Dense Matter in Neutron Star: Lessons from GW170817

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Abstract Neutron star merger event GW170817 sets an upper limit on the maximum mass of non-rotating neutron stars. Consequently, this event puts strong constraints on the dense matter equation of state (EoS). A comparative study of dense matter equations of state (EoSs) is presented here. It is found that the $\Lambda$ hyperon EoS $\phi$ (Banik, Hempel & Bandyopadhyay 2014) constructed within the framework of the density dependent hadron field theory is favoured.

1 Introduction

S. Chandrasekhar predicted the mass limit for the first family of compact astrophysical objects known as White Dwarfs [1]. Next L.D. Landau should be credited for his idea about the second family of compact objects, as 'giant nucleus' [2]. After the discovery of neutrons, it was realised that the second family might be neutron stars [3]. First pulsar was discovered in 1967 [4]. We are celebrating 50 years of the discovery of first pulsar in 2017. What could be a better celebration than finding the neutron star merger event GW170817 [5]. This stands out as a very important discovery in the history of mankind.

Neutron star merger event GW170817 was discovered both in gravitational waves and light. The gravitational wave signal was observed in LIGO detectors [5]. A short Gamma Ray Burst (sGRB) was recorded 1.7 s after the merger by the Fermi-GBM [6]. This, for the first time, established a link between a neutron star merger event and sGRB. Later electromagnetic signals in visible, ultra-violet and infra-red bands were detected from the ejected matter which formed a 'kilonova'.

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GW170817 is a boon to the nuclear astrophysics community because it allows to probe compositions and EoS in neutron star and r-process nucleosynthesis in the ejected neutron rich matter. The merger event provides crucial information about the remnant and neutron stars in the binary. The chirp mass is estimated to be $1.188^{+0.004}_{-0.002} \, M_\odot$. Assuming low spins as found from observations of neutron stars in our Galaxy, individual neutron star mass in the binary ranges $1.17$-$1.60 \, M_\odot$. The massive remnant formed in the merger has a mass $2.74^{+0.04}_{-0.01}$.

The outstanding question is what happened to the massive remnant formed in GW170817. The prompt collapse of it to a black hole is ruled out because large amount of matter was ejected. In this situation, either the remnant is a long lived massive neutron star or it collapsed to a black hole. Recent x-ray observation using the Chandra observatory indicates that the massive remnant might be a black hole.

It is possible to estimate the upper limit on the maximum maximum ($M_{\text{TOV}}^{\text{max}}$) of the non-rotating neutron star if the remnant becomes a black hole through delayed collapse. Different groups have determined the upper limit on $M_{\text{TOV}}^{\text{max}}$ from the multi-messenger observation of GW170817 as well as from numerical relativity. All these estimates converge to the same value of $\sim 2.16 \, M_\odot$ for the upper limit on $M_{\text{TOV}}^{\text{max}}$. It is already known from the observations of neutron stars that the most massive neutron star has a $2.01 \, M_\odot$ which sets the lower limit on $M_{\text{TOV}}^{\text{max}}$.

We organise the article in the following way. We introduce the density dependent hadron field theory and BHB EoS in Section 2. Results are discussed in Section 3. We conclude in section 4.

### 2 Equation of State for Neutron Star Matter

Equation of state is an important microphysical input for the study of core-collapse supernovae (CCSN), neutron stars and neutron star mergers. For CCSN and neutron star merger simulations, an EoS is a function of three parameters - density, temperature and proton fraction. These parameters vary over wide range of values. For example, density varies from $10^2$ – $10^{15} \, g/cm^3$, temperature from 0 to 150 MeV and proton fraction from 0 to 0.6. In this study, we focus on neutron star EoSs which are derived from the EoS constructed for CCSN and neutron star merger simulations. Particular, we describe here the BHB EoS and adopt the same for our calculation.

The compositions of matter in CCSN and neutron star changes with density, temperature and proton fraction. Below the saturation density ($2.7 \times 10^{14} \, g/cm^3$) and low temperature, nuclei and nuclear clusters are present and make the matter inhom-
Dense Matter in Neutron Star: Lessons from GW170817

In this case, non-uniform matter is made of light and heavy nuclei, nucleons and leptons in thermodynamic equilibrium. Matter above the saturation density is uniform. Several novel phases of matter such as hyperons, kaon condensate or quarks might appear at higher densities. We discuss both (non-)uniform matter in the following subsections.

