Kaluza-Klein on the Brane

G. Papadopoulos and P.K. Townsend

DAMTP, University of Cambridge,
Silver St., Cambridge CB3 9EW, U.K.

ABSTRACT

The M-theory interpretation of certain D=10 IIA p-branes implies the existence of worldvolume Kaluza-Klein modes which are expected to appear as 0-brane/p-brane bound states preserving 1/4 of the spacetime supersymmetry. We construct the corresponding solutions of the effective supergravity theory for $p = 1, 4$, and show that no such solution exists for $p = 8$. 

* Revised version
1. Introduction

There is now ample evidence that the IIA superstring theory is an $S^1$ compactification of an 11-dimensional supersymmetric quantum theory called M-theory. It was pointed out in [1,2] that this interpretation requires the presence in the non-perturbative IIA superstring theory of BPS-saturated particle states carrying Ramond-Ramond (RR) charge, corresponding to the Kaluza-Klein (KK) modes of D=11 supergravity, and it was argued that these should be identified with the IIA 0-branes. At the time, the only evidence for the required 0-branes was the existence of extreme electric ‘black hole’ solutions of the effective IIA supergravity theory [3], but their presence in the IIA superstring theory was subsequently confirmed by the interpretation of D-branes as the carriers of RR charge [4].

Actually, one needs not just the D-0-brane, for which the effective field theory realization is the extreme black hole of lowest charge associated with the first KK harmonic, but also a bound state at threshold in the system of $n$ D-0-branes for each $n > 1$, a prediction that has still to be confirmed although there is good evidence that it is true [5]. Assuming that these bound states exist, M-theory provides a KK interpretation of the D-0-branes of IIA superstring theory. However, as emphasized in [1], all the IIA p-branes must have a D=11 interpretation. Indeed, many of them can be interpreted as reductions, either ‘direct’ or ‘double’, of D=11 branes, i.e. M-branes. The cases of interest to us here are those IIA p-branes that that have a D=11 interpretation as (p+1)-branes wrapped around the compact 11th dimension. The massless worldvolume action for the D=10 p-brane is then a dimensional reduction on $S^1$ of the worldvolume action of the (p+1)-brane of M-theory. Thus, the D=11 interpretation of these D=10 p-branes requires the existence of massive particle-like excitations ‘on the brane’ that can be identified with the KK harmonics of the ‘hidden’ $S^1$. From the D=10 string theory perspective these excitations can only be BPS-saturated 0-brane/p-brane bound states [6]. Moreover, since the worldvolume KK states preserve 1/2 of the (p+1)-dimensional worldvolume supersymmetry and the p-brane preserves 1/2 of the spacetime su-
persymmetry, these ‘brane within brane’ states must preserve 1/4 of the spacetime supersymmetry [7]. The IIA p-branes for which we should expect to find such bound states are (i) the 1-brane, i.e. the fundamental IIA superstring, since this is a wrapped D=11 membrane, (ii) the D-4-brane, since this is a wrapped D=11 fivebrane, and possibly (iii) the D-8-brane, since it has been suggested [8] that the D-8-brane might be a wrapped D=11 ninebrane.

The required bound states are not difficult to identify in case (i). A fundamental string can end on a 0-brane; actually, charge conservation requires a 0-brane to be the end of at least two fundamental strings. Two such strings can be joined at their other ends to produce a closed string loop with a 0-brane ‘bead’. One could replace the 0-brane by a bound state of several 0-branes. Thus, the bound states needed for the KK interpretation of the IIA superstring as a wrapped D=11 membrane are an immediate consequence of the 0-brane bound states needed for the KK interpretation of the effective IIA supergravity theory. This is not so in cases (ii) and (iii) for which we need to find bound states of D-0-branes with D-4-branes or D-8-branes. The existence of such bound states is consistent with the ‘D-brane intersection rules’ [9,10] which allow, in particular, the possibility of a p-brane within a q-brane preserving 1/4 of the supersymmetry for $p = q \mod 4$. The issue of 0-brane/4-brane bound states has been discussed recently in the context of the D-brane effective action [6]. Here we investigate this question in the context of solutions of the effective IIA supergravity theory. We shall show that solutions representing 0-branes within p-branes preserving precisely 1/4 of the supersymmetry exist for $p = 1, 4$ but not otherwise (completing previous partial constructions by other methods [11]).

