The paper is organized as follows: the different energy functionals are briefly reviewed in section III as well as the ETF approximation used to calculate nuclear observables. In Section IV after discussing the overall performance of the ETF approximation on the Pb isotopic chain, we show our main results concerning the correlations between the different isovector observables and the empirical EoS parameters. Finally conclusions are drawn in section IV.

II. FORMALISM

A. Skyrme EDF

The most extensive calculations of nuclear observables and their correlations with EoS parameters have been performed using Skyrme EDF [31].

The nuclear Skyrme energy density is expressed in terms of local nucleon densities $n_q(r)$, kinetic energy densities $\tau_q(r)$...
and spin-orbit densities $J_q(r)$ defined by \[ n_q(r) = \sum_{\nu,s} \phi_q(r,s,q)^2 n^q_\nu, \]
\[ \tau_q(r) = \sum_{\nu,s} |\nabla \phi_q(r,s,q)|^2 n^q_\nu, \]
\[ J_q(r) = (-i) \sum_{\nu,s,s'} \phi^*_q(r,s',q) \nabla \phi_q(r,s,q) \times \langle s' | \sigma | s \rangle n^q_\nu. \tag{1} \]

where $\phi_q(r,s,q)$ represent the single-particle wave functions with orbital and spin numbers $\nu$ and $s$, $q = n, p$ indexes the nucleonic species and $n^q_\nu$ are the occupation numbers. The functional form of the EDF is generated by a mean-field calculation with an effective zero range momentum dependent pseudo-potential, augmented of a density dependent term. Standard pseudo-potentials, as the ones considered hereafter, depend on 10 parameters. The values of these parameters are typically determined by fits of experimental ground-state properties of spherical magic and semi-magic nuclei (e.g. binding energy, root mean square (rms) radius of the charge distribution, spin-orbit splitting, isotope shifts, surface thickness, breathing mode energy, etc.) and/or properties of symmetric nuclear matter (energy $E_{\text{sat}}$ and density $n_{\text{sat}}$ at saturation, compression modulus $K_{\text{sat}}$, symmetry energy $E_{\text{sym}}$) and/or equation of state of pure neutron matter as predicted by ab-initio models. These parameters vary largely from one Skyrme model to another. Properties of nuclear matter (NM) can be expressed analytically in terms of the same parameters $a$.

In the following, 17 Skyrme EDFs will be employed: SKa [32], SKb [32], Rs [33], SkMP [34], SLy2 [35], SLy9 [35], SLy4 [36], SLy230a [37], SkI2 [38], SkI3 [38], SkI4 [38], SkI5 [38], SkI6 [39], SKOp [40], SK255 [41], SKa [32], SKb [32], Rs [33], SkMP [34], SLy2 [35], SLy9 [35], SLy4 [36], SLy230a [37], SkI2 [38], SkI3 [38], SkI4 [38], SkI5 [38], SkI6 [39], SKOp [40], SK255 [41], KDE0v1 [42]. The extent to which they fulfill various constraints that have been obtained from experiment or microscopic calculations during the last decade [43] has been thoughtfully investigated in Ref. [44] in the context of unified equations of state for neutron star matter. Their values of saturation density of symmetric nuclear matter (SNM), energy per particle and compression modulus of symmetric saturated matter span relatively narrow ranges $0.1512 \leq n_{\text{sat}} \leq 0.1646$ fm$^{-3}$, $-16.33 \leq E_{\text{sat}} \leq -15.52$ MeV, $222.40 \leq K_{\text{sat}} \leq 271.5$ MeV, as these quantities are relatively well constrained. Larger domains are explored by the symmetry energy, $29.54 \leq E_{\text{sym}} \leq 37.4$ MeV, and, especially, its slope and curvature $44.3 \leq L_{\text{sym}} \leq 129.3$ MeV, $-127.2 \leq K_{\text{sym}} \leq 159.5$ MeV.

It is worthwhile to notice that the functional form of the Skyrme energy density leads to correlations between the different EoS parameters. Indeed 5 independent parameters govern the density dependence of the EDF (and 2 additional ones determine the density dependence of the effective masses). If the lowest order EoS parameters are fixed, namely $E_{\text{sat}}, n_{\text{sat}}, K_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}$, the higher order parameters can be analytically expressed as a function of those fixed quantities. In particular, Skyrme EDFs show a clear correlation between the slope $L_{\text{sym}}$ and the curvature $K_{\text{sym}}$ of the symmetry energy at saturation, which are a-priory independent EoS parameters.

