GLOBAL CONSTRAINTS ON KEY COSMOLOGICAL PARAMETERS

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Abstract

Data from Type Ia supernovae, along with X-ray cluster estimates of the universal baryon fraction and Big Bang Nucleosynthesis (BBN) determinations of the baryon-to-photon ratio, are used to provide estimates of several global cosmological parameters at epochs near zero redshift. We show that our estimate of the present baryon density is in remarkably good agreement with that inferred from BBN at high redshift, provided the primordial abundance of deuterium is relatively low and the Universe is flat. We also compare these estimates to the baryon density at \( z \approx 1100 \) as inferred from the CMB angular power spectrum.
1 Introduction

In the precision era of cosmology the accuracy in determining the key cosmological parameters will be limited by our ability to constrain systematic errors and identify observations which break the many degeneracies between the global cosmological parameters and those related to specific models of inflation and/or structure formation. Redundancy can also provide valuable probes of unanticipated systematic uncertainties and may serve to separate global from model dependent parameters. In particular, although precision measurements of the anisotropy of the cosmic microwave background (CMB) promise statistically accurate determinations of many cosmological parameters, their interpretation may be limited by the extent to which parameter degeneracies can be broken and systematic uncertainties can be constrained.

As a step in this direction, here we use current estimates of a restricted set of global cosmological parameters which are unaffected by “bias” (in mass versus light) and are independent of specific models of structure formation and specific theories of inflation to bound a variety of the key cosmological parameters. Such global cosmological constraints can then be employed in testing models of structure formation and inflation.

The choice of which observations may provide the best constraints on the global cosmological parameters is time-dependent and subjective (see, for example, Steigman, Hata & Felten 1999 (SHF); Bahcall et al. 1999). It is our goal here to minimize the observational input while maximizing the predicted output. To this end, we first utilize only the magnitude-redshift data from the type Ia supernovae (Perlmutter et al. 1997, 1999; Riess et al. 1997, 1998; Schmidt et al. 1998; for a recent review and extensive references, see Filipenko & Riess 2000). In a FRW cosmology with an equation of state limited to two components, matter with zero pressure ($p = 0$) and vacuum energy (or a cosmological constant $\Lambda$) with negative pressure, $p = -\rho$, the assumption of flatness, in concert with the SNIa data, constrain the matter density ($\Omega_M$) and the vacuum energy density ($\Omega_\Lambda$) (Goobar & Perlmutter 1995). Armed with $\Omega_M$ and $\Omega_\Lambda$ we determine a variety of the other key cosmological parameters such as the deceleration parameter ($q_0 = \frac{\Omega_M}{2} - \Omega_\Lambda$) and the dimensionless age of the Universe ($H_0 t_0$). Furthermore, since a non-BBN, non-CMB constraint on the baryon density is of great current interest, we use estimates of the universal baryon fraction ($f_B$) derived from X-ray observations of galaxy clusters to bound the zero-redshift baryon density (White
et al. 1993 (WNEF); Steigman & Felten 1995; SHF). This independent deter-
mination of the baryon density (at zero redshift) may be compared to the high redshift estimates inferred from BBN (see Olive, Steigman, & Walker 2000 and references therein) and from CMB anisotropy measurements (De Bernardis et al. 2000; Hanany et al. 2000; Lange et al. 2000; Jaffe et al. 2000 and further references therein). Next, knowledge of $\Omega_M$ and $\Omega_B$, along with the HST Key Project (Mould et al. 1999) constraint on the Hubble parameter permits us to bound other cosmological parameters such as the “shape” parameter $\Gamma$ (Peacock & Dodds 1993; Sugiyama 1995; Peacock 1997; SHF) and to estimate the present age of the Universe $t_0$.

We also explore a complementary approach by discarding the assumption of flatness ($k = 0$), and instead fixing the universal density of baryons from BBN. This along with estimates of the universal baryon fraction from the X-ray cluster data permits us to bound the total matter density, which may then be combined with the SNIa magnitude-redshift data to obtain constraints on the cosmological constant. With these constraints on $\Omega_M$ and $\Omega_\Lambda$ we proceed to bound the 3-space curvature ($\Omega_k \equiv 1 - (\Omega_M + \Omega_\Lambda)$), and the other global cosmological parameters such as $q_0$, $H_0 t_0$ (along with the age $t_0$), and $\Gamma$.