2.1 Non-uniform matter

Here the in-homogeneous matter is described by an extended version of the Nuclear Statistical Equilibrium (NSE) model that was developed by Hempel and Schaffner (HS) [16]. The extended NSE model takes into account interactions among nucleons, interaction of nuclei or nuclear clusters with the surrounding medium. Furthermore, the Coulomb interaction is considered.

Interactions among unbound nucleons are treated in the relativistic mean field (RMF) approximation using a density dependent relativistic hadron field theory. Nuclei are considered as classical particles described by the Maxwell-Boltzmann statistics. Binding energies of thousands of nuclei entering into the calculation are obtained from the nuclear mass data table [17]. When experimental values are not available, theoretically calculated values are exploited [18]. Medium modifications of nuclei or nuclear clusters due to the screening of Coulomb energies of background electrons as well as corrections due to excited states and excluded volume effects are taken into account in this calculation.

The total canonical partition function of the in-homogeneous matter is given by,

\[ Z(T, V, \{ N_i \}) = Z_{\text{nuc}} \prod_{A,Z} Z_{A,Z} Z_{\text{Coul}} . \]

where the first term gives the contribution of non-interacting nuclei, \( f_{\text{Coul}} \) corresponds to the Coulomb energy, the contribution of interacting nucleons \( f_{\text{nuc}} \) is multiplied by the available volume fraction of nucleons \( \xi \), \( n'_n \) and \( n'_p \) are local neutron and proton number densities and the last term goes to infinity when available volume fraction of nuclei \( \kappa \) is zero near the saturation density. The number density of nuclei is given by the modified Saha equation [13, 16],

\[ n_{A,Z} = \kappa g_{A,Z}(T) \left( \frac{M_{A,Z} T}{2\pi} \right)^{3/2} \exp \left( \frac{(A-Z)\mu_n^0 + Z\mu_p^0 - M_{A,Z} - E_{A,Z}^{\text{Coul}} - P_{\text{nuc}} V_{A,Z}}{T} \right) . \]
where the meaning of different quantities in the equation can be found from Ref. [13, 16]. Finally, the pressure is calculated as mentioned in Ref. [13, 16].

2.2 Density dependent field theory for dense matter

We calculate the EoS of uniform matter above the saturation density at finite temperature within the frame of a density dependent relativistic hadron field theory [12, 13]. In this case, the dense matter is made of neutrons, protons, hyperons and electrons. Being the lightest hyperons, Λ hyperons populate the dense matter first. Furthermore, heavier hyperons such as Σ and Ξ are excluded from this calculation because very little is known about their interaction in nuclear medium experimentally. The starting point here is the Lagrangian density for baryon-baryon interaction mediated by the exchange of σ, ω and ρ mesons. The interaction among Λ hyperons is taken into account by the exchange of φ mesons [13] as described by the Lagrangian density,

\[
\mathcal{L}_B = \sum_B \bar{\psi}_B \left( i \gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - g_{\phi B} \gamma_\mu \phi^\mu - g_{\rho B} \gamma_\mu \tau_B \cdot \rho^\mu \right) \psi_B \\
+ \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} \\
+ \frac{1}{2} \rho_{\mu \nu} \rho^{\mu \nu} - \frac{1}{4} \phi_{\mu \nu} \phi^{\mu \nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu \\
- \frac{1}{4} \rho_{\mu \nu} \cdot \rho^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^{\mu}. \tag{4}
\]