This correlation, which obviously affects the extrapolation of the EoS to super-saturation densities, is graphically illustrated in the top panel of Fig. 1 (open circles). Its Pearson correlation coefficient $C$ is $C(K_{\text{sym}}, L_{\text{sym}}) = 0.87$. It was recently shown that this correlation is observed in a large class of functionals and might therefore be physically founded [29, 26], even if its origin is not fully understood.

Another interesting non-trivial correlation is found between the effective nucleon mass at saturation, $m_{\text{sat}}$, and the isoscalar-like finite size parameter $C_{\text{fin}}$ (see section IIB and Ref. [29]). This correlation is illustrated in the bottom panel of Fig. 1. Its Pearson correlation coefficient is $C(m_{\text{sat}}, C_{\text{fin}}) = 0.88$. As already discussed in ref. [29], this correlation is probably induced by the parameter fitting protocol of Skyrme functionals. Indeed $m_{\text{sat}}$ and $C_{\text{fin}}$ are related to non-local terms in the EDF which have an opposite effect on the surface energy, and neither of them plays a role on the determination of EoS parameters: for a given set of EoS parameters a similar overall reproduction of binding energies over the nuclear chart can be obtained with compensating effects of the non-local terms.
B. Meta-modelling of the EDF

A theoretical calculation of a nuclear observable depends, besides the EoS, on the functional form assumed for the EDF as well as on the many-body technique employed. To assess the model dependence due to the functional form of the EDF, one should consider different families of models with similar values for the EoS parameters. To this aim, a meta-modelling technique was proposed in Ref. [28] and extended to finite nuclei EDF in Ref. [29]. Varying the parameters of the meta-modelling, a large number of EoS from different families of mean-field EDF can be generated. Moreover, novel density dependencies that do not correspond to existing functionals but do not violate any empirical constraint, can be also explored [28]. The inclusion of a single gradient term provides a minimal flexible EDF for finite nuclei, with performances on nuclear mass and radii comparable to the ones of full Skyrme functionals [29]. The exploration of the meta-modelling parameter space thus allows a full estimation of the possible model dependence of the extraction of EoS parameters from nuclear ground state observables, due to the choice of the EDF.

The potential energy per baryon is expressed as a Taylor expansion around saturation of symmetric nuclear matter in terms of the density parameter $x = (n - n_{sat})/(3n_{sat})$,

$$ e_{pot}(x, \delta) = \sum_{\alpha=0}^{N} (a_{\alpha 0} + a_{\alpha 2} \delta^2) x^{\alpha} \delta^2 u_{\alpha}(x), \quad (2) $$

where the functions $u_{\alpha}(x)$ represent a low density correction insuring a vanishing energy in the limit of vanishing density, without affecting the derivatives at saturation.

To correctly reproduce with a limited expansion order $N$ existing non-relativistic (Skyrme and ab-initio) and relativistic (RMF and RHF) EDFs up to total densities $n = n_{n} + n_{p} \approx 0.6$ fm$^{-3}$, and isospin asymmetries $\delta = (n_{n} - n_{p})/n$ ranging from symmetric matter $\delta = 0$ to pure neutron matter $\delta = 1$, the functional is supplemented by a kinetic-like term adding the expected $n^{2/3}$ dependence at low densities, as well as the contribution of higher orders in the $\delta$ expansion as:

$$ e_{kin}(x, \delta) = \frac{t_{FG}^{sat}}{2} (1 + 3x)^{2/3} \left[ (1 + \delta)^{5/3} \frac{m_{n}}{m_{n}} + (1 - \delta)^{5/3} \frac{m_{p}}{m_{p}} \right], \quad (3) $$

where $t_{FG}^{sat} = (3h^{2})/(10m) (3\pi^{2}/2)^{2/3} n_{sat}^{2/3}$ is the energy per nucleon of a free symmetric Fermi gas at nuclear saturation, $m$ stands for the nucleon mass and $m_{q}$ denote the effective mass of the nucleons $q = n, p$. For more details, see model ELMF in Ref. [28].