2 Non-BBN, Non-CMB Cosmological Constraints: Maximum Returns For Minimum Investment

Two major groups have mounted systematic investigations of the high-redshift SNIa magnitude-redshift relation, the “Supernova Cosmology Project” (SCP) of Perlmutter et al. (1997, 1999) and the “High-Z Supernova Search Team” (HZT) of Schmidt et al. (1998). S. Jha and the HZT have kindly made available to us the combined likelihoods and it is this joint data set we employ in our analysis. In our approach the computed quantities are the likelihood distributions for the various cosmological parameters (see Figures 2, 3, and 6). Since none of our resulting distributions are perfectly gaussian, we report our quantitative results in two ways. We quote results in the form $A^{+a_+}_{-a_-}$, where $A$ is the most likely value and the range from $A - a_-$ to $A + a_+$ defines a 68% confidence region bounded by equiprobable points. We also quote the full 95% confidence range.
Figure 1: The 68% (solid) and 95% (dotted) contours in the $\Omega_\Lambda - \Omega_M$ plane allowed by the magnitude-redshift relation inferred from the joint (HZT and SCP) SNIa data. Closed universes lie in the region to the right and above the $k = 0$ line while decelerating universes lie below and to the right of the $q_0 = 0$ line (dashed).

As is by now well known, the SNIa data identify a preferred region in the $\Omega_\Lambda - \Omega_M$ plane (see Figure 1) favoring an accelerating Universe ($q_0 < 0$). The SNIa contours avoid $\Omega_\Lambda = 0$, and are cut by the flatness relation ($k = 0$): $\Omega_k \equiv 1 - \Omega_\Lambda + \Omega_M = 0$. If flatness is imposed as a constraint, the SNIa degeneracy between $\Omega_M$ and $\Omega_\Lambda$ is broken and we find, in agreement with the SCP result of Perlmutter et al. (1999) and the HZT result of Filipenko & Riess (2000),

$$Flat \ (k = 0): \ \Omega_M \equiv 1 - \Omega_\Lambda = 0.28^{+0.08}_{-0.07}. \ \ \ (1)$$

The likelihood distribution for $\Omega_M$ is shown in panel $a$ of Figure 2. The corresponding 95% range, $0.15 \leq \Omega_M \leq 0.45$, is entirely consistent with independent estimates (e.g., Cole et al. 1997; Carlberg et al. 1997; Bahcall
Under the assumption of flatness the deceleration parameter is $q_0 = \frac{3\Omega_M - 1}{2}$ which, for the above value of $\Omega_M$, leads to $q_0 = -0.58^{+0.12}_{-0.10}$. The likelihood distribution is shown in panel $b$ of Figure 2; the corresponding 95% range is $-0.35 \geq q_0 \geq -0.77$. The Universe is accelerating.

A flat Universe has a dimensionless age (the age, $t_0$, in units of the “Hubble age”, $H_0^{-1}$) of $H_0t_0 = \frac{2}{3\Omega_M^2}\sinh^{-1}(\frac{\Omega_M}{\Lambda})^{1/2}$. We find $H_0t_0 = 0.96^{+0.07}_{-0.05}$, in excellent agreement with the SCP result $H_0t_0 = 0.96^{+0.09}_{-0.07}$ (Perlmutter et al. 1998) and that from the HZT, $H_0t_0 = 0.94^{+0.07}_{-0.05}$ (Riess et al. 1998; Filipenko & Riess 2000). The 95% range is $0.85 \leq H_0t_0 \leq 1.13$; the corresponding likelihood distribution is shown in panel $c$ of Figure 2. For

Figure 2: Likelihood distributions, normalized to unit maximum, for several global cosmological parameters as inferred from the SNIa data for a flat universe ($k = 0$). Panel $a$ shows the total (baryonic plus CDM) density parameter $\Omega_M$; panel $b$ shows the deceleration parameter, $q_0$; the age of the Universe in units of the Hubble age, $H_0t_0$, is shown in panel $c$; the shape parameter $\Gamma$ is shown in panel $d$. 
$H_0 = 71 \pm 6 \text{kms}^{-1}\text{Mpc}^{-1}$, (HST Key Project: Mould et al. 1999) the age of the Universe is $t_0 = 13.2^{+1.6}_{-1.3}$ Gyr; the 95% range, $10.8 \leq t_0(\text{Gyr}) \leq 16.7$, is in excellent agreement with that inferred from globular clusters (Chaboyer 2000; Chaboyer & Krauss 2000).

The shape parameter $\Gamma$ (Peacock & Dodds 1993; Sugiyama 1995; Peacock 1997; SHF) depends on $\Omega_M$, $\Omega_B$, $h \equiv H_0/100\text{kms}^{-1}\text{Mpc}^{-1}$, and the tilt parameter $n$. For $k = 0$ and $n = 1$ we find $\Gamma = 0.16^{+0.05}_{-0.04}$ and a 95% range of $0.08 \leq \Gamma \leq 0.26$, which has considerable overlap with that inferred from observations of large scale structure (Fisher, Scharf, & Lahav 1994; Webster et al. 1998; Eisenstein & Zaldarriaga 1999; Efstathiou & Moody 2000). The likelihood distribution for $\Gamma$ is shown in panel $d$ of Figure 2.