Here \( \psi_B \) denotes the baryon octets, \( \tau_B \) is the isospin operator and \( g_s \) are density dependent meson-baryon couplings. It is to be noted that φ mesons are mediated among Λ hyperons only. The pressure is given by [13],

\[
P = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 + \frac{1}{2} m_\phi^2 \phi^2 + \Sigma^r \sum_{B=n,p,\Lambda} n_B \\
+ 2T \sum_{i=n,p,\Lambda} \int \frac{d^3 k}{(2\pi)^3} \left[ \ln(1 + e^{-\beta(E_i - \nu_i)}) + \ln(1 + e^{-\beta(E_i + \nu_i)}) \right], \tag{5}
\]

where the temperature is defined as \( \beta = 1/T \) and \( E_i = \sqrt{(k^2 + m_i^2)} \). This involves the rearrangement term \( \Sigma^r \) [13, 19] due to many-body correlations which is given by

\[
\Sigma^r = \sum_B \left[ -g_{\sigma B} \sigma n_B^e + g_{\omega B} \omega_\alpha n_B + g_{\rho B} \tau_\beta \rho_\alpha n_B + g_{\phi B} \phi_\alpha n_B \right], \tag{6}
\]

where ' denotes derivative with respect to baryon density of species B.
The energy density is
\[ \varepsilon = \frac{1}{2}m_\sigma^2 \sigma^2 + \frac{1}{2}m_\omega^2 \omega_0^2 + \frac{1}{2}m_\rho^2 \rho_0^2 + \frac{1}{2}m_\phi^2 \phi_0^2 + 2 \sum_{i=n,p,\Lambda} \left( \frac{3}{\beta \beta(E^i - v_i)} + 1 + \frac{1}{e^{\beta(E^i + v_i)} + 1} \right). \] (7)

Parameters of the Lagrangian density are computed using available experimental data at the saturation density. Meson-nucleon couplings are determined by fitting the properties of finite nuclei using some functional forms of density dependent couplings [12]. This parameter set is known as the DD2. For vector meson couplings of \( \Lambda \) hyperons, we exploit the SU(6) symmetry relations whereas the scalar coupling is obtained from \( \Lambda \) hypernuclei data with a potential depth of \(-30\) MeV at the saturation density [20].

The descriptions of non-uniform and uniform matter are matched at the crust-core boundary in a thermodynamically consistent manner [13]. Charge neutrality and \( \beta \)-equilibrium conditions are imposed for neutron star matter.

![Fig. 1 Pressure versus energy density (EoS) is shown for the DD2, BHBA, and SFHo EoS models.](image)

### 3 Maximum Mass of Neutron Star

Here we discuss the results of our calculation. As discussed in the preceding section, we consider neutron star matter made of neutrons, protons, \( \Lambda \) hyperons and
electrons in the DDRH model. The EoS corresponding to nucleons only matter is denoted as the DD2 whereas the EoS of dense matter involving Λ hyperons is known as the BHBAφ. We also include the SFHo nuclear EoS of Steiner et al. in this discussion [21]. Figure 1 displays the EoSs (pressure versus energy density) corresponding to the DD2, BHBHAφ and SFHo models. It shows that the DD2 EoS is the stiffest among the three. Further we note that the SFHo EoS was softer over a certain region of energy density but becomes stiffer at higher densities than the BHBAφ. However, it follows from the structure calculation using the Tolman-Oppenheimer-Volkoff (TOV) equation that the overall SFHo EoS is softer compared with the BHBAφ EoS. Maximum masses of non-rotating neutron stars are 2.42, 2.11 and 2.06 M⊙ corresponding to the DD2, BHBAφ and SFHo EoS, respectively. All these EoSs are compatible with the observed 2 M⊙ neutron star [11].

We also compute the structures of rotating neutron stars using the LORENE library [22, 23]. Mass-radius relationships of (non)-rotating neutron stars are exhibited in Fig. 2. The sequences of non-rotating neutron stars (bottom curve) and uniformly rotating neutron stars (upper curve) at Keplerian frequencies are plotted for the DD2 EoS in the left panel and for the hyperon EoS BHBAφ in the right panel. Horizontal lines in both panels are fixed rest mass sequences. Those are denoted as normal and supramassive sequences. Rotating neutron stars evolve along those sequences keeping the total baryon mass conserved. The normal sequence finds its counterpart on the non-rotating star branch spinning down whereas neutron stars following the supramassive sequence would finally collapse into black holes. Any evolutionary sequence above the maximum mass rotating neutron star is known as
the hypermassive sequence and a neutron star in this sequence would be stabilised only by differential rotation before collapsing into a black hole in a few tens of milliseconds. Recently, it was demonstrated that the relation between the maximum mass ($M_{\text{max}}^{\text{Rot}}$) of the rotating neutron star at the Keplerian frequency and that ($M_{\text{max}}^{\text{TOV}}$) of the non-rotating neutron star satisfied a universal relation [24, 25]. This relation is given by [24]

$$M_{\text{max}}^{\text{Rot}} = 1.203 \pm 0.022M_{\text{max}}^{\text{TOV}}. \quad (8)$$

