Spin glasses are magnetic systems with conflicting and random interactions between individual spins. The denomination "spin glass" for the wide class of such materials which show a complex form of magnetic order at low temperatures dates from 1970 [1]; the same year and independently it was found that a dilute magnetic alloy has a sharp cusp in the low field a.c. susceptibility at a temperature $T_g$ [2], so defining a precise critical temperature despite the disorder. Spin glasses have been intensively studied ever since, theoretically, experimentally and numerically in the belief that they will provide a paradigm for a vast family of complex systems.

Numerical work has concentrated on the Edwards-Anderson version of a spin glass – a regular lattice of Ising spins coupled by random interactions. It turns out that this "theoretician’s spin glass" shows the same essential characteristics as the experimental systems, and is a perfect test-bed for checking out ideas on complex systems in general. The mean field (or infinite dimension) limit of this model is well understood [3]. However, in finite dimensions even this simplified version of a complex system turns out to be very devious, so that after 25 years of work there is still little consensus on the fundamental empirical facts, let alone the underlying theory.

At low temperatures there is a domain of irreversible, non-ergodic behaviour; we will discuss exclusively the dynamics in the other, ergodic, regime from high temperatures down to (or nearly down to) the freezing temperature $T_g$. This critical temperature can be defined operationally by the divergence of the relaxation time for large systems, or by invoking scaling laws. It is well known that at a standard second order transition one can define critical exponents which are related to each other and which have been shown, by the renormalization group theory, to obey universality rules. These state essentially that in a given dimension, all systems sharing a few basic parameters (such as the number of order parameter components) will have identical values of critical exponents. Physically this remarkable regularity arises because averaging over larger and larger volumes as the critical temperature is approached means that all microscopic distinctions between systems are blurred out.

There have been a number of numerical estimates of critical exponents in spin glasses. A first point which is clear is that the estimated exponents at a spin glass transition are very different from those at a standard second order transition. For instance, in dimension 3 the specific heat exponent $\alpha$ for an Ising ferromagnet is close to zero; in the Ising spin glass it is close to $-2$ [2]. It is then important to check if the universality rules hold at spin glass transitions as they do at canonical second order transitions. This turns out to be difficult to do in practice, mainly because the relaxation as the transition is approached becomes exceedingly slow, meaning that long and tedious numerical anneals must be used to make sure that the system is in equilibrium. As a result, simulation estimates of exponents have had substantial error bars associated with them.

We have used a combination of scaling rules which allow us in favourable cases to establish accurate values of the ordering temperatures together with the static exponent $\eta$ and the dynamic exponent $z$. Among these techniques is that of the scaling form of the non-equilibrium relaxation in going from complete disorder towards equilibrium, after a numerical quench to the critical temperature. This has the advantage of needing no preparatory anneal. It would be much more elegant if it were possible to establish the ordering temperature uniquely from such non-equilibrium measurements, as has been done in regular magnetic systems. Unfortunately up to now we have found no simple method of doing this.

The exponent values that we have estimated are more precise than but in good agreement with other simulation estimates [2], and with exponent values obtained from series methods [4]. The results clearly indicate that the exponents $\eta$ and $z$ for spin glasses in a given dimension change with the form of the random interactions. For instance, the exponents found with the binomial near neighbour interaction distribution $\pm J$ are not the same as those found with a Gaussian near neighbour interaction distribution. According to the universality rules,
such differences in the interactions should not be pertinent. One would expect, with a change in distribution, a change in the critical temperature but not a change in the exponents. We can note that in experimental spin glass systems the estimates for $\eta$ vary considerably from one material to another, again suggesting a breakdown of universality [10]. It seems that ordering at a spin glass transition cannot be simply classified with standard second order transitions. This is an important point, as the general renormalization argument would seem to be very robust so the breakdown is unexpected and goes against the conventional wisdom. The empirically observed non-universality implies that the physics of critical behaviour in spin glasses and other complex systems is much richer than in regular systems. It would be very useful to find a theoretical underpinning for this result, and to understand how the exponents change with the various control parameters such as the interaction distribution.