In summary, the SNIa data and the assumption of a flat Universe lead to a set of values for the global cosmological parameters $\Omega_M$, $\Omega_\Lambda$, and $q_0$, along with the large scale structure parameter $\Gamma$, which are consistent with other observational data and provide support for a Universe old enough to accommodate its oldest stars.

### 2.1 X-Ray Cluster Baryon Fraction and the Baryon Density

Rich clusters of galaxies, the largest collapsed systems in the Universe, provide an ideal laboratory for exploring the universal baryon fraction (cf. WNEF; SHF). The X-rays observed from the hot intracluster gas may be used to estimate both the baryonic mass and the total mass of the cluster. Following WNEF, SHF have written the baryon fraction, $f_B$, as

$$f_B = \frac{f_{HG}}{\Upsilon}(1 + \frac{h^{3/2}}{5.5}),$$

where $f_{HG}$ is the fraction of the total mass in the X-ray emitting hot gas, $\Upsilon$ is a baryon enhancement factor introduced by WNEF to account for the small offset between the universal and cluster baryon fractions, and the last term in eq. 2, taken from WNEF, accounts for the cluster baryons which are in stars rather than in the hot gas (while ignoring any possible contribution from baryonic cluster dark matter). From a variety of hydrodynamical cosmological simulations Frenk et al. (1999) find for the offset between the universal and cluster baryon fractions: $\Upsilon = 0.92 \pm 0.08$. 

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SHF used Evrard’s (1997) estimate of the hot gas fraction and chose \( f_{HG}h^{3/2} = 0.060 \pm 0.006 \). More recently, Mohr, Mathiesen and Evrard (1999) have summarized the observational results from an ensemble of clusters and also corrected for the effect of merger driven clumpiness (Mathiesen, Evrard & Mohr 1999) to find \( f_{HG}h^{3/2} = 0.075 \). Although their quoted formal statistical uncertainty is small, systematic errors likely dominate the error budget. For example, while the latter estimate has been obtained using the “isothermal beta model” as a total mass estimator, Evrard (Private Communication), preferring the virial theorem mass estimator, finds \( f_{HG}h^{3/2} = 0.056 \). In an attempt to account for this uncertainty, here we adopt the average of the Evrard (1997) and the Mathiesen, Evrard, & Mohr (1999) determinations, along with a correspondingly generous error estimate: \( f_{HG}h^{3/2} = 0.066 \pm 0.013 \). As gravitational lensing observations of X-ray clusters improve (see, for example, Tyson, Kochanski & Dell’Antonio 1998) this uncertainty should be reduced considerably. Observations of the Sunyaev-Zeldovich effect in clusters also promise more accurate determinations of the cluster hot gas fraction (see, for example, Grego et al. 2000).

The present ratio of baryons to (CMB) photons is parameterized by \( \eta_{10} \equiv 10^{10}n_B/n_\gamma \) which, for a present CMB temperature of 2.725 K (Mather et al. 1999), may be written in terms of the baryon density parameter \( (\Omega_B) \) and the Hubble parameter \((h)\) as,

\[
\eta_{10} = 274 \Omega_B h^2 = 274 (f_B h^2) \Omega_M. \tag{3}
\]

Combining the above X-ray cluster estimates with the HST Key Project determined Hubble parameter (Mould et al. 1999), we obtain the distribution for \( f_B h^2 \). We find \( f_B h^2 = 0.065^{+0.016}_{-0.015} \) and the 95% range is \( 0.037 \leq f_B h^2 \leq 0.099 \). Convolving this with the previously determined distribution (SNIa, \( k = 0 \)) for \( \Omega_M \) we find,

\[
Flat (k = 0) : \quad \eta_{10} = 4.8^{+1.9}_{-1.5} \quad (\Omega_B h^2 = 0.018^{+0.007}_{-0.005}). \tag{4}
\]