In the present work, we only consider subsaturation matter and, to avoid proliferation of unconstrained parameters, we limit the expansion to $N = 2$, which was shown to be enough to get a fair reproduction of nuclear masses [29]. The possible influence of higher order parameter is left for future work. When only average nuclear properties (e.g. binding energies and rms radii of neutron and proton distributions) are calculated, isoscalar and isovector finite-size and spin-orbit interactions can be fairly well described by a single isoscalar-like density gradient term [29] of the form $C_{fin}(\nabla n_{n} + \nabla n_{p})^2$. For the sake of convenience only this isoscalar density gradient will be considered in this work. Following Ref. [29], we also neglect the effective mass splitting between neutrons and protons. The meta-modelling parameters are then directly linked to the usual first and second order empirical parameters of the EoS by:

$$ a_{00} = E_{sat} - E_{FG}^{sat}(1 + \kappa_{sat}) \quad (4) $$
$$ a_{10} = -t_{FG}^{sat} (2 + 5\kappa_{sat}) \quad (5) $$
$$ a_{20} = K_{sat} - 2t_{FG}^{sat} (-1 + 5\kappa_{sat}) \quad (6) $$
$$ a_{02} = E_{sym} - \frac{5}{9}E_{FG}^{sat}(1 + \kappa_{sat}) \quad (7) $$
$$ a_{12} = L_{sym} - \frac{5}{9}E_{FG}^{sat}(2 + 5\kappa_{sat}) \quad (8) $$
$$ a_{22} = K_{sym} - \frac{10}{9}t_{FG}^{sat} (-1 + 5\kappa_{sat}) \quad (9) $$

where $\kappa_{sat} = m/m_{n}^{sat} - 1$.

Different EDF models for nuclei are generated by largely and evenly exploring the parameter space $\{P_{a}\} = \{ n_{sat}, E_{sat}, K_{sat}, E_{sym}, L_{sym}, K_{sym}, m_{n}^{sat}, C_{fin} \}$. For a given model, the ground state nuclear energies and radii are calculated in the extended Thomas Fermi approximation at second order, as detailed in the next section. We retain for the subsequent analysis only the models $\{ P_{a} \}$ which provide a fair description of the experimental binding energies of the spherically magic nuclei: (40,20), (48,20), (48,28), (58,28), (88,38), (90,40), (114,50), (132,50), (208,82) and charge radii of (40,20), (48,20), (58,28), (88,38), (90,40), (114,50), (132,50), (208,82). The absence of the nucleus (48,28) in the second list is due to the fact that its experimental charge radius is not yet available. We recall that this set of data represents the core of nuclear properties on which the parameters of many Skyrme interactions have been fitted. The limitation to spherical nuclei is obviously due to the simplifying spherical approximation of most approaches, including ours. Specifically, retained EDFs correspond to sets of parameters $\{ P_{a} \}$ which provide $\chi(B) \leq 5$ MeV and $\chi(R_{ch}) \leq 0.10$ fm. The minimum values here obtained for standard deviation of masses and charge radii are 2.7 MeV and, respectively, 2.07 · 10$^{-2}$ fm. As usual in the literature, the chi-square function is defined as

$$ \chi^{2}(X) = \sum_{i=1}^{N} (X_{ETF(i)} - X_{exp(i)})^{2}/N. $$

The accepted values of standard deviation on mass are typically one order of magnitude larger than the lowest value in the literature, 0.5 MeV, which corresponds to more than 2350 nuclei and has been obtained in the framework of a Hartree-Fock-Bogoliubov (HFB) mass model [45].

The variation domain of each parameter is obtained by considering the dispersion of the corresponding values in a large number of relativistic and non-relativistic mean-field models, see Ref. [28]. The precise frontiers of this domain depend on the number of models considered and their selection criteria, and is therefore somewhat arbitrary. However, a variation of the borders of the parameter space might affect the overall dispersion in the predictions of the meta-model, but not the
quality of the correlations among parameters and observables, which is the scope of the present work.

The domain considered for each parameter $P_a$ is reported in Table I in terms of average value and standard deviation. Good/poor experimental constraints on $n_{sat}$, $E_{sat}$ and $E_{sym}$ on one hand and $K_{sat}$, $L_{sym}$ and $K_{sym}$ on the other hand lead to narrow/wide variation domains of these variables.

As a first application of the meta-modelling, we can investigate the model dependence of the correlations among empirical parameters observed in the previous section for the Skyrme EDFs.