This result is consistent with the standard D=11 interpretation of all the type II p-branes for $p \leq 6$ but not with the interpretation of the IIA 8-brane as an $S^1$-wrapped M-theory ninebrane. A further argument against the M-theory ninebrane interpretation of the IIA 8-brane comes from consideration of a solution of IIA supergravity preserving 1/4 of the spacetime supersymmetry that represents a D-4-brane within a D-8-brane (the metric for this solution is already known [11];
here we present the complete solution). If there were an M-theory ninebrane it would be natural to interpret this 4-brane within 8-brane solution as a fivebrane within a ninebrane wrapped on $S^1$ along a fivebrane direction. However, if such an M-theory configuration were to exist it could also be reduced to a IIA 5-brane within an 8-brane but there does not exist any such solution of IIA supergravity preserving precisely 1/4 of the spacetime supersymmetry.

In what follows we shall use the notation $(q|q, p)$ to represent a q-brane within a p-brane preserving 1/4 of the supersymmetry. This is the special case of $(r|p, q)$, which we use to denote a solution representing an r-brane intersection of a p-brane with a q-brane. Thus, in this notation the supersymmetric solutions representing a 0-brane within a p-brane for $p = 1$ and $p = 4$ are $(0|0, 1)$ and $(0|0, 4)$. These solutions have magnetic duals, $(5|5, 6)$ and $(2|2, 6)$, respectively, whose existence is required by M-theory. To see this recall that the D=11 interpretation of the 6-brane is simply as a D=11 spacetime of the form $H_4 \times \mathbb{M}_7$ where $H_4$ is a particular (non-compact) hyper-Kähler manifold and $\mathbb{M}_7$ is 7-dimensional Minkowski spacetime [1]. Clearly, there is nothing to prevent the worldvolumes of either the D=11 membrane or fivebrane from lying within the $\mathbb{M}_7$ factor, and from the D=10 perspective this is a membrane or a 5-brane within a 6-brane. We shall show how the $(5|5, 6)$ and $(2|2, 6)$ can also be deduced from known intersecting M-brane solutions.

2. Branes within branes in IIA supergravity

As just explained, M-theory predicts the existence of a variety of IIA supergravity solutions preserving precisely 1/4 of the N=2 spacetime supersymmetry that represent ‘branes within branes’ (by ‘precisely’ we mean to exclude solutions preserving more than 1/4 of the supersymmetry). A summary of these predictions is as follows: we expect $(0|0, p)$ solutions for $p=1,4$ and possibly $p=8$, but not otherwise. We also expect the magnetic duals of $(0|0, p)$ for $p = 1, 4$, and a $(4|4, 8)$ solution.
That there are no \((0|0,2), (0|0,5)\) or \((0|0,6)\) solutions of IIA supergravity preserving 1/4 (as against any other fraction) of the supersymmetry follows from consideration of the projection operators associated with Killing spinors. A single p-brane solution is associated with a projection operator \(P_p\), of which precisely half the eigenvalues vanish, such that only spinors \(\kappa\) satisfying \(P_p\kappa = \kappa\) can be Killing. This accounts for the fact that such solutions preserve half the supersymmetry. Configurations representing a p-brane within a q-brane for \(p \neq q\) can also preserve some supersymmetry since \(P_p \) and \(P_q\) must either commute or anticommute. If \(P_p\) and \(P_q\) commute then the product \(P_pP_q\) is also a projector. In such cases one may find a supersymmetric solution preserving 1/4 of the supersymmetry, representing either two intersecting branes or a ‘brane within a brane’. If \(P_p\) and \(P_q\) anticommute then the matrix

\[
\alpha P_p + \beta P_q \quad (\alpha^2 + \beta^2 = 1)
\]