The likelihood distribution for \( \eta \) is shown in Figure 3. While the 95% range, \( 2.1 \leq \eta_{10} \leq 9.1 \) \((0.008 \leq \Omega_B h^2 \leq 0.033)\), is very broad, this determination of the baryon density (at zero redshift) is independent of, and complementary to, those from BBN \((z \approx 10^8)\) and the CMB \((z \approx 10^3)\). As will become clear in the next section (see, also, Fig. 3), this estimate of the
Figure 3: The likelihood distributions, normalized to unit maximum, for the baryon-to-photon ratio $\eta_{10} = 274\Omega_B h^2$. The solid curve is for the low-D BBN case; the dashed curve is for the high-D BBN case (see Sec. 3); the dotted curve is for the non-BBN case (SNIa & $k = 0$); the dot-dashed curve is the CMB inferred range for $\Omega_B h^2 = 0.032 \pm 0.005$ from Jaffe et al. (2000).

baryon density is in excellent agreement with that determined from BBN using the “low” deuterium abundance of Burles & Tytler (1998a,b; hereafter, BT). Although offset from the value of the baryon density consistent with BBN and the “high” deuterium abundance of Webb et al. (1997), given the large errors for both determinations there is still considerable overlap.

Fukugita, Hogan, and Peebles (1998) have attempted to inventory baryons at $z \approx 0$, finding a range (for $h \equiv 0.70$) $0.007 \lesssim \Omega_B \lesssim 0.041$. Although this range (corresponding to $1.0 \lesssim \eta_{10} \lesssim 5.5$) has considerable overlap with ours, there is a hint that some dark baryons may have escaped the Fukugita, Hogan, & Peebles (1998) inventory.

Measurements of the “large l” multipoles of the CMB temperature angular power spectrum can also probe the cosmological parameters. The recent
Boomerang (De Bernardis et al. 2000; Lange et al. 2000) and Maxima-1 (Hanany et al. 2000) CMB anisotropy results point to a “low” second acoustic peak. In the attempts to account for a second peak which is low relative to the first peak in the CMB anisotropy spectrum, there is some degeneracy between “global” cosmological parameters such as the baryon density, and model dependent parameters such as “tilt”. Nevertheless, all multi-parameter fits to the Boomerang and Maxima data appear to point to a “high” baryon density. For example, Jaffe et al. (2000) find for their flat model \((k = 0)\) which provides the best fit to the combined Boomerang, Maxima and COBE DMR data, \(\Omega_B h^2 = 0.032 \pm 0.005\). On the assumption of gaussian errors, (almost certainly wrong) the likelihood distribution for this result is plotted in Figure 3; note the significant overlap between the baryon density determined at present from the SNIa data and that from the CMB. It is a remarkable confirmation of the (current) standard model of cosmology that these two, independent determinations of the baryon density, “measured” at vastly different epochs in the evolution of the Universe, agree so well. Given the large uncertainties in each determination, the hint of a CMB challenge to the zero-redshift baryon density is only minor at present.

3 Using The BBN-Inferred Baryon Density

In this section we drop the assumption of flatness and replace it with estimates of the baryon density from BBN. Again we use the X-ray cluster inferred baryon fraction for an estimate of the universal baryon fraction, but this time we combine \(f_B\) with the BBN measure of \(\Omega_B h^2\) (utilizing the estimates of the primordial abundance of deuterium) to obtain an estimate of the overall matter density (see Figures 4 & 5):

\[
\Omega_M = \frac{\Omega_B}{f_B} = 0.057(1 \pm 0.22)\eta_{10},
\]

(5)

where we have accounted for the uncertainties in \(f_B\) and \(h\) as discussed above. Armed with the SNIa data and this BBN-related estimate of \(\Omega_M\), we proceed to determine the likelihood distributions for the global cosmological parameters \((\Omega_M, \Omega_A, \Omega_k, q_0, \Gamma, H_0 t_0)\).

It has long been known that deuterium provides an ideal baryometer (Reeves, Audouze, Fowler and Schramm 1976). Because deuterium has only
Figure 4: Same as Figure 1 with the 68% and 95% contours for $\Omega_M$ inferred from BBN and the X-ray cluster baryon fraction. The high-D, low-$\eta$ constraint bands (at the left) are in red/dashed, while the low-D, high-$\eta$ bands (at the right) are in blue/dotted.

been destroyed in the course of galactic evolution since the epoch of primordial nucleosynthesis (Epstein, Lattimer and Schramm 1976), any deuterium, observed anywhere in the Universe, provides a lower limit to its primordial abundance. In the high-redshift, low-metallicity environment of the QSO absorption-line systems the bulk of the gas is unlikely to have been processed through stars, so that the observed deuterium abundance is likely to be very nearly the primordial value. For two such systems BT derive a statistically accurate estimate for the primordial-D abundance: $(D/H)_P = 3.4 \pm 0.3 \times 10^{-5}$. The $\approx 8\%$ observational error is well matched to the comparable error in the BBN prediction (Hata et al. 1996). Due to the steep dependence of the BBN-predicted D-abundance on the baryon-to-photon ratio $\eta$, a 10% uncertainty in $(D/H)_P$ results in a baryon abundance determined to 6%. Combining the observational and theoretical errors in quadrature, the BT data lead to an es-
timate of the baryon abundance accurate to 7%. The likelihood distribution for $\eta_{10}$ for this low-D case is shown in Figure 3 for

$$ \text{Low} - D : \quad \eta_{10} = 5.1 \pm 0.4 \quad (\Omega_B h^2 = 0.019 \pm 0.001).$$

