Session Types in a Linearly Typed Multi-Threaded Lambda-Calculus

Hongwei Xi  Zhiqiang Ren  Hanwen Wu  William Blair
Boston University
(hwxi,aren,hwwu,wdblair}@cs.bu.edu

Abstract
We present a formalization of session types in a multi-threaded lambda-calculus (MTLC) equipped with a linear type system, establishing for the MTLC both type preservation and global progress. The latter (global progress) implies that the evaluation of a well-typed program in the MTLC can never reach a deadlock. As this formulated MTLC can be readily embedded into ATS, a full-fledged language with a functional programming core that supports both dependent types (of DML-style) and linear types, we obtain a direct implementation of session types in ATS. Compared to various existing formalizations of session types, we see the one given in this paper is unique in its closeness to concrete implementation. In particular, we report such an implementation ready for practical use that generates Erlang code from well-typed ATS source (making use of session types), thus taking great advantage of the infrastructural support for distributed computing in Erlang.

1. Introduction
In broad terms, a (dyadic) session is an interaction between two concurrently running programs, and a session type is a form of type for specifying (or classifying) sessions. As an example, let us assume that two programs P and Q are connected with a bidirectional channel. From the perspective of P, the channel may be specified by the following term sequence:

\[ \text{snd}(\text{int}) :: \text{rcv}(\text{int}) :: \text{rcv}(\text{bool}) :: \text{nil} \]

which means that an integer is to be sent, another integer is to be received, and finally the channel is to be closed. Clearly, from the perspective of Q, the channel should be specified by the following term sequence:

\[ \text{rcv}(\text{int}) :: \text{rcv}(\text{int}) :: \text{snd}(\text{bool}) :: \text{nil} \]

which means precisely the dual of what the previous term sequence does. We may think of P as a client who sends two integers to the server Q and then receives from Q either true or false depending on whether or not the first sent integer is less than the second one.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org.

Copyright © 2015 ACM 978-1-4503-3677-7/15/0011...
http://dx.doi.org/10.1145/2691398.2692874...

Figure 1. Some pseudo code in ML-like syntax

The session between P and Q is bounded in the sense that it contains only a bounded number of sends and receives. By introducing recursively defined session types, we can specify unbounded sessions containing indefinite numbers of sends and receives.

In Figure 1 we present some pseudo code showing a plausible way to implement the programs P and Q. Please note that the functions P and Q, though written together here, can be written in separate contexts. We use CH to refer to a channel available in a context and \( S \) and \( T \) for sending and receiving data via a given channel, and \( \text{channel}_\text{send} \) and \( \text{channel}_\text{recv} \) for sending and receiving data via a given channel. Let us now sketch a way to make the above pseudo code typecheck. We can assign the following type to \( \text{channel}_\text{send} \):

\[ (\langle \text{chan} \rangle \text{snd}(T) :: S) \gg \text{chan}(S), T \to 1 \]

where \( T \) stands for a type and \( S \) for a session type. Basically, this type means that calling \( \text{channel}_\text{send} \) on a channel of the type \( \text{chan}(S) :: T \) and a value of the type \( T \) returns a unit while changing the type of the channel to \( \text{chan}(S) \). Clearly, \( \text{chan} \) must be a linear type constructor for this to make sense. As can be expected, the type assigned to \( \text{channel}_\text{recv} \) should be of the following form:

\[ (\langle \text{chan} \rangle \text{rcv}(T) :: S) \gg \text{chan}(S)) \to T \]

which means that calling \( \text{channel}_\text{recv} \) on a channel of the type \( \text{chan}(S) :: T \) returns a value of the type \( T \) while changing the type of the channel to \( \text{chan}(S) \). As for \( \text{channel}_\text{close} \), it is assigned the following type:

\[ (\text{chan} \langle \text{nil} \rangle) \to 1 \]

indicating that calling \( \text{channel}_\text{close} \) on a channel consumes the channel (so that the channel is no longer available for use).

ATS [27] is a full-fledged language with a functional programming core based on ML that supports both dependent types.
We first present a multi-threaded lambda-calculus MTLC₀ equipped with a simple linear type system, setting up the basic machinery for further development. We then extend MTLC₀ to MTLCₙ in Section 3 with support for session types and establish both type preservation and global progress for MTLCₙ. We give interpretation to some linear logic connectives in Section 5 to facilitate understanding of session types, and briefly mention some issues on implementing session types. We next present a couple of interesting examples in Section 6 to illustrate programming with session types. Lastly, we discuss some closely related work in Section 8 and then conclude.

The primary contribution of the paper consists of a novel formalization of session-types. Compared to various existing formalizations of session types (e.g., [3, 6, 9, 10, 23, 24, 26]), we see this one being unique in its closeness to concrete implementation. Indeed, we report an implementation of session types ready for practical use that generates Erlang code from well-typed ATS source. The primary technical contribution of the paper lies in a simple and general approach to showing that concurrency based on session types is deadlock-free. For this, a novel notion of DF-reducibility (where DF stands for deadlock-freeness) is introduced.

2. MTLC₀ with Linear Types

We present a multi-threaded lambda-calculus MTLC₀ equipped with a simple linear type system, setting up the basic machinery for further development. The dynamic semantics of MTLC₀ can essentially be seen as an abstract form of evaluation of multi-threaded programs.

Some syntax of MTLC₀ is given in Figure 2. We use x for a lam-variable and f for a fix-variable, and sx for either a lam-variable or a fix-variable. Note that a lam-variable is considered a value but a fix-variable is not. We use rc for constant resources and c for constants, which include both constant functions cf and constant constructors cc. We treat resources in MTLC₀ abstractly and will later introduce communication channels as a concrete form.
of resources. The meaning of various standard forms of expressions in MTLC₀ should be intuitively clear. We may refer to a closed expression (that is, an expression containing no free variables) as a program.

We use \( T \) and \( \hat{T} \) for (non-linear) types and (linear) viewtypes, respectively, and refer \( \hat{T} \) to as a true viewtype if it is a viewtype but not a type. We use \( \delta \) and \( \hat{\delta} \) for base types and base viewtypes, respectively. For instance, \texttt{bool} is the base type for booleans and \texttt{int} for integers. For a simplified presentation, we do not introduce any concrete base viewtypes in MTLC₀. We assume a signature \( \Sigma \) for assigning a viewtype to each constant resource \( rc \) and a constant type (c-type) of the form \((\hat{T}_1, \ldots, \hat{T}_k) \Rightarrow \hat{T}\) to each constant.

We use \( \sigma \) and \( \hat{\sigma} \) for variables ranging over types and viewtypes, respectively, but we do not support explicit quantification over these variables until Section 4.

Note that a type is always considered a viewtype. Let \( \hat{T}_1 \) and \( \hat{T}_2 \) be two viewtypes. The type constructor \( \otimes \) is based on multiplicative conjunction in linear logic. Intuitively, if a resource is assigned the viewtype \( \hat{T}_1 \otimes \hat{T}_2 \), then the resource is a conjunction of two resources of viewtypes \( \hat{T}_1 \) and \( \hat{T}_2 \). The type constructor \( \Rightarrow \) is essentially based on linear implication \( \rightarrow \) in linear logic. Given a function of the viewtype \( \hat{T}_1 \rightarrow \hat{T}_2 \) and a value of the viewtype \( \hat{T}_1 \), applying the function to the value yields a result of the viewtype \( \hat{T}_2 \) while the function itself is consumed. If the function is of the type \( \hat{T}_1 \Rightarrow \hat{T}_2 \), then applying the function does not consume it. The subscript \( i \) in \( \rightarrow \) is often dropped, that is, \( \rightarrow \) is assumed to be \( \rightarrow \) by default. The meaning of various forms of types and viewtypes is to be made clear and precise when the rules are presented for assigning viewtypes to expressions in MTLC₀.

