Orbital Kondo effect modulated by off-diagonal orbital interference

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Abstract

We report the theoretical investigation of the orbital Kondo effect in an Aharonov-Bohm interferometer by slave-boson mean field approach. It is found that the present orbital Kondo effect can be tuned geometrically by the external magnetic flux. When the magnetic flux \( \varphi = (2n + 1)\pi \), the off-diagonal self-energy vanishes and the orbital Kondo problem can be exactly mapped onto the usual spin Kondo model. For a general \( \varphi \), the presence of the off-diagonal orbital wave function interference will modify the height and width of the orbital Kondo peak, but not change the position of the orbital Kondo peak. We also give an analytic expression of the flux-dependent Kondo temperature and find it decreases monotonously as the magnetic flux \( \varphi \) goes from \( (2n + 1)\pi \) to \( 2n\pi \), which means the Kondo effect is suppressed by the off-diagonal orbital interference process and becomes more easily destroyed by the thermal fluctuation. The flux-dependence conductance is also presented.

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Very recently, quantum dots have attracted a considerable interest because of their natural and potential applicability as basic blocks of solid-state quantum computers. One of the important problems in quantum dots is the Kondo problem, which exhibits the interplay between a localized spin of a quantum dot and delocalized electrons in leads. As a result, the local spin in quantum dot is screened by the coherent higher-order spin flip cotunneling process. Experimentally, a zero-bias peak of the differential conductance with a width given by the Kondo temperature $T_K$ emerges. In the last years, a great amount of works\cite{1, 2, 3, 4, 5, 6} was attributed to this problem. Especially, Wilhelm et al.\cite{7} proposed an original idea: by defining two electrostatically coupled quantum dots, the orbital structure of the wave functions acquires spin-like features, they can exactly map the orbital-Kondo problem onto the spin-Kondo problem. Using the four-terminal device, this idea was experimentally realized\cite{8} by the same group. Later, Lopez et al.\cite{9} have discussed the entanglement between charge and spin degrees of freedom when both interdot and intradot Coulomb interactions exist. In this work, we will consider the electron movement in a two-terminal Aharonov-Bohm interferometer\cite{10, 11, 12, 13, 14, 15, 16, 17, 18}, which has been widely used to investigate the electron coherence. The schematic diagram of the device is depicted in Fig.1. We suppose there are two quantum dots in the different orbital paths of Aharonov-Bohm interferometer, and there exists the Hubbard-type orbital interaction between two quantum dots. Note that the present model is also related to the recent so-called pseudospin Kondo correlation experiment\cite{19}, in which the off-diagonal orbital wave function interference information was neglected.

The model Hamiltonian we consider reads

$$H = \sum_{\alpha=L,R} \varepsilon_{\kappa\alpha} C_{\kappa\alpha}^\dagger C_{\kappa\alpha} + \sum_{i=u,d} \varepsilon_i d_i^\dagger d_i + U n_u n_d$$

$$+ \sum_{\kappa i} [T_{\kappa\alpha i} C_{\kappa\alpha}^\dagger d_i + T_{\kappa\alpha i}^* d_i^\dagger C_{\kappa\alpha}].$$

(1)

The first term stands for the Hamiltonians of the noninteraction leads $\alpha = L, R$ and $C_{\kappa\alpha}^\dagger (C_{\kappa\alpha})$ are the corresponding creation (annihilation) operators. The second term describes the Hamiltonian of the up and down quantum dots, and $d_i^\dagger (d_i)$ are the creation (annihilation) operators in the quantum dot $i$ with the discrete energy level $\varepsilon_i$. The third term is the interaction between the up and down quantum dots. The last one denotes the hopping Hamiltonian between the lead $\alpha$ and dot with the hopping matrix elements
\( T_{kL,u,d} = t_{L,u,d} \exp(\mp i\frac{\varphi}{4}) \), \( T_{kR,u,d} = t_{R,u,d} \exp(\pm i\frac{\varphi}{4}) \). Here \( \varphi \) is the magnetic flux. Note that we have neglected the spin degree of freedom of electrons, which contributes the simple factor 2 in the electronic current. We have also assumed that the hopping matrix elements \( T_{kL,u,d} \) are independent of the momentum index \( k \) in the following calculation.