As we have noted above, relaxation at the approach to a spin glass transition becomes extremely slow. It also becomes highly non-exponential. In a massive simulation Ogielski [1] measured accurately the relaxation of the autocorrelation function $q(t) = \langle S_i(t)S_i(0) \rangle$ for large samples of the ±J Ising spin glass in dimension 3. At the approach to a continuous transition one expects $q(t)$ to take the form $t^{-x}f(t/\tau)$ where $x$ is a combination of critical exponents and $\tau$ is a characteristic time which diverges at the critical temperature. Ogielski parametrised his results very succesfully using for the scaling function $f$ the stretched exponential form, $\exp[-(t/\tau)^\beta]$, with a temperature dependent stretched exponential (or Kohlrausch [11]) exponent $\beta$. The stretched exponential is used ubiquitously by the glass community for fitting experimental relaxation data. Having $\beta$ less than 1 can be interpreted as an indication of a distribution of relaxation times in the system. In Ogielski’s data, as the temperature approaches $T_g$ and $\tau$ diverges, $\beta$ tends to a value close to 1/3. Numerical data on different spin glasses are compatible with similar behaviour in each of the other cases [12,13]. However good the data, it is of course always necessary to extrapolate to estimate the limiting value of $\beta$ as $\tau$ diverges, which leads to a margin of uncertainty. Nevertheless the general tendency appears to be very similar in all the spin glasses which have been studied.

In experimental spin glass systems it is difficult to study the form of the relaxation for a range of temperatures above $T_g$ directly because of time scale problems, but muons can be used as local probes which are sensitive to the relaxation of the magnetic spins. The form of the decay of the muon polarization with time gives indirect information on the form of the spin relaxation, and it turns out that the muon time window is appropriate. Measurements have shown that in a standard metallic spin glass the spin relaxation is homogeneous and close to exponential only at temperatures of the order of 4 times $T_g$, and that the muon polarization decay pattern is compatible with an Ogielski form of spin relaxation for temperatures close to $T_g$ [14,15]. This means that the behaviour observed numerically in Ising spin glass simulations is mirrored in experimental, Heisenberg, spin glasses.

This pattern of behaviour is in fact even more general and is not confined to spin glasses alone. The relaxation in most structural glasses has a characteristic time which appears to diverge at a temperature $T_0$ somewhat below the ”glass temperature” $T_g$. (In the structural glass context the latter is conventionally defined as a temperature where the relaxation time scale is of the order of a few seconds. This is a convenient parameter for defining the practical onset of glassy as against liquid behaviour, but this glass $T_g$ is obviously not a thermodynamic temperature. $T_0$, the divergence temperature, is the equivalent of the spin glass freezing temperature). The relaxation data are well fitted by stretched exponentials, and for a large number of glasses $\beta(T)$ drops regularly as $T_0$ is approached, with $\beta$ tending to a value in the region close to 1/3 when the points are extrapolated to the temperature where the relaxation time becomes infinite [14,15]. Once again, extrapolation is obligatory, as the time needed to establish equilibrium before the start of the relaxation measurement exceeds laboratory time scales well before $T_0$ is attained. Nevertheless there is a striking similarity between the behaviour of the relaxation in the Ising spin glasses and in these structural glasses, two families of systems which have nothing in common on the microscopic level. This suggests that in these systems the special pattern of the dynamics as the freezing temperature is approached is a necessary consequence of the physics of the glassy transition.