There is no type constructor in MTLC₀ based on additive disjunction in linear logic denoted by \( \oplus \) (but such a type constructor is fully supported in ATS), and this omission is entirely for the sake of a simplified presentation. There are also multiplicative disjunction \( (\forall) \) and additive conjunction \( (\&\) in linear logic [3]. If we see viewtypes negatively in the sense that they are for classifying capabilities (spaces) of consuming (storing) resources, then \( \hat{T}_1 \forall \hat{T}_2 \) essentially means the capability (space) that joins two classified by \( \hat{T}_1 \) and \( \hat{T}_2 \). We can interpret \( \hat{T}_1 \& \hat{T}_2 \) as a choice to obtain any capability (space) that can be classified by either \( \hat{T}_1 \) or \( \hat{T}_2 \). There is no type constructor corresponding to \( \forall \in \text{ATS} \). As for \( \hat{T}_1 \& \hat{T}_2 \), we can use the following dependent type in ATS to replace it:

\[
\forall b : \texttt{bool}. \texttt{bool}(b) \Rightarrow \texttt{choose}(\hat{T}_1, \hat{T}_2, b)
\]

where \( \texttt{bool}(b) \) is a singleton type for the only boolean value equal to \( b \) and \( \texttt{choose}(\hat{T}_1, \hat{T}_2, b) \) equals \( \hat{T}_1 \) or \( \hat{T}_2 \) depending whether \( b \) is true or false, respectively.

There is a special constant function \texttt{thread\_create} in MTLC₀ for thread creation, which is assigned the following rather interesting c-type:

\[
\texttt{thread\_create} : (1 \rightarrow 1) \Rightarrow 1
\]

A function of the type \( 1 \rightarrow 1 \) is a procedure that takes no arguments and returns no result (when its evaluation terminates). Given that \( 1 \rightarrow 1 \) is a true viewtype, a procedure of this type may contain resources and thus must be called exactly once. The operational semantics of \texttt{thread\_create} is to be formally defined later.

A variety of mappings, finite or infinite, are to be introduced in the rest of the presentation. We use \( [i] \) for the empty mapping and \([i_1, \ldots, i_k \mapsto o_1, \ldots, o_k]\) for the finite mapping that maps \( i_k \) to \( o_k \) for \( 1 \leq k \leq n \). Given a mapping \( m \), we write \( \texttt{dom}(m) \) for the domain of \( m \). If \( i \notin \texttt{dom}(m) \), we use \( m[i \mapsto o] \) for the mapping that extends \( m \) with a link from \( i \) to \( o \). If \( i \in \texttt{dom}(m) \), we use \( m[i] \) for the mapping obtained from removing the link from \( i \) to \( m(i) \) in \( m \), and \( m[i := o] \) for \( (m[i])[i \mapsto o] \), that is, the mapping obtained from replacing the link from \( i \) to \( m(i) \) in \( m \) with another link from \( i \) to \( o \).

\[
\begin{align*}
p(rc) & = [rc] \\
p(c(e_1, \ldots, e_n)) & = p(e_1) \& \ldots \& p(e_n) \\
p(x) & = 0 \\
p(\emptyset) & = 0 \\
p(e_1 \& e_2) & = p(e_1) \& p(e_2) \\
p(\text{fst}(e)) & = p(e) \\
p(\text{snd}(e)) & = p(e) \\
p(\text{fix}(e)) & = p(e) \\
p(\text{app}(e_1, e_2)) & = p(e_1) \& p(e_2) \\
p(\text{lam}(x. e)) & = p(e)
\end{align*}
\]

We define a function \( p(\cdot) \) in Figure 3 to compute the multiset (that is, bag) of constant resources in a given expression. Note that \( \& \) denotes the multiset union. In the type system of MTLC₀, it is to be guaranteed that \( p(e_1) \) equals \( p(e_2) \) whenever an expression of the form \( \text{fix}(e_0, e_1, e_2) \) is constructed, and this justifies \( p(\text{fix}(e_1, e_2)) \) being defined as \( p(e_1) \& p(e_2) \).

We use \( R \) to range over finite multisets of resources. Therefore, \( R \) can also be regarded as a mapping from resources to natural numbers: \( R(rc) = n \) means that there are \( n \) occurrences of \( rc \) in \( R \). It is clear that we may not combine resources arbitrarily. For instance, we may want to exclude the combination of one resource stating integer 0 at a location \( L \) and another one stating integer 1 at the same location. We fix an abstract collection \texttt{RES} of finite multisets of resources and assume the following:

- \( \emptyset \in \texttt{RES} \)
- For any \( R_1, R_2 \in \texttt{RES} \), if \( R_1 \in \texttt{RES} \) and \( R_2 \subseteq R_1 \), where \( \subseteq \) is the subset relation on multisets.

We say that \( R \) is a valid multiset of resources if \( R \in \texttt{RES} \) holds.

In order to formalize threads, we introduce a notion of \texttt{pools}. Conceptually, a pool is just a collection of programs (that is, closed expressions). We use \( \Pi \) for pools, which are formally defined as finite mappings from thread ids (represented as natural numbers) to (closed) expressions in MTLC₀ such that 0 is always in the domain of such mappings. Given a pool \( \Pi \) and \( tid \in \texttt{dom}(\Pi) \), we refer to \( \Pi(tid) \) as a thread in \( \Pi \) whose id equals \( tid \). In particular, we refer to \( \Pi(0) \) as the main thread in \( \Pi \). The definition of \( p(\cdot) \) is extended as follows to compute the multiset of resources in a given pool:

\[
\rho(\Pi) = (\cup_{tid \in \texttt{dom}(\Pi)} p(\Pi(tid)))
\]

We are to define a relation on pools in Section 2.2 to simulate multi-threaded program execution.

### 2.1 Static Semantics

We present typing rules for MTLC₀ in this section. It is required that each variable occur at most once in an intuitionistic (linear) expression context \( \Gamma(\Delta) \), and thus \( \Gamma(\Delta) \) can be regarded as a finite mapping. Given \( \Gamma_1 \) and \( \Gamma_2 \) such that \( \text{dom}(\Gamma_1) \cap \text{dom}(\Gamma_2) = \emptyset \), we write \( (\Gamma_1, \Gamma_2) \) for the union of \( \Gamma_1 \) and \( \Gamma_2 \). The same notation also applies to linear expression contexts \( (\Delta) \). Given an intuitionistic expression context \( \Gamma \) and a linear expression context \( \Delta \), we can form a combined expression context \( (\Gamma, \Delta) \) if \( \text{dom}(\Gamma) \cap \text{dom}(\Delta) = \emptyset \). Given \( (\Gamma, \Delta) \), we may write \( (\Gamma, \Delta), x : T \) for either \( (\Gamma, x : T, \Delta) \) or \( (\Gamma, x : T, \Delta) \) (if \( T \) is actually a type).

We use \( \Theta \) for a substitution on type and viewtype variables:

\[
\Theta ::= [] | \Theta[x \mapsto T] | \Theta[\hat{a} \mapsto \hat{T}]
\]
Given a viewtype \( \hat{T} \), we write \( \hat{T}[\theta] \) for the result of applying \( \Theta \) to \( \hat{T} \), which is defined in a standard manner. Given a constant resource \( rc \), we write \( \text{SIG} \models rc : \hat{\sigma} \) to mean that \( rc \) is assigned the viewtype \( \hat{\sigma} \) in the signature \( \text{SIG} \). Given a constant \( rc \), we use the following judgment:
\[
\text{SIG} \models c : (\hat{T}_1, \ldots, \hat{T}_n) \Rightarrow \hat{T}_0
\]

By inspecting the rules in Figure 4 we can readily see that a closed value cannot contain any resources if the value itself can be assigned a type (rather than a linear type). More formally, we have the following proposition:

**Proposition 2.1.** Assume that \((\emptyset; \emptyset) \vdash v : T\) is derivable. Then \(\rho(v) = \emptyset\).

This proposition plays a fundamental role in the design of \(\text{MTLC}_0\) as the rules in Figure 4 are actually so formulated in order to make it hold.

The following lemma, which is often referred to as **Lemma of Canonical Forms**, relates the form of a value to its type:

**Lemma 2.2.** Assume that \((\emptyset; \emptyset) \vdash v : \hat{T}\) is derivable.

- If \(\hat{T} = \delta\), then \(v\) is of the form \(\text{cc}(v_1, \ldots, v_n)\).
- If \(\hat{T} = \delta\), then \(v\) is of the form \(\text{rc} or \text{cc}(v_1, \ldots, v_n)\).
- If \(\hat{T} = 1\), then \(v\) is \(\emptyset\).
- If \(\hat{T} = T_1 \cdot T_2\) or \(\hat{T} = T_1 \otimes T_2\), then \(v\) is of the form \(\langle v_1, v_2 \rangle\).
- If \(\hat{T} = T \rightarrow \tilde{T}_1 \otimes \tilde{T}_2\), then \(v\) is of the form \(\hat{T} lam \emptyset \vdash e\).