In order to deal with the inter-dot interaction, we use Coleman’s slave-boson mean field approach. Combining with Keldysh Green’s function, many authors have used slave-boson mean field to study all kinds of transport problems. In the \( U \to \infty \) limit, the localized electron operator \( d_i \) can be replaced by \( b^\dagger f_i \), where \( b \) and \( f_i \) being the standard boson and fermion annihilation operators describing the empty (\( n_u = 0, n_d = 0 \)) and singly occupied (\( n_u = 1, n_d = 0 \)) or (\( n_u = 0, n_d = 1 \)) states of up and down quantum dots. Since the dots are either empty or singly occupied in the \( U \to \infty \) limit, we have the following constriction condition

\[
\sum_{i=u,d} b^\dagger_i b + \sum_{i=u,d} f_i^\dagger f_i = 1. \quad (2)
\]

Therefore, we can obtain the effective Hamiltonian in slave-boson representation

\[
H_{\text{eff}} = \sum_{k\alpha=L,R} \varepsilon_{k\alpha} C_{k\alpha}^\dagger C_{k\alpha} + \sum_{i=u,d} \varepsilon_i f_i^\dagger f_i + \sum_{k\alpha i} [T_{k\alpha i} C_{k\alpha}^\dagger b_i + T_{k\alpha i}^\ast f_i^\dagger b_{k\alpha}] + \lambda [\sum_i f_i^\dagger f_i + b^\dagger b - 1]. \quad (3)
\]

Note that we have incorporated the constriction condition into the effective Hamiltonian by introducing a Lagrange multiplier \( \lambda \). In the mean-field approximation, \( b \) and \( b^\dagger \) are replaced by a real c number, \( b = b^\dagger = b_0 \). Because the slave-boson mean field is correct for Kondo regime, we have to restrict our nonequilibrium calculation to the low bias \( V << |\varepsilon_i| \). With these preparations, the key point for us is to determine the free parameters \( \lambda \) and \( b_0 \) self-consistently by the condition

\[
\frac{\partial \langle H_{\text{eff}} \rangle}{\partial \lambda} = \frac{\partial \langle H_{\text{eff}} \rangle}{\partial b} = 0, \quad (4)
\]

which gives the following two self-consistent equations

\[
- i \int \frac{dE}{2\pi} Tr G^<(E) + b_0^2 = 1, \quad (5)
\]

\[
\lambda b_0^2 = - \int \frac{dE}{2\pi} \sum_\alpha Tr \{ G^r(E) \Gamma_\alpha(E) f_\alpha(E) + \Gamma_\alpha(E) G^<(E)/2 \}. \quad (6)
\]
Once we have the parameters $b_0$ and $\lambda$, the present transport problem becomes noninteraction one with the renormalized quantum levels and $\varepsilon_i \rightarrow \varepsilon_i + \lambda$ and hopping matrix $T_{kai} \rightarrow b_0 T_{kai}$. Further, by using the Keldysh nonequilibrium Green’s functions, the electronic current can be calculated from the left lead,

$$I_L = q \left( \frac{d\hat{N}_L}{dt} \right)$$

$$= -iq \int \frac{dE}{2\pi} \text{Tr} \left\{ b_0^2 \Gamma_L(E) \ast \left[ (G^r(E) - G^a(E)) f_L(E) + G^<(E) \right] \right\}. \quad (7)$$

Where $f_\alpha(E) = \{ \exp[E - \mu_\alpha]/k_B T + 1 \}^{-1}$ is the Fermi distribution function and $[\Gamma(E)_{ij}] = 2\pi \sum_k T_{kai}^* T_{kaj} \delta(E - \varepsilon_k)$ is the linewidth function. The Green’s function $G^r_{ij, a, \leq}(E)$ is the Fourier transformation of $G^r_{ij, a, \leq}(t)$ with $G^r_{ij, a}(t) = \mp i\theta(\pm t)\langle \{ f_i(t), f_j^+(0) \} \rangle$ and $G^<(ij)(t) \equiv i\langle f_j^+(0)f_i(t) \rangle$.

In the following, we first calculate the retarded Green’s function $G^r(E)$ by making use of the Dyson equation,

$$G^r(E) = \frac{1}{E - H_{\text{eff}} - \Sigma^r(E)}, \quad (8)$$

where $\Sigma(E) = \Sigma^r_L(E) + \Sigma^r_R(E)$ with

$$\Sigma^r_L(E) = -\frac{i\Gamma(E) b_0^2}{2} \begin{pmatrix} 1 & \exp\left(\frac{i\varepsilon_0}{2}\right) \\ \exp\left(-\frac{i\varepsilon_0}{2}\right) & 1 \end{pmatrix},$$

$$\Sigma^r_R(E) = -\frac{i\Gamma(E) b_0^2}{2} \begin{pmatrix} 1 & \exp\left(-\frac{i\varepsilon_0}{2}\right) \\ \exp\left(\frac{i\varepsilon_0}{2}\right) & 1 \end{pmatrix}.$$
Eqs.(5)-(9) constitute a closed solution to the present transport problem, and we can calculate all kinds of physical quantities numerically. In the following, we set the Fermi level of the leads be zero, $2\Gamma = 1$ as the energy unit and both quantum dots have the same energy level $\varepsilon_u = \varepsilon_d = \varepsilon_0 < 0$. In addition, we assume the square symmetric bands for leads, that is, $\Gamma(E) = \Gamma \theta(W - |E|)$ with $W >> \max\{k_B T, \Gamma, qV, |\varepsilon_i|\}$. These parameters guarantee the system in the typical Kondo regime.