We have proposed a possible interpretation couched in terms of the morphology of phase space at temperatures above the glass transition [19,20]. Let us concentrate on the Ising systems. The total phase space of an N-spin Ising system consists of $2^N$ configurations which can be mapped onto the $2^N$ corners of a hypercube of dimension N. At finite temperature $T$ some of these configurations have such high energies that they are thermodynamically ”inaccessible”. The system performs a random walk among the low energy configurations that remain ”accessible”. Relaxation through successive single spin flips is simply a random walk where each step takes the system from one configuration to a near neighbour configuration on the hypercube. The difficult problem is to establish the morphology of the set of accessible configurations, given the Hamiltonian for the system. Suppose that in a glassy system we take an extreme limit and assume that a temperature $T$ corresponds to a fraction $p$ of accessible sites, distributed at random over the hypercube. The relaxation problem is now mapped onto a geometrical problem. As $p$ drops we will have the closed space equivalent of a percolation transition, with a single giant
cluster of sites existing down to a critical concentration $p_c$. Relaxation becomes a random walk among the sites of this giant cluster. We have argued [19,12], using the known behaviour of random walks on a percolation cluster in Euclidean space $^{22}$, that for the high dimension hypercube relaxation will approach the stretched exponential form $\exp[(-(t/\tau)^{1/3})]$ as $p_c$ is approached. Numerical simulations on this geometrical model confirm this conjecture $^{13,23}$.

Our explanation of the particular form of the relaxation as $\tau$ diverges at a glass transition is then the following. At the transition the accessible phase space splinters into many topologically disconnected and inequivalent pieces, like the small clusters below the critical concentration in a conventional percolation transition. At a temperature just above the critical temperature the accessible phase space is still topologically connected, but it has become so sparse and complicated (fractal-like) that the system takes a macroscopic time to explore all the nooks and crannies of this structure, in order to be able to make up its mind whether the attainable phase space is indeed topologically connected or not. As the random walk path is tortuous, the relaxation is strongly non-exponential, and the hypercube geometry argument leads us to a specific form of relaxation. We have discussed the Ising phase space explicitly, but any physical system has a phase space and the general image should carry over to other glassy transitions. This shattering of phase space at the transition is very different from the description of a second order transition in a regular system where at the ordering temperature phase space splits up into two (or a small number) of configuration clusters related to each other by symmetry. One can surmise that the non-universality of the critical exponents is also a consequence of the geometry of phase space, but the connection remains to be established.

We should at this point make a caveat. The discussion as it stands relates to glasses which are known as "fragile" glasses in the glass terminology, i.e. to glasses where $T_0$ is relatively close to $T_g$. In "strong" glasses, where the relaxation time grows following an Arrhenius law as the temperature is reduced to $T_g$, it appears to be the elementary step time which has increased to a macroscopic value at $T_g$, and it is impossible to probe close to $T_0$ (supposing it exists) in these glasses. $\beta$ remains close to 1 down to $T_g$ for these cases, which is entirely consistent with the general arguments we have given. In intermediate cases the extrapolation down to $T_0$ can only be made with a considerable degree of uncertainty. Relating the degree of fragility to the molecular structure of each glass remains a fundamental problem.

In conclusion, we have discussed numerical and experimental data on spin glasses and glasses. The results indicate empirically that the conventional universality rules are not obeyed at spin glass transitions. On the other hand, as the critical temperature (where the relaxation time scale diverges) is approached, the form of the relaxation in spin glasses or fragile structural glasses tends towards a stretched exponential with a Kohlrausch exponent close to 1/3. We have provided a general phase space image that could explain these observations.

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FIG. 1. The dynamic exponent $z$ and the critical exponent $\eta$ as functions of the critical temperature $T_g$ for four Ising spin glasses in dimension 4. The sets of interactions, from left to right, are exponential, gaussian, uniform and binomial.

FIG. 2. The Kohlrausch exponent $\beta$ for the relaxation of the autocorrelation function as a function of $T$, for the 3D Ising spin glass model with binomial interaction distribution. The points extrapolate to close to 1/3 at the critical temperature $T_g \simeq 1.175J$ (dashed line).

FIG. 3. The Kohlrausch exponent $\beta(T)$ as a function of $(\log(\tau(T)10^{13}))^{-1}$ obtained from dielectric relaxation measurements on a selection of polymers (PSF, PViBE, PE, PEI, PH and PMA) and on the 3D Ising spin glass model with binomial interaction distribution (+). The dashed line represents the practical limit ($\tau \sim 4$ months) for equilibrium measurements. All the sets of data are compatible with an extrapolated value of $\beta$ near 1/3 as $\tau$ goes to infinity (the lines through $\beta = 1/3$ are to guide the eye).