**Proof.** By inspection of the rules in Figure 4.

For each \(\theta\), we define the multiset \(\rho(\theta)\) of resources in \(\theta\) as follows:
\[
\rho(\theta) = \omega_{xf}[dom(\theta)]\rho(\theta[xf])
\]

Given an expression \(e\), we use \(e[\theta]\) for the result of applying \(\theta\) to \(e\), which is defined in a standard manner. We write \((\Gamma_1; \Delta_1) \vdash \theta : (\Gamma_2; \Delta_2)\) to mean that

- \(\text{dom}(\theta) = \text{dom}(\Gamma_2) \cup \text{dom}(\Delta_2)\), and
- \((\Gamma_1; \emptyset) \vdash \theta(\text{xf}) : \Gamma_2(\text{xf})\) is derivable for each \(\text{xf} \in \Gamma_2\), and
- there exists a linear expression context \(\Delta_1, x\) for each \(x \in \text{dom}(\Delta_2)\) such that \((\Gamma_1; \Delta_1) \vdash \theta(x) : \Delta_2(x)\) is derivable, and
- \(\Delta_1 = \cup_{x \in \text{dom}(\Delta_2)} \Delta_1, x, \hat{T}\).

The following lemma, which is often referred to as **Substitution Lemma**, is needed to establish the soundness of the type system of \(\text{MTLC}_0\):

**Lemma 2.3.** (Substitution) Assume \((\Gamma_1; \Delta_1) \vdash \theta : (\Gamma_2; \Delta_2)\) and \((\Gamma_1; \Delta_1) \vdash e : \hat{T}\). Then \((\Gamma_1; \Delta_1) \vdash e[\theta] : \hat{T}\) is derivable and \(\rho(e[\theta]) = \rho(e) \cup \rho(\theta)\).

**Proof.** By induction on the derivation of \((\Gamma_2; \Delta_2) \vdash e : \hat{T}\).

### 2.2 Dynamic Semantics

We present evaluation rules for \(\text{MTLC}_0\) in this section. The evaluation contexts in \(\text{MTLC}_0\) are defined below:

**eval. ctx. E ::=**

\[
\begin{align*}
| E & : [1] | c(\text{xf}, E, \text{xf}) | i\text{ff}(E, e_1, e_2) | \\
\langle E, E \rangle | \langle v, E \rangle | \text{let } (x_1, x_2) &= E \text{ in } e \text{ end } | \\
\text{fst}(E) | \text{snd}(E) | \text{app}(E, e) | \text{app}(v, E)
\end{align*}
\]

Given an evaluation context \(E\) and an expression \(e\), we use \(E[e]\) for the expression obtained from replacing the only hole \([1]\) in \(E\) with \(e\).

**Def. 2.4.** We define pure redexes and their redunqs as follows:

- \(\text{if}(true, e_1, e_2)\) is a pure redex whose redunq is \(e_1\).
- \(\text{if}(false, e_1, e_2)\) is a pure redex whose redunq is \(e_2\).
- \(\text{let } (x_1, x_2) = \langle v_1, v_2 \rangle \text{ in } e \text{ end }\) is a pure redex whose redunq is \(e[<x_1, x_2> \rightarrow v_1, v_2]\).
- \(\text{fst}(\langle v_1, v_2 \rangle)\) is a pure redex whose redunq is \(v_1\).
Lemma 2.2 is needed. Essentially, we can readily show that $\exists \text{ some tid}$ holds whenever $\Pi [\text{tid}]$ is a pure redex whose reduct is $e[x \mapsto v]$. If $\Pi [\text{tid}]$ is a value for some tid > 0, then this value must be $\bot$. So the rule (PR2) can be used to reduce $\Pi_1$. If $\Pi_1(tid)$ is of the form $E[e]$ for some redex $e$, then the rule (PR0) can be used to reduce $\Pi_1$.

By combining Theorem 2.7 and Theorem 2.8 we immediately conclude that the evaluation of a well-typed pool either leads to a pool that itself is a singleton mapping of the form $[0 \mapsto v]$ for some value $v$, or it goes on forever. In other words, MTLCo is type-sound.

3. Extending MTLCo with Channels

There is no support for communication between threads in MTLCo, making MTLCo uninteresting as a multi-threaded language. We extend MTLCo to MTLCo, with support for synchronous communication channels in this section. Supporting asynchronous communication channels is certainly possible but would result in a more involved theoretical development. We do support both synchronous and asynchronous session-typed communication channels in practice, though. In order to assign types to channels, we introduce session types as follows:

$$S ::= \text{nil} \mid \text{snd}(\hat{T}) : : S \mid \text{rcv}(\hat{T}) : : S$$

An empty session is specified by nil. When used to specify a positive channel, $\text{snd}(\hat{T})$ means to send onto the channel a value of the viewtype $\hat{T}$ and $\text{rcv}(\hat{T})$ means to receive from the channel a value of the viewtype $\hat{T}$. Dually, when used to specify a negative channel, $\text{snd}(\hat{T})$ means to send from the channel a value of the viewtype $\hat{T}$ and $\text{rcv}(\hat{T})$ means to receive onto the channel a value of the viewtype $\hat{T}$. After either sending or receiving is done, the channel is specified by $S$.

Formally, the dual of a session type is defined as follows:

$$\text{dual}(\text{nil}) = \overline{\text{nil}}$$

$$\text{dual}(\text{snd}(\hat{T})) = \text{rcv}(\hat{T}) : : \text{dual}(S)$$

$$\text{dual}(\text{rcv}(\hat{T})) = \text{snd}(\hat{T}) : : \text{dual}(S)$$

where $\overline{\text{nil}}$ is another constant session type denoting the dual of $\text{nil}$. Traditionally, $\text{nil}$ and $\overline{\text{nil}}$ are treated as the same constant in the study on session types. In our implementation, a positive channel specified by $\text{nil}$ awaits a message to close itself while a negative channel specified by $\overline{\text{nil}}$ is a positive channel specified by $\text{nil}$ and $\overline{\text{nil}}$ sends out such a message before closing itself.

Formally, we use $\text{chpos}(S)$ and $\text{chneg}(S)$ for a positive and negative channel specified by $S$, respectively. Though it is clear that $\text{chpos}(S) \land \text{chneg}(\text{dual}(S))$ equal $\text{chneg}(S)$ and $\text{chpos}(\text{dual}(S))$, respectively, we do not attempt for now to make use of this fact in our formalization of session types. In particular, there is currently no support for turning a positive channel into a negative channel or vice versa.

We use $\sigma$ as a variable ranging over session types. The function $\text{chneg} \_ \text{create}$ for creating a negative channel is assigned the following c-type:

$$\text{chneg} \_ \text{create} : (\text{chpos}(\sigma) \rightarrow 1) \Rightarrow \text{chneg}(\sigma)$$

Given a linear function of the type $\text{chpos}(S) \rightarrow 1$ for some $S$, $\text{chneg} \_ \text{create}$ essentially creates a positive channel and a negative channel that are properly connected, and then starts a thread for evaluating the call that applies the function to the positive channel, and then returns the negative channel. The newly created positive channel and negative channel share the same id.

The send and receive functions for positive channels are given the following c-types:

$$\text{send} : (\text{chpos}(\text{snd}(\hat{a}) : : \sigma), \hat{a}) \Rightarrow \text{chpos}(\sigma)$$

$$\text{rcv} : (\text{chpos}(\text{rcv}(\hat{a}) : : \sigma), \hat{a}) \Rightarrow \text{chpos}(\sigma) \land \hat{a}$$

Note that $\text{send}$ and $\text{rcv}$ correspond to the functions $\text{chanpos} \_ \text{send}$ and $\text{chanpos} \_ \text{rcv}$, respectively.
Dually, the send and receive functions for negative channels are given the following c-types:

\[ \text{send} : (\text{chneg}(\text{snd}(\tilde{a}) :: \sigma)) \to \text{chneg}(\sigma) \]
\[ \text{recv} : (\text{chneg}(\text{rcv}(\tilde{a}) :: \sigma)) \to \text{chneg}(\sigma) @ \tilde{a} \]

Note that send and recv correspond to the functions chneg\_send and chneg\_recv, respectively.

The functions close and close* for closing positive and negative channels, respectively, are given the following c-types:

\[ \text{close} : (\text{chpos}(\text{nil})) \to \text{I} \]
\[ \text{close*} : (\text{chneg}(\text{nil})) \to \text{I} \]

Note that close and close* can also be referred to as chpos\_close and chneg\_close, respectively.

