CHAOTIC BEHAVIOUR OF THE ROSSLER MODEL AND ITS ANALYSIS BY USING BIFURCATIONS OF LIMIT CYCLES AND CHAOTIC ATTRACTIONS

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Abstract: The behaviour of certain dynamical nonlinear systems are described in term as chaos, i.e., systems’ variables change with the time, displaying very sensitivity to initial conditions of chaotic dynamics. In this paper, we study archetype systems of ordinary differential equations in two-dimensional phase spaces of the Rossler model. A system displays continuous time chaos and is explained by three coupled nonlinear differential equations. We study its characteristics and determine the control parameters that lead to different behavior of the system output, periodic, quasi-periodic and chaos. The time series, attractor, Fast Fourier Transformation and bifurcation diagram for different values have been described.

Keywords: Chaos, Rossler model, attractor, bifurcation diagram.

1. Introduction

Term "chaos" is used to describe the behavior of certain dynamical nonlinear systems, i.e., systems' variables change with the time exhibiting very sensitivity to initial conditions of chaotic dynamics. This sensitivity of chaotic systems' behavior manifest exhibits almost random, which shows as an exponential evolution of perturbations in the initial conditions. [1]. Nonlinear systems appear in all domains of engineering, chemistry, physics, economics, biology and sociology. Paradigms of nonlinear chaotic systems contain neural network models, planetary climate prediction models, turbulence, data compression, nonlinear dynamical economics, mixing liquid with low power exhaustion, processing information, circuits and devices which have high performance, and preventing the collapse of systems' power [2]. Within the chaos and dynamical systems field for low dimensional systems, the Lorenz and the Rossler models are two paradigmatic problems that have been frequently studied. Most of the results have been expressed in models of three-dimensional, where, due to the limited phase-space only low-dimensional chaos can be noticed [3]. Chaotic systems are described by one direction of exponential spreading [4]. The three-dimensional Rossler system was originally conceived as a simple model for studying chaos. With only one nonlinear term it can be thought of as a simplification of the well-known Lorenz system and as a minimal model for continuous-time chaos [5].
Continuous chaos has been first described by E.N. Lorenz in a model of turbulence, under the name of deterministic nonperiodic flow [6]. The same model has lately been obtained to apply to lasers also, demonstrating the phenomenon of chaos in lasers [7]. The chaotic of dynamical behavior possible in nonlinear systems (for example, electronic) depends only on the set of state variables concerned [8]. The electronic circuit that has the simple physical approach is designed to emulate the system for investigating of a chaotic system. This approach has some evident features. Firstly chaotic electronic oscillators are generated and it can be shown on the oscilloscope and noticed quickly. Secondly this approach avoids the uncertainties arise from statistical errors and systematic in numerical emulations, for example the discretization and round-off errors in the numeral procedures or finite time approximation of a quantity that is properly described by an unlimited time integral [9]. An excellent instrument for the study of chaotic behavior is nonlinear electronic circuits. Some of these electronic circuits treat time as a discrete variable, using analog multipliers and sample-and-hold sub circuits to model iterated maps such as the logistic map [10]. In spite of their easiness of designing and ubiquity, electronic systems have rarely been made utilize of so far in experimental dynamics [8]. In this work we use a simple electronic system agree to simple third order differential equations to describe Rössler model. It is include only simple electronic elements such as operational amplifiers, resistors, and capacitors. Moreover, with small variations, they hold the possibility for very exact comparisons between experiment and theory.

2. Rössler Model

The Rössler model [7] are given by these three equations:

\[
\frac{dx}{dt} = -(y + z)
\]
\[
\frac{dy}{dt} = x + ay
\]
\[
\frac{dz}{dt} = b + xz - cz
\]

where \(a\), \(b\) and \(c\) are real parameters. The values firstly studied by Rössler were \(a \& b = 0.2\) and \(c=5.7\) and \(x\), \(y\) and \(z\) are the three variables which evolves with the time. The first two equations have linear terms that create oscillations in the variable \(x\) and \(y\). The last equation has only one nonlinear term \((xz)\) so the expected chaotic behavior is appeared from that the system.

Continuous chaos is minimal of this system for three reasons at least: its nonlinearity is minimum because it has single quadratic term, a chaotic attractor is generated with a one lobe, in contrast to the Lorenz attractor that has two lobes, and its phase-space has the minimum dimension three [11]. The chaotic behavior in Rössler model can be described by the time series and the trajectories in the phase space as shown in Figure 1 when the control parameter \(c\) is varied. When the parameter \(c\) changed, and keeping \(a\) and \(b\) fixed at \(a = 0.2\) and \(b = 0.2\). We noticed that the time series is periodic and the attractor is a limit cycle at \(c=2.3\) (Figure 1(a) and (b)) and it is period doubling and its attractor have two loops represented to two different amplitude in time series when \(c= 2.9\) as shown in Figure 2(a) and (b). More increasing in value of \(c\) we obtained another period-doubling bifurcation creates the 4-loop shown at 4.1. We noticed the dynamics exhibit chaotic behavior at \(c= 5.7\) when the time series shows different spikes with high and low amplitudes. The chaotic attractor is rather different from others attractors. It looks very strange (strange attractor). The FFT of this state demonstrates the signature of chaotic behavior, where the distribution is exponential decay (see Figure 4(a), (b), and (c)).
Figure.1 Numerical simulation results (a) time series of a system at $c=2.3$, (b) the corresponding attractor.