With this understanding of different evolutionary sequences respecting total baryon mass conservation, we discuss the fate of the massive remnant formed in merger event GW170817. The remnant could not be a hypermassive neutron star undergoing a prompt collapse to a black hole because a large amount of ejected matter was observed in the event [8]. This implies that the massive remnant existed for some duration. However, a long lived massive remnant is ruled out because of a sGRB sighted 1.7 s after the merger. It is inferred that the massive remnant collapsed to a black hole close to the maximum mass of a uniformly rotating sequence [9, 10]. This description might be intimately tied to the maximum mass of the non-rotating neutron star.

It is estimated from the observation of neutron star merger event GW170817 assuming low dimensionless spins for the neutron stars in the binary that the total binary mass was $\sim 2.74 \, M_\odot$. The mass loss from the merged object due to emissions of gravitational waves and neutrinos and ejected neutron rich matter amounts to $\sim 0.15 \pm 0.03 \, M_\odot$ [26]. Consequently, the mass of the remnant reduced to $\sim 2.6 \, M_\odot$. If we identify this mass of the remnant that might have collapsed into a black hole, with the maximum mass of the uniformly rotating neutron star at the Keplerian frequency i.e. $M_{\text{max}}^{\text{Rot}}$ of Eq. (8), an upper limit on the maximum mass of non-rotating neutron stars might be obtained [9]. It follows from Eq. (8) that the upper limit is $\sim 2.16 \, M_\odot$. It is already known from the observations of galactic pulsars that the lower limit on the maximum mass of non-rotating neutron stars is $2.01 \, M_\odot$. All these information put together lead to

$$2.01M_\odot \leq M_{\text{max}}^{\text{TOV}} < 2.16M_\odot. \quad (9)$$

Different groups converged almost to the same value of the upper limit from different analyses of GW170817 [8] [9] [10] [26].

### 3.1 Constraint on EoS

We discuss the implications of the lower and upper limits of the maximum mass on EoSs. Mass-radius relationships corresponding to the DD2, BHBA$\phi$ and SFHo EoSs are plotted in Fig. 3. The lower and upper limits on the maximum mass are also indicated by two horizontal lines. It is evident from the figure that the BHBA$\phi$ and SFHo EoSs are consistent with both limits of the maximum mass. But this is not case with the DD2 EoS because it fails to satisfy the upper limit. It is to be
Dense matter in neutron star

Fig. 3 Mass-radius relationships of non-rotating neutron stars are shown for the DD2, BHB $\Lambda\phi$ and SFHo EoSs. Horizontal lines denote the lower bound and upper bound on the maximum mass as given by Eq. (9).

noted that the DD2, BHB $\Lambda\phi$ and SFHo EoSs are being used for neutron star merger simulations by various groups [15, 26].