In MTLC\_chb, there are resource constants ch and ch\_r for positive and negative channels, respectively, where \( i \) ranges over natural numbers. For each \( i \), ch and ch\_r are dual to each other and their channel ids are \( i \). We use ch and ch\_r to range over ch and ch\_r, respectively, referring one as the dual of the other.

There are no new typing rules in MTLC\_chb over MTLC\_b. Given a session type \( S \), we say that the type chpos\( (S) \) matches the type chneg\( (S) \) and vice versa. In any type derivation of \( \Pi : T \) satisfying \( \rho(\Pi) \in \text{RES} \), the type assigned to a positive channel \( ch \) is always required to match the one assigned to the corresponding negative channel \( ch\_r \) of the same channel id. For evaluating pools in MTLC\_b, we have the following additional rules in MTLC\_ch:\n
\[ \Pi(tid_0) = E[\text{chneg}\_create(\lambda x.e)] \]
\[ \Pi \to \Pi[tid_0 := E[ch][tid \mapsto \text{app}(\lambda x.e,ch)]] \quad \text{(PR3)} \]
\[ \Pi(tid_1) = E_1[\text{close}(ch)] \quad \Pi(tid_2) = E_2[\text{close}(ch)] \]
\[ \Pi \to \Pi[tid_1 := E_1[\{]\{tid_2 := E_2[\}]] \quad \text{(PR4-clos)} \]
\[ \Pi(tid_1) = E_1[\text{send}(ch,v)] \quad \Pi(tid_2) = E_2[\text{send}(ch)] \]
\[ \Pi \to \Pi[tid_1 := E_1[ch][tid_2 := E_2[ch,v]]] \quad \text{(PR4-send)} \]
\[ \Pi(tid_1) = E_1[\text{recv}(ch,v)] \quad \Pi(tid_2) = E_2[\text{recv}(ch)] \]
\[ \Pi \to \Pi[tid_1 := E_1[ch][tid_2 := E_2[ch,v]]] \quad \text{(PR4-reccv)} \]

For instance, the rule PR4-send states: If a program in a pool \( \Pi \) is of the form \( E_1[\text{send}(ch,v)] \) and another of the form \( E_2[\text{send}(ch)] \), then this pool can be reduced to another pool by replacing these two programs with \( E_1[ch,v] \) and \( E_2[ch] \), respectively.

While Theorem 2.7 (Subject Reduction) can be readily established for MTLC\_b, Theorem 2.8 (Progress) requires some special treatment due to the presence of session-type primitive functions chneg\_create, close, close*, send, recv, send*, and recv*

A partial (ad-hoc) redex in MTLC\_chb is of one of the following forms: \( \text{close}(ch), \text{close*}(ch), \text{send}(ch), \text{recv}(ch), \text{send*}(ch), \text{recv*}(ch) \). Clearly, either a positive channel ch or a negative channel ch\_r is involved in each partial redex. We say that close(ch) and close\_r(ch) match, and send(ch,v) and send\_r(ch) match, and recv(ch) and recv\_r(ch,v) match. We can immediately prove in MTLC\_b that each well-typed program is either a value or of the form \( E[e] \) for some evaluation context \( E \) and expression \( e \) that is either a redex or a partial redex. We refer to an expression as a blocked one if it is of the form \( E[e] \) for some partial redex \( e \). We say two blocked expressions \( E_1[e_1] \) and \( E_2[e_2] \) match if \( e_1 \) and \( e_2 \) are matching partial redexes. Clearly, a pool containing two matching blocked expressions can be reduced according to one of the rules PR4-clos, PR4-send, and PR4-reccv.

Intuitively, a pool \( \Pi \) is deadlocked if \( \Pi(tid) \) for \( tid \in \text{dom} (\Pi) \) are all blocked expressions but there are no matching ones among them, or if \( \Pi(0) \) is a value and \( \Pi(tid) \) for positive \( tid \in \text{dom} (\Pi) \) are all blocked expressions but there are no matching ones among them. The following lemma states that a well-typed pool in MTLC\_chb can never be deadlocked:

**Lemma 3.1. (Deadlock-Freedom)** Let \( \Pi \) be a well-typed pool in MTLC\_chb such that \( \Pi(0) \) is either a value containing no channels or a blocked expression and \( \Pi(tid) \) for each positive \( tid \in \text{dom}(\Pi) \) is a blocked expression. If \( \Pi \) is obtained from evaluating an initial pool containing no channels, then there exist two thread ids \( tid_1 \) and \( tid_2 \) such that \( \Pi(tid_1) \) and \( \Pi(tid_2) \) are matching blocked expressions.

Note that it is entirely possible to encounter a scenario where the main thread in a pool returns a value containing a channel while another thread is waiting for something to be sent on the channel. Technically, we do not classify this scenario as a deadlocked one. There are many forms of values that contain channels. For instance, such a value can be a channel itself, or a closure-function containing a channel in its environment, or a compound value like a tuple that contains a channel as one part of it, etc. Clearly, any value containing a channel can only be assigned a true viewtype.

The primary technical contribution of the paper lies in the following presented approach to establishing Lemma 3.1. Let us use \( M \) for sets of (positive and negative) channels and \( M \) for a finite non-empty collection (that is, multiset) of such sets. We say that \( M \) is regular if the sets in \( M \) are pairwise disjoint and each pair of channels \( c \) and \( c\_r \) are either both included in the multiset union \( \bigcup(M) \) of all the sets in \( M \) or both excluded from it. Of course, \( \bigcup(M) \) is the same as the set union \( \bigcup(M) \) as the sets in \( M \) are pairwise disjoint.

Let \( M \) be a regular collection of channel sets. We say that \( M \) DF-reduces to \( M' \) via \( \alpha \) if \( M' \) contains a channel \( \alpha \) and \( M' \) contains \( \bigcup(M) \) for a finite \( \alpha \) and \( \bigcup(M) \). We say that \( M \) DF-reduces to \( M' \) via some ch. We may write \( M \leadsto M' \) to mean that \( M \) DF-reduces to \( M \). We say that \( M \) is DF-normal if there is no \( M' \) such that \( M \leadsto M' \) holds.

**Proposition 3.2.** Let \( M \) be a regular collection of channel sets. If \( M \) is DF-normal, then each set in \( M \) consists of an indefinite number of channel pairs \( ch \) and \( ch\_r \). In other words, for each \( M \) in a DF-normal \( M \), a channel \( ch \) is in \( M \) if and only if its dual \( ch\_r \) is also in \( M \).

**Proof** The proposition immediately follows from the definition of DF-reduction. ■

**Definition 3.3.** A regular collection \( M \) of channel sets is DF-reducible if either (1) each set in \( M \) is empty or (2) \( M \) is not DF-normal and \( M \) is DF-reducible whenever \( M \leadsto M' \) holds.

We say that a channel set \( M \) is self-looping if it contains both \( ch \) and \( ch\_r \) for some \( ch \). Obviously, a regular collection \( M \) of channel sets is not DF-reducible if there is a self-looping \( M \) in \( M \).

**Proposition 3.4.** Let \( M \) be a regular collection of channel sets. If \( M \leadsto M' \) and \( M' = M \setminus \{0\} \), then \( M' \) is also DF-reducible.

**Proof** Straightforwardly. ■

**Proposition 3.5.** Let \( M \) be a regular collection of channel sets. If \( M \leadsto M' \) and \( M' \) is DF-reducible, then \( M \) is also DF-reducible.

**Proof** Clearly, \( M \leadsto M' \) via some \( ch \). Assume \( M \leadsto M_1 \) via \( ch \) for some \( M_1 \) and \( ch\_r \). If \( ch \) and \( ch\_r \) are the same, then \( M_1 \) is DF-reducible as it is the same as \( M' \). Otherwise, it can be
readily verified that there exists M’ such that M \sim M’ via ch and M’ \sim M’ via ch_1. Clearly, the latter implies M’ being DF-reducible. Note that the size of M is strictly less than that of M. By induction hypothesis on M, we have M being DF-reducible. By definition, M is DF-reducible.

**Proposition 3.6.** Let M be a regular collection of channel sets that is DF-reducible. If M_1 and M_2 in M contain ch and ch, respectively, then M’ = (M_1 \cup M_2) ∪ (M_1, M_2) is also DF-reducible, where M_1’ = M_1\{ch\} and M_2’ = M_2\{ch\}.