Note the remarkable agreement between this BBN-determined baryon density and that obtained in the previous section using the X-ray cluster baryon fraction and the SNIa and $k = 0$ estimate (see Figure 3 for the corresponding likelihood distribution). The baryon abundance in the 10 – 15 Gyr old universe and that inferred for when the universe was only a few minutes old are virtually identical (as they should be!).

![Figure 5](image)

Figure 5: Same as Figure 4, but showing the 68\% and 95\% contours for the joint fit of the SNIa data with the BBN/X-ray cluster determined values of $\Omega_M$.

In contrast, a higher primordial deuterium abundance, $(D/H)_{P} = 20 \pm 10 \times 10^{-5}$ is inferred from HST observations of a different, lower redshift QSO absorption-line system (Webb et al. 1997). While the high deuterium inferred for this absorption system may well be contaminated by a low column density
hydrogen “interloper”, it could be that this higher abundance is primordial. If it were, then that found by BT would be low because the gas in their absorbing systems had been cycled through stars where the deuterium was destroyed. It would be surprising that the two high redshift systems are “evolved” while the one lower redshift system is not (or, is less evolved). Clearly more data will be crucial in resolving this question. Here we will attempt to use consistency with other cosmological parameters as a potential way to discriminate between the two possibilities (“low-D” and “high-D”). If the Webb et al. (1997) deuterium abundance is adopted, the corresponding baryon abundance is both lower and somewhat less accurately constrained

\[ \eta_{10} = 1.7 \pm 0.3 \quad (\Omega_B h^2 = 0.006 \pm 0.001). \]  

From Figure 3 it is clear that the overlap with the non-BBN estimate from the previous section is poor.

### 3.1 Global parameters from low-D

Here we adopt the BBN-determined baryon density corresponding to low-D and we use the X-ray cluster baryon fraction to estimate the total mass density. The corresponding 68%(95%) bands are shown in Figure 4 and the corresponding contours in the \( \Omega_\Lambda - \Omega_M \) plane are shown in Figure 5. Notice that the low-D determined contours are completely consistent with a flat Universe. The likelihood distributions for several global cosmological parameters are shown by the solid curves in panels \( a-f \) of Figure 6.

For the total (baryonic plus CDM) mass density we find \( \Omega_M = 0.26^{+0.09}_{-0.06} \); the 95% range is \( 0.16 \leq \Omega_M \leq 0.48 \). This is in excellent agreement with the \( k = 0 \) identified range from the SNIa data and with other, independent determinations of the mass density (Cole et al. 1997; Weinberg et al. 1999) providing support for BBN with low-D.

In this section we have dropped the restriction to a flat universe, thereby gaining an independent determination of the vacuum density (cosmological constant). The likelihood distribution for \( \Omega_\Lambda \) is shown by the solid curve in panel \( b \) of Figure 6. We find strong support for a non-zero value: \( \Omega_\Lambda = 0.74^{+0.17}_{-0.10} \); the 95% range is \( 0.35 \leq \Omega_\Lambda \leq 1.11 \).

Having removed the constraint of flatness we can now ask if low-D BBN is consistent with a flat Universe. It is clear from the solid curve in panel
c of Figure 6 that a flat universe ($\Omega_k = 0$) is completely consistent with the matter density determined from low-D BBN and the SNIa magnitude-redshift relation. Indeed, our best fit is for $\Omega_k = 0.00_{-0.24}^{+0.22}$ and the 95% range is $-0.53 \leq \Omega_k \leq +0.44$.

Given the similarity between the non-BBN parameters and those identified by low-D BBN, it is not surprising that here, too, we confirm an accelerating universe. The likelihood distribution for $q_0$ is shown by the solid curve in panel d of Figure 6. Our best fit is for $q_0 = -0.60_{-0.16}^{+0.18}$ and the 95% range is $-0.24 \geq q_0 \geq -0.91$.

