplicity)

\[
A(\{k_r\}, p_1, p_2) = g^M n^2 \sum_{\text{perms}} \left( \prod_{r=1}^{M} \int_0^{2\pi} d\sigma_r \right) \left. _{lc} \langle B, \eta_1, p_1 | \int_{\tau_M}^{\infty} d\tau_1 V_{1} e^{-\left(p_1^2 + N + \bar{N}\right)\tau_1} | B, \eta_2, p_2 \rangle_{lc}, \right. \tag{28}
\]

where \(V_r(\sigma_r, \tau_r, k_r)\) is the light-cone vertex for the emission of the \(r\)th on-shell state with transverse momentum \(k_r^i\) and polarization \(e^{MN}_r\) (where \(M, N\) label vector or spinor indices of the external states) and the sum is over all permutations of the ordering of the vertices in \(\tau\). The momenta in this expression are defined in a special frame in which \(k^+_r = 0\), so it will not describe the most general kinematic configuration when there are many external particles. Subject to this restriction, the amplitude (28) is the same as that obtained in the covariant approach after summing over spin structures.

The vanishing of the two-boundary amplitude (\(M = 0\)) in the case \(\eta_1 = \eta_2\) is here seen to be due to the integration over fermionic zero modes associated with space-time supersymmetry. The propagator for the \(\eta_1\) state is the \(M = 0\) amplitude with \(\eta_1 = -\eta_2\), which does not vanish. Using \((-1)^F B, \eta, p = | B, -\eta, p\) it is clear that the \(\tau\) boundary condition breaks supersymmetry – the open-string Grassmann coordinates circulating around the annulus have \(1/2\)-integer modes. The resulting momentum-space propagator has the form

\[
\tilde{G} \sim \int_0^{\infty} d\ell e^{-\alpha' \ell^2/2} \prod_{n=1}^{\infty} \frac{(1 + q^{2n})^8}{(1 - q^{2n})^8}, \tag{29}
\]

9
which is similar to (18) and is the Fourier transform of a position-space expression with a space-like singularity.

Amplitudes with two Dirichlet boundaries (labelled $\eta_1$ and $\eta_2$) and two or more external on-shell closed-string ground vertex operators ($M \geq 2$) do not necessarily vanish even if $\eta_1 = \eta_2$ since each vertex introduces four fermionic modes. From the earlier analysis it is evident that they may be expressed in terms of poles and multi-poles in position space. In order to exhibit these position-space singularities it is again convenient to represent the amplitude as an open-string loop diagram in which the circulating open string has end-points fixed at positions $y_1$ and $y_2 = y - y_1$,

$$A(\{k_r\}, y_1, y_2) \equiv \int \frac{dp_1}{(2\pi)^5} \frac{dp_2}{(2\pi)^5} e^{ip_1 \cdot y_1 + ip_2 \cdot y_2} A(\{k_r\}, p_1, p_2) \delta(p_1 + p_2 + \sum_{r=1}^{M} k_r) \quad (30)$$

where $L_0^\nu = y^\nu / 4\pi^2\alpha' + N_\alpha + N_S$ ($N_\alpha$ and $N_S$ are the level numbers in the Fock spaces of the bosonic and fermionic light-cone coordinates). In this expression the $y^2$ singularities (poles and multipoles) arise manifestly from the zeroes of $L_0^\nu$. When $\eta_1 = \eta_2$ the open-string Grassmann coordinates in $L_0$ have integer modes and the open-string supercharge is well-defined. In that case the leading singularity is on the light cone and there an infinite number of singularities inside the light-cone, separated by $4\pi^2\alpha'$ [2]. The behaviour of these amplitudes in the deep inelastic region (i.e., for large space-like momenta in the off-shell legs) is dominated by the light-cone singularities, leading to scaling with anomalous dimensions. The first singularity inside the light-cone gives an exponentially suppressed correction to the light-cone behaviour. In the limit $\alpha' \to \infty$ (or string tension $T \to 0$) the singularities inside the light-cone move to infinity, leaving just the light-cone singularity. If $\eta_2 = -\eta_1$ the Grassmann coordinates have half-integer modes, there is no supercharge in the open-string sector and the leading singularity is at $y^2 = 2\pi^2\alpha'$. These considerations generalize the comments outlined in [2],[1] in the context of the bosonic theory.

This article has been concerned with correlation functions of a very special class of BRST-invariant states, in which each external string is localized at a point in superspace. These states are complex scalar end-states of a string superfield. The correlation function of a pair of these states of opposite type (the ‘propagator’) is characterized by an infinite sequence of position-space singularities inside and on the light-cone as well as the space-like singularity at $y^2 = 2\pi^2\alpha'$, rather as in the bosonic theory. Correspondingly, the free propagator (or, equivalently, the equal-time commutator) possesses a singularity at space-like separations (which is presumably a sign of further space-like structure in the propagator of more general string states). Although the physical relevance of this observation is unclear it may be connected to questions of causality in superstring theory (for example, in the context of the arguments in [14]). The (‘supersymmetric’) correlation function of two end-states of the same type (i.e., from the same end of the space-time supermultiplet) vanishes, but the correlation functions describing the coupling of two end-states of the same type with two or more on-shell states are non-vanishing and exhibit the striking feature that the space-like position-space singularity cancels.
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