**Proof.** The proposition follows from a straightforward induction on the size of the set union ∪(M).

**Lemma 3.7.** Let M be a regular collection of n channel sets M_1, ..., M_n for some n ≥ 1. If the union ∪(M) = M_1 ∪ ... ∪ M_n contains at least n channel pairs (ch_1, ch_1), ..., (ch_n, ch_n), then M is not DF-reducible.

**Proof.** By induction on n. If n = 1, then M is not DF-reducible as M_1 is self-looping. Assume n > 1. If either M_1 or M_2 is self-looping, then M is not DF-reducible. Otherwise, we may assume that ch_1 \in M_1 and ch_1 \in M_2 without loss of generality. Then M DF-reduces to M’ via ch_1 for some M’ containing n - 1 channel sets. Note that ∪(M’) contains at least n - 1 channel pairs (ch_2, ch_2), ..., (ch_n, ch_n). By induction hypothesis, M’ is not DF-reducible. So M is not DF-reducible, either.

Given an expression e in MTLC_{ch}, we use ρ_C(e) for the set of channels contained in e. Given a pool Π in MTLC_{ch}, we use R_{ch}(Π) for the collection of R_{ch}(Π(tid)), where tid ranges over dom(Π).

**Lemma 3.8.** If R_{ch}(Π) is DF-reducible and Π evaluates to Π’, then R_{ch}(Π’) is also DF-reducible.

**Proof.** Note that R_{ch}(Π) and R_{ch}(Π’) are the same unless Π evaluates to Π’ according to one of the rules PR3, PR4-clos, PR4-send, and PR4-rev.

- For the rule PR3: We have R_{ch}(Π’) \sim R_{ch}(Π) via the newly introduced channel ch. By Proposition 3.6, R_{ch}(Π’) is DF-reducible.
- For the rule PR4-clos: We have that R_{ch}(Π’) is DF-reducible by Proposition 3.6.
- For the rule PR4-send: Let ch be the channel on which a value is sent when Π evaluates to Π’. Note that this value can itself be a channel or contain a channel. We have R_{ch}(Π) \sim M via ch for some M. So M is DF-reducible by definition. Clearly, R_{ch}(Π’) \sim M via ch as well. By Proposition 3.6, R_{ch}(Π’) is DF-reducible.
- For the rule PR4-rev: This case is similar to the previous one.

We are now ready to give a proof for Lemma 3.1.

**Proof.** Note that any channel, either positive or negative, can appear at most once in R_{ch}(Π), and a channel ch appears in R_{ch}(Π) if and only if its dual ch also appears in R_{ch}(Π). In addition, any positive channel ch being assigned a type of the form chpos(S) in the type derivation of Π for some session type S mandates that its dual ch be assigned the type of the form chneg(S).

Assume that Π(tid) is a blocked expression for each tid \in dom(Π). If the partial redex in Π(tid) involves a positive channel ch while the partial redex in Π(tid_2) involves its dual ch, then these two partial redexes must match. This is due to Π being well-typed. In other words, the ids of the channels involved in the partial redexes of Π(tid) for tid \in dom(Π) are all distinct. This simply implies that there are n channel pairs (ch, ch_1) in ∪(R_{ch}(Π)) for some n greater than or equal to the size of Π. By Lemma 3.7, R_{ch}(Π) is not DF-reducible. On the other hand, R_{ch}(Π) is reducible by Lemma 3.3 as Π evaluates to Π (in many step) and R_{ch}(Π_0) (containing only sets that are empty) is reducible. This contradiction indicates that there exist tid_1 and tid_2 such that Π(tid_1) and Π(tid_2) are matching blocked expressions. Therefore Π evaluates to Π’ for some pool Π’ according to one of the rules PR4-clos, PR4-send, and PR4-rev.

With Proposition 3.4, the case can be handled similarly where Π(0) is a value containing no channels and Π(tid) is a blocked expression for each positive tid \in dom(Π).

Please assume for the moment that we would like to add into MTLC_{ch} a function chneg_create2 of the following type:

( chneg_create2(\sigma_1, \sigma_2) ) \rightarrow Π \Rightarrow ( chneg(\sigma_1), chneg(\sigma_2) )

One may think of chneg_create2 as a slight generalization of chneg_create that creates in a single call two channels instead of one. Unfortunately, adding chneg_create2 into MTLC_{ch} can potentially cause a deadlock. For instance, we can easily imagine a scenario where the first of the two channels (ch_1, ch_2) returned from a call to chneg_create2 is used to send the second to the newly created thread by the call, making it possible for that thread to cause a deadlock by waiting for a value to be sent on ch_3. Clearly, Lemma 3.8 is invalidated if chneg_create2 is added.

The soundness of the type system of MTLC_{ch} tests upon the following two theorems (corresponding to Theorem 2.7 and Theorem 2.8).

**Theorem 3.9.** (Subject Reduction on Pools) Assume that r : Π_1 \rightarrow Π_2 such that r(\Pi_1) \in RES. Then r : Π_1 \rightarrow Π_2 is derivable.

**Proof.** The proof is essentially the same as the one for Theorem 2.7. The only additional part is for checking that the rules PR3, PR4-clos, PR4-send, and PR4-rev are all consistent with respect to the typing rules listed in Figure 4.

**Theorem 3.10.** (Progress Property on Pools) Assume that r : Π_1 \rightarrow Π_2 such that r(\Pi_1) \in RES. Then r : Π_1 \rightarrow Π_2 holds for some Π_2 such that r(\Pi_2) \in RES.

**Proof.** The proof follows the same structure as the one for Theorem 2.8. Lemma 3.1 is needed to handle the case where all of the threads (possibly excluding the main thread) in a pool consist of blocked expressions.

### 4. Additional Features for MTLC_{ch}

We briefly mention certain additional features for MTLC_{ch} that are to be used in some examples presented later.

#### 4.1 Bidirectional Forwarding

There is a special primitive function of the name chposneg_link for connecting a positive channel with a negative channel specified by the following session type:

chposneg_link : (chpos(\sigma), chneg(\sigma)) \Rightarrow Π
Given a positive channel and a negative channel, \textit{chposneg\_link} sends each value received from the positive channel onto the negative channel and vice versa. In other words, \textit{chposneg\_link} does bidirectional forwarding between these two channels. In practice, \textit{chposneg\_link} is often used to implement delegation of service. It can be readily verified that the two channels passed to a call to \textit{chposneg\_link} can never have the same channel id; if one is \texttt{ch1}, then the other must be \texttt{ch2} for some \texttt{ch1} distinct from \texttt{ch2}. Calling \textit{chposneg\_link} on a positive channel and its dual surely results in a deadlock. One of the evaluation rules for \textit{chposneg\_link} is given as follows:

\[
\Pi(id_1) = E_1[e_1] \quad \Pi(id_2) = E_2[e_2] \quad \Pi(id_3) = E_3[e_3]
\]

\[
\Pi \rightarrow \Pi[id_1 := E_1[ch_1]][id_2 := E_2[ch_2]][id_3 := E_3[ch_3]]
\]

where we have \(e_1 = \text{recv}(ch_1,v)\), \(e_2 = \text{chposneg\_link}(ch_1,\overline{ch_2})\), and \(e_3 = \text{recv}(ch_2)\). The other ones are omitted. It should be clear that Lemma 3.3 still holds after \textit{chposneg\_link} is added, and thus Lemma 3.4 still holds as well.

### 4.2 User-Defined Datatypes

The kind of (recursive) datatypes in ML (for tagged unions) can be added into MTLC\_\_a without any difficulty. In terms of theory, it is straightforward to support user-defined (recursive) session datatypes in MTLC\_\_a, allowing sessions of indefinite length to be specified. Essentially, all we need is to add folding/unfolding rules for handling recursive session types. As for implementation, we currently support recursive session types based on an indirect approach, which is illustrated through the examples presented in Section 6.

### 4.3 Quantification over Types

We can readily incorporate both universally and existentially quantified types into MTLC\_\_a. In ATS, there are predicative quantification and impredicative quantification. The former is for dependent types (of DML-style \([22, 30]\)) while the latter for parametric polymorphism. For instance, the example presented in Section 6.2 makes use of both parametric polymorphism and DML-style dependent types. In terms of theory, we can readily incorporate quantified session types into MTLC\_\_a. As for implementation, we have not yet attempted to add into ATS direct support for programming with quantified session types. Instead, we rely on an indirect approach to do so, which is illustrated in Section 5.