The only significant correlation that was found in the different models generated by the meta-modelling technique after application of the mass and radius filter, is the one between $E_{sym}$ and $L_{sym}$ parameters generated by the meta-modelling technique after application of the mass and radius filter, is the one between $E_{sym}$ and $L_{sym}$ on one hand and $K_{sym}$, $m_{sat}/m$ on the other hand lead to narrow/wide variation domains of these variables.

As a first application of the meta-modelling, we can investigate the model dependence of the correlations among empirical parameters observed in the previous section for the Skyrme EDFs.

This suggests that the origin of that correlation observed in the previous section for the Skyrme pseudo-potentials. This confirms that the Skyrme correlation comes from the physical constraint of mass reproduction, and is largely independent of the EDF model.

Conversely, only a poor correlation between $L_{sym}$ and $K_{sym}$ emerges from the meta-modelling after application of the mass constraint, see solid squares in the top panel of Fig. 1. This suggests that the origin of that correlation observed in different functionals [25, 26] is not due to the constraint of mass reproduction.

C. The Extended Thomas-Fermi approximation with parametrized density profiles

For a given EDF model, average properties of atomic nuclei can be reasonably well described within the Extended Thomas-Fermi (ETF) approximation [46]. In this work, we will limit ourselves to the second order expansion in $\hbar$ and to parametrized density profiles in spherical symmetry, such as to limit the number of variational parameters. Because of these approximations, the degree of reproduction of experimental data is not comparable to the one of dedicated fully quantal HFB calculations [45], and more realistic calculations will definitely have to be performed in order to determine EoS parameters in a fully quantitative way. Still, the complete exploration of the parameter space is not affordable with these more sophisticated many body techniques, and we believe that an ETF meta-modelling is sufficient to extract the correlations between EoS parameters and the neutron skin.

In the ETF framework, the energy of an arbitrary distribution of nucleons with densities $\{n_n(r), n_p(r)\}$ is given by the volume integral of the energy density according to:

$$E_{tot} = \int dr \left( e_{nuc}[n_n, n_p] + e_{Coul}[n_p] \right),$$

where the first term stands for the nuclear energy and the second for the electrostatic contribution.

At second-order in $\hbar$ expansion, the nuclear energy density functional writes

$$e_{nuc}[n_n, n_p] = \sum_{q=n,p} \frac{\hbar^2}{2m_q} \zeta_2 + e_{TF},$$

where $e_{TF}$ is the Thomas-Fermi approximation of the chosen nuclear EDF model, which can depend on local densities $n_q$ as well as on density gradients $\nabla n_q$ and currents $J_q$ and $\zeta_2$ is the (local and non-local) density dependent correction arising from the second order $\hbar$ expansion of the kinetic energy density operator.

The Coulomb energy density is expressed as [47],

$$e_{Coul}[n_p] = \frac{e^2}{2} n_p(r) \int \frac{n_p(r')}{|r - r'|} dr' - \frac{3e^2}{4} \left( \frac{3}{\pi} \right)^{1/3} n_p^{4/3}(r),(12)$$

where the Slater approximation has been employed to estimate the exchange Coulomb energy density.

The ground state is determined by energy minimization using parametrized neutron and proton distributions. For a generic nucleus with $N$ neutrons and $Z$ protons and under the simplifying approximation of spherical symmetry, these are customarily parametrized as Wood-Saxon (WS) density profiles, $n_{WS}(r) = n_{bulk,q} \frac{1 + \exp[(r - R_{WS,q})/a_q]}{1 + \exp[(r - R_{WS,q})/a_q]}$, (13)

where $n_{bulk,q}$ is linked to the central density of the $q = n, p$ distribution, and $R_{WS,q}^{q}$ and $a_q$ respectively stand for radius and diffusion parameters. With the extra condition of particle number conservation,

$$Z = 4\pi \int_0^\infty dr \: r^2 \: n_p(r), \: N = 4\pi \int_0^\infty dr \: r^2 \: n_n(r)$$

only four variables out of six are independent. In the variational calculation of the ground state, we make the choice of varying $\{n_{bulk,q}, a_q, q = n, p\}$, while $R_{WS,q}^{q}$ are obtained from Eq. (14).
The only experimental observables related to the distribution of matter are the root mean squared (rms) radius of the charge distribution and, with larger error bars, neutron skin thickness. Rms radius of the charge distribution is defined as the rms radius of the proton distribution corrected for the internal charge distribution of the proton $S_p=0.8$ fm,

$$\langle r_{ch}^2 \rangle^{1/2} = \left[ \langle r_p^2 \rangle + S_p^2 \right]^{1/2}. \quad (15)$$