is another projector with precisely half of its eigenvalues vanishing. In this case one can hope to find ‘brane within brane’ solutions preserving 1/2 the supersymmetry. An example of such a solution is the D=11 membrane within a fivebrane solution [12]; as shown in [13,14], this reduces to a \((2|2,4)\) solution of IIA supergravity preserving 1/2 the supersymmetry. Consideration of T-duality then implies the existence of \((0|0,2)\) solutions preserving 1/2 the supersymmetry*. Here we are interested in solutions preserving precisely 1/4 of the supersymmetry, so only those cases for which \(P_p\) and \(P_q\) commute are relevant. When both branes are D-branes one can show that \(P_p\) and \(P_q\) commute if and only if \(q = p \mod 4\), so that \((0|0,2)\) and \((0|0,6)\) solutions preserving 1/4 of the supersymmetry are immediately excluded, whereas \((0|0,4), (2|2,6)\) and \((4|4,8)\) are allowed, as is \((0|0,8)\). This D-brane rule says nothing about \((0|0,1)\) or \((0|0,5)\) since neither the IIA string nor the IIA 5-brane is a D-brane. It happens that \(P_1\) commutes with \(P_0\) whereas \(P_5\) does not, so a \((0|0,1)\) solution preserving 1/4 of the supersymmetry is allowed

* It has been pointed out to us independently by J. Maldacena and J. Polchinski that such a solution could be interpreted as a D-2-brane boosted in the 11th dimension. The solution has since been constructed [15].

5
whereas a \( (0|0,5) \) solution is not. A putative \( (5|5,8) \) solution preserving 1/4 of the supersymmetry is similarly ruled out. For the reason given earlier, this fact is evidence against the existence of a \( (0|0,8) \) solution. Thus, the projection operator analysis provides arguments both for and against the possibility of a \( (0|0,8) \) solution.

Leaving aside \( (0|0,8) \), we have now seen that the solutions not expected from M-theory considerations are indeed absent, while the solutions that M-theory requires to exist are permitted. We shall now show that all of the latter, among those mentioned above, not only exist but can be constructed from known intersecting M-brane solutions preserving 1/4 of the supersymmetry \([16,17,18,19,20]\) by means of the various dualities connecting M-theory with the IIA and IIB superstring theories. The relevant M-theory solutions can be obtained from the ‘M-theory intersection rules’ determining the allowed M-brane intersections together with the ‘harmonic function rule’ that allows one to write down the general solution. For example, the \( (0|0,1) \) solution of IIA supergravity can be deduced from the solution of D=11 supergravity associated with the intersection of two membranes at a point, i.e. \( (0|2,2)_M \). This is achieved by consideration of the ‘duality chain’

\[
(0|2,2)_M \rightarrow (0|1,2)^T \rightarrow (0|1,1_D)_B^T \rightarrow (0|0,1) ,
\]

where the subscript \( B \) indicates a solution of IIB supergravity and \( 1_D \) denotes the IIB D-string. In the first step one of the two D=11 membranes is wrapped around the 11th dimension; the corresponding D=10 solution being obtained by double-dimensional reduction. In the second step we T-dualize along a direction parallel to the IIA 2-brane to arrive at the IIB solution. A further T-dualization along one of the two directions determined by the D-strings leads to the required IIA solution.

To make clear the unambiguous nature of the derivation we shall give all the intermediate solutions for this example, while giving just the final result for the
examples to follow. Thus, we begin with the (0|2, 2) solution of D=11 supergravity
\[ ds^2 = U^{1/3} V^{1/3} \left[ -U^{-1} V^{-1} dt^2 + U^{-1} ds^2(E^2) + V^{-1} ds^2(E^2) + ds^2(E^6) \right] \]
\[ G_4 = -3 dt \wedge d(U^{-1} J_1 + V^{-1} J_2) \] (2.3)
where (in the terminology of [17]) \( U, V \) are harmonic functions of the overall transverse space \( E^6 \) and \( J_1 \oplus J_2 \) is a complex structure on the relative transverse space \( E^2 \oplus E^2 \). Double-dimensional reduction along one of the relative transverse directions results in the following (0|1, 2) solution of IIA supergravity:
\[ ds_{(10)}^2 = V^{1/2} \left[ -U^{-1} V^{-1} dt^2 + U^{-1} dx^2 + V^{-1} du^2 + ds^2(E^7) \right] \]
\[ e^{\frac{3}{4} \phi} = U^{-\frac{2}{3}} V^{\frac{1}{3}} \]
\[ F_4 = -3 dt \wedge d(V^{-1} J_2) \]
\[ F_3 = -3 dt \wedge dx \wedge dU^{-1} \] (2.4)
where \( x \) is the string coordinate. Next, using the T-duality rules of [21] (adapted to our conventions) to T-dualize along one of the directions of the 2-brane, we get the (0|1, 1)\(_B\) solution
\[ ds_{(10)}^2 = V^{1/2} \left[ -U^{-1} V^{-1} dt^2 + U^{-1} dx^2 + V^{-1} du^2 + ds^2(E^7) \right] \]
\[ e^{\frac{3}{4} \phi} = U^{-\frac{2}{3}} V^{\frac{1}{3}} \]
\[ F_3^{(2)} = -3 dt \wedge du \wedge dV^{-1} \]
\[ F_3^{(1)} = -3 dt \wedge dx \wedge dU^{-1} \] (2.5)
where \( \phi \) is the IIB dilaton. Finally, we transform (2.5) using T-duality along the \( u \) direction to get the following (0|1, 0) \( \equiv \) (0|0, 1) solution of IIA supergravity:
\[ ds_{(10)}^2 = V^{1/2} \left[ -U^{-1} V^{-1} dt^2 + U^{-1} dx^2 + ds^2(E^8) \right] \]
\[ e^{\frac{3}{4} \phi} = U^{-\frac{1}{3}} V^{\frac{1}{2}} \]
\[ F_3 = -3 dt \wedge dx \wedge dU^{-1} \]
\[ F_2 = -\frac{9}{2} dt \wedge dV^{-1} \] (2.6)