## 5. Interpreting Linear Logic Connectives

Unlike logic-based formalizations of session types \([22, 26]\), we have not introduced session type constructors that are directly based on or related to logic connectives in linear logic. In this section, we interpret in MTLC\_\_a some common linear logic connectives including multiplicative conjunction \(\otimes\), multiplicative implication \(\rightarrow\), additive disjunction \(\&\), and additive conjunction \(\oplus\). We are unclear as to whether multiplicative disjunction \(\mp\) can be handled at all. We also briefly mention the exponential connective ! at the end.

Note that the presented code is written in the concrete syntax of ATS, which is largely ML-like. We expect that people who can read ML code should have no great difficulty in following the presented ATS code as it makes only use of common functional programming features.

**M-Conjunction(\(\otimes\))**: Given two session types \(A\) and \(B\), a channel of the session type \(A \otimes B\) can be interpreted as one that inputs a channel specified by \(A\) and then behaves as a channel specified by \(B\).

**M-Implication(\(\rightarrow\))**: Given two session types \(A\) and \(B\), a channel of the session type \(A \rightarrow B\) can be interpreted as one that inputs a channel specified by \(A\) and then behaves as a channel specified by \(B\).

**A-Conjunction(\(\&\))**: Given two session types \(A\) and \(B\), a channel of the session type \(A \& B\) can be interpreted as one that inputs a boolean value and then behaves as a channel specified by either \(A\) or \(B\) depending on whether the boolean value is true or false, respec-

### Figure 5. Interpreting multiplicative conjunction

```fun
fserv_times{A,B:type}
(  chp: chpos(chsnd(chneg(A))::B)
  , chn_a: chneg(A), chn_b: chneg(B)
) : void = let
  //
  val () =
  chapos_send (chp, chn_a)
  //
  chaposneg_link (chp, chn_b)
end // end of [fserv_times]
```

### Figure 6. Interpreting (multiplicative) implication

```fun
fserv_implies{A,B:type}
(  chp: chpos(chrcv(chneg(A))::B)
  , fchn: chneg(A) <-clincloptr> chneg(B)
) : void = let
  val chn_b = fchn(chapos_recv(chp))
  in
  cloptr_free(fchn); chaposneg_link(chp, chn_b)
end // end of [fserv_implies]
```

\(B \otimes 26\). This interpretation is from the client’s viewpoint, meaning in MTLC\_\_a that \(A \otimes B\) should be defined as \(\text{snd}(\text{chneg}(A)) :: B\). Clearly, any reasonable interpretation for \(A \otimes B\) is expected to allow the construction of a channel of the type \(\text{chneg}(A \otimes B)\) based on two channels of the types \(\text{chneg}(A)\) and \(\text{chneg}(B)\) and vice versa. In Figure 5 a function \texttt{fserv\_times} is implemented to allow a channel of the type \(\text{chneg}(A \otimes B)\) to be built by the following call:

\[
\text{chneg\_create}(\lambda\text{chp} \Rightarrow \text{fserv\_times}(\text{chp}, \text{chn\_a}, \text{chn\_b}))
\]

where \texttt{chn\_a} and \texttt{chn\_b} are channels of the types \(\text{chneg}(A)\) and \(\text{chneg}(B)\), respectively, and the keyword \texttt{\_\_a} forms a linear function (that is to be called exactly once). The other direction (that is, obtaining \(\text{chneg}(A)\) and \(\text{chneg}(B)\) from \(\text{chneg}(A \otimes B)\)) is straightforward and thus skipped.

Of course, one can also interpret \(A \rightarrow B\) as \(\text{snd}(\text{chneg}(A)) :: \text{snd}(\text{chneg}(B)) :: \text{nil}\). With this interpretation, it should be obvious to see how \(\text{chneg}(A \rightarrow B)\) can be constructed based on \(\text{chneg}(A)\) and \(\text{chneg}(B)\) and vice versa.

**M-Implication(\(\rightarrow\))**: Given two session types \(A\) and \(B\), a channel of the session type \(A \rightarrow B\) can be interpreted as one that outputs a channel specified by \(A\) and then behaves as a channel specified by \(B\).

This interpretation is from the client’s viewpoint, meaning in MTLC\_\_a that \(A \rightarrow B\) should be defined as \(\text{rcv}^{\text{chneg}(A)} :: B\). If one has a function \(\text{fchn}\) that turns a negative channel specified by \(A\) into a negative channel specified by \(B\), then one can build as follows a negative channel specified by \(A \rightarrow B\):

\[
\text{chneg\_create}(\lambda\text{chp} \Rightarrow \text{fserv\_implies}(\text{chp}, \text{fchn}))
\]

where the function \texttt{fserv\_implies} is implemented in Figure 6. Note that the function \texttt{cloptr\_free} is called to explicitly free a linear function that has been called.

**A-Conjunction(\(\&\))**: Given two session types \(A\) and \(B\), a channel of the session type \(A \& B\) can be interpreted as one that inputs a boolean value and then behaves as a channel specified by either \(A\) or \(B\) depending on whether the boolean value is true or false, respec-
The quantifier is also in its scope. Essentially, the type assigned to a negative channel specified by some session type \( \text{adisj}(A, B) \) send 0 and 1 onto the channel, respectively. By now, it should be clearly that the following call turns a channel \( \text{chneg}(A) \) into one of the type \( \text{chneg}(\text{adisj}(A, B)) \):

\[
\text{chneg}_\text{create}(\text{llam}(\text{chp}) \Rightarrow \text{fserv}_\text{adisj}_\text{l}(\text{chp}, \text{chn}))
\]

Similarly, the following call turns a channel \( \text{chneg}(B) \) into one of the type \( \text{chneg}(\text{adisj}(A, B)) \):

\[
\text{chneg}_\text{create}(\text{llam}(\text{chp}) \Rightarrow \text{fserv}_\text{adisj}_\text{r}(\text{chp}, \text{chn}))
\]

Often, the kind of choice associated with \( A \odot B \) is referred to as internal choice as it is the server that determines whether \( A \) or \( B \) is chosen.

### A-Conjunction(&)

Given two session types \( A \) and \( B \), a channel of the session type \( A \odot B \) can be interpreted as one that outputs a boolean value and then behaves as a channel specified by either \( A \) or \( B \) depending on whether the boolean value is true or false, respectively. This interpretation is from the client’s viewpoint, meaning in \( \text{MTLC}_\text{c} \) that \( A \odot B \) should be defined as follows:

\[
\forall b : \text{bool}. \text{rcv}(b) :: \text{choose}(A, B, b)
\]

Note that the positive channel type \( \text{chpos}(\forall b : \text{bool}. \text{rcv}(\ldots) :: \ldots) \) should be interpreted as follows in this case:

\[
\exists b : \text{bool}. \text{chpos}(\forall rcv(b) :: \text{choose}(A, B, b))
\]

but the negative one \( \text{chneg}(\exists b : \text{bool}. \text{rcv}(\ldots) :: \ldots) \) is as follows:

\[
\forall b : \text{bool}. \text{chneg}(\forall rcv(b) :: \text{choose}(A, B, b))
\]

The code in Figure 8 essentially shows how a channel of the type \( \text{chneg}(A \odot B) \) can be built based on a value of the type \( \text{chneg}(A) \odot \text{chneg}(B) \). Note that \( \text{aconj} \) refers to & when called to form a session type. As & is not supported as a type constructor in ATS, we use the following one to represent it:

\[
\forall b : \text{bool}. \text{bool}(b) \rightarrow \text{choose}(\text{chneg}(A), \text{chneg}(B), b)
\]

The declared datatype \( \text{choose} \) is a linear one. The symbol “\( \odot \) in front of a linear data constructor like \( \text{choose}_\text{l} \) means that the constructor itself is freed after the arguments of the constructor are taken out. What \( \text{fserv}_\text{conj} \) does is clear: It checks the tag received on its first argument \( \text{chp} \) (a positive channel) and then determines whether to offer \( \text{chp} \) as a channel specified by \( A \) or \( B \).

So the kind of choice provided by \& is external: It is the client that decides whether \( A \) or \( B \) is chosen.

### Exponential(!)

Given a session type \( S \), we have a type \( \text{service}(S) \) that can be assigned to a value representing a persistent service specified by \( S \). With such a service, channels of the type \( \text{chneg}(S) \) can be created repeatedly. A built-in function \( \text{service}_\text{create} \) is assigned the following type for creating a service:

\[
\text{service}_\text{create} : (\text{chpos}(\sigma) \rightarrow 1) \Rightarrow \text{service}(\sigma)
\]

In contrast with \( \text{chneg}_\text{create} \) for creating a channel, \( \text{service}_\text{create} \) requires that its argument be a non-linear function (so that this function can be called repeatedly).