Neutron skin thickness is defined as the difference in the neutron-proton rms radii,

$$\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}, \quad (16)$$

and, as demonstrated in Ref. [24], it can be decomposed with good accuracy into a bulk contribution,

$$\Delta r_{np}^{bulk} = \sqrt{\frac{3}{5}} \left[ R_n^{WS} - R_p^{WS} \right] + \frac{\pi^2}{3} \left( \frac{a_n^2}{R_n^{WS}} - \frac{a_p^2}{R_p^{WS}} \right), \quad (17)$$

and a surface contribution,

$$\Delta r_{np}^{surf} = \sqrt{\frac{3 \pi^2}{5}} \left( \frac{a_n^2}{R_n^{WS}} - \frac{a_p^2}{R_p^{WS}} \right). \quad (18)$$

It is worthwhile to notice that each of these contributions depends on both WS radii and diffusivities of neutron and proton distribution.

III. RESULTS

A. Performance of the ETF approximation on experimental data

In order to visualize the overall performance of the ETF approximation, we consider in this section a single nuclear EDF model, namely the SLy4 [36] functional. We remind that the lot of data on which SLy4 [36] has been constrained includes binding energies and rms radii of doubly magic nuclei and the equation of state of pure neutron matter of Ref. [48]. The last constraint guarantees a correct behavior at high isospin asymmetry.

In terms of average standard deviation on masses and radii, we obtain for the considered pool of spherical nuclei $\langle \chi(B) \rangle = 4.9$ MeV and $\langle \chi(R_{ch}) \rangle = 4.1 \cdot 10^{-2}$ fm.

The results of total, i.e. nuclear plus electrostatic, energy minimization in the 4-dimensional space ($n_{bulk,n}, n_{bulk,p}, a_n, a_p$) are plotted in Figs. 2 and 3 for the isotopic chain of Pb as a function of the isospin asymmetry, $I = 1 - 2Z/A$. Two different methods are used to calculate the Coulomb energy. In one case it is calculated by accounting for the diffusivity of the proton distribution via eq. (12) ("self-consistent"). In the second, a uniformly charge distribution approximation is employed, which leads to $0.69Z^2/A^{1/3}$ ("approx."). The top and middle panels of Fig. 2 present the evolution of each of the four variational parameters as a function of $I$. The bottom panel presents the $I$-dependence of the WS radii on neutron and proton distributions, obtained from particle number conservation. We can notice the important effect of a self-consistent treatment of Coulomb in the determination of the density profiles. In particular, the obtained bulk densities and diffuseness parameters are in good agreement with fits of HF density profiles with the same EDF [49], which comforts us on the quality of the approximation.

We can also see that WS radii of neutron and proton distributions have similar values, though strongly dependent on $I$. This might suggest that the skin is mainly a surface effect for this calculation. However, this interpretation is not correct because the equivalent sharp radius $R_q = \left. \left( \int r^2 n_q(r) \right) \right|_{n_{bulk,q}}$ is different from the WS radius parameter, $R_{WS}$, and effectively depends on the diffuseness of the profile [24]. Moreover, as explicitly worked out in Ref. [50], the diffuseness parameter itself depends in a highly non trivial way both on the gradient terms of the EDF and on the bulk properties of
FIG. 3: ETF results corresponding to the ground state of Pb isotopes and SLy4 [36]. Binding energy per nucleon (top panel), rms radii of neutron and charge distributions (middle panel) and neutron skin thickness are plotted as a function of total isospin asymmetry. When available, experimental masses [51] and charge radii [52] are plotted as well. For neutron skin thicknesses of $^{208}$Pb the following experimental data are illustrated: $0.1515 \pm 0.0197$ fm [15], $0.156 \pm 0.021$ fm [16] and $0.3012 \pm 0.175$ $\text{fm}$ [13,53]. As in Fig. 2 two methods for calculating the Coulomb energy are considered. Neutron skin thickness decomposition into bulk and surface contributions according to eqs. (17) are represented on the bottom panel for the case in which the Coulomb energy is calculated self-consistently (open symbols).

