Refactoring Problem of Acyclic Extended Free-Choice Workflow Nets to Acyclic Well-Structured Workflow Nets

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SUMMARY A workflow net (WF-net for short) is a Petri net which represents a workflow. There are two important subclasses of WF-nets: extended free-choice (EFC for short) and well-structured (WS for short). It is known that most actual workflows can be modeled as EFC WF-nets; Acyclic WS is a subclass of acyclic EFC but has more analysis methods. Acyclic EFC WF-net refactoring problem can be checked in polynomial time. These results in the enhancement of the analysis power of acyclic EFC WF-nets.

key words: workflow net, refactoring, well-structured, soundness, branching bisimilarity

1. Introduction

A workflow net (WF-net for short) [1] is a Petri net [2] which represents a workflow. There are two important subclasses of WF-nets: extended free-choice (EFC for short) and well-structured (WS for short). It is known that most actual workflows can be modeled as EFC WF-nets; Acyclic WS is a subclass of acyclic EFC but has more analysis methods, e.g. heuristic computation of parallel degree [3].

In software engineering, code refactoring [4] has been attracting a great deal of attention in order to reduce the complexity of code. Code refactoring is to transform a source code to a new form without changing its external behavior. We try to introduce the concept of refactoring to the analysis of WF-nets. An acyclic EFC WF-net may be transformed to an acyclic WS WF-net without changing its external behavior. We name such a transformation acyclic EFC WF-net refactoring. If a given acyclic EFC WF-net is refactored to an acyclic WS WF-net, we can use the analysis methods of WS WF-nets to analyze the EFC WF-net. This results in the enhancement of the analysis power of EFC WF-nets.

In this paper, we give a formal definition of acyclic EFC WF-net refactoring problem. Next we give a necessary condition and a sufficient condition for solving the problem. We also show that those conditions can be checked in polynomial time.

2. Workflow Nets and Properties

A (labeled) WF-net is a labeled Petri net which represents a workflow. A labeled Petri net is a four tuple \( N=(P, T, A, \ell) \) with \( P \cup T=\emptyset \) and \( A \subseteq (P\times T)\cup(T\times P) \) are finite sets of places, transitions, and arcs, respectively. Each transition of a WF-net represents an action. Some actions can be observed, others cannot. The former and the latter are called external and internal, respectively. Actions are identified by label. Internal actions are labeled as a designated label \( \tau \). \( \ell : T\rightarrow A\cup{\tau} \) is a labelling function of transitions, where \( A \) denotes the set of all possible external labels.

Definition 1 (WF-net [1]): A labeled Petri net \( N=(P, T, A, \ell) \) is a (labeled) WF-net iff (i) \( N \) has a single source place \( p_I \) such that \( p_I = \emptyset \) and \( \forall p \in \{p_I\} : p\neq \emptyset \) and a single sink place \( p_O \) such that \( p_O = \emptyset \) and \( \forall p \in \{p_O\} : p\neq \emptyset \), where for a node \( x \in (P\cup T) \), \( \bullet x \) and \( \bullet^* x \) denote \( \{y | (y, x)\in A\} \) and \( \{y | (y, x)\in A\} \), respectively; and (ii) every place or transition is on a path from \( p_I \) to \( p_O \).}

Let \( N=(P, T, A, \ell) \) be a WF-net. We represent a marking of \( N \) as a bag over \( P \). A marking is denoted by \( M=\{p^{M_P} | p \in P, M(p)>0\} \), where \( M(p) \) denotes the number of tokens in \( p \). Let \( M_X \) and \( M_Y \) be markings. \( M_X \subset M_Y \) denotes that \( \forall p \in P : M_X(p)=M_Y(p) \). \( M_X \subsetneq M_Y \) denotes that \( \exists p \in P : M_X(p)<M_Y(p) \). A transition \( t \) is said to be firable in a marking \( M \) if \( M \models \bullet^* t \). Firing \( t \) in \( M \) results in a new marking \( M'=(M \cup (\bullet t)) \). This is denoted by \( M[N(t)M'] \). A marking \( M' \) is said to be reachable from a marking \( M \) if there exists a firing sequence of transitions transforming \( M \) to \( M' \). The set of all possible markings reachable from \( M \) is denoted by \( R(N, M) \).