### 6. Examples of Session-Typed Programs

We present some simple running examples in this section to further illustrate programming with session types. More practical examples (e.g. FTP and reversed FTP) are available but difficult to present here.

#### 6.1 Eratosthenes’s Sieve

We give an implementation of Eratosthenes’s sieve based on session-typed channels. In particular, we make use of channels
specifies by session lists that are of indefinite length. This example is essentially taken from SILL [8]. In Figure 9 we give three functions for unfolding channels specified by session lists: one for positive channels and two for negative channels. Strictly speaking, we should use the name co-lists (instead of lists) here as it is the client that decides whether the next element of a list should be generated or not.

For someone familiar with stream-based lazy evaluation, the code in Figure 10 and Figure 11 should be easily accessible. Given a channel chn of integers and an integer n0, the function ints_filter builds a new channel of integers that outputs, when requested, the first integer from chn that is not a multiple of n0. Given a channel chn of integers, the function sieve outputs, when requested, the first integer p0 from chn, and then applies ints_filter to chn and p0 to build a new channel, and then applies itself to the new channel recursively. Note that the code for ints_from is omitted, which returns a channel of all the integers starting from a given one.

6.2 A Queue of Channels

We give a queue implementation based on a queue of session-typed channels. This example is largely based on one in SILL. What is novel here mainly involves the use of DML-style dependent types to specify the size of each queue in the implementation.

Given a type a and an integer n, we use ssque(a,n) as a session type to specify a channel representing a queue of size n in which each element is of the type a. In Figure 12, there are four functions for unfolding channels specified by session queues: one for positive channels and three for negative channels.

The function queue_create creates a negative channel representing an empty queue. Based on the code for queue_create, we can see that a queue of size n is represented by a queue of n+1 channels (where the last one always represents an empty queue); each element in the queue is held in the corresponding channel (or more precisely, the thread running to support the channel); enqueuing an element is done by sending the element down to the last channel (representing an empty queue), causing this channel to create another channel (representing an empty queue); dequeuing is done by sending out the element held in the first channel (in the queue of channels) while the thread running to support the channel turns into one that does bidirectional forwarding.

Note that the implementation of queue_create never needs to handle dequeuing an empty queue or closing a non-empty queue as these operations are ill-typed, reaping typical benefits from (DML-style) dependent types.

7. Implementing Session-Typed Channels

As far as implementation is of the concern, there is very little that needs to be done regarding typechecking in order to support session types in ATS. Essentially, the entire effort focuses on implementing session-typed channels.
7.1 Implementation in ATS

The session-typed channels as presented in this paper are first implemented in ATS. The parties communicating to each other in a (dyadic) session run as pthreads. Each channel is represented as a record containing two buffers and some locking mechanism (i.e., mutexes and conditional variables); a positive channel and its negative dual share their buffers; the read buffer of a channel is the write buffer of its dual and vice versa. This implementation (of session-typed channels) is primarily done for the purpose of obtaining a proof of concept.

7.2 Implementation in Erlang

Another implementation of session-typed channels is done in Erlang. As the ML-like core of ATS can already be compiled into Erlang, we have now an option to construct distributed programs in ATS that may make use of session types and then translate these programs into Erlang code for execution, thus taking great advantage of the infrastructural support for distributed computing in Erlang. Each channel is implemented as a pair of processes; a positive channel shares with its dual the two processes: One handles read for the positive channel and write for its negative dual, and the other does the opposite. Implementing the functions chanpos_send/channeg_recv and chanpos_recv/channeg_send is straightforward. A significant complication occurs in our implementation of chanposneg_link, which requires the sender of a message to deliver it at its final destination (instead of having it forwarded there explicitly). For a straightforward but much less efficient implementation of chanposneg_link, one can just rely on explicit forwarding.

8. Related Work and Conclusion

Session types were introduced by Honda [9] and further extended [10, 20]. There have since been extensive theoretical studies on session types in the literature (e.g., [1, 3, 6, 11, 21, 24, 26]). However, there is currently rather limited support for practical programming with session types, and more evidence is clearly needed to show convincingly that session types can actually be employed cost-effectively in the construction of relatively large and complex programs. It is in this context that we see it both interesting and relevant to study implementation of session types formally.

There are reported implementations of session types in Java [5, 12, 13] and other languages (e.g., Python). However, these implementations are of a very different nature when compared to MTLCh. For instance, they mostly focus on session-typed functionalities being implemented rather than the (formal) correctness of the implementation of these functionalities.

There are also several implementations of session types in Haskell (e.g., [16, 17]), which primarily focus on obtaining certain
datatype chanpos_ssque
| chanpos_ssque_nil(a, nil, @) of ()
| {n:pos}
chanpos_ssque_deq(a, snd(a)::ssque(a,n-1), n) of ()
| {n:nat}
chanpos_ssque_enq(a, rcv(a)::ssque(a,n+1), n) of ()

extern fun
chanpos_ssque
{a:type}{n:nat}
(!chpos(ssque(a,n)) >> chpos(ss))
: #[ss:type] chanpos_ssque(a, ss, n)

extern fun
channeg_ssque_nil
{a:type}
(!chneg(ssque(a,0)) >> chneg(nil)): void

extern fun
channeg_ssque_deq
{a:type}{n:pos}
(!chneg(ssque(a,n)) >> chneg(snd(a)::ssque(a,n-1))
): void // end-of-function

and
channeg_ssque_enq
{a:type}{n:nat}
(!chneg(ssque(a,n)) >> chneg(rcv(a)::ssque(a,n+1))
): void // end-of-function

------------------------

Figure 12. Functions for unfolding session queue channels

Implement
queue_create
{a}({"void"}) = let
//
fun
fserv
{
chp: chpos(ssque(a,0))
): void = let
val opt = chanpos_ssque(chp)
in
//
case opt of
| chanpos_ssque_nil() =>
  chanpos_close(chp)
| chanpos_ssque_enq() => let
  val x0 = chanpos_recv{a}(chp)
in
  fserv2(x0, queue_create(), chp)
end // end of [chanpos_ssque_enq]
//
end // end of [fserv]
//
and
fserv2
{n:nat}
{
  x0: a
, chn: chneg(ssque(a,n))
, chp: chpos(ssque(a,n+1))
}: void = let
val opt = chanpos_ssque(chp)
in
//
case opt of
| chanpos_ssque_deq() => let
  val () = chanpos_send(chp, x0)
in
  channeg_link(chp, chn)
end // end of [chanpos_ssque_deq]
| chanpos_ssque_enq() => let
  val y = chanpos_recv{a}(chp)
  val () = channeg_ssque_enq(chn)
  val () = channeg_recv(chn, y)
in
  fserv2(x0, chn, chp)
end // end of [chanpos_ssque_enq]
//
end // end of [fserv2]
//
in
channeg_create(llam(chp) => fserv(chp))
end // end of [queue_create]

------------------------

Figure 13. Queue_create: creating an empty queue of channels

typeful encodings for session-typed channels. While the obtained encodings are shown to be type-correct, there is no provided mechanism to establish any form of global progress for them. As is explained in the case of chneg_create2, one can readily introduce potential deadlocks inadvertently without breaking type-correctness.

It is in general a challenging issue to establish deadlock-freeness for session-typed concurrency. There are variations of session types that introduce a partial order on time stamps \([19]\) or a constraint on dependency graphs \([3]\). As for formulations of session types (e.g., \([1, 26]\)) based on linear logic \([7]\), the standard technique for cut-elimination is often employed to establish global progress (which implies deadlock-freeness). In MTLC\(_{cb}\), there is no explicit tracking of cut-rule applications in the type derivation of a program (and it is unclear how such tracking can be done). In essence, the notion of DF-reducibility (Definition 3.3) is introduced in order to carry out cut-elimination in the absence of explicit tracking of cut-rule applications.

Probably, MTLC\(_{cb}\) is most closely related to SILL \([23]\), a functional programming language that adopts via a contextual monad a computational interpretation of linear sequent calculus as session-typed processes. Unlike in MTLC\(_{cb}\), the support for linear types in SILL is not direct and only monadic values (representing open process expressions) in SILL can be linear. For instance, one can readily construct linear data containing channels in ATS but this is not allowed in SILL. In terms of theoretical development, the approach to establishing global progress in SILL cannot be applied to MTLC\(_{cb}\) directly. Instead, we see Lemma 3.7 as a generalization of the argument presented in the proof of the theorem on global progress in SILL. Please see Theorem 5.2 \([23]\) for details.