Next we perform a comparative study of different EoSs. Particularly, we look at the nuclear matter properties of all EoSs such as the saturation density ($n_0$), binding energy ($E_0$), incompressibility ($K$), symmetry energy ($S$) and its density slope ($L$). The nuclear matter properties of eight EoSs are recorded in Table 1. The last row of the table gives experimental values of nuclear matter properties [27]. First five of those EoSs for example, Lattimer-Swesty 200 (LS200) [28], Skyrme Lyon (SLy) [29], Müller-Serot 1 (MS1) [30], Akmal-Pandharipande-Ravenhall 4 (APR4) [31] and hyperon EoS H4 [32] were used in the analysis of GW170817 [5] because all of them satisfy the lower limit on the maximum mass. It is to be noted that all are nucleons only EoSs except the H4 EoS. A closer look at the nuclear matter properties at the saturation density of first five EoSs throw up important information about their behaviour at higher densities. It is evident from the Table that one or more observables of nuclear matter in case of LS220, MS1, APR4 and H4 EoSs are not consistent with the experimental values. This leads to a very soft or stiff EoS in those cases. For example, high values of incompressibility ($K$) for the APR4 and H4 make them stiffer EoSs. The threshold for the appearance of hyperons is shifted to a lower density for a very stiff EoS leading to the large population of hyperons in dense matter and resulting in a lower maximum mass neutron star as it is happening in case of the H4 EoS. For the LS220 and MS1 EoSs, the density slope ($L$) of symmetry energy is much higher than the experimental value. As a result, the maximum mass for the MS1 EoS is higher than the upper limit of $2.16 M_\odot$. However, the interplay between a lower value of $K$ and higher value of $L$ for the LS220 EoS determines the maximum mass which falls well within the limits of Eq. (9). Though
the SLy EoS is consistent with the experimental values and observational limits on the maximum mass, it is a non-relativistic EoS and superluminal behaviour could be a problem in this case at very high density \( (5-8 n_0) \) \[31\]. We have already discussed the last three EoSs of the Table. The nuclear matter properties of the DD2, SFHo and BHBA\( \phi \) EoSs are in good agreement with the experimental values. However, it is concluded that the DD2 EoS is ruled out by Eq. (9). It is possible to further constrain EoSs using the measured tidal deformability from GW170817. Based on the tidal deformability of GW170817, the H4, APR4 and LS220 EoSs are excluded whereas the BHBA\( \phi \) EoS is consistent with GW170817 data \[33\].

### Table 1 Nuclear matter properties of different EoSs used in the analysis of GW170817 \[5\] and in this article are recorded here. Experimental values of saturation density \( (n_0) \), binding energy \( (E_0) \), incompressibility \( (K) \), symmetry energy \( (S) \) and its density slope \( (L) \) are listed in the last row of this table \[27\]. Maximum mass of non-rotating neutron stars corresponding to each EoS is also shown here. Lower bound on the maximum mass of non-rotating neutron stars is mentioned in the last row \[11\].

| EoS     | \( n_0 \) [fm\(^{-3}\)] | \( E_0 \) [MeV] | \( K \) [MeV] | \( S \) [MeV] | \( L \) [MeV] | \( M_{\text{max}} \) [M\(_\odot\)] |
|---------|--------------------------|----------------|-------------|-------------|-------------|-----------------|
| LS220   | 0.1550                   | 16.00          | 220         | 28.61       | 73.82       | 2.06            |
| SLy     | 0.160                    | 15.97          | 230         | 32.00       | 45.94       | 2.05            |
| MS1     | 0.1484                   | 15.75          | 250         | 35.00       | 110.00      | 2.77            |
| APR4    | 0.160                    | 16.00          | 266         | 32.59       | 58.46       | 2.19            |
| H4      | 0.153                    | 16.3           | 300         | 32.5        | 94.02       | 2.02            |
| DD2     | 0.1491                   | 16.02          | 243         | 31.67       | 55.04       | 2.42            |
| SFHo    | 0.1583                   | 16.19          | 245         | 31.57       | 47.10       | 2.06            |
| BHBA\( \phi \) | 0.1491                  | 16.02          | 243         | 31.67       | 55.04       | 2.11            |
| Exp.    | \( \sim 0.15 \)          | \( \sim 16 \)  | 240 \( \pm 10 \) | 29.0 – 32.7 | 40.5 – 61.9  | 2.01 \( \pm 0.04 \) |

### 4 Summary and conclusion

We have investigated the equations of state of dense matter within the framework of the density dependent relativistic hadron field theory. The nucleons only EoS is denoted as the DD2 whereas the \( \Lambda \) hyperon EoS is known as the BHBA\( \phi \). The neutron star merger event GW170817 gives an upper limit on the maximum mass of non-rotating neutron stars whereas the lower limit is already known from the observations of pulsars. The upper and lower limits severely constrain the EoS as we have found through a comparative study of eight EoSs and their nuclear matter properties. It is found that the BHBA\( \phi \) EoS is consistent with both limits of the maximum mass and the tidal deformability of GW170817.

**Acknowledgements** S.B. and D.B. gratefully remember the support and encouragement that they received always from Professor Walter Greiner.
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