B. Correlations between nuclear observables and parameters of nuclear matter

The correlation between the neutron skin thickness of $^{208}$Pb and $L_{\text{sym}}$ has been reported in the past years in many different studies based on density functionals [11,12,24], semi-classical approaches [56,57], as well as DM [56].

More recently, the existence of other correlations with various isovector modes of collective excitation was suggested, namely electric dipole polarizability [16–18], isovector giant dipole resonance (IVGDR) [58], isovector giant quadrupole resonance (IVGQR) [29], pygmy dipole resonance (PDR) [18,58,59], anti-analog giant dipole resonance (AGDR) [60–62]. A correct description of these modes demands a dynamical treatment in the framework of linear response theory and...
is beyond the purpose of this work. However, simplified expressions were proposed. An example in this sense is given by Ref. [63] which relates the electric dipole polarizability of a nucleus of mass number $A$ and isospin asymmetry $I$,

$$
\alpha_D = \frac{\pi e^2}{54} \frac{A(r^2)}{E_{sym}} \left( 1 + \frac{5}{3} \frac{E_{sym} - a_{sym}}{E_{sym}} \right),
$$

with the ground state symmetry energy in the local density approximation [24]

$$
a_{sym}(A) = \frac{4\pi}{A^{5/3}} \int_0^\infty dr r^2 n(r) \delta^2(r) e_{sym}(n(r)),
$$

where $e_{sym} = (1/2) \delta^2 e(n, \delta) / \partial \delta^2 |_{\delta=0}$ represents the local symmetry energy. Another example is offered by Ref. [64] which expresses the IVGDR energy constant in terms of the symmetry energy, saturation density and surface stiffness coefficient, $Q_{stiff}$, as,

$$
D = D_\infty / \sqrt{1 + 3E_{sym}A^{-1/3}/Q_{stiff}},
$$

where $D_\infty = \sqrt{8\hbar^2 E_{sym}/(mr^2)}$ and $r_0^3 = 3/(4\pi n_{sat})$. The surface stiffness coefficient measures the resistance of the asymmetric semi-infinite nuclear matter against separation of neutrons and protons to form a skin and is typically performed within HF or ETF approaches. Such calculations showed some sensitivity of $Q_{stiff}$ to the calculation procedure [65, 66] as well as significant correlations with the symmetry energy and its first and second order derivatives [57, 67]. Different approximation formulas have been proposed. Some of them express $Q_{stiff}$ in terms of a number of nuclear matter parameters and are based on fits of HF or ETF calculations performed using different EDFs. Within the Liquid Drop Model, Ref. [46] calculates $Q_{stiff}$ from calculations of finite nuclei disregarding the Coulomb interaction. In the present work we adopt the expression, $Q_{stiff} = 9E_{sym}A^{-1/3}/4(E_{sym}/a_{sym} - 1)$, obtained by equating the ground state symmetry energy given by eq. [20] with the corresponding DM expression [24]. For the case of $^{208}$Pb its accuracy is of the order of 10%, which leads to a relative error of 2% on the IVGDR energy constant of $^{208}$Pb calculated according to eq. [21]. This small uncertainty only marginally affects the correlation between the macroscopically derived IVGDR energy constant and various properties associated with the finite nuclei or the nuclear matter. However, more important distortions might come from the nature of the approximation itself, namely the use of macroscopic expressions in case of dynamical quantities. Such distortions apply to both $\alpha_D$ and $D$.

Another interesting observable, potentially linked to the isovector EoS parameters, is given by the difference between the proton radii $R_p = (r_p^2)^{1/2}$ of mirror nuclei [65, 66]. This observable has the interesting feature of being directly accessible from a variational calculation without any extra model assumption. Moreover, it is much more accessible experimentally than the neutron skin, which demands the measurement of the neutron distribution.