Also, MTLC\(_{cb}\) is related to previous work on incorporating session types into a multi-threaded functional language \([25]\), where a type safety theorem is established to ensure that the evaluation of a well-typed program can never lead to a so-called faulty configuration. However, this theorem does not imply global progress as a program that is not of faulty configuration can still deadlock.

As for future work, we are particularly interested in extending MTLC\(_{cb}\) with multi-party session types \([11]\). It will be very interesting to see whether the approach we use to establish global progress for MTLC\(_{cb}\) can be adapted to handling multi-party session types. We are also interested in studying session types in restricted settings. For instance, the number of channels allowed in a program is required to be bounded; channels (of certain types) may not be sent from one party to another; etc.
There are a variety of programming issues that need to be addressed in order to facilitate the use of session types in practice. Currently, session types are represented as datatypes in ATS, and programming with session types often involves writing boilerplate code (e.g., the code implementing functions like change_list, change_list_nil, and change_list_cons). In the presence of large and complex session types, writing such code can be tedious and error-prone. Naturally, we are interested in developing some meta-programming support for generating such code automatically. Also, we are in process of designing and implementing session combinators (in the spirit of parsing combinators) that can be conveniently called to assemble subsessions into a coherent whole.

References

[1] Luís Caires and Frank Pfenning. Session types as intuitionistic linear propositional. In CONCUR 2010 - Concurrency Theory, 21th International Conference. CONCUR 2010. Paris, France, August 31 - September 3, 2010. Proceedings, pages 222–236, 2010.

[2] Marco Carbone and Søren Debois. A graphical approach to progress for structured communication in web services. In Proceedings Third Interaction and Concurrency Experience: Guaranteed Interaction, ICE 2010, Amsterdam, The Netherlands, 10th of June 2010., pages 13–27, 2010.

[3] Giuseppe Castagna, Mariangiola Dezani-Ciancaglini, Elena Giachino, and Luca Padovani. Foundations of session types. In Proceedings of the 11th International ACM SIGPLAN Conference on Principles and Practice of Declarative Programming, September 7-9, 2009, Coimbra, Portugal, pages 219–230, 2009.

[4] Chiyian Chen and Hongwei Xi. Combining Programming with Theorem Proving. In Proceedings of the Tenth ACM SIGPLAN International Conference on Functional Programming, pages 66–77, Tallinn, Estonia, September 2005.

[5] Juliana Franco and Vasco Thudichum Vasconcelos. A concurrent language with refined session types. In Software Engineering and Formal Methods - SEFM 2013 Collocated Workshops: BEAT2, WS-FMDS, FM-RAIL-Bok, MoKMsD, and OpenCert, Madrid, Spain, September 23-24, 2013. Revised Selected Papers, pages 15–28, 2013.

[6] Simon J. Gay and Vasco Thudichum Vasconcelos. Linear type theory for asynchronous session types. J. Funct. Program., 20(1):19–50, 2010.

[7] Jean-Yves Girard. Linear logic. Theoretical Computer Science, 50(1):1–101, 1987.

[8] Dennis Griffith. SILL: A session-type functional programming language, 2015. Available at: https://github.com/ISANobody/sill11.

[9] Kohei Honda. Types for dyadic interaction. In CONCUR ’93, 4th International Conference on Concurrency Theory, Hildesheim, Germany, August 23-26, 1993, Proceedings, pages 509–523, 1993.

[10] Kohei Honda, Vasco Vasconcelos, and Makoto Kubo. Language primitives and type discipline for structured communication-based programming. In Programming Languages and Systems - ESOP ’98, 7th European Symposium on Programming, Held as Part of the European Joint Conferences on the Theory and Practice of Software, ETAPS ’98, Lisbon, Portugal, March 28 - April 4, 1998, Proceedings, pages 122–138, 1998.

[11] Kohei Honda, Nobuko Yoshida, and Marco Carbone. Multiparty asynchronous session types. In Proceedings of the 35th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, POPL 2008, San Francisco, California, USA, January 7-12, 2008, pages 273–284, 2008.

[12] Raymond Hu, Dimitrios Kouzapas, Olivier Pernet, Nobuko Yoshida, and Kohei Honda. Type-safe eventful sessions in Java. In ECOOP 2010 - Object-Oriented Programming, 24th European Conference, Maribor, Slovenia, June 21-25, 2010. Proceedings, pages 329–353, 2010.

[13] Raymond Hu, Nobuko Yoshida, and Kohei Honda. Session-based distributed programming in Java. In ECOOP 2008 - Object-Oriented Programming, 22nd European Conference, Paphos, Cyprus, July 7-11, 2008, Proceedings, pages 516–541, 2008.

[14] Graham Hutton. Higher-order functions for parsing. J. Funct. Program., 2(3):323–343, 1992.

[15] Robin Milner, J. Parrow, and David Walker. A calculus of processes, parts I and II. Information and Computation, 100:1–40 and 41–77, 1992.

[16] Matthias Neubauer and Peter Thiemann. An implementation of session types. In Practical Aspects of Declarative Languages, 6th International Symposium, PADL 2004, Dallas, TX, USA, June 18-19, 2004, Proceedings, pages 56–70, 2004.

[17] Riccardo Pucella and Jesse A. Tov. Haskell session types with (almost) no class. In Proceedings of the 1st ACM SIGPLAN Symposium on Haskell, Haskell 2008, Victoria, BC, Canada, 25 September 2008, pages 25–36, 2008.

[18] Rui Shi and Hongwei Xi. A Linear Type System for Multicore Programming. In Proceedings of Simposio Brasileiro de Linguagens de Programacao, Gramado, Brazil, August 2009.

[19] Eijiro Sumii and Naoki Kobayashi. A generalized deadlock-free process calculus. Electr. Notes Theor. Comput. Sci., 16(3):225–247, 1998.

[20] Kaku Takeuchi, Kohei Honda, and Makoto Kubo. An interaction-based language and its typing system. In PARLE ’94: Parallel Architectures and Languages Europe, 6th International PARLE Conference, Athens, Greece, July 4–8, 1994, Proceedings, pages 398–413, 1994.

[21] Bernardo Toninho, Luís Caires, and Frank Pfenning. Dependent session types via intuitionistic linear type theory. In Proceedings of the 13th International ACM SIGPLAN Conference on Principles and Practice of Declarative Programming, July 20-22, 2011, Odense, Denmark, pages 161–172, 2011.

[22] Bernardo Toninho, Luís Caires, and Frank Pfenning. Functions as session-type processes. In Foundations of Software Science and Computational Structures - 15th International Conference, FOSSACS 2012, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2012, Tallinn, Estonia, March 24 - April 1, 2012, Proceedings, pages 346–360, 2012.

[23] Bernardo Toninho, Luís Caires, and Frank Pfenning. Higher-order processes, functions, and sessions: A monadic integration. In Programming Languages and Systems - 22nd European Symposium on Programming, ESOP 2013, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2013, Rome, Italy, March 16-24, 2013, Proceedings, pages 350–369, 2013.

[24] Vasco T. Vasconcellos. Fundamentals of session types. Inf. Comput., 217:52–70, 2012.

[25] Vasco Thudichum Vasconcelos, António Ravara, and Simon J. Gay. Session types for functional multithreading. In CONCUR 2004 - Concurrency Theory, 15th International Conference, London, UK, August 31 - September 3, 2004, Proceedings, pages 497–511, 2004.

[26] Philip Wadler. Propositions as sessions. In ACM SIGPLAN International Conference on Functional Programming, ICFP ’12, Copenhagen, Denmark, September 9-15, 2012, pages 273–286, 2012.

[27] Hongwei Xi. Applied Type System (extended abstract). In post-workshop Proceedings of TYPES 2003, pages 394–408. Springer-Verlag LNCS 3085, 2004.

[28] Hongwei Xi. Dependent ML: an approach to practical programming with dependent types. Journal of Functional Programming, 17(2):215–286, 2007.

[29] Hongwei Xi. The ATS Programming Language System, 2008. Available at: http://www.ats-lang.org/.

[30] Hongwei Xi and Frank Pfenning. Dependent Types in Practical Programming. In Proceedings of 26th ACM SIGPLAN Symposium on Principles of Programming Languages, pages 214–227, San Antonio, Texas, January 1999. ACM press.