The above solutions, as others given below, depend on two independent harmonic functions, each of which is associated with a single p-brane. For simplicity,
let us suppose that each harmonic function has just one singularity (at the position of the brane). Clearly, one must further suppose that both harmonic functions have their singularities at the same location in order to be able to interpret the configuration as a ‘brane within brane’ solution associated with the long range fields of a bound state. The same solution could equally well represent the simple coincidence of two branes; the fact that solutions exist with two independent harmonic functions indicates that any bound state would be a bound state at threshold. It is a weakness of the effective field theory approach that it cannot distinguish between a bound state at threshold of two branes or their simple coincidence because both have the same long range fields. The evidence for bound states provided by the effective field theory is, therefore, not particularly strong. Nevertheless, when the harmonic functions in (2.6) are restricted in the way just described these solutions do give the long range fields of the KK modes that arise from the wrapping of the D=11 membrane on $S^1$ to give a D=10 string.

Before proceeding we pause to remark that the magnetic dual of the $(0|0, 1)$ solution can be found from the $(3|5, 5)_{M}$ solution of M-theory by the following duality chain:

$$(3|5, 5)_{M} \rightarrow (3|5, 4)_{T} \rightarrow (4|5, 5_{D})_{B} \rightarrow (5|5, 6),$$

where $5_{D}$ denotes the D-5-brane of the IIB theory (5 denoting the NS-NS 5-brane). In the second step we have T-dualized in a direction parallel to the IIA 5-brane, which is mapped to the IIB NS-NS 5-brane under this operation. The final $(5|5, 6)$ solution dual to $(0|0, 1)$ (which has been found previously by other means [18]), is

$$ds^2 = UV\frac{1}{2}[U^{-1}V^{-1}ds^2(\mathbb{M}^6) + V^{-1}dv^2 + ds^2(\mathbb{E}^3)]$$
$$e^{\frac{2}{3}\phi} = U^{\frac{1}{3}}V^{-\frac{1}{2}}$$
$$F_3 = 9dv \wedge *dU$$
$$F_2 = 27*dV,$$

where $U, V$ are harmonic functions on the Euclidean transverse space $\mathbb{E}^3$ and $*$ is the Hodge dual for $\mathbb{E}^3$. 