Figure.2 Numerical simulation results (a) time series of a system at $c=2.9$, (b) the corresponding attractor.

Figure.3 Numerical simulation results (a) time series of a system at $c=4.1$, (b) the corresponding attractor.
Poincaré originally developed bifurcation theory. It is utilized to describe specific variation in system’s behavior, in terms of the type and the number of solutions, under the change of one or more parameters on which the system depends [1]. The analysis was used to obtain the bifurcation diagram, which has a period-doubling route to chaos (Figure 5). When the value of \( c \) is changing from 0.7 to maximum value 8, the range of \( x \) scale in the time series is divided into various parts. In the range of \( 0.7 \leq c \leq 2.4 \) the dynamic is periodic. For \( 2.5 \leq c \leq 3.6 \), the period doubling behavior of the system is observed. At \( c \) value between 3.7 and 4.1 the system behaves as second period doubling behavior of the system, and finally, the chaos behavior region is starting from \( c \geq 4.2 \).

Figure 5. The Bifurcation diagram for the Rossler model. The intensity as a function of the control parameter \( c \) for the \( a \) and \( b \) values are constant and equal to 0.2.
3. Experimental Setup

The electronic circuit solves the 3 coupled differential equations of the Rössler attractor shown in figure 6, by replacing the nonlinear element by an analog multiplier chip. The circuits consist of resistors, capacitors, operational amplifiers and potentiometers (variable resistors). The output voltages $X$, $Y$ and $Z$ are registered with an oscilloscope which represents of the Rössler system. All op-amps powered by a ±12 V power source.

The experimental time series and attractor are shown in Figures 7 and 8 when the control parameter varied (variable resistor). The dynamic displays period doubling when the value of $R$ varied from (15-22.5) KΩ as shown in Figure 7. We notice the time series has two different amplitudes (see Figure 7(a)) and its attractor appears with two loops as represented in Figure 7(b), gradually increasing in control parameter the dynamics exhibit chaos and Rössler attractor obtained at 50 KΩ. Figure 8 a, b and c show respectively the time series of Rössler chaos, its attractor and corresponding FFT.

![Figure 6. Schematic of Rossler drive circuit using MultiSIM 13.0.](image)

![Figure 7. a) experimental results (a) time series of a system at $R=15$ KΩ, (b) the corresponding attractor.](image)
Figure 7 a) experimental results (a) time series of a system at R= 50 KΩ, (b) the corresponding attractor, (c) The corresponding FFT.

Conclusions
The Rössler model described continuous-time chaos, it exhibits a period-doubling route to chaos by increasing in the parameter c and keeping a and b fixed at 0.2. We obtained typical Rössler attractor at 5.7 of the value of c. in this paper we use bifurcation diagram to analyze the results, which it gives information about the behavior for all value of the parameter. Experimental design of the electronic circuit to study Rössler attractor is easier than Lorenz circuit because the third equation of the Rössler system has only a single nonlinear term.
References

[1] A. Sambas, M. Sanjaya W. S., and M. Mamat, 2013, Journal of Engineering Science and Technology Review 6 (4) 66-73.

[2] Qais H. Alsafasfeh, Mohammad S. Al-Arni, 2011, Circuits and Systems 2, 101-105.

[3] R. Barrio, F. Blesa, S. Serrano, 2009, Physica D 238, 1087_1100

[4] R. Barrio, M.A. Martinez, S. Serrano, D. Wilczak, 2015, Physics Letters A 379(38), 2300-2305.

[5] A E Botha and W Dednam, 2015, Proceedings of the 59th Annual Conference of the SAIP 59, 571.

[6] E N. Lorenz, 1963, J. Atmos. Sci. 20 (2), 130.

[7] O,E. Rössler, 1976, PHYSICS LETTERS, A 57,397.

[8] O.E. Rössler, 1983, Z. Naturforsch. 38a, 788.

[9] A T Azar and V Sundarapandian, 2015, Studies in Computational Intelligence, Springer-Verlag, Germany 581, ISBN 978-3-319-13131-3.

[10] K Kiers, D. Schmidt, J. C. Sprott, 2004, Am. J. Phys. 72, 503-509.

[11] P Gaspard, 2005, Encyclopedia of Nonlinear Science, Routledge, New York, pp. 808-811.