Figure 6: Likelihood distributions for several global cosmological parameters determined from the joint fit of the SNIa data, the X-ray cluster baryon fraction, and the BBN constraints on the baryon density from low-D (solid curves) and high-D (dashed curves). Panel a shows the total mass density parameter $\Omega_M$; panel b shows the density parameter associated with a cosmological constant $\Omega_\Lambda$; the “flatness” parameter $\Omega_k \equiv 1 - (\Omega_M + \Omega_\Lambda)$ is shown in panel c; the deceleration parameter $q_0$ is shown in panel d; in panel e is the dimensionless age $H_0t_0$; the shape parameter $\Gamma$ is shown in panel f.
The dimensionless age of the universe, $H_0 t_0$, is shown in panel $e$ of Figure 6. For this low-D universe we find $H_0 t_0 = 0.97^{+0.07}_{-0.05}$, with a 95% range of $0.88 \leq H_0 t_0 \leq 1.10$. Adopting the HST Key project value of $H_0 = 71 \pm 6$ kms$^{-1}$Mpc$^{-1}$ (Mould et al. 1999), we find for the age of the Universe $t_0 = 13.3^{+1.5}_{-1.3}$ Gyr with a 95% range extending from 11.0 Gyr to 16.5 Gyr.

Finally, we may predict the value of the shape parameter corresponding to this low-D case. The likelihood distribution is shown in panel $f$ of Figure 6. It corresponds to a somewhat low value $\Gamma = 0.15^{+0.05}_{-0.04}$, but the 95% range, $0.08 \leq \Gamma \leq 0.28$ is perfectly consistent with observational estimates (Peacock & Dodds 1994; Fisher, Scharf & Lahav 1994; Maddox, Efstathiou & Sutherland 1996; Webster et al. 1998; Eisenstein & Zaldarriaga 1999; Efstathiou & Moody 2000).

Very recently O’Meara et al. (2000) have reported the detection of deuterium in another high redshift, low metallicity QSO absorption system. The deuterium abundance they derive, $D/H = 2.54 \pm 0.23 \times 10^{-5}$, is some 3$\sigma$ smaller than the previous mean D/H for the “low-D” absorbers. Accounting for the dispersion among the three, low-D absorbers, O’Meara et al. suggest a revised primordial deuterium abundance: $D/H = 3.0 \pm 0.4 \times 10^{-5}$, which for BBN corresponds to a baryon-to-photon ratio of $\eta_{10} = 5.6 \pm 0.5$, or $\Omega_B h^2 = 0.020 \pm 0.002$. This small shift in the baryon density has minimal effect on the quantitative conclusions reached above. For example, while the 95% ranges for $\Omega_M$ and $\Omega_\Lambda$ shift upwards by approximately 0.03 and 0.04 respectively, that for $\Omega_k$ moves down by $\approx 0.05$, and the ranges of $q_0$, $\Gamma$, $H_0 t_0$, and $t_0$ are virtually unchanged.

In summary, the low-D BBN estimate of the baryon density, in combination with the X-ray cluster estimate of the baryon fraction and the SNIa data favors a flat ($k = 0$), accelerating ($q_0 < 0$) Universe whose age ($t_0 \geq 11$ Gyr) is consistent with estimates of the ages of the oldest stars and with a shape parameter ($0.1 \lesssim \Gamma \lesssim 0.3$) in agreement with estimates from observations of large scale structure.

### 3.2 Global parameters from high-D

If, in contrast to the above, the BBN-determined baryon density corresponding to high-D (Webb et al. 2000) is used, we find a very low matter (baryons plus CDM) density, $\Omega_M = 0.09 \pm 0.03$; the 95% range is $0.04 \leq \Omega_M \leq 0.17$. The corresponding vertical band for $\Omega_M$ is shown in Figure 4, the related
$\Omega_\Lambda - \Omega_M$ contours are shown in Figure 5, and the likelihood distribution for $\Omega_M$ is the dashed curve in panel a of Figure 6. Such a low value for the total mass density is in conflict with independent determinations (Bahcall et al. 1999) and with the comparison between models of structure formation and observations of large scale structure (Cole et al. 1997; Weinberg et al. 1999).

The corresponding likelihood distributions of the other global cosmological parameters are shown in Figure 6. It is clear from panel c that a flat Universe is very unlikely (95% range: $0.08 \leq \Omega_k \leq 0.78$); $\Omega_k \leq 0$ is excluded at the 99.3% confidence level if the high-D BBN-determined baryon density is assumed to provide a fair estimate of the present baryon density.