8
We turn next to the $(0|0, 4)$ case. This can be found from the following duality chain,
\[(0|2, 2)_M \rightarrow (0|2, 2)_T \rightarrow (0|1_D, 3)_B \rightarrow (0|0, 4), \tag{2.9}\]
where the first step is the direct reduction to D=10 of the D=11 solution. The resulting $(0|0, 4)$ solution is
\[
\begin{align*}
  ds^2 &= U^\frac{1}{2} V^\frac{1}{2} \left[ -U^{-1} V^{-1} dt^2 + V^{-1} ds^2(\mathbb{E}^4) + ds^2(\mathbb{E}^5) \right] \\
  e^{\frac{\phi}{2}} &= V^\frac{1}{2} U^{-\frac{1}{2}} \\
  F_4 &= 3 \star dU \\
  F_2 &= -\frac{9}{2} dt \wedge dV^{-1},
\end{align*}
\]
where $U, V$ are harmonic functions on $\mathbb{E}^5$ and $\star$ is now the Hodge dual for $\mathbb{E}^5$. The magnetic dual of this solution is $(2|2, 6)$, which can be found from the duality chain
\[(3|5, 5)_M \rightarrow (2|4, 4)_T \rightarrow (2|3, 5_D)_B \rightarrow (2|2, 6). \tag{2.11}\]
The final $(2|2, 6)$ solution is
\[
\begin{align*}
  ds^2 &= U^\frac{1}{2} V^\frac{1}{2} \left[ U^{-1} V^{-1} ds^2(\mathbb{M}^3) + U^{-1} ds^2(\mathbb{E}^4) + ds^2(\mathbb{E}^3) \right] \\
  e^{\frac{\phi}{2}} &= V^\frac{1}{2} U^{-\frac{1}{2}} \\
  F_4 &= -\frac{3}{2} \epsilon(\mathbb{M}^3) dV^{-1} \\
  F_2 &= 27 \star dU,
\end{align*}
\]
where $U, V$ are harmonic functions on $\mathbb{E}^3$ and $\star$ is the Hodge dual for $\mathbb{E}^3$.

We remark that both $(0|0, 4)$ and its magnetic dual $(2|2, 6)$ can also be found from $(1|2, 5)_M$ as follows:
\[(1|2, 5)_M \rightarrow (1|2, 4)_T \rightarrow (0|1_D, 3)_B \rightarrow (0|0, 4), \tag{2.13}\]
and
\[(1|2, 5)_M \rightarrow (1|2, 4)_T \rightarrow (1|1_D, 5_D)_B \rightarrow (2|2, 6). \tag{2.14}\]
The above solutions confirm the current D=11 interpretations of all IIA p-
branes for \( p \leq 6 \). In addition, the duality chain of (2.14) can be continued as follows:

\[
(2|2, 6)_T \rightarrow (3|3, 7)_B \rightarrow (4|4, 8)
\]

In principle, the 7-brane appearing in the penultimate solution is the D-7-brane. However, the 7-brane solution needed for this construction is the ‘circularly-symmetric’ 7-brane of IIB supergravity since, as shown in [8], it is this solution that is mapped to either the 6-brane or the 8-brane solution of \( S^1 \) compactified IIA supergravity. A further point is that the T-duality transformations to be used in the last link of the duality chain are the ‘massive’ ones of [8] connecting solutions of IIB supergravity with those of the massive IIA supergravity theory. Apart from these subtleties the construction proceeds as before, with the final result

\[
\begin{align*}
    ds^2 &= U^{1/2} V^{1/2} (U^{-1} V^{-1} ds^2(\mathbb{M}^5) + U^{-1} ds^2(\mathbb{E}^4) + dy^2) \\
    e^{2\phi} &= V^{-\frac{1}{2}} U^{-\frac{5}{6}} \\
    M &= \partial_y U \\
    F_4 &= 3\epsilon(\mathbb{E}^4) \partial_y V,
\end{align*}
\]

where \( U, V \) are harmonic functions of \( y \).

Finally, we return to the question of whether there exists a \((0|0, 8)\) solution which might represent KK modes in a possible M-theory ninebrane interpretation of the IIA 8-brane. If it exists we should be able to deduce it from M-theory. It cannot be so deduced from the intersecting M-brane solutions considered so far, but there exists a solution of D=11 supergravity preserving 1/4 of the supersymmetry that has been interpreted as the intersection of two M-theory fivebranes on a string, i.e. as a \((1|5, 5)\) solution [20]. Taking this solution as the starting point of the following duality chain

\[
(1|5, 5)_M \rightarrow (0|4, 4)_T \rightarrow (0|3, 5)_B \rightarrow (0|2, 6)_T \\
\rightarrow (0|1D, 7)_B \rightarrow (0|0, 8),
\]