There are two important subclasses of WF-nets: WS and EFC. A structural characterization of good workflows is that two paths initiated by a transition/place should not be joined by a place/transition. WS is derived from this structural characterization. To give the formal definition of WS, we introduce some notations. The Petri net obtained by connecting \( p_O \) with \( p_I \) via an additional transition \( r^* \) is called the short-circuited net of \( N \), denoted by \( \overline{N} \). An elementary circuit in \( \overline{N} \) is an elementary path from a node \( n_1 (\in P \cup T) \) to another node \( n_2 \) in \( N \) is said to be a handle of \( x \).

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A handle from a transition to a place is called a TP-handle. A handle from a place to a transition is called a PT-handle. A WF-net \( N \) is WS if there are neither TP-handles nor PT-handles in \( \overline{N} \). A WF-net is EFC if there exists \( p_Y \in P: p_Y \not\in \Delta(\overline{N}, \{p_I\}) \), where \( p_Y \) are called WF-nets. A WF-net is EFC if there exists \( \{p_I\} \) is live and safe. From Theorem 14 of Ref. [2], we can view \( \mathcal{N} \) as an acyclic WS WF-net. Let us consider an application example of this trivial algorithm. Figure 1 shows an acyclic EFC but non-WS WF-net \( N_1 \).

![Figure 1](image1.png)

**Fig. 1** An acyclic EFC but non-WS WF-net \( N_1 \).

3. Acyclic EFC WF-Net Refactoring Problem and Its Properties

3.1 Definition and Analysis

We first give a formal definition of acyclic EFC WF-net refactoring problem.

**Definition 4:** Acyclic EFC WF-net refactoring problem

- **Input:** Acyclic EFC WF-net \( N_X = (P_X, T_X, A_X, \ell_X) \), where every external action is unique in \( N_X \)
- **Output:** Acyclic WS WF-net \( N_Y = (P_Y, T_Y, A_Y, \ell_Y) \)
- **Constraints:** (i) \((N_X, \{p_I\}) \cup_b (N_Y, \{p_I\})\); (ii) every external action is unique in \( N_Y \).

Condition (ii) prohibits duplication of any external action. An external action is performed by resources (workers and/or machines). If the action is duplicated, it would share the resources with its duplicate. This makes it difficult that those resources are scheduled. Furthermore the action in the worst case is duplicated exponentially. This may disable refactoring from running in polynomial time.

There seems to exist a trivial algorithm for solving this problem. Let \( N \) be a sound EFC WF-net. It is known from Theorem 1 of Ref. [1] that \( N \) is live and safe. From Theorem 14 of Ref. [2], we can view \( N \) as an acyclic WS WF-net so that those source places and those sink places are respectively shared, we can obtain an acyclic WS WF-net. Let us consider an application example of this trivial algorithm. Figure 1 shows an acyclic EFC but non-WS WF-net, denoted by \( N_1 \). Applying the trivial algorithm to \( N_1 \), we can obtain an acyclic WS WF-net, denoted by \( N_1' \), which is shown in Fig. 2. Unfortunately, \( N_1' \) is not an answer of the refactoring problem because it does not satisfy Constraint (ii), i.e. external ac-

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1. A MG-component \( N_1 \) of a Petri net \( N \) is defined as a subnet generated by transitions in \( N_1 \), having the following two properties: (i) Each place in \( N_1 \) has at most one incoming arc and at most one outgoing arc; and (ii) A subnet generated by transitions is the net consisting of these transitions, all of their input and output places, and their connecting arcs. A Petri net is strongly connected if, for every pair of nodes \( n_1 \) and \( n_2 \), it contains a directed path from \( n_1 \) to \( n_2 \) and a directed path from \( n_2 \) to \( n_1 \).

11. A WF-net is marked graph, MG for short, if \( \{p, p\} = \{p_0\} = 1 \) and \( p \in P \setminus \{p_0\} \) or \( p \in P \setminus \{p_0\} \) and \( p \in P \setminus \{p_0\} \).
tions $\alpha$, $\beta$, $\gamma$, and $\zeta$ are not unique. Therefore the theoretical algorithm cannot solve the refactoring problem.

We give a decision problem related to the acyclic EFC WF-net refactoring problem.

**Definition 5: EFC-WF-REFACTORING**

**Instance:** Acyclic EFC WF-net $N_X = (P_X, T_X, A_X, \ell_X)$, where every external action is unique in $N_X$.

**Question:** Is there an acyclic WS WF-net $N_Y = (P_Y, T_Y, A_Y, \ell_Y)$ such that (i) $(N_X, [p^X_1]) \sim_b (N_Y, [p^Y_1])$; and (ii) every external action is unique in $N_Y$.