It is clear from Figure 6 that while the high-D, BBN determined values of $\Omega_\Lambda$, $q_0$, and $H_0t_0$ differ little from their low-D counterparts (at 95%; $0.15 \leq \Omega_\Lambda \leq 0.79$; $-0.10 \geq q_0 \geq -0.74$; $0.92 \leq H_0t_0 \leq 1.19$), the corresponding range for $\Gamma$ is very low ($0.01 \leq \Gamma \leq 0.10$), in conflict with observational estimates (Peacock & Dodds 1994; Fisher, Scharf & Lahav 1994; Maddox, Efstathiou & Sutherland 1996; Webster et al. 1998; Eisenstein & Zaldarriaga 1999; Efstathiou & Moody 2000).

In summary, in contrast to the low-D case considered above, the high-D estimate of the baryon density in combination with the X-ray cluster estimate of the baryon fraction and the SNIa data does not favor a flat ($k = 0$) Universe and prefers a low matter density ($\Omega_M \leq 0.17$) and a small value of the shape parameter ($\Gamma \leq 0.10$) which are in marginal conflict with estimates from observations of large scale structure. These latter data argue against high-D and favor the low-D estimate of the BBN-predicted primordial deuterium abundance.

4 Discussion

Data from observations of the SNIa magnitude-redshift relation identify a region in the $\Omega_\Lambda - \Omega_M$ plane consistent with a flat universe ($k = 0$), which disfavors a vanishing cosmological constant ($\Omega_\Lambda \neq 0$), and points towards an accelerating expansion ($q_0 < 0$) of the present universe. Imposing the assumption of flatness ($k = 0$) breaks the degeneracy between $\Omega_\Lambda$ and $\Omega_M$, identifying preferred values for each parameter separately, leading to well-defined predictions for a variety of other global cosmological parameters (see
Figures 1 & 2). In particular, we have supplemented the SNIa data with estimates of the cluster baryon fraction to relate the total mass density to the baryonic mass density, finding a value (determined at redshifts < 1) consistent with that suggested by BBN (fixed at redshifts > 10^8) for the case of a low primordial abundance of deuterium (η_{10} ≈ 5.1, Ω_B h^2 ≈ 0.019). Although the CMB-inferred value of the baryon density (Jaffe et al. 2000) has a large uncertainty, and the result is dependent on non-global cosmological parameters, it too is roughly consistent with this value (albeit on the high side by some 60%). In contrast, the BBN-determined value of the baryon density in the case of a high primordial abundance of deuterium (η_{10} ≈ 1.7; Ω_B h^2 ≈ 0.006) is inconsistent with both the SNIa – X-ray cluster result and that from the CMB anisotropy measurements. Our results are summarized in Figure 3 where we show the likelihood distributions for η_{10} for the cases considered here, as well as the estimate from the CMB anisotropy (Jaffe et al. 2000). It is clear that there is excellent overlap between the non-BBN SNIa range (assuming flatness) and the low-D BBN range. In contrast, high-D BBN combined with the X-ray cluster baryon fraction leads to a very low estimate of the total matter density which, when combined with the SNIa data, appears inconsistent with a flat universe.

In all cases considered here (non-BBN, low-D BBN, high-D BBN) the SNIa data strongly favor an accelerating Universe (q_0 < 0). For all cases the “age problem” has evaporated (H_0 t_0 ≈ 1, t_0 ≈ 13 – 14 Gyr). However, although both the non-BBN and the low-D BBN results predict a value for the shape parameter (Γ ≈ 0.2) consistent with observations (Peacock & Dodds 1994; Fisher, Scharf & Lahav 1994; Maddox, Efstathiou & Sutherland 1996; Webster et al. 1998), the high-D prediction (Γ ≈ 0.05) is low. All in all, our analysis supports a flat Universe (Ω_k ≈ 0), presently dominated by a non-zero cosmological constant (Ω_A ≈ 1 – Ω_M ≈ 0.7) or some other form of “dark energy”, and a baryon density consistent with that inferred from a comparison of BBN and the low value of the deuterium abundance found by BT (η_{10} ≈ 5; Ω_B h^2 ≈ 0.02). While this baryon density is roughly a factor of 3 larger than that inferred from BBN and the high deuterium value of Webb et al. (1997), it is within less than a factor of two from that derived from the CMB anisotropy data (Jaffe et al. 2000). Indeed, agreement between the baryon abundance at epochs in the evolution of the Universe (BBN, CMB, z ≈ 0) separated by some 10 – 14 orders of magnitude in time provides powerful support for the standard hot big bang cosmological model.
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References