The correlation between the proton radii differences in mirror nuclei and electric dipole polarizability on one hand and neutron skin thicknesses on the other hand has been addressed in Refs. [23, 68–70]. Ref. [64] focused on the nuclear symmetry energy dependence of the IVGDR energies by considering a series of Skyrme interaction potentials. The correlations between neutron skin thickness, electric dipole polarizability and IVGDR energy constant of $^{208}$Pb and proton radii difference for $A = 48$ mirror nuclei are investigated in Fig. 4.
both meta-modelling EDF and Skyrme functional. For completeness, Skyrme predictions corresponding to differences in the proton radii of $A = 50, 52, 54$ and $\Delta r_{np}^{(208\text{Pb})}$ are also plotted in the top panel. In the case $\Delta r_{\text{mirror}}$ vs. $\Delta r_{np}^{(208\text{Pb})}$, meta-modelling EDF lead to a strong correlation, with a Pearson correlation coefficient of 0.98. A moderate correlation is obtained for $D^{(208\text{Pb})}$ vs. $\Delta r_{np}^{(208\text{Pb})}$. A poor correlation is found between the dipole polarizability and neutron skin thickness. Skyrme functionals provide very similar results. Very strong correlations are obtained only between $\Delta r_{np}^{(208\text{Pb})}$ and proton radii differences in mirror nuclei with $A = 48, 50, 52, 54$. This result is in agreement with Refs. [68, 69].

The correlation between electric dipole polarizability and $\Delta r_{np}^{(208\text{Pb})}$ is loose, in agreement with Ref. [70]. Ref. [70] has actually evidenced that a much better correlation holds between $\Delta r_{np}$ and $(\alpha_{D} E_{\alpha})$, as expected from eq. (19). Finally, Skyrme functionals lead to medium strength correlations between IVGDR energy constant and neutron skin thickness of $^{208}\text{Pb}$. This result can be understood considering the $L_{\text{sym}}^{-1}$ and $K_{\text{sym}}^{-1}$ dependence of the $D$ quantity via $Q_{\text{stiff}}$.

We now turn to test the sensitivity of the observables to the different isovector parameters of the EoS. In a previous work [29], a full Bayesian analysis of the correlation matrix was performed, though with a more simplified version of the ETF meta-modelling, which did not include the self-consistent treatment of Coulomb nor the definition of $(n_{\text{bulk}}, n_{\text{bulk}}, d_{n}, d_{p})$ as independent variational variables. In that study, it was shown that the neutron skin is only sensitive to the $L_{\text{sym}}$ parameter. The present calculations, with a more sophisticated treatment of the ETF meta-modelling, confirm the results of our previous work.

The correlation between the neutron skin in $^{208}\text{Pb}$ and the $L_{\text{sym}}$ parameter is shown in the top panel of Fig. 5. The lower value of the correlation coefficient with respect to the results of Ref. [29] can be understood from the fact that the difference between the neutron and proton diffusivity was neglected in Ref. [29]. This value is also lower than the one corresponding to Skyrme functionals, as well as to the ones reported by most analyses in the literature using specific energy functionals [23, 24, 56, 57, 71]. The higher dispersion of the meta-modelling is due to the fact that the different EoS parameters are fully independent in the meta-modelling approach. As already observed in Ref. [29], though the EoS parameters are all influential in the calculation of nuclear masses and radii, the constraint on those quantities does not generate correlations among the EoS parameters because compensations can freely occur.

To demonstrate this statement, we have generated models with arbitrary fixed values of $K_{\text{sym}}$ fulfilling the same criteria imposed to the global set of models, see Section IIB. The resulting correlations are shown in Fig. 5 for three cases $K_{\text{sym}} = -100, 0, 100$ MeV. We can observe that the correlation between $^{208}\text{Pb}$ and $L_{\text{sym}}$ is greatly improved when $K_{\text{sym}}$ is fixed. In the case of Skyrme functionals, $K_{\text{sym}}$ can largely vary but its value is positively correlated to $L_{\text{sym}}$ because of the specific function form of the density dependent term in Skyrme interactions (see Figure 1(a)). As a consequence, the Skyrme results interpolate the more general meta-model ones and the correlation coefficient is only slightly less than those corresponding to meta-model EDF with fixed $K_{\text{sym}}$-values.

The bottom panel of Fig. 5 summarizes the analyses done above but for the correlation between the proton radii difference in $A = 48$ mirror nuclei and $L_{\text{sym}}$. The conclusions are similar: strong (poor) correlations exist in the case of Skyrme functionals and meta-modelling EDF with fixed $K_{\text{sym}}$-values (meta-modelling EDF with freely varying $K_{\text{sym}}$).