**3.2 Necessary Condition**

We give a necessary condition for EFC-WF-REFACTORING.

**Property 1:** Let $N_X$ be an acyclic EFC but non-WS WF-net whose every external action is unique. If $N_X$ is not sound then there is no acyclic WS WF-net $N_Y$ such that $(N_X, [p^X_1]) \sim_b (N_Y, [p^Y_1])$ and every external action is unique in $N_Y$.

**Proof:** Since $N_X$ is not sound, there is a dead marking besides $[p^X_1]$ in $(N_X, [p^X_1])$. The dead marking is denoted by $M^X_{\text{dead}}$. On the other hand, any acyclic WS WF-net is sound because its short-circuited net has neither TP-handles nor PT-handles. Since $N_Y$ is sound, there is no dead marking besides $[p^Y_1]$ in $(N_Y, [p^Y_1])$. Therefore there is no branching bisimulation relation $\mathcal{R}$ such that $\exists M^Y \in \mathcal{R}(N_Y, [p^Y_1]) : M_{\text{dead}}^X \not\subseteq \mathcal{R} M^Y$. Thus this property holds. Q.E.D.

This property implies that we cannot solve the refactoring problem if a given acyclic EFC WF-net is not sound. It is known from Corollary 1 of Ref. [1] that an EFC WF-net can be checked for soundness in polynomial time. Thus the above necessary condition can also be checked in polynomial time.

**3.3 Sufficient Condition**

Let us consider the net shown in Fig. 1. In this net, well-structuredness is lost on account of the structure composed of $p_3, p_4, t_4, t_5$. We focus on such a structure, named cross structure.

**Definition 6** (TP-cross and PT-cross structures): Let $N = (P, T, A, \ell)$ be an EFC WF-net.

- For places $p_1, p_2 \in (P, P)$ and transitions $t_1, t_2 \in (T, T)$, $(t_1, t_2, p_1, p_2)$ is a TP-cross structure if $[(t_1, p_1), (t_1, p_2), (t_2, p_1), (t_2, p_2)] \subseteq A$.
- For places $p_1, p_2 \in (P, P)$ and transitions $t_1, t_2 \in (T, T)$, $(p_1, p_2, t_1, t_2)$ is a PT-cross structure if $[(p_1, t_1), (p_1, t_2), (p_2, t_1), (p_2, t_2)] \subseteq A$.

We define a subclass of acyclic EFC but non-WS, named cross-bridged (CB for short). A CB WF-net intuitively has only one cross structure, which is a cut-set of the net. To give the formal definition of CB, we use an operator: Given two WF-nets $K$ and $L$, place refinement of a place $p$ of $K$ with $L$ yields a WF-net $N = K \otimes p L$, which is built as follows: $p$ is replaced in $K$ by $L$; transitions of $K$ become input transitions of the source place of $L$; and transitions of $K$ become output transitions of the sink place of $L$.

The formal definition of CB is given as follows:

**Definition 7 (cross-bridged; CB):** A WF-net $N$ is cross-bridged, CB for short, if $N$ is

- $((N_{TP}^{CB} \otimes (L_1 \otimes p_1) \otimes p_2) \otimes p_2, L_2) \otimes p_3, L_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}$, where $N_{TP}^{CB}$ is the net shown in Fig. 3, $(p_1, p_2, p_3, p_4)$ is the set of places in $N_{TP}^{CB}$, and $L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}$ are acyclic WS WF-nets.

- $((N_{TP}^{CB} \otimes (L_1 \otimes p_1) \otimes p_2) \otimes p_3, L_2) \otimes p_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}$, where $N_{TP}^{CB}$ is the net shown in Fig. 4, $(p_1, p_2, p_3, p_4)$ is the set of places in $N_{TP}^{CB}$, and $L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}$ are acyclic WS WF-nets.

Now we give a sufficient condition for EFC-WF-REFACTORING.

**Property 2:** Let $N_X$ be an acyclic EFC but non-WS WF-net whose every external action is unique. If $N_X$ is CB then there is an acyclic WS WF-net $N_Y$ such that $(N_X, [p^X_1]) \sim_b (N_Y, [p^Y_1])$ and every external action is unique in $N_Y$.

