On selection of optimal stop position and spatial orientation of mobile robot

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Abstract. The paper analyses the problem of finding optimal coordinates of a mobile robot equipped with a manipulator. An optimization criterion is proposed and validated, and a method for calculating most rational coordinates of the robot is described.

1. Introduction

According to the modern notions about the subject in question, robot is a reprogrammable automatic or automated machine with anthropomorphic (humanlike) elements [1, 2]. The likeness to humans are expressed in its movements, methods utilized for solving intellectual problems and behavior in general. On the whole, robots could be separated into two large categories – stationary and mobile ones [3–6]. The former have a fixed body, with only their arm tool moving, if at all. The latter move from one point of the workspace to another, and do it together with their body.

The movement of mobile robots could take place in various conditions and even various environments. They would perform various tasks which facilitate human work, or, in some cases, do what a person cannot due to restrictions of his/her physical and intellectual abilities [7, 8].

Specifics of the work for which mobile robots are developed often require designing special or specialized control systems which could realize the tasks of selecting a rational path of robot motion, controlling the velocity, etc. [9–12].

2. Optimization of robot positioning and orientation

Mobile robots equipped with manipulators are known to use those to perform various technological operations within their workspace both while they are moving or when they have stopped. In the latter case, it is necessary to choose a stop position which would be most rational, that is to properly orient the robot in space. So, one possible variant of solving the problem is proposed below.

Consider an XYZ coordinate system which relates to a robot and originates from the arm joint of its manipulator. Then, a criterion \( \Theta_p \) could be specified:

\[
\Theta_p = \sum_i \Theta_i P_i, \tag{1}
\]

where \( \Theta_i \) is a service coefficient of the manipulator [13, 14] at a certain working position \( A_i \) with the coordinates of \( x_i, y_i, z_i \), and \( P_i \) is a relative frequency or probability of performing specific operations at \( A_i \). Assume the frequencies \( P_i \) are known since they could be either a priori given (for example, for
industrial robots, those might be set on the basis of a statistical analysis of a technological process in which the robot is programmed to be used) or experimentally obtained (by running the robot through several testing cycles and, for every working point, counting the number of times the manipulator gripper gets there). Then, consider an $X'Y'Z'$ coordinate system which is rigidly connected to a workspace serviced by the robot and, at its stop position, would match the $XYZ$ system until the most rational position were found. Assume the working position $A_i$ in that coordinate system would have $x_i', y_i', z_i'$ coordinates and consider also those to be known since they could be determined using visual analyzers of the robot and applying known algorithms [15].

Let the robot move, within its service area, over $a$, $b$ and $c$ distances along the $X'Y'Z'$ axes and rotate, relative to those, at $\alpha$, $\phi$ and $\psi$ angles. Then the coordinates of any point $A_i$ in the $XYZ$ system would be defined using known methods of coordinate transformation as

$$
\begin{align*}
    x_i &= f_1(a, \alpha, \phi, \psi, x_i', y_i', z_i'), \\
    y_i &= f_2(b, \alpha, \phi, \psi, x_i', y_i', z_i'), \\
    z_i &= f_3(c, \alpha, \phi, \psi, x_i', y_i', z_i'),
\end{align*}
$$

with $\Theta_i$ at $A_i$ expressed as

$$
\Theta_i = f(x_i, y_i, z_i).$$

Knowing $x_i, y_i, z_i$ and $P_i$ and using equations (1) and (3), it is easy to calculate $\overline{\Theta}_p$, which corresponds to a spatial position of the robot according to its assumed $a, b, c, \alpha, \psi$. Since $\overline{\Theta}_p$ is essentially an average realizable service coefficient of the manipulator, it is obvious that the most rational position of the robot would be when $\overline{\Theta}_p = \max. Substituting various $a, b, c, \alpha, \phi, \psi$ into the expressions and, thus, modeling movements and rotations of the robot, one could find such values of those which would deliver the condition of a maximal $\overline{\Theta}_p$.

3. Example

Now, let us show how the proposed criterion could be used in an example of a robot equipped with an «ideal» manipulator similar to the one in [1]. Such a manipulator (figure 1) has three links connected via spherical joints, and, when $l_1 > l_2 > l_3$, its service coefficient is expressed as

$$
\Theta_i = \begin{cases} 
    \frac{(l_1 + R_i)^2 - (l_i - l_3)^2}{4l_iR_i} & \text{when } l_1 - l_2 - l_3 \leq R_i \leq l_1 - l_2 + l_3, \\
    1 & \text{when } l_1 - l_2 + l_3 \leq R_i \leq l_1 + l_2 - l_3, \\
    \frac{(l_1 + l_2)^2 - (R_i - l_3)^2}{4l_iR_i} & \text{when } l_1 + l_2 - l_3 \leq R_i \leq l_1 + l_2 + l_3,
\end{cases}
$$

where $R_i = \sqrt{x_i^2 + y_i^2 + z_i^2}$. Let the links be measured in some relative units as $l_1 = 3$, $l_2 = 2$, $l_3 = 1$, with the robot, as a simplification, assumed, with probabilities of $P_1 = 0.3$, $P_2 = 0.5$, $P_3 = 0.2$, to carry out assigned tasks only at three points $A_1, A_2, A_3$ with the coordinates of $x_1' = 1$, $x_2' = 3$, $x_1' = 5$ (in the same relative units) and $y_1' = y_2' = y_3' = z_1' = z_2' = z_3' = 0$. Obviously, under such conditions, the most rational position of the robot is achieved with any $\alpha, \phi, \psi$ and such $a, b, c$ which would provide the maximum:

$$
\overline{\Theta}_p = \Theta_1P_1 + \Theta_2P_2 + \Theta_3P_3,
$$
where \( \Theta_i = \begin{cases} \frac{(1 + R_i)^2 - 1}{4R_i} & \text{when } 0 \leq R_i < 2, \\ 1 & \text{when } 2 \leq R_i \leq 4, \\ \frac{25 - (R_i - 1)^2}{4R_i} & \text{when } 4 < R_i \leq 6, \end{cases} \) (6)

and \( R_i = \sqrt{(x_i' - a) + b^2 + c^2}. \) (7)

\[ 2 \leq (1-a)^2 + d^2 \leq 4, \]
\[ 2 \leq (3-a)^2 + d^2 \leq 4, \] (8)
\[ 2 \leq (5-a)^2 + d^2 \leq 4. \]