In Fig.2 we plot the local density of state ($LDOS \equiv -\frac{\text{Im}[G_r(E)]}{\pi} = -\frac{\text{Im}[G_d(E)]}{\pi}$) versus the energy with the different magnetic flux $\varphi$ and bare energy level $\varepsilon_0$. It is found that the $LDOS$ shows the well-known Kondo peaks at $E = 0$. To demonstrate the physics origin of the orbital Kondo peak, we first discuss the special case with the magnetic flux $\varphi = (2n+1)\pi$ [see also Fig.2(a)]. Since the off-diagonal retarded self-energy $[\Sigma^r_L(E) + \Sigma^r_R(E)]$ becomes zero at points $\varphi = (2n+1)\pi$, the orbital kondo problem is equivalent to the usual spin Kondo one. The physics of this orbital Kondo resonance results from the interesting co-tunneling process that is shown in Fig.3. When both of the quantum dot energy level $\varepsilon_i < 0$ and there is the strong Anderson interaction between two quantum dots, only one electron is occupied in one dot (we assume the down dot). Although the first-order tunneling is blocked, the higher-order tunneling process still happens: the first electron in the down quantum dot tunnels to the Fermi level of the right lead via the down arm, the second electron at the Fermi level of the left lead tunnels into the up dot via the up arm on a very short time scale $\sim \hbar/(\mu - \varepsilon_0)$. Next, the second electron repeats the process of the first electron via the up arm and another electron tunnels into the down dot via the down arm. At low temperature, a coherent superposition of all of this type co-tunneling process gives a narrow Kondo resonance peak in the $LDOS$ of both up and down dots. The position of the Kondo resonance is pinned at $E = \tilde{\varepsilon}_0 \equiv (\varepsilon_0 + \lambda) \to 0$ and the width is $\tilde{\Gamma} = \Gamma b_0^2$. The Kondo temperature is determined by $k_B T_K = \sqrt{\tilde{\varepsilon}_0^2 + \tilde{\Gamma}^2} \sim W \exp(-\frac{\pi|\varepsilon_0|}{\tilde{\Gamma}})$. In addition, we also find two features in Fig.2(a-d): (1) For a given magnetic flux $\varphi$, the width of Kondo resonance peak increases as the bare energy level $\varepsilon_0 \to 0$. This is because the empty state occupied probability increase when $\varepsilon_0$ approaches to zero, and thus $b_0$ becomes larger. (2) For a given bare energy level $\varepsilon_0$, the position of the Kondo resonance does not change with the magnetic flux. However, the width and height vary with the magnetic flux $\varphi$. We can understand these as follows: For a general $\varphi \neq (2n+1)\pi$, the off-diagonal orbital interference results in
the following diagonal retarded Green’s function for up or down quantum dot

\[ G_r(E) = \frac{E - \tilde{\varepsilon}_0 + i\tilde{\Gamma}}{(E - \tilde{\varepsilon}_0 + i\Gamma)^2 + \tilde{\Gamma}^2 \cos^2(\frac{\varphi}{2})}, \]  

which demonstrates that the position of the orbital Kondo resonance peak in LDOS is still at \( E = \tilde{\varepsilon}_0 \to 0 \), but the height and width of the peak are modified by the factors \( 1/\sin^2(\frac{\varphi}{2}) \) and \( \sin^2(\frac{\varphi}{2}) \), respectively. The corresponding Kondo temperature can be obtained from

\[ k_B T_K(\varphi) = \sqrt{\tilde{\varepsilon}_0^2 + \tilde{\Gamma}^2 \sin^4(\frac{\varphi}{2})} \]  

We must point out that \( T_K \) has a more complicated dependence of the magnetic flux \( \varphi \) due to \( \tilde{\varepsilon}_0 \) and \( \tilde{\Gamma} \) being still the implicit functions of \( \varphi \). In Fig.4, we demonstrate the Kondo temperature \( T_K(\varphi) \) decreases monotonously when the magnetic flux \( \varphi \) goes from \( (2n + 1)\pi \) to \( 2n\pi \). This means that the off-diagonal orbital wave function interference will suppress the orbital Kondo effect. In particular, when the magnetic flux \( \varphi \) is close to \( 2n\pi \), the Kondo temperature of the system becomes very small and the thermal fluctuation will more easily destroy the Kondo peak. Once the temperature is higher than Kondo temperature \( T_K(\varphi) \), the Kondo peak will suffer from a drift to the positions of the bare resonance. In Fig.5, we plot the conductance as a function of the external magnetic flux. It is found that although there are the same renormalized energy levels \( \tilde{\varepsilon}_0 \) for both up and down quantum dots which are very close to the Fermi energy of the leads, the destructive interference results in a greatly suppressed conductance for a general magnetic flux \( \varphi \). Especially, the conductance vanishes when \( \varphi = (2n + 1)\pi \). This anti-resonance phenomenon has been discussed in reference [16].