**Proof:** This proof consists of the following two cases: (i) $N_X$ is $((N_{TP}^{CB} \otimes (L_1 \otimes p_1) \otimes p_2) \otimes p_3, L_2) \otimes p_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}$, where $N_{TP}^{CB}$ is the net shown in Fig. 3, $(p_1, p_2, p_3, p_4)$ is the set of places in $N_{TP}^{CB}$, and $L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}$ are acyclic WS WF-nets. (ii) $N_X$ is $((N_{TP}^{CB} \otimes (L_1 \otimes p_1) \otimes p_2) \otimes p_3, L_2) \otimes p_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}$, where $N_{TP}^{CB}$ is the net shown in Fig. 4, $(p_1, p_2, p_3, p_4)$ is the set of places in $N_{TP}^{CB}$, and $L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}$ are acyclic WS WF-nets.

Case (i): Let us first consider a special case, i.e. $N_X$ is $N_{TP}^{CB}$. Assume that $N_Y$ is the WF-net $N_{WS}^{TP}$ shown in Fig. 5, $N_{WS}^{TP}$ is obviously acyclic WS. We must that $N_{WS}^{TP}$ satisfies Constraints (i) and (ii) of the acyclic EFC WF-net refactoring problem. In order to show that $N_{WS}^{TP}$ satisfies Constraint (i), we assume the following relation $\mathcal{R}$ between $R(N_{TP}^{CB}, [p_1])$ and $R(N_{WS}^{TP}, [p_1])$: $[p_1] \mathcal{R} [p_1], [p_1, p_2] \mathcal{R} [p_3], [p_1, p_2] \mathcal{R} [p_1, p_2], [p_0] \mathcal{R} [p_0]$. We show that $\mathcal{R}$ satisfies Condition (i) of branching bisimilarity. If $[p_1] \mathcal{R} [p_1]$ and $[p_1]N_{TP}^{CB} [p_1, p_2]$
then $[p_1][N^WS_{TP},\alpha][p_1]$ and $[p_1,p_2][R][p_3]$ hold. If $[p_1][R][p_1]$ and $[p_1][N^CB_{TP},\beta][p_1,p_2]$ then $[p_1][N^WS_{TP},\beta][p_3]$ and $[p_1,p_2][R][p_3]$ hold. If $[p_1,p_2][R][p_3]$ and $[p_1,p_2][N^CB_{TP},\gamma][p_0]$ then $[p_3][N^WS_{TP},\tau][p_1,p_2]$, $[p_1,p_2][N^WS_{TP},\tau][p_1,p_2]$, $[p_1,p_2][N^WS_{TP},\gamma][p_0]$, and $[p_0][R][p_0]$ hold. We show that $\mathcal{R}$ satisfies Condition (ii) of branching bisimilarity. If $[p_1][R][p_1]$ and $[p_1][N^WS_{TP},\alpha][p_1]$ then $[p_1][N^CB_{TP},\alpha][p_1,p_2]$ and $[p_1,p_2][R][p_1]$ hold. If $[p_1][R][p_1]$ and $[p_1][N^WS_{TP},\beta][p_1]$ then $[p_1][N^CB_{TP},\beta][p_1,p_2]$ and $[p_1,p_2][R][p_3]$ hold. If $[p_1,p_2][R][p_3]$ and $[p_3][N^WS_{TP},\tau][p_1,p_2]$ then $[p_1,p_2][N^WS_{TP},\tau][p_1,p_2]$ holds. If $[p_1,p_2][R][p_3]$ and $[p_1,p_2][N^WS_{TP},\gamma][p_0]$ then $[p_1,p_2][N^CB_{TP},\gamma][p_0]$ and $[p_0][R][p_0]$ hold. $\mathcal{R}$ obviously satisfies Condition (iii) of branching bisimilarity. Therefore $(N^CB_{TP},[p_1]) \sim_b (N^WS_{TP},[p_1])$ holds. And we can know from the structure of $N^WS_{TP}$ that $N^WS_{TP}$ satisfies Constraint (ii) of the refactoring problem, i.e. every external action is unique. Thus if $N_X$ is $N^CB_{TP}$ then this property holds.

Next let us consider a general case, i.e. $N_X$ is $(N^CB_{TP} \otimes p_0, L_1 \otimes p_0, L_2 \otimes p_0).$ Assume that $N_Y$ is $(N^WS_{TP} \otimes p_0, L_1 \otimes p_0, L_2 \otimes p_0).$ The net is acyclic WS because place refinement of a place in an acyclic WS WF-net with an acyclic WS WF-net yields an acyclic WS WF-net. Since $(N^CB_{TP},[p_1]) \sim_b (N^WS_{TP},[p_1])$ and $L_1, L_2, L_0$ are acyclic WS WF-nets, $((N^CB_{TP} \otimes p_0, L_1 \otimes p_0, L_2 \otimes p_0, L_0, [p_1])) \sim_b ((N^WS_{TP} \otimes p_0, L_1 \otimes p_0, L_2 \otimes p_0, L_0, [p_1]))$ holds. And we can know from the net structure that every external action is unique. Thus if $N_X$ is $((N^CB_{TP} \otimes p_0, L_1 \otimes p_0, L_2 \otimes p_0, L_0, [p_1]))$ then this property holds.