Bahcall, N. A., Ostriker, J. P., Perlmutter, S. & Steinhardt, P. J. 1999, Science, 284, 1481
Burles, S., & Tytler, D. 1998a;b, ApJ, 499, 699; ibid 507, 732 (BT)
Carlberg, R. G., Yee, H. K. C., & Ellingson, E. 1997, ApJ, 478, 462
Chaboyer, B. 2000, private communication
Chaboyer, B. & Krauss, L. 2000, in preparation
Cole, S., Weinberg, D. H., Frenk, C. S., & Ratra, B. 1997, MNRAS, 289, 37
De Bernardis, P. et al. 2000, Nature, 404, 955
Efstathiou, G. & Moody, S. J. 2000, preprint (astro-ph/0010478)
Eisenstein, D. J. & Zaldarriaga, M. 1999, preprint (astro-ph/9912149)
Epstein, R., Lattimer, J., & Schramm, D. N. 1976, Nature, 263, 198
Evrard, A. E. 1997, MNRAS, 292, 289
Filipenko, A. V., & Riess, A. G. 2000, preprint (astro-ph/0008057)
Fisher, K. B., Scharf, C. A., & Lahav, O. 1994, MNRAS 266, 219
Fukugita, M., Hogan, C. J., & Peebles, P. J. E. 1998, ApJ, 503, 518
Frenk, C. S. et al. 1999, ApJ, 525, 554
Grego, L. et al. 2000, preprint (astro-ph/0003083)
Goobar, A., & Perlmutter, S. 1995, ApJ, 450, 14
Hanany, S. et al. 2000, preprint (astro-ph/0005123)
Hata, N., Scherrer, R. J., Steigman, G., Thomas, D., & Walker, T. P. 1996, ApJ, 458, 637
Hernquist, L., Katz, N., Weinberg, D. H., & Miralda-Escudé, J. 1996, ApJ, 457, L51
Jaffe, A. H. et al. 2000, preprint (astro-ph/0007333)
Lange, A. et al. 2000, preprint (astro-ph/0005004)
Maddox, S. J., Efstathiou, G., & Sutherland, W. J. 1996, MNRAS, 283, 1227
Mather, J. C. et al. 1999, ApJ 512, 511
Mathiesen, B. Evrard, A. E., & Mohr, J. J. 1999, ApJ, 520, L21
McDonald, P., Miralda-Escudé, J., Rauch, M., Sargent, W. L. W., Barlow, T., & Cen R. 2000, preprint (astro-ph/0005333)
Mohr, J. J., Mathiesen, B., & Evrard, A. E. 1999, ApJ, 517, 627
Mould, J. R. et al. 2000, ApJ, 529, 786 (HST Key Project)
Olive, K. A., Steigman, G., & Walker, T. P. 2000, Physics Reports, 333-334, 389
O’Meara, J. M., Tytler, D., Kirkman, D., Suzuki, N., Prochaska, J. X., Lubin, D., & Wolfe, A. M. 2000, preprint (astro-ph/0011179)
Peacock, J. A. 1997, MNRAS, 284, 885
Peacock, J. A., & Dodds, S. J. 1994, MNRAS, 267, 1020
Perlmutter, S. et al. 1997, ApJ, 483, 565
Perlmutter, S. et al. 1999, ApJ, 517, 565
Rauch, M., Haehnelt, M. G., & Steinmetz, M. 1997, ApJ, 481, 601
Reeves, H., Audouze, J., Fowler, W. A., & Schramm, D. N. 1976, ApJ, 179, 909
Riess, A. G. et al. 1997, AJ, 114, 722
Riess, A. G. et al. 1998, AJ, 116, 1009
Schmidt, B. P. et al. 1998, ApJ, 507, 46
Steigman, G., & Felten, J. E. 1995, Spa. Sci. Rev., 74, 245
Steigman, G., Hata, N., & Felten, J. E. 1999, ApJ, 510, 564
Sugiyama, N. 1995, ApJS, 100, 281
Tyson, J. A., Kochanski, G. P., & Dell’Antonio, I. P. 1998, ApJ, 498, L107
Webb, J. K., Carswell, R. F., Lanzetta, K. M., Ferlet, R., Lemoine, M., Vidal-Madjar, A., & Bowen, D. V. 1997, Nature, 388, 250
Webster, M., Bridle, S. L., Hobson, M. P., Lasenby, A. N., Lahav, O., & Rocha, G. 1998, ApJ, 509, L65
Weinberg, D. H., Miralda-Escudé, J., Hernquist, L., & Katz, N. 1997, ApJ, 490, 564
Weinberg, D. H., Croft, R. A. C., Hernquist, L., Katz, N., & Pettini, M. 1999, ApJ, 522, 563
White, S. D. M., Navarro, J. F., Evrard, A. E., & Frenk, C. S. 1993, Nature, 366, 429 (WNEF)