The correlations of the dipole polarizability and IVGDR energy constant of $^{208}\text{Pb}$ with $L_{\text{sym}}$ are reported in Figure 6 for the meta-modelling and for the selected Skyrme functionals. As in Fig. 5 meta-modelling EoS with fixed values of $K_{\text{sym}} = -100, 0, 100$ MeV are also considered. As one may see, $\alpha_{D}^{(208\text{Pb})}$ and $D^{(208\text{Pb})}$ show less correlation with $L_{\text{sym}}$ than with $\Delta r_{np}^{(208\text{Pb})}$, when meta-modelling EDF are employed. At variance with this, Skyrme functionals provide for $\alpha_{D}^{(208\text{Pb})}$ and $D^{(208\text{Pb})}$ almost the same degree of correlation with $L_{\text{sym}}$ as with the neutron skin of $^{208}\text{Pb}$. A word of caution is nevertheless in order. The accurate calculation of these two dynamical quantities is possible only within the linear response theory. The Eqs. [19, 21] presently employed...
FIG. 6: Correlations between $L_{\text{sym}}$ and $\alpha_D^{(208}\text{Pb})$ (top) and $L_{\text{sym}}$ and $D^{(208}\text{Pb})$ (bottom). Numbers on r.h.s. of each plot correspond to Pearson correlation coefficients between the observables plotted on the axis. Upper (lower) values: meta-modelling (Skyrme). As in Fig. 5 meta-modelling EDF with fixed values of $K_{\text{sym}} = 100, 0, -100$ MeV are plotted as well.

rely on approximations and are, thus, expected to distort the sensitivity to nuclear matter EoS.

In Ref. [29] the isoscalar and isovector parameters of the meta-modelling EDF have been determined by fits of experimental binding energies of symmetric nuclei with masses $20 \leq A \leq 100$ and full isotopic chains of Ca, Ni, Sn and Pb. We have tested that the conclusions drawn above and the degrees of correlation remain the same if the pool of nuclei on which the parameters of the EDF are determined is replaced by the one considered in Ref. [29].

IV. CONCLUSIONS

In this paper, we have explored the influence of the different isovector empirical EoS parameters on some properties of atomic nuclei, namely neutron skin thickness, difference in proton radii of mirror nuclei, dipole polarizability and the IVGDR energy constant of $^{208}\text{Pb}$.

The analysis was done within a recently proposed meta-modelling technique [28, 29]. Varying the parameters of the meta-modelling, it is possible to reproduce existing relativistic and non-relativistic EDF, as well as to consider novel density dependencies which are not explored by existing functionals. With respect to our previous work Ref. [29], we have improved the ETF formalism employed to extract nuclear masses and radii out of a given EDF: the Coulomb interaction is consistently included in the variational procedure, and the bulk densities and diffuseness parameters of the density profiles are treated as independent variational parameters. These improvements allow a better description of nuclear radii and the nuclear skin. The correlation between this latter observable and the slope of the symmetry energy $L_{\text{sym}}$, already reported in numerous studies in the literature with different EDFs as well as many body techniques, is confirmed by our study.

However, we show that the quality of this correlation is considerably worsened if we allow independent variations of the curvature parameter $K_{\text{sym}}$ with respect to the slope $L_{\text{sym}}$, while this was not observed in previous studies probably because in most existing functionals such correlation exists. We conclude that it will be very important to constrain the curvature parameter with dedicated studies, in order to reduce the confidence intervals of EoS parameter and allow more reliable extrapolations to the higher density domain.

We have shown that the condition of a reasonable reproduction of nuclear masses and radii does not necessarily imply any strong correlation between $L_{\text{sym}}$ and $K_{\text{sym}}$. For this reason, it is possible that the existing correlation in the Skyrme EDF might be spuriously induced by the arbitrariness of the functional form, particularly the density dependent term. However, as suggested in Ref. [25, 26], such a correlation might also be physical and linked to the fact that Skyrme EDF are derived from a pseudo-potential which satisfies some basic physical properties, which is not the case of the more general meta-modelling. To answer this question, it will be important to evaluate and possibly constrain this correlation on ab-initio calculations of neutron matter [26]. This work is presently in progress.

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[72] The Pearson correlation coefficient between two variables $X$ and $Y$ is defined by $C(X,Y) = (\langle XY \rangle - \langle X \rangle \langle Y \rangle)/\sigma(X)\sigma(Y)$.