Case (ii): Let us first consider a special case, i.e. $N_X$ is $N^CB_{TP}.$ Assume that $N_Y$ is the WF-net $N^WS_{TP}$ shown in Fig. 6. $N^WS_{TP}$ is obviously acyclic WS. Assume the following relation $\mathcal{R}$ between $R(N^CB_{TP},[p_1])$ and $R(N^WS_{TP},[p_1]): [p_1][R][p_1], [p_1,p_2][R][p_2],$ and $[p_0][R][p_0].$ We can prove $(N^CB_{TP},[p_1]) \sim_b (N^WS_{TP},[p_1])$ in a similar way as Case (i). And we can know from the net structure that every external action is unique. Thus if $N_X$ is $N^CB_{TP}$ then this property holds.

Next let us consider a general case, i.e. $N_X$ is $((N^CB_{TP} \otimes p_0, L_1 \otimes p_0, L_2 \otimes p_0, L_0, [p_1]))$ and $[p_1][R][p_1]$ and $[p_1,p_2][R][p_2]$ and $[p_0][R][p_0].$ We can prove $(N^CB_{TP},[p_1]) \sim_b (N^WS_{TP},[p_1])$ in a similar way as Case (i). And we can know from the net structure that every external action is unique. Thus if $N_X$ is $((N^CB_{TP} \otimes p_0, L_1 \otimes p_0, L_2 \otimes p_0, L_0, [p_1]))$ then this property holds.

This property implies that we can solve the refactoring problem if a given acyclic EFC WF-net is CB.

Let us consider the computation complexity of checking the sufficient condition. We give an algorithm for checking whether a given acyclic EFC but non-WS WF-net is CB.

\textbf{Q.E.D.}

1° Find cross structures in $N.$ If two or more cross structures are found, output no and stop.
2° For each pair $(p_1,p_2) \in P \times P,$ if any path from $p_1$ to $p_2$ includes $p_1,$ and any path from $p_1$ to $p_0$ includes $p_2,$ then let $N'$ be a WF-net obtained by connecting all paths from $p_1$ to $p_2,$ apply $\leftrightarrow \text{Decision of Well-Structuredness} \Rightarrow \text{of Ref. [3] to } N'.$ If the result is yes, reduce the part corresponding to $N'$ in $N$ to a place.
3° If the resultant net is isomorphic with $N^WS$ or $N^WS$ then output yes and stop. Otherwise output no and stop.

Since $\leftrightarrow \text{Decision of Well-Structuredness} \Rightarrow \text{is a polynomial time algorithm}$, $\leftrightarrow \text{Decision of Cross-Bridgedness} \Rightarrow \text{can run in polynomial time obviously. Thus the above sufficient condition can be checked in polynomial time.}$

Let us consider the net $N_1$ shown in Fig. 1. We can transform $N_1$ to a net which is isomorphic to $N^CB_{TP}.$ This implies that $N_1$ is CB. We can know from the sufficient condition that we can solve the refactoring problem for $N_1.$ Figure 7 shows an answer of the problem.

\section{Conclusion}

In this paper, we have first given the formal definition of acyclic EFC WF-net refactoring problem. Next we have given a necessary condition and a sufficient condition for solving the problem. Then we have shown that those conditions can be checked in polynomial time. Finally we have illustrated that an instance of the problem can be solved with those conditions. Those conditions are only the first step to the refactoring problem. As the next step, we plan to investigate refactoring for the more general case. To promote this
plan, we first investigate decidability of the decision problem, EFC-WF-REFACTORING, related to the refactoring problem. If the problem is decidable then we would give a refactoring method for refactorable acyclic EFC WF-nets. Otherwise, we would look for a larger subclass of acyclic EFC refactorable to acyclic WS, and then give a refactoring method for the subclass. The method is intuitively to remove structures not allowed in acyclic WS WF-nets from a given EFC WF-net, guaranteeing branching bisimilarity.

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