Solving this system allows to determine ranges of \( a \) and \( d \) which belong to that region (figure 2):

Figure 1. Kinetic scheme of an «ideal» manipulator.

Assume, for example, \( b_2 + c_2 = d_2 = 0 \) and \( a = 0. \) Then \( \Theta_p \) would be 0.81. When \( d = 0 \) and \( a = 1, \) or \( d = 0 \) and \( a = 5, \) or \( d = 2 \) and \( a = 4, \) \( \Theta_p \) equals, correspondingly, to 0.85; 0.9 and 1. Obviously, \( \Theta_p \) is maximal in the latter case since all working points are covered by the full service of the manipulator. Thus, between four variants of robot positioning considered above, the most rational is the one with \( d = 2 \) and \( a = 4. \) Nonetheless, it is not the only way of obtaining \( \Theta_p = 1 \) — it is also the case for \( d = 2 \) and \( a = 2, \) \( d = 3 \) and \( a = 3, \) etc. A whole region of \( a \) and \( d \) values gives \( \Theta_p = 1; \) it is expressed by the following system of inequations:
\[
\begin{align*}
\sqrt{4-(1-a)^2} & \leq d \leq \sqrt{16-(5-a)^2} \quad \text{when } 1.5 \leq a \leq 2, \\
\sqrt{4-(3-a)^2} & \leq d \leq \sqrt{16-(5-a)^2} \quad \text{when } 2 \leq a \leq 3, \\
\sqrt{4-(3-a)^2} & \leq d \leq \sqrt{16-(1-a)^2} \quad \text{when } 3 \leq a \leq 4, \\
\sqrt{4-(5-a)^2} & \leq d \leq \sqrt{16-(1-a)^2} \quad \text{when } 4 \leq a \leq 4.5.
\end{align*}
\] (9)

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Region of rational positioning for the arm joint of the «ideal» manipulator.}
\end{figure}

Thus, to achieve an optimal positioning of the robot, the arm joint of its manipulator should be positioned by setting such \( a \) and \( d \) which would lie within any of two pairs of those intervals.

4. Conclusion
The solution of the problem considered here and formulated in the title allows a mobile robot which services a certain workspace to select a proper stop position and spatial orientation using only those means which are available to it. It also results in simplified programming of its operation and softens the requirements of accurate positioning of its work tool [16]. On practice, it is useful to make, based on the solution above, an autonomic programmable unit, similar to the one described in [17].

References
[1] Tsarenko V I 1990 \textit{Industrial robots} (Moscow: Higher School)
[2] Shurkov V N 1989 \textit{Basic production automation and industrial robots} (Moscow: Machine Building)
[3] Cherpakov B I and Velikovich V B 1989 \textit{Robot systems} (Moscow: Higher School)
[4] Liberman Y L and Kubatiev R R 2016 \textit{Robot system} (Russia, patent 160746, bul. 9, publ. 27.03.16)
[5] Liberman Y L and Ovchinnikova V A 2017 \textit{Mobile ground-based robot} (Russia, patent 167531, bul. 1, publ. 10.01.17)
[6] Liberman Y L and Antropov M S 2011 \textit{Robot cart} (Russia, patent 108356, bul. 26, publ. 20.09.11)
[7] Timofeev A V 1988 \textit{Adaptive robot systems} (Leningrad: Machine Building)
[8] Bragin V B, Voilov Y G and Zhabotinskiy Y D 1985 \textit{Sensing systems and adaptive industrial robots} (Moscow: Machine Building)
[9] Andrianov Y D, Gleizer L Y and Ignatiev M B 1984 \textit{Control systems for industrial robots}
[10] Liberman Y L and Letnev K Y 2017 *Special and specialized control systems for transporting machinery* (Ekaterinburg: For You Publishing)

[11] Liberman Y L and Shhekalev K A 2013 *System of robot-cart path control* (Russia, patent 125534, bul. 7, publ. 10.03.13)

[12] Liberman Y L and Biktashev D A 2013 *System of robot-cart positioning control* (Russia, patent 130540, bul. 21, publ. 27.07.13)

[13] Vinogradov I B, Kobrinskiy A E, Stepanenko Y A and Tyves L I 1969 *J. Machine Science* 3 17

[14] Vinogradov I B, Kobrinskiy A E, Stepanenko Y A and Tyves L I 1971 *J. Mechanics of Machines* 27-28 5

[15] Ignatiev M B, Kulakov F M and Pokrovskiy A M 1972 *Control algorithms for manipulating robots* (Leningrad: Machine Building)

[16] Liberman Y L and Novikov V A 2015 *System of robot-cart movement programmable control* (Russia, patent 150500, bul. 5, publ. 20.02.15)

[17] Liberman Y L and Teterina N A 2014 *Programmer for selecting optimal stop position of mobile robot* (Russia, software registration certificate 2014610099, publ. 09.01.14)