In summary, we have presented a flux-dependent orbital Kondo effect in an Aharonov-Bohm interferometer. The off-diagonal orbital wave function interference will modify the height and width of the orbital Kondo peak, but not change the position of the orbital Kondo peak. The analytic result of the flux-dependent Kondo temperature is also presented, and the numerical calculations show the Kondo peak is robust to the wide range of the magnetic flux \( \varphi \). Since this present orbital Kondo effect can be tuned by the geometric way, we hope this theoretical work will further stimulate the experimental interests in the orbital Kondo physics.
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[1] L.I. Glazman and M.E. Raikh, JETP Lett. 47, 452 (1988).
[2] T.K. Ng and P.A. Lee, Phys. Rev. Lett. 61, 1768 (1988).
[3] Y. Meir, N.S. Wingreen and P.A. Lee, Phys. Rev. Lett. 70, 2601 (1993).
[4] D. Goldhaber-Gordon et al., Nature 391, 156 (1998); D. Goldhaber-Gordon et al., Phys. Rev. Lett. 81, 5225 (1998).
[5] S.M. Cronenwett, T.H. Oosterkamp and L.P. Kouwenhoven, Science 281, 540 (1998).
[6] Q.F. Sun and H Guo, Phys. Rev. B 66, 155308 (2002).
[7] U. Wilhelm, J. Schmid, J. Weis and K.v. Klitzing, Physica E 9, 625 (2001).
[8] U. Wilhelm, J. Schmid, J. Weis and K.v. Klitzing, Physica E 14, 385 (2002).
[9] R. Lopez et al., Phys. Rev. B 71, 115312 (2005).
[10] A. Yacoby, M. Heiblum, D. Mahalu, and H. Shtrikman, Phys. Rev. Lett. 74, 4047 (1995).
[11] Y. Ji, M. Heiblum, D. Sprinzak, D. Mahalu, and H. Shtrikman, Science 290, 779 (2000); Y. Ji, M. Heiblum, and Hadas Shtrikman, Phys. Rev. Lett. 88, 076601 (2002).
[12] H. Aikawa, K. Kobayashi, A. Sano, S. Katsumoto and Y. Iye, Phys. Rev. Lett. 92, 176802 (2004); K. Kobayashi, H. Aikawa, S. Katsumoto and Y. Iye, Phys. Rev. Lett. 88, 256806 (2002).
[13] J. König and Y. Gefen, Phys. Rev. Lett. 86, 3855 (2001); J. König and Y. Gefen, Phys. Rev. B 65, 045316 (2002).
[14] A.W. Holleitner et al., Phys. Rev. Lett. 87, 256802 (2001).
[15] A. Levy Yeyati and M. Büttiker, Phys. Rev. B 52, R14360 (1995).
[16] B. Kubala and J. König, Phys. Rev. B 65, 245301 (2002).
[17] M.L. Ladron de Guevara, F. Claro and P.A. Orellana, Phys. Rev. B 67, 195335 (2003); P.A. Orellana, M.L. Ladron de Guevara and F. Claro, Phys. Rev. B 70, 233315 (2004).
Figure Captions

Fig. 1. Schematic diagram of our system.

Fig. 2. The LDOS of dependence of the energy for the different magnetic flux $\varphi$: (a) $\varphi = \pi$. (b) $\varphi = 3\pi/4$. (c) $\varphi = \pi/2$. (d) $\varphi = \pi/4$. The solid, dashed and dotted lines in (a)-(d) correspond to the different bare energy levels $\varepsilon_0 = -2.5, -3, -3.5$, respectively. Other parameters are set: $2\Gamma = 1, W = 100, \beta = 10^5$.

Fig. 3. The cotunneling process giving rise to the orbital Kondo resonance: (a) An electron in down dot tunnels into the right lead via the down AB arm, followed by another electron in the left lead tunnels into the up dot via the up AB arm. (b) An electron in up dot tunnels into the right lead via the up AB arm, followed by another electron in the left lead tunnels into the down dot via the down AB arm.

Fig. 4. Kondo temperature as the function of the magnetic flux $\varphi$. We have set $\varepsilon_0 = -3.5$ and other parameters are the same as those in Fig.2.

Fig. 5. The conductance versus the magnetic flux $\varphi$. We have set $\varepsilon_0 = -2.5$ and other parameters are chosen as those in Fig.2.
Fig. 3