we could apparently deduce the existence of the sought \((0|0, 8)\) solution. However,
the starting \((1|5, 5)\) solution has a quite different form from the other intersecting M-brane solutions. In particular, the two harmonic functions associated with each fivebrane are independent of the ‘overall transverse’ coordinate. On the other hand, consistency with the KK ansatz needed for the various T-duality steps in the above chain requires that both harmonic functions be independent of all the other coordinates. Therefore, the only acceptable starting solution for the above duality chain is the special case of the \((1|5, 5)\) solution for which both harmonic functions are constant; this is just the Minkowski vacuum which obviously preserves all the supersymmetry rather than just \(1/4\) of it. Of course, this shows only that a IIA \((0|0, 8)\) solution preserving precisely \(1/4\) of the supersymmetry cannot be obtained from a particular starting point. However, supposing such a solution to exist we could reverse the steps in the duality chain (2.16) to deduce the existence of a \((1|5, 5)\) solution of ‘conventional’ form for intersecting M-branes, i.e. with both harmonic functions depending only on the overall transverse coordinate. It is not difficult to see that there is no such solution because the associated 4-form field strength does not satisfy the field equation \(d \ast G = G \wedge G\). Thus, there is no \((0|0, 8)\) solution of the required type.

3. Conclusions

The interpretation of certain \(p\)-brane solutions of IIA supergravity as wrapped \((p+1)\)-branes of M-theory requires the existence of massive KK modes ‘on the brane’. In turn, this requires the existence (and in other cases, absence) of ‘brane within brane’ solutions of IIA supergravity preserving \(1/4\) of the supersymmetry. We have shown that the list of such solutions is compatible both with the current M-theory interpretations of the IIA \(p\)-branes with \(p \leq 6\), but not with an interpretation of the IIA 8-brane as an M-theory 9-brane.

Acknowledgments: GP thanks The Royal Society for a University Research Fellowship. We thank the organisers of the Benasque Centre for Physics in Spain,
where part of this work was done, Michael Douglas, Michael Green and Christopher Hull for discussions. We also thank Juan Maldacena and Joseph Polchinski for comments on an earlier version of this paper and especially Eric Bergshoeff, who pointed out a serious error in it.

REFERENCES

1. P.K. Townsend, Phys. Lett. **350B** (1995) 184.
2. E. Witten, Nucl. Phys. **B443** (1995) 85.
3. G.T. Horowitz and A. Strominger, Nucl. Phys. **B360** (1991) 197.
4. J. Polchinski, Phys. Rev. Lett. **75** (1995) 4724.
5. A. Sen, Phys. Rev. **D54** (1996) 2964.
6. M. Douglas, D. Kabat, P. Pouliot and S.H. Shenker, *D-branes and Short Distances in String Theory*, hep-th/9608024.
7. M. Douglas, *Branes within Branes*, hep-th/9512077.
8. E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, Nucl. Phys. **B470** (1996) 113.
9. J. Polchinski, S. Chaudhuri and C. Johnson, *Notes on D-branes*, hep-th/9602052.
10. M.B. Green and M. Gutperle, Nucl. Phys. **B476** (1996) 484.
11. K. Behrndt, E. Bergshoeff and B. Janssen, *Intersecting D-branes in Ten and Six Dimensions*, hep-th/9604168.
12. J.M. Izquierdo, N.D. Lambert and G. Papadopoulos and P.K. Townsend, Nucl. Phys. **B460**, (1996) 560.
13. G. Papadopoulos, *A brief guide to p-branes*, hep-th/9604068.
14. M.B. Green, N.D. Lambert, G. Papadopoulos and P.K. Townsend, Phys. Lett. **384B** (1996) 86.
15. A. Tseylin and J. Russo, *Waves, boosted branes and BPS states in M Theory*, hep-th/9611047.

16. R. Güven, Phys. Lett. **276B** (1992) 49.

17. G. Papadopoulos and P.K. Townsend, Phys. Lett. **380B** (1996) 273.

18. A. Tseytlin, Nucl. Phys. **B475** (1996) 149.

19. A. Tseytlin and I. Klebanov, Nucl. Phys. **B475** (1996) 179.

20. J.P. Gauntlett, D.A. Kastor and J. Traschen, Nucl. Phys. **B478** (1996) 544.

21. E. Bergshoeff, C.M. Hull and T. Ortín, Nucl. Phys. **B451** (1995) 547.